Research on Construction Sequence and Line Network Structure of Urban Rail Transit

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Abstract. With the construction of urban subway lines, the network characteristics of subways have begun to appear. Under certain objective conditions such as capital, manpower, and material resources, the construction sequence of subway lines ought to be different, so that priority construction of lines can maximize network benefits. In order to study the time sequence of urban subway line construction and the network effect of the line, the evaluation index of urban rail transit construction time sequence based on the complex network theory is constructed. The catastrophe progression theory is applied to avoid the subjectivity of the weighting of the primary indicators. The existing lines of Xi'an Metro are transformed into the L-space topology, and the relevant network indicators are calculated to judge the network characteristics of the subway. Through the calculation of the membership degree, the construction priority of each line is finally determined. The route planned for the third phase of the Xi'an Metro is taken as an example, the calculation results show that the small-world characteristics of the Xi'an Metro network are emerging and the network is in its infancy. The calculation results of the priority of the planned line construction sequence indicate that large-scale sports meeting and events have a decisive influence on the construction sequence of the subway line.

1. introduction
As an important public welfare transportation infrastructure, urban rail transit is one of the solutions to urban traffic congestion and convenience for citizens. The construction of urban rail transit is a huge system project, the timing of the construction of rail transit lines has an important influence on the structural optimization of the rail transit network. At present, the research methods of domestic and foreign scholars on the timing of urban rail transit construction can be mainly summarized into two categories:

The first is to determine the route based on the relevant benefits of the route. By establishing effective conditions and constraints, with the goal of maximizing the comprehensive benefits of the construction line, the construction timing is determined. Wang Wei[1] taking economic development as the goal, the construction time of Chongqing rail transit was analysed. However, it is difficult to obtain quantitative parameters, and it is difficult for indicators to fully and objectively reflect the influencing factors of the construction timing. The second is to determine the route based on the importance of nodes and lines. The multi-attribute decision-making model is used to quantify the relative value of different schemes in order to obtain a reasonable rail transit line construction scheme. The TOPSIS model is used to identify and judge the importance of urban rail transit lines to determine the construction sequence of urban rail
transit[2-3]; Li Yida[4] judges the construction timing of the line based on the lattice timing theory. However, because the selection of the index weight interval is mainly derived from the subjective knowledge of people, the final decision-making plan has the characteristics of vagueness and uncertainty.

Most of the literatures study the construction timing of urban rail transit from the importance of nodes and lines, and maximize the benefits of the lines. However, the rail transit network of most cities has shown network characteristics, therefor the above methods are difficult to measure the impact of new lines on the city urban subway network. Moreover, it is necessary to empower the grassroots indicators, but it is difficult to ensure their objectivity. The single-line construction sequence should be carried out under the guidance of the overall benefits of the network. Therefore, this paper focuses on the time sequence of the construction of related lines from the urban rail transit network. Through the quantification of basic indicators, the network structure characteristics of urban rail transit are studied. The catastrophe progression method is used to avoid the weighting of related indicators and reduce the subjectivity of weighting, so that the construction priority of related lines can be judged by the network indicators of the construction line.

2. Establishing of the construction priority index system

Since there are many factors affecting the construction timing of urban rail transit lines, involving factors such as technology, social benefits, and urban development. This paper refers to related literature[5-6], initially selects relevant indicators, and performs Pearson correlation analysis on the combed indicators. The indicators with higher correlation (R2=0.9) are removed. Then, the three aspects of network structure construction index system are shown in Table 1, which are used to evaluate the priority of urban rail transit network construction.

| Standard level | Index level | Data source | Index type |
|----------------|-------------|-------------|------------|
| [B₁] Network demand | C₁₁ Daily passenger turnover | Passenger flow forecast, | Positive |
| | C₁₂ Passenger flow intensity | Passenger flow forecast, | Positive |
| [B₂] Network benefits | C₂₁ Construction cost | Formula calculation | Negative |
| | C₂₂ Development coordination | Expert scoring | Positive |
| | C₂₃ Network efficiency | Formula calculation | Positive |
| [B₃] Network structure | C₃₁ Average path length | Formula calculation | Negative |
| | C₃₂ Average degree | Formula calculation | Positive |
| | C₃₃ Connectivity | Formula calculation | Positive |
| | C₃₄ Complexity | Formula calculation | Positive |
| | C₃₅ Clustering coefficient | Formula calculation | Positive |

2.1 Network requirements

2.1.1 Daily passenger turnover

The daily passenger turnover is the sum of passenger mileage during the statistical day and is the product of passenger volume and passenger distance.

