Jets of light hadrons via AdS/CFT correspondence

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Abstract.
Within the expanding fireball formed at heavy ion collision, jets are produced which probes the QGP. Analysis the energy loss of these energetic partons as they travel throw QGP may reveal extremely valuable information about the dynamics of the plasma and exhibit distinctive properties such as jet-quenching. The "AdS/CFT correspondence" which imposes the duality between the gauge theory and gravity is a novel tool provides valuable insight into the strongly coupled plasma. The most important result of AdS/CFT is calculating the value of shear viscosity to entropy density ratio which is in remarkable agreement with the hydrodynamics predictions. We study the energy loss rate of light quarks via AdS/CFT correspondence in both static and expanding plasma. In the hope of making contact with QGP physics, we propose a novel jet prescription based on the separation of hard and soft modes in the dual theory and test the AdS/CFT approach with the latest light hadron suppression data from CMS.

1. Introduction
The spectacular measurements from the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) provide compelling evidence that the matter produced in heavy ion collision is a deconfined state of QCD, Quark-Gluon Plasma (QGP) [1, 2, 3, 4], at temperatures above $\sim$160 MeV which appears to be nearly perfect, with an extremely low viscosity-to-entropy ratio $\eta/s \sim 1/4\pi$ [5, 6]. While lattice QCD is the proper tool for understanding the static equilibrium thermodynamics of such strongly coupled plasma, it does not allow us to calculate its dynamics evolution on heavy-ion collision.

Recently, a novel tool called "the AdS/CFT correspondence" [7, 8, 9, 10, 11] provide valuable insight into the strongly coupled plasma. This conjecture state that the $N = 4$ SYM theory in the large $N_c$ and large 't Hooft coupling is dual to classical supergravity on ten-dimensional $AdS_5 \times S^5$ geometry [7]. In order to study the theory at finite temperature, one can add black hole (BH) to the geometry [8] which yields to the AdS-Sch metric. Fundamental quarks are described by open strings moving in the 10d geometry. In the large $\lambda$ limit, the quantum fluctuation of string world sheet are suppressed and the dynamics of string are described by the classical string theory.

Jets are produced within the expanding fireball and probe the QGP. Analyses the energy loss of these energetic partons as they travel throw QGP may reveal extremely valuable information about the dynamics of the plasma and exhibit distinctive properties such as jet-quenching which can clearly be observed at RHIC [1, 2, 3, 4] and more recently LHC [12, 13, 14].

In this paper, we study the light quark jet energy loss in both gravitational background dual to static and expanding plasma. We propose a new prescription of jets in string theory based
on the separation of hard and soft sectors. We demonstrate that the light quark jet energy loss reveal the “Bragg peak” at late times. Then we compute the nuclear modification factor of jets $R_{AA}$, renormalise the quantity, and compare the results with the preliminary CMS data [15].

### 2. Light quark jet energy loss

According to the AdS/CFT correspondence [10], the $\mathcal{N}=4$ SYM theory at finite temperature is dual to a 10d black hole geometry with the AdS-Schwarzschild (AdS-Sch) metric as follows,

$$ds^2 = \frac{L^2}{u^2} \left[ -f(u) \, dt^2 + d\mathbf{x}^2 + \frac{du^2}{f(u)} \right],$$

where $f(u) \equiv 1 - (u/u_h)^4$ is the blackening factor and $L$ is the AdS curvature radius. Four dimensional Minkowski coordinates are denoted by $x_\mu$ and the coordinate $u$ is an inverse radial coordinate. So, the boundary of the AdS-Sch spacetime is at $u = 0$ and the event horizon is located at $u = u_h$. The temperature of the equilibrium SYM plasma relates to the event horizon as $T \equiv \frac{1}{(\pi u_h)}$. World sheet coordinates are as $\sigma^a$ where $t \equiv \sigma^0$ is denoted as the timelike world sheet coordinate, while the spatial coordinate is $\sigma \equiv \sigma^1$.

Fundamental representation quarks added to the $\mathcal{N}=4$ SYM theory are dual to open strings moving in the 10d geometry. Addition of a $\mathcal{N}=2$ hypermultiplet to the $\mathcal{N}=4$ SYM theory is performed by adding D7 branes to the 10d geometry [16]. These branes extend along the radial coordinate from the boundary at $u = 0$ down to maximal coordinate at $u = u_m$ as well as fill the whole 4d Minkowski space. Also, they wrap on $S^3$ from the $S^5$ sphere. The bare mass $M$ of quark is proportional to $1/u_m$ [17], so for massless quarks the D7 brane should fill the whole radial direction. Open strings that are attached to the D7 brane are dual to the quark-anti quark pairs on the field theory side. In the 5d geometry these strings can fall unimpeded toward the event horizon until their end points reach the radial coordinate $u_m$ where the D7 brane ends. Since for sufficiently light or massless quarks $u_m > u_h$, open string end points can fall into the horizon.

