Combining Box Counting-dimension with a Fuzzy Synthetic Evaluation Model Based Quantitative Evaluation on Soil and Water Erosion of Lake Dianchi Basin, China

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Abstract. Considering the various influencing factors of soil and water erosion in Lake Dianchi Basin, evaluation indices of soil and water erosion involving the fractal dimension of drainage network, the fractal dimension of terrain, elevation and slope are determined. The weight values of evaluation indices are given and the fuzzy synthetic evaluation model is established on the basis of the fuzzy mathematics and information diffusion principles. As a case study, the fuzzy synthetic evaluation model is applied to quantitatively evaluate soil and water erosion of Lake Dianchi Basin. The results show that the grade of soil and water erosion is at the secondary level and the indicator value of comprehensive evaluation indicator of soil and water erosion is 1.45 in Lake Dianchi Basin. The assessment results are in good agreement with the results of previous studies. The result indicates that the method is valid and evaluation system established is reasonable, and the fuzzy synthetic evaluation model can be regarded as a reference model for the quantitative evaluation of soil and water erosion.

1. Introduction

The analysis of coupling relationship between watershed landform and soil and water erosion can provide important information to reveal physical mechanisms of producing soil and water erosion, while the quantitative expression of watershed landform is difficult. The fractal dimension of drainage network and the fractal dimension of terrain were used as two aspects of describing the watershed geomorphological features. On the basis of DEM (Digital Elevation Model), land-surface parameters such as slope and elevation are extracted as the other two aspects of describing the watershed geomorphological features. Elevation, slope, the fractal dimension of drainage network and the fractal dimension of terrain were chose as evaluation indices of soil and water erosion, and fuzzy synthetic evaluation (FSE) model which had been applied throughout the world in decision-making and evaluation processes in imprecise environment [1] was used to quantitatively evaluate the situation of soil and water erosion.

2. Study region, data and methods

2.1 Study region and data
Lake Dianchi Basin (24°28’ to 25°28’ N, 102°30’ to 103°00’ E) is located in the middle of Yunnan-Guizhou Plateau in Southwest China. The entire basin has a total area of approximately 2,920 km², including part of Kunming City (the capital of Yunnan Province) and Songming, Chenggong, Jinning, and Xishan Counties. With a surface area of about 300 km² and maximum and mean depths of 10 and 4.4 m respectively, Lake Dianchi is the sixth largest fresh water highland lake in China. The geographic location of Lake Dianchi Basin (or the study area) is shown in figure 1.

For this study, 1: 5 000-scale topographic maps of Lake Dianchi Basin, a 1: 100 000-scale vector land-use map of Lake Dianchi Basin in 2005 and a 1: 250 000-scale raster DEM (grid size 100 m) of Lake Dianchi Basin were used as data source, which were provided by Institute of Remote Sensing Applications, Chinese Academy of Sciences.

2.2 Methods

2.2.1 Box counting-dimension

In some application fields, the box-counting dimension method is compared with other methods and has shown better results [2]. According to the definition of the box counting-dimension, the expression of the measurements of a fractal object is

\[ D_b = \lim_{r \to 0} \frac{\log N(r)}{\log r} \]  

Where \( r \) is the length of box side, and \( N(r) \) is the number of boxes that cover the fractal object. To calculate the box-counting fractal dimension of the fractal object, boxes with different side length are drawn during the calculation process and the number of the occupied boxes is counted for each box side length. The computation of \( N(r) \) is repeated by changing the length of the box side \( (r) \), so that \( r \) approaches zero. \( N(r) \) values and \( r \) are plotted on a log-log graph to derive the fractal dimension, e.g., the slope of the plot.

2.2.2 Fuzzy synthetic evaluation model

2.2.2.1 Evaluation criteria

The selection of evaluating criteria plays a key role in the evaluation process, and should be operational, indicative, and representative. The relevant study has showed that the bigger the fractal dimension value of drainage network is and the more serious soil and water erosion is. Similarly, this qualitative understanding can be applied to understand the fractal dimension value of terrain. Fractal dimensions of a terrain quantitatively describe the self-organized structure of the terrain geometry. The fractal dimension value of drainage network and the fractal dimension value of terrain are divided
according to the study results of He Longhua [3].

