Weak measurements of a large spin angular splitting of light beam on reflection at the Brewster angle

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Abstract: We reveal a large spin angular splitting of light beam on reflection at the Brewster angle both theoretically and experimentally. A simple weak measurements system manifesting itself for the built-in post-selection technique is proposed to explore this angular splitting. Remarkably, the directions of the spin accumulations can be switched by adjusting the initial handedness of polarization.

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OCIS codes: (240.3695) Linear and nonlinear light scattering from surfaces; (260.5430) Polarization; (240.0240) Optics at surfaces.

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1. Introduction

The spin Hall effect (SHE) of light manifests itself as a transverse spin-dependent splitting, when a spatially confined light beam passes from one material to another with different refractive index \[1,2\]. Recently, the transverse splitting has been reported at an air-prism interface via weak measurements \[3–5\]. On an air-semiconductor interface, the transverse splitting has been detected via ultrafast pump-probe techniques \[6\]. At an air-metal interface, the transverse splitting has been reported for the use of weak measurements and lock-in amplifying methods \[7,8\]. More recently, an in-plane spin-dependent splitting has been observed when a linearly polarized Gaussian beam impinges upon an air-prism interface \[9\]. The spin-dependent splitting is generally believed as a result of an effective spin-orbital coupling known as the influence of the intrinsic spin (polarization) on the trajectory, which produces transverse deflection of the spin. However, among these systems, the spin-dependent splitting is tiny and reaches just a fraction of the wavelength, limiting its future application.

In the present paper, we reveal a large spin angular splitting when a slightly elliptical polarization beam incidents at the Brewster angle. The reflected beam splits into two beams with opposite spin polarizations propagating at different angles. As a result, we can angularly separate the beam with different polarizations. It should be mentioned that the angular splitting is significantly different from that in previous works where the splitting is limited to the light intensity \[10–12\]. As an analogy of SHE in semiconductor microcavity \[13\], the directions of spin accumulations can be switched by adjusting the initial handedness of polarization.

Importantly, this large spin angular splitting is explored with an interesting simple weak measurements system which is similar to that of Ref. \[14\]. They propose that the coupling of a tightly focused optical beam to surface plasmon polaritons offers a natural weak measurements tool with a built-in post-selection. In our work, the combination of the slightly elliptical polarization incident beam with the reflected light beam at the Brewster angle also provides a built-in post-selection weak measurements technique for us. This simple weak measurements method shows significant difference from the previous works in which an additional post-selection is needed \[3,4,15\]. This spin angular splitting is also different from our previous work \[16\] where a linear polarization light beam incidents near the Brewster angle and a large spatial shift is observed. In addition, our work should be distinguished from the angular Imbert-Fedorov shift \[17,18\] whose angular splitting is in the orthogonal plane of incidence.

2. Theoretical analysis

The spin angular splitting is schematically shown in Fig. 1(a). The \(z\)-axis of the laboratory Cartesian frame \((x, y, z)\) is normal to the air-prism interface. We use the coordinate frames \((x_i, y_i, z_i)\) and \((x_r, y_r, z_r)\) to denote incidence and reflection, respectively. A left- or right-elliptically polarized light beam incidents at the air-prism interface. Here, we choose the long and short axis of the elliptical polarization beam along to the \(x_i\)- and \(y_i\)-axis, respectively. The elliptical polarization light beam can be decomposed into two orthogonal polarization components \(H\) and \(V\). It is noted that the mechanism of the elliptical polarization beam reflection on the prism at the Brewster angel acts as a built-in post-selection in which the \(H\) component is
mainly cut off and is equal to the $V$ component. That is to say, this mechanism takes the role of the second polarizer in the precious weak measurements technique. After reflection, the $H$ and $V$ components overlap and induce the large spin angular splitting. Additionally, the reflection coefficient of $H$ polarization component changes its sign across the Brewster angle, which means the induced total circular polarization reverses its handedness [Fig. 1(b) and 1(c)]. In other words, the two reflected beams are "colored" by different circular polarization. Selection of circular polarization in these beams is the post-selection procedure in the weak-measurement technique.

