A chiral SU(4) explanation of the $b \to s$ anomalies

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Abstract: We propose a variant of the Pati-Salam model, with gauge group $SU(4)_C \times SU(2)_L \times U(1)_{Y'}$, in which the chiral left-handed quarks and leptons are unified into a $4$ of $SU(4)_C$, while the right-handed quarks and leptons have quite a distinct treatment. The $SU(4)_C$ leptoquark gauge bosons can explain the measured deviation of lepton flavour universality in the rare decays: $\bar{B} \to \bar{K}^{(*)}\ell\ell$, $\ell = \mu, e$ (taken as a hint of new physics). The model satisfies the relevant experimental constraints and makes predictions for the important $B$ and $\tau$ decays. These predictions will be tested at the LHCb and Belle II experiments when increased statistics become available.

Keywords: B physics, unified model

ArXiv ePrint: 1809.xxxxx
1 Introduction

There is mounting evidence for a violation of lepton flavour universality (LFU) in flavour-changing neutral current processes $b \to s \ell \ell$ in recent measurements of $B$ decays [1–7]. The theoretically cleanest probes are the LFU ratios

$$R_{K^{(*)}} = \frac{\Gamma(B \to K^{(*)}\mu^+\mu^-)}{\Gamma(B \to K^{(*)}e^+e^-)}$$

(1)

which compare the decay rate $b \to s \ell \ell$ ratio between muons and electrons respectively. Hadronic uncertainties cancel out in the ratios as long as new physics effects are small [8–10]. The current experimental data shown in Table 1 indicates deviations of more than 2$\sigma$ for both LFU ratios $R_{K^{(*)}}$ separately. An effective field theory analysis including all $b \to s \ell \ell$ data in fact shows that the introduction of operators

$$O_9 = [\bar{s}\gamma^\mu P_L b][\bar{\mu}\gamma_{\mu}\mu]$$

$$O_{10} = [\bar{s}\gamma^\mu P_L b][\bar{\mu}\gamma_{\mu}\gamma_5\mu]$$

(2)

may improve the global fit by 4 − 5$\sigma$ [10–15]. In addition to the $R_K$ anomaly, there is some evidence for a deviation from standard model (SM) predictions in the muon $g−2$ measurements (see e.g. Ref. [16]) and also in charged-current semi-leptonic decays $b \to c\ell\bar{\nu}$ (see e.g. Ref. [17]). The experimental sensitivity is expected to significantly improve in the next few years: LHCb will acquire more data and the Belle II experiment is anticipated to start collecting data with the full detector soon.

The possibility that some or even all of these deviations might be a harbinger of new physics has been entertained in the literature, e.g. by introducing a new effective interaction of third-generation weak eigenstates [18], models of $Z'$ gauge bosons e.g. [19, 20] and leptoquarks e.g. [21, 22]. In this paper, we consider the possibility that the $b \to s \ell \ell$ anomalies are due to leptoquark gauge bosons in an $SU(4)$ gauge model extension of the standard model. Although various kinds of $SU(4)$ models have been considered in the context of the B-physics anomalies in several papers [23–31], we argue that the $b \to s \ell \ell$ anomalies can be simply explained in a rather particular Pati-Salam inspired chiral $SU(4)$ gauge model, which is the focus of this paper.

The paper is organised as follows. In Sec. 2 we introduce the model and discuss the relevant effective operators in Sec. 3. Our results are presented in Sec. 4 and we conclude in Sec. 5.

2 The Model

The Pati-Salam model [34] is a left-right symmetric model based on the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_R$ where both chiral left- and right-handed leptons are interpreted as the fourth colour of $(4, 2, 1), (4, 1, 2)$ fermion multiplets (the other three colours representing the quarks). In

|          | observed          | SM               | $q^2$ range        |
|----------|-------------------|------------------|--------------------|
| $R_K$    | $0.745^{+0.099}_{-0.074} \pm 0.036$ [1] | $1.0003 \pm 0.0001$ [32] | $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ |
| $R_{K^*}$| $0.69^{+0.11}_{-0.07} \pm 0.05$ [2]  | $1.00 \pm 0.01$ [33]    | $1.1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ |

Table 1: LFU ratios $R_{K^{(*)}}$, where we first list the statistical error and then the systematic.
the original version of the model, quite stringent limits on the $SU(4)$ symmetry breaking scale arises from various processes, especially two-body leptonic decays of mesons: $K \rightarrow \bar{\mu}e$, $B \rightarrow \bar{\mu}e$ etc.. These two-body rare decays are effectively enhanced over three-body processes because the $SU(4)$ leptoquark gauge bosons couple in a vector-like manner to the charged leptons, eliminating any helicity suppression.

