FAST TRACK COMMUNICATION

Timing analysis of two-electron photoemission

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Abstract

We predict a significant delay of two-electron photoemission from the helium atom after absorption of an attosecond XUV pulse. We establish this delay by solving the time-dependent Schrödinger equation and by subsequently tracing the field-free evolution of the two-electron wave packet. This delay can also be related to the energy derivative of the phase of the complex double-photoionization (DPI) amplitude which we evaluate by using the convergent close-coupling method. Our observations indicate that future attosecond time delay measurements on DPI of He can provide information on the absolute quantum phase and elucidate various mechanisms of this strongly correlated ionization process.

The attosecond streaking has made the characteristic timescale of electron motion in atoms experimentally accessible (Baltuska et al 2003, Kienberger et al 2004). Recent applications of this technique to atomic photoionization, both in the near-infrared (NIR) (Eckle et al 2008) and extreme ultraviolet (XUV) (Schultze et al 2010, Klünder et al 2011) spectral energy range, revealed a noticeable time delay between subjecting an atom to a short laser pulse and subsequent emission of the photoelectron. While in the NIR photon energy range such a delay can be related to nonadiabatic tunnelling (Yudin and Ivanov 2001), the XUV delay can be attributed to the energy-dependent quantum phase (de Carvalho and Nussenzveig 2002, Yakovlev et al 2010). This observation is particularly important as it allows for a complete characterization of the photoionization process in a so-called complete photoionization experiment (Cherepkov and Semenov 2004).

In an XUV time delay measurement (Schultze et al 2010), an attosecond XUV pulse sets the photoelectron in motion, while the phase-locked NIR streaking field is used to obtain the timing information. In our recent work (Kheifets and Ivanov 2010), we demonstrated that the XUV time delay can be determined accurately by solving the time-dependent Schrödinger equation (TDSE) and by tracing subsequent field-free evolution of the photoelectron wave packet without the explicit need for a streaking NIR field. In this work, we generalize this approach to the case of double photoionization (DPI). This non-trivial generalization requires considerable conceptual and computational efforts. First, we have to construct, and trace to large distances, a two-electron wave packet. Second, we have to relate our explicitly time-dependent approach with the standard characterization of the DPI process which is usually expressed in terms of a pair of symmetrized gerade and ungerade amplitudes (Briggs and Schmidt 2000). The moduli of these amplitudes and their relative phase can now be determined experimentally (Bolognesi et al 2003, Knapp et al 2005). In this communication, we demonstrate that additional information on the absolute individual phases of the DPI amplitudes can be obtained by an XUV time delay measurement. In our demonstration, we consider the helium atom driven by an XUV attosecond pulse with parameters similar to those employed in the attosecond streaking experiment on Ne by Schultze et al (2010). By solving the TDSE and by subsequently tracing the field-free evolution of the two-electron wave packet, we establish the apparent...
time zero’ when each of the two photoelectrons leaves the atom. This time depends sensitively on the photon energy and the energy sharing between the photoelectrons. Even though this timing information could be inferred implicitly from kinematically complete DPI experiments (Knapp et al 2002), it is only an XUV time delay measurement that can provide such information directly. This makes DPI of He an appealing process for future attosecond streaking experiments.

Individual attosecond streaking in a two-electron ionization process is most efficiently performed by directing emitted electrons parallel and perpendicular to the NIR field to respectively maximize and minimize its streaking effect (Smirnova et al 2005). To model such an experimental situation, we direct two photoelectrons to each other \( k_1 \perp k_2 \). We also distinguish the reference photoelectron, the one which would be streaked, and its spectator counterpart, which influences the reference photoelectron via their mutual Coulomb interaction. For simplicity of our analysis, both photoelectrons are kept in the same \( xz \) plane with the polarization vector of the XUV radiation directed along the \( z \)-axis.

