Efficient Pricing Technique for Resource Allocation Problem in Downlink OFDM Cognitive Radio Networks

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Abstract. In this paper, the problem of resource allocation in OFDM-based downlink cognitive radio (CR) networks has been proposed. The purpose of this research is to decrease the computational complexity of the resource allocation algorithm for downlink CR network while concerning the interference constraint of primary network. The objective has been secured by adopting pricing scheme to develop power allocation algorithm with the following concerns: (i) reducing the complexity of the proposed algorithm and (ii) providing firm power control to the interference introduced to primary users (PUs). The performance of the proposed algorithm is tested for OFDM-CRNs. The simulation results show that the performance of the proposed algorithm approached the performance of the optimal algorithm at a lower computational complexity, i.e., $O(N \log N)$, which makes the proposed algorithm suitable for more practical applications.

1. Introduction

Studies carried out by the Federal Communications Commission (FCC) have shown that conventional spectrum allocation technique is becoming insufficient to address the rapid development of wireless technologies, and there is a need for the development of an open spectrum allocation methodology to compensate spectrum underutilisation [1]. To meet this important demand, dynamic spectrum access (DSA) and cognitive radio (CR) have been proposed as smart techniques to resolve the problems related to the fixed spectrum approach. CR can be defined as a system, in which wireless devices have the capability to sense the spectrum which is belong to primary user (PU) and opportunistically access any unused bands based findings. However, the generated interference from CR-to-PU band must be at or below a defined threshold. OFDM technique is seen as a promising candidate for cognitive radio networks (CRNs) due to its reliability and flexibility in allocating the available resources among CRs [2-4]. In OFDM-based CRNs, the CR and PU band exist side by side, which results in mutual interference. This mutual interference is considered as a preventive factor that affects the performance of both networks [5]. The resource allocation problem in downlink non-cognitive radio networks has been widely examined, e.g., [6], [7]. However, the use of a conventional water-filling algorithm is not sufficient in case of CRNs, and an additional constraint must be proposed in the optimisation problem to control any negative effects of generated interference from CR to PU.

The problem of resource allocation in the context of multicarrier CRNs (MC-CRNs) has garnered much interest in the literature; see for example [9-10]. In contrast, game theory and supermarket theory have been used to provide efficient spectrum sharing among CRs and PUs. This is because such
techniques can properly define interaction and competition among players [11]. Moreover, there are a number of studies adopted game and supermarket theory in MC-CRNs to address the problem of resource allocation, as in [12–15]. In [12], a spectrum monopoly-market scenario based on non-cooperative game theory has been proposed for OFDM-based CRNs. The authors used a non-cooperative auction game to determine the optimal power for CRs in a leasing market scenario. However, convergence of the proposed game to the Nash equilibrium (NE), is a bit slow. A more complex scenario in MC-CRNs based on non-cooperative games has been considered in [13]. Instead of single-cell scenario, the authors proposed a resource allocation algorithm in multiuser, multi-cell MC-CRNs that proves to be a notable contribution compared to other studies in the literature. Resource allocation in MC-CRNs based on market games has been considered in [14]. Unlike [13], this study includes simple optimisation problem and adopts Blotto game to allocate resources among players while produces fair allocation in both uplink and downlink with fast convergence to the NE point.

In this paper, we consider more practical downlink set-up in OFDM-CRNs by adopting pricing scheme in the surplus function. The main objective is to maximise the downlink capacity of the CRNs at a lower computational complexity compared to that in [5]. Furthermore, the main contribution of this paper is that the pricing scheme adopted to provide flexibility in managing harmful interference to PUs rather than increasing the revenue of PUs. Hence, CRs have more freedom to utilise both active and non-active PU bands, considering that the total interference is at or below the defined threshold.

The remainder of this paper is organised as follows. In Section II, the system model, including the OFDM-based CR, system setting and interference model is presented. In Section III, we develop problem formulation. The downlink market model based on pricing scheme is illustrated in Section IV and includes subcarrier allocation algorithm, market power allocation, Nash equilibrium concept and the proposed power allocation algorithm. Simulation results of the proposed algorithm and the comparison study are given in Section V. Finally, we conclude this paper in Section VI.

