Rotational Doppler Effect in Magnetic Resonance

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We compute the shift in the frequency of the spin resonance in a solid that rotates in the field of a circularly polarized electromagnetic wave. Electron spin resonance, nuclear magnetic resonance, and ferromagnetic resonance are considered. We show that contrary to the case of the rotating LC circuit, the shift in the frequency of the spin resonance has strong dependence on the symmetry of the receiver. The shift due to rotation occurs only when rotational symmetry is broken by the anisotropy of the gyromagnetic tensor, by the shape of the body, or by magnetocrystalline anisotropy. General expressions for the resonance frequency and power absorption are derived and implications for experiment are discussed.

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I. INTRODUCTION

The term Rotational Doppler Effect (RDE) is used to describe a frequency shift encountered by a receiver of electromagnetic radiation when either the receiver or the source of the radiation are rotating. The effect is illustrated in Fig. 1. The frequency of the wave, \( \omega = 2\pi f \), measured at a given point in space, corresponds to the angular velocity of the rotation of the electric (magnetic) field due to the wave. If the receiver is rotating mechanically at an angular velocity \( \Omega \) about the axis parallel to the wave vector \( k \), than the frequency of the wave perceived by the receiver equals

\[
\omega' = \omega \pm \Omega.
\]  

The sign, plus or minus, depends on the helicity of the wave and the direction of the rotation of the receiver.

The RDE is less commonly known than the conventional Doppler effect. One reason is that it is more difficult to observe. Mössbauer technique provides the most sensitive method for the study of the frequency shift due to the conventional Doppler effect, \( \delta \omega = (v/c)\omega \) for \( v \ll c \). The limiting velocity has been a fraction of a millimeter per second and is due to the finite very small linewidth of gamma radiation, \( \delta \omega / \omega \sim 10^{-13} - 10^{-12} \). Such a small linewidth has even permitted observation of the transverse Doppler effect\textsuperscript{12} by performing Mössbauer experiment on a rotating platform. This effect, not to be confused with the RDE, consists of the frequency shift \( \delta \omega / \omega = -v^2/(2c^2) \) due to the relativistic time dilation for a receiver moving tangentially with respect to the source of the radiation. It is easy to see, however, that the frequency shift as little as \( \Omega / \omega \sim 10^{-13} - 10^{-12} \) due to the RDE would require angular velocity of the emitter or the receiver in the Mössbauer experiment on the order of a few MHz or even a few tens of MHz. The latter is still one-two orders of magnitude greater than the angular velocities of high-speed rotors used for magic angle spinning in NMR applications.

The RDE frequency shift caused by a rotating plate inserted into a beam of circularly polarized light was reported in Refs. \textsuperscript{3,4,7}. The RDE was predicted for rotating light beams\textsuperscript{8} and subsequently observed using millimeter waves\textsuperscript{9} as well as in the optical range\textsuperscript{10} (see Ref. \textsuperscript{11} for review). In solid state experiments the RDE has proved surprisingly elusive. Frequencies of the ferromagnetic resonance (FMR) are typically in the GHz range or higher, which is far above achievable angular velocities of mechanical rotation of macroscopic magnets. However, small magnetic particles in beams\textsuperscript{12} or in nanopores\textsuperscript{13}...
may rotate very fast. Eq. (1) was recently applied to the
analysis of the observed anomalies in the FMR data on
rotating nanoparticles14. The RDE may be especially
important for the NMR technology that uses rapidly spin-
ing samples. Frequency shifts of the quadrupole line in
the nuclear magnetic resonance (NMR) experiment with
a rotating sample were reported in Ref. 14 and analyzed
in terms of Berry phase14. It was never fully explained,
however, why such shifts do not persist in the NMR ex-
periments in which the angular velocity of the mag-
angle-spinning rotor with the sample often exceeds the
linewidth by an order of magnitude. Some hint to an-
swering this question can be found in Ref. 14 that stud-
ied the effect of the rotation on radiation at the atomic
level. The authors of this work correctly argued that
the RDE can only be seen in the radiation of atoms and
molecules placed in the environment that destroys rota-
tional symmetry.

