Research on Multi-Order Anti-Aliasing Filter in Bearing Profile Detection

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Abstract. In the process of bearing profile detection, an analog displacement sensor is used, but there is noise interference in the process of data acquisition. In order to suppress noise interference, the paper designs three-order and fifth-order anti-aliasing filters according to the actual interference signal source. The resistance value and capacitance value are calculated according to the transfer function. The real object is made by the porous simulation board. The comparison between the input and output signals shows that the fifth-order anti-aliasing filter is better than the third-order anti-aliasing filter. Experiments show that the higher order anti-aliasing filter has a certain suppression effect on the noise signal, but the effect is not obvious. The signal distortion would happen and the system would become more and more complicated. For this reason, the fifth-order anti-aliasing filter is finally selected, and the actual sampling is completed, and the effect reaches the requirements of the detection system.

1. Introduction

Bearings widely used in various industries have been designed with various types and different structures to implement different function and adapt to conditions of environments. As a result, the structural shape of the inner and outer rings of the bearing is complex and variable. Figure 1 shows the common structure of the inner and outer rings of the bearing (according to different structures, it is divided into categories of "0", "2", "3" and "7" respectively). As we know, the structural parameters of different types of bearings are significantly different, so many parameters need to be detected, however, the detected parameters are also discrepant. It is difficult to guarantee the accuracy and efficiency by using traditional detection methods due to the large demand and high precision of the profile detection of bearing inner and outer rings. This paper primarily introduces the principle of automatic detection of geometric parameters of bearing profile and how to reduce the influence of detection accuracy, which caused by thermal deformation of the detection system. The detected parameters mainly include inner ring diameter, outer ring diameter, raceway size, height size, taper size and etc.

Figure 1. The structure of bearing
According to the analysis of the detected parameters, the high-speed measurement method with multi-laser sensor is proposed to accomplish the detection of the non-circularity, outer diameter, inner diameter, height and taper of bearing inner and outer rings. The detection scheme as follows is determined on the basis of the requirement of actual test detection, as illustrated in Figure 2. It is used to compensate error of the placement of workpiece by symmetry with multi-sensor. Meanwhile, each laser sensor is moved to detected position through servo motor to make the detected workpiece situate the scope and detection range of laser sensor, and the grating ruler record the working position of each sensors. The center rotation axis drives the workpiece to rotate and the motor corresponding to the sensor to accomplish the scanning of the whole detected workpiece. The laser sensor continuously measures the shape of the detected workpiece, and the detection data is synchronously transmitted to the processor for processing to obtain a result of measurement. Synchronous automatic detection of multiple parameters of the inner and outer rings of the multi-type bearing is actualized thereby. The key to the detection system is to obtain real and stable data, thus the aliased noise signal must be processed. It is necessary to study the anti-aliasing filter of the detection system.

2. Anti-aliasing filter research

2.1. Anti-aliasing filter

The anti-aliasing filter (AAF) placed in front of a signal sampler (such as ADC, etc.) can satisfy the sampling theorem approximately or completely by limiting the bandwidth of a signal over a dominant band. The achievable AAF generally allows some aliasing, or attenuating some frequencies near the Nyquist limit. Therefore, the sampling of many practical data processing systems will be higher than the actual requirements to ensure that all the important frequencies can be reconstructed. This practice is called over-sampling.

AAF is widely used on analog-to-digital converters (ADC) in digital signal processing systems. Similar filters are used for reconstruction filters at the output of such systems, such as media players. The application of AAF at the output prevents radio frequency formation (the inverse process of aliasing, making the intra-frequency to mirror out of the frequency).
2.2. Butterworth filter

The Butterworth Filter (BF) is a type of basic filter first published by British engineer Stephen Butterworth in the British Journal of Radio Engineering in 1930. The BF is characterized by a maximum flat frequency-response curve in the passband without fluctuation, and a gradual drop to zero in the blocking band. On the Bode plot of the logarithmic to angular frequency of the amplitude, starting from a certain corner angular frequency, the amplitude gradually decreases with the increase of the angular frequency and tends to negative infinity.

