How to Post-Process Experimental Results from the Flange Bulging Test? Application to the characterization of a Zinc alloy.

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Abstract. Zinc alloys are used in a wide range of application such as electronics, automotive and building construction. Their various shapes are generally obtained by metal forming operation such as stamping. Therefore, it is important to characterize the material with adequate characterization tests. Sheet Bulging Test (SBT) is well recognized in the metal forming community. Different theoretical models of the literature for the evaluation of thickness and radius of the deformed sheet in SBT have been studied in order to get the hardening curve of different materials. These theoretical models present the advantage that the experimental procedure is very simple. But Koç et al. showed their limitation, since the combination of thickness and radius evaluations depend on the material. As Zinc alloys are strongly anisotropic with a special crystalline structure, a procedure is adopted for characterizing the hardening curve of a Zinc alloy. The anisotropy is first studied with tensile test, and SBT with elliptical dies is also investigated. Parallel to this, Digital Image Correlation (DIC) measures are carried out. The results obtained from theoretical models and DIC measures are compared. Measures done on post-mortem specimens complete the comparisons. Finally, DIC measures give better results and the resulting hardening curve of the studied zinc alloy is provided.

1. Introduction
It is now well recognized that material data in the plastic range, obtained with tensile test is not suitable for characterizing materials devoted to be formed by metal forming operations such as stamping for example. Moreover, it is important to define their forming limits under expansion loadings. The bulging test has been developed for giving more information on material behavior when the material is under biaxial loading. Within this test, carried out with circular die, the true stress-strain curve can be obtained. With elliptical dies, it is possible to study the material anisotropy and the Forming Limit Diagram (FLD), in its expansion domain, can be determined.

For post-processing experimental results of the bulging test, there are mainly two possibilities. One follows a very simple experimental route with the help of analytical models. Another one, based on DIC (Digital Image Correlation) systems, needs more time and more expensive device. The first one is certainly more convenient for SMEs (small and medium-sized enterprises) but it is essential to evaluate its validity.

The aim of the present study is to compare these two experimental routes for post-processing experimental results obtained by the bulging test performed on sheets, in the same spirit than other works from the authors presented in IDDRG 2014 and published in [1] for tube bulging test.

Section 1 presents the bulging test and the two possible experimental routes. Section 2 focuses on a zinc alloy used in the present study and the method used for the evaluation of the experimental procedures. The main results and discussions are given in Section 3.
2. Bulging test and experimental routes

2.1. Bulging test
The bulging test performed on sheet specimen allows studying the material behavior in the expansion range. The specimen is a disk cut in a sheet, which is clamped between two dies (Figure 1). The lower die presents a closed cavity in which a fluid is injected, creating an increasing pressure. The upper die is open and the sheet can freely deform under the pressure loading. The upper die can present different geometry, circular or elliptical, for varying the strain path (Figure 2).

During the bulging test, the sheet is subject to expansion and the strain and stress states are described by the following tensors, expressed in spherical coordinates:

\[
\varepsilon = \begin{bmatrix} \varepsilon_r & 0 & 0 \\ 0 & \varepsilon_\theta & 0 \\ 0 & 0 & \varepsilon_\phi \end{bmatrix}, \quad \sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_\theta & 0 \\ 0 & 0 & \sigma_\phi \end{bmatrix}
\]

and \( \varepsilon_r = \ln \left( \frac{t}{t_0} \right) \) (1)

where \( \varepsilon_\theta / \varepsilon_\phi = \beta \) and \( \sigma_\theta / \sigma_\phi = \alpha \), with \( \beta \) the strain path and \( \alpha \) the stress path.

The strain path \( \beta \) is linked to the elliptical die radii and the stress path \( \alpha \) can be defined by the anisotropic coefficients and the strain path:

\[
\alpha = \frac{a(\lambda)\beta + c(\lambda)}{b(\lambda) + c(\lambda)\beta}
\]

with:

\[
a(\lambda) = \frac{1}{4} [(F + G) + 2N - 2(F - G) \cos(2\lambda) + (F + G + 4H - 2N) \cos^2(2\lambda)]
\]

\[
b(\lambda) = \frac{1}{4} [(F + G) + 2N + 2(F - G) \cos(2\lambda) + (F + G + 4H - 2N) \cos^2(2\lambda)]
\]

\[
c(\lambda) = \frac{1}{4} [F + G - 2N - (F + G + 4H - 2N) \cos^2(2\lambda)]
\]

(3)

\( F, G, H \) and \( N \) are the Hill coefficients defined from the Lankford coefficients. \( \lambda \) represents the angle between the solicitation direction and the rolling direction;

The mechanical equilibrium (4) is expressed by the following equation from [2,3,4]:

\[
\frac{\sigma_\theta}{\rho_\theta} + \frac{\sigma_\phi}{\rho_\phi} = \frac{p}{t}
\]

(4)

By convention, the orientation \( \theta \) is along the largest die radius.
Figure 1. The sheet bulging test where $p$ is the internal pressure in the cavity, $t_0$ the initial thickness of the sheet, $t$ the current thickness at the pole, $\rho$ the radius of curvature, $h_d$ the bulge height, $r_c$ (20mm) and $r_f$ (5 mm) the cavity and fillet radius respectively.

