Observational Implications of Cosmological Event Horizons

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ABSTRACT

In a universe dominated by a small cosmological constant or by eternal dark energy with equation of state $w < -1/3$, observers are surrounded by event horizons. The horizons limit how much of the universe the observers can ever access. We argue that this implies a bound $N \sim 60$ on the number of e-folds of inflation that will ever be observable in our universe if the scale of the dark energy today is $\sim (10^{-3} \text{eV})^4$. This bound is independent of how long inflation lasted, or for how long we continue to observe the sky. The bound arises because the imprints of the inflationary perturbations thermalize during the late acceleration of the universe. They “inflate away” just like the initial inhomogeneities during ordinary inflation. Thus the current CMB data may be looking as far back in the history of the universe as will ever be possible, making our era a most opportune time to study cosmology.

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1 Introduction

Cosmological observations suggest that the expansion of the universe may be accelerating [1, 2, 3]. This may indicate that the universe contains a dark energy component with equation of state \( w = p/\rho \lesssim -2/3 \), comprising as much as 70% of the critical energy density, \( \rho_c \sim (10^{-3} eV)^4 \). The “usual suspects” for dark energy are either a cosmological constant or a time-dependent quintessence field [4, 5]. If the dark energy includes an eternal component obeying \( w < -1/3 \), the universe will continue to accelerate indefinitely. Any observer in it is surrounded by an event horizon. This limits the region of the universe accessible to observation, and raises interesting conceptual problems [6, 7, 8, 9].

Recently, the authors of [10] have argued that there is an incompatibility between the assumptions of the quantum field theory description of very long inflation and the bound on the number of states in the Hilbert space of an asymptotically de Sitter universe. Their conclusion is that in a universe dominated at late times by a positive, stable, cosmological constant, there is an upper limit on the number \( N \) of e-folds of inflation that can be described by a conventional quantum field theory. If inflation were longer, it would deposit so much matter in the universe that it would collapse to a big crunch at a time after reheating. However, long inflation was subsequently analyzed in [11], which found no bound on \( N \).

In this note we first briefly re-examine the arguments of [10]. Adhering to the covariant entropy bounds [12, 13] as did [11], we do not find a bound on how many e-folds the early inflation could have lasted in a universe which starts to accelerate forever. However, we do find a bound \( N \sim 60 \) on the number of e-folds that will ever be observable. If our universe accelerates forever, we will never see past the last 60 e-folds or so.

Further, we find that a universe which transitions to an eternally accelerating phase will contain the most information about the inflationary perturbations at the epoch of transition. Later observers will be able to observe less and less about the inflationary phase, because the fluctuations generated during inflation will cease reentering the horizon, and those that did reenter will be evicted again. The overall amplitude of the CMB will redshift, and more significantly the pattern of anisotropies will freeze in such a way that little new information will become available. Eventually, the CMB will redshift to a point where it is permanently contaminated by cosmological Hawking radiation.

Therefore, the current cosmological observations may already be looking as far back in the early universe as may ever be possible, making this a most opportune time to study cosmology. This provides an interesting new twist to the “Why Now?” problem: Why is now (± few current Hubble times) the best time to observe the signatures of early universe physics?

2 Counting E-folds

We first briefly review the argument of [10]\(^1\). Imagine that the early universe begins as a flat, inflating universe with Hubble parameter \( H_i \). This is a good approximation soon after the onset of inflation. Consider one initial Hubble patch, with a volume \( \sim 1/(H_i)^3 \). After \( N \) e-folds of inflation, the volume of this inflating patch has increased by a factor of \( e^{3N} \).

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\(^1\)Here we focus only on the case \( \kappa = p/\rho = 1 \) of [10], which gave the weakest bound.
Suppose that inflation then ends and the universe reheats, and the decay of the inflaton produces some local entropy density $\sigma$. In units where the Planck mass is set to unity, and assuming that the total entropy $S_r$ generated at reheating is simply the product of the entropy density and the volume, one obtains:

$$S_r = \sigma \frac{e^{3N}}{(H_i)^3}. \quad (1)$$

If there is a positive cosmological constant $\lambda$, the future of the universe will be either a big crunch or a de Sitter space with Hubble parameter $H_0 \sim \sqrt{\lambda}$, depending on the amount of energy released at the end of inflation. If the universe is to avoid a big crunch, [10] argues that the total entropy generated at the end of inflation must be bounded by the Gibbons-Hawking entropy of the final de Sitter space, which is given by the horizon area in Planck units. At sufficiently late times, this is the same as the horizon area of a single post-inflationary Hubble patch: $S_{GH} \simeq 1/(H_0)^2$. Requiring $S_r \lesssim 1/(H_0)^2$ yields

