Possible Evolution of the Pulsar Braking Index from Larger than Three to About One

H. Tong\textsuperscript{1,2} and F. F. Kou\textsuperscript{2}
\textsuperscript{1} School of Physics and Electronic Engineering, Guangzhou University, 510006 Guangzhou, China; htong_2005@163.com
\textsuperscript{2} Xinjiang Astronomical Observatory, Chinese Academy of Sciences, Urumqi, Xinjiang 830011, China

Abstract

The coupled evolution of pulsar rotation and inclination angle in the wind braking model is calculated. The oblique pulsar tends to align. The pulsar alignment affects its spin-down behavior. As a pulsar evolves from the magnetodipole radiation dominated case to the particle wind dominated case, the braking index first increases and then decreases. In the early time, the braking index may be larger than three. During the following long time, the braking index is always smaller than three. The minimum braking index is about one. This can explain the existence of a high braking index larger than three and a low braking index simultaneously. The pulsar braking index is expected to evolve from larger than three to about one. The general trend is for the pulsar braking index to evolve from the Crab-like case to the Vela-like case.

Key words: pulsars; general – pulsars: individual (PSR J1640-4631) – stars: neutron

1. Introduction

The pulsar braking index reflects the slow-down law of pulsars (Lyne et al. 2015), \( \nu \propto -\nu^4 \), where \( \nu \) and \( \nu' \) are the frequency and frequency derivative, respectively, and \( n \) is the so-called braking index. Previously, eight young pulsars had braking indices reported (Lyne et al. 2015). Their values are all smaller than three, lying between 0.9 and 2.84. The braking index may be very useful for discriminating between the different spin-down mechanisms of pulsars. A braking index of three is predicted by the magnetic dipole braking assumption. The possible explanations for a braking index smaller than three include the presence of fallback disks (Liu et al. 2014), an increasing magnetic field (Espinoza et al. 2011), an increasing pulsar inclination angle (Lyne et al. 2013), and particle outflow in the magnetosphere (e.g., wind braking of pulsars; Kou & Tong 2015).

There is marginal evidence for the evolution of the pulsar braking index (Espinoza 2013). Recently, the pulsar PSR J1640–4631 was reported to have a braking index larger than three: \( n = 3.15 \pm 0.03 \) (Archibald et al. 2016a). It is very challenging to understand the existence of a braking index that is both larger than three and smaller than three. The eight pulsars with braking indices smaller than three could be understood with the wind braking model (Ou et al. 2016). By considering the coupled evolution of rotation and inclination angle, it is shown that both the high and the low braking indices of pulsars can be reproduced in the wind braking model. The pulsar braking index is expected to evolve from larger than three to about one. The wind braking model is based on Kou & Tong (2015). The inclusion of inclination angle evolution is based on the prescription of Philippov et al. (2014).

2. Coupled Evolution of Rotation and Inclination Angle in the Wind Braking Model

2.1. The Wind Braking Model of Pulsars Considering the Evolution of the Inclination Angle

A pulsar is generally an oblique rotator. The rotational evolution equation is (Michel & Goldwire 1970; Philippov et al. 2014)

\[ I \frac{d\Omega}{dt} = K, \]  
\[ \frac{d\Omega}{dt} = -\frac{2\mu^2\Omega^3}{3c^3} \left( \sin^2\alpha + 3\kappa \frac{\Delta\phi}{\Delta \Phi} \right) = -\frac{2\mu^2\Omega^3}{3c^3} \eta, \]  

where \( I = 10^{45} \text{g cm}^2 \) is the moment of inertia, \( \Omega \) is the angular velocity, and \( K \) is the torque working on the star. For a spherical system, Equation (1) can be expressed as

\[ I \frac{d\Omega}{dt} = K_{\text{spinning}}, \]

\[ I \frac{d\Omega}{dt} = -K_{\text{alignment}}, \]

where \( \alpha \) is the angle between the magnetic axis and the rotational axis (i.e., the inclination angle), \( K_{\text{spinning}} \) is the torque to spin-down the pulsar, and \( K_{\text{alignment}} \) is the torque to align the rotational and magnetic axes.

