Manipulating waves by distilling frequencies: a tunable shunt-enabled rainbow trap

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Abstract
In this work, we propose and test a strategy for tunable, broadband wave attenuation in electromechanical waveguides with shunted piezoelectric inclusions. Our strategy is built upon the vast pre-existing literature on vibration attenuation and bandgap generation in structures featuring periodic arrays of piezo patches, but distinguishes itself for several key features. First, we demystify the idea that periodicity is a requirement for wave attenuation and bandgap formation. We further embrace the idea of ‘organized disorder’ by tuning the circuits as to resonate at distinct neighboring frequencies. In doing so, we create a tunable ‘rainbow trap’ (Tsakmakidis et al 2007 Nature 450 397–401) capable of attenuating waves with broadband characteristics, by distilling (sequentially) seven frequencies from a traveling wavepacket. Finally, we devote considerable attention to the implications in terms of packet distortion of the spectral manipulation introduced by shunting. This work is also meant to serve as a didactic tool for those approaching the field of shunted piezoelectrics, and attempts to provide a different perspective, with abundant details, on how to successfully design an experimental setup involving resistive-inductive shunts.

Keywords: wave manipulation, resonant shunts, metamaterials, rainbow trap, broadband filter

1. Introduction

Ever since Robert Forward’s paper on the electronic damping of vibrations in optical structures [2] (where he stated ‘We hope that the examples presented in this paper will […] lead to the realization that one does not need to find a purely mechanical solution to a mechanical vibration problem’), there has been extensive progress in the field of vibration control via shunted piezoelectric elements. Shunting is referred to as the act of connecting the electrodes of a piezoelectric element to a passive electrical circuit, which represents an electrical extension of the structure. A seminal work on the topic is that of Hagood and von Flotow [3], who presented a detailed theoretical formulation on shunting based on linear piezoelectric theory, and experimental results on the attenuation of a single mode of vibration via resistive and resistive-inductive (RL) circuits. While resistive circuits behave as broadband dampers, RL circuits allow to obtain pronounced and localized vibration attenuation; when connecting a RL in series to a piezo patch (which can be modeled as a capacitor in series with a voltage supply) we obtain a resonant RLC circuit, which behaves as an electrical dynamic damper which can be tuned by modifying the circuit parameters. In recent years, extensive efforts have been devoted to extending the effects of RL circuits to multiple modes of vibration [4–6] (multimodal vibration attenuation) and to the investigation of alternative passive circuits, such as the negative capacitor [7–9]. Moreover, with the discovery of
phononic crystals [10]—periodic structures with unconventional wave manipulation characteristics such as phononic bandgaps—researchers have begun to incorporate shunted piezoelectric patches in the architectures of these materials to obtain tunable, complex wave patterns. This has motivated an increasingly larger number of studies on structures involving periodic arrays of shunted piezos. Among these, we recall the pioneering contribution of Thorp et al [11], who paired the concepts of phononic crystals and shunting for the first time. It is also interesting how the authors hinted at the concept of relaxation of the periodicity to widen the bandgaps, invoking Anderson localization mechanisms, anticipating a discussion that has become central in recent phononic crystals literature [12]. Noteworthy is also the work of Airoldi and Ruzzene [13], who were the first to use a periodic structure with independent RL shunts to control waves, rather than to attenuate resonance peaks; their work—the first catering to problems encountered in metamaterial engineering applications—provided compelling results in terms of locally resonant bandgap generation. In parallel, researchers have also proposed more convoluted active and passive shunting strategies to enhance vibration attenuation. Examples are electrical networks placed in parallel to the mechanical network (where each patch is shunted to the neighboring patch instead of connected to ground) [14–16], enhanced resonant shunts [17] and amplifier-resonator feedback circuits [18]. While most of the studies involving periodic arrays of shunted piezos are centered around vibration attenuation and bandgap generation [11, 13–26], others have proposed shunted piezos as a mean to achieve tunable wave focusing in metamaterial architectures [27–29], and tunable waveguiding [30, 31].

