Sterile neutrinos as a solution to all neutrino anomalies *

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Abstract

The sterile neutrino solutions to different irregular results and observations in neutrino physics are studied. It is pointed out that introducing sterile neutrinos helps to solve simultaneously the observed anomalies. It is argued that sterile neutrinos can solve the conflict between dark matter neutrinos, LSND result and supernova nucleosynthesis. Other supernova constraints for sterile neutrinos are revised. Possibilities to avoid the big bang nucleosynthesis constraints for sterile neutrinos are explored. It is claimed that sterile neutrinos can solve the crisis in big bang nucleosynthesis. It is pointed out that sterile neutrinos can provide a consistent explanation to the anomalies observed at Karmen. It is claimed that the conversions to sterile states are consistent solutions to both the solar and the atmospheric neutrino problems, and cannot be ruled out by cosmological or astrophysical arguments. Models for the masses and the interactions of sterile neutrinos are reviewed.

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I. INTRODUCTION

Sterile neutrinos or neutrino-like objects are predicted in many grand unified theories. Supersymmetric models always involve many new neutral fermions. Although many of the new neutral fermions are not generally called neutrinos, being associated with some other particles, in many occasions they behave as sterile neutrinos, and may even mix with the known neutrinos. Here generically any fermion without standard model interactions mixing with the ordinary neutrinos is called a sterile neutrino.

I focus on sterile neutrinos that are light, i.e. have masses in the same scale as is allowed and expected for the known neutrinos ($m_{\nu_S} < 100$ MeV). There are several plausible ways to generate appropriate mass matrices, basically the small sterile neutrino mass could be due to similar mechanisms as that of the ordinary neutrinos, like see-saw or radiative mechanisms.

The purpose of this work is to find out whether the present astronomical and terrestrial observations allow, or even require, the existence of light sterile neutrinos. I will explore the sterile neutrino solutions to several irregular results, especially I will investigate whether the introduction of sterile neutrinos with appropriate properties helps to build consistent scenarios solving simultaneously all or most of the observed anomalies.

Astrophysical constraints for the properties of sterile neutrinos are reviewed, and their validity is discussed. I present some new limits for neutrino interactions involving sterile neutrinos, and revise some old. The ways to avoid the constraints are explored, to gain more parameter space for certain solutions.

It is argued that sterile neutrino solutions to both the atmospheric neutrino problem and the solar neutrino problem are consistent with other phenomena, for all plausible parameter ranges. Especially it is emphasized that they cannot be ruled out by cosmological arguments.

The anomalous results occupying the neutrino physicists are recalled in section II, and the sterile neutrino solutions to each of them are discussed, as well as the possibilities to solve simultaneously several anomalies, using the sterile neutrino degrees of freedom. The possible types of desired mass hierarchies are presented. It is also pointed out that introducing sterile neutrinos helps to build consistent scenarios accommodating the observations of LSND [1].

In section III several models appropriate for generating light sterile neutrinos are reviewed. Some specific types of mass matrices are investigated more thoroughly. Section IV is devoted to general phenomenological aspects of models involving sterile neutrinos. I list laboratory constraints for sterile neutrinos, and talk about different decay modes of a massive neutrino. Especially it is pointed out that the absence of the GIM mechanism can lead to a substantial decay rate by standard neutral currents.

In section V the role of sterile neutrinos in supernovae is considered. I show a way to avoid the limits for the $\nu_\mu \to \nu_e$ oscillations derived by requiring the supernova to be the origin of heavy elements [2,3]. The limits for the production of sterile neutrinos in the supernova core, by oscillations or by explicit interactions between sterile neutrinos and ordinary matter, are discussed. The limits for the neutrino decay involving sterile states are explored.

The section VI concentrates on the effects of the sterile neutrinos in the early universe. The possibilities to avoid the nucleosynthesis constraints [4–8] for the excitation of sterile states by oscillation or explicit interactions are discussed. However, at present the standard particle physics model gives such a poor fit to the nucleosynthesis model and the observations
of the light elements \[\] that it is questionable to use cosmological arguments to rule out any non-standard scenario. It is pointed out that decaying sterile neutrinos could save the nucleosynthesis. The role of sterile neutrinos as the dark matter of the universe is discussed at the end of this section.

Section \[\text{VII}\] treats the anomalies observed in Karmen \[\text{\[\text{[10]}\]}\]. It is shown that the sterile neutrino solution is not inconsistent with the supernova dynamics or the big bang nucleosynthesis. A new range of mixings is suggested, in deviation from \[\text{\[\text{[14]}\]}\].

Finally, in section \[\text{\[\text{[VIII]}\]}\] the results are summarized, and some other implications are discussed.

II. STERILE NEUTRINOS AND IRREGULAR RESULTS OF NEUTRINO PHYSICS

A. Anomalous observations

There has been, there is, and there will be several anomalies related to the neutrino physics. In the following I list some of them, and discuss briefly the possibility to solve them individually by sterile neutrinos.

1. Solar neutrino problem. All the four on-going solar neutrino experiments \[\text{\[\text{[12–15]}\]}\] have measured a deviation from the theoretical neutrino flux. Beside the canonical solutions it has been suggested that the deficit is due to the conversion of the emitted electron neutrinos to a sterile state, either by vacuum oscillation, resonant neutrino conversion \[\text{\[\text{[16]}\]}\], or spin flip \[\text{\[\text{[17]}\]}\]. The vacuum oscillation solution is not consistent with the recent data \[\text{\[\text{[18]}\]}\] while the resonant conversion solution can be fitted reasonably well \[\text{\[\text{[18,21,22]}\]}\] (See also \[\text{\[\text{[18,21,22]}\]}\]). The typical solutions require squared mass differences \(\left(4 \times 10^{-6} \ldots 10^{-5}\right) \text{eV}^2\), and mixing \(\sin^2 \theta \sim 4 \times 10^{-4} \ldots 4 \times 10^{-3}\). The spin flip solution also fits well \[\text{\[\text{[23]}\]}\], but it would require magnetic fields higher than usually believed to exist in the sun. Ignoring the results of one experiment increases the parameter range.

2. Atmospheric neutrino problem. The deficit of muon neutrinos produced in atmosphere has also been observed by several experiments \[\text{\[\text{[24–28]}\]}\]. The oscillation to sterile neutrinos has been generally considered as an inconsistent solution to this anomaly, because of the apparent conflict with the neutrino mixing bounds from the physics of the early universe \[\text{\[\text{[29]}\]}\]. Ignoring those bounds, the sterile neutrino solution would give as good a fit as the oscillation to tau neutrinos \[\text{\[\text{[30]}\]}\]. The masses considered are in the range \(10^{-3} \text{eV}^2 < \delta m^2 < 0.5 \text{eV}^2\), while the mixing should be quite large, \(\sin^2 \theta > 0.5\). The most favored region is close to maximal mixing with \(10^{-3} \text{eV}^2 < \delta m^2 < 3 \times 10^{-3} \text{eV}^2\).

3. Los Alamos neutrinos. The Liquid Scintillator Neutrino Detector (LSND) experiment has observed an excess of electron neutrinos, in a beam originally consisting of muon neutrinos \[\text{\[\text{[1]}\]}\]. The claimed signal is consistent with the oscillation of muon neutrinos to electron neutrinos, but the results are still too controversial to be conclusive. If it were a signal of neutrino mixing, it would imply squared mass differences from
0.2 eV$^2$ to 10 eV$^2$ and above, and a mixing $10^{-3} < \sin^2 2\theta < 0.04$, depending on the mass scale. For a wide range of parameters the results are consistent with other experiments (Karmen, BNL E776, Bugey) which, however, exclude a large part of the parameter space. Contrary to some previous reports of positive results for a neutrino mass from other experiments, the speculated mass scale is very natural, and even expected. Especially, the results allow neutrino masses in a cosmologically important scale. Hence it is well motivated to study the consequences of such masses, whether or not the preliminary results will be confirmed.

The Los Alamos results cannot be interpreted as a signal of sterile neutrinos. However, taken that the results are incompatible (or marginally compatible) with the solar neutrino and atmospheric neutrino solutions, a simultaneous solution to all three cases would require a fourth, necessarily sterile neutrino.

4. * Supernova neutrinos.* The supernova SN1987A did not show any anomalies with neutrinos, instead it gave us a possibility to gather information on the properties of neutrinos, especially giving constraints for the sterile neutrinos themselves. These constraints may restrict the sterile neutrino solutions to some anomalies.

The problem of the supernovae is that it is difficult to make them blow up: most simulations end up with a stalled explosion clearly contrary to all observations. The present status of modelling favors the scenarios where the neutrino radiation helps to blow up the envelope. It has been suggested that successive conversions to a sterile state and then back to active state would help to bring sufficiently energy to the outer regions to throw them away [31,32].

5. * Dark matter.* Neutrinos have been always considered as the main candidates for the invisible matter in the universe, required by several observations. The most popular solution to the formation of the structures in the universe employs a mixed dark matter model, where neutrinos with a mass of a few electronvolts provide some 30% of the energy density of the universe, the rest consisting mainly of heavier particles. Some simulations give the best fit with two neutrinos with a mass of 2.4 eV [33].

It has been speculated that some part of the dark matter would consist of sterile neutrinos [34,35]. It is noted that sterile neutrinos may behave differently than active neutrinos with the same mass, since their average energies may be different.

As an alternative to the mixed dark matter it has been suggested that one can do with only cold dark matter, if the onset of the structure formation is delayed, as would be caused by a massive particle decaying lately [36,37]. Here it is recalled that sterile neutrinos would either provide a candidate for that lately decaying particle, or provide a decay channel for an active neutrino.

