Non-linear extended MHD simulations of type-I edge localised mode cycles in ASDEX Upgrade and their underlying triggering mechanism

A. Cathey ∗1, M. Hoelzl1, K. Lackner1, G.T.A. Huijsmans2,3, M.G. Dunne1, E. Wolfrum1, S.J.P. Pamela4, F. Orain5, S. Günter1, the JOREK team6, the ASDEX Upgrade Team7, and the EUROfusion MST1 Team8

1Max Planck Institute for Plasma Physics, Boltzmannstr.2, 85748 Garching, Germany
2CEA, IRFM, 13108 Saint-Paul-Lez-Durance, France
3Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
4CCFE, Culham Science Centre, Abingdon, Oxon, OX14 3DB, United Kingdom
5Centre de Physique Thorique, Ecole Polytechnique, CNRS, France
6see https//www.jorek.eu for a list of current team members
7see the author list of H. Meyer et al. 2019 Nucl. Fusion 59 112014
8see the author list of B. Labit et al. 2019 Nucl. Fusion 59 0860020

Abstract

A triggering mechanism responsible for the explosive onset of edge localised modes (ELMs) in fusion plasmas is identified by performing, for the first time, non-linear magnetohydrodynamic simulations of repetitive type-I ELMs. Briefly prior to the ELM crash, destabilising and stabilising terms are affected at different timescales by an increasingly ergodic magnetic field caused by non-linear interactions between the axisymmetric background plasma and growing non-axisymmetric perturbations. The separation of timescales prompts the explosive, i.e. faster than exponential, growth of an ELM crash which lasts ∼500 µs. The duration and size of the simulated ELM crashes compare qualitatively well with type-I ELMs in ASDEX Upgrade. As expected for type-I ELMs, a direct proportionality between the heating power in the simulations and the ELM repetition frequency is obtained. The simulations presented here are a major step forward towards predictive modelling of ELMs and of the assessment of mitigation techniques in ITER and other future tokamaks.

1 Introduction

High-confinement mode (H-mode) [1] defines the standard operational scenario to achieve power amplification in ITER [2]. This operational regime hosts a steep pressure profile in the edge of the confined region which, in turn, drives a large toroidal current. Under such conditions, magnetohydrodynamic (MHD) instabilities called edge localised modes (ELM) can become excited and rapidly (within ∼0.1 to 1 ms) eject hot plasma towards the plasma facing components [3, 4, 5, 6]. The steep edge pressure profile together with the large toroidal current crash as a result, but begin to gradually recover until the process repeats itself, thus defining an ELM cycle. Type-I ELMs, the most pernicious type of such instabilities, repetitively expel between 5% and 15% of the plasma stored energy to the material surfaces. The associated heat fluxes pose significant concerns for next-step devices like ITER and must be completely avoided in a future reactor [7].

Resulting from the destructive potential inherent to type-I ELMs, and in order to produce physics-based predictions for future machines, substantial effort has been dedicated from experiment and theory to understand the underlying mechanisms that drive and trigger these instabilities [3, 4, 5, 6, 7, 8, 9, 10]. Nonlinear MHD simulations of ELMs in realistic tokamak geometry with various codes have played an increasingly important role in this regard [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. However, the simulations performed so far have had the shortcoming of modelling single ELM crashes by introducing arbitrary seed perturbations to unstable initial conditions [10]. Small differences in the chosen initial perturbations can have severe implications on the resulting dynamics and, therefore, results that depend on the amplitude and/or structure of the initial perturbations. Further, simulations that start from unstable profiles cannot answer how the plasma reached the unstable conditions in the first place.

We present for the first time non-linear MHD simu-
lations of multiple type-I ELM cycles. The simulated ELM repetition frequency is directly proportional to the heating source – also an important breakthrough. Additionally, a triggering mechanism for the explosive onset of the ELM is identified and described. The simulations shown here are a first of their kind in that they repetitively reproduce realistic ELM sizes with experimentally relevant timescales. Self-consistency of the perturbations that act as initial conditions for the ELMs is achieved because the perturbations retain a characteristic structure and a non-negligible amplitude determined by the last ELM – a feature of paramount importance for future studies regarding ELM triggering, suppression, and mitigation. Therefore, the work detailed here is an important step towards predictively studying the impact of natural type-I ELMs and the applicability – and robustness – of mitigation and suppression techniques to ITER.

