Method for Determination of $|U_{e3}|$ in Neutrino Oscillation

Appearance Experiments

Takaaki Kajita$^1$, Hisakazu Minakata$^2$ and Hiroshi Nunokawa$^3$

$^1$Research Center for Cosmic Neutrinos, Institute for Cosmic Ray Research, University of Tokyo, Kashiwa, Chiba 277-8582, Japan

$^2$Department of Physics, Tokyo Metropolitan University

1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, Japan

$^3$Instituto de Física Teórica, Universidade Estadual Paulista

Rua Pamplona 145, 01405-900 São Paulo, SP Brazil

Abstract

We point out that determination of the MNS matrix element $|U_{e3}| = s_{13}$ in long-baseline $\nu_\mu \rightarrow \nu_e$ neutrino oscillation experiments suffers from large intrinsic uncertainty due to the unknown CP violating phase $\delta$ and sign of $\Delta m_{13}^2$. We propose a new strategy for accurate determination of $\theta_{13}$; tune the beam energy at the oscillation maximum and do the measurement both in neutrino and antineutrino channels. We show that it automatically resolves the problem of parameter ambiguities which involves $\delta$, $\theta_{13}$, and the sign of $\Delta m_{13}^2$.

E-mail: kajita@icrr.u-tokyo.ac.jp

E-mail: minakata@phys.metro-u.ac.jp

E-mail: nunokawa@ift.unesp.br
I. INTRODUCTION

With the accumulating evidences for neutrino oscillation in the atmospheric [1], the solar [2] and the accelerator neutrino experiments [3], it is now one of the most important subjects in particle physics to explore the full structure of neutrino masses and the lepton flavor mixing. In particular, it is the challenging task to explore the relatively unknown (1-3) sector of the MNS matrix [4], namely, $\theta_{13}$, the sign of $\Delta m^2_{13}$ and the CP violating phase $\delta$. The only available informations to date are the upper bound on $\theta_{13}$ from the reactor experiments [5], and an indication for positive sign of $\Delta m^2_{13}$ by neutrinos from supernova 1987A [6]. Throughout this paper, we use the standard notation of the three flavor MNS matrix, in particular $U_{e3} = s_{13} e^{-i\delta}$, and define the neutrino mass-squared difference as $\Delta m^2_{ij} \equiv m^2_j - m^2_i$.

The long baseline $\nu_\mu \rightarrow \nu_e$ neutrino oscillation experiment is one of the most promising way of measuring $\theta_{13}$. In particular, it is expected that the JHF-Kamioka project which utilizes low energy superbeam can go down to the sensitivity $\sin^22\theta_{13} \approx 6 \times 10^{-3}$ [7]. A similar sensitivity is expected for the proposed CERN $\rightarrow$ Frejus experiment [8]. Although a far better sensitivity is expected to be achieved in neutrino factories [9], it is likely that the low energy conventional superbeam experiments are the ones which can start much earlier. Therefore, it is of great importance to examine how accurately $\theta_{13}$ can be determined in this type of experiments.

In this paper, we point out that determination of $\sin^22\theta_{13}$ by using only neutrino channel suffers from large intrinsic uncertainty of $\pm$ (30-70) % level due to the unknown CP violating phase $\delta$ and the undetermined sign of $\Delta m^2_{13}$. It should be noted that the intrinsic uncertainty exists on top of the usual experimental (statistical and systematic) errors. To overcome the problem of the intrinsic uncertainty, we suggest a new strategy for determination of $\theta_{13}$ by doing appearance experiments utilizing both antineutrino and neutrino beams. Our proposal is a very simple one at least at the conceptual level; tune the beam energy to the oscillation maximum and run the appearance experiments in both $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$ channels.

