On $G$–transitive version of perfectly meager sets

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Abstract. We study the $G$– invariant version of perfectly meager sets (a generalization of the notion of AFC′ sets). We find the necessary and sufficient conditions for the inclusion $AFC'_G \subseteq I$. In particular, we partially characterize for which groups $G$ of automorphisms of the Cantor space every $AFC'_G$ set is Lebesgue null.

1. Definitions and notation

We consider the Cantor space $2^\omega$ as a topological group (where $(x + y)(k) = x(k) + y(k) \mod 2$). By $2^{<\omega}$ let us denote the collection of all finite binary sequences: $2^{<\omega} = \{ f : n \to 2 \text{ where } n \in \omega \}$

For any $s \in 2^{<\omega}$ by $[s]$ denote the base open set determined by $s$: $[s] = \{ x \in 2^\omega : s \subseteq x \}$. Let Perf stand for the family of all perfect subsets of the space $2^\omega$. Recall that a proper collection of subsets of $2^\omega$: $\mathcal{I} \subseteq P(2^\omega)$ is called a $\sigma$-ideal iff it is closed under taking subsets and countable sums.

Throughout the paper we assume that every $\sigma$-ideal $\mathcal{I}$ contains all singletons: $\forall x \in X \{ x \} \in \mathcal{I}$.

Let $\mathcal{I} \subseteq P(2^\omega)$ be a $\sigma$-ideal. Define the following cardinal numbers:

Definition 1. $cov(\mathcal{I}) = \min\{ |A| : A \subseteq \mathcal{I} \wedge \bigcup A = 2^\omega \}$

and

$cof(\mathcal{I}) = \min\{ |A| : A \subseteq \mathcal{I} \wedge \forall Z \in \mathcal{I} \exists A \in A Z \subseteq A \}$.

Notice that we always have $cov(\mathcal{I}) \leq cof(\mathcal{I})$.

We assume that the reader is familiar with the basic concept of arithmetic of cardinal numbers. In particular, we need the notion of cofinality; recall that an uncountable cardinal number $\kappa$ is called regular iff $cf(\kappa) = \kappa$.

By $Hom(X)$ we denote the group of all homeomorphisms of the topological space $X$. We always assume that $G$ is a fixed subgroup of $Hom(2^\omega)$.

The following additional terminology will be useful in our proof.

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For an arbitrary $g \in G$ and $Q \in Perf$ we often abbreviate the image $g(Q) = \{gx : x \in Q\}$ and write simply $gQ$. Also for any $t \in 2^\omega$ and $A \subseteq 2^\omega$ we write $A + t = \{x + t : x \in A\}$.

We denote by $M(P)$ the collection of all first category sets on $P$, where $P \in Perf(X)$.

We use a letter $N$ to denote the sigma ideal of Lebesgue measure zero sets of $2^\omega$.

We denote by $\text{Trans}(2^\omega)$ the subgroup of all translations of $2^\omega$.

2. Introduction

Let us start with the following, classical definition:

**Definition 2.** A subset $S \subseteq 2^\omega$ is a Sierpiński set if, and only if, it is uncountable and has countable intersection with any set of measure zero.

Notice that under the assumption of Continuum Hypothesis there exists a Sierpiński set (see [9]) and, on the other hand, it is consistent that there is no Sierpiński set.

A special variation of the notion of a Sierpiński set is a $\kappa$-Sierpiński set with respect to the $\sigma$-ideal $I$, namely:

**Definition 3.** Suppose that $\kappa$ is a cardinal number and $\mathcal{I} \subseteq P(2^\omega)$ a $\sigma$-ideal. A set $X \subseteq 2^\omega$ is called a $\kappa$-Sierpiński set $X$ with respect to $\mathcal{I}$ iff $|X| = \kappa$ and $\forall A \in \mathcal{I} |A \cap X| < \kappa$.