As the driving XUV field, we construct a linearly polarized sech\(^2\)-shaped pulse with the carrier frequency \( \omega = 121 \text{ eV} \) and the peak field strength \( E_0 = 0.1 \text{ au} \). Such a pulse has a close to minimum time-bandwidth product and is characterized by the FWHM of \( \sim 250 \) as and \( \sim 8 \text{ eV} \) in the time and energy domains, respectively. The pulse is centred at \( t = 0 \), which we take as the physical ‘time zero’ in our model. The pulse is turned off outside the interval \( \pm T_1 \), where \( T_1 = 10T \) and \( T = 2\pi/\omega \) is the period of oscillation.

The time-dependent calculation of DPI of He was performed by a radial grid integration of the TDSE using the Arnoldi–Lanczos method (Park and Light 1986). Computational details can be found in our earlier work (Schuricke et al 2011). Specific to the present implementation, the angular variables were treated by partial wave expansion and the radial grid was composed of two intervals. On the first interval, a step was used with a number of points \( N = 200 \) and \( N = 500 \) were chosen. This resulted in a box size of \( R = 200 \text{ au} \). Possible reflection from the box boundary was eliminated by introducing an absorbing potential.

The field-free solution of the TDSE at \( t > T_1 \) is used to construct a two-electron wave packet \( \Psi_t(r_1, r_2, t) \) with the asymptotics corresponding to the given photoelectron momenta \( k_1, k_2 \). The wavepacket state is constructed from the solution of the TDSE by taking the projection

\[
\Psi_1(r_1, r_2, t) = \hat{P}_{k_1, k_2} \Psi(r_1, r_2, t),
\]

where the kernel of the projection operator is constructed as

\[
\{r_1', r_2'| \hat{P}_{k_1, k_2}|r_1, r_2\} = \int_{\Omega} \Psi_{q_1}(r_1) \Psi_{q_2}^*(r_2) \Psi_{q_2}^*(r_1')\]

\[
\Psi_{q_i}(r_i')^* dq_i'dq_2.
\]

Here \( \Psi_{q_i}(r_i), i = 1, 2 \), are one-electron scattering states with the ingoing boundary condition describing a photoelectron moving in the Coulomb field with \( Z = 2 \). The integration region is defined as \( \Omega = \Omega_1 \otimes \Omega_2 \), where \( \Omega_1 \) and \( \Omega_2 \) are spheres in momentum space centred around the momentum vectors \( k_1, k_2 \) so that \( |q_1 - k_1| < 0.25k_1 \).

The wavepacket state \( \Psi_1(t) \) can be expanded over the set of the double-contiuuma states of the He atom as

\[
\Psi_1(r_1, r_2, t) = \int dq_1 dq_2 f(q_1, q_2) \Psi_{q_1, q_2}^*(r_1, r_2) e^{-iEt},
\]

where \( E = q_1^2/2 + q_2^2/2 \). When both \( r_1 \) and \( r_2 \) are large,

\[
\Psi_{q_1, q_2}(r_1, r_2) \propto \exp[i(q_1 \cdot r_1 + q_2 \cdot r_2 + \gamma)],
\]

where \( \gamma \) is the Redmon logarithmic phase (Peterkop 1982). This leads to the following asymptotic expression:

\[
\Psi_1(r_{1,2} \to \infty, t > T_1) \simeq \int_{\Omega} dq_1 dq_2 f(q_1, q_2) \times \exp \left\{ i \left[ \arg f(q_1, q_2) + q_1 r_1 + q_2 r_2 + \gamma - E t \right] \right\}. \tag{5}
\]

The centre of the wave packet moves in such a way that its phase is stationary with respect to both \( q_1 \) and \( q_2 \) at the points \( k_1, k_2 \) of the centre of the wave packet:

\[
\dot{r}_i = k_i \left[ t - d \arg f(k_1, k_2)/dE_i - d \arg f(k_1, k_2)/dE_i \right]. \tag{7}
\]

The term containing the derivative of \( \gamma \) gives logarithmic (with \( i \)) corrections to the electron trajectory, which can thus be represented asymptotically as

\[
r_i(t) - k_it - r_i^*(t) \simeq k_it_0. \tag{8}
\]

Here \( t_0 = d \arg f(k_1, k_2)/dE_i \) are the time delays and \( r_i^*(t) \) are the known functions which vary logarithmically slowly with \( t \). The time offsets \( t_0 \) are respective delays relative to the time zero.

