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Prediction of the fundamental period of vibration of braced frame systems in irregular steel buildings

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Abstract: Braced frames are frequently used as lateral load resisting systems in steel buildings due to their cost-effectiveness and efficiency in resisting earthquake loads. Evaluating the seismic loads on those buildings requires estimating the fundamental period of vibration of the buildings at first. Most design codes use empirical formulas that depend only on the height of the buildings without considering the effect of bracing configurations or building irregularities. This paper aims to develop new equations to estimate the fundamental period of vibration of irregular steel buildings occupied with different bracing systems. For this purpose, 176 prototype buildings with different bracing configurations and irregularities have been selected and modeled using ETABS finite element program. Three types of irregularity have been considered: vertical, horizontal, and combined irregularity. Then, the fundamental periods of vibration have been estimated through analysis and optimal building design using nonlinear regression analysis. According to the findings, the configuration of the bracing system and the building irregularity influence the fundamental period of vibration of buildings of the same height.

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PUBLIC INTEREST STATEMENT

The Fundamental period of vibration is critical since it is one of the main parameters in estimating the seismic loads on structures. This study aims to investigate the effect of lateral load resisting systems and irregularities of building configuration on the fundamental period of vibration for steel structures and provide a new empirical equation to calculate the fundamental period. For this purpose, steel buildings with different lateral systems and irregularities were selected. One hundred and eight concentric braced frames (CBFs), and 68-eccentric braced frames (EBFs) were studied and analyzed by modal analysis. Based on this analysis, new empirical equations are derived for estimating the fundamental period, which considers the geometric irregularity and the bracing configuration for laterally bracing systems. Regression analysis with five variables was performed for fundamental period data. According to the findings, the configuration of the bracing system and the building irregularity influence the fundamental period of vibration of buildings having the same height.
where vertical and combined irregularities decrease the period values by about 20% for CBFs. This indicates the importance of incorporating new equations in calculating the fundamental period of these types of steel structures.

Subjects: Concrete & Cement; Waste & Recycling; Pollution

Keywords: steel buildings; fundamental period of vibration; irregularity; EBF; CBF; regression analysis

1. Introduction

The fundamental period of vibration, T, is one of the most important primary data required to calculate earthquake forces, whether using the equivalent static load method or the dynamic analysis in most design codes (Chopra & Goel, 2000; Gioncu & Mazzolani, 2013; Landolfo, 2018; Mazzolani & Piluso, 1996). Unfortunately, this period T cannot be determined until after the initial design of the buildings. Accordingly, design codes propose empirical formulas to estimate the period T based on material type, lateral load resisting system, and overall dimensions of the structures. Also, in steel structures occupied with concentrically braced frames (CBFs) and eccentrically braced frames (EBFs) as a lateral resisting system, design codes provide a separate expression for the period T for each bracing system. However, these expressions depend only on the height of the buildings without due consideration to the configuration of the bracing systems or building irregularities. Thus, the bracing geometry effect on period T is questionable, particularly in irregular steel buildings, which this paper addresses.

Several empirical formulas have been used to calculate the period T. These empirical formulas provided by most of the international design codes, and standards (ASEC 10–16, Eurocode 8, FEMA, and Egyptian Code ECP) are low-power formulas that are based on Rayleigh’s method and depending on the building height (H; Ruggieri et al., 2021) as follows:

\[ T = \alpha H^{\beta} \]  

(1)

Regression analysis is used to generate the parameters \( \alpha \) and \( \beta \) from the building’s database. Accordingly, there is no unique approach for predicting the value of T for all building geometries since it depends on the collected data from the literature. Besides, the collected data is few and extremely affected by numerical modeling assumptions (Uva et al., 2018).

The value of T is quite difficult to be predicted for all building geometries using one formula since it is empirically calculated based on real or numerical databases. Also, these data are affected by the time of construction, design philosophy and the construction site.

Several studies have been conducted on moment-resisting steel buildings to predict the fundamental period T (Elghazouli, 2010; Ferraioli et al., 2014). In addition, some studies have been carried out on CBFs and EBFs (Tremblay & Robert, 2001). In this paper, steel buildings occupied with CBFs and EBFs systems having different bracing configurations are studied to arrive at a new formula for these systems. Moreover, the effect of various building irregularities on period T is investigated.

1.1. Related work

Several attempts have been made to predict the fundamental period of vibration of buildings (Adeli, 1985; Amini et al., 2012; Cinitha, 2012; Hemmati & Kheyroddin, 2013; Hsiao, 2009; Pandit & Shinde, 2015; Ruggieri et al., 2021; Smith & Crowe, 1986; Yousef et al., 2010; Zalca, 2001. Nassani 2014) presented a new formula, based on the Rayleigh equation, to approximately calculate the period of vibration of steel buildings, considering the steel buildings’ connection rigidity, mass, and stiffness. To validate the proposed formula’s accuracy, Nassani presented several examples in
which the fundamental mode periods of various structures were calculated using the proposed formula and conventional empirical equation. Goel and Chopra (1997) developed a formula for estimating the fundamental period of vibration of reinforced concrete and moment-resisting frames using regression analysis. The study included a 42 buildings database for the moment-resisting frame of steel buildings. It was observed that calculated code periods are shorter than the measured periods from the recorded motions. For buildings up to 36 m, code formulas give approximately lower-bound values of measured period data; on the other hand, they result in 20–30% shorter than measured data for buildings taller than 36 m. It was concluded that the database must be expanded with the new earthquake data, and regression analyses should be repeated periodically on larger data sets. Chrysanthakopoulos et al. (2006) provided an approximate formula to determine the first three natural periods of vibration of plan steel unbraced and braced frames. The steel frame was modeled as an equivalent cantilever beam for which analytical expressions for the natural periods are available. Extensive parametric studies were conducted on 110 braced and unbraced frames analyzed with the finite element method to establish a formula that reflects the character of an equivalent cantilever beam. (Young, 2011) studied the fundamental period of MRFs with different irregularities. New equations were suggested for the moment frames considering vertical and horizontal irregularities. It was revealed that the proposed equation yields a more reasonable prediction than the ASCE 7–10 code equation. (Young & Adeli, 2014, 2016) addressed the fundamental period of CBFs and ECFs with building irregularities. A three-variable power model was formulated considering irregularities using statistical analysis. The proposed formulas have resulted in a better fit to the Rayleigh data than the empirical equation that depends on height only. Available period data for CBFs and EBFs were then used to validate the proposed equations. Gong et al. (2011) presented an empirical formula for evaluating the fundamental period of MRFs by analyzing a total of 36 such structures in California. All structures had recorded seismic response data from at least one seismic event. The buildings ranged from 3 stories to 18 stories, with all buildings having a total height of less than 80 m, the fundamental period was expressed in terms of the building height to the power of 0.6 and multiplied by 0.12.

