Charge-to-spin conversion of electron entanglement states and spin-interaction-free solid-state quantum computation

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Without resorting to spin-spin coupling, we propose a scalable spin quantum computing scheme assisted with a semiconductor multiple-quantum-dot structure. The techniques of single electron transitions and the nanostructure of quantum-dot cellular automata (QCA) are used to generate charge entangled states of two electrons, which are then converted into spin entanglement states using single-spin rotations only. Deterministic two-qubit quantum gates are also manipulated using only single-spin rotations with the help of QCA. A single-shot readout of spin states can be carried out by coupling the multiple dot structure to a quantum point contact. As a result, deterministic spin-interaction-free quantum computing can be implemented in semiconductor nanostructure.

I. INTRODUCTION

Using electron spins to implement quantum information and quantum computation in semiconductor nanostructure, as one of the most important technology developments in spintronics, has received tremendous attention in recent years. Prototypical quantum computation schemes based on electron spins have been proposed using gate voltage controlled1,3 and optically driven4,5,6,7,8 spin-spin coupling in semiconductor quantum dots. However, achieving a tunable spin-spin interaction with a sufficiently large strength (compared to the strength of the charge Coulomb interaction) is technically difficult. A spin-interaction-free mechanism for logical operations on electron spins is therefore more desirable. Many interaction-free schemes on measurement based quantum computing have recently been proposed9,10,11,12,13,14, but a robust, deterministic, and scalable spin-interaction-free solid-state quantum computing scheme in semiconductor nanostructure has yet to emerge.

In this paper, we propose an implementation of scalable spin quantum computation in semiconductor nanostructure without resorting to spin-spin coupling. We can generate a charge entangled state of two electrons using single electron transitions15,16 assisted by a semiconductor multiple-quantum-dot structure consisting of two double dots, called the quantum-dot cellular automata (QCA)17,18. The charge entangled state is then converted into a spin entangled state using only single-spin rotations. Spin-spin interaction is not required in this implementation, and deterministic two-qubit controlled gates can be easily manipulated as well. Therefore, deterministic and scalable spin-interaction-free quantum computation can be implemented in semiconductor nanostructure.

II. ARCHITECTURE OF THE SPIN-INTERACTION-FREE QUANTUM COMPUTER

The architecture of our scalable quantum computer is based on a semiconductor multiple-quantum-dot structure schematically shown in Fig. 1. Each shaded squared box in Fig. 1 is regarded as a unit cell. Each cell contains a qubit dot (the central black dot) surrounded by four ancilla dots (the white dots). The detailed structure of a unit cell is given in Fig. 2(a). The lines between quantum dots in the cell indicate the possibility of inter-dot transitions. We assume that each unit cell is charged with only one excess conduction electron19. The electrostatic potential energy ($\varepsilon_q$) of the excess electron in the qubit dot is low enough compared to the energy ($\varepsilon_a$) in the ancilla dots such that the electron sits initially in the qubit dot due to the Coulomb blockade effect [see Fig. 2(b), where $\varepsilon = \varepsilon_a - \varepsilon_q$]. Explicitly, we define quantum states of the excess electron in the qubit dot as $|S_i\rangle|e_i\rangle$ ($i = 1, 2, \cdots$), the charge states $|e_i\rangle$ are consid-
eroded as ancilla states, and the spin states $|S_i\rangle$ are chosen to be qubit states in Pauli basis, $|\uparrow\rangle = |0\rangle$ and $|\downarrow\rangle = |1\rangle$. A static uniform magnetic field can be applied to split the qubit states $|0\rangle$ and $|1\rangle$ by the Zeeman energy for qubit initialization.

Furthermore, the four ancilla dots within a unit cell are coupled to the qubit dot through gate voltages. The electron in each cell can be driven away from the qubit dot into ancilla dots only when a two-qubit controlled operation is performed, and will be forced to transit back as soon as the two-qubit operation has been completed. Such transitions are controlled using gate voltages $V_{i}^{LR}$, $V_{i}^{TB}$. For instance, by turning on the gate voltage $V_{i}^{LR}$, the electron will transit to a certain site ($C_i$ or $D_i$) of the right two ancilla dots [see Fig. 2(c)]. We denote the charge states of the electron sitting in the ancilla dots $A_i$, $B_i$, $C_i$, and $D_i$ as $|e_i^a\rangle$ with $a=A,B,C$ and $D$, respectively. The site dependence of electron spin state is negligible in this architecture. The effective Hamiltonian for the electron transition between the qubit dot and ancilla dots can take the form

$$H_i = \varepsilon(t)(|e_i^a\rangle\langle e_i^a| - |e_i\rangle\langle e_i|) + \Delta(|e_i^a\rangle\langle e_j^a| + |e_i\rangle\langle e_j^a|),$$

where $\varepsilon(t) = \varepsilon - V_{i}^{LR}(t)$ and $\Delta$ a tunneling coupling.

