A Multiresolution Independent Component Analysis for textile images

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Abstract. This paper aims to provide an efficient tool for pattern recognition in the fight against counterfeiting in textile design. As fabrics patterns to be protected can present numerous and various characteristics related to intensity or color feature but also to texture and relative scales features, we introduce a tool able to separate image independent components at different resolutions. The suggested "Multiresolution ICA" combines the properties from both wavelet transform and Independent Component Analysis.

1. Introduction
With the development of CAD (Computer Aided Design), the designers in a textile company may create nowadays several tens of designs per month. In this context, new solutions for protecting the fabrics patterns against illegal use are necessary. Indeed, as the copies have become frequent, the textile companies are highly interested to support researches for controlling the design activity. The increasing number of patterns and their content (CAD products) suggest, as solution, databases of textile numerical images. The creation of such databases generates two different kinds of problems. The first issue is to be able to define the images' characteristics (content, resolution etc.), in order to cover the patterns' high diversity; the second issue is to be able to verify if a new image of a pattern, suspected to be a copy, is already in the database. This latter demand necessitates databases with content based image retrieval (CBIR). In this case, methods specific to textile images analysis should be developed. Each created pattern has to be retrieved from images of manufactured textiles whatever are the features used for its physical generation: color, texture and size, all of them with a very large diversity. Let us notice that "drawing (pattern)" acts as a marker for of a given family of products. The evident independency between color and texture (different from woven texture) suggests to use these features separately for searching into the database. Another particularity, regarding this time the application, is the quality of the images used to question the database: they might be obtained in "hostile" conditions, e.g., sensed with digital cameras of mobile phones, in the bad lighting conditions of a shop, at different resolutions etc. Taking into account these various sensing conditions and the possible low image quality, the use of a single method of analysis is, certainly, not sufficient. In order to find solutions for a part of these problems, we have developed a hybrid method, combining two different transforms: the Wavelet Transform (WT) and the...
Independent Component Analysis (ICA). The WT is largely used in image processing due to its ability to perform a multiresolution analysis. The applications regard, among others, the texture recognition and the contour detection in images with a gradient lighting. The ICA is mainly used in source separation applications. In image processing, rather good results were obtained using ICA for face recognition. Combinations of WT and ICA have already been used in a series of image processing applications. In [1], by jointly using the multiresolution analysis and the ICA, the essential features and the structure of the palmprint images are better captured; in [2], the iris is more efficiently encoded by using Gabor wavelets and ICA, a similar combination is used in [3] for texture segmentation and better image classification is obtained in [4], by applying an ICA to wavelet coefficients in high frequency subbands. Our approach, introduced in this paper under the name of multiresolution ICA, combines properties from WT and ICA. Thus, by multiresolution ICA, it is possible to separate image independent components at different resolutions. Since the image features - the pattern and texture in the case of a textile image - may be more manifest in certain subbands, we expect better results in textile image recognition. The paper is organized as follows: Section 2 contains a basic description of WT and ICA, Section 3 presents the algorithm of multiresolution ICA and Section 4 is devoted to some theoretical considerations supporting the algorithmic choices. Some brief experiments done on jacquards, in Section 5, and the conclusions in Section 6 are completing the paper.

2. Related transforms

2.1. Wavelet Transform

The WT is a mathematical tool frequently used in image analysis. Like many other image transforms, it is separable: the image is transformed by applying 1-D WT, repeatedly, on rows and on columns. In the continuous case, the 1-D WT basis is obtained by translating and dilating a function $\psi$, called mother wavelet:

$$\psi_{a,b}(x) = |a|^{\frac{1}{2}} \psi\left(\frac{x-b}{a}\right)$$  \hspace{1cm} (1)

In eq. (1), the translations and the dilatations are given by parameters $b$ and $a$ respectively. The analysis currently used in practice is the dyadic WT, obtained for $a = 2$. Fig. 1a shows the db3 wavelet, translated and dilated by 2.

