SOME EXAMPLES OF SILTED ALGEBRAS OF DYNKIN TYPE

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Abstract. This paper studies silted algebras, namely, endomorphism algebras of 2-term silting complexes, over path algebras of Dynkin quivers. We will describe an algorithm to produce all basic 2-term silting complexes over the path algebra of a Dynkin quiver, and use this algorithm to compute some examples.

1. Introduction

Silting theory plays an important role in representation theory because it is closely related to many research fields, such as t-structures ([11][12][10]), cluster-tilting theory ([4]), and tilting theory ([1][2][6]). Silted algebras are defined as endomorphism algebras of 2-term silting complexes over hereditary algebras ([5]). According to a remarkable result of Buan and Zhou ([5]), if A is a silted algebra, then A either is a tilted algebra, or A is a strictly shod algebra, that is, A has global dimension 3 and any A-module has projective or injective dimension no greater than 1.

In this paper, a complete list of silted algebras is given for path algebras of certain Dynkin quivers. The main result is as follows:

(1) there is 1 silted algebra of the quiver $\circ \rightarrow \circ$ (tilted algebra of type $A_1$)
(2) there are 2 silted algebras of the quiver $\circ \rightarrow \circ$ (tilted algebra of type $A_2$) and $\circ \circ$ (tilted algebra of type $A_1 \oplus A_1$)
(3) there are 5 silted algebras of the quiver $\circ \rightarrow \circ \rightarrow \circ$, forming two families:
   (i) tilted algebras of type $A_3$:
       $\circ \rightarrow \circ \rightarrow \circ$, $\circ \rightarrow \circ \leftarrow \circ$, $\circ \leftarrow \circ \rightarrow \circ$, $\circ \leftarrow \circ \leftarrow \circ$
   (ii) tilted algebra of type $A_2 \oplus A_1$: $\circ \rightarrow \circ \circ$
(4) there are 15 silted algebras of the quiver $\circ \rightarrow \circ \rightarrow \circ \rightarrow \circ$, forming 3 families:
   (i) tilted algebras of type $A_4$, for details see Example 3.7 (I);
   (ii) tilted algebras of type $A_3 \oplus A_1$:
       $\circ \circ \circ \circ \circ$, $\circ \circ \circ \circ \circ \circ$, $\circ \circ \circ \circ \circ \circ$
   (iii) tilted algebras of type $A_2 \oplus A_2$: $\circ \rightarrow \circ \circ \rightarrow \circ$
(5) there are 13 silted algebras of the quiver

MSC2020: 16E35, 16G20.
Key words: silted algebra; tilted algebra; 2-term silting complexes; strictly shod algebra.
forming 4 families (see Example 3.10 for details on (i)(ii)(iii)):

(i) tilted algebras of type $D_4$;
(ii) tilted algebras of type $A_3 \sqcup A_1$;
(iii) tilted algebras of type $A_2 \sqcup A_1 \sqcup A_1$;
(iv) strictly shod algebra:

\[(s_1) \quad \circ \quad \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \circ\]

(6) there are 62 silted algebras of the quiver

forming 6 families (see Section 3.3.2 for details on (i)-(v)):

(i) tilted algebras of type $D_5$;
(ii) tilted algebras of type $D_4 \sqcup A_1$;
(iii) tilted algebras of type $A_4 \sqcup A_1$;
(iv) tilted algebras of type $A_3 \sqcup A_2$;
(v) tilted algebras of type $A_3 \sqcup A_1 \sqcup A_1$;
(vi) strictly shod algebras:

\[(s_2) \quad \circ \quad \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \circ\]
\[(s_3) \quad \circ \quad \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \circ\]
\[(s_4) \quad \circ \quad \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \circ\]
\[(s_5) \quad \circ \quad \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \circ\]

For a brief summary, all of these type $A$ silted algebras are tilted algebras, all but one of these type $D_4$ silted algebras are tilted algebras, and all but four of these type $D_5$ silted algebras are tilted algebras. Therefore, from these examples we obtain five strictly shod algebras $(s_1) - (s_5)$. 
In order to classify these silted algebras, we first classify the 2-term silting complexes. For this purpose, we develop an algorithm (Algorithm 3.1) based on the algorithm of Happel and Ringel classifying tilting modules. Given a Dynkin quiver $Q$, we can produce all the 2-term silting complexes by using this algorithm. Then we calculate the endomorphism algebra of each 2-term silting complex. For types $A_3$, $A_4$ and $D_4$, we actually classify silted algebras for all orientations, but we do not find strictly shod algebras except in (5) and (6) above.

The structure of this paper is as follows. In Section 2, we recall the definitions of tilting modules, tilted algebras, 2-term silting complexes and silted algebras. In Section 3, we describe the algorithm producing all 2-term silting complexes and calculate the concrete examples.

Throughout this paper, $K$ denotes an algebraically closed field and $D = \text{Hom}_K(\cdot, K)$ denotes the $K$-dual. All algebras will be finite-dimensional $K$-algebras, and all modules will be finite-dimensional right modules.

