Revealing topological phase in Pancharatnam-Berry metasurfaces using mesoscopic electrodynamics

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Relying on the local orientation of nanostructures, Pancharatnam-Berry metasurfaces are currently enabling a new generation of polarization-sensitive optical devices. A systematical mesoscopic description of topological metasurfaces is developed, providing a deeper understanding of the physical mechanisms leading to the polarization-dependent breaking of translational symmetry in contrast with propagation phase effects. These theoretical results, along with interferometric experiments, contribute to the development of a solid theoretical framework for arbitrary polarization-dependent metasurfaces.

I. INTRODUCTION

Pancharatnam-Berry (PB) metasurfaces, made of periodic arrangements of subwavelength scatterers or antennas, have been extensively studied over the last few years and are currently considered as a forthcoming substitute of bulky refractive optical components [1, 2]. The reflection and refractive properties of light at interfaces can be efficiently controlled by appropriately designing the phase profile of these surfaces [3]. Several applications of PB metasurfaces, ranging from coloring to the realization of multifunctional tunable/active wavefront shaping devices, have been proposed [4]. As a result of the fascinating degree of the wavefront manipulation offered by metasurfaces, this technology is currently bursting through the doors of industry, particularly driven by their potential application in redefining optical designs, such as lenses [5–8], holography [9–11], polarimetry [12–14] and a variety of broadband optical components, including free-form metaoptics [15–19].

Despite these applications, significant efforts are currently being made in deriving proper theoretical frameworks to guide the design of complex components. Most of the disruptive attempts in controlling light-matter interactions rely on a fully vectorial Maxwell’s equations, such as effective medium theories [20–22], and the comprehensive understanding of their polarization responses generally obtained using extensive numerical simulations, such as finite element method [23] or finite-difference time domain techniques [3, 24, 25], which often shows the quantitative simulation results but lacking of qualitative physical interpretations [26–28]. Another approach, Green’s function method and diffraction theory for gratings, provides partial interpretation of a few diffractive properties of metasurfaces. The generalized Snell’s law can be then understood as a maximum grating efficiency in a given diffraction order [29, 30]. However, a vectorial theoretical framework is still required to clearly explain why the generalized Snell’s law occurs in the cross-polarized transmitted fields in PB metasurface system in the -1st or 1st diffraction orders only. To overcome these difficulties, the concept of geometric phase (PB phase), which is responsible for the conversion of the polarization state in the linearly birefringent medium [32–36], is introduced. Several works have shown that the transmission matrix which describes the birefringent response can be separated into co-polarized and cross-polarized beams in the circular basis by applying the PB phase induced by the orientation of nano-antennas [31, 37, 38]. However, this approach does not originate from first principle derivation and is not capable of explaining other diffractive properties of PB metasurfaces, such as the connection between generalized Snell’s law and polarization conversion. Obviously, each of these approaches just capture a part of the whole physical mechanism. To fill the gap between these concepts and incomplete demonstrations, a theoretical framework is highly needed to interpret all the diffractive properties of PB metasurfaces in a precise and systematic way.

In this letter, we propose a systematic mesoscopic electrodynamical theory to study the polarization-dependent metasurface, showing that the transmission of a co-polarized beam only acquires global phase associated with the antenna response, called “the propagation phase delay”, while the transmission of a cross-polarized beam is sensitive to both PB and propagation phases. We extend this phase effect to a more general situation by decomposing the arbitrary polarization of a normally incident light in circular basis, showing that each eigenstate acquires an opposite phase delay due to the topological phase retardation associated with the PB phase (see Eq. 10). Furthermore, we derive a fully electrodynamical expression and conduct optical measurements to analyze and validate this theoretical framework describing the diffractive properties of topological phase gradient metasurfaces.

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including the physical mechanisms of the coexistence of the zero and nonzero phase gradient leading to the ordinary and generalized Snell’s law, and the universal principles of copolarization and cross-polarization transmission.

\[ P_j(z, \rho, \omega) = N_0 \sum_{m,n} \int d^2 \kappa f_{mn,j}(\phi_j) E(z, \kappa, \omega)e^{i\psi_{mn,j}}. \]

Here $\psi_{mn,j} = G_{mn} \cdot (\rho - j a_1) + \kappa \cdot \rho$ describes only the propagation phase, and the form-factor of the $j$-th element in the $mn$-th lattice unite cell is $f_{mn,j}(\phi_j) = \Omega(G_{mn})/[\pi(2N + 1) a_1 a_2]$ where $\Omega(G_{mn})$ is the Fourier transform of geometric shape factor $\Omega(\rho) = H(|x| \leq l_x/2) H(|y| \leq l_y/2)$ with Heaviside function $H(\text{condition}) = \{1, \text{when condition is true}; 0, \text{when condition is false}\}$. The momentum integration over $\kappa$ runs over $Q$ - the first Brillouin zone. The coefficient $N_0 = \frac{\mu_0}{\pi \varepsilon^2}$ includes the nanopillars material susceptibility $\chi_0$.

The translational symmetry of the metasurface dictates the form of the solution which is given by

\[ E(z, \rho) = \sum_{m,n} e^{iG_{mn} \rho} \int_Q d^2 \kappa E_{mn}(z, \kappa, e^{i\kappa \cdot \rho}). \]

Considering the thickness $l_z$ to be much smaller than the $xy$ dimension of the metasurface, we neglect the $E_z$ and $P_z$ components in the model. An incoming plane wave can be written as $E_{in}(z, \rho) = E_i e^{i k_i \cdot z + i \kappa \cdot \rho}$ where $k_i = \sqrt{\frac{\omega^2 n_i^2 - \kappa^2}{c}}$ with propagation condition $\omega n_i > c k_i$ and refractive index $n_i$.

The geometric anisotropy of the nanopillars can be taken into account by replacing the scalar susceptibility $\chi_0$ by the diagonal 2x2 susceptibility tensor. The tensor components along the $x$ and $y$ axes are given by $\chi_x$ and $\chi_y$, respectively. Therefore, for the rectangular nanopillar oriented along $x$ and $y$, the transmission matrix in momentum space is given by $\tilde{E} = \hat{T} \tilde{E}_i$, where the transmitted electric field in the momentum space is $\tilde{E} = \{\tilde{E}_x, \tilde{E}_y\}$, and the incident field is $\tilde{E}_i = \{\tilde{E}_{i,x}, \tilde{E}_{i,y}\}$. The transmission matrix then reads (see Section S2 in SM for details)

\[ \tilde{T} = \begin{pmatrix}
\tilde{t}_{xx} & \tilde{t}_{xy} \\
\tilde{t}_{yx} & \tilde{t}_{yy}
\end{pmatrix}, \]

where $\tilde{t}_{ij}, i,j = x,y$ defined in Eq. (S17) explicitly depends on the form-factor $f_{mn,j}$. For the element oriented along $x$ and $y$ axes, i.e. $\phi_j = 0$, the corresponding form-factor is $f_{mn,j}(\phi_j = 0) \equiv \sin(\frac{\pi n m}{2N+1}) \sin(\frac{\pi n m}{2N+1})/(m n n^4)$.

