Analysis of variable mass diffusivity in Maxwell’s fluid with Cattaneo-Christov and nonlinear stratification

Iffat Jabeen a, S. Ahmad b,*, Aisha Anjum c, M. Farooq d

a Department of Mathematics, COMSATS University Islamabad, Islamabad 44000, Pakistan
b Department of Mathematics, Riphah International University, Islamabad 44000, Pakistan
c Department of Mathematics, National University of Modern Languages, Islamabad, Pakistan
d Department of Pure and Applied Mathematics, The University of Haripur, Haripur, KPK, Pakistan

ABSTRACT

This investigation consists of Maxwell's fluid which describes rate type non-Newtonian fluid in best; it has great applications in engineering, technology and industry. Linear stretching sheet generates the flow in fluid. Flow momentum is measured with MHD effect. When Fourier's and Fick's laws are incorporated relaxation time factor then known as Cattaneo-Christov model which are implemented for heat and mass transport. The features of heat source or sink and non-linear type thermal stratification are employed with variable thermal conductivity. The features of chemical reaction and non-linear type solutal stratification are analyzed along with variable mass diffusivity. By the usage of boundary layer phenomenon in this problem, non-linear PDEs are achieved. These equations are transmuted into non-linear and non-homogeneous differential equations ramified with ordinary derivatives after applying similarity transformations. The most exclusive homotopic analysis method is used to get the analytic solutions of nonlinear and non-dimensional governing equations. The significant results of progressive parameters are dominant in this investigation. The arising parameters are examined in detail and results are shown graphically. It is found out that with the increment of time relaxation factor velocity, temperature and concentration profiles reduce. It increases the viscoelastic impacts related to stress relaxation time which makes viscoelastic materials more durable.

1. Introduction

Simple relationship between stress and strain is not enough to make difference between non-Newtonian fluids. They possess different properties according to the physical behavior of the fluids. They are of different categories as differential type, integral type or rate type. Maxwell's fluid falls in the category of rate type liquids, and stress relaxation and stress retardation show rate type non-Newtonian fluid models. It also possesses viscoelastic impacts in terms of the factor stress relaxation time, and it has vast application in automobiles, computers, mechanical engineering, medical and polymer industry. Hayat et al. [1] examined Hall impacts of Maxwell's fluid in porous medium. Khan et al. [2] elaborated viscoelastic fluid flow with Maxwell's model. Nadeem et al. [3] described Maxwell's fluid flow by accounting chemical reaction. Hayat et al. [4] described the characteristics of Brownian and thermophoretic diffusions in Maxwell's magnetized fluid flow under stretching phenomenon. Khan et al. [5] explained the effects of double stratification in Maxwell's fluid motion by considering variable fluid viscosity. Ahmad et al. [6] examined the heat and mass characteristics in motion of Maxwell's liquid by employing double stratification phenomenon. Zhang et al. [7] disclosed the Marangoni type convection in Maxwell's material flow under heating process. Gangadhar et al. [8] depicted the Hall and magnetic fields influence in second grade radiative nanomaterial flow caused by Riga sheet with convective heat process. Gangadhar et al. [9] stated the magnetization and convective heat processes in burgers’ liquid flow with auto-catalytic reactions. Gangadhar et al. [10] examined the magnetization characteristics in couple stress radiative liquid through paraboloid surface with bi-convective transport. Some more significant researches in this direction are illustrated in references [11, 12, 13, 14, 15].

* Corresponding author.
E-mail addresses: shakeel_oiiui@hotmail.com (S. Ahmad).

https://doi.org/10.1016/j.heliyon.2022.e11850

Received 18 October 2021; Received in revised form 19 January 2022; Accepted 16 November 2022

2405-8440/© 2022 The Author(s). Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
The phenomenon of heat transport has attracted very effective applications such as conduction of heat in drugs and muscular tissues, nuclear reactor cooling process, etc. In recent past, conduction law (Fourier’s law) [16] and mass diffusion law (Fick’s law) [17] have been attained much attraction as the heat and mass exchange while ignoring the anomaly occurred in these theories. Heat conduction law gives parabolic type expression which describes that heat transmuted instantly through whole object. In order to fix this ambiguity, Cattaneo [18] introduced the factor of relaxation time in order to overcome the anomaly, and supports the wavy transportation of heat with low speed. Later, Christov [19] made enhancement in Cattaneo law. Nadeem and Muhammad [20] described the darcian flow of double stratified liquid under modified heat flux. Results depict that relaxation time factor reduces the temperature. Anjum et al. [21] stated the generalized heat transport features in differential type fluid flow via stretchable Riga sheet accounting heat generation. Hayat et al. [22] addressed the irreversibility features in second grade hydromagnetic nanomaterial flow deformed by Riga sheet under modified double diffusion analysis. Loganathan et al. [23] discussed the modified double diffusion analysis in Maxwell’s radiative fluid motion over stretchable sheet with MHD effects. Zhang et al. [24] described the enthalpy impacts in polyacrylamide nanomaterial flow under Cattaneo-Christov type fluxes. Ramana et al. [25] disclosed the effects of nonlinear stretching phenomenon on Oldroyd-B hydromagnetic material flow under generalized heat flux model. Azam [26] illustrated the modified diffusion and activation energy phenomena in nonlinear type radiative motion of Maxwell MHD nanoliquid. It is witnessed that temperature goes incrementing upon increasing modified diffusion effects.