Aboelmaged et al. (2020) studied the effect of the lateral structural system on the fundamental period of vibration of steel buildings occupied with moment-resisting frames and bracing systems. Two hundred and twelve case studies were investigated using numerical analysis considering building irregularities. It was found that lateral systems considerably affect the fundamental period of steel structures. Harris et al. (2019) proposed a straightforward modification to the ASCE 7–16 fundamental period equation to incorporate change in system strength due to seismic categories. Eighteen archetype steel buildings with different seismic design resisting systems (MRFs, CBFs, and EBFs systems) were selected and designed for different risk categories. It was aimed to preserve the design conservatism consistent with the risk target among risk categories. It was revealed that the analytical fundamental period is affected by the variation of the building importance factor. Also, it was concluded that the code formula could be modified by dividing it by the root of two for structural steel systems.

Kwon and Kim (2010) presented an extensive review of the evolution of the ASCE code equations from the 1970s. In addition, a quantitative comparison of measured fundamental periods and estimated fundamental periods calculated from the code equations were conducted. They included 65 steel MRF buildings, 17 CBF buildings, and 8 EBF buildings in California, with a few exhibiting geometric irregularities. The authors determined that the code formula generally provides a conservative prediction for the fundamental period for MRF buildings of all heights ranging from single-story buildings to buildings of 215 m. When considering concentrically braced frames, the authors compared the measured periods with the estimated period from ASCE 7–10. For structures over 215 m, ASCE7-10 yielded a large underestimate, whereas for low- to medium-rise structures, the equation generally followed the lower bound of the measured data. The eight measured fundamental periods of EBFs were compared with the code equation. However, the measured data only reflected structures under 30 m and over 76 m, making a conclusion regarding the relationship between measured data and the code equation difficult. For the given data points,
it appeared that ASCE7-10 yields accurate results for structures under 30 m and generally under-estimates the period for structures over 76 m. Tremblay (2005) performed an analytical study to propose a simple expression for the fundamental period of CBFs buildings located in moderate and low seismic regions. Seven thousand five hundred twenty-four buildings were analyzed in this study. Available test and field data of building periods were compared to ones obtained with the analytical predictions. Building and design parameters were examined through a closed-form solution and an extensive parametric study. The simplified closed-form model concluded that large variations in structure periods are related to differences in seismic zones. Also, the fundamental period of concentrically braced frames varies with the frame geometry and the magnitude of the seismic design loads. Moreover, it was shown that period values of CBFs increase with the building height linearly, and as a lower-bound estimation, $T = 0.025$ of the building height is recommended.

Güneydin (2012) aimed to evaluate the fundamental periods of CBFs designed according to Euro code 8. A two-phase research study was performed. In the first phase, several CBFs were designed according to Euro code 8, and the accuracy of the empirical formulas was evaluated. A more accurate empirical equation was developed in the second phase based on the database formed in the first phase. This new empirical equation considers various factors such as the level of seismicity, soil conditions, gross dimension, and mass properties. In general, regardless of the type of design conducted, the final periods are longer than those obtained using the lower bound expression given in Eurocode 8.

1.2. Research significance

The dynamic approach and the equivalent horizontal force method used in seismic design codes worldwide require selecting seismic inputs based on the fundamental period of vibration. Thus, there is evidence that the fundamental period plays a decisive role in the relationship between seismic demand and capacity, thus assessing the structure's seismic performance.

Few studies have been conducted to predict the fundamental period of vibration of irregular steel buildings occupied with CBF and EBF, especially the effect of bracing system configurations. Field and test measurements have been used to formulate a simplified expression for estimating buildings' fundamental period reflecting its importance in seismic load calculations (Goel & Chopra, 1997). Some effort has been made to develop a simplified formulation suitable for certain buildings. The current study aims to provide the practical design engineer with a simple, approximate formula to calculate the fundamental period using building data input.

This study investigates 176 prototype buildings with various bracing systems analyzed and optimally designed using ETABS software. The bracing systems include both CBFs and EBFs with various configurations.

2. Available formulae for fundamental period adopted by current design codes and previous studies

The fundamental period of vibration $T$ for steel structures must be determined during the design stage to calculate earthquake base shear. Table 1 summarizes the value of $T$ used in different international codes and adopted in the literature.

Based on the listed code equations and formulation collected, it could be noticed that

- Code formulas are almost similar between the different codes.
- The building height is the main parameter in predicting the period of vibration.
- Irregularities in vertical and horizontal directions are not considered.
- Bracing system configuration has not been explicitly considered.
Table 1. Summary of the Codes formula of the fundamental period of vibration and previously adopted formulas for CBFs and EBFs

| Code                      | CBF                              | EBF                              |
|---------------------------|----------------------------------|----------------------------------|
| ASCE/SEI 7-16 (2016)      | $T = 0.0488H^{0.75}$            | $T = 0.0731H^{0.75}$            |
| Eurocode 8 (2004)         | $T = 0.05H^{0.75}$              | $T = 0.0731H^{0.75}$            |
| AS1176.4 (2007)           | $T = 0.0625H^{0.75}$            | $T = 0.075H^{0.75}$             |
| KBC (2016)                | $T = 0.049H^{0.75}$             | $T = 0.075H^{0.75}$             |
| Italian Code (2008)       | $T = 0.05H^{0.75}$              | $T = 0.05H^{0.75}$              |
| FEMA                     | $T = 0.0753H^{0.75}$            | $T = 0.086H^{0.75}$             |
| Indian Code (2016)        | $T = 0.085H^{0.75}$             | $T = 0.086H^{0.75}$             |
| SBC 301-CR-18 (2018)      | $T = 0.0731H^{0.75}$            | $T = 0.0488H^{0.75}$            |
| Iranian code of practice (2007) | $T = 0.05H^{0.75}$             | $T = 0.05H^{0.75}$              |
| Egyptian code of practice ECP (2012) | $T = 0.0488H^{0.75}$             | $T = 0.0753H^{0.75}$            |
| Young & Adeli, 2014 and Young & Adeli, 2016 | $0.036H^{0.65} \left(\frac{H}{D}\right)^{0.6} \left(\frac{D}{H}\right)^{0.35}$ | $0.042H^{0.65} \left(\frac{H}{D}\right)^{0.6} \left(\frac{D}{H}\right)^{0.35}$ |

Where $H_{av}$ and $D_{av}$ are the average height and dimension of the braced frame in the applied load direction.

3. Buildings prototypes

3.1. Description and design assumptions

The case study considered in this investigation is 176 braced frame buildings, which consist of 108 CBF and 68 EBF. The design of buildings was based on the Egyptian code of practice for steel construction ECP 201 (2018). Also, the buildings were modeled and optimally designed with variations in building height, the number of bays, and irregularity type.

