Kuhn Poker with Cheating and Its Detection

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Abstract—Poker is a multiplayer game of imperfect information and has been widely studied in game theory. Many popular variants of poker (e.g., Texas Hold’em and Omaha) at the edge of modern game theory research are large games. However, even toy poker games, such as Kuhn poker, can pose new challenges. Many Kuhn poker variants have been investigated: varying the number of players, initial pot size, and number of betting rounds. In this paper we analyze a new variant – Kuhn poker with cheating and cheating detection. We determine how cheating changes the players’ strategies and derive new analytical results.

Index Terms—game theory, Kuhn poker, cheating

I. INTRODUCTION

Poker is one of the most popular games studied among game theory researchers. The complex nature of the game provides challenging problems, however its complexity and size also make poker a difficult game to study. Kuhn poker, a toy poker game, can pose theoretical challenges at a much more manageable size. Multiple variants of Kuhn poker have previously been studied and optimal strategies determined; varying the number of players, the number of betting rounds, the initial pot size, etc [1]. A variant that has not been studied previously is the incorporation of cheating and its detection. What happens when one or both players cheat? How does cheating alter the players’ strategies?

II. PRELIMINARIES

A. Game Theory

Game theory is a theoretical framework for comprehending social interactions among competing players [2]. In some regards, game theory is the science of strategy: the optimal decision-making of independent and competing agents in a strategic setting. In game theory, the game itself serves as a model of an interactive decision making among rational players.

Game theory terms:

- Players: The game decision-maker
- Strategy: A player’s plan of action
- Payoff: The player payouts received from a particular outcome

III. INCORPORATING CHEATING AND ITS DETECTION

There are many ways in which cheating could occur in Kuhn poker. For example, a player could shuffle the deck, or otherwise arrange it, in such a way that all 6 two-card distributions are not equally likely. The cheating variation that we choose to analyze was having either player, or both players, look at the third/face-down card.

If either player looks at the face-down card, then that player knows who has the best hand. For the cheating player, it is a game of complete information. Note that cheating alone will not cause a player to win a hand – they may have a losing hand (e.g., the J). Note that bluffing against a cheating player will not be successful.

We analyzed several variations of our way of cheating:

1) Classic (no cheating by either player)
2) Player 1 (alone) cheats probabilistically
3) Player 2 (alone) cheats probabilistically
4) Players 1 and 2 both cheat probabilistically
5) Players 1 and 2 both cheat probabilistically and they both detect cheating probabilistically.

Probabilistic detection of cheating was also incorporated into the game. For example, “Only 10% of the time will player 1 determine if player 2 has cheated.” The reason for probabilistic detection is that there could be a cost associated with detection, so a player may only check for cheating some of the time. For a game that incorporates cheating detection, the rules differ slightly:

- If one player cheats and the other player detects it, then the round is played out per usual with betting and folding and then the detector wins, independent of the cards dealt or how the game played out.
- If both players cheat and both players detect that the other is cheating, then the round is played out per usual with betting and folding and then neither player wins, independent of the cards dealt or how the game played out. In this case the monies bet are returned.

### IV. Game Computation

#### A. Kuhn Poker Analytical Computation

For classical Kuhn poker there are a range of optimal strategies for player 1, all yielding the same optimal result [3]. While the strategy for player 2 is fixed, the optimal strategy for player 1 depends on a parameter "a", which can have any value between 0 and 1/3, inclusive. If player 1 is dealt a J, then player 1 bets (bluffs) with probability a; if player 1 is dealt a K, then player 1 bets with probability 3a. A complete description of both players’ optimal strategy is contained in Tables I and II; we call this the “fair strategy.”

Table III shows that the payoff to player 1 using the optimal strategy (including the value "a") is −1/18. Leftmost in Table III are the 6 equally possible ways that two cards can be dealt. Adjacent to that are the probabilistic amounts that each player can expect to win for that deal. We presume the first player never folds as their initial play, there is no benefit to this. The result shows that player 1 expects to win −1/18 (that is, a loss) each play of the game, and the result does not depend on the value of “a.”

Games 2 and 3 (defined in the last section) have one player cheating and the other player unaware of this cheating. In this case the natural question is “How much does the cheating player benefit?” The analysis is shown in

- Table IV where player 2 cheats 100% of the time, and player 1 plays their fair strategy
- Table V where player 1 cheats 100% of the time, and player 2 plays their fair strategy

where new optimal strategies were determined for the cheating player.

When player 2 cheats then player 1 expects to win −2/18 (that is, a loss) each play of the game. While the value of "a" did not make a difference in the fair game (with neither side cheating), it does make a difference when player 2 cheats. Hence, to minimize the loss against a possibly cheating opponent, player 1 should always play their optimal strategy with a = 0. When using a = 0, player 1 wins −2/3 (that is, a loss). This is a change (loss) of 11/18 compared to the fair game.
Alternately, player 1 could cheat and player 2 could be unaware of this. In this case (Table V), player 1 now expects to win \(7/18\) (that is, a gain) each play of the game. This is a change (increase) of \(8/18\) compared to the fair game.

