Dynamic Curvature Steering Control for Autonomous Vehicle: Performance Analysis

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Abstract. This paper discusses the design of dynamic curvature steering control for autonomous vehicle. The lateral control and longitudinal control are discussed in this paper. The controller is designed based on the dynamic curvature calculation to estimate the path condition and modify the vehicle speed and steering wheel angle accordingly. In this paper, the simulation results are presented to show the capability of the controller to track the reference path. The controller is able to predict the path and modify the vehicle speed to suit the path condition. The effectiveness of the controller is shown in this paper whereby identical performance is achieved with the benchmark but with extra curvature adaptation capabilities.

1. Introduction
The development of the lateral and longitudinal control are required for autonomous vehicle to follow the path successfully. The autonomous vehicle iteratively calculates the required actions in order to stay on the right path. For instance, steering wheel angle and throttle pedal are adjusted to accordingly when there is any changes in the path type.

Several approaches can be tracked focusing on the autonomous vehicle tracking control. Author in [1] proposed pure pursuit due to its simplicity. However, despite of the simplicity the vehicle dynamic is ignored by using this method. The controller is effective when the vehicle dynamic is neglected. It tends to failed during the sharp corner due to cut corner since the current vehicle speed is not considered into the algorithm [2].

The conventional PID technique has been investigated by researchers. The heading is used as feedback. However, the control design is proven to be unstable at high velocity and tends to cut corner [3]. Park et al. [4] has demonstrated the implementation of pure pursuit is improved compared to PID control. However, the technique is only work well in the straight path and normal curvature path.

Most of model based design control is based on assumption on linear model in order to ease the control formulation. The control method is assumed to operate in smooth and continuous path type. For instance, the model based control can be found at [5, 6].

RS Sharp et al. [7] introduces the multiple look-ahead preview point in their research. Each look-ahead has its own weight to the steering input whereby the longest look-ahead point carries the lowest weight to the steering input. The look-ahead introduced is a varying parameter based on the vehicle velocity and the vehicle preview time.
From the previous work done by MA Zakaria et al. in [8, 9] the nonlinear tracking control is introduced. The path tracking control is able to reduce the error in order to follow the desired path. However, the control is lacking on the information of the dynamic path curvature. The problem arises when the vehicle approaches a tight curvature where the control is unable to cope with the sudden curvature changes.

In order to solve the problem, a new path tracking strategy is proposed based on the preliminary work done by MA Zakaria et al [8, 9]. The control design is based on the geometrical technique and integrated with PID control for speed control purpose. The control make use of the vehicle and road parameter to generate the curvature profile of the path while lateral acceleration and cross-track error is used as feedback to the vehicle speed control in order to navigate the path smoothly. Furthermore, the curvature profile is used to estimate the look-ahead path and safe speed navigation. The effectiveness of the control is verified by simulation. Therefore, this proposed controller is the enhancement from the previous controller whereby the performance on the cornering is improved.

In this paper, the development of the curvature controller is discussed. The simulation results presented in this paper have shown that the controller is able to adapt the tracking based on the curvature of the path. The proposed controller offers a new opportunity to navigate the path with appropriate speed when approaching corner especially tight curve.

2. Vehicle Model

2.1. Kinematic Model

Consider the vehicle located at point x, y in global coordinate frame with heading direction, θ as shown in Figure 1. From x_d, y_d, θ_d which is reference point, the posture error could be obtained. The lateral error, heading error, longitudinal error can be obtained by matrix transformation from the global coordinate to the vehicle local coordinate. The general kinematic model is defined in Equation 1 whereby x_e, y_e, θ_e are the longitudinal error, cross-track error and heading error of the vehicle.

\[
\begin{bmatrix}
  x_e \\
  y_e \\
  \theta_e
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta_c & \sin \theta_c & 0 \\
  -\sin \theta_c & \cos \theta_c & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  x_d - x_c \\
  y_d - y_c \\
  \theta_d - \theta_c
\end{bmatrix}
\] (1)

