Turbulence anisotropy and coherent structures in electromagnetically generated vortex patterns

S. Kenjereš

1 Faculty of Applied Sciences, Delft University of Technology, Delft, The Netherlands
E-mail: s.kenjeres@tudelft.nl

Abstract. Numerical investigations addressing influence of the localised electromagnetic forcing on turbulent thermal convection of a weakly electrically conductive fluid in a wall-bounded rectangular enclosure are performed over a wide range of working parameters ($10^4 \leq Ra \leq 5 \times 10^5$, $Pr=7$). An asymmetrical electromagnetic forcing (EMF) is applied originating from combined effects of the imposed magnetic fields (originating from an array of $5 \times 7$ permanent magnets with $|b_0|_{\text{max}}=1$ T each, located beneath the lower thermally active wall) and electric fields (originating from two electrodes supplied with dc current of different intensities, $0 \leq I \leq 10$ A). Subgrid turbulent stress is modelled by electromagnetically extended Smagorinsky model and subgrid turbulent heat flux is represented by a simple gradient diffusion hypothesis. Simulations revealed two interesting findings: the electromagnetic forcing generated significant overall heat transfer increase (more than 500% for lower values of $Ra$) compared to its neutral case, and, the turbulence anisotropy was reduced in the central part of the enclosure.

1. Introduction

In present study we address electromagnetically generated three-dimensional vortex patterns in weakly conductive fluid (sea water). The vortex flow patterns were generated by combining arrays of permanent magnets and electrodes with supplied dc current inside a rectangular enclosure heated from below and cooled from above. Without the imposed electromagnetic forcing, this configuration mimics standard Rayleigh-Bénard convection case. We investigate to what extent the imposed electromagnetic forcing will affect flow, turbulence and heat transfer over a wide range of working parameters, $10^4 \leq Ra \leq 5 \times 10^5$. The strength of the electromagnetic forcing (EMF) is controlled by imposed dc current, which is in $0 \leq I \leq 10$ A range, enabling weak, intermediate or strong interactions with initially entirely thermally driven turbulent convective flow.

2. Equations and Numerical Method

The system of equations describing flow and heat transfer in turbulent regime of a weakly electrically conductive fluid subjected to combined effects of magnetic and electric fields can be
written as:

\[ \partial u_i = 0, \quad \partial b_i = 0, \quad \partial j_i = 0, \quad (1) \]
\[ \partial_t u_i + u_j \partial_j u_i = \partial_j (\nu \partial_j u_i - \tau_{ij}) - \frac{1}{\rho} \partial_i p + \sum f_i, \quad (2) \]
\[ \sum f_i = f_i^B + f_i^L = \beta g (T_0 - T) + \frac{1}{\rho} \epsilon_{ijk} j_j b_k, \quad (3) \]
\[ \partial_i \theta + u_j \partial_j \theta = \partial_j (a \partial_j \theta - \tau_{ij}), \quad (4) \]

where \( u_i, \theta, b_i \) and \( j_i \) are the velocity, temperature, magnetic field and total electric current density, respectively. In the momentum equation, two additional forces \( f_i \) are the thermal buoyancy \( f_i^B \) and electromagnetic force \( f_i^L \), respectively. Additionally, \( \rho \) is the fluid density, \( \nu \) kinematic viscosity, \( \alpha \) thermal diffusivity and \( \beta \) the thermal expansion coefficient. The total electric current densities are calculated as \( j = \sigma (e + u \times b) \), where \( \sigma \) is electric conductivity and \( e \) is the electric field. Note that for particular application, we have \( ||e||/||u \times b|| > 10^3 \), and the second term in equation for \( j \) can be omitted. The distributions of magnetic and electric field is calculated from simplified set of Maxwell’s equations by applying Biot-Savart law for electrodes and permanent magnets in the form of the analytical solutions for three-dimensional distributions of charges. This is sufficient since we have the one-way coupling between electromagnetic fields and velocity. The subgrid turbulent stress \( \tau_{ij} \) and turbulent heat flux \( (\tau_{ij}) \) are modelled by a turbulent viscosity model based on the electromagnetically extended version of the Smagorinsky model, Shimomura (1991), Kenjereš (2011). The system of equations Eqs.(1)-(4) is discretised and solved by a second-order accurate (in time and in space) finite-volume solver for structured geometries, Kenjereš (2008, 2011). The simulation domain - the rectangular enclosure heated from below and cooled from top, with adiabatic side walls - is represented by \( 182^2 \times 92 \) control volumes - clustered in the proximity of all walls. A special care is taken to properly resolve strong temperature and velocity gradients in the proximity of walls by placing 5-10 CV within boundary layers. A fully implicit second-order three-consecutive time steps scheme is used for time integration. The fully-convergent long-time statistics is obtained by averaging between \( 5 \times 10^4 \) (without electromagnetic forcing) and \( 7.5 \times 10^4 \) (with electromagnetic forcing) instantaneous fields.

