\textbf{D}³\textbf{C}²\textbf{-Net: Dual-Domain Deep Convolutional Coding Network for Compressive Sensing}

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\section*{Abstract}

Mapping optimization algorithms into neural networks, deep unfolding networks (DUNs) have achieved impressive success in compressive sensing (CS). From the perspective of optimization, DUNs inherit a well-defined and interpretable structure from iterative steps. However, from the viewpoint of neural network design, most existing DUNs are inherently established based on traditional image-domain unfolding, which takes one-channel images as inputs and outputs between adjacent stages, resulting in insufficient information transmission capability and inevitable loss of the image details. In this paper, to break the above bottleneck, we first propose a generalized dual-domain optimization framework, which is general for inverse imaging and integrates the merits of both (1) image-domain and (2) convolutional-coding-domain priors to constrain the feasible region in the solution space. By unfolding the proposed framework into deep neural networks, we further design a novel \textbf{D}ual-\textbf{D}omain \textbf{D}eep \textbf{C}onvolutional \textbf{C}oding \textbf{N}etwork (D³\textbf{C}²\textbf{-Net})\textsuperscript{1} for CS imaging with the capability of transmitting high-throughput feature-level image representation through all the unfolded stages. Experiments on natural and MR images demonstrate that our D³\textbf{C}²\textbf{-Net} achieves higher performance and better accuracy-complexity trade-offs than other state-of-the-arts.

\section{Introduction}

As a novel methodology of acquisition and reconstruction, compressive sensing (CS) aims to recover the original signal from a small number of its measurements acquired by a linear random projection \[1, 2\], which has been successfully used in many applications, such as single-pixel imaging \[3, 4\], accelerating magnetic resonance imaging (MRI) \[5\] and snapshot compressive imaging (SCI) \[6, 7, 8\].

Mathematically, given the original vectorized image \(x \in \mathbb{R}^N\) and a sampling matrix \(\Phi \in \mathbb{R}^{M \times N}\), the CS measurement of \(x\), denoted by \(y \in \mathbb{R}^M\) is formulated as \(y = \Phi x + n\), where \(n\) is the additive white Gaussian noise (AWGN) with standard deviation \(\sigma (\sigma = 0\) indicates ”noiseless”). The purpose of CS reconstruction is to infer \(x\) from its obtained \(y\). Considering \(M \ll N\), CS is a typical ill-posed inverse problem, whereby the CS ratio (sampling rate) is defined as \(\gamma = M/N\). Generally, conventional model-based CS methods reconstruct the latent clean image \(x\) by solving the following optimization problem:

\[
\min_x \frac{1}{2} \|\Phi x - y\|_2^2 + \lambda \phi(x), \tag{1}
\]

where \(\phi(\cdot)\) denotes a prior-regularized term with \(\lambda\) being the regularization parameter. For traditional CS methods \[9, 10, 11, 12, 13\], the prior term is usually hand-crafted sparsifying operator corresponding to some pre-defined transform basis, such as wavelet and discrete cosine transform (DCT) \[14, 15\]. Although these model-based methods enjoy the advantages of interpretability and

\textsuperscript{1}For reproducible research, the source code with pre-trained models of our D³\textbf{C}²\textbf{-Net} will be made available.
Figure 1: Illustration of the idea of convolutional coding. An image $x$ is represented by the combination (sum) of multiple image-level convolution results, i.e., $x = \sum_{i=1}^{C} d_i \ast \alpha_i$, where $d_i \in R^{h \times k}$ is the $i^{th}$ dictionary filter, $\alpha_i \in R^{h \times w}$ is the $i^{th}$ feature map, $\ast$ is the convolution operator and $C$ is the number of feature channels. The darker red (or blue) colors in each visualized filter correspond to the positive (or negative) entries with larger absolute values. Compared with the vanilla single-channel image data, this type of feature-level representation naturally enjoys higher capacity and flexibility.

strong convergence guarantees, they inevitably suffer from high computational complexity and the difficulty of choosing optimal transforms and hyper-parameters [16, 17].

