User Association With Maximizing Weighted Sum Energy Efficiency for Massive MIMO-Enabled Heterogeneous Cellular Networks

Tianqing Zhou, Zunxiong Liu, Dong Qin, Nan Jiang, and Chunguo Li

Abstract—In this letter, we design an association strategy to maximize the weighted sum energy efficiency (EE) for massive multiple-input and multiple-output (MIMO)-enabled heterogeneous cellular networks. Considering that the final formulated problem is in a sum-of-ratio form, we first need to transform it into a parametric nonfractional form, by which we can achieve its solution through a two-layer iterative algorithm. The outer layer searches the EE parameters and the multipliers associated with signal-interference-plus-noise ratio constraints using Newton-like method, and the inner layer optimizes the association indices using Lagrange multiplier method. Then, we give some convergence and complexity analyses for the proposed algorithm. Numerical results show that the proposed scheme significantly outperforms the existing one on the system throughput and network EE under a certain condition. In addition, we also investigate the impacts of the number of massive antennas and the transmit power of each pico base station on the association performance.

Index Terms—Heterogeneous cellular networks, massive MIMO, user association, SINR constraint, energy efficiency.

I. INTRODUCTION

HETEROGENEOUS cellular networks (HCNs) integrate macrocells and other small cells such as microcells, picocells and femtocells, which can eliminate the coverage holes and relieve the hot spots [1]. To further enhance the area spectral efficiency (SE), the massive multiple-input and multiple-output (MIMO) technology has been implemented at macro base stations (BSs), which simultaneously transmits some independent data streams to multiple users sharing the same resource via a large-scale antenna system [2]. However, the circuit power consumption increases with the number of massive MIMO antennas. Thus, the energy-efficient radio resource management should be an important topic for massive MIMO systems.

User association (UA) often assigns users to the different BSs available in the system, which is an indispensable element of radio resource management. So far, most researches on the UA mainly refer to the single antenna HCNs, such as [3] and [4], etc. The efforts in the literature toward UA in massive MIMO enabled HCNs are very few and still in infancy. In [5], authors design some association schemes with various objectives including (achievable) rate maximization, proportional fairness and the UA with resource allocation. In [6], authors consider an α-utility maximization association for massive MIMO wireless networks. Instead of system throughput in [5] and [6], authors in [2] take the energy efficiency (EE) as a key factor for the association design, and propose an energy-efficient association scheme to maximize the network-wide utility that is a function with respect to users’ EEs. To investigate the tradeoff between EE and SE, authors in [7] formulate a multi-objective optimization problem with joint UA and power coordination under proportional rate fairness.

In this letter, we take account of an energy-efficient association from a novel perspective. Unlike the efforts in [2] and [7], we design an association scheme to maximize the weighted sum EE under users’ SINR (signal-interference-plus-noise ratio) constraints, and finally achieve an association problem with a sum-of-ratio form. To solve this problem, we first need to transform it into a parametric nonfractional form, and then develop a two-layer iterative algorithm with guaranteed convergence. Specially, we search the EE parameters and the multipliers associated with SINR constraints using Newton-like method in the outer layer, and achieve the association indices using Lagrange multiplier method in the inner layer.

II. SYSTEM MODEL

Without loss of generality, we just consider the two-tier HCNs consisting of macro BSs (MBSs) and pico BSs (PBSs). Note that these BSs have primary differences in transmit power, propagation characteristics, backhaul, operation cost and ease-of-deployment. In such HCNs, any MBS implements M antennas but any PBS just utilizes one antenna [2]. Similar to the efforts in [2] and [7], we assume that all BSs use the same frequency band, any BS equally allocates the time-frequency resources for its associated users, and the proportional fairness scheduling [4] is employed. In addition, we also assume that any MBS can simultaneously transmit at most $S$ ($1 \ll S \ll M$) downlink data streams over the same frequency band, and linear zero-forcing beamforming is used for the massive MIMO downlink transmission [2], [7].

