A simple solution to the fine tuning problem of the cosmological constant

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Abstract: We show that the fine tuning problem of the cosmological constant can be solved in a conformal model, with explicit conformal symmetry breaking. We argue that the model has two different terms which essentially contribute to the cosmological constant. A local conformal transformation leads to a cancellation of these two terms among one another. Hence these produce no physical consequence. The model generates a small cosmological constant without requiring any fine tuning.

1 Introduction

The idea that conformal invariance [1–14] might solve the problem of fine tuning of the cosmological constant [15, 16] is very old [17, 18] and has attracted considerable interest in the literature [19–30]. A theory with conformal invariance does not permit a cosmological constant and hence might impose some constraint on its value. However due to conformal anomaly it is not clear that it is possible to maintain a small value of the cosmological constant at loop orders even if the action displays classical conformal invariance. Furthermore one requires some source of dark energy [31–33]. Hence the model has to provide its very small value without fine tuning.

It has been shown that conformal invariance can be implemented in the full quantum theory if we use a dynamical scale for regularization [8,19,22]. This is implemented by introducing a real scalar field in the model. The procedure has been called the GR-SI prescription in [21]. In this case the conformal symmetry is broken by a soft mechanism. It may be spontaneously broken [8,21,22] or broken by the background cosmic evolution [19,20]. This leads to a non-zero classical value of the real scalar field which provides a scale for regularization. One finds that the implications of conformal symmetry are maintained even in the full quantum theory. However the theory predicts renormalization group evolution of the coupling parameters despite being conformally invariant [21].

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The perturbation theory in the GR-SI prescription becomes more complicated involving additional scalar interaction terms. These additional terms make the model non-renormalizable. However the additional terms are suppressed by Planck mass and hence the problem is not more severe in comparison to the non-renormalizability of gravity \[30, 34, 35\]. Furthermore these additional terms are irrelevant if we ignore contributions due to the added real scalar field. Hence if we confine ourselves to a study of only the standard model fields, we recover the standard perturbation theory. We clarify that the details of the perturbation expansion in our case is not same to that in \[21\]. This is mainly because in \[21\] the authors break conformal invariance spontaneously. However in our proposal, described in the next section, it is broken by the background cosmic evolution.

Another problem with the model is that one of the allowed terms in the action has to be set to zero at each order in the perturbation theory. This is required in order to break conformal invariance spontaneously \[21\]. Else it is not possible to implement the GR-SI prescription. Alternatively one needs to maintain a very small value of the coupling constant corresponding to this additional term at each order in perturbation theory \[26, 36\]. The problem is again traced to the small value of the cosmological constant in comparison to other scales in the theory and hence the problem is not solved.

In a recent paper \[30\] we have shown that the fine tuning problem of cosmological constant gets partially resolved if we add small conformal symmetry breaking terms to the action. In this case we still demand that one of the terms in the conformal action is identically equal to zero despite it being allowed by the symmetry of the theory. Once this term is set to zero the conformal action predicts identically zero cosmological constant. We can add small symmetry breaking terms. The small values of these terms are preserved in perturbation theory since they receive zero contribution from the conformal action. We have shown in \[30\] that these symmetry breaking terms lead to the observed dark energy.

In the present paper we propose a model which has local conformal invariance. In this case also we introduce terms which break conformal invariance. We show that this model naturally solves the fine tuning problem of the cosmological constant, without having to arbitrarily set any symmetry preserving terms to zero. The implications of local conformal invariance have been extensively studied in literature \[37-51\]. The implications of global scale invariance have also been investigated \[52-59\].