Three category buildings with 5, 8 and 12 stories for each bracing system were selected. Each category consists of 12 building models. The floor plan of the buildings consists of five bays in the X-direction and 3, 6, and 8 bays in the Y-direction. The bay span in each direction is 6 m, and the story height is 3 m. The bays in Y-direction were divided by secondary beams every 2 m. Three types of irregularities were addressed based on the Egyptian standard; Vertical, horizontal, and combined irregularity, as shown in Figure 1. Three different configurations were used in the CBF

Figure 1. Types of irregularities in the selected buildings.

a. In-plan horizontal irregularity (H)  b. In-height vertical irregularity (V)  c. Combined irregularities (c)
Figure 2. Bracing system configurations.

system, while two configurations were used for the EBF system, as shown in Figure 2. These configurations are X, A, and D bracing for CBF; Z and K bracing for the EBF. The parameters used in investigated buildings are summarized in Table 2. The number of bays was selected to apply the setback that presents the defined irregularities in different codes. Also, the number of stories was selected to reflect both low- and high-rise buildings.

An equivalent load procedure has been used to calculate seismic loads on the selected buildings. Also, soil class C and response spectrum type 1 have been considered assuming a medium seismicity site, with a peak ground acceleration of 0.15 g, where g is the gravity acceleration. Also, we assumed uniform dead and live load on each floor where steel deck with 100 mm concrete slab and 1.5 kN/m² for ceiling and mechanical ducts included. Moreover, floor loads due to finishes have been accounted for. A 3 kN/m has been assigned on the outer beams for wall loads. In addition, we assigned a 3 kN/m² live load, the overloads for administration buildings, on all floors. All buildings have been designed considering steel-grade ST37 with a minimum yield stress of 240 N/mm² and tensile strength of 360 N/mm². Buildings were designed based on gravity and seismic load combinations to satisfy the ECP requirements. A list of European steel sections and some built-up sections was assigned during the analysis of the buildings. An iterative procedure was then used to obtain the optimum design sections from the assigned ones, which converged after many iterations.

3.2. Numerical modeling

A numerical model has been constructed for each building using the finite element (FE) software ETABS, where beams and columns are modeled as frame elements. Fixed columns base is assumed in the analysis. Also, linear analysis is assumed throughout the analysis of all buildings. Moreover, internal constraints have been considered using a rigid diaphragm at each floor level. It should be noted that the contribution of nonstructural elements to the building's lateral stiffness was not accounted for in the analysis of buildings. The following procedure is used with ETABS software to perform analysis and design: Add beams and columns to the auto-selection list, including European and built-up sections. ETABS selects the median section from the list for analysis and executes the design. After many iterations to merge the design and analysis sections, the program changes the sections to achieve the optimum design.

All model identification starts with frame type concentrically braced frame CBF or EBF followed by the number of Stories, then the number of adjustable bays in the Y-direction. In contrast, the type of irregularity abbreviation (V) is used for vertical irregularity, (H) for horizontal irregularity and (C) for combined irregularity. Also, the bracing configurations X, D, A, Z, and K are placed after the bracing type. For example, CBF-X-8-6-H means the concentrically braced frame of x-bracing configuration with eight stories in height, six bays in the y-direction, and the building have horizontal irregularity. In comparison, EBF-K-12-6-C means the eccentrically braced frame of K bracing configuration with 12 stories in height, six bays in the y-direction, and combined irregularities. Naming with no indication of irregularity type means regular geometry. Figure 3
and 4 demonstrate the model’s labels and configurations for eight-story buildings occupied by CBF and EBF. It is worth mentioning that the influence of nonstructural elements of the buildings is not considered in this study. The author believes this influence will have a minor impact on calculating the period of vibration for open steel structures with no infill walls.

4. Regression analysis and new proposed equations
Eigenvalue analysis is carried out for prototype buildings. The analysis results database, listed in Appendix A (Tables A1, A2, A3, A4, A5), is used as an input in the regression analysis to predict the
Table 2. Parameters of studied buildings for CBF and EBF system

| Number of stories | 5    | 8    | 12   |
|-------------------|------|------|------|
| Number of bays in |      |      |      |
| X-direction       | 5 @ 6.0 m |      |      |
| Number of bays in |      |      |      |
| Y-direction       | 3 @ 6.0 m | 6 @ 6.0 m | 8 @ 6.0 m |
| Building irregularities | H | V | C |
| CBF configuration | X | D | A |
| EBF configuration | Z | K |      |

Figure 4. EBFs prototype buildings with different configurations for eight-story buildings.

The fundamental period of vibration. The nonlinear regression is based on determining the parameter values that minimize the sum of the residual parameters iteratively (Brown 2001; Crowley & Pinho, 2010). The method uses the standard error of estimates (σ) and the coefficient of determination (R²) values for regression. The value of the coefficient of determination R² is defined as the proportion of variability in the data where the value of R² = 1.0 means that the curve passes through every data point, whereas R² = 0 means that the regression model does not describe the data any better than a horizontal line passing through the average of the database points. A zero standard deviation indicates that the regression model accurately describes the data.

Although Eq. 1 provides a simple approach to estimating the period of vibration, it has some conceptual constraints. It depends on the building height only without considering some other parameter that may be affecting the T value. Of these parameters that can be indicated in T calculation, in particular (schematically indicated in Figure 5):
Accordingly, a new equation is proposed which depends not only on the building height but also considers building irregularity parameters as follows:

$$T = \alpha H \left( \frac{B_1}{B_2} \right)^\beta_1 \left( \frac{A_{av}}{A} \right)_Y \left( \frac{A_{av}}{A} \right)_X$$

(3)

Where $\alpha$, $\beta_1$, $\beta_2$, $\beta_3$ are the regression parameters. The ratio of $B_1/B_2$ and the building height $H$ accounts for building stiffness, while the ratio of the average side area to the total area accounts for irregularities of the building.

