On the Contributions from Dilatonic Strings to the Flat Behaviour of the Rotational Curves in Galaxies

M. Leineker Costa\(^1\), A. L. N. Oliveira\(^2\) and M. E. X. Guimarães\(^3\)

1. Instituto de Física, Universidade de Brasília, Brasília-DF, Brazil
2. Departamento de Física, Universidade Estadual de Londrina, Londrina-PR, Brazil
3. Departamento de Matemática, Universidade de Brasília, Brasília-DF, Brazil

July 25, 2018

Abstract

We analyse the flat behaviour of the rotational curves in some galaxies in the framework of a dilatonic, current-carrying string. We determine the expression of the tangential velocity of test objects following a stable circular equatorial orbit in this spacetime.

1 Introduction:

The measurements of rotation curves in some galaxies show that the coplanar orbital motion of gas in the outer parts of the galaxies keeps a constant velocity up to several luminous radii\([1,2,3,4]\). The most accepted explanation for this effect is that there exists a spherical halo of dark matter which
surrounds the galaxy and account for the missing mass needed to produce the flat behavior of the rotational curves.

In this work, we would like to analyse this effect in the framework of a dilatonic, current-carrying cosmic string. In a number of papers [5-11], a current has been included in the internal structure of the cosmic string. The most noticeable consequence of a current-like effect is to modify the internal dynamics of cosmic strings in such a way that new states are reachable. Indeed, the breaking of the Lorentz boost invariance along the worldsheet allows rotating equilibrium configurations, called vortons, which, if they are stable, can overclose the universe, thereby leading to a catastrophe for the theory that predicts them [12]. Finally, inclusion of such an internal structure could drastically change the predictions of a cosmic string model [13] in the microwave background anisotropies [14]. In [15], it is shown that the long-range effect on a cosmologically relevant network of strings is vanishing on average, but that vorton-like states can be reached by microscopically small loops. Here, we would like address ourselves to a well-posed problem at the galactic scale.

In what follows, after having set the relevant gravitational theory and notations, we derive the geometry of an electrically charged cosmic string in the dilatonic theory of gravity in Sec. 2. Then, in the Sec. 3, we compute the tangential velocity for test particles imposing that its magnitude is independent of the radius. In doing this, we find a constraint equation for the metric coefficients of the dilatonic cosmic string. Unfortunately, as we will see later, we also find that the tangential velocity cannot be explained by a single string of the kind proposed in our model. Instead, in order to be compatible with the observed magnitude, one must have a bundle of \( N \sim 10^5 \) strings seeding a galaxy. With such a density, a cosmic string network would be dominating the universe, and its dynamics would be completely different. Section 4 summarizes our findings and discusses the relevant conclusions.
2 Exterior Solution for a Timelike Current-Carrying String:

2.1 The Model:

In this section we will mainly review the obtention of the gravitational field generated by a string carrying a current of timelike-type as presented in the Refs. [10].

We will concentrate our attention to superconducting vortex configurations which arise from the spontaneous breaking of the symmetry $U(1) \times U_{em}(1)$. Therefore, the action for the matter fields will be composed by two pairs of coupled complex scalar and gauge fields $(\varphi, B_{\mu})$ and $(\sigma, A_{\mu})$. Also, for technical purposes, it is preferable to work in the so-called Einstein (or conformal) frame, in which the scalar and tensor degrees of freedom do not mix:

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g} [R - 2\partial_{\mu}\phi \partial^{\mu}\phi] + \int d^4x \sqrt{-g} \left[ -\frac{1}{2}\Omega^2(\varphi)((D_{\mu}\varphi)^*D^{\mu}\varphi + (D_{\mu}\sigma)^*D^{\mu}\sigma) - \frac{1}{16\pi}(F_{\mu\nu}F^{\mu\nu} + H_{\mu\nu}H^{\mu\nu}) - \Omega^2(\varphi)V(|\varphi|,|\sigma|) \right],$$

(1)

where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$, $H_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}$ and the potential is suitably chosen in order that the pair $(\varphi, B_{\mu})$ breaks one symmetry $U(1)$ in vacuum (giving rise to the vortex configuration) and the second pair $(\sigma, A_{\mu})$ breaks the symmetry $U_{em}(1)$ in the core of the vortex (giving rise to the superconducting properties): $V(|\varphi|,|\sigma|) = \frac{\lambda_\varphi}{8}(|\varphi|^2 - \eta^2)^2 + f(|\varphi|^2 - \eta^2)|\sigma|^2 + \frac{\lambda_\sigma}{4}|\sigma|^4 + \frac{m^2}{2}|\sigma|^2$.