We theoretically analyze the spin angular splitting with a general beam propagation model. The reflected field $\tilde{E}_r$ is related to the incident angular spectrum $\tilde{E}_i$ by means of the relation

$$
\begin{bmatrix}
\tilde{E}^H_r \\
\tilde{E}^V_r
\end{bmatrix} = \begin{bmatrix}
r_p & k_0 (r_p + r_s) \cot \theta \\
k_0 (r_p + r_s) \cot \theta & k_0 \\
\end{bmatrix}^{-1} \begin{bmatrix}
\tilde{E}^H_i \\
\tilde{E}^V_i
\end{bmatrix}.
$$

Here, $H$ and $V$ represent horizontal and vertical polarization components, respectively. $\theta$ is the incident angle, $r_p$ and $r_s$ denote the Fresnel reflection coefficients for parallel and perpendicular polarizations, respectively. $k_0$ is the wave number in free space.

In the spin basis set, the incident angular spectrum for $H$ and $V$ polarizations can be written as: $\tilde{E}^H_i = (\tilde{E}_{i+} + \tilde{E}_{i-})/\sqrt{2}$ and $\tilde{E}^V_i = i(\tilde{E}_{i+} - \tilde{E}_{i-})/\sqrt{2}$. Here, $\tilde{E}_{i+} = (e_{ix} + i e_{iy})\tilde{E}_i/\sqrt{2}$ and $\tilde{E}_{i-} = (e_{ix} - i e_{iy})\tilde{E}_i/\sqrt{2}$ denote the left- and right-circular polarized (spin) components, respectively. We consider the incident beam with a Gaussian distribution and its angular spectrum can be written as

$$
\tilde{E}_i = \frac{w_0}{\sqrt{2\pi}} \exp\left[-\frac{w_0^2(k_{ix}^2 + k_{iy}^2)}{4}\right],
$$

where $w_0$ is the beam waist. The reflected angular spectrum can be obtained from Eq. (1). In the spin basis, $\tilde{E}^H_r = (\tilde{E}_{r+} + \tilde{E}_{r-})/\sqrt{2}$, $\tilde{E}^V_r = i(\tilde{E}_{r+} - \tilde{E}_{r-})/\sqrt{2}$. Here, $\tilde{E}_{r+} = (e_{rx} + i e_{ry})\tilde{E}_r/\sqrt{2}$ and $\tilde{E}_{r-} = (e_{rx} - i e_{ry})\tilde{E}_r/\sqrt{2}$, where $\tilde{E}_r$ can be obtained from the boundary conditions: $k_{ix} = -k_{rx}$ and $k_{iy} = k_{ry}$.
As for the elliptical polarization incident light beam, the Jones vector can be written as \((\cos \Delta, e^{i \phi} \sin \Delta)^T\). Here \(\Delta\) represents the azimuth angle (the angle between the crystal axis of wave plate and the \(x_i\)-axis) and \(\phi\) denotes the phase difference between the two polarization components \(H\) and \(V\). In the present study, we consider a elliptical polarization beam with its long and short axis along to the \(x_i\) and \(y_i\)-axis. Therefore the Jones vector will be simplified to \((\cos \Delta, \pm i \sin \Delta)^T\), representing the left- or right-elliptical polarization in the case of angle \(\phi = \pm \pi/2\). And we note here that the azimuth angle \(\Delta\) mentioned in the following is a tiny value allowing for a slightly elliptical polarization and its long axis along to the \(x_i\)-axis.

