Research Article

Computational Insights of Bioconvective Third Grade Nanofluid Flow past a Riga Plate with Triple Stratification and Swimming Microorganisms

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The goal of this study is to examine the heat-mass effects of a third grade nanofluid flow through a triply stratified medium containing nanoparticles and gyrostatic microorganisms swimming in the flow. The heat and mass fluxes are considered as a non-Fourier model. The governing models are constructed as a partial differential system. Using correct transformations, these systems are converted to an ordinary differential model. Ordinary systems are solved using convergent series solutions. The effects of physical parameters for fluid velocity, fluid temperature, nanoparticle volume percentage, motile microbe density, skin friction coefficients, local Nusselt number, and local Sherwood number are all illustrated in detail. When the values of the bioconvection Lewis number increase, the entropy rate also rises. The porosity parameter and modified Hartmann number show the opposite behaviour in the velocity profile.

1. Introduction

Researchers are interested in learning more about how to increase heat transmission because it is so important in design and business. Thermal transfer of convectional liquids such as ethylene glycol, water, and oil can be used in a variety of mechanical assemblies, electrical devices, and heat dissipates. Despite this, the thermal conductivity of these base fluids is weak. To counter this flaw, experts from several sectors are attempting to improve the heat conductivity of newly cited fluids by incorporating a unique type of nanosized particle into a new fluid known as “nanofluid,” see Choi [1]. Nanofluid flow on a flat surface was examined by Khan and Pop [2]. They see that the mass transfer gradient reduces for enhancing the thermophoresis parameter. Barnoon and Toghraie [3] analyze the impact of a non-Newtonian nanofluid on a porous medium. Natural convective flow of nanofluid past a heated porous plate was demonstrated by Ghalambaz et al. [4], and they concluded that the fluid velocity creases when increasing the thermophoresis parameter. Aziz and Khan [5] demonstrated the characteristics of natural convective flow of nanofluids over a plate. They identified that heat transfer reduced by the impact of Brownian motion parameter. The nanofluid flow over a thin needle was addressed by Ahmad et al. [6]. They proved that the Brownian motion parameter leads to suppressing the nanofluid concentration. Prasannakumara et al. [7] addressed the consequences of multiple slips of MHD Jeffery nanofluid past a surface. They detected that the thermal boundary layer thickness thickens when enriching the thermophoresis parameter.

The bioconvection phenomenon is a fluid dynamic mechanism that occurs in macroscopic convective fluid flow generated by a fluid density gradient established by collective swimming of microorganisms. Because of their motility, these bacteria are classified as chemotactic, oxytactic, or gyrotactic. Near the top of the fluid layer, these self-propelled motile bacteria clump together, forming a dense upper surface that is unstable or unstabilized. Bioconvection is used in a variety of industrial applications, including microbial improved oil recovery, sustainable fuel cell technologies, water treatment facilities, polymer synthesis,
and so on. The 2D radiative flow of tangent hyperbolic nanofluid past a Riga plate with gyrotactic microorganisms was disclosed by Waqas et al. [8]. They noted that the density of motile microorganisms decays when enriching the bioconvection Lewis number. Uddin et al. [9] portrayed the consequences of Stefan blowing of bioconvective flow of nanofluid past a porous medium. They see that the density of motile microorganisms enriches when strengthening the wall suction parameter. MHD flow of cross nanoparticles with gyrotactic motile microorganisms past wedge was scrutinized by Alshomrani et al. [10]. They noted that the motile microorganisms suppress when escalating the Peclet number. Muhammad et al. [11] developed the mathematical model for the unsteady MHD flow of Carreau nanofluids with bioconvection. flQ_hey detected that the density of local motile number depresses when enhancing the Peclet parameter. flQ_hey noted that the density of local motile number decays when enriching bioconvection Lewis number. Uddin et al. [9] portrayed the consequences of Stefan blowing of bioconvective flow of nanofluid past a porous medium. flQ_hey detected that the density of local motile number enriches when strengthening the bioconvection Lewis number. Uddin et al. [9] portrayed the consequences of Stefan blowing of bioconvective flow of nanofluid past a porous medium. flQ_hey detected that the density of local motile number depresses when enhancing the Peclet parameter. flQ_hey noted that the density of local motile number depresses when enhancing the Peclet parameter.

