Target tracking control of underactuated autonomous underwater vehicle based on adaptive nonsingular terminal sliding mode control

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Abstract
This article addresses the design of adaptive target tracking control for an underactuated autonomous underwater vehicle subject to uncertain dynamics and external disturbances induced by ocean current. Firstly, based on the line-of-sight method, the moving target tracking guidance strategy is designed, and the target tracking reference speed and reference angular velocity are given. According to the obtained reference speed and reference angular velocities, the reference control quantity is differentiated and filtered based on dynamic surface control. The target tracking controller is designed based on radial basis function neural network and nonsingular terminal sliding mode control and adaptive techniques. Lyapunov stability principle is utilized to ensure the asymptotic stability of the target tracking controller. Simulation of target tracking is carried out to illustrate the effectiveness of the proposed controller.

Keywords
Underactuated autonomous underwater vehicle, target tracking, model uncertainty, adaptive terminal sliding mode control, RBF neutral network

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Introduction
Autonomous underwater vehicles (AUVs) have been widely studied due to the potential civil and military application values.¹–³ As a key technology of AUV, target tracking control is very important to the task, such as underwater target reconnaissance and surveillance, intelligence collection, port defense, time-sensitive strike, underwater docking recovery, submarine pipeline maintenance, and halobios observation.⁴,⁵ Tracking an undersea moving object is frequently encountered in the unmanned underwater vehicle (UUV) applications, such as tracking a

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submarine. To achieve this, the UUV needs to follow the target in certain distance based on the sensor information.6

In recent years, there have been a number of significant robust control methods developed for tracking control, such as adaptive control,7 fuzzy logic control,8 backstepping control,9 and neural network (NN) method.10 A novel Lyapunov-based model predictive control framework was developed to improve the trajectory tracking performance by online optimization.11 An extended state observer is developed to estimate uncertainty induced by model dynamics and external disturbance in path-following task, where the controller is designed based on output-feedback methods.12

In practice condition, the AUVs kinematic and dynamic models are highly nonlinear and coupled, and most of the hydrodynamic parameters cannot be obtained or measured inaccurately, and there exist unknown and time-varying disturbances in actual ocean environment.13 For tackling the challenges mentioned previously, researchers have proposed various control methods.

Adaptive control is an effective method to deal with uncertainties, with little or no information of the bounds. Some output-feedback adaptive control approaches14–19 are proposed for underactuated AUVs. However, there is heavy calculation for higher order systems of these adaptive control approaches and they just compensate for constant and slowly varying parameters, which does not adapt to the complex actual condition. Hence, some robust adaptive technique, such as sliding mode control, is developed to attenuate the effects of external disturbances.

Bessa et al. propose a sliding mode control strategy mainly based on approach for remotely operated vehicle (ROV) depth regulation to cope with the external disturbances and uncertain model parameters.20 Matveev et al. propose a sliding mode-based navigation and guidance method to solve the problems of a unicycle-like robot passing through an environment with moving and deforming obstacles.21 In the problem of translational robotic systems trajectory tracking, sliding mode control is employed to reduce tracking errors dramatically and compensate implications of initial error, varying loading conditions, model uncertainties, and environmental disturbance.22

Liang et al. combine fuzzy logic theory with sliding mode methods to solve the problems of uncertain dynamics and external disturbances for underactuated AUV path following23 and trajectory tracking,24 and multiple underactuated unmanned surface vehicle (USV) path following.25,26

Joe et al. propose a second-order sliding mode control to stabilize an AUV in trajectory tracking with the unpredictable disturbance induced by waves and currents.27 Wang et al. propose a backstepping sliding mode controller based on a slow time-varying adaptive disturbance observer (BSMCTADO) to compensate for interferences caused by wall effect during near-wall following task.28 Qiao and Zhang propose an adaptive nonsingular integral terminal sliding mode control scheme, which does not require the bound information of the system uncertainty, however, the control just effective for first-order uncertain nonlinear dynamic system.29 Elmokadem et al. propose a robust control scheme using terminal sliding mode control to solve the horizontal plane trajectory tracking problem of underactuated AUV, which is effective for bounded disturbances.30 A terminal sliding mode controller is designed to improve the performance of trajectory tracking control and accelerate the converging rate, where an extended disturbance observer is employed to estimate current-induced uncertainties and an anti-windup technique is applied to weaken the influence of actuator saturation.31

