Attitude Tracking Control for Rigid Spacecraft With Parameter Uncertainties

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ABSTRACT In this paper, the attitude tracking control problem for rigid spacecraft is investigated in the presence of parameter uncertainties and external disturbances. An extended state observer is designed to estimate the parameter uncertainties and external disturbances. Then, by using the obtained estimation, a robust finite-time controller is proposed to achieve attitude tracking for the rigid spacecraft via the backstepping technique. The practical finite-time stability of the closed-loop system under the presented controller is proven by the Lyapunov stability theory. Finally, some simulation results are provided to illustrate the effectiveness and superiority of the proposed controller compared with some existing control schemes.

INDEX TERMS Rigid spacecraft, attitude tracking, extended state observer, practical finite-time stability, parameter uncertainties.

I. INTRODUCTION

The attitude tracking control for rigid spacecraft has already attracted a great deal of interests during the last decades. Various nonlinear control methods for attitude tracking have been developed for rigid spacecraft with priori-known model parameters, such as nonlinear feedback control [1], [2], optimal control [3], [4], robust control [5], [6], or their integration [7]–[9]. During the on-orbit operation of spacecraft, the inertia matrix may be uncertain since the mass properties of spacecraft could be changed by the movement of onboard payloads, the rotation of solar arrays and the consumption of fuel [10]. The change of the inertial matrix may be a potential factor that destabilizes the spacecraft system. Hence, the parameter uncertainties of the spacecraft should not be ignored when an attitude maneuver control system is designed.

Attitude tracking control of spacecraft systems with the parameter uncertainties and the external disturbances has been extensively studied. An adaptive finite-time controller was employed in [11] for the attitude tracking of a rigid spacecraft with external disturbances. A sliding mode finite-time technique combined with the model predictive control was adopted in [12] to achieve attitude tracking for a rigid spacecraft. The parameter uncertainties and disturbances were also taken into consideration in [12]. The aforementioned control scheme was designed under the assumption that the uncertain parameters of the inertial matrix are constants. In practical applications, the parameters in inertial matrix may be time-varying due to the complex motion inside or outside the spacecraft. From this perspective, an adaptive attitude tracking controller was given in [13] for spacecraft with uncertain time-varying inertia parameters. In [14], a fast terminal sliding mode was designed to solve the finite-time attitude tracking control problem for rigid spacecraft without the requirement of information about inertia uncertainties and external disturbances.

Although the methods mentioned above show certain robustness to parameter uncertainties and the external disturbances, in essence these methods can be regarded as a passive disturbance rejection technique. The performance of the spacecraft control systems based on such a technique will descend if the external disturbance changes. To overcome this deficiency, the active disturbance rejection technique based on the extended state observer (ESO) proposed by Han [15] has received wide attention in the field of spacecraft
attitude control. An ESO was applied in [16] to estimate the parameter uncertainties and the external disturbances without the requirement that the inertial matrix is a constant matrix. In [17], an ESO based adaptive sliding mode controller was designed for attitude stabilization of a rigid spacecraft with uncertain inertia parameters. Nevertheless, in [16] and [17], the external disturbances were supposed to satisfy a specific structure, which reduces the practicality of the controller to some extent. For the case where the disturbance has no particular structure but is bounded only, an adaptive quaternion feedback control associated with a linear extended state observer (LESO) was developed in [18] for attitude tracking control. An output feedback attitude tracking control law based on ESO and adaptive dynamic programming was investigated in [19] for rigid spacecraft with uncertainties and disturbances.

The control schemes in [16]–[19] can guarantee the attitude of closed-loop systems to be uniformly ultimately bounded rather than to be convergent in finite time. To ensure the spacecraft system with fast convergence rate and strong robustness to disturbances, the problem of finite-time attitude stabilization was investigated in [20] for rigid spacecraft. To solve the problem of attitude stabilization for rigid spacecraft with disturbances, a finite-time observer based output feedback attitude controller was designed in [21]. In [22], an ESO was proposed to estimate the specified uncertainties including actuator’s misalignments and parameter uncertainties. Furthermore, the uniformly ultimately boundedness in finite time of the ESO in [22] was ensured by Lyapunov analysis. In [23], a robust finite-time tracking controller based on ESO was presented for rigid spacecraft.

Motivated by existing work, a robust controller based on backstepping technique and finite-time control method is designed in this paper for attitude tracking of a rigid spacecraft in the presence of parameter uncertainties and external disturbances. The main contributions of this paper are as follows.

