Comparative Study on Multi-type Domes of Filament-Wound Composite Pressure Vessels

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Abstract. Filament-wound Composite Pressure Vessels (FCPV) have been used widely in the aerospace and commercial industries. Multi-type FCPVs have been applied to different mission scenes. In this paper, four types of domes (helical winding, helical winding ellipsoid, polar winding, polar winding ellipsoid) are studied based on a simplified parametric model, and a modified meridian computing method is employed for polar and helical winding domes after the inflection points. At last, four FCPV cases of different type of domes are calculated and discussed in detail. Results indicate that the parameters of the same winding pattern dome are similar, so it is suggested to use ellipsoid domes in prior for the convenience of fabrication and avoiding the inflection point at the meridian.

1. Introduction

Sourced from Solid Rocket Motor, Filament-wound Composite Pressure Vessels (FCPV) are widely applied in the aerospace and commercial industries, for the advantage of high strength and light weight. Most Studies of FCPV design and analysis have adopted the Netting Theory Assumption [1–3]. The forces, due to internal pressure, are resisted only by pure tension in the filament with no assistance from the resin. The path which has been considered by those investigators is Geodesic which is the greatest winding stability [4–5]. More recently, Classical Laminate Theory and Non-geodesic winding path are employed, which enlarge the design space and also take into account the behavior of resin matrix [6–8].

The main purpose of this paper is to compare the effects of winding pattern and meridian type of domes on the properties of FCPV, governing equations of multi-type domes are derived in detail, based on netting theory and geodesic winding. Four cases are also calculated, one for each type of domes, to compare and analyze the geometric and quality characteristic parameters.

2. Simplified Parametric Model

One simplified geometric model of FCPV body is illustrated in Figure 1. It consists of cylinder, front and rear domes, which are both filament-wound and subjected to internal pressure. The main parameters of the dome are meridian pattern, winding angle, wall thickness and fiber strength.
3. FCPV Design Method

3.1. Cylinder
As shown in Figure 2, one finite element is selected from cylinder, and fiber stress analysis along hoop direction and meridian direction \( Z \) is performed.

![Figure 2. Fiber force analysis of cylinder micro-cell](image)

According to membrane Eqs. (1) and fiber force Eqs. (2), the cylinder thickness in hoop and meridian directions can be expressed in Eqs. (3) [9–12].

\[
\begin{align*}
N_2 &= \frac{1}{2} R R_b \\
N_\theta &= R R_b
\end{align*}
\]

(1)

\[
\begin{align*}
T_z &= \sigma_f h_f \alpha \cos \theta \\
T_\theta &= \sigma_f h_f \sin^2 \theta + \sigma_b h_b
\end{align*}
\]

(2)

\[
\begin{align*}
\bar{h}_f &= \frac{R \bar{R}_b}{2 \sigma_f \cos \alpha_0} \\
h_f &= \frac{R \bar{R}_b}{2 \sigma_f} (2 - \tan^2 \alpha_0)
\end{align*}
\]

(3)

3.2. Dome
Three rules of dome design are employed as follows:
1) Rule 1: fibers symmetrical to the meridian;
2) Rule 2: winding angle \( \alpha \) of dome equator equals to winding angle \( \alpha_0 \) of cylinder;
3) Rule 3: fibers’ volume of each parallel section is constant.

The dimensionless dome governing Eqs. (4) can be derived from Rule 1 ~ Rule 3. It can be seen that there are four parameters: parallel radius \( \rho \), winding angle \( \alpha \), fiber stress \( \bar{\sigma} \) and dome thickness \( \bar{h} \), but there are only three equations. One additional condition is needed to solve the Eqs (4). According to different conditions, four types of domes are discussed in this paper: helical winding, helical winding ellipsoid, polar winding and polar winding ellipsoid.
3.2.1. Helical Winding Dome. Helical winding follows the geodesic path, with no consideration of fiber friction or slippage limits. However, it should be restricted to vessels with equal pole openings. The isotensoid condition is employed as shown in equation (6), and the winding angle is calculated from the Clairaut condition equation (7), so the governing equations are solvable.