The wavelet coefficients are obtained by projecting signal $u$ to be analysed on the wavelet basis (1):

$$WT_u(a,b) = \int_{-\infty}^{+\infty} u(x) \psi_{a,b}(x) dx$$ \hspace{1cm} (2)

The wavelets are functions with compact support, a property that gives to WT the ability of performing a local analysis of the signal. Indeed, due to the compactness, the coefficient resulted by projecting $u$, regards only that part of $u$ covered by the wavelet. The other important advantage of WT is the multiresolution capability [5]. This property, which is due to differently dilated wavelets, may be more easily explained in the frequency. In this domain, the mother wavelet $\psi$ is a band-pass filter. If we denote by $\Psi$ its Fourier Transform, then the spectrum of the dilated wavelet $\psi_{a,b}$ is given by the modulus of:

$$\Psi_{a,b}(f) = |a|^{\frac{1}{2}} (-a)e^{-j2\pi bf} \Psi(-af)$$ \hspace{1cm} (3)

which shows that the dilated $\psi_a$ is also a band-pass filter, but with a narrower band (the filter central frequency changes accordingly). Fig. 1b shows the band of db3 wavelet in three cases: not-dilated, dilated by 2 and dilated by 4.

Equation (3) shows that the WT is a multiband filtering: the short wavelets, with large bands, analyse the signal high frequencies and the long ones, with narrow bands, the low signal
frequencies. In the spatial domain, it is equivalent to say that the coefficients of short wavelets represent the signal’s fast variations and the coefficients of long wavelets, its slow variations. The coefficients obtained from wavelets of the same band (length) constitute the signal representation at a certain resolution. In the case of digital signals, the wavelets are numerical filters derived, like in the continuous setting, from a mother wavelet. The filter length determines the analysis resolution. For a fixed resolution, the WT is obtained by convolving the digital signal $u$ with the numerical wavelet, dilated correspondingly:

$$WT_u(a, b) = u \ast \psi_{a,b}$$  \hspace{1cm} (4)

where $\ast$ denotes the convolution. Defined like in (4), the WT of $u$ is approximately of the same length, at all resolutions. In the case of images, when applied on rows, the convolution (4) provides the subband of vertical details (at the considered resolution). It is an image of the same size as the analyzed image. The subband of horizontal details is obtained, similarly, by processing the image columns. The diagonal details subband is obtained by processing, sequentially, the image rows and columns. The data obtained by WT are almost decorrelated [6]. Like any transform using a pre-definite basis, the WT is suboptimal from this point of view and, consequently, the wavelet coefficients always preserve a small amount of the initial correlation of the analyzed signal.

2.2. Independent Component Analysis

The ICA is a method of analyse for random signals. By optimizing measures based on the fourth order statistics, ICA decomposes a random signal into a weighted sum of signals, having the property of being statistically independent. The mathematical model for ICA is [7]:

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{s}$$  \hspace{1cm} (5)

where $\mathbf{s}$ contains the set of independent signals, called independent components or, in certain applications, sources. In applications, the analyzed signal is provided by means of several particular realizations, assembled in $\mathbf{x}$, which is a matrix in ICA model (each row of $\mathbf{x}$ is a particular realization). The independent components are derived as single particular realizations
and are placed on the rows of \( s \). Thus, each particular realization in \( x \) is a weighted sum of the independent components in \( s \). The weights are contained in \( A \), called mixing matrix. The independent components (the rows of \( s \)) are all of unit variance and have an undetermined sign. The number of samples of each component is the same like for the particular realization in \( x \) (a natural constraint for having compatible dimensions in matrix product (5)). On the contrary, the number of independent components is a parameter that may be fixed by the user. From the standpoint of vector basis representations, the mathematical model (5) may be interpreted in two ways. One, already mentioned, considers the rows \( s_j \) as \( J \) basic sources. In this case, one has a basis of independent vectors, for representing rows \( x_k \). With such representations, the coefficients are elements from \( A \). It is an interpretation specific to source separation applications.

\[
x_k = \sum_{j=1}^{J} A_{kj} s_j.
\]  

(6)

The other interpretation considers that the basis vectors are columns \( A_j \) of \( A \). This basis provides a linear representation for columns \( x_i \) with coefficients that are, this time, from \( s \):

\[
x_i = \sum_{j=1}^{J} s_{ji} A_j
\]  

(7)

expressing each column of \( x \) as a linear combination of the columns of \( A \). Viewed in this way, ICA is similar to PCA, the difference consisting in that the latter provides decorrelated coefficients and the former independent ones. The ICA algorithms estimate both \( A \) and \( s \), by maximizing the components nongaussianity (one Gaussian component at maximum is allowed). The nongaussianity is a guarantee for the components' independence, since the sum of two non-Gaussian independent signals is always more Gaussian than each of them (a consequence of the Central Limit Theorem). The components provided by ICA may be more or less statistically independent. Their degree of independence depends on the nature of \( x \). There are many ICA algorithms. For our experiments, we have used FastICA [9], an algorithm that uses the negentropy as nongaussianity measure. The negentropy of a component is defined by [7]:

\[
J(S) = H_{\text{Gauss}}(S) - H(S)
\]  

