Nuclear shadowing at low photon energies

T. Falter, S. Leupold and U. Mosel
Institut für Theoretische Physik
Justus-Liebig-Universität Giessen
D-35392 Giessen, Germany
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Abstract

We calculate the shadowing effect in nuclear photoabsorption at low photon energies (1-3 GeV) within a multiple scattering approach. We avoid some of the high energy approximations that are usually made in simple Glauber theory like the narrow width and the eikonal approximation. We find that the main contribution to nuclear shadowing at low energies stems from $\rho^0$ mesons with masses well below their pole mass. We also show that the possibility of scattering in non forward directions allows for a new contribution to shadowing at low energies: the production of neutral pions as intermediate hadronic states enhances the shadowing effect in the onset region. For light nuclei and small photon energies they give rise to about 30% of the total shadowing effect.

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I. INTRODUCTION

At high energies the total cross section for the reaction of a hadronic projectile with a nucleus of mass number \(A\) is usually smaller than \(A\) times the nucleonic cross section. This phenomenon, called the shadowing effect, can be understood if the mean free path of the projectile inside the nucleus is smaller than the nuclear dimension. In that case the nucleons on the front side of the nucleus shadow the downstream nucleons which therefore do not contribute to the total nuclear cross section. Since the nucleonic cross section of photons is smaller than typical hadronic ones by a factor \(\alpha_{\text{em}}\) in order of magnitude it is at first sight astonishing that photonuclear reactions are shadowed at high energies. A qualitative explanation for the shadowing of high energy photons is given by their property to fluctuate into a hadronic state with the quantum numbers of the photon. This happens with a probability of order \(\alpha_{\text{em}}\). The distance that such a hadronic fluctuation travels, the so called coherence length, can be estimated from the uncertainty principle:

\[
l_h \approx \left| k - \sqrt{\nu^2 - m_{\text{h}}^2} \right|^{-1}
\]

where \(k\) denotes the momentum of the photon, \(\nu\) its energy and \(m_{\text{h}}\) the mass of the hadronic fluctuation \((k = \nu\) for real photons). If the coherence length becomes larger than the mean free path of the hadronic fluctuation inside the nucleus its total nuclear cross section and therefore also that of the photon is shadowed.

Quantitatively, the shadowing of nuclear photoabsorption can be understood within the simple Glauber model [1–3] by relating the nuclear photoabsorption cross section via the optical theorem to the nuclear Compton forward scattering amplitude. In order \(\alpha_{\text{em}}\) one gets two contributions to this amplitude (see Fig 1). The first one stems from forward scattering of the photon from a single nucleon inside the nucleus. Taking only this contribution into account leads to an unshadowed nuclear cross section. Its interference with the second amplitude in order \(\alpha_{\text{em}}\) causes the shadowing effect. Assuming that at high energies all scattering events are dominated in the forward direction (eikonal approximation) this second amplitude takes the following form: The incoming photon produces an intermediate hadronic state \(X\) at one nucleon without exciting the nucleus. Within the eikonal approximation this process is presumed to happen in the forward direction so that the hadronic state must have the quantum numbers of the photon. One therefore usually expects that these states are dominated by vector mesons. The intermediate hadronic state then scatters at fixed impact parameter through the nucleus and finally at some nucleon into the outgoing photon. One assumes that during the whole process the nucleus stays in its ground state and neglects processes with intermediate excitation of the nucleus. This so called multiple scattering approximation corresponds roughly to neglecting two body and higher nuclear correlations and is expected to be reasonable for high energies (see e.g. [4,5]).

Within the Glauber approach one can quantitatively understand nuclear shadowing of high energy photons (see e.g. [6,7] and references therein), but recent photoabsorption data [8] on C, Al, Cu, Sn and Pb in the energy range from 1 to 2.6 GeV display an early onset of the shadowing effect which some of the newer models [10–12] cannot explain. In [13] we
showed that one can understand the data within the simple Glauber model by taking the negative real part of the $\rho^0 N$ scattering amplitude into account. However, some of the assumptions that are made in this simple model, like the validity of the eikonal approximation and the neglect of the finite width of the $\rho^0$ meson become questionable at low energies. In the present work we therefore use a multiple scattering approach \[14–16\] that we consider as more reliable in the shadowing onset region. Within this approach one can account for the vacuum self-energy of the $\rho^0$ and include scattering processes in nonforward direction. We find that the latter allow for a new contribution to nuclear shadowing in photoabsorption at low energies, namely the production of $\pi^0$ as intermediate hadronic states. Since neutral pions cannot be produced in the forward direction without exciting the nucleus they do not contribute in calculations based on the eikonal approximation.

