COLOR SUPERCONDUCTING QUARK MATTER IN
COMPACT STARS

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Recent indications for high neutron star masses ($M \sim 2 M_\odot$) and large radii ($R > 12$ km)
could rule out soft equations of state and have provoked a debate whether the occurrence
of quark matter in compact stars can be excluded as well. We show that modern quan-
tum field theoretical approaches to quark matter including color superconductivity
and a vector meanfield allow a microscopic description of hybrid stars which fulfill the new,
strong constraints. For these objects color superconductivity turns out to be an essential
ingredient for a successful description of the cooling phenomenology in accordance with
recently developed tests. We discuss the energy release in the neutrino untrapping transition
as a new aspect of the problem that hybrid stars masquerade themselves as neutrino stars.
Quark matter searches in future generations of low-temperature/high-density
nucleus-nucleus collision experiments such as low-energy RHIC and CBM @ FAIR might
face the same problem of an almost crossover behavior of the deconfinement transition.
Therefore, diagnostic tools shall be derived from effects of color superconductivity.

Keywords: neutron stars, chiral quark model, color superconductivity

1. Introduction

Recently, observations of compact stars have provided new data of high accuracy
which put strong constraints on the high-density behaviour of the equation of state
of strongly interacting matter otherwise not accessible in terrestrial laboratories. In
particular, the high masses of $M = 1.96 \pm 0.09 / - 0.12 M_\odot$ and $M = 2.73 \pm 0.25 M_\odot$
obtained in very recent measurements on the millisecond pulsars PSR B1516+02B
and PSR J1748-2021B, respectively, together with the large radius of $R > 12$ km for
the isolated neutron star RX J1856.5-3754 (shorthand: RX J1856)\(^2\) point to a stiff equation of state at high densities. Measurements of high masses are also reported for compact stars in low-mass X-ray binaries (LMXBs) as, e.g., \(M = 2.0 \pm 0.1 \ M_{\odot}\) for the compact object in 4U 1636-536.\(^3\) For another LMXB, EXO 0748-676, constraints for the mass \(M \geq 2.10 \pm 0.28 \ M_{\odot}\) and the radius \(R \geq 13.8 \pm 0.18\) km have been derived.\(^4\) The status of these data is, however, unclear since the observation of a gravitational redshift \(z = 0.35\) in the X-ray burst spectra\(^5\) could not be confirmed thereafter despite numerous attempts.\(^6\)

Measurements of rotation periods below \(\sim 1\) ms as discussed for XTE J1739-285,\(^7\) on the other hand, would disfavor too large objects corresponding to a stiff EoS and would thus leave only a tiny window of very massive stars in the mass-radius plane\(^8,9\) for a theory of compact star matter to fulfill all above mentioned constraints.

