Suppressed $B \to PV$ CP asymmetry: CPT constraint

J.H. Alvarenga Nogueira$^b$, I. Bediaga$^a$, T. Frederico$^b$, P.C. Magalhães$^a$, J. Molina Rodriguez$^{**}$

$^a$Centro Brasileiro de Pesquisas Físicas, 22290-180, Rio de Janeiro, RJ, Brazil

$^b$Instituto Tecnológico de Aeronáutica, DCTA, 12228-900, São José dos Campos, SP, Brazil

(Dated: March 7, 2018)

Abstract

Charge Parity asymmetry in charmless B meson decays is a key issue to be understood. Many theoretical calculations have been performed using short distance factorization approaches which, in general, do not take into account the CPT invariance constraint. For each channel with CP violation there is an equal amount of CP asymmetry in another channel or other channels, with an opposite sign. This happens if these channels are coupled through final state interactions (FSI). In the specific process $B \to PV$, involving one pseudo-scalar and one vector particle in the final state, we argue that the CP asymmetry, inherent from a short distance mechanism, could be suppressed due to the CPT constraint. In this case, we propose a sensitive and practical experimental method to identify even a small CP asymmetry, which provides the values for $A_{CP}$ without the need for an amplitude analysis. This method, if applied directly to data, will enable to extract the CP asymmetry information in a model independent way and check to which extend the suggested suppression due to the CPT constraint is verified.

$^{**}$pmagalhaes@cbpf.br
I. INTRODUCTION

Direct charge parity (CP) asymmetry in charmless heavy-meson decays is a long and intriguing puzzle that has been widely studied since the seminal BSS paper [1] in the late 70’s. The idea, which became the BSS mechanism, is that the CP asymmetry comes from the interference between tree and penguin quark level diagrams owning different weak and strong phases. Nowadays, the most common concept used to compute branching fractions and CP asymmetries of charmless B meson decays is the factorization of the decay amplitude, which was started with the Naive factorization approach (NF) [2]. The breakthrough came in the 80’s by the seminal work of Lepage and Brodsky [3], which was the base to the main frameworks developed to study exclusive heavy meson decays. For the B meson, we recall that those main approaches are: QCD factorization (QCDF) [4], perturbative QCD (pQCD) [5] and soft-collinear effective theory (SCET) [6]. All of them are based on the factorization of the hadronic matrix elements, changing technically how this is done, i.e., the ordering of the scales involved, treatment of dynamical degrees of freedom and power counting. One example of the factorization approach with dressed hadronic form factors is given in [7]. There is also an alternative framework to factorization which considers the non-perturbative nature of the process based on the flavour $SU(3)$ symmetry [8].

In general these approaches considered only short distance amplitudes, without taking into account the constraint that CPT invariance imposes on the CP asymmetry at the hadronic level. For a long time, the common belief was that heavy meson B decays would produce many hadronic channels with a homogeneous momenta distribution over the phase space, from the allowed rescattering process to several hadronic channels. These possible FSI, involving two, three or more hadrons, induced the idea that CPT constraints would not be of practical use in charmless B decays [9]. However, the high sample of experimental data, first with BaBar and Belle and then with LHCb, shows that these multi-mesons rescattering processes are not dominant in charmless multi body B decays [10, 11].

Indeed, inspecting the LHCb data distribution of these decays [12], one can see that the events are placed basically around the Dalitz plot boards, dominated by low mass resonances. The fact that charmless three-body B decays populate regions close to the edge of the Dalitz plot, reinforces the idea that those processes are dominated by two-body interactions. In a recent theoretical paper [13] it was shown that these data distribution of charmless three-
body B decays is a consequence of the relatively small B meson mass. Following this paper, a homogeneous distribution in the Dalitz plot should be possible only for three-body decays of particles with masses that are more than five times larger than the B meson.

In the large compilation of B charmless processes presented in Ref. [14], one finds a quite reasonable agreement between theories based on factorization techniques and experimental results available for several branching fractions of $B \to PP$ and $B \to PV$ decays. However, the same agreement is not seen for the values of CP asymmetries in the same channels [14]. In fact, there are many discrepancies not only among the different models but also comparing them to the experimental CP asymmetry measurements. This situation is worse for $B \to PV$ decays. Nonetheless, it is important to note that there are some issues in these channels for both theoretical and experimental descriptions. In the latter, the processes $B \to PV$ are in fact three-body decays and consequently the observables are determined inside the complexity of a three-body phase-space.

