SAFEGUARDING UAV NETWORKS THROUGH INTEGRATED SENSING, JAMMING, AND COMMUNICATIONS

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ABSTRACT

This paper proposes an integrated sensing, jamming, and communications (ISJC) framework for securing unmanned aerial vehicle (UAV)-enabled wireless networks. The proposed framework advocates the dual use of artificial noise transmitted by an information UAV for simultaneous jamming and sensing of an eavesdropping UAV. Based on the information sensed in the previous time slot, an optimization problem for online resource allocation design is formulated to maximize the number of securely served users in the current time slot, while taking into account a tracking performance constraint and quality-of-service (QoS) requirements regarding the leakage information rate to the eavesdropper and the downlink data rate to the legitimate users. A channel correlation-based algorithm is proposed to obtain a suboptimal solution for the design problem. Simulation results demonstrate the security benefits of integrating sensing into UAV communication systems.

Index Terms— UAV, physical layer security, extended Kalman filter, resource allocation.

1. INTRODUCTION

Compared with traditional terrestrial networks, UAV-enabled wireless networks [1] have the potential to cover a larger area with a higher data rate, thanks to their high flexibility and mobility as well as the line-of-sight (LoS) dominated propagation channels to the ground terminals. It is expected that UAV-enabled wireless networks will play a key role in supplementing future cellular networks by providing communication services to rural and hot spot areas [2]. However, UAV communications are highly susceptible to eavesdropping as the associated leakage channels are also LoS dominated [3]. Therefore, the exchange of confidential information in UAV wireless networks has to be safeguarded. Numerous works studied secure resource allocation design for UAV-enabled wireless communication systems assuming the availability of perfect channel state information (CSI) of the eavesdropper channels [4–6]. For the case of imperfect CSI, the authors of [7,8] proposed robust and secure resource allocation schemes to enhance the physical layer security (PLS) of UAV communications. However, all these works [4–8] assumed static eavesdroppers and did not develop specific sensing schemes for the eavesdropper channels. In practice, if the eavesdropper has high-maneuverability, such as an eavesdropping UAV (E-UAV), guaranteeing communication security is very challenging and it is necessary to integrate leakage channel sensing/tracking into the system design. The general concept of sensing-aided PLS has been discussed in the recent article [9], but a corresponding practical scheme has not been reported. The authors in [10] studied PLS in dual-functional radar

Fig. 1: A downlink UAV communication system serving K GUs in the presence of an E-UAV.

communication (DFRC) systems, where the multi-user interference is designed to be constructive at the legitimate users, while disrupting the eavesdropper. A jamming-based PLS enhancing scheme has been proposed for DFRC systems in [11]. However, this scheme does not sense the eavesdropper channel.

In this paper, we propose an integrated sensing, jamming, and communications (ISJC) framework to guarantee PLS in UAV-enabled wireless networks. In particular, an information UAV (I-UAV) transmits confidential information to legitimate ground users (GUs) and jams the E-UAV with artificial noises (AN) to facilitate secure communications. At the same time, the I-UAV tracks the location and velocity of the E-UAV by reusing the reflected AN. Based on the sensing information obtained in the previous time slot, a channel correlation-based online resource allocation algorithm is developed to determine the user scheduling and precoding policy for the current time slot. Our simulation results demonstrate the benefits of integrating sensing, jamming, and communications for safeguard UAV networks.

2. SYSTEM MODEL

We consider a downlink UAV communication system where a hovering I-UAV serves as a base station broadcasting K independent confidential data streams to K legitimate single-antenna GUs in the presence of a flying E-UAV, cf. Fig. 1. The I-UAV is equipped with two uniform planar arrays (UPAs) for 3-dimensional (3D) transmit (Tx) and receive (Rx) beamforming, respectively, comprising $M_b \times M_b$ rows and $M_b \times M_b$ columns) antennas, and $M_e \times M_e$ rows and $M_e \times M_e$ columns) antennas. The E-UAV is equipped with a Rx UPA, comprising $M_e$ (rows and $M_e$ columns) antennas. The total service time $T$ is divided into $N$ equal-length time slots with a slot duration of $\delta$, i.e., $T = N \delta$. The locations of the I-UAV and GUs are denoted as $q_b = [x_b, y_b, z_b]^T$ and $q_k = [x_k, y_k, 0]^T$, $\forall k$, respectively, where $[.]^T$ is the transpose operation. The E-UAV flies along