In the left panel of Fig. 1 we show some of these modern observational constraints for masses and mass-radius relationships together with solutions of the Tolman-Oppenheimer-Volkoff (TOV) equations for nuclear EoS classified in three groups: (i) relativistic mean-field (RMF) approaches with non-linear (NL) self-interactions of the \(\sigma\) meson.\(^10\) In NL\(\rho\) the isovector part of the interaction is described only by a \(\rho\) meson, while the set NL\(\rho\delta\) also includes a scalar isovector meson \(\delta\) that is usually neglected in RMF models;\(^11\) (ii) RMF models with density dependent couplings and masses are represented here by four different models from two classes, where in the first one density dependent meson couplings are modeled so that a number of properties of finite nuclei (binding energies, charge and diffraction radii, surface thicknesses, neutron skin in \(^{208}\)Pb, spin-orbit splittings) can be fitted.\(^12\) D\(^3\)C has in addition a derivative coupling leading to momentum-dependent
nucleon self-energies and DD-F4 is modeled such that the flow constraint from heavy-ion collisions is fulfilled. The second class of these models is motivated by the Brown-Rho scaling assumption that not only the nucleon mass but also the meson masses should decrease with increasing density. In the KVR and KVOR models these dependences were related to a nonlinear scaling function of the $\sigma$-meson field such that the EoS of symmetric nuclear matter and pure neutron matter below four times the saturation density coincide with those of the Urbana-Argonne group. In this way the latter approach builds a bridge between the phenomenological RMF models and (iii) microscopic EoS built on realistic nucleon-nucleon forces. Besides the variational approaches (APR, WFF, FPS) such ab-initio approaches to nuclear matter are provided, e.g., by the relativistic Dirac-Brueckner-Hartree-Fock (DBHF) and the nonrelativistic Brueckner-Bethe-Goldstone approaches. Stiff EoS like D$^3$C, DD-F4, BBG and DBHF fulfill the demanding constraints for a large radius and mass, while the softer ones like NL$\rho$ don’t. It is interesting to note that the flow constraint shown in the right panel of Fig. 1 sets limits to the tolerable stiffness: it excludes the D$^3$C EoS and demonstrates that DD-F4, BBG and DBHF become too stiff at high densities above $\sim 0.55$ fm$^{-3}$. For a detailed discussion, see Ref.

A key question asked in investigating the structure of matter at high densities is how the quark substructure of hadrons manifests itself in the EoS and whether the phase transition to quark matter can occur inside compact stars. In Ref., Özel has debated that the new constraints reported above would exclude quark matter in compact star interiors reasoning that it would entail an intolerable softening of the EoS. Alford et al. have given a few counter examples demonstrating that quark matter cannot be excluded. In the following section we discuss a recently developed chiral quark model which is in accord with the modern constraints, see also.

2. Color superconducting quark matter: masquerade revisited

We describe the thermodynamics of the deconfined quark matter phase within a three-flavor quark model of Nambu–Jona-Lasinio (NJL) type, with a mean-field thermodynamic potential given by

$$\Omega_{MF}(T, \mu) = \frac{1}{8G_S} \left[ \sum_{i=u,d,s} (m_i^* - m_i)^2 - \frac{2}{\eta_{\sigma}} (2\omega_0^2 + \phi_0^2) + \frac{2}{\eta_{\sigma}} \sum_{A=2,5,7} |\Delta_{AA}|^2 \right]$$

$$- \int \frac{d^3p}{(2\pi)^3} \sum_{a=1}^{18} \left[ E_a + 2T \ln \left( 1 + e^{-E_a/T} \right) \right] + \Omega_l - \Omega_0 . \quad (1)$$

Here, $\Omega_l$ is the thermodynamic potential for electrons and muons, and the divergent term $\Omega_0$ is subtracted in order to assure zero pressure and energy density in vacuum ($T = \mu = 0$). The quasiparticle dispersion relations, $E_a(p)$, are the eigenvalues of
the hermitean matrix

$$\mathcal{M} = \begin{bmatrix} -\vec{\gamma} \cdot \vec{p} - \hat{m}^* + \gamma^0 \hat{\mu}^* & i\gamma_5 \tau_A \lambda_A \Delta_{AA} \\ i\gamma_5 \tau_A \lambda_A \Delta^*_{AA} & -\vec{\gamma}^T \cdot \vec{p} + \hat{m}^* - \gamma^0 \hat{\mu}^* \end{bmatrix},$$

(2)

in color, flavor, Dirac, and Nambu-Gorkov space. Here, $\Delta_{AA}$ are the diquark gaps. $\hat{m}^*$ is the diagonal renormalized mass matrix and $\hat{\mu}^*$ the renormalized chemical potential matrix, $\hat{\mu}^* = \text{diag}(\mu_u - G_S \eta_V \omega_0, \mu_d - G_S \eta_V \omega_0, \mu_s - G_S \eta_V \phi_0)$. The gaps and the renormalized masses are determined by minimization of the mean-field thermodynamic potential (1). We have to obey constraints of charge neutrality which depend on the application we consider. In the (approximately) isospin symmetric situation of a heavy-ion collision, the color charges are neutralized, while the electric charge in general is non-zero. For matter in $\beta$-equilibrium in compact stars, also the global electric charge neutrality has to be fulfilled. For further details, see. 24–27

