Recoilless Resonant Emission and Detection of Electron Antineutrinos: Mössbauer Antineutrinos

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      b) zero-point motion

IV) Requirements for successful experiment

V) Interesting experiments

VI) Conclusions
I) $\beta$-decay

I) Bound-state $\beta$-decay

J. N. Bahcall, Phys. Rev. 124, 495 (1961)

$$A(Z - 1) \rightarrow A(Z) + e^- + \bar{\nu}_e$$

Bound-state atomic orbit.
Not a capture of $e^-$ initially created in a continuum state (less probable).

Example:

$$^3H \rightarrow ^3He + e^- + \bar{\nu}_e$$

Atomic orbit in $^3$He

2-body process, $\bar{\nu}_e$ has a fixed energy:

$$E_{\bar{\nu}_e} = Q + B_z - E_R$$

where

$Q = (M_{Z-1} - M_Z)c^2$ end-point energy

$B_z$ binding energy

$E_R$ recoil energy

$^3He + e^-$ recoils
I) $\beta$-decay

Reverse process (absorption):

$$\bar{\nu}_e + A(Z) + e^- \rightarrow A(Z-1)$$

Example:

$$\bar{\nu}_e + ^3He + e^- \rightarrow ^3H$$

energy required for $\bar{\nu}_e$:

$$E_{\bar{\nu}_e} = Q + B_z + E_R'$$

$^3H$ recoils

Bound-state $\beta$-decay has a resonant character which is (partially) destroyed by the recoil in source and target.
## II) Example: $^3$H-$^3$He system

| Decay       | $E_{\nu_e}^{res}$ | $t_{1/2}$ | $B\beta / C\beta$ |
|-------------|--------------------|-----------|------------------|
| $^3$H $\rightarrow$ $^3$He | 18.60 keV | 1132 sec | $6.9x10^{-3}$ (80% ground state, 20% excited states) |

$^3$H (source) and $^3$He (target): gases at T=300K
→ thermal motion, Doppler energy profile

\[
FWMH(300K): 2\Delta = 2E_{\nu_e}^{res}(2k_BT/Me^2)^{1/2} \approx 0.16eV
\]

→ resonant spectral density $\rho \approx 10^6/0.16 \approx 6x10^6$

$2E_R \approx 0.12eV; \quad 2\Delta \approx 0.16eV$

→ reduced overlap

Recoil energy:

\[
E_R = \left(\frac{E_{\nu_e}^{res}}{2Mc^2}\right)^2 \approx 0.06eV
\]

Resonance cross section: $\sigma \approx 1x10^{-42}\text{ cm}^2$

To observe bound-state $\beta$-decay: 100-MCi sources ($^3$H) and kg-targets ($^3$He) would be necessary
III) Recoilless antineutrino emission and detection: Mössbauer neutrinos

1) Recoilfree fraction

Stop thermal motion!
Make \( E_R \) negligibly small!

\(^3\text{H}\) as well as \(^3\text{He}\) in metallic lattices:
freeze their motion \( \rightarrow \) no Doppler broadening.
\( M \rightarrow M_{\text{lattice}} \gg M \)
Leave lattice unchanged, leave phonons unchanged.

Energy of lattice with \( N \) particles:
\[
E_L = \sum_{s=1}^{3N} (n_s + 1/2) \eta \omega_s \quad (n_s = 0,1,2,\ldots)
\]
\( 3N \) normal modes

\[
E_L = \int_{0}^{\omega_{\text{max}}} \left( n(\omega) + 1/2 \right) \omega \cdot Z(\omega) d\omega \quad \text{with} \quad n(\omega) = 1/(\exp(\eta \omega / k_B T) - 1)
\]

\( Z(\omega) \cdot d\omega \): number of oscillators with frequency \( \omega \) between \( \omega \) and \( \omega + d\omega \)

Recoil energy:
\[
E_R = \frac{(E_{\text{res}})^2}{2 M c^2}
\]

zero-point energy
III) Recoilless antineutrino ...

