Non-equilibrium critical dynamics in disordered ferromagnets

Grégory Schehr\textsuperscript{1}, Raja Paul\textsuperscript{2}

\textsuperscript{1} Theoretische Physik, Universität des Saarlandes, 66041 Saarbrücken, Germany
\textsuperscript{2} BIOMS, IWR, Ruprecht-Karls-University Heidelberg, 69120 Heidelberg

E-mail: schehr@lusi.uni-sb.de

Abstract. We discuss some aspects of the non-equilibrium relaxational dynamics which occur after a quench at a disordered critical point. In particular, we focus on the violation of the fluctuation dissipation theorem for local as well as non-local observables and on persistence properties.

Although critical dynamics has been a subject of study for many years [1], it was rather recently recognized [2, 3] that they display interesting non-equilibrium features such as aging or violation of the Fluctuation Dissipation Theorem (FDT), which are commonly observed phenomena in more complex disordered or glassy phases [4]. Therefore, relaxational dynamics at pure critical point has been the subject of numerous analytical as well as numerical recent studies [5]. Interestingly, it has been proposed [3] that a non trivial Fluctuation Dissipation Ratio (FDR) $X$, originally introduced in the Mean Field approach to glassy systems, which generalizes the FDT to non equilibrium situations, is a new universal quantity associated to these critical points. As such, it has been computed using the powerful tools of RG, e.g. for pure $O(N)$ model at criticality in the vicinity of the upper critical dimension $d_{uc} = 4$ and for various dynamics [5].

On the other hand, critical dynamics display interesting 'global persistence' properties. Indeed, it has been shown [6] that the probability $P_c(t)$ that the global magnetization $M$ has not changed sign in the time interval $t$ following a quench from a random initial configuration, decays algebraically at large time $P_c(t) \sim t^{-\theta_c}$. In this context, analytical progress is made possible, thanks to the property that, in the thermodynamic limit, the global order parameter remains Gaussian at all finite times $t$. Indeed, for a $d$-dimensional system of linear size $L$, $M(t)$ is the sum of $L^d$ random variables which are correlated only over a finite correlation length $\xi(t)$. Thus, in the thermodynamic limit $L/\xi(t) \gg 1$, the Central Limit Theorem (CLT) asserts that $M(t)$ is a Gaussian process [6], for which powerful tools have been developed to compute the persistence properties [7, 8]. Remarkably, under the additional assumption that $M$ is a Markovian process, $\theta_c$ can be related to the other critical exponents via the scaling relation $\theta_c = \mu \equiv (\lambda_c - d + 1 - \eta/2)z^{-1}$, with $z$ and $\lambda_c$ the dynamical and autocorrelation [9, 10] exponent respectively, and $\eta$ the (static) Fisher exponent. Nevertheless, as argued in Ref. [6], $M$ is in general non Markovian and thus $\theta_c$ is a new exponent associated to critical dynamics. For the non conserved critical dynamics of pure $O(N)$ model, corrections to this scaling relation were indeed found to occur at two-loops order [11], in rather good agreement with numerical simulations in dimensions $d = 2, 3$ [12].
Characterizing the effects of quenched disorder on critical dynamics is a complicated task and indeed the question of how quenched randomness modifies these aging and persistence properties has been much less studied. In these notes, we address this question on one prototype of such disordered ferromagnets, the randomly diluted Ising model:

$$H = -\sum_{\langle ij \rangle} \rho_i \rho_j s_i s_j$$  \hspace{1cm} (1)

where \(s_i = \pm 1\) are Ising spins on a \(d\)-dimensional hypercubic lattice and \(\rho_i = 1\) with probability \(p\) and 0 with probability \(1 - p\). For the experimentally relevant case of dimension \(d = 3\) [13], for which the specific heat exponent of the pure model is positive, the disorder is expected, according to Harris criterion[14], to modify the universality class of the transition. For \(1 - p \ll 1\), the large scale properties of (1) at criticality are then described by the following \(O(1)\) model with a random mass term, the so-called Random Ising Model (RIM)[15]:

$$H^\psi[\varphi] = \int d^d x \left[ \frac{1}{2} (\nabla \varphi)^2 + \frac{1}{2} [\psi(x) + \varphi]^2 + \frac{g_0}{4!} \varphi^4 \right]$$  \hspace{1cm} (2)

where \(\varphi \equiv \varphi(x)\) is a coarse-grained spin variable and \(\psi(x)\) is a Gaussian random variable with \(\psi(x) \psi(x') = \Delta \delta^d(x - x')\), \(r_0\) and \(g_0\) being respectively the mass and the coupling constant.

1. **Non equilibrium dynamics of one time quantity : initial slip exponent.**
   
   We first focus on a numerical estimate of the initial slip exponent, which already carries the signature of a non-equilibrium situation on a one time quantity. In that purpose we study the relaxational dynamics associated to the lattice model (1) with periodic boundary conditions where the system is initially prepared in a random initial configuration with mean magnetization \(M_0 = 0.01\). At each time step, one site is randomly chosen and the move \(s_i \to -s_i\) is accepted or rejected according to Metropolis rule. One time-unit corresponds to \(L^3\) such time steps. In all subsequent times we measure the global magnetization \(M(t)\)

$$M(t) = \frac{1}{N_{\text{occ}}} \sum_i \rho_i s_i(t)$$  \hspace{1cm} (3)

where \(N_{\text{occ}}\) is the number of occupied sites for linear system sizes \(L = 8, 16, 32\) and 64. Finally data are averaged over \(8 \times 10^5\) (randomly diluted) samples for \(L = 8\) to \(10^4\) samples for \(L = 64\). From previous works [9] one expects a power law increase

$$M(t) \sim M_0 t^{\theta'}$$  \hspace{1cm} (4)

with \(\theta'\) the so called initial slip exponent. In the inset of Fig. 1, we show a plot of \(M(t)\) for \(p = 0.8\) and different system sizes. As expected [9], one observes an initial increase of \(M(t)\) until a time \(t_0\), which is an increasing function of \(L\) for the sizes considered here \(^1\) and compatible with the scaling \(t_0 \sim L^z\), and then decreases to zero (in that regime one expects \(M(t) \sim t^{-(d-2+\eta)/(2z)}\) although it is not clearly seen here). By computing \(M(t)\) for different values of \(p = 0.499, 0.6, 0.65\) and 0.8, we observe corrections to scaling, which are known to be strong in this model [17, 18]. Following Ref. [18, 19], we take them into account as \(M(t) = t^{\theta'} g_p(t)\) with \(g_p(t) = A'(p)[1 + B'(p)t^{-b}]\), where \(b = 0.23(2)\) has been determined previously [18, 19], \(A'(p), B'(p)\) being fitting parameters.

As shown in Fig. 1, one obtains a reasonably good data collapse of \(M(t)/g_p(t)\) vs. \(t\) for the different values of \(p\). After a microscopic time scale, one observes a universal power law increase (4), from which we get the estimate

$$\theta' = 0.10(2)$$ \hspace{1cm} (5)

which is in good agreement with a previous two-loops estimate \(\theta'_{2-\text{loops}} = 0.0868\) [20].

\(^1\) In the thermodynamic limit, one expects that it reaches the asymptotic value \(t_0 \sim M_0^{-1/(\theta' + (d-2+\eta)/(2z))}\) [9, 16].
2. Two time quantities: Aging and Violation of FDT.

Although non-equilibrium effects can already be observed in one time quantity such as the global magnetization $M(t)$, it is also interesting to study two-times, $t, t_w$ functions, which we now focus on.

