Cooperative Adaptive Control for Cloud-Based Robotics
Patrick M. Wensing and Jean-Jacques E. Slotine

Abstract—This paper studies manipulator adaptive control when knowledge is shared by multiple robots through the cloud. We first consider the case of multiple robots manipulating a common object through synchronous centralized update laws to identify unknown inertial parameters. Through this development, we introduce a notion of Ensemble Sufficient Richness, wherein parameter converge can be enabled through teamwork in the group. The introduction of this property and the analysis of stable adaptive controllers that benefit from it constitute the main new contributions of this work. Building on this original example, we then consider decentralized update laws and the influence of communication delays on this process. Perhaps surprisingly, these nonidealized networked conditions inherit the same benefits of convergence being determined through ensemble effects for the group. Simple simulations of a planar manipulator identifying an unknown load are provided to illustrate the central idea and benefits of Ensemble Sufficient Richness.

I. INTRODUCTION

With the dawn of Internet 4.0, challenges facing next-generation robots present the opportunity to benefit from dynamic global datasets through networking in the “Cloud” [1]. Sensing and computation may be shared through non-collocated agents distributed across wide spatial and geographic extents. However, the dynamic nature of data/experience shared through the cloud may present as many challenges as it does opportunities. Coupling between the complex networked updates to these datasets introduces interesting questions regarding stability of the updates and their corresponding effects on the dynamics of individual agents. In large, these considerations have been ignored in the literature, due in part to difficulties in stability analysis that arise from nonlinear effects in the dynamics of robots as well as the time delays that are an inevitable feature of interface with cloud-based data.

This paper makes steps toward addressing this challenge through considering a prototype problem of cooperative adaptive control through the cloud. Adaptive control of robots seeks to learn unknown parameters that affect closed-loop control outcomes while simultaneously ensuring stable trajectory tracking. This problem has a rich history [2]–[8] to cite just a few (see also [9]). This classical work has seen renewed interest (e.g., [10], [11]), due in part to the increasing availability of torque-controlled robotics platforms [12]–[15]. Here we treat the case of a team of manipulators working with a common object that has unknown parameters, as shown in Fig. 1. Such a case might occur with, for instance, multiple robots learning in tandem [16]. Although learning and adaptation are traditionally identified as separate in spirit, connections and commonalities (particularly for model-based sensorimotor learning) offer interesting future prospects.

The stability of networked adaptive control has been treated previously in variety of other contexts. Chung and Slotine studied synchronization of networked robots, wherein individual agents could include adaptation of their own parameters [8]. Schwager et al. [17] studied optimal collective sensing with kinematic robot models and shared adaptation to learn environmental features. The current work instead considers dynamic robot models, with shared adaptation under time delays. This later consideration is of particular interest within the context of cloud-based knowledge sharing.

In addressing the need to converge on a common representation of knowledge across agents, this current work shares a great deal in common with the literature on consensus phenomena [18], [19]. Here, we show the benefits of enforcing consensus in cooperative adaptive control. Namely, conditions on converge of shared knowledge to an optimum value, more technically described as conditions on sufficient richness of the learning signals, inherit relaxed requirements through teaming. The introduction of a concept of Ensemble Sufficient Richness is introduced, with its supporting analysis for cooperative adaptive control constituting the main contribution of this work.

The paper is structured as follows. Section II describes preliminary background on manipulator adaptive control. Section III discusses the case of cooperative adaptive control under idealized network conditions, illustrating the role of ensemble sufficient richness in parameter convergence. Sections IV and V then extend this analysis to handle decentralized and time-delayed communications, which may be more realistic in practice. Section VI discusses how modeling error may be fused with tracking errors to further improve the performance of the collaborative adaptive control laws. Section VII illustrates the central benefits of the theory through a simple example in simulation.
II. Preliminaries

In this section, we briefly introduce classical results in adaptive control for a single manipulator. We consider a traditional setting, wherein inertial parameters are identified while tracking a desired trajectory. We begin by recalling the framework of Direct Adaptive Control from [3], [6], which uses tracking error to estimate parameters.

