MODELING AND ANALYSIS OF A VEHICLE DYNAMICS AT FRONTAL IMPACT INTO THE RIGID BARRIER USING A SYSTEM IDENTIFICATION TOOLBOX

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The main objective of this paper is to create computer crash models (up to the level of the Kelvin model, where the coefficient of stiffness and damping is calculated by implementing appropriate numerical methods: Trust - Region, Lavenberg-Marquardt and so on) of measured real data that are obtained at the frontal impact of a vehicle into the rigid barrier. The process of modeling a vehicle crash can be done in two ways. One of them is related to CAE (Computer Aided Engineering) software including FEA. The other one (applied in this article) is based on the System Identification Toolbox, which contains MATLAB® functions, Simulink® blocks, and a special app for constructing models of dynamic systems from the measured input-output data.

KEYWORDS
Matlab, System identification toolbox, State-space model, Process model with transfer function, Polynomial model, Transfer function model, Kelvin model.

1 INTRODUCTION
Vehicle crash test is usually done in order to ensure safe design standards of a car software. Simulated crash tests can be performed and evaluated to the full extent of a real crash test. This means that due to simulated crash tests it is possible to significantly reduce the costs needed for performing real crash tests with real cars. Identification system concerns construction and validation of mathematical models from dynamic input-output data. In experiments the system reveals information about itself in terms of input and output measurements. System identification is routinely used in industry and by tools for modeling [Munzyazikwiwi 2013]. One of the most convenient and accessible solution is to use the System identification toolbox [Mathworks R2014a]. This toolbox is largely based on the works of Ljung [Ljung 1994] and it implements several common techniques used in the process of system identification. System identification toolbox enables us to create and use models of dynamic systems, which cannot be easily modeled from primary principles or specifications. We can use input-output data of the time- and frequency-domain to identify continuous-time and discrete-time transfer functions, process models, and state-space models. In a dynamic system, the values of the output signals depend on both, the instantaneous values of its input signals and also on the former behavior of the system. Models of dynamic systems are typically described by differential or difference equations, transfer functions, state-space equations, and pole-zero-gain models. System identification requires a model structure. A model structure is a mathematical relationship between those input and output variables which contain unknown parameters.

2 KELVIN MODEL
It contains a mass together with a spring and a damper connected in a parallel way. This model can be used for simulating the vehicle-to-barrier collision. In majority of cases the response of the system is underdamped. Therefore, we focus on this type of behavior. To obtain the parameters of the Kelvin model of the vehicle impact on the barrier from the actual tests, we use the Matlab Identification Toolbox.

Figure 1. Kelvin model [Huang 2002]

\( k \) – spring stiffness
\( c \) – damping coefficient
\( m \) – mass of the vehicle
\( v_0 \) – barrier initial impact velocity

3 GETTING DATA FROM THE IMPACT TEST
Data for the System identification Toolbox were obtained from a frontal impact test into the rigid barrier with full coverage at the speed of 56.17 km/h (15.6 m/s) according to NCAP (New Car Assessment Program) [NCAP 2017]. Honda Civic XL 2-door Coupe was chosen for providing the impact test. A measurement recording has been processed from an accelerometer that was firmly connected to the vehicle floor at the rear of the bodywork [Vlk 2003]. Accelerometer is a sensor of non-electric quantities. It converts detected quantity into an electrical signal which is subsequently processed and evaluated. Each impact test recording coming from accelerometer must be filtered by the CFC 60 filter (Channel Frequency Class) due to significant signal oscillations [Chicos 2006]. This type of filter is characterized by the following filter parameters: 3-dB limit frequency, 100 Hz. Stop damping, –30 dB. Sampling frequency, at least 600 Hz. It was designed primarily for measuring deceleration on car bodies. In order to perform the impact tests correctly, signal processing must be done under specific predetermined conditions. These standard conditions are written down in SAE J211-1: Instrumentation for Impact Test, Part 1, Electronic Instrumentation. Moreover, all physical quantities occurring in the impact test are specified there [Coufal 2012, Krenicky 2010].

