Monopole Zeros

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Abstract

Recently the existence of certain SU(2) BPS monopoles with the symmetries of the Platonic solids has been proved. Numerical results in an earlier paper suggest that one of these new monopoles, the tetrahedral 3-monopole, has a remarkable new property, in that the number of zeros of the Higgs field is greater than the topological charge (number of monopoles). As a consequence, zeros of the Higgs field exist (called anti-zeros) around which the local winding number has opposite sign to that of the total winding. In this letter we investigate the presence of anti-zeros for the other Platonic monopoles. Other aspects of anti-zeros are also discussed.

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1 Introduction

SU(2) BPS monopoles are topological soliton solutions of a Yang-Mills-Higgs gauge theory in three space dimensions. They are Bogomolny solitons, in that they attain a topological lower bound on the total energy, and so can be obtained as solutions of a first order equation (the Bogomolny equation) rather than the more general second order field equations. In this letter we shall mainly be concerned with static monopoles. The ingredients of the static theory are the Higgs field $\Phi$, and the gauge potential $A_i$, $i = 1, 2, 3$, both of which are $su(2)$-valued. The static theory can be defined by its energy density

$$E = -\frac{1}{2} \text{tr}(D_i \Phi)(D_i \Phi) - \frac{1}{4} \text{tr}(F_{ij}F_{ij}) \quad (1.1)$$

where $D_i = \frac{\partial}{\partial x_i} + [A_i, \cdot]$ is the covariant derivative and $F_{ij}$ the gauge field. Integrating the energy density over all $\mathbb{R}^3$ gives the energy $E$ of any configuration.

The boundary condition

$$\|\Phi\| \to 1 \quad r \to \infty \quad (1.2)$$

where $r = |x|$, $\|\Phi\|^2 = -\frac{1}{2} \text{tr} \Phi^2$, is imposed and may be thought of as a residual finite energy condition derived from a vanished Higgs potential.

The topological aspect arises because the Higgs field at infinity induces a map between spheres:

$$\Phi : S^2(\infty) \to S^2(1) \quad (1.3)$$

where $S^2(\infty)$ is the two-sphere at spatial infinity and $S^2(1)$ is the two-sphere of vacuum configurations given by $\{\Phi \in su(2) : \|\Phi\| = 1\}$. The degree of this map is an integer $k$, the winding number, which (in suitable units) is the total magnetic charge of the monopole. We shall refer to a monopole with magnetic charge $k$ as a $k$-monopole.

The Bogomolny bound

$$E \geq 8\pi |k|$$

gives a lower bound on the total energy of a configuration in terms of its winding number $k$. For each $k$ this bound can be attained, with the relevant configuration being a solution of the first order Bogomolny equation

$$D_i \Phi = \pm \frac{1}{2} \epsilon_{ijk}F_{jk} \quad (1.4)$$

where the lower sign corresponds to positive $k$, with solutions called monopoles, and the upper sign corresponds to $k$ being negative, where solutions are called anti-monopoles. We now choose to consider monopoles i.e. $k > 0$, and so fix the sign in (1.4) to be the lower one.

By topological considerations the total number of zeros of the Higgs field counted with multiplicity is $k$ for a $k$-monopole. These zeros need not be distinct, for example, axially symmetric monopoles exist for all $k \geq 2$ \[18, 15\] for which there is a single zero that has multiplicity $k$. When the zeros are distinct, and well separated, the $k$-monopole solution
has a natural interpretation as $k$ well-separated unit charge monopoles, each one centered at a zero of the Higgs field. Such solutions exist because physically there are no static forces between equally charged monopoles \[13\]. The moduli space of charge $k$ monopoles is $4k$ dimensional \[3\], and this is again consistent with the well-separated picture, where each individual charge one monopole has three position degrees of freedom and one internal phase.