6. * Anomalous ionization of interstellar hydrogen.* It has been suggested [38,39] that a neutrino ($\nu_\mu$ or $\nu_\tau$) with a mass of 27 eV would explain the anomalously large ionization of interstellar hydrogen by decaying to a lighter neutrino and a photon. The scenario is in agreement with the observed constraints for the ultraviolet radiation [40,41], but it may not fit the current understanding of the structure formation of the universe, without additional assumptions on the initial conditions.
The alternative of using sterile neutrinos as the decaying particles \[3, 4\] fits the astronomical observations as well, and is also consistent with the big bang nucleosynthesis. A sterile neutrino of 30 eV is, however, even worse as hot dark matter, but since its abundance may be lower it allows other coexisting particles to form the main part of the dark matter. Alternatively a non thermal production of the sterile states may improve their properties. In \[3, 4\] it was shown that models satisfying all the requirements can be built, in case the photons are emitted from the decay of one sterile state to another. A more economic alternative would be the decay of a sterile state to an active tau neutrino state. Again, models satisfying the primary conditions can be build, even a trivial one loop model may do. In the trivial models, however, the mixing between the tau neutrino and the sterile state would be large (\(\sin^2 \theta_{\tau} \sim 0.2\)), naively conflicting with the limits from the big bang nucleosynthesis.

7. **Karmen anomalies.** The preliminary results of the Karmen experiment display an unexpected time structure of the events \[7\]. These observations have already been interpreted as a signal for a sterile neutrino of 33.9 MeV, with a mixing to muon and electron neutrinos \[11\]. Below I will discuss some scenarios for such a particle, and present a more consistent choice of parameters.

8. **Crisis in Big Bang Nucleosynthesis.** The estimations of the abundances of the light elements in the universe constrain the effective number of the degrees of freedom during the nucleosynthesis. When expressed as a fit to the number of neutrino species the results suggest there being less than three neutrino species \[17, 3\], contrary to the LEP results [Eq. (17)] that state firmly the existence of three neutrino families. Despite possible systematic uncertainties in both the observations and the modelling, the results are quite worrying. Below I will discuss the possibility to improve the fit with sterile neutrinos.

9. **Origin of baryonic asymmetry.** The physics of neutrinos is closely related (see e.g. \[48–51\] and references therein) to the generation of the baryonic asymmetry of the universe. Usually neutrinos do not, however, any good for it, the interesting possibility that the oscillation to sterile neutrinos would be responsible for creating a lepton asymmetry \[52\] is shown to work in the opposite direction, washing out any deviation from the symmetric situation \[53\]. Also, requiring the baryonic asymmetry to survive the electroweak phase transition sets stringent constraints for models of neutrino masses, e.g. forbidding any Majorana masses above 10 eV \[54\]. Since the origin of the baryonic asymmetry is still poorly understood it is premature to draw any definitive conclusions for the properties of neutrinos.

Beside these there have been several irregular results from different beta decay experiments. At present the irregularities are not strong enough to point to new physics. Also many of the above anomalies may be due to erroneous interpretation of the measured data or defects in the theory describing the phenomenon.
B. Reconciling Los Alamos neutrinos with solar and atmospheric neutrinos

With the three left-handed neutrinos one can find a mass matrix solving simultaneously both the solar and the atmospheric neutrino problems. Typically that kind of solutions yield neutrino masses too low to have any cosmological significance. To generate masses in the dark matter scale, while insisting on both the solutions, one necessarily needs fine tuning which can be intelligently incorporated within a specific model. The new mass scale given by the Los Alamos experiment is completely incompatible with solar neutrinos, unless neglecting at least one experiment [55]. The scale is also disfavored by the atmospheric neutrino solutions. However, neglecting the results based on stopping/passing ratio, there could be a marginal possibility of having a muon neutrino of about 0.7 eV, the other two neutrinos being very light, so that the solar neutrino problem would be solved by the conversion from electron neutrino to tau neutrino, the atmospheric neutrino problem by oscillation from muon neutrino to tau neutrino, and the oscillation from muon neutrino to electron neutrino is then responsible for the LSND events. The required mass matrix would be very complicated, and intelligence is required generating it without fine tuning.

For any other mass hierarchy there is no way to fit with three neutrinos the Los Alamos mass scale without abandoning either solar or atmospheric neutrinos.

Sterile neutrinos have been suggested long ago as a solution to this schism. Before the LSND results there were around three possibilities to yield $4 \times 4$ mass matrices consistent with solar neutrino, atmospheric neutrino and hot dark matter requirements:

1. Muon and tau neutrino form a degenerated quasi-Dirac neutrino with mass in the dark matter scale. The atmospheric neutrino problem is solved by the oscillation from muon neutrino to tau neutrino. Electron neutrino is light, with mass less than $10^{-4}$ eV, and the solar neutrino problem is solved by electron neutrino converting to a sterile state with mass about $10^{-3}$ eV.

2. The solar neutrino problem is solved by oscillation from electron neutrinos to tau neutrinos which are light. The atmospheric neutrino deficit is due to muon neutrinos transforming to sterile neutrinos. These neutrinos form a quasi-Dirac neutrino in the dark matter scale.

3. The electron, muon and tau neutrinos are light, the mixings between them solve both the solar and the atmospheric neutrino deficits. The sterile neutrino solely provides the hot dark matter.

Of these, the second possibility is in imminent contradiction with the oscillation limits from the early universe, while the third one is strongly constrained by them.

Of the above three scenarios the first and the second are compatible with the new mass scale suggested by Los Alamos results, the third being disfavored.

The above arguments do not yet require more than one sterile state. However, depending on the model it may be more natural to have three. Moreover, more states may give more freedom to the theory, and, above all, they may help to avoid some constraints that have been put over the combined solutions.
With a second sterile state one gains another option for the transition schemes: The solar neutrino problem is solved by $\nu_e \rightarrow \nu_S$, these being light. The atmospheric neutrino problem is then due to $\nu_\mu \rightarrow \nu'_S$, these particles being quite degenerated, in the hot dark matter or LSND mass scale. The tau neutrino, and a possible third sterile state are left free, and can be used to avoid some conflicts with other phenomena.

III. NEUTRINO Mass Matrices WITH Sterile Neutrinos

A. Generalities

The approach to study light sterile neutrinos disagrees with the canonical view of the singlet states being very heavy. Although the masses of all neutral singlet states are arbitrary and independent in the SU(2)$\times$U(1) gauge model, it is more natural to expect them to be of some characteristic scale of the underlying theory. The lightness of the ordinary neutrinos, however, gives already evidence that very disparate mass scales can coexist, and we can expect the sterile neutrinos to be light for similar reasons as the ordinary neutrinos.

Apart from inconceivably small Yukawa couplings, there are three possibilities to generate naturally small neutrino masses, all thoroughly explored for the active sector: the see-saw mechanism, radiative mechanisms and a low scale vacuum expectation value for the higgs boson generating the masses. (See [56] for a recent review.) All these can be applied almost identically to the sterile sector, but we also want to generate a mixing between sterile and active neutrinos.

It is also possible that the final neutrino mass matrix is due to several different mechanisms. Such a hybrid model may include more degrees of freedom, allowing one to generate more or less naturally any kind of mass scenarios. Due to a large variety of possible neutrino mass matrices, in the following sections we take the neutrino masses and mixings as free independent parameters, with no region being theoretically excluded.

The number of sterile neutrino species is generally free. The most obvious choice is to assume that there is one per family, but some scenarios may require them two. It is also possible that sterile neutrinos appear outside the family structure, especially if they are originally non-neutrinos, in which case their number is more arbitrary.

Throughout this work, I denote the standard left-handed neutrino weak eigenstates by $\nu_L$; the right-handed states, as appearing in Dirac case, or in see-saw scenarios by $\nu_R$; and any exotic neutral singlet fermion as $\nu_S$. The mass eigenstates, with the dominant component being doublet or singlet state, respectively, are denoted by $\nu_A$ and $\nu_P$. In some occasions the mass eigenstates are labeled by numbers, not referring to their contents of weak eigenstates, so that $m_i < m_j$, if $i < j$. I use $\theta$ for the mixing between sterile and active states and $\phi$ for the mixing between active neutrinos. The angle $\eta$ is reserved for scalar boson mixing.

To protect the sterile neutrinos from having large masses, it is assumed that there is a new symmetry, denoted by S. Since there is a plethora of different possibilities to choose the symmetry group S, most of them behaving sufficiently similarly in the considered phenomena, I do not restrict to any particular symmetry group. Neither is fixed whether the symmetry S is global or local. The most obvious possibility of S being due to a mirror symmetry $[57,58]$ has been recently discussed in $[59,60]$. The known particles are not required to
carry any S-charge, but depending on the model they may do, possibly in a non-trivial way. The sterile neutrinos can then be given masses by breaking the S-symmetry spontaneously. In all cases the particles under consideration may not be exhaustive, the model may predict a zoo of other particles, too. Especially, in case of a local symmetry the cancellation of possible anomalies may require the existence of other particles.

It is most natural to expect the neutrinos to be Majorana particles. A Dirac kind of mass gives more freedom by not being constrained by neutrinoless double beta decay searches. However, it does not provide per se a new solution to any of the previously discussed anomalies, since the degeneracy of the masses prevents the oscillation to sterile states. One can break the degeneracy by inducing a splitting between the mass eigenstates by a small controllable Majorana mass term for either the active or the sterile sector. Since the protecting symmetries should be already broken such terms are protected only by the non-existence of scalars in appropriate representations. A small splitting leads to a quasi-Dirac case, favorable for certain solutions, especially to the atmospheric neutrino problem. In some cases the quasi-Dirac state may be also made of active neutrinos.

