On the Sample Complexity of End-to-end Training vs. Semantic Abstraction Training

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Abstract
We compare the end-to-end training approach to a modular approach in which a system is decomposed into semantically meaningful components. We focus on the sample complexity aspect, in the regime where an extremely high accuracy is necessary, as is the case in autonomous driving applications. We demonstrate cases in which the number of training examples required by the end-to-end approach is exponentially larger than the number of examples required by the semantic abstraction approach.

1 Introduction
The recent impressive empirical success of deep learning lead researchers to attempt building complicated systems by an end-to-end training procedure. That is, training examples, which are pairs of an input to the system and the desired output of the system, are generated, and a single artificial neural network is trained to mimic this input-output relationship. An alternative approach is to first break the system into sub modules, where each individual module has a clear semantic meaning. Second, each individual module is constructed either by using a machine learning approach (e.g., the module is trained in an end-to-end manner) or by relying on manual engineering. The choice of which option to use for each module is based on empirical success. We call this approach Semantic Abstraction.

To further demonstrate the two approaches, consider a simplified autonomous driving system, where a car driving in a highway should keep its lane and adapt its speed according to other vehicles. The input to the system is the sensory input (e.g., a video stream from a camera and a radar signal). The output is a two dimensional vector consisting of a steering command and an acceleration/deceleration command. The end-to-end approach will train a single deep network whose input is the sensory input and whose output is the two dimensional vector of control commands. The training examples are pairs of input-output to the whole system. In contrast, the semantic abstraction approach will break the system into several sub-modules. E.g., one module should detect vehicles based on the camera, another one should detect vehicles based on the radar, a third module should fuse the two sources of information. Other modules detect lanes, and other modules make high level driving policy decisions (e.g., “follow the car in front of you” or “be careful from the car on your right because it is likely to cut into your lane”). Finally, a low level control module provides the two dimensional vector of control commands.

Both approaches are far from being new. For example, the end-to-end approach to autonomous driving dates back to [3]. For a detailed description and references, see [1]. There, the end-to-end approach is called “behavior reflex” and the semantic abstraction approach is called “mediated perception”. We note that the term “semantic abstraction” is adapted from the “temporal abstraction” approach to reinforcement learning [5].

There are several advantages and disadvantages of the two approaches (see for example [2] [6] [1]). In this paper we focus on the amount of data required for the training process and for validating the quality of the learnt system. On one hand, the advantage of end-to-end training is that we do not need supervision for individual sub-modules of the system. On the other hand, as we formally show in the next section, in some situations the overall number of examples required by the end-to-end approach might be exponentially larger than the number of examples required by the semantic abstraction approach.
2 Main Results

Consider the problem of learning a system \( f \) that maps from a domain \( X \) into a domain \( Y \). Let \( \ell \) be a loss function that determines failures, namely, \( \ell(x, f(x)) = 1 \) if \( f \) fails on the input \( x \in X \) and \( \ell(x, f(x)) = 0 \) otherwise.

Following the standard PAC learning model (see for example [4] for a precise definition), we define the \((\epsilon, \delta)\)-sample complexity of learning to be the number of training examples required such that there exists a learning algorithm that with probability of at least \( 1 - \delta \) (over the random choice of the training examples) outputs a system \( \hat{f} \) for which \( \Pr[\ell(x, \hat{f}(x)) = 1] \leq \epsilon \). The probability is with respect to a random choice of \( x \) according to some (unknown) distribution \( D \) over the domain \( X \).

Assuming there is some perfect system (which fails with probability 0), classical VC theory (see again [4] for a reference) tells us that the sample complexity is (ignoring constants and logarithmic term) \( \text{VC}(\ell \circ F)/\epsilon \), where \( F \) is the set of systems we aim to learn, \( \ell \circ F = \{ x \mapsto \ell(x, f(x)) : f \in F \} \), and “VC” is the VC dimension.

One may argue that this is a worst-case bound (where the worst situation is w.r.t. the underlying distribution \( D \)), and for “real world” distributions the number of examples can be smaller. To make a stronger lower bound, suppose one already trained a system \( f \), and let us consider the simpler task of just distinguishing between the two cases \( \Pr[\ell(x, f(x)) = 1] \leq \epsilon \) or \( \Pr[\ell(x, f(x)) = 1] \geq 2\epsilon \). In other words, we are considering a “validation” task, determining if our system is good enough or not. Observe that if \( \Pr[\ell(x, f(x)) = 1] = 2\epsilon \) then the expected number of examples we need to observe in order to see a single failure is \( 1/(2\epsilon) \). Therefore, if the number of examples is significantly smaller than \( 1/(2\epsilon) \) we have no way to distinguish between the case \( \Pr[\ell(x, f(x)) = 1] = 2\epsilon \) and the case \( \Pr[\ell(x, f(x)) = 1] \leq \epsilon \). It follows that the sample complexity of this validation task is \( \Omega(1/\epsilon) \).

