Extinction at the Galactic Center Using Near- and Mid-infrared Broadband Photometry: A Twist on the Rayleigh–Jeans Color Excess Method

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Abstract

We present an extinction map of the inner ~15′ by 16′ of the Galactic center (GC) with map pixels measuring 5″ × 5″ using integrated light color measurements in the near- and mid-infrared. We use a variant of the Rayleigh–Jeans color excess (RJCE) method first described by Majewski et al. as the basis of our work, although we have approached our problem with a Bayesian mindset and dispensed with point-source photometry in favor of surface photometry, turning the challenge of the extremely crowded field at the GC into an advantage. Our results show that extinction at the GC is not inconsistent with a single power-law coefficient, β = 2.03 ± 0.06, and compare our results with those using the red clump (RC) point-source photometry method of extinction estimation. We find that our measurement of β and its apparent lack of spatial variation are in agreement with prior studies, despite the bimodal distribution of values in our extinction map at the GC with peaks at 5 and 7.5 mag. This bimodal nature of extinction is likely due to the infrared dark clouds that obscure portions of the inner GC field. We present our extinction law and map of the GC region using the point-source catalog of infrared sources compiled by DeWitt et al. The dereddening is limited by the error in the extinction measurement (typically 0.6 mag), which is affected by the size of our map pixels and is not fine-grained enough to separate out the multiple stellar populations present toward the GC.

Unified Astronomy Thesaurus concepts: Galactic center (565); Interstellar extinction (1093); Infrared excess (788); Infrared photometry (792)

Supporting material: data behind figure

1. Introduction

Interstellar extinction by dust has long been the bane of observers. Trumpler (1930) realized that some open clusters seemed smaller or dimmer than one might expect based on their angular size and number of stars. Trumpler assumed that open clusters were a sort of standard candle if one adjusted for the number of stars and angular extent on the sky, and so their photometric distance should correspond to their geometric distance, which he derived after assuming that the cluster’s diameters were roughly linear with the number of stars and calculating the distance based on the ratio of their geometric to apparent angular size. He found that more distant clusters were dimmer than expected, and concluded that astronomical extinction was the cause of this excess dimming. He estimated the extinction along the lines of sight to several open clusters by comparing photometric distances (via the distance modulus) to his derived geometric distances. He found that the extinction was consistent with a mean effect of 0.67 mag per kiloparsec of extinction. The source of astronomical extinction is now largely thought to be due to the scattering of light off of dust grains in the interstellar medium (ISM; Draine & Lee 1984; Weingartner & Draine 2001; Foster et al. 2013, and references therein).

These dust grains are believed to be a mix of primarily silicates and carbon-based molecules (including graphite-based molecules and more complex polycyclic aromatic hydrocarbons, or PAHs), both with and without a veneer of frozen volatiles such as water or methane ices. Draine & Lee (1984) modeled simple spherical silicate and graphite grains without an icy layer, which matched much of the observational data from optical to far-infrared wavelengths along many sight lines. However, broad emission features in the mid-infrared (MIR) did not fit simpler models; adding PAH molecules to the models brings them into better agreement with the data (Weingartner & Draine 2001; Draine & Li 2007). In particular, broad emission features commonly attributed to PAH emission occur centered at MIR wavelengths near 3.3, 6.2, 7.7, and 8.6 μm (Leger & Puget 1984; Allamandola et al. 1989; Rapacioli et al. 2005; Povich et al. 2007), among other emission features. The mechanism responsible is thought to be UV radiation reprocessed by PAH molecules and emitted at MIR wavelengths, principally via C–H bond stretching. At near-infrared (NIR) and optical wavelengths Rayleigh scattering dominates, as the typical particle size is much larger than the wavelength (R ≪ λ).5

Cardelli et al. (1989, henceforth CCM) were able to model extinction along several lines of sight to bright stars with a single-parameter fit:

\[
A_V = A_V(\alpha_V + b_V R_V)^{-1},
\]

where \( R_V \) is the slope of the extinction curve in the V band, defined as

\[
R_V = \frac{A_V}{E(B - V)},
\]

5 In the case of particles that are roughly the size of the wavelength of light (R ≈ λ), Mie theory is a more complete description of scattering.
aligned to each other. We generate an \((H - [4.5\mu m])\) color map from these surface brightness maps and use it as an input in a Bayesian framework to measure the extinction law toward the GC.

In Section 1.1 we have a brief discussion on filters and stellar color before introducing the RJCE method of estimating extinction in Section 1.2. Another method of measuring extinction, based on measuring red clump (RC) stars, is discussed in Section 1.3. We then delve into the assumptions behind near-infrared extinction laws in Section 1.4. In Section 2 we describe the data sets, first the near-infrared Infrared SidePort Imager (ISPI) image data in Section 2.1, followed by a discussion on the data reduction in Section 2.2. Similarly, Section 2.3 describes the Spitzer mid-infrared image data, while Section 2.4 describes how we reduced the Spitzer data. Section 2.5 describes how we build our color maps. In Section 2.6 we discuss the innovative approach we have taken to produce extinction maps using surface brightness, color map, and the RJCE. Then in Section 3 we show our extinction map, compare our results to the literature, and apply our extinction map to create dereddened color–magnitude diagrams derived from the ISPI point-source catalog. We conclude this work in Section 4 with a brief discussion on ideas for improvement and future work. We discuss our Bayesian approach, prior selection, and how it affects our results in Appendix.

### 1.1. Stellar Color and Filter Selection

Stars, to the first order, are blackbodies, but the range of intrinsic colors created by those blackbodies is broadened by the range of stellar effective temperatures, which are driven in turn by stellar masses, ages, and chemical compositions. Nonstellar emission from interstellar dust, such as PAH emission, features centered at 3.3, 6.2, 7.7, and 8.6 \(\mu m\).

Nonstellar emission features centered at 3.3, 6.2, 7.7, and 8.6 \(\mu m\) (Leger \\& Puget 1984; Allamandola et al. 1989; Rapacioli et al. 2005; Povich et al. 2007) coincide in wavelength space with the [3.6\(\mu m\), [6.8\(\mu m\), and [8.0\(\mu m\) Spitzer filters. Figure 1 shows side by side the \(H\)-band and [8.0\(\mu m\) images of the GC; the striking difference between the two images strengthens the strongly nonstellar origin of PAH emission, especially in comparison to the ratio of the absolute extinction in the \(V\) band to the color excess; the commonly adopted value of \(R_V\) in the Milky Way is 3.1 (CCM) but a range of 2 to 5.5 invoked for extreme environments (Foster et al. 2013). In Equation (1), \(a_\lambda\) and \(b_\lambda\) are piece-wise defined functions broken at 0.9 \(\mu m\). They are high-order polynomials in the optical bands, simplifying to a power law in the infrared with an index, \(\beta\), of about 1.61 (CCM; see also Mathis 1990; Foster et al. 2013, and references therein). We note others have found values of \(\beta\)~2 or more (Indebetouw et al. 2005) particularly in the direction of the Galactic center (GC; Nishiyama et al. 2006; Nogueras-Lara et al. 2018; Hosek et al. 2018). The infrared extinction equation is of the form

\[
A_\lambda \propto \lambda^{-\beta}
\]

where the wavelength range is approximately 0.9 \(\mu m < \lambda < 10 \mu m\). We discuss infrared extinction laws in Section 1.4 in more detail.

