Remark on $CP$-violating Polarization Asymmetry of $t \bar{t}$ via Anomalous Couplings to $\gamma$ and $Z$

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ABSTRACT

We calculate the $CP$-violating polarization asymmetry of $t \bar{t}$, $\delta \equiv [ \sigma(e\bar{e} \rightarrow t(+)\bar{t}(+)) - \sigma(e\bar{e} \rightarrow t(-)\bar{t}(-))] / \sigma(e\bar{e} \rightarrow t\bar{t})$, for the most general $t\bar{t}\gamma/Z$ couplings without dropping any non-standard contribution. We find that one term which is usually neglected increases with $s$ and will eventually become non-negligible at very high energy.

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The Standard Model (SM) of electroweak interaction has been very successful in describing various experimental data up to the scale of $O(M_{W,Z})$. Still many people believe in the existence of new physics at higher-energy scale which reduces the number of the free parameters in the SM. Since the top-quark couplings have not been studied in detail yet, there could be a room for new physics in them. In the near future, experiments of top-quark-pair production via $e^+e^-$ annihilation are expected to become possible at Next Linear Collider (NLC). Indeed, NLC will be a powerful tool for new-physics search, and provide us useful data for probing the top-quark couplings to the photon and $Z$ boson.

One of such studies will be a measurement of $CP$ violation through possible anomalous couplings. Since $t\bar{t}$ pairs are produced mainly through the vector-boson ($\gamma/Z$) exchange, the helicities of $t\bar{t}$ would be only $(+-)$ or $(-+)$ if top-quark mass were much smaller than $\sqrt{s}$. However, because the observed top mass is $173.9\pm5.2$ GeV [1], we can also expect $(++)$ and $(-\bar{t})$ combinations. This enables us to study $CP$ violation, because

$$\hat{C}\hat{P}|\mp\mp\rangle = |\pm\pm\rangle$$

and therefore $CP$ violation can be measured through the asymmetry

$$\delta \equiv \frac{N(-\bar{t}) - N(++)}{N(\text{all})}$$

where $N(\text{all}) \equiv N(++) + N(-\bar{t}) + N(\bar{t}+) + N(--)$.

On this theme have appeared so far many papers [2]–[7]. In those papers, products of non-standard parameters were usually neglected and only interference between the SM and non-SM terms was taken into account under the assumption that non-SM effects are tiny. At much higher energy, however, such neglected terms might come to play an important role. Therefore, in this short note we calculate the cross section of polarized top-pair productions via $e^+e^-$ starting from the most general form of top interaction and keeping those which are usually neglected. Then we study their effects on the above $CP$-violating asymmetry numerically.

In our calculation, we will assume the following $t\bar{t}\gamma/Z$ couplings:

$$\Gamma_{t\bar{t}\gamma/Z} = \frac{g}{2} \bar{u}(p_t) \left[ \gamma^\mu \{ A_v + \delta A_v - (B_v + \delta B_v)\gamma_5 \} + \frac{(p_t - \bar{p}_t)^\mu}{2m_t} (\delta C_v - \delta D_v\gamma_5) \right] v(p_t),$$

(2)
where \( g \) denotes the \( SU(2) \) gauge coupling constant, \( v = \gamma, Z \), and

\[
A_\gamma = \frac{4}{3} \sin \theta_W, \quad B_\gamma = 0, \quad A_Z = \frac{v_t}{2 \cos \theta_W}, \quad B_Z = \frac{1}{2 \cos \theta_W}
\]

with \( v_t \equiv 1 - (8/3) \sin^2 \theta_W \). \( \delta A_v, \delta B_v, \delta C_v \) and \( \delta D_v \) are parameters expressing non-standard effects.\(^1\)

On the other hand, we assume the standard form for the electron-positron couplings:

\[
\Gamma_{\gamma e\bar{e}}^\mu = -e \bar{v}(p_e)\gamma^\mu u(p_e), \tag{3}
\]
\[
\Gamma_{Ze\bar{e}}^\mu = \frac{g}{4 \cos \theta_W} \bar{v}(p_e)\gamma^\mu(v_e + \gamma_5)u(p_e), \tag{4}
\]

where \( v_e \equiv -1 + 4 \sin^2 \theta_W \).

