The geometry of the space of leaf closures of a transversely almost Kähler foliation

Robert A. Wolak

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Abstract

We study the geometry of the leaf closure space of regular and singular Riemannian foliations. We give conditions which assure that this leaf space is a singular symplectic or Kähler space.

In recent years physicists and mathematicians working on mathematical models of physical phenomena have realised that modelling based on geometric structures on now classical smooth manifolds is insufficient. In some cases more complicated topological spaces appear naturally and one would like to develop geometry of such spaces. One of the well-known examples is the orbit space of a smooth action of a compact Lie group. Such a space is a stratified pseudomanifold of Goresky-MacPherson, cf. [7, 9, 4]. This fact has been used to describe the topology and structure of the reduced space of the momentum map in the singular case, cf. [15].

The study of the Riemannian geometry of the orbit space of a smooth action of a compact Lie group has been initiated in [1]. One should also mention K. Richardson’s paper, cf. [13], in which the author demonstrates that any space of orbits of such an action is homeomorphic to the space of the closures of leaves of a regular Riemannian foliation, and, obviously, cf. [11], vice versa.

In this paper we initiate the study of the geometry of the space of leaf closures of a Riemannian foliation, both regular or singular. The first part is concerned with the space of leaf closures of an interesting class of regular Riemannian foliations - transversely almost hermitian, which includes transversely almost Kähler foliations. In this case the foliated manifold is presymplectic. The aim of this note is to describe the geometry of the leaf space \( M/F \); in other word to describe the way in which the Riemannian, almost complex and symplectic structures descend onto this leaf space.
One of the reasons of the study of such foliations is the fact that recently there have been a renewed interest in non-integrable geometries associated to Riemannian structures - in fact these geometries correspond to the choice of an additional geometric structure compatible with the Riemannian metric, cf. [6]. Almost Hermitian structures are one of the best known examples of such structures. Foliated (almost) Hermitian manifolds or foliated Kähler manifolds are of interest as they combine three foliated structures: a Riemannian, an almost complex and a symplectic ones. These structures appear quite naturally in geometry as Sasakian manifolds form a special class manifolds foliated by transversally Kähler 1-dimensional Riemannian foliations, cf. [20] and $\mathcal{K}$-manifolds give another example of such foliated manifolds, cf. [5].

1 Geometric structures on foliated manifolds

It is well-known that on a smooth manifold $M$ of dimension $m$
- the existence of a Riemannian metric $g$ is equivalent to the existence of an $O(m)$-reduction $B(M, O(m))$ of the frame bundle $L(M)$ of $M$;
- the existence of an almost complex structure $J$ is equivalent to the existence of a $GL(n, \mathbb{C})$-reduction $B(M, GL(n, \mathbb{C}))$ of the frame bundle $L(M)$ of $M$, $m = 2n$;
- the existence of a 2–form $\omega$ of maximal rank is equivalent to the existence of an $Sp(n)$-reduction $B(M, Sp(n))$ of the frame bundle $L(M)$ of $M$, $m = 2n$;

The almost complex structure $J$ is a complex structure iff
- the Nijenhuis tensor $N(J)(X, Y) = [X, Y] + J[X, JY] + J[JX, Y] + [JX, JY] = 0$;
or
- the structure $B(M, GL(n, \mathbb{C}))$ is integrable;
or
- the structure tensor of $B(M, GL(n, \mathbb{C}))$ vanishes identically.

The 2–form $\omega$ is closed iff
- the structure tensor of $B(M, Sp(n))$ vanishes identically;
or
- the $Sp(n)$-structure $B(M, Sp(n))$ is integrable.

Any two of the three groups $GL(n, \mathbb{C}), Sp(n)$ and $O(2n)$ intersect giving $U(n)$ - the maximal compact subgroup of $GL(n, \mathbb{C})$ and $Sp(n)$.  

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Any $U(n)$–reduction $B(M, U(n))$ of $L(M)$ assures the existence of a Riemannian metric $g$ and an almost complex structure $J$ such that

$$g(Ju, Jv) = g(u, v).$$

The associated 2–form $\omega(u, v) = g(Ju, v)$ is of maximal rank.

