Theoretical uncertainties in sparticle mass predictions from computational tools

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ABSTRACT: We estimate the current theoretical uncertainty in sparticle mass predictions by comparing several state-of-the-art computations within the minimal supersymmetric standard model (MSSM). We find that the theoretical uncertainty is comparable to the expected statistical errors from the Large Hadron Collider (LHC), and significantly larger than those expected from a future $e^+e^-$ Linear Collider (LC). We quantify the theoretical uncertainty on relevant sparticle observables for both LHC and LC, and show that the value of the error is significantly dependent upon the supersymmetry (SUSY) breaking parameters. We also present the theoretical uncertainty induced in fundamental scale SUSY breaking parameters when they are fitted from LHC measurements. Two regions of the SUSY parameter space where accurate predictions are particularly difficult are examined in detail: the large $\tan\beta$ and focus point regimes.

KEYWORDS: Supersymmetry Breaking, Beyond Standard Model, Supersymmetric Models
1. Introduction

Weak-scale supersymmetry (SUSY) is well motivated \cite{1,2} because it solves the technical hierarchy problem, removing ultra-violet quadratic divergent corrections to the Higgs mass. Viable weak-scale supersymmetry implies over a hundred “soft breaking” terms in the MSSM Lagrangian at the weak scale, making general analysis intractable. By making model-dependent assumptions on the origin of SUSY breaking for the MSSM fields, one often provides relations between the weak-scale SUSY breaking parameters, vastly reducing the available parameter space. Many theoretical schemes of SUSY breaking exist, the most popular of which can be classified by the mechanism that mediates SUSY breaking from a hidden sector to the MSSM fields. In this paper, we will use minimal supergravity (mSUGRA) \cite{3}, minimal anomaly mediation (mAMSB) \cite{4} and minimal gauge (mGMSB) \cite{5} mediation.

Supposing SUSY is discovered at a present or future collider, it will be a major challenge to measure the SUSY breaking parameters with good accuracy. In order to determine the free parameters, physical sparticle masses (or kinematical variables related to them) will be measured. These sparticle masses must be turned into SUSY breaking parameters, typically at some high scale. Provided enough information is collected, this will allow tests on the relations between the high-scale parameters. For example, a simple question already addressed \cite{6,7,8,9} is: do the gaugino masses unify, and if so, do they unify at the same scale as the gauge couplings? A double
affirmative would be strong support in favour of SUSY GUT-type schemes. On the other hand, there are many other possibilities: for example, string models can be non-universal [10], or unify at an intermediate scale.

There is a large literature upon expected empirical errors on the observables both a Linear Collider [11, 12, 13, 14, 15] and the LHC [16, 17, 18, 19]. It will be essential to know the theoretical uncertainty if we are to discriminate models of SUSY breaking. There are several studies of how empirical errors propagate into uncertainties on fundamental-scale SUSY breaking parameters in the literature [20, 21, 22, 16, 7, 23, 8, 9].

The deduction of high-scale SUSY breaking parameters from observables inevitably involves theoretical errors coming from the level of approximation used (e.g. neglected higher order terms), and it is these uncertainties that we study here. We use four modern codes to calculate MSSM spectra: ISAJET 7.64 [24], SOFTSUSY 1.71 [25], SPHENO 2.0 [26] and SUSPECT 2.101 [27]. We use the differences in results between the codes to define the current theoretical uncertainties. We have done our best to eliminate differences due to bugs by examining the relevant parts of codes in detail if there was an obvious large discrepancy. However, it would be unrealistic to claim that all of the codes are completely bug free. We therefore take a practical interpretation of ‘theoretical uncertainty’: after all, when fitting experimental data to SUSY breaking models, one must use one of the available computational tools. The precise implementation of the known higher-order corrections differs and has been found to produce significantly different results (for example, using different scales for parameters in the highest-order corrections). Therefore, certainly the differences in results between the codes are due (at least to a large part) to unknown higher-order corrections, matching more traditional notions of ‘theoretical uncertainty’.

Previously, there have been some studies of differences between various calculations of the MSSM spectrum. 2-5% differences in sparticle masses were noticed [28] between various particles along the Snowmass (SPS) [29] model lines 1a and 6 between the SOFTSUSY 1.2, ISAJET 7.51 and SUSPECT 2.0 programs. Model line 2 (the so-called focus point line, with very heavy scalar sparticles) was observed to show huge 30% differences in the masses of the weak gauginos. Differences in the spectra and branching ratios were also observed in [31] comparing ISAJET 7.58, SUSYGEN 3.00 and PYTHIA 6.2. Moreover, large 10% level differences between SOFTSUSY 1.3, ISAJET 7.58 and FeynSSG were observed [18] in the mSUGRA Post-LEP benchmarks [31]. Some of the benchmarks in some of the codes were found to not be consistent with electroweak symmetry breaking, unless one fiddled with the Standard Model inputs $m_t$ and $\alpha_s(M_Z)$. These were noticeably points E,F (focus-points) and K,M (high tan $\beta$ and high $m_0, m_{1/2}$ points). Likewise, the input parameters had to be adjusted in [31] to get similar spectra from ISAJET 7.51 and SSARD. Initial results highlighting the differences between the predictions in the focus-point and high tan $\beta$ regimes have already been presented by the current authors as a conference proceed-
We will present the main results of this last work here for completeness, updated to state-of-the-art calculations.

In this paper, we push these initial observations of theoretical uncertainties further by (i) comparing them to the expected experimental accuracies at the LHC and a future LC and (ii) examining their effect upon future empirical fits to fundamental-scale SUSY breaking schemes. We take expected statistical errors upon sparticle observables from previous studies of mSUGRA at the LHC \[^{[14]}\] at two of the LHC benchmark points. We then perform a fit to mSUGRA at each point with each of the four codes. The statistical precision of the fitted fundamental mSUGRA parameters may then be compared with the theoretical error by looking at the differences between the four fits. We next quantify and present the theoretical error in the coloured sparticle masses at the SPS points. We also re-examine the SPS points providing theoretical uncertainties on mass predictions for sparticles that may kinematically be accessed at a 500 GeV LC. Quantification of errors at particular points will give us an idea of their magnitude and whether or not they significantly depend upon the SUSY breaking scheme (or point).