In the wind braking model, the rotational energy is conserved by the magnetic dipole radiation and particle acceleration. Equation (2) can be written as (Xu & Qiao 2001; Kou & Tong 2015)

\[ \frac{d\Omega}{dt} = -\frac{2\mu^2\Omega_3}{3c^3} \left( \sin^2\alpha + 3\kappa \frac{\Delta\phi}{\Delta \Phi} \right) = -\frac{2\mu^2\Omega_3}{3c^3} \eta, \]

where \( \mu = 1/2BR^3 \) is the magnetic dipole moment (\( B \) is the magnetic field and \( R \) is the neutron star radius), \( c \) is the speed of light, \( \kappa \) means that the primary particle density \( 4 \) is \( \kappa \) times the Goldreich–Julian charge density \( \left( \text{Goldreich & Julian 1969} \right) \), \( \Delta\phi \) is the acceleration potential in the acceleration gap, and \( \Delta \Phi = \mu \Omega_2 / c^2 \) is the maximum potential for a rotating dipole (Ruderman & Sutherland 1975). Here \( \eta \) is a dimensionless function that can be viewed as the dimensionless spin-down torque. The expressions of \( \eta \) for different acceleration models are described in Table 2 of Kou & Tong (2015). The vacuum

\[ 8 \text{ For a nonspherical star, the torque may also lead the rotational axis to deviate from the rotational-magnetic plane (i.e., precession). For the sake of simplicity, a spherical system is assumed.} \]

\[ 4 \text{ In the wind braking model, all the particles injected into the magnetosphere from the acceleration region are defined as "primary particles." These are relative to the "secondary particles," which are generated subsequently and responsible for the radio emission.} \]
gap model (Ruderman & Sutherland 1975) is taken as an example: \( \Omega_{\text{NG}} = \sin^2 \alpha + 4.96 \times 10^2 \kappa B_{12}^{-8/7} \Omega^{-15/7} \), where \( B_{12} \) is the magnetic field in units of \( 10^{12} \) G. Compared with the previous wind braking model, a \( \cos^2 \alpha \) factor is omitted. Phenomenologically, \( \cos^2 \alpha \) is a weighting factor between the magnetic-dipole radiation and particle wind. Besides, considering the result of the magnetospheric simulations (Li et al. 2012), the \( \cos^2 \alpha \) factor may not appear in the particle wind component.

Because the two components of the spin-down torque are independent of the inclination angle when the rotational axis is vertical (\( \alpha = 90^\circ \)) and parallel (\( \alpha = 0^\circ \)), respectively, to the magnetic axis, the spin-down torque and the alignment torque are related as \( K_{\text{alignment}} = [K_{\text{spinning}}(0^\circ) - K_{\text{spinning}}(90^\circ)] \sin \alpha \cos \alpha \) (Philippov et al. 2014). Then, Equation (3) can be written as

\[
I_0 \frac{d\alpha}{dt} = \frac{2\mu^2 \Omega^3}{3e^3} \sin \alpha \cos \alpha. \tag{5}
\]

The form of alignment torque in the wind braking model is the same as that in the vacuum magnetosphere and similar to that in the magnetohydrodynamical (MHD) simulation (Philippov et al. 2014).

For the long-term evolution of pulsars, the effect of pulsar death should be considered (Zhang et al. 2000; Contopoulos & Spitkovsky 2006). A detailed treatment of pulsar death can be found in Kou & Tong (2015).

2.2. Coupled Evolution of Rotation and Inclination Angle

To compare with the calculations of the vacuum magnetosphere (i.e., magnetic dipole braking) and the MHD simulation, the fiducial initial period \( P_0 = 10 \) ms, initial inclination angle \( \alpha_0 = 60^\circ \), and magnetic field \( B = 10^{12} \) G are assumed. The same parameters are used for the wind braking model. In addition, \( \kappa = 100 \) is used in the wind braking model. The primary particle density of young pulsars is at least 80 times the Goldreich–Julian charge density in the vacuum gap model (Kou & Tong 2015; Ou et al. 2016). A much larger particle density than the Goldreich–Julian density in the pulsar magnetosphere is also found in other models and observations (see discussions in Kou & Tong 2015 and references therein). Figure 1 (top panel) shows the evolution of the pulsar inclination angle. The inclination angle tends to align in these models. Compared with the vacuum magnetosphere model, the evolution rate of the inclination angle in the plasma-filled magnetosphere (MHD simulation and wind braking model) is smaller, which means that the particle will delay the alignment of the pulsar inclination angle. Figure 1 (bottom panel) shows the angular velocity evolution. In the vacuum magnetosphere, the spin-down behavior tends to stop, and the angular velocity tends to be a constant value when the inclination angle is very small. In the MHD simulation and wind braking model, the angular velocity evolution tends to 0. This is due to the presence of an additional spin-down torque, even in the case of a very small inclination angle. The difference between the MHD simulation and the wind braking model is due to the different form of the spin-down torque (Equation (4) in this paper and Equation (16) in Philippov et al. 2014).