So the general picture appears to be that the Higgs field of a charge $k$ monopole has $k$ simple zeros, which may be thought of as the location of each monopole, and these zeros can coalesce as the monopoles merge. Although this picture has never been rigorously proved, it is widely accepted as true. Indeed there are at least three compelling reasons for believing the above. Firstly, all the known explicit monopole solutions do indeed have a Higgs field of this form. Secondly, in the analogous two dimensional case of abelian Higgs vortices at critical coupling, it has been proved that the total number of zeros of the Higgs field is bounded by the number of vortices \[10\]. Furthermore, in other models with topological solitons, such as the O(3) $\sigma$-model in the plane, the general static $k$-soliton solution can be given explicitly and a suitable field shown to have the above structure of zeros \[1\]. Finally, if the total number of Higgs zeros (ignoring multiplicities) were greater than $k$ then this would imply that zeros with negative multiplicity must exist, for the summed multiplicities to equal $k$. From now on we shall refer to a zero with a negative multiplicity as an anti-zero. An example of a configuration with an anti-zero is of course a single anti-monopole, which has $k = -1$. So it would seem highly unlikely that a monopole configuration with $k > 0$ could contain anti-zeros which were well-separated from other zeros, since we could interpret such a configuration as composed of monopoles and anti-monopoles. Since there are attractive forces between monopoles of opposite charge such a configuration could not saturate the Bogomolny energy bound.

Despite this wealth of circumstantial evidence, it was argued in a recent paper \[9\], with the aid numerical results, that a positive charge monopole solution exists which contains anti-zeros. In this letter we briefly review this result for the tetrahedral 3-monopole and then investigate the presence of anti-zeros for the remaining Platonic monopoles. Other aspects of anti-zeros will also be discussed, such as a signal for their occurrence and their relevance to skyrmions.

## 2 Zeros of Platonic Monopoles

Recently it has been shown that monopoles exist which have the symmetries of the Platonic solids \[4, 8\]. The actual monopole fields $\Phi, A_i$, were not calculated explicitly but rather a twistor approach was taken in which monopole solutions can be shown to be equivalent to certain algebraic objects, called spectral curves \[4\]. The spectral curves were explicitly found, from which the existence and symmetries of the monopoles follows. Using a numerical implementation of the twistor transform \[7\], the Higgs field and energy density of these monopoles can be computed. The results were displayed graphically in the form of a three-dimensional plot of a surface of constant energy density. For each of the four newly
discovered monopoles it was found that a surface of constant energy density resembled a Platonic solid. The results are summarized in Table 1, where we give the monopole charge $k$ and the Platonic solid it resembles. In each case the energy density takes its maximum values on the vertices of the relevant Platonic solid.

| Charge $k$ | Platonic solid |
|-----------|----------------|
| 3         | Tetrahedron    |
| 4         | Cube           |
| 5         | Octahedron     |
| 7         | Dodecahedron   |

Table 1. Charges of Platonic Monopoles

In these papers the Higgs field and its zeros were not studied, since if there are $k$ zeros then in each case the symmetry group acting implies that all $k$ zeros must be at the origin. For example, for the tetrahedral monopole $k = 3$ and if there are only three zeros then in order to arrange three points with tetrahedral symmetry all three points must be at the origin.

However, using the moduli space approximation [13, 16] the dynamics of $k$ slowly moving monopoles can be approximated by geodesic motion on the monopole moduli space $\mathcal{M}_k$. In [3] we presented a totally geodesic one dimensional submanifold of the 3-monopole moduli space, which contains on it the tetrahedral 3-monopole. This geodesic may therefore be interpreted in terms of the scattering of three monopoles which instantaneously form the tetrahedral 3-monopole. These results make it appear very unnatural (see [3] for more details) that there are three zeros at the origin and so we examined the Higgs field in more detail. Writing the Higgs field in terms of Pauli matrices as

$$\Phi = i\sigma_1 \varphi_1 + i\sigma_2 \varphi_2 + i\sigma_3 \varphi_3$$

(2.1)

we plotted the components $\varphi_1, \varphi_2, \varphi_3$ along the line $x_1 = x_2 = x_3 = L$, which goes through a vertex (at a negative value of $L$) and the center of a face (at a positive value of $L$) of the tetrahedron associated with the tetrahedral monopole. Fig 1. shows the results, and it is clear that along this line there are two points at which all the component of the Higgs field vanish. One point is the origin ($L = 0$) and the second occurs at a negative value of $L$, which indicates it is associated with a vertex of the tetrahedron, rather than a face. There are another three similar lines, going through the remaining vertices of the tetrahedron, and these were the only other lines along which Higgs zeros were found. So the result is that there are five Higgs zeros, one associated with each vertex of the tetrahedron and one at the origin. Since the monopole charge is three, then the zero at the origin must be an anti-zero. This can be checked numerically (see [3] for details of the scheme) by computing the winding number $Q(r)$, of the unit 3-vector

$$\psi = (\varphi_1, \varphi_2, \varphi_3) \frac{1}{\sqrt{\varphi_1^2 + \varphi_2^2 + \varphi_3^2}}$$