While CCM found that on average the value of \(R_V\) is 3.1, there is no reason a priori that \(R_V\) should be the same along all lines of sight due to the inhomogeneity of the Milky Way disk structure. Weigartner \\& Draine (2001) interpret the value of \(R_V\) as being linked to the physical size of the grains of dust, with larger grains having larger \(R_V\) values. This is an appealing model linking observations and theory.

The approach we have taken to measure the infrared extinction law at the GC is to use the Rayleigh–Jeans color excess (RJCE) method to determine the amount of extinction in map cells that are 5′ on a side. We do this by creating surface brightness maps in both the \(H\)-band and [4.5\(\mu m\)] bands that are well...
Figure 2. As in Figure 1, the images are centered on the GC. North is up and east is to the left. Left: the fully reduced ISPI H-band image of the GC. North is up and east is to the left. Note in particular the dark stripes running from right to left of the image (the “tiger stripes”); the lack of stars in the $H$ band indicate a very high extinction in those regions. The image is 17′ by 16′ (R.A. and decl., respectively) and is centered on the GC. Right: the Spitzer [4.5$\mu$m] band image of the GC. Black regions are data reduction artifacts and have zero counts. This image shows less evidence of extinction as there are no dark stripes to the right of the GC as there are in the H-band image. By eye several regions stand out as having an underdensity in stars, particularly the large region to the south and east and a slightly smaller region north and west of the GC.

Figure 2, which shows the [4.5$\mu$m] band image next to the H-band image. The [4.5$\mu$m] filter is the only Spitzer filter left not contaminated by PAH emission; we can and have assumed that the flux in this filter is largely stellar blackbody emission. We used the TRILEGAL model (Girardi et al. 2012) to generate several intrinsic color–color diagrams shown in Figure 3. The TRILEGAL model uses the Girardi et al. (2002) isochrones for a wide range of ages ($7 \leq \log(\text{age/yr}) \leq 10.15$), masses ($0.08 \leq M_\odot < 21$), and metallicities ($-1.5 \leq [\text{Fe/H}] \leq -0.18$), using a Chabrier initial mass function (IMF) to generate three stellar populations representing the disk, bulge, and halo. In Figure 3 main-sequence stars are blue and evolved stars (RC stars, discussed in Section 1.3, and red giant branch (RGB) stars) are red. Of note is the relatively tight distribution in both ($K_S - [4.5\mu m])_0$ and ($H - [4.5\mu m])_0$ regardless of stellar type; this is particularly true for evolved stars.

1.2. Rayleigh–Jeans Color Excess Method

The RJCE method (Zasowski et al. 2009; Nidever et al. 2012; Majewski et al. 2011) hinges on the fact that stars have a fairly uniform intrinsic color in the infrared. This allows us to assume that all stars have nearly identical color for a certain set of filters. For example, nearly all stars have an identical color ($H - [4.5\mu m])_0$, which is calculated by subtracting the magnitude in InfraRed Array Camera (IRAC) Channel 3 band (effective wavelength: 4.442 $\mu$m) from the $H$ band (effective wavelength: 1.664 $\mu$m), regardless of their age, metallicity, mass, and a host of other physical properties. Figure 4 shows an ($H - [4.5\mu m])_0$ versus ($J - [4.5\mu m])_0$ color–color diagram generated via the TRILEGAL model in order to derive the intrinsic ($H - [4.5\mu m])_0$ color scatter across the wide swathe of model stellar type and age. Earlier work by Majewski et al. (2011) found that the intrinsic color, ($H - [4.5\mu m])_0$, is about 0.08 mag for a wide range of stars, with a scatter of about 0.1 mag over F, G, and K stars.

We found that the scatter in ($H - [4.5\mu m])_0$ is at most 0.4 mag across all stellar types (B–M); if one excludes the O, B, and M dwarfs, we find that the scatter over the A, F, G, and K stars from the TRILEGAL model is about 0.1 mag, in good agreement with Majewski et al. (2011). This deliberate exclusion of the O, B, and M dwarfs is motivated by the fact that similarly, the hottest O and B dwarfs are the rarest stars in an imaging survey. Similarly, M dwarfs, while the most plentiful stars in the Galaxy, are also intrinsically the dimmest and therefore are insignificant in our ISPI and Spitzer images. Additionally, our technique averages over a number of stars in each map pixel, and this dilutes the effects of a single O, B, or M dwarf star in a pixel.

Other colors were considered by Majewski et al. (2011), but were discarded in favor of ($H - [4.5\mu m])$. A potential alternative color is ($K_S - [4.5\mu m])$, which exhibits a smaller scatter than ($H - [4.5\mu m])$. We opted to not use $K_S$ in order to preserve a more direct connection with Majewski et al. (2011). Also, in terms of spectroscopic follow-up measurements of extinction along the sight lines of single stars, the Apache Point Observatory Galactic Evolution Experiment (APOGEE) survey is a natural choice, but it is limited to $H$-band spectroscopy, and this makes using $H$-band photometry more appealing. Further, we note that at the GC, the contamination from PAH emission features at 3.3, 6.2, 7.7, and 8.6 $\mu$m is severe enough that only the [4.5$\mu$m] mid-infrared images are useful for determining color using starlight.

1.3. The Red Clump Extinction Method

A popular technique for estimating interstellar extinction using only near-infrared point-spread function (PSF) photometry is the RC method to estimate both extinction power-law exponents and extinction values (Nishiyama et al. 2006; Nogueras-Lara et al. 2018; Hosek et al. 2018, 2019).
We note that this method is solidly independent of our surface brightness approach. The RC method can be summed up thusly: if one assumes that RC stars all exhibit the same near-infrared colors, one can infer the amount of extinction based on the amount of reddening in the three near-infrared bands. This method assumes that because RC stars have just begun their helium-core burning phase and thus are producing the same amount of energy regardless of total stellar mass, core mass, or metallicity (and exhibit a small intrinsic color range as a result; Salaris & Girardi 2002) that they can be considered standard candles. With many thousands of color excess measurements of RC stars, one can build up a reddening law as well. However, by using only near-infrared bands and restricting the stars used to those with measurable J-band magnitudes, this method necessarily probes a shallower distance toward the GC than our surface photometry simply due to the extreme reddening present in the direction of the GC. We compare our results to other RC measurements of the near-infrared extinction law in Section 3.

### 1.4. Extinction Laws in the Infrared

Historically, extinction in the Milky Way has been measured in the optical along many lines of sight and then fit to a single parameter, $R_V$, from Equations (1) and (2) (Cardelli et al. 1989; Zasowski et al. 2009; Majewski et al. 2011; Nidever et al. 2012; Foster et al. 2013). In the near-infrared, extinction laws are well fit with a simple power law, which can be written as

$$\log_{10}\left(\frac{A_{\lambda}}{A_{K_S}}\right) = C - \beta \log_{10}[\lambda],$$

where $\beta$ is the slope of the extinction fit as a function of $\lambda$, and $\lambda$ is the effective wavelength of the filters used to make the measurements (Indebetouw et al. 2005). The constant, $C$, is the value of the equation at the limit $\lambda \rightarrow \infty$; the standard way of finding $C$ is to extrapolate the relative extinction values and is found to be about 0.6 (Indebetouw et al. 2005; Nishiyama et al. 2006). The relative extinction, $\frac{A_{\lambda}}{A_{K_S}}$, is found by measuring the color excess relative to $K_S$ and normalizing to one value of $K_S$. 