1) An ESO is designed to estimate the parameter uncertainties and the external disturbances in the spacecraft control system. Different from [16] and [23], the proof of practical finite-time stability of the presented ESO is given. Compared with [21] and [22], the observer in this paper has simpler structure for the sake of simplicity in implementation. It shows that the observer errors can be rendered to a region of zero in finite time.

2) Different from the results in [12], [23] and [19], the backstepping technique and finite-time control method are utilized to design the controller in this paper. The stability of the kinematics subsystem is guaranteed when the dynamics subsystem is stabilized.

3) A robust finite-time controller is presented to deal with the attitude tracking problem for a rigid spacecraft. Different from [17] and [18], the practical finite-time stability of the closed-loop system is ensured by the proposed controller with nonzero observer errors. The external disturbances can be attenuated under the proposed controller. The stability analysis is given in this paper, which illustrates that the tracking errors can converge to a neighbour of zero in finite time.

Notations: Throughout this paper, the notation $\otimes$ represents the Kronecker product of two matrices. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space. $I_n$ represents the $n$-dimensional identity matrix. For a real matrix $A$, we use $A^{-1}$ to denote its inverse. For a vector $a \in \mathbb{R}^n$, we use $a^T$ and $||a||$ to represent the transpose and the Euclidean norm of $a$, respectively. If $n = 3$, $a^\times$ is a skew-symmetric matrix in the following form

$$a^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}.$$  

$|b|$ denotes the absolute value of a scalar $b$. $c = O(\psi^r)$ means that $c$ is the $r$-th order infinitesimal with respect to the function $\psi$.

II. SPACECRAFT MODELING AND PROBLEM FORMULATION

A. SPACECRAFT MODELING

Consider a rigid spacecraft described by the following attitude kinematics and dynamics equations [22]:

\[ \begin{align*} 
\dot{\sigma} &= \frac{1}{4} M(\sigma) \Omega, \\
J\dot{\Omega} &= -\Omega^\times J\Omega + u + d, 
\end{align*} \tag{1} \]

with the Modified Rodrigues Parameters (MRPs) $\sigma = [\sigma_1 \sigma_2 \sigma_3]^T \in \mathbb{R}^3$, and

$$M(\sigma) = \left(1 - \sigma^T \sigma\right)I_3 + 2\sigma^\times + 2\sigma\sigma^T,$$

where $\Omega \in \mathbb{R}^3$ denotes the angular velocity, $u \in \mathbb{R}^3$ is the control torque, $d \in \mathbb{R}^3$ represents the external disturbance, and $J \in \mathbb{R}^{3 \times 3}$ denotes the total inertia matrix of the rigid spacecraft.

The desired attitude MRPs $\sigma_d$ and angular velocity $\Omega_d$ satisfy

$$\dot{\sigma}_d = \frac{1}{4} M(\sigma_d) \Omega_d. \tag{2}$$

By [24], the attitude tracking error between the actual attitude $\sigma$ and the desired attitude $\sigma_d$ can be defined as

$$\sigma_e = \sigma \otimes \sigma_d^{-1} = \frac{\sigma_d (\sigma^T \sigma - 1) + \sigma (1 - \sigma_d^T \sigma_d) + 2\sigma_d^\times \sigma}{1 + \sigma_d^T \sigma_d \sigma^T + 2\sigma_d^\times \sigma}.$$ 

Then, the tracking error system can be expressed by

\[ \begin{align*} 
\dot{\sigma}_e &= \frac{1}{2} M(\sigma_e) \omega, \\
J\dot{\omega} &= -(\omega + C\Omega_d)^\times J (\omega + C\Omega_d) \\
&\quad + J (\omega^\times C\Omega_d - \dot{C}\Omega_d) + u + d, \tag{3} 
\end{align*} \]

where

$$\omega = \Omega - C\Omega_d$$
is error angular velocity and
\[ C = I_3 - \frac{4(1 - \sigma_c^T \sigma_c)}{(1 + \sigma_c^T \sigma_c)^2} \sigma_c \times + \frac{8\sigma_c^T \sigma_c}{(1 + \sigma_c^T \sigma_c)^2}. \]

In the design of attitude control, we need the following assumptions.

**Assumption 1:** The inertial matrix \( J \) is a positive definite matrix in the form of \( J = J_0 + \Delta J \), where \( J_0 \) is a constant positive definite matrix and \( \Delta J \) is the differentiable uncertain part. In addition, there exists an unknown upper bound \( \tilde{J} > 0 \) such that
\[ \|J\| \leq \tilde{J}. \]

**Assumption 2:** The unknown external disturbance \( d \) is differentiable and satisfies
\[ \|d\| \leq \tilde{d}, \]
where \( \tilde{d} \) is an unknown positive constant.

