A fundamental question in nuclear physics is the role played by relativistic effects. At first sight, only rather moderate relativistic effects are expected, since the velocity of nucleons in nuclear matter reaches less than one fourth of the light velocity and the kinetic and potential energies are small as compared to the rest mass of the nucleons. However, considering a meson-exchange model for the NN interaction the weak single-particle potential occurs as a sum of a very attractive scalar field and a repulsive vector field, each of them of a size as large as several hundred MeV. This leads to a modification of the Dirac spinors for the nucleons in the nuclear medium and can be expressed in terms of an effective Dirac mass, which tends to decrease with increasing density\[1, 2, 3\]. The matrix elements of the NN interaction in the nuclear medium should be evaluated using these dressed Dirac spinors and therefore depend on the density of the matter considered. These relativistic effects lead to a successful microscopic description of the saturation properties of nuclear matter in Dirac-Brueckner-Hartree-Fock (DBHF) approach\[2, 4, 5\] without a need to include many-nucleon forces, which are required in non-relativistic investigations\[6\].

Apart from the division between relativistic and non-relativistic approaches, one can distinguish between investigations which are based on phenomenological interactions and those which are based on realistic NN interactions. The parameters of the former interactions have been adjusted to describe properties of isospin symmetric nuclear matter and of nuclei in the valley of \(\beta\) stability. These phenomenological models, such as Gogny\[7\], Skyrme\[8\], or relativistic mean field (RMF) approaches\[9\] provide a simple description of the mean field in terms of local single-particle densities. However, a disadvantage is that their predictive power may be rather limited, in particular for highly isospin asymmetric nuclear matter and nuclei far away from the line of \(\beta\) stability. The study of exactly these nuclei is of high interest with the forthcoming new generation of radioactive beam facilities like the future GSI facility FAIR in Germany and the RIA facility planned in the United States of America.

One the other hand, the microscopic approaches based on realistic NN interactions, like the Bonn interactions\[2\], have a high predictive power, which should give one confidence when the model is used in extreme cases like in a highly isospin asymmetric nuclear environment. These high-precision free-space NN interactions are adjusted to describe the experimental data of the NN interaction. However, the strong short range and tensor components of such realistic interactions make it inevitable to employ non-perturbative approximation schemes for the solution of the many-body problem. Therefore, calculations with such interactions are restricted to very light nuclei due to the dramatic increase of configurations with an increasing number of nucleons. Furthermore, such sophisticated calculations will not become feasible for heavier nuclei or the nuclear structures in the neutron star crust in the near future.

A possible way out of this problem is to restrict the nuclear structure calculation to the low momentum components by separating the low momentum and high momentum components of a realistic NN interaction by means of renormalization techniques\[10, 11, 12\]. If the cutoff \(\Lambda\) is appropriately chosen, i.e. around \(\Lambda = 2\) fm\(^{-1}\), the resulting low momentum interaction \(V_{\text{lowk}}\) will still describe the experimental data of the NN interaction up to the pion threshold. Moreover, a very attractive aspect is that this \(V_{\text{lowk}}\) interaction turns out to be independent of the underlying realistic interaction \(V\).

Using \(V_{\text{lowk}}\) in a calculation of nuclear matter, one obtains a binding energy per nucleon increasing with density in a monotonic way\[11, 13, 14\], unless three-body forces are added\[12\]. Thus, the emergence of a saturation point is prevented in symmetric nuclear matter. Attempts have been made to cure this problem and recently Siu et al.\[15\] suggested to increase the cutoff, account for the contribution of \(pphh\) ring diagrams and a modification of the NN interaction, the so-called Brown-Rho scaling\[16\]. Increasing the cutoff parameter leads to the necessity to account for correlation effects (\(pphh\) ring diagrams) in a non-perturbative way and thereby one of the nice features of the low momentum interaction, that they can be treated in a perturbative way, is lost.

Nuclear Saturation with Low Momentum Interactions.

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Relativistic effects are investigated in nuclear matter calculations employing renormalized low-momentum nucleon-nucleon (NN) interactions. It is demonstrated that the relativistic effects cure a problem of non-relativistic low-momentum interactions, which fail to reproduce saturation of nuclear matter. Including relativistic effects, one already obtains saturation in a Hartree-Fock calculation. Brueckner-Hartree-Fock calculations lead to a further improvement of the saturation properties. The results are rather insensitive to the realistic NN interaction on which they are based.

