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Pseudorapidity and transverse momentum dependence of flow harmonics in pPb and PbPb collisions

A. M. Sirunyan et al.*
(CMS Collaboration)

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Measurements of azimuthal angular correlations are presented for high-multiplicity pPb collisions at \( \sqrt{s_{\text{NN}}} = 5.02 \text{ TeV} \) and peripheral PbPb collisions at \( \sqrt{s_{\text{NN}}} = 2.76 \text{ TeV} \). The data used in this work were collected with the Compact Muon Solenoid (CMS) detector at the European Organization for Nuclear Research (CERN) Large Hadron Collider (LHC). Fourier coefficients as functions of transverse momentum and pseudorapidity are studied using the scalar product method; four-, six-, and eight-particle cumulants; and the Lee-Yang zero technique. The influence of event plane decorrelation is evaluated using the scalar product method and found to account for most of the observed pseudorapidity dependence.

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I. INTRODUCTION

High energy density matter with quark and gluon degrees of freedom, a state of matter known as the quark-gluon plasma (QGP), is created in relativistic heavy ion collisions at the Brookhaven National Laboratory (BNL) Relativistic Heavy Ion Collider (RHIC) and at the CERN LHC [1–6]. The energy density created in the initial heavy ion collision is azimuthally nonuniform as a consequence of the collision geometry and its fluctuations. Interactions among constituents in the QGP convert this nonuniformity into an observable anisotropy in the final-state particle momentum distribution. The azimuthal angle distribution of emitted particles can be characterized by its Fourier components [7]. In particular, the second and third Fourier components, \( v_2 \) and \( v_3 \), known as elliptic and triangular flows, respectively, most directly reflect the medium response to the initial collision geometry and its fluctuations [8]. The magnitudes of these components provide insights into the fundamental transport properties of the medium [9–11]. Two-particle correlations in the azimuthal angle (\( \phi \)) and pseudorapidity (\( \eta \)) differences between the two particles (\( \Delta \phi \) and \( \Delta \eta \)) have played a vital role in the observation of the azimuthal anisotropies [12–19]. These particle correlations are characterized by a pronounced structure at \( |\Delta \phi| \approx 0 \) extending over a large \( \Delta \eta \) range (referred to as the “ridge”). In collisions between two heavy nuclei, such as CuCu and AuAu collisions at RHIC [12–14] and PbPb collisions at the LHC [16–19], these long-range correlations are often attributed to the collective flow from a strongly interacting, expanding medium [20,21]. This is corroborated by multiparticle correlations, suggesting a hydrodynamic origin for the observed azimuthal anisotropies [22].

The lightest systems in which ridge-like structures have been observed include high-multiplicity final states in \( pp \) [23–27] and pPb [27–32] collisions at the LHC. Evidence of such long-range correlations is also observed at a nucleon-nucleon center-of-mass energy of \( \sqrt{s_{\text{NN}}} = 200 \text{ GeV} \) in \( pAu \) [33], \( dAu \) [34–36], and \( ^3\text{He}Au \) collisions [37] at RHIC. In pPb collisions, the overall strength of the correlation is observed so far to be significantly larger than in \( pp \) collisions, and is comparable to that found in peripheral PbPb collisions [38,39].

Both the ATLAS [40,41] and CMS [38] experiments have measured significant elliptic flow coefficients in pPb collisions at \( \sqrt{s_{\text{NN}}} = 5.02 \text{ TeV} \) using four-particle correlations based on the cumulant method [42]. The long-range correlations persist in measurements that study the correlation among six or more particles in pPb collisions [26,39,43] and in measurements of four-particle and six-particle correlations in \( pp \) collisions at \( \sqrt{s} = 13 \text{ TeV} \) [26,41]. Four-particle correlation measurements in the \( dAu \) system at \( \sqrt{s_{\text{NN}}} = 200, 62.5, 39, \) and 19.6 GeV by the PHENIX Collaboration and a six-particle correlation measurement by the same collaboration at \( \sqrt{s_{\text{NN}}} = 200 \text{ GeV} \) also find significant elliptic flow coefficients [44].

In combination, these measurements support a collective origin of the azimuthal correlations, and have raised the possibility that a QGP droplet might be formed in small-system collisions exhibiting fluidlike behavior [28–30,39,45]. If such a mechanism can be confirmed, it will significantly extend the range of system size for which the QGP medium is considered to exist. However, the origin of the ridge phenomenon in small collision systems is still being actively investigated. In addition to a hydrodynamic origin [45,46], possible alternative explanations include gluon saturation in the initial interacting state of the protons [47,48], multiparton interactions [49], and the anisotropic escape of partons from the surface of the interaction region [50].

To provide further constraints on the theoretical understanding of the azimuthal anisotropies in different collision

*Full author list given at the end of the article.

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systems, this paper presents results on the pseudorapidity and transverse momentum dependence of the flow harmonics in pPb and PbPb collisions. The $v_2$ coefficients are measured using the four-, six-, and eight-particle $Q$ cumulants [51], the Lee-Yang zeros (LYZ) [52], and the scalar product methods [53,54]. The $v_2$ coefficients, which result from fluctuations in the collision geometry, are studied with the scalar product method. Within the hydrodynamic picture, the longer lifetime of the medium on the Pb-going side in PbPb collisions is not at rest with respect to the laboratory frame. Massless particles emitted at $\eta_{\text{cm}} = 0$ in the nucleon-nucleon center-of-mass frame will be detected at $\eta = 0.465$ in the laboratory frame. Unless otherwise stated, all pseudorapidity reports in this paper are referred to with respect to the laboratory frame. A sample of $\sqrt{s_{NN}} = 2.76$ TeV PbPb data collected during the 2011 LHC heavy ion run, corresponding to an integrated luminosity of $2.3 \mu$b$^{-1}$, is also analyzed for comparison purposes. The triggers, event selection, and track reconstruction are identical to those used in Ref. [38].

In order to select high-multiplicity pPb collisions, dedicated high-multiplicity triggers were implemented using the CMS level-1 and high-level trigger (HLT) systems. The online track reconstruction at the HLT is based on the three layers of pixel detectors, and requires a track origin within a cylindrical region of length 30 cm along the beam axis and radius 0.2 cm perpendicular to the beam axis, centered at the nominal interaction point. For each event, the vertex reconstructed with the highest number of pixel tracks is selected. The number of pixel tracks ($N_{\text{tk}}$) with $|\eta| < 2.4$, $p_T > 0.4$ GeV/$c$, and a distance of closest approach to this vertex of 0.4 cm or less, is determined for each event. Several high-multiplicity ranges are defined with prescale factors that are progressively reduced until, for the highest multiplicity events, no prescaling was applied.

