Static Output Feedback Predictive Control for Cyber-Physical System under Denial of Service Attacks

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This paper addresses the static output feedback predictive (SOFP) control problem with cyber-physical system (CPS) subject to Denial-of-Service (DoS) attacks. The effects of DoS attacks are reasonably assumed to the bounded consecutive packet dropouts by considering the energy constraints of an attacker. Then, a novel predictive control sequence, in which only the latest successfully received output is employed, is designed to compensate such packet dropouts caused by DoS attacks. Furthermore, the stability criterion and predictive control design are carefully derived by using the switching Lyapunov functional approach and linear matrix inequality. Compared with the previous works, the proposed predictive control strategy can compensate arbitrary packet dropouts under DoS attacks while only the latest successfully received output is available. At last, a simulation example illustrates the effectiveness of the SOFP control strategy.

1. Introduction

The rapid development of CPS is attributed to the strong integration among computation, communication, and control technology, in which CPS has received considerable attention in the past decades. Taking advantage of low cost and flexible network architecture, it has been widely applied in some engineering fields such as smart grid, healthcare, and water/gas distribution and industrial process control [1–3].

Due to the capacity of connecting deeply integration physical plants and cyber elements in an unprecedented way, CPS offers ample opportunities for malicious threats to launch attacks. The applications of next-generation information technologies such as big data, cloud computing, and Internet of Things greatly provide performance improvements for physical systems but at the same time introduce more risks which make physical isolation more difficult to implement. Therefore, how to ensure the safe operation and preserve the control performance under malicious attacks are the basic security issues in CPS. In fact, CPS has realized more complex and high-risk industrial process control through the transmission of information in the heterogeneous network [4]. However, the vulnerability of open communication networks, as the key components of society safety-critical infrastructures in CPS, increases the severity of such malicious cyber-attacks in [5], which can menace the control systems. There have been an increasing number of cyber-attacks on power grids reported worldwide. For instance, a devastating cyber-attack on the power station brought down the information flow from the physical process to the remote management system, which plunges 225,000 people into blackout in Ukraine [6]. Besides, the “Stuxnet”, an advanced computer worm virus, intruded the nuclear facility and caused severe damage in Iran [7]. These facts show the serious economic loss and severe social detriment attacked by malicious network in CPS, which has attracted extensive attention of many scholars [8–10].
Obtained at the controller, the SOFP control strategy is proposed to deal with the security control problem. In contrast with the existing results, the main contributions of this paper can be summarised as follows:

1. A novel switching system model is established to characterize the security properties of CPS under DoS attacks. Different from [26], only limited output measurements are used to design the security controller in this paper.

2. Only the latest received measurements are used to design the proposed predictive control gains. Compared with traditional model predictive control methods, the proposed security control strategy will predict their future control gains rather than state prediction.

The remainder of this paper is organized as follows. Section 2 gives the problem formulations with the proposed SOFP control strategy by considering the energy-limited DoS attacks. Section 3 is presented in the security analysis, which infers the stability criterion to guarantee the security performance. The SOFP controller is designed in Section 4, and a simulation example is shown in Section 5 to illustrate the feasibility of the desired controller. And finally, Section 6 concludes this paper.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^m$ denote the $n$-dimensional and $m$-dimensional Euclidean space, respectively. $\mathbb{R}^{n \times m}$ is the set of $n \times m$ real matrices. The superscript "$T$" stands for matrix transposition. The notation $X > 0$ means that the matrix $X$ is real symmetric positive definite.

2. Problem Formulations

2.1. System Framework

The structure of SOFP control considered in this paper is shown in Figure 1, where the studied CPS is composed by the sensor, controller, buffer, and actuator. The SOFP control strategy against attack-induced severe packet dropout is that the controller receives all the measurement outputs from sensor and calculates the sequence of control inputs, which transmits to the buffer simultaneously. Then, the actuator selects the corresponding control value from $[u(t_0)^T, u(t_0 + 1)^T, \ldots, u(t_0 + r)^T]^T$ and delivers appropriate control inputs to the plant, which can compensate the arbitrary packet dropouts caused by DoS attacks.

