Isothermal magnetosheath electrons due to nonlocal electron cross talk

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Abstract Heating of the various plasma species at the Earth’s collisionless bow shock is not fully understood. Although the total amount of heating is constrained by the one-fluid Rankine-Hugoniot relations in terms of local plasma conditions, the partition of energy between, e.g., electrons and ions, is influenced by particle kinetics which are not considered in the Rankine-Hugoniot approach. Additionally, in this paper we demonstrate the impact of nonlocal effects. Here we model the effects of shock-heated electrons which traverse the magnetosheath to arrive at another point on the bow shock surface. We construct the distribution of electrons immediately downstream of each point on the bow shock from two populations: local solar wind electrons entering the magnetosheath for the first time and previously shocked electrons which have traversed the magnetosheath. We determine the self-consistent cross-shock potential at each point such that the resulting electron distribution gives a downstream plasma which is neutral and has zero parallel current. This leads to very little variation in the downstream electron temperature everywhere around the curved bow shock, in spite of large differences in the total (Rankine-Hugoniot) downstream temperature. Consequently, the partition of heating between electrons and ions at the shock, i.e., the equation of state, contains nonlocal contributions. We show that heating properties at the point where the shock is exactly perpendicular control electron heating throughout the rest of the shock surface.

1. Introduction

The high-speed flow of the solar wind is disrupted upstream of the Earth’s magnetopause by the bow shock, which slows and heats the solar wind plasma. The magnetosheath, which contains this shocked plasma, exhibits a number of interesting properties such as large differences in electron and ion temperatures, large ion temperature anisotropies, and highly non-Maxwellian velocity distributions for both electron and ion populations [Feldman et al., 1983]. Various plasma properties downstream of the bow shock may be determined using the Rankine-Hugoniot jump conditions [de Hoffmann and Teller, 1950; Burgess, 1995] which in addition to using Maxwell’s equations, impose conservation of mass, momentum, and energy across the shock. Several drawbacks to this approach exist, including the assumption of the unknown equation of state for the plasma (often by assuming an adiabatic index of 5/3), the neglect of kinetic phenomena, and the inability to distinguish between heating of the individual populations.

Heating at the bow shock is particularly challenging since the shock is collisionless. The heating mechanism is different for ions and electrons. Ion heating involves reflection of some fraction of the ion population at the shock by both the increasing magnetic field and electric cross-shock potential, followed by phase mixing and scattering to render the spread in velocities irreversible [Paschmann et al., 1982; Sckopke et al., 1983, 1990]. This process accounts for the large amount of heating, as well as the anisotropy of the ion distribution.

Heating of electrons on the other hand is both more subtle and more complex. This is in part due to the fact that the solar wind electron thermal velocity is larger than the bulk solar wind speed. Electrons traverse the shock in both directions, with both static and fluctuating electromagnetic fields playing a role to maintain overall charge neutrality and mediate the momentum and energy fluxes. Early work [Feldman et al., 1983; Scudder et al., 1986a, 1986b] emphasized the role of the DC cross-shock potential in accelerating the incoming portion of the upstream electron distribution, which creates a void in phase space, followed by unspecified turbulent scattering and trapping processes which irreversibly fill such voids.

In its simplest form, the length and time scales are sufficiently long that the electron motion in such DC fields remains adiabatic and energy conserving when viewed in the de Hoffmann-Teller frame in which the
bulk flow is field aligned [de Hoffmann and Teller, 1950; Goodrich and Scudder, 1984]. However, observations within the steepest ramp of the bow shock often reveal electric field spikes [Walker et al., 2004; Bale and Mozer, 2007] and very short scales [Bale et al., 2003; Schwartz et al., 2011] which might break the electron first adiabatic invariant and hence demagnetize the electron motion [Balikhin et al., 1993; See et al., 2013]. That demagnetization is also influenced by high-frequency turbulence within the shock [Bale et al., 2002; Hull et al., 2006; Wilson III et al., 2010]. There is also the suggestion that upstream whistler waves [Sandkvist et al., 2012; Hellinger et al., 2007; Lembege et al., 2004; Wilson III et al., 2012] precondition the electron population and promote nonadiabatic behavior.

Despite these various complications, the observations suggest that adiabatic motion in DC fields provides a zeroth-order description of the overall spread seen in the electron phase space distribution [Lefebvre et al., 2007]. It is this spread which manifests itself as the electron kinetic temperature, although as noted above the infilling of phase space voids and scattering are required to completely describe the irreversible heating. Our primary interest here is not in the details of electron behavior within the shock layer itself but the role of global-scale electron traversals of the entire magnetosheath.

