Tunable directional scattering is of paramount importance for operation of antennas, routing of light, and design of topologically protected optical states. For visible light scattered on a nanoparticle the directionality could be provided by the Kerker effect, exploiting the interference of electric and magnetic dipole emission patterns. However, magnetic optical resonances in small sub-100-nm particles are relativistically weak. Here, we predict inelastic scattering with the unexpectedly strong tunable directivity up to 5.25 driven by a trembling of small particle without any magnetic resonance. The proposed optomechanical Kerker effect originates from the vibration-induced multipole conversion. We also put forward an optomechanical spin Hall effect, the inelastic polarization-dependent directional scattering. Our results uncover an intrinsically multipolar nature of the interaction between light and mechanical motion. They apply to a variety of systems from cold atoms to two-dimensional materials to superconducting qubits and can be instructive to engineer chiral optomechanical coupling.

Scattering of light manifests itself in everyday life, fundamental science and device applications [1]. Elastic Rayleigh scattering governs the blue color of sky and sea. Inelastic Raman scattering is a workhorse of sensors. The ability to control the direction, frequency, and polarization of the scattered light is essential for optical devices. However, both the Rayleigh and Raman scattering usually have a symmetric emission pattern: the waves are symmetrically scattered in the two opposite directions, in particular, forward and backward [2, 3]. The asymmetry can be induced if the particle that scatters light moves. Then, the Doppler effect leads to a difference between the incident and scattered light frequencies [4], which depends on the scattering angle in a highly asymmetric way. As first noted by C.V. Raman himself [5], it vanishes for the forward-scattered wave and reaches maximum for the back-scattered one. The scattering cross-section depends on the angle between the incident light propagation direction and particle velocity, enabling cooling of atomic gases in optical molasses [6]. Still, the asymmetry of the emission intensity pattern remains small unless the particle velocity becomes comparable to that of light, which is realized, e.g., for Compton scattering of X-rays [7].

A simpler mechanism to achieve strong scattering directionality was proposed by M. Kerker [8]. Rather than using mechanical motion, it requires a scatterer to possess both electric (ED) and magnetic dipole (MD) susceptibilities. Since the electric field of these two modes is of the opposite spatial parity, their interference enables directional forward or backward scattering depending on the relative phase [9–11]. Thus, implementation of the Kerker effect requires magnetic response of the same strength as the electric one. At optical frequencies this becomes challenging since magnetic dipole transitions are intrinsically relativistically weak [12]. A promising recently emerged workaround is provided by submicron-size high-refractive-index dielectric nanoparticles [13–15] hosting both magnetic and electric Mie resonances. For instance, Huygens metasurfaces of Si nanodisks that transmit light forward changing its phase without reflection open new avenues for wavefront control at the nanoscale [16, 17]. However, optical Kerker effect for the particles smaller than the wavelength in the medium ~100 nm is still unfeasible.

Here, we uncover a deep nexus of the motion-induced scattering directionality and the Kerker effect. We put forward an optomechanical Kerker effect, where strong tunable directionality is achieved for light scattered by a small particle without any magnetic response that trembles in space. Our main concept is sketched in Fig. 1. The incident wave excites electric dipole polarization, that oscillates in time. Trembling of the electric dipole in the direction transverse to its polarization induces the loop electric current \( j \) with non-zero magnetic momentum \( m \) as well as the electric quadrupole (EQ) momentum. Interference of ED and MD+EQ contributions results in unidirectional scattering as shown in Fig. 1(b,c). While the idea to use motion-induced conversion of electric dipole to magnetic dipole seems straightforward, a naive expectation would be that the magnetic dipole is relativistically weaker than the electric one and their interaction between light and mechanical motion. They apply to a variety of systems from cold atoms to two-dimensional materials to superconducting qubits and can be instructive to engineer chiral optomechanical coupling.

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terference cannot result in any significant directionalitv. We found that magnetic and electric dipole components counterintuitively are of the same order when inelastic light scattering is considered. To demonstrate this, we have developed a novel theoretical framework of multipolar resonant optomechanics. It incorporates the effect of the resonant dispersion of the moving medium on the multipolar emission in a rigorous nonperturbative fashion and goes beyond previous approaches [18–20] restricted to non-resonant scatterers. Our predictions are quite general and apply both for particles and for thin layers, as shown in Figs. 1(b) and 1(c). We also put forward an optomechanical spin Hall effect, i.e. directional inelastic scattering of light depending on its circular polarization.

RESULTS

Directional inelastic backscattering

We start with the qualitative geometrical consideration to reveal a drastic difference in the angular patterns of elastic and inelastic scattering. Figure 2(a) sketches the plane wave with the frequency $\omega$ that is scattered on a small particle trembling at the frequency $\Omega$ along the incident light propagation direction. The incident light induces dipole polarization of the particle that then emits light in a different direction. The shift of the particle in the real space $u_z(t)$ gives rise to an additional time-dependent phase of the scattered light $\phi(\theta, t) = (1 - \cos \theta)(\omega/c)u_z(t)$, where the two terms are illustrated by cyan and magenta arrows in Fig. 2(a) and $\theta$ is the scattering angle. The electric field of the scattered wave reads $E'(\theta) = E_d(\theta) e^{i\phi(\theta, t)}$ with $E_d(\theta)$ being the electric field of the light scattered by the particle at rest. Taking the particle displacement in the form $u_z(t) = u_z e^{-i\Omega t} + c.c.$ and expanding the scattered field into series over $u_z$, one obtains harmonics at the frequencies $\omega + p\Omega$ with integer $p$. We suppose that the vibration amplitude is small. Then, the electric field of the harmonic at the initial light frequency $\omega$, that describes the elastic light scattering, coincides with $E_d(\theta)$. Its angular dependence is governed by the well known electric dipole radiation pattern that yields equal amplitudes of forward and backward scattering [21]. The linear-in-$u_z$ terms yield the harmonics at anti-Stokes- and Stokes-shifted frequencies $\omega \pm \Omega$ that describe inelastic scattering. Their intensities read

