Research on the parameter analysis of combined focusing of right Angle cone and projectile

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Abstract—In a sound wave measurement engineer, a combination focusing device of right angle cone and parabola was designed to increase the emission sound pressure, and by exploring and simulating the relationship between the position and size of the spherical focus area and the height of the right-angled cone, the position of the apex of the right-angled cone, the focal length of the parabolic and other parameters. The analytical relationship between each parameter and the sound pressure level of the spherical area focused at a certain position from the vertex of the right-angled cone is obtained.

1. introduction
Piston sound source is a planar vibrator that radiates sound waves by driving the BEM to reciprocate through a driving mechanism[1]. Acoustic focusing equipment can concentrate the sound energy in the focal area to obtain strong sound radiation. This is of great significance for studying the effective effects of physical, chemical and biological processes in strong ultrasonic fields. Commonly used concentrated focusing methods include: concave radiation focusing, acoustic waveguide focusing, plano-concave lens focusing, biconical focusing, and the combination of right-angled cone and parabolic focusing, etc.

In a sound wave measurement project, the reflection method is planned to be used, It is estimated that the sound pressure level is much lower than the measurable sound threshold of current acoustic sensors after multiple interface transmission and reflection. Especially the first media interface, when a sound wave is incident from a medium with a small acoustic resistance to a medium with a large acoustic resistance, the incident wave will cause great loss, the incident sound wave with higher sound pressure level is required to ensure the final sound pressure level after transmission and reflection for measurement. In the case of the sound pressure of the existing sound source, it is proposed to use a right-angled cone and a parabolic combination focusing device to focus the sound wave on the interface of the medium to enhance the incident sound pressure.

2. Combination focusing principle of right-angled cone and paraboloid
The sound source adopts a piston sound source, and the radiating plate is driven to reciprocate through a magnetostrictive transducer to generate radiated sound waves[2]. The sound wave focusing device
includes a right-angled cone and a rotating paraboloid. The sound waves are reflected to the inner surface of the paraboloid through the right-angled cone, and the paraboloid further reflects the sound waves and focuses on the spherical focusing area with point F as the center and r as the radius on the interface of the medium. In this paper, by exploring and simulating the relationship between the position and size of the spherical focus area, the height of the right-angled cone, the position of the apex of the right-angled cone, the focal length of the parabolic and other parameters, the parameters and the sound pressure level of the spherical area focused at a certain position from the apex of the right-angled cone.

Figure 1 Schematic diagram of the focusing principle of right-angled cone and parabolic combination (1 parabolic profile curve, 2 right angle cone \(4f\beta = x^2\), 3 vibrating diaphragm)

3. Parameter optimization

3.1. Related parameters

In order to facilitate the analysis, the coordinate system is established in Fig. 1: XOY, ZO’V. In the coordinate system XOY, the parabola vertex is located at the coordinate origin O, and the parabola focus is located at the point F (0, f). In the coordinate system ZO’V, the coordinate origin O'is located at the center of the diaphragm, the Z direction is the direction of sound wave propagation, the vertex of the right-angled cone is located at point A (Z, 0). Z is the first optimized parameter, and the focus point is located at F Click (H+Z, 0). To ensure that the incident sound waves are completely reflected by the right-angled cone, the lower opening radius \(r_0\) of the rotating paraboloid should be greater than or equal to the height R of the right-angled cone. Let H be the distance between the apex of the right-angled cone and the focal point F.

The sound intensity \(I^{[4]}\) in the spherical area with F as the center and r as the radius is shown in the following formula (1):

\[
I = \frac{(H + h) W}{2\pi R r^2 \sin 2\alpha}
\]  

(1)

Where W is the sound power radiated from the left to the cone, and \(\alpha\) is the angle of the reflected sound wave at the center of the rotating parabola. After derivation, the lower opening radius \(r_0\) and the upper opening radius \(r_1\) of the rotating paraboloid are shown in (2) and (3) respectively:

\[
r_0 = f - \frac{h^2}{4f}
\]  

(2)

\[
r_1 = f + \frac{H^2}{4f}
\]  

(3)

