Two-loop electroweak corrections to Higgs production at hadron colliders

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Abstract

We compute the two-loop electroweak corrections due to the top quark to the gluon fusion production cross section of an intermediate-mass Higgs boson. This result, together with the previously known contribution due to the light fermions, allows a complete determination of the two-loop electroweak corrections to this production cross section. For Higgs mass values above 120 GeV the top corrections have opposite sign of the light fermions contribution, however they are always smaller in size reaching at most 15\% of the latter. The total electroweak corrections to the gluon fusion production cross section for $115 \text{ GeV} \lesssim m_H \lesssim 2m_W$ range from 5\% to 8\% of the lowest order term.
The Higgs bosons, the still missing particle of the Standard Model (SM) responsible for the electroweak symmetry breaking, is actively searched for at the Tevatron and it is one of the main objectives of experimental program that will be carried out at the future Large Hadron Collider (LHC) which is supposed to span all the Higgs mass regions up to 1 TeV. The legacy of LEP has been a firm lower bound on the Higgs mass, $m_H > 114$ GeV [1], and at the same time, together with the information coming from SLD, a strong indirect evidence that the Higgs boson should be relatively light with a high probability for its mass to be below 200 GeV. A light Higgs is also required by the supersymmetric extensions of the SM, the SUSY models, which exhibit a definite prediction for the Higgs sector, namely that the lightest CP-even SUSY Higgs particle should not be heavier than 140-150 GeV at most [2].

At hadron colliders the main Higgs production mechanism is the gluon fusion process [3]. Therefore, a precise knowledge of this process is very important in order to put limits on the Higgs mass or, in case the Higgs is discovered, to compare the measured cross section with the Standard Model (SM) result. Given its relevance, the gluon fusion production mechanism has received in the recent years a large amount of theoretical work. Most of the attention has been paid to the calculation of the QCD corrections. The next-to-leading order (NLO) QCD corrections were first computed in the heavy top limit [4] and later keeping the full top mass dependence [5]. Next-to-next-to-leading corrections (NNLO) have also been computed, in the heavy top limit [6]. A recent discussion [7] on the residual theoretical uncertainty from perturbative QCD contributions, which includes also soft-gluon effects, estimates it to be below 10% for $m_H < 200$ GeV.

Electroweak corrections to the gluon fusion production mechanism were first considered in the heavy top limit and found to give a very small effect, below 1% [8]. Recently, the two-loop contribution due to the light fermions was discussed in Ref. [9]. That analysis shows that in the intermediate Higgs mass region, i.e., $114$ GeV $\lesssim m_H \lesssim 2m_W$, the contribution of the light fermions can be significant ranging from 4% to 9% of the lowest order term. Instead, above the $2m_W$ threshold the same analysis reports that the corrections are quite small.

Given the present status of the perturbative QCD uncertainty on the gluon fusion production cross section, it seems relevant to have a complete control of the electroweak corrections to this process at least in the intermediate Higgs mass region. This requires, besides the knowledge of the light fermions contribution, the result for the two-loop top corrections. The aim of this paper is to present such a contribution completing the calculation of the electroweak corrections to the $gg \rightarrow H$ process. Our investigation applies to values of the Higgs mass in the intermediate region.

The Higgs boson has no tree-level coupling to gluons; therefore the process $gg \rightarrow H$ is loop-induced. At the partonic level the cross section, not corrected by QCD effects, can be written as:

$$\sigma (gg \rightarrow H) = \frac{G_{\mu \alpha}^2}{512 \sqrt{2} \pi |G|^2},$$

where $G = G_{1l} + G_{2l} + \ldots$ and the lowest order one-loop contribution is only due to the top quark and is given by:

$$G_{1l} = -4 t_H \left[ 2 - (1 - 4 t_H) H \left( -r, -r; -\frac{1}{t_H} \right) \right],$$

where $t_H = G_{1l} + G_{2l} + \ldots$ and the lowest order one-loop contribution is only due to the top quark and is given by:

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$1$The contribution of the QED real corrections vanishes.

