Microwave measurements of the high magnetic field vortex motion pinning parameters in Nb$_3$Sn

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Abstract

The high frequency vortex motion in Nb$_3$Sn was analyzed in this work up to 12 T. We used a dielectric loaded resonator tuned at 15 GHz to evaluate the surface impedance $Z$ of a Nb$_3$Sn bulk sample (24.8 at.% Sn). From the field induced variation of $Z$, the high frequency vortex parameters (the pinning constant $k_p$, the depinning frequency $\nu_p$ and the flux flow resistivity $\rho_{ff}$) were obtained over a large temperature and field range; their field and temperature dependencies were analyzed. Comparison with other superconducting materials shows that high frequency applications in strong magnetic fields are also feasible with Nb$_3$Sn. In the present work, we report the first measurements about the microwave response in Nb$_3$Sn in strong magnetic fields.

Keywords: Nb$_3$Sn, surface impedance, microwave, high magnetic fields, vortex pinning parameters

(Some figures may appear in colour only in the online journal)

1. Introduction

Among superconducting (SC) materials, Nb$_3$Sn is currently one of the most used in technological applications due to its interesting superconductive and mechanical properties. Despite being a well-known material, new perspective applications of Nb$_3$Sn, such as superconductive radio frequency cavities (SRFC) [1, 2] also for magnetic environments [3–6] and improved magnets for new particles accelerators (e.g. the High Luminosity upgrade of LHC, or the Future Circular Collider FCC [7, 8]), are revamping the interest in Nb$_3$Sn characterization [9–11]. In fact, it is necessary to test this SC in the new challenging working conditions of these applications, to better understand how to improve its performances.

In particular, the interest in Nb$_3$Sn is increasing, the goal being to improve the high frequency performances in view of its use in SRFC particle accelerators. At present, the most used material for this application field is elementary Nb. However, the need to improve the performances of RF cavities and to achieve higher accelerating fields motivated the search for a new material. Nb$_3$Sn is a good candidate for this application but there is still a need to study why the predicted performances (i.e. superheating field) are still far from those experimentally obtained [12–16]. Local geometrical surface defects are often identified as being responsible for these low performances. A recent theoretical study has identified in the broadening of the density of states, in regions with higher pair-breaking scattering rates, a source of local heating and thus of decrease of the superheating field [17].

As reported above, different kinds of RF-cavities are expected to work in presence of moderate to high static magnetic fields. It its well known that superconductors, at high frequencies and in the presence of magnetic fields, can exhibit surface resistances comparable to those of normal conductors. In fact, under these conditions the main dissipative phenomenon is related to the vortex oscillations induced
by the impinging electromagnetic (e.m.) wave. For these applications, materials are searched with properties being quite different from those needed for the realization of standard SRFC cavities. Indeed, cavities optimized for zero static magnetic field require a pure superconductor with ideally no pinning centers to completely remove the trapped field after cooldown. However, in finite static magnetic fields, strong pinning is needed to avoid large oscillations of the fluxons. In particular, above the so called depinning frequency \( \nu_p \) the vortices move in the highly dissipative flux-flow regime [18–20]. Hence, the measurement of \( \nu_p \) in high magnetic fields is a discriminating parameter for the application of SC materials in experiments in dc magnetic fields.

In many power applications of Nb\(_3\)Sn, the knowledge of vortex pinning is essential. Even if microwave (\( \mu \omega \)) measurements do not directly yield design parameters for dc applications, they provide useful information about the pinning characteristics, in addition to those obtained by the dc characterization techniques. A better comprehension of the pinning phenomenon is only obtained by merging the different information given by different dynamical regimes [21] and \( \mu \omega \) can help to unveil new vortex pinning regimes [22].

Many aspects of the physics of Nb\(_3\)Sn have been already studied. For what concerns the high frequency regime, Nb\(_3\)Sn surface impedance \( Z \) measurements were performed in the 1–10 GHz range and allowed to observe deviations of the measured \( Z \) from the BCS theory and a particularly large gap \( \Delta_0/k_B T = 2.15 \) (being \( k_B \) and \( T \) the Boltzman constant and the temperature, respectively) [23–25]. The higher frequency behavior (at 87 GHz) was explored in [26] confirming the large superconductive gap in Nb\(_3\)Sn, \( 1.8 < \Delta_0/k_B T < 2.2 \). Despite the large \( \Delta_0 \), caused by a strong electron-phonon coupling in Nb\(_3\)Sn, a typical BCS signature on the conductivity temperature dependence (e.g. a large coherence peak in the real part [27]) was observed at 87 GHz [26, 28]. Since the experimentally determined penetration depth \( \lambda \) was shown to be close to the expected BCS value \( \lambda_{BCS} \) [29], the latter is often used when analyzing the experimental Nb\(_3\)Sn data [29]. The first \( Z \) measurements in Nb\(_3\)Sn at low magnetic fields (not larger than 12 mT) and in the non-linear region were presented in [30].

As it can be seen from the present description of the high frequency behavior of Nb\(_3\)Sn, no studies exist on the high frequency vortex motion regime in high static magnetic field. We present in this work a complete microwave (\( \sim 15 \) GHz) characterization of Nb\(_3\)Sn up to 12 T (with preliminary results reported in [31]) to provide new useful information about high frequency vortex motion physics in this SC. Thus, this work fills the gap of knowledge in the high frequency behavior of Nb\(_3\)Sn in high magnetic fields. In particular, the surface impedance \( Z(T,H) \) of a bulk Nb\(_3\)Sn polycrystalline sample is here measured with a dielectric loaded resonator (DR) [32] in zero field cooling (ZFC) condition at fixed temperature \( T \), and in field cooling (FC) condition at fixed applied magnetic field \( \mu_0 H \) values up to 12 T. Then, with a classical electrodynamics approach the complex resistivity \( \tilde{\rho}(T,H) \) is obtained and analyzed with the Coffey–Clem model [33] in order to obtain the complex vortex motion resistivity \( \tilde{\rho}_{vm}(T,H) \) of Nb\(_3\)Sn. Assuming negligible thermal phenomena, \( \tilde{\rho}_{vm} \) is only a function of the real flux flow resistivity \( \rho_{ff} \), the depinning frequency \( \nu_p \) and the measurement frequency \( \nu_0 \) [18]. Thus, \( \rho_{ff} \) and \( \nu_p \) are obtained resorting to literature values of the London penetration depth, which is a well known quantity in Nb\(_3\)Sn [23, 34, 35]. The measured \( \nu_p \) of bulk Nb\(_3\)Sn is remarkably high when compared with \( \nu_p \) in thin Nb films.

