Abstract Chameleon scalar field is a new model, which introduced to provide a mechanism for exhibiting accelerating universe. Chameleon field has several interesting aspects, such as field dependence on the local matter density. For this model we introduce a new kind of potential which has run away form and satisfies chameleon constraints. The results are acceptable in comparison with the other potentials which studied up to now.

Keywords Chameleon mechanism, New potential, Dark energy, Accelerating universe

1 Introduction

The observation that universe appears to be accelerating at present time has caused one of the greatest problem to modern cosmology. The recent cosmological observation suggests that the universe consist of about 24% cold dark matter and 76% dark energy (DE) [Fuji and Maeda 2003], while DE has a negative pressure, is used to explain the present cosmic acceleration. It is explicit that the nature of DE is unknown for researchers until now, but they can describe it by some candidates. On of that candidates is Cosmological constant ,Λ, but it has two well-known difficulties, the "fine tuning" problem and the "cosmic coincidence" problem [Steinhardt 1997]. Observational data indicates that \( \Omega_\Lambda = 0.763 \) and \( \Omega_m = 0.237 \), so large value of \( \Omega_\Lambda \) obviously predicts that the universe is accelerating today, rather than decelerating as had long been believed. The observation evidence tells us that rate of expansion in the high-z region is slower than that in our neighborhood. In this condition, where as variation of the \( \rho_\Lambda \) with respect to the time is equal to zero, this is provide a problem in cosmology, called fine taming, the quintessence (cosmon-field) solved this problem, by using coupling between scalar field and dark matter [Fuji and Nishioka 1984; Weinberg 1989]. There are several different theories, which have been proposed by people, to interpret the accelerating universe, such as, holographic DE model [Li 2004; Setare 2007a; Karami and Fehri 2010; Saaidi and Aghamohammadi 2011], agegraphic DE models [Cai 2007; Wei and Cai 2008; Karami et al. 2010; Setare 2007b] and scalar field models of DE, which including quintessence field [Nojiri and Odintsoz 2003], quintum field [Elazalde et al. 2004; Feng et al. 2005], phantom field [Caldwell et al. 2003] and many others. While the quantity of cosmological constant is non zero, the DE component is more generally modeled as quintessence mechanism. It is a scalar field rolling down a flat potential. In quintessence mechanism, the field has negative pressure and therefore acts to accelerate expansion [Weinberg 2008]. Khoury and Weltman (2003) have introduced another kind mechanism, which called chameleon mechanism. In this mechanism the scalar field acquire a mass whose magnitude depends on the local matter density [Brax et al. 2004]. Also it is a way to related an effective mass for scalar field \( \phi \). Scalar field is expansion field, and can be obtained from string theory [Orti'n 2004; Green et al. 2004]. Also the chameleon mechanism is a way to give an effective mass to a light scalar field via field self interaction and interaction between field and matter [Khoury and Weltman 2003].
By chameleon model they could detected fifth force that associated with potential energy. We exhibit chameleon behavior for a new kind of potential that have not \( \phi^4 \) form at quintessence model \( \text{[Waterhouse 2004]} \text{Upadhye et al. 2004} \), but has a run away form. Where the consequences of a run away potential for chameleon mechanism can play the role of DE, we select our potential in this category.

The scheme of the present paper is as follows: In sec. 2 we study the preliminary of chameleon mechanism, by using two potentials, as power law and exponential. In sec. 3 we introduce another potential, where has run away form. For this model we obtain several parameters which are useful in cosmology. One of that parameters is the matter density in earliest time. The last section is devoted to conclusion.

2 Preliminary

We consider the general action as

\[
S = \int d^4x \sqrt{-g} \left( \frac{M_{pl}^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right) - \int d^4x \sqrt{-\tilde{g}} \tilde{L}_m(\psi_m, g_{\mu\nu})
\]

where \( \phi \) is the chameleon scalar field and the potential \( V(\phi) \) has run away form and \( M_{pl} = (8\pi G)^{-\frac{1}{2}} = 2.44 \times 10^{18} \text{GeV} \), is the reduced planck mass. Each matter field, \( \psi_m \), coupled to a metric in Jordan frame, is related to the Einstein frame metric by a conformal transformation, \( g_{\mu\nu} = \epsilon^{\frac{2\beta}{M_{pl}}} g_{\mu\nu} \). Here \( \beta \) is the coupling constant without dimension. For the matter that described by pressureless perfect fluid we have \( \tilde{g}^{\mu\nu} \tilde{T}_{\mu\nu} = -\tilde{\rho} \), where

\[
\tilde{T}_{\mu\nu} = -\frac{2}{\sqrt{-\tilde{g}}} \frac{\partial \tilde{L}_m}{\partial g_{\mu\nu}}
\]

and \( \tilde{\rho} \) is the energy density. We need an effective potential to govern the dynamic of the chameleon field. It can be shown that \( \text{[Khoury and Weltman 2004]} \):

\[
\nabla^2 \phi = V_{eff,\phi}(\phi),
\]

where

\[
V_{eff}(\phi) := V(\phi) + \rho e^{\frac{\phi}{M_{pl}}},
\]