$$I'(\theta) \approx (\omega/c)^2 |u_z|^2 (1 - \cos \theta)^2 I_d(\theta), \quad (1)$$

where $I_d(\theta) \propto |E_d(\theta)|^2$ is the intensity of elastic dipole scattering. In stark contrast to elastic scattering, the inelastic scattering is strongly anisotropic. In forward-scattering geometry, the particle shift does not change the optical path. Thus, Stokes and anti-Stokes light intensities vanish for $\theta = 0$. The inelastic scattering is the most intensive in the backscattering geometry, $\theta = \pi$, when the optical path change is maximal.

Multipolar resonant inelastic scattering

The above geometric consideration predicting the inelastic scattering asymmetry has a crucial limitation: It is applicable only to the particle with the frequency-independent polarizability. Indeed, in the case of resonant optical response, the elastic scattering intensity $I_d$ strongly depends on the light frequency. Yet, it is completely unclear which frequency to choose in Eq. (1): either that of the incident light $\omega$ or that of the scattered light $\omega' = \omega \pm \Omega$.

To resolve this fundamental problem, we develop a rigorous theory of light interaction with a polarization of a trembling medium. We stress that the inelastic scattering considered here is caused by the motion-induced modulation of the interaction between light and scatterer, in contrast to conventional resonant Raman scattering that is due to modulation of the eigen energies of the scatterer itself. We give the general expression for inelastic scattering intensity Eq. (M17) in Methods, while here focus on the case of small trembling object described by the frequency-dependent electric dipole polarizability tensor $\alpha(\omega)$. In that case, the amplitude of inelastic scattering comprises two terms that are diagrammatically represented in Fig. 2(b) and (c). They show the vibration quantum (dashed line) being absorbed/emitted either before or after the medium polarization (bubble) is induced.
Therefore, the frequency dependence of polarizability can be exploited to tune \( d, m, \) and \( Q \) to the Kerker condition.

### Optomechanical Kerker effect

Now we analyze in detail the direction pattern Eq. (2) for light scattered on a vibrating particle with and without electric dipole polarizability \( \alpha(\omega) \). We focus on the anti-Stokes component at the frequency \( \omega' = \omega + \Omega \). Similar results for the Stokes component are obtained by inverting the sign of \( \Omega \) and complex conjugation of the particle displacement vector \( \mathbf{u} \).

For the electric field at the anti-Stokes-shifted frequency \( \omega' = \omega + \Omega \) at \( r \to \infty \) we get

\[
E'(r) = \frac{i\omega'^2\epsilon c^2}{e\sqrt{r}} \left[ \omega' (\mathbf{n}_0 \cdot \mathbf{u}) \alpha(\omega') \mathbf{E}_0 - \omega (\mathbf{n} \cdot \mathbf{u}) \alpha(\omega) \mathbf{E}_0 - \Omega (\mathbf{u} \cdot \mathbf{E}_0) \alpha(\omega') \mathbf{n}_0 - \omega (\mathbf{n} \cdot \mathbf{u}) \alpha(\omega) \mathbf{E}_0 \right]_{\perp},
\]

where \( \mathbf{n} = r/r, \mathbf{n}_0 \) and \( \mathbf{E}_0 \) are the propagation direction and electric field of the incident wave, subscript \( \perp \) indicates the perpendicular component with respect to \( \mathbf{n} \) should be taken, \( |E|_{\perp} = -\mathbf{n} \times (\mathbf{n} \times \mathbf{E}) \). The field Eq. (2) can be decomposed into electric dipole \( p \), quadrupole \( Q \), and magnetic dipole \( m \) contributions oscillating at the frequency \( \omega' \) with the amplitudes

\[
d = \frac{1}{c} \alpha(\omega') \left[ \omega' (\mathbf{n}_0 \cdot \mathbf{u}) \mathbf{E}_0 - \Omega \mathbf{u} \times (\mathbf{n}_0 \times \mathbf{E}_0) \right],
\]

\[
Q = 3\alpha(\omega) \mathbf{E}_0 \otimes \mathbf{u} + 3 \mathbf{u} \otimes \alpha(\omega) \mathbf{E}_0 - 2I (\mathbf{u} \cdot \alpha(\omega) \mathbf{E}_0),
\]

\[
m = \frac{1}{2} (\omega - \Omega) \left[ \alpha(\omega) \mathbf{E}_0 \otimes \mathbf{u} \right],
\]

where \( (\mathbf{a} \otimes \mathbf{b})_{ij} = a_i b_j \) and \( I \) is the identity matrix. All the multipole terms are of the same order, however, the induced electric dipole is proportional to the polarizability at the scattered light frequency \( \alpha(\omega') \) while electric quadrupole and magnetic dipole are determined by \( \alpha(\omega) \).