In section 2, we introduce the four state-of-the-art calculations we will use and their level of approximation: ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101. ‘Tricky’ regions of the MSSM parameter space, where it is difficult to make accurate predictions, are discussed in section 3: large \(\tan \beta\) and the focus-point regime. In section 4.1, we perform the LHC mSUGRA empirical fits and then quantify the theoretical error on squark and gluino masses for the SPS points. In section 4.2, we quantify theoretical errors on masses relevant for a 500 GeV Linear Collider. Finally, there are conclusions and an outlook in section 5.

2. The codes

We compare the latest versions of four public SUSY renormalisation group evolution (RGE) codes which we think constitute a representative sample of such programs: ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101. The basic principle of the SUSY mass spectrum calculation is the same in all programs: Gauge and Yukawa couplings are taken as input parameters at the electroweak scale. However, in the MSSM, in order to define them in the \(\overline{\text{DR}}\) scheme, from experimental observables, one must first subtract threshold corrections from sparticles. The sparticle spectrum is unknown at this stage and so to begin the calculation, some guess is made for the soft SUSY breaking parameters and spectrum. The MSSM parameters are then run to the high scale \(M_X\) by RGEs. At \(M_X\), boundary conditions are imposed on the SUSY breaking parameters. Couplings and SUSY parameters are then run back down to \(M_{\text{SUSY}} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}\) where the \(\mu\) and \(B\) MSSM Higgs potential parameters are set in order to give correct radiative electroweak symmetry breaking consistent with an input value of \(\tan \beta = v_2/v_1\) (\(v_1, v_2\) being the two Higgs fields’ vacuum expectation values).
values (VEVs)). The SUSY masses are calculated and radiative corrections are applied, and the parameters are run down to the electroweak scale. Finally, the whole process is iterated in order to obtain a stable solution.

An overview of which corrections are implemented in each of the four programs is given in Table 1. From this table we already expect some differences in the results due to the different levels of radiative corrections applied. In particular, ISAJET 7.64 has no finite radiative corrections to most of the sparticle masses. However, we note that even if each column of the table were identical, one could expect different numerical results from each of the codes. In practice, if a quantity is calculated at one-loop, one has the freedom (at one-loop accuracy) of using whatever scale one desires for parameters in the one-loop correction itself. The difference between using, for instance, pole or running masses or couplings derived in the MS or DR scheme in one-loop corrections is formally of higher-loop order, but leads to non-negligibly different results. Indeed, one can roughly estimate the effects of higher-loop terms by the difference in predicted masses by varying the scale of parameters in the highest loop included.

Let us now point out some of the differences in more detail. A subtle but important issue is the treatment of Yukawa couplings. In general, they are derived from the quark masses as

$$ h_t = \sqrt{2} m_t/v_2, \quad h_b = \sqrt{2} m_b/v_1, $$

where $m_t$ and $m_b$ are the running top and bottom quark masses in the DR scheme. Obviously, it makes a difference whether the VEVs $v_{1,2}$ are running (SOFTSUSY 1.71, SPHENO 2.0, SUSPECT 2.101) or not (ISAJET 7.64). It makes an important difference.

| RGEs   | ISAJET 7.64 | SUSPECT 2.101 | SOFTSUSY 1.71 | SPHENO 2.0 |
|--------|-------------|---------------|---------------|------------|
|        | 2−loop      | 2−loop        | 2−loop        | 2−loop     |
| VEVs   | not running | running (1−loop) |               |            |
|        | scalars at 1−loop |           |               |            |
| Yukawa cpl. |               |               |               |            |
| $h_t$  | full 1−loop  | full 1−loop   | full 1−loop   | full 1−loop |
| $h_b$  | full 1−loop  | full 1−loop   | full 1−loop   | full 1−loop |
| Higgs sector | 3rd gen. (s)fermions | complete 1−loop [33] | 2−loop [33, 34] |            |
|         | 1−loop [35]  | var. ops.     | 2−loop [36]   | 2−loop [37, 38] |
| SUSY masses | some corr. for $\tilde{\chi}^\pm$ | 1−loop approx. for $\Delta M_1, \Delta M_2, \Delta \mu$ | full 1−loop |            |
| $\tilde{t}$ |               | 1−loop approx. | full 1−loop   | full 1−loop |
| $\tilde{b}$ |               | 1−loop approx. | full 1−loop   | full 1−loop |
| $\tilde{g}$ |               |               |               |            |

**Table 1:** RGEs and radiative corrections implemented in ISAJET 7.64, SUSPECT 2.101, SOFTSUSY 1.71 and SPHENO 2.0.
how the $\overline{\text{DR}}$ masses are calculated from the pole or $\overline{\text{MS}}$ masses. We first discuss Standard Model threshold corrections, and afterwards the sparticle loop corrections.

- **ISAJET 7.64** takes 2-loop QCD corrections to $m_t$ into account, including the shift from the $\overline{\text{MS}}$ to the $\overline{\text{DR}}$ scheme as \[39\]

  $$m_t(M_t)^{\overline{\text{DR}}}_{\overline{\text{MS}}} = M_t \left[ 1 + \frac{5\alpha_s}{3\pi} + \left( 16.11 - 1.04 \left( 5 - \frac{6.63}{M_t} \right) \right) \left( \frac{\alpha_s}{\pi} \right)^2 \right]^{-1}, \tag{2.2}$$

  where $M_t$ is the top pole mass and $\alpha_s = \alpha_s(M_t)$ at 3-loops in the $\overline{\text{MS}}$ scheme. For the bottom quark mass, ISAJET 7.64 takes a hard-coded value of $m_b(M_Z)^{\overline{\text{DR}}}_{\overline{\text{MS}}} = 2.82$ GeV \[10\].