2.1. Analytical approach in $d = 4 - \epsilon$.

We study the relaxational dynamics of the Randomly diluted Ising Model in dimension $d = 4 - \epsilon$ described by a Langevin equation:

$$\eta \frac{\partial}{\partial t} \varphi(x, t) = -\frac{\delta H_\varphi}{\delta \varphi(x, t)} + \zeta(x, t)$$

where $\langle \zeta(x, t) \rangle = 0$ and $\langle \zeta(x, t)\zeta(x', t') \rangle = 2\eta T \delta(x - x')\delta(t - t')$ is the thermal noise and $\eta$ the friction coefficient. At initial time $t_i = 0$, the system is in a random initial configuration with zero magnetization $M_0 = 0$ distributed according to a Gaussian with short range correlations

$$[\varphi(x, t = 0)\varphi(x', t = 0)]_i = \tau_0^{-1}\delta^d(x - x')$$

Notice that it has been shown that $\tau_0^{-1}$ is irrelevant (in the RG sense) in the large time regime studied here [9] and it will be set to zero in the following. We focus on the correlation $C^q_{tt_w}$ in Fourier space and the autocorrelation $C_{tt_w}$

$$C^q_{tt_w} = \langle \varphi(q,t)\varphi(-q,t_w) \rangle \quad , \quad C_{tt_w} = \langle \varphi(x,t)\varphi(x,t_w) \rangle$$

and the response $R^q_{tt_w}$ to a small external field $f(-q,t_w)$ linearly coupled to $\varphi(q,t_w)$ as well as on the local response function $R_{tt_w}$ respectively defined, for $t > t_w$, as

$$R^q_{tt_w} = \frac{\delta\langle \varphi(q,t) \rangle}{\delta f(-q,t_w)} \quad , \quad R_{tt_w} = \frac{\delta\langle \varphi(x,t) \rangle}{\delta f(x,t_w)}$$

where $\tau$ and $\langle .. \rangle$ denote averages over disorder and thermal fluctuations respectively. In a previous work [19], using the exact renormalization group equation for the dynamical effective action, we have computed these functions $R^q_{tt_w}, C^q_{tt_w}$ up to one-loop order:

$$R^q_{tt_w} = q^{-2+\eta} \left( \frac{t}{t_w} \right)^\theta F_R(q^z(t-t_w)) \quad , \quad C^q_{tt_w} = T_c q^{-2+\eta} \left( \frac{t}{t_w} \right)^\theta F_C(q^z(t-t_w),t/t_w)$$

where $\theta$ and $\eta$ are defined in Eq. (6).
with (up to order $O(\sqrt{\epsilon})$): \( \eta = 0, z = 2 + \sqrt{\frac{6\epsilon}{53}} \) [21] and \( \theta = \frac{1}{2}\sqrt{\frac{6\epsilon}{53}} \) [22], \( \theta \) being related to the autocorrelation exponent \( \lambda_c \) through \( \theta = 1 + (d - 2 + \eta - \lambda_c)/z \). Notice that the scaling function \( F_R^q \) is a function of \( q^2(t-t_w) \) only (and therefore the superscript eq), which, although \( z \neq 2 \) is in agreement at this order with local scale invariance arguments [23]. In Ref. [19], we have computed analytically the scaling functions \( F_{R,C} \) at one-loop. And we will see later that some interesting information can be extracted from them (see in particular section 3). Here we only mention their asymptotic behaviors which may be relevant for our study. Defining \( v = qz(t-t_w) \) and \( u = t/t_w \) a first interesting scaling regime corresponds to \( u \gg 1 \), keeping \( v \) fixed. In that regime, one obtains an interesting relation

\[
F_C(v,u) = \frac{1}{u} F_{C,\infty}(v) + O(u^{-2})
\]

\[
F_{C,\infty}(v) = A_{C,\infty} v F_R^{eq}(v), \quad A_{C,\infty} = 2 + 2 \sqrt{\frac{6\epsilon}{53}}
\]

The first relation (11) is expected from general RG arguments [9], and it has also been checked for pure \( O(N) \) models [24]. The second one (12) is not predicted by such arguments, and it plays a crucial role in the computation of the FDR. We notice that it has also been obtained in the glassy phase of disordered elastic systems in \( d = 2 \) [25] as well as in the relaxational dynamics near the depinning transition [26]. Whether this relation holds for the present model at two-loops remains an open question.