(8)

where \( H(S) \) is the entropy of \( S \) and \( H_{\text{Gauss}}(S) \) is the entropy of a Gaussian random variable with the same covariance as \( S \). As, among all the possible distributions with a given covariance, Gaussian distributions have the highest entropy and the negentropy is always a nonnegative quantity. It is zero only for a Gaussian component. In [10] the sum of the negentropies of sources is used for texture classification. FastICA generates the independent components, by maximizing the negentropy of \( s \) rows. The algorithm is iterative: it starts from an initial \( A \), computes the projections of \( x \) columns on the basis in \( A \) for obtaining the components in \( s \) and estimates, from samples, their negentropy. Then, it modifies \( A \) in order to obtain, by a new projection, components with higher negentropy. The algorithm stops when the maximum is reached for the components negentropies. This procedure is preceded by \( x \) whitening and, during the algorithm, each iteration is completed by the orthogonalization of \( A \) columns. For negentropy estimation, FastICA uses a series of approximations, one of them being [7]:

\[
J(S) = k_1 \left( E \left[ S \cdot e^{-\frac{S^2}{2}} \right] \right)^2 + k_2 \left( E [||S||] - \sqrt{\frac{7}{8 \pi}} \right)^2
\]  

(9)

where \( k_1 = 36/(8\sqrt{3} - 9) \), \( k_2 = 1/(2 - 6/\pi) \). The solution provided by FastICA depends on the initial guess for \( A \). For different initial guesses, one may obtain different independent
components. For this reasons, in many applications, the algorithm has to be supplemented by some conditions in order to look for certain solutions only.

3. Multiresolution ICA

The transform proposed in this section concerns the image analysis. It is, essentially, a WT continued by an ICA. There are different ways to perform each of these transforms on images and the number of possibilities increases, by combining them. In our approach, the wavelet coefficients are obtained by a WT like in Section 2.1. and they are analyzed, subband by subband, by ICA. Let us suppose that we are interested in the analysis of the horizontal details, obtained by convolving image columns with the wavelet db3. In order to perform an ICA, one needs several subband particular realizations. In our approach, they are obtained with a sliding window, like in Fig. 2. The window slides vertically (the direction of WT for this subband) and, for each position, it provides a different particular realization.

![Figure 2. Sliding window successive positions (dashed line), for two particular realizations of horizontal details.](image)

The particular realizations are serialized, column by column, and placed on \( x \) rows. The window’s width may be equal to subband width, but its height should be shorter. The difference is imposed by the number of the particular realizations needed by ICA. For example, in the case of a subband consisting of 640x480 pixels, if ICA is performed on 8 particular realizations, the sliding window may be of maximum 633x480 pixels. In this situation, the particular realizations are, practically, equal to the wavelet subband. By construction, the matrix \( x \) has the particularity of being almost decorrelated on rows and columns. The reason is that both the columns and rows are fragments from subband columns, previously decorrelated by WT. As we shall see in the next section, this feature has important consequences on FastICA. In order to have the same property when the vertical details are analyzed, the window should slide horizontally and the particular realizations should be serialized row by row. For the diagonal details, which are decorrelated on both directions by WT, the choice for the sliding direction is not important. According to (5), the rows of \( s \) are independent components of the rows in \( x \). Since the rows of \( x \) are serialized images of the wavelet subband, it follows that the rows of \( s \) are also images. These images, representing the independent components of the wavelet subband, may be obtained by reshaping each row of \( s \) like a sliding window. The ICA of wavelet coefficient may proceed subband by subband and resolution by resolution. Our approach preserves thus, the multiresolution feature of WT.
4. Some properties of multiresolution ICA

4.1. Multiresolution ICA orthogonality

Generally, the mixing matrix in mathematical model (5) is not orthogonal. An exception is represented by the white data (x with uncorrelated rows). In this case, \( x \cdot x^\tau = I \) (where \( \tau \) denotes the transposition operation) and, consequently, the rows of \( A \) are orthogonal [7]:

\[
I = x \cdot x^\tau = A \cdot s \cdot (A \cdot s)^\tau = A \cdot s \cdot s^\tau \cdot A^\tau = A \cdot A^\tau
\] (10)

