Virtual Pion Scattering

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Abstract

We propose a theory which exploits the charge-exchange reactions \((^{3}\text{He},^{3}\text{H}\pi^{+})\) and \((p,n\pi^{+})\) as effective sources of virtual pions. We consider processes in which the creation of virtual pions is followed by conventional coupled-channel pion scattering to discrete nuclear states. This picture allows us to incorporate successful theories of pion scattering and utilize virtual pions as probes of the nuclear matter. For coherent pion production we clearly demonstrate that the shift of the coherent peak position in the excitation function of \(^{3}\text{He}-\text{A}\) relative to \(^{3}\text{He}-\text{N}\) scattering is determined entirely by the pion nucleus rescattering.
The effect that the presence of nuclear matter has on reactions of nucleons and mesons remains one of the most formidable open questions in nuclear physics. For many years pion beams have provided a unique probe for the study of nuclei with considerable success. Lately, the possibility has been raised that virtual pions may also be useful as a complementary probe [1]. Coherent pion production in nuclei has often been likened to elastic pion scattering [2] and may be utilized as an effective virtual pion source. It comprises of a charge-exchange reaction and the simultaneous creation of a pion, leaving the target nucleus in its ground or a low-lying excited state. Although it is a small channel relative to inclusive pion production, it has recently attracted a lot of attention [3–7]. Here we present a theory of coherent pion production which we believe is especially suitable for the study of the medium effects, and can shed light on the nature of the interaction between virtual pions and nuclear matter in this reaction.

One persistent feature established from measurements of cross sections in the charge-exchange reactions \((^{3}\text{He}, ^3\text{H}_\pi^+)\) and \((p,n\pi^+)\) is a downward shift in the transfer energy spectra of the \(^3\text{He}\)-nucleus and \(p\)-nucleus relative to the \(^3\text{He}\)-nucleon and \(p\)-nucleon reactions [3, 8–11]. Extensive theoretical work has covered many aspects of these interactions both in the ∆-hole model [4, 5, 7], with emphasis on the nuclear response function, as well as in the picture of [2, 6, 12] with emphasis on the elementary reaction. These models reproduce reasonably well the measured cross sections but do not reach consensus on the nature of the medium effects. In particular, none of the previous models provides an unambiguous interpretation of the observed energy shift, although the picture employed by [2] is in principle similar to ours. In our model the energy shift acquires a clear physical interpretation, and the connection between coherent pion production (to the g.s. and low-lying excited states of the target nucleus) and pion scattering (elastic and inelastic) is established quantitatively. Furthermore, our theory sets limits on the sensitivity that future coherent pion production experiments [13] must attain, and the sort of information one would hope to extract from them.

We employ a framework which views coherent pion production as a two-step process:
creation of a virtual pion, followed by scattering of the pion with nucleons in the target nucleus. The former is treated formally as a source term (effective virtual pion beam) while the latter is treated as pion scattering via a pion-nucleus optical potential. This is the main difference between our approach and the previous calculations \cite{4,6} where the main focus was on the $\Delta$ - hole dynamics determined by a meson exchange interaction with the exchange by $\pi$- and $\rho$-mesons. In our picture the $\Delta$ does not have explicit dynamical degrees of freedom, it is hidden in a resonant parametrization of pion-nucleus optical potential. Apart from resonant contribution the pion-nucleus optical potential has a contribution from s-wave pion nucleon scattering amplitude which is important at pion c.m. kinetic energy below 50 MeV.

We consider coherent pion production with $^3$He and p as probes. This formulation is concisely stated in the inhomogeneous coupled-channel Klein-Gordon equations, which we solve in coordinate space:

\[
\left( \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} - \frac{l_\beta (l_\beta + 1)}{r^2} - U_{l_\beta I_\beta I_\beta J} (r) + k_\beta^2 \right) R_{l_\beta I_\beta J}'(r) = \sum_\alpha U_{l_\beta I_\beta J}(r) R_{l_\alpha I_\alpha J}'(r) + \rho_{l_\beta I_\beta J}(r). \tag{1}
\]

Note that $l, I$ and $J$ are pion, nucleus and coupled-channel angular momenta and the indices $\alpha, \beta$ denote initial and final states involving real pions, whereas prime denotes the entrance channel with a virtual pion. The explicit separation of the source term $\rho_{l_\beta I_\beta J}(r)$ from the remaining (homogeneous) Klein-Gordon equation for pion-nucleus scattering enables us to disentangle the pion production from the pion interaction with the medium, and better understand how each component participates in the reaction. Similar approach has been successfully used first in nuclear reaction theory to describe the transfer reactions \cite{15}, and for the $\Delta$ - hole system in calculations of the $\Delta$-hole response function \cite{14}, and a pionic decay of the correlated $\Delta$-hole state leading to coherent pions \cite{4}.

