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Non Pharmaceutical Interventions for Optimal Control of COVID-19

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A B S T R A C T

Background and Objective: The outbreak of the current pandemic begun from the first individual of a 55-year-old from Hubei province in China, the disease instigated by the new coronavirus spreading across the world. Scientists presently speculate this coronavirus, SARS-CoV-2, originated in a bat and by one way or another jumped to another creature, potentially the pangolin, which at that point gave it to people. The ailment is currently spreading between individuals with no animal delegate. Researchers are struggling to follow the infection back to where it started to become familiar with its spread. In the event that, for example, specialists can locate the soonest cases, they might have the option to distinguish the creature have where the infection hides. In March and April 2020, researchers detailed that this virus created normally. Coronavirus has been become of the serious global phenomena in the recent years and has negative effects in the entire world health and economy. The virus is believed to have been associated with a host animal which human contracted. Subsequently, human-to-human infection began. Through migration as humans have become complex with easy mobility the disease has traveled to the entire continent. Now, numerous scientist are going on in the hope of obtaining medication and vaccination to prevent the spread of the disease and mortality of the disease. It is important that we obtain quantitative and qualitative information about the etiology of this disease which is crucial. Mathematical modeling is capable of providing qualitative information on many parameters that guides the decision making of health practitioners. In this work we focus the optimal control of COVID-19 with the help of Non Pharmaceutical Interventions (NPIs). To find the role of factors/parameters in the transmission of the syndrome we find $R_0$; the ratio of reproduction for the proposed model.

Methods: To find the role of parameters in the transmission of the syndrome we find $R_0$; the ratio of reproduction for the proposed model. On the basis of sensitivity indices of the parameters we apply Non Pharmaceutical Interventions(NPIs) to control the sensitive parameters and hence formulate the optimal control mode. With the help of Hamiltonian and Lagrangian we minimize the density of contaminated stuff and infected human population.

Results: We focus the optimal control of COVID-19 with the help of Non Pharmaceutical Interventions(NPIs). On the basis of sensitivity indices of the parameters we apply Non Pharmaceutical Interventions(NPIs) to control the sensitive parameters and hence formulate the optimal control model. The major NPIs are, \textit{STAY HOME, SANITIZER (wash hands), EARLY CASE DETECTION (PCR Test) and FACE MASK}. These NPIs helps in mitigation and reducing the size of outbreak of the disease.

Conclusions: We check the existence of the optimal solution for the system. At the end, Using matlab we produce numerical simulations for validation of results of control variables. The results demonstrate

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1. Introduction

Coronaviruses in fact represents a big family. This family causes different types of infections. The infection ranges from common cold/flu to the most severe infection like severe acute respiratory syndrome and MERS; middle east respiratory syndrome [1]. In December, 2019, the human population of Wuhan, China was affected by severe cases of pneumonia and respiratory problems. The correct etiology of the infection could not be traced that time. WHO reported the virus as a novel coronavirus (2019-nCoV). The disease was named as severe acute respiratory syndrome coronavirus-2 (SARS-CoV-2). The virus was first identified from a single individual. Subsequently the virus was verified in sixteen more cases [2,3].

It is expected that the virus might be bat origin [4], and the infection transmission might be initiated from a seafood market (Huanan Seafood Wholesale Market) of China [5,8]. By April 27, 2020, the density of the infected population has reached 2,908,527 and 203,332 deaths, world wide [9].

The sings symptoms of COVID-19 includes, coughing, breath shortening and fever. The most severe attack of the virus may cause death. In some cases the attack may result SARS (severe acute respiratory syndrome) or pneumonia. The symptoms of infection appears in 2-14 days [6].

The coronavirus (2019-nCoV) is genetically related to the coronavirus that caused the SARS-2003, however the diseases they caused are quite different [7]. The generic features and some clinical findings of the infection have been reported recently [8,10,11]. International air travel contributed the international spread of the infection. The infection has got global attention regarding its elimination and control [12].

The whole world is highly concerned with drastic future forecast of the disease. The scientists and researchers, therefore focus the development of mathematical model. The model not only helps estimating dynamics of the transmission of the virus but other important forecasts. Recent mathematical modeling includes [5,13–15]. These models mainly focused the spreading of coronavirus or basic reproduction number of coronavirus, ($R_0$). The authors followed intrinsic growth rate and the serial intervals. Wu et al in their study focus the forecasting and Newscasting of the novel coronavirus both nationally and internationally. The authors used Markov Chain Monte Carlo methods in their study [13].