### B. See-saw scenario

Taken that the see-saw mechanism is so beautiful for producing small neutrino masses it is most natural to keep it, and expand it to the sterile sector. Ignore the family mixing for a while, and assume that we have three neutrinos, $\nu_L$, $\nu_R$ and $\nu_S$. When the symmetries are unbroken, only $\nu_R$ can have a mass term (of Majorana type). Assume then that the S symmetry is broken spontaneously, similarly with the symmetry breaking of $SU(2) \times U(1)$. As a result of this there can arise a mass term connecting $\nu_R$ to $\nu_L$ and $\nu_S$, so that the tree level mass matrix looks like

$$ M = \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ a & b & C \end{bmatrix}, \quad (1) $$

in the $\nu_L, \nu_S, \nu_R$ basis. Typically it is assumed $a, b \ll C$. Similar models, from a different point of view, have been considered recently in [61,62], it has also been shown that this kind of mass matrices are possible in some supersymmetric scenarios [63].

Since the mass matrix (1) is singular, there is one massless state, defined as

$$ |\nu_1\rangle \simeq \sin \theta |\nu_L\rangle + \cos \theta |\nu_S\rangle. \quad (2) $$

Of the two massive eigenstates the combination

$$ |\nu_2\rangle \simeq \cos \theta |\nu_L\rangle - \sin \theta |\nu_S\rangle + \epsilon |\nu_R\rangle \quad (3) $$

is light, with a mass

$$ m_2 = \frac{a^2 + b^2}{C} = \epsilon^2 m_3, \quad (4) $$

while the state
\[ |\nu_3\rangle \simeq |\nu_R\rangle \]  

(5)

has a mass \( m_3 = C \). The neutrino mixings above are defined as

\[ \tan \theta = \frac{b}{a}, \]  

(6)

\[ \epsilon = \frac{\sqrt{a^2 + b^2}}{C}, \]  

(7)

where \( \epsilon \) is supposed to be very small.

When higher order corrections are taken into account the massless state may not remain massless any more. As long as the mass terms \( a \) and \( b \) are generated by different non-identical higgses there is no symmetry that can protect the masslessness. Hence the graphs \( \mathcal{G} \) give

\[
\delta m_{LL} = \frac{m_2 m_3^2}{32 \pi^2 v^2} \cos^2 \theta \left[ \left( \frac{M_1^2}{m_3^2 - M_1^2} \ln \frac{m_3^2}{M_1^2} - \frac{M_2^2}{m_3^2 - M_2^2} \ln \frac{m_3^2}{M_2^2} \right) \right. \\
- \sin^2 \eta \left( \frac{M_1^2}{m_3^2 - M_1^2} \ln \frac{m_3^2}{M_1^2} - \frac{M_2^2}{m_3^2 - M_2^2} \ln \frac{m_3^2}{M_2^2} \right) \left. \right] \\
+ \frac{g^2 m_2}{256 \pi^2 \cos^2 \theta \omega} \frac{m_3^2}{m_3^2 - M_2^2} \ln \frac{m_3^2}{M_2^2}
\]

(8)

and

\[
\delta m_{LS} = \frac{m_2 m_3^2}{64 v w \pi^2} \sin \eta \sin 2\theta \left( \frac{M_1^2}{m_3^2 - M_1^2} \ln \frac{m_3^2}{M_1^2} - \frac{M_2^2}{m_3^2 - M_2^2} \ln \frac{m_3^2}{M_2^2} \right) \]

(9)

where \( \eta \) is the mixing between the higgs particles, and \( v \) and \( w \) are the vacuum expectation values of the higgs fields, and \( M_1 \) and \( M_2 \) are higgs masses. The respective \( \delta m_{SS} \) entry depends on the details of the symmetry \( S \), it can be larger or smaller than the other entries.

Typically the electroweak corrections induce to the would-be massless state a mass

\[
m_1 \sim \left(10^{-3} \ldots 10^{-2}\right) m_2 \sin^2 2\theta,
\]

(10)

while the loops involving the sterile neutrinos may give smaller or larger contributions, depending on the model. Only if there is a strict symmetry between sterile and ordinary neutrinos do the different mass contributions cancel, conserving the masslessness.

In the three family basis we have still the freedom to choose the number of the sterile states. The most natural choice might be to assume that the above structure is replicated for all families. Then we would have totally six singlet neutrinos, and the mass spectrum could consist of three heavy states, three intermediate states and three very light states. The families may mix as they do in normal see-saw scenarios, with some more free parameters now. Non-trivial mass hierarchies are also possible, e.g. one can have the mass eigenstates
made mostly of muon and tau neutrinos to be lighter than that dominated by electron neutrinos. The mixings between active and sterile sectors are independent parameters, not tightly connected to the physical masses. However, it would be somewhat unnatural to expect mixing angles very close to \( \pi/4 \), unless there is some specific symmetry between standard and sterile sectors. If the number of sterile states is different, the mass spectrum will also change. For example, for only one \( \nu_S \), there emerges one almost massless state, and three light massive states.

C. Radiative mechanisms

With radiative mechanisms one does not need heavy fermions but instead new scalar particles are needed. The scalars, as well as the fermions, need to have well defined symmetry properties. In several ad hoc models, especially if requiring minimal particle contents, the symmetry assignments tend to be non-trivial.

The simplest mechanism to generate radiative masses to standard neutrinos is provided by the Zee model \cite{zee}. With one new singlet and another doublet, the one loop terms generate a mass matrix leading typically to one heavy quasi-Dirac state, and one light Majorana state. Hence this model gives naturally a mass spectrum favored by the solutions to the atmospheric neutrino deficit while keeping the lightest state (typically dominated by electron neutrino) sufficiently light, just waiting for a sterile neutrino to solve the solar neutrino problem. For natural values of the parameters one cannot obtain for ordinary neutrinos masses higher than 100 keV without introducing new fermions.

The mixing between ordinary and sterile neutrinos can be generated by the one loop graph presented in figure 2, with an ordinary charged lepton inside the loop. Here two new singly charged singlet scalars were introduced, one of which carries the same S quantum number as the singlet state, and the other may be the singlet appearing in the Zee model. The scalars mix, due to the spontaneous breaking of the symmetry, producing two mass eigenstates. The mixing term

\[
m = \frac{f h m_\ell}{32 \pi^2} \sin 2\eta \ln \frac{M_2^2}{M_1^2},
\]

where \( f \) and \( h \) are Yukawa coupling constants, can reach values up to MeV scale.

A Majorana mass term to the singlet states can be generated by the two loop graph \cite{two_loop}, with a new doubly charged singlet. The respective mass element is given by

\[
m_{SS} = \frac{f \zeta f D}{1024 \pi^4} \sin^2 2\eta I\{M\},
\]

where \( f \) and \( \zeta \) are Yukawa couplings, \( D \) is a scalar coupling constant of dimension of mass, and \( I\{M\} \) is a Feynman integral. For typical values of the parameters one can reach masses of \( \mathcal{O}(\text{keV}) \) if insisting staying in the electroweak mass scale. With other assumptions, or different particle contents, much larger masses are possible. One can also generate the sterile neutrino mass terms at one loop level, by introducing other particles with more complicated quantum number assignments.
With a more elaborated symmetry and particle contents one can obtain more specific mass matrices. Such scenarios have been considered abundantly in the literature, typically connected to multianomaly solutions.

One example is the model considered in [69–71]. It was based on the symmetry \( L_e + L_\mu - L_\tau \) (or \( L_e - L_\mu + L_\tau \)), with the sterile neutrino having lepton number \(-1/2\). Several scalar particles were introduced. The resulting neutrino mass spectrum consists of a quasi-Dirac state of mainly muon and tau neutrinos, the other two mass eigenstates being much lighter. The purpose of the model was to reconcile the hot dark matter (initially the 17 keV neutrinos) with solar and atmospheric neutrinos. Written before the LSND experiment, it fits their observations perfectly. Recently similar scenarios with different symmetries have been considered also by [72,73].

D. Tree level masses

A low vacuum expectation value of the higgs field generating the neutrino masses can allow the masses to arise at tree level. This way one avoids the need for additional fermions, or peculiarly charged scalars. The low scale of the symmetry breaking includes a theoretical caveat, by introducing another hierarchy problem. There are, however, ways to generate naturally low vacuum expectation values at a second order process.

The tree level Majorana masses for ordinary neutrinos have been considered in triplet higgs scenarios, appearing typically in left-right models. To generate a mixing to sterile neutrinos one needs a doublet higgs, necessarily different from the standard higgs generating the charged lepton masses. One can also have a singlet higgs inducing a Majorana mass term for singlet neutrinos. In that case there is no need for a mass term for the left-handed neutrinos, or the triplet field.

Since the symmetry breaking scale is typically connected to the mass of the higgs bosons, this kind of models often contain light scalars. That is not necessarily contradictory to any experiments, if the new scalars couple only to the sterile sector. Assuming the respective neutrino mass scales to extend below 1 eV, the natural mass scale for the scalars lies below MeV. However, to generate a 30 MeV neutrino the respective vacuum expectation value of the higgs field hardly can be lower.

E. Large mixing and large hierarchy

Some observations may hint for the existence of neutrinos at MeV scale. Moreover, in the scenarios explaining those observations it is profitable to have a large mixing between the heavy state, and another much lighter state. However, trivial mass matrices fail to give it, without fine tuning one cannot obtain mixings much larger than \( \sin^2 2\theta \sim 10^{-6} \) between neutrinos of 10 eV and 10 MeV.

Generally, models with specific symmetries can produce even massless states with a large mixing to massive states. One example is the above mentioned \( L_e - L_\mu + L_\tau \) symmetry, which can be applied also to sterile sector, like \( L_L - L_S + L_R \). That would lead to a mass matrix of the form
\[
\begin{pmatrix}
0 & 0 & A \\
0 & 0 & B \\
A & B & 0
\end{pmatrix}.
\]

(13)

This results in one massive Dirac neutrino state, and one massless Weyl state. As long as the given symmetry is strictly in force, radiative corrections cannot induce any mass to the massless state. A light mass term, as well as a splitting between the Dirac partners can be allowed by a small breaking of the symmetry. The considered mass matrix has the additional advantage that it is not restricted by neutrinoless double beta decay searches. Such mass matrices, with the massive component very heavy have been considered in [74], but here we want also the heavy state to be light enough. Hence it is feasible to expect the masses to be due to radiative mechanisms.