In Sec. II we give a brief description of the multiple scattering formalism as a method for calculating the nuclear Compton forward scattering amplitude from free scattering amplitudes. The calculated ratio $\sigma_{\gamma A}/A\sigma_{\gamma N}$ that we obtain within this model is presented in Sec. III. If one only considers vector mesons as intermediate hadronic states and accounts for the finite width of the $\rho^0$ meson the results are already in good agreement with the experimental data. In Sec. IV we show that the shadowing effect is enhanced at photon energies around 1 GeV by taking the contribution of the $\pi^0$ as an intermediate state into account. We summarize our results in Sec. V.

II. GLAUBER-GRIBOV MULTIPLE SCATTERING FORMALISM

To calculate the photoabsorption cross section of a nucleus with mass number $A$ we make use of the optical theorem. Assuming that the projectile scatters from individual nucleons inside the nucleus we can express the nuclear photoabsorption cross section in terms of multiple scattering amplitudes $\mathcal{A}^{(n)}$:

$$\sigma_{\gamma A} = \frac{1}{2m_N k} \text{Im} \sum_{n=1}^{A} \mathcal{A}^{(n)}, \quad (2.1)$$

where $k$ denotes the momentum of the photon and $n$ the number of nucleons that participate in each multiple scattering process. Our goal is to calculate the amplitudes $\mathcal{A}^{(n)}$ from free nucleonic scattering amplitudes $\mathcal{M}$:

$$\langle \vec{p}' \! ; \! \vec{k}' | iT | \vec{k} \! ; \! \vec{p} \rangle = (2\pi)^4 \delta^{(4)}(p + k - (p' + k')) \cdot i\mathcal{M}_{\gamma}(\vec{k}, \vec{p} \to \vec{k}', \vec{p}') \quad (2.2)$$

where $\vec{p}$ ($\vec{p}'$) and $\vec{k}$ ($\vec{k}'$) denote the momenta of the incoming (outgoing) nucleon and projectile respectively. We therefore have to express the bound nucleon states in terms of free nucleon states $|\vec{p}\rangle$ of momentum $\vec{p}$. The bound nucleons are treated nonrelativistically and we assume that they are described by nonrelativistic one-particle wave functions

$$\psi_\alpha(x) = \psi_\alpha(\vec{x}) e^{-iE_\alpha t}. \tag{2.3}$$

The probability amplitude for finding the momentum $\vec{p}$ in the bound nucleon state $|\psi_\alpha\rangle$ is given by the Fourier transform $\psi_\alpha(\vec{p})$ of (2.3) with the normalisation
\[
\int \frac{d^3 p}{(2\pi)^3} |\tilde{\psi}_\alpha(p)|^2 = 1. \tag{2.4}
\]

Therefore we can write for the bound nucleon states

\[
|\psi_\alpha\rangle = \int \frac{d^3 p}{(2\pi)^3} \tilde{\psi}_\alpha(p) \frac{1}{\sqrt{2E_p}} |\vec{p}\rangle
\approx \int \frac{d^3 p}{(2\pi)^3} \tilde{\psi}_\alpha(p) \frac{1}{\sqrt{2m_N}} |\vec{p}\rangle. \tag{2.5}
\]

The factor of \(\sqrt{2E_p}\) converts the relativistic normalization of \(|\vec{p}\rangle\langle\vec{p}| = 2E_p(2\pi)^3 \delta^3(\vec{p} - \vec{p}')\) (2.6) to the conventional normalization \(\langle \psi_\alpha | \psi_\alpha \rangle = 1\).

