NON-LOCALITY OF QUANTUM MECHANICS
AND THE LOCALITY OF GENERAL RELATIVITY

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Abstract

The conflict between the locality of general relativity, reflected in its space-time description, and the non-locality of quantum mechanics, contained in its Hilbert space description, is discussed. Gauge covariant non-local observables that depend on gauge fields and gravity as well as the wave function are used in order to try to understand and minimize this conflict within the frame-work of these two theories. Applications are made to the Aharonov-Bohm effect and its generalizations to non-Abelian gauge fields and gravity.

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1 Introduction

At present, the two most fundamental theories in physics are quantum theory and general relativity, which is a classical theory of the gravitational field. These two theories are incompatible. A major problem therefore is to unify these two theories in the sense that they are approximations of a deeper quantum theory of gravity. Perhaps the deepest level at which the incompatibility between the two theories is manifest is in the fact that general relativity is local whereas quantum mechanics is non-local. This is reflected in the formulation of general relativity geometrically in space-time, whereas quantum mechanics is formulated in Hilbert spaces whose elements (state vectors or wave functions) have non-local properties in space-time.

This fundamental incompatibility will be described in more detail in section 2. The translation group elements, which are non-local observables, are used to represent the non-local aspects of the quantum wave function, as shown for example in interference phenomena, through their expectation values. In section 3, the non-local quantum phenomena, such as the Aharonov-Bohm (AB) effect and its generalization to non-Abelian gauge fields, are described by means of gauge invariant non-local observables that are elements of the direct product of the translation group and the gauge group, following earlier work of Aharonov and the author. The gravitational AB effect around a cosmic string is considered in section 4. It is shown that this effect may be described as the change in the expectation value of a Poincare group element. This naturally leads to a description of the gravitational field by means of the Poincare group.

2 Locality versus Non-Locality

In a curved space-time, owing to the locality of all the interactions, it is not meaningful a priori to compare two tangent vectors $v$ and $v'$ at different space-time points $p$ and $p'$. But $v'$ may be parallel transported with respect to a
connection along a curve joining \( p' \) and \( p \) and the resulting vector \( v'' \) can be compared with \( v \). The different such vectors \( v'' \) corresponding to different choices of the curve are related by holonomy transformations (parallel transport around a closed curve). Thus ironically, the locality of general relativity leads us to consider holonomy transformations which appear to be non locally associated with an entire closed curve in space-time.

Similarly, in the presence of a gauge field, owing to the locality of the gauge field interaction, it is not possible to compare two internal vectors (which belong to a representation of the gauge group) at different points in space-time. The holonomy transformation in the case of the electromagnetic gauge field is

\[
u = \exp(-i \hat{q} \oint A_\mu dy^\mu)\]

where \( A_\mu \) is the 4-vector potential that acts on a wave function and \( \hat{q} \) is the charge operator. There is, however, a fundamental difference between the non-locality represented by (1) and the apparent non-locality of the holonomy transformation in a gravitational field. This is due to the soldering form, known to mathematicians as the canonical 1-form, in a gravitational field, which ‘solders’ the parallel transportable vectors to the tangent space at each point on space-time. Consequently, it is possible to relate the orientation of the vector that is parallel transported along a curve with the orientation of the tangent vector to the curve at each point on the curve, which cannot be done for the parallel transport of an ‘internal’ vector by means of the gauge field connection. In particular, while the holonomy transformation is unchanged if the enclosed ‘flux’ is changed by one ‘quantum’, the two situations may be distinguished in the gravitational case by the above mentioned comparison with the tangent vector, but they cannot be distinguished in the electromagnetic or gauge field case. This fundamental difference exists between gravitational and gauge fields even when the vector parallel transported is classical, because the soldering form is part of the gravitational field. But there is a further quantum non-locality that exists in the non-local nature of the wave function in both gravity and gauge.
This quantum non-locality may be seen already in the simple double slit experiment. Suppose the two slits are separated by a distance \( l \) in the \( y \)-direction and the quantum wave is moving in the \( x \)-direction that is normal to the two slits. Soon after the wave passes the double slit, it is a superposition of two wave packets at the two slits:

\[
\psi(x, y, z, t) = \frac{1}{\sqrt{2}} \{ \phi(x, y - l, z, t) + e^{i\alpha} \phi(x, y, z, t) \} \tag{2}
\]

