THE BUILT-IN FLEXIBILITY OF INCOME AND CONSUMPTION TAXES IN NEW ZEALAND

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This paper provides estimates of individual and aggregate revenue elasticities of income and consumption taxes in New Zealand, based on the 2001 tax structure and expenditure patterns. Using analytical expressions for revenue elasticities at the individual and aggregate levels, together with a simulated income distribution, values for New Zealand were obtained. Results using equi-proportional income changes suggest that the aggregate income and consumption tax revenue elasticities are both fairly constant as mean income increases, at around 1.3 and 0.95 respectively. This latter estimate assumes that increases in disposable income are accompanied by approximately proportional increases in total expenditure. If there is a tendency for the savings proportion to increase as disposable income increases, a somewhat lower total consumption tax revenue elasticity, of around 0.9, is obtained for 2001 income levels. However, non-equiproportional income changes are more realistic. Allowing for regression towards the geometric mean income reduces these elasticities, giving an elasticity for income and consumption taxes combined that is only slightly above unity. Examination of the tax-share weighted expenditure elasticities for various goods also revealed that, despite the adoption of a broad based GST at a uniform rate in New Zealand, the persistence of various excises has an important effect on the overall consumption tax revenue elasticity, especially for individuals at relatively low income levels.

I. Introduction

This paper provides estimates of the built-in flexibility, or revenue elasticity, of income and consumption taxes (GST and excise taxes) in New Zealand. Values at individual and aggregate levels are reported. These are obtained using convenient analytical expressions which have the advantage that they can be evaluated readily from relatively little information about the tax structure, income distribution and budget shares. Furthermore, the main factors affecting the size of the elasticities can be identified, using meaningful and easily interpreted decompositions of the revenue elasticities.\(^1\)

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\(^1\) The approach, involving explicit modelling of the tax structure, contrasts with the use of regression analyses, of time series data on tax revenues and income, which are sometimes used to produce aggregate elasticities. Some comparisons of aggregate income tax revenue elasticities based both on regressions and on tax-share weighted individual values are given in Giorno et al. (1995), although they do not include New Zealand.
Detailed official forecasts of tax revenues in New Zealand are of course frequently made, though for various reasons these do not always involve the explicit calculation of revenue elasticities. Few independent estimates for New Zealand appear to have been published; however, various elasticities are given by van den Noord (2000) for OECD countries. This neglect may reflect a perception that, in the presence of lower rates of inflation in recent years, a flattening of the income tax structure, and a broad-based consumption tax, fiscal drag is no longer significant. However, as the analysis below shows, the issue is more complex than this simple view would suggest. It is also useful to identify the various influences on the size of New Zealand tax revenue elasticities.

Section II sets out the relevant conceptual expressions for income and consumption tax revenue elasticities at the individual level. Subsection II ii provides some estimates based on the 2001 tax structure. Section III defines aggregate revenue elasticity expressions, with empirical estimates in subsection III ii. These estimates use the standard assumption that all incomes increase by the same proportion from year to year. Subsection III iii models the more realistic case of non-equiproportional income changes. The computation of aggregate elasticities requires information about the distribution of taxable income. One approach to income tax revenue forecasting is to use a purely numerical approach based on a large sample survey, or preferable longitudinal data on consecutive years, of taxable income. However, the aim of the present paper is to provide a modelling approach which provides an understanding of the determinants of revenue elasticities, and which requires only limited data which are available to all researchers (there are severe restrictions in New Zealand on access to individual data). It is then possible to examine the sensitivity of, say, income tax revenue elasticities to changes in the dynamic process of income changes from year to year, or the effects on consumption tax revenue elasticities of changes in expenditure patterns. Section IV draws some brief conclusions.

II. INDIVIDUAL REVENUE ELASTICITIES

This section examines tax revenue elasticities for individuals. The variation in individual elasticities with income provides a useful independent indication of the local progressivity of the tax structure, and of course the individual elasticities provide the basic components on which aggregate values are based. The appropriate formulae are given in subsection II i, and empirical estimates for the 2001 tax structure in New Zealand are reported in subsection II ii.

i) Elasticity formulae

Suppose \( T_{yi} \) denotes the income tax paid by individual \( i \) with a nominal income of \( y_i \). Changes in nominal income with respect to nominal tax allowances affect built-in flexibility. The revenue elasticity of the income tax with respect to a change in individual income, \( \eta_{T_{yi}, y_i} \), is defined as:

\[ \eta_{T_{yi}, y_i} = \frac{T_{yi} + \frac{\partial T_{yi}}{y_i} y_i}{T_{yi}} \]

van den Noord (2000, p. 19) gives elasticities of personal and indirect taxes, with respect to changes in GDP, of 1.2 for both sets of taxes in New Zealand. The value of 1.2 for indirect taxes (which appears to be based on regression analysis) seems unrealistically high, given the values reported in the present paper. But as mentioned below, independent numerical results confirm the modelling results reported below, in the case of income taxation.

For further details of elasticity formulae, see Creedy and Gemmell (2002, 2003).

The exception is where the tax function is homogeneous of degree one in income and allowances, and both are indexed similarly.

