NEUTRAL DENSE QUARK MATTER

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Abstract
The ground state of dense up and down quark matter under local and global charge neutrality conditions with $\beta$-equilibrium has at least four possibilities: normal, regular 2SC, gapless 2SC phases, and mixed phase composed of 2SC phase and normal components. The discussion is focused on the unusual properties of gapless 2SC phase at zero as well as at finite temperature.

Keywords: Local and global charge neutrality conditions, mixed phase, gapless color superconductivity

1. Introduction

Currently, our knowledge of sufficiently cold and dense matter is very limited. There are neither experimental data nor lattice data in this region. From the BCS theorem, it is speculated that if the matter is dense enough, the ground state of the deconfined quark matter at low temperature will be a color superconductor [1]. Recent studies show that dense quark matter has a rich phase structure, see Ref. [2] for reviews. At asymptotically high bayron densities, this phenomenon can be studied from first principles [3]. If all three colors of the three light quarks participate in the Cooper pairing, the ground state will be in the color-flavor-locking (CFL) phase [4].

In reality, we are more interested in the intermediate density region, where the color superconducting phase may exist in the interior of neutron stars or may be created in heavy ion collisions. Unfortunately, we have little knowledge about this region: we are not sure how the deconfinement and the chiral restoration phase transitions happen, how the QCD coupling constant evolves and how the strange quark behaves in dense matter, etc. Primarily, our current
understanding of the QCD phase structure in this region is based on assumptions.

In the framework of the bag model [5], or, in general, under the assumption that the strange quark mass is small [6], one can exclude the 2SC phase in the interior of compact stars when charge neutrality is considered.

However, there is another possibility, if the strange quark becomes light at a larger chemical potential than the $u, d$ quarks, there will be a density region where only $u, d$ quarks exist. Here, we focus on the dense quark matter composed of only $u, d$ quarks, assuming that the strange quark is too heavy to involve in the system. If bulk $u, d$ quark matter exists in the interior of compact stars, it should be neutral with respect to electrical as well as color charges [6–13]. Also, such matter should remain in $\beta$-equilibrium, i.e., the chemical potential for each flavor and color should satisfy the relationship

$$\mu_{ij,\alpha\beta} = (\mu_{ij} - \mu_e Q_{ij})\delta_{\alpha\beta} + 2/\sqrt{3}\mu_8 \delta_{ij} (T_8)_{\alpha\beta},$$

where $Q$ and $T_8$ are generators of $U(1)_{em}$ of electromagnetism and the $U(1)_8$ subgroup of the color gauge group. Satisfying these requirements imposes nontrivial relations between the chemical potentials of different quarks. We will see that these requirements play very important role in determining the ground state of dense $u, d$ quark matter.

The charge neutrality condition can be satisfied locally [6–11] or globally [12, 13]. In the following, we will firstly discuss the homogeneous phase when the charge neutrality is satisfied locally, then discuss the mixed phase when the charge neutrality condition is satisfied globally.

2. Local charge neutrality: homogeneous phase

2.1 Correct way to find the neutral ground state

To neutralize the electrical charge in the homogeneous dense $u, d$ quark matter, roughly speaking, twice as many $d$ quarks as $u$ quarks are needed, i.e., $n_d \simeq 2n_u$, where $n_{u,d}$ are the number densities for $u$ and $d$ quarks. This induces a mismatch between the Fermi surfaces of pairing quarks, i.e., $\mu_d - \mu_u = \mu_e = 2\delta\mu$, where $\mu_e$ is the electron chemical potential.

To get the ground state of the system, we need to know the thermodynamical potential. For simplicity, we use Nambu–Jona-Lasinio (NJL) model [14] to describe 2-flavor quark matter,

$$\mathcal{L} = \bar{q}i\gamma^\mu \partial_\mu q + G_S \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5 \bar{q})^2 \right] + G_D \left[ (\bar{q}^C \gamma^b \gamma_5 q) (\bar{q}^C \gamma^b \gamma_5 q) \right],$$

