Weighing of the Dark Matter at the Center of the Galaxy

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A promising method for measuring the total mass of the dark matter near a supermassive black hole at the center of the Galaxy based on observations of nonrelativistic precession of the orbits of fast S0 stars together with constraints on the annihilation signal from the dark matter particles has been discussed. An analytical expression for the precession angle has been obtained under the assumption of a power-law profile of the dark matter density. In the near future, modern telescopes will be able to measure the precession of the orbits of S0 stars or to obtain a strong bound on it. The mass of the dark matter necessary for the explanation of the observed excess of gamma radiation owing to the annihilation of the dark matter particles has been calculated with allowance for the Sommerfeld effect.

Significant advances have been achieved in recent years in the observations of stars gravitationally connected to the supermassive black hole SgrA\textsuperscript{*} at the center of the Galaxy. Several so-called S0 stars, which move at very high velocities (> 10\textsuperscript{3} km/s) in almost elliptic orbits around a very compact supermassive object, are observed in the infrared range\textsuperscript{1–4}. The mass of the dark matter necessary for the explanation of the observed excess of gamma radiation owing to the annihilation of the dark matter particles has been calculated with allowance for the Sommerfeld effect.

The existence of fast S0 stars provides a unique possibility of reconstructing the gravitational potential and measuring the mass distribution at the center of the Galaxy by fitting their orbits. The authors of\textsuperscript{1,8} performed a detailed multiparametric fitting of the orbits of several S0 stars and calculated the additional distributed mass with various exponents of the density profile. It was shown that the distributed mass within the orbit of the S0-2 star is no more than 3 – 4 % of the mass of the supermassive black hole. It is noteworthy that the expected measurement of the nonrelativistic precession of the orbit of the S0-2 star will allow either improving the indicated bound on the distributed dark mass by two or three orders of magnitude or determining this dark mass. We discuss and develop a method for studying the distribution of the dark matter at the center of the Galaxy by measuring the precession angle of orbits of S0 stars. For a number of particular cases, numerical calculations of the precession angle of orbits of S0 stars because of the extended mass distribution were performed\textsuperscript{14,19}. We obtained general analytical formulas for the precession of orbits of stars with a powerlaw profile of the dark matter; these formulas make it possible to easily determine the additional distributed mass from the measured precession angle.

An additional independent method for determining the distribution of the dark matter is the search for a possible annihilation signal from the center of the Galaxy. The explanation of the excess of a gamma signal with an energy of ~ 1 TeV from the center of the Galaxy observed by the HESS telescope by gamma annihilation of dark matter particles with allowance for constraints on the dynamics of stars for the case of the power-law density profile of the dark matter with a spike and an exponent as a free parameter was analysed in\textsuperscript{21}. The possibility of constraints on annihilation based on the dynamics of stars or precession was also mentioned in\textsuperscript{16}. We calculated (see Figs.\textsuperscript{1} and\textsuperscript{2}) the mass of the dark matter...
necessary for the explanation of the excess of gamma radiation from the center of the Galaxy detected recently by the Fermi-LAT space gamma telescope [22, 23]. In particular, we determined the dependence of the additional mass both on the profile of the central spike of the dark matter density and on the annihilation cross section of dark matter particles taking into account the Sommerfeld enhancement effect.

In the presence of a small correction $\delta U$ to the Newtonian potential of the black hole, the precession angle of the orbit of a probe particle (S0-2 star) in one turn is

$$\delta\phi = \frac{\partial}{\partial L} \left( \frac{2m}{L} \int_0^\pi r^2(\phi)\delta U d\phi \right).$$

(1)

Here, integration is performed with the trajectory of the particle in the form of an unperturbed elliptic orbit $r(\phi) = p(1 + e \cos \phi)^{-1}$, where $e$ is the eccentricity of the ellipse, $p = L^2/(GM_{BH}m) = a(1 - e^2)$ is the parameter of the orbit, $a$ is the major semiaxis, and $L$ is the conserved angular momentum of the star with the mass $m$. The observed parameters of the Kepler orbit of the S0-2 star: the eccentricity $e = 0.898 \pm 0.0034$, the radius of the pericenter $r_p = a(1 - e) = 0.585 \text{ mpc}$, and the radius of the apocenter $r_a = a(1 + e) = 9.42 \text{ mpc}$. We note that, in the case of relativistic precession, the orbit would rotate in the direction of the rotation of the star, but Newtonian precession (1) occurs in the opposite direction, i.e., $\delta\phi < 0$.

