An algorithm for controlling the visco-elasto-plastic model for studying shock interaction processes

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Abstract. In engineering practice, dynamic processes are crucial. Therefore, modeling and research of these processes are urgent tasks. To study the shock processes, a mechanorheological visco-elasto-plastic model was developed. An important parameter of the model is a force corresponding to the occurrence of plastic deformations. An improved algorithm and a new computer program for studying impact processes using the visco-elasto-plastic model with adjustable elastic-plastic transformations were developed. Differential model motion equations were built. The conditions of transformations were studied for the transition from elastic to plastic deformations, from the loading stage to the unloading one. Functions of the model were described, an algorithm of the model was developed and described.

1. Introduction
In engineering practice, dynamic processes (shock and vibration processes) are crucial. They describe and study the mechanics of interaction of machine parts and structural elements. At the same time, attention is paid to shock processes, since bodies experience large dynamic loads. They can cause large deformations and destroy machines and structures. Due to this, researchers simulate and study dynamic processes [1-26].

To study the impact interaction processes, Irkutsk National Research Technical University developed a mechanorheological visco-elasto-plastic model [27]. The model consists of two consecutive rheological blocks and two inertial elements \( m_1 \) and \( m_2 \) (figure 1). The viscoelastic-viscous block \( K_1 - C \) describes elastic deformations that occur under the influence of an external load and disappear when there is no load. The elasto-plastic block \( K_2 - f_2 \) simulates plastic deformations at the unloading stage. The body weight is taken into account by inertial elements.

Let us analyze the shock processes on the example of a spherical body. Resistance forces prevent deformations. The resistance force of viscoelastic deformations is determined by equations:

\[
\begin{align*}
N_1 &= F_V + F_{E1}, \\
F_V &= C( y_1 - y_2 )^2 ( y_1 - y_2 )^2, \\
F_{E1} &= K_1( y_1 - y_2 )^2.
\end{align*}
\]

where \( y_1, y_2, y_1', y_2' \) are movement and rate \( m_1 \) and \( m_2; \) \( K_1 \) is the stiffness coefficient of the elastic element; \( C \) is the viscosity coefficient of the viscous element.
The resistance force of elastic-plastic deformations is determined by equations:

\[ \begin{align*}
N_2 &= F_p + F_{E2} : N_2 \approx N_1, \\
F_{E2} &= K_2 y_2^{n_2} ; \\
F_p &= f_2 y_2^{n_3} + F_{ST}.
\end{align*} \]

\( F_{ST} \) is the force causing plastic deformations; \( K_2 \) is the stiffness coefficient of the elastic element; \( f_2 \) is the shear coefficient of the elasto-plastic block.

Variables \( y_1, y_1 \) describe the total deformation rate and value. Variables \( y_2, y_2 \) describe the plastic deformation rate and value. Therefore, the elastic deformation rate and value are determined as \( y_1 - y_2, y_1 - y_2 \).

The rheological elements (elastic, viscous, plastic) have non-linear characteristics, i.e., elastic, viscous and plastic deformation resistances are proportional to the rate and value of deformations \( (s_1, s_2, n_1, n_2, n_3) \). For elastic resistances, \( n_1 = n_2 = 3/2 \) [12-14] should be used; for viscous and plastic resistances, a value equal to 1 can be taken [12,13].

2. Problem statement
At the initial stage, a simplified version of the model was used. Elastic and plastic deformations occur simultaneously at the loading stage. This allowed us to develop and test a simpler model’s algorithm. A computer program was developed, and studies of the impact process [15] were carried out.

However, this simplification limits capabilities of the model and does not allow us to study impact processes. Therefore, for studying the shock process using a visco-elasto-plastic model with adjustable elastic-plastic transformations, an improved algorithm and a new computer program were developed [28].
3. The algorithm of the visco-elasto-plastic model with adjustable elastic-plastic transformations

The algorithm ensures the following sequence of blocks. At the initial stage, only the viscoelastic block \( K_1 - C \) operates. It describes elastic deformations of the body when it is loaded (Figure 2). The elasto-plastic block is in a non-deformable state until the impact interaction force \( N_1 \) reaches \( F_{ST} \). When the condition \( N_1 = F_{ST} \) is fulfilled, the elasto-plastic block comes into operation and both the blocks work sequentially. The loading stage is over when the shock interaction force reaches \( N_{max} \). The next stage is the unloading one.

When the body is unloaded, only elastic deformations disappear. Therefore, only the elastic-viscous block operates. The elasto-plastic block remains in a non-deformable state until the end of the impact process. The contact interaction stage ends when the normal reaction force \( N \) determined by equations (1) and (2), becomes equal to zero. The body bounces to height \( h \).

![Figure 2. The Model Transformation Scheme](image)

The differential equations of model’s transformations at the stage of contact interaction are as follows:

\[
\begin{align*}
    m_1 \dddot{y}_1 + C_1 \dot{y}_1 + y_1 s_1 + K_1 y_1 = -m_1 g, \quad (3) \\
    m_1 \dddot{y}_1 + C_1 (y_1 - y_2) + y_1 s_1 + K_1 (y_1 - y_2) = -m_1 g, \quad (4) \\
    m_2 \dddot{y}_2 + K_2 y_2 + f_2 y_2 + C_1 (y_2 - y_1) s_1 + K_1 (y_2 - y_1) = -m_2 g + F_{ST}. \quad (5)
\end{align*}
\]

Equation (3) describes elastic deformations at the loading and unloading stages. Equations (4) and (5) describe plastic deformations at the loading stage.