For solving the coordinated tracking problem of multi-AUVs with time-varying trajectory, a predictor-based neural dynamic surface controller is designed, where the predictor is utilized to recover the unmeasured velocities and identify the vehicle dynamics with little information of models.32 An adaptive NN controller is designed to deal with uncertainties of AUV induced by time-varying hydrodynamic damps, where the radial basis function (RBF) NN is utilized to estimate the unknown terms and adaptive law is introduced to optimize the estimation of the weight of NN.33 A neural adaptive controller is designed to deal with the unknown hydrodynamic parameters and time-varying environmental disturbances, and saturation functions are employed to compensate actuators’ saturation nonlinearity.34 A target tracking controller for AUV in 3-D space is designed, where NN and adaptive control are employed to deal with the problem of lacking prior information of vehicle dynamic model and external disturbances, and computational complexities are reduced by dynamic surface control approach.35

In this article, main contributions can be summarized as follows:

1. A target tracking methodology based on line-of-sight is proposed to obtain the reference velocities, and the kinematic stability is proved.
2. Dynamic surface control is employed to obtain the differential value of the reference velocities, and differential explosion problem is solved via command filtering by DSC approach.
3. For enhancing the robustness of an underactuated AUV against unknown model parameters and external disturbances, nonsingular terminal sliding mode control and RBF NN are employed to develop a target tracking controller, which ensures target tracking control converging fast and avoids buffeting problem of slide mode control.

Problem statement

AUV model

There are two commonly used coordinate systems to describe the motion of an AUV, as shown in Figure 1. One
The kinematic model of the AUV in the 3-D space can be described as follows\(^{13}\)

\[
\begin{align*}
\dot{\xi} &= u \cos \psi \cos \theta - v \sin \psi + w \sin \theta \cos \psi \\
\dot{\eta} &= u \sin \psi \cos \theta + v \cos \psi + w \sin \theta \sin \psi \\
\dot{\zeta} &= -u \sin \theta + w \cos \theta \\
\dot{\theta} &= g \\
\dot{\psi} &= \frac{r}{\cos \theta}
\end{align*}
\]

Here, \(\xi, \eta, \) and \(\zeta\) denote the position with coordinates in \{I\}, \(\theta\) denotes the pitch angle in \{I\}, and \(\psi\) denotes the yaw angle in \{I\}. \(u, v, w, q,\) and \(r\) denote the surge, sway, heave, pitch, and yaw velocities with coordinates in \{B\}.

The AUV considered in this article is assumed to be neutrally buoyant. Then, based on the momentum theorem and ignoring the rolling motion, the AUV’s dynamic model in the 3-D space is expressed as follows\(^{13}\)

\[
\begin{align*}
m_{11} \dot{u} &= m_{22} v r - m_{33} w q + d_u(u, v, w, q, r) + \tau_u - \tau_{eu}(t) \\
m_{22} \dot{v} &= -m_{11} u r + d_v(u, v, w, q, r) - \tau_{ev}(t) \\
m_{33} \dot{w} &= m_{11} u q + d_w(u, v, w, q, r) - \tau_{ew}(t) \\
m_{55} \dot{q} &= (m_{33} - m_{11}) u w + d_q(u, v, w, q, r) - m g h \sin \theta + \tau_q - \tau_{eq}(t) \\
m_{66} \dot{r} &= (m_{11} - m_{22}) u v + d_r(u, v, w, q, r) + \tau_r - \tau_{er}(t)
\end{align*}
\]

where

\[
\begin{align*}
m_{11} &= m - X_u, \quad m_{22} = m - Y_v, \quad m_{33} = m - Z_w \\
m_{55} &= I_y - M_q, \quad m_{66} = I_z - N_r
\end{align*}
\]

where \(m\) is the mass of the AUV, \(I_i\) and \(I_f\) are the moment of inertia about the \(x\) and \(z\) axes, \(X_u, Y_v, Z_w, M_q,\) and \(N_r\) are the inertia hydrodynamic coefficients of the AUV, \(h\) is the metacentric height of the AUV. The control input \(\tau_u\) is the force produced by thruster, \(\tau_q\) and \(\tau_r\) are the moment produced by elevators and rudders. \(\tau_{ei}(t), i = u, v, w, q, r,\) are used to describe the environmental force and moments acting on the vehicle. \(d_i(u, v, w, q, r), i = u, v, w, q, r,\) stand for the nonlinear dynamics of AUV.

**Control objectives**

To describe the target tracking control problem, we introduce three tracking variables, \(\delta\) represents the distance between the AUV and the target, \(\beta\) and \(\alpha\) represent bearing angles that the target relative to the AUV, as shown in Figures 2 and 3. In this article, we assume that these three variables can be measured by sensors equipped on the AUV.

