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Prediction of the Basic Creep of Normal and High-strength Concretes based on an Analytical Micromechanical Model

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Abstract

The main purpose of this paper is to predict analytically the concrete basic creep time-dependent deformation according to its composition and microstructure. Thus, concrete was modelled as a three-phase composite formed by aggregates surrounded by interfacial transition zones (ITZs) and embedded randomly in a hardened cement paste. First, the ITZ phase volume fraction within concrete was evaluated using an analytical formula that the authors had recently proposed (Zouaoui et al. 2017). Then, the specific basic creep deformation of concrete was evaluated analytically based on a four-sphere homogenization model where cement paste and ITZ have a linear viscoelastic behaviour whereas aggregates are assumed linear elastic.

The predictions of the proposed model show that cement paste basic creep and aggregates volume fraction and maximum packing density are the main parameters governing concrete basic creep. Moreover, these predictions show that aggregates gradation and ITZ thickness have secondary effects on concrete basic creep. Finally, the relevance and the validity of the proposed model have been discussed based on a comparison between its predictions and experimental results taken from literature and related to both normal and high-strength concretes.

1. Introduction

It is well known that when submitted to a sustained load for a long period of time, concrete and cementitious materials exhibit a time-dependent deformation called creep deformation. The latter can be decomposed in two parts: the basic creep and the drying creep. The drying creep is related to the time-dependent deformation coupled with the drying of concrete, whereas the basic creep is considered as a material intrinsic property which is dependent on concrete composition and microstructure (Jirasek and Bazant 2001; Bážant 2001). This time-dependant deformation is measured in sealed concrete specimens under isothermal conditions and without moisture exchange with the external environment. It can reach about 50% in the 2 to 3 months and about 90% in the 2 to 3 years after loading. Afterwards, the strain rate is almost negligible (Le Roy 1996; Jirasek and Bazant 2001; Bážant 2001; Gilbert and Ranzi 2011; Zhang et al. 2014; Le Roy et al. 2017).

In the literature, several researchers had carried out basic creep tests on normal-strength concrete (NSC) (Mamillan and Savin 1981; Bazant and Kim 1991; Benaisa et al. 1993; Granger 1995; Mekani 2011) and on high-strength concrete (HSC) (Collins 1989; Auperin et al. 1989; de Larrard and Le Roy 1992a; Le Roy 1996, 2017; Torrenti and Le Roy 2018). In general, the basic creep deformation of HSC is lower than that of NSC. In fact, concrete basic creep decreases when the water to cement ratio decreases (Granger 1995; Mekani 2011). Besides, Le Roy (1996) and Le Roy et al. (2017) had observed that HSC basic creep deformation decreases when silica fume to cement ratio increases. Moreover, it had been observed that concrete basic creep deformation decreases when aggregates volume fraction increases for the same cement paste (Granger 1995; Mekani 2011). However, Granger (1995) had found that concrete basic creep could significantly vary for the same cement type, water to cement ratio, aggregates size distribution and volume fraction. The main difference was the nature of aggregates.

Furthermore, several previous studies had focused on the micromechanical modelling of basic creep of concrete according to its composition and microstructure. For example, Le Roy and de Larrard had predicted analytically basic creep deformation of HSC based on a three-sphere homogenization model where HSC was modelled as a two-phase material made of a cement paste and aggregates (de Larrard and Le Roy 1992a, 1992b; Le Roy 1996). Lavergne et al. (2015) had studied numerically the effects of aggregates morphology and the interfacial transition zone (ITZ) on the basic creep of concrete modelled as a three-phase material (cement paste, ITZ and aggregates), and had shown that
aggregates size distribution and shape have negligible effects on concrete basic creep. Theses authors had also concluded that taking into account the ITZ in their numerical model helps to explain the difference between basic creep experimental results of concrete mixtures having similar compositions.