In what follows, we will write the general static metric with cylindrical symmetry corresponding to the electric case in the form

$$ds^2 = -e^{2\psi}dt^2 + e^{2(\gamma - \psi)}(dr^2 + dz^2) + \beta^2 e^{-2\psi}d\theta^2.$$  

(2)
where $\psi, \gamma, \beta$ are functions of $r$ only.

In order to solve the equations we will divide the space in two regions: the exterior region, $r \geq r_0$, in which only the electric component of the Maxwell tensor contributes to the energy-momentum tensor and the internal region, $0 \leq r < r_0$, where all matter fields survive. $r_0$ is the string thickness.

### 2.2 Form of the Line Element for the Exterior Region:

Due to the specific properties of the Maxwell tensor

$$T^\mu_\mu = 0 \quad \text{and} \quad T^\alpha_\nu T^\mu_\alpha = \frac{1}{4} (T^\alpha_\beta T^\mu_\beta) \delta^\mu_\nu$$  \hspace{1cm} (3)

we can find the metric through some algebraic relations called Rainich algebra \cite{15, 17}, which, for the electric case, have the form\footnote{In the scalar-tensor theories, these relations are modified by a term which depend on the dilaton \cite{9, 10}.}:

$$R^t_t = -R^\theta_\theta \quad R^t_t = R^r_r - 2 g^{rr} \phi \partial^2 \quad R^\theta_\theta = R^z_z$$

Therefore, the exterior metric for a timelike current-carrying string is:

$$ds^2_E = \left( \frac{r}{r_0} \right)^{2m^2-2n} W^2(r) (dr^2 + dz^2) + \left( \frac{r}{r_0} \right)^{-2n} W^2(r) B^2 r^2 d\theta^2 - \left( \frac{r}{r_0} \right)^{2n} \frac{1}{W^2(r)} dt^2$$  \hspace{1cm} (4)

where

$$W(r) \equiv \left( \frac{r}{r_0} \right)^{2n} + k \frac{1}{1 + k}$$  \hspace{1cm} (5)

The constants $m, n, k, B$ will be determined after the inclusion of the matter fields.
3 Stable Circular Geodesics Around the Cosmic String:

In this section we will derive the geodesic equations in the equatorial plane \((\dot{z} = 0)\), where dot stands for “derivative with respect to the proper time \(\tau\)”. First of all, let us re-write metric (4) in a more compact way:

\[
\begin{aligned}
ds^2 &= A(r)[dr^2 + dz^2] + B(r)d\theta^2 - C(r)dt^2, \\
\end{aligned}
\]

with

\[
\begin{aligned}
A(r) &= \left(\frac{r}{r_0}\right)^{2m^2-2n} W^2(r), \\
B(r) &= \left(\frac{r}{r_0}\right)^{-2n} W^2(r)\beta^2(r), \\
C(r) &= \left(\frac{r}{r_0}\right)^{-2n} W^{-2}(r). \\
\end{aligned}
\]

The Lagrangian for a test particle moving on this spacetime is given by:

\[
2\mathcal{L} = A(r)[\dot{r}^2 + \dot{z}^2] + B(r)\dot{\theta}^2 - C(r)\dot{t}^2
\]

and the associated canonical momenta, \(p_\alpha = \frac{\partial \mathcal{L}}{\partial \dot{x}^\alpha}\), are:

\[
\begin{aligned}
p_t &= -E = -C(r)\dot{t}, \\
p_\theta &= L = B(r)\dot{\theta}, \\
p_r &= A(r)\dot{r}, \\
p_z &= A(r)\dot{z}.
\end{aligned}
\]

