Validity of Landauer principle and quantum memory effects via collision models

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We study the validity of Landauer principle in the non-Markovian regime by means of collision models where the intracollisions inside the reservoir cause memory effects generating system-environment correlations. We adopt the system-environment correlations created during the dynamical process to assess the effect of non-Markovianity on the Landauer principle. Exploiting an exact equality for the entropy change of the system, we find the condition for the violation of the Landauer principle, which occurs when the established system-environment correlations become larger than the entropy production of the system. We then generalize the study to the non-equilibrium situation where the system is surrounded by many reservoirs at different temperatures. Our results, verified through collision models with Heisenberg-type interactions, suggest that the complexity of the environment does not play a significant role in the qualitative mechanisms underlying the violation of the Landauer principle under non-Markovian processes.

I. INTRODUCTION

The increasing abilities in the fabrication and characterization of nanoscale systems allow us to explore the fundamental laws of classical thermodynamics at the quantum level, which gives birth to the subject of quantum thermodynamics [1]. In the usual formulation of quantum thermodynamics, one typically makes Markovian approximations for an open system interacting with ideal heat baths. In this case, the environment is assumed to be able to relax to the equilibrium state at a time scale much faster than that of the system, which means the system is assumed to be able to relax to the equilibrium state at a time scale much faster than that of the system, which induces a monotonous one-way flow of information from the system to the environment. However, in many practical situations the Markov approximation is no longer valid [2–5], so that non-Markovian (memory) effects on the dynamical features of the quantum system have to be taken into account. Although considerable attention has been paid to the formulation of quantitative measures of non-Markovianity [6–11], to its experimental demonstration [12–17], and to the exploration of its origin [18,19], its role in affecting the thermodynamics properties of open quantum systems has so far remained little explored [20–25]. Effect of non-Markovianity in a logically irreversible processes has been investigated based on the Landauer principle [20,21], which gives a direct link between information theory and thermodynamics at both classical and quantum level [26–30]. It is known that the validity of Landauer principle breaks down in the case of non-Markovian environments [20,21]. By means of Landauer principle, it has been shown that the memory effects can control the amount of work extraction by erasure in the presence of realistic environments [22]. A non-exponential time behavior of heat flow in non-Markovian reservoir has been studied [23], where a bipartite system interacts dissipatively with a thermal reservoir in a cascaded fashion.

An efficient microscopic framework in simulating the non-Markovian dynamics is the collision model [31–52], which has also been recently used in the context of quantum thermodynamics [20,21,23,53,54]. The collision model assumes that the environment is a chain of N ancillas and the system of interest S interacts, or collides, with an ancilla at each time step. It has been shown that, when there are no initial correlations among the ancillas and no correlations are created between them during the process, a Lindblad master equation can be derived [32,53]. Collision models allow to recover the dynamics of non-Markovian (indivisible) channels by setting suitable system-environment conditions, for instance by introducing either correlations in the initial state of the ancillas or ancilla-ancilla collisions in the interval between two system-ancilla collisions. In other words, the non-Markovian dynamics can be achieved in the collision model when the system-environment interaction is mediated by the ancillary degrees of freedom. The collision models are also relevant to study some fundamental physical systems, such as the emission of an atom into a leaky cavity with a Lorentzian, or multi-Lorentzian, spectral density or a qubit subject to random telegraph noise [44]. The highly stable and configurable non-Markovian collision-based dynamics can be experimentally implemented with all-optical setups [55,56].

Albeit the Landauer principle has been shown to be violated in non-Markovian dynamics [20,21], the underlying mechanisms under this phenomenon remains to be understood. Here we address this aspect, elucidating the behavior of this thermodynamic principle in the context of quantum memory effects. In particular, we exploit collision models to study, in the non-Markovian regime, the validity of Landauer principle, finding the condition for its violation. In the employed models, the information of system S is firstly transferred to an ancilla of the reservoir via a system-ancilla collision, a part of which then goes to the nearest neighbor ancilla via the ancilla-ancilla intracollision so that the lost information of S can be recovered at the next system-ancilla collision. The reservoir intracollisions lead to system-reservoir correlations, which allow us to assess the influence of non-Markovianity on the Lan-
dauer principle. Following the framework of Ref. [57], we first provide an expression for the entropy change of the system which, apart from the original reversible and irreversible terms, exhibits an additional contribution due to the established system-reservoir correlations in terms of mutual information. We connect this expression to the Landauer principle and obtain the condition for its violation, which occurs when the established system-reservoir correlations are larger than the entropy production of the system. We moreover extend the study to the non-equilibrium case where the system is coupled with several different reservoirs. Also in this case, we show that the amount of correlations arising among the system and all the reservoirs during the dynamics determine whether the Landauer principle is satisfied or not. All these results are confirmed by means of collision models with Heisenberg-like coherent interactions.

