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The strong Langmuir turbulence spectra in weakly relativistic plasma

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ABSTRACT

The spectral characteristics of strong Langmuir turbulence in weakly relativistic plasma is analyzed. The temporal evolutions of the turbulence energy density spectra are obtained by performing the fast Fourier transformations on weakly relativistic Zakharov equations. The relativity parameter significantly influenced the Langmuir turbulence, in that a decrease in the relativity parameter lengthened the action time of wave-wave and wave-particle interactions, which is useful for studying the particles acceleration problem.

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I. INTRODUCTION

The ongoing question of strong Langmuir turbulence in plasma physics is of importance.1 Many researches on strong Langmuir turbulence have been well-established through both experimental data and numerical simulations.2 Turbulence is a coupling process in which the large-wave pattern is merged with the small-wave pattern that at the beginning of the dissipation.3 To describe strong Langmuir turbulence, Zakharov derived a set of nonlinear equations in 1972, known as the Zakharov equations (ZEs).4 The interaction of Langmuir waves and ion-acoustic waves can be described by ZEs; Nicholson and Goldman have investigated two-dimensional plasma by Zakharov equations.5,6 The ZEs incorporate significant modulational instability, which leads to Langmuir plasmon collapse. In this type of collapse, the large-wave pattern turns into a small-wave pattern containing multiple degrees of freedom, and is then termed strong Langmuir turbulence.4

As the technology evolves, the lasers involved are necessarily becoming stronger, reaching intensities of 1021 W cm−2. At these strengths, the relativistic effects become more important and must be considered in experiments involving high-energy laser plasma, relativistic electron beam plasma interactions, Thomson scattering, supernova remnants and gamma ray bursts.4−11 Many theoretical studies on strong Langmuir turbulence have generally been based on relativistic interactions. Freund et al. have used a multiple time-scale expansion method to describe the evolution of the relativistic Langmuir waves;12 Shukla et al. also presented a review of the relativistic nonlinear effects in plasma.13 Xiaolan Liu et al. have obtained the relativistic Zakharov equations.14 This paper researched the strong Langmuir turbulence spectra is following the relativistic Zakharov equations.

The strong Langmuir turbulence spectra have also been an active research area during the past few decades.15−17 The concept of turbulence spectra and energy flux distributed with respect to the spectra is fundamental. The wide-ranging interest in the turbulence spectra is not only motivated by the desire to gain a deeper understanding of SLT, but also because the spectra of the plasmons are believed to play a role in the turbulence acceleration by strong Langmuir plasma. Experimental and simulation results suggest that the saturated spectrum should not be of a Kolmogorov type, but rather it possesses a combination of power-law and exponential dependence on the wave number.16,17 Recent years, a series of these calculations is based on the non-relativistic Zakharov equations have been conducted by Mohamed, Sahu and G. I. de Olivera et al.10−12 Our study focused on weakly relativistic strong Langmuir turbulence and performed fast Fourier transformations (FFTs) to simulate the evolution of the turbulence spectra.

The remainder of this paper is organized as follows. In Sec. II, we begin with the relativistic Zakharov equations for strong Langmuir turbulence, and present the turbulence spectra of the three
waves through numerical simulation. The conclusions are given in Sec. III. This study will improve current understanding of strong Langmuir turbulence characteristics, which will enable further research on the laser, space and astrophysics plasma.

II. NUMERICAL SIMULATION

The relativistic Zakharov equations on describing the Langmuir wave and ion-acoustic wave is: \(^{14-16}\)

\[
\frac{\partial^2}{\partial t^2} \mathbf{v} - \nabla^2 \mathbf{n}^* (\mathbf{r}, t) = \left( \frac{m_e}{m_i} \right) \left( \frac{\omega_p^2}{\omega_p} \right)^2 \nabla^2 \left[ \mathbf{E}^*(\mathbf{r}, t) \right]^2, \tag{1}
\]

\[
\text{div} \frac{2i}{\omega_p} \left[ \frac{\partial \mathbf{E}^*(\mathbf{r}, t)}{\partial t} + \frac{3c^2}{5\omega_p^2} \mathbf{v} \cdot \nabla \mathbf{E}^*(\mathbf{r}, t) \right] = \text{div} \left[ \mathbf{E}^*(\mathbf{r}, t) \frac{n'(\mathbf{r}, t)}{n_0} \right]. \tag{2}
\]