In the offline analysis, hadronic collisions are selected by requiring a coincidence of at least one HF tower containing more than 3 GeV of total energy on either side of the intersection region. Only towers within $3.0 < |\eta| < 5.0$ are used in order to avoid the edges of the HF acceptance. The pPb interactions were simulated with both the EPOS LHC [58] and the HIJING 1.383 [59] event generators. The requirement of having at least one primary particle with total energy $E > 3.0$ GeV in each of the $\eta$ ranges $-5.0 < \eta < -3.0$ and $3.0 < \eta < 5.0$ is found to select 97–98% of the total inelastic hadronic cross section.

Events in the offline analysis are also required to contain at least one reconstructed primary vertex within 15 cm of the nominal interaction point along the beam axis ($z_{\text{vtx}}$) and within 0.15 cm transverse to the beam trajectory. At least two reconstructed tracks are required to be associated with the primary vertex. Beam-related background is suppressed by rejecting events for which less than 25% of all reconstructed tracks pass the track selection criteria for this analysis. The pPb instantaneous luminosity provided by the LHC in 2013 resulted in an approximately 3% probability of at least one additional interaction occurring in the same bunch crossing. Such pileup events become more significant as the event multiplicity increases. Following the procedure developed in Ref. [38] for rejecting pileup events, a 99.8% purity of single-interaction events is achieved for the pPb collisions belonging to the highest multiplicity class of this analysis.

The CMS “high-quality” tracks described in Ref. [60] are used in this analysis. Additionally, a reconstructed track is only considered as a candidate track from the primary vertex if the significance of the separation along the beam axis ($z$) between the track and the best vertex, $d_z/\sigma(d_z)$, and the significance of the track impact parameter measured transverse to the beam, $d_T/\sigma(d_T)$, are each less than 3. The

### II. THE CMS EXPERIMENT

A detailed description of the Compact Muon Solenoid (CMS) detector can be found in Ref. [56]. The results in this paper are mainly based on the silicon tracker detector and two hadron forward calorimeters (HF) located on either side of the tracker. Situated inside the 3.8-T field of a superconducting solenoid, the silicon tracker consists of 1 440 silicon pixel and 15 148 silicon strip detector modules. It measures charged particles within the range of $|\eta| < 2.4$ and provides an impact parameter resolution of $\approx 15 \mu$m and a $p_T$ resolution better than 1.5% at $p_T \approx 100$ GeV/$c$. Electromagnetic (ECAL) and hadron (HCAL) calorimeters are also located inside the solenoid and cover the range of $|\eta| < 3.0$. The HCAL has sampling calorimeters composed of brass and scintillator plates. The ECAL consists of lead-tungstate crystals arranged in a quasi-projective geometry. Iron–quartz-fiber Cherenkov HF cover the range $2.9 < |\eta| < 5.2$ on either side of the interaction region. The HF calorimeters, which are used in the scalar product analysis, are azimuthally subdivided into 20° modular wedges and further segmented to form $0.175 \times 10°$ (Δ$\eta$×Δ$\phi$) towers. The CMS detector response is determined through Monte Carlo (MC) studies using GEANT4 [57].

### III. EVENT AND TRACK SELECTION

The pPb data set corresponds to an integrated luminosity of 35 nb$^{-1}$. The beam energies were 4 TeV for protons and 1.58 TeV per nucleon for lead nuclei, resulting in $\sqrt{s_{NN}} = 5.02$ TeV. The beam directions were reversed during the run. The results from both beam directions are combined using the convention that the proton-going direction defines positive pseudorapidity. As a result of the energy difference between the colliding beams, the nucleon-nucleon center-of-mass frame in the pPb collisions is not at rest with respect to the laboratory frame. Massless particles emitted at $\eta_{\text{cm}} = 0$ in the nucleon-nucleon center-of-mass frame will be detected at $\eta = 0.465$ in the laboratory frame. Unless otherwise stated, all pseudorapidity reports in this paper are referred to with respect to the laboratory frame. A sample of $\sqrt{s_{NN}} = 2.76$ TeV PbPb data collected during the 2011 LHC heavy ion run, corresponding to an integrated luminosity of $2.3 \mu$b$^{-1}$, is also analyzed for comparison purposes. The triggers, event selection, and track reconstruction are identical to those used in Ref. [38].

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relative uncertainty in $p_T$, $\sigma(p_T)/p_T$, is required to be less than 10%. To ensure high tracking efficiency and to reduce the rate of incorrectly reconstructed tracks, only tracks within $|\eta| < 2.4$ and with $p_T > 0.3$ GeV/$c$ are used in the analysis. The entire PbPb data set is divided into classes of reconstructed track multiplicity, $N_{\text{trk}}^{\text{offline}}$, where primary tracks with $|\eta| < 2.4$ and $p_T > 0.4$ GeV/$c$ are counted. A different $p_T$ cutoff of 0.4 GeV/$c$ is used in the multiplicity determination because of the constraints on the online processing time for the HLT. The multiplicity classification in this analysis is identical to that used in Ref. [38], where more details are provided, including a table relating $N_{\text{trk}}^{\text{offline}}$ to the fraction of minimum bias triggered events.

The peripheral PbPb data collected during the 2011 LHC heavy ion run with a minimum bias trigger are also reanalyzed using the same event selection and track reconstruction as for the present PbPb analysis. A description of the 2011 PbPb data set can be found in Ref. [61]. The correspondence between the PbPb $N_{\text{trk}}^{\text{offline}}$ values and the total energy deposited in the HF [62], as characterized by a collision centrality, is given in Ref. [38], ranging from 67% centrality for $N_{\text{trk}}^{\text{offline}} = 120$ to 55% centrality for $N_{\text{trk}}^{\text{offline}} = 300$.

IV. ANALYSIS

A. Scalar product method

In previous publications, CMS has analyzed the elliptic [62] and higher order [63] flow coefficients for PbPb collisions at $\sqrt{s_{\text{NN}}} = 2.76$ TeV using the “traditional” event plane method [64]. It is now known that fluctuations in the participant geometry lead to $v_n$ coefficients that can vary event by event, with the average coefficients ($v_{n\bar{n}}$) being smaller than the corresponding root-mean-square values, $\sqrt{\langle v_n^2 \rangle}$. The $v_{n\bar{n}}$ values found using the traditional event plane method will fall somewhere between these two limits [54]. The scalar product method [53,54], which is used in this paper, avoids this ambiguity and gives results that correspond to $\sqrt{\langle v_n^2 \rangle}$ [54].

