CORS BAADE–WESSELINK DISTANCE TO THE LMC NGC 1866 BLUE POPULOUS CLUSTER

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ABSTRACT

We used optical, near-infrared photometry, and radial velocity data for a sample of 11 Cepheids belonging to the young LMC blue populous cluster NGC 1866 to estimate their radii and distances on the basis of the CORS Baade–Wesselink method. This technique, based on an accurate calibration of surface brightness as a function of \((U - B), (V - K)\) colors, allows us to estimate, simultaneously, the linear radius and the angular diameter of Cepheid variables, and consequently to derive their distance. A rigorous error estimate on radii and distances was derived by using Monte Carlo simulations. Our analysis gives a distance modulus for NGC 1866 of 18.51 \pm 0.03 mag, which is in agreement with several independent results.

Key words: distance scale – globular clusters: individual (NGC 1866) – Magellanic Clouds – stars: variables: Cepheids

Online-only material: color figures

1. INTRODUCTION

One of the main goals of modern cosmology is to determine the Hubble constant to an accuracy of 1%–2% (Freedman et al. 2011, and references therein). Determining the distance to the LMC is a critical step in the problem of determining the scale of the universe. It is, in fact, considered as a benchmark in determining of distances to other galaxies, being a useful place to compare and test different distance estimators. Therefore, any error in its distance contributes a substantial fraction to the uncertainty in the Hubble constant (Mould et al. 2000).

In the last two decades there have been improvements in the determination of the LMC distance both theoretically (e.g., red clump, tip of red giant branch, model fitting of variable stars light curves; Romaniello et al. 2000; Walker et al. 2001; Keller & Wood 2002, 2006; Bono et al. 2002; Marconi & Clementini 2005; Grocholski et al. 2007; Koerwer 2009), and observationally (e.g., parallaxes for Galactic Cepheids and RR Lyrae, eclipsing binaries; Popowski & Gould 1998; Whitelock & Feast 2000; Benedict et al. 2002, 2007, 2011; Fitzpatrick et al. 2002, 2003; Dall’Ora et al. 2004; Bonanos et al. 2011). The literature contains a huge number of estimated distance values ranging from 18.10 mag (Udalski 1998) to 18.80 mag (Groenewegen & Salaris 2003).

A direct measurement of the distance to the LMC can be obtained by using a version of the Baade–Wesselink technique called the surface brightness method (Gieren et al. 1997 and references therein). This method utilizes radial changes of the stellar surface brightness and the pulsational velocity in order to determine the linear radius and angular diameter of pulsating stars. Gieren et al. (2005) applied the near-infrared surface brightness technique to a sample of 13 Cepheids in the LMC and obtained a distance modulus of 18.56 \pm 0.04 mag. The same method has been applied recently by Storm et al. (2011b) who derived the distances of 36 Cepheids in the LMC and obtained a distance modulus of 18.45 \pm 0.04 mag.

In the present work, we face the problem of the LMC distance by using the CORS version of the Baade–Wesselink surface brightness technique (Cacciapuoti et al. 1981; Ripepi et al. 1997, 2000; Ruoppo et al. 2004; Molinaro et al. 2011), using recent photometric and spectroscopic data of a sample of Cepheids observed in the populous cluster NGC 1866. This object is

relation. From both the observational and theoretical perspective, some authors argue that this dependence is present (e.g., Freedman et al. 2001; Storm et al. 2003; Tammann et al. 2003; Sakai et al. 2004; Romaniello et al. 2005; Macri et al. 2006; Freedman & Madore 2011; Marconi et al. 2005; Bonito et al. 2008, 2010) even if the size and the sign of the effect are still disputed, while others suggest that the effect is small and perhaps ill defined (e.g., Groenewegen & Salaris 2003; Gieren et al. 2005; Storm et al. 2005, 2011b; Albier et al. 1999).
one of the few young (~10^8 yr; Brocato et al. 1989) LMC clusters that are close enough to allow for detailed observation of individual stars (see, e.g., Mucciarelli et al. 2011; Brocato et al. 2003, and references therein). Moreover, Storm et al. (2005) found that the NGC 1866 Cepheids are close to the average LMC Cepheid distance, so we will assume the distance to this cluster to be the distance of the LMC itself. To date, it is known that NGC 1866 harbors at least 23 Cepheids (Welch & Stetson 1993; Musella et al. 2006), the largest number among all the LMC clusters, and this makes it an excellent system for distance determination. Indeed, it provides a sizeable sample of variable stars contained in a limited volume and then all at the same distance. Furthermore, we assume that they are characterized by the same chemical composition, age, and reddening, so that we can derive their distances with no influence from differences in the above quantities among stars.

The first Baade–Wesselink distance to NGC 1866 was derived by Côté et al. (1991), obtaining the radii of seven variables in the field of NGC 1866 and using the $B - V$ color to calibrate the surface brightness. The final result, 18.6 ± 0.3 mag, is affected by a large error, probably due to the color index used, which is not a good surface brightness calibrator (Coulson et al. 1986).

Gieren et al. (1994), using the surface brightness modification of the classical Baade–Wesselink method, obtained the distances of four Cepheids in the field of NGC 1866. They used the $V - R$ color index and the radial velocities obtained from spectroscopic data taken at the Las Campanas 2.5 m du Pont reflector (Welch et al. 1991). The obtained distance modulus to NGC 1866 was 18.47 ± 0.20 mag, considering only three Cepheids (HV 12198, HV 12199, and HV 12203). The fourth, HV 12204, was excluded because the authors suspected it was not a member of the cluster.

Finally, using optical and near-infrared data, Storm et al. (2005) derived the distance to NGC 1866 by applying the surface brightness method to five Cepheids and found 18.30 ± 0.05 mag. We aim at improving their result by increasing the analyzed sample of Cepheids, thanks to new photometric and spectroscopic data, and by relying on an accurate calibration of the surface brightness obtained from grids of theoretical atmospheres.

