Cosmological models with bulk viscosity in presence of adiabatic matter creation and with $G$, $c$ and $\Lambda$ variables

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Abstract

Some properties of cosmological models with a time variable bulk viscous coefficient in the presence of adiabatic matter creation and $G$, $c$, $\Lambda$ variables are investigated in the framework of flat FRW line element. We trivially find a set of solutions through Dimensional Analysis. In all the studied cases it is found that the behavior of these "constants" is inversely proportional to the cosmic time.

1 Introduction.

Recently several models with FRW metric, where "constants" $G$ and Lambda are considered as dependent functions of time $t$ have been studied. For these models, whose energy-momentum tensor describes a perfect fluid, it was demonstrated that $G \propto t^\alpha$, where $\alpha$ represents a certain positive constant that depends on the state equation imposed while $\Lambda \propto t^{-2}$ is independent of the state equation (see [1]). More recently this type of model was generalized

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by Arbab (see [2]) who considers a fluid with bulk viscosity (or second viscosity in the nomenclature of Landau (see [3])). The role played by the viscosity and the consequent dissipative mechanism in cosmology has been studied by many authors. The heat represented by the great entropy per baryon in the cosmic background radiation provides a good indication of the early universe and a possible explanation of the huge entropy per baryon which is believed to have been generated by physical dissipative processes acting at the beginning of evolution. These dissipative processes may be responsible for the smoothing out of initial anisotropies.

In the models studied by Arbab constants $G$ and $\Lambda$ are substituted by scalar functions that depend on time $t$. The state equation that governs the bulk viscosity is: $\xi \propto \xi_0 \rho^\gamma$ where $\gamma$ is a certain indeterminate constant for the time being $\gamma \in [0, 1]$. Amongst all the possible $\gamma$ we are only going to concentrate on $\gamma = 1/2$ since, as pointed out by Golda et al. (see [4]) upon demonstrating that for an adequate election of the state equation the viscous models are topologically equivalent (structurally stable) to the classic FRW. Golda et al show that the bulk viscous fluids with FRW symmetries can structurally approximate the dynamics of the classic FRW. They found that approximation only takes place when the parameter of the bulk viscosity follows the state equation $\xi \propto \xi_0 \rho^{1/2}$. They seek spatially homogeneous and isotropic solutions with bulk viscosity that could be approximated in a structurally stable way to the dynamics that describe the classic FRW described by an ordinary perfect fluid. More specifically, they looked for such spatially homogeneous and isotropic solutions which after being disturbed by the dissipative parameter have a dynamics that is topologically equivalent to the classic FRW models. Therefore the only viscous models that can be approximated to the classic FRW in a structurally stable way are those that follow a state equation $\xi \propto \xi_0 \rho^{1/2}$ because of their viscous parameter.

The first authors that have studied the convenience of also considering the variation of "constant" $c$, the speed of light, as a scalar function depending on time $t$, to solve some of the problems that classic FRW models present, have been (see [5], [6]). Others (see [7] and [8]) demonstrate through a variational formulation that the resulting field equations continue being the Friedmann ones if a FRW metric is considered. In these papers it is not so much the accurate calculation of such a variation for $c$ that is taken into consideration as the examination of the benefits of a modification in the field equations. Thus, these models are able to explain the problem of the horizon and the
flatness of the universe, in the same way as the inflationary models.