In this paper we present a general discussion of the theoretical issues in the $B \to PV$ decays, arguing that regardless of the model chosen, the CPT constraint can play a crucial role in suppressing the CP asymmetry in these channels. We also propose a simple model to extract the CP asymmetry from these channels, avoiding the difficulty presented in Dalitz amplitude analyses, normally used to extract the values of $A_{CP}$ in these decays.

II. CPT AND CP ASYMMETRY SUPPRESSION IN $B \to PV$ DECAYS

In literature, the theoretical studies for $B \to PV$ decays are limited to low mass SU(3) vector particles: $\rho(770)$, $K^*(890)$ and $\phi(1020)$. Both $\rho(770)$ and $K^*(892)$ decays are restricted to one single two-body channel and the branching fraction for their decays are 100% to $\pi\pi$ and to $K\pi$, respectively [15]. The case of $\phi(1020)$ is slightly different, it decays mainly into $K\bar{K}$ but it has also a decay involving three pions through the $\pi\rho(770)$ channel with a branching fraction of 15% [15].

The isobar model, as well as K-matrix and other amplitude analyses, widely used by experimentalists for three-body heavy meson decays, considers the bachelor particle as a simple spectator of the process. In this quasi-two-body process or (2+1) approximation, resonances produced in heavy meson decays do not interact with the third particle. Within this scenario, the CPT constraint in $B \to PV$ processes, where $V = \rho(770)$ or $K^*(890)$,
suggests that there is no room to observe CP asymmetry in these channels. There is also the $\phi(1020)$ resonance, which could show CP asymmetry, but that does not seem possible due to the low contribution of tree diagrams to decays involving this vector particle. It is worth mentioning that the absence of final state interactions is a hadronic constraint and therefore, the impossibility to observe CP asymmetry in those processes is independent from the relative short distance contribution from tree and penguin diagrams.

There are at least three other possibilities for $B \to PV$ decays resulting in inelastic rescattering that can produce CP violation:

(i) rescattering from the pseudoscalar-vector like $PV \to P'X'$, where $X$ is a new particle or particles.

(ii) The $PV$ final state was not produced promptly but is a result of a rescattered process coming from another two-body decay channel, with the strong transition matrix related with the one in (i) by detailed balance, or time reversal invariance.

(iii) A three-body rescattering including the bachelor particle.

The CPT constraint demands that the total CP asymmetry distributed in different channels or phase-space regions coupled with the strong interaction add to zero. These processes (i)-(iii) are estimated to provide small contributions to the CP asymmetry distribution for the $B$ meson decay in different channels coupled by the strong interaction. The flux of CP asymmetry between channels/phase space regions is proportional to the S-matrix expectation value between the different states.

The probability calculation of a light-meson processes (i) or a transition amplitude for (ii) $PV \to P'X'$ or $P'V' \to PX$ should, in principle, start with the QCD theory, which is much beyond the scope of this work. Nevertheless, a rough estimation of the transition amplitude for a light-meson pair to scatter into a different pair of mesons at the $B$ mass is enough to exclude the importance of the coupling between two-body channels at such energies, as long as unitarity is respected. The off-diagonal S-matrix elements should have a magnitude lower than one due to unitarity. Its modulus square can be interpreted as the probability for the transition between the initial and final channels. Then, our task is to estimate the Lorentz invariant matrix element for the transition matrix associated to the inelastic scattering process.