Due to its numerous industrial and engineering uses, such as cooling nuclear reactors, power generation, cooling of electronic equipment, energy production, and many others, the process of heat transfer has gotten a lot of attention from modern scholars. Fourier [12] was the first to present the heat transfer law. However, this law has the disadvantage of producing a parabolic energy equation. To address this flaw, Cattaneo [13] rewrote the Fourier equation by including the relaxation time heat flux component. In addition, Christov [14] tweaked the Cattaneo model by incorporating thermal relaxation time and used the Oldroyd upper convective model. The heat transport analysis of 2D flow cross nanofluid Cattaneo–Christov theory was investigated by Salahuddin et al. [15], and they proved that concentration relaxation leads to downfall of the nanofluid concentration. Farooq et al. [16] examined the impact of MHD flow of radiative nanofluids with Cattaneo–Christov theory. They revealed that the fluid temperature diminishes when raising the thermal relaxation parameter. Thermally radiative flow of hybrid nanoliquids with Cattaneo–Christov heat flux theory was implemented by Waqas et al. [17].

Despite the fact that nanofluids have been widely investigated, the third grade nanofluid flow over a stretching sheet with entropy optimization was examined by Loganganathan et al. [18]. This study is extended with the effects of including the mixed convective flow of third grade nanofluids over a Riga plate with triple stratification and swimming microorganisms. The thermal radiative flow of third grade nanofluids containing microorganisms owing to the movement of the Riga plate is shown in this study to achieve this goal.

(i) The modified Fourier’s law is used to frame energy and nanoparticle concentration equations

(ii) The homotopy analysis method is used to compute the non-linear equations analytically

(iii) The results of the simulations might have unique implications in the fields of thermal processes, heat transfer industry, energy systems, nuclear systems, and so on.

2. Problem Development

For an incompressible fluid model with body forces, the continuity and motion equations are

\[
\begin{align*}
\text{div} \mathbf{v}^* &= 0, \\
\frac{d}{dt} \mathbf{v}^* &= \text{div} \mathbf{T} + \rho b + J + B,
\end{align*}
\]

where \( \rho \) is the “fluid density,” \( \mathbf{v}^* \) is the “velocity field,” \( b \) is the “body forces,” \( J \) is the “electric current,” and \( T \) is the “third-grade incompressible fluids Cauchy stress tensor” [19].

\[
T = -pI + \mu H_1 + A_1^* H_2 + A_2^* H_1^2 + \gamma_1 H_3 + 
+ \gamma_2 (H_1, H_2 + H_3 H_1) + \gamma_3 (tr H_1^2) H_1, 
\]

where \( \mu \), \( (H_1, H_2, H_3) \) and \( A_1^*, \gamma_i = “viscosity coefficient”, \) “kinematics tensors” and “material modulis”

\[
H_n = \frac{d}{dt} H_{n-1} + H_{n-1} L + (L)^T H_{n-1}, \quad n = 2, 3, \]

\[
L = \nabla \mathbf{v}^*,
\]

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}^* \cdot \nabla .
\]

The relationship between the Clausius–Duhem inequality and the thermodynamically compatible fluid is described by Fosdick and Rajagopal. [20].

\[
\mu \geq 0,
\]

\[
A_1^* \geq 0,
\]

\[
\gamma_1 = \gamma_3 = 0,
\]

\[
\gamma_3 \geq 0,
\]

\[
|A_1^* + A_2^*| \leq 2\sqrt{6\mu_3},
\]

\[
T = -pI + \mu H_1 + A_1^* H_2 + A_2^* H_1^2 + \gamma_3 (tr H_1^2) H_1.
\]
Pakdemirli [21] took into consideration the Boussinesq and normal boundary layer approximations. The representation of steady flow of third grade nanofluids containing motile microorganisms is assumed. The surface is linearly stretched via velocity \( u_w = ax \), in positive \( x \) direction in its own path. Moreover, the flow is considered along the sheet while \( v \) is perpendicular, and \( B_0 \) magnetic field is taken vertical to the flow direction. The wall temperature \( T_w \), wall concentration \( C_{w} \), and motile microorganisms’ wall concentration \( N_{w} \) are defined. Figure 1 portrays the flow geometry of the problem. The governing equations are extended from Loganathan et al. [18] as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\
\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{A_1^*}{\rho} \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} + \frac{3}{2} \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right) + 2A_2^* \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \\
+ 6 \frac{\beta_1^*}{\rho} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\rho - C_b}{\rho_x} u - \frac{1}{\rho_f} (1 - C_{co}) \rho_f \frac{\partial g(T - T_{co})}{\partial T} \\
- \frac{\partial (\rho_m \rho_p)}{\partial T} g(T - C_{co}) - (N - N_{co}) \frac{\partial g(T - T_{co})}{\partial T} + \frac{\pi f_j M_0}{8\rho} \exp \left( \frac{\pi}{a_1} y \right) \\
+ \left( \frac{\partial v}{\partial x} \frac{\partial^2 u}{\partial y \partial x} + \frac{\partial v}{\partial y} \frac{\partial^2 u}{\partial y \partial x} \right) - k_c (C - C_{co}), \\
\frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} - D_m \left( \frac{\partial^2 N}{\partial y^2} \right) = - \frac{b W_c}{(C_w - C_{co})} \frac{\partial}{\partial y} \left( N \frac{\partial C}{\partial y} \right). 
\]

