1. Introduction

In recent years, the adaptive control [1, 2] of nonlinear systems [3–7] is attracting more and more attention. As two main approximation tools, neural networks (NNs) and fuzzy logic systems (FLSs) [8, 9] are used to dispose some uncertain variables and unknown functions. For example, in [10], by using NNs, the neural network adaptive fault-tolerant control method under multiobjective constraints is designed. An effective finite-frequency $H_{-}/H_{\infty}$ fault detection method is proposed for the descriptor system in [11]. It is noted that the aforementioned studies neglect the influence of constraints on nonlinear systems.

In general, because of physical limitations and safety factors of the system, multifarious constraints exist in most practical systems, such as flexible string systems [12], flexible-joint robot manipulators [13], nonuniform gantry crane systems [14], and flexible aerial refueling hose [15]. The barrier Lyapunov function (BLF) is presented in [16] for strict feedback systems. Since then, as a main tool, the BLF plays a crucial role in dealing with constraints. Subsequently, full state constraints adaptive controller is designed based on BLF and backstepping technique in [17]. In order to handle the control problems of state-constrained nonlinear systems better, in [18, 19], the authors present an integral BLF. It is worth noting that none of the above articles involve time-varying constraints. In [20–22], the adaptive controller of time-varying full-state constraint is constructed. Thus, the constant constraints expand the time-varying constraints. However, there is a limitation in the adaptive control design based on BLF, i.e., the feasibility of the virtual controller, which will increase the cost of control design. Hence, how to eliminate the feasibility of the controller is an urgent problem. It is noted that few of them involve the feasibility conditions of intermediate controllers to marine vessel systems, which is also a challenge of this paper.

More recently, artificial intelligence has made impressive progress. More and more adaptive tracking control of marine vessel is widely studied. Subsequently, a number of significant results are proposed around the adaptive control design of the ship. The adaptive NN controller for ships is designed in [23]. Afterward, much effort has been made in addressing the situation where the ship system parameters are unknown. For the sliding mode method based on the model, an adaptive control approach is proposed in [24]. The precise recognition and learning control of marine ships...
under an unknown dynamic environment is further developed in [25]. Based on the above description, we propose an adaptive NN control method of marine vessel with time-varying output constraints.

In this article, we try our best to propose an adaptive NN control method for the marine vessel system with disturbance observer and time-varying output constraints. Furthermore, NNs are developed to deal with uncertain parameters and unknown functions. As is known to all, in the existing literature, this kind of marine vessel system with time-varying state constraints is rarely dealt with by removing the feasibility of virtual controller. A novel transformation function is proposed to ensure that the states do not violate the bounds. The main work and contributions of this article are as follows:

(1) As is known to all, most of the existing constraint control methods involve feasibility conditions. In this article, a new coordinate transformation is designed to resolve the time-varying position constraint of marine surface vessel without involving feasibility conditions. The coordinate transformation is used to get a control scheme that can avoid the feasibility condition, which is conducive to the realization of the control scheme.

(2) An exponential convergence disturbance observer is designed to deal with time-varying constraints and uncertain external disturbances, which are created by wind, waves, currents, and so on.

The specific organization of this paper is as follows. First, the dynamic model of the surface vessels is introduced. Then, the detailed procedure of BLF derivation for dealing with time-varying output constraints is given, in which the disturbance observer, the adaptive controller, and the adaptive laws are designed. Finally, simulation results verify the effectiveness of the proposed method.

2. Preliminaries

We consider the dynamics of 3 degree-of-freedom (3DOF) surface ships with model uncertainty and external disturbance. The dynamic model between the vessel’s position and velocity can be expressed as follows:

\[ \dot{\eta} = R(\eta) v, \]

where the vector \( \eta = [\eta_x, \eta_y, \eta_z]^T \in \mathbb{R}^3 \) stands for the output of earth-frame positions and heading, \( [\eta_x, \eta_y] \in \mathbb{R}^2 \) is the position of the vessel, and \( \eta_z \in \mathbb{R} \) denotes the heading, respectively. The vector \( v = [v_x, v_y, v_z]^T \in \mathbb{R}^3 \) represents the velocity vector of the vessel. The rotation matrix \( R(\eta) \) can be described as follows:

\[ R(\eta) = \begin{bmatrix} \cos \eta_z & -\sin \eta_z & 0 \\ \sin \eta_z & \cos \eta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R^{-1}(\eta) = R^T(\eta). \]