The first-order BF has a decay rate of 6 dB per octave and 20 dB per decade. The attenuation rate of the second-order BF is 12 dB per octave, and the attenuation rate of the third-order BF is 18 dB per octave. By analogy, the amplitude of the BF, the only filter whose amplitude to the angular frequency curve remains the same shape regardless of the order, and decreases monotonically to the angular frequency. However, the higher the order of the BF is, the faster the rate of amplitude attenuation in the blocking band is, while other filters have different shapes of amplitude to the angular frequency in the higher or lower order.

The amplitude-frequency characteristics of the n-order Butterworth low-pass filter:

$$G^2(\omega) = |H(j\omega)|^2 = \frac{G_0^2}{1 + \left(\frac{j\omega}{\omega_c}\right)^{2n}} \tag{1}$$

Where \(n\) is the order of the BF, \(H(j\omega)\) is the transfer function of the n-order BF, \(\omega_c\) is the cutoff frequency of the BF (the frequency at -3 dB) and \(G_0\) is the gain of the filter (the amplitude at 0 frequency).

If \(\omega_c=1\) rad/s and \(G_0=1\), the amplitude-frequency characteristics of the BF can be simplified as follows:

$$G(\omega) = \frac{1}{\sqrt{1 + \omega^{2n}}} \tag{2}$$

![Figure 4. Frequency response curve of a low-pass Butterworth filter with order 1 to 5 at cutoff frequency](image)

2.3. Butterworth low-pass filter design

The higher the order of the BF is, the closer its amplitude-frequency characteristic is to the ideal filter. In other words, as \(n\) increases, the amplitude in the passband approaches 1. When the transition domain narrows, the amplitude in the blocking band approached 0. At the same time, as the order increases, the number of circuit components also increases. The fundamental problem of filter design is to reduce the order as much as possible under the condition of meeting the filtering requirements.
As can be seen from the foregoing, the selected Butterworth low-pass filter has a cutoff frequency of 1 kHz and an amplification factor of less than or equal to -30 dB at a frequency of 2 kHz. Then the order of the filter can be obtained as follows:

\[ n = -0.05A_0 \times \log\left(\frac{\omega_b}{\omega_p}\right) \approx 4.98 \quad (3) \]

So the required Butterworth low-pass filter has an order of 5.

However, the specific circuit parameters is unable to be obtained only from the amplitude-frequency characteristic, and the transfer function of the specific filter is needed. Therefore, the Butterworth polynomial needs to be solved, as illustrated in Table 1.

**Table 1. Butterworth polynomial table**

| n  | N – order Butterworth polynomial |
|----|----------------------------------|
| 1  | \((s + 1)\)                       |
| 2  | \((s^2 + \sqrt{2}s + 1)\)         |
| 3  | \((s + 1)(s^2 + s + 1)\)          |
| 4  | \((s + 1)(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)\) |
| 5  | \((s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)\) |
| 6  | \((s^2 + 0.518s + 1)(s^2 + \sqrt{2}s + 1)(s^2 + 1.932s + 1)\) |
| 7  | \((s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)\) |

The required order of Butterworth low-pass filter is 5, and the transfer function of the normalized 5-order Butterworth low-pass filter is as following:

\[ H(s) = \frac{1}{(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)} \quad (4) \]

Now the transfer function of the fifth-order Butterworth filter can be certain, the circuit is designed as shown in transfer function (5). According to the foregoing, the circuit can be equivalent to a series connection of a first-order active low-pass filter and two second-order active low-pass filters (Sallen-Key topology), as illustrated in Figure 5.

