The d’-dibaryon in a colored cluster model

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We calculate the mass and structure of a \( J^P=0^-, T=0 \) six-quark system using a colored diquark-tetraquark cluster wave function and a nonrelativistic quark model Hamiltonian. The calculated mass is some 350 MeV above the empirical value if the same confinement strength as in the nucleon is used. If the effective two-body confinement strength is weaker in a compound six-quark system than in a single baryon, as expected from a simple harmonic oscillator model, one obtains \( M_{d'}=2092 \) MeV close to experiment.

INTRODUCTION

In pionic double charge exchange reactions on nuclei at 50 MeV and forward angles, there is considerable experimental evidence for a narrow resonance in the \( \pi NN \)-system with quantum numbers \( J^P=0^-, T=0 \). This resonance has been named \( d' \)-dibaryon. Experimentally, it has a mass of \( M_{d'}=2065 \) MeV, and a free hadronic decay width of approximately \( \Gamma_{d'} \approx 0.5 \) MeV [1].

In the present work, we investigate the mass and hadronic structure of a \( J^P=0^-, T=0 \) six-quark system in a colored diquark-tetraquark cluster model using the Resonating Group Method (RGM) [2]. This method determines the orbital configuration of the six-quark system dynamically, i. e. according to a given model Hamiltonian. Our microscopic approach allows to test the underlying assumption of the bag-string model [3], that the \( d' \) is a stretched diquark-tetraquark system. The bag-string model prediction for the \( d' \) mass \( M_{d'} \simeq 2100 \) MeV employs a single, non-antisymmetrized \( q^2 - q^4 \) dumbbell configuration. The present potential model description of the \( d' \) improves previous bag-string model calculations in the following respects: (i) the center of mass energy is exactly removed, (ii) the Pauli principle
for the whole six-quark system is respected, (iii) by virtue of the antisymmetrizer, the $q^3 - q^3$, $q^2 - q^4$, and $q^1 - q^5$ partition into colored clusters, as well as the $q^6$ compound state are automatically included. The model accurately reproduces the mass of the deuteron, which is the only established dibaryon.

A major purpose of this work is to study the effect of quark exchange interactions (Pauli principle) between the colored clusters on the mass and wave function of the $d'$. By comparing the RGM solutions with previous quark shell model results [4] employing a six-quark “bag” basis, we obtain additional information on the amount of clusterization in the system. We employ different confinement parametrizations and study how our results depend on the model of confinement. The central question is whether the present model supports a $J^P=0^-, T=0$ state with a mass compatible with experiment.

MODEL DESCRIPTION

The spontaneous breaking of chiral symmetry of low-energy QCD by the physical vacuum is responsible for the constituent quark mass generation, as well as for the appearance of pseudoscalar and scalar collective excitations of the vacuum ($\pi$ and $\sigma$ fields), that couple to the constituent quarks. The nonrelativistic quark model Hamiltonian for $n$-quarks with equal masses $m_q = 313$ MeV=$m_N/3$ (in SU$_F$(2)) contains therefore besides the residual one-gluon-exchange interaction, modelling asymptotic freedom at short distances, and besides the long-range effective two-body confinement potential, regularized one-pion- and one-sigma-exchange between constituent quarks. Several two-body confinement potentials, that differ in their radial dependence, have been considered [4]. As usual, the few parameters of the Hamiltonian are fitted to the nucleon and $\Delta$ ground state masses.

The six-quark wave function $|\Psi_{d'}\rangle = A(\chi_{L=1}(\vec{r}) \otimes |D\rangle \otimes |T\rangle)$ sketched in figure 1 is expanded in the cluster basis into the internal wave functions of the tetraquark $|T\rangle$ and diquark $|D\rangle$ clusters, respectively, and the relative wave function $\chi_{L=1}(\vec{r})$ between the two clusters, projected onto angular momentum $L=1$. The antisymmetrizer $A$ of Eq. (1), neglected in
previous calculations \cite{3}, contains the quark permutation operators $P_{ij}^{XSTC}$ in orbital (X), spin (S), isospin (T) and color-space (C)

$$\mathcal{A} = 1 - 8P_{46}^{XSTC} + 6P_{35}^{XSTC}P_{46}^{XSTC},$$

(1)

which ensures that the Pauli principle is respected for the whole six-quark system, and allows for a continuous transition from the compound $q^6$ six-quark state to the $q^2 - q^4$ clusterized state and back.