Since the metasurface consists of nanopillars rotated around $z$ axes with the constant incremental angle $\phi_j = \frac{\pi m}{2N+1}$ in Fig. 1, the corresponding rotation matrix $R(\phi_j)$.
is given by
\[
\hat{R}(\phi_j) = \begin{pmatrix}
\cos(\phi_j) & -\sin(\phi_j) \\
\sin(\phi_j) & \cos(\phi_j)
\end{pmatrix}.
\] (5)

According to superposition principle, the transmission matrix of the metasurface can be obtained by summing the contributions of individual nanopillars, given by \( \hat{E}(K) = \sum_j \hat{T}\sigma(\phi_j)\hat{E}_i \), where the rotation-dependent transmission matrix is given by \( \hat{T}\sigma(\phi_j) = \hat{R}(\phi_j) \hat{R}(\phi_j) \).

Using Pauli algebra for two-component polarization basis without explicit factorization of the additional propagation phase, the rotation-depending transmission matrix reads
\[
2\hat{T}\sigma(\phi_j) = (\hat{t}_{xx} + \hat{t}_{yy})\hat{I} + i(\hat{t}_{xy} - \hat{t}_{yx})\hat{\sigma}_z \\
+ (\hat{t}_{xx} - \hat{t}_{yy})(e^{2i\phi_j}\hat{\sigma}_+ - e^{-2i\phi_j}\hat{\sigma}_-) \\
+ i(\hat{t}_{xy} + \hat{t}_{yx})(e^{2i\phi_j}\hat{\sigma}_- - e^{-2i\phi_j}\hat{\sigma}_+). 
\] (6)

Here, \( \hat{\sigma}_\pm = (\hat{\sigma}_x \pm i\hat{\sigma}_y)/2 \) is the spin-flip operator and the extra phase term \( e^{\pm 2i\phi_j} \) can be understood as the PB phase term \([31, 37, 38]\).

The transmitted field in the coordinate space (the form of \( \hat{T} \) is listed in Eq. (S18)) can be consecutively written as \( E(z, \rho) = \sum_{mnj} F_{mn,j}(z, \rho)\hat{T}\sigma(\phi_j)\hat{E}_i(z, \rho) \), where the propagation factor is
\[
F_{mn,j} = e^{i\phi_{mn,j} \rho/2}e^{iKz}e^{-i2\phi_j} \\
= \frac{e^{i\phi_{mn,j} \rho/2}e^{iKz}}{(2N+1)a_1} \\
= \frac{e^{i\phi_{mn,j} \rho/2}e^{iKz}}{(2N+1)a_1},
\] (7)

where \( k_z = \sqrt{\omega^2 n_i^2/c^2 - K_x^2} \), \( K_x^2 = (G_{x,mn} + \kappa_x)^2 + (G_{y,mn} + \kappa_y)^2 \) with momentum vectors of incident light \( \kappa_i = \kappa_x\hat{e}_x + \kappa_y\hat{e}_y \).

A. Discussion of analytical results

Analogous to the Bragg scattering in solid crystals, constructive interference of the propagating wave on the sub-wavelength periodic structure changes the complex amplitude of the refracted and reflected waves due to cumulative scattering from different crystal planes (see Eq. (7)). The evanescent waves emerge when \( n \neq 0 \), whose momentum vectors satisfy \( \omega^2 n_i^2/c^2 - K_x^2 - K_y^2 < 0 \). Overall, the transmitted field in zeroth diffraction order \( n = 0 \) contains both effects of propagation and topological phases. The first line in Eq. (6) corresponding to the co-polarization component transmitted field contains only the propagation phase \( e^{i\phi_{mn,j} \rho/2} \) embedded in the propagation factor \( F_{mn,j} \). The second and the third lines in Eq. (6) yield the cross-polarization components which depend on both propagation and PB phases via \( e^{i\phi_{mn,j} \rho/2} \pm 2i\phi_j \). Due to the translation invariance, the PB phase of the individual nanopillars is distributed uniformly between 0 and \( 2\pi \) such that \( \sum_j e^{i2\phi_j} \approx 0 \) except \( m = 0, \pm 1 \), where

\( \Xi_j = -G_{mn} \cdot j a_1 \pm 2\phi_j \) is the total phase. For \( m = 0 \), only the PB phase-independent co-polarized component can be observed. By calculating the \( x \)-dependent part of propagation phase \( \psi_{mn,j} \), we can find this component corresponds to the conventional diffraction which follows the ordinary Snell’s law, \( n_i\sin(\theta') - n_i\sin(\theta) = 0 \) where \( \theta \) and \( \theta' \) are the incident and transmitted angles, respectively. For \( m = \pm 1 \), the PB phase cancels the \( j \)-dependent part of the propagation phase and only cross-polarized components are detectable since \( G_x \cdot \pm 1 a_1 j \equiv \pm 2\phi_j \). The remaining cross-polarization propagation phase given by \( \pm (\pm 2\pi / 2N + 1) + \kappa_1 x \) yields the generalized Snell’s law which governs the anomalous refraction. For the light beam propagating in the \( xz \) plane, the refraction angle is defined by \( \sin(\theta') = \frac{\cos(\theta'}{n_i} - \frac{\pm 2\pi}{(2N+1)a_1} + \kappa_1 x \).

Thus for \( x = 0, (2N + 1)a_1 \), one can obtain

\[
\sin(\theta') n_t - \sin(\theta) n_i = \pm \frac{\lambda}{(2N + 1)a_1}.
\] (8)

We now calculate the Fresnel coefficient and analyze the chiral transmission properties in the circular polarization (CP) basis: \( \sigma_\pm = (\hat{e}_x \cos(\theta') \pm i\hat{e}_y)/\sqrt{2} \) where \( \theta' \) is the refraction angle. Following the derivation shown in Section S3 and assuming the amplitude of the incident light is \( E^x = 1 \) \( s \) \( \hat{e}_y(x \mp 1) \), the transmitted light can be written as \( E = \sum_{mnj} (E_1 + E_2 e^{-i2\phi_j} + E_3 e^{i2\phi_j}) F_{mn,j} \), where amplitudes \( E_1, E_2, \text{ and } E_3 \) are given by

\[
E_1 = t_1 + E^s + t_1 E^{-s},
E_2 = t_2 - E^s + t_2 E^{-s},
E_3 = t_2 + E^s + t_2 E^{-s}.
\] (9)

Here, the coefficients \( t_1 \pm \) and \( t_2 \pm \) are given in Eq. (S25). The additional phase factor \( e^{i2\phi_j} \), the PB phase, originates not only from the geometric rotation of the nanopillar in the unit cell, but also from the polarization of light. In other words, the additional phase \( \pm 2\phi_j \) relies on the symmetry relation between the polarization of light and geometric nanostructures of metasurface rather than the specific coordinate system, which is a characteristic of topological phase. In order to make it more clear and easy to compare with existing research results, we discuss and summarize the selective transmission of cross-polarized beam for all possible chiral combinations of the input polarization and metasurface in Tab. 1. For the metasurface with clockwise rotating nanopillars depicted in Fig. 1B, right CP (RCP) incident light splits into RCP light and left CP (LCP) light, while LCP incident light splits into LCP light and RCP light. Furthermore, as we demonstrated here, the PB phase term \( e^{-i2\phi_j} \) of the output amplitude contributes to the effect of non-zero cross-polarized beam. If the entire metasurface is rotated counter-clockwise by \( \pi \) along \( z \) axis which means \( \phi_j = \pi / (2N + 1) \), the constant phase term is written as \( e^{i2\phi_j} \).

