DNLS: a Detection Method based on Normalized Short-Time Fourier Transform-Radon Transform for Low Frequency Sonar Pulse Signal

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ABSTRACT Under low-frequency background noise environments, due to the characteristics of poor stability and many interference targets of noise, the detection of unknown low-frequency sonar signals faces huge challenge. And sonar pulse signal detection methods based on time domain or frequency domain have the limitation of insufficient detection Signal-to-Noise Ratio (SNR). In order to improve the detection capability of weak sonar pulse signal in low frequency background noise environments, a Detection method based on normalized short-time Fourier transform-Radon transform for Low frequency Sonar pulse signal (DNLS) is proposed, which is a constant false alarm detection method in normalized short-time Fourier transform-Radon transform domain. In DNLS method, after the normalized short-time Fourier transform-Radon transformation, low-frequency noise energy to be dispersed into the entire transformation domain, and the sonar pulse signal energy containing the Linear Frequency Modulation (LFM) component is concentrated at a specific target point in the normalized short-time Fourier transform-Radon transform transformation domain, which can obtain a higher local SNR than the time-domain SNR. Moreover, the specific target point is distinguishable from the background noise, and the impulse signal detection decision is completed by constructing hypothesis test statistics on the target point data. The DNLS method solves the detection problems of low-frequency background such as poor stability, large fluctuations, and more interference. And the method of obtaining the test statistics of the constant false alarm detection, estimating the background noise and calculating the detection threshold is given. Extensive simulation results and actual data processing show that, under the simulation condition, in the minimum detection SNR of LFM, Continuous Wave(CW)-LFM and pulse trains of frequency modulated pulse signals with the same pulse width, compared with the dual-threshold constant false alarm rate energy detection method, the DNLS method is improved by 15dB, 13dB and 4dB, respectively. Under actual data conditions, in the detection of CW-LFM pulse signals with the same pulse width, compared with the double-threshold constant false alarm rate energy detection method, the DNLS method is improved by 5dB and 5.5dB, respectively. The data analysis results show that the DNLS method has very good detection performance for LFM, pulse trains of frequency modulated, CW-LFM and other sonar pulse signals at low SNR, and can effectively detect the sonar pulse signals under the background of strong ship radiated noise.

INDEX TERMS Sonar pulse signal; NSRT transform; Constant false alarm; Detection Threshold.

I. INTRODUCTION

In sonar signal processing research, detecting unknown pulse signals from Marine ambient noise is an important issue. With the extensive applications of feature processing techniques such as vibration and noise reduction, advanced propulsion, installation of anechoic tiles and special shape design, the noise level of medium and high frequency target radiation and target reflection intensity of submarine can be
greatly reduced. In order to improve the long-range detection capability of advanced submarines, the navies of various countries pay more and more attention to the development of low-frequency sonar equipment, and take low-frequency active detection as one of the main working modes in actual applications [1]. With the increase of shipping and other vessel activities, low-frequency Marine environmental noise has increased significantly compared with the past [2] [3]. However, under low-frequency background noise environments, due to the characteristics of poor stability and many interference targets of noise, the detection of unknown low-frequency sonar signals faces huge challenge [1] [4]. Sonar pulse signal detection methods based on time domain or frequency domain have the limitation of insufficient detection Signal-to-Noise Ratio (SNR). And when the noise or interference background is strong and the pulse signal is seriously polluted, the detection performance may be reduced [5].

The pulse types commonly used in modern active sonar have been developed into multiple pulse signal types such as Continuous Wave (CW), Linear Frequency Modulation (LFM), Hyperbolic Frequency Modulation (HFM), Pulse Trains of Frequency Modulated (PTFM), CW-LFM, CW-HFM, etc [6] [7]. In this paper, a Detection method based on Normalized Short-Time Fourier Transform (STFT)-Radon transform for Low frequency Sonar pulse signal (DNLS) is proposed, which is a constant false alarm detection method in Normalized STFT-Radon Transform (NSRT) domain. The aim of the DNLS method is to improve the detection performance of pulse signals in low frequency background noise. The DNLS method is based on the low-frequency background noise energy being dispersed into the entire transform domain after NSRT transformation, and in the NSRT transform domain, the energy of the sonar pulse signal in LFM signal will be concentrated at a specific target point. Therefore, a higher local SNR than the time-domain SNR can be obtained to detect impulse signals submerged in noise. On the other hand, due to the characteristics of poor stability and large fluctuation of low-frequency background noise, the changes in noise and interference intensity will lead to significant changes in false alarm probability and detection probability. Therefore, Constant False Alarm Rate (CFAR) processing technology is adopted in the background estimation processing, and adaptive estimation with high confidence is adopted for the background noise. Thus, the detection performance of pulse signal can be improved under certain control of false alarm probability. The main contributions of this paper are threefold:

1) In the traditional STFT-Radon transform method, the physical representation of each pixel in the transform domain is not uniform, so there is no data comparison significance among the pixel points. A new NSRT change method is proposed. In the new NSRT change method, the physical meaning of the pixel points in the transform domain is unified, and the values between the pixel points are comparative. Therefore, it can provide a new idea for sonar pulse signal automatic detection and other applications.

2) In order to solve the problems, such as the poor stability and large fluctuations in low-frequency background noise, and difficult estimation of low-frequency background noise power and detection threshold, a sonar pulse signal detection method based on NSRT transform is proposed. On the other hand, the method of constructing detection model, constructing test statistics and calculating adaptive inspection threshold is given. Therefore, the detection performance of low frequency pulse signals in non-stationary low frequency background is improved.

