RENORMALIZABILITY OF THE HEAVY QUARK EFFECTIVE THEORY

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Abstract

We show that the Heavy Quark Effective Theory is renormalizable perturbatively. We also show that there exist renormalization schemes in which the infinite quark mass limit of any QCD Green function is exactly given by the corresponding Green function of the Heavy Quark Effective Theory. All this is accomplished while preserving BRS invariance.
1 Introduction

Over the last few years there has been an enormous interest in the so called Heavy Quark Effective Theory (HQET) \[1\]. Hadrons made out of a heavy quark, such as $b$ or $c$, and either a light anti-quark (heavy-light meson) or two light quarks (heavy-light baryons) can be conveniently analyzed within the framework of the HQET. It exploits two symmetries which are not apparent in the standard QCD lagrangian when quark masses are small ($\approx \Lambda_{\text{QCD}}$): the spin and flavour symmetries. They come down to the statement that the dynamics of a heavy-light hadron is heavy-flavour and heavy-quark spin independent. These two symmetries can be combined into a larger one: the Isgur-Wise symmetry \[2\].

The phenomenological implications of the Isgur-Wise symmetry have been extensively discussed in the recent literature \[3\]. In this letter we address the field theoretical questions of whether the HQET is renormalizable or not and whether renormalization can be accomplished or not while preserving BRS invariance. In applications of the HQET, renormalizability is always assumed and even some renormalization constants have been computed under this assumption. However, as far as we know, a formal proof is still missing. These are not just academic questions since loop calculations are required in order to obtain the scaling (or large heavy-quark mass behaviour) of many phenomenologically relevant quantities, e.g. leptonic and semileptonic decay constants and form factors. Although the asymptotic behaviour of these and other parameters could in principle be obtained from standard QCD \[4\], the calculations are far more involved than the corresponding ones in the HQET and, to the best of our knowledge, no attempt has been made beyond one loop. This raises another important question: Does the HQET really provide the right (QCD based) large quark mass result beyond tree-level? We come back to this point below.

Our first goal is to prove that the HQET is renormalizable perturbatively to any order. What has to be shown is that it is possible to rescale the fields and coupling constants so that all Green functions involving only elementary fields are free of ultraviolet divergences without spoiling BRS invariance. (In fact, we would only need a proof for S-matrix amplitudes. However, these will be finite if Green functions have no divergences.) Thus, we choose a regularization scheme which preserves BRS invariance, e.g. dimensional regularization.

Throughout this letter we will assume that Weinberg’s convergence theorem \[6\] is satisfied even though Lorentz invariance is broken by the heavy quark propagator, $1/p \cdot v$. More explicitly, we will assume that if a 1PI graph $G$ has superficial degree of divergence $\delta(G)$, given by naive power counting, then its overall UV divergence is a polynomial in the external momentum of at most degree $\delta(G)$. We shall not attempt to prove this statement here, but rather note that the heavy quark propagator $1/p \cdot v$ is very similar to the principal-value prescribed pole $1/p \cdot n$ of axial gauges for which one can argue that Weinberg’s theorem holds \[7\]. Furthermore, a large variety of one- and two-loop calculations \[8\]
give support to this assumption.

Our second goal is to show that there exist renormalization schemes in both QCD and the HQET consistent with BRS invariance and such that any QCD Green function agrees with its HQET counterpart up to order $O(1/M)$. We shall refer to this as the matching. For instance, for the quark propagator one has

$$\langle \Psi \bar{\Psi} \rangle|_{\pm Mv+p} = \langle h^\mp \bar{h}^\mp |_p + O \left( \frac{1}{M} \right),$$