Then the phase gradient of cross-polarized beam changes its sign. It is clear that the sign of the phase gradient
is determined by the handedness of incident light and metasurface.

| Antenna rotation | Input | Output(order) | Phase gradient |
|------------------|-------|---------------|----------------|
| Clockwise        | σ_+   | σ_+ (−1)      | (2N+1)σ_1     |
|                  | σ_-   | σ_- (−1)      | (2N+1)σ_1     |
|                  | LP    | σ_- (−1)      | (2N+1)σ_1     |
|                  |       | σ_+ (−1)      | (2N+1)σ_1     |
| Counter-clockwise| σ_+   | σ_- (−1)      | (2N+1)σ_1     |
|                  | σ_-   | σ_+ (−1)      | (2N+1)σ_1     |
|                  | LP    | σ_- (−1)      | (2N+1)σ_1     |
|                  |       | σ_+ (−1)      | (2N+1)σ_1     |

LP denotes linear polarization.

For an arbitrary input polarization, we can decompose the normally incident light in the CP basis as $E_{(in)} = E_{||} = aσ_+ + bσ_-$ with $β = \sqrt{1 - α^2}$ and considering a normal incidence $θ' = 0$. The transmitted light can be then recast as

$$E = \sum_{j,m=\pm 1} (t_{||}F_{00,j}E_{||} + t_{\perp}F_{m0,j}M(φ_j)E_{\perp}), \quad (10)$$

where $t_{||} = \frac{t_{xx} + t_{yy}}{2}$, $t_{\perp} = \frac{t_{xx} - t_{yy}}{2}$, $E_{||} \cdot E_{\perp} = 0$, and $M(φ_j) = \begin{pmatrix} e^{i2φ_j} & 0 \\ 0 & e^{-i2φ_j} \end{pmatrix}$. The corresponding transmission of the co- and cross-polarized beams for an arbitrary polarization incident light are illustrated schematically in Fig. 1A. Depending on the combination of the incident polarization and geometric rotation of the nanopillar, a cross-polarized retardation with a positive or negative phase occurs, leading to self-constructive or self-destructive interference effects. Figure 1A indicates the relative phase retardation $δφ_{||}$, is a function of the interface lateral displacement $δ(x)$ between co- and cross-polarized beams.

**III. INTERFEROMETRIC MEASUREMENT OF THE TOPOLOGICAL PHASE**

Therefore, the PB phase results in the opposite phase delays on the orthogonal CP components. The relevant phenomena, such as generalized Snell’s law, arbitrary polarization holography [6, 31], optical edge detection [42] and the photonic spin Hall effect[43, 44], can be thus described using our theory. In the following, we focus on topological phase characterization using the polarization-dependent translational symmetry breaking measurement based on the Mach-Zehnder interferometer (MZI). The GaN-based PB metasurface is used as a 50/50 CP beam splitter in the performance of self-phase referencing. To better understand the design of the birefringent nanostructure, we theoretically calculate the co- and cross-polarized scattering amplitudes of an array of identical nanopillars as a function of the phase delay between $x$ and $y$ polarizations, i.e. tuning the phase difference of the diagonal elements of susceptibility tensor which represents the geometric anisotropy of the metasurface. As shown in Fig. 2A, the ratio of the co- and cross-polarized transmission amplitude reach 50/50 when the the phase difference of the diagonal elements of susceptibility tensor is $π/2$ or $3π/2$. In order to identify GaN nanopillars with $π/2$ or $3π/2$ phase delay between $x$ and $y$ polarizations, full wave numerical simulations is performed to extract the phase retardation between $E_x$ and $E_y$ components and also the

![Table I. Cross-polarized transmission for different combinations of the input polarization and metasurface.](image)

![Graph A: Calculated polarization conversion efficiency (blue), co-polarization transmission(red), of the subwavelength array of PB nanopillars as function of the delay between polarization eigenstates.](image)

![Graph B: Experimental measurements of the normalized transmission across a PB metasurface designed according to the guideline in (B) and (C) as functions of length and width of the nanopillars.](image)

![Graph C: Experimental measurements of the normalized transmission across a PB metasurface designed according to the guideline in (B) and (C) as functions of length and width of the nanopillars.](image)

![Graph D: Comparison between experiments and theory of the anomalous refraction efficiency as a function of the incident angle, where $I$ is the transmitted power. The parameters of the simulations are $a_1 = 500nm, a_2 = 400nm, l_x = 260nm, l_y = 85nm, l_z = 632.8nm, λ = 632.8nm, n_1 = 1.61 + 0.3i, n_2 = 1.2 + 0.001i$, and $χ_{x,y}$ (see Eq. (S8) in SM) with $ω_3 = 2.75$ PHz, $ω_1 = 1.71$ PHz, and $n_{eff} = 1.2 – 0.01i$ account for the Fresnel coefficient at the first interface (see Section S3.1 in SM for details).](image)
transmission efficiency as function of length and width of the nanopillars in Fig 2B and C. The white lines indicate the regions for which the phase delay between x and y polarizations is equal to $\pi/2$ and $3\pi/2$, needed to adjust amplitudes for the interferometric characterization of the PB phase. According to these theoretical prediction, dimensions of GaN nanopillars used were length $l_x = 260$ nm, $l_y = 85$ nm and height 800 nm. These dimensions generate phase retardation $3\pi/4$ between Ex and Ey components (see Section S4 for more details). We create the arrays of rotated nanopillars, each rotated by an angle $\pi/5$ from its neighboring element as indicated in Fig. 1. The whole metasurface is of the size 250$\mu$mX250$\mu$m array. The nanofabrication of metasurface was realized by patterning a 800 nm thick GaN thin film grown on a double side polished c-plane sapphire substrate via a Molecular Beam Epitaxy (MBE) RIBER system. The GaN nanopillars were fabricated using a conventional electron beam lithography system (Raith ElphyPlus, Zeiss Supra 40) process with metallic Nickel (Ni) hard masks through a lift-off process. To this purpose, a double layer of around 200 nm Poly(methyl methacrylate) (PMMA) resists (495A4 then 950A2) was spin-coated on the GaN thin film, prior to baking the resist at a temperature of 125 °C. E-beam resist exposition was performed at 20 keV. Resist development was realized with 3 : 1 Isopropyl Alcohol (IPA):Methyl isobutyl ketone (MIBK) and a 50 nm thick Ni mask was deposited using E-beam evaporation. After the lift-off process in the acetone solution for 4 hours, GaN nanopillar patterns were created using reactive ion etching (RIE, Oxford system) with a plasma composed of Cl2, CH4, Ar gases. Finally, the Ni mask on the top of GaN nanopillars was removed by using chemical etching with 1 : 2 solution of HCl:HNO3.