Acknowledgement: This paper is based on the master thesis of the author. She is deeply grateful to her supervisor Qunhua Liu and Dong Yang for their kind supervision, and she thanks Zongzhen Xie and Houjun Zhang for carefully reading the manuscript and pointing out an error. She acknowledges support by the National Natural Science Foundation of China No. 11671207.

2. Silted algebras

In this section, we recall the definitions of tilting modules, tilted algebras, 2-term silting complexes and silted algebras.

2.1 Tilted algebras.

**Definition 2.1.** [3, Chapter 6 Definition 2.1 and Chapter 6 Corollary 4.4] Let $A$ be an algebra. An $A$-module $T$ is called a tilting module if the following three conditions are satisfied:

- (T1) $\text{pd} T_A \leq 1$.
- (T2) $\text{Ext}^1_A(T, T) = 0$.
- (T3) $|T| = |A|$.

**Definition 2.2.** [3, Chapter 8 Definition 3.1] Let $Q$ be an acyclic quiver. An algebra $B$ is said to be tilted of type $Q$ if there exists a tilting module $T$ over the path algebra $A = KQ$ of $Q$ such that $B = \text{End}(T_A)$.

The global dimension of a tilted algebra is at most 2 [4, Proposition 3.2].

Let $Q$ be a Dynkin quiver and $A = KQ$. In this case the condition (T1) is automatic and both $A_A$ and $D(A_A)$ are tilting modules with endomorphism algebra $A$. According to the proof of [9, Proposition 2.1] by Happel and Ringel, we obtain the following algorithm to produce all basic tilting modules over $A$. 
Algorithm 2.3. Perform the following 3 steps for all non-empty subsets \( I \) of \( Q_0 \).

1. Let \( e = \bigoplus_{i \in I} e(i) \), \( P(I) = eA \), \( A(I) = A/\langle e \rangle \).
2. For each basic tilting \( A(I) \)-module \( N \) which, considered as an \( A \)-module, has no non-trivial injective direct summands, form the \( A \)-module \( M = P(I) \oplus \tau^{-1}A \).
3. For each \( A \)-module \( M \) obtained in (2), let \( m \in N \cup \{0\} \) be such that \( |\tau^{-m}A|M| = |A| \), and \( \tau^{-m}A \) has non-trivial injective direct summand. Form the \( A \)-modules \( \tau^{-p}A, 0 \leq p \leq m \).

Example 2.4. Let \( A \) the path algebra of the quiver \( 1 \overset{1}{\rightarrow} 2 \). Then the AR-quiver \( \Gamma(\text{mod} A) \) of \( \text{mod} A \) is of the form

We apply Algorithm 2.3

1. \( I = \{1\} \). The quiver of \( A(I) = A/\langle e \rangle \) has only one vertex 2. So \( \text{mod} A(I) \) has only one tilting module \( M_A = 0 1 \). This yields the tilting module

\[
T_A = P(I) \oplus \tau^{-1}(M_A) = P(1) \oplus \tau^{-1}(0 1) = D(A_A) = 1 1 \oplus 1 0.
\]

2. \( I = \{2\} \). The quiver of \( A(I) = A/\langle e \rangle \) has only one vertex 1. So \( \text{mod} A(I) \) has only one tilting module \( M_A = 1 0 \), which is injective.

3. \( I = \{1, 2\} \). In this case the tilting module is

\[
T_A = P(I) = P(1) \oplus P(2) = A_A = 1 1 \oplus 0 1.
\]

To summarise, \( A \) has two basic tilting modules: \( 1 1 \oplus 1 0 \) and \( 1 1 \oplus 0 1 \). Both endomorphism algebras are isomorphism to \( A \).

2.2 2-term silting complexes.

Definition 2.5. [5] Page 1] Let \( A \) be an algebra. Let \( P \) be a complex in the bounded homotopy category of finitely generated projective \( A \)-modules \( K^b(\text{proj} A) \). Then \( P \) is called silting if \( \text{Hom}_{K^b(\text{proj} A)}(P, P[i]) = 0 \) for \( i > 0 \), and if \( P \) generates \( K^b(\text{proj} A) \) as a triangulated category. Furthermore, we say that \( P \) is 2-term if \( P \) only has non-zero terms in degrees 0 and -1.
The following result is a corollary of [2, Theorem 3.2].

**Corollary 2.6.** Assume that $A$ is hereditary. Then any basic 2-term silting complex over $A$ is of the form $M \oplus P[1]$, where $P = eA$ for some idempotent $e$ of $A$, and $M$ is a basic tilting module over $A/\langle e \rangle$. Conversely, every complex of this form is a 2-term silting complex.

### 2.3 Silted algebras.

**Definition 2.7.** [5, Definition 0.1] Let $Q$ be an acyclic quiver. We call an algebra $B$ silted of type $Q$ if there exists a 2-term silting complex $M$ over $KQ$ such that $B \cong \text{End}_{K^b(\text{proj}KQ)}(M)$.

Tilted algebras are silted algebras, because (projective resolutions of) tilting modules are 2-term silting complexes.

**Theorem 2.8.** [5, Theorem 2.13] Let $A$ be a connected algebra. Then the following are equivalent:

1. $A$ is a silted algebra;
2. $A$ is a tilted algebra or a strictly shod algebra.

