A Hierarchical Model Predictive Tracking Control for Independent Four-Wheel Driving/Steering Vehicles with Coaxial Steering Mechanism

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Abstract. In this study, we apply a hierarchical model predictive control to omni-directional mobile vehicle, and improve the tracking performance. We deal with an independent four-wheel driving/steering vehicle (IFWDS) equipped with four coaxial steering mechanisms (CSM). The coaxial steering mechanism is a special one composed of two steering joints on the same axis. In our previous study with respect to IFWDS with ideal steering, we proposed a model predictive tracking control. However, this method did not consider constraints of the coaxial steering mechanism which causes delay of steering. We also proposed a model predictive steering control considering constraints of this mechanism. In this study, we propose a hierarchical system combining above two control methods for IFWDS. An upper controller, which deals with vehicle kinematics, runs a model predictive tracking control, and a lower controller, which considers constraints of coaxial steering mechanism, runs a model predictive steering control which tracks the predicted steering angle optimized an upper controller. We verify the superiority of this method by comparing this method with the previous method.

1. Introduction
Omni-directional mobile robots are useful for using in warehouse of factory or disaster site etc. since these robots can move in any direction quickly. Various kinds of steering mechanisms and wheels for omni-directional motion have been studied since each of them requires different control method. In previous studies using mecanum wheel, automatic operation method [2] and development of vibration suppression mechanism [3] were proposed. In addition, there are a lot of research with respect to wheel mechanisms of caster shape, where there are a development of high load bearing mechanisms [4] [5], adaptation for various environments [6], tracking trajectory control [7], balance control [8], modification for shape of mechanism [9], etc. In addition, there are methods using model predictive control implemented in such robots. Tracking trajectory control for an omni-wheel robot [10], compensating a friction of robot in trajectory tracking [11], etc. were proposed.

In this study, we deal with the independent four-wheel driving and steering vehicle (IFWDS) depicted in Fig. 1. This vehicle is equipped with coaxial steering mechanisms (CSM) composed of two steering joints on one axis. Figure 2 depicts CSM. These two joints have different features which are range of movement and rotational speed.
By combining these two joints, IFWDS achieves quick steering using specification of each joint. In our previous study, we proposed the model predictive tracking control considering steering range and tracking performance [12]. However, this method did not consider CSM. This vehicle includes a controller for CSM which has a first-order lag filter. It causes the delay of steering when the steering angle changes exponentially. Thus, the tracking performance of the vehicle is decreased when the vehicle maintaining its attitude moves on orthogonal trajectory, for example. We expect to prevent the decreasing of tracking performance by improving the delay of steering. In our other previous study, focusing just CSM, we proposed a model predictive steering control considering range of movement and rotational speed of two joints [13], and implemented this method for the vehicle [14] [15]. By these research of model predictive steering control, we achieve quick steering for CSM. We implemented the trajectory tracking control with model predictive steering control [16]. However, this tracking control predicts only a few predictive length interval.

To consider these controlled object simultaneously, we apply model predictive control for a system combining tracking control and steering control, where we expect that the two steering joints of CSM can perform preliminary action against future trajectory. However, the control cycle of controlled object should be set long since the computational effort increases depending on the number of dimension of system. In addition, the control frequency of steering control should be higher than that of tracking control. These problems may decrease tracking performance.

In this study, we propose a hierarchized model predictive control separating into tracking control and steering control. In the tracking trajectory control, the vehicle tracks the trajectory effectively based on long time prediction. In the steering control, CSM performs with preliminary motion based on predicted values given by tracking control. Therefore, we expect to achieve the improvement of tracking performance. By hierarchy, different band widths on each layer are considered so that the computational effort is decreased. In addition, the wheel parts can be modularized by hierarchy. In this paper, we conduct simulation using multi body dynamics engine to compare previous study [12] with proposed method, and verify the improvement of the tracking performance by proposed method.