where $\Psi (M)$ is the field (mass) of a heavy quark, i.e. $M \gg \Lambda_{\text{QCD}}$, and $h^\pm (h^-)$ is the effective field in the HQET corresponding to the particle (antiparticle) part of $\Psi$. In this letter we use the shorthand notation $\langle \cdots \rangle = \langle 0 | T \cdots | 0 \rangle$. Note that on the HQET side of equations such as (1), the momentum, $p$, of a heavy quark (antiquark) with velocity $v$ is actually its virtuality or “off-shellness”, defined by subtracting $Mv (-Mv)$ from the real momentum used on the QCD side. Assuming renormalizability of the HQET, Grin-stein [5] has shown that QCD and HQET Green functions match provided one chooses the appropriate counterterms, which turn out to depend logarithmically on $M$. Unfortunately, it is not clear if his approach preserves BRS invariance. We shall prove the following statement: By choosing our renormalization scheme in such a way that the heavy quark two-point functions in QCD and in the HQET agree up to order $O(1/M)$, BRS invariance ensures also the matching of the heavy quark-gluon vertex and, in turn, of any other Green function. Hence, our approach explicitly preserves the BRS invariance of the theory.

In applications, one usually works in the $\overline{\text{MS}}$ (or MS) scheme. From the preceding paragraph it follows that there must exist a coefficient $C(\log M)$ such that, for example,

$$\langle \Psi \bar{\Psi} \rangle|_{\pm Mv+p} = C(\log M) \langle h^\mp \bar{h}^\mp |_p + O(1/M).$$

Similar equations hold for any pair of Green functions. The coefficient $C^{1/2}(\log M)$ is just the finite wave function renormalization constant of the heavy quark that relates the $\overline{\text{MS}}$ scheme with the above-mentioned ones. It is important because it provides the scaling properties of the QCD Green functions at large $M$.

This letter is organized as follows: In sec. 2 we introduce the HQET lagrangian and discuss the BRS symmetry. In sec. 3 we briefly review the proof of renormalizability of QCD. We will follow very closely the proof given by Collins [9]. In sec. 4 we will show that the HQET is renormalizable. In sec. 5 we prove that matching of QCD and the HQET can be achieved while maintaining BRS invariance. Finally, sec. 6 will be devoted to comments and conclusions.
2 The HQET Lagrangian and BRS invariance

In terms of renormalized fields the lagrangian of the HQET, $\mathcal{L}^{\text{HQET}}$, has the form:

$$\mathcal{L}^{\text{HQET}} = \mathcal{L}^h + \mathcal{L}^{\text{QCD}_{\text{light}}},$$

where

$$\mathcal{L}^h = Z_h \bar{h}^+ (x) i \cdot D h^+ (x) - Z_h \bar{h}^- (x) i \cdot D h^- (x);$$

$$\mathcal{L}^{\text{QCD}_{\text{light}}} = -\frac{1}{4} Z_3 (G_{\mu}^a)^2 + Z_2 \bar{\psi} (i \partial - m_0) \psi - \frac{1}{2} (\partial \cdot A^a)^2 + \bar{c} D^\mu c.$$ (4)

For simplicity, only one heavy and one light quark flavour will be considered. Here,

$$G_{\mu}^a = \partial_\mu A_\mu^a - \partial_\nu A_\nu^a - X g_{\mu c} A_\mu^b c^c.$$ (5)

is the (gluon) field-strength tensor. The covariant derivatives in the defining and adjoint representations of $SU(N)$ are respectively:

$$D_\mu \chi = (\partial_\mu + ig \frac{X}{Z} t^a A_\mu^a) \chi; \quad \chi = \psi, h^\pm;$$

$$D_\mu c^a = (\partial_\mu + \frac{X}{Z} g c_{abc} t^b A_\mu^c).$$ (7)

In eqs.(3-7), $g$ is the coupling constant, $Z_3$, $Z_2$ and $Z_h$ are the wave function renormalization constants of the $A_\mu^a$ (gluon), $c^a$ (ghost), $\psi$ (light quark) and $h^\pm$ (effective heavy quark/antiquark) fields, whereas $X$ renormalizes the ghost-gluon vertex. Finally, $m_0$ is the bare (light) quark mass, $t^a$ are the generators of $SU(N)$ in its defining representation satisfying $\text{tr} (t^a t^b) = \frac{1}{2} \delta^{ab}$ and $c_{abc}$ are the structure constants defined through $[t^a, t^b] = ic_{abc} t^c$.

