Global gravitationally organized spiral waves and the structure of NGC 5247

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ABSTRACT
Using observational data, we build numerical N-body, hydrodynamical and combined equilibrium models for the spiral galaxy NGC 5247. The models turn out to be unstable as regards spiral structure formation. We simulate scenarios of spiral structure formation for different sets of equilibrium rotation curves, radial velocity-dispersion profiles and disc thicknesses and demonstrate that in all cases the simulated spiral pattern agrees qualitatively with the observed morphology of NGC 5247. We also demonstrate that an admixture of a gaseous component with a mass of about a few per cent of the total mass of the disc increases the lifetime of a spiral pattern by approximately 30 per cent. The simulated spiral pattern in this case lasts for about 3 Gyr from the beginning of the growth of perturbations.

Key words: instabilities – ISM: structure – galaxies: kinematics and dynamics – galaxies: spiral – galaxies: structure.

1 INTRODUCTION
Attempts to understand the phenomenon of spiral structure in galaxies have a long history. It has become clear nowadays that spiral structure is a density wave (or waves) propagating in a multicomponent stellar–gaseous disc. A universal mechanism for the generation of such spiral density waves that successfully explains the rich morphological variety of spiral galaxies does not yet exist. To describe the observed patterns, a few spiral-generation mechanisms are usually invoked. Some researchers treat the spiral structure as a transient phenomenon, so that the spiral pattern changes many times during galactic evolution (see e.g. Sellwood 2011 for references). Gerola & Seiden (1978) even suggested that spiral structure is merely a product of recent star formation, outlined by newborn stars.

On observational grounds, the problem of the origin of spiral structure has long been a topic of discussion. It has become clear nowadays that spiral structure is a density wave (or waves) propagating in a multicomponent stellar–gaseous disc. A universal mechanism for the generation of such spiral density waves that successfully explains the rich morphological variety of spiral galaxies does not yet exist. To describe the observed patterns, a few spiral-generation mechanisms are usually invoked. Some researchers treat the spiral structure as a transient phenomenon, so that the spiral pattern changes many times during galactic evolution (see e.g. Sellwood 2011 for references). Gerola & Seiden (1978) even suggested that spiral structure is merely a product of recent star formation, outlined by newborn stars.

In this paper, we continue our efforts to compare the properties of the observed spiral structure with theoretical predictions. In previous papers (Korchagin et al. 2000, 2005), we made such a comparison using a hydrodynamical approach to model the dynamics of galactic discs. In this paper we simulate three-dimensional collisional–gaseous models of galactic discs, aiming to determine the model parameters that most closely match the observed spiral pattern in the galaxy NGC 5247. We choose this galaxy for our comparison because it is well-studied observationally and its rotation curve as well as disc velocity dispersion and luminosity distribution have been accurately determined.
In the cases that have been studied, the admixture of a gaseous component increases the lifetime of the spiral pattern (Semelin & Combes 2002; Fux 1999). In general, however, the lifetime of spiral patterns in isolated galaxies and the role of gas remains an open issue that we address in this paper.

In Section 2 we discuss the observational properties of the galactic disc of NGC 5247. In Section 3 we present the basic equations and discuss the disc equilibrium. Section 4 presents the results of numerical simulations of a gaseous and collisionless stellar model of the galaxy NGC 5247. Section 5 presents the results of our simulations of stellar–gaseous models and in Section 6 we present some concluding remarks.

2 NGC 5247: CONSTRAINTS FROM OBSERVATIONAL DATA

NGC 5247, shown in Fig. 1, is a nearby spiral galaxy. It has a strong, well developed two-armed ‘grand-design’ spiral pattern and does not demonstrate any signs of interactions with other galaxies (Considere & Athanassoula 1988). The spiral pattern, however, looks somewhat lopsided. A low-amplitude spiral pattern is detected (Considere & Athanassoula 1988). The spiral pattern, however, does not demonstrate any signs of interactions with other galaxies.

NGC 5247, shown in Fig. 1, is a nearby spiral galaxy. It has a radial scalelength of 2.8 kpc, as determined by these authors. The inclination angle of NGC 5247 is rather uncertain. Measurements range from $i = 20^\circ$ (Zhao et al. 2006) to $i = 40^\circ$ (HyperLeda; Paturel et al. 2003). In this paper we adopt an inclination of $28^\circ$, which is close to an average from these references, and a distance to the galaxy of 17.4 Mpc (HyperLeda; Paturel et al. 2003).

The vertical scale of the disc ($z_0$) significantly influences its stability properties. Unfortunately, an accurate estimate of the vertical thickness of the galactic disc is not simple. Zhao et al. (2006) tackled this problem by using the Jeans equations applied along the vertical axis of the disc and estimated the disc thickness of NGC 5247 to be $z_0 = 1.5 \pm 0.6$ kpc. With this value, the ratio of the radial to vertical scalelength is $r_0/z_0 = 3.2^{+1.1}_{-0.8}$.