In this paper, the output of position and heading of marine vessel is constrained by

\[ -F_{k1}(t) \leq \eta \leq F_{k2}(t), \quad k = 1, 2, 3, \]

where

\[ F_{k1}(t) = [F_{11}(t), F_{21}(t), F_{31}(t)]^T, \]

\[ F_{k2}(t) = [F_{12}(t), F_{22}(t), F_{32}(t)]^T. \]

The relationship between the vessel’s velocity and the force acting on the vessel can be described by the dynamic model of vessel [26] as follows:

\[ M \ddot{v} + A(\eta)v + B(\eta)v + f(\eta) = \tau + g, \]

where \( g(t) = [g_1(t), g_2(t), g_3(t)]^T \) is unknown and time-varying external disturbance caused by wind and waves, and \( f(\eta) \) is unknown restorative force caused by the ship’s gravity and buoyancy. The control input is \( \tau = [\tau_1, \tau_2, \tau_3]^T \).

Besides, \( M = M^T \in \mathbb{R}^{3 \times 3} \) denotes the positive definite inertia matrix, \( A(\eta) \in \mathbb{R}^{3 \times 3} \) represents the torques of Coriolis and centripetal, and \( B(\eta) \in \mathbb{R}^{3 \times 2} \) stands for damping matrix. They are, respectively,

\[ M = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}, \quad A(\eta) = \begin{bmatrix} 0 & 0 & -a_{22}v_y - a_{23}v_y \\ 0 & 0 & a_{11}v_x \\ a_{22}v_y + a_{23}v_y & a_{11}v_x & 0 \end{bmatrix}, \quad B(\eta) = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \end{bmatrix}, \]

where

\[
\begin{align*}
    b_{11} &= -X_1 - X_{11}v_x - X_{11}v_x^2, \\
    b_{22} &= -Y_2 - Y_{22}v_y - Y_{22}v_y, \\
    b_{23} &= -Y_3 - Y_{23}v_y - Y_{33}v_y, \\
    b_{32} &= -N_2 - N_{22}v_y - N_{22}v_y, \\
    b_{33} &= -N_3 - N_{23}v_y - N_{33}v_y,
\end{align*}
\]

with \( a_{11}, a_{22}, a_{23}, a_{32}, a_{33}, X_1, X_{11}, X_{111}, Y_2, Y_{22}, Y_{222}, Y_3, Y_{33}, Y_{333}, N_2, N_{22}, N_{222}, N_3, N_{23}, N_{33} \) and \( N_{33} \) are design parameters.

Define \( x_1 = \eta \) and \( x_2 = v \), and then (1) and (5) can be transformed into the following equation:

\[
\begin{align*}
    \dot{x}_1 &= R(x_1)x_2, \\
    \dot{x}_2 &= M^{-1}(\tau + g - f(x_1) - A(x_2)x_2 - B(x_2)x_2).
\end{align*}
\]

The main task of this article is to design controller and adaptive laws so that the system output of vessel \( x_1 = [x_{11}, x_{12}, x_{13}]^T \) tracks the expected trajectory \( \tilde{\omega}_d = [\tilde{\omega}_{d1}, \tilde{\omega}_{d2}, \tilde{\omega}_{d3}]^T \) as effectively as possible. Meanwhile, all
signals are bounded in the closed-loop systems and their constraints are not violated forever.

As important approximation methods in nonlinear adaptive control, the radial basis function neural networks (RBFNNs) are usually utilized to deal with unknown and continuous functions. In this article, the RBFNNs are applied to approximate the unknown and continuous function $F(Z)$. The following relation holds:

$$F(Z) = \Theta^T \varphi(Z) + c(Z),$$  \hspace{1cm} (9)

where $c(Z)$ represents approximation error and satisfies $c(Z) \leq \varsigma$, which will be used in the following derivation. $Z \in \Omega_Z \subset R^l$ is the input vector, the optimal weight matrix is $\Theta^T \in R^l$, and $l \geq 1$ denotes the number of neurons. The known continuous basis function vector is $\varphi(Z) = [\varphi_1(Z), \varphi_2(Z), \ldots, \varphi_l(Z)]^T$ and $\varphi_i(Z) (i = 1, 2, \ldots, l)$ is the Gaussian function with

$$\varphi_i(Z) = \exp \left( \frac{\|Z - \mu_i\|^2}{\eta_i^2} \right), \quad i = 1, 2, \ldots, l,$$  \hspace{1cm} (10)

where $\mu_i = [\mu_{i1}, \ldots, \mu_{il}]^T, l > 1$ stands for the center of the receptive field, and the width of the Gaussian basis function is $\eta_i$.

