The recent high-quality Boomerang data allow to test many competing cosmological models. Here I present a seven-parameter likelihood analysis of dark energy models with exponential potential and explicit coupling to dark matter. Such a model is conformally equivalent to a scalar field with non-minimal coupling to gravity. So far, the constraints on a dark energy - dark matter coupling were extremely weak. The Boomerang data constrain the dimensionless coupling $\beta$ to be smaller than 0.1, an order of magnitude better than previous limits. In terms of the constant $\xi$ of non-minimally coupled theories, this amounts to $\xi < 0.01$.

On the other hand, Boomerang has not enough sensitivity to put constraints on the potential slope.

I. INTRODUCTION

In the last few years, a new reference model of structure formation has emerged, one in which 70% or so of the total matter content of the universe is in a form of dark energy. The main evidence for such a component is the supernovae Ia observations of Ref. [1] and [2], which can be explained by assuming an accelerated expansion due either to a cosmological constant or to a new matter component with equation of state

$$p = (w - 1)\rho$$

with $w \in (0, 0.6)$ [3]. This new component can also be modeled as a light scalar field with a potential that allows for a potential-dominated epoch as, e.g., an exponential potential or an inverse power law.

The evidence for the new component, sometimes denoted dark energy or "quintessence", has been reinforced by the recent Boomerang CMB observations that show a preference for a flat universe [6]. In fact, since a CDM density of $\Omega_c \in (0.2, 0.4)$ is in agreement with a host of independent observations, from cluster abundance to cluster X-ray temperature, lensing, velocity fields etc., we can conclude that a conspicuous fraction of the total energy density has to be unclustered (or weakly clustered) and with negative pressure, although the direct constraint on the amount of dark energy from CMB alone is weak.

The nature of this extra component is so far completely unknown. The simplest explanation, the cosmological constant, runs against the argument that, from a dimensional point of view, its value should be more than a hundred orders of magnitude smaller than expected, and tuned with astonishing precision in order to become dominant just today. The hypothesis of some fundamental scalar field has at least the advantage that one may hope to build a theory that explains the coincidence as a consequence of some fundamental principle. For instance, the inclusion of a coupling between dark matter and dark energy, as will be done in this paper, might explain why the two energy densities are comparable. Another motivation for considering a scalar field lies in the fact that it is premature to be too specific about the dark energy properties: a general scalar field includes as a limiting case the cosmological constant, but allows also to investigate less extreme effective equations of state.

In the same spirit of generality, I investigate the effect of an additional degree of freedom, represented by the coupling of the scalar field to ordinary matter. Such a coupling has been proposed and investigated in refs. [9] [10] [11] [12] [13] [14], and is equivalent, up to a conformal rescaling, to the classical Brans-Dicke coupling to gravity (for gravity coupling in the context of quintessence see refs. [15] [16] [17] [18] [19]). If the dark energy is coupled to dark matter alone (and not to baryons), the present constraints on the coupling are rather weak [20] [21] [13]; as a consequence, in [22] it was shown that the effects on the CMB turn out to be at the level of detectability already with the present data set.

Aim of this paper is to use the most recent CMB data, the Boomerang power spectrum [7], along with the COBE data (as reduced in [23]), to further constrain the coupled dark energy model. As particular cases, we will derive constraints on the pure cosmological constant model and on the uncoupled dark energy. A comparison of Boomerang with a different model of uncoupled dark energy is in [24].

In all the calculations of this paper a flat universe is assumed. This is of course a severe limitation, but the number of free parameters is already large enough to be at the limits of computational capabilities. Moreover, the hypothesis...
of quintessence has always been formulated in the context of flat models, in order to be consistent with the inflationary expectations.

While the work was almost completed the results of the Maxima experiment \cite{26,27,28} have been published. They seem to confirm the results of Boomerang, but have not been included in the present paper.

II. COUPLED DARK ENERGY

The model of coupled quintessence or dark energy has already been studied in detail in \cite{22}. Here I limit myself to a summary of its properties.