- **SOFTSUSY 1.71** and SPHENO 2.0 calculate both $h_t$ and $h_b$ at $Q = M_Z$. The $\overline{\text{DR}}$ top mass is related to the pole mass by 2-loop QCD \[33, 11\]:

  $$m_t(Q)^{\overline{\text{DR}}}_{\overline{\text{MS}}} = M_t \left[ 1 - \frac{\alpha_s}{3\pi} (5 - 3L) - \alpha_s^2 \left( 0.538 - \frac{43}{24\pi^2} L + \frac{3}{8\pi^2} L^2 \right) \right] \tag{2.3}$$

  where$^1$ $L = \ln(M_t^2/Q^2)$. For the $b$ quark, 3-loop relations \[42\] are used to calculate $m_b(M_t)^{\overline{\text{MS}}}_{\overline{\text{SM}}}$ from the $b$ pole mass, which is run to $M_Z$ by 3-loop RGEs \[43, 44\]. $m_b(M_Z)^{\overline{\text{MS}}}_{\overline{\text{SM}}}$ is then related to the $\overline{\text{DR}}$ mass by \[23\]

  $$m_b(M_Z)^{\overline{\text{DR}}}_{\overline{\text{SM}}} = m_b(M_Z)^{\overline{\text{MS}}}_{\overline{\text{SM}}} \left[ 1 - \frac{\alpha_s}{3\pi} - \frac{35\alpha_s^2}{72\pi^2} + \frac{3g_2^2}{128\pi^2} + \frac{13g_1^2}{1152\pi^2} \right] \tag{2.4}$$

  in SOFTSUSY 1.71. SPHENO 2.0 neglects the last two terms of eq. (2.4).

- **SUSPECT 2.101** uses 2-loop relations and 2-loop RGEs to derive $m_b(M_t)^{\overline{\text{MS}}}_{\overline{\text{SM}}}$ and $m_t(M_t)^{\overline{\text{MS}}}_{\overline{\text{SM}}}$ from the $b$ and $t$ pole masses. For the conversion to the $\overline{\text{DR}}$ scheme, eqs. (2.3) and (2.4) are applied (with $L = 0$ in eq. (2.3) since $Q = M_t$).

  Using $Q = M_t$ to define $m_t(Q)$ (ISAJET 7.64, SUSPECT 2.101) is in principle more accurate than using $Q = M_Z$ (SOFTSUSY 1.71, SPHENO 2.0) since then the QCD logs between $M_Z$ and $M_t$ are re-summed. We refer the reader to the respective manuals \[24, 25, 26, 27\] for more details.

  The next step is to include SUSY loop corrections. For $m_t$, all four programs apply full 1-loop SUSY corrections according to \[33\]. For $m_b$, the full 1-loop (ISAJET 7.64, SPHENO 2.0) or the leading (SOFTSUSY 1.71, SUSPECT 2.101) SUSY corrections are included. The $\tan \beta$ enhanced corrections to $m_b$ are re-summed as given in \[43\] in

$^1$After the publication of this paper, it was noted that eq. (2.3) incorrectly describes the 2-loop QCD corrections to the top mass: eq. (2.3) holds for $L = \ln(m_t^2(Q)/Q^2)$, not for $L = \ln(M_t^2/Q^2)$. SOFTSUSY 1.71 and SPHENO 2.0 therefore contain the incorrect formula, which ought to be

  $$m_t(Q)^{\overline{\text{DR}}}_{\overline{\text{SM}}} = M_t \left[ 1 - \frac{\alpha_s}{3\pi} (5 - 3L) - \alpha_s^2 \left( 0.876 - \frac{91}{24\pi^2} L + \frac{3}{8\pi^2} L^2 \right) \right] \tag{2.3a}$$

  for $L = \ln(M_t^2/Q^2)$. We thank D. R. T. Jones for drawing our attention to this issue.
all four programs, see [32]. Still, there are some important differences in the implementation of these corrections. For example, the $\tilde{g}\tilde{t}$ correction to $m_t$ is

$$
\left( \frac{\Delta m_t}{m_t} \right)_{\tilde{g}\tilde{t}} = -\frac{\alpha_s}{3\pi} \left\{ B_1(m_t^2, m_{\tilde{g}}, m_{\tilde{t}_1}^2) + B_1(m_t^2, m_{\tilde{g}}, m_{\tilde{t}_2}^2) + \frac{m_{\tilde{g}}}{m_t} \sin 2\theta_t \left[ B_0(m_t^2, m_{\tilde{g}}, m_{\tilde{t}_1}^2) - B_0(m_t^2, m_{\tilde{g}}, m_{\tilde{t}_2}^2) \right] \right\}. \quad (2.5)
$$

ISAJET 7.64 adds this correction at $m_t$, but using a scale $Q = \sqrt{m_{\tilde{g}} m_{\tilde{t}}}$, (this is done to avoid double counting of logarithmic corrections which are included via step functions in the RGEs) and $\alpha_s = \alpha_s(m_{\tilde{g}})$ in eq. (2.5), while SOFTSUSY 1.71 and SPHENO 2.0 calculate it at $Q = M_Z$ and SUSPECT 2.101 at $Q = M_t$. Accordingly, $m_t = m_t(Q)_{SM}$ in the term $m_{\tilde{g}}/m_t$ enters with different values in all four programs.

Due to differences in the inclusion of finite radiative corrections to sparticle masses, the gluino and stop masses in (2.5) vary from program to program. In particular, ISAJET 7.64 calculates the corrections to $m_{\tilde{g}}$ at $Q = m_{\tilde{g}}$ while SUSPECT 2.101 calculates them at $Q = M_Z$, which leads to quite different gluino masses. Analogous differences exist in the other contributions to $(\Delta m_t)^{\text{SUSY}}$ as well as in the calculation of $(\Delta m_b)^{\text{SUSY}}$.

Another comment is in order concerning $\alpha_s$. The value of $\alpha_s(M_Z)$ from experiment is given in the $\overline{\text{MS}}$ scheme. SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101 take $\alpha_s(M_Z)_{\overline{\text{MS}}}$ as input and convert it to the $\overline{\text{DR}}$ scheme [33]

$$
\alpha_s^{\overline{\text{DR}}}(M_Z) = \frac{\alpha_s(M_Z)_{\overline{\text{MS}}}}{1 - \Delta \alpha_s}, \quad (2.6)
$$

where

$$
\Delta \alpha_s = \frac{\alpha_s(M_Z)}{2\pi} \left[ \frac{1}{2} - \frac{2}{3} \ln \left( \frac{M_t}{M_Z} \right) - 2 \ln \left( \frac{m_{\tilde{g}}}{M_Z} \right) - \frac{1}{6} \sum_{\tilde{q}} \sum_{i=1,2} \ln \left( \frac{m_{\tilde{q}_i}}{M_Z} \right) \right]. \quad (2.7)
$$

In SUSPECT 2.101, the log terms are not added explicitly but included via threshold functions in the RGEs. Also ISAJET 7.64 re-sums the logs in (2.7) via step-function decoupling in the RGEs. The finite term, however, is not taken into account. The difference due to the finite term is small but relevant (1%). One could in principle interpret the input $\alpha_s(M_Z)$ in ISAJET 7.64 as already being the effective Standard Model $\overline{\text{DR}}$ value. Since, however, for some corrections ISAJET 7.64 effectively takes hard-wired values of $\alpha_s(M_Z)_{\overline{\text{MS}}} = 0.118$, we take the canonical input value of $\alpha_s(M_Z)_{\overline{\text{MS}}} = 0.118$ for all codes.