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Therefore, in the present work, we want to follow a different route: We include the relativistic effects discussed above and evaluate for each density the matrix elements of the bare $NN$ interactions using the Dirac spinors, which are appropriate for this density. In this work, the medium properties of the nucleons, which are used to dress these Dirac spinors, are obtained from the EoS presented in Refs. 3. From this density-dependent bare interaction we can then deduce a renormalized $V_{\text{lowk}}$ interaction, which will depend on the density, using the standard techniques.

For the construction of a low momentum potential, it is necessary to separate the low momentum and high momentum components of realistic interactions. In our work, the unitary-model-operator approach (UMOA) 17 is used to disentangle these parts. As in all well-known model space techniques, we define the projection operators $P$, which projects onto the low momentum subspace, and $Q$, which projects onto the complement of this subspace, the high momentum subspace. Furthermore, these operators $P$ and $Q$ satisfy, of course, the usual relations like $P + Q = 1$, $P^2 = P$, $Q^2 = Q$, and $PQ = 0 = QP$. The aim of the unitary-model-operator approach is now to define a unitary transformation $U$ in such a way that the transformed Hamiltonian does not couple $P$ and $Q$, which means

$$QU^{-1}HUP = 0$$

has to be fulfilled. It leads to an effective Hamiltonian

$$H_{\text{eff}} = h_0 + V_{\text{eff}},$$

which contains a starting Hamiltonian $h_0$ describing the one-body part of the two-body system and an effective interaction $V_{\text{eff}}$. This effective interaction is defined in terms of this unitary transformation as

$$V_{\text{eff}} = U^{-1}(h_0 + V)U - h_0,$$

with $V$ representing the bare $NN$ interaction. The unitary operator $U$ can be expressed as

$$U = (1 + \omega - \omega^\dagger)(1 + \omega^\dagger + \omega^\dagger \omega)^{-1/2}$$

with an operator $\omega$ fulfilling the relation $\omega = Q\omega P$ such that $\omega^2 = \omega^\dagger = 0$. This operator $\omega$ can be obtained by first solving the two-body eigenvalue equation

$$(h_0 + V)|\Phi_k\rangle = E_k|\Phi_k\rangle$$

and afterwards defining the matrix elements of $\omega$ using the eigenstates $|\Phi_k\rangle$ having the largest overlap with the low momentum space, the $P$-space 11. Next, the effective interaction $V_{\text{eff}}$ is calculated as described in 11, 18. In this way one obtains the effective Hamiltonian of Eq. 3, which contains the effective interaction $V_{\text{eff}}$. The eigenvalues, which are obtained by diagonalizing this effective Hamiltonian in the $P$-space, are identical to those, which are obtained in the diagonalization of the original Hamiltonian $H = h_0 + V$ in the complete space.

Next, this model-space scheme can be applied to the effective two-nucleon problem by considering for the basis states of the two-nucleon system the states identified by the relative momentum, its modulus and the corresponding partial wave. For a given partial wave, the states with a relative momentum smaller than a cutoff $\Lambda$ are identified as the states of the $P$-space. Therefore, applying the technique described above leads to the normal effective interaction $V_{\text{lowk}}$.

In order to address the question which role correlations beyond the simple mean-field or HF approximation play for the density dependent $V_{\text{lowk}}(\rho)$ interaction, the effects of $NN$ correlations are taken into account by means of the BHF approximation. In the BHF approach, one replaces the bare $NN$ interaction $V$, or in our case the effective interaction $V_{\text{eff}}$, used in the HF approximation by the $G$-matrix which obeys the Bethe-Goldstone equation.

In the two-particle center of mass frame, it takes for an effective interaction $V_{\text{eff}}$ with cutoff $\Lambda$ the form

$$G(k', k; \epsilon_k)_{\text{eff}} = V_{\text{eff}}(k', k) + \int_0^\Lambda dq V_{\text{eff}}(k', q)$$

$$\frac{Q_p}{2\epsilon_k - 2\epsilon_q + i\eta}G(q, k; \epsilon_k),$$

where $\epsilon_i$ with $i = k, q$ are single-particle energies. Furthermore, the Pauli operator $Q_p$ prevents scattering to occupied states and, therefore, restricts the intermediate states to particle states with momenta $q$ which are above the Fermi energy. In Eq. (6), we also take into account that $V_{\text{eff}}$ is designed for a model space with relative momenta smaller than $\Lambda$. Therefore, the integral in Eq. (6) is restricted to momenta $q$ below the cutoff $\Lambda$. In contrast, there is no upper integration limit in case of bare $NN$ interactions.