The line-of-sight velocity dispersion as a function of galactocentric radius for NGC 5247 has also been determined from observations (Bottema 1993; van der Kruit & Freeman 1986). The measurements by both groups agree with each other and show an approximately exponential decrease of the velocity dispersion with radius, which is typical for disc galaxies.

3 MODELLING NGC 5247: BASIC EQUATIONS

We assume that the potentials of the dark halo and the stellar bulge are static and remain axisymmetric. Such an assumption does not affect the dynamics of the galactic disc, because the contribution of high velocity dispersion components to spiral density waves is insignificant even if they involve most of the galactic mass (Marochnik et al. 1972). The galaxy does not have a large central bar, and we exclude from consideration those models with a bar that is considered to be an effective generator of spiral density waves (Korchagin & Shevelev 1981; Buta et al. 2009; Kaufmann & Contopoulos 1996). Secondly, observational data do not allow us to estimate the degree of asymmetry of spheroidal subsystems and the emergence of new free parameters greatly complicates the analysis. Moreover, an interaction with a live bulge or halo cannot be decisive.

To simulate the dynamics, we use a set of equations that self-consistently describes the behaviour of a stellar–gaseous disc in equilibrium with an external gravitational field $\Psi_{ext}$. The set of equations for a stellar disc consisting of $N$ gravitationally interacting particles is written as

$$\frac{d\mathbf{r}_i}{dt} = -\nabla \left[ \Psi_s(r_i) + \Psi_{ext}(r_i) + \Psi_h(r_i) \right], \quad i = 1, ..., N. \quad (1)$$

Here $\Psi_s$ is the gravitational potential of the stellar component of the disc, $\Psi_g$ is the potential caused by the gaseous component and $\Psi_{ext}$ is the combined gravitational potential of the galactic halo ($\Psi_h$) and the bulge ($\Psi_b$). Therefore $\Psi_{ext} = \Psi_b + \Psi_h$.

The set of equations describing the dynamics of the gaseous component is given by

$$\frac{\partial\rho_g}{\partial t} + \mathbf{u} \cdot \nabla \rho_g + \nabla p + \rho_g \nabla \left[ \Psi_g + \Psi_{ext} + \Psi_s \right] = 0, \quad (2)$$

$$\frac{\partial E}{\partial t} + \mathbf{u} \cdot \nabla [(E + p)\mathbf{u}] + \rho_g \mathbf{u} \cdot \nabla \left[ \Psi_g + \Psi_{ext} + \Psi_s \right] = 0, \quad (3)$$

where $\rho_g$, $p$ and $\mathbf{u}$ are the volume density, pressure and velocity vector of the gas. The energy density of the gas $E$ is given by the expression $E = \rho_g|\mathbf{u}|^2/2 + p/(\gamma - 1)$. We use an adiabatic equation of state $\rho_g = K_p|\mathbf{u}|^2$ for the gas, where the value of the adiabatic index is assumed to be $\gamma = 1.1$ and the constant $K_p$ was chosen so that the sound speed is equal to $c_s = 10$ km s$^{-1}$ in the central regions of the disc. The sound speed of the gas decreases slowly with radius, reaching approximately 8 km s$^{-1}$ at the periphery of the disc. Self-gravity is taken into account by the Poisson equation applied for the gaseous and stellar components:

$$\Delta \Psi_g = 4\pi G \rho_g, \quad (5)$$

$$\Delta \Psi_s = 4\pi G \rho_s. \quad (6)$$

To model the gravity of the halo, we use the potential

$$\Psi_h = \frac{G M_h}{C_h} \left\{ \ln(\xi) + \frac{\text{atan}(\xi)}{\xi} + \frac{1}{2} \ln 1 + \frac{1 + \xi^2}{\xi^2} \right\}. \quad (7)$$

Here $\xi = r/\alpha_h, \alpha_h$ is the scalelength of the halo potential, $M_h$ is the mass of the halo within radius $r_h = 20$ kpc and $C_h = \alpha_h(r_h/\alpha_h - \text{atan}(r_h/\alpha_h))$. Such a choice provides a constant rotation velocity at large radii $r > 2\alpha_h$, so we have a rotation curve typical for disc galaxies.