Recoilfree fraction $f$:

$$f = e^{-\left(\frac{E}{\eta}\right)^2 \cdot \langle x^2 \rangle} \quad \rightarrow \quad f < 1$$

$$f(T) = \exp \left\{ -\frac{E^2}{2Mc^2} \cdot \frac{1}{3N} \cdot \int_0^{\omega_{\text{max}}} \frac{Z(\omega)}{\eta \omega} \left[ \frac{2}{\exp \{ \eta \omega / k_B T \} - 1} + 1 \right] d\omega \right\}$$

Debye model:

$T \to 0$:  

$$f(T \to 0) = \exp \left\{ -\frac{E^2}{2Mc^2} \cdot \frac{3}{2k_B \Theta} \right\}$$

$f$ depends on: transition energy $E$  

mass $M$ of the atom  

Debye temperature $\Theta$

Example: $^3\text{H} - ^3\text{He}$

typically: $f(0) \approx 0.27$ for $\Theta \approx 800$K

Emission and absorption:

$$f^{^3\text{H}} \cdot f^{^3\text{He}} \approx 0.07 \quad \text{for } T \to 0$$
III) Recoilless antineutrino ...

Lattice expansion and contraction

$^3\text{He}$ and $^3\text{H}$ use different amount of lattice space. $^3\text{H}$ is more strongly bound than $^3\text{He}$. Will this cause lattice excitations?

The antineutrino picks up (delivers) most of the 18.6 keV transition energy. The energy of the electron is very small.

Adiabatic approximation: when $^3\text{He}$ forms, it pushes neighboring atoms away; lattice energy is “stored “ until it is released again when by antineutrino absorption a $^3\text{H}$ is formed.

→ Theoretical calculations
2) Linewidth

minimal width (natural width): $\Delta E^{nat} = \Gamma = \eta / \tau \quad \tau : \text{lifetime}$

$^3\text{H}: \tau = 17.81 \text{ y} \quad \Delta E^{nat} = \Gamma = 1.17 \cdot 10^{-24} \text{eV} \quad \text{(extremely narrow)}$

Two types of line broadening:

a) homogeneous broadening
   due to fluctuations, e.g. of magnetic fields

b) inhomogeneous broadening
   due to stationary effects, e.g. impurities, lattice defects

How big are these broadening effects?
III) Recoilless antineutrino ...

a) homogeneous broadening

Measurements: $^3$H (Pd), $^3$H (Ti-H), NbH

Typical relaxation times:
$T_2 \sim 2\text{ms}, 79\mu\text{s}$

$\Delta E \sim 8.6 \times 10^{-12}\text{eV} = \Gamma_H$
$\sim 7 \times 10^{12} \Gamma$

Magnetic interactions:
  a) $^3$H, $^3$He with atoms (nuclei) of metallic lattice
  b) $^3$H – $^3$H magnetic dipolar spin-spin interaction

Relaxation between the sublevels affects the lineshape and the total linewidth.

The linewidth is determined by the relaxation rate.
2) Linewidth

$^{109m}$Ag: gravitational spectrometer

$\Gamma \approx 1.2 \cdot 10^{-17} \text{ eV} \quad \tau \approx 40 \text{s}$

V.G. Alpatov et al., Laser Physics 17 (2007) 1067
Homogeneous Broadening: Magnetic Relaxation

Simplest magnetic relaxation model consists of a three-level system

Two lines of (almost) natural width;
With increasing $\Omega$, the lines broaden
→ effective lifetime
Typical for resonances in Ag and for the $^3$H/$^3$He system. For Ag:
$\Omega \sim 10^5$ s$^{-1}$ and $\Omega \sim 10$ s$^{-1}$

Intensity is distributed over a broad pattern, which extends over the total hf splitting $\Omega_0$ as suggested by the time-energy uncertainty principle

Motional narrowing: one line at the center of the hf splitting of practical natural width.
Stochastic frequency changes: between lines 1 and 2. Averaging process over short parts of the lifetime. Not for Ag and $^3$H/$^3$He.
Homogeneous Broadening: Frequency Modulation

M. Salkola and S. Stenholm, Phys. Rev. A 41, 3838 (1990)

\[ A \propto \sum_{k=-\infty}^{k=\infty} J_k^2(\eta) \frac{1}{\left[ (\Delta_0 / \Gamma) - k\xi \right]^2 + 1} \]

- \( \Delta_0 = \omega_0 - \omega \)
- \( \Gamma : \text{linewidth} \)
- \( \xi = \frac{\Omega_0}{\Gamma} \)

