Long-range universal quantum computation in a large-size coupled cavity array independent of cavity number

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Abstract
We present a new approach for implementing a universal $\sqrt{\text{swap}}$ gate between two spatially far apart sites connected by a large-size coupled cavity array as a quantum bus. The duration is only related to the parity of cavity number but independent of a specific number of cavities, thus it is possible to process quantum information in an arbitrary long distance in principle without time variation. Referring to the recent experimental progress on coupled cavity array, we also make an assessment of the scalability and take the cavity number $N = 5$ as an example to illustrate the robustness of our proposal via quantum process tomography.

Keywords: quantum Zeno dynamics, coupled-cavity array, universal quantum computation

(Some figures may appear in colour only in the online journal)

1. Introduction

One main task in quantum information processing (QIP) is to design a physical quantum computer that utilizes superposition and entanglement to outperform traditional computers by a far greater order of magnitude \cite{1–4}. In a quantum computer, the principle of universal quantum computation guarantees any quantum circuit can be synthesized by a series of single-qubit coherent rotations together with certain particular entangling operations \cite{5–7}. Thus considerable theoretical and experimental efforts have been devoted to simulation of two-qubit quantum unitary operation \cite{8–14} since the pioneering works accomplished in ion-trap systems \cite{15} and cavity quantum electrodynamics systems \cite{16, 17}.

The emergence of coupled cavity models have attracted much attention for they provide the means to overcome the problem of individual addressability and meet the requirement of distributed quantum computation, i.e. performing state transfer, entanglement generation, and quantum gate operations between two distant qubits. In general, a coupled cavity array consists of some optical cavities that permit photons to hop between neighboring cavities. As each cavity is doped with one or more atoms, many interesting physical phenomena are simulated, e.g. the strongly interacting polaritons are experimentally observed in a photonic crystal or coupled toroidal microcavities \cite{18}. Anisotropic Heisenberg spin-1/2 chains and higher spin chains are realized in \cite{19–21}. The transition from Mott state to superfluid state can be achieved via modulating the detuning between the hopping photon and the doped two-level system \cite{22}. Recently, two theoretical proposals for one-way quantum computation are also put forward \cite{23, 24}.

In this letter, we construct a large-size coupled cavity model to perform a two-qubit $\sqrt{\text{swap}}$ gate between two spatially far apart sites. This gate is a universal one and the operation time does not change with the increase in cavity number, i.e. we can carry out the quantum computation in an arbitrary distance in principle. This characteristic may greatly reduce the complexity for experiment control.
2. Physical model

The considered system consists of two A-type configuration qutrit trapped in the end sites of a coupled cavity array, as shown in Figure 1. Two low levels are used to encode qubit |0⟩ and |1⟩, respectively. The transition from |e⟩ to |0⟩ is coupled to the cavity mode with the coupling constant g, and the transition from |e⟩ to |1⟩ is driven by the classical fields with Rabi frequency \( \Omega \). The photon can hop between cavities with coupling strength \( J \), and \( J \) represents the corresponding one-photon detuning parameter. The Hamiltonian in the interaction picture reads

\[
(h = 1): H_I = \sum_{i=1,2} \Delta |e_i\rangle \langle e_i| + g \sum_{i=1,2} \left( a_i |e_i\rangle \langle 0| + a_i^\dagger |0\rangle \langle e_i| \right) + \sum_{i=1}^{N-1} J \left( a_i^\dagger a_{i+1} + a_i a_{i+1}^\dagger \right).
\]

For a two-qubit operation, the computation subspace is spanned by \(|00\rangle, |01\rangle, |10\rangle\rangle, where the cavities are assumed initially in the vacuum state |0⟩. Since the rotating wave approximation conserves the total number of excitations, we reclassify the above subspace according to a different excited number, i.e. \(|00\rangle, |01\rangle, |10\rangle\rangle, decoupled to the dynamical evolution of the system is termed ‘zero-excitation’ subspace; the ‘one-excitation’ subspace includes states with either one photon in the cavity or one atom in an excited state |e⟩; and the definition of ‘two-excitation’ subspace is made in a similar way. Before proceeding, we briefly introduce the concept of quantum Zeno dynamics [25] which enlightens us on this work. Suppose the dynamical evolution of a system is governed by the Hamiltonian \( H_K = H + KH_c \), where \( H \) is the Hamiltonian of the subsystem to be investigated, and \( H_c \) is an additional interaction Hamiltonian performing the ‘measurement’. \( K \) is the corresponding coupling strength. In a strong coupling limit \( K \to \infty \), the system is dominated by the effective Hamiltonian \( H_{eff} = \sum_{n} K_n P_n + P_n HP_n \), where \( P_n \) is the eigenprojection of the \( H_c \) belonging to the eigenvalues \( \eta_n \). In connection with the current model, the interaction between atoms and coupled cavity array plays the role of continuous measurements on the interaction between atoms and the classical fields. In what follows, we concentrate on the dynamical evolution of three other computation bases.