The measured \( \rho_{ff} \) is shown to exhibit a conventional Bardeen–Stephen behavior [36]. The scaling of the \( \rho_{ff} \) with the applied magnetic field allowed us to evaluate the upper critical field \( H_{c2}(T) \) down to \( 4 \) K. The so obtained \( H_{c2}(T) \) is well fitted by the Maki-de Gennes approximation [37, 38], as expected from other works [39].

Following [40] we extended the analysis of the high frequency vortex pinning characteristics in Nb\(_3\)Sn considering the contribution of the thermal creep: based on analytical constraints of the used equations and physical limits, a statistical approach is used to assess probability intervals of the evaluated pinning parameters.

The paper is organized as follows: in section 2 the high frequency vortex motion is briefly described, in section 3 the measurement method is presented, then the sample characteristics are reported in section 4. Finally the results are presented in section 5 and in section 6 a comparison of the \( \mu \omega \) performances of Nb\(_3\)Sn with those of MgB\(_2\) and YBa\(_2\)Cu\(_3\)O\(_7\)-\( \delta \) is performed.

2. Surface impedance of superconductors in the mixed state

The surface impedance \( Z \) is the complex physical quantity commonly used to describe the electromagnetic (e.m.) response of good conductors [41]. It is defined as the ratio \( Z = E_i/H_i \) [42], where \( E_i \) and \( H_i \) are respectively the electric and magnetic fields components parallel to the surface of the conductor. \( Z \) contains interesting information about the dissipative and energy storing effects of the material under study. For bulk materials, in the local limit, \( Z = \sqrt{\omega \mu_0 \tilde{\rho}} \) [42], where \( \omega = 2\pi \nu \) is the angular frequency of the impinging e.m. wave, \( \mu_0 \) is the vacuum magnetic permeability and \( \tilde{\rho} \) is the complex resistivity of the material. Since in this work we deal with a Nb\(_3\)Sn superconductive bulk sample in high magnetic field, \( \tilde{\rho} \) contains both the super/normal fluid complex charge transport and the vortex flow characteristics as presented in [33]. The first contribution is modeled by the two-fluid conductivity \( \sigma_2 = \sigma_1 - i \sigma_2 \) and the second by the complex vortex motion resistivity \( \rho_{vm} \), thus \( \tilde{\rho} = f(\sigma_2, \rho_{vm}) \). Far enough from the superconductive transition, where \( \sigma_2 >> \sigma_1 \), the normal fluid screening effect is weak enough to be neglected and \( Z \) is written as:

\[
Z \simeq \sqrt{\frac{i \mu_0 \nu \lambda^2}{\sigma_2^2 + i \rho_{vm}}};
\]

where \( \sigma_2 = 1/\omega \mu_0 \lambda^2 \), with \( \lambda \) the London penetration depth. When no external magnetic field is applied \( \rho_{vm} = 0 \) and

\[
Z \simeq \sqrt{-i \mu_0 \nu / \sigma_2} = i \omega \mu_0 \lambda.
\]

With high frequencies (microwaves) excitation and low e.m. field amplitude the vortices start oscillating around their
equilibrium positions (the pinning centres) as damped harmonic oscillators and their dissipative and reactive response depends on the pinning potential characteristics. Within the harmonic oscillator formalism, we can describe the Lorentz force due to the interaction between the microwave induced currents $J_{\mu\nu}$ and the magnetic flux quanta $\Phi_0$ as the driving force, the pinning effect as a linear elastic force $F_p = -k_p x$ (small oscillation), the non-equilibrium conversion between quasi-particles and condensate during the fluxons motion as a dissipative viscous drag force $F_{\text{drag}} = -\eta v$ and finally the thermal creep as a stochastic thermal force $F_\phi$

Thus, assuming massless fluxons [43] the dynamic equation of motion becomes:

$$J_{\mu\nu} \times \Phi_0 + F_\phi = k_p x + \eta v,$$

with $k_p$ the pinning constant, $x$ the fluxon displacement, $\eta$ the viscous drag coefficient and $v$ the fluxon velocity. The Coffey-Clem (CC) vortex motion resistivity $\rho_{vm}$ is then obtained [33, 40]:

$$\rho_{vm} = \rho_f \frac{\epsilon + i\nu/\nu_c}{1 + i\nu/\nu_c},$$

where $\rho_f = \Phi_0 B/\eta$ is the flux-flow resistivity, $B$ the magnetic flux density. In the London limit $B \simeq \mu_0 H$, with $H$ the applied magnetic field strength. The thermal creep contribution is taken into account by the adimensional creep factor $0 \leq \epsilon \leq 1$ [33, 40]. $\nu_c$ is the characteristic frequency of the vortex motion, marking the crossover between an elastic vortex motion regime ($\nu \ll \nu_c$) and an highly dissipative regime ($\nu \gg \nu_c$). When $\epsilon \to 0$, no flux creep exists and $\nu_c \to \nu_p$ with $\nu_p$ the depinning frequency, defined as $\nu_p = k_p/(2\pi\eta)$. In the case of small oscillations here relevant, $k_p$ is the pinning linear elastic constant which for rigid fluxons is a measure of the pinning well steepness [40, 44]. The $\epsilon \to 0$ limit is known in literature as the Gittleman-Rosenblum (GR) model [18]:

$$\rho_{vm,GR} = \rho_f B \frac{1}{\eta} \frac{1}{1 - i\nu/\nu_c}.$$  

In the high creep limit $\epsilon \to 1$ the fluxons behave as free fluxons due to thermal jumps and a free-flux flow regime takes place. The $\epsilon(U_0)$ and $\nu_c(\nu_p, U_0)$ dependencies on the creep activation energy $U_0$ are determined by the pinning potential shape and thus on the particular model used to describe the pinning profile [33, 45].

