The initial mass function of star clusters that form in turbulent molecular clouds

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ABSTRACT
We simulate the formation and evolution of young star clusters using the combination of smoothed particle hydrodynamics (SPH) simulations and direct \(N\)-body simulations. We start by performing SPH simulations of the giant molecular cloud (GMC) with a turbulent velocity field, a mass of \(4 \times 10^4\) to \(5 \times 10^6\) \(M_\odot\), and a density between \(\rho \sim 1.7 \times 10^3\) and \(170\) \(\mathrm{cm}^{-3}\). We continue the hydrodynamical simulations for a free-fall time-scale \((t_\text{ff} \simeq 0.83\) and \(2.5\) Myr), and analyse the resulting structure of the collapsed cloud. We subsequently replace a density-selected subset of SPH particles with stars by adopting a local star formation efficiency proportional to \(\rho^{1/2}\). As a consequence, the local star formation efficiency exceeds 30 per cent, whereas globally only a few per cent of the gas is converted to stars. The stellar distribution by the time gas is converted to stars is very clumpy, with typically a dozen bound conglomerates that consist of \(100\)–\(10^4\) stars. We continue to evolve the stars dynamically using the collisional \(N\)-body method, which accurately treats all pairwise interactions, stellar collisions and stellar evolution. We analyse the results of the \(N\)-body simulations when the stars have an age of 2 and 10 Myr. During the dynamical simulations, massive clusters grow via hierarchical merging of smaller clusters. The shape of the cluster mass function that originates from an individual molecular cloud is consistent with a Schechter function with a power-law slope of \(\beta = -1.73\) at 2 Myr and \(\beta = -1.67\) at 10 Myr, which fits to observed cluster mass function of the Carina region. The superposition of mass functions have a power-law slope of \(\lesssim -2\), which fits the observed mass function of star clusters in the Milky Way, M31 and M83. We further find that the mass of the most massive cluster formed in a single molecular cloud with a mass of \(M_g\) scales with \(6.1 M_g^{0.51}\) which also agrees with recent observation of the GMC and young clusters in M51.

Key words: methods: numerical – open clusters and associations: general – open clusters and associations: individual: Carina – galaxies: individual: M51, M31, M83 – galaxies: star clusters: general.

1 INTRODUCTION

Observed star-forming regions show filamentary or spumous structure, which appears to be a natural consequence of the star formation process (Andr\'e et al. 2010). Star-forming regions are thought to be the results of the gravitational collapse of giant molecular clouds (GMCs; McKee & Ostriker 2007, and references therein). Once the stars have formed they start to develop a wind, and then the first supernovae explosions occur a few million years later. These outflows cause the residual gas to be blown away. The stellar distribution as a consequence will be supervirial after all the gas is lost, but virial equilibrium is quickly re-established, after which the cluster will have lost some stars and the remaining bound stars eventually form a spherical and centrally concentrated distribution. There are numerous observed examples of such young clusters that are about to emerge from their parental molecular cloud (Lada & Lada 2003, and references therein).

The initial collapse of the molecular cloud is also driven by (magneto)hydrodynamical processes, radiation and chemical reactions, in contrast to the purely dynamical evolution of gas-deprived clusters. These processes have been studied extensively from an observational point of view (Keto & Wood 2006; Zinnecker & Yorke 2007; Zapata et al. 2008) and numerically (Bonnell, Clark & Bate 2008; Peters et al. 2010; Bate 2012; Federrath & Klessen 2012; Krumholz 2012; Krumholz, Klein & McKee 2012b; Krumholz &...
2 NUMERICAL METHODS

In our simulations, we combine a smoothed particle hydrodynamics (SPH) code with a direct N-body code. The method consists of three steps.

(i) Perform an SPH simulation of a turbulent molecular cloud for an initial free-fall time-scale.

(ii) Convert gas particles to stellar particles assuming a SFE, which depends on the local gas density, and remove the residual gas (SPH particles).

(iii) Turn on the direct N-body simulation for integrating the equations of motion of the stellar particles.

The initial conditions for SPH simulations are taken from an isothermal homogeneous gas sphere with a turbulent velocity field (Bonnell et al. 2003) with a spectral index of $k = -3$. The size and total mass for our standard model are $10 \text{ pc}$ and $4 \times 10^6 M_\odot$, respectively. The mean gas density is then $\sim 10 M_\odot \text{ pc}^{-3}(\sim 170 \text{ cm}^{-3})$ assuming that the mean weight per particle is $2.33m_H$ and the free-fall time $t_{ff} = 0.83 \text{ Myr}$.

2.1 The hydrodynamical collapse of the molecular cloud

We initialize the gas cloud by giving it zero total energy (potential plus kinetic). We perform additional simulations with a mean gas density of $\sim 10 M_\odot \text{ pc}^{-3}(\sim 170 \text{ cm}^{-3})$ by increasing the overall dimension of the system and adopting a total mass of $4 \times 10^3 - 2 \times 10^4 M_\odot$. With a mean gas density of $\sim 10 M_\odot \text{ pc}^{-3}$, $t_{ff} = 2.5 \text{ Myr}$. We adopt a gas temperature of 30K. We summarize the initial conditions of the simulations in Table 1. For each initial condition, we perform 1–3 runs with different random seeds for the generation of the turbulence in order to see the run-to-run variation.

The number of random seeds is indicated as ‘s’ in the table.

For our SPH simulations, we adopt $1 M_\odot$ per particle. The SPH softening length ($h$) is chosen such that $\rho h^3 = m_{nb}^\odot$ (Springel & Hernquist 2002), where $\rho$ is the density, $m$ and the particle mass, and $N$ is the target number of neighbour particles. We adopt $N_{nb} = 64$, with which the mass resolution is $\sim 100 M_\odot$. With this setup, the minimum scale we can resolve the Jeans instability is $h = 1.7 \text{ pc}$. These mass and size are comparable to the typical ($\sim 1 \text{ pc}$) size of embedded clusters (Lada & Lada 2003). Our SPH simulations therefore cannot resolve the formation of individual stars. The gravitational softening length is taken to be constant (approximately the smallest softening length encountered in the simulation) $\epsilon_{grav} = 0.1 \text{ pc}$ for the gas particles in order to improve energy and momentum conservation. The resolution of our simulations are lower than recent simulations which aim at simulating star formation in turbulent molecular clouds (Bonnell et al. 2011; Bate 2012). We motivate this relatively low resolution by our aim at reproducing the clumpy structure of star-forming regions, rather than the microscopic details of the star formation process. Of course, ideally we would like to resolve also the latter, but this is currently impractical, at least, and we think that our approach is reasonable and a considerable advance over adopting a simple virialized Plummer sphere (Plummer 1911) or King model (King 1966) to generate the initial realization for the gravitational N-body simulations.