4 ACHIEVED RESULTS AND THEIR DISCUSSION
As a matter of fact that for researchers it is important to know what happens mostly in the direction in which a car hits the rigid barrier, we have analyzed an \( x \) – direction (longitudinal). For the needs of this research, authors’ own
model of a vehicle was created (see Fig.2) in the Simulink programme, which is a graphical programming environment for modeling, simulating and analyzing multidomain dynamical systems. The main objective of this model is to process the measured data (unfiltered deceleration signal) in such a way that at the output of the Scope 5 block we would be able to observe the course of deformation of the chosen vehicle (Honda civic XL).

![Figure 2. Model of a vehicle (from deceleration to deformation)](image)

When data from Simulink’s own model were processed, they were imported into the system identification toolbox (input-deceleration output-deformation).

### 4.1 Linear models obtained due to Identification toolbox

**Transfer Function Models**

The general transfer function model has the following structure: \( Y(s) = \frac{num(s)}{den(s)}U(s) + E(s) \),

where \(Y(s)\), \(U(s)\) and \(E(s)\) represent the Laplace transforms of the output, input and error, respectively. Parameters \(num(s)\) and \(den(s)\) represent the numerator and denominator polynomials that define the relationship between the input and the output. The roots of the denominator polynomial are referred to as the model poles. The roots of the numerator polynomial are referred to as the model zeros.

- \(tf1\) - model (blue line) was created from the measured input-output data, see Fig. 4 (From input “Deceleration” to output “Deformation”).
- \(State-space\ model - ss1\): (green line) was created from the measured input-output data, see Fig.4 and has the following structure: \(dx/dt = Ax(t) + Bu(t) + Ke(t)\)
  
\[ y(t) = Cx(t) + Du(t) + e(t) \]

where \(y(t)\) represents the output at time \(t\), \(u(t)\) represents the input at time \(t\), \(x\) is the state vector, and \(e(t)\) is the white-noise disturbance. The System Identification Toolbox product estimates the state-space matrices \(A\), \(B\), \(C\), \(D\), and \(K\).

- \(The\ model\ n4s2\) - state-space model (pink line) was created from measured input-output data, see Fig. 4 (Matlab algorithm selects the model order automatically (in this case, 2)).

![Figure 4. Measured and model outputs](image)

**Polynomial ARX Models**

For a single-input/single-output system (SISO), the arx model structure looks like this:

\[ y(t) + a_1y(t-1) + \ldots + a_ny(t-n_y) = b_0u(t-n_u) + \ldots + b_nu(t-n_u+n_r) + e(t) \]

where \(y(t)\) represents the output at time \(t\), \(u(t)\) represents the input at time \(t\), \(n_r\) is the number of poles, \(n_u\) is the number of zeros plus 1, \(n_i\) is the input delay – the number of samples before the input affects the system’s output (called delay or dead time of the model), and \(e(t)\) is the white-noise disturbance.

- \(Name: arx441 - model\) (turquoise line) was created from the measured input-output data. Sample time: 0.0001 seconds, see Fig. 4.
- \(arxqs - model\) (yellow line) was created from the measured input-output data. Sample time: 0.0001 seconds, see Fig.4.
- \(Process\ model\ - P2U\) (red line) was created from the measured input-output data. Process model with a transfer function is shown in Fig. 4. A numerical search method is also very important for these models. In this particular case, the Levenberg-Marquardt method was used

\[ G(s) = \frac{K}{1+2*Ks*T*s+(T*s)^2} \]
where \( K = 8347.9 \text{ m/N} \)
\( T = 90.677 \text{ s} \)
\( K_{si} = 0 \)

Termination condition: Maximum number of iterations reached. Number of iterations: 20. Number of function evaluations: 83. Estimated using PROCEST on time domain data "honda". Fit to estimation data: 99.12%.