Next, the definition of practical finite-time stability is given.

**Definition 1** [25]: Consider a nonlinear system
\[ \dot{x} = h(x), \quad (4) \]
where \( x \) is the state vector. The solution of nonlinear system (4) is practical finite-time stable if for any initial condition \( x(t_0) = x_0 \), there exist \( \delta > 0 \) and \( T(\delta, x_0) < \infty \), such that
\[ \|x(t)\| \leq \delta, \quad \forall t \geq t_0 + T. \]

At the end of this section, some lemmas are provided.

**Lemma 1** [11]: Suppose \( m_1, m_2 > 0 \) and \( 0 < \lambda < 1 \), there holds
\[ (m_1 + m_2)^l \leq m_1^l + m_2^l. \]

**Lemma 2** [23]: Consider the system (4). Suppose \( V(x) \) is a \( C^1 \) smooth positive definite function, which is defined on \( U \subset \mathbb{R}^n \). There exists an area \( U_0 \subset \mathbb{R}^n \) such that any \( V(x) \) which starts from \( U_0 \subset \mathbb{R}^n \) can reach \( V(x) = 0 \) in finite time if
\[ \dot{V}(x) \leq \lambda_1 V(x) + \lambda_2 V(x) \leq 0, \]
where \( \lambda_1, \lambda_2 > 0 \) and \( 0 < \rho < 1 \). Furthermore, \( V(x) \equiv 0 \) can be reached after
\[ T_1 \leq \frac{1}{\lambda_1(1 - \rho)} \ln \frac{\lambda_2 V(x_0)}{\lambda_2}. \]

**Lemma 3** [25]: Consider the system (4). If \( V(x) \) defined in Lemma 2 satisfies
\[ \dot{V}(x) \leq -\lambda V^\rho(x) + \eta, \]
where \( \lambda, \eta > 0 \) and \( 0 < \rho < 1 \), then, the trajectory of the system (4) is practical finite-time stable. Besides, the state of the system is bounded in finite time as
\[ \lim_{t \to T_1} x \in \left\{ x \in \mathbb{R}^n | V(x) \leq \frac{\eta}{\theta} \right\}, \]

\[ \text{FIGURE 1. Attitude tracking control scheme for a rigid spacecraft.} \]

where \( 0 < \theta < \lambda \) and the time \( T_1 \) is bounded as
\[ T_1 \leq \frac{V^{1-\rho}(x_0)}{(\lambda - \theta)(1 - \rho)}. \]

**III. ATTITUDE TRACKING CONTROLLER DESIGN**

In this section, we intend to develop a control law to ensure the stability of the error system (3) with inertial parameter uncertainties and external disturbances. The block diagram of the proposed control scheme is shown in Fig.1. Notice that in the error system (3), the kinematics subsystem (3a) and the dynamics subsystem (3b) are indirectly stabilized only via the error angular velocity \( \omega \), which implies that the error system (3) is a nonlinear cascade system. As an efficient tool to stabilize a nonlinear cascade system, the backstepping technique is utilized to design a control law to stabilize the dynamics subsystem (3b) without destabilizing the kinematics subsystem (3a).

In what follows, a robust backstepping controller is proposed to accomplish attitude tracking for a rigid spacecraft. The design procedure can be divided into the following steps.

**Step 1:** Our task in this step is to stabilize the kinematics subsystem (3a). Consider the angular angular velocity \( \omega \) as a virtual control input \( \omega^* \). Then, we have the following lemma.

**Lemma 4:** The error MRPs \( \sigma_c \) of the kinematics subsystem (3a) is rendered to zero in finite time by the kinematics control law \( \omega = \omega^* \) with
\[ \omega^* = -K_1 \sigma_c - K_2 \sigma_c e_e (\sigma_c), \]

\[ \begin{align*}
\sigma_d &= \text{Transition Plant} \\
\Omega_d &= \text{Attitude Tracking Controller} \\
&= \text{Spacecraft Model} \\
\end{align*} \]
where $K_1$ and $K_2$ are positive-definite matrices in the form of $K_i = \text{diag}(k_{i1}, k_{i2}, k_{i3}), \ i = 1, 2,$ and $0 < r_1 < 1$. The function $\text{sig}'(a)$ is defined as

$$\text{sig}'(a) = [|a_1|' \text{sign}(a_1), \ldots, |a_n|' \text{sign}(a_n)]^T$$

with $a \in \mathbb{R}^n$.