Figure 2. Different geometries for the upper die with a major axis of 64mm and a minor axis of 16mm, 32mm et 48mm respectively.

2.2. Experimental Route N°1

The simplest experimental route consists of measuring the internal pressure ($p$ in Figure 1.) and the bulge height at the pole of the specimen ($h_d$ in Figure 1). A pressure sensor measures the pressure inside the cavity of the lower die and a displacement sensor placed at the center of the specimen measures the displacement at the pole (Figure 3.a). The resulting measure is a pressure – bulge height ($p, h$) curve illustrated in Figure 3.b. From the ($p, h$) curve, we can get the true stress-strain curve with the help of analytical models found in the literature [4-8]. All these analytical models are based on the mechanical equilibrium of an elementary volume which expression is, in case of circular free bulging window:

$$\frac{\sigma}{p} = \frac{p}{2t}$$

The pressure $p$ is measured, and we need to evaluate the curvature radii and the current thickness to get the stress. There are several models for the calculation of the curvature radius and the current thickness. Table 1 and Table 2 summarize the main equations essentially based on geometrical analysis of the bulging test with a circular free bulging window.

There are then several possible combinations to get the stress $\sigma$. Moreover, the evaluation of the current thickness at the pole permits to evaluate the components of the strain tensor.

A study performed on an AISI 201 [8] has shown that the best combination to get the flow stress curve was the calculation of the thickness with Kruglov’s [7] method and the calculation of the bulge radius with Panknin’s [4] approach. But we have no assurance that these recommendations are valid for
all types of materials, and we want to extend their possibilities to the analysis of experimental bulge test
done with elliptical dies.

![Experimental device](image1)

![Resulting measures](image2)

Figure 3. (a) Experimental device for experimental route n° 1 (b) Resulting measures: pressure –
bulge height curve (p, h).

Table 1. Models and equations for the evaluation of the curvature radius.

| Models         | Bulge radius |
|----------------|--------------|
| Hill [5]       | \( \rho = \frac{r_c^2 + h_d^2}{2h_d} \) |
| Panknin [4]    | \( \rho = \frac{(r_c + r_f)^2 - 2h_d r_f + h_d^2}{2h_d} \) |

Table 2. Models and equations for evaluation of the thickness.

| Models          | Thickness at dome apex |
|-----------------|------------------------|
| Hill [5]        | \( t = t_0 \left( \frac{1}{1 + \left( \frac{h_d}{r_c} \right)^2} \right)^2 - n \) |
| Chakrabarty [6] | \( t = t_0 \left( \frac{1}{1 + \left( \frac{h_d}{r_c} \right)^2} \right)^2 \) |
| Kruglov [7]     | \( t = t_0 \left( \frac{r_c}{\rho \arcsin \left( \frac{r_c}{\rho} \right)} \right)^2 \) |

2.3. Experimental Route N°2

Nowadays, the use of optical measurements is very common. In this case, the specimen is painted with
a speckle pattern black on white (Figure 4.a). Two CCD cameras film the evolution of the pattern during
the sheet bulging (Figure 4.b) and with image stereo correlation, it is possible to evaluate the field of
displacement in the sheet from which the strain field in the tangent plane of the sheet and the curvature radius can be calculated. From the plastic incompressibility, the current thickness can be computed.

\[ \text{Equation 4} \]

\[ r_0 = 0.20; \ r_{45} = 0.27; \ r_{90} = 0.50. \]

Zinc alloys, where the Zn is associated to Ti and Cu, are commonly used in building applications. A TiZn\(_{16}\) phase takes place with the shape of fillets where stress concentrates that limits its formability.

3. Material and Methods

3.1. Zinc alloy material
Zinc has a low melting temperature and is very easy to be shaped. With its hexagonal crystalline structure where the c/a ratio higher than the classical value, this material presents an important anisotropy. Its Lankford coefficients have been determined by tensile tests: \[ r_0 = 0.20; \ r_{45} = 0.27; \ r_{90} = 0.50. \] Zinc alloys, where the Zn is associated to Ti and Cu, are commonly used in building applications. A TiZn\(_{16}\) phase takes place with the shape of fillets where stress concentrates that limits its formability.

3.2. Comparison of the Experimental Routes
With the stereo correlation technique, the strain components and the current thickness are directly evaluated through the displacement field. For stress components, equation 4 is used and the curvature radii must be known. It is then important to compare the curvature radius and the current thickness obtained by the analytical models and the digital image processing with experimental measures. Bulging tests performed with elliptical dies have been stopped at different level of pressure loading.