2 The Basic Idea

The basic idea is very simple. We assume a model which has a conformal sector. A part of the action displays local conformal invariance \[2, 12, 14, 20, 36\], whereas the remaining part breaks this invariance. We consider a simple model consisting of gravity, a scalar field, \(\chi\), and the Weyl vector meson, \(S_\mu\). The action in 4 dimensions may be written as,

\[ S = S_C + S_{SB} \]  (1)
where $S_C$ is the conformal action,

$$S_C = \int d^4x \sqrt{-g} \left[ \frac{\beta}{8\lambda^2} \tilde{R} + \frac{1}{2} g^{\mu\nu} D_\mu \chi D_\nu \chi - \frac{1}{4} g^{\mu\rho} g^{\nu\sigma} \varepsilon_{\mu\nu\rho\sigma} - \frac{\lambda}{4} \chi^4 \right]$$  \hspace{1cm} (2)

Here $\tilde{R}$ is given by,

$$\tilde{R} = R + 6 f^2 g^{\mu\nu} S_\mu S_\nu + 6 f g^{\mu\nu} S_{\mu\nu},$$  \hspace{1cm} (3)

$\varepsilon_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu$ is the field strength tensor for $S_\mu$ and $D_\mu \chi = (\partial_\mu - f S_\mu) \chi$ is the covariant derivative with $f$ being the gauge coupling. The term, $S_{SB}$, breaks conformal invariance and is of the form,

$$S_{SB} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} m^2 \chi^2 + \Lambda + \frac{1}{2} m_1^2 S_\mu S_\mu \right] + ...$$  \hspace{1cm} (4)

Here $m$, $m_1$ are mass terms and $\Lambda$ the cosmological constant. We can choose the mass parameters as small as required. The conformal sector will not generate these terms nor the cosmological constant, $\Lambda$, at any order in perturbation theory. We clarify that here we shall use the regularization which does not break conformal invariance. This is called the GR-SI prescription in Ref. [21]. This is discussed in more detail below.

The conformal action displays invariance under the transformation,

$$g_{\mu\nu} \rightarrow \Omega(x)^2 g_{\mu\nu}, \chi \rightarrow \frac{\chi}{\Omega(x)}, S_\mu \rightarrow S_\mu - \frac{1}{f} \partial_\mu \ln(\Omega(x))$$  \hspace{1cm} (5)

where $\Omega(x)$ is the transformation parameter. This invariance is broken by $L_{SB}$.

We next explain how the cosmological constant problem appears in our model. For this purpose, we set the symmetry breaking terms to zero. Let us first assume that there exists a classical solution such that $\chi = \chi_0$ where $\chi_0$ is equal to a constant. A constant value solves the equations of motion, with $S_\mu = 0$ [20, 60–62]. It has also been shown that the solution is stable under perturbations [63]. The theory does not select any value of $\chi$ due to conformal invariance. We need to choose a value consistent with observations. We choose this constant to be,

$$\chi_0 = \frac{M_{PL}}{\sqrt{2\pi\beta}}$$  \hspace{1cm} (6)

A non-zero choice of $\chi_0$ leads to a soft breaking of the conformal symmetry. This value generates all the dimensional parameters in the theory, such as the Planck mass, $M_{PL}$, the electroweak symmetry and the Hubble parameter. In this case the term proportional to $\lambda$ acts as a cosmological constant. The scalar curvature for this solution becomes,

$$R = \frac{2\lambda}{\pi\beta^2} M_{PL}^2$$  \hspace{1cm} (7)

It is clear that we need a very small value of $\lambda$ in order to be consistent with observations. Hence this parameter might require acute fine tuning at each order in perturbation theory.

Within the framework of the action under consideration, it is not clear that a fine tuning is indeed present. This is because the quantum corrections can arise only due to gravity, the Weyl
meson and the scalar field $\chi$. The corrections due to $\chi$ and $S_\mu$ would depend on the parameters $\lambda$ and $f$. The parameter $\lambda$ is extremely tiny and $f$ is unknown. Hence one can always choose it to be small enough so that no fine tuning arises. One also might have corrections due to gravity. These are also likely to be suppressed. Large contributions would arise when we add standard model fields. We shall impose conformal invariance on the standard model action also and its masses would be generated by the non-zero value of $\chi$. As we shall see below, this sector can generate large corrections to $\lambda$ at higher orders in perturbation theory. These have to cancelled at each order leading to acute fine tuning.