The amplitude \(A^{(1)}\) for forward scattering of a photon with momentum \(\vec{k}\) and energy \(\nu(=|\vec{k}|)\) for real photons) from a single bound nucleon (see Fig. 2) then takes the form

\[
iA^{(1)} = \sum_{\alpha=1}^{\mathcal{A}} \int \frac{d^3 p}{(2\pi)^3} \tilde{\psi}^*_\alpha(p) i\mathcal{M}_\gamma(\vec{k},\vec{p} \rightarrow \vec{k},\vec{p}) \tilde{\psi}_\alpha(p) \tag{2.7}
\]

where \(\mathcal{M}_\gamma\) is the free invariant photon nucleon scattering amplitude. We use the local approximation and neglect the dependence of the amplitude \(\mathcal{M}_\gamma\) on the momentum \(\vec{p}\) of the incoming nucleon, i.e. we make the replacement

\[
\mathcal{M}_\gamma(\vec{k},\vec{p} \rightarrow \vec{k},\vec{p}) \rightarrow \mathcal{M}_\gamma(\vec{k},\vec{p}_0 \rightarrow \vec{k},\vec{p}_0). \tag{2.8}
\]

Since the momentum of the photon \(\vec{k}\) is much larger than the Fermi momentum of the bound nucleons, we set \(\vec{p}_0 \approx \vec{0}\) and the amplitude \(A^{(1)}\) takes the simple form

\[
iA^{(1)} = i\mathcal{M}_\gamma(\vec{k},\vec{p}_0 \rightarrow \vec{k},\vec{p}_0) \sum_{\alpha=1}^{\mathcal{A}} \int \frac{d^3 p}{(2\pi)^3} |\tilde{\psi}_\alpha(p)|^2 \tag{2.9}
\]

\[
= Ai\mathcal{M}_\gamma(\vec{k},\vec{0} \rightarrow \vec{k},\vec{0}) \tag{2.10}
\]

which leads to the unshadowed cross section \(A\sigma_{\gamma N}\) when used in Eq. (2.1).

The double scattering amplitude \(A^{(2)}\) is shown in Fig. 3 and corresponds to the process where the incoming photon produces a hadronic state \(X\) which propagates freely without further scattering to a second nucleon and there scatters into the outgoing photon. We assume that the nucleus stays in its ground state, i.e. we neglect nondiagonal contributions with an excited nucleus between the two scattering events (multiple scattering approximation). Hence there is no energy transfer to the two nucleons. If \(\vec{q}\) denotes the momentum transfer to the first nucleon the fact that we are dealing with the nuclear forward scattering amplitude fixes the momentum transfer to the second nucleon to \(-\vec{q}\). Using again the local approximation and introducing the nuclear formfactor
\[ F(\vec{q}) = \int d^3x e^{i\vec{q} \cdot \vec{x}} |\psi_\alpha(\vec{x})|^2 = \frac{1}{A} \int d^3x e^{i\vec{q} \cdot \vec{x}} n(\vec{x}) \] (2.11)

where \( n(\vec{x}) \) denotes the nucleon number density, the double scattering amplitude takes the simple form

\[ iA^{(2)} = A(A - 1) \int \frac{d^3q}{2m_N(2\pi)^3} \]
\[ \times \sum_X iM_{\gamma X}(\vec{q}) \frac{i}{\nu^2 - (\vec{k} - \vec{q})^2 - \tilde{m}_X^2 - \Pi_X(\nu^2 - (\vec{k} - \vec{q})^2)} \]
\[ iM_{X\gamma}(\vec{q}) \] (2.12)

Here \( \tilde{m}_X \) and \( \Pi_X \) denote the bare mass and the vacuum self energy of the intermediate state \( X \) and we used the following abbreviation for the invariant production amplitudes:

\[ iM_{\gamma X}(\vec{q}) = iM_{\gamma X}(\vec{k}, \vec{p}_0 \rightarrow \vec{k} - \vec{q}, \vec{p}_0 + \vec{q}) \] (2.13)

\[ iM_{X\gamma}(\vec{q}) = iM_{X\gamma}(\vec{k} - \vec{q}, \vec{p}_0 \rightarrow \vec{k}, \vec{p}_0 - \vec{q}) \] (2.14)

The nuclear formfactor \( F(\vec{q}) \) suppresses large values of \( q = |\vec{q}| \) and therefore favors light intermediate states. Though it is in principle possible to produce slow, massive \( \rho^0 \) mesons, their contributions are suppressed by the nuclear formfactor. The important contributions stem from light \( \rho^0 \) mesons having the energy of the photon \( (> 1 \text{ GeV}) \) so that the effect of Fermi motion becomes negligible. We therefore set again \( \vec{p}_0 \approx \vec{0} \). In Sec. III we will show that double scattering gives the leading contribution to nuclear shadowing.