Stratification occurs where fusion occurs between two or more fluids. Practically dual stratification appears collectively with the implementation of heat and mass mechanism. Free convection phenomenon in a stratified medium has great importance in fisheries management, in lower parts of ponds, rivers, lakes and oceans, heat reduction in solar ponds and power plant condensers [27]. Farooq et al. [28] discussed the analysis of stagnation point in nanomaterial flow with dual stratification and melting phenomenon. Ahmad et al. [29] explored the squeezing slip flow deformed by lower stretchable inclined surface with double stratified phenomenon. Ali et al. [30] depicted phenomenon of dual stratification in Maxwell’s nanoliquid flow deformed by stretchable inclined surface considering heat source/sink. Jabeen et al. [31] discussed the nonlinear double stratification analysis in Eyring-Powell material flow under modern diffusion and mixed convection. Malik et al. [32] described the Falkner-Skan type movement of nonlinear stratified Jeffrey liquid under the impact of point of stagnation and mixed convection. Khan et al. [33] examined the nonlinear stratified properties in convective motion of fluid caused by stretchable inclined surface. Shafiq et al. [34] explained the nonlinear stratification and convective heating analysis in squeezing non-darcian flow of fluid along with artificial neural network. Khalil et al. [35] examined the dual stratification features in flow of non-Newtonian magnetized fluid deformed by inclined stretchable surface under reactive features.

It appears that among thermal researchers, variable fluid characteristics under modern diffusion phenomenon have attracted significant attention. The goal of using such aspects in a Maxwell liquid is to depict accurately the transportation properties of the fluid. Therefore, in light of above literature, the survey discloses that the simultaneous role of variable thermal and solutal relaxation time factor along with non-linear dual phenomenon of stratification in the heat and mass transport inspection utilizing Cattaneo-Christov theory has not yet been analyzed. Owing to this fact, the specific aim of present study is to see the variable fluid features in heat and mass transportation under the modern double diffusion beside aspects of chemical reaction and heat generation/absorption. The physical analysis of hydromagnetic influences in the nonlinear stratified Maxwell fluid through a stretchable sheet is studied. Homotopic method [36, 37, 38, 39, 40] is employed in order to find convergent analytical solutions. The related distributions are discussed graphically through dominating parameters.

2. Problem modeling

Fig. 1 explicates the mechanism of the considered problem. Here, we investigate the steady motion of incompressible magneto-hydrodynamic Maxwell’s fluid caused by stretching phenomenon. Since the considered fluid motion is incompressible, hence system of Cartesian coordinate is employed. The components of velocity \((u, v)\) are selected along the \((x, y)\)-directions respectively. The sheet is moved with stretching velocity \(u_s(x)\). Chemical reaction and heat absorption/generation are also accounted. Further, nonlinear double stratification phenomenon is used to study heat and mass transport under the law of Cattaneo-Christov. For velocity field \(\mathbf{V} = \mathbf{V}[u(x, y), v(x, y)]\), the governing equations are defined in equations (1) and (2) as:

\[
\rho C_p \frac{\partial \mathbf{V}}{\partial t} = \nabla \cdot \mathbf{q} + \frac{\partial \mathbf{V}}{\partial t} \cdot \mathbf{V} - k \nabla T, \tag{1}
\]

\[
\nabla \cdot \mathbf{V} = 0, \tag{2}
\]

where, \(\mathbf{q}\) and \(\mathbf{j}\) represent fluxes related to heat and mass process which can be expressed for incompressible and steady fluid flow under the Cattaneo-Christov theory in equations (3) and (4) as:

\[
\mathbf{q} + \omega_1(T) \nabla \mathbf{q} - \mathbf{V} \nabla \mathbf{V} = -k(T) \nabla T, \tag{3}
\]

and

\[
\mathbf{j} + \omega_2(C) \nabla \mathbf{j} - \mathbf{V} \nabla \mathbf{V} = -D(C) \nabla C. \tag{4}
\]

here, \(\omega_1(T)\) denotes the variable thermal relaxation time factor, \(D(C)\) denotes varying mass diffusivity, \(\omega_2(C)\) denotes the variable solutal relaxation time factor, \(k(T)\) denotes varying thermal conductivity. The mathematical expressions for \(\omega_1(T), \omega_2(C), k(T)\) and \(D(C)\) are given in equations (5) and (6) as:

\[
\omega_1(T) = \frac{\alpha}{1 + \beta (T - T_\infty)}, \quad \omega_2(C) = \frac{\alpha^*}{1 + \beta_1 (C - C_\infty)}, \tag{5}
\]

\[
k(T) = k_\infty (1 + \varepsilon), \quad D(C) = D_\infty (1 + \varepsilon_1 \phi), \tag{6}
\]

where, \(\varepsilon, \varepsilon_1\) represent small scalar parameters called thermal and mass diffusivity parameters.

By the utilization of above equations under the boundary layer assumption, we get the equation (7) to equation (10) [5], [23]:

\[
\frac{\partial}{\partial x} + \frac{\partial}{\partial y} = 0. \tag{7}
\]
\[
\begin{align*}
\frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} + \lambda_1 \left( \frac{\partial^2 u}{\partial x^2} + \epsilon \frac{\partial^2 u}{\partial y^2} + 2\mu c \frac{\partial^2 u}{\partial x \partial y} \right) &= +v \left( \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \beta c}{\rho} \left( u + \lambda_1 \frac{\partial u}{\partial y} \right) \right), \\
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{1}{\rho C_p} \frac{\partial}{\partial y} \left( k(T) \frac{\partial T}{\partial y} \right) - \frac{\omega c}{1 + \beta(T - T_m)} \left[ u \frac{\partial^2 T}{\partial x \partial y} + \frac{\partial u}{\partial y} \frac{\partial T}{\partial y} + u \frac{\partial^2 T}{\partial y^2} + v \frac{\partial^2 T}{\partial y^2} \right] \\
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= \frac{\omega c}{1 + \beta_1(C - C_m)} \left[ \frac{\partial^2 C}{\partial x^2} + u \frac{\partial C}{\partial x} + \epsilon \frac{\partial C}{\partial y} + \frac{\partial^2 C}{\partial y^2} \right] \\
&+ \frac{\omega c \beta}{1 + \beta_1(C - C_m)} \left[ \frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial y \partial x} (C - C_m) + \frac{\partial C}{\partial y} \right] \\
&+ \frac{\omega c K_1}{1 + \beta_1(C - C_m)} \left[ \frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial y \partial x} (C - C_m) \right] \\
&+ K_1(C - C_m) + D_m(1 + \epsilon \phi) \frac{\partial^2 C}{\partial y^2} + D \frac{\partial C}{\partial y}.
\end{align*}
\]

