An extended wavelet approach of the money–output link in the United States

Mihai Mutascu¹,²,³ · Alexandre Sokic⁴

Received: 29 October 2021 / Accepted: 3 August 2022 / Published online: 14 August 2022
© The Author(s), under exclusive licence to Springer-Verlag GmbH Germany, part of Springer Nature 2022

Abstract
The paper examines the link between money and output in the U.S. by using wavelet analysis. The time span covers the period from 1960Q1 to 2021Q1. The main results evidence that money positively leads real output from the late 1960s to 1982, at medium frequency, with the interest rate playing an important role. In contrast, we reveal that real output negatively leads money, at the same medium frequency, but from the late 1990s to 2021. The COVID-19 pandemic generates significant co-movements in the short and medium frequencies, over the period from 2020 to 2021, with output negatively leading money. We underline that Federal Reserve monetary policy operating procedures play an essential role in explaining these findings. The results support using the federal funds rate operating procedure as countercyclical monetary policy measures and implementing unconventional monetary policy tools in times of the effective lower bound. Finally, no relationship between money and output is observed in the long term, while the short term (i.e. high frequency) reveals rather chaotic co-movements. The results remain robust to alternative wavelet tools, higher-frequency datasets, and a Hodrick–Prescott filtered quarterly sample.

Keywords Money · GDP · Growth · Interaction · Wavelets · U.S

Mihai Mutascu
mihai.mutascu@e-uvt.ro

Alexandre Sokic
alexandre.sokic@esce.fr

¹ Zeppelin University in Friedrichshafen, Am Seemooser Horn 20, 88045 Friedrichshafen, Germany
² Faculty of Economics and Business Administration, West University of Timisoara, 16, J. H. Pestalozzi St., 300115 Timisoara, Romania
³ LEO (Laboratoire d’Economie d’Orléans) and Labex Voltaire, CNRS FRE 2014, University of Orléans, Faculté de Droit d’Economie et de Gestion, Rue de Blois - B.P. 6739, 45067 Orléans, France
⁴ ESCE International Business School, OMNES Education, 10 rue Sextius Michel, 75015 Paris, France
1 Introduction

Determining whether monetary disturbances have played an essential role in real output fluctuations has been one of the most investigated research questions in macroeconomics. Thus far, that question remains an open empirical issue and is of primary importance for the conduct of monetary policy. The evidence of the relationship between money and output appears to be sensitive to different methodological approaches and varies over different time scales. Regarding the conduct of the monetary policy, the case of the Federal Reserve in the United States (U.S.) reveals that operating procedures have varied over the past 50 years, with alternating periods characterized by increased attention to either reserves or the federal funds rate as targets. In this context, the U.S. case is of particular interest. Money and output exhibit interesting dynamics over time, as Figs. 1 and 2 show. For reasons related to stylized facts, the M2 aggregate is preferred to M1 as M1 has been revised starting with May 2020 by including other liquid deposits).

Interestingly, the figures show that M2 and real GDP have sinusoidal evolutions, with three periods being prominent. Two episodes are characterised by high volatility (i.e. until the mid-1980s and since 2007), while one has low volatility coinciding with the period of the Great Moderation (i.e. from mid-1980s to the financial crisis of 2007–2008). Interesting is that an inverse connection between M2 and real GDP is observed from the late 1990s until 2021. More precisely, M2 and real GDP exhibited opposite dynamics during the whole interval. One noteworthy event is the period of the Great Recession, when real GDP collapsed from 2007 TO 2009 in parallel with

![Fig. 1 M2 and real GDP in the U.S. over 1960Q1–2021Q4 (annual quarter-to-quarter change). Source: performed based on a dataset offered by Federal Reserve Economic Data, Federal Reserve Bank of St. Louis, online database (FRED, 2022)
the increasing growth rate of the M2 monetary aggregate. This is clearly evidenced in 2009 when M2 reached a strong peak accompanied by a severe compressing of real GDP. Overall, M2 and GDP alternatively exhibit periods with both same-direction and opposite trends for the whole sample. Moreover, those signs seem to change when the dataset with different frequencies is considered (i.e. quarterly to annual frequency).

The empirical research on the relationship between money and output can be traced back to the influential empirical study of Friedman and Schwartz (1963). They found evidence of a positive correlation between money and output based on almost 100 years of U.S. data. They interpreted the positive correlation as evidence that money changes cause real output variations. Belongia and Ireland (2016) found that U.S. data over the 1967–2013 period reveal strong correlations between different monetary aggregates and real output, confirming Friedman and Schwartz’s (1963) findings for the prior 100 years. The first time-series econometric investigations started with Friedman and Meiselman (1963), giving rise to the famous St. Louis equations regressing nominal income on money. Friedman and Meiselman’s (1963) findings suggested a stable and significant relationship between output and money. Tobin (1970) was the first to raise the issue of causation by questioning the idea that a positive correlation between money and output could also mean that output might be causing money and not only the opposite, as initially interpreted Friedman and Schwartz (1963).