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where $t_H \equiv m_t^2/m_H^2$ and

$$H(-r, -r; x) = \frac{1}{2} \log^2 \left( \frac{\sqrt{x + 4} - \sqrt{x}}{\sqrt{x + 4} + \sqrt{x}} \right). \tag{3}$$

At the two-loop level the electroweak corrections to $\sigma (g g \to H)$ can be divided in two subsets: the corrections induced by the light (assumed massless) fermions and those due to the top quark, or

$$G_{2l} = G_{lf}^{2l} + G_{lt}^{2l}. \tag{4}$$

The two contributions are very different because the former one involves particles that do not appear in the one-loop calculation. In fact, at one loop, the light fermions do not contribute to the gluon fusion process because of their small coupling to the Higgs boson. Instead, at the two-loop level, the light fermions may couple to the $W$ or $Z$ bosons which in turn can directly couple to the Higgs particle. The light fermion contribution, $G_{lf}^{2l}$, contains also diagrams where the bottom quark, which is assumed massless, is present together with the $Z$ boson. The light fermion corrections have been computed exactly in Ref.\[9\] where the analytic result has been expressed in terms of Generalized Harmonic Polylogarithms.

To compute the top contribution, $G_{lt}^{2l}$, we find it convenient to employ the Background Field Method (BFM) quantization framework. The BFM is a technique for quantizing gauge theories\[10, 11\] that avoids the complete explicit breaking of the gauge symmetry. One of the salient features of this approach is that all fields are split in two components: a classical background field $\hat{V}$ and a quantum field $V$ that appears only in the loops. The gauge-fixing procedure is achieved through a non linear term in the fields that breaks the gauge invariance only of the quantum part of the lagrangian, preserving the gauge symmetry of the effective action with respect to the background fields. Thus, in the BFM framework some of the vertices in which background fields are present are modified with respect to the standard $R_\xi$ gauge ones. The complete set of BFM Feynman rules for the SM can be found in Ref.\[12\].

The advantage of this quantization scheme is that in the BFM Feynman gauge (BFG), the one-particle irreducible (1PI) contribution\[3\] and the reducible one, i.e. the Higgs wave function renormalization plus the expansion of the bare coupling $g_0/m_{w_0}$ ($g$ being the $SU(2)$ coupling), are separately finite. Concerning the latter, at the one-loop level $g_0/m_{w_0}$ can be related to the $\mu$-decay constant via

$$\frac{g_0}{m_{w_0}} = \frac{8 G_\mu}{\sqrt{2}} \left( 1 - \frac{1}{2} \left[ -\frac{A_{WW}}{m_W} + V + B \right] \right), \tag{5}$$

where $A_{WW}$ is the transverse part of the $W$ self-energy at zero momentum transfer and the quantities $V$ and $B$ represent the vertex and box corrections in the $\mu$-decay amplitude. In the BFG the combination

$$K_r = \frac{1}{2} \left[ \frac{A_{WW}}{m_W} - V - B + \delta Z_H \right] \tag{6}$$

is finite where $\delta Z_H$ is related to the Higgs field wave function renormalization through

$$H_0 = \sqrt{Z_H} H \simeq \left( 1 + \frac{1}{2} \delta Z_H \right) H. \tag{7}$$

\[2\] All the analytic continuations are obtained with the replacement $x \to x - i \epsilon$

\[3\] The top mass counterterm diagrams are needed to cancel divergencies in the two-loop irreducible corrections and, by definition, we include them in the 1PI contribution.
Figure 1: Examples of two-loop diagrams contributing to $gg \rightarrow H$.

We would like to stress that Eq. (6) is finite only in the BFG while in the standard $R_\xi$ Feynman gauge it shows some ultraviolet poles.

Explicitly, we have in the BFG and in units $\alpha/(4\pi s^2)$

$$K_r = N_c \frac{1}{w_t} \left[ -t_H - \frac{1}{8} + (t_H + 1) \sqrt{4t_H - 1} A(t_H) \right] + \frac{13 - 2\sqrt{3}\pi}{16w_H}$$

$$+ \frac{(3 + 4c^2) \log c^2}{8s^2} - \frac{3\log w_H}{8(1 - w_H)} + \frac{5 + 12z_H}{16c^2} + \frac{5 + 12w_H}{8}$$

$$- 3 \left( \frac{\sqrt{4w_H - 1}}{2} + \frac{2w_H^2}{\sqrt{4w_H - 1}} \right) A(w_H) - \frac{3}{2c^2} \left( \frac{\sqrt{4z_H - 1}}{2} + \frac{2z_H^2}{\sqrt{4z_H - 1}} \right) A(z_H), \quad (8)$$