With the rapid development of deep learning in recent years, many deep network-based image CS reconstruction methods have been proposed, generally divided into deep non-unfolding networks (DNUNs) and deep unfolding networks (DUNs). Treating CS reconstruction as a denoising problem, DNUNs directly learn the inverse mapping from the CS measurement $\Phi x$ to the original image $x$ through end-to-end networks [18, 19, 20, 21, 22, 23, 24], which seriously depend on careful tuning and lead to complex theoretical analysis. DUNs combine deep neural networks with optimization methods and train a truncated unfolding inference in an end-to-end fashion [25, 26, 27, 28, 29]. DUNs are composed of a fixed number of stages, and each stage corresponds to a iteration. Due to well-defined interpretability and superior performance, DUNs have become the mainstream for CS.

However, most existing DUNs are inherently designed based on traditional image-domain unfolding, where the input and output of each stage are one-channel images, with poor representation capacity, i.e., channel number reduction from multiple to one at the end of each stage, leading to inevitable limited feature transmission capability and the loss of image details [25, 26, 29, 27].

Recently, convolutional coding methods have been successfully adopted in DUNs [30, 31, 32]. As shown in Fig. 1, through the convolutional coding model, an image $x \in R^{h \times w}$ is represented as $x = D \ast \alpha = \sum_{i=1}^{C} d_i \ast \alpha_i$, where $\ast$ is the 2D convolution operator and $C$ is the number of channels; $D \in R^{C \times h \times k}$ is the convolutional dictionary and $d_i$ is the $i^{th}$ dictionary filter; $\alpha \in R^{C \times h \times w}$ is the feature map of image $x$ and $\alpha_i$ is the $i^{th}$ channel of $\alpha$. Taking the natural advantage of $\alpha$ being $C$-channel, these convolutional-coding-domain-based DUNs can transmit high-throughput information between stages. However, they only focus on specific tasks such as rain removal [31] and image denoising [32], lacking generalizability and flexibility.

To address the above issues, in this paper, we propose a Dual-Domain Deep Convolutional Coding Network, dubbed D$^3$C$^2$-Net, focusing on CS reconstruction. Specifically, we design a novel dual-domain unfolding framework, which resolves the lack of generalizability of existing methods, allows our D$^3$C$^2$-Net to transmit high-throughput information and inherits the advantages of image and convolutional-coding domain constraints. The proposed D$^3$C$^2$-Net can be viewed as an attempt to bridge the gap between convolutional coding methods and neural networks in the CS reconstruction problem, with the merits of clear interpretability and sufficient information throughput.

Our main contributions are three-fold: (1) We propose a novel generalized dual-domain optimization framework, which integrates the merits of both image-domain and convolutional-coding-domain priors to constrain the feasible solution space and can be easily generalized to other image inverse problems. (2) We design a new Dual-Domain Deep Convolutional Coding Network (D$^3$C$^2$-Net) for general CS reconstruction based on our proposed framework. Our D$^3$C$^2$-Net can transmit high-throughput feature-level image representation through all unfolded stages to capture sufficient features adaptively, thus recovering more details and textures. (3) Experiments on natural and MR image CS tasks show that our D$^3$C$^2$-Net outperforms existing state-of-the-art networks by large margins.
3.1 Convolutional-coding-inspired dual-domain formulation

As discussed above, different from existing image-domain-based DUNs, we draw inspiration from convolutional coding methods to enhance the information transmission capability. Figs. 2(a) and (b) show the architecture of the image-domain-based and convolutional-coding-domain-based DUN, respectively. One can observe that, the inherent design of image-domain-based DUNs that the one-channel image $x$ in Eq. (1) is taken as input and output of each stage greatly hampers the information transmission capability. Differently, taking the natural advantage of feature maps $\alpha$ being $C$ channel, convolutional-coding-based DUNs can transmit high-throughput information between stages. Notably, the prior term in Eq. (1) plays an essential role in reconstructing process because it can narrow the feasible region in the solution space. This idea leads to the integration of image-domain and convolutional-coding-domain priors as follows:

$$
\min_{D,z,\alpha} \frac{1}{2} \|\Phi z - y\|^2 + \frac{\mu z}{2} \|z - D \odot \alpha\|^2 + \lambda \psi(\alpha) + \tau \phi(z),
$$

where $z \in \mathbb{R}^{h \times w}$ is precisely an image, $\alpha \in \mathbb{R}^{C \times h \times w}$ is the feature map, $\phi(z)$ and $\psi(\alpha)$ are prior terms of image domain and convolutional-coding domain respectively, and $\mu z, \lambda$ and $\tau$ are trade-off parameters. We illustrate the advantages of dual-domain priors in Fig. 2(c). One can observe that the
introduction of dual-domain priors further constrains the feasible region in the solution space, leading to better reconstruction results than single-domain-based methods. Besides, compared with objective functions in [31] and [32] where the measurement matrix \( \Phi \) in \( y = \Phi x + n \) is specially the identity matrix \( I \), our method is more flexible and generalizable, and can be extended to other cases.

### 3.2 Dual-domain optimization framework

To simplify the overall optimization process, we collaboratively learn a universal \( D \) and the other network components through end-to-end training and solve \( z \) and \( \alpha \) in Eq. (2) iteratively as follows:

\[
\begin{align*}
    z^{(t)} &= \arg\min_z \frac{1}{2} \| \Phi z - y \|_2^2 + \frac{\mu_z}{2} \| z - D \odot \alpha^{(t-1)} \|_2^2 + \tau \phi(z), \\
    \alpha^{(t)} &= \arg\min_\alpha \frac{\mu_z}{2} \| D \odot \alpha - z^{(t)} \|_2^2 + \lambda \psi(\alpha).
\end{align*}
\]  

(3a) \hspace{1cm} (3b)

**Image-level optimization.** The image-domain optimization and the convolutional-coding-domain optimization are decoupled into Eqs. (3a) and (3b), respectively. The \( z \)-subproblem in Eq. (3a) can be solved through ISTA by iterating between the following two update steps:

\[
\begin{align*}
    r^{(t)} &= G_{\text{GDM}}(\alpha^{(t-1)}, z^{(t-1)}, D, \rho, \mu_z) \\
    &= z^{(t-1)} - \rho \left( \Phi^\top (\Phi z^{(t-1)} - y) + \mu_z (z^{(t-1)} - D \odot \alpha^{(t-1)}) \right), \\
    z^{(t)} &= G_{\text{PMN}}(r^{(t)}) = \text{prox}_{\tau \phi}(r^{(t)}) = \arg\min_{z^*} \frac{1}{2} \| z^* - r^{(t)} \|_2^2 + \tau \phi(z^*),
\end{align*}
\]

(4a) \hspace{1cm} (4b)

where \( G_{\text{GDM}} \) and \( G_{\text{PMN}} \) denote the gradient descent module (GDM) and proximal mapping network (PMN), respectively. Their structural details will be elaborated on in the next subsection.

**Feature-level optimization.** For the \( \alpha \)-subproblem in Eq. (3b), where \( \frac{\mu_z}{2} \| D \odot \alpha - z^{(t)} \|_2^2 \) is the data term, \( \psi(\alpha) \) is the prior term, and \( \lambda \) is a trade-off parameter. To separate the data term and the prior term, we apply the HQS algorithm, which tackles Eq. (3b) by introducing an auxiliary variable \( \tilde{\alpha} \), leading to the following objective function:

\[
\begin{align*}
    \min_{\alpha, \tilde{\alpha}} \frac{\mu_z}{2} \| D \odot \tilde{\alpha} - z^{(t)} \|_2^2 + \lambda \psi(\alpha) + \frac{\mu_\alpha}{2} \| \alpha - \tilde{\alpha} \|_2^2,
\end{align*}
\]