Assume that the sets of users and BSs are $\mathcal{K}$ with size $K = |\mathcal{K}|$ and $\mathcal{N} = \mathcal{N}_m \cup \mathcal{N}_p$ with size $N = |\mathcal{N}|$ respectively, and $\mathcal{N}_m$ and $\mathcal{N}_p$ are the ones of MBSs and PBSs respectively. Then, the SINR received by user $k$ from MBS $n \in \mathcal{N}_m$ is $\text{SINR}_{nk} = \frac{\kappa P_n g_{nk}}{\sum_{j \in \mathcal{N}_m \setminus \{n\}} P_j g_{jk} + \sigma_n^2}$, and the one from PBS $n \in \mathcal{N}_p$ is $\text{SINR}_{nk} = \frac{P_n g_{nk}}{\sum_{j \in \mathcal{N}_p \setminus \{n\}} P_j g_{jk} + \sigma_n^2}$ [2], [7]. Significantly, $P_n$ represents the transmit power of BS $n$; $g_{nk}$ denotes the channel.
gain between BS $n$ and user $k$; $\sigma_n^2$ is the noise power of BS $n$; $\kappa = (M - S + 1) / S$ is a scaling factor for any SINR$_{\text{th}}$ due to the array gain and equal power allocation from massive MIMO over the Rayleigh channel fading [2]. At last, the (effective long-term) data rate received by user $k$ from BS $n$ is $R_n^k = r_n^k / \sum_{i \in \mathcal{K}_n} x_{ni}$ if BS $n$ serves at least one user, $R_n^k = 0$ otherwise. Moreover, $r_n^k = \log_2 (1 + \text{SINR}_{nk})$ is the achievable rate received by user $k$ from MBS $n \in \mathcal{N}_o$; $r_n^k = \log_2 (1 + \text{SINR}_{nk})$ is the one received by user $k$ from PBS $n \in \mathcal{N}_p$ [2], [7]; $x_{nk}$ represents the association index, and it is 1 if user $k$ is associated with BS $n$, 0 otherwise. Significantly, $r_n^k$ reflects the channel condition between user $k$ and BS $n$, but it has not any practical meaning and is often used for the unbalanced (traditional) association design. $R_n^k$ can reflect user’s channel condition and resource consumption level, and thus it can be regarded as a practical data rate.

### III. Problem Formulation

To realize green communications and meet different EE requirements of users, we design an association scheme to maximize a weighted sum EE, and formulate it as

$$\max_{x, \omega} \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}_n} w_k E_{nk}$$

s.t. $C_1 : \sum_{n \in \mathcal{N}} x_{nk} = 1, \forall k \in \mathcal{K}$,

$C_2 : \sum_{n \in \mathcal{N}} x_{nk} \text{SINR}_{nk} \geq \tau_k, \forall k \in \mathcal{K}$,

$C_3 : x_{nk} \in \{0, 1\}, \forall n \in \mathcal{N}, \forall k \in \mathcal{K}$,  

(1)

where $x = \{x_{nk}, \forall n, \forall k\}$; $w = \{w_k, \forall k\}$, and $w_k$ is the priority of user $k$; $E_{nk} = x_{nk} R_n^k / (\rho_n p_n + p_n^0)$ is the EE of user $k$ at BS $n$; $\rho_n$ denotes the power amplifier coefficient of BS $n$; $\tau_k$ is the SINR threshold of user $k$; $p_n^0$ is the circuit power consumption of BS $n$ and $p_n^0 = \sum_{i=0}^{3} A_{i0} S^i + M \sum_{i=0}^{2} A_{1i} S^i$ for $n \in \mathcal{N}_o$, where $A_{i0}$ and $A_{1i}$ are the coefficients [2]; the constraint $C_1$ shows that some user can just be associated with only one BS; $C_2$ gives the minimal SINR requirements of associated users.