We will show that it not only solves the problem of intrinsic uncertainty mentioned above but also resolves the $(\delta - \theta_{13})$ two-fold ambiguity discussed in Ref. [10]. Furthermore, it does not suffer from possible ambiguity due to the unknown sign of $\Delta m^2_{13}$; the problem
first addressed in Refs. \cite{11,12}. We are aware that there are combined ambiguities to be resolved (even ignoring experimental uncertainties) to determine a complete set of parameters including $\delta$, $\theta_{13}$, and the sign of $\Delta m^2_{13}$, which are as large as four-fold \cite{14}. We take experimentalists’ approach to the ambiguity problem and try to resolve them one by one, rather than developing mathematical framework for the simultaneous solutions. The most important issue here is again to accurately determine $\theta_{13}$, because then all the combined ambiguities will be automatically resolved, as we will show below.

**II. INTRINSIC UNCERTAINTY IN DETERMINATION OF $\theta_{13}$ DUE TO CP VIOLATING PHASE**

Let us clarify how large uncertainty is expected for determination of $\theta_{13}$ due to our ignorance of $\delta$ in the $\nu_\mu \to \nu_e$ appearance experiment. To achieve intuitive understanding of the issue we use the CP trajectory diagram introduced in previous papers \cite{11,12}. Plotted in Fig. 1 are the CP trajectory diagrams in bi-probability space spanned by $P(\nu) \equiv P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}) \equiv P(\bar{\nu}_\mu \to \bar{\nu}_e)$ averaged over Gaussian distribution (see next paragraph) with three values of $\theta_{13}$, $\sin^2 2\theta_{13} = 0.05$ and 0.02 for $\Delta m^2_{23} > 0$ case and $\sin^2 2\theta_{13} = 0.064$ for $\Delta m^2_{23} < 0$ case. Since we assume $|\Delta m^2_{23}| \gg |\Delta m^2_{12}|$ the sign of $\Delta m^2_{23}$ is identical with that of $\Delta m^2_{13}$. (The fourth one with $\sin^2 2\theta_{13} = 0.04$ is for our later use.) The values of $\sin^2 2\theta_{13}$ for the second and the third trajectories are chosen so that the maximum (minimum) value of $\langle P(\nu) \rangle$ of the second (third) trajectory coincides with about 1.1%, the minimum value of $\langle P(\nu) \rangle$ of the first trajectory. The remaining mixing parameters are taken as the best fit value of the Super-Kamiokande (SK) and the K2K experiments \cite{15}, $|\Delta m^2_{23}| \equiv \Delta m^2_{\text{atm}} = 3 \times 10^{-3}$ eV$^2$, and the typical ones for the large mixing angle (LMA) MSW solar neutrino solution as given in the caption of Fig. 1.

While we focus in this paper on the JHF experiment with baseline length of 295 km, JAERI-Kamioka distance, many of the qualitative features of our results remains valid also for the CERN-Frejus experiment. Throughout this paper we take the neutrino energy

---

1 Our new strategy and these results were announced in the 8th Tokutei-RCCN workshop \cite{13}.
distribution of Gaussian form with width of 20% of the peak energy. Of course, it does not represent in any quantitatively accurate manner the effects of realistic beam energy spread and the energy dependent cross sections. But we feel that it is sufficient to make the point of this paper clear, illuminating our new strategy toward accurate determination of $\theta_{13}$.

Suppose that a measurement of appearance events gives us the value of the oscillation probability $\langle P(\nu) \rangle \simeq 1.1\%$. Then, it is obvious from Fig. 1 that a full range of values of $\sin^2 2\theta_{13}$ from 0.02 to 0.064 are allowed (even if we ignore experimental errors) due to our ignorance to the CP violating phase $\delta$ and the sign of $\Delta m^2_{13}$\(^2\). If we know that the sign is positive, for example, the uncertainty region would be limited to 0.02-0.05, which is still large.