With the boundary points
\[
\begin{align*}
  u &= u_w = ax, \\
  v &= 0, \\
  T &= T_w = T_0 + b_1 x, \\
  C &= C_w = C_0 + d_1 x, \\
  N &= N_w = N_0 + e_1 x, \text{ at } y = 0, \\
  u &\rightarrow 0, \\
  T &= T_{co} = T_0 + b_2 x, \\
  C &= C_{co}, \\
  N &= N_{co} = N_0 + e_2 x, \text{ as } y \rightarrow \infty.
\end{align*}
\]

Here, \( b_1, b_2, d_1, d_2, e_1, \) and \( e_2 \) are the dimensional constants, and \( T_0 \) and \( C_0 \) are the “reference temperature and concentrations,” respectively. \( u \) and \( v \) are the “velocity components” in \( x \) and \( y \) directions, \( \rho \) is the “fluid density,” \( \nu \) is the “fluid kinematics viscosity,” \( k_p \) is the “permeability of the porous medium,” \( C_b \) is the “drag coefficient,” \( T_0 \) is the “current density applied to the electrodes,” \( M_0 \) is the “magnetic property of the permanent magnets,” \( a_1 \) is the “magnets positioned in the interval separating the electrodes,” \( \sigma^* \) is the “Stefen-Boltzmann constant,” \( C_p \) is the “specific heat capacity of the fluid,” and \( k \) is the “thermal conductivity.”

Transformations are declared as follows:
The nonlinear governing equations are

\[
\begin{align*}
\eta &= f\left(\frac{d}{d\eta}\right)^{1/2}, \\
\psi &= (\alpha \eta)^{1/2} f(\eta), \\
u &= \frac{\partial \psi}{\partial y}, \\
u &= \frac{-\partial \psi}{\partial x}, \\
u &= -(u_w)^{1/2} f(\eta), \\
\theta &= \frac{T - T_\infty}{T_w - T_0}, \\
\phi(\eta) &= \frac{C - C_\infty}{C_w - C_0}, \\
\chi(\eta) &= \frac{N - N_\infty}{N_w - N_0}.
\end{align*}
\]

The nonlinear governing equations are

\[
\begin{align*}
f''' + f'' + f'' + \alpha_1 (2 f'' f''' - f'' f''') + (3 \alpha_1 + 2 \alpha_2) f^n \\+ 6 \beta \text{Re} f''' f^2 - P_m f' - Pr f'' + H_m \eta
\end{align*}
\]

\[
+ \lambda (\theta - N_\phi - R_\phi) = 0,
\]

\[
\left(1 + \frac{4}{3} R \frac{\partial}{\partial \eta}\right) \theta'' + Pr f \theta' - Pr S f'
\]

\[
- Pr T_1 \left[ f'' \theta + S f' - f' f'' \theta - S f f'' + f' \theta''\right]
\]

\[
+ Pr H g \theta + Pr N b \theta' \phi \theta + Pr N t \theta^2 = 0,
\]

\[
\phi'' + Le f \phi' - Le S f'
\]

\[
- Le T_2 \left[ f'' \phi + S f \phi' - f' \phi' + f' f'' \phi - S f f'' + f' \phi''\right]
\]

\[
+ \frac{N t}{N b} \eta'' - Le C r \phi = 0,
\]

\[
\chi'' + L_0 \left[ f \chi' - f' \chi\right] - L_0 S f'
\]

\[
- P_r [\phi' \chi + (\Omega + \phi) \chi'] = 0.
\]

