Application of Finite Difference Method in Image Reconstruction of Electrical Capacitance Tomography

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Abstract. Electrical capacitance tomography is to invert the medium distribution inside the object by using a limited set of measured capacitance values. In order to realize image reconstruction, the sensor sensitive field is solved by finite difference method under the condition of known sensor distribution and internal permittivity distribution of pipeline, and satisfactory results are obtained. According to the calculated sensitive field, the distribution image of the fluid on the fault plane is reconstructed by using the back-projection algorithm. Back-projection algorithm has the character of simple structure and fast speed of reconstructing images. However, the number of projections obtained by the capacitive tomography system is much smaller than the number of image pixels and the soft field characteristics of the imaging system make it difficult to accurately image, so only an approximate phase distribution image can be reconstructed.

1. Introduction
The technology of capacitance tomography (CT) is largely applied to medical science. Since the technology of CT has the advantage of reconstructing images by way of non-invading, the application field of CT is not limited in medical science. In the past decade, the application of CT in industrial field has made great progress. The application of CT in industrial field is called industrial process tomography (IPT) or briefly called process tomography (PT). The tomography of progress in industrial manufacturing is the same urgent as that of medical tomography in disease diagnoses. The technology of PT can get the distribution and movement of the materials in the container by way of non-invading and educe the model of process inside, which can be used as the basis of optimizing process design and operation, so PT is used more and more widely [1, 2]. After several years’ theoretical research and industry applications, the applying of PT to measurement of two-phase flow has gained preliminary achievement. In recent years many kinds of PT techniques based on electricity sensitivity have been developed, which is an important branch of PT mainly including electrical capacitance tomograph (ECT), electrical resistance tomography (ERT), electrical impedance tomograph (EIT) and electromagnetic tomography (EMT).

2. Practical observing system of ECT
A 12-electrodes system is designed in the experiment, which can provide 66 projections. The sensor of
the system used PVC pipe with 16cm outside diameter, 5mm thickness and 150cm length. 12 electrodes with 150cm length and 2cm width are symmetrically fixed on the surface of the PVC pipe. The dielectric in the experiment is air (corresponding with the continuous phase) and water (corresponding with the discrete phase). The water is filled in PVC pipe of 2cm diameter. In the process of image reconstruction, \( \varepsilon_i \) is the dielectric constant of water and its value is 81, \( \varepsilon_0 \) is the dielectric constant of air and its value is 1.

The sketch map of the cross section of the 12-electrodes ECT sensor designed is showed as the figure above.

In each measurement, one electrode is connected to positive voltage \( V_c \) and the other electrodes are connected to the ground so that the capacitance value \( C_{i,j} \) \( (i = 1,2,\cdots11, \ j = 1,2,\cdots12) \) between the positive electrode and the other electrodes can be measured in turn. The positive electrode is called source electrode and the electrodes that connected to the ground are called detection electrode.

![Figure 1. Structure of electrical capacitance tomography system](image)

Every electrode has three working state: exciting state, detecting state and free state. When measuring, if choosing the \( i \) th electrode as the exciting electrode, the \( (i+1) \) th electrode to 12th electrode are the detecting electrodes, then the 1st electrode to \( (i-1) \) th electrode are in free state. The process of measuring the capacitance value is first to choose 1st electrode as the exciting electrode, the 2nd electrode to 12th electrode as the detecting electrodes, to measure the capacitance values between 1st electrode and the other electrodes(11 values), then to choose 2nd electrode as the exciting electrode, the 3rd electrode to 12th electrode as the detecting electrodes, to measure the capacitance values between 2nd electrode and the other electrodes(10 values), finally to choose 11th electrode as the exciting electrode, the 12th electrode as the detecting electrodes, to measure the capacitance values between 11th electrode and the 12th electrode (1 value), then there are totally \( 2\cdot11\cdot12 = 66 \) data measured.