A. Direct Adaptive Control: Setup

Consider the standard manipulator equations for a rigid-body system with \( n_b \) bodies and \( n_j \) degrees of freedom

\[
\mathbf{H}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{\tau}
\]

where \( \mathbf{q} \in \mathbb{R}^{n_j} \) the vector of joint angles, \( \mathbf{H} \in \mathbb{R}^{n_j \times n_j} \) the mass matrix, \( \mathbf{C} \in \mathbb{R}^{n_j \times n_j} \) the Coriolis and centripetal terms, \( \mathbf{g} \in \mathbb{R}^{n_j} \) the generalized gravitation force, and \( \mathbf{\tau} \in \mathbb{R}^{n_j} \) the vector of actuator torques. It is well known that one can define a vector of inertial parameters \( \mathbf{a} \in \mathbb{R}^{10n_b} \) that collects unknown masses, first mass moments, and moments of inertia, and that the inverse dynamics defined by (1) are linear in these equations [3].

We consider the case of a smooth desired trajectory \( \mathbf{q}_d(t) \in \mathbb{R}^{n_j} \), and define the tracking error as

\[
\dot{\mathbf{q}} = \mathbf{q} - \mathbf{q}_d
\]

As a matter of notation, we denote reference velocities and accelerations

\[
\dot{\mathbf{q}}_r = \dot{\mathbf{q}}_d - \mathbf{A} \dot{\mathbf{q}}_r \quad \text{and} \quad \ddot{\mathbf{q}}_r = \ddot{\mathbf{q}}_d - \mathbf{A} \ddot{\mathbf{q}}
\]

If we further define

\[
\mathbf{s} = \dot{\mathbf{q}}_r = \dot{\mathbf{q}} - \dot{\mathbf{q}}_r = \dot{\mathbf{q}} + \mathbf{A} \ddot{\mathbf{q}}
\]

then regulation of \( \mathbf{s} \) to zero renders the sliding surface defined by

\[
\dot{\mathbf{s}} + \mathbf{A} \ddot{\mathbf{s}} = 0
\]

to be invariant. By design, the restriction dynamics on this surface themselves ensure \( \dot{\mathbf{q}} \) is regulated to zero. That is, if \( \mathbf{s} \to 0 \) then \( \ddot{\mathbf{q}} \to 0 \) due to the definition of \( \mathbf{s} \). Thus, with this strategy, tracking control has been reduced to the challenge of regulating \( \mathbf{s} \) to zero.

Original work in [3] introduced the adaptive control law

\[
\mathbf{\tau} = \dot{\mathbf{H}}(\mathbf{q}) \ddot{\mathbf{q}}_r + \dot{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}}_r + \mathbf{g}(\mathbf{q}) - \mathbf{K}_D \mathbf{s}
\]

where \( \dot{\mathbf{H}}, \dot{\mathbf{C}}, \dot{\mathbf{g}} \) are estimates of the structural components of the equations of motion based on estimated parameters \( \hat{\mathbf{a}} \). Letting the parameter error vector \( \hat{\mathbf{a}} = \mathbf{a} - \hat{\mathbf{a}} \), this choice of control law provides the following closed-loop dynamics

\[
\dot{\mathbf{H}} \dot{\mathbf{s}} + (\mathbf{K}_D + \mathbf{C}) \mathbf{s} = \mathbf{Y}(\dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r, \dot{\mathbf{q}}, \mathbf{q}) \mathbf{\hat{a}}
\]

where \( \mathbf{Y}(\dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r, \dot{\mathbf{q}}, \mathbf{q}) \) is the regressor [3] such that

\[
\mathbf{Y}(\dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r, \dot{\mathbf{q}}, \mathbf{q}) \hat{\mathbf{a}} = \mathbf{H}(\mathbf{q}) \ddot{\mathbf{q}}_r + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}}_r + \mathbf{g}(\mathbf{q})
\]

