Determination of trapped mode eigenfrequencies of an elastic plate with a strip-like delamination

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Abstract. Resonance scattering of Lamb waves by strip-like interface delamination is investigated. Trapped mode resonance frequencies of the plate with interface delamination are experimentally studied via the application of the Fourier transform to point-wise measurements along a line obtained using laser Doppler vibrometry. Eigenfrequencies are also calculated employing semi-analytical and mesh-based computational methods. A good agreement between experimentally extracted and numerically predicted trapped mode eigenfrequencies is demonstrated.

1. Introduction
Since composite structures are widely employed nowadays, effective methods for damage detection in them are necessary [1, 2]. Some techniques used in non-destructive testing and structural health monitoring are based on elastic guided waves because their reflection allows detecting hidden defects [3, 4, 5]. Therefore, damage detection methods can be improved by employing information on the scattering of guided waves by various obstacle types.

It has been shown in [6] that Lamb wave scattering by delamination-like obstacles when featured by resonance interaction can be used for damage identification. Resonance scattering by crack-like defect manifests itself in a localized wave motion at the defect's vicinity at its eigenfrequencies [7, 8, 9, 10]. This phenomenon is also known as trapped modes [11], scattering resonances [12] or local defect resonances [10, 13, 14]. The focus of the current study, considered as the further development of the ideas from ref. [6], is on the determination of trapped mode resonance frequencies of an interface strip-like delamination in an elastic laminate fabricated from different materials.

In this paper, delamination resonance scattering phenomena are investigated experimentally and numerically. For this purpose, several specimens made from aluminium and glass with two-sided epoxy tape as a couplant and introduced strip-like artificial delaminations are considered. The spectral analysis is applied to experimental transient B-scans and point-wise measurements obtained with scanning laser Doppler vibrometry (LDV) for resonance frequency extraction from the data (so called, S-scans). Some postprocessing relying on moving windowed Fourier
transform and averaging is proposed to increase the reliability of the proposed evaluation procedure. Developed semi-analytical and mesh-based computational models are used to compute eigenfrequencies and eigenforms. A good agreement between experimentally extracted and numerically predicted trapped mode eigenfrequencies is demonstrated.

2. Experimental setup

![Figure 1. Geometry of the problem.](image)

In order to investigate resonance scattering by a strip-like delamination several specimens were manufactured composing two plates of thicknesses \( h_1 \) and \( h_2 \) and of equal in-plane dimensions \( 600 \times 150 \text{ mm}^2 \). The plates were glued together using a two-sided epoxy tape of \( 0.05 \text{ mm} \) thickness, whereas a delamination was introduced as its absence in the rectangular area of \( l = 15 \text{ mm} \) width. The geometry of the specimen is shown in figure 1.

Wave motion in the sample is excited by a rectangular piezoelectric transducer \((10 \times 30 \times 0.25 \text{ mm}^3)\), which is glued on the surface of the plate parallel to its edges. The distance between two closest edges of the piezoelectric transducer and the delamination is \( 150 \text{ mm} \). The actuator is driven with a transient voltage \( p(t) \) applied at its electroded surface. Two pulse configurations \( p(t) \) are employed, i.e., in the form of either a broad-band rectangular pulse of short duration or a narrow-band Hann-windowed five-cycle sine burst, which spectrum is strongly concentrated near its central frequency \( f_0 \). The latter are generated by a Tektronix AFG 3022B arbitrary signal generator and are pre-amplified to the range \( 70 \text{ V-pp} \) by a NF HSA4101 external high-frequency power amplifier before being applied to the PWAS. The out-of-plane velocity field of propagating GWs is measured on the plate surfaces by means of a Polytec PSV-500 one-dimensional scanning LDV.