3. Tricky corners of SUSY parameter space

As a general point, when quantifying errors on predicted observables, we assume that the results from the codes follow a Gaussian probability distribution. This is a
priori unjustified, but we prefer it to quoting minimum and maximum values because we believe that the additional information included in the variance is desirable. For example, if three codes all provided identical results and one gave a result further away, we think that the true uncertainty ought to be less than the range of minimum-maximum results.

3.1 Large $\tan \beta$

Large $\tan \beta$ has always been recognised as a difficult case since it requires a thorough treatment of the bottom Yukawa coupling. Figure 1 shows $h_b$ of the four different programs as a function of $\tan \beta$ in the mSUGRA model. We see that SOFTSUSY 1.71 and SPHENO 2.0 agree very well on $h_b$, and there is also good agreement with ISAJET 7.64. Comparing only these programs we would assign a $\lesssim 3\%$ uncertainty on $h_b$ even for very large $\tan \beta$. The agreement with SUSPECT 2.101 is however not so good, and we find 4–8% uncertainty taking all four programs into account. The effect of re-summing the $\tan \beta$ enhanced SUSY loop corrections can be seen when comparing the solid and dotted lines in fig. 1, the solid line being the result of ISAJET 7.64, where the $(\Delta m_b)^{\text{SUSY}}$ corrections are resummed, and the dotted one being the result of ISASUSY 7.58, where this re-summation is not applied.

The bottom Yukawa coupling has its largest effect in the Higgs sector when it is large (at high $\tan \beta$): the evolution of $m_{H_1}^2$ is driven by $h_b$,

$$\frac{dm_{H_1}^2}{dt} \sim \frac{3}{8\pi^2} h_b X_b + \ldots, \quad X_b = (m_Q^2 + m_D^2 + m_{H_1}^2 + A_b^2),$$

(3.1)
Figure 2: Higgs boson masses as a function of tan \( \beta \), for \( m_0 = 400 \) Gev, \( m_{1/2} = 300 \) GeV, \( A_0 = 0 \), \( \mu > 0 \), \( M_t = 175 \) GeV; full (dotted) lines: ISAJET 7.64 (7.58), dashed: SOFTSUSY 1.71, dash-dotted: SPHENO 2.0, dash-dot-dotted: SUSPECT 2.101 (for \( h^0 \), the grey dash-dot-dotted line corresponds to SUSPECT 2.101 + FeynHiggsFast).

Figure 2 shows the Higgs boson masses obtained by the four programs as a function of tan \( \beta \) (the results obtained by ISAJET 7.58 are again shown as dotted lines in fig. 2). Let us first discuss the masses of \( A^0 \) and \( H^\pm \). For tan \( \beta = 10–50 \), we find differences in \( m_A \) and \( m_{H^\pm} \) of about 10–50 GeV, dominated over most of the

where \( t = \ln Q \), \( Q \) being the renormalisation scale. Differences in \( m_{H_1}^2 \) directly translate into the physical Higgs boson masses since

\[
m_A^2 = \frac{1}{c_{2\beta}} \left( m_{H_2}^2 - m_{H_1}^2 \right) + \frac{s_{\beta}^2 t_1}{v_1} + \frac{c_{\beta}^2 t_2}{v_2} - M_Z^2.
\]

Here \( m_{H_i}^2 = m_{H_0}^2 - t_i/v_i \), \( i = 1, 2 \), and \( t_{1,2} \) are the tadpole contributions. The self energies of \( Z \) and \( A \) have been neglected in eq. (3.2).

Here \( m_{H_0}^2 \) is the mass of the neutral Higgs boson, and \( t_i \) are the tadpole contributions. The difference in the masses of the neutral Higgs bosons is due to the difference in the tadpole contributions. The self energies of \( Z \) and \( A \) have been neglected in eq. (3.2).
tan \beta range by the difference between SPHENO 2.0 and SUSPECT 2.101. This has to be compared with differences of 100 GeV and more encountered with earlier versions, see for instance the dotted lines representing ISAJET 7.58. Assuming the error to be Gaussian, we now have \( \Delta m_{A,H^\pm} \simeq \pm 10 \) GeV at \( \tan \beta = 25 \) and \( \Delta m_{A,H^\pm} \simeq \pm 20 \) GeV at \( \tan \beta = 50 \). The bottom Yukawa coupling is, however, not the only source of differences in \( m_A \). Another source is, for example, whether one uses running or pole values for masses in the calculation of one-loop tadpoles \( t_{1,2} \). Also, the scale and scheme of parameters in the one-loop expressions for the tadpoles all vary. These differences are formally of higher order and indeed each program has a different approach.

The situation is somewhat different for the \( (h^0, H^0) \) system because here additional radiative corrections are necessary. It is well known that these involve a theoretical uncertainty on \( m_{h^0} \) of about 3 GeV \(^\text{[46]}\), evidence of which can be seen in fig. 2. For completeness we note that SUSPECT 2.101 offers various choices of Higgs mass calculations. In fig. 2, we have used its default \( m_{h^0} \) routine, i.e. \( \text{ichoice}(10) = 0 \), shown as a black dash-dot-dotted line. If we use instead SUSPECT 2.101 with FeynHiggsFast, \( \text{ichoice}(10) = 3 \), we get \( m_{h^0} \sim 115 \) GeV, shown as a grey dash-dot-dotted line in fig. 2. This will be relevant later in this paper when we discuss mSUGRA fits to LHC data.