The frequency dependence of polarizability can be exploited to tune \( d, m, \) and \( Q \) to the Kerker condition.

### Radiation pattern for elastic and inelastic light scattering by a particle

Figure 3 shows the radiation pattern of the light scattered on the particle trembling along the propagation direction of the linearly polarized incident wave. Panel (a) shows the usual elastic electric dipole scattering at the frequency \( \omega \), while panels (c-d) correspond to the inelastic scattering to the frequency \( \omega' \). Panel (c) shows the contribution of the first electric dipole term in Eq. (2) to the scattered field, while panel (d) corresponds to the second term in Eq. (2) and a combination of magnetic dipole and electric quadrupole radiation. The total scattering intensity is a superposition of the patterns shown in Fig. 3(c) and Fig. 3(d) with the coefficients \( \alpha(\omega') \) and \( \alpha(\omega) \), respectively. While the frequencies \( \omega \) and \( \omega' \) are close, the corresponding polarizabilities can differ strongly in the vicinity of the material resonance.

Figures 3(b) and (e) show the two limiting cases when \( \alpha(\omega') \approx \pm \alpha(\omega) \). In the non-resonant case, \( \alpha(\omega') = \alpha(\omega) \), the interference of electric dipole, magnetic dipole, and electric quadrupole radiation leads to the complete suppression of forward inelastic scattering, Fig 3(c). In the opposite case of strong frequency dependence of polarizability when \( \alpha(\omega') = -\alpha(\omega) \), Fig. 3(d), the inelastic backscattering vanishes.

Using Eq. (2) we evaluate the cross-section of the inelastic scattering for unpolarized light

\[
\frac{d\sigma}{d\Omega} = \frac{\omega^6}{2e^6} |\alpha(\omega)\mathbf{n}_0 - \alpha(\omega)\mathbf{n} \cdot \mathbf{u}|^2 [1 + (\mathbf{n}_0 \cdot \mathbf{n})^2],
\]

where \( d\sigma \) is the solid angle for scattered light direction. In the non-resonant case when \( \alpha(\omega') = \alpha(\omega) \) and \( \mathbf{n}_0 \parallel \mathbf{u} \) we recover the geometric optics result Eq. (1) with \( d\sigma \propto (1 - \cos \theta)^2 \) and suppressed forward scattering.

The asymmetry of the light scattering pattern can be quantified by the directivity \( D(\mathbf{n}) = 4\pi(d\sigma)^{-1}d\sigma/d\Omega \) [22]. In the considered geometry, \( \mathbf{u} \parallel \mathbf{n}_0 \), the directivity for forward \( (\mathbf{n} = \mathbf{n}_0) \) and backward \( (\mathbf{n} = -\mathbf{n}_0) \) scattering reads

\[
D(\pm \mathbf{n}_0) = \frac{15|\alpha(\omega') \mp \alpha(\omega)|^2}{10|\alpha(\omega')|^2 + 4|\alpha(\omega)|^2}.
\]
FIG. 4. Directivity of light scattered by a trembling resonant particle. Panels (a) and (b) show backward and forward directivity of non-polarized light depending on the incident light frequency \( \omega \) and trembling frequency \( \Omega \). Solid line shows the directivity equal to 3 that limits usual Kerker effect. (c) Degree of directivity \( [D(n_0) - D(-n_0)]/[D(n_0) + D(-n_0)] \).

For the non-resonant case when \( \alpha(\omega') = \alpha(\omega) \) the forward scattering is absent while the backward directivity reaches \( 30/7 \). The maximal value of forward (backward) directivity is \( 5.25 \) that is achieved when \( \alpha(\omega') = \pm (2/5) \alpha(\omega) \). Thus, the directivity of the optomechanical Kerker effect surpasses the limiting value of \( 3 \) for the classical Kerker effect, because the electric quadrupole contribution is additionally involved [11].

For numerical demonstration we consider the simplistic general model of the particle characterized by the resonant polarizability

\[
\alpha(\omega) = \frac{A}{\omega - \omega_x + i\Gamma},
\]

where \( A \) is a constant, \( \omega_x \) is the resonance frequency, and \( \Gamma \) is the resonance width. Such dependence corresponds to, e.g., exciton resonance in quantum dots and transitional metal dichalcogenide monolayers, electron transitions in cold atomic gases, plasmon resonance in graphene, resonances in atomic nuclei probed by Mössbauer \( \gamma \)-ray spectroscopy, superconducting resonators for radio-frequency electromagnetic field, see Discussion. Figures 4(a) and 4(b) show the color plots of the directivity of backward and forward anti-Stokes light scattering depending on the incident light frequency \( \omega \) and the trembling frequency \( \Omega \). In the dark areas bounded by the solid lines the directivity is larger than 3, which can be termed as a super-Kerker effect. For backward and forward scattering it is realized when initial and scattered light frequencies \( \omega \) and \( \omega' = \omega + \Omega \) are located on the same side or on the opposite sides of the resonance, respectively. Shown in the Fig. 4(c) is the degree of directivity \( [D(n_0) - D(-n_0)]/[D(n_0) + D(-n_0)] \). Red and blue colors indicate predominance of the backward and forward scattering, respectively. Degree of directivity reaches \( \pm 1 \) if \( \alpha(\omega') = \mp \alpha(\omega) \), that is realized at \( \Omega = 2(\omega_x - \omega) \gg \Gamma \) and \( \Omega \to 0 \), respectively.