with subjected conditions at boundary represented by equation (11):

\[
\begin{align*}
&\begin{align*}
\text{at } y = 0, \\
\text{at } y = \infty, \\
\text{at } y = \infty,
\end{align*}
\end{align*}
\]

here, \( u, v \) depict velocity component, \( \lambda_1 \) represents relaxation time factor, \( \sigma \) represents electric conductivity, \( \omega \) is solutal relaxation time coefficient, \( T_0 \) is reference temperature, \( \rho \) is density, \( K_1 \) is chemical reaction coefficient, \( D_m \) is mass diffusivity of surrounding, \( \omega \) is thermal relaxation time coefficient, \( T \) represents fluid temperature, \( C_p \) is fluid specific heat, \( \beta \) represents fluid concentration, \( u_m \) is stretching velocity, \( \omega \) defines thermal property, \( \beta_m \) is variable ambient concentration, \( Q_0 \) represents heat generation/absorption coefficient, \( \beta_m \) is magnetic field strength, \( T_m \) is wall temperature, \( \beta_m \) represents concentration property, \( v \) is fluid kinematic viscosity, \( C_m \) is wall concentration, \( C \) is reference concentration, \( T_m \) is variable ambient fluid temperature, \( k \) is thermal conductivity of surrounding, \( c, d, \), represent dimension constant.

By implementing equation (12):

\[
\begin{align*}
\xi &= \sqrt{\frac{\varepsilon}{T_0}} \sqrt{\frac{T}{T_0}}, \\
\zeta &= \frac{C - C_m}{C_m - C_0}, \\
\theta &= \frac{T - T_m}{T_m - T_0}, \\
\phi &= \frac{C - C_0}{C_m - C_0},
\end{align*}
\]

we get equation (13) to equation (15):

\[
\begin{align*}
&f''' + f'' - f' - 2 - M f' - k_2 \left( f^2 f'' + 2 f^2 f' f'' + M f f'' \right) = 0, \\
&\theta''(1 + \theta) + c \theta^{\prime 2} + \frac{Pr \left[ k_1(1 - \Theta) \left( f^2 f'' + 2 f^2 f' f'' + M f f'' \right) - k_2 Pr (1 - \theta) \left( f^{(n-1)} f' \left( S_1 + \Theta \right) + f^{(n-2)} f' \left( S_1 + \Theta \right) \right) \right]}{1 + k_1} \\
&\left[ \frac{v(n-1) f^{(n-2)} (S_1 + \Theta) f^2}{1 + k_2} \right] + \frac{v(n-1) f^{(n-2)} (S_1 + \Theta) f^2}{1 + k_2} + \frac{v(n-1) f^{(n-2)} (S_1 + \Theta) f^2}{1 + k_2} = 0, \\
&+ Pr k_3 (1 - 2 \Theta) \left[ \frac{v(n-1) f^{(n-2)} (S_1 + \Theta) f^2}{1 + k_2} \right] + \frac{v(n-1) f^{(n-2)} (S_1 + \Theta) f^2}{1 + k_2} + \frac{v(n-1) f^{(n-2)} (S_1 + \Theta) f^2}{1 + k_2} = 0.
\end{align*}
\]
\[ \phi'' + \varepsilon_1 \phi'' + \phi' f - n S c S f'' - n S c f' \phi + S c f \phi' = k_4 S c (1 - \phi \phi) + n (n - 1) f'' (S_2 + \phi) + n f'^2 (S_2 + \phi) + f' \phi'' \\
- S c k_4 (1 - \phi \phi) [2 \phi f' - \phi f] - S c k_4 \phi (1 - 2 \phi \phi) [n^2 f'^2 S_2 \phi + n^2 f''^2 \phi^2 + f'^2 \phi'^2] + S c k_4 \phi = 0. \] (15)

The corresponding dimensionless conditions at boundary represented by equation (16) are:

\[
\begin{align*}
& f(0) = 0, & \theta(0) &= 1 - S_1, & f'(0) &= 1, \quad \phi(0) = 1 - S_2, \\
& f(\infty) = 0, & \phi(\infty) &= 0, & \theta(\infty) &= 0,
\end{align*}
\] (16)

where \( k_2, k_3, k_4 \) represent Deborah number, thermal relaxation time parameter, solutal relaxation time parameter, respectively, \( S_1 \) represents thermal stratified parameter, \( M \) is magnetic parameter, \( S c \) denotes Schmidt number, \( Pr \) is Prandtl number is indicated by \( Pr \), chemical reaction parameter is denoted by \( kr \), \( S_2 \) represents solutal stratified parameter, heat generation/absorption parameter is represented by \( \delta, \theta, \phi \) represent temperature and concentration difference parameters respectively. Mathematically, these parameters can be represented in equation (17) as:

\[ Pr = \frac{v}{u} \quad kr = \frac{k_1}{c}, \quad k_2 = \Lambda_1 e, \quad k_3 = \omega c, \quad k_4 = \omega c e, \quad \phi = \beta_1 (C_{w-c} - C_0). \]

\[ M = \frac{\sigma \rho c^3}{\rho c e}, \quad S_1 = \frac{d_1}{d_1}, \quad S_2 = \frac{d_2}{d_1}, \quad \delta_1 = \frac{Q_0}{\rho c p c}, \quad \theta_1 = \beta(T_{w-c} - T_0), \quad S c = \frac{v}{D_0}. \] (17)