Consider three components, a scalar field $\phi$, baryons and CDM, described by the energy-momentum tensors $T_{\mu\nu(\phi)}$, $T_{\mu\nu(b)}$ and $T_{\mu\nu(c)}$, respectively. General covariance requires the conservation of their sum, so that it is possible to consider a coupling such that

\begin{align}
T^\mu_{\nu(\phi);\mu} &= (C_c T_{(c)} + C_b T_{(b)}) \phi_{,\nu}, \\
T^\mu_{\nu(c);\mu} &= -C_c T_{(c)} \phi_{,\nu}, \\
T^\mu_{\nu(b);\mu} &= -C_b T_{(b)} \phi_{,\nu}.
\end{align}

where $C_{c,b}$ are the coupling constants. This particular coupling is indeed obtained by conformally transforming a non-minimally coupled gravity theory or, following \cite{20}, by the Lagrangian (in units $G = c = 1$)

\begin{equation}
L_{tot} = -\frac{R}{2\kappa^2} + \frac{1}{2} \phi_{,\mu} \phi^{,\mu} - U(\phi) + L_c(e^{2C_c \phi} g_{\mu\nu}) + L_b(e^{2C_b \phi} g_{\mu\nu})
\end{equation}

where $\kappa^2 = 8\pi$ and where $L_{c,b}$ denote the Lagrangian of the dark matter and baryonic fields. Notice that our constant $C_c$ corresponds to the invisible coupling constant $\beta_I$ of \cite{24}. The radiation field (subscript $\gamma$) remains uncoupled, since $T_{(\gamma)} = 0$. We derive the background equations in the flat conformal FRW metric $ds^2 = a^2(-d\tau^2 + \delta_{ij}dx^i dx^j)$ assuming the exponential potential

\begin{equation}
U = Ae^{\kappa \phi}
\end{equation}

as proposed e.g. in \cite{29,10,30}. As anticipated, we will couple the dark energy scalar field to the dark matter only, putting $C_b = 0$. We call this choice dark-dark coupling. Generalizing \cite{32} we introduce the following variables (putting $H = \dot{a}/a$)

\begin{equation}
x = \kappa \dfrac{\dot{\phi}}{H \sqrt{6}}, \quad y = \kappa a H \sqrt{\dfrac{U}{3}}, \quad z = \kappa a H \sqrt{\dfrac{\rho_c}{3}}, \quad v = \kappa a H \sqrt{\dfrac{\rho_b}{3}}.
\end{equation}

Adopting the $e$-folding time $\alpha = \log a$ we can write the field, baryon and radiation conservation equation as a system in the four variables $x, y, z, v$ that depends on the parameters $\mu, \beta$

\begin{align}
x' &= \left(\dfrac{z'}{z} - 1\right) x - \mu y^2 + \beta(1 - x^2 - y^2 - z^2 - v^2), \\
y' &= \mu xy + y \left(2 + \dfrac{z'}{x}\right), \\
z' &= -\dfrac{z}{2} (1 - 3x^2 + 3y^2 - z^2), \\
v' &= -\dfrac{v}{2} (-3x^2 + 3y^2 - z^2)
\end{align}

where the prime denotes derivation with respect to $\alpha$, and where $\beta = C_c \sqrt{\dfrac{1}{2\kappa^2}}, \quad \mu = s \sqrt{\dfrac{1}{2\kappa^2}}$. The coupling constant $\beta$ can therefore be regarded as the ratio of the dark energy coupling strength to the gravitational strength. The dark-dark coupling $\beta$ is unconstrained by local experiments or by $\dot{G}/G$ measurements \cite{24}. Constraints can be derived only by global effects on the cosmological dynamics: for instance, from the requirement of a universe older than 10 Gyr Ref. \cite{20} found

\begin{equation}|eta| < 1\end{equation}
In the following we show that Boomerang puts a much more stringent constraint on the dark-dark coupling.

Neglecting the baryons, the system has a transient attractor (properly speaking, a saddle point) at the point $x_0 = 2/3, y_0 = 0, z_0 = 0$ on which the scale factor expands as $a \sim t^{3/(9 + k^2)}$ (here $t$ is the ordinary time defined as $dr = a(t)dt$), that is, slower than in a ordinary MDE. During this stage, found in Ref. [21], and denoted φMDE in [22], the scalar field kinetic energy and the dark matter have a constant density ratio. When the universe leaves the saddle, it reaches a global attractor if $\mu < 3$, at the point $x_0 = -\mu/3, y_0 = (1 - \mu^2/9)^{1/2}, z_0 = 0$ and the universe expansion follows the law $a \sim t^{3/\mu^2}$ mimicking a perfect fluid with equation of state $p = (w_\phi - 1)\rho$, with $w_\phi = 2\mu^2/9$. The expansion is accelerated only if $\mu < 3$. As long as the baryon component is small with respect to the CDM, the phase space of the system remains qualitatively the same.