2.1.2 Passenger flow intensity

Passenger flow intensity is an important indicator to measure the demand of rail transit passenger flow. It is the ratio of the average daily passenger flow of the line to the line's operating length. It reflects the size of the passenger flow demand of the line. The passenger flow intensity of the planned line can be predicted by the data of the existing line data. It plays an important role in the planning and implementation of rail transit lines. In the order of rail transit construction, theoretically, priority should be given to the construction of lines with high passenger flow.
2.2 Network benefits

2.2.1 Construction cost
The average construction cost of different rail transit lines is different. Under a certain total investment control, the lower the average construction cost of the general planning scheme, the stronger the implementability. The average construction cost of the line network is related to the length of the line on the one hand, and the construction period on the other. The construction length per unit time reflects the difficulty of the construction of urban rail transit projects. The unit construction time reflects the difficulty of construction. The construction cost defined in this paper includes investment cost and time cost, which reflects the difficulty of construction of the line while reflecting the benefits. It is calculated as follows:

\[ Q = \frac{q_i \times L}{l_i^2} \]  

In the formula: \( Q \): the average construction unit cost of the planned line network ((100 million yuan*year)/km); \( q_i \): total investment in rail transit line construction (100 million yuan); \( t_i \): the planned construction period of rail transit lines; \( l_i \): the planned construction length of rail transit lines (km).

2.2.2 Development coordination
There is a close relationship between urban rail transit and urban development. Rail transit is a product of urban residents’ traffic demand. Therefore, the construction of rail transit should be adapted to the urban spatial development direction, external transportation facilities, and major urban activities to meet the requirements of urban development and a large number of travel needs. This index data is obtained by the expert scoring method.

2.2.3 Network efficiency
Network efficiency \( E \) refers to the average value of the sum of the reciprocal distances between all pairs of nodes in the network, which is used to indicate the average difficulty of network information circulation. From the definition of network efficiency, it can be seen that network efficiency expresses the average closeness between all pairs of nodes in the network. The closer and shorter the distance between nodes in the network, the greater the network efficiency value. Network personnel and information circulation are facilitated as network efficiency increases. If the structure matrix of network \( G \) is denoted as \( a_{ij}(N \times N) \), the calculation formula is:

\[ E(G) = \frac{\sum_{i \neq j} d_{ij}}{N(N-1)} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}} \]  

In the formula: \( N \) is the number of nodes in the network; \( d_{ij} \) is the distance between nodes \( v_i \) and \( v_j \).

2.3 Network structure

2.3.1 Average path length
The average path length \( L \) is the average of the shortest distance between all pairs of nodes in the track network, and the shortest distance is the minimum number of edges that need to pass from node i to node j in the network:

\[ L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij} \]  

In the formula: \( N \) is the number of nodes, and \( V \) is the set of nodes in the network.
2.3.2 Average degree and degree distribution
The node degree \( k_i \) refers to the number of edges connected to the node, and the network average degree \( k \) refers to the average value of all node degrees in the orbital network. The node degree \( k_i \) and the network average degree \( K \) satisfy:

\[
K = \frac{1}{N} \sum_{i=1}^{N} k_i = \frac{1}{n} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}
\]  
(4)

The degree distribution represents the basic topological characteristics of the track network. It is essentially the probability distribution function of the node degree \( k_i \), which represents the probability of \( k \) connecting edges at node \( i \). Its value is equal to the number of nodes with node degree \( k_i \) and the number of all nodes in the network ratio.