The evolution of the pulsar braking index is shown in Figure 2. The braking index expected in the vacuum magnetosphere and MHD simulation is exactly three when not considering the evolution of the pulsar inclination angle (Philippov et al. 2014). Because of the inclination angle alignment, the braking index in the vacuum magnetosphere will always be larger than three, and the line evolves quickly. Due to the effect of particles in the magnetosphere, the braking index in the wind braking model evolves much more slowly and remains nearly three with time.
magnetosphere dominates the pulsar spin-down behavior, the pulsar braking index decreases. However, as the wind component begins to dominate the spin-down behavior, the pulsar braking index decreases. During the following long time, the braking index is smaller than three. The minimum braking index is 6/7 in the vacuum gap model (Ou et al. 2016). Generally, the minimum braking index is about one in the wind braking model.

Figure 3 shows the long-term evolution of the pulsars in the $P$–$P$ diagram. In the vacuum magnetosphere, the evolution line drops quickly before the group of rotation-powered pulsars. The line of the MHD simulation evolves down toward the right and through the rotation-powered pulsar group. In the wind braking model, the pulsar first evolves down to the right in the

index evolution in the MHD simulation is much gentler. However, the braking index in this case is also always larger than three (Arzamasskiy et al. 2015). Observations of braking indices smaller than three can be explained in the wind braking model. Furthermore, the inclination angle alignment also affects the braking index evolution in the wind braking model. According to the definition of pulsar braking index, its expression when considering the evolution of the inclination angle is

$$n = 3 + \frac{\Omega}{\eta} \frac{d\eta}{d\Omega} + \frac{\tau_c}{\tau_0} \frac{d\eta}{d\alpha},$$

where $\tau_c = -\frac{\Omega}{2\Omega}$ is the characteristic age and $\tau_0 = -\frac{\Omega}{2\Omega}$ is the evolution timescale of the inclination angle. An aligning inclination angle ($\tau_0 > 0$) leads to a larger braking index. The braking index increases to even larger than three during the early time, when magneto-dipole radiation dominates the spin-down torque. However, as the wind component begins to dominate the pulsar spin-down behavior, the pulsar braking index decreases. During the following long time, the braking index is smaller than three. The minimum braking index is 6/7 in the vacuum gap model (Ou et al. 2016). Generally, the minimum braking index is about one in the wind braking model.

Table 1

| $\alpha$ (°) | $\beta_{12}$ (10$^{12}$ G) | $\kappa$ | $\alpha_0$ (century$^{-1}$) | $P_0$ (ms) | $\alpha_0$ (°) |
|--------------|----------------|--------|----------------|-------------|----------------|
| 15           | 55             | 60     | -0.8           | 53          | 72             |
| 45           | 33             | 42     | -0.56          | 23          | 85             |
| 70           | 30             | 6      | -0.3           | 57          | 84             |

Note. $\alpha$ is the assumed present inclination angle, $\beta_{12}$ is the magnetic field in units of 10$^{12}$ G, and $P_0$ and $\alpha_0$ are the initial rotational period and inclination angle, respectively.

A high braking index ($>3$) is claimed for PSR J1640–4631 (Archibald et al. 2016a). Given the timing observations $\nu = 4.843$ s$^{-1}$, $\dot{\nu} = -2.28 \times 10^{-11}$ s$^{-2}$, $n = 3.15(3)$, and an assumed present inclination angle of PSR J1640–4631, the magnetic field, the particle density, and the evolution rate of the inclination angle can be calculated by Equations (4)–(6). Assuming a present age for PSR J1640–4631 (3000 yr for the present inclination angle of 45° and 70° and 2000 yr for 15°), the initial spin period and inclination angle can be calculated by integrating Equations (4) and (5). Table 1 shows the parameters of PSR J1640–4631 in the wind braking model. Comparably, the values of $\kappa$ are smaller. This means that the proportion of the particle wind of PSR J1640–4631 is relatively weak. The effect of the inclination angle alignment on the braking index cannot be covered by the particle wind effect. Hence, its braking index is larger than three. The present alignment rate of the inclination angle is $-0.56$ century$^{-1}$ if $\alpha_{\text{present}} = 45°$.