In some applications, the required \( \epsilon \) is extremely small. For example, in the autonomous driving application, we expect our system to run properly on many cars for many years, which yields an extremely large sample complexity. For an end-to-end system, it is unavoidable to require a sample of size \( \Omega(1/\epsilon) \) just for validating the system, and the sample complexity of training such a system is likely to be several orders of magnitude larger.

We next show that decomposing the problem into semantically meaningful sub-modules may lead to a significantly smaller sample complexity. Let \( g_1, \ldots, g_k \) be functions, where for every \( i \), \( g_i : X \to \{ 0, 1 \} \) is a boolean function that indicates a failure of some sub-module of our full system. For example, \( g_1 \) corresponds to a failure of a sub-module that prevents accidents with other vehicles, \( g_2 \) corresponds to a failure of a sub-module that prevents hitting pedestrians, and so on and so forth.

Let us first focus on some individual sub-module \( g_i \). To simplify the presentation, we omit the under-script, and aim at bounding \( \Pr[g] \), which is the probability of failure of this sub-module. As mentioned before, since we would like to be in a situation that \( \Pr[g] \) is extremely small (meaning that the performance of our sub-module is very good), it follows that the vanilla sample complexity of bounding \( \Pr[g] \) grows like \( 1/\Pr[g] \), which is excessively big.

However, if we make some prior assumptions, we can bound \( \Pr[g] \) using a much smaller number of examples. To motivate the idea, suppose that \( g \) indicates a failure of the sub-module that prevents accidents with other vehicles. Using a semantic abstraction approach, we will construct this sub-module by first having a module that detects vehicles anywhere in the image, and the second module will respond to detected vehicles only if they are in a dangerous position (e.g., immediately in front of us). Denote by \( z_1 \) the indicator of a mis-detection of a vehicle somewhere in the image and by \( z_2 \) the indicator of a vehicle being in a dangerous position. Then,

\[ \Pr[g] = \Pr[z_2] \Pr[z_1|z_2] . \]

We now introduce the prior assumption that \( \Pr[z_1|z_2] \leq \Pr[z_1] \). That is, the probability of mis-detection of a car which is in a dangerous position is at most the probability of mis-detection of a car in a general position. This is a reasonable assumption because cars in a dangerous position are close to us, and hence it is easier to detect them. Furthermore, rare type of cars, on which we are more likely to err, are more likely being on the side of the road then immediately in front of us. See Figure 1 for an illustration.

Under the above assumption, we obtain that

\[ \Pr[g] \leq \Pr[z_2] \Pr[z_1] . \]

We can continue the same line of thinking by having the vehicle detection system depends both on a camera-based system and a radar-based system. To generalize such arguments we rely on the following definition.
Figure 1: The probability of observing a rare type of vehicle somewhere in the image is much higher than the probability of observing such a vehicle immediately in front of us.

**Definition 1 (approximate independence)**  We say that $z_1$ is $c$-approximately independent of $z_2$ if

$$P[z_1 | z_2] \leq c P[z_1].$$

Relying on this definition, we have that if $g = z_1 \land \ldots \land z_T$, and each $z_t$ is $c$-approximately independent of $z_1, \ldots, z_{t-1}$, then

$$P[g] \leq c^T \prod_{t=1}^{T} P[z_t].$$

The advantage of the above is that if $(cP[z_t]) \ll 1$ then $P[g]$ decreases exponentially with $T$. At the same time, each $P[z_t]$ need not be excessively small to guarantee a very small bound on $P[g]$. For example, if $P[z_t]$ is order of $10^{-6}$, $c = 1.1$, and $T = 3$, we end up with $P[g] \leq 1.34 \times 10^{-18}$.

To bound each individual $z_t$, we rely on the following lemma, whose proof follows from Bernstein’s inequality (see Lemma B.10 in [4]):

**Lemma 1**  Consider flipping $m$ times a coin, whose probability to fall on “head” is $p$. Let $\hat{p}$ be the fraction of times the coin fell on “head”. Then, with probability of at least $1 - \delta$ over the $m$ flips we have

$$p \leq \hat{p} + \sqrt{2\hat{p} \log(1/\delta)/m} + 4 \log(1/\delta)/m.$$ 

In particular, if $\hat{p} = 0$ we have that $p \leq 4 \log(1/\delta)/m$.

