IMPLICATIONS OF PLANCK AND MAP MEASUREMENTS
ON SPARTICLE SPECTRA

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Future satellite (MAP and Planck), balloon and ground based experiments will
determine the basic cosmological parameters within a few percent. We examine
here the effect of this on constraining the SUSY parameter space for supergravity
R-parity conserving models (with \( \tan \beta \leq 25 \)) for the cases of \( \nu \)CDM and \( \Lambda \)CDM
cosmological models. For the \( \nu \)CDM (\( \Lambda \)CDM) models, the gluino mass is restricted
by \( m_{\tilde{g}} < \sim 720(540) \) GeV. In both cases, the cosmological constraints are sensitive to
non-universal SUSY soft breaking producing a lower bound \( m_{\tilde{g}} > \sim 400 \) GeV in some
regions and for the \( \nu \)CDM model, gaps in the allowed \( m_{\tilde{g}} \) range for other regions.
For gluino (neutralino) masses greater than 450(65) GeV, \( m_0 \) is constrained to be
small making squark and slepton masses generally light and determined mostly by
\( m_{\tilde{g}} \).

1 Introduction

One of the significant phenomena that has arisen with the development of
supersymmetry has been the deepening of the connection between particle
physics and cosmology. While this connection has always been present (e.g.
from the early nuclear synthesis calculations \( \Leftrightarrow \) supersymmetry (SUSY) now
offers models that reach up in energy to the GUT scale \( M_G \approx 10^{16} \) GeV (and
perhaps further to the Planck scale) and backwards in time to the very early
universe. In particular, models with R-parity invariance automatically predict
the existence of dark matter, and for a reasonable range of SUSY parameters, in
an amount comparable to what is observed astronomically. These models have
been waiting to be tested, and there are now a large number of experiments
that are currently on line or will be on line in the relatively near future that
will determine whether supersymmetry is a valid theory of nature.

While up to now the flow of information has been mostly from particle
physics to cosmology, the new satellite experiments \( \Leftrightarrow \) MAP and Planck as well
as about 25 balloon and ground based experiments can reverse this and can
give constraints on what might be expected at accelerators. These astronomical
experiments will measure the cosmological parameters with great accuracy.
One uses the parameter \( \Omega_i = \rho_i/\rho_c \) where \( \rho_i \) is the density of matter of type
and \( \rho_c = 3H^2/8\pi G_N \) (\( H \) = Hubble constant, \( G_N \) = Newton’s constant) to measure the mean amount of matter of type \( i \) in the universe. Current measurements give \( H = h \) (100 km s\(^{-1}\)Mpc\(^{-1}\)) with 0.5 \( \lesssim h \lesssim 0.75 \) (and hence \( \rho_c = 1.88 \times 10^{-29} h^2 \text{gm/cm}^3 \)). For a number of cosmological models one finds for cold dark matter (CDM) the range

\[
0.1 \leq \Omega_{CDM} h^2 \leq 0.4
\]  

(1)

and this range of CDM is the one assumed in many particle physics calculations. However, MAP and Planck measurements will greatly restrict this window, and impose strong constraints on the SUSY parameter space, and in this way restrict the SUSY mass spectrum. Thus astronomical measurements will impinge upon what is expected to be seen at accelerators.

2 Supergravity Models

In order to analyse dark matter predictions we need a well-defined SUSY model. We will use here supergravity grand unified models with gravity mediated SUSY breaking (at a scale \( \gtrsim M_G \)) in a hidden sector, and with gravity being the messenger of SUSY breaking to the physical sector.\(^3\) This gives rise to the following types of soft breaking masses at \( M_G \): \( m_0 \) (scalar soft breaking mass), \( m_{1/2} \) (gaugino soft breaking mass), \( A_0 \) (cubic soft breaking parameter), \( B_0 \) (quadratic soft breaking parameter), as well as non-universal soft breaking terms.\(^5\) In addition, a Higgs mixing parameter, \( \mu_0 \), for the two Higgs multiplets \( H_1 \) and \( H_2 \) (\( W^{(2)} = \mu_0 H_1 H_2 \)) can form naturally with \( \mu_0 \approx M_S \).