2. Controlled system

2.1. Independent four-wheel driving/steering vehicle

The simple model of IFWDS is depicted in Fig. 3. The $X - Y$ coordinate system is fixed in field, and the $x - y$ coordinate system is fixed in center of gravity (CoG) of the vehicle. In the
Figure 3. The simple model of IFWDS.

Figure 4. The movement of coaxial steering mechanism (for example wheel 3).

$X - Y$ coordinate, $(X_g, Y_g, \theta)$ is defined as the position of CoG and attitude of the vehicle. In the $x - y$ coordinate, $(u_x, u_y, u_\theta)$ is defined as the velocity of CoG. The state space equation of this vehicle is represented by

$$\frac{d}{dt} \begin{bmatrix} X_g \\ Y_g \\ \theta \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_\theta \end{bmatrix}. \quad (1)$$

Each wheel velocity and reference steering angle coincide with vector field when wheels do not skid. The $x$ and $y$ elements of wheel velocity $v_{x,i}$ and $v_{y,i}$ are represented by

$$v_{x,i} = u_x - w_i u_\theta, \quad (2)$$

$$v_{y,i} = u_y + l_i u_\theta, \quad (3)$$

where $i$, $w_i$ and $l_i$ are wheel number and position of each wheel in $x - y$ coordinate system, respectively. Width and length of the vehicle are represented by $2w$ and $2l$. Let the reference steering angle and wheel velocity be defined as $\delta_{i,r}$, then $v_{x,i}$ and $v_{y,i}$ are represented by

$$v_{x,i} = V_i \cos \delta_{i,r}, \quad (4)$$

$$v_{y,i} = V_i \sin \delta_{i,r}. \quad (5)$$

In this way, $V_i$ and $\delta_{i,r}$ are calculated from these equations as operation values.

2.2. Coaxial steering mechanism

The coaxial steering mechanism (CSM) which is depicted in Fig. 2 is equipped with two steering joints on one axis. These joints have different rotation speed and range of movement. The red area is steering angle of Joint No.1 which decides the neutral direction of Joint No.2 whose steering angle is represented by this blue area. The rotation speed of Joint No.1 should be
where \( x \) is represented by \( x \).

Figure 4 is the top view of CSM, and this figure depicts the movement of CSM. \( \delta_i \) is the reference steering angle for CSM. \( \delta_{i,1} \) formed by vehicle attitude and red arrow direction is Joint No.1 steering angle. \( \delta_{i,2} \) formed by red arrow direction and blue arrow direction is Joint No.2 steering angle.

The sum of these steering angle \( \delta_{i,1} + \delta_{i,2} \) are the net steering angle of CSM which converges to reference steering angle \( \delta_{i,r} \). Then, Joint No.1 which can steer widely converges to reference steering angle \( \delta_{i,r} \). While Joint No.2 converges to 0 and prepares for next steering. Deﬁning the angular velocity of Joint No.1 and No.2 as \( \omega_{i,1} \) and \( \omega_{i,2} \), respectively, the state space equation of CSM is represented by

\[
\frac{dx_{s,i}}{dt} = u_{s,i},
\]

where \( x_{s,i} = [\delta_{i,1}, \delta_{i,2}]^T \), \( u_{s,i} = [\omega_{i,1}, \omega_{i,2}]^T \).

3. Conventional method

3.1. Model predictive tracking control [12]

Model predictive tracking control is a control method which predicts the vehicle behavior of the future based on state space equation and calculates an optimal input to track the trajectory.

From equations (2), (3), (4) and (5), the vehicle velocity of CoG using wheel velocity and steering angle is represented by

\[
\begin{align*}
u_x &= \frac{1}{2} (V_1 \cos \delta_1 + V_2 \cos \delta_2), \\
u_y &= \frac{1}{2} (V_1 \sin \delta_1 + V_3 \sin \delta_3), \\
u_\theta &= \frac{1}{8w} (V_2 \cos \delta_2 - V_1 \cos \delta_1 + V_4 \cos \delta_4 - V_3 \cos \delta_3) \\
&+ \frac{1}{8l} (V_1 \sin \delta_1 - V_3 \sin \delta_3 + V_2 \sin \delta_2 - V_4 \sin \delta_4),
\end{align*}
\]

