FORWARD-BACKWARD MULTIPLICITY CORRELATIONS
AND LEAKAGE PARAMETER BEHAVIOUR IN
ASYMMETRIC HIGH ENERGY COLLISIONS

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Abstract

Continuing previous work, forward-backward multiplicity correlations are studied in asymmetric collisions in the framework of the weighted superposition mechanism of different classes of events. New parameters for the asymmetric clan distribution and for the particle leakage from clans in one hemisphere to the opposite one are introduced to effectively classify different classes of collisions. This tool should be used to explore forward-backward multiplicity correlations in AB and pA collisions in present and future experiments at RHIC and LHC.
1 Introduction: the weighted superposition mechanism of different classes of events

It has been found quite recently \cite{1} that the weighted superposition of different classes of events with negative binomial (NB) properties (events with and without mini-jets in $pp$ collisions, 2-, 3- or more-jets samples of events in $e^+e^-$ annihilation) reproduces, in the GeV energy range, the available experimental data on forward-backward (FB) multiplicity correlations of the two types of collisions; it is an intriguing result which sheds new light on long range properties in multiparticle dynamics and outlines the experimental consistency of the weighted superposition mechanism of different classes of events. In addition, the same superposition mechanism has been shown to provide in the GeV region an interesting phenomenological tool in order to describe the observed shoulder effect in $n$-charged particle multiplicity distributions (MD’s) and $n$-oscillations in the ratios of $n$-particle factorial moments to the $n$-particle factorial cumulant moments \cite{2, 3, 4}. Effects on which QCD has not (up to now) satisfactory predictions \cite{4}.

In the case of $pp$ collisions, essential part of the theoretical background in the GeV region has been the assumption that, in each substructure or component described by a negative binomial (Pascal) multiplicity distribution [NB(P)MD], independently produced clans (they follow a Poisson distribution) are binomially distributed in the two hemispheres and that logarithmically produced charged particles within each clan distribute themselves again binomially in the two hemispheres with an energy independent ‘leakage parameter.’ This parameter controls the number of particles, generated by clans in one hemisphere, falling in the opposite one and was determined in \cite{1} from the data at 63 GeV and 900 GeV. Accordingly, the correct reproduction in the GeV region of the experimentally observed increase with c.m. energy of the forward-backward multiplicity correlation strength as well as of the relation between the average charged particle multiplicity of particles lying in the backward hemisphere versus the number of charged particles lying in the forward hemisphere (and vice versa) support strongly our approach.

In the case of $e^+e^-$ annihilations, in addition, it should be pointed out that under the same assumptions of binomial distributions of clans and of particles generated by clans in the forward and backward hemispheres, the relatively small value of the forward-backward correlation strength in the total charged particle MD and the absence of FB multiplicity correlations in the separate 2- and 3-jet samples of events, measured by OPAL collaboration \cite{5}, have been also correctly reproduced. Forward-backward correlations in the total charged particle multiplicity distribution are indeed only due here to the superposition effect of the two samples of events. In our approach, in fact, forward-backward multiplicity correlations in the two separate sample of events, each of NB type, turn out to be zero, as no leakage is found from one hemisphere to the other in the two separate samples, and in the total sample resulting from the weighted superposition of two NB(P)MD we predict a correlation which coincides with the experimental one within experimental error \cite{1}.

The striking difference in FB charged particle multiplicity correlation strength for the total sample of events between proton proton collisions in the GeV energy domain and $e^+e^-$ annihilation at LEP energy outlines the deep link between FB multiplicity correlations and long range correlations, which are expected to be quite strong in the first case and relatively weak in the second one. In addition, the observed lack of FB multiplicity correlations in the two separate 2-jet and 3-jet samples of events in $e^+e^-$ suggests that the weak FB correlations seen in this reaction in the total sample of events are entirely due to the superposition of the two separate samples. This fact can be considered indeed an experimental evidence of the weighted superpo-
siton mechanism characteristic of our approach (justified up to now only on the basis of a quite successful description of collective effects in high energy energy phenomenology which QCD is unable at present to describe) and of the presence or absence of charged particle leakage from one hemisphere to the opposite one.

In terms of clan structure analysis of the NB behaviour for each class of events, it is clear that charged particle leakage as well as FB multiplicity and long range correlations are expected to be stronger when particle population per clan is larger, a phenomenon which usually goes together with the reduction of the average number of clans.

