Experimental investigations of density-induced segregation in binary granular mixtures under vertical vibrations

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Abstract. The objective of this work is aimed to investigate experimentally the influences of density ratio on segregation of cylindrical granular mixtures under vertical vibrations. Polytetrafluoroethylene, Polyamide, Polyethylene terephthalate, Balsa woods and Hinoki woods were used as the cylindrical particles, with 10 mm in diameter and 60 mm in length. The granular sample is composed of 1000 particles with two different materials with each type of 500 particles. All particles are periodically filled into the rectangular container with size 300 x 300 x 60 mm, as the initial condition. Density ratios are 2.73, 3.36, 5.27, and 13.63. The experimental operated by vertical shaker for 10 minute to achieve a steady state condition with same amplitude at 5 mm. The dimensionless vibration accelerations changed by frequency for 5 experimental, experiment the dimensionless vibration accelerations = 1, 2, 3, 4, and 5. The beginning state and the end state collected data by shooting the image. The segregation is considered by average height or the segregation coefficient. The result showed that for the density ratios of 2.73 and 3.36 for all dimensionless acceleration values, the segregation does not occur. On the other hand, for density ratio more than 5.27, the segregation occur by the way that the higher density particles move downward to the bottom of the sample, while the lower density particles move up to the top of the container. The segregation occurs when the dimensionless acceleration is more than 2 and the segregation is clearer when the dimensionless acceleration increases.

1. Introduction
Granular materials are omnipresent that can be found in our everyday lives and in nature. For example, sand, soil, pebble, floating dust in the air, and many layers of rock that overlaps below the crust, etc. These materials generally exhibit complex behaviors, i.e. sometimes behave like fluid and sometimes behave like solid. Such complex behaviors of granular materials are caused by the fact that these materials are constituted from grains or powders with a variety of size, shape, density, friction, and other physical properties. Several behaviors of granular media can be found in many industries, for instance a compaction of sand for a casting in the metal industry, a flow of grains inside the silo in the food industry, and a mixing in the manufacturing process of medicines in the pharmaceutical industry, etc. The other one interesting behavior is the segregation of materials. One of the well-known segregation phenomena is the Brazil Nut, effect [1-3]. This phenomenon provides the upward movement of the larger particles under vibrated granular bed. In practice, there are several factors inducing segregation, such as size, density, surface roughness [1-5]. Therefore, it can be seen that the segregation phenomena due to such factors are quite important in many industrial and engineering fields.
In many industries, the vibration is employed for several applications such as mixing, surface scrubbing, transport, and segregation. Particularly in the segregation, it is used for a qualification of the product, e.g., the product that does not have a required weight can be separated by means of vibration. It can be said that the density and the vibration are interesting factors to be investigated. Nevertheless, previous experimental and simulation studies regarding the density-induced segregation have been mostly performed with the granular mixtures composed of different particle sizes: Hong et al. [6] proposed a segregation diagram for the crossover from Brazil nut effect to reverse Brazil nut effect for the binary mixtures by means of molecular dynamics simulations. The similar segregation diagram for different vibration amplitudes was also proposed by Ciamarra et al. [7]. It can be seen that an unclear effect between density and size has been encountered. In the present work, we therefore aim at investigating the influence of density-induced segregation in binary granular mixture with same particle size subjected to different vertical vibrations.

2. Experimental setup

The cylindrical particles were made of Polytetrafluoroethylene, Polyamide, Polyethylene terephthalate, Ultra-high-molecular-weight polyethylene, Balsa woods and Hinoki woods. The particle diameter and length were about 10 mm and 60 mm [11], respectively. The tested configurations are presented in table 1. The granular mixtures consisted of 1000 particles, which were mixed between two species of density. Each species was composed of 500 particles. The particles were then periodically filled into the container, as shown in figure 1.

| Density ratio | Materials | Dense (Density in g/cm³) | Loose (Density in g/cm³) |
|---------------|-----------|--------------------------|--------------------------|
| 2.73          | 100       | 23.56                    |                          |
| 3.36          | 150       | 34.64                    |                          |
| 5.27          | 200       | 23.76                    |                          |
| 13.63         | 250       | 27.9                     |                          |

Figure 1. Example of a periodical filling into the container at an initial stage.

A vibrational testing machine designed for the present study is shown in figure 2. Note that this machine was completely installed on a standard base for the vibrational testing. The testing machine was consisted of an aluminum container covered by an acrylic sheet. The inside dimensions of the
container are 300x300x60 mm. The container was connected to a scotch yoke mechanism used for converting the rotational movement into the linear movement, i.e. a vertical sine wave. The vibration was provided by an electric motor, which was connected to an electronic inverter. This electronic inverter was employed to systematically adjust the vibrational frequency for specifying the dimensionless vibrational accelerations ($\Gamma$).