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**Figure 3.** Near-infrared and mid-infrared color–color diagrams. Stars that have evolved off the main sequence are red and main-sequence stars are blue. Note the tight relation of both $(K_S-[4.5\mu])$ and $(H-[4.5\mu])$. The overall scatter in the color of all stars is smaller in $(H-[4.5\mu])$ than in $(K_S-[4.5\mu])$. 

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**Figure 4.** TRILEGAL Color-Color Diagrams
relative extinction (in other words, making an assumption, or measurement, of the extinction law at two wavelengths; Rieke & Lebofsky 1985). Indebetouw et al. (2005) found that when incorporating mid-infrared measurements of RC stars (see Section 1.3), the infrared extinction is better fit with a second-order polynomial in log-space,

$$\log_{10} \left[ \frac{A_{\lambda}}{A_{K_S}} \right] = C - \beta_1 \log_{10}(\lambda) + \beta_2 (\log_{10}(\lambda))^2,$$

(5)

where $\beta_2$, $\beta_1$, and $C$ are fit using a weighted least-squares approach, and the $\log_{10}(\lambda)$ terms are calculated using the effective wavelength (in microns) of the filter in question. We opted to use Equation (4) for our analysis because most infrared extinction laws in the literature are linear (making direct comparisons with more complex laws difficult), and unless there is a pressing reason (theoretical motivations, for example), we prefer to use simpler models with fewer degrees of freedom. Based on earlier work (Indebetouw et al. 2005), we adopted a fixed value of $C = 0.60$. In contrast, Nishiyama et al. (2006), using the RC method, found that $\beta = 1.99 \pm 0.02$ and $C = 0.494 \pm 0.004$.

We use $(H - [4.5\mu])$ to calculate the extinction in $K_S$ ($A(K_S)$). Calculating the relationship between the color $(H - [4.5\mu])$, $\beta$, and $A(K_S)$ is straightforward. We begin with two equations, $(\log_{10} \left[ \frac{A_{\lambda}}{A_{K_S}} \right]_0)$ and $(\log_{10} \left[ \frac{A_{\lambda}}{A_{K_S}} \right]_1)$, and note that the color excess normalized by the extinction in $K_S$, $E(H-[4.5\mu]) / A(K_S)$, is the same as $A_H / A_{K_S}$. With some algebra, we can reduce the two equations to

$$E(H-[4.5\mu]) = \frac{A_H}{A_{K_S}} - \frac{A_{[4.5\mu]}}{A_{K_S}} \times 10^{C - \beta_1 \log_{10}(\lambda_H) - \beta_2 (\log_{10}(\lambda_H))^2}.$$  

In this equation, $\lambda_H$ and $\lambda_{[4.5\mu]}$ are simply the effective wavelength in microns of the filters (1.664 for $H$, 4.442 for $[4.5\mu]$). The intrinsic color, $(H - [4.5\mu])_0$, must also be subtracted from the color excess; its value is 0.08 mag (Majewski et al. 2011). Solving for $A_{K_S}$, we find

$$A_{K_S} = \frac{E(H-[4.5\mu]) - 0.08}{10^{C - \beta_1 \log_{10}(\lambda_H) - \beta_2 (\log_{10}(\lambda_H))^2}}.$$  

(6)

With our data we are able to fit $\beta$ as well as measure the extinction. The RJCE method thus requires only a measured $(H - [4.5\mu])$ value and Equation (6) with a suitable $\gamma$:

$$A_{K_S} = \gamma \left[ E(H-[4.5\mu]) - 0.08 \right].$$  

(7)

Plugging in the values for $C$ and $\beta$ from Indebetouw et al. (2005) and using our effective wavelengths, we calculate $\gamma$ to be 0.918, in agreement with Majewski et al. (2011), Zasowski et al. (2009), and Nidever et al. (2012). We used the simpler extinction power law for our work and found that we were able to independently fit for $\beta$ using our Bayesian approach, described in more detail in Appendix.

2. The Data Set

We use imaging data of the GC taken with the ISPI camera on the Cerro-Tololo International Observatory 4 m Blanco Telescope (van der Bliek et al. 2004) in 2005, for our $H$-band photometry. This data set was obtained and used by DeWitt et al. (2010) to find infrared counterparts to X-ray sources. DeWitt et al. (2010) generated a point-source catalog using DAOPHOT (Stetson 1987), first reducing the images using the Florida Analysis Tool Born Of Yearning for high-quality astronomical data (FATBOY) and then cross-checking their point-source catalog against the Two Micron All-Sky Survey (2MASS) point-source catalog (Skrutskie et al. 2006). We used the well-calibrated ISPI Point Source Catalog (henceforth IPSC) created by DeWitt et al. (2010) as ground-truth for our ISPI data reduction and calibration. We re-reduced the ISPI images for this work using a revamped version of FATBOY.
that is optimized for GPUs named superFATBOY (Warner et al. 2012).

For the 4.5μJ data we used the IRAC (Fazio et al. 2004) on board the Spitzer Space Telescope taken for the Galactic Legacy Infrared Midplane Survey Extraordinaire (GLIMPSE) survey (Benjamin et al. 2003; Churchwell et al. 2009). These data were downloaded using the website for the NASA Infrared Processing and Analysis Center (IPAC) Infrared Science Archive (IRSA), hosted at the Jet Propulsion Laboratory at the California Institute of Technology.9 We selected the 0′′6 images to assist with reducing the number of sources that are saturated.

2.1. The ISPI Data Set

The ISPI camera has a 10″ square field of view with a plate scale of 0″3 pixel−1 (van der Bliek et al. 2004). GC data were obtained in the J, H, and Ks bands on 2005 August 10. Four fields were observed, each using a four-point dither pattern with a dither step size of 20″ to cover a 10″3 field and to remove the detector’s cosmetic defects; the four overlapping fields were tiled to make a final field of view of 17′′ × 17′′.10 The centers of the four fields make a square 420″ on a side centered slightly south and to the east of the GC, at 17h 45m 39.9″, −28° 59′ 45.0″ (J2000). After the dither sequences were completed for all four fields, the telescope was centered to check for focus; as the focus stability was very good and required no adjustment, we effectively have an additional field from this data, albeit with a different exposure time than the four quadrant fields.

The individual frame exposure times in J, H, and Ks were 5 s, 3.2 s, and 3.2 s, respectively. The frame exposure times were kept short to prevent the saturation of the brightest stars in the field, or at least minimize the number of saturated pixels. Each of the four fields had a total exposure time of 200 s, 113 s, and 32 s for J, H, and Ks, respectively. The center field, made with the focus check frames, had a total of 60 s, 68 s, and 68 s for J, H, and Ks, respectively. We then made a master image out of all five fields, giving a rather deep Ks image of the center of the image in comparison to the corners.

After the four-point dither sequence was completed at each field, a less-crowded off-source field about 2° away was observed for sky background estimation. This off-sky field contained fewer bright sources than the GC field while still being close enough to give acceptable sky background estimates. After reducing the ISPI data with superFATBOY, we produced master images in J, H, and Ks roughly 17′ by 17′ in size. The depth of this master image varies across the 17′ field of view and bandpasses, and ranges from 200 to 468 s, 113 to 294 s, and 32 to 132 s for J, H, and Ks, respectively. The exposure time is deepest at the center of the field and shallowest at the corners.