Microwave measurements are particularly versatile since they allow to measure a obtain both of the microwave shape/steepness $k_p$ and of the free flux-flow resistivity $\rho_f$ which, particularly for bulk samples as in this case, would require high dc-current to be properly measured. In the following we describe how we obtain the vortex motion parameters in Nb$_3$Sn bulks.

3. Measurement system and method

In this section we briefly outline how $Z$ is obtained with our measuring system based on a dielectric loaded resonator (DR).
\[ \Delta Z(T', H) = G_c \frac{1}{Q(T', H)} - 2\nu_0(T', H) + \Delta bckg(T', H), \]

where \( G_c \approx 2700 \Omega \) is a geometrical factor evaluated with electromagnetic simulations and \( \Delta x(T', H) \) indicates a variation of \( x(T', H) \) parameter with respect to the reference value obtained with no applied magnetic field \( x(T', 0) \). \( \nu_0 \) is the reference resonance frequency at \( H = 0 \) T. Finally, \( bckg \) is a complex parameter which represents the response of the resonator itself. Since both the DR and the measurement system were carefully designed to operate in high magnetic fields, \( bckg \) is very weakly field dependent [32], with respect to the SC sample variation \( \Delta Z(T', H) \), thus we assume \( \Delta bckg(T', H) \sim 0 \).

Field cooling (FC) and zero field cooling (ZFC) measurements were performed and they are discussed in the next section. In FC condition the magnetic field was applied before cooling. After cooling down to \( \sim 6 \) K, the temperature was raised at a constant rate \( 0.1 \) K/min. In ZFC the sample was cooled without an externally applied magnetic field; when the target temperature was reached and stabilized within \( \pm 0.05 \) K, the magnetic field was swept at \( 0.3 \) T/min up to \( 12 \) T then down to -\( 12 \) T and back to \( 0 \) T and the reversible component isolated.

### 4. The sample

The NbSn sample platelet was obtained starting from a polycrystalline bulk piece sintered by Hot Isostatic Pressure (HIP) technique (2 kbar Argon pressure at 1250 °C for 24 h) at the University of Geneva [49]. After HIP, the NbSn bulk piece was cut into tiny platelets by means of spark erosion and each platelet was then polished with SiC grinding papers and submitted to ‘flash-anneal’ heat treatment (900 °C/10 min) for stress release.

Microstructural and magnetization analyses reveal an average grain size of \( \sim 20 \) \( \mu \)m, a composition very close to stoichiometry (24.8 at.%Sn) and a sharp superconducting transition at 17.9 K reflecting the high quality and homogeneity of these samples. Finally, from Rietveld refinement the lattice constant and the Bragg-Williams long-range order parameter have been estimated to be 5.291 Å and 0.98 respectively. Further details on the procedure and analysis can be found in [50]. A sample of approximate area of 30 mm\(^2\) was chosen for the present study.

### 5. Results

In this section we first show the \( Q \) and \( \nu_0 \) measurements to check the calibration process through the comparison of the obtained normal state sample characteristics with literature values. Then, we derive the vortex parameters under the common assumption of negligible thermal creep (i.e. Gittleman-Rosenblum (GR) model [18]): the use of the GR model is the standard analysis procedure [19, 20, 44, 51] so that it allows to easily compare the results on NbSn with other materials. Finally, in the last subsection, the contribution given by the flux motion is evaluated with a statistical analysis of the obtained data.

#### 5.1. Normal state

In figure 2 we show the measured variation \( \Delta 1/Q = Q(T, H)^{-1} - Q(T \to 0, H = 0)^{-1} \) and \( \Delta \nu_0/\nu_{ref} = (\nu_0(T, H) - \nu_{ref})/\nu_{ref} \) with \( \nu_{ref} = \nu_0(T = T_c) \) at 0 T, 2 T, 4 T, 8 T, 12 T in FC condition. Since below \( \sim 20 \) K the copper and sapphire losses do not depend on the temperature the height of the \( Q^{-1} \) transition (figure 2) can be assigned to \( \Delta R \) of NbSn. Thus, from equation (5) \( R_n = 94.6 \) mΩ with \( R_n \) the normal state surface resistance. From \( R_n \), the normal state resistivity is obtained from the normal skin effect as \( \rho_n = 2R_n^2/\omega \mu_0 \lambda = 14.8 \mu \Omega cm \).

An estimation of \( \rho_n \) based on the long range order parameter \( S \), yields \( \rho_n = 147(1-S^3) \mu \Omega cm \) [52]. The measured \( \rho_n \) corresponds to \( S = 0.97 \), perfectly in agreement with the measurement obtained with x-ray diffractometry on a sample from the same batch of our platelet [49].

Moreover, the obtained \( \rho_n \) matches well also with the atomic Sn content \( \beta \) of the sample, since with \( \rho_n = 14.8 \mu \Omega cm \) and from [53], \( \beta = 0.25 \) to be compared to our data \( \beta = 0.248 \). This excellent agreement between the measured \( \rho_n \) and the microscopic parameters measured on the sample from the same batch as ours represents a validation of the \( G_c \) estimation. Moreover, the composition of the bulk sample is also in agreement with the measured \( T_c \) and the generally accepted \( T_c(\beta) \) relation presented in [54]. This confirms that the Devantay data set is more descriptive of NbSn bulk samples behavior than Moore’s [55] as discussed in [53].