Equipartition of heating at the bow shock is clearly violated. Typically, the change in electron and ion temperatures across the shock differ such that, \( \Delta T_e > \Delta T_i \) [Thomsen et al., 1985; Schwartz et al., 1988] with the electron heating accounting for anything from 10% of the energy budget at high Mach numbers to 50% or more at subcritical shocks. Some systematic trends in this heating ratio have been observed, but it is largely not well understood. Clearly, to resolve this issue a knowledge of the dynamics of each species is needed, which requires knowledge of, for example, the local cross-shock potential. Electron heating due to shock potentials which vary with the change in the magnetic field has been examined by Hull et al. [2001].

Numerous factors may influence the properties of the bow shock. Different locations on the shock have different values of \( \theta_{bow} \), the angle between the solar wind magnetic field \( \mathbf{B} \) and the shock normal \( \mathbf{n} \), and \( U_n = |\mathbf{U} \cdot \mathbf{n}| \), the normal component of the solar wind bulk velocity \( \mathbf{U} \). Also, the number density \( n \) and temperature \( T \) of the solar wind is highly variable, resulting in differing Alfvén Mach numbers \( M_A = U_n \sqrt{\mu_0 m_p n / B} \) and plasma betas, \( \beta = 2 \mu_0 n k_B T / B^2 \), where \( m_p \) is the proton mass, \( \mu_0 \) the vacuum permeability, and \( k_B \) the Boltzmann constant.

In addition to these local parameters, nonlocal influences may also be important. Mitchell et al. [2012] have highlighted the possible influence of electron cross talk, in which electrons, due to their large thermal speed compared with the plasma bulk speed, travel in the magnetosheath from one place on the shock to another. This communication between different locations on the shock by electrons may result in the heating properties at any given location on the shock being influenced by many other, potentially very distant, points on the shock. In other words, electron heating is ultimately a nonlocal problem.

In this paper, we study the effects of electron cross talk on shock properties by following electron trajectories in the magnetosheath between different locations on the shock, and thereby reconstruct a portion of the downstream electron phase space distribution. We then find the cross-shock potential at each point required to ensure that the downstream number density conforms with the value predicted by the one-fluid polytropic Rankine-Hugoniot relations and that the parallel current density in the magnetosheath is zero.

We find that this enables us to determine the cross-shock potential as a function of position on the shock surface. The resulting magnetosheath electron temperature is highly uniform throughout the magnetosheath despite the fact that the total Rankine-Hugoniot heating based on local shock parameters varies considerably with location around the shock. Thus, the electron heating cannot be thought of, for example, as some fraction of the total plasma heating which depends only on the local upstream conditions.

The outline of this paper is as follows: in section 2 we present background material for the calculation, including electron traversals of the shock and propagation along a magnetic field line through the magnetosheath, as well as electron cross talk. In section 3 we discuss the algorithm used in the calculations, the results of which are presented in section 4. Conclusions are presented in section 5.

2. Mathematical Background

In this section we review some of the techniques used in the calculation of the electron distributions in the magnetosheath. We begin by discussing the kinematics of electrons at the bow shock, and how this gives rise to the non-Maxwellian electron velocity distributions in the magnetosheath (in section 2.1). We

\( \beta \) and plasma betas, \( \beta = 2 \mu_0 n k_B T / B^2 \), where \( m_p \) is the proton mass, \( \mu_0 \) the vacuum permeability, and \( k_B \) the Boltzmann constant.
then discuss electron propagation in the magnetosheath, and how this is implemented in our calculation (section 2.2). Finally, we discuss how the motion of electrons allows communication between different locations of the shock, and how this may affect electron heating (section 2.3).

2.1. Electron Shock Traversal

The change in an electron’s velocity as it crosses the bow shock is controlled by the electromagnetic fields within the shock. In reality, the shock fields show quasi-static DC components (foot, ramp, and overshoot) [Scudder et al., 1986a] together with short-scale, highly variable, and fluctuating fields [Bale et al., 2002; Mozer and Sundkvist, 2013, and many references therein]. The small-scale fluctuating fields are reputed to provide the particle scattering required to ensure that the shock is dissipative and irreversible. They may also introduce finite-Larmor radius influences which can lead to chaotic electron motion that does not preserve quantities (e.g., electron energy and magnetic moment) which are constants of the motion in smoother fields [Balikhin et al., 1993; See et al., 2013].

Our primary purpose in this paper is to demonstrate the consequences of electron traversals of the global magnetosheath, rather than the specifics of electron behavior within a fully realistic, time-dependent shock structure. To that end, we assume that the gross behavior of those electrons which cross the shock from the upstream solar wind can be represented by employing the simple, traditional view of the shock fields, namely static fields which preserve electron adiabatic invariants. This model is at least semiquantitatively consistent with observed electron evolution through the bow shock [Lefebvre et al., 2007] and reproduces the observed qualitative features of the phase space distributions well. We will see that the downstream electron distributions are also heavily influenced by electrons arriving from deeper within the magnetosheath so that in any case these transmitted solar wind electrons are only part of the story.