The outline of this paper is the following. In Sec. II focusing on a quantum system coupled to a single reservoir, we present the condition for the violation of Landauer principle in the non-Markovian process and its verification via a physical (collision) model. In Sec. III we study the non-equilibrium situation in which the system is surrounded by many reservoirs at different temperatures. We obtain the condition for the violation of Landauer principle in this case and demonstrate the result via a simple model involving two reservoirs. Finally, in Sec. IV we draw our conclusions.

II. LANDAUER PRINCIPLE IN A SINGLE NON-MARKOVIAN RESERVOIR

A. Condition for the violation of Landauer principle

We consider the information erasing process of a quantum system $S$ in contact with a reservoir consisting of a chain of $N$ identical ancillary qubits $R_1, R_2, \ldots, R_N$. The system $S$ and a generic reservoir qubit $R$ are described, respectively, by the Hamiltonians ($\hbar = 1$)

$$
\hat{H}_S = \omega_S \hat{S}_z^2/2, \quad \hat{H}_R = \omega_R \hat{R}_z^2/2,
$$

where $\hat{S}_z^\mu = |1\rangle_\mu \langle 1| - |0\rangle_\mu \langle 0|$ is the Pauli operator and $|0\rangle_\mu, |1\rangle_\mu$ are the logical states of the qubit $\mu = S, R$ with transition frequency $\omega_\mu$ (hereafter, for simplicity, we take $\omega_R = \omega_S = \omega$). Here, we focus on a non-Markovian reservoir through the introduction of interactions between two nearest-neighbor reservoir qubits $R_n$ and $R_{n+1}$ ($n = 1, 2, \ldots, N - 1$).

Our model is illustrated in Fig. 1 for the first two steps of collisions. After the interaction between $S$ and $R_1$, the intra-reservoir collision between $R_1$ and $R_2$ occurs and then the system shifts forward one site to interact with $R_2$. Therefore, the correlation between $S$ and $R_2$ has been established prior to their collision. Generally, the bipartite state $\rho_{SR}$ of $S-R_2$ is correlated before their collision for $n \geq 2$ and such correlation is characterized by the mutual information $I(\rho_{SR_n}) = S(\rho_{SR_n}) + S(\rho_{R_n}) - S(\rho_{SR})$, where $\rho_{SR_n} = \Tr_n \rho_{SR}$, $\rho_{R_n} = \Tr_S \rho_{SR_n}$ is the marginal state of $S$ ($R_n$) and $S(\rho) = -\Tr(\rho \log \rho)$ is the von Neumann entropy of $\rho$. The interaction between $S$ and $R_n$ is ruled by a unitary $\hat{U}_{SR_n}$, by which the state $\rho_{SR_n}$ is transformed to $\rho_{SR_n}' = \hat{U}_{SR_n} \rho_{SR_n} \hat{U}_{SR_n}^\dagger$. In the following, we label the marginal states of $S$ and $R_n$ after the collision as $\rho_{SR_n}' = \hat{U}_{SR_n} \rho_{SR_n} \hat{U}_{SR_n}^\dagger$. It is known that the total von Neumann entropy of $S-R_n$ under the unitary evolution is invariant, whether the state $\rho_{SR_n}$ is product or correlated: $S(\rho_{SR_n}) = S(\rho_{SR_n}')$. In contrast, the entropy of the system is in general a function of time, whose change $\Delta S_n$ by the interaction reads (see Appendix A for details)

$$
\Delta S_n = S(\rho_{SR_n}') - S(\rho_{SR_n}),
$$

where $D(\rho || \rho') = \Tr R \ln \rho - \Tr R \ln \rho'$ is the quantum relative entropy between two states $\rho$ and $\rho'$.

We assume both the reservoir and the system are initially prepared in the thermal states, namely, $\rho_R = e^{-\beta_R R_+}/Z_R$ for the reservoir and $\rho_S = e^{-\beta_S H_S}/Z_S$ for the system, with $\beta_R = 1/k_BT_R$ and $\beta_S = 1/k_BT_S$ the inverse temperatures ($k_B$ being the Boltzmann constant), and $Z_R$ and $Z_S$ the partition functions. In the process of evolution, though the system-environment correlation can be established between $S$ and $R_n$ before their interaction, the reduced state $\rho_{R_n}$ maintains the form of thermal state, i.e., $\rho_{R_n} = e^{-\beta_R H_n}/\Tr[e^{-\beta_R H_n}]$ with the $n$-dependent inverse temperatures $\beta_R$. In this case, we have $\Tr_n[(\rho_{SR_n}' - \rho_{SR_n}) \ln \rho_{R_n}] = \beta_R \Delta Q_n$ with $\Delta Q_n = \Tr_n[(\rho_{R_n} - \rho_{R_n}') H_R]$ denoting the heat flowing from reservoir element $R_n$ to the system $S$. Therefore, Eq. (2) can be written as

$$
\Delta S_n = D(\rho_{SR_n}' || \rho_{SR_n}') + \beta_R \Delta Q_n - I(\rho_{SR_n}).
$$