Situation depicted in Fig. 1 rather obviously leads to
the frequency shift by $\Omega$ when the emitter and the re-
ceiver are based upon LC circuits. This has been tested
by the GPS for the case of a receiving antenna mak-
ing as little as 8 revolutions per second as compared to
the carrier frequency of the electromagnetic waves in
the GHz range15. Eq. (1) has been also applied to the ex-
planation of the frequency shift encountered by NASA
in the communications with Pioneer spacecrafts16. One
essential difference between conventional and rotational
Doppler effects is that the first refers to the inertial sys-
tems while the second occurs in the non-inertial systems.
This prompted works that considered RDE in the con-
text of nonlocal quantum mechanics in the accelerated
frame of reference17. Relativity (or Galilean invariance
for $v \ll c$) makes the conventional Doppler effect quite
universal. As we shall see below, such a universality
should not be expected for the RDE. Indeed, the argu-
ment behind the RDE is based upon perception of a cir-
cularly polarized wave by a rotating observer. Through
the Larmor theorem18 the mechanical rotation of the sys-
tem of charges is equivalent to the magnetic field. Con-
sequently, when making the argument, one has to check
whether the resonant frequency of the receiver is affected
by the magnetic field. Resonant frequencies of LC cir-
cuits are known to be insensitive to the magnetic fields,
thus making the argument rather solid. On the contrary,
the frequency of the receiver based upon magnetic reso-
nance would be sensitive to the fictitious magnetic field
due to rotation, thus making the argument incomplete.

In this paper we develop a rigorous theory of the RDE
for magnetic resonance. We show that the frequency
shift due to rotation is always different from $\Omega$. Bro-
en rotational symmetry is required for the shift to have
a non-zero value, in which case the magnetic resonance
splits into two lines separated by $2\Omega$. For the electron
spin resonance (ESR) violation of the rotational sym-
metry would naturally arise from the anisotropy of the
gyromagnetic tensor. In a solid state NMR experiment
with a rotating sample, violation of symmetry would be
more common in the presence of the magnetic order that
provides anisotropy of the hyperfine interaction. For a
ferromagnetic resonance (FMR) the asymmetry comes
from the shape of the sample and from magnetocryst-
talline anisotropy. The paper is organized as follows.

The physics of spin-rotation coupling is reviewed in Sec-
tion III. Frequency shift of the ESR in a rotating crys-
tal with anisotropic gyromagnetic tensor is computed in
Section III. The effect of rotation on the NMR spectra
is discussed in Section IV. FMR in a rotating sample is
studied in Section V. Power absorption by the rotating
magnet is considered in Section VI. Section VII contains
some suggestions for experiment and discussion of possi-
bility application of the RDE in solid state physics.

II. SPIN-ROTATION COUPLING

In classical mechanics the Hamiltonian of the system
in a rotating coordinate frame is given by

$$H' = H - L \cdot \Omega. \quad (2)$$

Here $H$ is the Hamiltonian at $\Omega = 0$ and $L$ is the me-
chanical angular momentum of the system. For a system
of charges one can write

$$L = \frac{M}{\gamma}, \quad (3)$$

where $M$ is the magnetic moment and $\gamma$ is the gyro-
magnetic ratio. Eq. (2) then becomes equivalent to the
Hamiltonian,

$$H' = H - M \cdot B, \quad (4)$$

in the fictitious magnetic field,

$$B = \frac{\Omega}{\gamma}, \quad (5)$$

which is the statement of the Larmor theorem19.

Neither classical mechanics nor classical field theory
deals with the concept of a spin. The question then arises
whether Eq. (2) should contain spin $S$ alongside with the
orbital angular momentum $L$. Eq. (1) hints that since
the magnetic moment can be of spin origin this should
be the case. Also it is known from relativistic physics
that the generator of rotations is

$$J = L + S. \quad (6)$$

It should be, therefore, naturally expected that in the
presence of a spin Eq. (2) should be generalized as

$$H' = H - (L + S) \cdot \Omega. \quad (7)$$

In quantum theory this relation can be rigorously derived
in the following way. Rotation by an angle $\phi$ transforms
the Hamiltonian of an isolated system into20