It can be noticed that the $\phi$MDE depends on $\beta$, while the subsequent $\phi$-dominated epoch depends only on $\mu$. Therefore, as shown in [22], the possibility to set constraints on the coupling $\beta$ depends on the existence of the $\phi$MDE. This epoch is actually quite more general than appears from the discussion above: it exists in fact for any potential such that $U(\phi)$ and $dU/d\phi$ go asymptotically to zero, as for instance when $U$ is an inverse power law.

The coupled quintessence with exponential potential contains the case of pure cosmological constant ($\mu = 0, \beta = 0$), of uncoupled quintessence ($\beta = 0$) and is asymptotically equivalent to a perfect fluid with a constant equation of state $w_\phi = 2\mu^2/9$. The model is conformally equivalent to a non-minimal gravity 3-1 theory with Lagrangian term $-\frac{1}{2}\xi \dot{\psi}^2 R$ and potential $V(\psi) = \lambda \psi^n$ with $\xi = 2\beta^2/3$ and $n = 4 + \mu/\beta$ [10, 22, 23]. Therefore, a bound on $\beta$ amounts to constraining the non-minimal gravity coupling $\xi$. Notice that in the case we study here, in which the dark energy couples differently to baryons and to dark matter, there are two conformally related metrics in the Jordan frame (the frame in which gravity is coupled to the scalar field): the constant $\xi$ couples the scalar field to the Ricci scalar expressed in the metric in which the dark matter, rather than the baryons, follow the geodetics (see [21]).

The perturbation equations, derived in [24], are integrated by the use of a purposely modified CMBFAST code [24]. In addition to the scalar field, baryons, CDM and radiation, the code includes also massless neutrinos. I choose adiabatic initial conditions, as suggested by inflation. The initial conditions for the background equations are found for each set of parameters by trial and error so as to give today $\Omega_\text{b}, \Omega_\text{c}$ and $H_0$ as requested. This procedure is so time consuming with respect to the code without coupled scalar field that the assumption of flat space and a further reduction in the parameter space explained below turned out to be necessary.

### III. LIKELIHOOD ANALYSIS

Our theoretical model depends on two scalar field parameters and four cosmological parameters:

$$\beta, \mu, n_s, h, \Omega_\text{b}, \Omega_\text{c}$$

The remaining input parameters requested by the CMBFAST code are set as follows: $T_{\text{ cmb}} = 2.726 K, Y_\text{He} = 0.24, N_\text{e} = 3.04, \tau = 0$. The latter quantity specifies the optical depth to Thomson scattering, and is a measure of reionization. In the analysis of [2] this parameter was also included in the general likelihood and, in the flat case, was found to be compatible with zero at slightly more than 1$\sigma$. Moreover, in ref. [3] it is found that fixing $\tau = 0$ has only a minor effect on the other parameters. Therefore here, to further reduce the parameter space, I assume $\tau_c$ to vanish. Two other approximations with respect to [6] have been necessary: first, I did not include the cross-correlation between bandpowers because it is not available. Second, an offset log-normal approximation to the band-power likelihood has been advocated by [23] and adopted by [6], but the $x$-quantity necessary for its evaluation is not available. Since the offset log-normal reduces to a log-normal in the limit of small noise I evaluated the log-normal likelihood

$$-2 \log \mathcal{L}(\alpha_j) = \sum_t \frac{[Z_{t, t}(\ell_i; \alpha_j) - Z_{t, d}(\ell_i)]^2}{\sigma_t^2}$$

where $Z_t \equiv \log \hat{C}_t$, the subscripts $t$ and $d$ refer to the theoretical quantity and to the real data, $\hat{C}_t$ are the spectra binned over some interval of multipoles centered on $\ell_i$, $\sigma_t^2$ are the experimental errors on $Z_{t, d}$, and the parameters are denoted collectively as $\alpha_j$. A 10% calibration error is added to the experimental errors [6]. I also evaluated for comparison a likelihood gaussian in the variables $\hat{C}_t$, and found that the results do not change appreciably up to 2$\sigma$ from the peak.