Without the creation of system-environment correlations during the dynamical process, for instance in the Markovian
regime with no reservoir intracollisions, the mutual information 
\( I(\rho_{SR}) = 0 \) so that \( \Delta S_n = D(\rho^*_{SR} \| \rho_{SR}) + \beta \Delta Q_n \), with the two terms in the RHS being identified as the irreversible and reversible contributions to the system entropy change due to heat exchanges \([57] \). In the non-Markovian regime, however, the system-environment correlations in terms of the mutual information \( I(\rho_{SR}) \) provide an additional contribute to the change of system entropy.

We notice that, at variance with the case of initial system-environment correlations \([50] \), here we consider the problem of system entropy change from a different perspective, that is from the microscopic mechanism of dynamical memory effects. In fact, the term \( I(\rho_{SR}) \) in Eqs. \( 2 \) and \( 3 \) characterizes the system-environment correlations established within the dynamical process, whose amount is determined by the intra-environment collision strength and is thus a signature of the dynamical process, whose amount is determined by the non-Markovianity of thermal reservoir \([51] \). Therefore, instead of resorting to specific non-Markovian quantifiers \([6-11] \), we employ the established system-environment correlations as a figure of merit to reveal the quantitative condition for the validity of Landauer principle in the non-Markovian process. To this purpose, from Eq. \( 3 \), we obtain

\[
\beta_R \Delta Q_n = \Delta S_n + D(\rho^*_{SR} \| \rho_{SR}) - I(\rho_{SR}),
\]

where \( \Delta Q_n = -\Delta S_n \) is the heat dissipated to the reservoir and \( \Delta S_n = \Delta S_n \) denotes the entropy decrease of the system. We refer to the equality of Eq. \( 3 \) as Landauer-like principle in the non-Markovian regime. From Eq. \( 4 \), we can immediately see that, when \( I(\rho_{SR}) \leq D(\rho^*_{SR} \| \rho_{SR}) \), the usual Landauer principle holds, that is \( \beta_R \Delta Q_n \geq \Delta S_n \). In other words, the Landauer principle can still hold as long as the established system-environment correlations are smaller than such an upper bound. On the other hand, from Eq. \( 4 \) one finds that the condition

\[
I(\rho_{SR}) > D(\rho^*_{SR} \| \rho_{SR}),
\]

enables the violation of the Landauer principle during the dynamical process.

### B. Verification via a physical model

In the following, using the collision model depicted in Fig. \[1 \], we verify both the equality of Eq. \( 4 \) [or, equivalently, Eq. \( 3 \)] and the condition for the violation of the Landauer principle given in Eq. \( 5 \).

Among the possible choices for the interaction between \( S \) and a generic reservoir qubit \( R_n \), we focus on a Heisenberg-like coherent interaction described by the Hamiltonian

\[
\hat{H}_{\text{int}} = g(\hat{\sigma}_x^S \otimes \hat{\sigma}_x^{R_n} + \hat{\sigma}_y^S \otimes \hat{\sigma}_y^{R_n} + \hat{\sigma}_z^S \otimes \hat{\sigma}_z^{R_n}),
\]

where \( \hat{\sigma}_j^\mu (j = x, y, z) \) is the Pauli operator, \( g \) denotes a coupling constant and each collision is described by the unitary operator \( \hat{U}_{SR_n} = e^{-i\theta_n}, \) \( \tau \) being the collision time. By means of the equality

\[
e^{-i/2(\theta_{SR} + \theta_{SR}^* + \theta_{SR}^* + \theta_{SR})} = e^{-i/2(\cos \phi \hat{1} + i \sin \phi \hat{S})},
\]

where \( \hat{1} \) is the identity operator and \( \hat{S} \) the two-qubit swap operator such that \( \hat{S} |1\rangle_1 |0\rangle_2 = |0\rangle_1 |1\rangle_2 \) for all \( |\psi_1\rangle, |\psi_2\rangle \in \mathbb{C}^2 \), the unitary time evolution operator can be written as

\[
\hat{U}_{SR_n} = (\cos J) \hat{1}_{SR_n} + i(\sin J) \hat{S}_{SR_n},
\]

