A note on the newly observed $Y(4220)$ resonance

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BES III Collaboration has recently observed a vector resonance in the $\chi_{c0}\omega$ channel, at a mass of about 4220 MeV, named $Y(4220)$. Hints of a similar structure appear in the $h_c\pi^+\pi^-$ channel. We find that the two observations are likely due to the same state, which we identify with one of the expected diquark-antidiquark resonances with orbital quantum number $L = 1$. This assignment fulfills heavy quark spin conservation. The measured branching ratio of the $Y(4220)$ into $\chi_{c0}\omega$ and $h_c\pi^+\pi^-$ is consistent with the prediction for such a tetraquark state.

PACS numbers: 14.40.Rt, 12.39.Jh, 13.25.Gv

In a very recent paper, the BES III Collaboration reports the $e^+e^- \to \chi_{cJ}\omega \ (J = 0, 1, 2)$ production cross section as a function of $\sqrt{s}$ [1]. Hints of a resonant structure are present in the $\chi_{c0}\omega$ channel at $\sim 30$ MeV above threshold (i.e. at about 4220 MeV), whereas no evident structure appears in the $\chi_{c1,2}\omega$ channels. Some theoretical interpretations for these peak have been proposed [2]. BES Collaboration also reported the measurement of $e^+e^- \to h_c\pi^+\pi^-$ production cross section as a function of $\sqrt{s}$ [3]. Hints of structures not compatible with the $Y(4260)$ have been found [4]: in particular a narrow peak at a mass $\sim 4220$ MeV. In this mass region, many exotic charmonium-like states have been identified according to the diquark-antidiquark model [5] (for a review, see [6]). In particular, the latest model [7] predicts a tetraquark state, named $Y_0$, with quantum numbers $J^{PC} = 1^{--}$, and mass and decay modes compatible with a $Y(4220)$ resonance. The wave function of this tetraquark state contains both heavy quark spin states, so it can naturally decay into both $\chi_{c0}\omega$ and $h_c\pi^+\pi^-$ with no violation of the heavy quark spin. Since the Breit-Wigner parameters of the peaks measured in the two channels $\chi_{c0}\omega$ and $h_c\pi^+\pi^-$ are very similar, we test the hypothesis that the two observed structures may coincide.

We fit data with the same models (I and II in the following) considered in Refs. [1][4]. In the $h_c\pi^+\pi^-$ invariant mass distribution, we add to the BES dataset the experimental point $\sigma_{h_c\pi^+\pi^-}(4.17 \text{ GeV}) = (15.6 \pm 4.2) \text{ pb}$ by CLEO-c [8], with statistical and systematic errors added in quadrature. For the BES data, we take into account only statistical errors, since the systematic ones are common to all points and are not expected to modify the shape of the distribution.

Following model-I, we fit the $h_c\pi^+\pi^-$ and $\chi_{c0}\omega$ data with the sum of a Breit-Wigner and a pure phase-space background. To test our hypothesis, the mass and the width of the resonance are constrained to be the same in both channels. Thus, the fitting functions are:

$$\sigma_{h_c\pi^+\pi^-}(m) = A + \frac{B_{\omega}\sqrt{\phi_1}}{\sqrt{\text{PS}_3(m_0)}} BW(m, m_0, \Gamma) \bigg|_{\text{PS}_3(m)}^2,$$

$$\sigma_{\chi_{c0}\omega}(m) = C + \frac{D\sqrt{\phi_2}}{\sqrt{\text{PS}_2(m_0)}} BW(m, m_0, \Gamma) \bigg|_{\text{PS}_2(m)}^2,$$

where $m_0$ and $\Gamma$ are the mass and width of the resonance, $m$ is the invariant mass of the system, $BW(m, m_0, \Gamma) = (m^2 - m_0^2 + im\Gamma)^{-1}$, $B = \sqrt{12\pi\beta_0\Gamma_{ee}} \Gamma_{ee}$, $D = \sqrt{12\pi\beta_0\Gamma_{\chi_{c0}\omega}} \Gamma_{ee}$, and PS$_n$ is the n-body phase space. With this model, we get a mass of $4214 \pm 10$ MeV and a width of $50 \pm 21$ MeV. The $\chi^2/\text{DOF} = 17.81/15$, corresponding to a $\text{Prob}(\chi^2) = 27\%$.