B. Direct Adaptive Control: Convergence Analysis

Considering a Lyapunov function

\[
V = \frac{1}{2} \mathbf{s}^T \mathbf{H} \mathbf{s} + \frac{1}{2} \hat{\mathbf{a}}^T \mathbf{P} \hat{\mathbf{a}}
\]

shows that

\[
\frac{d}{dt} V = -\mathbf{s}^T \mathbf{K}_D \mathbf{s} + \mathbf{s}^T \mathbf{Y} \hat{\mathbf{a}} + \hat{\mathbf{a}}^T \mathbf{P} \hat{\mathbf{a}}
\]

(7)

Yet, since \( \dot{\mathbf{a}} = \mathbf{a} \), this suggests the following update law

\[
\hat{\mathbf{a}} = -\mathbf{P}^{-1} \mathbf{Y}^T (\dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r, \dot{\mathbf{q}}, \mathbf{q}) \mathbf{s}
\]

(8)

Under the update law (8), (7) simplifies to

\[
\frac{d}{dt} V = -\mathbf{s}^T \mathbf{K}_D \mathbf{s}
\]

Thus, it follows that \( \mathbf{s} \to 0 \) as \( t \to \infty \). Further, classical arguments [3] show that if the trajectories are sufficiently rich, then parameter errors \( \hat{\mathbf{a}} \to 0 \) as \( t \to \infty \) as well. This sufficient richness condition is phrased mathematically by requiring so-called persistency of excitation as defined below.

Definition 1 (Persistently Exciting). A time varying matrix-valued quantity \( \mathbf{W}(t) \) is said to be persistently exciting if there exists \( \alpha > 0 \), \( T > 0 \) such that

\[
1 \int_t^{t+T} \mathbf{W}^T(\tau) \mathbf{W}(\tau) d\tau \geq \alpha \mathbf{I} \quad \forall t
\]

Theorem 1 (Parameter Convergence of Single-Robot Direct Adaptive Control [20]). Consider the system (1) with the control law (4) and parameter update law (8). Then, if the regressor \( \mathbf{Y} \) is persistently exciting, the parameter errors \( \hat{\mathbf{a}} \) satisfy \( \hat{\mathbf{a}} \to 0 \) as \( t \to \infty \).

III. CONTRIBUTION: DESIGN AND ANALYSIS OF SYNCHRONOUS CLOUD-BASED UPDATE LAWS

A. Setup

This section considers the previous update laws in the case of cloud-based adaptation. We begin by treating a case of a shared estimate, wherein multiple systems are seeking to estimate the same parameters. Each system \( i \) may be tracking their own trajectories \( \mathbf{q}_i, \mathbf{d}(t) \) while manipulating separate copies of a common object with unknown parameters \( \mathbf{a} \). Although this represents a simple extension to the previous development, it will provide grounding for the extensions in Sections IV and V that build toward estimating parameters in a decentralized fashion with communication delays.

Using the same control laws (4) from the previous section, we consider the dynamics of \( n \) systems

\[
\dot{\mathbf{H}}_i \dot{\mathbf{s}}_i + (\mathbf{C}_i + \mathbf{K}_D) \mathbf{s}_i = \mathbf{Y}_i (\dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r, \dot{\mathbf{q}}, \mathbf{q}) \mathbf{a}, \quad i \in \{1, \ldots, n\}
\]

(9)

Note that while each system has its own tracking errors \( \mathbf{s}_i \), there is a common parameter error \( \hat{\mathbf{a}} \) since robots are initially assumed to be working with shared parameter information.

Toward analysis of this system, define

\[
\mathbf{s} = [\mathbf{s}_1^T, \ldots, \mathbf{s}_n^T]^T
\]

(10)

\[
\mathbf{Y}_R = [\mathbf{Y}_1^T, \ldots, \mathbf{Y}_n^T]^T
\]