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\[ \eta_{i,y_i} = \frac{dT(y_i)/dy_i}{T(y_i)/y_i} = \frac{mtr_i}{atr_i} \]

where \( mtr_i \) is the marginal tax rate and \( atr_i \) is the average tax rate faced by \( i \). Here, the first subscript of the revenue elasticity, \( \eta \), refers to the type of tax revenue considered (so that in the present case of the individual elasticity, the \( i \) subscript on this subscript is dropped for convenience) and the second subscript refers to the income (or tax base) that is considered to change. In a progressive tax structure, \( mtr_i > atr_i \) for all \( i \), so that \( \eta_{i,y_i} > 1 \). This elasticity is also a local measure of progressivity: it is the concept of liability progression defined by Musgrave and Thin (1948).

Consider an individual with gross income of \( y_i \) and facing a multi-step income tax function, such that if \( 0 < y_i \leq a_1 \), the tax paid is \( T_{y_i} = t_0 y_i \); if \( a_1 < y_i \leq a_2 \), tax paid is \( T_{y_i} = t_0 a_1 + t_1 (y_i - a_1) \); if \( a_2 < y_i \leq a_3 \), tax paid is \( T_{y_i} = t_0 a_1 + t_1 (a_2 - a_1) + t_2 (y_i - a_2) = t_2 y_i - a_1 (t_1 - t_0) - a_2 (t_2 - t_1) \), and so on. Hence if \( y_i \) falls into the \( k \)th tax bracket, so that \( a_k < y_i \leq a_{k+1} \), and \( a_0 = 0 \), income tax can be expressed for \( k \geq 1 \) as:

\[ T_{y_i} = t_k y_i - \sum_{j=k}^{k} a_j (t_j - t_{j-1}) = t_k (y_i - a_k') \]

where \( a_k' = \sum_{j=k}^{k} a_j (t_j - t_{j-1})/t_k \). Hence the tax paid under a multi-step function is equivalent to that paid with a single-step tax structure having a marginal rate, \( t_k \), imposed on the individual’s income in excess of an effective threshold of \( a_k' \).

If there is a range of income-related allowances, for example when there is tax relief for mortgage interest payments or pension contributions, it is necessary to introduce the concept of the income elasticity of effective thresholds, \( \eta_{d,y_i} \). Creedy and Gemmell (2003) show that, for the multi-step tax function, the individual elasticity is:

\[ \eta_{d,y_i} = 1 + \left( \frac{a_k'}{y_i - a_k'} \right) (1 - \eta_{d,y_i}) \]

This result shows that the individual revenue elasticity must exceed unity if \( \eta_{d,y_i} < 1 \). A positive value of \( \eta_{d,y_i} \) can be expected.\(^6\)

To derive the individual revenue elasticity for consumption taxes, define \( z_i \) as individual \( i \)'s net income, so that:

\[ z_i = y_i - T_{y_i} = a_k' t_k + y_i (1 - t_k) \]

Suppose a proportion, \( y_i \), of \( z_i \) is consumed, so that total consumption expenditure, \( m_i \), is \( y_i z_i \). In general, \( y_i \) can vary with \( z_i \) and hence with \( y_i \), and can exceed unity over some ranges of \( z_i \), as discussed below.

If the tax-exclusive \( ad \) \( valorem \) indirect tax rate imposed on the \( \ell \)th good (for \( \ell = 1, \ldots, n \)) is \( \nu_{i,\ell} \), the equivalent tax-inclusive rate is \( \nu_{i,\ell} = \nu_{i,\ell}/(1 + \nu_{i,\ell}) \). Define \( w_{i,\ell} \) as person \( i \)'s budget share of the \( \ell \)th good. The consumption tax paid by person \( i \) on good \( \ell \) can be written as:

\[ T_{\nu_{i,\ell}} = \nu_{i,\ell} w_{i,\ell} m_i = \nu_{i,\ell} w_{i,\ell} y_i z_i \]

\(^6\) For example, Creedy and Gemmell (2003) found that \( \eta_{d,y_i} \) takes values around 0.4 for the UK, but varies significantly over time in response to changes in the tax deductability of various income-related reliefs such as those for families, pensions and mortgages. The value of \( \eta_{d,y_i} \) is of course unlikely to exceed unity.

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It is required to obtain the consumption tax revenue elasticity for each good, that is, \( \eta_{\gamma_i,y_i} \). Writing \( m_{i\ell} = w_{i\ell} m_i \) as expenditure on the \( \ell \)th good, the following relationships are easily obtained:

\[
\begin{align*}
  m_{i\ell} &= w_{i\ell} m_i \quad \text{implies:} \quad \eta_{m_i,m_i} = 1 + \eta_{m_{i\ell},m_{i\ell}} \\
  m_i &= \gamma_i z_i \quad \text{implies:} \quad \eta_{m_i,z_i} = 1 + \eta_{\gamma_i,z_i} \\
  T_{u_i} &= \nu_i m_{i\ell} \quad \text{implies:} \quad \eta_{T_{u_i},m_{i\ell}} = 1
\end{align*}
\]

(6)

Differentiating (5) with respect to income, \( y_i \), and using the relationships in (6), it can be shown that:

\[
\eta_{\gamma_i,y_i} = (1 + \eta_{m_i,m_i})(1 + \eta_{\gamma_i,z_i}) \eta_{z_i,y_i}
\]

