Mathematical model of curvilinear motion of an active road train with electromechanical transmission

O I Chudakov¹, V A Gorelov¹, V A Gartfelder², L S Sekletina²

¹ Bauman Moscow State Technical University (BMSTU), 105005, Moscow, Russian Federation, 2nd Baumanskaya St., Bldg. 5, Block 1
² I.N. Ulianov Chuvash State University (Chuv SU), 428015, Cheboksary, Russian Federation, Moskovskiy pr.,15

Abstract. The paper presents a mathematical model of curvilinear motion of a two-unit road train consisting of a four-axle tractor with independent wheel drive and a three-axle semitrailer on non-yielding bearing surface. The model considers variation in the normal load on fifth-wheel coupling (FWC) of the tractor during motion and cornering resistance moment caused by friction between the semitrailer and FWC bolster. Using the developed software, we can examine the effectiveness of algorithms of steering systems of the tractor and trailer unit allowing for various traction combinations of road train units.

1. Introduction

Trucking plays an important role in the overall transportation system in most countries worldwide. The efficiency of trucking can be improved by increasing performance and reducing the cost of transportation [1,2]. The performance of vehicles is determined by the average speed and load capacity. Road trains have limited opportunities to increase the average speed, so the most promising way to enhance performance is to increase the load capacity that can be achieved, particularly, by using road trains [3,4].

The use of road trains offers great opportunities in industrial and agricultural areas with poor road network, i.e. driving off-road, which is especially relevant for our country. However, the use of road trains is often limited due to inadequate haulage capacity, when it comes to the transportation of heavy machinery, equipment and weapons in tough country road conditions [5]–[9]. Development of road trains with a sufficiently high level of traction dynamics is thus a pressing scientific and practical issue, which may be effectively solved by introducing an active drive of link element wheels [10]–[14].

2. Mathematical Model of Road Train Motion

The paper presents a mathematical model of planar motion of a two-unit road train. The subject of research is a road train with 120,000 kg gross vehicle weight consisting of a four-axle tractor with electromechanical transmission (EMT) and a three-axle semitrailer. Figure 1 shows the static distribution of vertical loads.
Figure 1. Static distribution of vertical loads along axes of fully loaded road train.

In the base case (road train with a passive semitrailer), each wheel drive is implemented using 60 kW traction electric motor (TEM). So, the total power of all TEMs of the tractor is 480 kW. This is the maximum power transmitted from the powertrain to TEMs. The power of active road train’s TEMs has been selected so as to have equal power-to-weight ratio in all cases.

Four power distribution cases are considered: in the first one, all power is transmitted to tractor wheels (base case), in the last one, 75% of the total power is transmitted to semitrailer wheels. Figure 2 shows power distribution among road train wheels.

The mathematical model of road train planar motion described in detail in [15] may be used to study the curvilinear motion of multi-unit vehicles with active trailer units. Longitudinal and transverse angles of relative motion of moving road train units along the horizontal base are not large. Therefore, the movement of each unit as a solid body is considered in the horizontal plane taking into account the lead angle in the direction of movement on flat non-yielding surface and consists of translational and rotational motion of the center of mass (Figure 3).
Figure 3. Calculation model of a two-unit road train.

The following coordinate systems are introduced to describe this road train motion case: fixed coordinates \(X'OY'\), the origin (point \(O\)) coincides with the start point of simulated route; moving coordinates \(X_1C_1Y_1\) and \(X_2C_2Y_2\) referenced to centers of mass (CM) of the tractor and semitrailer, respectively; coordinates \(X_iO_iY_i\) referenced to the \(i\)-th wheel of the \(j\)-th unit. Point \(O_i\) coincides with the projection of the \(i\)-th wheel center on the bearing surface, axis \(O_iX_i\) is directed along the projection of the wheel longitudinal plane on the bearing surface, \(O_iY_i\) – along the projection of wheel spindle.