where \( w_1 = w_3 = w, \ w_2 = w_4 = -w, \ l_1 = l_2 = l, \ l_3 = l_4 = -l \). The state space equation is represented by

\[
\dot{x}_t = f(x_t, u_t),
\]

where \( x_t = [X_g, Y_g, \theta, V^T, \delta^T]^T \), \( u_t = [\alpha^T, \omega^T]^T \) and

\[
f(x_t, u_t) = \begin{bmatrix}
(V_1 \cos \delta_1 + V_2 \cos \delta_2) \cos \theta - (V_1 \sin \delta_1 + V_3 \sin \delta_3) \sin \theta / 2 \\
(V_1 \cos \delta_1 + V_2 \cos \delta_2) \sin \theta + (V_1 \sin \delta_1 + V_3 \sin \delta_3) \cos \theta / 2 \\
(V_2 \cos \delta_2 - V_1 \cos \delta_1 + V_4 \cos \delta_4 - V_3 \cos \delta_3) / w \\
+ (V_1 \sin \delta_1 - V_3 \sin \delta_3 + V_2 \sin \delta_2 - V_4 \sin \delta_4) / l / 8 \\
\alpha \\
\omega
\end{bmatrix}.
\]

\( V \) and \( \delta \) is the vector which is summarized as the wheel velocity \( [V_1, V_2, V_3, V_4]^T \) and reference steering angle \( [\delta_1, \delta_2, \delta_3, \delta_4]^T \), respectively. The constraint with respect to state \( x_t \) is represented
by

\[
C(x_t) = \begin{bmatrix}
V_1 \cos \delta_1 - V_3 \cos \delta_3 \\
V_1 \sin \delta_1 - V_2 \sin \delta_2 \\
V_2 \cos \delta_2 - V_4 \cos \delta_4 \\
V_3 \sin \delta_3 - V_4 \sin \delta_4 \\
l(V_1 \cos \delta_1 - V_2 \cos \delta_2) + w(V_1 \sin \delta_1 - V_3 \sin \delta_3) \\
l(V_3 \cos \delta_3 - V_4 \cos \delta_4) + w(V_2 \sin \delta_2 - V_4 \sin \delta_4)
\end{bmatrix} = 0. \quad (12)
\]

When the steering range for each wheel is confined the maximum steering angle \( \delta \), the inequality constraints with respect to steering range of movement for each wheel is represented by

\[
g_i(x_t) = \delta_i^2 - \delta^2 \leq 0 \quad (i = 1, 2, 3, 4). \quad (13)
\]

Using these state equations and the constraints, IFWDS tracks a trajectory by model predictive control. The reference vector for state is defined as \( x_{t,r} = [X_{g,r}, Y_{g,r}, \theta_r, V_{1,r}, V_{2,r}, V_{3,r}, V_{4,r}, \delta_{1,r,U}, \delta_{2,r,U}, \delta_{3,r,U}, \delta_{4,r,U}]^T \). The index function is represented by

\[
J_t = \phi_t(x_t(t + T_{h,U}(t))) + \int_t^{t+T_{h,U}(t)} L_t(x_t(\tau), u_t(\tau), x_U(\tau))d\tau, \quad (14)
\]

where \( T_{h,U}(t) \) is the predictive interval in this method. \( \phi_t \) and \( L_t \) are the stage cost and terminal cost, respectively. These costs are represented by

\[
\phi_t(x_t(t), x_r(t)) = \frac{1}{2}(x_t(t) - x_r(t))^T S_f(x_t(t) - x_r(t)), \quad (15)
\]

\[
L_t(x_t(t), u_t(t), x_r(t)) = \frac{1}{2}(x_t(t) - x_r(t))^T Q(x_t(t) - x_r(t)) + \frac{1}{2} u_t(t)^T R u_t(t) + \frac{1}{2} C(x_t(t))^T W C(x_t(t)) + \frac{1}{\rho} \sum_{i=1}^{4} \log(-g_i(x_t(t)))), \quad (16)
\]

where \( S_f, Q, R \) and \( W \) are positive definite weight matrices, and \( \rho \) is a weight constant. To reduce the computational effort, the constraint \( C(x_t) \) is included in the index function. This stage cost made reduces the each element of it close to zero by setting the large weight matrix \( W \). The inequality constraints are included in the index function, and we consider these constraints by barrier function. In this study, we use Continuation/GMRES method [17] to implement model predictive control for these nonlinear state space equation.

