Cosmology with Clusters in the CMB

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Abstract. Ever since the seminal work by Sunyaev and Zel’dovich describing the distortion of the CMB spectrum, due to photons passing through the hot inter cluster gas on its way to us from the surface of last scattering (the so called Sunyaev-Zel’dovich effect (SZE)), small scale distortions of the CMB by clusters has been used to detect clusters as well as to do cosmology with clusters. Cosmology with clusters in the CMB can be divided into three distinct regimes: a) when the clusters are completely unresolved and contribute to the secondary CMB distortions power spectrum at small angular scales; b) when we can just about resolve the clusters so as to detect the clusters through its total SZE flux such that the clusters can be tagged and counted for doing cosmology and c) when we can completely resolve the clusters so as to measure their sizes and other cluster structural properties and their evolution with redshift. In this article, we take a look at these three aspects of SZE cluster studies and their implication for using clusters as cosmological probes. We show that clusters can be used as effective probes of cosmology, when in all of these three cases, one explores the synergy between cluster physics and cosmology as well take clues about cluster physics from the latest high precision cluster observations (for example, from Chandra & XMM–Newton). As a specific case, we show how an observationally motivated cluster SZ template can explain the CBI-excess without the need for a high $\sigma_8$. We also briefly discuss ‘self-calibration’ in cluster surveys and the prospect of using clusters as an ensemble of cosmic rulers to break degeneracies arising in cluster cosmology.

1. Introduction
Clusters of galaxies, observed mainly in optical and X-rays, and for the last 3 decades in SZ, have long been used as important cosmological probes. These observations in combination with analytical modeling and hydrodynamic simulations points at interesting physics associated with cluster evolution and appearance. With more observational bands coming routinely into use to look at clusters, multiple clusters surveys have been commissioned or planned and there is a new impetus to learn more about clusters and to use clusters for precision cosmology. The time is ripe for cluster cosmology.

Galaxy clusters are unique as astrophysical objects. They are the pinnacle of the structure formation hierarchy as the largest objects that have had time to collapse under the influence of their own gravity. They are also the largest objects to have undergone gravitational relaxation and entered into virial equilibrium. Cluster samples, most notably in X-rays, have lead to a good understanding of cluster physics, but have raised questions on plasma physics, properties of the dark matter, and feedback processes. Numerical hydrodynamical simulations have just now become able to reproduce these processes which illustrates the richness of physics of the intra-cluster medium. The baryonic component of clusters therefore contains a wealth of information about the processes involved in galaxy and cluster formation and in enriching and energizing
the cluster gas. However, to first approximation, clusters form a one parameter family whose properties are characterized by its mass. Precision cosmology with upcoming SZE surveys depends crucially in understanding how mass is related to the observable quantities. The currently precise X-Ray observations from *Chandra* & *XMM-Newton* help us in modelling the cluster gas properties, and with the numerical simulations, relating these properties to the underlying mass distribution.

The key concept which relates cluster numbers to cosmology is the excursion set formalism which allows to compute the cluster abundance from the dark matter power spectrum, assuming spherical collapse. In addition, the cluster correlation function can be computed, with a model for the strong cluster bias and the halo occupation distribution. For comparison to observations, both quantities need to be supplied with mass-observable relations, the most important of which are the baryon fraction and the mass-temperature/mass-flux relation. Moreover, when the clusters are unresolved, they contribute statistically to CMB anisotropies at arc-min scales (roughly $\ell > 2000$) and the amount of these SZE fluctuations depend on the cluster to cluster SZE imprint on the CMB sky as well as the background cosmology, most noticeably that on the value of amplitude of mass fluctuations given by $\sigma_8$. Finally, when the clusters are resolved, one can estimate the cluster size by observing it in both X-Ray and SZE and the ratio of this physical size to its angular size gives an estimate of the angular diameter distance at the cluster redshift, thus giving us another handle on cosmological parameter. This method has been traditionally used to measure the the Hubble constant $H_0$. In future, an ensemble of such clusters as physical rulers can be used to measure other cosmological parameters as well as to break degeneracies.

### 2. The Sunyaev-Zel’dovich Effect

The SZ effect is caused by the inverse Compton-scattering of photons when they interact with the hot electrons of the intra-cluster medium in a cluster of galaxies. The scattering boosts the energy of the CMB photons by $k_B T_e/m_e c^2$ per scattering on the average. Typically, for massive clusters of mass $\sim 10^{15} M_\odot$, the temperature anisotropy due to the SZ effect is about $10^{-4}$. The SZ effect leads to a decrease in the intensity of the CMB in the Rayleigh Jeans region and an increase at higher microwave frequencies. Thus the modified spectrum cuts the original CMB Planck spectrum at a frequency, $\nu_0$, which is typically 217 GHz.