2.3.3 Connectivity
Connectivity is the ratio of the number of edges \( m \) of the rail transit topology network to the maximum number of edges \( m_{\text{max}} \) of network theory. Connectivity is directly proportional to the degree of network development. With the increase in connectivity, the development of the network becomes higher; the travel of passengers becomes more convenient; and the network revenue increases. The calculation formula is:

\[
\gamma = \frac{m}{m_{\text{max}}} = \frac{m}{3N - 6}
\]  
(5)

2.3.4 Network complexity
Nodes of different networks have different network scales. Complexity \( B \) is the ratio of the actual number of edges \( m \) of the network to the number of stations \( N \) in the network, reflecting the complexity of different networks.

\[
B = \frac{m}{N}
\]  
(6)

2.3.5 Clustering coefficient
The clustering coefficient \( C_i \) characterizes the closeness of connections between network nodes. If node \( i \) has \( N_i \) nodes directly connected to it, assuming that all \( N_i \) nodes connected to node \( i \) are connected in pairs, then there are a total of \( \frac{1}{2} k_i(k_i-1) \) edges between these adjacent nodes, even if the maximum number of edges is reached. However, in the actual track network, the \( N_i \) adjacent nodes of node \( i \) are not necessarily connected in pairs. If only one edge is connected to each other, then the clustering coefficient of node \( i \) is defined as the number of edges between adjacent nodes and the maximum connection ratio of the number of sides. The clustering coefficient \( C \) of the entire subway network is the average of the clustering coefficients of all nodes or stations:

\[
C = \frac{1}{n} \sum_{i=1}^{n} C_i = \frac{1}{n} \sum_{i=1}^{n} \frac{2M_i}{N_i(N_i-1)}
\]  
(7)

3. Establishment of a catastrophe model of subway network
The construction timing of urban rail transit lines based on complex networks can be summarized as a mathematical problem: each line constructed will produce new benefits to the original subway network. Therefore, each line can be used as a variable and loaded on the original network in turn.

This paper studies the topological characteristics of urban rail transit network. The subway lines can be reached in both directions. In order to simplify the calculation, in the process of constructing the complex network topology model of the rail transit network, the rail transit network is abstracted as a non-directional and non-weighted network.

Step 1: abstract subway network topology model: graph theory is used to simplify the rail transit network. In the L-space topology, the actual subway station is defined as a node, and the connection is
defined according to whether there is a connection relationship between two adjacent stations in the same line.

Step 2: standardized processing: dimensionless processing methods are mainly divided into two processing methods: quantitative indicators and qualitative indicators: quantitative indicators are transformed into indicator values in the range of $[0, 1]$ according to the principle of "bigger is better", and qualitative indicators are determined by multiple experts through intervals scoring methods. In the comprehensive evaluation of multiple indicators, it mainly includes positive and negative indicators. The non-dimensional processing formula of the index is:

Positive indicators:  
$$x_i = \frac{x_i - \min\{x_i, \cdots, x_n\}}{\max\{x_i, \cdots, x_n\} - \min\{x_i, \cdots, x_n\}}$$  

Negative indicators:  
$$x'_i = \frac{\max\{x_i, \cdots, x_n\} - x_i}{\max\{x_i, \cdots, x_n\} - \min\{x_i, \cdots, x_n\}}$$

Step 3: calculate the mutation progression: cusp, dovetail, butterfly, Indian cottage catastrophe models are the most commonly used models in catastrophe progression evaluation method [7], and the calculation formula of the related model is as follows (14-16):

- Cusp catastrophe model:  
  $$\begin{cases} 
  x = \sqrt{u} \\
  x' = \frac{1}{2}\sqrt{v} 
  \end{cases}$$

- Swallowtail catastrophe model:  
  $$\begin{cases} 
  x = \sqrt{u} \\
  x' = \sqrt{v} \\
  x'' = \sqrt{w} 
  \end{cases}$$

- Indian cottage catastrophe model:  
  $$\begin{cases} 
  x = \sqrt{u} \\
  x' = \sqrt{v} \\
  x'' = \sqrt{r} \\
  x''' = \sqrt{t} 
  \end{cases}$$

The normalized formula is used for comprehensive quantitative recursive calculation to obtain the mutation level value of each control variable, and the total membership mutation function value is calculated by combining the "non-complementary" and "complementary" criteria.

