In this supplementary material we clarify some of the details from our manuscript on the trapping of a single vortex in a superconducting resonator and the interactions between trapped vortices and nonequilibrium quasiparticles.

**CONFIGURATION OF CRYOSTAT AND MEASUREMENTS**

The device is wire-bonded to a microwave board and enclosed in a brass box that is mounted on the cold-finger of an adiabatic demagnetization refrigerator (ADR). A 3K pulse-tube cooled stage cools the ADR magnet and also contains a support structure for mounting a superconducting Helmholtz coil and a cryogenic mu-metal can. The cold-finger is adjusted so that the sample is positioned at the center of the Helmholtz coil, which is used for applying the cooling fields for trapping vortices. The arrangement of the various components and their associated temperatures is shown in Fig. 1. We use a conventional configuration of cold attenuators on the coaxial driveline for exciting the resonance and we amplify the transmission signal with a cryogenic HEMT amplifier on the 3K stage followed by a room-temperature microwave amplifier.

![Schema of cryostat](image)

**ACCOUNTING FOR CURRENT DISTRIBUTION IN SIMULATION RESULTS**

In order to compare the simulation results in our Letter with the measured internal loss on the harmonic $1/Q_i(B)$, we account for the variation of the standing-wave current along the length of the resonator since $n_{qp}(x)$ is proportional to the local effective resistivity. By computing

$$\left(\frac{\int_{-L/2}^{L/2} I^2(x)n_{qp}(x)dx}{\int_{-L/2}^{L/2} I^2(x)dx}\right),$$

where $I(x)$ is a full period of a sine wave for the harmonic, and dividing by the $n_{qp}$ value that we extract from zero-field cooling, we can compare this with the measured $1/Q_i(B)$ for the harmonic, normalized by the average of $1/Q_i(B)$ for $B < B_{th}(8 \mu m)$.

**VARIATION IN LOSS WITH CRYOSTAT TEMPERATURE**

We have performed a similar measurement to Ref. [1] to confirm the presence of a significant density of nonequilibrium quasiparticles in our resonators due to pair-breaking radiation from warmer parts of the cryostat. By varying the temperature of the cryostat, separate from the cold-finger and sample, one can change the radiation power and spectrum that is influencing the resonator. For this test we used an identical chip from the same wafer as the one presented in our Letter with the same cryostat configuration. We cooled the resonator with no magnetic field applied with our Helmholtz coil to avoid trapping any vortices. By turning off the pulse-tube cooler with the sample at the base temperature, the pulse-tube stage warmed up, thus also warming the Helmholtz coil, magnetic shield, and 3K thermal shield. Even once these components reached 18K, the sample temperature increased no higher than 140 mK. We recorded $S_{21}$ along with the cryostat temperature during this warming process. In Fig. 2 we plot the loss $1/Q$ for the fundamental and harmonic resonance vs. the cryostat temperature. For both resonance modes, the loss increases significantly as the cryostat temperature rises. For a blackbody source with the full spectrum of radiation shining on the resonator, one would expect $1/Q \propto T_{cryostat}^2$ for the arguments put forth in
FIG. 2: (Color online) Measurements of $1/Q$ vs. cryostat temperature for zero-field cooling for (a) fundamental, (b) harmonic resonance. The temperature of the cold-finger and sample remained below 140 mK during the measurements. Dashed line is a guide to the eye for a quadratic dependence while the solid line corresponds to a linear dependence.

Ref. [1]. For increased levels of IR shielding surrounding the sample, the radiation spectrum can be cut off, leading to smaller exponents for the increase [1]. Our observed increase in $1/Q$ is closer to linear rather than quadratic, suggesting that our brass sample box that encloses our resonator chip provides some modest filtering of the IR radiation. Nonetheless, the immediate increase in $1/Q$ with $T_{cryostat}$ strongly suggests that nonequilibrium quasiparticles generated by stray IR radiation in our cryostat dominate the loss in our resonator measurements.

QUASIPARTICLE DIFFUSION SIMULATIONS FOR DIFFERENT VORTEX_POSITIONS

In our simulations of the quasiparticle diffusion, we include separate vortex-related terms in Eq. (1) in our Letter for each trapped vortex. From the analysis of the step features on the measurements of the fundamental resonance, we can extract the number of vortices in the central bulge region and coupling elbow for different cooling fields. However, we are unable to determine the precise location of each vortex within the bulge or elbow. Thus, for each vortex-number increment in our simulations, we have spaced the vortices evenly within each trapping region. In order to verify that our simulated reduction in quasiparticle density does not depend significantly on the detailed locations of each vortex in the distribution, we have chosen three example steps in the field-dependence from Fig. 4(b) of our Letter: (i) 2 vortices in the bulge and none in the elbow, (ii) 5 vortices in the bulge and none in the elbow, (iii) 7 vortices in the bulge and two in the elbow. For each case we have repeated the simulation for several different values of the intervortex spacing, within the constraints of the size of the bulge and elbow. Figure 3 shows the variation in the simulated normalized quasiparticle loss on the harmonic with intervortex spacing for each of these three cases with arrows indicating the spacing values that were used for the corresponding points in Fig. 4(b) of the Letter. There is no significant dependence on the intervortex spacing, thus we conclude that detailed knowledge of the vortex positions in the central bulge and coupling elbow is not necessary for our current modeling of the vortex-quasiparticle interactions.