1Mobile I-UAVs can further enhance PLS and will be considered in our future work.
Based on (1), the achievable data rate of GU $k$ is given by

$$ R_k[n] = \log_2 \left( 1 + \frac{\sum_{k' \neq k} u_{k'[n]} h_{b,k}^H w_{k'[n]} + h_{b,k}^H w_{k[n]} + \sigma_k^2}{\sigma_k^2} \right). $$

When E-UAV intercepts the information of GU, as a worst case, we assume it can mitigate the inter-user interference of the other GUs and is only impaired by the AN. Thus, the leakage information rate associated with GU $k$ in time slot $n$ is given by

$$ R_k^n = \log_2 \left( 1 + \frac{u_{k[n]} h_{b,k}^H w_{k[n]} + \sigma_k^2}{\sigma_k^2} \right). $$

In practice, the signal transmitted by I-UAV is partially received by the Rx antennas of E-UAV and is partially reflected by its body. The I-UAV is assumed to operate in the full-duplex mode, which allows it to transmit and receive signals simultaneously, while the received signal may suffer residual self-interference [13]. The corresponding echo received at I-UAV in time slot $n$ is given by

$$ \epsilon_k[n] = e_k[n] \sum_{k'=1}^{K} M_{k',k} \alpha_{k',n} a_{k',n} + v_{k,n}, $$

where the round-trip channel matrix is given by

$$ H_{be}[k] = 2d_{k}^{-1}[k] A_{M_e,k} M_{k'} \theta_{k',n} \phi_{k',n} a_{k',n} \tau_{k',n}. $$

Here, variables $\tau_{k,0}$ and $\nu_{k,0}$ denote the round-trip delay and Doppler shifts, respectively, $\epsilon_{k}[n] = \sqrt{\frac{\varphi}{4\pi\sigma_k^2}}$ denotes the reflection coefficient of E-UAV in time slot $n$, and $\varphi$ is the radar cross-section of E-UAV [14]. Vector $v_{k,n}(t) \sim \mathcal{CN}(0, \sigma_k^2 I_{M_e})$ captures both the background noise and the residual self-interference [13] at I-UAV, where $I_{M_e}$ denotes an $M_e \times M_e$ identical matrix. Note that clutter, such as the reflected signals of the GUs and the ground itself, is omitted here as it can be substantially suppressed by clutter suppression techniques [14] owing to its distinctive reflection angles and Doppler frequencies compared to the echoes from E-UAV [14]. Besides, we assume that the AOA is identical to the corresponding AOD at I-UAV in (6), which is reasonable when assuming a point target model [13] and reciprocal propagation.

### 3. E-UAV TRACKING

#### 3.1. Estimation Model of E-UAV

Based on the echoes in (5), different estimation methods can be used to estimate $\tau_{e}[n], \nu_{e}[n], \phi_{e}[n]$, and $\theta_{e}[n]$ [16]. One possible approach to estimate these parameters is the matched filter (MF) principle by exploiting the AN [16]:

$$ \{ \hat{\tau}_{e}[n], \hat{\nu}_{e}[n], \hat{\theta}_{e}[n], \hat{\phi}_{e}[n] \} = \arg \max_{\tau,\nu,\theta,\phi} \int_{\mathbb{R}} J_{b}(H_{be}[k], \theta, \phi, \tau_{e}(\nu, t), \nu_{e}(\nu, t) - \tau_{e}) e^{-j2\pi\nu t} dt. $$