So far, most of the experimental studies involving multiple patches bonded to the same substrate have been characterized by a uniform tuning of the circuit characteristics. Apart from the previously discussed work by Thorp et al [11], exceptions are the recent works of Wang and Chen [18] and Bergamini et al [16], who discussed the use of a superlattice method and diatomic unit cells, respectively, as strategies to extend the bandgaps to multiple frequencies. In both cases, different inductance values are connected to neighboring patches, while preserving some sort of periodicity. While modeling structures with periodically placed piezoelectric elements might present some convenient aspects, such as the possibility of using reduced models (i.e., a unit cell analysis) and the availability of additional frequency regions of Bragg attenuation, one might argue that, when dealing with resonant shunts, periodicity is not strictly needed for bandgap generation and wave attenuation. In fact, when locally resonant mechanisms are involved, the resonators’ placement throughout the medium does not affect the global formation of bandgaps [12, 32]. On the contrary, in some situations, it might be convenient to leverage some ‘organized disorder’ to enhance the wave attenuation performance. The attempt to extend the usually narrow frequency regions of resonance-based attenuation has received growing interest in the metamaterials community over recent years. An example is the work of Krödel et al [33] on metamaterial-enabled seismic isolation, where a wide locally resonant bandgap is created by employing resonators tuned at adjacent frequencies and displaying slightly overlapping spectra. The concept of trapping a wide spectrum of adjacent frequencies has been pioneered by work in optics, where the application to the control of visible light has inspired the name ‘rainbow trap’ [1, 34], and has recently been extended to the realm of acoustic waves [35–37].

In our work, we embrace these ideas of disorder and relaxation of the periodicity to demonstrate the wave attenuation capabilities of an array of randomly positioned piezoelectric elements shunted with non-uniform RL circuits. We demonstrate that a device capable of attenuating vibrations over a broad frequency range can be obtained by tuning different RL circuits to resonate at different frequencies, thus realizing a tunable rainbow trap for elastic wave manipulation. The behavior of our test system is probed with a transient chirp signal and monitored by measuring how its spectro-spatial characteristics are modified by the sequential activation of the shunting circuits. Moreover, we propose an additional angle to the problem, whereby we look at the effects of wave manipulation through the prism of wave packet distortion.

This paper is organized as follows. In section 2 we discuss in detail our experimental setup, including the specimen, the circuitry and the acquisition system. In section 3 we discuss the individual tuning of the circuits. In section 4 we report on the broadband wave attenuation enabled by the rainbow trap strategy. Finally, the conclusions of our work are drawn in section 5.

2. Experimental setup

In this section, we describe in detail the experimental framework used throughout this work. A comprehensive picture of the setup is shown in figure 1. We would like to point out that most of the circuitry solutions used here are inspired by the informative works of Viana and Steffen [5], and of Thomas, Ducarme and Déu [6, 38].

2.1. Specimen geometry and patch configuration

Our specimen is an Aluminum beam with eighteen PZT patches (STEMInC, part SMPL20W15T14R111) bonded according to a random spatial pattern, as shown in the schematic of figure 2. Note that the beam is instrumented with a dense population of patches. While this may be redundant for most wave control applications, it grants the possibility to test a variety of shunting sequences and to potentially investigate the effects of location, clustering and order. In our experiments, a maximum of seven patches will be activated simultaneously. Note that each patch has a slightly different value of capacitance; the average capacitance measured with a multimeter is \( C_{p,\text{ave}} = 2.137 \text{ nF} \). The main geometric dimensions of the specimens are the following: the length of the beam is \( L = 51.6 \text{ cm} \), the thickness of the beam is \( t = 0.26 \text{ cm} \), the width of the beam is \( b = 1.5 \text{ cm} \), the length...
of each patch is \( L_p = 2 \text{ cm} \), the thickness of a patch is \( t_p = 0.14 \text{ cm} \) and the width of the patch is \( b_p = 1.5 \text{ cm} \). Note that, for the sake of brevity, we did not report the distance between each pair of neighboring patches; it will be demonstrated that this parameter is essentially uninfluential with respect to the effects discussed in this article. The patches are bonded to the beam using a 2-part epoxy glue (3M Scotch-Weld 1838 B/A); details on how to properly bond the patches are reported in the supplementary data (SD) section. The sketch also shows the position of the actuator and of the input and output accelerometers.