Another form of a mass matrix leading to massless states mixing with massive states is

\[
\begin{pmatrix}
a a & ab & ac \\
ba & bb & bc \\
ca & cb & cc
\end{pmatrix}.
\]

(14)

which can be generalized to \(n\) dimensions. This results in one massive Majorana state, and \(n-1\) massless states. In some radiative scenarios such mass matrices can emerge naturally, at least approximately. Also here one requires a strict symmetry to keep the massless states really massless. It is difficult, however, to construct such a symmetry between doublet and singlet neutrino states, and it may not necessarily commute with the standard model symmetry group. A special case of the above mass matrix, with \(a = b = c\) is often called democratic (or communistic), because of an obvious analogy: in the initial basis all the components are equal, but in the physical eigenstate basis one component is everything and the rest are nothing.

**IV. PHENOMENOLOGY OF STERILE NEUTRINOS**

**A. New interactions**

The sterile neutrinos are assumed to have interactions among them, as determined by the S-symmetry. For global and local symmetries the interactions can be mediated by scalar bosons. If S is a local symmetry then there are also gauge interactions, mediated by new gauge bosons. The effective scale of the interactions is not fixed, it can be essentially higher or lower than that of the standard weak interactions, even though it is most natural to assume the same scale. Hence the interactions between the sterile neutrinos can be of the same magnitude or much stronger than the standard weak interactions.

If the interactions are mediated by a (gauge) boson with mass \(M\), the effective interaction strength is typically

\[
G_S \simeq \frac{g_s^2}{q^2 - M^2},
\]

(15)

where \(q\) is the momentum change, and \(g_s\) is the coupling constant of the boson to the sterile neutrinos (S-charge of the sterile neutrinos). For scalar mediated interactions it should be
replaced by the respective Yukawa coupling. For low values of $M$, the interactions at high energies are suppressed as $E^{-2}$. This can weaken the laboratory bounds discussed below. For a specific case $M \sim 100$ MeV, a resonance may occur at energies typical for neutrinos in the supernova core. A particle with such a mass is not necessarily contradictory to anything, if its couples mainly to the sterile sector. It is also heavy enough not to be produced excessively in many environments.

The (gauge) interactions in the sterile sector can lead to scattering processes like $\nu_p \nu_A \rightarrow \nu_p \nu_A$, $\nu_p \nu_A \rightarrow \nu_A \nu_A$, or $\nu_p \nu_A \rightarrow \nu_p \nu_p$. Such processes could also be mediated directly by scalar interactions, very likely to be present in several models. In model with radiative generation of masses, also interactions $\nu_p \ell^- \rightarrow \nu_A \ell^-$ or $\nu_p \ell^- \rightarrow \nu_p \ell^-$ are likely to occur. In some scenarios also the processes $\nu_p q \rightarrow \nu_p q$ are possible [75]. The sterile neutrino interactions can be theoretically almost as strong as the standard model charged current interactions. The considered scatterings can lead to the production (or destruction) of sterile neutrinos in several astrophysical environments, they can even cause a thermal or chemical equilibrium between sterile and active neutrinos.

B. Laboratory constraints

The laboratory bounds on the sterile neutrino mixing depend on the mass scale, the mixing of very heavy sterile neutrinos has been recently considered in [76-78]. For sterile neutrinos in the few MeV scale the most stringent limits come from searches of distortions in weak decays (see [79] and references therein). For example, for 10 MeV one has the constraint $\sin^2 2\theta_e < 4 \times 10^{-4}$, while for 33 MeV $\sin^2 2\theta_\mu < 0.008$ and $\sin^2 2\theta_e < 3 \times 10^{-6}$.

For relatively light neutrinos, the most relevant limits are from disappearance experiments. For electron neutrinos, with a sufficiently large mass difference, typically the reactor experiments give upper limits $\sin^2 2\theta_e < 0.14$ [80,81]. These limits are not valid for masses higher than $O$(MeV), also the mixing of neutrinos with $\delta m^2 < 0.008$ eV$^2$ is unconstrained. The BEBC accelerator experiment yields $\sin^2 2\theta_e < 0.07$ [82], for sterile neutrinos heavier than about 10 eV. The upper limit for the mixings to the muon neutrino is less stringent, typically $\sin^2 2\theta_\mu < 0.1$ for $3 \text{ eV}^2 < \delta m^2 < 1000 \text{ eV}^2$. On the other hand, the mixing between tau neutrinos and sterile neutrinos is practically unconstrained, for masses no higher than the upper limits for the tau neutrino mass.

For heavy masses a stringent limit for the mixing to electron neutrinos comes from neutrinoless double beta decay searches [83], that constrain the mixing of any massive Majorana neutrino to electron neutrino as

$$\sin^2 \theta < \left(\frac{1 \text{ eV}}{m_\nu}\right),$$

assuming different contributions not to cancel. For neutrinos heavier than 10 MeV the above limit is weakened by kinematical reasons, for a 30 MeV neutrino by no more than a factor of 2.

The laboratory experiments set also constraints for the new interactions involving sterile neutrinos. The constraint for the number of neutrinos from the $Z$ decay width [79]

$$N_\nu = 2.983 \pm 0.025$$

(17)
sets a bound for the effective four neutrino interactions [84]:

\[ G_{\text{eff}}(4\nu) < 4 \times 10^2 G_F. \]  

(18)

This limit applies also to interactions between active and sterile neutrinos, weakened by a factor of about 2. More stringent limits can be derived by considering the one-loop contribution to \( Z \to \nu_S \nu_S \). The graph depicted in Fig. 4, with an arbitrary fermion \( f \) in the internal line, has a decay width

\[ \Gamma_S = \left( \frac{g_f \xi_f \xi_f M^2}{16\pi^2 M^2} I \right)^2 \Gamma_\nu, \]  

(19)

where \( \Gamma_\nu \) is the decay rate to standard neutrinos, \( I \) is a Feynman integral, \( \xi_f \) is the coupling between \( f \) and \( \nu_S \), and \( g_f \) is the coupling of the fermion \( f \) to \( Z \). Assuming \( I \sim O(1) \), we obtain the (approximate) limit

\[ \frac{\xi_f \xi_f}{M} < 200 \frac{G_F}{g_f}. \]  

(20)

The limit does not apply for effective non-diagonal couplings \( G_S(ff') \) (for which there are much stronger limits from rare decays), neither for chirality flipping interactions of type \( G_S f \bar{f} \nu_S \nu_S \). Note that the limit given in [83] cannot be applied directly for sterile neutrinos. The above limits can be avoided by fine tuning. Since the neutrino mixing involves automatic fine tuning one cannot draw conclusions on mixing angles.

C. Neutrino decay

A heavy neutrino, composed of singlet and doublet weak eigenstates, is necessarily unstable. The decay can take place via standard charged or neutral currents, and there is no GIM mechanism to suppress the \( \nu_A - \nu_P \) matrix element. Decays \( \nu_A \to \nu'_A \) remain suppressed though not forbidden.

The Z exchange interactions lead to the decay mode \( \nu_2 \to \nu_1 f \bar{f} \), where \( f \) is any kinematically allowed fermion coupling to \( Z \). The neutrino lifetime can be expressed as

\[ \tau_\nu = \frac{1536 \pi^3}{N_f m^5 G^2 F \sin^2 2\theta} \approx \frac{1}{\sin^2 2\theta} \left( \frac{10 \text{ MeV}}{m_\nu} \right)^5, \]  

(21)

where \( N_f \) is the effective number of allowed final states, in the latter formula electrons and three types of neutrinos were assumed as possible final states. These lifetimes may allow the existence of heavy neutrinos, in the MeV scale, since they decay away sufficiently fast. On the other hand, a light neutrino in the dark matter range is essentially stable.

The decay to \( \nu_2 \to e^+ e^- \nu_1 \) can take place via both charged and neutral currents. For the typical values of parameters the neutral current reaction is the dominant one, its rate
being determined by the largest mixing while the charged current channel depends on the mixing to electron neutrinos. Hence the branching ratio of the electronic decay mode is

\[ B_e = 0.15. \] (22)

For the most interesting range of parameters the decays to other charged particles are all kinematically blocked.

Second order processes can induce a decay to photons. The standard electroweak corrections generate a transition magnetic moment

\[ \mu_\nu = 3 \times 10^{-12} \mu_B \sin 2\theta \left( \frac{m_\nu}{10 \text{ MeV}} \right). \] (23)

Since the radiative decay rate depends similarly on the mixing and on the mass as the three fermion decay via neutral currents, the decay rates are proportional to each others. The radiative decay is always weaker, with a branching ratio

\[ B_\gamma = 0.008. \] (24)

This is sufficiently large to cause observable effects for a large region of masses.

In models involving new charged scalars the transition moments may be enhanced. This is especially case in models where the neutrino masses are generated radiatively. Hence, depending on the details of the model, one can obtain

\[ \mu_\nu \sim 10^{-7} \mu_B \sin \theta \left( \frac{m_2}{10 \text{ MeV}} \right) \left( \frac{M}{50 \text{ GeV}} \right)^2, \] (25)

where \( M \) is the scale of scalar masses.