The generation of the initial conditions and the SPH simulations are performed with the Astronomical Multipurpose Software Environment (AMUSE; Pelupessy et al. 2013; Portegies Zwart et al. 2013). AMUSE is a follow-up of the earlier MUSE environment (Portegies Zwart et al. 2009), which was intended to be a general purpose framework for performing large scale astronomical simulations. In the AMUSE framework, we adopted the Python programming language to encapsulate the fundamental physics solvers, which are

1 see http://amusecode.org/.
Table 1. Models for hydrodynamical simulations ('s' indicates the random seeds for the turbulence).

| Model                  | Total mass $M_\odot$ | $N_\text{F}\_s$ | Radius $r_t/(\text{pc})$ | Density $\rho_\text{loc}/(\text{cm}^{-3})$ | Thermal energy $E_\text{t}/E_\text{k}$ | Temperature $T/(\text{K})$ |
|------------------------|----------------------|-----------------|---------------------------|------------------------------------------|------------------------------------|-----------------------------|
| m2M-d10-s18-30K        | $2 \times 10^6$      | $2 \times 10^6$ | 35.9                      | 170                                      | $1.1 \times 10^{-3}$               | 30                          |
| m1M-d100-s7-30K        | $1 \times 10^6$      | $1 \times 10^6$ | 13.4                      | $1.7 \times 10^3$                       | $8.4 \times 10^{-4}$               | 30                          |
| m1M-d100-s12-30K       | $1 \times 10^6$      | $1 \times 10^6$ | 13.4                      | $1.7 \times 10^3$                       | $8.4 \times 10^{-4}$               | 30                          |
| m1M-d100-s10-30K       | $1 \times 10^6$      | $1 \times 10^6$ | 13.4                      | $1.7 \times 10^3$                       | $8.4 \times 10^{-4}$               | 30                          |
| m1M-d10-s3-30K         | $1 \times 10^6$      | $1 \times 10^6$ | 28.5                      | 170                                      | $1.7 \times 10^{-3}$               | 30                          |
| m1M-d10-s4-30K         | $1 \times 10^6$      | $1 \times 10^6$ | 28.5                      | 170                                      | $1.7 \times 10^{-3}$               | 30                          |
| m400k-d100-s1-30K      | $4 \times 10^5$      | $4 \times 10^5$ | 10.0                      | $1.7 \times 10^3$                       | $1.6 \times 10^{-3}$               | 30                          |
| m400k-d100-s5-30K      | $4 \times 10^5$      | $4 \times 10^5$ | 10.0                      | $1.7 \times 10^3$                       | $1.6 \times 10^{-3}$               | 30                          |
| m400k-d100-s11-30K     | $4 \times 10^5$      | $4 \times 10^5$ | 10.0                      | $1.7 \times 10^3$                       | $1.6 \times 10^{-3}$               | 30                          |
| m400k-d100-s2-30K      | $4 \times 10^5$      | $4 \times 10^5$ | 21.0                      | 170                                      | $3.3 \times 10^{-3}$               | 30                          |
| m400k-d100-s3-30K      | $4 \times 10^5$      | $4 \times 10^5$ | 21.0                      | 170                                      | $3.3 \times 10^{-3}$               | 30                          |
| m100k-d100-s2-30K      | $1 \times 10^5$      | $1 \times 10^5$ | 6.2                       | $1.7 \times 10^3$                       | $3.9 \times 10^{-3}$               | 30                          |
| m100k-d100-s6-30K      | $1 \times 10^5$      | $1 \times 10^5$ | 6.2                       | $1.7 \times 10^3$                       | $3.9 \times 10^{-3}$               | 30                          |
| m100k-d100-s13-30K     | $1 \times 10^5$      | $1 \times 10^5$ | 6.2                       | $1.7 \times 10^3$                       | $3.9 \times 10^{-3}$               | 30                          |
| m40k-d100-s20-30K      | $4.1 \times 10^4$    | $4.1 \times 10^4$ | 4.6                      | $1.7 \times 10^3$                       | 0.0071                             | 30                          |
| m40k-d100-s21-30K      | $4.1 \times 10^4$    | $4.1 \times 10^4$ | 4.6                      | $1.7 \times 10^3$                       | 0.0071                             | 30                          |
| m40k-d100-s22-30K      | $4.1 \times 10^4$    | $4.1 \times 10^4$ | 4.6                      | $1.7 \times 10^3$                       | 0.0071                             | 30                          |

2.2 Forming the stars

We continue by analysing the resulting density distribution of the collapsed molecular gas cloud. In Fig. 2, we present a projected image of the density distribution of the gas at an age of 0.75 Myr ($\sim 0.9 t_{\text{ff}}$) after the start of the hydrodynamical simulation. The densest regions reached $\sim 10^6 M_\odot$ pc$^{-3}$, which is consistent to the results from earlier SPH simulations that included star formations through sink particles (Moeckel & Bate 2010).

The conversion of the SPH particles to stars was realized by adopting a local SFE, $\epsilon_{\text{loc}}$, which we calculate using

$$\epsilon_{\text{loc}} = \alpha_{\text{loc}} \sqrt{\frac{\rho}{10^3 (M_\odot \text{pc}^{-3})}} \left(\frac{10^8 M_\odot}{M_\odot}\right)^{\frac{1}{2}}$$

Here, $\rho$ is the local volume density, which is measured at the location of each individual SPH particle in the simulation. The coefficient, $\alpha_{\text{loc}}$, controls the SFE and is a free parameter in our simulations. With this prescription, the local SFE correlates with the instantaneous free-fall time of the gas via the square-root of the gas density. This assumption is motivated by recent results that indicate that the star formation rate scales with the free-fall time (Krumholz, Dekel & McKee 2012a). We adopted $\alpha_{\text{loc}} = 0.02$ for the models with $\rho_\text{c} = 100 M_\odot$ pc$^{-3}$, which is similar to what was obtained by Krumholz et al. (2012a). We chose a higher value of $\alpha_{\text{loc}} = 0.04$ for $\rho_\text{c} = 170 $ cm$^{-3}$, because in these cases the time for a part of the system to evolve from the moment when it reaches 1700 cm$^{-3}$ to the end of the hydrodynamical simulation is twice as long as the free-fall time of the models with 1700 cm$^{-3}$.

We replace the densest SPH particles with stellar particles by adopting the local SFE of equation (2). This resulted in the SFE in the dense regions ($\rho \geq 1.7 \times 10^4$ cm$^{-3}$ i.e. $10^3 M_\odot$ pc$^{-3}$), $\epsilon_4$, of $20$–$30$ per cent which is consistent with the observed SFE (Lada & Lada 2003). The residual gas is assumed to be ejected from the system. In Fig. 3, we present the stellar distribution that is obtained from the hydrodynamical simulation after the SPH particles have been converted to stars and the residual gas is removed.

The positions and velocities of the stars are identical to those of the SPH particles from which they formed. The mass of a star was written in high-performance compiled computer code. The flexibility of AMUSE makes it possible to perform simulations with one implementation of the numerical solver, and then repeat the same calculation with a different solver by changing one line in the AMUSE-PYTHON script.

As the engine for performing the hydrodynamical simulation we adopted the SPH code F$\_s$ (Hernquist & Katz 1989; Gerritsen & Icke 1997; pelupessy, van der Werf & Icke 2004; pelupessy 2005). Within F$\_s$, the internal time step is controlled internally, but we constrain this time step from within the framework to 0.025 Myr.

We stop the hydrodynamical simulation at $0.9 t_{\text{ff}}$. By this time, the volume density reaches $10^8$ cm$^{-3}$ ($10^6 M_\odot$ pc$^{-3}$) and the surface density reaches a value of 10 g cm$^{-2}$ ($10^3 M_\odot$ pc$^{-2}$). In Fig. 1, we present an example of the gas density distribution at the moment we stop the hydrodynamical calculations.

Figure 1. Gas surface density at an age of 0.9$t_{\text{ff}}$ for model m400k-d100-30K.
Cluster initial mass function

2.3 The dynamical evolution of the cluster

We now use the stellar masses, positions and velocities as initial realizations for our \( N \)-body calculations in which we study the dynamical evolution of the stellar system. For convenience, we associate the moment at which we start the \( N \)-body simulations with \( t = 0 \) Myr. The forces between each pair of stars is calculated directly, and the numerical integration of the equations of motion was performed using the sixth-order Hermite scheme (Nitadori & Makino 2008). The \( N \)-body code runs without softening and with a time step parameters \( \eta = 0.1-0.3 \). The energy error was less than 0.1 per cent for all simulations. It is small enough for obtaining a scientifically interpretable result in such \( N \)-body simulations (Portegies Zwart & Boekholt 2014).

If two stars approach each other closer than the sum of their radii, we resolve the collision by summing the mass and conserving the angular momentum. The stellar radius was calculated using the description in (Hurley, Pols & Tout 2000; Toonen, Nelemans & Portegies Zwart 2012) and for stars \( > 100 \, M_\odot \) we extrapolated the results (see Fujii et al. 2009, 2012, for the details). We only adopted very simple stellar evolution prescription in which a star turns into a black hole directly after the main sequence (Hurley et al. 2000).