(11)
where $Y_R$ denotes the regressors as being stacked row-wise. Similar to the classical direct case, consider a Lyapunov function with contributions from tracking errors $s_i$ and parameter errors $\hat{a}$

$$V = V_1 + V_2$$

$$V_1 = \frac{1}{2} \sum_{i=1}^{n} s_i^T H_i s_i$$

$$V_2 = \frac{1}{2} \hat{a}^T P \hat{a}$$

Examining the terms from the tracking error and the parameter vectors:

$$\frac{d}{dt} V_1 = -\sum_{i=1}^{n} s_i^T K_D s_i + s^T Y_R \hat{a}$$

$$\frac{d}{dt} V_2 = \hat{a}^T P \hat{a}$$

Thus, if cloud communication is used to accomplish a centralized update by aggregating the updates of all robots,

$$\dot{\hat{a}} = -P_i^{-1} Y_R^T s$$

$$= -P_i^{-1} \sum_{i=1}^{n} Y_i^T s_i$$

then again the Lyapunov function $V = V_1 + V_2$ satisfies

$$\frac{d}{dt} V = -\sum_{i=1}^{n} s_i^T K_D s_i$$

Similar to the single robot case, this provides that each $s_i \to 0$ as $t \to \infty$. While this may appear to simply generalize the result from the previous section, it illustrates marked benefits from teaming to satisfy sufficient richness conditions in ensemble.

**Definition 2** (Ensemble Persistently Exciting). A collection of $n$ time-varying matrix-valued quantities \(\{W_i(t), \ldots, W_n(t)\}\) is said to be persistently exciting in ensemble if there exists $T > 0$, $\alpha > 0$ such that

$$\frac{1}{T} \sum_{i=1}^{n} \int_{t}^{t+T} W_i^T(\tau) W_i(\tau) d\tau \geq \alpha I \ \forall t$$

Note that the condition for the collection \(\{Y_1(t), \ldots, Y_n(t)\}\) to be ensemble persistently exciting is a weaker condition than requiring any one $Y_i(t)$ to be persistently exciting on its own. Through this insight, we state the main result for this section.

**Theorem 2** (Parameter Convergence of Multi-Robot Direct Adaptive Control with a Common Estimate). Consider the set of systems (9) with the control laws (4) and centralized parameter update law (16). Then, if the regressors \(\{Y_1, \ldots, Y_n\}\) are ensemble persistency exciting, the parameter errors $\hat{a}$ satisfy $\dot{a} \to 0$ as $t \to \infty$.

**Proof.** Suppose that the regressors $\{Y_1, \ldots, Y_n\}$ are ensemble persistency exciting. Then there exists $\alpha > 0$, $T > 0$ such that

$$\frac{1}{T} \sum_{i=1}^{n} \int_{t}^{t+T} Y_i^T(\tau) Y_i(\tau) d\tau \geq \alpha I \ \forall t$$

which is equivalent to

$$\frac{1}{T} \int_{t}^{t+T} Y_R^T(\tau) Y_R(\tau) d\tau \geq \alpha I \ \forall t$$

Thus, $Y_R$ is persistently exciting, and it follows that $\dot{a} \to 0$ as $t \to \infty$. \qed

**Remark 1.** It should be noted that following comments as in [20], since each $q_i \to q_i$, ensemble sufficient richness of the regressors $Y_i(q_i, r_i, q_i, q)$ is equivalent to ensemble sufficient richness of the desired trajectories $Y_i(q_i, r_i, q_i, q, q_i, r_i, q_i, q)$. 

IV. CONTRIBUTION: ENSEMBLE SUFFICIENT RICHNESS FOR NETWORKED UPDATE LAWS

Although the previous section showed the benefits of teaming toward parameter convergence, the assumption of a centralized update law may be impractical. Instead, any updates in practice will occur through delayed communication channels, requiring each robot to operate with their own estimates in addition to time-delayed information from the cloud. We continue to build towards this case, first assuming decentralized updates without delays.

More specifically, suppose that instead of a shared parameter vector, that all robots instead carry along their own estimates $\hat{a}_i$ and update these estimates through coupling. For simplicity, all-to-all coupling is considered, however, the arguments that follow can be generalized to any fully-connected network topology. In the case of all-to-all coupling consider the system dynamics

$$H_i \dot{s}_i + (C_i + K_D) s_i = Y_i \hat{a}_i$$

$$\dot{\hat{a}}_i = -P_i^{-1} Y_i^T s_i - K \sum_{j=1}^{N} (\hat{a}_j - \hat{a}_i)$$

Algebraically, as in so-called quorum sensing [21, 22], this does not require communication between all robots, but rather simply requires a common average $\hat{a} = \frac{1}{N} \sum_{i=1}^{N} \hat{a}_i$ to be communicated to the group (through the server or the environment), with an equivalent update

$$\dot{\hat{a}} = -P_i^{-1} [Y_i^T s_i - K(\hat{a} - \hat{a}_i)]$$

Note that this common average is different from the common parameter vector in Section III since each system maintains its own estimate $\hat{a}_i$ in this case.