It is necessary to be reminded that Khoury and Weltman (2003 and 2004) had a good discussion on chameleon model at two different potentials, Ratra-Peebles and exponential as

\[
V(\phi) = M^{4+n} \phi^n,
\]

and

\[
V(\phi) = M^4 \exp \left( \frac{M^n}{\phi^n} \right),
\]

respectively. Note that, for large value of the field, only power law can tend to zero, see \( \text{[Affleck et al. 1983]} \text{Gusber and Khoury 2004]} \text{Huang et al. 2006} \) For further reviews. Also some researchers such as Brax (2004) and Waterhouse (2006) have considered chameleon model at different aspects for example DE, radion and chameleon cosmology. We are going to introduce another kind of potential which its results are acceptable and also satisfies chameleon condition.

3 The Model

We introduce a new potential such as

\[
V(\phi) = \frac{a + b(q\phi)^n}{1 + (q\phi)^n},
\]

where \( a \) and \( b \) are constants by dimension \( \text{Gev}^4 \), \( q \) is a constant by dimension \( \text{Gev}^{-1} \) and \( n \) is dimensionless real number.

This potential satisfies the constraints which emphasis in \( \text{[Waterhouse 2004]} \), and then the asymptotic behavior of \( V(\phi) \) is as

1. \( \lim_{\phi \to \infty} V(\phi) = b \),
2. \( \lim_{\phi \to 0} V(\phi) = a \),
3. \( V_{\phi}(\phi) \) is increasing and negative,
4. \( V_{\phi\phi}(\phi) \) is decreasing and positive.

The most advantage of this potential is capability of bring experiencing. Because one can obtain the constraints (1)...(4) by different class of \( a, b, q \) and \( n \). This potential has quintessence behavior because when \( \phi \) tend to infinity it converge to \( b \). Here \( b \) is very small and from this point of view, this potential has prefer to exponential potential which is studied in \( \text{[Bine’truy 2004]} \text{Brax et al. 2004} \). The theoretical results got by this potential can be very closed to observation evidence. Whereas this potential has chameleon behavior, so it causes cosmic acceleration. This potential has a run away form, therefore effective potential has a minimum, so that from Eq. (6), we have

\[
V_{eff,\phi}(\phi_{min}) = 0,
\]

therefore using (1) and (6), we have

\[
nq^n \phi_{min}^n (b - a) \left( 1 + (q\phi_{min})^n \right)^2 + \beta e^{\frac{\phi_{min}}{M_{pl}}} = 0,
\]

where the consequences of a run away potential for chameleon mechanism at two different potentials, Ratra-Peebles and exponential as
we can obtain the mass of the small fluctuations, $m_{\text{min}}$ as

$$m_{\text{min}}^2 = V,_{\phi}(\phi_{\text{min}}) + \rho \frac{\beta^2}{M_{\text{pl}}^2} e^{\frac{\beta \phi_{\text{min}}}{M_{\text{pl}}^2}},$$

(8)

so that by substituting [9] in (8) we have

$$m_{\text{min}}^2 = V,_{\phi}(\phi_{\text{min}}) \left( n - 1 + \frac{2n(q\phi_{\text{min}})^n}{1 + (q\phi_{\text{min}})^n} \right) + \rho \frac{\beta^2}{M_{\text{pl}}^2} e^{\frac{\beta \phi_{\text{min}}}{M_{\text{pl}}^2}}.$$  

(9)

Assuming that universe is just composed of dark matter and DE, so that by using the following data [Fuji and Maeda 2003; Waterhouse 2006]

$$\Omega_{\text{matter}} = 0.237, \quad \Omega_{DE} = 1 - \Omega_{\text{matter}} = 0.763,$$

$$\rho_{\text{matter}} = 1.04 \times 10^{-47} \text{GeV}^4, \quad \rho_{DE} = 3.34 \times 10^{-47} \text{GeV}^4,$$

and $\rho = e^{\frac{\beta \phi_{\text{min}}}{M_{\text{pl}}^2}} \rho_*$ in conformal transformation, we can rewrite Eq. (12) as

$$nq \phi_{\text{min}}(b - a) \left( 1 + (q\phi_{\text{min}})^n \right)^2 + \rho_m \frac{\beta^2}{M_{\text{pl}}^2} e^{\frac{\beta \phi_{\text{min}}}{M_{\text{pl}}^2}} = 0.$$  

