On the angular distribution of IceCube high-energy events

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The detection of high-energy astrophysical neutrinos of extraterrestrial origin by the IceCube neutrino observatory in Antarctica has opened a unique window to the cosmos that may help to probe both the distant Universe and our cosmic backyard. The arrival directions of these high-energy events have been interpreted as uniformly distributed on the celestial sphere. Here, we revisit the topic of the putative isotropic angular distribution of these events applying Monte Carlo techniques to investigate a possible anisotropy. A modest evidence for anisotropy is found. An excess of events appears projected towards a section of the Local Void, where the density of galaxies with radial velocities below 3000 km s\(^{-1}\) is rather low, suggesting that this particular group of somewhat clustered sources are located either very close to the Milky Way or perhaps beyond 40 Mpc. The results of further analyses of the subsample of southern hemisphere events favour an origin at cosmological distances with the arrival directions of the events organized in a fractal-like structure. Although a small fraction of closer sources is possible, remote hierarchical structures appear to be the main source of these very energetic neutrinos. Some of the events may have their origin at the IBEX ribbon.

1 Introduction

The IceCube neutrino observatory has announced the observation of very high-energy events that have been interpreted as the first robust evidence of neutrinos from extraterrestrial origin (Aartsen et al. 2013a,b,c, 2014a; Halzen 2014) other than those detected from the supernova 1987A (see, e.g., Arnett et al. 1989) or the Sun (see, e.g., Bellerive 2004; Bahcall 2005). A total of 37 events have been found in data collected from May 2010 to April 2013, an all-sky search over a period of 988 days, and their deposited energies range from \(\sim 30\) TeV to \(\sim 2\) PeV. Assuming an astrophysical origin for these detections, a number of possible sources have been suggested (see, e.g., Murase et al. 2013; Anchordoqui et al. 2014a; Joshi et al. 2014; Ahlers & Murase 2014). These include both Galactic and extragalactic sources (Halzen 2014) as well as others more exotic and rare (see, e.g., Bhattacharya et al. 2010).

Among the proposed Galactic sources we find the centre of the Galaxy (see, e.g., Razzaque 2013a), its halo (see, e.g., Taylor et al. 2014) with the Fermi Bubbles (see, e.g., Lunardini et al. 2014), and supernova remnants (see, e.g., Padovani & Resconi 2014). Suggested extragalactic sources range from active galactic nuclei (AGN; Winter 2013; Krauss et al. 2014; Padovani & Resconi 2014), radio (Becker Tjus et al. 2014) or starburst galaxies (Anchordoqui et al. 2014b; Liu et al. 2014) to gamma-ray bursts (GRBs; see, e.g., Razzaque 2013b). Decay of a particle much heavier than PeV can be another possible source (Ema et al. 2014). Dark matter annihilation has also been suggested as the origin of the most energetic ones (Zavala 2014) although they can be explained satisfactorily within the framework of the Standard Model of neutrino-nucleon interactions (Chen et al. 2014).

Whatever the source, the angular distribution of events is considered consistent with an isotropic arrival direction of neutrinos (Aartsen et al. 2013c, 2014a) and the majority of events detected by IceCube are believed to be from cosmological distances or, less likely, from the Galactic halo. In a recent update, Aartsen et al. (2014b) have found no evidence of neutrino emission from discrete or extended sources in four years of IceCube data. However, and although any distribution of halo sources is expected to be isotropic, the spatial distribution of galaxies (star-forming or quiescent) located at cosmological distances is far from uniform, being arranged in a hierarchical fashion (see, e.g., Pietronero 1987).