2 ELM phenomenology

Comparisons between theory and experiment have identified ELMs as the coupling of two MHD instabilities – the peeling mode and the ballooning mode. The peeling mode has a long wavelength (∥ to the magnetic field) and a low toroidal mode number. It is driven by the current density gradient and stabilized by the pressure gradient. Conversely, the ballooning mode is a short wavelength and high toroidal mode number instability driven by the pressure gradient, \( \nabla p \), on the bad curvature side, and stabilized by large current density \( j \) \([27, 28]\). At the edge of H-mode plasmas with large \( \nabla p \) and \( j \), these instabilities couple into peeling-ballooning (PB) modes and, if the stabilising/destabilising balance between \( \nabla p \) and \( j \) allows, cause an ELM crash.

Experimental analyses of ELMs often include linear ideal MHD simulations probing stability with respect to PB modes at different time points. These studies almost always find the pre-ELM crash profiles to be very near a so-called peeling-ballooning stability boundary. However, it is not clear whether the ELM onset occurs exactly when the stability boundary is crossed, and what is the role of non-linear interactions on the ELM onset. Linear simulations usually ignore non-ideal effects such as resistivity as well as plasma flows, both of which are known to affect the growth rates of MHD instabilities on astrophysical and laboratory plasmas \([29, 30, 31, 32, 33, 34, 35, 36, 37]\). In particular, the plasma flow, primarily determined by momentum input and by the ExB velocity, is known to have an important stabilising effect on pressure-gradient-driven ballooning modes, and therefore may move the PB stability boundary \([31, 32, 33, 34]\). In the edge of H-mode plasmas, the radial electric field is set by a dominant ion diamagnetic contribution (~\( \nabla p_i/n_i \), where \( n_i \) is the ion density) and a small \( v \times B \) contribution \([35]\).

The JOREK code \([39, 40]\), which solves the reduced visco-resistive single fluid MHD equations \([41, 42]\) in realistic divertor tokamak geometry, was developed in particular to study ELMs. Simulation results have already successfully captured many key characteristics of natural, triggered, and mitigated single ELM crashes in a qualitatively and quantitatively accurate manner \([19, 20, 21, 26, 22, 21, 23, 25]\). Furthermore, it has been possible to simulate small, repetitive, high-frequency ELM crashes \([43, 20, 25]\). Considering the stabilising effect of plasma flows (with the ion diamagnetic contribution to \( E_r \) \([21, 44]\)) was key to obtain cyclical dynamics and accurate divertor heat deposition \([43]\). Simulating type-I ELM cycles carries significant computational costs because of the need to resolve the short timescales of the ELM crash and the long timescales of the inter-ELM evolution \([10]\).

3 Type-I ELM cycles

The starting point of the simulation is a stable post-ELM crash equilibrium reconstruction of an AUG discharge obtained with CLISTE \([45]\). The plasma has low triangularity, high separatrix density \((n_{separatrix} \sim 0.4n_{GW})\), and no momentum input is considered. We impose heat and particle radial diffusion coefficients with an edge transport barrier together with heat and particle sources to build up a steep pressure profile. The radial diffusion coefficients and sources are static throughout the simulation time. These are used to account for physical effects beyond the scope of MHD. Namely, neoclassical and anomalous transport are represented through diffusion coefficients, and heating and fuelling through the source terms. Realistic Spitzer-H伯rm parallel heat diffusion is considered and the resistivity at the plasma edge is chosen within the experimental expectation of the neoclassical resistivity. With the increasing \( \nabla p \), the diamagnetic contribution to \( E_r \) and the bootstrap current develop self-consistently (we consider \( \nabla p_i = \nabla p/2 \) because the single fluid model used here does not distinguish \( T_i \) and \( T_e \)). The latter is built up by considering a source term through the Sauter formula \([46, 47]\).

The plasma core, which is also part of the simulation domain, is unstable to a 2/1 tearing mode. In order to simultaneously avoid interference between this mode with the cyclical dynamics of the ELMs and to reduce the computational cost, we include all even toroidal mode numbers between \( n = 0 \) and 12, i.e. simulate a half tokamak. Nevertheless, the triggering mechanism detailed here remains unchanged for a simulation with the entire toroidal mode spec-
trum. Including higher toroidal mode numbers leads to faster dynamics, but does not change the triggering mechanism or which toroidal mode numbers are most dominant. Using this increased toroidal resolution for the full 40 ms simulation time of fig. [1] is computationally not affordable for us at present. Non-axisymmetric perturbations of all the non-zero toroidal mode numbers allowed in the simulation are introduced at noise-level. Figure [1](a) and (b) show the time evolution of their magnetic energies.