Consider a discrete-time linear system described by

$$\begin{align*}
\begin{cases}
x(t + 1) = Ax(t) + Bu(t), \\
y(t) = Cx(t),
\end{cases}
\end{align*}$$

where $t + 1 \triangleq (t + 1)T$, $T$ represents the sampling period, $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the system state and control input, respectively, and $A, B$ are real matrices of appropriate dimensions.

To make the proposed method more suitable for practical network attacks, the DoS attacks behaviours considered energy constraints are presented as

$$H_n = \{h_n\} \cup [h_n, h_n + \tau],$$
where \( \tau \) is the attack duration and \( h_n \) is the instant transition time of the attack state.

As shown in Figure 2, “↑” denotes that the DoS is converted to the attack state and “↓” indicates that the DoS is the end of the attack process. When \( \tau \in \mathbb{R}_{\geq 0} \), the DoS launches a limited attack with the duration of \( \tau \) (\( \tau = 0 \) is a pulse attack only).

To clearly describe the energy-limited characteristics of DoS attacks in this paper, the following assumption is given.

**Assumption 1.** The maximum packet dropouts of consecutive DoS attacks are bounded with \( N \).

**Remark 1.** In practice, the attackers gradually run out of energy because of an inherent characteristic of energy constraints [27]. Based on this reliable fact, it is reasonable to consider that the packet dropouts of consecutive DoS attacks are bounded. Furthermore, to achieve predictive compensation for packet dropout, a data buffer is involved at the controller side to record recently successfully transmitted data packets.

### 2.2. Switching Model under DoS Attacks

Suppose that the controller latest received the value of process output \( y(t_h) \) at time \( t_h \), then the predictive control based on output feedback control law is given by

\[
u(t_h + \tau) = G_r y(t_h),
\]

where \( t_h + \tau \in (t_h + \tau)T \), \( t_h \) stands for the switching instant time, and \( \tau = 0, 1, \cdots, \sigma(t_h) \) is a time-varying switching signal which takes the value in a finite set \( \tau \in \mathbb{Z} \subseteq \{0, 1, \cdots, N\} \).

The packet-based transmission mechanism in CPS determines that the corresponding control input at time \( t_h, t_h + 1, \cdots, t_h + \tau \) is \( u(t_h), u(t_h + 1), \cdots, u(t_h + \tau) \), respectively. It is known that the neighboring two switching points have the following relation:

\[
t_{h+1} = t_h + 1 + \sigma(t_h), \quad t_0 = 0.
\]

Therefore, the evolution law of dynamics can be described by the following \( N + 1 \) cases:

**Case 0.** DoS-free (\( \sigma(t_h) = 0 \)):

\[
x(t_h + 1) = (A + BG_0C)x(t_h),
\]

\[
t_{h+1} = t_h + 1,
\]

\[
x(t_{h+1}) = x(t_h + 1) = \Phi_0 x(t_h),
\]

\[
\Phi_0 \triangleq A + BG_0C.
\]

**Case 1.** One-step packet dropout (\( \sigma(t_h) = 1 \)):

\[
x(t_h + 1) = (A + BG_0C)x(t_h),
\]

\[
x(t_h + 2) = Ax(t_h + 1) + BG_1Cx(t_h),
\]

\[
t_{h+1} = t_h + 2,
\]

\[
x(t_h + 1) = x(t_h + 2) = \Phi_1 x(t_h),
\]

\[
\Phi_1 \triangleq A\Phi_0 + BG_1C,
\]

\[\vdots\]

**Case N.** \( N \)-steps packets dropouts (\( \sigma(t_h) = N \)):

\[
x(t_h + 1) = (A + BG_0C)x(t_h),
\]

\[
x(t_h + 2) = Ax(t_h + 1) + BG_1Cx(t_h),
\]

\[\vdots\]

\[
x(t_{h+1} + N) = x(t_h + N) + BG_NCx(t_h),
\]

\[
t_{h+1} = t_h + N,
\]

\[
x(t_h + 1) = x(t_{h+1} + N + 1) = \Phi_N x(t_h),
\]

\[
\Phi_N \triangleq A\Phi_{N-1} + BG_NC.
\]

According to (5)-(7), model (1) with SOFP control strategy can be transformed into the following closed-loop system included \( N \)-steps packet dropouts:

\[
x(t_{h+1}) = \Phi_N x(t_h),
\]

where \( \Phi_i = A^{i+1} + \sum_{j=0}^{i} A^{j}BG_{i-j}C \). It illustrates the essential characteristics of the proposed SOFP control strategy, that is, the state is unchanged and the controller gain is changed.