Now, using the above vertices and propagators \( d_\gamma = 1/s, \ d_Z = 1/(s - M_Z^2) \), we can represent the invariant amplitude for \( e^+e^- \rightarrow t\bar{t} \) as follows:

\[
M(e\bar{e} \rightarrow t\bar{t}) = g_{\mu\nu}(d_\gamma \Gamma_{\gamma e\bar{e}}^\mu \Gamma_{\gamma \bar{t}t}^\nu + d_Z \Gamma_{Ze\bar{e}}^\mu \Gamma_{Z\bar{t}t}^\nu)
\]

\[
= C_{VV} \left[ \bar{v}_e \gamma_\mu u_e \cdot \bar{u}_t \gamma^\mu v_t \right] + C_{VA} \left[ \bar{v}_e \gamma_\mu u_e \cdot \bar{u}_t \gamma_5 \gamma^\mu v_t \right] + C_{AV} \left[ \bar{v}_e \gamma_5 \gamma_\mu u_e \cdot \bar{u}_t \gamma^\mu v_t \right] + C_{AA} \left[ \bar{v}_e \gamma_5 \gamma_\mu u_e \cdot \bar{u}_t \gamma_5 \gamma^\mu v_t \right] + C_{VS} \left[ \bar{v}_e \gamma_\mu u_e \cdot \bar{u}_t v_t \right] + C_{VP} \left[ \bar{v}_e \gamma_5 \gamma_\mu u_e \cdot \bar{u}_t \gamma_5 v_t \right] + C_{AS} \left[ \bar{v}_e \gamma_5 \gamma_\mu u_e \cdot \bar{u}_t \gamma_5 v_t \right] + C_{AP} \left[ \bar{v}_e \gamma_5 \gamma_\mu u_e \cdot \bar{u}_t \gamma_5 v_t \right], \tag{5}
\]

where

\[
C_{VV} = -\frac{ge}{2s} \left[ (A_\gamma + \delta A_\gamma) - v_e d' (A_Z + \delta A_Z) \right],
\]
\[
C_{VA} = -\frac{ge}{2s} \left[ \delta B_\gamma - v_e d' (B_Z + \delta B_Z) \right],
\]
\[
C_{AV} = -\frac{ge}{2s} d' (A_Z + \delta A_Z),
\]
\[
C_{AA} = -\frac{ge}{2s} d' (B_Z + \delta B_Z),
\]
\[
C_{VS} = -\frac{ge}{4m_t s} \left[ \delta C_\gamma - v_e d' \delta C_Z \right],
\]
\[
C_{VP} = \frac{ge}{4m_t s} \left[ \delta D_\gamma - v_e d' \delta D_Z \right],
\]

\(^1\)In fact, the most general form contains also a term proportional to \((p_t + p_{\bar{t}})^\mu\), but this term gives vanishing contribution in the limit of zero electron mass.
A straightforward calculation leads to the following differential cross section in which \( t \) and \( \bar{t} \) have spins \( s_+ \) and \( s_- \) respectively:

\[
\frac{d\sigma}{d\Omega} (e^+e^- \rightarrow t(s_+)\bar{t}(s_-)) = \frac{3\beta\alpha^2}{16s^3} \left[ D_V \left\{ 4m_t^2 s + (lq)^2 \right\} (1 - s_+s_-) + s^2 (1 + s_+s_-) + 2s(ls_+ ls_- - Ps_+ Ps_-) + 2lq(ls_+ Ps_- - ls_- Ps_+) \right] \\
+ D_A \left[ (lq)^2 (1 + s_+s_-) - (4m_t^2 s - s^2) (1 - s_+s_-) - 2(s - 4m_t^2)(ls_+ ls_- - Ps_+ Ps_-) - 2lq(ls_+ Ps_- - ls_- Ps_+) \right] \\
- 4 \text{Re}(D_{VA}) m_t \left[ s(Ps_+ - Ps_-) + lq(ls_+ + ls_-) \right] \\
+ 2 \text{Im}(D_{VA}) \left[ lq\epsilon(s_+, s_-, q, l) + ls_-\epsilon(s_+, P, q, l) + ls_+\epsilon(s_-, P, q, l) \right] \\
+ 4 E_V m_t lq(Ps_+ - Ps_-) \\
+ 4 \text{Re}(E_{VA}) \left[ 2m_t^2(ls_+ Ps_- - ls_- Ps_+) - lq s \right] \\
+ 4 \text{Im}(E_{VA}) m_t \left[ \epsilon(s_+, P, q, l) + \epsilon(s_-, P, q, l) \right] \\
+ D_S \frac{1}{m_t^2} [(lq)^2 + 4m_t^2 s - s^2] [(4m_t^2 - s)(1 - s_+s_-) - 2Ps_+Ps_-] \\
- D_P \frac{1}{m_t^2} [(lq)^2 + 4m_t^2 s - s^2] [s(1 + s_+s_-) - 2Ps_+Ps_-] \\
+ 4 \text{Re}(D_{SP}) \frac{1}{m_t} [(lq)^2 + 4m_t^2 s - s^2] (Ps_+ + Ps_-) \\
+ 2 \text{Im}(D_{SP}) \frac{1}{m_t} [(lq)^2 + 4m_t^2 s - s^2] \epsilon(s_+, s_-, P, q) \\
- \text{Re}(F_1) \frac{1}{m_t} [lq s(ls_+ - ls_-) - ((lq)^2 + 4m_t^2 s)(Ps_+ + Ps_-)] \\
+ 2 \text{Im}(F_1) \left[ s\epsilon(s_+, s_-, P, q) + lq\epsilon(s_+, s_-, P, l) \right] \\
+ 2 \text{Re}(F_2) s(Ps_+ ls_- + Ps_- ls_+) \\
- \text{Im}(F_2) \frac{s}{m_t} [\epsilon(s_+, P, q, l) - \epsilon(s_-, P, q, l)]
\[-2 \text{Re}(F_3) \, lq(Ps_+ \, ls_- + Ps_- \, ls_+) + \text{Im}(F_3) \, \frac{lq}{m_t} [\epsilon(s_+, P, q, l) - \epsilon(s_-, P, q, l)]
\]

