Classification of the Chinese Handwritten Numbers with Supervised Projective Dictionary Pair Learning

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Abstract—Image classification has become a key ingredient in the field of computer vision. To enhance classification accuracy, current approaches heavily focus on increasing network depth and width, e.g., inception modules, at the cost of computational requirements. To mitigate this problem, in this paper a novel dictionary learning method is proposed and tested with Chinese handwritten numbers. We have considered three important characteristics to design the dictionary: discriminability, sparsity, and classification error. We formulated these metrics into a unified cost function. The proposed architecture i) obtains an efficient classification error. We formulated these metrics into a unified cost function. The proposed architecture i) obtains an efficient sparse code in a novel feature space without relying on $\ell_0$ and $\ell_1$ norms minimisation; and ii) includes the classification error within the cost function as an extra constraint. Experimental results show that the proposed method provides superior classification performance compared to recent dictionary learning methods. With a classification accuracy of $\sim 98\%$, the results suggest that our proposed sparse learning algorithm achieves comparable performance to existing well-known deep learning methods, e.g., SqueezeNet, GoogLeNet and MobileNetV2, but with a fraction of parameters.

Index Terms—Deep Learning, Dictionary Learning, Sparse Coding.

I. INTRODUCTION

Handwritten number recognition has remained a challenging research topic within pattern recognition [1], [2], [3], still attracting many researchers [4], [5]. Traditional handwritten number recognition systems include the two important stages of feature extraction and classification. Researchers proposed a classifier based on LeNet-5 and support vector machine (SVM) for handwritten number recognition [6], [7]. Other relevant studies [8], involved utilizing Histogram of gradient (HOG) and SVM for feature extraction and classification, respectively. Results from the latter study suggest that HOG is an effective feature extraction method in handwritten number recognition. HOG was initially proposed for human detection [9]. Recently, deep learning-based approaches have been proposed to classify handwritten numbers [10], [11], [12]. Despite the superior performance of deep learning based approaches, they require a large scale data set to train and obtain features. Also, they require huge amount of computing resources to function at a reasonable speed. Conventional methods, in comparison, are more appropriate for images with lower resolution, such as, hand written image data [13].

In this paper, we focus on dictionary learning-based handwritten number classification. We present a Chinese handwritten number recognition system based on HOG features and dictionary learning. We refer to the proposed method as supervised projective dictionary pair learning (sDPL) because it utilizes the classification labels into the cost function. To quantitatively evaluate performance, we compare the efficiency of two learning-based methods, i.e., “dictionary learning” and “deep learning” methods. This work presents three major contributions in the preprocessing and classification of handwritten numbers data:

1) New HOG features have been considered to create more effective dictionaries than those in DPL;
2) The classification error term (associated to the class label matrix) is added as a prior knowledge to objective function;
3) The reconstruction error has been reduced for all classes by using two analysis and synthesis dictionaries related to classes.

Dictionary learning has achieved satisfactory results in pattern classification. The key success of these approaches originates from the fact that a sample from a class of interest can be efficiently represented as a sparse linear combination of other samples of the same class, however with a reduced intentionality [14]. The performance of sparse coding depends on the quality of the formed dictionary atoms. Obtained dictionary is crucial for generating the sufficient sparse representation, e.g., sparsity and grouping. Therefore, recent methods of dictionary learning attempt to adaptively design atoms that efficiently represent the input data, e.g., image data [15]. The preliminary approach for building dictionaries is to use all the data, however, this technique leads to huge dictionaries; impractical to optimize [16]. One alternative to this approach is to manually deselect data that does not provide informative features and start optimizing only the relevant low-dimensional data. Sparse coding was applied for face recognition [13] and brain signal classification [17].

A limitation associated with the above approach is that the size of the dictionary increases when adding more classes and that degrades the classification performance. To scale to large training sets, researchers have proposed methods that learn a dictionary by merging its atoms by optimizing a predefined objective function [18], [19]. This mechanism decreases the mutual information between the dictionary atoms and the class labels [18]. Additionally, it minimizes the loss of mutual information between the histogram of dictionary atoms over signal constituents [19]. Despite the acceptable performance of these dictionaries, they tend to be computationally expensive due to the feature merging stage.