Nevertheless, some features related to the dependence of concrete basic creep on its microstructure remain not well understood and sometimes controversial in the literature. In particular, the effects of aggregates size distribution and the ITZ volume fraction. Thus, the present study aims to investigate mainly these effects based on an analytical micromechanical approach where concrete is modelled as a three-phase material formed by aggregates surrounded by ITZs with variable thicknesses and embedded randomly in a hardened cement paste, and where cement paste and ITZs are linear viscoelastic whereas aggregates are linear elastic. Moreover, The ITZ volume fraction within concrete will be evaluated according to its microstructure based on an analytical model the authors had recently proposed in a previous paper (Zouaoui et al. 2017).

The present paper is organised as follows. In Section 2, the analytical formula that we had recently proposed to evaluate the ITZ volume fraction within concrete is briefly recalled. In Section 3, the four-sphere homogenization model proposed to evaluate the concrete basic creep time-dependent deformation is presented. In Section 4, a discussion on the proposed model is presented based on a sensitivity analysis of its predictions. In Section 5, the relevance and the validity of the proposed model are checked based on a comparison between its predictions and experimental data taken from literature and related to NSC and HSC. Finally, concluding remarks are given in the last section.

2. The analytical formula for the ITZ volume fraction proposed by Zouaoui et al. (2017)

Recall that the ITZ is the weakest zone located between aggregates and cement paste. Its thickness varies generally between 10 µm for HSC and 50 µm for NSC but it could reach in some concrete mixes 100 µm (Scrivener et al. 1988; Escadeillas and Maso 1991; Scrivenener and Nemati 1996; Mehta and Monteiro 2006). This interphase has generally a higher porosity compared to that of the bulk cement paste. Thus, it is characterised by weaker mechanical properties compared to those of the cement paste (Wang et al. 1988; Hashin and Monteiro 2002).

In order to evaluate the ITZ volume fraction within concrete according to its microstructure, the authors had proposed in a previous paper (Zouaoui et al. 2017) a three-phase discrete model for the concrete microstructure using the Voronoi tessellation (Aurenhammer and Klein 2000; Miled et al. 2012; Zouaoui et al. 2017; Naïja et al. 2018). In this section, we recall the analytical formula derived from this model for the ITZ volume fraction denoted \( f_{ITZ} \). In fact, to do this, we had considered a Representative Volume Element (RVE) of concrete containing a large number \( N \) of spherical aggregates with a total volume fraction \( g \) and aggregates maximum packing density \( g_{\text{max}} \) and which diameters \( D_{(k)} \leq D \leq D_{(l)} \) are ranging between \( D_{\text{min}} \) and \( D_{\text{max}} \), following the size distribution function in volume proposed by Funk and Dinger (1994), with \( m \) being a positive parameter ranging between 0 and 1 and when \( m \) increases, the percentage by volume of coarse aggregates within aggregates skeleton increases.

\[
W_i(D) = \frac{D^n_i - D^m_{\text{min}}}{D^n_{\text{max}} - D^m_{\text{min}}},
\]

Here, we had assumed that the ITZ local thickness is not uniform for all aggregates but it is a random variable depending on the local cement paste thickness available between each couple of neighboring aggregates and the latter varies according to aggregates size distribution. In this manner, the proposed model takes into account the overlapping between neighbouring ITZs and leads to a realistic evaluation of the ITZ volume fraction within concrete, which is found to not exceed 7% for NSC. The analytical formula proposed in Zouaoui et al. (2017) for \( f_{ITZ} \) can be presented as follows according to aggregates volume fraction \( (g) \), maximum packing density \( (g_{\text{max}}) \) and gradation \( (D_{\text{mic}}, D_{\text{max}}, \text{ and } m) \) and to the ITZ thickness \( (c) \).

When \( D_{\text{min}} > D^* \),

\[
f_{ITZ} = \frac{2 - m}{3 - m} \left( \frac{c}{k} \right), \quad \text{with} \quad \kappa = 1 - \frac{1}{2} \left( \frac{g_{\text{max}}}{g} \right)^{1/3} - 1,
\]

\[
A = \frac{2gm}{\left( D^n_{\text{max}} - D^m_{\text{min}} \right)^2},
\]

\[
B = \frac{4}{(3 - m)},
\]

\[
E = \frac{6}{D^n_{\text{max}} - D^m_{\text{min}}},
\]

\[
F = \frac{3}{D^n_{\text{max}} - D^m_{\text{min}}},
\]

When \( D_{\text{min}} \leq D^* \), \( f_{ITZ} \) is assumed invariant according to \( D_{\text{min}} \) and is given by the same analytical formula of Eq. (1) but in which \( D_{\text{min}} \) is replaced by \( D^* \).