The event plane angles can be expressed in terms of $Q$ vectors. For a perfect detector response, the $Q$ vector corresponding to the nth-order azimuthal asymmetry for a given event is defined as

$$Q_n = (Q_{nx}, Q_{ny}) = \left[ |\vec{Q}_n| \cos(n\Psi_n), |\vec{Q}_n| \sin(n\Psi_n) \right]$$

$$= \left[ \sum_{i=1}^{M} w_i \cos(n\phi_i), \sum_{i=1}^{M} w_i \sin(n\phi_i) \right], \quad (1)$$

where $M$ is the subevent multiplicity, $\phi_i$ is the azimuthal angle of the ith particle, $w_i$ are weighting factors, and the corresponding event plane angle is given as

$$\Psi_n = \frac{1}{n} \tan^{-1} \left( \frac{Q_{ny}}{Q_{nx}} \right). \quad (2)$$

Different weights $w_i$ are possible. For example, the $Q$ vectors with $w_i = 1$ relate to the azimuthal particle density, with $w_i = p_T,i$ to the transverse momentum distribution, and with $w_i = E_T,i$ to the transverse energy distribution. Since the $v_n(p_T)$ coefficients increase with $p_T$ up to $\approx 3$ GeV/$c$, the choice of either $p_T$ or $E_T$ weighting generally results in a better plane angle resolution than a unity particle weighting [64].

Expressed in terms of complex weighted $q$ vectors, where

$$q_n = \sum_{i=1}^{M} w_i e^{i n \phi_i} \quad (3)$$

and $W = \sum_{i=1}^{M} w_i$, the scalar product coefficients are found with

$$v_n [\text{SP}] = \frac{\langle q_n^* q_{n\bar{n}} \rangle}{\sqrt{\langle q_{n\bar{n}}^* q_{n\bar{n}} \rangle \langle q_{n\bar{n}}^* q_{n\bar{n}} \rangle}}. \quad (4)$$

In Eq. (4), the weighted average $\langle \rangle$ for vectors $q_{n\alpha}$ and $q_{n\beta}$ with total weights $W_\alpha$ and $W_\beta$, where $\alpha$ and $\beta$ correspond to the second subscripts (if present) on the $q$ vectors in Eq. (4), is given by

$$\langle q_{n\alpha} q_{n\beta}^* \rangle = \text{Re} \left[ \sum_{i=1}^{N_{\text{evt}}} W_\alpha W_\beta \langle q_{n\alpha} q_{n\beta}^* \rangle \right] / N_{\text{evt}}.$$

The value of $q_n$ in Eq. (4) is based on tracks within a specific $p_T$ and $\eta$ range for which the azimuthal asymmetry coefficient is being measured. Unit weights are used in Eq. (1) in this case.

In the HF detector regions, with $3 < |\eta| < 5$, the sums in Eq. (1) are taken over the towers and the weights are taken as the transverse energy deposited in each tower, with no restriction placed on the tower energy. For the tracker-based C plane, the sums are over the individual tracks with $0.3 < p_T < 3.0$ GeV/$c$ and the weights are taken as the corresponding $p_T$ values. The $Q$ vectors corresponding to event planes A, B, and C are “recentered” to account for nonuniformities in the detector response [64,65]. In recentering, the averages over all events of the $x$ and $y$ terms in Eq. (1) ($\langle Q_{nx} \rangle$ and $\langle Q_{ny} \rangle$) are subtracted on an event-by-event basis when calculating $\vec{Q}_n^{\text{recentered}}$. That is,

$$\vec{Q}_n^{\text{recentered}} = (Q_{nx} - \langle Q_{nx} \rangle, Q_{ny} - \langle Q_{ny} \rangle). \quad (6)$$

The value of $v_n$ in Eq. (4) is based on tracks within a specific $p_T$ and $\eta$ range for which the azimuthal asymmetry coefficient is being measured. In this case, unit weights are used in Eq. (1) and no recentering corrections are applied.

It has been noted recently [66-69], and experimentally confirmed by CMS [70], that the event plane angle should not be considered a global event observable. In the CMS study [70], the decorrelation between the event plane angles at pseudorapidity $\eta_A$ and $\eta_B$ is found to follow the functional form:

$$\cos[2(\Psi_n(\eta_B) - \Psi_n(\eta_A))] = e^{-F_n^0 |\eta_B - \eta_A|}, \quad (7)$$

where $F_n^0$ is the decorrelation strength.
Such a decorrelation can arise from fluctuations of the geometry of the initial-state nucleons and their constituent partons \([66–68]\). Previously it has been assumed that Fourier coefficients at pseudorapidity \(\eta_{\mathrm{ROI}}\), where ROI stands for “region of interest,” can be deduced using event plane angles found in a different pseudorapidity range (say, at \(\eta_A\)), with the caveat that a sufficient pseudorapidity gap is present to avoid short-range correlations. The event plane angle found at \(\eta_A\) is viewed as approximating a global participant plane angle set by the initial collision geometry and only differing from the ideal by its finite resolution, which, in turn, depends on both the number of particles used to define the angle and the azimuthal asymmetry at \(\eta_A\). The event plane resolution is accounted for in Eq. (4) by determining event planes in three separate regions of \(\eta\) and assuming that these planes reflect the same underlying geometry, only differing by their respective resolutions. The variation with pseudorapidity breaks this assumption and can have a significant effect on the harmonic coefficient values \(v_n\) deduced using either the traditional or scalar product methods.

Considering event plane decorrelation, each of the scalar products in Eq. (4) will be reduced by the decorrelation effect as indicated in Eq. (7). If the decorrelation strength \(F_\eta\) remains relatively constant as a function of the pseudorapidity gap between event planes, the \(v_n\) coefficient in the presence of decorrelation can be expressed in terms of the coefficient without decorrelation \(\bar{v}_n\) with

\[
v_n\{\text{SP}\} = \frac{\langle q_n\eta_{AN}\rangle e^{-F_\eta|\eta_{AN}-\eta_{ROI}|}}{\sqrt{\langle q_n\eta_{AN}\rangle^2 e^{-2F_\eta|\eta_{AN}-\eta_{ROI}|^2}}}
= \bar{v}_n\{\text{SP}\} e^{-F_\eta|\eta_{AN}-\eta_{ROI}|},
\]

where \(\eta_C\) is taken to fall between \(\eta_A\) and \(\eta_B\). Short-range, nonflow correlations, such as back-to-back dijets, resonance decay, etc., are again suppressed by having a pseudorapidity gap between \(\eta_{ROI}\) and \(\eta_A\).

For the “standard” analysis using a three subevent resolution correction where both the third subevent angle (\(\Psi^C_\eta\)) and the particles belonging to the region of interest are at midrapidity (\(\eta_{ROI} = \eta_C \approx 0\)), it follows that the decorrelation effect will not strongly influence the deduced Fourier coefficient \(v_n\). It can be noted that the same result is expected if a two-subevent resolution correction is used, as is commonly done for symmetric collision systems. However, if \(\eta_{ROI}\) is different from \(\eta_C\), the deduced \(v_n\) value will be reduced by the decorrelation effect.