Among the parameters that refer to the scalar field, we have already shown in [23] that $\mu$ has a negligible effect on the background solution at $z \gg 1$, since the equivalence time does not depend on it and, although $\mu$ sets the speed of the present accelerated expansion, this has only a minor effect on the perturbation spectrum at decoupling. This
will be confirmed below. Also, in [22] we have shown that the dynamics of the system is insensitive to the sign of \( \beta \),
since both the MDE and the final accelerated epoch do not depend on it. We will consider only \( \beta \geq 0 \).

In order to compare with the Boomerang analysis I assume uniform priors as in [7], with the parameters confined in
the range \( \beta \in (0,0.16) \), \( \mu \in (0.2,1) \), \( n_s \in (0.7,1.3) \), \( h \in (0.45,0.9) \), \( \Omega_b \in (0.01,0.2) \),
\( \Omega_c \in (0.1,0.9) \), with the further weak big-bang nucleosynthesis (BBN) constraint \( \Omega_b h^2 < 0.05 \) (and of course the condition \( \Omega_c + \Omega_b \leq 1 \)).
I evaluate the likelihood also using the most up-to-date BBN limit [36] [35], assuming a gaussian prior for \( \Omega_b h^2 \) with
mean 0.019 and 1\( \sigma \) error 0.002 (I refer to this as the strong BBN prior). The range of \( \mu \) includes all the values for
which there is acceleration at the present. A further parameter, the logarithm of the absolute normalization of the
\( C_\ell \) spectrum, is integrated out analytically with uniform prior. The same age constraints (> 10 Gyr) used in [7]
is adopted here. Note that here we do not have the problem of near-degeneracy of parameters that is considered in ref.
[7]. In fact, the degeneracy that exists for those values of \( \Omega_c, \Omega_b, h, \mu \) and \( \beta \) that give an identical angular-diameter
distance to the decoupling surface is completely removed by the combined constraints of flatness, BBN and the allowed
range of \( h \) and \( \mu \). We also checked that the limits on \( n_s \) and \( \beta \) are broad enough to contain the bulk of likelihood.

A grid of \( \sim 10,000 \) multipole CMB spectra \( C_\ell \) is used as a database over which I interpolate to produce the
likelihood function. Since both the COBE and the Boomerang data are in fact binned over intervals of multipoles, I
average in the same bandpowers the theoretical spectra for a correct comparison. Three cases will be distinguished:
pure \( \Lambda \) (\( \mu = \beta = 0 \)); dark energy (\( \beta = 0 \)); coupled dark energy. In Fig. 1 I report examples of multipole spectra
obtained varying \( \mu \) and \( \beta \) and fixing the other parameters: \( \{n_s, \Omega_b, \Omega_c, h\} = \{1, 0.05, 0.3, 0.7\} \). The quantity plotted
is actually \( \left( \ell (\ell + 1) C_\ell / 2 \pi \right)^{1/2} \mu \)K as customary. The strong decrease in amplitude of the acoustic peaks as \( \beta \) gets larger
than 0.1 depends on the fact that for these values the onset of the MDE occurs before the decoupling. During the
MDE the fluctuations smaller than the horizon grow less than during MDE, so that the fluctuations on the acoustic
scales are depressed relative to the larger scales. Moreover, the addition of the integrated Sachs-Wolfe effect at small
multipoles decreases the normalization of the intrinsic fluctuations at decoupling [18] [22].