If the symmetry \( S \) is global, its spontaneous breaking results in the existence of a massless Goldstone boson, to be denoted by \( J \). Since in case of sterile neutrinos we necessarily have neutrinos with different \( S \) charges, the coupling of the Goldstone boson to neutrinos is not diagonal in the mass basis. Hence the decay \( \nu_2 \to \nu_1 J \) can take place unsuppressed, leading typically to life times

\[ \tau_\nu \sim \frac{0.1 \text{ s}}{\sin^2 2\theta} \left( \frac{10 \text{ MeV}}{m_\nu} \right)^3 \left( \frac{w}{10^{10} \text{ GeV}} \right)^2, \] (26)

where \( w \) is the vacuum expectation value related to the breaking of the \( S \)-symmetry. Unless the scale \( w \) is very heavy, all relevant neutrinos are very unstable. To make a 10 eV neutrino stable one requires

\[ w > \sin 2\theta 10^{10} \text{ GeV}. \] (27)

Note, however, that the decay \( \nu_A \to \nu'_A J \) may be strongly suppressed.

Because of the neutrino mixing the sterile neutrino (gauge) interactions can cause the decay \( \nu_2 \to \nu_1 \nu_P \nu_P \). The decay rate can be much larger than that via neutral currents, with the lifetime given by

\[ \tau_\nu \sim \frac{1 \text{ s}}{\sin^2 2\theta} \left( \frac{G_F}{G_S} \right)^2 \left( \frac{10 \text{ MeV}}{m_\nu} \right)^5, \] (28)

depending on the details of the sterile sector. Hence one obtains more freedom for the scenarios requiring a rapid decay. Moreover, it can reduce the dangerous branching ratio to photons, since the sterile interactions may not contribute to the photon coupling.
V. STERILE NEUTRINOS IN A SUPERNOVA

A. Supernova nucleosynthesis

It has been claimed [2,3] that supernovae would put stringent constraints for the mixing from tau or muon neutrinos to electron neutrinos. The argument goes as following: Being emitted from their respective neutrinospheres, muon neutrinos have on average higher energies than electron neutrinos, electron antineutrinos having energies in between. A conversion from a muon neutrino to an electron neutrino would result in electron neutrinos having higher energies than electron antineutrinos, even though the fluxes of each species would be close to equal. Under such circumstances the layers containing heavy nuclei would turn proton-rich due to electron neutrino radiation, which would prevent the r-process. If one assumes that the r-process is the origin of the nuclei heavier than iron, such conversions should be disallowed. The derived limits [2] would restrict strongly the neutrino mixing in the dark matter mass range, and especially exclude the attractive scenario of having a massive muon neutrino as hot dark matter and explaining simultaneously the LSND events.

Previously many authors have suggested that an inverted mass spectrum ($m_{\nu_\mu} < m_{\nu_e}$) would solve the contradiction [86–88]. In this case there is no conversion to electron neutrinos, instead muon antineutrinos would convert to electron antineutrinos. It is disputable whether this is in agreement with the observations from SN1987A. The mass matrices implementing this scenario would be of quite peculiar form. Although there is nothing wrong in such scenarios, models leading to such a mass spectrum tend to be less attractive.

Here I focus on an alternative remedy using sterile neutrinos. Let us assume that we have a muon (or tau) neutrino with a mass $O(eV)$, and two other much lighter neutrinos, one of which electron type and the other sterile. The matter induced potentials for the electron neutrino and the muon neutrino, respectively, are given by

\[ V(\nu_e) = V_0(3Y_e - 1 + 4Y_{\nu_e}) , \]
\[ V(\nu_\mu) = V_0(Y_e - 1 + 2Y_{\nu_e}) , \]

where

\[ V_0 = 18 \text{ eV} \left( \frac{\rho}{5 \cdot 10^{17} \text{ kg/m}^3} \right) , \]

and $Y$ are the net abundances, relative to the nucleon density. The neutrino contribution is generally negligible outside the core. In the free streaming region it is further suppressed because most neutrinos fly to the same direction. It was assumed that no lepton flavor violation has occurred.

A level crossing can occur for both $\nu_\mu \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_S$ if the state consisting mainly of the muon neutrino is the heaviest state. For antineutrinos an inverted mass spectrum is required. The potential difference for the transition from a muon neutrino to a sterile neutrino is now smaller than that for the transition from a muon neutrino to an electron neutrinos, outside the strongly neutronized inner part. Consequently, the level crossing for the transition to sterile state occurs in an inner zone than that for the transition to electron neutrinos.
As a result of this level crossing scheme, a muon neutrino escaping from the protoneutron star may convert to a sterile state before reaching the muon to electron neutrino resonance zone. With the data of ref. [2], we can estimate that the transition is adiabatic for the considered range of parameters, if the mixing angle satisfies

\[ \sin^2 2\theta_\mu > 10^{-4} \ldots 10^{-3}. \]  

(32)

This condition is apparently in conflict with the cosmological bound, but taken that a partial transition is sufficient they may be marginally consistent. The resonance zones are situated very closely, and if they are too wide they overlap. One can estimate that the width of the resonances is narrower than the distance between the resonance zones if

\[ \sin^2 2\beta < 0.02, \]  

(33)

where \( \beta \) is the largest mixing angle. The suggested mixing from the Los Alamos experiment is below this limit in the dangerous mass range, hence the overlap depends on the mixing to the sterile neutrino. However, even when the sterile neutrino mixing would violate the above bound, to make the resonances overlap, it is expected that the neutrinos converted to electron neutrinos are always a minority, as long as the mixing to sterile states is not essentially smaller than the mixing between electron and muon neutrinos.

B. Sterile neutrino production in the core

For neutrino masses below 100 eV the conversion to sterile states in the core of the supernova is blocked by the strong matter effect. The above scenario thus does not conflict the present picture of the dynamics of the supernova. For sterile neutrinos with higher masses, relevant for the discussion in the cosmology section, one can rule out a certain area in the mass-mixing plane. So far only the transitions from electron neutrinos to sterile neutrinos have been considered seriously in the literature [89]. For \( \delta m^2 \gg 2 \times 10^9 \text{ eV}^2 \) one can exclude

\[ 10^{-8} < \sin^2 2\theta_e < 0.02, \]  

(34)

and for \( \delta m^2 \ll 2 \times 10^9 \text{ eV}^2 \) the excluded region is given by

\[ 3 \times 10^5 \text{ eV}^2 < |\delta m^2 \sin 2\theta_e| < 3 \times 10^8 \text{ eV}^2. \]  

(35)

The limits are very model dependent, the above constraints have been chosen to be conservative. Within specific models of the supernova interior one can obtain limits that are stronger by an order of magnitude. If a sterile neutrino has a large mixing with one active neutrino, above the excluded region, its mixings with other active neutrino species are not constrained.

For other neutrino species the respective limits may be different. If muon and tau neutrinos are excited only thermally, with a zero chemical potential, their abundances are much lower than those of electron neutrinos. The conversions to sterile neutrinos from non-degenerate neutrinos are not irrelevant, however, they may be equally or even more
dangerous. The reason is that the emission of the sterile neutrinos emerging from thermal neutrinos rapidly cools the protoneutron star, which, consequently, accelerates the emission of ordinary neutrinos. Hence, even though the energy carried out by sterile neutrinos never exceeds that carried by normal neutrinos, the cooling of the star is enough to upset the dynamics. The limits for the conversion to sterile states should be necessarily obtained by a self-consistent numerical simulation. So far this has been done for estimating the allowed Dirac mass of neutrinos [90], and the resulting limits are of the same order of magnitude compared as those for degenerate neutrinos.

Similarly one can constrain the exotic interactions producing sterile neutrinos. The limits depend on the target particle. For inelastic scattering from nucleons ($\nu_eN \rightarrow \nu_SN$) one can set the constraint [91]

$$G_S(N) < 10^{-4} G_F.$$  \hspace{1cm} (36)

Since electrons are very degenerate, their interactions are suppressed by Pauli’s rule, because most of the possible final states are occupied. The blocking factor $B_F \simeq \exp(-(E_f - \mu_e)/T)$, with $E_f$ the final electron energy, $\mu_e$ the electron chemical potential and $T$ the temperature of the core, may vary between $10^{-3}$ and 0.1 for temperatures between 10 and 40 MeV. Hence the respective limit for the scattering from electrons ($\nu_e e^- \rightarrow \nu_S e^-$) is weaker,

$$G_S(e) < 6 \times 10^{-4} G_F \ldots 6 \times 10^{-3} G_F.$$  \hspace{1cm} (37)

This limit is also more uncertain since it depends (exponentially) on the temperature of the core which is a dynamical variable. Even more uncertain are the limits for neutrino-neutrino scatterings. For a process ($\nu_e\nu_e \rightarrow \nu_e\nu_S$) or ($\nu_e\nu_e \rightarrow \nu_S\nu_S$) one may set

$$G_S(\nu_e) < 0.001 G_F \ldots 0.01 G_F,$$  \hspace{1cm} (38)

while for the process ($\nu_e\nu_\mu \rightarrow \nu_S\nu_\mu$), assuming non-degenerate muon neutrinos, the upper limit is about

$$G_S(\nu_\mu) < 0.001 G_F \ldots 0.02 G_F$$  \hspace{1cm} (39)

I refrain from giving limits for the transformation of non-degenerate neutrinos.

The same interactions, when sufficiently strong, can cause the sterile neutrinos to be trapped in the core. However, the sterile neutrinos can also be trapped by elastic scattering processes like $\nu_S e^- \rightarrow \nu_S e^-$. The respective production channel $e^- e^+ \rightarrow \nu_S \bar{\nu}_S$ is blocked because of a very low abundance of positrons, unless the core temperatures are very high, $T \gg 50$ MeV, or the electron chemical potential is reduced due to lepton number violation. We can estimate that the interaction is sufficiently strong to be in concordance with the observations if

$$G_S(N) > 0.1 G_F$$  \hspace{1cm} (40)

$$G_S(e) > 0.6 G_F \ldots 6 G_F$$  \hspace{1cm} (41)

$$G_S(\nu_e) > 1 G_F \ldots 10 G_F$$  \hspace{1cm} (42)

$$G_S(\nu_\mu) > 2 G_F \ldots 20 G_F.$$  \hspace{1cm} (43)
Also here the constraints for the lepton interactions involve more uncertainties, depending very strongly and non-linearly on the conditions in the core. The quoted limits are again conservative, to be safe values more stringent by an order of magnitude would be desirable.