The pseudorapidity-dependent decorrelation of event planes can occur through different mechanisms. Equation (8) assumes a Gaussian decorrelation characterized by a fixed \(F_\eta\) value. It is also possible for \(F_\eta\) to vary with \(\eta\), in which case the \(\eta\) dependence shown in Eqs. (7) and (8) would be more complicated. A simplified MC simulation was used to explore the two Gaussian spreading scenarios, corresponding to a fixed or \(\eta\)-dependent \(F_\eta\) factor. It was found that the input \(v_n\) values could be recovered by moving the \(\Psi^C_\eta\) event plane along with the particles of interest. An alternative source of decorrelation is the situation where rotation of the event plane angle results from a torque effect rather than a random spreading \([67]\). In this case, the MC simulations showed that moving the \(\Psi^C_\eta\) event plane does not fully correct for the decorrelation, although it does lead to results closer to the input values than is found by setting \(\eta_C = 0\). A comparison of the \(v_2\) and \(v_3\) results obtained with \(\eta_C = 0\) and with \(\eta_C = \eta_{ROI}\) might help in estimating the relative importance of the different types of decorrelation possible in heavy ion collisions. Event plane results using both of these assumptions for \(\eta_C\) are reported.

Two different reference event planes are used in the analysis: \(\mathrm{HF}^- (-5.0 < \eta < -3.0)\) and \(\mathrm{HF}^+ (3.0 < \eta < 5.0)\). The corresponding resolution correction factors are determined with the three subevent method where, for the \(\mathrm{HF}^+ (\mathrm{HF}^-)\) reference plane (A plane), the resolution correction is based on the \(\mathrm{HF}^- (\mathrm{HF}^+)\) event plane (B plane) as well as either the midrapidity tracker event plane, with \(-0.8 < \eta < 0.8\), or with even event planes that correspond to the pseudorapidity range of the ROI (C plane). Since analyses where the midrapidity event plane \(\eta_C\) is taken within \(-0.8 < \eta_C < 0.8\) and analyses where \(\eta_C = \eta_{ROI}\) are both presented, the convention is adopted of labeling results as “\(\eta_C = 0\)” or “\(\eta_C = \eta_{ROI}\),” respectively.

**B. Cumulant method**

If the particles emitted in a collision are correlated with a global reference frame, they will also be correlated with each other. The cumulant method explores the collective nature of the anisotropic flow through the multiparticle correlations. As the number of particles in the correlation study increases, the cumulant values will decrease if only part of the particle sample shares a common underlying symmetry, as would be the case for dijets. The flow harmonics are studied using the \(Q\) cumulant method \([51]\). The \(m\) particle (\(m = 2, 4, 6,\) or \(8\)) \(n\)th-order correlators are first defined by

\[
\langle\langle(2)\rangle\rangle \equiv \langle\langle\langle(4)\rangle\rangle\rangle,
\langle\langle(4)\rangle\rangle \equiv \langle\langle\langle(6)\rangle\rangle\rangle,
\langle\langle(6)\rangle\rangle \equiv \langle\langle\langle(8)\rangle\rangle\rangle,
\langle\langle(8)\rangle\rangle \equiv \langle\langle\langle\cdots\rangle\rangle\rangle,
\]

where \(\phi_i\) is the azimuthal angle of the \(i\)th particle and \(\langle\langle\cdot\cdot\cdot\rangle\rangle\) indicates that the average is taken over all \(m\)-particle combinations for all events. In order to remove self-correlations, it is required that the \(m\) particles be distinct. The unbiased estimators of the reference \(m\)-particle cumulants \([51]\), \(c_m\{m\}\), are defined as

\[
c_m\{4\} = \langle\langle(4)\rangle\rangle - 2\langle\langle(2)\rangle\rangle^2,
\]

\[
c_m\{6\} = \langle\langle(6)\rangle\rangle - 9\langle\langle(4)\rangle\rangle\langle\langle(2)\rangle\rangle + 3\langle\langle(4)\rangle\rangle^2,
\]

\[
c_m\{8\} = \langle\langle(8)\rangle\rangle - 16\langle\langle(6)\rangle\rangle\langle\langle(2)\rangle\rangle - 18\langle\langle(4)\rangle\rangle^2
+ 144\langle\langle(4)\rangle\rangle\langle\langle(2)\rangle\rangle^2 - 144\langle\langle(2)\rangle\rangle^4.
\]

The reference flow \(v_2\{m\}\) obtained by correlating the particles within the reference phase space of \(|\eta| < 2.4\) and \(p_T\) range of \(0.3 < p_T < 3.0\) \(\text{GeV}/c\) was presented in Ref. \([39]\)
using
\[ v_n[4] = \sqrt{-c_4[4]}, \]
\[ v_n[6] = \sqrt{c_6[6]/4}, \]
\[ v_n[8] = \sqrt{-c_8[8]/33}. \]  

(11)

The cumulant calculations are done using the code described in Ref. [71]. By replacing one of the particles in a correlator for each term in Eq. (9) with a particle from certain ROI phase space in \( p_T \) or \( \eta \), with the corresponding correlators denoted by primes, one can derive the differential \( m \)-particle cumulants as
\[ d_n[4] = \langle \langle 4 \rangle \rangle - 2 \langle \langle 2 \rangle \langle 2 \rangle \rangle, \]
\[ d_n[6] = \langle \langle 6 \rangle \rangle - 6 \langle \langle 2 \rangle \langle 4 \rangle \rangle + 3 \langle \langle 2 \rangle \langle 2 \rangle \langle 2 \rangle \rangle + 12 \langle \langle 2 \rangle \rangle^2, \]
\[ d_n[8] = \langle \langle 8 \rangle \rangle - 12 \langle \langle 2 \rangle \langle 6 \rangle \rangle - 4 \langle \langle 2 \rangle \langle 2 \rangle \rangle^2 - 18 \langle \langle 4 \rangle \rangle^2 + 72 \langle \langle 4 \rangle \rangle \langle \langle 2 \rangle \rangle + 72 \langle \langle 2 \rangle \rangle^3 \]
\[ = \langle \langle 4 \rangle \rangle^3 - 144 \langle \langle 2 \rangle \rangle^3. \]

(12)

Then the differential \( v_2[m](p_T, \eta) \) can be extracted as
\[ v_2[4](p_T, \eta) = -d_n[4]/(-c_4[4])^{3/4}, \]
\[ v_2[6](p_T, \eta) = \frac{d_n[6]}{4}/\left(\frac{c_6[6]}{4}\right)^{5/6}, \]
\[ v_2[8](p_T, \eta) = -\frac{d_n[8]}{33}/\left(-\frac{c_8[8]}{33}\right)^{7/8}. \]

(13)

An efficiency weight is applied to each track to account for detector nonuniformity and efficiency effects. For this analysis, the work of Ref. [71] was extended to allow for the explicit calculation of the differential \( Q \) cumulants for the first time.