**Proof:** By taking $\omega = \omega^*$ as the control input of the kinematics subsystem (3a), the kinematics subsystem (3a) can be written as

$$\dot{\sigma}_e = \frac{1}{4} M (\sigma_e) \omega^*. \quad (8)$$

For this system, consider the following Lyapunov function candidate

$$V_1 = 2b_0 \sigma_e^T \sigma_e, \quad (9)$$

where $b > 0$. From (8), the time derivative of $V_1$ is

$$\dot{V}_1 = 4b_0 \sigma_e^T \dot{\sigma}_e$$

$$= b_0 \sigma_e^T M (\sigma_e) \omega^*$$

$$= -bK_1 \left(1 + \sigma_e^T \sigma_e\right) \sigma_e^T \sigma_e$$

$$- bK_2 \left(1 + \sigma_e^T \sigma_e\right) \sigma_e^T \text{sig}'(\sigma_e). \quad (10)$$

Denote $k_i = \min \{k_{ij}, \ j = 1, 2, 3\}, \ i = 1, 2$. Then, we have

$$\dot{V}_1 \leq -b_k l \|\sigma_e\|^2 - b_k \|\sigma_e\|^2$$

$$= -\lambda_1 V_1 - \lambda_2 \dot{V}_1^\rho_1, \quad (11)$$

with

$$\lambda_1 = \frac{1}{2} k_1, \ \lambda_2 = \left(\frac{1}{2}\right)^{\frac{\rho_1}{2}} b^{-\frac{1}{2}} k_2,$$

$$\rho_1 = \frac{r_1 + 1}{2}, \quad \left(\frac{1}{2}, 1\right).$$

From the above relation, it is obvious that $\dot{V}_1$ is negative semi-definite. By Lemma 2, the error MRPs $\sigma_e$ converges to zero in finite time

$$T_1 \leq \frac{1}{\lambda_1 (1 - \rho_1)} \ln \frac{\lambda_1 V_1^{1-\rho_1}(\sigma_0) + \lambda_2}{\lambda_2}. \quad (12)$$

The proof of this lemma is completed. □

**Remark 1:** The item $-K_1 \sigma_e$ in the virtual angular input (7) is used to construct an output feedback for the kinematics subsystem (3a). This item ensures the convergence of the error MRPs $\sigma_e$. The item $-K_2 \text{sig}'(\sigma_e)$ is of help to guarantee the finite-time convergence of the kinematics subsystem (3a). The convergence rate can be changed by adjusting $K_2$ and $r_1$.

**Step 2:** In this step, a new error subsystem is established to ensure that the error angular velocity $\omega$ can track the virtual control input $\omega^*$. Define a new error variable $x$ as

$$x = \omega - \omega^*. \quad (13)$$

Then, one can obtain

$$\dot{x} = \dot{\omega} - \dot{\omega}^*$$

$$= F + J_0^{-1} u + G + \tilde{d}, \quad (14)$$

where

$$F = J_0^{-1} \left[-(\omega + C_0 \omega) \times J_0 (\omega + C_0 \omega) + J_0 (\omega^* C_0 \omega - C_0 \omega^*)\right] - \omega^*,$$

$$G = J_0^{-1} \left[-(\omega + C_0 \omega) \times \Delta J (\omega + C_0 \omega) + \Delta J^{-1} (\omega + C_0 \omega)^2 J (\omega + C_0 \omega) \right. - \Delta J^{-1} u\right],$$

$$\tilde{d} = J_0^{-1} \left(I_3 - \Delta JJ^{-1}\right) d.$$ \hfill (15)

Similarly to [26], denote $f \in \mathbb{R}^3$ as the total disturbance consisting of uncertainties and disturbance in the error dynamics (3b), which is in the form of

$$f = J_0^{-1} \left[-(\omega + C_0 \omega) \times \Delta J (\omega + C_0 \omega) \right.$$

$$- \Delta J \dot{\omega} \dot{\omega} + d\right]. \quad (16)$$

Then (14) can be rewritten as

$$\dot{x} = F + J_0^{-1} u + f. \quad (19)$$

It is worth noting that the total disturbance $f$ cannot be measured exactly in practical applications. To solve this problem, an ESO is introduced to estimate the total disturbance $f$ before we start to stabilize the error system (19).

