ON THE SHAPE OF CORE OVERSHOOTING IN STELLAR MODEL COMPUTATIONS, AND ASTEROSEISMIC TESTS

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Abstract. Slowly pulsating B stars (SPB) and γ Dor stars pulsate in high-order gravity (g-) modes. The frequencies of g-modes are sensitive to the detailed structure and evolution history of stars having convective cores. Receding convective cores in OB-type stars leave behind a chemically inhomogenous $\nabla \mu > 0$ radiative zone. Once a g-mode has radial nodes near the boundaries of these layers, the mode gets trapped and its period deviates from asymptotic period spacing. Careful study of such trapped modes allows constraining the extent of such layers by fitting individual pulsation frequencies. We employ 19 consecutive dipole g-modes of a very rich Kepler SPB pulsator, KIC 10526294, to demonstrate the power of mode trapping in B-stars in studying the thermal and chemical stratification in the overshooting layer.

1 Introduction

Once OB-type stars reach the zero-age main sequence (ZAMS), they immediately develop fully mixed convective cores, which already attains the largest possible mass and extent at ZAMS. During the evolution, the drop in center hydrogen ($X_c$) is accompanied by the decrease in radiative temperature gradient $\nabla_{\text{rad}}$. Consequently, the convective core shrinks monotonically. This leaves behind CNO enriched material and a steep increase in hydrogen abundance outside the boundary. Therefore, $\nabla_{\mu} = (\partial \ln \mu / \partial \ln p) > 0$; here, $\mu$ is the mean molecular weight, and $p$ is the total pressure. The boundary layer mixing induced by the overshoot of convective plumes into the stable radiative layers can partially mix the hydrogen, helium and metals outside the core, and reshape the $\nabla_{\mu}$ profile there.

The problem of the boundary layer mixing is still not understood well. Recently, Viallet et al. (2015) proposed that the overshooting mixing depends on the local Péclet number – defined as the local ratio of the thermal to viscous...
timescales – in the overshooting layer: Closer to the core boundary, the convective plumes are predicted to be quasi-adiabatic, increasing the size of the fully mixed core. Further away, the Péclet number drops steeply, and the overshooting layer becomes photon-dominated, and the mixing proceeds diffusively. These two influence the shape of $\nabla_\mu$ just outside the core, and it propagates directly into the pulsation equations through the definition of the Brunt-Väisälä frequency $N_{BV}^2$

$$N_{BV}^2 = \frac{g\delta}{H_p} \left( \nabla_{ad} - \nabla + \frac{\phi}{\delta} \nabla_\mu \right), \quad (1.1)$$

where $H_p$ is the local pressure scale height, $\nabla = (\partial \ln T / \partial \ln p)$ is the actual temperature gradient $\nabla_{ad}$ is the adiabatic temperature gradient, $\delta = (\partial \ln \rho / \partial \ln T)_{P,\mu}$ and $\phi = (\partial \ln \rho / \partial \ln \mu)_{P,T}$. The local thermal and compositional stratifications are encapsulated in $\nabla$ and $\nabla_\mu$ in Eq. (1.1).

The Brunt-Väisälä frequency is explicitly present in the adiabatic and non-adiabatic linearized oscillation equations (Unno et al. 1989; Aerts et al. 2010). Thus, even a slight modification to the $\nabla_\mu$ by mixing (through overshooting or extra diffusive mixing) in the radiative part of the star influences the eigenfunctions and eigenfrequencies, and exhibits measurable fingerprints in period spacing $\Delta P = P_{n+1} - P_n$, where $P_n$ is the period of a dipole g-mode of radial order $n$. Asteroseismology of heat-driven g-mode pulsators opens a direct window to study the physical properties of the fully-mixed core, and the partially homogeneous layer on top of the core (Miglio et al. 2008).

Recently, Pápics et al. (2014) studied the SPB star KIC 10526294, and identified 19 dipole g-modes from the triplet structure around each mode. Moravveji et al. (2015, MAP15) carried out a detailed forward seismic modelling of the dipole zonal frequencies, and Triana et al. (2015) inferred the internal rotation profile by inverting its rotational splittings. We demonstrate how trapping of high radial order g-modes in the $\nabla_\mu$-layer is used to confine the size of this layer. We also argue that our current treatment of the overshoot mixing requires a major revision.