3. Homotopic technique

While working with homotopy method [36, 37, 38, 39, 40], it is important to have initial approximation along with linear operators. Thus, the initiatory guesses given in equations (18)–(20) as:

\[
\begin{align*}
& f_0(\xi) = 1 - \exp(-\xi), \\
& \theta_0(\xi) = (1 - S_1) \exp(-\xi), \\
& \phi_0(\xi) = (1 - S_2) \exp(-\xi).
\end{align*}
\] (18–20)

Supporting linear operators expressed in equation (21) are:

\[
\begin{align*}
& L_f(f) = \frac{d^3 f}{d \xi^3} - \frac{d f}{d \xi}, \\
& L_\theta(\theta) = \frac{d^2 \theta}{d \xi^2} - \theta, \\
& L_\phi(\phi) = \frac{d^2 \phi}{d \xi^2} - \phi.
\end{align*}
\] (21)

which satisfy the specified properties given in equations (22)–(24):

\[
\begin{align*}
& L_f(D_1 + D_2 e^{i\xi} + D_3 e^{-i\xi}) = 0, \\
& L_\theta(D_4 e^{i\xi} + D_5 e^{-i\xi}) = 0, \\
& L_\phi(D_6 e^{i\xi} + D_7 e^{-i\xi}) = 0.
\end{align*}
\] (22–24)

here \( D_i \) \((i = 1, 2, 3, ..., 7) \) represent optional constants.

3.1. Zeroth-order problems

Here, equation (25) to equation (28) represents zeroth order deformation problem which is given as:

\[
\begin{align*}
(1 - q) L_f [f(\xi; q) - f_0(\xi)] &= q h_1 N_f [f(\xi; q)], \\
(1 - q) L_\theta [\theta(\xi; q) - \theta_0(\xi)] &= q h_2 \phi_0 [\theta(\xi; q), f(\xi; q)], \\
(1 - q) L_\phi [\phi(\xi; q) - \phi_0(\xi)] &= q h_3 \phi_0 [\phi(\xi; q), f(\xi; q)], \\
\phi'(0; q) &= 1, \quad f'(0; q) = 0, \quad \theta(0; q) = 1 - S_1, \quad \phi(0; q) = 1 - S_2, \quad f'(\infty; q) = 0, \quad \theta(\infty; q) = 0, \quad \phi(\infty; q) = 0.
\end{align*}
\] (25–28)

where, linear operators define in equations (29)–(31) as:

\[
\begin{align*}
L_f[f(\xi, q)] &= \frac{d^3 f}{d \xi^3} + f(\xi, q) \frac{d^2 f}{d \xi^2} - \left( \frac{d f}{d \xi} \right)^2 - M \left( \frac{d f}{d \xi} \right)^2 \\
&- k_2 \left[ \left( f(\xi, q) \right) \left( \frac{d f}{d \xi} \right) + 2 \left( \frac{d f}{d \xi} \right)^2 \right] + M \left( \frac{d f}{d \xi} \right) f(\xi, q).
\end{align*}
\] (29)
\[
\begin{align*}
\mathcal{H}_\theta \left[ \delta \theta (\zeta, q) \right] &= \frac{\delta^2 \delta \theta (\zeta, q)}{\delta z^2} (1 + \epsilon \delta \theta (\zeta, q)) + \epsilon \left( \frac{\delta f (\zeta, q)}{\delta z} \right)^2 + \delta \lambda \mathcal{H} \delta \theta (\zeta, q) \\
- n S_k \rho \frac{\delta f (\zeta, q)}{\delta z} - n P \frac{\delta f (\zeta, q)}{\delta z} \delta \theta (\zeta, q) + P \rho \frac{\delta f (\zeta, q)}{\delta z} \delta \theta (\zeta, q) \\
&= \left[ m(n - 1) \left( \frac{\delta f (\zeta, q)}{\delta z} \right)^2 (S_1 + \delta \theta (\zeta, q)) + n \left( \frac{\delta f (\zeta, q)}{\delta z} \right)^2 (S_1 + \delta \theta (\zeta, q)) \right] \\
- k_2 \rho \frac{1 - \delta \theta (\zeta, q) \delta \theta (\zeta, q)}{\delta z} f(\zeta, q) (S_1 + \delta \theta (\zeta, q)) \\
&= \left[ n^2 S_2 \delta \theta (\zeta, q) \left( \frac{\delta f (\zeta, q)}{\delta z} \right)^2 \right] \\
+ k_2 \rho \delta \theta (\zeta, q) (1 - 2 \delta \theta (\zeta, q) \delta \theta (\zeta, q)) f(\zeta, q) (S_1 + \delta \theta (\zeta, q)) \\
+ f(\zeta, q) f(\zeta, q) \left( \frac{\delta f (\zeta, q)}{\delta z} \right)^2 \right]
\end{align*}
\]

\[
\begin{align*}
\mathcal{H}_\phi \left[ \delta \phi (\zeta, q) \right] &= \frac{\delta^2 \delta \phi (\zeta, q)}{\delta z^2} + \epsilon \delta \phi (\zeta, q) + f(\zeta, q) \delta \phi (\zeta, q) + n S_2 \delta \phi (\zeta, q) \\
&= \left[ n(n - 1) \left( \frac{\delta f (\zeta, q)}{\delta z} \right)^2 (S_2 + \delta \phi (\zeta, q)) + n \left( \frac{\delta f (\zeta, q)}{\delta z} \right)^2 (S_2 + \delta \phi (\zeta, q)) \right] \\
- k_2 S_2 \phi (\zeta, q) \delta \phi (\zeta, q) f(\zeta, q) (S_2 + \delta \phi (\zeta, q)) \\
&= \left[ n^2 \phi (\zeta, q) \left( \frac{\delta f (\zeta, q)}{\delta z} \right)^2 \right] S_2 \\
- k_2 \rho \delta \phi (\zeta, q) f(\zeta, q) \phi (\zeta, q) \delta \phi (\zeta, q) \\
&= \left[ n^2 \phi (\zeta, q) \left( \frac{\delta f (\zeta, q)}{\delta z} \right)^2 \right] S_2 \\
+ f(\zeta, q) f(\zeta, q) \left( \frac{\delta f (\zeta, q)}{\delta z} \right)^2 \right]
\end{align*}
\]

Auxiliary parameters \( h_f, h_\theta, \) and \( h_\phi \) have value other than zero whereas \( q \) represents embedding parameter having values between 0 and 1.

