Research on pricing of electricity swing option

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Abstract: Electricity non-storability leads to relatively high fluctuation range of at-sight electricity price. It is necessary to apply electricity financial derivative tool for risk management and control. Electricity swing forward option pricing widely applied in electricity financial market is mainly studied in the paper. Swing option pricing is converted into linear complementary problem for solution through finite difference, discrete transaction time and price. Meanwhile, optimal exercise behavior of swing option purchaser is further combined for establishing an optimization model. Finally, the former model and algorithm are utilized for simulation pricing of electricity swing option through actual electricity futures.

1. Introduction
Swing option is also called acquisition payment option. Option holders can purchase underlying assets according to appointed price for many times within the holding period in the contract. However, the maximum and minimum quantities of the purchased underlying assets are preassigned. In addition, the maximum and minimum quantities of total purchased underlying assets within the whole period are also preassigned. In the exercise period, option holders can change (swing) the speed for purchasing underlying assets. The changing frequency is also limited under general condition.

The flexible delivery mode of electricity swing option can be regarded as the most important electricity financial derivative instrument with the most convenient delivery in electricity financial instrument contract, electricity swing option sellers mostly belong to large power factory generally. Purchasers are mostly public electricity dealers, they formulate retail price. However, swing option should be purchased for hedging aiming at electricity load and at-sight price risks.

Since option holders can make decision on swing option in the aspects of delivery quantity and exercise time according to own optimum strategy, swing option sellers also have the right to formulate the upper limit and lower limit of delivery price and delivery quantity, thereby it is complicated to price swing options.

Swing option pricing documents without consideration of subject matter include Eydeland and Geman (1998) as well as Davison and Anderson (2003), the value of holding one swing option is respectively held for swing option holders in document [1] and document [2] according to arbitrage-free principle, thereby pricing swing option. Swing option is considered into American options for many exercises in Thompson (1995) as well as Carmona and Touza (2006) document [3-4] from the perspective of swing option purchaser. Lari (2001) and Jaillet (2004) consider swing option pricing as a random dynamic planning problem from the perspective of purchaser in document [5-6], wherein underlying commodity price change is handled through binary tree or forest tree method. Longstaff and Schwartz (2001) as well as Ibanez (2004) respectively assumed that underlying commodity price obeys mean reversion model in document [7] and document [8], and least square Monte Carlo simulation method is utilized for solving swing option pricing on the basis.

Haarbrucker and Kuhn (2009) consider swing option pricing with object of at-sight electricity multi-stage random optimization of swing option purchase in document [9], wherein the optimum
value is equal to expected economic benefits brought by electricity swing option to holders, wherein random electricity price is handled through binary tree method. Broussev and Pflug (2009) consider the optimum decision-making of share option seller and share option purchaser in document [10] at the same time in the same year. The drawing of penalty function during option holder default is considered (purchasing quantity or purchasing frequency are not within the regulated scope). It is considered that the decision of share option seller and share option purchaser may affect the optimal decision of the opponent party, Broussev and Pflug (2014) consider electricity swing option pricing as a double-layer planning problem in document [11], wherein electricity swing option seller belongs to upper-layer decision-maker, who formulates different objective function according to market condition.

However, since electricity at-sight price has larger fluctuation, which can not be predicted accurately and easily, and at-sight electricity cash-sale direct delivery only accounts for small proportion of total actual delivery of electricity in the market at present. Both large power generation factories and electricity dealers may adopt financial derivative tools for reducing at-sight electricity price risk, and electricity futures are the most common which are developed more perfectly.

The subject matter of electricity futures is considered in the paper rather than at-sight electricity electricity swing option based on the above reason. Electricity futures contract under suitable delivery mode can be signed according to demands of both parties. It is closer to actual application on the one hand, it also makes up for at-sight electricity non-storability. Arbitrage-free principle can be used for pricing it. Purchaser optimal exercise behavior is combined for establishing an optimization model, thereby pricing electricity swing option as a whole.

2. Electricity swing forward option pricing model

2.1 Analysis on electricity swing forward option pricing

Since electricity swing options can be exercised for many times within the contract time, the time interval among exercises is limited, it is considered that each exercise can be implemented within averagely divided time interval in the paper for simple exercise. It is assumed that total contract duration is averagely divided into \( T \) stages, the stage is expressed with \( i \), \( t_i \) represents the time node, wherein \( i \in \{1, 2, \ldots, T\} \). Swing option purchaser must exercise once within each stage. However, he has the right to determine the exercise time within the time limit and delivery quantity during the exercise.

