Julio-Batalla, Jurgen; Petean, Jimmy
Nodal solutions of Yamabe-type equations on positive Ricci curvature manifolds. (English) Zbl 1482.53047
Proc. Am. Math. Soc. 149, No. 10, 4419-4429 (2021).

This article investigates nodal solutions of the Yamabe-type equation

$$-\Delta_g u + \lambda u = \lambda u^q,$$

on a closed Riemannian manifold $$(M, g)$$ of positive Ricci curvature, where $$\Delta_g$$ is the Laplace operator, while $$\lambda$$ and $$k$$ are real constants with $$\lambda > 0$$ and $$q > 1$$. The authors consider the case when there is a subgroup $$G$$ of the isometry group of $$g$$ that acts with cohomogeneity one. In this setting, it is proved that the above equation admits nodal solutions for any $$\lambda > 0$$ and $$q \in (1, \frac{n-m+2}{n-m-2})$$, where $$n$$ is the dimension of $$M$$ and $$m$$ is the minimum of the dimension of the orbits of $$G$$. This result is then applied to prove the existence of nodal solutions of the Yamabe equation on positive Einstein manifolds.

Reviewer: Gabriel Eduard Vilcu (Ploieşti)

MSC:
53C21 Methods of global Riemannian geometry, including PDE methods; curvature restrictions
53C25 Special Riemannian manifolds (Einstein, Sasakian, etc.)

Keywords:
Yamabe equation; cohomogeneity; Ricci curvature; Einstein manifold

Full Text: DOI arXiv

References:
[1] Ammann, Bernd; Humbert, Emmanuel, The second Yamabe invariant, J. Funct. Anal., 235, 2, 377-412 (2006) - Zbl 1142.53026
[2] Aubin, Thierry, Equations différentielles non linéaires et problème de Yamabe concernant la courbure scalaire, J. Math. Pures Appl. (9), 55, 3, 269-296 (1976) - Zbl 0336.53033
[3] Betancourt de la Parra, Alejandro; Julio-Batalla, Jurgen; Petean, Jimmy, Global bifurcation techniques for Yamabe type equations on Riemannian manifolds, Nonlinear Anal., 202, 112140, 23 pp. (2021) - Zbl 1455.58015
[4] Bettiol, Renato G.; Piccione, Paolo, Multiplicity of solutions to the Yamabe problem on collapsing Riemannian submersions, Pacific J. Math., 266, 1, 1-21 (2013) - Zbl 1287.53030
[5] Böhm, Christoph, Inhomogeneous Einstein metrics on low-dimensional spheres and other low-dimensional spaces, Invent. Math., 134, 1, 145-176 (1998) - Zbl 0965.53033
[6] Brendle, Simon, Blow-up phenomena for the Yamabe equation, J. Amer. Math. Soc., 21, 4, 951-979 (2008) - Zbl 1206.53041
[7] Clapp, Mónica; Fernández, Juan Carlos, Multiplicity of nodal solutions to the Yamabe problem, Calc. Var. Partial Differential Equations, 56, 5, Paper No. 145, 22 pp. (2017) - Zbl 1379.35131
[8] Clapp, Mónica; Ghimenti, Marco; Micheletti, Anna Maria, Solutions to a singularly perturbed supercritical elliptic equation on a Riemannian manifold concentrating at a submanifold, J. Math. Anal. Appl., 420, 1, 314-333 (2014) - Zbl 1295.35033
[9] del Pino, Manuel; Musso, Monica; Pacard, Frank; Pistoia, Angela, Torus action on $$(S^n, \omega)$$ and sign-changing solutions for conformally invariant equations, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5), 12, 1, 209-237 (2013) - Zbl 1267.53040
[10] Ding, Wei Yue, On a conformally invariant elliptic equation on $$(\mathbb{R}^n, \omega)$$, Comm. Math. Phys., 107, 2, 331-335 (1986) - Zbl 0608.35017
[11] Fernández, Juan Carlos; Petean, Jimmy, Low energy nodal solutions to the Yamabe equation, J. Differential Equations, 268,
Ghimenti, Marco; Micheletti, Anna Maria; Pistoia, Angela, Blow-up solutions concentrated along minimal submanifolds for some supercritical elliptic problems on Riemannian manifolds, J. Fixed Point Theory Appl., 14, 2, 503-525 (2013) · Zbl 1300.58008 · doi:10.1007/s11784-014-0168-1

[15] Grove, Karsten; Ziller, Wolfgang, Cohomogeneity one manifolds with positive Ricci curvature, Invent. Math., 149, 3, 619-646 (2002) · Zbl 1038.53034 · doi:10.1007/s002220200225

[16] Henry, Guillermo, Isoparametric functions and nodal solutions of the Yamabe equation, Ann. Global Anal. Geom., 56, 2, 203-219 (2019) · Zbl 1467.53045 · doi:10.1007/s10455-019-09664-x

[17] Henry, Guillermo; Madani, Farid, The equivariant second Yamabe constant, J. Geom. Anal., 28, 4, 3747-3774 (2018) · Zbl 1407.53035 · doi:10.1007/s12220-017-9978-x

[18] Henry, Guillermo; Petean, Jimmy, Isoparametric hypersurfaces and metrics of constant scalar curvature, Asian J. Math., 18, 1, 53-67 (2014) · Zbl 1292.53041 · doi:10.4310/AJM.2014.v18.n1.a3

[19] Kurepa, Aleksandra, Existence and uniqueness theorem for singular initial value problems and applications, Publ. Inst. Math. (Beograd) (N.S.), 45(59), 89-93 (1989) · Zbl 0677.34002

[20] Obata, Morio, The conjectures on conformal transformations of Riemannian manifolds, J. Differential Geometry, 6, 247-258 (1971/72) · Zbl 0236.53042

[21] Petean, Jimmy, Metrics of constant scalar curvature conformal to Riemannian products, Proc. Amer. Math. Soc., 138, 8, 2897-2905 (2010) · Zbl 1206.53042 · doi:10.1090/S0002-9939-10-10293-7

[22] Polack, Daniel, Nonuniqueness and high energy solutions for a conformally invariant scalar equation, Comm. Anal. Geom., 1, 3-4, 347-414 (1993) · Zbl 0848.58011 · doi:10.4310/CAG.1993.v1.n3.a2

[23] Schoen, Richard, Conformal deformation of a Riemannian metric to constant scalar curvature, J. Differential Geom., 20, 2, 479-495 (1984) · Zbl 0576.53029

[24] Schoen, Richard M., Variational theory for the total scalar curvature functional for Riemannian metrics and related topics. Topics in calculus of variations, Montecatini Terme, 1987, Lecture Notes in Math. 1365, 120-154 (1989), Springer, Berlin · doi:10.1007/BFb0089180

[25] Trudinger, Neil S., Remarks concerning the conformal deformation of Riemannian structures on compact manifolds, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4), 22, 265-274 (1968) · Zbl 0159.23801

[26] Wang, McKenzie Y.; Ziller, Wolfgang, Einstein metrics on principal torus bundles, J. Differential Geom., 31, 1, 215-248 (1990) · Zbl 0691.53036

[27] Yamabe, Hidehiko, On a deformation of Riemannian structures on compact manifolds, Osaka Math. J., 12, 21-37 (1960) · Zbl 0096.37201

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.