The amounts of soil and water erosion with relative stability in a certain slope range increase with slope gradient, but not in proportion to it. The denudation rate of soil and water erosion rises with the increase of undulating terrain elevation. That is, the amounts of soil and water erosion increase with the increase of elevation. On the basis of Standards for Classification and Gradation of Soil Erosion (SL190-96), the value of elevation and the value of slope are divided into three intervals which are consisted with the divided interval of fractal dimension value of drainage network and the divided interval of fractal dimension value of terrain (Table 1). On the basis of the above qualitative and quantitative analysis, an integrated evaluation criteria system is set up containing four factors: the fractal dimension value of drainage network, the fractal dimension value of terrain, the value of slope and the value of elevation, and the definition and description of each evaluation index are shown in table 1.

Table 1. The definition and description of each evaluation index

| Grade | The fractal dimension value of drainage network ($D_n$) | The fractal dimension value of terrain ($D$) | The value of elevation ($E$)/m | The value of slope ($S$)/° | Description |
|-------|------------------------------------------------------|---------------------------------------------|-------------------------------|--------------------------|-------------|
| Level 1 | $D_n=1.60$ | $D=1.60$ | $1880<E=2058$ | $0<S=5$ | Soil and water erosion is mild. |
| Level 2 | $1.60<D_n=1.89$ | $1.60<D=1.89$ | $2058<E=2316$ | $5<S=13$ | Soil and water erosion is serious. |
| Level 3 | $1.89<D_n=2.00$ | $1.89<D=2.00$ | $2316<E=2815$ | $13<S=62$ | Soil and water erosion is more serious. |

Table 1 showed that the evaluation criteria value is an interval, but the evaluation index belongs to which evaluating grade when its observation value falls to an interval. For example, the evaluating grade of soil and water erosion is at the secondary level when the fractal dimension value of drainage network is 1.89. However, the evaluating grade of soil and water erosion is the third level when the fractal dimension value of drainage network is 1.90. It is not reasonable that the difference between fractal dimension values of drainage network is only 0.1, but the evaluating grade of soil and water erosion varies from the secondary level to the third level. That will directly cause the distortion of evaluation result. On the basis of the information diffusion principles, the mid-value of the evaluation grade interval (Table 1) is obtained by using the statistical method so that evaluation criteria value changes from an interval to a concrete numerical point (Table 2). Then, the evaluation matrix will be reconstructed directly by the method in Ref. [4].

Table 2. The re-definition of each evaluation index value

| Value | Grade | Value | Grade | Value | Grade | Value | Grade |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.08  | 1     | 0.08  | 1     | 1969  | 1     | 2.50  | 1     |
| 1.75  | 2     | 1.75  | 2     | 2187  | 2     | 9.00  | 2     |
| 1.95  | 3     | 1.95  | 3     | 2566  | 3     | 37.50 | 3     |

2.2.2.2 Evaluation matrix

On the basis of prescribed evaluation criteria (Table 1 and Table 2), the membership functions of each evaluation factor toward evaluation criteria at each evaluating grade can be described quantitatively by a set of formulae as follows (Eqs. (2) - (4)).

$$
\mu_x(x) = \begin{cases} 
0, & x < a \\
\frac{x-a}{b-a}, & a \leq x < b \\
1, & b \leq x
\end{cases}
$$ (2)

$$
\mu_x(x) = \begin{cases} 
0, & x < a \\
\frac{x-a}{c-a}, & a \leq x < c \\
\frac{d-x}{d-c}, & c \leq x < d \\
1, & d \leq x
\end{cases}
$$ (3)

$$
\mu_x(x) = \begin{cases} 
0, & x < a \\
\frac{x-a}{b-a}, & a \leq x < b \\
\frac{d-x}{d-c}, & c \leq x < d \\
1, & d \leq x
\end{cases}
$$ (4)

Where $b=(b_1,b_2,...,b_n)$ is the grade value of arbitrary index in evaluation graded vector $B=(B_1,B_2,...,B_n)$, $x$ is the observational value of arbitrary index. The functions established Eq. (2), Eq. (3) and Eq. (4) are called information diffusion functions (IDF). The value of information diffusion
function corresponding to arbitrary index within an index vector will be obtained and labelled with \((a_{i1}, a_{i2}, \ldots, a_{ik})\) which are combined to an evaluation matrix expressed as Eq. (5):

\[
A = \begin{bmatrix}
a_{i1} & a_{i2} & \cdots & a_{ik} \\
a_{j1} & a_{j2} & \cdots & a_{jk} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nk}
\end{bmatrix}
\] (5)

The information diffusion function is the membership function of the observational value, and the evaluation matrix is also known as the fuzzy matrix.