![Figure 1: Magnetic energies of the non-axisymmetric perturbations rising and falling at each ELM crash in linear (a) and logarithmic (b) scales. The arbitrary seed perturbations at 10 ms lead to a critically different ELM crash with respect to the next three ELMs borne out of self-consistent perturbations. (c) Power incident on the inner and outer divertor tiles in time. The outer divertor receives \( \sim 59\% \) of the total power during the inter-ELM phase, and \( \sim 51\% \) during the ELM crash.](image)

As a PB stability boundary is crossed due to the simultaneously large \( \nabla p \) and \( j \), a low frequency ELM precursor phase begins with an \( n = 2 \) perturbation becoming unstable, as can be seen in fig. [1](b) at \( t \sim 12 \) ms. This perturbation non-linearly drives additional modes with larger \( n \) through three-wave interactions [48]. Accordingly, the growth rate of the driven modes corresponds to the sum of the driving modes and, therefore, the highest toroidal mode number usually is the fastest growing mode. The growth rate of the precursors increases with time, as expected when slowly driving the plasma across an instability boundary [49]. The existence of such low frequency, low-\( n \) precursor activity has been observed across different tokamaks [50, 51, 52, 53, 54, 55, 56]. These precursors cause moderate increases in the divertor incident power, fig. [1](c), and are qualitatively similar to experimentally observed slow increases lasting \( \gtrsim 1 \) ms prior to the ELM [57].

Thereafter, the \( n = 2 \) perturbation coupled mostly with \( n = 4 \) act together to modify the background axisymmetric plasma in sub-millisecond timescales and cause a gradual decrease of \( \nabla p \) and \( j \), and an even faster slowing down of the plasma flow. These timescales are shortened in simulations with higher toroidal mode numbers, but the faster slowing down of the plasma flow with respect to that of \( \nabla p \) and \( j \) remains present. After this initial decrease, an explosive growth phase begins. This marks the end of the precursor phase, and the onset of the first ELM crash phase which lasts \( \sim 1.5 \) ms. The same mechanism is responsible for all of the simulated ELMs. The sum of the magnetic energies of all \( n \) during the precursor and ELM crash phases is plotted against exponential and faster than exponential fitting functions in fig. [2] thereby showing the explosive nature of the ELM onset. The modification of the background axisymmetric plasma due to the precursors leads to a small reduction of the energy of the perturbations (cf. fig. [2] from 31.8 to 32.2 ms).

![Figure 2: Precursor phase and ELM crash for the third simulated ELM. The explosive onset of the ELM occurs when a phase with faster than exponential growth takes place. The sum of the magnetic energies is shown in black. The exponential and faster than exponential fitting functions are plotted in full red and dashed gray lines, respectively.](image)

Directly after the end of the ELM crash, \( \nabla p \) begins to gradually recover (which drives \( j \) and \( E_r \)) and excites inter-ELM modes with \( n \) mainly between 6 and 8 as seen in fig. [1](b) from roughly 18 to 21 ms, 26 to 27 ms, and 34 to 35 ms. Similar inter-ELM modes, with toroidal mode numbers between 5 and 8, have been observed in AUG [51] and KSTAR [58] (the latter were simulated with JOREK [25]). Afterwards, the amplitudes of the non-axisymmetric perturbations become several orders of magnitude weaker than those during the ELM crash, but over up to 10 orders of magnitude stronger than their arbitrary initial amplitudes before the first ELM. The weak perturbations become destabilized again when \( \nabla p \) and \( j \) are large enough to simultaneously excite PB modes and overcome the stabilising effect of the plasma flow. At this point the cycle repeats itself, and there is another precursor phase followed by an ELM crash. This second ELM crash expels roughly 6% of the plasma stored energy and lasts \( \sim 550 \) \( \mu \)s, which is
more than twice as fast as the first ELM, which expels \( \sim 11\% \) of the stored energy. The imposed diffusion coefficients, the applied heating power, and the particle source govern the timescale at which \( \nabla p \) grows. The pedestal build-up in reality results from dynamic anomalous and neoclassical transport, applied heating power and fuelling including neutrals recycling. Such dynamical effects go beyond the scope of this investigation. Nevertheless, we observe a direct dependency of ELM frequency with heating power, therefore bolstering the argument that type-I ELMs are simulated. Reducing the heating power by 15\% leads to a lower ELM repetition frequency, as shown in fig. 3. A thorough heating and fuelling scan is envisioned as future work.