It is firstly assumed that the electricity futures quantity of each delivery is one unit, and the analysis starts with the time section \([t_{T-1}, t_T]\) of the last stage \( T \) in order to better analyze electricity swing option pricing. If time section \([t_{T-1}, t_T]\) is considered only, swing option purchaser can determine to deliver one unit of electricity futures once at any time within the time section. Then, the price of the electricity swing option at \( t_{T-1} \) is equal to the price for solving one futures contract capable of advanced delivery. Since subject matter belongs to electricity futures, no-arbitrage principle can be applied for pricing it. Time section \([t_{T-1}, t_T]\) is considered only within the world with neutral risks, the Black-Scholes-Merton partial differential equation followed by the price of electricity swing forward option with unit of delivery quantity at time \( t_{T-1} \) is shown as follows:

\[
\frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial F^2} \sigma^2 F^2 = r V \quad (2.1)
\]

Then, the solution of the swing option price is similar to solution of American option pricing. American option is priced on the basis of Black-Scholes-Merton partial differential equation. It is considered in the paper that finite difference method is utilized for solving share option pricing, finite difference method belongs to the difference equation facilitating solution of Black-Scholes-Merton
partial differential equation. It can be transformed into a complementarity problem according to features of American option, and it can be solved through solving the algorithm of complementarity problem.

Document [12] is adopted as reference, formula (2.1) pricing can be written into the following partial differential inequality:

\[
\frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial F^2} \sigma^2 F^2 - rV \leq 0
\]

\[
V(\ F, t) - \Lambda(\ F, t) \geq 0
\]

(2.2)

\[
\left( \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial F^2} \sigma^2 F^2 - rV \right) (V(\ F, t) - \Lambda(\ F, t)) = 0
\]

The left end of partial differential inequality can be approximated as follows through difference approximation:

\[
-\frac{\partial V}{\partial t} - \frac{1}{2} \frac{\partial^2 V}{\partial F^2} \sigma^2 F^2 + rV \approx V(F + \delta F, t) \left( -\frac{1}{2} \sigma^2 F^2 \left( \frac{1}{(\delta F)^2} \right) \right) + V(F - \delta F, t) \left( -\frac{1}{2} \sigma^2 F^2 \left( \frac{1}{(\delta F)^2} \right) \right) + V(F + \delta F, t + \delta t) \left( \frac{1}{\delta t} + \sigma^2 F^2 (1 - \theta) \frac{1}{(\delta F)^2} \right) + V(F, t + \delta t) (F + \delta F, t + \delta t) \left( \frac{1}{\delta t} + \sigma^2 F^2 (1 - \theta) \frac{1}{(\delta F)^2} \right) + V(F - \delta F, t + \delta t) (F + \delta F, t + \delta t) \left( -\frac{1}{2} \sigma^2 F^2 (1 - \theta) \frac{1}{(\delta F)^2} \right)
\]

It can be transformed into next group of finite dimension linear complementary problem according to the above finite-difference approximation.

\[
0 \leq (V_{T-1}^l - \Lambda) \perp (MV_{T-1}^l + M) \geq 0, \ l = L - 1, L - 2, \cdots, 1, 0 \ (2.3)
\]

Wherein, \( \perp \) represents orthogonality of two vectors, and inner product is equal to 0. For example, \( x \perp y \) means \( x^T y = 0 \), \( V_{T-1}^l \) and \( \Lambda \) are respectively the following N-dimensional vectors:

\[
V_{T-1}^l \equiv \begin{pmatrix} V_{T-1,1}^l \\ \vdots \\ V_{T-1,N}^l \end{pmatrix} \quad \Lambda_{T-1}^l \equiv \begin{pmatrix} \Lambda_1 \\ \cdots \\ \Lambda_N \end{pmatrix}
\]

Wherein M is the matrix of \( N \times N \) as follows:
Elements in $M$ are respectively shown as follows:

\[ a_n = -\frac{1}{2} \sigma^2 n^2 \theta, \quad n = 1, \cdots, N \]

\[ b_n = r + \frac{1}{\delta t} + \sigma^2 n^2 \theta, \quad n = 1, \cdots, N \]

