Optimization of series-parallel connection of PV array to mitigate negative influence of partial shading conditions

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Abstract. Rapid development in the photovoltaic systems demanding more and more efficient solutions, not only in micro-fabrication techniques but in energy extraction methods as well. In recent decades large number of Maximum Power Point Tracking algorithms with different complexity accompanied by the capability to efficiently locate global maximum under partial shading were developed. However, a deeper look at the problem reveals that to further enhance the performance of the PV system under non-uniform irradiance there is a need for hardware intervention to the interconnect, or in other words adjustments to the panel configuration. In this paper, we propose a simple technique for optimizing the electrical scheme of the solar panel (connection of individual cells) by using a simple algorithm which disperses shadow across all structure. The method allows to mitigate partial shading effect on solar panels of any size and shape, therefore rise efficiency of energy extraction. Thus static optimization of the PV panel configuration disperses partial shading across all structure and ensures more synchronous functioning of the PV cells. The resulting structure not only capable of extracting more available power but, straighten ups power curve to loosen requirements for MPPT computations.

1. Introduction

Ever-increasing energy demand of the world needs to be fulfilled with renewable sources to save valuable resources of the planet. Solar energy is one of the means of achieving it, which posses distinctive advantages over competing alternatives: low maintenance cost, long lifetime, simplicity of manufacturing as well as the absence of moving parts. However, several challenges are preventing PV systems to be the energy of the future. One of the crucial factor which diminish output power of PV array is partial shading [1-6]. Dust collected on panel, bird droppings, passing clouds, and other standing objects create a barrier through which sunlight cannot pass. It was observed that under thus circumstances the performance of the PV system diminishes considerably and harvesting power becomes very difficult. Furthermore, shaded cells result in the creation of hot spots that deteriorate panel and reduces its lifetime. PV panel manufacturers often use bypass diodes to combat this issue. However, a completely efficient solution that minimizes the effect of partial shading is yet to come. Photovoltaic plants for industrial use are located in places where environmental conditions are ideal for harvesting sunlight. Nonetheless, panels for private use could be placed in areas where partial shading is unavoidable. Therefore, the issue of non-uniform solar irradiance is one of the major concerns in solar industry. One of the trick to pass around the destructive effect of partial shading is
to optimize the electrical interconnect of the PV array. In this paper, we propose a method to construct a partial shading resilient series-parallel configuration of PV array.

1.1. PV system decomposition
The theoretical limit of extracting energy from the PV cell is primarily set by material property (bandgap of semiconductor) [7]. However, in the PV system, not all that energy could be used due to non-ideality in power extraction techniques. The efficiency of the solar panel is further reduced under partial shading conditions. Orthogonalization of PV system into distinct modules, reveal several optimization options to boost performance under non-uniform solar irradiance [2]:
1. Development of MPPT algorithms able to locate the global maximum.
2. Modification of the PV array electrical scheme.
3. Alteration in the PV system architecture.
4. Reconfiguration of converter topology.

In the past decades, MPPT received more and more attention, and numerous algorithms with different complexity able to track global maximum under partial shading conditions were developed [8-10]. Recently, in literature, a rising interest in the optimization of the electrical interconnect of the PV array could be observed. The most common methods of schematic reconfiguration are presented below:
1. Static optimization of the PV array scheme – Serial Parallel (SP), Series (S), Total Cross Tied (TCT) configurations have different resilience toward partial shading [11-13]. Depending on the geometry of the shade, their performance varies. However, such configurations have little to no resistance to partial shading. Later, studies show that Sudoku [14-15] or Magic square formation [16] hold higher resilience to common patterns of shade.
2. Dynamic optimization of the PV array scheme – is the method of extracting maximum power based on the reconfiguration of the electrical scheme using switches [17-19]. Such a method allows to adaptively alter the configuration of the PV array based on the solar irradiance, therefore, to reduce mismatch effects actively. Among listed hardware solutions, the dynamic method offers a higher energy yield but requires complex hardware intervention and advanced control. Thus modifications would result in higher unit price as well as a reduced lifetime of the device due to a reduction in reliability.