**Optomechanical Spin Hall effect**

The interference of the electric and magnetic modes is known to give rise to a strong circular polarization of the scattered light upon excitation with a linearly polarized light [23, 24]. Conversely, photons with opposite circular polarizations scatter in different directions. This is termed as an optical spin Hall effect in analogy with the spin-dependent scattering of electrons in solids [25]. Here, we put forward an *optomechanical spin Hall effect*, i.e., inelastic polarization-dependent directional scattering on a trembling particle.

The two first terms in Eq. (2) are dominant and yield the scattered light with the same polarization as the incident. Optomechanical spin Hall effect results from the last two terms, that give a small correction of the order \( \Omega/\omega \) describing linear-to-circular polarization conversion. The circular polarization degree of the plane wave with the electric field amplitude \( E \) can be defined as \( P_c(n) = \lvert n \cdot (E \times E^*) \rvert / |E|^2 \). Substituting here the scattered wave from Eq. (2) we obtain for the case of non-polarized incident light

\[
P_c = \frac{\Omega}{\omega} \frac{2 n \times n_0}{1 + (n_0 \cdot n)^2} \cdot \text{Im} \left[ \frac{\alpha(\omega') + (n_0 \cdot n) \alpha(\omega)}{\alpha(\omega')n_0 - \alpha(\omega)n} \right] \cdot u,
\]

where we keep linear in \( \Omega/\omega \) terms only. Equation (7) indicates two possible origins of circular polarization: (i) the phase difference of the polarizabilities \( \alpha(\omega) \) and \( \alpha(\omega') \) and (ii) the phase difference of the components of the displacement vector \( u \). The first mechanism is likely to contribute in the vicinity of the material resonance where the phase of \( \alpha \) rapidly changes by \( \pi \). The second mechanism is realized even away from the resonances, however it requires the particle trembling around a circle or an ellipse rather than just along one axis.

Figure 5 shows by red and blue color the circular polarization degree of the anti-Stokes-scattered light for different particle trembling directions, indicated on the left, and different relations between polarizabilities at the frequencies of incident and scattered light, indicated on the
For the other linear polarization of the incident light (not shown) the conversion to circular polarization is absent. Therefore, even for the non-polarized incident light the circular polarization of scattered light persists and it is described by Eq. (7).

Now we turn to the second mechanism of the generation of circular polarization, that is realized for the plots on the yellow background. Figures 5(l)-(k) illustrate the circular polarization of the light scattered by the particle trembling around a circular trajectory in the plane perpendicular to the incident light direction. Then, the forward and backward scattered light reveal opposite signs of circular polarization, except for the case of Fig. 5(i) when the scattered wave is of the order of the small parameter $\Omega/\omega$ and linearly polarized, see Eq. (2). Figures 5(m)-(o) and (q)-(s) show the pattern of the circular polarization degree of the light scattered by the particle trembling around a circular trajectory in the plane parallel to the incident incident light propagation direction. The circular polarization sign depends on whether the light is scattered to the left or to the right with respect to the plane of trembling. Finally, if both optomechanical Spin Hall effect mechanisms are present, see plots on the purple background, their interplay leads to a strong asymmetry of both the scattering intensity pattern and the circular polarization pattern.

**Optomechanical Huygens surfaces**

A two-dimensional planar array of scatterers tuned to the Kerker condition is known to demonstrate no forward or backward scattering [16, 17, 27]. When the backscattering is suppressed it realizes a Huygens’ surface that transmits light without reflection and modifies only it phase. An optomechanical analog of the Huygens’ surface is a thin layer with a resonant dipole polarizability that trembles along its normal, see Fig. 6(a).

Since the flexural layer vibrations can possess an in-plane wave vector $\mathbf{q}$, the direction of scattered light can differ from that of the incident light. It is determined from the conservation of the in-plane wave vector component in the process of anti-Stokes (Stokes) scattering, $k' = k \pm \mathbf{q}$, where $k = (\omega/c) \cos \theta$ and $k' = (\omega'/c) \cos \theta'$ are the in-plane wave vectors of the incident and scattered light, $\theta$ and $\theta'$ are the angles between the light propagation direction and the layer normal. The electric field amplitudes of the anti-Stokes forward ($\leftarrow$) and backward ($\rightarrow$) scattered light are given by the diagrams of Fig. 2(b)-(c) and read $E^{\sigma\rightarrow\leftarrow(\rightarrow)}_0 = \sum_{\sigma'} S^{\sigma\rightarrow\leftarrow(\rightarrow)}_{\sigma\sigma'} E_0,\sigma' u_{\sigma',z}$, where $\sigma, \sigma'$ enumerates two polarizations, $s$ and $p$. The Jones matrix $S^{\sigma\rightarrow\leftarrow(\rightarrow)}_{\sigma\sigma'}$ with the elements (see Methods for

![Optomechanical Spin Hall effect](image_url)
calculation details)

\[
S_{ss}^{(\rightarrow)} = i \frac{\omega'}{c} \cos \theta \cos \phi \left[ r_s(\theta', \omega') \mp r_s(\theta, \omega) \right],
\]

\[
S_{ps}^{(\rightarrow)} = i \frac{\omega'}{c} \cos \theta \sin \phi \left[ r_p(\theta', \omega') \mp r_s(\theta, \omega) \right],
\]

\[
S_{sp}^{(\rightarrow)} = -i \frac{\omega'}{c} \sin \phi \left[ r_s(\theta', \omega') \mp r_p(\theta, \omega) \right],
\]

\[
S_{pp}^{(\rightarrow)} = i \frac{\omega'}{c} \cos \phi \sin \theta \cos \theta' \left[ r_p(\theta', \omega') \mp r_p(\theta, \omega) \right],
\]

