Experiment on basic performance of a simple waterwheel/windmill with a cross-stream axis

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Abstract. We conduct subsonic wind-tunnel experiments, in order to investigate the basic aerodynamic characteristics of an autorotating finite flat plate in uniform mainstream, which has a potential for unconventional waterwheels and windmills with very simple structures in addition to unusual mixers/diffusers. The present study focuses on both the effects of an aspect ratio $AR$ and a tip-speed ratio $\Omega^*$ upon autorotation of the flat plate with a low thickness-to-depth ratio $\lambda = 1.3 \times 10^{-2} \sim 2.0 \times 10^{-2}$. We carry out the measurements using a torque meter together with the synchronised measurements of the plate’s attack angle $\alpha$ for the analyses with phase-averaging technique. As a result, the (time-mean) torque coefficient $C_{TM}$ tends to decrease with increasing $\Omega^*$, and $C_{TM}$ tends to become large with increasing $AR$. (Time-mean) power coefficient $C_{PTM}$ shows that the present phenomenon is suitable for low-speed waterwheels and windmills.

1. Introduction

Our final purpose is to propose a new simple and durable waterwheel/windmill, and to optimise it for practical uses. This waterwheel/windmill is composed of a single rectangular flat plate rotating about a cross-stream axis. The waterwheel/windmill is expected to attain rather low efficiency, however is suitable for small, decentralised, environmentally-friendly, on-site and self-sufficient systems to generate power in disaster, isolated or remote regions/areas with energy independence and load’s flexibility.

The waterwheel/windmill is an application of autorotation; namely, a kind of self-sustained rotating motion known as ‘tumbling’ [1, 2]. The tumbling represents a motion with the axis perpendicular to mainstream’s direction. The tumbling is not only a purely-academic concern found as the falling motions of leaves and pieces of paper, but also a very important phenomenon in such actual aspects as aeronautical and space engineering, ballistics and meteorology in addition to mechanical engineering whose applications are unconventional renewable-energy converters like waterwheels and windmills [3, 4], smart flow-control devices [5], unusual mixers/diffusers and so on. Especially, the tumbling has a potential for new waterwheels and windmills with very simple structures in addition to mixers/diffusers. Until now, there have been some researches for the tumbling [6 – 20]. Unfortunately,
we have not obtained enough knowledge about basic performance of the tumbling in applications for such renewable converters. For example, it is necessary to specify torque and power as a converter in order to optimise it. Moreover, its aspect-ratio effect is important especially for renewable-energy converters from a practical point of view.

In the present study, we conduct a series of subsonic wind-tunnel experiments, in order to investigate the basic aerodynamic characteristics of a tumbling finite flat plate in uniform mainstream. Especially, we focus upon both the effects of an aspect ratio $AR$ and a tip-speed ratio $\Omega^*$ (or a reduced form of a rotating rate $n$) upon the tumbling of a finite flat plate, in addition to the effect of the Reynolds number $Re$. We should note that the flat plate is forced to rotate with a constant speed by an electric motor, because we can easily control $\Omega^*$ over a much wider range than free-rotation experiments or free-fall experiments. Of course, realised flow itself is the same. This is suitable for a fundamental approach in the first stage, as well as our previous studies [21, 22]. We carry out fluctuating-torque measurements using a torquemeter together with the synchronised measurements of the plate’s attack angle $\alpha$ which are used for phase-averaging analyses.