Section 2 contains a description of the photometric and spectroscopic data used in this work. The procedures followed to phase the light curve and the radial velocity curve, and to correct for reddening are described in Section 3. The CORS Baade–Wesselink method is introduced in Section 4, together with the procedure adopted to calibrate the surface brightness function using atmosphere models. Section 5 contains the derived Cepheid angular diameter and the linear radii. The distance to NGC 1866 and a comparison with other results from the literature are discussed in Section 6, while conclusions are contained in Section 7.

## 2. THE DATA

NGC 1866 harbors at least 23 Cepheids (Welch & Stetson 1993; Musella et al. 2006). Among them we selected those stars for which both near-infrared and radial velocity data were available. The final sample consists of 11 Cepheids whose data are presented in the next two sections and are resumed in Table 2.

### 2.1. Photometry

We used photometric data in the ultraviolet $U$ band, optical $B$, $V$, $I$ bands, and near-infrared $K$ band.

### 2.2. Radial Velocity

We used the radial velocity data from Storm et al. (2005, 2004) and Welch et al. (1991), obtained through the classical cross-correlation method. For three Cepheids, namely HV 12197, HV 12199, and We2, we used new data coming from the VLT/FLAMES data set by Mucciarelli et al. (2011; Appendix A). We note that our radial velocities are the first ones ever published for We2. The radial velocities for the three quoted Cepheids are listed in Table 1.

## Table 1

| Epoch (JD) | HV 12197 | HV 12199 | We2 |
|-----------|----------|----------|-----|
| 53280.77407 | 312.8 ± 0.8 | 320.3 ± 1.0 | 276.1 ± 1.2 |
| 53355.71398 | 287.8 ± 0.7 | 314.8 ± 1.1 | 285.6 ± 1.0 |
| 53355.75731 | 289.8 ± 1.1 | 315.0 ± 0.8 | 285.0 ± 1.0 |
| 53355.80046 | 289.8 ± 0.6 | 316.8 ± 0.9 | 282.6 ± 0.9 |
| 53357.60509 | 283.0 ± 0.7 | 319.8 ± 0.9 | 302.2 ± 1.0 |
| 53351.59229 | 293.5 ± 1.0 | 316.9 ± 1.0 | 284.6 ± 1.1 |
| 53351.64296 | 293.4 ± 1.1 | 318.1 ± 1.2 | 286.3 ± 0.8 |
| 53351.69311 | 294.1 ± 0.9 | 319.4 ± 1.0 | 288.1 ± 0.9 |
| 53376.61793 | 290.8 ± 0.8 | 282.1 ± 0.8 | 301.4 ± 1.0 |
| 53376.72984 | 292.7 ± 1.0 | 284.4 ± 0.9 | 302.9 ± 0.8 |
| 53377.73544 | 312.7 ± 1.1 | 311.7 ± 1.0 | 321.4 ± 0.9 |
| 53378.59190 | 298.1 ± 0.8 | 312.3 ± 1.0 | 282.1 ± 0.8 |

Notes. The new radial velocities in km s$^{-1}$ for HV 12197, HV 12199, and We2 are listed in the second, third, and fourth columns, respectively. The epochs of observations are in the first column.

The $U$-band data were obtained at Cerro Tololo Inter-American Observatory with 0.9 m, 1.5 m, and 4.0 m telescopes.

The optical data were taken at Very Large Telescope (VLT) with FORS1 imager and consist of 69 images in $B$, 90 images in $V$, and 62 images in $I$. Both ultraviolet and optical data were calibrated in the Johnson–Cousins photometric system. These VLT data have already been published in Musella et al. (2006). The complete $UBVRI$ data set used in this work will be published in a forthcoming paper (I. Musella et al., in preparation), including a detailed discussion of the procedure adopted to reduce the data and to address the crowding problem.

As for the near-infrared $K$ band, we used the data set described in Storm et al. (2005) and Testa et al. (2007). Furthermore, we used unpublished data collected by J. Storm for the Cepheids $V4$, $V7$, and $V8$, which were calibrated using the data by Testa et al. (2007) as a reference. The data from Testa et al. (2007) were calibrated in the LCO photometric system using the relations from Carpenter (2001). $K$-band data from Storm et al. (2005) were calibrated in the CIT photometric system.

To compare the photometric data with theoretical models (see Section 4.2), we transformed the infrared data from CIT and LCO into an SAAO photometric system using the simple transformation from Bessell & Brett (1988):

$$K_{SAAO} = K_{CIT} + 0.014 = K_{LCO} + 0.014,$$

where we also used the relation $K_{CIT} = K_{LCO}$ from Storm et al. (2005).
Figure 1. New radial velocity data for HV 12197, HV 12199, and We2 (filled circles) are shown together with those from Storm et al. (2005; open triangles) and Welch et al. (1991; open squares).

Table 2
Cepheids Considered in this Work

| Star    | Period (days) | Radial Velocity References                                   | Epochs (HJD) |
|---------|---------------|---------------------------------------------------------------|--------------|
| HV 12197| 3.143742      | Storm et al. (2005), this work(−1.25), Welch et al. (1991)   | 2452998.5638 |
| HV 12198| 3.522805      | Storm et al. (2004), Storm et al. (2005)(+2.8), Welch et al. (1991)(+1.1) | 2452947.7500 |
| HV 12199| 2.639181      | Storm et al. (2005), Welch et al. (1991), this work           | 2452993.6100 |
| HV 12202| 3.101207      | Storm et al. (2005), Welch et al. (1991)                      | 2452930.8922 |
| HV 12203| 2.954104      | Storm et al. (2005), Welch et al. (1991)                      | 2452972.6731 |
| V4      | 3.31808       | Storm et al. (2005), Welch et al. (1991)                      | 2452998.5651 |
| V6      | 1.944252      | Storm et al. (2005), Welch et al. (1991)                      | 2452976.7400 |
| V7      | 3.452075      | Storm et al. (2005)(−3.4), Welch et al. (1991)                | 2452915.9000 |
| V8      | 2.007157      | Storm et al. (2005), Welch et al. (1991)                      | 2452915.7700 |
| We2     | 3.054847      | This work                                                     | 2452990.6712 |
| We8     | 3.039855      | Welch et al. (1991)                                           | 2452995.5394 |

Notes. The Cepheids considered in this work are listed in the first column, their period is in the second column, and in the third column we list the source of the radial velocity data with the shift (km s\(^{-1}\)), if any, in parentheses. Finally, the epochs used to phase light curves and radial velocity curves are listed in the last column.