We are interested in studying the back-to-back jets so we consider the configurations in which the two endpoints of string move away from each other as the total spatial momentum of the string vanishes. By choosing the appropriate frame, one half of the string has a large spatial momentum in $x$ direction, and the other half of the string carries a large negative spatial momentum. We will limit our attention to strings which move in one direction in the $R^5$ space, $x$ direction, so the embedding function of string $X^\mu(\tau, \sigma)$ will be a map to $(t(\tau, \sigma), x(\tau, \sigma), u(\tau, \sigma))$. So, the profile of an open string which is created at a point in space at time $t = t_c$ is given by

$$t(0, \sigma) = t_c, \quad x(0, \sigma) = 0, \quad u(0, \sigma) = u_c. \quad (2)$$

By these conditions, the string created at time $t_c$ and by time evolution, the string evolves from a point into an extended object and the string endpoints fall toward the horizon.

The dynamics of string is governed by the Polyakov action

$$S_P = -\frac{T_0}{2} \int d^2 \sigma \sqrt{-\eta} \eta^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}. \quad (3)$$

Variation of the Polyakov action with respect to the embedding functions $X^\mu$ lead to the equation of motion as

$$\partial_a \left[ \sqrt{-\eta} \Pi^a_\mu \right] = \frac{1}{2} \sqrt{-\eta} \eta^{ab} \frac{\partial G_{\nu\rho}}{\partial X^\mu} \partial_a X^\nu \partial_b X^\rho, \quad (4)$$

where $\Pi^a_\mu$ are the canonical momentum densities associated to the string obtained from varying the action with respect to the derivatives of the embedding functions.
In order to study jets in AdS/CFT we need to define the proper objects in the dual string theory that corresponds to a jet, a slippery object even in field theory; jets are truly only defined by the algorithm used to measure them. Presumably the ideal way to compute jet observables in the dual theory is to compute the energy momentum tensor associated with a high-momentum probe and “run” a jet finding algorithm on the result. However, one can define a jet prescription in the AdS/CFT and calculate the rate of energy loss from the string profile itself.

The authors of [18] are motivated by the localization of the baryon density on the boundary which is of scale of order $\Delta x \sim 1/\pi T$ and defined jet as a part of string which is in the $\Delta x$ spatial distance from the endpoint. We called this as the “$\Delta x$ – prescription” of jet [18].

Figure 1. Illustration of the $\Delta x$ and $\Delta u$ prescriptions of a jet in the string theory; see text for details.

In this paper, motivated by the separation of energy scales in, e.g., thermal field theory, we propose rather a $\Delta u$ prescription which we believe will ultimately provide a closer approximation to the result of a more complete calculation, figure 1. Since the radial coordinate in the string theory sets an energy scale in the field theory, in our $\Delta u$ prescription the portion of the string above some cutoff $u = u^*$ in the radial direction is considered part of the jet; the portion of the string below the cutoff is considered part of the thermalized medium. By choosing any value of $u$ above the black hole horizon as the cutoff, we regain the natural result that a jet that is thermalized no longer has detectable energy or momentum.

We evaluate the energy loss rate of jet in the radial $\sigma = u$ parametrization for both prescriptions of jet and plot in figure 2(a,b). In order to define jet using the $\Delta x$ – prescription, we choose $\sigma_r(t)$ as $\Delta x = 0.3/\pi T$. Our result on the energy loss rate of light quark using the $\Delta u$ – prescription shows a Bragg peak at late times which means the explosive transfer of quark energy to the plasma at late times and is consistence with the previous works [18].

Since the quark-gluon plasma produced at ultra-relativistic heavy ion collision is an expanding and cooling medium, we study the light quark jet energy loss in a time dependent gravity dual to the boost invariant flow [19]. This geometry is similar to the static black hole geometry, but the location of the horizon moves in the bulk as $\tau^{1/3}$ where $\tau$ is the proper time. Also the temperature of plasma cools as $T(\tau) \tau^{-1/3}$.

We calculate the energy loss rate of light quark using both $\Delta x$ – prescription and $\Delta u$ – prescription of jet. We consider the initial temperature of plasma in JP metric $T_c$ the same
but the location of the horizon moves in the bulk as to the boost invariant flow [46]. This geometry is similar to the static black hole geometry, and cooling medium, we study the light quark jet energy loss in a time dependent gravity dual static and expanding plasma.

as the temperature of plasma in AdS-Sch metric. The result is shown in figure 2(c,d) which demonstrates that the behavior of light quark energy loss in the JP metric is same as the AdS-Sch metric, but the distance that quark traveled before thermalizing increases.

3. Nuclear Modification Factor of Jet

We calculate an approximation of the nuclear modification factor $R_{AA}$ for jet using our energy loss model of jet. We consider the contributions of both quark and gluon jets. We assume that the produced parton with initial energy $p_T^i$ loses a fraction of its energy $\epsilon$ with probability $P(\epsilon|p_T^i, L, T)$ as the final energy of parton is given by $p_T^f = (1 - \epsilon)p_T^i$. Also, we suppose that the AdS energy loss is approximately independent of the initial energy [20], and the gluons loss their energy by the factor of 2 in the large $N_c$ limit respect to the quarks.