The states \(|10\rangle, |0, \cdots, 0\rangle_c \) \( |01\rangle, |0, \cdots, 0\rangle_c \) belong to the ‘one-excitation’ subspace, after mapping the Hamiltonian of atom-cavity interaction to this subspace, we have a \((N+2) \times (N+2)\) matrix

\[
H_{\text{single}} = \begin{bmatrix}
\Delta_1 & g & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
g & 0 & J & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & J & 0 & J & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & J & 0 & J & \ldots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & J & 0 & J & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & 0 & J & 0 & \ldots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & J & 0 & g & \ddots & \ddots \\
0 & 0 & 0 & 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots
\end{bmatrix}.
\]

For convenience we assume \( g = J \), then equation (1) is equivalent to a tight-binding Hamiltonian with boundary impurities \( \Delta_1 \) and \( \Delta_2 \):

\[
H_{\text{TBH}} = \Delta_1 c_i^\dagger c_1 + \Delta_2 c_M^\dagger c_M + J \sum_{i=1}^{M-1} \left( c_{i+1}^\dagger c_i + c_i c_{i+1}^\dagger \right),
\]

where \( M = N + 2 \) throughout this paper unless otherwise specified. Unlike the homogeneous spin model, the impurities in site 1 and \( M \) break the translational symmetry, which leads to a complicated dynamical process. Fortunately, we may diagonalize the Hamiltonian (2) via the standard Green’s function technique [26]. Define \( H_{\text{TBH}} = H_{OM} + H_1 \), where

\[
H_{OM} = \Delta_2 c_M^\dagger c_M + J \sum_{i=1}^{M-1} \left( c_{i+1}^\dagger c_i + c_i c_{i+1}^\dagger \right) \quad \text{and} \quad H_1 = \Delta_1 c_1^\dagger c_1.
\]

Then the Green’s function of the total Hamiltonian \( H_{\text{TBH}} \) is
Figure 2. Plot of dispersion relation $E_k$ for a seven-site tight-binding Hamiltonian with two equal boundary impurities $\Delta$.

All eigenenergies are bounded in the band $[-2J, 2J]$ for zero defect. As the diagonal defect $|\Delta|$ increases, the two highest (lowest) eigenenergies move outside the band and become quasi-degenerate, and the other five eigenenergies approach the five-site tight-binding Hamiltonian without defects. All energies values are measured in units of $J = 1$ for simplicity.

\[ G = G_{0M} + G_{0M}|1\rangle \langle 1|G_{0M}, \]

where

\[ G_{0M} = G_0 + G_0 \frac{\Delta_2}{1 - \Delta_2 G_{0M}(M, M)} G_0 \]

corresponding to the Green's function of $H_{0M}$ and

\[ G_0 = \frac{2}{N+1} \sum_{i=j=1}^{M} \sum_{k=1}^{M} \sin \left[ \frac{k \pi}{M+1} \right] \sin \left[ \frac{k \pi}{M+1} \right] \frac{2J}{E_k - 2J \cos \left[ \frac{k \pi}{M+1} \right]} |i\rangle \langle j| \]

denotes the Green's function of the Hamiltonian without impurity. Now we can extract all information about the eigenvalues and eigenfunctions of $H_{TBH}$ from the above Green's functions, e.g. the poles of $G$ disclose the spectrum of $H_{TBH}$. In figure 2, we plot the dispersion relation of a seven-site tight-binding Hamiltonian \( (M = 7) \) with two equal boundary impurities $\Delta$ via solving the algebraic equation \( [1 - \Delta G_{0M}(1, 1)] = 0 \). Clearly, in the absence of impurity, all the eigenenergies are bounded in the band $[-2J, 2J]$. The increase of positive $\Delta$ leads the two highest eigenenergies to move outside the band and become quasi-degenerate.

\[ \frac{2}{N+1} \sum_{i=j=1}^{M} \sum_{k=1}^{M} \sin \left[ \frac{k \pi}{M+1} \right] \sin \left[ \frac{k \pi}{M+1} \right] \frac{2J}{E_k - 2J \cos \left[ \frac{k \pi}{M+1} \right]} |i\rangle \langle j| \]

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As $\Delta$ increases further, five other eigenenergies asymptotically approach the values of a five-site tight-binding Hamiltonian without impurity, and the variation of a negative $\Delta$ plays a similar role. To sum up, the effect of large impurities divide an $M$-site tight-binding Hamiltonian into two subspaces: one includes two $\Delta$-dependent eigenstates composed by end sites and the other consists of the middle \((M - 2)\) $\Delta$-independent eigenstates.