It should be pointed out that the occurrence of a larger average number of smaller size clans and of a smaller average number of larger size clans has a suggestive interpretation at parton level, in the framework of a two-step mechanism, and could be related to smaller and larger colour flow densities respectively. In a NB description of final charged particle MD for a single component (soft or semi-hard in \( pp \) collisions, 2- and 3-jet samples of events in \( e^+e^- \) annihilation), after using generalised local parton hadron duality (GLPHD), the first step of the parton production process is dominated by the \( A_q \rightarrow q + g (N_{\text{color}}, \epsilon) \) vertex and the average number of partonic clans corresponds to the average number of bremsstrahlung gluon jets (BGJ), which are effective independent intermediate gluon sources (IIGS), and the second step is controlled by the vertex \( A_g \rightarrow g + g (N_{\text{color}}, \epsilon) \), the gluon self-interaction vertex, whose increase corresponds to an enhancement of parton cascading from the IIGS [6, 7]. \( N_{\text{color}} \) is the number of colours and \( \epsilon \) the fixed cut-off regularization prescription of the theory.

Coming to the difference between \( e^+e^- \) and \( pp \) reactions, it is likely to expect that the observed increase of the average number of clans in \( e^+e^- \) with respect to the \( pp \) case is a consequence of a stronger activity of the first vertex with respect to the second one, and that the opposite will occur for the increase of the average number of partons per partonic clans. It is quite clear that the production of a large average number of partons per clan is a consequence of a longer cascading process, originated by IIGS generated at relatively high virtuality in regions where, being the coupling constant smaller, stronger colour flow between partons should be at work (a situation favoured in \( pp \) collisions). When the average number of partons per clan is relatively small, IIGS are expected to start to be effective at lower virtuality, their cascading becomes shorter and colour exchanges reduced with respect to the previous case (a situation favoured in \( e^+e^- \) annihilation). It seems therefore that the occurrence of stronger FB multiplicity correlations, long range correlations and related particle leakage enhancement from one hemisphere to the opposite one at hadron level are a specular image of stronger cascading from high virtuality IIGS and of larger colour exchange in this region at parton level.

We believe in fact that the understanding of FB multiplicity correlations at hadron level is a possible starting point in order to study new effects of colour quantum number exchanges in multiparticle dynamics.

Accordingly, we decided to continue our search initiated in Ref. [1], where FB correlations have been understood in the GeV region in symmetric reactions (like \( pp \) collisions and \( e^+e^- \) annihilation) and for symmetric definition of the hemispheres by assuming at hadron level:

a. NB behaviour for the (forward plus backward) MD of each component or substructure (class of events), i.e., clan structure analysis is assumed to be applicable to each of them.

The generalisation to the class of compound Poisson distributions (CPD) is of course possible: the generating function \( G_{\text{CPD}}(z) \) in this case can be written as follows

\[
G_{\text{CPD}}(z) = \exp \left\{ \bar{N}_{\bar{g}} [g_c(z) - 1] \right\}
\]
where $N_g$ is the average number of generalised clans and $g_c(z)$ the generating function of the MD of charged particles originated by a generalised clan (e.g., it is a logarithmic distribution when the MD for a single component is a NB(P)MD).

b. clans are therefore independently emitted and their distribution is Poissonian. In case of a generic CPD we talk, as previously stated, of generalised clans.

c. clans are binomially distributed in the forward and backward hemispheres (as are generalised clans).

d. logarithmically produced charged particles in each clan are also binomially distributed in the forward and backward hemispheres but with different probabilities $p$ and $q$ ($p + q = 1$, $p$ different from $q$); $p$ controls the leakage from one hemisphere to the other: $p = 1$ means that no particle leaks, while $p$ larger or equal to 0.5 and smaller than 1 indicates leakage. In case of a generic CPD, the $n$-charged particle MD is generated by $g_c(z)$, but $p$ and $q$ retain their meaning as described.

In order to perform our calculations in $pp$ collisions one extra assumption has been added, i.e., that the particle leakage parameter is constant throughout the GeV region for the two separate classes of events.

Possible scenarios in the TeV region based on extrapolations from data in the GeV region and on the weighted superposition mechanism of soft (without mini-jets) and semi-hard (with mini-jets) events have been indeed studied and predictions given on $n$-charged particle multiplicity distributions and on the ratios of $n$-particle factorial moments to the $n$-particle factorial cumulant moments general properties in the TeV region [8].

An attempt to predict the energy dependence of the forward-backward multiplicity correlation strength as well as $\bar{n}_B(n_F)$ vs $n_F$ general trends in the new energy range available at CERN with Alice detector has been considered [1]. The word is now to experiment which is supposed to test all these three sets of predictions at 14 TeV.