To maximize the objective function of (1), some users close to BSs may not need high transmit power and only far-away users need more transmit power to guarantee the required data rate. In addition, when the weight $w_k$ of some user $k$ takes a relatively small value (smaller than 1), the user may be forced to give up some high-EE BS and offloaded to the low-EE one with a potentially high-long term rate (supporting SINR requirement); when user’s weight utilizes a relatively large value (greater than 1), this user may have more chance to utilize high-EE BS than other users. Evidently, by properly setting the weight, the EEs achieved by users can be adjusted and users’ opportunities to select BSs can also be changed. That is to say, the weight $w$ can be used for meeting different EE requirements of users and also be used for guaranteeing user fairness.

### IV. Association Algorithm Design

To give a common design for all BSs, we consider a relaxation of EE and then reformulate the problem (1) into

$$\max_{X, \omega} \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}_n} w_k \bar{E}_{nk}, \text{ s.t. } C_1, C_2, C_3,$$

(2)

where $\bar{E}_{nk} = x_{nk} r_n^k / \left[ (\rho_n p_n + p_n^0) (1 + \sum_{i \in \mathcal{K}} x_{ni}) \right] \leq E_{nk}$. As we know, $E_{nk}$ should be 0 if BS $n$ has not any served user, which is consistent with $\bar{E}_{nk}$, i.e., $E_{nk} = \bar{E}_{nk} = 0$. In other cases, $E_{nk}$ often achieves a tight lower bound of $\bar{E}_{nk}$.

Certainly, the problem (2) also achieves a tight lower bound of (1). In addition, these two problems are nearly equivalent when the distributed users are so many that any BS has at least one served user, and the equivalence between them is higher if more users are served by any BS.

Next, the problem (2) can be equivalently converted into

$$\max_{X, \omega} F(X, \omega) = \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}_n} x_{nk} \bar{E}_{nk}$$

s.t. $C_1, C_2, C_3$,

$$C_4 : r_n^k \geq x_{nk} \bar{E}_{nk} (1 + \sum_{i \in \mathcal{K}} x_{ni}), \forall n, \forall k,$$

(3)

where $x_{nk} = (\rho_n p_n + p_n^0) / \omega_k$.

Similar to the operation in [8], the problem (3) can be transformed into a tractable form according to the following theorem.

**Theorem 1:** If $(\bar{X}, \bar{\omega})$ is the solution of (3), then there exist $\lambda$, such that $X$ satisfies the Karush-Kuhn-Tucker (KKT) conditions of the following problem for $\lambda = \bar{\lambda}$ and $\omega = \bar{\omega}$,

$$\max_{X} G(X) = \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}_n} (a_{nk} - x_{nk} \sum_{i \in \mathcal{K}} \lambda_{ni} a_{ni})$$

s.t. $C_1, C_2, C_3$.

(4)

In addition, $\bar{X}$ also satisfies the following system equations for $\lambda = \bar{\lambda}$ and $\omega = \bar{\omega}$.

$$\lambda_{nk} = x_{nk} / \left[ a_{nk} \left( 1 + \sum_{i \in \mathcal{K}} x_{ni} \right) \right], \forall n, \forall k,$$

(5)

$$a_{nk} = r_n^k / \left[ a_{nk} \left( 1 + \sum_{i \in \mathcal{K}} x_{ni} \right) \right], \forall n, \forall k.$$  

(6)

On the contrary, if $\bar{X}$ is a solution of (4) and satisfies the equations (5) and (6) for $\lambda = \bar{\lambda}$ and $\omega = \bar{\omega}$, $(\bar{X}, \bar{\omega})$ also satisfies the KKT conditions of (3).