Another interesting asymptotic behavior, relevant when we want to extract information on local quantity from (10), corresponds to \( v \gg 1 \), keeping \( u \) fixed. In that regime, \( F_{R,C}^{eq}(v) \) (as well as \( F_C(v,u) \)) decays algebraically. In particular

\[
F_{R,C}^{eq}(v) \sim v^{-2}, \quad v \gg 1,
\]

which is in sharp contrast with pure critical systems, e.g. pure \( O(N) \) models, where the decay is actually exponential [24].

From the response and correlation, it is interesting to compute the FDR \( X_{ttw}^q \) defined as [4, 24]:

\[
\frac{1}{X_{ttw}^q} = \frac{\partial_w C_{ttw}^q}{T R_{ttw}^q},
\]

such that \( X_{ttw}^q = 1 \) at equilibrium. From the scalings obtained above (10), one has directly \( X_{ttw}^q \equiv F_X(q^2(t-t_w),t/t_w) \). In particular, in the large \( t/t_w \) limit, keeping \( q^2(t-t_w) \) fixed, one has from (12)

\[
\lim_{u \to \infty} (X_{ttw}^q)^{-1} = 2 + 2 \sqrt{\frac{6\epsilon}{53}} + O(\epsilon)
\]

independently of \( v \), i.e. of (small) wave vector \( q \), which coincides of course with the asymptotic value for the \( q = 0 \) mode obtained in Ref. [27].

It is also interesting to study the limiting value of the FDR for different observables [28, 29, 30], and in particular to focus on local quantities. Therefore we compute the FDR associated to the autocorrelation \( C_{ttw} \) (8) and the local response \( R_{ttw} \) (9) which can be written as [24]:

\[
\frac{1}{X_{ttw}^{x=0}} = \frac{\partial_w C_{ttw}}{T R_{ttw}} = \frac{\int_q R_{ttw}^q (X_{ttw}^q)^{-1}}{\int_q R_{ttw}^q}
\]
For pure critical systems, $R^{q}_{ttw}$ decays exponentially for large $q^2 t$. And thus, in the limit $t \gg 1$, the integral over the Fourier modes $q$ in (16) is dominated by the $q = 0$ mode. One thus expects from this heuristic argument \cite{24} that,

$$\lim_{t_w \to \infty} \lim_{t \to \infty} X^{x=0}_{ttw} = \lim_{t_w \to \infty} \lim_{t \to \infty} X^{q=0}_{ttw}$$

We have, however, seen that for the dilute Ising model, the response function $R^{q}_{ttw}$ decays algebraically for $q^2 t \gg 1$ \cite{13} so that this heuristic argument does not hold. And therefore, this relation (17), if true at all, is far from trivial for the present model. To verify this, we need a direct computation in real space.

Having obtained the scaling functions associated to $c_{ttw}^{q}$ and $R_{ttw}^{q}$ for any Fourier mode $q$, we obtain $C_{ttw}$ and $R_{ttw}$ by Fourier transform \cite{19}. Due to the algebraic large $q$ behavior obtained previously \cite{13}, one obtains logarithmic corrections to scaling

$$R_{ttw} = \frac{K_d A_0^R + A_1^R \ln (t - t_w)}{2 (t - t_w)^{1+ (d-2+\eta)/z}} \left( \frac{t}{t_w} \right)^\theta, \quad C_{ttw} = \frac{K_d A_0^C + A_1^C \ln (t - t_w)}{t (t - t_w)^{(d-2+\eta)/z}} \left( \frac{t}{t_w} \right)^\theta F(t/t_w)$$

with $F(u) = 1/(1 + u) + \mathcal{O}(\epsilon)$ and where $A_0^R, A_1^C$ are non-universal amplitudes. From these expressions (18) one can then compute the local FDR under the form