Three gratings were designed and fabricated with different periodic arrangements of rotated nanopillars with periods 2, 2.9 and 4 $\mu$m, respectively. The refraction properties of these designed metasurfaces are measured as the experimental verification of theoretically predicted 50/50 PB metasurface beam splitter. The measurements have been realized using a conventional diffraction setup, comprising a Si-detector plugged into a lock-in amplifier to improve the detection signal to noise ratio. Acquiring the refracted signal as a function of the transmission angle, the detector rotates in a circular motion from $-30$ to 30 degrees. Spectral refraction response was obtained by sweeping the wavelength of a supercontinuum source coupled to a tunable single line filter in the range of 480 – 680 nm, by intervals of 20 nm. A linear polarizer followed by a quarter waveplate was utilized to select the state of the incident polarization. As shown in Fig. 2D (Fig. S5), the designed metasurface can stably realize the function of 50/50 beam splitter in the wavelength range of 480 – 680 nm. For normal LCP incident light, the zeroth order occurs at 0 degrees. Both diffracted -1st (dominant) and 1st orders (weak residual signals at opposite refraction angle) are a consequence of the PB phase gradient. The amplitudes of these two dominant co-CP and cross-CP remain 50/50 when the incident wavelength changes as shown in Fig. 2D. As shown in Fig. 2E, the experimentally measured transmission efficiency of cross-polarized beam has two well-resolved peaks around 15° and 48° which is in agreement with analytically predicted diffraction efficiency (red curve).

We have experimentally characterized the topological phase using a self-interferometric measurement in a MZI configuration, replacing a beam splitter by the metasurface as shown in Fig. 3A. Phase retardation of the anomalous refracted signal as a function of the lateral displacement of the metasurface, introduced by the shifting of metasurface along the phase gradient along the x axes, is recorded by monitoring the displacement of the interferogram fringes on a CCD camera after careful recombination and adjustment of the polarization handedness. The piezo stage controller is utilized to achieve minute translation of the metasurfaces as required for phase characterization in experiments discussed in Fig. 3B. In the present configuration, one arm of the MZI originates from the first order refraction from the metasurface. In addition to the anomalous refraction, the metasurface
imposes a phase
\[ \Phi_{\pm}(x) = G_{\pm10,x}\delta(x), \]  
which is proportional to the metasurface displacement \( \delta(x) \) along the phase gradient direction \( x \). We propose to experimentally measure this phase by recombining both arms on a beam splitter, and recording the resulting intensity profile as a function of the translation distance. The transmitted light of RCP/LCP incidence is
\[ I = \sum_j (t_|| + t_\perp)(1 + r \cos(\Phi_{\pm}(x))). \]  
Here \( I_|| = |\sum_j t_|||^2 \), \( I_\perp = |\sum_j t_\perp|^2 \) are the intensity of co- and cross-polarized transmission respectively, and \( r = \sqrt{t_||t_\perp}. \) Then the interference fringes displacement shown in right side of Fig. 3A provides indirect, yet unambiguous and conclusive measurement of the PB phase. As shown in Fig. 3B, we observe the linear phase variations with the wrapping periods equal to the PB phase of the metasurface in agreement with our theoretical result in Eq. (8).

**CONCLUSION**

In summary, we provide an in-depth analysis of topological PB metasurfaces by comparing experimental results obtained with spatially oriented subwavelength birefringent nanostructures, with a mesoscopic theory. This work, which demonstrates the origin of both controllable phase retardation effects, namely the propagation phase and the PB phase, is a first step in developing an intuitive understanding of topological and functional beam splitters for future applications in quantum optics and their implementations in relevant quantum information protocols based on metasurfaces, which is an important future research direction in this field [45–51].

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I. ELECTRODYNAMICS OF METASURFACE

A. Lattice model for light-metasurface interaction

The classical formulation of the light scattering problem is based on the Maxwell’s equations without external currents and charges \[1\]

\[
\nabla \cdot \mathbf{D} = 0, \tag{1}
\n\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \tag{2}
\n\nabla \cdot \mathbf{B} = 0, \tag{3}
\n\nabla \times \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}. \tag{4}
\]

We restrict ourself by considering non-magnetic media, so that the magnetic and diamagnetic fields are equal (the vacuum electric and magnetic permittivity are set to unity), \( \mathbf{B} = \mathbf{H} \). Following the standard Maxwell’s equations notations, we use the vector of polarization, \( \mathbf{D} = \mathbf{E} + 4\pi \mathbf{P} \). Therefore the system of Maxwell’s equations are given by Eq. (1) in the main text. Information about the matter is solely contained in the functional dependence of \( \mathbf{P} \) on \( \mathbf{E} \). The geometry of the system is chosen such that \( z \) axis of the laboratory coordinate system can be directed along the normal to the surface and \( \rho = (x, y) \) is a two-dimensional vector in the plane of metasurface. Particular dependence of \( \mathbf{P} \) on \( \mathbf{E} \) is considered in the following sections. When the metasurface is placed at the interface between two isotropic media with different refractive index, the corresponding \( z \)-dependent terms read

\[
\mathbf{P}(z, \rho, t) = \mathcal{H}(|z| \leq l_z/2) \mathbf{P}(\mathbf{E}) + \mathcal{H}(z < -l_z/2) \chi_i \mathbf{E}(z, \rho, t) + \mathcal{H}(z > l_z/2) \chi_i \mathbf{E}(z, \rho, t). \tag{5}
\]

Here and below Heaviside function \( \mathcal{H}(\text{condition}) = \{1, \text{when condition is true}; 0, \text{when condition is false}\} \). Then Eq. (1) in the main text for the domain \( z < -l_z/2 \) and \( z > l_z/2 \) describe the propagating waves with the effective speeds of light \( c_i = c/n_i \) and \( c_t = c/n_t \), respectively, where \( n_{i,t} = \sqrt{1 + 4\pi \chi_{i,t}} \) denotes the refractive index of substrates on both side (see Fig. (1)).