While the first ELM crash is, to a certain extent, governed by the arbitrary seed perturbations, the following crashes growing out of the self-consistent post-ELM state show different dynamics, in particular a faster and more violent ELM crash (clearly observed in fig. 4(c)). The difference between the seed perturbations before and after the first ELM crash is not limited to their amplitude, they also keep a PB mode structure at all times. In comparison to the first time the PB stability boundary was crossed, the precursors require less time to grow and affect the background plasma the subsequent times that the PB stability boundary was crossed, the precursors generally have a PB mode structure \([59, 60, 61, 62]\). Because of the discrepancies between the first (giant) and all the subsequent ELMs, in the following we focus on the latter to describe the triggering mechanism for the explosive onset of the ELM crash.

4 ELM triggering mechanism

By analyzing the simulation results we find that the influence of the precursors on the background axisymmetric plasma is responsible for the explosive ELM onset. The underlying mechanism relies on the existence of reconnection of magnetic field lines (taking place due to the non-zero resistivity) and on a separation of timescales between the responses of \( \nabla p \) and \( E_r \) to the enhanced transport by stochastic magnetic topology.

As the precursor amplitude becomes large enough \((\delta n_e/\rho_c \sim 1)\), the edge magnetic field starts to ergodize. Figure 4(a) shows magnetic field lines inside the separatrix closing in at the same flux surface where they started at 31 ms. One millisecond later, fig. 4(b) shows field lines that no longer necessarily arrive at the same flux surface where they started because axisymmetry is broken by the strong precursor activity.

![Figure 3: Magnetic energies of the non-axisymmetric perturbations for nominal (a) and 85\% nominal (b) heating. The ELM repetition frequency for (a) is \( f_{ELM} \approx 120 \) Hz, and it is reduced to \( f_{ELM} \approx 87 \) Hz. The nominal heating simulation is only performed until 40.9 ms.](image)

![Figure 4: Precursor phase and ELM crash showing (a)-(c) Poincaré plots of the magnetic field lines at 31, 32, and 33 ms respectively, and (d) time-evolving outboard midplane toroidally averaged pressure gradient. Precursor activity lasting roughly 1 ms starts at \( \sim 31.8 \) ms. We use the radial coordinate, \( \rho_{pol} = \sqrt{\psi_N} \) where \( \psi_N \) is the normalized poloidal magnetic flux equal to 0 in the magnetic axis and 1 at the separatrix, and the poloidal coordinate, \( \theta^* \) equal to 0 at the outboard midplane and \( -\pi/2 \) at the magnetic x-point.](image)
Therefore causing $\nabla p$ to change in the same manner, clearly shown in fig. 4(d). Since stochastic transport affects temperature gradients faster than it affects density, $\nabla T$ decreases faster than density does \cite{33}. As a result, $E_r$ decreases in a faster time scale than $\nabla p$ because the diamagnetic flow component of $E_r$ is inversely dependent on the local density - fig. 5. The second destabilising term, $j$, changes even slower than $\nabla p$ through current diffusion. We reiterate that the precursor timescales are faster when higher toroidal modes are considered and therefore we do not venture to compare the temporal dynamics to low frequency low-n precursors observed in experiment.

Figure 5: Time evolution of the outboard midplane axisymmetric ratio $E_r/\nabla p$. The ratio shows the balance between the stabilising $E_r$ and the destabilising $\nabla p$. It steadily decreases when the precursor phase begins at $\sim 31.6$ ms, therefore indicating increasingly unstable conditions which set the stage for the explosive ELM onset. The ratio increases again when the ELM crash ends at $\sim 33.2$ ms.

At first glance, the changes to the plasma caused by the precursors may seem stabilising. Namely, the decrease of $\nabla p$ and $j$ are, from the linear ideal MHD picture, stabilising effects. However, because of the decreasing stabilising/destabilising ratio ($E_r/\nabla p$), the overall effect is explosively destabilising, as shown in fig. 5 where four distinct phases can be observed.

The initial pre-ELM phase sustains a roughly constant $E_r/\nabla p$. The faster slowing down of the plasma flows with respect to $\nabla p$ marks the beginning of the precursor phase. During this phase $E_r/\nabla p$ quickly decrease (fig. 5), thereby leading to the explosive ELM onset (fig. 2) until it abruptly ends with the ELM crash at $\sim 32.8$ ms. The changes to the axisymmetric background during the precursor phase triggers PB modes to grow explosively and couple between one-another while at the same time making the ergodic region penetrate further inwards, as evidenced by the change from fig. 4(b) to (c). The ELM crash phase features losses due to the increasingly ergodic magnetic topology and from convective transport occurring in sub-millisecond timescales directly comparable to experimental observations \cite{2}. Finally, the recovery phase takes place once the ELM crash is concluded and during this phase $E_R/\nabla p$ returns to the pre-ELM state. Even though $E_r/\nabla p$ recovers in a sub-millisecond timescale after the crash, $E_r$ and $\nabla p$ individually require roughly 7 ms to return to the pre-ELM state.