3) In order to solve the problem of insufficient SNR in the detection of weak low-frequency sonar pulse signals in the time domain and frequency domain, the NSRT transform is employed to enhance the local detection SNR of sonar pulse signals with LFM components. Therefore, the detection ability of weak sonar pulse signals is improved.

4) In order to solve the problem that radiated noise from human moving objects will interfere with the traditional time domain sonar pulse signal detection, a method is proposed, which is based on the distinguishability of sonar pulse signal and human activity radiated noise interference in the NSRT transform domain, and can avoid interference when constructing test statistics in the NSRT transform domain. And thus, the influence of radiated noise interference on the detection performance of sonar pulse signal can be greatly reduced.

II. RELATED WORKS

In recent years, the research on weak signal detection method has made great progress, which has significantly improved the detection SNR of target signal. Wenshu Dai et al., [8] proposed a target detection method using target signal line spectrum. In this method, the characteristic of stable azimuth of the line spectrum component in the target radiation noise is used to detect weak targets. When the line spectrum at the frequency point is not obvious or the fluctuation is large, its performance is poor. A Subband Energy Detection (SED) algorithm is proposed in [9], which includes both subband peak energy detection and subband extreme energy detection. SED exploits the spatial coherence of the signal’s local maxima (“peaks”) and minima (“valleys”) compared to the randomness of noise to increase the quality of the broadband processing display. The simulation results showed that SED provides narrower contact traces and increased bearing resolution since only the energy of the peaks and valleys are summed. Moreover, there is also reduced smearing of acoustic energy over large azimuths and an improved ability to detect nearby contacts. According to the obvious low-frequency characteristics of the transient signal of the underwater high-speed target exiting the tube, the power-law transient signal detector is used to detect the transient signal based on the characteristics of the frequency domain [10]. The results shows that the method can detect underwater high speed target exceed tube noise effectively and reduce false alarm probability in a certain extent. Liang et al., [11] proposed a frequency domain constant false alarm detection.
method to detect weak CW pulse signals in low-frequency background noise, which has good detection performance. Ma et al., [6] pointed out that the main problem of frequency domain signal detection algorithm is that, when the active sonar transmits a single frequency signal, the echo of the target will have a certain Doppler frequency shift, which changes the frequency of the signal and affects the performance of the frequency domain detection algorithm. LFM pulse signal is a kind of commonly used sonar signal [12], which has both high delay and frequency shift resolution, and the delay resolution can be changed by adjusting the signal bandwidth. In active sonar signal processing, active sonar can use the known sonar signal to directly match filtering to obtain the best detection performance, while the pulse signal detected by sonar pulse signal is a non-cooperative signal, and the relevant information of pulse signal is unknown and thus the receiving gain cannot be improved by matched filtering. Tian [7] pointed out that time-domain energy detection is the best method to detect unknown Gaussian wideband signals from the time-domain Gaussian stationary background, and it has better robustness. In addition, Liang et al. [11] also proposed a double-threshold constant false alarm energy detection method based on the ideas of energy detection and multi-threshold detection. Liang pointed out that the double-threshold constant false alarm energy detection method under non-stationary background is better than the traditional time-domain energy detection method. Better detection performance. Xu et al. [13] compared the performance of the time-domain energy detection method and the double-threshold constant false alarm detection method, and pointed out that the detection ability and robustness of the double-threshold constant false alarm detection method are better than the time-domain energy detection method. Animesh [14] obtained the SNR according to the signal power detected by each cognitive node. By calculating the weight of SNR, the reliability of nodes with high SNR was enhanced, and the final result was obtained by the central node according to the corresponding fusion rules.

As can be seen from the above analysis, in the frequency domain processing method, the detection ability of line spectrum information in narrowband signal or broadband signal has been improved. However, there are still some problems to be solved for broadband signals. For broadband signals, the existing energy detection methods based on the beam domain do not make full use of the characteristics of the target signal, and mostly stay in the peak stacking based on the energy judgment. When the energy detection methods in the beam domain cannot detect the weak target, the post-processing algorithm cannot detect the weak target either [15]. Low frequency background noise has poor stationarity, large fluctuation and many interfering targets [1]. When the noise or interference background is strong and the pulse signal is seriously polluted, the detection performance of traditional energy detection methods based on time domain or frequency domain will be greatly affected.