We consider the power-law density profile of matter responsible for the correction $\delta U$ to the potential of the black hole:

$$\rho(r) = \rho_h \left( \frac{r}{r_h} \right)^{-\beta},$$

(2)

where $\rho_h$, $r_h$, and $\beta$ are the parameters. The corresponding total mass of the dark matter within the sphere with the radius $r$ is

$$M_{DM}(r) = \frac{4\pi \rho_h r_h^3}{3 - \beta} \left[ 3^{-\beta} - R_{min}^{3-\beta} \right],$$

(3)

where $R_{min}$ is the minimum radius to which the density profile given by Eq. (2) expands. The subsequent calculation of the precession angle of the orbit will be performed under the assumption that $R_{min} < r_p$ and $\beta < 3$, i.e., that most of the mass of the dark matter within the orbit is located near the apocenter $r = r_a$. We now determine the mass fraction of the dark matter within the orbit of the S0 star $\xi = [M_{DM}(r_a) - M_{DM}(r_p)]/M_{BH}$, which is significant for the subsequent analysis.

The correction to the potential in the case of the power-law profile given by Eq. (2) is

$$\delta U = \begin{cases} Ar^{2-\beta} + C_1 + C, & \beta \neq 2, \\ 4\pi G\rho_h r_h^3 \ln r + C_2 + C, & \beta = 2, \end{cases}$$

(4)

where $A = 4\pi G\rho_h r_h^3 m/[(3 - \beta)(2 - \beta)]$. The constant $C$ does not contribute to the precession angle $\delta\phi$ (because the corresponding contribution to integral (1) is proportional to $L$) and the term $\propto 1/r$ is responsible only for a small addition to the central mass and also does not contribute to the precession angle. The constants $C_1$ and $C_2$ can be represented in the form $C_{1,2} = GmM_{DM}(r_a)$, where $M_{DM}(r_a)$ is the total mass of the dark matter between the event horizon of the black hole and the radius of the apocenter of the star under consideration.
The calculation of the precession angle of the orbit of the star in the time of one turn around the black hole \( \delta \phi \) by Eqs. (1) and (4) gives an expression with two contiguous hypergeometric functions; with the use of the Gauss relations for contiguous functions, this expression is reduced to the following expression with one hypergeometric function \( _2F_1(a, b; c; z) \):

\[
\delta \phi = -\frac{4\pi^2 \rho_0 r_0^3 \beta^3 - \beta}{(1 - e)^{4 - \beta}} M_{\text{BH}}^2 \ _2F_1 \left( 4 - \beta, \frac{3}{2}; \frac{3}{2}; -\frac{2e}{1 - e} \right). \tag{5}
\]

To test this result, we also calculated the precession angle \( \delta \phi \) within standard perturbation theory with the use of the method of osculating elements \([13]\); the resulting expression coincides with Eq. (5). The precession angle \( \delta \phi \) given by Eq. (5) is negative at all allowed parameters.

The magnitude of the nonrelativistic precession angle given by Eq. (5) is in qualitative agreement with the numerical calculations of precession in \([14]-[19]\). Expression (5) for the precession angle at a small eccentricity of the orbit, \( e \ll 1 \), coincides with an accuracy of \( e^2 \) with the corresponding value calculated analytically by another method in \([20]\). However, at a large eccentricity, \( e \simeq 1 \), the precession angle calculated in \([20]\) changes sign to positive and diverges in the limit \( e \to 1 \). The formalism used in \([20]\) is possibly applicable only at \( e \ll 1 \), because the Newtonian precession angle \( \delta \phi \) should always be negative.

The function \( \delta \phi \) given by Eq. (5) is continuous at \( \beta = 2 \) (see Fig. 1). We use Eq. (5) to perform calculations for various density profiles of the dark matter. We calculate the function \( \delta \phi(\beta, \xi) \) in Eq. (5) and find the level line \( \delta \phi(\beta, \xi) = \delta \phi_{\text{obs}} \) with the value \( \delta \phi_{\text{obs}} \sim 0.01 \) maximum allowable by the observation data (see Fig. 2). The values \( \beta \) and \( \xi \) on this line indicate the parameters at which the observation results can be explained.

The excess of gamma radiation from a region with a density profile close to the power-law profile \( \rho \propto r^{-\gamma} \) (see \([22, 23]\)) is the cusp. Fields, Shapiro and Shelton \([23]\) showed that \( \beta = 2.36 \) for the adiabatically formed spike and this spike at \( \langle \sigma v \rangle = \text{const} \) would be a very bright source at the center of the Galaxy (see also \([24]\)). The annihilation signal from such a spike calculated in \([23]\) is a factor of about \( \sim 35 \) stronger than the signal from the extended region with excess of gamma radiation. Since such bright sources at the center of the Galaxy are absent, the existence of the adiabatic spike contradicts observations. It is stated in \([23]\) that the spike could be formed nonadiabatically or be destroyed. In this case, \( \beta < 2.36 \) and the contradiction could be removed.