It was noticed some time ago [35, 36] that variants of the Pati-Salam model can easily be constructed whereby the $SU(4)$ leptoquark gauge bosons couple in a chiral fashion to the quarks and leptons. Such chiral $SU(4)_C$ models are less constrained than the original Pati-Salam model, and $SU(4)$ symmetry breaking at the TeV scale can be envisaged. The particular model studied in Refs. [35, 36] featured leptoquark gauge bosons coupling to chiral right-handed quarks and leptons, a circumstance which is not well suited to explaining the $R_K$ anomaly. Here we aim to construct the simplest chiral $SU(4)$ model in which the leptoquark gauge bosons couple to quarks and leptons in a predominately left-handed manner.

The gauge symmetry of the model is $SU(4)_C \times SU(2)_L \times U(1)_{Y'}$, and the fermion/scalar particle content is listed in Table 2. The $SU(4)$ symmetry is broken by the vacuum expectation value (VEV) of the scalar $\chi$ at a high scale $(\langle \chi \rangle \equiv w \gtrsim 10 \text{ TeV})$, while the electroweak symmetry is broken by the VEVs of the scalars $\phi$ and $\Delta$, with $\sqrt{v^2 + u^2} \simeq 174 \text{ GeV}$ where $\langle \phi \rangle \equiv v$ and $\langle \Delta \rangle \equiv u$. The symmetry breaking pattern that results is

\[
SU(4)_C \times SU(2)_L \times U(1)_{Y'} \downarrow \langle \chi \rangle \\
SU(3) \times SU(2)_L \times U(1)_Y \downarrow \langle \phi \rangle, \langle \Delta \rangle \\
SU(3) \times U(1)_Q
\]

Here hypercharge $Y = T + Y'$ and electric charge $Q = I_3 + \frac{Y}{2}$. If we use the gauge symmetry to rotate the VEV of $\chi$ to the fourth component, then $T$ is the diagonal traceless $SU(4)$ generator with elements $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1)$. The Yukawa Lagrangian is

\[
\mathcal{L} = Y_u \tilde{Q}_L \phi u_R + Y_d \tilde{Q}_L \phi d_R + Y_N \tilde{u}_R \chi N_L + Y_E \tilde{d}_R \chi E_L + Y_e \tilde{Q}_L \Delta e_R \\
+ m_{\tilde{E}} \tilde{E}_R e_R + \frac{1}{2} m_N \tilde{N}_L N_L + h.c.,
\]

| fermion | $(SU(4)_C, SU(2)_L, U(1)_{Y'})$ | scalar | $(SU(4)_C, SU(2)_L, U(1)_{Y'})$ |
|---------|--------------------------------|--------|--------------------------------|
| $Q_L$   | $(4, 2, 0)$                    | $\phi$ | $(1, 2, 1)$                    |
| $u_R$   | $(4, 1, 1)$                    | $\chi$ | $(4, 1, 1)$                    |
| $d_R$   | $(4, 1, -1)$                   | $\Delta$| $(4, 2, 2)$                   |
| $E_L$   | $(1, 1, -2)$                   |        |                                |
| $e_R$   | $(1, 1, -2)$                   |        |                                |
| $N_L$   | $(1, 1, 0)$                    |        |                                |

Table 2: Particle content
where \( \tilde{\phi} \equiv i \tau_2 \phi^* \), and we have used bold face notation to label \( SU(4)_C \times 4 \) multiplets which contain the usual quarks plus a leptonic component. The generation index has been suppressed, and it is implicit that each of these components comes in three generations, i.e. \( u_R \equiv u^i_R = (u_R, c_R, t_R), d_R \equiv d^i_R = (d_R, s_R, b_R), \) etc.. The \( \chi \) field gives mass to the charged \( (\frac{2}{3}e) \) \( W' \) and neutral \( Z' \) gauge bosons along with the exotic charged \( E^-_{L,R} \) and neutral \( N_{L,R} \) fermions. The SM fields acquire mass via the \( \phi \) and \( \Delta \) fields.