(2.2)
corresponding to the normalized Higgs field on a two-sphere of radius $r$, centred at the origin. Note that by definition $Q(R) = k$, if $R$ is sufficiently large, so that all zeros of the Higgs field are contained within the ball of radius $R$ centred at the origin. Such a calculation gives $Q(0.2) = -1$ and $Q(1.0) = +3$, confirming that there is indeed an anti-zero at the origin.

Having briefly reviewed the results for the tetrahedral monopole we now go on to investigate the Higgs zeros of the other Platonic monopoles.

We begin with the cubic 4-monopole. First of all we explicitly prove that the Higgs field of the cubic monopole is zero at the origin. It is useful to give the details of this calculation, since it demonstrates the kind of work required to prove the results which the numerical evidence suggests.

Recall the ADHMN construction \[14, 6\] which is an equivalence between $k$-monopoles and Nahm data $(T_1, T_2, T_3)$, which are three $k \times k$ matrices which depend on a real parameter $s \in [0, 2]$ and satisfy the following:

(i) Nahm’s equation
\[
\frac{dT_i}{ds} = \frac{1}{2} \epsilon_{ijk}[T_j, T_k] \tag{2.3}
\]

(ii) $T_i(s)$ is regular for $s \in (0, 2)$ and has simple poles at $s = 0$ and $s = 2$,

(iii) the matrix residues of $(T_1, T_2, T_3)$ at each pole form the irreducible $k$-dimensional representation of SU(2),

(iv) $T_i(s) = -T_i^\dagger(s)$,

(v) $T_i(s) = T_i^\dagger(2 - s)$.

Finding the Nahm data effectively solves the nonlinear part of the monopole construction and is enough to prove existence of the monopole and compute its spectral curve. In fact this is how the spectral curves of the Platonic monopoles were calculated. However in order to calculate the Higgs field the linear part of the ADHMN construction must also be implemented. Given Nahm data $(T_1, T_2, T_3)$ for a $k$-monopole we must solve the ordinary differential equation
\[
(1_{2k} \frac{d}{ds} + 1_k \otimes x_j \sigma_j + iT_j \otimes \sigma_j) \mathbf{v} = 0 \tag{2.4}
\]
for the complex $2k$-vector $\mathbf{v}(s)$, where $1_k$ denotes the $k \times k$ identity matrix, $\sigma_j$ are the Pauli matrices and $\mathbf{x} = (x_1, x_2, x_3)$ is the point in space at which the Higgs field is to be calculated. Introducing the inner product
\[
\langle \mathbf{v}_1, \mathbf{v}_2 \rangle = \int_0^2 \mathbf{v}_1^\dagger \mathbf{v}_2 \, ds \tag{2.5}
\]
then the solutions of \((2.4)\) which we require are those which are normalizable with respect to \((2.3)\). It can be shown that the space of normalizable solutions to \((2.4)\) has (complex) dimension 2. If \(\vec{v}_1, \vec{v}_2\) is an orthonormal basis for this space then the Higgs field \(\Phi\) is given by

\[
\Phi = i \left[ \frac{\langle (s-1)\vec{v}_1, \vec{v}_1 \rangle - \langle (s-1)\vec{v}_1, \vec{v}_2 \rangle}{\langle (s-1)\vec{v}_2, \vec{v}_1 \rangle} \right].
\] (2.6)