Describes polarization conversion, \( r_s(\theta, \omega) \) and \( r_p(\theta, \omega) \) are the reflection coefficients for oblique incidence of \( s \)- and \( p \)-polarized light on the layer at rest, \( \phi \) is the angle between the in-plane wave vectors \( \mathbf{k} \) and \( \mathbf{k}' \). Similarly to the optomechanical Kerker effect for the trembling particle, the forward (backward) scattering on a trembling layer vanishes when \( r_s(\theta, \omega) \). The power of the anti-Stokes light scattered forward (backward) into the solid angle \( d\Omega \) by the unit area of the layer for the case of unpolarized excitation with the intensity \( I_0 \) reads

\[
\frac{dI'}{d\Omega} = \frac{\omega'^2 \cos^2 \theta'}{c^2} \frac{1}{2} \sum_{\sigma\sigma'} |S_{\sigma\sigma'}^{(\rightarrow)}|^2 |u_q|^2 I_0.
\]

Figures 6(b)-6(c) show the forward and backward scattered power for the case of normal incidence, \( \theta = 0 \), and constant \( |u_q|^2 \). Both plots feature two resonances indicated by dashed lines: the resonance for incident light at \( \omega = \omega_x \) and the resonance for scattered light at \( \omega + \Omega = \omega_x \). Since the vibration frequency \( \Omega \) increases with the vibration wave vector \( q = (\omega'/c) \sin \theta' \), the two resonances split with an increase of the scattering angle \( \theta' \). The main result of Figs. 6(b)-6(c) is that the forward scattering involving vibration with \( q = 0 \), i.e., for \( \theta' = 0 \), is suppressed while the backward scattering at \( \theta' = 0 \) is increased. The absence of the forward-scattered wave in the limit \( \Omega, q \to 0 \) has a clear physical interpretation: in the quasi-static picture, the shift of the layer as a whole affects the reflected plane wave but does not affect the transmitted one.

**DISCUSSION**

Now we discuss the possibilities for experimental realization of the optomechanical Kerker effect. The proof-of-principle observation of the suppression of the forward inelastic scattering in the nonresonant regime, Fig. 3(b), seems to be relatively straightforward for an arbitrary subwavelength particle. The only requirements are to operate in the far field regime and to avoid the internal deformations of the particle, so that it trembles as a whole. The true challenge is to realize a dynamical tunability between forward and backward scattering by exploiting resonance

**FIG. 7. Resonant optomechanical systems.** Shown is the tunability of resonant scatterers operating in different spectral ranges. When \( \Omega < \Gamma \) (blue area), the directional scattering can occur only in the backward direction. When \( \Omega \approx \Gamma \) (red area), one can realize both directional forward and backward scattering by a proper tuning of the incident light frequency and trembling frequency. Numerical parameters and references are given in Table I.
the resonance of particle polarizability. This requires the width of the resonance $\Gamma$ to be comparable with the frequency of vibrations $\Omega$, i.e., narrow resonances and high vibration frequencies. Table I presents an overview of various potential systems in different ranges of electromagnetic spectrum and Fig. 7 visualizes their tunability $\Omega/\Gamma$. Apparently, the highest degree of tunability can be attained by exploiting the Mössbauer resonances in the nuclei of crystals for $\gamma$-rays [38]. Namely, the linewidth can be as narrow as 0.5 MHz [36], while the GHz range hypersound waves in metals are available [37]. Further opportunities are provided by organizing planar cavities for synchrotron $\gamma$-radiation [39]. The opposite side of the electromagnetic spectrum, with sub 0.1 meV rather than 10 keV photon energies, is represented by the superconducting qubits [35]. They feature high quality factor resulting in potentially high tunability when being coupled to the ultrasound generator [33]. An apparent drawback of such setup is a relatively weak scattering efficiency due to the vibration amplitude being relatively small as compared to the electromagnetic wavelength, see Table I. The THz and optical frequency ranges are accessible by membranes made of graphene [29] and transition metal dichalcogenide monolayers [31], respectively. These platforms feature reasonable coupling strength but have limited tunability because of the relatively low frequency of the flexural vibrations ($\lesssim$ 0.1 GHz) as compared to the broad width of plasmonic or excitonic resonance. High tunability and strong optomechanical coupling efficiency for visible light can be realized by exploiting narrow resonances in cold atoms vibrating in an optical trap [34]. Alternatively, one could use excitonic resonances in artificial atoms, colloidal quantum dots [33].