This control method gives the reference steering angle. The steering control based on this is given in the following.

### 3.2. Frequency separated steering control [13]

We control CSM using given value \( \delta_i \) in tracking control as reference steering angle \( \delta_{i,r,L} \). In conventional steering control method, the reference steering angle is separated into two steering angles depend on frequency. CSM is controlled by using these two steering angles. The steering angle for Joint No.1 is the value filtered a low-pass filter (LPF) which is represented by

\[
\frac{d}{dt} \delta_{i,1} = -\frac{1}{T} \delta_{i,1} + \frac{1}{T} \delta_{i,r,L}, \quad (17)
\]
The steering angle $\delta$ is the angular velocity. In this study, the constraints for each steering joint of the CSM are set respectively. Moreover, this method can suppress the sudden movement of steering by evaluating the reference steering angle, and Joint No.1 and No.2 converge to reference steering angle and 0, with respect to steering angle. This index function is comprised of the error between reference the specification of each steering joint.

The index function in this control method is represented by

$$\delta_{i,2} = \delta_{i,r,L} - \delta_{i,1}. \quad (18)$$

For example, focusing on just wheel 1, Fig. 5 depicts the time transition of each steering joint angle when the reference steering angle is $\pi/2$. The red line, blue line and green line are Joint No.1 steering angle $\delta_{1,1}$, Joint No.2 steering angle $\delta_{1,2}$ and the net steering angles, respectively. The $\delta_{1,1}$ converge to reference steering angle $\delta_{1,r,L}$ gradually by LPF. $\delta_{1,2}$ increases quickly to the limit of the maximum steering angle by HPF. When the net steering angles arrives at the reference steering angle $\delta_{1,r,L}$, $\delta_{1,2}$ converge to zero according to increasing $\delta_{1,1}$. The time constant $T$ should be set large since Joint No.1 has large inertia moment. However, it causes the delay of steering. When the time constant $T$ is set small, $\delta_{1,1}$ increases exponentially. It causes the large load for Joint No.1.

3.3. Model predictive steering control [13]

The frequency separated steering control has the trade-off problem between the load and reaction rate. In our previous study, we proposed the model predictive steering control which considers the specification of each steering joint.

The index function in this control method is represented by

$$J_s = \phi_s(x_{s,i}(t + T_{h,L}), t + T_{h,L}) + \int_{t}^{t+T_{h,L}} L_s(x_{s,i}(\tau), u_{s,i}(\tau), \tau)d\tau, \quad (19)$$

where $T_{h,L}$ is the predictive interval in this method. $\phi_s$ and $L_s$ are the terminal cost and the stage cost, respectively. These costs are represented by

$$\phi_s(x_{s,i}(t), \delta_{i,r,L}(t), t) = \frac{1}{2} S_c(\delta_{i,1}(t) + \delta_{i,2}(t) - \delta_{i,r,L}(t))^2 + \frac{1}{2} S_1 \delta_{i,2}^2(t), \quad (20)$$

$$L_s(x_{s,i}(t), u_{s,i}(t), \delta_{i,r,L}(t), t) = \frac{1}{2} Q_c(\delta_{i,1}(t) + \delta_{i,2}(t) - \delta_{i,r,L}(t))^2 + \frac{1}{2} Q_1 \delta_{i,2}^2(t) + \frac{1}{2} R_0 \dot{\omega}_{i,1}^2(t) + \frac{1}{2} R_1 \dot{\omega}_{i,2}^2(t), \quad (21)$$