### 3.2 Focus point

For large \( m_0 \), the running of \( m_{H^2}^2 \) becomes very steep and very sensitive to the top Yukawa coupling:

\[
\frac{dm_{H^2}^2}{dt} \sim \frac{3}{8\pi^2} h_t X_t + \ldots, \quad X_t = (m_Q^2 + m_U^2 + m_{H^2}^2 + A_t^2). \quad (3.3)
\]

As a result, the \( \mu \) parameter given by

\[
\mu^2 = \frac{m_{H_1} - m_{H^2} \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2 \quad (3.4)
\]

becomes extremely sensitive to \( h_t \). This is visualised in fig. 3 where we show \( h_t \) and \( \mu \) as functions of \( m_0 \). The other parameters are \( m_{1/2} = 300 \) GeV, \( A_0 = 0 \), \( \tan \beta = 10 \) and \( \mu > 0 \). There is reasonable agreement on \( \mu \) up to \( m_0 \sim 1 \) TeV, and the differences observed for \( m_0 \lesssim 2 \) TeV are phenomenologically not so important. For larger values of \( m_0 \) we observe, however, large discrepancies between the four programs. These lead to completely different chargino/neutralino properties for very large \( m_0 \), and likewise to very different excluded regions, depending on which program is used. The situation is, however, already much better than the one reported in \(^\text{[32]}\), c.f. the dashed lines of ISASUSY 7.58 in fig. 3 (earlier versions of the other codes also gave quite different results).
In order to understand the behaviour of $\mu$ in fig. 3 it is useful to write eq. (3.4) in the form

$$\mu^2 \simeq c_1 m_0^2 + c_2 m_{1/2}^2 - 0.5 M_Z^2. \quad (3.5)$$

Approximate analytical expressions for $c_1$ and $c_2$ can be found e.g., in [47, 48]. For
\( A_0 = 0 \) and \( \tan \beta = 10 \) we get

\[
c_1 \sim \left( \frac{m_t}{156.5 \text{ GeV}} \right)^2 - 1, \quad c_2 \sim \left( \frac{m_t}{102.5 \text{ GeV}} \right)^2 - 0.52. \tag{3.6}
\]

Figure 4a shows contours of constant \( \mu \) in this approximation in the \((m_0, m_t)\) plane. Notice the fast increasing dependence on \( m_t \) for increasing \( m_0 \). For \( m_t \sim 156–157 \text{ GeV} \), \( \mu \) becomes almost independent of \( m_0 \). This is a signal of the actual focus point behaviour, which is defined as the insensitivity of \( m_{H_2} \) to its GUT scale value. Eq. 3.4 then implies that \( \mu \) is insensitive to \( m_0 \), since \( m_{H_1} \) appears with a suppression factor of \( \tan^2 \beta - 1 \). Interpreting \( m_t \) in eq. (3.6) as \( m_t(M_{\text{SUSY}}) = h_t(M_{\text{SUSY}}) v_2(M_{\text{SUSY}})/\sqrt{2} \), we can directly relate the \( m_0 \) dependence of \( \mu \) in fig. 4a to that of \( h_t \).

Some more comments are in order. Firstly, SUSPECT 2.101 has only 1–loop RGEs for scalar SUSY parameters. For large \( m_0 \), the 2–loop terms lead to \( O(10\%) \) correction and should thus be taken into account. Secondly, as already mentioned in section 3, the sparticle masses that enter the radiative corrections have different values in different codes. In particular, there are large differences in the gluino masses between SUSPECT 2.101 and the other codes, as illustrated in fig. 4b. Since the gluino mass enters \( h_t \) via the \( \tilde{g} \tilde{t} \) correction eq. (2.3), this may account for the different slope of \( h_t \) (and consequently of \( \mu \)) in SUSPECT 2.101 compared to the other programs, as evident in fig. 4.

It thus seems that for reliable results in the large \( m_0 \) region, a more complete calculation of the top Yukawa coupling at the 2–loop level is necessary. This should include finite radiative corrections to all sparticle masses at the full 1–loop level.

4. Comparison of theoretical and experimental uncertainties

4.1 Fits of mSUGRA parameters to LHC data

In the ATLAS TDR, a case study was made of fitting the mSUGRA model to possible measurements of six reference scenarios. We have re-analysed these fits for two of these scenarios, LHC Point 1 and Point 2. Here squarks and gluinos are produced with the dominant decays \( \tilde{g} \rightarrow q\tilde{q}_{L,R} \), \( \tilde{q}_L \rightarrow \tilde{\chi}_0 q \), \( \tilde{q}_R \rightarrow \tilde{\chi}_0 q \). The assumed measurements for the two points for low \( \mathcal{L} = 30 \text{ fb}^{-1} \) and high \( \mathcal{L} = 300 \text{ fb}^{-1} \) luminosity are given in table 2. They were estimated in ref. 14 by using ISAJET 7.34 and simulating the ATLAS experiment to determine expected empirical errors.

With each of the programs under discussion we have performed a \( \chi^2 \) fit of the mSUGRA parameters \( m_0 \), \( m_{1/2} \) and \( \tan \beta \) to the data of table 2, taking \( A_0 = 0 \)

\[ 2 \text{It is beyond the scope of the present paper to re-perform the experimental analysis in order to have the numbers in table 2 more up-to-date.} \]
and \( \mu > 0 \). The results are listed in tables 3 and 4. The quoted errors are at $1\sigma$ (68.3\% C.L.) from a simultaneous fit of all three parameters, i.e. \( \Delta \chi^2 = 3.53 \).

In case of SUSPECT 2.101, we have used its default option, ichoice(10) = 0, for the calculation of the $h^0$ mass. When linking SUSPECT 2.101 with FeynHiggsFast, ichoice(10) = 3, the results for $m_0$ and $m_{1/2}$ practically do not change. However, we get much lower values for $\tan \beta$: $\tan \beta = 1.7 \pm 0.1$ for Point 1, and $\tan \beta = 6.06 \pm 2.06$ for Point 2 at high luminosity.