5 Discussion and conclusions

We present, for the first time, simulations of realistic type-I ELM cycles in diverted tokamak geometry. Important differences in the modelled ELM crash dynamics (notably their size and duration) are observed with different initial seed perturbations. The first simulated ELM, with arbitrary seed perturbations, results in a longer ELM crash with more energy lost when compared to the subsequent ELM crashes with self-consistent seed perturbations. Since the seed perturbation depend on the non-linear dynamics of the previous ELM, we conclude that in order to use the present numerical tools to predictively assess the consequences of natural ELMs, or the applicability of existing ELM mitigation and suppression techniques to future tokamaks, it is necessary to model full ELM cycles.

From the simulation results we identify a non-linear electromagnetic triggering mechanism for the explosive ELM onset. During the precursor phase, an increasingly stochastic magnetic topology causes a decrease of $\nabla p$ and $j$ with an even faster slowing down of the plasma flows. Consequently, the stabilising effect of the plasma flows is rapidly lost and prompts an explosive ELM onset.

Given that a single fluid temperature was considered, the parallel heat transport resulting from the stochastic magnetic topology does not account for the separation in electron and ion timescales. We expect only the precursor phase duration to be modified as a result. Additionally, in experiments the inter-ELM evolution shows separate timescales between $T_e$ and $T_i$, which affects the diamagnetic contribution to $E_r$ \cite{33}. Therefore, separating the electron and ion temperature evolution is envisioned for future work. The diffusive transport of particles, and the ion and electron heat flux in the experiment is not determined by static diffusion coefficients like we have assumed here for simplicity. Future investigations into more accurate pedestal evolution are also of interest as they may shed light onto other inter-ELM modes and high mode number precursors.

Simulations with higher toroidal harmonics (all even modes until $n = 20$), or with the entire toroidal mode spectrum ($n = 0, 1, 2, ..., 12$), feature the same ranges of dominant toroidal mode numbers and same triggering mechanism with explosive onset, albeit with shorter precursor phases. Nonetheless, the observed non-linear triggering mechanism is robust to changes in the chosen toroidal mode numbers and to variations of the imposed inter-ELM evolution, i.e.
changes in heating power. In general, the simulated ELM crashes and precursors show characteristics that are qualitatively consistent with observed ranges of toroidal mode numbers, ELM sizes and duration, and divertor heat loads, to name a few. Finally, the ELM repetition frequency of the simulated ELMs shows a direct dependency to the applied heating power, as expected for type-I ELMs.

Acknowledgements

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training program 2014-2018 and 2019-2020 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission. This work used the MARCONI computer at CINECA under projects AUGJOR and ELM-UK.

References

[1] Fritz Wagner, G Becker, K Behringer, D Campbell, A Eberhagen, W Engelhardt, G Fussmann, O Gehre, J Gernhardt, G v Gierke, et al. Regime of improved confinement and high beta in neutral-beam-heated divertor discharges of the asdex tokamak. Physical Review Letters, 49(19):1408, 1982.

[2] EDA ITER et al. Mhd stability, operational limits and disruptions. Nuclear Fusion, 39(12 ITER physics basis):2251–2389, 1999.

[3] Hartmut Zohm. Edge localized modes (elms). Plasma Physics and Controlled Fusion, 38(2):105, 1996.

[4] EJ Doyle, RJ Groebner, KH Burrell, P Gohil, T Lehecka, NC Luhmann Jr, H Matsumoto, TH Osborne, WA Peebles, and R Philipona. Modifications in turbulence and edge electric fields at the l–h transition in the diii-d tokamak. Physics of Fluids B: Plasma Physics, 3(8):2300–2307, 1991.

[5] GTA Huysmans. Elms: Mhd instabilities at the transport barrier. Plasma Physics and Controlled Fusion, 47(12B):B165, 2005.

[6] Andrew Kirk, B Koch, Rory Scannell, HR Wilson, G Counsell, J Dowling, A Herrmann, RMWM Martin, M Walsh, et al. Evolution of filament structures during edge-localized modes in the mast tokamak. Physical review letters, 96(18):185001, 2006.

[7] T Eich, B Siegl, AJ Thornton, M Faitisch, A Kirk, A Herrmann, W Suttrop, et al. Elm divertor peak energy fluence scaling to iter with data from jet, mast and asdex upgrade. Nuclear Materials and Energy, 12:84–90, 2017.

[8] Anthony W Leonard. Edge-localized-modes in tokamaks. Physics of Plasmas, 21(9):090501, 2014.