The measured frequency response function of the beam (obtained using the signals recorded at the input and output accelerometers in response to a multitone excitation spanning the 5–10 kHz range) is shown in figure 3. The region highlighted in red represents the frequency interval of interest, i.e., the region where we want to test our wave attenuation strategy; note how this region, ranging from 7 to 8.8 kHz, deliberately spans multiple peaks of the beam response. The reason for selecting this frequency region will be explained in section 2.2.

### 2.2. Circuitry

Throughout this work, we resort to resistor-inductor (RL) shunting circuits. As shown by Hagood and von Flotow [3] and by Viana and Steffen [5], a piezoelectric patch (which acts as a capacitor with capacitance \( C_p \) in series with a voltage supply) placed in series with a RL circuit realizes a resonant RLC circuit and therefore behaves as a dynamic damper. In analogy with purely mechanical resonant systems, the electric dynamic damper resonates at a frequency that can be determined from the circuit parameters as:

\[
P_{\text{res}} = \frac{1}{2 \pi} \sqrt{\frac{1}{L C_p}}.
\]
Another major advantage of using this circuit is the fact that the value of the equivalent inductance can be modified by simply tuning one of the components. This feature is especially important in the context of our work, as tunability of the response is a key objective. In our case, as visible in the realization of the circuit on a breadboard shown in figure 4(b), the tunable component is the potentiometer corresponding to \( R_1 \), which can assume values in the 100–1000Ω range. The other circuitual components are: \( R_2 = 680 \, \Omega \), \( R_3 = 1000 \, \Omega \), \( R_4 = 1500 \, \Omega \), \( C = 0.22 \, \mu \text{F} \) (Mylar type), while, for the operational amplifiers, we select the OPA445 model (TI OPA445AP). Note that the amplifiers are powered by two DC power supplies (BK Precision 1667) arranged in series as to share the same ground and produce a voltage difference of ±30 V. Note that the ground terminal coming from the power supplies is used as reference ground for the circuits as well as for the patches. More details on grounding are reported in the SD section. Given these components, the circuit can assume inductance values ranging from 0.0485 to 0.485 H (from equation (2)). Since the average capacitance of the piezo patches is \( C_{\text{pave}} = 2.137 \, \text{nF} \), the piezo+RL circuits can be tuned to resonate between 4950 and 15600 Hz (from equation (1)).

It is important to point out that the choice of circuit components is not accidental. In fact, in order to harness the full attenuation capability of the shunting strategy, it is fundamental to reduce as much as possible the parasitic resistance of the circuit (i.e., the resistance that is inherent to the circuit components). The parasitic resistance can be measured experimentally as a function of the circuit components. In general, lower equivalent inductances lead to smaller values of the parasitic resistance; in light of this, working at high frequencies can be advantageous. For this reason, we decided to work in the 7–8.8 kHz frequency range. In any case, once the components are chosen and the circuits are assembled, it is suggested to test their electric response. Details on how to electrically test RL circuits are also discussed in the SD section.

2.3. Shunting and screaming

While the beam comprises eighteen randomly positioned patches, we only shunt seven of them (the results in this article refer to the activation of patches 1, 5, 8, 10, 12, 14 and 18, according to the numbering in figure 2). Working with a subset of patches strengthens our claim regarding the non-influence of the patch positioning. It is worth pointing out that increasing the number of patches working at the same time is not a simple task. This is due to the fact that shunting multiple patches can lead to an unstable behavior of the system. This is especially observed when the series resistors \( R_s \) (as previously mentioned, the circuit connected to each patch also includes a series resistor \( R_s \)) are zero or small. In an ideal scenario, it would be preferable not to use any series resistor, to enhance the attenuation behavior of each resonant circuit at its resonant frequency. However, without \( R_s \), spurious vibrations are transmitted to the beam specimen and the patches can be...
To generate and acquire the signals, we resort to a data acquisition system (NI cDAQ-9174), equipped with an output module (NI 9263) and an input module (NI 9215), and connected to a laptop running the LabVIEW software. The input signal is generated in LabVIEW, sent through the output module to a power amplifier (Brüel & Kjær Type 2718) and then fed to a shaker (Brüel & Kjær Type 4810), which transmits it to the beam specimen through a stinger. To span the frequency range of interest (highlighted in figure 3), we prescribe the signal shown in figure 5(a), i.e., a sine-modulated chirp with frequency content linearly increasing from 7 to 8.8 kHz in 2 ms. The frequency content of the signal is shown in figure 5(b).