The supernovae were assumed to be symmetric and therefore no natal kick was delivered to the compact remnant. By the end of the simulations (10 Myr), stars with a mass of \( \gtrsim 20 \, M_\odot \) reaches the end of their main-sequence lifetime and evolve to black holes.

3 CLUSTER MF

3.1 Cluster finding

At an age of 2 Myr and at 10 Myr, we interrupt the simulations to study the clumps in the stellar distribution. We identify these clumps as star clusters.

These clusters are detected using the HOP (Eisenstein & Hut 1998) clump finding algorithm (which is also incorporated in the AMUSE framework). The outer cut-off density (somewhat related to the density of an individual clump) was set to \( \rho_{\text{out}} = 4.5 \, M_\odot/(4\pi r_\text{h}^3) \), which is three times the half-mass density of the entire system. Other parameters in HOP are the number of particles to calculate the local density, for which we adopted \( N_{\text{dense}} = 64 \), the number of particles for neighbour search \( (N_{\text{hop}} = 64) \) and the number of particles of neighbours to determine for two groups to merge \( (N_{\text{merge}} = 4) \).

Because star clusters have a relatively high density contrasts compared to dark matter haloes, to which the method is applied with the default parameters, we adopted \( b\rho_{\text{cut}} \) for the saddle density.
threshold and 10\rho_{\text{cut}} for the peak density threshold. We do not include clusters with fewer than 64 stars in our analysis, because both the SPH simulations and the clump finding method used for analysis cannot resolve them. Such small clusters hardly ever exceed 100 M_\odot, which is consistent with the generally adopted minimum mass for a star cluster, regardless of the arbitrariness of this choice. We adopted these parameter because HOP was most successful in detecting all the clusters, but verified that changing the parameters does not influence our results qualitatively. The clumps detected are not necessarily bound, but we detected the members based on geometry. This might not be what a theorist normally would identify as a cluster, but from an observational point of view it is often hard to separate out the unbound stars from the bound stars. With the adopted method, we mimic an observational identification criterion.

After identifying all clusters, we determine their total mass and half-mass radius. In Fig. 4, we present two examples at 2 and at 10 Myr of the clusters identified using this procedure.

### 3.2 The star-cluster MF

For each simulation, we can now construct an MF of star clusters. In Fig. 5, we present the cumulative mass distribution for star clusters in our simulations. The offset scatter among the models is large, but the power of the MF is similar, irrespective of the model. We compare the obtained MFs with the Schechter function,

\[ \phi(M) = \frac{dN}{dM} \propto M^\beta \exp\left( -\frac{M}{M_{\text{cut}}} \right). \]  

Here, \( M \) is the mass of clusters and \( M_{\text{cut}} \) is the cut-off mass. Integration of equation (3) results in the cumulative MF, which has the form

\[ N(M) = \int_0^M \phi(M) dM = M^{\beta+1} \exp\left( -\frac{M}{M_{\text{cut}}} \right). \]  

We use equation (4) to fit (using least mean squares) the mass distribution of the clusters we obtained in each of our simulations. The average of the best-fitting parameter for all models together is \( \beta = -1.55 \pm 0.41 \) for the slope of the MF. The uncertainty in the fitting procedure for determining \( M_{\text{cut}} \) is very large.

Even though we were unable to determine a reliable value for \( M_{\text{cut}} \), we can use the mass of the most massive cluster (\( M_{c,\text{max}} \)) as \( M_{\text{cut}} \). Scaling equation (4) by assuming that \( M_{\text{cut}} = M_{c,\text{max}} \), the cumulative MF becomes

\[ N(M) = \frac{AM^\beta \exp\left( -\frac{M}{M_{c,\text{max}}} \right)}{M_{\text{cut}} \exp\left( -1 \right)}. \]  

Here, \( A \) is a factor and we scaled in such a way that \( N = A \) for \( M = M_{c,\text{max}} \). Fitting this function to the cluster MFs results in \( \beta = -1.71 \pm 0.18 \). In Table 3, we give for each model the maximum cluster mass and best-fitting parameters.

There appears to be a clear relation between the most massive cluster (\( M_{c,\text{max}} \)) and the initial total mass in gas (\( M_{g} \)), which we fitted using least squares to the form \( M_{c,\text{max}} = 6.3M_{g}^{0.51} \). In Fig. 6, we overplot the results of the simulations with the fitted function (black thick dashed line). To compare with observations, we overplot recent results of the mass of the most massive GMCs and star clusters in different regions in M51 (Hughes et al. 2013). If we consider that the most massive GMC forms the most massive star cluster, this observational result is directly comparable to our results. Interestingly, this relation is also quite consistent with the relation between the mass of the most massive star (\( M_{\text{max}} \)) and its host cluster (\( M_{c,\text{max}} \)) found in observations (Larson 2003; Pflamm-Altenburg, Weidner & Kroupa 2007), simulations (Bonnell, Vine & Bate 2004), and theoretical models (Elmegreen 2002). We make the connection between this relation and stellar scales by adopting a minimum stellar mass (brown dwarf) of 0.01 M_\odot which with a 50 per cent SFE must have formed from a gas cloud with a mass of 0.02 M_\odot. When we include this point as a boundary condition in the fitting procedure, we obtain \( M_{c,\text{max}} = 0.20M_{g}^{0.76} \) as a best fit to the data (including this artificial point). This result does not strongly depend on the value of the artificial point. If we adopt the stellar mass of 0.5 M_\odot and the parental gas mass of 2.15 M_\odot, which is obtained from the relation between the gas mass and the stellar mass in Bonnell et al. (2004), we obtain \( M_{c,\text{max}} = 0.28M_{g}^{0.74} \) using this empirical relation, we estimate the mass of the most massive star cluster in the Milky Way (MW) that formed from the most massive GMC. According to our analysis, the most massive GMC in the MW is about \( \sim 10^5 M_\odot \) (Murray 2011), and it could form a star cluster with a mass of \( \sim 3 \times 10^4 M_\odot \). This estimate for the most massive star cluster is consistent with that of Westerlund 1 (3 \times 10^5 M_\odot), RSGC01, 02, 03 (3–4 \times 10^4 M_\odot) and Arches (2 \times 10^4 M_\odot; Portegies Zwart et al. 2010). For clarity, we present in Fig. 6 the observed MW cluster and GMC.