The quark mass matrices are given by \( m_u = Y_u v \) and \( m_d = Y_d v \), while the charged and neutral lepton mass matrices are

\[
M_{e,E} = \begin{pmatrix} Y_u & m_d \\ m_1 & Y_E \end{pmatrix}, \quad M_N = \begin{pmatrix} 0 & m_u & 0 \\ m_u^T & 0 & Y_N w \\ 0 & Y_N^T w & m_N \end{pmatrix}.
\]

In defining these matrices we have adopted a basis \((e,E,L,R)\) \((\nu_L,N^c_L,N_L)\) where \(e_L,\nu_L\) are the fourth components of \( Q_L \) and \( E_R,N_R \) are the fourth components of \( d_R,\nu_R \). In the limit \( w \gg m_1,m_d \) (assumed in this paper) the charged lepton masses reduce to \( m_\ell \simeq Y_e u \) while the exotic charged leptons have mass \( M_E \simeq Y_E^L w \). Also, the \( W' \) leptoquark \( SU(4) \) gauge bosons couple chirally to the SM quarks and leptons. It is beneficial to explicitly write out the fermion multiplets. For the first generation we have

\[
Q_L = \begin{pmatrix} u_r & d_r \\ u_g & d_g \\ u_b & d_b \\ \nu & e \end{pmatrix}, \quad d_R = \begin{pmatrix} d_r \\ d_g \\ d_b \\ E \end{pmatrix}, \quad u_R = \begin{pmatrix} u_r \\ u_g \\ u_b \end{pmatrix}.
\]

Note that the active neutrino masses are generated via an inverse seesaw, and their observed sub-eV mass scale is compatible with a TeV scale VEV \( w \).

In this model the masses of the charged leptons arise from the VEV of the \( \Delta \) scalar, while the masses of the quarks result from the VEV of \( \phi \). In such a situation, consistent Higgs phenomenology requires the existence of a decoupling limit where the LHC Higgs-like scalar is identified with the lightest neutral scalar in the model. To see how this can arise consider the Higgs potential terms

\[
V(\chi, \phi, \Delta) = \lambda_1 (\chi^\dagger \chi - w^2)^2 + \lambda_2 (\phi^\dagger \phi - v^2)^2 + m_\Delta^2 \Delta^\dagger \Delta - m_{123} \Delta^\dagger \phi \chi - m_{123}^* \chi^\dagger \phi^\dagger \Delta.
\]

Here \( m_{123} \) is a trilinear coupling of dimensions of mass which, without loss of generality, we can take to be real. For \( \lambda_1, \lambda_2, m_\Delta > 0 \), and considering initially \( m_{123} = 0 \), the potential is minimised when \( \langle \chi^\dagger \chi \rangle = w^2, \langle \phi^\dagger \phi \rangle = v^2 \), and \( \langle \Delta \rangle = 0 \). Taking advantage of the gauge symmetry, the VEVs can be rotated into the real part of one of the complex components of \( \chi \) and \( \phi \): \( \langle Re : \chi_0 \rangle = w, \langle Re : \phi_0 \rangle = v \). In the non-trivial case where \( m_{123} \neq 0 \), a VEV is induced for the real part of \( \Delta_0 \)

\[
\langle Re : \Delta_0 \rangle \equiv u \simeq \frac{m_{123} w v}{m_\Delta^2}.
\]

In such a manner, \( u \ll v \) can naturally arise if \( m_{123} w / m_\Delta^2 \ll 1 \).