Step 4: calculation of total mutation membership function: according to the multi-objective fuzzy decision theory [8], for the same multi-objective system, when the evaluation index system is the larger the better, the larger the total mutation membership function value. Therefore a priority evaluation model for urban rail line construction based on fuzzy mutation can be established:

$$F_i = \frac{1}{3}(+R^j + S^j + T^j), \{i, j, k\} = \{2, 3, 4\}$$

In the formula: $F_i$ is the priority of urban rail transit line construction; $R$ is the sudden change membership function value of network demand; $S$ is the sudden change membership function value of network benefit; $T$ is the sudden change membership function value of network structure.

4. Case analysis

This paper takes Xi'an Urban Rail Transit Phase III construction planning line network [9] as an example. On the basis of existing lines in Xi'an (metro lines 5, 6, and 9 will be opened to traffic at the end of this year, excluding the airport intercity line), this paper construct a network topology map and use matlab and gephi to calculate the network of the new line structure and network efficiency. The specific values are shown in the basic data table of priority indicators in Table 2.

| Line 1 Phase 3 | 32.9450 | 0.5705 | 33.3762 | 7.5460 | 0.1038 | 15.6940 | 2.1050 | 0.3550 | 1.0526 |
|---------------|---------|--------|----------|--------|--------|----------|--------|--------|--------|
| Line 2 Phase 2 | 108.0807 | 2.2057 | 24.5086 | 7.3410 | 0.1064 | 15.0270 | 2.1070 | 0.3554 | 1.0536 |
| Line 8        | 1687.4432 | 1.3500 | 53.6004 | 8.6660 | 0.1164 | 13.3070 | 2.2370 | 0.3768 | 1.1183 |

Table 2. Basic data of priority index
After standardizing the data at the bottom level, this paper calculates the degree of membership by layer, and repeats step 2-3 to get the final priority of line construction. The calculation results are shown in Table 3.

| Line  | Phase 1 | Priority value |
|-------|---------|----------------|
| Line 10 | 150.4800 | 0.2416 |
| Line 14 | 65.2050 | 0.6848 |
| Line 15 | 134.2350 | 0.7437 |
| Line 16 | 37.7500 | 0.5000 |

The construction priority of each line in Xi'an Metro phase 3 Planning

| Line | Phase 1 | Phase 2 | Phase 3 |
|------|---------|---------|---------|
| Line 10 | 0.0000 | 0.6544 | 0.5122 |
| Line 15 | 0.6359 | 0.6488 | 0.6202 |

The construction timing of the third phase of Xi’an Metro is: Line 8>Line 14>Line 2 Phase 2>Line 1 Phase 3>Line 16>Line 15>Line 10 (the actual construction timing: Line 14>Line 8 Line>Line 2 Phase 2>Line 1 Phase 3>Line 16>Line 10>Line 15). From the calculation results of the model, in addition to the construction cost of Line 8, the construction priority of other indicators is relatively high. However, since the National Games in September 2021 will be held in Xi’an, Line 14 will pass through the Xi’an Olympic Sports Center, which connects the important crowded areas—the North Passenger Station. Therefore, the construction of Line 14 began as early as 2018, becoming the earliest line to start construction. Since the scores of the three indicators of the standard layer of Line 10 are relatively small, after standardization, the final priority calculation result is 0, therefore the construction sequence ranks last. Although the priority calculated by Line 10 is 0, according to the urban development plan of Xi’an, the "three-axis and two-belt" multi-center development and the north span strategy, the construction of Line 10 connects the Gaoling Group, the Equipment Industry Group, and the WeiHe River Ecological Belt, which greatly promoted the implementation of the plan and promoted regional integration. Therefore, the construction schedule of Line 10 has been advanced.

After standardizing the basic data of the network structure, as shown in Figure 1, from the perspective of the construction of the third phase of the Xi’an rail transit network, especially the construction of Metro Line 8, the network structure characteristics of the Xi’an Metro have undergone significant changes. The average degree and network connectivity of Xi’an Metro have been greatly improved, which greatly improves the accessibility of the subway line and the convenience of transfer.
the basis of the original line, and the construction of Line 15 increased the number of Hub nodes in the network by 2.

Since the mode node in the urban rail transit network topology is an ordinary node with a degree value of 2, the node with a degree of 1 is discarded when network statistics are performed, and the cumulative degree distribution curve is drawn. The power function is used to fit it as shown in Figure 3.

