Mathematical modelling and analysis of profile meter

DH WANG*

Shandong Vocational College of science & Technology, Weifang, Shandong, 261053, China
*Email: xiaorliu@36haojie.com

Abstract. The profiler measures the workpiece to reflect the contour curve shape of the surface of the workpiece. When measuring the same workpiece, the drawn contour curve will shift due to the different placement angle and horizontal position. In order to improve the measurement accuracy of the profiler, the paper proposes a new modelling and calculation method for the profiler. This method uses mathematical methods such as vectors and geometric relations to calculate parameters; then completes level correction by changing the coordinate system; finally, uses geometric methods and analogy inference methods to obtain the deviation between the calculated values of each parameter. The research results show that this method can realize the measurement of the profiler more accurately.

Keywords. Elementary function model; Regression model; Scatter plot; Reasoning from analogy

1. Introduction

Profilometer is a kind of two coordinate measuring instrument. It is composed of working platform, fixture, workpiece to be measured, probe, sensor and servo drive. The working principle of the contact profilometer is that the probe contacts the surface of the workpiece to be measured and slides at a constant speed. The sensor senses the geometric changes of the measured surface, samples in X and Z directions respectively, and converts them into electrical signals. After amplification and other processing, the electrical signal is converted into digital signal and stored in the data file. In order to simplify the problem, it is assumed that the contour of the workpiece to be measured is a plane curve composed of straight lines and arcs, and the data of the contour curve of the workpiece is given in the attachment.

Question 1: according to the data in table level in Annex 1, the parameter values of notch width, arc radius, distance between centers, arc length, horizontal segment length, oblique line segment length, angle between oblique line and horizontal line, and herringbone height are indicated.

Question 2: 1. For the data level correction in Annex I, down.
2. After correction, the task of problem 1 is completed.
3. The differences between the calculated values of the parameters in the two states were compared.

Question 3: 1. Calculate the tilt angle of workpiece 2 under 10 different conditions.
2. The parameter values of workpiece 2 are calculated (the same as problem 1).
3. Draw the complete contour curve of workpiece 2.

2. Problem analysis

There are many factors that affect the delineation of contour line. We focus on the influence of horizontal displacement and tilt angle and establish the mathematical model. (there are some problems in the contact profilometer, such as probe contamination, probe defect, inaccurate scanning position, etc. we eliminate the interference of these factors through the idealized model.)
For the first problem, we use Excel to process and integrate the data in Annex I, and draw a scatter diagram. We use Pythagorean theorem to calculate the radius of arc and the length of oblique line segment, and use vector to calculate the relevant angle. We also use mathematical methods such as geometric relationship to calculate the width of notch, the distance between centers, the length of circular arc, the length of horizontal line segment, the angle between oblique line and horizontal line The value of the herringbone height parameter.

For the second problem, we process, analyze and integrate the data from Annex I down, draw the scatter diagram, complete the horizontal correction by changing the coordinate system, and obtain the deviation between the calculated values of parameters through data operation and analogy reasoning.

For the third problem, we analyze and integrate the data of Annex II to obtain the scatter diagram of each group. According to the data and the drawn image, we can calculate the tilt angle, the parameter values, and draw the complete contour line of workpiece.

3. Assumptions of the model

Symbol description is shown in Table 1. In order to make the problem easier to understand, we make the following assumptions:

1. It is assumed that the contact profilometer has no problems such as probe contamination, probe defect and inaccurate scanning position.
2. It is assumed that the test frequency of the profilometer remains unchanged before and after the profilometer.
3. It is assumed that the worktable is flat and even.
4. Assume that the center angle of the arc in the figure is 180 degrees.

| Symbol | Meaning |
|--------|---------|
| $n_X$  | Notch width |
| $n_X'$ | Notch width after correction |
| $C_n$  | Distance between centers |
| $C_n'$ | Distance between centers after correction |
| $X_k$  | Length of horizontal line segment |
| $X_k'$ | Length of horizontal line after correction |
| $a_i$  | Chord length |
| $h$    | Arch height |
| $\beta$ | Angle between oblique line and straight line |
| $\theta$  | Workpiece inclination angle |
| $Z$    | Herringbone length |
| $H$    | Length of diagonal line segment |
| $\alpha$  | Center angle |
| $L_n$  | Arc length |
| $R_k$  | Radius |