\( \eta \approx 1 \Rightarrow \Omega \approx \Omega_0 \)

motional narrowing: \( \Omega \gg \Omega_0 \Rightarrow \eta \approx 0 \)

only center line at \( \omega_0 \) survives

\( \Omega_0 \gg \Omega \Rightarrow \eta \gg 1 \)

motional narrowing: not possible

Typical for resonances in Ag and for the \(^3\)H/\(^3\)He system. For Ag:
\( \Omega_0 \sim 10^5 \text{ s}^{-1} \) and \( \Omega \sim 10 \text{ s}^{-1} \)

\( \Omega_0 \cos S t \)

\( \omega \)

\( \omega_0 \)

\( \xi \)

\( \Delta_0 \)

\( \Gamma \)

\( \eta \)

\( \omega \)

\( \omega_0 \pm k\Omega \)

sum of Lorentzians, located at \( \omega=\omega_0 \pm k\Omega \)

\( \approx \Rightarrow \Omega \gg \Omega_0 \eta \approx 0 \)

many sidebands \( \rightarrow \) at \( \omega_0 \)

very little intensity
b) inhomogeneous broadening

In the best single crystals: \((1 + a)\Gamma \sim 10^{-13} \text{ eV corresp. to } 10^{11} \Gamma \) or larger

Both types of broadening reduce the resonant reaction intensity
III) Recoilless antineutrino ...

3) Relativistic effects

Second-order Doppler shift due to mean-square atomic velocity \(<V^2>\)

\[
\Delta t = \frac{\Delta t'}{\sqrt{1 - (V/c)^2}}
\]

Time-dilatation effect: moving system

stationary system

Frequencies:

\[
v = v' \sqrt{1 - (V/c)^2} \approx v' \left(1 - \frac{V^2}{2c^2}\right)
\]

Second-order Doppler shift:

\[
\Delta v = v - v' = -\frac{V^2}{2c^2}
\]

Reduction of frequency (energy)
III) Recoilless antineutrino …

Within the Debye model:

\[
\frac{\Delta E}{E} = \frac{9k_B}{16Mc^2} \left( \Theta_s - \Theta_t \right) + \frac{3k_B}{2Mc^2} \left[ T_s \cdot \frac{f \left( \frac{T_s}{\Theta_s} \right)}{\Theta_s} - T_t \cdot \frac{f \left( \frac{T_t}{\Theta_s} \right)}{\Theta_s} \right]
\]

Zero-point energy

If \( |T_s - T_t| = 1 \) degree \( \rightarrow \frac{\Delta E}{E} \approx 10^{-13} \) \( \rightarrow \Delta E \approx 200 \cdot \Gamma_{\text{exp}} \)

Low temperatures: \( T_s \approx T_t \approx 0 \) \( \rightarrow [\ldots] \approx 0 \) \( \text{However, zero-point energy remains!} \)

If \( |\Theta_s - \Theta_t| = 1 \) degree \( \rightarrow \frac{\Delta E}{E} \approx 2 \cdot 10^{-14} \) \( \rightarrow \Delta E \approx 40 \cdot \Gamma_{\text{exp}} \)

The Debye temperature for \(^3\text{H}\) has to be the same in source and target. The same holds for \(^3\text{He}\). The Debye temperatures of \(^3\text{H}\) and \(^3\text{He}\) in the metal matrix do not have to be equal.
IV) Requirements for experiment

A) Preparation of source and target

Source:
$^3$H chemically loaded into metals to form hydrides (tritides), e.g., Nb: in tetrahedral interstitial sites (IS).

Target:
$^3$He accumulates with time due to the tritium trick:

$$\text{Nb}^3\text{H}_x \xrightarrow{\text{time}=200\text{d}} \text{Nb}^3\text{H}_{x-y}^3\text{He}_y \xrightarrow{\text{remove}} \text{Nb}^3\text{He}_y$$

Remove $^3$H by isotopic exchange $^3\text{H} \rightarrow \text{D}$

R. S. Raghavan, hep-ph/0601079 v3, 2006
IV) Requirements for experiment

How much metal for source and target?