#### 5.2. Microwave vortex motion in NbSn

In order to isolate the flux motion response of NbSn, equation (5) is applied to the data shown in figure 2. In this way the temperature background contribution, which is particularly evident in \( \nu_0 \) measurements (figure 2), is removed. The same procedure is followed for the ZFC measurements shown in figure 3. In this case the variations of \( Q \) and \( \nu_0 \) are directly related to the sample \( \Delta Z \) since the resonator is made only with non magnetic materials. Then, the vortex motion resistivity \( \rho_{vm} = \rho_{vm}' + \rho_{vm}'' \) is obtained from the measured \( \Delta Z \), using equation (1), as:

\[ \rho_{vm}'(T, H) = 2\Delta R(T, H) \left( \lambda(T, 0) + \frac{\Delta X(T, H)}{\mu_0 \omega} \right), \]

\[ \rho_{vm}''(T, H) = -\Delta R(T, H)^2 + \left( \frac{\Delta X(T, H)}{\mu_0 \omega} + \lambda(T, 0) \frac{\Delta X(T, H)}{\mu_0 \omega} \right)^2 \]

\[ -\mu_0 \omega \lambda(T, H)^2. \]
Figure 2. (a) Variation $\Delta(1/Q)$ measured in field cooling condition at different fields (i.e. 0 T, 2 T, 4 T, 8 T, 12 T). (b) Variation $\Delta\nu_0/\nu_{ref}$ with measured in field cooling condition at different fields (i.e. 0 T, 2 T, 4 T, 8 T, 12 T). The temperature background of the resonator, which gives rise to the $\nu_0$ hump, is evident requiring the calibration procedure described in the text.

In order to reliably obtain $\rho_{vm}$, we calculate $\lambda$ from the well known values, as follows. It is known [29] that $\lambda$ in Nb$_3$Sn closely follows a BCS behavior [27] although stoichiometric Nb$_3$Sn exhibits similarities to strong coupling superconductors. It is then safe, following the common habit, to describe $\lambda$ with the BCS expression in equations (6), (7), with Debye temperature $\Theta_D = 230$ K [56] and superconducting energy gap $2\Delta = 3.77 k_B T_c$ [57]. Finally, it must be noticed that at high fluxons densities (in practice, just above the first penetration field), and in our measurement frequency band, the main reactive contribution is given by the vortex motion, thus equations (6), (7) are very weakly sensitive to $\lambda$. This was tested using as an alternative a simple two-fluid $\lambda(T)$ temperature dependence $1 - t^2$ and the discrepancies in the following analysis were well below 5%. For the field dependence we used a $1 - b^4$ superfluid fraction dependence with $b = B/B_c^2$ the reduced field. Similarly to the $T$ dependence, also the exact field dependence does not give significant changes on the final results.

From $\rho_{vm}$, with equation (4), $\rho_f(T, H)$ and $\nu_f(T, H)$ are directly obtained within the Gittleman–Rosenblum (GR) model. As previously discussed, the GR model assumes negligible thermal effects, thus it is more descriptive of the data far from the critical surface. Nevertheless, when thermal creep is not
negligible, the GR model provides a lower boundary for \( \nu_p \) and \( k_p \) \cite{40}. Thus, despite the model simplicity, the GR model is particularly useful for an estimation of the pinning parameters, as presented in the next subsections.

5.2.1. Flux-flow resistivity and viscous drag coefficient.

When a vortex moves, energy is lost by the non-equilibrium conversion of the condensate in quasi-particle on the onward vortex side and the restoring of the condensate in the back side \cite{27, 44}. Figure 4 shows \( \rho_{ff} \) as obtained by combining equations (6) and (7) and equation (4) as

\[
\rho_{ff} = \left( \rho_{vm}^2 + \rho_{vm}^{(t')^2} \right) / \rho_{vm}
\]

in ZFC. A good overlap is found with \( \rho_{ff} \) measured in FC conditions (see figure 4). The 15 K ZFC curve is not analyzed here since near the transition the unavoidable presence of flux creep prevents from performing the analysis here presented. For the same reason the derivation of the vortex parameters is estimated as possible only for \( T < 0.8T_{c2} \) where \( T_{c2} = T_c(H) \). Figure 4 correctly shows that \( \rho_{ff} \) increases when \( T \) and \( H \) increase. This is an expected behaviour since \( \rho_{ff} \propto \tau \). With \( \tau \) the quasiparticle scattering time in vortices core \cite{27} averaged on the Fermi surface. Moreover, figure 4 shows an almost perfect linear behavior \( \rho_{ff} \propto H \). This implies that \( \eta \) is field independent in agreement with both Tinkham \cite{27} and Bardeen-Stephen (BS) \cite{36} descriptions of the vortex dissipation phenomena. For \( T \ll T_c \), both theories give an equivalent description of the total viscosity \cite{27, 36}:

\[
\eta = \frac{\Phi_0B}{\rho_{ff}} \approx \frac{\Phi_0\mu_0H_{c2}}{\rho_{n}}.
\]

Equation (8) allows us to scale these curves with respect to \( H_{c2} \) once \( \rho_n(T) = 2R_n(T)^2/\omega\mu_0 \) is determined (section 5.1). Figure 5 shows the obtained good \( \rho_{ff} \) scaling that allows a reliable determination of the upper critical field \( H_{c2}(T) \) even above the maximum field reached. The obtained temperature derivative \( \mu_0dH_{c2}(T)/dT \mid_T \approx 2.2 \) T/K and the \( H_{c2}(T) \) data points which were directly observed, and/or obtained by the scaling procedure, are well fitted with the Maki-de Gennes (MG) approximation \cite{37, 38} (see figure 6) in agreement with the literature \cite{39}. The fact that the \( H_{c2} \) points obtained by the scaling procedure are well placed on the MG curve further validates the use of the BS model for Nb\(\text{Sn} \) and the scaling procedure. The MG model uses only two free par-
ameters: $T_c$ (measured) and the normal electrons diffusion coefficient $D$. In particular, it can be shown [39] that within the MG approximation $\mu_0 DH_2(T)/dT|_T = -4\Phi_0k_B/\pi^2hD$, with $k_B$ the Boltzmann constant and $h$ the reduced Planck constant. Thus, the fit contains only experimentally determined parameters. With the fit of the measured $H_2(T)$ we obtain $D \sim 5.0 \times 10^{-5}$ m² s⁻¹. We point out that $H_2(T)$ does not depend on the electron-phonon coupling, thus even if the simple MG approximation does not take the coupling strength into account (differently from the more complex Eliashberg theory) it can be reliably used in this case: it is shown in literature [39] that the MG model approximates well the Nb₃Sn $H_2(T)$ behavior in different samples (e.g. single crystal, thin films, bulk, wires) and with different Sn contents [39].