In the following we will assume that the gaugino masses are universal at \( M_G \) (as would be true to a good approximation for a simple GUT group). Over much of the parameter space \( \mu^2 \gg M_Z^2 \) which leads to the “scaling relations” for the charginos \( (\chi_{1 \pm}, i = 1, 2) \) and neutralinos \( (\chi_{0i}, i = 1 - 4) \):

\[
2m_{\chi_{10}} \approx m_{\chi_{1+}} \approx m_{\chi_{02}} \approx (\frac{3}{4} - \frac{1}{4})m_\tilde{g}.
\]

We assume here that the first two generations of squarks and sleptons are degenerate at \( M_G \) (to suppress flavor changing neutral currents) with common mass \( m_0 \) and parametrize the Higgs and third generation soft breaking masses at \( M_G \) as:

\[
m_{H_1}^2 = m_0^2(1 + \delta_1); \quad m_{H_2}^2 = m_0^2(1 + \delta_2)
\]

(2)

\[
m_{\tilde{q}_L}^2 = m_0^2(1 + \delta_3); \quad m_{u_R}^2 = m_0^2(1 + \delta_4); \quad m_{c_R}^2 = m_0^2(1 + \delta_5)
\]

(3)

\[
m_{\tilde{d}_R}^2 = m_0^2(1 + \delta_6); \quad m_{\tilde{l}_L}^2 = m_0^2(1 + \delta_7)
\]

(4)

Here \( q_L \equiv (\tilde{u}_L, \tilde{d}_L) \) is the \( L \) quark doublet, \( u_R \) the \( R \) up-squark singlet, etc. We limit our parameters to the domain \( m_0, m_\tilde{g} \leq 1 \text{ TeV}; \ |A_t/m_0| \leq \text{etc.} \)
7: tan β ≤ 25; |δ_i| ≤ 1 where A_t is the t-quark A-parameter at the electroweak scale. These represent a choice of “naturalness” conditions. Note that for tan β ≤ 25, δ_5, δ_6, δ_7 make only small contributions and can be neglected. (They would become important for larger tan β.)

The RGE then give for μ^2 at scale M_Z the result:

\[
\mu^2 = \frac{t^2}{t^2 - 1} \left[ \left\{ \frac{1 - 3D_0}{2} + \frac{1}{t^2} \right\} + \left\{ 1 - 2D_0 \right\} (\delta_3 + \delta_4) - \frac{1 + D_0}{2} \delta_2 + \frac{1}{t^2} \delta_1 \right] m_0^2
\]

\[+ \frac{t^2}{t^2 - 1} \left[ \frac{1}{2} \right] (1 - D_0) A_R^2 + C_β m_β^2 \right\} - \frac{1}{2} M_Z^2
\]

\[+ \frac{1}{22} t^2 + \frac{1}{1} \left( 1 + \frac{α_1}{α_G} \right) S_0 + \text{loop terms;} \quad t \equiv \tan β
\]

Here D_0 ≈ 1 - (m_t/200 sin β)^2, A_R ≈ A_t - 0.613m_β, S_0 = TrY m^2 (Y = hypercharge, m^2 = masses at M_G) and C_β is given in Ibañez et al. D_0 vanishes at the t-quark Landau pole and hence for m_t = 175 GeV is generally small: D_0 ≤ 0.23. (A_R is the residue at the Landau pole.) Note that the choice δ_2 < 0, δ_1 > 0 and δ_3 + δ_4 > 0 will increase the size of μ^2, while δ_2 > 0, δ_1 < 0 and δ_3 + δ_4 < 0 will decrease the size of μ^2. In the following we will see that μ^2 plays a key role in dark matter predictions, and the effects of these particular sign possibilities of δ_i will allow a qualitative understanding of the phenomena.