The spectral distortion of the CMB due to the SZ effect is very unique and is given by

$$\frac{\Delta T_{SZ}}{T_{CMB}} = f(x) y = f(x) \int n_e \frac{k_B T_e}{m_e c^2} \sigma_T \, dl$$

(1)

where $x = \frac{\hbar \nu}{k_B T_{CMB}}$, $y$ is the Compton $y$-parameter, $\sigma_T$ is the Thomson cross-section, $n_e$ is the electron number density and $T_e$ is the electron temperature.

$$f(x) = \left( \frac{x e^x + 1}{e^x - 1} - 4 \right) \left( 1 + \delta_{SZ}(x, T_e) \right),$$

(2)

$\delta_{SZ}(x, T_e)$ is the relativistic correction to the frequency dependence.

Integrating the SZ over the solid angle of the cluster, $d\Omega = dA/D_A^2$, where $D_A$ is the angular diameter distance gives

$$Y_{SZ} = \int_{\nu_0} \Delta T_{SZ} \, d\Omega \propto \frac{N_e \langle T_e \rangle}{d_A^2}$$

(3)

where $N_e$ is the total number of electrons in the clusters. The angular diameter distance $d_A(z)$ does not vary much at high redshifts.

For the same mass, clusters at higher redshift will have a higher temperature. An SZ survey thus has a mass threshold which has very little dependence on redshift. This is unlike XRay
surveys where the mass threshold increases continuously with redshift. Thus, a cluster is easier to find (and thus contributes) at higher redshift using SZE than XRay.

3. Cosmology with Cluster SZE power spectrum: Implications for the CBI & ACBAR observations

In this section, we take a look at the cause for the excess in CMB anisotropy at $l > 2000$ reported by CBI and ACBAR. At these angular scales, the primary CMB anisotropies are exponentially damped out and the cause of this excess has been attributed to Sunyaev-Zel’dovich distortions power spectrum ($\text{SZ-C}_\ell$) by an ensemble of galaxy clusters. This, then, gives us a strong probe of normalization of matter power spectrum, $\sigma_8$, since the $\text{SZ-C}_\ell$ has a strong dependence on $\sigma_8$ ($C_\ell \propto \sigma_8^{6-8}$). The CBI and ACBAR excess can thus be interpreted as that due to cluster SZ with a $\sigma_8 \sim 1$ which is inconsistent with the best fit WMAP $\sigma_8 \sim 0.8$. A number of hypothesis has been put forward to resolve this discrepancy ranging from extra contribution (e.g distortion from extra galactic point sources) to more esoteric (like explicit non-gaussianity in the matter fluctuations). The SZ effect depends on both cosmology and the cluster gas properties. We find that the cluster templates generally used do not give cluster global observables such as the observed mass to temperature scaling relation. To overcome this mismatch, we build a phenomenological model of cluster structure based on observations (where available) and simulations (as guide at large cluster radii). We find that with simple and reasonable assumptions about cluster temperature profile, the total gas fraction within cluster virial radius, the cluster boundary being at a radius greater than the virial radius and hydrostatic equilibrium for the cluster gas inside the dark matter halo, one can get a boost in the cluster SZ power spectrum for any particular $\sigma_8$. Thus, we can explain the CBI and ACBAR data to be due to cluster SZ with a $\sigma_8$ much closer to the WMAP value.

3.1. Modelling the cluster structure

3.1.1. The dark matter halo

The universal NFW density profile is given by

$$\rho_{\text{dm}}(x) = \frac{\rho_s}{x(1 + x)^2},$$

(4)

where $\rho_s$ is a scale density; $x = \frac{r}{r_s}$ and $r_s$ is a scale radius. The concentration parameter $c(M_{\text{vir}}, z)$ is defined by

$$c(M_{\text{vir}}, z) \equiv \frac{r_{\text{vir}}(M_{\text{vir}}, z)}{r_s(M_{\text{vir}}, z)} \approx \frac{10}{1 + z} \left[ \frac{M_{\text{vir}}}{M_*(0)} \right]^{-0.2},$$

(5)