where $q^C = Cq^T$ is the charge-conjugate spinor and $C = i\gamma^2 \gamma^0$ is the charge conjugation matrix. The quark field $q \equiv q_{ie\alpha}$ is a four-component Dirac spinor that carries flavor ($i = 1, 2$) and color ($\alpha = 1, 2, 3$) indices. The Pauli matrices
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are denoted by \( \vec{\tau} = (\tau^1, \tau^2, \tau^3) \), while \( (\varepsilon)^{ik} \equiv \varepsilon^{ik} \) and \( (\varepsilon^b)^{a\beta} \equiv \varepsilon^{a\beta b} \) are the antisymmetric tensors in the flavor and color spaces, respectively. We also introduce a momentum cutoff \( \Lambda \), and two independent coupling constants in the scalar quark-antiquark and scalar diquark channels, \( G_S \) and \( G_D \). We define \( \eta = G_D / G_S \), and \( \eta = 0.75 \) is the value from the Fierz transformation, but in principle, \( \eta \) is a free parameter.

In the mean-field approximation, the thermodynamical potential for \( u, d \) quarks in \( \beta \)-equilibrium with electrons takes the form \([7, 9]\):

\[
\Omega_{u,d,e} = \Omega_0 - \frac{1}{12\pi^2} \left( \mu_e^4 + 2\pi^2 T^2 \mu_e^2 + \frac{7\pi^4}{15} T^4 \right) + \frac{m^2}{4G_S} + \frac{\Delta^2}{4G_D} - \sum_a \int \frac{d^3p}{(2\pi)^3} \left[ E_a + 2T \ln \left( 1 + e^{-E_a/T} \right) \right],
\]

where \( \Omega_0 \) is a constant added to make the pressure of the vacuum zero, and the electron mass was taken to be zero, which is sufficient for the purposes of the current study. The sum in the second line of Eq. (2) runs over all (6 quark and 6 antiquark) quasi-particles. The explicit dispersion relations and the degeneracy factors of the quasi-particles read

\[
E_{ub}^\pm = E(p) \pm \mu_{ub}, \quad [\times 1] \quad (3)
\]
\[
E_{db}^\pm = E(p) \pm \mu_{db}, \quad [\times 1] \quad (4)
\]
\[
E_{\Delta}^\pm = E_{\Delta}(p) \pm \delta \mu. \quad [\times 2] \quad (5)
\]

Here we introduced the following shorthand notation: \( E(p) = \sqrt{p^2 + m^2} \) and \( E_{\Delta}(p) = \sqrt{[E(p) \pm \bar{\mu}]^2 + \Delta^2} \) with \( \bar{\mu} \equiv \mu - \mu_e/6 + \mu_8/3 \).

If a macroscopic chunk of quark matter is created in heavy ion collisions or exists inside the compact stars, it must be in color singlet. So in the following discussions, color charge neutrality condition is always satisfied.

Now, we discuss the role of electrical charge neutrality condition. If a macroscopic chunk of quark matter has nonzero net electrical charge density \( n_Q \), the total thermodynamical potential for the system should be given by

\[
\Omega = \Omega_{\text{Coulomb}} + \Omega_{u,d,e}, \quad (6)
\]

where \( \Omega_{\text{Coulomb}} \sim n_Q^2 V^{2/3} \) (\( V \) is the volume of the system) is induced by the repulsive Coulomb interaction. The energy density grows with increasing the volume of the system, as a result, it is almost impossible for matter inside stars to remain charged over macroscopic distances. So the bulk quark matter should also satisfy electrical neutrality condition, thus \( \Omega_{\text{Coulomb}}|_{n_Q=0} = 0 \), and \( \Omega_{u,d,e}|_{n_Q=0} \) is on the neutrality line. Under the charge neutrality condition, the total thermodynamical potential of the system is \( \Omega|_{n_Q=0} = \Omega_{u,d,e}|_{n_Q=0} \).
Here, we want to emphasize that: The correct way to find the ground state of the homogeneous neutral $u, d$ quark matter is to minimize the thermodynamical potential along the neutrality line $\Omega|_{n_Q=0} = \Omega_{u,d,e}|_{n_Q=0}$, not like in the flavor asymmetric quark system, where $\beta$-equilibrium is required but $\mu_e$ is a free parameter, and the ground state is determined by minimizing the thermodynamical potential $\Omega_{u,d,e}$.