The naive picture for two-body inelastic collision of two initial mesons is the annihilation of these two hadronic states to a quark-antiquark pair that propagates and recombine pro-
ducing the light-meson pair in the final state. The contribution from the intermediate state propagation of the quark pair in the Mandelstam variable $s$ is damped roughly by a factor $s^{-1}$. In addition, the breakup of a meson in a quark-antiquark pair has to bring a damping factor of $s^{-1}$, because in this case there is an imbalance in the relative momentum of the quarks by $\sim \sqrt{s}$. If in one of the mesons at the initial or final state the relative momentum of the pair is small then the other one should compensate with a large relative momentum, which is damped by the wave function of the meson. The asymptotic behaviour of the lowest Fock-state component of the light-cone wave function in $S-$wave decreases with the transverse momentum as $k_{\perp}^{-2}$ [16], which we naively associate with the invariant $s^{-1}$. Therefore, the Lorentz invariant matrix element of the transition operator should carry a damping factor of at least $\sqrt{s - s_{th}}/s^{3.5}$, where the threshold behaviour was included without changing the asymptotic behaviour in $s$.

According to the above discussion, the damping factor in the off-diagonal S-matrix element corresponding to $PV \rightarrow P'X'$ inelastic collision, is estimated as

$$S_{PV \rightarrow P'X'}(s) \sim N\sqrt{s/s_{th} - 1}/(s/s_{th})^\alpha,$$

with $\alpha = 3.5$. The S-matrix element cannot be larger than 1 due the unitarity constraint. By choosing the normalization factor as $N = \Lambda^6 = (1.24)^6$ in Eq.(1), the maximum value reaches $\sim 0.87$, when $\sqrt{s} = 1.08 \sqrt{s_{th}}$. An example for the application of the formula (1) is the $s-$wave isospin zero $\pi\pi \rightarrow KK$ cross section, which drops fast and is relevant below $\sqrt{s} \sim 1.6$ GeV [17]. This naive formula is consistent with one of the parametrizations presented in [18] for the inelasticity parameter $\eta(s) = \sqrt{1 - |S_{\pi\pi \rightarrow KK}(s)|^2}$. Using this simple formula with $\sqrt{s_{th}} = 2$ GeV, it results in $S_{PV \rightarrow P'X'}(m_B) \sim 0.014$, which provides a large suppression of the off-shell amplitude at the $B$ meson mass.

In process (iii), namely a three-body rescattering process, one of the pseudo-scalar meson scatters with the bachelor particle and, therefore, the CP asymmetry, can flow from one region of the three-body phase-space to another. At the amplitude level this one-loop process introduces new complex structures that could affect different partial waves. This effect was studied in the context of the charged $D^+ \rightarrow K-\pi^+\pi^+$ decay [19–21], and the results indicate that it was crucial to explain the observed experimental S-wave phase-shift. It was also shown that it is suppressed by a factor of about 20% with respect to the driving term. If we roughly estimate that it has a dependence on $s^{-1}$, then from the D decay to B
decay, one has a suppression factor \( \sim \left( \frac{1.89}{5} \right)^2 \) giving \( \sim 5\% \) for the three-body rescattering contribution with respect to the driving partonic amplitude. Recently, we performed a qualitative study [22] about the importance of FSI rescattering to \( B^+ \to \pi^- \pi^+ \pi^+ \) decay suggesting that the presence of hadronic loops shifts the P-wave phase near the threshold to below zero, and modifies the position of the \( \rho \)-meson peak as well as its width, in the Dalitz plot. Basically, the obtained contribution for the effect was a small percentage, but a more refined analysis which also considers the S-wave rescattering is being developed and should be released soon [23].

However, the amplitude analyses used to extract CP- asymmetry information from the data are model dependent and based on a \((2+1)\) approximation. Thus, there is still room for improvement as will be discussed in the following section.

III. MODEL INDEPENDENT METHOD TO EXTRACT \( A_{CP} \) IN \( B \to PV \) DECAYS

To avoid the dependence of the isobar model when extracting parameters of CP asymmetry in \( B \to PV \) processes, we propose a simple model independent experimental procedure to extract the \( A_{CP} \) from the data. The method explores the angular distribution of a vector resonance in the Dalitz plot and the property that the low mass vector meson is in general close to a scalar one, sharing the same region of the phase space. Thus, one can take a slice from the central mass of a light vector resonance that includes the interference with only one low mass scalar resonance along the other Dalitz variable. These scalar resonances can be the \( \sigma, \kappa, f_0(980) \) or even a non-resonant contribution.