Here, we revisit the topic of the possible isotropic angular distribution of IceCube high-energy events using a Monte Carlo approach. This paper is organized as follows. The characteristics of the data used in our study are summarized in Sect. 2. Section 3 presents our Monte Carlo-based angular distribution analysis; an area of the sky is identified as key to understand the most probable origin of the events studied here. The atmospheric background is further explored in Sect. 4. Our results are discussed within the context of the fractal structure of the Universe in Sect. 5. Conclusions are summarized in Sect. 6.
2 The data

The data used in this study appear in Supplementary Table 1 of Aartsen et al. (2014a). Thirty seven neutrino candidate events are included there, but event 32 was produced by a coincident pair of background muons from unrelated air showers and that made it impossible to reconstruct its arrival direction. This event will be excluded from our analysis as our initial working sample consists of 36 events (see Fig. 1). Twenty-eight of them had shower-like topologies, while the remaining 8 had visible muon tracks. Three events have deposited energies in excess of 1 PeV (in blue in Fig. 1). Event 14, with a deposited energy of nearly 1 PeV (the third highest in the sample), is positionally coincidental (within the angular errors but almost 1.5° from Sagittarius A*) with the location of the supermassive black hole of the Milky Way. Twenty-seven events have negative declination and, therefore, they arrived from the southern hemisphere. This subsample of 27 events is a time- and deposited energy-limited, and perhaps complete, sample.
an effect of the instrumental acceptance. However, the Earth rotation guarantees the uniform exposure of the detectors in right ascension (Aguilar et al. 2013). If we focus on the 27 southern events, there are 18 of them within the right ascension range $8^h$ to $20^h$ and this still translates into a somewhat significant $2.45\sigma$ departure from an isotropic distribution. Besides, this excess of neutrino candidate events towards the right ascension range $8^h$ to $20^h$ is consistent with the anisotropy in TeV cosmic rays identified by Schwadron et al. (2014).

In their study, Schwadron et al. (2014) have found a broad relative excess of cosmic rays between $\alpha \approx 8^h$ and $\alpha \approx 18^h$. The details of the observed anisotropy depend on the cosmic ray energy range considered. The relative excess in arrival direction is oriented towards the region of the sky that includes the downstream direction of the interstellar wind and the downfield direction of the Local Interstellar Magnetic Field (LIMF; Schwadron et al. 2014). These TeV cosmic ray anisotropies are related to the Interstellar Boundary Explorer (IBEX) ribbon (McComas et al. 2009). Neutrinos are not affected by the LIMF, but high-energy cosmic ray particles are and they can produce neutrinos when they interact with atomic nuclei creating unstable particles whose decay generates the neutrinos. As the parent high-energy cosmic ray particles can be deflected by magnetic fields, their original sources are almost impossible to trace. In this scenario, it is theoretically possible to produce extraterrestrial neutrinos at hundreds of AU’s in the outskirts of the solar system; in fact, the structure observed by the IBEX spacecraft is a bright, narrow ($\sim20^\circ$ wide) ribbon of energetic neutral atom emission that likely hints at the presence of an associated overdensity of matter. A putative production of neutrinos towards the edge of the solar system could further complicate the interpretation of the available data.

Figure 3 shows the location of the IBEX ribbon structure with respect to the Galactic equator and the IceCube events. The continuous yellow line follows the Local Interstellar Medium (LISM) magnetic equator as derived by Heerikhuisen et al. (2010). A $20^\circ$ wide band around the LISM magnetic equator appears in pink. The IBEX ribbon is found within that area. Some events appear to be aligned along the band. As the arrival distribution of high energy cosmic rays appears to be affected by perturbations induced in the local interstellar magnetic field by the heliosphere wake (see, e.g., Desiati & Lazarian 2012; Schwadron et al. 2014) some of the detected neutrinos may have been generated in that region.

Returning to the connection between neutrino candidate events and discrete sources, Bai et al. (2014) have provided robust evidence on the possible Sagittarius A* origin of some of the events observed by IceCube. In particular, event 25 coincides spatially as well as temporally with some transient activity observed in X-rays towards Sagittarius A* on 2012 February 9. Another notable spatial coincidence is the galaxy 2dFGRS TGS070Z303 and event 18 (a muon-track event), separated by just over 59 arcseconds. Galaxy 2dFGRS TGS070Z303 has a radial velocity of 37,866 km s$^{-1}$ (Colless et al. 2001). Also the galaxy 2dFGRS TGS280Z153 and event 10 (of shower-like topology) are separated by 2’18. Galaxy 2dFGRS TGS280Z153 has a radial velocity of 53,795 km s$^{-1}$ (Colless et al. 2001). An additional, perhaps significant, coincidence is the LINER AGN 2MASX J13550744-1309568 that is 5’16 from event 23, another muon-track event at coordinates $\alpha = 13^h\!9^m1, \delta = -13^\circ2^\prime$. AGN 2MASX J13550744-1309568 has a radial velocity of 24,924 km s$^{-1}$ (Mauch & Sadler 2007).