3. Analytical prediction of the basic creep time-dependent deformation of concrete

In order to take into account the ITZ as an additional phase of concrete, the three-sphere micromechanical model proposed initially by Le Roy and de Larrard for
concrete is extended hereafter by adding a fourth spherical layer surrounding the aggregate and representing
the ITZ (Fig. 1) (de Larrard and Le Roy 1992a, 1992b; Le Roy 1996).

Recall that Le Roy and de Larrard three-sphere model
is an extension of the well-known Hashin (1962) bi-sphere model in which concrete is considered as a two-phase isotropic composite formed by spherical aggregates having a volume fraction \( g \) and a maximum packing density \( g_{\text{max}} \) equal to 1 and surrounded each one by a cement paste spherical layer having a volume fraction \((1-g)\). In fact, the three-sphere model (de Larrard and Le Roy 1992a, 1992b; Le Roy 1996) takes into account the effect of aggregates maximum packing density \( g_{\text{max}} \),
which is smaller than 1 within concrete, by adding to the Hashin bi-sphere model a cement paste complement modelled by an another sphere placed in the center of the composite sphere as shown in Fig. 1 and representing the part of cement paste embedded between aggregates when their volume fraction \( g \) reaches \( g_{\text{max}} \).

Within the proposed four-sphere model, an additional spherical layer representing the ITZ phase and surrounding the aggregate layer is added to the original three-sphere model (Fig. 1). Moreover, its volume fraction within concrete \( f_{\text{ITZ}} \) is evaluated using the analytical formula proposed recently by Zouaoui et al. (2017) and recalled in Section 2.

\[
E_{c,0} = \frac{1+R_0}{1-R_0} E_{p,0}
\]

(2)

In the above equation, \( R_0 \) is given by the following.

\[
R_0 = \frac{E_{\text{max}}(1+\alpha)-f_{\text{ITZ}}(1-\alpha)}{E_{\text{max}}(1+\alpha)+f_{\text{ITZ}}(1-\alpha)}
\]

where \( E_{\text{max},0} \) above is given by,

\[
E_{\text{max},0} = \frac{(2-g_{\text{max}})E_{p,0} + g_{\text{max}}E_a}{g_{\text{max}}E_{p,0} + (2-g_{\text{max}})E_a}
\]

\[
E_a = \text{the aggregates elastic modulus, } E_{p,0} = \text{the cement paste elastic modulus and } \alpha \text{ is the ratio of the ITZ elastic modulus to the cement paste elastic modulus (} \alpha = \frac{E_{\text{ITZ},0}}{E_{p,0}} \text{).}
\]

Note that the above analytical formula [Eq. (2)] predicting the effective elastic modulus of concrete is obtained when assuming that the Poisson’s ratio is constant and equal to 0.2 for each concrete phase and therefore for concrete. This simplifying assumption was already adopted by Le Roy (1996) and Granger (1995) who had observed experimentally that the variation of the Poisson’s ratio of concrete is negligible both for elastic and basic creep deformations.

Then, the above linear elastic micromechanical model can be extended to the framework of non-aging linear viscoelasticity in order to obtain the concrete time-dependent modulus which characterizes both the instantaneous elastic and the time-dependent basic creep deformations, when concrete specimen is subjected to a constant and sustained uniaxial compressive stress applied since time \( t_0 \), under isothermal conditions and without drying. Indeed, assuming that cement paste and ITZ phases are linear viscoelastic whereas aggregates remains linear elastic, the concrete time-dependent modulus denoted \( E_c(t) \) at a given time \( t > t_0 \) will be given by the following expression which is a generalization over the time of Eq. (2):

\[
E_c(t) = \frac{1+R(t)}{1-R(t)} E_{p}(t)
\]