The global symmetries of $\mathcal{L}^{\text{HQET}}$, particularly the Isgur-Wise symmetry, have been discussed (see for instance [10]) and exploited extensively in the literature. It has also been shown from different approaches [11] that at tree-level the lagrangian $\mathcal{L}^{\text{HQET}}$ can be obtained from the full QCD lagrangian $\mathcal{L}^{\text{QCD}} = \mathcal{L}_\Psi + \mathcal{L}^{\text{QCD}_{\text{light}}}$, where

$$\mathcal{L}_\Psi = Z_\psi \bar{\Psi} (i \partial - M_0) \Psi$$

describes an ordinary quark field $\Psi$ with mass $M \gg \Lambda_{\text{QCD}}$ coupled to gluons through the covariant derivative $D_\mu$ defined as in (6). In the $\overline{\text{MS}}$ scheme one has $Z_\psi = Z_2$, however, if a mass dependent scheme is chosen, as we shall do to prove the matching in section 5, two independent wave function renormalization constants are required.

The lagrangian (2) is invariant under the (renormalized) BRS transformations

$$\delta_{\text{BRS}} \varphi = \delta_R \varphi \delta \lambda_R$$

where $\varphi$ is any basic field, $\delta \lambda_R$ is a Grassmann number and $\delta_R$ is
given by:

\[
\delta_R \psi(x) = -igXt^a \psi(x)c^a(x),
\]
\[
\delta_R A^a_\mu(x) = \bar{Z} \partial_\mu c^a(x) + X g\epsilon_{abc}c^b(x)A^c_\mu(x) = \bar{Z} D_\mu c^a,
\]
\[
\delta_R \bar{c}^a(x) = -\frac{1}{2} X g\epsilon_{abc}c^b(x)\bar{c}^c(x),
\]
\[
\delta_R \bar{c}^a(x) = \frac{1}{\xi} \partial \cdot A^a(x),
\]
\[
\delta_R h^{\pm}_v(x) = -igXt^a h^{\pm}_v(x)c^a(x).
\]

The full QCD lagrangian \( \mathcal{L}^{\text{QCD}} \) is also invariant under (9-12) and \( \Psi \) transforming as \( \psi \). This well-known result will be used in section 5.

Our goal is to prove that a clever choice of \( Z_3, Z_h, Z_2, \bar{Z}, X \) and \( m_0 \) suffices to render all Green functions of elementary fields, as well as the composite operators in (9–13), and (14) below, ultraviolet finite.

### 3 Renormalizability of QCD

The proof of renormalizability that we sketch out in this section requires that the regulated Green functions satisfy the Ward identities implied by BRS invariance. As already pointed out in the introduction, this is ensured by using dimensional regularisation (or another regulator that does not break BRS invariance).

Renormalizability of the standard QCD (light quark) sector of the HQET with lagrangian \( \mathcal{L}_{\text{light}}^{\text{QCD}} \) is proved by induction on the number of loops, \( N \). The assumptions are the following:

1. All 1PI Green functions of elementary fields as well as those with an insertion of the composite operators appearing in the BRS transformations (9-12) have successfully been made UV-finite at each order below \( N \) loops. This has been achieved by choosing the renormalization constants \( Z_2, \bar{Z}, Z_3, X \) and the bare mass \( m_0 \) appropriately.

2. As above for Green functions with insertions of the operator

\[
B^a_\mu(x) = \left( \frac{Z}{X} - 1 \right) \frac{1}{g} \partial_\mu \bar{c}(x)^a + \epsilon_{abc}c^b(x)A^c_\mu(x).
\]

This operator will be used to prove that \( \delta_R \psi \) (and \( \delta_R h^{\pm}_v \), in section 4), and the quark-gluon vertex are UV-finite.
Obviously 1 and 2 hold at tree level. It must be shown that they also hold at order $N$. Thus, we proceed to compute $N$ loop contributions. Since subdivergences have already been subtracted, only Green functions having non-negative superficial degree of divergence, $\delta(G) \geq 0$, may have an overall UV-divergence. These (potentially) UV-divergent Green functions are the quark, ghost and gluon self-energies; the ghost-gluon vertex; the gluon three- and four-point functions; some (1PI) insertions of the operators $\delta RA_{\mu}^a$, $\delta R\epsilon_a$, $B_{\mu}^a$, and $\delta R\psi$; and, finally, the quark-gluon vertex. Because of Weinberg’s theorem, their overall divergences are polynomials in the external momenta of degree $\delta(G)$. Hence, they may eventually be absorbed in the counterterm lagrangian implied by \((4)\) and the renormalization constants in \((3)\) \((12)\) \((14)\).