![FIG. 1. The colored diquark-tetraquark cluster model for the $d'$.](image)

**RESULTS AND DISCUSSION**

| Set | $M_D$ [MeV] | $M_T$ [MeV] | $M_{d'}$ [MeV] | $b_6$ [fm] | $M_{noQEX}$ [MeV] | $r_{d'}^{RGM}$ [fm] |
|-----|-------------|-------------|----------------|-----------|------------------|-------------------|
| I   | 643         | 1456        | 2440           | 0.75      | 2316             | 1.10              |
| II  | 621         | 1309        | 2092           | 0.95      | 2013             | 1.39              |
Table 1 shows our main results for two different treatments for the effective two-body confinement. If we assume that the Hamiltonian for baryons and dibaryons is the same, in particular that there exists a universal effective two-body confinement strength for both, baryons and dibaryons (set I), the mass of the $J^P=0^-, T=0$ state is with $M_{d'}=2440$ MeV nearly 400 MeV higher than suggested by experiment. The use of different radial functions for the confinement potential (e.g. linear or error-function) leads to some reduction of the predicted mass, but $M_{d'}$ is still 300-200 MeV higher than the experimental resonance position. A comparison of the results for the $d'$ mass with and without quark exchange ($M_{\text{noQEX}}$) show that the quark exchange interactions required by the Pauli principle contribute an additional energy of 80-120 MeV.

The comparison of the cluster model results with previous shell model results reveals an overall quantitative agreement for the $d'$ mass, the orbital structure, and the size parameter $b_6$ of the $d'$. We show in figure 2 that the $d'$ wave function does not display a pronounced clusterization but resembles rather closely the pure six-quark harmonic oscillator (H.O.) wave function given by the dot-dashed curve. The relative wave function has already died out at distances of about 2.5 fm between the clusters. The quark exchange diagrams enhance the radial color “attraction” between the two colored clusters.

![Figure 2](image_url)

**FIG. 2.** The relative RGM wave function between the tetraquark and diquark clusters with (Pauli-on) and without (Pauli-off) inclusion of the quark exchange diagrams for set I. The RGM wave functions are compared to a single six-quark shell model state (H.O.).
The rms radius in the relative cluster coordinate $r_d^\text{RGM}$ is in general smaller than the sum of the corresponding diquark and tetraquark radii. In other words, there is considerable overlap between the clusters. Therefore, the bag-string model assumption of inert colored clusters at the ends of a stretched bag is not satisfied in the present calculation. The Pauli principle, i.e. the fact that the quantum numbers of the $d'$ are incompatible with two colorless ground state nucleons, together with the confinement forces prevent large interquark distances and no distinct clusterization is observed.

At present, there is no theory of confinement. Although we do not know how to calculate the effective confinement strength for three- and compound six-quarks systems from first principles, we can gain some qualitative insight within the harmonic oscillator model of confinement. If we assume a universal confining mean field for any quark in a hadron (set II), one derives for the quadratic confinement, that the effective two-body confinement strength in the six-quark system $a_c^{(6)}$ is considerably smaller than in the baryon: $a_c^{(6)} \approx a_c^{(3)}/3$. Besides the dependence of the effective two-body confinement strength on the number of quarks and the color representation of the system, the larger characteristic hadronic size of a six-quark bag as compared to a baryon is mostly responsible for the weakening of the six-quark confinement. This assumption leads to a larger $d'$ and a mass $M_{d'} \approx 2092$ MeV close to the experimental $d'$ mass. Conversely, the empirical $d'$ mass may be interpreted as evidence for a weaker effective two-body confinement strength in a compound six-quark system.

Recently, we have calculated the decay width of the $d'$, showing that for a $d'$-mass of $M_{d'} \approx 2100$ MeV, as predicted by the weaker confinement hypothesis, the calculated pionic decay width is compatible with experiment.

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