Exact local correlations in kicked chains at light cone edges

Boris Gutkin\(^2\), Petr Braun\(^1\), Maram Akila\(^1,3\), Daniel Waltner\(^1\), Thomas Guhr\(^1\)
\(^1\)Fakultät für Physik, Universität Duisburg-Essen, Lotharstraße 1, 47048 Duisburg, Germany
\(^2\)Department of Applied Mathematics, Holon Institute of Technology, 58102 Holon, Israel
\(^3\)Fraunhofer IAIS, Schloss Birlinghoven, 53757 Sankt Augustin, Germany

We show that local correlators in a wide class of kicked chains can be calculated exactly at light cone edges. Extending previous works on dual-unitary systems, the correlators between local operators are expressed through the expectation values of transfer matrices \(T\) with small dimensions. Contrary to the previous studies, our results are not restricted to dual-unitary systems with spatial-temporal symmetry of the dynamics. They hold for a generic case without fine tuning of model parameters. The results are exemplified on the kicked Ising spin chain model, where we provide an explicit formula for two-point correlators near light cone edges beyond the dual-unitary regime.

**Introduction.** Spatially extended Hamiltonian systems with local interactions are paradigm systems in the field of many-body physics. On the experimental side, various aspects become ever better amenable to direct measurement \([1,4]\) whilst a recent burst of activities \([3,10]\) greatly improved our theoretical understanding. In the context to be addressed here, the outstanding importance of these systems is rooted in their spatiotemporal correlation of local observables which describe, in an often generic manner, experimentally accessible features of interacting many-body systems such as spectral statistics or transport properties \([2,11,12]\). The wealth of available results, unfortunately, covers systems which are either dynamically too simple, such as free or integrable ones, or too low in dimension, such as cat or baker maps. It is thus of paramount interest to find representatives of those systems capturing, on the one hand, the full complexity and, on the other hand, allowing for analytical treatment.

In this work we consider a class of systems admitting a number of different dynamical descriptions \([13]\). The standard one corresponds to the system evolution with respect to time, induced by the system Hamiltonian. Alternatively, one can consider evolution along one of the spatial directions. In this dual approach the corresponding coordinate takes on the role of time. The resulting dynamical system is generically a non-Hamiltonian one \([13,15]\). However, in some special cases it might happen that the dual spatial evolution is a Hamiltonian one, as well. The representatives of such systems, referred to as dual-unitary, can be found among coupled map lattices \([16,17]\), kicked spin chains \([18,21]\), circuit lattices \([22,24]\) and continuous field theories \([25]\).

Dual-unitary systems have recently attracted considerable attention \([18,30]\) due to their intriguing properties. On the one hand, these models generally exhibit features of maximally chaotic many-body systems. In particular, their spectral statistics are well described by the Wigner-Dyson distribution. They are insusceptible to many-body localisation effects even in the presence of strong disorder \([18,20]\). The entanglement has been shown to grow linearly with time and to saturate the maximum bound. On the other hand, dual-unitary models turned out to be amenable to exact analytical treatment. The growth of the entanglement entropy for kicked Ising spin chains (KIC) for certain types of initial states has been evaluated exactly in \([19]\) and their entanglement spectrum was found to be trivial \([23]\).

It has been recently shown that two-point correlations of local operators in dual-unitary quantum circuit lattices \([22,31]\) and kicked chains (KC) \([32]\) can be expressed exactly in terms of small dimensional transfer operators. The main goal of the present contribution is to demonstrate that, in fact, an analogous result holds in a much more general setting. We consider here KC built upon a pair of \(L \times L\) matrices \(u_1, u_2\). The model is defined for an arbitrary length \(N\) of the chain and an on-site Hilbert space dimension \(L\). It becomes a dual-unitary one when \(u_1, u_2\) are complex Hadamard matrices with all entries having the same absolute values.

In the body of the paper we show that correlators between local operators along the light-cone edges can be expressed through the expectation values of a transfer matrix \(T\) whose dimension is determined by \(L\) rather than \(N\). This result does not rely upon dual-unitarity and holds for generic model parameters. For the dual-unitary case the correlators, furthermore, vanish outside of the light-cone edges, in agreement with \([22,32]\). We illustrate our results on the example of KIC, where we provide an explicit formula for two-point local correlators at light-cone edges outside of the dual-unitary regime.

**Kicked chains (KC).** In this paper we consider cyclic chains of \(N\) locally interacting particles, periodically kicked with an on-site external potential. The system is governed by the Hamiltonian,

\[
H(t) = H_I + H_K \sum_{m=-\infty}^{+\infty} \delta(t - m),
\]  

(1)
with $H_I$, $H_K$ being the interaction and kick parts, respectively. The corresponding Floquet time evolution is the product of the operators, $U_I = e^{-iH_I}$ and $U_K = e^{-iH_K}$, acting on the Hilbert space $\mathcal{H}^\otimes N$ of the dimension $L^N$, where $\mathcal{H} = \mathbb{C}^L$ is the local Hilbert space equipped with the basis $\{|s\rangle, s = 1, \ldots, L\}$. We require that $H_I$ couples nearest-neighbour sites of the chain taking on a diagonal form in the product basis, $\{|s\rangle = |s_1\rangle|s_2\rangle\ldots|s_N\rangle\}$. The respective evolution is fixed by a real function $f_1$, 
\[
\langle s|U_I[f_1]|s'\rangle = \delta(s, s')e^{i\sum_{n=1}^{N} f_1(s_n, s_{n+1})},
\]
with $\delta(s, s') = \prod_{i=1}^{N} \delta(s_i - s'_i)$, and cyclic boundary condition $s_{N+1} \equiv s_1$. The second, kick part, is given by the tensor product 
\[
U_K[f_2] = \otimes_{i=1}^{N} u_2, \langle s|U_K[f_2]|s'\rangle = \prod_{i=1}^{N} \langle s_i|u_2|s'_i\rangle,
\]
where $u_2$ is a $L \times L$ unitary matrix with the elements $e^{i\frac{f_2(n,m)}{\sqrt{L}}}$ determined by a complex function $f_2$. Combining the two parts together we obtain the quantum evolution 
\[
U = U_I[f_1]U_K[f_2],
\]
acting on the Hilbert space of dimension $L^N$.