Source:

1 kCi of $^3$H (~100mg $^3$H): ~3g of Nb$^3$H
for NMR studies: 0.5 kCi $^3$H in 2.4g PdH$_{0.6}$

Target:

100mg of $^3$He implies ~100g of Nb$^3$H aged for 200 d
### IV) Requirements for experiment

#### B) Event rates for $^3$H – $^3$He recoilless resonant capture of antineutrinos

| Base line | $^3$H  | $^3$He  | Antineutrino capture per day | $R_\beta(\Delta t=65\text{d})$ per day |
|-----------|--------|---------|------------------------------|-------------------------------------|
| 5 cm      | 1 kCi  | 100 mg  | $\sim40 \times 10^3$        | $\sim40$                            |
| 10 m      | 1 MCi  | 1 g     | $\sim10^3$                   | $\sim10$                            |

$R_\beta(\Delta t)/\text{day}$: Reverse $\beta$-activity rate after growth period $\Delta t=65\text{d}=0.01\tau$

R. S. Raghavan, hep-ph/0601079 v3, 2006
IV) Requirements for experiment

C) Recoilless emission and detection of Mössbauer Anti-neutrinos

1) Recoilfree fraction \( f \):

Use low temperatures (liquid He) to make \( f \) large.

How are \(^3\)H and \(^3\)He bound in the metallic matrix? Can the lattice expansion and contraction be assumed to occur adiabatically?

Difficult problems. Inelastic neutron scattering experiments are necessary to determine the recoilfree fraction of \(^3\)H and \(^3\)He.
IV) Requirements for experiment

2) **Source and target** should be as similar as possible:

Use low temperatures (liquid He) to avoid effects caused by temperature drifts.

However:

Source contains $^3$H, whereas target contains mainly $^3$He.

Interaction (chemical bonds) of $^3$H and $^3$He with the metal atoms may be different because of different neighborhood

$$
\Theta_s \neq \Theta_t
$$
IV) Requirements for experiment

3) Linewidth

Two types of line broadening:

- **a) homogeneous broadening**
  - due to fluctuations, e.g. of magnetic fields, single-line spectrum highly unlikely

- **b) inhomogeneous broadening**
  - due to stationary effects, e.g. impurities, lattice defects, different atomic neighbors

Mössbauer Anti-neutrinos:

- **Energy width:** \( \Gamma_{\text{exp}} = 8.6 \cdot 10^{-12} \text{eV} \)
- **Cross section:** \( \sigma_{\text{res}} \approx 3 \cdot 10^{-33} \text{cm}^2 \)
V) Interesting experiments

1) Do Mössbauer neutrinos oscillate?

2) Determination of mass hierarchy and oscillation parameters
   $\Delta m^2_{32}$ and $\Delta m^2_{12}$: 0.6% and $\sin^2 2\theta_{13}$: 0.002

3) Search for sterile neutrinos

4) Gravitational redshift experiments (Earth).
V) Interesting experiments

1) Do Mössbauer neutrinos oscillate?

S.M. Bilenky et al., Phys. Part. Nucl. **38**, 117 (2007)
S.M. Bilenky, arXiv: 0708.0260
S.M. Bilenky et al., J. Phys. **G34**, 987 (2007)
E.Kh. Akhmedov et al., arXiv: 0802.2513; JHEP 0805 (2008) 005
S.M. Bilenky et al., arXiv: 0803.0527 v2
E.Kh. Akhmedov et al., arXiv: 0803.1424
S.M. Bilenky et al., arXiv: 0804.3409
V) Interesting experiments

2) If Mössbauer neutrinos do oscillate:

Ultra-short base lines for neutrino-oscillation experiments

Oscillatory term: \( \sin^2 \left( \frac{\pi L}{L_0} \right) \)

Oscillation length: 
\[
L_0 = 4\pi \eta c \frac{E}{|\Delta m^2|} \approx 2.480 \frac{E / MeV}{|\Delta m^2| / eV} \quad [m]
\]

A) Determination of \( \Theta_{13} \): \( E=18.6 \) keV instead of 3 MeV.