$M_{\text{vir}}$ is the virial mass and $M_*(0)$ is a solution to $\sigma(M) = \delta_c$ where $\delta_c$ is the threshold overdensity of spherical collapse at $z=0$ and $\sigma(M)$ is the variance of mass fluctuations on mass scale M. The virial radius is calculated using the spherical collapse model

$$r_{\text{vir}}(M_{\text{vir}}, z) \equiv \left[ \frac{M_{\text{vir}}}{(4\pi/3)\Delta_c(z)\rho_c(z)} \right]^{1/3},$$

(6)

where $\Delta_c(z)$ is a spherical overdensity of the virialized halo within $r_{\text{vir}}$ at $z$, in units of the critical density of the universe at $z$, $\rho_c(z)$. Here $\rho_s$ is defined as

$$\rho_s = c^3 \frac{M_{\text{vir}}}{4\pi r_{\text{vir}}^3 m(c)},$$

(7)

with $m(x) = \log(1 + x) - \frac{x}{1 + x}$.
The observed and adiabatic base model $M_{500} - T$ relation at $z=0$ (left panel) and $z=1$ (right panel). The observed scaling relation is constructed from XMM observations of clusters (Arnaud et al 2005). The theoretical relation depends on the modelling of the ICM gas density and temperature profiles, using cluster gas model of Komatsu & Seljak (2001). The temperature refers to the spectroscopic weighted temperature between $0.1R_{200} - 0.5R_{200}$ following the convention given in Rasia et al (2005). Notice that observed clusters are hotter at lower masses.

**3.1.2. ICM models and the cluster mass - temperature relation**   The mass temperature relation is the relation between the mass of a cluster and the average temperature. The mass of a cluster is not usually measured observationally and is connected to cluster observables through this relation. Hierarchical structure formation leads to a self-similar behaviour in clusters. Let $M_\delta$ is the mass within the radius $R_\delta$ inside which the mean mass density is $\delta$ times the critical density. A relation of the form $E(z)M_\delta = A(\delta)T^{3/2}$ is expected if the clusters are self similar and follow the equation of hydrostatic equilibrium. Here $T$ is a weighted mean of the temperature since clusters are not isothermal.

In reality clusters do not follow the self-similar relation fully. The entropy of the cluster gas is determined by gravitational processes as well as other non-gravitational processes that can influence the gas density and temperature. The comparison between observation and theory thus depends on which of these processes is taken into account in simulations/analytic calculations besides the definition of average temperature used.

Uncertainty in the normalisation of the M-T relation severely affects the cluster mass function measurements with XRay or SZE observations. This leads to a systematic uncertainty in cosmology parameter values derived from cluster temperatures.

The observed M-T relations of Arnaud et al, 2005 (using XMM – Newton) and Vikhlinin et al, 2006 (using Chandra) have slopes with the self-similar value when only hot clusters are considered($T>3.5$ keV). In general, observations show lower mass clusters to be hotter than expected from simulations (see Figure 1). Non-adiabatic simulations and analytic models that take into account cooling and galaxy feedback predict higher temperatures and thus lower normalisations. Arnaud et al find that non-adiabatic simulations are closer to their observed $M_{2500}$-T relation well. The simulations used a mass-weighted temperature $T_{m,2500}$ while Arnaud et al used a spectroscopic temperature which were supposed to be similar within $R_{2500}$. However greater discrepancies were found at $R_{500}$ and $R_{200}$ which is due probably to the different physical models used.

**Figure 1.** The observed and adiabatic base model $M_{500} - T$ relation at $z=0$ (left panel) and $z=1$ (right panel). The observed scaling relation is constructed from XMM observations of clusters (Arnaud et al 2005). The theoretical relation depends on the modelling of the ICM gas density and temperature profiles, using cluster gas model of Komatsu & Seljak (2001). The temperature refers to the spectroscopic weighted temperature between $0.1R_{200} - 0.5R_{200}$ following the convention given in Rasia et al (2005). Notice that observed clusters are hotter at lower masses.
3.1.3. Phenomenological cluster model  We start with ‘adiabatic base model’ (say, that given by Komatsu & Seljak (2001)) for two situations: (a) a polytropic temperature profile with cool core and (b) without a cool core. The cool core extends upto $0.1r_{500}$ and $T$ goes as $r^{0.4}$ within the cool core (Sanderson, Ponman & O’Sullivan, 2006).