The cumulative degree distribution curve can be approximated to a power-law distribution (Line 8 $R^2=0.8774$, Line 15 $R^2=0.8871$), indicating that the power-law distribution can well describe the degree distribution of a rail transit network, indicating that in a subway network. Most of them are ordinary nodes, and the number of transfer stations with higher degree values is less. The ratio of ordinary stations to the number of transfer stations is about 6:1.

The small world network has a small average path length and a high clustering coefficient. The judgment condition is: the average path length $L$ and clustering coefficient $C$ of the small world network should be greater than or equal to the average path length $L_R$ and clustering coefficient $C_R$ of the random network, namely: $L> L_R$, $C> C_R$. The calculation of $L$ and $L_R$ for each line of Xi'an Metro Phase III is shown in Table 4.

| Line   | Phase 1 | Phase 2 | Phase 3 |
|--------|---------|---------|---------|
| L      | 14.908  | 14.886  | 14.812  |
| LR     | 6.631   | 6.599   | 6.573   |

The calculation results show that $L> L_R$; however, because the urban rail stations are all set up on the line, it is difficult for the three adjacent stations to be in a triangular layout, resulting in the clustering coefficient $C=0$ of the Xi'an rail transit network, which cannot be compared with the clustering coefficient of the random network, but it does not show that its clustering characteristics are not notable [10]. The average path length increases linearly with the logarithm of the network scale, indicating that the Xi'an subway network is beginning to show a small world phenomenon.
5. Conclusion
(1) This paper applies the complex network theory to the timing of urban rail transit construction. It transforms the urban rail transit network into the L-space topology. The construction sequence analysis is carried out from the three aspects of network demand, network benefit and network structure. By decomposing the indicators by layer and using the catastrophe progression method, the subjectivity of the weighting of the indicators at the grassroots level can be avoided. Then, taking the planned construction of Xi'an Metro Line 3 as an example, the method proposed in this paper is used to calculate the construction timing priority. The case results show that major sports events, large-scale conferences, and crowded transportation hubs will advance the construction timing of related lines; this method can provide theoretical guidance for the sequence of urban rail transit construction.

(2) The power function fitting results indicate that the cumulative degree distribution of Xi'an subway network is a power-rate distribution, indicating that the topological structure of Xi'an rail transit network has small-world and scale-free characteristics, and network characteristics are beginning to appear. The calculation results of network complexity and network connectivity show that Xi'an subway network is still in the initial stage of development.

(3) This paper abstracts the network as an unauthorised network, and does not consider the influence of passenger flow on the subway network, when constructing the rail transit topology network. The next step is to use passenger flow as a weight to construct an urban rail transit topology model and study the network characteristics of urban rail transit, so that further study its construction sequence.

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References
[1] Wang W. (2007) Application Research of Chong Qing Railway Traffic Construct Sequence. D. Chongqing University, Chongqing.
[2] Guo Y Y, Liu P, Wu Y. (2013) Development of a Method for the Construction Schedule of Urban Rail Transit Lines. J. Journal of Wuhan University of Technology, 35(06): 75-80.
[3] Zhou X H, You Q, Ding Y T, Wang Z Q. (2015) Construction Sequence of Urban Rail Transit System Based on Improved TOPSIS Model. J. Railway Standard Design, 59(07): 79-84.
[4] Li Y D, Yan H F, Xi Z W. (2017) Construction Sequence of Urban Rail Transit System Based on Lattice Order Theory. J. Journal of Transportation Engineering and Information, 15(04): 53-60.
[5] Boccaletti S., Latora V., Moreno Y., Chavez M., Hwang D.-U. (2006) Complex networks: Structure and dynamics. Physics Reports, 424 : 175-308.
[6] Gao T Z, Chen K M, Li F L. (2018) Topology analysis of urban rail transit network. J. Journal of Chang'an University (Natural Science Edition), 38(03): 97-106.
[7] Thom R. (1977) Structural Stability, Catastrophe Theory, and Applied Mathematics. J. Siam Review, 19(2):189-201.
[8] Guo S C. (2009) Comparison and sequencing of fuzzy numbers based on the method of structured element. J. System Engineering Theory and Practice, 29(03): 106-111.
[9] Watts D J, Strogatz S H. (1998) Collective dynamics of ‘small-world’ networks. J. Nature, 393 (6684) :440-442.