Introducing the notion of Granger causality into the debate, Sims (1972) pioneered vector autoregression (VAR) models and suggested that money drove output on post-war U.S. data. Sims (1980) also found that the role of money was reduced when including the influence of a nominal interest rate. The result of causality between money and output appeared to be sensitive not only to the dimension of the model, but also to different methods. Eichenbaum and Singleton (1986) suggested that the specification of the regressions in log first differences rather than in log levels reduced the impact of money. Christiano and Ljungqvist (1988), and Stock and Watson (1989) found that either using growth rates of money, narrow monetary aggregates, or the
systematic treatment of the trend specification led to results increasing the predicting power of money on output. Thus far, many investigations have used the VAR methodology, but these appear to have produced mixed results (Canova and Menz, 2011).

The causality between money and output also appeared to vary over different time-periods. Using recursive methods, Thoma (1994) revealed instability in the relationship, indicating that the causality between money and output was time-varying. Friedman and Kuttner (1992, 1993) documented the breakdown of the relationship between money and output in the U.S. in the 1980s. A similar result has been obtained for U.S. data by Berger and Österholm (2009). The authors indicated that the causality between money and output ceased around Paul Volcker’s chairmanship of the Federal Reserve (Fed) in the early 1980s. They also found a reduction in the role of money after the Great Moderation of the mid-1980s. By contrast, Swanson (1998) suggested a stable causality between money and output across time. Using VARs with time-varying parameters, Ravn et al. (2005) found changing causality patterns between money and output.

Furthermore, as discussed by Lucas (1996) in his Nobel lecture, there is in mainstream economics the common belief that money is neutral in the "long-run" but that it can impact real output in the “short-run”. As stressed by Lucas (1996), although the nature of “long-run” and “short-run” is ambiguous and relative, it refers to the notion of time horizon or time–frequency of the causality. Therefore, the causality between money and output should be investigated on different time periods and time frequencies. The development of time-series econometrics and the wavelet approach allows performing such investigations (Ramsey and Lampart, 1998).

Our paper aims to contribute to the open empirical issue of the causality between money and output using the wavelet methodological approach to investigate different time-periods and different time frequencies. Thereby, the aim is also to account for the different monetary policy operating procedures employed by the Federal Reserve over the past 50 years. To the best of our knowledge, only four papers in that field follow the wavelet approach. Out of them, three are devoted to the case of the United States.

Ramsey and Lampart (1998) seem to open this outside mainstream economics topic by using the discrete wavelet method to split the study period into different time horizons. On this ground, they connected the nominal personal income with monetary aggregates having as the main target the income velocity of money in the U.S. over 1960–1994, with monthly frequency. The authors evidenced there is a significant variation between considered variables across scales.

In a different approach, Caraiani (2012) analysed the connection between money and output in the U.S. by using wavelet coherency as continuous wavelet tool. His span covers the period 1960–2010, having a quarterly frequency. Separately, the author linked the interest rate with output with the same method. The main results reflect a weak co-movement between output and money within the Great Moderation, and a stronger one during the Great Recession. Additionally, at business cycle frequencies, the interest rate is strongly connected with output over the whole 1970s and 1980s. Tiwari et al. (2018) use wavelet coherency and partial wavelet coherency to explore the relationship between output, money and interest in Japan. The targeted period is
1972–2017, with monthly frequency. They stressed that the output and money are strongly linked at all frequencies, while the coherence increases when controlling for interest rate. A very important finding is that the authors evidenced that the money leads the output variable over most of the time across frequencies.

In a recent paper, Habimana (2019) investigated the U.S. compared with Sweden’s case. He preferred the discrete wavelet approach and classical Granger causality for each of the five timescales identified. The dataset runs from January 1990 until December 2014 for the U.S., and from January 1998 until December 2013 in the case of Sweden. The interest rate is also considered in the pair ‘money–output’. The author found a strong association between interest and money in both countries but having a shorter time horizon in the U.S. than in Sweden. Further, Habimana (2019, p. 85) revealed that “at finest timescales, output Granger causes money in Sweden, whereas it is the other way around in the U.S. At long-time horizons, there is a feedback between money and output in both economies”.

Four main literature gaps devoted to the case of U.S. by using the wavelet can be identified. First, no papers offering the lead-lag status of ‘money–output’ co-movement by controlling for the interest rate have been found. Second, no other wavelet tool to examine the link as an alternative to the wavelet coherency method has been used. Third, no studies checking for robustness by using datasets with different frequencies have been identified. Finally, no papers clarify the ambiguous nature of ‘output—money’ relationship at different time horizons.

Our paper addresses all those gaps mentioned above, significantly contributing to the literature in the field, especially to the strand of the literature that uses the wavelet as a core methodology. On this ground, a battery of wavelet tools is considered (i.e. wavelet coherency, partial wavelet coherency and wavelet cohesion) to analyse the U.S. by covering the period from 1960Q1 to 2021Q4.

The main results reveal that money positively leads real output over the late 1960s to 1982, at medium frequency, with the interest rate playing an important role. In contrast, we evidence that real output negatively leads money, at the same medium frequency, but over the late 1990s to 2021. The COVID-19 pandemic generates significant co-movements on short and medium frequencies, over 2020–2021, with output negatively leading money. We underline that Federal Reserve monetary policy operating procedures play an essential role in explaining those findings. It is noteworthy that no co-movements are observed at the low frequency. The findings are robust under the alternative wavelet tool, a Hodrick–Prescott filtered quarterly sample and a sample with monthly frequency.

The rest of the paper is as follows: Sect. 2 describes the data and methodology, Sect. 3 presents the main results, and Sect. 4 checks for robustness. Finally, Sect. 5 concludes.
2 Data and methodology

2.1 Data

Two interest variables are considered to analyse the interaction between money and output in the U.S., in the time-frequency domain: money aggregates and GDP. The dataset covers the period from 1960Q1 to 2021Q1.