The problem we want to face in this paper, on the theory side, is the generalisation of the approach discussed in Ref. [1] for determining forward-backward multiplicity correlations properties for a single component, in more complex asymmetric reactions (like heavy ion AB and $pA$ collisions) and to provide a general framework for the study of forward-backward multiplicity correlations which includes symmetric reactions (like $pp$ and AA collisions and $e^+e^-$ annihilation) as a particular case.

Corner stones of our argument remain of course assumptions a, b, c, d.

They could hardly be abandoned in view of their success in giving a good phenomenological description of available experimental data on FB multiplicity correlations in symmetric reactions.

The generalisation of the approach to asymmetric reactions will concern therefore mainly how to implement, in the framework defined by assumptions a.b.c.d, the asymmetry of the reaction and the asymmetric definition of the forward and backward hemispheres.

In view of the lack of sound experimental data on asymmetric reactions and of the related analyses in terms of two or more component substructures both in full phase space and in rapidity intervals, the present paper should be considered as a stimulus to experimentalists of the new generation machines for a deeper analysis of total $n$-charged particles MD and a more satisfactory understanding of forward-backward multiplicity correlations as the c.m. energy increases and at fixed c.m. energy within rapidity intervals.
2 The general asymmetric case

Generalisation of the study performed in Ref. [1] on FB multiplicity correlations consists at the present stage of investigation in assuming that

A. the particle leakage percentage from the backward (B) to the forward (F) hemisphere, $q_B$, is different from the particle leakage percentage from F to B hemisphere, $q_F$, with $p_F + q_F = 1$ and $p_B + q_B = 1$, $p_F$ and $p_B$ being the corresponding percentages of particles not leaking from one hemisphere to the opposite one and remaining in the F and B hemisphere respectively; notice that in Ref. [1], $p_F = p_B = p$.

B. Poissonianly generated clans are asymmetrically (but binomially) distributed in the two hemispheres with the asymmetry parameter $r$ different from 1/2, $s$ being its complement to 1 in the opposite hemisphere; in Ref. [1], notice that $r = s = 1/2$ in addition to $p_F = p_B = p$.

2.1 The generating function

In general, the joint MD $P(n_F, n_B)$ for $n_F$ particles in the forward hemisphere and $n_B$ particles in the backward one is related to the global MD $P(n)$ though the probability distribution $f(n_F | n)$, which gives the probability to have $n_F$ F-particles when the total number of particles is $n$, as follows:

$$P(n_F, n_B) = P(n_F + n_B)f(n_F | n_F + n_B).$$

The generating function $G(z_F, z_B)$ for the joint distribution then satisfies

$$G(z_F, z_B) \equiv \sum_{n_F, n_B} z_F^{n_F} z_B^{n_B} P(n_F, n_B) = \sum_n z_B^n P(n) g_f(z_F/z_B; n),$$

where $g_f(z; n)$ is the generating function for $f(n_F | n)$; in case it is the binomial distribution with parameter $p$,

$$f(n_F | n) = \binom{n}{n_F} p^{n_F} (1-p)^{n-n_F},$$

then its GF is (defining $q \equiv 1 - p$):

$$g_f(z; n) = (q + pz)^n,$$

and one obtains a considerable simplification of Eq. (3):

$$G(z_F, z_B) = g(z_F p + z_B q),$$

where $g(z)$ is the generating function of the global distribution $P(n)$:

$$g(z) \equiv \sum_n z^n P(n).$$

Accordingly, we proceed now to calculate the GF for the general case. The formulae below are heavily based on previous work [1]. We consider a clan with logarithmic MD produced in
the F hemisphere and assume that each particle has the same probability \( p_F \) not to leak into the B hemisphere. Then the distribution at fixed number of particles is binomial, and we have immediately an application of the just-explained scheme: the GF for the joint distribution within a F-clan is thus

\[
g_{c,F}(z_F, z_B) = g_{\log}(z_F p_F + z_Bq_F); \quad (8)
\]

for a clan produced in the B hemisphere we have the corresponding GF:

\[
g_{c,B}(z_F, z_B) = g_{\log}(z_F q_B + z_Bp_B). \quad (9)
\]

Here the GF for the logarithmic distribution of parameter \( \beta \) is

\[
g_{\log}(z) \equiv \log(1 - z\beta) / \log(1 - \beta), \quad (10)
\]

where, in terms of standard NB parameters \( \bar{n} \) and \( k \),

\[
\beta = \frac{\bar{n}}{\bar{n} + k} \quad (11)
\]

and is related to the average number of particles per clan via the formula:

\[
\bar{n}_c = \frac{\beta}{(\beta - 1) \log(1 - \beta)} \quad (12)
\]

Recalling now Eq. (34) of [1], it is easy to calculate the GF of the joint distribution of F and B particles at given numbers \( N_F \) of F clans and \( N_B \) of B clans:

\[
g(z_F, z_B|N_F, N_B) = [g_{c,F}(z_F, z_B)]^{N_F} [g_{c,B}(z_F, z_B)]^{N_B}; \quad (13)
\]

due to the fact that all clans are by definition independent from each other, we can convolute the respective MD’s, which corresponds to multiply together the GF’s.