We2, we also estimated the systematic velocity \(V_r\) obtaining 301.4 km s\(^{-1}\) with an accuracy \(\sim 1\) km s\(^{-1}\), confirming its cluster membership.

Finally, we recall that from these spectroscopic data the iron content was also estimated following the approach described in Mucciarelli et al. (2011; see also Appendix A). The [Fe/H] values for the three Cepheids are \(-0.39 \pm 0.05\) dex for HV 12197, \(-0.38 \pm 0.06\) dex for HV 12199, and \(-0.43 \pm 0.05\) dex for We2. These values are fully consistent with the average iron content of the cluster, [Fe/H] = \(-0.43 \pm 0.01\) dex, obtained from the analysis of a sample of static stars in NGC 1866 (Mucciarelli et al. 2011).

3. DATA ANALYSIS

In this section, we describe the steps performed to phase the light and the radial velocity curves, as well as to estimate the reddening in the direction of NGC 1866.
Figure 2. Schematic plot of the procedure followed to estimate the reddening $E(B-V)$ from grids of theoretical models in the $(B-V)-(V-I)$ plane. The locus expected to be occupied by Cepheids is enclosed between the constant gravity and constant temperature curves (black and red lines, respectively). As an example, the representative point of the Cepheid V7 is out of the grids when it is not corrected for reddening. The values of $E(B-V)$ which make the point move along the reddening vector on the grids are selected (dashed line).

(A color version of this figure is available in the online journal.)

3.1. Construction of Light Curves and Radial Velocity Curves

Cepheid photometric data were phased as usual, i.e., requiring that the maximum light in the $B$ band occurs at phase zero, while the maximum in the other bands is shifted in phase as expected (see, e.g., Labhardt et al. 1997; Freedman 1988). The adopted epochs and periods are listed in Table 2. Since the data were obtained at different times, to estimate the $(V-K)$ and $(U-B)$ colors, it was necessary to first interpolate each light curve at the same phase points. This was achieved by means of a C code, written by one of the authors, performing smoothing spline interpolation. A different procedure has been performed to interpolate the $U$ band of HV 12197 and $K$ bands of We2 and We8. HV 12197 $U$-band light curve, in fact, was poorly sampled, while the $K$-band measurements for We2 and We8 were of lower quality with respect to the other variables. In these three cases, we performed the interpolation by using a template light curve. To construct the template we calculated the mean ratios $A(K)/A(V)$ and $A(U)/A(V)$ of the light-curve amplitudes in the $K$, $U$, and $V$ bands for all the stars of our sample showing well-covered light curves. Considering the $V$-band light curve of the star to be interpolated, we then calculated a “normalized” template light curve characterized by zero mean magnitude and an amplitude equal to one. Finally, the interpolation has been achieved by rescaling the template light curve, according to the estimated ratios, and shifting it to the mean magnitude of the investigated Cepheid.

To estimate the uncertainties of the interpolated magnitudes for all the Cepheids, we considered the rms of the residuals around the interpolated curve.

As for radial velocity curves, to obtain an accurate phasing with light curves, we used the same epoch and period as for the photometry.

As cited in Section 2.2, for all the Cepheids, except We8, we have measurements from different data sets. To combine them, we corrected for possible shifts in radial velocity (see Table 2). They were determined by interpolating the most accurate sample of radial velocities and then by estimating the median distance along the velocity axis between the fitted curve and the other radial velocity samples.

3.2. Reddening Determination

To correct the colors for extinction, we used the grids from models by Bessell et al. (1998; http://wwwuser.oat.ts.astro.it/castelli/colors/bcp.html). They provided models with different metallicity values and we linearly interpolated between solar ([Fe/H] = 0.0 dex) and subsolar ([Fe/H] = −0.5 dex) metallicity grids to obtain models at the metallicity estimated by Mucciarelli et al. (2011) and equal to [Fe/H] = −0.43 dex.

In this step we followed the technique introduced by Dean et al. (1978) to derive the reddening of Cepheids. According to this method, the reddening is derived by shifting the observed points in a color–color diagram onto their intrinsic unreddened position, which can be obtained either by using stars of known reddening or by using models. In particular, according to the models, the Cepheids occupy a narrow zone in the plane $(V-I)-(B-V)$ (Figure 2), which represents the reddened locus ($5000 \leq T_e \leq 6250$, $1.0 \leq \log g \leq 4.0$). Using this color–color plane, we plotted the point, whose coordinates are
Table 3

| Star    | $E(B - V)$  | $\delta E(B - V)$ |
|---------|-------------|-------------------|
| HV 12197 | ...         | ...               |
| HV 12198 | 0.102       | 0.018             |
| HV 12199 | 0.082       | 0.018             |
| HV 12202 | 0.03        | 0.02              |
| HV 12203 | 0.02        | 0.02              |
| V4       | 0.07        | 0.02              |
| V6       | 0.05        | 0.02              |
| V7       | 0.06        | 0.02              |
| V8       | 0.091       | 0.009             |
| We2      | 0.10        | 0.03              |
| We8      | 0.06        | 0.02              |

the mean observed colors of each Cepheid, and shifted it along the reddening vector, trying different values of $E(B - V)$ and selecting all the values which cause the star to lie on the grids. The mean of these values is an estimate of the $E(B - V)$, and the width of the range of selected values can be assumed as the uncertainty on the mean value. The procedure is shown in Figure 2 for the Cepheid V7, and the values of the uncertainty on the mean value. The procedure does not work for HV 12197 because its representative point is bluer than the grids along the reddening vector, trying different values of $E(B - V)$.