Letting

$$Y_B = \text{blkdiag}(Y_1, \ldots, Y_N)$$

$$P_B = \text{blkdiag}(P_1, \ldots, P_N)$$

denote the block diagonal collection of regressors and gains, and

$$\hat{a} = [\hat{a}_1^T, \ldots, \hat{a}_N^T]^T$$

the collection of estimates, the overall networked dynamics can be written as

$$H \dot{s} + (C + K_D) s = Y_B \hat{a}$$

$$\dot{\hat{a}} = -P_B^{-1} [Y_B^T s + L_K \hat{a}]$$
where

\[
L_K = \frac{1}{n} \begin{bmatrix}
    nK & -K & \cdots & -K \\
    -K & nK & \ddots & \vdots \\
    \vdots & \ddots & \ddots & -K \\
    -K & \cdots & -K & nK
\end{bmatrix}
\]

(24)

is a block Laplacian matrix \cite{23}. It can be shown that \( L_K \) is positive semi-definite, and \( L_K \hat{a} = 0 \) if and only \( \hat{a}_i = \hat{a}_j \) for all pairs \( i \) and \( j \) \cite{23}. These properties play a key role in the convergence analysis for this case.

\section{Analysis}

More formally, the Lyapunov function

\[
V = \frac{1}{2} s^T H s + \frac{1}{2} \hat{a}^T P \hat{a} = \sum_{i=1}^{N} \frac{1}{2} s_i^T H_i s_i + \hat{a}_i^T P_i \hat{a}_i
\]

(25)

has rate of change

\[
\dot{V} = -s^T K_D s - \hat{a}^T L_K \hat{a}
\]

(26)

Since it can easily be shown from the boundedness of \( V \) that \( \dot{V} \) is bounded, it follows that \( s \to 0 \) and \( L_K \hat{a} \to 0 \) as \( t \to \infty \), and thus each \( \hat{a}_i \) asymptotically converges to some fixed common estimate \( \bar{a} \) as well.

\section{Ensemble Sufficient Richness}

However, we see something deeper here, in that networked adaptation is afforded the same benefits of parameter excitation being determined in ensemble, despite the decentralized update law. Intuitively, as \( K \) becomes large, (19) enforces that each \( \hat{a}_i \approx \bar{a}_j \), approaching the centralized update law in the limit.

\section{Theorem 3 (Parameter Convergence of Multi-Robot Direct Adaptive Control with a Decentralized Update Law)}

Consider the system (18) with decentralized update laws (19). Then, if the regressors \( \{ Y_1, \ldots, Y_n \} \) are ensemble persistency exciting, the parameter errors \( \hat{a}_i \) all satisfy \( \hat{a}_i \to 0 \) as \( t \to \infty \).

\section{Proof}

Since \( \hat{a}_i \to \bar{a} \), it follows that \( Y_R \hat{a} \to 0 \). Yet, since each \( \hat{a}_i \to \bar{a} \), it follows that each \( Y_i (\bar{a} - a) = 0 \). Thus, \( Y_R (\bar{a} - a) = 0 \). From the proof of Theorem 2, \( Y_R \) persistently exciting is equivalent to ensemble persistency of excitation. Thus, since \( Y_R \) is persistently exciting, the only possibility is that \( \bar{a} = a \), and thus, all \( \hat{a}_i \to \bar{a} \).

As a key benefit, recall again that the condition of ensemble sufficient richness is a much weaker condition than a condition that any individual \( Y_i \) is persistently exciting. In summary, despite the decentralized law, parameter convergence is afforded the same excitation benefits through teaming as in the centralized case.