Let us estimate in a systematic way the uncertainty in the determination of $\theta_{13}$ due to the CP violating phase $\delta$. To do this we rely on perturbative formulae of the oscillation probabilities $P(\nu)$ and $P(\bar{\nu})$ which are valid to first order in the matter effect \[18\]. With relatively short baseline $\sim 300$ km or less the first-order formula gives reasonably accurate results. Ignoring $O(\sin^3 2\theta_{13})$ terms the formula can be written with use of the notation $\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{2E}$ ($L$ and $E$ denote baseline length and neutrino energy, respectively) in the form

$$P(\nu/\bar{\nu}) = P_{\pm} \sin^2 2\theta_{13} + 2Q \sin 2\theta_{13} \cos \left( \frac{\Delta_{13}}{2} \pm \delta \right)$$  \hspace{1cm} (1)

where

$$P_{\pm}(\Delta_{13}) = s_{23}^2 \left[ \sin^2 \left( \frac{\Delta_{13}}{2} \right) - \frac{1}{2} s_{12}^2 \Delta_{12} \sin (\Delta_{13}) \pm \left( \frac{2Ea}{\Delta m^2_{13}} \right) \sin^2 \left( \frac{\Delta_{13}}{2} \right) \mp \frac{aL}{4} \sin (\Delta_{13}) \right],$$  \hspace{1cm} (2)

$$Q = c_{12}s_{12}c_{23}s_{23}\Delta_{12} \sin \left( \frac{\Delta_{13}}{2} \right),$$  \hspace{1cm} (3)

\(^2\) It may be worth to remark the following: Low energy neutrino oscillation experiments with superbeams are primarily motivated as a result of the search for the place where CP violating effects are comparatively large and easiest to measure \[16\]. See e.g., \[17\] for works preceding to \[16\]. Unfortunately, this large effect of $\delta$ is the very origin of the above mentioned large intrinsic uncertainty in determination of $\theta_{13}$.
where \( a = \sqrt{2} G_F N_e \) denotes the index of refraction in matter with \( G_F \) being the Fermi constant and \( N_e \) a constant electron number density in the earth. The \( \pm \) signs in \( P_\pm \) refer to the neutrino and the antineutrino channels, respectively.

The maximum and the minimum of \( P(\nu) \) for given mixing parameters, neutrino energy and baseline is obtained at \( \cos \left( \delta + \frac{\Delta m^2_{ij}}{2} \right) = \pm 1 \). Then, the allowed region of \( \sin 2\theta_{13} \) for a given value of \( P(\nu) \), assuming blindness to the sign of \( \Delta m^2_{13} \), is given by

\[
\frac{\sqrt{Q^2 + P_+(\Delta_{13})P(\nu)} - |Q|}{P_+(\Delta_{13})} \leq \sin 2\theta_{13} \leq \frac{\sqrt{Q^2 + P_+(-\Delta_{13})P(\nu) + |Q|}}{P_+(-\Delta_{13})}
\]

(4)

In Fig. 2 presented is the allowed region of \( \sin^2 2\theta_{13} \) for a given value of measured oscillation probability \( P(\nu) \). Figures (a)-(c) correspond respectively to neutrino energies (a) \( E = 500 \) MeV, (b) \( 716 \) MeV (oscillation maximum), and (c) \( 1 \) GeV. One notices that the intrinsic uncertainty is large. It strongly depends on the value of \( \sin^2 2\theta_{13} \) and gradually decreases as \( E \) grows. Roughly speaking, it ranges between, \( \sim 45 \% (30 \%) \) at \( \sin^2 2\theta_{13} = 0.1 \) and \( \sim 80 \% (70 \%) \) at \( \sin^2 2\theta_{13} = 0.01 \) at \( E = 500 \) MeV (716 MeV). Notice that all the results shown in the plots in this paper were obtained by numerically solving the neutrino evolution equation assuming constant matter density without using the first-order formula.