We consider $\eta_D$ as a free parameter of the quark matter model, to be tuned with the present phenomenological constraints on the high-density EoS. Similarly, the relation between the coupling in the scalar and vector meson channels, $\eta_V$, is considered as a free parameter of the model. The remaining degrees of freedom are fixed according to the NJL model parameterization in table I of. 28 where a fit to low-energy phenomenological results has been made.

![Diagram](image)

**Fig. 2.** Same as Fig. 1 for hybrid EoS with a low density hadronic branch described by the DBHF approach and a high density quark matter branch obtained from a three-flavor NJL model with color superconductivity (diquark coupling $\eta_D$) and isoscalar vector meson coupling ($\eta_V$).

As a unified description of quark-hadron matter, naturally including a description of the phase transition, is not available yet, although steps in this direction have been suggested within nonrelativistic 29 and relativistic 30 models. We apply here the so-called two-phase description, being aware of its limitations. The nuclear matter phase is described within the DBHF approach and the transition to the quark matter phase given above is obtained by a Maxwell construction. In the right panel of Fig. 2 it can be seen that the necessary softening of the high density EoS in accordance with the flow constraint is obtained for a vector coupling of $\eta_V = 0.5$.
whereas an appropriate deconfinement density is obtained for a strong diquark coupling in the range $\eta_D = 1.02 - 1.03$. The resulting phase transition is weakly first order with an almost negligible density jump. Applying this hybrid EoS with so defined free parameters under compact star conditions a sequence of hybrid star configurations is obtained which fulfills all modern constraints, see the left panel of Fig. 2. In that figure we also indicate by a red dot the minimal mass $M_{DU}$ for which the central density reaches a value allowing the fast direct Urca (DU) cooling process in DBHF neutron star matter to occur, leading to problems with cooling phenomenology.\textsuperscript{31,32} Note that for a strong diquark coupling $\eta_D = 1.03$, the critical density for quark deconfinement is low enough to prevent the hadronic direct Urca (DU) cooling problem by an early onset of quark matter. For the given hybrid EoS, there is a long sequence of stable hybrid stars with two-flavor superconducting (2SC) quark matter, before the occurrence of the strange quark flavor and the simultaneous onset of the color-flavor-locking (CFL) phase which renders the star gravitationally unstable.\textsuperscript{22,33} Comparing the hybrid star sequences with the purely hadronic DBHF ones one can conclude that the former 'masquerade' themselves as neutron stars\textsuperscript{36} by having very similar mechanical properties. Besides for mass-radius relationships, the masquerade effect can be discussed also for the moment of inertia,\textsuperscript{22} which is becoming accessible to measurements in relativistic binary systems like the double pulsar J0737–3039, see.\textsuperscript{34,35}