In the wind braking model, the predicted frequency third derivative is $\ddot{\nu} \approx -10^{-32}$ s$^{-4}$, corresponding to a second braking index of $m = \nu^2 \ddot{\nu} / \nu^3 \approx 18$. It is consistent with the present upper limits of $|\ddot{\nu}| < 1.4 \times 10^{-30}$ s$^{-4}$ (Archibald et al. 2016a).

Figure 4 (top panel) shows the braking index evolution of PSR J1640–4631 with time. In the early time, the braking index increases to even larger than three because of the inclination angle alignment. As the particle wind begins to dominate the spin-down behavior, its braking index decreases. During the following long time, its braking index is smaller than three. Figure 4 (bottom panel) shows the braking index evolution of PSR J1640–4631 as a function of rotational period. Compared with other young pulsars, PSR J1640–4631 lies in the magneto-dipole radiation dominated case, and its present braking index is larger than three. But as the pulsar
spins down, the effect of the particle wind becomes more and more important, and its braking index decreases. The red line in Figure 3 shows the evolution of PSR J1640–4631 in the $P$–$P$ diagram.

2.4. Comparison with the MHD Simulations

The wind braking model considers the pulsar as an oblique rotator. The magnetic dipole moment has components that are both perpendicular and parallel to the rotational axis. The parallel component may be responsible for the particle acceleration (Ruderman & Sutherland 1975). The perpendicular component may be approximated by the magnetic dipole radiation. Therefore, as an educated guess, the pulsar spin-down torque (Equation (4)) is made up of two components (Xu & Qiao 2001). The key input is the particle component, which provides a natural link between the timing and emission properties of the pulsars. In the presence of a particle component, the pulsar braking index lies between three and one.

Considering the recent progress of MHD simulations of pulsar magnetospheres (Spitkovsky 2006; Kalapotharakos et al. 2012; Li et al. 2012; Contopoulos et al. 2014; Philippov et al. 2014), a general form of pulsar spin-down torque is

$$I \frac{d \Omega}{dt} = -k_1 \frac{\mu^2 \Omega^3}{c^3} (\sin^2 \alpha + \text{"particle term"})$$

$$= -k_1 \frac{\mu^2 \Omega^3}{c^3} \eta,$$

where $k_1$ is a numerical factor. Mathematically, the term proportional to $\sin^2 \alpha$ may be dubbed the “dipole term,” while the remaining term may be dubbed the “particle term.” In the wind braking model, the numerical factor is $k_1 = 2/3$. In the MHD simulations, the numerical factor is $k_1 \approx 1$ (Spitkovsky 2006; Philippov et al. 2014). However, according to the “new standard pulsar magnetosphere” (Contopoulos et al. 2014), the numerical factor is $k_1 \approx 0.82$. Particle-in-cell simulations also found a numerical factor smaller than one (Philippov et al. 2015), because particle acceleration means the presence of vacuum regions in the pulsar magnetosphere. Furthermore, the difference between the numerical factor of the wind braking model and the MHD simulations will mainly affect the magnetic field strength. Therefore, a different numerical factor will not affect the conclusions here.

In the wind braking model, the particle term is $3 \kappa \Delta \alpha / \Delta \Phi$. It is determined by the number of outflow particles and the acceleration potential. If the acceleration potential is assumed to be equal to the maximum acceleration potential, and the particle number density is assumed to be equal to the Goldreich–Julian density, then the particle term in the wind braking model is 3. Considering a numerical factor of 2/3, the particle term is 2. In the MHD simulation, the particle term is 1. Therefore, the particle term in the MHD simulations can be deduced from the wind braking model by assuming a maximum acceleration potential and a Goldreich–Julian particle density. The only difference is a factor of 2. In the resistive MHD simulations (Kalapotharakos et al. 2012; Li et al. 2012), the particle term is slightly modified; see Equation (13) in Li et al. (2012). It is similar to that in Contopoulos & Spitkovsky (2006), $\sin^2 \alpha + (1 - V_{\text{drop}}/V_{\text{pc}})$, except for a different combination of angular factor. If the acceleration potential is assumed to be equal to the maximum acceleration potential, the corresponding braking index is always equal to three. Considering the effect of pulsar death or inclination angle evolution, the braking index is larger than three. However, in physical acceleration models for pulsar magnetospheres, the acceleration potential is different from the maximum acceleration potential (Xu & Qiao 2001 and references therein). Recent observations and modeling also show possible evidence of a much higher particle density in the pulsar magnetosphere (Kou & Tong 2015 and references therein). By considering these two aspects, the corresponding expression of the wind braking model is obtained.