Therefore, the control objectives of this article are that we shall design control inputs \(\tau_u, \tau_q,\) and \(\tau_r\) to force the AUV to asymptotically track the target with all tracking variables converging to a neighborhood of zero and the controller should be with robustness against to uncertainties of model dynamics and unknown environmental disturbances. For avoiding collision with the target, a save distance between the AUV and the target is necessary, then, a positive threshold value \(\delta_0\) is introduced, and the control objective is changed as designing a controller to render tracking variables \((\delta - \delta_0), \beta\) and \(\alpha\) converge to a neighborhood of zero.

**Controller design**

**Kinematic controller design**

The position error between the follower AUV and target in \{I\} can be written as follows

\[
\begin{align*}
x_e &= \cos \theta \cos \psi(\xi_d - \xi_B) + \cos \theta \sin \psi(\eta_d - \eta_B) - \sin \theta(\zeta_d - \zeta_B) \\
y_e &= -\sin \psi(\xi_d - \xi_B) + \cos \psi(\eta_d - \eta_B) + \cos \theta(\zeta_d - \zeta_B) \\
z_e &= \sin \theta \cos \psi(\zeta_d - \zeta_B) + \sin \theta \sin \psi(\eta_d - \eta_B) + \cos \theta(\xi_d - \xi_B)
\end{align*}
\]

where \((\xi_B, \eta_B, \zeta_B)\) and \((\xi_d, \eta_d, \zeta_d)\) are the positions of the AUV and the target, respectively, in \{I\}.
Hence, the tracking variables $\delta$, $\beta$, and $\alpha$ are related to Cartesian errors $x_e$, $y_e$, and $z_e$ in $\{B\}$ by the following transformation

\[
\begin{align*}
\delta &= \sqrt{x_e^2 + y_e^2 + z_e^2} \\
\beta &= -\arctan\left(\frac{z_e}{\sqrt{x_e^2 + y_e^2}}\right) \\
\alpha &= \arctan\left(\frac{y_e}{x_e}\right)
\end{align*}
\] (4)

and

\[
\begin{align*}
x_e &= \delta \cos \beta \cos \alpha \\
y_e &= \delta \cos \beta \sin \alpha \\
z_e &= -\delta \sin \beta
\end{align*}
\] (5)

Differentiating equation (3) yields the following error dynamics in $\{B\}$

\[
\begin{align*}
\dot{x}_e &= -u - q z_e + ry_e + \chi_{1d} \\
\dot{y}_e &= -v - r(x_e + z_e \tan \theta) + \chi_{2d} \\
\dot{z}_e &= -w + q x_e + ry_e \tan \theta + \chi_{3d}
\end{align*}
\] (6)

where

\[
\begin{align*}
\chi_{1d} &= \dot{\xi}_d \cos \theta \cos \psi + \dot{\eta}_d \cos \theta \sin \psi - \dot{\zeta}_d \sin \theta \\
\chi_{2d} &= -\dot{\xi}_d \sin \theta \cos \psi + \dot{\eta}_d \sin \theta \sin \psi \\
\chi_{3d} &= \dot{\xi}_d \sin \theta \cos \psi + \dot{\eta}_d \sin \theta \sin \psi + \dot{\zeta}_d \cos \theta
\end{align*}
\] (7)

Then, the dynamics of tracking errors $x_e$, $y_e$, and $z_e$ can be transformed to the range and bearing angles dynamics by differentiating equation (4).

By differentiating $\delta$ in equation (4), one obtains

\[
\dot{\delta} = x_e \dot{x}_e + y_e \dot{y}_e + z_e \dot{z}_e
\] (8)

Substituting for $x_e$ and $\dot{x}_e$ from equation (6) yields

\[
\dot{\delta} = -u \cos \beta \cos \alpha - v \cos \beta \sin \alpha + w \sin \beta + \chi_{1d} \cos \alpha + \chi_{2d} \cos \beta \sin \alpha - \chi_{3d} \sin \beta
\] (9)

Let $\delta = -k_d(\delta - \delta_0)$, and replace $\dot{\delta}$ in equation (9), then the virtual surge control law is chosen as follows

\[
u_e = (\cos \beta \cos \alpha)^{-1} \left[ -k_d(\delta - \delta_0) - v \cos \beta \sin \alpha - w \sin \beta - \chi_{1d} \cos \beta \cos \alpha \right. \\
\left. - \chi_{2d} \cos \beta \sin \alpha - \chi_{3d} \sin \beta \right]
\] (10)

where $k_d > 0$ is a design parameter.