(7)

where \( \eta_{z_i,y_i} \) is the elasticity of disposable income, \( z_i \), with respect to \( y_i \). Three elasticities appear on the right hand side of (7). The first term in parentheses on the right hand side of (7) can be expressed in terms of \( e_i \), the total expenditure elasticity of demand for the \( \ell \)th good by person \( i \), since:

\[
e_i = 1 + \eta_{m_i,m_i}
\]

(8)

The last term in (7), \( \eta_{z_i,y_i} \), is the familiar measure of residual progression and can be written as:

\[
\eta_{z_i,y_i} = \frac{1 - mtr_i}{1 - atr_i}
\]

(9)

Combining (7), (8) and (9) it follows that:

\[
\eta_{\gamma_i,y_i} = e_i (1 + \eta_{\gamma_i,z_i}) \left( \frac{1 - mtr_i}{1 - atr_i} \right)
\]

(10)

Hence the consumption tax revenue elasticity for good \( \ell \) can be decomposed into three terms, reflecting the total expenditure elasticity for good \( \ell \), the way in which the proportion of disposable income consumed by \( i \) changes with income, and the degree of residual progression determined by individual \( i \)'s marginal and average income tax rates.

The consumption tax revenue elasticity for all goods combined, for person \( i \), \( \eta_{\gamma_i,y_i} \), can be obtained directly from the expression for the consumption tax paid on all goods, \( T_y \). Aggregating (5) over \( n \) goods gives:

\[
T_y = \sum_{\ell=1}^{n} T_{u_i}
\]

\[= m_i \sum_{\ell=1}^{n} \nu_i w_{i\ell}
\]

(11)

Differentiation of (11) therefore reveals that \( \eta_{\gamma_i,y_i} \) is given by:

\[
\eta_{\gamma_i,y_i} = (1 + \eta_{\gamma_i,z_i}) \left( \frac{1 - mtr_i}{1 - atr_i} \right) \left( \sum_{\ell=1}^{n} \frac{T_{u_i}}{T_y} e_i \right)
\]

(12)

The only difference, compared with the revenue elasticity for a single good in (10), is that the tax-share weighted expenditure elasticity appears in (12). To calculate the weighted elasticity, it is necessary to distinguish only between goods facing different \textit{ad valorem} tax rates.

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The elasticity of the consumption proportion with respect to income, $\eta_{i,v}$, also varies with incomes if saving rates vary across disposable income levels. While a non-proportional relationship is generally accepted for cross-sectional income differences and, to a lesser extent, for time-series changes over the short-term, changes in the consumption proportion over the long-run are probably best regarded as proportional.

Creedy and Gemmell (2003) allow for the possibility of a non-proportional relationship by using the specification:

$$m_i = a(z_i + b) \quad (13)$$

The over-spending at low income levels can be viewed in terms of the existence of transfer payments and consumption out of savings. For this case, it can be shown that $1 + \eta_{i,v,z}$ in (12) is equal to $z/(z + b)$. Hence for a proportional consumption function (including zero savings, where $a = 1, b = 0, 1 + \eta_{i,v,z} = 1$). The elasticity therefore depends on the three terms as follows:

$$\eta_{i,v,z} = \left( z \over z + b \right) \left( 1 - m_{tr} \right) \left( 1 - a_{tr} \right) \sum_{c=1}^C \left( T_{c,\ell} / T_{c,0} \right) e_{c,\ell} \quad (14)$$

The first two bracketed terms of equation (14) are less than or equal to unity, but the third component, shown in curly brackets, may exceed unity for some income levels and tax structures. However, $\eta_{i,v,z}$ tends towards unity as income increases. This is because all expenditure elasticities converge towards unity, along with the first two terms in (14), although the convergence may not be monotonic.\(^7\)

ii) Estimates of individual revenue elasticities

This subsection shows how individual revenue elasticities can be expected to vary across income levels in New Zealand. The New Zealand income tax structure in 2001 has marginal tax rates of 0.15, 0.21, 0.33, 0.39 applying above income thresholds of (NZ$) 0, 9500, 38000, and 60000. There is thus no initial tax-free allowance. These are the effective rates and thresholds, allowing for the existence of the Low-Income Rebate. These values are used to calculate effective allowances, $a_{c,\ell}$. In New Zealand there are virtually no deductions or allowances. Hence the set of tax thresholds does not change with individuals' incomes, and it is appropriate to set $\eta_{i,v,z} = 0$ in (3).

Consumption tax revenue elasticity calculations require estimates of the ad valorem-equivalent indirect tax rates. Most goods are taxed at the 12.5 per cent Goods and Services Tax (GST) rate. However some expenditures, such as rent and overseas travel, are exempt from GST. Furthermore a number of excise taxes produce very different effective tax rates on goods such as fuel, alcohol and tobacco. The consumption tax rates used are given in the Appendix.

It is necessary to have information about the total expenditure elasticities, at different expenditure levels, of the relevant commodity groups. It is not possible, even if separate income-unit data from budget studies were available, to produce precise individual values since estimates must be based on the cross-sectional variation in budget shares as total expenditure varies.\(^8\) The question therefore arises of the level of disaggregation to be used. The estimates reported here are based on an overall distribution of taxable income and use published budget

\(^7\) Creedy and Gemmell (2003a) illustrate this decomposition for the UK.

