Nonlinear characteristics in radio frequency nanoelectromechanical resonators

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Abstract. The nonlinear oscillations of nanoelectromechanical resonators have previously been studied both experimentally and analytically. Nanoresonators have achieved superior sensitivity and high quality factors in many applications. However, the linear operating range of nanoresonators is significantly limited because of the small dimensions and thus the linear regime of nanoresonators may be required to expand performance in various conditions. In order to increase the linear operating range, we proposed that proper adjustments of simultaneous application of drive and electrothermal power can be used to optimize the resonance performance, providing a wider linear range as well as to tune the resonance frequency. For a nanoresonator operated by simultaneous drive and electrothermal power, experimental data are theoretically supported using nonlinear damping and spring terms. In the transition between linearity and nonlinearity by proper combinations of ac drive and dc electrothermal power, the experimental data can be better fitted, by theoretical study, with newly derived nonlinear damping terms. We believe that better understanding of these effects

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with different ac/dc combinations on radio frequency oscillation is crucial for utilizing nanoresonators for various applications such as sensors, oscillators and filters.

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1. Introduction

Nanoelectromechanical resonators have been used for various electrical and mechanical devices such as sensors, filters and oscillators [1]–[3]. Extensive efforts are made to obtain high quality factors because the $Q$-factor directly affects the sensitivity. Micro- and nanoelectromechanical resonators have achieved extremely high $Q$-factors and sensitivity as filters and oscillators for radio frequency (RF) communications [4], zeptogram-level mass sensing [5], gas sensing [6], sub-attomewton force detection at milli-Kelvin temperature [7] or single-spin detection [8]. To keep the high $Q$-factor of resonance motion, stiffness and damping characteristics should be carefully studied [9] and also noise and dissipation issues in the various circumstances should be examined [10]–[12]. Furthermore, specific operating conditions require a deeper understanding of resonance properties in nano-scale devices.

Various applications benefit from a large dynamic range, which is the signal-to-noise ratio at resonance before nonlinearity occurs. Usually higher driving power provides a larger readout, which may improve the signal-to-noise ratio. However, large input can drive the resonator out of the range of linearity, and this limitation is more significant in the nanoresonators, because the smaller dimensions and higher $Q$-factor make it easier to put the resonance motion in the nonlinear regime [13, 14]. Accordingly, many applications will involve operation in the nonlinear regime. Thus understanding the trade-off between high sensitivity and a large dynamic range and controlling the nonlinearity will be essential for expanding the utility of nanoresonators.

There has been recent interest in nonlinearity effects on the micro- and nanoresonators. Previous work included characterization of critical amplitude [14], nonlinear response in the quantum regime [15], high driving power to obtain high signal level [16], bucking instabilities associated with the transverse mode [17], application of hysteresis as a stochastic sensor [18], synchronization in two nanoresonators coupled by a mechanical element [19], modal analysis of up to fifth order [20] and cancellation of the nonlinear effects to stay within the linear resonant range [21, 22]. The nonlinearity caused by the hardening spring effect has been addressed before [13, 14], but without involving nonlinear damping. Such nonlinear resonance can be explained by the critical amplitude, which determines the range of linear harmonic excitation.
In this work, we observe nonlinear motion at high ac driving power, and a shift in the resonance frequency. Then, as dc power increases, the oscillation amplitude is reduced, and the linear oscillatory behavior can be resumed. The effects of the hardening spring and nonlinear damping terms are analyzed in the measurements of Al/SiC nanoresonators. The experimental data obtained from electromagnetomotive transduction [23]–[25] are supported by the simulation results that explain nonlinear characteristics under different combinations of driving and electrothermal power.

2. Experiments

SiC has been considered a good material for high frequency nanoresonators [26, 27], because of its high tensile-strength-to-density and high acoustic velocity.

Single-crystal 3C-SiC films are grown on a silicon wafer from a mixture of propane, silane and hydrogen using a heteroepitaxial atmospheric pressure chemical vapor deposition (APCVD) reactor [28]. Propane (C\textsubscript{3}H\textsubscript{8}, 15% in hydrogen) and silane (SiH\textsubscript{4}, 5% in hydrogen) are used as precursors, and ultrahigh purity hydrogen is used as a carrier gas.

