Constraining Galactic cosmic-ray parameters with $Z \leq 2$ nuclei

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ABSTRACT

Context. The secondary-to-primary B/C ratio is widely used for studying Galactic cosmic-ray propagation processes. The $^2$H/$^4$He and $^3$He/$^4$He ratios probe a different Z/A regime, which provides a test for the “universality” of propagation.

Aims. We revisit the constraints on diffusion-model parameters set by the quartet ($^1$H, $^2$H, $^3$He, $^4$He), using the most recent data as well as updated formulae for the inelastic and production cross-sections.

Methods. Our analysis relies on the USINE propagation package and a Markov Chain Monte Carlo technique to estimate the probability density functions of the parameters. Simulated data were also used to validate analysis strategies.

Results. The fragmentation of CNO cosmic rays (resp. NeMgSiFe) on the interstellar medium during their propagation contributes to 20% (resp. 20%) of the $^3$He flux at high energy. The C to Fe elements are also responsible for up to 10% of the $^4$He flux measured at 1 GeV/n. The analysis of $^3$He/$^4$He (and to a lesser extent $^2$H/$^4$He) data shows that the transport parameters are consistent with those from the B/C analysis: the diffusion model with $\delta \sim 0.7$ (diffusion slope), $V_c \sim 20$ km s$^{-1}$ (galactic wind), $V_a \sim 40$ km s$^{-1}$ (reacceleration) is favoured, but the combination $\delta \sim 0.2$, $V_c \sim 0$, and $V_a \sim 80$ km s$^{-1}$ is a close second. The confidence intervals on the parameters show that the constraints set by the quartet data can compete with those derived from the B/C data. These constraints are tighter when adding the $^3$He (or $^2$H) flux measurements, and the tightest when the $^4$He flux is added as well. For the latter, the analysis of simulated and real data shows an increased sensitivity to biases. Using the secondary-to-primary ratio along with a loose prior on the source parameters is recommended to obtain the most robust constraints on the transport parameters.

Conclusions. Light nuclei should be systematically considered in the analysis of transport parameters. They provide independent constraints that can compete with those obtained from the B/C analysis.

Key words. methods: statistical – astroparticle physics – cosmic rays

1. Introduction

Secondary species in Galactic cosmic rays (GCRs) are produced during the CR journey from the acceleration sites to the solar neighbourhood by means of nuclear interactions of heavier primary species with the interstellar medium (ISM). Hence, they are tracers of the CR transport in the Galaxy (e.g., Strong et al. 2007). Studying secondary-to-primary ratios is useful because it factors out the “unknown” source spectrum of the progenitor, leaving $^2$H/$^4$He, $^3$He/$^4$He, B/C, sub-Fe/Fe – and recently $p/p$ (Putze et al. 2009; di Bernardo et al. 2010) – suitable quantities for constraining the transport parameters for species $Z \leq 30$.

Most secondary-to-primary ratios have $A/Z \sim 2$, and in that respect, $^3$He/$^4$He is unique because it probes a different regime and allows one to address the question of the “universality” of propagation histories. For instance, in an analysis in the leaky-box model (LBM) framework, Webber (1997) found that $^3$He/$^4$He data imply a similar propagation history for the light and heavier species (which was disputed in earlier papers). Webber also argued that the situation with regard to the $^2$H/$^4$He ratio is less clear, because the uncertainties on the measurements are large (mainly caused by instrumental and atmospheric corrections). H and He spectra are the most abundant species in the cosmic radiation, and thus $^2$H and $^3$He are the most abundant secondary species in GCRs. However, achieving a good mass resolution – especially at high energy – is experimentally challenging. This explains why the elemental B/C ratio received more focus both experimentally and theoretically (thanks to its higher precision data w.r.t. to the quartet data).

From the modelling side, after the first thorough and pioneering studies performed in the 60’s–70’s (Badhwar & Daniel 1963; Ramaty & Lingenfelter 1969; Meyer 1972; Mewaldt et al. 1976), the interest for the quartet nuclei somewhat stalled. Several updated analyses of the propagation parameters from the quartet were published whenever new data became available (see Table A.1 for references). However, very few dedicated studies were carried out in the 1980s (Beatty 1986; Webber et al. 1987) or 1990s (Webber 1990a; Seo & Ptuskin 1994; Webber 1997), and none in the first decade of this century. This is certainly related to the very slow pace at which new data became available in this period. Curiously, the most recent published data have not really been properly interpreted, i.e. for $^3$He/$^4$He data, IMAX92 (de Nolfo et al. 2000) and AMS-01 (Aguilar et al. 2011); and for $^3$He/$^4$He data, IMAX92 (Menn et al. 2000), SMILI-II (Ahlen et al. 2000), AMS-01 (Xiong et al. 2003), SMILI-II (Ahlen et al. 2000),
BESS98 (Myers et al. 2003), CAPRICE98 (Mocchiutti et al. 2003). Furthermore, almost all analyses have been performed in the successful but simplistic LBM except for a few studies\(^1\).

At the same time, the analysis of the B/C ratio has been scrutinised in greater detail. For instance, to replace the old usage of matching the data by means of an inefficient manual scan of the parameter space (e.g., Jones et al. 2001), more systematic scans were carried out (on the B/C and sub-Fe/Fe ratio) to get best-fit values as well as uncertainties on the parameters (Maurin et al. 2001; Lionetto et al. 2005; Evoli et al. 2008; di Bernardo et al. 2010). A recent improvement is the use of Markov Chain Monte Carlo (MCMC) techniques to directly access the probability-density function (PDF) of the GCR transport and source parameters (Putze et al. 2009, 2010, 2011; Trotta et al. 2011).

In this paper, we revisit the constraints set by the quartet nuclei and their consistency with the results of heavier nuclei. In the context of the forthcoming PAMELA and AMS-02 data on these ratios, we also discuss the strategy to adopt and intrinsic limitations of the transport parameters reconstruction. For that purpose, we take advantage of the data taken in the last decade as well as simulated data of any precision, and analyse them with an MCMC technique implemented in the USINE propagation code. This extends and complements analyses of the B/C ratio carried out (on the B/C and sub-Fe/Fe ratio) to get best-fit values as well as uncertainties on the parameters (Maurin et al. 2001; Lionetto et al. 2005; Evoli et al. 2008; di Bernardo et al. 2010). A recent improvement is the use of Markov Chain Monte Carlo (MCMC) techniques to directly access the probability-density function (PDF) of the GCR transport and source parameters (Putze et al. 2009, 2010, 2011; Trotta et al. 2011).

The paper is organised as follows. In Sect. 2, we briefly recall the main ingredients of the 1D diffusion model and the MCMC analysis. We also list the parameters that are constrained. The simulated data and their analysis are described in Sect. 3. The analysis of the real data is given in Sect. 4. We conclude in Sect. 5. Appendix A gathers the data sets and the updated cross-sections used in the quartet analysis.

2. MCMC technique, propagation and parameters

The MCMC technique and its use in the USINE propagation code is detailed in Putze et al. (2009) and summarised in Putze et al. (2010). The full details regarding the 1D transport model can be found in Putze et al. (2010). Below, we only provide a brief description.

2.1. An MCMC technique for the PDF of the parameters

The MCMC method, based on Bayesian statistics, is used to estimate the full distribution (conditional PDF) given some experimental data and some prior density for these parameters. Our chains are based on the Metropolis-Hastings algorithm, which allows to estimate the full distribution (conditional PDF) given some experimental data and some prior density for these parameters. The operator \(L\) (we omit the superscript \(j\)) describes the diffusion \(K(r,E)\) and the convection \(V(r)\) in the Galaxy, but also the decay rate \(\Gamma_{\text{rad}}(E) = 1/(\tau_{\text{UU}})\) if the nucleus is radioactive, and the destruction rate \(\Gamma_{\text{dest}}(r,E) = \sum_{\text{ISM}} n_{\text{ISM}}(r) \sigma_{\text{dest}}(E)\) for collisions with the ISM, in the form

\[
L(r,E) = -\nabla \cdot (K \nabla) + \nabla \cdot V + \Gamma_{\text{rad}} + \Gamma_{\text{dest}}.
\]

The coefficients \(b\) and \(c\) in Eq. (1) are first- and second-order gains/losses in energy, with

\[
b(r,E) = \left(\frac{dE}{dr}\right)_{\text{ion.coul}} - \frac{\nabla \cdot V}{3} E_k \left(\frac{m + E_k}{m + E_k}\right) + \frac{(1 + \beta^2)}{E} \times K_{pp},
\]

\[
c(r,E) = \beta^2 \times K_{pp}.
\]

In Eq. (3), the ionisation and Coulomb energy losses are taken from Mannheim & Schlickeiser (1994) and Strong & Moskalenko (1998). The divergence of the Galactic wind \(V\) gives rise to an energy-loss term related to the adiabatic expansion of cosmic rays. The last term is a first-order contribution in energy from reacceleration. Equation (4) corresponds to a diffusion in momentum space, leading to an energy gain. The associated diffusion coefficient \(K_{pp}\) (in momentum space) is taken from the model of minimal reacceleration by the interstellar turbulence (Osborne & Ptuskin 1988; Seo & Ptuskin 1994). It is related to the spatial diffusion coefficient \(K\) by

\[
K_{pp} \times K = 4 \left(\frac{V_d^2}{\delta (4 - \delta^2) (4 - \delta)}\right),
\]

where \(V_d\) is the Alfvénic speed in the medium.