From Figure 1, we can see the difference in determining the ground state for a charge neutral system and for a flavor asymmetric system. In Figure 1, at a given chemical potential $\mu = 400$ MeV and $\eta = 0.75$, the thermodynamical potential along the charge neutrality line $\Omega|_{n_Q=0}$ as a function of the diquark gap $\Delta$ is shown by the solid line. The minimum gives the ground state of the neutral system, and the corresponding values of the chemical potential and the diquark gap are $\mu_e = 148$ MeV and $\Delta = 68$ MeV, respectively. If we switch off the charge neutrality conditions, and consider the flavor asymmetric $u, d$ quark matter in $\beta$-equilibrium [15], the electrical chemical potential $\mu_e$ becomes a free parameter. At a fixed $\mu_e = 148$ MeV and with color charge neutrality, the thermodynamical potential is shown as a function of the diquark gap by the dashed line in Figure 1. The minimum gives the ground state of the flavor asymmetric system, and the corresponding diquark gap is $\Delta = 104$ MeV, but this state has positive electrical charge density, and cannot exist in the interior of compact stars.
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2.2 \( \eta \) dependent solutions

In the last subsection, by looking for the minimum of the thermodynamic potential along the charge neutrality line, we found the ground state for the charge neutral \( u, d \) quark system.

Equivalently, the neutral ground state can also be determined by solving the diquark gap equation together with the charge neutrality conditions. We visualize this method in Figure 2, with color neutrality always satisfied, at a given chemical potential \( \mu = 400 \) MeV. The nontrivial solutions to the diquark gap equation as functions of the electrical chemical potential \( \mu_e \) are shown by a thick-solid line (\( \eta = 0.75 \)), a long-dashed line (\( \eta = 1.0 \)), and a short-dashed line (\( \eta = 0.5 \)). It is found that for each \( \eta \), the solution is divided into two branches by the thin-solid line \( \Delta = \delta \mu \), and the solution is very sensitive to \( \eta \). Also, there is always a trivial solution to the diquark gap equation, i.e., \( \Delta = 0 \). The solution of the charge neutrality conditions is shown by a thick dash-dotted line, which is also divided into two branches by the thin-solid line \( \Delta = \delta \mu \), but the solution of the charge neutrality is independent of \( \eta \).

The cross-point of the solutions to the charge neutrality conditions and the diquark gap gives the solution of the system. We find that the neutral ground state is sensitive to the coupling constant \( G_D = \eta G_S \) in the diquark channel. In the case of a very strong coupling (e.g., \( \eta = 1.0 \) case), the charge neutrality line crosses the upper branch of the solution to the diquark gap, the ground state is a charge neutral regular 2SC phase with \( \Delta > \delta \mu \). In the case of weak coupling (e.g., \( \eta = 0.5 \)), the charge neutrality line crosses the trivial solution.
of the diquark gap, i.e., the ground state is a charge neutral normal quark matter with $\Delta = 0$. The regime of intermediate coupling (see, e.g., $\eta = 0.75$ case) is most interesting, the charge neutrality line crosses the lower branch of the solution of the diquark gap. We will see that this phase is a gapless 2SC (g2SC) phase with $\Delta < \delta \mu$, which is different from the regular 2SC phase, and has some unusual properties.

2.3 g2SC phase

In this subsection, we will explain why we call the color superconducting phase with $\Delta < \delta \mu$ the g2SC phase, and we will show some special properties of this phase.

**Quasi-particle spectrum**

It is instructive to start with the excitation spectrum in the case of the ordinary 2SC phase when $\delta \mu = 0$. With the conventional choice of the gap pointing in the anti-blue direction in color space, the blue quarks are not affected by the pairing dynamics, and the other four quasi-particle excitations are linear superpositions of $u_{r,g}$ and $d_{r,g}$ quarks and holes. The quasi-particle is nearly identical with a quark at large momenta and with a hole at small momenta. We represent the quasi-particle in the form of $Q(\text{quark}, \text{hole})$, then the four quasi-particles can be represented explicitly as $Q(u_r, d_g)$, $Q(u_g, d_r)$, $Q(d_r, u_g)$ and $Q(d_g, u_r)$. When $\delta \mu = 0$, the four quasi-particles are degenerate, and have a common gap $\Delta$. 