2.2. ISPI Calibration

As noted above, we used the superFATBOY data reduction code (Warner et al. 2012) to reduce the ISPI data. Written in Python, superFATBOY is massively parallelized to take advantage of NVIDIA’s Compute Unified Device Architecture (CUDA) Graphics Processing Unit (GPU) devices with hundreds to thousands of GPU cores.11 It is based on the CPU-only FATBOY-SLIM (FATBOY-Sans Lousy IRAF Mistakes) Python code with modifications geared toward making superFATBOY usable for any infrared or optical imaging or spectroscopy while depending on stable Python scientific packages like astropy and scpy. The algorithms used by superFATBOY are not instrument-specific (although modules that are instrument-driven can be incorporated) to reduce infrared data following standard data reduction algorithms, such as distortion correction, dark-subtraction, flat- fielding, bad pixel masking, sky background estimation and subtraction, and image stacking.

Sky background estimation and subtraction is one of the more delicate operations when reducing GC data. Because the infrared sky background levels change drastically in a stochastic manner over timescales of minutes, background estimation can be challenging; this is true even for uncrowded fields. In the GC, the source density is so high that we cannot get a clean estimate of the sky background using the scientific data.

The off-GC field used for sky background estimates fewer contained bright sources than the GC field while still being close enough to give decent sky background estimates. The ISPI data were reduced using superFATBOY’s off-source sky subtraction method. We made several iterations to find which sky background interpolation method worked best.12 In the end we ended up using a two-pass sky estimator algorithm to better remove the sources present in the off-source skies, and used the nebular flavor of scaling the background to the median of the science frames. Additionally we restricted the off-source frames that went into making the master sky background frames to be drawn from those off-source frames taken nearest in time to the science frames; in other words, each dither sequence used the off-source frames taken immediately after or before the sequence was completed. Ordinarily all the off-source sky frames go into making the master sky background image, but because the observations were distributed in time over the course of about an hour, the sky background could not be considered constant.

Once we had the reduced master images, we corrected and made uniform the astrometry of each image. In order to have a uniform astrometric solution across all of our images, we used a three-step process applied to each of the master images. By having a uniform astrometric solution, we were able to resample all of the images onto a common pixel grid. This is motivated by our desire to make color maps, which require the parent images to have the same pixel grid as much as is possible. We used Source Extractor (Bertin & Arnouts 1996), Scamp (Bertin 2006), and SWarp (Bertin et al. 2002) to correct the astrometry to a common pixel grid. The algorithm we used for astrometric correction is as follows:

1. Source Extractor: find the pixel coordinates of stars (we restricted Source Extractor to use bright but unsaturated stars) and record the plate scale of the image in a catalog.

9 The GLIMPSE website: http://irsa.ipac.caltech.edu/data/SPITZER/GLIMPSE.
10 To try and reduce the confusion between the four fields and the four-point dither pattern, we spell out numbers (e.g., one, four) when discussing fields and use numerals (e.g., 1, 4) when discussing the dither pattern.
11 superFATBOY can also be run in CPU mode, albeit with considerably slower run time in comparison to the GPU mode.
12 We used the ISPI data set as a testbed for the alpha version of superFATBOY, when results were in agreement with the previous data reduction performed by DeWitt (2011), we declared superFATBOY a success.
2. Scamp: compare the pattern of the stars’ locations from the Source Extractor catalog and, using an initial guess of the location on the sky provided by the FITS header, use the 2MASS point-source catalog to calculate both new astrometry and map any residual distortion present in the image.

3. SWarp: apply the astrometric and distortion correction from Scamp.

SWarp also provides the ability to set the output pixel scale. It uses a flux-conserving bicubic interpolation algorithm to resample the images onto the new grid. We used this function to make our output pixels $0.′′3$, or unchanged from the ISPI raw frame plate scale; we did this to simplify the map-making step later on. This process was repeated with the Spitzer data as described in Section 2.4.

With our reduced and astrometrically corrected images, we were able to make direct comparisons with the IPSC (DeWitt et al. 2010) and use this catalog to find the zero-point (ZP) magnitude of our newly reduced images and calibrate the photometry. First we made a Source Extractor catalog of the stars in our images. We then used the stars in our catalog with a signal-to-noise ratio $>500$ (where the signal and noise were taken from the Source Extractor columns “FluxBest” and “errFluxBest,” respectively) and, using the R.A. and decl. coordinates from our catalog, matched them to stars in the IPSC within $1′′$ on the sky. Of the 749 stars that were selected with a signal-to-noise ratio $>500$, 187 were rejected as having no counterpart in the IPSC. A further 15 were rejected for having two counterparts within $1′′$, leaving a total of 547 singly matched stars. We examined the curve of growth for singly matched stars to find the aperture size which appeared to capture all or most of the flux of each star and found that a $2′′$ aperture is where the curve of growth flattened out.

With the aperture set, we then found the instrumental magnitude for each star in our catalog by taking the log of the total flux in the $2′′$ aperture and multiplying by $−2.5$:

$$\text{instrumental magnitude} = −2.5 \log_{10}(\text{total flux}).$$ (8)

To find the ZP, we then took the magnitude from the IPSC and subtracted our instrumental magnitude:

$$\text{zero point} = \text{IPSC magnitude} − \text{instrumental magnitude}.$$ (9)

We found that our instrumental ZP magnitude in the $H$ band is best fit with a Gaussian with a mean of $22.26\text{mag}$ and a standard deviation of $0.09\text{mag}$. We do not quote the reduced error $(N^{−1/2} \times \sigma = (547)^{−1/2} \times 0.09\text{mag} = 0.004\text{mag})$ in order to be more conservative in our error estimation of our photometry.

Estimating the background at the GC is extremely difficult due to the high stellar density. Typically one uses an annulus centered around each point source with an inner radius of $10′′−20′′$ away, but at the GC, the crowding limit makes this approach worse than useless as each line of sight typically ends in another (albeit unresolved) star or stars. We do not perform background subtraction for the stars used to perform our ZP measurement because:

1. The stars we have selected for ZP measurement range in magnitude from $H = 10.5$ to $H = 12$. These stars are bright enough that the contribution from the background is not the dominant source of error. They are not bright enough to have saturated the detector, nor are they bright enough to be in the nonlinear regime of the detector.

2. The estimated magnitude errors for these stars in the IPSC is on average $0.014\text{magnitudes}$ with a standard deviation of $0.013\text{magnitudes}$.

We adopt the sum in quadrature of the standard deviation to our ZP fit and the average error from the IPSC stars as our estimated error in our photometry: $\sigma_{\text{phot}} = \sqrt{0.09^2 + 0.014^2} = 0.091 \approx 0.09$.

2.3. The Spitzer Data Set

We selected images from the GLIMPSE survey that covered as much as the ISPI field of view as possible. In the interest of time and given that we were limited to image stamps smaller than $600′′$ on a side, we downloaded a mosaic belonging to data set Spitzer #0013368832. This mosaic was created using 225 subimages of the GC and is $77′′5$ by $26′′8$ (Galactic longitude and latitude, respectively). However, at the Galactic longitude we care about ($|l| < 0.25$), this mosaic is only about $16′′$ high in latitude, meaning that we are unable to generate colors involving any of the mid-infrared bands for about $30′′$ from the top and bottom of our ISPI field.