We refer the reader to Appendix A of Putze et al. (2010) for the solution to Eq. (1) in the 1D geometry.

2.2. 1D Propagation model and parameters

The Galaxy is modelled to be an infinite thin disc of half-thickness \(h\), which contains the gas and the sources of CRs. The diffusive halo region (where the gas density is assumed to be equal to 0) extends to \(+L\) and \(-L\) above and below the disc. A constant wind \(V(r) = \text{sign}(z) \times V_c \times e_z\), perpendicular to the Galactic plane, is assumed. In this framework, CRs diffuse in the disc and in the halo independently of their position. Such semi-analytical models are faster than full numerical codes (GALPROP\(^2\) and DRAGON\(^3\)), which is an advantage for sampling techniques like MCMC approaches.

2.2.1. Transport equation

The differential density \(N_j\) of the nucleus \(j\) is a function of the total energy \(E\) and the position \(r\) in the Galaxy. Assuming a steady state, the transport equation can be written in a compact form as

\[
\frac{dN_j}{dE} + \frac{\partial}{\partial E} \left( b N_j - c \frac{\partial N_j}{\partial E} \right) = S_j.
\]

The operator \(L\) (we omit the superscript \(j\)) describes the diffusion \(K(r,E)\) and the convection \(V(r)\) in the Galaxy, but also the decay rate \(\Gamma_{\text{rad}}(E) = 1/(\tau_{\text{UU}})\) if the nucleus is radioactive, and the destruction rate \(\Gamma_{\text{dest}}(r,E) = \sum_{\text{ISM}} n_{\text{ISM}}(r) \sigma_{\text{dest}}(E)\) for collisions with the ISM, in the form

\[
L(r,E) = -\nabla \cdot (K \nabla) + \nabla \cdot V + \Gamma_{\text{rad}} + \Gamma_{\text{dest}}.
\]

The coefficients \(b\) and \(c\) in Eq. (1) are first- and second-order gains/losses in energy, with

\[
b(r,E) = \left(\frac{dE}{dr}\right)_{\text{ion.coul}} - \frac{\nabla \cdot V}{3} E_k \left(\frac{m + E_k}{m + E_k}\right) + \frac{(1 + \beta^2)}{E} \times K_{pp},
\]

\[
c(r,E) = \beta^2 \times K_{pp}.
\]

In Eq. (3), the ionisation and Coulomb energy losses are taken from Mannheim & Schlickeiser (1994) and Strong & Moskalenko (1998). The divergence of the Galactic wind \(V\) gives rise to an energy-loss term related to the adiabatic expansion of cosmic rays. The last term is a first-order contribution in energy from reacceleration. Equation (4) corresponds to a diffusion in momentum space, leading to an energy gain. The associated diffusion coefficient \(K_{pp}\) (in momentum space) is taken from the model of minimal reacceleration by the interstellar turbulence (Osborne & Ptuskin 1988; Seo & Ptuskin 1994). It is related to the spatial diffusion coefficient \(K\) by

\[
K_{pp} \times K = 4 \left(\frac{V_d^2}{\delta (4 - \delta^2) (4 - \delta)}\right),
\]

where \(V_d\) is the Alfvénic speed in the medium.

We refer the reader to Appendix A of Putze et al. (2010) for the solution to Eq. (1) in the 1D geometry.

\(^1\) See & Ptuskin (1994) used a 1D diffusion model with reacceleration, whereas Webber & Rockstroh (1997) relied on a Monte Carlo calculation; both studies conclude in a similar way (consistency with the gramage required for heavier species to produce the light secondaries). A preliminary effort based on the GALPROP propagation code was also carried out in Moskalenko et al. (2003).

\(^2\) http://galprop.stanford.edu/

\(^3\) http://www.desy.de/~maccione/DRAGON/
2.2.2. Free parameters of the analysis

The exact energy dependence of the source and transport parameters is unknown, but they are expected to be power laws of \( R = p c / Z e \) (rigidity of the particle).

The low-energy diffusion coefficient requires a \( \beta = v / c \) factor that takes into account the inevitable effect of particle velocity on the diffusion rate. However, the recent analysis of the turbulence dissipation effects on the transport coefficient has shown that this coefficient could increase at low-energy (Ptuskin et al. 2006; Shalchi & Büsching 2010). Following Maurin et al. (2010), it is parametrised to be

\[
K(E) = \beta^{n_s} \cdot K_0 R^\delta.
\]

The default value used for this analysis is \( \eta_T = 1 \). The two other transport parameters are \( V_c \), the constant convective wind perpendicular to the disc, and \( V_a \), the Alfvénic speed regulating the reacceleration strength (see Eq. (5)). The two models considered in this paper are given in Table 1.

The low-energy primary source spectrum from acceleration models (e.g., Drury 1983; Jones 1994) is also unknown. We parametrise it to be

\[
Q_{E_{\nu s}}(E) \equiv \frac{dQ}{dE_{k/n}} = q \cdot \beta^{n_s} \cdot R^{-\alpha},
\]

where \( q \) is the normalisation. The reference low-energy shape corresponds to \( q_s = -1 \) (to have \( dQ / dE \propto p^{-\alpha} \), i.e. a pure power-law.

The halo size of the Galaxy \( L \) cannot be solely determined from secondary-to-primary stable ratios and requires a radioactive species to lift the degeneracy between \( K_0 \) and \( L \). However, the range of allowed values is still very loosely constrained (e.g., Putze et al. 2010). Because the transport and source parameters can always be rescaled if a different choice of \( L \) were assumed (see the scaling relations given in Maurin et al. 2010), where \( \delta \) is shown not to depend on \( L \), we fixed it to \( L = 4 \) kpc. This will also facilitate the comparison of the results obtained in this paper with those of our previous studies (Putze et al. 2010, 2011).

3. MCMC analysis on artificial data sets

MCMC techniques make the scan of high-dimensional parameter spaces possible, such that a simultaneously estimation of transport and source parameters is possible (Putze et al. 2009). However, transport parameters are shown to be strongly degenerated for the B/C ratio data in the range 0.1–100 GeV/nuc (Maurin et al. 2010), and source and transport parameters are correlated (Putze et al. 2009, 2010). For GCR data in general, the fact that primary fluxes and secondary fluxes are not measured to the same accuracy\(^4\) can bias or prevent an accurate determination of these parameters: a simultaneous fit has been observed to be driven by the more accurately measured primary flux (Putze et al. 2011). This, although statistically correct, might not maximise the information obtained on the transport parameters. Therefore, several strategies can be considered when dealing with GCR data:

- a combined analysis of secondary-to-primary ratio and primary flux to constrain the source and transport parameters simultaneously;
- a secondary-to-primary ratio analysis only, either fixing the source parameters (i.e., using a strong prior), or using a loose prior;
- a primary flux analysis only, either fixing the transport parameters (i.e., using a strong prior), or using a loose prior.

In the literature, the strong prior approach has almost always been used to determine the transport or the source parameters. The question we wish to address is how sensitive the parameters of interest are to various strategies. This is the motivation for introducing artificial data, i.e. an ideal case study, as opposed to the case of real data where several other complications can arise (systematics in the data and/or the use of the incorrect propagation model or solar modulation model/level).

3.1. Sets of artificial data

To be as realistic as possible, we chose models that roughly reproduce the actual data points (see Fig. 4), but also match the typical energy coverage, number of data points, central value and spread (error bars) of the measurements\(^5\). To speed-up the calculation and for this section only, we assume that all \(^3\)He comes from \(^4\)He (see Sect. 4.1 for all relevant progenitors). No systematic errors were added, although they may set a fundamental limitation in recovering the cosmic-ray parameters. In practice, the statistical errors for the artificial data sets correspond to the sigma of the standard Gaussian deviations used to randomise the data points around their model value: \(^3\)He/\(^4\)He was generated with statistical errors of 10% while He fluxes were generated with 1% and 10% errors, to simulate the situation where primary fluxes are “more accurately” or “equally” measured (in terms of statistics) than the secondary-to-primary ratio.

The parameters of the two models used to simulate the data are listed in the two italic lines in Table 2, denoted Model II and Model III. They correspond to extreme values of the diffusion slope \( \delta \), but which still roughly fall in the range of values found for instance from the B/C analysis (Putze et al. 2010): for Model II with reacceleration only \( (V_c = 0) \), \( \delta \) is generally found to fall between 0.1 and 0.3, whereas for Model III with convection and reacceleration, \( \delta \) is generally found to fall into the 0.6–0.8 range (Jones et al. 2001; Maurin et al. 2010).