$M'$ is the matrix like $M$, and elements in the matrix are shown as follows:

\[ a_n' = -\frac{1}{2} \sigma^2 n^2 (1 - \theta), \quad n = 1, \cdots, N \]

\[ b_n' = -\frac{1}{\delta t} + \sigma^2 n (1 - \theta), \quad n = 1, \cdots, N \]

If exercise is carried out at the deadline time $T$, the price of swing option at time $T_{t-1}$ is equal to revenue function value, namely $V^L_{T_{t-1}} = \Lambda$. When electricity swing option seller gives a delivery price $K$, $\Lambda$ can be regarded as a known value, the linear complementary problem of formula (2.3) can be solved then through retrodicting time point $l = L-1, L-2, \cdots, 1, 0$ after dissociation, and $V^0_{T_{t-1}}$ can be obtained.

Actual time section $[0, T]$ of electricity swing option is analyzed at present. Similarly, total time is divided into $T$ stages, the price of swing option at time section $[t_i, t_{i+1}]$, $i \in \{0, 1, \cdots, T-1\}$ is considered at each stage. In the paper, finite difference method is used for converting Black-Scholes-Merton partial differential equation into linear complementarity constraint, and such practice has the following advantage that it can be directly applied to solution of derivative price capable of advanced exercise at multiple stages. It should be noted that the following formula is established:

\[ V^L_{i} = V^0_{i}, \quad i = 1, 2, \cdots, T \quad (2.4) \]

The $V^0_{i}$ obtained at each stage is just the $V^L_{i-1}$ of former stage, the $V^0_{i-1}$ of the former stage can be obtained through retrodicting. Therefore, $V^0_{0}$ can be obtained through $V^0_{1}$ each stage. Namely, it is assumed that purchaser can select any time of each stage for exercising the price of electricity swing option at the initial stage once under the condition that delivery quantity as one unit.

2.2 Behavior of share option purchaser

The estimation of any contract should contain optimal execution strategy of share option owners, including both hedgers and and speculators. They consider the objective of maximizing own profits. Since electricity futures price is random in the future, the simplest objective of function contract purchaser is maximization of expected benefits.
Swing option purchaser can fix the delivery price K of electricity futures contract of each unit in the future during contract signing. He has the right to determine the delivery quantity during each stage delivery in the future. It is assumed that the purchaser deliver electricity futures at each stage according to quantity \( z_i, \ i \in \{1, 2, 3, \cdots, T\} \), \( z_i \) represents the delivery quantity within time section \([t_{i-1}, t_i]\). In the actual financial market, delivery quantity should be determined within the former stage (generally one day) of actual delivery, namely the delivery quantity pf next stage should be determined by the purchase one stage in advance.

The delivery quantity of each stage has the limitation of maximum quantity and minimum quantity in the contract generally. Total quantity of each stage also has upper limit and lower limit. The constraint related to the delivery quantity can be expressed as follows:

\[
\underline{e}_i \leq z_i \leq \bar{e}_i \quad \forall \ i \in \{1, 2, 3, \cdots, T\} \quad (2.5)
\]

\[
\underline{E} \leq \sum_{i=1}^{T} z_i \leq \bar{E} \quad (2.6)
\]

Wherein formula (2.5) represents the upper limit and the lower limit of the delivery quantity \( z_i \) at stage i. Formula (2.6) represents the upper limit and the lower limit of total delivery quantity within the contract duration. The objective function of electricity swing option purchaser is combined, namely maximization of own expected benefits. It is further assumed that swing option seller has determined delivery price as K, the optimization of the purchaser can be described as follows for upward tendency share option contract:

\[
\max \sum_{i=1}^{T} e^{-r_{t_{i-1}}} z_i (V_{t_{i-1}}^0 - K)
\]

\[
s.t. \quad \underline{e}_i \leq z_i \leq \bar{e}_i \quad \forall \ i \in \{1, 2, 3, \cdots, T\} \quad (2.7)
\]

\[
\underline{E} \leq \sum_{i=1}^{T} z_i \leq \bar{E}
\]

