Gauge symmetry in the large-amplitude collective motion of superfluid nuclei†

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The adiabatic self-consistent coordinate (ASCC) method2) is a practical method for describing large-amplitude collective motion in atomic nuclei with superfluidity and an advanced version of the adiabatic time-dependent Hartree–Fock–Bogoliubov theory. In the application of the one-dimensional ASCC method, Hinohara et al.3) encountered numerical instability and found that it was caused by the symmetry of the basic equations of the ASCC method under a certain continuous transformation. This transformation involves the gauge angle \( \varphi \) and changes the phase of the state vector. In this sense, Hinohara et al. called it the “gauge” symmetry. They proposed a gauge-fixing prescription to remove redundancy associated with the gauge symmetry and successfully applied it to the shape coexistence phenomena in proton-rich Se and Kr isotopes.

We investigated this symmetry on the basis of the Dirac–Bergmann theory of constrained systems4,5). As is well known, the gauge symmetry is associated with constraints originating from the singularity of the Lagrangian. In the ASCC method, the linear term of the particle number \( n \) in the collective Hamiltonian can be regarded as a constraint, and it leads to the gauge symmetry.

In the ASCC method, we assume the following form of the state vector.

\[
|\phi(q,p,\varphi,n)\rangle = e^{-i\varphi\hat{N}}|\phi(q,p,n)\rangle = e^{-i\varphi\hat{N}}e^{i\hat{G}}|\phi(q)\rangle,
\]

with \( \hat{G}(q,p,n) = p\hat{Q}(q) + n\hat{\Theta}(q) \) and \( \hat{N} = \hat{N} - N_0 \). \( \varphi \) is the gauge angle conjugate to the particle number \( n = N - N_0 \) measured from a reference value \( N_0 \). The collective Hamiltonian is defined and expanded up to \( O(n) \) as below.

\[
\mathcal{H}(q,n) := \langle \phi(q,p,\varphi,n)|\hat{H}|\phi(q,p,\varphi,n)\rangle
= V(q) + \frac{1}{2}B(q)p^2 + \lambda n.
\]

This can be regarded as a system with the constraint \( n = 0 \), and \( \lambda \) is a Lagrange multiplier.

In Lagrange formalism, the Lagrangian corresponding to this (total) Hamiltonian is given by

\[
L = \frac{1}{2B(q^2)}(q^1)^2 - V(q^1, q^2),
\]

with \( \{q^1, q^2\} := \{q, \varphi\} \), and the rank of the Hessian \((\partial^2L/\partial q^i\partial q^j)\) is one. (We allowed the potential \( V \) to depend on \( q^2 = \varphi \) in order to make it easy to observe the number of the degrees of freedom.) Hence, this Lagrangian leads to one constraint, \( p_2 = \partial L/\partial q^2 = n = 0 \).

The time derivative of the constraint is given by \( \dot{n} = \{n, \mathcal{H}\} = -\partial_q V \). Thus, the constraint is preserved in time if \( V(q, \varphi) = V(q) \). Then, we have only one constraint \( n = 0 \), and it is a first-class constraint. From the above, it is clear that our system has one gauge degree of freedom.

It is known that a generator of a gauge transformation can always be written as a “linear combination” of the first-class constraints. We can write the generator \( G \) as \( G = \epsilon(q, p, \varphi, n, t) n \) with an infinitesimal function \( \epsilon \). This generator gives the gauge transformation of collective variables.

\[
\delta q = n\partial_q \epsilon \approx 0, \quad \delta p = -n\partial_q \epsilon \approx 0,
\]

\[
\delta \varphi = \epsilon + n\partial_n \epsilon \approx \epsilon, \quad \delta n = -n\partial_q \epsilon \approx 0.
\]

Here, the symbol \( \approx \) denotes weak equality. As \( \epsilon \) is an arbitrary function of \( (q, p, \varphi, n, t) \), in particular, of \( p \), the linear and higher-order terms of \( p \) are mixed only into \( \varphi \) by this gauge transformation. This is important for the adiabatic expansion in the ASCC method to make sense.

With the choice \( \epsilon = \alpha n \), we obtain

\[
\delta q = \alpha n, \quad \delta p = 0, \quad \delta \varphi = \alpha p, \quad \delta n = 0,
\]

which leads to the transformation of operators,

\[
\hat{Q} \to \hat{Q} + \alpha \hat{N}, \quad \hat{\Theta} \to \hat{\Theta} + \alpha \hat{P}.
\]

This is exactly the transformation found in Ref. 3), and thus, we confirmed that the symmetry discussed in Ref. 3) is a gauge symmetry.

In Ref. 1), the most general gauge transformation is discussed. While the equation of collective submanifold, from which the basic equations of the ASCC method (the moving-frame HFB & QRPA equations) are derived, is invariant under the most general gauge transformation, the gauge symmetry is partially broken by the adiabatic expansion at the level of the moving-frame HFB & QRPA equations. Above, we have considered the expansion of the collective Hamiltonian up to \( O(n) \). In Ref. 1), it is also shown that there is no gauge symmetry in the case where the collective Hamiltonian is expanded up to \( O(n^2) \).

References
1) K. Sato, Prog. Theor. Exp. Phys. 2015, 123D01
2) M. Matsuo, T. Nakatsukasa, and K. Matsuyanagi, Prog. Theor. Phys. 103, 959 (2000).
3) N. Hinohara, T. Nakatsukasa, M. Matsuo, and K. Matsuyanagi, Prog. Theor. Phys. 117, 27 (2007).
4) P. A. M. Dirac, Can. J. Math. 2, 129 (1950).
5) J. L. Anderson, P. G. Bergmann, Phys. Rev. 83, 1018 (1951).

† Condensed from the article in Ref. 1)

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