Polarized GPDs in pions and kaons electroproduction. Tranversity effects.

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Abstract. We analyze the electroproduction of pseudoscalar mesons within the handbag approach. To investigate these reactions, we consider the leading-twist contribution together with the transversity twist-3 effects that are crucial in the description of experimental data. Our results on the cross section are in agreement with experiment. We present our predictions for spin observables.

1 Introduction

We investigate the process of pseudoscalar meson leptonproduction (PML) at large $Q^2$ within the handbag approach, where the amplitudes factorize into a hard subprocess and soft part – Generalized Parton Distributions GPDs \cite{1}. The hard subprocess amplitudes are calculated by using the modified perturbative approach \cite{2} that takes into account quark transverse degrees of freedom as well as gluonic radiation condensed in a Sudakov factor.

The PML was analyzed in \cite{3,4}. It was shown that the leading-twist contribution determined by the polarized GPDs is not sufficient to describe processes of PML. The essential contributions from the transversity GPDs are needed to be consistent with experiment. Within the handbag approach, these twist-3 effects can be modeled by the transversity GPDs $H_T$, $E_T$, in conjunction with the twist-3 meson wave function.

In this report we study the cross sections of the pion leptonproduction in the HERMES and CLAS energy range on the basis of the model \cite{3,4}. Our results are in good agreement with experiment. We show that the transversity GPDs lead to a large transverse cross section for most reactions of the pseudoscalar meson production. Predictions for spin asymmetries in the pion leptonproduction are presented as well.

At the end, we present the model results for the cross section of the $K^+\Lambda$ leptonproduction, which is large due to the transversity contribution \cite{4}, and predictions for the spin asymmetry in this reaction.

2 Leptonproduction of pseudoscalar mesons

Hard exclusive PML amplitudes were studied on the basis of the handbag approach. The typical contributions are shown in Fig.1. In the left part of the graph we present

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the meson pole contribution which appears for the charge meson production. In Fig. 1 (right) the example of the handbag diagram is shown. In the leading twist the last contribution is expressed in terms of the polarized GPDs $\tilde{H}$ and $\tilde{E}$ whose parameterization can be found in [5].

Fig. 1. Examples of the graphs essential in PML. Left–pion pole and right–handbag contributions to the $\pi^+$ production.

The proton non- flip and helicity-flip amplitudes for longitudinally polarized photons $M^{M,0}_{+,0+}$, which dominates at large $Q^2$, can be written in the form:

$$M^{M,0}_{+,0+} \propto \sqrt{1 - \xi^2} \left[ -\frac{\xi(m_{N^i} + M_{N^f})Q^2}{1 - \xi^2} \frac{\rho_M}{t - m_M^2} + \langle \tilde{H}_M \rangle - \frac{\xi^2}{1 - \xi^2} \langle \tilde{E}_{n.p.}^M \rangle \right];$$

$$M^{M,0}_{-,0-} \propto \sqrt{-t} \left( (m_{N^i} + M_{N^f})Q^2 \frac{\rho_M}{t - m_M^2} + \xi \langle \tilde{E}_{n.p.}^M \rangle \right).$$

(1)

Here $M$- produced pseudoscalar meson, $N^i$-initial nucleon (proton), $N^f$-final barion (neutron, $\Lambda$, $\Sigma$). The corresponding amplitudes with transversally polarized photons are suppressed as $1/Q$.

The first terms in (1) appear for the charged meson production and are connected with the $M$ meson pole. The fully experimentally measured electromagnetic form factor of $M$ meson is included into $\rho_M$.