To evaluate the system response, we analyze the data recorded by the output accelerometer, i.e., the one which is further away from the excitation source. The signal recorded by this accelerometer (PCB Model 352A73) is conditioned with a signal conditioner (PCB Model 482A22), sent to the input module and displayed in LabVIEW. The postprocessing of the data is performed in MATLAB. To reduce the noise, the signal corresponding to each separate experiment is acquired seven times and averaged.

3. Circuit tuning and single-frequency distillation

Let us recall that our objective is to realize a tunable bandgap filter capable of operating over broadband spectra. This is the elastic-acoustic analog of a concept that, in optics and electromagnetism, has been originally termed ‘rainbow trap’. The idea consists of shunting a discrete number of patches and tuning each one individually at different adjacent frequencies, such that the corrections are felt over slightly overlapping frequency bands. Here, we demonstrate the concept using seven of the amenable eighteen patches (1, 5, 8, 10, 12, 14 and 18). The individual tuning of the patches is performed by monitoring real-time changes in the frequency content of the signal recorded at the output accelerometer. Figure 6(a) represents the frequency content of the signal in the reference scenario, in which all patches are short circuited. We can see that the spectrum of the applied chirp has morphed into the two-peak structure of figure 6(a), to reflect the presence of two resonances in the frequency response of the beam in the 7–8.8 kHz range (as shown in figure 3). The thick black curves in figures 6(b)–(h) represent the frequency content of the signals obtained by activating one patch at the time, superimposed on top of the response of the short-circuited beam (thin gray line). We can see that each shunt has a very localized behavior, being able to distill (subtract) the energy associated with a very narrow frequency band. In analogy with the spectrum of visible light rainbows, and in order to maintain a convenient and visually friendly taxonomy of the frequency sequence, we name each of the seven shunts as one of the seven colors of a discrete (Newtonian) rainbow: the RED (R) shunt resonates at \( f_R \approx 7260 \text{ Hz} \) (figure 6(b)), the ORANGE (O) at \( f_O \approx 7390 \text{ Hz} \) (figure 6(c)), the YELLOW (Y) at \( f_Y \approx 7520 \text{ Hz} \) (figure 6(d)), the GREEN (G) at \( f_G \approx 7600 \text{ Hz} \) (figure 6(e)), the BLUE (B) at \( f_B \approx 7700 \text{ Hz} \) (figure 6(f)), the INDIGO (I) at \( f_I \approx 7850 \text{ Hz} \) (figure 6(g)) and the VIOLET (V) at \( f_V \approx 8000 \text{ Hz} \) (figure 6(h)). These resonant frequencies are estimated by inspecting figures 6(b)–(h) and are highlighted by colored vertical lines. Also note that the R shunt corresponds to patch 14, the O to 8, the Y to 18, the G to 12, the B to 1, the I to 10 and the V to 5. By comparing the results produced by individual shunts, we can see that some of them cause stronger attenuation than others. In our opinion, this is not influenced by the position of the patch and it might be predominantly due to bonding imperfections (which could reduce the effective contact area of some of the patches).

4. The rainbow trap

In this section, we illustrate the result of the simultaneous activation of multiple patches and we provide a demonstration of our tunable electromechanical rainbow trap, analyzing its effects both on the frequency content of the signal and on the shape of the wavepacket.

4.1. Frequency response—bandgap effect

To activate the broadband trap, we short circuit all the patches, we turn on the DC power supplies and then we connect all the shunts one after the other, according to a pre-determined activation sequence. Note that any activation sequence is bound to produce the same net result, due to the linearity of our system; to prove this point, in the SD section, we report the results obtained with a different sequence, which indeed yields the same overall signal attenuation. For completeness,
we report the data recorded at each activation step. The frequency content of the signal recorded at the output accelerometer, for a R–I–O–B–V–G–Y activation sequence, is reported in figure 7. In figures 7(a)–(g), the colored vertical lines indicate the frequencies at which the shunts activated at each step of the sequence are tuned (the lines correspond to the tuning frequencies estimated in figure 6). As we go through the shunting sequence, the energy associated with the wave packet is gradually distilled: each shunt absorbs the energy corresponding to frequencies close to its resonance, until we reach the final attenuation stage shown in figure 7(h).