The rich consequences of the interplay of magnetic and electric response on the electromagnetic wave propagation are known at least for 50 years since the seminal work by Veselago on the media with negative permittivity and permeability [40, 41]. Still, the mutual effect of electric and magnetic resonances is very far from being completely understood. For example, it has been realized only quite recently that the interference and coupling of electric and magnetic resonances underpin bianisotropic photonic topological insulators [42, 43], where the light backscattering on disorder is suppressed. We expect that the proposed optomechanical Kerker and spin-Hall effects with trembling-induced magnetic response open a pathway to engineer chiral optomechanical coupling at nanoscale, expanding the chiral quantum optics [44, 45] to the optomechanical domain. Our results can be instructive for the design of nonreciprocal topological circuits [46, 47], where the disorder-robust propagation of light and sound is ensured by the time modulation of optical and mechanical properties [48–50].

| System                           | $\hbar\omega_{r}$ | $\Gamma/(2\pi)$ | $\Omega/(2\pi)$ | $\nu$ | Tunability $\frac{\Gamma}{\Omega}$ | Coupling $\frac{\omega_{r}}{\nu}$ |
|---------------------------------|-------------------|-----------------|-----------------|------|-------------------------------|-------------------------------|
| Plasmon in graphene [28–30]    | 0.1–1 eV          | 10 THz          | 0.1 GHz         | 10 mm| $10^{-3}$                      | 0.1                           |
| Exciton in TMID monolayers [30, 31] | 2 eV              | 20 GHz          | 0.1 GHz         | 10 mm| $5 \times 10^{-3}$             | 0.1                           |
| Exciton in colloidal QDs [32, 33] | 2 eV              | 400 MHz         | 20 MHz          | 200 mm| $5 \times 10^{-3}$             | 2                             |
| Cold atoms [34]                 | 1.5 eV            | 10 kHz          | 100 kHz         | 200 mm| 10                            | 1.5                           |
| Superconducting qubits [34, 35] | 20 $\mu$eV        | 0.1 MHz         | 20 MHz          | 200 mm| 200                           | $2 \times 10^{-3}$            |
| Resonance in nuclei [36, 37]    | 15 keV            | 0.5 MHz         | 10 GHz          | 10 nm| $2 \times 10^{-4}$             | 0.1                           |

TABLE I. Parameters of various resonant optomechanical systems

Methods

Polarization of trembling media

We consider the light with the frequency $\omega$ incident on a medium vibrating with the frequency $\Omega$. Medium motion is described by the displacement vector $\mathbf{u}(\mathbf{r}, t) = \mathbf{u}(\mathbf{r})e^{-i\omega t} + c.c.$ We suppose that the vibration amplitude is small and focus on the linear-in-$\mathbf{u}$ effect only, i.e. appearance of polarization at anti-Stokes- and Stokes-shifted frequencies $\omega \pm \Omega$.

Consider the unitary volume of medium that in the absence of vibration had coordinate $\mathbf{r}$. Its polarization at the time $t$ is determined by the electric field $\bar{E}(\mathbf{r}, t')$ that has acted on it in all previous moments of time $t' < t$,

$$\bar{P}(\mathbf{r}, t) = \int_{-\infty}^{t} dt' \chi(\mathbf{r}, t - t') \bar{E}(\mathbf{r}, t'),$$

(M1)

where $\chi(\mathbf{r}, \tau)$ is the dielectric susceptibility function. We do not account here for the possible change of susceptibility under medium deformation, because such photoelastic effect requires separate microscopic calculation. While photoelasticity may give dominant contribution to optomechanical coupling in resonant structures [51–54], it does not play any role in the effects we consider, where the objects move as a whole and deformation is absent. The electric field $\bar{E}(\mathbf{r}, t')$ in Eq. (M1) should be calculated in the reference frame that moves and rotates together with the considered medium volume. Keeping linear-in-$\mathbf{u}$ terms only we obtain

$$\bar{E} = E + \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial \tau}\right) E - \frac{\mathbf{rot} \mathbf{u}}{2} \times E + \frac{1}{c} \frac{\partial \mathbf{u}}{\partial \tau} \times B,$$

(M2)

where $E$ and $B$ are the electric and magnetic fields in the reference frame at rest, and all quantities are evaluated at the moment $t'$. Second term in the right-hand-side of Eq. (M2) stems from the fact that the electric field should be evaluated at the point $\mathbf{r} + \mathbf{u}(\mathbf{r}, t')$, third term accounts for the medium rotation, and the last term comes from the Lorentz transform.
Equation (M1) gives the polarization of the unitary volume of undeformed medium in the reference frame that moves and rotates with the medium. In the reference frame at rest, the polarization density reads

$$P = \tilde{P} - \left( u \frac{\partial}{\partial r} \right) \tilde{P} + \frac{\text{rot} \, u}{2} \times \tilde{P} - \tilde{P} \, \text{div} \, u, \quad (M3)$$

where the last term accounts for the difference between the deformed and undeformed unitary volumes. Additionally, the magnetization $\tilde{M} = -\partial u/\partial t \times \tilde{P}$ appears in the frame at rest due to the Lorentz transform.