In addition, a lot of work has been done in the field of time-frequency analysis. The time-frequency analysis method analyzes signals from the time domain and frequency domain. Compared with time domain or frequency domain analysis, time-frequency analysis can reflect the joint relationship between time and frequency, and can carry out a more comprehensive analysis of signals. The commonly used time-frequency analysis and detection methods include The Short-Time Fourier Transform (STFT), Wavelet Transform, (WT), Wigner-Ville Transform, Hilbert-Huang Transform (HHT), etc. Liu et al. [5] studied the interception detection and parameter estimation method of non-cooperative underwater acoustic pulse signal based on the time-frequency distribution of the signal. Yan et al. [16] used three time-frequency analysis methods, namely STFT, wavelet transform and Wigner-Ville transform, to detect underwater acoustic targets, and compared the results obtained by these three methods to analyze the advantages and disadvantages of different algorithms. Mallet [17] realized signal detection by wavelet decomposition and reconstruction, and proposed a new detection algorithm by processing the modulus of wavelet coefficient. In [18], the application of wavelet transform in underwater acoustic transient signal was analyzed. Yang et al. [19] improved the SNR of underwater acoustic signals by using wavelet de-noising. Han et al. [20] combined the Page-Test algorithm with wavelet transform, and proposed a Page-Test detector based on wavelet transform. According to the Wigner transform, the Wigner-Ville Distribution (WVD) analysis method is proposed [21], and the time-bandwidth product reaches the lower bound given by the Heisenberg uncertainty principle and has good time-frequency aggregation characteristics. In [22], researchers proposed a classical WVD method. But WVD has serious cross-term problems, and even becomes meaningless when the signal becomes complex. Gao et al. [23] demonstrated the application of Hilbert-Huang transform in underwater acoustic signal processing. Li et al. [24] applied The Hilbert-Huang transform to the detection of underwater acoustic transient signals. Wang et al. [25] analyzed the problems encountered in Hilbert-Huang transform and proposed and derived two de-noising methods based on empirical mode decomposition. However, there are still some problems that need to be solved: the wavelet transform problem lies in the choice of wavelet basis functions, and the performance of wavelet transform will be seriously affected when the improper basis function is selected in the non-cooperative sonar signal processing. WVD also has serious cross-term problems [21]. Hilbert-Huang transform also has some problems such as poor stability and limited engineering rapid calculation [25]. STFT [26] [5] is a classical analysis method, which is a time-frequency analysis method with good tolerance, simple calculation and small computation. However, in STFT, the time-frequency resolution is limited by the shape and width of the window function, so it can not get good time and frequency resolution at the same time. Academician Li [27] mentioned that in sonar engineering, transient LOFAR spectrum analysis based on frequency domain processing idea will be carried out for
ship radiated noise, and then the process of LOAFR spectrum analysis results will be displayed in time. Such processing effect is equivalent to time-frequency analysis of target signal using STFT method. The research of correlation method based on STFT has very good engineering application value in real-time sonar signal such as sonar pulse reconnaissance.

III. THEORETICAL ANALYSIS

For convenience, Table 1 summarizes the main parameters together with their significances in this paper.

A. STFT - RADON TRANSFORM

STFT is a very important tool to analyze signal characteristics in time domain and frequency domain simultaneously [26]. The definition of STFT can be calculated as:

\[
STFT(t, f) = \int s(t')h(t' - t)\exp(-jft')dt'
\]

(1)

where \( h(t) \), \( t \) and \( f \) represent time window function (hannning window and gaussian window are commonly used time window functions), the time and the frequency, respectively. The main idea of STFT is as follows:

On the basis of Fourier transform, a finite time window is added, which slides along the time axis over time, and the signal is assumed to be stationary (pseudo stationary) in the time window again and again, the spectrum of the signal at any moment can be obtained. At any time \( t \), \( STFT(t, f) \) can be considered as the “local spectrum” of the signal \( s(t) \) around the “analysis time”.

Radon transform is a projection transform of line integral [28], and it is an important tool in the field of image processing, such as navigation, physical and radiological science, chemistry, medicine and other technical fields.

Rotate the time-frequency plane coordinate \((t, f)\) counterclockwise by \( \theta \) Angle to get the new coordinate \((r, \theta)\), as shown in Figure 1. Integrate different \( r \) values parallel to the \( v \) axis, and the result is the Radon transform. The transformation relation between coordinates can be calculated by:

\[
\begin{align*}
    t &= r \cos \theta - v \sin \theta \\
    f &= r \sin \theta + v \cos \theta
\end{align*}
\]

(2)

For the time-frequency plane, for any two dimensional function \( x(t, f) \), the Radon transform \( R(r, \theta) \) of the \( \theta \) Angle can be expressed as

\[
R(r, \theta) = \int_{PQ} x(r \cos \theta - v \sin \theta, r \sin \theta + v \cos \theta) dv
\]

(3)

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(w_1, w_2)\delta(r - r_0)\delta(\theta - \theta_0)drdv,
\]

\[
0 \leq \theta_0 \leq \pi
\]

(4)

where \( r \) denotes the distance between line PQ and the origin of the coordinates, \( \theta \) denotes the angle of rotation, and \( w_1 = r \cos \theta - v \sin \theta, w_2 = r \sin \theta + v \cos \theta \).

When the two-dimensional function \( x(t, f) \) in Equation (3) is replaced by the signal \( STFT(t, f) \), the STFT-Radon Transform (SRT) \( R(r, \theta) \) of the signal can be computed as Equation (4), which is show in Figure 2.

\[
R(r, \theta) = \int_{PQ} STFT(w_1, w_2)dv
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} STFT(w_1, w_2)\delta(r - r_0)\delta(\theta - \theta_0)drdv
\]

\[
0 \leq \theta \leq \pi
\]

(4)

Where \( w_1 = r \cos \theta - v \sin \theta, w_2 = r \sin \theta + v \cos \theta \).

When the pulse signal feature is characterized, Radon transform can map the straight line in the time-frequency graph to the points in the transformation domain, which can achieve the effect of data reduction and focus. The commonly used sonar pulse signals are distributed according to the inherent time-frequency law in the time-frequency domain. The data accumulation ability of Radon transform can be used to analyze the time-frequency energy distribution and extract the relevant characteristic parameters.

B. IMPROVEMENT OF SNR IN THE TRANSFORM DOMAIN

The SNR is usually defined as the ratio of the average signal power to the average noise power. Let \( P_s \) denote the average power of a signal in the time domain, and we have

\[
P_s = \frac{1}{T} \int_0^T S^2(t)dt
\]

(5)

where \( T \) is the time length of signal \( S(T) \).