**Remark 1.** As usual, the neural network chosen in this article has some special properties. Firstly, with the increase of the number of neurons, approximation error $c(Z)$ will gradually decrease. Secondly, the approximate area of the NNS is uniformly covered by the receptive field $\mu_i$. Finally, based on the equation $\eta_i = \sqrt{2d_{max}/\sqrt{l}}$, we can get the width of the Gaussian function and $d_{max}$ denotes the maximal Euclid distance of the approximation region.

It is necessary to research the marine vessel position constraints because marine environmental disturbances are time-varying and marine conditions are complex. In [27], for the nonlinear system control algorithm based on BLF, the state constraint is converted into the constraint of tracking error, which increases the restriction on the initial conditions of the system. It should be noted that most of the previous research studies on constraints were carried out in simple cases. To deal with time-varying output constraints gracefully, we consider the following nonlinear state-dependent transformation (NSDT):

$$\mu_{ik} = \frac{x_{ik}(t)}{(F_{k1}(t) + x_{ik}(t))(F_{k2}(t) - x_{ik}(t))},$$  \hspace{1cm} (11)

where the initial states $x_{ik}(0) \in D_{ik}$, in which $k = 1, 2, 3$. From (11), it should be pointed that $\mu_{ik}$ tends to infinity when $x_{ik}$ is close to $D_{ik}$, for any initial condition $x_{ik}(0) \in D_{ik}$. That is

$$\mu_{ik} \rightarrow \pm \infty \text{ if and only if } x_{ik} \rightarrow -F_{k1} \text{ or } x_{ik} \rightarrow F_{k2}.$$  \hspace{1cm} (12)

Therefore, it is noted that for any $x_{ik}(0) \in D_{ik}$, if $\mu_{ik} \in [0, 1] \cup [0, 1]$, $t \geq 0$, then $x_{ik} \in D_{ik}$ can be guaranteed for any $t \geq 0$, that is only need $\mu_{ik}, k = 1, 2, 3$, are bounded, and time-varying output constraints will not be violated. In other words, the issue of output constraints attributed to making sure that $\mu_{ik}$ for $t \geq 0$ is bounded.

**Assumption 1.** Time-varying function $F_{k1}, \ k = 1, 2, 3$, and $F_{k2}, \ k = 1, 2, 3$, are bounded and continuous, as well as its first derivative is also bounded and continuous.

The time derivative of $\mu_{ik}$ is given as

$$\dot{\mu}_{ik} = \eta_{ik}\dot{x}_{ik} + y_{ik},$$  \hspace{1cm} (13)

where

$$\eta_{ik} = \frac{F_{k1}F_{k2} + x^2_{ik}}{(F_{k1} + x_{ik})^2(F_{k2} - x_{ik})^2},$$  \hspace{1cm} (14)

$$y_{ik} = -\frac{(\dot{F}_{k1}F_{k2} + F_{k1}\dot{F}_{k2})x_{ik} + (\dot{F}_{k1} - \dot{F}_{k2})x^2_{ik}}{(F_{k1} + x_{ik})^2(F_{k2} - x_{ik})^2},$$

for $k = 1, 2, 3$.

Then, (13) can also be rewritten as follows:

$$\dot{\mu}_{k} = \eta_{k}\dot{x}_{k} + y_{k},$$  \hspace{1cm} (15)

where

$$\mu_{k} = [\mu_{k1}, \mu_{k2}, \mu_{k3}]^T \in R, \quad k = 1, 2, 3,$$

$$\eta_{k} = \text{diag}[\eta_{k1}, \eta_{k2}, \eta_{k3}] \in R^{3 \times 3}, \quad k = 1, 2, 3,$$

$$y_{k} = [y_{k1}, y_{k2}, y_{k3}]^T \in R, \quad k = 1, 2, 3.$$  \hspace{1cm} (16)

In this paper, feasibility condition in the control design is removed. Accordingly, it is much better to implement control methods. Most of the existing vessel constraint methods dispose the constant state constraints. For comprehensive consideration, the adaptive NN control of time-varying output constraints is studied in this article.