Recall from [8, page 2] that an algebra $A$ is called shod (for small homological dimension) provided for each indecomposable $A$-module $X$, either $\text{pd}X_A \leq 1$ or $\text{id}X_A \leq 1$. It is known that $\text{gl.dim}A \leq 3$ [7, Proposition 2.2]. We call $A$ strictly shod if it is shod and $\text{gl.dim}A = 3$. It is known that tilted algebras are shod [7, Proposition 3.2].

The following lemma will be useful.

**Lemma 2.9.** An algebra $A$ is silted of type $Q$ if and only if $A^{op}$ is silted of type $Q^{op}$.

**Proof.** This is because $R\text{Hom}_A(? A)[1] : K^b(\text{proj}A) \rightarrow K^b(\text{proj}A^{op})$ is a triangle anti-equivalence and induces a bijection between the set of 2-term silting complexes over $KQ$ and that over $KQ^{op}$.

### 3. Examples of silted algebras of Dynkin type

In this section, we will describe an algorithm to produce all basic 2-term silt ing complexes over the path algebra of a Dynkin quiver, and use this algorithm to compute some examples.

Let $Q$ be a Dynkin quiver and $A = KQ$. Let $K[{-1,0}](\text{proj}A)$ be the full subcategory of $K^b(\text{proj}A)$ consisting of complexes concentrated in degrees -1 and 0. We will call the full subquiver of the AR quiver of $K^b(\text{proj}A)$ whose vertices belong to $K[{-1,0}](\text{proj}A)$ the AR quiver of $K[{-1,0}](\text{proj}A)$. It is obtained from the AR quiver of $\text{mod}A$ by properly gluing a copy of $Q$ from the right.
3.1 The algorithm.

Let $Q$ be a Dynkin quiver, and $A = KQ$. Due to Corollary 2.6 we have the following algorithm to produce all basic 2-term silting complexes over $A$.

**Algorithm 3.1.** We perform the following two steps for any subset $I$ of $Q_0$:

1. Let $e = \bigoplus_{i \in I} e(i)$ and $A(I) = A/(e)$.
2. For each basic tilting $A(I)$-module $M$ produced by Algorithm 2.3, form $T = M \oplus P[1]$ where $P = eA$.

**Observation 3.2.** Let $T = M \oplus P[1]$ be a 2-term silting complex over $A$, where $P \in \text{proj} A$ and $M \in \text{mod} A$. If $P = 0$ or $M$ has no non-trivial projective direct summands, then $\text{End}(T)$ is a tilted algebra of type $Q$.

Indeed, if $P = 0$, then $T = M$ is a tilting $A$-module; if $M$ has no no-trivial projective direct summands, then $\tau_A(T)$ belongs to $\text{mod} A$, and hence is a tilting $A$-module, so $\text{End}(T) \cong \text{End}(\tau_A(T))$ is a tilted algebra.

By Observation 3.2, we will divide silted algebras of type $Q$ into two classes:

(I) tilted algebras of type $Q$,

(II) $\text{End}(T)$, where $T = M \oplus P[1]$ is a 2-term silting complex such that $P \neq 0$ and $M$ has a non-zero projective direct summand over $A$. In other words, $T$ has direct summands both on the left border and on the right border of the AR quiver of $K^{[-1,0]}(\text{proj} A)$.

We remark that (I) and (II) may have overlaps. We are mainly interested in the silted algebras which are not tilted of type $Q$, especially the strictly shod algebras.

3.2 Examples of type $A$.

3.2.1 Type $A_1$.

**Example 3.3.** Let $A$ be the path algebra of the quiver $1 \rightarrow 2$. For a 2-term silting complex $M \oplus P[1]$, either $P = 0$, or $M = 0$. So $\text{End}_{K^b(\text{proj} A)}(M \oplus P[1])$ is isomorphic to $A$.

3.2.2 Type $A_2$.

**Example 3.4.** Let $A$ be the path algebra of the quiver

\[
\begin{array}{c}
1 \\
\downarrow \\
2
\end{array}
\]

Tilted algebras were already computed in Example 2.4. Thus below we apply Algorithm 3.1 to all non-empty subsets $I$ of $Q_0$.

The AR-quivar $\Gamma(K^{[-1,0]}(\text{proj} A))$ is

\[
\begin{array}{c}
1 \quad \cdots \\
0[1]
\end{array}
\]
(1) \( I = \{1\} \). \( A(I) = A/\langle e \rangle \) is given by the quiver \( \begin{array}{c} 1 \to 1 \end{array} \), which has only one tilting module \( M = 0 \to 1 \). The corresponding silting complex is

\[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet
\end{array}
\]

Its endomorphism algebra is given by the quiver \( \begin{array}{c} 0 \to 0 \end{array} \).

(2) \( I = \{2\} \). \( A(I) = A/\langle e \rangle \) is given by the quiver \( \begin{array}{c} 1 \to 1 \end{array} \), which has only one tilting module \( M = 1 \to 0 \). The corresponding silting complex is

\[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet
\end{array}
\]

It is clear that \( \tau T = A_A \).

(3) \( I = \{1, 2\} \). Then \( T = A_A[1], \tau T = D(A_A) \) and \( \text{End}(T) \cong A \).