Figure 2: Velocity profile for different values of \(\alpha_1, \alpha_2, Ha, P_w, Pe, Nr, Rb, \) and \(S_1.\)
The boundary conditions are specified in the following manner:

\[ f(0) = 0, \]
\[ f'(0) = 1, \]
\[ \theta(0) = 1 - S_1, \]
\[ \phi(0) = 1 - S_2, \]
\[ \chi(0) = 1 - S_3, \]
\[ f'(\infty) = 0, \]
\[ \theta(\infty) = 0, \]
\[ \phi(\infty) = 0, \]
\[ \chi(\infty) = 0, \]

The nondimensional variables are

\[ \frac{\chi}{\chi}, \quad \frac{f'}{f'}, \quad \frac{\theta}{\theta}, \quad \frac{\phi}{\phi}, \quad \frac{\chi}{\chi} \]

Material parameters = \( \alpha_1 = \frac{aA_1^*}{v}, \alpha_2 = \frac{aA_2^*}{v}, \beta = \frac{a\beta_1^*}{v} \),

Reynolds number (Re) = \( \frac{u_\infty x}{v} \),

Porous medium (\( P_m \)) = \( \frac{v}{k_p d} \),

Forchheimer number (Fr) = \( \frac{C_b}{\sqrt{k_p}} \)
Figure 4: Nanoparticle concentration profile for different values of $Ha, Rb, S_1, \lambda, Fr,$ and $P_m$.

Prandtl number ($Pr$) = $\frac{\nu}{\alpha}$

Radiation parameter ($Rd$) = $\frac{4\sigma T_\infty^4}{k^2}$

Heat generation parameter ($Hg$) = $\frac{Q_0}{\rho c_p}$

Heat thermal relaxation parameter ($\Gamma_1$) = $\lambda_T a$

Mass thermal relaxation parameter ($\Gamma_2$) = $\lambda_C a$

Mixed convection parameter ($\lambda$) = $\frac{\beta_T (1 - C_\infty) (T_w - T_0)}{a u_w}$

Buoyancy ratio parameter ($N_r$) = $\frac{(\rho_p - \rho_f) (C_w - C_0)}{\beta_T (1 - C_\infty) (T_w - T_0)}$

Bioconvection Rayleigh number ($R_b$) = $\frac{\gamma (N_w - N_0) (\rho_m - \rho_f)}{\beta_T (1 - C_\infty) (T_w - T_0)}$

Thermophoresis parameter ($N_t$) = $\frac{\tau D_T (T_w - T_0)}{\nu}$

Brownian motion parameter ($N_b$) = $\frac{\tau D_B (C_w - C_0)}{\nu}$

Lewis number ($Le$) = $\frac{\gamma}{D_B}$

Bioconvection Lewis number ($L_b$) = $\frac{\gamma}{D_m}$

Bioconvection Peclet number ($P_e$) = $\frac{b W_c}{D_m}$

Microorganisms concentration difference parameter ($\Omega$) = $\frac{N_w - N_0}{N_w - N_0}$

Thermal stratification parameter ($S_1$) = $\frac{b_2}{b_1}$

Mass stratification parameter ($S_2$) = $\frac{d_2}{d_1}$

Motile density stratification parameter ($S_3$) = $\frac{e_2}{e_1}$

(11)
Application of physical entities are

\[ C_f \text{Re}^{-0.5} = f''(0) + \alpha f''(0) f''(0) + \beta \text{Re}[f''(0)]^3, \]

\[ N_{\text{ux}} \text{Re}^{-0.5} = -1 + \frac{4}{3} \text{Rd}\theta'(0), \]

\[ Sh_x \text{Re}^{-0.5} = -\phi'(0), \]

\[ N_{nx} \text{Re}^{-0.5} = -\chi'(0). \]

\[ S_{\text{gen}}'' = \left\langle K_1 \left( \frac{\partial T}{\partial x} \right)^2 + \frac{16\sigma^* T^3}{3k^2} \left( \frac{\partial T}{\partial y} \right)^2 \right\rangle + \frac{\mu}{T_\infty} \left( \frac{\partial u}{\partial x} \right)^2 + \left\langle \frac{\partial N}{\partial y} \right\rangle + RD \left\langle \left( \frac{\partial C}{\partial x} \right)^2 + \left( \frac{\partial C}{\partial y} \right)^2 \right\rangle \]

\[ + \text{Rd} \left( \frac{\partial T}{\partial x} \right) \left( \frac{\partial C}{\partial x} \right) + \left( \frac{\partial T}{\partial y} \right) \left( \frac{\partial C}{\partial y} \right) + \frac{RD}{C_\infty} \left( \frac{\partial N}{\partial x} \right) + RD \left\langle \left( \frac{\partial T}{\partial x} \right) \left( \frac{\partial C}{\partial y} \right) \right\rangle, \tag{13} \]