2.4. Spitzer Calibration

The Spitzer images we downloaded were mosaicked and calibrated as part of the GLIMPSE survey. In order to guarantee excellent astrometric matching between our ISPI and Spitzer data, we used the Source Extractor → Scamp → SWarp algorithm described in Section 2.2, using Source Extractor to pull out bright but unsaturated stars from the Spitzer images. Instead of using the 2MASS catalog to correct the Spitzer astrometry (as we did for the ISPI image), we used a new star catalog created from running Source Extractor on the fully reduced and Scamped/ SWarped ISPI $H$-band image. We did this for two reasons. First, this allowed us to make a magnitude cut on the ISPI $H$-band Source Extractor catalog, our reasoning being that moderately bright stars in the $H$ band should be reasonably bright at mid-infrared wavelengths, without having to rely on the 2MASS point-source catalog (which is not as deep as our $H$-band image). Second, it also forced the Spitzer images to match the astrometry of the $H$-band image after we performed the Scamp → SWarp algorithm. We made the output plate scale match the IPSC plate scale ($0.′′3\text{pixel}^{-1}$), letting SWarp perform the resampling using its default (flux-conserving) settings.

The IRAC image values are in MJy sr$^{−1}$. To convert to Vega magnitudes, we simply use the following formula taken from Equation (4.19) in IRAC Instrument & Instrument Support Teams (2015):

$$m_{\text{Vega}} = −2.5 \log_{10}(\text{flux}) + 2.5 \log_{10}(\text{ZP}/C),$$ (10)

where the ZP depends on the IRAC channel (IRAC Instrument & Instrument Support Teams 2015, Table 4.9). Unlike the ZP for ISPI, IRAC’s ZP is the flux of a zeroth magnitude star in the Vega system, for $[4.5\mu]$, $C = 179.9 ± 2.6\text{Jy}$. The correction factor, $C$, in Equation (10) is a conversion factor from MJy sr$^{−1}$ to Jy pixel$^{−1}$. For the $0′′6$ by $0′′6$ pixels, $C = 8.461595\text{e}−6\text{Jy pixel}^{−1}/(\text{MJy sr}^{−1})$. For the Scamped/
SWarped image with $0\prime\prime.3$ by $0\prime\prime.3$ pixels, $C = 23.5045e-6Jy\,pixel^{-1}/(M\,Jy\,sr^{-1})$.

We need to perform an aperture correction on the surface brightness measured at each pixel. The Spitzer Handbook Section 4.11.3 gives the Spitzer surface brightness corrections (for $[4.5\mu]$), the fudge factor is 0.94 and at the end of that section notes that the correction factors should be good to 10%. In other words, the real surface brightness (SB) can be calculated by

$$\text{SB} = \text{sb} \times f$$  \hspace{1cm} (11)

where “sb” is the measured surface brightness in a pixel and $f$ is the correction fudge factor, which has the accuracy of about 0.1 magnitudes—comparable to the ISPI photometric (ZP) error, and, for the $[3.6\mu]$ and $[4.5\mu]$, $f$ is not significantly different from unity. The correction factor for $[4.5\mu]$ is 6% $\pm$ 10%, and when these are added in quadrature the result is about 10%. As a result, we have not applied the correction to our $[4.5\mu]$ photometry and instead adopt a blanket 10% error for our SB photometry error term.

2.5. Map Making

The next part of our analysis required us to build an $[H - 4.5\mu]$ color map from the astrometrically matched ISPI and Spitzer images. We used a map-making code written in Python by former University of Florida graduate student Daniel Gettings. The map_tools module was designed to create two-dimensional histograms with the bin size chosen by the user while maintaining full World Coordinate System (WCS) information. We generated maps with cells ranging in size from $5''$ to $60''$, effectively summing the pixel values in each image that fall in each map’s cell and converting that to an SB in Vega magnitudes. We note that map_tools does conserve flux geometrically, allotting the fractional value to map cells of pixels that partially fall on those cells.

We settled on using the $5''$ cell size for our maps because we wanted to use the smallest cells possible without partially resolving individual bright stars; given that the PSF of the IRAC $[4.5\mu]$ bandpass has an FWHM of about $1''1$, a $5'' \times 5''$ square aperture is about as small as we can go without single stars dominating a cell.

Pixels from the original images that straddle cell borders are a potential source of smearing in the sense that the PSFs of bright stars may fall across cell boundaries and contaminate the cell with perhaps a few pixels’ worth of light. However, given that the cells have an area of 25 arcsec$^2$, the contribution from a contaminated pixel is strongly diluted as the native pixel area is only 0.09 arcsec$^2$, a factor of almost 300 smaller than the map cells. Additionally, this contaminated light is not noise as it is still stellar in origin. In the case of a star centered more or less on a cell border, we note that the intensity measured in the map cells reflect the fact that there is a star whose PSF falls in all of the cells that it touches.

Once we generated our binned maps, creating color maps (e.g., $H-K_S$ or $H-[4.5\mu]$) was easy because all of the maps have identical cell and WCS coordinate grids.

2.6. The Innovation: Surface Brightness

Here we describe the RJCE method as we apply it to the GC region. We use the binned maps made as described in Section 2.5 to generate an $[H-[4.5\mu])$ color surface brightness map. This map acts as our measurement for each $5'' \times 5''$ cell. In aggregate, stars in any one particular cell have a $\langle H-[4.5\mu])_0 \rangle$ color of $0.08 \pm 0.1$; the errors originate in the intrinsic distribution of stellar color as shown in Figure 4 and discussed in Section 1.2. Thus, if we sum the colors of all sources (assumed to be stars), we can simply appropriate the intrinsic color and associated error as the expected color and color error of each cell. This allows us to take the measured color excess and assign an extinction value to each cell. Additionally, given the large amount of data (over 45,000 map cells), we can use a Bayesian approach to find values of $\beta$ and the extinction $A(K_S)$ given a distribution of extinction laws derived from varying $\beta$ and the observed color excess.

3. Results

In this section, we present our extinction map in Figure 5. This figure shows the $H$-band image, $[H-[4.5\mu])$ color map, $A(K_S)$ map, $\beta$ map, and the standard deviation of each pixel’s 29,000 realizations for both $A(K_S)$ and $\beta$. Examining the $H$-band image (upper left panel in Figure 5), there are dark regions that seem to have no stars. These regions persist across the $[H-[4.5\mu])$ color map, the $A(K_S)$ map, and the standard deviation in $A(K_S)$ map (the top row, middle right, and lower right panels of Figure 5), showing up as regions of high extinction and high standard deviation in the measured extinction. This bimodality of dark patches versus elsewhere in the $H$-band image and subsequent maps motivated us to examine the histograms of the extinction and $\beta$ values to look for evidence of bimodality and variation, respectively, the results of which are shown in Figure 6. We find that while there are typical and high regions of extinction, both are consistent with being described by a single power law. We discuss our results for the extinction $A(K_S)$, $\beta$, relative extinction values, and their comparison to previous work in Table A1 in the following subsections. The details of how we derived extinction and $\beta$ maps from the GC color map can be found in Appendix.

3.1. $A(K_S)$ at the GC

We find that the distribution of $A(K_S)$ is strongly double peaked, as demonstrated by Figure 5. This bimodality is also evident in the color histogram in Figure 6. Map cells with values of high extinction, $A(K_S) > 7.5$, are presented as a lower limit of the actual extinction; these map cells have large $H$-band magnitudes (meaning few stars or equivalently faint flux) and correspond to the darkest regions in the $H$-band image. It is possible that the extinction in these map cells is significantly higher than 7.5, but without higher flux in the $H$ band (or using a different color) we are insensitive to extinction higher than 7.5 due to the minimal $H$-band flux. This lack of flux in the $H$ band is imprinted upon the $\sigma_{K_S}$ map: the regions of highest extinction show the highest $\sigma_{K_S}$ as well.