However, these models don’t the discuss the origin(bat) and the route of spreading (seafood market). Chen et al extended this work. The authors in their work presented a comprehensive model of novel coronavirus [16].

We, in this study focus the formulation of more comprehensive mathematical model of COVID-19. The proposed model incorporates hospitalized class, Quarantined class and non contaminated stuff class. These classes, so far the author information, were not included before. Also till date non of the authors has addressed the Optimal Control model of COVID-19. We find the initial rate of disease transmission, $R_0$ and conduct the sensitivity test of $R_0$. We find the most sensitive parameters, regarding transmission, of the proposed model. On the basis of sensitivity indexes we propose Non Pharmaceutical control variables and formulate the optimal control model. We use numerical simulations to test the efficiency of applied NPI’s.

2. Model Formulation

The model incorporates human population and the stuff being open to the shedding of coronavirus. The human population is divided in seven sub-classes while the stuff is divided in two sub classes. The sub-classes of Human population are: susceptible humans class ($S$), the exposed humans class ($E$), symptomatic infected humans class ($I_1$), the quarantined class ($Q$), the hospitalized class $H$, asymptomatic infected humans class ($I_2$), and recovered humans class ($R$). $W_i$: denote the stuff required by the human class for its livelihood. $W_i$ denotes the surfaces/stuff shedded/stained with corona virus.

The susceptible human can catch infection from any of the infected class ($E, I_1, I_2, H$) and also from the contaminated stuff at different rates. A person who has contact with any of these infected classes is exposed to the attack of coronavirus. These exposed individuals are placed in infectious class $I_1$ if they develop sign symptoms of the disease and are quarantined ($Q$) otherwise. Some of the exposed individuals do not develop signs and symptoms and are silently infectious they are placed in class $I_2$. The infectious individuals with symptoms and those in quarantine are shifted to hospital for proper facilitation like PCR test and anti body tests. Most of the infected individual recover after 14 days however some of the infected individuals face problems in inhalation and are put on ventilators till recovery or death.

The mathematical model of the disease is given by the following set of coupled differential equations:

\[
\begin{align*}
S & = \Lambda - (\beta_1 (I_1 + c_2 I_2 + c_3 H + c_4 E) + \beta_2 W_i) S - \mu S \\
E & = (\beta_1 (I_1 + c_2 I_2 + c_3 H + c_4 E) + \beta_2 W_i) S - (\kappa_E + \mu) E \\
I_1 & = r_1 \kappa_E E - (k_1 + D_2 + \mu) I_1 \\
Q & = r_2 \kappa_E E - (k_0 + \mu) Q \\
H & = k_1 I_1 + k_0 Q - (r_1 + D_1 + \mu) H \\
R & = r_3 H + r_2 I_2 - \mu R \\
I_2 & = (1 - (r_1 + r_2)) \kappa_E E - (r_a + D_2 + \mu) I_2 \\
W_i & = A - (\xi_1 I_1 + \xi_2 I_2 + \xi_3 H) W_i - e_5 W_i \\
W_i & = (\xi_1 I_1 + \xi_2 I_2 + \xi_3 H) W_i - (e + e_5) W_i
\end{align*}
\]

The following Table (1) contains the values of the different parameters used in the model (1).

3. Model Analysis

Here, in this section 3 properties of the model; Disease Free Equilibrium, Invariant region and the Basic Reproduction Number are discussed.

