Graviton non-Gaussianities and Parity Violation in the EFT of Inflation

Lorenzo Bordin, Giovanni Cabass

\textsuperscript{a}School of Physics & Astronomy, University of Nottingham, University Park, Nottingham, NG7 2RD, UK
\textsuperscript{b}Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85741 Garching, Germany

E-mail: lorenzo.bordin@nottingham.ac.uk, gcabass@mpa-garching.mpg.de

Abstract. We study graviton non-Gaussianities in the EFT of Inflation. At leading (second) order in derivatives, the graviton bispectrum is fixed by Einstein gravity. There are only two contributions at third order. One of them breaks parity. They come from operators that directly involve the foliation: we then expect sizable non-Gaussianities in three-point functions involving both gravitons and scalars. However, we show that at leading order in slow roll the parity-odd operator does not modify these mixed correlators. We then identify the operators that can affect the graviton bispectrum at fourth order in derivatives. There are two operators that preserve parity. We show that one gives a scalar-tensor-tensor three-point function larger than the one computed in Maldacena, 2003 \cite{1} if $M_P^2 A_s/\Lambda^2 \gg 1$ (where $\Lambda$ is the scale suppressing this operator and $A_s$ the amplitude of the scalar power spectrum). There are only two parity-odd operators at this order in derivatives.
1 Introduction and summary of main results

Upcoming CMB experiments will target the primordial tensor-to-scalar ratio \( r \) to a sensitivity of \( \sigma_r \sim 10^{-3} \) [2, 3, 4, 5, 6, 7, 8, 9, 10]. If a detection of vacuum fluctuations of gravity is achieved, the way towards constraints on tensor non-Gaussianities, and then on the interactions of the graviton, will open. In contrast with the scalar sector, the situation for tensor perturbations is very constrained and, unless one considers exotic models with higher-spin degrees of freedom (see e.g. [11, 12, 13, 14, 15, 16]), it is hard to go beyond Maldacena’s result [1]. For instance, in [17] it was shown that, at leading order in slow roll, conformal invariance ensures that there are only two possible additional shapes beyond that first calculated in [1], with only one actually appearing in the graviton bispectrum.

The result can be circumvented if explicit couplings of the metric with the foliation are allowed. However, Ref. [18] showed that, even if such couplings are turned on, at leading (second) order in derivatives the graviton power spectrum and bispectrum are fixed to be those calculated in [1]. It is therefore interesting to extend this analysis beyond the leading order in...
derivatives. We will do this by means of the Effective Field Theory of Inflation (EFTI) \cite{19}, in which one can study in full generality the couplings of gravity with the fluctuations of the physical clock describing the slicing of spacetime during inflation.

Let us summarize our approach and our results. We look at an expansion in powers of $H/\Lambda$ around Einstein gravity, where the Hubble parameter $H$ is the typical energy scale of tensor fluctuations during inflation and $\Lambda$ is some energy scale controlling the size of higher-derivative operators. First, we focus on the contributions to the graviton two- and three-point functions $\langle \gamma \gamma \rangle$ and $\langle \gamma \gamma \gamma \rangle$ at next-to-leading order (NLO): these come from operators that carry three derivatives acting on the metric in unitary gauge. We identify two parity-odd operators that, starting quadratic in perturbations around an FLRW background, can correct both the tensor power spectrum and the tensor bispectrum. Only one gives a nonzero result on super-horizon scales. Moreover, there is an additional parity-even correction to the tensor bispectrum coming from an operator that starts cubic in perturbations.

We then focus on mixed correlators from the two parity-odd quadratic operators. We show that, at leading order in the slow-roll parameter $\epsilon = -\dot{H}/H^2$, neither of them modifies the scalar-tensor-tensor and the tensor-scalar-scalar bispectra.

We conclude with a discussion on the operators at next-to-next-to-leading order (NNLO) in derivatives. We show that only two parity-even operators (one starting quadratic and one cubic in perturbations) survive after integration by parts and field redefinitions. They involve direct couplings with the foliation. We compute the $\langle \zeta \gamma \gamma \rangle$ and $\langle \gamma \zeta \zeta \rangle$ three-point functions from the quadratic operator. It turns out that the latter is vanishing at late times, while the former is larger than the one computed in \cite{1} if the ratio $H^2/\Lambda^2$ between the Hubble rate and the scale suppressing this operator is larger than $\epsilon$ (equivalently, if $M_P^2 A_s/\Lambda^2 \gg 1$, being $A_s$ the amplitude of the scalar power spectrum). Finally, we show that there are two parity-odd operators starting cubic in perturbations.

The paper is organized as follows. In Section 2 we briefly review the results of \cite{18,20} and discuss the EFTI predictions for the correlators involving the graviton at leading order in derivatives. In Section 3 we look in more detail at the operators that enter at NLO in derivatives. We present the full results for the correlation functions in Section 4. In Section 5 we discuss the operators at fourth order in derivatives. We conclude in Section 6. In Appendix A we summarize our notation and conventions. Appendices B, C and D contain some details of the calculations carried out in the three main sections.

2 Redundant operators at leading order in derivatives

2.1 Field redefinitions and graviton bispectrum

To simplify the tensor sector as much as possible, one can perform field redefinitions that decay at late times. At leading order in derivatives the graviton action can be put in the Einstein-Hilbert form, and consequently both the graviton power spectrum and bispectrum are completely fixed by Maldacena’s result \cite{1,18}.

To go beyond this we will consider interactions with more than two derivatives acting on the metric. Stopping at fourth order in derivatives we can write our action as

$$S = S_0 + \sum I S_{\Lambda I} + \sum I S_{\Lambda^2 I},$$  \hspace{1cm} (2.1)
where

\[ S_0 = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left( R - \frac{2\dot{H}}{N^2} - 2(3H^2 + \dot{H}) \right), \quad (2.2a) \]

\[ S_{\Lambda_I} = M_P^2 \int d^4x \sqrt{-g} \frac{O_{1,I}}{\Lambda_I}, \quad (2.2b) \]

\[ S_{\Lambda^2_I} = M_P^2 \int d^4x \sqrt{-g} \frac{O_{2,I}}{\Lambda_I^2}. \quad (2.2c) \]

The operators \( O_{1,I} \) and \( O_{2,I} \) are constructed by combining the perturbation of the lapse function \( \delta N = N - 1 \), of the extrinsic curvature \( \delta K_{\mu\nu} = K_{\mu\nu} - H h_{\mu\nu} \) and of its trace \( \delta K = K - 3H \) (with \( h^\mu_{\nu} = \delta^\mu_{\nu} + n^\mu n_\nu \) being the projector on the hypersurfaces of constant time and \( n^\mu = (1, -N \vec{n})/N \) their normal vector), the 3-dimensional Riemann tensor \((3)R_{\rho\sigma\mu\nu}\), the ADM “acceleration” vector \( A^\mu = n^\nu \nabla_\nu n^\mu = h^{\mu\nu} \nabla_\nu \log N \), and the derivative of the lapse projected along the normal to the foliation \( V = n^\mu \nabla_\mu N \).

The coefficients \( \Lambda_I \) and \( \Lambda^2_I \) are in general time-dependent. In this paper we assume that their variation in time is small, suppressed by \( \varepsilon \) (we refer to [21] for a more detailed discussion about their time dependence). Consistently with this, we will assume an exact de Sitter background.

The rest of the paper studies what operators \( O_{1,I} \) and \( O_{2,I} \) affect the graviton bispectrum. As we explained in the introduction, all the operators that we will discuss involve direct couplings between the metric and the foliation. One might wonder if these couplings are slow-roll-suppressed. We can easily see that this is not the case by looking, for example, at a simple \( P(X, \phi) \) theory. In operators of the form \( X^n \) many of the legs can be evaluated on the background. This will give additional factors of \( \dot{\phi}^{1/2} \) that are large with respect to \( H \), i.e. the size of derivatives acting on field fluctuations around horizon crossing \( (H/\dot{\phi}^{1/2} = \mathcal{O}(10^{-2}) \) from the normalization of the scalar power spectrum). For a more detailed discussion, see e.g. [19,22,23].

### 2.2 Mixed tensor-scalar correlators

When discussing graviton non-Gaussianities it is also interesting to look at three-point functions involving both tensor and scalar modes. However, as it has been discussed in [20], these are not very constrained already at the two-derivative level, so it is difficult to make general statements for them. A simplification, however, occurs if we consider only those contributions that the nonlinear realization of time diffeomorphisms links to the modification of the graviton power spectrum. In the following we argue that these are the first one should constrain in case of a detection of the tensor power spectrum.

- Let us assume that a difference in the power spectra of left- and right-handed graviton helicities is detected, for example via a nonzero correlation of CMB \( E \)- and \( B \)-modes. We are then guaranteed that a parity-odd operator is present in the action \( S \) of Eqs. (2.1), (2.2), and we can go to look for a signal in observables like the \( EBB \) or \( BEE \) bispectra evaluated at configurations that would vanish if parity is conserved (see also [24] for a recent study of signatures in galaxy intrinsic alignments).