where \( J = 2g \tau \) is a dimensionless interaction strength between \( S \) and \( R_n \) which is supposed to be the same for any \( n = 1, 2, \ldots, N \). It is immediate to see that \( J = \pi/2 \) induces a complete swap between the state of \( S \) and that of \( R_n \). Thus, \( 0 < J < \pi/2 \) means a partial swap conveying the intuitive idea that, at each collision, part of the information contained in the state of \( S \) is transferred into \( R_n \). In the ordered basis \( B_{SR_n} = \{|00\rangle_{SR_n}, |01\rangle_{SR_n}, |10\rangle_{SR_n}, |11\rangle_{SR_n}\} \), \( \hat{U}_{SR_n} \) reads

\[
\hat{U}_{SR_n} = \begin{pmatrix}
e^{iJ} & 0 & 0 & 0 \\
0 & \cos J & i \sin J & 0 \\
0 & i \sin J & \cos J & 0 \\
0 & 0 & 0 & e^{iJ}
\end{pmatrix}
\]

The interaction between two nearest-neighbor reservoir qubits \( R_n \) and \( R_{n+1} \) is also described by an operation similar to that of Eq. \( 9 \), namely

\[
\hat{V}_{R_n R_{n+1}} = \begin{pmatrix}
e^{i\Omega} & 0 & 0 & 0 \\
0 & \cos \Omega & i \sin \Omega & 0 \\
0 & i \sin \Omega & \cos \Omega & 0 \\
0 & 0 & 0 & e^{i\Omega}
\end{pmatrix}
\]

where \( 0 \leq \Omega \leq \pi/2 \) is the dimensionless \( R_n R_{n+1} \) interaction strength being independent of \( n \).

After deriving \( \rho_{SR}^*, \rho_{SR}, \rho_{SR}^* \) and \( \rho_{SR}^* \), we can obtain all the terms appearing in the equality of Eq. \( 10 \) and are thus in the position to explore their dynamics with respect to the collision number \( n \). The equality of Eq. \( 10 \) is verified in Fig. \[2 \] by comparing its LHS and RHS in the dynamical process, which exhibit complete coincidence. Therefore, this equality supplies a reliable information measure for the heat dissipation of an information erasure process in a non-Markovian environment.

In Fig. \[2 \] though the initial temperature \( T_S \) of the system is set larger than \( T_R \) of the reservoir, we note that \( \Delta \tilde{O}_n \) can attain negative values indicating the backflow of heat from the reservoir to the system in the evolution for relatively large values of \( \Omega \) (e.g., \( \Omega = 1.3 \omega, 1.4 \omega \)). Actually, \( \Delta \tilde{O}_n \) can be divided into two different contributions

\[
\Delta \tilde{O}_n = \Delta \tilde{O}_n^{\text{dia}} + \Delta \tilde{O}_n^{\text{coh}},
\]

where

\[
\Delta \tilde{O}_n^{\text{dia}} = \omega \sin^2(J)(\rho_{33} - \rho_{22}), \quad \Delta \tilde{O}_n^{\text{coh}} = \omega \text{Im}(\rho_{23}) \sin(2J),
\]

are the heats determined, respectively, by the diagonal and coherent (off-diagonal) elements of the state \( \rho_{SR}^* \) of \( S - R_n \) before their collisions. In Eq. \( 12 \), we have used the ordered basis \( B_{SR_n} \) of \( S - R_n \) by setting \( \{|\tilde{1}\rangle = |00\rangle, \tilde{2}\rangle = |01\rangle, \tilde{3}\rangle = |10\rangle, \tilde{4}\rangle = |11\rangle \) and \( \rho_{kl} = \langle \tilde{k}\| \rho_{SR_n} \| \tilde{l} \rangle \) with \( k, l = 1, 2, 3, 4 \). The nonzero coherent term
\[ \rho_{23} \] of \( \rho_{SR} \), is a direct witness of the correlations between \( S \) and \( R \), which in turn gives the correlation-dependent heat \( \Delta Q_n^{\text{coh}} \). For relatively small values of \( \Omega \), the established system-environment correlations are weak, so the contribution \( \Delta Q_n^{\text{dia}} \) plays a major role in determining the behavior of the total heat \( \Delta Q_n \). Differently, when \( \Omega \) is sufficiently large the behavior of \( \Delta Q_n \), especially its transition from positive to negative values, is mainly determined by the contribution \( \Delta Q_n^{\text{coh}} \), as highlighted in Fig. 3.