The dynamics of a road train unit is described by the following set of equations:

\[
\begin{align*}
    a_{xj} &= \frac{dV_{sj}}{dt} - \alpha_{xj} \cdot V_{sj} = \frac{1}{m_j} \left( \sum_{i=1}^{n} R_{xji} - m_j \cdot g \cdot \sin \alpha - P_{xj} + F_{sj} \right); \\
    a_{yj} &= \frac{dV_{sj}}{dt} + \alpha_{yj} \cdot V_{sj} = \frac{1}{m_j} \left( \sum_{i=1}^{n} R_{yji} - P_{yj} + F_{sj} \right); \\
    J_{zj} \frac{d\omega_{zj}}{dt} &= \sum_{i=1}^{n} M_{xji} - \sum_{i=1}^{n} R_{xji} \cdot y_{xji} + \sum_{i=1}^{n} R_{yji} \cdot x_{xji} + M_j; \\
    V_{xj}' &= \frac{dx_j'}{dt} = V_{sj} \cdot \cos \theta_j - V_{yj} \cdot \sin \theta_j; \\
    V_{yj}' &= \frac{dy_j'}{dt} = V_{sj} \cdot \sin \theta_j + V_{yj} \cdot \cos \theta_j; \\
    \omega_{xj} &= \frac{d\theta_j}{dt},
\end{align*}
\]

where \(j\) is the road train unit number; \(m_j\) is the mass of the \(j\)-th unit (kg); \(J_{zj}\) is the inertia moment of the \(j\)-th unit relative to \(C_jZ\) axis (kg/m²); \(V_x\) and \(V_y\) are longitudinal and transverse components of the velocity vector of the unit CM; \(a_x\) and \(a_y\) are longitudinal and transverse components of the acceleration vector of unit CM (absolute derivative of the unit CM velocity vector); \(\omega_{zj}\) is the angular velocity vector of the \(j\)-th unit (s⁻¹); \(\theta\) is the steering angle of the \(j\)-th unit relative to \(OX'\) axis (rad); \(x', y'\) are fixed coordinates of the unit center of mass (m); \(R_{xji}\) and \(R_{yji}\) are longitudinal and transverse components of soil interaction force acting on the \(i\)-th wheel of the \(j\)-th unit (N); \(\alpha\) is the bearing surface inclination.
angle (rad); \( P_{xw} \) and \( P_{yw} \) are longitudinal and transverse components of air resistance force (N); \( M_{jwi} \) is the cornering resistance moment of the \( i \)-th wheel of the \( j \)-th unit; \( F_{jfi} \) is the hitch force acting along \( C_jX_j \) axis (N); \( F_{ji} \) is the hitch force acting along \( C_jY_j \) axis (N); \( M_j \) is the moment applied to the \( j \)-th unit from forces acting in the hitch (Nm).

The last three equations in the set (1) establish the connection between moving and fixed coordinates. An individual set of equations is created for each unit according to (1). In this case, the traction combination of units may vary, i.e. trailer unit may be either active (with torque applied to wheels) and passive. To implement various traction combinations of units, the mathematical model of road train motion is complemented by equations describing the dynamics of a particular type of transmission.

In general, the dynamics of EMT with individually driven wheels can be described by the following set of equations:

\[
\begin{align*}
(J_{en} + J_G) \cdot \dot{\omega}_{en} &= M_{en} - M_{CE}; \\
(J_{wji} + J_{ji} \cdot U_{wg}^2) \cdot \dot{\omega}_{wji} &= M_{wji} - M_{Cji},
\end{align*}
\]

where \( J_{en}, J_G, J_T \) are the inertia moments of ICE crankshaft, generator shaft, shaft of the \( i \)-th TEM (kg/m²); \( \dot{\omega}_{en} \) is the angular acceleration of ICE shaft (s⁻²); \( M_{en} \) is ICE torque (Nm); \( M_{CE} \) is the resistance moment applied to ICE shaft (Nm). \( U_{wg} \) is the wheel gear ratio; \( \dot{\omega}_{wji} \) is the angular acceleration of the \( i \)-th wheel of the \( j \)-th unit (s⁻²); \( M_{wji} \) is the torque applied to the \( i \)-th wheel of the \( j \)-th unit (Nm); \( M_{Cji} \) is the motion resistance moment applied to the \( i \)-th wheel of the \( j \)-th unit (Nm).