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*Figure 3.* The quasi-particle dispersion relations at low energies in the 2SC phase (left panel) and in the g2SC phase (right panel).
If there is a small mismatch ($\delta\mu < \Delta$) between the Fermi surfaces of the pairing $u$ and $d$ quarks, the excitation spectrum will change. For example, we show the excitation spectrum of $Q(u_r, d_g)$ and $Q(d_g, u_r)$ in the left panel of Figure 3. We can see that $\delta\mu$ induces two different dispersion relations, the quasi-particle $Q(d_g, u_r)$ has a smaller gap $\Delta - \mu$, and the quasi-particle $Q(u_r, d_g)$ has a larger gap $\Delta + \mu$. This is similar to the case when the mismatch is induced by the mass difference of the pairing quarks [16].

If the mismatch $\delta\mu$ is larger than the gap parameter $\Delta$, the lower dispersion relation for the quasi-particle $Q(d_g, u_r)$ will cross the zero-energy axis, as shown in the right panel of Figure 3. The energy of the quasi-particle $Q(d_g, u_r)$ vanishes at two values of momenta $p = \mu^-$ and $p = \mu^+$ where $\mu^\pm = \bar{\mu} \pm \sqrt{(\delta\mu)^2 - \Delta^2}$. This is why we call this phase gapless 2SC (g2SC) phase. An unstable gapless CFL phase has been found in Ref. [16], and a similar stable gapless color superconductivity could also appear in a cold atomic gas or in $u, s$ or $d, s$ quark matter when the number densities are kept fixed [17].

As one would expect, far outside the pairing region, $p \simeq \bar{\mu}$, the quasi-particle dispersion relations are similar to those in the 2SC phase. Also, around $p \simeq \bar{\mu}$, the quasi-particle $Q(u_r, d_g)$ resembles the dispersion relations with that in the regular 2SC phase. The most remarkable property of the quasi-particle spectra in the g2SC phase is that the low energy excitations ($E \ll \delta\mu - \Delta$) are very similar to those in the normal phase represented by solid lines. The only difference is that the values of the chemical potentials of the up and down quarks $\mu_{ur} = \mu_{ug}$ and $\mu_{dg} = \mu_{dr}$ are replaced by the values $\mu^-$ and $\mu^+$, respectively. This observation suggests, in particular, that the low energy (large distance scale) properties of the g2SC phase should look similar to those in the normal phase.

**Finite temperature properties**

In a superconducting system, when one increases the temperature at a given chemical potential, thermal motion will eventually break up the quark Cooper pairs. In the weakly interacting Bardeen-Copper-Schrieffer (BCS) theory, the transition between the superconducting and normal phases is usually of second order. The ratio of the critical temperature $T_c^{BCS}$ to the zero temperature value of the gap $\Delta_0^{BCS}$ is a universal value [18]

$$r_{BCS} = \frac{T_c^{BCS}}{\Delta_0^{BCS}} = \frac{\gamma_E}{\pi} \approx 0.567,$$

where $\gamma_E \approx 0.577$ is the Euler constant. In the conventional 2SC phase of quark matter with equal densities of the up and down quarks, the ratio of the critical temperature to the zero temperature value of the gap is also the same as in the BCS theory [19]. In the spin-0 color flavor locked phase as well as in
Figure 4. The temperature dependence of the diquark gap (solid line) and the value of \( \delta \mu \equiv \mu_e/2 \) (dashed line) in neutral quark matter. For comparison, the diquark gap in the model with \( \mu_e = 0 \) and \( \mu_s = 0 \) is also shown (dash-dotted line). The results are plotted for \( \mu = 400 \text{ MeV} \) and \( \eta = 0.75 \).

The spin-1 color spin locked phase, on the other hand, this ratio is larger than the BCS ratio by the factors \( 2^{1/3} \) and \( 2^{2/3} \), respectively. These deviations are related directly to the presence of two different types of quasi-particles with nonequal gaps [20].