Now we check the condition for the violation of the Landauer principle given in Eq. (5) by comparing the values of \( I(\rho_{SR}) \) and \( D(\rho'_{SR} || \rho'_{SR}) \) for different intra-reservoir collision strengths \( \Omega \). As shown in Fig. 4(a), for relatively small values of \( \Omega \) (e.g., \( \Omega = 1.2\omega \)), \( I(\rho_{SR}) \) is smaller than \( D(\rho'_{SR} || \rho'_{SR}) \) in the whole dynamical process. This means that \( \beta_{\nu_R} \Delta Q_n^{\text{coh}} \) remains larger than \( \Delta S_n \), implying that the Landauer principle still holds although there exist nonzero established system-environment correlations at later times (mem-
or eory effects). By contrast, as shown in Fig. 4(b) and (c), relatively large values of $\Omega$ (e.g., $\Omega = 1.3\omega$, $1.4\omega$) can make $I(\rho_{SR_n})$ supersede $D(\rho'_{SR_n} \parallel \rho_{SR_n})$ during some time intervals of the evolution when the Landauer principle is thus violated, being $\beta_n \Delta Q_n < \Delta S_n$.

III. LANDAUER PRINCIPLE IN MULTIPLE NON-MARKOVIAN RESERVOIRS

The previous results obtained for a single non-Markovian reservoir leaves now open the question whether they remain qualitatively the same for a composite environment of multiple reservoirs, the only difference emerging from the quantitative side. In this section we investigate this aspect. In the following, we generalize the results obtained in a single reservoir to multiple reservoirs, where the quantum system $S$ is coupled to $M$ finite-size heat reservoirs, i.e., $R_1^{(1)}, R_2^{(1)}, ..., R_M^{(1)}$, with each one consists of $N$ identical qubits $R_1^{(m)}, R_2^{(m)}, ..., R_N^{(m)}$. The system and generic reservoir qubit are still described by the Hamiltonians given in Eq. 1. The model with $M = 2$ is illustrated in Fig. 5 for the first two steps of collisions.

To begin with, the system $S$ collides with the first qubits $R_1^{(1)}, R_1^{(2)}, ..., R_1^{(M)}$ in all the $M$ reservoirs and then the intra-reservoir collisions of $R_1^{(1)}$, $R_2^{(1)}$, $R_1^{(2)}$, $R_2^{(2)}$, ..., $R_1^{(M)}$, $R_2^{(M)}$ occurs leading to the constructions of correlations among the various parts $S$, $R_1^{(1)}$, $R_2^{(1)}$, ..., $R_2^{(M)}$ prior to their collisions. Generally, the state $\rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}}$ of $S$, $R_1^{(1)}$, $R_2^{(1)}$, ... $R_2^{(M)}$ are correlated before their collisions for $n \geq 2$ and such correlations are characterized by the mutual information $I(\rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}}) = S(\rho_{SR_n}) + \sum_{m=1}^{M} S(\rho_{R_m^{(m)}}) - S(\rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}})$ with $\rho_{SR_n}$ and $\rho_{R_m^{(m)}}$ the marginal states of $S$ and $R_m^{(m)}$. Governed by the unitary actions $\hat{U}_{S,R_1^{(1)}}, \hat{U}_{SR_2^{(1)}}, ..., \hat{U}_{SR_2^{(M)}}$ for the interactions of $S$, $R_1^{(1)}$, $R_2^{(1)}$, ..., $R_2^{(M)}$, the state $\rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}}$ is transformed to $\rho'_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}} = \hat{U}_{S,R_1^{(1)}} \hat{U}_{SR_2^{(1)}} \hat{U}_{SR_1^{(2)}} \hat{U}_{SR_2^{(2)}} \hat{U}_{SR_1^{(M)}} \hat{U}_{SR_2^{(M)}}$. The marginal states of $S$ and $R_m^{(m)}$ after the collisions are labeled as $\bar{\rho}_S = \text{Tr}_{R_1^{(1)}R_2^{(1)}...R_2^{(M)}} \rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}}$ and $\bar{\rho}_{R_m^{(m)}} = \text{Tr}_{S} \rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}}$ with $\bar{\rho}_{R_m^{(m)}}$ the reservoir qubits other than $R_m^{(m)}$. By the fact that the total von Neumann entropy of the system and reservoirs under the unitary actions retains invariant, namely, $S(\rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}}) = S(\rho'_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}})$, we can derive the entropy change $\Delta S_n$ of the system as (see Appendix B for details)

$$\Delta S_n = S(\bar{\rho}_S') - S(\bar{\rho}_S),$$

$$= D(\bar{\rho}_S' \parallel \bar{\rho}_S) \prod_{m=1}^{M} \rho_{R_m^{(m)}}$$

$$+ \sum_{m=1}^{M} \text{Tr}_{R_m^{(m)}} \left( \rho_{R_m^{(m)}} - \rho_{R_m^{(m)}}' \right) \ln \rho_{R_m^{(m)}} - I(\rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}}),$$