The second terms in (1) represent the handbag contribution to the PML amplitude. The $\langle \tilde{F} \rangle$ in (1) is a convolution of GPD $\tilde{F}$ with the hard subprocess amplitude $\mathcal{H}_{0\lambda,0\lambda(x,\xi,\tau, Q^2)}$:

$$\langle \tilde{F} \rangle = \sum_\lambda \int_{-1}^1 d\tau d^2b \mathcal{H}^{0\lambda,0\lambda}_{0\lambda}(x, \xi, \tau, Q^2, b) \exp[-S(\tau, b, Q^2)].$$

(2)

Here $\tau$ is the momentum fraction of the quark that enters into the meson.
quark contribution it looks like

\[ f_i(\beta, \alpha, t) = h_i(\beta, t) \frac{3}{4} \frac{(1 - |\beta|)^2 - \alpha^2}{(1 - |\beta|)^3}. \]  

(3)

The functions \( h \) are determined in the terms of PDFs and are parameterized in the form

\[ h(\beta, t) = N e^{\text{out} \beta - \alpha(t)} (1 - \beta)^n. \]  

(4)

Here the \( t \)-dependence is considered in a Regge form and \( \alpha(t) \) is the corresponding Regge trajectory. The parameters in (4) are obtained from the known information about PDFs \( [8] \) e.g., or from the nucleon form factor analysis \( [9] \).

We calculate the leading-twist amplitudes together with the meson pole contribution on the basis of (1). Unfortunately, these terms are insufficient to describe experimental data at low \( Q^2 \). We can demonstrate this using the \( A_{UT} \) asymmetry in the \( \pi^+ \) lepton production as an example.

![Fig. 2. \( A_{UT}^{\sin(\phi_s)} \) asymmetry of the \( \pi^+ \) production. Dashed line- leading twist contribution. Solid line- model results, including twist-3 effects. Data are from HERMES \( [10] \).](image)

This asymmetry is expressed in terms of interference of \( M_{0-;++} \) and proton non-flip amplitude \( [3] \):

\[ A_{UT}^{\sin(\phi_s)} \propto \text{Im}[M_{0-;++}^* M_{0+;++}]. \]  

(5)

The leading twist contributions cannot explain this asymmetry –see Fig. 2. A new twist-3 contribution to the \( M_{0-;++} \) amplitude, which is not small at \( t' \sim 0 \), is needed. We estimate this contribution to \( M_{0-;++} \) by the transversity GPD \( H_T \) in conjunction with the twist-3 pion wave function in the hard subprocess amplitude \( H \) \( [4] \). We have

\[ M_{0-;++}^{M,\text{twist-3}} \propto \int_{-1}^{1} d\vec{x} H_{0-;++}(\vec{x}, ...) \left[ H_T^M + ...O(\xi^2 E_T^M) \right]. \]  

(6)

The \( H_T \) GPD is connected with transversity PDFs as

\[ H_T^a(x, 0, 0) = \delta^a(x); \quad \text{and} \quad \delta^a(x) = C N_T^a x^{1/2} (1 - x) [q_a(x) + \Delta q_a(x)]. \]  

(7)

Here \( a \) is a quark flavor. We parameterize the PDF \( \delta \) by using the model \( [11] \). The double distribution \( [5] \) is used to calculate GPD \( H_T \).
The amplitude $M_{0^{++},++}$ is extremely important in analyzes of PML as well. The transversity twist-3 contribution to this amplitude is determined by $\bar{E}_T$ GPDs and has the form \[4\] similar to \[6\] 

$$M_{0^{++},++} \propto \sqrt{-t} \int_{-1}^{1} d\tau \mathcal{H}_{0^{-,-}\tau} \bar{E}_T^M.$$  

(8)

The hard scattering subprocess amplitude $\mathcal{H}_{0^{-,-}\tau}$ in \[8\] is the same as in \[6\].

At the moment, the information on $\bar{E}_T$ is very poor. Some results were obtained only in the lattice QCD \[12\]. The lower moments of $\bar{E}_u$ and $\bar{E}_d$ were found to be quite large, have the same sign and a similar size. At the same time, $H_u$ and $H_d$ are different in sign. For the pion production we have the following contribution to GPDs \[13\] 

$$F(\pi^+) = F^{(3)} = F^u - F^d,$$

$$F(\pi^0) = 2/3 F^u + 1/3 F^d.$$  

(9)

From these equations we find an essential compensation of the $\bar{E}_T$ contribution to the $\pi^+$ amplitude but $H_T$ effects are not small there. For the $\pi^0$ production we have the opposite case – $\bar{E}_T$ contributions are large but $H_T$ effects are smaller.