\section{CONTRIBUTION: ENSEMBLE SUFFICIENT RICHNESS WITH TIME DELAYS}

Treating the desired case of cloud-based teaming with delay, assume the same updates as previously under a uniform communication delay \( T \). Again, the arguments that follow can be generalized to the case of non-uniform delays and for application to any fully-connected network topology. The case of all-to-all coupling with uniform delay is considered for simplicity of exposition. In this case, the updates take the form

\[
H_i \hat{s}_i + (C_i + K_D) s_i = Y_i \hat{a}_i
\]

(27)

\[
\hat{a}_i = -P_i^{-1} \left[ Y_i^T s_i - \frac{K}{n} \sum_{j=1}^{n} (\hat{a}_j (t-T) - \hat{a}_i) \right]
\]

(28)

It would seem reasonable that such delays would disrupt parameter convergence and even perhaps interfere with tracking. However, we will show that the proposed update laws are robust to time-delayed communications.

\section{Analysis}

Let \( G = \left( \frac{K}{n} \right)^2 \) such that \( K \to GG^T \). Similar to \cite{24} we consider the updates through an alternative notation inspired by wave variables \cite{25}. Let

\[
v_i = G^T \hat{a}_i \quad u_i = v_i (t-T)
\]

and define

\[
\tau_{ji} = u_j - v_i
\]

(30)

Then the update laws can be alternately expressed as

\[
\hat{a}_i = -P_i^{-1} \left[ Y_i^T s_i - G \sum_{j=1}^{n} \tau_{ji} \right]
\]

(31)

Consider the modified Lyapunov function

\[
V = \frac{1}{2} \sum_i \sum_j \left( s_i^T H_i s_i + \hat{a}_i^T P_i \hat{a}_i + \int_{t-T}^{t} v_j^T v_j dt \right)
\]

which, roughly, accounts for the parameter errors stored in the transmission delay through the additional integral.

The rate of change of this Lyapunov function is then

\[
\dot{V} = \sum_i \sum_j \left( -s_i^T K_D s_i + \int_{t-T}^{t} v_j^T v_j dt \right)
\]

Noting that each \( u_j^T u_i \) can be expressed via (30) as

\[
u_j^T u_j = v_j^T v_i + 2 v_j^T \tau_{ji} + \tau_{ji}^T \tau_{ji}
\]

(32)

It follows that

\[
\dot{V} = \sum_i \sum_j \left( -s_i^T K_D s_i - \int_{t-T}^{t} \tau_{ji}^T \tau_{ji} dt \right)
\]

(33)

Thus each \( s_i \to 0 \) and \( \tau_{ji} \to 0 \) as \( t \to \infty \). From (31) it can be seen that each \( \hat{a}_i \to 0 \) asymptotically. This does not
necessarily mean that each \( \hat{a}_i \) converges in general. However, here, since

\[
\begin{align*}
\mathbf{u}_j &= \mathbf{v}_i + \boldsymbol{\tau}_{ji} = \mathbf{v}_j(t - T), \quad \text{and} \\
\mathbf{u}_i &= \mathbf{v}_j + \boldsymbol{\tau}_{ij} = \mathbf{v}_i(t - T)
\end{align*}
\] (34) (35)

it follows that each \( \hat{a}_i \) is periodic with period \( 2T \). Since \( \hat{a}_i \rightarrow 0 \) the only possibility for this periodic trajectory is that each \( \hat{a}_i \) converges to a constant. Further, since each \( \boldsymbol{\tau}_{ij} \rightarrow 0 \), every pair \( \hat{a}_i \rightarrow \hat{a}_j \). Thus, all the estimates \( \hat{a}_i \) must converge to some common constant \( \boldsymbol{\pi} \).

**Theorem 4** (Parameter Convergence of Multi-Robot Direct Adaptive Control with Time-Delayed Decentralized Updates). Consider the system (27) with decentralized update laws (28). Then, if the regressors \( \{ \mathbf{Y}_1, \ldots, \mathbf{Y}_n \} \) are ensemble persistency exciting, the parameter errors \( \hat{a}_i \) all satisfy \( \hat{a}_i \rightarrow 0 \) as \( t \rightarrow \infty \).