4.2 Obtained parameters for Kelvin model

Based on the discussion above and Fig. 4, we can say that the measured reference output best fits the estimated output of the model in the following order: model tf1 = 99.78%, model ss1 = 99.22%, model P2U = 99.12%. Therefore, we will deal with the Process Model with the P2U (Investigation of real crash data) transfer function and the use of the Identification toolbox in more detail.

This identification toolbox allows us to obtain the parameters of the system according to the input and output data. As an example we are going to use the model of the second order differential equation (second order oscillating element). The forcing factor is the external force over mass (acceleration) – initial conditions (velocity \( v_0 = 15.6 \text{ m/s} \) and displacement \( x_0 = 0 \text{ m} \)).

Equation of the second order oscillating element is as follows [Kowal 2003]:

\[
T^2 \frac{d^2 X(t)}{dt^2} + 2 \zeta T \frac{dX(t)}{dt} + X(t) = Kx(t)
\]  

(1)

where \( X(t) \) – output and \( x(t) \) – input.

Providing the Laplace transform of equation (1) with initial conditions \( v_0 = 15.6 \text{ m/s} \) and \( x_0 = 0 \text{ m} \) we get:

\[
T^2 s^2 Y(s) - v_0 + 2 \zeta T s Y(s) + Y(s) = KX(s)
\]  

(2)

Therefore the transfer function of the system given by (2) is:

\[
G(s) = \frac{Y(s)}{X(s)}
\]  

(3)

From the equation of motion of the Kelvin model we know:

\[
m \frac{d^2 y(t)}{dt^2} + c \frac{dy(t)}{dt} + ky(t) = mu(t)
\]  

(4)

input \( u(t) \) is an deceleration.

By taking Laplace transform of (4) with the same initial conditions as previously we obtain the following transfer function:

\[
G(s) = \frac{Y(s)}{U(s)}
\]  

(5)

Both equations, (3) and (5) describe the same model. Therefore, they equal each other if the following is true:

\[
T = \sqrt{\frac{m}{K}}, \ K = \frac{m}{K_{\xi}}, \ \xi = \frac{c}{2m \omega_n}, \ \omega_n = \sqrt{\frac{K}{m}}
\]  

(6)

With this findings we proceed to Identification Toolbox and select the appropriate type of estimation. An underdamped system with two poles has been chosen as a matter of fact that this research is based on Kelvin model. Parameters obtained for Process model are shown in Fig. 5.

Figure 5. Identification toolbox - results

Several parameters, namely, time constant \( T \), damping coefficient \( \xi \) and gain \( K \) are the same for both models – reference , oscillating element model - Process model P2U and Kelvin model . It means that as soon as we get the input and the output data of the system and set the required initial conditions, we can determine precisely the numeric values of coefficients k1 and c1 of the Kelvin model according to relations (6). It is done through the computational kernel of the Matlab or through the Simulink model of blocks [Murcinkova 2013] . (E.g.: It is possible to determine the value of time and dynamic crush \( C \) for the Kelvin model and compare them with the values that result from the model output - Process model P2U or compare them from values up to the level of the measured output).

For this Process model with the Lavenberg-Marquardt numerical method, we can see a high fit value of up to 99.12%. To calculate the stiffness coefficient k1 and the damping coefficient c1 for the Kelvin model we can use relations in (6). Then we get: \( k_1 = 0.17 \text{ N/m} \), \( c_1 = 0 \text{ Ns/m} \), \( m=1422 \text{ kg} \).

The problem is low values of the stiffness coefficient k1 and the damping coefficient c1 to zero which result in other problem with the low value of the forces in the spring (in Fig. 6). Therefore, the same Process model with the transfer function but different numerical method seems to be more suitable. We used the Trust - Region reflective Newton numerical method with the following output. Process model P2U with transfer function:

\[
G(s) = \frac{K}{(1+2*K_{si}*Ts+(Ts)^2)}
\]

,where \( K = 2.7769 \text{ m/N} \)
\( T = 1.6569 \text{ s} \)
\( K_{si} = 0.12152 \)

Termination condition: Maximum number of iterations or number of function evaluations reached. Number of iterations: 21. Number of function evaluations: 22. Estimated using PROCEST on time domain data "honda". Fit to estimation data: 99.02%.