The different coordinate transformation is introduced as follows:

$$z_1 = x_1 - \beta_d,$$  \hspace{1cm} (17)

$$z_2 = x_2 - \alpha_1,$$  \hspace{1cm} (18)

with

$$\beta_d = [\beta_{d1}, \beta_{d2}, \beta_{d3}]^T \in R,$$

$$\beta_{dk} = \frac{\varpi_{dk}(t)}{(F_{k1}(t) + \varpi_{dk}(t))(F_{k2}(t) - \varpi_{dk}(t))}, \quad k = 1, 2, 3,$$  \hspace{1cm} (19)

where $\alpha_1$ is the intermediate controller and it will be defined later. It should be clear that $\beta_{dk}$ is bounded in the set of $D_{dk}$.

**Assumption 2.** The expected tracking signal $\varpi_{dk}, \ k = 1, 2, 3$, is bounded, and the derivative of $\varpi_{dk}$ with respect to time is also bounded.

Then, the time derivative of $\beta_d$ is given as follows:

$$\dot{\beta}_d = \eta_d \dot{\varpi}_d + \rho_d,$$  \hspace{1cm} (20)

where
\[ \eta_d = \text{diag}\{\eta_{dk}\} \in \mathbb{R}^{3\times 3}, \ k = 1, 2, 3, \]
\[ \rho_{dk} = [\rho_{d1}, \rho_{d2}, \rho_{d3}]^T \in \mathbb{R}, \ k = 1, 2, 3, \]
\[ \omega_{dk} = [\omega_{d1}, \omega_{d2}, \omega_{d3}]^T \in \mathbb{R}, \ k = 1, 2, 3, \]
\[ \eta_{dk} = \frac{F_{k1}F_{k2} + \omega_{dk}^2}{(F_{k1} + \omega_{dk})^2 (F_{k2} - \omega_{dk})^2}, \]
\[ \rho_{dk} = \frac{-(\dot{F}_{k1}F_{k2} + F_{k1}\dot{F}_{k2})\omega_{dk} + (\dot{F}_{k1} - \dot{F}_{k2})\omega_{dk}^2}{(F_{k1} + \omega_{dk})^2 (F_{k2} - \omega_{dk})^2}, \]
for \( k = 1, 2, 3 \), and according to the definition of \( \eta_{dk} \) and \( \rho_{dk} \), we know that \( \eta_{dk} \) and \( \rho_{dk} \) are known and computable and will be used in the design of the controller in the later.

**Remark 2.** For uncertain vessel nonlinear system, to make sure that the state constraints are never violated their bounded, the virtual controller must satisfy the feasibility condition:
\[ -F_{k1} \leq \alpha_{k1} \leq F_{k2}, \quad k = 1, 2, 3, \] (22)
where \( \alpha_{k1} \) is the virtual controller and \( -F_{k1} \) and \( F_{k2} \) are smooth functions. Although the virtual controller \( \alpha_{k1} \) is related to state variables, it also depends on other design parameters. The feasibility conditions of (22) must be satisfied, and only in this way can the control scheme be implemented effectively. However, it is unrealistic and even difficult to find such parameters.

**Remark 3.** It should be pointed that according to the actual need, the external disturbances are required to be bounded. Note that all of these boundary conditions are not necessary to implement adaptive control, and they can only be used in analysis.