We next argue that this fine tuning can be removed by a suitable choice of symmetry breaking terms. The basic point is that the conformally invariant sector does not give any contribution to the symmetry breaking terms, at any order in perturbation theory. Hence we may choose $m$ to be arbitrarily small, without requiring fine tuning.

We next make a conformal transformation, Eq. 5. Under this transformation, $S_C$ remains unchanged. However $S_{SB}$ becomes,

$$S_{SB} = \int d^4x \sqrt{-g} \left[ - \frac{1}{2} m^2 \Omega^2 \chi^2 + \Omega^4 \Lambda + \frac{m^2}{2} g^{\mu\nu} (S_\mu - \frac{1}{f} \partial_\mu \ln \Omega) (S_\nu - \frac{1}{f} \partial_\nu \ln \Omega) \right]$$ (8)

We next choose the transformation parameter, $\Omega(x)$, so that the terms, proportional to $\lambda$ and $\Lambda$ cancel one another. We require,

$$\Omega^4 = \frac{\lambda \chi^4}{4 \Lambda}$$ (9)

With this reparametrization, the relevant terms in the full action can be written as,

$$S = \int d^4x \sqrt{-g} \left[ \beta \chi^2 \tilde{R} - \frac{1}{2} m^2 \chi^4 \sqrt{\frac{\lambda}{4 \Lambda}} \right] + ...$$ (10)

Now the quadratic coupling parameter is equal to $m^2 \sqrt{\lambda/\Lambda}$. Since we can choose $m$ to be arbitrarily small, the effective cosmological constant does not suffer from the standard fine tuning problem. It might be simplest to assume a value of $\Lambda$, which is not too small. A comparatively large value of $\Lambda$ does not lead to large quantum corrections in the matter sector. It will only contribute to processes involving graviton exchanges. Such quantum gravity contributions are not well understood and are likely to be suppressed. In any case the main purpose of the present paper is to show that matter contributions to the cosmological constant do not lead to fine tuning. Here we shall ignore quantum gravity corrections.

Before proceeding further, we point out that we do require the conformal invariance to be applicable even at the quantum level. This is implemented by the GR-SI prescription [8,19–22]. The conformal invariance is broken by quantum effects since the procedure of regularization requires introduction of a mass scale. In the present case, however, we maintain conformal invariance by using a dynamical mass scale, $\chi$, for regularization [8,19–22]. This is permissible as long as the classical value of the field, $\chi$, is not equal to zero. In this case conformal invariance is maintained in the full quantum theory. For completeness, we next display the action in $d$ dimensions in order to explicitly show how the mechanism may be implemented in the full
quantum theory. The conformal action in $d$ dimensions may be written as,

$$S_C = \int d^d x \sqrt{-g} \left[ \frac{\beta \chi^2}{8} \tilde{R} + \frac{1}{2} g^{\mu\nu} D_\mu \chi D_\nu \chi - \frac{1}{4} g^{\mu\rho} \varepsilon^\nu_\sigma \varepsilon^\rho_\sigma (\chi^2)^\delta - \frac{1}{4} \lambda \chi^4 (\chi^2)^{-\delta} \right]$$  \hspace{1cm} (11)

where $\delta = (d - 4)/(d - 2)$ and

$$\tilde{R} = R + 4^{d-1} g^{\mu\nu} + f^{d-2} S^\mu S_\mu.$$  \hspace{1cm} (12)

This action is invariant under the conformal transformation,

$$\chi \rightarrow \frac{\chi}{\Omega(x)}, g_{\mu\nu} \rightarrow \Omega^b(x) g_{\mu\nu}, S_\mu \rightarrow S_\mu - \frac{1}{f} \partial_\mu \ln(\Omega(x)), A_\mu \rightarrow A_\mu, \Psi \rightarrow \Psi / \Omega^c(x)$$  \hspace{1cm} (13)

where, $b = 4/(d - 2)$ and $c = (d - 1)/(d - 2)$. Here $\Psi$ is a fermion field and $A_\mu$ any additional vector field we may introduce into the model. Choosing this regularization scheme it is clear that we will not generate any terms in the symmetry breaking part of the action at any order in the perturbation theory. Hence the mechanism proposed above works in the full quantum framework.