To summarise, there are 2 silted algebras of \( \begin{array}{c} 0 \to 0 \end{array} \), forming two families:

(i) tilted algebra of type \( A_2 \): \( \begin{array}{c} 0 \to 0 \end{array} \)

(ii) tilted algebra of type \( A_1 \amalg A_1 \): \( \begin{array}{c} 0 \to 0 \end{array} \)

More precisely, we have the following table:

| silted algebras | 2-term silting complexes | tilted type |
|-----------------|--------------------------|-------------|
| \( \begin{array}{c} 0 \to 0 \end{array} \) | \( \begin{array}{c} 0 \to 0 \end{array} \) \( \begin{array}{c} 0 \to 0 \end{array} \) \( \begin{array}{c} 0 \to 0 \end{array} \) \( \begin{array}{c} 0 \to 0 \end{array} \) \( \begin{array}{c} 0 \to 0 \end{array} \) \( \begin{array}{c} 0 \to 0 \end{array} \) | \( A_2 \) |
| \( \begin{array}{c} 0 \to 0 \end{array} \) \( \begin{array}{c} 0 \to 0 \end{array} \) | \( \begin{array}{c} 0 \to 0 \end{array} \) | \( A_1 \amalg A_1 \) |

**3.2.3 Type \( A_3 \).**

According to [13, Theorem 1], there are 14 basic 2-term silting complexes, 5 of which are tilting modules. Up to isomorphism there are three quivers of type \( A_3 \). Due to Lemma 2.9, we classify silted algebras for two of them.

**Example 3.5.** Let \( A \) be the path algebra of the quiver

\[
\begin{array}{c}
\circ \\
\circ \\
\circ
\end{array}
\]

\[
\circ \to \circ \to \circ
\]
(I) The AR-quiver $\Gamma(\text{mod}A)$ of $\text{mod}A$ is of the form

```
\begin{array}{ccc}
0 & 1 & 1 \\
\downarrow & \downarrow & \downarrow \\
0 & 1 & 0 \\
\downarrow & \downarrow & \downarrow \\
0 & 0 & 1
\end{array}
```

We apply Algorithm 2.3 to obtain the following table of tilted algebras

| tilted algebras | tilting modules |
|-----------------|-----------------|
| $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \circ \circ \circ$ |
| $\circ \rightarrow \circ$ | $\circ \bullet$ |
| $\circ \rightarrow \circ$ | $\bullet \circ$ |
| $\circ \rightarrow \circ$ | $\circ \circ \bullet$ |

(II) The AR-quiver $\Gamma(K^{-1,0}(\text{proj}A))$ is

```
\begin{array}{ccc}
1 & 1 & 1 \\
\downarrow & \downarrow & \downarrow \\
0 & 1 & 0 \\
\downarrow & \downarrow & \downarrow \\
0 & 0 & 1
\end{array}
```

Below we apply Algorithm 3.1 to all non-empty subsets $I$ of $Q_0$.

(1) $I = \{1\}$. $A(I) = A/\langle e \rangle$ is given by the quiver $\circ \rightarrow \circ \rightarrow \circ$. $A(I)$ has two basic tilting modules: $M_1 = 0 \ 1 \ 0 \oplus 0 \ 1 \ 1$ and $M_2 = 0 \ 0 \ 1 \oplus 0 \ 1 \ 1$.

(i) For $M_1 = 0 \ 1 \ 0 \oplus 0 \ 1 \ 1$, the corresponding silting complex is

```
\begin{array}{ccc}
\circ & \circ & \circ \\
\bullet & \circ & \circ \\
\end{array}
```

Its endomorphism algebra is given by the quiver $\circ \rightarrow \circ \circ$.

(ii) For $M_2 = 0 \ 0 \ 1 \oplus 0 \ 1 \ 1$, the corresponding silting complex is

```
\begin{array}{ccc}
\circ & \circ & \circ \\
\bullet & \circ & \circ \\
\end{array}
```

Its endomorphism algebra is given by the quiver $\circ \rightarrow \circ \circ$.

(2) $I = \{2\}$. $A(I) = A/\langle e \rangle$ is given by the quiver $\circ \circ \rightarrow \circ \circ$. $A(I)$ has only one basic tilting module: $M_1 = 1 \ 0 \ 0 \oplus 0 \ 0 \ 1$. The corresponding silting complex is

```
\begin{array}{ccc}
\circ & \circ & \circ \\
\bullet & \circ & \circ \\
\end{array}
```
Its endomorphism algebra is given by the quiver $\circ \rightarrow \circ \circ$.

(3) $I = \{1, 2\}$. $A(I) = A/I\langle e \rangle$ is given by the quiver $\circ$. $A(I)$ has only one basic tilting module: $M_1 = 0 0 1$. The corresponding silting complex is

$\circ \circ \circ \circ \circ \circ$.

Its endomorphism algebra is given by the quiver $\circ \rightarrow \circ \circ$.