Next, i) we choose $Z_2$ and $m_0$ to cancel the UV divergences of the quark two-point function; ii) we choose $Z_3$ to render the gluon self energy finite (a BRS identity must be used to check that UV-divergences do not show up in the longitudinal piece); iii) $\tilde{Z}$ is chosen to cancel the part of the divergence of the ghost self-energy and iv) $X$ is chosen to make the ghost-gluon vertex finite (note that only the colour tensor structure $f_{abc}$ is allowed by charge conjugation). At this point, all renormalization constants have been fixed and one must check that the UV-divergences of the remaining Green functions automatically cancel against the relevant contributions of the renormalization constants we have just determined. This is accomplished through the use of BRS identities. One proceeds orderly (for each step requires the conclusions of the preceding ones) as follows: v) check that the operator $\delta R A_{\mu}^a$ is finite; vi) using (v) and some BRS identity, check finiteness of the gluon three- and four-point functions; vii) show that no UV-divergence arises in insertions of $\delta R\epsilon_a$; viii) as above for $B_{\mu}^a$; ix) as above for $\delta R\psi$. Finally, use (i, ix) and some identities to prove that x) the quark-gluon vertex is also UV-finite. This completes the proof of renormalizability. (For details, see ref.\([1]\).)

4 Renormalizability of the HQET

Now, we turn to the HQET. An important simplification arises from the observation that loops of only heavy quark propagators never occur. This is due to the fact that in $\mathcal{L}^{\text{HQET}}$ heavy quarks, $h_{\nu}^+, \bar{h}_{\nu}^+$, do not couple to heavy antiquarks, $h_{\nu}^-, \bar{h}_{\nu}^-$. Consequently, the renormalization of Green functions with no external effective heavy quark legs (entirely described by $\mathcal{L}_{\text{light}}^{\text{QCD}}$) remains as in ordinary QCD. Note that this statement shows that steps (i–x) of the previous section can be carried out exactly as for the lagrangian \((4)\) if no Green function with external heavy quark fields is considered. Among those Green functions involving $h_{\nu}^\pm$ there are only four superficially divergent: the quark two-point function, $\langle h_{\nu}^\pm(x)\bar{h}_{\nu}^\pm(0)\rangle$; the quark-gluon vertex, $\langle h_{\nu}^\pm(x)\bar{h}_{\nu}^\pm(0)A_{\mu}(y)\rangle$, as well as $\langle \delta R h_{\nu}^\pm(x)\bar{h}_{\nu}^\pm(0)c^\nu(y)\rangle$ and $\langle h_{\nu}^\pm(x)\delta R \bar{h}_{\nu}^\pm(0)c^\nu(y)\rangle$. To the induction assumptions of section 3, one has to append the finiteness of these Green functions at each order below $N$ loops. One has also to add
to the previous list of steps the following: i') choose $Z_h$ to cancel the UV-divergences in the heavy-quark two-point function; ix') using the appropriate identities, check that the operators $\delta_R h_v^\pm$ and $\delta_R \bar{h}_v^\pm$ are finite; finally, x') show that no UV-divergence appears in the (heavy) quark-gluon vertex. This, we carry out in this section.