C. Lee-Yang zero method

The LYZ method [52] allows for a direct study of the large-order behavior by using the asymptotic form of the cumulant expansion to relate locations of the zeros of a generating function to the azimuthal correlations. This method has been employed in previous CMS PbPb and pPb analyses [39,62,63]. The \( v_2 \) harmonic averaged over \( 0.3 < p_T < 3.0 \) GeV/c is found for each multiplicity bin using an integral generating function [17]. Similar to the cumulant methods, a weight for each track is implemented to account for detector-related effects. Anisotropic flow is formally equivalent to a first-order phase transition. As a result, the first zero of the generating grand partition function can be viewed as an anisotropic flow of the final-state system.

The integrated flow for the harmonic \( n \) is the average value of the flow \( Q \)-vector projected onto the unit vector with angle \( n\Phi_R \).
\[ v_n^{\text{int}} \equiv \langle Q_n \cos(n\Phi_R) + Q_n \sin(n\Phi_R) \rangle = \langle Q_n^{\text{bk}} \rangle, \]  

(14)

where \( \Phi_R \) is the actual reaction-plane angle. Since \( \Phi_R \) is not an observable, the LYZ method is used to obtain an estimate of this quantity. In the present analysis, a complex product generating function is first defined as
\[ G_n^\theta (i\tau) = \langle \langle g_n^\theta (i\tau) \rangle \rangle = \left\{ \prod_{j=1}^{M} [1 + i \tau w_j \cos(n(\phi_j - \theta))] \right\}, \]

(15)

where \( M \) is the event multiplicity, \( \phi_j \) and \( w_j \) are, respectively, the azimuthal angle and the weight of the \( j \)th particle, \( \langle \rangle \) is taken over all events, and \( \theta \) is chosen to take discrete values within the range \([0, \pi/n]\) as
\[ \theta = \frac{k \pi}{n\theta}, \quad k = 0, 1, 2, \ldots, n_\theta - 1. \]

(16)

The number of projection angles is set to \( n_\theta = 5 \) to get the average values. This number was found in the previous CMS studies to achieve convergence of the results [39,62,63].

To calculate the yield-weighted integrated flow, \( G_n^\theta (r) \) is evaluated for many values of the real positive variable \( r \). Plotting the modulus |\( G_n^\theta (r) \)| as a function of \( r \), the integrated flow is directly related to the first minimum \( r_0^\theta \) of the distribution, with
\[ v_n^{\beta, \text{int}} \langle \infty \rangle \equiv \frac{j_{01}^n}{r_0^\beta}, \]

(17)

where \( j_{01} \approx 2.405 \) is the first root of the Bessel function \( J_0(x) \). The quoted results involve a final average over different \( \theta \) values, with
\[ v_n^{\beta} \equiv \frac{1}{n_\theta} \sum_{\theta=0}^{n_\theta-1} v_n^{\beta, \text{int}} \langle \infty \rangle. \]

(18)

After the integrated flow coefficient \( v_n^{\beta} \) is determined, the \( p_T \)- and \( \eta \)-dependent \( v_2 \) [LYZ] values are found using
\[ \frac{v_0^{\beta, \text{int}}}{v_0^{\beta}} = \frac{\langle g_0^\beta (i\tau_0^\beta) \cos(n(\phi - \theta)) \rangle}{1 + r_0^\beta w_j \cos(n(\phi_j - \theta))}, \]

(19)

The average \( \langle \cdots \rangle_\theta \) in the numerator is taken over the particles in the ROI. The average in the denominator is over all particles with \( 0.3 < p_T < 3.0 \) GeV/c and \( \eta \) < 2.4. Again, the final results involve an average over the different \( \theta \) values
\[ v_n = \frac{1}{n_\theta} \sum_{\theta=0}^{n_\theta-1} v_n^{\beta}. \]

(20)

D. Systematic uncertainties

The systematic uncertainties resulting from the track selection and efficiency, from the vertex position, and from the pileup contamination contribute to all three methods (scalar product, cumulant, and LYZ). The effects of track quality requirements were studied by varying the track selection requirements, \( d_T/\sigma(d_T) \) and \( d_T/\sigma(d_T) \), from 2 to 5, and \( \sigma(p_T)/p_T \) from 5% to the case where this requirement is not applied. A comparison of the results using efficiency correction tables from EPOS and HIJING MC event generators was made to study the tracking efficiency uncertainty. By comparing the results from different event primary vertex positions along the beam direction, with \( |z_{\text{vtx}}| < 3 \) cm and \( 3 < |z_{\text{vtx}}| < 15 \) cm, it is possible to investigate the uncertainties.
coming from the tracking acceptance effects. The effects of pileup events were studied by looking at events where there was only one reconstructed vertex. The experimental systematic effects are found to have no significant dependence on $N_{\text{trk}}^{\text{offline}}$, $p_T$, or $\eta$.

The $v_2$ systematic uncertainties associated with the PbPb collision results were found to be comparable for the three methods ($\approx 3\%$), with contributions from the track selection and efficiency ($1\%$–$2\%$), the vertex position ($1\%$–$2\%$), and pileup effects ($< 1\%$). Similar uncertainties are found for $p$Pb collisions based on both the cumulant and scalar product methods. For the LYZ $p$Pb results, a more conservative uncertainty of $11\%$ is quoted based on the large statistical uncertainties associated with the corresponding systematic studies.

In addition, a comparison was done between the results for the two different beam directions. For the event plane analysis, the $p$-side and Pb-side HF detectors used to determine the event plane angles are switched by changing the beam direction. Based on this study, where the small magnitude of the $v_3$ coefficient limits the statistical significance of the systematic studies, a larger, conservative systematic uncertainty is assigned to the $v_3$ results of $10\%$. The overall systematic uncertainties are summarized in Table I, and shown as gray boxes in the figures.

The multiparticle cumulant and LYZ analyses are expected to be relatively insensitive to nonflow effects. For the scalar product method, however, the nonflow effects can become significant as the differential particle density decreases, as is the situation for the lower $N_{\text{trk}}^{\text{offline}}$ ranges and for higher $p_T$ values. Also, the nonflow effects become more significant as the gap between the primary event plane ($\eta_A$) and the region of interest ($\eta_{\text{ROI}}$) becomes small. In this paper, the nonflow influence on the scalar product results is viewed as part of the physics being explored and is not taken as a systematic uncertainty.

![Graph](attachment:image.jpg)
V. RESULTS

We first explore the transverse momentum dependence of \( v_2 \) and \( v_3 \) in \( pPb \) and \( PbPb \) at comparable particle multiplicities. The \( v_2 \) values were found using the scalar product, \( m \)-particle cumulant, and LYZ methods, denoted as \( v_2[SP] \), \( v_2[m] \), and \( v_2[LYZ] \), respectively, while \( v_3 \) was found using only the scalar product method.