(4) If $3 \in I$, then $\tau T$ is a tilting module. We list all such $T$ below:

Note that the four silted algebras in the above (1), (2) and (3) are isomorphic, so there are 5 silted algebras of type $\circ \rightarrow \circ \rightarrow \circ$, forming two families:

(i) tilted algebras of type $A_3$:

\[
\begin{align*}
&\circ \rightarrow \circ \rightarrow \circ, \quad \circ \rightarrow \circ \leftarrow \circ, \quad \circ \leftarrow \circ \rightarrow \circ, \\
&\circ \leftarrow \circ \rightarrow \circ
\end{align*}
\]

(ii) tilted algebra of type $A_2 \uplus A_1$: $\circ \rightarrow \circ \circ$

Example 3.6. Let $A$ be the path algebra of the quiver

\[Q = \begin{array}{c}
\circ \\
\circ \\
\circ \\
\circ \\
\circ
\end{array}\]

The AR-quiver of $K[-1,0](\text{proj} A)$ is

(I) We first apply Algorithm 2.3 to produce all tilting modules and compute the corresponding tilted algebras. These are the tilted algebras of type $Q$.

| No. | tilted algebras | tilting modules |
|-----|-----------------|-----------------|
| (1) | $\circ \rightarrow \circ \rightarrow \circ$ | $\circ \circ \circ$ |
| (2) | $\circ \rightarrow \circ \rightarrow \circ$ | $\circ \circ \circ$ |
| (3) | $\circ \rightarrow \circ \rightarrow \circ$ | $\circ \circ \circ$ |

(II) We apply Algorithm 3.1 to all non-empty subsets $I$ of $Q_0$. Due to Observation 3.2, we only list below the endomorphism algebras $\text{End}_{K^b(\text{proj} A)}(T)$, where $T = M \oplus$
$P[1]$ is a 2-term silting complex with $M \neq 0$ and $P \neq 0$ (i.e. $T$ has direct summands both on the left border and the right border of the AR quiver)

| No. | silted algebras | 2-term silting complexes | tilted type  |
|-----|----------------|--------------------------|-------------|
| (4) | $\circ \rightarrow \circ \circ$ | $\bullet \circ \circ \bullet \circ \circ$ | $A_2 \amalg A_1$ |
| (5) | $\circ \circ \circ$ | $\bullet \circ \circ \bullet \circ \circ$ | $A_1 \amalg A_1 \amalg A_1$ |
| (6) | $\circ \circ \circ \circ \circ \circ \circ$ | $\bullet \circ \circ \bullet \circ \circ$ | $A_3$ |

To summarise, there are 6 silted algebras of type $Q$, forming 3 families:

(i) tilted algebra of type $A_3$: (1) – (3), (6);
(ii) tilted algebra of type $A_2 \amalg A_1$: (4);
(iii) tilted algebra of type $A_1 \amalg A_1 \amalg A_1$: (5).

### 3.2.4 Type $A_4$.

According to [13, Theorem 1], there are 42 basic 2-term silting complexes, 14 of which are tilting modules. Up to isomorphism there are four quivers of type $A_4$. Due to Lemma [2.9] we classify silted algebras for three of them.

**Example 3.7.** Let $A$ be the path algebra of the quiver

$$Q = \begin{array}{cccc}
1 & 2 & 3 & 4 \\
\circ & \circ & \circ & \circ
\end{array}$$

The AR-quiver of $K[1,0]\langle\text{proj }A\rangle$ is

(I) Tilted algebras of type $Q$ are
(II) Silted algebras of the form $\text{End}_{K^b(\text{proj} A)}(M \oplus P[1])$ ($P \neq 0, M \neq 0$) are
To summarise, there are 15 silted algebras of type $Q$, forming 3 families:

(i) tilted algebras of type $\tilde{A}_4$: (1) – (10);
(ii) tilted algebras of type $\tilde{A}_3 \sqcup \tilde{A}_1$: (11) – (15);
(iii) tilted algebras of type $\tilde{A}_2 \sqcup \tilde{A}_2$: (14).

**Example 3.8.** Let $A$ be the path algebra of the quiver

$$Q = \begin{array}{cccc}
1 & \rightarrow & 2 & \leftarrow 3 & \rightarrow & 4
\end{array}$$

The AR-quiver of $K[-1,0](\text{proj} A)$ is

(I) Tilted algebras of type $Q$ are

| No. | tilted algebras | tilting modules |
|-----|----------------|----------------|
| (1) | $\circ \rightarrow \circ \rightarrow \circ \rightarrow \circ$ | $\bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet$ |
| (2) | $\circ \rightarrow \circ \rightarrow \circ \leftarrow \circ$ | $\bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet$ |
| (3) | $\circ \rightarrow \circ \rightarrow \circ \rightarrow \circ$ | $\bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet$ |
| (4) | $\circ \rightarrow \circ \rightarrow \circ \rightarrow \circ$ | $\bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet$ |
| (5) | $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet$ |
| (6) | $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet \cdot \bullet$ |
(II) Silted algebras of the form $\text{End}_{K^b(\text{proj}A)}(M \oplus P[1])$ ($P \neq 0$, $M \neq 0$) are