**Proof.** Since each \( \hat{a}_i \rightarrow \boldsymbol{\pi} \) for some fixed common constant \( \boldsymbol{\pi} \), convergence of each \( \hat{a}_i \rightarrow 0 \) follows through identical arguments to the proof of Theorem 3. \( \square \)

VI. EXTENSIONS VIA COMPOSITE ADAPTATION

Beyond using the tracking errors to drive adaptation, any modeling errors linear in the unknown parameters can be used to drive parameter update laws. Composite adaptive control is a method to increase the convergence rate of the direct adaptive control by fusing information related to these modeling errors. We address a number of potential inclusions of these composite updates to improve the performance of cloud-based networked adaptive control.

A. Composite Adaptive Control: Filtered Torques

One possible addition is to filter the dynamics (4)

\[
\frac{\lambda}{p + \lambda} \tau = \frac{\lambda}{p + \lambda} \mathbf{Hq} + \mathbf{Cq} + \mathbf{g}
\] (36)

where \( p \) the Laplace variable. Writing the right hand side as a convolution integral with the impulse response of the low-pass filter, integration by parts can be used to yield

\[
\frac{\lambda}{p + \lambda} \tau = \lambda \mathbf{Hq} - \frac{\lambda}{p + \lambda} \left[ \lambda \mathbf{Hq} + \mathbf{C}^T \mathbf{q} - \mathbf{g} \right]
\] (37)

Since all entries of the right hand side are linear in \( \mathbf{a} \), one can construct a matrix \( \mathbf{W} \), dependent on the time histories of \( \mathbf{q} \) and \( \dot{\mathbf{q}} \) such that

\[
\frac{\lambda}{p + \lambda} \tau = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}}) \mathbf{a}
\] (38)

Letting the modeling error \( \mathbf{e} = \mathbf{W} \hat{\mathbf{a}} - \lambda/(p + \lambda) \tau \), then

\[
\mathbf{e} = \mathbf{W} \hat{\mathbf{a}}
\] (39)

To explore the effects of including composite adaptation, consider again the networked systems without time delay:

\[ \mathbf{H}_i \dot{s}_i + (\mathbf{C}_i + \mathbf{K}_D) s_i = \mathbf{Y}_i \hat{a}_i \] (40)

\[ \dot{\hat{a}}_i = -\mathbf{P}_i^{-1} \left[ \mathbf{Y}_i^T \mathbf{s}_i + \mathbf{W}_i^T \mathbf{e}_i - \frac{\mathbf{K}}{n} \sum_{j=1}^{N} (\dot{\hat{a}}_j - \hat{a}_i) \right] \] (41)

Through using composite adaptation, the Lyapunov function (25) has rate of change

\[ \dot{V} = -\hat{a}_i^T \mathbf{L}_K \hat{a}_i - \sum_{i=1}^{n} (s_i^T \mathbf{K}_D s_i + \hat{a}_i^T \mathbf{W}_i^T \mathbf{e}_i) \] (42)

\[ = -\hat{a}_i^T \mathbf{L}_K \hat{a}_i - \sum_{i=1}^{n} (s_i^T \mathbf{K}_D s_i + \mathbf{e}_i^T \mathbf{e}_i) \] (43)

which is more negative in comparison to the direct case (26). Note that the only property used here is \( \mathbf{e}_i = \mathbf{W}_i \hat{a}_i \), and thus one can pursue other error measures that are linear in the parameters. In a sense, the terms \( \mathbf{e}_i^T \mathbf{e}_i = \hat{a}_i^T \mathbf{W}_i^T \mathbf{W}_i \hat{a}_i \) can be viewed as adding positive semidefinite terms to the diagonal of the block Laplacian \( \mathbf{L}_K \), which may be positive definite on average in the case of persistency of excitation for each \( \mathbf{W}_i \).

