The reliability of rural water distribution systems in relation to the layout of the pipework within the network

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Abstract
In this paper, the $K$ readiness index was adopted in order to evaluate the reliability of the rural water distribution system; complete systems were assessed using the partial survey method. Hydraulic calculations were performed, followed by reliability evaluation with our own NSW programme. In addition, the lengths of the ring and branched sections of the analysed water distribution systems were calculated and 256 computational variants were obtained. Taking into account the values of the $K$ readiness index and the cable lengths, the influence of the pipework system on the level of reliability of a water distribution system with a pumping-hydrophore structure was determined. In the case of a branched system, a moderately strong negative correlation was identified between reliability and the length of pipework.

Keywords: rural water distribution system, reliability, stationary index of readiness $K$

Streszczenie
W niniejszej pracy do oceny niezawodności wiejskiego systemu dystrybucji wody przyjęto wskaźnik gotowości $K$, a ocenę całych systemów prowadzono metodą przeglądu częściowego. Wykonano obliczenia hydrauliczne, a następnie niezawodnościowe z pomocą autorskiego programu NSW. Wyliczono długości przewodów części pierścieniowej i rozgałęzionej analizowanych systemów dystrybucji wody. Uzyskano 256 wariantów obliczeniowych. Biorąc pod uwagę uzyskane wartości wskaźnika gotowości $K$ oraz długości przewodów, starano się ustalić wpływ układu przewodów na poziom niezawodności systemu dystrybucji wody o układzie pompowo-hydroforowym. W przypadku przewodów rozgałęzionych uzyskano średnio silną ujemną korelację między wskaźnikiem niezawodności $K$ a długością rurociągów.

Słowa kluczowe: wiejski system dystrybucji wody, niezawodność, stacjonarny wskaźnik gotowości $K$
1. Introduction

The water distribution system is one of the most important and expensive elements of the water supply system and its design is a decisive factor in the reliability of delivering water to consumers. The primary task of a water distribution system is to provide an uninterrupted supply of water at the right level of pressure and for the minimum cost of system construction and operation. Designing a water distribution system for a given terrain topography and within given spatial planning restrictions requires various technical solutions to be considered. The decision to build a water supply system is characterised by the following features: it is irreversible; it is non-routine; it requires huge financial investment; significantly, it changes the living conditions of the given community. These features make the design stage extremely important. The longevity of the water distribution system for many years of reliable water supply in the future is a decisive factor.

In practice, design variants are predominantly analysed from technical and economic standpoints, but reliability aspects are also increasingly taken into consideration nowadays.

2. Characteristics of rural water distribution systems

Rural water distribution systems are characterised by many features that distinguish them from urban systems, hence the need for them to be analysed separately from technical and reliability perspectives. These features are as follows [1, 2]:

▶ high fluctuations in hourly water consumption in the countryside,
▶ low water pressure required by consumers in rural settlements compared to the pressure required in cities,
▶ an extensive water supply network in rural areas which differs from the grid system for more compact urban developments,
▶ there is no water intake from many water pipes,
▶ the predominant share of the branched structure of the water supply network,
▶ no mains water pipes characteristic of the urban water distribution systems in the structure of the rural water network – transit pipework is more common, as is the case with group water supply systems.

The main components of the water distribution systems are the pipework and fittings, such as valves, reducers and air vents. The route of the pipework is mostly adapted to the rural traffic system. In rural conditions, water pipes rarely exceed diameters of 200 mm. In terms of maintaining pressure, the most common solution in Poland is the pump-hydrophore system.

3. Review of the research on the reliability of water distribution systems

Research on the reliability of water distribution systems has been ongoing for many years [3–6]. The theoretical basis for the application of the analysis of reliability in water supply issues is discussed in, inter alia [7–10]. A comparative analysis of reliability indicators for
rural and municipal water supply systems is presented in articles [11–13]. The reliability of water supply and sewage infrastructure in Poland is discussed in study [14]. The subject of the reliability of rural water supply systems was discussed in works [15–17].