If the structure tensor of $B(M, U(n))$ vanishes identically, then the structure tensors of the corresponding $B(M, GL(n, \mathbb{C}))$ and $B(M, Sp(n))$ vanish identically, thus the associated almost complex structure $J$ is integrable and the associated 2–form $\omega$ is closed. Therefore such a structure is a Kähler structure.

In the foliated case we have the following. Let the foliation $\mathcal{F}$ be given by a cocycle $\mathcal{U} = \{U_i, f_i, g_{ij}\}$ modelled on a manifold $N_0$ of dimension $2q$, i.e.

i) $\{U_i\}$ is an open covering of $M$,

ii) $f_i: U_i \rightarrow N_0$ are submersions with connected fibres defining $\mathcal{F}$,

iii) $g_{ij}$ are local diffeomorphisms of $N_0$ and $g_{ij} \circ f_j = f_i$ on $U_i \cap U_j$.

The manifold $N = \coprod f_i(U_i)$ we call the transverse manifold of $\mathcal{F}$ associated to the cocycle $\mathcal{U}$ and the pseudogroup $H$ generated by $g_{ij}$ the holonomy pseudogroup (representative) on the transverse manifold $N$.

It is not difficult to verify, cf. [16, 17], that the following conditions a, b, c and d are equivalent:

a) On the manifold $N$ there exist

i) a positive definite symmetric tensor $\overline{g}$;

ii) an almost complex structure $\overline{J}$ such that:

for any $u, v \in TN$ $\overline{g}((\overline{J}u, \overline{J}v) = \overline{g}(u, v);

for any $h \in H$ and any $u, v \in TN$ $\overline{g}(dh_u, dh_v) = \overline{g}(u, v)$;

for any $h \in H$ $dh\overline{J} = \overline{J}dh$;

then the 2–form $\overline{\omega}$, $\overline{\omega}(u, v) = \overline{g}(\overline{J}u, v)$ for any $u, v \in TN$, is $H$-invariant and of maximal rank. Therefore $(N, \overline{g}, \overline{J})$ is an almost hermitian manifold and elements of the holonomy pseudogroup are holomorphic isometries.

b) On the normal bundle $N(M, \mathcal{F})$ there exist

i) an almost complex structure $J$,

ii) a positive definite symmetric tensor $g$ such that

for any $u, v \in N(M, \mathcal{F})$ $g(Ju, Jv) = g(u, v)$;

for any $X \in T\mathcal{F}$ $L_X g = 0$ $L_X J = 0$;

Between these three structures we have the following relations (on the foliated level), cf. [3] for the non-foliated case.
1) If a base-like 2–form \( \omega \) of maximal rank and a foliated almost complex structure are given such that

\[
\omega(Ju, Jv) = \omega(u, v) \]

\[
\omega(Ju, u) > 0 \quad \forall u \in N(M, F), \quad u \neq 0,
\]

then \( g(u, v) = \omega(u, Jv) \) is a foliated positive definite Riemannian metric on \( N(M, F) \).

2) If a foliated Riemannian metric \( g \) and a foliated almost complex structure \( J \) are given on \( N(M, F) \) such that \( g(Ju, Jv) = g(u, v) \) for any \( u, v \in N(M, F) \), then \( \omega(u, v) = g(Ju, v) \) is a non–degenerate foliated 2–form of maximal rank, perhaps not closed.

3) If a foliated Riemannian metric \( g \) and a foliated 2–form \( \omega \) of maximal rank on \( N(M, F) \) are given, then there exists a compatible foliated almost complex structure \( J \) on \( N(M, F) \). However, in general, the Riemannian metric \( \tilde{g}(u, v) = \omega(u, Jv) \) is different from the initial Riemannian metric \( g \).

To prove this fact we have to mimick the considerations in \([3, pp.68-69]\).

c) Let \( L(M, F) \) be the frame bundle of \( N(M, F) \). \( L(M, F) \) admits a foliated reduction, cf. \([11, 16, 17]\), to the structure group \( U(q) \).

d) The frame bundle \( L(N) \) admits a \( U(q) \)-reduction which is \( \mathcal{H} \)-invariant.