(3)

where \( R(t) \) and \( E_{\text{max}} \) above are given by,

\[
E_{\text{max}}(t) = \frac{g_{\text{max}}(1+\alpha)-E_a(1-\alpha)}{1+\alpha)(E_{\text{max}}(t)+E_a)}
\]

\[
E_{\text{max}}(t) = \frac{(2-g_{\text{max}})E_{p}(t) + g_{\text{max}}E_a}{g_{\text{max}}E_{p}(t) + (2-g_{\text{max}})E_a}
\]

where \( \alpha \) is the ratio of the ITZ time-dependent modulus \( E_{\text{ITZ}}(t) \) to the cement paste time-dependent modulus \( E_{p}(t) \). This parameter is assumed to be constant over the time.
time since the ITZ time-dependent modulus is very difficult to measure over the time and also in order to not complicate further the proposed model. Whereas, the cement paste time-dependent modulus $E_{p}(t)$ can be determined experimentally with a laboratory uniaxial compressive basic creep test carried out on a cement paste specimen. It can be also expressed as follows:

$$E_{p}(t) = \frac{1}{J_{p}(t_{0}, t)}$$

(4)

where $J_{p}(t_{0}, t)$ is the cement paste uniaxial basic creep compliance function which represents the instantaneous elastic deformation plus the basic creep deformation at time $t$ caused by a unitary uniaxial compressive stress ($\sigma_{0} = 1$ MPa) applied on the cement paste specimen since time $t_{0}$. This function can be expressed as follows [Eq. (5)] based on the generalized Kelvin-Voigt model (Schiessel et al. 1995) classically used for solid materials:

$$J_{p}(t_{0}, t) = \frac{1}{E_{p,0}} + \sum_{i=1}^{n} \frac{1}{E_{p,i}} \left(1 - e^{-\frac{t}{\tau_{i}}}\right)$$

(5)

where $E_{p,0}$ is the elastic modulus of the cement paste measured in the beginning of the uniaxial basic creep test (at $t_{0}$), $E_{p|t_{0} \leq t \leq n}$ is the elastic modulus associated to element $i$ ($1 \leq i \leq n$) belonging to the Kelvin-Voigt chain and $\tau_{i}$ is its characteristic time. All these parameters could be determined or fitted experimentally based on the basic creep test.

Finally, the concrete "specific" basic creep deformation denoted $\varepsilon_{sc}(t)$ and representing the concrete basic creep deformation at time $t$ caused by a unitary uniaxial compressive stress ($\sigma_{0} = 1$ MPa) applied on concrete specimen since time $t_{0}$, will be given by Eq. (6):

$$\varepsilon_{sc} = \varepsilon_{tot} - \varepsilon_{c}(t_{0}) = \frac{1}{E_{c,0}} \left(1 - \frac{1}{E_{c}(t)} \right)$$

(6)

where $\varepsilon_{tot}(t)$ and $\varepsilon_{c}(t_{0})$ are respectively the total concrete deformation at time $t$ and its instantaneous elastic deformation at time $t_{0}$ caused by a unitary uniaxial compressive stress ($\sigma_{0} = 1$ MPa) applied on concrete specimen since time $t_{0}$. In this analytical formula [Eq. (6)], $E_{c,0}$ and $E_{c}(t)$ are expressed in MPa and are evaluated by respectively Eq. (2) and Eq. (3) in which $f_{ITZ}$ is evaluated by Eq. (1).

4. Sensitivity analysis on the specific basic creep deformation of concrete

It is well established experimentally that the creep of cement paste is the main source of concrete creep (Neville and Brooks 1987; Le Roy 1996). However, in this part, the effects of different parameters related to aggregates and ITZ on the concrete specific basic creep deformation are studied using the proposed four-sphere model.

In this sensitivity analysis, the same cement paste was used. The latter is issued from Le Roy (1996) study and corresponds to a normal-strength cement paste (NSP) which is denoted by Le Roy as “P50” and is characterized by a water to cement ratio (W/C) of 0.5 and an elastic modulus $E_{p|0}$ of 13.5 GPa. The viscous parameters of the generalized Kelvin-Voigt model used here to describe the linear viscoelastic behaviour of this cement paste are presented in Table 1. These parameters have been identified based on a comparison between the cement paste P50 specific basic creep experimental deformations over the time (Le Roy 1996) and the theoretical deformations given by the generalized Kelvin-Voigt model with 7 viscous elements, as shown in Fig. 2.