For the cubic monopole \(k = 4\) and the Nahm data is explicitly known \(\mathbb{I}\). Writing \(v = (v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8)^t\) then \((2.4)\) becomes the set of equations

\[
\begin{align*}
\dot{v}_1 + x_3 v_1 + (x_1 + i x_2) v_2 + (4y + 3x) v_1 + 20y v_8 &= 0 \\
\dot{v}_2 - x_3 v_2 + (x_1 - i x_2) v_1 - (4y + 3x) v_2 + 2\sqrt{3}(-2y + x) v_3 &= 0 \\
\dot{v}_3 + x_3 v_3 + (x_1 + i x_2) v_4 + 2\sqrt{3}(-2y + x) v_2 + (-12y + x) v_3 &= 0 \\
\dot{v}_4 - x_3 v_4 + (x_1 - i x_2) v_3 + (12y - x) v_4 + 4(3y + x) v_5 &= 0 \\
\dot{v}_5 + x_3 v_5 + (x_1 + i x_2) v_6 + 4(3y + x) v_4 + (12y - x) v_5 &= 0 \\
\dot{v}_6 - x_3 v_6 + (x_1 - i x_2) v_5 + (-12y + x) v_6 + 2\sqrt{3}(-2y + x) v_7 &= 0 \\
\dot{v}_7 + x_3 v_7 + (x_1 + i x_2) v_8 + 2\sqrt{3}(-2y + x) v_6 - (4y + 3x) v_7 &= 0 \\
\dot{v}_8 - x_3 v_8 + (x_1 - i x_2) v_7 + 20y v_1 + (4y + 3x) v_8 &= 0.
\end{align*}
\] (2.7)

where dot denotes differentiation with respect to \(s\)

\[
\begin{align*}
x(s) &= \frac{\kappa}{5} \left( -2\sqrt{\wp(\kappa s)} + \frac{1}{4} \frac{\wp'(\kappa s)}{\wp(\kappa s)} \right) \\
y(s) &= \frac{\kappa}{20} \left( \sqrt{\wp(\kappa s)} + \frac{1}{2} \frac{\wp'(\kappa s)}{\wp(\kappa s)} \right)
\end{align*}
\] (2.8) (2.9)

with \(\kappa\) a known constant and \(\wp\) the elliptic function satisfying

\[
\wp'^2 = 4\wp^3 - 4\wp.
\] (2.10)

Here prime denotes differentiation with respect to the argument. The constant \(\kappa = \Gamma(1/4)^2/\sqrt{8\pi}\) is such that the real period of the elliptic function is \(2\kappa\).

We wish to calculate the Higgs field at the origin so we set \(x_1 = x_2 = x_3 = 0\). Then the first and last equations decouple from the rest, so we may look for a solution with \(v_2 = v_3 = v_4 = v_5 = v_6 = v_7 = 0\) to give

\[
\begin{align*}
\dot{v}_1 + (4y + 3x) v_1 + 20y v_8 &= 0 \\
\dot{v}_8 + (4y + 3x) v_8 + 20y v_1 &= 0.
\end{align*}
\] (2.11)

The symmetry of this system allows the reduction \(v_8 = v_1\) which brings us to the single equation

\[
\dot{v}_1 + (24y + 3x) v_1 = 0.
\] (2.12)
Now
\[ 24y + 3x = \frac{3\kappa}{4} \frac{\phi'}{\phi} = \frac{3}{4} \frac{\phi'}{\phi} \quad (2.13) \]
so the equation is
\[ \dot{v}_1 + \frac{3}{4} \frac{\phi'}{\phi} v_1 = 0 \quad (2.14) \]
with solution
\[ v_1 = A \phi^{-3/4} \quad (2.15) \]
where \( A \) is a constant. The properties of the elliptic function \( \phi \) are such that \( v_1 \) is finite for \( s \in [0, 2] \). Hence we have our first unit norm solution
\[ \hat{v}_1 = B^{-1} \phi^{-3/4} (1, 0, 0, 0, 0, 0, 0, 1)^t \quad (2.16) \]
where \( B \) is the constant
\[ B^2 = 2 \int_0^2 \phi^{-3/2} \, ds. \quad (2.17) \]
In a similar way the fourth and fifth equations in (2.4) decouple to give
\[ \hat{v}_2 = B^{-1} \phi^{-3/4} (0, 0, 0, 1, 1, 0, 0, 0)^t. \quad (2.18) \]
Substituting these solutions into (2.6) we have that
\[ \Phi = i2B^{-2} \phi^{3/2} \int_0^2 (s - 1) \phi^{-3/2} \, ds. \quad (2.19) \]
However, \( \phi \) is symmetric on the real line about its half period, so the integrand in (2.19) is antisymmetric about \( s = 1 \), and hence \( \Phi = 0 \). So finally, we have proved that the Higgs field of the cubic monopole has a zero at the origin. To find the Higgs field at points other than the origin requires a similar calculation, but it is more involved, since for a general point the set of equations will no longer decouple in a simple way. This is why it is a difficult task to prove that the tetrahedral 3-monopole has five zeros, and instead we rely on numerical results.