$$\hat{H}' = \exp \left[ \frac{i}{\hbar} (L + S) \cdot \phi \right] \hat{H} \exp \left[ -\frac{i}{\hbar} (L + S) \cdot \phi \right]. \quad (8)$$
To the first order on a small rotation \( \phi \) one obtains
\[
\hat{\mathcal{H}}' = \hat{\mathcal{H}} - \frac{i}{\hbar} (L + S) \cdot [\hat{\mathcal{H}}, \phi],
\]
where we have taken into account that for an isolated system \( J \) is conserved, that is \( L + S \) commutes with \( \hat{\mathcal{H}} \). This equation becomes Eq. (7) if one takes into account the quantum-mechanical relation
\[
\Omega = \frac{d\phi}{dt} = \frac{i}{\hbar} [\hat{\mathcal{H}}, \phi]
\]
and replaces operator \( \Omega \) by its classical expectation value. For an electron Eq. (7) can be also formally derived as a non-relativistic limit of the Dirac equation written in the metric of the rotating coordinate frame. To elucidate the physical meaning of this, we consider the resulting equation of motion for the term \(-i\hbar \sigma \cdot \hat{\mathcal{H}}\), where \( \sigma \) are the Pauli matrices.

The effect of rotations on various magnetic resonances is considered in the next sections.

In this Section we consider an electron in a rotating crystal or in a rotating quantum dot characterized by the anisotropic gyromagnetic tensor, \( g_{ij} \). The effect of local rotations due to transverse phonons on the width of the ESR has been studied in Ref. 28. Here we are interested in the effect of the global rotation on the ESR frequency. To deal with the stationary states we shall assume that the axis of rotation \( \Omega \) is parallel to the applied magnetic field \( B \) and will compute the energy levels of the electron as measured by the observer rotating together with the system. In the rotating frame the spin Hamiltonian of the electron is
\[
\hat{\mathcal{H}}' = \frac{1}{2} \mu_B g_{ij} \sigma_i B_j - \frac{1}{2} \hbar \sigma \cdot \Omega.
\]
Positive sign of the first (Zeeman) term is due to the negative gyromagnetic ratio \( \gamma \) for the electron (\( \mu_B = h\gamma \)) being the Bohr magneton.

The geometry of the problem is illustrated in Fig. 2. In the rotating frame the solid matrix containing the electron is stationary. It is convenient to choose the coordinate axes of that matrix along the principal axes of the tensor \( g_{ij} \). Then \( g_{ij} \) is diagonal,
\[
g_{ij} = g_i \delta_{ij},
\]
represented by three numbers, \( g_x, g_y, \) and \( g_z \) that can be directly measured when the system is at rest. Eq. (14) then becomes
\[
\hat{\mathcal{H}}' = \frac{1}{2} [(\mu_B g_x B_x - \hbar\Omega_x) \sigma_x + (\mu_B g_y B_y - \hbar\Omega_y) \sigma_y + (\mu_B g_z B_z - \hbar\Omega_z) \sigma_z].
\]

Diagonalization of this Hamiltonian with the account of the fact that \( \Omega \) was chosen parallel to \( B \) gives the following energy levels of \( \hat{\mathcal{H}}' \):
\[
E_{\pm} = \pm \frac{1}{2} \mu_B B \left[ \sum_{i=x,y,z} \left( g_i - \frac{\hbar\Omega_i}{\mu_B B} \right)^2 n_i^2 \right]^{1/2}
\]
Here \( n \) is the unit vector in the direction of the axis of rotation,
\[
n = \frac{\Omega}{|\Omega|} = \frac{B}{B}.
\]

In practice, the angular velocity of the mechanical rotation will always be sufficiently small to provide the condition \( \hbar \Omega \ll \mu_B B \). Contribution of the rotation to the ESR frequency in the rotating frame,
\[
\hbar \omega_{ESR} = E_+ - E_-,
\]
will, therefore, be small compared to the ESR frequency
\[
\hbar \omega_{ESR} = \mu_B B (g_x^2 n_x^2 + g_y^2 n_y^2 + g_z^2 n_z^2)^{1/2}
\]
unperturbed by rotation. Expanding Eq. (17) to the first order in \( \Omega \) one obtains
\[
\omega_{ESR}' = \omega_{ESR} - \kappa \Omega,
\]
\[
\kappa = \frac{g_x n_x^2 + g_y n_y^2 + g_z n_z^2}{\sqrt{g_x^2 n_x^2 + g_y^2 n_y^2 + g_z^2 n_z^2}}.
\]
Here $\Omega$ can be positive or negative depending on the direction of rotation.