\[-\text{Re}(F_4) \, \frac{s}{m_t} [lq(Ps_+ + Ps_-) - (s - 4m_t^2)(ls_+ - ls_-)]
\]

\[-2 \text{Im}(F_4) \, [Ps_+\epsilon(s_-, P, q, l) + Ps_-\epsilon(s_+, P, q, l)]
\]

\[+ 2 \text{Re}(G_1) \, \{4m_t^2s + (lq)^2 - s^2\} (1 - s_+s_-) - 2s Ps_+Ps_-
\]

\[+ lq(ls_+ Ps_- - ls_- Ps_+)]
\]

\[-\text{Im}(G_1) \, \frac{lq}{m_t} [\epsilon(s_+, P, q, l) + \epsilon(s_-, P, q, l)]
\]

\[-\text{Re}(G_2) \, \frac{s}{m_t} [(s - 4m_t^2)(ls_+ + ls_-) - lq(Ps_+ - Ps_-)]
\]

\[-2 \text{Im}(G_2) \, [Ps_+\epsilon(s_-, P, q, l) - Ps_-\epsilon(s_+, P, q, l)]
\]

\[-\text{Re}(G_3) \, \frac{lq}{m_t} [lq(Ps_+ - Ps_-) - (s - 4m_t^2)(ls_+ + ls_-)]
\]

\[-2 \text{Im}(G_3) \, lq\epsilon(s_+, s_-, q, l)
\]

\[+ 2 \text{Re}(G_4) \, [(s - 4m_t^2)(Ps_+ \, ls_- - Ps_- \, ls_+) + 2lq \, Ps_+Ps_-]
\]

\[+ \text{Im}(G_4) \, \frac{1}{m_t}(s - 4m_t^2)[\epsilon(s_+, P, q, l) + \epsilon(s_-, P, q, l)] \right],
\]

where \( \beta \equiv \sqrt{1 - 4m_t^2/s} \), \( P \equiv p_e + p_e(-p_t + p_t) \), \( l \equiv p_e - p_e \), 
\( q \equiv p_t - p_t \), the symbol \( \epsilon(a, b, c, d) \) means \( \epsilon_{\mu\nu\rho\sigma}a^\mu b^\nu c^\rho d^\sigma \) for \( \epsilon_{0123} = +1 \), and 
\( D_V = (s^2/e^4)(|C_{VV}|^2 + |C_{AV}|^2), \quad D_A = (s^2/e^4)(|C_{VA}|^2 + |C_{AA}|^2), \)
\( D_{VA} = (s^2/e^4)(C^*_{VV}C_{VA} + C^*_{AV}C_{AA}), \)

\( E_V = 2(s^2/e^4)\text{Re}(C^*_{AV}C_{VV}), \quad E_A = 2(s^2/e^4)\text{Re}(C^*_{AA}C_{VA}), \)
\( E_{VA} = (s^2/e^4)(C^*_{VV}C_{AA} + C^*_{AV}C_{VA}), \)

\( D_S = m_t^2(s^2/e^4)(|C_{VS}|^2 + |C_{AS}|^2), \quad D_P = m_t^2(s^2/e^4)(|C_{VP}|^2 + |C_{AP}|^2), \)
\( D_{SP} = m_t^2(s^2/e^4)(C^*_{VS}C_{VP} + C^*_{AS}C_{AP}), \)