For the g2SC phase, the typical results for the default choice of parameters \( \mu = 400 \text{ MeV} \) and \( \eta = 0.75 \) are shown in Figure 4. Both the values of the diquark gap (solid line) and the mismatch parameter \( \delta \mu = \mu_e/2 \) (dashed line) are plotted. One very unusual property of the shown temperature dependence of the gap is a nonmonotonic behavior. Only at sufficiently high temperatures, the gap is a decreasing function. In the low temperature region, \( T \leq 10 \text{ MeV} \), however, it increases with temperature. For comparison, in the same figure, the diquark gap in the model with \( \mu_e = 0 \) and \( \mu_s = 0 \) is also shown (dash-dotted line). This latter has the standard BCS shape.

Another interesting thing regarding the temperature dependences in Figure 4 appears in the intermediate temperature region, \( 22.5 \leq T \leq 37 \text{ MeV} \). By comparing the values of \( \Delta(T) \) and \( \delta \mu \) in this region, we see that the g2SC phase is replaced by a “transitional” 2SC phase there. Indeed, the energy spectrum of the quasi-particles even at finite temperature is determined by the same relations in Eqs. (3) and (5) that we used at zero temperature. When \( \Delta > \delta \mu \), the modes determined by Eq. (5) are gapped. Then, according to our standard classification, the ground state is the 2SC phase.

It is fair to say, of course, that the qualitative difference of the g2SC and 2SC phases is not so striking at finite temperature as it is at zero temperature. This difference is particularly negligible in the region of interest where temperatures \( 22.5 \leq T \leq 37 \text{ MeV} \) are considerably larger than the actual value of the
smaller gap, \( \Delta - \delta \mu \). However, by increasing the value of the coupling constant slightly, the transitional 2SC phase can be made much stronger and the window of intermediate temperatures can become considerably wider. In either case, we find it rather unusual that the g2SC phase of neutral quark matter is replaced by a transitional 2SC phase at intermediate temperatures which, is replaced by the g2SC phase again at higher temperatures.

It appears that the temperature dependence of the diquark gap is very sensitive to the choice of the diquark coupling strength \( \eta = \frac{G_D}{G_S} \) in the model at hand. This is not surprising because the solution to the gap equation is very sensitive to this choice. The resulting interplay of the solution for \( \Delta \) with the condition of charge neutrality, however, is very interesting. This is demonstrated by the plot of the temperature dependence of the diquark gap calculated for several values of the diquark coupling constant in Figure 5.

The most amazing are the results for weak coupling. It appears that the gap function could have sizable values at finite temperature even if it is exactly zero at zero temperature. This possibility comes about only because of the strong influence of the neutrality condition on the ground state preference in quark matter. Because of the thermal effects, the positive electrical charge of the diquark condensate is easier to accommodate at finite temperature. We should mention that somewhat similar results for the temperature dependence of the gap were also obtained in Ref. [21] in a study of the asymmetric nuclear matter, and in Ref. [22] when number density was fixed.

The numerical results for the ratio of the critical temperature to the zero temperature gap in the g2SC case as a function of the diquark coupling strength \( \eta = \frac{G_D}{G_S} \) are plotted in Figure 6. The dependence is shown for the most interesting range of values of \( \eta = \frac{G_D}{G_S}, 0.68 \leq \eta \leq 0.81 \), which allows the g2SC stable ground state at zero temperature. When the coupling gets weaker
in this range, the zero temperature gap vanishes gradually. As we saw from Figure 5, however, this does not mean that the critical temperature vanishes too. Therefore, the ratio of a finite value of $T_c$ to the vanishing value of the gap can become arbitrarily large. In fact, it remains strictly infinite for a range of couplings.

3. **Global charge neutrality: mixed phase**

We have discussed the homogeneous 2-flavor quark matter when charge neutrality conditions are satisfied locally, and found that the local charge neutrality conditions impose very strong constraints on determining the ground state of the system.

On the other hand, one can construct mixed phase when charge neutrality conditions are satisfied globally. Inside mixed phases, the charge neutrality is satisfied “on average” rather than locally. This means that different components of mixed phases may have non-zero densities of conserved charges, but the total charge of all components still vanishes. In this case, one says that the local charge neutrality condition is replaced by a global one. There are three possible components: (i) normal phase, (ii) 2SC phase, and (iii) g2SC phase.