- Let us then consider the parity-conserving scenario. The fact that at leading order in derivatives the tensor power spectrum is univocally fixed by the Hubble rate leads to a “consistency relation” between the tensor tilt and the tensor-to-scalar ratio, \( r = -8n_\tau \). At
higher orders, instead, the amplitude of \( \langle \gamma \gamma \rangle \) depends also on other EFT coefficients: the “consistency relation” will be broken by an amount controlled by \( H^2/\Lambda^2 \). Hence, a detection of \( r \neq -8n_t \) implies the presence of higher-derivatives operators. We should therefore look for their imprint in observables that probe \( \langle \zeta \gamma \gamma \rangle \) and \( \langle \gamma \zeta \zeta \rangle \): these will be crucial to determine \( \varepsilon \) since they give a handle on the combination \( H^2/\Lambda^2 \).

3 Operators involving the graviton at NLO

3.1 Parity-even operators

Let us first focus on operators that do not break parity. It is not possible to write down a parity-even correction to the tensor power spectrum at third order in derivatives [18, 20, 25]; at this order we have only the cubic operator \( \delta K^\alpha \gamma^\rho \delta K_{\rho \alpha} \supset \dot{\gamma}_{ij} \dot{\gamma}_{jk} \dot{\gamma}_{ki} / 8 \) [20], i.e.

\[
S_{\Lambda_1} = M_p^2 \int \mathrm{d}^4 x \sqrt{-g} \frac{\delta K^\mu \delta K_\nu \delta K_\rho \delta K^\rho_\mu}{\Lambda_1} .
\]  

(3.1)

This operator will contribute to the graviton bispectrum.

Notice that, naïvely, there is an additional operator at this order in derivatives. Consider the Gauss-Bonnet term \( R^2 - 4R^\sigma_\nu R_{\sigma \nu} + R^\sigma_\mu \rho \sigma_\mu \nu R^\rho_\sigma \mu \nu \): in four dimensions, it is equal to the four-divergence of a current, i.e.

\[
\mathcal{G}B \equiv R^2 - 4R^\sigma_\nu R_{\sigma \nu} + R^\sigma_\mu \rho \sigma_\mu \nu R^\rho_\sigma \mu \nu = \nabla_\mu G^\mu .
\]  

(3.2)

This current \( G^\mu \) is not a four-vector. However, the fact that the Gauss Bonnet combination is independent of the chosen coordinate system tells us that \( G^\mu \) transforms as such under \( \int \mathrm{d}^4 x \sqrt{-g} \). More precisely, \( \int \mathrm{d}^4 x \sqrt{-g} G^0 \) is invariant under spatial diffeomorphisms and transforms as the zeroth component of a four-vector under time diffeomorphisms. It is therefore a legit object to be added to the EFTI action.

A manageable expression for \( G^\mu \) in terms of metric components and Christoffel symbols is not easy to derive [26]. This obscures a bit the fact that, indeed, \( G^0 \) is build from the usual geometric objects employed in the EFTI. Let us show it. We focus on gravitons only,\(^1\) so that the relation \( a^3 \nabla_\mu G^\mu = \partial_\mu (a^3 G^\mu) \) holds. At quadratic order in \( \gamma_{ij} \), and assuming a constant Hubble parameter for simplicity, the Gauss-Bonnet term contains the following structures:

\[
a^3 \mathcal{G}B \supset (-a^3 H \dot{\gamma}_{ij} \dot{\gamma}_{ij}), \ (-aH \partial_\kappa \dot{\gamma}_{ij} \partial_\kappa \dot{\gamma}_{ij}), \ (2a \dot{\gamma}_{ij} \partial^2 \gamma_{ij}), \ \partial_i (a^3 G^i) .
\]  

(3.3)

In the first combination we recognize the four-divergence \( \nabla_\mu (-4H \delta K_{\rho \sigma} \delta K^{\rho \sigma \mu}) \). Using the perturbative expressions for \( (3)R \) and \( (3)R_{ij} \) we recognize \( \nabla_\mu (4H (3)R_{\nu}^\mu) \) and \( \nabla_\mu (8 \delta K_{\rho \sigma} (3)R_{\rho \sigma}^\mu \nu) \) in the second and third combinations, respectively. The first two are lower-derivative operators: one can deal with them with the field redefinitions discussed in Section 2.1. The third one can be integrated by parts to give lower-derivative operators and operators involving scalar modes (see e.g. Eq. (C.1) of Appendix C).

At cubic order in fluctuations we would find that \( G^0 \) contains also a term of the form \( \delta K^\alpha^\gamma \delta K_\gamma^\rho \delta K_{\rho \alpha} \delta K^{\mu} \). In summary this tells us that there is no need to consider this operator in the EFTI action.

\(^1\)We do this only to simplify the calculations: the same conclusion holds if we include also scalars.
3.2 Parity-odd operators

We can now focus on parity-odd operators. At third order in derivatives there are three operators that, in principle, contribute to \( \langle \gamma \gamma \rangle \). These are\(^2\)

\[
S_{\Lambda_2} = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \frac{1}{\Lambda_2} \frac{e^{\mu\nu\rho\sigma} n_\mu}{N} D_\nu \delta K_{\rho\lambda} \delta K_{\lambda\gamma},
\]

\[
S_{\Lambda_3} = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \frac{1}{\Lambda_3} e^{0ijk} \left( \frac{(3)^l_{\imath m} \partial_j (3)^m_{\jmath k}}{2} + \frac{(3)^l_{\imath m} (3)^m_{\jmath n} (3)^n_{\kappa l}}{3} \right),
\]

\[
S_{\Lambda_4} = M_P^2 \int d^4x \sqrt{-g} \frac{1}{\Lambda_4} e^{0\alpha\beta\gamma} \left( \frac{\Gamma^\sigma_{\alpha\nu} \partial_\beta \Gamma^\nu_{\gamma\sigma}}{2} + \frac{\Gamma^\sigma_{\alpha\nu} \Gamma^\nu_{\beta\lambda} \Gamma^\lambda_{\gamma\sigma}}{3} \right).
\]

The volume form \( e_{\mu\nu\rho\sigma} \) (\( e^{\mu\nu\rho\sigma} \)) is equal to \( \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} \) (\( -\epsilon_{\mu\nu\rho\sigma} / \sqrt{-g} \)), where \( \epsilon_{\mu\nu\rho\sigma} \) is the Levi-Civita symbol such that \( \epsilon_{0ijk} = \epsilon_{ijk}, \epsilon_{0123} = 1 \). The operator in Eq. (3.4b) is the Chern-Simons term for the metric on the hypersurfaces of constant time. The operator in Eq. (3.4c), instead, is the zeroth component of the current \( K^\mu \) that satisfies

\[
\nabla_\mu K^\mu = \frac{1}{4} e^{\mu\alpha\beta\gamma} R^\sigma_{\rho\alpha\beta} R^0_{\sigma\mu\nu},
\]

where

\[
K^\mu = 2 e^{\mu\alpha\beta\gamma} \left( \frac{\Gamma^\sigma_{\alpha\nu} \partial_\beta \Gamma^\nu_{\gamma\sigma}}{2} + \frac{\Gamma^\sigma_{\alpha\nu} \Gamma^\nu_{\beta\lambda} \Gamma^\lambda_{\gamma\sigma}}{3} \right).
\]

The contributions to the tensor power spectrum of the two operators of Eqs. (3.4a), (3.4b), and the contribution to the scalar-tensor-tensor three-point function of the third one, were computed respectively in [18] and [27, 28] (see also [29] for a discussion about the contribution to \( \langle \gamma \gamma \rangle \) of \( S_{\Lambda_4} \)).

Notice that, similarly to \( G^\mu \), \( K^\mu \) is not a four-vector. However one can again show that \( K^0 \) is a legit operator of the EFTI: its transformation properties under \( \int d^4x \sqrt{-g} \) are the same as those of the zeroth component of a four-vector. Since \( \int d^4x \sqrt{-g} K^0 \) is invariant under spatial diffeomorphisms one might wonder if it can be decomposed in more “fundamental” building blocks via the \( 3 + 1 \) splitting of spacetime, as we did with \( G^0 \). This is indeed the case: in Appendix B we show that, when we consider tensor modes only, it is always possible to write \( S_{\Lambda_4} \) as a combination of \( S_{\Lambda_3} \) and \( S_{\Lambda_4} \) (including scalar modes would require additional operators to fully decompose \( S_{\Lambda_4} \), but we are not interested in them in this paper). For this reason we will not consider \( S_{\Lambda_4} \) in the following.