where $D(\rho'_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}} \parallel \rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}})$ is the irreversible entropy production of the system [57]. Here, we still assume both the system and reservoirs are initially prepared in the thermal states but the reservoirs may have different temperatures, namely, for the $m$-th reservoir its state $\rho_{R_m^{(m)}} = e^{-\beta_m H_S} / \text{Tr}[e^{-\beta_m H_S}]$ with $\beta_m = 1/k_B T_m$ the inverse temperature. In the process of evolution, though the system-reservoir correlations can be established between $S$ and $R_m^{(m)}$ before their interactions, the reduced states $\rho_{R_m^{(m)}}$ still retain the forms of thermal states, i.e., $\rho_{R_m^{(m)}} = e^{-\beta_m H_S} / \text{Tr}[e^{-\beta_m H_S}]$ with the $n$-dependent inverse temperatures $\beta_m^{(n)}$. In this case, we have $\text{Tr}_{R_m^{(m)}} \left( \rho_{R_m^{(m)}}' - \rho_{R_m^{(m)}} \right) \ln \rho_{R_m^{(m)}} = \beta_m^{(n)} \Delta Q_n^{(m)} + \Delta Q_n^{(m)} = \text{Tr}_{R_m^{(m)}} \left( \rho_{R_m^{(m)}}' - \rho_{R_m^{(m)}} \right) H_B$ denoting the heat flowing from reservoir qubit $R_m^{(m)}$ to the system $S$. Therefore, the Eq. 13 is reformulated as

$$\Delta S_n = D(\rho'_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}} \parallel \rho_S') \prod_{m=1}^{M} \rho_{R_m^{(m)}} + \sum_{m=1}^{M} \beta_m^{(n)} \Delta Q_n^{(m)}$$

$$- I(\rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}}).$$

Without the construction of system-reservoir correlations in the dynamical process, for instance, in the Markovian regime with $\Omega = 0$, the mutual information $I(\rho_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}}) = 0$ so that we have $\Delta S_n = D(\rho'_{SR_1^{(1)}R_2^{(1)}...R_2^{(M)}} \parallel \rho_S') \prod_{m=1}^{M} \rho_{R_m^{(m)}} + \sum_{m=1}^{M} \beta_m^{(n)} \Delta Q_n^{(m)}$ with the first term and the other $M$ terms in RHS being identified as the irreversible and reversible contributions, respectively, to the system entropy change due to
The initial temperatures of the system and the two reservoirs are chosen as $T_S = 2\omega$, $T_r^{(1)} = 3\omega$ and $T_r^{(2)} = \omega$, respectively. Here, we assume the same strengths $\Omega$ for the intracollisions of $R_n^{(1)}-R_n^{(1)}$ and $R_n^{(2)}-R_n^{(2)}$ and the same coupling constants $J = 0.1\omega$ for $S-R_n^{(1)}$ and $S-R_n^{(2)}$.

We now connect the equality (14) to the Landauer principle in the presence of multiple non-Markovian reservoirs. From Eq. (15), we obtain that

$$\sum_{m=1}^{M} \beta_n^{(m)} \Delta Q_n^{(m)} = \Delta S_n + D \left( \rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{S'} \prod_{m=1}^{M} \rho'_{R_n^{(m)}} \right).$$

where $\Delta Q_n^{(m)} = -\Delta Q_n^{(m)}$ is the heat dissipated to $R_n^{(m)}$ and $\Delta S_n = -\Delta S_n$ denotes the entropy decrease of the system. We call the equality (15) Landauer-like principle in the multiple non-Markovian reservoirs. From Eq. (15), we can immediately obtain that when $I(\rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{S'}, \prod_{m=1}^{M} \rho'_{R_n^{(m)}}) \leq D \left( \rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{S'} \prod_{m=1}^{M} \rho'_{R_n^{(m)}} \right)$, the Landauer principle still holds, namely, $\sum_{m=1}^{M} \beta_n^{(m)} \Delta Q_n^{(m)} \geq \Delta S_n$. At the same time, the condition for the violation of the Landauer principle is obtained as

$$I(\rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{S'}, \prod_{m=1}^{M} \rho'_{R_n^{(m)}}) > D \left( \rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{S'} \prod_{m=1}^{M} \rho'_{R_n^{(m)}} \right).$$

Based on the model with $M = 2$ as illustrated in Fig. 5, we verify the equality (15) and the condition (16) for the violation of Landauer principle. For the interactions between $S$ and reservoir qubits as well as intra-interactions between reservoirs qubits, we still adopt the Heisenberg-like interaction described by the Hamiltonian (6). Specifically, we apply

$$\hat{H}_{SR_n^{(1)} R_n^{(1)}} = \hat{H}_{SR_n^{(2)} R_n^{(2)}} \equiv \hat{H}_{SR}, \text{ in Eq. } (9) \text{ for the interactions of } S-R_n^{(1)} \text{ and } S-R_n^{(2)} \text{, while } \hat{V}_{R_n^{(1)} R_n^{(1)}} = \hat{V}_{R_n^{(2)} R_n^{(2)}} \equiv \hat{V}_{R \rightarrow S \rightarrow R} \text{ in Eq. } (10) \text{ for the interactions of } R_n^{(1)}-R_n^{(1)} \text{ and } R_n^{(2)}-R_n^{(2)}. \text{ A comparison between the LHS and RHS of the equality of Eq. } (15) \text{ is shown in Fig. 6, where one can see their complete coin-