Essentially the numerical scheme [7] solves the linear differential system (2.4), extracts an orthonormal basis for the normalizable solutions and performs the required integrations to obtain the Higgs field.

Returning to a numerical investigation of the cubic monopole we plot, in Fig 2. (the solid curve), the norm squared of the Higgs field \( \|\Phi\|^2 \) along the line \( x_1 = x_2 = x_3 = L \). This line goes through two vertices of the cube associated with the cubic monopole. It is not easy to see exactly where this function is zero, so we also plot in Fig 3. the component \( \phi_2 \) (solid curve). From this plot it seems relatively clear that the cubic monopole has a zero of the Higgs field only at the origin, and not at points associated with the vertices of a cube. Calculation of the Higgs field along other lines, for example through the centre of a face, did not reveal any other zeros. So it would seem that the cubic monopole is not like the tetrahedral monopole, and does not possess anti-zeros. Supporting evidence comes
from a calculation of the winding number around the origin, which gives $Q(0.1) = +4$, in agreement with the cubic monopole having just four zeros, which are all located at the origin.

Having demonstrated that the tetrahedral monopole appears to have an anti-zero, it may now seem surprising that the cubic monopole has no anti-zeros. However, there are several reasons why we might expect the cubic monopole to be unlike the tetrahedral monopole. The first is obvious, in that if the cubic monopole had a zero at each vertex then, since it has charge four, there would have to be four anti-zeros at the origin i.e. an anti-zero with local winding -4. This requirement of multiple anti-zeros is not impossible, but it does seem a little contrived.

A second reason comes from the study of monopole scattering, which can be addressed using the moduli space approximation, as mentioned earlier. There is a totally geodesic one dimensional submanifold of $\mathcal{M}_3$ which contains the tetrahedral monopole [9]. The Nahm data associated with this submanifold involves the family of elliptic curves

$$y^2 = 4x^3 - 3(a^2 - 4)^{2/3}x - 4$$  \hspace{1cm} (2.20)

where $a \in \mathbb{R}$ is a parameter, such that $a = \pm 2$ gives the tetrahedral monopole. Now there are points on the geodesic which correspond to three well-separated unit charge monopoles, so we know that at such points the corresponding monopole configuration can have no anti-zeros. Since the tetrahedral monopole has anti-zeros there must be special ‘splitting points’ along the geodesic at which anti-zeros appear/disappear. The discriminant of the elliptic curve (2.20) is

$$\Delta = 27a^2(a^2 - 8)$$  \hspace{1cm} (2.21)

which vanishes at the three points $a = 0, \pm \sqrt{8}$. The numerical results are consistent with the conjecture that the splitting points occur at these three parameter values at which the elliptic curve is singular.

There is a geodesic in $\mathcal{M}_4$ which contains the cubic monopole [7, 17] and is associated with the family of elliptic curves

$$y^2 = 4x^3 - 4x + 12a^2$$  \hspace{1cm} (2.22)

where $a \in (-3^{5/4}\sqrt{2}, +3^{5/4}\sqrt{2})$. The discriminant of this elliptic curve is

$$\Delta = 16(4 - 3^5a^4)$$  \hspace{1cm} (2.23)

which never vanishes for $a$ in its allowed range. Hence, if the above conjecture is correct, then there are no splitting points along this geodesic. Since the geodesic contains points which correspond to four well-separated unit charge monopoles, then this implies that the cubic monopole has no anti-zeros. In Fig 2. and Fig 3. we plot $\|\Phi\|^2$ and $\varphi_2$ (dashed lines) along the line $x_1 = x_2 = x_3 = L$ for two monopole configurations corresponding to two other points on the geodesic in $\mathcal{M}_4$. From these (and similar) plots it can be seen that in each case there is only one zero along this line, which can be tracked as it moves in from
infinity, through the origin and back out to infinity in the opposite direction along the line. There is no signature of any splitting of zeros taking place.