Figure 1. Left: optical image of the NGC 5247 galaxy (Eskridge et al. (2002)). Right: H II regions outlining the spiral arms.
A King model with a cut-off a density distribution at large radii $r > r_b^m$ is used to model the potential of a stellar bulge:

$$\Psi_b = -\frac{G M_b}{C_b r} \ln \left( \frac{r}{r_b} + \sqrt{1 + \left( \frac{r}{r_b} \right)^2} \right),$$

where $r_b$ is the bulge scalelength, $M_b$ is the bulge mass within $r_b^m$ and

$$C_b = \ln \left( \frac{r_b^m}{r_b} + \sqrt{1 + \left( \frac{r_b^m}{r_b} \right)^2} \right) - \frac{r_b^m/r_b}{\sqrt{1 + (r_b^m/r_b)^2}}.$$  

The observed rotation curve of the disc can be reproduced by varying the parameters of the dark halo ($M_h, a_h$), stellar bulge ($M_b, r_b$) and stellar disc ($M_d, r_d, z_0$).

To analyse the behaviour of a one-component collisionless model, we assume that the gas density $\rho_g$ is equal to zero and solve equation (6) using a TREE code (Barnes & Hut 1986) adapted to the parallel calculations. The number of particles used in our collisionless models ranges from $N = 10^6$ to $10^7$. To build a self-consistent model of a stellar–gaseous disc, the set of hydrodynamical equations for a gaseous disc rotating in a self-consistent gravitational potential is added to the equations of motion for a stellar disc.

We apply a total variation diminishing (TVD) monotonic upwind scheme for conservation laws (MUSCL) type scheme in a cylindrical coordinate system (van Leer 1979) to solve equations (2)–(4). To preserve conservation laws at a grid level, a finite-volume approximation of the variables is used:

$$(q)_i^{+1/2} = q_i + 0.25[(k+1)D_+ + (k-1)D_-],$$

where $(q)_i$, $(q)_i^{+1/2}$ are the conserved variables at the border of the $i$th cell and corresponding at its centre; $D_+ = \text{minmod}(d_+, b d_+), D_- = \text{minmod}(d_-, b d_-)$, where $d_+, d_-$ are the variations of the conserved variables to the left and to the right of a cell border. The parameter $b = (3-k)/(1-k)$ determines an order of approximation in space. We use the value $k = 1/3$, corresponding to the third order of spatial approximation. The function

$$\text{minmod}(x, y) = \frac{\text{sign}(x) + \text{sign}(y)}{2} \min(|x|, |y|)$$

is used to reconstruct the discontinuous functions between the nodes of the computational grids.

Interaction between the stellar and gaseous components of the disc occurs due to gravity. Similarly to the one-component models, such interactions are computed using a TREE code.

At the beginning of the simulations, the stellar disc is in an equilibrium in the radial and the vertical directions, with its density given by

$$\rho_s = \rho_{g0} \exp(-r/r_a) A(z/z_0),$$

where $r_a, z_0$ are the radial and vertical scales of the stellar disc. Function $A(z)$ determines a vertical distribution of density of the stellar disc: $A(z) = \text{ch}^{-2}(z/z_0)$.

The equilibrium of the stellar disc in the vertical direction is determined by solving the Jeans equation:

$$\rho_s \frac{d^2 \rho_s}{dz^2} + \frac{\rho_s}{c_s^2} \left( \frac{d \rho_s}{dz} \right)^2 + \frac{\rho_s}{c_s^2} \frac{d^2 c_s^2}{dz^2} + 4\pi G \rho_s^2 (\rho_s + \rho_{e0} + \rho_{w0}(z)) + \frac{\rho_s}{c_s^2} \frac{d}{dz} E_w = 0,$$

where $E_w = -\frac{1}{4\pi G r} \frac{\partial^2 \varphi}{\partial r^2}$.

Here $c_s$ is the stellar velocity dispersion in the vertical direction, $\alpha_{\rho} = (\mu/w)$ is the result of averaging the product of the radial $u$ and vertical $w$ velocity components and $\rho_{e0}$ is the total density of the halo and the bulge.

By using a Jeans equation in the radial direction, we determine the rotational velocity of the stellar disc in the disc plane $z = 0$ (Khoperskov et al. 2010):

$$V_r^2 = (\langle v \rangle)^2 = V_c^2 + c_s^2 \left( 1 - \frac{c_s^2}{c_t^2} \right) + \frac{r}{\rho_s c_t^2} \frac{\partial (\rho_s c_t^2)}{\partial r} + \frac{r}{\rho_s c_s^2} \frac{\partial \rho_s}{\partial z},$$

where $V_c$ is the circular velocity of a test particle in an axisymmetric field $\Psi$, i.e. $V_r^2/r = -\partial \Psi/\partial r$, $c_t/c_s = 2\Omega/k, \Omega = V/r$ and $k = \sqrt{4\Omega^2 [1 + r d\Omega/(2\Omega dr)]}$ is the epicyclic frequency.

The gaseous disc extends in our models beyond the optical radius of the stellar disc to the distance $r = 16 r_d$. The density distribution of the gaseous disc is modelled by the expression

$$\rho_g = \rho_{g0} \left[ 1 - r/(16 r_d) \right] B(z),$$

where the function $B(z)$ is determined by the balance of the gravity force and the gas pressure gradient. With such a distribution, the gas density decreases a few times within the optical radius and goes to zero in the ghost zones of the computational grid at $r = 16 r_d$.

