Design-Parameters Setup for Power-Split Dual-Regime IVT

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Abstract. To analyze the working possibilities of power-split infinitely variable transmissions (IVTs) it is necessary to follow a systematic approach. The method proposed in this paper consists of generating a block diagram of the transmission and then, based on this diagram, to derive the kinematics and dynamics equations of the transmission. For an actual numerical case, the derived equations are used to find characteristic values of the transmission components (gear and chain drives, planetary units) necessary to calculate the speed ratios, the speeds, torques and powers acting on the shafts and coupling (control) elements, and even to estimate the overall efficiency of the transmission.

1. Introduction
Although known for some time, the continuously variable transmissions (CVTs) began to be used in mass-produced automotive-vehicle only in the second half of the 20th century. As their name indicates, these transmissions have the ability to change the speed ratio (transmission ratio) continuously, [1], [2], [3], versus the conventional transmissions where speeds are changed in steps, and therefore discontinuously. The part of the transmission which allows a continuous gear-ratio change is named variator and represents the core element of any CVT.

The variator is a convertor (transformer) of rotational mechanical energy. It receives at its input shaft a power flow with certain angular speed and moment (torque) and delivers at its output shaft another power flow with possible different speed and moment. Thus, the variator is able to change, in an automatic or controlled way, its speed ratio (transmission ratio), while the output torque will be equal to the product of input torque, transmission ratio and efficiency.

Considering the working principle (the type of energy used to modify the parameters of the power between input and output), friction, hydrodynamic, hydrostatic and electric variators can be identified.

While the acronym CVTs refers mainly to transmissions employing mechanical friction variators, the aspects presented in this paper are also applicable to continuously variable transmissions with hydrodynamic, hydrostatic or electric variators.

Nowadays, the mechanical friction CVTs have important market share of the total mechanical transmissions, with Japan reaching about 40% of the total. A steady increase of CVT use has occurred during the last decades [4].

The main factors responsible of the rapid growth of CVT use in automobiles are as follows:

- the increased demand for environment protection and fuel saving by the automotive industry;
- the recent improvements in steel and traction oils;
- the advances in electronic control systems;
- the emergence of new and innovative ideas on new transmissions or hybrid propulsion systems layouts.

The present work is an attempt to help in the respect of this last item, i.e. to validate the viability of a new transmission layout and to permit the adoption of the primary kinematic and dynamic parameters of the design.

To analyze the working possibilities of such CVT it is necessary to follow a systematic approach. The method proposed in this paper consists in realizing first a block diagram of the transmission and then, based on that diagram, to derive the governing equations of the transmission kinematics and dynamics functioning. On the base of some adopted design data, the next step consists in the use of the derived equations to find characteristic values of the transmission components (gear and chain drives, planetary units), necessary to start the design. Finally, the functional equations will be used to calculate the speed ratios, the speeds, torques and powers acting on the shafts and coupling (control) elements, and even to estimate the overall efficiency of the transmission.

2. Components of the Multi-Regime CVTs

In some cases, a hydrostatic or an electric variator can solely represent the entire CVT, because:
- the variator input (respectively the pump or the generator) can be connected at the engine and the variator output (the hydrostatic or electric motor) can drive the wheels directly;
- the variator permits to reverse the vehicle direction of movement (both the forward and reverse driving are allowed);
- the variator is able to adapt enough the vehicle speed- and tractive force, ensuring inclusively the vehicle start-up, in order to cover all driving conditions; which means that the CVT represented by a hydrostatic or an electric variator is in fact an infinitely variable transmission (IVT).

Because the others batch-produced variator-types (the hydrodynamic torque convertors and the mechanical friction variators) are not able to invert the driving direction or to ensure a sufficient spread of the speed ratios, in addition to the variator, there is the need for other functional modules to obtain a CVT [4]. These are driving invertor, start-up device, gear or chain drives to adapt the speed, a parking brake, and a control system to change the speed ratio, together with a hydraulic power source for lubrication, cooling and actuation. Moreover, if the drive axle is placed on the same side of the vehicle as the engine, the CVT will also include the axle subassemblies i.e. the final drive and the differential. Consequently, the resultant CVT can become quite complex.