By differentiating $\beta = -\arctan(z_e/\sqrt{x_e^2 + y_e^2})$ in equation (4), one obtains

\[
\dot{\beta} = -q \cos \alpha - r \tan \theta \sin \alpha + \sin \beta \cos \alpha (u - \chi_{1d})/\delta + \sin \beta \sin \alpha (v - \chi_{2d})/\delta + \cos \beta (w - \chi_{3d})/\delta
\] (11)

Let $\tilde{\beta} = -k_p \beta$, then the virtual pitch control law is chosen as follows

\[
q_e = (\cos \beta \cos \alpha)^{-1} \left[ k_p \beta - r \tan \theta \sin \alpha + \sin \beta \cos \alpha (u - \chi_{1d})/\delta \right. \\
\left. + \sin \beta \sin \alpha (v - \chi_{2d})/\delta + \cos \beta (w - \chi_{3d})/\delta \right]
\] (12)

where $k_p > 0$ is a design parameter.

By differentiating $\alpha = \arctan(y_e/x_e)$ in equation (4), one gets

\[
\dot{\alpha} = r (\tan \theta \tan \beta \cos \alpha - 1) + (u - \chi_{1d}) \sin \alpha/\delta \cos \beta - q \tan \beta \sin \alpha + (v - \chi_{2d}) \cos \alpha/\delta \cos \beta
\] (13)

Let $\dot{\alpha} = -k_h \alpha$, then the virtual heading control law is considered:
where $k_\alpha > 0$ is a design parameter.

A Lyapunov function is chosen as follows

$$V_1 = \frac{1}{2} \left[ (\delta - \delta_0)^2 + \alpha^2 + \beta^2 \right]$$

Differentiating equation (15) yields

$$\dot{V}_1 = (\delta - \delta_0) \delta + \alpha \dot{\alpha} + \beta \dot{\beta}$$

According to the chosen virtual control law, equation (16) can be written as follows

$$\dot{V}_1 = -k_\epsilon (\delta - \delta_0)^2 - k_\alpha \alpha^2 - k_\beta \beta^2 \leq 0$$

Consequently, the parameters $k_\epsilon$, $k_\alpha$, and $k_\beta$ can be chosen to ensure the tracking errors $x_\epsilon$, $y_\epsilon$, and $z_\epsilon$ converge to a specified compact set around 0.

**Dynamic controller design**

To avoid the computation complexity, virtual control laws $u_\epsilon$, $q_\epsilon$, and $r_\epsilon$ are passed through the following first-order filter.\textsuperscript{36}

$$\begin{cases} \nu u(t) + u_c(t) = u_\epsilon(t) \\ u(t) = u_c(0) \end{cases}$$

$$\begin{cases} \nu q(t) + q_c(t) = q_\epsilon(t) \\ q(t) = q_c(0) \end{cases}$$

$$\begin{cases} \nu r(t) + r_c(t) = r_\epsilon(t) \\ r(t) = r_c(0) \end{cases}$$

where $u_\epsilon(t)$, $q_\epsilon(t)$, and $r_\epsilon(t)$ are filtered virtual control law and $\nu u$, $\nu q$, and $\nu r$ are positive design parameters. Then, the following are defined

$$\begin{cases} u_e(t) = u(t) - u_c(t) \\ q_e(t) = q(t) - q_c(t) \\ r_e(t) = r(t) - r_c(t) \end{cases}$$

and the following error equations are obtained from equation (2)

$$\begin{cases} m_{11} \dot{u}_e = -f_u + \tau_u - \tau_{eu}(t) \\ m_{55} \dot{q}_e = -f_q + \tau_q - \tau_{eq}(t) \\ m_{66} \dot{r}_e = -f_r + \tau_r - \tau_{er}(t) \end{cases}$$

Choosing the following integral sliding mode surface

$$\begin{cases} S_u = \int_0^t u_e dt + \frac{P_u}{\mu_u} \left| q_u \right| q_u \text{ sign}(q_u) \\ S_q = \int_0^t q_e dt + \frac{P_q}{\mu_q} \left| q_q \right| q_q \text{ sign}(q_q) \\ S_r = \int_0^t r_e dt + \frac{P_r}{\mu_r} \left| q_r \right| q_r \text{ sign}(r_r) \end{cases}$$

where $\mu_i > 0$, $P_i$, $q_i$ are positive odd number, and $1 < \mu_i < 2$, $i = u, q, r$.