Notice that if $\mathcal{T}$ is a $\sigma$-ideal (which contains singletons) and $\kappa = \text{cof}(\mathcal{I}) = \text{cov}(\mathcal{I})$ then there exists a $\kappa$-Sierpiński set $X$ with respect to $\mathcal{I}$.

Recall the classical definition of perfectly meager sets (called also always of the first category sets):

**Definition 4.** A set $X$ of $2^\omega$ is a perfectly meager (AFC) set iff for every $P \in Perf$, $X \cap P$ is a first category set in the topology of $P$.

The following notion of sets was first defined in [5] and then it has been studied most extensively in papers [6] and [7].

**Definition 5.** A set $X \subseteq 2^\omega$ is an AFC′-set if for each perfect set $P$ there exists an $\mathcal{F}_\sigma$-set $F$ such that $X \subseteq F$ and for each $t \in 2^\omega$, $(F + t) \cap P$ is a first category set in the topology of $P$.

Notice that the notion AFC′ is a strengthening of the classical perfectly meager sets.

The following notion was first defined by Karel Prikry: (see [3], introduction):

**Definition 6.** A set $X \subseteq 2^\omega$ is called strongly meager (SFC) iff for every measure zero set $A \subseteq 2^\omega$, there exists $t \in 2^\omega$, such that $(X + t) \cap A = \emptyset$. 
Notice that K. Prikry conjectured that the collection of strongly meager sets form a $\sigma$-ideal but it turned out that it is consistent that strongly meager sets are exactly the countable sets (see [3]) and that it is consistent that even the sum of two strongly meager sets need not be strongly meager set (see [2]).

It is known (see for example [5] and [7]), that $\text{AFC}' \subseteq \text{AFC}$ and every strongly meager set is an $\text{AFC}'$ set.

It is also known (see [8]) that every Sierpiński set is strongly meager.

We can summarize all these inclusions in Fig. 1

$$\text{Sierpiński set} \quad \longrightarrow \quad \text{SFC} \quad \longrightarrow \quad \text{AFC}' \quad \longrightarrow \quad \text{AFC}$$

**Figure 1. Basic relations.**

Let us define the main notion of this article.

**The $\text{AFC}'_G$ - sets**

Suppose that $G$ is a subgroup of $\text{Hom}(2^\omega)$ and let $X$ be an arbitrary subset of $2^\omega$.

**Definition 7.** Suppose that $X \subseteq 2^\omega$. We write $X \in \text{AFC}'_G$ iff for every $Q \in \text{Perf}$ there exists $F \supseteq X$, $F \in F_\sigma$ such that $\forall g \in G gQ \cap F \in \mathcal{M}(gQ)$.

This notion is a natural generalization of the notion of $\text{AFC}'$ sets.

**Remarks:**

It is obvious that

$$\text{AFC}'_{\text{Trans}(2^\omega)} = \text{AFC}', \quad \text{AFC}'_{\{id\}} = \text{AFC}, \quad \text{AFC}'_{\text{Hom}(2^\omega)} = [2^\omega]^{\leq \omega}.$$ 

It is also evident that if $G_1 \subseteq G_2$, then $\text{AFC}'_{G_1} \supseteq \text{AFC}'_{G_2}$.

All inclusions are summarized in Fig. 2 (where arrows denote inclusions).

$$\text{AFC}'_{\text{Hom}(2^\omega)} \quad \longrightarrow \quad \text{AFC}'_{\text{Trans}(2^\omega)} \quad \longrightarrow \quad \text{AFC}'_{\{id\}}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$[2^\omega]^{\leq \omega} \quad \text{AFC}' \quad \text{AFC}$$

**Figure 2. Relations between various versions of perfectly meager sets.**

Let us define:
Definition 8. Suppose that $\mathcal{I}$ is a $\sigma$-ideal of subsets of the space $2^\omega$.

We say that a group $G \leq \text{Hom}(2^\omega)$ has the $(Em)_{\mathcal{I}}$ property iff there exists a perfect set $Q \in \text{Perf}$ such that for each $P \in \text{Perf} \setminus \mathcal{I}$ there exists $g \in G$ such that $P \cap gQ \not\in M(gQ)$.

