Effect of Pore-Filler on the Propagation of Rayleigh Waves in Quasi-Saturated Porous Medium

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Effect of Pore-Filler on the Propagation of Rayleigh Waves in Quasi-Saturated Porous Medium

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Abstract. The propagation of Raleigh waves at free boundaries of solid-quasi-saturated porous media and gas-quasi-saturated porous media is compared in this paper. Based on the equivalence theory, a small amount of solid particles embedded in pore-water of solid-quasi-saturated porous media, or small amount of gas embedded in pore-water of gas-quasi-saturated porous media, together with the pore-water, are equivalent to a new pore-fluid. The dispersion relation modeled by Biot theory is derived, to study the influence of the pore-filler and saturation on the phase velocity of body and Rayleigh waves. Numerical analysis is carried out on pore-space gas hydrate-bearing sediments and slight gas-bearing sediments. It is found that the types of pore-filler have great influence on the phase velocity. The presence of gas hydrate in the pore slightly increases the velocity of P1-wave and S-wave, but has little effect on the Rayleigh wave. But even a small amount of gas in the pore can significantly reduce the velocity of P1-wave. The velocity of S-wave is almost independent of saturation in gas-quasi-saturated porous media, and under the influence of P1-wave, the velocity of Rayleigh wave decreases rapidly and then tends to stabilize with the decreases of saturation. The effect of the type and saturation of the filler is influenced by the dry-frame stiffness.

1. Introduction
Rayleigh wave is a surface wave that propagates along the surface and interferes with depth, resulting from the interference of compressional and shears waves at the free surface [1]. Compared with body waves, Rayleigh waves have a wide range of applications in many fields of engineering, such as near-surface non-destructive testing (SASW method), earthquake engineering, etc., due to their high carrying energy and slow attenuation [2-3]. It is well known that connecting pores are very common in geological materials. Most of early studies on Rayleigh waves were based on the classical elasticity theory. In many circumstances, they cannot reflect the pore characteristics of materials such as soils and rocks.

In 1956, Biot [4] established a fluid-saturated two-phase theory describing the propagation of waves in porous media and the theory can well simulate wave propagation in fluid-saturated soils and rocks. Based on the Biot theory, the propagation of Rayleigh waves in porous media were studied by johns [5], Tajuddin [6], Sharma and Gogna [7], Zhang [2, 8]. In fact, the pore space of geological material may not be occupied by a liquid. For example, the pore space of unsaturated soil is occupied by the mixture of a liquid and a rarefied gas, and the pore space of gas hydrate-bearing sediment and frozen soil is occupied by the mixture of a liquid and another solid. Three-phase or multi-phase
models were used to study the propagation characteristics of waves in the medium [9-11]. The increasing number of phases increases the computational complexity of the dispersion equation of Rayleigh waves, which reduces the applicability of the theory.

In gas-quasi-saturated three-phase medium or solid-quasi-saturated three-phase medium, the gas or solid in pore spaces is surrounded by water and not in direct contact with the sediment particles because the volume fraction of water is high. The material in the pores is equivalent to a fluid phase, and the research results based on the Biot theory will greatly simplify the problem. Using the simplified model, Yang [3, 12] studied the reflection of the SV-wave and the propagation of the Raleigh wave at the free interface of in gas-quasi-saturated three-phase medium, and Ma [13] studied the reflection and transmission of bottom simulating reflectors in gas hydrate-bearing sediments.

To the best of the authors’ knowledge, the influence of pore-filler on Rayleigh waves is of fundamental importance, but there have been few studies. In particular, there has been no relevant research on the effect of solid fillers. Based on this, the equivalent medium theory is adopted in this paper. Based on the Rayleigh wave dispersion relation equation of Biot theory, the influence of the type and content of pore filler on the Rayleigh wave are discussed.

2. Rayleigh waves in fluid-saturated porous media
Fluid-saturated porous media are assumed to be a continuum consisting of solid frame with connected pores, and the pores are fully filled by water. The volume ratio of the pore space is defined as porosity, \( \phi \). Two displacement vectors, \( u_s^i \) and \( u_w^i \), describe the motions of the multiphase of the solid and the water, respectively. The isotropic stress-strain relations porous fluid-saturated media can be expressed as [4]

\[
\sigma_{ij}^s = (A\varepsilon + Q\varepsilon)\delta_{ij} + 2N\xi_{ij}, \ s = \varepsilon + R\varepsilon
\]  

(1)

Where \( \sigma_{ij}^s \) are stress tensor of the solid; \( s \) is average fluid stress; \( \varepsilon = u_s^i \), \( \varepsilon = u_w^i \) and \( \xi_{ij} = 0.5(u_s^{i,j} + u_s^{j,i}) \). \( A, Q, R, \) and \( N \) are four elastic constants determined by the elastic moduli of the dry-frame and the pore fillers, and can be expressed as [14]

\[
N = \frac{3(1-2v_b)}{2(1+v_b)}K_b, \quad A = K_b - 2N/3 + \frac{(1-\phi - K_b/K_s)^2}{(1-\phi - K_b/K_s) + \phi K_s/K_t}K_s
\]