The viscous parameters of Le Roy (1996) normal strength cement paste P50.

| $\tau_{i}$ (days) | $E_{ip}$ (cement paste) |
|------------------|------------------------|
| 0.002            | 2663.66                |
| 0.02             | 2572.92                |
| 0.2              | 165.04                 |
| 2                | 47.66                  |
| 20               | 16.09                  |
| 200              | 8.29                   |
| 2000             | 70.00                  |

4.1 Effect of aggregates volume fraction

Figure 3 shows that increasing aggregates volume fraction ($g$) reduces significantly concrete specific basic creep deformation. This important effect of aggregates volume fraction on concrete basic creep had been already observed and reported by Counto (1964), Hobbs (1971) and Le Roy (1996).

4.2 Effect of aggregate maximum packing density

To study the effect of aggregates maximum packing density ($g_{max}$) on concrete specific basic creep deformation, $g_{max}$ has been varied between 0.75 and 0.85. It is found that increasing the aggregates maximum packing density $g_{max}$ increases considerably the specific basic creep deformation of concrete as shown in Fig. 4.

![Fig. 2 Experimental (Le Roy 1996) and theoretical specific creep deformations of cement paste P50 according to the relative time (t - t0).](image-url)
4.3. Effect of aggregates size distribution
As shown in Fig. 5, when aggregates maximum diameter \((D_{\text{max}})\) increases, concrete specific basic creep deformation decreases slightly. In the same tendency, increasing aggregates minimum diameter \((D_{\text{min}})\) results in a more slight decrease of concrete specific basic creep deformation as shown in Fig. 6.

Moreover, the effect of the power \(m\) of aggregates size distribution function used here and which is originally proposed by Funk and Dinger (1994) is also negligible on the specific basic creep deformation of concrete as shown in Fig. 7. Lavergne et al. (2015) had also found numerically that aggregates size distribution has a small effect on concrete basic creep. Thus, we can conclude that according to the proposed model, aggregates gradation has a negligible effect on concrete basic creep.

4.4 Effect of the ITZ time-dependent modulus
The ITZ time-dependent modulus, \(E_{\text{ITZ}}(t)\), has a small effect on the specific basic creep deformation of concrete. In fact, Fig. 8 shows that the latter decreases slightly when the ITZ modulus increases, i.e., when \(\alpha\) increases.
4.5 Effect of the ITZ thickness

As shown in Fig. 9, the ITZ thickness (c) has also a negligible effect on the specific basic creep deformation of concrete, even that the latter increases slightly when the ITZ thickness increases. This effect had been numerically studied by Lavergne et al. (2015) who had found also that concrete basic creep increases slightly when the ITZ thickness increases.

Thus, according to this sensitivity analysis of the proposed model analytical predictions, it can be concluded that aggregates volume fraction (g) and maximum packing density (g_{max}) are the main microstructural parameters governing concrete basic creep in addition to the cement paste basic creep. All other parameters related to aggregates gradation (D_{min}, D_{max} and m) and to the ITZ, i.e., E_{ITZ}(t) and c, have secondary and even very slight effects on concrete basic creep deformation.

4.6 Comparison with other micromechanical models

A comparison between the predictions in terms of concrete specific basic creep deformation predictions given by three classical micromechanical models with those of the proposed four-sphere model is presented in Fig. 10. The first model is the classical Hashin bi-sphere model (without ITZ) (Hashin 1962), the second is Le Roy and de Larrard three-sphere model (without ITZ) (de Larrard and Le Roy 1992a, 1992b; Le Roy 1996) and the last is the Generalized Self Consistent (GSC) model (with ITZ) which had been extended to three phase elastic materials and applied to concrete by Li et al. (1999) and by Zouaoui et al. (2017). Note that for this comparison, conducted in the framework of linear viscoelasticity by following the same methodology presented previously in Section 3, the same cement paste “P50” presented above has been considered for the four models.