$$X^{x=0}_{ttw} = F_X(t/t_w)$$

$$F_X(u) = 2 \left( \frac{u^2 + 1}{(u + 1)^2} + \sqrt{\frac{6 \epsilon}{53}} \left( \frac{u - 1}{u + 1} \right)^2 \right) + \mathcal{O}(\epsilon)$$

where $F_X(u)$ is a monotonic increasing function of $u$ : it interpolates between 1, in the quasi-equilibrium regime for $u \to 1$, and its asymptotic value for $u \to \infty$ given by

$$\lim_{t_w \to \infty} \lim_{t \to \infty} (X^{x=0}_{ttw})^{-1} = \lim_{t_w \to \infty} \lim_{t \to \infty} (X^{q=0}_{ttw})^{-1} = 2 + \sqrt{\frac{6 \epsilon}{53}} + \mathcal{O}(\epsilon)$$

which shows explicitly, at order $\mathcal{O}(\sqrt{\epsilon})$ that the asymptotic FDR for both the global and the local magnetization are indeed the same (17).

2.2. Monte Carlo study : Autocorrelation functions.

Let us next present results from our Monte Carlo simulations of the relaxational dynamics of the randomly diluted Ising model (1) in dimension $d = 3$, which, as previously (see section 1) were done on $L^3$ cubic lattices with periodic boundary conditions. The system is initially prepared in a random initial configuration with zero magnetization, and at each time step, the $L^3$ sites are sequentially updated \cite{2} according to Metropolis rule. Here we present our numerical data for $p = 0.8$ (other values of $p$ are discussed in \cite{19}). We compute the spin-spin autocorrelation function defined as

$$C_{ttw} = \frac{1}{L^3} \sum_i \overline{s_i(t) s_i(t_w)}$$

Fig. 2 shows the autocorrelation function $C_{ttw}$ as a function of $t - t_w$ for different values of the waiting time $t_w = 2^4, 2^5, 2^6, 2^7$ and $2^8$ at $p = 0.8$. One observes a clear dependence on $t_w$, which indicates a non-equilibrium dynamical regime. The scaling form obtained from the RG analysis (18) suggests, discarding the logarithmic correction, to plot $t_w^{(1+\eta)/z} C_{ttw}$ as a function

\footnote{We have checked, in the present case of diluted Ising systems, that the random update gives the same results at large times.}
of \( t/t_w \). Taking the values \( \eta = 0.0374 \) from Ref. [31] and \( z = 2.62 \) from Ref. [18], we see in Fig. 3 that, for \( p = 0.8 \), one obtains a good collapse of the curves for different \( t_w \). As shown in Fig. 3, \( t_w^{(1+\eta)/z}C_{ttw} \) decays as a power law, which allows to estimate the value of the decaying exponent \( \lambda_c/z \):

\[
\frac{\lambda_c}{z} = 1.05 \pm 0.03
\]  

We have checked [19], taking carefully corrections to scalings, that this decaying exponent (23) is actually independent of the dilution factor \( p \), which supports universality of the long-time non-equilibrium relaxation in this model. In addition, using \( z = 2.6 \), our numerical estimates of \( \theta'(5) \) and \( \lambda_c \) (23) are consistent with the scaling relation \( \lambda_c = d - z\theta' \).

3. Persistence properties.

Let us now focus on persistence properties of the dilute Ising model at criticality. Defining the global magnetization \( M(t) \) as in Eq. (3) we are interested in the disorder averaged probability \( \overline{P_c}(t) \) that the magnetization has not changed sign in the time interval \( t \) following the quench.