Our mechanism is reminiscent of the standard Peccei-Quinn mechanism [64] to solve the strong CP problem. In this case the $\theta$ term in the action is cancelled by making a chiral transformation which generates an additional term due to chiral anomaly. Since we are free to make a chiral transformation the $\theta$ term does not lead to any physical consequences. However in detail the two mechanisms are different. In the present case we cancel the cosmological constant term, which explicitly breaks conformal symmetry, by making a local conformal transformation. We also point out that the symmetry preserving part of the action is not anomalous.

### 2.1 Model with two real scalar fields

We next repeat the above calculation by adding an additional real scalar field, $\phi$, which may be representative of the Higgs field. We add the following additional terms to the action,

$$S_M = \int d^d x \sqrt{-g} \left[ \frac{1}{8} \beta_1 \phi^2 \tilde{R} + \frac{1}{2} g^{\mu\nu} D_\mu \phi D_\nu \phi - \frac{\lambda_1}{4} (\phi^2 - \lambda_2 \chi^2)^2 \right]$$  \hspace{1cm} (14)

Having added these terms the fine tuning problem of the conformal sector becomes apparent. The vacuum value, $\phi_0$, of $\phi$, is expected to be of order 250 GeV. We expect the coupling $\lambda_1$ to be of order unity in order to fit the mass of the Higgs particle. Quantum corrections may, therefore, generate a large contribution to the term proportional to $\lambda$. As we have shown above, this effectively leads to a huge cosmological constant. This may be cancelled by adding suitable counterterms, leading to an acute fine tuning problem. However these additional terms do not give any symmetry breaking contributions. Hence the mass, $m$, does not require any fine tuning. We can consistently choose it to be as small as desired. We again make a suitable conformal transformation to cancel the terms proportional to $\lambda$ and $\Lambda$ among themselves. This effectively leaves behind a small contribution to the cosmological constant, corresponding to the second
term on the right hand side of Eq. [10]

We next explicitly display the classical solution, which breaks conformal symmetry after inclusion of the field, $\phi$. For $S_\mu = 0$ and $\phi$ and $\chi$ constant, the Klein-Gordon equations of motion become,

$$\frac{\beta R}{4} = \lambda \chi^2 - \lambda_1 \lambda_2 (\phi^2 - \lambda_2 \chi^2)$$

$$\frac{\beta_1 R}{4} = \lambda_1 (\phi^2 - \lambda_2 \chi^2)$$

(15)

Hence we obtain the solution [20],

$$\phi_0^2 = \chi_0^2 \left[ \lambda_2 + \frac{\lambda \beta_1}{\lambda_1 (\beta + \lambda_2 \beta_1)} \right]$$

$$R = \frac{4\lambda \chi_0^2}{(\beta + \beta_1 \lambda_2)}$$

(16)

where $\chi_0$ is a constant. The Einstein’s equations are also consistent with these. As before, the theory does not pick a value for $\chi_0$. We choose it to obtain the right value of the gravitational constant. We find,

$$\chi_0^2 = \frac{M_{PL}^2}{2\pi \beta + \beta_1 \left[ \lambda_2 + \frac{\lambda \beta_1}{\lambda_1 (\beta + \lambda_2 \beta_1)} \right]}$$

(17)

The model we have discussed is renormalizable as long as we consider quantum corrections which include only the Standard Model fields. The gravitational action breaks renormalizability. The quantum corrections involving the scalar field, $\chi$, also break renormalizability in the GR-SI prescription. This is due to the presence of higher order terms in the action obtained by expanding $(\chi^2)^\delta$ in Eq. [11] However such non-renormalizable terms are suppressed by Planck mass [34, 35] and hence the break down of renormalizability is not very serious. The problem is similar to that in the case of gravity. The Weyl meson is also intrinsically coupled to $\chi$ and gravity and hence its contributions to the action may not be renormalizable. This requires further study.