Assuming that the interactions are mediated by a boson that is massless or sufficiently light (less than about 200 MeV) the above limits can be written as

\[ \xi_{\nu_S} \xi_N > 10^{-8}, \]
\[ \xi_{\nu_S} \xi_{\nu_e} > 10^{-6}, \]

where \( \xi_f \) is now a diagonal Yukawa coupling constant with the fermion \( f \). Also these limits involve uncertainties by an order of magnitude.

For very heavy neutrinos the transition is blocked kinematically. Since the electron neutrinos have chemical potentials about 150–200 MeV just after the bounce, the bounds for the conversion from degenerate neutrinos are valid for neutrino masses up to 100 MeV, above it the limits are weaker. Masses higher than 300 MeV are safe. On the other hand, for the transitions from thermal, nondegenerate neutrinos the kinematical barrier depends on the temperature which may vary between 8 MeV and 80 MeV. A mass of few tens of MeV may still be dangerous, while more than 100 MeV can be already considered safe. The conversions will stop when the temperatures drop low enough, for the 33 MeV neutrino the temperature of the core would be driven to below 5 MeV, assuming free emission, which may hardly be an acceptable value.

Consider then the possible effects due to resonant transitions between electron neutrinos and sterile neutrinos. The respective potential (29) may cross the zero several times in the vicinity of the core, where the electron abundance may vary above and below 1/3, during the deleptonisation process [92]. This can lead to successive conversions and reconversions between electron neutrinos and sterile neutrinos. The adiabaticity condition depends strongly on the model of the supernova evolution, and can be also substantially affected by the conversions themselves. Typically conversions can occur for masses above 10 eV, or even smaller if neutrinos have a large magnetic moment. These conversions may confuse the observable neutrino spectrum, thus affecting the conclusions made from the observed neutrinos. They may also change the dynamics of the explosion, even reviving the explosion [92]. Now there appears also the possibility of a successive transition \( \nu_e \rightarrow \nu_s \rightarrow \nu_\mu \), plausible for the eV scale masses, especially as suggested by the LSND results. If all the conversions are sufficiently effective, it may result to a beam of high energy muon neutrinos. Since the current experiments are quite insensitive to muon neutrinos we cannot make any definitive conclusions from the observations of SN1987A for such transitions.

C. Flavor conversions and internal deleptonisation

For large neutrino masses it is possible that flavor changing oscillations among active neutrinos become effective in the core [93] (See also [94,95]). Again, for masses below 100 eV the conversions are blocked, but above that the conversions may be rapid enough to cause a chemical equilibrium between neutrino species, depending on the mixing [32]. The transmutation time scale can be estimated to be
\[ t_r \sim \frac{10^{-8}}{\sin^2 2\phi_m} \quad (46) \]

where \( \phi_m \) is the matter mixing angle. Here the transition time is determined by the collisions of neutrinos with the background particles via charged current. Such interactions can involve as final states electrons, in which case the Pauli blocking suppresses the cross section, or muons, provided the total energy exceeds the muon mass threshold (105 MeV). The reactions leading to muons have initially larger available phase space, until a degenerate sea of muons is formed. Neutral currents, being flavor blind, do not affect the neutrino flavor conversion, except when the effective neutrino mixing changes in the collision because of an energy change \[95\].

A conversion time of 0.1 s can be reached for neutrino masses above 40 keV if the mixing satisfies

\[ \sin^2 2\phi \sim 10^{-7}, \quad (47) \]

and for neutrinos lighter than 40 keV, respectively,

\[ \delta m^2 \sin 2\phi \sim 10^6 \text{ eV}^2. \quad (48) \]

Because of the non-linear dependence on energy and temperature, the above results involve an uncertainty of an order of magnitude. To reach a full equilibrium one may need much more time, since there may be other additional transitions to the initial state or from the final state. In many occasions a partial transition may be sufficient to cause significant effects.

As long as muon and tau neutrinos have equal distributions it makes no sense to consider conversions between them. However, in case there is a flavor conversion from electron neutrinos to one of them, they are no longer equal. For the transitions between muon and tau neutrinos there is initially no such a large potential difference to inhibit the transitions. Note that charged current interactions can distinguish muon and tau neutrinos even if the matter is symmetric between them, when the neutrino energies are above the muon mass but below the tau mass. Hence conversions among muon and tau neutrinos with time scale of 0.1 s may be caused approximately by the mixing \[17\]. However, for \( \delta m^2 < 500 \text{ eV}^2 \) the oscillation length exceeds the typical mean free path which damps the oscillation. Hence, in that case, to obtain the given transition time one needs

\[ \sin^2 2\varphi (\delta m^2)^2 \sim 0.01 \text{ eV}^4, \quad (49) \]

assuming no potential difference between muon and tau neutrinos. However, if the conversion rate between muon and tau neutrinos is smaller than the conversion from electron neutrinos to either of them, it cannot maintain the equilibrium and a neutrino potential appears that can slow the conversion (or accelerate, if there is a resonance) for small \( \delta m^2 \). Also excited muons may contribute to the effective potential. Moreover, several models generate a potential difference between muon and tau neutrinos even in flavor neutral medium at higher orders in perturbation theory \[96\].

A neutrino being in chemical equilibrium with the electron neutrino has the same chemical potential and is equally abundant as it. The limits for the mixing between this neutrino
and a sterile neutrino are then the same as those for electron neutrinos. Note that the chemical equilibrium may change substantially the conditions in the core \[93\], as well as the dynamics of the shock wave, so that one should treat the results with some caution.

Flavor equilibrium can be also induced by exotic flavor changing currents, with a strength

\[ G_{\text{FCNC}}(N) > 10^{-4} G_F, \]
\[ G_{\text{FCNC}}(\nu) > 10^{-2} G_F, \]

the first bound being valid for interactions with nucleons and the last one for neutrino-neutrino interactions.

A large transition magnetic moment may cause an equilibrium between electron neutrinos and antineutrinos of another flavor. For this, one needs a magnetic moment of \(10^{-11} \mu_B\), obtainable in some radiative models.

A simultaneous occurrence of both flavor and spin flavor transitions may result in a gross internal deleptonisation of the core. The lepton number violation can also occur because of a Majorana mass: the neutral current scattering can cause substantial helicity flip (“neutrino-antineutrino transition”) for Majorana masses higher than 20 keV. If any neutrino in flavor equilibrium with electron neutrinos has such a mass, this will lead to the deleptonisation. A majoron interchange process \(\nu_e \nu_e \rightarrow \bar{\nu}_e \bar{\nu}_e\) would have similar effects if the majoron coupling satisfies \(\xi > 5 \times 10^{-6}\).

The internal deleptonisation would invalidate all the limits previously derived for neutrino conversions. Under such circumstances, the chemical potentials of all neutrino flavors would be driven to zero. This would also accelerate the neutronization process \(e^- p \rightarrow n \nu_e\). As a consequence, the temperature will rise rapidly, which then changes the densities and the pressure. The effects for the dynamics would be so drastic that it is impossible to say anything quantitative without a new numerical simulation.

D. Neutrino decay in supernovae

The observed supernovae put restrictions on the decay of the emitted neutrinos, and these apply to sterile neutrinos as well. Especially the radiative decay of a heavy neutrino between the supernova surface and the earth is strongly constrained. The non-observation of any gamma-ray pulse \[97, 99\] sets a bound

\[ Y_R B_\gamma < 3 \times 10^{-10}, \]

where \(B_\gamma\) is the branching ratio to photons, and \(Y_R\) is the abundance of the heavy state in the surface of the star, relative to stable standard neutrinos. For the standard electroweak decay modes with sterile neutrinos, discussed in section IV, the branching ratio to photons is never sufficiently low to satisfy the above limit, hence their abundance must be diluted kinematically, or they must decay inside the star. Only if the neutrinos are much heavier than 100 MeV is the kinematical dilution sufficient alone. For lighter neutrinos a very small fraction of neutrinos is allowed to survive up to the surface, so that for neutrino masses about or less than 30 MeV the life time should satisfy
\[ \tau_{\nu} < 10 \text{s} \left( \frac{m_{\nu}}{30 \text{ MeV}} \right). \]  

The above limit is stronger, by about a factor of 2, if the neutrinos interact more weakly, because of a smaller mixing for instance, so that they are emitted from a hotter region closer to the center. Note that there is no solution with \( \nu < 8 \text{ MeV} \).

The decay inside the star may deposit energy to outer regions which may then have some consequences on the explosion itself. For the standard model decay channels the energy transfer may be very effective since 10% of the decay products involve electrons or positrons. Taken that the observed kinetic energies are about 1% of the total energy released, a much larger energy transfer would be inconsistent unless the absorbed energy is immediately transformed to new neutrinos. In some scenarios it may be very profitable to bring new energy to outer regions, helping the star to explode. However, for the life times of 0.1 s – 10 s the energy may be transited too out (beyond 10 000 km from the center). To deliver the energy on a region of about 1000 km, a lifetime of \( O(5 \text{ ms}) \) is required. The total energy deposited to the envelope depends on the initial flux of the sterile state which is typically from 0.1 to 10 times the flux of standard neutrinos, depending on the interactions of the sterile state.