Other approaches involve jointly learning the dictionary and classifier using an optimized objective function. K-singular
value decomposition (K-SVD) method was utilized to train the dictionary \[^{20}\]. This method has been applied to a variety of image processing problems, including infilling missing pixels and image compression. The authors in \[^{21}\] proposed a method for dictionary learning that jointly learns the classifier parameters and dictionary for face recognition. A method called label consistent K-SVD (LC-KSVD) was proposed to learn a discriminative dictionary for sparse coding \[^{22}\], \[^{23}\]. Introducing labels and classification error to the objective function has leveraged the performance of LC-KSVD method.

More recently \[^{24}\], projective dictionaries pair learning (DPL) was proposed where two types of dictionaries were introduced, namely, analysis dictionary (for generating discriminative code by linear projection) and synthesis dictionary (for reconstructing the data). We used this method in a brain-computer interfacing application \[^{25}\]. In addition, we proposed an extension to the DPL method, called incoherent dictionary pair learning (InDPL) for the classification of Chinese handwritten numbers \[^{26}\].

Here, we present a Chinese handwritten numbers recognition system based on histogram of oriented gradients (HOG) features and labeled projective dictionary pair learning. In this method, the synthesis and analysis dictionaries are used to calculate the sparse codes. We include the class labels in the learning process but instead of using \(\ell_0\) and \(\ell_1\) regularisers, we propose a novel linear projection method that provide optimum trade-off between sparsity and grouping effect.

II. Methods

The proposed method comprises four stages: 1) preprocessing, 2) HOG feature extraction, 3) sDPL execution, and 4) classification. In the first stage, we enhance the quality of collected images. Then, the HOG features are extracted by taking the orientation histograms of edge intensity in local regions. In the third stage, the features of training samples construct the dictionary columns (a.k.a atoms). Supervised projective dictionary pair learning extracts the features before the fourth stage, in which Chinese numbers are classified.

A. Image Dataset

We use the handwriting Chinese numbers database to analyse the effectiveness of our method. This open source database was launched recently in our earlier work \[^{27}\]. The database contains 15,000 handwritten numbers from 100 Chinese nationals studying at Newcastle University, UK. Each participant wrote the 15 numbers of Figure 1 10 times.

B. Image preprocessing

The pre-processing phase includes two parts; image enhancement and noise removal. Initially, the scanned images

![Fig. 1: Chinese numbers and corresponding English numbers.](image1)

![Fig. 2: Example result of image pre-processing step for Chinese number 100. (A) the original grayscale image of size 64\(\times\)64; (B) binarized image using Otsu’s method; (C) cropped image; (D) down-sized image. The images are made negative for ease of presentation.](image2)

![Fig. 2: Example result of image pre-processing step for Chinese number 100. (A) the original grayscale image of size 64\(\times\)64; (B) binarized image using Otsu’s method; (C) cropped image; (D) down-sized image. The images are made negative for ease of presentation.](image2)

...
be a label matrix for input samples \( X \), and \( W \) encompasses classifier parameters. By considering (1) and (3) for constructing function \( \Psi \), to estimate \( P^*, D^*, W^* \) the following cost function can be proposed:

\[
\begin{align*}
\min_{P,D,W} & \quad \sum_{i=1}^{Q} \| X_i - D_i P_i X_i \|_F^2 + \lambda_1 \| P_i X_i \|_F^2 \\
& + \lambda_2 \| H_i - W_i P_i X_i \|_F^2 \\
& \text{s.t.} \| d_i \|_2^2 \leq 1
\end{align*}
\]

(4)

where \( X_i \) denotes the complementary data matrix \( X_i \) and \( H = \{ H_1, \ldots, H_q, \ldots, H_Q \} \) and \( H_q \in \mathbb{R}^{Q \times K} \) is the label matrix corresponding to an input sample \( X_q \). The below example shows the values of \( H_2 \) using four samples and three classes:

\[
H_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.
\]

The proposed sDPL algorithm is summarized in Algorithm 1. The equation (4) is generally non-convex and cannot be simultaneously solved for all variables. By considering variable matrix, \( A = PX \) in the objective function to calculate \( P^*, D^*, W^* \), and \( A^* \) becomes:

\[
\begin{align*}
\min_{P,D,W,A} & \quad \sum_{i=1}^{Q} \| X_i - D_i A_i \|_F^2 + \lambda_1 \| P_i X_i \|_F^2 \\
& + \lambda_2 \| H_i - W_i P_i X_i \|_F^2 + \lambda_3 \| P_i X_i - A_i \|_F^2 \\
& \text{s.t.} \| d_i \|_2^2 \leq 1
\end{align*}
\]

(5)

where \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) are the regularization parameters. For the optimization, equation (5) can be alternated between the following steps.