The model we have presented solves the problem of fine tuning of the cosmological constant. It has also been shown that at one loop the mass of the $\phi$ field, identified with the Higgs, is stable under quantum corrections [21, 65]. The model does contain an additional parameter, $\lambda_2$, whose value has to be chosen very small. This is related to the standard gauge hierarchy problem where one has to maintain a low electroweak scale in the presence of other very high GUT or Planck mass scales. In the toy model under consideration $\lambda_2$ may not require fine tuning at loop orders since quantum corrections to this parameter are very small. However as we add additional heavier fields to the theory which may couple to the Higgs field the standard fine tuning problem associated with gauge hierarchy problem will appear [66]. This problem has a standard solution. One has to either invoke supersymmetry or dynamical symmetry breaking or some other mechanism to stabilize the scalar sector. We do not see any problem with implementing such a mechanism within the present framework. However we postpone a
Our proposal has a sector which maintains local conformal invariance while another which breaks it. One might argue that this may create some problems in the quantum theory. The main problem associated with such a breakdown of local invariance is absence of renormalizability. This may not even be an issue for us since the gauge symmetry being broken is abelian. For example, a U(1) invariant theory which includes explicit vector meson mass terms in the action is renormalizable provide the vector field couples to a conserved current [67]. In any case, as we have discussed above, our model is not renormalizable due to the use of GR-SI prescription. However, this is not a serious problem in the sense discussed above.

We finally point out that the introduction of a Weyl meson in the theory is elegant but not necessarily required. We may instead chose a particular form of the action, discussed recently in Ref. [47]. In this case local conformal invariance is maintained by introducing just one scalar field without introducing the Weyl meson. Here the vector field is identified with $S_\mu \sim \partial_\mu \ln \chi$.

3 Including Standard Model fields

The conformal model including the standard model fields has been given earlier [25]. For completeness we present the action here in $d$ dimensions. It is given by,

$$S = S_C + S_{SM} + S_{SB}$$

(18)

Here $S_C$ is given by Eq. 2. The symmetry breaking action, $S_{SB}$, is same as that given in Eq. 4 where one may add more terms as may be required. The Standard Model action, $S_{SM}$, is given by,

$$S_{SM} = \int d^d x \sqrt{-g} \left[ \frac{\lambda_1}{4} \mathcal{H}^\dagger \mathcal{H} + g^\mu_\nu (D_\mu \mathcal{H})^\dagger (D_\nu \mathcal{H}) - \frac{1}{4} g^\mu_\nu g^{\alpha\beta} (A^i_\mu \alpha A^i_\nu \beta) 
+ B_{\mu \alpha} B_{\nu \beta} + G^j_{\mu \nu} G^j_{\mu \nu}) (\chi^2)^{2\delta - \lambda (\mathcal{H}^\dagger \mathcal{H})^2 (\chi^2)^{-\delta}} \right] + S_{fermions},$$

(19)

where $\mathcal{H}$ is the Higgs multiplet and $G^j_{\mu \nu}, A^i_{\mu \nu}$ and $B_{\mu \nu}$ and $E_{\mu \nu}$ are the standard field strength tensors for the $SU(3), SU(2)$ and $U(1)$ gauge fields. We point out that these gauge fields remain unchanged under conformal transformation. The fermion action, $S_{fermions}$, is given by

$$S_{fermions} = \int d^d x e \left( \overline{\Psi}_L i \gamma^\mu D_\mu \Psi_L + \overline{\Psi}_R i \gamma^\mu D_\mu \Psi_R \right) - \int d^d x \ e (g_Y \overline{\Psi}_L \mathcal{H} \Psi_R (\chi^2)^{-\delta/2} + h.c.),$$