$$N \lesssim \frac{2}{3} \ln \frac{H_i}{H_0} \sim 85, \quad (2)$$

when $\sigma \sim H_i$, $\kappa = 1$, $H_i \lesssim 10^{14}GeV$ and $H_0 \sim 10^{-33}eV$ [10]. Varying the parameter $\kappa$ one finds qualitatively similar bounds. When the inequality (2) is violated, [10] conclude that the universe would eventually end in a big crunch, despite the fact that standard FRW evolution would indicate otherwise.

Figure 1: Causal patch of an observer in a universe where inflation and reheating are followed by eternal accelerated expansion. The symbols designate: PH and FH - past and future sections of the event horizon, AH - apparent, or Hubble, horizon, RS - reheating surface, and LS - a future oriented light sheet, which intersects both the event horizon and the infinitely inflated future. Black arrows are the worldlines of the entropy released at the end of inflation.

The situation is shown in Fig 1., where we depict the conformal spacetime diagram for a universe which undergoes early inflation, reheats and goes through the era of radiation and matter domination, and finally accelerates forever. Note that the entropy generated by reheating is uniformly deposited along the reheating surface RS, which corresponds to a
spatially flat slice at the moment when inflation ends, at least in a region of size $\sim e^N/H_i$. The size of this region can greatly exceed the Hubble scale $1/H_0$ during the late acceleration era if $N$ is large. After a long inflation, an initial Hubble patch will expand to fill a volume much larger than the volume inside the event horizon, and therefore the reheating surface will extend well past the future Hubble horizon $1/H_0$ and the event horizon (see Fig. 1.). Computing the entropy as in equation (1) corresponds to counting all the entropy along the reheating surface, including the parts both inside and outside the causal patch. Thus if inflation is long the entropy (1) may exceed greatly the entropy that an observer in an eternally accelerating universe will ever see. Banks and Fischler argue [10] that this implies an incompatibility between the assumptions of the conventional QFT picture of inflation and the bound on the number of states in the Hilbert space of an asymptotically de Sitter universe. They claim this means that the predictions about observable consequences of models with a large number of e-folds cannot be trusted.

Here we take the point of view that in backgrounds with horizons, the entropy contents of the spacetime conform to the covariant entropy bounds of [12, 13]. With this assumption, the existence of a horizon constrains only the entropy inside the final Hubble volume to be less than the area of its horizon. It does not restrict the entropy deposited outside of it. This bound is clearly not violated in our universe, as the entropy of the observable universe is many orders of magnitude below the horizon area. To an observer inside the causal patch in Fig. 1., the portion of the reheating surface RS which is outside the event horizon lies beyond her causal future, in a sense. She will never see that inflation ended there and that any entropy has been released, or for that matter, that inflation even happened there.

Outside of the event horizon, then, we should constrain the entropy on the part of the reheating surface there using the lightsheet labelled LS in Fig. 1. According to the covariant bound [13], the entropy that crosses any segment of a lightsheet is bounded by a maximum of the area along this segment, in Planck units. However, because LS intersects the infinitely inflated future at the top of the diagram, and so its maximal area diverges, the covariant entropy bound does not give an interesting constraint. It can easily accommodate the entropy released after arbitrarily long inflation. For a more detailed discussion of these issues, see [11]. Problems with adding up entropy stored on large and arbitrarily chosen spacelike surfaces have been noticed before [12, 13, 14], and they are consistently resolved by the application of the covariant bound.

3 How Many E-folds Can We See?

In this section we argue that if in the future the spacetime undergoes eternal acceleration, the event horizon limits the total number of e-folds that we can ever observe to the last $N \sim 60$ or so. Recall that we study inflation by observing temperature and density contrasts on the sky. The contrasts at larger scales correspond to fluctuations that were produced earlier in inflation. In order to solve the horizon and flatness problems, inflation must have lasted at least $N \sim 60$ e-folds [15]. Quantum fluctuations during this stage are imprinted on the curvature via a mechanism closely related to gravitational particle production, and are

\[2\text{We agree, however, that it is unclear how to think about the entropy deposited in this large volume from the viewpoint of any given observer.}\]
subsequently stretched by inflation to super (Hubble) horizon scales [16, 17]. Once there, they “freeze out”, i.e. their amplitude approaches a constant set by the horizon crossing condition, and their wavelength scales with the particle horizon, $\lambda(t) = \lambda_0 a(t)/a_0$. What happens to them next depends on the subsequent evolution of the universe. If inflation ends and reheating occurs, the Hubble horizon starts growing linearly in time, while the wavelength stretches more slowly, as $\lambda \sim a(t)$. If the vacuum energy is zero, this situation will persist indefinitely, and after a long enough time the Hubble horizon catches up with the perturbation (see the left panel of Fig. 2), after which the perturbation “melts”; i.e., it begins to oscillate and to seed structure formation via the Jeans instability [16, 17]. A

