Can HI gas trace the matter density distribution linearly on large scales?

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It is customary to assume that HI gas traces the matter density distribution linearly on large scales. However, nonlinear effects which modulate HI gas in halos might spoil this. We employ three approaches to generate the mock HI density from an N-body simulation at low redshifts, and demonstrate, for the first time, that the HI linearity assumption breaks down at $k > 0.1 \, h \, Mpc^{-1}$, generically, except for a “sweet-spot” redshift near $z=1.2$. In addition, the HI bias scales approximately linearly with redshift for $z \lesssim 3$. Our findings are independent of models.

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Introduction.— Neutral hydrogen (HI) atoms, which are expected to be contained in halos at low redshifts ($0.5 \lesssim z \lesssim 3$), produce 21-cm line radiation that can be observed [1]. The 21-cm intensity mapping experiments, e.g., Tianlai [2], CHIME [3], HIRAX [4], BINGO [5], and SKA [6], which survey the HI mass distribution in very large volumes, provide a promising way to constrain the expansion history and structure formation in the Universe, thereby unveiling the nature of dark energy.

These 21-cm intensity mapping experiments, despite low angular resolutions, can be used to detect large-scale features in the cosmological density field, particularly the baryon acoustic oscillations (BAO) [7,8]. The success of this approach, however, depends on how accurate HI gas traces the matter density fluctuations at the BAO scale ($\sim 100 \, h^{-1} \, Mpc$). In general, the power spectrum of the HI gas distribution is related to that of the underlying matter through a bias relation, $P_{\text{HI}}(k) = b_{\text{HI}}^2 P_m(k)$, where $b_{\text{HI}}$ is the bias factor. It is, therefore, necessary to understand the bias factor, $b_{\text{HI}}$, in particular its scale dependence, before one can use $P_{\text{HI}}(k)$ to infer the distribution of mass in the universe.

After cosmic reionization, most HI gas is expected to be in galaxies, thanks to their high density and low temperature, while the neutral fraction in the intergalactic medium is very low, about $10^{-5}$. Furthermore, fluctuations in the ionization field are not expected to affect the HI power spectrum on large scales [4]. Thus, the distribution of the HI gas may be understood in terms of its relation with galaxies, or with dark matter halos in which galaxies reside. Gas and star-formation processes can, in principle, change the HI-gas distribution in dark matter halos, and potentially introduce non-linear bias in the relationship between HI gas and dark matter. The nonlinearity, albeit at small scales (i.e., the size of halos), might spoil the HI linearity assumption even on large scales, because of mode coupling on different scales.

Previous studies of HI bias either employed oversimplified HI-halo mass relation (similar to the fitting formula in [10]) applied to N-body simulations [11,14], or modelled the HI gas using hydrodynamic simulations, such as IllustrisTNG [13]. However, the volumes of gas simulations, typically $\lesssim (100 \, h^{-1} \, Mpc)^3$, are usually too small to be valid on BAO scales.

Given its importance, here we study the relationship between HI-gas and dark matter on large scales, using three – empirically, numerically, and observationally oriented, respectively – approaches to model HI gas in halos of different masses, and using halos in a large N-body simulation to construct the HI-gas distribution on large scales. Our simulation volume, $(500 \, h^{-1} \, Mpc)^3$, is sufficiently large so that the finite box effect on the power spectrum and bias is negligible on BAO scales [10]. The use of different models for HI-gas in dark matter halos also allows us to draw generic conclusions that are independent of our ignorance about the details of galaxy formation in dark matter halos.

Mocking the HI gas distribution.— Our HI mock data is constructed from the results of a large-scale, high-resolution N-body simulation “ELUCID” [17] of the $\Lambda$CDM universe (performed with the L-Gadget code, a memory-optimized version of Gadget-2 [18]) in a comoving volume of $500 \, h^{-1} \, Mpc$ on each side using $3072^3$ particles. We refer the readers to Ref. [17] for details of this.
To find halos, we use the FoF algorithm with a linking length of 0.2 times the mean particle separation. The SUBFIND algorithm [13] is employed to resolve the sub-structures (i.e. subhalos) in each FoF halo and to build the merger trees. We adopt an empirical model [21] to construct the star formation histories of galaxies in those halos with masses above $10^{10} h^{-1} M_\odot$ (about 30 N-body particles). To fully trace the star formation history, we develop a Monte Carlo method to append unresolved progenitors to the leaf-halos of the merger tree [21]. The HI gas is then assigned to halos with masses above $10^{10} h^{-1} M_\odot$ using a star formation model [22, 27] that provides the full information about the star formation history. Finally, the HI gas is smoothed onto grids to compute the HI power spectrum. The key components of our method are detailed below. The background cosmology is consistent with that given by the WMAP five-year data [21]: $\Omega_m = 0.258, \Omega_\Lambda = 0.742, \Omega_b = 0.044$, $h = 0.72, n_s = 1.0$ and $\sigma_8 = 0.8$.