The momentum-dependent \( v_2(p_T) \) results in the region \( |\eta| < 2.4 \) for \( pPb \) and \( PbPb \) collisions are shown in Fig. 1. The scalar product values, shown separately for the \( p \)- and \( Pb \)-going event planes, are found to be significantly higher than the multiparticle cumulant (\( v_2[4] \), \( v_2[6] \), and \( v_2[8] \)), and Lee-Yang zero (\( v_2[LYZ] \)) results. The two-particle correlations (\( v_2[2] \)) and lower order cumulant (\( v_2[4] \)) measurements shown in the figure are from Ref. [38]. As will be discussed when presenting the yield-weighted integral \( v_2 \) values, the greater values found for \( v_2[SP] \) and \( v_2[2] \) suggest a significant, and expected, contribution of fluctuations in the initial-state geometry to these results.

In the range of \( p_T < 2 \text{ GeV/c} \), there is very little difference between the \( v_2[SP] \) results obtained with the \( p \)- and \( Pb \)-going side event planes. However, at higher transverse momenta, the \( p \)-going event plane leads to systematically larger values. This behavior suggests that the nonflow contribution has a larger effect on the high-\( p_T \) \( v_2 \) values based on the \( p \)-going side event plane. Monte Carlo simulations using the HIJING event generator support a nonflow component to the \( v_2 \) signal that increases almost monotonically with \( p_T \). In situations where both the event plane angle and the \( Q \) vector associated with the region of interest are based on small numbers of particles, the nonflow behavior can be significant. It is also possible that the \( p_T \)-dependent event-plane decorrelation effects might be different on the \( Pb \)- and \( p \)-going sides.

In contrast to Fig. 1, which uses an \( \eta \) region that is symmetric in the laboratory frame, Fig. 2 compares the \( v_2(p_T) \) results for symmetric pseudorapidity ranges in the center-of-mass frame. The laboratory frame results for the range of \( 2.0 < \eta < 2.4 \) correspond approximately to the center-of-mass range of \( 1.6 < \eta_{c.m.} < 2.0 \) and are obtained with respect to the event plane found on the \( Pb \)-going side with \( -5.0 < \eta < -3.0 \), as indicated with the notation \( v_2 \{ \text{Pb-SP} \} \). Similarly, the range of \( -1.6 < \eta < -1.2 \) approximately corresponds to \( -2.0 < \eta_{c.m.} < -1.6 \). Here the results are obtained with respect to the event plane found on the \( p \)-going side with \( 3.0 < \eta < 5.0 \), as indicated with the notation \( v_2 \{ \text{p-SP} \} \). The measured values are shown separately with \( \eta_C = 0 \) and \( \eta_{ROI} \). The reference event plane used in each case corresponds to the more distant HF detector. In the region with \( 1.5 < p_T < 3.0 \text{ GeV/c} \), the enhancement observed on the \( Pb \)-going side \( (2.0 < \eta_{c.m.} < 1.6; \text{p-SP}) \) with \( \eta_C = 0 \) (top row) is reduced by taking \( \eta_C = \eta_{ROI} \) (bottom row). This dependence on \( \eta_C \) suggests the presence of event plane decorrelation.

Further evidence for event plane decorrelation is seen by comparing the pseudorapidity dependence of the yield-weighted \( v_2 \) values for \( 0.3 < p_T < 3.0 \text{ GeV/c} \). This is shown in Figs. 3 and 4 for the \( pPb \) and \( PbPb \) collisions, respectively. The top row in each figure shows the scalar product results with \( \eta_C = 0 \) and the bottom row with \( \eta_C = \eta_{ROI} \). For the \( pPb \) collisions, results are shown separately over the full pseudorapidity range of the CMS tracker using the HF event planes on the \( p \)- and \( Pb \)-going side of the collision. For the
symmetric PbPb collisions, the results using the HF$^+$ and HF$^-$ event planes are shown separately. The yield-weighted elliptic flow coefficients for PbPb collision are found to be $\approx 20\%$ larger than for pPb collisions. In the absence of decorrelation effects, the choice of $\eta_C = 0$ or $= \eta_{ROI}$ would be expected to result in similar distributions. In previous PbPb studies [62,63], taking $\eta_C = 0$, the $v_2(\eta)$ values with $\eta < 0$ were reported using the event plane with $3.0 < \eta < 5.0$, and the values with $\eta > 0$ were reported using the event plane with $-5.0 < \eta < -3.0$, thus achieving the largest possible gap in pseudorapidity. Before accounting for an increasing decorrelation of event planes with an increasing pseudorapidity gap,
the $v_2$ values based on $p$-going and Pb-going side event planes ($p$Pb collisions) or HF$^+$ and HF$^-$ event planes (PbPb collisions) show different pseudorapidity dependences, with the values decreasing as the gap with the reference event plane increases. This reference event plane dependence largely disappears once a correction is applied for decorrelation effects, with the corrected $v_2$ values showing very little pseudorapidity dependence. The resulting boost invariance is consistent with the azimuthal dependence being determined by the initial-state geometry. For the $p$Pb collisions, the results with $2.0 < \eta < 2.4$ determined using the $p$-going side reference event plane are systematically higher in each of the $N_{\text{off}}^{\text{offline}}$ ranges. This is consistent with the reduced multiplicity associated with this $\eta$ region, allowing for an increased influence of nonflow effects.

The current results suggest that event plane decorrelation effects might be significant in trying to understand the pseudorapidity dependence of the flow coefficients. The results with $2.0 < \eta < 2.4$ determined using the $p$-going side reference event plane are systematically higher, suggesting the possible influence of nonflow effects.

Expanding on the results in Figs. 3 and 4, which show only $v_2$ from the scalar product method, the yield-weighted average $v_2$ values for all of the analysis methods are shown in Fig. 5. It is interesting to note that the pseudorapidity dependence is almost flat for the scalar product calculations where $\eta_C = \eta_{\text{ROI}}$. This is in contrast to the scalar product results for $\eta_C = 0$ and for the higher order particle correlation analyses, where the $v_2$ values at larger pseudorapidities are significantly smaller. It is only for the scalar product analysis with $\eta_C = \eta_{\text{ROI}}$ that a partial accounting for the event plane decorrelation behavior is achieved. Both the cumulant and LYZ analyses employ integral reference flows based on the full range of the CMS tracker and thus are not able to account for decorrelation effects. There is an apparent asymmetry as a function of pseudorapidity for the LYZ results for the two highest $N_{\text{off}}^{\text{offline}}$ ranges, with a larger $v_2$ signal observed on the Pb-going side event plane. Although this asymmetry appears to be larger than that found for the cumulant or scalar product analyses, the large statistical uncertainties make a direct comparison difficult.

It can be seen from Fig. 5 that the PbPb results for a given $N_{\text{off}}^{\text{offline}}$ range are consistently higher than the corresponding $p$Pb results. This likely reflects the very different collision geometries for the two systems, with the elliptic flow for PbPb collisions being influenced by the lenticular-shaped overlap region developed in noncentral collisions of two Pb nuclei. In a later discussion, this result
will be contrasted with a similar comparison for the $v_3$ harmonic.