Following Arbab’s line, the author of this article (see [9]) attempts to generalize the models with $G, c$ and $\Lambda$ variables through the consideration of a flow with bulk viscosity. In this type of models, (the structurally stable $\gamma = 1/2$) inspite of considering viscous fluids, that involve a natural production of entropy (irreversible processes), it demonstrates that the entropy continued being constant (as in the classic FRW). In this paper we consider both mechanisms of matter production and mechanisms of entropy. To attempt to solve this problem we consider in this paper so much mechanisms of matter production as of entropy. The problem of matter creation in the universe has been studied by many authors following Prigogine and co-workers pioneering ideas (see [10]). We will follow Lima et al’s work. (see ([11]). This problem in the bulk viscous fluids framework has been studied by Desikan ([12]). The general idea is the following: to consider the universe as an open system, the creation of matter itself generates entropy and thus the second law of the thermodynamics can be incorporated into the field of equations. In the case of open systems the number of particles in a given volume is not fixed and though we consider that the transformation is adiabatic, it is demonstrated that the entropy is growing when the matter creation acts as an internal energy mechanism. In this paper we calculate the variation of "constants" $G, c$ and $\Lambda$ in the framework of a model with FRW symmetries, with $k = 0$ i.e. flat, with adiabatic matter creation and bulk viscosity as separate irreversible processes. We consider the processes of adiabatic matter creation (see [11]) to attempt to solve the problem of entropy. We wish to draw attention to how the use of the Dimensional Analysis (D.A.) (see [13]) enables us to find the solution to our model in a trivial way. We do not want to carry out a meticulous study of the cosmological implications of the results obtained for each of the parameters on which the solution depends ($\omega, \beta, \gamma$) (postponing this paragraph for a subsequent article) but simply to demonstrate them. We will show that for an adequate election of such parameters we obtain the same behaviour for the principal quantities as those obtained for the classic FRW models. The behaviour obtained for the "constants" is that for these parameters they always vary in a way inversely proportional to the time. These results are compared to those of other authors.

The paper is organised as follows: In the second section the governing equations of our model will be shown and considerations on the followed dimensional method will be made. In the third section we will make use of
the D.A. (Pi theorem) to obtain a solution to the principal magnitude that appears in the model and finally in the fourth section we will present two particular cases of the obtained solutions together with some conclusions.

2 The model.

For a flat universe $k = 0$ with FRW symmetries i.e. we assumed homogeneity and isotropy and therefore there will not be spatial variations of the ”constants” $G, c$ and $\Lambda$ solely temporary. We suppose equally that our fluid is bulk viscous (second viscosity) and we consider mechanisms of creation of matter. With these suppositions the equations that govern the model are as follows:

\[
-2 \frac{f''}{f} - \frac{(f')^2}{f^2} = \frac{8\pi G(t)}{c^2(t)} (p + p_c) + c^2(t) \Lambda(t)
\]  

(1)

\[
\frac{(f')^2}{f^2} = \frac{8\pi G(t)}{3c^2(t)} \rho + c^2(t) \Lambda(t)
\]  

(2)

\[
n' + 3nH - \psi = 0
\]  

(3)

where $n$ measures the particles number density, $\psi$ is the function that measures the matter creation, $H = f'/f$ represents the Hubble parameter ($f$ is the scale factor that appears in the metrics), $p$ is the thermostatic pressure, $\rho$ is energy density and $p_c$ is the pressure that generates the matter creation.

The creation pressure $p_c$ depends on the function $\psi$. For adiabatic matter creation this pressure takes the following form ([11]):

\[
p_c = \left[ \frac{\rho + p}{3nH} \right]^{\psi}
\]  

(4)

The state equation that we will use is the known expression

\[
p = \omega \rho
\]  

(5)

where $\omega = \text{const.} \, \omega \in [0, 1]$ physically realistic equations, making in this way that the energy-momentum tensor $T_{ij}$ verify the energy conditions.

We need to know the exact form of the function $\psi$ the one which is determined from a more fundamental theory than involves quantum processes. We assumed that this function continues the following law:

\[
\psi = 3\beta nH
\]  

(6)
here we are following to Lima et al (see [11]) for other treatment [12] while Prigogine et al [10] follows this other law $\psi = \kappa H^2$ where $\beta$ is a dimensionless constant (if $\beta = 0$ then there is matter creation since $\psi = 0$) that it is given presumably by models of particles physics of matter creation. Physically one hope that the most interesting situations emerge during phases in those which i.e. $\beta \approx 1$ will be of the order of unity.