### 3.2. m-th order problem

Here, equation (32) to equation (35) represents \( m \)-th order deformation problem which is given as:

\[
\begin{align*}
L_f \left[ f_m (\zeta) - x_n f_{m-1} (\zeta) \right] &= h_f \mathcal{R}_f (\zeta), \\
L_\theta \left[ \theta_m (\zeta) - x_n \theta_{m-1} (\zeta) \right] &= h_\theta \mathcal{R}_\theta (\zeta), \\
L_\phi \left[ \phi_m (\zeta) - x_n \phi_{m-1} (\zeta) \right] &= h_\phi \mathcal{R}_\phi (\zeta), \\
f_m'(0) = 0, \quad f_m(0) = 0, \quad \theta_m(0) = 0, \quad \phi_m(0) = 0, \\
f_m'(\infty) = 0, \quad \theta_m(\infty) = 0, \quad \phi_m(\infty) = 0,
\end{align*}
\]

where, linear operators for \( m \)-th order define in equations (36)–(39):

\[
\begin{align*}
\mathcal{R}_f (\zeta) &= f_m'' + \sum_{k=0}^{m-1} f_{m-k-1} f_k' - \sum_{k=0}^{m-1} f_{m-k-1} f_k' - M \sum_{k=0}^{m-1} f_{m-k-1} f_k \\
- k_2 \left[ 2 \sum_{k=0}^{m-1} f_{m-k-1} \sum_{n=0}^{k} f_{k-n} f_n + M \sum_{n=0}^{m-1} f_{m-n-1} f_k \right]
\end{align*}
\]
\[
R_m^w(\xi) = \theta_m'' + \epsilon \sum_{k=0}^{m-1} (\theta_m''_{m-1-k} \theta_k) + \epsilon \sum_{k=0}^{m-1} \theta_m''_{m-1-k} \theta_k' + 6 \theta \theta_m
\]

\[
-n Pr S_1 f_{m+1}' - n Pr \sum_{k=0}^{m-1} (f_{m-1-k} \theta_k) + Pr \sum_{k=0}^{m-1} (\theta_{m-1-k} f_k)
\]

\[
-k_3 Pr (1 - \theta \theta_{m-1}) \left[ \begin{align*}
&\frac{n(n-1)S^{m-1}_{k=0} f'_{m-1-k} f_k' + nS^{m-1}_{k=0} f''_{m-1-k} f_k' + nS^{m-1}_{k=0} f''_{m-1-k} f_k'}{+ n(n-1) \sum_{k=0}^{m-1} f'_{m-1-k} \sum_{k=0}^{m-1} f''_{m-1-k} \theta_k + n \sum_{k=0}^{m-1} f'_{m-1-k} \sum_{k=0}^{m-1} f''_{m-1-k} \theta_k} \\
&- S_1 \sum_{k=0}^{m-1} (f''_{m-1-k} f_k) - \sum_{k=0}^{m-1} f''_{m-1-k} \sum_{k=0}^{m-1} f_k \theta_k \sum_{k=0}^{m-1} \theta_k \sum_{k=0}^{m-1} f_k \theta_k
\end{align*} \right]
\]

\[
S c k \left[ (1 - \phi \phi_{m-1}) \left[ \begin{align*}
&\frac{2 \sum_{k=0}^{m-1} (f'_{m-1-k} \theta_k) - \sum_{k=0}^{m-1} (f'_{m-1-k} \theta_k)}{\phi_k' + 2 \sum_{k=0}^{m-1} (f'_{m-1-k} \theta_k)}
\end{align*} \right]
\]

To find the solutions, the following expression can be written in equations (40)–(42):

\[
f(\xi; 0) = f_0(\xi), \quad f(\xi; 1) = f(\xi),
\]

\[
\theta(\xi; 0) = \theta_p(\xi), \quad \theta(\xi; 1) = \theta(\xi),
\]

\[
\phi(\xi; 0) = \phi_0(\xi), \quad \phi(\xi; 1) = \phi(\xi),
\]

and when \(q\) takes value 0 then initial approximations \(f_0(\xi), \theta_p(\xi)\) and \(\phi_0(\xi)\) arises and hence final solutions get for value 1. By utilizing \(q = 1\) and series of Taylor we get equations (43)–(45):

\[
f(\xi) = f_n(\xi) + \sum_{n=1}^{\infty} f_n(\xi),
\]

\[
\theta(\xi) = \theta_n(\xi) + \sum_{n=1}^{\infty} \theta_n(\xi),
\]

\[
\phi(\xi) = \phi_n(\xi) + \sum_{n=1}^{\infty} \phi_n(\xi).
\]

The appropriate solutions \(f_n, \theta_n\) and \(\phi_n\) of equations (43) to (45) corresponding to \((f''_m, \theta''_m\) and \(\phi''_m\) are given in equations (46)–(48):

\[
f_m(\xi) = D_1 + D_2 \exp(\xi) + D_3 \exp(-\xi) + f''_m(\xi),
\]

\[
\theta_m(\xi) = D_4 \exp(\xi) + D_5 \exp(-\xi) + \theta''_m(\xi),
\]

\[
\phi_m(\xi) = D_6 \exp(\xi) + D_7 \exp(-\xi) + \phi''_m(\xi).
\]
\[ \phi_m(\xi) = D_b \exp(\xi) + D_\gamma \exp(-\xi) + \phi_\ast^m(\xi). \]  
\hspace{10cm} (48)