In order to sum over the clan MD, let us remember that we have assumed that clans are Poisson distributed and independent of each other, thus the joint distribution can be written as:

\[
P(N_F, N_B) = \frac{\bar{N}^{N_F+N_B}}{(N_F+N_B)!} e^{-\bar{N}} \left( \frac{N_F+N_B}{N_F} \right)^{N_F} r^{N_F} (1 - r)^{N_B}, \quad (14)
\]

where \( \bar{N} = \bar{N}_F + \bar{N}_B \) is the average number of clans, and \( r = \bar{N}_F / \bar{N} \) is the fraction of clans emitted in the F hemisphere. The corresponding GF is

\[
G(z_F, z_B) = \exp \left\{ \bar{N}r [z_F + (1 - r)z_B - 1] \right\}. \quad (15)
\]

We can now perform the last step in the calculation, again exploiting the general properties of the binomial distribution:

\[
g(z_F, z_B) = \sum_{N_F} \sum_{N_B} g(z_F, z_B|N_F, N_B) P(N_F, N_B) = G \left( g_{c,F}(z_F, z_B), g_{c,B}(z_F, z_B) \right). \quad (16)
\]

\[
g(z_F, z_B) = \exp \left\{ r\bar{N} [g_{\log}(z_F p_F + z_Bq_F) - 1] \right\} \exp \left\{ (1 - r)\bar{N} [g_{\log}(z_F q_B + z_Bp_B) - 1] \right\}. \quad (17)
\]

Notice that the above formula is valid for the NB(P)MD, but can easily be extended, as anticipated, to any compound Poisson distribution, provided all correlations are exhausted within a (generalised) clan: it is sufficient to replace \( g_{\log} \) with the appropriate GF within a clan, \( g_c(z) \).
2.2 The correlation strength and the marginal distributions

We can now use the above result to deduce the correlation strength $b$:

$$b = \frac{\langle n_F n_B \rangle - \bar{n}_F \bar{n}_B}{[(\langle n_F^2 \rangle - \bar{n}_F^2)(\langle n_B^2 \rangle - \bar{n}_B^2)]^{1/2}},$$

(18)

since everything can be read from the GF:

$$\langle n_F n_B \rangle = \left. \frac{\partial^2 g}{\partial z_F \partial z_B} \right|_{z_F = z_B = 1}$$

(19)

$$\bar{n}_i = \left. \frac{\partial g}{\partial z_i} \right|_{z_F = z_B = 1}$$

(20)

with $i = F, B$.

The average number of F-particles at fixed number of B-particles is also easily obtained though differentiation, since the MD of $n_F$ at fixed $n_B$ is given by

$$p(n_F | n_B) = \frac{p(n_F, n_B)}{p_{\text{marg}}(n_B)} = \frac{p(n_F, n_B)}{\sum_{n_F} p(n_F, n_B)},$$

(21)

where $p_{\text{marg}}(n_B)$ is the marginal distribution in the B hemisphere; the GF is

$$g(z_F | n_B) \equiv \sum_{n_F} z_F^n p(n_F | n_B) = \left. \frac{\partial^n B}{\partial z_B^n} g(z_F, z_B) \right|_{z_B = 0}$$

(22)

being $g(1, z_B)$ the GF of $p_{\text{marg}}$. The first moment is obtained from the GF by differentiating once:

$$\bar{n}_F(n_B) \equiv \left. \frac{\partial g(z_F | n_B)}{\partial z_F} \right|_{z_F = 1} = \left. \frac{\partial^n B}{\partial z_F^n} \frac{\partial}{\partial z_F} g(z_F, z_B) \right|_{z_B = 0, z_F = 1}$$

(23)

Analogous formulae for the B hemisphere quantities can be easily obtained in the same way. The corresponding marginal distributions are $g(z_B = z, 1)$ and $g(1, z_F = z)$:

$$g(z, 1) = \exp \left\{ r \bar{N} [g_{\log}(z p_F + q_F) - 1] \right\} \exp \left\{ s \bar{N} [g_{\log}(z q_B + p_B) - 1] \right\};$$

(24)

$g(1, z)$ can be obtained from Eq. (24) by interchanging parameter $p_i$ with $q_i$ ($i = F, B$).

