Timeholes and remnants in dilaton gravity

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ABSTRACT: We propose a family of dilaton gravity models possessing regular black holes called timeholes with interiors connecting separate asymptotically flat regions. We demonstrate that inner Cauchy horizons are stable given certain initial conditions. We study causal structure and evaluate thermodynamic properties of timeholes using Euclidean methods. Extremal timeholes have zero temperature and can be considered as remnants. We speculate that quantum filed fluctuations can dissolve event horizons in case of timeholes providing a possible resolution to information paradox.
1 Introduction

Existence of black holes poses one of the most die-hard riddles in theoretical physics. According to quantum field theory black holes evaporate into dust of thermal radiation [1]. This semiclassical result conflicts with the quantum mechanical postulate of unitary evolution [2].

The Gauge/String duality make us believe this contrariety is apparent and black hole evaporation is nothing but a sophisticated scattering process [3]. Nevertheless, a conclusive proof is absent. On the contrary, the AMPS-firewall argument sharpen the information paradox as a no-go theorem reading one of the commonly-believed statements, namely purity of Hawking radiation, validity of the local quantum field theory beyond horizon scale, or “non-dramatic” horizon, should be omitted [4].

This claim is not resistant to possible loopholes. There was recently a disclosure of so-called “islands” by using replica wormholes to evaluate unitary form of the Page curve during black hole evaporation [5]. This achievement stimulated a revival of interest in low-dimensional models of gravity which are perfect playgrounds to verify these new ideas [6].
Two-dimensional gravity is considered to be renormalizable suggesting that a solution to the information loss problem is analytically tractable at least in principle. It also allows us to leave aside unimportant higher-dimensional complications. Holography provides a connection to condensed matter models like the SYK spin chain [7], so it can be potentially verified even in the lab experiments [8].

In this paper we investigate dilaton gravity models which are modifications of the prolific CGHS model [9],

$$S_{\text{CGHS}} = 
\int d^2x \sqrt{-g} \left( e^{-2\phi} R + 4e^{-2\phi} \left( (\nabla \phi)^2 + \lambda^2 \right) - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right), \quad (1.1)$$

describing interactions of the metric $g_{\mu\nu}$, dilaton $\phi$, and $N$ massless scalar fields $f_i$. The classical CGHS model is exactly solvable but it is not singularity-free, and adding one-loop corrections from the scalar fields does not resolve this issue [10, 11]. It was conjectured that quantum gravity effects should resolve singularity. Instead of considering complicated full dynamics at large curvatures one may apply a “phenomenological” limiting condition $R_{\mu\nu\rho\sigma}^2 < \Lambda^2$ with some dimensionful Planck scale parameter $\Lambda$ [12]. Next, one can modify the gravity action (1.1) so that singularity is dynamically avoided. By implementing the limiting curvature condition as a constraint one obtains models where gravitational collapse stops with the de Sitter core’s formation [13, 14].

In this paper we modify the gravity action (1.1) to make the bounce happen instead of the de Sitter core. Therefore, a given class of models contains regular black holes similar to the Bardeen black holes as exact vacuum solutions. These regular black holes called timeholes\(^1\) have event horizons and oscillate in global time connecting infinite number of asymptotically flat regions. The curvature is finite and restricted to a spacetime region with the strong coupling.

In the considered dilaton gravity models there is a threshold mass $M_{\text{ext}}$ corresponding to the extremal timeholes. Non-extremal timeholes have masses $M > M_{\text{ext}}$ and solutions with $M < M_{\text{ext}}$ are horizonless. Timeholes are more or less stable against matter perturbations but it can lead to the mass inflation phenomenon [16] at the certain choice of initial condition.

We evaluate the temperature and the entropy of timeholes using Euclidean methods. The extremal timeholes have zero temperature and do not evaporate, and can be considered as remnants. Mean field approximation reads the remnants are stable but it seems likely they undergo quantum decay. An infinitesimal amount of energy

\(^{1}\)This term was introduced firstly in Ref. [15].
is necessary to dissolve the event horizon completely and all the matter fallen into a
would-be interior returns back after the end of evaporation.

The paper is organized as follows. In Sec. 2 we find static classical solutions in the
proposed class of models. Sec. 3 addresses inclusion of the matter fields. And in Sec. 4
we discuss gathered results and illuminate some prospects.