2.2.2.3 The principle of data synthesis

When the evaluation matrix \((A)\) and the weighted vector \((W)\) are given, fuzzy synthetic evaluation can be performed following equation 6:

\[
G = W \ast A = (g_1, g_2, g_3)
\] (6)

Where the \(*\) is the multiply operation.

According to the principle of maximum membership degree, the grade of soil and water erosion is expressed by Eq. (7):

\[
G_{\text{max}} = \max\{G\} = \max\{(g_1, g_2, g_3)\}
\] (7)

Here, the maximum value of three values \((g_1, g_2, g_3)\) determines the grade of soil and water erosion. If \(G_{\text{max}}\) equals to \(g_k\), the grade of soil and water erosion is the \(k\)th level. (Note: If there are two equal maximum values, the one near the second maximum value should be chosen.)

The indicator value of fuzzy synthetic evaluation of soil and water erosion is defined as eq.(8):

\[
G_{\text{max}} = \frac{\sum_{i=1}^{m} g_i^l}{\sum_{j=1}^{m} g_j}
\] (8)

3. Results and discussion

3.1 The fractal characteristics of drainage network

Drainage network analysis in the framework of surface deformation is of great significance as they are the recorders of tectonic and erosional processes. The drainage structure of Lake Dianchi Basin is obtained by using the eight flow directions (D8) approach and DEM raster data and land-use vector data. Drainage structure data was determined through a sequence of operations involving flow directions, watershed boundaries, modeled river and lake network and sub-watersheds. Comparing with 1: 50 000-scale topographic maps of Lake Dianchi Basin, a 1000-cell threshold (10 km²) was chosen for the upstream points of the network through trialing different threshold value without erasing any redundant information in order to obtain the drainage network of Lake Dianchi Basin (Figure 2).

Figure 2 showed that the drainage network of Lake Dianchi Basin. In fact, the only major differences between the variable approaches of obtaining drainage structure came from the threshold value which led to a less ramified modeled network.

The conversion from vector data to raster data is similar to use the box having a certain size to cover an image in ArcGIS software. The number of grid (or occupied boxes) is calculated through checking attribute table in ArcGIS platform. The fractal dimension value of drainage network (Figure 3) was obtained by using a moving box of variable size \((r)\) on an image and counts the number of drainage pixels within the box \(N(r)\).

The fractal dimension value of drainage network is 1.07 in Lake Dianchi Basin (Figure 3), which is the same as the conclusion that the fractal dimension is greater than one-dimensional topological dimension and less than two-dimensional spatial dimension. This indicates the developmental stage of landscape in Lake Dianchi Basin is in the stage of infancy [3] and are allied with regions of high susceptibility to surface deformation. The Lake Dianchi is surrounded by hills or mountains, and Lake Dianchi Basin belongs to the accumulation of special landscape environment. Therefore, the fractal dimension value of drainage network of Lake Dianchi Basin is smaller.
3.2 The fractal characteristics of terrain

The topographic contour could be taken as the reduced form of the three-dimensional cube watershed landform morphology [5] and the contour map of Lake Dianchi Basin (Figure 4) is created on the basis of DEM.

The fractal dimension of terrain was also calculated by applying the box counting-dimension method and the value is 0.93 in Lake Dianchi Basin (Figure 5).

3.3 The result of fuzzy synthetic evaluation

1: 250 000-scale raster DEM (grid size 100 m) of Lake Dianchi Basin was used to extract land-surface parameters such as slope (Figure 6) and elevation (Figure 7) of Lake Dianchi Basin in ArcGIS platform.