Before proceeding, let us discuss whether it is possible to remove these operators via field redefinitions. For simplicity we work perturbatively in \( 1/\Lambda_1 \). Since these operators are third-order in derivatives, and the variation of \( S_0 \) carries at least two derivatives due to the variation of the Einstein-Hilbert action, we are forced to consider field redefinitions that carry at most one derivative. Moreover, these field redefinitions must include the volume form \( e^{\mu\nu\rho\sigma} \) since \( S_0 \) is parity-conserving. It is then straightforward to realize that we cannot write down any redefinition \( g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu} \) that would keep the metric a symmetric tensor.

The signs and overall numerical factors in Eqs. (3.4) are chosen to reproduce the actions used in [18, 27]. More precisely, Eqs. (3.4a), (3.4b) reproduce Eq. (21) of [18] upon identification of their \( \alpha/\Lambda, \beta/\Lambda \) with \( 1/\Lambda_2, 1/\Lambda_3 \), and for \( \Lambda_4 = M_{\text{CS}} \) we match the action used in [27] at leading order in slow roll.
the invariance of the EFTI action under spatial diffeomorphisms, it is enough to consider \( \xi^\mu = Fn^\mu \) [20]. Given that the Einstein-Hilbert action does not vary (thanks to the Bianchi identity), we are allowed to take \( F \) to be at most of second order in derivatives. Even so, we cannot construct, from the building blocks of the EFTI, a function \( F \) that contains \( e^{\mu \nu \rho} \) while remaining at second order in derivatives.

We conclude this section by checking if there are parity-odd operators at cubic order in perturbations and third order in derivatives that contribute to \( \langle \gamma \gamma \gamma \rangle \). It is straightforward to convince ourselves that there is no such operator. Let us work in term of the graviton fluctuations. The presence of \( \epsilon_{ijk} \sim e^{\mu \nu \rho} \eta_{\mu} \) allows us to consider also \( \partial_k \gamma_{ij} \sim D_0 \partial K_{\mu \rho} \) in addition to \( \partial^2 \gamma_{ij} \sim (3)R_{\mu \nu} \) and \( \gamma_{ij} \sim \partial K_{\mu \nu} \). We also want three powers of \( \gamma_{ij} \) to contract with \( \epsilon_{ijk} \). However there are too many indices that would remain free unless we add additional spatial derivatives or scalar modes through the ADM acceleration vector \( A^\mu \) (for the same reason, we still cannot consider \( \partial_k \partial_l \gamma_{ij} \sim (3)R_{\rho \sigma \mu \nu} \) as a building block).

4 Graviton non-Gaussianities at NLO

In this section we compute the super-horizon correlation functions via the in-in formalism. Before doing that, however, it is worth to estimate the size of these new contributions with respect to the non-Gaussianities coming from the minimal action \( S_0 \). This is especially important for the mixed scalar-tensor-tensor and tensor-scalar-scalar three-point functions. Indeed we expect that they are slow-roll-enhanced with respect to those of \([1]\). In Section 4.2 we will see that, at variance with this expectation, the mixed three-point functions from \( S_{\Lambda_I} \), \( I = 2, 3 \), vanish at leading order in slow-roll parameters.

4.1 Estimates

Let us first review the non-Gaussianities from the minimal action \( S_0 \). It is simpler to work in flat gauge: let us reintroduce the Stueckelberg field \( \pi \) via \( t \to t + \pi \) (see Appendix A for details).

At leading order in slow roll, on super-horizon scales \( \zeta \) and \( \pi \) are related by \( \zeta = -H \pi \). After reintroducing \( \pi \) we can solve for the constraints: they are \( dN = \varepsilon H \pi, a^{-2} \partial^2 N_\pi = -\varepsilon H \dot{\pi} \) [1, 30]. At this point we can estimate the size of non-Gaussianities by comparing the cubic vertices to the quadratic Lagrangians for \( \pi \) and for \( \gamma_{ij} \). The quadratic action for the Stueckelberg field is of order \( \varepsilon \): schematically, \( S_0|_{\pi \pi} \sim \varepsilon \pi \pi \). The quadratic action for \( \gamma_{ij} \) is instead \( S_0|_{\gamma \gamma} \sim \gamma \gamma \). What about the cubic vertices?

- We first focus on the three-graviton vertex. It comes from the three-Ricci scalar \((3)R \) [1, 17].

Hence we have \( S_0|_{\gamma \gamma \gamma} \sim \gamma \gamma \gamma \). Since \( S_0|_{\gamma \gamma} \sim \gamma \gamma \), we find \( \langle \gamma \gamma \gamma \rangle_0 \sim \langle \gamma \gamma \rangle_0 \langle \gamma \gamma \rangle_0 \).

- At leading order in slow roll the cubic vertex with one scalar and two gravitons comes from plugging the constraints in the Einstein-Hilbert action (which, being invariant under all diffeomorphisms, does not contain the \( \pi \) field by itself). We then have at most \( S_0|_{\pi \gamma \gamma} \sim \varepsilon \pi \gamma \gamma \): comparing with \( S_0|_{\gamma \gamma} \sim \gamma \gamma \) we find that \( \langle \gamma \gamma \rangle_0 \) is of order \( \varepsilon \langle \gamma \gamma \rangle_0 \langle \gamma \gamma \rangle_0 \).

- The cubic vertex with two scalars and one graviton comes from the term \(-M_0^2 \dot{H}/N^2 \) in \( S_0 \), which contains \( g^{\mu \nu} \nabla_\mu \pi \nabla_\nu \pi \sim -a^{-2} \gamma_{ij} \partial_\pi \partial_\pi \). From this we see that \( S_0|_{\pi \pi} \sim \varepsilon \pi \pi \pi \): since \( S_0|_{\pi \pi} \sim \varepsilon \pi \pi \pi \), we find \( \langle \pi \pi \pi \rangle_0 \sim \langle \pi \pi \rangle_0 \langle \pi \pi \rangle_0 \).

We are now in the position to do the same estimates for \( S_{\Lambda_I} \). For \( S_{\Lambda_1} \) we care only about the graviton bispectrum. Since \( \partial K^{\alpha \gamma} \partial K_\rho \partial K_{\rho \sigma} \) is simply \( \dot{\gamma}_{ij} \dot{\gamma}_{jk} \dot{\gamma}_{ki}/8 \) at cubic order in \( \gamma_{ij} \), the
Table 1. Expected size of three-point functions for $S_{\Lambda_i}$, $I = 2, 3$, with respect to Maldacena’s ones. We see that an enhancement by $1/\epsilon$ is expected in both $\langle \zeta \gamma \gamma \rangle$ and $\langle \gamma \zeta \zeta \rangle$. As we discuss in Section 4.2, the actual in-in calculation shows that such enhancement is absent.

| $S_{\Lambda_2}$ | $H/\Lambda_2$ | $H/\Lambda_2 \epsilon$ | $H/\Lambda_2 \epsilon$ |
|-----------------|----------------|------------------------|------------------------|
| $S_{\Lambda_3}$ | $H/\Lambda_3$ | $H/\Lambda_3 \epsilon$ | $H/\Lambda_3 \epsilon$ |

estimate is very simple: we expect that $\langle \gamma \gamma \gamma \rangle_{\Lambda_2} / \langle \gamma \gamma \gamma \rangle_0$ is of order $H/\Lambda_1$. What about the parity-odd operators? Let us first consider their contribution to $\langle \gamma \gamma \gamma \rangle$. Also in this case we have that, for both operators, the cubic vertex with three gravitons carries an extra derivative, leading to a simple $H/\Lambda_I$ suppression.

Things are a bit more tricky for correlators involving scalar modes. The reason is that we first need to study how the solution for the constraints is modified by the presence of the new operators (we emphasize that we work perturbatively in $1/\Lambda_I$ also in the solution of the constraint equations for $\delta N$ and $N_L$). We will study the two operators separately, and summarize the results in Tab. 1.

$S_{\Lambda_2}$

The Stueckelberg trick for the operator of Eq. (3.4a) is reviewed in Appendix D. All mixings of the $\pi$ field with the constraints vanish due to the presence of $\epsilon^{\mu \nu \rho \sigma} n_\mu$. Moreover, it is straightforward to see that the modification of the quadratic action for $\delta N$ and $N_L$ is not slow-roll-enhanced with respect to the contribution coming from $S_0$. Therefore the solution of the constraints in terms of $\pi$ is the same as Maldacena’s one up to a change of order $H/\Lambda_2$ in their normalization. I.e. we have, schematically, that

$$\delta N, N_L \sim \epsilon \pi \left( 1 + \frac{H}{\Lambda_2} \right).$$

Let us then focus on the cubic vertices containing both $\gamma_{ij}$ and $\pi$. Since $S_{\Lambda_2}$ is not invariant under time diffeomorphisms we expect these vertices to be present even before we plug in the solution for the constraints. This is shown explicitly in Appendix D. As expected, these vertices give (schematically) $S_{\Lambda_2}\pi\gamma\gamma \sim (H/\Lambda_2) \pi \gamma \gamma$, $S_{\Lambda_2}\pi\pi\pi \sim (H/\Lambda_2) \pi \pi \pi$. Hence, at leading order in slow roll and $H/\Lambda_2$ we can forget about $\delta N$ and $N_L$, and just take $N = 1$, $N^i = 0$ throughout (we emphasize that, had there been a modification of the constraints at order $H/\Lambda_2$ that was not slow-roll-suppressed, we should have worried about having to substitute the constraints in $S_0$ as well, in order to capture all the relevant interactions).