Figure 2: Evolution of the wavelengths of some typical inflationary perturbations in the causal patch in a universe without (left panel) and with (right panel) event horizons. In the left panel, all fluctuations eventually reenter the Hubble horizon. In the right panel, in the case a), a fluctuation is stretched outside of the Hubble horizon during inflation, remains there for a time, then reenters during a matter dominated era after inflation, and eventually gets expelled out of the horizon once more during the final stage of acceleration. In the case b), the fluctuation could have reentered about now, but the late acceleration pushes it back out. In the case c), the late acceleration prevents the fluctuation from ever reentering the Hubble horizon.

patient observer in such a universe would be able to see arbitrarily far back into inflation: the longer she waits, the earlier the fluctuations she sees were created.

However, if at some time the post-inflationary universe begins to accelerate and continues to do so forever, there will be event horizon as in the right panel of Fig. 2. In this case a (huge!) part of the global spacetime is permanently inaccessible to any given observer. The evolution of inflationary perturbations is very different in this case. Depending on when they are produced, inflationary fluctuations could either (see the right panel of Fig. 2): a) reenter the Hubble horizon during matter domination, and then eventually be expelled again in the future, b) in the marginal case, have a wavelength which equals the Hubble horizon size at about the time when the universe begins to accelerate again, or c) never reenter, and remain outside the Hubble horizon forever after their eviction from it during early inflation.

Inflationary fluctuations that reenter the horizon produce small curvature perturbations on the background geometry, generating a distribution of gravitational potential wells. As a result, an observer can gain information about inflation by examining the structures which
form by the accretion of matter in these gravitational wells, and by observing anisotropies induced by these wells in the temperature of the thermal photons released during reheating.\(^3\) As time goes on, an observer in an accelerating universe will notice a gradual loss of the information about inflation. She will observe a lack of new structures at the largest scales, because the inflationary fluctuations stop reentering after the onset of late acceleration. Further, she will notice that the structures that have already begun forming start to disperse, as fluctuations at sub-horizon scales get stretched out to larger and larger distances. Eventually all the inflationary fluctuations which re-entered during radiation and matter domination will be pushed out of the Hubble horizon, whose interior will be smoothed out again (at least on large scales).

However, the photons which comprise the CMB originate on the slice (i.e. a sphere) of the last scattering surface (RS) which is separated from the observer by null geodesics (labelled PT, for photon trajectory, in Fig. 2.). The inflationary fluctuations are imprinted on them en route to the observer via the Sachs-Wolfe effect, and appear as a distribution of hot and cold spots on the last scattering surface. In a decelerating universe, the radius of this last scattering sphere grows without bound, the pattern of spots changes, and new information about inflation continues to become available over time. Eventually, if one continued to observe the pattern of anisotropies in the CMB, the entire history of the inflationary period would (in principle) be available.

In a universe which accelerates, the last-scattering sphere asymptotes to the size of the event horizon at the time of last scattering, which is finite. As the acceleration continues, waiting a given period of time will correspond to a smaller and smaller change in the size of the last-scattering sphere. Therefore, the pattern of anisotropies in the CMB will “freeze” after the transition to future acceleration, first on the largest scales, and then on shorter and shorter scales. Continuing to observe after the beginning of the late acceleration will not reveal any information about periods of inflation earlier than those that have already been seen, and will at best slightly improve the data on the already visible period.\(^4\) The acceleration freezes an ever-fainter image of one slice of the last scattering surface on the sky, for a very long time.

Eventually, however, even this information will be erased. Spacetimes with event horizons contain Hawking particles, and as the cosmological expansion advances, the CMB cools until it reaches a point where the number of CMB photons counted by an observer drops below the number of Hawking photons. After this time, any information in the CMB will be masked by the “noise” in this cosmological Hawking radiation. Asymptotically the bath of Hawking particles will completely overwhelm the CMB. Some implications of the loss of a record of the last stages of inflation for astronomy have been discussed in \([18, 19, 20]\).