The procedure does not work for HV 12197 because its representative point is bluer than the grids along the $V - I$ color. A possible explanation for this behavior may be the presence of a companion which can affect the color of the Cepheid, especially in optical bands (Szabados 2003). In order to reduce the effect of outliers, we used the median of the derived reddening values, obtaining $E(B - V) = 0.06$ mag and an rms of 0.02 mag, consistent with the value typically adopted for NGC 1866 (see Storm et al. 2005 and references therein).

To calculate the extinction in each observational band we used $E(B - V) = 0.06$ mag, $R_V = 3.3$ (Feast & Walker 1987) and the law by Cardelli et al. (1989), obtaining $A_U = 0.303$ mag, $A_B = 0.261$ mag, $A_V = 0.198$ mag, $A_I = 0.120$ mag, and $A_K = 0.02$ mag.

### 4. THE CORS BAADE–WESSELINK METHOD

In the following sections we give a brief description of the complete CORS Baade–Wesselink method used to derive the linear radius of Cepheid stars and of the procedure followed to calibrate the surface brightness function, which is at the base of the complete CORS technique. Moreover, the surface brightness allows us to estimate the star angular diameter and consequently the distance $d$ which is the factor linking the linear radius $R$ and the angular diameter $\theta$ according to $R = (1/2)d\theta$.

#### 4.1. Theoretical Background

The original CORS method (Caccin et al. 1981) is a realization of the classical Baade–Wesselink method (Wesselink 1946) used to derive the radius of pulsating stars.

Starting from the surface brightness function

$$S_V = m_V + 5 \log \theta,$$

where $\theta$ is the angular diameter (in mas) of the star and $m_V$ is the apparent visual magnitude, it is straightforward to obtain the basic CORS equation by differentiating the phase ($\phi$), multiplying the result by a generic index ($C_{ij}$) and integrating along the pulsational cycle:

$$q\int_0^1 \ln \left\{ R_0 - pP \int_{\phi_0}^{\phi} v(\phi')d\phi' \right\} C_{ij}d\phi - B + \Delta B = 0, \quad (3)$$

where $q = 5/\ln 10$, $P$ is the period, $v$ is the radial velocity, and $p$ is the radial velocity projection factor, which correlates radial and pulsational velocities according to $R'(\phi) = -p \cdot P \cdot v(\phi)$. The last two terms, $B$ and $\Delta B$, are

$$B = \int_0^1 C_{ij}(\phi)m_V'(\phi)d\phi$$

$$\Delta B = \int_0^1 C_{ij}(\phi)S_V'(\phi)d\phi. \quad (5)$$

Equation (3) is an implicit equation in the unknown radius $R_0$ at an arbitrary phase $\phi_0$. The radius at any phase $\phi$ can be obtained by integrating the radial velocity curve. The main characteristic in Equation (3) is the estimate of the $\Delta B$ term as it contains the surface brightness function. Typically, the $\Delta B$ term has a small value ($10^{-3}$ to $10^{-4}$; Onnembo et al. 1985) and in the original Baade–Wesselink method it is neglected (see Caccin et al. 1981). However, the Cepheid radii estimated by including the $\Delta B$ term (complete CORS method) in Equation (3) are more accurate than those based on the original Baade–Wesselink method (Molinaro et al. 2011 and references therein).

#### 4.2. The Surface Brightness Calibration

To use the complete CORS technique, it is necessary to calibrate the surface brightness function. Here we describe only the main steps of the procedure followed and refer the reader to Molinaro et al. (2011) and references therein for more mathematical details.

Assuming the validity of the quasi-static approximation (Onnembo et al. 1985), for the Cepheid atmosphere, it is possible to express any photometric quantity as a function of effective temperature $T_e$ and gravity $\log g$. As a consequence, considering the surface brightness $S_V$ and the two generic colors $C_{ij}$ and $C_{kl}$, they can be expressed as $S_V = S_V(T_e, \log g)$ and $C_{ij} = C_{ij}(T_e, \log g)$, $C_{kl} = C_{kl}(T_e, \log g)$. If the last two equations can be inverted, then it is possible to express effective temperature and gravity as a function of the two colors and, consequently, the surface brightness becomes

$$S_V = S_V(T_e(C_{ij}, C_{kl}), \log g(C_{ij}, C_{kl})). \quad (6)$$

In general, the invertibility condition is not valid over the entire parameter space, since the same pair of colors trace different pairs of gravity and temperature. However, after an appropriate choice of the colors $C_{ij}$ and $C_{kl}$ and of their range of variability, it is possible to find a local invertibility condition. Using grids of models provided by Bessell et al. (1998; http://wwwuser.oat.ts.astro.it/castelli/colors/bcp.html) and interpolated at the correct metallicity values, we succeeded in inverting the previous equations for $(U - B)$ and $(V - K)$ colors and for the range of parameters typical of Cepheids with period $P \sim 3$ days (i.e., $0.5$ dex $\leq \log g \leq 4.5$ dex and $5250 \leq T_e \leq 6750$ K).

Figure 3 shows the selected region of the theoretical grids in the $(V - K)(U - B)$ color–color plane containing the loops of all the analyzed Cepheids. To make the figure clearer, we show only...
Figure 3. Color–color loops of five Cepheids from our sample plotted on the theoretical grids from Bessell et al. (1998). For clarity only a selected sample of objects is plotted (see the text for details). The locus of constant temperature (dashed lines) and of constant gravity (continuous lines) are also plotted.

(A color version of this figure is available in the online journal.)

Data for a limited sample of objects. The loops outlined by HV 12198 and HV 12202 are representative of those drawn by the majority of Cepheids, with the exception of We2 and HV 12197. The loops of these two objects show extreme positions, with the bluest and the reddest \((U - B)\) color, respectively. This occurrence could be explained by invoking the binarity of the target or, more likely, the presence of one or more blending objects. In this case, depending upon the angular separation between the Cepheid and the blended companions, relative to the radius of the seeing disk, it is possible to oversubtract or undersubtract the light of the companion. Similarly, the loop of HV 12203 seems too extended (especially along the \((U - B)\) direction) with respect to the average of “normal” Cepheids.