5.2.2. Pinning constant. The pinning constant, shown in figure 7, is obtained by combining equations (6) and (7) and equation (4), $k_p = 2\pi\Phi_0B_0/\rho_p$. As presented in section 2 this parameter in the limiting case of rigid vortices is a measure of the steepness of the pinning potential wells. The obtained $k_p$ correctly decreases when the temperature and the magnetic field are increased due to a reduction of the pinning efficiency. Figure 7 shows that even at $\mu_0H = 8$ T and $T \sim 8$ K, $k_p > 10$ kN m⁻². This value indicates an enhanced pinning efficiency in Nb₃Sn as compared to that of Nb films. In fact, in the latter, the $k_p$ literature value is assessed to be about an order of magnitude smaller than that of Nb₃Sn at $t = 0.5$ in a 40 nm thick film [58] and even smaller in a 30 nm thick film at $t = 0.86$ [59, 60]. A $k_p$ value similar to that of Nb₃Sn was also observed on pristine bulk MgB₂ where, at 1 T and at 10 K, $k_p \sim 11$ kN m⁻² [61]. Higher $k_p$ are observed in cuprates, e.g. $k_p \approx 75$ kN m⁻² at $t = 0.5$ and $\mu_0H = 0.5$ T in 100 nm YBa₂Cu₃O₇-δ thin film added with BaZrO₃ inclusions [62], and $k_p$ up to 100 kN m⁻² attained even at much higher $t \sim 0.74$, at $\mu_0H = 0.5$ T, in 200 nm YBa₂Cu₃O₇-δ thin film added with Ba₂YNbO₆+Ba₂YTaO₆ inclusions [63, 64].

The temperature dependence $k_p(T)$ is shown in figure 7: $k_p$ decreases steadily with the temperature indicating that no matching-field effects take place. The temperatures for which $k_p = 0$, corresponding to the complete vanishing of the pinning effect, are obtained through a linear extrapolation of the high temperature region of the curves in figure 7. The corresponding points are reported on the phase diagram of figure 6. These points mark the depinning line as obtained by the microwave technique. As it can be seen, the complete flattening of the pinning potential arises very close to $H_c2$.

The ZFC measured $k_p(H)$ at 4 K is shown in the inset of figure 7 to follow the power law dependence $k_p \propto H^{\alpha}$ with $\alpha = -0.47$. This behavior is expected in the collective pinning regime where for conventional superconductors one expects $\alpha = -0.5$ [18, 44, 65]. In fact, this field dependence indicates that, even at low temperature, vortices in Nb₃Sn are not individually pinned but a bunch of vortices is bounded around weak pins, thus the vortices concentration is higher than that of the pinning centres. We indicate this pinning regime as collective pinning according to [44]. In this configuration the fluxons interact with each other, and thus the pinning properties are strongly dependent on the fluxons density and the pinning strength decreases with the field. In this regime $k_p$ is no more a direct measure of the single pinning centre strength but it is a statistical average of the contribution given by several pinning centres and vortices. This means that in principle there is still room of improvement for enhanced $k_p$ values in Nb₃Sn samples engineered for high field and high frequency applications (e.g. RF cavities for dark matter research [4]). In fact an upper limit for $k_p$ can be estimated in the single-vortex pinning regime by assuming vortices individually pinned by cylindrical defects of diameter 2$\xi$, being $\xi$ the coherence length, oriented parallel to the applied magnetic field. In this case the condensation energy (per unit length) in the vortex core $k_pH_c2\xi^2$, with $H_c$ the thermodynamic critical field, is equal to the maximum pinning elastic energy (per unit length) $k_pH_c2\xi^2$. Hence, the maximum $k_p^{\text{max}} \approx 0.25\mu_0H_c2$ [44] can be assessed in this ideal core pinning configuration. Using the literature value $\mu_0H_c(0) \sim 0.52$ T [66] for stoichiometric Nb₃Sn, $k_p^{\text{max}} \sim 50$ kN m⁻² is obtained. This $k_p$ upper limit is near to that measured on 100 nm film pristine YBCO [62].

5.2.3. Depinning frequency. Finally, the depinning frequency $\nu_p = k_p/(2\pi f)$ is discussed in this section. We show in figure 8(a) the depinning frequency $\nu_p$ measured in FC condition at $\mu_0H = \{2, 4, 8, 12\}$ T and obtained with equation (3). We can see that it is almost constant at low enough temperature (i.e. for $T/T_c < 0.7$) and it sharply decreases approaching the depinning line as described in section 5.2.2. In figure 8(b) the $\nu_p(H)$ field dependence at $T = \{6, 7.5, 9\}$ K is shown. We note that at the lower $T$ $\nu_p(H)$ starts to decrease above $\mu_0H = 4$ T, while at lower fields it tends to saturate at $\nu_p \sim 6.5$ GHz. The measured values are quite large also at high fields, $\nu_p > 4$ GHz at 12 T and low $T$ which is larger than that exhibited by thin Nb films. It is known that $\nu_p$ is strongly dependent on the sample

![Graph of k_p(H, T) in FC condition at 2 T, 4 T, 8 T, 12 T. The sparse empty cycles come from the ZFC measurements. In the inset k_p(H) at 4 K is shown in a log-log plot to highlight the power dependence k_p \propto H^{\alpha}, with \alpha = -0.47, typical of the collective pinning regime.](image-url)
Figure 8. (a) The depinning frequency $\nu_p$ measured in FC condition at different fields (i.e. 2 T, 4 T, 8 T, 12 T) obtained with the GR model. The shown data are smoothed and the standard deviation of the data scattering represented by the shadowed areas. (b) The depinning frequency $\nu_p$ dependence on the applied magnetic field $\mu_0 H$ at 6.0 K, 7.5 K, 9.0 K. The dashed line is a guide for the eye.