We suppose that the production spectrum can be approximately by a power law [20], with slowly varying with respect to $p_T$, then we may find a simple equation for the jet nuclear modification factor as follows,

$$R_{AA}^{R\rightarrow jet}(p_T) = \left\langle \int d\epsilon P(\epsilon|p_T, L, T) \left(1 - \epsilon\right)^{n_R(p_T)-1} \right\rangle.$$  \hspace{1cm} (5)

For an absolutely uniform nucleus that is a 1D line, the geometric average is carried out as an integral over a line of production points with a parton that propagates through the line. In this case, $R_{AA}^{R\rightarrow jet}(p_T)$ is obtained from the below line integral [20]

$$R_{AA}^{R\rightarrow jet}(p_T) = \int_{0}^{L_{\text{max}}} \frac{dl}{L_{\text{max}}} \left(1 - \epsilon_R(p_T, l, T)\right)^{n_R(p_T)-1}.$$  \hspace{1cm} (6)

We calculate $R_{AA}$ of jet by using our results of jet energy loss in both AdS-Sch and JP metric. Our results are shown in figure 3. The purple curve shows the $R_{AA}$ obtained from the AdS-Sch metric, while the blue curve is the $R_{AA}$ obtained from the JP metric. The AdS/CFT results of jet energy loss show an over suppression of jets in both static and expanding plasma.

The point-like initial condition falling string that we consider here is dual to creation of a pair of quark-antiquark which fly away each other in the strongly coupled plasma, interact and lose their energy. We expect that jets produced in the pp collision do not loss their energy. So, we consider the falling string with the same initial conditions in $AdS_5$ metric. Our results show that the string falls in the empty $AdS_5$. So, jet loses its energy in the vacuum! We calculate the $R_{AA}$ for a falling string in $AdS_5$ metric and show in figure 3 (a) in red curve.

Figure 2. The instantaneous energy loss of a light quark jet as a function of time in the AdS-Sch (a,b) and JP metrics (c,d) in the $\Delta x$ prescription and $\Delta u$ prescription. The normalization constant $E_0 = 100$ GeV is the initial energy of the jet, which has a virtuality of 175 GeV$^2$, and $T = 350$ MeV is the temperature of the plasma.
Since the definition of $R_{AA}$ measured the difference between the medium and vacuum effects on jet, we define a renormalized $R_{AA}$ in AdS/CFT as

$$R_{AA}^{\text{renorm}} = \frac{R_{AA}^{\text{medium}}}{R_{AA}^{\text{AdS}_5}}$$

(7)

We plot the renormalized $R_{AA}^{\text{renorm}}$ for jets in both AdS-Sch and JP metric in figure 3 (b) and compare with the CMS preliminary data for the most central Pb-Pb collision at $\sqrt{s} = 2.76$ TeV per nucleon [15]. A suppression factor of 0.5 for high $p_T$ jets is observed in central Pb-Pb collision in comparison to the pp collisions. Our results also show surprisingly agreement with CMS preliminary results on jet $R_{AA}$.

4. Conclusions

In this paper we have purposed a novel prescription of jet in the context of string theory and AdS/CFT correspondence. We have defined jet as a part of a falling string which lies above the radial coordinate $u^*$ near the event horizon. Our motivation is defining the jet as hard partons. The instantaneous energy loss of light quark is identified with the energy flux from the point at $u(\tau, \sigma_\kappa) = u^*$. We have shown that using the $\Delta u$ prescription of jet, the light quark energy loss exhibit the Bragg peak again at late times in both static and expanding plasmas. This late time behavior of jet energy lost implies that after traveling substantial distances through the plasma, the thermalization of light quark ends with a large amount of energy transferring to the plasma which is similar to the energy loss rate of a fast charge particle moving through ordinary matter.

We considered a brick of plasma and calculated the nuclear modification factor of jet in both AdS-Sch and JP metric. We assumed that the temperature of plasma around 350 MeV at AdS-Sch metric and at initial time in JP metric. Our results show an aver suppression of jet of order of ten respect to the data. We investigated that it is because of the falling string setup at AdS space. In fact, $R_{AA}$ of jet using the falling string in $AdS_5$ which is dual to jets in vacuum is not one, even though it is less than one. We introduced a renormalized $R_{AA}^{\text{renorm}}$ by dividing the
Figure 4. The maximum stopping distance of jet versus the virtuality of quark $Q^2$. Our result show a nontrivial dependency of the thermalization time to the initial condition of the string.

$R_{AA}$ in the medium to the $R_{AA}$ in the vacuum. Surprisingly, our ratio shows good agreement with the experimental data on the $R_{AA}^{et}$ of most central Pb-Pb collision at LHC figure 3 (b).

On the other hand, the light quark energy loss is highly depends on the initial conditions of falling string figure 4. The only way to determine the energy loss of a jet precisely in strongly coupled regime is solving the gravitational bulk-to-boundary problem. One can solve Einstein’s equations for the perturbation in the 5d geometry due to the presence of the string and according to the bulk to boundary map interpret the near boundary behavior of the metric perturbation as the perturbation in the SYM energy-momentum tensor by the presence of jet which will be left for the future work.

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