Remarks:
One can prove that $\text{Trans}(2^\omega)$ does not have the $(Em)_{\mathcal{N}}$ property.

Without loss of generality we may assume that in Definition 8 $P$ is only closed set such that $P \not\in \mathcal{I}$.

We will start with the following theorem.

Theorem 1. Let $\mathcal{I}$ be an arbitrary $\sigma$-ideal of subsets of $2^\omega$ such that $\forall x \in 2^\omega \{x\} \in \mathcal{I}$.

Moreover, let $G \leq \text{Hom}(2^\omega)$ be a subgroup of $\text{Hom}(2^\omega)$ with the property $(Em)_{\mathcal{I}}$.

Then we have: $\text{AFC}'_G \subseteq \mathcal{I}$.

Proof. Let $X \subseteq 2^\omega$ be a set such that $X \not\in \mathcal{I}$. By the definition of the notion $(Em)_{\mathcal{I}}$ there is a perfect set $Q$ such that for each closed $E \not\in \mathcal{I}$ we have $\exists g \in G \ E \cap gQ \not\in M(gQ)$.

Let $F \subseteq 2^\omega$ be an $F_\sigma$ set such that $X \subseteq F$. We have

$$F = \bigcup_{n < \omega} F_n,$$

where $cl(F_n) = F_n$, so there exists $n_0 < \omega$ such that $F_{n_0} \not\in \mathcal{I}$. Now there exists $g \in G$ such that $F_{n_0} \cap gQ$ is not meager in $gQ$. So we conclude, that $X$ is not an AFC$'_G$ set. \hfill \Box

The implication given in Theorem 1 is reversible under some additional set theoretical assumptions. Indeed, we have the following theorem.

Theorem 2. Let us assume like in Theorem 1 that $\mathcal{I}$ is an arbitrary $\sigma$-ideal of subsets of $2^\omega$ such that $\forall x \in 2^\omega \{x\} \in \mathcal{I}$ and $G \leq \text{Hom}(2^\omega)$ is a subgroup of $\text{Hom}(2^\omega)$. Moreover, assume that

(1) $\text{cof}(\mathcal{I}) = \text{cov}(\mathcal{I}) \leq \text{non}(\text{AFC}'_G)$,
(2) $\forall P \in \text{Perf} \setminus \mathcal{I} \exists |C| \leq \omega 2^\omega \setminus (P + C) \in \mathcal{I}$,
(3) $\text{Trans}(2^\omega) \subseteq G$.

Then the following conditions are equivalent:

(1) $\text{AFC}'_G \subseteq \mathcal{I}$
(2) $G$ fulfills $(Em)_{\mathcal{I}}$.

Proof. Theorem 1 gives us immediately the implication $(2) \Rightarrow (1)$.

Now suppose that $G$ fulfills $\neg (Em)_{\mathcal{I}}$. Since $\kappa = \text{cof}(\mathcal{I}) = \text{cov}(\mathcal{I})$ and $\mathcal{I}$ contains singletons we conclude that there exists a $\kappa$-Sierpiński set $X$ with respect to $\mathcal{I}$ (see Def. 3). Let $Q \in \text{Perf}$ be arbitrary. From the assumption
\neg (Em)_\mathcal{I} \text{ there exists a perfect set } P \text{ such that } P \notin \mathcal{I} \text{ and } \forall g \in G gQ \cap P \in \mathcal{M}(gQ). \text{ Pick a countable set } C \subseteq 2^\omega \text{ such that } 2^\omega \setminus (C + P) \in \mathcal{I}.