3. Results

The configuration of set-up is shown in Fig. 1 where distributions of the vertical magnetic field component are depicted in the central vertical and in a horizontal plane close to the bottom wall. Imprints of an array of 5 × 7 permanent rectangular magnets of opposite polarities are clearly visible. The magnetic forcing itself is not enough to affect the motion of a fluid of relatively low electric conductivity. This is achieved by supplying dc current through electrodes located on left- and right-side walls of the enclosure - both extending along entire depth of the enclosure (\( 0 \leq y \leq 0.6 \) m). The combined effects of magnets and electric currents generate electromagnetic force that is active in the proximity of the bottom wall. By varying strength of the imposed dc current, different forcing intensities can be generated - ranging from weak (I=0.1 A), intermediate (I=1 A) to strong (I=10 A) forcing. Without imposed electromagnetic forcing (EMF), we have a classical case of a fully developed turbulent thermal convection (Rayleigh-Bénard convection).

The long-term averaged velocity and temperature fields are used for showing characteristic flow patterns in vertical and horizontal planes, Figs. 2,3. The weak electromagnetic case (I=0.1 A) shows presence of large coherent convective cells in the central part of enclosure, two small corner rolls, and initial appearance of the rolls in the horizontal boundary layer, Fig. 2a. The situation is very different for intermediate EMF case (I=1 A), where central rolls are not present
anymore and an array of smaller vortical cells is generated in the proximity of the bottom wall, Fig. 2b. From horizontal slices, it is clear that even for a weak imposed dc current of \(I=0.1\) A, well defined vortical structures are generated within the horizontal boundary layer along the lower wall, Fig. 3a-left. Imprints of these vortical structures are also visible in the central horizontal plane, but to a lesser extent due to still dominant thermal buoyancy forcing in the central part of the enclosure, Fig. 3a-right. At the same location, distribution of temperature indicates a strong vertical updraft in the upper part of the enclosure (red colour) (above \(y=0.3\) m line) whereas the cold downdrafts (blue colour) are squeezed to regions in the proximity of the side walls. Contours of the vertical vorticity component (\(\omega_z\)) are shown in Fig. 4. Stronger dc current generates well-defined and self-sustained vortical structures in the central horizontal plane, Fig. 4-right. These structures mirror exactly the underlying geometrical distribution of the permanent magnets - confirming the EFM dominant flow regime.