The size of the intrinsic uncertainty must be compared with the statistical and the systematic errors which are expected in the actual experiments. A detailed estimation of the experimental uncertainties is performed for the JHF experiment by Obayashi [19] assuming the off-axis beam (OA2) [7] and running of 5 years. The results strongly depend upon \( \theta_{13} \). We quote the case of three typical values; \( \sin^2 2\theta_{13} = 0.1 + 0.018 - 0.014, 0.03 + 0.010 - 0.007 \), and \( 0.01 + 0.007 - 0.006 \). The errors include not only statistical but also systematic ones. We implemented these errors in Fig. 2b which is drawn with the similar energy as the peak energy of OA2 beam (\( \sim 780 \) MeV). We should note, however, an important difference between Fig. 2 and the plot in [19]; the abscissa of Fig. 2 is the Gaussian averaged probability, whereas the corresponding axis of the plot in [19] is the number of events. Therefore, we tentatively determined the location of errors in Fig. 2 so that the center of the error bars coincide with the center of the allowed band of \( \sin^2 2\theta_{13} \). Keeping this difference in mind, we still feel it informative for the readers to display the expected experimental uncertainties in Fig. 2b for comparison.
Therefore, the intrinsic uncertainty due to $\delta$ and undetermined sign of $\Delta m_{13}^2$ is larger than the expected experimental errors in most of the sensitivity region for $\theta_{13}$ in the experiment. We note that the experimental errors are dominated by the statistical one in phase I of the JHF-SK neutrino project and hence it should be improved by a factor of $\sim 10$ in two years of running in the phase II with a megaton water Cherenkov detector [7]. Thus, the intrinsic uncertainty completely dominates over the experimental ones if one stays only on the neutrino channel.

III. POSSIBLE WAY OUT AND THE RELATIONSHIP WITH $\theta_{13} - \delta$ AMBIGUITY

Let us discuss possible ways out of the uncertainty problem in the determination of $\theta_{13}$. It is tempting to think about seeking better resolution by adding more informations. A natural candidate for such possibilities in this line of thought is to do additional appearance experiment $\bar{\nu}_\mu \to \bar{\nu}_e$ using antineutrino beam. While it strengthens constraints, it does not completely solve the uncertainty problem even if we ignore the experimental errors. It is due to the inherent two-fold ambiguity which exists in simultaneous determination of $\delta$ and $\theta_{13}$ as has been pointed out by Burguet-Castell et al. [10]. While their discussion anticipates applications to neutrino factory, the issue of the two-fold ambiguity is in fact even more relevant to our case because of the large effect of $\delta$ as we saw in the previous section.

The existence of the two-fold $(\theta_{13} - \delta)$ ambiguity is easy to recognize by using the CP trajectory diagram. We show in Fig. 1 by a dash-dotted curve another trajectory drawn with $\sin^2 2\theta_{13} = 0.04$ which has two intersection points with the solid curve trajectory with $\sin^2 2\theta_{13} = 0.05$. Suppose that measurements of neutrino and antineutrino oscillation probabilities $P(\nu)$ and $P(\bar{\nu})$ had resulted into either one of the two intersection points. Then, it is clear that we have two solutions, for positive $\Delta m_{13}^2$, $(\sin^2 2\theta_{13}, \delta) = (0.04, 0.65\pi)$ and $(0.05, 0.35\pi)$ for the upper intersection point, and $(\sin^2 2\theta_{13}, \delta) = (0.04, 1.4\pi)$ and $(0.05, 1.7\pi)$ for the lower intersection point. Similar two-fold $(\theta_{13} - \delta)$ ambiguity also exists for negative $\Delta m_{13}^2$ which however is not shown in Fig. 1. In other word, we can draw two different CP trajectories which pass through a point determined by given values of $P(\nu)$ and
This is the simple pictorial understanding of the \((\theta_{13} - \delta)\) two-fold ambiguity which is uncovered and analyzed in detail in [10]. We will show in the next two sections that the ambiguity is automatically resolved by our proposal.

**IV. NEW STRATEGY FOR DETERMINATION OF \(\theta_{13}\)**

We now present our new strategy for determination of \(\theta_{13}\) which avoids the problem of the large intrinsic uncertainty. It is intuitively obvious from the CP trajectory diagram displayed in Fig. 1 that if one can tune the experimental parameters so that its radial thickness (which measures the \(\cos \delta\) term in Eq. (1)) vanishes then the two-fold ambiguity is completely resolved. It occurs if we tune the beam energy at the oscillation maximum so that \(\Delta_{13} = \pi\) as is clear from Eq. (1).