Let us now quantify our bound. First, we recall the derivation of the minimum number of e-folds necessary to solve the horizon and flatness problems \([15]\). Let us again assume that we begin with one Hubble patch of homogeneous space. Inflation must then produce a sufficiently large number \(N\) of e-folds such that this initial patch evolves into a region the size of the present Hubble horizon size, \((H_0)^{-1} \sim (10^{-33} \text{eV})^{-1}\). The wavelengths of perturbations

\(^3\)For simplicity we ignore the difference between the reheating surface and the last scattering surface here.

\(^4\)We would like to thank Gil Holder for discussions on this point.
grow in time according to
\[ \lambda(t) = \lambda_0 \frac{a(t)}{a(t_0)}. \] (3)

Taking \( t_0 \) to be \( O(\text{today}) \) \( (t_0 \sim 10^{10} \text{ years}) \), we are interested in the largest scale observable now, namely\(^5\) \( \lambda_0 = 1/H_0 \). A horizon scale perturbation originated during inflation at some time \( t_b < t_0 \), when its wavelength was the inflationary Hubble size, \( \lambda(t_b) = 1/H(t_b) \). Hence,
\[ a(t_b)H(t_b) = a_0H_0. \] (4)

Equation (4) is the usual horizon crossing condition [17] in a slightly unconventional form. Approximating the inflating phase as de Sitter space with a constant Hubble scale \( H_i \) and using the flat slicing, we have \( a(t) = a_e \exp(H_i(t - t_e)) \) for times during inflation, where the subscript \( e \) refers to the end of inflation. Evaluating this at a time \( t_b \) during inflation and substituting into (4) yields
\[ N \equiv H_i(t_e - t_b) = \ln \left( \frac{a_eH_i}{a_0H_0} \right). \] (5)

After inflation, the universe grew by a factor of about \( a_0/a_e \sim T_e/T_0 \), where \( T_e \) is the reheating temperature and \( T_0 \sim 10^{-3} eV \) the current CMB temperature. Taking this ratio to be about \( 10^{26} - 10^{28} \) and the scale of inflation to be \( H_i \lesssim 10^{14} GeV \), one finds \( N \sim 60 \), with some sensitivity on the reheating temperature, the scale of inflation et cetera, which we will ignore here (see [15]).

![Figure 3: On the left, evolution of the comoving Hubble scale \( a(t)H(t) \) for a universe which inflates, followed by radiation and matter domination; on the right, the same graph for a universe that enters a late-time accelerating phase.](image)

To make this equation more transparent, we plot the comoving Hubble scale \( a(t)H(t) \) for a universe without any late epoch of cosmic acceleration in the left panel of Fig. 3. Initially it grows exponentially because of inflation. Subsequently, it decreases as a small negative power of \( t \), because after reheating the universe decelerates. For example, if the universe is dominated by matter with an equation of state \( p = w\rho \), \( a(t)H(t) \) scales as \( t^{-(1+3w)/(3(1+w))} \).

\(^5\)We denote quantities evaluated at time \( t = t_0 \) with a subscript 0.
which is decreasing for \( w > -1/3 \). As \( a(t)H(t) \) decreases, it scans through more and more values of the comoving momentum \( k = 1/\lambda_0 \), which means that those scales reenter the horizon. Thus, regardless of how large a scale \( \lambda_0 \) is, if the universe decelerates forever and \( a(t)H(t) \) continues to decrease, at some time this scale will reenter the Hubble horizon.

On the other hand, if the universe accelerates in the future, the comoving Hubble scale \( a(t)H(t) \) begins to grow again at late times, as we can see by setting \( w < -1/3 \) in the scaling law given above. At a time \( t_f \), where \( f \) stands for final, when the comoving Hubble scale equals its value at reheating, the very last perturbation generated during inflation will be pushed back out of the horizon. Indeed, after \( t_f \) no inflationary perturbations will remain in the Hubble horizon and no new structure will form from the seeds generated by inflation\(^6\) (see the right panel of Fig. 3.). The time \( t_f \) is defined by the equality

\[
a(t_f)H(t_f) = a(t_e)H_i, \tag{6}
\]

where \( t_e \) is the time at reheating. The value of \( t_f \) depends on the equation of state of the dark energy in a way we calculate below.