3 Direct Detection of Dark Matter

We review briefly in this section the direct detection of dark matter particles in the Milky Way incident on a terrestrial detector. The density of such matter is estimated at ρ_{DM} ≈ 0.3 GeV/cm^3 with an impinging velocity of v_{DM} ≈ 300 km/s. Calculation of detector event rates proceeds in two steps. One first calculates the expected χ_0^1 relic density of Ω_{χ_0^1}h^2, left after annihilation in the early universe. The size of Ω_{χ_0^1}h^2 varies as one moves across the SUSY parameter space. In particular, for m_{χ_0^1} ≤ 65 GeV (or by the scaling relations, m_β ≤ 450 GeV) the annihilation cross section is dominated by the s-channel Z and h poles and requires sensitive treatment. For m_{χ_0^1} ≥ 65 GeV, the t-channel squark and slepton poles become dominant. These two regimes will show up below in the detector event rates.

One first restricts the SUSY parameter space so that Ω_{χ_0^1}h^2 falls within the allowed window of the cosmological model under consideration. In addition, one restricts the parameter space so that accelerator bounds are obeyed. One then calculates the scattering of incident Milky Way χ_0^0 by quarks in a nuclear
target within the above restricted parameter space, giving rise to detector event rates $R$ measured in $\text{events/kg d}$.

4 Cosmological Models

Current astronomical measurements have sufficient uncertainty that they allow for a large variety of cosmological models. However, the Planck and MAP satellites will be able to determine the basic cosmological parameters, $H$, $\Lambda$, $\Omega$ etc. to $(1-10)\%$ accuracy by measurements of the deviations $\Delta T$ from the CMB temperature $T_0 = (2.728 \pm 0.002) ^\circ K$. Different cosmological models predict different angular correlations of the CMB deviations allowing for experimental determination of the cosmological model. We consider here two possible cosmological models that might result from such measurements, and examine what consequences these might have on the sparticle spectra, and hence on accelerator searches for supersymmetry.

(i) $\nu$CDM Model. If neutrinos have masses of $O(eV)$ they could furnish the hot dark matter (HDM) of the universe, $\Omega_\nu$. In addition there is baryonic dark matter ($B$) and CDM (the neutralinos $\chi^0_1$) such that the total $\Omega = 1$. As an example we assume that the satellite measurements determine central values to be $\Omega_\nu = 0.2$, $\Omega_B = 0.05$, $\Omega_{CDM} = 0.75$ and a Hubble constant of $h = 0.62$. Using estimates of the errors with which these quantities can be measured by the Planck satellite, we find

$$\Omega_{CDM} h^2 = 0.288 \pm 0.013 \quad (6)$$

This shows the remarkable accuracy future determinations of cosmological parameters are capable of when compared with the current knowledge of Eq. (1).

(ii) $\Lambda$CDM Model. As a second model we assume the existence of a cosmological constant $\Lambda$ along with baryonic and CDM, and assume that Planck has determined the central values of these parameters to be $\Omega_B = 0.05$, $\Omega_{CDM} = 0.40$, $\Omega_\Lambda = 0.55$, and $h = 0.62$. Using the estimates of the errors in the Planck measurements, one finds here that

$$\Omega_{CDM} h^2 = 0.154 \pm 0.017 \quad (7)$$

While the above models are only two possible examples, they represent bounds on what might actually exist. The important point is that the future balloon and satellite measurements will be able to determine these parameters well, and the above models represent limits within current knowledge.
5 νCDM Model