In Fig. 3 (left), we show our results \[3\] for the unseparated cross section of the $\pi^+$ production which describes fine HERMES data \[14\]. The $\sigma_L$ and $\sigma_T$ are shown as well. The longitudinal cross section determined by leading-twist contribution dominates at small momentum transfer $-t < 0.2 \text{GeV}^2$. At larger $-t$ we find a not small transverse cross section where the $H_T$ contribution is visible. In Fig. 3 (right), our results for the cross section of the $\pi^0$ production are presented which are very different from the $\pi^+$ process. The transverse cross section

$$\sigma_T \propto |M_{0^{++},++}^{M,\text{twist} - 3}|^2 + |M_{0^{-,-}++}^{M,\text{twist} - 3}|^2,$$  

(10)

where the $\bar{E}_T$ and $H_T$ contributions are important \[3\] dominates. At small momentum transfer the $H_T$ contribution is visible and provides a nonzero cross section. At larger
−t′ ∼ 0.2GeV² the $E_T$ contribution is essential and gives a maximum in the cross section.

The longitudinal cross section which is expected to play an important role is much smaller with respect to the transverse cross section $\sigma_T$. The essential contributions to the $\sigma_T$ cross section are determined by the twist-3 $H_T$ and $E_T$ effects and decreases quickly with $Q^2$. At quite large $Q^2$ the leading-twist effects will dominate.

![Fig. 4. Left: $A_{UT}$ asymmetry of the $\pi^+$ production at HERMES energies. Right: $A_{UL}$ asymmetry of the $\pi^+$ production at HERMES. HERMES data are shown [10]. Dashed line-results without transversity $H_T$ effects.](image)

In Fig. 4, we demonstrate that the transversity $H_T$ effects are essential in asymmetries of the $\pi^+$ production. When we omit the $H_T$ contributions, asymmetries change drastically.

In Fig 5 (left), our prediction for the $\pi^0$ production in the CLAS energy range [15] is shown together with experimental data [16]. Our results are close to the experimental data and definitely show the same dip in the unseparated cross section at low momentum transfer, as was observed for HERMES—see Fig.3 (right). We present in this plot the interference $\sigma_{LT}$ and $\sigma_{TT}$ cross sections too. The value of $\sigma_{LT}$ is quite small, compatible with zero. The $\sigma_{TT}$ cross section is negative and large. Note that the $E_T$ contribution to $\sigma_T$ and $\sigma_{TT}$ cross sections is strongly correlated. The fact that we describe the CLAS data for both cross sections quite well can be an indication of observation of large transversity effects at CLAS. However, the definite conclusion on the importance of transversity effects in the $\pi^0$ cross section can be made only if the data on the separated $\sigma_L$ and $\sigma_T$ cross section will be available experimentally and $\sigma_T$ will be much larger than $\sigma_L$. Probably, such a study can be performed at JLAB12.

In Fig. 5 (right), we analyze the transversity effects in the ratio of the $\eta/\pi^0$ cross section at CLAS energies. The two parameterizations of $H_T$-GPDs [4] are presented there. Different combinations of the quark contributions to these processes lead to the essential role of $H_T$ effects in this ratio at small $-t < 0.2$GeV². At larger momentum transfer large $E_T$ effects in the $\pi^0$ production found in the model lead to a rapid decrease of the $\eta/\pi^0$ cross section ratio with $t$-growing. At $-t > 0.2$GeV² this ratio becomes close to $\sim 0.3$, which was confirmed by CLAS [17].

In Fig.6 (left), we present our results for the moments of the $A_{UT}$ asymmetry in the $\pi^0$ production at HERMES. The predicted asymmetries are large and can give additional information on transversity effects in this reaction. In Fig.6 (right), we show the $A_{LU}$ asymmetry in the pion production at HERMES. $A_{LU}(\pi^+)$ is large because of the pion-pole contribution in this channel. The predicted $A_{LU}$ asymmetry
in the π^0 production is small. Measurement of this asymmetry at HERMES can give information on the nonpole term of £^{M}_{n.p.} in (1).