With the full eigenenergies at hand, we are able to write down all the eigenfunctions using the Lippman–Schwinger equation: \( |E_k\rangle = |E_k^0\rangle + (E_k - H_0)^{-1} |E_k^0\rangle \), where \( |E_k^0\rangle \) corresponds to the eigenstate for the unperturbed Hamiltonian $H_0$, and \( |E_k\rangle \) indicates the unnormalized eigenstate for the full Hamiltonian. In the language of Green’s function, this equation can be reformulated as $|E_k\rangle = \{ 1 - G_0(E_k) \} V^{-1} |E_k^0\rangle$, where $V = \Delta_1 c_1^c + \Delta_2 c_5^c c_{M}$ in our scheme and

\[
|E_k^0\rangle = \frac{\sqrt{2}}{\sqrt{M+1}} \left\{ \sin \left[ \frac{k \pi}{M+1} \right], \sin \left[ \frac{2k \pi}{M+1} \right], \ldots, \sin \left[ \frac{(M-1)k \pi}{M+1} \right], \sin \left[ \frac{ Mk \pi}{M+1} \right] \right\}
\]

Finally we obtain the standard eigenfunctions of equation (2) as $|E_k\rangle = \frac{1}{\sqrt{\langle E_k | E_k \rangle}} \left\{ f_1^k, f_2^k, \ldots, f_{M-1}^k, f_M^k \right\}$, where $f_i^k = \langle i | E_k \rangle$ describes the probability amplitude of the site $|i\rangle$ in the eigenvector $|E_k\rangle$. Since the system is mirror symmetric, the eigenvectors must be alternately symmetric or antisymmetric, i.e. $\langle i | E_k \rangle = (-1)^{k+i} \langle i | E_k \rangle$ for an ascending arrangement of $E_k$, where $i$ means the mirror-conjugate site of $i$.

Most interestingly, we find that two relations exist in the current model which are independent of the specific ratio between $\Delta$ and $J$, i.e.

\[
\sum_{k=1}^{M} f_k^1 \left\langle E_k \right| E_k \rangle = (-1)^{M-1} \frac{1}{2\Delta} E_k
\]

Equation (3) is the crucial result throughout this letter because it determines the interaction form of two end sites. Explicitly, suppose the Rabi frequencies $|\Omega_1|$ and $|\Omega_2|$ are much weaker compared with the values of $|\langle E_k | E_k \rangle|$, we will always have an effective dipole–dipole interaction between $|10\rangle_0|0, \cdots, 0\rangle_0$ and $|01\rangle_0|0, \cdots, 0\rangle_0$, with a fixed strength $\frac{\Omega_1 \Omega_2}{2\Delta}$ regardless of the length of the chain, so is the case for the stark shifts of two states.

For the computation basis $|11\rangle_0|0, \cdots, 0\rangle_0$, it is a good approximation to expand the corresponding atom-cavity Hamiltonian into the single excitation subspace due to the weak excitation $|\Omega_1|, |\Omega_2|$, the stark shift of state $|11\rangle_0|0, \cdots, 0\rangle_0$ can then be cancelled from the above two uncorrelated block matrices. Therefore, the effective Hamiltonian governing the evolution of whole system reduces to

\[
H_{\text{eff}} = (-1)^{\frac{N}{2}+1} \frac{\Omega_1 \Omega_2}{2\Delta} |10\rangle_0 \langle 01| + \text{H. c.}
\]

\[
+ \frac{\Omega_1^2}{2\Delta} |10\rangle_0 \langle 10| + \frac{\Omega_2^2}{2\Delta} |01\rangle_0 \langle 01|,
\]

where we have discarded the term of cavities for they all stay in the vacuum state. Note equation (4) is derived only under the assumption $|\Omega_1|, |\Omega_2| \ll |\langle E_k | E_k \rangle|$, which has no restriction on the relation between $\Delta$ and $J(x)$. The dipole–dipole interaction between two end sites are caused by summation of $k$ independent virtual-photon-induced Raman transitions. To achieve the two-qubit $\sqrt{\text{SWAP}}$ gate within the shortest time, the related parameters should satisfy $(-1)^{\frac{N}{2}+1} \Omega_1 \Omega_2 = \Omega_1^2 = \Omega_2^2$, which signifies $\left(-1\right)^{\frac{N}{2}+1} \Omega_1 \Omega_2 = \Omega_2 = \Omega_1$. In the left panel of figure 3 we numerically simulate the fidelity of the state transforming
versus Rabi frequency for simplicity. The performance of the two-qubit swap gate is evaluated via the definition of average fidelity [28, 29]