As already suggested for the $p_T$-dependent results, the difference between the scalar product and two-particle correlations results, as compared to the higher order correlation studies, is likely to reflect initial-state fluctuation effects. Event-by-event fluctuations in the location of the participant nucleons can have a large and method-dependent influence on the harmonic coefficients \cite{72,73}. Expressing the fluctuations in terms of the azimuthal anisotropy in the participant plane $v$, where the harmonic number is suppressed, the magnitude of the fluctuations is given by $\sigma_v^2 \equiv \langle v^2 \rangle - \langle v \rangle^2$. To leading order in $\sigma_v$ \cite{73}, two- and four-particle correlations are affected differently, with

$$v[2]^2 = \langle v^2 \rangle = \langle v \rangle^2 + \sigma_v^2$$

and

$$v[4]^2 = (2\langle v^2 \rangle - \langle v^4 \rangle)^{1/2} \approx \langle v \rangle^2 - \sigma_v^2.$$  \hspace{1cm} (21)

Multiparticle correlations with more than four particles are expected to give results similar to those of four-particle correlations. Fluctuations affect the scalar product and two-particle correlations in a similar manner. The difference between the scalar product and higher order cumulant results therefore reflects the initial-state fluctuations.

Using Eqs. (21) and (22), the fluctuation ratio $\sigma_v/\langle v \rangle$ can be calculated as

$$\frac{\sigma_v}{\langle v \rangle} = \sqrt{\frac{v[2]^2 - v[4]^2}{v[2]^2 + v[4]^2}} = \sqrt{\frac{v[2][SP]^2 - v[4][SP]^2}{v[2]^2 + v[4]^2}}.$$  \hspace{1cm} (23)

This ratio is shown in Fig. 6 for the $p$Pb and PbPb collisions in different $N_{\text{trk}}^{\text{off}}$ ranges. The $v_2[SP]$ results with $\eta_C = 0$ are used in the calculations since the $v_2[4]$ results are expected to be affected by decorrelation effects. The fluctuation component is found to be significantly larger for the $p$Pb collisions as compared to the PbPb results. A small (15–20\%) increase in the ratio is found for both the $p$Pb and PbPb systems as the $N_{\text{trk}}^{\text{off}}$ range increases. The $p$Pb system also shows an increase in the ratio as the pseudorapidity increases.

The results presented here can be used to evaluate in more detail previous CMS analyses which suggest a significant pseudorapidity dependence of the $v_2$ coefficient of $p$Pb collisions, with a larger “flow” signal on the Pb-going side \cite{74}. That study was based on a two-particle correlation analysis and focused on the ratio $v_2(\eta)/v_2(\eta = 0)$. Since the Ref. \cite{74}
analysis does not take into account decorrelation effects, it is most closely related to the scalar product analysis with $\eta_C = 0$ and to the multiparticle correlation measurements based on the integral flow coefficients found using an extended range of the CMS tracker acceptance. The Ref. [74] results are compared to the scalar product and four-particle cumulant results in Fig. 7. Agreement is found among these measurements. The scalar product results with $\eta_C = \eta_{ROI}$, also shown in Fig. 7, fall off more slowly when moving away from midrapidity.

To explore further the possible asymmetry in the pseudorapidity-dependent $v_2$ results of Fig. 5 for the pPb system, Fig. 8 shows the ratios of the yield-weighted integral values on the p- and Pb-going sides at comparable center-of-mass pseudorapidity for pPb collisions. The results are shown for the scalar product analyses with $\eta_C = 0$ and $\eta_{ROI}$ and for the four-particle cumulant analysis. Also shown are the comparable results from the Ref. [74] analysis. For the pPb results where decorrelation effects are not taken into account (i.e., $v_2[SP, \eta_C = 0]$ and $v_2[4]$), the Pb-going side values are significantly larger. The asymmetry between the Pb-going and p-going sides largely disappears when decorrelation effects are taken into account. A small asymmetry continues to be present when decorrelation effects are considered (i.e., $v_2[SP, \eta_C = \eta_{ROI}$), although it needs to be recognized that the procedure of moving the $\eta_C$ range with $\eta_{ROI}$ is not expected to fully account for these effects if a torque-effect decorrelation is present; there may be some additional influence of nonflow

![FIG. 8. Ratio of the p- to Pb-going side $v_2$ coefficients at comparable $\eta_{c.m.}$ values for pPb collisions. The two-particle correlation results (labeled “2-part”) are from Ref. [74]. The reference HF event plane is the one furthest from the particles of interest.](image)

![FIG. 9. (Top) The $v_3$ values from the scalar product method for pPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV with $\eta_C = 0$. (Bottom) Same, but with $\eta_C = \eta_{ROI}$. The notations p-SP and Pb-SP indicate the pseudorapidity side of the reference event plane and correspond to the p- and Pb-going directions, respectively. Pseudorapidities are given in the laboratory frame. Systematic uncertainties are indicated by the gray boxes.](image)
effects when the $\eta$ gap between the $\eta_C$ and either the $\eta_A$ or $\eta_B$ event planes becomes small.

In contrast to the second-order Fourier coefficients discussed above, triangular flow, corresponding to the $v_3$ Fourier harmonic, is believed to arise from fluctuations in the participant geometry in collisions of heavy nuclei. It is interesting to see how this behavior extends to the very asymmetric $p$-$Pb$ system. Figure 9 shows the scalar product results for the $p$-$Pb$ collisions at $\sqrt{s_{NN}} = 5.02$ TeV with $\eta_C = 0$ (top) and $\eta_C = \eta_{ROI}$ (bottom), respectively, as a function of $\eta$. Yield-weighted $v_3$ values with $0.3 < p_T < 3.0$ GeV/$c$ are shown. A pronounced jump in $v_3$, which becomes smaller with increasing $N_{\text{offline}}$, is observed for $\eta > 2$ when using the $p$-going side reference event plane. This could be due to nonflow effects when the ROI is close to the reference event plane. For the Pb-going side reference event plane, a similar, but much smaller effect, may be present when taking $\eta_C = \eta_{ROI}$.

A small pseudorapidity dependence is seen in the $v_3(\eta_C = \eta_{ROI})$ results, with the values becoming smaller on the $p$-going side. This might suggest a changing level of fluctuations driving the triangular flow signal. The pseudorapidity dependence appears to become less significant as $N_{\text{offline}}$ increases.

Figure 10 shows the corresponding scalar product results for the PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with $\eta_C = 0$ (top) and $\eta_C = \eta_{ROI}$ (bottom). The $v_3$ values are found to increase with increasing $N_{\text{offline}}$ for both systems, as previously observed when the $\eta$ gap between the $\eta_C$ and either the $\eta_A$ or $\eta_B$ event planes becomes small.