(10)

By getting the parameters of the Eq. (5) as

$$a = 1.1 \times 10^{-12} \text{GeV}^4, \quad b = 2.0 \times 10^{-48} \text{GeV}^4,$$

$$q = 2.05 \times 10^{20} \text{GeV}^{-1}, \quad n = 0.9, \quad \beta = 9,$$

we obtain $\phi_{\text{min}} = 1.1775 \times 10^{18} \text{GeV}$. We have drown $V(\phi), V_{,\phi}(\phi)$ and $V,_{\phi\phi}(\phi)$, by these constants for more introduction. From figure (1) it is seen that this potential satisfies the constraints of Bine'etruly 2000. Our definition, gives $V(\Phi)$ the dimensions of an energy density, therefore, according to Weinberg 2008; Waterhouse 2006, one can define DE density, as

$$\rho_{DE} = V(\phi_{\text{min}}) = \frac{a + b(q\phi_{\text{min}})^n}{1 + (q\phi_{\text{min}})^n},$$

by making use of $\phi_{\text{min}}$ and other constants $a, b, q$ and $n$, we can obtain density of DE as $\rho_{DE} = 3.34 \times 10^{-47} \text{GeV}^4$. This result exactly is equalled with main quantity which is brought in Waterhouse 2006. Now we want obtain $m_{\text{min}}$ for this model. Assuming the matter is the atmosphere of the earth with $\rho = 4 \times 10^{-21} \text{GeV}^4$, using relation [9] one can compute $m_{\text{min}}$ as

$$m_{\text{min}} = 4.03 \times 10^{-24} \text{GeV}.$$  

We should note that $m_{\text{min}}$ is small fluctuation around minimum. However, we obtain the relation between $V_{,\phi}(\phi)$ and momentum-energy tensor as

$$T^{\mu\nu} = -\frac{2}{\sqrt{-\gamma}} \partial_{m} \gamma^{m \nu}.$$  

(11)

In Jordan frame we have $\bar{T}^{\mu\nu} \bar{\gamma}_{\mu\nu} = 3\bar{p} - \bar{\rho}$, by substituting $\bar{p} = \omega \bar{\rho}$ we have $\bar{T}^{\mu\nu} \bar{\gamma}_{\mu\nu} = (3\omega - 1)\bar{\rho}$. But in Einstein frame we obtain

$$T^{\mu\nu} = \bar{T}^{\mu\nu} e^{\frac{\beta \phi}{M_{\text{pl}}^2}},$$  

(12)

from relation [12] we have $\bar{T}^{00} \bar{\gamma}_{00} = -\bar{\rho}$, this means that $\omega = 0$. Eventually we can obtain

$$\bar{T}^{00} = \bar{\rho} e^{-\frac{\beta \phi}{M_{\text{pl}}^2}},$$  

(13)

by substituting [13] in Eq. (10), we have

$$\bar{T}^{00} = \frac{M_{\text{pl}}}{\beta} \left( \frac{nq \phi_{\text{min}}(a - b)}{(1 + (q\phi_{\text{min}})^n)^2} \right),$$  

(14)

by making use of the value of $\phi_{\text{min}}$ and other relevant constant we obtain $\bar{T}^{00} = 6.5 \times 10^{-48} \text{GeV}^4$ for this model. This is another result which is agree with other works.

Now we want focus on the chameleon behavior in the earlier universe by $\omega = -1$, of course note that in earlier universe, we use

$$V_{\text{eff}}(\phi) = a + b(q\phi)^n + \rho c e^{-\beta q \phi/M_{\text{pl}}^2},$$  

(15)

we define $\Omega_m$ as

$$\Omega_m = \frac{\rho_m}{\rho_c} e^{-\beta q \phi_{\text{min}}/M_{\text{pl}}^2},$$

where $\rho_c = 3H^2 M_{\text{pl}}^2$. Therefore from Eq. (10) we obtain

$$\frac{m_{\text{min}}^2}{H^2} = \frac{3\beta \Omega_m M_{\text{pl}}}{\phi_{\text{min}}} \left( 1 - n + \frac{2n(q\phi_{\text{min}})^n}{1 + (q\phi_{\text{min}})^n} \right) + 3\beta^2 \Omega_m,$$