\( \Delta m_{23}^2 \) observed with atmospheric neutrinos

Chooz experiment: \( \sin^2 2\Theta_{13} \leq 2 \cdot 10^{-1} \)

Oscillation base line: \( L_0/2 \sim 9.3 \) m

Base line \( L \) of 9.3 m instead of 1500 m
V) Interesting experiments

B) Mass hierarchy and oscillation parameters

To determine mass hierarchy:
Measure $\Delta m^2$ in reactor-neutrino and muon-neutrino (accelerator long-baseline) disappearance channels to better than a fraction of 1%

For $\sin^2 2\theta_{13}=0.05$ and 10 different detector locations one can reach uncertainties:
in $\Delta m^2_{31}$ and $\Delta m^2_{12}$: 0.6%,
in $\sin^2 2\theta_{13}$: 0.002
V) Interesting experiments

3) Search for conversion of \( \overline{\nu}_e \rightarrow \nu_{sterile} \)

LSND experiment: \( \Delta m^2 \approx 1eV^2 \) and \( \sin^22\theta \sim 0.1 \) to 0.001

(largely excluded by MiniBooNE)

Possibility: \( \overline{\nu}_e \rightarrow \nu_{sterile} \)

V. Kopeikin et al.: hep-ph/0310246

Test: Disappearance experiment with 18.6 keV antineutrinos

\[ \text{Oscillation length } L_0 \sim 5\text{cm!} \]

\[ \text{Ultra-short base line, difficult to reach otherwise} \]
4) Gravitational redshift experiments (Earth)

Gravitational redshift: \[ \frac{\delta E}{E} = \frac{gh}{c^2} \]

Experimental linewidth: \[ \Gamma_{\text{exp}} = \Delta = 8.6 \cdot 10^{-12} eV \]

\[ \Delta = \frac{\eta \omega}{c^2} gh_{\Delta} \quad \text{where} \ h_{\Delta} \ \text{is height corresponding to 1 experimental linewidth} \]

\[ h_{\Delta} \approx 4.25 m \]

Can not be used to determine the neutrino mass

Gravitational spectrometer
VI) Conclusions

1) Recoilless resonant emission and detection of antineutrinos: 
   $^3$H – $^3$He system is the prime candidate.

2) Experiment is very difficult:
   a) Recoilfree fraction might be smaller than expected.
      Adiabatic lattice expansion and contraction?
   b) Temperature difference between source and target (temperature shift)
   c) Different Debye temperatures in source and in target (chemical shift)
   d) Homogeneous and inhomogeneous broadening of linewidth

3) If successful, very interesting experiments become possible:
   a) Do Mössbauer neutrinos oscillate?
   b) Mass hierarchy and accurate determination of oscillation parameters
   c) Search for sterile neutrinos (LSND experiment)
   d) Gravitational redshift experiments (Earth).
Extra slides
Earlier papers:

W. M. Visscher, Phys. Rev. 116, 1581 (1959)

W. P. Kells and J. P. Schiffer, Phys. Rev. C 28, 2162 (1983)

More recent papers:

R. S. Raghavan, hep-ph/0601079 v3, 2006

W. Potzel, Phys. Scrip. T127, 85 (2006);

S. M. Bilenky, F. von Feilitzsch, and W. Potzel,
J. Phys. G: Nucl. Part. Phys. 34, 987 (2007);

E. Kh. Akhmedov, J. Kopp, and M. Lindner, 0802.2513 (hep-ph)
I) $\beta$-decay

1) Usual $\beta$-decay

$$A(Z-1) \rightarrow A(Z) + e^- + \bar{v}_e$$

neutron transforms into a proton

occupy states in continuum

3-body process: $e^-, \bar{v}_e$ show (broad) energy spectra

Maximum $\bar{v}_e$ energy: $E_{\bar{v}_e}^{\text{max}} = Q$

where $Q = (M_{Z-1} - M_Z)c^2$

is the end-point energy
I) $\beta$-decay

Resonance cross section

$$\sigma = 4.18 \cdot 10^{-41} \ g_0^2 \cdot \frac{\rho \left( \frac{E_{\text{res}}}{\bar{v}_e} \right)}{ft^{1/2}} \ [cm^2]$$

$$g_0 = 4\pi \left( \frac{\eta}{mc} \right)^3 |\Psi|^2 \approx 4 \left( \frac{Z}{137} \right)^3$$

for low Z, hydrogen-like $\psi$

$m$: electron mass

$|\psi|^2$: probability density of e in A(Z)

$$\rho \left( \frac{E_{\text{res}}}{\bar{v}_e} \right)$$: resonant spectral density, i.e., number of $\bar{v}_e$ in an energy interval of 1MeV around $E_{\text{res}}$.