![Figure 1](image.png)

In this paper, we propose a method of reconfiguring the electrical interconnect of the SP PV array (figure 1). By the offered technique, an array of any size and shape could be restructured to make it more resilient to partial shading (by static optimization). The rearranged structure relaxes requirements for the MPPT algorithm as it smooths the power curve preventing generation of numerous local maximums. Other modules of the PV system will not be touched. However, the derived optimization procedure for interconnect is independent of other modules, and resulting PV arrays could be integrated into any PV system.
1.2. Model of a PV cell

The starting point of our analysis is a basic building block of the PV system - PV cell. Several mathematical models for understanding the electrical behavior of PV cells exist. In this paper, we focused on the widely accepted PV model as shown in figure 2. More complex models [20] considering recombination current and correction factors to shunt resistance, as well as a model constructed by artificial neural network [21] were also studied and could be used for the simulation with minor changes. However, for the optimization of the electrical interconnect, it would be more important to see possible outcomes of the partial shading to the PV system and pick algorithm accordingly rather than putting efforts on determining the most accurate model. Therefore, the focus was kept on a simpler model. The performance of the PV cell was simulated on MATLAB and results of single diode solutions could be seen from figure 3.

\[ I = I_L - I_d - I_p \]  

(1)

where \( I \) – output current of the cell (A); \( I_L \) – current generated by solar irradiance (A); \( I_d \) – current passing through the diode (A); \( I_p \) – current dissipated in shunt resistor (A).

Rewriting equation (1) would give:

\[ I = I_L + I_0 \left( e^{\frac{q(V+IR_s)}{n k T}} - 1 \right) - \frac{V + IR_s}{R_{sh}} \]  

(2)

where \( q = 1.602 \cdot 10^{-19} \text{C} \) (electronic charge); \( k = 1.380 \cdot 10^{-23} \text{J/K} \) (Boltzmann constant); \( n \) – is ideality factor; \( I_0 \) – is diode reverse saturation current (A); \( R_{sh} \) – shunt resistance (\( \Omega \)); \( R_s \) – series resistance (\( \Omega \)).

Current generated by the solar irradiance was taken as shown in [22].

\[ I_L = \frac{G}{G_{ref}} \left[ I_{L,ref} + \mu_{I,sc} (T_c - T_{c,ref}) \right] \]  

(3)

where \( G \) – solar irradiance (W/m²); \( T \) - temperature (°C); \( \mu_{I,sc} \) – temperature coefficient of short circuit current (A).

If the reference parameters of the solar panel are not known, the simulation could be performed by parameter estimation methods based on the data from the PV manufacturers (models constructed empirically) [20-24]. However, it is not always possible to do so as not all manufacturers provide the necessary information. Therefore, single diode computation was performed on imaginary PV cell table 1. Simulation results for various temperature and irradiance ranges are presented in figure 3.

| Parameters | Values (explanation) |
|------------|----------------------|
| \( I_{L,ref} \) | 7.53 A |
| \( I_{0,ref} \) | 82.689 nA |
| \( E_{g,ref} \) | 1.121 eV (*Band gap of semiconductor*) |
| \( R_s,ref \) | 0.094 \( \Omega \) |
| \( R_{sh,ref} \) | 15.72 \( \Omega \) |
| \( \mu_{I,sc} \) | 3 mA/°C |
From a single diode numerical computations, the influence of environmental factors on the PV cell could be observed. This dependence changes over time due to the degradation of materials, therefore, altering the property. However, data is key in the further analysis as operation range under different temperatures and solar irradiance are very important information of efficient array structure. From figure 3 it could be already observed how non-uniform temperature or irradiance might disrupt synchronous functioning of the PV array. Furthermore, from Figure 3c significance of bypass diodes becomes obvious, as shaded cells will block all current in the string [25].

![I-V and P-V characteristics of PV cell at various temperature and irradiance ranges: (a) I-V under different temperatures (1000 W/m²); (b) P-V under different temperatures (1000 W/m²); (c) I-V under different solar irradiance (20 °C); (d) P-V under different solar irradiance (20 °C).](image)

**Figure 3.** I-V and P-V characteristics of PV cell at various temperature and irradiance ranges: (a) I-V under different temperatures (1000 W/m²); (b) P-V under different temperatures (1000 W/m²); (c) I-V under different solar irradiance (20 °C); (d) P-V under different solar irradiance (20 °C).

### 1.3. PV array

In the previous section, electrical response of the PV cell to the changes in irradiance as well as to the temperature was presented. As a next step, the electrical performance of the PV array at a series-parallel (SP) configuration was simulated. Computation was conducted on (20×20) PV arrangement (figure 4). In other words, 20 elements connected in series were connected in parallel to the other 20 modules of the same size. Solar irradiance was inputted separately to each cell. In principle performance of an array of any dimension could be represented by the following procedure.
Sum of voltage drop across the cells connected in series is equal to the output voltage (Kirchhoff’s voltage law):

\[ V_a = \sum_{i=1}^{n} V^i \]  

where \( V_a \) – output voltage of the PV array (V); \( V^i \) – voltage drop across the functioning cell (V).