4. The establishment and solution of the model

4.1 Establishment and solution of the first problem model

The level data of Annex I table is converted into scatter plot by using Excel directly to get the figure 1:
In order to simplify the analysis, the data were processed and integrated to obtain the scatter plot Fig 2:

The width of notch can be obtained from Figure A2 as follows:

\[
X_1 = 53 - 48 = 5 \\
X_3 = 60 - 56 = 4 \\
X_5 = 67 - 63 = 4 \\
X_7 = 82 - 71 = 11 \\
X_{11} = 102 - 93 = 9 \\
X_{13} = 114 - 105 = 9
\]

It can be seen from figure A2 that the length of horizontal line segment is:

\[
X_2 = 56 - 53 = 3 \\
X_4 = 63 - 60 = 3 \\
X_6 = 71 - 67 = 4 \\
X_8 = 83 - 82 = 1 \\
X_{10} = 93 - 90 = 3 \\
X_{12} = 105 - 102 = 3
\]

According to figure A2, the height of herringbone line is as follows:

\[
Z = 1.76 - 0.84 = 0.92
\]
According to Fig. A2 and Pythagorean theorem, the length of oblique line segment is as follows:

\[ H^2 = \left( \frac{82 - 72}{2} \right)^2 + 0.92^2 \]

Therefore \( H = 5.0839 \).

According to figure A2 and vector angle formula:

\[ \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|a| |b|} \]

The angle between the straight line and the oblique line can be obtained as follows:

\( \beta_1 = 100.7^\circ, \beta_2 = 101^\circ, \beta_3 = 97^\circ, \beta_4 = 99^\circ, \beta_5 = 97.6^\circ, \beta_6 = 100^\circ, \beta_7 = 159.5^\circ, \beta_8 = 158^\circ. \)

According to Figure A2 and the model hypothesis, by using the formula \( L = \pi r \), the length of the circle can be obtained as follows:

\( L_1 = 1.73485, L_2 = 1.57, L_3 = 1.256, \)
\( L_4 = 11.0057, L_5 = 2.85062, L_6 = 11.4296. \)

From Figure A2, it can be concluded that the distance between the centers of circles is:

\( C_1 = 58 - 50.8 = 7.2, C_2 = 65 - 58 = 7 \)
\( C_3 = 85.3 - 65 = 20.3, C_4 = 88 - 85.3 = 2.7 \)
\( C_5 = 97.5 - 88 = 9.5, C_6 = 110 - 97.5 = 12.5 \)

Arc radius formula: \( R^2 = \left( \frac{a}{2} \right)^2 + \left( R - h \right)^2 \), by processing the data locally and substituting it into the arc radius formula, we can get the following results:

\( R_1 = 0.5625, R_2 = 0.52857, R_3 = 0.4, \)
\( R_4 = 3.505, R_5 = 0.9083, R_6 = 3.64, R_7 = 4.16. \)

4.2 Establishment and solution of the second problem model

Directly use Excel to convert the down data of Annex I table into scatter plot, as follow the figure 3:

![Figure 3](image-url)

Figure 3 convert the down data of Annex I table
Horizontal correction:

According to figure B1, the offset angle is as follows: \( \tan \theta = \frac{1.4 - (-7.86)}{119 - 48} = 0.1304 \), according to the calculation, it can be concluded that \( \theta = 7.4^\circ \). Therefore, the correction can be completed by rotating the coordinate axis clockwise about the origin by \( 7.4^\circ \).

Solution of various parameters:

For notch width, it can be seen from figure B1 and Pythagorean theorem that:
\[
\begin{align*}
X_1' &= \sqrt{4^2 + 2.8^2} = 4.9, \\
X_3' &= \sqrt{3.2^2 + 2.2^2} = 3.9, \\
X_7' &= \sqrt{9.8^2 + 4.7^2} = 10.9
\end{align*}
\]

The conclusion is as follows: \( X_n' = X_n \cos \theta \).