A decay to only neutrinos or other neutral objects outside the core is harmless for the dynamics. A decay to freely escaping particles inside the supernova core, instead, may be very dangerous, leading to a rapid energy flow out of the star. Hence one may exclude (relativistic) lifetimes less than about 0.1 s. Note that this limit is more conservative than that given in [100]. For very short lifetimes there are necessarily new interactions between active and passive neutrinos, and these may be actually much more dangerous, though in extreme case they may evade the bound by trapping.

VI. COSMOLOGY OF STERILE NEUTRINOS

A. Big bang nucleosynthesis

A sterile neutrino with a substantial mixing to active neutrinos would keep in thermal equilibrium with the active universe. This might contradict the present results about the big bang nucleosynthesis and the observed amounts of light elements. The most recent limit for the effective number of neutrinos \( N_{\nu} < 3.6 \) [17] forbids an additional light sterile neutrino, but allows an additional light scalar boson. This bound should be treated with some caution, however, since actually the best fit for the effective number of neutrinos lies deeply in the unphysical region: Ref. [17] obtained \( N_{\nu} = 2.17 \pm 0.27 \pm 0.42 \), while in [9] it was claimed

\[ N_{\nu} = 2.0 \pm 0.3, \]  

excluding \( N_{\nu} = 3 \) at 99.7% confidence level. This hardly can be a statistical fluke, but may rather hint that there is something that is not fully understood yet. The poor consistency of the standard model detracts from the credibility of the bounds for non-standard physics.

The constraints for the additional number of neutrinos have been used to bound the mixing between active and sterile neutrinos [3,5]. These limits, taken at their face values
rule out the sterile neutrino interpretation of the atmospheric neutrino deficit with a clear margin. They do not yet contradict the sterile neutrino solution to the solar neutrino problem, but they would be detrimental to some scenarios discussed above. Let us next study some scenarios to avoid such limits.

An apparently trivial way to avoid the oscillation to a sterile state is to assume that the mixing, or the splitting, arises only after the neutrino decoupling. This could be arranged, if the sterile neutrino masses and mixings, or the splitting, are due to a spontaneous breaking of a symmetry which occurs at temperatures below $O(1 \, \text{MeV})$. In the symmetric phase, at high temperatures, the neutrino oscillation is then inactive, and the sterile states could be excited only by interactions explicitly coupling to both sterile and active sectors, needed to generate the neutrino mixing. However, the constraints for such interactions are respectively very strong, and it is not self-clear that those can be consistently satisfied without releasing the initial presumptions. In fact, most scenarios fail to do this. If the interactions occur via a heavy state, as in see-saw scenarios the limits can be satisfied by construction. Another possibility is that neutrinos are initially Dirac particles, the splitting being induced in the late phase transition. In that case all the new interactions concern only the sterile particles that are sufficiently decoupled.

Another remedy is to induce a large effective potential due to interactions mediated by majorons, or other light particles $^{101}$. For a certain range of parameters that may relax the constraints $^{102}$. Also, a large $\nu_S - \bar{\nu}_S$ asymmetry might induce a substantial potential term for the sterile neutrinos, blocking partially the transitions.

A more attractive escape may be provided by a heavy neutrino. The density of a sufficiently heavy neutrino can be diluted kinematically, by pair annihilation, so that it would not count as a full neutrino species. For standard interactions one needs masses above 30 MeV. The recent measurement for the tau neutrino mass ($m_{\nu_\tau} < 24 \, \text{MeV}$) $^{103}$ closes this window for tau neutrinos. Nevertheless, this scenario allows the existence of heavy sterile neutrinos, provided they have sufficiently strong interactions. Note that this is a remedy for only the nucleosynthesis limit, for other cosmological purposes the heavy neutrinos must be essentially unstable, unless they are heavier than several GeV.

Also a rapid decay may reduce the density sufficiently, and perhaps more naturally $^{104}$. A decay to standard particles would not have any other side effects for the nucleosynthesis, if it is sufficiently rapid. A decay to invisible particles may help if the the heavy particle is already sufficiently diluted.

Other than only deleting one neutrino species, the neutrino decay can be used to modify the onset of nucleosynthesis, to solve the conflict with the result ($^{54}$). It has been shown $^{105}$ that an unstable neutrino with mass of the order of 1 MeV, and a lifetime from 0.1 s to 10 s, decaying to electron neutrinos (or light mixed neutrinos with a substantial component of $\nu_e$) would balance the effect on the neutron to proton ratio. The reason for this is that the resulting electron neutrinos, with energies higher than thermal, would keep the neutron to proton ratio longer in equilibrium, thus reducing the neutron fraction. Since one then needs less neutron decay to fit the observed neutron fraction, the expansion of the universe in the considered times could be accelerated by additional energy density. With certain values of the parameters as many as 16 new neutrino-like degrees of freedom would be allowed. More than only allowing new objects, this mechanism would indeed give a better fit to the observations, solving the apparent conflict between big bang nucleosynthesis and standard
particle physics model.

Previously mainly tau neutrinos have been considered as the heavy state, since it is the only known neutrino allowed to have a sufficiently heavy mass. As the sterile neutrinos may liberate the tau neutrino from the other duties, such a possibility remains very viable. Furthermore, the sterile neutrinos open a valid channel for the decay of tau neutrinos, namely $\nu_\tau \rightarrow \nu_p \nu_\tau \bar{\nu}_e$, via standard neutral currents, with the life times \cite{21} being in the required region. Unavoidably the decay products would include also other than electron neutrinos since the neutral currents are flavor blind. There should, however, be sufficiently many electron neutrinos to do the task.

Alternatively we can assume the heavy state to be mostly a sterile neutrino. This choice actually gives us more liberty, since the density of the sterile neutrino can be considered phenomenologically as an independent degree of freedom. As above, the heavy state decays via neutral currents as $\nu_P \rightarrow \nu_A \nu_A \bar{\nu}_A$, with the life time as above. For $m = 10$ MeV and $\sin^2 2\theta = 0.02$ the life time is about 10 s, just within the correct magnitude. Defining the allowed parameter range would require a more thorough analysis of the output on nucleosynthesis, most likely involving numerical simulation, and it depends also on the full particle spectrum of the model. This remains out of the scope of this work; obviously there is sufficiently freedom to make an acceptable fit.

The relatively high mixings required for the fast decay may be consistent with the limits from the supernovae, Eq. (34). Nevertheless, it may be theoretically difficult to build a natural scenario giving large mixings with large mass differences, and an appropriate family hierarchy. One remedy for the problem of a single heavy state with large mixing is to assume the daughter neutrino to be also quite heavy. This is, of course, only a partial solution since one has then to get rid of the daughter neutrino also which is no easier, even though there are less restrictions for the decay channel.

The decay may also take place via some new interactions. A sufficiently strong interaction in the sterile sector may similarly cause a rapid decay, the majority of the decay products being also most likely sterile. A scalar interchange process can also lead to electrons. In models with a global symmetry the decay to massless Goldstone bosons may be very rapid. In typical models without coincidental suppression, to obtain a lifetime of 0.1 s for a 20 MeV neutrino decaying to electron neutrino, the scale of the symmetry breaking should be no more than $10^6$ GeV, not to violate the limits from neutrinoless double beta decay.

\section*{B. Dark matter and structure formation}

A lately decaying particle may pose some problems for the initiation of the structure formation of the universe in some models. However, in some other models, notably the formerly popular cold dark matter scenario, this may be exactly what is needed to make it work, since the decay may result in the increase in the energy density of relativistic particles which, consequently, delays the beginning of the matter dominated era \cite{36–39}. For a massive neutrino one needs a lifetime

$$\tau_{\nu} \simeq \frac{50 \text{ s}}{Y_{\nu}^2 \left(\frac{10 \text{ MeV}}{m_\nu}\right)^2}, \quad (55)$$
where $Y_\nu$ is the abundance of the heavy neutrino relative to the standard neutrinos, after its decoupling. For the mass ranges typically considered, from 100 eV to few MeV, the standard model neutral current decay mode is insufficient. Hence one has to introduce new interactions, a decay to a massless majoron is typically appropriate. On the other hand, for several MeV masses the neutral currents do well if the mixing is sufficiently large.

A sterile neutrino heavier than few electronvolts may provide a substantial part of the mass of the universe. Since we assumed the sterile neutrinos to mix with the ordinary neutrinos they are likely to be excited during the evolution of the early universe, so that their present number densities are not negligible, unless they decay sufficiently fast to ordinary matter. Whenever the mixing between active and sterile neutrinos is within the forbidden region of ref. \[6\], the sterile neutrinos may be almost as abundant as the active ones. Moreover, the interactions the sterile neutrinos under consideration necessarily have with ordinary matter may also be responsible for exciting them. These interactions may be specially strong in models with radiative generation of neutrino masses. In scenarios with heavy neutrinos, a decay $\nu_2 \rightarrow \nu_1 \nu_1 \bar{\nu}_1$ may increase the number of $\nu_1$ above the usual neutrino densities. Hence, a sterile neutrino with a mass from 10 eV to 1 keV may close the universe, heavier neutrinos must be unstable.

The effect of the relic sterile neutrinos on the formation of the structures may be different from that of ordinary neutrinos. Since the sterile neutrinos may be produced in a different way, their temperatures may be also different, or they may even have nonthermal distributions. The particles emerging from the decay of a non-relativistic particle necessarily have average energies higher than given by the thermal distribution. Hence quite a heavy particle (a few hundred electronvolts) can behave as hot dark matter. On the other hand, the particles decoupled very early are colder than the rest of the universe, typically a particle decoupling around the electroweak phase transition has a present day temperature half of that of the visible universe. As a curiosity, one can build scenarios where the very same particle provides both the hot and the cold dark matter, hence giving a more literal sense to the expression.

Apart from some other dark matter candidates, the sterile neutrinos lack the dissipative mechanisms needed to make them coalesce in galaxies. Hence they are not optimal to form the galactic halos, even if their masses would be large enough (more than 30 eV) to avoid the phase space constraints.