For the observational M-T relation we use the $M_{500}$-$T$ relation of Vikhlinin et al(2006).They use an M-T relation of the form $E(z)M_{\delta} = A(\delta)(T/5keV)^{\alpha}$. Here $\alpha$ is the slope and $A(\delta)$ is the normalisation. The slope and normalisation they obtain are $\alpha = 2.89 \pm 0.15$ and $A(\delta) = 1.58 \pm 0.11 \times 10^{15}M_{\odot}$. They have used the spectroscopic temperature calculated between $0.15r_{500}$ and $r_{500}$.

There are differences between the observed and the theoretical $M_{500} - T$ relations. The observed temperature is higher than the theoretical temperature at lower masses. This relation reverses at the higher masses. Thus one can get the ratio of the spectroscopic temperatures at each mass and redshift and call this a ‘normalizing factor’. This normalising factor is then used to normalize the base model temperature to the observed temperature. The equation of

Figure 2. Gas density profiles for mass $10^{14}h^{-1}M_{\odot}$ at two different redshift for $z = 0.01$ (lower set) and $z = 1$ (upper set) for the phenomenological model. The cool core clusters are shown in dashed line and non cool core clusters in solid line. The x-axis is $R/R_{VIR}$.

Figure 3. Gas temperature profiles for mass $10^{14}h^{-1}M_{\odot}$ at two different redshift for $z = 0.01$ (lower set) and $z = 1$ (upper set) for the phenomenological model. The cool core clusters are shown in dashed line and non cool core clusters in solid line. The x-axis is $R/R_{VIR}$. 
hydrostatic equilibrium is then solved to give the new density / temperature profiles.

We use gas mass-normalisation to normalize the gas density profiles. This is done by requiring that the total gas mass within the cluster up to a certain 'normalising radius', which we chose to be twice the virial radius, is given by $\frac{\Omega_b}{\Omega_m} M_{\text{vir}}$.

One can now find the new average temperature and 'normalising factor' and thus solve the equation of hydrostatic equilibrium iteratively. The iterations converge to profiles shown in the figures 2 and 3. The density after iterations thus increases with respect to its original value in the central regions and decreases in the outside regions. The pressure profiles also behave in a similar fashion. These final pressure profiles are then used to calculate the expected SZE templates on the sky and hence the SZE power spectrum. As a sanity check for our method we also calculate the SZE scaling relations between SZE total flux observed with $r_{2500}$ as a function of mass and temperature and compare them with the observed scaling relations got by Bonamente et al (2007). We see that we our X-Ray observations motivated cluster SZ model matches the SZE observations quite well. This is shown in figure 4.

![Figure 4](image_url)

**Figure 4.** Comparison of the observed SZE scaling relation given by Bonamente et al (2008) and our simple model of clusters. The left panel shows the SZE flux-temperature scaling and the right panel shows the SZE flux-mass scaling.

### 3.2. Modelling the SZ power spectrum

The SZ power spectrum can be derived from either simulations or analytically. The SZ power spectrum from various hydrodynamical numerical simulations agree within a factor of two. The resolution of the simulations however affects the power at small angular scales while the size of the simulations affects the power at large scales. Analytic computations are based on the cluster mass function and the cluster model. For a cluster, the Compton $y$ parameter is defined as

$$\frac{\Delta T(\theta)}{T_{\text{cmbr}}} = g(x)y(\theta), \quad (8)$$

Expanding the two point angular correlation function into Legendre polynomials one gets,

$$\langle \frac{\Delta T}{T_{\text{cmbr}}} (n) \frac{\Delta T}{T_{\text{cmbr}}} (n + \theta) \rangle = \frac{1}{4\pi} \sum_{\ell} C_{\ell} P_{\ell}(\cos \theta). \quad (10)$$
When discrete sources are considered, $C_\ell = C_\ell^{(P)} + C_\ell^{(C)}$. Here, $C_\ell^{(P)}$ is the contribution from the Poissonian noise and $C_\ell^{(C)}$ is from the correlation among clusters. $C_\ell^{gg} \equiv C_\ell/g^2(x)$ which is independent of frequency.

The frequency independent $C_\ell$ s are thus given by

$$C_\ell^{gg(P)} = \int_0^{z_{\text{dec}}} \frac{dV}{dz} \int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn(M,z)}{dM} |y_t(M,z)|^2,$$

$$C_\ell^{gg(C)} = \int_0^{z_{\text{dec}}} \frac{dV}{dz} \int_{M_{\text{min}}}^{M_{\text{max}}} dM \Phi_\ell(M,z)^2,$$

where $V(z)$ and $r(z)$ are the comoving volume and the comoving distance. $\Phi_\ell(M,z)$ is the ‘biased temperature function’:

$$\Phi_\ell(M,z) \equiv \frac{dn(M,z)}{dM} b(M,z) y_t(M,z),$$

where $y_t$ is the angular Fourier transform of $y(\theta)$. The cluster correlation function $P_\ell(k, M_1, M_2, z)$ is related to the matter power spectrum via $P_\ell(k, M_1, M_2, z) = b(M_1, z) b(M_2, z) D^2(z) P_m(k, z = 0)$.