3.1. Analytical approach in \( d = 4 - \epsilon \)

In a previous publication [32], we have shown that the Exact renormalization group equation allows to describe the time evolution of the magnetization by an effective Gaussian process \( \tilde{M}(t) \):

\[
\partial_t \tilde{M}(t) + \sigma(t)\tilde{M}(t) = -\int_0^t dt_1 \Sigma_{tt_1} \tilde{M}(t_1) + \tilde{\zeta}(t)
\]

\[
\Sigma_{tt'} = -\frac{1}{2} \sqrt{\frac{6c}{53}} \gamma(t - t')^2, \quad \sigma(t) = -\int_{t_i}^t dt_1 \Sigma_{tt_1}
\]

where \( \gamma(x) = (x + \Lambda_0^{-2})^{-1} \), \( \Lambda_0 \) being the UV cutoff, and \( \tilde{\zeta}(t) \) is an effective disorder induced Gaussian noise with zero mean and correlations \( \langle \tilde{\zeta}(t)\tilde{\zeta}(t') \rangle_{\text{eff}} = 2T\delta(t - t') + D_{tt'} \) with:

\[
D_{tt'} = \frac{T_c}{2} \sqrt{\frac{6c}{53}} \left( \gamma(t - t') - \gamma(t + t') \right)
\]

The idea is then to compute \( \overline{P_c}(t) \) as the persistence probability of the process \( \tilde{M}(t) \). The rhs of Eq. (24) clearly indicates that this process is non-Markovian. However, as \( \Sigma_{tt'} \) (25) as
well as \(D_{\mu}(26)\) are of order \(O(\sqrt{\epsilon})\), one can use the perturbative computation of \(\theta_c\) around a Markov process initially developed in Ref. [8] and further studied in the context of critical dynamics in Ref. [11]. In that purpose [7], let us introduce the normalized Gaussian process

\[
m(t) = \frac{\hat{M}(t)}{\sqrt{\langle \hat{M}^2(t) \rangle_{\text{eff}}}},
\]

where \(\langle \ldots \rangle_{\text{eff}}\) stands for an average over the Gaussian noise \(\tilde{\zeta}(t)\). Let \(T = \ln t\), then \(m(T)\) is a stationary Gaussian process and its persistence properties are obtained from the autocorrelation function:

\[
\langle m(T)m(T_w) \rangle_{\text{eff}} = e^{-\mu(T-T_w)} A(e^{T-T_w}) \quad , \quad A(x) = \left[ 1 + \frac{1}{4} \sqrt{\frac{6\epsilon}{53}} \left( x \log \frac{x-1}{x+1} - \log \frac{x^2-1}{4x^2} \right) \right]
\]

with \(\mu = (\lambda_c - d + 1 - \eta/2)/z\). Under this form (27), one can use the first order perturbation theory result of Ref. [11] to obtain the one-loop estimate:

\[
\Delta \equiv \theta_c - \mu = \left\{ \frac{6\epsilon}{53} \frac{\sqrt{2} - 1}{2} + O(\epsilon) \right\} = 0.06968... \quad \text{in} \quad d = 3,
\]

where \(\mu\) is the value corresponding to a Markov process. The second term in Eq. (28) is the first correction due to the non-Markovian nature of the dynamics. Interestingly, this correction is entirely determined by the non-trivial structure of the scaling function \(A(x)\), which is directly obtained from \(F_c(v,u)\) (10). Notice also that, at variance with the pure \(O(N)\) models [6, 11], these corrections in presence of quenched static disorder already appear at one-loop order.

3.2. Numerical simulation in \(d = 3\)

We now turn to the results from our Monte Carlo simulations of the relaxational dynamics of the randomly diluted Ising model (1) in dimension \(d = 3\), which were performed on \(L^3\) cubic lattices with periodic boundary conditions. The system is initially prepared in a random initial configuration where up and down spins are randomly distributed on the occupied sites, mimicking a high-temperature disordered configuration before the quench. The initial magnetization \(M_0\) is thus a random variable of zero mean. At each time step, one site is randomly chosen and the move \(s_i \rightarrow -s_i\) is accepted or rejected according to Metropolis rule. One time unit corresponds to \(L^3\) such time steps. The exponent \(\theta_c\) is measured numerically for cubic lattices of linear size \(L = 8, 16, 32\) and \(64\). After a quench to \(T_c\) from the initial random configuration each system evolves until the global magnetization first change sign. \(\mathcal{P}_c(t)\) is then measured as the fraction of surviving systems at each time \(t\), over a number of samples which varies from \(2 \times 10^5\) for \(L = 8\) to \(2 \times 10^4\) for \(L = 64\).