The conservation principle carries us to the following expression:

$$\rho' + 3(\omega + 1)\rho \frac{f'}{f} = (\omega + 1)\rho \frac{\psi}{n}$$

Integrating the equation (7) we obtain the following relationship between energy density and the radius of the universe and what is more important the constant of integration necessary for our subsequent calculations:

$$\rho = A_{\omega,\beta} f^{-3(\omega+1)(1-\beta)}$$

where $A_{\omega,\beta}$ is the constant of integration that depends on the state equation that is considered i.e. of the constant $\omega$ and of the constant $\beta$ that measures the matter creation.

The effect of the bulk viscosity in the equations is shown replacing $p$ by $p - 3\xi H$ where $\xi$ follow the law $\xi = \xi_0 \rho^\gamma$ (see [14], [2] and [12]). This last state equation, in our opinion, it does not verify the homogeneity principle by this reason we modify it by:

$$\xi = k_\gamma \rho^\gamma$$

where the constant $k_\gamma$ causes that this equation yes will be dimensionally homogeneous for any value of $\gamma$.

The dimensional analysis that we followed needs to make the following distinctions, we need to know beforehand the set of fundamental quantities together with that of unavoidable constants (in the nomenclature of Barenblatt are designated as governing parameters). In this case the single one fundamental quantity that appears in the model is the cosmic time $t$ as can be deduced with facility of the homogeneity and isotropy supposed for the model. The unavoidable constants of the model are the constant of integration $A_{\omega,\beta}$ that depends on the state equation $\omega$ and of the mechanisms of matter creation $\beta$ and the constant $k_\gamma$ that controls the influence of the viscosity in the model.
In a previous work ([13]) was calculated the dimensional base of this type of models, being this \( B = \{L, M, T, \theta\} \) where \( \theta \) represents the dimension of the temperature. The dimensional equation of each one of the governing parameters are:

\[
[t] = T \quad [A_{\omega,\beta}] = L^{3(\omega + 1)(1-\beta) - 1} M T^{-2} \\
[k_\gamma] = L^{\gamma - 1} M^{1-\gamma} T^{2\gamma - 1}
\]

All the derived quantities or governed parameters in the nomenclature of Batrenblatt we will calculate them in function of these quantities (the governing parameters), that is to say, in function of the cosmic time \( t \) and of the two unavoidable constants \( k_\gamma \) and \( A_{\omega,\beta} \) with respect to the dimensional base \( B = \{L, M, T, \theta\} \).

### 3 Solutions through D.A.

We go to calculate through dimensional analysis D.A. i.e. applying the Pi Theorem, the variation of \( G(t) \) in function of \( t \), the speed of light \( c(t) \), the energy density \( \rho(t) \), the radius of the universe \( f(t) \), the temperature \( \theta(t) \), the entropy \( S(t) \) and the entropy density \( s(t) \), the viscosity coefficient \( \xi(t) \), the particle number density \( n(t) \propto f^{-3} \) and finally the variation of the cosmological ”constant” \( \Lambda(t) \).

The dimensional method carries us to (see [13])

### 3.1 Calculation of \( G(t) \)

As we have indicated above, we are going to accomplish the calculation of the variation of \( G \) applying the Pi theorem. The quantities that we consider are: \( G = G(t, k_\gamma, A_{\omega,\beta}) \). with respect to the dimensional base \( B = \{L, M, T, \theta\} \).

We know that \( [G] = L^3 M^{-1} T^{-2} \)

\[
\begin{pmatrix}
G & t & k_\gamma & A_{\omega} \\
L & 3 & 0 & \gamma - 1 & 3(\omega + 1)(1-\beta) - 1 \\
M & -1 & 0 & 1 - \gamma & 1 \\
T & -2 & 1 & 2\gamma - 1 & -2
\end{pmatrix}
\]
we obtain a single monomial that leads to the following expression for $G$

$$G \propto A_{\omega,\beta}^{\frac{2}{3(\omega+1)(1-\beta)}} k_n^{\frac{2+3(\omega+1)(1-\beta)}{3(\omega+1)(1-\beta)(\gamma-1)}} t^{-4-\left[\frac{2+3(\omega+1)(1-\beta)}{3(\omega+1)(1-\beta)(\gamma-1)}\right]} \quad (10)$$