The relevant study has showed that the area of the most serious soil and water erosion is distributed on the elevation of 1986m to 1996m and the slope of 15° to 20° in watershed scale in China. That is, the elevation of 1996m and the slope of 20° are the threshold to easier produce soil and water erosion. Therefore, observational value such as the fractal dimension value of drainage network (1.07), the fractal dimension value of terrain (0.93), the value of elevation (1996m) and the value of slope (20°) in Lake Dianchi Basin were applied to the calculation of fuzzy synthetic evaluation (FSE) model.
Therefore, index vector can be expressed as Eq. (9).

\[(\text{The fractal dimension value of drainage network, the fractal dimension value of terrain, the value of elevation, the value of slope}) = (1.07, 0.93, 1996, 20)\]  

(9)

Then, evaluation matrix of index vector (Eq. (9)) corresponding to evaluating grade is shown in Table 3.

| Evaluating grade | 1     | 2     | 3     |
|------------------|-------|-------|-------|
| The fractal dimension value of drainage network | 0.595 | 0.405 | 0     |
| The fractal dimension value of terrain        | 0.507 | 0.493 | 0     |
| The value of elevation                         | 0     | 0.876 | 0.124 |
| The value of slope                             | 0     | 0.386 | 0.614 |

The evaluation matrix of index vector about the evaluation criteria is:

\[A = \begin{bmatrix}
0.595 & 0.405 & 0 \\
0.507 & 0.493 & 0 \\
0 & 0.876 & 0.124 \\
0 & 0.386 & 0.614
\end{bmatrix}\]  

(10)

The weight values of four evaluation indices are determined by Delphi method [6], and then the weight vector of the above evaluation vector is:

\[W = \{0.34, 0.26, 0.18, 0.22\}\]  

(11)

Evaluation vector that the weight vector combined with the evaluation matrix do synthetic operation can be expressed as Eq. (12).

\[G = W \cdot A = (0.334, 0.508, 0.158)\]  

(12)

The value of comprehensive evaluation indicator of soil and water erosion in Lake Dianchi Basin is determined through maximum membership degree method:

\[G_{\text{max}} = \max \{0.334, 0.508, 0.158\} = 0.508\]  

(13)

The results show that the grade of soil and water erosion is at the secondary level and the value of comprehensive evaluation indicator is 1.45 in Lake Dianchi Basin. That is, soil and water erosion of Lake Dianchi Basin is serious which is in good agreement with the results of previous studies [7].

4. Conclusions

The evaluation result is in good agreement with the results of previous studies. This indicates the evaluation method is valid and the established evaluation system is reasonable, and the fuzzy synthetic evaluation model can be regarded as a reference model for the quantitative evaluation of soil and water erosion.
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References
[1] Ulubeyli, S. 2017. Industry-wide competitiveness assessment through fuzzy synthetic evaluation: the case of cement industry. Journal of Business Economics and Management, 18(1), 35-53.
[2] Kemal Ihsan Kilic, Rahib Hidayat Abiyev. 2011. Exploiting the synergy between fractal dimension and lacunarity for improved texture recognition. Signal Processing 91, 2332-2344.
[3] He Longhua, Zhao Hong. 1996. The fractal dimension of river networks and its interpretation. Scientia Geographica Sinica 16(2), 124-128. (in Chinese)
[4] Zhang Jiguo. 2007. The model of fuzzy comprehensive evaluation based on information diffusion principle. Statistics and Decision 20, 155-156. (in Chinese)
[5] Yoshitaka Kimori, Eisaku Katayama, Nobuhiro Morone, Takao Kodama. 2011. Fractal dimension analysis and mathematical morphology of structural changes in actin filaments imaged by electron microscopy. Journal of Structural Biology 176, 1-8.
[6] Antonio Chamorro, Francisco J. Miranda, Sergio Rubio, Victor Valero. 2012. Innovation and trends in meat consumption: An application of the Delphi method in Spain. Meat Science 92, 816-822.
[7] Li Wenwen, Jiao Yizhi, Guan Yi, Xie Mingshu. 2009. Analysis on Agricultural Non-point Source Pollution Caused by Soil and Water Loss in Dianchi Watershed and Its Countermeasures. Journal of Anhui Agri. Sci. 37 (26), 12679-12680, 12694. (in Chinese)