(5)

where \( \mu_\alpha \) is the penalty parameter for the distance between \( \alpha \) and \( \tilde{\alpha} \). The above Eq. (5) can be also solved iteratively as follows:

\[
\begin{align*}
    \tilde{\alpha}^{(t)} &= \arg\min_{\alpha^*} \frac{1}{2} \| D \odot \alpha^* - z^{(t)} \|_2^2 + \frac{\mu_\alpha}{2} \| \alpha^* - \alpha^{(t-1)} \|_2^2, \\
    \alpha^{(t)} &= \arg\min_{\alpha^*} \frac{1}{2} \| \alpha^* - \tilde{\alpha}^{(t)} \|_2^2 + \beta \psi(\alpha^*),
\end{align*}
\]

(6a) \hspace{1cm} (6b)

where \( \eta = \mu_\alpha / \mu_z \) and \( \beta = \lambda / \mu_\alpha \). For solving the Eq. (6a), the Fast Fourier Transform (FFT) can be utilized by assuming the convolution is carried out with circular boundary conditions. Let \( D = \mathcal{F}(D) \), \( z^{(t)} = \mathcal{F}(z^{(t)}) \), and \( \mathcal{A}^{(t-1)} = \mathcal{F}(\mathcal{A}^{(t-1)}) \), where \( \mathcal{F} \) denotes the 2D FFT. Following [32], we apply the data-term solving module (DTSN), leading the following closed-form solution:

\[
\tilde{\alpha}^{(t)} = G_{\text{DTSN}}(\alpha^{(t-1)}, z^{(t)}, D, \eta) = \frac{1}{\eta} \mathcal{F}^{-1} \left( \mathcal{H}^{(t)} - D \circ \left( \frac{D \odot \mathcal{H}^{(t)}}{\eta + (D \odot D) \uparrow C} \right) \right),
\]

(7)

where \( \circ \) is the Hadamard product, \( X \odot Y = \sum_{i=1}^C X_i \odot Y_i, X \uparrow C \) expands the channel dimension of \( X \) to \( C, \odot \) is the Hadamard division, \( \mathcal{F}^{-1}(\cdot) \) denotes the inverse of FFT, \( D \) denotes the complex conjugate of \( D \), and \( \mathcal{H}^{(t)} \) is defined as \( \mathcal{H}^{(t)} = D \circ (z^{(t)} \uparrow C) + \eta \mathcal{A}^{(t-1)} \).

For solving the Eq. (6b), we apply a prior-term solving network (PTSN) to estimate \( \alpha^{(t)} \) as follows:

\[
\alpha^{(t)} = G_{\text{PTSN}}(\tilde{\alpha}^{(t)), \beta},
\]

(8)

and the structural design of PTSN will be presented in the following.
As discussed above, the unfolding optimization consists of an image-domain optimization subproblem (i.e., Eq. (3a)) and a convolutional-coding-domain optimization subproblem (i.e., Eq. (3b)). Mapping the unfolding process into a deep neural network, we propose our D₃C²-Net, which alternates between the image domain block (IDB) and the convolutional coding domain block (CCDB). Fig. 3 illustrates the overall architecture of D₃C²-Net with T stages, whereby the recovered result \( \hat{x} \) is obtained by \( \hat{x} = D \odot \alpha^{(T)} \). It can be seen that the proposed D₃C²-Net can transmit \( C \)-channel high-throughput information between every two adjacent stages. Fig. 4(a) gives more details about each stage. As shown in Fig. 4(a), each IDB is composed of a gradient descent module (GDM in Eq. (4a)) and a proximal mapping network (PMN in Eq. (4b)), while each CCDB is composed of a data-term solving module (DTSN in Eq. (7)) and a prior-term solving network (PTSN in Eq. (8)). Besides, for hyper-parameters \( \{ \rho, \mu, \eta, \beta \} \), inspired by [54] and [32], we adopt a hyper-parameter network (HPN) to predict them for each stage. Fig. 4(b) illustrates the architectures of the sub-networks, including InitNet, PMN, PTSN and HPN. More details are shown below.