Doubly clamped nanoscale beams are fabricated from a 3C-SiC-grown silicon substrate by the standard e-beam lithography process. Poly methyl methacrylate (PMMA) resist (495 kDa, 4% in anisole) is coated and then baked at 170 °C. The pattern is defined by e-beam lithography and developed in a methyl isobutyl ketone (MIBK) : isopropyl alcohol (IPA) (1 : 3) solution at 7 °C for 20 s. In order to improve conductivity for magnetomotive transduction, a metallic layer is deposited using a thermal evaporator, and the liftoff process is followed. For the samples used in this work, Al was used as the additional metallic layer. Using the reactive ion etching (RIE) process with a CF\textsubscript{4} /O\textsubscript{2} (in the ratio 9 : 1) gas mixture at the operating conditions of 300 mTorr and 3 min, the SiC-grown silicon wafer is etched anisotropically and isotropically. Then clamped–clamped nano-beams are fully suspended for magnetomotive excitation as shown in the inset of figure 1.

The resonators fabricated are 12.0 \(\mu\)m long and 100 \(\mu\)m wide. The resonators were tested in a table-top vacuum chamber at room temperature (around 1 torr) under an adjustable moderate magnetic field B (1.0 Tesla). RF excitation and measurement were performed with a network analyzer (Agilent E5071C). An ac drive of a few tens of \(\mu\)V was applied through the beam in a vertical magnetic field. This RF current through the beam caused the lateral vibration. The induced electromotive force (EMF) voltage was detected as the oscillatory motion of the beam cut the magnetic field. To optimize the electrical readout, two identical resonators were fabricated side by side to form an RF resistance bridge [3], as shown in figure 1(a). The drive signal passes through a 180° power splitter before being applied to each end of the beam. In this way, the center arm of the device is on virtual ground until one of the beams is in resonance; thus the background noise can be reduced significantly. The suspended nanoresonators shown in figures 1(b) and (c) are connected to one detecting port of the network analyzer through an amplifier and two drive-ports with power reduction [3].

From the values of the detected resonance frequency and the resonance frequency computed at zero stress state, the effective stress over the beam can be extracted [3]. dc voltage up to 100 mV is simultaneously applied in parallel with the RF driving voltage. This dc power heats the beam and changes the stress distribution on the beam. As a result, the resonance frequency is shifted back to the harmonic resonance frequency [29, 30].

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3. Dynamics of nonlinear resonators

This study is concerned with lateral vibration of the doubly clamped beam as shown in figure 1. Let $A$ denote the cross-sectional area and $L$ the length of the beam, and the current through the beam is

$$I_{dt}(t) = I_{dc} + I_{ac} \cos \omega t. \quad (1)$$

Here $I_{dc}$ and $I_{ac}$ denote the dc and ac components of the current and $\omega$ is the driving frequency.

The lateral displacement $y(x, t)$ of the beam can be described by Euler beam theory:

$$- \frac{\partial^2}{\partial x^2} \left[ EI \frac{\partial^2 y}{\partial x^2} \right] + v(x, t) = \rho A \frac{\partial^2 y}{\partial t^2}, \quad (2)$$

where $E$ denotes Young’s modulus, $\rho$ is the density, $I = tw^3/12$ is the cross-sectional moment of inertia of the beam with width $w$ and thickness $t$, and $v(x, t)$ is the transverse force per unit length.

With the terms due to the axial road $T_f$ and the electromagnetic force under the magnetic field of strength $B$, the following equation is obtained from (2):

$$EI y_{xxxx} - \left( T_f + \frac{EA}{2L} \int_0^L y_x^2 \, dx \right) y_{xx} + \rho Ay_{tt} + \alpha \left( 1 + \beta y_t^2 \right) y_t = BI_{dt}(t). \quad (3)$$

The last term on the left-hand side represents the damping force: $\alpha$ is the damping coefficient and $\beta$ is the relative magnitude of cubic damping compared with linear damping.
For the doubly clamped beam, the boundary conditions \( y(0) = y(L) = 0 \) and \( \frac{\partial y}{\partial x}(0, t) = \frac{\partial y}{\partial x}(L, t) = 0 \) are imposed. Let us use the following form to approximate the first resonant mode [13]:

\[
y(x, t) = y(t) \sqrt{\frac{2}{3}} \left( 1 - \cos \frac{2\pi x}{L} \right)
\]  

and then substitute (4) into the left-hand side of (3), which we denote by \( e \). Application of the Galerkin method [31]