Finally, we combine Eqs. (M1)–(M3) and evaluate the current $\tilde{j} = \partial \tilde{P}/\partial t + \text{rot} \, \tilde{M}$ in the reference frame at rest. The relation between the current $\tilde{j}$ at the anti-Stokes-shifted frequency $\omega' = \omega + \Omega$ and the vector potential $\tilde{A}$ of light at the initial frequency $\omega'$ in the $k$-space assumes the form

$$j_{k'}(\omega') = \sum \delta \Pi_{k',k} A_{k}(\omega),$$

where we use the gauge with zero scalar potential,

$$\delta \Pi_{k',k} = \omega'^2 \chi_{k'q-k} (\omega') A_{k+q,k}(\omega', \omega) + \tilde{A}^T_{k',k} (\omega', \omega) \omega^2 \chi_{k'q+k}(\omega'),$$

$$\chi_q(\omega) = \iint \chi(r, r') e^{i\omega q r - i\omega q r'} d^3 r d^3 r',$$

and $u_q$ are the Fourier transforms of $[\varepsilon(r, \omega) - 1]/(4\pi)$ and $u(r)$, respectively, superscript T denotes matrix transposition, and

$$A_{k+q,k}(\omega', \omega) = i(u_q \cdot k) - i\omega' - \omega u_q \otimes q - q \otimes u_q.$$

To calculate the polarization at the Stokes-shifted frequency $\omega' = \omega + \Omega$ one should change in the above equations the sign of $\Omega$ and replace $u_q$ with $u_{-q}$. The quantity $\delta \Pi$ is the correction to the polarization operator caused by the medium vibration. The two terms in the right-hand side of Eq. (M3) can be represented diagrammatically as shown in Figs. M1(a) and M1(b). The wavy line corresponds to a photon, dashed line is a vibration, the bubble stands for polarization operator, and solid dot represents optomechanical interaction $A_k$.

**Light scattering on trembling objects**

Here we describe how the amplitude of inelastic light scattering on a trembling object of an arbitrary shape can be calculated. We use $c = 1$ for simplicity. The full amplitude can be represented as a sum of four terms diagrammatically shown in Fig. 2(b)-(c) and Fig. M1(c)-(d). They correspond to the medium polarization, described by the dressed polarization operator $\mathcal{P} = \Pi(1 + D\Pi)^{-1}$, accounted before, after, or both before and after the optomechanical interaction.

Note that photon Green’s function $D_k(\omega) = -4\pi(1 - k \otimes k/\omega^2)/(\omega^2 - k^2)$ is $k$-even while the optomechanical interaction $A$ is $k$-odd in the absence of the last term describing medium rotation, see Eq. (M5). In the main text, we consider the small object characterized by the wavevector-independent dressed polarization operator $\mathcal{P}_{k',k}(\omega) = \omega^2 \alpha(\omega)$. In such case, the summation over the phonon wave vectors in the intermediate states of the diagrams Fig. M1(c) and (d) yields zero. The scattering amplitude is then given by the remaining diagrams Fig. 2(b) and (c).

In the general case all the diagrams of Fig. M1 contribute to the total scattering matrix element that reads

$$M_{k'k} = A_{k'} \cdot (1 + \mathcal{P}D)(\Pi^T + \Pi A)(1 + D\mathcal{P})A_k.$$

where $A_k = \sqrt{2\pi/\omega} e^{i k \cdot r}$ and $A_{k'} = \sqrt{2\pi/\omega'} e^{i k' \cdot r}$ are the vector potentials of the incident and scattered photons, $e_0$ and $e'$ are their polarizations. Introducing the field distributions of incident and scattered photons in the form $A_0(r) = (1 + D\mathcal{P})A_k$ and $A'_{\omega}(r) = (1 + D\mathcal{P}^T)A'_{k'}$ we finally obtain the scattering amplitude per solid angle $d\Omega$

$$\frac{d\Sigma_{k'k}}{d\Omega} = \frac{i\omega'^2}{(2\pi)^2} \int \Pi^T(r, \omega') \mathcal{A}_{\omega}(r) \cdot A_0(r) \cdot \mathcal{A}_{\omega}(r) \cdot \Pi(r, \omega) A_0(r) \cdot \Pi(r, \omega) A_0(r) d^3 r.$$

Here, the operator $\Pi$ with the components

$$\Pi_{\alpha\beta}(\omega', \omega) = \left( u \cdot \frac{\partial}{\partial r} \right) \delta_{\alpha\beta} - \frac{\omega' - \omega}{\omega'} \frac{\partial}{\partial r} \delta_{\alpha\beta} + \frac{1}{2} \frac{\omega}{\omega'} \varepsilon_{\alpha\beta\gamma} (\text{rot} \, u) \gamma,$$

is the optomechanical interaction operator Eq. (M5) in the coordinate representation and the polarizability operator can be readily expressed via the dielectric permittivity as $\Pi(r, \omega) = \omega^2 [\varepsilon(r, \omega) - 1]/4\pi$. When the time-inversion symmetry holds the dielectric permittivity tensor is symmetric, so $\Pi^T = \Pi$. Then the distributions $A_0(r)$ and $A'_{\omega}(r)$ can be calculated as the fields induced in the system by the light incident with wave vectors $k$ and $-k'$, respectively.