Integral heat transfer coefficient (\(\text{Nu}\)) at the lower horizontal wall for different values of \(10^4 \leq \text{Ra} \leq 5\times10^9\) and different applied dc currents (\(0\leq I \leq 10\) A) are shown in Fig. 5. Three distinct regimes can be distinguished: the EMF dominant region - for low Ra and strong
Figure 2. Temperature contours and streamtraces in the central vertical (x-z) plane: (a.) weak electromagnetic forcing (EMF), I=0.1 A; (b.) intermediate EMF, I=1 A; Ra=10^7, Pr=7.

dc currents, transitional region - for intermediate Ra and strong EMF, and, finally, thermal buoyancy dominant flow regimes - for high Ra. It can be seen that a good agreement is obtained for a neutral case (no EMF imposed) with a DNS results of Verdoold et al. (2008) proving validity of employed LES and its sufficient the near-wall resolution. For low Ra, the EMF significantly enhances integral heat transfer (more than 500% increase) because of dramatic flow and turbulence structure reorganisation compared to its neutral state. This level of the heat transfer enhancement can be compared to other forcing methods, for example with rotation, where the maximum values obtained were about 20% - for very similar range of working parameters, Zhong et al. (2009). The presence of the electromagnetically generated vortical structures within horizontal boundary layer particularly contribute to this dramatic heat transfer increase. For transitional regime, both EMF and thermal buoyancy are of same importance, and deviations from the pure EMF start to appear. For Ra≥10^9, thermal buoyancy effects are so strong that imposed EMF does not change significantly overall heat transfer.

The long-term and spatially averaged (in horizontal plane) vertical profiles of turbulent kinetic energy and of the turbulent stresses are shown in Fig. 6. It can be seen that EMF brings significant increase of the TKE and introduces asymmetrical distributions due to the non-symmetrical forcing (only along the bottom wall), Fig. 6a. In the EMF dominant regime, there is only weak influence of the background thermal convection, e.g. TKE profiles for Ra=10^9 and 10^8 for I=10 A show just small differences. The redistribution of the energy among fluctuating velocity components is depicted in Fig. 6b. For neutral case, the horizontal components reach their peak values in the proximity of the horizontal walls, while the vertical component reach its peak in the enclosure centre. The EMF introduces significant increase of the horizontal fluctuations in the proximity of the bottom wall, while the horizontal components are just slightly different in the proximity of the top wall - compared to the neutral state. In addition, this increase of the horizontal components of the turbulent stress surpasses its vertical component indicating significant re-channelling of the energy redistribution. To portray levels of turbulence anisotropy for neutral and EMF cases, invariants of turbulent stresses are analysed, Fig. 7. The
Figure 3. (a.) Streamtraces and temperature contours of long-term averaged fields in the central horizontal plane (-left) and in the proximity of the bottom wall (-right), Ra=10^7, Pr=7, applied dc current of I=0.01 A. (b.) same as for (a.) only now for I=1 A.

second- and third-invariant of the turbulent stress are defined as $A_2 = a_{ij}a_{ji}$ and $A_3 = a_{ij}a_{jk}a_{ki}$, where $a_{ij} = \frac{\langle u_iu_j \rangle}{k} - \frac{2}{3} \delta_{ij}$ is anisotropy tensor. The Lumley’s flatness parameter, defined as $A = 1 - \frac{9}{8} (A_2 - A_3)$, is unity in isotropic turbulence, and is zero in two-component turbulence (where one normal stress vanishes). This parameter is used to evaluate levels of turbulence isotropy for the neutral and EMF case, for two considered values of Ra=10^6 and 10^9, as depicted in Fig. 7. For Ra=10^6 case, due to significantly stronger mixing in the enclosure centre generated by applied EMF, flatness parameter shows values close to $A=1$, indicating almost perfectly isotropic turbulence, Fig. 7a. This is in contrast to neutral case where turbulence is still highly anisotropic ($A=0.6$). For Ra=10^9 case, the EMF again reduces levels of turbulence anisotropy in the central part of the enclosure ($0.2 \leq z/H \leq 0.8$) - but to lesser extent compared to lower value of Ra=10^6.
Figure 4. The vertical vorticity contours ($\omega_z$) in the central horizontal plane: EMF case with $I=0.01$ A (-left), EMF case with $I=1$ A (-right).

Figure 5. Integral heat transfer coefficient ($Nu$) at the lower horizontal wall for different $Ra=10^4 - 5 \times 10^9$ and different strengths of the imposed dc current $I=0, ..., 10$ A.

