Laser-driven electron storage rings

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Abstract. Advanced accelerator concepts usually address linear acceleration schemes. Storage rings, however, are often superior to linear machines regarding repetition rate, stability and efficiency. The radiative energy loss per turn in an electron storage ring is compensated by radiofrequency resonators with a wavelength of the order of 1 meter, which corresponds to the spacing between consecutive potential wells, so-called buckets, and results in a bunch length around 1 centimeter or several 10 picoseconds. As an alternative, longitudinal focusing could be performed by a laser wave co-propagating with the electrons in an undulator. Considering a continuous-wave carbon dioxide laser beam as an example, the bucket spacing would be 10.6 micrometer with a bunch length in the femtosecond range. The paper discusses chances and limitations of such a laser-driven storage ring concept with steady-state femtosecond bunches.

1. Introduction

Advanced accelerator concepts such as laser-plasma acceleration having reached multi-GeV electron energies (e.g., [1]) or schemes using dielectric structures (e.g., [2, 3]) aim at linear acceleration of charged particles after which the beam fulfills its purpose, e.g., the emission of synchrotron radiation in the case of electrons, and is subsequently dumped. Using advanced schemes to inject beam into conventional storage rings is also under discussion. The primary aim is to reduce the footprint and cost of the accelerating structure compared to radiofrequency (RF) resonators with electric fields of several 10 MV/m (pulsed) or 1 MV/m (continuous).

Storage rings have a number of advantages over single-pass linear machines in which the whole particle energy ends up in a beam dump. In a storage ring with a circumference of 100 m and a beam lifetime of 10 h, as an example, each accelerated particle returns on average $10^{11}$ times and only the energy lost by radiation emission (in the case of electrons) has to be restored. The current of the circulating charge is enhanced by a revolution frequency in the MHz range. In the case of electrons, synchrotron radiation provides longitudinal and transverse damping on a typical time scale of $10^{-2}$ s, resulting in a very stable beam. Measuring the position of particle bunches and acting back on the same bunches allows feedback systems to further stabilize the beam against motion caused by external sources, e.g., ground vibration or electrical hum leaking through magnet power supplies, or by impedance-induced collective instabilities.

In electron storage rings, the accelerating structure restores the energy lost by synchrotron radiation while providing longitudinal focusing which results in equidistant potential wells (buckets) with a spacing equal to an RF wavelength of the order of 1 m (Fig. 1, top). Replacing the RF system by an advanced acceleration scheme with much smaller wavelength would not
significantly reduce the size of the ring, which is limited by the bending magnetic fields. The minimum circumference is \( C_{\text{min}} \) = 11.6 \( \cdot \) \( E \) [GeV] for a relativistic beam energy \( E \) and a magnetic field of \( B = 1.8 \) T provided by normal-conducting coils or permanent magnets. Superconducting magnets with a field up to 10 T would allow for a more compact design but also increase the radiation losses. Additional space is needed to accommodate focusing elements, insertion devices, RF cavities, injection, diagnostics etc. increasing the circumference by a factor of 2 (compact racetrack-type ring, e.g. [4]) to 5 or more (typical synchrotron light source [5]).

A candidate for an alternative acceleration scheme in storage rings is the interaction of electrons with a co-propagating laser wave in an undulator [6] similar to an inverse free-electron laser (Fig. 1, bottom). Other schemes are less likely to work. Plasma wakefields, for example, require a gas which would drastically reduce the beam lifetime by elastic scattering or bremsstrahlung emission. For a rough estimate, consider a required gas pressure of 10 hPa in a 1 cm long channel. Along a circumference of 100 m, the average pressure would be \( 10^{-3} \) hPa, which exceeds typical storage ring vacuum by 6 orders of magnitude. Furthermore, plasma wakefield as well as dielectric laser-driven acceleration would not provide a homogeneous accelerating gradient over a typical beam cross section of several 100 \( \mu \)m. Laser-electron interaction in an undulator, on the other hand, is successfully employed in seeded free-electron lasers (FELs) [7] and in short-pulse schemes for storage rings [8] like femtoslicing [9] or coherent harmonic generation [10]. In contrast to these applications, a laser beam replacing the RF system of a storage ring has to provide a sufficiently homogeneous electric field in all directions, and the laser repetition rate has to match the bunch rate.