A final indication that the tetrahedral and cubic monopoles have a different structure in their zeros comes from an analogy with another kind of topological soliton, the skyrmion. Numerical evidence suggests [2] that the minimal energy 3-skyrmion and the minimal energy 4-skyrmion resemble a tetrahedron and a cube respectively, so there is some similarity between monopoles and skyrmions. Using instanton generated skyrmions [11] these kinds of configurations were investigated in more detail, and it was found that the tetrahedral skyrmion has regions in which the baryon density is negative, but no such regions were found for the cubic skyrmion. For skyrmions the baryon density is the quantity which when integrated over all $\mathbb{R}^3$ gives the number of skyrmions i.e. it is the topological charge density. For an anti-skyrmion the baryon density is negative, so for a skyrmion configuration with positive topological charge to have a region in which the baryon density is negative is analogous in the monopole context to a region containing an anti-zero. Hence, if the tetrahedral monopole contains anti-zeros, but the cubic monopole does not then this is yet again another parallel between monopoles and skyrmions.

Having looked for anti-zeros in the tetrahedral and cubic monopoles and finding apparently different answers for each, it is by no means clear what the situation will be for the other two Platonic monopoles. Note that for the octahedron, since the charge is five, a zero at each vertex would imply that only a single anti-zero is required at the origin. Thus in this respect the octahedral monopole is like the tetrahedral monopole rather than the cubic monopole, and is a candidate for anti-zeros. Fig 4. shows a plot of $\|\Phi\|^2$ for the octahedral monopole, along the line $x_1 = x_2 = 0, x_3 = L$, which passes through two vertices of the associated octahedron. This clearly suggests that there are three zeros along this line. There are two other similar lines, so we find that the octahedral monopole has a zero on each of the six vertices of the octahedron and an anti-zero at the origin. This conclusion is supported by a winding number calculation which gives $Q(3.0) = +5$ and $Q(0.1) = -1$.

Numerical results for the dodecahedral 7-monopole are not as conclusive as for the other three Platonic monopoles, but seem to suggest that it is like the cubic 4-monopole in not possessing anti-zeros. This would seem the most acceptable result, since if the dodecahedral monopole had anti-zeros in the same manner as the tetrahedral and octahedral monopole then this would require multiple anti-zeros (in fact thirteen) at the origin.

It would clearly be desirable to test the conjecture, relating splitting points to singular elliptic curves, with other examples. In particular it would be instructive if Nahm data could be found which corresponds to a geodesic in $\mathcal{M}_5$ that includes the octahedral monopole. The conjecture implies that the associated family of elliptic curves should contain singular curves. Two appropriate one-dimensional totally geodesic submanifolds of $\mathcal{M}_5$ are known [8, 9], but unfortunately the computation of the associated Nahm data appears not to be a tractable problem. However, a more suitable candidate does appear to exist and is obtained by imposing tetrahedral symmetry on five monopoles. This should be investigated as it could provide a simple counter example to prove the conjecture false, if it could be shown that such a geodesic exists and its associated family of elliptic curves
contained no singular curves.

3 Conclusion

In this letter we point out that it appears that monopole solutions exist which saturate the Bogomolny energy bound and yet which have more zeros of the Higgs field than number of monopoles. We refer to such spurious zeros as anti-zeros, since they have a local winding which has opposite sign to the total charge of the monopole. Whether such monopole configurations could be interpreted as BPS monopole anti-monopole states is not yet known, since such an interpretation would require a local definition of magnetic charge density (because the zeros and anti-zeros are close together). At present no useful definition exists, since the standard definition relies upon a consideration of the asymptotic field far from the monopole where the non-abelian symmetry is broken to an abelian symmetry which can be identified with electromagnetism.

Some discussion on a signature for the appearance of anti-zeros has been given, and a conjecture made relating this to the singular behaviour of certain elliptic curves. Further work needs to be made on checking this conjecture with other examples, on proving that anti-zeros do exist, and on finding indications for their existence in other approaches, such as rational maps and spectral curves.
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Figure Captions

Fig 1. Components of the Higgs field for the tetrahedral monopole.

Fig 2. The square of the Higgs field for the cubic monopole (solid line) and two other configurations (dashed lines) on the same geodesic.

Fig 3. As Fig 2. but for the component $\varphi_2$.

Fig 4. The square of the Higgs field for the octahedral monopole.
Fig. 1

Fig. 2

components of the Higgs field