For an additive white Gaussian noise with zero mean and variance \( \sigma_n^2 \), the average noise power is \( P_N = \sigma_n^2 \), and then the average SNR \( SNR_{average} \) can be calculated as

\[
SNR_{average} = \frac{P_S}{P_N} = \frac{1}{T} \int_0^T S^2(t)dt \sigma_n^2
\]

(6)
As can be seen from Figure 3(a), for the wideband signal submerged in the additive white Gaussian noise with zero mean value and variance, the signal cannot be distinguished from the noise in the time domain or frequency domain due to its small average signal power and low SNR. Through time-frequency transformation, the energy of random noise is distributed to the whole time-frequency domain. While the energy of signal is usually concentrated in the limited time interval and frequency range, and the signal submerged in noise can be more easily identified in the time-frequency domain. For the LFM component in the sonar pulse signal, its main energy is distributed over time on a straight line in the time-frequency domain. When Radon transform is used to sum the signal energy in the time-frequency diagram along the straight direction, the LFM signal can be found at a lower SNR than that in the time-frequency domain, as shown in Figure 3.

For a LFM signal \( s(t) \), the length of signal is \( T \), the signal bandwidth is \( B \), the power spectral density of signal is \( P_s \), the white noise signal is \( n(t) \), and the power spectral density of noise is \( \sigma_n^2 \), then the SNR \( SNR_{in} \) can be calculated as
\[
SNR_{in} = P_s / \sigma_n^2
\]  
(7)

As shown in Figure 4, STFT is applied to the signal with noise, and the sliding window length is \( N \). The frequency dimension is divided into \( N/2 \) subfrequency bands, and the signal bandwidth corresponding to each subfrequency band is \( \Delta B = 2B/N \). The signal length is \( T \), and the time dimension is divided into \( K \) segments, each of which is \( \Delta T = T/K \). If the power spectral density of LFM signal remains unchanged, the power spectrum of white noise is expanded to the entire time-frequency domain, then the output SNR \( SNR_{stft} \) of a single pixel of STFT transform domain can be calculated as \([29][30]\)
\[
SNR_{stft} = \frac{B T SNR_{in}}{\Delta B \Delta T} = \frac{NK}{2} SNR_{in}
\]  
(8)

Then, Radon transform is carried out on the STFT time-frequency graph. Radon transform is to sum up the discrete data along the straight line direction. The amount of accumulated data is expressed as \( Q = M_1 / |\cos \theta| \) or

\[ Q = M_1 / \sin \theta \]  
(9)

\[ Q = M_1 / |\cos \theta| \]  
(10)

\[ Q = M_1 / \sin \theta \]  
(11)

\[ Q = M_1 / \sin \theta \]  
(12)

\[ Q = M_1 / |\cos \theta| \]  
(13)

\[ Q = M_1 / \sin \theta \]  
(14)

\[ Q = M_1 / |\cos \theta| \]  
(15)

\[ Q = M_1 / \sin \theta \]  
(16)

\[ Q = M_1 / |\cos \theta| \]  
(17)

\[ Q = M_1 / \sin \theta \]  
(18)
\( Q = \frac{M_2}{|\sin \theta|} \). Let \( Q_{\text{max}} \) denote the maximum number of accumulates under ideal conditions, and we have

\[ Q = \sqrt{M_1^2 + M_2^2} \quad (9) \]

where \( M_1 \) represents the number of points on the time axis of the Radon transform image, and \( M_2 \) represents the number of points on the frequency axis of the Radon transform image. Let \( \text{SNR}_{\text{Radon}} \) denote Radon transform output, and we have

\[ \text{SNR}_{\text{Radon}} = Q \text{SNR}_{\text{stft}} \quad (10) \]

Let \( \text{SNR}_{\text{SRT}} \) denote the improvement effect of STFT-Radon Transform domain \([29] [30]\), and \( D_S \) denote the improvement gain. And we have

\[ \text{SNR}_{\text{SRT}} = \frac{QNK}{2} \text{SNR}_{\text{sn}} \quad (11) \]

\[ D_S = 10 \log_{10} \frac{QNK}{2} \quad (12) \]

From the above analysis, it can be seen that a pulse signal in noise is more easily detected in the transform domain.

**C. CONSTANT FALSE ALARM DETECTION IN THE NSRT TRANSFORM DOMAIN**

Although STFT-Radon transform method can distinguish the pulse signal submerged in noise in the transform domain. In the STFT-Radon transformation result, the STFT time-frequency domain integration length corresponding to each pixel is different, and the physical meaning of each pixel is not uniform, and there is no data comparison meaning between each pixel. Therefore, there is no engineering value of using STFT-Radon transform to automatically detect pulse signal.

A new Normalized STFT-Radon Transform (NSRT) method is introduced in this paper. In NSRT method, the physical meaning of each pixel in the transformation domain is unified. Let \( \text{NR}(r_0, \theta_0) \) denote the NSRT transform, and
we have

\[ NR(r_0, \theta_0) = \frac{1}{N(r_0, \theta_0)} \int_{PQ} STFT(w_{10}, w_{20})\,dv \]

\[ = \frac{1}{N(r_0, \theta_0)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} STFT(w_1, w_2)\delta(r - r_0)\delta(\theta - \theta_0)\,dr\,dv \]

\[ 0 \leq \theta_0 \leq \pi \]  

where \( N(r_0, \theta_0) \) is the number of STFT pixels in the time-frequency domain accumulated by NSRT transformation at the coordinate point \((r_0, \theta_0)\). The physical meaning of \( NR(r_0, \theta_0) \) is, in the STFT time-frequency domain, the energy distribution density on a straight line that deviates from the origin of the plane coordinate by \( \theta_0 \) angle and the distance from the origin is \( r_0 \), and \( w_{10} = r_0 \cos \theta_0 - v \sin \theta_0, \n w_{20} = r_0 \sin \theta_0 + v \cos \theta_0, \n w_1 = r \cos \theta - v \sin \theta, \n w_2 = r \sin \theta + v \cos \theta. \)