**Step 3:** In this step, an ESO is designed to estimate the total disturbance $f$. Consider $f$ as an extended state variable of the ESO. That is, the system governed by (19) can be regarded to have two states $x_1 = x$ and $x_2 = f$. Let $\hat{f}(t) = g(t)$. Then, the system (19) can be reconstructed as

$$\left\{ \begin{array}{l}
\dot{x}_1 = F + J_0^{-1} u + x_2, \\
\dot{x}_2 = g(t).
\end{array} \right. \quad (20a)$$

Denote $z_1, z_2$ as the outputs and $e_1 = x_1 - z_1, e_2 = z_2 - z_2$ as the estimation errors of the observer. According to [15], an ESO can be designed as follows

$$\left\{ \begin{array}{l}
\dot{z}_1 = F + J_0^{-1} u + z_2 + b_1 e_1, \\
\dot{z}_2 = b_2 e_1.
\end{array} \right. \quad (21a)$$

where $b_1$ and $b_2$ are the observer gains to be chosen.

Before giving the the convergence of the ESO, the following assumption is needed.

**Assumption 3:** $g(t) = [g_1(t) \ g_2(t) \ g_3(t)]^T$ is bounded.

$g_i(t), \ i = 1, 2, 3,$ satisfy the following inequality

$$|g_i(t)| \leq \bar{g}, \quad i = 1, 2, 3,$$

where $\bar{g}$ is an unknown positive constant.

**Remark 2:** Assumption 3 indicates that the total disturbance is differentiable and its first-order time derivative has unknown upper bound. In practice, the change rate of model uncertainties and external disturbances can not be infinite when a spacecraft is under control. Therefore, Assumption 3 is reasonable for rigid spacecraft in practical applications. Similar assumptions can also be found in some related literatures [16], [18] and [23].
Referring to the results in [27], the following useful lemma can be stated.

**Lemma 5:** Consider the system (20) with an ESO (21), for a given constant \( a_0 > 1 \), there exist two constants \( c_1, c_2 > 0 \) and a finite time \( T \) such that the estimation errors \( e_i = [e_{i1} \ e_{i2}]^T, \ i = 1, 2, \) satisfy

\[
|e_{ij}(t)| \leq c_i, \ \forall t \geq T, \ i = 1, 2, j = 1, 2, 3,
\]

with \( c_i = O \left( \frac{1}{\omega_0^r} \right), \ r = 3 - i \).

**Proof:** The observer error dynamics can be expressed as

\[
\begin{align*}
\dot{e}_1 &= \dot{x}_1 - \dot{z}_1 = -b_1 e_1 + e_2, \quad (22a) \\
\dot{e}_2 &= \dot{x}_2 - \dot{z}_2 = -b_2 e_1 + g(t), \quad (22b)
\end{align*}
\]

Define a coordinate transformation as

\[
e_i = [e_{i1} \ e_{i2}]^T = \left[ \begin{array}{c} e_{i1} \\ e_{i2} \end{array} \right]^T, \ i = 1, 2, 3.
\]

Let

\[
A = \left[ \begin{array}{cc} -a_1 & 1 \\ -a_2 & 0 \end{array} \right], \quad B = \left[ \begin{array}{c} 0 \\ 1 \end{array} \right],
\]

where \( A \) is a Hurwitz matrix by choosing proper \( a_1, a_2 > 0 \). In view of (22a) and (22b), the observer gains are chosen as \( b_1 = a_1\omega_0 \) and \( b_2 = a_2\omega_0 \), respectively. Then, we get

\[
\dot{e}_i = \omega_0 A e_i + B g_i(t), \quad i = 1, 2, 3.
\]

The solution to (24) is

\[
e_i(t) = e^{\omega_0 At} e_i(0) + \int_0^t e^{\omega_0 A(t-\tau)} B g_i(\tau) d\tau, \quad (25)
\]

with \( i = 1, 2, 3 \).

Denote

\[
e^{\omega_0 At} = \left[ \begin{array}{cc} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{array} \right].
\]

Since \( A \) is a Hurwitz matrix, there exists a finite time \( T \) such that

\[
|\alpha_{ij}| \leq \frac{1}{\omega_0^r}, \ \forall t \geq T, \ i, j = 1, 2.
\]

Besides, let

\[
e^{\omega_0 At} e_i(0) = \left[ \begin{array}{c} B_1 \\ B_2 \end{array} \right] = \left[ \begin{array}{c} \alpha_{11} e_{i1}(0) + \alpha_{12} e_{i2}(0) \\ \alpha_{21} e_{i1}(0) + \alpha_{22} e_{i2}(0) \end{array} \right],
\]

with \( i = 1, 2, 3 \). Then, one can have

\[
|\beta_i| \leq \frac{\mu_1}{\omega_0^r}, \ \forall t \geq T, \ i = 1, 2, 3,
\]

where

\[
\mu_1 = \max \{|e_{i1}(0)| + |e_{i2}(0)|, \ i = 1, 2, 3\}.
\]

