Spectral correlations of phase modulated entangled photons

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Abstract. We investigate spectral properties of entangled photons in EPR-pair, which interacts with phase modulators. In three mode approximation we got the exact analytical expressions for their common state evolution and calculated probabilities of coincidence between detectors clicks.

1. Introduction
Phase modulation (PM) process of quantized radiation have being actively investigated in last decades due to the development of optical communication systems. Quantum transmission lines require the possibility to control and shape wave packets at the single-photon level. Classical theory of modulation does not fit for this problem since it operates with infinity number of photons. A new theory of electromagnetic field modulation process was needed.

The foundation in this area was formed by works [1], [2], [3] and [4], where different approaches were suggested. It was shown in [3] that PM allows the complete control of single-photon wave packets. Nonclassical correlations between spectral modes of modulated wave packets is now at the most interest. Nonlocal spectral properties of entangled photon pairs were investigated in [6, 7]. In [8] the protocol of joint spectrum tunable control was suggested. Experimental study of group velocity dispersion effect on an entangled two-photon wave packet was reported in [9]. Statistical properties of field quantum state interacted with PM were studied in detail in [4].

Here using effective Hamiltonian method we investigate analytically modulation process of entangled photons in the system similar to that was studied in [2]. Restricted by three mode approximation we analyze probabilities of single and double clicks of detectors, monitoring single photon wave packets at each of three choosing modes. In our scheme polarization entangled photons (with horizontal and vertical states of polarizations) are generated due to spontaneous down-conversion process (see fig. (1)) and transmitted to the detector part with optical waveguides in use. Passing through these waveguides photons interacts with PM’s $PM_1$ and $PM_2$ setting in each channel. Just after this interaction the photons are detected. The aim of this work is to obtain analytical formulas for joint probabilities of detectors clicks and calculate conditional states of modes in one channel using the measurement data from another.

2. The model of PM
According to [4] Hamiltonian of phase modulation process with phase $\varphi$ may be written as

$$H = \omega_{opt}N + \Omega A_0 + \frac{2\gamma}{3} \left( e^{-i\varphi} e^{-i\tilde{\Omega}t} A_+ + e^{i\varphi} e^{i\tilde{\Omega}t} A_- \right),\tag{1}$$
Figure 1. a Scheme of optical phase modulation process in the cavity; b Nonlocal phase modulation of polarization-entangled photons pair generated by nonlinear crystal (NC). Each photon is transmitted by optical waveguide and interacts with phase modulator. After that the probability of detectors clicks coincidence is monitored

where $\omega_{mod}$ is the frequency of modulated cavity field, $\tilde{\Omega}$ is the modulation frequency, $\Omega$ is frequency difference between adjacent optical modes and $\gamma$ is mode interaction parameter. The operators $A_+$, $A_-$, $A_0$ and $N$ are defined as

$$A_+ = \sum_{m=-1}^{0} f(m) a_m a_{m+1}^\dagger, \quad A_- = \sum_{m=-1}^{0} f(m) a_m^\dagger a_{m+1},$$

$$A_0 = \sum_{m=-1}^{1} m a_m a_m^\dagger, \quad N = \sum_{m=-1}^{1} a_m a_m^\dagger. \quad (3)$$

Here $m \in \{-1, 0, 1\}$ is the index of mode relative to the center the cavity spectrum (with mode number $\tilde{m}$), $f(m)$ is overlap function of the interacted modes. For our three mode approximation we assumed $f(m) = \sqrt{(2+m)(1-m)}$, which allows to $A_+$, $A_-$, $A_0$ to be generators of $SU(2)$ algebra:

$$[A_0, A_\pm] = A_\pm, \quad [A_+, A_-] = 2A_0. \quad (4)$$

Also it is worthwhile to note that $N$ commute with all these generators, which follows form the fact that spectrum of $N$ is degenerate. Then evolution operator corresponded to Hamiltonian (1) may has the form

$$U(t) = e^{-i\left(\tilde{\Omega} t + \omega \tilde{m} N + A_0\right)} e^{-i\frac{\gamma}{3} \left( A_+ + A_- \right)} e^{-iQ t} e^{iQ (\tilde{m} N + A_0)}, \quad (5)$$

where $Q$ is the operator of "quasienergy"

$$Q = \omega (\tilde{m} N + A_0) + \frac{2\gamma}{3} (A_+ + A_-), \quad (6)$$

and $\omega = \Omega - \tilde{\Omega}$. Operator $Q$ may be diagonalized by unitary transformation $U_0$

$$U_0 Q U_0^\dagger = 2\Gamma A_0 + \omega \tilde{m} N, \quad (7)$$

$$U_0 = \exp\left(-\frac{\pi}{4} \left( A_+^\dagger - A_-^\dagger \right) \right), \quad (8)$$

where we use resonance condition $\omega = 0$ and $\Gamma = \left|\frac{2\gamma}{3}\right|$. 