(2a)

\[
Q = \frac{\phi(1-\phi - K_b/K_s)}{(1-\phi - K_b/K_s) + \phi K_s/K_t}, \quad R = \frac{\phi^2}{(1-\phi - K_b/K_s) + \phi K_s/K_t}
\]

(2b)

Where \( K_b \) and \( v_b \) are the bulk modulus and poisson ratio of dry-frame; \( K_s \) and \( K_t \) are the bulk moduli of solid grains and fluid, respectively. With the help of Helmholtz decomposition, the displacements on two-dimensional plane can be decomposed into

\[
u_s^\alpha = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_w^\alpha = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x}
\]

(3)

Where \( \alpha = s, f \). The governing equations of the compressional and shear waves on two-dimensional plane can be written as [4, 14]

\[
\begin{align*}
\rho_1\phi_{i}^s + \rho_2\phi_{i}^f + b (\phi_s - \phi_t) &= P \nabla^2 \phi_{i} + Q \nabla^2 \phi_t, \\
\rho_1\phi_{i}^s + \rho_2\phi_{i}^f &= Q \nabla^2 \phi_{i} + R \nabla^2 \phi_t,
\end{align*}
\]

(4)
Where $\rho_{11} = (1 - \phi)\rho_s - \rho_{12}$, $\rho_{12} = \phi \rho_f$, and $\rho_{22} = \gamma (1 - \phi) \rho_f$ are the dynamic mass coefficients, $b$ is the coefficient of dissipation, and are given as, $b_{12} = \eta_f \phi \gamma \kappa_f$. Constant $\rho_s$ and $\rho_f$ are the densities of the solid material and pore-fluid; $\gamma$ is the coefficient for induced inertia by solid-fluid interaction, taken as 0.5 in this paper; $\eta_f$ is the absolute viscosity of the fluid; $\kappa_f$ is the intrinsic permeability of frame; $\phi_s$, $\phi_f$, $\psi_s$, and $\psi_f$ are the compression potentials and the shear potentials corresponding to the solid and the fluid. The potentials of Rayleigh waves can be expressed as

$$\left\{\varphi_s, \varphi_f\right\} = \sum_{i=1}^{2} \left\{A_i, \delta_{i}, A_i\right\} \exp(-\gamma_{p}z) \exp\left[i(k_{x}z - \omega t)\right]$$

$$\left\{\psi_s, \psi_f\right\} = \left\{B_{s}, \delta_{s}, B_{s}\right\} \exp(-\gamma_{s}z) \exp\left[i(k_{x}z - \omega t)\right]$$

Where $\delta_{p}$ and $\delta_{s}$ are the amplitudes ratio of different phases for different body waves; $l_{p}$ and $l_{s}$ are the wavenumber corresponding to different body waves.

$$\gamma_{p} = \text{p.v.}\sqrt{k_{x}^{2} - l_{p}^{2}}, \quad \gamma_{s} = \text{p.v.}\sqrt{k_{x}^{2} - l_{s}^{2}}$$

Where p.v. represents the principal value of the square root of a complex number. The calculation of the wave number and the amplitude ratio of each phase corresponding to different body waves are found in [14]. The boundary conditions at free boundary in the present work can be expressed as

$$\sigma_{x}^s + s = 0, \quad \sigma_{x}^s = 0, \quad s = 0$$

Substituting Eq. (3) and (5) into Eq. (1) and (7), the dispersion equations at free boundary based on Biot theory can be obtained as

$$\text{Det}\left[\mathbf{M}\right] = 0$$

Where $m_{11} = [A + Q + (Q + R) \delta_{p}] \ell_{p} - 2N \gamma_{p} \psi_{s}, m_{12} = [A + Q + (Q + R) \delta_{p}] \ell_{p} - 2N \gamma_{p} \psi_{s}, m_{13} = 2i N \gamma_{s} k_{R}, m_{21} = 2i N \gamma_{p} k_{R}, m_{22} = 2i N \gamma_{p} k_{R}, m_{23} = N (\gamma_{s}^{2} + k_{h}^{2}), m_{31} = [Q + R \delta_{p}] \ell_{p}, m_{32} = [Q + R \delta_{p}] \ell_{p}, m_{33} = 0$.