The coefficients for the Kelvin model (6) are as follows: \( k_1=518.5 \text{ N/m} \), \( c_1=207.3 \text{ Ns/m} \). The result is shown in Fig. 6. The better the result is, the closer to real conditions the force in the spring gets [Hlavac 2018].

Having the parameters of the Kelvin model we investigate its response using the Simulink diagram or the Matlab simulation program (used here) with the initial velocity \( v_0=15.6 \text{ m/s} \) a \( d_0=0\text{m} \).
Thus, when comparing the output of the Kelvin model (Fig. 6) with the reference, Process model (respectively with the measured output) (Fig. 4) it can be seen that the dynamic crush in both models differs by the order of centimeters. However, it is achieved at the same time. Unsuitable numerical methods are Gauss-Newton method - fitting 0.0825% and Gradient method has fitting 0.0826%.

**5 CONCLUSION**

For the purpose of this work, an authors’ own model (see Fig. 1) was created in Matlab-Simulink. In order to get better results, the model was created in such a way that it was capable of filtering off (Lowpass CFC 60 filter) the native acoustic signal obtained from the accelerometer.

Figure 4 shows measured and estimated model outputs that reconstruct the vehicle crash with small errors in terms of dynamic crush. Time of dynamic crush obtained from the acoustic signal obtained from the accelerometer.

The fit between the two curves (measured output - first curve and estimated model output - second curve) is computed in such a way that 100% means a perfect fit, and 0% indicates a poor fit. This means that in our case the measured output best fits the model output in the following order: model tf1 = 99.78%, model s1 = 99.22%, model P2U = 99.12%, model n42 = 98.97%, model arx441 = 94.71% and finally model arx48 = 94.70%.

Identification Toolbox allows us to obtain the Process model with the P2U transfer function according to the input and output data, the parameters of which also depend on the numerical method search. It is namely time constant T, damping coefficient ξ and gain K [Jurko 2012].

The most suitable numerical methods include: the Lavenberg-Marquardt numerical method, in which a fit value is up to 99.12% and the Trust - Region reflective Newton numerical method with a fit value up to 99.02%.

Based on the fact that both models – reference, Process model and Kelvin model are the same, we have calculated the values of k1, c1 for the Kelvin model. When using the Trust - Region reflective Newton numerical method, mentioned parameters reach the following values: k1 = 518.5 N/m, c1 = 207.30 Ns/m. These numerical values represent the best result. It is a situation when the force in the string is very close to real conditions and difference in dynamic crush for both models is very little (a few centimeters). Moreover, the dynamic crush is reached at the same time of 0.08 seconds (see Fig. 4 and Fig. 6).

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**Figure 6.** Simulation result resulting from Kelvin model for parameters k1=518 N/m and c1=207 Ns/m and vehicle mass m=1422kg

| Position x | t [s] |
|------------|-------|
| 0.05       | 0.05  |
| 0.1        | 0.1   |
| 0.15       | 0.15  |
| 0.2        | 0.2   |
| 0.25       | 0.25  |
| 0.3        | 0.3   |
| 0.35       | 0.35  |

| Speed v1  | t [s] |
|-----------|-------|
| 0.05      | 0.05  |
| 0.1       | 0.1   |
| 0.15      | 0.15  |
| 0.2       | 0.2   |
| 0.25      | 0.25  |
| 0.3       | 0.3   |
| 0.35      | 0.35  |

| Force in spring k1 | t [s] |
|--------------------|-------|
| 0.05               | 0.05  |
| 0.1                | 0.1   |
| 0.15               | 0.15  |
| 0.2                | 0.2   |
| 0.25               | 0.25  |
| 0.3                | 0.3   |
| 0.35               | 0.35  |
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