The resulting deformed sheets have been cut along the largest die radius. The thickness and the curvature radius have been measured with a non-contact optical measurement machine. The corresponding bulge height is also measured. The evolution of the curvature radius and of the current pole thickness with the bulge height is compared and experimental measures are overlaid. The resulting stress-strain curves are also compared.

4. Results and Discussion

4.1. Circular die
The bulging test is first carried out with a circular die having a 20 mm radius (\(r_c\)). Due to the circular symmetry of the test, we consider the major and minor directions corresponding to the rolling and transverse directions.

The test is run on one hand by measuring \(h_M\) with a displacement sensor and on the other hand by measuring the strain field in the sheet with the use of the stereo correlation system. Each test has been repeated three times to verify the repeatability of the experimental results.

For the analytical analysis, the thickness has been evaluated with Hill approach and with the combinations Kruglov-Hill and Panknin-Kruglov. For the curvature radius, the methods proposed by Hill and Panknin have been applied.

With the DIC system, the VIC 3D software has been used to evaluate the curvature radius and the pole thickness based on the current coordinates of the speckle pattern.
From the $h_d$ measure, the use of analytical models leads to different values for the strain, the curvature radius and the pole thickness. These different results are compared with the one obtained with the DIC system.

Figure 5 shows the comparison between the evaluated curvature radius and the pole thickness obtained by the analytical models and the DIC system. Figure 5a shows clearly the difficulty for analytical models to represent the results obtained with the DIC system, even if all the methods converge for the largest bulge height $h_d$. It is particularly remarkable that the DIC system permits to distinguish the curvature along the major and minor directions revealing the anisotropy of the studied zinc alloy. The analytical models cannot reveal this anisotropy. From the Figure 5b, it is obvious that the combination Kruglov-Hill is the best and gives the same results than the VIC system. It is then different from the conclusions done on an AISI 201 in [8].

4.2. **Elliptical die**
We present the results obtained with the elliptical die with dimensions ratio (major axis/minor axis) of 0.75. The specimen is cut along the major and minor axis, and the curvature radius and the thickness are measured along the two cut specimen by using the equation of the Tables 1 and 2.

The results obtained for the curvature radius along the major axis are given in Figure 6a. It is clear that the analytical models are not able to give an efficient evaluation of the curvature radius. Hill approach represents well the radius at the beginning of the test, as the Panknin model is better for higher bulge height. The resulting curvature radius obtained by post-processing digital images of the speckle pattern with the VIC 3D software is definitively better.

Comparisons have been done for the pole thickness evaluation in Figure 6b. Very large difference can be observed between the different methods. Again, the result obtained with the stereo-correlation technique is better and coincide with the measures done on post-mortem specimen. For the test with the elliptical die which dimensions are major axis/minor axis of 0.75, it is found that the mean value between the results obtained along the minor and the major axes with the combination Kruglov-Hill overlaps the one obtained with the stereo-correlation. Unfortunately, this remark cannot be generalized to other elliptical die geometries (ratio of 0.25 and 0.5).

4.3. **Determination of the stress-strain curve**
Finally, we illustrate in Figure 7 the implication of a poor post-processing of the sheet bulging test on the resulting hardening curve obtained with circular die. Kruglov-Panknin method leads to an underestimation of the forming possibilities of the zinc alloy sheet under biaxial loading, as Hill’s equations will lead to its overestimation.

At this stage, the stereo-correlation technique is the more appropriate method for getting the hardening curve of the studied zinc alloy.
Figure 5. Comparison of (a) the curvature radius along the major axis and (b) the pole thickness obtained by analytical models and with the stereo-correlation technique for bulging test in circular die.

Figure 6. Comparison of (a) the curvature radius along the major axis and (b) the pole thickness obtained by analytical models and with the stereo-correlation technique for bulging test in elliptical die (ratio 0.75). The symbol represents the measures done on post-mortem specimen.

Figure 7. Implication on the resulting hardening curve.
5. Conclusion
The bulging test is recommended for getting efficient material data in the plastic range for performing predictive finite element simulations of metal forming operations such as stamping or deep-drawing. The post-processing of the experimental results of such a test is more complicated. Analytical models have been developed in the past to get the strain and stress components; they are convenient for SMEs because the experimental procedure is very simple. But the present work presents important limitations of these models for characterizing the studied zinc alloy.

The analytical models cannot reveal the anisotropy. During bulging test in circular die, due to anisotropy, the curvature radius is different in the rolling and the transverse directions. As analytical models are based on geometrical considerations where only the radius of the free bulging zone is considered, they lead to the same curvature while it can be observed with the naked eye on the bulged samples, that they are different along the rolling and transverse directions. For the elliptical die, the analytical models lead to different values for the curvature radius and the pole thickness. Again, as they are based on geometrical considerations, different reference dimensions (minor or major axis) for the die are considered leading to two different values for the pole thickness, which is not physically possible. In conclusion, for anisotropic materials such as the studied zinc alloy, stereo correlation has to be preferred for post-processing experimental results of the bulging test.

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