**Proof:** By introducing Lagrange multipliers $\lambda = \{\lambda_{nk}, \forall n, \forall k\}$ for the constraint $C_4$ of (3), we can achieve a Lagrange function with respect to $C_4$, i.e.,

$$\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}_n} \left( x_{nk} \omega_{nk} + \lambda_{nk} \left[ r_n^k - x_{nk} \omega_{nk} \left( 1 + \sum_{i \in \mathcal{K}} x_{ni} \right) \right] \right)$$

$$L = (X, \omega, \lambda).$$

Since $(\bar{X}, \bar{\omega})$ is the solution of (3), there exist $\bar{\lambda}$ such that partial KKT conditions of (3) can be written as

$$\frac{\partial L}{\partial \omega_{nk}} = \bar{\lambda}_{nk} - a_{nk} \bar{\lambda}_{nk} \left( 1 + \sum_{i \in \mathcal{K}} x_{ni} \right) = 0,$$

(8)

$$\frac{\partial L}{\partial \lambda_{nk}} = \bar{\lambda}_{nk} \left[ r_n^k - a_{nk} \omega_{nk} \left( 1 + \sum_{i \in \mathcal{K}} x_{ni} \right) \right] = 0.$$

(9)

Evidently, the condition $\sum_{i \in \mathcal{K}} x_{ni} + 1 > 0$ is met for any $n$. Combining with (8) and (9), we can easily obtain

$$\bar{\lambda}_{nk} = x_{nk} / \left[ a_{nk} \left( 1 + \sum_{i \in \mathcal{K}} x_{ni} \right) \right], \forall n, \forall k,$$

(10)

$$\bar{\omega}_{nk} = r_n^k / \left[ a_{nk} \left( 1 + \sum_{i \in \mathcal{K}} x_{ni} \right) \right], \forall n, \forall k.$$  

(11)
It is easy to know that the (8) and (9) are also the KKT conditions of the following problem for \( \lambda = \bar{\lambda} \) and \( \omega = \bar{\omega} \).

\[
\max_{\mathbf{X}} H (\mathbf{X}) = \mathcal{L} (\mathbf{X}, \lambda), \text{ s.t. } C_1, C_2, C_3. \tag{12}
\]

Evidently, the problem (12) can be easily simplified into (4). Thus, the first conclusion of Theorem 1 holds. Similarly, the contrary conclusion can also be easily proved. \( \Box \)

Theorem 1 shows that the feasible solution of (3) can be searched using subgradient method \([9]\), i.e.,

\[
\text{maximize its association objective. Next, the solution of dual problem can be searched using subgradient method \([9]\), i.e.,} \\
 \max_{\mu} \mathcal{L} (\mathbf{X}, \mu, \omega, \lambda), \text{ s.t. } C_1, C_2, C_3, \tag{14}
\]

and the dual problem of (4) is \( \min_{\mu \geq 0} \mathcal{L} (\mu) \), where \( a \geq b \) if \( \alpha_{nk} \geq \beta_{nk} \) for any \( n \) and \( k \).

When the dual optimal solution \( \mu^* \) is given, the problem (14) can be easily solved by obeying the following rule:

\[
n^* = \arg \max_{n \in \mathcal{X}} \left\{ \alpha_{nk} + \mu_k \text{SINR}_{nk} - \alpha_{nk} \sum_{i \in \mathcal{X}} \lambda_{ni} \omega_{ni} \right\}. \tag{15}
\]

The rule (15) shows that any user \( k \) selects some BS \( n^* \) to maximize its association objective. Next, the solution of dual problem can be searched using subgradient method \([9]\), i.e.,

\[
\mu_k^{t+1} = \left[ \mu_k^t - \xi \left( \sum_{n \in \mathcal{X}} x_{nk}^t \text{SINR}_{nk} - t_k \right) \right]^+, \quad \forall k, \tag{16}
\]

where \( [z]^+ = \max \{z, 0\} ; \xi \) represents a small enough stepsize.

