Quintessence Energy and Dissipation

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Abstract. The combined effect of a dissipative fluid and quintessence energy can simultaneously drive an accelerated expansion phase at the present time and solve the coincidence problem of our current Universe. A solution compatible with the observed cosmic acceleration is succinctly presented.

1. Introduction

In recent years observation of Type Ia supernovae has lend strong support to an accelerated expansion of the Universe at present time $H_0$. This unexpected feature could be explained by resorting to a small cosmological constant $\Lambda$, which on the one hand would provide enough negative pressure to account for this acceleration, and on the other hand would contribute an energy density of the same order of magnitude than the energy density of the matter content (baryonic plus dark) of today’s Universe -say $\rho_m \simeq 0.3$ and $\rho_\Lambda \simeq 0.7$. This seemingly straightforward “solution” poses, however, a serious problem. Since $\rho_m$ redshifts as $a^{-3}$ while $\Lambda$ is a constant, why they turn out to be comparable today? i.e., why we happen to live in a very special (and rather short) phase of cosmic expansion? This is the so–called coincidence problem.

To avoid it a new form of dark energy (quintessence energy, or Q–matter) has been introduced. This energy corresponds to a scalar field $\phi$ that slowly rolls down its potential with the key property of having a negative pressure. In some respects it mimics the scalar field that supposedly drove inflation at the very early Universe -see e.g. [3]. All these models -formulated for spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) universes- overlook the fact that the matter component of the Universe (baryonic and non–baryonic) might not be very well approximated by a perfect fluid since, in general it should behave as a dissipative fluid and therefore it must have a non–equilibrium pressure that might be non–negligible.

In this short report we shall show that the combination of quintessence energy (an evolving scalar field with negative pressure) and a perfect matter fluid cannot simultaneously drive the current accelerated expansion and solve the coincidence problem. However, when the matter fluid is no longer assumed
perfect this difficulty disappears altogether for open and flat FLRW universes \[4\]. At this point it is fair to say, however, that attempts to solve the coincidence problem have been also made by using scalar–tensor theories of gravity rather than general relativity \[5\]. We will not deal with these here.

2. Quintessence plus perfect fluid

The stress–energy tensor of a perfect fluid (normal matter, i.e., baryonic and non–baryonic) plus a scalar field reads

\[
T_{ab} = (\rho_m + \rho_\phi + p_m + p_\phi)u_a u_b + (p_m + p_\phi)g_{ab} \quad (u^a u_a = -1). \tag{1}
\]

The equation of state for the normal matter is

\[p_m = (\gamma_m - 1)\rho_m,\]

with the baryotropic index lying in the range \(1 < \gamma_m < 2\), and

\[
\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi). \tag{2}
\]

A corresponding equation of state for the scalar field can be written as \(p_\phi = (\gamma_\phi - 1)\rho_\phi\), or what it is the same

\[
\gamma_\phi = \frac{\dot{\phi}^2}{\left(\frac{\dot{\phi}^2}{2}ight) + V(\phi)}, \tag{3}
\]

where for non-negative potentials \(V(\phi)\) one has \(0 \leq \gamma_\phi \leq 2\). However, the scalar field can be properly interpreted as quintessence if the restriction \(\gamma_\phi < 1\) is met.

The Einstein equations for any FLRW universe take the form

\[
\Omega_m + \Omega_\phi + \Omega_k = 1, \tag{4}
\]

\[
\dot{\Omega} = \Omega (\Omega - 1) (3\gamma - 2) H, \tag{5}
\]

where \(\Omega = \Omega_m + \Omega_\phi, \quad \Omega_m = \rho_m / \rho_c, \quad \Omega_\phi = \rho_\phi / \rho_c\), with \(\rho_c = 3H^2\) the critical density, and \(\Omega_k = -k / (aH)^2\). As usual \(H = \dot{a} / a\) denotes the Hubble factor, and \(k\) the spatial curvature index.