The polarization in Eq. (1) vanishes in vacuum, and has non-zero values inside the media, which represents the thin layer of metasurface. We consider a classical linear response model for the metasurface

\[
\mathbf{P}(z, \rho, t) = \chi(\rho) \mathbf{E}(z, \rho, t).
\]

The Fourier transformation of the fields over \( \rho \) are defined as follows:

\[
\mathbf{E}(z, \kappa, \omega) = \int d^2 \rho dt \mathbf{E}(z, \rho, t) e^{-i\kappa \cdot \rho - i\omega t}, \quad \mathbf{E}(r, t) = \frac{1}{(2\pi)^2} \int d^2 \kappa d\omega \mathbf{E}(z, \kappa, \omega) e^{i\kappa \cdot r - i\omega t}. \tag{6}
\]

Polarization is expressed in terms of the field,

\[
\mathbf{P}(z, \kappa, \omega) = \frac{1}{(2\pi)^2} \int d^2 \kappa \tilde{\chi}(\kappa - \kappa') \mathbf{E}(z, \kappa', \omega), \quad \tilde{\chi}(\kappa) = \int d^2 \rho \chi(\rho) e^{-i\kappa \cdot \rho}. \tag{7}
\]

The linear susceptibility function \( \chi(\rho) \) is a periodic function of coordinates. It can be represented as a sum of the

\[
\chi(\rho) = \sum_{\mathbf{k}} \chi_{\mathbf{k}} e^{i\mathbf{k} \cdot \rho}.
\]
susceptibilities of the individual primitives shifted and rotated in $xy$ plane. The reflection and transition is therefore defined by the vectors of the reciprocal lattice $G$, where the function $\chi(\Delta \kappa)$ reaches its maxima.

B. Metasurface with translational symmetry

Since the length of the antenna is in the sub-wavelength scale and the metasurface is translation-invariant without considering the rotation of the antenna, as shown in Fig. 2, the susceptibility function $\chi(\rho)$ can be represented by

$$\chi(z, \rho) = \chi_0 \sum_{mn} \sum_{j=-N}^{N} \Omega(\rho - g_{Mm+j,n}), \quad \chi_0 = \begin{pmatrix} \chi_x & 0 \\ 0 & \chi_y \end{pmatrix},$$

where $N = (M - 1)/2$ with the number of elements in the unit cell $M$, and the susceptibilities of the nanopillar along $x$ and $y$ directions are $\chi_x = \left[4\pi \omega - 4\pi \omega_0 (1 - \sin^2(\theta)/n_{eff}^2)^{-1/2}\right]^{-1}$, $\chi_y = \left[4\pi \omega - 4\pi \omega_1 (1 - \cos^2(\theta)/n_{eff}^2)^{-1/2}\right]^{-1}$ with different resonance frequencies $\omega_0$ and $\omega_1 [2]$, effective refractive index of the nanopillar $n_{eff}$ and incident angle $\theta$. The indicator function $\Omega(\rho)$ describes the basic geometric primitive, which has a rectangular shape:

$$\Omega(\rho) = \mathcal{H}(|x| \leq l_x/2)\mathcal{H}(|y| \leq l_y/2).$$
The lattice translation vector \( \mathbf{q}_{m,n} = ma_x + na_y \) with lattice primitive translation vectors \( a_x = (2N + 1)a_1 e_x \) and \( a_y = a_2 e_y \). 2D spatial Fourier transformation of Eq. (8) is

\[
\hat{\chi}(\mathbf{k}) = \chi_0 \sum_m e^{-i\mathbf{k}a_2} \sum_{j=-N}^{N} \tilde{\Omega}_{mnj}(\mathbf{k}) e^{-i((2N+1)m+j)\mathbf{k}a_1},
\]

where \( \tilde{\Omega}_{mnj}(\mathbf{k}) \) is the Fourier transform of \( \Omega(\mathbf{q} - \mathbf{q}_{Mm+nj}) \). Since the rotation of antenna in one unite cell is treated by the rotation of the transmission matrix later by the form \( \hat{T}(\phi_j) = \hat{R}(\phi_j) \hat{T}(\phi_j) \) in the main text, all the \( \tilde{\Omega}_{mnj}(\mathbf{k}) \) with different indexes \( m, n, j \) are same and the subscripts \( mnj \) will be omitted below. By using the Poisson summation formula \( \sum_{n'=-\infty}^{\infty} F(k - 2\pi n'a) = \frac{1}{2\pi a} \sum_{n=-\infty}^{\infty} \hat{F}(\frac{n}{2\pi a}) e^{-i kn/a} \) where \( \hat{F} \) is the Fourier transformation of \( F \), Eq. (10) reduce to

\[
\hat{\chi}(\mathbf{k}) = \sum_{m',n'} \delta \left( k_x - \frac{2\pi n'}{(2N+1)a_1} \right) \delta \left( k_y - \frac{2\pi n'}{a_2} \right) F_{m'n'},
\]

where \( F_{m'n'} = \frac{\chi(2\pi)^2}{(2N+1)a_1 a_2} \sum_{j=-N}^{N} \tilde{\Omega}(\mathbf{k}) e^{-ij\mathbf{k}a_1} \). Taking the inverse Fourier transform, we obtain

\[
\hat{\chi}(\mathbf{\rho}) = \frac{\chi_0}{(2N+1)a_1 a_2} \sum_{m',n'} \sum_{j=-N}^{N} \tilde{\Omega}(\mathbf{G}_{m'n'}) e^{i\mathbf{G}_{m'n'} \cdot (\mathbf{\rho} - j\mathbf{a}_1)}
\]

where the reciprocal lattice vector \( \mathbf{G}_{m'n'} = \frac{2\pi m'}{(2N+1)a_1} e_x + \frac{2\pi n'}{a_2} e_y \). Substitution of the result (11) into (7) produces the series for the polarization vector

\[
P(z, \mathbf{\kappa}, \omega) = \frac{\chi_0}{(2N+1)a_1 a_2} \sum_{m',n'} \sum_{j=-N}^{N} \int d^2 \mathbf{\kappa}' \delta (\mathbf{\kappa} - \mathbf{\kappa}' - \mathbf{G}_{m'n'}) e^{-i\mathbf{G}_{m'n'} \cdot \mathbf{P}(z, \mathbf{\kappa}')} \tilde{\Omega}(\mathbf{\kappa} - \mathbf{\kappa}') E(z, \mathbf{\kappa}', \omega).
\]

By taking inverse Fourier transform of Eq. (12), polarization vector is given by Eq. (2) in the main text as a sum over the Brillouin zones. To define the nonuniform part of the system of Maxwell’s equations, we assume that the incident light has the form \( E(z, \mathbf{\rho}, t) = E_0 e^{i(k_z z + i\mathbf{k} \cdot \mathbf{\rho} - \omega t)} \) where \( k_z = \sqrt{\omega^2 - \mathbf{k}^2} \) with the conditions \( \omega n_i > c k_z \). Since there are no other temporal characteristics except the \( \omega_i \), all the time-derivatives can be replaced by the multiplication by \(-i\omega_i\). We seek for the solution of the Maxwell’s equations (1) in the form of Eq. (4) in the main text.