We conclude that player 2 is more motivated to cheat than player 1 is. That is, the benefit to player 2, when player 2 cheats, exceeds the benefit to player 1, when player 1 cheats.

**B. Kuhn Poker Programmatic Computation**

The optimal strategy for Kuhn poker can also be solved programatically using software tools such as Gambit [4]. Gambit is an open source library of game theory tools for the construction and analysis of finite extensive and strategic games. The Gambit GUI displays the extensive tree form of the game as shown in Figure 1. Some information about this tree:

- The player actions are shown on the tree edges.
- The top/root node in green is a chance node, it has 6 links representing the 6 possible card dealings.
- Player 1 actions are in red, player 2’s are in blue.
- Figure 1 displays the information sets (ISs) as dotted lines – these are the states that a player cannot distinguish between with the available knowledge.
- Kuhn poker has 24 decision nodes, of which 12 are distinct after IS consideration.

![Fig. 1. Game tree representation in Gambit](image)

After the game was created in extensive form, Gambit solved it. In the fair case (no cheating), the result are the same as the solutions previously shown in Table I, with “a” equal to zero.

The way to implement our extensions to the classic version of Kuhn poker is to build on the tree in Figure 1. Specifically, a game in which only player 1 cheats probabilistically is shown in Figure 2. For this Figure:

- P1C is the path from the root node when player 1 does cheats (with probability \(p\)).
- P1N is the path from the root node when player 1 does not cheats (with probability \(1 - p\)).

![Fig. 2. Representation of Kuhn poker when player 1 cheats probabilistically](image)

There are lines drawn between the boxed images, these represent the IS connections. For example, player 2, who is not cheating, cannot determine from the information available to them if they are on the left or right branch of the tree in Figure 2. Hence, the actions taken by player 2 when player 1 bets, cannot be different. Stated differently, player 2 does not know if player 1 is cheating or not.

The analogous game when only player 2 cheats is to replace the labels in Figure 2 from “P1C(\(p\))” to “P2C(\(q\))” (that is, player 2 cheats with probability \(q\)) and from “P1N(1 - \(p\))” to “P2N(1 - \(q\))”. To implement a game where both players 1 and 2 cheat probabilistically, we merely need to extend the height of the game tree one more level, as shown in Figure 3.

![Fig. 3. Representation of Kuhn poker when players 1 and 2 both cheat probabilistically](image)

In this case, the ISs are connected across all four subfigures of the bottom layer, depending on what a player knows when it is their turn. Note that the same result is obtained if the player 2 options are followed by the player 1 options (as shown in Figure 3) or if the order is reversed and the first options are for player 1. Adding cheating detection is performed in the same manner as adding cheating.

Figure 4 adds cheating detection by player 1. In this case, player 1 detects cheating on the left branch (case “P1D” which occurs with probability \(r\)) and does not on the right branch (case “P1F” which occurs with probability \(1 - r\)). Once again, there are horizontal lines connecting the ISs. Additionally, when we incorporated cheating detection, we also changed the game payoffs. We implemented the rule.
that a player caught cheating always lost, even if the other player had the lower card. When both players cheat, it is a draw.

Fig. 4. Representation of Kuhn poker when both players cheat and player 1 can detect cheating

Unsurprisingly, we can also create the tree representing the game when each player cheats probabilistically and each player can also probabilistically detect cheating by the other player, as shown in Figure 5. We solved this game using Gambit, there are approximately 1000 nodes when both cheating and cheating detection were incorporated.

We used python to create the tree in Figure 5 and specify all the information sets. Gambit determined the optimal game strategy and the corresponding payoffs, using the linear program algorithm.

The Gambit calculations for the payoff to player 1, when only one player cheats, are shown in Tables VI and VII. Unlike the analytical calculations shown in Section IV-A, the programatic calculations include players adapting to cheating. For example, player 1 may not know that player 2 is cheating, but based on player 2’s actions, player 1 changes their strategy with respect to the fair play strategy. Therefore, the incorporation of cheating also influences the non-cheating player’s game play.

When player 2 cheats and player 1 adapts to the cheating, player 1 expects to win $-\frac{1}{18}$, this is a change (loss) of $\frac{1}{18}$ compared to the fair game. When player 1 cheats and player 2 adapts to the cheating, player 1 expects to win $\frac{5}{9}$, this is a change (gain) of $\frac{3}{18}$ compared to the fair game. In this case, when players adapt to cheating, player 1 is more motivated than player 2 to cheat. This finding is opposite of what was seen in Section IV-A where player 2 was more motivated to cheat.

V. Results

We investigated the effects of cheating and its detection on the 5 games described in section III. Game 1 is the classic, fair play, version of Kuhn poker. This version has been well studied, and our numerical results reproduce the known results for the parameter value $a = 0$.