Note that we exploit the AN rather than the user signals for sensing as using user signals would require the I-UAV to beamform the information-bearing signals towards the E-UAV, which increases the risk of information leakage. Instead, using AN for both sensing and jamming is a win-win strategy for secrecy applications. Analyzing the estimation variances associated with (7) is a challenging task. According to [15] [16], assuming the independence of the AN and the user signals, i.e., $\sum_{k=0}^{K} s_{k}(n,t) a_{e_{k}(n,t)} dt \approx 0, \forall k$, the estimation variances of $\tau_{e}[n], \nu_{e}[n], \phi_{e}[n]$, and $\theta_{e}[n]$ can be modeled by $\sigma_{\tau_{e}[n]}^2 = c_{\tau_{e}}/SNR, \sigma_{\nu_{e}[n]}^2 = c_{\nu_{e}}/SNR, \sigma_{\phi_{e}[n]}^2 = c_{\phi_{e}}/SNR$, and $\sigma_{\theta_{e}[n]}^2 = c_{\theta_{e}}/SNR$, respectively, where the MF output signal-to-noise ratio (SNR) is given by

$$ SNR = \frac{16\pi^2 d^2_{e}(n)}{16\pi^2 d^2_{e}(n)}. $$

Parameters $c_{\tau_{e}}, c_{\nu_{e}}, c_{\phi_{e}}, c_{\theta_{e}} > 0$ are determined by the specific adopted estimation methods [15] and $G_{MF}$ is the MF gain, which is proportional to the number of transmit symbols in a time slot.
3.2. Measurement Model for E-UAV

The measurement model characterizes the relationship between the observable parameters and the hidden state, which is the key for inferring the state of E-UAV. In particular, considering the positions of both I-UAV and E-UAV in Fig. 1, the measurement model associated with $r_{iU}[n], r_{eU}[n], \phi_{iU}[n],$ and $\theta_{eU}[n]$ are given by

$$\begin{align*}
\hat{r}_{iU}[n] &= 2d_{iU}[n] - q_{iU}[n] + v_{r_{iU}[n]}, \\
\hat{r}_{eU}[n] &= 2d_{eU}[n] - q_{eU}[n] - q_{bU}[n] + v_{r_{eU}[n]}, \\
\sin \hat{\theta}_{eU}[n] &= \frac{x_{eU}[n] - x_{bU}[n]}{\sqrt{x_{eU}[n] - x_{bU}[n] + y_{eU}[n] - y_{bU}[n]^2}} + v_{\sin \theta_{eU}[n]}, \\
\cos \hat{\theta}_{eU}[n] &= \frac{y_{eU}[n] - y_{bU}[n]}{\sqrt{x_{eU}[n] - x_{bU}[n] + y_{eU}[n] - y_{bU}[n]^2}} + v_{\cos \theta_{eU}[n]}, \\
\sin \hat{\phi}_{eU}[n] &= \frac{x_{eU}[n] - x_{bU}[n]}{\sqrt{x_{eU}[n] - x_{bU}[n] + y_{eU}[n] - y_{bU}[n]^2}} + v_{\sin \phi_{eU}[n]}, \\
\cos \hat{\phi}_{eU}[n] &= \frac{y_{eU}[n] - y_{bU}[n]}{\sqrt{x_{eU}[n] - x_{bU}[n] + y_{eU}[n] - y_{bU}[n]^2}} + v_{\cos \phi_{eU}[n]},
\end{align*}$$

where $d_{iU}[n], d_{eU}[n]$ are the predicted distance, azimuth AOD, and elevation AOD, respectively.

3.4. Tracking E-UAV via EKF

3.2. Measurement Model for E-UAV

According to the predicted state, one can predict the effective channel between I-UAV and E-UAV in time slot $n$ as follows,

$$\begin{align*}
\hat{h}_{iU}[n] &= \frac{d_0}{d_{iU}[n] - 1} A_{\phi_{iU}[n], \phi_{eU}[n]} (\hat{\theta}_{iU}[n] - 1, \phi_{eU}[n] - 1),
\end{align*}$$