Then, taking the ratios $S_{\Lambda_2}\pi\gamma\gamma / S_0\gamma\gamma$ and $S_{\Lambda_2}\pi\pi\pi / S_0\pi\pi$ we see that we should expect both $\langle \zeta \gamma \gamma \rangle_{\Lambda_2} / \langle \zeta \gamma \gamma \rangle_0$ and $\langle \gamma \zeta \zeta \rangle_{\Lambda_2} / \langle \gamma \zeta \zeta \rangle_0$ to be of order $(H/\Lambda_2) \epsilon^{-1}$.

$S_{\Lambda_3}$

The three-dimensional Chern-Simons term is the simplest of the two operators. In this case we can work directly in $\zeta$ gauge: indeed, we see that in Eq. (3.4b) the lapse and shift constraints do not appear. First, this tells us that the solution of $\delta N$ and $N_L$ in terms of $\zeta$ is unchanged:
after we plug them into $S_0$ to get the quadratic action for $\zeta$ we can forget about them. Using the same logic as we did for $S_{\Lambda 2}$, we then expect that the $\langle \zeta \gamma \gamma \rangle$ and $\langle \gamma \zeta \zeta \rangle$ three-point functions are suppressed by $H/\Lambda_3$ but enhanced by $1/\varepsilon$ with respect to those from $S_0$.

### 4.2 Calculation via in-in formalism

After these estimates, we are ready to look at the full results of the in-in calculation for these correlation functions.

Before doing that let us briefly address the single-field consistency relations. Since the operators we consider are higher-derivative ones we are guaranteed that there is no modification of the squeezed correlation functions at order $1/q^3$ or $1/q^2$ ($q$ being the long mode). The consistency relations are then trivially satisfied if we take into account how the new operators modify the normalization of the de Sitter modes for the $\pi$ and $\gamma$ fields once we compute the non-Gaussianities from $S_0$.

#### $S_{\Lambda 1}$

We start from $S_{\Lambda 1}$. For the correction to the graviton bispectrum we find

\[
\Delta \langle \gamma_{k_1}^{s_1} \gamma_{k_2}^{s_2} \gamma_{k_3}^{s_3} \rangle' = - \frac{H^4}{\Lambda_1} \frac{3}{M_3^4 k_1 k_2 k_3 K^3} \epsilon_{i_1}^{s_1}(k_1) \epsilon_{j_2}^{s_2}(k_2) \epsilon_{k_3}^{s_3}(k_3),
\]

where $K = \sum_i k_i$ and $\epsilon_i^s(k)$ are the graviton polarization tensors defined in Eq. (A.5). Comparing with Eq. (4.15) of [1] we see the expected $H/\Lambda_1$ suppression.

#### $S_{\Lambda 2}$

Let us move to $S_{\Lambda 2}$. The details of the calculation of the flat-gauge action are contained in Appendix D. The in-in calculation shows that $\langle \gamma \gamma \gamma \rangle$ vanishes super-horizon scales, as does $\langle \gamma \gamma \rangle$. We have also checked that at leading order in slow roll the corrections to $\langle \zeta \gamma \gamma \rangle$ and $\langle \gamma \zeta \zeta \rangle$ vanish: there is no $1/\varepsilon$ enhancement with respect to Maldacena’s result.

The only contribution of Eq. (3.4a) is to add a field-dependent phase to the wavefunction of the universe on the boundary of de Sitter spacetime.\(^3\) While the operator does not contribute to the vacuum expectation values of scalar and tensor fluctuations, it can therefore affect the scalar product between the vacuum and another state.

#### $S_{\Lambda 3}$

This operator gives a fractional correction to the power spectrum of the two helicities $\gamma_k^s$ of the graviton proportional to $\lambda_s H/\Lambda_3$ ($s = \pm$ and $\lambda_{\pm} = \pm 1$) [18,27,28,32].

Interestingly there is also a nonzero modification of the graviton bispectrum (the parity-breaking operator $WWW^*$ considered in [17,33,34] does not affect $\langle \gamma \gamma \gamma \rangle$ since it contributes to the wavefunction of the universe via a pure phase). It is

\[
\Delta \langle \gamma_{k_1}^{s_1} \gamma_{k_2}^{s_2} \gamma_{k_3}^{s_3} \rangle' = 2\pi \frac{H^4}{\Lambda_3} \frac{I_{s_1 s_2 s_3}(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3} + 2 \text{ cyclic permutations},
\]

\(^3\)This can be seen by taking the imaginary part of Eq. (A.7) after dropping the commutator. See [31] for a discussion of parity-violating signatures in the scalar sector, and how local interactions only give corrections to the phase of the wavefunction (and are then unobservable in correlators) at leading order in slow roll.
where we have defined (we suppress the momentum arguments of the polarization tensors to ease the notation, \( e^s = \epsilon^s_{ij}(k) \), and use \([\cdot]\) to denote the trace)

\[
\mathcal{I}^{s_1 s_2 s_3}(k_1, k_2, k_3) = - [e^{s_2} \cdot e^{s_3}] k_2 \cdot e^{s_1} \cdot k_3 \left( \frac{8}{3} \lambda_{s_1} k_1 + \frac{1}{2} \lambda_{s_2} k_2 + \frac{1}{2} \lambda_{s_3} k_3 \right) \\
+ k_2 \cdot e^{s_3} \cdot e^{s_1} \cdot e^{s_2} \cdot k_3 \left( - \lambda_{s_1} k_1 + \frac{1}{2} \lambda_{s_2} k_2 + \frac{1}{2} \lambda_{s_3} k_3 \right) \\
+ [e^{s_1} \cdot e^{s_3} \cdot e^{s_2}] k_2 \cdot k_3 (\lambda_{s_2} k_2 + \lambda_{s_3} k_3) - i \epsilon_{ijk} \epsilon_{kmn} \epsilon_{ilm} k_2^l k_3^m \\
+ k_3 \cdot e^{s_1} \left( \left( \frac{4}{3} \lambda_{s_1} k_1 + \frac{1}{2} \lambda_{s_2} k_2 \right) e^{s_3} \cdot e^{s_2} - (2 \leftrightarrow 3) \right) \cdot k_1 .
\]

(4.4)

Using the expression for the polarization tensors given in Appendix A, cf. Eq. (A.5), it is straightforward to check that the term in the bispectrum proportional to \( i \epsilon_{ijk} \) is real.

What about the mixed correlation functions involving scalars? They vanish identically: indeed, integrating by parts and using the antisymmetry of \( \epsilon_{ijk} \) one can show that the vertices \( S_{\lambda_3} |_{\zeta \gamma \gamma} \) and \( S_{\lambda_3} |_{\gamma \zeta \zeta} \) are zero. Therefore \( \langle \zeta \gamma \gamma \rangle \) and \( \langle \gamma \zeta \zeta \rangle \) are not enhanced in slow roll with respect to Maldacena’s ones.

5 Operators at NNLO in derivatives

In this section we play the same game as in Section 3 but focusing on operators at fourth order in derivatives.

5.1 Parity-even operators

First, let us consider parity-even operators that start quadratic in perturbations. These have been identified in [20]: we recap them here for convenience of the reader. Working with the graviton \( \gamma_{ij} \), the only structures we can write are \( \tilde{\gamma}_{ij} \tilde{\gamma}_{ij} \), \( \gamma_{ij} \partial^2 \gamma_{ij} \) and \( \partial^2 \gamma_{ij} \partial^2 \gamma_{ij} \). They correspond to

\[
\tilde{\gamma}_{ij} \tilde{\gamma}_{ij} \sim n^\rho \nabla_\rho \delta K^{\mu \nu} n^\sigma \nabla_\sigma \delta K_{\mu \nu} , \quad (5.1a)
\]

\[
\gamma_{ij} \partial^2 \gamma_{ij} \sim (3) R_{\mu \nu} n^\rho \nabla_\rho \delta K^{\mu \nu} , \quad (5.1b)
\]

\[
\partial^2 \gamma_{ij} \partial^2 \gamma_{ij} \sim (3) R_{\rho \sigma} (3) R_{\mu \nu} , \quad (3) R_{\rho \sigma} (3) R_{\mu \nu} , \quad (5.1c)
\]

where numerical factors and scale factors are neglected for simplicity. We then move to operators starting cubic in perturbations. There are only two structures that we can write, i.e. \( \gamma_{ij} \gamma_{jk} \partial^2 \gamma_{ki} \) and \( \gamma_{ij} \gamma_{kl} \partial_k \partial_l \gamma_{ij} \). The corresponding operators are

\[
\gamma_{ij} \gamma_{jk} \partial^2 \gamma_{ki} \sim (3) R_{\rho \sigma} \delta K^{\mu \nu} \delta K_{\rho \sigma} , \quad (5.2a)
\]

\[
\gamma_{ij} \gamma_{kl} \partial_k \partial_l \gamma_{ij} \sim (3) R_{\rho \sigma} \delta K^{\rho \mu} \delta K^{\sigma \nu} . \quad (5.2b)
\]