![Fig. 3. The effect of neutrino untrapping on hybrid star configurations: a mass defect of 0.04 $M_\odot$ occurs in the transition from neutrino trapping case (solid red line, $Y_e = 0.4$) to untrapping (dashed black lines, $Y_e = 0$) at fixed baryon mass $M_B$, independent of the presence of a quark core for masses above the threshold indicated by an asterix. The blue rectangle corresponds to the constraint from the lighter companion (star B) in the double pulsar PSR J0737-3039, see.\textsuperscript{39}](image-url)
In these proceedings we add a new aspect to this masquerade discussion: the energy release in the neutrino untrapping transition of a cooling protoneutron star (PNS). The process is pictured such that the PNS formed in a supernova collapse is hot enough ($T_{\text{PNS}} \sim 30\ldots50$ MeV) to trap neutrinos since their mean free path is shorter than the size of the star. In this era the fast neutrino cooling cannot proceed from the volume but only from the surface. Until the neutrino opacity temperature ($T_{\text{opac}} \sim 1$ MeV) is reached, the neutrinos take part in the $\beta$-equilibration processes and a finite lepton fraction is established. We will assume $Y_{\text{le}} = 0.4$ corresponding to a neutrino chemical potential $\mu_\nu \sim 200$ MeV in the PNS core. For a discussion of hot hybrid PNS, see.\textsuperscript{37} In the untrapping transition at $T \sim T_{\text{opac}}$, the neutrinos decouple from the $\beta$-equilibrium so that $Y_{\text{le}} \to 0$ and the equilibrium configuration gets readjusted with a new gravitational mass while naturally, the baryon number is conserved in this process. From Fig.\textsuperscript{3} one can estimate the mass defect (energy release) due to neutrino untrapping to $\Delta M = 0.04 M_{\odot}$, almost independent of the mass of the configuration and even of its structure: neutron stars and hybrid stars result in the same energy release! This is another aspect of the masquerade effect. Note that the neutrino untrapped configurations fulfill the extended $M_G - M_B$ constraint,\textsuperscript{38} given by the rectangle in the right panel of Fig.\textsuperscript{3}. This constraint is derived from the lighter companion (star B) in the double pulsar J0737–3039 (B).\textsuperscript{39}

3. Unmasking compact star interiors

To unmask the neutron star interior and to disentangle a quark matter core from a hadronic one, might therefore require observables based on transport properties, strongly modified from normal due to the color superconductivity. It has been suggested to base tests of the structure of matter at high densities on analyses of the cooling behavior\textsuperscript{32,40,41} or the stability of fastly rotating stars against r-modes.\textsuperscript{42,43} It has turned out that for these phenomena the fine tuning of color superconductivity in quark matter is an essential ingredient. The result of these studies suggests that on the one hand unpaired quark matter results in too fast cooling (same holds for pure two-flavor color superconductivity (2SC) due to one unpaired (blue) color) and on the other, complete pairing with large gaps (CFL phase, $\Delta_{\text{CFL}} \sim 100$ MeV) would result in too slow cooling,\textsuperscript{44,45} r-mode instability\textsuperscript{42} and gravitational collapse.\textsuperscript{22,33} Viable alternatives for quark matter thus have to be color superconducting, with all quark modes paired, but a few modes (at least one) should have a small gap of $\Delta_X \sim 1$ MeV, desirable with a decreasing density dependence, as suggested for the $2\text{SC}+X$ phase.\textsuperscript{46,47} Unfortunately, there is no microscopic model supporting the $2\text{SC}+X$ pairing pattern yet. A suitable candidate, however, could be the recently suggested single flavor pairing in the isotropic spin-1 color superconductivity phase (iso-CSL),\textsuperscript{48,49} see also.\textsuperscript{50} It has been shown that the neutrino emissivity and bulk viscosity of the iso-CSL phase fulfills constraints from cooling and r-mode stability.\textsuperscript{51}
4. Conclusions

In this contribution it is shown that modern quantum field theoretical approaches to quark matter including color superconductivity and a vector meanfield allow a microscopic description of hybrid stars which fulfill the new, strong constraints for high masses, large radii, sufficiently fast cooling evolution and absence of r-mode instabilities. The deconfinement transition in the resulting stiff hybrid equation of state is weakly first order so that signals of it have to be expected due to specific changes in transport properties governing the rotational and cooling evolution caused by the color superconductivity of quark matter. This conclusion holds analogously for the investigation of quark deconfinement in future generations of nucleus-nucleus collision experiments at low temperatures and high baryon densities, such as CBM @ FAIR. The extrapolation of the hybrid EoS constrained here from compact star physics results in almost crossover behavior at the deconfinement transition, so that promising observable effects for its diagnostics shall be based on (precursor) effects of color superconductivity.

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