In Fig. 4 we present the results of \(\mathcal{P}_c(t)\) for \(p = 0.8\) and for different lattice sizes. According to standard finite-size scaling [6], one expects the scaling form \(\mathcal{P}_c(t) = t^{-\theta_c} f(t/L^z)\), where \(z\) is the dynamical exponent. Keeping the rather well established value of \(z = 2.62(7)\) [18, 19] fixed, \(\theta_c\) is varied to obtain the best data collapse. The final scaled plot is shown in the inset of Fig. 4. This allows for a first estimate of the exponent \(\theta_c\)

\[
\theta_c = 0.35 \pm 0.01
\]

We have also computed the persistence probability for systems quenched from random configuration with a small initial magnetization \(M_0 = 0.001\). The number of up \(N_{\text{up}}\) and down \(N_{\text{down}}\) spins are thus: \(N_{\text{up}} = (1 + M_0)/2N_{\text{occ}}\) and \(N_{\text{down}} = N_{\text{occ}} - N_{\text{up}}\). First we randomly distribute the \(N_{\text{up}}\) up spins in the occupied sites of the lattice and then fill up the rest with down spins. As noticed previously [12] this protocol allows to reduce the statistical noise and thus to study larger system sizes (this however renders the finite size scaling analysis more subtle [12]). In Fig. 5, we plot the persistence probability for system size \(L = 100\) (the data have been averaged over \(2 \times 10^4\) ensembles) for \(p = 0.499, 0.6, 0.65\) and \(0.8\). The study of larger system
size allows to reduce the corrections to scaling. Indeed, although in the short time scales, the straight lines have slightly different slopes which depends upon $p$, at later times the slopes varies from $0.36(1)$ for $p = 0.8$ to $0.35(1)$ for $p = 0.499$: this confirms the $p$-independent value of $\theta_c$ obtained previously Eq. (29).

In order to compare this numerical value (29) with our one-loop calculation (28) one needs an estimate for $\mu$. Such an estimate is needed not only for the sake of this comparison but also to characterize quantitatively non Markovian effects. The argument mentioned in the introduction, relying on the CLT, which says that the global magnetization is, in the thermodynamic limit, a Gaussian variable is also valid in the presence of disorder, and we have checked it numerically. Therefore, a finite difference $\Delta$ (28) is the signature of a Non Markovian process. Because of the relatively big error bars on $\lambda_c$ and $z$, we propose, alternatively, to express $\mu$ in terms of the initial slip exponent $\theta'$ [9], using $\lambda_c = d - z\theta'$, as $\mu = -\theta' + (1 - \eta/2)/z$. Using our previous estimate for the initial slip exponent $\theta' = 0.10(2)$ (5), this gives $\mu_{\text{num}} = 0.27(3)$ and our numerical estimate

$$\Delta_{\text{num}} = 0.08 \pm 0.04$$

which is in good agreement with our previous one-loop estimate (28). We also notice that these deviations from a Markov process are slightly larger than for the pure case [12].

4. Conclusion

In conclusion, we have studied, analytically using the Exact renormalization group equation as well as numerically, different aspects of non-equilibrium dynamics at a random critical point. Interestingly, concerning the violation of FDT, although the heuristic argument of Ref. [24] does not hold for the present model, we have shown explicitly that the limiting FDR for the local and global magnetization do coincide (21). And it would be interesting to compare this perturbative calculation with numerical simulations. In addition, in view of recent studies [19, 33, 34, 35, 36], it would be also interesting to study this FDR for the relaxational dynamics following a quench from a completely ordered initial condition. Finally, concerning the persistence properties, we have shown that the RG calculation, together with the perturbative methods of Ref. [8, 11] allows for a rather precise estimate of $\theta_c$. And it would be interesting to extend this kind of approach deep inside the glassy phase of disordered systems.
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