3.3. Convergence analysis

HAM conveniently facilitates in order to provide convergent series solution. It is highly dependent upon auxiliary parameters. Hence, \( h \)-curve has drawn. It is witnessed from Fig. 2, the acceptable values of \( h_f, h_\theta, h_\phi \) parameters are \(-1.8 \leq h_f \leq -0.2, -1.7 \leq h_\theta \leq -0.5 \) and \(-0.8 \leq h_\phi \leq -0.2\).
4. Discussion

Variations in concentration, temperature and velocity fields against pertinent parameters are reported in this portion. Fig. 3 indicates the decrementing impact of velocity field \( f'(\xi) \) with increment of Deborah number \( k_d \). As Deborah number contains relaxation time factor and when relaxation time factor increases then greater Deborah number will appear which produces resistance in fluid flow and consequently, velocity becomes smaller. Maxwell’s fluid will turn out to be viscous fluid if \( k_d \) is zero. Fig. 4 depicts the decreasing profile of velocity field \( f'(\xi) \) due to dominant magnetic parameter \( M \). Physically, with the increment of \( M \), density of fluid increases and resistive forces appear which slows down the velocity profile. Fig. 5 indicates the increasing characteristics of velocity field \( f'(\xi) \) against higher heat generation parameter \( \delta_1 \). As the heat generation parameter \( \delta_1 \) increases extra heat is engaged by the fluid particles which show elevation in velocity distribution. Fig. 6 reports temperature field against Prandtl number \( Pr \). It confirms that with the greater values of \( Pr \), thermal diffusivity becomes small which declares reduction in temperature. Fig. 7 highlights the reducing trend of temperature field \( \theta(\xi) \) for greater parameter \( k_3 \). It is shown that with the increment of \( k_3 \), thermal boundary layer and temperature become smaller. Physically, \( k_3 \) depends on thermal relaxation time factor. When value of \( k_3 \) increases then thermal relaxation time factor increases due to which temperature decreases. If \( k_3 = 0 \) then Fourier’s law will be achieved. Fig. 8 explains the declining field of temperature against stratification parameters \( S_1 \). Physically, low temperature is a result of decaying difference among surface temperature and ambient fluid temperature. Thus, temperature declines. Fig. 9 discloses the difference parameter \( \theta_l \) impacts on temperature \( \theta(\xi) \). Here, temperature and related thickness of layer grows when \( \theta_l \) is increased. Physically, the difference among sheet temperature and layer temperature of fluid grows and as a consequence, temperature raises. Fig. 10 states the decreasing trend of concentration field \( \phi(\xi) \) for higher Schmidt number \( Sc \). Dominating \( Sc \) reduces the mass diffusion coefficient and consequently, concentration distribution reduces. Fig. 11 examines concentration field against solutal stratified parameter \( S_2 \). Concentration reduces for greater \( S_2 \). Physically, difference in surface concentration and ambient concentration diminishes when \( S_2 \) is enlarged. Thus, concentration field diminishes. Competency of concentration difference parameter \( \phi_l \) on liquid concentration \( \phi(\xi) \) is highlighted in Fig. 12. It reports that concentration and related layer thickness maximizes when \( \phi_l \) is incremented. In fact, when difference between wall concentration and reference concentration of fluid increases then concentration field is increased. Fig. 13 discloses the reactive parameter \( kr \) impacts on concentration field. Enlarge reactive (constructive) parameter grows the field of concentration. For higher \( kr \), more fluid particles take a part in making product. Thus, concentration of liquid grows. Fig. 14 discloses the behavior of solutal relaxation time parameter \( k_4 \) on concentration. It shows that with an increment of \( k_4 \), concentration field decreases. Physically, \( k_4 \) appears due to the presence of solutal relaxation time factor. As \( k_4 \) increases the solutal relaxation time factor which slows down the concentration field. If value of \( k_4 \) goes zero in Cattaneo-Christov model then Fick’s law will be recovered. Fig. 15 depicts the growing profile of temperature \( \theta(\xi) \) due to dominant thermal conductivity parameter \( \epsilon \). Physically, with the increment of \( \epsilon \), thermal conductivity of fluid increases which enhances the temperature field. Fig. 16 reports concentration field against mass diffusivity parameter \( \epsilon_1 \). It confirms that with the greater values of \( \epsilon_1 \), mass diffusivity becomes larger which declares enhancement in concentration. It should be stated here that the current results are acquired analytical technique. For the aim of comparison, good agreement can be witnessed between our analytical outcomes demonstrated in Fig. 3 and those which is obtained numerically in Fig. 4 by Shahid [41]. Regarding this, it may be pointed out that our exact outcomes and numeric solutions obtained in [41] are in good agreement.

5. Summary

The phenomenon of nonlinear stratification is incorporated in natural and industrial processes, for instance, volcanic related flow system, oceanography, mixture of heterogeneous in food related industries, agriculture field, density related atmosphere stratification, reservoirs and rivers, etc. Hence, such mechanism requires analyzing. In this study, inspection of modified heat and mass transport on steady hydromagnetic flow of Maxwell liquid under varying fluid characteristics is discussed. Chemical reaction along with nonlinear stratification and heat source/sink are incorporated. It is unveiled that the process of dual stratification diminishes the temperature and concentration and consequently, heat transport rate reduces which is very significant in heating/cooling processes. It is also witnessed that heat generation/absorption and reactive aspects are widely utilized in industry related applications where warming/cooling processes are involved such as missiles, gas turbines, nuclear plants, spaceships, etc. Hence, temperature and concentration fields enhance when heat generation/absorption and reactive parameters are increased respectively.
Fig. 6. Inspection of Pr on $\theta(\zeta)$.