B. Composite Adaptive Control: Filtered Energy

Another possible choice of composite error injection follows from considering the rate in change of system energy [4], [26]

\[ \dot{q}^T \tau = \frac{d}{dt} \left[ \frac{1}{2} q^T \mathbf{Hq} \right] + q^T \mathbf{g} \]

Again, filtering both sides, taking the impulse response of the filter convolution and applying integration by parts, it can be shown that

\[ \frac{\lambda}{p + \lambda} \dot{q}^T \tau = \frac{\lambda}{2} \dot{q}^T \mathbf{Hq} - \frac{\lambda}{p + \lambda} \left[ \frac{\lambda}{2} \dot{q}^T \mathbf{Hq} - \dot{q}^T \mathbf{g} \right] \]

Similar to before, the right hand side can be formed from the time histories of \( q, \dot{q} \) and is linear in parameters \( a \). Thus, there exists a modified filtered regressor \( \mathbf{W}(q, \dot{q}) \) such that

\[ \frac{\lambda}{p + \lambda} \dot{q}^T \tau = \mathbf{W} a \] (44)

\[ \mathbf{e} = \mathbf{W} \hat{\mathbf{a}} - \frac{\lambda}{p + \lambda} \dot{q}^T \tau = \mathbf{W} \hat{\mathbf{a}} \] (45)

Thus all of the previous results hold with the filtered energy regressor in place of the filtered torques regressor.

More broadly, the update law (41) could be employed with heterogenous choices for the extra data fused at each node. That is, some robots might use filtered energy for additional fusion, while others use the filtered torques. The main message is that this additional fusion can only help the convergence process, and that this positive effect is shared through the network. We state this result more formally:

**Theorem 5** (Parameter Convergence of Multi-Robot Composite Adaptive Control with Decentralized Updates). Consider the system (27) with decentralized update laws (28). Then, if the regressors \( \{ \mathbf{Y}_1, \ldots, \mathbf{Y}_n, \mathbf{W}_1, \ldots, \mathbf{W}_n \} \) are ensemble persistency exciting, the parameter errors \( \hat{a}_i \) all satisfy \( \hat{a}_i \rightarrow 0 \) as \( t \rightarrow \infty \).

**Remark 2.** Note that while composite adaptive control was proposed in the late 80s, applications were originally lacking due to computation requirements. However, efficient recursive algorithms for composite adaptation [27] and
increased computing power since original development make the approach increasingly viable.

VII. SIMULATION RESULTS

To show the benefits and validity of the new theory, this section reports on a simple set of simulations. These simulations are meant to illustrate the central benefits of Ensemble Sufficient Richness through the simplest essence. We consider two planar 3-DoF robots as depicted in Fig. 2 attempting to identify a common unknown load. The case of no teaming when $K = 0$ is shown in Fig. 3. Other parameters are set as $\Lambda = 4 \, \text{s}^{-1}$, $K_D = 4 \, \text{N/s/m}$. Due to the trajectories followed, the first robot is unable to identify the rotational inertia of the unknown load, while the second robot cannot identify the unknown mass of the load.

Although the trajectories of neither robot are sufficiently rich, when considered together, there is enough information content in the combined signals to satisfy ensemble sufficient richness conditions. Fig. 4 shows this case with coupling gain $K = 5$ and a communication time delay of 0.25 s. The parameters converge to their true values despite delays thanks to benefits of collaboration in this case. Note that the mass temporarily goes negative in both examples, highlighting the potential advantage of exploiting known physical consistency constraints [28]–[30] to be included in the adaptation laws. This inclusion represents an interesting area for future improvements.

VIII. CONCLUSION

This paper has treated the case of cooperative adaptive control for robot manipulators akin to collaboration through the cloud. We have introduced new decentralized parameter update laws and proved their convergence through the introduction of a new characterization of ensemble sufficient richness. With the update laws proposed, this property holds robustly with time-delayed communication and can be extended to improve convergence rates through the inclusion of composite adaptation. Although the work has assumed uniform delays and an all-to-all network topology, the results hold more generally for any fully-connected network and under non-uniform communication delays. Future work will consider extensions to underactuated systems and the inclusion of physical consistency constraints into the adaptive update laws.

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