Localization of Elastic Layers by Correlated Disorder

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The equilibrium behavior of a system of elastic layers under tension in the presence of correlated disorder is studied using functional renormalization group techniques. The model exhibits many of the features of the Bose glass phase of type II superconductors induced by columnar defects, but may be more directly applicable to charge density waves, incommensurate magnetic phases, stacked membranes under tension, vicinal crystal surfaces, or superconducting “vortex–chains”. Below five dimensions, an epsilon expansion for the stable zero temperature fixed point yields the properties of the glassy phase. Transverse to the direction of correlation, the randomness induces logarithmic growth of displacements. Displacements are strongly localized in the correlation direction. The absence of a response to a weak applied transverse field (transverse Meissner effect) is demonstrated analytically. In this simple model, the localized phase is stable to point disorder, in contrast to the behavior in the presence of dislocations, in which the converse is believed to be true.

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Recently, considerable progress has been made in understanding the behavior of elastic media in the presence of randomness. Examples include single flux lines in a dirty superconductor [12], interfaces in random magnets [3], charge density waves [4], and the vortex glass phase of bulk superconductors [5,6]. The experimental work of Civale et. al. [9] has demonstrated the feasibility of creating superconducting samples with correlated (columnar)

disorder. One of the most striking aspects of the resulting localized Bose glass phase [10,11] is the existence of a finite critical mismatch angle $\theta_c$ between the applied field and the correlation direction, such that the flux lines remain parallel to the correlation axis for $\theta < \theta_c$. In this paper, an analogous localized phase is studied analytically near 5 dimensions for a somewhat simplified model which may describe other systems of interest. The pulling–away from the correlation axis for $\theta > \theta_c$ is discussed in Ref. [12].

Consider a system made up of a stack of layers (for general $d$, these will be oriented manifolds of dimension $d-1$, for instance interfaces) which may fluctuate in the perpendicular direction, but may not pass through one another (see Fig. 1). Such a model could describe a charge density wave, the domain walls of an incommensurate striped phase in a magnet [3], a stack of membranes under tension, or, for $d = 2$, a set of steps on a miscut crystal surface [4]. Another possible realization is suggested by recent observations of “vortex–chains” in YBCO [5], which should fluctuate as layers defined by the chain and magnetic field directions. The displacement of the $k^{th}$ layer, $u_k$, is defined by $x_{k,1} \equiv k\ell + u_k(x_2, \ldots, x_{d-1}, z)$, where $\ell$ is the average layer separation, and $x_1$ is taken to be the layering axis. The last coordinate $x_d \equiv z$ has been distinguished as the direction of correlations for the random potential. The use of a displacement field neglects dislocations, which will be important in many systems beyond some length scale. Taking the continuum limit $u_k(x_2, \ldots, x_{d-1}, z) \rightarrow u(x, z)$, the Hamiltonian is

$$H = \int d^{d-1}x dz \left\{ \frac{K}{2} (\nabla u)^2 + \frac{K}{2} (\partial_z u)^2 + h \partial_z u + V_C(u, x) + V_P(u, x, z) \right\},$$

where the $x_1$ coordinate has been rescaled to remove the anisotropy of the in-layer elasticity, and a momentum cut-off $\Lambda \sim 1/\ell$ is implicitly included in the $x$ direction.
(but not in $z$) due to the discreteness of the layers. A non-zero $h$ represents a force tending to tilt the layers, such as that caused by a change in the applied field in a superconductor, or by tilted boundary conditions. Only the response to a small $h$ will be considered here. $V_C$ and $V_P$ are random potentials describing, respectively, columnar and point disorder. By including both types of pinning, it is possible to study the competition between vortex-glass and Bose-glass like phases in this dislocation-free model [10]. Because of the periodicity of the stack of layers, these potentials must be periodic functions of $u$. For weak disorder, the fixed point potentials will have Gaussian distributions, with the correlation functions

$$\langle V_C(u,x)V_C(u',x') \rangle = R_C(u-u')\delta(x-x'),$$  \hspace{1cm} \text{(2)}

$$\langle V_P(u,x,z)V_P(u',x',z') \rangle = R_P(u-u')\delta(x-x')\delta(z-z').$$  \hspace{1cm} \text{(3)}

Choosing $\ell = 2\pi$, periodicity implies $R_{C,P}(u + 2\pi) = R_{C,P}(u)$.

To understand the behavior on long wavelengths, a renormalization group (RG) analysis was performed using the methods of Ref. [1]. Lengths are rescaled parallel and perpendicular to the disorder, according to $x \rightarrow b x$ and $z \rightarrow b^2 z$. The displacement cannot be rescaled due to periodicity. The cut-off is kept fixed by integrating out modes with $x$ momenta in a shell $\Lambda/b < p < \Lambda$ (momenta in the $z$ direction are unrestricted). Formally, this procedure can be carried out by expanding the partition function in $V_C$ and $V_P$ and performing the functional integrals order by order. The resulting terms are functions of the remaining modes, and upon reexponentiation yield flows of the interactions in the Hamiltonian. Since $u$ is dimensionless, however, $R_C$ and $R_P$ cannot be taken to have the $u^4$ form commonly encountered in RG studies of phase transitions.