The vanishing of the normal Nijenhuis tensor \( N(J)(X,Y) = [X,Y] + J[JX,Y] + J[X,JY] - [JX,JY] = 0 \mod F \) for any \( X, Y \in N(M, F) \), ensures that the almost complex structure on \( N(M, F) \) is a complex one. This condition is equivalent to the vanishing of the Nijenhuis tensor \( N(J) \) of the induced almost complex structure \( J \) on the transverse manifold \( N \).

The same is true for the corresponding reduction \( B(N, Sp(q)) \) and the 2–form \( \varpi \). The 2-form \( \omega(u, v) = g(Ju, v) \) is a basic 2–form inducing on the normal bundle \( N(M, F) \) a non-degenerate 2–form.

Let \( B(M, Sp(q); F) \) be the \( Sp(q) \)-reduction of \( L(M; F) \) corresponding to \( \omega \). The fact that the form is base–like translates itself into the condition "foliated reduction". Therefore the condition "the structure tensor vanishes identically" is equivalent to "\( d\omega = 0 \)". The same is true for the corresponding reduction \( B(N, Sp(q)) \) and the 2–form \( \varpi \); the reduction should be holonomy invariant, cf. \([16, 17]\).
2 The regular case: the topology of the leaf closure space $M/\mathcal{F}$

An almost transversely hermitian foliation is, in particular, a Riemannian foliation, therefore the manifold $M$ admits the following stratification, cf. [11].

Let $k$ be any number between 0 and $n$. Define

$$\Sigma_k = \{ x \in M : x \in L_\alpha, \ dim L_\alpha = k \}.$$  

The leaves of $\mathcal{F}$ in any $\Sigma_k$ are of the same dimension but they can have different holonomy. P. Molino demonstrated that connected components of these subsets are submanifolds of $M$ and that $\Sigma_k \subset \bigcup_{i \leq k} \Sigma_i$. For some $i$ the sets $\Sigma_i$ may be empty. Let $k_0$ be the maximum dimension of leaves of $\mathcal{F}$. Then the set $\Sigma_{k_0}$ is open and dense in $M$. It is called the principal stratum.

In many respects the foliation $\mathcal{F}|_{\Sigma_i}$ behaves as a RF on a compact manifold, in particular, the closure of any its leaves is in $\Sigma_i$. There we can define

$$\Sigma_{ij} = \{ x \in \Sigma_i : x \in L \in \mathcal{F}, \ dim L = j \}.$$  

Each $\Sigma_{ij}$ is a submanifold of $\Sigma_i$. The closures of leaves of $\mathcal{F}$ induce a regular RF $\mathcal{F}_{ij}$ of compact leaves on $\Sigma_{ij}$. The holonomy group of each $L$ is finite, and there is a finite number of types of the holonomy groups. Therefore each set $\Sigma_{ij}$ decomposes into a finite number of submanifolds

$$\Sigma_{p_j \alpha} = \{ x \in L \in \mathcal{F} : \ dim L = p, \ dim L = p, h(L, x) \in \alpha \}$$

where $h(L, x)$ is the holonomy group of $L$ in $\Sigma_{p_j}$ and $\alpha$ is conjugacy class of finite subgroups of $O(q_{pj})$ where $q_{pj}$ is the codimension of $\mathcal{F}$ in $\Sigma_{p_j}$.

In this way, we have obtained a stratification $\mathcal{S} = \{ \Sigma_\gamma \}$ of $(M, \mathcal{F})$ into submanifolds on which both $\mathcal{F}$ and $\mathcal{F}$ define regular Riemannian foliations and the foliation $\mathcal{F}$ is without holonomy. The stratification introduced above is slightly finer than the stratification defined by M. Pierrot in [12]. On each stratum $S_\alpha$ of $\mathcal{S}$ the foliation $\mathcal{F}$ defines a compact RF without holonomy, so the natural projection on the corresponding stratum $\mathcal{S}_\alpha$ of $\mathcal{S}$ is a locally trivial fibre bundle.