$ft^{1/2}$ value: reduced half-life of decay

$ft^{1/2} \approx 1000$: super-allowed transition

L.A. Mikaélyan, et al.: Sov. J. Nucl. Phys. 6, 254 (1968)
What does this mean for the effective values $\Theta_s$ and $\Theta_t$?

The differences of these SOD values in source and target have to be the same. In a practical experiment this means:

The Debye temperature for $^3\text{H}$ has to be the same in source and target. The same holds for $^3\text{He}$. The Debye temperatures of $^3\text{H}$ and $^3\text{He}$ in the metal matrix do not have to be equal.
Phonon density of states
III) Recoilless antineutrino ...

B. Balko, I. W. Kay, J. Nicoll, J. D. Silk, and G. Herling, Hyperfine Interactions 107, 283 (1997).
### Candidates for recoilless neutrino absorption

**TABLE I. Candidates for recoilless neutrino absorption.**

| Nuclide | \( Q \) (keV) | \( \tau \) (yr) | \( f_R \) \(^{a}\) (10\(^{-4}\)) | \( \alpha \) (10\(^{-16}\)) | \( \gamma \) | \( \sigma_{\text{eff}} \) (10\(^{-36}\) cm\(^2\)) | \( \sigma_{\text{eff}}/\tau \) \(^{b}\) |
|---------|----------------|----------------|----------------------------|----------------|--------|---------------------------|----------------|
| \(^{3}\text{H}\) | 18.6 | 12.3 | 0.40 | 200\(^{c}\) | 8 | 0.1 | 1.0 |
| \(^{63}\text{Ni}\) | 68 | 92 | 0.07 | 1 | 1 | 10\(^{-9}\) | 10\(^{-9}\) |
| \(^{93}\text{Zr}\) | 60 | 1.5 \times 10^6 | 0.18 | 1 | 7 \times 10^{-5} | 10\(^{-12}\) | 10\(^{-16}\) |
| \(^{107}\text{Pd}\) | 33 | 6 \times 10^6 | 0.62 | 1 | 2 \times 10^{-5} | 10\(^{-11}\) | 10\(^{-16}\) |
| \(^{151}\text{Sm}\) | 76 | 90 | 0.11 | 1 | 1 | 10\(^{-9}\) | 2 \times 10^{-9} |
| \(^{171}\text{Tm}\) | 97 | 1.9 | 0.04 | 1 | 50 | 5 \times 10^{-9} | 3 \times 10^{-7} |
| \(^{187}\text{Re}\) | 2.6 | \(4 \times 10^1\) | 1.0 | 1000\(^{d}\) | 10^{-9} | 2 \times 10^{-7} | 10^{-15} |
| \(^{193}\text{Pt}\) | 61 | 50 | 0.29 | 1 | 2 | 3 \times 10^{-8} | 8 \times 10^{-8} |
| \(^{157}\text{Tb}\) | 58 | 150 | 0.29 | 0.4\(^{d}\) | 0.7 | 2 \times 10^{-9} | 10^{-9} |
| \(^{163}\text{Ho}\) | 2.6 | 7000 | 1 | 73\(^{d}\) | 0.01 | 7 \times 10^{-3} | 1 \times 10^{-4} |
| \(^{179}\text{Ta}\) | 115 | 1.7 | 10^{-2} | 0.5\(^{d}\) | 60 | 10^{-10} | 6 \times 10^{-9} |
| \(^{205}\text{Pb}\) | 60 | 1.4 \times 10^7 | 0.3 | 8\(^{d}\) | 10^{-5} | 10^{-11} | 10^{-16} |

\(^{a}\) Recoilless fraction calculated for effective Debye temperatures assuming that the nuclei are imbedded in \( W \), and that the simple approximations in the text are valid.

\(^{b}\) Normalized to 1.0 for \(^{3}\text{H}\).

\(^{c}\) From Ref. 4.

\(^{d}\) Estimated from atomic wave function calculations of the relevant shells.