\(^8\) It must, as always, be acknowledged that the cross-sectional variations may not necessarily reflect the adjustments to total expenditure changes that would take place over time.
share data derived from average expenditures for a range of goods and income groups, from the 2000–2001 Household Expenditure Survey (HES); further details of the method used are discussed in the Appendix. These are for all households combined, rather than considering different household types separately; however, the methods could be applied to more disaggregated data, where available.

Finally, consumption function parameters, $a$ and $b$ in (13) are required. In view of the considerable difficulty in obtaining reliable information about savings functions, three consumption function cases were examined. These are the no savings case with $a = 1, b = 0$, the proportional savings case where $a = 0.95, b = 0$, and the non-proportional case with $a = 0.85, b = 3000$. The proportional case assumes that 95 per cent of disposable income is spent, while the non-proportional case implies an average propensity to consume of 0.95 at NZ$30,000, which is a little above the arithmetic mean income level.\footnote{The use of a single set of $a$ and $b$ parameters again reflects the high level of aggregation used here, where no attempt is made to distinguish different transfer payments according to household composition.}

Figure 1 shows how the income and consumption tax revenue elasticity (all goods) varies across income levels. This displays the standard property whereby the income tax elasticity generally declines as income rises, with discrete jumps taking place as individuals cross the tax thresholds, reflecting the sharp increase in the marginal rate of income tax.

For the consumption tax elasticity, two examples of the proportional and non-proportional cases are shown. As shown in subsection II i, the shapes of these profiles reflect the three combined effects of the progressivity of the income tax, saving habits and differing expenditure elasticities across goods (combined with their associated ad valorem rates). Income tax progressivity tends to induce a ‘mirror image’ effect in the consumption tax profile via changes in disposable incomes. For example, discrete declines are evident in the consumption tax elasticity profiles in Figure 1 at the income tax thresholds and, at higher income levels, elasticities tend to rise.
However, at lower income levels consumption tax revenue elasticities also decline, rather than showing a mirror image of the decline in the income tax elasticity. This arises because of the dominant effect of declines in the tax-share weighted expenditure elasticities. These elasticities are shown in Figure 2, which reveals substantial declines in tax-share weighted expenditure elasticities for vehicles (mainly fuel) and tobacco as incomes increase from relatively low levels. Since these expenditures face especially high tax rates, changes in these tax-share weighted elasticities dominate changes in the weighted average at low income levels. Figure 1 also reveals that the elasticities produced by the non-proportional consumption function relationship are generally slightly lower, by about 0.1 to 0.2 percentage points, than the proportional equivalents.\textsuperscript{10}

### III. Aggregate Revenue Elasticities

This section examines aggregate tax revenue elasticities, which are the most relevant from the point of view of tax forecasting and planning, along with possible automatic stabilisation properties of the tax structure. Subsection III i presents the basic formulae required and subsection III ii reports results for New Zealand in the case of equiproportional income changes. The implications of non-equiproportional income changes are examined in subsection III iii.

\textit{i) Elasticity formulae}

Aggregate revenue elasticities can readily be expressed as a tax-weighted sum of the individual elasticities. Letting $T_i$ and $Y$ denote respectively total income tax revenue and total income, the aggregate income tax revenue elasticity is:

\[\frac{dT}{dY} = \sum T_i \frac{dT_i}{dY} \]

\textsuperscript{10} Larger differences are found for the aggregate elasticities below.

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Evaluation of the aggregate elasticity therefore requires, in addition to information about the income distribution (for computation of the income tax shares $T_y/T_T$), knowledge of the extent to which individuals’ incomes change when aggregate income changes, reflected in the term $\eta_{y,y}$, where the condition $\frac{1}{N}\sum_{i=1}^{N} \eta_{y_i,y} = 1$ must hold. A typical simplifying assumption is that all incomes increase by the same proportion, so that $\eta_{y,y} = 1$ and the aggregate income tax revenue elasticity is a simple tax-share weighted average of individual values.

The aggregate consumption tax elasticity can be calculated, following (15), using:

$$\eta_{y,y} = \sum_{i=1}^{N} \left( \frac{T_{y_i}}{T_y} \right) \eta_{y_i,y,y}$$

where $T_y$ is aggregate consumption tax revenue. Furthermore, since total revenue is $T = T_y + T_v$, the elasticity of total revenue with respect to aggregate income can be found as a tax-share weighted average of the income and consumption tax revenue elasticities.

**ii) Estimates of aggregate revenue elasticities**

This subsection uses the analytical expressions in subsection 3.1, along with the assumption of equiproportional income changes, to examine how aggregate revenue elasticities vary with aggregate income levels in New Zealand.

As mentioned in Section I, one approach would be to use detailed information, in the form of a large data set containing data on individual taxable incomes. However, the method used here (since such individual data are available only to a highly restricted group of users in New Zealand) is to parameterise the distribution, based on grouped income distribution data, and then to produce a simulated distribution of incomes by taking random draws from the fitted distribution.