![FIG. 2: (a) The nanostructure of the unit cell $i$. The gate voltages $V_{i}^{LR}$ and $V_{i}^{TB}$ control the electron transitions among dots inside the cell, (b) and (c) show the electrostatic potential energies of the electron at different dots in the cell, without and with applying a gate voltage $V_{i}^{LR}$, respectively.](image)

Based on the above architecture, one can find that the two double-dot pairs (e.g., $C_iD_i$ and $A_jB_j$ in Fig. 3) between the qubit dots of two neighboring cells form a QCA. QCA was originally proposed as a transistorless alternative to digital circuit devices at nanoscale. Recently, semiconductor QCA has been fabricated from GaAs/AlGaAs heterostructures and from buried dopants as well. Due to the Coulomb repulsion, when a QCA is charged with two electrons, the excess electrons in the two cells are transited into the QCA consisting of the double-dot pairs $C_iD_i$ and $A_jB_j$, and occupy one of the two polarized states due to the Coulomb repulsion. To be specific, let the two electrons occupy the polarized state $|\uparrow\rangle = |e_i^C e_j^B\rangle$, that is,

$$\Psi_0 = |S_iS_j\rangle|e_i^C e_j^B\rangle.$$

By turning on a positive voltage $V_{i}^{LR}$ and a negative voltage $-V_{i}^{LR}$ to lower the on-site energy of the dots $C_iD_i$ and $A_jB_j$, the excess electrons in the two cells are transited into the QCA, and the two polarized states controlled by gate voltages acting on the two double-dot pairs. The electron tunneling between different cells is forbidden by a built-in sufficiently high energy barrier between the two neighboring cells.

![FIG. 3: Quantum mechanically, the four quantum dots (the dotted square boxes) between two qubit dots of the neighboring cells form a coherent QCA.](image)

### III. CHARGE-TO-SPIN CONVERSION OF ELECTRON ENTANGLEMENT STATES

A key to manipulate two-qubit controlled operations in this architecture is the charge-to-spin conversion of two-electron entanglement states. We shall use the QCA structure to generate a charge entangled state via single electron transitions and then convert it into a spin-entangled state using single-spin rotations only. Explicitly, consider a pair of neighboring unit cells, e.g. the $i$th and the $j$th cells (see Fig. 3). The initial state of the two excess electrons is given as

$$\Psi_0 = |S_iS_j\rangle|e_i^C e_j^B\rangle.$$

This manipulation is reliable in current experiments with passage time of a few tens of picoseconds or less. A numerical simulation of electron transitions based on Eq. (1) is presented in Fig. 4 (also refers to). To generate a charge entangled state through the QCA structure, we may adjust the external bias $E_{bias}$ to make the two polarized states degenerate (i.e., $E_{bias} = -\omega$), and then apply a gate voltage pulse (with a pulse time
At the dots $|\Psi_2\rangle = U^{DA}\Psi_1\rangle = e^{-iH_{QCA}\tau_p}|\Psi_1\rangle = |S_iS_j\rangle - \frac{1}{\sqrt{2}}(e^{C_jB_j} - i|e^{D_jA_j}\rangle). \quad (5)