We firstly take left-elliptical polarization incident light beam as an example and the right-elliptical polarization can be obtained in the similar way. The Jones vector of the left-elliptical polarization is \((\cos \Delta, i \sin \Delta)^T\). Therefore, according to Eqs. (1) and (2), we can obtain the reflected angular spectrum:

\[
\tilde{E}_r = \frac{r_p \cos \Delta}{\sqrt{2}} \left[ (1 + i \tan \Delta \delta_{ry} + \eta) \tilde{E}_{r+} + (1 + i \tan \Delta \delta_{ry} - \eta) \tilde{E}_{r-} \right].
\]

Here, \(\delta_{ry} = (1 + r_s/r_p) \cot \theta_i/k_0\) and \(\eta = ik_{ry} \delta_{ry} + r_s \tan \Delta/r_p\). At any given plane \(z_r = \text{const.}\), the transverse displacement of field centroid compared to the geometrical-optics prediction is given by

\[
\delta_{\pm} = \int \int \tilde{\xi}_{\pm} \tilde{\xi}_{r_{\pm}}^* \frac{d k_{rx}}{k_{0}} dk_{ry},
\]

where \(\tilde{\xi}_{\pm} = r_p \cos \Delta (1 + i \tan \Delta \delta_{ry} \pm \eta) \tilde{E}_{r\pm}\). We note that there needs a theoretical correction and the higher-order terms should be taken into account when the beam is incident near the Brewster angle [16]. By making use of a Taylor series expansion based on the arbitrary angular

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![Fig. 2. (a) Experimental setup for characterizing the large spin angular splitting on reflection at the Brewster angle. The light source is a 17mW linearly polarized He-Ne laser at 632.8nm (Thorlabs HRP170); Prism with refractive index \(n = 1.515\) (BK7 at 632.8nm); Lens, lens with effective focal length: 50mm; HWP, half-wave plate (for adjusting the intensity); QWP1 and QWP2, quarter-wave plates; GLP1 and GLP2, Glan Laser polarizers; Here, QWP2 together with GLP2 allow for measuring the Stokes parameter \(S_3\); K, knife edge (The purpose of the knife is to produce a single spin accumulation so that only one spin component can be detected in the CCD); CCD, charge-coupled device (Coherent LaserCam HR). The inset: The incident beam is preselected in the left- or right-elliptical polarization state by GLP1 whose optical axis make angles \(\Delta\) or \((-\Delta)\) with \(x_i\)-axis. Here, we choose \(\Delta=0.5^\circ\).](image-url)
spectrum component, \( r_p \) and \( r_s \) can be expanded as a polynomial of \( k_{ix} \):

\[
r_{p,s}(k_{ix}) = r_{p,s}(k_{ix} = 0) + k_{ix} \left[ \frac{\partial r_{p,s}(k_{ix})}{\partial k_{ix}} \right]_{k_{ix}=0} + \sum_{j=2}^{N} \frac{k_{ix}^j}{j!} \left[ \frac{\partial^j r_{p,s}(k_{ix})}{\partial k_{ix}^j} \right]_{k_{ix}=0}.
\] (5)

Using this method, we can obtain the theoretical shift of the single circular polarization component induced by angular splitting in the case of left-elliptical polarization:

\[
\delta_{\pm} = \pm \frac{2z_r r_s \frac{\partial r_p}{\partial \theta_i}}{k_0 R} \left[ (k_0 R + \csc^2 \theta_i - 1) \sin 2\Delta + \csc^2 \theta_i - 1 \right] \cos 2\Delta - \left[ 2k_0 R r_s^2 + \left( \frac{\partial r_p}{\partial \theta_i} \right)^2 \right] (k_0 R + \csc^2 \theta_i + \cot^2 \theta_i \sin 2\Delta - 1).
\] (6)

Here \( R = k_0 w_0^2 / 2 \), \( w_0 \) is the beam waist and \( z_r \) is the propagation distance. It should be noted that the reflected light beam will experience a spatial shift in the case of linear polarization and an angular displacement according to the elliptical polarization. In this work, we only consider the elliptical polarization in which the large spin angular splitting is explored.