To illustrate the method we begin with a simple situation where \( B^\pm \to h^\pm (V \to h^+ h^-) \) decay (h=\( \pi \) or K) receives contributions only from one vector resonance (V) and a constant nonresonant (NR) amplitude. Generically, one could represent the total amplitudes for \( B^+ \) and \( B^- \) charge conjugate decays as:

\[
M^+ = a_+^V e^{i\delta^V} F^{BW}_V \cos \theta(s_\perp, s_\parallel) + a_{nr}^+ e^{i\delta_{nr}} F^{NR}, \tag{2}
\]

\[
M^- = a_-^V e^{i\delta^V} F^{BW}_V \cos \theta(s_\perp, s_\parallel) + a_{nr}^- e^{i\delta_{nr}} F^{NR}, \tag{3}
\]

where the \( F^{NR} \) is a real and scalar non-resonant amplitude, and \( \delta_{\pm} \) contains both the fixed weak and strong phases. The vector resonance V is described by a Breit-Wigner (BW)
function, $F^{BW}_V$, that depends on $s_\parallel = (p_{h_+} + p_{h_-})^2$, one of the invariant variables at Dalitz plot,

$$F^{BW}_V(s_\parallel) = \frac{1}{m_V^2 - s_\parallel - i m_V \Gamma_V(s_\parallel)}, \quad (4)$$

and $\Gamma_V(s_\parallel)$ is the energy dependent relativistic width. The vector amplitude has an additional strong phase, inherent to the BW form, and a spin 1 factor, proportional to $\cos \theta(s_\perp, s_\parallel)$. The angle $\theta$ is defined as the helicity angle between the bachelor and center of mass of the two particles produced by the resonance. For a vector resonance in the $s_\parallel$ channel, the cosine of the helicity angle is given by [24]

$$\cos \theta(s_\perp, s_\parallel) = \frac{(M_B^2 - s_\parallel - M_{h_+}^2)(s_\parallel + M_{h_+}^2 - M_{h_-}^2) + 2 s_\parallel(M_{h_+}^2 + M_{h_-}^2 - s_\perp)}{\sqrt{\lambda(M_B^2, s_\parallel, M_{h_+}^2)} \sqrt{\lambda(s_\parallel, M_{h_+}^2, M_{h_-}^2)}}, \quad (5)$$

where $\lambda(x, y, z)$ is the Kallen function and $\sqrt{\lambda(x, y, z)} = \left[ x - (\sqrt{y} + \sqrt{z})^2 \right] \left[ x - (\sqrt{y} - \sqrt{z})^2 \right]$. We investigate the behaviour of $\cos \theta(s_\perp, s_\parallel)$ for values of $s_\parallel \approx m_V^2$, where $V = \rho(770)$, $K^*(892)$ and $\phi(1020)$. We found that the values of $\cos \theta$ remain stable as a function of $s_\perp$ around the center value of $s_\parallel = m_V^2$, i.e., within a region of about the width of the resonance. Therefore, the helicity angle can be assumed to be a function of only $s_\perp$: $\cos \theta(s_\perp, m_V^2 \pm \delta_m) \approx \cos \theta(s_\perp, m_V^2)$.

The CP asymmetry is obtained from the ratio of subtracting/adding the square modulus of the $B^+$ and $B^-$ amplitudes:

$$|\mathcal{M}_+|^2 \mp |\mathcal{M}_-|^2 = [(a_+^V)^2 \mp (a_-^V)^2]|F^{BW}_V|^2 \cos^2 \theta(s_\perp, s_\parallel) + [(a_+^{nr})^2 \mp (a_-^{nr})^2]|F^{NR}|^2$$

$$+ 2 \cos \theta(s_\perp, s_\parallel)|F^{BW}_V|^2|F^{NR}|^2 \times$$

$$\left\{ (m_V^2 - s_\parallel) [a_+^V a_+^{nr} (\cos \delta^V - \delta_\perp^{nr}) \mp a_-^V a_-^{nr} \cos (\delta^V - \delta_-^{nr})] \right.$$

$$- m_V \Gamma_V \left[ a_+^V a_+^{nr} (\sin \delta^V - \delta_\perp^{nr}) \mp a_-^V a_-^{nr} \sin (\delta^V - \delta_-^{nr}) \right] \right\}. \quad (6)$$

A formula similar to Eq. (6), but integrated in $s_\perp$, was used in [11] to explore different types of CP asymmetries in charmless three-body $B$ decays. The formula was applied in the fitting of the integrated experimental distributions released by the LHCb experiment [12]. Here, instead, we propose to look for the distribution on the $s_\perp$ variable around the resonance mass, i.e., $s_\parallel \approx m_V^2$.