The form of the \( n \)-fold scattering amplitude \( A^{(n)} \) can be read off from Fig. 4. Using the same approximations as before one gets

\[ iA^{(n)} = \frac{A!}{(A - n)!} \prod_{i=1}^{n-1} \left[ \int \frac{d^3q_i}{2m_N(2\pi)^3} \right] F(\vec{q}_1) ... F(\vec{q}_n) i\mathcal{V}^{(n)}(\{\vec{q}_i\}) \] (2.15)

with

\[ i\mathcal{V}^{(n)}(\{\vec{q}_i\}) = \sum_{X_1} iM_{\gamma X_1}(\vec{q}_1) \frac{i}{\nu^2 - (\vec{k} - \vec{q}_1)^2 - \tilde{m}_{X_1}^2 - \Pi_{X_1}(\nu^2 - (\vec{k} - \vec{q}_1)^2)} \]
\[ iM_{X_1X_2}(\vec{q}_2) \]
\[ \times \frac{i}{\nu^2 - (\vec{k} - (\vec{q}_1 + \vec{q}_2))^2 - \tilde{m}_{X_2}^2 - \Pi_{X_2}(\nu^2 - (\vec{k} - (\vec{q}_1 + \vec{q}_2))^2)} \]
\[ \times ... iM_{X_{n-1}X_2}(\vec{q}_n) \] (2.16)

and

\[ \vec{q}_n = - \sum_{i=1}^{n-1} \vec{q}_i \] (2.17)

\[ M_{\alpha\beta}(\vec{q}_i) = M_{\alpha\beta}(\vec{k} - \sum_{j=1}^{i-1} \vec{q}_j, \vec{0} \rightarrow \vec{k} - \sum_{j=1}^{i} \vec{q}_j, \vec{q}_i) \] (2.18)
Using the eikonal approximation within this approach corresponds to the neglect of any dependence of $\mathcal{V}^{(n)}$ on the momentum transfer $\vec{q}_{\perp,i}$ transverse to the initial photon direction, i.e. setting $\vec{q}_{\perp,i} = 0$ in Eq. (2.16) (see e.g. [15]). At high energies this is certainly a good approximation, so that for high energies and in the large $A$ limit this multiple scattering series can be summed up and reduces to the simple Glauber formula used in [13] if one neglects the widths of the vector mesons. To describe photoabsorption also at lower energies in the following we will relax the eikonal approximation by allowing nonforward scattering events.

III. BEYOND THE EIKONAL APPROXIMATION

The nuclear formfactor $F(\vec{q})$ suppresses large momentum transfer $\vec{q}$. Hence at very high energies the scattering processes tend predominantly into the forward direction, justifying the usage of the eikonal approximation. In the eikonal limit the intermediate hadronic states $X_i$ in (2.16) must have the quantum numbers of the photon since they are produced without nuclear excitation. With decreasing energy nonforward scattering becomes more important. In this section we present a calculation that goes beyond the eikonal approximation but we still restrict the intermediate hadronic states $X_i$ to the light vector mesons $V = \rho^0, \omega, \phi$ that are the only possible intermediate states in the eikonal limit. Note that the nuclear formfactor suppresses the production of more massive states in the energy regime we are interested in. In Sec. IV we will see that at photon energies around 1 GeV it becomes also important to have a $\pi^0$ as an intermediate state. In the following we will neglect nondiagonal contributions where a state $X_i$ scatters into a different state $X_j$.

For $\omega$ and $\phi$ mesons we use constant self energies such that:

$$m_{\omega,\phi}^2 + \text{Re}\Pi_{\omega,\phi} = m_{\omega,\phi}^2$$

$$\text{Im}\Pi_{\omega,\phi} = -m_{\omega,\phi} \Gamma_{\omega,\phi},$$

with the physical masses $m_\omega$ and $m_\phi$ and the widths $\Gamma_\omega$ and $\Gamma_\phi$ taken from [17]. For the $\rho^0$ we use a momentum dependent vacuum self energy $\Pi_{\rho}(p^2)$ [18].

The nuclear formfactor $F(q)$ is calculated via Eq. (2.11) assuming a Woods-Saxon distribution

$$n(\vec{r}) = \frac{\rho_0}{1 + \exp \left[ \frac{\vec{r} - \vec{R}}{a} \right]}$$

with the parameters listed in Tab. I.