**VII. KARMEN ANOMALIES**

The preliminary results of the Karmen experiment \[10\] on the number of events as a function of the time after the pion collision show an unexpected bump at about 3.6 $\mu$s. The results are not yet statistically significant, to achieve that one needs more years of data taking. However, it is interesting to speculate on the possible particle physics origin of such a bump.

An evident cause would be a heavy particle produced via the two body decay of the pion to a muon. That particle would then decay to something visible inside the detector. The velocity of the hypothesized particle is determined to be $v = 0.017_{-0.005}^{+0.007}c$, and kinematically one can obtain for its mass a value of 33.9 MeV \[10\]. Since the number of detectable events
is a function of both the production and the decay rates of the new particle, one can make for $\tau > 10 \mu s$ the fit

$$B_P \sim \frac{3 \times 10^{-11}}{B_{\text{vis}}} \left( \frac{\tau}{1 \text{ s}} \right),$$  \hspace{1cm} (56)$$

where $B_P$ is the branching ratio to the heavy particle in pion decay, $\tau$ is the lifetime of that particle, and $B_{\text{vis}}$ is the fraction of the decays of the new particle to something visible. No known particle fits the measurements.

In [11] it was suggested that the culprit is a sterile neutrino, produced with the branching ratio

$$B_P = 7 \times 10^{-3} \sin^2 2\theta_{\mu}.$$ \hspace{1cm} (57)$$

Assuming it to decay by charged currents as $\nu_P \rightarrow e^- e^+ \nu_e$ they obtained for its mixings with the standard neutrinos the range $10^{-9} < \sin^2 2\theta_e < 2.5 \times 10^{-7}$ and $3 \times 10^{-5} < \sin^2 2\theta_{\mu} < 8 \times 10^{-3}$. These values are inconsistent with the constraints from both the supernova and the early universe. The authors of [11], however, ignored the visible neutral current decay modes.

Generally the dominant decay channel goes via neutral currents, $\nu_P \rightarrow \nu_A f \bar{f}$, where $f$ can be a light neutrino or an electron. The life time can be estimated

$$\tau_{\nu} \simeq \frac{2 \times 10^{-3} \text{ s}}{\sin^2 2\theta},$$ \hspace{1cm} (58)$$

where $\theta$ is the largest mixing to any active neutrino. (If the mixings are equal, then one should use the sum of the respective squared mixing components instead). The branching ratio to electrons is $B_{\text{vis}} = 0.15$. Thus the equation (58) gives a relation

$$\sin^2 2\theta_{\mu} \sin^2 2\theta \simeq 6 \times 10^{-11}.$$ \hspace{1cm} (59)$$

Assuming both the production and the decay to be determined by the same mixing angle to muon neutrino, one obtains a unique solution $\sin^2 2\theta_{\mu} = 8 \times 10^{-6}$, and $\tau = 220 \text{ s}$, which is inconsistent with supernova observations and dangerous for cosmology. If the production and decay depend on a different mixings, we have two free parameters, and a large range of lifetimes is possible. For instance, a life time of one seconds can be obtained with the mixing angles $\sin^2 2\theta_{\mu} = 3 \times 10^{-8}$ and $\sin^2 2\theta_{\tau} = 2 \times 10^{-3}$. Requiring a large mixing, e.g. $\sin^2 2\theta_{\tau} > 0.02$, as suggested by (34), one obtains

$$\tau_{\nu} < 0.1 \text{ s},$$ \hspace{1cm} (60)$$

and

$$6 \times 10^{-11} < \sin^2 2\theta_{\mu} < 3 \times 10^{-9}.$$ \hspace{1cm} (61)$$

Additional invisible decay modes would not change the given solutions (59) for the mixing angles.

The mass and the range of the lifetimes of the sterile neutrino suggested by Karmen experiment are very interesting from a cosmological point of view. Naively, the result (59)
is contradictory to the cosmological limits for the mixing with sterile neutrinos. However, the solution itself provides the ingredients to avoid the given limits by a rapid decay. The mass and the lifetime of the heavy neutrino fall into the range where they naturally provide the early decaying particle needed to solve the conflict with the nucleosynthesis.

These results can also be in agreement with the considerations of neutrino emission from supernova, including the limits for the radiative decay. For the required mixing the sterile state would be trapped in the core. The neutrinos escaping the core, with the suggested properties, may deliver dangerously much energy in the outer regions of the star, but for sufficiently short lifetimes it may be used for profit, to help blowing up the envelope. Note also that the emitted flux of the heavy neutrinos may be reduced kinematically, especially if their interactions with matter are sufficiently strong (large mixing). Any additional invisible decay mode would reduce the transferred energy.

Other solutions can be found with non-standard interactions. In such a case the mixings can be minimal allowed by (59), $\sin^2 2\theta \sim O(10^{-5})$. The new interactions should then trap the neutrinos in the core of the supernova, and also cause the decay of the heavy neutrino. To obtain lifetimes shorter than 10 s, consistent with supernova and early universe considerations, at least if the decay products are active neutrinos, one needs a nondiagonal effective coupling $G_S > 0.01G_F$.

VIII. CONCLUSIONS

The sterile neutrinos can provide a solution to the solar neutrino deficit, the atmospheric neutrino problem, the missing matter of the universe, the anomalous ionization of interstellar hydrogen, the explosion of a supernova, the conflict between a part of the parameter space suggested by the LSND experiment and the supernova nucleosynthesis, the crisis of the big bang nucleosynthesis, and the anomalies observed in the Karmen experiment. All these problems can also be solved individually without sterile neutrinos, either by ordinary neutrinos or by some other objects. It is also possible that some, if not all, anomalies are due to misconception in underlying theory or uncontrolled phenomena in the experimental apparatus.

The standard electroweak model with three massive neutrinos allows only a simultaneous solution of two or three of the above problems, unless excessive fine tuning is applied. By introducing three sterile neutrinos one can solve almost all of the above problems simultaneously, as well as fit the LSND results.

The existence of light sterile neutrinos is not itself neither theoretically inconsistent nor unnatural. Several effective models can generate a small mass to sterile neutrinos, with a mixing to active neutrinos. In this sense, the expression sterile neutrino can refer to any exotic fermion without standard model interactions, with a mixing to standard neutrinos.

The sterile neutrino solution to the atmospheric neutrino problem is not contradictory to any observation, and can be taken as a viable alternative. In many models it is more natural that the favored quasi-Dirac state is made of a doublet and a singlet components than that it were made of two active neutrinos. Hence, one can build a scenario of using the sterile neutrinos to solve both solar and atmospheric neutrino problems, and muon neutrino to electron neutrino oscillation to solve the LSND results.
It was found that the conversion of a muon neutrino to a light sterile state in the supernova can solve the apparent conflict between the large mass solutions of the LSND results and the supernova nucleosynthesis. This does not conflict any bounds derived from SN1987A observations, or any other dynamical consideration. It was also recalled that the sterile neutrinos can help to blow up the supernova, either due to a reconversion to active state, or by decaying to electromagnetically interacting particles.

Limits for the production of sterile neutrinos in the supernova core were reconsidered, and some new limits were presented. These limits are modified in case of equilibrium between neutrino flavors, or neutrino helicity states. The latter may result in an internal deleptonisation of the core which upsets its dynamics completely. In such a case all the present considerations for neutrinos in supernovae would cease to be valid.

The limits for light sterile neutrinos in the early universe can be evaded by introducing more sterile neutrinos. An early decaying heavy neutrino, with mass O(10 MeV), may solve the nucleosynthesis anomaly, and may also allow new additional light neutrino states. To obtain the required lifetime it is enough that the heavy mass eigenstate is composed significantly of both active and sterile weak eigenstates. It is emphasized that one cannot rely very much on the constraints from the early universe for new physics anyway, since the observed abundances of the light elements are inconsistent with the standard particle physics model and the current models for the big bang nucleosynthesis.

The sterile neutrino solution to the Karmen anomalies fits the observed results. It was found to be consistent with cosmology, and may even improve the nucleosynthesis. It is also in agreement with the observations of SN1987A, possibly helping the star to blow up. The presented parameter range differs from the previous speculations. Non-standard interactions may expand the allowed ranges for the mixing angles.

The suggested solution for Karmen anomalies, together with other requirements, may lead to an extraordinarily shaped mass matrix. The large mixing with a heavy neutrino generally implies the other state also to be quite heavy. A general $2 \times 2$ mass matrix without fine tuning would imply the mass of the lighter state in the MeV range, assuming the heavy state to be 33 MeV, while the seesaw like mass matrix (II) may predict masses of the order of 17 keV. Only models involving a specific symmetry with active and sterile neutrinos could produce mass matrices with a strong hierarchy without fine tuning. Alternatively, if the decay occurs by explicit non-standard interactions between active and sterile neutrinos there is no need for large neutrino mixing.

The sterile neutrinos may provide part or all of the missing matter of the universe. Due to the large variety of possibilities for their masses and origins, they can act as either hot or cold dark matter, or even both. The decaying neutrino scenarios can result in relatively massive neutrinos being hot, while an early decoupled neutrino is colder. The relic density of sterile neutrinos may be lower or higher than that of ordinary neutrinos. A radiative decay of relic sterile neutrinos may be the cause for the anomalous ionization of the interstellar hydrogen, which does not require any new physics beyond what was considered here.
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FIGURES

FIG. 1. The radiative corrections to the neutrino masses in the extended see-saw scenario.

FIG. 2. The graphs generating a mixing between doublet and singlet neutrinos.

FIG. 3. The graph generating a Majorana mass term for singlet neutrinos.

FIG. 4. The one-loop graph inducing the $Z$ decay to sterile neutrinos.
Figure 1