### Table 2. Models for N-body simulations.

| Model          | Total mass \( M_*(M_\odot) \) | \( N_\text{particles} \) | Virial ratio \( Q_{\text{vir}} \) | SFE (Global) | SFE (Dense) |
|----------------|---------------------------------|--------------------------|---------------------------------|--------------|-------------|
| m2M-dl0-s18-30K| \( 1.0 \times 10^5 \)          | 99546                    | 3.9                             | 0.050        | 0.39        |
| m1M-dl0-s7-30K | \( 1.1 \times 10^5 \)          | 109952                   | 6.1                             | 0.11         | 0.26        |
| m1M-dl0-s12-30K| \( 9.4 \times 10^4 \)          | 94464                    | 2.2                             | 0.096        | 0.23        |
| m1M-dl0-s3-30K | \( 7.8 \times 10^4 \)          | 78201                    | 0.66                            | 0.079        | 0.54        |
| m1M-dl0-s4-30K | \( 5.7 \times 10^4 \)          | 56681                    | 2.2                             | 0.058        | 0.41        |
| m400k-dl0-s3-30K| \( 3.0 \times 10^4 \)          | 30487                    | 2.4                             | 0.074        | 0.21        |
| m400k-dl0-s5-30K| \( 2.6 \times 10^4 \)          | 24211                    | 4.8                             | 0.059        | 0.16        |
| m400k-dl10-s11-30K| \( 4.0 \times 10^4 \)        | 39933                    | 1.5                             | 0.074        | 0.26        |
| m400k-dl10-s8-30K| \( 1.9 \times 10^4 \)          | 19236                    | 2.8                             | 0.047        | 0.39        |
| m400k-dl10-s9-30K| \( 2.0 \times 10^4 \)          | 19672                    | 2.6                             | 0.047        | 0.36        |
| m100k-dl0-s2-30K| \( 1.3 \times 10^4 \)          | 12833                    | 0.58                            | 0.13         | 0.34        |
| m100k-dl0-s6-30K| \( 4.6 \times 10^3 \)          | 4572                     | 7.8                             | 0.045        | 0.14        |
| m100k-dl0-s6-30K| \( 4.6 \times 10^3 \)          | 4572                     | 7.8                             | 0.045        | 0.14        |
| m100k-dl0-s10-30K| \( 8.0 \times 10^3 \)          | 7987                     | 1.9                             | 0.079        | 0.22        |
| m40k-dl10-s20-30K | \( 2.9 \times 10^3 \)          | 2866                    | 1.7                             | 0.079        | 0.21        |
| m40k-dl10-s21-30K| \( 3.2 \times 10^3 \)          | 3175                     | 1.9                             | 0.077        | 0.21        |
| m40k-dl10-s22-30K| \( 2.1 \times 10^3 \)          | 2073                     | 2.0                             | 0.051        | 0.16        |
When we adopt the relation $M_{\text{cut}} = M_{\text{max},c} = 0.20M_0^{0.76}$, equation (5) still fits satisfactorily to the simulations, which then results in $\beta = -1.73 \pm 0.17$ and $A = 0.64 \pm 0.29$. And adopting $M_{\text{cut}} = M_{\text{max},c} = 6.3M_0^{0.51}$ results in $\beta = -1.75 \pm 0.17$ and $A = 0.56 \pm 0.32$. The values of $A$ and $\beta$ for each model are presented in Table 3. We ignore models in which fewer than five clumps were detected, because the resulting statistics becomes unreliable. This fit is presented as the dashed lines in Fig. 5 (indicated with 'fitted model'). This relation is slightly shallower than the observed power-law ($\beta \approx -2$) MF for massive clusters in Galactic disc and starburst galaxies (Portegies Zwart et al. 2010, and references therein). We discuss this in Section 4. Note that the run-to-run variation in our simulations is relatively large. The mass of the most massive cluster, for example, varies by about an order of magnitude. Model m40k-d100-30K models are an extreme case (see blue squares in Fig. 5).

We also compared our simulation results with the observed young-cluster MF in the MW (data from Lada & Lada 2003; Piskunov et al. 2008; Portegies Zwart et al. 2010), and in the Carina region (data from Feigelson et al. 2011), which fits $\beta = -1.66 \pm 0.01$. This curve is presented in Fig. 5 as the solid red curve. The observed MF is consistent with our models for $M_\gamma \simeq 10^6M_\odot$.

In our simulations, the SFE and consequently the maximum mass of a star clusters, depends slightly on the initial density of the molecular cloud. The total stellar mass correlates with $\sqrt{\rho}$, because we assumed the local SFE to depend on the local gas density. As a consequence, the SFE is highest in the densest regions.

### 3.3 The secular evolution of the cluster MF due to mergers

In our simulations, clusters form hierarchically; more massive clusters form from repeated mergers. Gravitationally bound small stellar clumps form quickly after the residual gas is removed. Each clump corresponds to one of the density peaks in the parental gas distribution (see Fig. 2). In the first few Myr, the clumps grow in mass primarily by accreting smaller structures, but by an age of about 2 Myr, the cluster population has almost established itself. At that moment, $\sim 30$ per cent of the stars belong to a cluster (see $M_{\text{cl}}/M_\gamma$ in Table 3), and the number of clusters drops in 10 Myr to $\sim 80$ per cent.

The most massive clusters tend to accrete some smaller clusters. For example in model m400k-d100-s1-30K, the number of clumps drops from 15 at 2 Myr to 9 at an age of 10 Myr. During this phase, the distance between clusters increases because initially the entire system is unbound, and the merger process stops in due time. In addition, the smallest clusters that are still around at 2 Myr have disappeared by an age of 10 Myr due to evaporation by relaxation process.

We now investigate the location where the member stars of the clusters formed. In Fig. 7, we have coloured the initial position of stars that at an age of 10 Myr belong to one cluster. Stars located near each other within a few pc typically merge to one cluster. The density peaks that at 10 Myr still belong to a cluster are presented in the bottom panel of Fig. 7. We identify several density peaks, each of which eventually (at an age of 10 Myr) corresponds to one cluster. From the figure, we see that a minimum gas density...
Figure 5. Cumulative cluster MF obtained from the simulations with a temperature of 30K when the stars had an age of 2 Myr. The colours represent different masses of initial molecular clouds: cyan, green, magenta, blue and yellow indicate $M_g$ of $2 \times 10^6$, $10^6$, $4 \times 10^5$, $10^5$ and $4 \times 10^4 \, M_\odot$, respectively. The dashed curves give the fitted MF (see equation 5). We adopt $A = 0.64$, $\beta = -1.73$ and $M_{c,\text{max}} = 0.20M_\odot^{0.76}$. The red dashed curve is the MF of the MW young clusters (<3 Myr) within 1 kpc from the Sun (indicated by $D < 1 \, \text{kpc}$). The data is from (Lada & Lada 2003; Piskunov et al. 2008; Portegies Zwart, McMillan & Gieles 2010). We here assumed that all the (embedded) clusters in Lada & Lada (2003) are younger than 3 Myr. Red thick curve is the cluster MF of the Carina region (Feigelson et al. 2011).

Table 3. The results of simulations at $t = 2 \, \text{Myr}$. $M_{c,\text{cl}}/M_\odot$ is the stellar mass fraction which belongs to clusters. $M_{c,\text{max}}$ is the mass of the most massive clusters formed in the simulations. $N_c$ is the number of clusters. $\beta_1$ is the power of the fitted cluster MF adopting the value of $M_{c,\text{max}}$ obtained from the simulations. $\beta_2$ and $A$ are the power and the factor of the fitted cluster MF (equation 5) but with $M_{c,\text{max}} = 0.20M_\odot^{0.76}$. The fitting results exist only for models with $N_c > 4$. Averaging the results, we obtain $\beta_1 = -1.71 \pm 0.18$, $\beta_2 = -1.73 \pm 0.17$ and $A = 0.64 \pm 0.29$.