**Nomenclature**

- $AR$: Aspect ratio, $\equiv s/d$
- $d$: Depth of plate [m]
- $f$: Frequency [Hz]
- $f_d$: Dominant frequency [Hz]
- $g$: The gravitational acceleration [m/s$^2$]
- $n$: Rotating rate [Hz]
- $P$: Power [W]
- $Re$: Reynolds number, $\equiv U_\infty d/\nu$
- $s$: Span of plate [m]
- $t$: Thickness of plate [m]
- $T$: Torque [Nm]
- $U_\infty$: Mean velocity of uniform mainstream [m/s]
- $\alpha$: Attack angle [deg.]
- $\lambda$: Thickness-to-depth ratio, $\equiv t/d$
- $\rho$: Density [kg/m$^3$]
- $\nu$: Kinetic viscosity [m$^2$/s]
- $\tau$: Time [s]
- $\Omega^*$: Tip-speed ratio (or reduced rotating rate), $\equiv n \pi d/U_\infty$
- $C_T$: Torque coefficient, $\equiv 4T/\rho U_\infty^2sd^2$
- $C_P$: Power coefficient, $\equiv C_T \Omega^*$

**Subscripts**

- PA: Phase-averaged
- TM: time-mean
- maxTPA: Maximum phase-averaged torque in a range of $\alpha = 0^* - 180^*$
2. Experimental method

2.1. Model and parameters

Figure 1 shows the present model, together with the Cartesian coordinate system O-xyz. The model is a finite thin flat plate, to be strict, a rectangular-cross-section prism, which rotates in uniform flow with a velocity \( U_\infty \). And the model is forced to rotate by an electric motor. The direction of model’s rotation can be arbitrary for the symmetric present model. So, we suppose that the direction is always clockwise as shown in Figure 1 through the present study.

We define aspect ratio \( AR \) by \( s/d \), where \( s \) and \( d \) denote the span and the depth of the flat plate, respectively. \( \lambda \) is a geometrical parameter specifying the cross section of the flat plate. We define thickness-to-depth ratio \( \lambda \) by \( t/d \), where \( t \) denotes the thickness of the flat plate.

In the present study, we suppose four non-dimensional control parameters; namely, (1) the aspect ratio \( AR \), (2) a tip-speed ratio \( \Omega^* \), (3) the Reynolds number \( Re \) and (4) the thickness-to-depth ratio \( \lambda \). We define \( \Omega^* \) by \( n \pi d / U_\infty \), where \( n \) denotes the rotating rate of the flat plate. We define \( Re \) by \( U_\infty d / \mu \), where \( U_\infty \) and \( \mu \) denote the mean velocity of uniform mainstream and the kinetic viscosity of fluid, respectively.

The tested range of a geometrical parameter \( AR \) is 0.67 – 5.0. The tested value of \( \Omega^* \) varies from 0 to 2.2. The tested value of \( Re \) varies from \( 5.0 \times 10^3 \) to \( 2.0 \times 10^4 \). The tested range of another geometrical parameter \( \lambda \) is \( 2.0 \times 10^{-2} – 5.0 \times 10^{-2} \), which is small enough to be regarded as a thin flat plate.

2.2. Experimental Arrangement

Figure 2 shows a schematic diagram of the present experimental apparatus. In the uniform mainstream issued from a wind-tunnel nozzle (1), the model (2) rotates through a supporting shaft (3) and a torquemeter (4) at the model’s rotating rate \( n \), driven by a DC electric motor (5). The supporting shaft is extended to another outside of the wind-tunnel test section, where the supporting shaft is connect to a disc with a target (6) for a laser optical sensor (hereinafter, referred to as LOS) (7), in order to detect the rotating rate \( n \). For the measurement of flow velocity, we use a hot-wire anemometer (hereinafter, referred to as HWA). A hot-wire probe (8) of the HWA is placed on a traverser (9). The main analogue circuit of the HWA consists of a DC-power-supply unit (10), a CTA unit (11), a lineariser unit (12), and a RMS unit (13). All the raw analogue signals from the torquemeter and the HWA are recorded by a PC with an A/D converter, together with the synchronised measurements of the LOS in order to carry out phase-averaging analyses.