Figure 9. Lower limit of the creep activation energy $U_{0,\text{min}}(T, H)$ in FC at 2 T, 4 T, 8 T and 12 T. We show an almost perfect scaling of the $U_{0,\text{min}}(T, H)$ curves in the inset. The continuous line in the inset is the fit realized with the normalized $U_0 \propto H^2 \xi_n$.

thickness in Nb films: $\nu_p \sim 20$ GHz in 10 nm Nb film at 0.2 T and 5 K [58], $\nu_p \sim 5$ GHz in 60 nm Nb film at 0.6 T [59] and it falls to 1 GHz for 160 nm films in 0.2 T and 5 K [58]. In Nb the increase of $\nu_p$ with the lowering of the film thickness was attributed to the dominant effect of the surface pinning centres [58]. This effect is masked in thicker samples due to the increased volume interested by the weaker volume pinning in Nb [58]. Moreover, it is well known that the main contribution to pinning in Nb$_3$Sn [67–70] as in other intermetallic compounds [71, 72] and metals as Nb [73–75] is given by the grain boundaries and that the pinning efficiency is inversely proportional to the average grains size. Since in Nb$_3$Sn the grain size can be reduced by lowering the sample thickness [69], it is reasonable to expect that for thin Nb$_3$Sn samples $\nu_p$ could reach very high values. This opens the possibilities to interesting RF applications of Nb$_3$Sn films also in presence of high magnetic fields. Moreover, assuming a Nb$_3$Sn sample engineered with a sufficiently high defects density to firmly remain in the single vortex pinning regime, from the previously calculated $k_{p,\text{max}}$, a theoretical upper limit $\nu_{p,\text{max}} \sim 16$ GHz can be expected in bulk samples.

Considering other superconducting materials, it comes out that the obtained values at 4 T are comparable with those measured in a MgB$_2$ thin film in the same $H$-$T$ region [76]. On the other hand, it must be noticed that FeSe$_{0.5}$Te$_{0.5}$ and YBa$_2$Cu$_3$O$_{7-\delta}$ performances are still far, in fact at 12 K and 0.6 T $\nu_p \sim 22$ GHz and >40 GHz, respectively in 300 nm and 240 nm thick FeSe$_{0.5}$Te$_{0.5}$ films [77, 78] while $\nu_p \sim 50$ GHz in 100 nm thick YBa$_2$Cu$_3$O$_{7-\delta}$ films at 72 K [62]. For a more complete comparison, thin Nb$_3$Sn films should be characterized in the same conditions to experimentally verify the increase of $\nu_p$ with the reduction of the sample thickness. Despite of this, from this study, it is shown that bulk Nb$_3$Sn could remain a good choice for applications that work at not too high frequencies (e.g. radio frequency cavities for axions detection [79]) and for which the use of a metallic and wieldy material is an important requirement.

5.2.4. Evaluation of the thermal creep contribution. We complete this work by providing an estimate of the thermal creep contribution to the evaluation of the vortex parameters through a statistical analysis according to [40]. We derive the maximum creep factor $\epsilon_{\text{max}}$ and the lower limit for the activation energy $U_{0,\text{min}}$. We then derive a confidence interval for the characteristic frequency $\nu_c$ (we recall that when creep is taken into account, the characteristic frequency is no longer $\nu_p$, but $\nu_c$, see equation (3) and equation (4)).
The maximum creep factor \( \epsilon_{\text{max}} = 1 + 2r^2 - 2r\sqrt{1 + r^2} \), with \( r = \rho_{\text{eff}}/\rho_{\text{ms}} \), is obtained from analytical constraints \([40]\). Then, the corresponding minimum activation energy is determined with the CC model, in the scenario of a periodic pinning potential, since \( \epsilon = (U_0(U_0(T, B)/(2k_B T))^{-2} \) \([33]\) with \( b_0 \) the modified Bessel function of first kind.

The lower limit for the activation energy \( U_{0, \text{min}}(T, H) \) is shown in figure 9. The data show a non-monotonic temperature dependence with a peak height which becomes smaller, and moves at lower \( T \), as the field is increased. This behavior is expected for \( U_0 \) since at higher \( H \) values the pinning becomes less effective. Since \( U_0 \propto H^2 \xi^4 \) \([80]\), then the \( U_0 \) temperature dependence can be evaluated from \( H_c = \Phi_0/(\mu_0 \sqrt{2\pi H \xi}) \) and using the BCS \( \lambda \) previously used for the data analysis and \( \xi = \sqrt{\Phi_0/(2\pi \mu_0 H_c)} \) \([27]\) with the measured \( H_c \) from figure 6. The \( n = 0, 1, 2, 3 \) parameter depends on the relevant length scale for the pinning energy: it indicates the dimensions of the correlated volume of the fluxons bunch that is thermally activated. From the theoretical behavior for \( U_0 \), the observed non-monotonic trend can be obtained only with \( n = 3 \). Keeping this value for \( n \), a tentative comparison between the theoretical curve and the experimental data, arbitrarily scaled with the constraint \( U_{0, \text{min}} < U_0 \) for each \( T \), is reported in the inset of figure 9. Assuming that the temperature behavior \( U_{0, \text{min}}(T) \) reflects \( U_0(T) \), \( n = 3 \) is an indication that in this sample the vortices correlated volume has a length scale \( \xi \) along the three spatial directions. It must be noticed that the peak in \( U_{0, \text{min}} \) is narrower than what expected from the theory and also that the lower temperature \( U_{0, \text{min}} \) dependence does not saturate to a finite value but it is a linear function of \( T \). The narrow peak and the increase of \( U_0 \) with the temperature was already observed in other superconductors \([78, 80, 82]\). This discrepancy with respect to the theory was justified introducing pinning models that included junctions and non-homogeneities \([81, 83, 86]\).