The potential advantage of a laser-driven electron storage ring would neither lie in a smaller footprint nor in a higher accelerating gradient, but in a novel bunch structure given by the laser wavelength \( \lambda_L \). Synchrotron radiation pulses with a length of \( \sigma_z \approx \lambda_L/10 \) can be employed for, e.g., pump-probe applications with very high time resolution. In addition, these bunches give rise to coherent emission at wavelengths around \( \lambda \approx \sigma_z \) and longer with an intensity proportional to the number of electrons squared like in FELs.

The scheme discussed here is one version of steady-state microbunching (SSBM) as proposed in [11] which can either mean a sustained generation of extremely short bunches, where the laser replaces the RF system as discussed above, or performing and undoing microbunching at every turn in a conventional storage ring. Another option would be sustained microbunching in addition to RF buckets [12] to make sure that electrons with excursions beyond the micro-bucket separatrix are not lost from the beam.

Possible lasers for sustained microbunching range from low-gain FELs with wavelengths of the order of 1 mm [12] to frequency-multiplied lasers in the ultraviolet regime [13]. Given the problem of isochronicity in a storage ring, a carbon dioxide laser with \( \lambda_L = 10.6 \) \( \mu \)m [14] is still an ambitious but not unrealistic scenario which will be discussed in the following.
2. Driving an electron beam with a carbon dioxide laser

Carbon dioxide (CO$_2$) lasers operating at several wavelengths between 9.1 and 10.9 $\mu$m are based on mature technology [15]. Efficient high-power continuous-wave (cw) and Q-switched lasers are commercially available for scientific, industrial and medical purposes. In accelerator physics, frequency-stabilized CO$_2$ lasers are used, for example, to measure the energy and energy spread of electron beams with high precision by Compton backscattering [16].

Replacing a conventional RF system by a CO$_2$ laser would result in almost $10^7$ buckets along a ring circumference of 100 m. With the bunch length being a fraction of the bucket size, such a storage ring would emit synchrotron radiation pulses with a duration of a few femtoseconds and a repetition rate of $2.8 \cdot 10^{13}$ s$^{-1}$. Single femtosecond pulses could be generated by filling all buckets using a conventional injection scheme and then removing selected bunches by energy modulation with a laser of shorter wavelength, where the modulation amplitude would exceed the energy acceptance of the CO$_2$ laser buckets.

A simplified spectrum of radiation emitted by a 1 GeV beam in a 1.5 T bending magnet is shown in Fig. 1 (red curve). The fact that pulses with a temporal spacing of $t_0$ result in a frequency comb with harmonics of $f_0 = 1/t_0$ is usually ignored and not easily detected at a spacing corresponding to typical RF waves [17]. With a laser wavelength of 10.6 $\mu$m and a temporal spacing of 35 fs, however, the 28 THz comb structure is more prominent. Furthermore, the radiation power at frequency $\omega$ is given by [18]

$$P(\omega) = P_e(\omega)N_e + P_e(\omega)N_e(N_e - 1)g^2(\omega),$$  

(1)

where $P_e(\omega)$ is the power emitted by a single electron and $N_e$ is the number of electrons. The formfactor $g^2(\omega)$ is the square of the Fourier transform of the temporal distribution. The second term $\sim N_e^2$ describes coherent emission and is shown in Fig. 1 (blue curve) for bunches with a spacing of $\lambda_L = 10.6 \mu$m, a length of $\sigma_z = \lambda_L/10$ (rms), and $N_e = 2.2 \cdot 10^4$, corresponding to a beam current of 100 mA with evenly filled buckets over a circumference of 100 m. The coherent emission increases the total radiation power by about 1% in this example.

The unusual parameters of a laser-driven beam may lead to unexpected collective instabilities. Here, only a quick look is taken at the current threshold of the longitudinal microwave instability driven by coherent synchrotron radiation (CSR). In [19], the threshold condition is given in terms
of a CSR strength parameter

\[ S_{CSR} = \frac{e^2}{4\pi\varepsilon_0 m_0 c^2} \frac{N_e}{2\pi \nu_s \gamma \sigma_\delta} R^{1/3} \sigma_z^{-4/3} = 0.5 + 0.12 \cdot \Pi_s, \]  

where \( R \) is the bending radius in dipole magnets, \( \nu_s \) is the synchrotron tune, \( \gamma = E/(m_0c^2) \) with the electron rest mass \( m_0 \) is the Lorentz factor, and \( \sigma_\delta \) is the relative spread (rms). Neglecting the shielding parameter \( \Pi_s \) for very short bunches and using the relation \( 2\pi \nu_s = \alpha_c \sigma_\delta C / \sigma_z \) [18] with momentum compaction factor \( \alpha_c \) and ring circumference \( C \) yields a threshold of