**Remark 4.** If \( l \) and \( m \) are the nonnegative real numbers, \( a > 1, b \) is nonzero real number, and \( 1/a + 1/b = 1 \). Then, we can get
\[ \text{Im} \leq \frac{m^a}{a} + \frac{m^b}{b}. \] (23)

In this article, the following disturbance observer is constructed to deal with unknown disturbance:
\[ \bar{g} = y + K_o Mx_2, \]
\[ \dot{y} = -K_o y - K_o [-A(x_2)x_2 - B(x_2)x_2 - f(x_1)] + \tau + K_o Mz_2, \] (24)
where \( K_o \in \mathbb{R}^{3\times 3} \) is an observer gain positive definite matrix, \( y \) is the introduced intermediate variable, and the definition of \( \bar{g} \) is given in the following equation:
\[ \dot{\bar{g}} = \dot{y} + K_o Mx_2 \]
\[ = -K_o y - K_o [-A(x_2)x_2 - B(x_2)x_2 - f(x_1) + \tau + K_o Mz_2] \]
\[ + K_o [\tau + g - A(x_2)x_2 - B(x_2)x_2 - f(x_1)] \]
\[ = K_o [g - (y + K_o Mz_2)] + K_o (g - \bar{g}). \]
Then, differentiating \( \bar{g} \), it yields
\[ \dot{\bar{g}} = \dot{y} - \bar{g} = \dot{g} - K_0 (g - \bar{g}) = \dot{g} - K_0 \bar{g}, \] (26)
where \( \bar{g} = g - \bar{g} \) represents estimation error vector and \( \bar{g} = [\bar{g}_1, \bar{g}_2, \bar{g}_3]^T \), vector \( g = [g_1, g_2, g_3]^T \) denotes disturbance estimation, and vector \( \bar{g} = [\bar{g}_1, \bar{g}_2, \bar{g}_3]^T \) is external disturbance. Define \( \|\bar{g}(t)\| \leq C_g \leq \infty \), where \( C_g \) is a positive constant.

The detailed design steps are as follows:

**Step 1:** the following Lyapunov function is chosen:
\[ V_1 = \frac{1}{2} z_1^T z_1. \] (27)

Based on \( z_1 = \mu_i - \beta_d \) in (17), and the definition of system (8), then taking derivation of \( z_1 \), one obtains
\[ \dot{z}_1 = \dot{\mu}_i - \dot{\beta}_d \]
\[ = \eta_1 R(x_1)z_2 + y_1 - \eta_d \dot{\omega}_d - \rho_d \]
\[ = \eta_1 R(x_1)z_2 + \eta_1 R(x_1)\alpha_1 + y_1 - \eta_d \dot{\omega}_d - \rho_d. \] (28)

Based on (28), the time derivative of \( V_1 \) is given by
\[ \dot{V}_1 = z_1^T \dot{z}_1 \]
\[ = z_1^T (\eta_1 R(x_1)z_2 + \eta_1 R(x_1)\alpha_1 + y_1 - \eta_d \dot{\omega}_d - \rho_d). \] (29)

The virtual controller is designed as follows:
\[ \alpha_1 = R^T(x_1)\eta_1^{-1} (-K_1z_1 - y_1 + \eta_d \dot{\omega}_d + \rho_d). \] (30)

Substituting (30) into (29), we have
\[ \dot{V}_1 = -z_1^T K_1 z_1 + z_1^T \eta_1 R(x_1)z_2. \] (31)

**Step 2:** the Lyapunov function is designed as follows:
\[ V_2 = \frac{1}{2} z_2^T Mz_2 + \frac{1}{2} \bar{g}^T \bar{g} + \frac{1}{2} \sum_{i=1}^{3} \theta_i^T \Gamma_i^{-1} \theta_i, \] (32)
where \( \bar{\theta} = \theta - \hat{\theta} \) denotes the NN weight error, \( \hat{\theta} \) is the estimation of weight matrix \( \theta \), and \( \Gamma_i = \Gamma_i^T \) is positive definite matrix.

Taking the derivative of \( z_2 \) in (18), and based on 
\[ \dot{x}_2 = M^{-1} (\tau + g - f(x_1) - A(x_2)x_2 - B(x_2)x_2) \],
it yields
\[ \dot{z}_2 = x_2 - \dot{\hat{\theta}}_1 = M^{-1} (\tau + g - f(x_1) - A(x_2)x_2 - B(x_2)x_2) - \dot{\hat{\theta}}_1. \]  
(33)

Substituting (33) into (32), it leads to
\[ \dot{V}_2 = z_2^T M \dot{z}_2 + g^T \dot{g} + \sum_{i=1}^{3} \Lambda_i^{-1} \dot{\theta}_i \]
\[ = z_2^T (\tau + g - f(x_1) - A(x_2)x_2 - B(x_2)x_2 - M \dot{\hat{\theta}}_1) \]
\[ + g^T \dot{g} + \sum_{i=1}^{3} \Lambda_i^{-1} \dot{\theta}_i. \]  
(34)