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W. P. Kells and J. P. Schiffer, Phys. Rev. C 28, 2162 (1983)
IV) Consequences ...

³He generated in Nb: 
c1: concentration in interstitial sites for different temperatures and times. The He in the T-free absorber below 200K is almost all interstitial.

R.S. Raghavan: 
hep-ph/0601079 
revised v3; calculations: Sandia Natl. Lab., USA
**IV) Consequences** …

### Table 1. He transport parameters in NbT at 200K

| M\_1T\_1 | E1 eV  | E2 eV  | E3 eV  | D/cm\(^2\) s |
|----------|--------|--------|--------|---------------|
| M=Nb     | 0.9\(^a\) | 0.13\(^b\) | 0.43\(^b\) | 1.1E-26\(^c\) |

\(^a\) Ref. 7; \(^b\) Ref. 9; \(^c\) Assumes tritium pre-exponential D\(_0\) (ref. 6)

### Table 2. Theoretical (Ref. 7) EST & ZPE for T and \(^3\)He in Nb interstitial sites (IS)

| Site | EST (eV) | ZPE (eV) |
|------|----------|----------|
|      | \(T\)    | \(He\)   | \(T\)    | \(He\)   |
| TIS  | -0.133   | -0.906   | 0.071    | 0.093    |
| OIS  | -0.113   | -0.903   | 0.063    | 0.082    |

### Table 3. Nearest neighbor (NN) Displacements(%) and measured\(^6\) activation energies E\(_{ac}\) (eV) in NbIS (Ref. 7)

| 1\(^{st}\) NN Displacement | 2\(^{nd}\) NN Displacement |
|----------------------------|----------------------------|
| H  | D  | T  | H  | D  | T  |
| TIS | 4.1 | 3.9 | 3.9 | -0.37 | -0.36 | -0.35 |
| OIS | 7.7 | 7.5 | 7.4 | 0.2  | 0.19 | 0.19 |
| E\(_{ac}\)\(^6\) | 0.106 | 0.127 | 0.135 |

Little difference between Deuterium and Tritium

- 6 TIS
- 3 OIS

EST: self-trapping energy
ZPE: zero-point energy

- theoretical
- experimental activation energies
V) Interesting experiments

1) Do Mössbauer neutrinos oscillate? Different approaches to neutrino oscillations

CC weak process, \[ |\nu_l\rangle = \sum_k U_{lk}^* |\nu_k\rangle \] U: unitary PMNS matrix Pontecorvo, Maki, Nakagawa, Sakata

Transition probability: \[ P(\nu_l \to \nu_{l'}) = \left| \sum_{k=1}^3 U_{l'k} e^{-i\Delta m_{lk}^2 \frac{L}{2E}} U_{lk}^* \right|^2 \]

For only two flavors: \[ P(\nu_a \to \nu_b) = \sin^2 2\Theta \cdot \sin^2 (\pi L / L_0) \]

Amplitude: \( \sin^2 2\Theta \) Oscillatory term: \( \sin^2 (\pi L / L_0) \)

Oscillation length: \( L_0 = 4\pi \eta c \frac{E}{|\Delta m^2|} \approx 2.480 \frac{E}{MeV} \frac{E}{|\Delta m^2|/eV} \) [m]
V) Interesting experiments

Question: What will be the state of the neutrino after some time (at some distance L)?

A) Evolution in time

Schrödinger equation for evolution of any quantum system:

\[ i \frac{\partial |\psi(t)\rangle}{\partial t} = H |\psi(t)\rangle \quad \rightarrow \quad |\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle \]

\[
P(v_i \rightarrow v_{i'}) = \left| \sum_{k=1}^{3} U_{l',k} e^{-i(E_k - E_i)t} U_{lk}^* \right|^2
\]

No matter what the neutrino momenta are!

If \( E_k = E_i \), there will be no neutrino oscillations: \( P(v_i \rightarrow v_{i'}) = \delta_{l'l} \)

The neutrino state is stationary

If \( E_k \) are different, neutrino state is non-stationary.