In order to obtain a pure fluid torque \( T \), we conduct a preliminary measurement using a torquemeter, where the supporting shaft without the model rotates in the same conditions with the fluid-torque measurement. By this preliminary measurement, we estimate the torque related with mechanical friction of the present supporting system. So, this torque is deducted from the torquemeter’s raw signal, in order to get the net torque which is approximately equal to the fluid torque. Through all the fluid torque measurements, \( d \) is fixed to 150 mm.

Figure 1. Model: a thin flat plate rotating in uniform flow, together with a coordinate system (x, y, z).
3. Results and discussion

3.1. Torque’s waveform & dominant frequency $f_d$

Figures 3 shows two typical examples of the waveforms of fluid torque; namely, time series of phase-averaged torque $T_{PA}$ (for $AR = 3.0$, $Re = 4.0 \times 10^4$ and $d = 150$ mm) with less high-frequency random noises. The abscissa is the attack angle $\alpha$ of model, instead of time $\tau$. Figures (a) and (b) represent the results for $\Omega^* = 0.1$ and 0.9, respectively.

At first, we see figure (a). We can see that the fluid torque $T_{PA}$ fluctuates with one dominant period of $1/f_d$ which corresponds to half a plate’s rotation. Actually, the Fourier transform of this waveform indicates a remarkable spectrum peak at a frequency $f_d$ $2n$. In addition, the time series of $T_{PA}$ likely involves the higher harmonics of $2n$ with less high-frequency random noises, and tends to be distorted from a sinusoid.

Next, we see figures (b). Again, we can see that the fluid torque $T_{PA}$ fluctuates with one dominant period which corresponds to half a plate’s rotation. Actually, the Fourier transform of this waveform indicates a remarkable spectrum peak at a frequency $f_d$ of $2n$, as well as figure (a). However, the time series of $T_{PA}$ is closer to a sine wave than that in figure (a).

As a result, most of all the waveforms $T_{PA}$ including Figure 3 are characterised by one dominant period $1/f_d$ corresponding to half a plate’s rotation (or one remarkable spectrum peak at $f_d/n = 2.0$). This is contrast to the velocity waveforms in wake [21, 22].

![Experimental apparatus.](image)

**Figure 2.** Experimental apparatus.

![Waveform examples.](image)

**Figure 3.** Typical examples of waveform: time series of phase averaged torque $T_{PA}$ (for $AR = 3.0$, $Re = 4.0 \times 10^4$ and $d = 150$ mm). The abscissa is the attack angle $\alpha$ of model, instead of time $\tau$. 
3.2. Torque

Figure 4 shows the influence of the Reynolds number $Re$ upon (time-mean) torque $T_{TM}$: namely, torque coefficient $C_{TM}$ is plotted against the tip-speed ratio $\Omega^*$ for various $Re$’s (for $AR = 3.0$ and $d = 150$ mm). In this figure, the quasi-steady theory is also shown for reference: namely, $C_{TM} = 0$ at $\Omega^* = 0$. To be exact, the ordinate denotes the time-mean value $C_{TM}$ of torque coefficient. (For example, $C_{TM} > 0$ in Figure 3(a), and $C_{TM} < 0$ in Figure 3(b)). As $\Omega^*$ increase from zero to about 0.2, $C_{TM}$ increases from zero. In contrast, as $\Omega^*$ increases over about 0.2, $C_{TM}$ decreases. Then $C_{TM}$ attains the maximum at $\Omega^* = 0.2$, and crosses zero at $\Omega^* \approx 0.5$ which is close to our free-flight experiments [17]. As a result, $C_{TM}$ is positive (or the model can autorotate) at $\Omega^* \leq 0.5$, and negative (or the model’s rotation is damped) at $\Omega^* \geq 0.5$. We can see that the above behaviour of $C_{TM}$ is independent of $Re$: namely, the influence of $Re$ upon torque is negligible in a range of $Re \geq 2.0 \times 10^4$.