2 Deformations of CGHS model

2.1 General LDV model

We describe the class of dilaton gravity models possessing the linear dilaton vacuum
(LDV),

\[ \phi = -\lambda r , \quad R = 0 , \quad ds^2 = -dt^2 + dr^2 , \]

as a solution of field equations. One finds it is provided by the action,

\[ S_{LDV} = \int d^2 x \sqrt{-g} \left( W(\phi) R + W''(\phi) \left( (\nabla \phi)^2 + \lambda^2 \right) \right) + S^m , \]

where primes denote derivatives of \( W(\phi) \) with respect to its argument and \( S^m \) is an
action for some matter which we ignore in this Section. The choices \( W(\phi) = e^{-2\phi} \) and
\( W(\phi) = e^{-2\phi} - N\phi/48\pi \) correspond to the CGHS and RST [11] models respectively.

By varying Eq. (2.2) with respect to \( \phi \) and \( g^{\mu\nu} \) one derives field equations,

\[ W'(\phi) R = 2 W''(\phi) \Box \phi + W'''(\phi) \left( (\nabla \phi)^2 - \lambda^2 \right) , \]

\[ g_{\mu\nu} \left( W''(\phi) (\nabla^2 \phi - \lambda^2) + 2 W'(\phi) \Box \phi \right) - 2 W'(\phi) \nabla_\mu \nabla_\nu \phi = T^m_{\mu\nu} , \]

where \( T^m_{\mu\nu} = (-2/\sqrt{-g}) \delta S^m / \delta g^{\mu\nu} \) is the matter stress tensor. Eqs. (2.3), (2.4) have
the one-parametric set of vacuum solutions,

\[ ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} , \quad \phi = -\lambda r , \quad f(r) = 1 + \frac{M}{\lambda W'(\phi)} . \]

where \( M \) is an integration constant corresponding to the ADM-mass for asymptotically
flat spacetimes. Global properties of the solution (2.5) are determined by a form of the
function \( W'(\phi) \).

If \( f(r) > 0 \) for all \( r \) it defines a spacetime without event horizons with configuration
resembling a kink from field theory. If \( f(r) \) changes its sign the equation \( f(r_h) = 0 \)
determines a position of the event horizons.

The Ricci curvature is given by formula \( R = -\partial_r^2 f(r) \) for the Schwarzschild
ansatz (2.5). Therefore, a singularity occurs if \( W'(\phi_s) = 0 \) and \( W''(\phi_s) \neq 0 \) at
$r = -\phi_s/\lambda$. This observation allows us to construct models (2.2) possessing non-singular black holes with event horizons by explicit choice of $W(\phi)$. In the next Section we provide a concrete example.

Let us calculate the thermodynamic properties of black holes in the models (2.2). The solution contributing to the partition function is a Euclidean continuation of Eq. (2.5),

$$ds_E^2 = f(r)dt_E^2 + \frac{dr^2}{f(r)}, \quad 0 \leq t_E < \beta_H,$$

which is periodic in imaginary time $t_E = it$ and has no conifold singularity on the event horizon $r = r_h$. The last condition fixes the imaginary time period which is the inverse Hawking temperature $\beta_H = T_H^{-1} = 4\pi/f'(r_h)$. Relating the event horizon position with the black hole mass by $M = -\lambda W'(\phi_h)$ one finds the temperature

$$T_H = \frac{\lambda^2 W''(\phi_h)}{4\pi M}. \quad (2.7)$$

It appears that the extremal black hole has zero temperature because $W''(\phi_h) = 0$ in this case. This is a signature of a possible remnant at the end of black hole evaporation.

The black hole entropy is given by

$$S_{BH}(M) = \int_{M_{ext}}^{M} \frac{dM}{T_H(M)} = 4\pi W(\phi_h) - 4\pi W(\phi_{h,ext}), \quad (2.8)$$

where we had taken into account that the lightest black hole has vanishing entropy by fixing limits of integration.

### 2.2 Example: sinh-CGHS model

Let us fix $W(\phi) = -2 \sinh(2\phi)$ in Eq. (2.2), namely consider the action

$$S_{\text{sinh}} = -2 \int d^2x \sqrt{-g} \sinh(2\phi) \left(R + 4(\nabla \phi)^2 + 4\lambda^2\right), \quad (2.9)$$

which describes two copies of the CGHS model plus corrections at large $|\phi| = \lambda|r|$. Left ($\phi > 0$) and right ($\phi < 0$) copies interpolate smoothly near the “core” region of proper size/duration $\simeq \lambda^{-1}$ across the line $\phi = 0$.