2.1. The variator

Over time, numerous CVT designs, with different type of variators, have been introduced in motor vehicle production. Some electro-mechanical or hydro-mechanical transmissions for heavy-vehicles contain a variator which is an electric generator-motor assembly [1] or a hydrodynamic torque convertor [1], [5]. A successful CVT with a hydrostatic variator used in tractor is presented in reference [6].

The friction variators (traction drives) used today in motor-vehicles are generally of two types [1], [2], [3]: pulleys variators and toroidal variators.

At the pulleys variators the axes of the input and output shafts are parallel and the two shafts are rotating in the same direction [7], [8]. At the toroidal variators the input and output shafts are coaxial and the two shafts may rotate either in the opposite direction (single-roller full-toroidal [9], or single-roller semi-toroidal [10]) or in the same direction in other designs (double-roller full-toroidal [11] or VariGlide [12]). The keeping or inversion of the rotation direction at the output shaft with respect to the input shaft is indicated on the symbols presented in the Table 1 through a plus (+) or minus (-) sign.

The efficiency of the previously presented variators depends on many factors, from which the most important are the instantaneous speed ratio and input torque [1], [4].
Table 1. Suggested symbols to be used in the construction of the CVTs block diagrams

| Parallel axes variator (pulleys variator) | Coaxial variator (toroidal variator) | Electric or hydrostatic variator | Electric generator or hydrostatic pump | Electric or hydrostatic motor | Clutch or brake |
|------------------------------------------|-------------------------------------|---------------------------------|---------------------------------------|-------------------------------|----------------|
| ![Parallel axes variator](image1)        | ![Coaxial variator](image2)        | ![Electric or hydrostatic variator](image3) | ![Electric generator or hydrostatic pump](image4) | ![Electric or hydrostatic motor](image5) | ![Clutch or brake](image6) |

2.2. The mechanical drives with constant speed ratio
The devices used to reduce or increase the speed with a fix step are gear- or chain drives (Table 1). While the chain drives are maintaining the same direction of rotation of the input and output shafts (indicated by the plus sign), the spur gears will invert the rotation in the case of exterior meshing (-), but will keep the rotation way in the case of interior meshing (+).

2.3. The control devices
The clutches or brakes used to change the CVT working regime will be represented in the block diagrams by a switch sign (Table 1). If the switch is on (clutch or brake engaged) the torque will be transmitted. If the switch is off (clutch or brake disengaged) the torque will be interrupted.

2.4. The planetary units
A planetary unit (epicyclic train) is a mechanism (generally with gears or sometimes with revolution bodies) having three coaxial shafts and two degrees of freedom. Combining two or more such elementary planetary mechanisms, a planetary (epicyclic) gearset will be obtained which may have three or more degrees of freedom.

Further, the three central elements (shafts) of the planetary unit will be indicated by the indices \( h, x \) and \( y \) (where \( h \) indicates the planet carrier and \( x \) and \( y \) the other shafts).

The next equation due to Willis [1], [13],

\[
\omega_x - i_0 \omega_y - (1 - i_h) \omega_h = 0 \quad \text{(equivalent with: } \frac{\omega_x - \omega_h}{\omega_y - \omega_h} = i_0 = (-1)^p \frac{z_y}{z_x} \text{)}
\]

will be used in the study of the planetary unit kinematics. Here \( p \) is the number of planet wheels groups (normally, \( p=1 \) or \( p=2 \)), \( z_x \) and \( z_y \) are the teeth numbers of the gear wheels \( x \) and \( y \) respectively, and \( i_0 \) is known as the internal speed ratio of the planetary unit (the speed ratio from the wheel \( x \) to the wheel \( y \) when the carrier is blocked: \( \omega_h=0 \)).

Assuming negligible friction and stationary working regime (constant speeds for all three shafts) [1], the dynamics of the planetary unit will follow equations (2) and (3) below:

\[
M_x + M_y + M_z = 0 \quad \text{(2)}
\]

which is obtained applying the principle of moment equilibrium for the whole planetary unit, and

\[
i_h M_x + M_y = 0 \quad \text{(equivalent with: } \frac{M_y}{M_x} = -i_h = (-1)^{p-1} \frac{z_y}{z_x} \text{)} \quad \text{(3)}
\]

which is derived applying the principle of moment equilibrium for the planet wheels.

The most widely used planetary units in the constructions of automotive transmissions consist of a sun wheel \( s \), a ring wheel \( r \), a planet carrier \( h \) and more planet wheels. Noting with \( k = \frac{z_y}{z_x} = |i_h| \)
the positive ratio of the outer and inner wheels teeth-numbers, from the equations (1), (2) and (3) one can obtain particular equations for the most common planetary units (Table 2).