A proposal for the method of determining the index of readiness for simple water supply subsystems was discussed in study [18]. Article [19] presents the methodology for determination of readiness index $K$ using a multi-stage decomposition process. Study [20] describes a procedure for analysing the reliability of the water distribution systems during interruptions to supply resulting from the disconnection of water pipe sections for reasons of maintenance or repair, taking into account the probability of failure. In study [21], the theory of entropy was proposed for the evaluation of the reliability of the system. Entropy was used as a substitute measure of reliability and described the degree of fragmentation of the water flow between the source and the nodes in the water distribution system.

In paper [22], a methodology was presented to estimate the hydraulic reliability of the water distribution system; this can be defined as the probability that the system can provide the required flow rate at the required pressure. Because of the random nature of future water demand, along with the required pressure and the condition of the pipework, the reliability of future distribution systems is uncertain. Using the ‘Monte Carlo’ method, a simulation model with three basic components was developed, these components are the generation of random numbers, hydraulic network simulations and reliability calculations. Another simulation model for calculations of the reliability of the network and pumping stations was proposed in paper [23]. In study [24], a reliability assessment model was discussed; this combined the selection of pipe diameters and tank heights. An assessment of the reliability of water distribution systems along with an analysis of network gateway locations is described in articles [25, 26].

Hydraulic calculation methods are supplemented with various optimisation methods and reliability assessments [27–34].

A description of the management of the water distribution system with regard to its reliability is provided in works [35, 36]. Article [37] describes the situation encountered in many under-developed countries where the water supply system is unable to supply water to all consumers simultaneously. In such a situation, the reliability of the system is improved by optimising the water supply through the proper management of the operation and the introduction of maintenance schedules. Aspects of reliability and economics are described in articles [38–40]. In article [41], a model describing the probability of water shortage is proposed. Studies simulating simultaneous occurrences of water shortages were conducted. It was demonstrated that a single failure within the water distribution system ($k = 1$) has a considerable effect on the degree of severity vis-à-vis the scarcity of water. The model can be helpful in determining service standards for the amount of water supplied to consumers.
4. **A study on the reliability of rural water distribution systems from the point of view of the pipework**

In study [42], the necessity to create two basic models for assessing operational reliability within rural water systems led to the following suggestions:

- a model of node sub-systems, the capture, treatment, pumping, storage and regulation of water pressure,
- a linear subsystem that is a water distribution model.

The above division is related to the need for various methods of assessing the reliability of the node and pipework elements of a rural water supply system. Water distribution sub-systems are characterised by their complex reliability structure, which can be represented as follows:

- graphically, as in a block diagram,
- as a function of the structure, such as in a structure table or an analytical record,
- the full range of minimal ability paths, or inability paths.

Reliability in rural water distribution systems may be determined by one of the following methods [43, 44]:

- via the ‘simple decomposition’ method,
- via the ‘complete decomposition’ method,
- via the ‘partial decomposition’ method,
- via the ‘complex decomposition’ method,
- via minimum suitability paths, or non-suitability paths,
- via the ‘matrix’ method,
- via the ‘mass service’ method.

The ‘simple decomposition’ method consists of the consecutive transformation of structures which change the system into a number of sub-systems, each made up of simple structures.

The ‘complete decomposition’ method (also known as the ‘system state review’, ‘complete review’, or ‘operational state review’) consists of the decomposition of all suitable and non-suitable areas within the system.

The ‘partial decomposition’ method or the ‘partial review’ method is the same as the ‘complete decomposition’ method except that it is limited to the occurrence of one or more simultaneous failures. In the case of a rural water distribution system limited to one failure, it is possible to assess its reliability in detail.

The ‘complex decomposition’ method is based on the fact that the division of a system consisting of \( n \)-elements is made with respect to a selected group consisting of \( k \) elements: \( 1 < k < n \). \( 2^k \) sub-systems containing \( n-k \) elements are obtained. Decomposition is performed until sub-systems with simple structures are formed.