The general vacuum solution (2.5) becomes one with

$$f(r) = 1 - \frac{M}{4\lambda \cosh(2\lambda r)}, \quad (2.10)$$

The metric component (2.10) is plotted in Fig. 1a for different values of the mass parameter $M$. 

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One finds the Ricci scalar scales uniformly with mass $M$. It concentrates inside the core reaching the maximum positive value $\sqrt{2/3} \cdot \lambda M$ at the borders and the minimum negative value $-\lambda M$ at the core’s center, see in Fig. 1b.

One notes from Eq. (2.10) there is a threshold mass $M_{\text{ext}} = 4\lambda$. If $M < M_{\text{ext}}$ the metric (2.10) describes the gravitational kink depicted in Fig. 1c. If $M > M_{\text{ext}}$ the spacetime describes the non-extremal regular black hole with outer and inner event horizons at $r = \pm r_h$,

$$r_h = \frac{1}{2\lambda} \text{arcosh} \left( \frac{M}{M_{\text{ext}}} \right),$$

so that $f(\pm r_h) = 0$. For the extremal black hole one has $r_h = 0$.

Substituting $W(\phi) = -2\sinh(2\phi)$ into Eqs. (2.7), (2.8) one finds the black hole entropy and temperature,

$$S_{\text{BH}} = \frac{2\pi}{\lambda} M \sqrt{1 - \frac{M_{\text{ext}}^2}{M^2}} \quad \text{and} \quad T_H = \frac{\lambda}{2\pi} \sqrt{1 - \frac{M_{\text{ext}}^2}{M^2}}.$$

For large black holes the temperature approaches a value $\lambda/2\pi$ in agreement with the CGHS model limit.

The Schwarzschild coordinates $(t, r)$ in Eqs. (2.5), (2.10) are geodesically incomplete and describe the exterior region at $r > r_h$. In order to study the global structure of the eternal black hole one needs to perform the coordinate extension.
Let us introduce new coordinates,

\[ T = \sqrt{g(r)} \sinh(2\pi T_H t), \quad R = \sqrt{g(r)} \cosh(2\pi T_H t), \]  

(2.13)

where

\[ g(r) = \frac{(1 + \frac{M_{\text{ext}}}{M}) \tanh(\lambda r) - 2\pi T_H / \lambda}{(1 + \frac{M_{\text{ext}}}{M}) \tanh(\lambda r) + 2\pi T_H / \lambda} \exp(4\pi T_H r). \]  

(2.14)

The function \( g(r) > 0 \) in the exterior region \( r > r_h \), so that the Schwarzschild coordinates \( (t, r) \) cover a quadrant \( R > |T| \). Applying the coordinate transformation (2.13) one finds the metric (2.5), (2.10) in a form

\[ ds^2 = \frac{f(r)}{4\pi^2 T_H^2 g(r)} (-dT^2 + dR^2), \]  

(2.15)

where a conformal factor \( \propto f(r)/g(r) \) is positive everywhere except at the inner horizon \( r = -r_h \).
The metric (2.15) can be analytically continued into the interior region if one redefines coordinates \((T, R)\) as
\[
T = \sqrt{-g(r)} \cosh(2\pi T^H t) , \quad R = \sqrt{-g(r)} \sinh(2\pi T^H t) ,
\] (2.16)
so that \((T, R)\) cover the entire spacetime patch with \(r > -r_h\).

This patch is still not geodesically complete. The global spacetime consists of the infinite number of patches with the interior regions matched onto each other. The lines \(\phi = \text{const}\) belonging to \(T^+\)-region of \(i\)-th patch and \(T^-\)-region of \((i+1)\)-th patch can be identified by a map
\[
V_{i+1} = -\frac{\kappa}{V_i} , \quad U_{i+1} = -\frac{1}{\kappa U_i} ,
\] (2.17)
where \(V_i = T_i + R_i, U_i = T_i - R_i\) are the light-cone coordinates on the \(i\)-th patch, and \(\kappa\) is a residual parameter corresponding to respective shifts of the identified patches along the lines \(\phi = \text{const}\). The resulting Penrose diagram is presented in Fig. 2. Unlike traversable wormhole observers can travel in one time direction, hence we refer to this spacetime as the timehole.