Games 2–4 focus primarily on incorporating cheating into Kuhn poker. We analyzed these cases for cheating probabilities ranging from 0 (never cheat) to 1 (always cheat) for each player.

In this analysis, the game was fully adaptive; equivalently each player knew the probability of the other player’s cheating. The computational results are shown in Figure 6, for which the axes are the probability that each player cheats and the height is the expected payoff to player 1.

There are several observations:

1) The payoff surface has several bi-linear patches – we have not yet been able to analytically explain this.

2) The 4 marked corners are special cases, the values below are the expected payoff to player 1:
   a) Point 1: fair game, neither player cheats: $-\frac{1}{18}$
   b) Point 2: player 1 cheats, player 2 does not: $\frac{1}{9}$
   c) Point 3: player 2 cheats, player 1 does not: $-\frac{1}{9}$
   d) Point 4: both players cheat: 0

3) When both players cheat (Point 4), the payoff surface is flat and equal to zero for a range of cheat values for both players 1 and 2. For example, when both players cheat 90% of the time, the expected payoff is zero. It is surprising that if either player changes their likelihood of cheating from 90% to 89% or 91%, there is no change in the payoff.

| cards dealt | player 1 | player 2 | player 1 winnings | player 2 winnings |
|-------------|----------|----------|------------------|------------------|
| K J         | -\frac{1}{18} | 1 | 1 | \frac{1}{9} |
| Q J         | 1 | \frac{10}{9} | 1 | \frac{1}{9} |
| Q K         | 1 | \frac{5}{9} | 1 | \frac{1}{9} |
| J K         | \frac{1}{9} | 1 | 1 | \frac{3}{18} |
| J Q         | 1 | \frac{1}{9} | 1 | \frac{10}{9} |

Table VI: The value of the game to player 1 when player 2 cheats and player 1 adapts

| cards dealt | player 1 | player 2 | player 1 winnings | player 2 winnings |
|-------------|----------|----------|------------------|------------------|
| K J         | 1 | -\frac{1}{18} | 1 | 1 |
| K Q         | \frac{5}{9} | 1 | 1 | \frac{1}{9} |
| Q J         | 1 | \frac{1}{9} | 1 | \frac{1}{9} |
| Q K         | 1 | \frac{10}{9} | 1 | \frac{1}{9} |
| J K         | \frac{3}{18} | 1 | 1 | \frac{1}{9} |
| J Q         | \frac{1}{9} | 1 | 1 | \frac{1}{9} |

Table VII: The value of the game to player 1 when player 1 cheats and player 2 adapts
Fig. 6. The value of the game to player 1 when both players 1 and 2 are probabilistically cheating

Fig. 7. The value of the game to player 1 when both players are probabilistically detecting

Note that the payoff to player 1, when only one player cheats, is different from the payoff in Section IV-A. Section IV-A had one player cheating, while the other naively played fairly. In this case, there is full adaption to the cheating (“I know that you are cheating p% of the time.”) Game 5 incorporated both cheating and detecting cheating. We implemented multiple iterations of game 5, varying the probability of cheating and the probability of detecting cheating. The version of most interest was when both players always cheated and both players’ detection probabilities varied from 0 to 1. In this case, player 1 payoffs are shown in Figures 7. The axes represent the probability of each player detecting cheating and the payoffs are color coded (higher payoffs = yellow, lower payoffs = blue). These figures show that player 1 benefits more from detecting cheating than player 2 does. Even in a fair game, player 1 is at a disadvantage by betting first. Therefore, player 1 has more to gain by detecting cheating.

VI. Conclusion

Kuhn poker is a toy poker game involving two players and three cards; it is zero sum. Each player is dealt one card, while the third card is face down. Classic Kuhn poker is a game of imperfect information, each player only knowing their own card. In this paper we analyzed Kuhn poker when the game allowed cheating: one or both players peaked at the face down card.

In the analytical analysis, we assumed only one player was cheating and the other player was using the “fair” (non-cheating) strategy; this is the non-adaptive approach. In this case player 2 was more motivated to cheat than player 1. We also analyzed cheating in a fully adaptive situation, where the non-cheating player knew how likely the cheating player was to cheat. In this case, player 1 was more motivated to cheat than player 2. The main results are (the values below are the expected payoff to player 1):

1) When neither player cheats: $-\frac{1}{18}$
2) When player 1 cheats (alone, non-adaptive): $\frac{7}{18}$
3) When player 1 cheats (alone, adaptive): $\frac{2}{18}$
4) When player 2 cheats (alone, non-adaptive): $-\frac{12}{18}$
5) When player 2 cheats (alone, adaptive): $-\frac{2}{18}$
6) If both players cheat “enough”: 0

The incorporation of cheating detection within Kuhn poker was also analyzed. Either players could probabilistically detect if the other player was cheating. In this case, when one player always cheats and the other player detects cheating probabilistically, player 1 gains more by detecting than player 2 does. This is due to the fact that Kuhn poker is asymmetric, player 1 is at a disadvantage by having to bet first.

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