The usefulness of this procedure is that it enables us to identify the signature of $\cos \theta(s_\perp, m_V^2)$ when looking at the amplitude distribution in $s_\perp$. By doing that, one can
relate the cosine signatures to a specific type of CP asymmetry source. Inspecting Eq. (6), one notes that the first two terms are associated to the direct CP asymmetry, created from BSS mechanism. The former is related to the vector resonance and is proportional to \( \cos^2 \theta(s_\perp, m_V^2) \), whereas the second is constant and associated to the scalar NR amplitude. The last two terms in Eq. (6) are proportional to \( \cos \theta(s_\perp, m_V^2) \) and related to the interference between the NR amplitude and the vector resonance. This type of CP asymmetry has two contributions: one associated to the real part of the vector amplitude and the other to the imaginary.

In other words, the coefficients \( (a_V^\pm)^2 \) tracks the \( \cos^2 \theta(s_\perp, m_V^2) \), which is related to a CP asymmetry from the BSS mechanism on the vector meson decay amplitude. The coefficients \( a_V^\pm \) tracks the linear representation of the \( \cos \theta(s_\perp, m_V^2) \) and is related to a CP asymmetry produced from the FSI interference. Finally, the coefficient \( (a_{NR}^\pm)^2 \) represents the possibility of CP asymmetry produced by BSS mechanism in the NR amplitude, related to the second term in Eq. (6). All these coefficients can be obtained directly from data by fitting the square amplitudes with a quadratic function on \( \cos \theta(s_\perp, m_V^2) \).

The constraint of CPT applied to the CP asymmetry in the hypotheses of no three-body rescattering contributions, as discussed in the previous section, imply that the integral over the phase space of the asymmetry, computed with Eq. (6), should vanish as discussed in detail in [11]. Concerning the linear term in \( \cos \theta \), independently of the context it will vanish after integration over the phase-space once it is an odd-function. Therefore, CPT implies that the integration over the phase-space of the first two terms of Eq. (6) must be zero. The most probable solution is that the coefficients from \( B^+ \) and \( B^- \) are the same. However, they could also be different and compensate upon integration over the phase-space.

An important product of this method is the possibility of extracting the values of \( A_{CP} \) in a direct measurement of the CP asymmetry inherent to the BSS mechanism, within a model independent approach. From the quadratic coefficients of the fit one directly obtains the \( A_{CP} \) without needing a model for the amplitude:

\[
A_{CP}^V = \frac{(a_V^+)^2 - (a_V^-)^2}{(a_V^+)^2 + (a_V^-)^2}.
\]  

(7)

The inclusion of one low mass scalar resonance like \( \sigma \), \( \kappa \) or \( f_0(980) \), does not change the main features of this method. In this case, the charmless three-body B decay amplitudes are given by replacing the NR amplitude for a Breit-Wigner denoted as \( F_{BW}^S \). The scalar
resonance has an inherent strong phase that will interfere with the vector amplitude. The CP asymmetry considering the scalar resonance results in

\[ |\Delta M|^2 = |M_+|^2 - |M_-|^2 = \left[ (a_V^2 - a_V^2) |F_{V}^{BW}|^2 \cos^2 \theta + [(a_S^2 - a_S^2)] |F_{S}^{BW}|^2 + 2 \cos \theta |F_{V}^{BW}| |F_{S}^{BW}| \right] \times \left\{ [(m_V^2 - s)(m_S^2 - s) - m_Y \Gamma_Y m_S \Gamma_S][a_V^2 a_S^2 \cos(\delta_V - \delta_S) - a_V^2 a_S^2 \cos(\delta_V + \delta_S)] \right. \\
\left. - [m_Y \Gamma_Y (m_S^2 - s) - m_S \Gamma_S (m_Y^2 - s)] [a_V^2 a_S^2 \sin(\delta_V - \delta_S) - a_V^2 a_S^2 \sin(\delta_V + \delta_S)] \right\}. \tag{8} \]

Comparing this formula with Eq. (6), one notes that the coefficients related to BSS mechanism are the same and the interference term is also proportional to \( \cos \theta(s_{\perp}, m_Y^2) \). Therefore, the amplitudes can be parametrized by the same quadratic function of \( \cos \theta(s_{\perp}, m_Y^2) \) as in the previous example.