3.1. Invariant Region

The time derivative of the state variables denote the change that occur in density of the concerned compartment and hence are taken non negative. Similar is the case of parameters. The total human population denoted by $\tilde{N}$ and the total stuff denoted by $W$ and are obtained by adding all the compartments of the concerned population. $\tilde{N}$ and $W$ are given by:

\[
\begin{align*}
\dot{\tilde{N}} & = \Lambda - \mu N - D_2 (I_1 + I_2) - D_1 H, \\
\dot{\tilde{W}} & = A - (\xi_1 I_1 + \xi_2 I_2 + \xi_3 H) W_i - e_5 W_i
\end{align*}
\]
\[ W = A - e_5 W - e W_i. \]  \hspace{1cm} (3)

From the above equations we have:

\[ N \leq N(0)e^{-\mu t} + \frac{A}{\mu} (1 - e^{-\mu t}) \Rightarrow N \leq \frac{A}{\mu} \text{ when } t \to \infty. \]

And

\[ W \leq W(0)e^{-\epsilon t} + \frac{A}{\epsilon X} (1 - e^{-\epsilon t}) \Rightarrow W \leq \frac{A}{\epsilon X} \text{ when } t \to \infty. \]

Using reference [23], we claim the following result:

**Proposition 3.1.** Given the region \( \Omega \) defined by

\[ \Omega = \left( S, E, I_1, Q, H, R, I_2, W_1, W_2 \right) \in R^8 N \leq \frac{A}{\epsilon X}, W \leq \frac{A}{\epsilon X}. \]

is positively invariant domain, also the model is epidemiologically and mathematically well posed as all the trajectories are forward bounded.

\[ F = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}; \text{ the jacobian of the column matrix } \mathbf{f}. \]

\[ F = \begin{pmatrix}
    a\beta_1c_4A \\
    a\beta_1A \\
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0
  \end{pmatrix}
\]

and

\[ v = \begin{pmatrix}
    v_1 \\
    v_2 \\
    v_3 \\
    v_4 \\
    v_5 \\
    v_6 \\
    v_7 \\
    v_8 \\
    v_9
  \end{pmatrix}
\]

The column entries of matrix \( v \) denotes the individuals that enter the infected class or leave the infected class, excluding those coming from susceptible class.

\[ V = \frac{\partial \mathbf{v}}{\partial \mathbf{w}}; \text{ the jacobian of the matrix } \mathbf{v}. \]

\[ V = \begin{pmatrix}
    -\left(\kappa E + \mu\right) \\
    r_1\kappa E \\
    r_2\kappa E \\
    (1 - (r_1 + r_2))\kappa E \\
    0 \\
    0 \\
    \xi_1W \\
    0 \\
    0 \\
    \xi_2W
  \end{pmatrix}
\]

The rate of spread of infection after its initial outbreak is called disease's Reproduction number. It is denoted by \( R_0 \) and is find by the following formula of generation matrix \([22, 24, 25]\), \( R_0 = \rho(-FV^{-1}) \), \( \rho \) being the spectral radius.

Here

\[ f = \begin{pmatrix}
    f_1 \\
    f_2 \\
    f_3 \\
    f_4 \\
    f_5 \\
    f_6 \\
    f_7 \\
    f_8 \\
    f_9
  \end{pmatrix} = \begin{pmatrix}
    \beta_1(1 + c_2I_2 + c_3H + c_4E + \beta_2W_1)S \\
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0
  \end{pmatrix}
\]

The column entries of matrix \( f \) denotes the newly infected individuals.

\[ R_0 = \frac{c_4a\beta_1\Lambda}{\mu(\kappa E + \mu)} + \frac{r_1\kappa_1a\beta_1\Lambda}{\mu(\kappa E + \mu)(k_1 + D_2 + \mu)} + \frac{1 - (r_1 + r_2)k_2\delta a\beta_1\beta_2\Lambda}{\mu(\kappa E + \mu)(k_1 + D_2 + \mu)} + \frac{a\beta_1c_3\Lambda}{\mu(\kappa_E + \mu)(k_1 + D_2 + \mu)} \]
3.3. Sensitivity analysis of $R_0$

The sensitivity of $Z$ with respect to the parameter $K$ is defined as [22].

$$\frac{\partial Z}{\partial K} = \frac{\partial Z}{\partial K}.$$

The following Table (2) contains the sensitivities of the indices of the model is given below:

3.4. Control variables on the basis Sensitivity indices

To control the further transmission of the disease in the community, we use sensitivity test to check the absolute value of index of a parameter. Greatest the absolute value of the index of parameter, highest would be role of the parameter in disease transmission. All the parameters appearing in $R_0$ have impact on the initial rate of corona transmission. However the control of some of these parameters is beyond human control, like the incubation period etc.