Through the substitutions,

\[
\xi = \frac{2\sqrt{\pi}}{3} k_d \mathbf{r}, \quad \tau = \frac{2\mu}{\omega_p^2 \pi} \mathbf{E}(\xi, \tau) = \frac{\sqrt{3} \mathbf{E}^*(\mathbf{r}, t)}{4\sqrt{\pi} m_0 T_e},
\]

\[
n = \frac{3}{4\mu} n', \quad \mu = \frac{m_e}{m_i}, \quad \alpha = \frac{c^2}{3\sqrt{\pi} \gamma},
\]

we can now rewrite Eq. (1) and Eq. (2) in the dimensionless form

\[
\left( \frac{\partial^2}{\partial t^2} - \nabla^2 \right) n = \frac{\mu}{\alpha} \nabla^2 |\mathbf{E}|^2, \tag{4}
\]

\[
i \frac{\partial}{\partial \tau} \mathbf{E} + \frac{3}{5} \sqrt{\alpha} \nabla \mathbf{E} - \frac{\mu}{\sqrt{\alpha}} \mathbf{n} \mathbf{E} = 0, \tag{5}
\]

where \(\mathbf{E} = E_x \mathbf{i} + E_y \mathbf{j} + E_z \mathbf{k} \), \(\mathbf{E} \) is the Langmuir wave envelope, \(\alpha \) is the relativistic parameter, \(c \) is the speed of light, and \(v_T \) is the thermal velocity of electron, \(\mu = m_e/m_i, \) \(m_i \) is the mass of ion, \(m_e \) is the mass of ion, \(t \) is time, \(n \) is the dimensionless low-frequency density, \(T_e \) is electron temperature. And the Langmuir wave satisfies:

\[
\nabla \times \mathbf{E} = 0. \tag{6}
\]

The component equations of Eq. (5) are

\[
i \frac{\partial}{\partial \tau} E_x + \frac{3}{5} \sqrt{\alpha} \left( \frac{\partial^2}{\partial x^2} E_x + \frac{\partial^2}{\partial x \partial y} E_y + \frac{\partial^2}{\partial x \partial z} E_z \right) - \frac{\mu}{\sqrt{\alpha}} n E_x = 0, \tag{7}
\]

\[
i \frac{\partial}{\partial \tau} E_y + \frac{3}{5} \sqrt{\alpha} \left( \frac{\partial^2}{\partial x \partial y} E_x + \frac{\partial^2}{\partial y^2} E_y + \frac{\partial^2}{\partial y \partial z} E_z \right) - \frac{\mu}{\sqrt{\alpha}} n E_y = 0, \tag{8}
\]

\[
i \frac{\partial}{\partial \tau} E_z + \frac{3}{5} \sqrt{\alpha} \left( \frac{\partial^2}{\partial x \partial z} E_x + \frac{\partial^2}{\partial y \partial z} E_y + \frac{\partial^2}{\partial z^2} E_z \right) - \frac{\mu}{\sqrt{\alpha}} n E_z = 0. \tag{9}
\]

By considering all the physical parameters are two dimensions and three components (only dependent on \(x \) and \(y \)), Eqs. (7)–(9) can be written as:

\[
i \frac{\partial}{\partial \tau} E_x + \frac{3}{5} \sqrt{\alpha} \left( \frac{\partial^2}{\partial x^2} E_x + \frac{\partial^2}{\partial x \partial y} E_y \right) - \frac{\mu}{\sqrt{\alpha}} n E_x = 0, \tag{10}
\]

\[
i \frac{\partial}{\partial \tau} E_y + \frac{3}{5} \sqrt{\alpha} \left( \frac{\partial^2}{\partial x \partial y} E_x + \frac{\partial^2}{\partial y^2} E_y \right) - \frac{\mu}{\sqrt{\alpha}} n E_y = 0, \tag{11}
\]

\[
i \frac{\partial}{\partial \tau} E_z + \frac{3}{5} \sqrt{\alpha} \left( \frac{\partial^2}{\partial x \partial z} E_x + \frac{\partial^2}{\partial y \partial z} E_y \right) - \frac{\mu}{\sqrt{\alpha}} n E_z = 0. \tag{12}
\]

Assume the ion-acoustic movements are subsonic, which means \(\frac{\partial^2}{\partial \tau^2} \ll \nabla^2 \). Therefore, Eq. (4) and Eq. (5) will change into

\[
n = -\frac{\mu}{\alpha} |\mathbf{E}|^2. \tag{13}
\]

\[
i \frac{\partial}{\partial \tau} \mathbf{E} + \frac{3}{5} \sqrt{\alpha} \nabla \mathbf{E} - \frac{\mu}{\sqrt{\alpha}} |\mathbf{E}|^2 \mathbf{E} = 0. \tag{14}
\]