The data scenario is more complex because there are also resonances placed in a crossed channel, that are functions of \( s_{\perp} \), and can interfere in a non-trivial form with the distribution of \( \cos \theta(s_{\perp}, m_Y^2) \), producing other sources of CP asymmetry. However these interferences are localized at lower masses (in general below 5 GeV^2). Taking into account the big phase space accessible for charmless three-body B decays, even if we exclude this interference region, there is still a large region to perform the analysis and extract the \( A_{VCP}^V \) measurements from the fit parameters, with good resolution and limited errors.

IV. VIABILITY OF METHOD THROUGH FAST MONTE CARLO SIMULATION

In this section, we apply the previously presented method for the \( B^\pm \rightarrow \pi^\pm \pi^+ \pi^- \) decay generated by using Toy Monte Carlo (MC) pseudo-experiments. The samples for these studies were simulated from the results obtained in the BABAR experiment [25], by fitting the data with an isobar model which includes: the vector resonances \( \rho(770) \) and \( \rho(1450) \), the scalar \( f_0(1370) \), the tensor \( f_2(1270) \) and a flat nonresonant contribution. In their fitting results, they had obtained a CP asymmetry in all the resonance channels considered, including the NR contribution. Specifically for the channel we are interested in, \( B^\pm \rightarrow \rho(770)\pi^\pm \), they found an \( A_{CP}^{\rho(770)} = 18\% \pm 7\% \).

In order to show the viability of the method employed to identify the CP asymmetry signatures, we have considered two different scenarios for the Toy MC: one of them using
the BABAR inputs for magnitude and phase, and a second one where we only modify
the magnitudes for \( \rho(770) \) in order to produce an \( A_{CP}^{\rho(770)} = 0 \). For both situations we have
generated 1000 samples by using the Laura++ [26] package, each sample with 20,000 events.
By applying the method we aim, in the first case, to reproduce the \( A_{CP}^{\rho(770)} = 18\% \) obtained
by Babar, whereas, for the second case, we should find \( A_{CP}^{\rho(770)} = 0 \).

The procedure to calculate the \( A_{CP} \) is straightforward: we choose a 50 MeV mass window
around the vector resonance \( \rho(770) \), integrating it in the parallel Dalitz variable, and then
fit the binned distributions of \( B^+ \) and \( B^- \) histograms in the orthogonal Dalitz variable with
a quadratic polynomial function of \( \cos \theta(s_\perp, m_V^2) \) in the interval of 5 to 23.5 GeV\(^2\). Following
that, we calculate the \( A_{CP} \) by applying the quadratic coefficients previously obtained from
both fitted curves, and replacing its values into Eq. (7). It is worth mentioning that the
choice of the fitting interval excludes some interference effects. As we have argued before,
these effects result from the \( \rho(770) \) inference with the crossed channel, which produces an
extra CP asymmetry that does not come from the BSS mechanism. Therefore, it is important
to exclude these regions.

In short, with this procedure we find \( A_{CP} = 0.177 \pm 0.03 \) and \( A_{CP} = 0.0 \pm 0.03 \) for the
two cases under study. The results for the fit with the quadratic function are given in Fig. 1
and the values for the quadratic coefficients are summarized in Table I. We do not report
linear and constant fit parameters, once they do not add relevant information to the aims
of our study.

