Observations on the Space of Four Dimensional String and $M$ Theory Vacua$^1$

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Abstract

The space of four dimensional string and $M$ theory vacua with non-Abelian gauge symmetry, chiral fermions and unbroken supersymmetry beyond the electroweak scale appears to be a disconnected space whose different components represent distinct universality classes of vacua. Calculating statistical distributions of physical observables a la Douglas therefore requires that the distinct components are carefully accounted for. We highlight some classes of vacua which deserve further study and suggest an argument which may serve to rule out vacua which are small perturbations of supersymmetric $AdS_4$.

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String/M theory appears to have many vacua. Many known examples of these vacua do not correspond to universes with just four macroscopic spacetime dimensions, an $SU(3) \times SU(2) \times U(1)$ gauge symmetry at the electroweak scale, chiral fermions or many other observed properties of our universe. For example, eleven dimensional Minkowski spacetime, the $E_8 \times E_8$ heterotic string compactified to six dimensions on a $K3$ surface, or Type IIB string theory on $AdS_5 \times S^5$ with $N$ units of flux are strongly believed - because of various strong-weak coupling dualities - to be exact vacua.

This is not a problem for the theory if there does exist a suitably stable “correct vacuum”. The main problem for theory is to demonstrate the existence of such a vacuum. Indeed, there is no good reason that we know of that implies one vacuum in a quantum theory of gravity should be preferred over another and the proliferation of incorrect string vacua suggests that we may never have such a reason. However, a proper understanding of cosmology in string/M theory may, ultimately, lead to selection rules [1].

Given the lack of any known selection rules, we therefore need to understand the space of realistic string/M theory vacua. To make progress with such an understanding clearly requires choices. For example, in field theory one can conceive of many extensions of the standard model beyond the electroweak scale. This is reflected in string theory also: there may be vacua with classically broken supersymmetry at the compactification scale or vacua with supersymmetry unbroken at high energies. Similarly, some stable vacua may have large extra dimensions others small extra dimensions.

Furthermore, we do not have a complete list of all vacua and even if we did would presumably not have time to wade through the list and single out all of those that agree with the standard model at the electroweak scale and below. In view of these issues, we will consider vacua which have some basic properties that we will deem essential to all realistic vacua. This is the practical viewpoint proposed in [1].

We will make the following choices: four macroscopic spacetime dimensions, non-Abelian gauge symmetry (with rank $\geq$ four), chiral fermions and unbroken supersymmetry at some scale above $M_{\text{ew}}$.

Remarks. 1. The first three choices are justified by experiment. 2. Whilst the existence of supersymmetry is purely conjectural, in addition to the standard reasons for including it, there is a more practical reason: string vacua with classically broken supersymmetry are poorly understood and may even be unstable.

We will denote the space of string/M theory vacua with these properties as $\mathcal{M}$. At the present time we have no idea how big $\mathcal{M}$ is. Of course, the set of known examples of string/M theory vacua in $\mathcal{M}$ provide us with a lower bound on the number of points it contains. One aim of the present paper is
to present evidence that \( \mathcal{M} \) is a disconnected space - disconnected in a very physical sense; namely that there exist points in \( \mathcal{M} \) whose physics is of a different universality class to other points in \( \mathcal{M} \). A second aim is to suggest that the space is “larger” than our currently known examples would tend to suggest.

A third point is that the discussion here serves to put the recent work of Douglas et. al. [1, 2] into a broader context: one needs to know what fraction of \( \mathcal{M} \) the known vacua represent before one can say with confidence that a physical property is statistically favoured. We will return to this point later.

What do we know about vacua in \( \mathcal{M} \)? There are many examples of vacua in \( \mathcal{M} \). A more or less complete list of known points in \( \mathcal{M} \) can be classified as follows, in no particular order

a. \( E_8 \times E_8 \) or \( SO(32) \) heterotic strings on Calabi-Yau threefolds, \( Z_{CY} \)
b. \( M \) theory on \( Z_{CY} \times \mathbb{C}^1 \)
c. Type IIA orientifolds on \( Z_{CY} \) with D6-branes
d. Type IIB orientifolds on \( Z_{CY} \) with D(odd)-branes
e. \( F \)-theory on Calabi-Yau 4-folds.
f. \( M \) theory on \( G_2 \)-holonomy manifolds with singularities.
g. Non-geometric CFT’s where the extra dimensions are represented by a SCFT without a geometric limit.
h. Freund-Rubin compactifications of \( M \) theory on singular Einstein manifolds with \( R > 0 \).