Define \( v(t) = [v_1(t) \ v_2(t)]^T \), which has the following form

\[
v(t) = \frac{\tilde{g}}{\omega_0} \int_0^t e^{\omega_0 A(t-\tau)} B d\tau.
\]

Solving (28), one can attain that

\[
v(t) = \frac{\tilde{g}}{\omega_0^2} \left( -A^{-1} B + A^{-1} e^{\omega_0 At} B \right).
\]

From the definitions of \( A, B \) and \( e^{\omega_0 At} \), one has

\[
A^{-1} B = \left[ \begin{array}{c} -\frac{1}{a_2} \\ -\frac{1}{a_2} \end{array} \right],
\]

\[
A^{-1} e^{\omega_0 At} B = \left[ \begin{array}{c} -\frac{1}{a_2} a_2 \\ \frac{a_1}{a_2} \end{array} \right].
\]

Then, (29) can be rewritten as

\[
v(t) = \frac{\tilde{g}}{\omega_0^2} \left[ \begin{array}{c} \frac{1}{a_2} (1 - \alpha_{22}) \\ \frac{1}{a_2} \alpha_{12} + \frac{a_1}{a_2} (1 - \alpha_{22}) \end{array} \right].
\]

Apparently, the elements in \( v(t) \), satisfy

\[
v_j(t) \leq \frac{\tilde{g} \mu_2}{\omega_0^2}, \ j = 1, 2,
\]

where

\[
\mu_2 = \max \left\{ \frac{1}{a_2} (1 - \alpha_{22}), \ \frac{\mu_1 + \frac{\tilde{g} \mu_2}{\omega_0^2}}{\omega_0^2}, \ \alpha_{12} + \frac{a_1}{a_2} (1 - \alpha_{22}) \right\}.
\]

Denote \( p_i(t) = [p_{i1}(t) \ p_{i2}(t)]^T, \ i = 1, 2, 3, \) as

\[
p_i(t) = \int_0^t e^{\omega_0 A(t-\tau)} B g_i(\tau) d\tau, \ i = 1, 2, 3.
\]

Substituting \( g_i(t) \leq \tilde{g}, \ i = 1, 2, 3, \) from Assumption 3, and (31) into (32) yields that

\[
p_{ij}(t) \leq v_j(t) \leq \frac{\tilde{g} \mu_2}{\omega_0^2}, \ \forall t \geq T,
\]

with \( i = 1, 2, 3, j = 1, 2 \).

In view of (27) and (33), (25) can be rewritten as

\[
|e_{ij}(t)| \leq \frac{\mu_1 + \frac{\tilde{g} \mu_2}{\omega_0^2}}{\omega_0^{3-i}}, \ \forall t \geq T, \ i = 1, 2, 3, j = 1, 2.
\]

From (23), the observer errors \( e_1 \) and \( e_2 \) satisfy the following inequalities,

\[
|e_{ij}(t)| \leq \frac{\mu_1 + \frac{\tilde{g} \mu_2}{\omega_0^2}}{\omega_0^{3-i}} = c_i,
\]

for all \( t \geq T, \ i = 1, 2, j = 1, 2, 3 \). Obviously, \( c_i = O \left( \frac{1}{\omega_0^r} \right), \ i = 1, 2 \). Therefore, this completes the proof of Lemma 5. \( \square \)

Notice that if the observer gains \( b_1 \) and \( b_2 \) are properly chosen, the observer outputs \( z_1 \) and \( z_2 \) converge to actual states \( x \) and \( f \), respectively. In order to counteract the total disturbance \( f \), the observer output \( z_2 \) is used as a compensation signal in the control law.
Step 4: Based on the first three steps, we aim to stabilize the error system (19) with the total disturbance $f$ estimated by the ESO (21). The controller can be designed as

$$
u = -J_0 \left[ b \left( 1 + \sigma_e^T \sigma_e \right) \sigma_e + F + z_2 \\
+ K_3 x + K_4 \sin \sigma_e^T (x) \right],$$

(35)

where $K_3$ and $K_4$ are positive-definite matrices in the form of $K_i = \text{diag} (k_{i1}, k_{i2}, k_{i3}), \ i = 3, 4,$ and $0 < r_2 < 1$. The stability of the closed-loop system under the control law (35) based on the ESO (21) can be stated by the following theorem.

Theorem 1: Consider the system (3) in the presence of inertial parameter uncertainties and external disturbances, the closed-loop system can be ensured to be practical finite-time stable under the control law (35) based on the ESO (21).