### 2. System Model

#### 2.1. OFDM-based Cognitive Radio: System Setting

In this paper, a hybrid network in an underlay spectrum-sharing scenario has been considered. A network composed of one CRN-based OFDM coexists with primary user network (PUN) in the same geographical area, and sharing same spectrum, as shown in Figure 1 (a). Single-cell, OFDM-based CR scenario has been assumed in this research. CRN consists of M CRs, denoted by \( m = \{1, 2, ..., M\} \), sharing licensed band with L PUs. Moreover, the spectrum band is divided into N subcarrier, denoted by \( N = \{1, 2, ..., N\} \), with a bandwidth represented by \( \Delta f \). The distribution of the frequency among users is supposed to be a side-by-side distribution, as shown in Figure 1 (b).

The frequency bands, denoted as \( B_1^{PU}, B_2^{PU}, ..., B_L^{PU} \), are utilised by PUs, i.e., active band, while the remainder of the bands represent the non-active bands, where PUs are not transmitting. The channel gains of the links shown in Figure 1 can be defined as follows. \( g_{ij}^{PU} \) denotes channel gain of interference link from CRBS to the \( i \)th PU on the \( j \)th subcarrier. The superscript \( (c) \) refers to CR. The channel gain of interference link from the \( i \)th PU’s transmitter to the \( m \)th CR over the \( j \)th subcarrier is denoted by \( y_{j,m}^{PU} \), where the superscript \( (P) \) refers to PU. Finally, \( h_{i,m} \) denote the \( i \)th subcarrier fading gain from the CRBS to the \( m \)th user.
2.2. Interference Model

2.2.1 Interference generated by CR’s signal

A common problem in CR hybrid networks is the generated interference from CRs to PUs. Hence, interference must be well controlled to avoid an unacceptable performance degradation in primary user network (PUN) to achieve feasible coexistence between CRs and PUs. The total available bandwidth is divided into $N$ OFDM subcarriers, as shown in Figure 1 (b). The power spectrum density of the $i^{th}$ OFDM subcarrier can be defined as [9, 10]

$$
\phi_i(f) = P_i T \left( \sin \frac{\pi f T}{\pi T} \right),
$$

where $T$ is the OFDM symbol duration and $P_i$ is the total power produced by the $i^{th}$ subcarrier. The interference generated by the $m^{th}$ CR on the $i^{th}$ subcarrier to the $l^{th}$ PU, denoted by $I^{PU}_{m,i} (d_i, P_i)$, can be defined as the integral of power spectrum density (PSD) of the $i^{th}$ subcarrier across the $l^{th}$ PU band. Mathematically speaking, the interference introduced by the CR’s signal can be modelled according to [9]

$$
I^{PU}_{m,i} (d_i, P_i) = \int_{d_i - r_i/2}^{d_i + r_i/2} |g_{i,l}(f)|^2 \phi_i(f) df = P_i \Omega_i,
$$

where $d_i$ is the distance between the $i^{th}$ subcarrier and the $l^{th}$ PU’s spectrum band, i.e., $B_l$, and $\Omega_i$ is the interference component of the $i^{th}$ subcarrier.
2.2.2 Interference generated by PU signal

The interference generated by the lth PU to the mth CR on the ith OFDM subcarrier, denoted by $I_{l,j}^{CR_m}$, can be defined as the integral of the PSD of the lth PU over the ith subcarrier which can be modelled according to [9]

$$I_{l,j}^{CR_m} = \int_{f_{d,j} - M/2}^{f_{d,j} + M/2} |Y_{l,j}|^2 \phi_l(f) df,$$

where $\phi_l(f)$ is the power spectrum density of the lth PU signal.