Let us choose a stratum $\Sigma_\alpha \in \mathcal{S}$. The foliation $\mathcal{F}$ is regular on $\Sigma_\alpha$, so we have the following orthogonal splitting of the bundle $TM|\Sigma_\alpha$:

$$(*) \quad TM = T\mathcal{F} \oplus Q_1 \oplus Q_2 \oplus Q_3$$

where $T\mathcal{F} \oplus Q_1 = \mathcal{F}$ and $T\mathcal{F} \oplus Q_1 \oplus Q_2 = T\Sigma_\alpha$. 

2.1 Symplectic structure

A foliation $\mathcal{F}$ is transversally symplectic if it admits a basic closed 2-form $\omega$ of maximal rank. Therefore the codimension of $\mathcal{F}$ is even, say $2q$. In particular, any transversally almost Kähler foliation is transversally symplectic.

If $\mathcal{F}$ is transversely symplectic, then $C^\infty(M, \mathcal{F}) = C^\infty(M, \mathcal{F}) = C^\infty(M/\mathcal{F})$ admits the structure of a Poisson algebra.

The form $\omega$ projects on the holonomy invariant form $\overline{\omega}$, which is a symplectic form of $N$. The symplectic form $\overline{\omega}$ defines a Poisson structure $\{,\}_N$ on $N$ which assigns to two $H$-invariant functions an $H$-invariant function. Therefore this Poisson structure lifts to a Poisson structure $\{,\}_B$ on the set of basic functions $C^\infty(M, \mathcal{F})$. The basic functions are the same for both foliations $\mathcal{F}$ and $\mathcal{F}$, therefore $\{,\}_B$ is a Poisson structure on the set $C^\infty(M, \mathcal{F})$. This algebra of smooth functions can be considered as the smooth structure on the singular space $M/\mathcal{F}$, which is compatible with the definition of a smooth structure on the orbit space, cf. [15].

We would like to prove that our stratified pseudomanifold $M/\mathcal{F}$ is a symplectic stratified space, cf. [15].

Let $\mathcal{F}$ be transversally almost Kähler for the Riemannian metric $g$, the almost complex structure $J$ and the symplectic form $\omega$ on the normal bundle $N(M, \mathcal{F})$. If $J(T\mathcal{F}/T\mathcal{F}) \subset T\mathcal{F}/T\mathcal{F}$, then $M/\mathcal{F}$ is a singular symplectic space.

It is well-known that the closures of leaves of a Riemannian foliations are the orbits of the commuting sheaf of this foliation, cf. [11], which is defined using the bundle of transverse orthonormal frames. The local vector fields of the commuting sheaf are local Killing vector fields of the induced Riemannian metric. In our case we can refine the definition, cf. [18] [17]. The compatible foliated Riemannian and symplectic structures define a foliated $U(q)$-reduction $B(M, U(q); \mathcal{F})$ of the bundle $L(M, \mathcal{F})$ of transverse frames, i.e. the frames of the normal bundle $N(\mathcal{F})$. The group $U(q)$ is of type 1, so the foliation $\mathcal{F}_1$ of the total space of $B(M, U(q); \mathcal{F})$ is transversely parallelisable (TP) and the closures of leaves form a regular foliation with compact leaves. The projections of these leaves onto $M$ are the closures of leaves of $\mathcal{F}$. From the general theory of TP foliations, cf. [11], we know that these closures are the orbits, in the foliated sense, of local vector fields commuting with global foliated vector fields, in particular with vector fields of the transverse parallelism. These vector fields are the lifts to $B(M, U(q); \mathcal{F})$ of local foliated vector fields on $(M, \mathcal{F})$, which are infinitesimal automorphisms of the transverse $U(q)$-structure, so they preserve both the transverse Riemannian metric, almost complex structure and the associated 2–form.
We shall look at transverse symplectic and holomorphic structures and check whether some of their components project onto the leaf space $M/\mathcal{F}$ and verify what structures they induce.