4. ONLINE RESOURCE ALLOCATION DESIGN

4.1. Problem Formulation

In time slot $n-1, \forall n$, the resource allocation design is formulated as the following optimization problem:

$$\begin{align*}
\max_X & \sum_{k=1}^{K} u_{k}[n] \\
\text{s.t.} & \quad C1: \sum_{k=1}^{K} u_{k}[n] \|w_k[n]\|^2 + \|w_{6-5}[n]\|^2 \leq P_{\text{max}}, \\
& \quad C2: \hat{R}_k[n] \geq u_k[n] R_{\text{min}}, \forall k, \\
& \quad C3: \max \hat{R}_k[n] \leq u_k[n] R_{\text{Leakage}}, \forall k, \\
& \quad \Delta h[n] = 1, \forall n-1 \in \mathbb{R}^{6 \times 6}. \\
\end{align*}$$

where $X = \{u_k[n] \in \{0, 1\}, w_k[n], w_{6-5}[n]\}$. The objective function $\sum_{k=1}^{K} u_{k}[n]$ represents the number of GUs that can be served securely. In [18], $P_{\text{max}}$ in C1 is the maximum transmit power, $R_{\text{min}}$ in C2 the minimum data rate for the selected GUs, and $\Delta h[n] = 1$ in C3 is the maximum tolerable tracking MSE for the location and velocity of E-UAV. $\Delta h[n]$ in C3 is the maximum allowable leakage rate associated with the selected GUs in the presence of channel prediction uncertainty.

4.2. Proposed Solution

Now, introducing six auxiliary variables $t_i \geq 0, i = \{1, \ldots, 6\}$, to bound the main diagonal entries of $C_{\alpha}[n]$, i.e., $C_{\alpha}[n]_{ii} \leq t_i$. C4 in (18) becomes $C4: \sum_{i=1}^{6} t_i \leq \text{MSE}_{\max}$. The problem in (18) is then equivalently transformed as follows

$$\begin{align*}
\min_{X, t_i} & \sum_{k=1}^{K} u_{k}[n] \\
\text{s.t.} & \quad C1-C4, C5: \left[\frac{C_{\alpha}[n]_{ii}}{e_t[i]} \right]_{t_i} \geq 0.
\end{align*}$$
lem in (20) is still infeasible, the I-UA V transmits only AN for
\begin{equation}
\text{(20)}
\end{equation}
becomes feasible. If all GUs are de-selected and the prob-
\text{bility relaxation (SDR) approach} \cite{21}. Hence, we propose a channel
\text{correlation-based user scheduling strategy. Without loss of gener-
ality, all GUs are indexed in descending order of the correlation
coefficients between \(h_{u_k}[n-1]\) and \(h_{u_k}[n]\), \(\forall k\). In general, the
higher the channel correlation, the higher the risk of information
leakage. Then, we first select all GUs for service, i.e., \(u_k[n] = 1\), \(\forall k\). If the resulting problem in (20)
is infeasible, we de-select GUs one-by-one in descending order of their channel correlations until
(20) becomes feasible. If all GUs are de-selected and the problem
in (20) is still infeasible, the I-UA V transmits only AN for
jamming and sensing adopting maximum ratio transmission, i.e.,
\(w_k[n] = 0\), \(\forall k\), and \(w_e[n] = h_{u_k}[n-1]\| \sqrt{p_{\text{max}}} \). The details of the
resulting algorithm are omitted here due to space limitation.

5. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed ISJC
scheme via simulations. The main simulation parameters are as
follows: \(K = 10\), \(\delta = 1\) s, \(N = 92\), \(p_{\text{max}} = 30\) dBm, \(R_{\text{min}} = 5\)
\text{bit/s/Hz}, \(R_{\text{Leakage}} = 0.01\) bit/s/Hz, \(G_{\text{MF}} = 10^4\), \(\sigma_e^2 = \sigma_k^2 = -99\)
dBm, \(\delta_e = 0.1\) m², and \(M_{k}\) = \(M_{e}\). Note that a higher \(M_{\text{SEmax}}\) implies that a poorer tracking performance is tolerable by the
considered system. Thus, we consider \(M_{\text{SEmax}} \in [5, 26]\). The locations
of I-UA V and the K GUs and the unknown trajectory of E-UA V are
illustrated in Fig. 3. The modeling parameters, including \(c_{\text{SE}}, c_{\text{FM}}, c_{\text{GUS}}, c_{\text{GU}},\)
and \(Q_{\alpha_e}\), are initialized numerically assuming random
walks of the I-UA V and E-UA V, respectively.

