Curiosities and counterexamples in smooth convex optimization. (English) Zbl 07606025 Math. Program. 195, No. 1-2 (A), 553-603 (2022)

Summary: Counterexamples to some old-standing optimization problems in the smooth convex coercive setting are provided. We show that block-coordinate, steepest descent with exact search or Bregman descent methods do not generally converge. Other failures of various desirable features are established: directional convergence of Cauchy’s gradient curves, convergence of Newton’s flow, finite length of Tikhonov path, convergence of central paths, or smooth Kurdyka-Łojasiewicz inequality. All examples are planar. These examples are based on general smooth convex interpolation results. Given a decreasing sequence of positively curved $C^k$ convex compact sets in the plane, we provide a level set interpolation of a $C^k$ smooth convex function where $k \geq 2$ is arbitrary. If the intersection is reduced to one point our interpolant has positive definite Hessian, otherwise it is positive definite out of the solution set. Furthermore, given a sequence of decreasing polygons we provide an interpolant agreeing with the vertices and whose gradients coincide with prescribed normals.

MSC: 90C25 Convex programming

Keywords: convex programming; smooth convex counterexamples; interpolation of decreasing convex sequences; Bregman methods; block-coordinate methods; exact line search

Full Text: DOI arXiv

References:

[1] Alvarez, F.; Bolte, J.; Brahic, O., Hessian Riemannian gradient flows in convex programming, SIAM J. Control. Optim., 43, 2, 477-501 (2004) · Zbl 1077.34050 · doi:10.1137/S0363012902419977
[2] Alvarez, D.F., Pérez, C.J.M.: A dynamical system associated with Newton’s method for parametric approximations of convex minimization problems. Appl. Math. Optim., 38, 193-217 (1998)
[3] Auslender, A., Optimisation Méthodes Numériques (1976), Paris: Masson, Paris · Zbl 0326.90057
[4] Auslender, A., Penalty and barrier methods: a unified framework, SIAM J. Optim., 10, 1, 211-230 (1999) · Zbl 0953.90045 · doi:10.1137/S1052623497324825
[5] Aubin, J.-P.; Cellina, A., Differential Inclusions: Set-Valued Maps and Viability Theory (1984), Berlin: Springer, Berlin · doi:10.1007/978-3-642-69512-4
[6] Bauschke, H.H.; Bolte, J.; Teboulle, M., A descent lemma beyond Lipschitz gradient continuity: first-order methods revisited and applications. Math. Oper. Res., 42, 2, 330-348 (2016) · Zbl 1346.90625 · doi:10.1287/moor.2016.0817
[7] Bauschke, H.H.; Combettes, P.L., Convex Analysis and Monotone Operator Theory in Hilbert Spaces (2011), New York: Springer, New York · Zbl 1218.47001 · doi:10.1007/978-1-4419-9467-7
[8] Beck, A., First-Order Methods in Optimization (2017), Philadelphia: SIAM, Philadelphia · Zbl 1384.65033 · doi:10.1137/1.9781611974997
[9] Beck, A.; Teboulle, M., Mirror descent and nonlinear projected subgradient methods for convex optimization, Oper. Res. Lett., 31, 3, 167-175 (2003) · Zbl 1046.49007 · doi:10.1016/S0167-6377(02)00231-6
[10] Beck, A.; Tetruashvili, L., On the convergence of block coordinate descent type methods, SIAM J. Optim., 23, 4, 2037-2060 (2013) · Zbl 1297.90113 · doi:10.1137/120887679
[11] Bertsekas, D.P., Scientific, A., Convex Optimization Algorithms (2015), Belmont: Athena Scientific, Belmont
[12] Bolte, J.; Daniilidis, A.; Ley, O.; Mazet, L., Characterizations of Łojasiewicz inequalities: subgradient flows, talweg, convexity, Trans. Am. Math. Soc., 362, 6, 3319-3363 (2010) · Zbl 1202.26026 · doi:10.1090/S0002-9947-09-05048-X
[13] Bolte, J.; Nguyen, TP; Peyronnet, J.; Suter, BW, From error bounds to the complexity of first-order descent methods for convex functions, Math. Program., 165, 2, 471-507 (2017) · Zbl 1373.90076 · doi:10.1007/s10107-016-1091-6
[14] Bolte, J.; Teboulle, M., Barrier operators and associated gradient-like dynamical systems for constrained minimization problems, SIAM J. Control Optim., 42, 4, 1266-1292 (2003) · Zbl 1051.49010 · doi:10.1137/S0363012902410861
[15] Borwein, JM; Li, G.; Yao, L., Analysis of the convergence rate for the cyclic projection algorithm applied to basic semialgebraic convex sets, SIAM J. Optim., 24, 1, 498-527 (2014) · Zbl 1296.41011 · doi:10.1137/130919052