According to the Kirchhoff’s current law, current at the output is sum of all currents generated in each series connection (module):

\[ I_a = \sum_{i=1}^{n} I^i_s \]  

where \( I_a \) – output current of the PV array (A); \( I^i_s \) – current passing through the cells connected in series (A).

Using the data from the PV cell model and applying it to the scheme in Figure 4 several test scenarios of partial shading were simulated figure 5. The results are presented in figures 6-8. Depending on the shape of the shade, P-V curves change dramatically.

![Figure 4. SP electrical scheme.](image)

![Figure 5. Example irradiances inputed to the PV array.](image)
Simulation Assumptions:
1. Voltage drop across the functioning cells connected in series are same [26].
2. Temperature across the PV array is uniform. (Temperature of each cell could be specified as well.
However, to keep it simpler, it was decided to keep uniform temperature (20 °C) distribution across
the panel.)

Partial shading of the PV panel regions prevents to harvest all the available power. Such an event is taking
place because of shaded modules (strings of cells connected in series) that go out from the operation range
earlier. Thus mismatched functioning could be seen from figure 6. The SP configuration is not able to extract
all the power produced by the PV array.
2. Optimization and simulations

2.1. Problem formulation
The problem was simplified for simulation so that each PV cell represents an element in a matrix. A procedure that repositions elements (permanently) to minimize mismatch effects, needs to be derived (to prevent the early reaching of open-circuit voltage due to shaded cells). In the SP configuration, to mitigate the negative effect of shaded cells, the array needs to be dispersed in a way that each module (a string of series-connected cells) contain an equal number of shaded elements. To achieve mismatch reduction cells were relocated to the furthest distance relative to original nearest neighbors. In other words, the distance between original neighbors is the controlling parameter, which needs to be maximized for all elements in a uniform manner. This would ensure spread of all elements across the panel surface and any hypothetical shaded region will include elements from all the matrix (maximum diversity). Bluntly searching for a whole state space would require an analysis of \((n \times n)!\) combinations which is not feasible and will require computational time more than the age of the universe [27]. The combinatorics problem formulated in this paper although with some differences resembles the maximum diversity problem. State of the art algorithms for solving it includes greedy randomized adaptive search procedure (GRASP) [28], as well as the implementation of memetic and tabu search algorithm [29], etc. Any of the mentioned heuristics methods cannot be directly applied as nearly all problems need some sort of customization of the algorithm [30]. We attempted to find close to the optimal solution by setting manageable constraints and designing a very simple hybrid evolutionary algorithm (EA) with the ability of local search to rearrange shade across all circuitry. Such a class of EA often called memetic algorithms [30]. The choice fell on the memetic algorithm due to its straightforward structure, yet powerful local search ability.

For the simplicity of simulation and to not deviate from previous sections all the computations were performed on \(n \times n\) \((n = 20)\) matrix. Number of elements in the matrix is \(N = n^2 = 400\). For each element \(i\) (from 1 to \(N\)) there are set of neighbors located \((d)\) distance away.

Figure 8. Estimated P–V curve for the irradiance cases shown in figure 5 before rearrangements.
\[ A^d = \{a_{ij}\}, \quad 1 \leq d \leq (n-1), \quad 1 \leq i \leq N, \quad n, N \in \mathbb{Z} \]  

where \( d \) (Chebychev distance) could be found from standard (Euclidean) coordinates by the equation (7) and \( j \) is size of the neighbor set.

\[ d(x, y) := \max_i (|x_i - y_i|). \quad (7) \]

For instance, \( A^1_i = \{a^S_{i,1}, a^N_{i,2}, a^E_{i,3}, a^W_{i,4}, a^{SE}_{i,5}, a^{SW}_{i,6}, a^{NE}_{i,7}, a^{NW}_{i,8}\} \) (figure 9a) is set of nearest neighbors or elements located one Chebychev distance away in the original matrix. Chebychev space was used for the convenience, while other coordinate systems are suitable as well. If the shape of the PV cell is not square (hexagonal, triangular ... etc.) or if there is a non PV element in the panel, sets would change but optimization algorithm should still be able to solve it.