→ time-energy uncertainty relation holds:

\[ \Delta E \cdot \Delta t \geq 1 \]

\( \Delta t \) is the time interval during which the state of the system is significantly changed

If \( E_k \neq E_i \), the uncertainty relation takes the form:

\[ (E_k - E_i) \cdot t \approx \frac{\Delta m_{lk}^2}{2E} t \]
V) Interesting experiments

B) Evolution in time and space

Mixed neutrino state at space-time point \( x = (t, \mathbf{x}) \):

\[
|\nu_l\rangle_x = \sum_{k=1}^{3} e^{-ip_k t} U^*_{lk} |\nu_k\rangle \quad \longrightarrow \quad P(\nu_l \rightarrow \nu_{l'}) = \left| \sum_{k=1}^{3} U_{lk} e^{-i(p_k - p_{l'}) t} U^*_{lk} \right|^2
\]

with \( (p_k - p_1) = \frac{E_k^2 - E_1^2}{E_k + E_1} t - (p_k - p_1)L \) and \( E_i^2 = p_i^2 + m_i^2 \)

a) \( t \approx L \quad \longrightarrow \quad (p_k - p_1)x \approx \frac{\Delta m_{1k}^2}{2E} L \) oscillatory phase

b) neutrinos: different masses have the same energy

\[ \longrightarrow \text{neutrino state is stationary} \]

\[ \longrightarrow p_k \neq p_i : \quad (p_k - p_i)x = \frac{\Delta m_{1k}^2}{2E} L \quad \quad P(\nu_l \rightarrow \nu_{l'}) = \left| \sum_{k=1}^{3} U_{lk} e^{-i\Delta m_{1k}^2 \frac{L}{2E}} U^*_{lk} \right|^2 \]
Mössbauer neutrinos:

Energy width: \( \Gamma_{\text{exp}} = 8.6 \cdot 10^{-12} \text{eV} \)

\[
(E_3 - E_2) \approx \frac{\Delta m^2_{23}}{2E} \approx 6.5 \cdot 10^{-8} \text{eV} \quad \Delta m^2_{23} \text{ observed with atmospheric neutrinos}
\]

\[\Delta \approx \Delta = \eta \pi [\text{m}]\]

Mössbauer neutrinos take a long time to change significantly

Time-energy uncertainty: Extremely long “oscillation “ length

Determination of \( \Theta_{13} \): \( E = 18.6 \text{ keV} \) instead of \( 3 \text{ MeV} \).

Chooz experiment: \( \sin^2 2\Theta_{13} \leq 2 \cdot 10^{-1} \) \quad Oscillation base line: \( L_0/2 \approx 9.3 \text{ m} \)

b) \( \Delta m^2_{12} \) observed with solar neutrinos

\[
(E_2 - E_1) \approx \frac{\Delta m^2_{12}}{2E} \approx 2.1 \cdot 10^{-9} \text{eV}
\]

Amplitude: \( \sin^2 2\Theta_{12} \approx 0.82 \)

Oscillation base line: \( L_0/2 \approx 300 \text{ m} \)

Oscillation length: \( L_0 = 4\pi\eta c \frac{E}{|\Delta m^2|} \approx 2.480 \frac{E}{\text{MeV}} \frac{E}{\text{MeV}} / \text{eV} [\text{m}] \)
5) Real-time, $^3$H-specific signal of $\overline{\nu}_e$ resonance

a) sudden change of the magnetic moment from -2.1nm ($^3$He)$\rightarrow$+2.79nm ($^3$H)

transient (~0.1ms) magnetic field which couples to electron moment of $^3$H via hyperfine interaction

Read-out by SQUID

b) new electrons appear in the Nb bands when $^3$H is formed. These electrons cause additional specific heat that grows linearly with $^3$H concentration.

detectable by ultra-sensitive (micro)-calorimeters?
Red(blue)shift \(^{67}\)ZnO-Mössbauer exp.

Gravitational redshift

Difference in height: 1m in gravitational field of Earth

Gravitational blueshift

Accuracy: \((\Delta E/E) \leq 1 \times 10^{-18}\)

W. Potzel et al., Hyp. Interact. 72, 197 (1992)
Gravitational Redshift Experiment

Fig. 3. Basic set-up of the Mössbauer gravitational redshift experiment. Two transmission experiments are carried out simultaneously: through the reference absorber and through the main absorber. A piezoelectric drive moves the source sinusoidally with respect to both absorbers.