\[ \langle 1 \rangle \Rightarrow (1 + \text{i}/2)|01\rangle + (1 - \text{i}/2)|10\rangle \]

for \( N = 5 \) \( (\text{square}) \), \( N = 29 \) \( (\text{circle}) \), and \( N = 99 \) \( (\text{triangle}) \). The time is reported in the unit of \( T = \pi\lambda/(2f^2) \). The maximal fidelity is obtained at \( T \) for all cases, where the parameters are set as \( d = J = 100/3\Omega \). (Right panel) Average fidelity \( F(t) \) versus Rabi frequency \( \Omega \) and the evolution time \( T \) for a five-coupled cavity system. In each group, the black curve represents an ideal case, while the red and the blue lines are the corresponding decoherence parameters \( \kappa = \gamma = 0.01J \) and \( \kappa = \gamma = 0.1J \), respectively.

3. Effect of decoherence

In the current model, the master equation of the whole system can be expressed by the Lindblad form [27]

\[ \dot{\rho} = -i[H, \rho] - \sum_{j=1}^{N^2} \frac{\kappa}{2} \left( a_j a_j^\dagger \rho - 2a_j^\dagger a_j \rho + \rho a_j^\dagger a_j \right) - \sum_{j=1}^{N^2} \frac{\gamma j^d}{2} \left( \sigma_{j^d}^\dagger \rho - 2\sigma_{j^d} \rho \sigma_{j^d}^\dagger + \rho \sigma_{j^d}^\dagger \sigma_{j^d} \right), \]

where \( \kappa \) denotes the decay rate of the cavity, \( \gamma j^d \) represents the branching ration of the atomic decay from level \( |e_j\rangle \) to \( |g_j\rangle \) and we assume \( \gamma_g = \gamma_a = \gamma / 2 \) for simplicity. The performance of the two-qubit swap gate is evaluated via the definition of average fidelity [28, 29]

\[ F(e, U_{\text{swap}}) = \frac{\sum_{j \in \{0, 1\}} U_{\text{swap}} U_{\text{swap}}^\dagger (U_j) + d^2}{d^2(d + 1)}, \]

where \( d = 4 \) for two qubits and \( U_j \) being the tensor of Pauli matrices II, IX, IY, ···, ZZ. \( U_{\text{ideal}} \) being the ideal swap gate and \( \epsilon \) being the trace-preserving quantum operation obtained through our scheme. In the right panel of figure 3, we illustrate the average fidelity \( F(t) \) for a 5-coupled cavity system under decoherence with three kinds of strength: \( \kappa = 0, \kappa = 0.01J \) and \( \kappa = 0.1J \). In the ideal case, the fidelities are 99.97%, 99.69% and 98.80% corresponding to \( \Omega = 0.01J, \Omega = 0.03J \) and \( \Omega = 0.05J \), which agree well with our previous statement. Although the fidelity decreases with the increase of \( \kappa \), a relatively high fidelity is still available in the range \( \kappa < 0.1J \). To completely characterize the dynamical process, we give the quantum process tomography of the swap gate in figure 4 with the parameters \( \gamma, \kappa, \epsilon \sim (2.5 \times 10^3, 1.6 \times 10^7, 4 \times 10^5) \) Hz referred to in a recent experiment about large-scale arrays of ultrahigh-\( Q \) coupled nanocavities [30]. In the modified Pauli basis \( Y \rightarrow -iY \), the overlap of the chi matrix between our scheme and the ideal one is 99.32%, which confirms the effectiveness of our assumption further. It should be pointed out that the current model is not limited to the realization of a swap gate only. Equation (3) constructs the building block for cavity number-independent long-range quantum information processing, thus other forms of two-qubit
gates such as a CNOT gate and conditional Z gate can also be implemented via modulating the classical fields acting on end atoms. For an even number cavity, we find a similar relation as

\[
\sum_{k=1}^{M} \frac{|f_k|^2}{E_k} = \sum_{k=1}^{M} \frac{|f_k|^2}{E_k} = \frac{\Delta}{\Delta^2 - F^2} \]

which means remote quantum computation can be implemented in this case without a time change either.

4. Summary

In summary, we have presented a scheme for long-range universal quantum computation. This scheme utilizes a large-size coupled cavity array as a medium to induce the Raman coupling between two remote end sites, and the interaction time is independent of a specific number. The virtually excited photon process makes the scheme more robust against the typical decoherence parameters in cavity quantum electrodynamics. We hope that our work may be useful for quantum information processing in the near future.

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