Based on this comparison, we can see that the three-sphere model (without ITZ) is the stiffest model whereas the GSC model (with ITZ) is the softest model since it gives the highest basic creep deformation. The latter is almost the double of that given by the proposed four-sphere model (from 64 days of loading) as shown in Fig. 10. Thus, the GSC model which we had been used in Zouaoui et al. (2017) to predict the elastic modulus of concrete seems to not be always suitable to predict concrete basic creep deformation or time-dependent modulus due to its excessive softness when it is extended to viscoelasticity.

Moreover, despite the Hashin bi-sphere model does not consider the ITZ phase, it is softer than the three-sphere model (without ITZ) and also than the proposed four-sphere model (with ITZ). This is due to the fact that Hashin bi-sphere model does not take into account the real aggregates maximum packing density in concrete (g_{max} < 1) which results in softer composite inclusions compared to those of the three-sphere model and also to those of the proposed four-sphere model.

Finally, it can be concluded based on this comparison that the proposed four-sphere model gives a specific basic creep deformation which is slightly greater than that given by the three-sphere model but much lower than that predicted by both the Hashin bi-sphere and the GSC models.

5. Validation of the proposed model

In order to validate and to evaluate the relevance of the proposed four-sphere model, its predictions have been compared with various experimental data taken from literature and related to both normal strength (NSC) and high strength (HSC) concretes.

5.1 Validation of the proposed model predictions over the time

In order to validate the proposed model predictions in terms of concrete specific creep deformation over the time, a comparison between model predictions and experimental data of Le Roy (1996) and Granger (1995) has been conducted.

Experimental results from Le Roy (1996) are related respectively to a NSC made with a the normal strength cement (NSP) paste “P50” presented above and to a HSC made with a high strength cement paste (HSP) denoted by Le Roy as “P38”, which is characterized by
a (W/C) ratio of 0.38, 10% of silica fume (S/C = 0.1) and an elastic modulus of 19 GPa. Moreover, these two concretes are made from two similar aggregates skeletons as presented in Table 2.

Moreover, two NSC from Granger (1995) experimental study are used in this comparison. These two concretes have practically the same cement paste denoted hereafter as “P48” and having a (W/C) ratio of 0.48. The difference between Granger two concretes is only the origin or the mineralogical nature of used aggregates, which are from different regions (Flamanville region, Paluel region).

The power $m$ related to aggregates size distribution is assumed constant for all concretes and equal to 0.5. This assumption is justified since the sensitivity analysis presented in Section 4 has shown that this parameter has negligible effect on concrete basic creep deformation.

Note also that the ITZ thickness has been fixed to 50 µm for NSC and to 30 µm for HSC (since W/C = 0.38) and its time-dependent modulus was assumed to be the half of the cement paste modulus (i.e., $\alpha = 0.5$). These assumptions related to the ITZ phase will have very little effects on the following comparison since the sensitive analysis conducted on the proposed model predictions has shown that the ITZ has very slight effect on concrete basic creep deformation.

The viscous parameters of the generalized Kelvin-Voigt model used to describe the viscoelastic behaviour of cement pastes $P50$, $P38$ and $P48$ are summarized in Table 3. These parameters have been identified based on a comparison between each cement paste specific basic creep experimental deformations over the time (Le Roy 1996; Granger 1995) and the theoretical deformations given by the generalized Kelvin-Voigt model with 7 viscous elements, as shown in Fig. 11.

Figure 12 shows a comparison between the analytical predictions of the proposed four-sphere model and the experimental results of Le Roy NSC made with cement paste $P50$. Thus, it can be concluded that the proposed model gives a very good prediction of the specific basic creep deformation over the time of Le Roy NSC.

Figure 13 shows a comparison between the analytical predictions of the proposed four-sphere model and the experimental results of Le Roy HSC made with cement paste $P38$. Thus, it can be concluded that the proposed model gives also a good prediction of the specific basic creep deformation over the time of Le Roy HSC.

Finally, Fig. 14 shows also a relatively good agreement (on average) between the analytical predictions of the proposed four-sphere model and the experimental results of Granger two NSC made with cement paste $P48$ and two different types of aggregates (from Flamanville region, Paluel region).
The gap observed here can be due to the aggregates type effect on concrete basic creep which is taken into account within the proposed model only through their elastic modulus value, whereas their mineralogical nature may also have an effect on concrete basic creep as it had been already observed and mentioned by Granger (1995).