To obtain the significance of the $Y(4220)$, we perform a likelihood ratio test: we repeat the fit according to a pure phase-space background hypothesis, i.e. forcing $B = D = 0$. The $\Delta\chi^2/\Delta\text{DOF}$ with respect to the full fit is $131/6$, which rejects the pure background hypothesis with a significance $>10\sigma$.

By comparing the Breit-Wigner amplitudes in the two channels, we get the ratio:

$$\frac{B(Y(4220) \to \chi_{c0}\omega)}{B(Y(4220) \to h_c\pi^+\pi^-)} = 8.5 \pm 4.8 \pm 1.9$$

$^1$ $\sigma_f(m)$ indicates the cross section $\sigma(e^+e^- \to f)$ at $\sqrt{s} = m$. 
where the second error is the quadrature sum of the systematic uncertainties of 15% for $\sigma_{\chi c\pi^0\omega}$ and 18% for $\sigma_{h_c\pi^+\pi^-}$. In this way, we consider the two BES datasets to have statistically independent systematics, which leads to a conservative estimate of the error.\footnote{We remark that the ratio in Eq. (3) is compatible with the ratio of the branching fractions of Ref. \cite{1} and \cite{4}, once the value for $B(Y \rightarrow h_c\pi^+\pi^-) \times \Gamma_{ee}$ in Ref. \cite{4} is corrected by a typo of one order of magnitude \cite{9}.}

According to model-II, the background is parametrized by a broad Breit-Wigner:

\[ \sigma_{h_c\pi^+\pi^-}(m) = \left| \frac{B_1}{\sqrt{\text{PS}_3(m_1)}} \text{BW}(m,m_1,\Gamma_1) + \frac{B_2 e^{i\phi_1}}{\sqrt{\text{PS}_3(m_2)}} \text{BW}(m,m_2,\Gamma_2) \right|^2 \text{PS}_3(m), \]

\[ \sigma_{\chi c\omega}(m) = \left| C + \frac{D e^{i\phi_2}}{\sqrt{\text{PS}_2(m_1)}} \text{BW}(m,m_1,\Gamma_1) \right|^2 \text{PS}_2(m). \]

With this model, we get a mass of $4232 \pm 7$ MeV and a width of $36 \pm 17$ MeV. The $\chi^2/\text{DOF} = 6.4/13$, corresponding to a $\text{Prob}(\chi^2) = 93.2\%$. In this case, the significance of the signal is $>9\sigma$. This model yields to a branching fraction ratio of $B(Y(4220) \rightarrow \chi c\omega)/B(Y(4220) \rightarrow h_c\pi^+\pi^-) = 12 \pm 11 \pm 2.8$.

Even though model-II fits data better, the presence of two peaks with so different widths and amplitudes appears unlikely. The broad Breit-Wigner peak just acts as a more effective (but less plausible) parameterization of the background although more data at masses higher than 4.5 GeV are needed to discriminate experimentally between the two models. The fitted values of mass and width of the $Y(4220)$ according to two models are not in statistical agreement since the two data samples are the same. Nonetheless, we will assume that the best estimates of the $Y(4220)$ mass and width come from the model-I fit since it is sounder from the physical point of view. In any case, the conclusions of our study would be the same considering the results of the fit to model-II.

Ref. \cite{10} proposed that exotic resonances are due to a Feshbach mechanism, \textit{i.e.} a resonance appears in an open channel (molecular) because of the hybridization with a closed channel (discrete level of a tetraquark Hamiltonian).
The width of these resonances can be evaluated to be \( \Gamma = A \cdot g_Y \) needed to clarify the resonant structures in \( \pi\pi \rightarrow \chi_Y \). Effective couplings in the two channels point at \( \sqrt{s} \) threshold, and we parametrize the matrix elements by enforcing Lorentz invariance and discrete symmetries.