Within this framework of adiabatic electron behavior, it is convenient to work in a frame of reference, known as the de Hoffmann-Teller frame (HTF), in which the shock is at rest and the bulk flow velocity, $U$, is parallel (or antiparallel) to the magnetic field $B$. In this frame, the electric field is dominated by the electron pressure gradient and acts to ensure overall charge neutrality by preventing hot magnetosheath electrons from escaping into the upstream region. More importantly, our assumption of adiabatic electron behavior means that we do not need to consider the details of the shock structure. We need to know the jump in magnetic field across the shock, which we calculate using the Rankine-Hugoniot relations, together with the jump in electrostatic potential, $\Delta \Phi^H$, in the HTF, which we treat as an unknown parameter.

We make use of the peculiar velocity of an electron $w = v - U$ where $v$ is the electron’s velocity and split velocities into components parallel and perpendicular to the magnetic field. In the HTF, by definition $U \parallel \equiv 0$, so that $w \parallel = v \parallel$. Conservation of the first adiabatic invariant demands that

$$\frac{w_{d\perp}^2}{B_d} = \frac{w_{u\perp}^2}{B_u} . \quad (1)$$

Furthermore, from energy conservation (in the HTF)

$$\frac{m_e}{2} \left[ (w_{d\parallel} + U_d^H)^2 + w_{d\perp}^2 \right] + e\Delta \Phi^H = \frac{m_e}{2} \left[ (w_{u\parallel} + U_u^H)^2 + w_{u\perp}^2 \right] . \quad (2)$$

Using (1), this gives an expression for $w_{d\parallel}$

$$w_{d\parallel} = -U_d^H \pm \left[ (w_{u\parallel} + U_u^H)^2 \right. + w_{u\perp}^2 \left. \right]^\frac{1}{2} \left[ 1 - \frac{B_d}{B_u} \right] + \frac{2e\Delta \Phi^H}{m_e} \right]^\frac{1}{2} , \quad (3)$$

where the sign is chosen so that $v_{d\parallel}^H$ and $v_{u\parallel}^H$ have the same sign.

Using Liouville’s theorem, the phase space density following an electron trajectory which has peculiar velocity $w_u = (w_{u\parallel}, w_{u\perp})$ before crossing the shock and $w_d = (w_{d\parallel}, w_{d\perp})$ after will be the same. That is, the phase space distributions of electrons upstream $f_u$ and downstream $f_d$ of the shock satisfy

$$f_d(w_{d\parallel}, w_{d\perp}) = f_u(w_{u\parallel}, w_{u\perp}) . \quad (4)$$
Equation (4) enables us to construct those portions of $f_d$ that are connected by electron trajectories originating upstream. We shall focus here only on these incoming electrons.

The magnetosheath electron distributions described above have regions of phase space, however, which are inaccessible from the upstream [Feldman et al., 1983; Scudder et al., 1986a], which is physically unsatisfactory. Various processes then take place to fill these voids in phase space [Scudder et al., 1986a]. Observationally, this gives rise to flattop distributions, which are largely isotropic in pitch angle. We introduce below a prescription for creating this flattopped feature.

We do not attempt to model these void-filling processes in detail. They will involve nonadiabatic and scattering mechanisms which lead to a violation of Liouville's theorem and to the shock irreversibility which must occur to increase the entropy. However, the envelope of the electron phase space distribution is of primary concern here, and this simple model enables us to treat the electron shock traversal in a uniform and self-consistent way within the global context of interest.

2.2. Propagation of Electrons Through the Magnetosheath

We shall need to follow the motion of electrons through the magnetosheath to some distant location. To do so, we make two assumptions. Firstly, we assume that electron propagation is scatter free. Mitchell et al. [2012] show that the electron distributions observed at widely separated positions within the sheath along the same field line are not inconsistent with this assumption. The present work would actually be unchanged if scattering occurs, e.g., within the flattopped portions of the distribution, provided that, as observed, the overall shape of the distribution is maintained. Secondly, since the electrons in which we are interested travel close to the bow shock, we assume that the magnetic field and bulk velocity in the magnetosheath are the same as those predicted by the Rankine-Hugoniot jump conditions at the nearby shock. Following the technique of Mitchell et al. [2012] enables us to map out in particular the magnetic field lines along which an electron travels.