3. Problem Formulation

3.1. Protecting PUs

CRs are allowed to share the spectrum with PUs in underlay scenario, and this could result in poverty in the performance of PUN. The expected degradation can be avoided by introducing stringent interference and power constraints. Moreover, adopting pricing techniques can also be used to maintain good QoS in primary network. Let $I_{th}$ represents the maximum tolerable interference by PU and that $\bar{P}$ is the upper limit power budget constraint for CR across all subcarriers. Let $s_{i,m}$ be the subcarrier indicator allocation, where $s_{i,m} = 1$ if and only if the given subcarrier belongs to the mth user and $s_{i,m} = 0$ otherwise. Moreover, the aggregate interference at the lth PU can be mathematically defined according to (4).

$$\sum_{m=1}^{M} \sum_{i=1}^{N} s_{i,m} p_i^m \Omega_i \leq I_{th}, \forall l \in \{1,2,...,L\}. \tag{4}$$

Furthermore, the transmission power for all CRs in the networks should be less than the maximum power budget of CRBS, i.e., $\bar{P}$. Therefore, the total power constraint can be represented mathematically as (5)

$$\sum_{m=1}^{M} \sum_{i=1}^{N} s_{i,m} p_i^m \leq \bar{P}, \tag{5}$$

If both interference and power constraints are satisfied, then CRs can use the spectrum of PUN and thus increase the spectral efficiency.

3.2. The Optimization Problem

Assuming that $p_i^m$ is the transmit power of the mth CR on the ith OFDM subcarrier. Therefore, the signal-to-interference-plus-noise ratio (SINR) for the ith subcarrier can be formulated as

$$\gamma_i^m = \frac{P_i^m |h_{i,m}|^2}{\delta_{AWGN} + \sum_{l=1}^{L} I_{l,i}^{CR_m}}, \tag{6}$$

The total interference can be calculated according to $I_{Tot} = \delta_{AWGN} + \sum_{l=1}^{L} I_{l,i}^{CR_m}$, where $\delta_{AWGN}$ is the mean variance of the additive white Gaussian noise (AWGN) and $I_{l,i}^{CR_m}$ is the interference introduced by the lth PU to the mth CR on the ith subcarrier. Furthermore, the interference factor, i.e., $I_{l,i}^{CR_m}$, which is presented in (3), is assumed to be the superposition of a large number of independent random variable components, i.e., $\sum_{l=1}^{L} I_{l,i}^{CR_m}$. Thus, using central limit theorem, the interference factor ($I_{l,i}^{CR_m}$) can be model as AWGN. Note that this assumption is used different works in the literature [4, 9]. Therefore, the transmission rate of the ith subcarrier utilised by the mth CR can be modelled as follows:
\[ R_i^m = \Delta f \log_2 (1 + \gamma_i^m). \]  \hfill (7)

The main objective is to maximise the total capacity of the CRN by considering interference constraint in (4) and total power constraint in (5). Hence, the problem of resource allocation in OFDM-based downlink CRNs can be formulated as follows:

\[
\begin{align*}
\text{P1:} & \quad \max_{p_i^m} (R_i^m), \\
\text{s.t.} & \quad C1: s_{i,m} \in \{0,1\}, \quad \forall i, m, \\
& \quad C2: \sum_{m=1}^M s_{i,m} \leq 1, \quad \forall i, \\
& \quad C3: \sum_{m=1}^M \sum_{i=1}^N s_{i,m} p_i^m \gamma_i^m \leq I_{th}, \forall l \in \{1,2,\ldots,L\}, \\
& \quad C4: \sum_{m=1}^M \sum_{i=1}^N s_{i,m} p_i^m \leq \bar{P}, \\
& \quad C5: p_i^m \geq \bar{P} \quad \forall n \in \{1,2,\ldots,N\}. 
\end{align*}
\]  \hfill (P1)

C2, in (P1) ensures that any given sub-channel should be allocated to one and only one user, and \( \bar{P} \) in C5 refers to the minimum transmission power for CR. The optimisation problem in (P1) includes both binary variables, e.g., C1 in (P1), and continuous variables. Thus, it is a hard problem to solve. Moreover, the maximum data rate in downlink set-up is achieved if each subcarrier is allocated to the user with the best channel gain for that subcarrier, as shown in [7]. Moreover, in the downlink CRN, the CRBS has a common interference factor for all of the available CRs in the network, i.e., the value of the interference component (\( \Omega_i^l \)) is independent of the CR. Consequently, the proof presented in [7] is valid for the CRN scenario as well.