| Model       | $M_{c,\text{cl}}/M_\odot$ | $M_{c,\text{max}}(M_\odot)$ | $N_c$ | $\beta_1$ | $\beta_2$ | $A$   |
|-------------|-----------------------------|--------------------------------|-------|-----------|-----------|-------|
| m2M-d10-s18-30K | 0.31                        | $3.1 \times 10^4$       | 61    | -1.68 ± 0.02 | -1.60 ± 0.08 | 0.93 ± 0.15 |
| m1M-d100-s7-30K | 0.43                        | $9.5 \times 10^3$       | 51    | -1.65 ± 0.02 | -1.64 ± 0.02 | 1.13 ± 0.10 |
| m1M-d100-s12-30K | 0.40                        | $8.9 \times 10^3$       | 50    | -1.74 ± 0.02 | -1.73 ± 0.02 | 0.72 ± 0.05 |
| m1M-d10-s3-30K | 0.42                        | $2.4 \times 10^4$       | 30    | -2.16 ± 0.04 | -2.14 ± 0.04 | 0.90 ± 0.01 |
| m1M-d10-s4-30K | 0.33                        | $2.5 \times 10^3$       | 45    | -1.93 ± 0.04 | -2.00 ± 0.03 | 0.26 ± 0.03 |
| m400k-d100-s1-30K | 0.33                        | $3.0 \times 10^3$       | 15    | -1.53 ± 0.05 | -1.55 ± 0.05 | 0.73 ± 0.12 |
| m400k-d100-s5-30K | 0.40                        | $2.7 \times 10^3$       | 16    | -1.59 ± 0.07 | -1.62 ± 0.07 | 0.82 ± 0.16 |
| m400k-d100-s11-30K | 0.37                        | $7.4 \times 10^3$       | 14    | -1.64 ± 0.04 | -1.58 ± 0.04 | 0.78 ± 0.11 |
| m400k-d100-s8-30K | 0.38                        | $1.4 \times 10^3$       | 18    | -1.63 ± 0.05 | -1.71 ± 0.04 | 0.50 ± 0.06 |
| m400k-d100-s9-30K | 0.36                        | $2.1 \times 10^3$       | 17    | -1.71 ± 0.06 | -1.75 ± 0.06 | 0.46 ± 0.08 |
| m100k-d100-s2-30K | 0.41                        | $5.0 \times 10^3$       | 4     | -           | -           | -     |
| m100k-d100-s6-30K | 0.35                        | $3.9 \times 10^2$       | 7     | -1.46 ± 0.20 | -1.81 ± 0.22 | 0.38 ± 0.17 |
| m100k-d100-s13-30K | 0.45                        | $8.5 \times 10^2$       | 11    | -1.51 ± 0.08 | -1.60 ± 0.08 | 0.08 ± 0.16 |
| m40k-d100-s20-30K | 0.35                        | $4.9 \times 10^2$       | 4     | -           | -           | -     |
| m40k-d100-s21-30K | 0.28                        | $3.2 \times 10^2$       | 2     | -           | -           | -     |
| m40k-d100-s22-30K | 0.28                        | $8.5 \times 10^2$       | 2     | -           | -           | -     |
of \( \sim 10^9 \, M_\odot \, pc^{-3} \) is required to form a cluster which survives for 10 Myr. This seems to be related the SFE-law we adopted (see equation 2). With a gas density of \( 1.6 \times 10^2 \, M_\odot \, pc^{-3} \), the SFE \( \sim 0.5 \), and the mass in regions with a density \( \gtrsim 10^2 \, M_\odot \, pc^{-3} \) are dominated by stars; as a consequence these regions easily survive the gas expulsion.

The relationship between the maximum cluster mass and molecular cloud mass does not change appreciably from 2 to 10 Myr. In Fig. 6, we plot the results obtained from our simulations and lines for \( M_{c,max} = 6.3 \, M_\odot^{0.51} \) and \( M_{c,max} = 0.20 \, M_\odot^{0.76} \).

In Fig. 8, we present the mass distribution of star clusters as a function of time. The slope of the cluster MF becomes shallower between 2 and 10 Myr, even though the smaller number of clusters at later age. At \( t = 10 \) Myr, the slope of the cluster MF becomes \( \beta = -1.56 \pm 0.14 \) for the simple fitting to equation (4). If we assume that \( M_c = M_{c,max} \) obtained from the simulations, we obtain \( \beta = -1.53 \pm 0.16 \). Assuming that \( M_{c,max} = 0.20 \, M_\odot^{0.76} \), the slope of the cluster MF becomes \( \beta = -1.67 \pm 0.30 \). The best-fitting parameters for each model are summarized in Table 4. The flatter slope is a natural consequence of the evaporation of the smallest clusters and hierarchical merging, in which lower mass clumps merge to form massive clumps. After 10 Myr clusters stop merging, because the entire system is unbound (see virial ratio of the system in Table 2).

Figure 6. The relation between the mass of the most massive cluster \( (M_{c,max}) \) and the total mass of the molecular cloud \( (M_g) \) or between the mass of the most massive star \( (\text{m}_{\text{max}}) \) and the total mass of its host cluster \( (M_c) \). Black dots and triangles indicate the results of our simulations with 30K for \( t = 2 \) and 10 Myr, respectively. Thick dashed line indicates the fitted function to our result \( (M_{c,max} = 6.3 \, M_\odot^{0.51}) \), and thick full line is also fitted but forced to at \( m_c = 0.01 \, M_\odot \) at \( M_g = 0.02 \, M_\odot \) \( (M_{c,max} = 0.20 \, M_\odot^{0.76}) \). Red pluses indicate the observed relation between the mass of the most massive GMCs and clusters in different environments in M51 (Hughes et al. 2013). Star indicates the most massive cluster and GMC in the MW. The colour thin dashed, dash–dotted, full and dotted lines give the relation between the mass of the most massive star as a function of its host cluster mass; \( m_{c,max} = 0.27 \, M_\odot^{0.25} \) (Elmegreen 2002), \( m_{c,max} = 1.2 \, M_\odot^{0.43} \) (Larson 2003), \( m_{c,max} = 0.3 \, M_\odot^{0.51} \) (Bonnell et al. 2004) and \( m_{c,max} = 0.4 \, M_\odot^{0.67} \) (Pflamm-Altenburg et al. 2007), respectively (see also Weidner, Kroupa & Bonnell 2010). The length of each line is proportional to the mass range from which the relation is obtained.

Figure 7. Projected position of stars after the residual gas has been removed. Each colour identifies the cluster to which the star belongs at an age of \( 10 \) Myr. The data is from model m400k-d100-s1-30K. We used the same colours as in Fig. 4. The projected stellar density is presented as a grey-scale. In the bottom panel, we show the density distribution along one dimension, and gives the same data as is presented in Fig. 2, but for SPH particles which are converted to stars. The colours are the same as the top panel. Red, green, blue, cyan and magenta clusters are relatively massive, and they are 2800, 5300, 690, 360 and 230 \( M_\odot \), respectively.

Figure 8. Same as Fig. 5, but for \( t = 10 \) Myr. We adopt \( A = 0.63 \), \( \beta = -1.67 \) and \( M_{c,max} = 0.20 \, M_\odot^{0.76} \) for the fitting MF (see equation 5).
4 TOTAL CLUSTER MF IN DISC GALAXIES

In order to compare our simulations with the observed cluster MF in a galactic environment, we assume the GMC MF to follow a power-law down to 100 $M_\odot$. We ignore less massive clouds because they are unable to form sufficiently massive clusters to compare with the observations. For the MW, we adopt a power of $-1.45$ (Planck Collaboration XXIII 2011) and for M31 we adopt $-0.9$ (Kirk et al. 2015). For M82, we also adopt a power of $-1.45$, because there is insufficient data to properly fit the distribution. We assume that each individual GMC forms a conglomerate of clusters that follow equation (5). Here, we adopt $\beta = -1.73$ and $A = 0.64$ for $t = 2$ Myr, $\beta = -1.67$ and $A = 0.63$ for $t = 10$ Myr, and $M_{c,\text{max}} = 0.20M_\odot$ for both. As a consequence, a distribution of GMCs forms a superposition of multiple cluster MFs; one for each GMC in the galaxy. The MFs for all young clusters in such a galaxy is presented in Fig. 9, which we derived using the fitting functions at 2 Myr (dashes) and at 10 Myr (dotted curves). We assume that the total mass of the molecular gas is $2 \times 10^7 M_\odot$ and $8 \times 10^5 M_\odot$ for M31 and M83, respectively. In this calculation, we do not allow any GMC to exceed half the total gas mass in the galaxy. Observationally, the total mass of the molecular gas in each galaxy is $3.6 \times 10^7 M_\odot$ for M31 (Nieten et al. 2006) and $2.5 \times 10^6 M_\odot$ for M83 (Crosthwaite et al. 2002). For young clusters in the MW and within 1 kpc of the Sun, we derive the total mass in molecular clouds of $10^6 M_\odot$ in order to match the cluster MF. This indicates that the total mass in molecular clouds within 1 kpc is $\sim 10^6 M_\odot$.