While the details will be provided elsewhere \cite{16}, let us briefly discuss Eq. (1): The pion-nucleus optical potential

\[
U_{l_\beta I_\beta J}(r) = U_0 \delta_{\alpha \beta} + < l_\beta I_\beta J | V_{\pi A} | l_\alpha I_\alpha J > \tag{2}
\]
has been applied successfully for elastic, inelastic and charge exchange pion-nucleus scattering (with the homogeneous Klein-Gordon equation). The diagonal component includes medium corrections which have been shown to be well-described in terms of pion absorption, the Lorentz-Lorenz effect, and Pauli correlations for low pion energies (see [17,18] and references therein), while near resonance it includes phenomenological corrections from fits to elastic, inelastic and double-charge exchange pion-nucleus data (see [19] and references therein). The pion-nucleus transition operator $V_{\pi A}$ is derived microscopically from the pion-nucleon $t$-matrix, summed over all the valence nucleons and weighed by one-body transition densities from the shell model. Its matrix elements between pion-nucleus coupled-channel states, as in Eq. (2), are the off-diagonal transition matrix elements.

The advantages of this formalism – as explained in Refs. [17,18] – include taking into account the internal and external distortions of the pion waves, avoiding the closure approximation by explicitly including the excitation energy of all the nuclear states, and using nuclear structure input from shell-model calculations. All these features were shown to be important in pion charge-exchange scattering. In particular, the external pion distortions were necessary for the physical behaviour of the model while the other improvements accounted for corrections.

For the case at hand, of equal importance is the evaluation of the source term $\rho_{I,J,J'}(r)$. We derive a microscopic coordinate-space operator for coherent pion production, which includes corrections due to the Fermi motion of the nucleons in the target nucleus, starting from the $t$-matrix [10]

$$t_{\pi N \rightarrow \pi NN} = 4\pi \frac{\hbar c}{m_\pi^2} f_{\pi NN} f_{\pi N}^2 (t'_{N \Delta} - E_{\text{res}})^2 \left( \begin{array}{c} \Lambda'_{\pi} - m_\pi^2 \\ \Lambda_{\pi}^2 - t \end{array} \right) \left( \begin{array}{c} (\hat{\sigma} \cdot \hat{q}) (\hat{S}^\dagger \cdot \hat{q}) + (\hat{\sigma} \times \hat{q}) \cdot (\hat{S}^\dagger \times \hat{q}) \\ \omega - E_{\text{res}} + i \Gamma(\omega) \end{array} \right) \left( \begin{array}{c} \hat{S} \cdot \hat{k} \\ \hat{T}^\dagger \cdot \hat{T} \end{array} \right),$$

where $t'_{N \Delta} = 0.6$ [4] and $\Lambda'_{\pi} = 650$ MeV [4,20]. We choose the particular model for pion production amplitude via $\Delta$ in order to be able to compare the results with the previous calculations [4,5]. We stress that Eq. (3) must be interpreted as purely phenomenological,
which however roughly reproduce the main observable features of the elementary reaction $NN \rightarrow N\Delta$, the ratio of spin-transverse and spin-longitudinal cross-sections close to 2:1, and weak energy dependence at intermediate energies.

With the coherent pion production operator as described we obtain the source term $ho_{l'J'J}(r)$ using a distortion factor for the projectile (in the case of $^3$He) in the eikonal approximation following the treatment of Refs. [5,21].

We have applied this model for calculations of coherent pion production (program MEGAPI [16]) with a $^3$He probe on a $^{12}$C nucleus to the ground state (g.s.) as well as, for the first time, to the lowest-lying $2^+$ and $1^+$ states (Fig 1). As the resolution of future experiments will be sufficient to distinguish between the g.s. and low-lying excited states, these results are instructive [13]. All our calculations were performed with harmonic oscillator wavefunctions for the nucleons, except for one (dashed) in Fig 1a with Hartree-Fock wavefunctions [22], in order to test the sensitivity of our model to the nuclear density input. The one-body transition densities which we used both in the source as well as in the pion scattering terms of Eq. (1) were obtained from the shell-model code OXBASH [23] with the Cohen-Kurath interaction. We also show calculations with p as a probe on $^{12}$C and $^{40}$Ca to the g.s. (Fig 2). Canonical calculations to the g.s. include the source term (production) and the diagonal piece of the pion-nucleus optical potential Eq. (2) (pion rescattering) with all its higher-order medium corrections as described earlier, plus pion rescattering via intermediate excited states via the off-diagonal piece of potential. For the excited states, canonical calculations include direct contributions from the source to the g.s. as well as the excited states, followed by rescattering via the diagonal pion-nucleus optical potential and medium corrections, and scattering from the g.s. to excited states via the off-diagonal component of Eq. (2).