4. Downlink Market Model

4.1. Definition and Subcarrier Allocation

The joint solution to the problem of resource allocation (RA) in multicarrier-based CRNs is, in general, an NP-hard problem. Instead, the problem can be decomposed and solved in two steps, subcarrier and power steps, by many suboptimal algorithms presented in the literature, e.g., [16]. Once the subcarrier is allocated in the first step, the problem of RA can then be virtually addressed as a single-user optimisation problem. A common subcarrier allocation algorithm is adopted, similar to [5], to allocate the subcarrier to CRs, as shown in Algorithm 1.

| Algorithm 1. Subcarrier to CR Allocation |
|-----------------------------------------|
| 1- Initialisation Step: \( s_{i,m} = 0 \) \quad \forall i,m \ |
| 2- SA Step: \quad For \( i = 1 \) to \( N \) do \ |
| \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad m = \arg \max_{m} \{h_{i,m}\}; \quad s_{i,m} = 1 \ |
| End \ |

Moreover, the channel gain can be determined from the subcarrier allocation steps according to
\[ h_i = \sum_{m=1}^{M} s_{i,m} h_{i,m} \]  

(8)

4.2. Market Power Allocation (MPA)

In power allocation sub-problem, the pricing technique is adopted from supermarket theory to further simplify (P1). This can be achieved by relaxing interference constraint in (P1) and letting the pricing scheme in the utility function manage the interference generated to PUs. The market power allocation (MPA) can be defined as follows MPA \( \{M, \{a_{i}^{\text{action}}\}, \{S^e(\bullet)\}\} \), where \( M = \{1,2,\ldots,M\} \) is a finite set of decision makers (i.e., players or CR nodes); \( \{a_{i}^{\text{action}}\} \) is the action space, i.e., power to subcarrier allocation; and \( S^e(\bullet) \) is the surplus function linked to each CR node in the network, where the superscript \( (e) \) refers to CR. Assuming that the distributed power strategy over the available subcarriers, is a compact convex set bounded by minimum and maximum power budgets denoted by \( \hat{P}_i = [\tilde{P}, \tilde{P}] \).

Mathematically speaking, the strategy space can be modelled as

\[
\hat{P} = \left\{ P_i : \sum_{i=1}^{N} p_i \leq \tilde{P}, \tilde{P} \leq p_i \leq \tilde{P}, \forall i \in N \right\}.
\]

(9)

Without loss of generality, we assume that \( \tilde{P} = 0 \).

4.3. Design of Surplus Function

A commonly chosen surplus function in market theory is the transmission rate for a given link in the network. The surplus function can be described as a function of action chosen by player \( m \) on the \( i \)th subcarrier, i.e., \( (a_{i}^{\text{action}}) \), and actions chosen by all of the players in the game, except those of player \( m \), i.e., \( \{a_{i}^{\text{action}}\} \). Mathematically speaking, the surplus function can be formulated as follows:

\[
S^e_{i,m}(P_{i,m},P_{-i,m}) = \sum_{i=1}^{N} \log_2 \left( 1 + \frac{p_i^m}{\Omega_i} \right) - p_i^m \sum_{i=1}^{N} \alpha_i \Omega_i^i.
\]

(10)

where \( S^e_{i,m}(\bullet) \) is the net surplus function for the \( m \)th player, which can be described as the difference between the rate of the user and the pricing function. Knowing that \( \alpha_i \) is the price-control factor that manages the generated interference to the PUs. Accordingly, pricing technique has been used to decrease the complexity of (P1) by relaxing interference constraint (C3). By adopting Algorithm 1 and surplus function in (10), problem (P1) can be reformulated as follows:

\[
P2: \max_{\hat{P}} (S^e_{i,m}),\]

s.t.

\[
C1: \sum_{i=1}^{N} \hat{P}_i \leq \tilde{P},
\]

\[
C2: \hat{P}_i \geq \tilde{P} \quad \forall i \in \{1,2,\ldots,N\}.
\]

(P2)

Any solution to the above market model is a solution that obtains the NE.

**Theorem 1:** At least one NE exists in the market model, i.e., (P2), if it satisfies the following conditions:

1) The action strategy profile (i.e., \( \hat{P}_i \)) is a closed, bounded, convex subset.

2) The surplus function \( S^e_{i,m}(\bullet) \) is a continuous, quasi-concave function over the strategy set.