Step 1): Fix \( D, W, P \) and update \( A \): when fixing \( D, W, P \) to update \( A \), we omit the terms independent of \( A \) from (5):

\[
A^* = \arg\min_A \sum_{i=1}^{Q} \| X_i - D_i A_i \|_F^2 \\
+ \lambda_2 \| H_i - W_i P_i X_i \|_F^2 \\
+ \lambda_3 \| P_i X_i - A_i \|_F^2.
\]

(6)

This equation is convex and differentiable. After taking derivative with respect to \( A \) and equating it to zero we have:

\[
A^* = \frac{(D_i^T D_i + W_i^T W_i + \lambda_3 I)^{-1}}{(D_i^T X_i + \lambda_2 W_i^T H_i + \lambda_3 P_i X_i)}.
\]

(7)

Step 2: Fix \( D, W, A \) and update \( P \):

\[
P^* = \arg\min_P \sum_{i=1}^{Q} \lambda_1 \| P_i X_i \|_F^2 + \lambda_3 \| P_i X_i - A_i \|_F^2.
\]

(8)

We follow the same procedure as for \( A \) after solving the equation (8), \( P^* \) can be calculated with:

\[
P^* = (\lambda_3 X_i X_i^T + \lambda_1 X_i^T X_i^T + \gamma I)^{-1}(\lambda_3 A_i X_i^T)
\]

(9)

where \( \gamma \) is a small number to prevent division by zero.

Step 3: Fix \( P, D, A \), and update \( W \) with (10):

\[
W_i^* = (A_i A_i^T + \gamma I)^{-1}(H_i A_i^T).
\]

(10)
Algorithm 1: Proposed sDPL pseudo-code.

Input: Training samples for Q classes $X = [X_1, ..., X_Q]$, $m$, $\lambda_1$, $\lambda_2$, $\lambda_3$, $\gamma$

1. Initialize $D_0$ and $P_0$ as random matrix and calculate $A_0$ in equation (7) and $W_0$ in equation (10), $t = 0$
2. While not converge do
   3. $t = t + 1$
   4. For $i = 1 : k$
      5. Update $A_k^{(t)}$ by equation (7)
      6. Update $P_k^{(t)}$ by equation (9)
      7. Update $W_k^{(t)}$ by equation (10)
   8. Update $D_k^{(t)}$ by equation (11)
9. End
10. Output: $P^*$, $D^*$, $W^*$

Step 4: Fix $P, A, W$ and update $D$: Obtaining $D$ by using Alternating Direction Method of Multipliers (ADMM) algorithm [29] is as follows:

$$D^{(r+1)} = \min_D \sum_{i=1}^Q \|X_i - D_i A_i\|_F^2 + \rho \|D_i - S_i^{(r)} + T_i^{(r)}\|_F^2$$

$$S^{(r+1)} = \min_S \sum_{i=1}^Q \rho \|D_i^{(r+1)} - S_i^{(r)} + T_i^{(r)}\|_F^2$$

$$T^{(r+1)} = T^{(r)} + D_i^{(r+1)} - S_i^{(r+1)}.$$ (11)

E. Classification

Upon the completion of training with the labeled data in the proposed dictionary learning method, we obtain a learned synthesis dictionary $D$, analysis dictionary $P$ and transformation matrix $W$ for every class. Using $P^*$, $D^*$ and $W^*$ from the training stage, a class label for testing a typical input $X_t$ is obtained via:

$$\text{Class}(X_t) = \arg\min_i \|X_t - D_i P_i X_t\|_F^2 + \|H_i - W_i P_i X_t\|_F^2.$$ (12)

III. EXPERIMENTAL RESULTS

In this section, we conduct extensive experiments to evaluate the effectiveness of the proposed method. Hence, the classification results of proposed method on Chinese handwritten numbers database are presented first. Then, we compare the results of our proposed method with those obtained from several deep learning architectures as state-of-the-art machine learning techniques.