| No. | silted algebras | 2-term silting complexes | tilted type |
|-----|-----------------|--------------------------|-------------|
| (7) | ![Diagram](7) | ![Diagram](7) | $A_4$ |
| (8) | ![Diagram](8) | ![Diagram](8) | $A_4$ |
| (9) | ![Diagram](9) | ![Diagram](9) | $A_4$ |
| (10) | ![Diagram](10) | ![Diagram](10) | $A_4$ |
| (5) | ![Diagram](5) | ![Diagram](5) | $A_4$ |
| (6) | ![Diagram](6) | ![Diagram](6) | $A_4$ |
| (11) | ![Diagram](11) | ![Diagram](11) | $A_3 \amalg A_1$ |
| (12) | ![Diagram](12) | ![Diagram](12) | $A_3 \amalg A_1$ |
| (13) | ![Diagram](13) | ![Diagram](13) | $A_3 \amalg A_1$ |
| (14) | ![Diagram](14) | ![Diagram](14) | $A_3 \amalg A_1$ |
| (15) | ![Diagram](15) | ![Diagram](15) | $A_2 \amalg A_2$ |
| (16) | ![Diagram](16) | ![Diagram](16) | $A_2 \amalg A_1 \amalg A_1$ |
| (17) | ![Diagram](17) | ![Diagram](17) | $A_1 \amalg A_1 \amalg A_1 \amalg A_1$ |

To summarise, there are 17 silted algebras of type $Q$, forming 5 families:

(i) tilted algebras of type $A_4$: (1) – (10);
(ii) tilted algebras of type $A_3 \Pi A_1$: (11) – (14);
(iii) tilted algebras of type $A_2 \Pi A_2$: (15);
(iv) tilted algebras of type $A_2 \Pi A_1 \Pi A_1$: (16);
(v) tilted algebras of type $A_1 \Pi A_1 \Pi A_1 \Pi A_1$: (17).

Example 3.9. Let $A$ be the path algebra of the quiver

\[ Q = \begin{array}{cccc}
\circ & \rightarrow & \circ & \rightarrow & \circ & \rightarrow & \circ \\
1 & 2 & 3 & 4
\end{array} \]

The AR-quiver of $K^{[-1,0]}(\text{proj}A)$ is

(I) Tilted algebras of type $Q$ are

| No. | tilted algebras | tilting modules |
|-----|----------------|----------------|
| (1) | $\begin{array}{cccc}
\circ & \rightarrow & \circ & \rightarrow & \circ & \rightarrow & \circ \\
1 & 2 & 3 & 4
\end{array}$ | $\begin{array}{c}
\bullet \\
\bigcirc
\end{array}$ |
| (2) | $\begin{array}{cccc}
\circ & \rightarrow & \circ & \rightarrow & \circ & \rightarrow & \circ \\
1 & 2 & 3 & 4
\end{array}$ | $\begin{array}{c}
\bullet \\
\bigcirc
\end{array}$ |
| (3) | $\begin{array}{cccc}
\circ & \rightarrow & \circ & \rightarrow & \circ & \rightarrow & \circ \\
1 & 2 & 3 & 4
\end{array}$ | $\begin{array}{c}
\bullet \\
\bigcirc
\end{array}$ |
| (4) | $\begin{array}{cccc}
\circ & \rightarrow & \circ & \rightarrow & \circ & \rightarrow & \circ \\
1 & 2 & 3 & 4
\end{array}$ | $\begin{array}{c}
\bullet \\
\bigcirc
\end{array}$ |
| (5) | $\begin{array}{cccc}
\circ & \rightarrow & \circ & \rightarrow & \circ & \rightarrow & \circ \\
1 & 2 & 3 & 4
\end{array}$ | $\begin{array}{c}
\bullet \\
\bigcirc
\end{array}$ |
| (6) | $\begin{array}{cccc}
\circ & \rightarrow & \circ & \rightarrow & \circ & \rightarrow & \circ \\
1 & 2 & 3 & 4
\end{array}$ | $\begin{array}{c}
\bullet \\
\bigcirc
\end{array}$ |
| (7) | $\begin{array}{cccc}
\circ & \rightarrow & \circ & \rightarrow & \circ & \rightarrow & \circ \\
1 & 2 & 3 & 4
\end{array}$ | $\begin{array}{c}
\bullet \\
\bigcirc
\end{array}$ |
(II) Silted algebras of the form $\text{End}_{K^{\text{proj}}}(M \oplus P[1])$ ($P \neq 0, M \neq 0$) are

| No. | silted algebras | 2-term silting complexes | tilted type |
|-----|----------------|-------------------------|-------------|
| (8) | $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \bullet \bullet \bullet \bullet$ | $A_3 \perp A_1$ |
| (9) | $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \bullet \bullet$ | $A_3 \perp A_1$ |
| (10) | $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \bullet \bullet$ | $A_3 \perp A_1$ |
| (11) | $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \bullet \bullet \bullet$ | $A_2 \perp A_2$ |
| (12) | $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \bullet \bullet \bullet \bullet \bullet$ | $A_2 \perp A_1 \perp A_1$ |
| (13) | $\circ \bullet \circ \rightarrow \circ \circ$ | $\bullet \bullet \bullet \bullet \bullet$ | $A_3 \perp A_1$ |
| (14) | $\circ \rightarrow \circ \rightarrow \circ \rightarrow \circ$ | $\bullet \bullet \bullet \bullet \bullet$ | $A_4$ |
| (15) | $\circ \rightarrow \circ \rightarrow \circ \rightarrow \circ$ | $\bullet \bullet \bullet \bullet \bullet$ | $A_4$ |
| (16) | $\circ \bullet \circ \rightarrow \circ \circ$ | $\bullet \bullet \bullet \bullet \bullet$ | $A_4$ |
| (6) | $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \bullet \bullet \bullet \bullet$ | $A_4$ |
| (7) | $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \bullet \bullet \bullet \bullet$ | $A_4$ |