The standard deviation map of $A(K_S)$ shows quite a bit of structure. The typical standard deviation values range in value from 0.5–1.0 mag. The tiger stripes are distinct, as are patchy sections to the north and the west (upper right), which we interpret as evidence that these regions are so highly extinguished that there are not many stars in those $H$-band map cells.

Given the obvious spatial grouping of high-extinction map cells, we were curious to see if (1) there was a preferred spatial
Figure 5. Maps of the GC. All of the images have been aligned to show the same field which spans approximately $17' \times 16'$. North is up, east is to the left. The northwest corner (upper right) is at 17h 44m 56.1s, −28d51m29.6s (J2000). The center of the field of view for all images is 17h 45m 39s, −28d 59m 45s (J2000). These images and maps are available online as FITS images. Top left: the GC in the $H$ band. Top right: $(H-[4.5\mu])$ color map, binned at $5'' \times 5''$ pixels. Middle left: mean $\beta$ map. The scale goes from 2.01 (black) to 2.06 (white). Here we interpret the apparent lack of structure in our map of the mean $\beta$ values as consistent with a lack of spatial variation of the extinction law. Middle right: mean extinction $A(K_s)$ map. The scale goes from 0.3 (black, low extinction) to 9.5 (white, high extinction). Bottom left: standard deviation of $\beta$ map. The scale goes from 0.28 (black) to 0.295 (white). The standard deviation of the $\beta$ values is tight and shows no obvious spatial structure, which is consistent with the interpretation of no spatial variation of the extinction law at the GC. Bottom right: standard deviation in the $A(K_s)$ map. The scale goes from 0.4 (black) to 0.95 (white). Note that the areas of highest extinction (the black cells in middle right panel) have elevated (brighter blue or white) standard deviations, indicating map cells with very few stars in the $H$ band or incomplete $[4.5\mu]$ data.

(The data used to create this figure are available.)
scale in the extinction features, and (2) if there was a difference in the spatial scale distribution of the typical and high extinction cells. We ran a two-dimensional cross-correlation analysis on our extinction map to look for peaks in the distribution of the extinction at various spatial scales. We made binary maps created by setting map cells with $A_{\lambda}$ distributions of the extinction at various spatial scales. We made analysis on our extinction map to look for peaks in the extinction cells. We ran a two-dimensional cross-correlation in the spatial scale distribution of the typical and high extinction regimes. We found the mean extinction, one with a mean of about 4 and standard deviation of 1, and the second with a mean of about 8 with a standard deviation of about 0.75; the vertical dotted lines are at $A(K_S) = 4$ and 8. Right: histogram of the mean $\beta$ values. The peak of the distribution of the mean $\beta$ is 2.029 and the standard deviation is 0.002. Despite the bimodal distribution of extinction, there is no evidence of a bimodal extinction law.

This may indicate that the tiger stripes are closer to Earth than the GC, as we would expect foreground stars to fill in the tiger stripes; this was not pursued further.

We find that the value of $\beta$ does not vary spatially over our entire mapping region of $\sim 17'' \times 16'$. We found the mean value of $\beta$ for each pixel and show the distribution in Figure 6. The mean $\beta$ is 2.029, with $\sigma_{\beta} = 0.002$ being the standard deviation of the distribution of mean per-pixel $\beta$ values over the entire GC region we examined. Individual cells have a rather large $\sigma_{\beta}$ of about 0.3, but assuming Poisson statistics, we reduce the error on $\beta$ to 0.06 per pixel ($0.3/\sqrt{25} = 0.06$). We use 25 as the number of map cells in this case because we are only using 25 cells in each pymc3 run. While the global behavior of $\beta$ is self-consistent, we only used 25 map cells in our partial pooling model for any given run, and we choose to err on the conservative side when propagating errors; Appendix details our Markov Chain Monte Carlo (MCMC) data analysis and why we settled on using 25 map cells in our runs.

We conclude that $\beta = 2.029 \pm 0.002$, where 0.002 is the error of the mean $\beta$ values, and 0.06 the error on individual cells in each set of 25. Adding the errors in quadrature, we find that $\beta = 2.029 \pm 0.06$. This value of $\beta$ is consistent within 1$\sigma$ with many of the previous studies using only near-infrared point-source photometry that found $\beta \geq 2$ in the direction of the GC (Indebetouw et al. 2005; Nishiyama et al. 2006, 2009; Fritz et al. 2011).

We compare our values of $\beta$, $A(J)/A(K_S)$, $A(H)/A(K_S)$, and $C$ to six selected works on extinction at the GC in Table A1. Indebetouw et al. (2005) found that $C = 0.61 \pm 0.04$, $\beta_1 = 2.22 \pm 0.17$, and $\beta_2 = 1.21 \pm 0.23$ if one used Equation (5). Much of the literature quotes Indebetouw et al. (2005) to have a power-law index of $\beta = 1.65$ or 1.66 (Nishiyama et al. 2006; Nidever et al. 2012). Despite repeated readings of this paper, we were unable to find a single mention of this value or indeed any attempt to fit their data using Equation (4). However, using the relative extinction values in this paper one can find $\beta$ by fitting their relative extinction values to $A(\lambda) \propto \lambda^{-\beta}$, which is perhaps where $\beta = 1.66$ originates. The inconsistency in the cited $\beta$ value in the literature is unclear but might be due to authors’ varied preferences in fitting routines. Our value of $\beta$ is consistent with
Table A1

Near-infrared Extinction Law Comparison

| Publication               | $\beta$           | $\alpha(J)/A(K)$ | $\alpha(H)/A(K)$ | C       |
|--------------------------|-------------------|-------------------|-------------------|---------|
| Indebetouw et al. (2005) | $2.22 \pm 0.17, \beta \approx 1.66^a$ | 2.5 $\pm$ 0.15  | 1.55 $\pm$ 0.08  | 0.61 $\pm$ 0.04 |
| Nishiyama et al. (2006)  | 1.99 $\pm$ 0.02  | 3.02 $\pm$ 0.004 | 1.73 $\pm$ 0.01  | 0.494 $\pm$ 0.006 |
| Nishiyama et al. 2009    | 2.23 $\pm$ 0.23$^b$ | 3.02 $\pm$ 0.04  | 1.73 $\pm$ 0.03  | ...$c$ |
| Fritz et al. (2011)      | 2.11 $\pm$ 0.06  | ...               | 1.74 $\pm$ 0.14$^d$ | ...$e$ |
| Nogueras-Lara et al. (2018) | 2.31 $\pm$ 0.03$^f$ | ...             | ...               | ...$g$ |
| Hosek et al. (2019)      | 2.38 $\pm$ 0.15$^g$ | 3.14 $\pm$ 0.07  | 2.04 $\pm$ 0.04  | ...$h$ |
| This work                | 2.03 $\pm$ 0.06  | 2.57 $\pm$ 0.03  | 1.42 $\pm$ 0.04  | 0.60$^i$ |

Notes.

$a$ Indebetouw et al. (2005) uses a polynomial extinction law as in Equation (5), but one can fit a simple power law to their data and get $\beta \approx 1.66$ as is often quoted in the literature.

$b$ We use the value of $\beta$ as given in Fritz et al. (2011).

$c$ We use “...” to denote where cited papers do not explicitly give the values used in this table and are not straightforwardly derived.