Few observations are in order. Firstly, according to Eq. (22), the frequency shift for the observer rotating together with the sample containing the electron is never zero. Secondly, when the rotation is about one of the principal axes of the gyromagnetic tensor, Eq. (22) gives $\kappa = 1$, so that the frequency shift for the rotating observer is exactly $\Omega$. The ESR occurs when the frequency $\omega'$ of the circularly polarized electromagnetic wave perceived by the rotating observer and given by Eq. (11) coincides with $\omega'_{ESR}$. If the rotation is about one of the principal axes of $g_{ij}$, then $\kappa = 1$ and the angular velocity $\Omega$ cancels exactly from the equation $\omega' = \omega'_{ESR}$ for the polarization of the wave that corresponds to $\omega' = \omega - \Omega$, thus, resulting in no RDE frequency shift for an experimentalist working in the laboratory frame. For the opposite polarization of the wave, corresponding to $\omega' = \omega + \Omega$, the shift in the rotationally invariant case formally equals $2\Omega$. However, such photons would have their spin projection in the direction opposite to the one necessary to produce the spin transition. They can be absorbed only when the rotational symmetry is broken so that the electron spin in the direction of the wave vector is no longer a good quantum number (see Section IV).

**IV. FREQUENCY SHIFT OF THE NUCLEAR MAGNETIC RESONANCE DUE TO ROTATION**

Let us consider a nuclear spin $I$ in the magnetic field parallel to the axis of rotation of the sample. It is clear from the previous section that the mechanical rotation combined with the rotationally invariant Zeeman interaction of the nuclear magnetic moment with the field,

$$\hat{H}' = -\gamma_n g_n I \cdot B - I \cdot \Omega,$$

(with $\gamma_n > 0$ and $g_n$ being nuclear gyromagnetic ratio and gyromagnetic factor, respectively) are not sufficient to produce the RDE. Isotropic hyperfine interaction with an atomic spin $S$ of the form $-A I \cdot S$ would not change this either. However, an anisotropic hyperfine interaction,

$$\hat{H}_{hf} = -A_{ij} I_i S_j,$$

in principle, can do the job. If there is a ferromagnetic order in the solid, then $S$ develops a non-zero average, $\langle S \rangle$. Replacing $S_j$ in Eq. (24) with $\langle S \rangle$ and adding the hyperfine interaction to Eq. (23), one obtains

$$\hat{H}' = -\gamma_n g_n I \cdot B - A_{ij} I_i \langle S_j \rangle - I \cdot \Omega.$$

To work with the stationary energy states in the rotating frame, we shall assume that all three vectors $B$, $\langle S \rangle$, and $\Omega$ are parallel to each other. Let us study the case of $I = 1/2$. Choosing the coordinate axes along the principal axes of tensor $A_{ij} = A_i \delta_{ij}$, it is easy to see that $\hat{H}'$ is equivalent to the Zeeman Hamiltonian,

$$\hat{H}' = -\frac{1}{2} \mu_n \left[ g_x \sigma_x B_x + g_y \sigma_y B_y + g_z \sigma_z B_z \right]$$

with an effective gyromagnetic tensor whose principal values are given by $(i = x, y, z)$

$$g_i^{eff} = g_n + B_i^{eff} / B + \frac{\hbar \Omega}{\mu_n B},$$

where we have introduced the nuclear magneton, $\mu_n = \hbar \gamma_n$, and the hyperfine field, $B_i^{eff}$, with components

$$B_i^{eff} = \frac{h A_i \langle |S| \rangle}{\mu_n}.$$  

The energy levels of the Hamiltonian (26) are

$$E_{\pm} = \pm \frac{1}{2} \mu_n B \left[ \sum_{i=x,y,z} \left( g_i^{eff} \right)^2 n_i^2 \right]^{1/2},$$

where $n = B / B$.

