Zero Cross-Talk Regimes in Dually Modulated Mutually-Coupled Nano-lasers

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Abstract. The modulation properties of dually modulated mutually-coupled nano-lasers have been analyzed using rate equations which include the Purcell cavity-enhanced spontaneous emission factor \( F \) and the spontaneous emission coupling factor \( \beta \). Analysis of the dynamical response of modulated mutually-coupled nano-lasers reveals the existence of regimes of zero cross-talk wherein the response of one nano-laser is not impacted by the dynamics of the other nano-laser. The availability of zero-cross talk regimes is seen to offer opportunities for exploitation in photonic integrated circuits.

1. Introduction
Mutually coupled lasers have been investigated for many decades [1]. Activity on mutually coupled semiconductor lasers also has long antecedents [2], [3] with significant effort having been given to identifying regimes of synchronization and instabilities [4]-[7]. Optical injection is well-known as a means for enhancing the modulation bandwidth of semiconductor lasers [8] and in recent work modulation bandwidth enhancement in mutually-coupled monolithically integrated laser diodes has been reported [9]. Semiconductor nano-lasers [10], [11] are of interest not least for their potential for inclusion in photonic integrated circuits. Fabricating nano-lasers is rather challenging and, as far as we are aware, no experimental studies of their dynamical properties have been published. In this context, it is considered appropriate to explore nano-laser dynamics using a generic model rather than limiting the analysis to any specific nano-laser structure. It is foreseen that such analysis may offer perspectives for future experimental exploration of nano-lasers as they become available.

In early work, the impact of Purcell enhanced spontaneous emission on the modulation performance of nano-LEDs and nano-lasers [12] was examined. In addition to [13], [14] a number of recent investigations of the dynamical performance of nanolasers have been undertaken. The behaviour of optically pumped nanolasers has been studied including the role of the spontaneous emission factor, \( \beta \), in achieving single mode operation of nanolasers [12]. Ding et al. explored how the dynamics of electrically pumped nanolasers are impacted by \( F \) and \( \beta \) [15]. Theoretical work has also been reported on the control of dynamical instability in such lasers [16].
Previous investigations of the influence of $F$ and $\beta$ shows that modulation bandwidth of up to 60 GHz can be achieved for metal clad nano-lasers [17]. In later work on the effect of optical injection in nanolasers, it has been identified the modulation bandwidth of nanolaser can approach 90 GHz [18].

The theme of the present paper is the impact of direct current modulation on the dynamics of mutually-coupled nano-lasers. The paper is structured as follows. The nano-laser dynamical model is instructed in section 2. Results given in section 3 delineate the main dynamical behavior which arises when mutually-coupled nanolasers are subject to different modulation frequencies, wherein modulation responses are calculated. Finally, in section 4, conclusions are drawn based on the results obtained.

2. Nano-laser Dynamical Model

A schematic diagram of mutually coupled nano-lasers where they are modulated with different modulation frequencies is shown in Figure 1.

![Schematic diagram of modulated mutually-coupled semiconductor nano-lasers.](image)

It is underlined that the Purcell factor and the spontaneous emission coupling factor impact the spontaneous emission rate as shown in Eqs. (1) and (2) below. Specifically it is pointed out that for Purcell factors greater than unity an effective reduction in the carrier lifetime will result. Similarly an increase of the spontaneous emission coupling factor towards unity also causes an effective reduction of the carrier lifetime. In contrast, the phase Eq. (3) is dependent on the laser gain and hence is not affected by the enhanced spontaneous emission.

\[
\frac{dS_{nI}}{dt} = \Gamma \left[ \frac{F \beta N_{nI}(t)}{\tau_n} + G_n \left( N_{nI}(t) - N_{th} \right) S_{nI}(t) \right] + 2 \frac{\kappa_{nj}}{\tau_n} \sqrt{S_{nI}(t)S_{nI}(t - \tau_{ny})} \cos(\theta_{nI}(t))
\]

(1)

\[
\frac{dN_{nI}}{dt} = \frac{I_{nI}}{eV_b} \frac{N_{nI}(t)}{\tau_n} \left( F \beta + \left( 1 - \beta \right) \right) - G_n \left( N_{nI}(t) - N_{th} \right) S_{nI}(t)
\]

(2)

\[
\frac{d\phi_{nI}}{dt} = \frac{\alpha}{2} \Gamma G_n \left( N_{nI}(t) - N_{th} \right) + \Delta \omega - \frac{\kappa_{ny}}{\tau_n} \sqrt{S_{nI}(t - \tau_{ny})} S_{nI}(t) \sin(\theta_{nI}(t)) \]

(3)

\[
\theta_{nI}(t) = \pm \Delta \omega t + \omega_{a} \tau_{ny} + \phi_{nI}(t) - \phi_{II}(t - \tau_{ny})
\]

(4)

\[
I_{nI}(t) = I_{dc} \left( 1 + h_{m} \sin(2\pi f_{m,nI} t) \right)
\]

(5)