Because of the symmetries of this particular spacetime, the quantities \(E\) and \(L\) are constants for each geodesic and, because this spacetime is static, the Hamiltonian, \(\mathcal{H} = p_\alpha \dot{x}^\alpha - \mathcal{L}\), is also a constant. Combining this information
with the restriction of a motion in an equatorial plane, we arrive to the
following equation for the radial geodesic:
\[ \dot{r}^2 - A^{-1}[E\dot{t} - L\dot{\theta} - 1] = 0 . \] (9)

In this work, we will concentrate on stable circular motion. Therefore,
we have to satisfy three conditions simultaneously. Namely:

- \( \dot{r} = 0 \);
- \( \frac{\partial V(r)}{\partial r} = 0 \), where \( V(r) = -A^{-1}[E\dot{t} - L\dot{\theta} - 1] \);
- \( \frac{\partial^2 V(r)}{\partial r^2} |_{ext} > 0 \), in order to have a minimum.

Consequently, we have:
\[ E\dot{t} - L\dot{\theta} - 1 = 0 \]
\[ \frac{\partial}{\partial r} \left\{ A^{-1}[E\dot{t} - L\dot{\theta} - 1] \right\} = 0 . \] (10)

Expressing \( \dot{t} \) and \( \dot{\theta} \) in terms of the constant quantities \( E \) and \( L \), respectively,
we can get their expressions \(^2\):
\[ E = C\sqrt{\frac{B'}{B'C - BC'}} , \]
\[ L = B\sqrt{\frac{C'}{B'C - BC'}} . \] (11)

where prime means “derivative with respect to the coordinate \( r \)”.\(^2\)

Recalling that the angular velocity of a test particle moving in a circular
motion in an orbital plane is \( \Omega = \frac{d\theta}{dt} = \frac{\dot{\theta}}{\dot{t}} \), we have:
\[ \Omega = \sqrt{\frac{C'}{B'}} . \] (12)

\(^2\)It is remarkable that our conclusions are independent on the metric coefficient \( A \).
We are now in a position to compute the tangential velocity of the moving particles in a circular orbit in the equatorial plane. From now on, we will follow the prescription established by Chandrasekhar in [18]. Let us re-express the metric (6) in terms of the proper time $\tau$, as $d\tau^2 = -ds^2$:

$$d\tau^2 = C(r)dt^2 \left\{ 1 - \frac{A}{C} \left[ \left( \frac{dr}{dt} \right)^2 + \left( \frac{dz}{dt} \right)^2 \right] - \frac{B}{C} \left( \frac{d\theta}{dt} \right)^2 \right\}, \quad (13)$$

and comparing with the expression $1 = C(r)(u^0)^2[1 - v^2]$, where $u^0 = dt/d\tau$, we can easily obtain the spatial velocity $v^2$:

$$v^2 = (v^{(r)})^2 + (v^{(z)})^2 + (v^{(\theta)})^2, \quad (14)$$

from which we can obtain all the components of the spatial velocities. However, we are particularly interested in the tangential component $v^{(\theta)}$:

$$v^{(\theta)} = \sqrt{\frac{B}{C}} \left( \frac{d\theta}{dt} \right) = \sqrt{\frac{B}{C}} \Omega. \quad (15)$$

In order to have stable circular orbits, the tangential velocity $v^{(\theta)}$ must be constant at different radii at the equatorial plane. Therefore, we can impose:

$$v^{(\theta)} = \sqrt{\frac{BC'}{B'C}} = v^{(\theta)}_c = \text{const}. \quad (16)$$

At this point we would like to notice the following theorem [1]:

**Theorem:** The tangential velocity of circular stable equatorial orbits is constant iff the coefficient metrics are related as\(^3\)

$$C^{1/2} = \left( \frac{r}{r_0} \right)^l, \quad (17)$$

provided $l = \frac{\omega^{(\theta)}}{1 + v^{(\theta)}_c}$.