The physical scalar content consists of electrically charged \( 5/3 \) and \( 2/3 \) coloured leptoquark scalars, a singly charged scalar, \( \Delta^+ \), three neutral scalars, \( \tilde{\chi}_0 / \sqrt{2} = Re : \chi_0, \tilde{\phi}_0 / \sqrt{2} = Re : \phi_0, \)
\[ \Delta_0 / \sqrt{2} = Re : \Delta_0, \] and a pseudo scalar, \[ \Delta_0' / \sqrt{2} = Im : \Delta_0. \] In the limit \( w^2 \gg v^2 \), the \( \chi_0 \) scalar decouples and the two remaining neutral scalars mix so that their physical mass eigenstates take the form

\[
\begin{align*}
    h &= \cos \beta \tilde{\phi}_0 + \sin \beta \tilde{\Delta}_0 \\
    H &= -\sin \beta \tilde{\phi}_0 + \cos \beta \tilde{\Delta}_0
\end{align*}
\]

where \( \sin \beta \simeq m_{123} w / (m_{\Delta}^2) = u / v \) in the decoupling limit \( m_{\Delta}^2 \gg m_{123} w \). In this limit it is easy to check that the lightest scalar, \( h \), has Higgs-like coupling to the SM particles. This result would hold for the most general Higgs potential so long as a decoupling regime as described is considered [37]. The scalar \( h \) can thus be identified with the Higgs-like scalar discovered at the LHC [38, 39].

Finally, the model features an unbroken global \( U(1)_B \) baryon number symmetry. As with the standard model, this global symmetry is not imposed but appears as an accidental symmetry of the Lagrangian. However, unlike the standard model, the unbroken baryon global symmetry does not commute with the gauge symmetries, and is generated by

\[
B = \frac{B' + T}{4}.
\]

Here, we have introduced the generator, \( B' \), which commutes with the gauge symmetries, and is defined by the charges: \( B'(Q_L, u_R, d_R, \chi, \Delta) = 1, B'(E_L, e_R, N_L, \phi, G) = 0 \) (\( G \) is the set of gauge fields). With \( B \) defined as above, one can easily check that \( U(1)_B \) is an unbroken symmetry of the Lagrangian (i.e. \( B \langle \chi \rangle = B \langle \Delta \rangle = B \langle \phi \rangle = 0 \)). The \( U(1)_{B'} \) is also a symmetry of the Lagrangian, but is not independent of the gauge symmetries and \( U(1)_B \).

### 3 Effective operators

The relevant new physics contributions to the anomalies and possible constraints are most efficiently described by the effective Lagrangian

\[
L_{\text{eff}} = \frac{4G_F \alpha_{em}}{\sqrt{2} \pi \alpha_{em}} \sum_{q, q', \ell, \ell'} V_{iq} V_{i\ell}^* \sum_{i=9,10} \left( C_i^{qq' \ell \ell'} O_i^{qq' \ell \ell'} + C_i^{qq' \ell \ell'} O_i^{qq' \ell \ell'} \right) + \text{h.c.},
\]

where \( O_i \) denotes operators with two down-type quarks and two charged leptons

\[
\begin{align*}
O_{9}^{qq' \ell \ell'} &= (\bar{q}_\gamma \mu P_L q') (\bar{\ell}_\gamma \mu \ell') \\
O_{10}^{qq' \ell \ell'} &= (\bar{q}_\gamma \mu P_L q') (\bar{\ell}_\gamma \mu \gamma_5 \ell').
\end{align*}
\]

In the above, \( G_F \) denotes the Fermi constant, \( \alpha_{em} = 1/127.9 \) the fine-structure constant evaluated at the electroweak scale, \( V_{ij} \) are CKM mixing matrix elements, \( q^{(i)} \) are down-type quark fields, \( \ell^{(i)} \) denotes charged leptons and \( P_{L,R} = (1 \pm \gamma_5)/2 \) are the chiral projection operators.

The relevant \( SU(4) \) gauge interactions with the fermions, together with the leptoquark gauge boson mass term, are given by

\[
L = \frac{g_8}{\sqrt{2}} K_{ij} W^\nu_j \bar{d}_i \gamma^\mu P_L \ell_j + \frac{g_8}{\sqrt{2}} K_{ji} W^\nu_i \bar{\ell}_j \gamma^\mu P_L \ell_j - m_{W'}^2 W'^\mu W'^\mu
\]

where \( g_8 \) is the \( SU(4) \) gauge coupling constant. Here we have defined \( \ell \) to include the three charged SM leptons and the three heavy exotic charged lepton mass eigenstates, i.e. \( \ell = e, E \). This means
that \( K_{ij} \) is in general a \( 3 \times 6 \) matrix which satisfies the unitarity condition \( KK^\dagger = 1_{3\times3} \), where \( 1_{3\times3} \) is the \( 3 \times 3 \) unit matrix.