[9] HR Wilson and SC Cowley. Theory for explosive ideal magnetohydrodynamic instabilities in plasmas. Physical review letters, 92(17):175006, 2004.

[10] GTA Huysmans, CS Chang, N Ferraro, L Sugiyama, Francois Waebroeck, XQ Xu, Alberto Loarte, and Shimpei Futatani. Modelling of edge localised modes and edge localised mode control. Physics of Plasmas, 22(2):021805, 2015.

[11] LE Sugiyama and HR Straus. Magnetic x-points, edge localized modes, and stochasticity. Physics of Plasmas, 17(6):062505, 2010.

[12] LE Sugiyama. Intrinsic stochasticity in fusion plasmas. Physica Scripta, 86(5):058205, 2012.

[13] NM Ferraro, Stephen C Jardin, and PB Snyder. Ideal and resistive edge stability calculations with m 3 d-c 1. Physics of Plasmas, 17(10):102508, 2010.

[14] P. B. Snyder, H. R. Wilson, and X. Q. Xu. Progress in the peeling-ballooning model of edge localized modes: Numerical studies of nonlinear dynamics. Physics of Plasmas, 12(5):056115, 2005.

[15] XQ Xu, BD Dudson, PB Snyder, MV Umansky, HR Wilson, and T Casper. Nonlinear elm simulations based on a nonideal peeling–ballooning model using the bout++ code. Nuclear Fusion, 51(10):103040, 2011.

[16] PW Xi, XQ Xu, and PH Diamond. Phase dynamics criterion for fast relaxation of high-confinement-mode plasmas. Physical Review Letters, 112(8):085001, 2014.

[17] GS Xu, QQ Yang, N Yan, YF Wang, XQ Xu, HY Guo, R Maingi, L Wang, JP Qian, XZ Gong, et al. Promising high-confinement regime for steady-state fusion. Physical review letters, 122(25):255001, 2019.

[18] JR King, AY Pankin, SE Kruger, and PB Snyder. The impact of collisionality, flr, and parallel closure effects on instabilities in the tokamak pedestal: Numerical studies with the nimrod code. Physics of Plasmas, 23(6):062123, 2016.
[19] GTA Huysmans, Stanislas Pamela, Emiel Van Der Plas, and Pierre Ramet. Nonlinear mhd simulations of edge localized modes (elms). *Plasma Physics and Controlled Fusion*, 51(12):124012, 2009.

[20] GTA Huysmans and A Loarte. Non-linear mhd simulation of elm energy deposition. *Nuclear Fusion*, 53(12):123023, 2013.

[21] F Orain, M Bécoulet, G Dif-Pradalier, G Huysmans, S Pamela, E Nardon, C Passeron, G Latu, V Grandgirard, A Fil, et al. Non-linear magnetohydrodynamic modeling of plasma response to resonant magnetic perturbations. *Physics of Plasmas*, 20(10):102510, 2013.

[22] M Hoelzl, GTA Huysmans, F Orain, FJ Artola, S Pamela, M Becoulet, D van Vugt, F Liu, S Futatani, A Lessig, et al. Insights into type-i edge localized modes and edge localized mode control from jorek non-linear magnetohydrodynamic simulations. *Contributions to Plasma Physics*, 58(6-8):518–528, 2018.

[23] S Futatani, G Huysmans, A Loarte, LR Baylor, N Commaux, TC Jernigan, ME Fenstermacher, C Lasnier, TH Osborne, and B Pegourié. Non-linear mhd modelling of elm triggering by pellet injection in diii-d and implications for iter. *Nuclear Fusion*, 54(7):073008, 2014.

[24] Marina Bécoulet, François Orain, GTA Huysmans, Stanislas Pamela, P Cahyna, Matthias Hoelzl, Xavier Garbet, Emmanuel Franck, Eric Sonnendrücker, Guillem Dif-Pradalier, et al. Mechanism of edge localized mode mitigation by resonant magnetic perturbations. *Physical review letters*, 113(11):115001, 2014.

[25] M Bécoulet, M Kim, G Yun, S Pamela, J Morales, Xavier Garbet, GTA Huysmans, C Passeron, O Février, M Hoelzl, et al. Non-linear mhd modelling of edge localized modes dynamics in kstar. *Nuclear Fusion*, 57(11):116059, 2017.

[26] S Pamela, T Eich, Lorenzo Frassinetti, B Sieglin, S Saarela, M Huysmans, M Hoelzl, M Becoulet, F Orain, S Devaux, et al. Non-linear mhd simulations of elms in jet and quantitative comparisons to experiments. *Plasma Physics and Controlled Fusion*, 58(1):014026, 2015.