In the same way one constructs the dual evolution acting on the Hilbert space of dimension $L^T$ by exchanging $N \leftrightarrow T$ and $f_1 \leftrightarrow f_2$: 
\[
\hat{U} = U_I[f_2]U_K[f_1].
\]
The following remarkable duality relation \[14\] holds between their traces for any integers $T, N$: 
\[
\text{Tr} \ U_T = \text{Tr} \ \hat{U}^N.
\]
In contrast to the original evolution, $\hat{U}$ is a non-unitary operator, in general. However, if 
\[
\langle n|u_1|m\rangle = \frac{e^{i\frac{f_1(n,m)}{\sqrt{L}}}}{\sqrt{L}}, \langle n|u_2|m\rangle = \frac{e^{i\frac{f_2(n,m)}{\sqrt{L}}}}{\sqrt{L}},
\]
are $L \times L$ complex Hadamard matrices (i.e., unitary matrices which matrix elements have the same absolute value) the dual operator, $\hat{U}$ is unitary as well. We refer to such models as dual-unitary. Note that in the dual-unitary case both $f_1, f_2$ are real. A wide family of such models, referred as dual-unitary Fourier transform chains (FTC) were constructed in \[32\] for each $L$ by fixing $n_1, u_2$ to be the unitary discrete Fourier transform multiplied on both sides by arbitrary diagonal unitary and permutation matrices. The correlators in dual-unitary KC have been studied in \[32\]. Here we are primarily focused on general KC models with no demand of dual-unitarity.

**Correlations between local operators.** Let $(q_1, q_2), \ (q_3, q_4)$ be two pairs of matrices acting on the on-site Hilbert space $\mathcal{H}$. We define the corresponding many-body operators 
\[
\Sigma_{n_1} = \underbrace{1 \otimes \ldots \otimes 1}_{n_1-1} \otimes q_1 \otimes q_2 \otimes \underbrace{1 \otimes \ldots \otimes 1}_{N-n_1-1}
\]
\[
\Sigma_{n_2} = \underbrace{1 \otimes \ldots \otimes 1}_{n_2-1} \otimes \underbrace{1 \otimes \ldots \otimes 1}_{N-n_2-1} \otimes q_3 \otimes q_4 \otimes \underbrace{1 \otimes \ldots \otimes 1}_{N-n_2-1},
\]
supported at the sites $n_1, n_1 + 1$ and $n_2, n_2 + 1$ of the chain, respectively. In what follows we consider the two-point correlator: 
\[
C(n, t) = L^{-N} \text{Tr} \ U^T \Sigma_{n_1} U^{t-\Sigma_{n_2}},
\]
where we assume $n = n_2 - n_1 > 0, t > 0$. By translation symmetry of the model, we can set $n_1 = 1, n_2 = n_1 + 1$ without loss of generality.

The above correlation function can be written in the form of the partition function, 
\[
C(n, t) = \frac{1}{L^{Nt}} \sum_{\{s_{mk} \mid (m,k) \in L_1\}} e^{-iF(\{s_{mk}\})} \left[ \prod_{(m,k) \in L_2} \delta(s_{mk}, s_{m,2t+k-1}) \right] D(s_{n_1}, \ldots, s_{n_2t}),
\]
where the last factor, 
\[
D = \langle s_{n_1, 2t}| q_1^\dagger |s_{n_1,1}\rangle \langle s_{n_1, 2t}| q_2^\dagger |s_{n_1,1}\rangle \langle s_{n_2, t}| q_3^\dagger |s_{n_2, t+1}\rangle \langle s_{n_2, t}| q_4^\dagger |s_{n_2, t+1}\rangle
\]
$q_1^\dagger = u_2 q_1 u_2^\dagger$, $q_2^\dagger = u_2 q_2 u_2^\dagger$, depends on the eight lattice sites, $L_0 = \{(n_1, k), (n_1 + 1, k) \mid k = 1, 2t \} \cup \{(n_2, k), (n_2 + 1, k) \mid k = t, t+1 \}$ corresponding to the location of the observables and the function $F(\{s_{mk}\})$ is given by eq. \[32\] in the supplementary material. The above sum runs over $2t \times N$ sites of the lattice $L_1 = \{(m, k) \mid k = 1, \ldots, 2t, m = 1, \ldots, N\}$ while the product in \[11\] is, furthermore, restricted to the subset $L_2 = \{(m, k) \mid k = 1, t, t+1, 2t, m = 1, \ldots, N\} \setminus L_0$.