The highest sensitivity indices are: $\kappa_1 = 1.9, \beta_1 = 0.85, \mu = -0.9999, A = 0.14, \beta_2 = 0.1454, k_1 = -0.75, e_X = -0.1817$ and $\varepsilon = -0.1091$.

$\kappa_1$ is the incubation period of COVID-19 in human class. Though the sensitivity index of $\kappa_1$ is high but the control of incubation period of a disease is beyond the human control. Therefore we can’t intervene it. Similarly, $\mu$, the natural death rate of human has got high sensitivity index and is inversely proportional to $R_0$. That is increase in $\mu$ by 10% would decrease $R_0$ by 9.9%. However this increase is impractical and we can’t intervene. A, the stuff/food supply to market and $e_X$, the expiry period of food/stuff, effect the transmission of the disease. However this effect is not significant due to low sensitivity indices of the concerned parameters. Also this intervention would effect the supply and demand ratio of the market. Perhaps no government can afford to disturb the supply and demand ratio of the market.

$\beta_1$, the transmission probability of the infection from infected human to susceptible human, through direct contact or communication, has got the sensitivity of index 0.85. So a decrease of 50% in $\beta_1$ would reduce transmission rate by 42%. Control of $\beta_1$ directly effect the exposed class by reducing its density and it indirectly reduce the shedding coefficients of $I_1$ and $I_2$.

Case detection period, $k_1$, has got high sensitivity index and is inversely proportional to $R_0$. That is, quick case detection significantly reduce the transmission rate of disease. The sensitivity index of $\varepsilon$, the life time of virus, is low, however we introduce control variable, the sanitizer, here because sanitizer is more easily accessible intervention. Similar is the case of $\beta_2$, the chances of catching infection from contaminated stuff.

We address here the parameters, $K_i$; the detection period, $\beta_1$; social contacts, $\beta_2$; rate of catching infection from contaminated stuff/food products and $\varepsilon$; the life time of coronavirus. We introduce control variables $h_1$; stay home, $h_2$; use mask and wash hands, $h_3$; testing labs, and $h_4$; sanitizer, to control the sensitive parameters of the model.

4. Optimal Control Model

Based on the above control variables we propose the following optimal control model.

$$\begin{align*}
\dot{S} &= A - (\beta_1(1 - h_1)(I_1 + c_2I_2) + c_1H) + c_4E) + \beta_2(1 - h_2)W_1)S - \mu S \\
\dot{E} &= (\beta_1(1 - h_1)(I_1 + c_2I_2) + c_1H) + c_4E) + \beta_2(1 - h_2)W_1)S - (\kappa_1 + \mu)E \\
\dot{I}_1 &= \tau_1\kappa_1E - (\kappa_1(1 + h_1) + D_2 + \mu)I_1 \\
\dot{Q} &= \tau_2\kappa_1E - (\kappa_1 + \mu)Q \\
\dot{H} &= k_1(1 + h_1)I_1 + k_2Q - (\tau_1 + D_1 + \mu)H \\
\dot{R}_1 &= \tau_1H + r_1I_2 - \mu R \\
\dot{I}_2 &= (1 - (\tau_1 + r_2))k_1E - (\tau_1 + D_2 + \mu)I_2 \\
\dot{W}_1 &= A - (\xi_1I_1 + \xi_2I_2 + \xi_3H)W_1 - (e_X)W_1 \\
\dot{W}_2 &= (\xi_1I_1 + \xi_2I_2 + \xi_3H)W_1 - ((\varepsilon(1 + h_4)) + e_X)W_1 \\
\end{align*}$$

To minimize/reduce the density of infected human population and contaminated stuff we define an objective function as:

$$J(h_1, h_2, h_3, h_4) = \int_0^T (a_1I_1 + a_2I_2 + a_3W_1 + \frac{1}{2}(g_1h_1^2(t)$$

$$+ g_3h_3^2(t) + g_3h_2^2(t) + g_4h_4^2(t))dt$$

The weight constants of the symptomatic infectious human, $I_1$ and asymptomatic infectious human population, $I_2$ are $a_1$ and $a_2$. The weight constants of contaminated stuff is $a_3$.