C. Thin metasurface limit

In zeroth approximation, we assume that the surface polarization is caused by the incident light only, so that \( P_{\parallel} = \chi E_{\parallel 0} e^{i(k_z z + i\mathbf{\kappa} \cdot \mathbf{\rho})} \) and

\[
\frac{\partial^2}{\partial z^2} E_{\parallel} + \nabla_\rho^2 E_{\parallel} + \frac{\omega_0^2 n_i^2}{c^2} E_{\parallel} = -4\pi \nabla_\rho (\nabla_\rho \cdot P_{\parallel}) - 4\pi \nabla_\rho \frac{\partial}{\partial z} P_z - 4\pi \frac{\omega_0^2}{c^2} P_{\parallel}.
\]

The following equations are written for the \( e^{i(G_{mn} + \mathbf{\kappa}) \mathbf{\rho}} \) components of above equations. All the \( G_{mn} + \mathbf{\kappa} \) spatial components of the \( \hat{E}, \hat{P} \) have the same subscripts \( m \) and \( n \) which will be further omitted. The corresponding thin polarization component \( \mathcal{H}(|z| \leq l_z/2) P(E) \) is

\[
\mathcal{H}(k_z) P_{\parallel}(k_z, \mathbf{\kappa}) = \frac{\sin(\xi l_z)}{4\pi \xi} \chi_0 f_{mn} E_{\parallel, i} \delta(\mathbf{\kappa}_i - \mathbf{\kappa}) \simeq \frac{l_z}{4\pi} \chi_0 f_{mn} E_{\parallel, i} \delta(\mathbf{\kappa}_i - \mathbf{\kappa}) E_{\parallel, i} \mathbf{\kappa},
\]

where \( f_{mn} = \sum_{j=-N}^{N} f_{mn,j} e^{-ijG_{mn} a_1} \), \( 2\xi = k_{zi} - k_z \), \( E_{\parallel, i} \) is the \( x, y \) amplitudes of incident light. When the incident angle or the thickness of the antenna \( l_z \) is small which corresponds to \( \xi \simeq 0 \) or \( l_z \simeq 0 \), then \( \frac{\sin(\xi l_z)}{\xi} \simeq l_z \). Considering
the thickness of the metasurface to be small, we neglect the $E_z$ and $P_z$ components in the model and obtain

$$\frac{1}{2\pi} \left( \frac{\omega_i^2 n_z^2}{c^2} - K_{||}^2 - k_z^2 \right) \tilde{E}_x(k_z, \kappa) = \left( K_x^2 - \frac{\omega_i^2}{c^2} \right) \tilde{P}_x + K_y K_z \tilde{P}_y,$$

(15)

$$\frac{1}{2\pi} \left( \frac{\omega_i^2 n_z^2}{c^2} - K_{||}^2 - k_z^2 \right) \tilde{E}_y(k_z, \kappa) = \left( K_y^2 - \frac{\omega_i^2}{c^2} \right) \tilde{P}_y + K_x K_z \tilde{P}_x,$$

(16)

where $K_x = G_{mn,x} + \kappa_x$, $K_y = G_{mn,y} + \kappa_y$, $K_{||} = (G_{mn,x} + \kappa_x)^2 + (G_{mn,y} + \kappa_y)^2$, $\tilde{E}_y(k_z, \kappa), \tilde{E}_y(k_z, \kappa)$ are function of $k_z, \kappa$ need to be calculated, $\tilde{P}_|| = i_z \chi_0 f_{mn} E_{||,i} \delta(\kappa - \kappa_i)$.

II. REFLECTION AND REFRACTION FROM METASURFACES WITH TRANSLATIONAL SYMMETRY

A. Transmission matrix

We can now investigate how the light is transmitted through the metasurface. In the present model, we assume that, for a typical design, individual primitives are spaced sparsely which eliminates a possibility of the inter-element interactions. In this case, one can calculate the transmission through the individual primitive and then sum over all the elements of the unit cell and the entire metasurface. Consider the rectangular primitive with perfectly aligned sides along x and y coordinates such that the long side $l_y$ is along the $y$-axis and the short side $l_x$ is along the $x$-axis.

We can thus recast Eqs. (15) - (16) in terms of the transmission matrix $\mathbf{T}$ given by Eq. (4) in the main text (Due to $e^{-iG_{mn,a}t}$ is included in the propagation phase $\psi_{mn,j}$, we omitted it in here.):

$$\begin{pmatrix}
\tilde{E}_{x,m,n} \\
\tilde{E}_{y,m,n}
\end{pmatrix} =
\begin{pmatrix}
t_{xx} & \tilde{t}_{xy} \\
\tilde{t}_{yx} & t_{yy}
\end{pmatrix}
\begin{pmatrix}
E_{xi} \\
E_{yi}
\end{pmatrix},$$

where the elements of transmission matrix are given by

$$\tilde{t}_{xx} = \frac{\tilde{C}}{C'} (K_x^2 - \omega_i^2) \chi_x,$$

$$\tilde{t}_{xy} = \frac{\tilde{C}}{C'} (K_x K_y) \chi_y,$$

$$\tilde{t}_{yx} = \frac{\tilde{C}}{C'} (K_x K_y) \chi_x,$$

$$\tilde{t}_{yy} = \frac{\tilde{C}}{C'} (K_y^2 - \omega_i^2) \chi_y.$$

(17)

Here $\tilde{C'} = -k_z^2 + \frac{\omega_i^2 n_z^2}{c^2} - K_{||}^2$, $\tilde{C'} = 2\pi l_z f_{mn,j} \delta(\kappa_i - \kappa), \kappa_i < G_{i,1} = \frac{2\pi}{(2N+1)a_1} e_x + \frac{2\pi}{a_2} e_y$. In the physical space, the transmission matrix $\mathbf{T}$ can be written as

$$t_{xx} = \frac{C}{C'} (K_x^2 - \omega_i^2) \chi_x,$$

$$t_{xy} = \frac{C}{C'} (K_x K_y) \chi_y,$$

$$t_{yx} = \frac{C}{C'} (K_x K_y) \chi_x,$$

$$t_{yy} = \frac{C}{C'} (K_y^2 - \omega_i^2) \chi_y.$$

(18)

where $C' = \sqrt{\frac{\omega_i^2 n_z^2}{c^2} - K_{||,\kappa=\kappa_i}^2}$, $C' = il_z f_{mn,j}/4\pi|\kappa=\kappa_i$. 
III. DERIVATION OF THE FRESNEL COEFFICIENT