5.2 Integration by parts and field redefinitions

Are some of these operators redundant? First we consider the operator \( (3) R_{\rho \sigma} (3) R_{\mu \nu} \) in Eq. (5.1c). We can rewrite it in terms of lower-derivative operators and the remaining operators in Eqs. (5.1), (5.2) using the 3 + 1 decomposition of the Riemann tensor (summarized in
Appendix D) and the fact that the Gauss-Bonnet combination of Eq. (3.2) is a total derivative. Then, in Appendix C we show that

\[(3) R_{\mu\nu} n^{\rho} \nabla_{\rho} n^{\sigma} D_{\mu} K_{\rho\sigma} = 2 D_{\sigma} K^{\rho\sigma} D_{\mu} K_{\rho\sigma} + 2 (3) R^{\rho}_{\lambda \mu \sigma} K^{\lambda \sigma} K_{\rho \mu} + K^{\rho \sigma} D_{\mu} D_{\rho} K_{\sigma\sigma} + \text{boundary terms} + \text{lower-derivative operators} + \text{operators involving scalar modes}, \tag{5.3}
\]

where the first term on the right-hand side starts at fourth order in $\gamma_{ij}$. This tells us that it is sufficient to consider $(3) R_{\mu\nu} n^{\rho} \nabla_{\rho} \delta K^{\mu\nu}$ in the action up to a redefinition of the coefficients of the cubic operator of Eq. (5.2b) and of lower-derivative operators.

Let us discuss the field redefinitions. Starting at linear level in fluctuations we have only

\[g^{\mu\nu} \to g^{\mu\nu} + c_{(3)R} (3) R^{\mu\nu} + c_{K} n^{\rho} \nabla_{\rho} \delta K^{\mu\nu}, \tag{5.4}\]

where $c_{(3)R}$ and $c_{K}$ have the dimensions of a length squared. Using the projection of the Einstein tensor on the hypersurfaces of constant time, i.e.

\[h_{\sigma}^\gamma h_{\nu}^\delta G_{\gamma\delta} = (3) R_{\mu\nu} + K K_{\sigma\nu} + n^{\rho} \nabla_{\rho} K_{\sigma\nu} - \frac{1}{2} (3) R h_{\sigma\nu} + \frac{1}{2} K h_{\sigma\nu} - \frac{1}{2} K_{\alpha\beta} K^{\alpha\beta} h_{\sigma\nu} - h_{\sigma\nu} \nabla_{\rho} (K n^\rho) + \text{terms involving } A^\mu, \tag{5.5}\]

we see that the variation of the Einstein-Hilbert action under Eq. (5.4) contains the three operators $n^{\rho} \nabla_{\rho} \delta K^{\mu\nu} n^{\sigma} \nabla_{\sigma} \delta K_{\mu\nu}$, $\delta K^{\mu\nu} n^{\rho} \nabla_{\rho} (3) R_{\mu\nu}$ and $(3) R^{\rho\sigma}(3) R_{\rho\sigma}$. We choose to remove the first two: this is simply because the last one is easier to deal with once scalar modes are considered since it does not contain the constraints.

Let us conclude this section by studying the field redefinitions for the cubic operators. It is straightforward to see that we can generate the operator $(3) R_{\mu\nu} \delta K^{\mu}_{\rho} \delta K^{\nu}_{\sigma}$ from the first term in Eq. (5.5) with the field redefinition

\[g^{\mu\nu} \to g^{\mu\nu} + c_{K} \delta K^{\mu}_{\rho} \delta K^{\nu}_{\sigma}. \tag{5.6}\]

On the other hand, it is not possible to generate the operator $(3) R_{\rho\sigma\mu\nu} \delta K^{\mu\nu} \delta K^{\sigma\nu}$ with any field redefinition since the Riemann tensor carries too many indices. We then conclude that at fourth order in derivatives there are only two parity-even operators that are not redundant, and only one of them modifies the tensor power spectrum.

5.3 Parity-odd operators

Let us now move to operators that break parity. At quadratic order in perturbations we have only two structures, i.e. $\epsilon_{ijk} \partial_{i} \delta \gamma_{jl} \delta \gamma_{ik}$ and $\epsilon_{ijk} \partial_{i} \delta \gamma_{jl} \partial_{2} \gamma_{ik}$. At the covariant level, these correspond to the operators

\[\epsilon_{ijk} \partial_{i} \delta \gamma_{jl} \delta \gamma_{ik} \sim \frac{e_{\mu \rho \sigma} n_{\mu}}{N} D_{\nu} \delta K_{\rho \lambda} n^{\delta} \nabla_{\delta} \delta K^{\lambda}_{\sigma}, \tag{5.7a}\]

\[\epsilon_{ijk} \partial_{i} \delta \gamma_{jl} \partial_{2} \gamma_{ik} \sim \frac{e_{\mu \rho \sigma} n_{\mu}}{N} D_{\nu} \delta K_{\rho \lambda} (3) R^{\lambda}_{\sigma}. \tag{5.7b}\]

There is only one option when we move to operators starting cubic in perturbations. If we want to remain with four derivatives the antisymmetry of $\epsilon_{ijk}$ forbids us to use the three-Ricci tensor. Moreover, we still cannot use $(3) R_{\rho\sigma\mu\nu}$ since we would remain with a free index on $\epsilon_{ijk}$.
unless we introduce scalar modes. The only structure we can build is 
\[ \epsilon_{ijk} \dot{\gamma}^i_{lm} \partial_j \dot{\gamma}^m_{nk} \]. 
At the covariant level this is
\[ \epsilon_{ijk} \dot{\gamma}^i_{lm} \partial_j \dot{\gamma}^m_{nk} \sim e^{\mu\nu\rho\sigma} n_\mu \delta K_\lambda \delta K_\gamma \partial_j \delta K_{\gamma\sigma}. \] 

Let us first discuss integration by parts. It is straightforward to see that the cubic operator of Eq. (5.8) is not a total derivative. On the other hand, the two operators of Eqs. (5.7) are total divergences at quadratic order in \( \gamma_{ij} \). In Appendix C we show that we can remove the operator of Eq. (5.7a) at all orders in perturbations, up to a redefinition of the coefficients of lower-derivative operators and of the operator of Eq. (5.8).

One might wonder if the same holds for the quadratic operator involving the three-Ricci tensor, cf. Eq. (5.7b). We have a reason to believe that this is not the case. Indeed, showing that the operator is a total divergence at quadratic order in \( \gamma_{ij} \) explicitly requires to use the linear-order relation \( [3] R_{ij} = -\partial^2 \gamma_{ij} / 2 \), that does not have an analogue at the nonperturbative level. This is only a hint: a definitive proof would be, e.g., computing the equations of motion for this operator (see [35], for example). In the rest of this paper we take this to be an operator that starts cubic in \( \gamma_{ij} \).

We conclude by discussing field redefinitions. In order to remove these cubic operators we would need a symmetric tensor involving \( e^{\mu\nu\rho\sigma} \) that starts quadratic in perturbations and carries two derivatives acting on the metric. Similarly to what happened at third order in derivatives, it is not possible to build such a tensor.

### 5.4 Three-point functions involving scalars

In this section we compute the scalar-tensor-tensor and tensor-scalar-scalar three-point functions from \( [3] R_{\mu\nu}[3] R^{\mu\nu} \). The action is
\[ S = S_0 + M_p^2 \int d^4x \sqrt{-g} \frac{[3] R_{\mu\nu}[3] R^{\mu\nu}}{\Lambda_1^2}. \] 

This operator gives a correction to the power spectrum of the graviton proportional to \( H^2 / \Lambda_1^2 \), as shown in [20]. Similarly to the discussion in Section 4.1, we expect that the ratio of the \( \langle \zeta \gamma \gamma \rangle \) and \( \langle \gamma \zeta \zeta \rangle \) three-point functions with those coming from \( S_0 \) is enhanced by \( 1/\epsilon \). In fact, the explicit calculation reveals that this holds only for the former: the latter vanishes at late times.