where $Q_c$, $Q_1$ and $R_0$, $R_1$ in the stage cost are the weight constants with respect to steering angle and angular velocity, respectively. $S_c$ and $S_1$ in the terminal cost are the weight constants with respect to steering angle. This index function is comprised of the error between reference and net steering angle, steering angle of Joint No.2 and angular velocity. The CSM can track the reference steering angle, and Joint No.1 and No.2 converge to reference steering angle and 0, respectively. Moreover, this method can suppress the sudden movement of steering by evaluating the angular velocity. In this study, the constraints for each steering joint of the CSM are set $|\dot{\omega}_{i,1}| \leq \pi/2 \text{rad/s}$, $|\dot{\omega}_{i,2}| \leq 1.84 \text{rad/s}$, $|\delta_{i,1}| \leq \pi/2 \text{rad}$ and $|\delta_{i,2}| \leq \pi/6 \text{rad}$.
4. Proposed method

4.1. Hierarchical model predictive tracking control

In our previous study [12], we implemented the model predictive tracking control and the frequency separated steering control and control separately. An advantage of separating these method is that we can set the different control cycle for each controlled object which has different bandwidth for control. When the control cycle are unified for these control objects, the tracking performance is deteriorated; the control frequency of steering control is higher than that of tracking control. For this reason, the method setting the different control cycle for each controlled object is reasonable.

To maintain this advantage, we propose the hierarchized model predictive control separated into tracking and steering control. This method uses the predicted steering angle calculated by model predictive tracking control as reference steering angle for model predictive steering control. We expect that delay of steering and tracking performance are improved by this method. In addition, the wheel parts can be modularized. The upper controller uses as a main body to control the vehicle and the lower controller is an exchangeable module depending on operating environment. In this way, we expect that general versatility of vehicle is enhanced.

4.2. Control flow

We depict the system block diagram of hierarchical model predictive tracking control in Fig. 6. This flow follows as:

(i) The upper controller computes the each wheel velocity $V_i$ and reference steering angle $\delta_i$ by model predictive tracking control, and obtains the predicted reference steering angle $\delta_i[k] (k = 0, 1, ..., N_U)$ for lower controller.

(ii) The predicted reference steering angle is converted by interpolated steering angle since the number of predictive step and predictive length is different between upper and lower controller.

(iii) The lower controller computes the steering angles $\delta_{i,1}$ and $\delta_{i,2}$ for each wheel by model predictive steering control.

(iv) The lower controller inputs the steering angle $\delta_{i,1}$, $\delta_{i,2}$ and wheel velocity $V_i$ to CSM and the upper controller computes again. (Go back to (i))

Where, $N_U$ and $N_L$ are the predictive steps and length of upper and lower controller, respectively.

4.3. Converting the predicted steering angle between upper and lower controller

When the lower controller calculates steering angle using reference steering angle calculated in upper controller, the difference of number of predictive steps and length have to be considered. Then, the process (ii) shown by Fig. 6 converts the reference steering angle. Figure 7 depicts the interpolation. First, the predicted steering angle calculated in upper controller is interpolated linearly (blue line). Next, the model predictive steering control refers the predicted reference steering angle on interpolated line. In this way, this system considers the difference predictive steps and length between upper and lower controller.

4.4. Motion of steering by prediction

CSM can perform preliminary action by using predicted steering angle calculated from upper controller. For example, Fig. 8 depicts the motion of CSM when IFWDS maintaining its attitude moves on orthogonal trajectory. This flow follows as:

(i) The vehicle predicts the next steering timing, and Joint No.1 steers to the direction of reference trajectory preliminarily and smoothly. Joint No.2 compensates for steering angle of Joint No.1.
(ii) Model Predictive Tracking Control

Calculation of reference for lower controller

\[ V[0] \quad \delta[k] \quad \delta[i][k] \quad V[i][0] \quad x_t, rt \]

(iii) Model Predictive Steering Control

\[ \delta_{i,r,L}[j] \quad (j = 0, 1, ..., N_L) \]

\[ \delta_{i,1}[0] \quad \delta_{i,2}[0] \]

(iv) Coaxial Steering Mechanism

**Figure 6.** The system block diagram of hierarchical model predictive tracking control.

**Figure 7.** Calculation of reference steering angle for lower controller.

**Figure 8.** The steering motion of coaxial steering mechanism using predicted value.

(ii) Joint No.2 steers quickly to the direction of reference trajectory using wide steering range as possible.