To summarise, there are 16 silted algebras of type $Q$, forming 4 families:

(i) tilted algebras of type $A_4$: (1) – (7),(14) – (16);
(ii) tilted algebras of type $A_3 \perp A_1$: (8) – (10),(13);
(iii) tilted algebras of type $A_2 \perp A_2$: (11);
(iv) tilted algebras of type $A_2 \perp A_1 \perp A_1$: (12).
3.3 Examples of type $\mathbb{D}$.

3.3.1 Type $\mathbb{D}_4$.

According to [13, Theorem 1], there are 50 basic 2-term silting complexes, 20 of which are tilting modules. Up to isomorphism there are four quivers of type $\mathbb{D}_4$. Due to Lemma 2.9 we classify silted algebras for two of them.

**Example 3.10.** Let $A$ be the path algebra of the quiver

![Quiver Diagram](image)

The AR-quiver of $K[-1,0](\text{proj}A)$ is

![AR-quiver Diagram](image)

(I) Tilted algebras of type $Q$ are

| No. | tilted algebras | tilting modules |
|-----|----------------|-----------------|
| (1) | ![Tilted Algebras](image) | ![Tilting Modules](image) |
| (2) | ![Tilted Algebras](image) | ![Tilting Modules](image) |
(II) Silted algebras of the form $\text{End}_{\mathcal{K}^\text{proj}}(M \oplus P[1])$ ($P \neq 0$, $M \neq 0$) are

| No. | silted algebras | 2-term silting complexes | tilted type |
|-----|-----------------|--------------------------|-------------|
| (8) | $\circ \rightarrow \circ \rightarrow \circ$ | $\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \b
(iii) tilted algebras of type $\mathbb{A}_2 \coprod \mathbb{A}_1 \coprod \mathbb{A}_1$: (11);
(iv) the strictly shod algebra $\circ \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \quad : (13)$.

**Example 3.11.** Let $A$ be the path algebra of the quiver

![Quiver Diagram]

The AR-quiver of $K[-1,0](\text{proj}A)$ is

![AR-quiver Diagram]

(I) Tilted algebras of type $Q$ are

| No. | tilted algebras | tilting modules |
|-----|----------------|-----------------|
| (1) | $\circ \rightarrow \circ$ | $\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet$ |
| (2) | $\circ \rightarrow \circ$ | $\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet$ |
| (3) | $\circ \rightarrow \circ$ | $\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet$ |
To summarise, there are 11 silted algebras of type $Q$, forming 4 families:

(i) tilted algebras of type $D_4$: (1) – (7), (9);
(ii) tilted algebras of type $A_3 \coprod A_1$: (8);
(iii) tilted algebras of type $A_2 \coprod A_1 \coprod A_1$: (10);
(iv) $A_1 \coprod A_1 \coprod A_1 \coprod A_1$: (11).

(II) Silted algebras of the form $\text{End}_{K^b(\text{proj} A)}(M \oplus P[1])$ ($P \neq 0$, $M \neq 0$) are
3.3.2 Type $\mathbb{D}_5$.

Let $A$ be the path algebra of the quiver.

\[
Q = \begin{array}{c}
1 \\
\downarrow \\
3 \\
\downarrow \\
2 \\
\end{array} \quad \begin{array}{c}
3 \\
\rightarrow \\
4 \\
\rightarrow \\
5 \\
\end{array}
\]

According to [13] Theorem 1], there are 182 basic 2-term silting complexes, 77 of which are tilting modules. The AR-quiver of $K^{[-1,0]}(\text{proj} A)$ is

(I) Tilted algebras of type $Q$ are:

\[
\begin{array}{c}
\circ \rightarrow \circ \rightarrow \circ \rightarrow \circ \rightarrow \circ \rightarrow \circ \rightarrow \circ \rightarrow \circ \\
\circ \rightarrow \circ \rightarrow \circ \rightarrow \circ \\
\circ \rightarrow \circ \rightarrow \circ \\
\circ \rightarrow \circ \\
\end{array}
\]
(II) Silted algebras of the form \( \text{End}_{K^b(\text{proj} A)}(M \oplus P[1]) \) \((P \neq 0, M \neq 0)\) are