### Table 1

| \( C \) | \( (0.3 \pm 2.4) \times 10^{-4} \text{ GeV}^{-1} \) |
| \( (B_{h_c \pi^+\pi^-} \times B_{c\eta})_1 \) | \( (5.8 \pm 3.7) \times 10^{-9} \) |
| \( (B_{h_c \pi^+\pi^-} \times B_{c\eta})_2 \) | \( (6.9 \pm 1.9) \times 10^{-9} \) |
| \( (B_{\chi_{c0}\omega} / B_{h_c \pi^+\pi^-})_1 \) | \( 12 \pm 11 \) |
| \( m_1 \) | \( (4231.5 \pm 7.4) \text{ MeV} \) |
| \( \Gamma_1 \) | \( (36 \pm 17) \text{ MeV} \) |
| \( m_2 \) | \( (4303 \pm 17) \text{ MeV} \) |
| \( \Gamma_2 \) | \( (208 \pm 72) \text{ MeV} \) |
| \( \phi_1 \) | \( (10 \pm 56)^\circ \) |
| \( \phi_2 \) | \( (-1 \pm 540)^\circ \) |
| \( \chi^2/\text{DOF} \) | \( 6.37/13 \) |

Figure 2. Combined fits of \( \chi_{c0}\omega \) and \( h_c \pi^+\pi^- \) data: model-II. The purple disk in the right panel is the CLEO-c data point at \( \sqrt{s} = 4.17 \) GeV.

To further check the predictions within the tetraquark model, we compute the ratio in Eq. (6). Assuming that the effective couplings in the two channels \( \chi_{c0}\omega \) and \( h_c \pi^+\pi^- \) are of the same order of magnitude, the nonresonant three-body decay width \( \Gamma(Y \rightarrow h_c \pi^+\pi^-) \) would be negligible. However, the same analysis of \( h_c \pi^+\pi^- \) final state showed a resonance, dubbed \( Z_c'(4020) \), in the \( e^+e^- \rightarrow Z_c'(4020) \rightarrow h_c \pi^+\pi^- \) process. We assume that the decay \( Y \rightarrow h_c \pi^+\pi^- \) is dominated by this intermediate resonance. On the other hand, we will not include an intermediate \( Z_c(3900) \rightarrow h_c \pi^+ \) channel, since the signal \( Z_c(3900) \rightarrow h_c \pi^+ \) is not significant. We also consider the contribution of a \( \pi\pi \) resonance, in particular \( Y \rightarrow h_c \sigma \rightarrow h_c \pi^+\pi^- \). However, with increased statistics, a detailed Dalitz analysis will be needed to clarify the resonant structures in \( Y(4220) \rightarrow h_c \pi^+\pi^- \).

We parametrize the matrix elements by enforcing Lorentz invariance and discrete symmetries,

\[
\langle \chi_{c0}(p) \omega(\eta, q)|Y(\lambda, P)\rangle = g_\chi \eta \cdot \lambda, \tag{6a}
\]

\[
\langle Z'_c(\eta, q) \pi(p)|Y(\lambda, P)\rangle = g_Z \eta \cdot \lambda, \tag{6b}
\]

\[
\langle h_c(\eta, q) \sigma(p)|Y(\lambda, P)\rangle = g_h \varepsilon_{\mu\nu\rho\sigma} \eta^\mu \lambda^\nu P^\rho q^\sigma, \tag{6c}
\]

where \( g_Z, g_h \) and \( g_\chi \) are effective strong couplings with dimension of a mass. Hence, the decay widths in narrow
width approximation \[11\] are:

\[
\Gamma (Y(4220) \rightarrow \chi_{c0}\omega) = \frac{1}{3} \frac{g^2}{8\pi M^2}\left(3 + \frac{p^2(M_Y, m_{\chi}, m_{\omega})}{m^2}\right), \tag{7a}
\]

\[
\Gamma (Y(4220) \rightarrow Z_{c}^{\pm}\pi^{\mp} \rightarrow h_{c}\pi^{\pm}\pi^{-}) = 2 \frac{g^2}{3\pi M^2} \int_{(m_\pi + m_\chi)^2}^{(M_Y - m_{\pi})^2} ds p^*(M_Y, \sqrt{s}, m_{\pi}) \left(3 + \frac{p^2(M_Y, \sqrt{s}, m_{\pi})}{m^2}\right) \times \frac{m_s \Gamma Z}{\pi (s - m_Z^2) + m_s \Gamma Z} p^3(\sqrt{s}, m_{\pi}, m_z) \frac{m_s^3}{s^{3/2}} B(Z_{c}' \rightarrow h_{c}\pi), \tag{7b}
\]

\[
\Gamma (Y(4220) \rightarrow h_{c}\sigma \rightarrow h_{c}\pi^{\pm}\pi^{-}) = \frac{1}{3} \frac{g^2}{8\pi M^2} \int_{m_\sigma^2}^{m_{\pi}} ds p^*(M_Y, \sqrt{s}, m_{\pi}) \frac{2p^2(M_Y, \sqrt{s}, m_{\pi})}{m_s^2 + p^2(M_Y, \sqrt{s}, m_{\pi})} \times \frac{m_s \Gamma_{\sigma}}{\pi (s - m_\sigma^2) + m_s \Gamma_{\sigma}^2} p^*(\sqrt{s}, m_{\pi}, m_{\sigma}) \frac{m_s}{\sqrt{s}} B(\sigma \rightarrow \pi^{\pm}\pi^{-}), \tag{7c}
\]

where \(p^*(m_1, m_2, m_3)\) is the decay 3-momentum in the \(m_1\) rest frame. In Eq. \[7\], the factor of 2 takes into account the incoherent sum over the two charged resonances, being the interference numerically negligible.

For the sake of simplicity, since we are not able to resolve the details of the lineshape within our large uncertainties, we considered the \(\sigma\) resonance to be described by a Breit-Wigner distribution with mass and width \(M_\sigma = (475 \pm 75)\) MeV, \(\Gamma_\sigma = (550 \pm 150)\) MeV. To obtain the branching ratio \(B(Z_{c}' \rightarrow h_{c}\pi)\), we assume the total width of \(Z_{c}'\) to be saturated by the observed decay modes into \(h_{c}\pi\) \[3\] and \(D^*D^*\) \[12\]. We use the BES measurements of production cross sections

\[
\sigma(e^{+}e^{-} \rightarrow Z_{c}^{\pm}\pi^{\mp} \rightarrow h_{c}\pi^{\pm}\pi^{-}) = (7.4 \pm 1.7 \pm 2.1 \pm 1.2) \text{ pb}, \tag{8}
\]

\[
\sigma(e^{+}e^{-} \rightarrow (D^*D^*)^{\pm}\pi^{\mp}) = (137 \pm 9 \pm 15) \text{ pb}, \tag{9}
\]

and of the cross sections ratio

\[
R = \frac{\sigma(e^{+}e^{-} \rightarrow Z_{c}^{\pm}\pi^{\mp} \rightarrow (D^*D^*)^{\pm}\pi^{\mp})}{\sigma(e^{+}e^{-} \rightarrow (D^*D^*)^{\pm}\pi^{\mp})} = 0.65 \pm 0.09 \pm 0.06, \tag{10}
\]

to estimate the branching ratio

\[
B(Z_{c}' \rightarrow h_{c}\pi) = (8.0 \pm 3.6)\%. \tag{11}
\]

The branching fraction \(B(\sigma \rightarrow \pi^{\pm}\pi^{-})\) can be assumed to be \(\simeq \frac{2}{3}\) via isospin symmetry. The effective strong couplings \(g_{h}, g_{\chi}, g_{Z}\) in Eq. \[7\] are unknown and should be fitted from data.