For simplicity, it is assumed here that the two wave packets are the same except for the phase difference \( \alpha \). However, \textit{no local experiments done on the non overlapping wave packets emerging from the slits could determine this phase difference}. For example, the expectation values of the local variables \( p^n \), where \( n \) is any positive integer, give no information about \( e^{i\alpha} \). (This is easily verified from \( p^n = (-i\hbar \frac{\partial}{\partial y})^n \) in the coordinate representation.) But

\[
<\psi|\exp(-i\frac{pl}{\hbar})|\psi> = \frac{e^{i\alpha}}{2} \tag{3}
\]

This \( e^{i\alpha} \) is of course observed when the two wave packets overlap subsequently to produce interference fringes. This justifies treating the translational group element \( s = \exp(-i\frac{pl}{\hbar}) \) as an observable, and \( s^\dagger \) was called modular momentum by Aharonov et al \[3, 4\]. The reason why \( <\psi|\exp(-i\frac{pl}{\hbar})|\psi> \) contains more information than all the \( <\psi|p^n|\psi> \) is because \( |\psi> \) vanishes in a region between the wave packets and therefore \( <\psi|\exp(-i\frac{pl}{\hbar})|\psi> \) is not analytic. However, the latter is well defined through the action of \( \exp(-i\frac{pl}{\hbar}) \) on the momentum space wave function of \( |\psi> \) by multiplication.

A great importance of the observable \( s \) lies in the fact that it determines whether the state is a pure state or a mixture. Suppose we have a mixture of the above mentioned localized wave packets, denoted \( |\psi_1> \) and \( |\psi_2> \), instead of the above pure state \( |\psi> = \frac{1}{\sqrt{2}}(|\psi_1> + |\psi_2>) \). The density matrix of this mixture is \( \rho = \frac{1}{2}(|\psi_1><\psi_1| + |\psi_2><\psi_2|) \). Now \( \rho \) gives the same results for
the observation of all local observables as the density matrix $\rho_\psi = \vert \psi > < \psi \vert$ of the pure state, i.e.

$$\text{tr}(\rho \hat{A}) = \text{tr}(\rho_\psi \hat{A}) = \frac{1}{2} (\langle \psi_1 | \hat{A} | \psi_1 \rangle + \langle \psi_2 | \hat{A} | \psi_2 \rangle) \quad (4)$$

for every local observable $\hat{A}$. However, $\text{tr}(\rho s) = 0$, whereas $\text{tr}(\rho_\psi s) = e^{i\alpha}/2$, which is the same as eqn. (3) above. Here $\text{tr}(\rho_\psi s) \neq 0$ because $\rho_\psi$ is the density matrix of a coherent superposition of the two wave packets unlike $\rho$. Hence $s$ may be used to represent the quantum coherence between the two wave packets.

This remark becomes important when we probe the gravitational field by means of quantum mechanical particles, instead of the classical particles that Einstein used to obtain his principle of equivalence. If we probe the classical gravitational field using a particle whose state is $\vert \psi >$, and measure $\vert \psi >$ by means of local observables, this changes it into a mixture or is indistinguishable from a mixture. This is because the interaction of a classical gravitational field is local, which enables it to be incorporated into the space-time geometry. In this way we determine the classical gravitational field and the associated space-time geometry. We need a non-local observable, such as the translational group element $s = \exp(-ip\ell)$ in order to probe the quantum coherence of the gravitational field. Also, the backreaction of $\vert \psi >$ that represents the superposed wave packets requires quantum gravity, which should be non-local because of the above non-local aspect of $\vert \psi >$. Hence, although non-local operators cannot be observed by present day experiments because of the locality of the interactions, their study may be useful in understanding the quantum aspects of the gravitational field. This requires that $s$ should be generalized to incorporate the change in space-time geometry due to the gravitational field which will be attempted in section 4. But in the next section we shall consider the generalization of $s$ to gauge fields, which are easier than the gravitational field because they do not require modification of the Minkowski space-time and it associated symmetries.
3 Non Locality of the Aharonov-Bohm Effect

The Aharonov-Bohm (AB) effect [5], which is a non local interaction between the wave function and the electromagnetic field, gives important information about both the field and the wave function. It shows, on the one hand, that the complete description of the electromagnetic field are given by holonomy transformations (1) associated with arbitrary closed curves. The field strength $F_{\mu\nu}$ has too little information in a multiply connected region, while the phase $\bar{q} \frac{\hbar}{\pi} \oint A_\mu dy^\mu$ has too much information about the electromagnetic field because it may be changed by $2\pi n$ ($n$ is an integer), without observable consequences [6], as mentioned in the previous section. Since (1) is an element of the $U(1)$ group, the AB effect is telling us that the electromagnetic field is a $U(1)$ gauge field. On the other hand, the AB effect also shows the non local quantum coherence of the wave function discussed in the previous section. It is therefore natural to try to describe this effect using quantities that depend on both the wave function and the field, as will be done shortly.