For each element expected distance between its nearest neighbors is described by equation (8)

\[ \mu_i = \sum_{j=1}^{8} \frac{1}{8} d_{i,j}, \quad (8) \]

While mean distance between original neighbors for whole matrix is:

\[ \mu = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{8} \frac{1}{8} d_{i,j} = \frac{1}{N} \sum_{i=1}^{N} \mu_i \quad (9) \]

Our goal is to maximize the equation (9) keeping uniformity of equation (8). Condition for uniform spread of matrix elements is dictated by spread of distance (standard deviation) which needs to be minimized:

\[ \sigma = \sqrt{\frac{\sum_{i=1}^{N} (\mu_i - \mu)^2}{N}} \quad (10) \]

Furthermore, additional condition for uniformity is minimum distance below which no neighbor should be located:

\[ \{b^{(1,2,...m)}_i \mid b \notin A^1_i\}, \quad 1 \leq i \leq N, \quad m, N \in \mathbb{Z} \quad (11) \]

In other words, to keep uniform dispersion of elements from each other we set a minimum distance \( m \) and make sure that no neighbors are located closer than the value set. In order for search to be successful more elements need to satisfy the condition in equation (11) and ensure decrease in spread as well as increase in mean distance equation (12).

\[ \sigma' \leq \sigma \quad \mu' \geq \mu \quad (12) \]

Here \( \sigma' \) and \( \mu' \) is solution offered at each iteration of the algorithm. To avoid boundary conditions of non equal number of neighbors computation were performed on toroid (figure 9) and results were transformed back to original coordinates.
2.2. Perturbation schedule

The problem formulation Equation (8-9) resembles maximum diversity problem. However, the difference and the major challenge which arises is each time element is relocated it has an effect on all the matrix, reshaping the whole structure creating an immense amount of possible combinations. To tackle this issue we attempted to narrow down the search by segmenting perturbation schedule. Sets described equation (6) mimics orbits of each element and different repelling strategies are used in each stage of perturbation (table 2).

Table 2. Algorithm structure.

| Algorithm | 1: Generate Initial solution $X$ |
|-----------|----------------------------------|
|           | 2: Generate a neighbor solution around $X$ with perturbation schedule. Perturbation rules: |
|           | **Stage 1:** |
|           | Perturb according to Equation (13-15). |
|           | **Stage 2:** |
|           | Perturb according to Equation (16-18). |
|           | 3: Evaluate: If the neighbor solution is better - update $X$. |
|           | 4: Return to step 2 until termination criteria or maximum number of iterations is reached. |
|           | 5: End |

**Stage 1:** In this stage, all the matrix elements repeal nearest neighbors in a certain range to the further distance. In other words, all the closest elements in the original matrix which are currently in the range of $m$ are randomly redistributed through ought the matrix $(\forall i \in \{1,2,...N\})$ equation (13-15) is performed randomly selecting $i$ till it is not performed for all elements). Using such a perturbation rule can reshape the solution significantly in a single iteration. The result is accepted only if all conditions set in equation (11-12) are satisfied therefore ensuring the formation of desired final topology. Structures formed in this stage are subject to change and as better solutions are found and more elements are located further than midstance away the formations are less susceptible to change and the perturbation schedule is changed to the 2nd stage.

$$B^{l(1,2,m)}_i = \{b_1, b_2, ..., b_j\} \quad b_j \in A^1_l$$
$$B^{l(d_1)}_i = \{b'_1, b'_2, ..., b'_j\} \quad b'_j \notin A^1_l$$
$$B^{l(1,2,m)}_i = \{B^{l(1,2,m)}_i \setminus \{b_j\}\}, \quad B^{l(d_1)}_i = \{B^{l(d_1)}_i \setminus \{b'_j\}\},$$
$$\text{append} (\{b_j\} \text{ to } B^{l(d_1)}_i), \quad (\{b'_j\} \text{ to } B^{l(1,2,m)}_i)$$

**Stage 2:** As long as no neighbors are left within m distance away search space is narrowed. Perturbation is continued only by selecting elements randomly and shifting nearest neighbors further away. Similar to previous stage solutions are accepted IFF requirements set in equation (11-12) are satisfied. As perturbation rule is applied to a single element and its neighbors at a time, each iteration takes significantly less time in this stage. Furthermore, the matrix cannot be reshaped significantly in single step anymore as search space is significantly narrowed.

$$B^{l(m+1)}_i = \{b_1, b_2, ..., b_j\} \quad b_j \in A^1_l$$
$$B^{l(d_2)}_i = \{b'_1, b'_2, ..., b'_j\} \quad b'_j \notin A^1_l$$
$$B^{l(m+1)}_i = \{B^{l(m+1)}_i \setminus \{b_j\}\}, \quad B^{l(d_2)}_i = \{B^{l(d_2)}_i \setminus \{b'_j\}\},$$
$$\text{append} (\{b_j\} \text{ to } B^{l(d_2)}_i), \quad (\{b'_j\} \text{ to } B^{l(m+1)}_i)$$