The pressure of the main three phases of two-flavor quark matter as a function of the baryon and electrical chemical potentials is shown in Figure 7 at $\eta = 0.75$. In this figure, we also show the pressure of the neutral normal quark and gapless 2SC phases (two dark solid lines). The surface of the g2SC phase extends only over a finite range of the values of $\mu_e$. It merges with the pressure surfaces of the normal quark phase (on the left) and with the ordinary 2SC phase (on the right).
Figure 7. At $\eta = 0.75$, pressure as a function of $\mu \equiv \mu_B/3$ and $\mu_e$ for the normal and color superconducting quark phases. The dark solid lines represent two locally neutral phases: (i) the neutral normal quark phase on the left, and (ii) the neutral gapless 2SC phase on the right. The appearance of the swallowtail structure is related to the first order type of the phase transition in quark matter.

It is interesting to notice that the three pressure surfaces in Figure 7 form a characteristic swallowtail structure. As one could see, the appearance of this structure is directly related to the fact that the phase transition between color superconducting and normal quark matter, which is driven by changing parameter $\mu_e$, is of first order. In fact, one should expect the appearance of a similar swallowtail structure also in a self-consistent description of the hadron-quark phase transition. Such a description, however, is not available yet.

From Figure 7, one could see that the surfaces of normal and 2SC quark phases intersect along a common line. This means that the two phases have the same pressure along this line, and therefore could potentially co-exist. Moreover, as is easy to check, normal quark matter is negatively charged, while 2SC quark matter is positively charged on this line. This observation suggests that the appearance of the corresponding mixed phase is almost inevitable.

Let us start by giving a brief introduction into the general method of constructing mixed phases by imposing the Gibbs conditions of equilibrium [23, 24]. From the physical point of view, the Gibbs conditions enforce the mechanical as well as chemical equilibrium between different components of a mixed phase. This is achieved by requiring that the pressure of different components inside the mixed phase are equal, and that the chemical potentials ($\mu$ and $\mu_e$) are the same across the whole mixed phase. For example, in relation
Figure 8. At \( \eta = 0.75 \), pressure as a function of \( \mu \equiv \mu_B/3 \) and \( \mu_e \) for the normal and color superconducting quark phases (the same as in Figure 7, but from a different viewpoint). The dark solid line represents the mixed phase of negatively charged normal quark matter and positively charged 2SC matter.

to the mixed phase of normal and 2SC quark matter, these conditions read

\[
P^{(NQ)}(\mu, \mu_e) = P^{(2SC)}(\mu, \mu_e),
\]

\[
\mu = \mu^{(NQ)} = \mu^{(2SC)},
\]

\[
\mu_e = \mu_e^{(NQ)} = \mu_e^{(2SC)}.
\]

It is easy to visualize these conditions by plotting the pressure as a function of chemical potentials (\( \mu \) and \( \mu_e \)) for both components of the mixed phase. This is shown in Figure 8. As should be clear, the above Gibbs conditions are automatically satisfied along the intersection line of two pressure surfaces (dark solid line in Figure 8).

Different components of the mixed phase occupy different volumes of space. To describe this quantitatively, we introduce the volume fraction of normal quark matter as follows: \( \chi_{2SC}^{NQ} \equiv V_{NQ}/V \) (notation \( \chi_A^{A} \) means volume fraction of phase A in a mixture with phase B). Then, the volume fraction of the 2SC phase is given by \( \chi_{NQ}^{2SC} = (1 - \chi_{NQ}^{2SC}) \). From the definition, it is clear that \( 0 \leq \chi_{2SC}^{NQ} \leq 1 \).

