Vortices in superconducting strips: interplay between surface effects and the pinning landscape

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Vortices in a narrow superconducting strip with a square array of pinning sites are studied. The interactions of vortices with other vortices and with external sources (applied magnetic field and transport current) are calculated via a screened Coulomb model. The edge barrier is taken into account and shown to have an important role on the system dynamics. Numerical simulations in this approach show that the field dependent magnetic moment presents peaks corresponding to history dependent configurations of the vortex lattice. Some effects of the edge barrier on the I-V characteristics are also reported.

1. Introduction

There is a growing interest in the microscopic structure of the vortex lattice (VL) in superconducting films with artificially fabricated columnar defects. In the presence of a perpendicular magnetic field and zero current this structure depends on how strong is the pinning force produced by the defects with respect to the internal elastic forces of the VL. This results in two possible situations: commensurate and incommensurate lattice, depending on the pinning landscape being strong enough to determine the VL structure or not. Many experimental [1] and theoretical [2,3] works have reported maxima in the critical current $j_c$ and field dependent magnetic moment $m(H)$ in the points of the phase diagram where a commensurate state takes place.

When a finite current stronger than $j_c$ is applied, an additional ingredient governs the structural evolution of the VL. In the center of mass (CM) frame, the vortices interact with a time dependent pinning potential, which is determined by the CM velocity. As the driving current is increased from $j_c$, a variety of dynamical phases is possible. But commensurate ordered lattices are only possible at very high driving forces, when the pinning potential is averaged in the direction of motion becoming a static washboard potential. Recently, Carneiro [4] has shown by numerical simulations that at a high enough temperature this ordered state is reached at a finite driving force.

Frequently, these phenomena have been investigated under the assumption of infinite sample, i.e. no surface or geometrical effects are taken into account. As it is well known, surface effects can play an important and even a dominant role on the irreversible properties of a superconducting specimen. In the critical state approach, Kuznetsov et al. [5] estimated the enhancement due to edge effects of hysteresis in magnetization of thin strips with strong pinning. The penetration of flux in finite sample are also described by the geometric barrier model [6,7], which has shown to be successful in describing the irreversible properties of flat high-$T_c$ single crystals [8] at high temperatures. All these continuum models however are not able to determine structural details of the VL and their relevance in the macroscopic response.

In the present work, we propose a simple qualitative description of individual vortices dynamics in superconducting strips. The vortex-vortex interactions and the edge barrier are calculated in the frame work of the screened Coulomb model [8]. With a change of scale the problem may be mapped into the well known problem of an infinite slab under a parallel field. The calculated $m(H)$ presents a series of maxima corresponding to matching of the VL in the pinning struc-
ture and a strong asymmetry of the upward (virgin) and downward branches, which characterize the surface effects. We also study the dynamics of the VL when a transport current is applied across the strip. The strong current inhomogeneity leads to different dynamics in different regions of the strip.

2. Screened Coulomb model and dynamics

We describe the motion of vortices in a superconducting strip of thickness \( d \) much smaller than the London penetration depth \( \Lambda \). The geometry and coordinate system are depicted in Fig. 1. The vortex equation of motion, appropriate for overdamped dynamics, is

\[
\eta \nabla_k = -\nabla U_p(r_k) + \Gamma_k(t),
\]

where \( \nabla_k \) is the instantaneous terminal velocity of vortex \( k \), \( \nabla U_p(r_k) \) is the total Lorentz Force, \( U_p \) is the pinning potential and \( \Gamma_k(t) \) is a Gaussian Langevin force acting on vortex \( k \) at instant \( t \). The Lorentz force acting on a vortex line is defined by the total sheet current \( J = \int j \, dz \) probed by this vortex, \( \mathbf{F}_L = \phi_0 \mathbf{J} \times \hat{z} \). A difficult task is to precisely determine an expression for \( \mathbf{F}_L \) in this system. Recently, Brandt has generalized the continuum theory to account for the penetration depth and numerically calculated the stream lines of current of vortices in thin films and washers in the small \( \Lambda \) limit.

A useful procedure is to write the sheet current in terms of a scalar potential, the stream function \( g(x, y) \) satisfying the zero divergence of the current \( \nabla \cdot \mathbf{J} = 0 \): \( \mathbf{J}(x, y) = \nabla \times \hat{z}g = -\hat{z} \times \nabla g \). Note that there is a close relation between the stream lines probed by a vortex and the energy of this vortex. This is clear when we rewrite the Lorentz force as \( \mathbf{F}_L = \phi_0 \mathbf{J} \times \hat{z} = -\phi_0 \nabla \mathbf{g} \). With the help of the Biot-Savart law, the London equation for a vortex located at \( r_k \) in the \( xy \) plane of a thin strip under a perpendicular field \( H \) may be written as

\[
\Lambda \nabla^2 g(r) - \int d^2 r' K(r, r')g(r') = H - \frac{\phi_0}{\mu_0} \delta(r - r_k), \tag{1}
\]

where \( K(r, r') \) is a kernel with a suitable cutoff to account for the finite sample thickness. We propose a simple way of calculating \( g \), using a screened Coulomb model approach, suitable for describing thin-film vortices with \( \Lambda \) comparable to the system dimension.