Money is alternatively captured by the M1 and M2 monetary aggregates, as seasonally adjusted stocks, both of them expressed in billions of dollars. M1 represents the currency, demand deposits at commercial banks, and other checkable deposits. M2 includes M1 plus savings deposits (including money market deposit accounts), small-denomination time deposits less individual retirement account and Keogh balances at depository institutions, and balances in retail money market funds less individual retirement account and Keogh balances. Noteworthy is the change in measuring M1 adopted by the Federal Reserve in May 2020 that led to include savings deposits in M1. Since M2 includes M1, this change does not impact the size of the M2 monetary aggregate. The quarterly frequency form of the data is obtained based on the raw monthly frequency data, officially available, using the quarterly average.

Output is measured by GDP volume in billions of dollars as seasonally adjusted real GDP, while the monthly dataset is interpolated based on quadratic assumption.

Additionally, the interest rate is entered to control the magnitude of the ‘money–output’ interaction via the effective Fed Funds rate, in percentages. Like money variables, the interest rate is aggregated into quarterly frequency based on monthly data, officially available, by considering the quarterly average. The interest rate is taken into account to satisfy the conditions of New Keynesian models as the monetary aggregates register significant real economic effects (Woodford, 2003; Caraiani, 2012).

The dataset is collected from Federal Reserve Economic Data, Federal Reserve Bank of St. Louis, online database (FRED, 2022). In order to ensure a high level of volatility, a quarter-to-quarter annual percentage change is considered for all variables.

It is noteworthy that in the time-frequency domain, stationarity is not a mandatory condition, as Aguiar-Conraria et al. (2008, p. 2877) suggested. They underline that the wavelet is used “to quantify the degree of linear relation between two non-stationary time-series in the time-frequency domain”.

In the classical time domain, two non-stationary time-series integrated by the same order can reflect a stationary linear combination, referred to as cointegration. More precisely, the variables can exhibit a long-run equilibrium towards which their behaviour converges over time. Otherwise, the stationarity characteristic of a variable supposes that its mean, variance or covariance are constant over time. Unlike with cointegration approaches, stationarity is a mandatory condition to avoid any bias in estimations (e.g. non-stationary time-series can generate changing parameters).

Compared with time-series methods, the time-frequency domain reveals a different approach. Herein, time-series are seen as a signal. The frequency remains constant over time for a stationary signal, not adding much more information when the signals are interacted. By considering non-stationary signals, the wavelet is “able to yield information on the spatio-temporal organization between two time-series, or in other
words how two time-series evolve one against the other”. (Issartel et al., 2015, p. 2). In other words, if two time-series do not exhibit similar dynamics, the wavelet can support this by quantifying how different they are and what is the time lag between them from the behavioural point of view.

That is why making quarter-to-quarter corrections should not be seen as a “re-alization of a stationarity process with mean zero” (Percival and Walden, 2000, p. 319) but rather a way to increase the volatility of series. Therefore, the volatility in time-frequency seems to be a sort of “compromise” related to the property of non-stationarity.

2.2 Methodology

The targeted methodology to analyse the link between money and output is based on wavelets as time–frequency domain method. This approach offers several significant advantages compared to the classical one, as Mutascu (2018, p. 444) claims: “(1) offers short-, medium- and long-run frameworks; (2) details the interaction between variables across different frequencies over time; and (3) shows the lead-lag and cyclical vs. countercyclical status of the nexus”.

The tool’s core is the wavelet function having zero mean and infinite energy (e.g. the Haar, Morlet, Mexican hat, Paul or Daubechies wavelets). The Morlet wavelet is widely used in related empirical studies, offering "a good balance between time and frequency localization" (Grinsted et al., 2004, p. 563). The Morlet wavelet function \( \psi_0(\eta) \) is as follows:

\[
\psi_0(\eta) = \pi^{-\frac{1}{4}} e^{i\omega_0 \eta} e^{-\frac{1}{2} \eta^2},
\]

where \( \eta \) is the dimensionless ‘time’ parameter, \( \omega_0 \) denotes the dimensionless frequency parameter, and \( i = \sqrt{-1} \). The admissibility condition proposed by Farge (1992) is ensured by setting the dimensionless frequency to 6.

The wavelet transformation of series is the next step in this methodology. Herein, the time-series are converted in both time and frequency based on the Morlet wavelet, namely the continuous wavelet transformation (CWT). Unlike discrete wavelet transformation (DWT), which is characteristic for time-series treatment, the CWT is specific for feature-extraction purposes (Tiwari et al., 2013), as in our case.

For a given discrete time-series \( \{x_n\} \), the wavelet transformation has this form:

\[
W_n^x(s) = \frac{\delta t}{\sqrt{s}} \sum_{n'=0}^{N-1} x_{n'} \psi^* \left( \frac{(n' - m) \delta t}{s} \right), \text{wherem = 0, 1, ..., N - 1,}
\]

where \( \delta t \) represents the time spacing with a scale \( s \) (with \( n = 0...N-1 \)). More precisely, \( x \) is multiplied with \( \psi^* \) over \( r \) in each of its \( n \) levels, with a time-translated lag \( m \). Each complete iteration is repetitively computed by augmenting the scale \( s \). In this case, the wavelet sequentially decompresses as scale \( s \) increases, having the propensity to capture lower frequency (i.e. wavelet is compressed for high \( s \)).
For two transformed time-series $x = \{x_n\}$ and $y = \{y_n\}$, the co-movement between them is depicted by the wavelet coherency, including phase difference (Torrence and Campo, 1998; Grinsted et al., 2004; Ng and Chan, 2012).