3 Matter considerations

3.1 Core stability?

One may wonder if a singularity appears in the core \(|\phi| \lesssim 1\) in response to a perturbation by infalling matter. We consider one of the massless scalar fields \(f_i \equiv f\) in Eq. (1.1) as a source of gravity. Notice that the scalar field \(f\) becomes ghost-like in the left region \((\phi > 0)\) because of negative relative sign in front of the stress tensor,
\[
T_{\mu\nu} = \nabla_\mu f \nabla_\nu f - \frac{1}{2} g_{\mu\nu}(\nabla f)^2 ,
\] (3.1)
on the r.h.s of Eq. (2.4). As an effect the matter appears to anti-gravitate for asymptotic observer on the left side of the spacetime.

Ingoing wave packet \(f(v)\) admits an exact solution with the Vaidya metric,
\[
ds^2 = -F(v,r)dv^2 + 2dvdr ,
\]
\[
F(v,r) = \left( 1 - \frac{\mathcal{M}(v)}{4\lambda \cosh(2\lambda r)} \right) , \quad \partial_v \mathcal{M}(v) = (\partial_v f(v))^2 ,
\] (3.2)
where \(\mathcal{M}(v)\) is the Bondi mass, see Fig. 3. The Ricci scalar \(R = -\partial^2 F(v,r)\) is always finite so we conclude the core is classically stable.

We need to remark that classical stability of the core does not mean automatically the singularity is absent in the full quantum regime. Indeed, the core region lives at
the strong coupling as $W(\phi) \to 0$ and the quantum corrections can drastically modify its internal structure.

One may wonder if something peculiar happens already after inclusion of the one-loop corrections from the matter fields [18]. It was argued that the classical linear dilaton vacuum is unstable in the CGHS model. The semiclassical static solutions were found with the dilaton field bouncing off the strong coupling region [19].

It is intriguing if the similar thing would happen in the models with timeholes so that the strong coupling problem is avoided. Unfortunately, we found this is not the case, see Appendix A for details. Therefore, this problem remains opened for future researches.

In this paper we assume that singularity never appears. A minisuperspace approximation of the RST model suggests there is a bouncing behaviour [20]. The similar picture is motivated by loop quantum gravity (LQG) [21]. Nevertheless, there can be severe obstacles to adopt such a picture without hesitation, namely LQG provides generic mechanisms which may turn the singularity into an acausal “euclidean core” free of infinite curvature but impeding deterministic evolution [22].

### 3.2 Mass inflation?

There is a widespread opinion that the regular black holes with Cauchy horizons are unstable because of the mass inflation phenomenon [16]. This effect is related to exponential accumulation of matter near inner horizons leading to formation of the spacelike singularity.

**Figure 3.** The Vaidya solution (3.2) with a wave packet $\partial_v f(v) = 10\lambda \text{sech}(\lambda v)$ (blurry horizontal strip). Dark lines are outgoing null geodesics. The inner and outer horizons are correspondingly a future and past attractors for null geodesics.
This instability may not be necessarily present [23, 24] but it was demonstrated that charged black holes in the CGHS model suffer from the mass inflation both at classical and semiclassical levels [25–27]. Therefore, it becomes important to check whether the inner Cauchy horizons are stable against matter perturbations in our case.

We assume that timeholes are large $M \gg M_{\text{ext}}$ and ignore evaporation effects. We approximate spacetime with the CGHS model solution in the regions far from the core at $|\phi| \lesssim 1$. The general CGHS solution with choice of the metric $ds^2 = -e^{2\phi}dvdw$ is

$$e^{-2\rho} = e^{-2\phi} = -\lambda^2 vu + g(v) + h(u),$$

$$g(v) = \frac{1}{2} \int_0^v dv' \int_{v'}^{+\infty} dv'' (\partial_v f(v''))^2,$$

$$h(u) = -\frac{1}{2} \int_{-\infty}^u du' \int_{u'}^{-\infty} du'' (\partial_u f(u''))^2,$$

where $g(v), h(u)$ are functions depending on the matter content [9].

One takes

$$g(v) \simeq \frac{M}{2\lambda} - \frac{g_{\infty}}{(\lambda v)^{2\alpha}}, \quad \alpha > 0,$$

which corresponds to a power–law tail $f(v) \simeq f_0 \cdot (\lambda v)^{-\alpha}$ as $v \to +\infty$. After passing the core region the wave packet distorts accordingly to Eq. (2.17),

$$f(v) \mapsto f_0 \cdot (-\lambda v)^\alpha,$$

on the future side of the timehole.

