Study on lateral stiffness and parametric sensitivity of high speed elevator traction wire rope

Chuang Xu¹, Ruijun Zhang¹*, Qing Zhang¹, Xiujuan Qi¹

¹Shandong Jianzhu University, School of Mechanical and Electronic Engineering, Jinan, China

*Corresponding author: zhangruijun@sdjzu.edu.cn

Abstract-In the real service environment of high-speed traction elevator, the horizontal sway of towing wire rope caused by the center of gravity deviation and airflow disturbance seriously affects the comfort of passengers in the elevator. In view of this problem, this paper first establishes the equivalent model of the flexible traction wire rope of high-speed elevator. Secondly, based on the energy method, the lateral stiffness calculation formula of the towing wire ropes is derived, and then compares the stiffness calculation formula derived from the classical stiffness calculation formula to verify the correctness of the proposed stiffness calculation formula. Last but not least, In MATLAB, the regularity of tension force and traction wire rope lift length on the minimum lateral stiffness value and minimum stiffness position is explored. The results show that the tension force is positively correlated with the minimum stiffness value, negatively related to the minimum stiffness position, and the lifting length of the traction wire rope is negatively related to the minimum stiffness value, and the minimum stiffness position is positively correlated.

1. Introduction

In the actual operation of the high-speed traction elevator, due to the center of gravity bias and airflow disturbance and other reasons, the traction wire rope connecting the car system leads to the problem of lateral skew[1-2], which aggravates the vibration problem of the car system, seriously affecting the comfort of passengers and operation safety of the elevator[3-5]. It can be seen from the references[6-7] that the horizontal swing of the traction wire rope is related to the horizontal stiffness of the traction wire rope itself, and the horizontal stiffness of the traction wire rope is an important factor to ensure the safe operation of the elevator car. Therefore, it is of great significance to study the horizontal stiffness of the traction wire rope to reduce the swing of the traction wire rope and thus reduce the vibration of the car system.

Therefore, this paper first establishes the equivalent model of the flexible lift system of high-speed traction elevator, and further deduces the formula of the lateral stiffness, minimum stiffness position and minimum stiffness value of the traction wire rope based on the energy method. Then, the derived horizontal stiffness calculation formula and the horizontal stiffness classic calculation formula are compared to verify the correctness of the derived formula. Finally, the rule of the change of the minimum stiffness with the tensioning force and the hoisting length of the wire rope and the position of the minimum stiffness with the tensioning force and the hoisting length of the wire rope are analyzed by Matlab software simulation, providing a theoretical basis for reducing the deflection of the traction wire rope.
2. Research on lateral stiffness of traction wire rope based on energy method

2.1. Establishment of equivalent model of flexible lifting system

Among them, \( L \) is the maximum lift length of the traction wire rope.

Considering that the traction wire rope is flexible and has complex mechanical properties, it is necessary to make equivalent simplification in order to study its lateral stiffness. The traction wire rope is simplified to a continuous string, and the car system is simplified to a heavy block with mass \( m \). The simplified equivalent model of the lifting system is shown in Fig. 1. Considering that the traction wire rope is flexible and has complex mechanical properties, it is necessary to make equivalent simplification in order to study its lateral stiffness. The traction wire rope is simplified to a continuous string, and the car system is simplified to a heavy block with mass \( m \). The simplified equivalent model of the lifting system is shown in Fig. 1.

2.2. The derivation of the formula for calculating the horizontal stiffness of the flexible traction wire rope

Further, according to the equivalent model of the lifting system shown in Fig. 1, the stress diagram of the traction wire rope can be obtained in Fig. 2.
Among them, T is the tension force exerted by the traction wire rope, F is the transverse force exerted by the traction wire rope, U is the transverse deflection, L₁ is the vertical distance between the stress point F and the center of mass of the cage system.

According to Fig. 2, the calculation formula of lateral stiffness of the traction wire rope is derived based on the energy method.

Equation (1): The total potential energy of system

\[ E = \int_{0}^{l} T(x) \, d\varepsilon = \int_{0}^{l} [T + \rho g x] \, d\varepsilon \]  

Which, \( \rho \) is traction wire rope line density, \( g \) is gravitational acceleration, \( \varepsilon \) is the disturbance of strain.