![Graph](attachment:image1.png)

**FIG. 6.** (Color online) The LHS (circle) and RHS (square) of Eq. (15) against the collision number $n$. The initial temperatures of the system and the two reservoirs are chosen as $T_S = 2\omega$, $T_r^{(1)} = 3\omega$ and $T_r^{(2)} = \omega$, respectively. Here, we assume the same strengths $\Omega$ for the intracollisions of $R_n^{(1)}-R_n^{(1)}$ and $R_n^{(2)}-R_n^{(2)}$ and the same coupling constants $J = 0.1\omega$ for $S-R_n^{(1)}$ and $S-R_n^{(2)}$.

heat exchanges [57]. In the non-Markovian regime, however, the system entropy change will also be contributed by the system-reservoir correlations in terms of the mutual information $I(\rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{R_n^{(1)}}).$

We now connect the equality (14) to the Landauer principle in the presence of multiple non-Markovian reservoirs. From Eq. (14), we obtain that

$$\sum_{m=1}^{M} \beta_n^{(m)} \Delta Q_n^{(m)} = \Delta S_n + D \left( \rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{S'} \prod_{m=1}^{M} \rho'_{R_n^{(m)}} \right).$$

where $\Delta Q_n^{(m)} = -\Delta Q_n^{(m)}$ is the heat dissipated to $R_n^{(m)}$ and $\Delta S_n = -\Delta S_n$ denotes the entropy decrease of the system. We call the equality (15) Landauer-like principle in the multiple non-Markovian reservoirs. From Eq. (15), we can immediately obtain that when $I(\rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{S'}, \prod_{m=1}^{M} \rho'_{R_n^{(m)}}) \leq D \left( \rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{S'} \prod_{m=1}^{M} \rho'_{R_n^{(m)}} \right)$, the Landauer principle still holds, namely, $\sum_{m=1}^{M} \beta_n^{(m)} \Delta Q_n^{(m)} \geq \Delta S_n$. At the same time, the condition for the violation of the Landauer principle is obtained as

$$I(\rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{S'}, \prod_{m=1}^{M} \rho'_{R_n^{(m)}}) > D \left( \rho'_{S R_n^{(1)} R_n^{(1)}}, \rho'_{S'} \prod_{m=1}^{M} \rho'_{R_n^{(m)}} \right).$$

Based on the model with $M = 2$ as illustrated in Fig. 5, we verify the equality (15) and the condition (16) for the violation of Landauer principle. For the interactions between $S$ and reservoir qubits as well as intra-interactions between reservoirs qubits, we still adopt the Heisenberg-like interaction described by the Hamiltonian (6). Specifically, we apply
cidence implying the validity of equality of Eq. (15). In Fig. 7(a), (b) and (c), we check the condition of Eq. (16) that leads to Landauer principle violation in the presence of two non-Markovian reservoirs by comparing the values of $I(\rho_{SR(i)}; R_2)$ and $D\left(\rho'_{SR(i)} \parallel \rho'_{SR(j)}\right)$ for different intra-collision strengths $\Omega$. As displayed in Fig. 7(a), for relatively small values of $\Omega$ (e.g., $\Omega = 1.2\omega$), the amount of $I(\rho_{SR(i)}; R_2)$ is always less than $D\left(\rho'_{SR(i)} \parallel \rho'_{SR(j)}\right)$ in the whole dynamical process, so that the quantity $\sum_{n=1}^{2} \beta_n \Delta Q_n^{(m)}$ stays larger than $\Delta S_n$: this means that the Landauer principle still holds although there exist nonzero system-environment correlations. By contrast, as shown in Fig. 7(b) and (c), larger values of $\Omega$ (e.g., $\Omega = 1.4\omega$, $1.45\omega$) can make $I(\rho_{SR(i)}; R_2)$ exceed $D\left(\rho'_{SR(i)} \parallel \rho'_{SR(j)}\right)$ in some time intervals of the evolution during which the Landauer principle is violated with $\sum_{n=1}^{2} \beta_n \Delta Q_n^{(m)} < \Delta S_n$. We notice that, although there are quantitative differences with the case of a single non-Markovian reservoir, the qualitative results remain the same.