Remarks. 1. Vacua in classes [a], [c], [d] can be defined perturbatively with a world-sheet super conformal field theory (SCFT) and as such are expected to share many properties with those in class [g]. 2. Under certain conditions, one can add fluxes to some of the vacua listed here [3]. By definition, this should be taken implicitly. For example, vacua in class [c] means all \( F \)-theory vacua with supersymmetric fluxes.

Dualities Between Classes.

Vacua from a given class may be dual to vacua in another class. In fact, vacua from classes [a] through to [f] can be related by dualities. For example, all of the [c] vacua are also [f] vacua; many of the [d] vacua are limits of [c] vacua; when the \( G_2 \)-holonomy manifolds in [f] vacua are \( K3 \)-fibered, they are dual to vacua of class [a]. So, for vacua from the first six classes listed, non-perturbative equivalences are known to occur. As such, many of these vacua
share many similar properties. For example, the classical four dimensional vacuum is flat spacetime.

More importantly, there are vacua in this list which are apparently not dual to other vacua in this list.

For example, if the Calabi-Yau fourfolds in $[c]$ or the $G_2$-holonomy manifolds in $[f]$ are not $K3$-fibered then they are apparently not dual to the heterotic string on a Calabi-Yau threefold.

In fact if we assume that all dualities between string/M theory vacua originate from fundamental known dualities between vacua with 16 or more supercharges, then it is clear that there exist points in $\mathcal{M}$ not dual to other points in $\mathcal{M}$.

This already indicates the disconnectedness of $\mathcal{M}$. An important assumption of [1] is that vacua of type $[d]$ represent a reasonable enough fraction of vacua in $\mathcal{M}$ so that statistical analyses of such vacua give a reasonable picture of the statistics of $\mathcal{M}$ itself. However if, as we have suggested, vacua exist which are not dual to $[d]$ vacua, we need to have a good idea about how many such vacua there are. For vacua from classes $[a]$ through to $[f]$ this amounts to knowing for example, how many Calabi-Yau fourfolds or $G_2$-holonomy manifolds there are which are not $K3$-fibered. In the case of non-dual Calabi-Yau vacua, this might not represent a problem for such analyses, since it seems reasonable to suppose that statistics based on a single Calabi-Yau will not vary considerably from Calabi-Yau to Calabi-Yau. However, non-dual $G_2$-holonomy vacua may have different statistics to $[d]$ vacua.

Let us also note that, dualities aside, very distinct universality classes could also be contained within a single type of vacuum. Consider for instance the $G_2$-holonomy vacua described in our paper in [3]. Properties of such vacua depend upon a topological invariant $c_2$, the imaginary part of a complex Chern-Simons invariant. If $c_2$ is large, these vacua have large extra dimensions, if it is small they have small extra dimensions. We also remind the reader that constructing $G_2$-holonomy manifolds is technically difficult, so at present we have no idea if there are more vacua large $c_2$ vacua than those with small $c_2$.

We will now go on to discuss vacua of class $[h]$ and describe several features of these which suggest that they are in a completely different universality class from the rest. We also emphasise that there are large numbers of $[h]$ vacua. If, as we suggest, these vacua are truly distinct from the other types, then it is crucial for a statistical analysis of the physics of $\mathcal{M}$ to compare $[h]$ vacua with, say, $[c]$ vacua.
Freund-Rubin Vacua

Thus far, we have only discussed vacua in classes \([a] - [g]\). A Freund-Rubin vacuum \([4]\) can be described classically as a solution to d=11 supergravity in which the extra dimensions form a compact Einstein manifold \((X, g_7)\) with positive scalar curvature and four dimensional spacetime is anti de Sitter. The background is supersymmetric if \((X, g_7)\) admits a Killing spinor. Quantum mechanically the Freund-Rubin vacuum might be defined as the three-dimensional conformal field theory residing on the world volume of a \(N\) M2-branes at the tip of a cone with base \(X\) \([5, 6]\).