The wavepacket function is plugged into the one-electron density function

\[
\rho(r, t) = \int |\Psi_1(r_1, r_2, t)|^2 \left[ \delta(r_{1}) + \delta(r_{2}) \right] dr_1 dr_2. \tag{9}
\]

The maxima of this density function are then traced to determine the trajectories of both photoelectrons which, at large distances, can be approximated by equation (8).

As an illustration of this procedure, we consider a DPI process in which one photoelectron escapes with energy \( 32 \text{ eV} \) along the \( z \)-axis and another with energy 10 eV along the \( x \)-axis, thus sharing the excess energy of 42 eV. The snapshot of the electron density in the \( xz \) plane corresponding to the moment of time \( t = 14T \) (four field cycles elapsed after the end of the XUV pulse) is shown in the top panel of figure 1. The figure clearly exhibits the two well-formed maxima corresponding to the centre of the wave packet \( \Psi_t(r_1, r_2, t) \) propagating in the \( x \) and \( z \) directions. A sequence of such snapshots is taken with an interval of \( 2T \) and the maxima of the electron density are traced in time. With the known logarithmic function \( r_i^*(t) \), this procedure defines
respectively. Top: the electron density plot in the $xz$ plane at $t = 14T$. Bottom: trajectories of both photoelectrons as functions of time measured in numbers of field cycles. The straight lines visualize the free propagating of both photoelectrons which are exhibited in the bottom panel of figure 2. The error bars indicate the uncertainty of the fitting procedure. 

The inherent asymmetry of the CCC method in its treatment of the photoelectrons is clearly visible. For example, the two Coulomb phases $\delta_l(Z = 1)$ and $\delta_l(Z = 2)$ in equation (12), which substantially affect the overall phase of the DPI amplitude, are quite different. This physically corresponds to the shielding approximation with the faster photoelectron, labelled 1, asymptotically seeing the singly-charged ion and the slower one, labelled 2, the doubly charged ion. 

Knowing the asymptotics of the radial orbitals

$$R_{kl} \propto \sin[kr + \delta_l(k) + 1/k \ln(2kr) - \pi/2]$$

and applying the usual saddle-point approximation, we arrive to equation (6) with the following definition of the DPI amplitude:

$$f(k_1, k_2) = \sum_{l_1l_2} D_{ll_1}(k_1, k_2)Y_{l_1l_2}^l(\hat{k}_1, \hat{k}_2).$$

The time delay is related to the energy derivative of the phase of the DPI amplitude

$$t_{02} = \mathrm{d} \arg f(k_1, k_2)/\mathrm{d}E_j$$

taken over from the corresponding photoelectron energy. Due to an inherent asymmetry of the CCC methods, the photoelectrons are always distinguished as ‘fast’ $E_1 > E/2$ and ‘slow’ $E_2 \leq E/2$, where $E$ is the excess energy above the DPI potential. With this restriction, neither of photoelectrons can serve as the reference electron in the full excess energy range. In the following, we choose the photoelectron 2 to be the reference one. We compute the time delay $t_{02}$ which is defined for $E_2 \leq E/2$, and continue it analytically past the mid-excess energy point $E_1 > E/2$. 

Figure 1. Time evolution of the two-electron wave packet for 32 eV and 10 eV photoelectrons propagating along the $z$ and $x$ axes, respectively. Top: the electron density plot in the $xz$ plane at $t = 14T$. Bottom: trajectories of both photoelectrons as functions of time measured in numbers of field cycles. The straight lines visualize the free propagating $k_1(t - t_0)$ and $k_2(t - t_0)$. 

the trajectories $r_i(t) - r'_i(t)$ for both photoelectrons which are exhibited in the bottom panel of figure 2. The raw data, shown by the points, are fitted with the straight lines $k_i(t - t_0)$ and $k_i(t - t_0)$ which visualize the free propagation. The intersection of these straight lines with the abscissa gives the corresponding time delays $t_0$. We ran an analogous simulation with the directions of the fast and slow photoelectrons being swapped and obtained very similar time delay figures. We also explored other energy sharing cases. Thus obtained time delay data are collated in the bottom panel of figure 3. The error bars indicate the uncertainty of the fitting procedure. 