Following \([23]\), we write the density of the dark matter in the spike in the form of Eq. (2), where \( r_h = GM_{\text{BH}}/v_c^2 \sim 1.7 \) pc is the radius of the action of the black hole, \( v_c = 105 \pm 20 \) km s\(^{-1} \) is the observed standard deviation of velocities at the distance \( \sim 1 \) pc from the center of the Galaxy, and the density \( \rho_0 \) is determined by matching Eq. (2) with the density given by Eq. (5) at the radius \( r_h \).

The minimum radius \( r_{\text{ann}} \) is determined by the annihilation of particles in the time of existence of the spike (see \([23]\)). This quantity depends on the parameters of particles and the density distribution. According to the calculations in \([23]\), the best fit of the gamma spectrum is obtained at \( m = 35 \) GeV and \( \langle \sigma v \rangle = 1.7 \times 10^{-26} \) cm\(^3\) s\(^{-1} \); in this case, \( r_{\text{ann}} \sim r_{\text{ann}} = 3 \times 10^{-3} \) pc. It was shown in \([23]\) that contradiction with the bright point source is absent if \( \beta = \gamma_h \approx 1.8 \). In this case, the mass of the dark matter within the orbit of the S0-2 star is \( M_{\text{DM}} \approx 45 M_\odot \). Noticeable dynamic effects (precession of the orbit of the star, etc.) should absent at such a small mass.

It was assumed above that \( \langle \sigma v \rangle = \text{const} \). However, in a number of models of the dark matter \( \langle \sigma v \rangle \) can depend on \( v \). The velocities \( v \) of particles increase when approaching the black hole. The dependence \( \langle \sigma v \rangle \) of on \( v \) can significantly affect annihilation. Owing to high Kepler velocities near the black hole, the annihilation signal from the center can be reduced, conserving the extended signal from the region \( \sim 10^6 \). In particular, \( \langle \sigma v \rangle \) depends on \( v \) in models involving the Sommerfeld enhancement effect \([23, 27]\). Sommerfeld enhancement is possible if a dark matter particle is a member of a multiplet of states with close masses, between which coannihilation occurs,

where \( R_c = \max\{r_p, r_{\text{ann}}\} \), \( r_{\text{ann}} \) being the possible inner edge of the distribution of the dark matter associated with its annihilation (see below). Quantity \( \chi \) is much smaller than the value accessible for the constraints by the dynamics of stars. However, in the presence of the central black hole, the indicated extrapolation is invalid, because the density profile should be significantly modified by the gravity of the black hole.
e.g., in the model of neutralino with the dominance of Higgsino. The gain $R$ owing to the Sommerfeld effect is determined from the relation $\langle \sigma v \rangle = R \langle \sigma v \rangle_0$, where

$$R = \frac{\pi \mu}{b} \left(1 - e^{-\pi \mu/b}\right)^{-1}. \quad (9)$$

Here, $\mu = \text{const}$ and $b = v/c$. We consider a quite general case where the cross section in the corresponding region of the parameters can be approximated by the power-law dependence

$$\langle \sigma v \rangle = \langle \sigma v \rangle_0 \left(\frac{v_0}{v}\right)^{\eta}, \quad (10)$$

where $\langle \sigma v \rangle_0 = \text{const}$ and $v_0 = \text{const}$. Power-law dependence (10) was considered in [30] in the calculation of the annihilation of the dark matter in self-gravitating bunches. The model with $\langle \sigma v \rangle = \text{const}$ and the model with Sommerfeld enhancement at $\pi \mu/b \ll 1$ correspond to particular cases $\eta = 0$ and $\eta = 1$, respectively.

The radius $r_{\text{ann}}$ at which the maximum density $\rho$ of the dark matter limited by the annihilation effect is reached is determined from the condition

$$n\langle \sigma v \rangle t_g \sim 1, \quad (11)$$

where $n = \rho/m$ and $t_g \sim 10^{10}$ yr is the age of the density peak around the black hole. For cross section (10), we obtain

$$r_{\text{ann}} = r_h \lambda^{-1/2}, \quad (12)$$

where $\lambda \equiv \rho_0 \langle \sigma v \rangle_0 t_g / m$.