The marginal distribution of Eq. (24) is the product of the GF’s of two NB(P)MD’s with characteristic NB parameters $\{\bar{n} r p_F, kr\}$ and $\{\bar{n} s q_B, ks\}$ respectively as can be seen immediately by noticing that

$$g(z, 1) = \left\{ 1 + \frac{\bar{n} r p_F}{k r} (1 - z) \right\}^{-kr} \left\{ 1 + \frac{\bar{n} s q_B}{k s} (1 - z) \right\}^{-ks};$$

(25)
\( g(1, z) \) can be defined in an analogous way.

\( g(z, 1) \) and \( g(1, z) \) are the GF’s of the MD’s obtained by convoluting the MD’s for particles generated by F (B) clans which stay in the F (B) hemisphere and do not leak in the opposite hemisphere with the MD’s for particles generated by B (F) clans which are leaking in the F (B) hemisphere.

Although \( g(z, 1) \) (and \( g(1, z) \)) is the product of the GF’s of NB(P)MD’s it is not the GF of a NB(P)MD. This consideration notwithstanding, it is interesting to remark that \( g(z, 1) \) (and \( g(1, z) \)) is an infinitely divisible distribution (IDD), a fact which allows to define the generalised clan concept, as already remarked.

In fact Eq. (24) can be rewritten as follows

\[
g(z, 1) = \exp \left\{ \bar{N} \left[ r g_{\log}(zp_F + q_F) + s g_{\log}(zp_B + p_B) \right] - \bar{N} \right\}
\]

i.e., as

\[
g(z, 1) = \exp \left[ \bar{N}(A(z) - 1) \right]
\]

with \( A(z) = [rg_{\log}(zp_F + q_F) + sg_{\log}(zp_B + p_B)] \) and \( A(0) \) different from zero. This last remark implies that the probability of generating zero particles is different from \( e^{-\bar{N}} \). Therefore the probability of generating empty clans is also different from zero, a result in contrast with the standard clan definition (each clan contains at least one particle), definition which we would like of course to enforce.

In order to do that, let us add and subtract in the exponent of Eq. (27) the term \( A(0) \), i.e., we rewrite Eq. (27) as follows

\[
g(z, 1) = \exp \left\{ \bar{N}[A(z) - A(0) + A(0) - 1] \right\}.
\]

From Eq. (28) one obtains the compound Poisson distribution belonging to the class of IDD

\[
g(z, 1) = \exp \left\{ \bar{N}_g[G_g(z) - 1] \right\},
\]

with \( \bar{N}_g = [1 - A(0)]\bar{N} \) and \( G_g(z) = [A(z) - A(0)][1 - A(0)]^{-1} \).

Accordingly, one can define the average number of generalised clans, \( \bar{N}_g \), in terms of the standard NB parameters \( \bar{n} \) and \( k \), and of \( p_F \) and \( p_B \):

\[
\bar{N}_g = \bar{N}[1 - A(0)] = \bar{N}\{1 - [rg_{\log}(q_F) + sg_{\log}(p_B)]\}
\]

\[
= - \ln g(0, 1) = rk \ln(\bar{n} + k - \bar{n}q_F) + sk \ln(\bar{n} + k - \bar{n}p_B) - k \ln k.
\]

The backward marginal multiplicity distribution and related properties can be be obtained from the forward one by interchanging parameter \( p_i \) with \( q_i \) (i=F,B).

3 The cases of partially removed symmetry

3.1 Symmetry for clans only \((r = s = 1/2, p_F \neq p_B, q_F \neq q_B)\)

The symmetry of the reaction can be broken partially by assuming that binomially distributed clans go fifty per cent in the F hemisphere and fifty per cent in the B hemisphere \((r = s = 1/2)\) but particle leakage from clans in F to B hemisphere, \( q_F \), and particle leakage from clans in B
to F hemisphere, \( q_B \), (as their complements \( p_B \) and \( p_F \) with \( p_F + q_F = 1 \) and \( p_B + q_B = 1 \)) are different. In other words, assumption B from the previous section is valid but A is not.