We have
\[
X = \left[ 2^\omega \setminus (P + C) \right] \cap X \cup \left[ (P + C) \cap X \right].
\]

Since \( 2^\omega \setminus (P + C) \in \mathcal{I} \) we obtain \( \left[ 2^\omega \setminus (P + C) \right] \cap X \subsetneq \kappa \). Moreover, if \( c \in C \) and \( g \in G \), then \( hQ \cap P \in \mathcal{M}(hQ) \), where \( h \in G \) is defined by \( h(x) = g(x) - c \). Hence \( gQ \cap (P + c) \in \mathcal{M}(gQ) \), thus \( gQ \cap (P + C) \in \mathcal{M}(gQ) \) for each \( g \in G \).

Since \( \kappa \leq \text{non}(\text{AFC}_G') \) we obtain \( [2^\omega \setminus (P + C)] \cap X \in \text{AFC}_G' \), so there exists \( E \in F_\sigma, E \supseteq X \setminus (P + C) \) such that \( \forall g \in G gQ \cap E \in \mathcal{M}(gQ) \). Finally, define \( E^* = E \cup (P + C) \). It is easy to see that \( X \subseteq E^* \) and \( \forall g \in G gQ \cap E^* \in \mathcal{M}(gQ) \). Hence \( X \in \text{AFC}_G' \) and the proof is completed, since \( X \) does not belong to \( \mathcal{I} \). \( \square \)

Unfortunately, we don’t know whether this theorem is true under weaker assumptions. Thus we think that the following question may be of some interest.

**Question 3.** Can we prove the equivalence from Theorem 2 under weaker assumptions?

For any \( \mathcal{F} \subseteq \text{Perf} \) let us define the following cardinal coefficient:

**Definition 9.** \( Em(\mathcal{F}, G) = \min\{ |G| : G \subseteq \text{Perf} \wedge \forall P \in \mathcal{F} \exists g \in G \exists Q \subseteq gQ \subseteq P \} \)

Let us formulate a characterization of the property \((Em)\) in terms of the coefficient \( Em(\mathcal{F}, G) \).

Assume that \( G \) has the property that for each \( x \in 2^\omega \) the orbit \( Gx \) is dense in \( 2^\omega \). Then the following conditions are equivalent:

1. \( G \) fulfills \((Em)_\mathcal{I}\);
2. \( |Em(\text{Perf} \setminus \mathcal{I}, G)| \leq \aleph_0 \).

We will need the following technical lemma (folklore for the group \( G = \text{Trans}(2^\omega) \)):

**Lemma 1.** If \( G \leq \text{Hom}(2^\omega) \) is a group such that for each \( x \in 2^\omega \), \( Gx \) is dense in \( 2^\omega \), then for every sequence \( \langle Q_n \rangle \) of perfect subsets of \( 2^\omega \) there exists a perfect \( P \in \text{Perf} \) such that \( \forall n \in \omega \exists g \in G gQ \cap P \notin \mathcal{M}(P) \).

**Proof.** Let \( v_k = [(0, \ldots, 0, 1)] \) (0 \( k \) times). For each \( k \) choose \( x_k \in Q_k \) and \( g_k \in G \) such that \( g_k x_k \in V_k \). Define \( P = \bigcup_{k \in \omega} g_k Q_k \cap V_k \), then \( P \) is a perfect set and if \( k \in \omega \) then \( g_k Q_k \cap P \supseteq g_k Q_k \cap V_k \notin \mathcal{M}(P) \). \( \square \)

**Proof.** \( 1 \rightarrow 2 \)

Assume that \( G \) has the \((Em)_\mathcal{I}\) property, i.e. there exists \( Q \in \text{Perf} \) such that \( \forall P \in \text{Perf} \setminus \mathcal{I} \exists g \in G P \cap gQ \notin \mathcal{M}(gQ) \). Let us define perfect sets: \( \mathcal{G} = \{ Q \cap [s] : Q \cap [s] \neq \emptyset \wedge s \in 2^{<\omega} \} \). Then \( |\mathcal{G}| \leq \aleph_0 \) and if \( P \in \text{Perf} \setminus \mathcal{I} \) then there exists \( g \in G \) such that \( P \cap gQ \notin \mathcal{M}(gQ) \), so \( P \cap gQ \supseteq W \cap gQ \neq \emptyset \) for
some open set $W$. Then $g^{-1}[W] \cap Q \neq \emptyset$ so there exists $Q_1 \in G$ such that $Q_1 \subseteq g^{-1}[W] \cap Q$. Hence $g[Q_1] \subseteq W \cap g[Q] \subseteq P \cap g[Q]$. This proves (2).