IV. CONCLUSION

In conclusion, we have studied by means of collision models the validity of Landauer principle in a non-Markovian process caused by intracollisions of reservoir ancillary qubits. We have utilized the system-environment correlations formed during the dynamical process to assess the effect of non-Markovianity (memory effects) on Landauer principle. We first consider the situation with the system interacting with a single thermal reservoir. By connecting an exact equalization for $\Delta S_n$ of the system because of the interaction with the reservoir ancilla $R_n$. The derivation is as follows

$$\Delta S_n = S(\rho_{SR_a}) - S(\rho_{SR}) = S(\rho'_{SR_a}) - S(\rho_{SR}) = -Tr_{SR_a} \rho'_{SR_a} \ln \rho'_{SR_a} + Tr_{SR_a} \rho_{SR} \ln \rho_{SR} - Tr_{R_a} \rho'_{SR_a} \ln \rho'_{SR_a} + Tr_{R_a} \rho_{SR} \ln \rho_{SR} = -Tr_{SR_a} \rho'_{SR_a} \ln \rho'_{SR_a} + Tr_{SR_a} \rho_{SR} \ln \rho_{SR} - Tr_{R_a} \rho'_{SR_a} \ln \rho'_{SR_a} + Tr_{R_a} \rho_{SR} \ln \rho_{SR} = -Tr_{SR_a} \rho'_{SR_a} \ln \rho'_{SR_a} + Tr_{SR_a} \rho_{SR} \ln \rho_{SR} - Tr_{SR_a} \rho'_{SR_a} \ln \rho'_{SR_a} + Tr_{R_a} \rho_{SR} \ln \rho_{SR} = -D\left(\rho'_{SR_a} \parallel \rho_{SR_a}\right) + Tr_{SR_a} \rho_{SR} \ln \rho_{SR} - Tr_{R_a} \rho'_{SR_a} \ln \rho'_{SR_a} - I(\rho_{SR_a})$$

(App 1)

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Appendix A: Derivation of Eq. (2)

In this appendix we report the detailed steps which lead to the expression given in Eq. (2) for the entropy change $\Delta S_n$ of the system because of the interaction with the reservoir ancilla $R_n$. The derivation is as follows

$$\Delta S_n = S(\rho_{SR_a}) - S(\rho_{SR}) = -Tr_{SR_a} \rho'_{SR_a} \ln \rho'_{SR_a} + Tr_{SR_a} \rho_{SR} \ln \rho_{SR} - Tr_{R_a} \rho'_{SR_a} \ln \rho'_{SR_a} + Tr_{R_a} \rho_{SR} \ln \rho_{SR} = -D\left(\rho'_{SR_a} \parallel \rho_{SR_a}\right) + Tr_{SR_a} \rho_{SR} \ln \rho_{SR} - Tr_{R_a} \rho'_{SR_a} \ln \rho'_{SR_a} - I(\rho_{SR_a})$$

(App 1)

Appendix B: Derivation of Eq. (13)

In this appendix we give the calculations which allow us to obtain the expression of Eq. (13) for the entropy change $\Delta S_n$ of the system due to the interaction with the multiple reservoir ancillas $R_{n1}, R_{n2}, \ldots, R_{nM}$. The derivation is as follows
\[ \Delta S_n = S(\rho'_S) - S(\rho_S) = S(\rho'_S) - S(\rho_{SR}^{R(1)R(2)\ldots R(n)}) + \sum_{m=1}^{M} S(\rho_{R}^{m}) - I(\rho_{SR}^{R(1)R(2)\ldots R(n)}) \]

\[ = S(\rho'_S) - S(\rho_{SR}^{R(1)R(2)\ldots R(n)}) + \sum_{m=1}^{M} S(\rho_{R}^{m}) - I(\rho_{SR}^{R(1)R(2)\ldots R(n)}) \]

\[ = -\text{Tr}_{S}\rho'_S \ln \rho'_S + \text{Tr} \rho'_S \ln \rho'_S - \sum_{m=1}^{M} \text{Tr}_{R} \rho^{m}_{R} \ln \rho^{m}_{R} + \sum_{m=1}^{M} \text{Tr}_{R} \rho^{m}_{R} \ln \rho^{m}_{R} \]

\[ = -\text{Tr}_{S}\rho'_S \ln \rho'_S + \text{Tr} \rho'_S \ln \rho'_S - \sum_{m=1}^{M} \text{Tr}_{R} \rho^{m}_{R} \ln \rho^{m}_{R} + \sum_{m=1}^{M} \text{Tr}_{R} \rho^{m}_{R} \ln \rho^{m}_{R} \]

\[ = \ldots \]

\[ = D \left( \rho'_S \parallel \rho_S \right) + \sum_{m=1}^{M} \text{Tr}_{R} \left( \rho^{m}_{R} - \rho_{R}^{m} \right) \ln \rho^{m}_{R} - I \left( \rho_{SR}^{R(1)R(2)\ldots R(n)} \right) . \] (B1)

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