Figure 5 shows the influence of the aspect ratio $AR$ upon (time-mean) torque $T_{TM}$: model’s torque coefficient $C_{TM}$ plotted against the tip-speed ratio $\Omega^*$ for various $AR$’s (for $Re = 4.0 \times 10^4$ and $d = 150$ mm), together with $C_{TM}$ of the Savonius windmill by Hau (2000). In this figure, the quasi-steady theory is also shown for reference: namely, $C_{TM} = 0$ at $\Omega^* = 0$. The result at $AR = 3.0$ is the same as Figure 4. Symbols for $AR = 0.67$ and 2.0 represent the ensemble mean of 2 trials. Symbols for $AR = 3.0$ represent the ensemble mean of 3 trials. Error bars for $AR = 0.67, 2.0$ and 3.0 correspond to the maximum and minimum $C_{TM}$’s. Solid line is approximate curve of symbols which are combinations of both parabolas and straight lines. To conclude, We can see the influence of $AR$ upon $C_{TM}$, in contrast to that of $Re$ in Figure 4. The result for $AR = 2.0$ is almost the same as that for $AR = 3.0$, qualitatively and quantitatively. For $AR = 1.0$, as $\Omega^*$ increases from zero to about 0.1, $C_{TM}$ increases from zero. In contrast, as $\Omega^*$ increases over about 0.1, $C_{TM}$ decreases. Then, $C_{TM}$ attains the maximum for $\Omega^* = 0.1$, and crosses zero for $\Omega^* \approx 0.3$. As a result, $C_{TM}$ is positive for $\Omega^* \leq 0.3$, and negative for $\Omega^* \geq 0.3$. For $AR = 0.67$, as $\Omega^*$ increases from zero, $C_{TM}$ monotonically decreases from zero. As a result, $C_{TM}$ is always negative (or the model never autorotates), being independent of $\Omega^*$. In summary, the influence of $AR$ upon torque is negligible for $AR \geq 2.0$. For $AR \geq 2.0$, the torque tends to be small (or tends to suppress autorotation) with decreasing $AR$. Of course, the above aspect ratio influence represents the degree of two/three dimensionality of flow which should be discussed by flow visualisation and so on in future. Complementally speaking, torque produced by the present model could be smaller than that by the Savonius windmills even for large $AR$.

3.3. Power

Figure 6 shows the influence of the aspect ratio $AR$ upon (time-mean) power $P_{TM}$: namely, power coefficient $C_{PTM}$ plotted against the tip-speed ratio $\Omega^*$ for various $AR$’s (for $Re = 4.0 \times 10^4$). To be exact, the ordinate denotes the time-mean value $C_{PTM}$ of power coefficient. In this figure, the quasi-steady theory is also shown for reference: namely, $C_{PTM} = 0$ at $\Omega^* = 0$.

According to $C_{TM}$, it shown in Figure 5, $C_{PTM}$ shows similar behaviours with $C_{TM}$, because $C_{PTM}$ is given by $C_{TM}$ $\Omega^*$. As can be seen, the influence of $AR$ upon power is negligible for $AR \geq 2.0$. For $AR \geq 2.0$, the power tends to be small (or tends to suppress autorotation) with decreasing $AR$. $C_{PTM}$ is approximated by parabolic curves for $\Omega^* \geq 0.5$, because $C_{TM}$ is approximately by straight line for $\Omega^* \geq 0.5$ in Figure 5. In addition, $C_{PTM}$ tends to become large with increasing $AR$. Then, the maximum power coefficient $C_{PTM_{\text{max}}}$ tends to become large with increasing $AR$. And, the peak power tip-speed ratio $\Omega^*_{\text{CPTM_{max}}}$, where $C_{PTM}$ attains the maximum $C_{PTM_{\text{max}}}$, tends to become large with increasing $AR$. As well, the neutral tip-speed ratio $\Omega^*_{\text{CPTM=0}}$, where $C_{PTM}$ crosses zero, coincides with $\Omega^*_{\text{CTM=0}}$ and $\Omega^*_{\text{CPTM=0}}$ tends to become large with increasing $AR$.