\[
\begin{align*}
\tan^2 \alpha &= \frac{\rho \beta}{1 + \beta^2} \\
\delta &= \frac{\rho (1 + \beta^2)^{1/2} \h}{h \cos \alpha} \\
\h &= \frac{1}{\rho \cos \alpha}
\end{align*}
\] (4)

\[
\begin{align*}
\rho &= \frac{r}{R} \\
\xi &= \frac{Z}{R} \\
\kappa &= \frac{h}{R} \\
\sigma &= \frac{\rho (1 + \rho^2)^{1/2}}{\xi}
\end{align*}
\] (5)

As shown in Figure 3, while calculating the equations, the direction of meridian curvature changed at inflection point where \( r = \sqrt{15} \rho_0 \). At point \( r = \sqrt{2} \rho_0 \), meridian and parallel curvature radius are equal and maximum which is

\[ R_\varphi = (R_\varphi)_{\text{max}} = \frac{R}{\sin 2\alpha_c} \] (7)

Therefore, part of dome meridian after constant curvature point is replaced with a circle arc [13], and the arc radius is \((R_\varphi)_{\text{max}}\).

![Figure 3. Inflection point and modified meridian of helical winding dome](image)

3.2.2. Helical Winding Ellipsoid Dome. Ellipsoid dome is usually utilized for the convenience of fabrication, Eqs. (8) and Eqs. (9a) ~ (9c). The meridian equations of ellipsoid dome are given, as illustrated in Figure 4, so the governing Eqs. (10a) ~ (10b) are solvable. It is important to note that this kind of dome is not isotensoid, fiber stress is changeable as show in equation (10a).
3.2.3. Polar Winding Dome. Polar winding covers the vessel completely up to the poles. It is only for short vessels with the total length \( L \leq 2D \) where \( D \) is the vessel diameter. Two geometry equations can be got by analysing the finite element from the dome [14], as shown in Figure 5. Substitution of Eqs. (11) and (12) into (4), after some arrangements, leads Eqs. (13).

Similarly, there is also an inflection point at dome meridian curve, where winding angle \( \alpha = 54.7^\circ \). The same method which replaces the part of meridian with an arc after the constant coverture point is used to solve this problem.

\[
\begin{align*}
\rho^2 + m^2 \xi^2 &= 1 \quad (8) \\
\rho(\xi) &= \sqrt{1 - m^2 \xi^2} \quad (9a) \\
\dot{\rho}(\xi) &= -\frac{m^2}{{\sqrt{1 - m^2 \xi^2}}} \quad (9b) \\
\ddot{\rho}(\xi) &= -\frac{m^2}{(1 - m^2 \xi^2)} \sqrt{1 - m^2 \xi^2} \quad (9c)
\end{align*}
\]

\[
\begin{align*}
\tilde{\sigma} &= \frac{\rho^2 (1 + \rho^2)^{1/2}}{(\rho^2 - \rho_0^2)^{3/2}} \quad (10a) \\
\sin \alpha &= \frac{\rho_0}{\rho} \quad (10b) \\
\dot{\rho} &= -\frac{m^2 \xi^2}{\sqrt{1 - m^2 \xi^2}} \quad (10c) \\
\ddot{r} &= \frac{1}{\rho \cos \alpha} \quad (10d)
\end{align*}
\]
3.2.4. Polar Winding Ellipsoid Dome. The fourth type of dome is polar winding ellipsoid dome. Governing Eqs. (14) are almost the same with Eqs. (13), but replacing the meridian equation with elliptical one.