3.2 Calculation of $c(t)$

$c(t) = c(t, k_\gamma, A_{\omega,\beta})$ where $[c] = LT^{-1} \implies$

$$c(t) \propto A_{\omega,\beta}^{\frac{1}{3(\omega+1)(1-\beta)}} k_n^{\frac{1}{3(\omega+1)(1-\beta)(\gamma-1)}} t^{-1-\left[\frac{1}{3(\omega+1)(1-\beta)(\gamma-1)}\right]} \quad (11)$$

3.3 Calculation of energy density $\rho(t)$

$\rho = \rho(t, k_\gamma, A_{\omega,\beta})$ with respect to the dimensional base $B$, where $[\rho] = L^{-1}MT^{-2}$

$$\rho \propto \frac{1}{k_n^{\frac{1}{n-1}}} t^{\frac{1}{\gamma-1}} \quad (12)$$

we observe that this relationship shows us that energy density does not depend neither on the state equation $\omega$ nor on the mechanisms on creation of matter i.e. does not depend on the constant $A_{\omega,\beta}$ solely on the viscosity of the fluid.

3.4 Calculation of the radius of the universe $f(t)$.

$f = f(t, k_\gamma, A_{\omega,\beta})$ where $[f] = L \implies$

$$f \propto A_{\omega,\beta}^{\frac{1}{3(\omega+1)(1-\beta)}} k_n^{\frac{1}{3(\omega+1)(1-\beta)(\gamma-1)}} t^{\frac{1}{3(\omega+1)(1-\beta)(\gamma-1)}} \quad (13)$$

We can observe that:

$$q = -\frac{f''}{(f')^2} = -\left[3\beta(\omega+1)(\gamma-1) - 3n(\omega+1) + 3\omega + 4\right]$$

$$H = \frac{f'}{f} = -\left(\frac{1}{3(\omega+1)(1-\beta)(\gamma-1)}\right) \frac{1}{t}$$
3.5 Calculation of the temperature \( \theta(t) \).

\[ \theta = \theta(t,k,\omega,\beta) \] where \( k_B \) is the Bolzmann constant : \( [\theta] = \theta \) and \( [k_B\theta] = L^2MT^{-2} \)

\[ k_B\theta \propto A_{\omega,\beta}^{-\frac{1}{4}} k_n^{\frac{1-\gamma(1-\beta)}{(\omega+1)(1-\beta)}} t^{-\left[\frac{1-\gamma(1-\beta)}{(\omega+1)(1-\beta)}(\gamma-1)\right]} \] (14)

3.6 Calculation of the entropy \( S(t) \):

\[ S = s(t,k,\omega,\beta) \] where \( a \) is the radiation constant. \( [S] = L^2MT^{-2}\theta^{-1} \)

\[ S \propto A_{\omega,\beta}^{-\frac{1}{4}} k_n^{\frac{1-\gamma(1-\beta)}{(\omega+1)(1-\beta)}} t^{-\left[\frac{1-\gamma(1-\beta)}{(\omega+1)(1-\beta)}(\gamma-1)\right]} a^{\frac{1}{4}} \] (15)

3.7 Calculation of the entropy density \( s(t) \):

\[ s = s(t,k,\omega,\beta) \] where \( a \) is the radiation constant. \( [s] = L^{-1}MT^{-2}\theta^{-1} \)

\[ s \propto A_{\omega,\beta}^{0} k_n^{\frac{3}{4}} t^{-\left[\frac{3}{4}\gamma(1-\beta)\right]} a^{\frac{1}{8}} \] (16)

3.8 Calculation of the viscosity coefficient \( \xi(t) \):

\[ \xi = \xi(t,k,\omega,\beta) \] where \( [\xi] = L^{-1}MT^{-1} \)

\[ \xi \propto k_n^{\frac{1}{\gamma}} t^{-\frac{1}{\gamma}} \] (17)