Proof: Consider the following Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2} x^T x,$$

(36)

where $V_1$ is given in (9). The time derivative of $V_2$ is

$$\dot{V}_2 = \dot{V}_1 + x^T (F + J_0^{-1} u + f)$$

$$= -b \left( 1 + \sigma_e^T \sigma_e \right) \sigma_e^T \omega^*$$

$$+ x^T \left[ b \left( 1 + \sigma_e^T \sigma_e \right) \sigma_e + F + J_0^{-1} u + f \right].$$

(37)

Denote $k_i = \min \{ k_{ij}, j = 1, 2, 3 \}, \ i = 3, 4$. Then, substituting the control law (35) into (37), yields

$$\dot{V}_2 \leq -b k_1 \| \sigma_e \|^2 - b k_2 \| \sigma_e \|^{r_1+1} - k_3 \| x \|^2$$

$$- k_4 \| x \|^{r_2+1} + x^T e_2.$$  

(38)

Utilizing some basic inequalities, we can obtain

$$x^T e_2 \leq \frac{1}{2 \gamma^2} \| x \|^2 + \frac{\gamma^2}{2} \| e_2 \|^2,$$

(39)

where $\gamma$ is a constant satisfying $\gamma > \frac{1}{\sqrt{2k_1}} > 0$. Taking (34) and (39) into (38), yields

$$\dot{V}_2 \leq -b k_1 \| \sigma_e \|^2 - b k_2 \| \sigma_e \|^{r_1+1} - \left( k_3 - \frac{1}{2 \gamma^2} \right) \| x \|^2$$

$$- k_4 \| x \|^{r_2+1} + \frac{3 \gamma^2}{2} c_2^2.$$  

(40)

Then, the relation (40) can be expressed as

$$\dot{V}_2 \leq -b k_2 \| \sigma_e \|^{r_1+1} - k_4 \| x \|^{r_2+1} + \frac{3 \gamma^2}{2} c_2^2$$

$$= -\lambda_2 \left( 2 b \| \sigma_e \|^2 \right)^{\frac{r_1+1}{2}} - 2 \frac{r_2+1}{2} k_4 \left( \frac{1}{2} \| x \|^2 \right)^{\frac{r_2+1}{2}}$$

$$+ \frac{3 \gamma^2}{2} c_2^2.$$  

(41)

Let

$$\lambda_3 = \min \left\{ \lambda_2, \frac{2^{r_1+1}}{\sqrt{2}} k_4 \right\},$$

$$\rho_2 = \min \left\{ r_i + 1, i = 1, 2 \right\},$$

$$c_3 = \frac{3 \gamma^2}{2} c_2^2,$$

then the time derivative of $V_2$ can be rewritten as

$$\dot{V}_2 \leq -\lambda_3 \left( 2 b \| \sigma_e \|^2 \right)^{\rho_2} - \lambda_3 \left( \frac{1}{2} \| x \|^2 \right)^{\rho_2} + c_3.$$  

(42)

Since $0 < r_i < 1, \ i = 1, 2$, we have $\frac{1}{2} < \rho_2 < 1$. By Lemma 1, the following inequality can be acquired

$$\dot{V}_2 \leq -\lambda_3 V_2^{\rho_2} + c_3.$$  

(43)

By Lemma 3, it can be concluded that the state $x$ is ultimately bounded in a compact residual set

$$D = \left\{ x \in \mathbb{R}^3 | V_2^{\rho_2} (x) \leq \frac{c_3}{\theta_0} \right\},$$

(44)

where $0 < \theta_0 < \lambda_3$. That is,

$$\lim_{t \rightarrow T_2} x \in D,$$

(45)

and the time $T_2$ satisfies

$$T_2 \leq \frac{\dot{V}_2^\rho (x_0)}{(\lambda_3 - \theta_0) (1 - \rho_2)}.$$  

(46)

Therefore, the practical finite-time stability of the closed-loop system (3) with the ESO (21) under the controller (35) is guaranteed. Thus, this proof is completed. \qed

Remark 3: The residual set $D$ (44) is a compact for any positive constants $c_3, \theta_0$ and $\rho_2$. That is, the global boundedness is guaranteed in the presence of bounded disturbances with unknown bounds, no matter what values these constants are chosen. From (44) and (45), it is obvious that the upper bounds of $V_2 (t)$ and $x (t)$ can be reduced by decreasing the value of $c_3$ and increasing the value of $\theta_0$ properly.

Remark 4: Notice that the parameter $c_3$ is related to the upper bounds of the estimation error $e_2$ and the adjustable parameter $\gamma$. Hence, we can reduce the size of the residual set $D$ by improving the accuracy of the ESO and decreasing the value of $\gamma$. Furthermore, the performance of the control system for the rigid spacecraft will be improved.