FIG. 1. Collapse development of Langmuir wave when \(\alpha = 0.5\): (a) \(\tau = 30\), \(E_{max}^2 = 64.1\); (b) \(\tau = 340\), \(E_{max}^2 = 118.3\); (c) \(\tau = 520\), \(E_{max}^2 = 634.1\).
The components of the Langmuir wave envelope are set as

\[ E_x = c_x + id_x, \quad E_y = c_y + id_y, \quad E_z = c_z + id_z, \quad (15) \]

where \( c_x, c_y, c_z, d_x, d_y, d_z \) are real numbers. Eqs. (10)–(12) can be written as

\[
\frac{\partial}{\partial \tau} c_x + \frac{3}{5} \sqrt{\alpha} \left( \frac{\partial^2 d_x}{\partial x^2} + \frac{\partial^2 d_y}{\partial x \partial y} \right) + \frac{\mu^2 \sqrt{\alpha}}{\alpha^2} |E|^2 d_x = 0, \quad (16)
\]

\[
\frac{\partial}{\partial \tau} d_x - \frac{3}{5} \sqrt{\alpha} \left( \frac{\partial^2 c_x}{\partial x^2} + \frac{\partial^2 c_y}{\partial x \partial y} \right) - \frac{\mu^2 \sqrt{\alpha}}{\alpha^2} |E|^2 c_x = 0, \quad (17)
\]
FIG. 4. Turbulence spectrum of Langmuir wave when $\alpha = 0.5$, (a) $\tau = 30$, (b) 430, (c) 520.

\[
\frac{\partial c_y}{\partial \tau} + \frac{3}{5} \sqrt{\alpha} \left( \frac{\partial^2 d_y}{\partial y^2} + \frac{\partial^2 d_y}{\partial x \partial y} \right) + \frac{\mu^2 \sqrt{\alpha}}{\alpha} E^2 d_y = 0, \quad (18)
\]

\[
\frac{\partial d_y}{\partial \tau} - \frac{3}{5} \sqrt{\alpha} \left( \frac{\partial^2 c_y}{\partial y^2} + \frac{\partial^2 c_y}{\partial x \partial y} \right) - \frac{\mu^2 \sqrt{\alpha}}{\alpha} E^2 c_y = 0, \quad (19)
\]

\[
\frac{\partial c_z}{\partial \tau} + \frac{\mu^2 \sqrt{\alpha}}{\alpha^2} |E|^2 d_z = 0, \quad (20)
\]

\[
\frac{\partial d_z}{\partial \tau} - \frac{\mu^2 \sqrt{\alpha}}{\alpha^2} |E|^2 c_z = 0. \quad (21)
\]

FIG. 5. Turbulence spectrum of Langmuir wave when $\alpha = 1$, (a) $\tau = 20$, (b) $\tau = 300$, (c) $\tau = 380$.

FIG. 6. Turbulence spectrum of Langmuir wave when $\alpha = 5$, (a) $\tau = 10$, (b) $\tau = 160$, (c) $\tau = 200$. 

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A weakly relativistic kinetic treatment of strong Langmuir turbulence with fast Fourier transformations (FFTs) is proposed to investigate the evolution of the turbulence spectra. Then, we have studied the effects of the relativity parameters on the evolution. The results may be summarized as follows:

Figs. 1(a), (b), and (c) show the Langmuir evolution process under different times. The collapse phenomenon of wave on these strong fields is given by Figs. 1(a), (b), and (c). When the evolution time is larger than $\tau_0=1000$, $L_0=500$. The instantaneous strength values of Langmuir waves are given in Figs. 4–6. When the evolution time is larger than $\tau_0=1000$, $L_0=500$. The instantaneous strength values of Langmuir waves are given in Figs. 4–6. When the evolution time is larger than $\tau_0=1000$, $L_0=500$.

The instantaneous strength values of Langmuir waves are given in Figs. 4–6. When the evolution time is larger than $\tau_0=300$, $\tau=300$, and $\tau=520$ the time goes on, the energy of Langmuir wave is transferred from the small wave number area into the large one, and demonstrate the filamentation of Langmuir wave, that is to say, different peak values for the different large wavenumber regions, which is the typical characteristic of strong Langmuir turbulence. Figs. 4–6 show that the spectra of strong Langmuir turbulence for the case of the relativistic parameter $\alpha_0=0.5$, $\alpha=1$, and $\alpha=5$. Obviously, the relativistic parameters have a great influence on the Langmuir turbulence. In the physical sense, the stronger the relativistic parameter is, the longer the action time of wave-particle is, and there will be a longer evolution time of collapse, and the turbulence will be more tempestuous.

Our numerical simulations on the weakly relativistic turbulence spectra are advantageous to research the particles acceleration problem, which will be discussed in the future.

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