(iii) Joint No.2 converges to zero as increasing Joint No.1 steering angle when the net steering angle arrives at the reference direction.

This method reduces the load for Joint No.1 since it optimizes inputs considering constraint of angular velocity. Thus, the proposed controller can steer to the direction of reference trajectory quickly and widely and reduce the load by preliminary action of Joint No.1.
Table 1. The parameters of model predictive tracking control.

|  |  |
|---|---|
| $Q$ | diag(500, 500, 50, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01) |
| $R$ | diag(0.2, 0.2, 0.2, 0.1, 0.1, 0.1, 0.1) |
| $S_f$ | diag(500, 500, 50, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01) |
| $W$ | diag(500, 500, 500, 5000, 5000) |
| $\rho$ | 100 |

Table 2. The parameters of model predictive steering control.

|  |  |
|---|---|
| $Q_1$ | 400 |
| $Q_e$ | 10000 |
| $S_1$ | 400 |
| $S_e$ | 10000 |
| $R_1$ | 500 |
| $R_2$ | 5 |

5. Simulation
5.1. Simulation condition
We verified the tracking performance for IFWDS using Open Dynamics Engine which is multibody dynamics simulator. The size of IFWDS is 0.5 m square and the weight is 10.6 kg. Fig. 9 depicts the reference trajectory. The equation of the reference trajectory is follow as:

\[
U_{X,r} = V \\
U_{Y,r} = 0 \\
U_{\theta,r} = 0 \\
(0 \leq t < 5s)
\]

\[
U_{X,r} = 0 \\
U_{Y,r} = V \\
U_{\theta,r} = 0 \\
(5s \leq t < 10s)
\]

\[
U_{X,r} = V \\
U_{Y,r} = 0 \\
U_{\theta,r} = 0 \\
(10s \leq t < 15s)
\]

where the vehicle velocity $V$ is 0.25 m/s. In this trajectory, each wheel makes a similar movement since the vehicle does not change the attitude. Table 1 shows the parameters of model predictive tracking control. The parameters of Continuation/GMRES method [17] used in this control are $T_f = 2s$, $k_{\text{max}} = 10$, $\alpha = 1.5$, $\zeta = 55.6$, and number of predictive step $N_U$ is 10. The predictive length is $T_{h,U}(t) = T_f(1 - e^{-\alpha t})[s]$ and it converge from $T(0) = 0$ to $T_f$. Table 2 shows the parameters of model predictive steering control. To track the reference steering angle, a weight constants $Q_e$, $S_e$ with respect to error between reference and net steering angle were set the large values. In addition, to suppress the sudden movement of steering for Joint No.1, a weight constant $R_1$ with respect to $\omega_1$ is set the large value. The predictive length $T_{h,L}$ is 1s, where the number of predictive step $N_L$ is 20 and time increment $dt$ is 50 ms. We compare the proposed method with conventional method combined the model predictive tracking control and frequency separated steering control. The time constant $T$ used in frequency separated steering control is 1s.

5.2. Simulation result
5.2.1. Comparing with conventional method Figure 10 depicts the travel track at the each method. The blue and red lines mean the conventional and proposed method, respectively. In the conventional method, the vehicle overshoots the reference trajectory at the corners. On the other hand, the proposed method reduces the overshoot. Next, Fig.11 depicts the time transition of the tracking error at the each method. The proposed method is superior to the conventional method; the tracking error of X and Y direction in proposed method is smaller and converges faster than conventional method. Table 3 shows the maximum tracking error of each method in time range of 5s-10s and 10s-15s. The proposed method reduces the maximum tracking error by 58.4\% of the conventional method. Next, Fig. 12 depicts the time transition of steering angle for CSM. We show just the data of front-left wheel since each CSM of the vehicle makes a similar movement in this trajectory. In the conventional method, $\delta_{1,1}$ is slow to react for
Figure 9. The reference trajectory for simulation.