The usual way of explaining the AB effect is by means of the gauge invariant holonomy transformation (1) associated with closed curves, which depends only on the external field. However, gauge invariant quantities associated with open curves that depend on both the field and the wave function were used previously to study the AB effect in superconductors [7]. An example of such a quantity for a particle with charge $q$ at a given time $t$ is [7]

$$z(x, t, \xi) = \psi^*(x + \xi, t) \exp\left\{\frac{i}{\hbar} \int_{x}^{x+\xi} A(y, t) \cdot dy\right\} \psi(x, t) \quad (5)$$

It is easy to verify that (5) is invariant under the gauge transformation

$$\psi'(x) = \exp\left\{\frac{i}{\hbar} \Lambda(x)\right\} \psi(x), \quad A'(x) = A(x) + \nabla \Lambda(x)$$

where $x$ stands for $(x, t)$, which will also be denoted by $x^\mu$. At present, for simplicity, the integral in (5) is taken along the straight line joining $(x, t)$ and $(x + \xi, t)$, although the gauge invariance of $z(x, t, \xi)$ holds more generally for
integral along any piece-wise differentiable curve joining these two points.

To construct an observable corresponding to the gauge invariant quantities \( z \), consider the transformation \( \psi'_{\ell}(x) = f_{\ell}(x)\psi(x) \), where

\[
f_{\ell}(x) = \exp\left(-\frac{i}{\hbar}p \cdot \ell\right) \exp\left\{\frac{i}{\hbar}\int_{x}^{x+\ell} A(y, t) \cdot dy\right\}
\] (6)

It follows that

\[
<\psi| f_{\ell}|\psi> = <\exp\left(-\frac{i}{\hbar}p \cdot \ell\right)\psi| \exp\left\{\frac{i}{\hbar}\int_{x}^{x+\ell} A(y, t) \cdot dy\right\} \psi>
= \int d^3x\psi^*(x + \ell, t) \exp\left\{\frac{i}{\hbar}\int_{x}^{x+\ell} A(y, t) \cdot dy\right\} \psi(x, t)
\] (7)

The integrand of the last expression is the same as the \( z \) given by (5), which is therefore more general than (6). Since (5) is gauge invariant, so is (6). Therefore, (6) is gauge covariant.

Consider now the translational group element \( \exp\left(-\frac{i}{\hbar}p \cdot \ell\right) \). This operator determines the quantum coherence of a wave function, as discussed in the previous section. This is shown dramatically in the AB effect \( \| \) due to interference of two wave packets around a solenoid. It was shown by Aharonov et al \( \| \) that while there is no exchange of \( p^n \) for any positive integer \( n \), there is an exchange of \( \exp(ip\ell) \) between the solenoid and the electron. To see this, consider a wave that is a superposition of two localized wave packets, as in \( \| \), which pass on opposite sides of the solenoid, with \( \alpha \) now being the AB phase caused by the solenoid. Now, while \(< \psi|p^n|\psi> \) is independent of \( \alpha \), \(< \psi|\exp\left(-\frac{i}{\hbar}p \cdot \ell\right)|\psi> \) depends on \( \alpha \) for appropriate choice of \( \ell \) as shown by (5). Therefore, while \(< \psi|p^n|\psi> \) is unaffected by the AB phase, \(< \psi|\exp\left(-\frac{i}{\hbar}p \cdot \ell\right)|\psi> \) is changed by it. This difference is due to the non locality of the quantum wave which, as mentioned in the previous section, exists due to the quantum coherence of the two wave packets.