For $d = 5 - \epsilon$, the behavior of the system is dominated by a non-trivial zero-temperature fixed point. The scale changes yield the eigenvalues $\lambda_T = -\theta = -2 - \chi + O(\epsilon)$ and $\lambda_{R_P} = -\chi + O(\epsilon)$. Naively, $\chi = 1 + O(\epsilon)$, but this point will be returned to later. Regardless of the value of $\chi$ (so long as it is positive), $T$ and $R_P$ are formally irrelevant, so I will begin by working directly at $T = R_P = 0$.

At zero temperature, the computation of the partition function reduces to the optimization of the hamiltonian. At each step of the RG, the lowest energy configuration of the modes in the shell is found as a function of the low momentum modes, which are held fixed (see, e.g. Ref. [1]). The minimum of $H$ (Eq.4) clearly satisfies $\partial_u u = 0$ exactly. The renormalization of $R_C$, therefore, must be identical to the case of point disorder in $d - 1$ dimensions. In addition, the statistical Galilean invariance of the $d - 1$ dimensional model [17] guarantees that temperature is only trivially renormalized by the scale changes. There is, however, a non-trivial renormalization of $K$, since the iterative minimization does not throw out ex-citation information until it is on a scale smaller than the cut-off. Defining the force-force correlation function $\Delta_C(u) \equiv -R_C''(u)$, the RG equations for $b = e^{\lambda t}$ to lowest non-trivial order are

$$\frac{dK}{dt} = \left(2 - 2\chi - \Delta_C''(0)/(8\pi^2)\right)K,$$  \hspace{1cm} \text{(4)}

$$\frac{d\Delta_C}{dt} = \epsilon \Delta_C - \frac{1}{8\pi^2} \left(\Delta_C'' + \Delta_C(\Delta_C - \Delta_C(0))\right),$$  \hspace{1cm} \text{(5)}

where, for simplicity, I have taken $K = 1$. Eq.3 is obtained assuming analyticity of $\Delta_C$, and will be corrected later. Eq.4 has been derived previously by a number of other authors [15,16,19], and has the stable $2\pi$-periodic fixed point solution

$$\Delta_C(u) = \frac{4\pi^2\epsilon}{3} \left[(u - \pi)^2 - \pi^2/3\right],$$  \hspace{1cm} \text{for} \ 0 < u < 2\pi.$$  \hspace{1cm} \text{(6)}

This fixed point solution leads [19] to logarithmic displacement fluctuations. First order perturbation theory in $R_C$, evaluated at the fixed point (Eq.5), gives the $O(\epsilon)$ result

$$\langle (\tilde{u}(x) - \tilde{u}(0))^2 \rangle \sim \frac{2\pi^2\epsilon}{9} \ln |x|,$$  \hspace{1cm} \text{(7)}

where $\tilde{u}(x)$ is defined as the $z$ average of $u(x,z)$, and the angular brackets denote both thermal and disorder averaging. The fixed point function has a slope discontinuity when $u = 0$, so that $\Delta_C'(0) = -\infty$! Note that the procedure employed in the near-threshold dynamic problem of Ref. [1] for handling this singularity does not apply here due to the different physics of force–free equilibrium. The divergence implies that the feedback of $\Delta_C(u)$ to the elastic terms (Eq.4) must be re-analyzed. With no assumptions on the analyticity of $\Delta_C(u)$, the term generated by the RG which led to this equation takes the form

$$\frac{dH}{dt} \bigg|_{\text{elastic}} = -\frac{1}{2} \frac{\Lambda^3}{16\pi^2K^{1/2}} \int d^{d-1}x dz dz' \Delta_C(u(z) - u(z'))$$

$$\times \exp \left[\frac{\Lambda}{K^{1/2}} |z - z'|\right].$$  \hspace{1cm} \text{(8)}

The exponential decay for large separations justifies a gradient expansion of $u(z)$ near $z'$, which combined with the small $u$ behavior $\Delta_C(u) \approx \frac{4\pi^2\epsilon}{9} |(2\pi/3 - \pi |u| + u^2/2)$ yields a new term in the Hamiltonian

$$\Delta H = \frac{\sigma}{2} \int d^{d-1}x dz |\partial_u u|. \hspace{1cm} \text{(9)}$$

Eq.8 is replaced by the pair of equations

$$\frac{d\sigma}{dt} = (2 - \chi)\sigma + \frac{\pi\Lambda\epsilon K^{1/2}}{3},$$  \hspace{1cm} \text{(10)}

$$\frac{dK}{dt} = (2 - 2\chi - \epsilon/3)K,$$  \hspace{1cm} \text{(11)}

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The new elastic term has a direct physical meaning. Consider the response of the system to an infinitesimal field $h$. In the absence of disorder, the minimum of the Hamiltonian as a function of $\partial_z u$ is shifted over by an amount linear in $h$, resulting in a response $\theta \equiv \partial_z u \propto h$. This will be true regardless of the magnitude of $K$. If the elastic term has the form of $\sigma|\partial_z u|$, however, the minimum will remain at $\partial_z u = 0$ for all $h < K$. This implies the existence of a finite threshold force, below which the layers remain locked in their localized positions.