The Euler equation uniquely determines the rotational speed of the gaseous disc:

$$\frac{V_r^2}{r} = -\frac{1}{\rho_g} \frac{\partial p}{\partial r} - \frac{\partial}{\partial r} \left[ \Psi_g + \Psi_s + \Psi_{ex} \right].$$

We build a set of models for a stellar and a stellar–gaseous disc of the galaxy NGC 5247 such that the model-predicted quantities like the rotation curve and velocity dispersion are consistent with the observational data (and the uncertainties associated with the observations). In the following sections, we explore the model parameter space and discuss the dependence of the morphology of the spiral pattern on the rotation curve, the stellar velocity dispersion, the vertical thickness of the stellar disc and the mass ratio of gaseous to stellar components of the galactic disc.

### 4 DISC EQUILIBRIUM

#### 4.1 Rotation curve

A small inclination angle of the galaxy influences the reconstruction of its rotation curve significantly. Using the Tully–Fisher relation, Patsis, Grosbol & Holstein (1997) found the maximum rotational velocity of the galaxy to be $V_{\text{max}} = 205 \text{ km s}^{-1}$. Other measurements give the values $V_{\text{max}} = 165 \pm 20 \text{ km s}^{-1}$ (Bottema 1993) and $V_{\text{max}} = 300 \text{ km s}^{-1}$. Following the paper of Contopoulos & Grosbol (1986), we build a set of rotation curves that have quasi-solid rotation in the central regions of the disc and are flat in its outer regions. Such a shape of rotation curve is typical for a large spiral galaxy. To build more realistic models, we take into account the gravitational potentials of the halo, bulge and galactic disc. The parameters of the potentials and corresponding properties of the rotation curves are listed in Table 1. To take into account observational uncertainties in our knowledge of the rotational curve of the galaxy, we vary the rotational velocity of the disc. Namely, model A has the largest rotational velocity, B the middle one and model C the lowest rotational velocity. The resulting rotation curves are shown in Fig. 2. A large spread in the rotational velocities reflects an uncertainty in observational data for this galaxy.
Table 1. The parameters of one-component (N-body) and two-component models. \( c_{r0} \) is the radial velocity dispersion in the centre of the disc, \( V_{\text{max}} \) is the maximum value of the rotation velocity, \( M_g/M_\text{s} \) the percentage of gas in the one-component model is equal to zero.

| Model name | \( c_z/c_r \) | \( c_{r0} \) (km s\(^{-1}\)) | \( z_0 \) (kpc) | \( V_{\text{max}} \) (km s\(^{-1}\)) | \( M_g/M_\text{s} \)  |
|------------|----------------|----------------|----------------|----------------|----------------|
| A1         | 0.43           | 120            | 1.5            | 245            | 0              |
| A2         | 0.6            | 92             | 1.5            | 245            | 0              |
| A3         | 0.8            | 70             | 1.5            | 245            | 0              |
| B1         | 0.43           | 120            | 1.5            | 200            | 0              |
| B2         | 0.6            | 92             | 1.5            | 200            | 0              |
| B3         | 0.8            | 70             | 1.5            | 200            | 0              |
| C1         | 0.43           | 120            | 1.5            | 153            | 0              |
| C2         | 0.6            | 92             | 1.5            | 153            | 0              |
| C3         | 0.8            | 70             | 1.5            | 153            | 0              |
| D1         | 0.8            | 70             | 2.2            | 153            | 0              |
| D2         | 0.8            | 70             | 0.9            | 153            | 0              |
| D3         | 0.8            | 70             | 1.5            | 153 0.01      | 0              |
| E1         | 0.8            | 70             | 1.5            | 153 0.05      | 0              |
| E2         | 0.8            | 70             | 1.5            | 153 0.1       | 0              |
| E3         | 0.8            | 70             | 1.5            | 153 0.2       | 0              |

Figure 2. Rotation curves for one-component models. The maximum values of the rotation curves are equal to 245 km s\(^{-1}\) (Model A), 205 km s\(^{-1}\) (Model B) and 153 km s\(^{-1}\) (Model C).

4.2 Velocity dispersion

To constrain the radial component of the velocity dispersion of the stellar disc \( c_r \), we use observed line-of-sight velocity dispersion data of NGC 5247 (Fig. 3). For the nearly face-on orientation of NGC 5247, the line-of-sight velocity dispersion is close to the component of the velocity dispersion in the direction perpendicular to the disc.

To mimic the observational uncertainties in our knowledge of the velocity dispersion, we choose three profiles as shown in Fig. 4. In this figure, index 1 corresponds to the hottest disc and index 3 corresponds to the model with the lowest velocity dispersion.