**Proof:** It is straightforward to prove that the above conditions are satisfied for the following reasons:
1) Because the strategy profile, as defined in (9), is defined by a minimum and maximum power, the first condition is readily satisfied.
2) To prove that the second condition is also satisfied, we have to show that the given price-based utility function is quasi-concave in the strategy profile.

**Lemma 1:** The surplus function given in (10) is concave in the defined power strategy profile.

**Proof:** To show that the second condition is true, we have to solve the following set of equations:

\[
\frac{\partial^2 S_{i,m}}{\partial^2 p_i^m} < 0. \quad \text{In the following, we have} \quad \frac{\partial^2 S_{i,m}^c}{\partial^2 p_i^m} = - \frac{|h_{i,m}|^2}{\left(I_{Tot}^2 + p_{i,m}|h_{i,m}|^2\right)^2},
\]

which is <0.

Because both conditions have been proved, the NE exists in the proposed MPA. ■

### 4.4. Proposed Power Algorithm

In this section, optimal power allocation is derived according to the following proposition.

**Proposition 1:** If power is allocated to subcarriers by maximising (P2), then the optimal power strategy across the subcarriers is given by

\[
\hat{p}_i = \left[\frac{1}{\phi(\lambda + \sum_{L} \alpha_i \Omega_i^l)} \left(\lambda \hat{h}_i + \hat{p}_i \hat{h}_i^2\right) - \frac{I_{Tot}}{|\hat{h}_i|^2}\right]^+ \quad (11)
\]

**Proof:** The solution of (P2) can be easily found by adopting the Lagrangian technique. Thus, the Lagrangian related to (P2) can be defined as follows:

\[
\Gamma(\hat{p}_i) = \sum_{i=1}^{N} \log_2 \left(1 + \frac{\hat{p}_i |\hat{h}_i|^2}{\lambda \sum_{L} \alpha_i \Omega_i^l} - \hat{p}_i \sum_{L} \alpha_i \Omega_i^l - \lambda \left(\sum_{i=1}^{N} \hat{p}_i - \tilde{P}\right)\right) + \sum_{i=1}^{N} \hat{p}_i \beta_i
\]

(12)

where \(\beta\) and \(\lambda\) are non-negative Lagrange multipliers. Because \(\beta_i \hat{p}_i = 0\) and \(\beta_i \geq 0\), we obtain \(\hat{p}_i = 0\). Therefore, the optimal solution for the power allocation can be written as follows:

\[
\hat{p}_i = \left[\frac{1}{\lambda \sum_{L} \alpha_i \Omega_i^l} - \frac{I_{Tot}}{|\hat{h}_i|^2}\right]^+ \quad (13)
\]

where \([\bullet]^+ = \max(0, \bullet)\), \(\lambda\) is the Lagrangian multiplier. The power strategy algorithm in (13) utilises a single Lagrangian multiplier, in contrast to the use of the two Lagrangian multipliers in [5]. Thus, the complexity of the power strategy algorithm is reduced, which is one of the advantages behind adopting pricing mechanism. The Lagrangian multiplier \((\lambda)\) can be calculated by substituting (13) into (P2-C1), i.e., \(C1: \sum_{i=1}^{N} \hat{p}_i \leq \tilde{P}\), accordingly the Lagrangian multiplier \((\lambda)\) can be determined according to (14).
\begin{equation}
\lambda = \frac{|N|}{\psi_j \left( \tilde{P} + \sum_{i=1}^{N} I_{Tot}^i \right) - \sum_{i=1}^{L} \alpha_i \Omega_{avg}^i} \tag{14}
\end{equation}

where \( \psi_j \) is the factor that compensates for replacing interference component by the average interference component, which can be modelled as follows, \( \psi_j = \frac{\Omega_j I_{Tot}^j}{|h_i|^2} + I_{th} \). Based on the above derivations and analysis, the MPA algorithm is proposed to determine the optimal power, which is denoted by \( \hat{p}_i \). Furthermore, the procedure of MPA is shown in Algorithm 2.