This superposition state is indeed a maximally entangled charge state. Note that the current technique enables us to have nearly identical dots for electronic tunneling \cite{27,28}. As long as no observable difference reflected in tunneling, the dots in our design are not required to be fully identical. Meanwhile, if the pulse duration $\tau_p$ (in picoseconds, see the discussion later) can be accurate to femtoseconds, a very high fidelity of Eq. (5), see the discussion later) can be accurate to femtoseconds and picoseconds and femtoseconds \cite{27,28}. Explicitly, consider the initial spin state of the two electrons, $|S_iS_j\rangle = |\uparrow\uparrow\rangle$ \cite{27,28}. Applying two spin rotations on the electrons sitted at the dots $D_i$ and $A_j$, respectively, $U^{DA} = R^D_x(\theta) \otimes R^D_z(\varphi)$, where $R_k(\theta) \equiv \exp(-i\theta\sigma_k/2), k = x, y, z$, the corresponding two-electron spin state at the dots $D_i$ and $A_j$ becomes $U^{DA}|01\rangle|e^{C_jB_j}\rangle = -i|10\rangle|e^{D_jA_j}\rangle$, while the spin state at the dots $C_i$ and $B_j$ remains unchanged: $U^{DA}|01\rangle|e^{C_jB_j}\rangle = |01\rangle|e^{C_jB_j}\rangle$. After the spin rotation operations, we turn off the gate voltages $V^{LR}_i$ and $-V^{LR}_j$.

The two electrons in the QCA are transited back into the qubit dots $i$ and $j$, namely, the electron charge states return to the initial states,

$$|\Psi_3\rangle = U^{DA}_S|\Psi_2\rangle = \frac{1}{\sqrt{2}}([10]|e^{C_jB_j}\rangle - |01\rangle|e^{D_jA_j}\rangle)$$

$$= (V^{LR}_i - V^{LR}_j)_{\text{off}} \frac{1}{\sqrt{2}}([01] - |10\rangle)|e_je_j\rangle. \quad (6)$$

As a result, the electron charge entangled state has been converted completely into the spin entangled state, the Bell state $|\psi^\rangle$. This is the implementation of charge-to-spin conversion of a two-electron entanglement state. Repeating the process of Eqs. (5) with different initial spin states $|10\rangle, |00\rangle$, and $|11\rangle$, we obtain

$$|\Psi_3\rangle = U^{DA}_S U^{QCA}\tau_p |\Psi_0\rangle = UC_{\text{CNOT}}|\Psi_0\rangle. \quad (8)$$

It is easy to check that Eq. (8) gives explicitly the CNOT gate as an example. Instead of using the spin rotation $U^{DA}_S$ in Eq. (6), we should apply a single spin rotation on each dot $C_i, D_i, A_j, and B_j$ in the QCA with the rotation operator $U^{DA}_{C} = R^C_x(\pi) \otimes R^C_y(\pi) \otimes R^C_z(\varphi)$ to rotate the corresponding electron spin state in $|\Psi_2\rangle$. Using the same process of Eqs. (5) with the above replacement of the spin rotations, we have

$$|\Psi_3\rangle = (V^{LR}_j - V^{LR}_i)_{\text{off}} \frac{1}{\sqrt{2}}([01] - |10\rangle)|e_je_j\rangle.$$
otherwise, the electron will transit to the dots $C_i$ and $D_i$ and then return to the qubit dot after the pulse if it is in the state $|1\rangle$. Thus, one can read-out qubit states by measuring charge current changes through the QPC channel. Such a measurement has been experimentally demonstrated in semiconductor dots and theoretically studied extensively.

The possible imperfection in the above manipulation may come from decoherence and inaccurate operations on electron states. The effect of gate voltage pulses on the electrically floating double dots involves an abrupt change in the confinement potential and modifies $\Delta, \omega$ as well as $\varepsilon, \gamma$ in Eqs. (1,2). Combining with the noises from the electron-phonon interaction and piezoelectric coupling results in a typical charge decoherence time $T_2 \sim 1 - 10$ ns in GaAs dots. The decoherence could not affect the transition of Eq. (1) since the tunneling only occurs near the resonance region $\varepsilon(t) \sim 0$ in picoseconds (see Fig. 4). For the operation in (3), the tunneling coupling inside the QCA should be controlled with the requirement, $Ke^2/r < \gamma < Ke^2/d$, where $K = 6.9 \times 10^4 N \cdot m^2 / C^2$ for GaAs, $e$ is the electron charge, and $r = \sqrt{2d}$ with $d$ the spacing between $A_j$ ($C_i$) and $B_j$ ($D_i$). Direct calculation for $d \sim 50 - 100$ nm shows that $\gamma$ is of the order of terahertz. Accordingly the operation in (3) could also be done within picoseconds, much shorter than the charge decoherence time in double dots.