\[ N_e = 0.5 \cdot \frac{4\pi\varepsilon_0}{e^2 - \alpha_c \sigma_\delta^2} \frac{E}{C} \left( \frac{\sigma_z}{R} \right)^{1/3} \]  

electrons per bunch for the microwave instability which would increase the bunch length as well as the energy spread. For \( C = 100 \, \text{m} \), \( \sigma_z = 1 \, \mu\text{m} \), and \( \sigma_\delta = 10^{-3} \), the momentum compaction factor should be of the order of \( 10^{-6} \). Assuming \( E = 1 \, \text{GeV} \) and a magnetic field of 1.5 T, the bending radius is \( R = 2.2 \, \text{m} \), resulting in \( N_e = 2.7 \cdot 10^5 \) or a threshold current of 0.13 \( \mu\text{A} \) per bunch. With all \( 9.4 \cdot 10^6 \) buckets filled, the total current would exceed 1 A. Another unusual feature of the large number of buckets is the existence of \( 9 \) modes in the beam spectrum, however, is same as in conventional storage rings, that is, the revolution frequency.

Another collective phenomenon in storage rings is the Touschek effect, i.e., the change of energy due to electron-electron scattering that limits the beam lifetime [21]. The loss rate is primarily given by the electron density which, for a given total current and an even fill of all buckets, scales with the ratio of bunch separation and bunch length which is not fundamentally different for laser-driven storage rings. Yet another collective effect is the trapping of residual gas atoms ionized by the beam and acting on trailing bunches [21]. The bunch charge in laser-driven storage rings is very low but the bunch spacing is extremely small. In a one-dimensional model, the two effects cancel, but with \( \lambda_L \) being smaller than typical transverse beam dimensions, a more detailed investigation of this issue is mandatory. At a first glance, the beam in a laser-driven storage ring appears to be not more prone to collective effects than in a conventional machine, but verification of this statement needs further studies.

3. Laser-based energy modulation

In analogy to RF phase focusing in electron storage rings, the modulation amplitude \( E_m \) of a laser wave co-propagating with the electron beam in an undulator [8] should exceed the radiative energy losses \( W \) per turn by a so-called overvoltage factor \( q \) of, say, five or more in order to ensure sufficient beam lifetime and stability [18]:

\[ W[\text{keV}] = 88.5 \frac{E^4[\text{GeV}]^4}{R[\text{m}]} \quad E_m = -e \int_{s=0}^{N_U \lambda_U} \mathcal{E}_L(s) x'(s) ds \quad q = \frac{E_m}{W} \geq 5. \]  

In addition to the previously defined symbols, \( N_U \) is the number of undulator periods, \( \lambda_U \) is the period length, \( \mathcal{E}_L = \sqrt{2I/\varepsilon_0} \) is the electric field provided by a laser with intensity \( I \) (power per area), and \( x'(s) \equiv dx/ds \) is the horizontal angular coordinate of an electron at longitudinal position \( s \) assuming a horizontally planar undulator. When the undulator wavelength

\[ \lambda = \frac{\lambda_U}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right) \]  

with undulator parameter \( K = 93.4 \cdot \lambda_U[m]B_U[T] \) and magnetic field \( B_U \) equals the laser wavelength \( \lambda_L \), the electron slips with respect to the laser field by one wavelength per undulator.
period and $E_m$ is maximized (the term 'undulator' is used here instead of 'wiggler' even for $K \gg 1$). Figure 3 (left) shows the energy loss $W$ and the modulation amplitude $E_m$ as function of electron energy $E$. For $K \gg 1$, the amplitude of the angular oscillation $x'_{\text{max}} = K/\gamma$ as given by Eq. (5) is nearly independent of $E$, and so is $E_m$ according to Eq. (4). Optimizing the laser Rayleigh length for maximum energy modulation and assuming $N_U = 17$ undulator periods with $\lambda_U = 25$ cm, the laser beam size is $\sigma_x(s) = \sigma_y(s) = 0.6 \text{mm} \sqrt{1+(s/0.5 \text{m})^2}$ (rms), which is large compared to the electron beam with a typical emittance of a small ring (e.g., $10^{-8}$ rad m) and a beta function of a few meter. With a cw laser power of 1 kW, the amplitude of the electric field is 670 kV/m at the waist and 150 kV/m at both undulator ends, and with $x'_{\text{max}} = 0.013$ a modulation amplitude of 9.3 keV is obtained. The overvoltage condition $q \geq 5$ is met for $E = 515$ MeV. For this energy and wavelength $\lambda = 10.6 \mu$m, the magnetic field of the undulator is 0.56 T, as shown in Fig. 3 (right).