The ‘minimum suitability or non-suitability path’ method consists of transforming the real system into one equivalent parallel-to-serial or serial-to-parallel system illustrated by a logical scheme.

‘Matrix’ methods are used in the case of dependent damages.

‘Mass handling’ methods are used for homogeneous systems.
In the case of water distribution sub-systems, there is a close relationship between the technical solution used and its reliability rating. The process of assessing the reliability of a water distribution system under fixed flow conditions consists of the following steps [16]:

- The hydraulic calculation of the water distribution system can be carried out in a number of ways, using various numerical calculation programmes that take into account sectional expenditures, the type of network structure and the possibility of using temporary reservations, or in other words, the co-operation of those networks with tanks or reservoirs.
- The simulation of failures consisting of the disconnection of successive sections of water pipes and hydraulic calculations for the existing conditions assuming failure for peak hour water demand.
- Observations are conducted of the values of calculated flows and loss of pressure within the nodes. In the event of the pressure dropping below the required value in the i-th step of the simulation resulting in a lack of water supply to the recipient, a correction of the diameter of the i-th section must be made. Faced with the unsuitability of such a solution, it would be necessary to change the structure of the water distribution system.
- Final hydraulic calculations should be made after correcting the diameters or the structure.
- Evaluation should be undertaken of the reliability of the water distribution network using the ‘partial decomposition’ method.

A reliability analysis for all variants of the distribution system was undertaken on thirty-three rural water systems located in the former voivodeships of Białystok, Toruń and Bydgoszcz.

For simulation purposes, the location of the water supply stations and the structure of the water distribution system were changed. Hydraulic calculations were made for all variants. Calculation results were automatically uploaded by our own NSW software – NSW being ‘niezawodność systemów wodociągowych’ (water supply system reliability) – used to calculate the reliability level of individual variants using the ‘partial review’ method. The NSW programme also defined the length of the water supply pipes along the network branches and in the rings [16].

The stationary index of readiness $K$, which determines the probability of the system retaining its suitability to work at any point in time $T$, was used as a criterion for assessing the level of reliability. This probability is expressed by the following formula:

$$K = \frac{T_p}{T_p + T_n}$$

where:

$T_p$ – is the average working time of the water supply system [d],

$T_n$ – is the average renewal time of the water supply system [d].

It follows that the readiness index $K$ depends both on the likelihood of the system becoming impaired and on its renewability. The readiness index characterises the reliability of renewable facilities.
The partial review method consists of making a statement regarding only the most probable elementary states of the system. In the case of rural water distribution systems, only one failure of the water supply pipeline was accepted \((k = 1)\). Simulations of failures of particular sections of the network pipelines were performed in order to determine the state of efficiency or disability for each elementary state.

In order to distinguish the above two states, \(N_{gr}\) was adopted, which was calculated on the basis of the formula \([15]\):

\[
N_{gr} = \frac{L_z}{L_c - L_t} \geq 10\% \tag{2}
\]

where:
- \(L_z\) – maximum distance between shut-off valves, which was adopted at 300 [m],
- \(L_c\) – total length of the water supply network [m],
- \(L_t\) – length of transit wires [m].

The basis for calculating the reliability of the rural water distribution system was the values of the readiness index \(K_i\) of water supply pipes given for a 1 km length for different pipeline diameters and materials that were adopted from work \([15]\) and included in Table 1.

Table 1. The values of the readiness index \(K(L_i)\) for water pipes in rural water distribution systems for a 1 km length of water main \([15]\)

| Diameter and type of water pipe | \(K(L_i)\) |
|---------------------------------|-----------|
| cast iron pipelines            | 0.9975    |
| steel pipelines                | 0.9967    |
| PVC pipelines \(\varnothing 90 \text{ mm}\) | 0.9980    |
| PVC pipelines \(\varnothing 110 \text{ mm}\) | 0.9981    |
| PVC pipelines \(\varnothing 160 \text{ mm}\) | 0.9983    |

The readiness index \(K(L_i)\) for particular sections of the pipelines, depending on their length was calculated from the formula \([15]\):

\[
K(L_i) = K(L_i)^{(L_i/1000)} \tag{3}
\]

where:
- \(K(L_i)\) – readiness index for the \(i\)-th section of the water pipe depending on the length,
- \(L_i\) – the length of the \(i\)-th section of the water pipe.