The vehicle’s motion is depending on the velocity and yaw rate velocity and defined as Equation 2

\[
\begin{bmatrix}
  \dot{x} \\
  \dot{y} \\
  \dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta_c & 0 & 0 \\
  \sin \theta_c & 0 & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  v \\
  \psi
\end{bmatrix}
\] (2)

and from Equation 2 the position to the global coordinate frames are

\[
\begin{bmatrix}
  x_c \\
  y_c \\
  \theta_c
\end{bmatrix} =
\begin{bmatrix}
  \int_{0}^{t} v \cos(\int_{0}^{t} \psi dt) \\
  \int_{0}^{t} v \sin(\int_{0}^{t} \psi dt) \\
  \int_{0}^{t} \psi dt
\end{bmatrix} +
\begin{bmatrix}
  x_0 \\
  y_0 \\
  \theta_0
\end{bmatrix}
\] (3)

Equation 3 is converted into discrete time during the implementation for simulation and experiment.
2.2. Dynamic Vehicle Model

In this paper, the dynamic model is derived from the nonlinear model. The reader is directed [10] for the details of the nonlinear model used. The model parameter is derived from the actual autonomous vehicle platform, IREV as in Figure 2. The IREV is equipped with sensors such as odometers, steering encoder, IMU (Inertial Measurement Unit) and laser range finder.

Figure 2: IREV, Autonomous Vehicle Prototype Used in the Research

Under normal cornering, when the slip angles is below than 5 degrees, a linear relationship between the slip angle and lateral force denoted by \( f_y \) can be obtained. Hence, the cornering stiffness can be calculated as Equation 4. Thus the approximation nominal linear model is [11],

\[
C_\alpha = \frac{f_y}{\alpha}
\]  

Thus, the linear IREV parameters are defined in Table 1.

Table 1: Vehicle Model Parameters

| Parameter | Value | Description |
|-----------|-------|-------------|
| \( l_f \) | 0.7 m | distance of center of gravity to the front axles |
| \( l_r \) | 0.6 m | distance of center of gravity to the rear axles |
| \( m \) | 460 kg | vehicle mass |
| \( J_z \) | 3000 \( kg.m^2 \) | moment of inertia |
| \( C_r \) | 47617 \( N/rad \) | rear cornering stiffness coefficient |
| \( C_f \) | 42170 \( N/rad \) | front cornering stiffness coefficient |
3. Lateral Controller

3.1. Steering Controller

In this paper, the steering controller consists of the heading correction and lateral error correction. The steering controller is defined as

\[ \delta_n = K_1 \theta_e + \frac{K_2 y_e}{V_x} + K_3 y_e + K_4 \sum_{i=1}^{n} y_e \cdot \Delta t \]  

(5)

where

\[ \theta_e = \theta_d - \theta_c \]  

(6)

\[ \delta_{\text{min}} \leq \delta \leq \delta_{\text{max}} \]  

(7)

The steering control is improved version from MA Zakaria et al. [20] where in Equation 7, \( \delta \) is the steering wheel angle, \( K_1 \) is the heading tuning gain, \( \theta_e \) is the heading error, \( K_2 \) is the lateral control tuning gain, \( y_e \) is the cross-track error, and \( V_x \) is the vehicle longitudinal speed. In this paper, the integral action has been added to improve the steady state error. The cross-track error, \( y_e \) of the vehicle is obtained as in Equation 1 in vehicle model section.

3.2. Look Ahead Selection

The look ahead is selected so that it is proportional with vehicle velocity. The varying look-ahead from RS Sharp et al. [7] is used. This value will be used to calculate the future prediction point as in [8]. The look-ahead distance value is updated iteratively based on vehicle speed, \( V_x \) for each sample time, \( \Delta t \).

\[ L = V_x \cdot \Delta t \]  

(8)

4. Longitudinal controller

In this paper, the longitudinal controller is obtained from the derivation of the path curvature and vehicle curvature. First, the road curvature, \( \kappa_{\text{road}} \) and vehicle curvature, \( \kappa_{\text{vehicle}} \) is calculated iteratively during the movement of the vehicle.

The curvature of the path and vehicle can be calculated iteratively as

\[ \kappa = \frac{\Delta \theta}{\Delta S} \]  

(9)

where \( \theta \) is the orientation and \( S \) is the path length.