As in electron motion across the shock, we assume conservation of the first adiabatic invariant (1), which is reasonable here due to the large macroscopic scale lengths in the magnetosheath compared to the electron gyroradius, and to our assumption that electron scattering here is not important. Given the time-stationary nature of our approach and the large length scales associated with the magnetosheath, the only electric field here is the motional one, i.e., $\mathbf{E} = -\mathbf{U} \times \mathbf{B}$ which has a negligible influence on the energy of an electron. Apart from the corresponding $\mathbf{E} \times \mathbf{B}$ drift $\mathbf{v}_D = \mathbf{E} \times \mathbf{B}/B^2$, we neglect other particle drifts, such as the...
Figure 3. Electron phase space distributions in the bulk flow frame calculated at $y_V = 10.05 R_E$, $\theta_{Bn} = 78^\circ$, and $\Delta \Phi_B = 265$ V immediately downstream of the bow shock. The bottom half (pitch angles between $0^\circ$ and $180^\circ$) shows the composite distribution made up of cross-shock potential-accelerated solar wind electrons (red portion with pitch angles $0^\circ$–$30^\circ$) and electrons which have traversed the magnetosheath from elsewhere on the bow shock (yellow portion with pitch angles $90^\circ$–$180^\circ$). Note the hole in phase space at pitch angles $90^\circ$ which is due to regions of phase space that are inaccessible from either the solar wind or farther downstream. Our method “flattens” this distribution by cutting the solar wind peak and filling in the holes to a constant height $F$, resulting in the pitch angle distribution shown in the top portion of the figure. This step involves iterating to find the values of the potential and height $F$ to match the physical requirements of density compression and zero electric current (see text). Slices of these distributions are shown in Figure 4. Model results shown here and in the following phase space figures are taken for the case $T_{90} = 100$ eV.

Gradient and curvature drifts, which are smaller. In some general frame in which the global shock is at rest, such as the GSE frame, energy conservation gives

$$ (w_{\parallel} + U_{\parallel}^2)^2 = \left( w_{\parallel 0} + U_{\parallel 0}^2 \right)^2 + w_{\perp 0}^2 \left( 1 - \frac{B}{B_0} \right) + U_{\perp 0}^2 - U_{\perp 0}^2, $$

where the subscript 0 denotes a quantity at some earlier point on the electron’s trajectory.

The trajectory finding technique allows two types of reflection. When the magnetic field $B$ becomes sufficiently large that $v_{\parallel}^2$ would become negative, the electron suffers magnetic mirroring, in which case $v_{\parallel}$ is replaced by $-v_{\parallel}$. Additionally, magnetosheath electrons arriving at the shock may have insufficient energy to overcome the cross-shock electric potential (in the local de Hoffmann-Teller frame). In this latter case the HTF parallel velocity is multiplied by $-1$. We note here that some electrons reflect multiple times before arriving at the point of the bow shock under investigation.

This trajectory finding technique allows magnetosheath electrons which arrive at the bow shock to be traced backward to the original location at which they crossed the bow shock. Specifically, if an electron approaches the shock in the magnetosheath with peculiar velocity $w_d = (w_{d\parallel}, w_{d\perp})$, its trajectory can be traced backward until the point at which it crossed the shock is reached, and its peculiar velocity at that point $w_{d0} = (w_{d0\parallel}, w_{d0\perp})$ may be found using (5). The electron’s velocity before it entered the magnetosheath $w_u = (w_{u\parallel}, w_{u\perp})$ may be found in terms of $w_{d0}$ using (1) and (3). This means we find $w_u$, the electron’s velocity before it entered the magnetosheath, in terms of $w_d$, the peculiar velocity after it crossed the bow shock and traveled some large distance (e.g., $\sim 20$ Earth radii) through the magnetosheath and encountered the shock a second time. The phase space density at this final location then satisfies Liouville’s theorem, so that

$$ f_u(w_{d\parallel}, w_{d\perp}) = f_{d0}(w_{d0\parallel}, w_{d0\perp}) = f_d(w_{d\parallel}, w_{d\perp}). $$

The velocity distribution of all electrons moving toward some point on the shock from within the magnetosheath can be found using this method.
Figure 4. Slices of the magnetosheath electron distributions shown in Figure 3 along (solid line) and perpendicular (dashed line) to the magnetic field. (bottom) Two electron populations: the cross-shock potential-accelerated solar wind peak (visible as the solid curve in the right-hand portion of the panel) and electrons arriving at the point in question from other points on the bow shock after traversing the magnetosheath (left-hand portion). Also shown as the thin dotted curve is the upstream isotropic kappa distribution used in the model (note that in the upstream region the peak of the distribution lies below 10 eV). (top) The electron distribution after our “flattening” process (see text). The dotted curve here shows the parallel cuts of magnetosheath distributions measured by the Cluster 2 spacecraft on 13 January 2008 at 00:30.