Table 2. Characteristics of the most used planetary units in the CVTs design

| PU type                       | PU kin. sch. | PU block diag. | \( i_h \) | Torque ratio |
|-------------------------------|--------------|----------------|----------|-------------|
| Single-planet epicyclic unit  | ![Image](image1.png) | ![Image](image2.png) | \( -\frac{z_r}{z_s} \) | \( \frac{M_r}{M_s} = k = \frac{z_r}{z_s} \) |
| (spur gears)                  |              |                |          |             |
| Double-planet epicyclic unit  | ![Image](image1.png) | ![Image](image2.png) | \( +\frac{z_r}{z_s} \) | \( \frac{M_c}{M_s} = 1 + k = 1 + \frac{z_r}{z_s} \) |
| (spur gears)                  |              |                |          |             |
| Single-planet epicyclic unit  | ![Image](image1.png) | ![Image](image2.png) | \( -1 \)  | \( \frac{M_{s2}}{M_{s1}} = k = 1 \) |
| (bevel gears)                 |              |                |          |             |

The moments applied to the planetary-unit shafts are considered positive for the driving ones, and negative for the resistant ones (in fulfilment of equation 2). Due to its three shafts, in a planetary unit the magnitude (absolute value) of one moment will be always equal with the sum of the other two moment magnitudes. In the symbols presented in the third column of the Table 2, the area associated with one central element (carrier \( c \), sun \( s \) and ring \( r \)) is equal with the summed areas of the other two. Also, arrows will be used to indicate the input and output moments (the orientation of the power flows).

Some friction variators, as Dana’s VariGlide toroidal variator [12], can be realized (and used) in two configurations: as a normal variator, with one degree of freedom, or as a planetary unit, with two degree of freedom. In this second case, the design includes three rotating shafts (the balls-axes support is rotating also). Thus, this mechanical device can have two inputs and one output for the torque or vice-versa, one input and two outputs. For that reason, the variator behaves similarly to a planetary mechanism (the internal speed ratio \( i_h \) of which may be changed, unlike the gear planetary mechanism which maintains constant ratio). This constructive solution was therefore named CVP (Continuously Variable Planetary) variator.

3. Multi-Regime Power-Split CVTs

Due to their possibility to generate more speed ratio or to join or split power flows, the planetary units are widely included in the construction of mechanical transmissions, CVTs being no exception. For example, the CVTs with a single power flow use commonly a planetary unit as inversor [4], allowing not only the vehicle forward-driving, but also the reverse driving, generally with a smaller speed. Its control devices (normally a clutch and a brake) may be used also as starting device.

A power-split configuration of a CVT realizes firstly the splitting of the power flow in two or more mechanical paths and finally the re-joining of these flows at the transmission output. Such a design has several important advantages:

- bigger CVT power input for the same variator power loading capability;
- bigger overall efficiency of the CVT;
- the possibility to obtain, at a certain variator speed ratio, a particular functioning situation named geared-neutral condition.
At the geared-neutral (also named powered-neutral), the transmission is engaged and ensures an infinity speed-ratio, at which the vehicle speed remains zero, while the engine is running under certain load, determined by external forces acting on the vehicle (like grade and aerodynamic forces). That means a continuous and smooth speed change it is possible from forward driving, passing through stop, to reverse driving (and vice-versa) – the vehicle can shuttle (can switch its movement between forward and reverse) without any actuation of the CVT clutch or brake [14].

To obtain the “geared-neutral”, the planetary unit must be placed at the transmission output (as summation device). If the planetary unit is placed at the input, an opposed functional case can be obtained: speed ratios with very small absolute values (inclusive zero). Named „geared zero” [15], this case corresponds to the working situation in which the transmission is engaged (under power), the vehicle is moving, but the engine is stopped. That situation may be used to stop and to restart the engine during driving, for example at the hybrid propulsion systems (the start/stop function).

A multi-regime transmission is able to change its power flows structure by engaging or disengaging, in an appropriate mode, some control elements i.e. clutches, brakes, dog clutches or one-way clutches. In the case of such CVT, the variator needs to pass (“sweep”) more times through its speed-ratio range in order to cover the larger range necessary for the transmission (for example, two times for a two-regime CVT, as is presented in Figure 1, bottom side).