A correlation between the period $T$ of CBF-X bracing and EBFs-A bracing with building height $H$, $B_1/B_2$, $(A_{av}/A)_Y$, and $(A_{av}/A)_X$ is shown in Figure 6. The value of $T$ ranges from 0.5 sec to 2.5 sec for CBFs X-bracing configuration. In comparison, it ranges from 1.0 sec to 3.0 sec for EBFs K-bracing configuration, and this difference is mainly due to the stiffness of the X-bracing relative to the K-bracing. Also,

4.1. Concentrically braced frames

The multiple power regression model in Eq (3) has been used to develop the period $T$ for CBF having different bracing configurations in Table 3. The results are shown in Eq (4–6) for the X, D, and A bracing systems, respectively.
Figure 6. Fundamental period of vibration for CBFs-X bracing and EBF-K bracing with building height (H), side ratio ($B_1/B_2$) and side areas ratios ($A_{avg}/A_y$, and ($A_{avg}/A_x$).

\[ T(X \text{ - Bracing}) = 0.06H^{1.0} \left( \frac{B_1}{B_2} \right)^{0.2} \left( \frac{A_{avg}}{A} \right)_y^{0.8} \left( \frac{A_{avg}}{A} \right)_x^{0.8} \]  

(4)

\[ T(D \text{ - Bracing}) = 0.066H^{1.0} \left( \frac{B_1}{B_2} \right)^{0.2} \left( \frac{A_{avg}}{A} \right)_y^{0.8} \left( \frac{A_{avg}}{A} \right)_x^{0.8} \]  

(5)
The multiple power regression model in Eq (3) has been used to develop the period \( T \) for CBF with different bracing configurations. The results are shown in Eq (4a–6b) for the X, D, and A bracing systems, respectively.

\[
T(A - Bracing) = 0.058 H^{1.0} \left( \frac{B_1}{B_2} \right)^{0.2} \left( \frac{A_{ov}}{A} \right)^{0.8} \left( \frac{A_{ov}}{A} \right)_X^{0.8}
\]

\[
T(X - Bracing) = 0.06 H^{1.0} \left( \frac{B_1}{B_2} \right)^{0.2} \left( \frac{A_{ov}}{A} \right)^{0.8} \left( \frac{A_{ov}}{A} \right)_X^{0.8}
\]

\[
T(D - Bracing) = 0.066 H^{1.0} \left( \frac{B_1}{B_2} \right)^{0.2} \left( \frac{A_{ov}}{A} \right)^{0.8} \left( \frac{A_{ov}}{A} \right)_X^{0.8}
\]

\[
T(A - Bracing) = 0.058 H^{1.0} \left( \frac{B_1}{B_2} \right)^{0.2} \left( \frac{A_{ov}}{A} \right)^{0.8} \left( \frac{A_{ov}}{A} \right)_X^{0.8}
\]

Regression parameters, coefficient of determination \( R^2 \), and standard errors of estimate \( \sigma \) have been evaluated to assess the accuracy of the proposed equations. These parameters are shown in Table 3, where the coefficient of determination \( R^2 \) presents the proportional between the database variability and the correctness of the suggested models, and \( \sigma \) indicates the variance between the database and the predicted results. The values \( R^2 \) are very close to one, which means that the suggested models predict the database results well. On the contrary, the values of \( \sigma \) are close to zero, indicating a good approximation of the suggested models to the database results.

### 4.3. Eccentrically braced frames

The multiple power regression model in Eq (3) has also been used to develop the period \( T \) for EBF with different bracing configurations. The results are shown in Eq (7) and Eq (8) for the K and Z bracing, respectively.

\[
T(K - bracing) = 0.07 H^{1.0} \left( \frac{B_1}{B_2} \right)^{0.25} \left( \frac{A_{ov}}{A} \right)^{0.3} \left( \frac{A_{ov}}{A} \right)_X^{0.3}
\]

\[
T(Z - bracing) = 0.64 H^{0.46} \left( \frac{B_1}{B_2} \right)^{0.04} \left( \frac{A_{ov}}{A} \right)^{0.7} \left( \frac{A_{ov}}{A} \right)_X^{0.7}
\]

In addition, regression parameters, coefficient of determination \( R^2 \) and standard errors of estimate \( \sigma \) are listed in Table 4. Their values, as per CBFs, indicate that the proposed equation correlates well with the database results.
5. Results and discussion

5.1. Comparison of proposed equations with code formulas
For verification purposes, the proposed equation results have been compared with the results provided by the ASCE 7-16 and ECP 2012 codes, as illustrated in Fig. 5 and 6 for CBFs and ECBFs, respectively. This way of validation is used to check for any overfitting issues in the proposed models. It could be seen from the comparison that the proposed models perform in a similar trend to the results of the technical codes. For all bracing configurations, the period \( T \) from the proposed models is higher than the code’s value; this is due to the nonstructural element not being considered in the finite element modeling of the prototype buildings, which, if considered, will add stiffness to the buildings. These results are in line with the work by Young and Adeli (2014), where their proposed equation values were also higher than the values of the codes due to the non-inclusion of nonstructural elements. Thus, the proposed model is considered an upper bound to the adopted values used in design codes.

5.2. Adaptation of the proposed equations and validation of the proposed equations
Using completely different new steel buildings occupied with CBF and EBF system is the best way to validate the current regression model of Eq (4–8). However, this approach is quite hard since the authors do not have other analogous data to the one studied. Alternatively, regression model results were intended to be compared with measured database records. The problem is that the available records are for existing steel buildings occupied with nonstructural elements that are not included in the prototype buildings modeling. Accordingly, the validation was performed in two stages. First, the best-fit equation is reduced by comparing the proposed equation with real measured period data from Kwon and Kim (2010) for CBFs with X-bracing systems. Several trials are performed to adapt a new equation conforming to the measured data, Eq (9). The adapted equation was compared with measured data, as shown in Figure 7. Second, the results of the adapted Eq (9) are compared with ambient vibration test results of Mirtaheri and Salehi (2018) as shown in Table 5 where \( \Delta T \) values present the percentage difference between record data and adapted equation. It could be noted that the difference is small for building 1 and 2 while the difference is high for building 3. This high difference may be due to the lack of information on the tested buildings, like irregularities.

\[
T(X - Bracing) = 0.028H^{1.0} \left( \frac{B_1}{B_2} \right)^{0.2} \left( \frac{A_{ov}}{A} \right)^{0.8} \left( \frac{A_{ov}}{A} \right)^{0.8}
\]  

5.3. Effect of bracing configurations and irregularities results
Figure 8 illustrates the results of the best fit of the Rayleigh period database from Eq (4–6) for the various CBFs bracing configurations. Similarly, Figure 9 shows the results from Eq (7) and Eq (8) for EBFs bracing configurations. The database values are scattered vertically because of the irregularities of the selected buildings, and the regression model results are overlapped with the database values. For CBFs, the A-shape bracing presents the lower fundamental period for all building heights, while the D-bracing presents the higher one, indicating that the A bracing is stiffer than the X and D bracing systems. For EBFs, the D-bracing has a longer period than K-bracing, indicating

\[
\text{Table 4. Regression coefficients for EBFs system}
\]

| CBF system | \( \alpha \) | \( \beta \) | \( \beta_1 \) | \( \beta_2 \) | \( \beta_3 \) | \( R^2 \) | \( \sigma \) |
|------------|-----------|-----------|-----------|-----------|-----------|--------|--------|
| K-bracing  | 0.07      | 1.0       | 0.25      | 0.3       | 0.3       | 0.92   | 0.174  |
| Z-bracing  | 0.64      | 0.64      | 0.04      | 0.7       | 0.7       | 0.93   | 0.096  |
that the latter system is much stiffer. Also, period T for EBFs has a longer period than the CBFs for all bracing configurations, which is in line with the code formulas.