3. Quasi-saturated porous medium

Equivalent theory is provided in this section to equate two phases in the pores as a new fluid phase. First, for a solid-quasi-saturated porous medium containing a small amount of solid, a small amount of solid is embedded in the pore water and coordinated with pore water deformation. The bulk modulus and density of the mixed phase can be expressed as

$$\frac{1}{K_{f}} = \frac{s_{s}}{K_{w}} + \frac{1 - s_{s}}{K_{h}}, \quad \rho_{f} = s_{s} \rho_{w} + (1 - s_{s}) \rho_{h}$$

Where $s_{s}$ is saturation, i.e., the volume of pore water accounts for the percentage of pore volume; $K_{w}$ and $K_{h}$ are the bulk moduli of the water and pore-filler; $\rho_{w}$ and $\rho_{h}$ are the density of the water and pore-filler. Then for a gas-quasi-saturated porous medium containing a small amount of gas, the bulk modulus and density of the mixed phase can be expressed as [3, 12, and 15]
\[
\frac{1}{K_F} = \frac{1}{K_w} + \frac{1-s}{p_{ab}}, \rho_F = s \rho_w + (1-s) \rho_g
\]

(10)

Where \( p_{ab} \) is the absolute pore pressure; \( \rho_g \) is the density of the of gas.

4. Numerical results and discussions

In this section, gas hydrate-bearing sediments (herein referred to as Model A) and gas-bearing sediments (herein referred to as Model B) are taken as examples to analyze the effects of pore-filler and saturation on body and Rayleigh waves in quasi-saturated porous elastic media with different stiffness of dry-frame. The parameters in sediments are as follows: \( \phi = 0.35, \rho_s = 2650 \text{ kg/m}^3, K_s = 38.7 \text{ GPa}, \rho_w = 1000 \text{ kg/m}^3, K_w = 2.25 \text{ GPa}, \kappa_s = 1.0 \times 10^{-11} \text{ m}^2, \eta_w = 1.8 \times 10^{-3} \text{ Pa s}. \) The parameters of gas hydrate in Model A are as follows: \( \rho_h = 920 \text{ kg/m}^3, K_h = 3.32 \text{ GPa}. \) The parameters of gas in Model B are given as follows: \( \rho_g = 1.2 \text{ kg/m}^3. \) It is noteworthy that the Poisson ratio of dry-frame \( \nu_b \) is taken to be 0.3, and bulk modulus of dry-frame \( K_b \) is taken to be \( 0.2K_s, 0.02K_s, \) and \( 0.002K_s \) depending on the degree of consolidation of sediments.

When the saturation \( s_r \) is taken to be 1, it corresponds to a fluid-saturated two-phase medium. As the saturation decreases, the content of gas hydrate (in Model A) or gas (in Model B) increases. Figures 1(a)-(c) illustrate the effect of saturation on the phase velocities of P1-wave, S-wave, and Rayleigh wave for two models with different dry-frame stiffness. It is observed that when the saturation is 1, both models have the same velocity. The small change in saturation makes the velocity of the P1-wave rapidly decrease in the Model B, and slightly increase in model A. In both models, the velocity of the S-wave is insensitive to changes in saturation, and the velocity of S-wave increases slightly in model A. The velocity of Rayleigh wave is insensitive to the change of saturation in Model A. However, with the decreases of saturation, the velocity of Rayleigh wave decreases rapidly and then tends to stabilize in Model B.

Compared with the three images, it is found that the velocity of P1-wave, S-wave and Rayleigh wave are controlled by the dry-frame stiffness. A medium with high stiffness of the dry-frame has the high velocity, and behaves like a classical elasticity medium. And the influence of saturation increases as the stiffness of dry-frame decreases.

Figure 1(d) shows the effect of saturation on the ratio of the velocity of Rayleigh to S-wave \( (V_R / V_s) \), a very important parameter in the classical elasticity theory. It is observed that the ratio is almost unaffected by saturation in Model A. But in Model B, even a small amount of gas will reduce the ratio rapidly, and eventually tend to 0.927 (A ratio corresponding to the calculation result when poisson ratio in classical elasticity theory equals to poisson ratio of dry-frame in Biot theory).

The presence of gas rapidly reduces the bulk modulus of the equivalent fluid, weakening the effect of the pore fluid on the frame and making it behave more like a classical elasticity medium. The bulk modulus of the hydrate is slightly larger than that of the pore water, which slightly increases the effect of the equivalent fluid, but slightly increases the phase velocity, but the effect is not significant.
5. Conclusion

The effect of gas-hydrate or gas in the pore of quasi-saturated porous medium on the velocity of body and Rayleigh waves are investigated by a two-phase theory and an equivalent medium theory. Based on the previous discussion, the following points are of notice:

1. The velocity of S-wave is mainly controlled by the stiffness of the dry-frame, and the presence of pore-fillers, especially gas, has little effect on the shear wave velocity.

2. The velocity of P1-wave is controlled by the stiffness of the dry-frame and pore-fillers. The presence of gas-hydrate in the pore can slightly increase the P1-wave velocity, but even a small amount of gas can significantly reduce the velocity of P1-wave.

3. As the Rayleigh wave is formed by the interference of P1, P2, and S-waves, which is mainly controlled by the S-wave. The presence of gas-hydrate has little influence on the phase velocity of Rayleigh waves. However, with the decreases of saturation, the velocity of Rayleigh wave decreases rapidly and then tends to stabilize in gas-quasi-saturated porous media.

4. The stiffness of dry-frame has a significant effect on the analysis results, and the influence of pore-filler and saturation increases as the stiffness of dry-frame decreases.

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