3 A Monte Carlo-based angular distribution analysis

Let us consider a sample of $n$ events observed projected on the surface of a sphere, the sky in our case. We want to study the possible non-uniform spatial distribution of these events. In order to do that, we generate random points on the surface of a sphere using an algorithm due to Marsaglia (1972). The positions of these points on the sky have right ascension in the range (0$^h$, 24$^h$) and declination in the range (-90$^\circ$, 90$^\circ$). We perform $10^7$ experiments, each one including $n$ events, randomly distributed on the sky. In our histograms, the adopted number of bins is $2 n_p^{1/3}$, where $n_p$ is the number of pairs. We compute the angular distance, $d$, between two given points (or events) using the equation

$$\cos d = \cos(\alpha_1 - \alpha_2) \cos \delta_1 \cos \delta_2 + \sin \delta_1 \sin \delta_2,$$

where...
\((\alpha_1, \delta_1)\) and \((\alpha_2, \delta_2)\) are the equatorial coordinates of both points. Then count the number of pairs per bin and divide by the number of experiments performed.

The black continuous curves in Fig. 4 show the expected distribution of separations between events on the sky if they are uniformly distributed. The most probable separation is \(\pi/2\) radians or 90°. The distribution associated with the data in Supplementary Table 1 of Aartsen et al. (2014a) is far from regular (top panel) but only one bin deviates \(\sim 2\sigma\) \((2.25\sigma)\) from a uniform distribution. Here, we use the approximation given by Gehrels (1986) when \(n_p < 21\) to compute the error bars as Poisson statistics applies: \(\sigma \sim 1 + \sqrt{0.75 + n_p} \quad (\sigma = \sqrt{n_p} \) otherwise), where \(n_p\) is the number of pairs. In principle, all the events are uncorrelated and distributed according to Poisson statistics. The addition of two independent Poissonian variables (considering terrestrial and extraterrestrial events, see above) is still Poissonian.

Figure 4 top panel, does not take into account the fact that a fraction of the events (11 out of 37) likely have terrestrial origin. Only the extraterrestrial neutrinos are of interest here, i.e. the sample must be decontaminated. Background rejection is implemented following the conclusions obtained by Aartsen et al. (2014a), and also using an algorithm in which we assume that they will have deposited energies under 100 TeV and that (approximately and based on Fig. 2 in Aartsen et al. 2014a) 8 background events have energies in the range 30–53.3 TeV, 2 in the range 53.3–76.7 TeV, and 1 in the range 76.7–100 TeV. In addition, we rank events based on how high is their probability of being part of an isotropic distribution. At the energies of interest here, the angular distribution of atmospheric neutrinos is expected to be very nearly isotropic. We compute the separation distributions for all the possible combinations (more than 600 million) of 11 events and compare with that of an isotropic distribution of the same size. Those combinations of 11 events with root-mean-square deviation from the isotropic behaviour < 0.0047 are selected and the number of times each event appears is counted. Statistically speaking, those with the highest number of counts are more likely to be part of an isotropically distributed subsample. Using the conclusions from Aartsen et al. (2014a) and the events ranked as described, we conclude that the most probable background events are: 1, 3, 9, 18, 27, 28, 29, 31, 32 and 37. These events appear as red empty circles in Fig. 4 (for a different approach to the issue of background rejection see Padvani & Resconi 2014).