Dark matter haloes do not cluster in the same way as the mass density field. The linear bias factor $b(M,z)$ is used to quantify the difference. Here, $D(z)$ is the linear growth factor for density fluctuations. For beams comparable to the size of the richest clusters, the Poissonian approximation is valid. For very large beams however the cross-correlation becomes important. In our work, we neglect the correlation term since they are only important at $l < 200$ whereas we are interested at $l > 2000$.

Our results for the SZE $C_\ell$ for our cluster model is shown in figure 5. As can be seen right away, due to our improved modeling of the cluster structure, the SZE imprint of each cluster is greater than previous estimates (and also agreeing with both X-Ray and SZE observations) and hence we need a lower $\sigma_8$ than needed before to account for CBI arc-min scale measurements of the CMB distortion. Due to relatively more contribution from the smaller masses, our $\sigma_8$ scaling of the SZE $C_\ell$ goes as $\sigma_8^{1.6}$ in the $\ell$-range of interest. We find that we need $\sigma_8 \sim 0.9$ to explain the CBI results. This is within $3\sigma$ of the WMAP best fit value of $\sigma_8 = 0.817$ whereas a previously estimated $\sigma_8 \sim 1$ to explain the CBI results is $7\sigma$ away from the WMAP value. In this current article, we do not show the modifications when one accounts for additional pressure support of the virial mass by the dispersion velocity of the gas. When one takes this into account, the gas pressure (and hence density) is lower at higher radii. However, to conserve the total gas mass fraction, as seen be observations and simulation, the density is higher at inner radii. The resulting SZE $C_\ell$’s are higher and hence one would need an even smaller $\sigma_8$ to explain the CBI observations.

4. Cosmology with SZE detected galaxy clusters in a survey

4.1. The Cluster Redshift Distribution

The observed cluster redshift distribution in a survey is the comoving volume per unit redshift and solid angle $dV/dz d\Omega$ times the comoving density of clusters $n_{\text{cls}}$ with masses above the survey detection limit $M_{\text{lim}}$. This can be written as

$$\frac{dN}{dz d\Omega} = \frac{dV}{dz d\Omega} = \frac{c}{H(z)} d_A^2(z) (1 + z) \int_{M_{\text{lim}}(z)}^{\infty} dM \frac{dn}{dM},$$

where $dn/dM$ is the cluster mass function, $H(z)$ is the Hubble parameter as a function of redshift and $d_A$ is the angular diameter distance. The expansion history of the universe is
Figure 5. The SZE $C_\ell$ for our phenomenological model of cluster structure for different values of $\sigma_8$. Inlaid are the CBI and ACBAR results for arc-min scale CMB anisotropy. cc & ncc refer to cool core and non cool core clusters. The WMAP estimate of $\sigma_8 = 0.817$. Notice that we need $\sigma_8 \sim 0.9$ to explain the CBI excess.

given by $H(z) = H_0 E(z)$, where $H_0$ is the Hubble parameter and the parameter $E(z)$ describes its evolution such that $E^2(z) = \Omega_M (1+z)^3 + (1-\Omega_M-\Omega_{DE}) (1+z)^2 + \Omega_E (1+z)^{3(1+w)}$. The expansion history of the universe bears the signature of dark energy and affects both the comoving volume as well as the growth of structures.

Other than the volume and the growth dependence, the survey yield depends sensitively on the mass limit. This is connected to the survey flux limit $f_{lim}$ over which an instrument is capable of detecting clusters. The connection between these two are through the mass to the integrated SZ flux-mass or central SZ decrement-mass relations for SZE surveys and depend on both cosmology as well as cluster physics. In figure 6, we show the expected cluster counts for different cosmology and cluster physics.

4.2. Self-Calibration and Synergy of cluster physics and cosmology

A few years back people realized that a sufficiently large survey allows one to measure cosmological parameters and constrain the cluster mass–observable relation simultaneously if one assumes perfect knowledge of the redshift evolution of galaxy cluster structure. In the subsequent years, different authors have developed this very important idea and many calculations have been made underscoring the importance of incorporating information from multiple observables into future cluster surveys, and they demonstrate that cluster surveys are essentially self–calibrating– containing enough information to solve for the mass–observable relation at every redshift and constrain cosmological parameters.