We explain below in more detail how it occurs and then discuss by what kind of quantity \(\theta_{13}\) is determined. In the following discussion we assume that the mixing parameters \(|\Delta m_{13}^2| \simeq |\Delta m_{23}^2| \equiv \Delta m_{\text{atm}}^2\), \(\Delta m_{12}^2 \equiv \Delta m_{\odot}^2\), \(\theta_{23}\), and \(\theta_{12}\) are accurately determined by the time of the experiment. It is not so unrealistic assumption in view of the array of experiments ongoing (SK, SNO, K2K, KamLAND), on schedule (Borexino, MINOS, OPERA), or in planned (JHF). For example, the uncertainty in measurement of \(\theta_{23}\) is expected to be \(\delta(\sin^2 2\theta_{23}) \simeq 0.01\) in JHF phase I [7].

We note that the oscillation probabilities (4) can be written as

\[
P(\nu) = A \cos \delta + B \sin \delta + C_+
\]

\[
P(\bar{\nu}) = A \cos \delta - B \sin \delta + C_-
\]

where \(A = Q \cos \left(\frac{\Delta_{13}}{2}\right)\), \(B = -Q \sin \left(\frac{\Delta_{13}}{2}\right)\), and \(C_\pm = P_\pm \sin^2 2\theta_{13}\) in the present approximation. It is easy to show from this expression that CP trajectory diagram is elliptic in the approximation that we are working [11]. (In fact, it is the case for all the known perturbative formulae.) Given (4) it is simple to observe that the CP trajectory is a straight line at the oscillation maximum, \(A = 0\); the equation obeyed by the oscillation probabilities is given as \(P(\nu) + P(\bar{\nu}) = C_+ + C_-\). Moreover, the first order matter effect cancels in \(C_+ + C_-\), leaving the vacuum peace of \(P_\pm\). Therefore, the slope of the straight-line CP trajectory is the same as that in vacuum, and the matter effects affects only on the maximum and the minimum
points of the straight line in $P(\nu)$ and $P(\bar{\nu})$ coordinates. Thus, once a set of values of $P(\nu)$ and $P(\bar{\nu})$ is given by the experiments, one can determine $C_+ + C_-$ as the segment of the “CP straight line” in the diagram, and hence $\sin^2 2\theta_{13}$ to which $C_+ + C_-$ is proportional. Thus, measurement of $P(\nu)$ and $P(\bar{\nu})$ at the oscillation maximum implies determination of $\theta_{13}$ without suffering from any uncertainties due to unknown value of $\delta$ and the sign of $\Delta m^2_{13}$.

In Fig. 3 we present the thinnest trajectories with the tuned value of the energy $E = 760$ MeV for $L = 295$ km (JAERI-Kamioka distance) with $\sin^2 2\theta_{13} = 0.05$ and 0.02, by taking the other mixing parameters given in the caption of Fig. 1. The energy would be $E = 716$ MeV if we sit on the oscillation maximum. It arises because the contributions from higher and lower energy parts around the peak energy do not completely cancel because of the extra $1/E$ factor in the $\cos \delta$ term for symmetric Gaussian beam width. Thus, we need slightly higher energy to have the thinnest trajectory. It should be noted, however, that the feature highly depends upon the specific beam shape, and will also be affected by the fact that the cross section has an extra approximately linear $E$ dependence.

The slightly different slope of the straight-line trajectories of positive and negative $\Delta m^2_{13}$ indicates the higher order matter effect. This effect must be (and can be) taken into account when one try to determine $\theta_{13}$ following the method proposed above.