\[
\int_0^L e \left( 1 - \cos \frac{2\pi x}{L} \right) \, dx = 0
\]
yields

\[
\ddot{y} + 2\gamma_1 \dot{y} + \omega_0^2 y + \gamma_3 \dot{y}^3 + \alpha_3 y^3 = H(1 + \gamma \cos \omega t),
\]

where the resonant frequency is

\[
\omega_0^2 = \left( \frac{2\pi}{L} \right)^4 \left[ \frac{E I}{3\rho A} \left( 1 + \frac{T_f}{E I} \left( \frac{L}{2\pi} \right)^2 \right) \right]
\]

and \( \gamma_1 = \alpha / 2 \rho A, \gamma_3 = (\alpha / \rho A)(35/18)\beta, \alpha_3 = (E / 18\rho)(2\pi / L)^4, \) \( H = \sqrt{2/3}(B / \rho A)I_{dc} \) and \( h = (I_{ac} / I_{dc}) \). Here, \( \gamma_3 \) represents the nonlinear damping and \( \alpha_3 \) the hardening cubic spring effect.

The secular perturbation method [31] is applied to solve (5). Assuming that \( \gamma_1, \gamma_3, \alpha_3 \) and \( HH \) are all small, we can rewrite (5) as the following, with a small parameter \( \epsilon \):

\[
\ddot{y} + \omega_0^2 y = -2\epsilon \gamma_1 \dot{y} - \epsilon \gamma_3 \dot{y}^3 - \epsilon \alpha_3 y^3 + H + \epsilon H \gamma \cos \omega t.
\]

Instead of using the frequency of the excitation \( \omega \) as a parameter, we introduce a detuning parameter \( \sigma \) that quantitatively describes the nearness of \( \omega \) to \( \omega_0 \):

\[
\omega = \omega_0 + \epsilon \sigma.
\]

To use the method of multiple scales, we express the solution in terms of different time scales as

\[
y(t; \epsilon) = y_0(T_0, T_1) + \epsilon y_1(T_0, T_1) + \cdots,
\]

where \( T_0 = t \) and \( T_1 = \epsilon t \). We also express the excitation in terms of \( T_0 \) and \( T_1 \) as

\[
\epsilon \gamma \cos \omega t = \epsilon \gamma \cos(\omega_0 T_0 + \sigma T_1).
\]

Substituting (8) and (9) into (7), we obtain a polynomial in \( \epsilon \). Equating the coefficients of \( \epsilon^0 \) and \( \epsilon \) on both sides, we obtain

\[
D_{0,0}^2 y_0 + \omega_0^2 y_0 = H,
\]

\[
D_{0,0}^2 y_1 + \omega_0^2 y_1 = -2\gamma_1 D_{0,0} y_0 - \gamma_3 (D_{0,0} y_0)^3 - 2D_0 D_{1,0} y_0 - \alpha_3 y_0^3 + H \gamma \cos(\omega_0 T_0 + \sigma T_1),
\]

where \( D_0 = \partial / \partial T_0 \), and \( D_1 = \partial / \partial T_1 \).

The general solution of (10) can be written as

\[
y_0 = \frac{H}{\omega_0^2} + C(T_1) e^{i\omega_0 T_0} + C(T_1) e^{-i\omega_0 T_0}.
\]
We substitute \( y_0 \) into (11) and collect the terms with \( e^{i\omega_0 T_0} \). Secular terms will be eliminated from the particular solution of (11) if we choose \( C \) to be a solution of

\[
2\omega_0 \left[ \frac{dC}{dT_1} + (i\gamma_1 + \Delta\omega_0)C \right] + 3(\alpha_3 + i\gamma_3\omega_0^3)C^2 \tilde{C} = \frac{HH}{2} e^{i\omega T_1},
\]

where \( \Delta\omega_0 = 3\alpha_3 H^2/(2\omega_0^3) \) denotes a small change from the linear natural resonance frequency \( \omega_0 \).