Over the last few years, there has been quite a bit of progress in looking at the prospect of using ‘self-calibration’ in cluster studies. Below, we list the different approaches:
Figure 6. Cluster redshift counts for South Pole Telescope survey having a flux limit of 5mJy. The solid line is for fiducial LCDM model; the green long dashed line is for 10% increase in the value of $\sigma_8$; the blue short-dashed line for similar increase in $\Omega_M$; the purple dotted line is for dark energy model with equation of state given by $w(a) = -0.8 + 0.3a$; the brown triple dashed line shows effect due to 20% change in normalization of SZ flux-mass scaling reln; and the yellow dotted line shows the number counts for flux limit of 8mJy. The figure is taken from Aghanim, Majumdar & Silk (2008).

- Limited mass follow-up (using full hydro equilibrium/weak lensing)
- Using shape of mass-function in redshift slices
- Using the cluster power spectrum and $P(k)$ oscillations
- Adding information from counts-in-cell
- Time or flux slicing of survey: using shape of $dndz$
- For SZ surveys, adding SZ rms distortions to number counts
- Scatter is self-calibrated using both $dndz$ and mass (flux) binning
- Having a subset of clusters observed in both SZ & Xray (assumption on cluster structure crucial)

A more detailed discussion of some of the methods, and SZE surveys in general, can be found in Majumdar (2005) and Aghanim, Majumdar & Silk (2008). In the next section, we describe in a little more detail the last item in the list of ‘self-calibration’ methods.

5. Using SZE resolved clusters as an ensemble of cosmological rulers
As mentioned above, breaking the degeneracies between cosmological parameters as well as those between cluster observables and cosmology is crucial for obtaining tight constraints from clusters as cosmological probes. Recently, angular diameter distance measurements have been done on various clusters using X-Ray scaling relations, as well as, a combination of X-Ray and SZ observations. The angular diameter distance $d_A$, provides us “standard rulers” in measuring...
cosmological distances (see Reese et al. (2002)). Accurate measurement of $d_A$ offers strong constraints on the equation of state of dark energy $q = p/\rho$.

A few years back, Molnar et al (2004) discussed constraints on models with parameters $(\Omega_m, w, h)$, and $(\Omega_m, \Omega_\Lambda, h)$ using simulations of angular diameter distance measurements to clusters of galaxies and showed that the degeneracies in cosmological parameters from this technique are similar to those from SNe Ia, and therefore they are complementary to constraints from redshift distribution of clusters. This is not surprising, since the luminosity distance, which is utilized in the SNe studies, is closely related to the angular diameter distance. We have explored this possibility in greater detail. To do so, we create mock measurements of the angular diameter distances, $d_A$ which are obtained using the combined observations from SZ and X-Ray surveys, taking actual surveys that are commissioned. To these, we add the cosmological constraints due to the change in the number density of clusters as a function of redshift, $dN/dz$. We also include the effects of uncertainties on the cluster observables which contributes to the uncertainties in the determination of cosmological parameters.

5.1. Estimating the angular diameter distance using joint SZE & X-Ray observations

The angular diameter distance $d_A$ of a cluster is traditionally calculated by assuming a spherical $\beta$ model to describe the Intra Cluster Medium (ICM). In this model the ICM is assumed to be radially symmetric. The model is inaccurate when there are asphericities as a result of which there are always inherent uncertainties in the measurement of $d_A$. The electron number density in an isothermal $\beta$ model has the following profile:

$$n_e(r) = n_{e0} \left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta/2},$$

where $n_e$ is the electron number density, $r$ is the radius from the center of the cluster, $r_c$ is the core radius of the ICM while $\beta$ is the power law index.