Let us begin considering $\langle h_v^\pm \bar{h}_v^\pm \rangle$ in momentum space. Weinberg’s theorem tells us that the renormalization prescription can be chosen so that the overall divergence of the 1PI portion of $\langle h_v^\pm \bar{h}_v^\pm \rangle$ is a polynomial in the external residual momenta or “off-shellness” and the light quark mass of degree $k$. For such a renormalization prescription, the overall divergence can only be proportional to $k \cdot v (1 \pm \eta)/2$ and $m (1 \pm \eta)/2$, $k$ being the residual external momentum and $m$ the mass of the light quark propagating in the loops. Obviously, we can get rid of the divergence proportional to $k \cdot v$ by adjusting $Z_h$. However, there is no counterterm in $L^b$ to cancel a divergence proportional to the light quark mass. Thus, we must show that this divergence is absent. This is easily seen by noting that the $m$ dependence of $\langle h_v^\pm \bar{h}_v^\pm \rangle$ comes only from light quark loops. The corresponding traces of $\gamma$-matrices are seen to be an even function of $m$ and, hence, no UV divergence proportional to $m$ can arise. This completes step (i').

Now we turn to (ix'). There is only one 1PI diagram with an insertion of $\delta_R h_v^\pm$ in which UV-divergences can occur:

$$\langle \delta_R h_v^\pm(x) \bar{h}_v^\pm(y) \rangle^{1PI}. \tag{15}$$

To prove that (15) is actually UV-finite, we consider $\langle \delta h_v^\pm(x) \bar{h}_v^\pm(y) \square c^a(z) \rangle$. Clearly, UV-finiteness of this Green function ensures UV-finiteness of (15). One can show that

$$\langle \delta_R h_v^\pm(x) \bar{h}_v^\pm(y) \square c^a(z) \rangle = -1 \frac{1}{X} \left\langle \delta_R h_v^\pm(x) \bar{h}_v^\pm(y) \frac{\delta S}{\delta c^a(z)} \right\rangle - \frac{1}{2} \xi g \langle \delta_R h_v^\pm(x) \delta_R \bar{h}_v^\pm(y) c_{ade} c^d(z) c^a(z) \rangle - g \langle \delta_R h_v^\pm(x) \bar{h}_v^\pm(y) \partial^\mu B^a_\mu(z) \rangle, \tag{16}$$

where $S$ is the complete action, $S = \int d^D x L^{HQET}$. In order to obtain (15) we have computed $\partial^\mu B^a_\mu$ from (14). Next, by (12) we have written $\partial^\mu A^a_\mu$ as $\xi \delta_R c^a$. Finally, we have used the nilpotence property $\delta_R [\delta_R h_v^\pm] = 0$ and the identity $\delta_R \langle \delta_R h_v^\pm \bar{h}_v^\pm c_{ade} c^d c^e \rangle = 0$ to “integrate by parts” $\langle \delta_R h_v^\pm \delta_R (c_{ade} c^d c^e) \rangle$ into its final form in (16).

Each of the three terms of (15) is now shown to be finite: 1) Using the anti-ghost equation of motion [9], one can easily show that

$$-1 \frac{1}{X} \left\langle \delta_R h_v(x) \bar{h}_v(y) \frac{\delta S}{\delta c^a(z)} \right\rangle = g t^a \delta(x - z) \langle h_v(x) \bar{h}_v(y) \rangle. \tag{17}$$

Since the heavy quark two-point function is UV-finite by (i'), so is the first term of (15). 2) The superficial degree of divergence of the second Green function of (15) is $\delta(G) = -1$. 3) The superficial degree of divergence for the third Green function of (15) is $\delta(G) = -1$. Therefore, the overall divergence of the 1PI contribution is $-\frac{1}{2} \xi g$. This is absorbed by a counterterm in $L^b$.
It is finite because it is 1PI and subdivergences have already been canceled according to the induction assumptions. 3) Finally, we must consider the last term of (16), whose decomposition in 1PI parts is shown in figure 1. For simplicity we have amputated the external quark leg, which has previously seen to be UV-finite. Graph (c) has \( \delta(G) = -1 \), so it is finite. Adding graphs (a) and (b) one obtains

\[
(a) + (b) = \langle \delta_R h^\pm_v \bar{h}^\pm_v \bar{c}^a \rangle^{(N-1)} \langle c^b \partial^\mu B^a_{\mu} \rangle^{(N)},
\]

where the superscript \((L)\) indicates that contributions up to \(L\) loops are included. Note that the \(N\)-loop contribution from \( \langle c^b \partial^\mu B^a_{\mu} \rangle \) does not appear in (18) because no zeroth order graph of \( \langle c^b \partial^\mu B^a_{\mu} \rangle \) exists. The first factor in (18) is finite by the induction assumption, while the second factor is also finite by step (viii). This completes step (ix').