To evaluate the impact of finite flux creep on the estimate of the vortex parameters, one should know the statistical distribution of the activation energies. Although we can set \( U_{0, \text{min}} \) from the data, a full knowledge of the statistical distribution is not available. We then model the distribution of \( U_0 \) as a rectangular (uniform) distribution, and we seek for an estimate of the maximum (cutoff) \( U_{0, \text{max}} \). The latter is determined consistently with the models used in this analysis. In particular, in the ideal case one can assume that the measured \( k_p \) is not dependent on the fluxon displacement (i.e. that the pinning wells have perfectly parabolic profiles) and that the wells maximum width is \( 2l \). In this case the maximum elastic pinning energy is \( U_{0, \text{max}} = \frac{1}{4} k_p \xi^4 l \) where \( l \) is the length of the effective pinning along the direction of the applied magnetic field \([44]\). According to the indication \( n = 3 \) (figure 9), \( l \sim \xi \), then we set \( l \sim \xi \). The coherence length \( \xi \) is obtained from the previously determined \( H_c \) (figure 6), \( \xi = \sqrt{\Phi_0/(2\pi \mu_0 H_c)} \) \([27]\) and from the GR model \( k_p \) a first estimation of \( U_{0, \text{max}} \) is obtained. Actually, since the creep is now taken into account, the \( U_{0, \text{max}} \) estimation can be enhanced with a recursive approach: once \( U_{0, \text{max}} \) is obtained from the GR \( k_p \), it can be used to calculate the creep factor \( \epsilon \) to be used in equation (3), thus a new \( k_p \) can be obtained with the CC model. This in turns fixes \( \epsilon \) and a refined value for \( k_p \) can be evaluated from the measured data and the fixed \( \epsilon \) with the CC model. In this way \( U_{0, \text{max}} \) is evaluated several times until \( (k_p^{i+1} - k_p^i)/k_p^i < 0.01 \), with \( i \) the iteration number (e.g. at \( T = 9 \) K and \( \mu_0 H = 2 \) T the problem converges in 5 steps). The \( U_{0, \text{max}} \) obtained from the last iteration is used for the statistical analysis now presented.

With \( U_0 \) taken to follow a rectangular distribution between \( U_{0, \text{min}} \) and \( U_{0, \text{max}} \), we recalculate the pinning parameters using equation (3) with 1000 \( \epsilon \) values randomly extracted from the \( U_0 \) distribution previously built. We show as an exemplary case the creep contribution evaluated on \( \nu_{c}(t = 0.5, \mu_0 H = 2 \) T) with \( t = T/T_c \). We focus on \( \nu_{c} \), since it is the cross-over frequency between the low frequency vortex elastic motion and the high frequency dissipative region. Thus in case of creep \( \nu_{c} \) is the parameter of interest for technological applications. The cumulative probability distribution of the characteristic frequency \( \nu_{c} \) is shown in figure 10. The expected value \( E[\nu_{c}^{CC}] = 7.4 \) GHz is about 20 % larger than the numerical value for \( \nu_{c} \) within the GR model in the same condition. At \( t = 0.5 \) and \( \mu_0 H = 2 \) T, we evaluated with the CC model also the \( \rho_f \) distribution obtained \( E[\rho_f^{\text{CC}} - \rho_f^{\text{GR}}]/\rho_f^{\text{GR}} \sim 3 \) % Whereas, at \( t = 0.5 \) and \( \mu_0 H = 12 \) T, where the creep phenomenon is more intense, the discrepancy between the CC and GR valued parameters is about 35 % on \( \nu_{c} \) and 6 % on \( \rho_f \).

As expected from \([40]\), one can notice that the GR evaluation gives a lower boundary for \( \nu_{c} \), \( \rho_f \) and \( k_p \). In particular, even if neglecting the thermal creep brings to an underestimation of the material characteristic frequency \( \nu_{c} \), a drastic difference is not expected from that obtained with the more complete CC model. This means that, for the design of perspective RF/high frequency applications of Nb3Sn, the \( \nu_{c} \) shown in section 5.2.3 can be treated as the worst RF performance of Nb3Sn, while the expected values shown in this section can be used as a more realistic estimation of \( \nu_{c} \). Whereas,
sections, YBCO performances are still better than those of smaller gap is suppressed. Finally, as shown in the previous µ advantage for large kµ. It can be assessed that the GR determination can be considered reliable since even at the highest creep rate its shift is modest (i.e. ≤6%).

6. Vortex parameters comparison with other SCs

In this section a brief comparison between the mixed-state microwave properties of Nb3Sn and those of other technologically interesting superconductors (i.e. MgB2 and YBa2Cu3O7−δ) is provided.

In order to keep the comparison as meaningful as possible, we compare data obtained mainly by our group on MgB2 and YBCO with the same technique and with the use of the same physical model (e.g. the GR model). The MgB2 data were obtained on bulk pristine and doped samples at 16.5 and 26.7 GHz up to 1.2 T. Further details on the MgB2 characterization are shown in [61]. Concerning YBCO, the parameters used for the comparison come from several literature results on thin films [21, 22, 62-64], commercial coated conductors [87, 88] and single crystals [89]. The vortex parameters are linked to each other by vµ = kµ/(2πην) and η ∝ ρη−1 (see equation (8)). It is then useful to investigate the parametric plots as reported in figures 11 and 12. In figure 11 the comparison at μ0H = 1 T and T = 10 K is shown on the plane pµ = νµ. From this, one can notice that Nb3Sn shows the lowest νµ of the three SC materials. Despite of this, it must be noticed that the large MgB2 νµ comes from its particularly large pµ as shown in [61]. Thus, despite the larger νµ, the µw losses are smaller in Nb3Sn with respect to those in MgB2, even above νµ. In particular, in MgB2 pµ exhibits a non conventional Bardeen-Stephen behavior [61, 90] due to the presence of the weak superconductive π-band. This makes MgB2 advantageous for µw applications in the mixed state only in particular conditions, e.g. below the field values for which the smaller gap is suppressed. Finally, as shown in the previous sections, YBCO performances are still better than those of metallic SCs. On the other hand, the practical use of YBCO in large-scale µw applications like cavities is hindered by the difficulties in the deposition on continuous, and possibly non-planar, surfaces. It must be noticed that in Nb3Sn νµ is high enough for applications like the dark matter cavity detectors [79], thus despite its lower νµ it retains its importance for µw applications.