3. Model solution method

3.1 Linear complementary problem solution

The complementarity problem solution property of electricity swing option pricing given in the paper is analyzed firstly. \( x = V_{t_{T-1}}^{t_1} - \Lambda \) and \( q = Mx + M\Lambda + M V_{t_{T-1}}^{t_1} \) are transformed for more intuitive explanation, it should be noted that \( V_{t_{T-1}}^{t_1} \) is known within time section \([t_{i-1}, t_{i+1}]\) due to retrodicting solution. The problem (2.3) can be simplified into the following problem then:

\[
x \geq 0, \quad Mx + q \geq 0, \quad x^T (Mx + q) = 0 \quad (3.1)
\]

The existence and uniqueness of the solution are proved next aiming at the above problems. The following lemma is proposed in document [13]:

lemma3.1: is one square matrix A belongs to a diagonal donimant matrix, and the diagonal element elements in A are positive, A is a P matrix.

The above-mentioned matrix \( A = (a_{ij}) \) is a diagonal donimant matrix, and it is equal to the following established formula:

\[
|a_{ii}| > \sum_{j \neq i} |a_{ij}|, \quad i = 1, 2, \cdots, n
\]

However, one square matrix belongs to a P matrix, and it means that all of its principal minors are positively. The following conclusions of P matrix in the work [14] are established:
Proposition 3.1: If one square matrix \( A \in \mathbb{R}^{n \times n} \) belongs to a P matrix, linear complementary problem is shown as follows;

\[
0 \leq x \perp Ax + q \geq 0
\]

Any \( q \in \mathbb{R}^n \) has its own unique solution.

Theorem 3.1: M matrix in the complementarity problem transformed from electricity swing option pricing model of the paper belongs to a P matrix, and the transformed complementarity problem has its unique solution.

Evidence: Firstly, it is obvious that matrix M diagonal element s are positive. Therefore, it is concluded according to lemma (3.1) that it is only necessary to prove matrix M as a diagonal dominant matrix in order to prove matrix M as a P matrix. It should be noted that, 

\[
\begin{align*}
|b_n| - |a_n| &= r + \frac{1}{\delta t} + \sigma^2 \frac{\partial}{\partial t} - \frac{1}{2} \sigma^2 \frac{\partial^2}{\partial t^2} = r + \frac{1}{\delta t} + \frac{1}{2} \sigma^2 \theta \\
|b_n| - 2 |a_n| &= r + \frac{1}{\delta t} + \sigma^2 n^2 \frac{\partial}{\partial t} - \sigma^2 n^2 \frac{\partial^2}{\partial t^2} = r + \frac{1}{\delta t}, \quad n = 2, \ldots, N - 1 \\
|b_N| - |a_N| &= r + \frac{1}{\delta t} + \sigma^2 N^2 \frac{\partial}{\partial t} - \frac{1}{2} \sigma^2 N^2 \frac{\partial^2}{\partial t^2} = r + \frac{1}{\delta t} + \frac{1}{2} \sigma^2 N^2 \theta
\end{align*}
\]

Obviously, the value of the above formula is larger than 0. Therefore, matrix M belongs to a P matrix. It can be concluded according to lemma (3.1) that complementarity problem transformed from electricity swing option pricing model in the paper has unique solution.

Since coefficient matrix M has many high-quality properties. In addition, the scale of linear complementary problem is larger and larger with smaller and smaller difference step length. Therefore, when the linear complementary problem in the paper is solved, though interior point method, smoothing Newton method, etc. have excellent astringency and accuracy, the computation is too large aiming at the problem, therefore it is not applicable. Solution concept of the paper is mainly shown as follows: fixed point algorithm is firstly approximated, solution of linear complementary problem is transformed into solution of a group of large-scale linear equations with fixed point. Then, it can be transformed into solution of large-scale linear equations through over-relaxation iterative method.

4. Numerical experiment

In the paper, the electricity swing option is priced with the model and algorithm established in the paper through actual electricity futures data provided by Chicago Mercantile Exchange. It is considered that one electricity futures contract in transaction at present searched from CME official website is selected (including the transaction duration of underlying futures at New York Electricity Exchange lasts for one month, 5 megawatts of electricity futures base load electricity futures can be delivered for each future). Total duration is also 30 days a month (namely \( T = 30 \)), the futures can be delivered once a day (namely they can be divided into 30 stages, \( T = 30 \)). It is assumed that at least one and no more than three electricity futures are required for each delivery quantity. Total delivery quantity should be between 40 and 60 electricity swing options.