Since \( [3] R_{\mu\nu}[3] R^{\mu\nu} \) does not contain the lapse and shift constraints, being built exclusively from the connection coefficients of the three-dimensional covariant derivative, it is easier to carry out the calculation in \( \zeta \) gauge. The resulting three-point function is
\[ \Delta \langle k_1 \gamma_{k_2} \gamma_{k_3} \rangle = \frac{H^2}{\Lambda_1^2} \frac{H^4}{8 \pi M_p^2 k_1^3 k_2^3 k_3^3 K_5} \left\{ \left( T_a(k_1, k_2, k_3) + T_b(k_1, k_2, k_3) \right) [e^{s_2} \cdot e^{s_3}] + T_c^{s_2, s_3}(k_1, k_2, k_3) \right\}, \] 

\[ (5.10) \]

\[ ^4 \text{Notice that there are other cases of operators that are total divergences at a finite order in perturbations but not at all orders. One example is } [3] R. \]

\[ ^5 \text{The computation in [20] was missing the piece } \delta N [3] R_{\mu\nu}[3] R^{\mu\nu} \text{ in the cubic action } (\delta N = \dot{\zeta}/H). \]
where

$$\mathcal{T}_a(k_1, k_2, k_3) = \frac{3k_1^4}{2} \left( k_1^4 + 10k_1^3k_2 + 14k_1^2(k_1^2 - k_2^2) + 20k_1^2k_2k_3 + 30k_2^2k_3(k_1 - k_2) - 6k_2^2k_3^2 \right), \quad (5.11)$$

$$\mathcal{T}_b(k_1, k_2, k_3) = -k_2^3 \left( 15k_1^2k_2^2(k_1 + k_3) + 3k_1^3k_2^2 + 60k_1^2k_2k_3 + 45k_1k_3^2(k_1 - k_3) + 11k_1^2k_2k_3^2 + 10k_2^2k_3^2(k_1 + k_3) + 2k_2^3k_3^2 + 40k_1k_2k_3^2 + 8k_2k_3^3 \right), \quad (5.12)$$

and

$$\mathcal{T}_{c_{s2,s3}}(k_1, k_2, k_3) = 4K \left( k_1^3 + 4k_1^2(k_2 + k_3) + (4k_1 + k_2 + k_3)(k_2^2 + 3k_2k_3 + k_3^2) \right) \times \left( k_2^2 (k_1 \cdot e^{s_2} \cdot e^{s_3} \cdot k_1) + (k_1 \cdot k_2)(k_1 \cdot e^{s_2} \cdot e^{s_3} \cdot k_2) \right). \quad (5.13)$$

### 6 Conclusions

In this paper we studied the perturbative non-Gaussianities of the graviton in the framework of single-field inflation. When couplings of the metric to the foliation are allowed, deviations from Einstein gravity are present already at next-to-leading order in derivatives.

At order $H/\Lambda$ the split in the graviton helicities can be traced to a single operator, that is the Chern-Simons term on the hypersurfaces of constant clock. This operator modifies the graviton bispectrum, but at leading order in slow roll it does not correct the mixed correlators $\langle \zeta \gamma \gamma \rangle$ and $\langle \gamma \zeta \zeta \rangle$. There is an additional operator that breaks parity but does not affect super-horizon correlators (it gives only a correction to the phase of the wavefunction of the universe). Finally, an operator starting cubic in metric perturbations gives the leading parity-conserving correction to the graviton bispectrum.

At order $H^2/\Lambda^2$ there are no parity-odd modifications of the power spectrum and a single parity-even one. Of the three operators starting cubic in perturbations, one conserves parity and two do not. The operator that modifies the power spectrum affects $\langle \zeta \gamma \gamma \rangle$ as well: this new contribution is more sizable than Maldacena’s result if the ratio $H^2/\Lambda^2$ is larger than the slow-roll parameter $\varepsilon$ (equivalently, using the normalization $A_s$ of the scalar power spectrum, if the scale $\Lambda$ is much lower than $M_P \sqrt{A_s}$). We argue that this makes this operator a prime target if tensor modes are detected.

In light of this, it would be interesting to see if it is possible to disentangle the signature of this operator in $\langle \zeta \gamma \gamma \rangle$ from the contributions of the operators with two derivatives. Given that the latter effectively come only from the modification of the constraint equations, this amounts to study what are the most general solutions for $\delta N$ and $N_L$ at leading order in derivatives.

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A Notation and conventions

In this appendix we summarize our notation and conventions for convenience of the reader. The unitary-gauge line element is

\[ ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) , \]  

(\!(A.1)\!)

where

\[ h_{ij} = a^2 e^{2\zeta}(\gamma^i)_{ij} , \quad \gamma_{ii} = 0 , \quad \partial_t \gamma_{ij} = 0 . \]  

(\!(A.2)\!)

We write the shift function as

\[ N_i = h_{ij} N^j = D_i N_L + h_{ij} N^j_T , \quad D_i N^i_T = 0 . \]  

(\!(A.3)\!)

Notice that, at linear order in perturbations, this definition has an overall \( a^{-2} \) with respect to that of [1]. The transverse part of the shift does not play a role in this work (vector modes are not excited since we work perturbatively around the minimal single-clock action).

The coordinate transformation from unitary gauge to flat gauge, i.e. the reintroduction of the Stueckelberg field \( \pi \), is defined as \( t_\zeta = t_\pi + \pi(t_\pi, \mathbf{x}) \) [19]: we will use the shorthand \( t \to t + \pi \) for this transformation.

Our decomposition of scalar and tensor modes is

\[ \zeta(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \zeta_k(t)e^{ik \mathbf{x}} , \]  

(A.4a)

\[ \gamma_{ij}(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \sum_{s=\pm} \epsilon_{ij}^s(k) \gamma^s_k(t)e^{ik \mathbf{x}} , \]  

(A.4b)

and similarly for the Stueckelberg field \( \pi(t, \mathbf{x}) \). Here, the traceless polarization tensors \( \epsilon_{ij}^s \) satisfy \( k_i \epsilon_{ij}^s(k) = 0 \). The property \( ik \epsilon_{ijlm}\epsilon_{im}^s(k) = \lambda_s k \epsilon_{ij}^s(k) \), \( \lambda_s = \pm 1 \), is also repeatedly used. To check that the term with \( i \epsilon_{ijk} \) in Eqs. (4.3), (4.4) is real one can use the following expression for \( \epsilon_{ij}^s(k) \):

\[ \epsilon_{ij}^s(k) = \frac{(\hat{u}_i \hat{u}_j - \hat{v}_i \hat{v}_j) - i(\hat{u}_i \hat{v}_j + \hat{v}_i \hat{u}_j)}{\sqrt{2}} , \]  

(A.5)

where \( \hat{v} \) and \( \hat{u} = \hat{k} \times \hat{v}/|\hat{k} \times \hat{v}| \) are unit vectors orthogonal to \( k \).

Primes on correlation functions denote that we have removed a factor of \((2\pi)^3\) times the Dirac delta function of momentum conservation, and we will drop the time argument on the Fourier modes when we look at late-time correlation functions.

The in-in master formula for the vacuum expectation value \( \langle \mathcal{O}(t) \rangle = \langle \Omega | \mathcal{O}(t) \Omega \rangle \) of an operator \( \mathcal{O}(t) \) (for example \( \mathcal{O}(t) = \gamma^s_k(t)\gamma^s_{k'}(t) \) for the graviton power spectrum) is (see e.g. [36, 37] for a review)

\[ \langle \mathcal{O}(t) \rangle = \left\langle \Omega \left| \left( T e^{-i \int_{-\infty}^t dt' H_I(t')} \right)^\dagger \mathcal{O}(t) \left( T e^{-i \int_{-\infty}^t dt' H_I(t')} \right) \right| \Omega \right\rangle , \]  

(A.6)

where \( H_I \) is the interaction Hamiltonian and the rotation \( -\infty(1 - i \epsilon) \) projects onto the free vacuum. All our calculations of correlation functions will stop at tree level. Switching to conformal time and focusing on the late-time limit \( \eta \to 0 \), the formula above then reduces to

\[ \langle \mathcal{O}(0) \rangle = i \int_{-\infty}^0 d\eta a(\eta) \langle 0 [H_I(\eta), \mathcal{O}(0)] 0 \rangle . \]  

(A.7)
In this appendix we show how to relate the four-dimensional Chern-Simons term to the three-dimensional one and the operator of $S_{\Lambda_4}$. For simplicity we focus on tensor modes only. First, consider the coordinate basis $\partial_\mu^\alpha$ dual to $dx^\alpha_\mu$. If we neglect scalar modes, we have

$$K_{\mu\nu} = \nabla_\mu n_\nu = -\nabla_\mu dx^0_\nu, \quad (B.1a)$$

$$K_\mu^\nu = \nabla_\mu n^\nu = \nabla_\mu \partial_0^\nu. \quad (B.1b)$$

In components, we then find that $\Gamma_{\mu\nu}^0 = K_{\mu\nu}$ and $\Gamma_{\mu0}^\rho = K_{\mu}^\rho$. Using the fact that for a projected tensor the upper temporal indices vanish, and the lower temporal indices vanish as well if we put the shift vector to zero, we find that $\Gamma_{00}^{\rho0} = 0$ and $\Gamma_{00}^{\rho0} = 0$. Finally, with similar manipulations one can show that $\Gamma_{ij}^k = (3)\Gamma_{ij}^k$.