The unknown function is defined by
\[ F_2(Z_2) = -A(x_2)x_2 - B(x_2)x_2 - M \dot{\hat{\theta}}_1 - f(x_1), \]  
(35)
where \( F_2(Z_2) \) is an unknown and continuous function and \( Z_2 = [x_1^T, x_2^T, a_1^T, a_2^T]^T \) are the inputs of the NNs. With the help of the NN approximation capability, the unknown function can be rewritten as follows:
\[ F_2(Z_2) = \theta^T \varphi(Z) + \zeta(Z). \]  
(36)

The actual controller \( \tau \) is constructed as follows:
\[ \tau = -R^T (x_1) \eta_1^T z_1 - K_z z_2 - g - \bar{g}^T \varphi(Z). \]  
(37)

Substituting (37) and (36) into (34), one obtains
\[ \dot{V}_2 \leq -z_2^T K_z z_2 - z_2^T \eta_1^T z_1 + z_2^T \bar{g} + z_2^T \zeta(Z) \]
\[ + z_2^T \varphi(Z) + \bar{g}^T \bar{g} - \sum_{i=1}^{3} \Lambda_i^{-1} \dot{\theta}_i. \]  
(38)

Substituting (26) into (38), we have
\[ \dot{V}_2 \leq -z_2^T K_z z_2 - z_2^T \eta_1^T z_1 + z_2^T \bar{g} + z_2^T \zeta(Z) \]
\[ + z_2^T \varphi(Z) + \bar{g}^T \bar{g} - \sum_{i=1}^{3} \Lambda_i^{-1} \dot{\theta}_i \]
\[ \leq -z_2^T K_z z_2 - z_2^T \eta_1^T z_1 + z_2^T \bar{g} + z_2^T \zeta(Z) \]
\[ + z_2^T \varphi(Z) + \bar{g}^T \bar{g} - \sum_{i=1}^{3} \Lambda_i^{-1} \dot{\theta}_i. \]  
(39)

The adaptive laws are selected as follows:
\[ \dot{\hat{\theta}}_i = \Gamma_i (\varphi(Z) z_{2,i} - \delta_i \dot{\theta}_i), \quad i = 1, 2, 3, \]  
(40)
where \( \delta_i > 0, i = 1, 2, 3 \) is the design parameter.

Using Young’s inequality, we have the following inequalities:
\[ z_2^T \bar{g} \leq h_1 z_2^T z_2 + \frac{1}{4h_1} \]  
\[ \bar{g}^T \bar{g} \leq h_2 \bar{g}^T \bar{g} + \frac{1}{4h_2} C g \]  
\[ z_2^T \zeta(Z) \leq \frac{1}{2} z_2^T z_2 + \frac{1}{2} \]  
(41)
\[ \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i = \frac{3}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i, \]  
where \( h_1 \) and \( h_2 \) are the design parameters.

Substituting all the above inequalities into (39), we have
\[ \dot{V}_2 \leq -z_2^T K_z z_2 - z_2^T \eta_1^T z_1 + z_2^T \bar{g} + \frac{1}{2} z_2^T z_2 + \frac{1}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i \]
\[ + \frac{1}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i + \frac{1}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i \]  
(42)
\[ \leq \frac{1}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i. \]  
(43)

Consider the Lyapunov function as follows:
\[ V = V_1 + V_2 \]
\[ = \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_2^T M z_2 + \frac{1}{2} \bar{g}^T \bar{g} + \frac{1}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i. \]  
(44)