The partition function \[11\] allows for an instructive graphical representation illustrated on fig. \[4\] As we show in the supplementary material, on the light cone edge, $n = t$ it can be considerably simplified by eliminating most of the variables, $s_{n,t}$ provided that $N > 2t$. The remaining lattice sum contains only the variables along the light cone edges as shown on fig\[2\]. The resulting expression can be represented in the form of the expectation value 
\[
C_t = C(t, t) = \langle \Phi_{q_1, q_2} | T^{-2} | \Phi_{q_3, q_4} \rangle,
\]
of the transfer operator $T$,

$$
\langle \eta \eta | T | \eta' \eta' \rangle = \frac{1}{L^3} \left| \sum_{i=1}^{L} e^{i(f_1(\eta_1) + f_2(\eta_2) + f_2(\eta_3) + f_2(\eta_4))} \right|^2 , \quad (14)
$$

acting on the small space $\mathcal{H} \otimes \mathcal{H}$. The left $\Phi_{\eta_1 \eta_2}$ and the right $\Phi_{\eta_3 \eta_4}$ vectors are defined as

$$
\langle \eta \eta | \Phi_{\eta_1 \eta_2} \rangle = \frac{1}{L^3} \sum_{a,\bar{a},b=1}^{L} \Gamma_{a \bar{a}}^b \langle a | q_3 | \bar{a} \rangle \langle b | q_4 | b \rangle , \quad (15)
$$

$$
\langle \Phi_{\eta_1 \eta_2} | \eta \eta' \rangle = \frac{1}{L^3} \sum_{a,\bar{a},b=1}^{L} \Gamma_{a \bar{a}}^{b'} \langle a | q_3 | \bar{a} \rangle \langle b | q_4^{*} | b \rangle , \quad (16)
$$

where

$$
\Gamma_{a \bar{a}}^b = e^{i(f_1(\eta_1) - f_1(\eta_2) + f_2(\eta_3) - f_2(\eta_4))} ,
$$

$$
\Gamma_{a \bar{a}}^{b'} = e^{i(f_1(\eta_1) - f_1(\eta_2) + f_2(\eta_3) - f_2(\eta_4))} .
$$

It is easy to check that $T$ is doubly stochastic i.e., satisfies

$$
\sum_{\nu=1}^{L} \sum_{\eta'=1}^{L} \langle \nu \eta | T | \eta' \eta' \rangle = \sum_{\nu=1}^{L} \sum_{\eta'=1}^{L} \langle \nu \eta | T | \eta' \eta' \rangle = 1 .
$$

This implies that the spectrum of $T$ is contained within the unit disc with the largest eigenvalue, $\mu_1 = 1$. The left (resp. right) eigenvector corresponding to $\mu_1$ are given by the choice $q_3 = q_4 = 1$ (resp. $\bar{q}_3 = \bar{q}_4 = 1$). For typical system parameters the correlators between traceless observables decay exponentially with the rates determined by the second eigenvalue $\mu_2$, $|\mu_2| \leq |\mu_1|$ of $T$ having the largest absolute value after $\mu_1$. Eq. (13) can be also used to evaluate correlations between strictly local observables in KC by setting $q_1 = 1, q_4 = 1$. While $\Phi_{\eta_1 \eta_2} = \Phi_{\eta_3 \eta_4} = 0$ for traceless $q_2, q_3$ in dual-unitary KC, these vectors do not vanish generically, implying non-trivial correlations $C_t$ between strictly local operators in a general KC.

It is important to emphasize that (13) holds for any KC model (1) and does not require dual-unitarity. In essence, any KC of this type is solvable, as far as, local correlators are restricted to the light cone edge. What makes dual-unitary case special is that $C(n,t)$ is zero there for traceless $q_1$’s if $n \neq t$ and $N > 2t$. As has been pointed out in [22], this can be understood in a simple intuitive way. Since the speed of information propagation in KC (1) equals one, the correlator of operators (8,9) with traceless $q_1$’s must vanish outside of the light cone. What leaves the light cone edges $|t| = |n|$ as the only possible places on the space-time lattice where non-trivial correlations might arise. Accordingly, for dual-unitary models we have

$$
C(n,t) = \delta(n,t) C_t , \quad (17)
$$

where $C_t$ is given by eq. (13).

The above results can be straightforwardly extended to systems with spatial-temporal disorder, where the local functions $f_1, f_2$ depend on the lattice sites. In such a case the transfer operator $T^{t-2}$ in eq. (13) is substituted with the product of local “gate” operators $T_i \prod_{j \neq i} T_{j-2}$, where each $T_i$ is determined by the functions $f_1, f_2$ at the point $(i,i)$ of the spatial-temporal lattice. For a sub-family of dual-unitary, FTC models introduced in [32] all matrices $T_i$ are diagonalized by one and the same unitary transformation. As a result, the decay expo-
ponents of the correlators \( C_{\alpha\beta}(n,t) \) in the disordered FTC are just given by the averages of the local exponents.

**KIC model.** Below we illustrate our results on the example of KIC model providing a minimal, \( L = 2 \), realisation of the KC model [1]. The KIC evolution is governed by the Hamiltonians:

\[
H_j = \sum_{n=1}^{N} J \hat{\sigma}_n^x \hat{\sigma}_{n+1}^x + h \hat{\sigma}_n^z, \quad H_K = b \sum_{n=1}^{N} \hat{\sigma}_n^x, \tag{18}
\]

where \( \hat{\sigma}_n^x = 1 \otimes \cdots 1 \otimes \sigma^x \otimes 1 \otimes \cdots 1 \) and \( \hat{\sigma}_n^y = 1 \otimes \cdots 1 \otimes \sigma^y \otimes 1 \otimes \cdots 1 \).

For the sake of simplicity of exposition we set \( b = \pi/4 \) with \( b \) and \( J \) being arbitrary. For this choice of parameters eq. (13) gives (see supplementary material) for the correlator (10) at \( n = t, N > 2t \):

\[
C_t = C_{\alpha\beta}^\delta (\sin^2 2J \cos 2h)^t, \tag{19}
\]

where the prefactors \( C_{\alpha\beta}^\delta \) depend on the operators \( q_1 = \sigma^\alpha, q_2 = \sigma^\beta, q_3 = \sigma^\gamma, q_4 = \sigma^\delta \). Specifically, \( C_{yz}^z = 1, C_{xz}^x = \tan^2 2h, C_{yx}^y = C_{zy}^z = -\tan 2h \) and zeroes for all other spin combinations. The dual-unitary case corresponds to \( J = b = \pi/4 \) leading by (17) to \( C(n,t) = \delta(n,t)C_{\alpha\beta}^\delta (\cos 2h)^t \), the result obtained in [32].