In the same way, the length of horizontal line segment after correction can be obtained as follows:
\[
X_2' = 2.9, X_4' = 2.9, X_6' = 3.9
\]

The conclusion is as follows: \( X_k' = X_k \cos \theta \).

According to the imaging principle of profilometer and the analysis of Annex I, the radius of the arc of the contour line obtained by the same workpiece under different angles and different displacements is constant, and then the length of the arc is constant, and the height of the herringbone line is constant.

To sum up, the distance between the centers of the circle after correction follows: \( C_n' = C_n \cos \theta \).

### 4.3 Establishment and solution of the third problem model

Through the analysis and integration of the data in Annex II by Excel, we can get the following results from the geometric relationship and Pythagorean theorem:

\[
\begin{align*}
\tan \theta_1 &= 2.166 \Rightarrow \theta_1 = 65^\circ \\
\tan \theta_2 &= 2.058 \Rightarrow \theta_2 = 65^\circ \\
\tan \theta_3 &= 1.85 \Rightarrow \theta_3 = 62^\circ \\
\tan \theta_4 &= 1.96 \Rightarrow \theta_4 = 62^\circ \\
\tan \theta_5 &= 1.744 \Rightarrow \theta_5 = 56^\circ \\
\tan \theta_6 &= 1.635 \Rightarrow \theta_6 = 58.2^\circ \\
\tan \theta_7 &= 1.824 \Rightarrow \theta_7 = 61^\circ
\end{align*}
\]

From the angle of inclination and geometric relationship, the width of notch is obtained as follows: \( X_1 = 16 \)

From the angle of inclination and geometric relationship, the length of the horizontal line segment can be obtained as follows: \( X_2 = 6 \)

From the angle of inclination and geometric relationship, the radius and length of the arc are obtained as follows:
\[
R_1 = 10, L_1 = \pi R_1 = 31.4
\]

According to vector angle formula:
\[
\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}|| \cdot ||\mathbf{b}||}
\]
The angle between the straight line and the oblique line can be obtained as follows: \( \beta = 93.4^\circ \).

After processing each group of data in Annex II, the complete data of workpiece II are summarized according to the tilt angle, and then the complete contour line is drawn, as shown in Figure 4:

![Figure 4 complete data of workpiece II](image)

5. Summary

Based on the principle of profilometer, this paper analyzes the parameters of the image. In the process of solving and analyzing the model, we use a lot of mathematical formulas and geometric relations to support the established mathematical model. Using Excel to simplify the data processing, the data information and formula are verified each other, which makes the results more meticulous. However, there are some problems in the contact profilometer, such as probe contamination, probe defects, inaccurate scanning position and so on, which can not be ignored, and the center angle of circular arc in the figure is also uncertain. We should carefully select the instruments needed for measurement, so as to minimize the influence of the instrument itself on the error, so as to make the trajectory curve closer to the ideal model. We should use more mathematical software to provide more powerful data support for the established mathematical model, and make full use of the data information provided by its attachments to make the results more accurate, so as to achieve the purpose of reducing errors.

6. References

[1] Gao H, Li QX, Xue SF and Gu XY 1995 Journal of instruments and meters 02 pp 213-218
[2] Han ZG 2009 Mathematical modeling method and its application Beijing: Higher Education Press
[3] Wu CP 2012 On solving mathematical problems by mathematical analogy mathematics teaching research (Journal) 10(6) pp 512-516
[4] Wang LF and Xu SB 1990 Introduction to analytic hierarchy process Beijing: China Renmin University Press
[5] Zhang SH 2010 Mathematical modeling Science Press
[6] Li Y and Gao JY 2009 Application of optimization method in mathematical modeling Department of mathematics Cangzhou Teachers College Hebei 061001.
[7] Jiang QY, Xie JX and Ye J 2018 Mathematical model (5th Edition) Beijing: Higher Education Press
[8] Liu JG and Xu L 1992 A new method of surface roughness measurement 02 pp 414-419