Hudgins et al. (1993) offer one of the first versions of wavelet coherency (WTC), namely cross-wavelet spectrum (XWT), with this form:

$$\text{W}_{xy}^n = \text{W}_x^n \text{W}_y^n^*,$$

where the $\text{W}_x^n$ and $\text{W}_y^n$ denote the CWT of $x$ and $y$, while $|\text{W}_{xy}^n|$ is the cross-wavelet spectrum.

As the XWT “can show strong peaks even for the realisation of independent processes suggesting the possibility of spurious significance tests” (Aguiar-Conraria and Soares, 2011, p. 649), an improved version is proposed to fix those issues, being named WTC. It has the smoothing operator $S$ for both time and scale:

$$R_n(s) = \frac{|S(s^{-1}\text{W}_{xy}^n(s))|}{S(s^{-1}|\text{W}_x^n|)^{\frac{1}{2}} S(s^{-1}|\text{W}_y^n|)^{\frac{1}{2}}}.$$  

(4)

The WTC also allows to identify the phase angle or phase difference $\phi_{x,y}$ for two transformed time-series $x = \{x_n\}$ and $y = \{y_n\}$, as follows:

$$\phi_{x,y} = \tan^{-1}\left(\frac{\Im\{\text{W}_{xy}^n\}}{\Re\{\text{W}_{xy}^n\}}\right) \text{ and } \phi_{x,y} \in [-\pi, \pi],$$

(5)

where $\Re$ and $\Im$ represent the real and imaginary part of a complex number, respectively. Herein, $x$ leading $y$ when $\phi_{x,y} \in [0, \frac{\pi}{2}]$, the series being considered in phase. Otherwise, $y$ leads $x$ when $\phi_{x,y} \in [-\frac{\pi}{2}, 0]$. The series are considered in anti-phase for $\pi$ or $-\pi$ phase difference. In this case, $x$ leads $y$ when $\phi_{x,y} \in [-\pi, -\frac{\pi}{2}]$, while $y$ leads $x$ for $\phi_{x,y} \in [\frac{\pi}{2}, \pi]$, respectively.

Finally, the interaction between variables can be controlled by following the multiple wavelet coherency (MWC) and partial wavelet coherency (PWC) proposed by Mihanović et al. (2009).

The MWC considers the influence of more than two explanatory variables ($x_1, x_2, ... x_n$) on a dependent one ($y$), while the PWC connects two time-series $y$ and $x_1$, after removing the influence of the third one $x_2$ (i.e. $x_2$ is seen as control determinant).

For two transformed variables $x_1 = \{x_{1n}\}$ and $y = \{y_n\}$, with $x_2 = \{x_{2n}\}$ as control, the MWC has this form:

$$\left(RM_{n}^{y_{x_1} x_2}\right)^2 = \left(RM_{n}^{y_{x_1}}\right)^2 + \left(RM_{n}^{y_{x_2}}\right)^2 - 2Re\left(RM_{n}^{y_{x_1} x_2} RM_{n}^{y_{x_2} x_1}\right) \frac{1 - \left(RM_{n}^{y_{x_2} x_1}\right)^2}{1 - \left(RM_{n}^{y_{x_2} x_1}\right)^2}.$$  

(6)
Further, after removing the influence of the third variable $x_2$, the MWC becomes PWC, as follows:

$$\left( R P_n^{y,x_1x_2} \right)^2 = \frac{\left| R P_n^{y,x_1} - R P_n^{y,x_2} \right|^2}{\left[ 1 - (R P_n^{y,x_2})^2 \right] \left[ 1 - (R P_n^{y,x_1x_2})^2 \right]}.$$  (7)

In our study, the WTC, MWC and PWC routines\(^1\) are followed for the ‘money–output’ link by alternatively using M1 and M2 aggregates, with the interest rate as the control variable.

### 3 Findings

The WTC of co-movement between M1 and GDP and M2 and GDP is plotted in Figs. 3 and 4.

At high frequency (i.e. the short term\(^2\)), up to 4 quarters band of scale, the results seem to be rather idiosyncratic for both plots. A noteworthy episode is registered over 2008–2009, when M1 is out of phase regarding GDP. However, the lead-lag status is not conclusive, as the arrows generally are parallel with the X-axis. The COVID-19 pandemic period (i.e. 2020–2021) highlights an episode when both M1 and M2 are out of phase related to real GDP at high frequency. The lead-lag status indicates that real GDP negatively impacts money in this period (i.e. the arrows are pointed to the left and up). This result reveals the endogeneity of the money supply in line with the conventional Federal Reserve monetary policy operating procedure prevailing at that time. The federal funds interest rate was the main policy instrument.

Expecting a major recession to come, the Federal Open Market Committee (FOMC) met twice in March 2020, deciding to reduce the federal funds interest rate to its effective lower bound range of 0–0.25% (Clarida et al., 2021). This monetary policy response led to an increase in non-borrowed reserves, positively impacting monetary aggregates. Moreover, hitting the effective lower bound as early as March 2020, the Federal Reserve deployed all its non-conventional monetary policy options. Those include the forward guidance, liquidity provision, and large-scale asset purchases programmes (quantitative easing) to support the economy. Therefore, the Federal Reserve monetary policy response to the pandemic crisis increased the money supply.