3. Phase modulation of entangled state

Here it is assumed EPR state of photons polarization at the input of the modulation scheme. Denoting horizontal and vertical state of polarization by $H$ and $V$ correspondingly this state may be written as

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}[(010)_1|000\rangle_{1V}|000\rangle_{2H}|010\rangle_{2V} + |000\rangle_{1H}|010\rangle_{1V}|010\rangle_{2H}|000\rangle_{2V}], \quad (9)$$

where $|ijk\rangle_{n,\alpha} = |i\rangle_1|j\rangle_m|k\rangle_r$, $i,j,k \in \{0,1\}$, $l,m,$ and $r$ are labels of modes corresponded to indexes $-1(l), 0(m)$ and $1(r)$, $n \in \{1,2\}$ is number of the channel and $\alpha \in \{H,V\}$ is state of polarization.

State of the photon pair after interaction with PM’s is taken by the formula

$$|\Psi_{PM}\rangle = U_2(t)U_1(t)|\Psi_0\rangle = \frac{1}{\sqrt{2}}[(000)_{2H}|\Phi\rangle_{2V}|\Phi\rangle_{1H}|000\rangle_{1V} +$$

$$+|\Phi\rangle_{2H}|000\rangle_{2V}|000\rangle_{1H}|\tilde{\Phi}\rangle_{1V}]. \quad (10)$$

Here $U_n(t)$ is evolution operator for $n$-th phase modulator and

$$|\Phi\rangle_{n,\alpha} = U(t)|010\rangle_{n,\alpha} = \left(ie^{-i\tilde{\varphi}_n}\sin(2\Gamma t)\right)|000\rangle_{n,\alpha} +$$

$$+ \left(\sqrt{2}e^{-3i\tilde{\varphi}_n}\cos(2\Gamma t)\right)|010\rangle_{n,\alpha} +$$

$$+ \left(ie^{-2i\tilde{\varphi}_n}\sin(2\Gamma t)\right)|001\rangle_{n,\alpha}. \quad (11)$$

Thus probabilities of detection photon in certain mode are given by

$$P_l = P_r = |\sin(2\Gamma t)|^2, \quad P_m = 2|\cos(2\Gamma t)|^2. \quad (12)$$

4. Joint probabilities of detection events

In Table 1 we present correlations between detectors clicks for each pair of modes in two channels. Namely, probability to get the clicks from detectors $D_x$ and $D_y$, where $x \in \{l_1, m_1, r_1\}$ and $y \in \{l_2, m_2, r_2\}$ may be calculates as

$$P_{x,y} = \left\langle a_x^\dagger a_x a_y^\dagger a_y \right\rangle = \langle \Psi_{PM}|a_x^\dagger a_x a_y^\dagger a_y |\Psi_{PM}\rangle. \quad (13)$$

For instance, if detector $D_{l_1}$ clicks in the first channel, one can obtain probability distribution to get detector click in the second channel. This distribution showed in the first column of Table 1. These probabilities as a functions of time is presented on fig. 2.

|       | $l_1$       | $m_1$       | $r_1$       |
|-------|-------------|-------------|-------------|
| $l_2$ | $\frac{1}{4}\sin^4(2\Gamma t)$ | $\frac{1}{4}\sin^2(2\Gamma t)\cos^2(2\Gamma t)$ | $\frac{1}{4}\sin^4(2\Gamma t)$ |
| $m_2$ | $\frac{1}{2}\sin^2(2\Gamma t)\cos^2(2\Gamma t)$ | $\cos^4(2\Gamma t)$ | $\frac{1}{2}\sin^2(2\Gamma t)\cos^2(2\Gamma t)$ |
| $r_2$ | $\frac{1}{4}\sin^4(2\Gamma t)$ | $\frac{1}{4}\sin^2(2\Gamma t)\cos^2(2\Gamma t)$ | $\frac{1}{4}\sin^4(2\Gamma t)$ |

Table 1. Matrix of joint probabilities for $D_x$ and $D_y$ detectors clicks.

In particular, one can find from here that conditional probabilities to detect photon in sideband modes $r$ and $l$ are equal which allow to determine if another part of EPR-pair is detected in central mode or in one of the sideband modes.
Figure 2. Joint probabilities as functions of time: solid green line is for $P_{m,l} = P_{m,r} = P_{l,m} = P_{r,m}$, dashed blue line is for $P_{l,l} = P_{r,l} = P_{r,r}$ and dashed-pointed red line is for $P_{m,m}$.

5. Conclusion

In the present work we investigated analytically process of phase modulation of entangled photons. Using effective Hamiltonian approach and three mode approximation we studied conditional state of modes and spectral properties of one photon after detection its counterpart in another channel. We found that the scheme under consideration in some cases does not allow separation between sidebands. There are correlations between spectra of two entangled photons, but using detection result from one channel one can’t separate statistical which mode was detected at another channel. Also we found that proposed measurement scheme drops out phase of modulator and phase difference between two modulators. Spectral correlations of two entangled photons may likely used for quantum key distribution protocol, but it requires more improved measurement scheme.

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