The effect of possible blending on the radius calculation has been analyzed in Section 5.2.

Within the plotted ranges of colors we have been able to fit the relations \(\log T_e = \log T_e(V - K, U - B)\), \(\log g = \log g(V - K, U - B)\) by means of polynomials. The fitted surfaces are shown in Figures 4 and 5 and all the mathematical details are given in Appendix B. The rms around the fitted relations amount to 0.00013 and 0.03 dex for \(\log T_e\) and \(\log g\), respectively.

Using the fitted relations for temperature and gravity, we have derived the surface brightness from the following equation:

\[
S_V = 42.207 - 10.0 \log T_e - BC,
\]

where the constant only depends on the bolometric absolute magnitude of the Sun, the solar constant, and on the Stefan–Boltzmann constant (Fouqué & Gieren 1997), while BC is the bolometric correction calculated as a function of the temperature and gravity through a polynomial fit (see Appendix B).

Figure 4 shows the fitted surface \(BC = BC(\log T_e, \log g)\) together with the models from Bessell et al. (1998).

As a test of our calibration, we have analyzed the relation \(F_V = F_V(V - K)\), where \(F_V = 4.2207 - 0.15S_V\) is the surface brightness parameter. Figure 7 shows the selected Cepheids plotted in the \(F_V - (V - K)\) plane together with the relation obtained by Kervella et al. (2004) using interferometric measurement of nine Galactic Cepheids. The errors for the color \((V - K)\) have been estimated by considering the scatter around the interpolated light curves, as explained in Section 3.1, while those for surface brightness have been estimated by means of...
simulations, as described in Appendix C. The plot shows a small discrepancy between the fitted relation and the surface brightness of NGC 1866 Cepheids, which seems to be systematically brighter than the expected values from the relation by Kervella et al. (2004). Although the observed systematic shift is included within the uncertainty on reddening, it is important to stress that the relation by Kervella et al. (2004) has been obtained from Galactic Cepheids, which are more metal-rich than those in NGC 1866. Therefore, the observed shift can be due to metallicity differences since less metallic Cepheids are more luminous than more metal-rich ones. Therefore, a metallicity dependence for the surface brightness relation is not ruled out.

5. DERIVATION OF THE RADIUS

This section contains the procedure followed to estimate the radii of the selected Cepheids and a comparison with other results present in the literature. Here we also justify the value of the projection factor $p$ chosen in our analysis.

5.1. The Projection Factor

One of the most uncertain parameters in the Baade–Wesselink method is the projection factor $p$, which converts radial velocity into pulsational velocity. It depends on both the physical structure of the stellar atmosphere and the way the radial velocity is measured. Furthermore, there is an open discussion about its dependence on the period and/or pulsational phase. The question has been considered by many authors and the reader can find an exhaustive review in Barnes (2009) and Storm et al. (2011a).

As mentioned before, the radial velocities used in this work have been derived with the cross-correlation method. In a recent work, Nardetto et al. (2009) achieved a period-dependent value of the projection factor to correct radial velocity obtained by means of cross-correlation. Using their relation, namely $p = 1.31 - 0.08 \log g$, and the mean value of the period of our sample (once first overtones are fundamentalized according to Feast & Catchpole 1997), we calculated a projection factor value $p = 1.27$. This is the same value found by Groenewegen (2007) by using Cepheids with interferometric angular diameters and Hubble Space Telescope (HST) parallaxes. Furthermore, it allows us to be consistent with Molinaro et al. (2011), where they compared the two values $p = 1.36$ and $p = 1.27$ and found that the latter is the favored one.

5.2. Radius Calculation

Using the photometric data and the radial velocities discussed in Section 2, we are able to derive the mean radius for each star of our sample by using a FORTRAN 77 code. This performs a fit of the $V$ magnitude curve, the $(V - K)$, $(U - B)$ color curves, and the radial velocity curve using a Fourier fit, with a number of harmonics fixed interactively by the user. Then it solves the CORS, Equation (3), for the radius at an arbitrary phase both with and without the $\Delta B$ term. The mean radius is, finally, calculated by integrating the radial velocity curve twice. Our results are listed in Table 4. It contains the linear radii (in solar units) obtained from the CORS method and the angular diameters (in mas) calculated from the calibrated surface brightness, Equation (7), for the selected sample of Cepheids in NGC 1866 analyzed in this work. The uncertainties on both the parameters are also listed and are obtained from Monte Carlo simulations as described in Appendix C. Note that for HV 12203 we provide only the radius obtained without the $\Delta B$ term (29.5 $R_\odot$). Indeed, the inclusion of this term produces an anomalously small radius value (25.3 $R_\odot$), as a consequence of the peculiar loop of HV 12203 in the color–color plane (see Section 4.2), that, in turn, generates an unusually large value of the $\Delta B$ term ($\sim 10^{-2}$). The anomalous width of the HV 12203 loop has been considered in the estimate of the error for the angular diameter for this object.

Table 4 also contains the linear radii obtained by other authors for the stars in common with our sample. Their values have been rescaled to our projection factor value. Our results are systematically larger than those by Storm et al. (2005) and Gieren et al. (1994), whereas they are consistent, within the uncertainties, with those by Côté et al. (1991), with the exception of HV 12199 which has a highly undetermined radius in their work.

As noted in Section 4.2, the color–color loops of We2 and HV 12197 are systematically shifted in the $(U - B)$ with respect to the locus occupied by all of the other Cepheids on the grids of models in the $(U - B)$, $(V - K)$ plane. This could be due to the effect of overestimating or underestimating the flux in blue-ultraviolet bands, due to the difficult photometric analysis of crowded regions. To estimate the possible effect of blending on the derived radii, i.e., how the presence of a blue unresolved
Figure 7. Selected Cepheids of NGC 1866 (black points) plotted in the plane $F_V - (V - K)$ together with the surface brightness color relation obtained by Kervella et al. (2004; solid line).