But \( \alpha \) in the above analysis is gauge dependent. Consider therefore the operator \( f'_{\ell} = \exp\{i(-p + qA(t)) \cdot \ell\} \) that is obtained by replacing the canonical
momentum $p$ in the translation with the gauge-covariant kinetic momentum $p - q\hat{A}(t)$, where $\hat{A}(t)$ is a time-dependent operator in the Schrödinger picture that acts on the Hilbert space according to the rule

$$\hat{A}(t)\psi(x) = A(x, t)\psi(x)$$  \hspace{1cm} (8)$$

Here $\psi(x)$ is any element of the Hilbert space of the particle and $A(x, t)$ is the usual classical vector potential in space-time. If the electromagnetic field is also quantized then $A(x, t)$ would be the expectation value of a time-independent Schrödinger operator representing the quantized field with respect to a Schrödinger coherent state of the field at time $t$. For this reason, I shall always let $f'\ell$ act on the Schrödinger wave function of the particle at time $t$, because the latter wave function multiplies the coherent state wave function of the field at the same time in the present approximation. Thus this restriction may be deduced from a deeper theory in which all degrees of freedom are quantized. Then the wave function resulting from the action of $f'\ell$ is gauge-covariant owing to the fact that $-p + q\hat{A}(t)$ is gauge-covariant.

Also, $f'\ell = f\ell$  \hspace{1cm} (8), i.e. for every wave function $\psi$,

$$\exp\left\{\frac{i}{\hbar}(-p + q\hat{A})\cdot\ell\right\}\psi(x) = \exp\left\{-\frac{i}{\hbar}p\cdot\ell\right\}\exp\left\{iq\frac{\hbar}{\ell}\int_{x}^{x+\ell} A(y, t) \cdot dy\right\}\psi(x)$$  \hspace{1cm} (9)$$

To prove this, note that (9) is valid in an axial gauge in which $\hat{A}(t)\cdot\ell = 0$. Also, both sides of (9) are gauge-covariant, as shown earlier. Hence, (9) is valid in every gauge.

More generally, define the operator $g_{\hat{\gamma}}$ that acts on the Hilbert space according to

$$g_{\hat{\gamma}}\psi(x) = \exp(-\frac{i}{\hbar}p\cdot\ell)\exp\left\{i\frac{q}{\hbar}\int_{\gamma} A(y, t) \cdot dy\right\}\psi(x)$$  \hspace{1cm} (10)$$

where $\hat{\gamma}$ is any piece-wise differentiable curve in $\mathbb{R}^3$ and $\gamma$ is a curve in physical space that is congruent to $\hat{\gamma}$ and joins an arbitrary point $x$ to $x + \ell$. Here, $\ell$
is independent of \( x \) but depends on \( \hat{\gamma} \). In the special case when \( \hat{\gamma} \) is a straight line, \( g_{\hat{\gamma}} \) is the same as \( f_\ell \). Also, for any piece-wise differentiable curve \( \hat{\gamma} \), \( g_{\hat{\gamma}} \) can be shown to be gauge-covariant in the same way as \( f_\ell \) was shown to be gauge-covariant. From now onwards, the \(^\wedge\) over \( \gamma \) will be dropped, for simplicity, if no confusion will arise.

Consider now the AB effect \([5]\) in which a wave packet is split by a beam splitter into two wave packets that go past on two sides of a solenoid (figure 1).

![Figure 1](image-url)

Figure 1. A wave packet is split at the beam splitter M into two wave packets A and B that go around the solenoid S and interfere at I. When the imaginary curve \( \gamma \) crosses the solenoid, the gauge invariant expectation value of \( g_{\gamma} \) changes.

The wave function of the particle is

\[
\psi(x, t) = \frac{1}{\sqrt{2}} \{ \psi_1(x, t) + \psi_2(x, t) \} 
\]

(11)

where \( \psi_1 \) and \( \psi_2 \) are the wave functions of the two wave packets. Then we may write

\[
\psi_1(x, t) = \exp \left\{ \frac{i q}{\hbar} \int_{\gamma_1}^{x} A(y, t) \cdot dy \right\} \psi_{10}(x, t),
\]

\[
\psi_2(x, t) = \exp \left\{ \frac{i q}{\hbar} \int_{\gamma_2}^{x} A(y, t) \cdot dy \right\} \psi_{20}(x, t)
\]