(i) Star formation history. For resolved halos with $M_h \geq 10^{10} h^{-1} M_\odot$, we follow the empirical model for star formation rate ("SFR") as described in Ref. [20] (their "Model III"). The SFR of a central galaxy is assumed to depend only on the mass of its host halo, $M_h$, and redshift $z$, $\text{SFR}(M_h, z) = \varepsilon f_h M_h / \tau (X + 1) \alpha [(X + R)/(X + 1)]^\beta (X/(X + R))^{\gamma}$. Here $\varepsilon$ is the overall efficiency, $f_h = \Omega_h/\Omega_m$ is the cosmic baryon fraction, $\tau = 1/(10 H_0) (1 + z)^{-3/2}$ describes the dynamical timescale of halos at a redshift $z$, the variable $X \equiv M_h / M_c$ where $M_c$ is a characteristic mass scale. Other variables are parametrized as $\alpha = \alpha_0 (1 + z)^{\alpha'}$, and $\gamma = \gamma_0$ if $z < z_c$, or, otherwise, $\gamma = (\gamma_a - \gamma_b) (z + 1)^{\gamma'} + \gamma_b$. The free parameters ($\varepsilon, R, M_c, \alpha_0, \alpha', \beta, \gamma_0, \gamma_a, \gamma_b, \tau, \gamma_c$) can be found by fitting the observed galaxy stellar mass functions and a composite local cluster conditional galaxy luminosity function at the $z$-band, as shown in Ref. [20] (their Table 3). For unresolved halos with $M_h < 10^{10} h^{-1} M_\odot$, Monte Carlo trees are adopted to extend their assembly histories down to $10^9 h^{-1} M_\odot$ [21].

This model [20] assumes that, during galaxy mergers, the SFR is under exponential decay in satellite galaxies where the gas can be stripped. As such, the HI gas is dominated by the contributions from central galaxies. While this may not be true for big halos (cf. [13]), we neglect the HI gas from satellite galaxies, for simplicity.

With empirical star formation and merger models, we can trace the mass growth of each central galaxy from its merger tree, and obtain its stellar mass $M_*$. For a given halos mass, the stellar mass $M_*$ may not be the same in different halos because of their different merger histories.

(ii) Star formation model. To connect the surface density of SFR $\Sigma_s$ and that of gas mass $\Sigma_g$, we follow the star formation model developed in Ref. [22, 27], in which $\Sigma_s = f_{H_2} \epsilon_g \Sigma_g / t_g$, where $\epsilon_g = 0.01$, $t_g = 31 [\Sigma_g / (M_\odot pc^{-2})]^{-0.25}$ Myr. Assuming that the gas is cold and comprised of H$_2$ and HI, the H$_2$ fraction is given by $f_{H_2} = 1 - \frac{3}{5} \left(\frac{\Sigma_g}{\Sigma_{HI}}\right)$ if $s \leq 2$, or otherwise 0. The variable $s = \ln(1 + 0.61 x + 0.01 y^2)/(0.67 x')$, where $\tau = 320 c Z_0 \Sigma_g / (g cm^{-2})$, and the clumping factor $c = 1.0$. To estimate the gas phase metallicity relative to the solar one, $Z_0$, we adopt the average metallicity-stellar mass relation from the FIRE simulation [27], $\log Z_0 = 0.35 \log (M_*/M_\odot) - 10 + 0.93 \exp(-0.43 z) - 0.74$. The radiation field parameter $\chi$ is estimated as

$\chi = 72 G_0 / n_{\text{CNM}}$, where $G_0 = \Sigma_s / \Sigma_{HI} = 2.5 \times 10^{-3} M_\odot pc^{-2} Myr^{-1}$, and $n_{\text{CNM}}$ is the density of cold neutral medium (CNM) in $cm^{-3}$. In molecular-rich regions, the CNM density is $n_{\text{CNM,hydro}} \approx \Sigma_g / (M_\odot pc^{-2})$, while in molecular-rich regions, the CNM density is $n_{\text{CNM,2p}} = 72 G_0 / (10^{3.41} M_\odot pc^{-2})$. In general, $n_{\text{CNM}} = \max(n_{\text{CNM,2p}}, n_{\text{CNM,hydro}})$.