The calculations for $^{12}$C are consistent with the available data (see e.g. Ref. [3]). Our findings concerning the interaction of the virtual pion with the nucleus are contained compactly in Fig 1c: The dashed curve is a calculation where the optical potential of Eq. (2) has been omitted. It is equivalent to “removing” the nuclear medium, except for a residual
presence as a density of finite volume of the target nucleus in the source term. In this case the cross-section to the g.s. is proportional to the target formfactor squared. The target formfactor is a Fourier component $\rho(q-k)$ of the density present in the source term. Here $q$ and $k$ are the incoming momentum transfer from the projectile and the outgoing pion momentum. The formfactor defines the pion angular distribution width at high pion momentum. At low pion momentum the formfactor suppresses the pion yield due to momentum mismatch which is the largest at low pion energy. The dot-dashed curve includes pion rescattering via a lowest-order pion-nucleus optical potential without the higher order medium corrections discussed earlier. This results in a dramatic reduction of the total cross section near the $\Delta$ resonance—corresponding to $T_\pi \sim 160$ MeV, $\omega \sim 300$ MeV—where p-wave pion scattering dominates, and a small enhancement at lower pion energies, in the range of s-wave dominance. The reduction comes from a resonant pion absorption with a subsequent incoherent $\Delta$ decay. The incoherent decay width of the $\Delta$ is large compared to the decay width into a single coherent channel and this reduces the yield of coherent pions near the $\Delta$ peak. Thus, the downward shift in this approach is determined mainly by the resonance absorption mechanism. This interpretation of the peak position differs from previous calculations [4,6] where the peak position was determined by the $\Delta$-nucleus dynamics. Finally, the solid curve is a canonical calculation as described earlier, with a pion-nucleus optical potential and higher order medium corrections as determined from pion-nucleus scattering. The difference between the dot-dashed and the solid curves reflects the details required of future coherent pion production experiments in order to probe the higher order corrections of the pion-nucleus optical potential, and potential differences between real and virtual pionic probes.

In summary, we have presented a theory of coherent pion production which utilizes this reaction as an effective virtual pion source and probes the nature of the interaction between virtual pions and the nucleus. We have established the close connection between coherent pion production and pion scattering by showing the considerable change in the energy spectra of $^3$He-A and p-A relative to $^3$He-N and p-N as a result of pion nucleus rescattering. The
explicit separation of pion production from rescattering in our theory provides a handle for
the study of the higher-order medium effects in the production amplitude, which illustrate
the range of possible differences between real and virtual pionic probes.

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FIGURES

The ($^3$He,$^3$Hπ+) calculations, for $T_{^3He} = 2$ GeV and $\theta_{^3H} = 0$ deg: a) Angular distribution for charge-exchange on a $^{12}$C target left in the g.s. and transfer $\omega = 260$ MeV, with a Woods-Saxon (solid) and Hartree-Fock (dashed) density. b) As before but for the lowest-lying $2^+$ (solid) and $1^+$ (dashed) states in $^{12}$C. c) The $\omega$-transfer dependence of the total cross section for the g.s. transition in $^{12}$C without rescattering of the coherent pion (dashed), with rescattering via a lowest-order pion-nucleus optical potential (dot dashed), and with rescattering via the full pion-nucleus optical potential including higher-order medium effects (solid). d) Full calculations of the $\omega$-transfer dependence for the $2^+$ (solid) and $1^+$ (dashed) states in $^{12}$C. The irregularity of the spectra at 260 - 280 MeV comes from use of different parametrization of pion-nucleus optical potential at low and high pion energy.

The (p,nπ+) calculations, for $T_p = 800$ MeV and $\theta_n = 0$ deg, for charge-exchange transitions to the ground state: a) Angular distribution for transfer $\omega = 260$ MeV on a $^{12}$C target. b) As before but for $^{40}$Ca. c) $\omega$-transfer dependence for charge-exchange without rescattering (dashed), and with rescattering via the full pion-nucleus optical potential (solid) on a $^{12}$C target. d) As in (c) but for $^{40}$Ca.