(2) $\Rightarrow$ (1).

Next we give an useful characterization of the property $(Em)_N$.

**Theorem 4.** Let $G$ be a subgroup of $\text{Hom}(2^\omega)$ which contains the subgroup $\text{Trans}(2^\omega)$. The following two conditions are equivalent:

1. $\neg (Em)_N$,
2. For every $Q \in \text{Perf}$ and for every $\epsilon > 0$ there exists an open set $U$, such that $\mu(U) < \epsilon$ and $\forall g \in G gQ \cap U \neq \emptyset$

**Proof.** (1) $\Rightarrow$ (2)
Assume that $\forall Q \in \text{Perf} \exists P \in \text{Perf} \forall g \in G gQ \cap P \in \mathcal{M}(gQ)

Let $Q \in \text{Perf}$ be any perfect set and let $\epsilon > 0$. Pick a perfect set $P$, $\mu(P) > 0$ such that $\forall g \in G gQ \cap P \in \mathcal{M}(gQ)$. We can find finite $C \subseteq 2^\omega$ such that $\mu(2^\omega \setminus (C + P)) < \epsilon$. Now put $U = 2^\omega \setminus (C + P)$.

By way of contradiction suppose that there exists $g \in G$ such that $gQ \cap U = \emptyset$. Then $gQ \subseteq C + P$, hence there exists $c_0 \in C$ and an open set $I$ such that $\emptyset \neq I \cap gQ \subseteq P + c_0$. Define $h(x) = g(x) - c_0$, obviously $h \in G$. Next, $hQ = gQ - c_0$ thus $\emptyset \neq hQ \cap (I - c_0) \subseteq P$, which is a contradiction with $hQ \cap P \in \mathcal{M}(hQ)$.

(2) $\Rightarrow$ (1)
Assume (2). Let $R$ be any perfect set. Let $\{I_m\}_{m<\omega}$ be an enumeration of all basic clopen sets of $2^\omega$. Let

$$\epsilon_m = \frac{1}{2^m + 2}.$$ 

For any $m < \omega$ we choose, using the assumption (2), an open set $U_m$ such that

$$\forall g \in G R \cap I_m \neq \emptyset \Rightarrow U_m \cap g(R \cap I_m) \neq \emptyset$$

and $\mu(U_m) < \epsilon_m$. This can be done, since $I_m \cap R$ is a perfect or an empty set.

Now put

$$U = \bigcup_{m<\omega} U_m.$$ 

We see that

$$\mu(U) \leq \sum_{m<\omega} \frac{1}{2^m + 2} \leq 2 \cdot \frac{1}{4} < 1.$$ 

Define $F = 2^\omega \setminus U$, then we have $\mu(F) > 0$ so choose a perfect $P \subseteq F$ of positive measure.

Let $g \in G$ and $I_{m_0}$ be given such that $R \cap I_{m_0} \neq \emptyset$. 

Now \(U_{m_0} \cap g(R \cap I_{m_0}) \neq \emptyset\). Moreover, since \(U_{m_0} \cap P = \emptyset\) we obtain that \(g(R \cap I_{m_0}) \not\subset P\). This means that (1) holds. \(\square\)

Notice that in the proof of implication (2) \(\Rightarrow\) (1) we did not use the assumption that \(\text{Trans}(2^\omega) \leq G\).

In the next part we will prove theorems about relations between \(\text{AFC}'_G\) and different classes of peculiar small sets of the real line.

**Theorem 5.** Assume that \(G\) is a subgroup of \(\text{Hom}(2^\omega)\) which contains \(\text{Trans}(2^\omega)\). If \(G\) fulfills \(\neg(Em)_\mathcal{N}\), then every strongly meager set is an \(\text{AFC}'_G\) set.