(iii) Disk size. We assume the gas surface density follows an exponential profile: $\Sigma_g(r) = \Sigma_0 e^{-r/R_d}$. We assume the gas disk to stellar disk size ratio $R_d/R_*= 3.3$ which fits best with the gas mass fraction in local galaxies [28] (c.f. $R_d/R_* = 2.6$ in [24]). The stellar disk size at $z \approx 0.1$ is estimated as $R_* (M_*) = R_0 (M_*/M_\odot)^{m} [1 + (M_*/M_\odot)^{m}]^{(b-c)/m}$, where $c = 0.18$, $b = 0.52$, $\gamma = 1.8$, $R_0 = 10^{0.72} kpc, M_0 = 10^{10.44} M_\odot$. The disk size evolves with redshift as $R_*(z, M_*) = R_*(M_*) [(1 + z)/1.1]^{-0.44}$.

(iv) HI halo mass relation. For a fixed stellar mass $M_*$, a given value of disk central density $\Sigma_0$ determines $\Sigma_g(r)$ at some radius in the disk. The aforementioned star formation model is employed to solve for $\Sigma_*(r)$ numerically from $\Sigma_g(r)$, which gives the HI surface density $\Sigma_{HI}(r)$. By integrating over the disk, we can find a correlation between the SFR and the HI mass for a central galaxy, given $M_*$. For each halo, we compute the SFR using the aforementioned empirical model, and $M_*$ from halo merger history. Finally, the HI mass is computed by interpolation using its correlation with SFR.

In Figure 1 we show the average HI-halo mass relation for central galaxies at $z = 0$. Our results ("LK model") are compared with predictions from the Illustris-TNG simulation [14] ("TK model") and the results from updated ALFALFA observations [31] ("AH model", only available at $z = 0$). All results agree well for low-mass halos ($M_h < 10^{10} h^{-1} M_\odot$). We checked that this agreement holds well at higher redshifts ($0 < z < 3$) between LK and TK models. For massive halos, nevertheless, our model underestimates the HI mass, for two possible reasons. Firstly, the HI mass in the TK model includes the contributions from both central and satellite galaxies, while both our model and the AH model only consider those from the central galaxies. Secondly, our empirical model might underestimate the SFR for massive halos. However, the contribution of HI gas from massive halos is generally negligible due to the sharp decrease of the halo mass function towards the massive end.

(v) HI Power spectrum. The HI mass in each halo
is smoothed onto a uniform grid with 1024³ cells, and we compute the HI power spectrum from the FFT. We only keep the power spectrum for wavenumber less than a quarter of Nyquist number (k < 1.57h Mpc⁻¹) to avoid the alias effect. The mass-weighted HI shot noise is estimated by shuffling HI gas randomly and then subtracted from the raw power spectrum. The linearity assumption can be tested using the bias \( b_{\text{HI}}(k) = [P_{\text{HI}}(k)/P_m(k)]^{1/2} \).

**Results.**— In Figure 2 we show the HI bias from different HI-halo mass relations at \( z = 0 \) as well as the halo bias. In all three models, the HI bias remains a constant at large scales for \( k \lesssim 0.1 \, h \, \text{Mpc}^{-1} \), i.e. we confirm that, generically, HI gas is indeed a linear biased tracer at the first BAO peak. However, the linearity assumption begins to break down at the second BAO peak. To test whether this break-down scale relies on the halo resolution in our simulation, we vary the minimum halo mass from \( 10^{10}\, h^{-1} \, M_\odot \) to \( 10^{11}\, h^{-1} \, M_\odot \), and find that while the amplitude of HI bias depends on the halo mass cutoff similar to that of the halo bias, the linearity break-down scale is almost unchanged. Also, to test the effect of satellite galaxies, we estimate the HI mass from satellite galaxies, which are assigned at the center of subhalos, using the LK model at \( z = 0 \), and find that the shape of the HI power spectrum cannot be changed by satellite galaxies on large scales until \( k \gtrsim 1 \, h \, \text{Mpc}^{-1} \).

In Figure 3 we find that, for both LK and TK models, while the HI density at small scales is suppressed with respect to the linear bias near \( z = 0 \), the HI bias becomes enhanced at higher redshifts. The competition between two opposite effects results in the evolution of HI bias at small scales. HI gas can be held only inside halos after cosmic reionization, and halo mass density fluctuations are always enhanced at small scales (before corrected for shot noise). On the other hand, HI mass is suppressed (i.e. \( dM_{\text{HI}}/dM_h \) turns small) in large halos, which decreases the HI density fluctuations at small scales with respect to the level of fluctuations caused by halos. The HI suppression effect is stronger at low redshifts because more massive halos are formed. When these two effects are balanced out, there exists a transition time when the HI bias is constant and linear down to some small scales. In the LK model, we find that this “sweet-spot” redshift is near \( z = 1.2 \) with the linearity break-down at \( k \approx 0.5 \, h \, \text{Mpc}^{-1} \). In the TK model, the transition takes place at \( z \approx 1 \). As such, the “sweet spot” redshift may contain the information of the HI-halo mass relation.