V. COMMENTS ON THE RELATIONSHIP WITH $\theta_{13} - \delta$– SIGN OF $\Delta M^2_{13}$ AMBIGUITIES

We now show that the $(\theta_{13} - \delta)$ ambiguity is automatically resolved by tuning neutrino energy at the oscillation maximum. It must be the case because two straight-line trajectories with the same slope do not have intersection points. For our purpose, it suffices to work with oscillation probability at a fixed monochromatic beam energy because averaging over a finite width complicates the formalism and may obscure the essence of the problem. It can be shown [10] that the difference between the true ($\theta_{13}$) and the false ($\theta'_{13}$) solutions of $\theta_{13}$ for a given set of $P(\nu)$ and $P(\bar{\nu})$ is given under the small $\theta_{13}$ approximation by

$$\theta_{13}' - \theta_{13} = -\frac{\sin \delta - z \cos \delta}{1 + z^2} \frac{2Q}{P_- - P_+} \sin \left( \frac{\Delta_{13}}{2} \right)$$

(6)
where

\[ z = \frac{P_+ + P_-}{P_+ - P_-} \tan \left( \frac{\Delta_{13}}{2} \right) \]  

(7)

Hence, the difference vanishes at the oscillation maximum, \( \Delta_{13} = \pi \), which means \( z \to \infty \). Thus, no \((\theta_{13} - \delta)\) ambiguity exists at the oscillation maximum as expected.

It should be emphasised that our strategy of tuning beam energy at the oscillation maximum is not affected by the ambiguity correlated with the sign of \( \Delta m^2_{13} \) which is discussed in Ref. [11]. It is because the matter effect split the straight-line CP trajectories of positive and negative \( \Delta m^2_{13} \) toward the direction of the line itself in first order of the matter effect. The possible correction comes from higher order matter effect which is small in the relatively short baseline of the JHF (as well as the CERN \( \rightarrow \) Frejus) experiment, as shown in Fig. 3. The effect can be easily taken care of in the actual determination of \( \theta_{13} \).

VI. CONCLUDING REMARKS

In this paper, we proposed a new strategy for accurate determination of \( \theta_{13} \) without suffering from the intrinsic ambiguity due to unknown value of \( \delta \). That is, tune the beam energy at the thinnest CP trajectory and do the measurement both in neutrino and antineutrino channels. We have shown that our new strategy completely resolves the ambiguities in the determination of \( \theta_{13} \) due to \( \delta \) and due to the sign of \( \Delta m^2_{13} \) within the experimental accuracy attainable in such experiments.

One of the proposal which could be extracted from the strategy described in this paper is a possibility of having \( \bar{\nu}_\mu \) beam as early as possible. It would be the promising option for the case of relatively large \( \sin^2 2\theta_{13} \), say, within a factor of 2-3 smaller than the CHOOZ bound. In this case, the \( \nu_\mu \to \nu_e \) appearance events can be easily established in a few years of running of next generation neutrino oscillation experiments. Then, the uncertainties in determination of \( \theta_{13} \) would be greatly decreased by switching to \( \bar{\nu}_\mu \) beam rather than just running with the \( \nu_\mu \) beam.

What would be the implication of our strategy to the determination of \( \delta \)? The tuning of
beam energy at thinnest trajectory in fact also provides a good way of measuring $\delta$. The ambiguity ($\delta \to \pi - \delta$), however, is unresolved and it would necessitate supplementary measurement either by using “fattest” trajectory configuration [11], or by second detector with different baseline distance [11]. We should emphasize that once $\theta_{13}$ is measured accurately there is no more intrinsic ambiguities in determination of $\delta$. We have explicitly shown that $(\delta - \theta_{13})$ ambiguity is resolved. The only ambiguity which would survive (from the viewpoint of determination of $\delta$) would be the accidental one that arises in a correlated way ($\delta - \text{sign of $\Delta m^2_{13}$}$), which is nothing but the remnant of $(\delta \to \pi - \delta)$ degeneracy in vacuum [11]. But it is also resolved by either one of the two second measurements mentioned above.