We make the following ansatz for the elastic $VN$ scattering amplitudes, motivated by the angular distribution of $\rho^0$ in photoproduction data [20]:

$$\mathcal{M}_V(s, t, m) = 8\pi m_N f_V(\vec{0}, k_V(s, m)) e^{\frac{1}{2} Bt}$$

(3.4)
where $k_V$ denotes the momentum of the vector meson with invariant mass $m$ in the rest frame of the nucleon at invariant energy $\sqrt{s}$. For $t = 0$ this expression yields exactly the invariant forward scattering amplitude which is related to the total $VN$ cross section via the optical theorem:

$$\sigma_{VN} = \frac{1}{2m_Nk_V} \text{Im}M(s, t = 0, m)$$

$$= \frac{4\pi}{k_V} \text{Im}f_V(\bar{0}, k_V(s, m)).$$

(3.5)

(3.6)

The slope parameter in the considered energy region can be estimated from $\rho^0$ photoproduction data [20]:

$$B \approx 6 \text{ GeV}^{-2}.$$

(3.7)

We use the $\rho^0 N$ forward scattering amplitude $f_\rho(\bar{0}, k_V)$ from [21]. In [13] we already showed that the negative real part of $f_\rho$ enhances the shadowing effect at low energies and leads to a higher effective mass of the $\rho^0$ meson in nuclei. Also in the present calculation the real part of $f_\rho$ turns out to be important. We set

$$f_\omega(\bar{0}, k_\omega) = f_\rho(\bar{0}, k_\omega)$$

$$f_\phi(\bar{0}, k_\phi) = \frac{i}{4\pi} k_\phi \sigma_\phi,$$

(3.8)

(3.9)

assuming the total $\phi N$ cross section $\sigma_\phi = 12$ mb [3]. We relate the photoproduction amplitude of a vector meson of invariant mass $m$ to the elastic $VN$ scattering amplitudes via the vector meson dominance model (VDM) and use the formfactor

$$F(m^2, m^2_V) = \frac{\Lambda^4}{\Lambda^4 + (m^2 - m^2_V)^2}.$$  

(3.10)

with a cutoff parameter $\Lambda = 1.2$ GeV [22] to describe the off-shell behavior of the photoproduction amplitude for a broad vector meson:

$$\mathcal{M}_{\gamma V}(s, t, m) = \mathcal{M}_{\gamma V}(s, t, m) = \frac{e}{g_V} \mathcal{M}_V(s, t, m) F(m^2, m^2_V).$$

(3.11)

Here the first equality is given by detailed balance. The coupling constants $g_V$ are taken from model I of Ref. [5]. Note that expression (3.11) is not based on experimental data. It is an off-shell extrapolation for the production amplitude which in the on-shell case reproduces the experimental $\rho^0$ and $\omega$ photoproduction data [20]. A low value of $\Lambda$ suppresses the production of vector mesons that are far off their mass shell. However, the formfactor (3.10) is only important in the case of a $\rho^0$ as an intermediate state since contributions to nuclear shadowing originating from off-shell $\omega$ and $\phi$ are already strongly suppressed by their propagators in (2.12) because of their much smaller widths.

The photon nucleon cross section $\sigma_{\gamma N}$ is approximated for each nucleus with mass number $A$ and proton number $Z$ by the isospin averaged cross section
\[ \sigma_{\gamma N} = \frac{Z\sigma_{\gamma p} + (A-Z)\sigma_{\gamma n}}{A}, \] (3.12)

fitting the data on \(\sigma_{\gamma p}\) and \(\sigma_{\gamma n}\) in the considered energy region.