Differentiating equation (24) and utilizing equation (22) yields

$$\begin{cases} \dot{S}_u = -\lambda u \text{ tanh}(\gamma_u S_u) - k_u S_u \\ \dot{S}_q = -\lambda_q \text{ tanh}(\gamma_q S_q) - k_q S_q \\ \dot{S}_r = -\lambda_r \text{ tanh}(\gamma_r S_r) - k_r S_r \end{cases}$$
where \( \lambda_i, k_i > 0 \) represent the controller gains, \( i = u, q, r; \)
\( \gamma_i \) are positive, \( i = u, q, r. \)

The control input can be selected as

\[
\begin{align*}
\tau_u &= -\mu_u \frac{q_u}{p_u} m_{11} |u| e_u - \lambda_u \tanh(\gamma_u S_u) - k_u S_u + f_u \\
\tau_q &= -\mu_q \frac{q_q}{p_q} m_{55} |q| e_q - \lambda_q \tanh(\gamma_q S_q) - k_q S_q + f_q \\
\tau_r &= -\mu_r \frac{q_r}{p_r} m_{66} |r| e_r - \lambda_r \tanh(\gamma_r S_r) - k_r S_r + f_r
\end{align*}
\]

(27)

Consider the following Lyapunov function:

\[
V_2 = \frac{1}{2} (S_u^2 + S_q^2 + S_r^2)
\]

(28)

Differentiating equation (28) and substituting equation (27) into equation (25) yield

\[
\dot{V}_2 = S_u \dot{S}_u + S_q \dot{S}_q + S_r \dot{S}_r
\]

\[
= -S_u \frac{1}{p_u} \left[ \frac{p_u}{q_u} \right] |u| e_u - \frac{1}{m_{11}} [\lambda_u \tanh(\gamma_u S_u) + k_u S_u + \tau_{eu}]
\]

(29)

To satisfy the sliding condition with minimum control effort, \( \dot{V}_2 = SS \leq 0 \) is required, the parameters \( \lambda_i \) and \( k_i \) should be chosen as

\[
\begin{align*}
\lambda_i &= -\tau_{ei} \tanh(\gamma_i S_i)^{-1} + \varepsilon_{i1} \\
k_i &= -\varepsilon_{i1} \tanh(\gamma_i S_i)^{-1} + \varepsilon_{i2}
\end{align*}
\]

(30)

where \( \varepsilon_{i1} \) and \( \varepsilon_{i2} \) are positive, \( i = u, q, r. \)

Since \( \lambda_i \) and \( k_i \) cannot be computed without the knowledge of disturbance \( \tau_{ei}, \) we utilize an adaptive tuning rule to update those parameters. Let \( \tilde{\lambda}_i \) and \( \tilde{k}_i \) be the estimated values of \( \lambda_i \) and \( k_i \) respectively, then

\[
\begin{align*}
\dot{\tilde{\lambda}}_i &= \lambda_i - \tilde{\lambda}_i \\
\dot{\tilde{k}}_i &= k_i - \tilde{k}_i
\end{align*}
\]

(31)

where \( \tilde{\lambda}_i \) and \( \tilde{k}_i \) are the estimation errors. The Lyapunov function can be modified as

\[
V_3 = V_2 + \sum_{i=u, q, r} \left( \frac{\gamma_i^2}{2}\tilde{\lambda}_i^2 + \frac{\gamma_i^2}{2}\tilde{\lambda}_i^2 \right)
\]

(32)

where \( \gamma_{\lambda i} > 0, \gamma_{ki} > 0, i = u, q, r. \)

Hence, according to equation (29), can be expressed as follows

\[
\dot{V}_3 = \sum_{i=u, q, r} S_i \frac{1}{\mu_i} \frac{p_i}{q_i} |i| e_i \left[ \lambda_i \tanh(\gamma_i S_i) - \tilde{\lambda}_i S_i - \tau_{ei} \right] + \sum_{i=u, q, r} \frac{p_i}{q_i} \left[ \lambda_i \tanh(\gamma_i S_i) - \tilde{\lambda}_i S_i - \tau_{ei} \right]
\]

(33)

Using equation (31), equation (33) can be written as follows

\[
\dot{V}_3 = \sum_{i=u, q, r} S_i \frac{1}{\mu_i} \frac{p_i}{q_i} |i| e_i \left[ \lambda_i \tanh(\gamma_i S_i) + \tilde{k}_i S_i - \lambda_i \tanh(\gamma_i S_i) - k_i S_i - \tau_{ei} \right]
\]