\(^3\)We point out to the reader that, for the sake of clarity, we have changed the notation with respect to the original paper [1]. The Theorem, of course, is unchangeable.
This theorem implies that the line element in the equatorial plane must be:

\[ ds^2 = -\left(\frac{r}{r_0}\right)^{2l} dt^2 + \left(\frac{r}{r_0}\right)^{-2l} \left[ \left(\frac{r}{r_0}\right)^{2m^2} dr^2 + B^2 r^2 d\theta^2 \right] . \]  

(18)

This form clearly is not asymptotically flat and neither describes a spacetime corresponding to a central black hole. Therefore, we can infer that it describes solely the region where the tangential velocity of the test particles is constant, being probably joined in the interior and exterior regions with other metrics, suitably chosen in order to ensure regularity in the asymptotic limits.

Let us notice, however, that this metric has the form which has been found in [10], after identifying \( l \) with the appropriate constant parameters which depend on the microscopic details of the model. The calculations are straightforward but lengthy. We will skip here the details and provide directly the results. For the details of these calculations, we refer the reader to Refs. [10]. For this particular configuration, consisting of an electrically charged dilatonic string, we have:

\[ l = 2G_0 \alpha(\phi_0)[U + T + I^2] , \]  

(19)

where \( U, T \) and \( I^2 \) are the energy per unit length, the tension per unit length and the current of the string, respectively. \( \alpha(\phi_0) \) measures the coupling of the dilaton to the matter fields. For cosmic strings formed at GUT scales, \( G_0[U + T + I^2] \sim 3 \times 10^{-6} \), and for a coupling \( \alpha(\phi_0) \) which is compatible with present experimental data [19], \( \alpha(\phi_0) < 10^{-3} \), the parameter \( l \) (and, thus, the tangential velocity \( v_c^{(\theta)} \)) seems to be too small. The observed magnitude of the tangential velocity being \( v_c^{(\theta)} > 3 \times 10^{-4} \) cannot be explained by a single dilatonic current-carrying string in this case. As argued by Lee in the Ref. [20], if a bundle of \( N \) cosmic strings formed at GUT scales seeded one galaxy, then the total magnitude of the tangential velocity would be \( N v_c^{(\theta)} \).

\footnote{Here, again, we have adapted the notation for the sake of clarity.}
In our case, to be compatible with the astronomical observations, one must have a bundle of $N \sim 10^5$ strings seeding a galaxy. With such a density, a cosmic string network would be dominating the universe, and its dynamics would be completely different. The only situation where such a high number of strings could be possible is at much lower energy scales (electroweak scale, say) but then of course the energy scale is far too low to have any relevance for structure formation.

If, on the other hand, one supposes that one single string could explain the observed values of the tangential velocity, one should therefore impose that such a string is formed at Planck scales, which at the moment seems not quite realistic conclusion as well.

4 Conclusions:

The measurements of rotation curves in galaxies show that the coplanar orbital motion of gas in the outer parts of the galaxies keeps a constant velocity up to several luminous radii. The most accepted explanation for this effect is that there exists a spherical halo of dark matter which surrounds the galaxy and account for the missing mass needed to produce the flat behavior of the rotational curves.

In the Ref. [1], it has been shown that in a static, axially symmetric spacetime, a sufficient and necessary condition in order to have a flat behavior for the rotational curves in galaxies is that the metric assumes the form (18). This form clearly is not asymptotically flat and neither describes a spacetime corresponding to a central black hole. Therefore, we can infer that it describes solely the region where the tangential velocity of the test particles is constant, being probably joined in the interior and exterior regions with other metrics, suitably chosen in order to ensure regularity in the asymptotic limits.

In previous papers [10], we have found a metric corresponding to the
spacetime generated by an electrically charged dilatonic string which possesses the form (18), with appropriate parameters which are related to the microscopic details of the model (19). The observed magnitude of the tangential velocity cannot be explained by a single dilatonic current-carrying string in this case. However, if we consider that a bundle of $N$ cosmic strings formed at GUT scales seeded one galaxy, then the total magnitude of the tangential velocity would be $Nv_c(θ)$. In our case, to be compatible with the astronomical observations, one must have a bundle of $N \sim 10^5$ strings seeding one galaxy! The only situation where such a high number of strings could be possible is at much lower energy scales (electroweak scale, say) but then the energy scale is far too low to have any relevance for structure formation.

