Parameters and Ratios of Metal Cylindrical Mesh Shells

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Abstract. Finite element modeling, calculation and analysis of metal cylindrical mesh shells are carried out. The choice of the surface topology is justified and the recommended ratios of overall dimensions are adopted. The criteria for determining the stability of the grid are given. The limits of permissible curvature and the main geometric parameters are obtained. The effect of increasing the rigidity and height of the structure on the work of the mesh surface is analyzed. Patterns of changes in nodal deviations and internal force factors of vulnerable areas of the shell are revealed. The results of changes in the shape and features of the critical deformation of the system are presented.

1. Introduction

In construction, metal cylindrical mesh shells can be widely used to cover various objects. They are formed from rod elements and operate in a complex stress-strain state [1-11]. For them, some elements almost do not work, others are completely overloaded. As a result, there are zones with large forces in the elements and significant movements of the nodes. An increase in internal force factors and deformation parameters is a consequence of an increase in the overall dimensions of the shells, dangerous load application schemes, and the vulnerability of the design solution. The publications [4, 5, 9, 11, 13, 20] indicate that the main measures aimed at reducing the impact of these indicators are mainly constructive solutions. Most often, preference is given to the use of additional elements that serve as a means of strengthening and increasing the load-bearing capacity of cylindrical mesh surfaces. Thus, it is possible to achieve the most complete use of the strength of the material, the desired redistribution of forces, acceptable rigidity and stability of the shells [12-20]. Some proposals for schemes for their amplification are found in the literature, but in an incomplete and unrealized form. Therefore, the purpose of the article is to perform modeling, calculation and analysis of the stress-strain state of possible variants of metal cylindrical mesh shells, depending on the structural, geometric and force factors.

2. Parametric ratios

The development of some reticular surface shells is known from the literature [2, 6, 10]. There is also information that each grid has distinctive properties and works differently. A comparison of the features of the work of mesh surfaces indicates the feasibility of choosing a scheme with longitudinal and transverse elements and descending diagonals. This scheme is chosen as the main one and ensures maximum efficiency of the decision made.

The width $B$ of the grid, or the distance between the reference nodes in the direction of the arc, is a...
function of the radius $R$, height $H$, and the angle $\alpha$ of the circumscribed circle of the shell:

$$B = f(R, H, \alpha).$$ \hspace{1cm} (1)

Here the width $B$ and length $L$ of the shell are fixed. The remaining geometric parameters are changed.

It should be noted here that when forming cylindrical mesh surfaces for metal shells, it is proposed to choose a width $B$ at a level of no more than 21 m.

Of course, this indicator is dictated by economic and constructive considerations and does not imply the presence of additional elements. However, there are already quite acceptable results of shell calculations beyond the specified value. In particular, the linear dimension $B$ was recently adopted to be larger and equal to 24 m.

And according to studies of geometric parameters depending on the rigidity of the structure, the length of the shell is directly related to the width and is within the specified limits $L = (1.2...1.4)B$. Similarly, the width $B$ of the structure is a dependent quantity that is 3...5 times larger than the height $H$.

In accordance with the current trends of shaping and the specified ratios of geometric parameters, the most suitable overall dimensions of the shell $B \times L = 24 \times 30$ m, are taken, which correspond to the specified limits and make a small difference between them, as evidenced by the resulting expression $L = 1.25B$.

Panels along the width and in the direction of the length of the shell provide the formation of rectangular cells.

For the production of the elements, S235 class steel was selected. The load-bearing elements are assigned a thin-walled seamless tubular profile.