Figure 5 in the Appendix shows the New Zealand grouped income distribution in 2001, and discusses the application of a lognormal distribution to summarise the data. It was found that a mean and variance of the logarithms of incomes of $\mu = 9.85$ and $\sigma^2 = 0.7$ provide a reasonable approximation to parameterise a lognormal income distribution. These values imply an arithmetic mean income of $26,903.13$. Each aggregate revenue elasticity is obtained using a simulated population of 20,000 individuals, drawn at random from the distribution. As the results reported here assume that all incomes increase by the same proportion, the relative dispersion of incomes remains constant as incomes change over time. The non-equiproportional case is examined below.

Figure 3 shows aggregate elasticity profiles for income and all consumption taxes (for the non-proportional consumption case), and for total tax revenues, as incomes increase over a

11 Some studies use an entirely numerical approach, by imposing small income increases on each individual in the data set and examining the resulting tax changes, rather than using explicit formulae such as those given above.
12 In Giorno et al. (1995), aggregate income tax revenue elasticities were obtained by fitting lognormal distributions, using information for each country about only the ratios of the first and ninth deciles to the median income. Values of individual elasticities were computed at 16 points on the income distribution. An aggregate elasticity was obtained as the ratio of average (income weighted) marginal rates to that of average (income weighted) average tax rates. (The weights were obtained from the first moment distribution of the associated lognormal distribution). Unfortunately the ratio of averages is not equivalent to the average of ratios, which is the required measure.
13 In the lognormal case, the arithmetic mean income is derived as $\bar{y} = \exp \{ \mu + \sigma^2/2 \}$. 

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wide range of average income levels. This wide range, either side of the 2001 average, has been chosen to examine the sensitivity of the elasticities to extremes, even though revenue forecasts over such an effectively long time period are most unlikely to be made on the assumption of fixed thresholds.

A strong feature of these profiles is that the elasticities are relatively stable, despite the wide range of average incomes considered, involving a substantial movement of individuals from the lower income tax ranges to a situation in which a significant proportion of the income distribution faces the highest marginal income tax rate. The very slight tendency of the income tax elasticity to rise and then decline at higher average income levels is associated with this systematic upward movement of individuals through the thresholds as average income increases. The income and consumption tax elasticities are slightly below 1.3 and 0.9 respectively throughout the range.

Figure 4 shows how consumption tax elasticities depend on assumed saving behaviour. Revenue elasticity estimates are noticeably higher for the proportional consumption and ‘no savings’ cases (profiles A and B), but decline more rapidly as income rises, compared to the non-proportional case in profile C. For example, at mean income levels of around $30,000, elasticities of about 1.0 and slightly below 0.9 are obtained from profiles A and C respectively. Figure 4 also suggests that the effect on the revenue elasticity of ignoring savings is not substantial provided, when income increases, the proportion of income consumed remains approximately constant.

Figure 3. Aggregate tax revenue elasticities

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14 The 20k values were selected randomly from an initial distribution with a lower mean of logarithms than the 2001 distribution. The average income increase, with a fixed variance of logarithms of income, was achieved by increasing all incomes by a fixed proportion each year. The non-equiproportional case involves a more complex process of income change, as shown below.

15 For the highest average income shown, about one quarter of taxpayers are above the top threshold.

16 This result for the aggregate income tax revenue elasticity is consistent with the value obtained by Bell (2003), which uses a numerical approach with a very large sample of individual data from tax records, to which equi-proportional increases are applied.
iii) Non-equiproportional income changes

This subsection relaxes the assumption of equiproportionate income changes, used in the previous subsection and in the vast majority of studies. In line with the present approach of using parametric specifications at a fairly high level of aggregation, subsubsection a) presents a function to describe the systematic variation in $\eta_{y_i}$ with $y_i$. Subsubsection b) presents revised aggregate elasticities based on estimates of the dynamic specification.

a) A specification

It is convenient to specify a functional form for the variation in $\eta_{y_i}$ with $y_i$. A suitable form, involving just one parameter, is:

$$\eta_{y_i} = 1 - (1 - \beta)(\log y_i - \mu)$$

(17)

where $\mu$ is the mean of logarithms of income (the logarithm of geometric mean income). This means that if $\beta < 1$ and $y_i$ is less than geometric mean income, the elasticity, $\eta_{y_i}$, is greater than unity, and vice versa, so that (17) involves equalising changes. If $\beta > 1$, income changes are disequalising. This specification can thus be used to examine the sensitivity of aggregate revenue elasticity measures to variations in the standard assumption of $\eta_{y_i} = 1$.18

In examining changes in total income arising from non-equiproportionate changes, according to equation (17), it is also useful, when increasing the 20,000 simulated incomes from one year to the next, to impose random proportionate income changes, in addition to the systematic equalising or disequalising tendency reflected in $\beta$. Without such changes, annual income

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17 In the case of lognormal income distributions, this is also the median income.