Substituting (3.5) into Eq. (3.3) and using $R = -2\Box \rho$ one obtains the Ricci scalar,

$$R \simeq 4\lambda^2 e^{2\phi} \left( \frac{M}{2\lambda} + (2\alpha + 1)g_{\infty}(-\lambda v)^{2\alpha} + \frac{E_{\text{out}}(u)}{2\lambda} \right. + \frac{2\alpha + 1}{2\alpha - 1} \frac{2\alpha g_{\infty}}{\lambda} (-\lambda v)^{2\alpha - 1} \partial_v h(u) \right),$$

which to be finite at the Cauchy horizon if $\alpha > 1/2$. Otherwise, the outgoing wave packet crossing the Cauchy horizon at $v = 0$ triggers a singularity formation.

### 3.3 Remnants?

In the adiabatic approximation the Hawking radiation from timeholes is approximately thermal described by a black body spectrum. The radiation flux can be related to a mass change rate by the 2D Stefan–Boltzmann law, see Appendix B for derivation,

$$\frac{dM}{dt} = -\frac{\pi}{12} F_H^2(M),$$

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where temperature $T_H(M)$ is given by Eq. (2.12).

One integrates out Eq. (3.7) and obtains

$$M(t) + \frac{M_{\text{ext}}}{2} \log \left( \frac{M(t) - M_{\text{ext}}}{M(t) + M_{\text{ext}}} \right) = M_0 - \frac{\lambda^2 t}{48\pi},$$

where $M_0 \gg M_{\text{ext}}$ was assumed. Early times are characterised by the linear regime of evaporation corresponding to the CGHS model according to Eq. (3.8). After loosing the bulk of its mass timeholes’ evaporation slows down,

$$M \simeq M_{\text{ext}} \left( 1 + \exp \left( -\frac{\lambda^2 t}{24\pi M_{\text{ext}}} \right) \right),$$

so it takes infinite amount of time to settle down to the extremal limit. This configuration seems to be stable and can be regarded as a remnant, see in Fig. 4a. Possibility for remnants in the CGHS model was previously advocated in Ref. [28].

One suspects it is unreliable to draw conclusion about stability of the remnants from the semiclassical picture because a precise dynamics at the late times (3.9) should depend on the strong coupling behaviour.

Indeed, one conjectures the thermal fluctuations of the fields near the event horizon can destroy the remnant. A characteristic decay time can be estimated from thermodynamic principles as follows. Recalling Einstein theory of fluctuations,

$$\langle M \rangle = -\frac{1}{Z} \frac{\partial Z}{\partial \beta}, \quad \langle M^2 \rangle = \frac{1}{Z} \frac{\partial^2 Z}{\partial \beta^2}, \quad \Rightarrow \quad \langle (\Delta M)^2 \rangle = -\frac{\partial \langle E \rangle}{\partial \beta},$$

one finds using Eq. (2.12) a deviation

$$\delta M = M_{\text{dec}} - M_{\text{ext}} = \sqrt{\langle (\Delta M)^2 \rangle} = \frac{\lambda^2}{M_{\text{ext}}} O(1)$$

from the mean field result (3.9) assuming $\delta M \ll M_{\text{ext}}$. Substitution into Eq. (3.9) gives an estimate,

$$t_{\text{dec}} \simeq \frac{48\pi}{\lambda^2} \frac{M_{\text{ext}}}{M_{\text{ext}}} \log \left( \frac{M_{\text{ext}}}{\lambda} \right),$$

for the expected remnant lifetime.

Hereby we propose a scenario for the entire evolution. The timehole approaching the extremal state is subjected to stochastic quantum field fluctuations which cause transition from collapsing into expanding near-extremal timehole. This state releases the would-be interior matter content into the same region of spacetime rendering unitary evolution and then relaxes to a gravitational kink state of unspecified mass, see Fig. 4b.
Figure 4. Penrose diagrams for (a) the evaporating timehole formed from gravitational collapse of matter in the semiclassical approximation and (b) the proposed scenario for decaying remnant in the end of evaporation. Arrows represent Hawking radiation.

4 Discussion

We propose simple two-dimensional dilaton gravity models with non-singular black holes and Minkowski spacetime as a trivial solution. The properties of non-trivial vacuum solution depends on its mass. There are gravitational kinks ($M < M_{\text{ext}}$), extremal ($M = M_{\text{ext}}$) and non-extremal ($M > M_{\text{ext}}$) timeholes. The gravitational kinks has the same causal structure as the Minkowski spacetime. The eternal timeholes resemble charged black holes with separate asymptotically flat regions connected by one-way interiors.