In the original version of the wind braking model (Xu & Qiao 2001), the dimensionless torque is $\eta = \sin^2 \alpha + \ldots \times \kappa \cos^2 \alpha$. An additional angular factor $\cos^2 \alpha$ is present. This kind of combination is also found in Contopoulos & Spitkovsky (2006). In previous works on the wind braking of pulsars, the possible evolution of the pulsar inclination angle is not considered, and the inclination angle is constant. Since the particle density and inclination angle always appear as $\kappa \cos^2 \alpha$, for a constant inclination angle, there is degeneracy between the particle number density $\kappa$ and $\cos^2 \alpha$.
(Rogers & Safi-Harb 2017). If there is no \( \cos^2 \alpha \) in the \( \eta \) function, all the previous results can be obtained by replacing the corresponding particle density \( \kappa \) with \( \kappa \cos^2 \alpha \). This kind of ambiguity is already known when applying the wind braking model to intermittent pulsars (Li et al. 2014). In order to remain consistent with previous works, the \( \cos^2 \alpha \) factor is kept. When not considering the evolution of the pulsar inclination angle, this ambiguity will not affect the physical results. According to the relation between pulsar spin-down and alignment (Philippov et al. 2014), the inclination angle evolves to decrease the spin-down torque. If there is a \( \cos^2 \alpha \) in the dimensionless torque, then (1) in the dipole radiation dominant case \( (\eta \text{ is dominated by the dipole term, which is proportional to } \sin^2 \alpha) \), the inclination will decrease with time; and (2) as the pulsar evolves, the particle component begins to dominate \( (\eta \text{ is dominated by the particle term, which is proportional to } \cos^2 \alpha) \), and the inclination angle will increase with time. However, statistically, the pulsar inclination angle tends to decrease with time (Lyne & Manchester 1988; Tauris & Manchester 1998). Therefore, compared with the observations of the pulsar inclination angle, the possible \( \cos^2 \alpha \) factor may not appear.

The evolution of the inclination angle in the wind braking model (Equation (5)) is done in analogy with that of magnetic dipole braking and MHD simulations. Apart from a different numerical factor, the alignment equation in the magnetic dipole case and the MHD case can be viewed as the same (Equations (7) and (15) in Philippov et al. 2014). This is because they have the same dipole term: the one proportional to \( \sin^2 \alpha \). The term independent of the inclination angle in the spin-down torque does not contribute to the alignment torque. Since the wind braking model has the same dipole term, it is possible that the equation for the inclination angle evolution is also the same. Furthermore, according to the prescription of Equation (33) of Philippov et al. (2014), Equation (5) can be deduced when assuming that the coefficients A and B are independent of the inclination angle. Even if A or B depends on the inclination angle, that angle will always decrease with time for the spin-down torque of Equation (4). The resulting change is only quantitative. At an early age, when the magnetic dipole radiation dominates the spin-down torque for a decreasing inclination angle, the corresponding braking index may also be larger than three.

It is tempting to combine the MHD simulations with some amount of particle outflow. Then, the dimensionless spin-down torque will be of the form \( \eta = 1 + \sin^2 \alpha + \text{particle term} \). For young pulsars with a measured braking index, the inclination angle may not have decreased significantly; see Figure 1. For an inclination angle of 45° in the particle wind dominated case, the particle term will be significantly larger than the dipole term, which is proportional to \( \sin^2 \alpha = 1/2 \). Then, if the dipole term is replaced by the MHD results, the particle term will also be larger than the MHD term, which is proportional to \( 1 + \sin^2 \alpha = 3/2 \). Therefore, mathematically, such a combination will not affect the braking index evolution of young pulsars. Numerical calculations confirm this analysis. Furthermore, when the particle acceleration is introduced in some way in magnetospheric simulations (Kalapotharakos et al. 2012; Li et al. 2012; Philippov et al. 2015), the combination “MHD + particle term” is not found. The main modification is

the term “1” in the MHD simulations (Li et al. 2012). This may tell us that term 1 is associated with particle accelerations. Therefore, the results of resistive magnetospheric simulations are not in strong support of such a combination. However, the combination cannot be ruled out at present. The study of the pulsar braking index is not sensitive enough to discriminate between the two cases of “dipole + particle wind” and “MHD + particle wind.” Both a \( \sin^2 \alpha \) and a \( 1 + \sin^2 \alpha \) term will result in a braking index of three. Other pulsar observations may help to solve this problem, e.g., intermittent pulsars.