### 3. Stability Analysis

In this section, the security analysis based SOFP strategy is given with some mathematical derivations. The following necessary definition and lemmas are introduced.
Definition 1. If there are positive scalars $c$ and $\lambda < 1$ such that the following inequality,
\[ \|x(t)\| \leq c \lambda^t \|x(0)\|, \]  
holds, the CPS (1) is said to be exponentially stable, where $x(0) \in \mathbb{R}^n$ is an arbitrary initial value.

Lemma 1 (see [27]). For an arbitrary matrix $\Psi \in \mathbb{R}^{m \times n}$ and an arbitrary vector $x \in \mathbb{R}^n$, the following inequality,
\[ \lambda_{\min} \|x\| \leq \|\Psi x\| \leq \lambda_{\max} \|x\|, \]  
holds, where $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum singular value and the maximum singular value, respectively, of $\Psi$.

Based on the above, the following theorem gives criteria for system exponentially stable with arbitrary switching characteristics under DoS attacks.

Theorem 1. For some given scalars $0 < \lambda_1 < 1$, $\mu > 0$, if there exist matrices $P \succ 0$, such that the following inequalities,
\[ -\lambda_1 P_i + \Phi_i^T P_i \Phi_i < 0, \]  
\[ \mu \beta \sigma \in \{0, 1, \cdots, N\}, \]  
\[ \rho \equiv \max \{\lambda_1 \mu | i \in \mathbb{Z}\} < 1, \]  
hold, then the system (8) will be exponentially stable with the decay rate $\frac{2\mu}{1-\rho}$.

Proof. Choose the following Lyapunov function:
\[ V_{\sigma(t)}(t_h) = x^T(t_h)P_{\sigma(t)}x(t_h). \]  

Therefore, it is derived from (14) at time $t_{h+1}$ that
\[ V_{\sigma(t)}(t_{h+1}) = x^T(t_{h+1})P_{\sigma(t)}x(t_{h+1}). \]  

Then, pre- and post-multiplying inequality (11) by $x^T(t_h)$ and $x(t_h)$, one has
\[ \Phi_{\sigma(t)}x(t_h) \preceq P_{\sigma(t)}x(t_h) - \lambda_{\sigma(t)}x^T(t_h)P_{\sigma(t)}x(t_h) < 0. \]  

Substituting the function (14) and (15) into (16), we have,
\[ V_{\sigma(t)}(t_{h+1}) < \lambda_{\sigma(t)} V_{\sigma(t)}(t_h). \]  

Similarly,
\[ V_{\sigma(t_{h+1})}(t_{h+2}) < \lambda_{\sigma(t_{h+1})} V_{\sigma(t_{h+1})}(t_{h+1}). \]  

Suppose that one-step packet dropout at time $t_{h+1}$ and $t_{h+2}$ caused by DoS attacks, respectively. Thus, we obtain that
\[ V_{\sigma(t_{h+1})}(t_{h+1}) = x^T(t_{h+1})P_{\sigma(t_{h+1})}x(t_{h+1}), \]  
\[ V_{\sigma(t_{h+2})}(t_{h+2}) = x^T(t_{h+2})P_{\sigma(t_{h+2})}x(t_{h+2}). \]  

Utilizing (12) and (18) together leads to
\[ V_{\sigma(t_{h+1})}(t_{h+2}) < \mu V_{\sigma(t_{h+1})}(t_{h+1}) < \mu \lambda_{\sigma(t_{h+1})} V_{\sigma(t_{h+1})}(t_{h+1}) \]  
\[ < \mu \lambda_{\sigma(t_{h+1})} \mu V_{\sigma(t_{h+2})}(t_{h+2}) < \mu \lambda_{\sigma(t_{h+1})} \mu \lambda_{\sigma(t_{h+2})} V_{\sigma(t_{h+1})}(t_{h+1}). \]  

Define $\rho = \mu \lambda_{\sigma(t_{h+1})}$, then the above law (21) can be written as
\[ V_{\sigma(t_{h+1})}(t_{h+2}) < \rho V_{\sigma(t_{h+1})}(t_{h+1}) < \rho^2 V_{\sigma(t_{h+1})}(t_{h+1}) < \cdots \]  
\[ < \rho^{t+1} V_{\sigma(t_{h+1})}(t_h) < \rho^{t+2} V_{\sigma(t_{h+1})}(t_h). \]  

Then, it is concluded from (13) and (14) that
\[ \lim_{t \to \infty} x(t_h) = 0. \]  

which implies that the system will be stable in $\{x(t)\}$.