It is interesting to note that not only the central values but also the size of the errors can be quite different. Similarly, the minimum $\chi^2$, which is a measure of the quality of the fit, can show large variations. To make the comparison easier and to visualise correlations and non-Gaussian effects, we show in Figs. 5 and 6 contours of 68\% and 95\% C.L. for the fits of Points 1 and 2 in the ($m_0$, $m_{1/2}$) and ($m_0$, $\tan \beta$) planes. The values of $\Delta \chi^2$ used for these confidence levels are based upon a simultaneous two-parameter fit, i.e. $\Delta \chi^2 = 2.23$ and 5.99. The third parameter, $\tan \beta$ or $m_{1/2}$, is always fixed to its best-fit value. As one can see, the error ellipses have only little or even no overlap. We therefore conclude that the theoretical uncertainty is about the same size as the statistical one in the fitted quantities.

One might expect that the main source of these differences is the theoretical uncertainty on $m_h$. This is indeed the case for the determination of $\tan \beta$. However, $m_h$ has only little influence on the fit of $m_0$ and $m_{1/2}$. This becomes clear when using e.g., SUSPECT 2.101 with different routines for the Higgs mass calculation. In fig. 6a, we show the results of SUSPECT 2.101 + FeynHiggsFast, ichoice(10) = 3, as dashed contours in addition to those obtained with its default Higgs mass routine (ichoice(10) = 0, solid contours). In fig. 6b, we have omitted the default SUSPECT 2.101 results to avoid confusion of the many lines. They would look similar to the SPHENO 2.0 contours but centred at $m_0 = 450$ GeV and without an upper limit on $\tan \beta$. We thus conclude that the uncertainties in $m_0$ and $m_{1/2}$ mainly come from the differences in the programs pointed out in Sects. 2–3, and not from different Higgs mass calculations.

We next address the question of how the theoretical uncertainty depends on the

| Quantity     | Low-L         | High-L         |
|--------------|---------------|----------------|
| $m_h$ (Point 1) | 95.4 $\pm$ 1.0 GeV | 95.4 $\pm$ 1.0 GeV |
| $m_h$ (Point 2) | 115.3 $\pm$ 1.0 GeV | 115.3 $\pm$ 1.0 GeV |
| $m_{1/2}^{\max}$ | 758.3 $\pm$ 25 GeV | 758.3 $\pm$ 25 GeV |
| $m_{\tilde{q}_R}$ | 959 $\pm$ 40 GeV | 959 $\pm$ 15 GeV |
| $m_{\tilde{g}}$ | 1004 $\pm$ 25 GeV | 1004 $\pm$ 12 GeV |
| $m_{\tilde{t}_1}$ (Point 1) | none | 647 $\pm$ 100 GeV |
| $m_{\tilde{t}_1}$ (Point 2) | none | 713 $\pm$ 100 GeV |

Table 2: Possible LHC measurements for Point 1 and Point 2, from [16].
SUSY parameter point. Copious quantities of squarks and gluinos are expected to be produced at the LHC, leading to a fairly precise measurement of their masses, particularly if $m_{\chi^0_1}$ is determined accurately by a LC. In table 3 we compare the $\tilde{g}$, $\tilde{u}_L$, $\tilde{u}_R$ and $\tilde{t}_1$ masses obtained by the four programs for the Snowmass (SPS) points \cite{29}. Assuming a Gaussian distribution, we quote the variance of these masses as the theoretical error, i.e. $\delta m_X = \sqrt{\frac{1}{N-1} \sum_i [(m_X)_i - \overline{m_X}]^2}$ where $\overline{m_X}$ is the mean of $(m_X)_i$ and $N = 4$ in our case. We make the following observations: (i) the absolute theoretical uncertainty (in GeV) varies from point to point; (ii) the typical relative uncertainty in mSUGRA and mGMSB scenarios in generic (i.e. not tricky) regions of parameter space is about 2–5%; (iii) in some cases, in particular in focus point

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
\multicolumn{5}{c}{Point 1, Low-L} \\
\hline
$\chi^2_{\text{min}}$ & $m_0$ [GeV] & $m_{1/2}$ [GeV] & $\tan \beta$ \\
ISAJET & 0.10 & 490 ± 135 & 424 ± 25 & 1.77 ± 0.21 \\
SOFTSUSY & 9.30 & 280 ± 246 & 425 ± 32 & 1.60 ± 0.03 \\
SPHENO & 0.02 & 373 ± 175 & 436 ± 26 & 2.10 ± 0.15 \\
SUSPECT & 0.32 & 411 ± 116 & 410 ± 20 & 2.08 ± 0.16 \\
\hline
\end{tabular}
\begin{tabular}{lcccc}
\multicolumn{5}{c}{Point 1, High-L} \\
\hline
$\chi^2_{\text{min}}$ & $m_0$ [GeV] & $m_{1/2}$ [GeV] & $\tan \beta$ \\
ISAJET & 0.57 & 496 ± 61 & 424 ± 12 & 1.77 ± 0.20 \\
SOFTSUSY & 11.66 & 356 ± 78 & 422 ± 12 & 1.60 ± 0.03 \\
SPHENO & 0.27 & 370 ± 82 & 436 ± 12 & 2.10 ± 0.15 \\
SUSPECT & 1.79 & 422 ± 67 & 409 ± 13 & 2.08 ± 0.15 \\
\hline
\end{tabular}
\caption{Fit to LHC measurements for Point 1.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\multicolumn{5}{c}{Point 2, Low-L} \\
\hline
$\chi^2_{\text{min}}$ & $m_0$ [GeV] & $m_{1/2}$ [GeV] & $\tan \beta$ \\
ISAJET & 0.03 & 523 ± 129 & 424 ± 24 & 6.55 ± 2.37 \\
SOFTSUSY & 0.08 & 414 ± 140 & 419 ± 23 & 4.65 ± 0.76 \\
SPHENO & 0.19 & 405 ± 167 & 437 ± 26 & \gtrsim 7 \\
SUSPECT & 0.06 & 444 ± 114 & 409 ± 18 & \gtrsim 7 \\
\hline
\end{tabular}
\begin{tabular}{lcccc}
\multicolumn{5}{c}{Point 2, High-L} \\
\hline
$\chi^2_{\text{min}}$ & $m_0$ [GeV] & $m_{1/2}$ [GeV] & $\tan \beta$ \\
ISAJET & 0.14 & 521 ± 58 & 424 ± 11 & 6.52 ± 2.30 \\
SOFTSUSY & 0.33 & 411 ± 68 & 419 ± 11 & 4.63 ± 0.88 \\
SPHENO & 1.08 & 394 ± 80 & 438 ± 13 & \gtrsim 7 \\
SUSPECT & 0.20 & 450 ± 64 & 408 ± 11 & \gtrsim 7 \\
\hline
\end{tabular}
\caption{Fit to LHC measurements for Point 2.}
\end{table}
Figure 5: Fit to LHC measurements for Point 1 with ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101 for $\mathcal{L} = 300$ fb$^{-1}$. Shown are contours of 68% and 95% C.L. for each program in the ($m_0$, $m_{1/2}$) and ($m_0$, $\tan\beta$) planes, with the third parameter fixed to its best fit value, c.f. table 3.