The cost of different interventions in disease control is represented by the term:

$$\frac{1}{2}(g_1h_1^2(t) + g_2h_2^2(t) + g_3h_3^2(t) + g_4h_4^2(t)).$$

Let us denote the control function by $J_1$ then we need to find $(h_1^*, h_2^*, h_3^*, h_4^*)$ so that

$$J_1 = J(h_1^*, h_2^*, h_3^*, h_4^*)$$

subject to the system (4), that is

$$J_1 = \min J(h_1, h_2, h_3, h_4), (h_1, h_2, h_3, h_4) \in D).$$

$D$ denoted the set of control variables and is defined as:

$$D = \{(h_1, h_2, h_3, h_4) | h_i(t) \text{ is lebesgue measurable on } [0, T], 0 \leq h_i(t) < 1, i = 1, 2, 3, 4\}.$$
4.1. Solution of the proposed optimal control model

In this Section, we consider the control system (4) with initial conditions at \( t = 0 \) to investigate the existence of the control problem. For this we use [27] and the results there in.

We claim that the solution of the state system (4) is bounded. Because the control variable is bounded lebesuge measurable and the initial conditions are non-negative.

Next we prove that the system has optimal solution. For this we introduce Lagrangian and Hamiltonian as:

\[
L(t) = a_1I_1 + a_2I_2 + a_3W_t + \frac{1}{2} (g_1h_1^2 + g_2h_2^2 + g_3h_3^2 + g_4h_4^2),
\]

(6)

and

\[
\mathbb{H}(t) = L(t) + \ell_1 \frac{dS}{dt} + \ell_3 \frac{dE}{dt} + \ell_4 \frac{dQ}{dt} + \ell_5 \frac{dH}{dt} + \ell_6 \frac{dR}{dt} \]

(7)

where \( \mathbb{H}(t) \) is meant for the minimal value of the Lagrangian.

**Theorem 4.1.** The set \((h_1^*, h_2^*, h_3^*, h_4^*)\) of optimal control, minimize \( J \) over \( D \), subject to the initial conditions specified at \( t = 0 \).

**Proof.** Since both state and control variables are non-negative. So using [28], the objective function is convex in control variables \( h_i, i = 1, 2, 3, 4 \).

Clearly from def, \( D \) is closed and convex.

Further, since the optimal system is bounded hence the set of optimal control is compact. And the control variables \( h_1, h_2, h_3, h_4 \) occur as quadratic. Therefore the integrand in (5) is convex on control set \( D \).

Also, all \( Y, \) for \( Y \in \Omega \), are bounded. So We can find a constant \( \dot{\sigma} > 1 \) and positive numbers \( \omega_1 \) and \( \omega_2 \) such that

\[
J(h_1, h_2, h_3, h_4) \geq \omega_1(|h_1|^2, |h_2|^2, |h_3|^2, |h_4|^2) \dot{\sigma} - \omega_2.
\]

Which completes the proof of existence of an optimal control. □

Next we characterize the control variables of the proposed model. We use Pontryagin Maximum Principle [29]; stated here for convenience.

\[ M = (S^*, E^*, I_1^*, Q^*, H^*, R^*, I_2^*, W_1^*, W_2^*)^T \]

be the states associated with control variables \((h_1^*, h_2^*, h_3^*, h_4^*)\).

Let \((y, h)\) be the solution of (4), for \( y \in \Omega \) and \( h = (h_1^*, h_2^*, h_3^*, h_4^*) \) then \( \exists \) some vector function

\[
\ell = (\ell_1, \ell_2, ..., \ell_n)
\]

so that:

\[
\frac{dy}{dt} = \frac{\partial \mathbb{H}(t, y, h, \ell)}{\partial \ell}
\]

\[
0 = \frac{\partial \mathbb{H}(t, y, h, \ell)}{\partial h}
\]

\[
\ell' = -\frac{\partial \mathbb{H}(t, y, h, \ell)}{\partial y}
\]

For necessary conditions on \( \mathbb{H} \) we prove the following result.