**Fig. 1** Possible transposition of the variator speed ratio into the CVT overall speed ratio [14]

The variant of two-regimes CVT with power-split realized in the high-range speed ratios (Figure 1, bottom right) has the advantage of a higher efficiency on the most driving time (at medium and high vehicle speeds), but the necessity of a starting device still exists. The other variant, with the power-split attained at low vehicle speeds (Figure 1, bottom left), eliminates the need of a starting device and ensures excellent drivability in difficult terrains, but with a penalty of the vehicle overall fuel efficiency.

Despite of their added complexity, IVT with three or even four regimes are studied [16], because the power-split architecture may have key benefits, providing an increase in tractive effort simultaneously with higher efficiency and durability.

**4. Requirements for the Multi-Regime CVTs**

Even if currently there are only few power-split CVTs implemented in motor-vehicle series-production, a large number of such solutions (based on many variator types and planetary units) have been studied by researchers [10], [9], [15], [17], [18], [19].
The packaging, kinematic, dynamic and control possibilities of the adopted variator types will influence the layout of the transmission and, consequently, its suitability for different vehicle categories (front-wheel drive, rear-wheel drive or all-wheel drive vehicles). For example, the variator positions of the shafts or number of working cavities will impose different implementations of the other transmission components. The variator speed-ratio spread, capable torque, efficiency variations or reaction time are also very important aspects of the possible power-split designs.

The diminish of the number, together with the dimensional minimization and functional optimization of the control elements (couplings, brakes, dog clutches or one-way clutches) are also major concerns of the design engineers.

Even aspects which seem to be of minor importance are studied, as the avoidance (as much as possible) of the situations in which the variator shafts may stop rotating or the rotation way is reversed.

The existent variators present different control possibilities: in some designs the speed ratio it is controlled, while in others the torque is controlled, which makes one variator or the other more suitable for a specific application or driving situation. For the friction variators, a second action of control is oriented towards the contact force or towards the slip between the discs and rollers. All these aspects represent big problems for the engineers.

5. Example of CVT Design Parameters Setup
A constructive solution of a double-roller full-toroidal IVT, suggested by the Australian company Ultimate Transmissions on its website [11], will be analyzed to setup a hypothetic transmission.

In the left side of the Figure 2 it is presented the kinematic scheme drawn by the authors. That was the base for the construction of the power-flows diagram, shown in the same figure.

Fig. 2 Kinematic scheme (left) and power-flows diagram (right) of a dual-mode power-split IVT, based on double-roller full-toroidal variator

The power flow splits at the level of the input shaft. The two flows are joined again at the summation planetary unit. That has a single group of planet wheels.

Three exterior-meshing spur gears \( G_L, G_H \) and \( G_F \) and the bevel-gear axle-differential are also necessary for this transmission. Two clutches \( C_L \) and \( C_H \) ensure the two working regimes.

Obtained when \( C_L \) is off (disengaged) and \( C_H \) is on (engaged), the High regime is characterized by a single power flow and is used for medium and high vehicle speeds. The power passes through the
input shaft, the dual-roller full-toroidal (DFT) variator, the gears $G_H$ and $G_F$ (the final drive) and has the output to the axle differential.

In the Low regime, obtained when $C_L$ is on and $C_H$ is off, the power has two different paths. That regime should ensure geared neutral and low speed driving, both in forward and reverse.

A third situation is also possible: a true neutral, when both clutches are disengaged. The situation when both clutches are engaged is not permitted.

The first ratio-sweep moves from the variator high gear (overdrive) to low gear (the IVT Low regime), and then, after the switchover which disconnects the planetary gear set, the variator moves from low to high.

The first aspect to be verified is if the vehicle moves forward in the CVT High regime. Considering that the engine is mounted in the vehicle right side, the (+) sign of the variator (caused by the double roller) and the two (-) signs indicate that the vehicle drive-wheels will rotate in the same way as the engine. That means the condition is ensured.

5.1. The initial data
Before starting the full transmission set-up, some initial values must be known.

From the calculation of the vehicle dynamical performances, the next transmission overall speed-ratios are already established: $i_{TV}=3.64$, the ratio at which the vehicle will obtain its maximal speed; $i_{To}=1.952$, the minimum ratio for forward driving (overdrive, necessary for cruising with the legal speed on high-ways, at low engine RPM); $i_{Ti}=-6.25$, the ratio for the maximal speed in reverse.