### InitNet

InitNet takes the concatenation of \( x^{\text{init}} \) and \( \gamma \) as input to obtain a feature map initialization \( \alpha^{(0)} \), where \( x^{\text{init}} = \Phi^T y \), and \( \gamma \) is the CS ratio map generated from \( \gamma \) with a same dimension as \( x \). It consists of two convolutional layers (\( \text{Conv1}(\cdot) \) and \( \text{Conv2}(\cdot) \)). The former one receives 2-channel inputs and generates \( C \)-channel outputs with ReLU activation. InitNet is formulated as:

\[
\alpha^{(0)} = \mathcal{G}_{\text{InitNet}}(x^{\text{init}}, \gamma) = \text{Conv2}(\text{ReLU}(\text{Conv1}(\text{Concat}(x^{\text{init}}, \gamma)))).
\]  

### PMN

PMN solves the proximal mapping problem \( \text{prox}_{\tau \varphi}(r^{(t)}) \). It consists of two convolutional layers (\( \text{Conv1}(\cdot) \) and \( \text{Conv2}(\cdot) \)) and two residual blocks (\( \text{RB1}(\cdot) \) and \( \text{RB2}(\cdot) \)), which generate residual outputs by the structure of Conv-ReLU-Conv. Specifically, \( \text{Conv1}(\cdot) \) takes one-channel \( r^{(t)} \) as input and generates \( C \)-channel outputs. Then two \( \text{RB}(\cdot) \)s are used to extract deep representation. Finally, \( \text{Conv2}(\cdot) \) outputs the result by feature conversions from \( C \)-channel to one-channel under a residual learning strategy. Accordingly, PMN can be formulated as:

\[
z^{(t)} = \mathcal{G}_{\text{PMN}}^{(t)}(r^{(t)}) = r^{(t)} + \text{Conv2}(\text{RB2}(\text{RB1}(\text{Conv1}(r^{(t)})))).
\]  

### PTSN

PTSN takes the concatenation of \( \bar{z}^{(t)} \) and \( \bar{\beta}^{(t)} \) as input to learn the implicit prior on feature map \( \alpha \), where \( \bar{\beta}^{(t)} \) is generated from \( \beta^{(t)} \) as \( \gamma \) does. It consists of one convolutional layer (\( \text{Conv1}(\cdot) \)) and two residual blocks (\( \text{RB1}(\cdot) \) and \( \text{RB2}(\cdot) \)). The convolutional layer receives \( (C + 1) \)-channel inputs and generates \( C \)-channel outputs. Residual learning strategy is applied. PTSN is formulated as:

\[
\alpha^{(t)} = \mathcal{G}_{\text{PTSN}}^{(t)}(\bar{z}^{(t)}, \bar{\beta}^{(t)}) = \bar{\alpha}^{(t)} + \text{RB2}(\text{RB1}(\text{Conv1}(\text{Concat}(\bar{z}^{(t)}, \bar{\beta}^{(t)})))).
\]  

### HPN

HPN takes CS ratio map \( \gamma \) as input and predicts hyper-parameters for each stage. It consists of two \( 1 \times 1 \) convolutional layers with Sigmoid as the first activation function and Softplus as the last, ensuring all hyper-parameters are positive. HPN can be formulated as:

\[
(\rho^{(t)}, \mu^{(t)}, \eta^{(t)}, \beta^{(t)}) = \mathcal{G}_{\text{HPN}}^{(t)}(\gamma) = \text{Softplus}(\text{Conv2}(\text{Sigmoid}(\text{Conv1}(\gamma)))).
\]
To balance the performance and efficiency, we choose \( k \) the default filter size of each dictionary filter is set to be 5, the number of feature maps \( C \) is set to be 64, and the stage number \( T \) is set to 6. The number of filters in \( \Phi_j \) and the recovery network \( \Theta \) is 10. The learning rate starts from \( 1 \times 10^{-4} \) and decays a factor by 0.1 after \( 1.6 \times 10^5 \) and \( 2.4 \times 10^5 \) iterations. The default filter size \( k \) of each dictionary filter is set to be 5, the number of feature maps \( C \) is set to be 64, and the stage number \( T \) is set to be 8. The number of filters in \( \Phi \) is determined by the number of feature maps \( i.e., \) same as \( C \). The selection of \( k, C \) and \( T \) is discussed in Section 4.2.