In this model the Wilson coefficients for the effective four-fermion interaction after integrating out the heavy \( W' \) mediator and using the appropriate Fierz rearrangement to collect quark and lepton bilinears are

\[
C_9^{q\ell q\ell'} = -C_{10}^{q\ell q\ell'} = \frac{\sqrt{2} \pi^2 \alpha_s}{V_{tq} V_{tq}^* \alpha_{em}} \frac{K_{q\ell} K_{q\ell'}^*}{G_F m_{W'}^2},
\]

where \( \alpha_s = g_s^2(m_{W'})/4\pi \). Typically, limits from lepton flavour violating Kaon decays are more stringent than those from \( B \) meson decays, and this constrains the possible flavour structure of the theory. In order to satisfy these constraints, and to explain the \( R_{K^{(*)}} \) anomaly, a particular structure of the \( K \) matrix is suggested. Considering only the first 3 columns of the general \( K \) matrix, i.e. the part relevant to quark-SM lepton interactions, we adopt the limiting case:

\[
K = \begin{pmatrix}
0 & 0 & 1 \\
cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0
\end{pmatrix}.
\]

In general, the zero elements need not be exactly zero, but for the \( m_{W'} \), \( \theta \) values of interest for the \( R_{K^{(*)}} \) measurements are constrained from lepton flavour violating Kaon decays to be relatively small \((\lesssim 0.1)\).

### 4 Results & discussion

With the ansatz Eq. (15) it is straightforward to evaluate the \( W' \) leptoquark gauge boson contributions to the \( R_{K^{(*)}} \) anomaly. The model has the distinctive feature that both \( b \to s\ell\ell \) and \( b \to s\ell\ell \) processes receive corrections of approximately the same magnitude, but with opposite sign. One consequence of this is that modifications to the angular distributions are anticipated in both muon and electron channels. However, it is noteworthy that the muon channel is experimentally advantageous over the electron channel due to improved resolution.

The favoured region of parameter space for the model is identified using the \texttt{flavio} package [40] and tree-level analytical estimations where appropriate. The \( B \to K^{(*)}\mu^+\mu^- \), \( B \to K^{(*)}\mu^+\mu^- \) rates are used to determine the \( R_K \) and \( R_{K^*} \) ratios for a given \( m_{W'} \), leptoquark mass and \( \theta \) mixing angle, with the \( C_9 \) and \( C_{10} \) coefficients detailed in Eq. (14). Additionally, we calculate \( BR(B^+ \to K^+\mu^-e^+) \) and \( BR(B^+ \to K^+e^-\mu^+) \) values. The \( 1\sigma \) and [90\% C.L.] favoured parameter region is defined by the \( m_{W'}, \theta \) values which satisfy \( R_K = 0.745 \pm 0.097 \) \([R_{K'} = 0.745 \pm 0.159] \), \( R_{K^*} = 0.69 \pm 0.12 \) \([R_{K^{*}} = 0.69 \pm 0.20] \) and also satisfy the current 90\% C.L. experimental limits \( BR(B^+ \to K^+\mu^-e^+) < 1.3 \times 10^{-7} \) and \( BR(B^+ \to K^+e^-\mu^+) < 9.1 \times 10^{-8} \) [41]. It turns out that the favoured region, defined in the way we have done, is not currently constrained by any other process.

A plot of the allowed model parameters is shown in Figure 1. From that figure it is clear that the favoured range of \( \theta \) is approximately between \([-\frac{\pi}{2}, 0] \) or \([\frac{\pi}{2}, \pi] \) and \( m_{W'}/\text{TeV} \) between [12, 31]. The identical nature of the two adjacent regions can be understood as follows. Under the transformation \( \theta \to \theta + \pi \), \( \sin \theta \to -\sin \theta \), \( \cos \theta \to -\cos \theta \), and the leading order amplitudes for
Figure 1: The favoured parameter regions compatible with the current experimental limits from $B^+ \rightarrow K^+ \mu^- e^+$, $B^+ \rightarrow K^+ e^- \mu^+$. Shown are the 1σ (blue) and 90% confidence level (red) bands suggested by the measured $R_K$ and $R_{K^*}$ ratios.