The plots of WTC disclose two interesting results at the medium frequency (i.e. medium-term), between 4 and 32 quarters range of scale. First, money and real GDP are in phase for both M1 and M2, essentially over 1982. The M1-related plot indicates that M1 positively leads real GDP over the period from 1979 to 1982 (i.e. the arrows are pointed to the right and up). Similarly, Fig. 4 shows that M2 positively drives real GDP at medium frequency from the late 1960s to 1982 (i.e. the arrows are pointed to the right and up). The Keynesian expansionary fiscal policies conducted by the U.S. administration in the late 1960s and early 1970s were accompanied by an

---

\(^1\) The estimations are employed based on Matlab codes freely offered by Grinsted et al. (2004), with corrections and extensions of Ng and Chan (2012).

\(^2\) The time horizons are purely conventional.
expansionary monetary policy promoted by Federal Reserve to stimulate aggregate demand. This boosted the real GDP but neglected the inflationary pressures. Monetary expansions conducted by accommodating increases in non-borrowed reserves led to lower interest rates. Lower interest rates led to higher aggregate demand and real GDP, restoring money-market equilibrium in line with the liquidity effect that is well-known in monetary economics. From 1979 to 1982, the Federal Reserve adopted a non-borrowed reserves operating procedure to fight inflation pressures. Non-borrowed reserves were adjusted to achieve a targeted growth rate for the money supply (Walsh, 2017). As a result, the federal funds interest rate adjusted to shifts in reserve demand. The adjustment in short-term interest rates led to real GDP changes. For example, if the money stock was growing faster than targeted, the non-borrowed reserves target would be adjusted downward, leading to a higher federal funds rate and slower real output growth.

Second, and most interestingly, at medium frequency, money and real GDP are out of phase, essentially from the late 90s to 2021, when real GDP negatively drives money. Figure 3 indicates that real GDP co-moves M1 with a negative sign over 2001–2003, 2006–2009 and 2020–2021 (i.e. the arrows are pointed to the left and up). Similarly, Fig. 4 reveals that real GDP negatively drives M2 at medium frequency.
An extended wavelet approach of the money–output link…

Fig. 4 WTC ‘M2—GDP’ in U.S., for 1960Q1–2021Q12 (annual quarter-to-quarter). Note: (1) The thick black contour reflects the 5% significance being estimated from Monte Carlo simulations with phase randomized surrogate series (i.e. removing the trends, correcting the mismatch between start and end points and their related first derivatives, and subtracting the mean to avoid any influence of different mean values). The cone of influence (COI) appears as a lighted shadow, here the edge effects might distort the picture; (2) The power range goes from blue (low power) to yellow (high power); (3) The phase difference between the two series is shown by the arrows. The variables are in phase when the arrows are oriented to the right (positively linked). Money is leading when the arrows are oriented to the right and up, while GDP is leading when the arrows are pointed to the right and down. (4) The variables are out of phase when the arrows are oriented to the left (negatively linked). GDP is leading when the arrows are oriented to the left and up, while money is leading when the arrows are pointed to the left and down. (5) The variables have a cyclical effect in the phase and anti-cyclical effect in the anti-phase. (6) The X-axis symbolizes the time-period, while the Y-axis denotes the frequency as $2\pi/\text{period}$

over 2003–2021 (i.e. the arrows are pointed to the left and up). This outcome relates to the reverse causation argument of Tobin (1970), stating that output might be causing money. The endogenous nature of money is likely to be particularly relevant when the monetary authority employs a short-run interest rate as its main policy instrument. Over 2003–2021, the Federal Reserve employs a federal funds rate operating procedure where the federal funds interest rate is the key monetary policy instrument. In this case, non-borrowed reserves adjust to the federal funds interest rate target, thereby accounting for endogenous monetary aggregates. Within a federal funds rate operating procedure, a Taylor rule can account for the monetary policy response to the state of the economy. Taylor (1993) is one of the earliest to model the federal funds interest rate target as a function of inflation and the real output gap. According to that federal funds rate-setting behaviour, with everything else staying equal, when real output growth accelerates, the federal funds rate target should be raised. In turn, it would require a decline of non-borrowed reserves and then lead to a fall in the money supply. Inversely, supposing everything else equal, the rule would require a cut in the federal funds rate target when economic growth slows down. This would require an increase in non-borrowed reserves that would positively impact the money supply.
Taylor-type rules are commonly used to represent countercyclical monetary policy in empirical Dynamic Stochastic General Equilibrium (DSGE) models within the New Keynesian theoretical framework (Woodford, 2003). However, the global financial crisis of 2007–2008 led to major adjustments in the Federal Reserve’s monetary policy operating procedure. Between September 2007 and December 2008, the FOMC cut the federal funds rate target from 5.25% to zero, hitting the effective lower bound, where it remained until December 2015. Therefore, from 2008 U.S. monetary policy can no longer be represented in terms of a Taylor rule for setting the policy interest rate. To deal with the crisis and severe recession, the Federal Reserve developed and implemented unconventional tools of monetary policy involving forward guidance, liquidity provision, and large-scale asset purchase programmes (quantitative easing). These unconventional monetary policy responses led to a substantial monetary expansion accounting for output driving money negatively. Facing the pandemic crisis in March 2020, the Federal Reserve reacted in the same way after hitting the effective lower bound for its main policy rate. It deployed its kit of non-conventional monetary policy tools to support the economy. Once again, the Federal Reserve monetary policy responded to the pandemic crisis by increasing the money supply.