Figures 7 shows the comparison on the power coefficient $C_{PTM}$ of the present flat plates with some other windmills by Hau [23]. As the other windmills, we consider the Savonius, the Darrieus, the Dutch and a three-blades windmills. The Savonius windmill and Darrieus windmill are the vertical-axis windmills. And, the Dutch windmill and the three-blade windmill are the horizontal-axis windmills. In general, the vertical-axis windmill has some advantages as (1) we need no control for
wind-direction fluctuation, and as (2) we can install such heavy devices as an electric generator near the ground. Then, its whole structure can be simple and its manufacturing cost can be low. Among them, the Savonius windmill consists of two semicircle blades in order to obtain the initial torque easily. Then, the Savonius windmill has the closest structure to the proposed windmill made with a flat plate.

Figure 4. Influence of Reynolds number \(Re\) upon (time-mean) torque \(T_{TM}\): torque coefficient \(C_{TM}\) against tip-speed ratio \(\Omega^*\) for various \(Re\)’s (for \(AR = 3.0\) and \(d = 150\) mm).

Figure 5. Influence of aspect ratio \(AR\) upon (time-mean) torque \(T_{TM}\): torque coefficient \(C_{TM}\) against tip-speed ratio \(\Omega^*\) for various \(AR\)’s (for \(Re = 4.0 \times 10^4\) and \(d = 150\) mm), together with \(C_{TM}\) of Savonius windmill by Hau (2000). Symbols for \(AR = 0.67\) and 2.0 represent the ensemble mean of 2 trials. Symbols for \(AR = 3.0\) represent the ensemble mean of 3 trials. Error bars for \(AR = 0.67, 2.0\) and 3.0 correspond to the maximum and minimum \(C_{TM}\)’s.
As shown in Figure 5, the flat plate with $AR = 3.0$ at low $\Omega^*$ indicates worse torque characteristics than the Savonius windmill. In a range at $\Omega^* \geq 1.4$, $C_T$ for the flat plate with $AR = 3.0$ becomes larger than the Savonius windmill. However, $C_T$ of the flat plates with $AR < 3.0$ always shows worse torque characteristics than the Savonius windmill.

In Figure 7, the peak value $C_{P_{MAX}}$ of the power coefficient of the Savonius windmill becomes larger than those of any flat plates with $AR \leq 3.0$.

**Figure 6.** Influence of aspect ratio $AR$ upon (time-mean) power $P_{TM}$: power coefficient $C_{PTM}$ against a tip-speed ratio $\Omega^*$ for various $AR$’s (for $Re = 4.0 \times 10^4$ and $d = 150$ mm).

**Figure 7.** Comparison on power coefficient $C_{PTM}$ between the present result and other windmills (Hau, 2000).
Figure 8. Border for neutral torque (and neutral power) with $C_{TM} = 0$ for $Re = 4.0 \times 10^4$ on the $\Omega^*\times AR$ plane, together with a stability diagram of wake modes for $Re = 3.0 \times 10^3 - 5.0 \times 10^4$ (Kinoshita et al., 2015). The wake modes are characterised by $f_d/n = i$ or $1/i$ with $i = 1, 2, 3, \ldots$, where $f_d$ denotes the dominant frequency of wake velocity.

Figure 9. Contours the attack angle $\alpha_{max,TPA}$ of model with the maximum phase-averaged torque in a range of $\alpha = 0^\circ - 180^\circ$ for $Re = 4.0 \times 10^4$ on the $\Omega^*\times AR$ plane, together with a stability diagram of wake modes for $Re = 3.0 \times 10^3 - 5.0 \times 10^4$ (Kinoshita et al., 2015). The wake modes are characterised by $f_d/n = i$ or $1/i$ with $i = 1, 2, 3, \ldots$, where $f_d$ denotes the dominant frequency of wake velocity.
3.4. Aspect-ratio effect upon torque and power