18 For non-equiproportional changes, the aggregate revenue elasticity can now be less than 1; for example it is zero if the only incomes which increase are below a tax-free threshold and none crosses the threshold.
inequality changes too rapidly. The specification in (17) is consistent with the following dynamic process. Let \( y_{it} \) denote individual \( i \)'s income in period \( t \), and let \( \mu_t \) denote the mean of logarithms in period \( t \), with \( g_t = \exp(\mu_t) \) as the geometric mean. The generating process can be written as:

\[
y_{i2} = \left( \frac{y_{i1}}{g_1} \right) ^ \beta \exp(\mu_2 + u_i)
\]

(18)

where \( u_i \) is \( N(0, \sigma_u^2) \). Equation (18) can be rewritten as:

\[
(\log y_{i2} - \mu_2) = \beta (\log y_{i1} - \mu_1) + u_i
\]

Hence the variance of logarithms of income in period 2, \( \sigma_2^2 \), is given by:

\[
\sigma_2^2 = \beta^2 \sigma_1^2 + \sigma_u^2
\]

(20)

The variance of logarithms is therefore constant when \( \sigma_u^2 = \sigma_1^2 (1 - \beta^2) \).

\textit{b) Aggregate elasticities}

Estimation of equation (19) was carried out for a range of pairs of consecutive years during the 1990s, using information from large samples of IR5 and IR3 filers. The results suggest a relatively stable value of \( \beta \) of around 0.85. This reflects a substantial degree of regression towards the (geometric) mean; indeed, with no random component of income change this would have the effect of halving income inequality in as little as three years. If \( \beta = 0.85 \) is combined with \( \sigma_u^2 = 0.194 \), the variance of logarithms of income remains constant over time. These values produce an aggregate income tax revenue elasticity, at 2001 mean income, of about 1.11. For the proportional and non-proportional consumption functions respectively, the aggregate consumption tax revenue elasticities are 0.93 and 0.83, giving corresponding total tax revenue elasticities of 1.05 and 1.03 respectively.

Regression towards the geometric mean therefore reduces the aggregate revenue elasticities. This arises because, for those above the geometric mean income level, the value of \( \eta_{y,Y} \eta_{y,Y} \) is reduced, and \textit{vice versa} for those below the geometric mean. The aggregate elasticity, from (15), is a tax-shares weighted average of these terms, and in view of the fact that \( T(y)/T_Y \) increases as \( y \) increases, the lower values of \( \eta_{y,Y} \eta_{y,Y} \) at the upper income levels dominate.

To give some idea of the sensitivity of results to the variation in \( \beta \), consider a value of \( \beta = 0.9 \), which requires \( \sigma_u^2 = 0.133 \) for a stable degree of income inequality. The aggregate income tax elasticity, again at 2001 mean income, is now 1.17, while the consumption tax elasticities are 0.96 and 0.85 for proportional and non-proportional consumption functions (giving total revenue elasticities of 1.11 and 1.07).

\footnote{In general, the variance of logarithms of income increases if the regression coefficient, \( \beta \), exceeds the correlation between log-incomes in the two periods. If \( \beta < 1 \), the variance of logarithms of incomes eventually becomes stable at \( \sigma^2/(1 - \beta^2) \). On dynamic income specifications, see Creedy (1985).}

\footnote{The regressions were kindly carried out by Matthew Bell.}

\footnote{The degree of regression is expected to be lower for separate cohorts or age groups. Part of the overall regression reported here arises from the systematic component of change associated with age-earnings profiles.}

\footnote{This value is in fact similar to that estimated for IR3 filers, though the values obtained for IR5 filers were lower, at about 0.1. Given that these samples do not constitute all taxpayers, and in practice inequality is relatively stable, it is appropriate here to model a stable variance of logarithms.}

\footnote{Again, using a purely numerical approach to income tax elasticities, using a large sample of longitudinal income data, Bell (2003) found a very similar elasticity.}

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IV. Conclusions

This paper has examined the revenue responsiveness properties of New Zealand income and consumption taxes, based on the 2001 tax structure and expenditure patterns. Using analytical expressions for revenue elasticities at the individual and aggregate levels, together with a simulated income distribution, values for New Zealand were obtained. Treating income growth as equiproportionate, these suggest that the aggregate income and consumption tax revenue elasticities are both fairly constant as mean income increases, at around 1.3 and 1.0 respectively. This latter estimate assumes that increases in disposable income are accompanied by approximately proportional increases in total expenditure. Allowing for non-equiproportionate income growth reduces revenue elasticities to around 1.1 (income tax) and 0.93 (consumption taxes). If there is a tendency for the savings proportion to increase as disposable income increases, a somewhat lower total consumption tax revenue elasticity, of around 0.85 – 0.90, is obtained at mean income levels which approximate current levels in New Zealand. Examination of the tax-share weighted expenditure elasticities for various goods also revealed that, despite the adoption of a broad based GST at a uniform rate in New Zealand, the persistence of various excises has an important effect on the overall consumption tax revenue elasticity, especially for individuals at relatively low income levels.

Appendix: Further Details of the Elasticity Computations

Expenditure elasticities

Expenditure elasticities were obtained using the published summary table of average expenditures over a range of income groups in the 2001 NZ Household Economic Survey (HES), obtained from http://www.stats.govt.nz/. This table divides all households in the sample into $K = 11$ income groups. Within each group the budget shares for each of $n = 58$ commodity groups were obtained (by dividing average expenditure in each category by average total expenditure).$^{24}$ Denote the arithmetic mean total expenditure of the $k$th group by $m_k$ ($k = 1, \ldots, K$) and the budget share of the $i$th commodity group and $k$th total expenditure group by $w_{ki}$ ($i = 1, \ldots, n$).