Based on the mentioned-above analyses, we can easily give a two-layer iterative algorithm (i.e., Algorithm 1) to solve the problem (3). In this algorithm (Association with Maximizing Weighted Sum Energy Efficiency, AMWSEE), the and are updated using Newton-like method in the outer layer and is decided by employing the rule (15) in the inner layer. In addition, we also give the definitions of some functions as follows:

\[
\phi_{nk} (\lambda_{nk}) = \alpha_{nk} \lambda_{nk} \left( 1 + \sum_{i \in \mathcal{X}} x_{ni} \right) - x_{nk}, \quad \forall n, \forall k, \tag{17}
\]

\[
\varphi_{nk} (\omega_{nk}) = \alpha_{nk} \omega_{nk} \left( 1 + \sum_{i \in \mathcal{X}} x_{ni} \right) - r_{nk}, \quad \forall n, \forall k, \tag{18}
\]

\[
\chi_{nk} = \frac{1}{\alpha_{nk} \left( 1 + \sum_{i \in \mathcal{X}} x_{ni} \right)}, \quad \forall n, \forall k. \tag{19}
\]

In Algorithm 1, \( t_1 \) and \( t_2 \) are the iteration indices of outer and inner loops respectively, \( T_1 \) and \( T_2 \) represent the maximal numbers of iterations of outer and inner loops respectively, and the steps 6 and 11 normalize the multipliers to ensure that the Lagrange functions (7) and (13) are bounded respectively.

**Lemma 1:** Inner and outer loops of Algorithm 1 are convergent.

### Algorithm 1 AMWSEE

1. **Initialization:** Let \( t_1 = 1, t_2 = 1, \zeta \in (0, 1), \epsilon \in (0, 1); \) initialize \( \mathbf{X}^0, \chi^1, \mu^2 \) and \( \omega^1. \)
2. **Repeat (Outer Loop)**
   3. **Repeat (Inner Loop)**
      4. Any user selects some BS according to the rule (15).
      5. Update multiplier \( \mu_k^{t_2+1} \) using (16) for any \( k. \)
      6. \( \mu_k^{t_2+1} = \mu_k^{t_2+1} \sum_{i \in \mathcal{X}} \mu_k^{t_2+1}; \quad t_2 = t_2 + 1. \)
      7. Until \( \mathcal{G} (\mathbf{X}) \) converges or \( t_2 = T_2. \)
      8. If the following conditions are satisfied, then stop the algorithm.
         Otherwise, go to step 9.
      9. Find the smallest \( m \) among \( m \in \{0, 1, 2, \ldots \} \) satisfying
         \[
         \sum_{n \in \mathcal{X}} \sum_{k \in \mathcal{K}} \left| \phi_{nk} (\lambda_{nk}^t - \zeta m \chi_{nk} \varphi_{nk} (\lambda_{nk}^t)) \right|^2 + \sum_{n \in \mathcal{X}} \sum_{k \in \mathcal{K}} \left| \phi_{nk} (\omega_{nk}^t - \zeta m \chi_{nk} \varphi_{nk} (\omega_{nk}^t)) \right|^2 \leq (1 - \epsilon \zeta_m) \sum_{n \in \mathcal{X}} \sum_{k \in \mathcal{K}} \left[ \left| \phi_{nk} (\lambda_{nk}^t)^2 + \left| \phi_{nk} (\omega_{nk}^t)^2 \right| \right| \right],
         \]
   10. Update \( \lambda \) and \( \omega \) using Newton-like method:
      \[
      \lambda_{nk}^{t+1} = \lambda_{nk}^t - \zeta m \chi_{nk} \varphi_{nk} (\lambda_{nk}^t), \quad \forall n, \forall k,
      \]
      \[
      \omega_{nk}^{t+1} = \omega_{nk}^t - \zeta m \chi_{nk} \varphi_{nk} (\omega_{nk}^t), \quad \forall n, \forall k,
      \]
      \[
      \chi_{nk}^{t+1} = \chi_{nk}^t + \sum_{n \in \mathcal{X}} \sum_{k \in \mathcal{X}} \lambda_{nk}^t \chi_{nk}^t; \quad t_1 = t_1 + 1.
      \]
      11. Until \( F (\mathbf{X}, \omega) \) converges or \( t_1 = T_1. \)