Figure 2: Expectation for (a) $BR( B^+ \rightarrow K^+ \mu^- e^+)$ (b) $BR( B^+ \rightarrow K^+ e^- \mu^+)$ for the favoured parameter region identified in Figure 1. The black dashed lines correspond to the current experimental 90% C.L. upper bounds on these branching fractions.

$b \rightarrow s \ell \ell$ (which are proportional to $\sin \theta \cos \theta$) are invariant. Also the amplitudes for the decay processes, $B^+ \rightarrow K^+ \mu^- e^+$, $B^+ \rightarrow K^+ e^- \mu^+$, are proportional to $\sin^2 \theta$ and $\cos^2 \theta$ respectively, and are also invariant under $\theta \rightarrow \theta + \pi$. It should be noted that the $R_{K(s)}$ anomalies on their own can potentially have $m_{W'} < 12$ TeV, but the low mass cut-off is acquired due to the $B^+ \rightarrow K^+ e^\mp \mu^\pm$ decay constraints.

For each point in the favoured region shown in Figure 1 we can calculate the expected rates for the rare $B^+ \rightarrow K^+ \mu^- e^+$ and $B^+ \rightarrow K^+ e^- \mu^+$ processes. The result of this exercise is shown in Figure 2. Note that $B^+ \rightarrow K^+ \mu^- e^+$ probes $\sin^2 \theta \approx 1$, while $B^+ \rightarrow K^+ \mu^+ e^- \mu^+$ probes $\cos^2 \theta \approx 1$, and thus these two decay channels are complimentary.

In addition to further improvements to $B^+ \rightarrow K^+ \mu^\pm e^\mp$ there are a number of other ways to
test this model. In the remainder of this paper we focus on making predictions for various rare decays that directly involve the new physics invoked in explaining the \( R_{K^{(*)}} \) anomalies. We first consider the rare tau lepton decays: \( \tau \rightarrow K_s \ell, \ell = e, \mu \). The decay rate for the \( \tau \rightarrow K_s \ell \) process is calculated to be

\[
\Gamma(\tau \rightarrow K_s \ell) = \frac{f_K^2 \alpha_s^2 \pi (m_\tau^2 - m_{K_s}^2)^2 |K_{s\ell}|^2 |K_{d\ell}|^2 + |K_{s\tau}|^2 |K_{d\ell}|^2}{64 m_{W'}^2 m_\tau}.
\] (16)

Here, \( m_{K_s} \approx 497.7 \) MeV and \( f_K \approx 156.1 \) MeV are the \( K_s \) meson mass and decay constant respectively, and we have set the final state lepton mass to zero in the above calculation. With the ansatz, Eq. (15), we have \( K_{s\ell} = \cos \theta, K_{s\mu} = \sin \theta, K_{d\tau} = 1, K_{d\ell} = 0 \). Using the experimentally observed decay width, \( \Gamma(\tau \rightarrow \text{all}) \approx 2.27 \times 10^{-12} \) GeV, the branching fraction,

\[
\text{BR}(\tau \rightarrow K_s \ell) = \frac{\Gamma(\tau \rightarrow K_s \ell)}{\Gamma(\tau \rightarrow \text{all})}
\]

can then be obtained. Our results are shown in Figure 3.