### 4.2 Ablation study

In this section, we first discuss the selection of filter size \( k \) of \( \Phi \), the number of feature maps \( C \), and the number of stages \( T \). Then we investigate the contribution of each domain in our dual-domain network. All the experiments are performed with CS ratio \( \gamma = 30\% \).

#### Dictionary filter size \( k \)

We first explore the effects of dictionary filter size \( k \in \{3, 5, 7\} \). As shown in Fig. 5(a), the recovery performance is improved with a larger \( k \) while the inference time increases. To balance the performance and efficiency, we choose \( k = 5 \) in our default \( D^3C^2 \)-Net setting.
We observe that PSNR rises as T with MADUN, which demonstrates the effectiveness of dual-domain constraints. To demonstrate the generality of D, the combination of image and convolutional-coding domain priors introduces intermediate results as auxiliary information to introduce intermediate results as auxiliary information to transmit between stages without changing the idea of image-domain-based unfolding. As shown in Table 1, compared with MADUN, D uses fewer parameters while improving PSNR by 1.31dB on Urban100 with γ = 10%, which validates the stronger learning capability of D from our dual-domain unfolding principle.

4.4 Application to compressive sensing MRI

To demonstrate the generality of D, we directly extend it to the practical problem of CS-MRI reconstruction, which aims at restoring MR images from a small number of under-sampled data.
Table 2: Average PSNR (dB) and SSIM performance comparisons on Set11 [21] and Urban100 [60] datasets with five different levels of CS ratios (or sampling rates). We compare our D³C²-Net with five prior arts. The best and second best results are highlighted in red and blue colors, respectively.

| Dataset       | Methods                     | CS Ratio γ       |
|---------------|-----------------------------|------------------|
|               |                             | 10%  | 20%  | 30%  | 40%  | 50%  |
| Set11 [21]    | CSNet⁺ [23]                 | 28.34/0.8580    | 31.66/0.9203  | 34.30/0.9490  | 36.48/0.9644  | 38.52/0.9749  |
|               | SCSNet [24]                 | 28.52/0.8616    | 31.82/0.9215  | 34.64/0.9511  | 36.92/0.9660  | 39.01/0.9769  |
|               | OPINE-Net⁺ [26]             | 29.81/0.8904    | 33.43/0.9392  | 35.99/0.9596  | 38.24/0.9718  | 40.19/0.9800  |
|               | AMP-Net [59]                | 29.40/0.8779    | 33.33/0.9345  | 36.03/0.9586  | 38.28/0.9715  | 40.34/0.9804  |
|               | MADUN [28]                  | 29.89/0.8982    | 34.09/0.9478  | 36.90/0.9671  | 39.14/0.9769  | 40.75/0.9831  |
|               | D³C²-Net (Ours)             | 30.80/0.9061    | 34.64/0.9512  | 37.41/0.9684  | 39.49/0.9773  | 41.29/0.9836  |
| Urban100 [60] | CSNet⁺ [23]                 | 23.96/0.7309    | 26.95/0.8449  | 29.12/0.8974  | 30.98/0.9273  | 32.76/0.9484  |
|               | SCSNet [24]                 | 24.22/0.7394    | 27.09/0.8485  | 29.41/0.9016  | 31.38/0.9321  | 33.31/0.9534  |
|               | OPINE-Net⁺ [26]             | 25.90/0.7979    | 29.38/0.8902  | 31.97/0.9309  | 34.27/0.9548  | 36.28/0.9697  |
|               | AMP-Net [59]                | 25.32/0.7747    | 29.01/0.8799  | 31.63/0.9248  | 33.88/0.9511  | 35.91/0.9673  |
|               | MADUN [28]                  | 26.23/0.8250    | 30.24/0.9108  | 33.00/0.9457  | 35.10/0.9639  | 36.69/0.9746  |
|               | D³C²-Net (Ours)             | 27.54/0.8464    | 30.98/0.9161  | 34.06/0.9522  | 36.11/0.9676  | 37.89/0.9771  |