The raw values of these budget shares cannot be used to obtain elasticities because sampling variations (particularly for low and high income groups) give rise to negative elasticities. Regressions were carried out of the form:

$$w_{ki} = a_{ik} + b_{ik} \log(m_k) + \frac{c_k}{m_k}$$

(21)

for each commodity group, $i$. In addition to generally providing a good fit, this specification has the advantage that weights based on the estimated parameters add to unity.$^{25}$ The smoothed budget shares were then used to calculate the total expenditure elasticities.

Differentiating (21), and dropping the $k$ subscript, gives:

$$\frac{d w_i}{d m} = \frac{b_i y - c_i}{m^2}$$

(22)

$^{24}$ Several commodity groups were excluded on the grounds that they more closely represented savings rather than expenditure. The ratio of averages is of course not the same as the average budget share (though earlier experiments using data for individual households showed that the differences were minor).

$^{25}$ However, it does not guarantee that the predicted weights always lie in the range $0 < w < 1$, though in practice this was not a serious problem; a few negative values at low total expenditure levels for some goods were set to zero and the other shares were adjusted accordingly to maintain the adding-up requirement.
so that \( w_i \) unequivocally falls as \( m \) rises if \( b_i < 0 \) and \( c_i > 0 \); or if \( c_i < 0 \), so long as \( m > c_i/b_i \).

Alternatively, the share rises as income rises (that is, the income elasticity exceeds 1) if \( b_i > 0 \) and \( c_i < 0 \); or if \( c_i > 0 \), so long as \( m > c_i/b_i \).

The coefficient estimates are reported in Tables I and II. The required expenditure elasticities were obtained using:

\[
e_{ki} = 1 + \frac{dw_{ki}}{dm} \frac{m_k}{w_{ki}}
\]

with \( dw/dm \) taken from differentiation of (21).

**Income distribution**

The grouped frequency distribution of taxable income in New Zealand for 2000–2001 is given in Table III. This distribution is for income from all sources, and covers employed and self employed individuals 15 years and older. A histogram of this distribution is shown in Figure 5, where a second mode right at the bottom of the distribution (below $2000 per year) is evident.
### Table II  Budget share regressions

| Category                              | $a$   | $b$   | $c$   | $R$-squared |
|---------------------------------------|-------|-------|-------|-------------|
| Overseas travel                      | −0.47099 | 0.07091 | 28.55522 | 0.787953    |
| Road vehicles                        | 0.291782 | −0.02883 | −31.0849 | 0.652508    |
| Vehicle ownership expenses           | 0.625611 | −0.0743 | −42.92999 | 0.569541    |
| Private transport costs n.e.c.       | −0.01354 | 0.002442 | 1.589636 | 0.058705    |
| Tobacco products                     | 0.166991 | −0.0214 | −8.79842 | 0.620878    |
| Alcohol                              | −0.12924 | 0.021882 | 7.49086 | 0.629366    |
| Medical goods                        | −0.02832 | 0.004974 | 3.036628 | 0.105698    |
| Toiletries and cosmetics             | 0.00956 | 0.000191 | −0.92042 | 0.395319    |
| Personal goods                       | 0.032463 | −0.00257 | −3.42718 | 0.58933     |
| Pets, racehorses and livestock       | 0.120291 | −0.01498 | −7.75913 | 0.470165    |
| Stationery and office equip          | 0.070482 | −0.00593 | −5.10796 | 0.308231    |
| Leisure goods                        | 0.019047 | 0.000662 | 4.02311 | 0.757771    |
| Recreational vehicles                | 0.011469 | −0.00076 | −2.11383 | 0.635397    |
| Goods n.e.c.                         | −0.0119 | 0.002321 | 0.9711 | 0.140835    |
| Health services                      | −0.12371 | 0.020146 | 11.10986 | 0.545345    |
| Personal services                    | −0.10878 | 0.015507 | 9.209476 | 0.352852    |
| Educational and tuition              | 0.026106 | −0.00101 | 1.068944 | 0.17518     |
| Accommodation services               | −0.07752 | 0.011855 | 4.266511 | 0.679455    |
| Fin, insurance and legal             | 0.097855 | −0.00857 | −2.83561 | 0.185043    |
| Vocational services                  | −0.00239 | 0.000818 | −0.75566 | 0.896757    |
| Leisure services                     | −0.03519 | 0.009307 | 0.15564 | 0.86498     |
| Services n.e.c.                      | 0.061889 | −0.00639 | −7.72831 | 0.46135    |
| Outgoings n.e.c.                     | 0.207629 | −0.02421 | −17.3681 | 0.078868    |

**Figure 5.** Income distribution

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These very low annual incomes are likely to be associated with part time casual work by teenagers, or small amounts of interest income accruing to non-beneficiaries. Their (tax-share) contribution to the aggregate revenue elasticity is obviously negligible and there is nothing to be gained by modelling this mode in the present context. For this reason, the pragmatic solution was adopted of adjusting the frequencies in the bottom two groups of the distribution, as shown by smaller marked blocks for those income classes.\footnote{Such bimodal distributions can in fact be modelled using a mixture distribution comprising a weighted average of lognormal and exponential distributions; see Bakker and Creedy (1999).}

The resulting distribution can then be modelled using a unimodal lognormal distribution, as discussed above, whose two parameters can be obtained directly as sample values of the mean and variance of logarithms. Further support for the lognormal is provided by the fact that the implied arithmetic mean value (using the properties of the lognormal mentioned earlier) was found to be close to the arithmetic mean calculated directly from the distribution.