Observational data of external galaxies show that the ratio of the vertical velocity dispersion to the radial velocity dispersion is approximately constant along the radius of the galactic disc and varies from galaxy to galaxy within \( c_z/c_r = 0.3-0.8 \) (Korchagin et al. 2000; Khoperskov et al. 2010). In our models we choose this ratio equal to \( c_z/c_r = 0.43; 0.6; 0.8 \) in the central regions of the galaxy.

4.3 Disc thickness

A direct measurement of the vertical scale of the disc of NGC 5247 is impossible. Zhao et al. (2006) solved this problem by applying the Jeans equation in the direction perpendicular to the collisionless stellar disc. They found the value of the disc vertical scale to be \( z_0 = 1.5 \pm 0.6 \) kpc. Taking into account this result, we additionally consider two variations of model C3 with the larger thickness of the disc equal to \( z_0 = 2.1 \) kpc and the lower disc scalelength \( z_0 = 0.9 \) (model D3).

A variation of the thickness of the stellar disc is taken into account in a set of models D, with thickness of the disc decreasing from model D1 to model D3.

5 ONE-COMPONENT MODELS

5.1 Hydrodynamical model

The simplest model to study the dynamics of an unstable galactic disc is the hydrodynamical approximation. In this approximation, a stellar–gaseous gravitating disc is represented by a gravitating compressible fluid. There are some arguments justifying the hydrodynamic approximation for a description of stellar discs: Marochnik (1966), Hunter (1979) and Sygnet, Pellat & Tagger (1987) demonstrate that the behaviour of perturbations in collisionless discs can be
of the surface density $\Sigma_1$ formed in the disc quantitatively, we use a Fourier decomposition.

The rotation curve significantly influences the dynamics of a collisionless disc. Fig. 8 shows the evolution of the Fourier harmonics $A_m$ in three models with different rotation curves. In the fast-rotating model A3, the disc is stable due to the presence of a massive halo. The model with the fastest disc rotation therefore cannot explain the observed spiral pattern in the galaxy NGC 5247. In model B3 (middle frame) the perturbations grow slowly with the most unstable mode two-armed spiral. Within approximately 3 Gyr from the beginning of simulations, the wave amplitude reaches about 2–5 per cent of the disc unperturbed values, which is significantly lower than the amplitude of the observed spiral pattern. Apparently, models A and B cannot explain the observed spiral structure of the galaxy NGC 5247.

Let us examine model C, which has a more massive stellar disc compared with models A and B. The minimum value of the Toomre parameter for a stellar disc that has stability parameter $Q$ larger then unity:

$$Q_{TS} = \frac{K_0 c_s}{\pi G \Sigma} \geq 1.$$  \hspace{1cm} (13)

If condition (13) does not hold for the whole disc, then the perturbations grow significantly during a short evolution time and the parameters of the disc change within 1–2 disc rotations. We exclude such cases from our consideration.

5.2 Collisionless models

Although the hydrodynamical approximation reproduces the morphology of NGC 5247 qualitatively, the lifetime of the spiral pattern is short. After approximately 0.3 Gyr the spiral pattern is destroyed in these hydrodynamical models and they no longer resemble the observed spiral structure. The mechanism that destroys the spiral structure is not completely understood. Using a hydrodynamical model, Laughlin, Korchagin & Adams (1997) showed that a non-linear spiral mode causes a hollowing out of the surface density profile in the vicinity of a corotation resonance. This stops the spiral growth and eventually destroys the spiral itself. For collisionless discs, the mechanism remains unclear. We therefore explore collisionless models to study to what extent the collisionless nature of a galactic disc increases the lifetime of the spirals.

Using the distributions shown in Fig. 7, we build a set of models for a stellar disc that has stability parameter $Q$ larger then unity:

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Figure 8. The amplitudes of the Fourier harmonic for models with different rotation curves. Left frame: model A3; central frame: model B3; right frame: model C3. Parameters of the models are given in Table 1.

Figure 9. Evolution of the surface density in numerical simulations of collisionless discs with different rotation curves. 

$Q$ parameter indicates that this model is gravitationally unstable. Similarly to model B, there is a well-defined two-arm global spiral structure that extends to about four scalelengths in the disc. The amplitude of the perturbations is much higher compared with the other cases. During the initial stages ($t < 0.5 \times 10^9$ yr) one can see a three-arm spiral and the spiral pattern is a superposition of two- and three-armed modes. However, in the non-linear stage a two-armed pattern dominates (Fig. 9). Fig. 10 shows the typical dependence of the angular velocity of a two-armed spiral pattern generated in model C3. Between approximately 0.2 and 0.7 Gyr, when exponential growth of a spiral perturbation is observed, the spiral perturbation has an angular velocity approximately equal to $22 \text{ km s}^{-1} \text{kpc}^{-1}$. After 0.8 Gyr, non-linear saturation starts: the angular velocity decreases to $18 \text{ km s}^{-1} \text{kpc}^{-1}$ and remains approximately constant during the rest of simulations.