To solve (13), we write \( C \) in the following form:

\[
C = \frac{a}{2} e^{i(\phi + \Delta\omega_0 t)},
\]

where \( a \) and \( \phi \) are real. Inserting (14) into (13), we obtain

\[
\omega_0 \frac{da}{dt} + \omega_1 i\gamma_1 a - \omega_0 a \frac{d\phi}{dt} + \frac{3}{8} \alpha_3 a^3 + \frac{3}{8} i\gamma_3\omega_0^3 a^3 = \frac{HH}{2} e^{i[\phi + (\omega - \omega_0 - \Delta\omega_0) t]}.
\]

Let \( \Delta\omega = (\omega - \omega_0 - \Delta\omega_0) \) and separate (15) into real and imaginary parts:

\[
-\omega_0 \frac{d\phi}{dt} + \frac{3}{8} \alpha_3 a^3 = \frac{HH}{2} \cos(\phi - \Delta\omega t),
\]

\[
-\omega_0 \frac{da}{dt} + \gamma_1 a - \frac{3}{8} i\gamma_3\omega_0^3 a^3 = \frac{HH}{2} \sin(\phi - \Delta\omega t).
\]

At steady state, \( a \) and \( \phi - \Delta\omega t \) will tend to constant values; then \( d\phi/dt \to \Delta\omega \). By combining (16) and (17), we obtain

\[
9(\alpha_3^2 + \gamma_3^2\omega_0^6)a^6 + 48\omega_0(\gamma_1\gamma_3\omega_0^3 - \Delta\omega\alpha_3)a^4 + 64\omega_0^2a^2(\Delta\omega^2 + \gamma_1^2) = 16H^2h^2.
\]

At the onset of hysteresis, i.e. at the critical amplitude \( A_c \), we have \( d\Delta\omega/d(a^2) = 0 \), \( d^2\Delta\omega/d(a^2)^2 = 0 \) for jumps in amplitude and bistability, respectively. Demanding these conditions on (18) yields the critical value of \( \Delta\omega \):

\[
\Delta\omega_c = \frac{\gamma_1 \omega_0}{\sqrt{3}} \frac{q + 3}{1 - q}
\]

and the critical amplitude:

\[
A_c^2 = \frac{4}{3\sqrt{3}} \frac{\gamma_1\omega_0}{\alpha_3} \frac{1}{1 - q} = \frac{4}{3\sqrt{3}} \frac{\gamma_1\omega_0}{\alpha_3 - \sqrt{3}\gamma_3\omega_0^3},
\]

where \( q = \sqrt{3}\gamma_3\omega_0^3/\alpha_3 \), and \( \gamma_1 = \omega_0/2Q \), where \( Q \) is the \( Q \)-factor.

Tension in the beam after the dc heating is reduced from \( T_0 \) to

\[
T_f = T_0 - kEA V^2_{dc},
\]

where \( k \) is defined using the specific heat conductivity \( C_p \), the mass of the resonator \( m \), and the resistance \( R \) as follows:

\[
k = \frac{(L/\pi)^2}{mC_pR}.
\]

Thus the natural frequency \( \omega_0^2 \) in equation (5) decreases in proportion to \( V^2_{dc} \).
4. Results and discussion

Figure 2 shows the frequency response of a nanoresonator at four different settings of driving voltages. When the driving voltage $V_{ac}$ is at a low level (e.g. 10 $\mu$V), the oscillation stays in the linear regime. With increasing $V_{ac}$, the resonance frequency $\omega_0$ and the measured $V_{EMF}$ increase. As $V_{ac}$ increases further, the increased displacement creates an additional tension, which acts as hardening stiffness. With the resonance amplitude exceeding its critical amplitude $A_c$, the resonator shows nonlinearity as shown in figure 2 for 10, 20, 30 and 40 $\mu$V.

As illustrated in figure 2, the nanoresonator can be easily driven out of the linear regime due to its small dimensions. By applying additional dc power simultaneously with ac drive, the oscillation in such a nonlinear regime can be brought back to the linear regime. Figure 3 shows the amplitude of $V_{EMF}$ while increasing dc power at a fixed ac drive. The amplitude decreases with increasing $V_{dc}$. It can be seen that the $V_{EMF}$ at $V_{dc} = 200$ mV is about 50% of that when $V_{dc} = 0$ mV. While the amplitude of the oscillation decreases with increasing dc voltage, its critical amplitude $A_c$ increases [13]. Therefore, when the amplitude is lowered down below the $A_c$, a transition back to the linear regime occurs. In our experiment shown in figure 3 at the fixed $V_{ac} = 30$ $\mu$V, this transition occurs when $V_{dc}$ is between 100 and 150 mV.