where $N_c$ is the color factor, $s^2 \equiv \sin^2 \theta_W$, $c^2 = 1 - s^2$, $w_t \equiv m_t^2/m_W^2$, $w_H \equiv m_H^2/m_H^2$, $z_H \equiv m_Z^2/m_H^2$ and

$$A(x) = \arctan \frac{1}{\sqrt{4x - 1}}. \quad (9)$$

The two-loop top contribution to the gluon fusion production cross section can be written as:

$$G_t^{2l} = K_r \mathcal{G}_t^{1l} + \mathcal{G}_{1PI}^{2l}, \quad (10)$$

where $\mathcal{G}_{1PI}^{2l}$ contains the the two-loop 1PI corrections.

To compute $\mathcal{G}_{1PI}^{2l}$ we notice that the diagrams contributing to it can be naturally organized in two classes: i) diagrams with a triangular fermionic loop as well as top mass counterterm diagrams, that can be classified as corrections to the one-loop amplitude, like the one shown in Fig.(1a); ii) diagrams in which the Higgs does not couple directly to the top, Fig.(1b). We notice that in the BFG the two sets of diagrams are separately finite and equal to zero for vanishing Higgs mass.

To evaluate both kind of graphs we make the observation that, taken the bottom quark massless, some diagrams seem to have a cut at $q = 0$, see Fig.(2a), $q$ being the momentum carried by the Higgs. However, this cut is actually not present because of the helicity structure of the diagram. In fact, the Higgs should couple to one left-handed and one right-handed bottom quark, therefore the one-loop amplitude on the right-hand side of the dashed line in Fig.(2a) is non-zero only when the bottom quarks have opposite helicities, while in the tree amplitude on the left-hand side the helicity is conserved along the quark line. Since helicities cannot match, no cut develops at $q = 0$. In this situation, the first cut in these diagrams appears at $2m_H$ (see Fig.(2b)). Therefore, the evaluation of $\mathcal{G}_{1PI}^{2l}$ for Higgs mass in the intermediate region can be obtained by computing the relevant diagrams employing an ordinary Taylor expansion in the variable $h_{4w} \equiv q^2/(4m_W^2)$. 

\[4\]
Figure 2: a) The possible intermediate helicities of the massless fermions in a two-loop cut diagram. b) Amplitudes corresponding to a cut at $q = 2m_W$.

We follow this strategy to analytically evaluate the 1PI amplitudes up to $h_4^4$ terms. However, because the complete expression is quite long, we report explicitly only the leading term. We find in units $\alpha/(4\pi s^2)$

$$G_{1PI}^{2l} = \frac{1}{w_t} \left[ 1 + \frac{2t_H + 1}{4(4t_H - 1)} + \frac{t_H(2t_H - 5)}{2(4t_H - 1)^2} \log t_H - \frac{t_H(2t_H^2 + 1)}{2(4t_H - 1)^2} \phi \left( \frac{1}{4t_H} \right) \right] - \frac{28}{9} + \frac{7}{18c^2}$$

$$+ \frac{1}{w_H} \left[ \frac{w_t - 1 - \log w_t}{3(w_t - 1)^2} + \frac{5z_t - 2}{12(z_t - 4)^2} + \frac{46 - 19z_t}{6(z_t - 4)^3} \log z_t + \frac{z_t^2 - 6}{2z_t(z_t - 4)^3} \phi \left( \frac{z_t}{4} \right) \right]$$

$$+ O(h^4) \ ,$$

(11)

where $z_t = m_Z^2/m_t^2$ and

$$\phi(z) = 4 \sqrt{\frac{z}{1 - z}} Cl_2(2 \arcsin \sqrt{z}) \ ,$$

(12)

where $Cl_2(x) = \text{Im} \text{Li}_2(e^{ix})$ is the Clausen function.