Figure 10. The vehicle trajectory of each method in simulation (Conv. vs Prop.).

Figure 11. The time transition of tracking error (Conv. vs Prop.).

Figure 12. The time transition of steering angle in wheel 1.

reference since it passed low-pass filter which is set a large time constant. To compensate this delay, \( \delta_{1,2} \) steer quickly. However, it does not have enough range to steer to reference steering angle by the constraint of movable range. Then, the vehicle decreases the tracking trajectory performance since the net steering angle does not track the reference steering angle. In addition, with respect to \( \delta_{1,r,L} \) at the around 5 s - 7 s and 10 s - 12 s, the upper controller requires to steer more widely for CSM since the vehicle seeks to correct the reference trajectory by position feedback. On the other hand, in the proposed method, \( \delta_{1,1} \) steers smoothly before reference steering direction; \( \delta_{1,2} \) begin to steer to opposite direction. Around 5 second, \( \delta_{1,2} \) begin to steer to reference steering direction quickly. Therefore, CSM tracks the reference steering angle and improve the trajectory tracking performance by proposed method.
Table 3. The maximum tracking error.

| Time range | Conv. method | Prop. method | Ideal steering vehicle |
|------------|--------------|--------------|------------------------|
| X direction |
| 5 s ≤ t < 10 s | 0.110 m | 0.0458 m | 0.0440 m |
| 10 s ≤ t < 15 s | −0.0356 m | −0.0251 m | −0.0229 m |
| Y direction |
| 5 s ≤ t < 10 s | −0.0373 m | −0.0257 m | −0.0231 m |
| 10 s ≤ t < 15 s | 0.106 m | 0.0450 m | 0.0435 m |

Figure 13. The vehicle trajectory of each method in simulation (ideal steering vehicle vs Prop.).

Figure 14. The time transition of tracking error (ideal steering vehicle vs Prop.).

5.2.2. **Comparing with ideal steering vehicle** We conduct a simulation using the same trajectory for ideal steering vehicle which has no constraint of movable range and rotational speed. This vehicle can steer to reference steering angle without delay of steering. Figure 13 and Figure 14 depict the tracking error of trajectory tracking between the ideal steering vehicle and the proposed method and the time transition of the tracking error, respectively. In Fig. 13, the ideal steering vehicle overshoots 44.0 mm. The weight of these model are so heavy that these overshoot at a corner by inertia force. The proposed method performs the similar action with the ideal steering vehicle. Table 3 shows the maximum tracking error of the ideal steering vehicle in time range of 5 s–10 s and 10 s–15 s. The difference between the proposed method and the
ideal steering vehicle is up to 2.6 mm. It is smaller than the overshoot. For these reason, the proposed method performs an enough capability of tracking trajectory.

6. Conclusion
In this study, we focused the independent four wheel driving/steering vehicle equipped with CSM. In our previous study, we proposed a model predictive tracking control for the vehicle. The steering control of this method includes a first-order lag filter. It causes a delay of steering when the steering angle changes exponentially. In our other previous study, focusing single CSM, we proposed the model predictive steering control considering range of movement and rotational speed for two joints. In this study, we proposed the hierarchical model predictive tracking control combining the model predictive tracking control and the model predictive steering control. We can consider the different bandwidth between tracking control and steering control by this method. In addition, the steering controller can utilize the predicted value which is optimized in the tracking control as target angle; CSM perform with preliminary motion based on prediction in the tracking control. A delay of steering is reduced and tracking performance is improved. We conducted the simulation using multi-body dynamics to compare a tracking performance of the conventional method with that of a proposed method and verified the availability of a proposed method. In this paper, we have not yet consider the different band-width explicitly. Our future work is to consider it.

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