Using $\epsilon_{ijk} = -\epsilon_{ijlk}/\sqrt{-g}$, in Eq. (3.4c) we have

$$S_{\Lambda_4} = -M_p^2 \int d^4x \frac{1}{\Lambda_4} \epsilon_{ijk} \left( \frac{\Gamma_{ij}^\rho \partial_\rho \Gamma_{kl}^\sigma}{2} + \frac{\Gamma_{il}^\rho \Gamma_{jk}^\sigma \Gamma_{kl}^\lambda}{3} \right). \quad (B.2)$$

Dropping the overall constant $-M_p^2/\Lambda_4$ for simplicity, and expanding the Einstein summation, we isolate the three-dimensional Chern-Simons term plus five additional terms. Three of these terms involve two powers of the extrinsic curvature and one of the connection coefficients of the three-dimensional covariant derivative. Using the antisymmetry of $\epsilon_{ijk}$, they add to give

$$-\int d^4x \epsilon_{ijkl} K_{ij} (3)\Gamma_{kl}^m K_{mk}. \quad (B.3)$$

The two remaining terms (that involve the extrinsic curvature and its spatial derivative) are equal to each other after integration by parts. Their sum is

$$\int d^4x \epsilon_{ijkl} K_i (3)\Gamma_{jl}^i K_{kl}. \quad (B.4)$$

Using the fact that the term $-(3)\Gamma_{jk}^m K_{ml}$ in the covariant derivative $D_j K_{kl}$ vanishes once contracted with $\epsilon_{ijk}$, we arrive exactly at the operator of Eq. (3.4a) up to irrelevant factors.

In this appendix we review how to derive Eq. (5.3) and how to integrate by parts the parity-odd operator of Eq. (5.7a). As a warm-up let us derive the equation \cite{38}

$$\lambda(t) (3)R_{\mu\nu} K^{\mu\nu} = \frac{\lambda(t)}{2} (3)RK + \frac{\dot{\lambda}(t)}{2N} (3)R + \text{boundary terms}. \quad (C.1)$$

We rewrite $(3)R^{\mu\nu} K_{\mu\nu}$ as

$$(3)R^{\mu\nu} K_{\mu\nu} = \frac{1}{2} (3)R^{\mu\nu} L_n h_{\mu\nu}$$

$$= \frac{1}{2} L_n (3)R - h_{\mu\nu} L_n (3)R^{\mu\nu} \quad (C.2)$$

$$= \frac{1}{2} \nabla_\rho (3)R^{\rho\nu} - \frac{1}{2} (3)RK - h_{\mu\nu} L_n (3)R^{\mu\nu}. \quad (C.2)$$
We recognize exactly the second operator of Eq. (5.1b), and we can rewrite the first term as
\[ \Delta^3 R_{\mu \nu} = -\frac{1}{2} D_\rho D^\rho \Delta h_{\mu \nu} + \frac{1}{2} h^\rho_\mu D_\nu D^\rho h_{\mu \rho} + \frac{1}{2} h^\rho_\nu D^\rho D_\mu \Delta h_{\rho \sigma} . \]  
(C.3)
we get
\[ (3) R_{\mu \nu} K_{\mu \nu} = \frac{1}{2} (3) RK - \frac{1}{2} \nabla_\rho (3) R h^\rho_\mu + h_{\mu \nu} \Delta^3 R_{\mu \nu} \Delta h_{\rho \sigma} = 2K_{\rho \sigma} , \]  
(C.4)
where the last term is a total spatial divergence. It can be neglected under \( \int d^4x \sqrt{-g} \) up to terms involving scalar modes.

We can manipulate the operator \((3) R_{\mu \nu} n^\rho \nabla_\rho K^{\mu \nu}\) in a similar way (notice that \( n^\rho \nabla_\rho K^{\mu \nu}\) is equal to \( n^\rho \nabla_\rho K^{\mu \nu}\) up to terms suppressed by \( \varepsilon \) and terms involving \( A^\sigma \)). First, we write
\[ (3) R_{\mu \nu} n^\rho \nabla_\rho K^{\mu \nu} = (3) R_{\mu \nu} \mathcal{L}_n K^{\mu \nu} + 2(3) R_{\mu \nu} K^{\mu \rho} K^\rho_\nu . \]  
(C.5)
Expanding \( K_{\mu \nu} = H h_{\mu \nu} + \delta K_{\mu \nu}\) we see that the second term on the right-hand side generates lower-derivative operators and modifies the coefficient of the cubic operator of Eq. (5.2a). What about the first term? We perform the same integrations by parts that led to Eq. (C.4).

The new piece we have is \(-K^{\mu \nu} \mathcal{L}_n (3) R_{\mu \nu}\). Using Eq. (C.3) with \( \Delta h_{\rho \sigma} = 2K_{\rho \sigma}\), it is equal to
\[ -2K^{\rho \sigma} D_\rho D_\sigma K^\rho_\sigma + K^{\rho \sigma} D^\sigma D^\rho D_\mu K_{\mu \sigma} . \]  
(C.6)
We recognize exactly the second operator of Eq. (5.1b), and we can rewrite the first term as
\[ K^{\rho \sigma} D_\rho D_\sigma K^\rho_\sigma = K^{\rho \sigma} D_\rho D_\sigma K^{\mu \rho} + K^{\rho \sigma} [D_\rho, D_\sigma] K^\mu_\rho \]
\[ = D_\sigma (K^{\rho \sigma} D_\mu K^\rho_\mu) - D_\rho K^{\rho \sigma} D_\sigma K^\rho_\mu + (3) R_{\rho \sigma} K^{\lambda \rho} K^\rho_\sigma - (3) R^\rho_\lambda D_\rho K^{\lambda \sigma} K^\sigma_\rho . \]  
(C.7)
The last two terms modify the coefficients of the two cubic operators of Eqs. (5.2) and of lower-derivative operators, while the second term starts at fourth order in perturbations if we consider only tensor modes.

We conclude this appendix by showing that the operator \( e^{\mu \nu \rho \sigma} n_\mu D_\nu \delta K_{\rho \lambda} n^\lambda \nabla_\delta \delta K^\lambda_\sigma / N\) can also be removed. Again, we drop scalar modes throughout. First, we use the relation
\[ n^\delta \nabla_\delta K^\lambda_\sigma = \mathcal{L}_n K^\lambda_\sigma . \]  
(C.8)
With this relation we rewrite the operator as
\[ \frac{e^{\mu \nu \rho \sigma} n_\mu}{N} D_\nu \delta K_{\rho \lambda} n^\delta \nabla_\delta \delta K^\lambda_\sigma = \frac{e^{\mu \nu \rho \sigma} n_\mu}{N} D_\nu K_{\rho \lambda} n^\delta \nabla_\delta K^\lambda_\sigma \]
\[ = \mathcal{L}_n \left( \frac{e^{\mu \nu \rho \sigma} n_\mu}{N} D_\nu K_{\rho \lambda} K^\lambda_\sigma \right) - K^\lambda_\sigma \mathcal{L}_n \left( \frac{e^{\mu \nu \rho \sigma} n_\mu}{N} D_\nu K_{\rho \lambda} \right) . \]  
(C.9)
In the first term on the right-hand side we recognize the operator of \( S_{\Lambda_2} \) after integrating by parts. What about the second term? Using the relations \( \mathcal{L}_n n_\mu = A_\mu \) and \( \mathcal{L}_n e^{\mu \nu \rho \sigma} = -K e^{\mu \nu \rho \sigma} \) we see that we only care about the Lie derivative of \( D_\nu K_{\rho \lambda}\). When the Lie derivative acts on \( K_{\rho \lambda}\) we get \(-e^{\mu \nu \rho \sigma} n_\mu D_\nu K_{\rho \lambda} n^\delta \nabla_\delta K^\lambda_\sigma / N\) (notice the minus sign) plus terms starting cubic in perturbations that are reabsorbed by the operator of Eq. (5.8). Finally, using the relation
\[ \Delta^3 \Gamma^\alpha_\mu \Delta h_{\rho \sigma} = -D^\alpha K_{\mu \rho} + D_\rho K^\alpha_\mu + D_\mu K^\alpha_\rho \]  
(C.10)
for the (projected) variation of the Christoffel symbols in \( D_\nu\), we see that when \( \mathcal{L}_n \) acts on \( D_\nu\) we again get terms that “renormalize” the coefficient of the operator of Eq. (5.8).
D Stueckelberg trick for $\epsilon_{ijk}D_iK_{jl}K'_l$

In this appendix we show how to perform the Stueckelberg trick for the operator in $S_{\Lambda^2}$. Let us define $O_{\Lambda^2} = 2O_{1,2}$, cf. Eqs. (2.2b), (3.4a), as

$$O_{\Lambda^2} = \frac{\epsilon_{\mu\nu\rho\sigma}}{N}n_\mu D_\nu\delta K_{\rho\lambda}\delta K^\lambda_\sigma .$$

(D.1)

Since $\Lambda^2$ is constant in time we do not need to consider it when we introduce the $\pi$ field.