3. The boundary value problem

The time-harmonic motion with the angular frequency \( \omega = 2\pi f \) of each layer \( j = 1,3 \) in a composite laminate plate \(-H \leq x_2 \leq 0\) is governed by the Lamé equations

\[
(\lambda^{(j)} + 2\mu^{(j)})\nabla \left( \nabla \cdot \mathbf{u}^{(j)} \right) - \mu^{(j)}\nabla \times \left( \nabla \times \mathbf{u}^{(j)} \right) + \rho^{(j)}\omega^2 \mathbf{u}^{(j)} = 0,
\]

which can be written in terms of the displacement vector \( \mathbf{u}^{(j)}(\mathbf{x}) \), Lamé elastic constants \( \lambda^{(j)} \), \( \mu^{(j)} \) and mass density \( \rho^{(j)} \). The surfaces of the plate \( x_2 = -H \) and \( x_2 = 0 \) as well as the surfaces
of the delamination $\Omega$ are assumed stress-free
\[
\sigma_{j2}(x_1, x_2, 0) = \sigma_{j2}(x_1, x_2, -H) = 0
\]
extcept the domain of the contact between the plate and the piezoelectric transducer. Here $\sigma_{ij}$ is the stress tensor related with displacements $u_i$ by the Hooke’s law. The material properties of elastic materials considered in the computations are given in table 1.

| Material   | Poisson’s ratio $\nu$ | Shear modulus $\mu$ [GPa] | Young’s modulus $E$ [GPa] | $\lambda$ | Density $\rho$ [kg·m$^{-3}$] | SV-wave $c_T$ [m·s$^{-1}$] | P-wave $c_L$ [m·s$^{-1}$] |
|------------|-----------------------|---------------------------|--------------------------|---------|-----------------------------|--------------------------|--------------------------|
| Aluminium  | 0.33                  | 26.3                      | 70                       | 50.5    | 2700                        | 3098.15                  | 6816.15                  |
| Two-sided tape | 0.43                 | 0.227273                  | 0.65                     | 1.3961 | 930                         | 494.348                  | 1801.87                  |
| Glass      | 0.25                  | 27.66                     | 69.15                    | 27.66   | 2770                        | 3159.99                  | 5473.27                  |

To determine eigenfrequencies of the boundary-value problem, two methods have been used. The boundary integral equation method [15, 16] relies on integral representations for the wave-field scattered by the crack in the form of the convolution of the corresponding Green matrices with unknown crack opening displacement $\Delta u$. In this case, the complex-valued resonance poles in the frequency plane or eigenfrequencies $f_n$ are spectral points of the integral operator $A$ describing stresses induced by the delamination. Numerically, they are approximated by the zeros of the determinant of the linear algebraic system to which the boundary integral equation is reduced via Galerkin method.

The finite element method (implemented with COMSOL Multiphysics 5.5) can be also applied employing the perfectly matched layers (PML) to simulate an open or unbounded waveguide. The comparison of the results computed using both methods shows a good agreement.

4. Resonance frequencies extraction from experimental data
To extract the values of trapped mode eigenfrequencies from a line scan made by laser vibrometer, the Laplace transform can be applied to the part of the signal measured on the plate surface. It is preferable to excite the piezoactuator with a broad-band voltage $p(t)$, e.g., in the form of 0.5 $\mu$s rectangular pulse, see [6] for more details. However, the direct application of the Laplace transform does not reveal localization phenomena. Therefore, the spectral analysis of the “tail” of the transient signal measured at the damage, i.e.

\[
\mathcal{L}[v](x_1, f) = \int_{t_1}^{t_2} v(t)e^{i2\pi ft}dt, \tag{1}
\]

should be performed. An example of the application of (1) to the out-of-plane velocities $v(x_1, t)$ measured by the LDV along the line $x_2 = 0$ for the specimen composed of 2 mm Aluminium and 3 mm Glass plates is demonstrated in figure 2 (a). Two high peaks near 80 kHz are clearly visible on the surface $|\mathcal{L}[v](x_1, f)|$. This allows to determine the second eigenfrequency, whereas the third and fourth eigenfrequencies cannot be determined in such a straightforward manner. To meet this problem, an averaging technique is proposed. This approach takes into account that information about different eigenfrequencies are distributed in different parts of the signal.
Figure 2. The absolute value of $|L[v](x_1, f)|$ for different number of averages: $M = 1$ (a), $M = 5$ (b), $M = 9$ (c). From the B-scan at the surface of aluminium of the specimen 2 mm Aluminium — 3 mm Glass; $a = 0.64\mu$s.