It can be obtained from the strain - displacement relationship and the geometric relationship shown in Fig. 2.

\[ \varepsilon = \frac{1}{2} \left( \frac{\partial u}{\partial x} \right)^2 = \frac{1}{2} \left( \frac{u}{L - L_1} \right)^2 \]  

Substituting Equation (2) into Equation (1) can be obtained,

\[ \int_{0}^{l} T(x) \, d\varepsilon = \int_{0}^{l} [T + \rho g x] \left[ \frac{1}{2} \left( \frac{\partial u}{\partial x} \right)^2 \right] \]  

\[ \frac{1}{2} \frac{T + \rho g L}{L - L_1} + \frac{T}{L} u^2 \]  

Equation (4): The elastic potential energy of the trailing wire rope

\[ E = \frac{1}{2} K u^2 \]  

The lateral equivalent stiffness of the traction wire rope can be obtained by combining the vertical (3) and equation (4). The express is

\[ K = \frac{L(T + \rho g l)}{L_1(L - L_1)} \]  

According to the geometric relationship shown in Fig. 2, by taking the derivative of Equation (5) and setting it equal to 0, \( l_{\text{min}} \) of the minimum stiffness position of the traction wire rope can be obtained.

The express is

\[ l_{\text{min}} = \frac{T - \sqrt{T(T - \rho g l)}}{\rho g} \]  

The equation of minimum stiffness value is

\[ K_{\text{min}} = \frac{L(\rho g)^2 \sqrt{T(T + \rho g L)}}{(\sqrt{T(T + \rho g L)} - T)(T + \rho g L - \sqrt{T + \rho g L})} \]  

3. Verification of the calculation formula of lateral stiffness of flexible traction wire rope

| L₁ (m) | This paper deduces the result of calculation formula (N/m) | The classical formula calculates the result (N/m) |
|-------|--------------------------------------------------------|---------------------------------|
| 100   | 164.2                                                  | 165.4                           |
| 110   | 175.6                                                  | 176.9                           |
| 120   | 188.8                                                  | 190.1                           |
| 130   | 203.3                                                  | 204.5                           |
| 140   | 221.5                                                  | 222.7                           |
| 150   | 244.7                                                  | 245.9                           |
| 160   | 271.6                                                  | 272.9                           |
| 170   | 305.3                                                  | 306.7                           |
| 180   | 348.7                                                  | 348.8                           |
| 190   | 406.6                                                  | 407.9                           |
| 200   | 487.7                                                  | 488.9                           |
In order to verify the correctness of the formula for calculating the lateral stiffness of the traction wire rope derived in this paper, this section compares the calculated results of the formula with those of the classical formula.

The equation of the classical transverse stiffness calculation formula is

\[ K = \frac{Qa(1+\alpha)}{L \ln\left(\frac{1+\alpha}{1+\xi a}\right) \ln(1+\xi a)} \]  

(8)

The calculation results of equation (5) and equation (8) are shown in Table I. It can be seen that the error of the calculation results is within 3%. Therefore, the calculation formula of lateral stiffness proposed in this paper is correct and reasonable.

4. Parametric sensitivity analysis

Taking a high-speed traction elevator with a running speed of 5m/s as an example, the influence law of tension force and hoisting length of traction wire rope on the minimum stiffness value of traction wire rope and the position of the minimum stiffness value is analyzed. The main simulation parameters are selected as shown in Table II.

| Parameter                          | Numerical Value | Parameter                          | Numerical Value |
|-----------------------------------|-----------------|-----------------------------------|-----------------|
| Drag wire rope nominal diameter   | 16mm            | Traction wire rope line density    | 9.85 kg/m       |
| The effective length of the traction wire rope | 250m            | The tension of the car to the trailing wire rope | 2e5N           |
| Elastic modulus of traction wire rope E | $8 \times 10^{10}$ N/m$^2$ | Maximum running speed | 5m/s           |

According to formula (6) and formula (7), the tensioning force and hoisting length of the traction wire rope are the main factors affecting the minimum stiffness value and the position of the minimum stiffness. Therefore, this section explores the changing law of the minimum stiffness value and the position of the minimum stiffness by changing the size of the tensioning force and the hoisting length of the traction wire rope.