From (31) and (42), we can obtain
\[ \dot{V} \leq -z_1^T K_z z_1 + z_2^T \varphi(Z) z_2 - z_2^T \eta_1^T z_1 + z_2^T \bar{g} + z_2^T \zeta(Z) \]
\[ + h_1 z_2^T z_2 + \frac{1}{2} z_2^T \bar{g} + \frac{1}{2} h_2 \bar{g}^T \bar{g} - g^T K_2 g - \frac{3}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i \]
\[ \leq -z_1^T K_z z_1 - z_2^T \eta_1^T z_1 + \frac{1}{2} \bar{g}^T \bar{g} - g^T K_2 g - \frac{3}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i \]
\[ \leq \frac{1}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i + \frac{1}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i \]  
(45)
\[ \leq \frac{1}{2} \sum_{i=1}^{3} \delta_i \bar{\theta}_i^T \bar{\theta}_i - CV + D, \]  
(46)
where

\[
C = -2 \min \left[ \lambda_{\min}(K_1), \lambda_{\min}(K_2M^{-1}) - h_1\lambda_{\max}(M^{-1}) - \frac{1}{2}\lambda_{\max}(M^{-1}), \lambda_{\min}(K_0) - \frac{1}{4h_1} - h_2, \frac{1}{2} \delta \lambda_{\min}(\Gamma_1) \right],
\]

\[
D = \frac{1}{2} \sum_{i=1}^{3} \delta \|\theta_i\|^2 + \frac{1}{4h_2} C_y + \frac{1}{2} \delta,
\]

with \( \lambda_{\max}(\cdot) \) and \( \lambda_{\min}(\cdot) \) standing for the largest and minimum eigenvalue of a matrix, respectively.

**Theorem 1.** Consider the marine vessel nonlinear system with unknown time-varying disturbances (8) under the virtual controller (30), actual controller (37), and adaptive laws (40). Then, we have the following: (1) error signal undulates in a small neighborhood close to zero; (2) all signals in the closed-loop system are SGUUB; and (3) in the meantime, the output signal will not violate the time-varying boundary.

**Proof.** According to (44), we can obtain

\[
0 \leq V(t) \leq \frac{D}{C} + \left( V(0) - \frac{D}{C} \right) e^{-Ct},
\]

\[
\|z_i\| \leq \sqrt{\frac{2D}{C} + 2 \left( V(0) - \frac{D}{C} \right) e^{-Ct}}.
\]

From (44), it is clear that the signals \( z_1, z_2, \bar{\theta}_i, i = 1, 2, 3, \) and \( \bar{\gamma} \) are SGUUB. Since \( \bar{\theta}_i, i = 1, 2, 3 \) is bounded by \( \bar{\theta}_i = \theta_i - \bar{\theta}_i \) with \( \theta_i \) being also bounded. As well, \( b \) is bounded. Because \( \bar{\theta}_i \) is bounded in the set \( D_k, \) from \( z_i = \mu_i - \bar{\theta}_i, \) we can infer that \( \mu_i \in L_{\infty}, k = 1, 2, 3. \) Based on (12), for any initial condition \( x_{ik}(0) \in D_{ik}, k = 1, 2, 3, \) one deduces that \( x_{ik}(t) \) is bounded. Because \( \eta_{d1}, \eta_{d2}, \omega_{d}, \) and \( \omega_{d} \) are bounded, gradually, it is obtained that the virtual controller \( \alpha_1 \) is also bounded. According to \( z_2 = x_2 - \alpha_1, \) we can infer that \( x_2 \) is bounded.

\[ \square \]

### 3. Simulation Example

In this section, to illustrate the effectiveness of the proposed method, we conduct the simulation on the model of Cybershers II, which is a 1 : 70 scale supply vessel replica setup in a marine control laboratory in the Norwegian University of Science and Technology [28]. The following expected trajectory is considered:

\[
\begin{align*}
\omega_{d1} &= -0.2 + 0.04 \sin(0.5t), \\
\omega_{d2} &= -0.1 + 0.02 \cos(0.2t), \\
\omega_{d3} &= 0.002 \sin(t),
\end{align*}
\]

(47)

where the matrices are chosen as follows:

\[
M = \begin{bmatrix}
20 & 0 & 0 \\
0 & 23.6 & -0.8 \\
0 & -0.8 & 2.76
\end{bmatrix},
\]

\[
A = \begin{bmatrix}
0 & 0 & 23.6v_y - 0.8v_y \\
0 & 0 & 20v_x \\
23.6v_y + 0.8v_y & -20v_x & 0
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
B_1 \\
B_2 \\
B_3
\end{bmatrix},
\]

where

\[
B_1 = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
\]

\[
B_2 = \begin{bmatrix}
0 \\
0.3578 + 34.5653v_y + 5.21v_y \\
0.674v_y + 3.76v_y - 0.1079
\end{bmatrix},
\]

\[
B_3 = \begin{bmatrix}
0 \\
-1.052 - 5.0437v_y - 0.13v_y \\
0.674v_y + 3.76v_y - 0.1079
\end{bmatrix},
\]