Increasing the building bays from 3 to 8 increases period T’s values by up to 20% in CBFs for all configurations, as depicted in Figure 10. The ratio of increase reaches 30% in the case of the EBFs of the K-bracing system for the same increase in bays, as seen in Figure 11. In contrast, an increase in the number of bays in EBFs of Z-bracing has little effect on the period T.

5.4. Effect of building irregularities
Figures 12, 13, 14 show the effect of horizontal, vertical and combined irregularities on the period T of CBFs for buildings with different bays. The same is shown in Figures 15, 16, 17 for EBFs. There is no variation in T values due to horizontal irregularities in CBFs buildings with 3 and 6 bays and EBFs for all bracing geometry; however, the values of period T in eight-bay buildings of CBFs, having horizontal irregularities are about 10% lower than the regular ones for buildings of 36 m high. This can be explained by the fact that both regular and horizontal irregular building has similar parameters.

On the other hand, the vertical and combined irregularities impact the values of T, as can be revealed from Figures 12, 13, 14, where the values of T are 20% less than the corresponding values of regular buildings for all CBFs configurations. This decrease is only 10% in the case of EBFs, as illustrated in Figures 15, 16, 17.

6. Limitation of the study
The authors believe it is important to discuss the limitations of the proposed equations for the fundamental period of vibration as follows:

- Medium seismicity ground acceleration of 0.15 g is used in the design of the prototype buildings. This means other equations may be generated for other seismic zones.
Figure 8. Comparison of the proposed T with building with code equation for CBF’s buildings.
Nonstructural elements are not considered in buildings modeling, which, if considered, may result in stiffer buildings and a lower fundamental period of vibration.

7. Conclusions
This research proposes new equations for predicting the fundamental period of vibration of CBF and EBF having different bracing configurations and irregularities. A group of 108 CBFs and 68 EBFs buildings prototype occupied with different bracing configurations have been optimally designed and analyzed using modal analysis. The fundamental periods of the buildings have been utilized
for developing new formulations by employing nonlinear regression analysis. Accordingly, the following conclusions can be adapted:

1. The proposed equations in this research provide a fundamental period $T$ close to the calculated from modal and Rayleigh equation.

2. The horizontal irregularity has little effect on the fundamental period of vibration for both CBFs and EBFs for all configurations except for CBFs of 8 bays, where a 10% decrease in period is noticed.

3. The combined irregularity in buildings presents results lower than in regular buildings; this difference appears in CBFs more than in EBFs.

4. Codes equations results are conservative compared with the current model for all bracing systems. However, the proposed model values are in the same trend as the previous
Figure 12. Effect of irregularities on the period T for CBFs with three bays.

Figure 13. Effect of irregularities on the period T for CBFs with six bays.

Figure 14. Effect of irregularities on the period T for CBFs with eight bays.
The nonstructural elements’ stiffness which did not consider in this research, may lead to longer period results.

(5) Increasing the building bays increases 20% and 30% on T’s values for CBFs and EBFs systems, respectively.

(6) The proposed model can be used in predicting the fundamental period of vibration, and thus a conceptual design of steel buildings having CBFs or EBFs as lateral load resisting systems performed accordingly.
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Notation
The following symbols are used in this paper:
CBFs = concentric braced frames
EBFs = Eccentric braced frames
H = building height
Wi = weight of the structure at level i
δi = lateral displacement at level i
B1 = building width in the direction of period T
B2 = building width in a perpendicular direction of period T
Aov = average side area in T direction
A = total side area
R2 = coefficient of determination
α, βi, and ββ = regression parameters
FM = finite element model

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### Appendix

#### Table A1. Database of CBFs with X-bracing

| Building type | Height | B2/B1 | $A_{avg}/A_Y$ | $A_{avg}/A_X$ | $T_{Rayleigh}$ | $T_{Eq. \ (4)}$ |
|---------------|--------|-------|----------------|----------------|----------------|-----------------|
| CBF-X-5-3    | 15.00  | 0.60  | 1.00           | 1.00           | 0.74           | 0.81            |
| CBR-X-5-3-C  | 15.00  | 0.60  | 0.72           | 1.10           | 0.58           | 0.68            |
| CBR-X-5-3-V  | 15.00  | 0.60  | 0.89           | 1.11           | 0.71           | 0.80            |
| CBF-X-5-6    | 15.00  | 1.20  | 1.00           | 1.00           | 0.92           | 0.93            |
| CBR-X-5-6-C  | 15.00  | 1.20  | 0.76           | 1.03           | 0.76           | 0.77            |
| CBR-X-5-6-H  | 15.00  | 1.20  | 0.94           | 1.06           | 0.92           | 0.93            |
| CBR-X-5-6-V  | 15.00  | 1.20  | 0.63           | 1.00           | 0.76           | 0.65            |
| CBF-X-5-8    | 15.00  | 1.60  | 1.00           | 1.00           | 0.99           | 0.99            |
| CBR-X-5-8-C  | 15.00  | 1.60  | 0.71           | 1.09           | 0.79           | 0.81            |
| CBR-X-5-8-H  | 15.00  | 1.60  | 0.88           | 1.10           | 0.95           | 0.96            |
| CBR-X-5-8-V  | 15.00  | 1.60  | 0.80           | 1.00           | 0.82           | 0.83            |
| CBF-X-8-3    | 24.00  | 0.60  | 1.00           | 1.00           | 1.29           | 1.30            |
| CBR-X-8-3-C  | 24.00  | 0.60  | 0.75           | 1.10           | 1.07           | 1.12            |
| CBR-X-8-3-H  | 24.00  | 0.60  | 0.89           | 1.11           | 1.27           | 1.29            |
| CBR-X-8-3-V  | 24.00  | 0.60  | 0.83           | 1.00           | 1.08           | 1.12            |
| CBF-X-8-6    | 24.00  | 1.20  | 1.00           | 1.00           | 1.49           | 1.49            |
| CBR-X-8-6-C  | 24.00  | 1.20  | 0.79           | 1.06           | 1.29           | 1.29            |
| CBR-X-8-6-H  | 24.00  | 1.20  | 0.94           | 1.06           | 1.48           | 1.50            |
| CBR-X-8-6-V  | 24.00  | 1.20  | 0.80           | 1.00           | 1.29           | 1.25            |
| CBF-X-8-8    | 24.00  | 1.60  | 1.00           | 1.00           | 1.56           | 1.58            |
| CBR-X-8-8-C  | 24.00  | 1.60  | 0.74           | 1.09           | 1.30           | 1.33            |
| CBR-X-8-8-H  | 24.00  | 1.60  | 0.79           | 1.10           | 1.51           | 1.61            |
| CBR-X-8-8-V  | 24.00  | 1.60  | 0.83           | 1.00           | 1.35           | 1.37            |
| CBF-X-12-3   | 36.00  | 0.60  | 1.00           | 1.00           | 1.96           | 1.95            |
| CBF-X-12-3-C | 36.00  | 0.60  | 0.75           | 1.10           | 1.68           | 1.68            |
| CBF-X-12-3-H | 36.00  | 0.60  | 0.89           | 1.11           | 1.96           | 1.93            |
| CBF-X-12-3-V | 36.00  | 0.60  | 0.83           | 1.00           | 1.73           | 1.69            |
| CBF-X-12-6   | 36.00  | 1.20  | 1.00           | 1.00           | 2.21           | 2.24            |
| CBF-X-12-6-C | 36.00  | 1.20  | 0.79           | 1.06           | 1.92           | 1.94            |
| CBF-X-12-6-H | 36.00  | 1.20  | 0.94           | 1.06           | 2.22           | 2.24            |
| CBF-X-12-6-V | 36.00  | 1.20  | 0.83           | 1.00           | 1.93           | 1.94            |
| CBF-X-12-8   | 36.00  | 1.60  | 1.00           | 1.00           | 2.52           | 2.37            |
| CBF-X-12-8-C | 36.00  | 1.60  | 0.74           | 1.09           | 2.01           | 2.00            |
| CBF-X-12-8-H | 36.00  | 1.60  | 0.79           | 1.10           | 2.39           | 2.12            |
| CBF-X-12-8-V | 36.00  | 1.60  | 0.83           | 1.00           | 2.13           | 2.05            |
### Table A2. Database of CBFs with D-bracing