The effective Lagrangian that induces modifications to the \( R_K \) ratio also modifies the two-body \( B_s \) decays: \( B_s \rightarrow \mu^- \mu^+ \) and \( B_s \rightarrow e^- e^+ \). These decays also arise in the standard model, and so it is useful to compute the ratio

\[
R(B_s \rightarrow \ell^- \ell^+) \equiv \frac{\Gamma(B_s \rightarrow \ell^- \ell^+) / \Gamma_{SM}(B_s \rightarrow \ell^- \ell^+)}{(1 + R_K)/2,} (17)
\]

where the numerator, \( \Gamma(B_s \rightarrow \ell^- \ell^+) \), includes the new physics \( (W') \) contributions as well as the standard model contribution. In this model we expect \( R(B_s \rightarrow \mu^- \mu^+) \approx (1 + R_K)/2, \) and \( R(B_s \rightarrow e^- e^+) \approx (3 - R_K)/2. \) In Figure 4 we have calculated the predictions for \( R(B_s \rightarrow \ell^- \ell^+) \). A comparison of the experimental values [41] with the SM predictions [42] shows that the \( R(B_s \rightarrow \mu^- \mu^+) \) ratio inferred from measurement is \( R(B_s \rightarrow \mu^- \mu^+) = 0.7 \pm 0.3 \). This value is consistent with what we would expect given the central values of \( R_K \) and \( R_{K^*} \), but of course the current error is too large to rigorously test this model. In Figure 4 we have also shown the predicted branching ratios \( BR(B_s \rightarrow \mu^- e^+) \) and \( BR(B_s \rightarrow e^- \mu^+) \), together with the 90\% C.L. upper bound \( BR(B_s \rightarrow e^+ \mu^-) < 1.1 \times 10^{-8} \).

We have briefly looked at the \( \mu \rightarrow e \gamma \) radiative decay. This decay arises at one-loop level, with virtual down-type quarks and \( W' \) gauge boson propagators in the loop. Making use of the general
Figure 4: Expectation for (a) $R(B_s \to \mu^- \mu^+)$ (b) $R(B_s \to e^- e^+)$ (c) $BR(B_s \to \mu^- e^+)$ (d) $BR(B_s \to e^- \mu^+)$ for the favoured region of parameter space identified in Figure 1.

calculation given in Ref. [43], we show that the first two terms in the $m_b^2/m_W^2$ expansion vanish: the first one due to unitarity and the second one

$$\Gamma(\mu \to e\gamma) \simeq \frac{9 \alpha_em_\mu^4m_\mu^5 (2Q_b + Q_{W'})^2 \sin^2\theta \cos^2\theta}{256m_W^8}$$

is proportional to $(2Q_b + Q_{W'})^2$ and thus vanishes as the charge assignments in this model satisfy $Q_b = -1/3$ and $Q_{W'} = 2/3$. Hence we do not expect the $\mu \to e\gamma$ process to be important in this model.

5 Conclusion

We have proposed a Pati-Salam variant $SU(4)$ theory, with gauge group $SU(4)_C \times SU(2)_L \times U(1)_{Y'}$, which is capable of explaining the $R_K$ and $R_{K^*}$ anomalies via new gauge interactions. The model is consistent with experimental constraints, including the stringent limits on $B^+ \to K^+ \mu^- e^+$ and $B^+ \to K^+ e^- \mu^+$ decays. In this model, the chiral left-handed fermions are arranged in a similar fashion to the original Pati-Salam model, i.e. with leptons making up the fourth colour, while the chiral right-handed fermions are treated quite differently. The model features $SU(4)$ symmetry breaking via the introduction of a $SU(4)$ scalar multiplet $\chi$ with a VEV $w \gtrsim 10$ TeV and
electroweak symmetry breaking via scalars $\phi$ and $\Delta$ with VEVs that satisfy $\sqrt{v^2 + u^2} \simeq 174$ GeV. In addition to new scalar particles, the model contains new charged $\left(\tfrac{2}{3}e\right)$ $W'$ and neutral $Z'$ gauge bosons along with heavy exotic charged $E_{L,R}$ and neutral $N_{L,R}$ fermions. The charged leptoquark gauge bosons $W'$ couple in a chiral manner to the familiar quarks and leptons and can thereby interfere with SM weak processes. The theory makes predictions for $B^+ \to K^+ \mu^- e^+$, $B^+ \to K^+ e^- \mu^+$, $\tau \to K_\mu \ell$, $B_s \to \mu^- e^+$, as well as the highly suppressed $B_s \to \mu^- e^+$ and $B_s \to e^- \mu^+$ processes. These predictions can be tested at the LHCb and Belle II experiments when increased statistics become available.

Acknowledgements

This work has been supported in part by the Australian Research Council.

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