Despite a significant difference in the values of radial velocity dispersion used in our numerical simulations, the spiral patterns are quite similar, as can be seen by comparison of models C1, C2 and C3 in Fig. 11. On a qualitative level, one can conclude that the morphological properties of spiral structure are independent of the variation of the radial velocity dispersion, within observational errors. The growth rates, however, are different, ranging from 0.7 Gyr in the coldest disc to 1.5 Gyr for the model with the largest velocity dispersion (Fig. 12).

The value of the disc vertical scale used in our simulations is close to the value estimated by Zhao, Peng & Wang (2004) of 1.1–2.5 kpc, implying a large range of values to be explored. Fig. 13 shows that the spiral patterns in collisionless cases D1–D3 are similar and independent of the disc thickness.

Taking into account the third dimension, i.e. the disc thickness, is an important requirement in disc dynamics to simulate the density growth in spiral arms correctly. Vertical motions lead to a decrease in the growth rate of an unstable spiral wave. Under similar conditions, the amplitude of the spiral wave grows faster in a thinner disc (Fig. 13). In the model with a relatively large scalelength of $z_0 = 2.1$ kpc, the structure of the spiral arms and their lifetimes are qualitatively similar to the behaviour of model C3. However, the disc with small thickness is significantly more unstable, resulting in a rapid growth of multi-armed perturbations and formation of a flocculent spiral pattern.

In general, the lifetime of the global two-armed spiral pattern in our collisionless models is about 1.5 Gyr. As was mentioned, a gaseous component in the disc increases the lifetime of the spiral structure. In the following section we will consider this effect in detail.
tial density distribution with a radial scalelength of \( r \) dispersion on disc equilibrium. The stellar disc has an exponen-
in their radial velocity dispersions and the influence of the velocity
curves of stellar and gaseous components is caused by a difference

tivities for a stellar–gaseous disc. The difference between the rotation
profile. Fig. 7 shows initial equilibrium distributions of basic quan-
tities for a stellar–gaseous disc. The difference between the rotation

For a two-component gravitating disc, the stability criterion needs
to be modified. A few such modifications of the stability parameter
have been suggested in the literature. A simple stability criterion
has been proposed by Wang & Silk (1994):

\[
\frac{1}{Q_{ws}} = \frac{1}{Q_s} + \frac{1}{Q_g}.
\]

Here, \( Q_s \) and \( Q_g \) are the values of the Toomre \( Q \) parameter for
the stellar and gaseous components respectively. Romeo & Wiegert
(2011) demonstrated that such an approach is not suitable for three-
dimensional systems. Romeo & Wiegert (2011) suggested an ex-
pression for the effective stability criterion that takes into account
the finite thicknesses of stellar and gaseous discs:

\[
\frac{1}{Q_{rw}} = \left\{ \begin{array}{ll}
W/T_s Q_s & T_s Q_s \geq T_g Q_g; \\
1/T_s Q_s & W/T_s Q_g, T_s Q_g \geq T_g Q_s;
\end{array} \right.
\]

Here \( W = 2 c_s c_r (c_s^2 + c_r^2) \), \( T_s = 0.7 + 0.8 (c_s/c_r) \), and \( T_g =
0.7 + (c_s/c_r)^2 \). The radial dependence of the stability parameters
\( Q_s, Q_g \) and \( Q_{rw} \) is shown in Fig. 14.

The evolution of the surface-density perturbations in a stellar
disc in models with different admixtures of gas is shown in Fig. 15.
During \( t < 1 \) Gyr, a spiral pattern is formed in all models. Later on
\( (t > 1 \) Gyr), the spiral wave dissipates in the purely collisionless
model and becomes indistinguishable from the background of low-
scale density perturbations. A small gas admixture with a mass of
a few per cent of the total mass of the disc significantly increases
the lifetime of the spiral structure and the spiral pattern exists up to
3 Gyr. If the mass of the gas is increased to one-tenth of the total
mass of the disc, then a strong gravitational instability is created,
leading to the formation of a complex, highly transient multi-armed
spiral structure. The disc changes significantly from its initial state
during about 0.5 Gyr. In such a case, star formation should be taken
into account, which is beyond the scope of our paper.

Self-consistent stellar–gaseous models allow reproduction of a
long-lived two-armed spiral pattern. We find, however, similarly
to Elmegreen & Thomasson (1993) and Thomasson et al. (1990),
that the morphology of the spiral pattern evolves with time, with
the overall lifetime of the spirals being about 10 disc rotations. In
comparison, lifetime of spirals in the one-component models do not
exceed 1.5 Gyr.