Since the charge decoherence time is only a few nanoseconds short, a very fast and elaborately operated single-spin rotation is required for single spin rotations in Eqs. (4) and (5). Recent experiments demonstrated that optical tipping pulses with a frequency below the band gap of the semiconductor nanostructure can create an effective magnetic field in the order of 20 T via the optical Stark effect, which can induce substantial rotations of electron spins at the femtosecond scale. Meanwhile, spin-flip Raman transitions using the adiabatic process of two ultrafast laser pulses can also fully control single-spin rotations in semiconductor quantum dots at picosecond or femtosecond scale. These optical controls of single spin rotations are technically very supportive for a practical implementation of the present scheme.

As an overall decoherence analysis, recent measurements in GaAs and In(Ga)As quantum dots have shown a long spin relaxation time ($T_1 \sim 1 - 20$ ms), the spin decoherence time ($T_2$) in GaAs dots caused by the complicated nuclear spin fluctuation is about $10^{-11}$-s. A lower bound on the spin coherence time exceeding 1 ms has also been established using spin-echo techniques on two electrons in double dots, while the charge decoherence time is a few ns and maybe up to 200 nanoseconds in isolated silicon double dots. If the manipulation of single electron transitions and single-spin rotations can be completed within picoseconds to femtoseconds, the implementation of spin-interaction-free quantum computation with quantum error correction is reliable in experiments.

V. SUMMARY

In summary, without resorting to spin-spin coupling, we have proposed a deterministic and scalable spin quantum computing scheme assisted by a semiconductor multiple-quantum-dot structure. Spin-interaction-free solid-state quantum computing is a big challenge, in principle. In this scheme, we are able to achieve such an implementation basically relying on the charge-to-spin conversion of electron entanglement states with the help of the QCA. The QCA structure offers an intrinsic charge coupling of two electrons, which is more effective, completely deterministic, and scalable in comparison with the measurement based quantum computing scheme in semiconductor nanostructure. Since spin couplings are much weaker than the charge Coulomb interaction, such a spin-interaction-free quantum computing has the advantage of being robust against the technical difficulties of electronically or optically generating tunable spin-spin couplings. The present scheme only involves gate voltage controls of electron transitions and the optical manipulation of spin coherence in semiconductor dots. These techniques are currently reliable in experiments. Therefore, the spin-interaction-free quantum computing can be realized practically in semiconductor nanostructure.