Supporting nodal joints are solved in a hinged version with the condition of ensuring the geometric immutability of the spatial structure:

$$X = Y = Z = 0; \quad X = Z = 0; \quad Z = 0.$$ \hspace{1cm} (2)

The own weight is determined depending on the change in the structure of the grid surface based on the assigned stiffness parameters of the elements from the available program range. The snow and wind load is calculated for the conditions of urban development and is applied to the structure in the form of concentrated nodal forces

$$F_1 = qA_1 = 0.25qh; \quad F_2 = qA_2 = 0.5qh; \quad F_3 = qA_3 = qhl,$$ \hspace{1cm} (3)

where $q$ – the result of the sum of its own weight, the action of snow and wind; $A_1$, $A_2$, $A_3$ – cargo areas in individual sections of the mesh surface; $l$, $h$ – the dimensions of the cells in the length and arc of the shell, respectively.

3. Forming a mesh surface
The initial positions considered in this way are the basis for the formation of the shell. Using the method of surface rotation, the necessary finite element models are constructed. In figure 1 shows the sequence of shell formation using SCAD software.
4. Materials and methods of research
In the process of creating models and subsequent calculations, the influence of the density of the arrangement of elements within the limits of ensuring the stability of the structure is analyzed, on the basis of which the possible degrees of filling of the grid surface are estimated.

5. Results and discussion
Based on the results of the analysis, the aimlessness of sealing the grid with additional cells is determined, especially when additional elements are introduced. As a result, based on the test examples, a change in the surface density is assumed, and the change in the height of the shell is controlled by checking its stability in accordance with the D. T. Wright condition

\[ a^2/Ri = a^2 8H/\left(B^2 + 4H^2\right) i < 9, \]  

where \( a \) – the size of a rectangular cell directed towards the generator; \( i \) – the radius of inertia of the cross-section of the rod.

The sensitivity of the grid surface is determined by the change in the angle \( \alpha \) of the described circle of the shell, the radius of curvature \( R \) and height \( H \), and is reflected by the number \( n \) of half-waves in the direction of the arc and the coefficient \( \lambda \) of stability reserve:

\[ n = [2, 4]; \lambda \in [0, \Lambda], \]  

where \( \Lambda \) – the specified value that the parameter \( \lambda \) can reach.

Along the arc and in the direction of the generator, 12 cells were taken (a smaller number of them failed the stability test, and a larger number led to unjustified material costs).

Thus, the limits of possible curvature \((1/R = 0.059...0.076)\) and the initial parameters of the shell geometry are determined (shown in table 1) by searching for the feasibility of using the specified reinforcement method.

| Model | \( \alpha (\degree) \) | \( R \) (m) | \( H \) (m) |
|-------|----------------|---------|---------|
| 1     | 90             | 16.97   | 4.97    |
| 2     | 100            | 15.66   | 5.60    |
| 3     | 110            | 14.65   | 6.25    |
| 4     | 120            | 13.86   | 6.93    |
| 5     | 130            | 13.24   | 7.64    |

The obtained restriction of the angle \( \alpha \) is revealed from the stability test condition (4) and the results of determining the deformation parameters.

In addition to previous studies [11] and despite different design schemes, the rationality of using
shells with curvature $1/R = 0.068\ldots0.076$ at the initial value of the angle $\alpha = 110^\circ$ is revealed.

The basis for establishing rational parameters was a comparison of dangerous movements of nodes recorded in shell models. In particular, the maximum numerical values from the first to the third model are in the vertical direction. But the fourth and fifth models showed the greatest nodal movements in the direction of width.

The regularity of changes in dangerous movements of nodes at different geometric parameters is fixed, as shown in figure 2.

![Figure 2](image-url)

**Figure 2.** Dynamics of changes in the maximum vertical (a) and horizontal (b) displacements of the shell model nodes.

Basically, under the condition of gradual lifting of the shell, the greatest movements of the nodes in the vertical direction tended to decrease, and no significant changes were detected in the horizontal direction. The exception was observed at the level of the fifth scheme vertically and at the level of the fourth scheme horizontally. This circumstance, when forming the structure, requires limiting the angle to $120^\circ$.

Of course, the noted deformation indicators were the result of the work of the shell elements within the framework of changing geometric parameters.