**Proof:** In the outer loop of Algorithm 1, \( \lambda \) and \( \omega \) are updated using Newton-like method with a linear convergence rate. When \( \zeta_m = 1 \), the update of \( \lambda \) and \( \omega \) in the step 10 reduces to Newton method with a quadratic convergence rate. In addition, the convergence of outer loop of Algorithm 1 can be also proven by using a similar method used in \([8]\). It is easy to find that the function (13) and its derivative should be bounded with respect to \( \mu \). According to the results of subgradient method \([2], [9]\), we can easily know that the inner loop will converge to its dual optimum. \( \Box \)

Next, we will give some complexity analyses for the proposed algorithm. Since \( \sum_{i \in \mathcal{X}} x_{ni} \) can be calculated before running the step 4, we can easily know that the inner loop has a computation complexity of \( O (T_2 NK) \). Similarly, the step 8 has a complexity of \( O (NK) \) since \( \sum_{i \in \mathcal{X}} x_{ni} \) can be achieved before performing the step 8. Considering that the step 9 needs to find the smallest \( m \), we can deduce that such a step should have a complexity of \( O (m + 1) NK \). As for other steps of outer loop, it is easy to find that they have a complexity of \( O (NK) \). Thus, the computation complexity of outer loop is \( O (m + 1) T_1 NK) \), where \( m \) often takes a relatively small integer number. In general, the computation complexity of Algorithm 1 is \( O (\max \{T_1 T_2 NK, (m + 1) T_1 NK\}). \)

### V. Numerical Simulation

We assume that the maximal transmit power of MBS and PBS are 46 dBm and 30 dBm respectively, the circuit power of PBS is 13.6 W, the power amplifier coefficients of MBS.
and PBS are 4 and 2 respectively, the noise power spectral density is -174 dBm/Hz, the system bandwidth is 10 MHz, \( A_{00} = 4, \ A_{10} = 4.8, \ A_{20} = 0, \ A_{30} = 2.08 \times 10^{-8}, \ A_{01} = 1, \ A_{11} = 9.5 \times 10^{-8} \) and \( A_{21} = 6.25 \times 10^{-8} \) [2]. In HCNs, the pathloss models are \( l_{nk} = 128.1 + 37.6 \log 10 (d_{nk}) \) and \( l_{nk} = 140.7 + 36.7 \log 10 (d_{nk}) \) for MBS and PBS respectively, where \( d_{nk} \) is the distance (in km) between user \( k \) and BS \( n \). Meanwhile, we also consider a log-normal shadowing with a standard deviation of 8 dB.

Next, we will investigate the performance of proposed association (AMWSEE), and meanwhile introduce another energy-efficient association (Association with Maximizing Sum Utility, AMSU [2]) for comparison.

Fig. 1(a) and Fig. 1(b) show the impacts of maximal transmit power of each PBS (MTPP) on the average rates (ARs) and the average EEIs (AEEIs) under different numbers of MBS antennas (NMAs) for various associations respectively, where AR and AEE represent the average of effective (long-term) rates and the one of EEIs for all associated users respectively. Although the increment of transmit power of PBS can increase the effective rates of pico users (users associated with PBSs), it can also decrease the effective rates of macro users (users associated with MBSs) due to the increased interference. Since the number of pico users is often far smaller than the one of macro users, the AR may decrease with MTPP in Fig. 1(a).

In view of the fact that the signal strength of MBSs increases with NMA and MBSs have more served users than PBSs, the AR may increase with NMA in Fig. 1(a).