**Theorem 4.2.** The necessary condition for control \( h = (h_1^*, h_2^*, h_3^*, h_4^*) \) to be optimal with corresponding state \( M \) is that \( \exists \) adjoint variables:

\[
\ell_i \text{ for } i = 1, 2, ..., 9 \text{ satisfying:}
\]

\[
\ell_i^* \text{ for } i = 1, 2, ..., 9
\]

with transversality conditions or boundary conditions

\[
\ell_i(t_{end}) = \ell_i(T) = 0 \text{ for } i = 1, 2, ..., 9.
\]

And the optimal controls \( h_1^*, h_2^*, h_3^*, h_4^* \) are given by

\[
\begin{align*}
    h_1^* &= \max \left\{ \left( \ell_1 - \frac{2}{g_1} S \left( h_1 + c_1 I_2 + c_2 H + c_4 E \right) \right) \right\}, 0 \\
    h_2^* &= \max \left\{ \left( \ell_2 - \frac{W_2 \beta S}{g_2} \right) \right\}, 0 \\
    h_3^* &= \max \left\{ \left( \ell_3 - \frac{h_3}{g_3} \right) \right\}, 0 \\
    h_4^* &= \max \left\{ \left( \ell_4 W_4 \right) \right\}, 0
\end{align*}
\]

**Proof.** First we differentiate \( \mathbb{H} \) given in (7), with respect to the state variables involved in the model. As a result we get the system

\[
\frac{d\ell_i}{dt} = 0, \quad \frac{d\ell_1}{dt} = 0, \quad \frac{d\ell_2}{dt} = 0, \quad \frac{d\ell_3}{dt} = 0.
\]

Solving this system we obtain the results shown in (10)-(13). □

Here, the second derivative of Lagrangian \( L \) with respect to \( h_1^*, h_2^*, h_3^*, h_4^* \) is +ve. This show that the optimal control is maximum at control \( h_1^*, h_2^*, h_3^*, h_4^* \).

We put these values in system (4) and propose the following optimal control model.

\[
\begin{align*}
    S^* &= \Lambda - ((1 - h_1^*) \beta_1 I_1^* + c_1 I_2^* + c_3 H^* + c_4 E^*) \\
    E^* &= ((1 - h_1^*) \beta_1 I_1^* + c_1 I_2^* + c_3 H^* + c_4 E^*) \\
    H^* &= (k_1 H^*) S^* - (k_1 E^*) \\
    I_1^* &= r_1 k_2 E^* - (k_1 (1 + h_1^*) + D_2 + \mu) I_1^* \\
    Q^* &= r_2 k_2 E^* - (k_5 Q^*) \\
    H^* &= (k_1 H^*) I_2^* + k_3 Q^* - (r_1 + D_1 + \mu) H^* \\
    R^* &= r_1 H^* + r_2 I_2^* - \mu R^* \\
    I_2^* &= (1 - (r_1 + r_2) k_2 E^* - (r_1 + D_2 + \mu) I_2^* \\
    W_1^* &= A - (\xi_1 I_1^* + \xi_2 I_2^* + \xi_3 H^*) W_1^* - (c_1 + h_4^*) W_1^* \\
    W_2^* &= (\xi_1 I_1^* + \xi_2 I_2^* + \xi_3 H^*) W_2^* - (c_1 + h_4^*) W_2^*
\end{align*}
\]
with
\[
H^* = a_1 I_1 + a_2 I_2 + a_3 W^*_d \\
+ \frac{1}{2} \left( g_1 \left( \text{maximum} \left\{ (\ell_1 - \ell_2) S g_1 (I_1 + C_1 J_2 + C_3 H + C_4 E), 1 \right\}, 0 \right) \right)^2 \\
+ g_2 \left( \text{maximum} \left\{ (\ell_1 - \ell_2) W H S, 1 \right\}, 0 \right)^2 \\
+ g_3 \left( \text{maximum} \left\{ (\ell_5 - \ell_1) d_i, 1 \right\}, 0 \right)^2 \\
+ g_4 \left( \text{maximum} \left\{ (\ell_6 W_h), 1 \right\}, 0 \right)^2 \\
+ \ell_4 \frac{dW^*_d}{dt} + \ell_5 \frac{dH^*}{dt} + \ell_6 \frac{dE^*}{dt} + \ell_7 \frac{dI^*_1}{dt} + \ell_8 \frac{dW^*_1}{dt} + \ell_9 \frac{dW^*_2}{dt} \\
(15)
\]

5. Numerical simulations

We solve the optimality system equation (14), numerically using RK4 method. The method solve the state system (4), forward in time and the adjoint system (8), backward in the time and the system (9), the controls are updated continuously using system (10) to (13). The process is continued until the results at the consecutive iterations are too closed. For detail [30], we assign the following values to weight constants. $a_1 = 0.0181, a_2 = 0.010, a_3 = 0.1, g_1 = 0.081, g_2 = 0.01, g_3 = 0.1$ and $g_4 = 0.008$. We have used $S = 2000, E = 10, I_1 = 10, Q = 5, H = 5, R = 5, I_2 = 15, W^*_d = 4000$ and $W_1 = 100$.