SIMULATIONS WITH DIFFERENT QUASIPARTICLE_DIFFUSION_CONSTANTS

There have been many investigations of quasiparticle dynamics in Al films at low temperatures and the effective quasiparticle diffusion constants $D$ reported in the literature can be influenced by multiple factors. The normal metal diffusion constant $D_n$ directly affects $D$ and there is a range of reported values of $D_n$ for Al films, including 49 cm$^2$/s [2] and 140 cm$^2$/s [3]. Of course, such variations can be caused by different electronic mean free paths depending on the film quality in the various exper-
iments. Also, $D$ will be reduced from $D_n$ depending on the quasiparticle energy: $D(E) = D_n(1 - (\Delta/E)^2)^{1/2}$ [4], as we described in our Letter. However, even after accounting for the reduction in $D$ due to the quasiparticle energy, there is evidence that the effective $D$ is typically reduced further still [5]. For our Al film, we estimate $D_n = 150 \text{cm}^2/\text{s}$ based on the measured resistivity at 4 K of 0.5 $\mu\Omega \cdot \text{cm}$. In order to account for the anomalous reduction described in Ref. [5], we used $D_n = 60 \text{cm}^2/\text{s}$, combined with an estimate for the approximate quasiparticle energy as described in the Letter, to determine $D$ for the simulations presented in Fig. 4(b). We then explored the sensitivity of our model to the value of $D$ used in the simulations by running our simulations from Fig. 4(b) for two other values of $D_n$: 30 and 150 $\text{cm}^2/\text{s}$ (Fig. 4). In each case, we adjusted the values of $\Gamma_R$ and $\Gamma_v$ to give the best agreement between the simulated curve and the normalized measured loss vs. field for the harmonic; the resulting values are listed in the caption to Fig. 4. For smaller $D_n$, we obtained the best match to the data for smaller $\Gamma_R$ and larger $\Gamma_v$. The resulting values for $\Gamma_R$ for all three cases are well within the range reported by others for Al films [4] and all three resulting $\Gamma_v$ values are also quite consistent with typical electron-phonon scattering rates for Al thin films at low temperatures [6, 7]. Thus, our model of the quasiparticle diffusion and interaction with vortices is able to provide a reasonable description of our experimental measurements over a range of parameters for quasiparticle dynamics, consistent with the variation in values for quasiparticle dynamics in Al films reported in the literature.

**THRESHOLD COOLING FIELDS FOR VORTEX TRAPPING**

The relationship between the width of a superconducting strip and the value of $B_{th}$ was studied in Ref. [8] with field-cooling followed by imaging of the vortex distributions. The extracted values of $B_{th}$ for strips of different width $w$ were found to be in reasonable agreement with the expression

$$B_{th} = \frac{2\Phi_0}{\pi w^2} \ln \left( \frac{w}{4\xi} \right),$$

where $\Phi_0 = h/2e$ is the superconducting flux quantum and $\xi$ is the coherence length at the temperature at which the vortices freeze into their respective pinning sites [8, 9]. From our measurements of $1/Q_i(B)$ in our Letter, we have extracted values of $B_{th}$ for $w = 3, 6, 8 \mu\text{m}$ for the three characteristic widths in the different regions of our resonator and we plot these values in Fig. 5. Because this is a rather narrow range of $w$ to compare with Eq. (1), we have chosen to include some previously unpublished $B_{th}$ data from our lab on some other Al resonators with a different geometry, but a wider range of

![FIG. 4](image-url) (Color online) Measured $1/Q_i(B)$ for harmonic, normalized by average of $1/Q_i$ below threshold field (points); simulations of normalized quasiparticle loss on harmonic for different parameters (solid line): (a) $D = 30 \text{cm}^2/\text{s}$, $\Gamma_R = 20 \mu\text{m}^3/\text{s}$, $\Gamma_v = 7 \times 10^6 \text{s}^{-1}$; (b) $D = 60 \text{cm}^2/\text{s}$, $\Gamma_R = 30 \mu\text{m}^3/\text{s}$, $\Gamma_v = 3.5 \times 10^6 \text{s}^{-1}$; (c) $D = 150 \text{cm}^2/\text{s}$, $\Gamma_R = 40 \mu\text{m}^3/\text{s}$, $\Gamma_v = 2 \times 10^6 \text{s}^{-1}$. Note: the plot in (b) is identical to Fig. 4(b) in the Letter and is repeated here for comparison with the other simulations.

![FIG. 5](image-url) (Color online) Plot of threshold cooling fields $B_{th}$ for different width segments on resonator from Letter (filled circles) and $B_{th}(w)$ values for quarter-wave uniform-width resonators from separate device as discussed in text (open squares). Curve corresponds to Eq. (1) for $\xi = 250 \text{nm}$. 

linewidths. This other chip contained four quarter-wave coplanar waveguide resonators with uniform-width center conductors, similar to the device in Ref. [10], with widths \( w = 10, 12, 18, 26 \, \mu m \). Also, the thickness of the Al film on this other chip was 150 nm, although it is not clear what role, if any, film thickness plays in determining \( B_{th} \). By analyzing the \( 1/Q_v \) measurements for this chip, we have extracted \( B_{th} \) for the four resonators of different widths and we include this data in Fig. 5 with the \( B_{th}(w) \) points extracted from the measurements in our Letter. We then include a curve corresponding to Eq. (1) by adjusting \( \xi \). We find that for \( \xi = 250 \, nm \), we obtain decent agreement with the measured \( B_{th}(w) \) points, although the curve is not a perfect match to the data. Deviations between the measurements and the predicted dependence of Eq. (1) could be due to a variety of reasons, as discussed in Ref. [8], such as variations in the details of the vortex freezing process between the strips of different widths. Also, for some of our features, such as the 6 \( \mu m \) and 8 \( \mu m \) regions of our resonator, the finite length of these regions may change the details of Eq. (1) as well. Nonetheless, the general trend of \( B_{th} \) is clear and vortices trap at higher threshold fields for narrower superconducting traces. Thus, our scheme for making resonators with variable widths of the center conductor allows for the control of vortex-trapping locations along the resonator length.

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