Period Ratio Sculpting Near Second-Order Mean-Motion Resonances

A new, independent method for constraining the population eccentricity distribution of multi-planet systems

Highly-eccentric planets produce a very narrow peak near the second-order resonance locations when time-averaged

Lack of significant peaks in Kepler data can constrain the eccentricity distribution of the population

Eccentricity and Resonances

The second-order resonance examined here is symmetrical in shape and wider at higher eccentricities. The period ratio distribution, when time-averaged, should reveal a sharp, narrow peak at the exact resonance location, provided the length of the averaging period is longer than the typical libration period. This peak is stronger for more eccentric pairs.

Demonstration via Simulations

We create a set of generated planet pairs near the 3:1 and 5:3 resonances, with properties similar to those of planets observed by Kepler except with an arbitrarily wide range of eccentricities. After applying a stability cut, we integrate (using REBOUND) the remaining pairs for 3.5 years and time-average the period ratios. As shown in the figure below, the sharp peak is indeed revealed as expected.

Constraining Eccentricity

Using the library of generated planet pairs, we can construct period ratio distributions for subsamples with a given eccentricity distribution. Comparing these subsamples to the distribution in the same region in the Kepler data allows us to constrain the eccentricity of planet pairs near these resonances in the Kepler population.

Upper Limit for Kepler Eccentricity

Rayleigh scale parameter

95% confidence

σ = 0.245 (3:1)
σ = 0.095 (5:3)

Selected References: Fabrycky, D. C., Lissauer, J. J., Ragozzine, D., et al. 2014, ApJ, 790, 146; Hadden, S. 2019, AJ, 158, 238; Murray, C. D., & Dermott, S. F. 1999, Solar系统 dynamics; Rein, H., & Liu, S.-F. 2012, A&A, 537, A128