3. Weak measurements system

Next, we focus our attention on the experiment. Figure 2 illustrates the experimental setup. A Gaussian beam generated by a He-Ne laser is preselected as a slightly elliptical polarization state by GLP1 and QWP1. By choosing the focal length of lens, we can obtain the desired beam waist in reflection. When the beam impinges onto the prism interface, the reflected beam is angularly separated into two opposite spin components. The prism is mounted to a rotation stage allowing for precise control of the incidence at the Brewster angle. It should be noted that the reflected mechanism discussed above can be seen as a built-in post-selection amplified technique in which the angular splitting is significantly amplified. To detect the angular splitting, a knife edge is applied to achieve a distribution of single spin component. We use a CCD to measure the centroid of the spin accumulation after the knife edge. Using additional QWP2 and GLP2, we can measure the Stokes parameter \( S_3 \) which reveals the circular polarization state of the angular splitting [13].

![Fig. 3. (a) Presection and postselection of polarizations give rise to an amplified spin angular splitting. (b) Theoretical and experimental results of two spin components induced by the spin angular splitting for beam waist \( w_0 = 18.66 \mu m \). We measured the \( z_r \) from the beam waist. The incident light beam is left-elliptical polarization for \( \Delta = 0.5^\circ \) from the \( x_i \)-axis and incidents at the Brewster angle \( \theta_i = 56.57^\circ \).](image)
The amplifying mechanism of spin angular splitting is schematically shown in Fig. 3(a). The incident beam is preselected in the elliptical polarization, and then postselected in the circular polarization state when it reflects on the prism at the Brewster angle. In our measurement, we first choose the lens with focal length $f = 50\,\text{mm}$ to generate beam waist $w_0 = 18.66\,\mu\text{m}$ and make the incident light beam at the Brewster angle by modulating the GLP1 along to the $x_i$-axis. Then we select the incident light beam as slightly left-elliptical polarization by modulating the $\Delta = 0.5^\circ$ from the $x_i$-axis. Limited by the large holders of the knife edge and diaphragm, the angular splitting at small propagation distance are not measured. We measure the displacements every 10mm from 150mm to 300mm [Fig. 3(b)]. The detected splitting value reaches about $1500\,\mu\text{m}$ at the plane of $z_r = 300\,\text{mm}$. The solid lines represent the theoretical predictions. The experimental results are in good agreement with the theory without using parameter fit.

To obtain a clear physical picture, it is necessary to analyze the polarization distribution of the reflected light beam. The Stokes parameter $S_3$ is introduced to describe the circular polarization state of the spin angular splitting. Here, $S_3 = +1$ or -1 represents the left- or right-circular polarization. Figure 4(a) and 4(b) illustrate the theoretical polarization distribution of reflected light beam considering the different left- and right-elliptical polarization incident beam. We can clearly see that, with the incident beam elliptical polarization state changing from left (right) to right (left), the spin angular splitting will reverse the direction. As an analogy of SHE in electronic system [13], the directions of spin accumulations can be switched by the initial handedness of polarization.

We also carry out another experiment to measure the polarization distribution described by Stokes parameter $S_3$. The experimental setup is similar to the first experiment. A new Glan laser polarizer (GLP2) and a new quarter-wave plate (QWP2) are added behind the prism. The last Glan laser polarizer, quarter-wave plate and CCD establish a general experimental system for measuring polarization distribution. By rotating the GLP2 to two angles and holding the QWP2 along to the $y_r$-axis, we can conclude the Stokes parameter $S_3$ from the intensity distributions on CCD. The rotation angles are $45^\circ$ and $135^\circ$, the deviations from the configuration of the
The experimental results shown in Fig. 4(c) and 4(d) are in good agreement with the theoretical calculation.

4. Conclusions

In conclusion, we have revealed a large spin angular splitting on reflection at the Brewster angle. The detected splitting reaches about 1500 µm at z_r = 300 mm. As an analogy of SHE in semiconductor microcavity [13], we are able to switch the directions of the spin accumulations by adjusting the initial handedness of spin states. This phenomenon can be interpreted from the inversion of horizontal electric field vector across the Brewster angle. Importantly, we propose a simple weak measurements system offering an interesting built-in post-selection technique to explore this angular splitting, which will provide us a new method on weak measurements technique.

Acknowledgments

We are sincerely grateful to the anonymous referees, whose comments have led to a significant improvement of our paper. This research was partially supported by the National Natural Science Foundation of China (Grants Nos. 61025024 and 11074068).