In Fig. 5 we compare the calculated ratio \(\sigma_{\gamma A} / A\sigma_{\gamma N}\) for several nuclei with experimental data [8,9]. The dotted line represents the calculation using Eq. (2.1) and including terms up to double scattering \((n=2)\). The inclusion of triple scattering \((n=3)\) leads to the solid curve. The result including \(A^{(4)}\) lies between the two curves because (2.1) is an alternating series as can be seen by neglecting the real part of the amplitudes \(M\) in (2.16). Thus at low energies the calculation up to triple scattering is already a good approximation. The dominant contribution to shadowing stems from the double scattering term. At higher energies the terms with \(n > 3\) become important. More than 90% of the shadowing is caused by the \(\rho^0\) meson as an intermediate particle because its photoproduction amplitude and its width are larger than those of \(\omega\) and \(\phi\). At small energies light \(\rho^0\) mesons \((m \ll m_{\rho})\), albeit suppressed by the propagator in (2.12), play the dominant role because the nuclear formfactor favors small invariant masses. This can be verified by choosing a smaller value for \(\Lambda\) in (3.10) or a smaller \(\rho^0\) width in the propagator of (2.12) which in both cases suppresses contributions from lighter \(\rho^0\). In the limit of vanishing \(\rho^0\) width there will be no \(\rho^0\) contributions to shadowing below the \(\rho\) photoproduction threshold \(\nu_{th} \approx 1.1\) GeV and the contribution above threshold will be strongly suppressed by the nuclear formfactor. This can also be understood in terms of the coherence length \(l_h\). Lighter \(\rho^0\) mesons have a larger coherence length as can be seen from Eq. (1.1). Therefore their interactions are shadowed more than those of on-shell \(\rho\) mesons. Note that \(l_h\) is connected directly to the momentum transfer \(|\vec{q}|\): \(l_h = |\vec{q}|^{-1}\). For heavier nuclei our result is in good agreement with the data. The shadowing at very small photon energies \((\nu < 1.25\) GeV) is enhanced by the inclusion of the \(\pi^0\) as will be shown in Sec. IV. The result within the eikonal approximation, i.e. setting \(\vec{q}_{\perp,i} = 0\) in Eq. (2.16), leads to a \(\sim 10\%\) stronger shadowing in the considered energy region in case of the vector mesons as intermediate states only. Note that additional shadowing stemming from intermediate \(\pi^0\) is not possible in the eikonal approximation.

At the end of this section we want to discuss briefly the effect of \(NN\) correlations. In our simple Glauber calculation [13] the results were very sensitive to two body correlations which were needed to describe the experimental data. The reason for this was the narrow width approximation that allowed only for \(\rho^0\) mesons with mass \(m = m_{\rho} = 770\) MeV as intermediate states and led to a large momentum transfer in the production process, especially at low photon energies. In the present work the main contribution to shadowing stems from the lighter \(\rho^0\) that are connected with a momentum transfer much smaller than the characteristic momentum \(q_c = 780\) MeV [23] at which correlations become important. Therefore any correlations between the nucleons can be omitted in the present work.

**IV. \(\pi^0\) CONTRIBUTION**

If one does not restrict the scattering events to the forward direction it becomes possible to produce neutral pions without exciting the nucleus. In this process the helicity of the
nucleon at which the $\pi^0$ is produced is not allowed to change. Decomposing the $\pi^0$ photoproduction amplitude on the free nucleon into helicity amplitudes $H_i$, one can construct the invariant photoproduction amplitude without helicity change of the nucleon:

$$\mathcal{M}_{\gamma N}^{\text{coh}}(s, t) = \sqrt{2s} 8\pi (H_1(s, t) + H_4(s, t)).$$  \hspace{1cm} (4.1)

The helicity amplitudes are calculated using the partial wave analysis from Arndt et al. \cite{25}. For each nucleus we use the isospin averaged amplitude $\mathcal{M}_{\gamma N \rightarrow \pi^0 N}^{\text{coh}}$ in analogy to (3.12). The production in forward direction is forbidden by angular momentum conservation. Therefore a large transverse momentum transfer is necessary to produce the $\pi^0$ at high energies. Since large momentum transfers are suppressed by the nuclear formfactor, we expect the $\pi^0$ not to contribute to shadowing at high energies.