Acknowledgements:

The authors would like to thank Prof. E. R. Bezerra de Mello for very interesting comments and suggestions. M. Leineker Costa would like to thank CAPES for a grant. A. L. Naves de Oliveira and M. E. X. Guimarães would like to thank CNPq for a support. M. E. X. Guimarães would like to thanks the High Energy Sector of the Abdus Salam International Center for Theoretical Physics for hospitality during the preparation of this work.

References

[1] T. Matos, D. Núñez, F. Siddartha Guzmán and E. Ramírez, Gen. Rel. Grav. 34, 283 (2002).

[2] V. C. Rubin, N. Thonnard and W. K. Ford, Ap. J. 225, L107 (1978); Ap. J. 238, 471 (1980).

[3] M. Persic and P. Salucci, Ap.J. Suppl. Ser. 99, 501 (1995).
[4] M. Persic, P. Salucci and F. Stel, M. N. R. A. S. 281, 27 (1996).

[5] P. Peter, Class. Quantum Grav. 11, 131 (1994).

[6] E. Witten, Nucl. Phys. B249, 557, (1985).

[7] B. Carter, Phys. Lett. B 224, 61 (1989); 228, 466 (1989); Nucl. Phys. B 412, 345 (1994); see also B. Carter, P. Peter, and A. Gangui, Phys. Rev. D 55, 4647 (1997).

[8] P. Peter, Phys. Rev. D 45, 1091 (1992); 46, 3335 (1992).

[9] C. N. Ferreira, M. E. X. Guimarães, and J. A. Helayël-Neto, Nucl. Phys. B581, 165 (2000).

[10] A. L. N. Oliveira and M. E. X Guimarães, Phys. Lett. A 311, 474 (2003); A. L. N. Oliveira and M. E. X Guimarães, Phys. Rev. D 67, 123514 (2003).

[11] M. C. B. Abdalla, A. A. Bytsenko and M. E. X. Guimarães, Mod. Phys. Lett. A 19, 2445 (2004); V. C. Andrade, A. L. Naves de Oliveira and M. E. X. Guimarães, Proc. Sci. WC2004(2004)034.

[12] R. L. Davis, Phys. Rev. D 38, 3722 (1988); R. L. Davis and E. P. S. Shellard, Nucl. Phys. B323, 209 (1989); B. Carter, Ann. N.Y. Acad.Sci. 647, 758 (1991); B. Carter, Phys. Lett. B 238, 166 (1990); in Proceedings of the XXXth Rencontres de Moriond, Villard–s–Ollon, Switzerland, 1995, edited by B. Guiderdoni and J. Tran Thanh Van (Editions Frontières, Gif–sur–Yvette, 1995); R. Brandenberger, B. Carter, A. C. Davis, and M. Trodden, Phys. Rev. D 54, 6059 (1996); B. Carter, A. C. Davis, ibid 61, 123501 (2000).

[13] A. Riazuelo, N. Deruelle, and P. Peter, Phys. Rev. D 61, 123504 (2000).
[14] C. L. Bennett et al., Ap.J.Suppl. 148, 97 (2003); D. N. Spergel et al., Ap.J.Suppl. 148, 175 (2003).

[15] P. Peter, M. E. X. Guimarães and V. C. Andrade, Phys. Rev. D 67, 123509 (2003).

[16] L. Witten, in Gravitation, ed. L. Witten (Wiley, New York, 1962).

[17] M. A. Melvin, Phys. Lett. 8, 64 (1964).

[18] S. Chandrasekhar, in “Mathematical Theory of the Black Holes” (Clarendon Press Oxford, 1983).

[19] Th. Damour, Astrophys. Space Sci. 283, 445 (2003).

[20] T. H. Lee, Mod. Phys. Lett. A 19, 2929 (2004).