Equation (13) was changed by using the boundary layer approximation.

\[ S_{\text{gen}}'' = \left\langle K_1 \left( \frac{\partial T}{\partial x} \right)^2 + \frac{16\sigma^* T^3}{3k^2} \left( \frac{\partial T}{\partial y} \right)^2 \right\rangle + \frac{\mu}{T_\infty} \left( \frac{\partial u}{\partial x} \right)^2 + \left\langle \frac{\partial N}{\partial y} \right\rangle + RD \left\langle \left( \frac{\partial C}{\partial x} \right)^2 + \left( \frac{\partial C}{\partial y} \right)^2 \right\rangle \]

\[ + \text{Rd} \left( \frac{\partial T}{\partial x} \right) \left( \frac{\partial C}{\partial x} \right) + \left( \frac{\partial T}{\partial y} \right) \left( \frac{\partial C}{\partial y} \right) + \frac{RD}{C_\infty} \left( \frac{\partial N}{\partial x} \right) + RD \left\langle \left( \frac{\partial T}{\partial x} \right) \left( \frac{\partial C}{\partial y} \right) \right\rangle, \]

\[ N_T = \frac{K_1}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 + \frac{16\sigma^* T^3}{3k^2} \left( \frac{\partial T}{\partial y} \right)^2 = "\text{Entropy contribution due to heat transmission}" \]

\[ N_f = \frac{\mu}{T_\infty} \left( \frac{\partial u}{\partial y} \right)^2 = "\text{Entropy contribution due to fluid friction}" \]

\[ N_C = \frac{RD}{C_\infty} \left( \frac{\partial C}{\partial y} \right)^2 + RD \left( \frac{\partial T}{\partial y} \right) \left( \frac{\partial C}{\partial y} \right) = "\text{Entropy contribution due to mass transmission}" \]

\[ S_{\text{gen}}'' = N_T + N_f + N_C + N_m. \]

The typical entropy generation rate $S_0''$ is given by
As a consequence, the dimensionless entropy generation number may be calculated by using the following formula:

$$E_G = \frac{S'_0}{S_0}.$$  \hspace{1cm} (15)

As a result, the total entropy generation number has the corresponding dimensionless form:

$$E_G = \text{Re} \left( 1 + \frac{4}{3} R_d \right) \theta'^2 + \text{Re} \frac{B_r}{\Omega} f''^2 + \text{Re} \left( \frac{\zeta}{\Pi} \right)^2 \lambda \phi'^2$$

$$+ \text{Re} \left( \frac{\zeta}{\Pi} \right) \lambda \phi' \theta' + \text{Re} \lambda \chi' + \text{Re} \lambda \chi' \theta'.$$  \hspace{1cm} (17)

Expression of the Bejan number is

$$Be = \frac{N_T + N_C + N_m}{E_G}.$$  \hspace{1cm} (18)

### 4. Homotopy Solutions

The governing equations are solved analytically by applying the HAM scheme [18, 22–32]. In this regard, initially, we fix the initial approximation

$$f_0(\eta) = \left[ 1 - e^{-\eta} \right],$$

$$\theta_0(\eta) = \left[ 1 - S_1 e^{-\eta} \right],$$

$$\phi_0(\eta) = \left[ 1 - S_2 e^{-\eta} \right],$$

$$\chi_0(\eta) = \left[ 1 - S_3 e^{-\eta} \right].$$  \hspace{1cm} (19)

