COSMOLOGICAL BACKREACTION AND THE FUTURE EVOLUTION OF AN ACCELERATING UNIVERSE

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We investigate the effect of backreaction due to inhomogeneities on the evolution of the present universe within the Buchert framework. Our analysis shows how backreaction from inhomogeneities in the presence of the cosmic event horizon causes the current acceleration of the Universe to slow down in the future and even lead in certain cases to the emergence of a future decelerating epoch.

Keywords: Dark Energy; Cosmic Backreaction; Large Scale Structure.

1. Introduction

The present acceleration of the Universe is well established observationally, but far from understood theoretically, although there is no dearth of innovative ideas. In recent times there is an upsurge of interest on studying the effects of inhomogeneities on the expansion of the Universe and several approaches have been developed to facilitate this and it has been argued that backreaction from inhomogeneities from the era of structure formation could lead to an accelerated expansion of the Universe.

2. The Backreaction Framework

In the framework developed by Buchert for a compact spatial domain $D$ the scale-factor, $a_D(t) = \left(\frac{|D|_g}{|D|_D}\right)^{1/3}$, encodes the average stretch of all directions of the domain, where $|D|_g$ is the volume of $D$. Using the Einstein equations, with a pressure-less fluid source, we get the following equations

\begin{align}
3\frac{\dot{a}_D}{a_D} &= -4\pi G \langle \rho \rangle_D + Q_D + \Lambda \\
3H_D^2 &= 8\pi G \langle \rho \rangle_D - \frac{1}{2} \langle R \rangle_D - \frac{1}{2} Q_D + \Lambda
\end{align}

Here the average of the scalar quantities on the domain $D$ is defined as, $\langle f \rangle_D(t) = \frac{1}{|D|_g} \int_D f d\mu_g$ and where $\rho$, $R$ and $H_D$ denote the local matter density, the Ricci-scalar, and the domain dependent Hubble rate $H_D = \dot{a}_D/a_D$ respectively. The kinematical backreaction $Q_D = \frac{2}{3} \left( \langle \theta^2 \rangle_D - \langle \theta \rangle_D^2 \right) - 2\sigma_D^2$ where $\theta$ is the local expansion rate and $\sigma^2 = 1/2 \sigma_{ij} \sigma^{ij}$ is the squared rate of shear.

The “global” domain $D$ is assumed to be separated into subregions and following Ref. 7 we work with only two subregions. Clubbing those parts of $D$ which consist of initial overdensity as $M$ (called ‘wall’), and those with initial underdensity as $E$
(called ‘void’), such that \( D = \mathcal{M} \cup \mathcal{E} \), one obtains

\[
\frac{\ddot{a}_D}{a_D} = \lambda_M \frac{\ddot{a}_M}{a_M} + \lambda_E \frac{\ddot{a}_E}{a_E} + 2 \lambda_M \lambda_E (H_M - H_E)^2
\]  

(3)

Here \( \sum \lambda = \lambda_M + \lambda_E = 1 \), with \( \lambda_M = |\mathcal{M}|/|D| \) and \( \lambda_E = |\mathcal{E}|/|D| \), and \( a_M \), \( H_M \) and \( a_E \), \( H_E \) are the scale factors and Hubble parameters of the \( M \) and \( E \) subdomains respectively.

### 3. Effect of event horizon

We consider the universe once the present stage of acceleration sets in and try to see the effect of backreaction in the presence of the cosmic event horizon (first presented in Ref. 8). We can write the equation of the event horizon \( r_h \), to a good approximation by

\[
r_h = a_D \int_{t}^{\infty} \frac{dt'}{a_D(t')}
\]

(4)

The void-wall symmetry of Eq. (3) ensures that the conclusions are similar whether one chooses to define the event horizon with respect to the wall or with respect to the void.

We assume that the scale-factors of the regions \( \mathcal{E} \) and \( \mathcal{M} \) are, respectively, given by \( a_E = c_E t^\alpha \) and \( a_M = c_M t^\beta \) where \( \alpha \), \( \beta \), \( c_E \) and \( c_M \) are constants. Since an event horizon forms, only those regions of \( D \) that are within the event horizon are causally accessible to us. Therefore we have to introduce an apparent volume fraction of \( \mathcal{M} \) which is defined as \( \lambda_{M_h} = \frac{|\mathcal{M}_h|}{4\pi r_h^3} = \frac{c_M^3}{r_h^3} \), where \( c_M^3 = 3c_M|\mathcal{M}_h|/4\pi \) is a constant. Normalizing the total accessible volume in the presence of the horizon we can write \( \lambda_{E_h} = 1 - \lambda_{M_h} \), where \( \lambda_{E_h} \) is the apparent volume fraction of the sub-domain \( \mathcal{E} \). It hence follows that global acceleration equation (3) is now given by

\[
\frac{\ddot{a}_D}{a_D} = \frac{c_M^3}{r_h^3} \left( \frac{\beta}{t^2} - 1 \right) + \left( 1 - \frac{c_M^3}{r_h^3} \right) \left( \frac{\alpha}{t^2} - 1 \right) + 2 \frac{c_M^3}{r_h^3} \left( 1 - \frac{c_M^3}{r_h^3} \right) \left( \frac{\beta}{t} - \frac{\alpha}{t} \right)^2
\]

(5)

The current acceleration of the Universe ensures the formation of the event horizon, so \( r_h \) defined by (4) will be finite valued, thus enabling us to rewrite (4) as

\[
r_h = \frac{\dot{a}_D}{a_D} r_h - 1
\]

(6)

We can therefore solve numerically the set of coupled differential equations (5) and (6) by using as an ‘initial condition’ the observational constraint \( q_0 = -0.7 \), where \( q_0 \) is the current value of the deceleration parameter.
4. Discussions and Conclusions

In Fig. 1 the curves (i) and (iii) are for the case when an event horizon is included, and curves (ii) and (iv) correspond to the case without an event horizon. We see that whether $\alpha < 1$ or $\alpha > 1$, the acceleration always becomes negative in the future when we include the event horizon (curves (i) and (iii)), whereas the acceleration only becomes negative for $\alpha < 1$ when we don’t include the horizon in our calculations (curve (ii)). We also see that the deceleration is much faster when we include the event horizon, the reason for that could be that the inclusion of the event horizon somehow decreases the available volume of the underdense region $\mathcal{E}$ which causes the overdense region $\mathcal{M}$ to start dominating much earlier and leads to global deceleration much more quickly.

Our results indicate the fascinating possibility of backreaction being responsible for the slowing down of the current acceleration and in some cases cause a transition to a future decelerated era, no matter what the cause of the current acceleration.

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