To obtain a prediction within the diquark-antidiquark model, we assume that a tetraquark couples universally to any charmonia, \textit{i.e.} that the strong effective couplings are equal to a universal constant times a factor depending on heavy quark spin content \[7\] \[13\].

In the \(|s_{c\bar{c}}, s_{qq}\) basis, we have:

\[
|Y(4220)\rangle = \frac{\sqrt{3}}{2} |0, 0\rangle - \frac{1}{2} |1, 1\rangle, \\
|Z_{c}'\rangle = \frac{1}{\sqrt{2}} (|1, 0\rangle + |0, 1\rangle) \tag{12}
\]

and we recall

\[
|h_{c}\rangle = |s_{c\bar{c}} = 0\rangle, \quad |\chi_{c,J}\rangle = |s_{c\bar{c}} = 1\rangle. \tag{13}
\]

Hence, we get \(g_h : g_{\chi} = \langle Y|h_{c}\rangle : \langle Y|\chi_{c,J}\rangle = \sqrt{3} : 1\). The estimate of the ratio \(g_{Z} : g_{h}\) deserves a separate comment. The decay \(Y(4220) \rightarrow Z_{c}'\pi\) is an hadronic transition between tetraquark states. With the additional assumption that the dynamics of tetraquark transitions is the same as that of tetraquark-charmonium decays, one could get \(g_{Z} : g_{h} = \langle Y|Z_{c}'\rangle : \langle Y|\chi_{c,J}\rangle = \frac{\sqrt{3} - 1}{2}\sqrt{3} : \frac{1}{2} \simeq 0.52\). This result is potentially affected by large corrections. Comparisons with new tetraquark candidates decays will allow us to probe the validity of this assumption, and evaluate the errors properly. That said, an order-of-magnitude estimate is given by the ratio:

\[
\frac{\Gamma (Y(4220) \rightarrow \chi_{c0}\omega)}{\Gamma(Y(4220) \rightarrow Z_{c}^{\pm}\pi^{\mp} \rightarrow h_{c}\pi^{\pm}\pi^{-})} = 29 \pm 10, \tag{14a}
\]

\[
\frac{\Gamma (Y(4220) \rightarrow \chi_{c0}\omega)}{\Gamma(Y(4220) \rightarrow Z_{c}^{\pm}\pi^{\mp} \rightarrow h_{c}\pi^{\pm}\pi^{-})} = 29 \pm 10, \tag{14a}
\]
while if we add an intermediate $\sigma$ resonance, the ratio decreases to

$$\frac{\Gamma (Y(4220) \rightarrow \chi_c \omega)}{\Gamma (Y(4220) \rightarrow (h_c \sigma, Z'_c \pi) \rightarrow h_c \pi^+ \pi^-)} = 19 \pm 8 \pm 3,$$

(14b)

where the three resonances are summed incoherently, to be compared with Eq. (3). The errors in Eq. (14) are due to the experimental uncertainty on masses, widths and branching fractions of the intermediate resonances. The second error in (14b) estimates the systematic uncertainty from neglecting interferences. We stress that we are not considering the error on the couplings.

In conclusion, the structures seen by BES III in $h_c \pi^+ \pi^-$ and $\chi_c \omega$ can be explained within the diquark-antidiquark model. Our estimate in the case of a decay involving only $Z'_c$ resonances is in agreement with the experimental value [3] at 2$\sigma$ level; the agreement improves if a broad resonance is added in the $\pi^+ \pi^-$ channel. This hypothesis can be verified by a detailed Dalitz analysis of $Y(4220) \rightarrow h_c \pi^+ \pi^-$ when more data from BES III will be available.

ACKNOWLEDGMENTS

We wish to thank L. Maiani and V. Riquer for many interesting comments and discussions.

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