3. Distribution of Sensitivity Field

The method of calculating the sensitivity field is first to get the electric potential difference between the nodal points, then to use formula \( C_{i,j} = \frac{Q_{i,j}}{U_{i,j}} \) to calculate the corresponding capacitance value \( C_{i,j}(k) \) \( (i, j = 1,2,\cdots,12) \). And then to measure the capacitance values when the pipe is filled with water and when the pipe is filled with air. Alternately to set the dielectric constant in the \( k \) th unit in the cross section of the pipe as \( \varepsilon_2 \) and that of the other units as \( \varepsilon_1 \). Finally to calculate the sensitivity values of every electrode pair in the unit according to the formula below.
\[ S_{i,j}(k) = \mu(k) \ast \frac{C_{i,j}(k) - C_{i,j}^{\text{empty}}}{(C_{i,j}^{\text{full}} - C_{i,j}^{\text{empty}})(\varepsilon_i - \varepsilon_0)} \]  

\( C_{i,j}(k) \) is the capacitance value when the dielectric constant in the \( k \) th unit is \( \varepsilon_1 \), and the dielectric constant in the other units is \( \varepsilon_0 \). \( C_{i,j}^{\text{empty}} \) and \( C_{i,j}^{\text{full}} \) is the capacitance value when the pipeline is respectively filled with air and water, \( \varepsilon_1 \) is the dielectric constant of relatively large electric constant dielectric, \( \varepsilon_0 \) is the dielectric constant of relatively small electric constant dielectric, \( \mu(k) \) is the ratio of the area of the total cross section of pipeline and the area of the unit [5].

The sensitivity field of the sensor is obtained by disposing the data measured according to the method stated above.

![Figure 2.a. The sensitivity field of the electrodes neighboring (the 1th electrode and the 2th electrode)](image)

![Figure 2.b. The sensitivity field of the electrodes separated by an electrode (the 1th electrode and the 3th electrode)](image)

![Figure 3.a. The sensitivity field of the electrodes separated by two electrodes (the 1th electrode and the 4th electrode)](image)

![Figure 3.b. The sensitivity field of the electrodes separated by three electrodes (the 1th electrode and the 5th electrode)](image)
The figures from figure 9 to figure 14 above are three-dimension figure of the sensitivity field. In these figures, X - Y plane is the cross section of ECT sensor, Z-axis is the relative sensitivity.

From these figures, some conclusions can be made.

1) The sensitivity field of ECT sensor is non-linear, even in some place there is negative sensitivity, therefore, the capacitance-measuring field is a kind of soft field.

2) The sensitivity field of electrodes neighboring is close to the electrodes and appeared as a single peak. The amplitude of the peak is relatively large; the sensitivity field of electrodes not neighboring appears as the shape of saddle and the amplitude of the peak are relatively small. The sensitivity in the center part of the saddle is relatively weak and the sensitivity is relatively strong close to the two electrodes.

3) The sensitivity field of ECT sensor appeared as a kind of area distribution. The change of capacitance values concentrates in the area of two electrodes towards. Outside this area the change of capacitance value is very small. Closer to the electrodes, the change of the capacitance value is more evident. Consequently the center part of the sensor is a low sensitivity area.

4) Farther the distance of the two electrodes is, weaker the sensitivity field of them is.

5) As to different pairs of electrodes, the positive sensitivity area locates in different area; the area except the positive area is negative area.

4. Electric capacitance tomography

After having obtained the sensitivity field and having measured the capacitance values when there is water in the pipeline, the dielectric distribution in the pipeline can be calculated according to the formula below.

\[
G(k) = \sum_{i=1}^{11} \sum_{j=1}^{12} \frac{C_{i,j} - C_{i,j}^\text{empty}}{(C_{i,j}^\text{full} - C_{i,j}^\text{empty})(\varepsilon_i - \varepsilon_0)} \times \frac{S_{i,j}(k)}{\sum_{m=1}^{11} \sum_{n=1}^{12} S_{m,n}(k)}
\]  

(2)

In the formula above, \( S_{i,j}(k) \) is the sensitivity that the \( k \)th image element corresponds with the electrode pair \((i, j)\), \( C_{i,j}^N \) is the normalized value of the measured capacitance value between electrode \( i \) and electrode \( j \).

The process of ECT is showed as the flow chart below.
In the condition of software and hardware being well prepared, the data is measured. There are totally three models in the experiment. Model 1 is the pipe filled with stratified flow; Model 2 is the pipe filled with Kernel Streaming; Model 3 is the pipe filled with annular flow; The data measured of model 1 is as the Table below shows.