It is revealed that in each model the gap between the parameters of the maximally stretched and maximally compressed rods changes. Most models revealed the excess of the greatest tensile forces over the greatest compressive forces. The exception was the fourth model, in which the maximum value of the longitudinal force with some excess over the maximum tensile force was obtained by a compressed rod.

In general, when assessing the distribution of forces on the surface, it should be noted that a characteristic change in the sign from tension to compression is recorded in the weakly loaded rods of the upper edge of the support faces. In the fifth model, only two rods equidistant from the ends of the shell did not respond to this change. In the diagonal elements of the lower faces at the same distance from the extreme arcs, the support trusses led to a similar change in forces. The first three models also showed compression in the diagonal elements of the next truss on both slopes. The results obtained generally provided an acceptable distribution of forces over the surface.

Calculations of the load-bearing capacity of the models showed the fulfillment of the conditions of the limit states.

Meanwhile, the change in these parameters slightly affected the size of the cells, which led to the use of different load values and, as a result, to a significant (by $54.9\%$) change in the value of the reserve factor $\lambda$.

The shape of the possible occurrence of a critical state of the structure in the case of support trusses for the first four specified models showed three half-waves ($n = 3$) along the arc and one half-wave
(n = 1) in the direction of the generator. However, the fifth model showed a more stable character. The height of this model provided an increase in the rigidity of the shell, which led to an increase in the number of half-waves in width.

In figure 3 shows the deformations of one of the shells based on the results of a possible excess of the design load.

Based on the results of the calculations, three configurations of the curvature of the mesh surfaces were obtained.

\[ n = 3 \text{ in the global system } \mathbf{1} \]
\[ m = 1 \text{ in the global system } \mathbf{1} \]
\[ \text{along the arc} \]
\[ \text{in the direction of the generator} \]

![Figure 3](image)

**Figure 3.** Example of changing the geometry of a finite element shell model with parameters \( \alpha = 110^\circ, R = 14.65 \text{ m}, f = 6.25 \text{ m} \) along the arc and in the direction of the generator.

Of these, the first is illustrated by deflections in the support sections, the second by dangerous vertical movements in the ridge zone, and the third by simultaneous deformations in the upper and lower parts of the structure, as shown in figure 4.

![Model 1](image)
![Model 3](image)
![Model 5](image)

**Figure 4.** Shape curvature configurations of the three shell models.

For models 2 and 4, the shapes of the surface changes are identical to the shapes of the corresponding models 1 and 3. Geometrically, such schemes have different angles \( \alpha \) of the circumscribed circle of the shell, radii of curvature \( R \), and heights \( f \). Model 5 showed a completely different variation of the arc with a high potential of the original structure. Further lifting or reducing the height of the shell is not rational, since the first violates the technological efficiency, and the second leads to large nodal movements.

6. Conclusions

Finite element modeling, calculation and analysis of metal cylindrical mesh shells are carried out. The parameters of rational use of structures are determined. The assessment of the existing trends in the formation of the structure within the limits of ensuring its stability is carried out. The choice of the surface topology is justified, the recommended ratios of overall dimensions \( L/B = 1.25, B = 3...5f \) are adopted and the corresponding calculation models are formed.

The criteria for determining the stability of the grid are given. The mandatory fulfillment of the conditions of the limit states and the sensitivity to loss of stability are taken into account. The limits of permissible curvature \( 1/R = 0.068...0.076 \) and a set of basic geometric parameters \( \alpha, R, f \) for creating optimal structures are obtained. The effect of increasing the rigidity and height of the structure on the work of the mesh surface is analyzed.
The characteristic patterns of changes in nodal deviations and internal force factors of vulnerable areas of the shell are found. There is little difference between moving nodes in dangerous directions. A change in the sign of the forces from tension to compression in insufficiently loaded rods is revealed. The most complete use of the strength of the material is achieved. The results of changing the shape of the shell on the basis of data on the possible excess of the design load are presented. Weak points and features of the critical deformation of the system are found.

7. References
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