Electronically Tunable Fractional Order All Pass Filter

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Abstract: In this paper, an electronically tunable fractional order all pass filter (FOAPF) based on operational transconductance amplifier (OTA) is presented. It uses two OTAs and single fractional order capacitor (FC) of non-integer order $\alpha$ to provide FOAPF of $\alpha$ order. Two different values of $\alpha$, in particular 0.5 and 0.9, for FC are taken for investigation. The functionality of the proposal is verified through SPICE simulations using TSMC 0.18 $\mu$m Complementary Metal Oxide Semiconductor (CMOS) process parameters. Simulated and theoretical frequency and time domain responses are found to be in close agreement.

Keywords: All pass filter, fractional order element, fractional order filter, OTA

1. INTRODUCTION

All pass filters (APF) are widely used in phase shifting application for analog signal processing system [1]-[6] where they pass all frequencies over the desired frequency range with a predictable phase shift. Several active first order filter realization based on various active building blocks such as current differencing buffer amplifier (CDBA) [6], [7], current conveyor (CC) [8], universal voltage conveyor (UVC) [9], [10], second generation current conveyor (CCII) [3], [11]-[13], third generation current conveyor (CCIII) [14], second generation current controlled conveyor (CCCII) [15], differential voltage current conveyor (DVCC) [4], [16]-[21], differential difference current conveyor (DDCC) [22], [23], dual-X second generation current conveyor (DXCCII) [24], [25], fully differential CCII [5], [26], [27], current controlled conveyor transconductance amplifier (CCCTA) [28], operational transconductance amplifiers (OTAs) [9], [20], [29], [30], voltage differencing invertingbuffered amplifier (VDIBA) [31], voltage differencing transconductance amplifier (VDTA) [32] are available in literature. It is observed that the active blocks with OTA or in particular having OTA block are useful for achieving electronic tuning of phase response of APF.
Research interest in fractional order filters (FOFs) is growing recently due to inherent extra degree of freedom and possibility of embedding concept of scaling effect on frequency which leads to increase design flexibility. Their design equations are generalized from the theory of classical filter. The FOFs make use of fractional order element (FOE) whose behavior is approximated by Oustaloup Recursive approximation [33], Carlson approximation [33], Matsuda approximation [33], Chareff approximation [33], Continued Fraction Expansion (CFE) [33], Modified Oustaloup [34] and El-Khazali reduced order approximations [35]. Based on the finite element approximation of FOE, the implementation of fractional order capacitor (FC) may be obtained by a structure approximating half order FC as presented via semi-infinite RC trees in [36] or by other combinations of RC ladder networks [37] such as cross RC ladder network, domino ladder, tree structure etc. The method outlined in [38] gives structure for approximating FC of any order.

This work presents electronically tunable fractional order all pass filter (FOAPF). Two FOAPFs [39], [40] are available in open literature to the best of authors’ knowledge but these lacks in electronic tuning feature. The remaining part of this paper is organized as follows. Section 2 is divided into three subsections which include FOE, OTA and realization of FOAPF. Section 3 deals with theoretical and simulation results and finally conclusion is placed in section 4.

2. Proposed Circuit

2.1 Fractional Order Element (FOE)

The impedance function of an FOE (denoted as F) may be expressed as \( Z_F(s) = a s^\alpha \) [41], where \( \alpha \) represents non-integer value and is termed as fractional order. The FOE impedance function can be represented in terms of magnitude (\( |a s^\alpha| \)) and phase (\( \alpha \pi / 2 \)) functions. It may be noted that though, its magnitude function is frequency dependent but phase is independent of frequency and has a constant value for a given fractional order. Therefore, the FOE is also termed as a constant phase element (CPE). The FOE is not commercially available though, its behavior can be simulated by finding an appropriate rational approximation [42] of its impedance function. The FOE is versatile element and is used to generate various basic structures such as fractional order capacitor (FC), fractional order inductor (FI), differentiator [43], [44], integrator [43], [44] inverted-L type [42] and the derived RC and RL circuits as imaginary impedance [45] in fractional domain and is found in designing of various fractional order analog circuits.

2.2 Operational Transconductance Amplifier (OTA)

The circuit symbol of OTA is given in Fig. 1. It processes differential voltage and provides output current as

\[
I_o = \pm g_m (V_+ - V_-)
\]

(1)
Here the transconductance gain $g_m$ of OTA can be characterized by

$$g_m = \sqrt{\mu_n C_{ox} \frac{W}{L} I_b}$$

(2)

Where $\mu_n$, $C_{ox}$ and $W/L$ are the electron mobility of NMOS, gate oxide capacitance per unit area and transistor aspect ratio respectively. This expression shows that the value of $g_m$ depends on bias current ($I_b$) which allows electronic tunability of the circuit parameters. Fig. 2 shows MOS circuit realization of OTA [46] for the simulation.

2.3 Realization of Fractional Order All Pass Filter (FOAPF)

Fig. 3(a) shows FOAPF based on OTA which is generalized by replacing traditional capacitor with FC in structure [47]. The impedance function of FC gives magnitude as $1/(\omega C)$ and phase as $-\alpha \pi / 2$. The behavior of FC is approximated by CFE method and then realized with the RC ladder network of Fig. 3(b).
A single component of FC having non-integer order $\alpha$ and capacitor value $C$ is chosen for designing of FOAPF. The impedance function of FC gives its magnitude as $1/|\omega^\alpha C|$ and phase as $-\alpha \pi / 2$.