In the rate equations including the modulation, the subscripts ‘I’ and ‘II’ represent laser I and laser II respectively. $S(t)$ is the photon density and $N(t)$ is the carrier density, $\phi(t)$ is the phase of the laser, $\theta(t)$ is the phase of injection laser. $\Gamma$ is the confinement factor; $\tau_n$ and $\tau_p$ are the radiative carrier lifetime and photon lifetime respectively. $G_n$ is the differential gain that takes into account the effect of group velocity, $N_{th}$ the transparency carrier density, $\epsilon$ is the gain saturation factor and $\alpha$ is the linewidth enhancement factor. $I_{dc} = I_{th}$ is the dc bias current, where $j$ is the normalized injection current; $I_{th}$ is the threshold current ($I_{th} = (F\beta + (1 - \beta)) N_{th} V_a / \tau_n$), $V_a$ is the volume of the active region $e$ is the electron charge and $N_{th}$ ($N_{th} = N_{th} + 1/\Gamma G_n \tau_n$) is the threshold carrier density. $\Delta \omega$ is the angular frequency detuning between laser I and laser II. $\tau_{ny} = D/c$ is the injection delay, where $D$ is the distance between laser I and laser II, $c$ is the speed of light in free space. $\tau_{m} = 2nL/c$ is the round-trip time in of the laser cavity, where $L$ is the cavity length and $n$ is group refractive index. The mutually-coupled optical injection into the laser I and laser II is controlled by the injection fraction, $\kappa_{nj}$, which is related to the injection parameter [20]. Sinusoidal direct current modulation of the lasers included in Eq. (2) is characterised by a modulation frequency,
$f_{m1}$ or $f_{m2}$, for the laser I and laser II, and the corresponding depth of modulation $h_m$. The values of the nano-lasers device parameters used in the simulations are provided in ref. [22].

3. Spectral Response

3.1. Spectral Characteristics

Figure 2 shows the response when nano-laser I is modulated at the relatively high frequency of 50 GHz whilst nano-laser II is modulated at 10 GHz. This is an example of a bi-directionally isolated zero cross-talk state where no frequency component derived from one nano-laser appears in the other nano-laser. As shown in Figure 3 a non-reciprocal zero-cross state can be accessed by changing a combination of such experimentally-accessible parameter as the injection coupling, the depth of modulation or the laser bias current.

From Figure 3(a) it is seen that nano-laser I, which is subject to the higher modulation frequency, exhibits spectral features of nano-laser II at harmonics of 10 GHz, viz., 20 GHz, 30 GHz and 40 GHz. However it is apparent from Figure 3(b) that no spectral component from nano-laser I appears in the spectrum nano-laser II. That is, one laser displays the signature of the modulation of the other but not vice versa. The practical significance of the zero cross-talk regime is that it permits independent modulation of the coupled lasers.
This zero cross-talk behaviour can also be found for relative lower modulation frequency, such as \( f_{m1} = 7 \) GHz and \( f_{m2} = 3 \) GHz. The results are shown in the Fig. 4, where the bias currents are twice threshold and the modulation depth is 0.6. We notice that even for such modulation depths as 0.1, bi-directionally isolated zero cross-talk can also be observed if injection strength is \( \kappa_{\text{inj}} = 0.1 \times 10^{-3} \). With other parameters unchanged, if the injection strength increases to say \( \kappa_{\text{inj}} = 0.3 \times 10^{-3} \), the bi-directionally isolated behavior requires a larger modulation depth: greater than or equal to 0.2.

![Fig. 4 Photon density time series and FFT at \( I_I = I_{II} = 2I_{th}, m=0.6 \) and \( \kappa_{\text{inj}} = 0.1 \times 10^{-3} \). (a) nano-laser I with \( f_{m1} = 7 \) GHz, (b) nano-laser II with \( f_{m2} = 3 \) GHz.](image)

3.2. Modulation Response

Having identified two general classes of zero cross-talk modulation regimes, attention is now focused on the direct current modulation response of dually modulated nano-lasers in these regimes.

The modulation response index, \( \eta \) defines as:

\[
\eta = \frac{S_{\max}(t) - S_{\min}(t)}{S_{\text{mean}}(t)}
\]

where \( S(t) \) is photon density of nano-laser in modulation. By altering modulation frequency, \( f_{m1} \) or \( f_{m2} \), the corresponding response index values are obtained.

As a function of the laser bias current when the modulation frequency of nano-laser I is varied from 10 GHz to 50 GHz while the modulation frequency for nano-laser II is maintained as 10 GHz. The results given in Figure 5(a) show that the modulation index for nano-laser II increases with increasing bias current but then saturates. The same qualitative behavior is seen for nano-laser II in Figure 4(b) albeit with the different modulation frequency leading to a different modulation index. The salient feature of Figure 5(b) is that the response curves for different modulation frequencies are over-layered thereby explicitly demonstrating the consequences of zero cross-talk.

In fact, strong driving bias currents or deep modulation depth or weak injection coupling strength makes each of the modulated mutually-coupled nanolasers independent of each other. However non-linear cross talk will occur if use is made of strong injection coupled power or a very low modulation depth or relatively small bias currents. A more detailed discussion of these aspects is given in [22].
Theoretical analysis shows that dually modulated mutually coupled nano-lasers can display a range of interactions depending on the bias current, injection coupling and the modulation depth. Zero cross-talk regimes have been identified for which the modulation response of the lasers have been calculated. The facility to individually address nano-lasers is expected to find ready applications.

Acknowledgments
This work is supported by The Sêr Cymru National Research Network in Advanced Engineering Materials, the International Science & Technology Cooperation Program of China (2014DFA50870), The National Natural Science Foundation of China (61527819, 61475111, 61601319), The Natural Science Foundation of Shanxi Province(201601D202043).

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