3.2. Strategies to analyse the data

To test the impact on the reconstruction of the transport (\( \eta_T, K_0, \delta, V_a \), and \( V_c \)) and/or source (\( \alpha \) and \( \eta_s \)) parameters, we tested

\(^4\) Statistical uncertainties are smaller for primary fluxes (more abundant than secondary fluxes), but the latter are more prone to systematics than ratios (e.g. secondary-to-primary ratios used to fit transport parameters).

\(^5\) The uncertainty on the H and He fluxes is a few percent (for the recent PAMELA data, Adriani et al. 2011) and 20–50% for the \(^3\)He/\(^4\)He ratio.

\[ L \leq 2 \text{ nuclei} \]
Table 2. MCMC analysis on simulated data.

| Option (param): data | $\eta_T$ | $K_0 \times 10^2$ (kpc$^{-2}$ Myr$^{-1}$) | $\delta$ | $V_T$ (km s$^{-1}$) | $V_0$ (km s$^{-1}$) | $\alpha$ | $\eta_T$ | $\chi^2_{\text{best/d.o.f.}}$ |
|----------------------|----------|---------------------------------|---------|-----------------|-----------------|--------|---------|-----------------|
| **Model II**         |          |                                 |         |                 |                 |        |         |                 |
| 1: $^3\text{He}/^4\text{He}+^4\text{He}$ | 10.3$^{+1.96}_{-1.89}$ | 0.185$^{+0.34}_{-0.35}$ | (--) | 72.7$^{+6.5}_{-7.6}$ | 2.29$^{+0.09}_{-0.10}$ | 0.78$^{+0.13}_{-0.17}$ | 0.93 |
| 2: $^4\text{He}/^4\text{He}$ | 10.2$^{+2.00}_{-2.31}$ | 0.192$^{+0.31}_{-0.33}$ | (--) | 73.0$^{+7.2}_{-8.2}$ | 2.29$^{+0.03}_{-0.04}$ | 0.88$^{+0.14}_{-0.15}$ | 1.08 |
| 3: $^3\text{He}/^4\text{He}$ | 9.7$^{+3.1}_{-3.4}$ | 0.199$^{+0.40}_{-0.44}$ | (--) | 68.0$^{+4.4}_{-4.6}$ | 2.70$^{+0.40}_{-0.40}$ | 1.5$^{+0.9}_{-1.0}$ | 0.87 |
| 4: $^3\text{He}/^4\text{He} +$ src = prior | 9.0$^{+1.1}_{-1.0}$ | 0.2$^{+0.15}_{-0.16}$ | (--) | 73.3$^{+5.5}_{-5.6}$ | 2.70$^{+0.58}_{-0.58}$ | 1.5$^{+0.9}_{-1.0}$ | 1.08 |
| **Model III**        |          |                                 |         |                 |                 |        |         |                 |
| 1: $^3\text{He}/^4\text{He}+^4\text{He}$ | 1.66$^{+0.42}_{-0.52}$ | 1.5$^{+0.40}_{-0.47}$ | 0.51$^{+0.33}_{-0.78}$ | 18.4$^{+17.0}_{-10.0}$ | 54.1$^{+18.0}_{-4.9}$ | 2.29$^{+0.99}_{-1.7}$ | 1.43 |
| 2: $^4\text{He}/^4\text{He}$ | 1.42$^{+2.88}_{-1.44}$ | 0.66$^{+0.97}_{-0.93}$ | 0.725$^{+0.10}_{-0.13}$ | 19.7$^{+0.36}_{-0.41}$ | 37.1$^{+9.2}_{-3.3}$ | 2.34$^{+0.30}_{-0.17}$ | 1.00 |
| 3: $^3\text{He}/^4\text{He}$ | 1.49$^{+0.40}_{-0.40}$ | 0.71$^{+0.93}_{-0.93}$ | 0.71$^{+0.14}_{-0.14}$ | 18.3$^{+1.66}_{-1.61}$ | 40.7$^{+2.2}_{-2.2}$ | 2.70$^{+0.58}_{-0.58}$ | 0.83 |
| 4: $^3\text{He}/^4\text{He} +$ src = prior | 1.34$^{+0.29}_{-0.10}$ | 0.37$^{+0.30}_{-0.16}$ | 0.65$^{+0.29}_{-0.17}$ | 20.3$^{+1.33}_{-4.09}$ | 42.2$^{+1.53}_{-11.6}$ | [1.8, 2.5] | [–2, +2] | 0.96 |
| **Model III: analysis with Model II** |          |                                 |         |                 |                 |        |         |                 |
| 1: $^3\text{He}/^4\text{He}+^4\text{He}$ | 13.8$^{+1.6}_{-1.8}$ | 0.21$^{+0.8}_{-0.6}$ | 0.51$^{+0.33}_{-0.78}$ | 18.4$^{+17.0}_{-10.0}$ | 54.1$^{+18.0}_{-4.9}$ | 2.29$^{+0.99}_{-1.7}$ | 1.43 |
| 2: $^4\text{He}/^4\text{He}$ | 11.4$^{+1.5}_{-1.88}$ | 0.263$^{+1.8}_{-3.5}$ | 0.85$^{+0.4}_{-0.3}$ | 18.4$^{+17.0}_{-10.0}$ | 54.1$^{+18.0}_{-4.9}$ | 2.29$^{+0.99}_{-1.7}$ | 1.43 |
| 4: $^3\text{He}/^4\text{He} +$ src = prior | 20.7$^{+20.7}_{-2.2}$ | 0.089$^{+0.21}_{-0.18}$ | (--) | 107$^{+12.8}_{-2.8}$ | 2.70$^{+0.58}_{-0.58}$ | 1.5$^{+0.9}_{-1.0}$ | 1.08 |
| **Model II: analysis with Model III** |          |                                 |         |                 |                 |        |         |                 |
| 1: $^3\text{He}/^4\text{He}+^4\text{He}$ | 1.37$^{+0.1}_{-0.1}$ | 4.0$^{+0.3}_{-0.4}$ | 0.29$^{+0.3}_{-0.4}$ | 17.2$^{+0.9}_{-0.0}$ | 73.5$^{+7.2}_{-1.7}$ | 2.21$^{+1.8}_{-0.5}$ | 0.80$^{+0.12}_{-0.29}$ | 0.97 |
| 2: $^4\text{He}/^4\text{He}$ | 0.87$^{+0.92}_{-0.1}$ | 7.4$^{+1.9}_{-1.9}$ | 0.25$^{+0.3}_{-0.3}$ | 8.6$^{+0.9}_{-0.9}$ | 76.3$^{+5.5}_{-0.5}$ | 2.23$^{+0.4}_{-0.4}$ | 0.68$^{+0.7}_{-1.0}$ | 1.1 |
| 4: $^3\text{He}/^4\text{He} +$ src = prior | 0.43$^{+0.39}_{-0.07}$ | 4.9$^{+0.49}_{-0.7}$ | 0.24$^{+0.50}_{-0.31}$ | 18.4$^{+12.0}_{-15.9}$ | 69.9$^{+12.0}_{-18.0}$ | [1.8, 2.5] | [–2, +2] | 1.05 |

Notes. For each model considered (II or III), the first line (in italic) gives the value of the transport parameters used to generate the artificial data (the halo size is set to L = 4 kpc). Each subsequent line corresponds to the MCMC-reconstructed values (most probable value, and relative uncertainties corresponding to the 68% CI) based on a given parameters/data option (see Sect. 3.2). The last column gives the value of the best $\chi^2$/d.o.f. configuration found (corresponding to the curves shown in Fig. 1). A value in square brackets corresponds to the fixed value of the parameter for the analysis. An interval in square brackets corresponds to the prior used for the analysis (the posterior PDF obtained is then close to the prior).

the following combinations (data set|model parameters) for the analysis.

$^3\text{He}/^4\text{He} + $ He data

- Option 1 ($\sigma_{\text{He}} = 10\%$): transport + source;
- Option 2 ($\sigma_{\text{He}} = 1\%$): transport + source;
- Option 2' ($\sigma_{\text{He}} = 1\%$): transport (source = “true” value);

$^3\text{He}^4\text{He}$ ratio only

- Option 3: transport + source;
- Option 3': transport (source = “true” value);
- Option 4: transport (source = weak prior).

We find that the He data alone cannot constrain the transport parameters (not shown here), in agreement with the Putze et al. (2011) results (strong degeneracy between $\alpha$ and $\delta$, but also with $K_0$, $V_T$, and $\eta_T$).