Let us consider the case of small $\Omega$. Making the series expansion of Eq. (29) one obtains to the first order on $\Omega$

$$\omega_{NMR}^i = E_+ - E_- = \omega_{NMR} + \kappa \Omega$$

with $\kappa$ given by

$$\kappa = \frac{\sum_{i=x,y,z} \left( g_n + B_i^{eff} / B \right) n_i^2}{\sqrt{\sum_{i=x,y,z} \left( g_n + B_i^{eff} / B \right)^2 n_i^2}}.$$  

In the case of the isotropic hyperfine interaction, $B_i^{eff} = B_x^{eff} = B_y^{eff} = B_z^{eff}$ (that is, $A_x = A_y = A_z$), Eq. (31) gives $\kappa = 1$. Same situation occurs when the direction of the field and the axis of rotation coincide with one of the
principal axes of the tensor of hyperfine interactions. For arbitrary rotations Eq. (31) gives \( \kappa \to 1 \) when \( B \gg B_{hf} \), making the frequency shift defined by \( \omega' = \omega_{NMR}' \) negligible for the polarization (\( \omega' = \omega + \Omega \)) that is predominantly absorbed due to the selection rule. Is is likely, therefore, that a significant RDE in the NMR can be observed only in magnetically ordered materials, in the field comparable or less than the hyperfine field, for rotations about axes that do not coincide with the symmetry axes of the crystal. If these conditions are satisfied, and the width of the resonance is not very large compared to \( \Omega \), the NMR produced by linearly polarized waves would split into two lines of uneven intensity separated by \( 2\Omega \). In fact, the existing experimental techniques permit observation of this effect (see Section VII).

V. FREQUENCY SHIFT OF THE FERROMAGNETIC RESONANCE DUE TO ROTATION

We now turn to the rotating ferromagnets. We begin with a simplest model of ferromagnetic resonance studied by Kittel. In this model one neglects the effects of magnetocrystalline anisotropy and considers a uniformly magnetized ferromagnetic ellipsoid in the external magnetic field \( B = \mu_0 H \) (with \( \mu_0 \) being the magnetic permeability of vacuum). The energy density of such a ferromagnet is determined by its Zeeman interaction with the external field and by magnetic dipole-dipole interactions inside the ferromagnet:

\[
\mathcal{H} = \mu_0 \left[ -M \cdot H + \frac{1}{2} N_{ij} M_i M_j \right].
\]  

(32)

Here \( M \) is the magnetization and \( N_{ij} \) is tensor of demagnetizing coefficients. The principal axes of \( N_{ij} \) coincide with the axes of the ellipsoid. Choosing the coordinate axes along the principal axes and taking into account that for a ferromagnet

\[
M^2 = M_x^2 + M_y^2 + M_z^2 = M_0^2
\]

(33)
is a constant, one can rewrite Eq. (32) as

\[
\mathcal{H} = -\mu_0 \left[ M \cdot H + \frac{1}{2} (N_x - N_z) M_x^2 + \frac{1}{2} (N_y - N_z) M_y^2 \right],
\]  

(34)

where we have omitted unessential constant. For, e.g., an infinite circular cylinder \( N_x = N_y = 1/2, N_z = 0 \). In general, for an ellipsoid elongated along the \( Z \)-axis one has \( N_x - N_z > 0, N_y - N_z > 0 \), so that in the absence of the field the minimum of Eq. (34) corresponds to \( M \) in the \( Z \)-direction. This will still be true in the external field if the latter is applied in the \( Z \)-direction, which is the case we consider here. Note that a finite field is always needed to prevent the magnet from breaking into magnetic domains.

The FMR frequency, \( \omega_{FMR} \), can be obtained from either classical or quantum mechanical treatment. Classically, it is the frequency of the precession of \( M \) about its equilibrium direction. To find \( \omega_{FMR} \) one should linearize the equation,

\[
\frac{dM}{dt} = \gamma M \times B^{(eff)}, \quad B^{(eff)} = -\frac{\delta\mathcal{H}}{\delta M},
\]

(35)

around \( M = M_0 e_z \) (\( \gamma < 0 \) being the gyromagnetic ratio). The answer reads

\[
\omega_{FMR} = \sqrt{\omega_x^2 + \omega_y^2},
\]

(36)

where

\[
\omega_x = |\gamma| [(B + (N_x - N_z)\mu_0 M_0)]
\]

\[
\omega_y = |\gamma| [(B + (N_y - N_z)\mu_0 M_0)].
\]

(37)

To study the RDE we should now solve the same problem in the coordinate frame rotating about the \( Z \)-axis at an angular velocity \( \Omega \). In the presence of rotation the Hamiltonian becomes

\[
\mathcal{H}' = \mathcal{H} - \frac{M}{\gamma} \cdot \Omega.
\]

(38)