**Table 1.** The capacitance value measured when the pipe filled with water is in the center of the sensor

| $l$ (pF) | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 |
|----------|----|----|----|----|----|----|----|----|----|----|
| -2       | 33.4 |    |    |    |    |    |    |    |    |    |
| -3       | 23.7 | 31.7 |    |    |    |    |    |    |    |    |
| -4       | 20.1 | 21.1 | 32.1 |    |    |    |    |    |    |    |
| -5       | 18.5 | 17.7 | 21.2 | 31.3 |    |    |    |    |    |    |
| -6       | 20.2 | 17.9 | 18.6 | 22.1 | 32.2 |    |    |    |    |    |
| -7       | 19.3 | 29.5 | 16.2 | 19.1 | 21.7 | 30.4 |    |    |    |    |
| -8       | 23.6 | 19.1 | 16.1 | 16.6 | 18.7 | 21.3 | 30.1 |    |    |    |
| -9       | 18.9 | 22.7 | 18.5 | 20.7 | 17.0 | 19.1 | 20.3 | 31.5 |    |    |
| -10      | 19.4 | 19.5 | 21.3 | 20.0 | 15.9 | 16.8 | 17.0 | 20.8 | 31.7 |    |
| -11      | 22.8 | 20.3 | 18.6 | 15.3 | 19.3 | 19.1 | 14.8 | 16.9 | 21.7 | 31.5 |
| -12      | 32.5 | 21.8 | 18.3 | 16.7 | 17.8 | 18.9 | 14.7 | 16.2 | 18.8 | 22.1 |

The images of model 1 to model 3 are reconstructed according to the sensitivity field and the capacitance values measured as the figures show below.
Table 2. The image reconstructed when a pipe filled with water is in different place of the sensor

|                     | stratified flow | Kernel Streaming | annular flow |
|---------------------|-----------------|------------------|--------------|
| the image before    |                 |                  |              |
| reconstructed       |                 |                  |              |
| the image after     |                 |                  |              |
| reconstructed       |                 |                  |              |

In the Table above, the black background means air and the white part means water. The effect of image reconstructing is better when there is only a pipe filled with water placed in the sensor than that of when there are two pipes filled with water placed in the sensor. From the image reconstructed the conclusion can be made that the sensitivity function is ideal because the image reconstructed according to the sensitivity field and LBP algorithm roughly reflected the position and the shape of the object measured. The position of the object measured in the image reconstructed approximately corresponds with the practical position, which proves that the sensitivity field calculated is right. But the edges in the images are distorted and do not correspond with the practical object. This is because that the image element discretized in the cross section of the sensor is rough which leads to being difficult to approach round object on the one hand and the number of projection data that ECT system can obtain is far less than that of the image element of the images on the other hand. Therefore, it is only possible to reconstruct approximate image. What’s more, the “soft field” character of the measuring field and the non-uniform characteristic of the electric field makes the discrete-phase dielectric close to the electrodes enhance the degree of non-linear of the measuring field, which reduced the accuracy of image reconstructed. While the discretized discrete-phase dielectric in the center of the sensor has less effect on the measuring field, so when the pipe filled with water is in the center of the sensor, the image reconstructed is better and close to the practical object.

5. Conclusions

In the process of understanding the knowledge of ECT and writing programs, the conclusions can be made that it is flexible to use the method of numerical analysis to carry out the forward calculation of ECT. The boundary condition should be discretized according to practical conditions. In this paper FDM was used to carry out the forward calculation of the electromagnetic field and obtain the sensitivity field of the electromagnetic field. The image of the flow in the pipeline was reconstructed and the result is satisfied.

The technology of ECT is one of the process tomography that is applied and studied most often. The difficulties of ECT lie in the four aspects as below.

1) ECT involves the measurement of very small capacitance value so it is demanded that the sensor should have high sensitivity and good interference-resisting characteristic.

2) Another difficulty is the “soft field” characteristic of the capacitance-measuring field. That is to say that the distribution of sensitivity is influenced by the distribution of the dielectric measured.

3) Due to the limitation of the size of the pipe measured, it is impossible to install many electrodes on the surface of the pipe. The number of independent data measured is limited accordingly.
At present, the ECT system still has the shortcomings of big error and low quality of images reconstructed. Only through the improvements stated above, may the shortcomings be overcome. As a result, the ECT system will be applied more widely in industrial online monitoring and control.

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