**Light scattering by a vibrating resonant layer**

We derive here the amplitudes of light scattering on a vibrating resonant layer, Eq. (8). The layer is described by the remaining diagrams Fig. 2(b) and (c).
by a dielectric susceptibility tensor with the components
\( \chi_{\alpha\beta}(r, \omega) = \delta_{\alpha\beta}\delta(z)(\chi(\omega)) \) and
\( \chi_{zz} = \chi_{xx} = \chi_{xy} = 0 \). Here
\( \alpha, \beta = x, y \) are the in-plane coordinates and
\( z \) is the layer normal. Using the Green’s function
\[
D_{\alpha\beta}(k, z, \omega) = \frac{2\pi i}{k_z} \left( \delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{\omega^2} \right) e^{i k_z |z|},
\]  
(M9)
where \( k = (k_x, k_y) \) is the in-plane wave vector and
\( k_z = \sqrt{\omega^2 - k^2} \), the dressed polarization operator
of the layer \( \mathcal{P} = \mathbf{\Pi}(1 - D) \) can be evaluated. We find
\[
\mathcal{P}_{\alpha\beta}(k, z, \omega) = \frac{\omega^2 \chi(\omega) \delta(z)}{1 - 2\pi i \omega \chi(\omega) / k_z} \left( \delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{\omega^2} \right)  
+ \frac{\omega^2 \chi(\omega) \delta(z)}{1 - 2\pi i k_z \chi(\omega) / k^2}. 
\]  
(M10)
and \( \mathcal{P}_{zz} = \mathcal{P}_{\alpha z} = \mathcal{P}_{\beta z} = 0 \). The amplitude of
cohherent light reflection from the layer is given by
\( r = e^{i\omega t} \cdot (2\pi i \mathbf{P} / k_0) e_0 \). For \( s \)- and \( p \)-polarized light we obtain
\[
r_s(k, \omega) = \frac{2\pi i \omega^2 \chi(\omega) / k_z}{1 - 2\pi i \omega \chi(\omega) / k_z}, \]
(M11)
\[
r_p(k, \omega) = \frac{2\pi i k_z \chi(\omega) / k^2}{1 - 2\pi i k_z \chi(\omega) / k^2}. \]
(M12)
To calculate the amplitudes of light scattered by a layer
vibration, we use the approach described in the previous section
of Methods. First, we calculate the vector potential distribution
induced by a photon at the frequency \( \omega \) incident from
\( z \rightarrow -\infty \) with the in-plane wave vector \( k \) and the polarization
electron,
\[
\mathbf{A}_{\omega, k}(r) = \sqrt{\frac{2\pi}{\epsilon_0 k_0}} e^{i k_z z} \left\{ e \left[ e_k e_z + \frac{\omega^2}{k_z^2} p(k, \omega) e_t \right. 
+ r_p(k, \omega) e_t + \text{sign} z \cdot r_p(k, \omega) e_x \left. \right] e^{i k_z |z|} \right\}, \]
(M13)
where \( e = e_t + e_z + e_x \) with \( e_x \parallel z \) being the out-of-plane component of the light polarization vector \( e \), and
\( e_t \parallel k \), \( e_x \perp k \) being its in-plane components. The layer polarization
induced by the photon reads
\[
\mathbf{\Pi}(r, \omega) \mathbf{A}_{\omega, k}(r) = -\frac{ik_0 \chi(z)}{2\pi \omega} \left[ r_s(k, \omega) e_t + \frac{\omega^2}{k_z^2} p(k, \omega) e_t \right], \]
(M14)
and then calculate \( \mathbf{A}(\omega', \omega) \mathbf{A}_{\omega, k}(r) \). Keeping in mind that
according to Eq. (M7) the result is to be multiplied by
\( \mathbf{\Pi}(r, \omega) \mathbf{A}_{\omega, k}(r) \), only needed are the in-plane components
of \( \mathbf{\Pi}(\omega', \omega) \mathbf{A}_{\omega, k}(r) \) at \( z = 0 \). They read
\[
\mathbf{\Lambda}(\omega', \omega) \mathbf{A}_{\omega, k} = i \sqrt{\frac{2\pi}{\omega}} \left[ k_z (e_t + e_l) + \frac{\Omega}{\omega} \frac{k_z^2}{k_z} e_l 
- \frac{\omega^2}{k_z^2} e_x q \right] u \cdot e^{i(k+q) \cdot r}, \]
where the ellipsis replaces the out-of-plane component. Note
that when evaluating the action of the optomechanical
interaction operator Eq. (M8), in the last term describing the layer
rotation we used \( \mathbf{u} = 2(\partial u_x / \partial y - \partial u_y / \partial x, 0) \). The factor
2 arises from the contribution of \( \partial u_x / \partial y (\alpha = x, y) \) that are
non-zero even when \( u_x = 0 \) at \( z = 0 \).

Finally, we calculate the backward and forward photon
scattering amplitudes
\[
R_{\omega} = \frac{i}{k_0^2} u_z \left[ \mathbf{\Pi} \mathbf{A}_{\omega', \omega} \right. \left. \mathbf{A}(\omega', \omega) \mathbf{A}_{\omega, k} \right] dz. \]
(M16)
We make use of Eqs. (M14 and (M15) and take into account
that for the photon incident from \( z \rightarrow +\infty \), the in-plane
components of \( \mathbf{\Lambda} \mathbf{A} \) have opposite signs, while \( \Pi \mathbf{A} \) is the same. Finally we obtain
\[
R_{\omega} = \frac{i}{k_0^2} u_z \left[ k_z (e_t + e_l) + \frac{\Omega}{\omega} \frac{k_z^2}{k_z} e_l 
\pm k_z e_t + \frac{\omega^2}{k_z^2} e_x q \right] u \cdot e^{i(k+q) \cdot r}, \]
where the quantities without (with) prime refer to the initial
(scattered) photon. Multiplying the result by the factor
\( \sqrt{\omega^2 / \omega^2} \) to relate electric fields rather than the photon amplitudes
and considering \( e \) and \( e' \) that represent \( s \) and \( p \) polarizations, we recover Eq. (8) of the main text.

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Author contributions

A.V.P. developed the theoretical model, both A.V.P. and A.N.P. contributed to the discussion of the results and writing the manuscript.