Routine analysis of Fig. 3(a) provides following transfer function

$$T(s)_{\text{FAPF}} = \frac{s^\alpha - g_{m1}/C}{s^\alpha + g_{m2}/C}$$  \hspace{1cm} (3)

The pole-zero plot of classical first order filter in complex s-plane allocates zero at $s = \omega_{b1}$ and pole at $s = -\omega_{b2}$. Consequently s-plane transformation into fractional domain ($s^\alpha$) yields zero at $s^\alpha = \omega_{b1}^\alpha$ and pole at $s^\alpha = -\omega_{b2}^\alpha$.

Setting $s = \omega e^{j\pi/2} (= j\omega)$, $\omega_{b1}^\alpha = g_{m1}/C$ and $\omega_{b2}^\alpha = g_{m2}/C$, the frequency response of (3) becomes

$$T(j\omega)^\alpha = \frac{(\omega e^{j\pi/2})^\alpha - (\omega_{b1})^\alpha}{(\omega e^{j\pi/2})^\alpha + (\omega_{b2})^\alpha}$$  \hspace{1cm} (4)

The magnitude and phase of FOAPF may be expressed by (5) and (6) respectively

$$|T(j\omega)| = \left| \frac{\omega^{2\alpha} + \omega_1^{2\alpha} - 2\omega^\alpha \omega_1^\alpha \cos (\alpha \pi / 2)}{\omega^{2\alpha} + \omega_2^{2\alpha} + 2\omega^\alpha \omega_2^\alpha \cos (\alpha \pi / 2)} \right|$$  \hspace{1cm} (5)

$$\angle T(j\omega) = \tan^{-1} \left[ \frac{\omega^\alpha \sin (\alpha \pi / 2) / \omega_1^\alpha \cos (\alpha \pi / 2) - \omega_2^\alpha}{\omega^\alpha \sin (\alpha \pi / 2) / \omega_2^\alpha \cos (\alpha \pi / 2) + \omega_1^\alpha} \right]$$  \hspace{1cm} (6)

It has been seen [40] that the gain at $\omega = \omega_p$ (at which $\angle T(j\omega) = \pm \pi / 2$) has minima if $\alpha < 1$, maxima if $\alpha > 1$ and flat if $\alpha = 1$. In this work the behavior of FC is emulated through RC ladder network as shown in Fig. 3(b) which is based on 4th order CFE approximation.

3. Circuit Simulation

The proposed FOAPF is functionally verified through SPICE simulation wherein CMOS based schematic of OTA [46] is used with 0.18 $\mu$m TSMC CMOS process parameters. Simulations are carried out for two different FCs of fractional orders 0.5 and 0.9 respectively. The FCs of 1$\mu$F each are designed around a centre frequency of 1 kHz. To obtain electronic tunability of FOAPF parameters the $g_m$ of OTAs is controlled through bias currents ($I_{b1}$ and $I_{b2}$) variation. The supply voltages are taken as $\pm 1.8$ V. The frequency responses of the designed FOAPF, as obtained through SPICE simulation (solid lines) and theoretical (dash lines) results are depicted in Figs. 4 and 5 respectively. The theoretical results are calculated from (5) and (6) and are plotted using MATLAB program. In the responses of Figs. 4 and 5 the pole frequencies ($\omega_{b1} = \omega_{b2}$) of 2.5 krad/s, 10 krad/s, 22.5 krad/s, 40 krad/s are chosen for $\alpha = 0.5$ whereas for $\alpha = 0.9$ the pole frequencies are fixed at 500 rad/s, 750 krad/s, 1 krad/s, 1.25 krad/s.

It may be observed from various responses that the simulation and theoretical results are quite close for a wide range of frequencies.
Fig. 5 FOAPF (a) magnitude (b) phase responses for order of FC is $\alpha=0.9$

The time domain response of proposed filter is shown in Fig. 6 where inputs (i) 10 mV amplitude and 1 kHz frequency sinusoidal signal for order $\alpha = 0.5$ as shown in Fig. 6(a) and (ii) 100 mV amplitude and 100 Hz frequency sinusoidal signal for order $\alpha = 0.9$ as shown in Fig. 6(b); are applied to the filter and output is about -$90^0$ phase shifted. In addition, their X-Y plot (Lissajous pattern) with -$90^0$ phase shift is also illustrated in Fig. 7 which verifies the circuit as a phase shifter.

Fig. 6: Time domain response of FOAPF having (a) $\alpha = 0.5$ and (b) $\alpha = 0.9$

Fig. 7: (a) and (b) Lissajous (XY) pattern of Fig. 4 (a) and (b)
4 CONCLUSION

This paper proposes an electronically tunable FOAPF by generalizing first order all pass filter into fractional domain where tuning operation is executed through bias currents of OTAs. The phase shift operation is also tuned electronically. It is shown that all pass filter has more flexibility in shaping the filter response when generalizing into fractional domain. The frequency and time domain responses of FOAPF of order 0.5 and 0.9 are depicted which are close to theoretical results.

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