As has been explained above, in the dual-unitary case all two-point correlators

\[
C_{\alpha\beta}(n,t) = \frac{1}{2N} \text{Tr} \left( U^{-t} \hat{\sigma}_{n+1}^{\alpha} U^t \hat{\sigma}_{1}^{\beta} \right), \tag{20}
\]

\( \alpha, \beta \in \{ x, y, z \} \) between local spin operators vanish identically for \( t > 0, N > 2t \).  For a general KIC, away from the self-dual regime, the correlators (20) are non-zero, in general, and can be evaluated at \( n = t = 1, N > 2t \) by using eq. (13). To this end we set \( q_1 = 1, q_4 = 1 \) and \( q_2 = \sigma^\gamma, q_3 = \sigma^\beta \) which yields for \( t > 1 \)

\[
C_{\alpha\beta}(t-1,t) = \langle \Phi_{x_1x_2} | T^{t-2} \Phi_{x_3x_4} \rangle. \tag{21}
\]

For \( b = \pi/4 \) and general \( J \) a straightforward evaluation of (21) leads to

\[
C_{\alpha\beta}(t-1,t) = C_{\alpha\beta}(\cos 2h \sin^2 2J)^t \cot^2 2J \tag{22}
\]

with the coefficients given by

\[
C_{xz}^x = 1, C_{zx}^z = \tan 2h, C_{zy}^y = \tan^2 2h,
\]

and by zeroes for other \( \alpha, \beta \) pairs. Note that for all \( n \geq t \) the correlator \( C_{\alpha\beta}(n,t) \) vanishes. For \( n = t \) this result can be obtained by the substitution \( q_2 = 1, q_4 = 1 \) and \( q_1 = \sigma^\alpha, q_3 = \sigma^\beta \) into (13). Since \( \Phi_{x_1x_2} = 0 \), on gets immediately \( C_{\alpha\beta}(t,t) = 0 \).

For a larger \( n > t \), the same answer follows straightforwardly from the fact that speed of information propagation in KIC is one.

The correlators (19) (22) decay exponentially with the rates \( \cos 2h \sin^2 2J \), except the cases where \( \frac{2n}{2} = \frac{2t}{2} \in \mathbb{Z}, \frac{2h}{2} \in \mathbb{Z} \), see fig. 3. For these parameters KIC corresponds to well known cases of integrable classical 2-d Ising spin model with complex parameters [33, 35].

**Conclusions.** We derived an analytic formula, relating correlators \( C(n,t) \) between operators with two point support for \( n = t \) (light cone edge) to the expectation values of a transfer operator with small dimensions. The result holds for a sufficiently long generic KC and does not require fine-tuned system parameters. For the subfamily of dual-unitary KC this allows for a full characterization of the correlator behavior, as \( C(n,t) = 0 \) for \( n \neq t \) in this case. We illustrated these results on the example of KIC, where an explicit expression for correlations between strictly local operators has been obtained also next to the light cone edge at \( n = t - 1 \).

Our study clarifies the role of dual-unitarity with regard to the model solvability. The fact, that local correlators in the vicinity of the light cone edge can be expressed in terms of a small dimensional transfer operator is due to the locality of the system interactions. On its own it does not require dual-unitarity of the system dynamics. The dual-unitarity is only essential to ensure that correlators of traceless operators vanish outside of the line \( n = t \). For a general model one has \( C(n,t) = 0 \) only for \( n > t \). The above results allow for several generalizations. First, models with a larger range of interactions can
be treated in a similar manner. For systems with $r$-point interactions, $H_I = \sum_{i=1}^{N} |f_i(s_{1+i}, \ldots, s_{r+i})$ the correlations at the light cone edge $n = rt$ can be expressed through transfer operators of the dimension $L' \times L'$. Second, in the present work we restricted our considerations to correlators between operators with two-point support. An analogous result holds for correlations between operators with a larger support, i.e., $\sum_{n,t}^{(l)} = I_1 \otimes \cdots \otimes I \otimes q_{k+1} \otimes \cdots \otimes q_{k+l} \otimes I_1 \otimes \cdots \otimes I$. In general, the correlators $\langle \hat{\Sigma}_{0}^{(l)} \hat{\Sigma}_{n}^{(l)} \rangle$ can be evaluated through expectation values of transfer operators $T_I$ with the dimensions $L' \times L'$. By using this, the correlators $C(n, t)$ in (10) can be evaluated above the light cone edge $t = n + l$, $l > 0$ as well. To this end one fixes all $q_i$ in $\sum_{n,t}^{(l)}$ to $I$, except $q_l, q_{n+l}$. The price to pay is in the dimension of the transfer operators - the dimension of $T_I$ increases exponentially with $l$. Finally, it is worth of noticing that for even $N$ and even propagation times $t$ the correlators (10) can be mapped, in principle, upon correlators of a circular lattice with a special gate operator $U_{gate}$, provided by (26) in the supplementary material. It seems to be very plausible that an analogue of our main result (13) holds for a general circular lattice, as well.

Acknowledgements

We thank T. Prosen for useful discussion. One of us (B.G.) acknowledges support from the Israel Science Foundation through grant No. 2089/19.