Likewise the evolution equation for the scalar field \(\ddot{\phi} + 3H\dot{\phi} + V' = 0\) can be recast as

\[
\dot{\Omega}_\phi = [2 + (3\gamma - 2) \Omega - 3\gamma_\phi] \Omega_\phi H, \tag{6}
\]

where \(\gamma\) is the average baryotropic index defined by

\[
\gamma = \gamma_m \Omega_m + \gamma_\phi \Omega_\phi. \tag{7}
\]

The combined measurements of the cosmic microwave background temperature fluctuations and the distribution of galaxies on large scales suggest a flat or nearly flat Universe \[3\]. Hence the interesting solution of \(3\) at late times is \(\Omega = 1\) (i.e., \(k = 0\)), and so we discard the solution \(\Omega = 0\) as incompatible with observation. The solution \(\Omega = 1\) will be asymptotically stable for expanding
universes provided that condition $\frac{\partial \dot{\Omega}}{\partial \Omega} < 0$ holds in a neighborhood of $\Omega = 1$, and this implies $\gamma < 2/3$. Hence the matter stress violates the strong energy condition (SEC) $\rho + 3p \geq 0$ and as a consequence the Universe accelerates its expansion, i.e., $3\ddot{a}/a = -(\rho + 3p)/2 > 0$.

Since the mixture of Q–matter and perfect dark matter fluid must violate the SEC, $\gamma_\phi$ must be low enough. Namely, because of $\gamma < 2/3$, $\gamma_m \geq 1$, and $\gamma_\phi < \gamma_m$, equation (7) implies $\gamma_\phi < \gamma$. Then, introducing $\Omega = 1$ in equation (6) we obtain

$$\dot{\Omega}_\phi = 3(\gamma - \gamma_\phi)\Omega_\phi H,$$

and therefore $\dot{\Omega}_\phi > 0$, i.e., $\Omega_\phi$ will grow until the constraint (1) is saturated, giving $\Omega_\phi = 1$ in the asymptotic regime. Thus the matter fluid yields a vanishing contribution to the energy density of the Universe at large times. This implies that a flat FLRW universe driven by a mixture of normal perfect fluid and quintessence matter cannot both drive an accelerated expansion and solve the coincidence problem. Therefore some other contribution must enter the stress–energy tensor of the cosmic fluid. A sensible choice is a negative pressure arising from the dissipative character of the matter component. It is worth mentioning that in deriving the above result neither $\gamma_m$ nor $\gamma_\phi$ were restricted to be constants.

3. Quintessence plus dissipative fluid

The only dissipative pressure that may enter the stress–energy tensor is a scalar pressure $\pi$ which has to be semi–negative definite for expanding fluids to comply with the second law of thermodynamics. In this light the stress–energy tensor keeps the same form as (1) but with $p_m$ replaced by $p_m + \pi$. Now equation (4) remains in place but

$$\dot{\Omega} = \Omega (\Omega - 1) \left[ 3 \left( \gamma + \frac{\pi}{\rho} \right) - 2 \right] H,$$

and

$$\dot{\Omega}_\phi = \left\{ 2 + \left[ 3 \left( \gamma + \frac{\pi}{\rho} \right) - 2 \right] \Omega - 3\gamma_\phi \right\} H\Omega_\phi,$$

substitute equations (5) and (6), respectively. The energy conservation equation of the normal matter is $\rho_m' + 3(\gamma_m + \pi/p_m) \rho_m H = 0$. Owing to the presence of the dissipative pressure $\pi$ the constraint $\gamma < 2/3$ does not longer have to be fulfilled for the solution $\Omega = 1$ of equation (4) to be stable. Likewise, inspection of (10) shows that when $\Omega = 1$ one can have $\dot{\Omega}_\phi < 0$ just by choosing the ratio $\pi/\rho$ sufficiently negative. Thereby the constraint (4) allows a nonvanishing $\Omega_m$ at large times. By contrast tracker fields based models (valid only when $\Omega_k = 0$) predict that $\Omega_m \to 0$ asymptotically.

A fixed point solution of equation (4) is $\Omega = 1$. Note that equations (4) and (10) have fixed point solutions $\Omega_m = \Omega_{m0}$ and $\Omega_\phi = \Omega_{\phi0}$, respectively, when the partial baryotrophic indices and the dissipative pressure are related by
\[ \gamma_m + \frac{\pi}{\rho_m} = \gamma_\phi = -\frac{2H}{3H^2}. \]

(11)

Then, the smaller \( \gamma_\phi \), the larger the dissipative effects. Let us investigate the requirements imposed by the stability of these solutions. From (9) we see that \( \gamma + \pi/\rho < 2/3 \) must be fulfilled if the solution \( \Omega = 1 \) is to be asymptotically stable. This condition, together with (11), leads to the additional constraint on the viscosity pressure \( \pi < \left(\frac{2}{3} - \gamma_m\right)\rho_m \), which must be negative. Also by virtue of (8) and the first equality in (11) we obtain from last relationship that \( \gamma_\phi < 2/3 \).