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in Ref. [38]. However, contrary to what is found for the $v_2$ coefficients, the $v_3$ values are very similar for the $p$Pb and PbPb systems in a given $N_{\text{offline}}$ range. In order to show the system dependence of $v_2$ and $v_3$ more directly, Fig. 11 shows scalar product results with $\eta_C = \eta_{\text{ROI}}$ for both the $p$Pb and PbPb systems. The $v_3$ values, believed to result almost entirely from initial geometry fluctuations, are almost the same for the two systems. The $v_2$ values are still likely to reflect the lenticular shape of the collision geometry in the PbPb system, leading to larger $v_2$ coefficients than seen for the $p$Pb system. The PbPb $v_2$ values are also found to increase with increasing event activity, reflecting the additional contribution of the changing collision overlap geometry.

VI. SUMMARY

The pseudorapidity and transverse momentum dependencies of the elliptic flow $v_2$ coefficient are presented for $p$Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV and for peripheral PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV based on scalar product, multiparticle cumulant, and Lee-Yang zero analyses. The data are obtained using the CMS detector. The $\eta$ dependence of the triangular flow $v_3$ coefficient is also presented based on the scalar product analysis. For the first time, $p_T$- and $\eta$-dependent cumulant results are presented based on six- and eight-particle correlations. The results provide detailed information for the theoretical understanding of the initial-state effect and final-state evolution mechanism.

All methods lead to a similar $\eta$ dependence for the $v_2$ harmonic across the pseudorapidity range studied. The scalar product results are consistently higher than the corresponding multiparticle correlation behavior, with the $v_2[4]$, $v_2[6]$, $v_2[8]$, and $v_2[LYZ]$ having comparable magnitude. An analysis of fluctuations suggests their greater influence in the system formed in $p$Pb as compared to that in the PbPb collisions. No significant pseudorapidity dependence is found for the fluctuation component, although there is a small increase in the level of the fluctuations with increasing $N_{\text{offline}}$ in both the $p$Pb and PbPb systems. The boost invariance indicated by the decorrelation-corrected results confirms that the flow signal develops very early in the collision and thus reflects the initial-state geometry.

A method is presented to account for the possible decorrelation of the event plane angle with an increasing $\eta$ gap between two regions of pseudorapidity. The results suggest that most of the $\eta$ dependence observed using the different methods might be a consequence of the decorrelation effect. Earlier results exploring the $\eta$ dependence of elliptic flow in heavy ion collisions may need to be reassessed based on the presence of such decorrelation effects.

Only a small difference is found for the $v_2$ coefficients on the Pb- and $p$-going sides for the $p$Pb collisions once decorrelation effects are considered. This is in contrast to a previous study, in which the decorrelation effects were not considered and where a larger $v_2$ value was found on the Pb-going side. If the decorrelation effects are not considered, as is the case with the current cumulant, LYZ, and scalar product analysis with $\eta_C = 0$, good agreement is found with the previous results. When decorrelation effects are considered, there appears to be very little longitudinal dependence of the flow coefficients near midrapidity.

The yield-weighted $v_2$ results of $p$Pb and PbPb collisions at comparable values of $N_{\text{offline}}$ show a similar $\eta$ dependence, with the heavier system values being about 20% higher than for $p$Pb collisions. No significant difference is observed for the PbPb $v_2$ values as compared to $p$Pb collisions, suggesting that the $v_3$ results are solely a consequence of fluctuations in the initial-state participant geometry.

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University of Colorado Boulder, Boulder, Colorado, USA
Cornell University, Ithaca, New York, USA
Fermi National Accelerator Laboratory, Batavia, Illinois, USA
University of Florida, Gainesville, Florida, USA
Florida International University, Miami, Florida, USA
Florida State University, Tallahassee, Florida, USA
Florida Institute of Technology, Melbourne, Florida, USA
University of Illinois at Chicago (UIC), Chicago, Illinois, USA
The University of Iowa, Iowa City, Iowa, USA
Johns Hopkins University, Baltimore, Maryland, USA
The University of Kansas, Lawrence, Kansas, USA
Kansas State University, Manhattan, Kansas, USA
Lawrence Livermore National Laboratory, Livermore, California, USA
University of Maryland, College Park, Maryland, USA
Massachusetts Institute of Technology, Cambridge, Massachusetts, USA
University of Minnesota, Minneapolis, Minnesota, USA
University of Mississippi, Oxford, Mississippi, USA
University of Nebraska–Lincoln, Lincoln, Nebraska, USA
State University of New York at Buffalo, Buffalo, New York, USA
Northeastern University, Boston, Massachusetts, USA
Northwestern University, Evanston, Illinois, USA
University of Notre Dame, Notre Dame, Indiana, USA
The Ohio State University, Columbus, Ohio, USA
Princeton University, Princeton, New Jersey, USA
University of Puerto Rico, Mayaguez, Puerto Rico, USA
Purdue University, West Lafayette, Indiana, USA
Purdue University Northwest, Hammond, Indiana, USA
Rice University, Houston, Texas, USA
University of Rochester, Rochester, New York, USA
The Rockefeller University, New York, New York, USA
Rutgers, The State University of New Jersey, Piscataway, New Jersey, USA
University of Tennessee, Knoxville, Tennessee, USA
Texas A&M University, College Station, Texas, USA
Texas Tech University, Lubbock, Texas, USA
Vanderbilt University, Nashville, Tennessee, USA
University of Virginia, Charlottesville, Virginia, USA
Wayne State University, Detroit, Michigan, USA
University of Wisconsin–Madison, Madison, Wisconsin, USA

a Also at Vienna University of Technology, Vienna, Austria.
b Also at State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing, China.
c Also at Universidade Estadual de Campinas, Campinas, Brazil.
d Also at Universidade Federal de Pelotas, Pelotas, Brazil.
e Also at Université Libre de Bruxelles, Bruxelles, Belgium.
f Also at Joint Institute for Nuclear Research, Dubna, Russia.
g Also at Helwan University, Cairo, Egypt; Zewail City of Science and Technology, Zewail, Egypt.
h Also at Fayoum University, El-Fayoum, Egypt; British University in Egypt, Cairo, Egypt.
i Also at Helwan University, Cairo, Egypt.
J Also at Université de Haute Alsace, Mulhouse, France.
k Also at Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia.
l Also at Tbilisi State University, Tbilisi, Georgia.
m Also at CERN, European Organization for Nuclear Research, Geneva, Switzerland.
Also at RWTH Aachen University, III. Physikalisches Institut A, Aachen, Germany.
o Also at University of Hamburg, Hamburg, Germany.
p Also at Brandenburg University of Technology, Cottbus, Germany.
q Also at Institute of Nuclear Research ATOMKI, Debrecen, Hungary.
r Also at MTA-ELTE Lendület CMS Particle and Nuclear Physics Group, Eötvös Loránd University, Budapest, Hungary.
s Also at Institute of Physics, University of Debrecen, Debrecen, Hungary.