[1] I. Bloch, J. Dalibard, and W. Zwerger, “Many-body physics with ultracold gases,” Rev. Mod. Phys., vol. 80, pp. 885–964, Jul 2008.
[2] G. D. Mahan, Condensed Matter in a Nutshell, vol. 8. Princeton University Press, 2011.
[3] M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, “Observation of many-body localization of interacting fermions in a quasirandom optical lattice,” Science, vol. 349, no. 6250, p. 842845, 2015.
[4] J. Simon, W. S. Bakr, R. Ma, M. E. Tai, P. M. Preiss, and M. Greiner, “Quantum simulation of antiferromagnetic spin chains in an optical lattice,” Nature, vol. 472, no. 7344, p. 307312, 2011.
[5] T. Engl, J. Dujardin, A. Argüelles, P. Schlagheck, K. Richter, and J. D. Urbina, “Coherent backscattering in fock space: A signature of quantum many-body interference in interacting bosonic systems,” Phys. Rev. Lett., vol. 112, p. 140403, Apr 2014.
[6] R. Dubertrand and S. Miller, “Spectral statistics of chaotic many-body systems,” New Journal of Physics, vol. 18, p. 033009, mar 2016.
[7] P. Ponte, Z. Papic, F. Huveneers, and D. A. Abanin, “Many-body localization in periodically driven systems,” Phys. Rev. Lett., vol. 114, p. 140401, Apr 2015.
[8] Y. Y. Atas and E. Bogomolny, “Spectral density of a one-dimensional quantum Ising model: Gaussian and multi-Gaussian approximations,” Journal of Physics A: Mathematical and Theoretical, vol. 47, p. 335201, aug 2014.
[9] J. P. Keating, N. Linden, and H. J. Wells, “Spectra and Eigenstates of Spin Chain Hamiltonians,” Communications in Mathematical Physics, vol. 338, p. 81102, Aug 2015.
[10] S. Czischek, M. Gätttner, M. Oberthaler, M. Kastner, and T. Gasenzer, “Quenches near criticality of the quantum Ising chain—power and limitations of the discrete truncated Wigner approximation,” Quantum Science and Technology, vol. 4, p. 044006, oct 2018.
[11] A. Altland and B. D. Simons, Condensed Matter Field Theory. Cambridge University Press, 2 ed., 2010.
[12] J. P. Sethna, Statistical Mechanics: Entropy, Order Parameters and Complexity, Great Clarendon Street, Oxford OX2 6DP: Oxford University Press, first edition ed., 2006.
[13] M. Akila, D. Waltner, B. Gutkin, and T. Guhr, “Particle-time duality in the kicked Ising spin chain,” J. Phys. A, vol. 49, p. 375101, 2016.
[14] M. Akila, D. Waltner, B. Gutkin, P. Braun, and T. Guhr, “Semiclassical identification of periodic orbits in a quantum many-body system,” Phys. Rev. Lett., vol. 118, p. 164101, 2017.
[15] M. Akila, B. Gutkin, P. Braun, D. Waltner, and T. Guhr, “Semiclassical prediction of large spectral fluctuations in interacting kicked spin chains,” Ann. Phys., vol. 389, pp. 250–282, 2018.
[16] B. Gutkin and V. Osipov, “Classical foundations of many-particle quantum chaos,” Nonlinearity, vol. 29, pp. 325–356, 2016.
[17] B. Gutkin, L. Han, R. Jafari, A. K. Saremi, and P. Cvitanović, “Linear encoding of the spatiotemporal cat map,” 2019. arXiv:1912.02940.
[18] B. Bertini, P. Kos, and T. Prosen, “Exact spectral form factor in a minimal model of many-body quantum chaos,” Phys. Rev. Lett., vol. 121, p. 264101, 2018.
[19] B. Bertini, P. Kos, and T. Prosen, “Entanglement spreading in a minimal model of maximal many-body quantum chaos,” Phys. Rev. X, vol. 9, p. 021033, 2019.
[20] P. Braun, D. Waltner, M. Akila, B. Gutkin, and T. Guhr, “Transition from quantum chaos to localization in spin chains,” 2019. arXiv:1902.06265.
[21] R. Pal and A. Lakshminarayan, “Entangling power of time-evolution operators in integrable and nonintegrable many-body systems,” Phys. Rev. B, vol. 98, p. 174304, Nov 2018.
[22] B. Bertini, P. Kos, and T. Prosen, “Exact correlation functions for dual-unitary lattice models in 1+1 dimensions,” Phys. Rev. Lett., vol. 123, p. 210601, Nov 2019.
[23] S. Gopalakrishnan and A. Lamacraft, “Unitary circuits of finite depth and infinite width from quantum channels,” Phys. Rev. B, vol. 100, p. 064309, 2019.

[24] B. Bertini, P. Kos, and T. Prosen, “Operator entanglement in local quantum circuits I: Maximally chaotic dual-unitary circuits,” 2019. arXiv:1909.07407.

[25] J. Avan, V. Caudrelier, A. Doikou, and A. Kundu, “Lagrangian and hamiltonian structures in an integrable hierarchy and space-time duality,” Nuclear Physics B, vol. 902, pp. 415 – 439, 2016.

[26] L. Piroli, B. Bertini, J. I. Cirac, and T. Prosen, “Exact dynamics in dual-unitary quantum circuits,” 2019. arXiv:1911.11175.

[27] Z. Krajnik and T. Prosen, “Kardar-Parisi-Zhang physics in integrable rotationally symmetric dynamics on discrete space-time lattice,” 2019. arXiv:1909.03799.

[28] T. Zhou and A. Nahum, “The entanglement membrane in chaotic many-body systems,” 2019. arXiv:1912.12311.

[29] D. Goyeneche, D. Alsina, J. I. Latorre, A. Riera, and K. Žyczkowski, “Absolutely maximally entangled states, combinatorial designs, and multiunitary matrices,” Phys. Rev. A, vol. 92, p. 032316, Sep 2015.