3.3. Analysis of the artificial data

In a first step, we used the MCMC technique to estimate the best-fit parameters. The $^3\text{He}/^4\text{He}$ ratio and $^4\text{He}$ flux are shown for model II (crosses) and the corresponding simulated data (pluses) in Fig. 1. When both the $^3\text{He}/^4\text{He}$ and $^4\text{He}$ data are included in the fit (options 1 and 2, red dotted and red solid lines), the initial flux (crosses) is perfectly recovered for $^4\text{He}$, and very well recovered for $^3\text{He}/^4\text{He}$. When the fit is only based on $^3\text{He}/^4\text{He}$ (options 3 and 5, magenta dashed and blue solid lines), the initial flux is obviously not recovered (unless the source parameters are set to the true value as in option 3), but the $^3\text{He}/^4\text{He}$ ratio is consistent with the data. Unsurprisingly, the associated $\chi^2$/d.o.f. values (last column of Table 2) are close to 1.
The MCMC analysis allows us to go further because it provides the PDF of the parameters from which the most probable value and confidence intervals (CIs) are obtained. The results are gathered in Table 2 and Fig. 2. The various panels of the latter represent the PDFs for transport and source parameters for each “option” for Model II. (For clarity the correlation plots are not shown.) From these plots, some arguments are in favour of a simultaneous use of the secondary-to-primary ratio and the primary flux (here, $^{3}\text{He}/^{4}\text{He}$ and $^{4}\text{He}$), but not all.

### 3.3.1. Advantages from a simultaneous analysis (ratio + flux)

A simultaneous analysis ($^{3}\text{He}/^{4}\text{He}$ + $^{4}\text{He}$) gives more stringent constraints on the transport parameters than an analysis of the secondary-to-primary ratio (compare the PDFs for the red curves and blue curves in Fig. 2 respectively, for $K_0$, $\delta$, and $V_a$). This partly comes from the observed correlations between transport and source parameters\(^6\) (e.g., Putze et al. 2009). Option 2' and option 3' with fixed source parameters show that the better CIs on the transport parameters come from the information contained in primary fluxes\(^7\). The same conclusions hold true for Model III ($V_0 \neq 0$), although with larger relative uncertainties because of the two extra transport parameters of the model ($\eta_T$ and $V_c$).

### 3.3.2. What if the wrong model is used?

As an illustration, we analysed data simulated from model III ($V_0 \neq 0$) with model II ($V_c = 0$) and vice versa (lower half of Table 2). If we force $V_c = 0$ (while $V_{\text{true}} = 18$ km s\(^{-1}\)), the diffusion slope decreases to a low value $\delta \sim 0.2$ ($\delta_{\text{true}} = 0.7$), while the Alfvénic speed increases to a high value $V_a \sim 100$ km s\(^{-1}\) ($V_a^{\text{true}} = 41$ km s\(^{-1}\)). The higher $\chi^2_{\text{best}}$/d.o.f. value with respect to the one obtained fitting the correct model easily disfavours this model. The second test (simulated with II, analysed with III) indicates whether allowing for more freedom in the analysis (two additional free parameters $\eta_T$ and $V_c$) affects the recovery of the parameters. The values of $\delta_{\text{true}} = 0.2$ and $V_a^{\text{true}} = 70$ km s\(^{-1}\) are recovered, while the others are systematically offset but less than $3\sigma$ away from their true value. In this simple example, adding extra parameters is not a problem because the $\chi^2_{\text{best}}$/d.o.f. still favours the minimal model. However, with real data (see

\(^{6}\) More stringent constraints on the source parameters (from more precise data) lead to more stringent constraints on the transport values: option 1 ($\sigma_{\text{true}} = 10\%$) vs. option 2 ($\sigma_{\text{true}} = 1\%$).

\(^{7}\) Note that the lack of constraints on the source parameters for option 4 confirms that the secondary-to-primary ratio is only marginally sensitive to the source parameters (e.g., Putze et al. 2011).
Sect. 4.3 and the B/C analysis of Putze et al. 2010; Maurin et al. 2010) it is so far impossible in this situation to conclude whether the correct model is used because of the possible problem of multimodality and biases from systematics (see below).

3.3.3. Drawbacks of a simultaneous analysis

For Model II, but even more for Model III (which has more free transport parameters), a possible worry is the presence of multimodal PDF distributions, which more often happens for the simultaneous analysis. An example of multimodality is the analysis with Model II (i.e. \( V_d = 0 \)) of data simulated with Model III, which corresponds to a local minimum of the true Model III parameters. This is of no consequence for the ideal case, but real data may suffer from systematics errors and/or the inappropriate solar modulation model may be chosen. In that case, the true minimum can be displaced, or turned into a local minimum (and vice versa). Measurements over the past decades showed that primary fluxes are more prone to systematics than secondary-to-primary ratios. Primary fluxes are also more sensitive to solar modulation than ratios. For these reasons, the use of secondary-to-primary data only (option 4) for the analysis, although they perform less well in obtaining stringent limits on the transport parameters, is expected to be more reliable and robust.

3.4. Recommended strategy for analysing real data

The most robust approach to determine the transport parameters (and their CIs) is to analyse the secondary-to-primary ratio using a loose but physically-motivated prior on the source parameters (option 4). This has the advantage of taking into account the correlations between the source and transport parameters. The simultaneous analysis is mandatory to obtain the source parameters (option 1 or 2). It also yields more information on the transport parameters, but the primary fluxes can bias their determination if it suffers from systematics. We recommend such an analysis to be performed in addition to the direct secondary-to-primary ratio analysis, in order to obtain the following diagnosis: if the range of values for the transport parameters from both analyses are

- inconsistent, it indicates that the values and CIs obtained for the source parameters are biased or unreliable;
- consistent, the selected propagation model may be the correct one, and the source parameters are then the most probable ones for this model. However, the CIs on the transport parameters are very likely to be underestimated if the error bars on the ratio are much larger than those on the primary fluxes.

Obviously, our analysis does not cover the range of all systematics when dealing with real data. A more systematic analysis – e.g. covering a wider family of propagation models, several solar modulation models, several sources of systematics in the data – goes far beyond the scope of this paper. Note that some of these effects are likely to be energy dependent, which complicates the analysis even more. With the successful installation of the AMS-02 detector on the ISS and its expected high-precision data, these problems are bound to gain importance.

4. Constraints from the quartet data

We now apply the MCMC technique to the analysis of real data. We emphasise that for the artificial data, we assumed the \(^3\)He to come solely from the \(^4\)He fragmentation to speed up the calculation. Based on our new compilation for the cross-section formulae (see Appendix B), we included the contributions from \( A > 4 \) CR parents, checking which parents are relevant (Sect. 4.1). After determining the heaviest parent to consider in the calculation, we then move on to the result of the MCMC analysis (Sect. 4.2), and those from our best analysis (Sect. 4.3).

4.1. Fractional contributions

At first order, the contribution to the \(^2\)H and \(^3\)He secondary production from \( Z > 4 \) nuclei is proportional to the source term \( S_j \) (see Eq. (1)). For a secondary contribution, the source term is proportional to the primary flux of the parents (which have been measured by many experiments), and to the production cross-section. Normalised to the production from \(^4\)He, we have

\[
\text{Rel}_P \propto \frac{S_j}{S_{^4\text{He}}} \propto \frac{\Phi_P}{\Phi_{^4\text{He}}} \times \gamma_j^P,
\]

where \( P \) is the CR projectile, \( S \) is the secondary fragment considered, and \( \gamma_j^P \) (see Eq. (B.3)) is the production cross-section.

### Table 3. Estimated fractional contribution of projectile \( A > 4 \) to the \(^2\)H and \(^3\)He fluxes.

| P   | \( A_P \) | \( \frac{\Phi_P}{\Phi_{^4\text{He}}} \) | \( \gamma_j^P \) | \( \gamma_j^{^2\text{He}} \) | \( \gamma_j^{^3\text{He}} \) |
|-----|---------|-------------------------------|----------------|-------------------|-------------------|
| He  | 4       | 1.0                           | (…)            | (…)               | 60.1              | 86.0              |
| C   | 12      | \( 3.3 \times 10^{-2} \)    | 5.5            | 2.3               | 7.5               | 3.6               |
| N   | 14      | \( 7.4 \times 10^{-3} \)    | 6.6            | 2.4               | 2.0               | 0.8               |
| O   | 16      | \( 3.4 \times 10^{-2} \)    | 7.8            | 2.6               | 10.7              | 4.1               |
| F   | 19      | \( 5.2 \times 10^{-4} \)    | 9.6            | 2.8               | 0.2               | 0.1               |
| Ne  | 22      | \( 5.1 \times 10^{-3} \)    | 11.4           | 3.1               | 2.4               | 0.8               |
| Na  | 23      | \( 8.8 \times 10^{-4} \)    | 12.1           | 3.2               | 0.4               | 0.1               |
| Mg  | 24      | \( 6.7 \times 10^{-3} \)    | 12.7           | 3.3               | 3.4               | 1.0               |
| Al  | 26      | \( 1.1 \times 10^{-3} \)    | 14.0           | 3.5               | 0.6               | 0.2               |
| Si  | 28      | \( 5.5 \times 10^{-3} \)    | 15.3           | 3.7               | 3.4               | 1.0               |
| P   | 31      | \( 1.8 \times 10^{-4} \)    | 17.3           | 4.0               | 0.1               | <0.1              |
| S   | 32      | \( 1.0 \times 10^{-3} \)    | 17.9           | 4.1               | 0.7               | 0.2               |
| Cl  | 35      | \( 1.8 \times 10^{-4} \)    | 20.0           | 4.5               | 0.1               | <0.1              |
| Ar  | 36      | \( 3.1 \times 10^{-4} \)    | 20.6           | 4.6               | 0.3               | 0.1               |
| K   | 39      | \( 2.1 \times 10^{-4} \)    | 22.7           | 5.0               | 0.2               | <0.1              |
| Ca  | 40      | \( 6.1 \times 10^{-4} \)    | 23.4           | 5.1               | 0.6               | 0.1               |
| Sc  | 45      | \( 1.0 \times 10^{-4} \)    | 27.0           | 5.8               | 0.1               | <0.1              |
| Ti  | 48      | \( 3.4 \times 10^{-4} \)    | 29.2           | 6.2               | 0.4               | 0.1               |
| V   | 51      | \( 1.8 \times 10^{-4} \)    | 31.4           | 6.6               | 0.2               | 0.1               |
| Cr  | 52      | \( 3.6 \times 10^{-4} \)    | 32.1           | 6.8               | 0.5               | 0.1               |
| Mn  | 55      | \( 3.0 \times 10^{-4} \)    | 34.3           | 7.2               | 0.4               | 0.1               |
| Fe  | 56      | \( 3.7 \times 10^{-3} \)    | 35.1           | 7.4               | 5.3               | 1.3               |
| Ni  | 58      | \( 2.4 \times 10^{-4} \)    | 36.6           | 7.7               | 0.4               | 0.1               |
As seen from Table 3, the most important contributions from primary species heavier than He (Z > 2) are C and O, followed by Mg and Si and finally Fe. The total contribution of these species amounts to ∼35% for ²H and ∼11% for ³He, but mixed species (such as N) or less abundant species also contribute to ∼5%.