Note added:

While this paper was being written, we became aware of the paper by Barger et al. [21] whose results partially overlaps with ours. However, most of the ambiguities discussed in the paper will be gone once $\theta_{13}$ is determined accurately, as we noted above.

ACKNOWLEDGMENTS

We thank Takashi Kobayashi and Yoshihisa Obayashi for valuable informative correspondences on low energy neutrino beams, detector backgrounds, and $\theta_{13}$ sensitivity in the JHF experiment. This work was supported by the Brazilian funding agency Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), and by the Grant-in-Aid for Scientific Research in Priority Areas No. 12047222, Japan Ministry of Education, Culture, Sports, Science, and Technology.

3 Tuning of beam energy at the oscillation maximum itself has been proposed before for differing reasons from ours. First of all, it is preferred experimentally because it maximizes disappearance of $\nu_\mu$ as well as the number of electron appearance events [7]. The tuning of beam energy to the oscillation maximum for measurement of CP violating phase $\delta$ was proposed by Konaka for the purpose of having maximal CP-odd ($\sin \delta$) term at the energy [20].
REFERENCES

[1] Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. B335 (1994) 237; Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81 (1998) 1562; ibid. 85 (2000) 3999.

[2] Homestake Collaboration, K. Lande et al., Astrophys. J. 496 (1998) 505; SAGE Collaboration, J. N. Abdurashitov et al., Phys. Rev. C 60 (1999) 055801; GALLEX Collaboration, W. Hampel et al., Phys. Lett. B 447 (1999) 127; Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 86 (2001) 5651; ibid. 86 (2001) 5656; SNO Collaboration, Q. R. Ahmed et al., Phys. Rev. Lett. 87 (2001) 071301.

[3] K2K Collaboration, S. H. Ahn et al., Phys. Lett. B 511 (2001) 178; J. E. Hill, for the K2K Collaboration, Talk presented at Snowmass 2001, Snowmass, Colorado, June-July, 2001, hep-ex/0110034.

[4] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

[5] CHOOZ Collaboration, M. Apollonio et al., Phys. Lett. B 420 (1998) 397; ibid. B 466 (1999) 415; The Palo Verde Collaboration, F. Boehm et al., Phys. Rev. D 62 (2000) 072002.

[6] H. Minakata and H. Nunokawa, Phys. Lett. B 504 (2001) 301.

[7] Y. Itow et al., The JHF-Kamioka neutrino project, hep-ex/0106019.

[8] J. J. Gomez-Cadenas et al. (CERN working group on Super Beams Collaboration), hep-ph/0105297.

[9] A. Cervera, A. Donini, M. B. Gavela, J. J. Gomez Cadenas, P. Hernandez, O. Mena and S. Rigolin, Nucl. Phys. B579 (2000) 17 [Erratum-ibid. B593 (2000) 731]; V. Barger, S. Geer, R. Raja and K. Whisnant, Phys. Rev. D 63 (2001) 113011; C. Albright et al., hep-ex/0008064.

[10] J. Burguet-Castell, M.B. Gavela, J.J. Gomez-Cadenas, P. Hernandez and O. Mena, Nucl. Phys. B 608 (2001) 301.
[11] H. Minakata and H. Nunokawa, JHEP 0110 (2001) 001 [hep-ph/0108085].

[12] H. Minakata and H. Nunokawa, Talk presented at The 3rd International Workshop on Neutrino Factories Based on Muon Storage Rings (NuFACT01), Tsukuba, Japan, May 24-30, 2001, hep-ph/0111130.

[13] H. Minakata, Talk at 8th Tokutei-RCCN Workshop on Future Neutrino Oscillation Experiments, ICRR, Kashiwa, Japan, November 9, 2001.

[14] H. Minakata and H. Nunokawa, Talk presented at 7th International Workshop on Topics in Astroparticle and Underground Physics (TAUP2001), Laboratori Nazionali del Gran Sasso, Italy, September 8-12, 2001, hep-ph/0111131.