Table 4

| Star     | $(R \pm \delta R)$ $(R_\odot)$ | $(\theta \pm \delta \theta)$ (μ-arcsec) | S05       | G94       | C91       |
|----------|---------------------------------|----------------------------------------|-----------|-----------|-----------|
| HV 12197 | 31.6 ± 1.7                      | 5.58 ± 0.08                            | 23.9 ± 0.6| ...       | ...       |
| HV 12198 | 33.1 ± 1.3                      | 6.00 ± 0.06                            | 27.5 ± 0.4| 28.3 ± 2.6| 38.3 ± 6.3|
| HV 12199 | 27.9 ± 1.1                      | 4.96 ± 0.08                            | 23.0 ± 1.0| 24.6 ± 3.1| 56.1 ± 19.1|
| HV 12202 | 29.6 ± 0.5                      | 5.78 ± 0.08                            | 26.3 ± 0.9| ...       | 25.2 ± 4.4|
| HV 12203 | 29.5 ± 0.7a                     | 5.22 ± 0.11                            | 26.1 ± 1.1| 24.2 ± 4.4| 25.9 ± 4.9|
| V4       | 30.2 ± 1.3                      | 5.74 ± 0.04                            | ...       | ...       | 29.3 ± 8.1|
| V6       | 31.8 ± 1.3                      | 5.10 ± 0.16                            | ...       | ...       | ...       |
| V7       | 31.7 ± 2.0                      | 5.92 ± 0.08                            | ...       | ...       | ...       |
| V8       | 31.2 ± 2.3                      | 5.18 ± 0.06                            | ...       | ...       | ...       |
| We2      | 30.7 ± 1.1                      | 5.68 ± 0.12                            | ...       | ...       | ...       |
| We8      | 28.8 ± 1.0                      | 5.40 ± 0.10                            | ...       | ...       | ...       |

Notes. The mean linear radii (second column) and angular diameters (third column) of all the selected Cepheids are listed with their uncertainties estimated from Monte Carlo simulations, as described in the text. The linear radii, expressed in solar units, listed in the remaining columns have been obtained by the following authors: Storm et al. (2005, S05), Gieren et al. (1994, G94), and Côté et al. (1991, C91).

Value obtained without the $\Delta B$ term (see the text).

close companion affects the $(U - B)$ color of the quoted Cepheids, we decided to shift the two extreme loops described by We2 and HV 12197 along the $(U - B)$ direction, to match the region of the grids occupied by most of the Cepheids. This color shift is of the order of ±0.08–0.09 mag. We then recalculated the radii of these two Cepheids with the CORS method, obtaining as a result a difference in radii smaller than 1% and 4% for We2 and HV 12197, respectively. Similarly, for the angular diameters the change was 0.3% and 0.5%, respectively. Therefore, our result is robust against the effect of contamination of the flux of Cepheids by undersubtracted or oversubtracted companions.

This is due to the fact that blending affects only the value of the $\Delta B$ term, which typically has a small value (see Section 4.1) and influences the radius at most by about 5% on average7 (see Molinaro et al. 2011 and references therein). In any case, we will take into account this possible source of systematic uncertainty by adding it to the random errors on the radius and surface brightness (as obtained in Appendix C).

Note that this is not the case for the Cepheid HV 12203 quoted above whose loop causes a $\Delta B$ term approximately one order of magnitude larger than the typical ones, and in turn, a difference of ∼15% in the radii.

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Using the results of our procedure, we investigated how the selected Cepheids are located with respect to the Galactic period–radius relation recently obtained by Molinaro et al. (2011). A visual comparison is shown in Figure 8 and it is evident that the Cepheids in NGC 1866 lie on the same period–radius relation as the Galactic Cepheids. As a test, we have fitted the equation \( \log R = a + b \log P \) to the sample including the 26 Galactic Cepheids by Molinaro et al. (2011) and the stars analyzed in the present work, with the exception of the first overtone pulsators (V6 and V8). The fitting relations, excluding and including the \( \Delta B \) term, are the following:

\[
\begin{align*}
\log R &= (1.120 \pm 0.019) + (0.723 \pm 0.019) \log P \quad (8) \\
\log R &= (1.110 \pm 0.015) + (0.746 \pm 0.015) \log P, \quad (9)
\end{align*}
\]

respectively. The inclusion of the Cepheids belonging to NGC 1866 extends the period–radius relation in the direction of short periods, giving stronger constraints on the coefficients of the fit. It is important to stress that the equation with the \( \Delta B \) term remains almost unchanged with respect to that found by Molinaro et al. (2011), but with smaller uncertainties on the coefficients of the period–radius relation. Concerning the case without the \( \Delta B \) term, the fitting equation is slightly different with respect to that by Molinaro et al. (2011), but in any case is consistent within the uncertainties.

6. DISTANCE TO NGC 1866

Using the values of the angular diameter derived from the surface brightness and the CORS Baade–Wesselink linear radius, we have estimated the distance to NGC 1866 by using the simple equation \( d(\text{kpc}) = 2R(U\ A)/\theta(\text{mas}) \), where \( R \) and \( \theta \) are the mean linear radius and the angular diameter, respectively.

The typical procedure used in other works (Fouqué & Gieren 1997; Storm et al. 2005, 2011a) consists of matching the curves of the linear radius and the angular diameter, giving the same result. In particular, we have performed a correction for the eventual phase shift between the two curves and then calculated the distance as the parameter that minimizes the quantity \( \sum (R(\phi_i) - (d/2)\theta(\phi_i))^2 \), where the index \( i \) runs over the phases from 0.0 to 0.8. A phase cut has been performed to avoid the influence from possible shocks in the stellar atmosphere close to the minimum radius (see, e.g., Storm et al. 2004).

As an example of radial curve matching, we report the case of We2. In particular, Figure 9 shows the linear radius of We2 as a function of the phase and the curve of the angular diameter corrected for phase shift and multiplied for the best distance value obtained from the minimization of the function cited above. It is evident from the plot that the two curves deviate at phases larger than 0.8, which have been excluded in the matching procedure.