Notice that the system should not only be stable in the discrete regions $\{x(t)\}$ but also converge to the subset $\{x(t_{d,h})\}$ after $d$ sampling periods.

Therefore, by Lemma 1, we obtain by induction that the following inequality holds:
\[ \|x(t_{d,h})\| \leq \xi \|x(t_h)\|, \]  
where $\xi \equiv \max \{\lambda_1 \mu | i = 0, 1, \cdots, N-1\}$.

It means that the upper bounded value of $\|x(t_{d,h})\|$ is $\xi \|x(t_h)\|$. Thus,
\[ \lim_{t \to \infty} x(t_{d,h}) = 0. \]  

Based on the above analysis, the closed-loop switching system (8) tends to be exponentially stable and then the exponential decay rate is obtained.

It is deduced from (19) that
\[ V_{\sigma(t_{h+1})}(t_{h+2}) = x^T(t_{h+2})P_{\sigma(t_{h+2})}x(t_{h+2}). \]  

Further, it is easy to see that
\[ V_{\sigma(t_{h+2})}(t_{h+2}) = x^T(t_{h+2})P_{\sigma(t_{h+2})}x(t_{h+2}) \geq \eta_1 \|x(t_{h+2})\|^2. \]  

It is derived from (22) that
\[ \eta_1 \|x(t_{h+2})\|^2 \leq \rho V_{\sigma(t_{h+1})}(t_{h+1}) < \rho^2 V_{\sigma(t_{h+1})}(t_{h+1}) < \cdots \]  
\[ < \rho^{t+2} V_{\sigma(t_{h+1})}(t_h) \leq \rho^{t+2} \eta_2 \|x(0)\|^2, \]  
where $\eta_1$ and $\eta_2$ are the minimum singular value and the maximum singular value, respectively, of $P$.

Therefore, it is easy to know from (28) that
\[ \|x(t_{h+2})\| \leq \sqrt{\eta_2} \eta_1 \|x(0)\|^2. \]  

Substituting (24) into (29), it can be found that
\[ \|x(t_{d,h+2})\| \leq \xi \sqrt{\eta_2} \eta_1 \|x(0)\|^2. \]  

Meanwhile, $t_{d,h+2}$, $t_{h+2}$, and $t+2$ have the following relations:
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\[ t_{h+2} \leq (N + 1)(t + 2). \]  
\[ t_{d,h+2} \leq (N + 1)(t + 2) + N \leq (N + 2)(t + 2), \]  
where \( h + 2 \geq N. \)

It is clearly deduced from (31) and (32) that

\[ \frac{t_{h+2}}{(N + 1)} \leq t + 2. \]  

Because of \( \rho < 1, \) the following inequalities hold.

\[ \|x(t_{h+2})\| \leq (\frac{t_{h+2}}{(N + 1)} \eta_1 t_{h+2})^{1/2} \|x(0)\|^2, \]  
\[ \|x(t_{d,h+2})\| \leq \xi (\frac{t_{d,h+2}}{(N + 1)} \eta_1 t_{d,h+2})^{1/2} \|x(0)\|^2, \]  
where \( h + 2 \geq N. \)

Finally, we can further obtain from (34) and (35) that, for an arbitrary instant time \( t, \) the following inequality,

\[ \|x(t)\| \leq \xi (\frac{t}{(N + 1)} \eta_1 t)^{1/2} \|x(0)\|^2, \]  
holds. The proof is thus completed. \( \square \)

4. Control Design of SOFP

In this section, the SOFP control sequence based on Theorem 1 is derived below.