5.2 Validation of the proposed model predictions after 1000 days

The proposed four-sphere model predictions have been confronted hereafter with a set of experimental data taken from literature (Pirtz 1968; Auperin et al. 1989; Granger 1995; Le Roy 1996; Persson 1998; Brooks 2005) at a fixed basic creep time of 1000 days ($t = 1000$ days). However, the problem with concrete basic creep tests results available in the literature is often the lack of the corresponding cement paste basic creep tests results. Thus, the cement paste time dependent modulus after 1000 days and for a loading age ($t_0$) of 28 days, $E_p(1000) = 1/J_p(28, 1000)$, will be evaluated using the following empirical formula [Eq. (7)] for $J_p(28, 1000)$ proposed by Le Roy (1996),

$$J_p(28,1000) = \frac{R_c^{28}}{60} \left[ \frac{5.5}{1 + 0.15} \left( \frac{W/C}{1 + 1.74e^{-1}} \right)^{0.5} \right]$$

where $R_c^{28}$ is the cement strength class (MPa).

### 5.2.1 Validation of the proposed model on NSC

The proposed four-sphere model predictions have been confronted with experimental results related to 13 NSC taken from literature (Pirtz 1968; Granger 1995; Le Roy 1996; Brooks 2005) and made from different cement pastes with $W/C$ ranging between 0.48 and 0.8, aggregates types with $E_g$ ranging between 55 and 75 GPa and gradations with $D_{max}$ ranging between 19 and 38.1 mm. The experimental data related to these NSC and to the proposed model parameters and predictions are summarized in Table 4. Figure 15 shows a good agreement between the four-sphere model predictions and the experimental results related to the thirteen NSC presented above, in terms of concrete specific basic creep deformation value after 1000 days.

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Table 4: Experimental data of various NSCs reported by Pirtz (1968), Granger (1995), Le Roy (1996) and Brooks (2005) together with comparison with the proposed model predictions.

| Data set | $g$ | $g_{max}$ | $D_{min}$ (mm) | $D_{max}$ (mm) | $E_g$ (GPa) | $W/C$ | $S/C$ | $R_{c28}$ (MPa) | $c$ (mm) | $f_{ITZ}$ (%) | $\varepsilon_{sc}$ ($10^{-6}$ MPa$^{-1}$) |
|----------|-----|-----------|----------------|----------------|-------------|-------|------|----------------|--------|-------------|------------------|
| Dworshak Dam, reported by Pirtz (1966) | 0.695 | 0.861 | 0.08 | 38.1 | 60 | 0.56 | 0 | 45 | 0.05 | 3.93 | 59.18 | 48.81 |
| Reported by Granger (1995) | 0.623 | 0.842 | 0.1 | 25 | 61 | 0.557 | 0 | 50 | 0.05 | 5.32 | 57.50 | 63.53 |
| | 0.687 | 0.836 | 0.1 | 20 | 70 | 0.543 | 0 | 49 | 0.05 | 5.12 | 50.00 | 42.25 |
| | 0.624 | 0.842 | 0.1 | 25 | 65 | 0.577 | 0 | 55 | 0.05 | 5.31 | 58.00 | 58.95 |
| | 0.667 | 0.842 | 0.1 | 25 | 55 | 0.480 | 0 | 55 | 0.05 | 4.91 | 38.00 | 40.28 |
| | 0.664 | 0.842 | 0.1 | 25 | 65 | 0.480 | 0 | 55 | 0.05 | 4.94 | 22.00 | 40.06 |
| Reported by Le Roy (1996) | 0.705 | 0.870 | 0.08 | 20 | 75 | 0.500 | 0 | 60 | 0.05 | 5.47 | 27.50 | 33.63 |
| Concrete using North Notts gravel, reported by Brooks (2005) | 0.595 | 0.842 | 0.08 | 20 | 70 | 0.80 | 0 | 45 | 0.05 | 6.19 | 136.50 | 111.57 |
| | 0.634 | 0.842 | 0.08 | 20 | 70 | 0.67 | 0 | 45 | 0.05 | 5.85 | 81.60 | 77.52 |
| | 0.663 | 0.842 | 0.08 | 20 | 70 | 0.58 | 0 | 45 | 0.05 | 5.54 | 59.90 | 57.39 |
| | 0.674 | 0.842 | 0.08 | 20 | 70 | 0.54 | 0 | 45 | 0.05 | 5.41 | 49.80 | 50.14 |
| | 0.689 | 0.842 | 0.08 | 20 | 70 | 0.50 | 0 | 45 | 0.05 | 5.21 | 41.70 | 42.32 |