Similar to the STFT-Radon transform result of LFM signal, the results of the NSRT transform of sonar pulse signal containing LFM component will show a peak at the coordinates of \((r, \theta)\) and \( \theta \neq 90^\circ \) in the transformation domain, as shown in Figure 5, and it is distinguishable from background noise. The impulse signal submerged in noise can be observed by looking for the spike peak in the transform domain. When the pulse signal energy at the peak is used as the test statistic, the historical data at the peak is used to estimate the detection background, and the pulse signal can be effectively detected through a reasonable threshold setting.

The detection method based on NSRT is a binary hypothesis detection. The observation result is either a signal containing noise or a noise. According to Naiman-Pearson criterion, under a certain false alarm probability, the detection probability of the pulse signal is maximized. Under low frequency background noise environments, the traditional detection method based on fixed threshold is not suitable for detecting unknown signals in the environment with unstable statistical characteristics. This is because low-frequency background noise has the characteristics of poor stability and large fluctuation. Therefore, the noise changes will cause significant changes in false alarm probability and detection probability.

In this paper, the CFAR processing method [31] is introduced in the transformation domain detection. The adjacent units of the test unit are used to estimate the statistical characteristics of the background noise, and the automatic detection threshold is set based on the estimation to determine whether the test unit contains pulse signals, as shown in Figure 6. And thus, the detection threshold can be adjusted adaptively according to the fluctuations of low-frequency background noise to improve the detection performance of pulse signals.

In general, the marine background noise received by pas-
sive sonar can be regarded as band-limited Gaussian white noise after being band-limited [7]. After NSRT transform of noise data of different groups, the transformation result

\[ UR_{r,\theta}(x) \]

at the same pixel point position \((r, \theta)\) can be considered to approximately conform to Gaussian distribution [7] [32] [33], as shown in Figure 7. On the other hand, \( UR_{r,\theta}(x) \) is the normalized STFT-Radon transform results of different batches of data at coordinate point \((r, \theta)\), and its probability density function \( f(x) \) can be calculated as

\[ f(x) = \frac{1}{\sigma_n \sqrt{2\pi}} \exp\left(-\frac{(x - u_n)^2}{2\sigma_n^2}\right) \]  

(14)

where \( \sigma_n \) is the mean square error of noise and \( u_n \) is the mean of noise.

Let \( S_T \) and \( P_{fa} \) represent Gaussian distribution detection threshold and false alarm probability, respectively. And we have

\[ P_{fa} = \int_{S_T}^{\infty} \frac{1}{\sigma_n \sqrt{2\pi}} \exp\left(-\frac{(x - u_n)^2}{2\sigma_n^2}\right) dx \]  

(15)

It can be seen from Equation (15) that the false alarm probability is related to both the mean value \( u_n \) and mean square error \( \sigma_n \). Substituting \( y = \frac{x - u_n}{\sigma_n} \) into the Equation (15), we can obtain

\[ P_{fa} = \int_{\hat{V_T}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy = 1 - \phi(V_T) \]  

(16)

where \( V_T \) is the threshold weighting coefficient, and the random variable \( y \) follows \( N(0, 1) \) uniform distribution. It can be seen from Equation (16) that false alarm probability is independent of background noise. The threshold \( S_T = \hat{u}_n + V_T\hat{\sigma}_n \) is not constant, and it changes with the background.

Figure 8 shows the flow diagram of constant false alarm detection in NSRT transform domain. The points below the CFAR threshold \( \hat{u}_n + V_T\hat{\sigma}_n \) are considered as background noise, and all other points above the CFAR threshold \( \hat{u}_n + V_T\hat{\sigma}_n \) are considered as signal overlay background noise. The constant false alarm detection of the NSRT transform domain can be described as follows
\[ N R_{r_{\text{max}}, \theta_{\text{max}}} \begin{cases} & H_1 \quad U + V_T Z \quad (17) \\
& H_0 \quad \frac{1}{L} \sum_{i=1}^{L} N R_{r, \theta} (x_i) (18) \end{cases} \]

where \( U \) is the estimated sample mean and \( Z \) is the estimated sample mean square error, and we have

\[ Z = \sqrt{\frac{1}{L-1} \sum_{i=1}^{L-1} [N R_{r, \theta} (x_i) - U]^2} (19) \]

where \( L \) denotes the reference sliding window length.

Figure 9 shows the flow chart of using NSRT to detect unknown LFM pulse signals in the ocean background noise, and the detailed steps are described as follows:

**Step 1:** NSRT transform processing
According to the specific performance requirements of pulse signal detector, the NSRT transformation processing parameters were determined, and the NSRT transformation processing was carried out on the sample data.

**Step 2:** Test statistics extraction
The peak point of the NSRT transform domain is the suspected target point, the target point coordinate is \( (r_{\text{max}}, \theta_{\text{max}}) \), and the target point data is \( N R_{r_{\text{max}}, \theta_{\text{max}}} \). The target point is used as the test statistic for the test decision.

**Step 3:** Historical background data extraction at the target point
Determine the reference sliding window length of the C-FAR detection background noise estimation as \( L \), and extract the \( L \) groups of historical background data of the target point \( (r_{\text{max}}, \theta_{\text{max}}) \) in the **Step 2**.