3.9 Calculation of the cosmological constant: \( \Lambda(t) \).

\[ \Lambda = \Lambda(t,k,\omega,\beta) \] where \( [\Lambda] = L^{-2} \)

\[ \Lambda \propto A_{\omega,\beta}^{\frac{1}{4}} k_n^{\frac{2}{4}} t^{-\left[\frac{2}{4}\gamma(1-\beta)(\gamma-1)\right]} \] (18)

4 Different cases.

All the following cases can be calculated without difficulty. But as we have indicate in the first section we are going to centre our attention only in those models that follow the law \( \xi = k_\gamma \rho^{1/2} \) i.e. \( \gamma = (1/2) \) that corresponds to
models that are topologically equivalent to the classic FRW. We are going to study two models with $\gamma = 1/2$, one with $\omega = 1/3$ that corresponds to a universe with radiation predominance and other with $\omega = 0$ corresponding to a universe with matter predominance.

4.1 $\gamma = 1/2$ and $\omega = 1/3$

$$G \propto A_{\omega}^{\frac{1}{2}} k_n^{-2(1-\beta)} t^{-2+\frac{1}{2(1-\beta)}}$$
$$c \propto A_{\omega}^{\frac{3}{4}} k_n^{-2(1-\beta)} t^{-\frac{1}{2(1-\beta)}}$$
$$\Lambda \propto A_{\omega}^{\frac{1}{2}} k_n^{-1(1-\beta)} t^{-\frac{1}{2(1-\beta)}}$$

if $\beta = 0 \Rightarrow G \propto t^{-1}, c \propto t^{-1/2}, \Lambda \propto t^{-1}$

$c \propto t^{-1/2}$ also has been obtained by Barrow [7] and Troiskii [8], with respect to the rest of the quantities we have obtain the same behaviour that Lima et al. [11]

$$\rho \propto k_B t^2$$
$$k_B \theta \propto A_{\omega}^{\frac{1}{2}} k_n^{-2(1-\beta)} t^{-2+\frac{1}{2(1-\beta)}}$$
$$f \propto A_{\omega}^{\frac{3}{4}} k_n^{-2(1-\beta)} t^{-\frac{1}{2(1-\beta)}}$$
$$a^{-1} S \propto A_{\omega}^{\frac{3}{2}} k_n^{-\frac{3\beta}{2(1-\beta)}} t^{-\frac{3\beta}{2(1-\beta)}}$$
$$a^{-\frac{3}{2}} s \propto A_{\omega}^{\frac{3}{4}} k_n^{-\frac{3\beta}{2}} t^{-\frac{3\beta}{2}}$$
$$\xi \propto k_B^2 t^{-1}$$
$$\xi \propto t^{-1}$$

Si $\beta = 0 \Rightarrow \theta \propto t^{-1/2}, f \propto t^{1/2}, S \propto t^0 = \text{const.}$

With respect to the thermodynamic behavior, the matter creation formulation considered here is a clear consequence of the nonequilibrium thermodynamic in presence of a gravitational field. We see that the $\beta$ parameter works in the opposite sense to the expansion, that is, reducing the cooling rate with respect to the case where there is no matter creation. A very meaningful result is the fact that the spectrum of this radiation cannot be distinguished from the usual blackbody spectrum at the present epoch (see [11]). Therefore models with adiabatic matter creation can be compatible with the isotropy currently observed in the spectral distribution of the background radiation.

We observe equally, that the obtained model is clearly irreversible (classic
FRW is reversible). We want also to express the fact that all the important thermodynamic quantities of the classic FRW models are recovered if we made $\beta = 0$. (see [13])

$$f \propto t^{1/2}, \quad \rho \propto t^{-2}, \quad \theta \propto t^{-1/2}, \quad S = \text{const.}, \quad s \propto t^{-3/2}$$

Finally it is interesting to stick out that the presented model may significantly alter the predictions that make the classic FRW on the abundance of elements. Such result puts a possible limitation to the values that could take the $\beta$ parameter.

