Electron - Dark Matter Scattering in an Evacuated Tube

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The light dark matter model can explain both the primordial abundance of dark matter and the anomalous 511 keV gamma-ray signal from the galactic center. This model posits a light neutral scalar, $\chi$, with a mass in the range $1 \text{ MeV} < M_\chi < 10 \text{ MeV}$, as well as a light neutral spin-1 boson, $U$, which mediates the annihilation channel $\chi\chi \rightarrow e^+e^-$. Since the dark matter particle is light, its number density is relatively large if it accounts for a local dark matter density of $\rho = 0.3 \text{ GeV/cm}^3$. We consider an experiment in which a low-energy, high-current electron beam is passed through a long evacuated tube, and elastic scattering of electrons off dark matter particles is observed. The kinematics of this process allow a clean separation of the signal process from scattering off residual gas in the tube, and also a direct measurement of $M_\chi$.

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I. INTRODUCTION

The nature of dark matter is one of the most interesting questions of modern science. The canonical explanation from particle physics posits a massive, weakly-interacting particle. However, a mass on the order of 100 GeV is not strictly necessary, and a model based on the idea of light dark matter particles [1, 2] has been proposed by Boehm, Fayet and others [2, 3]. This model can also explain the anomalous 511 keV gamma-ray signal from the galactic center observed by INTEGRAL [1, 2] and earlier balloongborne devices [1, 2].

The light dark matter model (LDM) posits a light neutral scalar particle, $\chi$, with mass in the range $1 \text{ MeV} < M_\chi < 10 \text{ MeV}$ [3], as well as a light neutral spin-1 boson, $U$, which mediates the annihilation channel $\chi\chi \rightarrow e^+e^-$. Some versions of the model also include a heavy charged fermion, $F^\pm$ [3], but this particle plays no role in the present study. It is also possible for the light dark matter particle to be a fermion, but for our present estimates we will assume that it is a scalar.

Particle physics and astrophysical data constrain the coupling constants of electrons and of $\chi$ to the $U$-boson as a function of $M_\chi$ and $M_U$. Following Fayet [2],

$$|C_\chi f_\chi| \approx 10^{-6} \frac{M_U^2 - 4M_\chi^2}{M_\chi (1.8 \text{ MeV})} \sqrt{B_{\text{ann}}}$$

where $C_\chi$ and $f_\chi$ are the $U$-$\chi$ and $U$-$e^-$ coupling constants, respectively, and $B_{\text{ann}}$ is the fraction of all $\chi\chi$ annihilations which result in an $e^+e^-$ final state. We take $B_{\text{ann}}^\text{ee} = 1$ in the present study.

If the annihilation channel $\chi\chi \rightarrow e^+e^-$ exists, then the scattering channel $e^-\chi \rightarrow e^-\chi$ also exists with a cross-section that is directly related to the annihilation cross-section. We compute the scattering cross-section on the basis of the LDM model, and describe a conceptual experiment to observe this process. The scattering rate depends on the local dark matter density, which we take to be $\rho = 0.3 \text{ GeV/cm}^3$. Even with an ultra-high vacuum, a large background from atomic scattering persists, but this background can be eliminated on the basis of the kinematics of the scattered electron. In a previous study [2], we investigated the production of $\chi$-particles in low-energy $e^-$ $p$ scattering. We also computed the contribution of $U$-bosons to rare pion decay [10]. The present “evacuated tube” experiment would confirm and extend the knowledge gained from the $e^-$ $p$-scattering experiment.

II. SCATTERING CROSS SECTIONS

We consider elastic scattering of relativistic electrons off quasi-stationary scalar dark matter particles, $\chi$, via the $t$-channel exchange of a neutral spin-1 boson, $U$. We notate the four-vectors as follows: $p_1$ is the incoming electron, $p_2$ is the $\chi$ particle before collision, $p_3$ is the outgoing electron, and $p_4$ is the outgoing $\chi$. Let $E'$ be the energy of the incoming electron in the lab frame, and $E'$ be the outgoing energy; as long as $E \gg m_\chi$ we can neglect the electron mass and write $p_1 = (E,0,0,E)$ and $p_3 = (E',E'\sin \theta,0,E'\cos \theta)$, which defines the electron scattering angle $\theta$.