The disturbance vector is given as follows:

\[
g(t) = \begin{bmatrix}
1.4 + 2 \sin(0.02t) + 1.5 \sin(0.1t) \\
-0.9 + 2 \sin\left(0.02t - \frac{\pi}{6}\right) + 1.5 \sin(0.3t) \\
-\sin\left(0.09t + \frac{\pi}{3}\right) - 4 \sin(0.01t)
\end{bmatrix}.
\]

The initial state variables are selected as \( x_1 = [x_{11}, x_{12}, x_{13}]^T = [0.004, 0.019, 0]^T \) and \( x_2 = [x_{21}, x_{22}, x_{23}]^T = [0.4, 0.05, 0]^T. \) The state variables \( x_{11}, x_{12}, \) and \( x_{13} \) satisfy \(-F_{11} < x_{11} < F_{11}, -F_{12} < x_{12} < F_{12}, \) and \(-F_{13} < x_{13} < F_{13} \) with \( F_{11} = 1 + 0.1 \sin(t), F_{12} = 1 + 0.1 \sin(t), F_{13} = 1 + 0.1 \sin(t), F_{22} = 1 + 0.1 \sin(t), F_{23} = 0.3 + 0.1 \sin(t) \) and \( F_{23} = 0.3 + 0.1 \sin(t). \)

The initial value of the observer is selected as \( \bar{\gamma}(0) = [0, 0, 0]^T. \) The design parameters are given as \( \delta = [0.001, 0.001, 0.001]^T. \) The parameter matrices are given as \( K_1 = 50I, K_2 = 100I, K_3 = 200I, \) and \( K_0 = \text{diag}[2, 2, 2]. \) The
control gains are set as \( K_1 = \text{diag}[0.001, 0.1, 0.1] \) and \( K_2 = \text{diag}[200, 200, 0.02] \).

Figures 1–11 show the simulation results. Figures 1–3 show the output signal \( x_1 \), reference signal \( \varpi_{d1} \), and constraints interval \( F_{k1} \) and \( F_{k2} \) with \( k = 1, 2, 3 \), which indicates that the tracking trajectory is perfect, and the state variable does not violate the constraint interval. Figure 4 indicates the trajectory of the state variable \( x_2 \). Figures 5 and 6 are given to illustrate the trajectories of tracking error \( z_1 \) and \( z_2 \), and it can be seen that the error is small enough. The trajectory of actual controller is displayed in Figure 7. From Figure 8, it indicates that the disturbance observer is constructed to deal with the external disturbance better. Meanwhile, Figures 9 and 10 are used to explain the trajectory of adaptive laws. We can clearly see that the adaptive laws are bounded. Figure 11 shows the trajectory of virtual controller. The results show that the adaptive control method is effective.
Figure 3: Trajectory of $\dot{\omega}_{d3}$, $x_{13}$, and constraint intervals.

Figure 4: Trajectory of velocity vector $x_2$.

Figure 5: Trajectories of errors $z_1$. 
Figure 6: Trajectories of errors $z_i$.

Figure 7: Trajectories of control input $\tau$.

Figure 8: Trajectories of external disturbances $g_1$, $g_2$, and $g_3$ and their estimations $\hat{g}_1$, $\hat{g}_2$, and $\hat{g}_3$. 
Conclusion

The adaptive NN control method is studied for vessel nonlinear systems with disturbance observer and time-varying output constraints in this paper. Compared with the existing research on vessel, the method studied in this article is more practical. First, the feasibility condition of virtual controller is eliminated by introducing nonlinear state correlation function and new coordinate transformation. Only in this way, we can deal with control design gracefully. Then, with the help of NNs, the unknown and continuous function is approximated to simplify the design of the controller. Next, the disturbance observer is constructed to dispose the disturbances. Finally, the control performance is tested through simulation examples. In the future work, we will apply the constraint form of this paper.
to the position and speed constraints of flexible manipulator.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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