In Fig. 2 (panels a–d, solid lines) I plot the likelihood function for the parameters \( n_s, h, \Omega_b, \Omega_c \) (marginalizing
over all the other parameters) in the case of pure \( \Lambda \) and weak BBN prior. They are reasonably well in agreement with
the analysis of [7] (their case P10). The means and variances for the pure \( \Lambda \) model are

\[
\begin{align*}
n_s &= 0.96 \pm 0.06, \quad h = 0.73 \pm 0.09, \quad \Omega_b = 0.056 \pm 0.016, \quad \Omega_c = 0.5 \pm 0.2, \\
\end{align*}
\]

while the best estimates (peaks) are \( n_s = 0.975, h = 0.7, \Omega_b = 0.05, \Omega_c = 0.3 \) in agreement with [7] (notice that Ref.
[7] quotes the likelihood maximum for clear detections, e.g. for \( \Omega_b \) and \( n_s \), and the mean in the other cases). For the
case of uncoupled dark energy, I plot in Fig. 2 the likelihood functions in short dashed lines, plus in the panel e the
new parameter \( \mu \). As expected, there is almost complete degeneracy in the direction of \( \mu \), and the results of pure \( \Lambda \)
remain very similar. If the Boomerang errorbars were reduced to one third, the likelihood would begin to show some
preference for higher values of \( \mu \), as shown in the same panel e (dotted line). The problem of a high baryon content
from Boomerang (see e.g. ref. [37]) is not alleviated by this model of dark energy.

The likelihood functions for the coupled dark energy case are plotted in Fig. 2 as long-dashed lines. Due to the
degeneracy along \( \mu \) we can simplify the analysis fixing this parameter to any value in the relevant range; we put
\( \mu = 0.25 \). It turns out that the previous results are quite robust also with respect to the coupling, except for a shift
of \( n_s \) toward smaller values (\( n_s = 0.88 \pm 0.06 \)). In the panel f I plot the coupling \( \beta \) : at the 96.8\% c.l. we obtain

\[
|\beta| < 0.1
\]

Assuming the strong BBN prior we obtain the likelihood functions shown as dot-dashed lines in the same Fig. 2
(shown only for the coupled dark energy case for clarity; the other cases are similar). As expected, smaller \( \Omega_b \) and
smaller \( h \) than before are now acceptable. Also, \( \Omega_c \) moves to smaller values, in order not to decrease excessively the
first peak. This effect, along with the decrease of \( n_s \) in panel b, is observed also in [7] when the strong BBN is imposed.
The likelihood for the exponential slope \( \mu \) (not shown) becomes quite flatter with the strong BBN. Interestingly, \( \beta \)
now peaks around 0.05, because this value raises somewhat the first peak, compensating for the low baryon content.
The limit [41] remains valid.

**IV. CONCLUSIONS**

The most interesting conclusion that can be drawn from our analysis is that the coupling \( \beta \), between dark matter and
dark energy has to be smaller than 0.1, an order of magnitude better than previously [21], independently of the
BBN constraint. Although I derived this limit in the particular case of an exponential potential, it extends to a much
larger class of theories since the MDE is dominated by the kinetic, rather than potential, energy of the scalar field,
as will be shown in another paper,
In terms of the coupling constant $\xi$ of non-minimal gravity theories we get roughly $\xi < 0.01$ at the 96.8% c.l. The likelihood of the other cosmological parameters is robust with respect to the addition of the coupling. The potential parameter $\mu$, which sets the effective equation of state of the dark energy at the present, is not well constrained by the present CMB, since its effect is very recent on the cosmological time scale.

It is of course possible to add further constraints from large-scale structure, supernovae Ia and from other CMB experiments. In [22] a constraint similar to [11] was found from cluster abundance, for a particular choice of the other parameters. Future data, especially high-resolution, high-coverage experiments on CMB, have the potential to strengthen the limit by at least another order of magnitude.

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FIG. 1. Top panel: spectra $C_\ell^\ast \equiv (\ell(\ell+1)C_\ell/2\pi)^{1/2} \mu K$ of uncoupled dark energy for several values of $\mu$ (solid: $\mu = 0$; dots: $\mu = 0.7$; short dashes: $\mu = 1.4$; long dashes: $\mu = 2.1$). Bottom panel: spectra $C_\ell^\ast$ of coupled dark energy for several values of $\beta$ (solid: $\beta = 0$; dots: $\beta = 0.08$; short dashes: $\beta = 0.1$; long dashes: $\beta = 0.12$)
FIG. 2. Marginalized likelihood functions. Solid lines: pure Λ model. Short-dashed lines: dark energy. Long-dashed lines: coupled dark energy. Dot-dashed lines: coupled dark energy imposing the strong BBN prior. In panel e the dotted line represents the likelihood for µ if the Boomerang errorbars were reduced to one third.