\( F_1 = 2m_t(s^2/e^4)(C^*_{VV}C_{VP} + C^*_{AV}C_{AP}), \)
\( F_2 = 2m_t(s^2/e^4)(C^*_{VV}C_{AP} + C^*_{AV}C_{VP}), \)
\[ F_3 = 2m_t(s^2/e^4)(C_{VA}^* C_{VP} + C_{AA}^* C_{AP}), \]
\[ F_4 = 2m_t(s^2/e^4)(C_{VA}^* C_{AP} + C_{AA}^* C_{VP}), \]
\[ G_1 = 2m_t(s^2/e^4)(C_{VV}^* C_{VS} + C_{AV}^* C_{AS}), \]
\[ G_2 = 2m_t(s^2/e^4)(C_{VV}^* C_{AS} + C_{AV}^* C_{VS}), \]
\[ G_3 = 2m_t(s^2/e^4)(C_{VA}^* C_{VS} + C_{AA}^* C_{AS}), \]
\[ G_4 = 2m_t(s^2/e^4)(C_{VA}^* C_{AS} + C_{AA}^* C_{VS}). \]

\( D_{S,P,SP} \) are coefficients which are usually neglected and the other coefficients are defined the same way as in ref. [6].

The asymmetry \( \delta \) can be calculated by using the above differential cross section:
\[ \delta = \frac{-2\beta \text{Re}[F_1 - \beta^2(s/m_t^2)D_{SP}]}{(3 - \beta^2)V + 2\beta^2A - 2\beta^2\text{Re}(G_1) + \beta^2(s/m_t^2)(\beta^2S + D_P)}. \]  

If we keep only \( V, A \) and \( F_1 \) terms, this \( \delta \) agrees with the one calculated in [3]–[5]. We find that the term including \( D_{SP} \) in the numerator, which consists of \( \delta C_v \) and \( \delta D_v \), is proportional to \( s \). This means there is a possibility that \( D_{SP} \) contributes greatly to \( CP \) violation at very high energy, depending on the size of \( \delta C_v \) and \( \delta D_v \).

Let us show the difference between our calculations and usual calculations visually in Fig.1 and Fig.2, where we use as input data \( m_t = 173.9 \text{ GeV} \) [4], \( M_Z = 91.1867 \text{ GeV} \) [8] and \( \sin^2\theta_W = 0.2315 \) [8], and assume as examples all the real and imaginary parts of \( \delta A_v, \cdots, \delta D_v \) in eq.(2) to be 0.01 in Fig.1 and 0.1 in Fig.2. These figures show there is no difference around \( \sqrt{s} = 500 \text{ GeV} \), but our \( \delta \) decreases rapidly for higher \( \sqrt{s} \).

Our calculations here are exact as long as we can treat the anomalous couplings as constant parameters. Some comments are necessary, however, on this point before going to summary. If these couplings come from some new physics at a higher-energy scale \( \Lambda \), then our results are applicable only for \( \sqrt{s} < \Lambda \). One plausible way to estimate this \( \Lambda \) will be given by the effective-lagrangian approach [9]. According to it, \( \delta C_v \) and \( \delta D_v \) are both \( O(m_t^2/g\Lambda^2) \) [10], which leads to \( \Lambda \sim \)
Figure 1: $CP$-violation asymmetry $\delta$ via our calculations (solid line) and the usual calculations (dotted line) assuming all the non-SM parameters to be 0.01.

Figure 2: $CP$-violation asymmetry $\delta$ via our calculations (solid line) and the usual calculations (dotted line) assuming all the non-SM parameters to be 0.1.

$m_t/\sqrt{g}\delta_v (\delta_v = \delta C_v$ or $\delta D_v)$ and roughly 2.5 TeV (0.8 TeV) for $\delta_v = 0.01$ (0.1). Therefore our results, especially the one in Fig. 2 must be considerably affected if this approach describes the nature correctly, but in that case all the other usual calculations also lose their validity anyway.

To summary, we studied contribution of the products of non-standard parameters, which are usually neglected, to $CP$ violation in $e^+e^- \rightarrow t\bar{t}$ assuming the most general $t\bar{t}\gamma/Z$ couplings. We showed there is a possibility that usual approximate calculations may fail to give accurate results at very high $\sqrt{s}$. We considered top productions at NLC in this paper, but the same discussion holds also for those at hadron colliders [11] if we replace $e^+$ and $e^-$ with the light quarks (and add the gluon-fusion diagram). Finally, we cannot detect $t\bar{t}$ directly in actual experiments. If there are no or only tiny anomalous terms in $tbW$ couplings, it is easy to derive, e.g., the final-lepton-energy distributions [3, 5, 6]. In order to study new-physics effects consistently, however, we should study the decay process the same way as we did here for the production process. We would like to do it elsewhere.
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