$d$ We derived the $\alpha(J)/A(K)$ for Fritz et al. (2011) from their given $A(H)$ and $A(K_\text{s})$ values.

$e$ See Section 3.2 for a discussion of the $\beta$ value of Nogueras-Lara et al. (2018) compared to ours.

$f$ Hosek et al. (2019) recalculated the extinction curves from Hosek et al. (2018) using new photometric ZPs in some of the filters used in their earlier work, and note that there is no significant deviance from a power-law fit longward of 1.25 $\mu$m.

$g$ We fixed the intercept value in our analysis to be 0.60.

$h$ Their power-law index, $β_1$, is consistent with our $\beta$ value at the 1σ level.

$i$ Comparing our results to Nishiyama et al. (2006) shows an interesting mix of agreement and disagreement. Our $\beta$ values are consistent at the 1σ level, but our relative extinction and power law intercepts are substantially inconsistent. The difference is in large part due to the different intercept values we use for our respective power laws, and our methods for measuring the extinction are also quite different. Nishiyama et al. (2006) performed point-source photometry on stars in the GC, which naturally restricted them to stars that are more likely on the near side of the GC, whereas we used surface brightness photometry and took advantage of the severe crowding at the GC. Effectively, the last photosphere of extinction that we measure is closer to the GC than the stars Nishiyama et al. (2006) were able to perform point-source photometry on in $J$, $H$, and $K_\text{s}$.

Fritz et al. (2011) give $A(H)$ and $A(K_\text{s})$ as separate values, so we include their value of $A(H)/A(K_\text{s})$ in Table A1. We conclude that we are consistent within 1.8σ with Fritz et al. (2011).

Hosek et al. (2019) examines the extinction toward the Arches Cluster. Their value of $\beta$ is within 1.7σ of ours, but their relative extinction values are significantly higher than what we find (5.8σ and 36σ in $A(J)/A(K_\text{s})$ and $A(H)/A(K_\text{s})$, respectively. This may be due to a different dust distribution toward the Arches Cluster environment. Other previous works do not explicitly give relative extinctions or intercept values ($C$).

5.8σ and 36σ in $A(J)/A(K_\text{s})$ and $A(H)/A(K_\text{s})$, respectively. This may be due to a different dust distribution toward the Arches Cluster environment. Other previous works do not explicitly give relative extinctions or intercept values ($C$).

4. Conclusions and Future Work

We have presented an extinction map of the inner $16.5' \times 17' \times 17'$ region of the Galaxy using $H$ and $[4.5\mu]$ surface brightness photometry. We have found the infrared extinction law at the GC to be consistent with being spatially invariant despite the obvious bimodal distribution of extinction values, and that the power-law index $\beta = 2.03 \pm 0.06$. Our derived extinction law is in agreement with many recent studies, although it is in tension with Nogueras-Lara et al. (2018). Additionally, our extinction map can be used as a first

Notes.

$^a$ The extinction laws in Hosek et al. (2018) were revised slightly in Hosek et al. (2019); see their Appendix B), and reduced the complexity of their extinction law longward of 1.25 $\mu$m.

$^b$ The extinction laws in Hosek et al. (2018) were revised slightly in Hosek et al. (2019); see their Appendix B), and reduced the complexity of their extinction law longward of 1.25 $\mu$m.
approximation to the extinction toward any new transient infrared sources that are lacking in spectroscopic data.

Majewski et al. (2011) note that other near-infrared and mid-infrared color combinations are worth consideration for extinction mapping, such as $(K_s - [3.6\mu])$. As noted previously, broad emission features commonly attributed to PAH emission occurs at mid-infrared wavelengths near 3.3, 6.2, 7.7, and 8.6\micron, limiting the usefulness of mid-infrared bands other than $[4.5\mu]$ in determining extinction. Given the width of the Spitzer IRAC filters, this means that the only filter not subject to PAH contamination is $[4.5\mu]$. This implies that $(K_s - [4.5\mu])$ is a viable choice for future color mapping. The other filters present an opportunity to test PAH emission as a tracer for extinction by correlating PAH maps derived from other mid-infrared bands to the extinction map.

Future work includes making and folding in $(K_s - [4.5\mu])$ color maps to the MCMC analysis, and due to the doubling of data points for each map cell, an exploration of more complex near-infrared extinction laws (higher-order power laws, spatially variations on small scales) can be undertaken. Dereddening the IPSC and other photometric data sets is also in the future.

We gratefully acknowledge Dan Gettings for sharing his map tools code, for teaching us how to use it, and many fruitful conversations as to the nature of measurements and modeling in a Bayesian framework. Our reviewer went above and beyond in their insightful and helpful comments, and this work was made stronger and more approachable because of their effort. All mistakes, from grammatical to technical, remain the fault of the authors.

This research made use of astropy, a community-developed core Python package for Astronomy (Astropy Collaboration et al. 2013, 2018). This work is based in part on observations made with the Spitzer Space Telescope, which is operated by the Jet Propulsion Laboratory, California Institute of Technology under a contract with NASA.

Facilities: CTIO, Spitzer (IRAC).

Software: TRILEGAL (Girardi et al. 2012); DAOPHOT (Stetson 1987); superFATBOY (Warner et al. 2012); Source Extractor (Bertin & Arnouts 1996); Scamp (Bertin 2006); SWarp (Bertin et al. 2002); pymc3 (Salvatier et al. 2016); astropy (Astropy Collaboration et al. 2013, 2018).

Appendix
Choosing Priors and pymc3

We chose to use a Bayesian approach to our modeling of extinction in order to take advantage of the large amount of measured color excess data. We used the Python package pymc3 (Salvatier et al. 2016), which is a Bayesian statistical modeling suite that uses a flexible MCMC approach to modeling. A discussion of the machinery of pymc3 and Bayesian statistics is beyond the scope of this paper, but we will discuss how we constructed our Bayesian model of extinction in this section and note that pymc3 uses a highly adaptive, gradient-based sampler, which frees us of having to implement our own sampler code.

The basic idea of our model is simple: find $A(K_s)$ given the observed color excess $(H - [4.5\mu])$ and expected $(H-[4.5\mu])_b$, and find the range of $\beta$ values that are consistent with the data.

### Table A2

| Parameter        | Type* | Mean | $\sigma$ | Lower | Upper |
|------------------|-------|------|----------|-------|-------|
| $(K_s)$          | Uniform* | ...  | ...      | 0.01  | 15    |
| $\sigma_{(K_s)}$| HalfNormal* | 0    | 0.5      |       |       |
| $\beta$          | Uniform | ...  | ...      | 1.5   | 2.5   |

Notes.
* The type column signifies what sort of predefined class of the variable for the parameter definitions in pymc3.
* In the case of a Uniform distribution (which is continuous), we give lower and upper bounds.
* The HalfNormal class is a continuous normal distribution with a mean of zero and restricted to only positive values; these properties make the HalfNormal useful as an error term.