[15] G. L. Fogli, E. Lisi, and A. Marrone, hep-ph/0110089.

[16] H. Minakata and H. Nunokawa, Phys. Lett. B495 (2000) 369; Nucl. Instrum. Meth. A472 (2001) 421; J. Sato, Nucl. Instrum. Meth. A472 (2001) 434; B. Richter, hep-ph/0008222.

[17] J. Arafune and J. Sato, Phys. Rev. D55 (1997) 1653; J. Arafune, M. Koike and J. Sato, Phys. Rev. D56 (1997) 3093 [Erratum *ibid.* D 60 (1999) 119905]; H. Minakata and H. Nunokawa, Phys. Rev. D57 (1998) 4403; Phys. Lett. B413 (1997) 369; K. Dick, M. Freund, M. Lindner and A. Romanino, Nucl. Phys. B 562 (1999) 29; O. Yasuda, Acta. Phys. Polon. B 30 (1999) 3089; M. Koike and J. Sato, Phys. Rev. D61 (2000) 073012; [Erratum *ibid.* D 62 (2000) 079903].

[18] J. Arafune, M. Koike and J. Sato, in Ref. [17].

[19] Y. Obayashi, Talk at 8th Tokutei-RCCN Workshop on Future Neutrino Oscillation Experiments, ICRR, Kashiwa, Japan, November 9, 2001 and private communications.

[20] A. Konaka, Talk at JHF-SK Nu Workshop, Tsukuba, Japan, May 30-31, 2001, http://neutrino.kek.jp/jhfnu/workshop2/ohp/konaka2.pdf.

[21] V. Barger, D. Marfatia, and K. Whisnant, hep-ph/0112119.
FIG. 1. CP trajectory diagrams showing the contours of $\langle P(\nu) \rangle \equiv \langle P(\nu_\mu \rightarrow \nu_e) \rangle$ and $\langle P(\bar{\nu}) \rangle \equiv \langle P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \rangle$ as a function of $\delta$. The Gaussian energy distribution of neutrino beam with $\langle E \rangle = 0.5$ GeV with width $\sigma = 0.1$ GeV is assumed and the baseline length is taken as $L = 295$ km. The mixing parameters are fixed to be $\Delta m^2_{23} = \pm 3 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{23} = 1.0$, $\Delta m^2_{12} = 6.2 \times 10^{-5} \text{ eV}^2$, $\tan^2 \theta_{12} = 0.35$. We take the matter density as $\rho = 2.8 \text{ g/cm}^3$ and the electron fraction as $Y_e = 0.5$. /
(a) \(\nu_\mu \rightarrow \nu_e\)

(b) \(\nu_\mu \rightarrow \nu_e\)

(c) \(\nu_\mu \rightarrow \nu_e\)

\(<E>=0.5 \text{ GeV}\)

\(<E>=0.716 \text{ GeV}\)

\(<E>=1 \text{ GeV}\)

\(\Delta m^2_{23} > 0\)

\(\Delta m^2_{23} < 0\)

FIG. 2. Allowed region of \(\sin^2 2\theta_{13}\) is shown as a shaded strip for given values of \(\langle P(\nu_\mu \rightarrow \nu_e) \rangle\) (given in \%) assuming Gaussian energy distribution of neutrino beam centered at \(\langle E \rangle =\) (a) 0.5, (b) 0.716, and (c) 1.0 GeV with 20 \% width \(\sigma\) of \(\langle E \rangle\) for \(L = 295\) km. If the sign of \(\Delta m^2_{23}\) is known, the allowed region is within the solid \(\Delta m^2_{23} > 0\) and the dashed \(\Delta m^2_{23} < 0\) lines. The other mixing parameters and the matter density are taken as in Fig. 1.
FIG. 3. The thinnest CP trajectories for a tuned peak energy for \( \sin^2 2\theta_{13} = 0.05 \) and 0.02.

The beam profile, the mixing parameters and the matter density are taken as in Fig. 1.