The plots of WTC for both M1 and M2 disclose no co-movement between money and real output at low frequency (i.e. long-term), for more than 32 quarters range of scale. This result is in line with the well-known proposition in monetary economics about the neutrality of money in the long-run. On this point, our results depart from those of Habimana (2019).

Figures 5 and 6 reveal the MWC plots of co-movement between M1 and GDP, and M2 and GDP, respectively, by entering the interest rate as the control variable.

![Fig. 5 MWC 'M1—GDP' in U.S., for 1960Q1–2021Q12—interest as control variable (annual quarter-to-quarter). Note: (1) The colour code denotes the intensity of correlations, going from blue (low correlation) to yellow (high co-movement intensity); (2) The X-axis symbolizes the time-period, while the Y-axis denotes the frequency as $2\pi$/period](image)
The plots underline the determinant role of interest on the ‘money–output’ interaction, its influence significantly augmenting the ‘money–output’ co-movement as the yellow areas strongly extend in the plots. The finding underlines the vital implications of the interest rate for M1 and M2 money aggregates and output.

Further, by removing the influence of the interest rate, Figs. 7 and 8 show the PWC of co-movement between M1 and GDP, and M2 and GDP, respectively.

The removal of the interest rate reveals that the co-movement is significant only on the right side, for both M1—and M2-related plots. This suggests that the interest rate is crucial for the ‘money–output’ link, especially over the period from 1960 to 1984. After removing the interest rate influence, the remaining yellow areas indicate that the ‘money–output’ nexus is not sensitive to this control. In other words, the co-movement remains solid irrespective of the control influence. This is particularly the case for the periods around 2009 and 2020–2021, when the effective lower bound was reached and non-conventional monetary policy measures were implemented. Otherwise, the co-movement is very sensitive to control when the yellow areas denoting the ‘money–output’ link disappear. This strongly supports the idea that the interest rate is the ‘basis’ of ‘money–output’ co-movement.

Overall, compared with similar wavelet approaches in the field devoted to the U.S., our results confirm Caraiani’s (2012) results by highlighting the role of the interest rate in the ‘money–output’ link during the 1960 and 1970s. Unlike these findings, we evidence that money leads output over the same time period, while output drives money at the end of the 1990s and beginning of the 2000s. Also as a novelty, we find that no relationship between money and output is registered at low frequency (i.e.
Fig. 7 PWC 'M1—GDP' in U.S., for 1960Q1-2021Q12—interest as control variable (annual quarter-to-quarter). Note: (1) The colour code denotes the intensity of correlations, going from blue (low correlation) to yellow (high co-movement intensity); (2) The X-axis symbolizes the time-period, while the Y-axis denotes the frequency as $2\pi/\text{period}$.

Fig. 8 PWC 'M2—GDP' in U.S., for 1960Q1–2021Q12—interest as control variable (annual quarter-to-quarter). Note: (1) The colour code denotes the intensity of correlations, going from blue (low correlation) to yellow (high co-movement intensity); (2) The X-axis symbolizes the time-period, while the Y-axis denotes the frequency as $2\pi/\text{period}$.
The results are inconsistent with Habimana (2019), who used a visibly shorter dataset by mixing the DWC method with the classical Granger causality test.

4 Robustness check

The robustness check is performed by following (i) the wavelet cohesion tool as an alternative method to WTC in order to examine the previous co-movement results, (ii) a sample with monthly frequency helping to replicate all the wavelet tools used previously, and finally (iii) analyzing a Hodrick–Prescott (HP) filtered sample with quarterly frequency to the main quarterly scenario.

(i) The wavelet cohesion (WC) method is developed by Rua (2010), trying to offer more information about the phase difference status of two time-series. The co-movement is measured through $\rho_{x_n y_n}$ as a real number over the range $[-1, 1]$. Exclusively following the real part of wavelet cross-spectra, it has this form:

$$\rho_{x_n y_n} = \frac{\Re(W_n^x W_n^y)}{\sqrt{|W_n^x|^2 |W_n^y|^2}}$$

Given its characteristics, the WC captures both positive and negative co-movements between two variables. The results of WC co-movement between M1 and GDP, and M2 and GDP, respectively, are illustrated in Figs. 9 and 10. Both the M1- and M2-related plots reinforce the WTC results, highlighting two essential episodes, at medium frequency, between 4 and 32 quarters range of scale. In the first episode, the variables are in phase (i.e. positive co-movement, suggested by yellow colour area), over the period 1960–1984, while in the second one, the variables are out of phase (i.e. negative co-movement, indicated by blue colour area), for the period 1984–2021. Additionally, the negative co-movements during the COVID-19 crisis are clearly evidenced over 2020–2021, at high and medium frequencies (i.e. short and medium-terms). Although some links are observed at the low frequency (i.e. long-term), at more than 32 quarters range of scale, they are not robust to WTC outputs.

(ii) A sample with monthly frequency is constructed in parallel to check for robustness, having quarterly frequency. The period is related to 1960M1–2021M12.