The mathematical model considers variation in the normal load on fifth-wheel coupling (FWC). Variation in FWC normal load significantly affects traction and dynamic properties of the road train, because the weight on its driving axles depends on it. The calculation is based on the following principle: semitrailer is treated as a seven-bearing system, i.e. the seventh response from FWC (\( R_{fwz} \)) is factored in addition to six normal responses under semitrailer wheels (\( R_{i2j} \)):

\[
\begin{align*}
R_{fwz} - R_{i21} &= (x_{i2} - x_{w21}) \cdot k \cdot \tan \phi + (y_{i2} - y_{w21}) \cdot k \cdot \tan \psi = 0; \\
R_{i21} - R_{i22} &= (x_{w21} - x_{w22}) \cdot k \cdot \tan \phi + (y_{w21} - y_{w22}) \cdot k \cdot \tan \psi = 0; \\
&\vdots \\
R_{i25} - R_{i26} &= (x_{w25} - x_{w26}) \cdot k \cdot \tan \phi + (y_{w25} - y_{w26}) \cdot k \cdot \tan \psi = 0; \\
\sum_{i=1}^{6} R_{i2j} + R_{fwz} &= m_2 \cdot g \cdot \cos \alpha;
\end{align*}
\]

where \( i = 1...6 \) is the semitrailer wheel number; \( x_{i2}, y_{i2} \) are FWC coordinates relative to semitrailer CM (m); \( x_{w2i}, y_{w2i} \) are coordinates of the \( i \)-th wheel relative to semitrailer CM (m); \( k \) is the suspension stiffness coefficient (Nm); \( \phi, \psi \) are trim and roll angles (deg); \( m_2 \) is the semitrailer mass (kg); \( h_{z2} \) is the height of semitrailer CM; \( a_{z2}, a_{y2} \) is the acceleration of semitrailer CM (m/s²); \( M_{fz2i} \) is the rolling resistance moment of the \( i \)-th wheel (Nm); \( P_{xfw}, P_{yfw} \) are longitudinal components of FWC force (N); \( h_{fw} \) is the height of FWC (m).

The normal load on FWC affects the cornering resistance moment caused by friction between the semitrailer and FWC bolster. The friction model described in [16] is used to calculate this resistance moment.

### 3. Research results

The mathematical model of the active road train with EMT is implemented in Matlab/Simulink dynamic mathematical simulation environment. The effect of trailer wheel activation on road train dynamics was assessed using the results of simulation of going into corner and subsequent turning with a fixed position.
of the steering wheel and constant speed. Figure 4 shows simulation results for two power distribution cases (100/0 and 25/75).

Figure 4. Road train trajectories for two cases of power distribution between the units:

1. – 100/0; 2. – 25/75.

As can be seen from Figure 4, road train trajectories differ significantly. The turning radius of the tractor with passive semitrailer is 19.18 m, but it is 17.05 m with active semitrailer. The main reason for differences is the variation of wheel slip angles when torque is applied to semitrailer wheels. Figure 5 shows the comparison of wheel slip angles of active and passive semitrailers (for wheels 1, 3 and 6). Figure 6 shows the variation of tractor trajectory curvature for four cases of power distribution between the tractor and semitrailer.
Figure 5. Variation of wheel slip angles of active and passive semitrailers.

Figure 6. Variation of tractor trajectory curvature for various power distribution cases: 1 – 100/0; 2 – 75/25; 3 – 50/50; 4 – 25/75.
4. Conclusions
Based on computational experiments, we can conclude that the semitrailer wheel activation has a significant effect on the road train trajectory. The mathematical model can be used in designing automatic active drive control systems and solving a scientific and real-world problem of power distribution optimization using various electronic control systems, including those based on the hitch force factor analysis.

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