2 Simplified picture of mixing in B stars

MAP15 compared the exponentially decaying ($D_{ov} = D_{conv} \exp[-2(r-r_{core})/f_{ov}H_p]$) and the step-function ($D_{ov} = D_{conv}$ for $r_{core} \leq r \leq \alpha_{ov}H_p$) overshooting prescriptions in their extensive seismic modelling of KIC 10526294. Note that $\alpha_{ov} \approx 10f_{ov}$. The mixing induced by convective overshoot is assumed diffusive (Zhang 2013; Viallet et al. 2015) with an exponentially decaying behaviour (Herwig 2000) (Fig.1a) or step-function behaviour (Fig.1b). We also assume that the thermal stratification in the overshoot layer is purely radiative $\nabla = \nabla_{rad}$. Clearly, $N_{BV}^2$ (red dashed) shows a steeper rise in the step-overshoot model. Fig.1c shows the observed versus modelled $\Delta P$, where the first five modes agree very well with observations, and the other thirteen deviate significantly from the fit. We argue the reason for this is our reasonable ability in confining the width of the $\mu$-gradient layer, and at the same time our poor understanding of the thermal and chemical stratification in the overshooting layer.
The overshooting properties of KIC 10526294

Fig. 1. The convective (blue), overshooting (grey) and extra diffusive mixing (orange) profiles in the best models of KIC 10526294, using exponential (a) and step-function (b) overshooting prescriptions. See Table 4 in MAP15 for the mass and extent of each zone. The dashed lines show the Brunt-Väisälä $N_{BV}$ (Eq.1.1) profiles. (c) The observed (grey) versus modelled (black) period spacing from the best seismic model (with $f_{ov} = 0.017$).

3 Mode trapping

Fig. 2 presents the behaviour of the rotation kernels $K_{n,\ell}$ (defined in Aerts et al. (2010), Eq. 3.356) versus enclosed mass. The bottom panels show the run of the Brunt-Väisälä frequency $N_{BV}$, the Lamb frequency $S_{\ell}$ and the mode frequency in logarithmic scale. The $\mu$-gradient zone corresponds to the local peak in $N_{BV}$ around 0.85 $M_\odot$. The shortest period mode (left) is perfectly trapped in the $\mu$-gradient layer, and has a sizable amplitude there (Dziembowski and Pamyatnykh, 1991). The longest period mode (right) still has a large amplitude in the $\mu$-gradient layer, in addition to a significant amplitude in the diffusive overshooting layer below the $\mu$-gradient zone. Thus, the eigenfunctions of the latter mode are sensitive to the treatment of overshooting (Figs 1a and 1b). The frequencies and period spacings associated to highest-order g-modes of KIC 10526294 (Fig. 1c) do not perfectly match the observation. This manifests that our underlying assumptions about the thermal and compositional structure of the overshooting layer build in 1D stellar structure codes need a major revision.

The current assumption in 1D stellar models is that the temperature gradient $\nabla$ changes abruptly from fully adiabatic in the convective core to fully radiative just above the convective core, where the convective plumes overshoot into the radiative regions. However, this picture is not supported by 3D simulations of Viallet et al. (2013) (although performed for the conditions of deep convection in red giant envelopes). It is possible that the behaviour of the temperature gradient in the overshooting layer is quasi-adiabatic, up to some pressure scale heights from the core boundary, where it efficiently mixes the materials and adds to the size of the fully mixed core. Such a prescription is similar to the classical step-function overshoot. An extension of this quasi-adiabatic layer can be photon-dominated, and an exponentially decaying diffusive mixing can be applicable there. Therefore, a more complicated 1D stellar evolution models should be built, hopefully
Fig. 2. The rotation kernels for the shortest period mode $n = -14$ (Left) and for the longest period mode $n = -32$ (Right) chosen from the best model of KIC 10526294. The profile of the Brunt-Väisälä frequency $N_{BV}$ and the Lamb frequency are shown to guide the eyes. The convective core is highlighted in blue.

with improved treatment of convection (Zhang [2013], Arnett et al. [2015]) to provide better inputs for asteroseismic inferences. Indeed, the one-to-one frequency matching with observations can discriminate between the most plausible convective/overshoot prescription.

Acknowledgement. EM thanks the support from the Belgian Federal Science Policy Office (BELSPO) grant n°246540, the Marie Curie IIF grant n°623303, and the Research Council of KU Leuven grant GOA/2013/012.

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