(20)

where $e = \det(e_a^\mu), e_a^\mu e_b^\nu \eta_{ab} = g_{\mu \nu}, \gamma^\mu = e_a^\mu \gamma^a, e_a^\mu$ is the vielbein and $a, b$ are Lorentz indices. Here $\Psi_L$ and $\Psi_R$ represent the left and right handed projections of the fermion field and $g_Y$ is a Yukawa coupling. The gauge covariant derivatives of fermion fields are same as in the standard model. The Weyl meson does not couple to fermions [14]. It also does not couple to
the standard model gauge fields. The gauge covariant derivative of the Higgs field, however, gets a contribution from the Weyl meson. Hence \( D_\mu \mathcal{H} = (D'_\mu - f S_\mu) \mathcal{H} \), where \( D' \) represents the standard model gauge covariant derivative. In Eq. 20 we have shown only a representative term of the fermion field. Additional terms corresponding to different Yukawa couplings and different families can be added analogously.

It is clear from the action that the mechanism presented in section 2 can be implemented in this case also. We again make a suitable conformal transformation to cancel the cosmological constant with the \( \lambda \chi^4 \) term in the action. The classical solution involving the Higgs multiplet and \( \chi \) is similar to that given section 2.1. The solution is given explicitly in [36]. The solution generates a non-zero classical value for \( \chi \) and the Higgs field. If we are only interested in the standard model processes we can simply ignore the Weyl meson and set \( \chi \) equal to its classical value in the action. The model reduces to the standard model. These additional fields, \( \chi \) and the Weyl meson, give rise to the dark components. We have already demonstrated in this paper that the \( \chi \) field consistently accommodates the cosmological constant. The Weyl meson is a potential candidate for dark matter since it does not couple to the standard model fields besides the Higgs [14]. Its implications as a dark matter candidate have been explored in the literature [36,46,49,62]. If we assume that the gauge coupling \( f \) is of order unity then the Weyl meson mass is of order of the Planck mass. In applications to collider and cosmology a wide range of this coupling parameter has been explored in earlier papers [36, 46, 49, 62]. However this requires further study. In particular, earlier papers did not take into account the symmetry breaking sector we have introduced in this paper. Furthermore it may be interesting to consider other dark matter candidates within the framework of our model.

4 Conclusions

We have presented a simple mechanism which solves the fine tuning problem of the cosmological constant. It is based on local conformal invariance but also requires terms which break this symmetry. We show that by a suitable conformal transformation we can cancel the cosmological constant in the symmetry breaking sector with a quadratic scalar field term in the symmetry preserving sector. This leaves behind a small term which effectively leads to the physical cosmological constant. Its small value is maintained by choosing very small parameters in the symmetry breaking sector. These do not require fine tuning since they do not get any contribution from the symmetry preserving sector at loop orders. The model is very economical since it only introduces one additional real scalar, \( \chi \), and the Weyl meson. It is also possible to construct a model without the Weyl meson. This will involve only one additional real scalar field besides the standard model fields. The model uses the GR-SI prescription for renormalization. The perturbation theory in this case is more complicated. However if we ignore the contributions due to \( \chi \) and the Weyl meson, it reduces to the standard perturbation theory. The contributions due to \( \chi \) break renormalizability of the model. However such terms are suppressed by Planck mass and comparable to contributions due to quantum gravity. Hence this problem is only as serious as the problem of non-renormalizability of gravity. Our proposal solves the fine tuning problem of the cosmological constant due to potentially large contributions arising from the
matter fields.

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