Therefore, (1) is substituted by

$$L[v](x_1, f) = \sum_{k=1}^{M} t_2 + ka \int_{t_1 + ka}^{t_2 + ka} v(x_1, t)e^{i2\pi ft} dt,$$

(2)

where $a$ is the shift of the windows in time, $M$ is the number of averages. The improvement of the procedure of the determination of peaks corresponding to eigenfrequencies of trapped modes via the application of (2) is shown in figure 2 (b), (c).

5. Numerical analysis

Figure 3 demonstrates surfaces results of the application of (2) to B-scans measured at both surfaces of two specimens: 2 mm Aluminium / 3 mm Glass (figure 3 (a), (b)) and 3 mm Aluminium / 2 mm Glass (figure 3 (c), (d)). One can see periodically situated peaks localized at the same frequencies. Centers of narrow frequency ranges, where wave motion localization is clearly visible, can be identified as resonance frequencies. It should be noted that resonance frequencies are more accurately identified at the glass-side of the specimen surface. The results of the determination of resonance frequencies $f_n$ numerically and experimentally are summarized in table 2.

To demonstrate that trapped mode phenomena can be clearly seen at the determined frequencies a B-scan has been made at the glass surface of the specimen 2 mm Aluminium / 3 mm Glass, see figure 4. In the measurement, Hann-windowed five-cycle sine burst with the central frequency $f_0 = f_2 = 82$ kHz has been used for actuator excitation. One can see a strong localization above the delamination, which becomes weaker if the central frequency is not equal to one of eigenfrequencies.
Figure 3. The absolute value of \(|L[v](x_1, f)|\) obtained from B-scans at the surface of aluminium ((a), (c)) and glass ((b), (d)) of the specimen 2 mm Aluminium / 3 mm Glass ((a), (b)) and 3 mm Aluminium / 2 mm Glass ((c), (d)); \(a = 0.64 \mu s, M = 9\).

| Eigenfrequency | 2 mm Aluminium / 3 mm Glass | 2 mm Aluminium / 3 mm Glass |
|----------------|-----------------------------|-----------------------------|
|                | Aluminium side | Glass side | Aluminium side | Glass side |
| \(f_1\)        | Experiment    | 45.6       | 31            | 30            |
|                | Simulation    |            |               |               |
| \(f_2\)        | Experiment    | 80         | 82.5          | 76            | 76            |
|                | Simulation    | 78         | 147           | 140           | 140           |
| \(f_3\)        | Experiment    | 145        | 150           | 150           |
|                | Simulation    | 217        | 217           | 217           | 219           |
| \(f_4\)        | Experiment    | 240        | 295           | 279.5         |
|                | Simulation    |            |               |               |
| \(f_5\)        | Experiment    | 292        | 283           |
|                | Simulation    |            |               |

Table 2. Eigenfrequencies for two specimens considered.

6. Conclusions
Resonance scattering of Lamb waves by strip-like interface delamination is investigated. Trapped mode resonance frequencies of the plate with interface delamination are experimentally studied via the application of the Fourier transform to point-wise measurements along a line obtained using LDV. Proposed data postprocessing increases the reliability of the experimental results. Eigenfrequencies and eigenforms are also calculated employing semi-analytical and mesh-based computational methods. A good agreement between experimentally extracted and numerically predicted trapped mode eigenfrequencies is demonstrated. Since these frequencies strongly depend on the obstacle geometry and location within the waveguide thickness, this technique could have a potential for damage identification in a combination with other methods.
Figure 4. The absolute value of $|L[v](x_1, f)|$ obtained from B-scans at the surface of aluminium ((a), (c)) and glass ((b), (d)) of the specimen 2 mm Aluminium / 3 mm Glass ((a), (b)) and 3 mm Aluminium / 2 mm Glass ((c), (d)); $a = 0.64 \mu s$, $M = 9$.

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