Since a square orthogonal matrix of dimension \( n \) has only \( n(n-1)/2 \) freedom degrees, the property (10) is used by several ICA algorithms in order to reduce the problem complexity (only half of \( A \) elements should be estimated in this case). The algorithms of this class have, in the beginning, a whitening step. FastICA works similarly: \( x \) is whitened by PCA and the independent components are extracted from white data. In multiresolution ICA, the PCA is useless and it may be removed from FastICA. Indeed, as we have shown in Section 3, by construction, the rows of \( x \) are uncorrelated. Therefore, in order to have white data, it is sufficient to normalize \( x \) rows by their standard deviation. Moreover, since \( x \) rows are serialized sliding windows from the same subband, their standard deviation is equal to the subband standard deviation. Consequently, in the case of data consisting of wavelet coefficients, the whitening stage may be replaced by the simple normalization of \( x \) by its standard deviation. It is the approach that we have used in developing the multiresolution ICA. As the wavelet coefficients in \( x \) are not perfectly decorrelated, the solution found by FastICA - without PCA - will be probably less optimal from the statistical independence point of view. However, with our approach, by accepting a loss of independence, we get the advantages of a less intensive computation (contrary to PCA, WT has a fast algorithm) and of an orthogonal basis (FastICA, without PCA, provides always a mixing matrix with orthogonal columns).

4.2. Multiresolution ICA as a band-pass filtering

In Section 2.1, we have shown that WT is a multiband filtering. The details on a subband are the result of filtering the image by a wavelet having a given bandwidth and orientation. Since in multiresolution ICA, the wavelet coefficients are transformed by ICA, it is useful to know how ICA modifies their frequency band and how this effect may be controlled. Due to its particular configuration, the projection of \( x \) on the vector basis in \( A \) (according to ICA model (5)) is equivalent to convolving the wavelet subband by the columns of \( A \). Consequently, the independent components may be seen as the result of filtering the wavelet subband by the filters in \( A \). The columns’ spectra of the mixing matrix \( A \) may give, therefore, the answer to the above question. As explained in Section 2.2, depending on the initial guess for \( A \), FastICA may provide different solutions. In the following, we shall show these spectra in two particular situations. By developing the multiresolution ICA, our intention was to obtain a hybrid transform, joining properties from both WT and ICA. For this reason, in order to constraint FastICA to look for a solution close to wavelets, we have given, as initial guess for \( A \), a part of the the wavelet basis. With this approach, for two independent components and db3 wavelet, the initial guess for analysing the subbands at the highest resolution should be:

\[
A = \begin{pmatrix}
0.00 & 0.00 & 0.04 & 0.12 & -0.19 & -0.65 & 1.14 & -0.47 \\
0.04 & 0.12 & -0.19 & -0.65 & 1.14 & -0.47 & 0.00 & 0.00
\end{pmatrix}
\] (11)

The wavelet on the columns of \( A \) is the same as the one used for obtaining the highest resolution subbands and the columns are shifted such to have orthogonality (this prevents FastICA to modify too much \( A \), by orthogonalization). At next resolution, since the wavelet is dilated by 2, the columns of \( A \) will be approximately twice longer and shifted by four. For more relevance, we give in Fig. 4 the spectra of ICA basis obtained for 2nd resolution initial...
guess, for the test image "Arabesque" (Fig. 3). It is, certainly, a basis specific to the analyzed image (ICA is based on data statistics); however, it worth to note the filters similarity and the fact that they are reduced wavelet band versions.

Given ICA filters in Fig. 4, the independent components should have a rather similar aspect. The two components, reshaped and displayed in Fig. 5, confirm our assumption. Their approximate negentropies, estimated by (9), are also close: 2.01 and 1.98.

Despite their similar aspect, the two independent components in Fig. 5 should be far from being identical. The spatial representation of ICA filters (Fig.6) explains the components’ apparent similarity: ICA filters are rather similar and shifted like wavelets.

Another way to impose a wavelet-like orthogonal family as initial guess for $A$ consists in combining a wavelet and a low-pass filter (like the couple scale function - wavelet, used in multiresolution analysis). The law-pass filter may be, for instance, a Gaussian with an appropriate variance (in order to have almost orthogonality). In the case of Flower image, as the low frequencies were already removed by WT, the ICA filter derived from the Gaussian, succeeds at saving the lower frequencies in the wavelet subband (Fig.7b).

The independent components in Fig. 7 were obtained for an initial guess constituted of a Gaussian with the standard deviation of 50 and db3 wavelet dilated by two. The ICA filters’ frequency response (continuous line), plotted against those of the filters in the initial guess (dotted lines), are shown in Fig. 8. The filter derived from the Gaussian is a low-pass filter. The other one is, like the originating wavelet, a pass-band filter. Opposite to the filters derived for "Arabesque" (Fig. 4), its band recovers the upper part of the wavelet pass-band, by letting the lower part for to the Gaussian like ICA filter. This particular frequency response is imposed.