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A proper calculation of these fractional contributions involves the full solution of the propagation equation, taking into account energy gains and losses, total inelastic reactions, and convection. Based on the propagation parameters found to fit the current data best (see next section), we show in Fig. 3 the fractional contribution of A > 4 nuclei as a function of energy for the full calculation. It confirms the previous figures, but with a residual energy dependence (itself depending on the species) hitting a plateau above ∼100 GeV/n. The difference can be mostly attributed to a preferential destruction of heavier nuclei at low energy. Note that for ²H production, the coalescence of two protons (long-dashed pink curve) contributes up to 40% of the total at ∼1 GeV/n energy (peak of the cross-section, see Fig. B.3). Depending on the precision reached for the data, it is important to include the CNO contribution (e.g. Ramaty & Lingenfelter 1969; Jung et al. 1973b; Beatty 1986), but also all contributions up to Ni.

Finally, the fragmentation of CNO can also affect the ¹H and ⁴He primary fluxes. The peak of contribution occurs at GeV/n as secondary fluxes drop faster than primary fluxes with energy. Figure 3 shows this contribution to be ≤10% for ³He. With the high precision measurement from PAMELA and the even better measurements awaited from AMS-02, this will need to be further looked into in the future.

4.2. MCMC analysis: test of several data combinations

Given the accuracy of current data (see Fig. 4), we must take into account the contribution from all parent nuclei at least up to ³⁰Si. In the rest of the analysis, we use PAMELA data for He (Adriani et al. 2011), as they overcome all others in the ∼GeV–TeV range in terms of precision. Before giving our final results, and to complement Sect. 3.2, we discuss the appropriate choice of data to consider here, in order to get the best balance between robustness and reliability for the ³H and ³He-related analyses.

4.2.1. Simulated vs. real data

We start by comparing the results obtained with the simulated and the actual data set. To avoid lengthy comparisons of numbers, we limit ourselves to Model III (where we also fix ηf to its default value, i.e. 1). The obvious difference with the simulated data is that we no longer have access to the true source parameters (automatically excluding options 2 and 4 discussed in Sect. 3.2). For the simultaneous analysis using He PAMELA data – the precision of which is ∼1% –, we recover similar values and CIs for the parameters (compare option 2 in Table 2 and the first three lines of Table 4). The second row of Table 4 is based on a subset of He data: high-energy data points are discarded because they show departure from a single power-law (Ahn et al. 2010; Adriani et al. 2011), whereas low-energy data points are discarded because of their sensitivity to solar modulation, which is presumably too crudely described by the Force-Field approximation used here. The χ²/d.o.f. value (first row) shows that the model has difficulties to perfectly match the high precision PAMELA He data over the whole energy range. The analysis of ³He/⁴He ratio using a prior on the source parameters (option 4 in

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Fig. 3. Fractional contributions to the propagated ²H fluxes (top panel), ³He fluxes (middle panel) and ⁴He fluxes (bottom panel) as a function of E_{kin} from A > 4 CR parents. For ⁴He, the primary contribution is also considered.

\[ f^{P\rightarrow S}_{A\rightarrow S} = \frac{\text{Rel}^{P\rightarrow S}_{A\rightarrow S}}{\sum_{P=4\text{He}} \text{Rel}^{P\rightarrow S}_{A\rightarrow S}} \]  

(9)

As seen from Table 3, the most important contributions from primary species heavier than He (Z > 2) are C and O, followed by Mg and Si and finally Fe. The total contribution of these species amounts to ∼35% for ²H and ∼11% for ³He, but mixed species (such as N) or less abundant species also contribute to ∼5%.

\[ \text{CR parents:} \quad \begin{aligned} ²\text{He} & \quad ³\text{He} & \quad ⁴\text{He} \\ \text{H, He, O, Si, Zn} & \quad \text{H, He, O, Si} & \quad \text{H} \\ \text{CR contributions:} \quad \begin{aligned} \text{Prim, Li, O, Si, Zn} & \quad \text{Prim, Li, O} & \quad \text{4He (primary)} \\ \end{aligned} \end{aligned} \]

\[ \sum \text{CR} + \text{H/He} \rightarrow {²H, ³He} \]

\[ \sum \text{CR} + \text{H/He} \rightarrow {³He} \]

\[ \sum \text{CR} + \text{H/He} \rightarrow {⁴He} \]
We partially repeated the analysis for $^2$H in the last three rows of Table 2 and third line of Table 4) gives higher CIs for the transport parameters. The result from the $^3$He analysis (third line) and that based on the partial He data (second line), and their discrepancy with the results of the analysis based on the full He data set (first line) confirms our suspicion that high-precision measurements for primary fluxes can bias the transport parameter determination.

### 4.2.2. Adding the secondary $^3$He flux in the analysis

Replacing He by $^3$He in the simultaneous analysis (4th and 5th line) affects the determination of the transport parameters even more. This is not surprising since $^3$He data are not all consistent with one another (see Fig. 4). The bias is stronger when taking into account the recently published AMS-01 data (Aguilar et al. 2011). If both $^3$He and He are taken into account, the much better accuracy of the He PAMELA data with respect to the $^3$He data amounts to a smaller weight of the latter in the analysis.

### 4.2.3. $^2$H/$^4$He vs. $^3$He/$^4$He

We partially repeated the analysis for $^2$H in the last three rows of Table 2. The data are so inconsistent with each other for $^2$H/$^4$He (see Fig. 4) that we were forced to use at least the $^2$H flux (whose data points are also markedly inconsistent with each other). Even so, the results are not reliable. PAMELA and AMS-02 have the capability to improve greatly the situation, but in the meantime, we are forced to include He as well in the analysis (next-to-last row in the table). The transport parameter values from the $^2$H/$^4$He analysis are mostly consistent with those from the $^3$He/$^4$He analysis, but are likely to suffer from similar biases (see the previous paragraph). Reducing the energy range of He data is not even possible for the $^2$H analysis (last line in the table), because the obtained results are not reliable.

### 4.3. MCMC analysis: “best” results

Taking into account the specifics of the actual data (previous section), our “best” analysis is based on the most relevant combinations of data for $^2$H and $^4$He:

- the $^3$He/$^4$He analysis (with a prior for the source parameters) gives robust and conservative results for the transport parameters. The result from the $^3$He/$^4$He analysis is more sensitive to biases, but using an energy sub-range for He data is expected to limit them;
- owing to the paucity of $^2$H data, the $^2$H/$^4$He+$^3$H+$^4$He (full energy-range for He) analysis is the only reliable option, although it probably suffers from biases.

The corresponding most probable values and CIs are gathered in Table 5, and the corresponding envelopes for $^3$He/$^4$He, $^2$H, $^3$He/$^4$He, and $^1$H/$^4$He are given in Fig. 4. We also re-analysed the $B/C$ ratio according to our “best-analysis” scheme (B/C alone with a prior for the source parameters or B/C + C). The results are reported in Table 5, where the results obtained in Putze et al. (2010) for fixed source parameters are also reproduced; we note that the new strategy yields results that agree better with those of the quartet analysis (e.g., the transport parameters $\delta$ and $\kappa_0$ are shifted by more than 30% for Model II), which again demonstrates its usefulness.