During the simulation, the NSW program determines the water deficiency \(N_i\) during the failure of each \(i\)-th section of the water pipe. The values of \(N_i\) were compared with \(N_{gr}\).

If \(N_i > N_{gr}\), it was assumed that the system was in a state of disability. Otherwise, when \(N_i < N_{gr}\), the system was in a state of efficiency. For all failures of pipelines in which the system was considered to be operational, a partial index of readiness \(K_i\) was calculated \([15]\):

\[
K_i = [1 - K(L_i)] \cap K(L_j) \quad i = 1, 2, 3, n \quad j \neq i \tag{4}
\]
where:

- $K_i$ – a partial readiness indicator for the failure of the i-th section of the water pipe,
- $K(L_j), K(L_i)$ – readiness indicators for individual pipeline sections including their length $L$, calculated according to formula (3),
- $i, j$ – sections of water pipes.

If the water distribution system was in a state of disability, $K_i = 0$ was assumed. The index of readiness $K$ for the rural water distribution system was calculated by summing the values of the partial index of readiness $K_i$ for the failure of the next sections of water supply pipelines and the entire network in an undamaged state.

A total of 261 variants of rural water distribution systems were analysed with respect to their reliability. The analysis was simplified on the assumption that only staple or sectional expenditure and water uptake is proportional to the length of expenditure of the pipework at $L$, which, in the case of rural water distribution systems, does not seem to be too simplistic. Expenditures for transit pipework were assumed to be zero [16].

As a result of studies on the influence on the level of reliability of the water distribution system structure, the following dependencies were established for those systems with a pumping-hydrophore structure [16]:

- in the case of water distribution systems with a branched structure, the value of the stationary index of readiness $K$ decreases, depending on the total length of the pipework according to the following formula:

$$K = 0.999858 - 1.05736 \cdot 10^{-6} \cdot L$$  \hspace{1cm} (5)

$$r = -0.909107$$  \hspace{1cm} (6)

where in formulas (5–10):

- $K$ – stationary index of readiness,
- $L$ – total length of the pipework, range 2000–40 000 [m],
- $r$ – Pearson correlation coefficient.

- for systems with a mixed structure, this relationship assumes the following form:

$$K = 1.00204 - 5.42144 \cdot 10^{-6} \cdot L$$  \hspace{1cm} (7)

$$r = -0.764953$$  \hspace{1cm} (8)

- branch pipework in rural water distribution systems:

$$K = 1.00076 - 7.15538 \cdot 10^{-7} \cdot L$$  \hspace{1cm} (9)

$$r = -0.882719$$  \hspace{1cm} (10)

- in the case of pipework built into rings, no relationship between the length of the pipework and the value of the stationary index of readiness was found.
The obtained results show that reliability decreases with an increase in the length of the pipework. With the help of equations, it is possible to predetermine whether a given water distribution system will require additional reliability solutions before carrying out basic calculations.

5. Summary and conclusions

This paper discusses reliability issues in rural water distribution systems. Based on hydraulic calculations and the reliability of selected rural water distribution systems, the interrelationship was demonstrated between reliability levels, the pipework for the water supply system and the length of that pipework. In the case of manifolds, fairly strong relationships were found indicating a decrease in reliability, which is associated with the increase in the length of such pipework. There is no dependency on the pipework in the rings – this may be due to the small amount of such pipework found in rural group waterworks where branched systems predominate. On this basis, it can be stated that when designing water distribution systems, it is desirable to aim to shorten manifolds in order to increase reliability; however, it should be remembered that any correction to waterworks systems should be preceded by hydraulic analysis.

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