From the technological point of view figure 11 shows a useful comparison of the µw most interesting parameters of these SCs at fixed working conditions (i.e. at μ0H = 1 T and T = 10 K). However, it can be useful to compare the different SCs also with a more physical approach, evaluating kµ and η at the same reduced temperature t = T/Tc and field b = B/Bc2. This comparison is shown in figure 12. The higher kµ in YBCO is caused by the single fluxon pinning in this kind of SC [44], while in both Nb3Sn and MgB2 it was shown that the vortex system is in the collective pinning regime [61]. However, it must be noticed that the kµ,max here theorized for Nb3Sn corresponds to the lower limit for kµ in YBCO. Thus, in theory if it would be possible to optimize Nb3Sn with artificial pinning centres effective at microwaves, the high frequency performances of Nb3Sn could be expected to be near that of YBCO. Finally, the viscous drag coefficient η = Φ0b/ρη ∝ t > shows that the quasi-particles scattering time in the fluxons cores is particularly reduced in MgB2 because of the high normal carriers density coming from the suppressed π-band [61, 91]. In Nb3Sn η is about 3 times smaller than the lowest η value in YBCO, thus even if for kµ and νµ there is still room for improvement, the Nb3Sn microscopic properties would still limit the high frequency dissipation in this material.

The comparison between the Nb3Sn, MgB2 and YBa2Cu3O7−δ µw vortex parameters shows that Nb3Sn exhibits intermediate performances between those of the other two SCs. However, the possibility of increasing kµ with artificial pinning centres optimized to work at µw in order to extend the single pinning regime in Nb3Sn, can be particularly interesting.

Figure 11. Comparison between the µw vortex parameters (pµ and νµ) of Nb3Sn, MgB2 and YBa2Cu3O7−δ at μ0H = 1 T and T = 10 K.

Figure 12. Comparison between the pinning constant kµ and the viscous drag coefficient η of Nb3Sn, MgB2 and YBa2Cu3O7−δ at t = 0.5 and b = 0.2.
to obtain a metallic superconductor with improved \( \mu_W \) pinning characteristics, close to that of YBCO.

7. Summary

A polycrystalline bulk Nb\(_3\)Sn sample was characterized at 15 GHz in order to study the high-frequency vortex motion in high magnetic fields up to 12 T. The measurements were performed with a dielectric loaded resonator both in field cooling and zero field cooling conditions. The obtained normal state material parameters matched with the literature values. The upper critical field \( H_{c2}(T) \) was evaluated at the higher temperatures directly up to 12 T while at the lower temperatures \( H_{c2}(T) \) was obtained by the scaling of the flux flow resistivity \( \rho_{ff} \) in a self-consistent way: the scaling procedure based on the Bardeen-Stephen (BS) model \([36]\) \( \rho_{ff}/\rho_n \sim H/H_{c2} \) gave a \( H_{c2}(T) \) in agreement with the expected, from literature \([39]\), Maki-de Gennes (MG) behavior \([37, 38]\) confirming both the validity of the scaling itself and the conventional BS behavior of bulk Nb\(_3\)Sn. The Nb\(_3\)Sn depinning frequency \( \nu_p \) reached rather high values, above 4 GHz even at 12 T and low \( \nu \), indicating that Nb\(_3\)Sn is suitable for radio frequency low loss applications up to few GHz in bulk form. Since \( \nu_p \) strongly decreases with the film thickness, as shown in Nb, higher \( \nu_p \) values can be expected in Nb\(_3\)Sn thin films. The pinning constant \( k_p(T, H) \) was found to decrease when the temperature and the field are increased due to the reduction of the pinning efficiency. In particular, a field dependence typical of the collective pinning scenario (i.e. \( k_p \propto H^{-0.5} \)) was shown. Despite the collective pinning, \( k_p > 10 \text{ kN m}^{-2} \) for \( H \leq 8 \text{ T and } T \leq 8 \text{ K} \) which is about 10 times greater the values found in thin Nb films \([58]\). An estimation of the maximum \( k_{p,max} \sim 50 \text{ kN m}^{-2} \), corresponding to single-vortex core pinning, shows that an alternative path to higher \( \nu_p \) (from higher \( k_p \)) might arise from appropriate defect engineering. In fact, only in the last years the necessity to operate at high frequencies and high magnetic fields emerged, while no particular material studies were undertaken for optimizing the superconductive properties in these harsh working conditions.

Finally, we provided an analysis of the impact of the thermal activation on the vortex motion parameters through a statistical method and using the Coffey-Clem (CC) model \([33]\). The creep activation energy \( U_0 \) distribution was modeled with a uniform distribution with upper and lower bound estimated consistently with the models and results already obtained. Although the results obtained with the Gittleman-Rosenblum (GR) model \([18]\) (assuming negligible creep) represent a lower limit for both \( \nu_p \) and \( t_\nu \), we obtained that \( \nu_p \), evaluated with the GR model, can be considered a reliable determination while for \( t = 0.5 \) the expected value \( E[\nu_\nu]_{\text{CC}} \sim 1.2\nu_{p,GR} \) at 2 T and \( E[\nu_\nu]_{\text{CC}} \sim 1.4\nu_{p,GR} \) at 12 T.

This work represents, to our knowledge, the first report of the microwave response in Nb\(_3\)Sn at high fields. The results here obtained are encouraging for the use of Nb\(_3\)Sn in RF in high fields, although further optimization of the pinning can be needed for the specific requirements of high-frequency applications.

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