The linear operator is

$$\hat{L}_f = \frac{d^3 f}{d \eta^3},$$

$$\hat{L}_\theta = \frac{d^2 \theta}{d \eta^2} - \theta,$$

$$\hat{L}_\phi = \frac{d^2 \phi}{d \eta^2} - \phi,$$

$$\hat{L}_\chi = \frac{d^2 \chi}{d \eta^2} - \chi.$$  \hspace{1cm} (20)

with the property

$$\hat{L}_f \left[ \Psi_{11} e^\eta + \Psi_{12} e^{-\eta} \right] = 0,$$

$$\hat{L}_\theta \left[ \Psi_{21} e^\eta + \Psi_{22} e^{-\eta} \right] = 0,$$

$$\hat{L}_\phi \left[ \Psi_{31} e^\eta + \Psi_{32} e^{-\eta} \right] = 0,$$

$$\hat{L}_\chi \left[ \Psi_{41} e^\eta + \Psi_{42} e^{-\eta} \right] = 0.$$  \hspace{1cm} (21)
Here, $\Psi_i \{i = 1 - 9\}$ are the arbitrary constants. After utilizing the jth order HAM, we get

$$
f_j(\eta) = f_j^*(\eta) + \Psi_1 e^{\eta} + \Psi_2 e^{-\eta},
$$

$$
\theta_j(\eta) = \theta_j^*(\eta) + \Psi_3 e^{\eta} + \Psi_4 e^{-\eta},
$$

$$
\phi_j(\eta) = \phi_j^*(\eta) + \Psi_5 e^{\eta} + \Psi_6 e^{-\eta},
$$

$$
\chi_j(\eta) = \chi_j^*(\eta) + \Psi_7 e^{\eta} + \Psi_8 e^{-\eta}.
$$

Here, $f_j^*(\eta), \theta_j^*(\eta), \phi_j^*(\eta),$ and $\chi_j^*(\eta)$ are the particular solutions.

The HAM includes the auxiliary parameters $(h_f, h_0, h_2, h_3, h_4, h_5),$ and these are the responsible for solution convergence.

5. Convergence Analysis

The convergence values are of $h_f, h_0, h_2, h_3,$ and $h_4,$ are plotted in Figure 1. The range of convergence is $-0.4 \leq h_f \leq -0.1, -0.5 \leq h_0, h_2 \leq -0.1, -0.5 \leq h_3 \leq 0.0,$ and $-0.55 \leq h_4 \leq -0.2.$ Table 1 observes $f^n(0), \theta^n(0), \phi^n(0),$ and $\chi^n(0)$ for the 15th order of estimation. The convergence range of the current solution is $h_f = 0.35$ and $h_0 = h_2 = h_3 = h_4 = -0.30.$

6. Results and Discussion

This section focused on the effects of divergent physical factors on fluid velocity, fluid temperature, nanoparticle volume fraction, motile microbe density, skin friction coefficients, local Nusselt number, and local Sherwood number. Table 1 provides the validation of the present analysis with previously published results [18, 22]. From this comparison, we found that the current computation is an optimum one.

In this section, we focused on the variations of fluid velocity, fluid temperature, nanoparticle volume fraction, motile microorganism density, skin friction coefficients, local Nusselt number, and local Sherwood number for divergent physical parameters. Figures 2(a)–2(d)) provide the impact of $\alpha_1, \alpha_2, Ha, P_m, \alpha_1, N_r, R_b,$ and $S_1$ on the velocity profile. It is detected that the fluid velocity enriches when escalating the quantity of $\alpha_1, \alpha_2, Ha,$ and $P_m,$ and it downsalls when enhancing the quantity of $P_m, N_r, R_b,$ and $S_1.$ Physically, the modified Hartmann number leads to strengthening the external electric field, and this causes to increase the fluid velocity. The temperature variations of $Ha, R_b, S_1, \lambda, Fr,$ and $P_m$ are presented in Figures 3(a)–3(c)). It is seen that the fluid temperature escalates when raising the quantity of $R_b, Fr,$ and $P_m,$ and the opposite behaviour was attained when varying the values of $Ha, S_1,$ and $\lambda.$ Figures 4(a) and
4(b)) portray the consequences of \( R_b, S_2, \Gamma_2, \) and \( \lambda \) on the concentration profile. It is concluded that the fluid concentration increases when rising the quantity of \( R_b \), and it reduces when strengthening the values of \( S_2, \Gamma_2, \) and \( \lambda \). The microorganism profile for various values of \( \text{Pe}, \Omega, \text{Nr}, \) and \( L_b \) is illustrated in Figures 5(a) and 5(b) and found that the microorganism profile suppresses when enhancing the \( \text{Pe}, \Omega, \) and \( L_b \) quantities, and it escalates when escalating the values of \( \text{Nr} \).