Besides the simplicity of the method proposed, the results for the Toy MC study are
very satisfactory. The values for \( A_{CP} \) obtained from the fitting coefficients agree with the
correspondent input values within the statistical errors. We also observe that although we
restrict our study to a small part of the three-body B phase space, the \( A_{CP} \) errors of these
samples are competitive with the ones extracted by an amplitude analysis with a similar
sample. Thus, our main conclusion is that this method is reliable and easy to be applied to
data.
FIG. 1: Left: fit to a random sample of binned distributions of $B^+$ and $B^-$ histograms generated with $A_{CP}^{(770)} = 0.18$ (top) and $A_{CP}^{(770)} = 0.0$ (bottom). Right: the distributions of the main value for $A_{CP}$ obtained from the fits to the 1000 pseudo-experiments with 20000 events each, simulated with: the BABAR results (top), and $A_{CP}^{(770)} = 0.0$ (bottom).

| $A_{CP}$ | 0.18 | 0.0 |
|----------|------|-----|
| $a_2^-$  | 1.98 ± 0.075 | 1.99 ± 0.077 |
| $a_2^+$  | 1.41 ± 0.063 | 1.99 ± 0.076 |

TABLE I: Central values for the quadratic coefficients resulting from the fits of the 1000 samples generated with the given $A_{CP}$ with the quadratic function on $\cos \theta(s_\perp, m_{V}^2)$. The values for $A_{CP}$ can be extracted from the method by including the coefficients $a_2^-$ and $a_2^+$ in Eq. (7).
V. FINAL REMARKS

Although short distance approaches in theoretical calculations indicate a substantial contribution to the direct CP asymmetry in some charmless $B \to PV$ decays, CPT constraints can suppress these possible contributions. Indeed, there are two experimental features involving these decays that reinforce those constraints. First, the hadronic three-body final state observables from charmless B decays have their events placed on the edge of the Dalitz plot. This favours a (2+1) approximation with two-body resonances plus a spectator particle. Second, the elastic dominance in the $\rho(770)$ and $K^*(890)$ resonances suppresses the coupling of these channels through FSI. Combining these two features in the limit where the (2+1) approximation would be exact, one can reasonably consider that there is no possibility to have a direct CP violation in $B \to PV$ channels, since there is no inelastic rescattering to satisfy the CPT constraint. However, there is still room for inelastic rescattering contributions that could generate a direct CP violation from the BSS mechanism, even though they are suppressed when compared with short distance calculations.

In order to perform $A_{CP}$ measurements for several charmless $B \to PV$ decays, we have proposed a model independent procedure to extract these experimental values. The method used some important charmless three-body B decay properties, such as the large phase space, the relatively low mass of $\rho(770)$, $K^*(890)$ and $\phi(1020)$ resonances, and their exclusive interference with scalar amplitudes along all of the phase spaces. The Toy Monte Carlo simulation, based on experimental amplitude analysis results, proved the utility of the proposed method, which succeeded in extracting the correct values of $A_{CP}$ with limited errors. Moreover, the information one extracts from this method directly confronts several theoretical predictions for $A_{CP}$ [14].

It is worth noting that this method has been successful in its purpose, which consists in measuring directly and model independently the $A_{CP}$ in $B \to PV$ processes, avoiding the complexity of amplitude analyses for three-body decays. The latter is a tool to access the resonance branching fractions - beyond the reach of our method - providing a complete description of three-body decays once it deals with many other resonance structures and their interferences in a model dependent way. Besides this, both approaches can be complementary. One can see that the method we presented here is very sensitive in detecting interferences in the crossed channel, that are functions of $s_\perp$, as we can see in our toy Monte
Carlo studies. This feature can be used to investigate the possibility of new high mass resonances in the $s_{\perp}$ invariant mass, which can only be confirmed - including their specific properties - through the use of the amplitude analysis technique.

Furthermore, it would be important to investigate the possibility of the rescattering of double charm B decays into light pseudoscalar mesons as proposed by Wolfestein[27], in order to explain part of the possible dynamics of direct CP violation involving charmless three-body B decays.

Acknowledgments

We wish to thank Fernando Ferreira Rodrigues for the fruitful discussions. We also thanks the support from Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) of Brazil. The work of J.H.A.N. was supported by the grant #2014/19094-8 from São Paulo Research Foundation (FAPESP).