If we focus on the 26 events (or 325 pairs) which are most probably of extraterrestrial origin, we obtain the bottom panel in Fig. 4. For this decontaminated sample, the difference between the theoretical (isotropic) value and the actual one is always < 2.5\(\sigma\). However, the range 0 to \(\pi/2\) radians contains 195 pairs, outnumbering by 65 the 130 pairs in the range \(\pi/2\) to \(\pi\) radians; this is a 3.6\(\sigma\) departure from an isotropic distribution, where \(\sigma = \sqrt{n_p/2}\) is now the standard deviation for binomial statistics. Therefore, the irregular distribution on the celestial sphere is different from that of a sample of points uniformly scattered, i.e. an anisotropy exists. An excess of events appears to be located towards the equatorial coordinates \((18^h7^m, -23^\circ)\); this has already been pointed out by Aartsen et al. (2013c). This region is relatively far from the Galactic disk, the densest part of the Milky Way Galaxy, and towards the Local Void (Tully & Fisher 1987). This area of the sky has a confirmed low local density of galaxies with radial velocities below 3,000 km s\(^{-1}\) (see, e.g., Fig. 2 in Nasonova & Karachentsev 2011). This region of low density extends up to distances of 40 to 60 Mpc centred at least 23 Mpc from the Milky Way (Tully et al. 2008) and it may be even larger, perhaps connecting with the Microscopium/Sagittarius void (Kraan-Korteweg et al. 2008). The geometric centre of the Local Void is found towards the equatorial coordinates \((19^h0^m, +3^\circ)\), starting at the edge of the Local Group (Nasonova & Karachentsev 2011). The large grey empty circle in Fig. 4 signals the approximate location and projected extension of the Local Void. Taking into account the error bars, it encloses nearly 25% of all the IceCube events.

On the other hand, the Galactic disk is the brightest source of gamma rays in the sky, can the excess of events pointed out above have their origin in the disk? To test this hypothesis we count the number of IceCube events within

![](image)
Our previous analysis has not taken into consideration the positional errors associated with the events, in particular to those of shower topology, which are very large, much larger than those typical of optical astrometry. It is unlikely that the dominant source of these high-energy neutrinos is in or near the Galactic disk. This is somewhat consistent with results obtained by Aartsen et al. (2014a); in their work, it is found that the correlation with the Galactic plane is not significant, with a probability of 2.8% for a width of the disk of ±7.5°. Restricting the analysis to the range of right ascension 16h to 22h does not show any significant changes and that also makes the Galactic bulge an unlikely source. Results are equivalent for the full sample and the decontaminated one (see Table 1 in boldface).

Table 1 Number of events detected by IceCube as a function of the galactic latitude, \( b \), or distance from the Galactic plane in steps of 5° and the expected number of events derived from an isotropic model (see the text) with its associated error. The difference between the observational data and the isotropic model in units of \( \sigma \) is also given. Results for the decontaminated sample appear in boldface. See Fig. 4 for a reference on the various \( |\Delta b| \) regions.

| \( |\Delta b| (\degree) \) | Isotropic (±\( \sigma \)) | IceCube data | Difference (in units of \( \sigma \)) |
|------------------|-----------------|--------------|------------------|
| 5                | 3.14±2.97       | 3             | 0.05             |
| 10               | 2.27±2.74       | 2             | 0.10             |
| 15               | 4.51±3.29       | 8             | 1.06             |
| 20               | 9.32±4.17       | 15            | 1.36             |
| 25               | 6.73±3.73       | 11            | 1.14             |
| 30               | 8.89±4.10       | 12            | 0.75             |
| 35               | 15.21±4.99      | 18            | 0.56             |
| 40               | 10.98±4.42      | 14            | 0.68             |
| 50               | 18.00±5.33      | 20            | 0.38             |
| 100              | 13.00±4.71      | 16            | 0.64             |

The atmospheric background

The background produced by atmospheric neutrinos is of diffuse nature. These atmospheric neutrinos result from cosmic rays interacting with the atmosphere. So far, our analysis has assumed that this background follows a mathematically isotropic model as the theory predicts. However, in the calculations carried out by the IceCube collaboration, the significance of their results is evaluated by comparing with a distribution obtained by performing yet another analysis over a sample of only-background data sets that are obtained by scrambling real data in right ascension. In other words, right ascension is randomized while keeping the original values in declination. This is done to take into account that the detector has anisotropic acceptance, i.e. its instrumental response function is unknown. This approach assumes that the unknown instrumental response function does not change over time, i.e. it remains the same over the entire search period. Figure 5 shows that under this approach the separation distribution of the 27 southern events and that of an average of 10⁷ only-background data sets are compatible, i.e. the angular distribution of the events is isotropic. However, deviations close to 1.5\( \sigma \) are observed.