The forward-backward joint charged particle multiplicity GF for one single component turns out to be

\[
g(z, 1) = \exp \left\{ \frac{1}{2} \bar{N} [g_{\log}(z_F p_F + z_B q_F) - 1] \right\} \exp \left\{ \frac{1}{2} \bar{N} [g_{\log}(z_B p_B + z_F q_B) - 1] \right\},
\]

and for the corresponding marginal B distribution \((z_B = 1, z_F = 1)\)

\[
g(z, 1) = \exp \left\{ \frac{1}{2} \bar{N} [g_{\log}(p_F + z q_F) - 1] \right\} \exp \left\{ \frac{1}{2} \bar{N} [g_{\log}(z p_B + q_B) - 1] \right\}.
\]

In terms of standard NB parameters Eq. (32) becomes

\[
g(z, 1) = \left[ 1 + \bar{n} p_F (1 - z) \right]^{-k/2} \left[ 1 + \bar{n} q_B (1 - z) \right]^{-k/2}
\]

\[
= \left[ 1 + \bar{n} k (1 - z) + \bar{n}^2 q_F q_B (1 - z) \right]^{-k/2}.
\]

The symmetric definition of the two hemispheres is removed here in one component by assuming that particle leakage parameters in the forward and backward hemispheres are different.

The GF of the forward marginal multiplicity distribution thanks to the quadratic term as already shown in the general case is no more a NB(P)MD, although it is the product of the GF’s of two NB(P)MD’s with characteristic parameters \( \{ \frac{1}{2} \bar{n} p_F, \frac{1}{2} k \} \) and \( \{ \frac{1}{2} \bar{n} q_B, \frac{1}{2} k \} \) with asymmetric average charged particle multiplicities. The GF of the marginal distribution becomes the GF of a NB(P)MD in some special cases, i.e., for \( q_B = 0 \), corresponding to no particle leakage from the backward to the forward hemisphere, and for \( \bar{n} \ll k \), a situation which occurs in the Poissonian limit for average charged particle multiplicity much less than the \( k \) parameter and almost coinciding with the average number of clans. The GF \( g(z, 1) \), although not NB, is still an infinitely divisible distribution, as expected.

3.2 Symmetry for particles within clans only \((r \neq s \neq 1/2, p_F = p_B, q_F = q_B)\)

Another way to remove partially the symmetry is to use assumption A without B, i.e., to assume that binomially distributed clans are not symmetrically subdivided between F and B hemispheres, but particle leakage from clans in one hemisphere to the other is the same \((r \neq s \neq 1/2\) with \( r + s = 1 \), and \( p_F = p_B = p, q_F = q_B = q \) with \( p + q = 1 \), and \( p \) larger or equal than 1/2 and smaller than 1).

The FB joint particle multiplicity distribution GF for one component, \( g(z_F, z_B) \), becomes in this case

\[
g(z_F, z_B) = \exp \left\{ \bar{N} r [g_{\log}(z_F p + z_B q) - 1] \right\} \exp \left\{ \bar{N} s [g_{\log}(z_B p + z_F q) - 1] \right\},
\]

and the corresponding forward marginal charged particle MD GF

\[
g(z, 1) = \exp \left\{ \bar{N} r [g_{\log}(zp + q) - 1] \right\} \exp \left\{ \bar{N} s [g_{\log}(zq + p) - 1] \right\},
\]

which in terms of standard NB parameters of the total charged particle MD of the component under investigation is

\[
g(z, 1) = \left[ 1 + \bar{n} p (1 - z) \right]^{-kr} \left[ 1 + \bar{n} q (1 - z) \right]^{-ks}.
\]
The forward marginal MD GF, although again not of NB type, is the product of the GF’s of two NB(P)MD’s with parameters \( \{ \bar{n}_{rp}, kr \} \) and \( \{ \bar{n}_{qs}, ks \} \) respectively. The two sets of parameters characterise the group of non-leaking particles lying in the F hemisphere and those which leak from the backward hemisphere to the forward one. The backward marginal distribution can be easily obtained from the above equations by interchanging parameter \( r \) with \( s \).

### 4 The symmetric case \((r = s = 1/2, p_F = p_B, q_F = q_B)\)

In this case, assumptions A and B are both rejected. The formula studied in [1] for the joint charged particle MD GF in the symmetric case follows from Eq. (17) by taking \( p_F = p_B = p \), \( q_F = q_B = q \) with \( p \) different from \( q \), \( p + q = 1 \), and \( r = s = 1/2 \).

We get

\[
g(z_B, z_F) = \exp \left\{ \frac{1}{2} \bar{N} \left[ \log(z_F p + z_B q) - 1 \right] \right\} - 1 = k \left[ \frac{k + \bar{n}(1 - p z_B - q z_F)}{k + \bar{n}(1 - p z_F - q z_B)} \right]^{k/2} \tag{37}\]

and for the corresponding forward marginal distribution \( g(z) \)

\[
g(z, 1) = [1 + \bar{n} q/(1 - z)]^{-k/2} [1 + \bar{n} p/(1 - z)]^{-k/2} = \left\{ 1 + \frac{\bar{n}}{k} (1 - z) + pq \left[ \frac{\bar{n}}{k} (1 - z) \right]^2 \right\}^{-k/2} \tag{38}\]

In conclusion, Eq. (17) can be considered the wanted generalisation of Eq. (37) to the case of asymmetric reactions and an asymmetric definition of the forward and backward hemispheres.