To relate the time delay to the phases of the DPI amplitudes, we employ the lowest order perturbation theory (LOPT), with respect to the field, on the basis of channel states of the convergent close-coupling (CCC) method (Kheifets and Bray 1996). This basis is composed of the products of a Coulomb wave $\chi_k$ (for the faster of the two photoelectrons) and a positive energy pseudostate $\phi_i$ (for the slower photoelectron) which is obtained by diagonalizing the He$^+$ Hamiltonian in a truncated Laguerre basis. Using this basis, we can expand the time-dependent wavefunction in the following form:

$$\Psi(r_1, r_2, t) = -i \sum_i \sum_k \int d^3k \langle \chi_k | D | \Phi_i \rangle \chi_k(r_1) \phi_i(r_2) \times e^{-iE_i t} \tilde{\mathcal{E}}(E_i - E_0).$$

Here, $\mathcal{E} = \int_{-\infty}^{\infty} e^{i\omega t} \mathcal{E}(\tau) \, \mathrm{d}\tau$ is the Fourier transform of the XUV field, $D$ is the two-electron dipole operator, $E_i = k_i^2/2 + \epsilon_i$ and $k_i^2/2 \geq \epsilon_i > 0$. By projecting the positive energy pseudostate onto the matching energy Coulomb wave $k_2^2/2 = \epsilon_{n_2}$, we restore the continuum normalization and phase and write a two-electron wave packet in the form of a partial wave expansion:

$$\Psi_i(r_1, r_2, t) = i \sum_{l_1l_2} \int d^3k_1 \tilde{D}_{l_1l_2}(k_1, k_2) \times R_{l_1l_2}(r_1) R_{l_2l_2}(r_2) Y_{l_1l_2}^l(\hat{r}_1, \hat{r}_2) e^{-iE_i t} \tilde{\mathcal{E}}(E - E_0).$$

Here, $Y_{l_1l_2}^l$ is a bipolar harmonic and $E = k_1^2/2 + k_2^2/2$. The two-electron dipole matrix element is defined as

$$\tilde{D}_{l_1l_2}(k_1, k_2) = (-)^{l_1+l_2} g_{l_1l_2}(Z = 1) + h_{l_1l_2}(Z = 2)$$

$$\times D_{l_1l_2}(k_1n_2) | l_2k_2 \parallel l_1n_2$$

with $D_{l_1l_2}(k_1n_2)$ to be found by integrating the bare dipole matrix element with the half-on-shell $T$-matrix (Kheifets and Bray 1996). 

The inherent asymmetry of the CCC method in its treatment of the photoelectrons is clearly visible. For example, the two Coulomb phases $\delta_l(Z = 1)$ and $\delta_l(Z = 2)$ in equation (12), which substantially affect the overall phase of the DPI amplitude, are quite different. This physically corresponds to the shielding approximation with the faster photoelectron, labelled 1, asymptotically seeing the singly-charged ion and the slower one, labelled 2, the doubly charged ion. 

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...
while the angle of the fast photoelectron $\theta_1$ is fixed and the slow electron angle $\theta_2$ is variable. In the bottom panel, directions of the photoelectrons are swapped and the angle of the slow photoelectron is fixed at $\theta_2 = 0^\circ$, while the angle of the fast photoelectron $\theta_1$ varies.

An example of the phase plot of the DPI amplitude is given in figure 2. In the $xz$ plane, the geometry of the two-electron escape is fully defined by the two azimuthal angles $\theta_1$, $\theta_2$. In the top panel of figure 2, the phase $\arg f(E_1, E_2)$ is plotted for the slow electron energies $E_2 = 6, 10$ and $30$ eV and the variable angle $\theta_2$ whereas the fast electron energy $E_1 = 40$ eV and its angle $\theta_1 = 0^\circ$ are fixed. In the bottom panel, directions of the photoelectrons are swapped and the angle of the slow photoelectron is fixed at $\theta_2 = 0^\circ$, while the angle of the fast photoelectron $\theta_1$ varies. The phases of the DPI amplitudes displayed in figure 2 can be used to obtain the timing information of the two complementary processes in which the slow reference photoelectron is directed along with (bottom panel) and perpendicular to (top panel) the XUV field.