2.3. Electron Cross Talk

The electron distribution just inside the magnetosheath may be divided into two populations: (1) Electrons which have just crossed the shock and are therefore moving deeper into the magnetosheath and (2) electrons which crossed the shock at some distant point and traveled through the magnetosheath before encountering the shock for a second time at the point in question. The phase space density of the latter population of electrons is controlled by the shock at the distant locations where they originally entered the magnetosheath, as described in section 2.2. The electrons entering the magnetosheath for the first time are controlled by the local cross-shock potential and obey (4).

The point at which a magnetic field line first encounters the shock cannot receive electrons from other locations. At this point the shock is exactly perpendicular, i.e., \( \theta_{\text{int}} = 90^\circ \). As the field line convects along with the solar wind, at any one time it threads the shock at two points. The points which the field line has previously threaded then define a curve on the shock surface in the \( \mathbf{V} - \mathbf{B} \) plane (see Figure 1). Any electrons arriving at the current-threading points from within the magnetosheath must have entered the magnetosheath at some point on this curve. We see therefore that as the field line convects away from the perpendicular point, the electron population tied to that field line is influenced by a growing portion of the bow shock. In the following section we make use of this idea to develop an algorithm to calculate \( \Delta \Phi^H \) and the downstream electron temperature \( T_{\text{de}} \) as a function of position on the shock surface in the \( \mathbf{V} - \mathbf{B} \) plane.

3. Calculation Methodology

As described in the previous section (2.3), a magnetic field line that intersects the bow shock will in general thread the shock in two locations. As sketched in Figure 2, any electrons which encounter the shock at either of these two points from within the magnetosheath, and which therefore may be able to influence \( \Delta \Phi^H \) at these points, must have originated on the curve of influence shown as the black portion of the bow shock in Figure 2. In this section we outline the algorithm used to calculate \( \Delta \Phi^H \) and thence the...
Figure 5. (a) The bow shock surface in the \( \mathbf{V} - \mathbf{B} \) plane in which \( \Delta \Phi^H \) was calculated. Black and gray curves are on opposite sides of the perpendicular point. (b) The downstream electron temperature as a function of position, where the downstream distribution at the perpendicular point has a prescribed temperature \( T_{90} \) of 160 eV, 100 eV, and 50 eV, corresponding to full, dashed, and dotted curves, respectively. The thick curve shows the total temperature predicted by the Rankine-Hugoniot relations. (c) The ratio of the change in electron temperature over the shock to the change in total temperature (where the latter is found using Rankine-Hugoniot temperature). (d) The cross-shock potential for each case is shown.

between the threading points, and be close to points at which \( \Delta \Phi^H \) has previously been determined.

5. Using the previously determined values, we interpolate \( \Delta \Phi^H \) at the crossing points \( A_i \). Using the procedure outlined in section 2.2, we can now map forward from the upstream distribution, through all such crossing points \( A_i \) and through the magnetosheath to construct the phase space distribution of electrons approaching the shock from within the magnetosheath at the threading point \( A \). An example pitch angle distribution of these particles can be seen in Figure 3. The arriving magnetosheath particles form the yellow patch on the left side of the bottom half of Figure 3.

6. We now have the phase space distribution of the electrons arriving at \( A \) from within the magnetosheath. Next we need to consider electrons crossing the shock at \( A \) and entering the magnetosheath for the first time. We use Liouville mapping (equation (4)) from the upstream distribution to initiate this process. At this stage, the value of the cross-shock potential \( \Delta \Phi^H \) is an unknown and free parameter. The action of the potential \( \Delta \Phi^H \) accelerates the peak of the solar wind distribution into the magnetosheath and creates a hole or void in phase space at energies < \( e \Delta \Phi^H \) and shaped by the influence of magnetic moment.
conservation on electron trajectories [Scudder et al., 1986a]. These features can be seen in the right-hand portion of Figure 3 (bottom). We fill this hole by cutting $f_d(v)$ at some height $F$, mimicking the observed flattop nature of magnetosheath electrons and filling the hole to height $F$. This height represents a second free parameter in this step of our method. This process yields the pitch angle distribution shown in Figure 3 (top portion), which displays strong similarities to observed magnetosheath electron distributions. Figure 4 shows corresponding slices of the magnetosheath distributions prior to cutting/filling (bottom) and after cutting and filling holes to a height $F$ (top). The values of $F$ and $\Delta \Phi_H$ are now chosen so that, after this “flattening” exercise, (a) the number density agrees with the one-fluid Rankine-Hugoniot relations and (b) the electric current density in the magnetosheath satisfies $J_\| = 0$.

7. Steps 4 to 6 are repeated for the other threading point $B$.

8. We now go back to task 3 and repeat the procedure, thereby sequentially moving the threading points further away from $\theta_{bn} = 90^\circ$ and building a map of $\Delta \Phi_H$ as a function of position. In this way, we follow the evolution of the electron population on this particular field line and determine self-consistently the cross-shock potential and downstream electron distributions around the shock surface.