Ground Truth CSNet⁺ SCSNet OPINE-Net⁺ AMP-Net MADUN D³C²-Net

Figure 6: Visual comparisons on recovering an image named “Barbara” from Set11 [21] dataset with CS ratio γ = 30% (top) and an image from Urban100 [60] dataset with CS ratio γ = 10% (bottom).
Table 3: Average PSNR/SSIM performance comparisons on testing brain MR images with eight recent methods. The best and second best results are highlighted in red and blue colors, respectively.

| Methods            | 10%  | 20%  | 30%  | 40%  | 50%  |
|--------------------|------|------|------|------|------|
| Hyun et al. [20]   | 32.78/0.8385 | 38.85/0.9383 | 40.65/0.9539 | 42.35/0.9662 |
| Schlemper et al. [38] | 34.23/0.8921 | 38.85/0.9539 | 42.63/0.9724 | 44.19/0.9794 |
| ADMM-Net [45]      | 34.42/0.8968 | 38.62/0.9383 | 42.58/0.9726 | 44.19/0.9796 |
| RDN [39]           | 34.59/0.8968 | 38.62/0.9383 | 42.58/0.9726 | 44.19/0.9796 |
| CDDN [40]          | 34.63/0.9002 | 38.59/0.9474 | 42.59/0.9725 | 44.15/0.9795 |
| ISTA-Net + [25]    | 34.65/0.9038 | 38.67/0.9480 | 42.65/0.9727 | 44.24/0.9798 |
| MoDL [43]          | 35.18/0.9091 | 38.51/0.9457 | 40.97/0.9636 | 42.38/0.9705 |
| MADUN [28]         | 36.15/0.9237 | 39.44/0.9542 | 41.48/0.9666 | 43.06/0.9746 |
| D\(^3\)C\(^2\)-Net (Ours) | 36.48/0.9289 | 39.66/0.9558 | 41.59/0.9671 | 43.14/0.9748 |

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Figure 7: Visualizations for analyzing the learned dictionary \(D\) and feature maps \(\alpha\) including (a) learned global dictionary filters \(d_i\) in every four channels, whose values are distributed in \([-0.17, 0.23]\), (b) feature maps \(\alpha_9, \alpha_{25}, \alpha_{41}\) and \(\alpha_{57}\), (c) low-frequency information \(d_{25} \ast \alpha_{25}\) and the complementary (d) high-frequency information \(\sum_{i \neq 25} d_i \ast \alpha_i\), with applying \(D^3C^2\)-Net to the image named “Cameraman” from Set11 [21] with \(\gamma = 30\%\). \(D^3C^2\)-Net learns diverse dictionary filters through end-to-end optimization with the recovery network trunk and obtains better image representations than prior arts with clearly separating the (c) low- and (d) high-frequency components.

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5 Conclusion

Inspired by convolutional coding methods, we first propose a generalized dual-domain unfolding framework which combines the merits of both image-domain and convolutional-coding-domain priors to constrain the feasible region in the solution space. Compared with most existing convolutional coding methods, on the one hand, our framework adopts deep priors rather than traditional sparsity [30, 51, 52, 53] to better leverage the learning capability of deep neural networks. On the other hand, our framework is more generalizable, while existing deep convolutional coding methods for image restoration are exceptional cases where the degradation matrix \(\Phi\) is the identity \(I\) [31, 32]. Based on our proposed framework, we further design a novel Dual-Domain Deep Convolutional Coding Network for compressive sensing (CS) imaging, dubbed \(D^3C^2\)-Net. Compared with most existing CS DUNs [25, 26, 27, 29], our \(D^3C^2\)-Net transmits high-throughput feature-level representation through all stages and captures sufficient features adaptively. Extensive CS experiments on both natural and MR images demonstrate that \(D^3C^2\)-Net outperforms state-of-the-art network-based CS methods with large accuracy margins and lower complexities. In the future, we will extend our generalizable unfolding framework and \(D^3C^2\)-Net to more inverse imaging tasks and video applications.
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