It is interesting to notice how the activation of some patches causes the frequency content of the signal to be stretched, compressed and deformed, apparently deploying some of the energy from the corrected region to a neighboring frequency interval. For example, this is visible in figure 7(a), where the RED circuit causes frequencies around 7.7 kHz to become more favorable than in the reference case. Figure 7(h) allows to fully appreciate the broadband wave attenuation capability of our rainbow trap; it also confirms how the periodicity of the patch placement and the homogeneity of the shunts characteristics are not required in a transient wave propagation problem involving RL shunts. This is due to the fact that the traveling packet, which has local support, propagates along the beam and is therefore forced to interact sequentially with all the patches, thus feeling the effect of each correction.

Moreover, due to its short wavelength characteristics, the wave has the opportunity to impart significant deformation to each patch. In contrast, in vibration attenuation problems (steady-state conditions), the possibility for the patches to undergo significant levels of strain at the frequencies corresponding to the peaks that we intend to suppress is directly related to the modal characteristics of the structure. In such scenario, the arrangement of patches would need to be carefully designed.

4.2. Time domain considerations—packet distortion

Most of the existing literature on RL shunts focuses on spectral effects and frequency manipulation. In this work, we attempt to provide an additional angle to the problem, by monitoring how resonant shunts can affect the morphological characteristics of a traveling signal. Since our working signal is a chirp, with a frequency content that evolves across the packet, it is interesting to explore how the effects of specific frequency distillations are recorded in different (predictable) zones of the signal (e.g. front or tail) and how they manifest as localized modifications of the packet envelope. In figure 8(a), we report the wavepacket recorded when all the patches are short circuited. In the 0–3 ms interval, we can
identify a first packet, corresponding to the signal traveling away from the source, here termed forward (FWD) packet. The oscillations starting approximately at 3 ms, on the other hand, correspond to the signal reflected by the clamped end of the beam and traveling back towards the source, here termed backward (BWD) packet. We can see that, even without shunting, the FWD packet already features some distortion with respect to the prescribed signal (shown in figure 5(a)), due to the inherent dispersive characteristics of the beam and its bimodal response in the frequency region of interest. In the interest of clarity, we limit the inspection to the FWD portion of the signal, although similar considerations can be made for the reflected wave. In the time interval of interest, we can recognize two distinct zones with modified features. Between 0.8 and 1.8 ms the wave is attenuated, as highlighted by the amplitude reduction of the wave crests. This conforms to the fact that the frequency content of this portion of the packet lives in the neighborhood of the resonance frequency of the RED shunt. In contrast, an increase in the crests amplitude indicates that the wave is amplified in the 1.8–3 ms interval. This observation is consistent with the global picture provided by the spectrum of figure 7(a): while the signal is dynamically damped in the neighborhood of the natural frequency of the resonant shunt, it is locally amplified at frequencies above resonance, which are here located towards the tail of the packet. A complete description of the effects of the other steps in the activation sequence is reported in the SD section. In summary, due to the activation of each shunt, we observe a localization of the amplitude correction effects in the signal which is consistent with the position of the frequencies in the chirp time history. This could open avenues towards the design of structures featuring RL shunts that could be programmed to impart desired morphological characteristics and engineered distortion patterns to the envelope of broadband signals.

Figure 7. Behavior of the rainbow trap in the frequency domain. The trap is activated following the sequence R–I–O–B–V–G–Y. (a)–(g) Results after each of the seven activation steps. The thin gray profiles in the background represent the frequency content of the signal recorded when the patches are all short circuited. (h) Final spectrum of the signal, obtained when all seven shunts are activated.
5. Conclusions

To summarize, in this work we have demonstrated the wave attenuation capabilities of a random array of piezo elements bonded to a waveguide (beam) and shunted with RL circuits. By letting the shunts resonate at adjacent frequencies, we obtained broadband attenuation of a traveling wavepacket. We have revised the concept of ‘organized disorder’ in the context of shunted piezoelectrics, thus adding a layer to the outstanding versatility of shunted piezo elements and their applicability in the design of tunable metamaterials.

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