[27] JW Connor, RJ Hastie, HR Wilson, and RL Miller. Magnetohydrodynamic stability of tokamak edge plasmas. *Physics of Plasmas*, 5(7):2687–2700, 1998.

[28] PB Snyder, HR Wilson, JR Ferron, LL Lao, AW Leonard, TH Osborne, AD Turnbull, D Mossessian, M Murakami, and XQ Xu. Edge localized modes and the pedestal: A model based on coupled peeling–ballooning modes. *Physics of Plasmas*, 9(5):2037–2043, 2002.

[29] AH Glasser, JM Greene, and JL Johnson. Resistive instabilities in general toroidal plasma configurations. *The Physics of Fluids*, 18(7):875–888, 1975.

[30] JF Drake, TM Antonsen Jr, AB Hassam, and NT Gladd. Stabilization of the tearing mode in high-temperature plasma. *The Physics of fluids*, 26(9):2509–2528, 1983.

[31] PH Diamond, PL Similon, TC Hender, and BA Carreras. Kinetic theory of resistive ballooning modes. *The Physics of fluids*, 28(4):1116–1125, 1985.

[32] BN Rogers and JF Drake. Diamagnetic stabilization of ideal ballooning modes in the edge pedestal. *Physics of Plasmas*, 6(7):2797–2801, 1999.

[33] RJ Hastie, Peter J Catto, and JJ Ramos. Effect of strong radial variation of the ion diamagnetic frequency on internal ballooning modes. *Physics of Plasmas*, 7(11):4561–4566, 2000.

[34] GTA Huysmans, SE Sharapov, AB Mikhailovskii, and W Kerner. Modeling of diamagnetic stabilization of ideal magnetohydrodynamic instabilities associated with the transport barrier. *Physics of Plasmas*, 8(10):4292–4305, 2001.

[35] M Velli and AW Hood. Resistive ballooning modes in line-tied coronal fields. *Solar physics*, 106(2):353–364, 1986.

[36] M Swisdak, M Opher, JF Drake, and F Alouani Bibi. The vector direction of the interstellar magnetic field outside the heliosphere. *The Astrophysical Journal*, 710(2):1769, 2010.

[37] W Fundamenski, Volker Naulin, T Neukirch, Odd Erik Garcia, and J Juul Rasmussen. On the relationship between elm filaments and solar flares. *Plasma Physics and Controlled Fusion*, 49(5):R43, 2007.

[38] M Cavedon, T Pütterich, Eleonora Viezzer, FM Laggner, A Burckhart, M Dunne, R Fischer, A Lebschy, F Mink, U Stroth, et al. Pedestal and r profile evolution during an edge localized mode cycle at asdex upgrade. *Plasma Physics and Controlled Fusion*, 59(10):105007, 2017.
[39] GTA Huysmans and O Czarny. Mhd stability in x-point geometry: simulation of elms. Nuclear fusion, 47(7):659, 2007.

[40] Olivier Czarny and Guido Huysmans. Bézier surfaces and finite elements for mhd simulations. Journal of computational physics, 227(16):7423–7445, 2008.

[41] HR Strauss. Reduced mhd in nearly potential magnetic fields. Journal of Plasma Physics, 57(1):83–87, 1997.

[42] Emmanuel Franck, Matthias Hölzl, Alexander Lessig, and Eric Sonnendrücker. Energy conservation and numerical stability for the reduced mhd models of the non-linear jorek code. arXiv preprint arXiv:1408.2099, 2014.

[43] F Orain, Marina Becoulet, GTA Huijsmans, G Dif-Pradalier, M Hoelzl, J Morales, X Garbet, E Nardon, S Pamela, C Passeron, et al. Resistive reduced mhd modeling of multi-edge-localized-mode cycles in tokamak x-point plasmas. Physical review letters, 114(3):035001, 2015.

[44] JA Morales, Marina Becoulet, X Garbet, F Orain, G Dif-Pradalier, M Hoelzl, S Pamela, GTA Huijsmans, P Cahyna, A Fil, et al. Edge localized mode rotation and the nonlinear dynamics of filaments. Physics of Plasmas, 23(4):042513, 2016.

[45] PJ Mc Carthy. Analytical solutions to the grad–shafranov equation for tokamak equilibrium with dissimilar source functions. Physics of Plasmas, 6(9):3554–3560, 1999.

[46] Olivier Sauter, Clemente Angioni, and YR Lin-Liu. Neoclassical conductivity and bootstrap current formulas for general axisymmetric equilibria and arbitrary collisionality regime. Physics of Plasmas, 6(7):2834–2839, 1999.