(34)
Substituting for \( \lambda_i \) and \( k_i \) from equation (30) yields

\[
\dot{V}_3 = \sum_{i=u,q,r} S_i \frac{p_i}{\mu_i} \left| q_i \right| \frac{p_i}{\mu_i} - 1 \left[ \dot{x}_i \tanh(\gamma_S i) + \dot{k}_i S_i - \varepsilon_{12} S_i + \sum_{i=u,q,r} \left( - \frac{\gamma_{ik}}{2} \right) \right]
\]

\[
= - \sum_{i=u,q,r} S_i \frac{p_i}{\mu_i} \left| q_i \right| \frac{p_i}{\mu_i} - 1 \varepsilon_{12} S_i + \sum_{i=u,q,r} \left[ \dot{x}_i S_i - \frac{p_i}{\mu_i} \left| q_i \right| - \frac{\gamma_{ik}}{2} \right]
\]

\[
+ \sum_{i=u,q,r} \left( \dot{k}_i S_i - \frac{p_i}{\mu_i} \left| q_i \right| - \frac{\gamma_{ik}}{2} \right)
\]

Using the properties of trace, if

\[
\begin{cases}
\dot{x}_i = \gamma_{ij} \frac{p_i}{\mu_i} \frac{p_i}{\mu_i} - 1 \tanh(\gamma_S i) S_i \\
\dot{k}_i = \gamma_{ki} \frac{p_i}{\mu_i} \frac{p_i}{\mu_i} - 1 S_i^2
\end{cases}
\]

then

\[
\dot{V}_3 = - \sum_{i=u,q,r} \varepsilon_{12} \frac{1}{\mu_i} \left| q_i \right| \frac{p_i}{\mu_i} \frac{p_i}{\mu_i} - 1 S_i^2 \leq 0
\]

Since the nonlinear dynamics \( a_i(u, v, w, q, r) \) in equation (23) are always unknown, we cannot compute the control input value from equation (27). For solving the mentioned problem, RBF NN technique is employed to approximate the nonlinear dynamics due to its simplicity.

For an arbitrary continuous function \( f(x) : \mathcal{V} \rightarrow \mathcal{V} \), there exists an RBF NN as follows:

\[
f(x) = W \phi(x) + e_w(x)
\]

where \( f(x) \) denotes the input vector and \( f(x) = \left[ f_1(x), \ldots, f_p(x) \right]^T, x \in \mathcal{V}, p \) is the number of output nodes; \( W \in \mathcal{V}^{p \times \ell} \) represents weights matrix of the NN, \( \ell \) is the number of hidden nodes; \( e_w \) denotes the approximation error vector and \( e_w = \left[ e_{w1}, e_{w2}, \ldots, e_{wp} \right]^T \), where \( e_w \leq B_w, \forall x \in U \subset \mathcal{V} \), and \( B_w > 0 \) is an unknown constant; \( \phi(x) = [\sigma_1(x), \ldots, \sigma_{\ell}(x)]^T \), where \( \sigma_k(x) \) is the \( k \)th Gaussian basis function as

\[
\sigma_k(x) = \exp \left\{ -\frac{(x - \mu_k)T(x - \mu_k)}{\sigma_k^2} \right\}, \quad k = 1, 2, \ldots, \ell
\]

where \( \mu_k = [\mu_{k1}, \mu_{k2}, \ldots, \mu_{k\ell}]^T \) is the center vector and \( \sigma_k \) is standard deviation.

According to the RBF NN approximation above, equation (23) can be written as follows:

\[
f_i = W_i \sigma(x_i) + e_{wi}, \quad i = u, q, r
\]

Let \( W_i \) be the estimated value of \( W_i \), then the control input is changed as

\[
\begin{align*}
\tau_u &= -\mu_u q_u m_{1u} |u| - \lambda_u \tanh(\gamma_u S_u) - k_u S_u + \dot{W}_u \sigma(x_u) \\
\tau_q &= -\mu_q q_q m_{1q} |q| - \lambda_q \tanh(\gamma_q S_q) - k_q S_q + \dot{W}_q \sigma(x_q) \\
\tau_r &= -\mu_r q_r m_{1r} |r| - \lambda_r \tanh(\gamma_r S_r) - k_r S_r + \dot{W}_r \sigma(x_r)
\end{align*}
\]

and

\[
\dot{W}_i = W_i - \dot{W}_i, \quad i = u, q, r
\]

where \( \dot{W}_i \) is the estimation error.