4. Conclusions

The high-resolution numerical simulations are performed for a range of Rayleigh numbers ($10^4 \leq Ra \leq 5 \times 10^9$) and different strengths of the imposed dc current ($0 \leq I \leq 10$ A), while the strength of the magnetic field was constant (permanent magnets with $|B_0| = 1$ T). The numerical simulations revealed that EMF forcing generated more than five-fold increase of the wall-heat transfer for low- and intermediate-range of Rayleigh numbers ($10^4 \leq Ra \leq 10^6$). This is significantly higher when compared to the rotating turbulent Rayleigh-Bénard convection for similar range or Ra numbers, where maximum of a rather modest 20% increase is reported, Zhong et al. (2009). In contrast to the generally accepted view that electromagnetic forcing
Figure 6. The long-time and spatially (in horizontal plane) averaged vertical profiles of: (a.) turbulent kinetic energy for $10^6 \leq Ra \leq 10^9$, $Pr=7$, $I=0$ (-solid lines) and $10$ A (-dashed lines); (b.) the turbulent stresses without (solid lines) and with (dashed lines) imposed electromagnetic forcing $I=1$ A, $Ra=10^8$, $Pr=7$.

will suppress velocity fluctuations and will increase anisotropy of turbulence, we demonstrated that localised forcing can enhance turbulence isotropy of thermal convection compared to its neutral state.

References

HONJI H., OHKURA M. & IHEHATA Y. 1997 Flow patterns of an array of electromagnetically-driven cellular vortices. *Exp. Fluids* 23, 141.

KENJEREŠ S. & HANJALIČ K. 2007 Numerical simulation of a turbulent magnetic dynamo. *Phys. Rev. Lett.* 98(10), 104501.
Figure 7. The vertical profiles of the second ($A_2 = a_{ij}a_{ji}$), third ($A_3 = a_{ij}a_{jk}a_{ki}$) invariants of the turbulent stresses and of Lumley’s flatness parameter ($A = 1 - 9(A_2 - A_3)/8$) (where $a_{ij} = u_iu_j/k - 2/3\delta_{ij}$): (a.) $Ra=10^6$, (b.) $Ra=10^9$, without (-solid lines) or with electromagnetic forcing of $I=1$ A (-dashed lines).

KENJEREŠ, S. 2008 Electromagnetic enhancement of turbulent heat transfer. Physical Review E 78(6), 066309.

KENJEREŠ, S. 2011 Electromagnetically driven dwarf tornados in turbulent convection. Physics of Fluids 23(1), 015103.

LARDEAU, S., FERRARI, S. & ROSSI L. 2008 Three-dimensional numerical simulation of electromagnetically driven multi-scale shallow layer flows: Numerical modelling and physical properties. Physics of Fluids 20, 127101.

ROSSI, L., VASSILICOS, J. C. & HARDALUPAS, Y. 2006 Multiscale laminar flows with turbulentlike properties. Phys. Rev. Lett. 97(14), 144501.

ROSSI, L., VASSILICOS, J. C. & HARDALUPAS, Y. 2006 Electromagnetically controlled multi-scale flows. J. Fluid Mech. 558, 207.

SHIMOMURA Y. 1991 Large eddy simulation of magnetohydrodynamics turbulent channel flows under a uniform magnetic field. Physics of Fluids 3, 3098.
Verdoold J., van Reewijk M., Tummers, M. J., Jonker H. J. J. & Hanjalić K. 2008 Spectral analysis of boundary layers in Rayleigh-Bénard convection. Phys. Rev. E 77, 016303.

Zhong, J. Q., Stevens, R. J. A. M., Clerx, H. J. H., Verzicco, R., Lohse, D. & Ahlers, G. 2009 Prandtl-, Rayleigh-, and Rossby-Number Dependence of Heat Transfer in Turbulent Rotating Rayleigh-Bénard Convection. Phys. Rev. Lett. 102, 044502.