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1 D. Loss and D. P. DiVincenzo, Phys. Rev. A 57, 120 (1998).
2 B. E. Kane, Nature (London) 393, 133 (1998).
3 G. Burkard, D. Loss, and D. P. DiVincenzo, Phys. Rev. B 59, 2070 (1999); D. P. DiVincenzo, D. Bacon, J. Kempe, G. Burkard, and K. B. Whaley, Nature (London) 408, 339 (2000).
4 A. Imamoglu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. 83, 4204 (1999).
5 C. Piermarocchi, P. Chen, L. J. Sham, and D. G. Steel, Phys. Rev. Lett. 89, 167402 (2002).
6 F. Troiani, E. Molinari, and U. Hohenester, Phys. Rev. Lett. 90, 206802 (2003).
7 E. Pazy, E. Biolatti, T. Calarco, I. D’Amico, P. Zanardi,
F. Rossi and P. Zoller, Europhys. Lett. 62 175 (2003); M. Feng, I. D’Amico, P. Zanardi, and F. Rossi, Europhys. Lett. 66, 14 (2004).
8 A. Nazir, B. W. Lovett, S. D. Barrett, T. P. Spiller, and G. A. D. Briggs, Phys. Rev. Lett. 93, 150502 (2004); B. W. Lovett, A. Nazir, E. Pazy, S. D. Barrett, T. P. Spiller, and G. A. D. Briggs, Phys. Rev. B 72, 115324 (2005).
9 R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. 86, 5188 (2001).
10 E. Knill, R. Laflamme, and G. J. Milburn, Nature 409, 46 (2001).
11 C. W. J. Beenakker, D. P. DiVincenzo, C. Emary, and M. Kindermann, Phys. Rev. Lett. 93, 060501 (2004).
12 H. A. Engel and D. Loss, Science 309, 586 (2005).
13 Y. L. Lim, A. Beige, and L. C. Kwek, Phys. Rev. B 72, 115324 (2005).
14 W. G. van der Wiel, S. De Franceschi, J. M. Elzerman, T. Fujisawa, S. Tarucha, and L. P. Kouwenhoven, Rev. Mod. Phys. 75, 1 (2003).
15 T. Fujisawa, T. Hayashi, and S. Sasaki, Rep. Prog. Phys. 69, 759 (2006).
16 C. S. Lent, P. D. Tougaard, W. Porod, and G. H. Bernstein, Nanotechnology 4, 49 (1993); P. D. Tougaard and C. S. Lent, J. Appl. Phys. 75, 1818 (1994).
17 A. O. Orlov, I. Amlani, G. H. Bernstein, C. S. Lent, and G. L. Snider, Science 277, 928 (1997); I. Amlani, A. O. Orlov, G. Toth, G. H. Bernstein, C. S. Lent, G. L. Snider, ibid. 284, 289 (1999).
18 M. Ciorga, A.S. Sachrajda, P. Hawrylak, C. Gould, P. Zawadzki, S. Jullian, Y. Feng, and Z. Wasilewski, Phys. Rev. B 61, R16315 (2000); J. M. Elzerman, R. Hanson, J. S. Greidanus, L. H. Willems van Beveren, S. D. Franceschi, L. M. K. Vandersypen, S. Tarucha, and L. P. Kouwenhoven, ibid. 67, 161308 (2003).
19 J. Gorman, D. G. Hasko, and D. A. Williams, Phys. Rev. Lett. 95, 090502 (2005).
20 T. Hayashi, T. Fujisawa, H. D. Cheong, Y. H. Jeong, and Y. Hirayama, Phys. Rev. Lett. 91, 226804 (2003).
21 S. Gardelis, C. G. Smith, J. Cooper, D. A. Ritchie, E. H. Linfield, and Y. Jin, Phys. Rev. B 67, 033302 (2003).
22 J. H. Cole, A. D. Greentree, C. J. Wellard, L. C. L. Hollenberg, and S. Prawer, Phys. Rev. B 71, 115302 (2005).
23 G. Toth, and C. S. Lent, Phys. Rev. A 63, 052315 (2001).
24 M. Forre, J. P. Hansen, V. Popsleva, and A. Dubois, Phys. Rev. B 74, 165304 (2006).
25 M. Xiao, I. Martin, E. Yablonovitch and H. W. Jiang, Nature 430, 435 (2004); Y. Kato, R. C. Myers, D. C. Driscoll, A. C. Gossard, J. Levy, D. D. Awschalom, Science 299, 1201 (2003).
26 N. V. Vitanov, T. Halfmann, B.W. Shore, and K. Bergmann, Annu. Rev. Phys. Chem. 52, 763 (2001).
27 J. A. Gupta, R. Knobel, N. Samarth, D. A. Awschalom, Science 292, 2458 (2001).
28 J.M. Elzerman, R. Hanson, L. H. Willems van Beveren, B. Witkamp, L. M. K. Vandersypen and L. P. Kouwenhoven, Nature 430, 431 (2004); R. Hanson, B. Witkamp, L. M. K. Vandersypen, L. H. W. van Beveren, J. M. Elzerman, and L. P. Kouwenhoven, Phys. Rev. Lett. 94, 196802 (2005).
29 S. A. Gurvitz, Phys. Rev. B 56, 15215 (1997).
30 M.T. Lee and W.M. Zhang, Phys. Rev. B 74, 085325 (2006).
31 P. Chen, C. Piermarocchi, L. J. Sham, D. Gammon, and D. G. Steel, Phys. Rev. B 69, 075320 (2004).
32 T. Fujisawa, D. G. Austing, Y. Tokura, Y. Hirayama, and S. Tarucha, Nature 419, 278 (2002); M. Kroutvar, Y. Ducommun, D. Heiss, M. Bichler, D. Schuh, G. Abstreiter and J. J. Finley, ibid. 432, 81 (2004).
33 F.H.L. Koppens, J. A. Folk, J. M. Elzerman, R. Hanson, L. H. W. van Beveren, I. T. Vink, H. P. Tranitz, W. Wegscheider, L. P. Kouwenhoven, and L. M. K. Vandersypen, Science 309, 1346 (2005); M.V. Gurudev Dutt, J. Cheng, B. Li, X. Xu, X. Li, P. R. Berman, D. G. Steel, A. S. Bracker, D. Gammon, S. E. Economou, R. B. Liu, and L. J. Sham, Phys. Rev. Lett. 94, 227403 (2005).
34 J. R. Petta, A. C. Johnson, J. M. Taylor, E. A. Laird, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Science 309, 2180 (2005).