The temperature change due to the SZ effect in this model is given as,

$$\Delta T = f_{(x,T_e)} T_{CMB} d_A \int d\zeta \sigma T n_e \frac{k_B T_e}{m_e c^2} \left(1 + \theta^2 / \theta_c^2\right)^{(1-3\beta)/2},$$

where $T_{CMB} = 2.728$ K is the temperature of the CMB radiation, $k_B$ is the Boltzmann constant, $\sigma_T$ is the Thompson cross section, $m_e$ is the mass of the electron, $c$ is the speed of light, $\Delta T_0$ is the thermodynamic SZE temperature decrement/increment at the center of the ICM, $\theta$ is the angular radius in the plane of the sky and $\theta_c$ the corresponding angular core radius, and the integration is along the line of sight $l = d_A \zeta$. The function $f_{(x,T_e)}$ encodes the frequency dependence of the thermal SZE, with $x = h\nu/k_B T_{CMB}$ and is given by,

$$f_{(x,T_e)} = \left(\frac{e^x + 1}{e^x - 1} - 4\right) \left[1 + \delta_{SZE}(x, T_e)\right],$$

where $\delta_{SZE}(x, T_e)$ is the relativistic correction to the frequency dependence. In the non-relativistic and Rayleigh-Jeans (RJ) limits, $f_{(x,T_e)} \rightarrow -2$. This correction decreases the magnitude of $f_{(x,T_e)}$ by < 5% (typically 3%).

The X-ray surface brightness is

$$S_x = \frac{1}{4\pi (1+z)^4} d_A \int d\zeta n_e n_H \Lambda e_H$$

$$= S_{X0} \left(1 + \frac{\theta^2}{\theta_c^2}\right)^{(1-6\beta)/2},$$
where $S_X$ is the X-ray surface brightness in cgs units ($\text{erg s}^{-1} \text{ cm}^{-2} \text{ arcmin}^{-2}$), $z$ is the redshift of the cluster, $n_H$ is the hydrogen number density of the ICM, $\Lambda_{eH}$ is the X-ray cooling function of the ICM in the cluster rest frame in cgs units ($\text{erg cm}^3 \text{ s}^{-1}$) integrated over the redshifted ROSAT band, and $S_{X_0}$ is the X-ray surface brightness at the center of the cluster. The normalizations, $\Delta T_0$ and $S_{X_0}$, used in the fit include all of the physical parameters and geometric terms that come from the integration of the $\beta$ model along the line of sight.

One eliminates $n_e$ (noting that $n_H = n_e \mu_e / \mu_H$ where $n_j \equiv \rho / \mu_j m_p$ for species $j$) to solve for the angular diameter distance,

$$d_A = \left( \frac{\Delta T_0}{S_{X_0}} \right)^2 \left( \frac{m_e c^2}{k_B T_{ce}} \right)^2 \Lambda_{eH} \mu_e / \mu_H \frac{4 \pi^{3/2} f^2_{(x,T)}}{f_{(x,T)}} \frac{T_{CM}^2 \sigma_T^2 (1+z)^4}{\Gamma(3/2) \Gamma(3/2 - 1/2) \Gamma(3/2 - 1/2) \Gamma(3/2)}$$

where $\Gamma(x)$ is the Gamma function.

The most important contribution to uncertainties in $d_A$ arise from $\Delta T_0$, $S_{X_0}$, $\beta$ and $\theta_c$. $S_{X_0}$ is affected by the background cosmic X-Ray sources, while the Radio point sources affect the $\Delta T_0$ measurements.

5.2. Adding $dNdZ$ from future surveys to $d_A$

To the measurements of $d_A$ one need to add measurements of clusters from surveys. We consider the following cluster surveys:

(i) A 4000 deg. sq. wide survey with a sensitivity of 8mJy.
(ii) A deep survey with area coverage of 200 deg. sq. and a sensitivity of 5mJy.
(iii) A very large scale survey of area coverage of 30,000 deg. sq. and a sensitivity of 100 mJy.

Since, to get $d_A$ we need some of these clusters to be observed in X-Ray as well, we need to look at X-Ray surveys also. The clusters observed in the overlap of these SZE and X-Ray surveys can be used for our purpose. In particular we compute the constraints on the cosmological parameters for wide and deep surveys. For this we consider the following:

(i) An all sky survey with $f_X = 4 \times 10^{-13} \text{ erg cm}^{-2} \text{ s}^{-1}$.
(ii) A 20000 square degree wide survey with $f_X = 8.25 \times 10^{-14} \text{ erg cm}^{-2} \text{ s}^{-1}$.
(iii) A 200 square degree deep survey with $f_X = 2 \times 10^{-14} \text{ erg cm}^{-2} \text{ s}^{-1}$.

The above surveys are a fair sample of all the SZE and X-Ray surveys going on at present or funded.