Figure 8 shows the border for neutral torque (and then, for neutral power) with \( C_{TM} = 0 \) for \( Re = 4.0 \times 10^4 \) on the \( \Omega^* - AR \) plane, together with a stability diagram of wake modes for \( Re = 3.0 \times 10^3 - 5.0 \times 10^4 \) [21]. As well, Figure 9 shows the contours the attack angle \( \alpha_{\text{max}}^{TPA} \) of model with the maximum phase-averaged torque in a range of \( \alpha = 0^\circ - 180^\circ \) for \( Re = 4.0 \times 10^4 \) on the \( \Omega^* - AR \) plane, together with a stability diagram of wake modes for \( Re = 3.0 \times 10^3 - 5.0 \times 10^4 \) [21]. The wake modes are characterised by \( f_d/n = i \) or \( 1/i \) with \( i = 1, 2, 3, \ldots \), where \( f_d \) denotes the dominant frequency of wake velocity.

In addition, Figures 8 and 9 show the result at \( AR = 40 \) by Suzuki et al. (1980) [24]. We can see that their result agrees well with our ones at \( AR \geq 4.0 \). This suggests both (1) high reliability of our experiments and (2) a negligible \( AR \) effect at \( AR \geq 4.0 \).

To conclude, Figure 8 reveal that torque characteristics is difficult to be linked with wake characteristics. Actually, in the figure, the corresponding \( AR \) of the border increases with increasing \( \Omega^* \), in contrast to the curves with \( f_d/n = \text{constant} \).

On the other hand, we can also find out any delicate links between torque and wake. For example, in Figure 9, the corresponding \( AR \) of the contours decreases with increasing \( \Omega^* \), as well as the curves with \( f_d/n = \text{constant} \).

4. Conclusions

In the present study, we conduct a series of subsonic wind-tunnel experiments, in order to investigate the basic aerodynamic characteristics of a tumbling finite flat plate in uniform mainstream. Especially, we focus upon both the effects of an aspect ratio \( AR \) and a tip-speed ratio \( \Omega^* \) which is a reduced form of a rotating rate \( n \) upon the tumbling of a finite flat plate, in addition to the Reynolds number \( Re \). We should note that the plate is forced to rotate with a constant speed, because we can easily control \( \Omega^* \) over a much wider range than free-rotation or free-fall experiments. This is suitable as a fundamental approach in the first stage. We carry out fluctuating-torque measurements using a torquemeter together with the synchronised measurements of the plate’s attack angle \( \alpha \) which are used for phase-averaging analyses. The tested range of \( AR, \Omega^* \text{ and } Re \) are \( 0.67 - 5.0, 0.1 - 1.5, 2.0 \times 10^4 - 1.0 \times 10^5 \), respectively. The tested range of another geometrical parameter a depth-to-width ratio \( \lambda \) concerning the plate’s cross section is \( 1.3 \times 10^{-2} - 2.0 \times 10^{-2} \), which is small enough to be regarded as thin. As a result, we have revealed the effects of \( AR, \Omega^* \text{ and } Re \) upon the tumbling of a flat plate. Torque coefficient \( C_T \) monotonically decreases with increasing \( \Omega^* \), and that tends to become large with increasing \( AR \). Then, the value of \( \Omega^* \) where \( C_T \) crosses zero tends to become large with increasing \( AR \). Both \( C_T \) and \( C_P \) depend upon \( AR \) and \( \Omega^* \), but does not almost depend upon \( Re \) at \( Re \geq 2.0 \times 10^5 \). The fluid torque \( T_{PA} \) fluctuates with one dominant period which corresponds to half a plate’s rotation. In contrast to the velocity waveforms in wake [21, 22], the influence of \( AR \) upon torque is negligible for \( AR \geq 2.0 \). For \( AR \geq 2.0 \), the torque tends to be small. Torque characteristics is difficult to be linked with wake characteristics.

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