So far we have investigated the transmission properties of the metasurfaces assuming that the incident and transmitted light are polarized linearly. We therefore have to transform the solution to the CP basis \( \sigma_{\pm} = (e_x \cos(\theta') \pm ie_y)/\sqrt{2} \), where \( \theta' \) is the refraction angle (see Fig. 1). For an ordinary operator \( \hat{A} \) in the linear polarization basis, we define an operator \( \hat{A} \) in the CP basis \( \hat{A}^{(\sigma_x,\sigma_z)}(\sigma_x,\sigma_z) \hat{A} \). For instance, the rotation operator \( \hat{R}(\phi_j) \) defined in Eq. (6) can be recast in the circular polarization basis as an operator \( \hat{R}(\phi_j) \) defined as follows

\[
\hat{R}(\phi_j) = \begin{pmatrix}
\frac{e^{i\phi_j} (2 - \sec \theta' - \cos \theta') + e^{-i\phi_j} (2 + \sec \theta' + \cos \theta')}{4} & \frac{e^{i\phi_j} (\sec \theta' - \cos \theta') + e^{-i\phi_j} (- \sec \theta' + \cos \theta')}{4} \\
\frac{e^{-i\phi_j} (\sec \theta' + \cos \theta') + e^{i\phi_j} (- \sec \theta' - \cos \theta')}{4} & \frac{e^{-i\phi_j} (2 - \sec \theta' - \cos \theta') + e^{i\phi_j} (2 + \sec \theta' + \cos \theta')}{4}
\end{pmatrix}.
\]  

(19)

Similarly the transmission matrix \( \hat{T} \) in Eq. (5) takes the form

\[
\hat{T} \equiv \begin{pmatrix}
t_{yy}-t_{xx}-it_{yx} \cos \theta'+it_{yx} \sec \theta' \quad t_{yy}-t_{xx}-it_{yx} \cos \theta'-it_{yx} \sec \theta' \\\nt_{yy}+t_{xx}+it_{yx} \cos \theta'+it_{yx} \sec \theta' \quad t_{yy}+t_{xx}+it_{yx} \cos \theta'-it_{yx} \sec \theta'
\end{pmatrix}.
\]  

(20)

We then consider the in plane transmission condition \( t_{xy} = 0, t_{yx} = 0 \). In the CP basis the corresponding operator reads

\[
\hat{T}(\phi_j) = \begin{pmatrix}
t'_{1j} & t'_{2j} \\
t'^*_{2j} & t'^*_{1j}
\end{pmatrix},
\]  

(21)

where

\[
t'_{1j} = \frac{4(t_{xx} + t_{yy}) + e^{i2\phi_j}(t_{xx} - t_{yy})(\cos \theta' - \sec \theta') + e^{-i2\phi_j}(t_{xx} - t_{yy})(\sec \theta' - \cos \theta')}{8},
\]

\[
t'_{2j} = \frac{e^{-i2\phi_j}(t_{xx} - t_{yy})(2 - \sec \theta' - \cos \theta') + e^{i2\phi_j}(t_{xx} - t_{yy})(2 + \sec \theta' + \cos \theta')}{8}.
\]

The CP light with the incident angle \( \theta \) can be recast as \( \{ \frac{1}{2} (\pm 1 + \sec \theta' \cos \theta), \frac{1}{2} (\mp 1 + \sec \theta' \cos \theta), 0 \} \). The \( E_{mn,j} \) component of the refracted light then reads

\[
E_{mn,j} = \begin{pmatrix}
E_{+1} + E_{+2}e^{-i2\phi_j} + E_{+3}e^{i2\phi_j} \\
E_{-1} + E_{-2}e^{-i2\phi_j} + E_{-3}e^{i2\phi_j}
\end{pmatrix} F_{mn,j},
\]  

(22)

where

\[
E_{+1} = \frac{(t_{xx} + t_{yy})(\cos \theta \sec \theta' \pm 1)}{4}, \quad E_{+2} = \frac{(t_{xx} - t_{yy})(\cos \theta \pm 1)(\sec \theta' - 1)}{8},
\]

\[
E_{+3} = \frac{(t_{xx} - t_{yy})(\cos \theta' \mp 1)(\sec \theta' + 1)}{8},
\]

\[
E_{-1} = \frac{(t_{xx} + t_{yy})(\cos \theta \sec \theta' \mp 1)}{4}, \quad E_{-2} = \frac{(t_{xx} - t_{yy})(\cos \theta \pm 1)(\sec \theta' + 1)}{8},
\]

\[
E_{-3} = \frac{(t_{xx} - t_{yy})(\cos \theta' \mp 1)(\sec \theta' - 1)}{8}.
\]

(23)

Consider the general refraction law, the refraction angle satisfies \( \sin \theta' = \frac{m \lambda}{n_{a1} \tau_{1,r} + \frac{n_1 \sin \theta}{n_{a1}}} \) where \( m = 0, \pm 1 \). Using the
field amplitudes in the circular polarization basis $E^\pm = (e_x \cos(\theta') \pm i e_y) / \sqrt{2}$, which yields the general result:

$$E = \sum_{mnj} (E_1 + E_2 e^{-i \phi_j} + E_3 e^{i \phi_j}) F_{mn,j}$$

with $E_1, E_2, E_3$ can be written as Eq. (9) where

$$t_{1\pm} = \frac{(t_{xx} + t_{yy})(\cos \theta \sec \theta' \pm 1)}{4},$$

$$t_{2\pm\pm} = \frac{(t_{xx} - t_{yy})(\cos \theta \pm 1)(\sec \theta' \pm 1)}{8},$$

$$E^\pm = (e_x \cos(\theta') \pm i e_y) / \sqrt{2}.$$  \hspace{1cm} (25)

Since $l_y << \lambda$ and $l_x < \lambda$, then $t_{yy} \approx \chi_y$ can be neglected. In order to get a more clear and simper analytic expression without effect physical picture, we discuss a 1D model in the following. Consider $t_{xx} = \frac{1}{2} \left| f_{mn,j} \right| l_z (\frac{K_z^2 - \omega^2}{c^2}) \approx t_{xx}' \cos \theta'$

where $t_{xx}' = -\frac{1}{2} \sum_{m,n,j} l_z (\frac{K_z^2 - \omega^2}{c^2})$ and the refractive indexes in two sides of the metasurface are close $n^i = n^t \approx 1$, then $E_1, E_2, E_3$ can be written as

$$E_1 = t_{1+} E^s + t_{1-} E^{-s},$$

$$E_2 = t_{2+} E^s + t_{2+} E^{-s},$$

$$E_3 = -t_{2-} E^s - t_{2-} E^{-s},$$

where

$$t_{1\pm}' = \frac{1}{4} t_{xx}' (\cos \theta \pm \cos \theta'),$$

$$t_{2\pm\pm}' = \frac{1}{8} t_{xx}' (\cos \theta \pm 1)(\cos \theta' \pm 1).$$

It is clear that the amplitude of different components satisfy $|t_{1+}'| > |t_{2-}'|, |t_{2+}| > |t_{1-}'|, |t_{2-}'|$, when the incident and refracted angles are small. The transmission light is in the form of

$$E = \sum_{mnj} (t_{1+} E^s + t_{2+} E^{-s} e^{-i \phi_j}) F_{mn,j}.$$  \hspace{1cm} (27)

Combining with the constant phase term $e^{2i \phi_j}$ in $F_{mn,j}(z, \rho)$ (antennas are rotated in a clockwise direction in Fig. 2), for RCP incident light $(s = 1)$, only $m = 0, 1$ order components can be refracted; for LCP incident light $(s = -1)$, only $m = 0, -1$ order components can be transferred due to the averaging out of the phase components $\sum_j e^{\pm i \phi_j} \approx \sum_j e^{\pm i \phi_j} \approx 0$.