| Building type | Height | B2/B1 | $(A_{avg}/A)_Y$ | $(A_{avg}/A)_X$ | $T_{Rayleigh}$ | $T_{Eq. (5)}$ |
|---------------|--------|-------|----------------|-----------------|----------------|----------------|
| CBF-D-5-3    | 15.00  | 0.60  | 1.00           | 1.00            | 0.85           | 0.89           |
| CBR-D-5-3-C  | 15.00  | 0.60  | 0.72           | 1.10            | 0.67           | 0.74           |
| CBR-D-5-3-H  | 15.00  | 0.60  | 0.89           | 1.11            | 0.91           | 0.89           |
| CBR-D-5-3-V  | 15.00  | 0.60  | 0.80           | 1.00            | 0.70           | 0.75           |
| CBF-D-5-6    | 15.00  | 1.20  | 1.00           | 1.00            | 1.04           | 1.03           |
| CBR-D-5-6-C  | 15.00  | 1.20  | 0.76           | 1.03            | 0.85           | 0.85           |
| CBR-D-5-6-H  | 15.00  | 1.20  | 0.94           | 1.06            | 1.04           | 1.03           |
| CBR-D-5-6-V  | 15.00  | 1.20  | 0.63           | 1.00            | 0.86           | 0.71           |
| CBF-D-5-8    | 15.00  | 1.60  | 1.00           | 1.00            | 1.09           | 1.09           |
| CBR-D-5-8-C  | 15.00  | 1.60  | 0.71           | 1.09            | 0.88           | 0.89           |
| CBR-D-5-8-H  | 15.00  | 1.60  | 0.88           | 1.10            | 1.16           | 1.05           |
| CBR-D-5-8-V  | 15.00  | 1.60  | 0.80           | 1.00            | 0.93           | 0.91           |
| CBF-D-8-3    | 24.00  | 0.60  | 1.00           | 1.00            | 1.40           | 1.43           |
| CBR-D-8-3-C  | 24.00  | 0.60  | 0.75           | 1.10            | 1.14           | 1.23           |
| CBR-D-8-3-H  | 24.00  | 0.60  | 0.89           | 1.11            | 1.35           | 1.42           |
| CBR-D-8-3-V  | 24.00  | 0.60  | 0.83           | 1.00            | 1.16           | 1.24           |
| CBF-D-8-6    | 24.00  | 1.20  | 1.00           | 1.00            | 1.58           | 1.64           |
| CBR-D-8-6-C  | 24.00  | 1.20  | 0.79           | 1.06            | 1.35           | 1.42           |
| CBR-D-8-6-H  | 24.00  | 1.20  | 0.94           | 1.06            | 1.58           | 1.65           |
| CBR-D-8-6-V  | 24.00  | 1.20  | 0.83           | 1.00            | 1.36           | 1.62           |
| CBF-D-8-8    | 24.00  | 1.60  | 1.00           | 1.00            | 1.81           | 1.74           |
| CBR-D-8-8-C  | 24.00  | 1.60  | 0.74           | 1.09            | 1.48           | 1.67           |
| CBR-D-8-8-H  | 24.00  | 1.60  | 0.79           | 1.10            | 1.74           | 1.56           |
| CBR-D-8-8-V  | 24.00  | 1.60  | 0.83           | 1.00            | 1.54           | 1.50           |
| CBF-D-12-3   | 36.00  | 0.60  | 1.00           | 1.00            | 2.16           | 2.15           |
| CBF-D-12-3-C | 36.00  | 0.60  | 0.75           | 1.10            | 1.83           | 1.84           |
| CBF-D-12-3-H | 36.00  | 0.60  | 0.89           | 1.11            | 2.16           | 2.12           |
| CBF-D-12-3-V | 36.00  | 0.60  | 0.83           | 1.00            | 1.88           | 1.85           |
| CBF-D-12-6   | 36.00  | 1.20  | 1.00           | 1.00            | 2.50           | 2.66           |
| CBF-D-12-6-C | 36.00  | 1.20  | 0.79           | 1.06            | 2.10           | 2.14           |
| CBF-D-12-6-H | 36.00  | 1.20  | 0.94           | 1.06            | 2.44           | 2.47           |
| CBF-D-12-6-V | 36.00  | 1.20  | 0.83           | 1.00            | 2.13           | 2.13           |
| CBF-D-12-8   | 36.00  | 1.60  | 1.00           | 1.00            | 2.57           | 2.61           |
| CBF-D-12-8-C | 36.00  | 1.60  | 0.74           | 1.09            | 2.09           | 2.20           |
| CBF-D-12-8-H | 36.00  | 1.60  | 0.79           | 1.10            | 2.46           | 2.33           |
| CBF-D-12-8-V | 36.00  | 1.60  | 0.83           | 1.00            | 2.20           | 2.26           |
| Building type | Height | B2/B1 | (A_{avg}/A)_Y | (A_{avg}/A)_X | T_{Rayleigh} | T_{Eq. (6)} |
|--------------|--------|-------|---------------|---------------|-------------|-------------|
| CBF-A-5-3    | 15.00  | 0.60  | 1.00          | 1.00          | 0.638       | 0.79        |
| CBR-A-5-3-C  | 15.00  | 0.60  | 0.72          | 1.10          | 0.503       | 0.65        |
| CBR-A-5-3-H  | 15.00  | 0.60  | 0.89          | 1.11          | 0.732       | 0.78        |
| CBR-A-5-3-V  | 15.00  | 0.60  | 0.80          | 1.00          | 0.517       | 0.66        |
| CBF-A-5-6    | 15.00  | 1.20  | 1.00          | 1.00          | 0.856       | 0.90        |
| CBR-A-5-6-C  | 15.00  | 1.20  | 0.76          | 1.03          | 0.689       | 0.74        |
| CBR-A-5-6-H  | 15.00  | 1.20  | 0.94          | 1.06          | 0.844       | 0.90        |
| CBR-A-5-6-V  | 15.00  | 1.20  | 0.63          | 1.00          | 0.697       | 0.63        |
| CBF-A-5-8    | 15.00  | 1.60  | 1.00          | 1.00          | 1.075       | 0.96        |
| CBR-A-5-8-C  | 15.00  | 1.60  | 0.71          | 1.09          | 0.738       | 0.78        |
| CBR-A-5-8-H  | 15.00  | 1.60  | 0.88          | 1.10          | 0.903       | 0.93        |
| CBR-A-5-8-V  | 15.00  | 1.60  | 0.80          | 1.00          | 0.780       | 0.80        |
| CBF-A-8-3    | 24.00  | 0.60  | 1.00          | 1.00          | 1.211       | 1.26        |
| CBR-A-8-3-C  | 24.00  | 0.60  | 0.75          | 1.10          | 0.998       | 1.08        |
| CBR-A-8-3-H  | 24.00  | 0.60  | 0.89          | 1.11          | 1.119       | 1.24        |
| CBR-A-8-3-V  | 24.00  | 0.60  | 0.83          | 1.00          | 1.043       | 1.09        |
| CBF-A-8-6    | 24.00  | 1.20  | 1.00          | 1.00          | 1.494       | 1.44        |
| CBR-A-8-6-C  | 24.00  | 1.20  | 0.79          | 1.06          | 1.29        | 1.25        |
| CBR-A-8-6-H  | 24.00  | 1.20  | 0.94          | 1.06          | 1.48        | 1.45        |
| CBR-A-8-6-V  | 24.00  | 1.20  | 0.80          | 1.00          | 1.32        | 1.21        |
| CBF-A-8-8    | 24.00  | 1.60  | 1.00          | 1.00          | 1.56        | 1.53        |
| CBR-A-8-8-C  | 24.00  | 1.60  | 0.74          | 1.09          | 1.27        | 1.29        |
| CBR-A-8-8-H  | 24.00  | 1.60  | 0.79          | 1.10          | 1.49        | 1.37        |
| CBR-A-8-8-V  | 24.00  | 1.60  | 0.83          | 1.00          | 1.32        | 1.32        |
| CBF-A-12-3   | 36.00  | 0.60  | 1.00          | 1.00          | 1.93        | 1.89        |
| CBF-A-12-3-C | 36.00  | 0.60  | 0.75          | 1.10          | 1.61        | 1.62        |
| CBF-A-12-3-H | 36.00  | 0.60  | 0.89          | 1.11          | 1.89        | 1.87        |
| CBF-A-12-3-V | 36.00  | 0.60  | 0.83          | 1.00          | 1.64        | 1.63        |
| CBF-A-12-6   | 36.00  | 1.20  | 1.00          | 1.00          | 2.24        | 2.17        |
| CBF-A-12-6-C | 36.00  | 1.20  | 0.79          | 1.06          | 1.88        | 1.88        |
| CBF-A-12-6-H | 36.00  | 1.20  | 0.94          | 1.06          | 2.21        | 2.17        |
| CBF-A-12-6-V | 36.00  | 1.20  | 0.83          | 1.00          | 1.90        | 1.87        |
| CBF-A-12-8   | 36.00  | 1.60  | 1.00          | 1.00          | 2.35        | 2.29        |
| CBF-A-12-8-C | 36.00  | 1.60  | 0.74          | 1.09          | 2.00        | 1.93        |
| CBF-A-12-8-H | 36.00  | 1.60  | 0.79          | 1.10          | 2.25        | 2.05        |
| CBF-A-12-8-V | 36.00  | 1.60  | 0.83          | 1.00          | 2.00        | 1.98        |
### Table A4. Database of EBFs with K-bracing