The coordinates of the intersection point between the midpoint of the right-angled cone and the parabola is:
The sound pressure of the piston sound source at a certain point on the central axis is shown in (5) [3]:

\[ P = 2\rho_0 c_0 u_a \sin \frac{k}{2} (R - z)e^{\frac{jw - k}{2}(R+z)} \]  

(5)

In this acoustic wave measurement project, the radius of the vibrating diaphragm of the piston sound source is set to \( a \), and the wavelength of the sound wave is \( \lambda \), assuming \( Z>2a \). At this time, the argument of the sine function can be expressed in series as shown in (6):

\[ \sin \frac{k}{2} (R - z) \approx \sin \frac{ka^2}{4z} = \sin \frac{\pi z_g}{2z} \]  

(6)

Among them:

\[ z_g = \frac{a^2}{\lambda} \]

Further simplify the sound pressure function to obtain (7):

\[ P = 2\rho_0 c_0 u_a \sin \left(\frac{\pi z_g}{2z}\right)e^{\frac{jw - k}{2}(R+z)} \]  

(7)

Sound pressure amplitude:

\[ P_a = 2\rho_0 c_0 u_a \sin \left(\frac{\pi z_g}{2z}\right) \]

When the medium is air, the average sound energy density is shown in (8) [3]:

\[ \text{Average sound energy density} = \frac{1}{2} \rho_0 c_0 u_a^2 \]  

(8)
\[ \overline{p} = \frac{p^2_a}{2 \rho_l c_0^2} = \frac{p^2_e}{\rho_l c_0^2} \]  

(8)

When the medium is air, the average sound power is shown in equation(9) \cite{3}:

\[ \overline{W} = \overline{\rho} c_0 S \]  

(9)

As shown in Figure.2, take a section of truncated cone from the right-angled cone. When the height of the truncated cone is small enough, it can be approximated that the sound wave radiation surface is a rectangle with a width of \( \sqrt{2} \rho c_0 \) and a length of \( h_2 \). The average sound power can be approximately regarded as the average sound energy contained in the aluminum cuboid with the bottom area \( \sqrt{2} \rho c_0 dh_2 \) and height \( c_0 \). According to the formula (8), the formula (9) can be rewritten as (10) formula:

\[
\begin{align*}
\overline{W} &= 2 \sqrt{2} \pi \rho c_0 dh_2 \sqrt{2} \pi \frac{p^2_a}{\rho_l c_L} dh_2 \\
&= 4 \sqrt{2} \pi \rho_l c_L^2 u_a^2 \sin^2 \left( \frac{\pi z}{2z} \right) dh_2 \\
&= \frac{\rho_l c_L}{\rho_l c_L} \]  

(10)

Among them, \( \rho_l, c_L \) is the density of aluminum-iron alloy and the propagation speed of sound waves in it.

![Figure 2](image)

Figure 2  Schematic diagram of the relationship between the upper cone height and the surface area of a right-angled cone

Substituting equation (7) into equation (10) to obtain the sound power of the piston sound source radiated to the aluminum cone is shown in (11):

\[ W = \int_0^r 4 \sqrt{2} \rho_l c_L^2 u_a^2 \sin^2 \left( \frac{\pi z}{2z} \right) dh_2 \]  

(11)

According to formula (1), formula (12) is obtained, the sound intensity of the right-angled cone and parabolic combined focusing device in the spherical area with radius \( r \) on the interface of the medium \cite{4}:

\[ I = \frac{(H + h) W}{2 \pi R r^2 \sin 2\alpha} = (H + h) \int_0^R 4 \sqrt{2} \rho_l c_L u_a^2 \sin^2 \left( \frac{\pi z}{2z} \right) dh_2 \]  

(12)
\[
\int_{z}^{z_{R}} \frac{4\sqrt{2} \rho \varepsilon_{L} u_{s}^{2} \sin \left(\frac{z_{R}}{2x}\right)}{R r^{2} \sin 2\alpha} (x-z) \, dx
\]

(12)

The sound intensity is the average sound power per unit area, and the sound intensity at the interface of the medium is obtained according to formula (11) as shown in formula (13):

\[
I = \frac{W}{S} = \bar{c}_{0} = \frac{P_{ef}^{2}}{\rho_{0} c_{0}}
\]

(13)