It is easy to see that for \( \Omega = \Omega e_z \) this effectively adds \( \Omega / \gamma \) to the external field. Consequently, the FMR frequency in the rotating frame becomes

\[
\omega_{FMR}' = \sqrt{\omega_{FMR}'^2 + \omega_{\Omega}^2},
\]

(39)

with

\[
\omega_x' = |\gamma| \left[ B + \frac{\Omega}{\gamma} + (N_x - N_z)\mu_0 M_0 \right]
\]

\[
\omega_y' = |\gamma| \left[ B + \frac{\Omega}{\gamma} + (N_y - N_z)\mu_0 M_0 \right].
\]

(40)

Our immediate observation is that for a symmetric ellipsoid \( (N_x = N_y) \)

\[
\omega_{FMR}' = \omega_{FMR} - \Omega,
\]

(41)

so that the RDE frequency shift determined by the equation \( \omega' = \omega - \Omega = \omega_{FMR}' \) is exactly zero. For an asymmetric ellipsoid \( (N_x \neq N_y) \), expanding Eq. (39) into a series on \( \Omega \) one obtains to the first order

\[
\omega_{FMR}' = \omega_{FMR} - \kappa \Omega,
\]

(42)

with

\[
\kappa = \frac{1}{2} \left( \sqrt{\frac{\omega_x'}{\omega_y'}} + \sqrt{\frac{\omega_y'}{\omega_x'}} \right).
\]

(43)

It is easy to see that \( \kappa \geq 1 \). At large fields, \( B \gg \mu_0 M_0 \), equations (37) and (43) give \( \kappa \to 1 \), that is, no frequency shift due to the RDE. Sizable frequency shift of the FMR observed in the laboratory frame due to the rotation of
the sample should occur only at \( B \) not significantly exceeding \( \mu_0 M_0 \) and only in a sample lacking the rotational symmetry.

One can easily generalize the above approach to take into account any type of the magnetocrystalline anisotropy. The formulas look especially simple in the case of the orthorhombic anisotropy (\( \beta \) to the Hamiltonian of the magnet, with \( \beta \) to the equilibrium spin of the magnet, \( S_0 \), is antiparallel to its equilibrium magnetic moment \( M_0 \).)

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VI. POWER ABSORPTION BY A ROTATING MAGNET

For non-relativistic rotations the radiation power absorbed by the magnet should be the same in the laboratory frame and in the rotating frame. Calculation in the rotating frame is easier. We shall assume that the dimensions of the sample are small compared to the wavelength of the radiation, so that the field of the wave at the position of the ferromagnet is nearly uniform. The geometry studied below is illustrated in Fig. 3. Within the model of Eq. 38, the rotating magnet placed in the field of a circularly polarized wave feels the oscillating magnetic field that can be represented by a complex function

\[
h(t) = h_0 e^{\pm i\omega t}, \quad \omega' = \omega \mp \Omega
\]

giving the components of the field as

\[
h_x = \text{Re}(h), \quad h_y = \text{Im}(h).
\]

Here \( h_0 \) is the complex amplitude of the wave, \( \pm \) sign in Eq. 46 determines the helicity of the wave, while the sign of \( \Omega \) determines the direction of rotation of the magnet. Due to the wave the magnetization acquires a small ac-component \( m(t) \) (whose real and imaginary parts represent \( m_x \) and \( m_y \), respectively),

\[
m(t) = \hat{\chi}(\omega) h(t),
\]

where \( \hat{\chi} \) is the susceptibility tensor. The absorbed power is given by

\[
P = \pm i\mu_0 \omega' h_0^* (\hat{\chi} - \hat{\chi}^\dagger) h_0.
\]

The problem has, therefore, reduced to the computation of the susceptibility in the rotating frame. The latter can be done by solving the Landau-Lifshitz equation,

\[
\frac{dM}{dt} = \gamma M \times B^{(\text{eff})} - \frac{\eta}{M_0} |\gamma| M \times [M \times B^{(\text{eff})}],
\]

in the rotating frame, that is, with \( B^{(\text{eff})} = -\delta \mathcal{H}'/\delta M \) and

\[
\mathcal{H}' = \mathcal{H} - \frac{M}{\gamma} \cdot \Omega - M \cdot h.
\]

The parameter \( \eta \) in Eq. 50 is a dimensionless damping coefficient that is responsible for the width of the FMR in the absence of inhomogeneous broadening.