[30] S. A. Rather, S. Aravinda, and A. Lakshminarayan, “Creating ensembles of dual unitary and maximally entangling quantum evolutions,” 2019. arXiv:1912.12021.

[31] A. L. Pieter W. Claeys, “Maximum velocity quantum circuits,” 2020. arXiv:2003.01133.

[32] B. Gutkin, P. Braun, M. Akila, D. Waltner, and T. Guhr, “Local correlations in dual-unitary kicked chains,” 2020. arXiv:2001.01298.

[33] C. N. Yang and T. D. Lee, “Statistical theory of equations of state and phase transitions. i. theory of condensation,” Phys. Rev., vol. 87, pp. 404–409, Aug 1952.

[34] T. D. Lee and C. N. Yang, “Statistical theory of equations of state and phase transitions. ii. lattice gas and ising model,” Phys. Rev., vol. 87, pp. 410–419, Aug 1952.

[35] V. Matveev and R. Shrock, “On properties of the ising model for complex energy/temperature and magnetic field,” Journal of Physics A: Mathematical and Theoretical, vol. 41, p. 135002, mar 2008.
SUPPLEMENTARY MATERIAL

Relation to circuit lattices

For the sake of comparison it is instructive to observe a connection between quantum kicked chains considered in this work and circuit lattices. Such a connection can be established when both the chain length $N$ and the propagation times $t$ are even. It is straightforward to see that the quantum evolution operator $U^{2t}$ for even times can be cast into the form

$$U^{2t} = U_0^t U_{\text{circ}}^t (U_0^t)^\dagger.$$  \hfill (23)

Here, the operator $U_0^t$ corresponds to the even “half of the interaction”:

$$\langle s | U_0^t | f_1 \rangle | s' \rangle = \delta(s, s') e^{i \sum_{n=1}^{N/2} f_1(s_{2n}, s_{2n+1})},$$  \hfill (24)

and the evolution $U_{\text{circ}}$ has the form

$$U_{\text{circ}} = T U_\epsilon^t U_\kappa U_0^t T^\dagger U_\kappa U_0^t,$$  \hfill (25)

where $T$ is the circular shift operator on a lattice of $N$ sites. Note that $U_{\text{circ}}$ has a special structure, characteristic to circuit lattice evolution, see e.g., \cite{22}. The role of the unitary gate operator is fulfilled here by

$$U_{\text{gate}} = u_1^t (u_2 \otimes u_2) u_1^t,$$  \hfill (26)

where the diagonal matrix

$$\langle s_1 s_2 | u_1^t | s'_1 s'_2 \rangle = \delta(s_1, s'_1) \delta(s_2, s'_2) e^{i f_1(s_1, s_2)}$$

is a restriction of $U_0^t$ to two adjacent lattice sites.

By eq. (25) we find for the two-point correlator

$$\text{Tr} \left( (U_t^t Q_1 U^{-t}_t) Q_2 \right) = \text{Tr} \left( U_{\text{circ}}^t \hat{Q}_1 U_{\text{circ}}^{-t} \hat{Q}_1 \right),$$  \hfill (27)

where $\hat{Q}_i = (U_0^t)^i Q_i U_0^t$. Since $U_0^t$ couples two neighbouring sites, any strictly local operator with one-point support in the kicked model corresponds to a two site operator of the respective circuit model.

Graphical method for evaluation of correlators

Correlation function between a number of local observables in the Flouquet KC \cite{1} can be written in the form of partition function,

$$Z = \frac{1}{\mathcal{L}_1} \sum_{\{s_{m,k} | (m,k) \in \mathcal{L}_1\}} \left[ e^{-\mathcal{F}(\{s_{m,k}\})} \prod_{(m,k) \in \mathcal{L}_2} \delta(s_{m,k}, s_{m,1-k+2t}) \right] D(z_1, \ldots, z_n),$$  \hfill (28)

where the last factor, $D$ depends on a finite number of lattice sites, $\mathcal{L}_0 = \{z_1, \ldots, z_l\}$ corresponding to location of the observables. The above sum, in general, runs over a subset $\mathcal{L}_1$ of sites from the $2t \times N$ lattice $\mathcal{L}_{N \times 2t} = \{(m,k) | k = 1, \ldots, 2t, m = 1, \ldots, N\}$ while the product in (28) is, furthermore, restricted to a subset $\mathcal{L}_2 \subseteq \mathcal{L}_1$. In what follows we will distinguish between three type of points $(m,k) \notin \mathcal{L}_0$ of the spatial-temporal lattice $\mathcal{L}_{N \times 2t}$ and introduce the corresponding symbolic notation for lattice sites:

- **Type 1:** $(m,k) \notin \mathcal{L}_1$ i.e., there is no summation over the variables $s_{m,k}, s_{m,1-k+2t}$ in the partition function. The sites of this type are depicted by empty circles $\{\circ\}$.

- **Type 2:** $(m,t) \in \mathcal{L}_2$ i.e., there is summation over the variables $s_{m,k}, s_{m,1-k+2t}$ coupled by the term $\delta(s_{m,k}, s_{m,1-k+2t})$. The sites of this type are depicted by full red circles $\{\bullet\}$. 

• Type 3: \((m, k) \in \mathcal{L}_1 \setminus \mathcal{L}_2\) i.e., there is summation over uncoupled variables \(s_{m,k}, s_{m,1-k+2t}\). The sites of this type are depicted by full black circles \(\bullet\).