It is clear, both from the physical interpretation above and from the form of Eq.11 and Eq.13, that the simple expectation of $O(\epsilon)$ corrections to $\chi$ will not hold. Eq.11 suggests a value of $\chi = 2$, in accord with a simple "random walk" picture of excursions from the localized ground state. A careful analysis, however, must continue the RG procedure after the new elastic term has been generated. While this cannot affect the renormalization of $\Delta_C$, it will probably affect the $z$-dependent portions of the Hamiltonian. It is not known whether a consistent treatment of such a non-analytic elastic term is possible even to $O(\epsilon)$. Justification of the value $\chi = 2$, even to lowest order, requires a detailed investigation of the zero temperature RG along the lines of that performed in appendix C of Ref.4. Such an analysis is not available at present.

Because the vanishing of the tilt response requires $\Delta'_C(0) = -\infty$, the weak field behavior is sensitive to corrections from formally irrelevant operators such as temperature and point disorder. To safely conclude the stability of the correlated phase, the corrections will now be studied in more detail. These operators yield additional renormalizations of $\Delta_C$:

$$
\frac{d\Delta_C}{dl} = \frac{\Lambda}{16\pi^2K^{1/2}} \left[ \Lambda^2T + \frac{1}{2}\Delta_P(0) \right] \Delta_C'.
$$

Because of the presence of such terms, analytic behavior of $\Delta_C(u)$ will persist within a narrow boundary layer around $u = 0$. Since temperature and uncorrelated disorder are irrelevant operators, the width $w$ of the boundary layer decreases under the RG (note that since $\theta > |\lambda_{R_P}|$, point disorder will dominate this width at long length scales). Although this rounding is not a property of the fixed point value of $\Delta_C$, the rapid divergence of $\Delta'_C(u)$ in the absence of the irrelevant operators (see, e.g. Ref.4) indicates that the size of the smoothed region is determined by the terms in Eq.12.

As the RG iterates to longer length scales, $\Delta_C(u)$ sharpens up near the origin. From Eq.13, the jump in slope across the boundary layer, where $\Delta_C(u)$ must match its fixed point value, is $O(\epsilon)$. Since this change in slope must be accommodated over a width $w$, the curvature $\Delta'_C(0) \sim -w$. Equating the terms in Eq.13 to those in Eq.12 gives the scaling of the boundary layer width

$$
w(l) \sim \frac{\Lambda}{\epsilon K^{1/2}} \left[ \Lambda^2T(l) + \frac{1}{2}\Delta_P(0;l) \right].
$$

FIG. 2. A schematic illustration of the columnar disorder correlation function $\Delta_C(u)$. For $|u-2\pi| \gg w$, $\Delta_C(u)$ has the simple form of Eq.6. For $|u-2\pi| \ll w$, $\Delta_C(u)$ is rounded, with curvature of $O(\epsilon/w)$.

$\Delta_C(u)$ is illustrated in Fig.3. From the structure of Eq.13 it is clear that (for small gradients) the generated terms take the form

$$
\frac{dH}{dl}_{\text{elastic}} = \epsilon \Lambda^2 \int d^{d-1}x dz f \left[ K^{1/2} / \Lambda |\partial_z u| \right],
$$

where $f(u) \sim |u|$ for $w \ll |u| \ll 1$, but is smooth for $|u| \ll w$.

Tilting for small $h$ can only occur for $\theta$ within the smooth boundary layer. Although Eq.14 is valid only while feedback from $\sigma$ can be neglected, it does demonstrate that this boundary layer width, $w$, vanishes exponentially as a function of length scale. Since the average tilt $\theta$ is a bulk ($q = 0$) quantity, the boundary layer does not contribute. By contrast, it is precisely the small boundary-layer in Eq.14 which leads to thermal creep in weakly driven interfaces and charge-density waves24. The only important effect of the interaction between point and columnar disorder in this model is to decrease the relevance of point disorder, both through an increase in $\chi$ and through terms from $\Delta_C$ feeding into the RG equation for $\Delta_P$ (not shown here).

In conclusion, the system of elastic layers studied here forms a disordered-dominated phase in the presence of correlated randomness. Despite the reduction in the number of components of the displacement field and the neglect of dislocations, the model exhibits analytically many of the properties of the Bose glass. The physical properties are governed by a zero temperature fixed point, at which the layers are completely parallel to the direction of correlation. Due to this localization, the tilt response to an applied transverse field vanishes below some finite threshold field, corresponding to the "transverse Meissner effect" of Ref.10. The fluctuations of the layers perpendicular to the correlations grow logarithmically, with a universal prefactor. In contrast to the
Bose glass case [21], the system is stable against uncorrelated disorder. It thus appears that dislocations are necessary to make point disorder relevant in the localized phase. Naive arguments suggest a simple “random walk” scaling, $\chi = 2$, for the low-lying excitations, although it is quite likely that this result will be corrected by non-trivial renormalizations. The analysis of these excursions from the ground state and the extension to more components remain interesting open problems.

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