Fig. 7. Inspection of $k_3$ on $\theta(\zeta)$.

Fig. 8. Inspection of $S_1$ on $\theta(\zeta)$.

Declarations

Author contribution statement

Iffat Jabeen: Performed the experiments; Wrote the paper.
S. Ahmad: Conceived and designed the experiments.
Aisha Anjum: Analyzed and interpreted the data.
M. Farooq: Contributed reagents, materials, analysis tools or data.

Funding statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.
Fig. 9. Inspection of $\theta_i$ on $\theta(\xi)$.

Fig. 10. Inspection of $Sc$ on $\phi(\xi)$.

Fig. 11. Inspection of $S_2$ on $\phi(\xi)$.

Data availability statement

No data was used for the research described in the article.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.
Fig. 12. Inspection of $\phi_\xi$ on $\phi(\zeta)$.

Fig. 13. Inspection of $k_r$ on $\phi(\zeta)$.

Fig. 14. Inspection of $k_4$ on $\phi(\zeta)$.

References

[1] T. Hayat, N. Ali, S. Asghar, Hall effects on peristaltic flow of a Maxwell fluid in a porous medium, Phys. Lett. A 363 (5–6) (2007) 397–403.

[2] M. Khan, S.H. Ali, C. Fetecau, H. Qi, Decay of potential vortex for a viscoelastic fluid with fractional Maxwell model, Appl. Math. Model. 33 (5) (2009) 2526–2533.

[3] S. Nadeem, S. Ahmad, N. Muhammad, M.T. Mustafa, Chemically reactive species in the flow of a Maxwell fluid, Results Phys. 7 (2017) 2607–2613.

[4] T. Hayat, S. Ahmad, M.I. Khan, A. Ahsani, Simulation of ferromagnetic nanomaterial flow of Maxwell fluid, Results Phys. 8 (2018) 34–40.

[5] M. Khan, M.Y. Malik, T. Salshuddin, S. Saleem, A. Hussain, Change in viscosity of Maxwell fluid flow due to thermal and solutal stratifications, J. Mol. Liq. 288 (2019) e110970.

[6] S. Ahmad, M.N. Khan, S. Nadeem, Mathematical analysis of heat and mass transfer in a Maxwell fluid with double stratification, Phys. Scr. 96 (2) (2020) e025202.

[7] Y. Zhang, Y. Zhang, Y. Bai, B. Yuan, L. Zheng, Flow and heat transfer analysis of a Maxwell-power-law fluid film with forced thermal Marangoni convective, Int. Commun. Heat Mass Transf. 121 (2021) e105062.

[8] K. Gangadhar, M.A. Kumari, A.J. Chamkha, EMHD flow of radiative second-grade nanofluid over a Riga plate due to convective heating: revised Buongiorno’s nanofluid model, Arab. J. Sci. Eng. 46 (2021) 1–11.

[9] K. Gangadhar, M.A. Kumari, M.V. Subba Rao, K. Alnafaise, I. Khan, M. Andaleem, Magnetization for Burgers’ fluid subject to convective heating and heterogeneous-homogeneous reactions, Math. Probl. Eng. 2022 (2022) e2747676.
\[ S_1 = 0.2, k_2 = 0.1, \Pr = 2.5, S_2 = 0.1, \]
\[ \Theta = 0.2, k_4 = 0.8, n = 1, k_3 = 0.1, \delta = 0.1, \]
\[ \epsilon = 0.1, \phi = 0.1, \kappa = 0.1, \mathrm{Sc} = 2.5, \]
\[ M = 0.8 \]

\[ \epsilon = 0.1, 0.4, 0.7, 0.9 \]

**Fig. 15. Inspection of \( \epsilon \) on \( \theta(\xi) \).**

\[ k_3 = 0.1, S_1 = 0.1, S_2 = 0.1 \]
\[ k_4 = 0.2, k_2 = 0.1, n = 1, \mathrm{Sc} = 0.1 \]
\[ \epsilon = 0.1, \phi = 0.1, k = 0.1, \Pr = 2.5 \]
\[ \delta = 0.1, M = 0.2 \]

**Fig. 16. Inspection of \( \epsilon_1 \) on \( \phi(\xi) \).**

[10] K. Gangadhar, P. Manasa Seshakumari, M. Venkata Subba Rao, A.J. Chamkha, Biconvective transport of magnetized couple stress fluid over a radiative paraboloid of revolution, Proc. Inst. Mech. Eng., E J. Process Mech. Eng. (2022).

[11] K. Gangadhar, T. Kannan, P. Jayalakshmi, Magnetohydrodynamic micropolar nanofluid past a permeable stretching/shrinking sheet with Newtonian heating, J. Braz. Soc. Mech. Sci. Eng. 39 (11) (2017) 4379–4391.

[12] M. Venkata Subba Rao, K. Gangadhar, P.L.N. Varma, A spectral relaxation method for three-dimensional MHD flow of nanofluid flow over an exponentially stretching sheet due to convective heating: an application to solar energy, Indian J. Phys. 92 (12) (2018) 1577–1588.

[13] K. Gangadhar, A.J. Chamkha, Entropy minimization on magnetized Boussinesq couple stress fluid with non-uniform heat generation, Phys. Scr. 96 (9) (2021) e095205.

[14] K. Gangadhar, K.B. Lakshmi, T. Kannan, A.J. Chamkha, Stefan blowing on chemically reactive nano-fluid flow containing gyrotactic microorganisms with leading edge accretion (or) ablation and thermal radiation, Indian J. Phys. 95 (2021) 1–14.

[15] K. Gangadhar, M.A. Kumari, M. Venkata Subba Rao, A.J. Chamkha, Oldroyd-B nanoliquid flow through a triple stratified medium submerged with gyrotactic bioconvection and nonlinear radiations, Arab. J. Sci. Eng. 47 (2022) 1–13.