Having this notation at hand, we can uniquely encode a partition function of the type \((28)\) by filling nodes \((m,k)\) of the lattice \(\mathcal{L}_1 \setminus \mathcal{L}_0\) with symbols drawn from the alphabet \(\{\circ, \bullet\}\), see figs. 1, 2. Thanks to the unitarity of the operator \(u_2\) a simple graphical method for calculation of partition functions like \((28)\) can be developed. To this end we establish “contraction rules” for sites of \(\mathcal{L}_{N \times 2t} \setminus \mathcal{L}_0\). Let \((m,k)\) be a site of the type II such that three of its neighbours are of the type II, and the forth one of the type III. It can be easily shown that after summation over \(s_{m,k}, s_{m,1-k+2t}\) variables the fourth site becomes of the type II as well, while \((m,k)\) becomes of the type I, see fig. 4. Indeed, whenever \((m-1, k), (m+1, k), (m, k), (m, k-1) \in \mathcal{L}_2\) we have for sum over \(s_{m,k}, s_{m,1-k+2t}\) variables in eq. \((28)\)

\[
\frac{1}{L} \sum_{s_{m,k}} \sum_{s_{m,1-k+2t}} e^{-i(f_2(s_{m,k}, s_{m,k+1})-f_2(s_{m,1-k+2t}, s_{m,1-k+2t}))} \delta(s_{m,k}, s_{m,1-k+2t}) = \delta(s_{m,k+1}, s_{m,1-k+2t}).
\]

In an analogous way one can obtain all other contraction rules illustrated on fig. 4. Note that the above contraction rules are akin of the operator “fusion rules” introduced in [22].

Obviously, each contraction leads to removing of two summation variables from the sum \((28)\) without changing its form. As a result, by consecutive applications of the contraction rules the initial partition function can be reduced to the state where the vast majority of the summation variables are excluded from the sum \((28)\). The remaining sum can be then represented with the help of a transfer operator of a small dimension, independent of \(N\).

Correlations between operators with two-point support

Here we consider the two point correlator,

\[
C(n, t) = L^{-N} \text{Tr} U_t^{n_1} U_t^{-n_2}, \quad n = n_2 - n_1,
\]

FIG. 4: The figure illustrates contraction rules for lattice sites \((m, t)\) belonging to the set \(\mathcal{L}_2\). The four figures above correspond to the case where three out of four neighbours of \((m, t)\) belong to the set \(\mathcal{L}_2\). The bottom figure illustrates the case where all four neighbours belong to \(\mathcal{L}_2\).
where operators $\Sigma_{n_1}, \Sigma_{n_2}$ are given by eqs. (15). As the first step, we cast (30) into the form of partition function for a classical statistical model. Specifically, we have

$$C(n,t) = \frac{1}{N^L} \sum_{\{s_m,k\in1...L\}} e^{-i\mathcal{F}(s_m,k)} \langle s_{n_1,2t}|q_1^t|s_{n_1,1}\rangle \langle s_{n_1,2t}|q_2^t|s_{n_1,1}\rangle \langle s_{n_2,t}|q_3^t|s_{n_2,t+1}\rangle \langle s_{n_2,t}|q_4^t|s_{n_2,t+1}\rangle \times \prod_{m\neq n_1,n+1}^N \delta(s_{m,2t},s_{m,1}) \prod_{m\neq n_2,n+2}^N \delta(s_{m,t},s_{m,t+1}),$$

where

$$\mathcal{F} = \sum_{k=1}^{t} \sum_{m=1}^{N} f_1(s_{m,k},s_{m+1,k}) - f_1(s_{m,k+t},s_{m+1,k+t}) + f_2(s_{m+1,k},s_{m,k}) - f_2(s_{m,k+t},s_{m+1,k+t}).$$

We will consider (30) at the cone light border $n = t$. It is instructive to represent $C_t = C(n = t, t)$ in the form of a partition function. The initial expression is shown in a graphic form on fig. (1). The summation variables $s_{m,k}$ are excluded one by one by applying the contraction rules, see fig. (4). For $N > 2t$ the elimination of $s_{m,k}$ variables can be continued up to reaching the stage illustrated by the figure (2). Here the remaining summation variables (shown in red and black) are located along the one dimensional strip only, which reduces the whole problem to calculation of a quasi one-dimensional partition function. By using the transfer operator (14), it can be cast into the form of the expectation value

$$C_t = \langle \Phi_{s_1,s_2}|T^{t-2}|\Phi_{s_1,s_2}\rangle,$$

where the left $\Phi_{s_1,s_2}$ and the right $\Phi_{s_1,s_2}$ vectors are defined by (15) (16), respectively.

**Application to KIC model**

The KIC model provides a minimal realisation of the model (1) with $L = 2$. The KIC evolution is governed by the Hamiltonians:

$$H_1 = \sum_{n=1}^{N} J \hat{\sigma}_n^x \hat{\sigma}_{n+1}^x + h \hat{\sigma}_n^z, \quad H_K = b \sum_{n=1}^{N} \hat{\sigma}_n^z,$$

$$\hat{\sigma}_n^\alpha = \mathbb{1} \otimes \cdots \otimes \mathbb{1} \otimes \sigma^\alpha \otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1},$$

where $\sigma_n^\alpha, \alpha = x, y, z$ are Pauli matrices. For the sake of simplicity of exposition we restrict our considerations to $b = \pi/4$ and arbitrary $J, h$. Note that the dual-unitary case corresponds to $J = b = \pi/4$. The resulting evolutions $U_K, U_t$ take the form (4) with the functions

$$f_1 = -Jmn - \frac{h}{2}(m + n), \quad f_2 = \frac{\pi}{4}(mn - 1),$$

$m, n = \pm 1$, defining the two unitary matrices $u_1, u_2$:

$$u_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i(J+h)} & e^{iJ} \\ e^{iJ} & e^{-i(J-h)} \end{pmatrix}, \quad u_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}.$$