### 5 Behaviours of \( \bar{n}_F(n_B) \) vs. \( n_B \) and of \( \bar{n}_B(n_F) \) vs. \( n_F \)

In this section we examine the relation between the average number of particles in one hemisphere, \( \bar{n}_F(n_B) \) or \( \bar{n}_B(n_F) \), at fixed value of the number of particles, respectively \( n_B \) or \( n_F \), in the other hemisphere, according to Eq. (23).

Let us start by recalling that the von Bahr-Ekspong theorem [9] implies, since we assume the total MD to be of NB type, that there is no linearity in \( \bar{n}_B(n_F) \) vs \( n_F \) (and in \( \bar{n}_F(n_B) \) vs \( n_B \)) unless the MD for F-particles (and for B-particles!) at fixed total number of particles is binomial, in which case the GF is simply:

\[
\exp \left[ \log(z_F p + z_B q) - 1 \right]. \tag{39}\]

Notice that it does not make sense to distinguish \( p_F \) from \( p_B \) in this case; however, if one did not distinguish F-clans from B-clans, i.e., if each clan emitted the same fraction \( p \) in one hemisphere, then in Eqs. (8) and (9) above one would put \( p_F = q_B = p \) and Eq. (17) would reduce to Eq. (39).

In the following the symmetry in the clan distribution is contrasted with the asymmetric clan distribution. In addition, leakage parameters for the two components are taken either equal or different. The resulting pictures of forward-backward multiplicity correlations are shown...
Figure 1: Behaviours of $\bar{n}_F(n_B)$ vs. $n_B$ (dashed lines) and of $\bar{n}_B(n_F)$ vs. $n_F$ (solid lines) for $r = s = 1/2$ and values of the leakage parameters as indicated. In order to illustrate a practical case, we have taken for the NB parameters $\bar{n} = 39.5$ and $k = 7.0$, corresponding to the soft component of $pp$ collisions at 14 TeV in the scenarios examined in Ref. [8]. In panes where only the solid line is visible, it means the dashed line coincides with it.

in Fig.s 1 and 2: of course our choice of the involved parameters is arbitrary. The word is again to experiment on symmetric collisions ($r = s = 1/2$) in order to understand eventual deviations from identity of particle leakage parameters from clans in the forward and backward hemispheres and to experiment on asymmetric collisions ($r \neq s \neq 1/2$) in order to measure the different leakage parameters in the two hemispheres. All together, expected different values of $r$ and $s$ as well as $p_F$ and $p_B$ parameters could lead to a new intriguing classification, in terms of FB multiplicity correlations, of high energy collisions and their substructures.

5.1 The case $r = s = 1/2$ (symmetry in the clan distribution)

We start by examining the case in which $r = 1/2$, illustrated in Figure 1. When $p_F = p_B = 1/2$, there is perfect linearity in $\bar{n}_B(n_F)$ versus $n_F$. This is in agreement with the mentioned theorem, because here we are saying that, within each clan, particles are binomially distributed in F and B with the same probability $1/2$, thus the fact that particles are produced in clans does not make any difference. When $1/2 < p_F = p_B < 1$, on the contrary, the fact that the production happens in two steps becomes again important and the relation between $\bar{n}_B(n_F)$ and $n_F$ is non-linear. Linearity is recovered again in the limiting case of no correlations ($p_F = p_B = 1$), see below.
When \( p_F \neq p_B \), the symmetry is lost because the particles leaking from the F hemisphere into B are not compensated (if \( p_F < p_B \)), or are over-compensated (if \( p_F > p_B \)), by those leaking from B into F. Thus one obviously finds that \( \bar{n}_B(n_F) \) vs \( n_F \) is not the same as \( \bar{n}_F(n_B) \) vs \( n_B \). Indeed the two marginal distributions differ from each other, but can be obtained one from the other by exchanging \( p_F \) with \( p_B \). In general for \( p_F > p_B \) there is less leakage from F than from B: particles prefer to stay in the F hemisphere thus one has the line of \( \bar{n}_F(n_B) \) vs \( n_B \) above \( \bar{n}_B(n_F) \) vs \( n_F \). Vice versa, the opposite is true for \( p_F < p_B \).