Substituting \( M = M_0 e_x + m \) into Eq. 50 and solving for \( \hat{\chi} \) one obtains for the power

\[
P_{\pm} = \frac{1}{2} \eta |\gamma| M_0 h_0^2 f_\pm(\omega'),
\]

where

\[
f_\pm = \frac{\omega'^2 [2(\omega'^2 - \omega_{\text{FMR}}^2) \pm 2\omega'(\omega_x' + \omega_y') + (\omega_x' + \omega_y')^2]}{(\omega'^2 - \omega_{\text{FMR}}^2)^2 + \eta^2 \omega'^2 (\omega_x' + \omega_y')^2}.
\]
We have computed the frequency shift of the magnetic resonance due to rotation of the sample. The effect of rotation on the ESR, NMR, and FMR has been studied. We found that it is, generally, quite different from the rotational Doppler effect reported in other systems. The differences stem from the observation that the spin of an electron or an atom would be insensitive to the rotation of the body as whole if not for the relativistic spin-orbit coupling. Even with account of spin-orbit interactions the spin would not simply follow the rotation of the body but would exhibit more complex behavior described by the dynamics of the angular momentum. Everyone who watched the behavior of a gyroscope in a rotating frame could easily appreciate this fact.

We found the following common features of the magnetic resonance in a rotating sample.

- If the spin Hamiltonian is invariant with respect to the rotation, then the rotation of the body has no effect on the frequency of the resonant absorption of a circularly polarized electromagnetic wave.
- As the rotational invariance is violated, the absorption line shifts. The shift is different from the angular velocity of rotation, $\Omega$. It depends on the degree of violation of the rotational symmetry. The frequency shift goes to zero when the symmetry is restored.
- In the case of a linearly polarized radiation a second resonance line emerges, separated by $2\Omega$ from the first line. The intensity of that line depends on the degree of violation of rotational symmetry. It disappears when the rotational symmetry is restored.

ESR and FMR measurements are usually performed in the GHz range, with the width of the resonance being sometimes as low as a few MHz. Currently available small mechanical rotors can rotate as fast as 100 kHz, which, nevertheless, is still low compared to the linewidths of ESR and FMR. Note, however, that the position of the ESR or FMR maximum can be determined with an accuracy of a few hundred kHz. It is then not out of question that under appropriate conditions the RDE frequency shift and the splitting of the resonance can be observed in high precision ESR and FMR experiments even when the rotation frequency is significantly lower than the linewidth. Since anisotropy of the sample is needed to provide rotational asymmetry, the measurements should be performed on single crystals. Crystals with significant anisotropy of the gyromagnetic tensor should be selected for ESR experiments. When the magnetocrystalline anisotropy is weak, the RDE in FMR can be induced by the asymmetric shape of the sample alone due to the anisotropy of dipole-dipole interactions. Even in this case, however, a single crystal would be preferred to provide a narrow linewidth. Same applies to experiments on RDE in solid state NMR. The NMR frequency range is much lower than that used in ESR and FMR experiments. The width of the NMR line can be as low as a few kHz, that is, well below the available rotational angular velocities. The key to the observation of RDE in a solid state NMR must be the use of a crystal having magnetic order and strong anisotropy of the hyperfine interaction.

A separate interesting question is magnetic resonance in small magnetic particles that are free to rotate. Particles of size in the nanometer range can easily be excited into rotational states with $\Omega$ of hundreds of MHz. Contrary to the rotational quantum states of molecules that have been studied for decades, analytical solution of the problem of a quantum-mechanical rotator does not exist even without a spin. Presence of the spin interacting with a mechanical rotation complicates this problem even further. Rigorous solution has been recently found for the low energy states of a rotator that can be treated as a two-state spin system. General solution is very difficult to obtain. In the case when a particle consists of
a large number of atoms, one can develop a semiclassical approximation in which $\Omega$ is replaced with $L/I$ (with $I$ being the moment of inertia). This suggests that the magnetic resonance in nanoparticles that are free to rotate would split into many lines related to the quantization of $L$. Some evidence of this effect has been recently found in the FMR studies of magnetic particles in nanopores\textsuperscript{13}. Rapid progress in measurements of single magnetic nanoparticles\textsuperscript{31} may shed further light on their quantized rotational states and related spin resonances.

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