Figure 6: Fit to LHC measurements for Point 2 with ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101 for $\mathcal{L} = 300$ fb$^{-1}$. Shown are contours of 68% and 95% C.L. for each program in the ($m_0$, $m_{1/2}$) and ($m_0$, $\tan\beta$) planes, with the third parameter fixed to its best fit value, c.f. table 4. In case of SUSPECT 2.101, the solid lines are for its default $m_h$ routine and the dashed lines for $m_h$ calculated with FeynHiggsFast.

and mAMSB scenarios, the relative uncertainty is larger, about 5–10%; (iv) in any case, the theoretical error is of the same order of magnitude as the experimental one.

4.2 Linear Collider measurements

At a high-luminosity $e^+e^-$ Linear Collider, one expects to measure chargino, neu-
Table 5: Gluino and squark masses in GeV for the SPS benchmark points, and their theoretical uncertainties. The theoretical uncertainty is displayed in bold type face and is calculated as described in the text.

| mass | code      | 1a | 1b | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|------|-----------|----|----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\tilde{g}$ | ISAJET    | 607| 936| 794 | 794 | 732 | 719 | 718 | 944 | 835 | 1296|
|      | SOFTSUSY  | 614| 949| 802 | 946 | 743 | 730 | 729 | 964 | 852 | 1306|
|      | SPHENO    | 594| 917| 782 | 914 | 719 | 705 | 704 | 940 | 836 | 1232|
|      | SUSPECT   | 626| 964| 870 | 959 | 761 | 730 | 742 | 986 | 902 | 1395|
| error|           | 13 | 20 | 40  | 19  | 18  | 12  | 16  | 22  | 31  | 67  |
| $\tilde{u}_L$ | ISAJET    | 536| 835| 1532| 817 | 730 | 642 | 640 | 858 | 1079| 1233|
|      | SOFTSUSY  | 549| 851| 1582| 831 | 753 | 657 | 662 | 876 | 1083| 1291|
|      | SPHENO    | 565| 876| 1563| 859 | 764 | 676 | 674 | 910 | 1127| 1314|
|      | SUSPECT   | 570| 886| 1595| 867 | 775 | 681 | 680 | 910 | 1138| 1502|
| error|           | 15 | 23 | 27  | 19  | 18  | 18  | 26  | 30  | 116 |
| $\tilde{u}_R$ | ISAJET    | 520| 807| 1529| 788 | 714 | 622 | 626 | 830 | 1033| 1242|
|      | SOFTSUSY  | 569| 884| 1592| 866 | 774 | 681 | 679 | 914 | 1142| 1297|
|      | SPHENO    | 548| 847| 1552| 828 | 746 | 655 | 659 | 880 | 1080| 1266|
|      | SUSPECT   | 550| 852| 1585| 832 | 754 | 656 | 662 | 880 | 1092| 1492|
| error|           | 20 | 32 | 29  | 32  | 25  | 24  | 22  | 35  | 45  | 114 |
| $t_1$ | ISAJET    | 379| 633| 947 | 621 | 523 | 236 | 476 | 774 | 951 | 998 |
|      | SOFTSUSY  | 398| 658| 974 | 645 | 544 | 232 | 497 | 813 | 987 | 951 |
|      | SPHENO    | 398| 658| 964 | 646 | 545 | 248 | 497 | 813 | 982 | 986 |
|      | SUSPECT   | 410| 676| 1004| 663 | 560 | 243 | 513 | 831 | 1015| 1140|
| error|           | 13 | 18 | 24  | 17  | 16  | 7   | 15  | 24  | 26  | 83  |

It turns out that the uncertainty in the LSP mass in mSUGRA and mGMSB scenarios is typically a few hundred MeV, depending on the parameter point. An exception is the focus point scenario (SPS2) where $\delta m_{\tilde{\chi}_1^0} = 1.4$ GeV. For $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^\pm$, we find uncertainties of about 1–3 GeV in mSUGRA and mGMSB scenarios. (Here note that with earlier program versions, especially with ISASUSY 7.51–7.63, we had discrepancies of 50% and more for focus point scenarios.) For the sleptons we find typical uncertainties of 1–2 GeV. We note that for SPS5, the lighter stop would be accessible, with $m_{\tilde{t}_1} = 235.6, 232.3, 248.4$ and 242.6 GeV for ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101, respectively, corresponding to an error of 7 GeV, c.f. table 3. In the mAMSB scenario, SPS 9, we have much larger uncertainties of $\sim 8$ GeV for $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_1^\pm}$. This is due to the fact that SOFTSUSY 1.71 has 2-loop GUT-scale boundary conditions for mAMSB, while the other programs have only 1-loop boundary conditions. If we enforce 1-loop boundary
conditions for all four programs the error decreases to 3 GeV. We therefore expect that if all programs were to use 2-loop boundary conditions the error would be better than 3 GeV.

By comparing table 5 with tables 6, 7 we see that the present theoretical uncertainty in mass predictions is significantly smaller for weakly interacting sparticles than for strongly interacting ones. This is expected, since the former have smaller threshold corrections. However, the errors in tables 6, 7 are still larger by up to an order of magnitude than the expected experimental accuracies at an $e^+e^-$ Linear Collider. Moreover, the differences in the masses produce cross sections and branching ratios that differ by a few per-cent. While this is not a problem for determining SUSY-breaking parameters at the low scale, it will be relevant when relating them to GUT-scale parameters in order to test their unification and to determine the sources of SUSY breaking.

5. Conclusions

If sparticles are detected at future colliders, measurements of their properties (and confirmation that they are in fact sparticles) will take place. The question following from this line of investigation will be: what can we learn about SUSY breaking from these measurements? Is there a unification of certain SUSY breaking parameters, and is it possible to distinguish various SUSY breaking models?