As we have mentioned above, spacetimes with event horizons contain Hawking particles, with a characteristic temperature\(^7\) given by the Gibbons-Hawking formula \( T_H = H/2\pi \). This temperature does not redshift in the usual way, because the Hawking radiation is continuously replenished by quantum fluctuations, rather than being a remnant of an earlier hot big bang. If the universe is accelerating, the CMB temperature \( T_{CMB} \) will eventually redshift to a point where it is equal to \( T_H \sim H(t) \). If \( T_e \) was the reheating temperature, under adiabatic evolution the temperature at any later time is related to it by \( T_{CMB}(t) = T_e a(t_e)/a(t) \). Thus the temperatures of the CMB and the Hawking particles obey \( T_{CMB}(t_T) = T_H \) at a time \( t_T \) when

\[
a(t_T)H(t_T) = a(t_e)T_e. \tag{7}
\]

If reheating were perfectly efficient, the reheating temperature would be related to the Hubble scale at the end of inflation by \( T_e \sim \sqrt{H_i} \) (recall that we have set the Planck mass equal to unity). Thus, since \( H_i < 1 \), \( T_e > H_i \), and \( t_T > t_f \). In practice, however, the reheating temperature is model dependent\(^8\). It is possible that there are some models where the ordering of \( t_f \) and \( t_T \) is reversed. But in this case, for \( t > t_T \) the CMB photons will be outnumbered by Hawking photons, and it would be impossible to extract any information about inflation from their fluctuations.

Therefore if the cosmic acceleration never ends, only those inflationary fluctuations with comoving momenta in the interval \( a(t_b)H(t_b) \leq k \leq a(t_e)H(t_e) \) will ever be observable. An observer will never be able to see much past the last 60 e-folds of inflation, however patient she may be. Further, the information which was accessible to her will be lost after the time \( t_T \), the value of which depends on the equation of state of the dark energy (\( t_T \sim t_f \)). It can be

\( ^{6} \)In reality, the information about the primordial inflation encoded in the shortest scales generated during inflation will already be strongly contaminated by the nonlinear effects occurring in the intervening period between \( t_e \) and \( t_f \), such as galaxies, clusters etc. We are ignoring this contamination here.

\( ^{7} \)This is certainly correct for a positive cosmological constant (\( w = -1 \)). For quintessential universes, we believe there is a similar effect \( [7] \), but we are not aware of a precise calculation of it.

\( ^{8} \)As a result, the right-hand side of the equation (7) should really read \( a(t_T)T_k \), and this quantity may evolve slightly differently. However, the differences will all be model dependent and confined to short scales, and so we will ignore this here.
found by solving (7), eqs. (4) and (5), and the scaling \( a(t)H(t) \sim a_0 H_0 (t/t_0)^{- (1+3w)/(3(1+w))} \) when \(-1 < w < -1/3\):

\[
t_T \sim 10^{78(1+w)/(1+3w)} t_0 .
\]

In the limit \( w \to -1/3 \) the time diverges, as expected since for \( w \geq -1/3 \) the event horizon and the Hawking particles disappear, and the information about early inflation survives and remains available to an investigation by a patient observer. The limit \( w \to -1 \) is simpler to determine by directly substituting \( a(t_T)H(t_T) = a_0 H_0 \exp(H_0 t_T) \), which yields

\[
t_T \sim \frac{60}{H_0} .
\]

Hence if the dark energy is a small cosmological constant, the record of early inflation will be lost in about a trillion years (see also [18, 19, 20] for astronomical implications of these time scales).

4 Summary

In closing, we note that because of the Grishchuk-Zel’dovich effect [22] the perturbations at superhorizon scales may have a weak effect on observable structures. The absence of large perturbations at the current horizon scale implies that the momentum space scale below which the perturbation power spectrum may change significantly must be about 500 times lower than the current Hubble scale \( H_0 \) [23, 24, 25]. A more recent analysis of the WMAP data improves this limit to be about 400 times lower than the Hubble scale [26]. This allows us to probe the period of inflation slightly more than 60 e-folds from the end, giving us some information about perhaps 8 more.

To conclude, having assumed the covariant entropy bounds [12, 13] we have found no limitations on the number of allowed e-folds of inflation in universes dominated at late times by dark energy. However, we have found that eternal dark energy with \( w < -1/3 \) does prevent us from ever measuring inflationary perturbations which originated before the ones currently observable. Further, it slowly degrades the information stored in the currently observable perturbations. This allows us to re-formulate the “Why Now?” problem in a novel and interesting way: why are we living in the time at which we can see back to the earliest scales? In other words, why would the number of e-folds required to solve the horizon problem and explain the observed large scale homogeneity, isotropy and flatness of the universe also be the maximum number of e-folds which we will ever be able to observe?

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