In the following figures we present the effect of proposed non pharmaceutical interventions. Since the initial rate of disease transmission, $R_0$, is very high. Therefore the density of susceptible population reduces very soon in the initial period of few days. And in the same period the density of exposed class reaches the peak as shown in Figs. (1) and (2). The figures show that with in small period of 5-10 days, 1800 out 2000 susceptible humans catch infection. The proposed non pharmaceutical interventions helps in control and reduction of disease but in particular it helps to flatten the curve of infection. Curve flattening means to reduce the patient burden in the hospitals and quarantine centers as shown in Figs. (3)-(5). Fig. (8) shows that recovery with out control is high than that of with control. The reason is that our control variables are non pharmaceutical and therefore recovery is infect a ratio. That is about 80% to 95% of total infected individuals recover. So greater the number of infected individuals, greater would be the number of recoveries. Face mask and sanitizer are not only useful in self defence but also play better role in control of density of contami-
**Fig. 3.** The effect of interventions on the density of symptomatic infectious class.

**Fig. 4.** The fig represents the density of the quarantine class with and without control variables.

**Fig. 5.** The fig represents the burden of patients in hospitals with and without control variables.
**Fig. 6.** The fig represents the recovered human population with and without interventions.

**Fig. 7.** The fig represents the effect of interventions on asymptomatic human class. The difference of the graphs denotes the effectiveness of control variables.

**Fig. 8.** The fig represents the effect of interventions upon the density of contaminated stuff.
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Fig. 9. The fig represents the intervention ‘STAY HOME’.

Fig. 10. The fig represents the intervention ‘FACE MASK’.

Fig. 11. The fig represents the intervention ‘QUICK CASE DETECTION’ with help of laboratory tests.
nated stuff by reducing the shedding coefficients $\xi_1$, $\xi_2$ and $\xi_3$, as shown in Fig. (6).

6. Conclusion

In this work we tried to seek the optimal control of COVID-19 pandemic. The sensitivity test of the reproduction number shows that non of the parameters and particularly easily commendable parameters, have got the dominant role in disease transmission. Therefore the control of this disease is comparably tough but not disappointing. We have introduced four control variables in the transmission concerned parameters; $\beta_1$, the transmission probability of the infection from infected human to susceptible human, Case detection period, $\tau$, $\epsilon$, the life time of virus and $\beta_2$, the contact with non-living contaminated items.

The disease is not so dangerous in the sense that it takes about 14 days in its recovery from mild cases and 30–40 days in severe cases with disease induced mortality rate 1% to 4%. However the disease transmission rate is no doubt alarming as shown by the Fig. (2). The figure demonstrate that if there is no control (variables/interventions), 1800 out 2000 susceptible individuals may be infected (exposed) in very short period. As such a circumstances no agency fighting against COVID-19 could be successful due to its limited resources. The same is the problem with hospitalized and quarantined class as shown in Figs. (4) and (5). No hospital nor quarantined center can accommodate such a huge number of infected individuals.

However the implementation of all the strategies(control variables, $h_2$, $h_3$ and $h_4$) at the same time for at least 50 days may esuriently reduce the number of new cases. Figs. (2), (5) and (4) in the presence of active disease interventions the density of exposed (infected) individual will be reduced from 1800 to 650 individuals and consequently reducing the burden of hospitalization to 260 and quarantine to 170 only. Also considerable decrease is observed in the contaminated class as shown in Fig. (8).

It is worth mentioning to keep the non pharmaceutical interventions continue till the elimination of the pandemic. Otherwise the new outbreaks will occur, and due to highest rate of disease transmission, will infect the community once again.

Declaration of Competing Interest

The authors declare that they have no competing interests

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