A. Derivation of the transmission efficiency

In order to describe experimental observations, the details of theoretical model are presented as follows. As shown in the Fig. 3, the transmission of the light traverses three medium: the air, the substrate, and the metasurface.

We first consider light transmission through the air/substrate interface. Refraction angle satisfies ordinary Snell’s law: $\frac{\sin(\theta' )}{\sin(\theta )} = \frac{n}{n^s}$ where $n^a$ and $n^s$ are the refractive indexes of air and substrate, respectively. The $x$ and $y$ components of the electric field in the substrate satisfy the Fresnel formula

$$E_x = \frac{2 \cos(\theta)^2}{\sqrt{2}|\cos(\theta) + n_x \cos(\theta')|}, \quad E_y = \frac{i 2 \cos(\theta)}{\sqrt{2}|\cos(\theta') + n_y \cos(\theta)|}. $$

After passing through the substrate light interacts with the metasurface where the refraction is described by the generalized Snell’s law and the amplitude obeys Eq. (10) in the main text. In order to model the structural birefringence
of the rectangular nano pillars we define the x and y components of the susceptibility tensor of GaN nanopillar according to already defined after Eq. (S8). This functional form of the susceptibility Cartesian components with two resonant frequencies $\omega_0$ and $\omega_1$ [2], which agrees with angle resolved photoluminescence experiments of the birefringent material. In addition, by expanding the angle-dependent susceptibility components in a Taylor series, one can obtain an angular dispersion similar to that observed in THz metasurfaces [3], where the phenomenological amplitudes in the expansion represent inter-particle coupling strengths. The transmission through the metasurface is further described by our mesoscopic model (see Eq. (10)), which allows us to obtain the final anomalous refraction efficiency.

IV. DESIGN OF THE POLARIZATION DEPENDENT BEAM SPLITTER

The interferometric measurement between normal and anomalous refraction imposes that the scattering properties of the interface should contain both zero and first diffraction order with similar amplitude and polarization. The latter can be addressed by inserting a quarter wave plate into the path of one of the diffracted beams. The former condition requires tuning the antenna scattering parameter. The amplitude and phase responses of the nanopillars forming the PB metasurface are related to the length and width of the nanopillar with a constant height of 800 nm. To maximize the PB metasurface efficiency, the antenna should maximally convert the polarization from the opposite orthogonal circular polarization, thus introducing a phase shift between ordinary and extraordinary axis of a nanopillar [4]. However, in contrast to the previous PB metasurface works seeking for the high performance devices [5], we are herein interested in quantifying the PB phase from the self-referenced interferometric measurements. Proper design is achieved by considering birefringent nanopillar introducing $\pi/2$ or $3\pi/2$ phase retardation, as indicated in Fig. 2A. To account for both tapering and to compensate for anisotropic etching, we choose the nanopillar size corresponding to $3\pi/4$ phase shift. To quantify the phase-shift of the light transmitted through the GaN nanopillars, we perform the electromagnetic simulations of the light transmission through a subwavelength array of nanopillars arranged in a square lattice. The metasurface consists of GaN nanopillar array on Sapphire ($\text{Al}_2\text{O}_3$) substrate. The refractive index for GaN has the following Sellmeier like relation [6]:

$$n_0(\lambda) = \sqrt{1 + \frac{A_0\lambda^2}{\lambda^2 - (\lambda_G^0)^2} + \frac{B_0\lambda^2}{\lambda^2 - (\lambda_H^0)^2}},$$

$$n_e(\lambda) = \sqrt{1 + \frac{A_e\lambda^2}{\lambda^2 - (\lambda_G^e)^2} + \frac{B_e\lambda^2}{\lambda^2 - (\lambda_H^e)^2}}.$$ 

Here, $A_0 = 0.213$, $B_0 = 3.988$, $A_e = 0.118$, $B_e = 4.201$, $\lambda_G^0 = 350\text{nm}$, $\lambda_H^0 = 153\text{nm}$, $\lambda_G^e = 173.5\text{nm}$. For Sapphire ($\text{Al}_2\text{O}_3$), refractive index relation is referred from [7]. For $\lambda = 632.8\text{nm}$, the design wavelength, $n = 1.766$. GaN has very small birefringence, which we decided to neglect as a first approximation. To avoid diffraction both in free space and in the substrate, we arranged the spacing between the elements with a subwavelength period of 320 nm. The simulations are performed using the FDTD using plane wave sources at 632.8$\text{nm}$ wavelength polarized along $x$ and $y$ axis, impinging at normal incidence satisfying the perfectly matched layer (PML) conditions in the direction
of the light propagation subject to periodic boundary conditions along all the in-plane directions. The use of PML boundary conditions in the propagation direction results in an open space simulation while in-plane periodic boundary conditions mimic a subwavelength array of the identical nanostructures.

In figure 4, the polarization-conversion efficiency of a single GaN nanopillar is obtained by FDTD simulation. A meta-atom is impinged with two plane waves sources (x and y-polarized) of varying wavelength from 480 nm to 680 nm with the interval of 20 nm. The other simulation conditions are kept same as in the design simulation. To perform interference measurements as described in the main text, the design of the element is chosen to diffract 50% of the incident light on cross polarization.

FIG. 4. Polarization conversion efficiency of a single GaN nanopillar.

V. INTERPRETATION OF THE INTERFEROMETRIC EXPERIMENTS

Note, that Eq. (12) shows that MZI detects the displacement phase. Technically the displacement phase is a propagation phase, also known as detour phase, since the measurement involves translational motion alone. However as has been already pointed out in the discussion following Eq. (7), the propagation phase contains two parts, one of which cancels out the PB phase yielding non vanishing first diffraction order. The detour phase is therefore a remaining part of the propagation phase which enters the Snell’s law. Note, however, that due to the translational invariance and uniform distribution of PB phases in the unit cell, the magnitude of the displacement phase is equivalent to the PB phase. This equivalence is not accidental and follows directly from the metasurface design itself, and therefore can be controlled at will. The MZI measurements thus provides although indirect yet unambiguous and conclusive measurement of the PB phase.

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FIG. 5. left: Measured angular deflection efficiency as a function of the incident wavelength for $\sigma_+$ incident polarization, to be compared with Fig 2D. Right: deflection efficiency as a function of the wavelength.