In summary, our solution rests on one free parameter, $T_{90}$, and adjusts two parameters, $\Delta \Phi_H$ and $F$, to match zero-current and density compression conditions at successive points on the shock in the $\mathbf{V} - \mathbf{B}$ plane. For this purpose, we employ one-fluid polytropic ($\gamma = 5/3$) Rankine-Hugoniot relations. These relations are clearly a substitute for a more complete description of the plasma equation of state. However, they provide an indication of the gross downstream state. We also ignore complications of internal shock structure and dynamics. To the extent that our main conclusion that electron heating is nonlocal holds, it is clear that formulating a better equation of state and incorporating more complex shock structure while retaining our global perspective would be challenging.

4. Results

In this section we examine the results of the calculation outlined in section 3. We begin by discussing the downstream temperature predicted by these calculations in section 4.1. In section 4.2 we examine the predicted cross-shock potential. Finally, in section 4.3, we examine a scenario with a different upstream geometry to see the effects of such a variation.

4.1. Electron Temperature

In this and the following section (4.2) we examine electron heating using a shock with the following parameters: the upstream bulk velocity measured in the GSE frame is $\mathbf{U} = [-530, 30, 0]$ km s$^{-1}$, magnetic field $\mathbf{B} = [-3, 3, 4]$ nT, number density $n = 4.5$ cm$^{-3}$, and temperature $T_i = T_e = T = 3 \times 10^5$ K (26 eV). These values correspond to solar wind conditions on 13 January 2008 at 00:30. Parallel cuts of the magnetosheath

Figure 6. (a) Downstream electron temperature $T_{de}$ and (b) cross-shock potential $\Delta \Phi_H$ versus the jump in ram energy $\frac{1}{2} m_e (U_{un}^2 - U_{dn}^2)$ for the solar wind conditions used in Figure 5. The different curves show results for calculations with the same values of $T_{90}$ as in Figure 5. Note that for given solar wind conditions, there are usually two points along the bow shock which have the same jump in ram energy. The fact that the curves in this figure are double valued reveals that in general those two points do not have the same conditions or heating.
Figure 7. As in Figure 6 showing (a) $T_{de}$ and (b) $\Delta \Phi^H$ but versus the local shock geometry $|\cos \theta_{Bn}|$.

by the electron temperature and Rankine-Hugoniot temperature displayed in Figure 5b. We see that the fraction of the total heating that goes to electrons shows considerable variation. This is due to the large change in the total amount of heating, which in turn depends upon the incident ram energy, while the downstream electron temperature shows little variation with position. By implication, the ion heating must vary around the shock to make up the difference between total and electron heating.

Figure 8. As in Figure 4 but taken at a location close to the tangent point.
The relationship between the electron temperature and the change in ram energy across the shock (using the Rankine-Hugoniot value for the downstream velocity) is displayed in Figure 6a. We see little relationship between $T_{de}$ and the jump in ram energy. This is particularly evident at ram energy jumps of 600–700 eV and 950–1100 eV, where curves for the same $T_{90}$ are double valued, with quite different $T_{de}$. We conclude, therefore, that when we look globally at the heating around the bow shock for given solar wind conditions, the downstream electron temperature is not strongly controlled by the variation in the local upstream ram energy due to the local orientation of the shock.

We would expect that the downstream electron temperature at the tangent point (our only free parameter) is controlled by its local incident ram energy. This reconciles to some extent the apparent discrepancy between the relatively weak dependence of $\Delta T_e$ on ram energy discussed above and earlier studies indicating a primary statistical correlation between the electron heating and the change in ram energy [Thomsen et al., 1987; Schwartz et al., 1988].

Figure 7a shows $T_{de}$ plotted against $|\cos \theta_{bn}|$. Although the change in $T_{de}$ is small, there does appear to be some systematic relationship between these two quantities, with the electron temperature being slightly larger near the tangent point. This variation may in part be understood by noting that as a magnetic field line in the magnetosheath convects downstream, $\theta_{bn}$ at each of the threading points decreases from 90°, while the length of the portion of the field line inside the magnetosheath increases. We therefore expect some cooling of the electron population tied to the field line and trapped in the magnetosheath by the cross-shock potential at the bow shock, as suggested by Feldman et al. [1983].

### 4.2. Cross-Shock Potential

The approximately constant electron temperature resulting from electron cross talk is achieved by variations in the cross-shock potential as a function of position. We examine the cross-shock potential in this section, and its relationship to the ram energy and $\theta_{bn}$.

Figure 5d shows the cross-shock potential as a function of position on the bow shock surface for the calculations described in section 4.1. For all values of $T_{90}$ we see that $\Delta \Phi_H$ increases sharply near the tangent point, while further from the tangent point the potential tends toward some constant value. This constant value increases with $T_{90}$.