After inserting $f_1, f_2$ into eq. (14) we obtain:

$$T = \frac{1}{2} \begin{pmatrix} \cos^2 h_+ & \sin^2 h & \sin^2 h_+ & \cos^2 h \\ \sin^2 h_+ & \cos^2 h & \cos^2 h_+ & \sin^2 h \\ \sin^2 h & \cos^2 h_+ & \cos^2 h & \sin^2 h_+ \\ \cos^2 h & \sin^2 h_+ & \sin^2 h & \cos^2 h_+ \end{pmatrix}.$$
where \( h_+ = h + J - \pi/4, h_- = h - J + \pi/4 \). The four eigenvalues of \( T \) are
\[
\mu_1 = 1, \quad \mu_2 = \cos 2h \sin^2 2J, \quad \mu_3 = 0, \quad \mu_4 = 0.
\]
As a result, the \( n \)-th power of \( T \) is given for \( n > 1 \) by
\[
T^n = \mu_1^n \Phi_2 \otimes \Phi_2 + \Phi_1 \otimes \Phi_1
\]
with \( \Phi_1 = \frac{1}{2}(1,1,1,1)^T \) being the eigenvector of \( T \) for the leading eigenvalue \( \mu_1 \) and
\[
\Phi_2 = \frac{1}{c + d}(c, -c, -d, d)^T, \quad \Phi_2 = \frac{1}{c + d}(c, -d, -c, d).
\]
c = \cos 2h_+ + \cos 2h, d = \cos 2h_- + \cos 2h, are the left and right eigenvectors corresponding to \( \mu_2 \).

**Four-point correlators.**

To evaluate correlators note that the operators \( u^2 q_1 u_d^1 q_d \) contribute only diagonal elements into \( \langle \Phi_1| \Phi_1 \rangle \). In the case of KIC model this means that only the spin combinations, \( \Sigma_{\alpha n_1} = \delta^\alpha_{n_1} \delta_{n_1+1}, \Sigma_{\beta n_2} = \delta^\beta_{n_2} \delta_{n_2+1} \) for \( \alpha = y, \delta = z \) might have \( C_{ij} \neq 0 \). By using the representation \( (37) \) we have for the correlator \( (13) \)
\[
C_{t t} \equiv C(t, t) = \mu_2^{-2} \langle \Phi_{y y} \Phi_{z z} | \Phi_2 \rangle \langle \Phi_2 | \Phi_{y y} \Phi_{z z} \rangle + \langle \Phi_{y y} \Phi_{z z} | \Phi_1 \rangle \langle \Phi_1 | \Phi_{y y} \Phi_{z z} \rangle,
\]
where the vectors \( \Phi_{y y} \Phi_{z z}, \Phi_{y z} \Phi_{z y} \) are calculated by \( (15, 16) \). Explicitly, they are given by
\[
\Phi_{y y} \Phi_{z z} = \Phi_{y z} \Phi_{z y} = \sin 2J \left( \begin{array}{c} -\sin(2h - 2J) \\ \sin(2h - 2J) \\ -\sin(2h + 2J) \\ \sin(2h + 2J) \end{array} \right), \quad \Phi_{y z} \Phi_{z y} = \cos 2J \left( \begin{array}{c} -\cos(2h - 2J) \\ -\cos(2h + 2J) \\ \cos(2h - 2J) \\ \cos(2h + 2J) \end{array} \right).
\]

After inserting \( (38, 40) \) into \( (39) \) we obtain
\[
C_{t t} = C_{\alpha \beta}^\gamma \cos 2h \sin^2 2J^t,
\]
where prefactors, \( C_{\alpha \beta}^\gamma \) are given by
\[
C_{y z}^{y z} = 1, \quad C_{x z}^{x z} = \tan^2 2h, \quad C_{y z}^{y z} = \frac{C_{y z}^{y z}}{C_{x z}^{x z}} = -\tan 2h
\]
while zeroes for all other spin combinations.

**Two-point correlators.**

By using the representation \( (37) \) we have for the correlator \( (21) \)
\[
C^{\alpha \beta}(t - 1, t) = \mu_2^{-2} \langle \Phi_{1 \sigma \sigma} \Phi_{2} | \Phi_2 \rangle \langle \Phi_2 | \Phi_{1 \sigma \sigma} \rangle + \langle \Phi_{1 \sigma \sigma} \Phi_{2} | \Phi_1 \rangle \langle \Phi_1 | \Phi_{1 \sigma \sigma} \rangle,
\]
where the vectors \( \Phi_{1 \sigma \sigma}, \Phi_{1 \sigma i} \) can be calculated by \( (15, 16) \). Explicitly, they are given by
\[
\Phi_{\sigma 1} = \frac{\cos 2J}{2}, \quad \Phi_{\sigma i 1} = \frac{\cos 2J}{2}, \quad \Phi_{\sigma 1} = -\frac{\sin(2h - 2J)}{\sin(2h + 2J)}, \quad \Phi_{\sigma 1} = \frac{\sin(2h - 2J)}{\sin(2h + 2J)}, \quad \Phi_{\sigma i 1} = 0,
\]
and \( \Phi_{\sigma 1} = \Phi_{1 \sigma \sigma}, \Phi_{\sigma i 1} = \Phi_{1 \sigma i}, \Phi_{\sigma 1} = \Phi_{1 \sigma \\). After substitution of \( (44) \) into eq. \( (43) \) one has
\[
C^{\alpha \beta}(t - 1, t) = C^{\alpha \beta} \cos 2h \sin^2 2J^t \cot^2 2J
\]
with the coefficients given by
\[
C_{x z}^{x z} = 1, \quad C_{x y}^{x y} = C_{x z}^{x z} = \tan 2h, \quad C_{z y}^{z y} = \tan^2 2h,
\]
and zeroes for all other \( \alpha, \beta \) combinations.