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**Algorithm 2. Market Power Allocation (MPA)**

**Algorithm**

a) Initialisation step: \( N, M, \tilde{P}, P, \Omega_j^i, I_{Tot}^i \)

b) Set the value of \( \alpha \) such that the interference constraint is satisfied.

c) Call Pricing-based Water-filling Algorithm
   • Find \( \lambda \) according to (14)
   • Set \( \hat{p}_i \) according to (11)


d) Declare the power \( \hat{p}_i \).

End

---

5. Simulation Results

5.1. Simulation Setting

The simulations are implemented according to the system model shown in figure 1. The multicarrier networks consist of four (4) CRs, i.e., \( M = 4 \), and the total number of subcarriers is assumed to be \( N = 16 \). The values of \( \Delta f, \tilde{P} \) and \( I_{th} \) are assumed to be 0.3125 MHz, 1 W and 1 mW, respectively. The thermal noise is assumed to be \( 10^{-6} \). The subcarrier fading gains \( h \) and \( g \) are assumed to be the outcome of independent, identically distributed (i.i.d) Rayleigh distributed random variables with unity mean. Furthermore, the channel gains are assumed to be perfectly known by the CRBS. In addition, all the results have been compiled over 1000 iterations.

5.2. Deployment Model and Simulation Results

The simulations are implemented according to the general system model shown in figure 1. However, to enable a fair comparison, only one active PU band is assumed, i.e., \( L=1 \) in the deployment setting, as shown in figure 3. An underlay coexistence scenario has been considered in this deployment, as defined in Figure1. Thus, the CRs are allowed to transmit over both active and non-active bands considering that the generated interference to PU must be less than the defined threshold \( I_{th} \).

![Figure 3. Deployment Scenario](image_url)
The achievable capacity of CRs against the interference induced to PU band in the given deployment scenario using \textit{MAP} and \textit{PI algorithm} for different interference constraints is plotted in figure 4.

![Figure 4. Achieved capacity vs. allowable interference threshold for OFDM-CRNs](image)

Note that, figure 4 shows that for a given interference threshold, the proposed \textit{MPA} algorithm almost approaches the \textit{optimal} algorithm scheme. Moreover, the capacity of the \textit{MPA} algorithm is slightly less than that of the \textit{PI-algorithm} scheme. The net interference induced to PUs using the \textit{MPA} and \textit{PI algorithm} is plotted in figure 5.

![Figure 5. Generated interference introduced to the PU vs. interference threshold](image)

Note that the net interference induced by \textit{MPA} exhibits more flexibility in terms of interference management compared to \textit{PI algorithm} and satisfies the predefined interference threshold, which makes \textit{MPA algorithm} more suitable for practical scenarios. The strict management of interference induced to PU band resulted from varying the pricing factor in the adopted pricing scheme, which can be varied.
until the interference constraint is affordable to PU band. In contrast, in PI algorithm, the parameters are pre-defined in the power algorithm, which resulted in a fixed interference component that does take advantage of all of allowable interference to PU.

5.3. Computational Complexity
Note that by adopting pricing scheme, the computational complexity of the proposed MPA algorithm is reduced compared to PI-algorithm presented in [5]. This is because the proposed MPA algorithm utilises only one water-filling algorithm based on pricing scheme; therefore, the computational complexity is \( O(N \log(N)) \), where \( N \) is the number of subcarriers. In contrast, the PI-algorithm performs three complex steps, i.e., two water-filling algorithms and power level readjustment step; therefore, the computational complexity is \( O(N \log(N)) + O(L) \), where \( L \) in the number of PU. Moreover, the proposed MAP algorithm exhibits a much lower computational complexity compared to that of the optimal algorithm, which is \( O(N^3) \).

6. Conclusion
A market-based power allocation method in an underlay downlink-cognitive radio network termed as MPA has been proposed in this paper. The pricing scheme has been used to provide flexibility in allocating the power budget to available subcarriers. Hence, CRs can utilise all allowable interference produced by PU. The existence and uniqueness of the NE have been proven mathematically. Moreover, the proposed algorithm offers a higher capacity compared to the previously tested algorithm in the context of OFDM-CRNs. Furthermore, the computational complexity of the proposed algorithm is reduced from \( O(N \log(N) + \eta N) + O(L) \) to \( O(N \log(N)) \), which makes the proposed algorithm more suitable for practical implementation.

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