The degree of vibration use the dimensionless vibrational acceleration ($\Gamma$) was defined as $\Gamma = \frac{r \omega^2}{g}$, where $g$ is the gravitational force, $r$ is the vibrational amplitude, $\omega = 2\pi f$ is the vibrational frequency. In this study, the vibrational amplitude was fixed at 5 mm, while the vibrational frequencies were progressively changed in order to obtain the values of $\Gamma = 1$-5. The container was subjected to a vertical vibration under sinusoidal waves.

![Figure 2. Drawing of a vibrational testing machine.](image)

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This experiment was taken a photo at initial stage and final stage at 10 minutes. All photos were used open-source software named “ImageJ” [12] for finding vertical positions of each centroid particle. Giving the $z_i$ is the height of particle $i$ with the distance from the bottom container to the vertical final position and $N_t$ is the total number of the all particles inside the container, the average height of particle can be determined by

$$z_a = \frac{1}{N_t} \sum_{i=1}^{N_t} z_i$$  \hspace{1cm} (1)

Segregation indicator used in this study is the segregation coefficient ($H$). This coefficient is employed to statistically characterize the degree of segregation of the binary mixture in a static stage, which is given by

$$H = 2 \left( \frac{z_a - z_b}{z_a + z_b} \right)$$  \hspace{1cm} (2)

where $z_a$ is the average height of lower density particles and $z_b$ is the average height of higher density particles. The factor 2 is a multiplication factor arising from the equal solid fraction of each particle species which limits the average height of particles in such species to $0.25 < z_a < 0.75$ and thus $-1 < H < 1$. It must be noted that in each density ratios at the initial stage, the segregation coefficient approaches
to zero, as shown in figure 1. This implies that a homogeneous system is observed. Note that the higher density particles move downward to the bottom is initially observed when $H > 0$, while the higher density particles move up to the top of the container found when $H < 0$.

3. Results

Experimental results of segregation in binary granular mixture under vertical vibration are presented in this section. The results are separated into 2 parts: 1) effect of the dimensionless vibrational acceleration on the segregation coefficient and 2) effect of density ratio on the segregation coefficient.

3.1. The effect of the dimensionless vibrational acceleration

In general, the particles commonly arranged to have the highest density. The densest packing is the hexagonal packing or honeycomb [13]. Figure 3 shows the hexagonal packing at an equilibrium system. This packing prevents potentially the particle movement. Nevertheless, the equilibrium system created from the hexagonal packing can be broken by using the vertical vibration, i.e. this vibration practically produces a void space that the particles can be moved into it. In this study, the level of the vertical vibration is related to the dimensionless vibrational acceleration as mentioned above. It means that different values of the $\Gamma$ are able to create different amount of the void space. As a result, it can be seen that the segregation coefficient increases when the dimensionless vibrational acceleration is increased, as presented in figure 4. More clearly, the packing zone for $\Gamma = 2$ is larger than that for $\Gamma = 5$, as illustrated in figure 5. This is a reason why the segregation coefficient for $\Gamma = 2$ is higher than that for $\Gamma = 5$. This result was also found by previous studies as in refs. [7, 10].

![Hexagonal packing](image)

**Figure 3.** Hexagonal packing.
Figure 4. The segregation coefficient as a function of the dimensionless vibrational acceleration at the final stage.

Figure 5. Examples of the destroying packing of particles under different values of $\Gamma$ at 180 seconds: a) $\Gamma=2$ and b) $\Gamma=5$. 
3.2. The effect of the density ratio
The results of the segregation in binary mixtures due to different density ratios are shown in figure 6. It is clearly seen that the segregation coefficient increases with the density ratio. It must be noted that the particles whose have a higher density move downward to the bottom of the container. This can be explained by the momentum theory, i.e. the under the same external applied loading, particles whose have a lower density can move upward in the direction that opposites the gravity, better than the particles whose have a higher density. This result was also observed by previous works [4, 7, 14].

![Figure 6. The segregation coefficient as a function of the density ratio at the final stage at 10 minutes](image)

4. Conclusion
The influence of density ratios on segregation phenomena in binary granular mixtures under vertical vibrations is experimentally investigated in this study. The ratios between particles whose have a higher density and particles whose have a lower density are 2.73, 3.36, 5.27, and 13.63, respectively. The particles are periodically placed into the rectangular container. The samples are then shaken in the vertical direction with five different dimensionless vibrational accelerations (Γ=1-5). The experimental results show that the segregation coefficient increases when the dimensionless vibration acceleration is increased, i.e. the segregation phenomenon occurs when the dimensionless vibrational acceleration is higher than 2 for the density ratio that segregation. This segregation is clearer when the dimensionless vibration accelerations are increased. Furthermore, the segregation coefficient increases when the density ratio is increased. The segregation does not occur for the density ratios of 2.73 and 3.36. It is observed when the density ratio is higher than 5.27.

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