\[
\sin \theta = \rho_e + \xi \tan \alpha_0
\]  

\[
\tan \alpha = \rho \frac{\dot{\theta}}{\sqrt{1 + \rho^2}}
\]  

\[
\tan^2 \alpha = \frac{[\tan \alpha - \rho (\rho_e + \tan \alpha_0)]^2}{(1 + \rho^2) [\rho^2 - (\rho_e + \tan \alpha_0)^2]}
\]  

\[
\bar{\rho} = \left[ \frac{[\tan \alpha - \rho (\rho_e + \tan \alpha_0)]^2}{(1 + \rho^2) [\rho^2 - (\rho_e + \tan \alpha_0)^2]} - 2 \right]^{1/2}
\]  

\[
\bar{H} = \frac{1}{\rho} \left[ 1 + \frac{[\tan \alpha - \rho (\rho_e + \tan \alpha_0)]^2}{(1 + \rho^2) [\rho^2 - (\rho_e + \tan \alpha_0)^2]} \right]^{1/2}
\]  

\[
\bar{\sigma} = \rho^2 \left[ 1 + \rho^2 + \frac{[\tan \alpha - \rho (\rho_e + \tan \alpha_0)]^2}{\rho^2 - (\rho_e + \tan \alpha_0)^2} \right]^{1/2}
\]

4. Results & Discussion

Four cases are calculated in this paper, the material is T700/Epoxy resin. As shown in Table 1, the outer radiuses are all 500mm. For Ex.1 and Ex.2, the cylinder length is 800mm, front-polar and rear-polar radiuses are respectively 150mm and 250mm. For Ex.3 and Ex.4, the cylinder length is 2000mm, front-polar and rear-polar radiuses are the same 250mm. Ellipsoid ratio of Ex.2 and Ex.4 is 1.4.

| No. | Dome Types          | Cylinder Outer Radius(mm) | Cylinder Length(mm) | Front-Polar Radius(mm) | Rear-Polar Radius(mm) | Ellipsoid Ratio |
|-----|---------------------|---------------------------|---------------------|------------------------|------------------------|-----------------|
| Ex.1| Polar Winding       | 500                       | 800                 | 150                    | 250                    | -               |
| Ex.2| Polar Winding       | 500                       | 800                 | 150                    | 250                    | 1.40            |
|     | Ellipsoid           |                           |                     |                        |                        |                 |
| Ex.3| Helical Winding     | 500                       | 2000                | 250                    | 250                    | -               |
| Ex.4| Helical Winding     | 500                       | 2000                | 250                    | 250                    | 1.40            |
|     | Ellipsoid           |                           |                     |                        |                        |                 |
Figure 6. Polar winding dome geometry

Figure 7. Meridian r-z

Figure 8. Winding Angle α-z

Figure 9. Thickness h-z

Figure 10. Fiber Stress σ-z
It can be found that dimensions of four rear domes are the same. The results are discussed as follows:

1) Meridians of Ex.1, 2&4 are almost the same, and dome height is about 305mm; while dome height of Ex.3 is the largest 332mm. It is because of the difference of winding patterns. It can also be seen that the meridians of Ex.1 & Ex.3 are continuous before and after inflection point, so the method which replaces parts of meridians with circle arc is reasonable.

2) The axis y of Figure 8 & Figure 9 are winding angle and thickness respectively. It can be seen that results of the same winding pattern are similar. Winding angle and thickness of helical winding is larger than ones of polar winding.

3) Fiber stress of the same winding pattern is similar, but there are un-continuous points at the curves of Ex.1&Ex.3. From the fiber stress equation (4b), stress is the functions of winding angle, meridian and its first-order derivative, and only meridian’s first-order derivative is un-continuous at inflection point, so that is why fiber stress curves un-continuous. It is suggested to utilize Lamination Theory instead of Netting Theory to get more accurate fiber stress.

5. Conclusion
The conclusion of this paper is as follows:

1) Based on a simplified geometry parametric model, cylinder and dome are systematically studied;

2) Governing equations of two Filament-wound patterns and four dome types are derived in detail. A modified meridian computing method is supplied for polar and helical winding domes after the inflection point;

3) Results of four rear domes are compared in detail. Parameters of the same winding pattern are similar. Ellipsoid domes are preferred to utilize under the same conditions.

The future work is suggested as below:

1) Non-geodesic Winding will be utilized to enlarge the design space for FPCV without limitation of polar radius and cylinder length;

2) Laminated method or other FEA method will be studied to obtain more accurate fiber stress.
results.

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