Theorem 2. For given scalars \( 0 < \lambda_1 < 1, \mu > 0, \) if there exist matrices \( X, \Omega_i, \) and \( P_i > 0, \) such that the following inequalities,

\[ \begin{bmatrix} -\lambda_i \Omega_i & * \\ \Xi & -X - X^T + \Omega_i \end{bmatrix} < 0, \]  
\[ \lambda_1 \Omega_i < \mu \rho, \]  
\[ \rho \leq \max [\lambda, \mu] \leq 1, \]  
where \( \Xi = (A^{i+1} + \sum_{i=0}^{1} A^i B G_{i-1} C) X, \) hold, then the system (8) will be exponentially stable with the decay rate \( 2(N+1)\sqrt{\rho}. \)

Proof. According to the stability condition of the discrete-time linear system, for matrices \( A, P > 0, \) the following inequality,

\[ A^T P A - P < 0, \]  
holds, if and only if there exists a matrix \( \Psi \) such that

\[ \begin{bmatrix} \Psi^T & \Psi - \Psi^T + P \end{bmatrix} < 0. \]  

Therefore, the inequality (11) will be held if there exists a matrix \( \Psi \) such that the following inequality,

\[ \begin{bmatrix} -\lambda_i P_i & * \\ \Psi \Phi_i & -\Psi - \Psi^T + P_i \end{bmatrix} < 0, \]  
holds. Based on this fact, the controller can be easily obtained below.

Define \( X = \Psi^{-1} \) and \( \Omega_i = X^T P_i X. \) Then pre- and post-multiplying inequality (42) by \( \text{diag}[\Psi^{-T}, \Psi^{-T}] \) and \( \Omega_i = X^T P_i X \) (notice from (41) that the matrix \( \Psi \) is invertible), one has

\[ \begin{bmatrix} -\lambda_i \Omega_i & * \\ \Phi_i X & -X - X^T + \Omega_i \end{bmatrix} < 0. \]  

Substituting (5)–(7) into (43),

\[ \begin{bmatrix} -\lambda_i \Omega_i & * \\ A^{i+1} + \sum_{i=0}^{1} A^i B G_{i-1} C & X & -X - X^T + \Omega_i \end{bmatrix} < 0. \]  

It can be found that the inequalities (44) and (37) in Theorem 2 are equivalent.

Meanwhile, the inequality (38) in Theorem 2 is derived by pre- and post-multiplying \( X^T, X \) for \( P_a < \mu \rho, \forall a, b \in Z. \) This completes the proof.

However, the above inequalities still cannot be solved due to the coupling nonlinear item \( ABGCX. \) In order to deal with such items, Theorem 3 transformed nonlinear item is presented.

Theorem 3. For given scalars \( 0 < \lambda_1 < 1, \epsilon > 0, \) if there exist matrices \( X, \Omega_i, \) and \( P_i > 0, \) full rank matrix \( M, \) and any matrix \( V \) of appropriate dimensions such that the following inequalities,

\[ \begin{bmatrix} -\lambda_i \Omega_i & * \\ \Xi & -X - X^T + \Omega_i \end{bmatrix} < 0, \]  
\[ \begin{bmatrix} \epsilon & * \\ \Omega & -M C - C X \end{bmatrix} < 0, \]  
where \( \Xi = A^{i+1} X + \sum_{i=0}^{1} A^i B V_{i-1} C, \tau = i - l, \) hold, then the system (8) secured by \( G_r = V_r M^{-1} \) will be exponentially stable with the decay rate \( 2(N+1)\sqrt{\rho}. \)

Proof. It is deduced from (44) that \( C = M^{-1} C X. \) By replacing \( G_r; C \) and \( V_r; C, \) we can easily obtain the above result with \( G_r = V_r M^{-1} \) which completes this proof. \( \square \)

5. Simulation Example

In this section, an inverted pendulum control system is presented to illustrate the proposed security method with the SOFP control strategy. The plant model is described as
\[ u = M \frac{d^2y}{dt^2} + m \frac{d^2}{dt^2} (y + l \sin \phi), \]
\[ mg l \sin \phi = m \frac{d^2}{dt^2} (y + l \sin \phi)l \cos \phi, \]

and the inverted pendulum system is shown in Figure 3.