**Step 4:** Background noise estimation
The noise background mean \( U \) and mean square error \( Z \) were estimated from the target historical background data.

**Step 5:** Detection threshold calculation
According to the estimated mean value \( U \) and mean square error \( Z \) of the noise background, the detection threshold \( U + V_T Z \) was calculated.

**Step 6:** Detection and decision
The target data \( N R_{r_{\text{max}}, \theta_{\text{max}}} \) is compared with the threshold \( U + V_T Z \) to determine whether the data \( x(t) \) contains pulse signals.

When \( N R_{r_{\text{max}}, \theta_{\text{max}}} \geq U + V_T Z \), that is, \( H_1 \) hypothesis is true, the sonar pulse signal is detected.

**Step 7:** Background noise data iteration
When \( N R_{r_{\text{max}}, \theta_{\text{max}}} < U + V_T Z \), it means that no pulse signal is detected in this test. Iterative update of background noise should be carried out. The earliest set of data in group \( L \) is deleted from the background noise data, and the transformation result \( N R(r, \theta) \) is put into the background noise library. When \( N R_{r_{\text{max}}, \theta_{\text{max}}} \geq U + V_T Z \), then the background noise data of \( L \) groups remains unchanged.

**IV. SIMULATION AND VERIFICATION**

Different active sonar platforms have different designs of signal pulse widths. Typical sonar pulse signal widths have a length of a few tenths of a second, a few seconds, or even tens of seconds.

Several groups of simulation data were used to verify the detection performance of the NSRT transform constant false alarm detection algorithm, in which the sampling rate was normalized to 1, the sliding window of STFT adopted a Chebyshev window of length \( L = 1024 \), and the repetition degree was 973.

Figure 10 gives the time-frequency diagrams of LFM, PTFM and CW-LFM combined signal. And the time domain amplitudes of the three signals are the same. The pulse width of LFM signal is 3s, and the normalized frequency band is \([0.17,0.33]\). PTFM is composed of 6 identical LFM sub-pulses, the pulse width of each sub-pulse is 0.5s, and the frequency band is normalized to \([0.17,0.33]\). In the CW-LFM combination signal, the pulse width of LFM signal is 2.4s, and the normalized frequency band is \([0.17,0.33]\). The CW signal pulse width is 0.5s, and the normalized center frequency is 0.17. The interval between the two groups of signals is 0.1s.

Let the reference sliding-window length \( L \) of NSRT transform domain C-FAR detection background noise estimation be 8, the false alarm probability \( P_{fa} \) is 0.00508, and the threshold weighting coefficient \( V_T \) is 2.57. According to the simulation data, we conduct 1000 times Monte-Carlo experiments on the LFM signal, PTFM signal and CW-LFM combined signal, and we obtain the variation of detection probability \( P_d \) with SNR for three types of signals NSRT transform domain C-FAR detection, as shown in Figure 11 (a). Dual-threshold C-FAR energy detection is a time-domain C-FAR detection method with strong detection performance [11] [13]. Figure 11(b) shows the detection probability curve of dual-threshold C-FAR energy detection for LFM, PTFM and CW-LFM signals, where the number of fusion samples \( N_c \) of dual-threshold detection is 48, and the second-level threshold \( R \) value is 10. The first level of detection is C-FAR energy detection with the square integral length \( M \) of 2, where the reference sliding window length is 48 and the threshold weighting coefficient \( T_E \) is 0.1004.

In DNLS method, \( P_d \) is equal to 0.9 as the performance evaluation standard of the detector. It can be seen from Figure 11(a) that, for all three kinds of pulse signals, the constant false alarm detector in the NSRT transformation domain has
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(a) LFM signal

(b) PTFM signal

(c) CW-LFM combined signal

FIGURE 10: Time-frequency diagrams of LFM, PTFM and CW-LFM combined signals)

a good detection performance, and the lowest detection SNR for LFM signals with a pulse width of 3s even reached -10dB. On the other hand, the detection performance of complex pulse signal is related to the length of LFM signal. In Figure 11(a), the LFM length of the CW-LFM combined signal is 2.4s, and the minimum detection SNR of the CW-LFM combined signal reaches -8dB. The LFM length of PTFM signal is 0.5s, and the lowest detection SNR of PTFM signal is also 1dB.

The detection performance of the time-domain CFAR detection sonar pulse signal is related to the time-domain waveform amplitude and SNR, and has nothing to do with the signal type. It can be seen from Figure 11(b) that the detection probability curves of the three types of simulated sonar pulse signals of the same amplitude almost overlap. By comparing the detection probability curves of the three types of pulse signals in Figure 11(a) and Figure 11(b), it can be seen that, compared with the dual-threshold CFAR energy detection method, the minimum detection SNR of the NSRT transform domain constant false alarm detection method for LFM, CW-LFM and PTFM pulse signals with the same pulse width is improved by about 15dB, 13dB and 4dB, respectively.

The above analysis results show that the detection performance of weak sonar pulse signals can be greatly improved by using NSRT transform domain constant false alarm detection technology. The detection performance of complex pulses such as CW-LFM combined signals depends on the LFM component, and the longer the LFM component is, the better the detection performance is.

V. SEA TRIAL DATA PROCESSING

The sea trial data were used to verify the constant false alarm detection performance of the NSRT transform under the ocean background noise, where sampling rate was normalized to 1, and the sliding window of the STFT adopted a Chebyshev window. The Chebyshev window length $L$ equals to 1024 and the repetition degree is 973.