Choosing a Lyapunov function

\[
V_4 = \sum_{i=u,q,r} \frac{1}{2} S_i^2 + \sum_{i=u,q,r} \left( \dot{W}_i^T \dot{W}_i + \frac{\dot{W}_i^2}{\gamma_{wi}} + \frac{\dot{k}_i^2}{\gamma_{ki}} \right)
\]

Differentiating equation (43), and substituting equation (41) into equation (25), \( V_4 \) can be described as follows:
Using equations (31) and (42), equation (44) can be written as follows:

\[
\dot{V}_4 = \sum_{i\in\{u,q,r\}} S_i \frac{1}{\mu_i} \frac{p_i}{q_i} \left[ -\hat{\lambda}_i \text{tanh}(\gamma_i S_i) - \hat{k}_i S_i + \tilde{W}_i \sigma(x_i) - \tau_{ei} - f_i \right] \\
+ \sum_{i\in\{u,q,r\}} S_i \frac{1}{\mu_i} \frac{p_i}{q_i} \left[ \hat{\lambda}_i \text{tanh}(\gamma_i S_i) - \hat{k}_i S_i - \tilde{W}_i \sigma(x_i) \right] \\
+ \sum_{i\in\{u,q,r\}} \left( -\frac{\tilde{W}_i \tilde{W}_i^T}{\gamma_{wi}} - \frac{\hat{\lambda}_i \hat{\lambda}_i}{\gamma_{ii}} - \frac{\hat{k}_i \hat{k}_i}{\gamma_{ki}} \right) \\
+ \sum_{i\in\{u,q,r\}} S_i \frac{1}{\mu_i} \frac{p_i}{q_i} \left[ -\tau_{ei} - f_i - \hat{\lambda}_i \text{tanh}(\gamma_i S_i) - k_i S_i + W_i \sigma(x_i) \right]
\]

Using equations (31) and (42), equation (44) can be written as follows:

\[
\dot{V}_4 = \sum_{i\in\{u,q,r\}} S_i \frac{1}{\mu_i} \frac{p_i}{q_i} \left[ \hat{\lambda}_i \text{tanh}(\gamma_i S_i) + \hat{k}_i S_i - \tilde{W}_i \sigma(x_i) \right] \\
+ \sum_{i\in\{u,q,r\}} \left( -\frac{\tilde{W}_i \tilde{W}_i^T}{\gamma_{wi}} - \frac{\hat{\lambda}_i \hat{\lambda}_i}{\gamma_{ii}} - \frac{\hat{k}_i \hat{k}_i}{\gamma_{ki}} \right) \\
+ \sum_{i\in\{u,q,r\}} S_i \frac{1}{\mu_i} \frac{p_i}{q_i} \left[ -\tau_{ei} - f_i - \hat{\lambda}_i \text{tanh}(\gamma_i S_i) - k_i S_i + W_i \sigma(x_i) \right]
\]

Substituting equations (30), (36), and (40) into equation (45) yields

\[
\dot{V}_4 = -\sum_{i\in\{u,q,r\}} \varepsilon_{i2} \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right| S_i^2 - \sum_{i\in\{u,q,r\}} \tilde{W}_i S_i \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right| \sigma(x_i) + \frac{\tilde{W}_i^T}{\gamma_{wi}} \\
- \sum_{i\in\{u,q,r\}} S_i \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right| e_{wi} \leq -\sum_{i\in\{u,q,r\}} \varepsilon_{i2} \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right| S_i^2 \\
- \sum_{i\in\{u,q,r\}} \tilde{W}_i \left| S_i \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right| \right| \sigma(x_i) + \frac{\tilde{W}_i^T}{\gamma_{wi}} - \sum_{i\in\{u,q,r\}} \left| S_i \right| \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right| B_W
\]

Using the properties of trace, if

\[
\dot{W} = -\gamma_{wi} \sigma^T(x_i) S_i \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right|^{\mu_i - 1}, i = u, q, r
\]

where \(\gamma_{wi} > 0\) are design parameters

\[
x_u := [u, v, w, q, r, \dot{u}_c]^T, x_q := [u, w, q, 0, \dot{q}_c]^T,
\]

and \(x_r := [u, v, r, \dot{r}_c]^T\)

then

\[
\dot{V}_4 = -\sum_{i\in\{u,q,r\}} \varepsilon_{i2} \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right| S_i^2 - \sum_{i\in\{u,q,r\}} S_i \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right| e_{wi} \leq -\sum_{i\in\{u,q,r\}} \varepsilon_{i2} \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right| S_i^2 - \sum_{i\in\{u,q,r\}} \left| S_i \right| \frac{1}{\mu_i} \frac{p_i}{q_i} \left| \tilde{\epsilon}_i \right| B_W
\]

Since \(\varepsilon_{i2}, \mu_i, p_i,\) and \(q_i\) are all positive, and \(\left| \tilde{\epsilon}_i \right|^{\mu_i - 1} \geq 0,\)

\(|S_i| \geq 0, B_W > 0, \dot{V}_4 \leq 0.\) It is easily understood that
sliding mode can be achieved in finite time. Hence, we can conclude that under the proposed tracking control law of equations (40), (36), and (47), the tracking error will converge to neighborhood of the origin without any singularity.