**Tax rates**

*Ad valorem* tax-exclusive indirect tax rates are required for the same commodity groups as listed in Tables I and II. In most cases the appropriate rate is simply the GST rate of 0.125. For

\begin{table}
\centering
\begin{tabular}{|r|c|c|c|c|c|}
\hline
\textbf{Range} & \textbf{no.} & \textbf{Average} & \textbf{Range} & \textbf{no.} & \textbf{Average} \\
\textbf{(000s)} & & \textbf{Income} & \textbf{(000s)} & & \textbf{Income} \\
\hline
\text{-1} & 93,213 & 409.83 & 30–31 & 41,890 & 30,414.00 \\
\text{1–2} & 61,912 & 1,452.57 & 31–32 & 34,770 & 31,507.18 \\
\text{2–3} & 41,329 & 2,521.56 & 32–33 & 44,251 & 32,477.18 \\
\text{3–4} & 44,486 & 3,544.49 & 33–34 & 38,084 & 33,533.93 \\
\text{4–5} & 36,385 & 4,552.98 & 34–35 & 33,849 & 34,534.91 \\
\text{5–6} & 43,644 & 5,437.80 & 35–36 & 30,837 & 35,526.43 \\
\text{6–7} & 32,442 & 6,580.17 & 36–37 & 27,936 & 36,512.18 \\
\text{7–8} & 55,677 & 7,495.72 & 37–38 & 34,878 & 37,548.02 \\
\text{8–9} & 67,115 & 8,371.65 & 38–39 & 31,082 & 38,498.93 \\
\text{9–9.5} & 66,408 & 9,267.31 & 39–40 & 25,273 & 39,483.37 \\
\text{9.5–10} & 42,052 & 9,707.22 & 40–41 & 27,987 & 40,563.92 \\
\text{10–11} & 143,426 & 10,669.95 & 41–42 & 30,355 & 41,480.31 \\
\text{11–12} & 89,621 & 11,513.70 & 42–43 & 17,037 & 42,551.03 \\
\text{12–13} & 74,977 & 12,571.58 & 43–44 & 19,855 & 43,478.16 \\
\text{13–14} & 80,137 & 13,429.31 & 44–45 & 20,373 & 44,647.87 \\
\text{14–15} & 163,260 & 14,396.08 & 45–46 & 15,027 & 45,453.50 \\
\text{15–16} & 53,649 & 15,422.00 & 46–47 & 17,132 & 48,471.26 \\
\text{16–17} & 52,664 & 15,658.82 & 47–48 & 15,720 & 49,700.59 \\
\text{17–18} & 38,984 & 17,472.11 & 48–49 & 28,662 & 51,011.51 \\
\text{18–19} & 48,558 & 18,469.33 & 49–50 & 26,642 & 52,891.96 \\
\text{19–20} & 46,773 & 19,566.15 & 50–52 & 12,519 & 54,980.98 \\
\text{20–21} & 47,134 & 20,556.02 & 52–54 & 19,947 & 56,876.89 \\
\text{21–22} & 37,598 & 21,524.06 & 54–56 & 24,379 & 59,208.08 \\
\text{22–23} & 37,181 & 22,572.31 & 56–58 & 19,203 & 59,208.08 \\
\text{23–24} & 35,388 & 23,539.56 & 58–60 & 19,203 & 59,208.08 \\
\text{24–25} & 32,699 & 24,518.45 & 60–65 & 52,194 & 62,407.42 \\
\text{25–26} & 31,564 & 25,449.60 & 65–70 & 24,748 & 67,319.34 \\
\text{26–27} & 30,796 & 26,333.18 & 70–80 & 43,913 & 74,481.48 \\
\text{27–28} & 37,842 & 27,465.45 & 80–90 & 22,783 & 85,128.34 \\
\text{28–29} & 35,810 & 28,866.43 & 90–100 & 19,963 & 94,708.39 \\
\text{29–30} & 35,872 & 29,622.28 & Over 100 & 47,218 & 182,074.25 \\
\hline
\end{tabular}
\caption{Grouped distribution of taxable income: New Zealand}
\end{table}
rent and overseas travel, the rate is zero. In other cases, particularly where excises are involved, the computation of an effective *ad valorem* rate is complicated by the use of unit taxes, in combination with GST, and by the need to consolidate a wide variety of goods into a single category. The non-zero rates differing from the standard GST rate are as follows: road vehicles 0.07054; vehicle ownership expenses 0.58642; tobacco products 2.39845; alcohol 0.46819; recreational vehicles 0.0625; financial, insurance and legal services 0.0625; and expenditure not elsewhere included 0.23. For details on the computation of these rates, see Young (2002).

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