At the stage of elastic deformations, the normal response is determined by formula

\[
N_i = C y_1 s_1 + K_1 y_1. \quad (6)
\]

At the stage of plastic deformations, the normal response is determined by formula:

\[
N_i = K_2 y_2 + f_2 y_2 + F_{ST}. \quad (7)
\]

The calculation is performed with step \( dt \) using the Runge-Kutt method. The impact time is determined by formula

\[
T = dt \cdot i, \quad (8)
\]

where \( i \) is the number of steps.

The condition for the transition from elastic to plastic deformations is as follows:

\[
N_i > F_{ST} \quad (9)
\]

The condition for the end of the loading stage and the beginning of the unloading stage is as follows:
The condition for the end of the contact interaction stage is as follows:

\[ N_{i-1} > N_i \]  

(10)

The condition for the end of the contact interaction stage is as follows:

\[ N_i = 0 \]  

(11)

An algorithm for studying the shock process is as follows (figure 3):

1. Block 1 forms source data.

2. Block 2 sets initial conditions: initial time \( t = 0 \), initial deformation \( y = 0 \), initial rate \( y' = y'\), initial shock interaction force (normal reaction force) \( N = 0 \).

3. Block 3 calculates values of parameters \( K_1, K_2, f_2, C \).

4. Block 4 calculates current time and numbers of calculation steps at the elastic deformation stage.

5. Block 5 calculates dynamic parameters of the impact process using the viscoelastic model: elastic deformation \( y_1 \), deformation rate \( y_1' \) by equation (3), normal response force \( N_i \) by equation (6).

6. Blocks 6 and 16 check the fulfillment of condition (11). If the condition is not fulfilled, the calculation is performed at the next calculation step. When the condition is fulfilled, the calculation is completed at the stage of impact interaction. The transition to the stage of model rebound takes place.

7. Blocks 7 and 12 determine the end of the loading stage. If condition (10) is fulfilled, the normal response \( N_i \) decreases, the unloading stage is over, and the calculation is performed at the next step of the unloading stage. The control function is transferred to block 4 or block 14. If condition (10) is not fulfilled, the normal response force \( N_i \) continues to increase, the model is loading, and the control function is transferred to block 8 or block 13.

8. Block 8 determines \( N_{max} \) and time corresponding to the maximum value of the impact interaction force \( T_{max} \). It transfers the control function to block 9.

9. Block 9 determines the end of the elastic deformation stage when loading the model. If condition (9) is fulfilled, \( N_i \) exceeds \( F_{st} \) corresponding to the occurrence of plastic deformations. The elastic deformation stage is completed, the elastic-plastic deformation stage begins and the control function is transferred to block 10. If condition (9) is not fulfilled, \( N_i \) does not exceed \( F_{st} \), corresponding to the occurrence of plastic deformations. The control function is transferred to block 4 which performs calculations at the elastic deformation stage.

10. Block 10 calculates current time and a calculation step number at the elastic-plastic deformation stage.

11. Block 11 calculates dynamic parameters of the impact process at the plastic deformation stage: deformations and deformation rates \( y_1, y_1', y_2, y_2' \) using equations (4) and (5), normal response forces \( N_i \) using equation (7). The control function is transferred to block 12.

12. Block 12 determines the end of the loading stage. If condition (10) is fulfilled, \( N_i \) decreases, the loading stage is over, and the calculation is performed at the next step of the unloading stage. The control function is transferred to block 14. If condition (10) is not fulfilled, \( N_i \) continues to increase, the model is loading, and the control function is transferred to block 13.

13. Block 13 determines \( N_{max} \) and time corresponding to the maximum value of impact interaction force \( T_{max} \). The control function is transferred to block 10.

14. Block 14 calculates current time and a calculation step number at the stage of elastic deformation during unloading the model. The control function is transferred to block 15.

15. Block 15 calculates dynamic parameters of the shock process using the viscoelastic model at the unloaded stage: elastic deformation \( y_1 \), deformation rate \( y_1' \) by equation (3), normal response force \( N_i \) by equation (6). The control function is transferred to block 16.
Figure 3. The scheme of the algorithm
16. Block 16 checks the fulfillment of condition (11). If the condition is not fulfilled, the calculation is performed at the next calculation step during impact interaction. The control function is transferred to block 14. When the condition is fulfilled, the calculation is completed and the transition to the model bound stage is carried out. The control function is transferred to block 17.

17. Block 17 determines rebound height \( h \) and flight time \( T_F \). The control function is transferred to block 18.

18. Block 18 generates output data on the impact process: time of the impact interaction \( T \) by equation (8); the maximum value of the impact interaction force \( N_{\text{max}} \); time corresponding to the maximum value of impact interaction force \( T_{n_{\text{max}}} \); rebound height \( h \) (flight time before the second impact).

Based on this algorithm, a computer program was developed [28] and some studies [29-31] were carried out. The program allowed us to study the effect of elastic, viscous and plastic properties of a material on the dynamics of impact interaction of a spherical body with a surface under various elastic-plastic transformations of the mechanorheological model that describe the transition of the material from the elastic state to the elasto-plastic one. The program can be used to calculate dynamic interaction parameters: impact time, impact interaction force, deformation value and rate, rebound height.

4. Conclusion

For studying the impact process using a visco-elasto-plastic model with adjustable elastic-plastic transformations, an improved algorithm and a new computer program were developed. Differential equations of model motion were presented. The conditions of model transformation during the transition from elastic to plastic deformations, from the loading stage to the unloading one were described. The functions of each block were described, the operation algorithm was developed and described.

The practical application of the program can improve accuracy and reliability of shock process simulation. It aims at developing methods for determining physical and mechanical properties of materials using the shock methods. Knowledge of mechanical properties of materials is required when solving various research problems by modeling vibration and shock processes. An important task is adaptation of the model to real shock processes, which requires the development of appropriate methods and techniques. The task can be solved using the research results.

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