We initialized the parameters $m, \lambda_1, \lambda_2, \lambda_3, D_0, P_0, A_0$ and $W_0$ for our methods. In all experiments, we choose $m, \lambda_1, \lambda_2$ and $\lambda_3$ by 10-fold cross validation on each dataset. We then employed random initialization for both $D$ and $P$ for each class. Then, these parameters are used to compute the initial $A_0$ in equation (7) and consequently $W_0$ in equation (10).

We computed the performance of the proposed method with using three different validation approaches: conventional, between-subjects and within-subjects. In conventional cross validation, we applied 10-fold cross validation for all images in the database. In the between-subject cross validation, we considered all data from one person as test set and data of all other persons were used as training set. We repeated this process for each participants handwriting. In the within-subject cross validation, we consider n-th sample from all the participants or people for testing set and the remaining samples were used as training set. This process was repeated 10 times.

Table I shows the obtained results for the proposed method on Chinese handwritten numbers classification. From this table, we can see our proposed method outperforms result reported in [26] where an incoherent DPL (InDPL) was used. The average accuracy rate for Chinese numbers classification is 98.39% which is higher than InDPL method [26]. Our proposed penalty terms, i.e. classification labels, in addition to new HOG features, have dramatically enhanced the performance of our dictionaries.

Table II shows the confusion matrix on Chinese handwritten numbers where it is notable that misclassification occurred between classes of 13 and 11 that refers to 1000 and 10 from Chinese numbers, respectively. This is due to the semantical similarities between these digits. Next, we compare the proposed method with other classifiers such as k-nearest neighbor (kNN) and the original DPL [24]. The comparison results are shown in Figure 5. The accuracy of classification using the proposed method here is higher compared to using same features.

A. sDPL versus deep learning

For completeness, we compared our method with powerful deep learning models: that provide state-of-the-art performance in the field of machine vision. In this experiment, we selected three well-established platforms, namely, GoogLeNet [30], MobileNetV2 [31], and SqueezeNet [32]. To maximise the performance of these models, we used the fully-optimised version of the above models that were pre-trained on the very large ImageNet database [33]. The overall performance of sDPL was 98.53% which is comparable to GoogleNet (99.83%), MobileNetV2 (98.55%), and SqueezeNet (98.53%). The results also show that our method is more robust in recognising complex Chinese digits, e.g., number 9 and number 12; compared to CNN-based models. In the supplementary materials, we provide further details regarding class performances of each of the used deep learning methods.
Fig. 4: Confusion matrix representation as a result of performing conventional cross-validation experiment using sDPL for Chinese handwritten database. Horizontal and vertical axis show the target (true) and output classes, respectively. The diagonal values show correct classification accuracy, and off-diagonals indicate misclassification associated to each target class.

Fig. 5: Comparison of classification accuracy for three methods, namely, sDPL, DPL, and kNN, using different cross-validation settings.

B. Optimization performance

Figure A shows the optimization process of objective function values for 10 iterations. The value of cost function in (5) against the number of iterations are represented in this graph. As expected, the objective function value decreases monotonically and quickly.

To evaluate the effect of dictionary size on overall performance, we conducted an experiment with conventional 10-fold cross validation running at several dictionary sizes. Figure B shows the results of classification accuracy for proposed method and DPL, against different dictionary sizes (number of atoms $m$) for $D$. The results indicate that our method is robust against different dictionary sizes and achieves a stable accuracy for atom numbers more than 150. Hence, in order to preserve high performance we chose $m = 340$ for all of the experiments where the classification is at highest level as seen from Fig. B.

IV. Conclusions

We proposed a new supervised projective dictionary pair learning approach. Unlike most existing dictionary learning methods which use $\ell_0$-norm and $\ell_1$-norm to calculate sparse code, our approach is able to calculate sparse code by linear...
projection. We tested it with classifying Chinese handwritten numbers. The experimental results show that our approach yields excellent classification performance verified that were higher than that with conventional methods. Crucially, unlike deep learning methods, our method runs on computers with modest specifications; it runs all the data locally and it does not require GPU devices or cloud processing; two standard mechanisms for running deep learning models. Finally, GoogLeNet, MobileNetV2, and SqueezeNet, require 7, 3.5 and 1.24 million parameters respectively; the proposed model requires the fine-tuning of only eight parameters.

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