Figure 12 shows the time-frequency diagram of actual data of LFM and CW-LFM combined signals. The pulse width of LFM signal is 0.5s, the initial frequency $f_1$ is normalized to 0.27, and the frequency band is normalized to [0.27,0.33]. In the CW-LFM combination signal, the LFM signal pulse width is 0.4s, and the frequency band is normalized to [0.06,0.12]. The CW signal frequency pulse width is 0.1s, and the center frequency $f_0$ is normalized to 0.2. There is no interval between these two groups of signals.

Figure 13 shows the performance comparison between CFAR detection in the transformation domain and CFAR detection in the time domain. Let the reference sliding-window length $L$ of NSRT transform domain CFAR detection background noise estimation be 8, the false alarm probability $P_{fa}$ is 0.00508, and the threshold weighting coefficient $V_T$ is 2.57. According to the actual data of LFM and CW-LFM combined signals, the NSRT transform constant false alarm detection processing is carried out, and the change of detection probability $P_d$ of constant false alarm detection in the NSRT transformation domain of the two kinds of signals with SNR is statistically obtained, as shown in Figure 13(a).

Figure 13(b) shows the detection probability curve of dual-threshold CFAR energy detection for CW-LFM combined signals, where the number of fusion samples $N_r$ of dual-
threshold detection is 250, and the second-level threshold \( R \) value is 33. The first level of detection is CFAR energy detection with the square integral length \( M \) of 2, where the reference sliding window length is 48 and the threshold weighting coefficient \( T_E \) is 0.1004.

Taking \( P_d \) equal to 0.9 as the detector performance evaluation standard. It can be seen from Figure 13(a) that the constant false alarm detector in the NSRT transformation domain has a good detection performance for both LFM and CW-LFM combined signals in the actual processing, and the lowest detection SNR for LFM signal with pulse width of 0.5s reaches -1dB. The false alarm probability for both kinds of signals is less than the preset value of \( 10^{-3} \).

The detection performance of complex pulse signals such as combined signals is related to their LFM component length. In Figure 13(a), the LFM signal with a pulse width of 0.5s has a minimum detection SNR of -1dB, while the LFM length of CW-LFM combined signal is 0.4s, and its minimum detection SNR also reaches -0.5dB.

By comparing the detection probability curves of CW-LFM combined signals in Figure 13(a) and Figure 13(b), it can be seen that, compared with the dual-threshold CFAR energy detection method, the minimum detection SNR of CW-LFM pulse signals with the same pulse width by the constant false alarm detection method in NSRT transform domain is improved by about 5dB.

The above analysis results show that the detection performance of weak sonar pulse signals can be improved to a large extent by using NSRT transform domain constant false alarm detection technology, where the detection performance of LFM signals is the best. The detection performance of complex pulses such as CW-LFM combined signals depends on the LFM component. The longer the LFM component is, the better the detection performance will be.
which the line spectrum is related to the propulsion system, propeller and auxiliary engines. Continuous spectrum [15] reflects the ability distribution of random noise in noise signals. A large number of measurement results show that the continuous spectrum of ship noise has a peak value. Different ship types have different upper limit of peak frequency, but they are all between 200Hz ∼ 400Hz. When the frequency is lower than the upper limit of the spectral peak frequency, the frequency spectrum increases with the frequency to be relatively flat, and it occupies most of the energy of the radiated noise. Line spectra [15] reflect the energy distribution of periodic noises in noise signals, most of which are distributed at low frequencies (below 800Hz), and the frequency and amplitude of the noise line spectrum of different human activities are not the same.

Figure 14 shows the noise spectrum diagram of a class of human moving objects at different speeds. It can be seen that the average power spectrum of ship radiated noise includes both continuous line spectrum and discrete frequency line spectrum. It can be seen that the average power spectrum of noise radiated by human moving objects has both continuous line spectrum and discrete frequency line spectrum.

In APPENDIX, Appendix Figure 1 ∼ Appendix Figure 5 show the normalized frequency spectrum and time-frequency spectrum of radiated noise of several kinds of human activity objects. As can be seen from the spectrum and time-frequency diagrams of Appendix Figure 1 ∼ Appendix Figure 5, the most concentrated part of energy in the spectrum and time-frequency diagrams is the line spectrum component and the peak value of the continuous spectrum of the radiated noise of the human moving object. For sonar pulse signal detection, on the one hand, the continuous spectrum noise of human activity object raises the detection background noise level in energy. On the other hand, line spectrum and continuous spectrum characteristics interfere with the feature identification of sonar pulse signal. Therefore, if the linear spectral composition and the distribution law of the peak position of continuous spectrum can be used to avoid it effectively, the influence of radiated noise of human activity objects interference on the detection of sonar pulse signal can be greatly weakened, and the detection performance of pulse signal under the background of interference can thus be improved. Generally speaking, the radiated noise of far-field underwater human marine moving objects received by passive sonar can still be regarded as band-limited Gaussian white noise. After the NSRT transform of different batches of noise data, the transformation result $UR_{x\theta}(r,\theta)$ at the same pixel point position $(r, \theta)$ can be considered as approximately obeying Gaussian distribution.

Figure 15 shows the pulse signal NSRT transformation domain information under strong interference. According to the above analysis, the results of the NSRT transform of sonar pulse signal containing LFM component show a peak at the coordinate $(r, \theta)$ and $\theta \neq 90^\circ$ of the transformation domain. After the NSRT transform, the linear spectral component of human objects radiate noise presents an interference peak.
FIGURE 14: Spectrum diagram of noise of human moving objects at two different speeds.