(12)
where $\gamma_1$ and $\gamma_2$ are variable curves outside the solenoid that begin at a point $x_0$ on the beam splitter, and $\psi_{10}$ and $\psi_{20}$ are the unperturbed solutions of Schrödinger’s equation in the absence of $A$. It is easily verified that $\psi_1$ and $\psi_2$ are solutions of Schrödinger’s equation with $A$ minimally coupled. Suppose the displacement between the centers of the wave packets is $\ell$. Then

$$
<\psi|g_\gamma|\psi> = <\exp\left\{\frac{i}{\hbar}\mathbf{p} \cdot \ell\right\}\psi|\exp\left\{\frac{q}{\hbar} \int_{\gamma \times x} A(y, t) \cdot dy\right\}|\psi>
$$

$$
= \int d^3x \psi^*(x + \ell, t) \exp\left\{\frac{q}{\hbar} \int_{\gamma \times x} A(y, t) \cdot dy\right\} \psi(x, t)
$$

$$
= \int d^3x \psi_1^*(x + \ell, t) \exp\left\{\frac{q}{\hbar} \int_{\gamma \times x} A(y, t) \cdot dy\right\} \psi_2(x, t)
$$

$$
= \int d^3x \psi_{10}^*(x + \ell, t) \exp\left\{\frac{q}{\hbar} \oint_{\gamma_0} A(y, t) \cdot dy\right\} \psi_{20}(x, t) (13)
$$
on using (12), where $\gamma_0$ is the closed curve formed by $\gamma_2, \gamma$ and the reverse of $\gamma_1$.

As the two wave packets move past the solenoid, $\gamma$ crosses the solenoid. Then, $\exp\left\{\frac{q}{\hbar} \oint_{\gamma_0} A(y, t) \cdot dy\right\}$ changes from 1 to $\exp\left\{i\frac{q}{\hbar}\Phi\right\}$, where $\Phi$ is the magnetic flux enclosed by the solenoid. This has been shown for the special case when $\gamma$ is a straight line joining the wave packets in a special gauge [10]. But the present gauge invariant treatment which is valid for arbitrary $\gamma$ has the following advantages: A) There is nothing special about the time when the imaginary straight line joining the wave packets crosses the solenoid, because for other choices of the curve $\gamma$, this curve would cross the solenoid at different times. B) Even at a given time $t$, $\gamma$ may be chosen to be on either side of the magnetic flux, which correspond to $\exp\left\{\frac{q}{\hbar} \oint_{\gamma_0} A(y, t) \cdot dy\right\} = 1$ or $\exp\left\{i\frac{q}{\hbar}\Phi\right\}$.

Consider a circular superconducting ring enclosing the magnetic flux $\Phi$ that has $\ell$ as a diameter. The ring is interrupted by a Josephson junction on one side of this diameter. Then $\gamma$ may be chosen to be through the semi-circular ring on
either side of this diameter. For these two choices, $< \psi | g_\gamma | \psi >$ would take two possible values that differ by the factor $\exp(i \frac{\Phi}{\hbar})$. This difference is responsible for the Josephson current through the Josephson junction. To summarize, the usual AB effect described in fig. 1 as well as the AB effect in a superconducting ring may be understood as due to the change in $< \psi | g_\gamma | \psi >$ that depends on the open curve $\gamma$. But this change depends on the holonomy transformation (1) associated with a closed curve.

4 Generalization to Non-Abelian Gauge Fields and the Gravitational Aharonov-Bohm Effect

The results in the previous section naturally generalize to non Abelian gauge fields [9]. For an arbitrary gauge field, (10) is generalized to

$$
\hat{g}_\gamma \psi(x) = \exp(-i \frac{\hbar}{\bar{p}} \cdot \ell) \left( P \exp \left\{ i \frac{\bar{g}_0}{\hbar} \int_{\gamma} T_k A^k(y, t) \cdot dy \right\} \psi(x) \right)
$$

(14)

where $T_k$ generate the gauge group, and $P$ denotes path ordering, which is necessary because $T_k A^k$ at different points do not commute in general. In the special case when $\gamma$ is a straight line,

$$
\exp \left\{ i \frac{\hbar}{\bar{p}} (\bar{p} + g_0 T_k \hat{A}^k \cdot \ell) \right\} \psi(x) = \exp(-i \frac{\hbar}{\bar{p}} \cdot \ell) \left( P \exp \left\{ i \frac{\bar{g}_0}{\hbar} \int_{\gamma} T_k A^k(y, t) \cdot dy \right\} \psi(x) \right)
$$

(15)

This result may be proved, as in the Abelian case, using the gauge covariance of both sides of (15) and noting that (15) holds in the axial gauge in which there is no contribution from the non Abelian vector potential. Also, this result can be generalized to an arbitrary (piece-wise differentiable) curve $\hat{\gamma}$, i.e.