We considered a particular model approximated by the CGHS model except for the central core region of size $\lambda^{-1}$. This core stable against perturbations by classical infalling matter. We found that mass inflation on the Cauchy horizons can happen at certain choice of initial conditions.

Euclidean methods allows to calculate temperature and entropy of the timeholes. The extremal timeholes are supposed to have zero temperature and entropy hence they can be considered as remnants.

We argue that remnants are unstable because of the thermal fluctuations of mass
$\delta M \simeq T_H$. Infinitesimally small change can completely dissolve the event horizon. This scenario of evaporation is analogous to one proposed for regular black holes in Refs. [29, 30]. Loop quantum gravity motivates the similar picture of evaporation for real black holes [31].

Let us discuss some prospects for future research. Next step will be further development of the S-matrix approach, e.g. in the spirit of ’t Hooft’s black hole ansatz [32].

Previously, we calculated semiclassical scattering amplitudes using regularization method in the CGHS model with a reflecting boundary $\phi = \phi_0$ and a massive pointlike particle as matter [33], see also Refs. [34, 35] for more details on this model. Obtained results are consistent with unitarity but analogous calculation for complete theory with quantum matter fields is still missing.

The same method can be applied to the sinh-CGHS model in attempt to find the regular solution contributing to the path integral. One may expect that given initial and final quantum states, namely high-energy collapsing wave packet and outgoing low-energy Hawking radiation, regularization method produces a horizonless spacetime envisioned in Sec. 3.3.

Additional important question concerns the “islands” method for calculating the entanglement entropy [6]. It seems reasonable to connect the replica wormholes with the saddle-point solutions contributing to scattering amplitudes. Models with non-singular black holes are perfectly suited for this task.

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A Quantum kinks in sinh-CGHS

In this Appendix we investigate static solutions in the model (2.9) with the one-loop corrections from $N$ matter fields. It is accounted by adding the Polyakov action [18], which can be recast in the local form,

$$S_{\text{one-loop}} = \int d^2x \sqrt{-g} \left( -\frac{1}{2} (\nabla \chi)^2 + \sqrt{\frac{N}{48\pi}} \chi R \right),$$

(A.1)

by using an auxiliary field $\chi$ representing contribution from all $N$ scalar fields given vacuum background state. By varying Eq. (A.1) one finds the field equation,

$$\Box \chi = -\sqrt{\frac{N}{48\pi}} R,$$

(A.2)
With choice of the metric with the line element $ds^2 = e^{2\alpha(r)}(-dt^2 + dr^2)$ one derives differential equations,

\begin{align}
W''(\phi(r))\alpha''(r) + W''(\phi(r))\phi''(r) + \frac{1}{2}W'''(\phi(r))((\phi'(r))^2 - \lambda^2 e^{2\alpha(r)}) &= 0, \quad (A.4) \\
\frac{N}{24\pi}\alpha''(r) + W'(\phi(r))\phi''(r) + W''(\phi(r))((\phi'(r))^2 - \lambda^2 e^{2\alpha(r)}) &= 0, \quad (A.5)
\end{align}

for unknown functions $\phi(r), \alpha(r)$. Initial conditions are fixed by asymptotic behaviour of fields.

Let us consider Eq. (2.2) with $W(\phi) = e^{-2\phi} - a \cdot e^{2\phi}$ interpolating between the CGHS ($a = 0$) and sinh-CGHS ($a = 1$) models. We solved Eqs. (A.4), (A.5) numerically on Mathematica using the implicit Runge-Kutta method. We present results in Fig. 5.

Numerical solution with $a = 0$ reproduces quantum kinks obtained in Ref. [19]. Analogous picture arises in the sinh-CGHS models if the number of scalar fields is

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{(a) Set of numerical solutions interpolating between the CGHS and sinh-CGHS models. Mass $M = 10^{-3} \cdot M_{\text{ext}}$ as seen by asymptotic observer and number of fields $N = 1$. (b) Transition from classical kink of mass $M = 0.5 \cdot M_{\text{ext}}$ to quantum kink in the sinh-CGHS model with growing number of quantum fields $N$.}
\end{figure}
comparably large. In both cases the Ricci scalar $R = -2e^{-2\alpha} \partial^2 \alpha$ diverges as $r \to -\infty$ and the quantum kink has singularity at the left infinity. Regular solutions do not have bouncing behaviour of dilaton field.