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Fig. 13: Comparison between the analytical predictions of the proposed model and the experimental results of HSC made with cement paste P38 (Le Roy 1996).

Fig. 14: Comparison between the analytical predictions of the proposed model and the experimental results of two NSCs with aggregates from the Flamanville and Paluel regions Granger (1995).
5.2.2 Validation of the proposed model on HSC

The proposed four-sphere model predictions have been also confronted with experimental results related to 13 HSC made all from high strength cement pastes characterized by low ($W/C$) values (ranging between 0.25 and 0.419) and containing silica fume. The experimental data related to these HSC and the proposed model parameters and predictions are summarized in Table 5.

Figure 16 shows a satisfying agreement between the four-sphere model predictions and the experimental data related to these thirteen HSC in terms of concrete specific basic creep deformation value after 1000 days, which confirms the validity and the relevance of the proposed model.

### Table 5: Experimental data of HSCs (Auperin et al. 1989; Le Roy 1996; Persson 1998) and comparison with the proposed model predictions.

| Data set            | $g$    | $g_{\text{max}}$ | $D_{\text{min}}$ (mm) | $D_{\text{max}}$ (mm) | $E_g$ (GPa) | $W/C$ | S/C | $R_{\text{C28}}$ (MPa) | $c$ (mm) | $f_{\text{ITZ}}$ (%) | $\varepsilon_{sc}$ (10^-6 MPa^-1) |
|---------------------|--------|-------------------|------------------------|------------------------|-------------|-------|-----|------------------------|---------|--------------------|-------------------------------------|
| Auperin et al. (1989) | 0.662  | 0.87              | 0.08                   | 0.87                   | 0.380       | 0.08  | 0.1 | 0.60                   | 0.01    | 2.65               | 14.00                               |
| Le Roy (1996)       | 0.715  | 0.87              | 0.08                   | 0.87                   | 0.330       | 0.1   | 0.60 | 0.01                   | 2.37    | 11.00              | 10.09                               |
| Persson (1998)      | 0.710  | 0.87              | 0.08                   | 0.87                   | 0.329       | 0.1   | 0.60 | 0.01                   | 2.27    | 9.50               | 9.08                                |

6. Conclusions

In this paper, the concrete basic creep deformation has been evaluated analytically in the framework of non-aging linear viscoelasticity based on a four-sphere micromechanical model which is an extension of the three-sphere model proposed initially by Le Roy and de Larrard for concrete and which does not account for the ITZ phase. Indeed, within the proposed model, concrete is modelled as a three-phase composite formed by aggregates surrounded by ITZs and embedded randomly in a hardened cement paste. The latter (i.e., cement paste and ITZ) are assumed linear viscoelastic whereas aggregates are linear elastic. Besides, the ITZ volume fraction within concrete has been evaluated using an ana-
lytical formula that the authors had recently proposed in (Zouaoui et al., 2017). The proposed micromechanical approach has allowed predicting concrete specific basic creep time-dependent deformation according to its composition and microstructure. Then, based on a sensitivity analysis conducted on the proposed model predictions, it has been shown that aggregates volume fraction and maximum packing density are the main microstructural parameters governing concrete basic creep in addition to the cement paste basic creep and that all other parameters related to aggregates gradation and to the ITZ have secondary and even very slight effects on concrete basic creep deformation.

Finally, the validity and the relevance of the proposed model have been checked and proven based on a comparison between its predictions and various experimental data taken from literature and related to both normal strength and high strength concretes.

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