Because of the cut-off in the Schechter MF, the power-law slope of the superposed MF is $\leq -2$. This value is consistent with cluster MFs observed in nearby galaxies (Portegies Zwart et al. 2010). The solid curves in Fig. 9 indicate the observed MF for young clusters in the MW (within a distance of $D = 1$ kpc from the Sun), M31 and M83. With ‘young’, we here indicate clusters with an age comparable to the typical free-fall time of the GMC in each galaxy. If we consider that GMCs collapse on this time-scale and form stars, the free-fall time-scale of GMCs would be associated with the star formation time-scale, as was suggested in Krumholz et al. (2012a). The typical free-fall time-scale estimated from observations is $\sim 25$ Myr for M31 and $\sim 70$ Myr for M83 (Krumholz et al. 2012a). These values are much longer than the free-fall time-scales in our models, but we consider that the final SFE does not change much even if the initial free-fall time-scale is longer. For the MW, we adopted a typical value for disc galaxies of 30 Myr. Our model for the total cluster MF agrees with the observed cluster MF in the MW, M31 and M83.

We have not considered the evolution of star clusters beyond 10 Myr. At these later ages, the cluster-disruption process is expected to continue to change the shape of the cluster MF (Bastian et al. 2012), in particular by stellar mass loss, internal two-body relaxation and the external influences of the galactic tidal field. Some of the discrepancies between our simulations and the observation are probably caused by our adopted limited time-scale of 10 Myr, and by ignoring the global potential of the parent galaxy.

## 5 DISCUSSION

### 5.1 The self-similarity between the stellar MF and the cluster MF

The similarity between the cluster MF and the stellar MF seems to originates from the self-similar structure of GMC, as was also discussed in Elmegreen (2002). From an observational perspective, the relation between the self-similar structure of GMCs and star-forming regions has been suggested by Elmegreen & Falgarone (1996), Elmegreen & Scalo (2004) and Sánchez et al. (2010). As can be seen in Fig. 7, more massive clusters tend to form in denser regions. If the resolution of our simulation was sufficiently high to resolve the formation of individual stars, we expect to see spikes in the density distribution which correspond to individual stars. We then would expect that more massive stars form in more massive and denser regions. The most massive star will then be born in the most massive cluster in the system. The birth of the most massive star cluster as well as the most massive individual star is then limited by the highest mass and densest GMCs in the MW. A similar argument was discussed in Weidner & Kroupa (2005) and Weidner et al. (2010).

| Model                  | $M_{c,\text{max}}(M_\odot)$ | $N_\ast$ | $\beta_1$   | $\beta_2$ | $A$       |
|------------------------|-----------------------------|----------|-------------|------------|-----------|
| m400k-d100-s1-30K      | $5.3 \times 10^3$       | 9        | $-1.41 \pm 0.05$ | $-1.35 \pm 0.06$ | $0.84 \pm 0.16$ |
| m400k-d100-s5-30K      | $2.3 \times 10^3$       | 14       | $-1.60 \pm 0.05$ | $-1.58 \pm 0.04$ | $0.83 \pm 0.10$ |
| m100k-d100-s2-30K      | $5.2 \times 10^3$       | 4        | $-1.75 \pm 0.08$ | $-2.07 \pm 0.08$ | $0.20 \pm 0.03$ |
| m100k-d100-s6-30K      | $4.6 \times 10^2$       | 5        | $-1.58 \pm 0.05$ | $-1.57 \pm 0.05$ | $0.20 \pm 0.03$ |

Table 4. The results of simulations at 10 Myr. Averaging the results, we obtain $\beta_1 = -1.53 \pm 0.16$, $\beta_2 = -1.67 \pm 0.30$ and $A = 0.63 \pm 0.30$. Here, we assume $M_{c,\text{max}} = 0.20M_\odot^{0.76}$. The observational data is from Bastian et al. (2010). The solid curves in Fig. 9 indicate the observed MF for young clusters in the MW (within a distance of $D = 1$ kpc from the Sun), M31 and M83. With ‘young’, we here indicate clusters with an age comparable to the typical free-fall time of the GMC in each galaxy. If we consider that GMCs collapse on this time-scale and form stars, the free-fall time-scale of GMCs would be associated with the star formation time-scale, as was suggested in Krumholz et al. (2012a). The typical free-fall time-scale estimated from observations is $\sim 25$ Myr for M31 and $\sim 70$ Myr for M83 (Krumholz et al. 2012a). These values are much longer than the free-fall time-scales in our models, but we consider that the final SFE does not change much even if the initial free-fall time-scale is longer. For the MW, we adopted a typical value for disc galaxies of 30 Myr. Our model for the total cluster MF agrees with the observed cluster MF in the MW, M31 and M83.
In our simulations, we obtain the relation between GMC mass and the maximum cluster mass of $M_{c,\text{max}} = 6.3M_\odot^{0.51}$ or $M_{c,\text{max}} = 0.20M_\odot^{0.76}$. If we apply these relations to the MW, in which the most massive GMC is $10^7 M_\odot$ (Murray 2011), it can form at most a cluster of a few $10^5 M_\odot$, which again forms a single star of at most $\sim 100 M_\odot$. Note that even if the most massive stars are born in the regions with the highest density, this location is not necessarily associated with the most massive cluster. In this sense, we consider that the $m_{\text{max}} - M_c$ relation is a statistical result, as was suggested earlier by Elmegreen (2006), Bastian, Covey & Meyer (2010) and Bressert et al. (2010).

Although we observe a similarity between the largest structures in the system; stellar MF and cluster MF, the slopes are somewhat different. In order to follow the formation of individual stars, higher resolution simulations of more massive clusters with the appropriate physics would be required. Upon performing such simulations, we anticipate that the MF of individual stars would have a slope consistent with the Salpeter (Salpeter 1955) slope, as was suggested by (Bate, Bonnell & Bromm 2003; Bonnell et al. 2003). In Fig. 10, we present a schematic picture of the formation of a cluster (left) down to individual stars (right), as we perceive it from our simulations. In the simulations where individual stars remain unresolved clusters can form, but by increasing the resolution more fine-structure in the density distribution of the gas will appear. Those density peaks are associated with individual stars, and groups of peaks are associated with clusters of stars. If the peak density exceeds the critical density for star cluster formation ($\rho_{c,\text{cl}} \sim 10^4 M_\odot pc^{-3}$ in our simulation), the cluster survives for at least 10 Myr. Some clusters will merge to a larger and more massive cluster. If we were able to resolve individual star formation, we would resolve the cluster formation peaks to ensembles of peaks that correspond to individual stars.

### 5.2 Higher temperature simulations

We adopt a temperature of 30K for our standard models, but we also perform a series of simulations with a thermal energy of 1 per cent of the kinetic energy in order to see the effect of the temperature. With this setup, the gas temperature exceed 100 K. The other parameters are the same as models with 30K. The initial conditions of the high-temperature models are summarized in Table 5.