From the inspection of figure 2, we see that the phases of the DPI amplitudes depend sensitively on the mutual photoelectron orientation. However, the spacing between the various $E_2$ phase curves does not change significantly with the variable photoelectron angle near $\theta_2 \approx 90^\circ$. So the energy derivative of the phase of the DPI amplitude, and hence the effective time delay, do not change very much with the relative orientation of the photoelectrons. This is an important observation which allows one to bin together the attosecond streaking data from various orientations of the spectator photoelectron, thus increasing the data collection efficiency.

The vertical line in both panels of figure 2 marks the mutual angle of the photoelectrons $\theta_{12} = 90^\circ$. The phases in both panels are very similar for this orientation which means that the time delay of the reference photoelectron does not depend significantly on its orientation relative to the XUV field. We have already acknowledged this fact when analysing the TDSE time delay results.

Weak sensitivity of the time delay to the field orientation can be understood from the general parametrization of the DPI amplitude (Briggs and Schmidt 2000):

$$f(k_1, k_2) = [\cos \theta_1 + \cos \theta_2] M^\theta(E_1, E_2, x) + [\cos \theta_1 - \cos \theta_2] M^u(E_1, E_2, x).$$

Here $x = \cos \theta_{12} = \cos(\theta_2 - \theta_1)$ and the complex gerade $M^\theta$ and ungerade $M^u$ amplitudes possess the exchange symmetry $M^{\theta/\mu}(E_1, E_2) = \pm M^{\mu/\theta}(E_2, E_1)$. Even for the most severe energy sharing $E_2 \ll E_1$, the gerade amplitude is still strongly dominant $|M^\theta| \gg |M^u|$ (Kheifets and Bray 2002). Therefore, unless the photoelectrons are anti-parallel and the kinematic factor accompanying the gerade amplitude tends to zero, its contribution is dominant and it is $M^\theta$ that determines the overall DPI phase. This means that the timing measurement at perpendicular photoelectron orientation can only deliver the $M^\theta$ phase. An analogous measurement for $M^u$ would require the anti-parallel orientation which is not practicable for individual photoelectron streaking. However, since the relative phase of $M^{\theta/\mu}$ can be determined independently,
knowing the $M^\theta$ phase will immediately deliver the missing phase of $M^\varphi$.

In the top panel of figure 3, we show a cumulative phase plot $\arg f(E_1, E_2)$ for various combinations of the reference and spectator electron energies taken at the perpendicular orientation $\theta_{12} = 90^\circ$. The raw CCC data, marked by the points, are only available for the $E_2 \leq E_1$. To obtain the phases across the whole excess energy range, we fit the raw CCC data with a rational function and continue it analytically past the mid-excess energy point. The energy derivative of this function, calibrated in units of time delay, is presented in the bottom panel of figure 3. The values of time delay obtained from the solution of the TDSE are marked by dots. We observe that TDSE and CCC results for the time delays are quite close. The major deviation between predictions of the two models is at the top end of the energy scale where the CCC results are somewhat below the TDSE. This difference exceeds the error bars which accounts for uncertainty in the trajectory tracing.

The time delay of the reference photoelectron varies rapidly with its energy but depends more weakly on the energy of the spectator electron. When the energies of both electrons are low, the time delay is particularly large, in excess of 100 as. In the opposite limit of large reference electron energy, the time delay becomes small. More importantly, it does not vary significantly with the spectator electron energy. Physically, this regime corresponds to the shake-off mechanism of DPI in which the fast photoelectron absorbs the whole of the photon energy and angular momentum and the slow photoelectron is subsequently shaken off into the continuum (Knapp et al 2002). The fast photoelectron leaves the atom without any significant delay, but emission of the slow photoelectron is delayed considerably. This delay becomes particularly large when the energy of both photoelectrons is small and they are able to interact for a long time. In this regime, the main mechanism of DPI is the knock-out process in which the primary photoelectron impinges on the ion and knocks out the secondary electron into the continuum.