Results for the energy per nucleon of symmetric nuclear matter as a function of the density $\rho$ obtained from HF and BHF calculations are displayed in Fig. 11. In all our calculations we use a cutoff $\Lambda$ of 2 fm$^{-1}$ and the Bonn $A_2$ interaction has been employed for the calculations displayed in this figure. The conventional method, which ignores the medium modifications of the Dirac spinors, leads to results with the well known features: The HF calculation using $V_{\text{lowk}}$ does not exhibit a minimum in the energy as a function of the density. This absence of saturation is one of the major problems of calculations for nuclear matter employing $V_{\text{lowk}}$. This problem cannot be cured by the inclusion of correlations beyond the HF approximation, e.g. by means of the BHF approximation.

These rather weak effects of the $NN$ correlations be-
Beyond mean-field can be explained by the fact that only intermediate two-particle states with momenta below the cutoff $\Lambda$ can be taken into account in Eq. \([6]\), which is not the case for bare $NN$ interactions. The $NN$ correlation effects even vanish at high densities due to the lack of phase space for these correlations and energies resulting from BHF calculations approach those determined in the HF approximation.

The inclusion of relativistic effects by calculating the underlying Bonn A interaction in terms of dressed Dirac spinors, however, drastically changes this picture. The HF calculation for the density dependent $V_{\text{lowk}}(\rho)$ interaction already leads to a minimum in the energy as a function of the density. Furthermore, this HF calculation yields a saturation density in the neighborhood of the empirical region of saturation, but it yields too little binding energy, i.e., the energy is about -11 MeV per nucleon. This result for the binding energy can be improved by taking into account the effects of $NN$ correlations within the BHF approximation. Hence, the BHF calculation for the density dependent $V_{\text{lowk}}(\rho)$ interaction yields about 2 MeV more binding in the region of saturation than the corresponding HF calculation due to the additional correlations from $NN$ states. In short, the mean-field calculations using a density dependent $V_{\text{lowk}}(\rho)$ can already lead to reasonable results and the $NN$ correlations beyond mean-field are rather weak, in particular compared to mean-field calculations using bare $NN$ interactions \([13]\).
An attractive feature of the use of a standard $V_{lowk}$ interaction in HF or BHF calculations is that the corresponding results are rather insensitive to the realistic interaction model on which the $V_{lowk}$ interaction is based. Therefore, we want to explore to which extent this insensitivity applies in nuclear matter calculations employing the density dependent $V_{lowk}(\rho)$ interaction. The HF calculations for density dependent $V_{lowk}(\rho)$ interactions based on different realistic interactions qualitatively shows the same behavior, although quantitatively some difference in the binding energy strength exists as can be seen in Fig. 2. The HF calculation using Bonn A as underlying interaction yields about 2.2 MeV stronger binding in the saturation point than the one using Bonn C. The same observations and conclusions about the sensitivity on the underlying realistic interaction can be made for the BHF calculations in Fig. 3. This dependence of the calculated energy on the underlying $NN$ interaction is much weaker than the corresponding difference of 8 MeV per nucleon obtained in non-relativistic BHF calculations using directly Bonn A and C potentials. The model-dependence of the relativistic $V_{lowk}$ interactions, however, seems to be slightly stronger than the corresponding model-dependence in non-relativistic calculations using the modern potentials, which fit the $NN$ data with high precision. Here, one must keep in mind, however, that the potentials Bonn A, B and C did not fit the data very well, especially for heavier nuclei or for the neutron star crust in the near future. However, one of the major problems of these calculations employing an effective low momentum interaction is the absence of saturation, unless three-body forces are added. Therefore, relativistic effects are introduced by dressing the Dirac spinors of the underlying realistic interaction on which the low momentum interaction is based. In this way a density dependent effective interaction $V_{lowk}(\rho)$ is obtained. Employing this density dependent $V_{lowk}(\rho)$ interaction, one already obtains a saturation point in the HF calculation.

The effects of the $NN$ correlations introduced by the BHF approximation, although rather weak, leads to a further improvement of the saturation properties. Hence, this leads to the nice feature that HF calculations using a density dependent $V_{lowk}(\rho)$ can lead to reasonable results and $NN$ correlations beyond mean-field are rather weak. This opens the door for studying finite nuclei in calculations which are based on a realistic $NN$ interaction, treating correlation effects in a perturbative manner.

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