Figures 6(a)–6(d) display the consequences of \( \text{Ha}, \text{Rd}, \Gamma_1, \) and \( L_b \) on the entropy generation profile. It is seen that the entropy generation diminishes near the plate and upturns away from the plate for varying the \( \text{Ha} \) and \( \Gamma_1 \) values, and the opposite behavior occurs for enhancing the \( \text{Rd} \) values. In addition, the \( L_b \) leads to enrich the entropy generation. The changes of the Bejan number for different values of \( \text{Fr}, S_1, \Gamma_2, \) and \( \text{Pe} \) are presented in Figures 7(a)–7(d) and seen that the Bejan number upturns near the plate and downfalls away from the plate for changing the \( \Gamma_2 \) and \( s \) values. The quite opposite trend attains for varying the \( \text{Fr} \) values. The \( S_1 \) values lead to reduce the Bejan number.

Fig. 8(a) reveals the collective effect of \( \text{Ha} \) and \( \lambda \) on \( [\text{Nu}]_x \) with other parameters are kept fixed heat transfer rate \( [\text{Nu}]_x \) is abridged with growing amounts of both \( \text{Ha} \) and \( \lambda \). Figure 8(b) explores the graphical assessment of Sherwood number \( \text{Sh}_x \) against the variations in \( \text{Cr} \) and \( \Gamma_1 \) with other parameters are retained fixed. The Sherwood number \( \text{Sh}_x \) is improved with the enhancement in \( \text{Cr} \) and \( \Gamma_2 \). Figure 8(c)
Table 1: Comparison of $C_f Re^{-0.5}$ for the limiting conditions. $P_m = Fr = Hm = \lambda = N_s = R_b = 0$.

| $\alpha_1$ | $\alpha_2$ | $\beta$ | $Re$ | Imtiaz et al. [22] | Loganathan et al. [18] | Present |
|------------|------------|---------|------|-------------------|------------------------|---------|
| 0.0        | 0.1        | 0.1     | 0.1  | 0.04605           | 0.04605                | 0.04605 |
| 0.1        | 1.06680    | 1.06680 | 1.7470 | 1.12010           | 1.12010                | 1.12010 |
| 0.2        | 1.17470    | 1.17470 | 1.17470 | 1.17470           | 1.17470                | 1.17470 |
| 0.0        | 0.1        | 0.1     | 0.1  | 1.01830           | 1.01830                | 1.01830 |
| 0.1        | 1.06290    | 1.06290 | 1.06290 | 1.06680           | 1.06680                | 1.06680 |
| 0.2        | 1.01830    | 1.01830 | 1.01830 | 1.01830           | 1.01830                | 1.01830 |
| 0.1        | 0.0        | 0.1     | 0.1  | 1.06680           | 1.06680                | 1.06680 |
| 0.0        | 1.06290    | 1.06290 | 1.06290 | 1.06680           | 1.06680                | 1.06680 |
| 0.1        | 1.06680    | 1.06680 | 1.06680 | 1.06680           | 1.06680                | 1.06680 |
| 0.2        | 1.01830    | 1.01830 | 1.01830 | 1.01830           | 1.01830                | 1.01830 |

7. Conclusions

In this article, we analyse the performance of heat-mass effects of third grade nanofluid flow through a triply stratified medium with swimming of nanoparticles, and gyrostatic microorganisms are swum into this flow. The non-Fourier heat and mass flux’s theory were used to frame the energy and nanoparticle concentration equations. The reduced models were analytically solved by applying the HAM scheme. The major outcomes are summarized as follows:

(i) The fluid velocity enhances when raising the modified Hartmann number, and it suppresses for a larger quantity of the thermal relaxation parameter.

(ii) The fluid temperature rises when enhancing the Forchheimer number and downfalls when increasing the bioconvection parameter.

(iii) The fluid concentration decays when strengthening the solutal relaxation time and stratification parameters.

(iv) The microorganism profile reduces when improving the quantity of $P_e, \Omega$, and $L_b$, and it escalates when escalating the values of $Nr$.

(v) The entropy rate is enhanced for higher values of the heat thermal relaxation parameter and bioconvection Lewis number.

(vi) The Bejan number diminishes for the solutal thermal relaxation parameter, thermal stratification, and bioconvection Peclet number.

Data Availability

“The raw data supporting the conclusion of this report will be made available by the corresponding author without undue reservation.”

Conflicts of Interest

The author declares that there are no conflicts of interest.

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