6 TWO-COMPONENT MODELS

In our two-component models, we take into account the gravita-
tional interaction between the gaseous and stellar components of
the disc. The effects of star formation or exchange of matter be-
tween stellar and gaseous subsystems are neglected. It is known
that gas has a strong destabilizing effect on a gravitating disc even
if its surface density is much smaller than the surface density of
the stellar disc. We build our stellar–gaseous models based on the
one-component model C3 and by varying the total gas mass within
the radius of the stellar disc. Namely, we analyse the models with
the relative mass of a gaseous component equal to

\[ M_g/M_\star = [0.01; 0.05; 0.1; 0.2]. \]

For a two-component gravitating disc, each subsystem has its own
spatial density distribution, rotation curve and velocity-dispersion
profile. Fig. 7 shows initial equilibrium distributions of basic quantities
for a stellar–gaseous disc. The difference between the rotation
curves of stellar and gaseous components is caused by a difference
in their radial velocity dispersions and the influence of the velocity
dispersions on disc equilibrium. The stellar disc has an exponen-
tial density distribution with a radial scalelength of \( r_g = 4.8 \) kpc.

The velocity dispersion of the stellar disc in the direction perpen-
dicular to the disc decreases from 70 km s\(^{-1}\) in its central regions
to about 20 km s\(^{-1}\) at the disc periphery. The gaseous disc is cold
compared with the stellar one. Its velocity dispersion is close to
10 km s\(^{-1}\) in the disc’s central regions and changes slowly within the
stellar disc.

Figure 11. Evolution of the perturbed surface density of the stellar disc in
simulations with different radial velocity-dispersion profiles.

Figure 12. Fourier harmonics in models with different radial velocity dispersions: left frame, \( c_s/c_r = 0.8 \); central frame, \( c_s/c_r = 0.6 \); right frame, \( c_s/c_r = 0.43 \).
Figure 13. Evolution of surface-density perturbations in a stellar disc in numerical simulations with different thicknesses of stellar disc: D3, $z_0 = 0.9$ kpc; D2, $z_0 = 1.5$ kpc; D1, $z_0 = 2.2$ kpc.

Figure 14. Radial distribution of the Toomre parameter: gaseous, stellar and a stellar–gaseous disc for model E2.

Fig. 16 shows the azimuthal density distribution at $r = 4.9$ kpc in the gaseous and stellar components. The figure clearly demonstrates formation of shock fronts in the gaseous component, located at the inner edge of the stellar arms.

A shock front is formed when the supersonic flow of interstellar gas passes through the density wave of the spiral arm (Roberts 1969). The distribution of gas along the azimuth indicates the position of the shock wave on the inner edge of the stellar density wave in our simulations. In the presence of a strong shear flow, the system of shock fronts is unstable to hydrodynamical instabilities (Wada & Koda 2004; Khoperskov et al. 2011). We cannot reveal such instability due to the low resolution of our simulations.

Fig. 17 shows the azimuthal dependence of the stellar surface density at different radii in the disc. As one can judge from the figure, the amplitude of the spiral perturbation decreases with radius in the stellar component. The shift in phase of the density maxima with radius reflects the spiral nature of the perturbations.

7 DISCUSSION

7.1 Toomre’s parameter

Sellwood (2011) reviewed observational as well as theoretical arguments and came to the conclusion that spiral patterns are short-lived. A qualitative counterargument against this picture is that if spirals
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One of his arguments is that the theory of spiral modes requires galactic discs to be dynamically cool. Namely, Toomre’s axisymmetric stability parameter should lie in the range $1 \leq Q \leq 1.2$. As a test of the applicability of a modal approach in such systems, Sellwood simulated the dynamics of a disc that has a $Q$ parameter equal to unity everywhere except the disc’s central regions, where the $Q$ parameter rises steeply. Via this experiment, Sellwood demonstrated that such a low value of $Q$ parameter supports vigorous collective responses that change the disc properties during a short dynamical time.

We repeat the experiment of Sellwood in three-dimensional simulations, assuming that the disc of the galaxy NGC 5247 has $Q = 1$ everywhere except its central regions. We find a qualitative agreement of our simulations with the results of Sellwood (2011). During approximately one disc rotation, a complex multi-armed transient spiral structure is formed that causes disc heating and quick dissipation of the spiral waves. Fig. 18 shows the radial profile of disc radial velocity dispersion at different moments of time. After violent disc heating, it comes to a stationary stage.

The assumption about a radial profile of Toomre’s $Q$ parameter invoked by Sellwood is not related to spiral galaxies. It is known from direct measurements that the velocity dispersion in galactic discs decreases exponentially with radius and has a larger scale-length compared with the scalelength of the surface brightness (or density). In combination with the fact that the discs are nearly flat and self-gravitating in the vertical direction, this leads to a bell-like shape of Toomre’s $Q$ parameter rising towards the centre of a disc and on its periphery. With such a $Q$ profile, discs are globally unstable and support spiral structure during tens of galactic rotations.