### 4.3.1. Universality of the transport parameters

If we focus on the transport parameters, we note that combinations involving the $^2$H/$^4$He, $^3$He/$^4$He, or $B/C$ ratio yield broadly consistent transport parameter values, be it for Model II or $^3$He/$^4$He.
Table 5. Most probable values and CIs for Models II and III (with $L = 4$ kpc) for our “best” analysis (see Sect. 4.3).

| Data | $K_0 \times 10^2$ (kpc$^2$ Myr$^{-1}$) | $\delta$ | $V_c$ (km s$^{-1}$) | $V_a$ (km s$^{-1}$) | $\alpha$ | $\eta_3$ | $\chi^2_{K_0 / \Delta K_0}$ d.o.f. |
|------|---------------------------------|-------|-----------------|-----------------|-------|-------|-----------------|
| $^3\text{He}^4\text{He}$ | 15.0$^{+0.5}_{-0.5}$ | 0.29$^{+0.02}_{-0.03}$ | 166$^{+11}_{-7}$ | [1.8, 2.5] | [−2, +2] | 3.3 |
| $^3\text{He}^4\text{He} + ^3\text{He} + ^4\text{He}$ | 7.0$^{+0.3}_{-0.2}$ | 0.31$^{+0.03}_{-0.02}$ | 74$^{+4}_{-3}$ | 2.36$^{+0.03}_{-0.05}$ | 0.43$^{+0.08}_{-0.09}$ | 4.6 |
| $^3\text{He}^4\text{He} + ^4\text{He} + ^4\text{He}$ | 14.8$^{+0.5}_{-0.5}$ | 0.06$^{+0.03}_{-0.03}$ | 44$^{+5}_{-8}$ | 2.66$^{+0.03}_{-0.05}$ | 0.70$^{+0.05}_{-0.05}$ | 5.1 |
| B/C | [Putze et al., 2010] | 9.7$^{+0.3}_{-0.2}$ | 0.23$^{+0.06}_{-0.05}$ | 73$^{+2}_{-1}$ | $\alpha + \delta = 2.65$ | −1 | 4.7 |
| B/C | [this paper] | 6.2$^{+0.4}_{-0.4}$ | 0.35$^{+0.01}_{-0.01}$ | 80$^{+2}_{-1}$ | [1.8, 2.5] | [−2, +2] | 1.5 |
| B/C + C (all) | [this paper] | 6.5$^{+0.1}_{-0.1}$ | 0.31$^{+0.06}_{-0.06}$ | 57$^{+2}_{-1}$ | 2.340$^{+0.005}_{-0.008}$ | 0.96$^{+0.04}_{-0.04}$ | 13.9 |
| B/C + C (HEAO) | [this paper] | 6.3$^{+0.1}_{-0.1}$ | 0.35$^{+0.04}_{-0.04}$ | 78$^{+2}_{-2}$ | 2.250$^{+0.015}_{-0.016}$ | 1.48$^{+0.08}_{-0.08}$ | 12.8 |

| Model III |
|------------|
| $^3\text{He}^4\text{He}$ | 0.5$^{+0.4}_{-0.3}$ | 0.67$^{+0.04}_{-0.10}$ | 27.4$^{+0.9}_{-1.3}$ | 41$^{+14}_{-18}$ | [1.8, 2.5] | [−2, +2] | 2.9 |
| $^3\text{He}^4\text{He} + ^3\text{He} + ^4\text{He}$ | 1.0$^{+0.3}_{-0.2}$ | 0.64$^{+0.03}_{-0.05}$ | 23.5$^{+0.5}_{-1.0}$ | 54$^{+5}_{-3}$ | 2.37$^{+0.03}_{-0.03}$ | 0.03$^{+0.04}_{-0.04}$ | 1.6 |
| $^3\text{He}^4\text{He} + ^4\text{He} + ^4\text{He}$ | 3.2$^{+0.8}_{-0.1}$ | 0.50$^{+0.08}_{-0.10}$ | 27.1$^{+2.5}_{-1.8}$ | 72$^{+9}_{-16}$ | 2.41$^{+0.08}_{-0.12}$ | 0.39$^{+0.07}_{-0.06}$ | 5.5 |
| B/C | [Putze et al., 2010] | 0.46$^{+0.06}_{-0.06}$ | 0.80$^{+0.04}_{-0.04}$ | 18.0$^{+0.4}_{-0.4}$ | 38$^{+2}_{-2}$ | $\alpha + \delta = 2.65$ | −1 | 1.5 |
| B/C | [this paper] | 0.46$^{+0.10}_{-0.08}$ | 0.82$^{+0.05}_{-0.05}$ | 18.3$^{+0.3}_{-0.3}$ | 40$^{+5}_{-3}$ | [1.8, 2.5] | [−2, +2] | 0.9 |
| B/C + C (all) | [this paper] | 0.57$^{+0.05}_{-0.05}$ | 0.80$^{+0.02}_{-0.01}$ | 17.4$^{+0.2}_{-0.2}$ | 36$^{+1}_{-1}$ | 2.260$^{+0.007}_{-0.009}$ | 0.24$^{+0.03}_{-0.03}$ | 5.2 |
| B/C + C (HEAO) | [this paper] | 0.33$^{+0.06}_{-0.06}$ | 0.93$^{+0.05}_{-0.05}$ | 18.2$^{+0.3}_{-0.2}$ | 35$^{+2}_{-2}$ | 2.31$^{+0.019}_{-0.026}$ | 1.9$^{+0.1}_{-0.2}$ | 2.0 |

Notes: $^{(1)}$ Excluding PAMELA He point below 5 GeV/n and above 183 GeV/n. An interval in square brackets corresponds to the prior used for the analysis (the posterior PDF obtained is close to the prior). The B/C results are based on IMP7-8 (Garcia-Munoz et al. 1987), Voyager 1&2 (Lukasiev et al. 1999), ACE-CRIS (George et al. 2009), HEAO-3 (Engelmann et al. 1990), Spacelab (Mueller et al. 1991), AMS-01 (Aguilar et al. 2011), and CREAM (Ahn et al. 2008), shown to be the most compatible data for a B/C analysis (Putze et al. 2009).
or Model III\(^{10}\). Regardless of the actual propagation model, we conclude that these results hint at the universality of CR transport for all species. Another important result is that the constraints set by the quartet data on the transport parameters can compete with those set by the B/C ratio, so that the quartet data should be a prime target for AMS-02.

4.3.2. Model II (\(\delta \sim 0.3\)) or Model III (\(\delta \sim 0.7\))?

According to Sect. 3.2, comparing the results of the secondary-to-primary ratio analysis with those of the combined analysis (ratio + primary flux) gives an indication of their robustness. Table 5 shows that the results for the diffusion slope \(\delta\) is very robust, regardless of the model considered. A more detailed comparison shows that for \(^3\)He-related constraints, the transport parameter values for Model II are inconsistent with each other at the 3\(\sigma\) level, whereas the 68% CIs overlap with one another (but for \(V_c\)) for Model III, hence slightly favouring the latter (\(\delta \sim 0.7\)). The comparison of the \(\chi^2_{\text{min}}/\text{d.o.f.}\) values also tends to favour model III. Hence, although the value \(\delta \sim 0.7\) seems favoured, we cannot exclude a pure reacceleration model (\(V_c = 0\)) with \(\delta \sim 0.3\) yet. Moreover, as shown in Maurin et al. (2010), many ingredients of the propagation models can lead to a systematic scatter of the transport parameters larger than the width of their CIs. Data at higher energy for any secondary-to-primary ratio are mandatory to conclude on this question.

4.3.3. Source spectrum

The present analysis is more general than that used in Putze et al. (2011), where the transport parameters were fixed. Although it is not the main focus of this paper, we remark that the values of the source slope \(\alpha\) from the B/C + C analysis are consistent with those found in Putze et al. (2011), strengthening the case of a universal source slope \(\alpha\) at the \(-5\%\) level. For the quartet values, \(\alpha_{4\text{He}}\) is broadly consistent with Putze et al.’s analysis (based on AMS-01, BESS98 and BESS-TeV data for He). However, the results for the source parameters depend on the choice of data sets and energy range considered. This indicates that for \(\lesssim 1\%\) accuracy data, either the model for the source is inappropriate, or the solar modulation model is faulty, or some systematics exist in the measurements. The AMS-02 data will help to clarify this question.

5. Conclusion

We have revisited the constraints set on the transport (and also the source) parameters by the quartet data, i.e. \(^1\)H, \(^2\)H, \(^3\)He, and \(^4\)He fluxes, but also the secondary-to-primary ratios \(^2\)H/\(^4\)He and \(^3\)He/\(^4\)He. This extends and complements a series of studies (Putze et al. 2009, 2010, 2011) carried out with the USINE propagation code and an MCMC algorithm. The three main ingredients on which the analysis rests are

- A minute compilation of the existing quartet data and survey of the literature, showing that the most recent/precise data (AMS-01, BESS93–98, CAPRICE98, IMAX92, and SMILI-II) have not been considered before this analysis.

- We performed a systematic survey of the literature for the cross-sections involved in the production/survival of the quartet nuclei. This has lead us to propose new empirical production cross-sections of \(^2\)H, \(^3\)H, and \(^3\)He, valid above \(\sim 30\) MeV/n for any projectile on p and He (we also updated inelastic cross-sections).

- We extensively used artificial data sets to assess the reliability of the derived CIs of the GCR transport and source parameters for various combinations of data/parameter analyses.