In a Freund-Rubin vacuum, non-Abelian gauge symmetry can emerge from one of two sources: a) isometries of \(g_7\) or b) from ADE-singularities supported along a 3-manifold \(Q \subset X\), analogously to the case of \(G_2\)-holonomy manifolds \([8]\).

In order for a supersymmetric Freund-Rubin vacuum with non-Abelian gauge symmetry to represent a point in \(\mathcal{M}\) it must also admit chiral fermions. In \([7]\) we demonstrated that such points exist. The mechanism for chiral fermions is identical to that of the \(G_2\)-holonomy vacua \([9]\): conical singularities in \(X\) of various types.

The Freund-Rubin vacua in \(\mathcal{M}\) which were studied in \([7]\) have various properties which suggest that they are generally not dual to any of the vacua in \(\mathcal{M}\) of types \([a]\) to \([g]\).

The vacua discussed in \([7]\) roughly come in a two parameter family labelled by two integers \((N, k)\). \(N\) is the unit of Freund-Rubin flux, which is dual to the number of M2-branes characterising the holographically dual conformal field theory. \(N\) controls the four-dimensional cosmological constant, which in Einstein frame is of order

\[
\lambda \sim \frac{m_p^4}{N^{\frac{4}{3}}} \tag{1}
\]

This formula demonstrates that \(\lambda\) can be parametrically small in Planck units. This is also true for the supersymmetric \(AdS\) vacua discussed in \([3]\).

\(k\) is proportional to the rank of the gauge group, which in the examples considered in \([7]\) is \(SU(k)^3\).

Both \(N\) and \(k\) are free parameters of the supergravity solution and can, at least classically, take any positive integral value.

Vacua of types \([a] - [g]\) have a gauge group whose rank is typically not that large eg less than 100. Consider a Freund-Rubin vacuum with, say, \(k = 10^8\). It is difficult to imagine that it could be dual to, say a Calabi-Yau compactification with branes. Of course, if the gauge group were confined at
low energies, then perhaps $k$ is dual to a flux quantum number in Calabi-Yau compactification. But the gauge group need not be confined in general.

Another aspect of the vacua discussed in [7] is that the supergravity solution is extremely simple compared to Calabi-Yau or $G_2$-holonomy vacua. This is because one can more or less explicitly write the spacetime metric whereas this is much more difficult for compact, simply connected Ricci flat manifolds. This is reflected by the fact that the 7-manifold $X$ of extra dimensions is much simpler topologically than a Calabi-Yau or $G_2$-holonomy space. For instance, the examples discussed in [7] are manifolds fibered by a circle over $S^3 \times S^3$ and the circle degenerates in particular ways along spherical submanifolds of $S^3 \times S^3$ (these are the places at which $X$ has singularities supporting gauge fields and chiral fermions). These examples are almost certainly not $K3$-fibered lending further evidence, albeit mathematical, to the proposition that they are not dual to other classes of vacua.

**Other Classes of Vacua?**

Thus far we have only considered the known types of vacua in $\mathcal{M}$ and have offered some arguments that up to non-perturbative dualities there are many distinct universality classes. In our opinion, it is only a matter of time before new vacua in $\mathcal{M}$ are found. This is just a technical problem of generally solving the conditions for unbroken supersymmetry.

**Cutting $\mathcal{M}$ down to Realistic Vacua.**

Ultimately we would like to consider only those vacua in $\mathcal{M}$ which agree with the standard model at low energies. Again we face the difficulty of not knowing all vacua. So we will have to choose questions which are more general. We will begin by discussing a simple physical objection to Freund-Rubin vacua. Then we will point out that this objection in fact quite general and applies to many vacua in $\mathcal{M}$ including some of those discussed in [3]. The argument might be used to rule out certain vacua which are small perturbations of supersymmetric $AdS_4$. We will emphasise that until we understand supersymmetry breaking and other corrections to the theory that these arguments are not strong enough to rule out more interesting vacua in $\mathcal{M}$.