Carbon dioxide lasers with cw power in the kW range are commercially available and using an impedance-matched enhancement cavity, the power can be further increased significantly, for example, by a factor 20 for round-trip losses of 5% in the cavity [22]. This example with not very demanding laser and undulator parameters shows that a sufficient laser-induced energy modulation can be achieved for electron beam energies around 1 GeV. Higher power allows to increase the laser beam radius and thus improve the homogeneity of the electric field.

4. Storage ring design
Phase focusing of electrons with a CO$_2$ laser requires isochronicity on a level below 1 $\mu$m for a typical circumference of 100 m, which is beyond the capabilities of existing storage rings. Quasi-isochronous lattices have been considered for a long time, e.g., [23, 24, 25]. For small values of the momentum compaction factor, its dependence on the energy deviation $\delta \equiv \Delta E/E$

$$\alpha_c(\delta) = \alpha_c^{(0)} + \alpha_c^{(1)} \delta + \alpha_c^{(2)} \delta^2 + \ldots$$

becomes significant. While $\alpha_c^{(1)}$ is controlled by sextupole magnets, some storage rings dedicated to low-$\alpha_c$ operation are equipped with octupole magnets to manipulate $\alpha_c^{(2)}$, for example, the Metrology Light Source (MLS) in Berlin [26].

However, even tuning $\alpha_c$ to the order of 10$^{-6}$ is not sufficient to equalize the individual electron trajectories on the sub-$\mu$m level. To first order and considering only the horizontal plane, the longitudinal rms deviation from the ideal path is $\sigma_z = \left(r^2_{51}\sigma_{x}^2 + r^2_{52}\sigma_\gamma^2 + r^2_{56}\sigma_\delta^2\right)^{1/2}$ with

![Figure 3](image-url)
the rms values $\sigma_i$ of horizontal phase space ($i = x, x'$) and relative energy ($i = \delta$) distributions
and the matrix elements
$$
\begin{align*}
    r_{51} &= \int_0^C \frac{C_x(s)}{R(s)} ds \\
    r_{52} &= \int_0^C \frac{S_x(s)}{R(s)} ds \\
    r_{56} &= \int_0^C \frac{D_x(s)}{R(s)} ds.
\end{align*}
$$

Here, $C_x(s)$ and $S_x(s)$ are solutions of the homogeneous equation of motion and $D_x(s)$ is the
dispersion function. The effect of transverse motion on path length was discussed in [27]
resulting in $\sigma_{tr}^2 = \sqrt{\varepsilon_x H_x}$, where $\varepsilon_x$ is the horizontal emittance and $H_x$
is the so-called chromatic invariant. One option is a multibend achromatic lattice to keep the dispersion small which is the strategy
pursued for novel low-emittance storage rings [5, 28].

Yet another source of non-isochronicity is given by the random nature of synchrotron radiation
emission [29]. It is not enough to minimize $\alpha_c$ for the total storage ring but also the partial
momentum compaction factor integrating $D_x(s)/R(s)$ over a fraction of the ring must be
considered which leads to path length variations by photon emission at arbitrary positions $s$. A
storage ring design optimized for SSBM was presented in [30]. A proof-of-principle experiment
to verify some of the design ideas took place at the MLS in Berlin [31], performing laser-induced
energy modulation and observing the residual microbunching after one turn.

Finally, Fig. 4 shows the basic idea of an isochronous storage ring combined with a
ring resonator fed by a cw CO$_2$ laser. Here, laser-electron interaction takes place at four
equidistant undulators, which not only increases the modulation amplitude but also mitigates the
isochronicity issue by a factor of four. A set of mirrors for each quadrant optimizes transverse
position and angle of the laser beam and allows to adjust the laser phase at the respective
interaction point as well as the cavity length. As an intermediate step between RF- and
laser-driven storage rings, a demonstrator ring could be equipped with X-band resonators at
a wavelength around 3 cm for coherent emission of radiation in the THz regime, as proposed in
[32], and to study isochronicity and stability issues for future SSMB beams.

5. Summary

Electron storage rings driven by a laser rather than by an RF wave are potentially a novel type
of 'advanced' accelerator providing ultrashort radiation pulses at very high repetition rate. Here,
the example of a CO$_2$ laser resulting in a bunch rate up to 28 THz was considered. The main
unresolved problem is the required isochronicity on the sub-μm level, but also other issues such
as collective effects need to be investigated for this completely unexplored parameter regime.
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