The Bayesian part comes in because we have an intrinsic color spread $(\sigma_{(H-[4.5\mu])})$ an expected distribution of $\beta$ values, and a limit on what the extinction can be (e.g., negative $A(K_s)$ values are not physical, although nonthermal emission from PAH or dust is a possible source of bluer-than-expected color). We can also leverage partial pooling, which uses a common distribution of errors or estimated values in comparing multiple data points, to better estimate the extinction law. What this means in our context is that we drew test values for our variables, like $\beta$, from a single distribution for each realization. A mild complication is that sharp increases in extinction can be due to infrared dark clouds (IRDCs; Rathborne et al. 2006), which at the distance of the GC (8.5 kpc) are typically about 10$^8$, or two map cells, in size and can have extinctions as high as $A(K_s)$ of 10 to 15. This means that, depending on the extinction law used, IRDCs can have absolute visible extinction(A$_{V}$) values in excess of 100 magnitudes. The tiger stripes, filamentary structures apparent to the right of the center of the H-band image (Figure 2), are very dark, indicating extremely high extinction typical of IRDCs.

We designed our model with the parameters listed in Table A2. The model itself is simple. For each pixel we draw an $A(K_s)$, $\beta$, and $\sigma_{(K_s)}$ from their respective distributions. We then calculate $\gamma$ as defined in Equation (7), $1/(10^{0.6-\beta \log_{[H]} - 10^{0.6-\beta \log_{[4.5\mu]}}}$, and use that value to calculate the model color excess $M$ based on the extinction and $\gamma$ values, $M(H-[4.5\mu]) = \gamma * A(K_s) + 0.08$.

We then compare $M$, the model color excess, to the measured $E(H-[4.5\mu])$ color, using $M$ as the mean and $\sigma_{(M)}$ as the standard deviation of a normal distribution (e.g., we assume Gaussian errors to our measured values). The model is then fed into pymc3, which we initialized with the No U-Turn Sampler (or NUTS), an auto-adaptive Hamiltonian sampler written expressly for pymc3. We set the sampler to run 30,000 realizations, and initialized it with 100,000 samples that use autodifferential variational inference (ADVI) to find and define the gradient for the parameter space of the model; this both finds the initial values and sets the step size for each parameter appropriately. We recorded the traces using the HDF5 file format and analyzed them with the python package pytables and our custom-written code.

A.1. Splitting up the Data

A significant constraint for pymc3 and the other partial pooling MCMC code is that it runs progressively slower with larger data sets. We found that running our model with the full

[^http://www.astropy.org]: http://www.astropy.org
The data set was impossible (each realization, or run, in our MCMC took approximately one CPU day). We explored two ways of splitting up the data into subsets; while this diminishes the power of Bayesian partial pooling, we were able to complete 30,000 realizations for each realization in less than 2 CPU weeks as opposed to 30,000 CPU days.

The first way we split the data was spatially. We grouped map cells into subsets of 100, gridding up our map into 10 cell × 10 cell parental boxes. While computationally efficient and conceptually straightforward, we found that the standard deviations of the extinction values of these parental boxes showed stark boundary differences at the borders. Neighboring parental boxes had notably different means and standard deviations. This clearly indicated that we were enforcing an artificial structure in our calculations of the extinction at the GC.

We looked more closely at our data, looking specifically at each of the 10 cell × 10 cell parental boxes and the star counts from the IPSC within each box. When we plotted the number of stars present in the IPSC in $K_s$ in the parental boxes versus the color of each pixel (assigning the number of stars in the larger 10 × 10 parental boxes to each pixel in that box), we found that the data appeared to cluster nicely into two regimes by eye. Smaller parental boxes of 5 × 5 map cells showed similar clustering (as shown in Figure A1, left panel) and when we looked at the median $(H – K_s)$ of all stars from the IPSC in the parental boxes versus the color of each pixel, we found similar clustering (Figure A1, right panel).

There are three clusters apparent in Figure A1:

1. There are a few map cells that contain zero stars in the IPSC;
2. Most map cells have stars and have an $H – [4.5\mu]$ color $< 8.5$; and
3. The remainder cells have a color $H – [4.5\mu] > 8.5$.

As such, we elected to cut the data into two populations split at $(H – [4.5\mu])$ color $= 8.5$, with the A population having an $(H – [4.5\mu])$ color $\leq 8.5$ and the B population having an $(H – [4.5\mu])$ color $> 8.5$.

A.2. Turning the Crank

After dividing up the data into two populations, we then randomly selected 25 map cells of either A or B populations and ran the pymc3 code, recording each realization in HDF5 files. Each run (consisting of 25 randomly selected cells from solely either the A or B populations) took approximately 15 minutes using one CPU core. We then recombined the data by reading in each HDF5 file, recording the mean and standard deviation of the last 29000 realizations for $A(K_s)$ and $\beta$ for the first 1000 realizations we discarded to ensure that we used realizations that were no longer using the initial values for our variables, a practice known as burn-in. The total computation time for this analysis was slightly over 2 CPU weeks.

We performed two sets of cross-checks to verify that we were using both enough map cells and using enough realizations in our MCMC chains. We ran the data independently of each other for our checks with 100 total cells each: one set used a sample of 100 cells and the other used four samples of 25 cells each. The four samples of 25 cells were drawn randomly from the sample of 100 cells in order to isolate any possible selection effects from the cross-checks we performed. The differences were taken on a per-pixel basis, which is why we chose to draw the 4 smaller samples of 25 cells each from the larger sample of 100 cells.

A.2.1. Sensitivity to Number of Realizations

To check if we had a good convergence of our MCMC chains, we ran chains of 100,000 realizations for both the 4 sets of 25 map cells each and the set of 100 cells. We compared the results of the 4 smaller samples of 25 cells against the larger sample of 100 cells. We found that some variation of the mean $A(K_s)$ did occur at the $\sim 0.05\sigma_{K_s}$ level, which is not unexpected given the quasi-random sampling method of pymc3’s NUTS sampler. Similar variations occurred for $\beta$. Figure A2 shows two panels. The first panel shows the difference in $A(K_s)$ between the group of 100 pixels versus the four groups of 25 pixels. There are slight differences between the sets of realizations, however, the variation of both $A(K_s)$ and $\beta$ is small and consistent with 0. As a result, we conclude that using 30,000 realizations in our MCMC modeling is sufficient for our purposes.

A.2.2. Sensitivity to the Sample Size

As a test to see if we were using too small a sample of pixels per draw (25), we ran tests with a sample of 100 pixels for 30,000 realizations and compared it to four groups of 25 pixels each.

![Figure A1](image-url)

Figure A1. Clustering of map cells. Left: number of stars detected in $K_s$ in the IPSC within each $25' \times 25'$ parental box vs. the $(H – [4.5\mu])$ color of each pixel in that parental box. Note that there are two seemingly distinct clusters of points separated at about $(H – [4.5\mu]) = 8.5$. Right: same as above, but with the median $(H – K_s)$ color of stars within each parental box (the photometry taken from the IPSC) vs. the $(H – [4.5\mu])$ color of each pixel in that parental box.
We found that the sample of 100 pixels did have slight but statistically insignificant difference in the means of $A(K_s)$ on a per-pixel basis, as shown in Figure A2. The results for $\beta$ were similarly insensitive to sample size.

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**Figure A2.** pymc3 cross-checks. Note the y-axis range covers only 25 millimag. Left: the mean $A(K_s)$ values for 4 sets of 25 randomly chosen pixels using 30,000 runs minus the mean $A(K_s)$ values for the same 100 pixels as 1 set using 100,000 runs. The red horizontal line shows the average difference while the black vertical line represents ±1 standard deviation of the difference in the means. Right: as above, but this time comparing the mean $A(K_s)$ values for 4 sets of 25 randomly chosen pixels using 30,000 realizations minus the mean $A(K_s)$ values for the same 100 pixels as 1 set using 30,000 realizations.