We assume that the $U$-boson has a purely vector coupling to the $e^-$, so that the $U$-$e^-$ vertex factor is $f_\nu \gamma^\mu$. Similarly, the Feynman rule for the $U$-$\chi$ vertex is $C_\chi (p_2 + p_4)$. Boehm and Fayet [1] give the amplitude for $\nu$-$\chi$ scattering,

$$|M|^2 = \frac{C_\chi^2 f_\nu^2}{(t - M_U^2)} (s - u)^2 + (4M_\chi^2 - t)),$$

where $f_\nu$ is the $U$-$\nu$ coupling. This expression coincides with the spin-averaged amplitude for $\chi$-$e^-$ scattering, in the ultra-relativistic limit, with $f_\nu$ replaced by $f_e$. In the lab frame, $t = -4EE'\sin^2(\theta/2)$ and $s - u = 4M_\chi E'$, so
with our notation,

$$
\langle |M|^2 \rangle = \frac{16C_\chi^2 f_\chi^2}{(4EE'\sin^2(\theta/2) + M_\chi^2)^2} \times \\
(M_\chi^2 E^2 + E^2E'^2 \sin^4(\theta/2) - EE'M_\chi^2 \sin^2(\theta/2)) \cdot
$$

For a $2 \rightarrow 2$ scattering process with a relativistic incident particle, the differential cross-section is given by

$$
\frac{d\sigma}{d\Omega} = \langle |M|^2 \rangle \left( \frac{E'}{8\pi M_\chi E} \right)^2. \tag{2}
$$

For $M_\nu = 10$ MeV and $M_\chi = 2$ MeV, Eq. (1) gives $|C_\chi f_\chi| \approx 2.3 \times 10^{-5}$. Taking $E_{\text{beam}} = 100$ MeV and integrating Eq. (2) numerically, we obtain a total cross-section of 138 pb.

III. CONCEPTUAL DESIGN

Our concept for a practical experiment is as follows. A low-energy, high-current electron beam is passed through a long evacuated tube. The energy of the beam is $E = 100$ MeV and the average current is 50 kA. There is no need for the beam to be well-focused, nor is the time structure important. The length of the tube is taken to be 100 m, and its radius, 2.5 cm. An ultra-high vacuum is established in the tube, at the level of $10^{-12}$ Torr, which has been achieved already in the laboratory. There would be about $3 \times 10^4$ molecules per cm$^3$ to be compared to $O(100)$ dark matter particles. The ultra-high vacuum tube is encased within a tube of radius 20 cm, with a lesser vacuum of $10^{-6}$ Torr, which contains and supports all the instrumentation. See Fig. 1 for a drawing.

![Fig. 1: drawing of a repeated structure consisting of two Si detector planes and a calorimeter module, through which the thin tube at ultra-high vacuum is threaded. The beam enters at the left. The outer tube is held at a more ordinary vacuum and supports the detector modules. This drawing shows approximately one-fiftieth of the entire experiment. Example trajectories for a scattered electron and dark matter particle are shown.](image)

The kinematics of the outgoing electron would be measured with the following apparatus. Two silicon pixel detector plates, spaced 5 mm apart, would be placed perpendicular to the beam line, followed by a circular array of calorimeter elements. This arrangement would be repeated every 50 cm. If an electron passes through the pair of detector plates and the calorimeter array, the two position measurements at the silicon plates would provide a measurement of $\theta$, while the calorimeter would measure $E'$. This arrangement minimizes the effects of multiple scattering, since the path of the outgoing electron is nearly normal to the plane of the plates.

The acceptance of this detector apparatus is limited by the thickness of the calorimeters, since electrons exiting the inner tube directly under a calorimeter array would miss both silicon detectors. To reduce this “dead area” it is advantageous to make the calorimeters as thin as possible while still providing an energy measurement up to 100 MeV. Keeping only $\theta > 10^\circ$ and assuming a calorimeter thickness of 30 cm, we find a geometric acceptance of about 37%.

The signal rate will be far smaller than the background rate, even for a vacuum of $10^{-12}$ Torr, because the “target” of dark matter particles is at least a factor hundred thinner than the background, and the scattering cross section is orders of magnitude smaller. Hence, the ability to measure $\theta$ and $E'$ accurately is crucial.

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Measurement errors on $\theta$ come from multiple scattering and the position resolution of the Si plates. Assuming ultra-thin wafers with a thickness of 100 $\mu$cm, an effective thickness for multiple scattering of 0.002$X_0$, and a coordinate measurement error of 50 $\mu$m on both the $x$- and $y$-coordinates, the RMS error on $\theta$ would be in the $1–3^\circ$ range.