Substituting (13) into (12) to obtain (14) the effective sound pressure of the spherical focus area with F as the center and radius r:

\[
P_{ef}^{2} = (2H-R) \int_{z}^{z_{R}} \frac{4\sqrt{2} \rho \varepsilon_{L} \rho_{0} c_{0} u_{s}^{2} \sin \left(\frac{z_{R}}{2x}\right)}{R r^{2} \sin 2\alpha} (x-z) \, dx
\]

(14)

The sound pressure level (15) [3] is:

\[
SPL = 20 \log \frac{P_{ef}}{P_{ref}} (dB) *
\]

(15)

Where \( P_{ref} \) is the reference sound pressure, \( P_{ref} = 2 \times 10^{-5} \text{Pa} \).

3.4. Optimization model

Finally, the analytical form and constraint conditions of the objective function of the optimization problem are shown in formula (16):

\[
\begin{cases}
\max P_{ef}^{2} = (2H-R) \int_{z}^{z_{R}} \frac{4\sqrt{2} \rho \varepsilon_{L} \rho_{0} c_{0} u_{s}^{2} \sin \left(\frac{z_{R}}{2x}\right)}{R r^{2} \sin 2\alpha} (x-z) \, dx \\
\text{s.t.} \\
\quad z > 2a \\
\quad z \in [0.5z_{F}, z_{R}] \\
\quad 0 < R \leq 2a
\end{cases}
\]

(16)

4. The simulation

Assuming that the distance from the vertex \( z \) of the right-angled cone to the focal point \( F \) is \( H=10 \text{m} \). The radius of the diaphragm \( a=40 \text{mm} \), and the wavelength of the sound wave \( \lambda =13.6 \text{mm} \). \( z_{a} = a / \lambda^{2} = 117.643 \text{mm} \). According to formulas (2), (3), (4), (6), the main parameters of the focusing device are calculated as shown in Table 1.

| Serial number | Right angle cone height \( R \) (mm) | \( H \) (m) | \( f \) (m) | \( r_{0} \) (mm) | \( r_{1} \) (mm) | \( 1 / \sin 2 \alpha \) | Remarks |
|---------------|----------------------------------|----------|-------|-------|-------|-----------------|--------|
| 1             | 32                               | 10       | 5,000 | 32    | 40    | 624.45          | -      |

Assuming that the right-angled cone is made of aluminum-iron alloy. The density is \( \rho_{a} = 2700 \text{kg/m}^{3} \), the sound wave propagation speed is \( c_{L} = 5040 \text{m/s} \), and the experiment requires that the sound pressure amplitude of the vibrating diaphragm is 6.3 MPa and the wave velocity is \( u_{s} = P_{s} / \rho_{L} c_{L} = 0.46296 \text{m/s} \).
When the temperature is $20^\circ C$, the air density is $\rho_0 = 1.21 \text{ kg/m}^3$ and the sound propagation speed is $c_0 = 344 \text{ m/s}$.

The relationship between the surface sound pressure level of the spherical focusing area ($r=10\text{mm}$) near the focal point and the height of the right-angled cone, the position of the apex of the right-angled cone and the surface sound pressure level of the focusing area is shown in Figure 3.

Figure 3 The relationship between the surface sound pressure level of the spherical ($r=10\text{mm}$) near the focal point and the height of the right-angled cone and the position of the apex of the right-angled cone.

5. conclusion

It can be seen from Figure 3 that the greater the height of the right-angled cone, the greater the sound pressure level in the focus area when the right-angled cone apex position is optimal. The heighter of the right-angled cone should be made as high as possible when the project permits.

When $Z$ is constant, $R$ is small, and the sound pressure level changes drastically with $R$. The larger of $R$ is, the more smooth the sound pressure level changes with $R$. When the position of the apex of the right-angled cone is less than $0.5Z_g$, the sound pressure level in the focus area changes drastically, and the deviation of the apex of the right-angled cone will seriously affect the sound pressure level. In order to reduce the sound pressure level loss caused by the processing error and the deviation of the optimal apex position of the right-angled cone, the height of the right-angled cone should be greater than $a/2$, and the apex position of the right-angled cone should be greater than $0.5Z_g$.

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