5.3. Prospects of combining cluster rulers with number counts

We find that a combination of eROSITA (i.e survey number 2 in X-Rays) with that of SZ survey case (ii) gives the best overlap. We obtain a mock distribution of 100 clusters using a mass limit cutoff of $\sim 8 \times 10^{14} h^{-1} M_{\odot}$. We then bin them into 15 uniformly distributed bins between redshift of $0 < z < 1$. Fisher analysis was performed on this realistic distribution of clusters in redshift space. For $d_A$, the corresponding Fisher matrix was obtained from the covariance matrix obtained using a best-fit from actual data and then extrapolating to our case of 100 clusters.

The implications of using a realistic distribution of clusters in redshift space is indicated in the Figure 7. In this figure, the confidence curves from $d_A$ and number counts turn out to
Table 1. The table shows 1σ uncertainties in the determination of Ω_m, w_0 and σ_8 obtained from a mock d_A survey, number count observations and their combination.

|               | only d_A | only dN/dz | d_A + dN/dz |
|---------------|----------|------------|-------------|
| ΔΩ_m         | 0.53     | 0.018      | 0.015       |
| Δw_0         | 1.1      | 0.47       | 0.17        |
| Δσ_8         | -        | 0.058      | 0.029       |

Figure 7. Left: Confidence limits at 1σ and 2σ for the constraints in Ω_m – w_0 plane. The dash-dot curves are constraints obtained from a mock d_A survey of 100 clusters upto z = 1, while the dashed (blue) curves are constraints from a 4000 deg. dN/dz survey in SZ. The inner solid (black) line shows the corresponding combined constraints. Right: Confidence limits at 1σ and 2σ for the constraints in Ω_m – σ_8 plane. The dashed (blue) curves are constraints from a 4000 deg. dN/dz survey in SZ, while the inner solid (black) line shows the combined constraints from number counts and a d_A survey.

be almost orthogonal in Ω_m - σ_8. This greatly reduces the errors in determination of these parameters when the both the constrains are combined. We also see that adding the fisher matrix form d_A causes a rotation of the confidence curves and also causes a significant reduction in the uncertainties in the determination of Ω_m and σ_8. The net effect of the two surveys in d_A measurements and dN/dz is shown in Table 1.

6. Discussion and conclusions
We have looked at three different scenarios of using clusters in the CMB as cosmological probes: first when they are unresolved and contributed to SZE power spectrum, second when cluster are detected using their SZE flux in surveys and third when the structural properties (like size) are used in well resolved clusters.

We look at the cause for the excess in CMB anisotropy at l > 2000 reported by CBI and ACBAR. The CBI and ACBAR excess can thus be interpreted as that due to cluster SZ with a σ_8 ∼ 1 which is inconsistent with the best fit WMAP σ_8 ∼ 0.8. We point out that the cluster templates generally used does not give cluster global observables such as the observed mass to temperature scaling relation. To overcome this mismatch, we have built a phenomenological
model of cluster structure based on observations (where available) and simulations (as guide at large cluster radii). Within this model, with simple and reasonable assumptions about cluster temperature profile, the total gas fraction within cluster virial radius, the cluster boundary being at a radius greater than the virial radius and hydrostatic equilibrium for the cluster gas inside the dark matter halo, one can get a boost in the cluster SZ power spectrum for any particular $\sigma_8$. Thus, we could explain the CBI and ACBAR data to be due to cluster SZ with a $\sigma_8$ much closer to the WMAP value.

We have also shown that in the presence of ‘self-calibration’, cluster survey is a powerful technique for studying the nature of the dark energy, as well as, any other characteristics of the universe that affects the expansion history, growth of density perturbations or the nature of the transfer function (through $P_{\delta}(k)$). In addition, cluster surveys provide cosmological constraints from different physical properties of the universe than other techniques (i.e. surveys probe the growth of density perturbations whereas SNIa distance measurements only probe the expansion history) and hence enhance our understanding of the evolving universe.

Finally, we have shown that by using $d_A$ measurements from 100 clusters distributed in redshift space in a realistic way, assuming realistic errors in the determination of errors in $d_A$ and taking priors from WMAP 5 data, we are able to determine the cosmological parameters - $\Omega_m$, $w_0$ and $\sigma_8$ accurately. We demonstrate that with the right choice of future surveys having a high sensitivity in detecting clusters both in X-Ray and SZ, these parameters can be measured with an accuracy of a few percent - 5% for the matter density of the Universe $\Omega_m$, 17% for the equation of state of dark energy $w_0$ and 4% for $\sigma_8$ which characterizes the fluctuations in the matter density.

To sum up, we have just entered the golden age of cluster cosmology with SZE clusters.

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