4.1 Analysis of the influence of tensioning force and hoisting length of traction wire rope on the minimum stiffness value

![Figure 3. Relation curve of minimum stiffness value and tension force](image-url)
Fig. 3 shows the relation curve between the minimum stiffness value of the traction wire rope and the tension force of the hoisting traction wire rope. It can be seen from Fig. 3 that, when the lifting stiffness of the traction wire rope is constant, the minimum stiffness value of the traction wire rope increases with the increase of the tension force on the traction wire rope, and there is a positive correlation between them; as the increase of tension force can increase the minimum stiffness value of the traction wire rope, it can be known that the greater the tension force is, the greater the lateral stiffness of the traction wire rope will be. The greater the transverse stiffness of the traction wire rope, the smaller the transverse deflection. Therefore, when the traction wire rope has a certain lifting length, the tension can be appropriately increased to control the transverse deflection of the traction wire rope. However, the tension force does not increase indefinitely, so while increasing the transverse stiffness to reduce the transverse deflection of the traction wire rope, the maximum tension force should be selected within a reasonable range to reduce the transverse deflection of the car system.

![Figure 3. The relation curve between the minimum stiffness value and the lifting length](image)

Fig. 4 shows the relation curve between the minimum stiffness of the traction wire rope and the hoisting length of the traction wire rope. According to Fig. 4, when the tension force is constant, with the increase of the hoisting length of the traction wire rope, the minimum stiffness value of the traction wire rope decreases and there is a negative correlation between them; the minimum stiffness of the traction wire rope can increase by reducing the hoisting length of the traction wire rope, but the hoisting length of traction wire rope is not infinitely decreasing, therefore, while increasing the lateral stiffness and reducing the lateral deflection of the traction wire rope, the shortest hoisting length of the traction wire rope should be selected in a reasonable range to achieve the goal of reducing the car deflection.

4.2. Analysis of the influence of tensioning force and hoisting length of traction wire rope on the minimum stiffness position

The relation curve between the position of the minimum lateral stiffness of the traction wire rope and the tension force is shown in Fig. 5. As can be seen from Fig. 5, when the hoisting length of the traction wire rope is fixed, with the increase of the tension force exerted on the traction wire rope, the minimum stiffness position will gradually approach the lower position in the middle of the traction wire rope, showing a negative correlation between them. Because the lower end of the traction wire rope is strengthened by the constraint of tensioning force, the minimum stiffness position of the lifting traction wire rope gradually moves down. In order to change the gradual downward movement of the minimum stiffness position, the traction wire rope stabilizer can be installed near the minimum stiffness position.
Fig. 6 shows the curve of the relation between the minimum stiffness position and the length of the trailing wire rope. As can be seen from Fig. 6, when the tension force is constant, the minimum stiffness position moves up with the increase of lifting length, which is positively correlated with each other. Under different hoisting length of the traction wire rope, the minimum stiffness position appears in the position below the middle of the traction wire rope instead of the middle position. This is because the mass of the traction wire rope is equivalent to the increase of the tension force during the operation of the elevator, so that the minimum stiffness position appears in the lower middle position. In order to reduce the transverse deflection of the towing rope, the towing rope stabilizer can be installed near the minimum stiffness position.

5. Conclusion
Based on the energy method, this paper deduces the calculation formula of the lateral stiffness of the traction wire rope, and on this basis, studies the law of the influence of tensioning force and the hoisting length of the traction wire rope on the minimum lateral stiffness value and the minimum stiffness position.

(1) Based on the energy method, the formula for calculating the lateral stiffness of the traction wire rope is derived, and the correctness of the formula is verified by comparing with the results of the classical formula for calculating the lateral stiffness. The formula is simple, which provides a theoretical basis for the design and calculation of the lateral stiffness of the traction wire rope in engineering practice.

(2) When the hoisting length of the traction wire rope is fixed, the tension force is positively correlated with the minimum lateral stiffness value. The tension force is negatively correlated with the minimum stiffness position.
(3) When the tension force is constant, the hoisting length of the traction wire rope is negatively correlated with the minimum lateral stiffness value, and positively correlated with the minimum stiffness position.

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