[1] M. Bander, D Silverman, and A. Soni, Phys. Rev. Lett. 43, 242 (1979).
[2] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C 29, 637 (1985); M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C 34, 103 (1987).
[3] G. P. Lepage and S. J. Brodsky, Phys. Rev. D 22, 2157 (1980).
[4] M. Beneke, G. Buchalla, M. Neubert, and C. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); Nucl. Phys. B 591, 313 (2000).
[5] Y. Keum, H. Li, and A. Sanda, Phys. Lett. B 504, 6 (2001); Phys. Rev. D 63, 054008 (2001).
[6] C. Bauer, S. Fleming, and M. Luke, Phys. Rev. D 63, 014006 (2000); C. Bauer, S. Fleming, D. Pirjol, and I. Stewart, Phys. Rev. D 63, 114020 (2001); C. Bauer and I. Stewart, Phys. Lett. B 516, 134 (2001); C. Bauer, D. Pirjol, and I. Stewart, Phys. Rev. D 65, 054022 (2002).
[7] A. Furman, R. Kamiński, L. Leśniak and B. Loiseau, Phys. Lett. B 622, 207 (2005); B. El-Bennich, A. Furman, R. Kamiński, L. Leśniak and B. Loiseau, Phys. Rev. D 74, 114009 (2006); B. El-Bennich, A. Furman, R. Kamiński, L. Leśniak, B. Loiseau, B. Moussallam, Phys. Rev. D 79, 094005 (2009); A. Furman, R. Kamiński, L. Leśniak, P. Zenczykowski, Phys. Lett. B 699, 102 (2011); J.-P. Dedonder, A. Furman, R. Kaminski, L. Lesniak, B. Loiseau, Acta
[8] D. Zeppenfeld, Z. Phys. C 8, 77 (1981); L. L. Chau and H. Y. Cheng, Phys. Rev. Lett. 56, 1655 (1986); L. L. Chau and H. Y. Cheng, Phys. Rev. D 36, 137 (1987); M. J. Savage and M. B. Wise, Phys. Rev. D 39, 3346 (1989) [Erratum-ibid. D 40, 3127 (1989)]; L. L. Chau, H. Y. Cheng, W. K. Sze, H. Yao and B. Tseng, Phys. Rev. D 43, 2176 (1991) [Erratum-ibid. D 58, 019902 (1998)].

[9] I. I. Bigi, Front. Phys. 10, 101203 (2015).

[10] I. Bediaga, T. Frederico, and O. Lourenço, Phys. Rev. D 89, 094013 (2014).

[11] J. H. Alvarenga Nogueira, I. Bediaga, A. B. R. Cavalcante, T. Frederico and O. Lourenço, Phys. Rev. D 92, 054010 (2015).

[12] LHCb Collaboration, R. Aaij et al, Phys. Rev. Lett. 112, 011801 (2014).

[13] S. Krankl, T. Mannel and Javier Virto, Nucl. Phys. B 899, 247 (2015).

[14] H.-Y. Cheng, C.-W. Chiang, and A.-L. Kuo Phys. Rev. D 91, 014011 (2015).

[15] K.A. Olive et al. (Particle Data Group), Chin. Phys. C 38, 090001 (2014) and 2015 update.

[16] X. Ji, J.-P. Ma, and F. Yuan, Phys. Rev. Lett. 90, 241601 (2003).

[17] G. Grayer et al., Nucl. Phys. B 75, 189 (1974).

[18] J. R. Peláez and F. J. Ynduráin, Phys. Rev. D 71, 074016 (2005).

[19] P. C. Magalhães, M. R. Robilotta, K. S. F. F. Guimarães, T. Frederico, W. de Paula, I. Bediaga, A. C. dos Reis, C. M. Maekawa, and G. R. S. Zarnauskas, Phys. Rev. D 84, 094001 (2011).

[20] K. S. F. F. Guimarães, O. Lourenço, W. de Paula, T. Frederico and A. C. dos Reis, JHEP 08, 135 (2014).

[21] P.C. Magalhães and M.R. Robilotta, Phys.Rev. D 92, 094005 (2015)

[22] I. Bediaga and P.C. Magalhães, arXiv:1512.09284 [hep-ph].

[23] Work in progress.

[24] E. Byckling and K. Kajantie, “Particle Kinematics”, John Willey and Sons (1973).

[25] BABAR Collaboration, Phys. Rev. D 79, 072006 (2009).

[26] T. Latham, AIP Conf. Proc. 1735, 070001 (2016)

[27] L. Wolfenstein, Phys. Rev. D 43, 151 (1991).