A rapid change of the time delay as a function of the streaking electron energy may make an experimental comparison with the present calculation quite difficult. To address this issue, we performed a Gaussian convolution of the CCC results with the bandwidth function of the XUV pulse. We found that for the photoelectron energy above 10 eV, this convolution does not change our calculations in any appreciable way.

In conclusion, we perform the timing analysis of the two-electron emission from the He atom which is subjected to an attosecond XUV pulse. We employ an explicit time-dependent treatment of the DPI process by seeking solution of the TDSE. We complement this procedure by the LOPT treatment which allows us to connect the time delay with the energy-dependent phase of the DPI amplitude, the latter being evaluated within the CCC method. This opens up a possibility of a complete DPI experiment in which both the magnitudes and phases of the symmetrized DPI amplitudes can be determined. Such an experiment will require an attosecond streaking measurement on one of the two photoelectrons which can be performed in a close to perpendicular orientation of the photoelectrons. To our best knowledge, except for a very recent report (Emmanouilidou et al 2010), this is the first practical attosecond measuring scheme suggested for a DPI process.

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References

Baltuska A et al 2003 Attosecond control of electronic processes by intense light fields Nature 421 611
Bolognesi P et al 2003 A procedure to extract the complex amplitudes of He photo-double ionization from experimental data J. Phys. B: At. Mol. Opt. Phys. 36 L241
Briggs J S and Schmidt V 2000 Differential cross sections for photo-double-ionization of the helium atom J. Phys. B: At. Mol. Opt. Phys. 33 R1–48
Cherepov N A and Semenov S K 2004 On a complete experiment on photoionization of atoms J. Phys. B: At. Mol. Opt. Phys. 37 1267
de Carvalho C A A and Nussenzveig H M 2002 Time delay Phys. Rep. 364 83–174
Eckle P et al 2008 Attosecond ionization and tunneling delay time measurements in helium Science 322 1525–9
Emmanouilidou A, Staudte A and Corkum P B 2010 The two-electron-attosecond streak camera for time-resolving intra-atomic collisions New J. Phys. 12 103024
Kheifets A S and Bray I 1996 Calculation of double photoionization of helium using the convergent close-coupling method Phys. Rev. A 54 R995–7
Kheifets A S and Bray I 2002 Symmetrized amplitudes of the helium-atom double photoionization Phys. Rev. A 65 022708
Kheifets A S and Ivanov I A 2010 Delay in atomic photoionization Phys. Rev. Lett. 105 233002
Kienberger R et al 2004 Atomic transient recorder Nature 427 817–21
Klünder K et al 2011 Probing single-photon ionization on the attosecond time scale Phys. Rev. Lett. 106 143002
Knapp A et al 2002 Mechanisms of photo double ionization of helium by 530 eV photons Phys. Rev. Lett. 89 033004
Knapp A et al 2005 Photo double ionization of helium 100 eV and 450 eV above threshold: III. Gerade and ungerade amplitudes and their relative phases J. Phys. B: At. Mol. Opt. Phys. 38 645
Park T J and Light J C 1986 Unitary quantum time evolution by iterative Lanczos reduction J. Chem. Phys. 85 5870–6
Peterkop R 1982 On the asymptotic form of wavefunction for the ionisation problem J. Phys. B: At. Mol. Phys. 15 L751
Schulzke M et al 2010 Delay in photoemission Science 328 1658–62
Schuricke M, Zhu G, Steinmann J, Simeonidis K, Ivanov I, Kheifets A, Grum-Grzhimailo A N, Bartschat K, Dorn A and Ullrich J 2011 Strong field ionization of lithium Phys. Rev. A 83 023413
Smirnova O, Yakovlev V S and Ivanov M Y 2005 Use of electron correlation to make attosecond measurements without attosecond pulses Phys. Rev. Lett. 94 213001
Yakovlev V S, Gagnon J, Karpowicz N and Krausz F 2010 Attosecond streaking enables the measurement of quantum phase Phys. Rev. Lett. 105 073001
Yudin G L and Ivanov M Y 2001 Physics of correlated double ionization of atoms in intense laser fields: quasistatic tunneling limit Phys. Rev. A 63 033404