In this case, the raw quarterly variable GDP has been transformed in the monthly frequency series by using the quadratic estimation method. The rest of the variables (i.e. M1, M2 and the interest rate) are directly offered with monthly frequency. All variables are treated as monthly annual percentage changes. The same Federal Reserve Economic Data, Federal Reserve Bank of St. Louis, online database (FRED, 2022) is the data source. The variables have the same description as in the previous sample. By replicating all previous wavelet tools, including the WC one, the results are presented in Figs A1–A8 (Appendix). The plots reinforce the quarterly frequency results, which remain robust under both monthly and quarterly frequencies.

---

3 WC plots are performed based on Matlab code developed by Rua (2010).
Fig. 9 WC 'M1—GDP' in U.S., for the period 1960Q1–2021Q12 (annual quarter-to-quarter). Note: (1) The colour code shows the intensity of co-movement, going from blue (negative co-movement) to yellow (positive co-movement); (2) The X-axis symbolizes the time-period, while the Y-axis denotes the frequency as $2\pi$/period

Fig. 10 WC 'M2—GDP' in U.S., for the period 1960Q1–2021Q12 (annual quarter-to-quarter). Note: (1) The colour code shows the intensity of co-movement, going from blue (negative co-movement) to yellow (positive co-movement); (2) The X-axis symbolizes the time-period, while the Y-axis denotes the frequency as $2\pi$/period
(iii) An HP filtered sample with quarterly frequency is also employed for comparison with annual quarter-to-quarter results, as both approaches capture the cyclical components of the series. The new dataset covers 1960Q1–2021Q4. Herein, M1, M2 and the interest rate are estimated by considering their quarterly average, while real GDP is extracted based on its volume and real quarterly growth rates (FRED, 2022). By sequentially computing all previous wavelet tools, the results are presented in Figs. A9–A16 (Appendix). The plots fully align with the main quarterly results, with only a few co-movement areas disappearing in the short-term at the middle of the analyzed period. The bias comes from the HP filter limit spuriously affecting the dynamic differences between the data in the middle and those at the end of the sample (Hamilton, 2018).

5 Conclusions

The paper analyses the interaction between money and output in the U.S. by using a wavelet methodology. The dataset covers the period from 1960Q1 to 021Q1. The interest rate is entered to control the ‘money–output’ link. The wavelet-based analysis reveals that the co-movement between money and output appears to be varying across different time-periods and different frequencies.

The empirical investigation comes up with two interesting results at the medium frequency (i.e. over the medium-term), between 4 and 32 quarters range of scale. The first finding is related to the late 1960s to 1982, with money appearing to drive real GDP positively. This is consistent with an exogenous money supply as a vector of expansionary monetary policies and non-borrowed operating procedures. It contributes to validating the well-known liquidity effect in monetary economics. The result is in line with the findings of Caraiani’s (2012) wavelet-based investigation, highlighting the role of interest rate in the ‘money–output’ link during the 1960s and 1970s. Second and more interesting, real GDP appears to negatively drive money from the late 1990s to 2021. This outcome reveals an endogenous money supply resulting from the federal funds rate operating procedure adopted by the Federal Reserve over that period. In this case, the federal funds interest rate is the key monetary policy instrument, and a Taylor rule can account for the monetary policy response to the state of the economy. Also illustrative is the Federal Reserve’s remarkable monetary policy response to the Great Recession when the FOMC cut the federal funds rate target from 5.25% to zero between 2007 and 2008, thereby increasing the money supply. This finding supports the use of Taylor-type rules in empirical DSGE models within the New Keynesian theoretical framework (Woodford, 2003). The finding of real output negatively driving the money supply is also particularly relevant in periods when the effective lower bound prevailed, with the federal funds rate being essentially zero. Then, the Federal Reserve complemented its traditional monetary policy response with a non-conventional toolkit to deal with major recessions like the Great Recession of 2007–2009 or the most recent COVID-19 recession in 2020. These policy responses to recessions contributed to increase the money stock in a significant manner. For the specific pandemic period 2020–2021, it is noteworthy that the empirical results reveal that real GDP drives money in anti-phase also at high frequency (i.e. in the short-term).
The empirical analysis uncovers no co-movement between money and real output at low frequency (i.e. long-term), for more than 32 quarters range of scale. This result is in line with the well-known proposition in monetary economics about the neutrality of money in the long run. On this particular point, our results depart from those of Habimana (2019), who used a shorter sample by combining the DWC method with the classical time-domain tool.

Also noteworthy is the fact that we quasi-clarified the antinomy ‘short-long’ term regarding the link between output and monetary aggregates. The short-term (i.e. high frequency) reveals rather idiosyncratic co-movements, while no evidence of connections is found in the long-term.

Regarding policy implications, our results support the use of the federal funds rate operating procedure as countercyclical monetary policy measures and the implementation of non-conventional monetary policy tools in times when the effective lower bound is reached. Special attention requires the long-term (i.e. lower frequency), as no connection between money and output is evidenced here.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s00181-022-02294-6.

Funding No funds, grants, or other supports were received.

Declaration

Conflict of interest We confirm that we do not have any conflict of interest.