We consider first universal soft breaking where by Eqs. (3)-(4), the δ₁ = 0. Fig. 1 plots the maximum and minimum event rates for Xe detector as a function of the gluino mass [m_{\tilde{g}} \cong (7 - 8)m_{\chi_1^0}] by the scaling relations for a 1 std band of Eq. (3). One may compare this with Fig. 2 of where only the broad band of Eq. (3) is imposed. One sees a reduction in event rates in the region m_{\tilde{g}} \lesssim 450 \text{ GeV} (i.e. m_{\chi_1^0} \lesssim 65 \text{ GeV}), the Z and h pole dominated region in the early universe annihilation, and a corresponding increase for the higher masses. More significant is the appearance of forbidden regions, i.e. gaps, in the allowed values of m_{\tilde{g}} in the region m_{\tilde{g}} \gtrsim 500 \text{ GeV} and m_{\tilde{g}} \gtrsim 600 \text{ GeV}. In examining the effects of non-universal soft breaking, we note the correlation that when \mu^2 is decreased (which by Eq. (5) occurs for \delta_2 = 1 = -\delta_1) the event rate \dot{R} is increased and when \mu^2 is increased (\delta_2 = -1 = -\delta_1) \dot{R} is decreased. Fig. 2 shows these effects for the latter case, and also that the forbidden regions below m_{\tilde{g}} = 500 \text{ GeV} are significantly widened. For the alternate possibility (\delta_2 = 1 = -\delta_1), the event rates increase. The maximum rates for this case are at the sensitivity of the current NaI detectors, and detectors with a sensitivity of \dot{R} \gtrsim 10^{-3} \text{ events/kg d} would cover the entire parameter space. The gaps of the previous case have now disappeared. However, m_{\tilde{g}} is now bounded from below i.e. m_{\tilde{g}} \gtrsim 420 \text{ GeV}. In all of these cases there is
also an upper bound on $m_{\tilde{g}}$ of $m_{\tilde{g}} \lesssim 720 \text{ GeV}$. Thus cosmological constraints bound the allowed gluino mass range, and are sensitive to the non-universal soft breaking.

The large $t$-quark mass causes large $L - R$ mixing in the stop ($mass)^2$ matrix, making the light eigenvalue, $m_{\tilde{t}_1}^2$, small. In general $m_{\tilde{t}_1}$ is governed by the size of $m_0$ and in the light neutralino domain $m_0$ can be large, i.e. $m_{\tilde{t}_1}$ can rise to $\approx 600 \text{ GeV}$. However, for heavy neutralinos ($m_{\tilde{\chi}_1^0} \lesssim 65 \text{ GeV}$, $m_{\tilde{g}} \lesssim 450 \text{ GeV}$) the early neutralino annihilation is governed by the $t$-channel sfermion poles, and in order to get sufficient annihilation for Eq. (6) to hold, $m_0$ must be small i.e. $m_0 \lesssim 200 \text{ GeV}$. One finds in this domain that $m_{\tilde{t}_1}$ is generally an increasing function of $m_{\tilde{g}}$ with mass ranging from 250 GeV to 500 GeV.

6 $\Lambda$CDM Model

Here $\Omega_{\chi^0} h^2$ obeys Eq. (7) which even more sharply restricts the SUSY parameter space. Fig. 3 shows the maximum and minimum expected event rates for a Xe detector for different values of $\delta_i$. While one has the same general behavior as in the $\nu$CDM model, there are interesting differences that distinguish the two. Thus for this case for the 1 std window of Eq. (7), one finds a much tighter upper bound of $m_{\tilde{g}} \lesssim 520 \text{ GeV}$. There is also a lower bound $m_{\tilde{g}} \gtrsim 400 \text{ GeV}$ for $\delta_2 = 1 = -\delta_1$ (dashed curve), sharply restricting the gluino
Figure 3: Maximum and minimum event rates for a xenon CDM detector vs. gluino for \( \Lambda_{CDM} \) model (1 std window), \( \mu > 0 \), for universal soft breaking (solid curve) and non-universal soft breaking \( \delta_1 = 1 = -\delta_2 \) (dotted curve), \( \delta_1 = -1 = -\delta_2 \) (dashed curve). From Ref. [16].

The squarks and sleptons are again constrained to be relatively light in the domain \( m_{\tilde{\chi}} \gtrsim 420 \text{ GeV} \) since here Eq. (7) requires \( m_{\tilde{\chi}} \lesssim 100 \text{ GeV} \). Thus \( m_{\tilde{t}_1} \) ranges from \((200 - 400) \text{ GeV}\) in this domain and for the first two generations of squarks, \( m_{\tilde{q}} \) ranges from \(400 \text{ GeV}\) to \(500 \text{ GeV}\) and approximately scales with \( m_{\tilde{g}} \). The lightest slepton, \( \tilde{e}_R \) can have a mass as low as \((85 - 90) \text{ GeV}\) in this region, at the edge of detectability of LEP 190.