Figures 4 and 5 show comparisons between the measured data and theoretical results computed for the same condition with the experiment. The squared curves are measured responses, and the solid curves represent a set of solutions of (18) computed at each frequency. Parameters in (5) were calculated with the data in table 1 using the relationships in (6). $T_f$ and $\alpha$ were estimated from $\omega_0$ and $Q$-factor of the experimental result and adjusted to fit the experimental results. Table 2 shows each parameter value that is used for simulations. The overall resistance $R$ of the resonator was set to 80–150 $\Omega$. ac and dc input current values are calculated from input voltages and the overall resistance $R$. 


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Figure 3. Amplitude of $V_{\text{EMF}}$ at 30 $\mu$V ac with varying dc voltage from 0 to 200 mV. The arrow in the inset indicates the trend of recovery with increasing dc power. The oscillation is brought back to the linear regime from the nonlinear behavior. As the applied dc voltage increases, resonance frequency $\omega_0$ and $Q$-factor decrease.

Figure 4. Comparison of the experimental resonance peak with theoretical computation when the driving power increases. The resonance frequency shifts to the right side as the ac voltage increases. (a) The resonance with the driving power of 5 $\mu$V presents the linear peak. (b) The driving power of 20 $\mu$V shows the nonlinear peak. (c) The driving power of 40 $\mu$V presents severe nonlinearity.

In figure 4, the experimental data from figure 2 is fitted with the nonlinearity simulation, under the driving power of 5–40 $\mu$V. The linear resonance is fitted by the simulation pretty well as shown in figure 4(a). However, as the driving power increases, the difference between the
Figure 5. Comparison of the experimental resonance peak with theoretical computation when the 30 µV drive is supplied with electrothermal recovery power. (a) The resonance at 0 mV electrothermal power presents the nonlinear peak, (b) the resonance at 100 mV electrothermal power is recovered a little, but still in the nonlinear mode, and (c) the linear resonance mode is fully recovered at 200 mV electrothermal power.

Table 1. The properties of the resonator and the setup.

| Properties              | Value          |
|-------------------------|----------------|
| Beam width              | 100 nm         |
| Beam thickness          | 30 nm          |
| Beam length             | 12 µm          |
| Young’s modulus (SiC/Al)| 410 GPa/68 GPa |
| Density (SiC/Al)        | 3200 kg m⁻³/2700 kg m⁻³ |
| Magnetic field          | 1.1 Tesla      |

Table 2. Parameter values used for simulations to fit the experimental results.

| Parameter       | Value                  |
|-----------------|------------------------|
| $T_f$ (µN)      | $0.507–0.517$          |
| $\alpha$        | $0.937–10.624 \times 10^{-5}$ |
| $\beta$         | $2$                    |
| $R(\Omega)$     | $80$                   |
| $T_f$ (µN)      | $0.448–0.506$          |
| $\alpha$        | $0.452–11.734 \times 10^{-5}$ |
| $\beta$         | $2$                    |
| $R(\Omega)$     | $150$                  |

experiment and simulation is larger in the frequency range right above the resonant frequency, where the amplitude suddenly drops from the peak value.

Higher driving power tends to increase the nonlinearity in resonance characteristics. The data under the high driving voltage of 30 µV are taken from figure 3. As the resonator is tuned by electrothermal power up to 200 mV, the experimental data are presented along with theoretical computation in figure 5. As commented on above, the severe nonlinear portion in figure 5(a)
Figure 6. (a) The tensile force of the resonator increases with increasing driving power. (b) The damping coefficient $\alpha$ and the beam force of the resonator when the electrothermal power is applied. As the electrothermal power increases, the damping coefficient increases, but the beam force decreases parabolically.

is not accurately fitted. But as the dc power increases, the response is recovered to the linear mode, and the fitting becomes more accurate. All these results are not matched well unless the nonlinearity damping term $\beta$ is adjusted carefully, and here it is set to 2.

The tensile force on the beam is a crucial factor that decides the resonance performances such as the critical amplitude, dynamic range and quality factor. This force of the beam changes as the driving power changes and the frequency is tuned by thermal power. The higher driving power causes strain hardening, which induces higher stress concentration as shown in figure 6(a), whereas higher electrothermal power lowers the stiffness of the beam, resulting in lower force distribution as shown in figure 6(b).