The execution of the algorithm was done by the flow given in figure 10. $d_1$ and $d_2$ in equation (13-18)
were chosen as \{5, 6, 7\} and \{m + 2, m + 3\} where \(m \leq \frac{n}{2} - 3\). \(m\) is incremented as soon as all nearest neighbors are shifted to \(m\) distance away. Thus however doesn’t necessarily fix values. We experimented with several \(d_1\) and \(d_2\) values and selected the ones which had desirable convergence speed and could produce enough diversity of solutions. With our setup of \(n = 20\) matrices, the best result received in toroid was shifting all the neighbors to at least 5 Chebychev distance away and achieving \(\mu = 7.64\), \(\sigma = 1.48\). The result of the simulation could be seen from figure 11. The final structure after rearrangements resembles Sudoku arrangement as the probability of neighbors being in the same row, column, and diagonal is minimized. Thus any shaded region would contain elements throughout the whole matrix, due to dispersion of neighbors in a uniform manner.

![Figure 10. Computation is executed according to the flow shown above.](image)

![Figure 11. Simulations results.](image)
3. Results and discussion
For different test case scenarios shown in figure 5, after rearrangements of PV cells confident increase in maximum power was detected. In real operating conditions it is impossible to predict the shape and size of the shadow. Therefore, different shadow patterns were experimented. It has to be noted that it is physically impossible to disperse some of the shadow patterns uniformly across the circuitry due to size limitations. Nevertheless, such a pattern also had to be tested to see the efficiency of the method. The results of simulations are shown in figures 12-13. In all cases, more synchronous functioning of the PV array was observed, and I-V, P-V curves had fewer peaks compared to the regular SP interconnect. Numerical results are shown in table 3.

![Figure 12. Estimated I–V curve for the irradiance cases shown in figure 5 after rearrangements.](image)

![Figure 13. Estimated P – V curve for the irradiance cases shown in figure 5 after rearrangements.](image)
Table 3. Maximum Power results for irradiance cases shown in Figure 5 after rearrangements.

| Case | Original [kW] | After rearrangement [kW] |
|------|---------------|--------------------------|
| 1.   | 22.74         | 22.74                    |
| 2.   | 19.33         | 20.57                    |
| 3.   | 13.21         | 16.01                    |
| 4.   | 12.95         | 14.49                    |
| 5.   | 16.48         | 19.82                    |
| 6.   | 14.11         | 17.77                    |
| 7.   | 12.05         | 14.80                    |
| 8.   | 7.27          | 9.24                     |
| 9.   | 13.75         | 17.45                    |
| 10.  | 17.13         | 18.04                    |
| 11.  | 15.85         | 17.23                    |
| 12.  | 9.08          | 12.17                    |
| 13.  | 15.42         | 15.46                    |
| 14.  | 12.83         | 14.96                    |
| 15.  | 11.37         | 14.14                    |
| 16.  | 11.37         | 13.12                    |

Adjusting positions of PV cells come with a substantial cost of complex wiring patterns. The issue could be partially compensated by the introduction of central connection unit circuitry to configure all the cells respectively. However, the same challenge arises for all the dynamic electrical configuration optimization techniques with an additional reduction in reliability [25] due to the installation of switches. Therefore, selecting an optimization technique should be done according to the priorities (cost, efficiency, fabrication complexity, etc.) set for the specific application.

4. Conclusion

In this paper optimization of an SP electrical configuration of PV cells was discussed. The developed method could be applied to a PV array of any shape and dimensions as there is no need for symmetry or size restrictions. Furthermore, if there are non-PV elements present in the panel method can take it into account by specifying initial boundary conditions. Although it is not an exact solution, it was observed that our heuristic approximation for simple shadow shapes increases power yield up to 20%, and up to 10 - 15% for more complex patterns. Numerical computations from a basic building block of PV array until the simulation of a whole panel were performed on MATLAB. Interconnect optimization takes place according to the algorithm which disperses shadow across all circuitry to mitigate the effect of electrical mismatch. The results show a promising increase in power harvesting and more synchronous functioning of PV cells under partial shading. Furthermore, the resulting power curve was simpler, therefore making it easier for the MPPT algorithm to find a global maximum.

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