Also, there are known the variator minimum and maximum ratios: $i_{Vo}=0.427$ (overdrive) and $i_{Vu}=2.343$ (underdrive) [11]. From these, the variator ratio spread (or domain or range) $D_V$ results as

$$D_V = \frac{i_{VH}}{i_{Vo}} = 5.487 \tag{4}$$

and that allows to find the transmission ratio at which the two regimes are changed (switched over)

$$i_{Te} = i_{To}D_V = 10.711. \tag{5}$$

Another initial calculation permits to find the variator ratio at which the vehicle reaches its maximal speed:

$$i_{Vv} = \frac{i_{Tv}}{i_{Vo}} = 0.796. \tag{6}$$

5.2. The transmission ratios
Studying the power-flows diagram from Figure 2, it can be observed the possibility to write three equations (two equations for the speed ratios in the transmission High and Low regimes and the equation of Willis for the summation planetary unit). Because the number of design unknowns is four (the intern ratio of the planetary unit $i_0$ and the speed ratios $i_L$, $i_H$, $i_F$ of the gears), that means it is necessary to adopt one value. In this case, the ratio of the final drive is adopted, $i_F=-3.76$, to maintain a certain compatibility with the conventional transaxles of the vehicle.

Because the overall transmission ratio in the High regime is

$$i_T = i_VF, \tag{7}$$
in the particular case of the minimum ratio for forward driving $i_{To}$, when the variator works with its highest ratio $i_{Vo}$, it results the ratio of the gear $G_H$:

$$i_H = \frac{i_{Tv}}{i_{Vo}F} = -1.216, \tag{8}$$

In the transmission Low regime, the overall ratio is

$$i_T = i_PF, \tag{9}$$
where $i_{PS} = \frac{\omega_{in}}{\omega_r}$ is the speed ratio of the power-split part.

Observing from the Figure 2 that
\[
\frac{\omega_c}{i_L} = \frac{\omega_s}{i_V i_H}
\]
and introducing in the equation of Willis
\[
\omega_s - i_o \omega_r - (1 - i_0) \omega_c = 0
\]
it results
\[
i_{PS} = \frac{i_0}{i_V i_H - i_0 i_L}.
\]
where the ratios $i_0$ and $i_L$ are unknown.

Particularizing for the extreme ratios of the Low regime, corresponding to the variator ratios $i_{V_o}$ and $i_{V_u}$, it obtains
\[
\frac{i_0}{i_{V_o} i_H - i_0 i_L} = \frac{i_{PS}}{i_F} = 1.662; \quad \frac{i_0}{i_{V_u} i_H - i_0 i_L} = \frac{i_{PS}}{i_F} = -2.849.
\]
Solving this last system of two equations, it obtain the internal ratio of the planetary unit,
\[
i_0 = \frac{\frac{1}{i_{V_o} i_H - i_0 i_L}}{i_{PS}} = -1.653,
\]
and the ratio of the gear $G_L$,
\[
i_L = \frac{1 - i_0}{i_{V_u} i_H - i_0 i_L} = -2.849.
\]

The last element needed to finalize the setup of the IVT main parameters is the variator ratio at which the geared-neutral will be obtained. For that, the ratio $i_{PS}$ must have an infinite value, which is possible only if the denominator in the equation (12) will be equal with zero. Putting this condition, the geared-neutral variator ratio is obtained:
\[
i_{V_{gn}} = \frac{i_L}{i_{H}(1 - i_0)} = 0.883
\]

**Fig. 3** Ratio spread of the IVT, compared with common CVT- and 5-gear manual transmissions

A graphical representation of the resultant IVT kinematic characteristics (eliminating the contribution of the final drive ratio $i_F$) is presented in **Figure 3**. Here it can be seen how the two regimes are distributed and also the important advantage of a larger ratio domain, if one compare the
IVT with a 5-speed gearbox and with a normal CVT (excluding also the influence of the final drive ratio).

5.3. The shaft torques

The calculation of the torques (torsional moments) acting on the IVT shafts are obtained starting from the schemes in Figure 2 and on the equations in the first line of Table 2.