To appreciate the convergence of the series, we show in table 1 the numerical values of the individual terms in the expansion of $G_{1PI}^{2l}$. We have further separated them in the two contributions corresponding to the diagrams of type i), labeled “fer”, and ii) labeled “bos”. The input values chosen in the table are: $m_t = 178$ GeV, $m_W = 80.4$ GeV and $m_Z = 91.18$ GeV. As expected, the “fer” contribution shows a very good convergence for Higgs masses in the intermediate region. In fact, in these diagrams the first cut actually occurs at $q = 2m_t$. In the same Higgs mass region, the “bos”part has a worse convergence. However, taking the last term in the expansion as error of the calculation we find that for $m_H = 160$ GeV we can assign to the “bos” contribution an uncertainty of 9%, which translates into a 5% uncertainty on the total top correction. Clearly, in the case of lower Higgs mass the uncertainty will be reduced.

Our result on the top corrections can be put together with the result of Ref.[9] on the light fermion contribution to obtain a complete prediction of the two-loop electroweak correction to the gluon fusion process for Higgs mass values in the intermediate region. In table 2, we compare the numerical values of the two-loop corrections to the amplitude due to a light fermion generation with that of the third generation. Concerning the latter, we have divided it in two parts. The first number corresponds to diagrams not containing the top, which have been computed exactly in Ref.[9]. The second one instead corresponds to $G_{1PI}^{2l}$, as calculated in this paper. The table shows that the contribution of the third generation is at most 20% of that of a light fermion. We notice that, in the units used in the table, the result of the large $m_t$ limit \[ amounts to
Table 1: Numerical values of the $h_{4w}^n$ terms in $G_{1PI}^{2l}$ in units $\alpha/(4\pi s^2)$ and their total. The label “fer” refers to diagrams of type i) while “bos” to type ii) ones.
Table 2: Contributions to the amplitude, in units $\alpha/(4\pi s^2)$, of one massless (from Ref.\cite{9}), and of the third generation and total relative correction to the cross section. The third generation contribution has been divided into a topless part (from Ref. \cite{9}, left number) and the rest (Eq.\!(10), right number).

$-m_t^2/(6m_W^2) = -0.82$. This number does not approximate at all the total correction, which is actually dominated by the light fermion contribution, and it is also quite different from the result of $G^2_t$ even in the case of low Higgs mass. Concerning $G^2_t$, we notice that the two terms in Eq.\!(10) tend to cancel each other, cf. table 1 and 2. In table 2 we also report the result for $m_H = 160$ GeV, namely in the mass region close to the $2m_W$ threshold that actually requires a more refined analysis. In fact, close to the threshold the factor $K_t$ in the top corrections behaves as $1/\sqrt{1-4w_H}$ for $w_H$ close to $1/4$, cf. Eq.\!(5). This unphysical singularity is related to the opening of the $2W$ channel in the wave-function renormalization of the Higgs \cite{13} and it is a signal that in this region a first order treatment of the $W$-boson propagator is inadequate. A way to eliminate this threshold singularity is to employ the definition of the mass and of the width of a particle from the complex-valued position of its propagator’s pole \cite{14}. Following this procedure we replace $m_W^2$ with $m_W^2 - i\Gamma_W m_W$ in the $A(w_H)/\sqrt{4w_H - 1}$ term appearing in Eq.\!(5). In the same table we have also reported the total electroweak relative corrections $\delta_{ew}$ to the Higgs production cross section $\sigma \equiv \sigma_0(1 + \delta_{ew})$, where $\sigma_0$ is the lowest order result.

The result we have derived for $\sigma(gg \to H)$ can be easily translated into a result for the corrections to the partial decay width $\Gamma(H \to gg)$ recalling the relation

$$\Gamma (H \to gg) = \frac{8m_H^3}{\pi^2} \sigma (gg \to H). \quad (13)$$

In conclusion, we have computed the top contribution to $\sigma(gg \to H)$ for an intermediate-mass Higgs particle completing the calculation of the two-loop electroweak correction to this important production cross section. In this Higgs mass range the electroweak corrections are dominated by the light fermion contribution computed in Ref.\cite{9}. The top contribution has opposite sign to it, apart from the Higgs mass region close to the present experimental lower bound, but in any case it is much smaller in size reaching at most 15% of the light fermion contribution.
To complete the calculation of the electroweak corrections to $\sigma(gg \to H)$ for any value of the Higgs mass the top corrections for $m_H > 2m_W$ are still needed. However, for this Higgs mass values the light fermion corrections are quite small and we do not expect the top corrections to have a size significantly different from the $m_H < 2m_W$ case. Thus, the total electroweak correction to $\sigma(gg \to H)$ for $m_H > 2m_W$ is probably quite small.

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