**B Hawking radiation from Weyl anomaly**

We revisit here calculation of the renormalized energy-momentum tensor of the massless scalar field in two dimensions. One recalls the expectation value of its trace is anomalous,

$$\langle \tilde{T}_\mu^\mu \rangle \psi = -\frac{R}{24\pi}, \quad (B.1)$$

given a quantum state of the matter.

For the metric with a line element $ds^2 = -f(r) dv du$ where $r$ is an implicit function of $v - u$ one finds

$$\langle T_{vv} \rangle \psi = \frac{1}{96\pi} \left( f''(r)f(r) - \frac{1}{2} (f'(r))^2 \right) + g_\psi(v), \quad (B.2)$$

$$\langle T_{uu} \rangle \psi = \frac{1}{96\pi} \left( f''(r)f(r) - \frac{1}{2} (f'(r))^2 \right) + h_\psi(u), \quad (B.3)$$

where $g_\psi(v)$ and $h_\psi(u)$ are function determined by the quantum state.

The Unruh state is given by

$$g_{\text{Unruh}}(v) = 0, \quad h_{\text{Unruh}}(u) = \frac{\lambda^2}{48\pi} \left( 1 - \frac{M^2}{M^2_{\text{ext}}} \right), \quad (B.4)$$

where we used Eq. (2.10). This corresponds to the Hawking flux from the collapsing timehole with $M > M_{\text{ext}}$. Calculation shows $\langle T_{uu} \rangle_{\text{Unruh}}$ is regular at the event horizon and future infinity where it gives a black body radiation flux with the temperature (2.12).

**References**

[1] S. W. Hawking, *Particle Creation by Black Holes*, Commun. Math. Phys. 43 (1975), 199-220 [erratum: Commun. Math. Phys. 46 (1976) 206].

[2] S. W. Hawking, *Breakdown of Predictability in Gravitational Collapse*, Phys. Rev. D 14 (1976) 2460-2473.

[3] D. Harlow, *Jerusalem Lectures on Black Holes and Quantum Information*, Rev. Mod. Phys. 88 (2016) 015002 [arXiv:1409.1231 [hep-th]].
[4] A. Almheiri, D. Marolf, J. Polchinski and J. Sully, *Black Holes: Complementarity or Firewalls?*, JHEP 02 (2013) 062 [arXiv:1207.3123 [hep-th]].

[5] G. Penington, *Entanglement Wedge Reconstruction and the Information Paradox*, JHEP 09 (2020) 002 [arXiv:1905.08255 [hep-th]].

[6] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian and A. Tajdini, *Replica Wormholes and the Entropy of Hawking Radiation*, JHEP 05 (2020) 013 [arXiv:1911.12333 [hep-th]].

[7] D. A. Trunin, *Pedagogical introduction to the Sachdev–Ye–Kitaev model and two-dimensional dilaton gravity*, Usp. Fiz. Nauk 191 (2021) no.3, 225-261 [arXiv:2002.12187 [hep-th]].

[8] D. I. Pikulin and M. Franz, *Black Hole on a Chip: Proposal for a Physical Realization of the Sachdev-Ye-Kitaev model in a Solid-State System*, Phys. Rev. X 7 (2017) no.3, 031006 [arXiv:1702.04426 [cond-mat.dis-nn]].

[9] C. G. Callan, Jr., S. B. Giddings, J. A. Harvey and A. Strominger, *Evanescent black holes*, Phys. Rev. D 45 (1992) no.4, R1005 [arXiv:hep-th/9111056 [hep-th]].

[10] J. G. Russo, L. Susskind and L. Thorlacius, *Black hole evaporation in (1+1)-dimensions*, Phys. Lett. B 292 (1992) 13-18 [arXiv:hep-th/9201074 [hep-th]].

[11] J. G. Russo, L. Susskind and L. Thorlacius, *The Endpoint of Hawking radiation*, Phys. Rev. D 46 (1992) 3444-3449 [arXiv:hep-th/9206070 [hep-th]].

[12] M. A. Markov, *Problems of a perpetually oscillating universe*, Annals Phys. 155 (1984) 333-357.

[13] V. P. Frolov, M. A. Markov and V. F. Mukhanov, *Black Holes as Possible Sources of Closed and Semiclosed Worlds*, Phys. Rev. D 41 (1990) 383.

[14] V. P. Frolov and A. Zelnikov, *Two-dimensional black holes in the limiting curvature theory of gravity*, JHEP 08 (2021) 154 [arXiv:2105.12808 [hep-th]].