The damping coefficient $\alpha$ is another important parameter related to the quality factor and resonance frequency as in equation (23). It is difficult to estimate accurately the value of the damping coefficient $\alpha$ from the plots containing the severe hysteresis region such as figure 5(a). On the other hand, when nonlinearity becomes minor, which is affected by recovering effects by electrothermal power, the damping coefficient $\alpha$ can be estimated from the quality factor in figure 5(c). As the thermal power increases, the damping coefficient increases due to softening associated with the recovery from the hardening effects of overdrive

$$\alpha = \frac{\pi f_0}{2\rho A Q}.$$  

Figure 7 shows that the resonance frequency $\omega_0$ also decreases as we had expected in section 3. $Q$ also decreases. $\gamma_1 = 2\pi f_0/2Q$, the linear damping term in (5), increases with increasing $V_{dc}$ according to the trends of $f_0$ and $Q$.

Both the $Q$-factor and $\omega_0 = 2\pi f_0$ decrease with increasing $V_{dc}$. The results on $f_0$ and $Q$ in figure 7 yield that $\gamma_1 = 2\pi f_0/2Q$, the linear damping term in (5), increases with increasing $V_{dc}$. 

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Figure 7. Resonance properties as dc voltage varies: both the resonance frequency \( f_0 \) and \( Q \)-factor decrease as \( V_{dc} \) increases.

\( Q \) will decrease. Thus, we can infer that increasing \( \gamma_1 \) has a more crucial effect than decreasing \( f_0 \). Since the nonlinear damping ratio term \( \beta \) is proportional to \( \gamma_1 \), the larger value of \( \beta \) would make \( A_c \) larger in (20). This shows that the nonlinear damping term can be one of the key parameters for understanding nonlinear resonance characteristics.

It is presumed that the nonlinear damping ratio will vary depending on the intrinsic or extrinsic properties. The damping properties are significantly dependent on the resonator fabrication. The metal layer deposition process onto the SiC–Si substrate is crucial [32], and selection of parameters such as the deposition rate and temperature is especially important. This may induce different structural effects on the clamping loss, surface loss and frictions between layers [33]. The intrinsic mechanical properties such as Young’s modulus and density are also important factors [27, 34]. In addition, experimental conditions contribute significantly to the performance of mechanical excitation because the dissipation (\( Q^{-1} \)) is directly affected by surroundings.

In this study, experimental data are approximated by the critical value of the relative nonlinear damping ratio \( \beta = 2 \). Various intrinsic or extrinsic properties of resonator experiments will affect the specific value of \( \beta \). Especially with smaller dimensions of devices, the effects of these properties may prevail. In a future study, these effects on the resonator oscillation in magnetomotive transduction with moderate conditions shall be analyzed for higher applicability.

High quality factors are related to low bandwidth, because \( \Delta f \approx \frac{\omega_0}{2\pi Q} \). This induces the narrow linear region of oscillation, which makes it difficult to maintain the performance over a desired frequency range. Especially when a high drive power is applied to obtain a higher level of signal or when high stress is applied to improve the resonance frequency, we observe that the nonlinearity effect becomes more significant. Therefore, nonlinearity issues should not be neglected when designing high-performance nanoresonators. In this study, this effect was explained in terms of the critical amplitude by theoretical analysis using the nonlinear spring and damping terms. This analysis supported the experiments, which were performed in moderate conditions under room temperature and rough vacuum, and helps to comprehend the complicated RF oscillation of nanoresonators.
Nonlinear damping in nanoresonators has not been well understood before. This research allows us to better predict the complicated resonance phenomena in magnetomotive transduction for more practical applications.

5. Conclusions

We have presented the measurements of oscillations of Al/SiC nanoelectromechanical resonators at a room temperature setup with a moderate vacuum and magnetic field. At high driving voltages, the resonators showed nonlinear behavior. By increasing electrothermal dc power, their nonlinear motion has been brought back to the linear regime. We have explained the nonlinear characteristics under different ac and dc power with theoretical simulation results, which include the effects of the hardening spring and nonlinear damping terms. In order to analyze the transition between nonlinear and linear regimes as dc power varies, the critical amplitude and dynamic range for the nanoresonator have been computed from the derived equations of motion. Understanding nonlinear phenomena under simultaneously applied ac and dc power can be used to control the nonlinearity of nanoresonators and improve their applicability.

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