The linear HI bias (i.e. the constant HI bias averaged over large scales) increases with redshift, as shown in Figure 4. We find an interesting feature in LK and TK models that, generically, the HI bias varies *approximately* linearly with redshift. This linear relation is almost exact between \( z = 1 \) and 2, with error < 10% for \( z < 1 \) and error < 15% for \( 2 < z < 3 \). This can be understood as follows. The linear HI bias can be written as \( b_{\text{HI},\text{linear}}(z) = [D_{\text{HI}}(z)/D_m(z)] b_{\text{HI},\text{linear}}(1) \), where \( D_{\text{HI}} \) and \( D_m \) are the linear growth functions of the HI and matter density fluctuations, respectively, i.e. \( D_{\text{HI}}(z) = [P_{\text{HI}}(z)/P_{\text{HI}}(1)]^{1/2} \) and \( D_m(z) = [P_m(z)/P_m(1)]^{1/2} \). As shown in the insets of Figure 4, the HI density power spectrum varies slightly with redshift, i.e. \( D_{\text{HI}}(z) \approx 1 \), which is consistent with the similar evolution of cosmic HI abundance with redshift [13]. On the other hand, in a matter-dominated universe, the matter growth function scales

![Figure 1](https://via.placeholder.com/150)

**FIG. 1.** The average HI-halo mass relation derived from different models at \( z = 0 \). Our results using the empirical SFR model [21] and the star formation model [22] are dubbed “LK model” (blue), which stands for “Lu et al. + Krumholz et al. model”. The results using IllustrisTNG simulation (their gas data) [12] and the same star formation model are dubbed “TK model” (magenta), which stands for “IllustrisTNG + Krumholz et al. model”. The results using updated measurements of ALFALFA survey and HOD model [31] with error bars are dubbed “AH model” (green), which stands for “ALFALFA data + HOD model”.

![Figure 2](https://via.placeholder.com/150)

**FIG. 2.** The bias \( b(k) = \sqrt{P(k)/P_m(k)} \) of halo mass density fluctuations (red) and HI mass density fluctuations derived from the LK (blue), TK (magenta), and AH (green) models at \( z = 0 \) with respect to the matter density fluctuations, with shot-noised corrected (thick solid lines) and uncorrected (thin solid lines). The dashed lines indicate the constant linear bias. The dot-dashed vertical lines mark the wavenumbers of the first (black) and second (grey) BAO peaks.
as $D_m(z) \propto (1 + z)^{-1}$. Altogether these two effects lead to the linear scaling relation $b_{\text{HI, linear}}(z) \propto (1 + z)$, which we find is generic. There are two reasons why this relation is not exactly linear. Firstly, at low redshifts of $z < 1$, $D_m(z)$ is suppressed when dark energy kicks in. Secondly, the HI power spectrum has small, non-monotonic, evolution with redshift. For diagnostic purpose, we consider a case in which $D_H(z) = 1$ exactly, but $D_m(z)$ takes the value from linear perturbation theory (including the effect of dark energy). We found that the prediction of linear HI bias in this case agrees with the results in both models. More precisely, the former can be slightly higher than the latter with < 15% error. This is consistent with the fact that the HI power spectrum reaches its maximum at $z \approx 1 - 2$, with the values at $z = 0$ and 3 about 20% lower than the maximum.

Other than those generic results above, however, the value of the linear HI bias can be model-dependent. Figs. 3 and 4 show that generally the TK model predicts a higher value of linear HI bias than the LK and AH models. This difference might be attributed to the contributions of HI abundance from massive halos.

**Conclusions.**—In this Letter, we test directly from an N-body simulation the validity of the HI bias linearity assumption at the BAO scale for 21-cm intensity mapping experiments at low redshifts. The LK, TK, and AH models represent empirically, numerically, and observationally oriented approaches, respectively, for the estimation of HI mass. They predict different HI-halo mass relations. However, our findings are generic, as follows.

We confirm that density fluctuations of HI gas inside halos are a linear biased tracer of total dark matter density fluctuations on large scales, generically, at $k \lesssim 0.1 \text{ h Mpc}^{-1}$. It is unbiased, in principle, to measure the first BAO peak with upcoming 21-cm intensity mapping surveys.

We find an interesting transition between HI bias being suppressed and enhanced at small scales. At this “sweet spot”, the HI bias is linear down to small scales, e.g. with the linearity break-down scale $k \approx 0.5 \text{ h Mpc}^{-1}$ at $z \approx 1.2$. This may have twofold impacts. On one hand, a much larger number of modes down to small scales can be taken to put more stringent constraints on cosmological parameters. On the other hand, the exact redshift of this epoch might be sensitive to the HI-halo mass relation. Therefore, pinning down the sweet spot redshift observationally may place constraints on the star formation model.

We also find that the linear HI bias is an approximately linear function of redshift for $z \leq 3$. This may make cross-checks between different redshifts more useful for calibration and foreground removal.
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