First consider the principal stratum $\Sigma_0$. The splitting of $TM$ reduces itself to $T\mathcal{F} \oplus Q_1 \oplus Q_2$ as $\Sigma_0$ is an open and dense subset $M$. Since $J(Q_1) \subset Q_1$, then $J(Q_2) \subset Q_2$. Therefore as the splitting is $g$-orthogonal, it is also $\omega$-orthogonal and the transverse symplectic form $\omega$ can be written as $\omega = \omega^{2,0} + \omega^{0,2}$ where $\omega^{2,0}$ and $\omega^{0,2}$ are homogeneous components with respect to the splitting. Locally, the subbundle $Q_1$ is spanned by foliated Killing vector fields $X$ such that $L_X \omega = 0$. Their flows preserve the splitting so

$$L_X(\omega^{2,0} + \omega^{0,2}) = L_X \omega^{2,0} + L_X \omega^{0,2}$$

and $L_X \omega^{2,0} = 0$ and $L_X \omega^{0,2} = 0$.

The restrictions of both forms to $Q_1$ and $Q_2$, respectively, are of maximal rank. Therefore to prove that $\omega^{0,2}$ is a transverse symplectic form for the foliation $\mathcal{F}$ on $\Sigma_0$, it is sufficient to demonstrate that $d\omega^{0,2} = 0$.

In fact, $0 = d\omega = d\omega^{2,0} + d\omega^{0,2} = d_1 \omega^{2,0} + d_2 \omega^{2,0} + \partial \omega^{0,2} + d_1 \omega^{0,2} + d_2 \omega^{0,2}$.

Thus $d_1 \omega^{2,0} = 0$, $d_2 \omega^{2,0} = 0$, $\partial \omega^{0,2} + d_1 \omega^{0,2} = 0$, $d_2 \omega^{0,2} = 0$.

Therefore it remains to prove that $d_1 \omega^{0,2} = 0$. Now, for any vector field $X$ of the commuting sheaf

$$L_X \omega^{0,2} = 0 = i_X d\omega^{0,2} + d_i X \omega^{0,2} = i_X d\omega^{0,2} = i_X d_1 \omega^{0,2} + i_X d_2 \omega^{0,2} = i_X d_1 \omega^{0,2}.$$

As these vector fields span the subbundle $Q_1$ we obtain $d_1 \omega^{0,2} = 0$, and hence $d\omega^{0,2} = 0$. Therefore our foliation $\mathcal{F}$ is transversely symplectic for the $2$-form $\omega^{0,2} = \tilde{\omega}$. Thus the projection $\pi_0 : \Sigma_0 \rightarrow \Sigma_0$ projects the $2$-form $\tilde{\omega}$ to a symplectic form on $\Sigma_0$, which we denote by the same letter.

Let $\Sigma_0$ be any stratum of $(M, \mathcal{F})$. $\mathcal{F}$ induces a regular foliation of no holonomy.

In [19] we have proved that global i.a. of $\mathcal{F}$ are tangent to the strata and that the module $X(M, \mathcal{F})$ of these global vector fields is transverse to $\mathcal{F}$ in each stratum. If $X$ is an i.a., so is $JX$. Therefore each stratum is $J$-invariant, i.e.

$$J(Q_1 \oplus Q_2) \subset Q_1 \oplus Q_2,$$

hence the splitting (*) over any stratum is also $\omega$-orthogonal. In this case the standard reasoning ensures that the $2$-form $\omega_\alpha = i_\alpha^* \omega$, where $i_\alpha$ is the inclusion of the stratum $\Sigma_\alpha$ into $M$, is a transverse symplectic form of the foliated manifold $(\Sigma_\alpha, \mathcal{F})$. Having proved that the same considerations as for
the principal stratum demonstrate that each stratum of the stratification $\mathcal{S}$ is a symplectic manifold.

Let $\pi_\alpha : \Sigma_\alpha \to \Sigma_\alpha$ be the local trivial fibre bundle defining the foliation $\mathcal{F}$ on $\Sigma_\alpha$. Clearly, the mapping $\pi_\alpha$ is a morphism of the Poisson algebras $C^\infty(\Sigma_\alpha, \mathcal{F}), \{,\}_\alpha$ and $C^\infty(\Sigma_\alpha), \{,\}_\alpha$, where the Poisson brackets $\{,\}_\alpha$ and $\{,\}_\alpha$ are defined by $\omega_\alpha$ and $\bar{\omega}_\alpha$, respectively.