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References

1. E. M. Burbidge, G. R. Burbidge, W. A. Fowler and F. Hoyle, Rev. Mod. Phys. 29, 547 (1957); P. J. E. Peebles, Ast. J. 146, 542 (1966); R. V. Wagoner, W. A. Fowler and F. Hoyle, Ap. J., 148, 3 (1967).
2. http://map.gsfc.nasa.gov/
http://astro.estec.esa.nl:80/SA-general/Projects/Cobras/cobras.html
3. A. H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49,
970 (1982). For reviews see P. Nath, R. Arnowitt and A. H. Chamseddine, *Applied N=1 Supergravity* (World Scientific, Singapore, 1984); H. P. Nilles, *Phys. Rep.* 110, 1 (1984); R. Arnowitt and P. Nath, Proc. of VII J.A. Swieca Summer School ed. E. Eboli and V. O. Rivelles (World Scientific, Singapore, 1994).

4. R. Barbieri, S. Ferrara and C. A. Savoy, *Phys. Lett. B* 119, 343 (1982); L. Hall, J. Lykken and S. Weinberg, *Phys. Rev. D* 27, 2359 (1983); P. Nath, R. Arnowitt and A. H. Chamseddine, *Nucl. Phys. B* 227, 121 (1983).

5. S. K. Soni and H. A. Weldon, *Phys. Lett. B* 126, 215 (1983); V. S. Kaplunovsky and J. Louis, *Phys. Lett. B* 306, 268 (1993).

6. R. Arnowitt and P. Nath, *Phys. Rev. Lett.* 69, 725 (1992). P. Nath and R. Arnowitt, *Phys. Lett. B* 289, 368 (1992).

7. K. Inoue et al. *Theor. Phys.* 68, 927 (1982); L. Ibañez and G. G. Ross, *Phys. Lett. B* 110, 227 (1982); L. Alvarez-Gaumé, J. Polchinski and M. B. Wise, *Nucl. Phys. B* 221, 495 (1983); J. Ellis, J. Hagelin, D. V. Nanopoulos and K. Tamvakis, *Phys. Lett. B* 125, 2275 (1983); L. E. Ibañez and C. Lopez, *Nucl. Phys. B* 233, 545 (1984); L.E. Ibañez, C. Lopez and C. Muñios, *Nucl. Phys. B* 256, 218 (1985).

8. P. Nath and R. Arnowitt, *Phys. Rev. D* 56, 2820 (1997).

9. For a review see G. Jungman, M. Kamionkowski and K. Greist, *Phys. Rep.* 267, 195 (1995); E. W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading, 1989).

10. K. Greist and D. Seckel, *Phys. Rev. D* 43, 3191 (1991); P. Gondolo and G. Gelmini, *Nucl. Phys. B* 360, 145 (1991).

11. R. Arnowitt and P. Nath, *Phys. Lett. B* 299, 58 (1993); 303, 403 (1993) (E); P. Nath and R. Arnowitt, *Phys. Rev. Lett.* 70, 3696 (1993); H. Baer and M. Brhlick, *Phys. Rev. D* 53, 597 (1996); V. Barger and C. Kao, hep-ph/9704403.

12. For a review see Ref.[9].

13. A. Kosowsky, M. Kamionkowski, G. Jungman and D. Spergel, *Nucl. Phys. Proc. Suppl.* 51B, 49 (1996).

14. S. Dodelson, E. Gates and A. Stebbins, *Ap. J.* 467, 10 (1996).

15. A. Bottino et al, *Phys. Lett. B* 402, 113 (1997).

16. P. Nath and R. Arnowitt, hep-ph/9801xxx.