| Building Type | Height | B2/B1 | (A_{avg}/A)_Y | (A_{avg}/A)_X | T_{Rayleigh} | T_{Eq. (7)} |
|---------------|--------|-------|--------------|--------------|--------------|-------------|
| EBF-K-5-3     | 15.00  | 0.60  | 1.00         | 1.00         | 1.229        | 0.92        |
| EBF-K-5-3-C   | 15.00  | 0.60  | 0.72         | 1.10         | 1.162        | 0.86        |
| EBF-K-5-3-H   | 15.00  | 0.60  | 0.89         | 1.11         | 1.246        | 0.92        |
| EBF-K-5-3-V   | 15.00  | 0.60  | 0.80         | 1.00         | 1.228        | 0.86        |
| EBF-K-5-6     | 15.00  | 1.20  | 1.00         | 1.00         | 1.268        | 1.10        |
| EBF-K-5-6-C   | 15.00  | 1.20  | 0.76         | 1.03         | 1.178        | 1.02        |
| EBF-K-5-6-H   | 15.00  | 1.20  | 0.94         | 1.06         | 1.273        | 1.10        |
| EBF-K-5-6-V   | 15.00  | 1.20  | 0.63         | 1.00         | 1.180        | 0.96        |
| EBF-K-5-8     | 15.00  | 1.60  | 1.00         | 1.00         | 1.295        | 1.18        |
| EBF-K-5-8-C   | 15.00  | 1.60  | 0.71         | 1.09         | 1.178        | 1.09        |
| EBF-K-5-8-H   | 15.00  | 1.60  | 0.88         | 1.10         | 1.253        | 1.17        |
| EBF-K-5-8-V   | 15.00  | 1.60  | 0.80         | 1.00         | 1.187        | 1.10        |
| EBF-K-8-3     | 24.00  | 0.60  | 1.00         | 1.00         | 1.599        | 1.48        |
| EBF-K-8-3-C   | 24.00  | 0.60  | 0.75         | 1.10         | 1.435        | 1.40        |
| EBF-K-8-3-H   | 24.00  | 0.60  | 0.89         | 1.11         | 1.579        | 1.47        |
| EBF-K-8-3-V   | 24.00  | 0.60  | 0.83         | 1.00         | 1.463        | 1.40        |
| EBF-K-8-6     | 24.00  | 1.20  | 1.00         | 1.00         | 1.590        | 1.76        |
| EBF-K-8-6-C   | 24.00  | 1.20  | 0.79         | 1.06         | 1.534        | 1.67        |
| EBF-K-8-6-H   | 24.00  | 1.20  | 0.94         | 1.06         | 1.607        | 1.76        |
| EBF-K-8-6-V   | 24.00  | 1.20  | 0.80         | 1.00         | 1.506        | 1.64        |
| EBF-K-8-8     | 24.00  | 1.60  | 1.00         | 1.00         | 1.711        | 1.89        |
| EBF-K-8-8-C   | 24.00  | 1.60  | 0.74         | 1.09         | 1.601        | 1.77        |
| EBF-K-8-8-H   | 24.00  | 1.60  | 0.79         | 1.10         | 1.689        | 1.81        |
| EBF-K-8-8-V   | 24.00  | 1.60  | 0.83         | 1.00         | 1.577        | 1.79        |
| EBF-K-12-3    | 36.00  | 0.60  | 1.00         | 1.00         | 2.116        | 2.22        |
| EBF-K-12-3-C  | 36.00  | 0.60  | 0.75         | 1.10         | 1.975        | 2.10        |
| EBF-K-12-3-H  | 36.00  | 0.60  | 0.89         | 1.11         | 2.104        | 2.21        |
| EBF-K-12-3-V  | 36.00  | 0.60  | 0.83         | 1.00         | 1.997        | 2.10        |
| EBF-K-12-6    | 36.00  | 1.20  | 1.00         | 1.00         | 2.833        | 2.64        |
| EBF-K-12-6-C  | 36.00  | 1.20  | 0.79         | 1.06         | 2.636        | 2.50        |
| EBF-K-12-6-H  | 36.00  | 1.20  | 0.94         | 1.06         | 2.719        | 2.64        |
| EBF-K-12-6-V  | 36.00  | 1.20  | 0.83         | 1.00         | 2.762        | 2.50        |
| EBF-K-12-8    | 36.00  | 1.60  | 1.00         | 1.00         | 3.072        | 2.83        |
| EBF-K-12-8-C  | 36.00  | 1.60  | 0.74         | 1.09         | 2.771        | 2.66        |
| EBF-K-12-8-H  | 36.00  | 1.60  | 0.79         | 1.10         | 2.970        | 2.72        |
| EBF-K-12-8-V  | 36.00  | 1.60  | 0.83         | 1.00         | 2.803        | 2.68        |
### Table A5. Database of EBFs with Z-bracing