In order to perform the Stueckelberg trick, we first rewrite $O_{\Lambda^2}$ in a way that involves as few tensors that are not invariant under time diffeomorphisms as possible. First, notice that the definition of Eq. (D.1) is equivalent to

$$O_{\Lambda^2} = \frac{\epsilon_{\mu\nu\rho\sigma}}{N}n_\mu D_\rho K_{\nu\lambda}D_\lambda n_\sigma .$$

(D.2)

Then, we use the Codazzi-Mainardi relation

$$h^\nu_\alpha h^\lambda_\beta h^\rho_\gamma h^d_\delta R^e_{\alpha\beta\gamma\delta} = D^\nu K_{\rho\lambda} - D_\rho K^\nu_\lambda$$

(D.3)

together with the antisymmetry of $\epsilon_{\mu\nu\rho\sigma}$ to rewrite $O_{\Lambda^2}$ as

$$O_{\Lambda^2} = \frac{1}{2} \frac{\epsilon_{\mu\nu\rho\sigma}}{N} h^b_\lambda n^e R^e_{\mu\rho\gamma} \nabla^\gamma n_\sigma .$$

(D.4)

Finally, employing the antisymmetry of $R_{e\rho\beta\gamma}$ in $bc$ we arrive at

$$O_{\Lambda^2} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\lambda} R_{\rho\sigma\mu\nu}n_\alpha n^\nu \nabla^\mu \left( \frac{n_\beta}{N} \right) .$$

(D.5)

The advantage of Eq. (D.5) is that we only need to know how $n_\mu$ and $N$ transform under $t \rightarrow t + \pi$. Using that $n_\mu = -N\partial_\mu t$ and that $g^{00} = g^{\mu\nu}\partial_\mu t\partial_\nu t = -1/N^2$ we have

$$N \rightarrow \tilde{f} \cdot N ,$$

$$n_\mu \rightarrow \tilde{f} \cdot (n_\mu - N\nabla_\mu \pi) \equiv \tilde{f} \cdot p_\mu ,$$

$$\tilde{f} = \frac{1}{\sqrt{1 + 2Nn^\mu \nabla_\mu \pi - N^2g^{\mu\nu}\nabla_\mu \pi\nabla_\nu \pi}}$$

(D.6a)

(D.6b)

(D.6c)

and

$$O_{\Lambda^2} \rightarrow \frac{\tilde{f}^2}{2} \epsilon^{\alpha\beta\gamma\lambda} R_{\rho\sigma\mu\nu}p_\alpha p^\nu \nabla^\mu \left( \frac{p_\beta}{N} \right) .$$

(D.7)

First, let us confirm that $O_{\Lambda^2}$ does not couple $\pi$ to the constraints. Using $(^3R_{\nu[\beta\rho\sigma]} = 0$, the only second-order term is of the form $\epsilon^{\alpha\beta\gamma\lambda} n_\alpha D_\rho \delta K_{\sigma\mu}D^\mu D_\beta \pi \propto \epsilon_{ijk}D_j\delta K_{kl}D^l\pi$. Since at this order $\delta K_{kl} = -H\delta N h_{kl} - D_{(k}N_{l)} = -H\delta N h_{kl} - D_kN_l$ (thanks to the fact that $D_i$ is torsionless and that $N_i = D_iN_L$), we remain with

$$\epsilon_{ijk}D_j\delta K_{kl}D^l\pi \supset -\epsilon_{ijk}D_jD_kN_lD^l\pi .$$

(D.8)

At this order in perturbations we can exchange $D_jD_k$ with $D_kD_j$, hence this term vanishes.

Since there is no additional mixing of $\pi$ with the constraints, the correction to Maldacena’s solution for $\delta N$ and $N_L$ enters only at order $\varepsilon H/\Lambda$. According to the discussion in Section 4.1,
at leading order in slow roll we can then put \( N = 1 \), \( N^i = 0 \) in \( \pi \) gauge when discussing non-Gaussianities involving scalar modes. Eqs. (D.6b), (D.6c), (D.7) then become

\[
p_{\mu} = \bar{f} \cdot (n_{\mu} - \nabla_{\mu} \pi) , \tag{D.9a}
\]

\[
\bar{f} = \frac{1}{\sqrt{1 + 2\pi - \pi^2 + h_{\mu\nu} D_{\mu} \pi D_{\nu} \pi}} \tag{D.9b}
\]

and

\[
O_{\Lambda_2} \rightarrow \frac{f^2}{2} e^{\alpha \beta \rho \sigma} R_{\rho \sigma \mu \nu} p_{\alpha} p_{\nu} \nabla^{\mu} p_{\beta} . \tag{D.10}
\]

In terms of metric fluctuations, up to cubic order and assuming a constant Hubble rate we then find that \( O_{\Lambda_2} \) is equal to \( \bar{f}^2/2 \) times the sum of three terms:

1. from the Gauss relation \( h^c_{\rho} h^d_{\sigma} h^a_{\mu} h^b_{\nu} R_{cdab} = \frac{1}{3} R_{\rho \sigma \mu \nu} + 2 K_{[\rho \sigma]} K_{[\mu \nu]} \) we get

\[
a - a^{-2} e^{b_{ij}(3)} R_{ijkl} \partial_{i} \partial_{j} \pi (\gamma_{li} - 2a^{-2} \partial_{i} \partial_{j} \pi) - H e^{b_{ij}(3)} \partial_{i} \partial_{j} \pi \partial_{i} \partial_{j} \pi ; \tag{D.11}
\]

2. from the Codazzi-Mainardi relation we have

\[
2(1 + \pi) \frac{2}{3} e^{b_{ij}(3)} D_{k} \delta K_{jm} ((1 + \pi) \delta K_{jm} - D_{m} D_{j} \pi) + 2 e^{b_{ij}(3)} \partial_{i} \partial_{j} \pi \partial_{i} \partial_{j} \pi - H e^{b_{ij}(3)} \partial_{i} \partial_{j} \pi \partial_{i} \partial_{j} \pi ; \tag{D.12}
\]

3. from the Ricci equation \( h^c_{\rho} h^d_{\sigma} h^a_{\mu} h^b_{\nu} R_{cdab} = -K_{\rho \sigma} K_{[\mu \nu]} + D_{\mu} A_{\nu} - n^{\delta \nu} \nabla_{\delta} K_{\mu \nu} + 2 n_{[\rho} K_{\sigma]} A^{\sigma} + A_{\rho} A_{\mu} \) we get

\[
e^{b_{ij}(3)} \left( a^2 H \delta_{i j} + a^2 \frac{2}{3} \delta_{i j} \right) \partial_{i} \partial_{j} \pi (2a^{-2} \partial_{i} \partial_{j} \pi - \gamma_{ij}) . \tag{D.13}
\]

Here \( e^{b_{ij}(3)} = -\epsilon_{ij} / \sqrt{-g} \) and \( \bar{f} = 1 - \pi + 2\pi^2 - a^{-2} \partial_{i} \partial_{j} \pi / 2 \). In Eq. (D.12) we have that

\[
D_{k} \delta K_{jm} = \partial_{k} \delta K_{jm} - (3) \delta_{km} \delta K_{j} , \tag{D.14a}
\]

\[
D_{m} D_{j} \pi = a^{-2} (e^{-\gamma})_{mn} \partial_{n} \partial_{j} \pi - (3) \delta_{km} \delta K_{j} ; \tag{D.14b}
\]

\[
(3) R_{ijkl} = a^2 \left( \partial_{l} (3) \delta_{mn} - (3) \partial_{m} (3) \delta_{lk} \right) ; \tag{D.14c}
\]

\[
\delta K_{jm} = \frac{a^2 d(e^{-\gamma})_{jm}}{2} ; \tag{D.14d}
\]

\[
\delta K_{mi} = \frac{1}{2} (e^{-\gamma})_{mn} \frac{d(e^{-\gamma})_{ni}}{dt} ; \tag{D.14e}
\]

\[
(3) \delta_{ij} = \frac{-\partial_{k} \gamma_{ij} + \partial_{l} \gamma_{jk} + \partial_{m} \gamma_{ki}}{2} . \tag{D.14f}
\]

From these formulas we see that the operator \( O_{\Lambda_2} \) contains direct interactions of the graviton with \( \pi \), i.e. there are \( \pi \gamma \gamma \) and \( \gamma \pi \pi \) vertices even in absence of scalar metric fluctuations.
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