**Table 5.** Models for hydrodynamical simulations with higher temperatures (‘s’ indicates the random seeds for the turbulence).

| Model         | Total mass $M_g$ ($M_\odot$) | N of particles $N_g$ | Radius $r_g$ (pc) | Density $\rho_g$ ($cm^{-3}$) | Thermal energy $E_t/E_k$ | Temperature $T$ (K) |
|---------------|-------------------------------|---------------------|-------------------|-------------------------------|--------------------------|---------------------|
| m5M-d10-s1-490K | $5 \times 10^6$ | $5 \times 10^6$ | 49.0              | 170                           | 0.01                     | 490                 |
| m1M-d100-s1-360K | $1 \times 10^6$ | $1 \times 10^6$ | 134               | $1.7 \times 10^3$            | 0.01                     | 360                 |
| m1M-d100-s6-360K | $1 \times 10^6$ | $1 \times 10^6$ | 134               | $1.7 \times 10^3$            | 0.01                     | 360                 |
| m1M-d100-s7-360K | $1 \times 10^6$ | $1 \times 10^6$ | 134               | $1.7 \times 10^3$            | 0.01                     | 360                 |
| m1M-d10-s1-170K  | $1 \times 10^6$ | $1 \times 10^6$ | 28.5              | 170                           | 0.01                     | 170                 |
| m1M-d10-s4-170K  | $1 \times 10^6$ | $1 \times 10^6$ | 28.5              | 170                           | 0.01                     | 170                 |
| m1M-d10-s1-190K  | $4 \times 10^5$ | $4 \times 10^5$ | 10.0              | $1.7 \times 10^3$            | 0.01                     | 190                 |
| m400k-d100-s1-190K | $4 \times 10^5$ | $4 \times 10^5$ | 21.0              | 170                           | 0.01                     | 92                  |
| m400k-d100-s2-190K | $4 \times 10^5$ | $4 \times 10^5$ | 6.2               | $1.7 \times 10^3$            | 0.01                     | 78                  |
| m400k-d100-s5-190K | $4 \times 10^5$ | $4 \times 10^5$ | 4.5               | $1.7 \times 10^3$            | 0.01                     | 43                  |
Table 6. Models for $N$-body simulations based on higher temperature molecular clouds.

| Model                   | Total mass $M_\star(M_\odot)$ | Number of particles $N_\star$ | Virial ratio $Q_{vir}$ | SFE (Global) $\epsilon$ | SFE (Dense) $\epsilon_d$ |
|-------------------------|---------------------------------|-------------------------------|-------------------------|--------------------------|----------------------------|
| m5M-d10-s1-490K         | $1.6 \times 10^5$               | 155 972                       | 5.4                     | 0.032                    | 0.25                       |
| m1M-d100-s1-360K        | $5.8 \times 10^4$               | 57 642                        | 3.0                     | 0.058                    | 0.20                       |
| m1M-d100-s6-360K        | $4.1 \times 10^4$               | 40 510                        | 7.6                     | 0.041                    | 0.13                       |
| m1M-d100-s7-360K        | $6.9 \times 10^4$               | 68 901                        | 1.0                     | 0.070                    | 0.19                       |
| m1M-d10-s1-170K         | $3.1 \times 10^4$               | 31 023                        | 4.6                     | 0.032                    | 0.18                       |
| m1M-d10-s4-170K         | $3.6 \times 10^4$               | 36 224                        | 3.7                     | 0.037                    | 0.30                       |
| m400k-d100-s1-190K      | $3.2 \times 10^4$               | 31 868                        | 2.8                     | 0.078                    | 0.26                       |
| m400k-d100-s2-190K      | $3.0 \times 10^4$               | 30 496                        | 1.5                     | 0.075                    | 0.22                       |
| m400k-d100-s5-190K      | $2.5 \times 10^4$               | 25 419                        | 2.8                     | 0.062                    | 0.19                       |
| m400k-d10-s1-90K        | $1.5 \times 10^4$               | 15 124                        | 3.2                     | 0.037                    | 0.35                       |
| m100k-d100-s1-80K       | $6.4 \times 10^3$               | 6474                          | 1.4                     | 0.063                    | 0.20                       |
| m40k-d100-s1-40K        | $2.5 \times 10^3$               | 2498                          | 2.0                     | 0.062                    | 0.19                       |

The structures obtained from the hydrodynamical simulation are considerably smoother because the fragmentation scale is larger at a higher gas temperature. In Fig. 11, we present the gas surface density at the end of the hydrodynamical simulation for model m400k-d100-190K, which is almost indistinguishable from m400k-d100-30K, but which has a much higher gas temperature of 190K. The density peaks shown in Fig. 12 are less pronounced than in the 30K models (see Fig. 2). Although the number of small clumps seen as density peaks in Fig. 2 are smaller in the high-temperature simulations, the eventual stellar distributions at 2 or 10 Myr, such as the shape of cluster MF obtained from the $N$-body simulations was not much affected by the temperature. In Table 6, we summarize the initial conditions for $N$-body simulations.

The evolution of the stellar system for model m400k-d100-190K is shown in Fig. 13. The position and mass of the clusters formed at 10 Myr in m400k-d100-190K are different than those resulting from model m400k-d100-30K. The stellar composition of the most massive clusters and their initial locations in space is hardly affected by the different temperatures. In Fig. 14, we present, for model m400k-d100-190K, the stellar positions that are contained in the five most massive clusters at an age of 10 Myr.

We investigated the cluster MF for these models at $t=2$ and 10 Myr. The resulting cluster MFs are shown in Figs 15 and 16 for $t=2$ and 10 Myr, respectively. We also fit the results to equation (5). The fitted $\beta$ and $A$ for each model are summarized in Tables 7 and 8. When we adopt the actual values of the most massive clusters as $M_{\star,max}$, we obtain $\beta = -1.52 \pm 0.21$ for the high-temperature...
Cluster initial mass function

Figure 13. The snapshots of the N-body simulation for model m400k-d100-190K.

Figure 14. Projected position of stars after the residual gas has been removed. Each colour identifies the cluster to which the star belongs at an age of 10 Myr. The data is from model m400k-d100-190K. We used the same colours as in Fig. 4. The stellar surface density is presented as a grey-scale. In the bottom panel, we show the density distribution along one dimension, and gives the same data as is presented in Fig. 2, but for SPH particles which are converted to stars. The colours are the same as the top panel. Red, green, blue, cyan and magenta clusters are relatively massive, and they are 4100, 3400, 760, 920 and 123 $M_\odot$, respectively.

Figure 15. Same as Fig. 5, but for models with $>30$ K. The colours represent different masses of initial molecular clouds: orange, green, magenta, blue and yellow indicate $M_g$ of $5 \times 10^6$, $10^6$, $4 \times 10^5$, $10^5$ and $4 \times 10^4$ $M_\odot$, respectively. The dashed curves give the fitted MF (see equation 5). We adopt $A = 0.60$, $\beta = -1.54$ and $M_{c,max} = 0.20 M_\odot^{0.76}$.

models, but $\beta = -1.62 \pm 0.22$ when we include the 30K models. The mass of the most massive clusters are not much different from those for 30K models. The relation between the initial gas mass and the mass of the most massive cluster is $M_{c,max} = 26 M_\odot^{0.39}$ at $t = 2$ Myr only for high-temperature models. By adding artificial points at $M_{c,max} = 0.01 M_\odot$ and $M_g = 0.02 M_\odot$, we obtain $M_{c,max} = 0.17 M_\odot^{0.73}$ at $t = 2$ Myr. If we assume $M_{c,max} = 0.20 M_\odot^{0.76}$ (the same as that obtained in 30K models), we obtain the averaged power-law slope of the fitting function of $\beta = -1.52 \pm 0.22$ and $-1.37 \pm 0.20$ at $t = 2$ and 10 Myr, respectively. These slopes are slightly shallower than those in the low-temperature simulations, but both values are consistent within the uncertainty. We also obtain $A = 0.60 \pm 0.29$ and $0.78 \pm 0.34$ at $t = 2$ and 10 Myr. With these values, the total cluster MF for galaxies obtained from our model is consistent with the observations (see Fig. 17).

The high temperature we assumed here (>100 K) is not realistic for a model of molecular clouds. The main clusters formed from initial conditions with a high temperature, however, are not much
different from those formed from molecular cloud with a realistic temperature. In low-temperature models, there are more clumps in a region which finally merge to a cluster. The clumps merge within a local dynamical time-scale irrespective of the gas temperature. The clumpy structure is lost quickly in the merger process, mainly due to violent relaxation. The formation of massive cluster is therefore not affected by the unrealistically high temperature of the gas. This result is practical when applying this same method to a cluster formation simulations on a galactic scale. In environments where temperature of \( \gtrsim 100 \) K are reached, which is a quite typical threshold temperature of star-forming regions in galaxy simulations, the temperature is sufficiently low to result in clustered star formation. In those cases, we do not necessarily resolve the low-temperature and high-density regions.