The $\pi^0$ contribution to $\sigma_{\gamma A}$ is calculated using Eq. (2.1) and (2.12):

$$\Delta\sigma_{\gamma A}^{(\pi^0)} = -\frac{A(A-1)}{2m_Nk} \Re \int \frac{d^3q}{2m_N(2\pi)^3} F(\vec{q})i\mathcal{M}_{\gamma N \rightarrow \pi^0 N}^{\text{coh}}(s, t)\tilde{D}_{\pi^0}(|\vec{k}|^2 - (\vec{k} - \vec{q})^2)i\mathcal{M}_{\pi^0 N \rightarrow \gamma N}^{\text{coh}}(s, t)F(-\vec{q}) \hspace{1cm} (4.2)$$

where we have introduced the effective propagator

$$\tilde{D}(p^2) = \frac{i}{p^2 - m_{\pi^0}^* - i m_{\pi^0}^* \Gamma_{\pi^0}^*}. \hspace{1cm} (4.3)$$

This propagator takes the multiple $\pi^0 N$ scattering into account. Therefore equation (4.2) is not merely the double scattering contribution with an intermediate $\pi^0$ but includes all higher order scattering terms within the eikonal form of the propagator (4.3). This is equivalent to the propagation of the intermediate $\pi^0$ in an optical potential \cite{13}. One could in principle also use the free propagators and sum up all multiple scattering terms for the $\pi^0$ to obtain the same result, but this is numerically more tedious. The use of the eikonal approximation for the intermediate $\pi^0$ scattering is justified because the $\pi^0$ has the energy of the initial photon and therefore large momentum. To make the same approximation for the intermediate vector mesons is not possible since for the energies of interest vector mesons are much slower because of their larger mass. The momentum dependent effective mass and the collisional broadening of the $\pi^0$ in the nucleus is related to the $\pi^0 N$ forward scattering amplitude $f_{\pi^0}$ \cite{26}:

$$m_{\pi^0}^* = m_{\pi^0}^2 - 4\pi \Re f_{\pi^0}(\vec{0}, k_\pi)\rho_N \hspace{1cm} (4.4)$$

$$m_{\pi^0}^* \Gamma_{\pi^0}^* = 4\pi \Im f_{\pi^0}(\vec{0}, k_\pi)\rho_N. \hspace{1cm} (4.5)$$

The average nucleon number density

$$\rho_N = \frac{1}{A} \int d\vec{r}^3 n(\vec{r})^2 \hspace{1cm} (4.6)$$

is determined separately for each nucleus using Eq. (3.3) and the parameters listed in Tab. I. For $\pi^0$ momenta $k_\pi$ up to 1.65 GeV ($\sqrt{s} \leq 2$ GeV) we take the partial waves from the SM95 solution of Ref. \cite{27} to construct the $\pi^0 N$ forward scattering amplitude $f_{\pi^0}$. For higher $\pi^0$
moments we use the KA84 solution [27]. Note that $m^{*2}_\pi$ as well as $m^{*}_\omega\Gamma^{*}_\pi$ are small compared to the squared energy of the $\pi^0$ that enters the propagator in (4.3). Therefore our results are not very sensitive to the in-medium changes of the $\pi^0$.

Taking the $\pi^0$ contribution into account leads to an enhancement of the shadowing effect at small energies as can be seen in Fig. 6. The solid line represents the old result including only $\rho^0$, $\omega$ and $\phi$ as intermediate states and including terms up to triple scattering in the multiple scattering series. The dashed line shows the result one gets by including the $\pi^0$ as an intermediate hadronic state. At small energies and light nuclei the $\pi^0$ contribution (4.2) to nuclear shadowing is about 30%. Here the $\rho^0$ photoproduction, even in forward direction, is suppressed by the nuclear formfactor because of its much larger mass. Because of angular momentum conservation the lighter $\pi^0$ cannot be produced in the forward direction without exciting the nucleus. The necessary transverse momentum transfer increases with rising photon energy. Therefore the nuclear formfactor suppresses the $\pi^0$ contribution to nuclear shadowing at high energies. This suppression is larger for the heavier nuclei as one would expect from the $A$ dependence of the nuclear formfactor. This reduces the $\pi^0$ contribution to shadowing in the case of Pb to about 10% at small energies.