In broad agreement with previous studies, (e.g. Ramaty & Lingenfelter 1969; Beatty 1986), we find that the fragmentation of CNO contributes significantly to the \(^2\)H flux (\(\sim 30\%\)) above a few GeV/n energies (\(^3\)He fragmentation is the dominant channel for the \(^2\)H and \(^3\)He fluxes). Nevertheless, we provided a much more detailed picture, showing in particular that heavy nuclei (\(8 < Z \leq 30\)) contribute up to 10% for \(^3\)He (20% for \(^2\)H) at high energy. We also provided an estimate of the secondary fraction to the \(^4\)He flux. By definition, the secondary contribution has a steeper spectrum than the primary one and therefore quickly becomes negligible at high energy. This secondary contribution peaks at a few GeV/n, and amounts to \(-10\%\) of the total flux (\(-7\%\) up to O fragmentation, \(\sim 2\%\) from elements heavier than O), which is already a sizeable amount given the \(-1\%\) precision reached by the PAMELA data (Adriani et al. 2011). For \(^1\)H, the knowledge of the multiplicity of neutron and proton produced by the interaction of all elements on the ISM is required to calculate its secondary content precisely.

Simulated data have allowed us to check several critical behaviours. Firstly, the He flux is obviously useful (and required) to constrain the source parameters, but it has also been found to bring significant information on the transport parameters: fitting a secondary-to-primary ratio plus a primary flux brings more constraints than just fitting the ratio (even when source parameters are fixed). Secondly, we have checked that a model with more free parameters (than those used to simulate the data) is able to recover the correct values. However, our analysis has also strongly hinted that adding the primary flux He biases the determination of the transport parameters if systematics (which are usually more important in primary fluxes than in ratios) are present, and/or if the wrong model is used. For this reason, when dealing with measurements, we recommend to always compare the result from the secondary-to-primary ratio + primary flux analysis to that of the secondary-to-primary ratio using a loose but physically motivated prior on the source parameters.

The analysis of real data has shown that quartet data slightly favour a model with large \(\delta \sim 0.7\) (with \(V_c \sim 20\) km s\(^{-1}\) and \(V_a \sim 40\) km s\(^{-1}\)), but that a model with small \(\delta \sim 0.2\) (with \(V_c \sim 0\) and \(V_a \sim 80\) km s\(^{-1}\)) cannot be completely ruled out. Better quality data, and especially data at higher energy are required to proceed. The conclusions are similar and the range of transport parameters found are consistent with those obtained from the B/C analysis (Jones et al. 2001; Putze et al. 2010; Maurin et al. 2010)\(^{11}\). This strongly hints at the the universality of the GCR transport for all nuclei. Furthermore, we have shown that the analysis of the light isotopes (and the already very good precision on He) is as constraining as the B/C analysis (similar range of CIs).

The several difficulties that we pointed out in this analysis could be solved by using better data. However, it is more likely

\(^{10}\) The most significant difference is for the \(^4\)He/\(^4\)He+\(^2\)H+He analysis, which is inconsistent in both models and clearly unreliable for Model II (\(\delta \sim 0\)). For model III, B/C and \(^4\)He/\(^4\)He-related constraints are roughly in the same region but are located at several \(\sigma\) from each other (they are consistent with each other for Model II).

\(^{11}\) Note that we did not attempt to combine the results of different secondary-to-primary ratios (\(^2\)H/\(^4\)He, \(^3\)He/\(^4\)He, B/C, sub-Fe/Fc, \(\beta/\rho\)). This is left for a future study, for which a Bayesian evidence could be used to better address (in a Bayesian framework) the crucial question of model selection.
that the interpretation of future high-precision data will require
the development of refined models for the source spectra and/or
transport and/or solar modulation. For instance, the force-field
approximation for solar modulation is already too crude to pre-
cisely match the PAMELA He data. The forthcoming AMS-02
data at an even better accuracy will definitively pose interesting
new challenges.

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Appendix A: Cosmic-ray data

Deuterons and $^3$He fluxes are very sensitive to the modulation
level, whereas ratios are less affected. The exact value for the
solar modulation level $\phi$ is uncertain. For instance, the values
given in the seminal papers can differ greatly from those esti-
ated by Casadei & Bindi (2004) (in order to match the electron
and positron fluxes of various experiments, see their Table 1), or
from those reconstructed from the neutron monitors (Usoskin
et al. 2002). This difference may arise because the latter analy-
sis correctly solves the Fokker-Plank equation of GCR transport
in the heliosphere, whereas most papers rely on the widely used
force-field approximation, which is known to fail for a strong
modulation level $\phi \geq 1000$ MV (e.g., Usoskin et al. 2002). In this
analysis, we did not attempt to go beyond this force-field approx-
imation, because speed is of essence for our MCMC analysis.
We relied mostly on the force-field effective modulation param-
eter $\phi$ necessary to reproduce the data (as quoted in the seminal
paper), but these values are slightly adjusted to give overlapping
fluxes when all data are demodulated and plotted together. Given
the uncertainty on the data, the large uncertainty on $\phi$, and be-
cause the most probable region of parameter space is constrained
by the $^2$H/$^4$He and $^1$He/$^4$He ratio (rather than the best fit to the
$^2$H and $^4$He fluxes), we feel that it is a safe procedure until high-
precision data from PAMELA of AMS-02 are available.

The demodulated interstellar (IS) fluxes for $^2$H and $^3$He are
shown in the left panels of Fig. 4, whereas the here quoted top-of-atmosphere (TOA) ratios for $^2$H/$^4$He and $^3$He/$^4$He are
shown in its right panels. The references for the data are given in
Table A.1.

Appendix B: Cross-sections

This appendix summarises the production and destruction cross-
sections employed for the quartet nuclei in this paper.

B.1. Elastic and inelastic cross-sections

All reaction cross-sections were taken from the parametrisations
of Tripathi et al. (1999), except for the $p$H reaction cross-section.
The latter is evaluated from $\sigma_{pp}^{inel} = \sigma_{tot}^{el} - \sigma_{pp}^{el}$, where the total
and elastic cross-sections are fitted to the data compiled in the
PDG12. Note also that for $^4$He + $^4$He, we had to renormalise
Tripathi et al. (1999) formulae by a factor 0.9 to match the low-
energy data.

Our parametrisations (lines) and the data (symbols) are shown in Fig. B.1 for reaction on H and He. Note that we re-
lied on Tripathi et al. (1997) for any other inelastic reaction.

B.2. Light nuclei production: Nuc + $p$ 

The light nuclei $^3$He and $^4$H are spallative products of cos-
mic rays interacting with the ISM. The total secondary flux
is obtained from the combination of production cross-sections
and measured primary fluxes. In principle, all nuclei must be
considered, but the ISM and GCRs are mostly composed of
$^1$H and $^4$He, making the reactions involving these species
dominant. The decreasing number of heavier species is bal-
danced by their higher cross-section. In several studies (e.g.
Ramaty & Lingenfelter 1969; Jung et al. 1973b; Beatty 1986),
it was found that the CNO+$^1$H$_{ISM}$ reactions contribute up to
$\approx 30\%$ of the $^4$He flux above GeV/n energies. The reverse re-
action $^4$He+$^1$CNO$_{ISM}$ mostly produces fragments at lower en-
ergies, making them irrelevant for CR studies in the regime
$\gtrsim 100$ MeV/n. Note that $^3$H is also produced in these reactions,
but it decays in $^3$He with a life time (12.2 years) that is short
with respect to the propagation time. All tritium production is
thus assimilated to $^3$He production, but the cross-sections for this
fragment are also provided below.

The energy of the fragments roughly follows a Gaussian dis-
bution (e.g. Cucinotta et al. 1993). Its impact on the secondary
flux was inspected for the B/C analysis by Tsao et al. (1995),
where an effect $\leq 10\%$ was found, compared to the straight-ahead
approximation, in which the kinetic energy per nucleon of the
fragment equals that of the projectile. The precision sought for
the cross-sections is driven by the level of precision attained by the
CR data to analyse. Given the large errors on the existing
data, the straight-ahead approximation is sufficient for this anal-
ysis. However, future high-precision data (e.g. from the AMS-02
experiment) will probably require a refined description.

B.2.1. $^4$He + $p$ → $^2$H, $^3$H, and $^3$He

Recent and illustrative reviews on $^4$He+$^1$H reaction and the pro-
duction of light fragments are given by Bildsten et al. (1990),
Cucinotta et al. (1993), and Blinov & Chadeyeva (2008). As
said earlier, we are only interested in the total inclusive produc-
tion cross-section, not in all the possible numerous final states
(see, e.g., Table 3 of Blinov & Chadeyeva 2008). We adapted
the parametrisation of Cucinotta et al. (1993), which separately
considers the break-up and stripping (for $^4$He and $^2$H) cross-
sections. The former reaction corresponds to the case where the
helium nucleus breaks up, leading to coalescence of free nucleons
into a new nucleus. The latter happens via the pickup re-
anction where the incident proton tears a neutron or a proton off
the helium nucleus. The two reactions and their sum are shown
along with the experimental data in Fig. B.2.