Noting the torques acting on the input shafts of the variator and gears with \( M_V, M_H, M_L \) respectively, and the torques acting on the shafts of the planetary unit with \( M_c, M_s, M_r \), a system of six equations may be written:

\[
\begin{align*}
M_{in} + M_L + M_V &= 0, && M_c = M_L i_L \eta_L, \\
M_s &= M_H i_H \eta_H^q, && M_H = M_V i_V \eta_V^q, \\
M_c + M_s + M_r &= 0, && M_r = -i_0 \eta_0^q M_s
\end{align*}
\]

In these equations, the values \( \eta_V, \eta_H, \eta_L \) and \( \eta_0 \) represent respectively the efficiencies of the variator, gears \( G_H \) and \( G_L \) and planetary unit (when only the frictions proportional with the input torques are considered). Also, the exponent \( q=\pm 1 \) will consider the direction of the power flow in the branch of the variator (\( q=+1 \) for normal flow; \( q=-1 \) for inversed flow, when the power loop appears), while the exponent \( s=\pm 1 \) keep account of the direction of power flow through the planetary unit (\( s=+1 \) if the sun wheel rotates faster as the ring wheel i.e. \( |\omega_s|>|\omega_r| \)).

Solving the equations system (17), the torques \( M_V, M_H, M_L, M_c, M_s, M_r \) can be expressed as functions of the input torque \( M_{in} \). For example, the variator input torque and the output torque of the transmission power-split part are:

\[
\begin{align*}
M_V &= M_{in} \frac{i_L \eta_L \eta_V^q}{\text{den}} \\
M_r &= M_{in} \frac{-i_0 \eta_0^q i_V \eta_V^q i_H \eta_H^q}{\text{den}} = M_V (-i_0 \eta_0^q i_H \eta_H^q)
\end{align*}
\]

where the denominator is

\[
\text{den} = (1 - i_0 \eta_0^q i_V \eta_V^q i_H \eta_H^q - i_L \eta_L)
\]

Using the equations (18), in the Figure 4 it is presented how many times the variator input torque and the output torque of the transmission power-split part (the ring wheel torque) are increased with respect to the transmission input torque. As can be seen, at the variator ratio \( i_{Vgn}=0.883 \) (geared neutral), the gain is theoretically infinite. Negative values are obtained for \( i_V<i_{Vgn} \). That means also that, while the vehicle moves in reverse, the power flow in the variator branch is normal, not inversed as is presented in the Figure 2 (no power loop in this case).

![Fig. 4 Torques amplification vs. variator ratio (IVT “Low” regime)](image)
In the Figure 5 it can be observed how big the variator input torque $M_V$ is if compared with the output torque $M_r$ of the transmission power-split part. As can be seen, the variator torque exceeds the output torque only when the vehicle moves fast in reverse. The discontinuity of the curve is explained by the change of the power flow in the variator branch.

5.4. The efficiency
The efficiency of the IVT will be calculated in different ways for the two working regimes. While in the High regime (with a single power path) the overall efficiency is simply

$$\eta_T = \eta_V\eta_H\eta_F$$

in the Low regime, characterized by two power paths, the overall efficiency is

$$\eta_T = \eta_{PS}\eta_F = \frac{M_r}{M_\text{in}}\frac{\omega_r}{\omega_\text{in}}\eta_F = \frac{M_r}{M_\text{in}}\frac{1}{i_{PS}}\eta_F$$

In the Figure 6 it is represented the variation of the theoretical efficiency of the IVT for the entire domain of the transmission ratios. The calculation was performed considering the next constant values for the component efficiencies: $\eta_V=0.955$, $\eta_H=0.98$ and $\eta_0=0.965$. [1], [13].

In this hypothetical case of IVT, the power split layout of the Low regime ensures an efficiency at reduced (forward and reverse) vehicle speeds which is superior to the vehicles equipped with a gearbox or a normal CVT using as starting device a friction clutch or even hydrodynamic torque converter. This conclusion was also presented in reference [14].

6. Conclusions
The graphical representations of the power flows with changing structures in complex CVTs are useful in a better understanding and analysis of the behavior of the transmission. In the present paper, the
authors have shown how such diagrams allow a simplification of the writing of the functional equations.

On the base of these equations it is then possible to setup the main characteristics of the transmission components. A practical example illustrating the procedure was also shown in the last part of the paper.

The method presented here can be useful also for the analysis of epicyclic automatic transmissions and even for hybrid propulsion systems that use planetary units.

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