![Figure 5. The circuit of fifth-order Butterworth low-pass filter](image)

The transfer function of the circuit is as (5):

\[ H(s) = \frac{1}{(R_1C_1s + 1)} \times \frac{1}{(C_2C_3R_2R_3s^2 + C_3(R_2 + R_3)s + 1)} \times \frac{1}{(C_4C_5R_4R_5s^2 + C_5(R_4 + R_5)s + 1)} \quad (5) \]

According to the above formula, the appropriate resistance and capacitance are selected to obtain the circuit diagram shown in Figure 6.
The actual transfer function of the circuit is as (6):

\[ H(s) = \frac{1}{(R_1C_1s + 1)} \times \frac{1}{(C_2C_3R_2R_3s^2 + C_3(R_2 + R_3)s + 1)} \times \frac{1}{(C_4C_5R_4R_5s^2 + C_5(R_4 + R_5)s + 1)} \]  

(6)

According to Figure 7, the frequency at -3.01dB is 6270, so the cutoff frequency is:

\[ f_c = \frac{6270}{2 \times 3.1416} \approx 997.9 \, \text{Hz} \]  

(7)

3. Butterworth low-pass filter experiment

In order to compare the effect of the 5th-order Butterworth low-pass filter designed in the previous section, a third-order Butterworth low-pass filter is designed. The parameters are consistent with the fifth-order Butterworth filter with 1KHz cutoff frequency. The design process of the third-order Butterworth low-pass filter is similar to the fifth-order Butterworth filter, which will not be described here.
To this end, an experimental platform was built with TDS1002 oscilloscope, linear stabilized power supply, switching power supply, multimeter and laser ranging sensor.

![Figure 8. The schematic of third-order Butterworth low-pass filter](image)

**Figure 8.** The schematic of third-order Butterworth low-pass filter

After the output of the laser distance measuring sensor is connected in parallel with the precision resistance, the current signal can be converted into a voltage signal, which is connected to the input of the third-order BF and the fifth-order BF. Then an oscilloscope is used to observe the waveforms of input and output of the filter.

The results of the experiment of the third-order and fifth-order Butterworth low-pass filter are shown in followings:

![Figure 9. Butterworth low-pass filter experimental platform](image)

**Figure 9.** Butterworth low-pass filter experimental platform

(a): input  
(b): output

**Figure 10.** Third-order Butterworth low-pass filter input and output waveform (T=2.5ms)
According to the experimental results, the signal fluctuation attenuation is about 75% of the past after passing the third-order Butterworth low-pass filter, and about 50% of the past after passing the fifth-order Butterworth low-pass filter.

From the scale of oscilloscope 1us, the glitch of the signal is significantly reduced, and it becomes much smoother after the signal passes through the Butterworth low-pass filter. It can be concluded from the experimental results that the Butterworth low-pass filter has a significant improvement on the signal quality, and the filtering effect of the fifth-order filter is much better than that of the third-order filter.
4. Experimental data processing

In the bearing detection system, the signals output by the triangular laser sensor and the spectral confocal sensor are processed with a fifth-order Butterworth filter, sampled by a 16-bit ADC built in the TMS320F28377D digital signal processor, and analyzed by a digital signal processor.

In order to verify the effect of the fifth-order BF, two sets of experiments were conducted. The output signals of one set of sensors were directly sampled by the 16-bit ADC built in the digital signal processor, and the output signals of the other set of sensors were processed with the filter and then sampled by the built-in 16-bit ADC built in the digital signal processor. The Figure 14 and Table 2 show the results of the experiments as followings:

![Figure 14. Original and filtered sampled data line chart](image)

Table 2. Data comparison table before and after filtering

| Data Sources     | Channel | Standard Deviation | Range |
|------------------|---------|--------------------|-------|
| Using Filter     | Channel 1 | 135.92             | 478   |
|                  | Channel 2 | 231.62             | 758   |
|                  | Channel 3 | 221.59             | 771   |
| Not Using Filter | Channel 1 | 2.3979             | 8     |
|                  | Channel 2 | 1.8329             | 5     |
|                  | Channel 3 | 2.6458             | 8     |

5. Conclusions

It can be seen intuitively from Figure 14 that the fluctuation of the filtered data is significantly reduced before filtering. The data resampled after filter processing is smaller in two orders of magnitude than the data directly sampled without filter processing, measured by standard deviation and range, as shown in Table 2. In other words, the fifth-order Butterworth filter has obvious smoothing and filtering effect on the data sampling, so that the sampled data is stable and reliable.

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