Finiteness of \( \delta_R \bar{h}^\pm_v \) can be proved by the same method.

Step (x') is an immediate consequence of (ix') and the identity \( \delta_R \langle h^\pm_v \bar{h}^\pm_v c^a \rangle = 0 \), which can be written as

\[
\frac{1}{\xi} \langle h^\pm_v \bar{h}^\pm_v \partial^\mu A^a_{\mu} \rangle = \langle h^\pm_v \delta_R \bar{h}^\pm_v c^a \rangle - \langle \delta_R h^\pm_v \bar{h}^\pm_v c^a \rangle.
\]

This Ward identity shows that the contraction of the heavy quark gluon vertex (1PI, after subtraction of the UV-finite external propagators) with the momentum of the gluon leg has no UV divergence. Since by power counting the divergence of the vertex can only be proportional to \( v_\mu (1 \pm \gamma) / 2 \), the vertex itself is finite. This completes the proof of renormalizability of the HQET.

## 5 Matching

In the previous section we have shown that the HQET is renormalizable. As already mentioned, this is precisely the main assumption used by Grinstein in [5] to show that the Green functions of \( \mathcal{L}_{\text{HQET}} \) and those of \( \mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{QCD}}^\text{light} + \mathcal{L}_{\text{QCD}}^\text{heavy} \) match for large \( M \). Again, the proof in [5] (and also here) is by induction on \( N \). It is assumed that matching holds at each order below \( N \) loops for any Green function of the type considered in the induction assumptions 1 and 2 of section 3. Note that at tree-level any Green function matches by construction [11]. Then, as shown in [5], the matching at order \( N \) holds for graphs with negative superficial degree of divergence. Thus, we need only be concerned with the matching of \( \langle \Psi \bar{\Psi} \rangle, \langle \Psi \bar{\Psi} A^a_{\mu} \rangle, \langle \delta_R \Psi \bar{\Psi} c^a \rangle \) and \( \langle \bar{\Psi} \delta_R \bar{\Psi} c^a \rangle \).

Let us begin by borrowing from [5] the following result: it is always possible to modify the counterterm in \( Z_\Psi, Z_h \) and \( M_0 \) by an UV-finite amount in such a way that the quark two-point functions match, i.e, eq. (17) is satisfied, at \( N \) loops. One can check that the readjusted renormalization constants and \( M_0 \) become functions of \( \log M \). If BRS invariance
is to be preserved, one cannot change the value of the remaining counterterm in (8) (see also eq. (7)). Therefore, we must show that the QCD and QHET quark-gluon vertexes match automatically.

First, we shall check that

\[
\langle \delta R \bar{\Psi} \bar{\Psi} \bar{c}^a \rangle \big|_{\pm M v + p} - \langle \delta R h_v^\pm \bar{h}_v^\pm c^a \rangle \big|_p = O \left( \frac{1}{M} \right).
\]

(20)

Since the identities obtained from (16), (17) and (18) by the replacement \( h_v^\pm \rightarrow \Psi \) also hold, we only need to prove the matching of the Green functions on the right hand side of (16) with their \( \Psi \) counterparts (in momentum space). The matching of \( \langle \delta R h_v^\pm \bar{h}_v^\pm \delta S/\delta c^a \rangle \) and \( \langle \delta R \bar{\Psi} \bar{\Psi} \delta S/\delta c^a \rangle \) is guaranteed by the matching of the quark two-point functions (1) and by the \( \Psi \)- and \( h_v \)-version of (17), which in momentum space read

\[
- \frac{1}{X} \left( \delta R \chi \bar{\chi} \frac{S_0}{\delta c^a} \right) = g t^a \langle \chi \bar{\chi} \rangle, \quad \chi = h_v^\pm, \Psi.
\]

(21)