Simulation results

To verify the effectiveness and robustness of the proposed target tracking controller, numerical simulations are carried out with an AUV named WL-4, as shown in Figure 4. The AUV is driven by a stern thruster and manipulated by elevators and rudders arranged in a cross. Parameters of the AUV are provided in Table 1.

The other dynamic parameters of the AUV are assumed to be unknown completely. Ocean environmental disturbances are assumed to be steady ocean current, and the velocity is $V_{\text{current}} = (0.5, 0, 0)$ m/s.

In this simulation, controller parameters are set to $\delta_0 = 4$ m, $k_s = 1.5$, $k_f = k_u = 4.8$, $\kappa_i = 0.01$, $\gamma_{li} = \gamma_{hi} = 0.2$, $\gamma_{Wi} = \gamma_l = 10$, $\mu_q = \mu_r = 0.4$, $\mu_u = 2$, $p_i = 9$, and $q_i = 7$, where $i = u, q, r$. The initial position and orientation of the AUV are given by $(\xi_0, \eta_0, \zeta_0)_{\text{AUV}} = (95, 95, 95)$ m and $(\varphi_0, \theta_0, \psi_0) = (0, 0, 0)$ rad, and the initial velocity is $(u_0, v_0, w_0) = (0, 0, 0)$ m/s. The target’s initial

Table 1. Parameters of the WL-4 AUV.

| Parameter   | Value |
|-------------|-------|
| $m$         | 83.7 kg |
| $h$         | 11.8 mm |
| $I_x$       | 22.27 kg · m$^2$ |
| $I_y$       | 22.16 kg · m$^2$ |
| $X_u$       | -6.76 kg |
| $Y_u$       | -102.8 kg |
| $Z_u$       | -106.8 kg |
| $N_u$       | -27.04 kg · m$^2$ |
| $M_u$       | -27.84 kg · m$^2$ |

AUV: autonomous underwater vehicle.

Figure 4. The WL-4 AUV. AUV: autonomous underwater vehicle.

Figure 5. AUV and target trajectories. (a) 3-D space, (b) horizontal plane, and (c) vertical plane. AUV: autonomous underwater vehicle.
position is \((\xi_0, \eta_0, \zeta_0)_d = (100, 100, 100)\) m and the motion is generated by the following timing laws:

\[
\begin{align*}
\xi_d &= 100 + 0.5t \\
\eta_d &= 100 + 50\sin(0.01t) \\
\zeta_d &= 100 + 50\sin(0.01t)
\end{align*}
\]

Simulation results are shown in Figures 5 to 10, which shows that AUV successfully track the target with a distance of \(\delta_0 = 4\) m in the presence of model uncertainties and ocean disturbance. Figure 5 shows the trajectories of AUV and target in 3-D space and in horizontal and vertical planes. Although the
**Figure 8.** Control efforts: (a) force produced by thruster, (b) moment produced by elevators, and (c) moment produced by rudders.

**Figure 9.** Estimation of nonlinear dynamics: (a) surge force, (b) pitch moment, and (c) yaw moment.
AUV’s trajectory is different from the target’s, the tracking is stable. Figure 6 shows the linear and angular velocities of AUV during tracking. Figure 7 shows that range and bearing angle tracking errors, that is, \((\delta - \delta_0)\), \(\alpha\) and \(\beta\), asymptotically converge to small neighborhood around zero. Figure 8 shows the required control efforts of longitudinal thrust, pitching moment, and yawing moment of AUV under the action of steady ocean current. It can be observed from Figures 8 and 9 that the thrust is larger in the initial stage, and then remains near 10 N. Figure 10 shows that the adaptive tune gains work well under the perturbation of steady currents.

**Conclusion**

In this article, an adaptive target tracking controller based on nonsingular terminal sliding mode control is designed for underactuated AUV. Unknown dynamics of the AUV are approximated by RBFNN. Adaptive tuning algorithm is employed to update gains of sliding mode controller and weights of the NN, which makes the system more robust to model uncertainties and external disturbances. Lyapunov method is utilized to examine the stability of the target tracking control system, which proves that all control inputs are bounded and tracking errors could converge to a small neighborhood of zero in finite time. In the end, simulation is carried out, and the results illustrate that the proposed controller is effective for underactuated AUV applications.

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