For the measurement of $E'$, we considered the high-resolution electromagnetic calorimeter deployed in the BaBar experiment. This state-of-the-art device employs CsI crystals at least 16$X_0$ deep, and the light is collected by a pair of photo-diodes. The energy resolution is better than 5.1% for $E' > 50$ MeV.

IV. RESULTS AND DISCUSSION

We can achieve a clear discrimination of signal and background by exploiting the relation between scattering angle and outgoing electron energy,

$$
E' = \frac{E}{1 + (E/M)(1 - \cos \theta)}, \tag{3}
$$

where $M$ is the mass of the particle off which the electron scatters. For scattering off a nucleus, $M \gg E$, so $E' \approx E$ for any scattering angle. However, since $M_\chi < E$, $E'$ has a significant variation with $\theta$, as shown in Fig. 2. Except at zero scattering angle, which is inaccessible in the experiment, there is a dramatic difference in $E'$ for backgrounds (such as He) and the signal, for a given $\theta$.

We sketch a simple analysis as follows. The apparent scattering angle is given by the position measurements from the Si plates. If we take $\theta > 10^\circ$, then we would expect $E' \approx 100$ MeV for background and $E' < 65$ MeV for signal, depending on $M_\chi$. Both the signal and background cross sections fall rapidly with $\theta$, so the event dis-
distributions will be concentrated near 10°. Thus, naively, the \( E' \) distribution would consist of a large narrow peak at \( E' \approx E \) for the background, and a much smaller, broader distribution in the 10 – 70 MeV range for signal. The exact shapes of these distributions will depend on the resolution in \( \theta \) and \( E' \). A simple estimate based on \( M_\chi = 2 \) MeV gives, for \( \theta = 10^\circ, E' = 57 \) MeV, \( \sigma_\theta = 1^\circ \) and \( \sigma_{E'} = 2.5 \) MeV. For \( \theta = 45^\circ, E' = 6 \) MeV, \( \sigma_\theta = 3^\circ \) and \( \sigma_{E'} = 0.5 \) MeV (nominal). Thus, measurement errors do not significantly distort the kinematic distribution for the signal, and the success of the search depends mainly on the rejection power against the background.

The main cut between signal and background comes from the measurement of \( E' \). Given the excellent performance of the BaBar calorimeter, one can expect a rejection power sufficient to select the signal events from above the tail of the elastic scattering distribution in \( E' \). For example, a cut \( E' < 65 \) MeV corresponds to nominally \( > 7 \sigma \) from the background peak, and should reject essentially all of the background events - the fraction of events remaining assuming a simple Gaussian resolution function is incalculably small \( (O(10^{-26})) \). A firm estimate of the realistic rejection power would require detailed prototype studies, which is beyond the scope of this paper.

Taking \( \theta > 10^\circ \), the ratio of integrated cross section is \( \sigma_\chi/\sigma_A = 1.5 \times 10^{-8} \), where \( \sigma_\chi \) is \( \int_{10^\circ}^{90^\circ} d\sigma \) for \( M_\chi = 2 \) MeV, and \( \sigma_A \) is the corresponding quantity for a He nucleus. Thus, a calorimeter which can distinguish \( E' < 65 \) MeV from \( E' > 99 \) MeV at the level of \( 10^{-7} \) will be sufficient.

Even if the background can be eliminated, one must obtain a sufficient signal size to establish discovery and to measure the properties of the dark matter particle. With the machine and detector parameters listed in Section III for \( M_\chi = 2 \) MeV and \( M_U = 10 \) MeV, and running for \( 10^7 \) s (about 120 days), the signal yield would be about 53 events. The yield drops rapidly with \( M_\chi \) but is relatively insensitive to \( M_U \).

Eq. 3 shows that \( E' \) at a given \( \theta \) is directly correlated with \( M_\chi \), which allows us to measure \( M_\chi \) given \( E \) and measurements of \( E' \) and \( \theta \). For the stated measurements above, we find that the resolution on \( M_\chi \) would be in the 15% – 20% range, for \( 1 < M_\chi < 8 \) MeV. Thus a sample of just twenty events could provide a measurement of \( M_\chi \) better than 5%.

V. SUMMARY

We described an experiment to find light dark matter particles in a long evacuated tube by observing scattered electrons and measuring their angles and energies precisely. Scattering from residual beam gas is eliminated by virtue of these kinematics. For a sufficiently long tube and intense incoming electron beam, some tens of events could be collected, allowing a good measurement of the dark matter particle mass, and of the rate. This experiment would be ambitious but not impossible; one would perform this experiment after an initial observation in elastic electron-proton scattering as detailed in Ref. 9.

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