The velocities of particles near the black hole are $v(r) \approx (GM_{\text{BH}}/r)^{1/2}$ at $r < r_h$. Let $3 - 2\beta + \eta/2 < 0$. This condition is satisfied for the parameters considered below. The corresponding rate of annihilation of the dark matter in the range of radii from $r_1$ to $r_2$ under the condition $r_1 \ll r_2$ can be written in the form

$$N = 4\pi \int_{r_1}^{r_2} r^2 dr \rho^2(r)m^{-2} \langle \sigma v \rangle_0 v = \frac{4\pi \rho_0^2 r_h^{2\beta} \langle \sigma v \rangle_0 v_0^2 \eta^{3-2\beta+\eta/2}}{m^2 (GM_{\text{BH}})^{\eta/2} (2\beta - 3 - \eta/2)}. \quad (13)$$

The authors of [23] where $\langle \sigma v \rangle = \text{const}$ was accepted, found the parameters of the power-law profiles of the cusp and spike that ensure the absence of contradiction with the bright point source at the center. We assume that the integral annihilation signals from the cusp and peak in model (10) are the same as in [23]. This assumption allows a simple calculation without a detailed fit of the observed excess of gamma radiation. The main signal in the cusp is generated at $r \sim r_h$, where $v \sim v_c$. Hence, fixing the parameter $v_0 \equiv v_c$, we obtain $\langle \sigma v \rangle_0 = 1.7 \times 10^{-26}$ cm$^3$ s$^{-1}$ as in [23]. The density profile in the peak in our case differs from that used in [23] because of the dependence $v(r)$. We determine the density profile in the peak at $\langle \sigma v \rangle \propto v^{-\eta}$ taking into account the above assumptions. Equating the annihilation rate given by Eq. (13) in the spike at $\beta = \gamma_a = 1.8$ and $\langle \sigma v \rangle = \langle \sigma v \rangle_0$ to corresponding rate (13) at arbitrary values $\beta$ and $\langle \sigma v \rangle = \langle \sigma v \rangle_0 v_0^2/\sigma v$, we obtain the nonlinear equation

$$\frac{x}{x + 3 - \beta} \ln + \ln(xr) = 0, \quad (14)$$

where

$$\beta = \frac{3}{2} + \frac{\eta}{4} + \frac{x}{2}, \quad x = \frac{1}{2\gamma_a - 3} \left(\frac{r_h}{r_{\text{ann}}}\right)^{2\beta - 3}, \quad (15)$$

and $\lambda$ is determined from Eq. (12). We solve numerically Eq. (14) with respect to $x$ and, then, calculate the mass of the dark matter by Eq. (8). The results of the calculations are shown in Fig. 3. The equality $r_{\text{ann}} = r_p$ is reached at $\eta = 0.6$ and the inequality $r_{\text{ann}} < r_a$ is always valid under the accepted conditions. Thus, in Eq. (8), $R_n = r_p$ at $\eta < 0.6$ and $R_n = r_{\text{ann}}$ at $\eta > 0.6$. The adiabatic density profile $\beta = 2.36$ is reached at $\eta = 3.13$.

In particular, the mass of the dark matter $M_{\text{DM}}$ within the orbit of the S0-2 star in particular cases $\eta = 0, 1, 3.13$, and 3.5 is 45$M_\odot$, 144$M_\odot$, 1.8$\times$10$^3M_\odot$, and 2.8$\times$10$^3M_\odot$, respectively. These values correspond to the values $\xi = 1.2 \times 10^{-5}$, 3.6 $\times$ 10$^{-5}$, 4.4 $\times$ 10$^{-4}$, and 6.9 $\times$ 10$^{-4}$ and $\beta = 1.8, 1.9, 2.36$, and 2.4, respectively. These values are upper bounds on the possible $\xi$ and $\beta$ values. In the first two cases, the distributed mass of the dark matter is still too small to affect dynamic effects (see Fig. 3). At the same time, the real prospect of the measurement of the additional mass of the dark matter from the precession of S0 stars appears already at $\eta > 3$.

The currently existing observation accuracy is still insufficient for the measurement of the precession angle of fast S0 stars and the distributed invisible mass. However, there is a high probability of reaching in the near future the accuracy required either for the measurement of the precession angle or for the determination of a
strong bound, which in turn will make it possible to impose stringent dynamic constraints on the additional dark mass. If the invisible mass is attributed to annihilating particles, the observation of the annihilation signal from the center of the Galaxy provides additional possibilities for the calculation of the distributed mass or for the determination of bounds on it. According to Fig. 3 at $\langle \sigma v \rangle = \text{const}$ and even with Sommerfeld enhancement $\langle \sigma v \rangle \propto 1/v$, the dynamics of stars still cannot give constraints on annihilation, because the mass of the dark matter within the orbit of the S0-2 star in these cases is very small. At the existing accuracy, the dynamics of stars and annihilation are independent. However, if the annihilation cross section depends on the velocity with a large exponent $\eta > 3$ in Eq. (10), the mass of the dark matter can be significant. In this case, joint constraints could be obtained in the near future from the dynamics of stars and from the data on gamma radiation from the center of the Galaxy.

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