We examine first the relationship between $\Delta \Phi_H$ and the jump in ram energy, displayed in Figure 6b. As was the case with the electron temperature, we see no clear relationship between these two quantities and,
Figure 10. As in Figure 6 but for the magnetic field orientation used for the calculations shown in Figure 9.

Indeed, find that two points on the bow shock with the same change in ram energy often have significantly different cross-shock potentials. This demonstrates the strong influence of nonlocal effects due to electron cross talk, coupled to a lesser extent to the different magnetic field geometries at these two locations. At first sight, this result contradicts previous work [e.g., Hull et al., 2000] showing a correlation between $\Delta \Phi_H$ and local changes in ram energy. However, those estimates of $\Delta \Phi_H$ are based typically on electron fluid considerations and hence linked directly to observed electron heating, with correlations resulting from the statistics of shock crossings under varying solar wind conditions. By contrast, the lack of correlation in Figure 6b corresponds to variations around the bow shock under a single set of fixed solar wind conditions and to the inclusion of kinetic aspects not contained within the fluid treatment. There would be an underlying correlation amongst $\Delta \Phi_H$, $\Delta T_e$, and $\Delta U_n^2$ if we raised the value of $U_u$. There is also a positive dependence of both $\Delta \Phi_H$ on $T_{90}$ which can be seen in Figure 5.

What is not clear is whether this apparent discrepancy with previous work is a consequence of the electron fluid determination of the potential jump or the simplicity of the global model we have constructed.

Figure 7b, on the other hand, does show a relationship between $\Delta \Phi_H$ and $\cos \theta_B$. In particular, close to the tangent point we see that $\Delta \Phi_H \sim |\cos \theta_B|^{-1}$. We shall see that this is not inconsistent with (2).

We consider the point in phase space at the center of the distribution, i.e., $w_{d\perp} = 0, w_{d\parallel} = 0$. This trajectory maps to $w_{d\perp} = 0, w_{d\parallel} = e V_{thd}$, where $V_{thd}$ is the downstream thermal speed and $e \sim O(1)$ parametrizes the trajectory. Then (2) can be written

$$\Delta \Phi_H = \frac{m_e}{2e} \left[ 2e V_{thd} U_{d\parallel} + e^2 V_{thd}^2 + U_{d\perp}^2 - U_{d\parallel}^2 \right].$$

(7)

Rankine-Hugoniot considerations (see (9) and (10)) reveal that neither of the terms $e^2 V_{thd}^2$ nor $\Delta (U_d^2) = U_{d\parallel}^2 - U_{d\perp}^2$ diverge as $\theta_B \rightarrow 90^\circ$. The first term in brackets does diverge, however, giving

$$\Delta \Phi_H|_{\theta_B \rightarrow 90^\circ} \approx \frac{m_e e V_{thd} U_{d\parallel}}{e}.$$

(8)

The normal and tangential components of $U_{d\parallel}$ are

$$U_{d\parallel} = -\frac{|U_{\parallel}|}{r},$$

(9)

and

$$U_{d\perp} = -|U_{\parallel}| \tan \theta_B \left( \frac{M_A^2 - \cos^2 \theta_B}{M_A^2 - r \cos^2 \theta_B} \right).$$

(10)

where $r$ is the shock density compression ratio. Therefore,

$$|U_{d\parallel}|^2 = \frac{U_{\parallel}^2}{r^2} + U_{\parallel}^2 \tan^2 \theta_B \left( \frac{M_A^2 - \cos^2 \theta_B}{M_A^2 - r \cos^2 \theta_B} \right)^2.$$

(11)
Thus, as $\theta_{Bn} \rightarrow 90^\circ$, $|U_H^d|^2$ diverges as

$$|U_H^d|^2 \rightarrow \frac{U_{un}}{\cos \theta_{Bn}}.$$  \hspace{1cm} (12)

This, together with (8) shows that $\Delta \Phi^H \sim |\sec \theta_{Bn}|$ near $\theta_{Bn} = 90^\circ$, consistent with Figure 7b.

Although the potential increases significantly near the tangent point, our results (e.g., Figure 5) suggest that the electron temperature is much less than these values would imply if, as has been previously assumed [Schwartz et al., 1988], the spread in velocity or flattop edge corresponded directly to the value of the HTF potential.

In Figure 8 we show cuts of the distributions near to the tangent point in the same format as those under more oblique geometries shown in Figure 4. Although some aspects are more extreme near the tangent point, the overall characteristics are qualitatively and quantitatively similar at the two locations. Although the electrons suffer a much larger potential drop near the tangent point, the HTF transformation velocity plays a much bigger role there so that first term in equation (7) breaks the simple relation between the electron spread and HTF potential.