Using the same model we calculate the cross section and spin asymmetry for the K^+Λ production. To estimate proton- hyperon transition GPDs we use the SU(3) flavor symmetry model [13]

\[ H_T(p \rightarrow A) \sim [2H_T^u - H_T^d - H_T^s]. \]  

Due to different signs of \( H_T^u \) and \( H_T^d \) we find a quite large \( H_T \) effect here. In this reaction, the kaon pole contribution should be much smaller with respect to the π^+ case. The details of calculations can be found in [4]. The large transversity \( H_T \) effects in the \( K^+A \) channel provide the large σ_T cross section without a forward dip which dominated with respect to \( σ_L \), see Fig. 7 (left). In Fig. 7 (right), we show our predictions for moments of the \( A_{UT} \) asymmetry in this channel. The sin(φ_s) moment of asymmetry determined by the \( H_T \) transversity contribution is quite large.

**Fig. 5.** Left: π^0 production in the CLAS energy range together with the data. Dashed-dotted line- \( σ_T + εσ_L \), dashed line-\( σ_{LT} \), dashed-dotted- \( σ_{T^T} \). Right: \( η/π^0 \) production ratio in the CLAS energy range together with preliminary data.

**Fig. 6.** Left: Moments of the \( A_{UT} \) asymmetries at HERMES for the π^0 production. Right: The predicted \( A_{LU} \) asymmetry in the π^+ and π^0 production at HERMES.
Fig. 7. Left: The $K^+\Lambda$ production cross sections at HERMES energies. Right: Predicted moments of $A_{UT}$ asymmetries for $K^+\Lambda$ channel at HERMES.

3 Conclusion

We calculate the PML amplitude within the handbag approach, in which the amplitudes factorize into hard subprocesses and GPDs [1]. The hard subprocess amplitudes were calculated within the modified perturbative approach [2] where quark transverse degrees of freedom and the gluonic radiation, condensed in a Sudakov factor were taken into account.

At leading-twist accuracy the PML reactions are sensitive to the GPDs $\tilde{H}$ and $\tilde{E}$ which contribute to the amplitudes for longitudinally polarized virtual photons. This contribution should be predominated at large $Q^2$. Unfortunately, now experimental data on these reactions are available at small photon virtualities.

We observed that the experimental data on pseudoscalar meson leptoproduction at low $Q^2$ also require contributions from the transversity GPDs, in particular, from $H_T$ and $E_T$. Within the handbag approach the transversity GPDs are accompanied by a twist-3 meson wave function. At HERMES and COMPASS energies the twist-3 $E_T$ effects produce a large transverse cross section $\sigma_T$ [4] which exceeds substantially the leading twist longitudinal cross section for most reactions with the exception of the $\pi^+$ and $\eta'$ channels.

The indication of large transversity effects are available now at CLASS. They observe a large unseparated and large negative $\sigma_{T\pi^0}$ cross section which can be described in our model by large transversity $E_T$ effects. Essential $H_T$ and $E_T$ effects are predicted at the ratio $\eta/\pi^0$ cross section. Large $E_T$ effects in the $\pi^0$ production predict that for $-t > 0.2\text{GeV}^2$ this ratio should be close to $\sim 0.3$, which was confirmed by CLAS [17].

Nevertheless the experimental separation of the $\sigma_L$ and $\sigma_T$ cross section in the $\pi^0$ electroproduction is important. If it is found that $\sigma_T$ is much larger than the $\sigma_L$ cross section, this will be a definite demonstration of observation of transversity effects in this reaction. We hope that it can be done at JLAB12. Essential $H_T$ effects in the $K^+\Lambda$ channel were predicted.

We describe well the cross section and spin observables for various PML. Thus, we can conclude that the information on GPDs discussed above should be not far from reality. Future experimental results at COMPASS, JLAB12 can give important information on the role of transversity effects in these reactions.
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