References

Aguiar-Conraria L, Azevedo N, Soares MJ (2008) Using wavelets to decompose the time-frequency effects of monetary policy’. Physica A 387:2863–2878
Aguiar-Conraria L, Soares MJ (2011) Oil and the macroeconomy: using wavelets to analyze old issues’. Empirical Economics 40:645–655
Belongia MT, Ireland PN (2016) Money and output: Friedman and Schwartz revisited. J Money Credit Bank 48(6):1223–1266
Berger H, Österholm P (2009) Does money still matter for U.S. output? Econ Lett 102:143–146
Canova F, Menz T (2011) Does money matter in shaping domestic business cycles? An international investigation. J Money, Credit Bank 43(4):577–607
Caraiani P (2012) Money and output: New evidence based on wavelet coherence. Econ Lett 116:547–550
Christiano LJ, Ljungqvist L (1988) Money does Granger-cause output in the bivariate money-output relation. J Monet Econ 22:217–235
Clarida RH, Duygan-Bump B, Scotti C (2021) The COVID19 Crisis and the Federal Reserve’s Policy Response, Finance and Economics Discussion Series 2021-035, Washington: Board of Governors of the Federal Reserve System
Eichenbaum M, Singleton KJ (1986) Do equilibrium real business cycle theories explain postwar U.S. business cycles?. In: NBER maroeconomics annual 1986, 91–146. Cambridge, MA: MIT Press
Farge M (1992) Wavelet transforms and their applications to turbulence. Annu Rev Fluid Mech 24:395–457
FRED (2022). Federal Reserve Economic Data, Federal Reserve Bank of St. Louis, online database. Accessed in February 2022.
Friedman BM, Kuttner KN (1992) Money, income, prices and interest rates. Am Econ Rev 82(3):472–492
Friedman BM, Kuttner KN (1993) Another look at the evidence on money-income causality. J Econom 57:189–203
Friedman M, Meiselman D (1963) The relative stability of monetary velocity and the investment multiplier in the United States: 1897–1958. In: Commission on money and credit: stabilization policies, pp 165–268. Englewood Cliffs, NJ: Prentice Hall
Friedman M, Schwartz A (1963) A Monetary History of the United States, 1867–1960. Princeton University Press, Princeton
Grinsted A, Moore SJ, Jevrejeva C (2004) Application of the cross wavelet transform and wavelet coherence to geophysical time series. Nonlinear Process Geophys 11:561–566
Habimana O (2019) Wavelet multiresolution analysis of the liquidity effect and monetary neutrality. Comput Econ 53:85–110
Hamilton JD (2018) Why you should never use the Hodrick-Prescott filter. Rev Econ Stat 100(5):831–843
Hudgins L, Friehe C, Mayer M (1993) Wavelet transforms and atmospheric turbulence. Phys Rev Lett 71(20):3279–3282
Issartel J, Bardainne T, Gaillot P, Marin L (2015) The relevance of the cross-wavelet transform in the analysis of human interaction - a tutorial. Front Psychol 5:1566
Lucas R (1996) Nobel lecture: monetary neutrality. J Polit Econ 104(4):661–682
Mihanović H, Orlić M, Pasic Z (2009) Diurnal thermocline oscillations driven by tidal flow around an island in the Middle Adriatic. J Mar Syst 78:S157–S168
Mutascu M (2018) A time-frequency analysis of trade openness and CO2 emissions in France. Energy Policy 115:443–455
Ng EKW, Chan JCL (2012) Geophysical applications of partial wavelet coherence and multiple wavelet coherence. J Atmos Ocean Technol 29:1845–1853
Percival D, Walden A (2000) Wavelet methods for time series analysis. Cambridge University Press, Cambridge
Ramsey JB, Lampart C (1998) Decomposition of economic relationships by timescale using wavelets. Macroecon Dyn 2(1):49–71
Ravn MO, Psaradakis Z, Sola M (2005) Markov Switching causality and the money-output relationship. J Appl Econom 20(5):665–683
Ria A (2010) Measuring co-movement in the time-frequency space. J Macroecon 32(2):685–691
Sims CA (1972) Money, income and causality. Am Econ Rev 62(4):540–542
Sims CA (1980) Comparisons of interwar and postwar business cycles. Am Econ Rev 70(2):250–257
Stock JH, Watson MW (1989) Interpreting the evidence on money-income causality. J Econom 40(1):161–181
Swanson NR (1998) Money and output viewed through a rolling window. J Monet Econ 41(3):455–474
Taylor JB (1993) Discretion versus policy rules in practice. In: Carnegie Rochester Conference Series on Public Policy vol 39, issue 1, pp 195–214
Thoma MA (1994) Subsample instability and asymmetries in money-income causality. J Econom 64(2):279–306
Tiwari A, Mutascu M, Andries A (2013) Decomposing time-frequency relationship between producer price and consumer price indices in Romania through wavelet analysis. Econ Model 31:151–159
Tiwari A, Olayeni O, Sherafatian-Jahromi R, Sodik O (2018) Output gap, money growth, and interest rate in Japan: evidence from wavelet analysis. Arthankiti-J Econ Theory Pract 1–12
Tobin J (1970) Money and income: post hoc ergo proctor hoc? Quart J Econ 84(2):301–317
Torrance C, Compo GP (1998) A practical guide to wavelet analysis. Bull Am Meteor Soc 79:605–618
Walsh CE (2017) Monetary theory and policy. The MIT Press, Cambridge
Woodford M (2003) Interest and prices: foundation of a theory of monetary policy. Princeton University Press, Princeton

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.