To complete the proof that $M/\mathcal{F}$ is a singular symplectic space we have to show that for any $\Sigma_\alpha \in \mathcal{S}$ the inclusion $i_\alpha : \Sigma_\alpha \to M/\mathcal{F}$ is a Poisson morphism, i.e.

$$\forall f, g \in C^\infty(M/\mathcal{F}) \quad \{f, g\}_B|_{\Sigma_\alpha} = \{f|_{\Sigma_\alpha}, g|_{\Sigma_\alpha}\}_\alpha.$$

In fact,

$$\{f, g\}_B|_{\Sigma_\alpha} = \omega(X_f, X_g)|_{\Sigma_\alpha} = \omega_\alpha(X_f, X_g) = \omega_\alpha(X_f|_{\Sigma_\alpha}, X_g|_{\Sigma_\alpha}) = \{f|_{\Sigma_\alpha}, g|_{\Sigma_\alpha}\}_\alpha,$$

as vector fields $X_f, X_g$ are tangent to strata.

The almost complex structure $J_2$ on $Q_2$

$$g(J_2 v, w) = \omega(v, w)$$

for any $v, w \in Q_2$ is the restriction of the almost complex structure $J$ to $Q_2$ and therefore its Nijenhuis tensor $N_{J_2}$ is equal to 0.

### 3 Singular transversely almost Kähler foliations

The notion of a singular Riemannian foliation was introduced by Pierre Molino in [10], see also [11]. However, we are not aware of any notion of an almost complex structure adapted to a singular foliation.

**Definition 1.** A $(1,1)$-tensor field $J \in \text{Hom}(TM, TM)$ is a foliated almost complex structure iff

1) $J(T\mathcal{F}) \subseteq T\mathcal{F}$;

2) for any $i.a$ $X$ of $\mathcal{F}$ the vector field $JX$ is also an $i.a.$ of $\mathcal{F}$;

3) for any $i.a$ $X$ of $\mathcal{F}$ $J^2 X = -X \mod \mathcal{F}$.

**Properties**

1) In the regular case any transverse almost complex structure can be extended to a foliated almost complex structure.

2) If $J(T\mathcal{F}) \subseteq T\mathcal{F}$, then the strata of $(M, \mathcal{F})$ are $J$-invariant provided that the foliation $\mathcal{F}$ is Riemannian.
It is a simple consequence of (i) of the definition and of the fact that
global i.a. of $\mathcal{F}$ are transverse to the closures of leaves in the strata, cf. [19].

3) On any stratum $\Sigma_\alpha$ the foliations $\mathcal{F}$ and $\mathcal{F}$ are regular. Then the
tensor field $J_\alpha = J|T\Sigma_\alpha$ is well defined and induces transverse almost complex
structures for $\mathcal{F}|\Sigma_\alpha$.

A foliated Riemannian metric $g$ and a foliated almost complex structure $J$ are said to be compatible if $g(JX, JY) = g(X, Y)$ for any vectors $X, Y \in TM$. Then on any stratum $\Sigma_\alpha$ the induced Riemannian metric $g_\alpha$ and the induced foliated almost complex structure $J_\alpha$ are compatible. Then the 2-
form $\omega_\alpha(X, Y) = g_\alpha(J_\alpha X, Y)$ for $X, Y \in T\Sigma_\alpha$ is of maximal rank. Moreover, $J_\alpha$ induces a transverse almost complex structure for $\mathcal{F}|\Sigma_\alpha$.

**Definition 2.** A singular foliation $\mathcal{F}$ on $M$ is said to be transversely almost Kähler if it admits a foliated Riemannian metric $g$ and a foliated almost complex structure, which are compatible, and such that on any stratum $\Sigma_\alpha$ of the associated stratification of $M$ the 2-forms $\omega_\alpha$ are closed. Such a structure is called transversely Kähler if the induced almost complex structures are transversely integrable for $\mathcal{F}_\alpha$.

**Theorem 1.** Let $\mathcal{F}$ be a transversally almost Kähler singular foliation on a compact manifold $M$. Then the space of leaf closures $M/\mathcal{F}$ is an almost Kähler singular space.

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