The electricity swing option is priced according to the model and algorithm proposed in the following paper mainly. Firstly, parameters \( r, \sigma \) and \( \theta \) should be firstly determined during model establishment. \( r \) belongs to risk-free interest rate, and U.S. 10-year Treasury yield is generally chosen, namely \( r \approx 2.54\% \). \( \sigma \) represents the volatility of the random process followed by electricity futures price. In the paper, it is assumed that the transaction month of electricity swing option is the whole April according to
CME history data and seasonal characteristics (30 days). The volatility of underlying electricity futures price of future electricity swing option is estimated through the volatility weight of April in the former years. The estimated volatility is \( \sigma_1 = 32.55\% \).

Secondly, differential parameter \( \theta \) can be selected, Crank-Nicolson difference method is selected according to document [14], namely \( \theta = 1/2 \) is selected.

Now, electricity futures price and time difference step length are considered. It is discovered through the historical data that electricity futures price can not be lower than $15 per unit of futures in April generally, which can not be higher than $30. Currently, electricity futures price difference steps \( N = 100 \), difference step length \( \delta F = 0.15 \), time difference steps \( L = 24 \) and time difference step length \( \delta t = 1/24 \) are selected, namely each day is divided into 24 hours for consideration.

Element \( a_n \) and element \( b_n \) in M matrix can be concluded with the assumption of the above parameters:

\[
a_n = -\frac{1}{4} 0.3255^2 n^2, \quad n=1, \cdots, N
\]

\[
b_n = 0.0254 + 24 + \frac{1}{2} \sigma^2 0.3255^2 n^2, \quad n=1, \cdots, N
\]

Similarly, elements \( a_n' \) and \( b_n' \) in matrix can be obtained as follows:

\[
a_n' = -\frac{1}{4} 0.3255^2 n^2, \quad n=1, \cdots, N
\]

\[
b_n' = -24 + \frac{1}{2} 0.3255^2 n^2, \quad n=1, \cdots, N
\]

It is assumed that seller optimal delivery price is \( K = 22 \), it is delivered into problem (2.3). Since there are more selected price difference steps, the obtained \( V_0 \) belongs to a 100 1-row matrix, they are not listed in the paper. CME data show that the initial electricity futures price is $19.73, the delivery unit electricity swing option price is about $23.7 corresponding with the electricity futures price range in \( V_0 \). The optimum purchase quantity of the purchaser in each stage is considered, the electricity swing option price is finally obtained, namely about $2004.

In addition, different time difference step lengths are also selected in the paper, seasonal influence is not considered. The price of electricity swing option is obtained through calculation as follows according to the volatility \( \sigma_2 \approx 43.74\% \) obtained in the former months:

| Difference step length | 1 day | 4 hours | 1 hour | 30 min |
|------------------------|-------|---------|--------|--------|
| \( \sigma_1 \) | 2328.3692 | 2396.8280 | 2004.1351 | 1458.8847 |
| \( \sigma_2 \) | 3344.8208 | 3171.2810 | 2441.8448 | 1578.7603 |

It can be found that the price of electricity swing option is constantly modified and decreased with the decrease of difference step length and the increase of difference step number. The volatility is greater, electricity swing the option price is larger under the same step length, and it conforms to the actual situation. Therefore, the overall price in the numerical experiment is higher because the volatility of the adopted electricity futures price is higher. It can be concluded that the price for entering one electricity swing option is higher in summer an winter (season with higher volatility), which is also in line with the actual situation.
5. Summary and prospect
In the paper, since the subject matter is electricity swing option pricing of electricity futures, electricity swing option pricing is converted into linear complementary problem for solution through finite difference method differential transaction duration and electricity futures price. In addition, analysis on purchaser's optimal exercise behaviors is combined for pricing electricity swing option more completely. It can be found that the optimal decisions of buyers and sellers influence each other, double-layer planning model can be further established for pricing electricity swing option. Meanwhile, it is not necessary to strictly control the the purchase quantity of electricity swing option purchaser at each stage within a given scope in reality. It should not be higher than the upper limit or lower than the lower limit. However, certain liquidated damages should be paid. Therefore, the optimum behavior of the purchase can be further considered as no limitation of the transaction amount. However, the objective contains mathematical planning problem of penalty function.

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