The most accurate set of data (upward blue empty trian-
gles) are from the experiments set up in ITEP and LHE JINR
(Aladashvill et al. 1981; Glagolev et al. 1993; Abdullin et al.
1994; summarised in Blinov & Chadeyeva 2008). Their higher
energy data point (Glagolev et al. 1993) is a conservative es-
timate because the more than, or equal to, six-prong reactions
are not detailed (see Table 3 of Blinov & Chadeyeva 2008; and
Table 4 of Glagolev et al. 1993). To take into account that possi-
bility, we considered an error of a few mb in the plots of Fig. B.2.
Let us consider each product of interest in turn.

$^3$He production. The stripping cross-section data (d and $^3$He in
the final state) are well-fitted by Eq. (130) of Cucinotta et al.
(1993). However, the Griffiths & Harbison (1969) and Jung et al.
(1973a) values are $\approx 30\%$ below the other data. Indeed, for the
latter (filled stars) the break-up cross-section is higher than other

12 http://pdg.lbl.gov/

13 Solar modulation also ensures that only species created at energies
$\gtrsim$ GeV/n in matter.
data, it may be that the end products are misreconstructed (in this or the other experiments). Nevertheless, the sum of the two – which is the one that matters – is consistent in all data. Note that we slightly modified the break-up cross-section provided by Cucinotta et al. (1993) to better fit the high-energy data points. For the latter, all data are consistent with each other, except for the high-precision ITEP data at 200 MeV/n.

$^2$H production. The stripping cross-section is the same as for $^3$He (d and $^3$He in the final state). The high-energy break-up cross-section data (LHE JINR and Webber 1990b) are inconsistent. We decided to rescale the Webber data to take into account the fact that in his preliminary account of the results (Webber 1990b), the total inelastic cross-section is smaller than that given in a later and updated study (Webber 1997). Still, the agreement...
between the two sets is not satisfactory. The other high-energy data point is the Innes (1957) experiment, and it suffers large uncertainties and maybe systematics (it is for n + 4He reaction, and the data point is provided by Meyer (1972) who relied on several assumptions to derive it). The ITEP/LHE JINR data being the best available, we replaced the formula for the 2H breakup of Cucinotta et al. (1993) by a form similar to that given for 3He, but we changed the parameters to fit the high-energy points.

3H production. There is only break-up for the Cucinotta et al. (1993) 3H production. The data broadly agree with each other, except for the Nicholls et al. (1972) point (open plus). Again, we adapted the Cucinotta et al. (1993) parametrisation to better fit the ITEP/LHE JINR data.

B.2.2. 3He + p → 2H (breakup) and p + p → 2H (fusion)

There are two other channels for producing 2H from light nucleus reactions, and they are shown in Fig. B.3 along with the data. The first one is from 3He (break-up and stripping). The CR flux of the latter is less abundant than the 4He flux. With a ratio of ~20% at 1 GeV/n (decreasing at higher energy) and similar production cross-sections (~30–40 mb), this is expected to contribute the same fraction at GeV/n energies, and then to become negligible > 10 GeV/n. The second channel is the 2H coalescence from two protons. The cross-section is non-vanishing only for a very narrow energy range. Even if the cross-section is 10 times smaller than for the other channels, the fact that CR protons are
Table B.1. References for the light nuclei production cross-sections (references for 4He projectiles are given in Fig. B.2).

| Proj. | Frag. | #Data | \( E_{\text{kin}} \) | Ref. |
|-------|-------|-------|----------------|------|
| N, O, Fe | \(^3\)H | 3 | 2.2 | Fireman (1955) |
| C, N, O, Mg, Al, Fe, Ni, Ag, Sn, Pb | \(^3\)H | 26 | 0.45–6.2 | Currie (1959); Currie et al. (1956) |
| C | \(^4\)H | 4 | 0.225–0.73 | Honda & Lal (1960) |
| Mg, Al | \(^3\)He | 2 | 0.54 | Bieri & Rutsch (1962) |
| C, O, Mg, Al, Si, V, Cr, Mn, Fe, Ni, Cu, Ag, Pb, Bi | \(^4\)He, \(^3\)He | 38 | 0.225–5.7 | Goebel et al. (1964) |
| CNO | \(^4\)H, \(^3\)H, \(^3\)He | 14 | 0.02–7.5 | Ramaty & Lingenfelter (1969) |
| C, O, Si | \(^4\)H, \(^3\)He | 12 | 0.6–3.0 | Kruger & Heymann (1973) |
| Si, Mg | \(^3\)He | 33 | 0.02–3.06 | Walton et al. (1976) |
| Mg | \(^3\)He | 6 | 0.015–0.07 | Puffer (1979) |
| C, O | \(^4\)H, \(^3\)H, \(^3\)He | 10 | 1.05–2.1 | Olson et al. (1983) |
| Ag | \(^3\)He | 1 | 0.48 | Green et al. (1984) |
| Mg, Al, Si | \(^3\)He | 3 | 0.6 | Michel et al. (1989) |
| Mg, Al, Si, Fe, Ni | \(^3\)He | 21 | 0.8–2.6 | Michel et al. (1995) |
| Mg, Al, Si | \(^3\)He | 33 | 0.015–1.6 | Leya et al. (1998) |
| C | \(^3\)He | 3 | 1.87–3.66 | Korejwo et al. (2000, 2002) |
| Pb | \(^3\)He | 22 | 0.04–2.6 | Leya et al. (2008) |
| Fe, Ni | \(^3\)He | 55 | 0.022–1.6 | Ammon et al. (2008) |

Several data exist for the production of light isotopes from nuclei \( A \geq 12 \) on H (see Table B.1). The most complete sets of data in terms of energy coverage are for the projectiles C, N, and O (\( \langle A \rangle = 14 \)), the group Mg, Al, and Si (\( \langle A \rangle = 26 \)), and the group Fe and Ni (\( \langle A \rangle = 57 \)). They are plotted in Fig. B.4 (top panels and bottom left panel). The solid lines correspond to an adjustment (by eye), rescaled from the \( \sigma_{\text{breakup}}^4\text{He} \rightarrow f \) cross-section (because heavy projectiles do not give \( A = 3 \) fragments in the stripping process). Because \( \sigma_{\text{prod}}^3\text{He} \approx \sigma_{\text{prod}}^4\text{He} \), no distinction was made for the fit (\(^3\)He data are scarce and do not influence the conclusions drawn from these three groups of nuclei). The following parametrisation

\[
\sigma_{\text{prod}}^4\text{He}(E_{\text{kin}}, A_P) = \gamma_P^F \cdot f(E_{\text{kin}}, A_P) \cdot \sigma_{\text{breakup}}^4\text{He}(E_{\text{kin}}),
\]

with

\[
f(E_{\text{kin}}, A_P) = \begin{cases} 
\frac{E_{\text{kin}}}{1.5 \text{ GeV/n}} \cdot \frac{0.8}{\sqrt{E_{\text{kin}}}} & \text{if } E_{\text{kin}} < 1.5 \text{ GeV/n}, \\
1 & \text{otherwise};
\end{cases}
\]

proves to fit the three groups of data well for energies greater than \( \sim 30 \text{ MeV/n} \). Thanks to the \( f(E_{\text{kin}}, A_{\text{proj}}) \) factor, there is no other energy dependence in the \( \gamma_P^F \) factor, so that the latter can be determined from the data points at any energy. The bottom right panel of Fig. B.4 shows the measured mean value and dispersion\(^\text{14}\) as a function of \( A \), from which we obtained:

\[
\gamma_P^{3\text{He}} = \gamma_P^{3\text{H}} = 1.3 \left[ 1 + \left( \frac{A_P}{25} \right)^{1.5} \right],
\]

\[
\gamma_P^{4\text{He}} = 0.28 A_P^{1.2}.
\]

The set of formulae (B.1), (B.2), and (B.3) completely define the Proj+p production cross-sections for the light fragments.

B.3. Proj\(A_{\geq 4} + 4\text{He} \rightarrow \) \(^2\)H, \(^3\)H, and \(^3\)He

Data for \( T + 4\text{He} \) where the target \( T \) is heavier than \( p \) are scarce. In a compilation of Davis et al. (1995), the authors find that the

\^\text{14}\) In practice, for a given \( A \), the mean and dispersion are calculated from all existing points above a projectile energy 150 MeV/n.
3He production scales as $A_{T}^{1.31}$ (based on four data points with $A_T \geq 7$). This is the scaling we employed for the 3H and 2H production as well.

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Fig. B.4. $\text{Proj}(A_{\text{proj}}) + p \rightarrow \text{^3He}$, $\text{^4He}$, and $\text{^3He}$ cross-sections for $\text{Proj} = \text{C, N, O (top left), Proj} = \text{Mg, Al, Si (top right)}$, and $\text{Proj} = \text{Fe, Ni (bottom left)}$. The bottom right correspond to the $\gamma_p$ factor (see Eq. (B.1) and text for explanations). The references for the data are gathered in Table B.1.