| Building type | Height | B2/B1 | \((A_{avg}/A)_Y\) | \((A_{avg}/A)_X\) | \(T_{Rayleigh}\) | \(T_{Eq. (7)}\) |
|---------------|--------|-------|-----------------|-----------------|----------------|----------------|
| EBF-Z-5-3     | 15.00  | 0.60  | 1.00            | 1.00            | 1.90           | 1.85           |
| EBF-Z-5-3-C   | 15.00  | 0.60  | 0.72            | 1.10            | 1.44           | 1.58           |
| EBF-Z-5-3-H   | 15.00  | 0.60  | 0.89            | 1.11            | 1.79           | 1.84           |
| EBF-Z-5-3-V   | 15.00  | 0.60  | 0.80            | 1.00            | 1.69           | 1.58           |
| EBF-Z-5-6     | 15.00  | 1.20  | 1.00            | 1.00            | 1.88           | 1.90           |
| EBF-Z-5-6-C   | 15.00  | 1.20  | 0.76            | 1.03            | 1.69           | 1.61           |
| EBF-Z-5-6-H   | 15.00  | 1.20  | 0.94            | 1.06            | 1.91           | 1.91           |
| EBF-Z-5-6-V   | 15.00  | 1.20  | 0.63            | 1.00            | 1.58           | 1.38           |
| EBF-Z-5-8     | 15.00  | 1.60  | 1.00            | 1.00            | 2.01           | 1.93           |
| EBF-Z-5-8-C   | 15.00  | 1.60  | 0.71            | 1.09            | 1.54           | 1.61           |
| EBF-Z-5-8-H   | 15.00  | 1.60  | 0.88            | 1.10            | 1.90           | 1.87           |
| EBF-Z-5-8-V   | 15.00  | 1.60  | 0.80            | 1.00            | 1.62           | 1.65           |
| EBF-Z-8-3     | 24.00  | 0.60  | 1.00            | 1.00            | 2.30           | 2.24           |
| EBF-Z-8-3-C   | 24.00  | 0.60  | 0.75            | 1.10            | 1.73           | 1.96           |
| EBF-Z-8-3-H   | 24.00  | 0.60  | 0.89            | 1.11            | 2.19           | 2.22           |
| EBF-Z-8-3-V   | 24.00  | 0.60  | 0.83            | 1.00            | 1.90           | 1.97           |
| EBF-Z-8-6     | 24.00  | 1.20  | 1.00            | 1.00            | 2.29           | 2.30           |
| EBF-Z-8-6-C   | 24.00  | 1.20  | 0.79            | 1.06            | 2.10           | 2.03           |
| EBF-Z-8-6-H   | 24.00  | 1.20  | 0.94            | 1.06            | 2.31           | 2.30           |
| EBF-Z-8-6-V   | 24.00  | 1.20  | 0.83            | 1.00            | 2.12           | 2.02           |
| EBF-Z-8-8     | 24.00  | 1.60  | 1.00            | 1.00            | 2.28           | 2.33           |
| EBF-Z-8-8-C   | 24.00  | 1.60  | 0.74            | 1.09            | 1.78           | 2.00           |
| EBF-Z-8-8-H   | 24.00  | 1.60  | 0.79            | 1.10            | 2.20           | 2.11           |
| EBF-Z-8-8-V   | 24.00  | 1.60  | 0.83            | 1.00            | 1.99           | 2.05           |
| EBF-Z-12-3    | 36.00  | 0.60  | 1.00            | 1.00            | 2.78           | 2.63           |
| EBF-Z-12-3-C  | 36.00  | 0.60  | 0.75            | 1.10            | 2.25           | 2.30           |
| EBF-Z-12-3-H  | 36.00  | 0.60  | 0.89            | 1.11            | 2.62           | 2.61           |
| EBF-Z-12-3-V  | 36.00  | 0.60  | 0.83            | 1.00            | 2.61           | 2.31           |
| EBF-Z-12-6    | 36.00  | 1.20  | 1.00            | 1.00            | 2.78           | 2.70           |
| EBF-Z-12-6-C  | 36.00  | 1.20  | 0.79            | 1.06            | 2.37           | 2.39           |
| EBF-Z-12-6-H  | 36.00  | 1.20  | 0.94            | 1.06            | 2.62           | 2.71           |
| EBF-Z-12-6-V  | 36.00  | 1.20  | 0.83            | 1.00            | 2.47           | 2.38           |