**Proof.** Let \(X\) be a strongly meager set and let \(Q\) be an arbitrary perfect set. Since \(\neg(Em)_\mathcal{N}\) we obtain that there exists a perfect set \(P\) such that \(\mu(P) > 0\) and \(\forall g \in G g(Q) \cap P \in \mathcal{M}(g(Q))\). Let \(C \subseteq 2^\omega\) be a countable set such that \(2^\omega \setminus (P + C) \in \mathcal{N}\). Then there exists \(x_0\) such that \((x_0 + X) \cap [2^\omega \setminus (P + C)] = \emptyset\), so \(x_0 + X \subseteq P + C\), hence \(X \subseteq P + C - x_0\). Let \(g \in G\) be an arbitrary and let \(c \in C\). Define \(h \in G\) by \(h(x) = g(x) - c + x_0\). Then \(h(Q) \cap P \in \mathcal{M}(h(Q))\), hence \((g(Q) - c + x_0) \cap P \in \mathcal{M}(g(Q) - c + x_0)\), thus \(g(Q) \cap (P + c - x_0) \in \mathcal{M}(g(Q))\). Since \(c \in C\) was taken arbitrary, we conclude that \(g(Q) \cap (P + C - x_0) \in \mathcal{M}(g(Q))\). This implies that \(X \in \text{AFC}'_G\), since \(P + C - x_0 \in \text{F}_\sigma\). \(\square\)

**Remark:**

This implication is reversible under CH. Namely:

**Theorem 6.** Suppose that \(G \leq \text{Hom}(2^\omega)\) and assume that \(G\) has the \((Em)_\mathcal{N}\) property. Moreover, assume CH. Then there exists a strongly meager set \(X \subseteq 2^\omega\) such that \(X \notin \text{AFC}'_G\).

**Proof.** Let \(X \subseteq 2^\omega\) be arbitrary Sierpiński set. Then \(X\) is strongly meager ([8]). From the \((Em)_\mathcal{N}\) property we obtain that there exists \(Q \in \text{Perf}\) such that

\[
\forall P \in \text{Perf} \setminus \mathcal{N} \exists g \in G P \cap g(Q) \notin \mathcal{M}(g(Q)).
\]

Suppose that \(E\) is an \(\text{F}_\sigma\)-set such that \(X \subseteq E\). Since \(X \notin \mathcal{N}\) it follows that \(E \notin \mathcal{N}\). Hence there exists \(P \in \text{Perf} \setminus \mathcal{N}\) such that \(P \subseteq E\).

Therefore \(\exists g \in G P \cap g(Q) \notin \mathcal{M}(g(Q))\), hence \(E \cap g(Q) \notin \mathcal{M}(g(Q))\). This yields \(X \notin \text{AFC}'_G\), which finishes the proof. \(\square\)

**Corollary 1.** Assume that \(\text{cov}(\mathcal{N}) = \text{cof}(\mathcal{N})\) and \(\text{cov}(\mathcal{N})\) is a regular cardinal. Let \(G \leq \text{Hom}(2^\omega)\) and assume that \(\text{Trans}(2^\omega) \leq G\). Then the following conditions are equivalent:

1. \(G\) has the \((Em)_\mathcal{N}\) property.
2. \(\text{AFC}'_G \subseteq \mathcal{N}\).
Proof. The implication (1) ⇒ (2) follows immediately from Theorem 1. Assume ¬(Em)_{\mathcal{N}}. Since cov(\mathcal{N}) = cof(\mathcal{N}), there exists a cof(\mathcal{N}) – Sierpiński set. By Lemma 8.5.4 from [1] if there exists a \kappa – Sierpiński set and cf(\kappa) = \kappa > \omega, then every set of size < \kappa is strongly meager. Hence by Theorem 5 we conclude that non(AFC'_{G}) ≥ cof(\mathcal{N}) thus all assumptions of Theorem 2 are satisfied. \square

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