Combining the above lemma with the union bound, we obtain our main result:

**Corollary 1**  Consider a function $g(x) = z_1(x) \land \ldots \land z_T(x)$, and assume that for every $t$, $z_t$ is $c_t$ independent of $z_1, \ldots, z_{t-1}$. Let $x_1, \ldots, x_m$ be $m$ random examples and denote $\hat{p}_t = |\{i : z_t(x_i) = 1\}|/m$. Then, for every $\delta \in (0,1)$, with probability of at least $1 - \delta$ we have

$$P[g] \leq \prod_{t=1}^{T} c_t \left( \hat{p}_t + \sqrt{2\hat{p}_t \log(T/\delta)/m} + 4 \log(T/\delta)/m \right).$$

The interesting fact about the above corollary is that the upper bound on $P[g]$ can be much smaller than $1/m$. That is, it may be the case that we do not see even a single failure of $g$ on our $m$ training examples, yet we can guarantee a strong bound on the probability of failure of $g$. 

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So far we have shown how to bound the probability of a failure of a single sub-module of our full system. We now get back to estimating the performance of our full system, namely, bounding \( \mathbb{P}[\ell(x, f(x))] \). Unlike the case of bounding \( \mathbb{P}[g_j(x)] \), where we made the strong assumption that \( g_j \) is an AND of several events, for the full system we do not impose such a strong assumption on \( \ell(x, f(x)) \). We only rely on the very mild assumption that the probability of our system to fail given that all of its sub-modules work properly is at most \( 1/2 \). The following lemma bounds the failure probability of the entire system in terms of the failure probability of each sub-module and a residual term.

**Lemma 2** Assume that \( \mathbb{P}[\neg g_1(x) \land \ldots \land \neg g_k(x)] \geq 0.5 \), then

\[
\mathbb{P}[\ell(x, f(x))] \leq 2 \sum_{j=1}^{k} \mathbb{P}[g_j(x)] + \mathbb{P}[h(x) | \neg g_1(x) \land \ldots \land \neg g_k(x)].
\]

**Proof** To simplify the notation denote by \( h(x) = \ell(x, f(x)) \). Using the law of total probability we can write

\[
\mathbb{P}[h(x)] = \mathbb{P}[g_1(x)] \mathbb{P}[h(x) | g_1(x)] + \mathbb{P}[\neg g_1(x)] \mathbb{P}[h(x) | \neg g_1(x)]
\leq \mathbb{P}[g_1(x)] + \mathbb{P}[h(x) | \neg g_1(x)]
\leq \mathbb{P}[g_1(x)] + \mathbb{P}[g_2(x) | \neg g_1(x)] + \mathbb{P}[h(x) | \land_{i \leq 2} \neg g_i(x)]
\leq \left( \sum_{j=1}^{k} \mathbb{P}[g_j(x) | \land_{i < j} \neg g_i(x)] \right) + \mathbb{P}[h(x) | \land_{i \leq k} \neg g_i(x)]
\]

Using the inequality \( \mathbb{P}[A|B] = \frac{\mathbb{P}[A \land B]}{\mathbb{P}[B]} \leq \frac{\mathbb{P}[A]}{\mathbb{P}[B]} \) and the assumptions in the lemma, the claim follows.

To interpret the lemma, recall that we have previously shown how to upper bound \( \mathbb{P}[g_j(x)] \), and we expect these terms to be exponentially small. Therefore, their sum is also likely to be rather small. The last term in the lemma is the probability of a system failure given that all of the sub-modules worked properly. In a sense, such failures account for cases which are beyond our control (e.g., in the context of autonomous driving, such a failure can be due to a sudden tire explosion). We assume that the probability of such events is close enough to zero.

To summarize, we have shown how utilizing prior knowledge and decomposing our system into semantically meaningful sub-modules enable us to upper bound the failure probability of our entire system by a quantity which is much smaller than \( 1/m \). This is impossible for an end-to-end system, where we must see order of \( 1/\epsilon \) examples in order to ensure that the probability of failure is below \( \epsilon \).

**References**

[1] Chenyi Chen, Ari Seff, Alain Kornhauser, and Jianxiong Xiao. Deepdriving: Learning affordance for direct perception in autonomous driving. In *Proceedings of the IEEE International Conference on Computer Vision*, pages 2722–2730, 2015.

[2] James J Gibson. *The ecological approach to visual perception: classic edition*. Psychology Press, 2014.

[3] Dean A Pomerleau. Alvinn: An autonomous land vehicle in a neural network. Technical report, DTIC Document, 1989.

[4] S. Shalev-Shwartz and S. Ben-David. *Understanding Machine Learning: From Theory to Algorithms*. Cambridge University Press, 2014.

[5] Richard S Sutton, Doina Precup, and Satinder Singh. Between mdps and semi-mdps: A framework for temporal abstraction in reinforcement learning. *Artificial intelligence*, 112(1):181–211, 1999.

[6] Shimon Ullman. Against direct perception. *Behavioral and Brain Sciences*, 3(03):373–381, 1980.