The objection to vacua of type $[h]$ is this. The classical solution is such that the cosmological constant of anti de Sitter space is of the same order of magnitude (but with opposite sign) as that of the 7-manifold of extra dimensions. This implies that the masses of Kaluza-Klein modes are of the same order as the fluctuations of the graviton in the $AdS_4$. Then, if the $AdS_4$ were large enough to contain our universe the Kaluza-Klein modes would be too light to have escaped detection today. However, this argument has two flaws.
Firstly, it is possible that the radii of the extra dimensions be parametrically smaller than the radius of $AdS_4$ as was demonstrated in [7]. In this case there is a mass gap between $m_{kk}$ and $m_{AdS}$. However, in practice it is probably impossible to produce an Einstein manifold whose scalar curvature is equal to that of a round 7-sphere but whose volume is exponentionally smaller which is what is required for a Freund-Rubin vacuum with $\lambda \sim 10^{-120} m_p^4$ and large enough Kaluza-Klein masses.

The second flaw is simply that the argument is based on the behaviour of the classical solution. Quantum corrections, which are poorly understood in this context, or even supersymmetry breaking classical corrections might give a vacuum which is compatible with a large universe and suitably massive Kaluza-Klein modes. An example of the latter might be obtained by additional fluxes through $X$, raising the energy of the vacuum.

In any case, these physical arguments against Freund-Rubin vacua are actually symptomatic of many vacua in $\mathcal{M}$. The reason for this is that in a four dimensional supergravity theory with superpotential $W$ and Kahler potential $K$, unbroken supersymmetry requires that $\partial_i W + \partial_i KW = 0$, where the derivative is with respect to all fields. If the theory contains $p$ fields this is $p$ equations for $p$ fields. Typically, with $\mathcal{N} = 1$ supersymmetry the superpotential will be non-zero and hence we expect many supersymmetric vacua to exist. Since $W$ will typically be non-zero in the vacuum\(^2\) we have a supersymmetric $AdS_4$ vacuum in which $\lambda = -3e^K|W|^2$ in Planck units. In vacua with fluxes and/or quantum corrections accounted for, this can give rise to $AdS$ vacua whose cosmological constant is much less than one in Planck units [3]. However, if one considers such examples in which $|\lambda_{AdS}| \sim 10^{-120} m_p^4$ then typically the masses of Kaluza-Klein modes are too small. This is simply because the potential in Einstein frame decreases at large volume. This is due to the $e^K$ factor being small $^3$.

An exception to this argument could be provided by vacua in which the supersymmetric critical point exists because of instanton contributions to the superpotential. If $W$ is consequently very small in the vacuum - a condition which does not require particularly large volume - then one would have cosmologically large $AdS$ space with massive enough Kaluza-Klein modes. Supersymmetric KKLT vacua, examples of which have recently been constructed [10], or $G_2$-holonomy vacua with fluxes and instanton contributions included, would be natural places to look for such examples.

So whilst this argument might serve to rule out vacua which are small\(^2\)setting it to zero is an additional condition, generically incompatible with the first $p$ constraints.

\(^3\)K is quite generally of the form $-lnVol(X)^a$ for positive $a$
perturbations of supersymmetric $AdS$, small superpotentials and vacua in which high degree cancellations to the supersymmetric vacuum energy occur can not be falsified by it.

In any case, supersymmetry breaking and other corrections to the potential clearly need to be understood better before we can safely disregard significant numbers of points in $\mathcal{M}$.

In summary: there appear to be a large number of four dimensional string and $M$ theory vacua with non-Abelian gauge groups, chiral fermions and unbroken supersymmetry above $M_{\text{ew}}$. Among these are distinct universality classes whose contributions to the distributions of physical observables must be carefully accounted for in the proposal of [1]. Non-$K3$-fibered $G_2$-manifolds and Freund-Rubin vacua are two notable classes which deserve further study. From general supergravity considerations there are a significant number of supersymmetric $AdS_4$ vacua. Vacua which are a small perturbation of these in which the Kahler potential is large and negative and the superpotential is not small might be ruled out on the grounds that the extra dimensions are too large.

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