6 SUMMARY

We performed \( N \)-body simulations of ensembles of young star clusters. The initial conditions of our dynamical simulations are obtained from smoothed particles hydrodynamical simulations of turbulent molecular clouds. Both calculations, the collapsing molecular cloud and the gravitational dynamical simulations, are performed individually and are separated. As a result, our calculations are not self-consistent, but there is a natural causality.

In our approach we start with the hydrodynamical simulation, which we continue for about a free-fall time-scale. We subsequently analyse the distribution of the gas and assign stellar mass to individual SPH particles. In this procedure, mass is locally not conserved, but globally it is. Only those SPH particles that have a local density in excess of the star formation threshold are assumed to form star. This procedure of adopting a simple density threshold results in a local (core) SFE of about 30 per cent, but only about 1 per cent of the total mass in gas is converted to stars. Individual stars are assigned the positions and velocities of the SPH particles from which they

| Model          | \( M_{\text{c,min}}(M_\odot) \) | \( M_{\text{c,max}}(M_\odot) \) | \( N_c \) | \( \beta_1 \) | \( \beta_2 \) | \( A \) |
|----------------|---------------------------------|---------------------------------|--------|----------|----------|-------|
| m5M-d10-s1-490K | 0.33 \( 1.0 \times 10^4 \)     | 27 \( -1.61 \pm 0.03 \)         | \(-1.63 \pm 0.03 \) | 0.37 \( \pm 0.05 \) |
| m1M-d10-s1-360K | 0.33 \( 8.4 \times 10^3 \)     | 14 \( -1.43 \pm 0.16 \)         | \(-1.43 \pm 0.06 \) | 0.65 \( \pm 0.15 \) |
| m1M-d10-s6-360K | 0.34 \( 4.3 \times 10^3 \)     | 14 \( -1.58 \pm 0.05 \)         | \(-1.62 \pm 0.05 \) | 0.57 \( \pm 0.09 \) |
| m1M-d10-s7-360K | 0.39 \( 7.0 \times 10^3 \)     | 29 \( -1.52 \pm 0.02 \)         | \(-1.52 \pm 0.02 \) | 1.24 \( \pm 0.10 \) |
| m1M-d10-s1-170K | 0.26 \( 5.2 \times 10^3 \)     | 7 \( -1.39 \pm 0.16 \)          | \(-1.41 \pm 0.05 \) | 0.46 \( \pm 0.09 \) |
| m1M-d10-s4-170K | 0.21 \( 2.4 \times 10^3 \)     | 9 \( -1.23 \pm 0.04 \)          | \(-1.40 \pm 0.08 \) | 0.69 \( \pm 0.16 \) |
| m400k-d100-s1-190K | 0.29 \( 3.8 \times 10^3 \)    | 14 \( -1.59 \pm 0.05 \)         | \(-1.45 \pm 0.05 \) | 0.76 \( \pm 0.12 \) |
| m400k-d100-s2-190K | 0.35 \( 9.1 \times 10^3 \)   | 10 \( -1.78 \pm 0.15 \)         | \(-1.76 \pm 0.15 \) | 0.19 \( \pm 0.09 \) |
| m400k-d100-s5-190K | 0.37 \( 1.5 \times 10^3 \)   | 14 \( -1.87 \pm 0.07 \)         | \(-1.85 \pm 0.07 \) | 0.29 \( \pm 0.06 \) |
| m400k-d10-s1-90K | 0.31 \( 1.3 \times 10^3 \)    | 7 \( -1.16 \pm 0.06 \)          | \(-1.35 \pm 0.08 \) | 0.81 \( \pm 0.17 \) |
| m100k-d100-s1-80K | 0.31 \( 1.3 \times 10^3 \)   | 3 \( - \)                     | \(- \)          | \(- \)           |
| m400k-d100-s1-40K | 0.28 \( 2.1 \times 10^2 \)    | 4 \( - \)                     | \(- \)          | \(- \)           |

Table 8. The results of simulations at \( t = 2 \) Myr. Averaging the results, we obtain \( \beta_1 = -1.36 \pm 0.20 \), \( \beta_2 = -1.37 \pm 0.20 \) and \( A = 0.78 \pm 0.34 \) for high-temperature models and \( \beta_1 = -1.43 \pm 0.21 \), \( \beta_2 = -1.46 \pm 0.27 \) and \( A = 0.73 \pm 0.34 \) for all models including 30K models. Here, we assume \( M_{\text{c,max}} = 0.20 \text{ } M_\odot^{0.76} \).

| Model          | \( M_{\text{c,max}}(M_\odot) \) | \( N_c \) | \( \beta_1 \) | \( \beta_2 \) | \( A \) |
|----------------|---------------------------------|--------|----------|----------|-------|
| m1M-d10-s1-360K | 9.4 \( 10^4 \)                  | 9 \( -1.31 \pm 0.07 \)         | \(-1.55 \pm 0.04 \) | 0.55 \( \pm 0.08 \) |
| m1M-d10-s6-360K | 4.0 \( 10^3 \)                  | 14 \( -1.43 \pm 0.14 \)        | \(-1.22 \pm 0.07 \) | 0.79 \( \pm 0.18 \) |
| m1M-d10-s1-170K | 2.9 \( 10^3 \)                  | 5 \( -1.08 \pm 0.05 \)         | \(-1.37 \pm 0.05 \) | 0.88 \( \pm 0.17 \) |
| m1M-d10-s4-170K | 2.2 \( 10^3 \)                  | 12 \( -1.22 \pm 0.03 \)        | \(-1.07 \pm 0.11 \) | 1.28 \( \pm 0.49 \) |
| m400k-d100-s1-190K | 4.1 \( 10^3 \)               | 8 \( -1.24 \pm 0.08 \)         | \(-1.23 \pm 0.08 \) | 1.18 \( \pm 0.30 \) |
| m400k-d100-s2-190K | 9.3 \( 10^3 \)               | 8 \( -1.70 \pm 0.09 \)         | \(-1.68 \pm 0.17 \) | 0.20 \( \pm 0.11 \) |
| m400k-d100-s5-190K | 4.9 \( 10^3 \)               | 13 \( -1.41 \pm 0.05 \)        | \(-1.49 \pm 0.08 \) | 0.61 \( \pm 0.15 \) |
| m100k-d10-s1-80K | 1.2 \( 10^3 \)                  | 4 \( - \)                     | \(- \)          | \(- \)           |
| m40k-d100-s1-40K | 3.2 \( 10^2 \)                  | 3 \( - \)                     | \(- \)          | \(- \)           |
were generated. These mass, positions and velocities are adopted as the initial conditions for the gravitational dynamics simulations.

The distribution of stars resulting from this procedure show a strong hierarchical structure. The distribution of the mass of the stellar clumps are consistent with the Schechter function. In the first 2 Myr, after the stars formed, the MF of clusters resembles a power law with a slope of $\approx -1.73$. This slope becomes shallower with time because of hierarchical merging, to reach a slope of $\approx -1.67$ at an age of 10 Myr.

The shape of the cluster MF in our simulations is consistent with the one observed in active star-forming regions in the MW, such as in the Carina region. We find a relation between the mass of the GMC and the most massive cluster: $6.3M_\odot^{0.51}$ (or $0.20M_\odot^{0.76}$ if we assume that this relation continues down to the stellar mass scale). This relation is consistent with the observed cluster distribution in M51 (Hughes et al. 2013), and it is similar to the observed relation between the most massive star and the total mass of the cluster (Weidner et al. 2010). We therefore conclude that star-forming regions and star-cluster-forming regions have a self-similar structure down to the formation of individual stars.

Using our simulations, we estimate the global galactic cluster MF in the MW, M31 and M83. We satisfactory fitted the distribution of cluster masses for each of these galaxies. The galactic cluster MFs we obtained from our model have a power of $\lesssim -2$, which is consistent with observed cluster MF nearby galaxies.

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