[15] M. Visser and D. Hochberg, *Generic wormhole throats*, Annals Israel Phys. Soc. 13 (1997) 249 [arXiv:gr-qc/9710001 [gr-qc]].

[16] E. Poisson and W. Israel, *Inner-horizon instability and mass inflation in black holes*, Phys. Rev. Lett. 63 (1989) 1663-1666.

[17] W. Y. Ai, *Nonsingular black hole in two-dimensional asymptotically flat spacetime*, Phys. Rev. D 104 (2021) no.4, 044064 [arXiv:2006.07962 [hep-th]].

[18] A. M. Polyakov, *Quantum Geometry of Bosonic Strings*, Phys. Lett. B 103 (1981) 207-210.

[19] B. Birnir, S. B. Giddings, J. A. Harvey and A. Strominger, *Quantum black holes*, Phys. Rev. D 46 (1992) 638-644 [arXiv:hep-th/9203042 [hep-th]].
[20] R. G. Daghigh, M. D. Green and G. Kunstatter, Quantum mechanics of the interior of the Russo-Susskind-Thorlacius black hole, Phys. Rev. D 98 (2018) no.12, 124017 [arXiv:1807.02461 [gr-qc]].

[21] A. Ashtekar, Black Hole evaporation: A Perspective from Loop Quantum Gravity, Universe 6 (2020) no.2, 21 [arXiv:2001.08833 [gr-qc]].

[22] M. Bojowald, Information loss, made worse by quantum gravity?, Front. in Phys. 3 (2015) 33 [arXiv:1409.3157 [gr-qc]].

[23] V. I. Dokuchaev, Mass inflation inside black holes revisited, Class. Quant. Grav. 31 (2014) 055009 [arXiv:1309.0224 [gr-qc]].

[24] A. Bonanno, A. P. Khosravi and F. Saueressig, Regular black holes with stable cores, Phys. Rev. D 103 (2021) no.12, 124027 [arXiv:2010.04226 [gr-qc]].

[25] R. Balbinot and P. R. Brady, Inside two-dimensional black holes, Class. Quant. Grav. 11 (1994) 1763-1773.

[26] J. S. F. Chan and R. B. Mann, Mass inflation in (1+1)-dimensional dilaton gravity, Phys. Rev. D 50 (1994) 7376-7384 [arXiv:gr-qc/9406021 [gr-qc]].

[27] A. V. Frolov, K. R. Kristjansson and L. Thorlacius, Global geometry of two-dimensional charged black holes, Phys. Rev. D 73 (2006) 124036 [arXiv:hep-th/0604041 [hep-th]].

[28] A. Almheiri and J. Sully, An Uneventful Horizon in Two Dimensions, JHEP 02 (2014), 108 [arXiv:1307.8149 [hep-th]].

[29] S. A. Hayward, Formation and evaporation of regular black holes, Phys. Rev. Lett. 96 (2006) 031103 [arXiv:gr-qc/0506126 [gr-qc]].

[30] V. P. Frolov, Information loss problem and a 'black hole' model with a closed apparent horizon, JHEP 05 (2014), 049 [arXiv:1402.5446 [hep-th]].

[31] E. Bianchi, M. Christodoulou, F. D’Ambrosio, H. M. Haggard and C. Rovelli, White Holes as Remnants: A Surprising Scenario for the End of a Black Hole, Class. Quant. Grav. 35 (2018) no.22, 225003 [arXiv:1802.04264 [gr-qc]].

[32] C. R. Stephens, G. ’t Hooft and B. F. Whiting, Black hole evaporation without information loss, Class. Quant. Grav. 11 (1994) 621-648 [arXiv:gr-qc/9310006 [gr-qc]].

[33] M. Fitkevich, D. Levkov and S. Sibiryakov, Semiclassical S-matrix and black hole entropy in dilaton gravity, JHEP 08 (2020), 142 [arXiv:2006.03606 [hep-th]].

[34] M. Fitkevich, D. Levkov and Y. Zenkevich, Exact solutions and critical chaos in dilaton gravity with a boundary, JHEP 04 (2017) 108 [arXiv:1702.02576 [hep-th]].

[35] M. Fitkevich, D. Levkov and Y. Zenkevich, Dilaton gravity with a boundary: from unitarity to black hole evaporation, JHEP 06 (2020) 184 [arXiv:2004.13745 [hep-th]].