The estimated trajectory of E-UA V is shown in Fig. 3. The
\text{corresponding posterior tracking MSE,} \(\| \hat{e}_k[n] - \alpha_e[n] \|^2\), in each
time slot is shown in the upper half of Fig. 4. We observe that a
smaller \(M_{\text{SEmax}}\) leads to a lower tracking MSE and a more ac-
curate trajectory estimation. In fact, a smaller \(M_{\text{SEmax}}\) imposes a
core stringent tracking MSE constraint for resource allocation
design and thus more power and spatial degrees of freedom are
allocated for sensing. To provide more insights, we define K
\text{projection points} on the unknown trajectory for which E-UA V
has the smallest 3D distance w.r.t. the GUs, respectively. Comparing
Fig. 3 and Fig. 4 we observe a high tracking MSE around some
projection points, such as \(n=64\) and \(n=73\), where E-UA changes
directions, since the constant velocity model assumption for the
EKF is less accurate in those points. Note that in Fig. 4, the poste-
rior tracking MSE in some time slots is higher than \(M_{\text{SEmax}}\) while
the predicted MSE is guaranteed to be smaller than \(M_{\text{SEmax}}\). In
fact, \(M_{\text{SEmax}}\) only limits the predicted tracking MSE for resource
allocation design while the actual tracking MSE, which is not ac-
cessible to the I-UA V at the time of resource allocation design,
might be much higher due to the state evolution model mismatch
and the measurement uncertainty in (10). Furthermore, the aver-
age number of scheduled GUs versus \(M_{\text{SEmax}}\) is shown in the
lower half of Fig. 4. The performance of a baseline separate
jamming and sensing scheme, where a dedicated sub-slot is used for
sensing and the remaining time is used for communications and
jamming, is also illustrated for comparison. We can observe a
higher number of scheduled GUs for the proposed scheme com-
pared to the benchmark scheme. This is because the dual use of
AN in the proposed scheme preserves the system resources and
provides more flexibility for resource allocation design. This un-
derlines the advantage of integrating sensing, jamming, and com-
munications for improving secrecy performance. It can be fur-
ther observed that both a small \(M_{\text{SEmax}}\) and a large \(M_{\text{SEmax}}\) re-
sult in a small number of securely served GUs for both schemes.
For a small \(M_{\text{SEmax}}\), more resources are needed for sensing, and
thus, fewer GUs can be scheduled. This implies that enhancing the
tracking performance is not always beneficial for communications
as it requires more system resources. Increasing \(M_{\text{SEmax}}\) relaxes
the tracking performance constraint C4 in (18) and thus results in
a larger objective value. However, a too large \(M_{\text{SEmax}}\) in a given
time slot leads to unreliable eavesdropper channel prediction in
the next time slot, and thus, a lower jamming efficiency. Hence,
the number of securely served GUs is ultimately limited by the
channel prediction uncertainty in the large \(M_{\text{SEmax}}\) regime. In
fact, choosing \(M_{\text{SEmax}}\) properly is critical for maximizing the
secrecy communication performance. For example, for the consid-
ered scenario, \(M_{\text{SEmax}} = 14\) is optimal for the proposed scheme.

6. CONCLUSIONS

In this paper, we proposed a novel ISJC framework and an on-
line resource allocation design for securing UAV-enabled down-
link communications. The dual use of AN enables the I-UA V
to concurrently jam the E-UA V and sense the corresponding chan-
nels efficiently. Through simulations, we demonstrated that inte-
grating sensing, jamming, and communications improves the se-
crecy performance, while choosing a suitable value for the toler-
able tracking MSE is critical for balancing tracking and secrecy
performance.
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