The distances in kpc of the Cepheids analyzed in the present work are listed in Table 5. The corresponding distance moduli, \( \mu = m - M = -5 + 5 \log[d(\text{pc})] \), are also reported in the same table and are plotted in Figure 10 as a function of the period.

In our analysis, we exclude the two overtone pulsators, whose distances are clearly discrepant (apparently more distant at a 3\( \sigma \) level) with respect to the other stars considered (see Figure 10). Since the Cepheids in NGC 1866 are all expected to be at nearly the same distance, this occurrence could be due to...
Figure 9. Example of matching between the linear radius curve and the angular diameter curve for the Cepheid We2.

(A color version of this figure is available in the online journal.)

Figure 10. Distance moduli ($\mu$), with error bars, of fundamental Cepheids (empty circles) and first overtones (full points) are plotted as a function of the period ($P$) expressed in days. Our best distance estimate (solid line) is plotted together with the results by other authors (point-dashed lines): Storm et al. (2011b, S11), Storm et al. (2005, S05), Walker et al. (2001, W01), Gieren et al. (1994, G94), and Côté et al. (1991, C91).

(A color version of this figure is available in the online journal.)
an imperfect calibration of the surface brightness function of overtones throughout the grids of model atmosphere. Further analysis is required about on this point.

Using a weighted statistic of the nine remaining Cepheids, we obtain a distance of $50.3 \pm 0.6$ kpc corresponding to $18.51 \pm 0.03$ mag in the distance modulus and an rms of 0.09 mag. However, if we do not consider the weight, then the mean distance becomes $51.1 \pm 0.6$ kpc, equal to a distance modulus $\mu = 18.54 \pm 0.03$ mag with an rms of 0.07 mag. Since we are confident that our estimate of errors for each single Cepheid is correct, we consider the weighted result as our best value for the distance to NGC 1866.

The distance modulus obtained in this work is 0.37 mag larger than that obtained by Storm et al. (2005) and (Storm et al. 2011b) rescaled to our projection factor. The distance to NGC 1866 has also been derived by using other methods. As an example, Walker et al. (2001) used HST V and I photometry of stars in NGC 1866 to apply the main-sequence fitting technique, obtaining a distance equal to 18.35 $\pm$ 0.05 mag, which is shorter than our result. On the contrary, in a recent work Salaris et al. (2003) obtained the distance to NGC 1866 from the red clump technique. Their analysis gave a distance modulus of 18.53 $\pm$ 0.07 mag in agreement with our result.

### Table 5

| Star    | $(d \pm \delta d)$ (kpc) | Distance Modulus (mag) |
|---------|--------------------------|------------------------|
| HV 12197| 53.2 $\pm$ 3.0           | 18.63 $\pm$ 0.12       |
| HV 12198| 52.2 $\pm$ 2.0           | 18.59 $\pm$ 0.08       |
| HV 12199| 53.1 $\pm$ 2.3           | 18.62 $\pm$ 0.10       |
| HV 12202| 48.1 $\pm$ 1.0           | 18.41 $\pm$ 0.05       |
| HV 12203| 52.6 $\pm$ 1.7           | 18.61 $\pm$ 0.07       |
| V4      | 49.5 $\pm$ 2.2           | 18.47 $\pm$ 0.10       |
| V6      | 58.5 $\pm$ 2.9           | 18.83 $\pm$ 0.11       |
| V7      | 50.3 $\pm$ 3.2           | 18.51 $\pm$ 0.14       |
| V8      | 56.3 $\pm$ 4.2           | 18.75 $\pm$ 0.17       |
| We2     | 51.0 $\pm$ 2.0           | 18.54 $\pm$ 0.09       |
| We8     | 50.2 $\pm$ 2.1           | 18.50 $\pm$ 0.09       |

Note. Obtained from the CORS Baade–Wesselink linear radius and the angular diameters derived by the surface brightness.

As for the projection factor, we used the appropriate value $p = 1.27$, obtained from a recent $p$-factor–log $P$ relation introduced by Nardetto et al. (2009) and already adopted in Molinaro et al. (2011).

The linear radii obtained from the CORS technique are larger than those derived by Gieren et al. (1994) and Storm et al. (2005) for some of the stars in our sample. On the contrary, we are in good agreement with the results obtained by Côté et al. (1991).

We also studied how the selected Cepheids place with respect to the period–radius relation obtained by Molinaro et al. (2011) for Galactic Cepheids. Our analysis shows that the Cepheids in NGC 1866 follow the same linear relation as the Galactic ones.

A weighted statistical analysis gives a distance modulus of NGC 1866 equal to 18.51 $\pm$ 0.03 mag and an rms of 0.09 mag.

Finally, the distance obtained in this work is in agreement with the converging value of 18.50 mag for the LMC obtained by many authors and described in the Introduction.

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### APPENDIX A

#### SPECTROSCOPIC DATA

The new spectroscopic measurements for the three Cepheids HV 12197, HV 12199, and We2 were secured with VLT/FLAMES under the program 074.D-0305. The observations were performed in the GIRAFFE mode and employed three different setups, namely HR 11 (with a coverage between 5597 and 5840 Å), HR 12 (5821–6146 Å), and HR 13 (6126–6405 Å), with spectral resolutions ranging from 19,000 to 24,000. Five exposures were secured for HR 11, four for HR 12, and three for HR 13 (all the exposures are of 3600 s each).

Each exposure was analyzed independently. The spectra were reduced using the girBLDRS pipeline developed at the Geneva Observatory\(^8\) including bias subtraction, flat-fielding, wavelength calibration with a reference Th–Ar lamp and the final extraction of each spectrum. The accuracy of the zero point in the wavelength calibration was checked by measuring the position of some sky-emission lines, compared with their rest-frame positions taken from the sky-emission lines atlas by Osterbrock et al. (1996). No relevant shift was detected in the emission lines position, confirming the reliability of the adopted wavelength solution.

The girBLDRS pipeline performs, in its last step, the measurement of the radial velocity through the classical cross-correlation technique (Tonry & Davis 1979). Heliocentric corrections were applied to each radial velocity. The typical uncertainty in the derived velocity of each exposure is about 0.8 km s$^{-1}$.