(16)

for investigating the cosmology behavior we consider two regimes,

1. $\phi_{\text{min}} \geq b^+$
2. $\phi_{\text{min}} \gg b^+$

For $\phi_{\text{min}} \geq b^+$ regime, where $(q\phi_{\text{min}})^n \approx (10^{17})^n$ is very larger than one in denominator, and $M_{\text{pl}} / \phi_{\text{min}} \approx 10^{39}$, so that we can rewrite Eq. (16) as

$$\frac{m_{\text{min}}^2}{H^2} \approx 3\beta \Omega_m (n + 1) \times 10^{39},$$  

(17)
it is well known for $\Omega_m > 10^{-28}$ we have $\frac{m^2_{min}}{H^2} \gg 1$. We can obtain the similar result for the case which we have coupling constant cosmology only. In this case, equation of state is $P = -\rho$ this means that $\omega = -1$, therefore according to Eq.(9) one can obtain the similar result by replacing $4\beta$ instead of $\beta$. So that we have

$$\frac{m^2_{min}}{H^2} = \frac{12\beta \Omega_{vac} M_{pl}}{\phi_{min}} \left( (1 - n) + \frac{2n(q\phi_{min})^n}{1 + (q\phi_{min})^n} \right) + 48\beta^2 \Omega_{vac},$$

(18)

therefore we can obtain $\frac{m^2_{min}}{H^2} \simeq 48\beta^2 \Omega_{vac} \gg 1$, as

$$\Omega_{vac} = \frac{\rho_{vac} e^{\frac{4\beta \phi_{min}}{\rho_c}}}{\rho_c}.$$

For $\phi_{min} \gg b^\frac{1}{4}$ regime, from Eq.(16), we have

$$\frac{m^2_{min}}{H^2} \simeq \frac{3\beta \Omega_m M_{pl}}{\phi_{min}} (n + 1),$$

(19)

By using the definitions of $\Omega_m$, we obtain

$$\frac{m^2_{min}}{H^2} \simeq \frac{3\beta \rho_m e^{\frac{4\phi_{min}}{\rho_c}}}{\rho_c \phi_{min}} (n + 1),$$

(20)

using $\rho_c = 3H_0^2M_{pl}^2$, and Eq.(20), we have

$$\rho_m = \frac{m^2_{min} \phi_{min} M_{pl}}{(n + 1)e^{\frac{4\phi_{min}}{\rho_c}}},$$

(21)

By making use of the value of $m_{min}$ and other introduced constant, we can arrive at

$$\rho_m = 0.4 \times 10^{-16} Gev^4.$$  

This result is an estimate for matter density in earliest universe. Note that in present time $\rho_m \simeq 10^{-47} Gev^4$ therefore our estimate says that the density falls off in proportion to the volume of the universe. Also this condition is for earliest time, and $\phi_{min}$ increase with time, so that the matter density is diluted. We see that by this potential with out complex computation the results are satisfied.

4 Conclusion

In this paper, we introduced a potential which satisfy the chameleon mechanism conditions. We obtain a situation that this potential is become run away form. It is notable that this kind of potential has quintessence behavior, because when $\phi$ tend to infinity it converge to $b$ which is very small and nearly equal to zero. This properties prefer our model to exponential type. By making use of this kind of potential, we have obtained the DE density, minimum mass of scalar field and $\phi_{min}$ as $\rho_{DE} = 3.34 \times 10^{-47} Gev^4$, $m_{min} = 4 \times 10^{-21} Gev^4$ and $\phi_{min} = 1.1775 \times 10^{18} Gev$ respectively. It is seen that these obtained values are in comparison with observational data. Also, we have investigated two regimes of $\rho_m$ on the present and earliest time. We have found $\rho_m = 0.4 \times 10^{-16} Gev^4$ for matter density in early time. Our results are in accordance with other articles (Brax et al. 2004; Khoury and Weltman 2004; Waterhouse 2006).
Fig. 1 (a) is $V(\phi)$, (b) is $V_{,\phi}(\phi)$ and (c) is $V_{,\phi\phi}(\phi)$. In these figures we are use these constant, $a = 1.1 \times 10^{-12} \text{Gev}^4$, $b = 2.0 \times 10^{-18} \text{Gev}^4$, $q = 2.05 \times 10^{20} \text{Gev}^{-1}$, $n = 0.9$ and $\beta = 9.$
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