Based on the above inequality, the initial state variables of the system can be defined as

\[ x_1 = y, \quad x_2 = \phi, \quad x_3 = \dot{y}, \quad x_4 = \dot{\phi}. \]

To make the description simpler, the inverted pendulum control system takes the following parameters, which are given in Table 1.

### Table 1: Parameters of the inverted pendulum system.

| Symbol | Meaning                        | Value  |
|--------|--------------------------------|--------|
| \( M \) | Mass of the cart              | 1.378 kg |
| \( m \) | Mass of the pendulum          | 0.051 kg |
| \( l \) | Length of the pendulum        | 0.25 m  |
| \( g \) | Acceleration of gravity       | 9.8 m/s² |
| \( \phi \) | Angle from the upright position | —     |

Figure 3: Single inverted pendulum system.

Figure 4: Distribution of DoS attacks.

1: DoS attacks exist; 0: No DoS attacks

| Jammed time | Time (s) |
|-------------|----------|
| 0           | 0        |
| 0.5         | 2        |
| 1           | 4        |
| 2           | 6        |
| 3           | 8        |

Table 1: Parameters of the inverted pendulum system.
Let the sampling period \( T = 0.01 \text{s} \), then the discrete-time model of the inverted pendulum is given as

\[
\begin{aligned}
    x(k + 1) &= Ax(k) + Bu(k), \\
    y(k) &= Cx(k),
\end{aligned}
\]  

(49)

where

\[
\begin{bmatrix}
    A \\
    B \\
    C
\end{bmatrix} =
\begin{bmatrix}
    0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 \\
    0 & 0 & 29.43 & 0
\end{bmatrix},
\]

(50)

\[
\begin{bmatrix}
    0 & 1 & 0 & 3
\end{bmatrix}^T,
\]

\[
\begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0
\end{bmatrix}.
\]
In the simulation settings, by selecting $\delta = 0.85$, $\mu = 1$, $\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = 0.9$. The initial condition is set to be $x_0 = \begin{bmatrix} 10 & 0 & 0 & 0 \end{bmatrix}^T$ and the simulation time is chosen as $t \in [0 \ 10]$. Then, the stabilization control law in Theorem 3 can be resorting to LMI toolbox in MATLAB. Suppose that the maximum number of packet dropouts is $N = 3$ under the worst attacks in this simulation example. Therefore, the corresponding gain $G = \begin{bmatrix} -3.7808 & 1.9434 \end{bmatrix}$ is obtained. Meanwhile, the distribution of DoS attacks is shown in Figure 4.

**Case I. DoS-free case:**

The state responses of the system (49) with the designed controller under DoS-free case are shown in Figure 5, in which the stability of the studied is verified. It is worth noting that the angle value in Figures 5–7 has been reduced by one tenth in order to make a more intuitive comparison between the angle and position curves.

**Case II. DoS attacks case:**

In the second scenario, the designed controller in Case I is still used. Under the DoS attacks in Figure 4, the state responses of the system (49) are depicted in Figure 6. It is evident that the angle and position states of the inverted pendulum system are not convergent, in which state responses are presented in a worse performance. Thus, one can see that the switching system is unstable when there are no SOFP control inputs to confront uncertain packet dropouts caused by DoS attacks.

**Case III: DoS attacks migration with SOFP**

The third scenario considered the SOFP control strategy. In such case, the corresponding output feedback gain $G$ against the worst attacks is employed to ensure system stability and maintain the desired control performance. Similarly, we can obtain the following in Figure 7.

Based on the angle and position curves, the proposed method is effective, as the control strategy demonstrates that the closed-loop system is stable with bounded packet dropouts. One can see that the system performance is better than the one without predictive control. As a result, the proposed packet-based compensation control method has certain robustness and security.

According to the simulation examples shown above, it can be summarised that the designed controller is stable against DoS attacks and the feasibility of the proposed designing method is verified.

6. Conclusion

In this paper, a novel predictive control strategy is proposed to cope with packet dropouts caused by DoS jamming attacks. Firstly, the discrete-time switched linear control system is formulated to characterize the properties of CPS under DoS attacks. Then, the stability criterion is derived, and the predictive control sequences have been given by LMIs. Finally, the corresponding simulation example results have shown the validity of the SOFP control method.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this article.
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