In Fig. 1(b), the AEE initially increases with MTPP, but then it may decrease with this parameter when the transmit power of PBSs is high enough. As we know, the increased transmit power can improve the experience of pico users by enhancing the transmitted signal strength in the low power domain, but it then degrades the experience because of stronger and stronger intra-tier interference in the high power domain. Since the MBSs often have a far higher transmit power than PBSs and the transmit power of the former is often far larger than the effective rates of users associated with it, the change of effective rates of macro users may not have an evident influence on the EEs of macro users. Thus, the change of transmit power of PBSs mainly affect the EEs of pico users. Evidently, the AEE may initially increase with MTPP due to the increased signal strength, but then it may decrease with this power due to the increased interference and transmit power. As we know, the power consumption scales with NMA, and thus the AEE may decrease with NMA.

In addition, the strategy AMWSEE has a higher AR and AEE than strategy AMSU. Although the users in the former can set their priorities to guarantee the EE fairness, we take \( w_k = 1 \) without loss of generality for any user \( k \). To guarantee the user fairness, the latter may have to degrade its AR and AEE.

Fig. 2(a) and Fig. 2(b) show the convergence of outer and inner loops of Algorithm 1 respectively, where \( t \) is the iteration index. As shown in Fig. 2, these two loops of Algorithm 1 can converge in a fairly high speed. It means that the Algorithm 1 has a relatively high convergence rate and thus can be well implemented in the real system.

### VI. Conclusion

In this letter, we design an energy-efficient association scheme from a novel perspective, and formulate it as a weighted sum EE maximization problem under users’ SINR constraints. To solve this problem, we develop a two-layer iterative algorithm with guaranteed convergence, and then give some convergence and complexity analyses for it. Numerical results show that the proposed scheme has a higher system throughput and network EE than the existing one under a certain condition. In addition, we also investigate the impacts of NMA and MTPP on the association performance.

### References

1. A. Ghosh et al., “Heterogeneous cellular networks: From theory to practice,” IEEE Commun. Mag., vol. 50, no. 6, pp. 54–64, Jun. 2012.
2. D. Liu, L. Wang, Y. Chen, T. Zhang, K. Chai, and M. Elkashlan, “Distributed energy efficient fair user association in massive MIMO enabled HetNets,” IEEE Commun. Lett., vol. 19, no. 10, pp. 1770–1773, Oct. 2015.
3. Q. Ye, B. Rong, Y. Chen, M. Al-Shalash, C. Caramanis, and J. G. Andrews, “User association for load balancing in heterogeneous cellular networks,” IEEE Trans. Wireless Commun., vol. 12, no. 6, pp. 2706–2716, Jun. 2013.
4. K. Shen and W. Yu, “Distributed pricing-based user association for downlink heterogeneous cellular networks,” IEEE J. Sel. Areas Commun., vol. 32, no. 6, pp. 1100–1113, Jun. 2014.
5. Y. Xu and S. Mao, “User association in massive MIMO HetNets,” IEEE Syst. J., vol. 11, no. 1, pp. 7–19, Mar. 2017.
6. D. Bethanabhotla, O. Y. Bursalioglu, H. C. Papadopoulos, and G. Caire, “Optimal user-cell association for massive MIMO wireless networks,” IEEE Trans. Wireless Commun., vol. 15, no. 3, pp. 1835–1850, Mar. 2016.
7. Y. Hao, Q. Ni, H. Li, and S. Hou, “Energy and spectral efficiency tradeoff with user association and power coordination in massive MIMO enabled HetNets,” IEEE Commun. Lett., vol. 20, no. 10, pp. 2091–2094, Oct. 2016.
8. Y.-C. Jong, (May 2012). An Efficient Global Optimization Algorithm for Nonlinear Sum-of-Ratios Problem. [Online]. Available: http://www.optimizationonline.org
9. S. Boyd, L. Xiao, and A. Mutapcic. (Oct. 2003). Subgradient Methods. [Online]. Available: http://web.mit.edu/6.976/www/notes/subgrad_method.pdf