Figure 3. Scans of two Jacquard patterns created by "Tissages de Charlieu" (640x480 pixels, 8bits/pixel): "Arabesque" (a) and "Flowers" (b).

(a) (b)
Figure 4. The frequency responses of the two ICA filters (continuous line) superimposed to the frequency response of the db3 wavelet (dotted line), at $2^n$ resolution, for image "Arabesque".

Figure 5. The vertical details and its two independent components (IC), at 2nd resolution (upper left corner of "Arabesque" image): wavelet subband (a), first IC (b) and second IC (c).

Figure 6. ICA filters (a) and db3 wavelets in the initial guess (b), at 2nd resolution, for image "Arabesque".
by Flower spectrum, which is different from that of "Arabesque". It must be noted that the frequency responses of the two ICA filters are complementary in the wavelet pass-band filter (an expression of uncorrelation). They perform a subband analysis of the wavelet subband.

The independent components in Fig. 8 are not anymore similar. The Gaussian based filter has provided an image that contains mainly the jacquard pattern, while the wavelet based one has retrieved the woven texture. This has been possible because, at this resolution, the pattern and the texture of Flower image are two independent features. Consequently, by using an initial guess constituted of a Gaussian and a wavelet, at appropriate resolution, the multiresolution ICA may arrive to separate rather well the jacquard pattern and texture. In our example, this was more obvious at second resolution but, depending on textile sample or image sensor, the separation may be possible at various resolutions and across more scales.

5. Towards an image recognition application
We have developed the multiresolution ICA in order to have a method capable to recognize textile images sensed in difficult conditions. In the previous section, we have shown that, by
using a couple of a Gaussian and a wavelet, it is possible to separate the pattern from the woven texture. This suggests that the pattern or texture could be also separated from some noise specific to difficult sensing conditions, in order to have more accurate textile image recognition. In the following, we shall investigate the multiresolution ICA ability to extract textile pattern and texture from noisy images. The noisy images were simulated by adding to a test image, Gaussian white noise of standard deviation varying in the range \([0, 100]\). For each noisy image, we have extracted the independent components at the first and second resolutions and we have compared them with the corresponding components of the test image. In order to measure their similarity, we have used the symmetrical Kullback divergence (which has the properties of a distance) as in [8]. Fig. 9 shows the components’ Kullback distance vs. the noise standard deviation, in the case of vertical details of Flower. The distance between the normalized wavelet coefficients (of variance equal to unity) is also plotted.

![Figure 9](image)

**Figure 9.** Pattern component (continuous line), texture component (dashed line) and wavelet coefficients (dotted line), for image ”Flower”: 1st resolution (a), 2nd resolution (b) and 2nd resolution, zoom on low distances (c).

For both components, the distance increases with the noise intensity, showing its presence in texture as in pattern components (a natural consequence of white noise theoretically infinite frequency band). However, it is interesting to note that the distance between the pattern components is almost everywhere lower than the wavelet coefficients’ one. At second resolution, the texture distance has a rapid increase, probably because of the large amount of noise captured by this component. Fig. 10 shows the two independent components for a noise of standard deviation equal to 30. The jacquard pattern is still visible in the independent component retrieved by the Gaussian based filter.

6. Conclusions
The Multiresolution ICA, proposed in this paper, combines in a single transform the properties of WT and ICA, two types of analysis largely used in image and signal processing. As, by multiresolution ICA, it is possible to retrieve image independent components at various resolutions, the recognition applications might largely benefit from our approach. The multiresolution ICA algorithm that we have used is a modified FastICA. Although it combines two transforms, the multiresolution ICA algorithm is not more complex than FastICA. On the contrary, by replacing PCA by WT, we have obtained a multiresolution ICA that is faster than FastICA (unlike WT, the PCA has no fast algorithm). The constraints imposed to the mixing matrix are determinant in obtaining an appropriate solution. We have developed multiresolution ICA in order to use it in CBIR databases for textile images. The tests, done on jacquard images, have shown that multiresolution ICA is capable to separate the pattern and the fabrics texture, at certain resolutions, and to provide thus two specific features for database queries.
Figure 10. The vertical details and its two independent components, at 2nd resolution, for noisy "Flower" image (noise with standard deviation equal to 30): wavelet subband (a), IC retrieved by a Gaussian based filter (b) and IC retrieved by wavelet based filter (c).

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