In this large-\( \Lambda \) limit, the screening length \( \Lambda_c \) is identified with the smaller of \( \Lambda \) and some linear dimension of the sample. Therefore, for thin strips with \( \Lambda > \Lambda_c \), we use a nonhomogeneous Helmholtz equation instead of solving the exact, although much more complicated, Eq. 1. Using the boundary condition \( g = 0 \) at the edges, this problem is similar to that of a superconducting slab under a parallel field. Here, we add periodic boundary conditions (with periodicity \( L \)) in the \( y \) direction. Solving for a single vortex, one obtains the vortex-vortex interaction energy:

\[
V_{vv} = \epsilon \sum_{m=1}^{\infty} \frac{\cosh \alpha_m (|\hat{y}_i - \hat{y}_j| - L/2)}{\alpha_m \sinh \alpha_m L/2} \times
\]

\[
\left( \sin \frac{m\pi \hat{x}_i}{W} \sin \frac{m\pi \hat{x}_j}{W} \right), \tag{2}
\]

where \( \alpha_m = \sqrt{1 + (m\pi/W)^2} \), \( \epsilon = \phi_0^2/\mu_0\Lambda_c W \). The tilded length quantities are normalized by \( \Lambda_c \). With a suitable cutoff to account for the vortex
core, one obtains the vortex self-energy:
\[ V_{\text{self}} = \epsilon \sum_{m=1}^{\infty} \cosh \alpha_m (\xi - L/2) \sinh \alpha_m L/2 \sin \frac{m \pi \tilde{x}_i}{W}, \]  
(3)
where \( \xi \) is the coherence length. Now, solving for the external field \( H \), we have
\[ V_H = \frac{\phi_0 H}{\Lambda_c} \left[ \cosh (\tilde{x}_i - \tilde{W}/2) - 1 \right]. \]  
(4)

Equations (3) and (4) form the edge barrier for flux entry. Thus, the Lorentz force on a vortex \( i \) due to a vortex \( j \) and an external field \( H \), for instance, is given by \(-\nabla_i V(r_i, r_j)\) where \( V(r_i, r_j) = V_{\text{ev}}(r_i, r_j) + V_{\text{self}}(r_i) + V_H(r_i)\).

3. Periodic pinning: static aspects

In the above framework, we simulated \( m(H) \) measurements on a thin strip with \( W = 3\Lambda \). A square lattice of columnar defects (SLCD) is represented by a periodic arrangement of Gaussian wells with radius \( r_p = 0.045\Lambda \) and period \( a_p = 0.3\Lambda \). The depth of the wells is varied from \( U_0 = 0 \) to \( U_0 = \epsilon \). The simulations are carried out using the same procedure as in Ref. [10]: at each time step a vortex is attempted to nucleate in some random point in a line which is a distance \( \xi \) from one of the strip edges. Here we take \( \xi = 0.01 \). The magnetic moment was calculated using \( \mathbf{m} = \frac{1}{2} \int d^2 \mathbf{r} \times \mathbf{J} = \frac{1}{2} \int d^2 \mathbf{r} \cdot \nabla g \), which for the chosen gauge of \( g = 0 \) at the edges is simply \( \int d^2 \mathbf{r} g \).

In Fig. 1 is shown the first \( m(H) \) half cycle for \( U_0 = 0.5 \epsilon \). This curve depicts features of both bulk pinning due to regularly arranged defects and edge effects. Matching peaks are present, but behave differently in the upward (virgin) and downward branches. As shown in Fig. 2, these peaks correspond to history dependent states of commensuration. In upward field, the edge barrier imposes a vortex free region which produces vacant pinning sites even in the second matching configuration. In downward field, these states are similar to those expected for infinite samples, no vortex free region is present. This points up the different dynamics of flux entrance and exit.

Figure 2. Snapshots of the vortex lattice (closed circles) in the matching peaks depicted in Fig. 1: (A) \( H = 14.5 \) (upward branch), (B) \( H = 10.5 \) (downward), (C) \( H = 23 \) (upward), and (D) \( H = 20.5 \) (downward). The hollow circles represent the pinning lattice.

4. Periodic pinning: dynamical aspects

We have also simulated measurements of the time averaged CM velocity of the VL as a transport sheet current \( J_t \) was varied. The current is applied along the \( y \) direction and the contacts are considered to be far away from the region studied, in such a way to guarantee zero current divergence in this region. The current distribution in the strip width may be calculated from the Helmholtz equation for \( g_j \) with the condition that \( g_j \) must be an odd function and that \( g_j(W) - g_j(0) = J_t W \) (current continuity). Thus the potential energy due to \( J_t \) is
\[ V_j = \frac{J_t W \sinh (\tilde{x}_i - \tilde{W}/2)}{2 \sinh \tilde{W}/2}. \]  
(5)

The simulation procedure is the same as described in the preceding section. We divide the strip in three regions of width \( \Lambda_c \). We computed the time-averaged CM velocity \( v_{\text{CM}} \) in each re-
Figure 3. Time-averaged CM velocity $v_{CM}(J_t)$ in the central third of the strip for pinning strengths $U_0 = 0, 0.25, 0.5, 0.75,$ and $1.0 \epsilon$ (from top to bottom). Panels: density-density correlation functions for $U_0 = \epsilon$ and different current values (arrows).

5. Summary

We have described the dynamics of vortices in narrow superconducting strips in the screened Coulomb model approach. We calculated the vortex-vortex interaction energy and the interaction with an edge barrier. In this scenario, we carried out simulations on vortex motion in field ramp measurements of the magnetic moment and I-V characteristics using Langevin dynamics. The $m(H)$ curves evidenced the strong influence of the edge barrier in the VL configuration in upward field, where the first matching states are not fully accomplished in the entire sample. For the I-V characteristics, we observed that the edge barrier represents an effective pinning for the VL as expected, and ordered motion of the VL is disturbed by the random penetration of vortices. A more detailed study will be reported elsewhere.

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