FIGURE 15: Pulse signal NSRT transform domain information under strong interference (SNR=-8dB)

FIGURE 16: Time-frequency diagram of fishing boat radiated noise

at the coordinate \((r, \theta)\) and \(\theta \neq 90^\circ\) of the transformation domain. Therefore, it is only necessary to adopt two-dimensional maximum search in the transformation domain to avoid the influence of line spectral interference, to find the peak at \(\theta \neq 90^\circ\) in the transformation domain, and then pulse signals annihilated in the human activity object noise interference can be observed. When pulse signal energy at the peak is taken as the test statistic, and the historical data at the peak is used to estimate the detection background. The pulse signal can be effectively detected by setting reasonable threshold.

Let me see that the time-frequency diagram under the radiated noise of the fishing boat. The performance of the NSRT transform constant false alarm detection algorithm in sonar pulse signal detection under strong interference background is verified by actual data. The sampling rate is normalized to 1. The sliding window of STFT adopts a Chebyshev window with length \(L\) of 1024, and the repetition degree is 973. The strong interference is the radiated noise of a fishing boat, and the noise time-frequency information is shown in Figure 16.

Figure 17 shows the time-frequency diagram of the actual data of LFM and CW-LFM combined signals under strong interference background. The pulse width of LFM signal is 0.5s, the initial frequency \(f_l\) is normalized to 0.27, and the frequency band is normalized to \([0.27,0.33]\). In the CW-LFM combination signal, the LFM signal pulse width is 0.4s, and the frequency band is normalized to \([0.06,0.12]\). The CW signal frequency pulse width is 0.1s, and the center frequency \(f_0\) is normalized to 0.2. There is no interval between the two groups of signals.

Figure 18 shows the constant false alarm detection performance of three kinds of signals in the NSRT transformation domain under strong interference. The reference sliding window length \(L\) estimated by NSRT transform domain CFAR
for background noise detection under strong interference is 8. The false alarm probability $P_{fa}$ is 0.00508. The threshold weighting coefficient $V_T$ calculated by looking up the table is 2.57. According to the actual data, the LFM and CW-LFM combined signals were detected for 1000 times by the normalized STFT-Radon transform constant false alarm, and the change of detection probability $P_d$ of CA-CFAR detection of the two kinds of signals with SNR under strong interference was statistically obtained.

Taking $P_d$ equal to 0.9 as the performance evaluation standard of detector. It can be seen from Figure 18(a) that the constant false alarm detector in the NSRT transformation domain has a very good detection performance for LFM and CW-LFM combined signals in the strong interference background under the actual environments. The lowest detection SNR for LFM signal with pulse width of 0.5s reaches -6dB. The false alarm probability of the two kinds of signals is less than the preset value of $10^{-5}$.

The detection performance of complex pulses such as combined signals is related to the length of pulse LFM. In Figure 18(a), the lowest detection SNR of LFM signal with pulse width of 0.5s is -6dB, while the LFM length of CW-LFM combined signal is 0.4s, and the lowest detection SNR of LFM signal also reaches -1dB.

Now, we compare the detection probability curves of the CW-LFM combined pulse signal in Figure 18(a) and 18(b). Taking the detection probability $P_d$ equal to 0.9 as the evaluation standard, it can be seen that the minimum detection SNR of CW-LFM pulse signals with the same pulse width by the NSRT transform domain constant false alarm detection method under strong interference background is about 5.5dB higher than that of the dual-threshold CFAR energy detection method.

Based on the above analysis results, it can be seen that the use of DNLS method can greatly improve the detection performance of weak sonar pulse signals. Under the interference of strong target radiation noise, the DNLS method still has a strong detection ability for sonar pulse signals containing LFM signals, and the detection ability is better than that of time-domain detection methods such as double-threshold CFAR energy detection.

VII. CONCLUSION
In this paper, a Detection method based on NSRT for Low frequency Sonar pulse signal (DNLS) is proposed, which is a constant false alarm detection method in the NSRT transform domain. In the NSRT transform domain, the local SNR is higher than that in the time domain or frequency domain, and it is distinguishable from the background noise. The detection decision of pulse signal is completed by constructing hypothesis test statistics on target point data. On the other hand, DNLS method solves the detection problems of low frequency background, such as poor stationarity, large fluctuation and more interference. In addition, the corresponding test statistics acquisition method, background noise estimation method and constant false alarm detection threshold calculation method are given. Simulation analysis and actual data processing results show that the performance of DNLS method is better than that of the conventional fixed threshold detection and time domain constant false alarm detection methods, and it has great engineering application value.

ACKNOWLEDGMENT
This work was supported in part by the Shandong Smart Ocean Ranch Engineering Technology Collaborative Innovation Center, in part by the research fund for high-level talents of Qingdao Agricultural University (NO.1119048), in part by the Shandong Agricultural Science and Technology Service Project (NO.2019FW037-4), in part by Shandong Technology Innovation Guidance Program (NO.2020LYXZ023), in part by Horizontal Project (NO.20193702010792), in part by Experimental technical project of Qingdao Agricultural University (NO.SYJK18-01), and in part by Ministry of Education Industry-University Cooperation Collaborative Ed-
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VIII. APPENDIX

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Appendix Figure 1: Radiated noise of small-sized objects in human activities.

Appendix Figure 2: Radiated noise of medium-sized objects in human activities 1.
Appendix Figure 3: Radiated noise of medium-sized objects in human activities 2.

Appendix Figure 4: Radiated noise of medium-sized objects in human activities 3.
Appendix Figure 5: Radiated noise of underwater objects in human activities.