The average electrical charge density of the mixed phase is determined by the charge densities of its components taken in the proportion of the corresponding volume fractions. Thus,

\[
n_{e}^{(MP)} = \chi_{2SC}^{NQ} n_{e}^{(NQ)}(\mu, \mu_e) + (1 - \chi_{2SC}^{NQ}) n_{e}^{(2SC)}(\mu, \mu_e).
\]
If the charge densities of the two components have opposite signs, one can impose the global charge neutrality condition, \( n_e^{(MP)} = 0 \). Otherwise, a neutral mixed phase could not exist. In the case of quark matter, the charge density of the normal quark phase is negative, while the charge density of the 2SC phase is positive along the line of the Gibbs construction (dark solid line in Figure 8). Therefore, a neutral mixed phase exists. The volume fractions of its components are

\[
\chi_{2SC}^{NQ} = \frac{n_e^{(2SC)}}{n_e^{(2SC)} - n_e^{(NQ)}},
\]

(12)

\[
\chi_{NQ}^{2SC} \equiv 1 - \chi_{2SC}^{NQ} = \frac{n_e^{(NQ)}}{n_e^{(NQ)} - n_e^{(2SC)}}.
\]

(13)

After the volume fractions have been determined from the condition of the global charge neutrality, we could also calculate the energy density of the corresponding mixed phase,

\[
\varepsilon^{(MP)} = \chi_{2SC}^{NQ}\varepsilon^{(NQ)}(\mu, \mu_e) + (1 - \chi_{2SC}^{NQ})\varepsilon^{(2SC)}(\mu, \mu_e).
\]

(14)

This is essentially all that we need in order to construct the equation of state of the mixed phase.

So far, we were neglecting the effects of the Coulomb forces and the surface tension between different components of the mixed phase. In a real system, however, these are important. In particular, the balance between the Coulomb forces and the surface tension determines the size and geometry of different components inside the mixed phase.

In our case, nearly equal volume fractions of the two quark phases are likely to form alternating layers (slabs) of matter. The energy cost per unit volume to produce such layers scales as \( \sigma^{2/3}(n_e^{(2SC)} - n_e^{(NQ)})^{2/3} \) where \( \sigma \) is the surface tension [25]. Therefore, the quark mixed phase is a favorable phase of matter only if the surface tension is not too large. Our simple estimates show that \( \sigma_{\text{max}} \leq 20 \text{ MeV/fm}^2 \). However, even for slightly larger values, \( 20 \leq \sigma \leq 50 \text{ MeV/fm}^2 \), the mixed phase is still possible, but its first appearance would occur at larger densities, \( 3\rho_0 \leq \rho_B \leq 5\rho_0 \). The value of the maximum surface tension obtained here is comparable to the estimate in the case of the hadronic-CFL mixed phase obtained in Ref. [26]. The thickness of the layers scales as \( \sigma^{1/3}(n_e^{(2SC)} - n_e^{(NQ)})^{-2/3} \) [25], and its typical value is of order 10 fm in the quark mixed phase. This is similar to the estimates in various hadron-quark and hadron-hadron mixed phases [25, 26]. While the actual value of the surface tension in quark matter is not known, in this study we assume that it is not very large. Otherwise, the homogeneous gapless 2SC phase should be the most favorable phase of nonstrange quark matter [9].
Under the assumptions that the effect of Coulomb forces and the surface tension is small, the mixed phase of normal and 2SC quark matter is the most favorable neutral phase of matter in the model at hand with $\eta = 0.75$. This should be clear from observing the pressure surfaces in Figs. 7 and 8. For a given value of the baryon chemical potential $\mu = \mu_B / 3$, the mixed phase is more favorable than the gapless 2SC phase, while the gapless 2SC phase is more favorable than the neutral normal quark phase.

4. Conclusion

Dense $u, d$ quark matter under local and global charge neutrality conditions in $\beta$-equilibrium has been discussed.

Under local charge neutrality condition, the homogeneous neutral ground state is sensitive to the coupling constant in the diquark channel, it will be in the regular 2SC phase when the coupling is strong, in the normal phase when the coupling is weak, and in the g2SC phase in the case of intermediate coupling. The low energy quasi-particle spectrum in g2SC phase contains four gapless and only two gapped modes, and this phase has rather unusual properties at zero as well as at finite temperature.

Under global charge neutrality condition, assuming that the effect of Coulomb forces and the surface tension is small, one can construct a mixed phase composed of positive charged 2SC phase and negative charged normal quark matter.

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