| No. | silted algebras | 2-term silting complexes | tilted type |
|-----|----------------|--------------------------|-------------|
| (1) | -              | -                        | \( \mathbb{D}_5 \) |
| (2) | -              | -                        | strictly shod |
| (3) | -              | -                        | strictly shod |
| No. | silted algebras | 2-term silting complexes | tilted type |
|-----|----------------|--------------------------|------------|
| (4) | ![Diagram](image1) | ![Diagram](image2) | strictly shod |
| (5) | ![Diagram](image3) | ![Diagram](image4) | $\mathbb{D}_4 \oplus A_1$ |
| (6) | ![Diagram](image5) | ![Diagram](image6) | $A_4 \oplus A_1$ |
| (7) | ![Diagram](image7) | ![Diagram](image8) | $A_4 \oplus A_1$ |
| (8) | ![Diagram](image9) | ![Diagram](image10) | $A_4 \oplus A_1$ |
| (9) | ![Diagram](image11) | ![Diagram](image12) | $A_3 \oplus A_2$ |
| (10) | ![Diagram](image13) | ![Diagram](image14) | $A_3 \oplus A_2$ |
| (11) | ![Diagram](image15) | ![Diagram](image16) | $A_3 \oplus A_2$ |
| (12) | ![Diagram](image17) | ![Diagram](image18) | $A_3 \oplus A_1 \oplus A_1$ |
| (13) | ![Diagram](image19) | ![Diagram](image20) | $A_3 \oplus A_1 \oplus A_1$ |
| (14) | ![Diagram](image21) | ![Diagram](image22) | $A_3 \oplus A_1 \oplus A_1$ |
| No. | silted algebras | 2-term silting complexes | tilted type |
|-----|----------------|--------------------------|------------|
| (15) | ![Diagram](image1) | ![Diagram](image2) | $A_3 \amalg A_1 \amalg A_1$ |
| (16) | ![Diagram](image3) | ![Diagram](image4) | $D_5$ |
| (17) | ![Diagram](image5) | ![Diagram](image6) | $D_5$ |
| (18) | ![Diagram](image7) | ![Diagram](image8) | $D_5$ |
| (19) | ![Diagram](image9) | ![Diagram](image10) | $D_5$ |
| (20) | ![Diagram](image11) | ![Diagram](image12) | strictly shod |
| (21) | ![Diagram](image13) | ![Diagram](image14) | $D_4 \amalg A_1$ |
| (22) | ![Diagram](image15) | ![Diagram](image16) | $D_4 \amalg A_1$ |
| (23) | ![Diagram](image17) | ![Diagram](image18) | $D_4 \amalg A_1$ |
| (24) | ![Diagram](image19) | ![Diagram](image20) | $D_4 \amalg A_1$ |
| (25) | ![Diagram](image21) | ![Diagram](image22) | $D_4 \amalg A_1$ |
To summarise, there are 62 silted algebras of type $Q$, forming 6 families:

(i) tilted algebras of type $D_5$: (I), (1), (16) − (19); 
(ii) tilted algebras of type $D_4 \amalg A_1$: (5), (21) − (26), (27); 
(iii) tilted algebras of type $A_4 \amalg A_1$: (6), (7), (8); 
(iv) tilted algebras of type $A_3 \amalg A_2$: (9), (10), (11); 
(v) tilted algebras of type $A_3 \amalg A_1 \amalg A_1$: (12) − (15); 
(vi) strictly shod algebras: (2), (3), (4), (20).

REFERENCES

[1] T. Aihara, O. Iyama. Silting mutation in triangulated categories. J. Lond. Math. Soc., 2012, 85(3): 633-668. 
[2] T. Adachi, O. Iyama, I. Reiten. $\tau$-tilting theory. Compos. Math., 2014, 150(3): 415-452. 
[3] I. Assem, A. Skowronski, D. Simson. Elements of the Representation Theory of Associative Algebras: Volume 1: Techniques of Representation Theory. 2006. 
[4] A. B. Buan, I. Reiten, H. Thomas. Three kinds of mutation. J. Algebra, 2011, 339(1): 97-113. 
[5] A. B. Buan, Y. Zhou. Silted algebras. Adv. Math., 2016, 303: 859-887. 
[6] A. B. Buan, Y. Zhou. A silting theorem. J. Pure Appl. Algebra, 2016, 220(7): 2748-2770. 
[7] F. U. Coelho. Shod algebras. IME-USP, 2001, 5(1):25-61. 
[8] F. U. Coelho, M. Lanzilotta. Algebras with small homological dimensions. Manuscripta Math., 1999, 100(1): 1-11. 
[9] D. Happel, C. M. Ringel. Construction of tilted algebras. In Representations of algebras. Springer, Berlin, Heidelberg. 1981, 125-144. 
[10] B. Keller, P. Nicolas. Cluster hearts and cluster tilting objects, work in preparation. Talk notes based on this work are available at [http://www.iag.uni-stuttgart.de/LatAGeoAlg/activities/t-workshop/Nicolas Notes.pdf](http://www.iag.uni-stuttgart.de/LatAGeoAlg/activities/t-workshop/Nicolas Notes.pdf). 
[11] B. Keller, D. Vossieck. Aisles in derived categories. Bull. Soc. Math. Belg. Sr. A., 1998, 40(2): 239-253. 
[12] S. Koenig, D. Yang. Silting objects, simple-minded collections, t-structures and co-t-structures for finite-dimensional algebras. Doc. Math., 2014, 19(1): 403-438. 
[13] M. A. A. Obaid, S. K. Nauman, W. M. Fakieh, C. M. Ringel. The numbers of support-tilting modules for a Dynkin algebra. J. Integer Seq., 2014, 18(10).