The measurement of the iron content of the three Cepheids was performed following the same approach already employed in Mucciarelli et al. (2011) concerning the chemical analysis of a sample of stars in NGC 1866. In each spectrum we identify a number of reliable, unblended iron lines (typically 10–15 in each spectrum), deriving their abundance through the $\chi^2$-minimization of the observed line profile with a grid of synthetic spectra computed with different Fe abundances. The

\(^8\) http://girbldrs.sourceforge.net/
Table 6
Coefficients of the Polynomial Equations Obtained from the Interpolation of Temperature, Gravity, and Bolometric Correction

| $a_1$  | $a_2$  | $a_3$  | $a_4$  | $a_5$  | $a_6$  | $a_7$  | $a_8$  | $a_9$  | $a_{10}$ |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|----------|
| 3.9858 ± 0.0019 | -0.208 ± 0.005 | 0.055 ± 0.004 | 0.110 ± 0.008 | -0.031 ± 0.004 | 0.057 ± 0.008 | -0.029 ± 0.007 | -0.0093 ± 0.0011 | -0.071 ± 0.007 | -0.121 ± 0.004 |
| $b_1$  | $b_2$  | $b_3$  | $b_4$  | $b_5$  | $b_6$  | $b_7$  | $b_8$  | $b_9$  | $b_{10}$ |
| 5.51 ± 0.09 | -4.61 ± 0.18 | 25.9 ± 1.6 | -14.0 ± 0.8 | 24.5 ± 1.5 | -13.2 ± 1.3 | 2.65 ± 0.10 | -21.7 ± 1.4 | -19.4 ± 0.8 |
| $c_1$  | $c_2$  | $c_3$  | $c_4$  | $c_5$  | $c_6$  | $c_7$  | $c_8$  | $c_9$  | $c_{10}$ |
| -221.0 ± 7.4 | 114.9 ± 4.0 | -14.9 ± 0.5 | -21.4 ± 1.8 | 2.9 ± 0.2 | -0.106 ± 0.009 | 0.00160 ± 0.00016 | 0.39 ± 0.03 | 40.2 ± 3.4 |
computation of the synthetic spectra was performed by means of the SYNTHE code (Kurucz 1993) coupled with the ATLAS9 model atmospheres.

APPENDIX B

THE FIT OF THEORETICAL GRIDS

Here we give the explicit mathematical expression for the surfaces obtained by interpolating log $T_e$ and log $g$ as a function of the two colors ($U - B$) and ($V - K$), and which describe the bolometric correction BC as a function of log $T_e$ and log $g$:

$$\log T_e = a_1 + a_2(V - K) + a_3(V - K)^2 + a_4(V - K)(U - B) + a_5(V - K)^2(U - B) + a_6(V - K)(U - B)^2 + a_7(U - B)^3 + a_8(V - K)^3 + a_9(U - B)^2 + a_{10}(U - B)$$

(B1)

$$\log g = b_1 + b_2(V - K)^2 + b_3(V - K)(U - B) + b_4(V - K)^2(U - B) + b_5(U - B)^3 + b_6(V - K)^3 + b_7(U - B)^2 + b_8(U - B)$$

(B2)

$$BC = c_1 + c_2 \log T_e + c_3(\log T_e)^2 + c_4(\log T_e)(\log g) + c_5(\log T_e)^2(\log g) + c_6(\log T_e)(\log g)^2 + c_7(\log g)^3 + c_8(\log g)^2 + c_9 \log g.$$  

(B3)

The coefficients $a_i, b_i, c_i$ of the previous equations are listed in Table 6. The rms of the previous relations are 0.00013 dex, 0.0007 dex, and 0.0012, respectively.

APPENDIX C

ERROR ESTIMATE ON RADIi THROUGH MONTE CARLO SIMULATIONS

To investigate how the errors in the various parameters involved in the CORS method influence the linear radius estimation, we have performed Monte Carlo simulations. In particular, we have considered the uncertainties for the colors ($V - K$) and ($U - B$) on the reddening $E(B - V)$, and the error in the phase matching between the photometric and radial velocity curves. We have run 1000 simulations for each of the cited parameters. They consist of varying each analyzed parameter using random numbers extracted from a Gaussian distribution with an rms equal to the error on the parameter itself. For all of the extracted displacements the radius is recalculated and the rms of the obtained values is assumed as the uncertainty in the radius due to the error on the analyzed parameter. As for the colors ($V - K$) and ($U - B$), we have derived their errors from the scatter around the interpolated light curves. Here we do not give all of the derived errors in the colors, but only remind the reader that typical values are $\sim 0.03$ mag for both the ($V - K$) and ($U - B$), except for a few cases with errors of $\sim 0.06$ mag because of fewer sampled light curves in the $K$ and/or $U$ bands. For the three Cepheids HV 12197, HV 12203, and We2, an additional systematic error has been considered for the ($U - B$) color, leading to a total uncertainty of the order of 0.085–0.10 mag. The effect of reddening on the radius estimation has been analyzed by considering the error of 0.02 mag obtained as described in Section 3, while for the phase matching we have chosen a typical error of 0.01 in phase. This value has been chosen to be conservative because the error on the phase matching, due to the uncertainty on the period propagated between the epoch of photometry and that of spectroscopy, is typically smaller. Furthermore, we have taken into account the fact that the star HV 12202 is a spectroscopic binary (Welch et al. 1991) and consequently its photometry and radial velocity can be influenced by the flux and the motion of the companion, respectively. To be conservative, we have doubled the errors on the photometry and the radial velocity data. The total effect of the various errors on the radius is always less than 5%, with the exception of V7 and V8, for which it is $\sim 7\%$ (Table 4).

Monte Carlo simulations have also been used to estimate the errors for the surface brightness. As it depends on the two colors ($V - K$) and ($U - B$) through the equations given in Appendix B, we have run 1000 simulations for each color following the same procedure described before. The derived error for the surface brightness has been used to calculate the uncertainty of the angular diameter, obtained from Equation (7) and reported in Table 4.

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