In summary, the importance of a particle outflow is stressed in the wind braking model. This particle term will result in a braking index smaller than three. Compared with the results of MHD simulations, there may be many assumptions in the wind braking model. However, these uncertainties will not affect the final conclusions. If future pulsar magnetosphere simulations can model the particle acceleration and injection into the magnetosphere more physically, the basic assumptions of the wind braking model can be tested.

3. Discussion and Conclusion

Lyne & Manchester (1988) analyzed the polarization information of hundreds of pulsars. The statistical studies showed that the inclination angle distribution is uniform for young pulsars but aligned for older pulsars (Lyne & Manchester 1988). By studying two groups of pulsar polarization data, Tauris & Manchester (1998) concluded that the pulsar inclination angle tends to align. The inclination angle evolution in the wind braking model is consistent with these observations. Lyne et al. (2013) proposed that the inclination angle of the Crab pulsar is increasing at a rate of \( 0.62 \pm 0.03 \text{ century}^{-1} \). On the one hand, an increasing inclination angle is one of the possible explanations for the steady increase in the separation of the main pulse and interpulse of the Crab pulsar (Lyne et al. 2013). On the other hand, the increasing inclination angle may be caused by the pulsar precession in the nonspherical case (Arzamasskiy et al. 2015).

There are other works that explain the observed high braking index of PSR J1640–4631. The braking index is larger than three in the plasma-filled magnetosphere because of the inclination angle alignment (Eksi et al. 2016; Figure 2 in this paper). The possible gravitational wave emission also leads to a high pulsar braking index (de Araujo et al. 2016; Chen 2016). However, these works only try to explain the braking index observations that are larger than three.

In the wind braking model, the braking index in the early time can be larger than three when considering the possible evolution of the inclination angle. At later time, it will evolve from about three to about one. Therefore, the general expectation is that pulsars with a higher braking index should be younger than those with a lower braking index. The pulsar braking index should evolve from that of a PSR J1640–4631–like case (larger than three), to that of a Crab-like case (about three), to that of a Vela-like case (about one). There is marginal evidence for the braking index evolution (Espinoza 2013). At a later time, the effect of pulsar death leads the pulsar to the death valley (Figure 3). A very large braking index may be expected in this case. However, for these old pulsars, the fluctuation in the magnetosphere dominates the frequency second derivatives.

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9 For old pulsars, such a combination will provide an additional spin-down torque when the inclination angle becomes very small.

8 The braking index can be as high as five, depending on the parameter space; see Figure 4.
and the observed $\dot{v}$ may be dominated by the fluctuation of the magnetosphere (i.e., timing noise; Ou et al. 2016).

The possible evolution of the pulsar braking index discussed above is for long-term evolutions. Two kinds of short-term evolutions of the pulsar braking index are possible (Kou et al. 2016). One is a discrete change of particle density. For a higher particle density, the pulsar is expected to have a higher spin-down rate and a lower braking index. The second is a secular change of particle density. An increasing particle density will result in a lower braking index while not affecting the spin-down rate significantly. The second case may correspond to the lower braking index of PSR J1846–0258 (Archibald et al. 2015). The state change and smaller braking index of PSR B0540–69 (Marshall et al. 2015, 2016) are consistent with the first case. The much smaller braking index of PSR B0540–69 than predicted may be due to secular changes in the particle density similar to those of PSR J1846–0258. A general prediction in the wind braking model is that when the magnetospheric activities are stronger, the braking index will be smaller. A smaller braking index for PSR J1119–6127 is expected after its outburst (Archibald et al. 2016b).

In conclusion, both a braking index higher than three and the small braking index of pulsars can be obtained in the wind braking model by including the evolution of the inclination angle. No additional braking mechanism is required. The pulsar braking index is expected to evolve from larger than three to about one. Future observations of more sources will help to clarify the possible long-term and short-term evolutions of the pulsar braking index.

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Note added. During the reviewing process, more pulsars with braking index measurements were reported. Gamma-ray timing of a young pulsar found a braking index of $n = 2.598$ (Clark et al. 2016). Vela-like glitching pulsars generally have braking indices of $n \leq 2$ (Espinoza et al. 2017). These observations are consistent with the prediction in the wind braking model; i.e., the braking index evolves from the Crab-like case to the Vela-like case.

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