Remark 5: From (45) and (46), it can be seen that the value of the parameter $\theta_0$ is related to the upper bound of $x (t)$ and the convergence time $T_2$. Apparently, the larger the bound is, the shorter the convergence time becomes, and vice versa.
Therefore, a trade-off between the bound and the convergence time should be considered.

**IV. SIMULATION RESULTS**

Consider the rigid spacecraft (1) with the nominal inertia matrix [16]

\[
J_0 = \begin{bmatrix}
20 & 1.2 & 0.9 \\
1.2 & 17 & 14 \\
0.9 & 1.4 & 15
\end{bmatrix} \text{ kg} \cdot \text{m}^2.
\]
The control input under the SMC in [16], FTC in [20] and the RFTC (35). The uncertainty item $J$ is

$$J = \text{diag}(\sin(0.1t), \ 2\sin(0.2t), \ 3\sin(0.3t)) \text{kg} \cdot \text{m}^2.$$ 

The external periodic disturbance is assumed as

$$d(t) = \begin{bmatrix} 0.2 + \cos(0.1t) \\ 0.5 + \cos(0.2t) \\ -0.2 + \sin(0.1t) \end{bmatrix} \text{Nm}.$$ 

The initial attitude condition described by MRP is

$$\sigma(0) = [-0.1579, 0.1368, -0.0947]^T,$$

and the initial attitude angular velocity is $\Omega(0) = [0 \ 0 \ 0]^T \text{rad/s}$. The desired MRP is $\sigma_d(0) = [0 \ 0 \ 0]^T$ and the desired angular velocity is

$$\Omega_d(t) = \begin{bmatrix} 0.05 \sin\left(\frac{2\pi t}{30}\right) \\ 0.05 \cos\left(\frac{2\pi t}{30}\right) \end{bmatrix} \text{rad/s}.$$ 

Besides, the initial values of the ESO are assumed to be

$$z_i = [0.01 \ 0.01 \ 0.01]^T, \ i = 1, 2.$$ 

The parameters of the controller (35) are chosen as

$$K_1 = \text{diag}(2.5, \ 1.8, \ 2.0), \ K_2 = \text{diag}(0.6, \ 0.8, \ 0.7), \ K_3 = \text{diag}(0.4, \ 0.1, \ 0.1), \ K_4 = \text{diag}(0.5, \ 1.2, \ 1.2), \ b = 0.1, \ r_1 = 0.8, \ r_2 = 0.6.$$ 

The gains of the ESO (21) are $b_1 = 20, \ b_2 = 300$ (with $a_1 = 2, \ a_2 = 3$ and $\omega_0 = 10$).

Fig.2-Fig.4 plot the states in the error system (3) under the proposed robust finite-time controller (RFTC) (35). The sliding-mode controller (SMC) in [16] and the finite-time controller (FTC) in [20] are also considered for comparison purpose. The time responses of the error attitude variables MRP and the error angular velocity are shown in Fig.2 and Fig.3, respectively. Obviously, the RFTC can render error MRP and error angular velocity to the neighbour of zero.
before 10s, while the states converge in about 15s under the SMC and the FTC. Meanwhile, the error system (3) with the RFTC has smaller overshoot than the other two controllers. The corresponding control torques are described in Fig.4. It seems that the RFTC costs less control torque and has smaller steady error than the other two controllers. Therefore, it can be concluded that the RFTC (35) has a better performance compared with the SMC in [16] and the FTC in [20].

Fig.5 intuitively shows that the system (1) achieves the desired attitude within 6s under the RFTC (35). In addition, the performance of the observer to estimate the total disturbance \( f \) is presented in Fig.6. It is clear that the estimation state \( z_2 \) converges to the total disturbance \( f \) in one second. Fig.7 and Fig.8 illustrate the time responses of the observer errors of the ESO (21). These results further demonstrate that the RFTC (35) with the ESO (21) can guarantee the practical finite-time stability of the closed-loop system and maintain satisfactory performance for the rigid spacecraft (1) when there exist inertial parameter uncertainties and external disturbances simultaneously.

V. CONCLUSION

In this paper, a robust finite-time controller is proposed to solve the attitude tracking control problem for the rigid spacecraft in the presence of parameter uncertainties and external disturbances. With the idea of estimating and compensating the system model uncertainties and external disturbances, an ESO is introduced in the control scheme. Then, an observer based finite-time controller is designed to ensure that the closed-loop system has fast convergence rate, insensitivity to parameter uncertainties and robustness to external disturbances. Moreover, the practical finite-time stability of the closed-loop system is guaranteed under the proposed control scheme. Simulation results show the effectiveness and superiority of the developed control law.

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