$$
g_\gamma \psi(x) = P \exp \left\{ i \frac{\hbar}{\bar{p}} \int_{\hat{\gamma}} (-\bar{p} + g_0 T_k \hat{A}^k) \cdot d\ell \right\}
$$

(16)

where $P$ denotes path ordering. The result (16) implies that the operators $\{ g_\gamma \}$ form a group, which in general is infinite dimensional.
Also, the AB effect described in fig. 1 may be generalized to the case when the flux through the cylinder is due to an arbitrary gauge field. The wave function is given by (11), where now
\[
\psi_1(x, t) = P \exp \{ i \frac{q_0}{\hbar} \int_{\gamma_1} T_k A^k(y, t) \cdot dy \} \psi_{10}(x, t),
\]
\[
\psi_2(x, t) = P \exp \{ i \frac{q_0}{\hbar} \int_{\gamma_2} T_k A^k(y, t) \cdot dy \} \psi_{20}(x, t),
\]
(17)

Then,
\[
< \psi | g_\gamma | \psi > = \int d^3 x \psi_{10}^\dagger(x + \ell, t) P \exp \{ i \frac{q_0}{\hbar} \int_{\gamma_0} T_k A^k(y, t) \cdot dy \} \psi_{20}(x, t)
\]
(18)

where \( \gamma_0 \) is the closed curve, as defined before, beginning and ending at \( x_0 \). The gauge invariance of this expression follows from the fact that in (17) the parallel transport operators (path ordered integrals) begin at \( x_0 \). Therefore, under a local gauge transformation \( U(x, t) \), the unperturbed wave functions \( \psi_{10}(x) \) and \( \psi_{20}(x) \) must undergo the global gauge transformation \( U(x_0, t) \) in order that \( \psi_1(x) \) and \( \psi_2(x) \) transform covariantly to \( U(x) \psi_1(x) \) and \( U(x) \psi_2(x) \), respectively. The AB effect in the superconducting ring may be generalized to the AB effect due to an arbitrary gauge field flux that arises from the generalization of the Josephson effect to a non Abelian gauge theory \[7\]. In both these cases, as discussed in the previous section for the \( U(1) \) gauge theory, the generalized AB effect arises from the change in \( < \psi | g_\gamma | \psi > \) due to the crossing of the flux by \( \gamma \), where now \( g_\gamma \) is given by (14).

Extending these results to the gravitational case, however, is difficult because of the fundamental locality of the gravitational field due to space-time curvature \[2\]. To this end, consider the gravitational AB effect due to a cosmic string. In particular, in the experiment described in fig. 1, the solenoid may be replaced by a cosmic string. Outside the cosmic string, the curvature and torsion vanish, which is analogous to the vanishing of the electromagnetic field strength outside the solenoid. Define a Hilbert space \( \mathcal{H} \) to consist of \( L^2 \) functions that have

12
support outside the cosmic string. Now generalize $g_\gamma$ to include the gravitational field \cite{11, 12}:

$$g_\gamma = P \exp \left( \frac{i}{\hbar} \int_\gamma \left( \theta_\mu^a P_a + \frac{1}{2} \omega_\mu^{ab} M^{ba} + A_\mu^j T_j \right) dx^\mu \right),$$

(19)

where $\theta_\mu^a$ is the soldering form mentioned in section 2, $\omega_\mu^{ab}$ is the linear connection form, and $P_a$ and $M^{ba}$ generate the space-time translations and Lorentz transformations in the Hilbert space $\mathcal{H}$. Hence,

$$<\psi|g_\gamma|\psi> = \int d^3x \psi_{10}^\dagger (x + \ell, t) g_{\gamma_0} \psi_{20} (x, t)$$

(20)

For simplicity, suppose that there is no gauge field present. It was shown \cite{13} that the gravitational phase shift is determined by $g_{\gamma_0}$ when $\gamma_0$ encloses the cosmic string. The part of $g_{\gamma_0}$ that contributes to the phase shifts are symmetries of the space-time geometry of the cosmic string. So, again the change in $<\psi|g_\gamma|\psi>$ due to $\gamma$ crossing the cosmic string (which makes $\gamma_0$ that was defined below \cite{13} enclose the cosmic string) determines the gravitational AB effect in this case. The generalization of the above results to an arbitrary gravitational field will be discussed elsewhere.

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