4.3. Effects of Interplanetary Field Geometry

In this section we present results of a calculation similar to that in sections 4.1 and 4.2, however with a different interplanetary magnetic field ($B_u = [-1, -3, 4] \text{ nT}$), with other parameters $n_u$, $T_u$ and $U_u$ unchanged.

The results of this calculation are presented in Figure 9, which may be compared with Figure 5. We see qualitatively similar behavior in both calculations: (i) the downstream electron temperature is approximately constant, with a small increase near the tangent point, (ii) the fraction of the total heating that goes to electron heating is smallest behind the nose due to the combination of the constant electron temperature, and the large increase in total temperature at the nose, and (iii) the cross-shock potential increases sharply near $\theta_{Bn} = 90^\circ$ and tends toward a constant value away from the tangent point.

Figure 10 shows the relationship between $\Delta \Phi^H$ and $T_{de}$ and the jump in ram energy. Similarly, the dependence upon $\theta_{Bn}$ is displayed in Figure 11. We see that, as in the previous results shown in Figure 7, there appears to be a dependence of both $\Delta \Phi^H$ and $T_{de}$ on $|\cos \theta_{Bn}|^{-1}$ near $\theta_{Bn} = 90^\circ$. As before, this is due to electron cross talk creating an approximately isothermal electron temperature along the magnetic field line. We see no clear relationship between either $T_{de}$ or $\Delta \Phi^H$ and the local ram energy, which is consistent with the nonlocal influences on shock heating properties due to electron cross talk.

5. Conclusion

Calculations determining the motion of electrons in the magnetosheath between different locations on the bow shock have enabled us to construct a model demonstrating the nonlocal influences on electron heating at the curved bow shock. This calculation requires assumptions about the shape of the downstream electron distribution and downstream electron temperature near the point on the shock where $\theta_{Bn} = 90^\circ$. We make use of the one-fluid Rankine-Hugoniot shock jump conditions to calculate the downstream fields and also the jump in plasma density. Once these assumptions have been made, then electron...
cross talk combined with the condition that the plasma carries zero parallel electric current imposes a value on the cross-shock potential jump, \( \Delta \Phi^{\parallel} \), around the shock (in the same \( V - B \) plane). This is the manifestation of the phase space influence of electrons arriving from deeper in the magnetosheath and hence which suffered shock “heating” elsewhere. These electrons set a scale in velocity which must be balanced by incoming solar wind electrons to maintain zero current. In our model these incoming electrons are treated via adiabatic particle motion and Liouville’s theorem in static shock fields.

It would be possible, perhaps, to parameterize the influence of fluctuating fields and nonadiabatic behavior to incorporate such effects into this overall global framework. Such complications would still need to adjust to match the zero-current condition and hence match the velocity scale imposed by magnetosheath electrons. We have seen that this scale is set initially by electron heating at the tangent point. Thus, we would expect that a more sophisticated, and largely ad hoc, treatment of the details at the shock itself would not change our primary qualitative conclusions.

The primary result of this work is that the electron temperature is highly uniform along a magnetic field throughout the magnetosheath. This means that, to a large extent, electron heating at the point where \( \theta_{\text{bs}} = 90^\circ \) controls the electron heating throughout the rest of the shock. A small amount of cooling is observed as the field line travels deeper into the magnetosheath. This is qualitatively consistent with adiabatic cooling of electrons trapped in the magnetosheath as the length of the portion of the field line between the two bow shock threading points increases [Feldman et al., 1983], although the continuous addition of hot electrons at the shock limits the cooling.

The small variation in \( \Delta T_e \) is quite different to the very large variation around the bow shock in total plasma heating, as demanded by the Rankine-Hugoniot relations. This demonstrates how electron cross talk controls the downstream electron temperature, with processes near \( \theta_{\text{bs}} = 90^\circ \) determining the temperature at other locations on the shock. The nonlocal nature of the electron heating problem implies that the ion heating is also influenced by nonlocal effects in order that the total heating matches the overall shock energy balance. Although ion heating is usually stronger [Thomsen et al., 1987; Schwartz et al., 1988], the details are governed by the shock electromagnetic fields which are in turn supported by the electron physics. Ion dynamics, including the reflection of incident solar wind ions and the subsequent development of anisotropies, are sensitive to the details of the shock field structure. Thus, kinetic processes and heating, both ion and electron, at the bow shock, and by extension all curved shocks, cannot be predicted purely on the basis of the traditional local shock parameters.

We report separately [Mitchell and Schwartz, 2013] observational signatures consistent with the constant (along field lines) magnetosheath temperature. That work also reveals that the plasma parameters at the tangent point under varying solar wind conditions order the observed magnetosheath electron temperature much better than the local values. Future work should attempt to determine what governs the electron heating properties at the tangent point, which will therefore lead to an understanding of electron heating along the rest of the bow shock.

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