Vital ingredients for answers to these questions will be the amount and precision of empirical measurements made at the particular colliders [11, 13, 14, 15, 16, 18]. Another vital ingredient will be the precision with which we can relate the experimentally observed quantities at the TeV scale with the fundamental physics at the high-energy scale. Accurate extrapolations require multi-loop results for the RGEs and the related threshold corrections. In this paper, we have addressed the question: what is the current theoretical uncertainty associated with determining fundamental-scale SUSY parameters? To this end, we compared four public state-of-the-art MSSM spectrum calculations: ISAJET 7.64, SOFTSUSY 1.71, SPHENO 2.0 and SUSPECT 2.101, taking the spread of their results as a measure of the to-date uncertainty. Although this does not correspond to the usual notion of ‘theoretical uncertainty’, it is pragmatic in the sense that (at least) one of the available calculational tools will be used to perform fits if and when the relevant data arrives. The uncertainty was shown to be largest in certain tricky corners of parameter space: the focus point region and high tan $\beta$. However, even in these regions, comparison with previous versions of the codes [22] shows that the theoretical uncertainty has significantly improved. Sparticle masses in these regions are particularly sensitive to the values of the Yukawa couplings (especially the top Yukawa for the focus point, and the bottom Yukawa for the high tan $\beta$ regime). Slightly different treatments of top and bottom masses can lead to large differences in mass predictions. It is there-
Table 6: Differences in predicted masses in GeV for the SPS points 1a to 5 (mSUGRA points). Only sparticles with masses that kinematically can be produced at a 500 GeV Linear Collider are displayed. The theoretical uncertainty is displayed in bold type face and is calculated as described in the text.

We used previous LHC estimates of expected empirical errors at two benchmark points to perform separate fits to mSUGRA with the four different codes. The parameters resulting from the four calculations show a difference comparable to the statistical error upon them, showing that theoretical uncertainties must be taken into account. We then went on to quantify the theory uncertainty associated with squark and gluino masses for the SPS benchmark points. The theory uncertainty was also quantified for parts of the MSSM spectrum that would be kinematically...
### Table 7: Differences in predicted masses in GeV for the SPS points 6 to 9 (6: non-minimal SUGRA, 7+8: mGMSB, 9: mAMSB). Only sparticles with masses that kinematically can be produced at a 500 GeV Linear Collider are displayed. The theoretical uncertainty is displayed in bold type face and is calculated as described in the text.

| point | code     | $M_{\tilde{\chi}_1^0}$ | $M_{\tilde{\chi}_2^0}$ | $M_{\tilde{\chi}_1^\pm}$ | $M_{\tilde{\nu}_e}$ | $M_{\tilde{\nu}_\tau}$ | $M_{\tilde{\tau}_1}$ | $M_{\tilde{\tau}_2}$ |
|-------|----------|--------------------------|--------------------------|--------------------------|---------------------|----------------------|----------------------|----------------------|
| 6     | ISAJET   | 190.0                    | 218.1                    | 215.7                    | –                   | 236.8                | 227.8                | –                    |
|       | SOFTSUSY | 190.2                    | 220.6                    | 217.9                    | –                   | 240.9                | 232.8                | –                    |
|       | SPHENO   | 191.2                    | 225.3                    | 222.3                    | –                   | 237.6                | 229.1                | –                    |
|       | SUSPECT  | 190.2                    | 222.2                    | 219.6                    | –                   | 240.9                | 232.2                | –                    |
|       | error    | 0.3                      | 1.7                      | 1.6                      | –                   | 1.3                 | 1.4                 | –                    |
| 7     | ISAJET   | 162.4                    | 268.0                    | –                         | 248.7               | 248.3                | 127.3                | 261.1                | 119.9 | 263.3 |
|       | SOFTSUSY | 163.6                    | 263.5                    | –                         | 247.2               | 246.9                | 126.4                | 259.3                | 120.5 | 261.2 |
|       | SPHENO   | 163.4                    | 271.1                    | –                         | 251.6               | 251.3                | 131.0                | 265.3                | 123.8 | 267.3 |
|       | SUSPECT  | 163.6                    | 262.2                    | –                         | 246.9               | 246.6                | 127.8                | 259.4                | 121.6 | 261.4 |
|       | error    | 0.3                      | 2.4                      | 1.2                      | –                   | 1.2                 | 1.6                 | 1.0                  | 1.6   |
| 8     | ISAJET   | 137.4                    | 254.6                    | –                         | –                   | 175.7                | –                    | 168.9                | –     |
|       | SOFTSUSY | 138.4                    | 261.2                    | –                         | –                   | 175.4                | –                    | 169.9                | –     |
|       | SPHENO   | 139.2                    | 266.1                    | –                         | –                   | 180.3                | –                    | 173.5                | –     |
|       | SUSPECT  | 140.0                    | 263.5                    | –                         | –                   | 177.6                | –                    | 171.8                | –     |
|       | error    | 0.6                      | 2.8                      | 1.2                      | –                   | 1.3                 | 1.2                 | –                    |
| 9     | ISAJET   | 174.8                    | –                        | 175.0                    | –                   | –                   | –                    | –                    | –     |
|       | SOFTSUSY | 196.7                    | –                        | 196.7                    | –                   | –                   | –                    | –                    | –     |
|       | SPHENO   | 168.0                    | –                        | 168.4                    | –                   | –                   | –                    | –                    | –     |
|       | SUSPECT  | 167.3                    | –                        | 167.3                    | –                   | –                   | –                    | –                    | –     |
|       | error    | 7.9                      | 7.9                      | –                        | –                   | –                   | –                    | –                    | –     |
dependent GUT-scale threshold corrections \cite{33,54} are expected from (for example) heavy coloured triplets. If the combined theoretical and empirical accuracy is significantly better than 1\%, then observables at colliders could be used to measure these threshold corrections.

Fortunately, theoretical errors in sparticle mass predictions are certainly not static. There has been much progress reducing them recently (especially in the Higgs and electroweak symmetry breaking sectors, see for example \cite{38,46,55,56,34}). We expect this trend to continue, which, as our present results indicate, is desirable if we are to disentangle SUSY breaking from experimental observables.

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