4.2 $\gamma = 1/2$ and $\omega = 0$:

$$G \propto A_{\omega}^{\frac{2}{3(1-\beta)}} k_n^{-\frac{2}{3(1-\beta)}} t^{-\frac{2}{3(1-\beta)}}$$

$$c \propto A_{\omega}^{\frac{1}{3(1-\beta)}} k_n^{\frac{-2}{3(1-\beta)}} t^{-\frac{1}{3(1-\beta)}}$$

$$\Lambda \propto A_{\omega}^{-\frac{2}{3(1-\beta)}} k_n^{\frac{4}{3(1-\beta)}} t^{\frac{-4}{3(1-\beta)}}$$

Si $\beta = 0 \Rightarrow G \propto t^{-2}, c \propto t^{-1/3}, \Lambda \propto t^{-4/3}$

$c \propto t^{-1/3}$, this result also it is obtained by Barrow (see [4]) and Petit ([8]) but not by Troiskii ([5]). The rest of the quantities coincides with the model presented by Petit, except for energy density, since Petit considers that the mass also should vary.

$$\rho \propto k_n^{2} t^{-2} \quad \rho \propto t^{-2}$$

$$f \propto A_{\omega}^{\frac{1}{3(1-\beta)}} k_n^{-\frac{2}{3(1-\beta)}} t^{\frac{2}{3(1-\beta)}}$$

if $\beta = 0 \Rightarrow f \propto t^{2/3}$

$$\xi \propto k_n^2 t^{-1} \quad \xi \propto t^{-2/3}$$

this model with $\beta = 0$ is very similar to a FRW with matter predominance.

5 Conclusions.

We have solved through D.A. a flat model i.e. the sectional curvature of the 3-space is zero, homogeneous and isotropic i.e. we admitted symmetries type FRW. The energy-momentum tensor is described by a fluid with bulk viscosity in the one which furthermore we envisage mechanisms so much of
creation of matter as of entropy and in the one which the classics "constants" $G$, $c$ and $\Lambda$ are considered as variable.

The envisaged cases here show the following behavior for such "constants"

\[ G \propto t^{-1} \quad c \propto t^{-1/2} \quad \Lambda \propto t^{-1} \]

for a model with $(\gamma = 1/2, \omega = 1/3$ y $\beta = 0)$ and

\[ G \propto t^{-2/3} \quad c \propto t^{-1/3} \quad \Lambda \propto t^{-4/3} \]

for a model with $(\gamma = 1/2, \omega = 0$ y $\beta = 0)$ an equal behavior for the rest of the quantities that are observed in the classic FRW.

Several problems have emerged during the development of the article.

1. Even though we have supposed the classic constants of the physics as variable have arisen (and result us indispensable) two constants, $k_\gamma$ and $A_{\omega,\beta}$ that have clear physical meaning and without those which we cannot arrive to solve our model. Will be these two characteristic "constants" of our model also universal?

2. For the calculation of the thermodynamic quantities as the temperature and the entropy we have used the "constant" $k_B$ that we have supposed constant and the "constant" of radiation $a$ that also we have supposed constant, but if $a$ is constant (observe that $a \propto \left(\frac{k_B^4}{c^3 \hbar^3}\right)$) then the only one possibility that we have is to make $\hbar \propto c^{-1}$ that is equals to say that $\hbar \propto t^{1/2}$ or $\hbar \propto t^{1/3}$ depending on the model.

3. If we abandon the characteristic value of $\gamma = 1/2$ we prove without difficulty that for $\gamma > 1/2$ the "constant" $G$ varies in a proportional way to the time instead of inversely proportional to the time as have obtained here. But we would need of some evidence or physical rigorous reasoning to take similar values of $\gamma$. The only one possibility to obtain $G$ and $c$ constant with $\gamma = 1/2$ is imposing a physically unrealistic condition $\omega = -1/3$.

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