5 Discussion

After being emitted, neutrinos move unaffected by gas, dust or magnetic fields. The somewhat significant excess observed towards the equatorial coordinates (18°, 7°, –23°) is an important clue to the origin of the observed events as their associated neutrinos have travelled very little distance (if their sources are Galactic) or across mostly empty space in their vicinity.
The analogous separation distribution for an uniformly distributed sample across a semisphere of ous green line shows the average of pairs is displayed as a continuous black line. The continuous 1 of Aartsen et al. (2014a), the number of pairs is bars, top panel) for the 27 events located towards the south-

Separation distribution (red line/points with error 6 de la Fuente Marcos & de la Fuente Marcos: On the angular distribution of IceCube events
nal leg, the Local Void (if they are of extragalactic ori-

The gray curve in Fig. 6, top panel, shows the distribution of uniformly distributed points, each one is centre for an associated arc of radius R/λ. Within each arc we consider na additional uniformly distributed points, each one is centre for an associated arc of radius R/λ². Within each arc and on the surface of the unit semisphere, we generate na uniformly distributed points. The approach is similar to the one described by Soneira & Peebles (1978).

In our case, and in order to have 27 points (events) per experiment, we consider na = 3, R in the range ~0 to ~2π and λ ∈ (0, 2). After performing a large number of tests, each one consisting of 10⁷ experiments, the best fit to the southern hemisphere data is found for R = π and λ = 0.9. The gray curve in Fig. 6 top panel, shows the distribution of separations for our fractal-like model. Although unable to reproduce the asymmetric behaviour at separations in the range 0.5-1.5 radians (top panel), its deviations from the actual data are not statistically significant and its overall agreement improves for the average separation distribution of synthetic events (blue curve in Fig. 6 top panel). These
deviations are always smaller than those from a uniform distribution. In order to find out how significant the differences between the two models (fractal-like versus isotropic) are and because the sample is rather small, we use the likelihood formula due to Saha (1998). The average value of the $W$ estimator for the fractal-like model is 6 times greater than the one for the isotropic model; therefore, it is statistically more robust. The distribution of events over the southern hemisphere is almost certainly anisotropic.

6 Conclusions

In this paper, we have studied the angular distribution of IceCube high-energy events trying to answer the astrophysically relevant question of whether or not their spread is isotropic to conclude that the observed events are very probably not evenly distributed on the sky. Differences between a uniform angular distribution and the observed one are indeed marginally significant at the $2\sigma$ level or better. Our main findings can be summarized as follows:

- The angular distribution of IceCube high-energy events exhibits a modest but statistically significant anisotropy.
- There is no statistically significant excess of events projected towards the Galactic disk.
- The observed distribution of events appears to be incompatible with an origin in the Milky Way halo.
- A relatively significant excess of events appears projected towards the Local Void. This suggests either a Galactic origin or one well beyond the Local Group for these events.
- The angular distribution of events towards the southern hemisphere appears to be anisotropic and it is statistically more compatible with an origin in a distant, fractal-like organized structure than with the currently assumed isotropicity.

We must stress that our results are based on small number statistics. However, they are compatible with well-established results for GRBs, namely that short GRBs are distributed anisotropically (Balázs et al. 1998). Anisotropy is also observed for the highest energy cosmic rays (e.g. Abreu et al. 2012; Schwadron et al. 2014). Because of the small size of the current data set compared to those of short GRBs or high-energy cosmic rays, the results of our study are necessarily limited in scope as only very large effects can be reliably identified. Although a small fraction of closer sources is possible (perhaps those putatively related to the IBEX ribbon), remote hierarchical structures appear to be the main source of these energetic neutrinos.

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