One can notice that when \( p_F = 1 \), \( \bar{n}_F(n_B) \) vs \( n_B \) is a straight line, and when \( p_B = 1 \), \( \bar{n}_B(n_F) \) vs \( n_F \) is linear. The fact is that when \( p_F = 1 \), clans that fall in the F hemisphere do not contribute to the B hemisphere. The only relation between \( n_B \) and \( \bar{n}_F \) is given by B clans leaking to the F hemisphere. And this relation is by definition binomial for each clan: the number of clans is Poisson distributed, thus the overall is still binomial with the same parameter, as was shown above. Indeed, the GF with \( p_F = 1 \) is:

\[
g(z_F, z_B) = \exp \left\{ r\bar{N}\left[ g_{\log}(z_F) - 1 \right] \right\} \exp \left\{ (1 - r)\bar{N}\left[ g_{\log}(z_B + z_B p_B) - 1 \right] \right\}; \quad (40)
\]

applying Eq. (23), the first term gives just the constant \( r\bar{n}_F \) while the second one is of the same type of Eq. (39) and thus also gives a linear behaviour. Notice that this happens independently of the value of \( r \). A corresponding result can be obtained for \( p_B = 1 \) by appropriately exchanging the roles of F and B.

5.2 The case \( r \neq s \neq 1/2 \) (asymmetry in the clan distribution)

When \( r \) increases over 1/2 (e.g., in Figure 2 the case \( r = 3/4 \) is illustrated), the excess of clans in the F hemisphere implies an increase of \( \bar{n}_F(n_B) \) vs \( n_B \) and a decrease of \( \bar{n}_B(n_F) \) vs \( n_F \), enlarging the differences between the curves when \( p_F > p_B \) and reducing them when \( p_F < p_B \) (\( \bar{n}_F(n_B) \) vs \( n_B \) can even become larger than \( \bar{n}_B(n_F) \) vs \( n_F \), depending on the values of \( r \), \( p_F \) and \( p_B \)). Exchanging \( p_F \) with \( p_B \) does not restore the symmetry. If \( p_F = p_B = 1/2 \), then the value of \( r \) has no influence on the outcome: if particles within each clan have a 50\% chance of going into the other hemisphere, it makes no difference if clans prefer one hemisphere over the other, and the two curves coincide again. This is not true if \( p_F = p_B > 1/2 \). Finally, in agreement with the remark at the end of the last subsection, we notice that even in the case \( r > 1/2 \) when \( p_F = 1 \), \( \bar{n}_F(n_B) \) vs \( n_B \) is a straight line, and when \( p_B = 1 \), \( \bar{n}_B(n_F) \) vs \( n_F \) is linear.

5.3 Asymmetry in the average number of particles per clan

All the above has been studied in the framework of a NB(P)MD describing in one component the forward plus backward MD. However, there are low energy data on pA collisions in which NB behaviour has been found to hold separately in the forward and in the backward hemisphere, with different NB parameters \([10]\). Because clans are independently emitted, the only correction to make to our formulae is to allow for different average numbers of particles per clan in the two hemispheres: in other words, the parameter \( \beta \) of the logarithmic distributions in Eq.s (8) and (9) will be different, say \( \beta_F \) and \( \beta_B \). Now, we immediately get

\[
g(z_F, z_B) = \exp \left\{ r\bar{N}\left[ g_{\log}(z_F + z_B p_F; \beta_F) - 1 \right] \right\} \exp \left\{ s\bar{N}\left[ g_{\log}(z_B + z_B p_B; \beta_B) - 1 \right] \right\}
\]

\[
(41)
\]
in place of Eq. (17). The product of these two NB(P)GF is not a NB(P)MD, but it is still, by construction, an infinitely divisible distribution.

6 Conclusions

It has been shown that assuming different particle leakage percentages \( p_B \neq p_F \) for binomially generated particles from clans in one hemisphere to the opposite one and asymmetric \( r \neq s \) distribution in the two hemispheres of binomially generated clans, a general formula for the generating function of the joint \((n_F, n_B)\)-charged particle multiplicity distribution for each class of events (or substructure) can be obtained when the total MD GF is of NB type. The formula reduces to that discussed in Ref. [1] for \( p_B = p_F \) and \( r = s = 1/2 \). Of particular interest are also the cases in which the symmetry is only partially removed (assumption A without B and B without A). All above-mentioned results, although explicitly derived for substructures of NB type, can be easily generalised to any discrete infinitely divisible MD. This search is relevant for the study of forward-backward multiplicity correlations in non-identical heavy ion and in proton-nucleus collisions. Accordingly, the newly introduced particle leakage and asymmetry parameters can be considered as effective indices classifying different classes of collisions.
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