We note also that if real galaxies were to have a $Q$-parameter profile close to that assumed by Sellwood, they would be in a phase of violent evolution and would have a very short dynamical time. This would be difficult to reconcile with the number of observed spiral galaxies and their lifetimes. Density-wave theory (the disc’s instability towards global spiral modes) does not require the $Q$ parameter to be in a narrow range $1 \leq Q \leq 1.2$, as was demonstrated by a number of examples in hydrodynamical simulations by Korchagin et al. (2000, 2005) and in collisionless models by Khoperskov et al. (2007).

7.2 Resonances

After the work of Toomre (1981), the swing amplification of perturbations in a differentially rotating gravitating disc is regarded as one of the key mechanisms that amplifies spirals. Swing amplification works in the presence of a shearing disc. The orbital clock is faster in the interiors of galactic discs, and the mechanism runs more rapidly at smaller radii. The important part of the swing amplification mechanism is a propagation of waves in a radial direction with the group velocity. A propagating wave reflects from the centre and/or from a corotation region, changing from trailing to leading and simultaneously changing the sign of the group velocity, and serves, according to Toomre, as ‘fresh grist to a swing amplified mill’ to operate continuously in a disc.

In this mechanism, the inner Lindblad resonance is considered as the most important in the generation of spiral density waves. This resonance must be shielded to prevent a fierce damping of the waves. To check the importance of the inner Lindblad resonance in the generation of spirals, we performed an artificial experiment with our hydrodynamical model, imposing an absorption region in the inner regions of the disc where all perturbations are zeroed out. This cuts the feedback loop and destroys amplification of spiral perturbations. We performed a series of two-dimensional simulations of an unstable two-dimensional gaseous disc with such an artificially imposed ‘absorption ring’, where all perturbations are forced to be zero and the macroscopic characteristics of the disc are set to be unperturbed in a smooth way. Fig. 19 shows angular velocity, epicyclic frequency and positions of resonances for such a disc. As one can see, the model does not harbour the inner Lindblad resonance, so an absorption ring was imposed artificially to mimic its influence.

Fig. 20 shows the result of such simulations. On the right panel of Fig. 20 is shown a two-armed spiral pattern developing in an unstable ‘template’ gravitating disc. The left panel shows the results of simulations when an absorption ring has been imposed close to the disc centre. The disc remains unstable and a two-armed spiral is developing out of noise perturbations. We find that the spiral

Figure 17. Azimuthal layers of stellar-component surface density for various values of radius. From top to bottom: $r = \{2.88; 4.8; 6.72; 8.64\}$ kpc for model E2.

Figure 18. Radial distribution of the radial velocity dispersion at different times: the dotted line is the initial statement ($t = 0$), the dash–dotted line is the distribution after one rotation period ($t = T$), the full line is the stationary distribution ($t/T = 3–10$).
perturbations develop in the disc independently of the position of the absorption ring, be it inside or outside the corotation resonance. An instructive experiment is to impose the absorption ring at corotation. The behaviour of perturbations changes dramatically in this case (Fig. 21). The growth of a two-armed spiral is totally suppressed and a multi-armed spiral starts to grow, with a four-armed spiral prevailing. These experiments illustrate that the qualitative picture of spiral growth by swing amplification is not necessarily as relevant as once thought. Instead, the corotation resonance may be more important in spiral generation.

The issue of transient spirals is currently a topic of heated debate. Zhang (1996, 1998, 1999) finds that galaxies undergo a secular evolution that leads to a redistribution of the disc matter and heats the disc stars so they gradually rise above the galactic plane in the disc central regions. Both effects are natural ingredients of the presence of a spiral density wave. Stars migrate inside and outside the corotation region and a more and more centrally concentrated density distribution is achieved with time, together with the build-up of of an extended outer envelope. A similar effect was found in hydrodynamical simulations by Laughlin et al. (1997).

8 CONCLUSION
We have built one-component collisionless and hydrodynamical models as well as multicomponent stellar–gaseous models of the disc galaxy NGC 5247. The models are unstable to $m = 2$ global modes that form a two-armed spiral pattern with amplitudes of about 10–20 per cent, qualitatively similar to observations.

Our simulations show that in purely collisionless models the lifetime of a spiral structure is about a few galactic rotations and it does not last longer than 1 Gyr. An admixture of a gaseous component with a mass of a few per cent of the mass of the stellar component significantly increases the lifetime of the spiral structure. In the
Figure 21. The figure shows the results of simulation when an absorption ring has been imposed at the corotation radius. Note that, unlike Fig. 20, the growth of a two-armed spiral is completely suppressed due to the presence of the absorption ring and a multi-armed spiral starts to form.

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