In the special case of a spatially flat universe (\( \Omega = 1 \)), the stability of the solutions \( \Omega_m \) and \( \Omega_\phi \) may be studied directly from (10). Setting \( \Omega_\phi = \Omega_\phi + \omega \), with \( |\omega| \ll \Omega_\phi \), and using (7) it follows that

\[ \dot{\omega} = 3\Omega_m \left( \gamma_m - \gamma_\phi + \frac{\pi}{\rho_m} \right) H (\Omega_\phi + \omega). \]

(12)

Accordingly the solution \( \Omega = 1, \Omega_\phi = \Omega_\phi \) is stable for the class of models that satisfies \( \psi \equiv \gamma_m - \gamma_\phi + (\pi/\rho_m) < 0 \) and \( \psi \to 0 \) for \( t \to \infty \). Note that this coincides with the attractor condition (11).

To study the stability of the solutions \( \Omega_m \) and \( \Omega_\phi \) when \( k \neq 0 \) we introduce the parameter \( \epsilon \equiv \Omega_m/\Omega_\phi \). As it turns out its evolution is governed by

\[ \dot{\epsilon} = -\frac{3H\epsilon}{\Omega_\phi} \left[ \frac{2H}{3H^2} + \gamma_m + \frac{\pi}{\rho_m} + \left( \frac{2}{3} - \gamma_m - \frac{\pi}{\rho_m} \right) \Omega_k \right], \]

(13)

and perturbing this expression about the solution \( \epsilon = \epsilon_0 \sim \mathcal{O}(1) \), (i.e., using the ansatz \( \epsilon = \epsilon_0 + \delta \) with \( |\delta| \ll 1 \)) we obtain with the help of (11)

\[ \dot{\delta} = -\frac{3}{\Omega_\phi} \left( \frac{2}{3} - \gamma_\phi \right) \Omega_k H (\epsilon_0 + \delta) \]

(14)

near the attractor. For \( \Omega_k > 0 \) (negatively spatially curved universes) the ratio \( (\Omega_m/\Omega_\phi)_0 \) is a stable solution. For \( \Omega_k < 0 \) one has to go beyond the linear perturbative regime and/or restrict the class of models as in the spatially flat case to determine the stability of the solution.

We would like to stress that by large times we mean times after the cosmological perturbations evolved into the nonlinear regime. Thus the structure formation scenario will not be spoiled by the quintessence field.

Recently there have been claims that CDM should not be a perfect fluid because it ought to self-interact (with a mean free-path in the range 1 kpc \( \leq l \leq 1 \) Mpc) if one wishes to explain the structure of the halos of galaxies. It is not unreasonable to think that this same interaction lies at the root of the dissipative pressure \( \pi \) at cosmological scales. Bearing in mind that \( l = 1/n\sigma \), with \( n \) the number density of CDM particles and \( \sigma \) the interaction cross section, a simple estimation reveals that at such scales \( l \) is lower than the Hubble distance \( H^{-1} \) and accordingly the fluid approximation we are using is valid.

There exist a handful of solutions with the desired asymptotic properties -see [4] for details. By way of example we just mention \( a \simeq t^{-2/\lambda} \) with
\[ \lambda = \frac{1}{2} \left\{ - (3 \gamma + \nu) + \left[ (3 \gamma - \nu)^2 + 36 \gamma_m v^2 \Omega_m \right]^{1/2} \right\}, \quad (15) \]

where \( \nu \) denotes the number of interactions between CDM particles in a Hubble’s time and \( v \) the speed of the dissipative signals.

We may conclude by stressing that both acceleration and coincidence can be satisfactorily explained by a combination of quintessence and dissipative dark matter. For these models attractor solutions exist with very interesting properties: an accelerated expansion, spatially flatness and a fixed ratio of quintessence to dark matter energy density. In consequence, the quintessence scenario becomes more robust when the dissipative effect of the nonequilibrium pressure arising in the CDM fluid is allowed into the picture.

In [3] we presented specific models with an ample region in the space of out–of–equilibrium thermodynamic parameters satisfying observational constraints in the asymptotic attractor regime which our Universe may well be approaching. In a future research we shall aim to improve these constraints by analysis of the luminosity distance–redshift relation for type Ia supernovae and simulations of structure formation that include dissipative effects.

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