The matching of \( \langle \delta R h_v^\pm \delta R \bar{h}_v^\pm c_{ade} \bar{c}^d \bar{c}^e \rangle \) and \( \langle \delta R \bar{\Psi} \delta R \bar{\Psi} c_{ade} \bar{c}^d \bar{c}^e \rangle \) holds because both have negative superficial degree of divergence. Finally, we must prove that \( \langle \delta R h_v^\pm \bar{h}_v^\pm B_{\mu}^a \rangle \) and \( \langle \delta R \bar{\Psi} \bar{\Psi} B_{\mu}^a \rangle \) also match. This is apparent from figure 1 since graphs of the type (c) have \( \delta (G) = -1 \), whereas from (18), graphs (a) and (b) are seen to involve \( \langle \delta R h_v^\pm \bar{h}_v^\pm c_{\mu}^b \rangle_{1PI} \) and \( \langle \delta R \bar{\Psi} \bar{\Psi} c_{\mu}^b \rangle_{1PI} \) at \( N - 1 \) loops at most, so they match by the induction assumption. It is important to recall that the external quark legs have been amputated in figure 1. This causes no trouble, for the quark propagators are already known to match. Similarly, one can show that the matching of the terms involving \( \delta R \bar{\Psi} \) and \( \delta R \bar{h}_v^\pm \) also holds.

We now turn to the quark-gluon vertex. One can show [5] that

\[
\langle \Psi \bar{\Psi} A_{\mu}^a \rangle \big|_{\pm M v + p} - \langle h_v^\pm \bar{h}_v^\pm A_{\mu}^a \rangle \big|_p = S_v^\pm (p) C_v^\pm v_\mu S_v^\pm (p) + O \left( \frac{1}{M} \right).
\]

(22)

Where \( S_v^\pm (p) = (1 \pm \gamma_5) / 2 p \cdot v \) is the effective heavy quark propagator and the coefficient \( C_v^\pm \) is UV-finite, dimensionless and dependent only on \( \log M \). For simplicity we have amputated the gluon leg in (22). It must be shown that \( C_v^\pm \) vanishes. This is easily seen from (19), the Ward identity

\[
\frac{1}{\xi} \langle \Psi \bar{\Psi} \bar{c}^a \rangle = \langle \Psi \delta R \bar{\Psi} \bar{c}^a \rangle - \langle \delta R \bar{\Psi} \bar{\Psi} \bar{c}^a \rangle
\]

(23)

and the matching of the Green functions on their right hand sides. This completes the proof.
6 Conclusions and discussion

We have shown that the HQET is renormalizable perturbatively. Weinberg’s theorem and BRS invariance play an essential role in the proof. Weinberg’s theorem is assumed to hold for the HQET, as several one- and two-loop calculations seem to indicate, while BRS invariance is explicitly preserved at any order in perturbation theory. We have also shown that the HQET provides the infinite mass limit of QCD for heavy quarks near mass-shell. Again, BRS invariance is crucial in our proof. If the $\overline{\text{MS}}$ scheme is chosen, QCD and the HQET match in the following sense: 

$$\langle \Psi \bar{\Psi} \cdots \rangle |_{\overline{\text{MS}}^{\pm M_v+\vec{p}}} = C(\log M) \langle h^\pm \bar{h}^\pm \cdots \rangle |_{\overline{\text{MS}}}^{\pm} + O(1/M),$$

where the dots stand for any string of (light) fields and $C^{1/2}(\log M)$ is a UV-finite wave function renormalization constant of the heavy quark fields.

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Figure captions

**Fig. 1.** Decomposition of $\langle \delta_R h_\psi^\pm(x) \bar{h}_\psi^\pm(y) \partial^\mu B_\mu^{a}(z) \rangle$ in 1PI portions. For simplicity the external quark leg has been amputated. The Feynman rules for the two different insertions of $\partial^\mu B_\mu^{a}$ can be read off from the corresponding two terms of (14). The same decomposition holds in QCD by substituting $\Psi$ for $h_\psi^\pm$. 
This figure "fig1-1.png" is available in "png" format from:

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