The value of these curvature is used by the curvature function, \( f(\kappa_{\text{road}}, \kappa_{\text{vehicle}}) \) to generate the speed reduction based on the current curvature value. Based on Equation 10, the new desired speed, \( V_d \) is reduced from the vehicle current speed, \( V_x \) minus the curvature function. The Equation 10 prevents the vehicle movement from cutting the corner of the path.

\[ V_d = V_x - f(\kappa_{\text{road}}, \kappa_{\text{vehicle}}) \]  

(10)

The new calculated value of the desired speed, \( V_d \) is sent to the PID controller module to adjust the throttle pedal position in order to follow the desired speed, \( V_d \) as in Equation 11,
\[ V_e = V_d - V_x \]  

\[ U_v = K_p V_e + K_i \sum V_e \Delta t + K_d \frac{\Delta V_e}{\Delta t} \]

(11a)  
(11b)

where \( V_e, U_v, K_p, K_i, K_d \) are the velocity error, pedal control output, proportional gain, integral gain, and derivative gain. Since the curvature is calculated iteratively, the threshold region can be added to the road curvature to define the curvature type which is straight line, normal corner and sharp corner.

\[ |\kappa_{road}| = \begin{cases} 
  \text{straight} & \text{if } 0 < \kappa_{road} < \kappa_{normal} \\
  \text{normal} & \text{if } \kappa_{normal} < \kappa_{road} < \kappa_{sharp} \\
  \text{sharp} & \text{if } \kappa_{road} > \kappa_{sharp} 
\end{cases} \]

(12)

5. Results and Discussions

In this section, the controller is compared with the preview steering control by RS Sharp et al. in [7]. The formulation is rewritten as in Equation 13. Three look-ahead preview point is selected and the tuning gain is defined as in Table 2. The vehicle speed is set constant at 1 m/sec using preview control theory. However, for curvature controller, the vehicle speed is varying based on the curvature profile generation in Equation 10.

\[ \delta = K\psi \cdot e_\psi + K_1 \cdot e_1 + \sum_{i=2}^{i=n} K_i \cdot e_i \]

(13)

Table 2: Preview Control Tuning Gain

| Gain | Value |
|------|-------|
| \( K_1 \) | 10 |
| \( K_2 \) | 10 |
| \( K_3 \) | 5 |
| \( K_\psi \) | 1 |

The tuning gain for curvature steering control from Equation 5 is defined as in Table 3.

Table 3: Curvature Control Tuning Gain

| Gain | Value |
|------|-------|
| \( K_1 \) | 1.0 |
| \( K_2 \) | 0.9 |
| \( K_3 \) | 2 |
| \( K_4 \) | 0.5 |

The actual road parameters are taken as a reference path into the simulation environment. From the trajectory response in Figure 3, both controller are able to track the path successfully. By using preview control, the steering response is adjusted based on the multiple look ahead. The gain is tuned so that the higher index error in Equation 13 carries least weight. This results a proper cornering is taken during the corner.

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In this paper, different approach is used for the proposed controller. The speed is reduced based on the curvature that the vehicle navigates. When the curvature value is high, the speed reduction is high to ensure the vehicle is able to track the path successfully. Figure 4 shows the vehicle speed adjustment based on the road curvature. From the Figure 4, the speed is inversely proportional with road curvature magnitude.

The lateral error response for both controller are observed. From Figure 5, both controller is able to track the path with small lateral error which is below 0.1 meter error. $e_1, e_2, e_3$ in Figure 5 are the lateral error for each of the look-ahead selection point using preview steering control.
In Figure 6, the road curvature, $\kappa_{\text{road}}$ and vehicle curvature, $\kappa_{\text{vehicle}}$ are calculated iteratively. The identical response between both curvature profiles indicate that the vehicle is on the reference path. The response will be different when the vehicle’s trajectory is away from the reference path.

6. Conclusion
From the result, the curvature steering controller is able to track the path successfully with small error. The ability of the controller to estimate the upcoming curvature type provides a useful feedback to control the vehicle speed to appropriate value accordingly. This serve a better control to vehicle especially during cornering maneuver. The proposed controller can be enhanced by using the integration with the vehicle model observer in order to have a better vehicle’s response estimation.

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