[47] Olivier Sauter and Clemente Angioni. Erratum: neoclassical conductivity and bootstrap current formulas for general axisymmetric equilibria and arbitrary collisionality regime [phys. plasmas 6, 2834 (1999)]. Physics of Plasmas, 9(12):5140–2839, 2002.

[48] Isabel Krebs, Matthias Hoelzl, Karl Lackner, and Sibylle Günter. Nonlinear excitation of low-n harmonics in reduced magnetohydrodynamic simulations of edge-localized modes. Physics of plasmas, 20(8):082506, 2013.

[49] James D Callen, Chris C Hegna, Bradley W Rice, Edward J Strait, and Alan D Turnbull. Growth of ideal magnetohydrodynamic modes driven slowly through their instability threshold: Application to disruption precursors. Physics of Plasmas, 6(8):2963–2967, 1999.

[50] T Kass, S Günter, M Maraschek, W Suttrop, H Zohm, and ASDEX Upgrade Team. Characteristics of type i and type iii elm precursors in asdex upgrade. Nuclear fusion, 38(1):111, 1998.

[51] Felician Mink, Elisabeth Wolfrum, Marc Maraschek, Hartmut Zohm, László Horváth, Florian M Laggner, Peter Manz, Eleonora Viezzer, Ulrich Stroth, et al. Toroidal mode number determination of elm associated phenomena on asdex upgrade. Plasma Physics and Controlled Fusion, 58(12):125013, 2016.

[52] AF Mink, M Hoelzl, E Wolfrum, F Orain, M Dunne, A Lessig, S Pamela, P Manz, M Maraschek, GTA Huijsmans, et al. Nonlinear coupling induced toroidal structure of edge localized modes. Nuclear Fusion, 58(2):026011, 2017.

[53] N Oyama, K Shinohara, Y Kamada, Y Miura, T Oikawa, and S Takeji. Collapse of density pedestal by giant elm on jet-60u. Plasma physics and controlled fusion, 43(5):717, 2001.

[54] CP Perez, HR Koslowski, GTA Huysmans, TC Hender, P Smeylulders, B Alper, E De La Luna, RJ Hastie, L Meneses, MFF Nave, et al. Type-i elm precursor modes in jet. Nuclear fusion, 44(5):609, 2004.

[55] R Maingi, CE Bush, ED Fredrickson, DA Gates, SM Kaye, BP LeBlanc, JE Menard, H Meyer, D Mueller, N Nishino, et al. H-mode pedestal and power threshold studies in nstx. Nuclear Fusion, 45(9):1066, 2005.

[56] A Kirk, D Dunai, M Dunne, G Huijsmans, S Pamela, M Becoulet, JR Harrison, J Hillesheim, C Roach, and S Saarelma. Recent progress in understanding the processes underlying the triggering of and energy loss associated with type i elms. Nuclear Fusion, 54(11):114012, 2014.

[57] Th Eich, A Herrmann, P Andrew, A Loarte, et al. Power deposition measurements in deuterium and helium discharges in jet mkiid divertor by ir-thermography. Journal of nuclear materials, 313:919–924, 2003.

[58] JE Lee, GS Yun, M Kim, Jisan Lee, Woochang Lee, Hyeon Keo Park, Calvin W Domier, NC Luhmann Jr, WH Ko, et al. Toroidal mode number transition of the edge localized modes in the kstar plasmas. Nuclear Fusion, 55(11):113035, 2015.
[59] GL Jackson, J Winter, TS Taylor, KH Burrell, JC DeBoo, CM Greenfield, RJ Groebner, T Hodapp, K Holtrop, EA Lazarus, et al. Regime of very high confinement in the boronized diii-d tokamak. Physical review letters, 67(22):3098, 1991.

[60] MFF Nave, P Smeulders, TC Hender, PJ Lomas, B Alper, P Bak, B Balet, JP Christiansen, S Clement, HPL De Esch, et al. An overview of mhd activity at the termination of jet hot ion h modes. Nuclear fusion, 37(6):809, 1997.

[61] AV Chankin, N Asakura, T Fukuda, A Isayama, K Itami, Y Kamada, H Kubo, Y Miura, T Nakano, N Oyama, et al. Influence of plasma–wall interactions on the behaviour of elms in jt-60u. Journal of nuclear materials, 313:828–833, 2003.

[62] GTA Huysmans, TC Hender, and B Alper. Identification of external kink modes in jet. Nuclear fusion, 38(2):179, 1998.

[63] John Wesson. Tokamaks; 4th ed. International series of monographs on physics. Oxford Univ. Press, Oxford, 2011.