V. SUMMARY

In this work we have presented a description of nuclear shadowing in photoabsorption at low energies that is more realistic than the simple Glauber model. We use a multiple scattering expansion to express the nuclear forward Compton amplitude in terms of nucleonic scattering amplitudes. From that the total nuclear cross section is calculated via the optical theorem. The main contribution to nuclear shadowing results from the double scattering term. Here the incoming photon produces an intermediate hadronic state at one nucleon that scatters on a second nucleon into the outgoing photon. In contrast to the simple Glauber model we also allow for off-shell vector mesons as intermediate states and avoid the eikonal approximation that is usually made in such calculations. We find that very light $\rho^0$ mesons below their pole mass $m_\rho = 770$ MeV give the biggest contribution to nuclear shadowing at low energies. This is reasonable because these light states have the largest coherence length among the intermediate vector meson states. By including only the light vector mesons $\rho$, $\omega$ and $\phi$ as intermediate states in the multiple scattering series, we can already reproduce the photoabsorption data for heavy nuclei. At low energies the shadowing effect is enhanced by intermediate $\pi^0$. Since a $\pi^0$ has no spin it cannot be produced in the forward direction without excitation of the nucleus. Therefore $\pi^0$ contribution to nuclear shadowing in photoabsorption is absent in any calculation based on the eikonal approximation. In the present work we have shown that at low energies intermediate $\pi^0$ are responsible for about 10-30% of the total shadowing effect. At high energies and for heavy nuclei their contribution is suppressed by the nuclear formfactor.
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FIG. 1. The two amplitudes that contribute in order $\alpha_{em}$ to the nuclear Compton forward scattering amplitude in the Glauber model. The left amplitude alone would lead to an unshadowed cross section. The right one gives rise to nuclear shadowing.

\[
\sum_{\alpha} \gamma (\nu, \vec{k}) N_{\alpha} A \rightarrow A-1 \rightarrow A \gamma
\]

FIG. 2. First contribution to the photon-nucleus forward scattering amplitude in order $\alpha_{em}$: Forward scattering of the incoming photon from a bound nucleon $N_{\alpha}$. This amplitude corresponds to the left amplitude in Fig. 1 and leads to an unshadowed cross section.

\[
\sum_{\alpha} \sum_{X} \sum_{\alpha' \neq \alpha} \gamma (\nu, \vec{k}) (\nu, \vec{k} - \vec{q}) N_{\alpha} X N_{\alpha'} N_{\alpha'} \rightarrow A \rightarrow A-2 \rightarrow A \gamma
\]

FIG. 3. Second contribution to the photon-nucleus forward scattering amplitude in order $\alpha_{em}$: The photon produces a hadron $X$ at nucleon $N_{\alpha}$. This hadron propagates freely to a second nucleon $N_{\alpha'}$ where it scatters into the outgoing photon. We assume that the nucleus stays in its ground state during the whole scattering process, i.e. there is no energy transferred to the bound nucleons.

\[
\sum_{X_i} \sum_{\alpha_1} \sum_{\alpha_2 \neq \alpha_1} \ldots \gamma (\nu, \vec{k}) (\nu, \vec{k} - \vec{q}_i) (\nu, \vec{k} - \vec{q}_1, \ldots, \vec{q}_n) \rightarrow N_{\alpha_i} \rightarrow N_{\alpha_i} \rightarrow \ldots \rightarrow N_{\alpha_n} \rightarrow A \rightarrow A-n \rightarrow A
\]

FIG. 4. General form of $\mathcal{A}^{(n)}$ for $n \geq 1$. 

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FIG. 5. Calculated ratio $\sigma_{\gamma A}/A\sigma_{\gamma N}$ as a function of the photon energy $\nu$. Only the light vector mesons $\rho^0$, $\omega$ and $\phi$ are taken into account as intermediate states. The dotted curve represents the calculation up to double scattering, the solid line also includes the corrections from triple scattering. The experimental data are taken from: $\bullet$[8], $\blacksquare$[9].
FIG. 6. Calculated ratio $\sigma_{\gamma A}/A\sigma_{\gamma N}$ as a function of the photon energy $\nu$. The solid line corresponds to the calculation up to triple scattering involving only the vector mesons. The dashed line represents the calculation including the $\pi^0$ contribution to nuclear shadowing $\langle 12 \rangle$. The experimental data are taken from: $\bullet [8]$, ■ [9].
TABLES

|       | $^{12}$C | $^{27}$Al | $^{63}$Cu | $^{120}$Sn | $^{208}$Pb |
|-------|---------|---------|---------|---------|---------|
| $R$/fm | 2.209 | 3.090 | 4.313 | 5.513 | 6.755 |
| $a$/fm | 0.479 | 0.478 | 0.477 | 0.476 | 0.476 |
| $\rho_0$/fm$^{-3}$ | 0.182 | 0.177 | 0.167 | 0.159 | 0.154 |

TABLE I. Parameters used in the Woods-Saxon distribution (3.3).