[16] J.B.J. Fourier, G. Darboux, Théorie analytique de la chaleur, vol. 504, Didot, Paris, 1822.

[17] A. Fick, Ueber diffusion, Ann. Phys. 170 (1) (1855) 59–86.

[18] C. Cattaneo, Sulla conduzione del calore, Atti Semin. Mat. Fis. Univ. Modena 3 (1948) 83–101.

[19] C.I. Christov, On frame indifferent formulation of the Maxwell–Cattaneo model of finite-speed heat conduction, Mech. Res. Commun. 36 (4) (2009) 481–486.

[20] S. Nadeem, N. Muhammad, Impact of stratification and Cattaneo-Christov heat flux in the flow saturated with porous medium, J. Mol. Liq. 224 (2016) 423–430.

[21] A. Anjum, N.A. Mir, M. Farooq, M. Javed, S. Ahmad, M.Y. Malik, A.S. Alshomrani, Physical aspects of heat generation/absorption in the second grade fluid due to Riga plate: application of Cattaneo-Christov approach, Results Phys. 9 (2018) 955–960.

[22] T. Hayat, F. Shah, A. Adeel, Cattaneo-Christov double diffusions and entropy generation in MHD second grade nanofluid flow by a Riga wall, Int. Commun. Heat Mass Transf. 119 (2020) e104824.

[23] K. Loganathan, N. Alesa, N. Namgyel, T.S. Karthik, MHD flow of thermally radiative Maxwell fluid past a heated stretching sheet with Cattaneo-Christov dual diffusion, J. Math. 2021 (2021) e5562667.

[24] Y. Zhang, Y. Zhang, Y. Bai, L. Zheng, Heat and mass transfer analysis of polyacrylamide nanofluid with specific enthalpy effect, Case Stud. Therm. Eng. 26 (2021) e101060.

[25] K. Venkata Ramana, K. Gangadhar, T. Kannan, A.J. Chamkha, Cattaneo–Christov heat flux theory on transverse MHD Oldroyd-B liquid over nonlinear stretched flow, J. Therm. Anal. Calorim. 147 (3) (2022) 2749–2759.

[26] M. Azaam, Effects of Cattaneo-Christov heat flux and nonlinear thermal radiation on MHD Maxwell nanofluid with Arrhenius activation energy, Case Stud. Therm. Eng. 34 (2022) e102048.

[27] K.T. Yang, J.L. Novotny, Y.S. Cheng, Laminar free convection from a nonisothermal plate immersed in a temperature stratified medium, Int. J. Heat Mass Transf. 15 (5) (1972) 1097–1109.

[28] M. Farooq, M. Javed, M.I. Khan, A. Anjum, T. Hayat, Melting heat transfer and double stratification in stagnation flow of viscous nanofluid, Results Phys. 7 (2017) 2296–2301.

[29] S. Ahmad, M. Farooq, M.I. Khan, A. Anjum, Slip analysis of squeezing flow using doubly stratified fluid, Results Phys. 9 (2018) 527–533.

[30] A. Ali, M. Nazir, M. Awais, M.Y. Malik, Stratification phenomenon in an inclined rheology of UCM nanomaterial, Phys. Lett. A 383 (18) (2019) 2201–2206.

[31] I. Jabeen, M. Farooq, M. Rizwan, U. Rullah, S. Ahmad, Analysis of nonlinear stratified convective flow of Powell-Eyring fluid: application of modern diffusion, Adv. Mech. Eng. 12 (10) (2020) 1–10.

[32] H.T. Malik, M. Farooq, S. Ahmad, Significance of nonlinear stratification in convective Falkner-Skan flow of Jeffrey fluid near the stagnation point, Int. Commun. Heat Mass Transf. 120 (2021) e105032.
[33] Q. Khan, M. Farooq, S. Ahmad, Convective features of squeezing flow in nonlinear stratified fluid with inclined rheology, Int. Commun. Heat Mass Transf. 120 (2021) e104958.

[34] A. Shafiq, A.B. Çolak, T.N. Sindhu, T. Muhammad, Optimization of Darcy-Forchheimer squeezing flow in nonlinear stratified fluid under convective conditions with artificial neural network, Heat Transf. Res. 53 (3) (2022) 67–89.

[35] K.U. Rehman, W. Shatanawi, M.Y. Malik, Heat transfer and double sampling of stratification phenomena in non-Newtonian liquid suspension: a comparative thermal analysis, Case Stud. Therm. Eng. 33 (2022) e101934.

[36] S. Liao, Homotopy Analysis Method in Nonlinear Differential Equations, Higher Education Press, Beijing, 2012, pp. 153–165.

[37] S. Liao, Advances in the Homotopy Analysis Method, World Scientific, Amazon, 2014.

[38] S. Ahmad, M. Farooq, M. Rizwan, B. Ahmad, S.U. Rehman, Melting phenomenon in a squeezed rheology of reactive rate type fluid, Front. Phys. 8 (2020) 108.

[39] M. Waqas, N. Akram, Z. Asghar, M.M. Gulzar, M.A. Javed, An improved Darcian analysis for chemically reacted Maxwell liquid toward convectively heated moving surface with magnetohydrodynamics, J. Therm. Anal. Calorim. 143 (3) (2021) 2069–2074.

[40] S.U. Rehman, N.A. Mir, M. Farooq, N. Rafiq, S. Ahmad, Analysis of thermally stratified radiative flow of Sutterby fluid with mixed convection, Proc. Inst. Mech. Eng., Part C, J. Mech. Eng. Sci. 236 (2) (2022) 934–942.

[41] A. Shahid, The effectiveness of mass transfer in the MHD upper-convected Maxwell fluid flow on a stretched porous sheet near stagnation point: a numerical investigation, Inventions 5 (4) (2020) 64.