ABSTRACT

We find that the mass ratio \( q \) in quiescent black hole (BH) X-ray transients is tightly correlated with the ratio of the double-peak separation (DP) to the full width half maximum (FWHM) of the H\(_\alpha\) emission line, \( \log q = -6.88 - 23.2 \log(DP/FWHM) \). This correlation is explained through the efficient truncation of the outer disk radius by the 3:1 resonance with the companion star. This is the dominant tidal interaction for extreme mass ratios \( q = M_2/M_1 \lesssim 0.25 \), the realm of BH (and some neutron star) X-ray transients. Mass ratios can thus be estimated with a typical uncertainty of \( \approx 32\% \), provided that the H\(_\alpha\) profile used to measure DP/FWHM is an orbital phase average. We apply the DP/FWHM–\( q \) relation to the three faint BH transients XTE J1650–500, XTE J1859+226, and Swift J1357–0933 and predict \( q = 0.026^{+0.038}_{-0.007}, 0.049^{+0.023}_{-0.012} \) and \( 0.040^{+0.003}_{-0.005} \), respectively. This new relation, together with the FWHM–\( K_2 \) correlation presented in Paper I, allows the extraction of fundamental parameters from very faint targets and, therefore, the extension of dynamical BH studies to much deeper limits than was previously possible. As an example, we combine our mass ratio determination for Swift J1357–0933 with previous reported values to yield a BH mass of \( 12.4 \pm 3.6 \) M\(_\odot\). This confirms Swift J1357–0933 as one of the most massive BH low-mass X-ray binaries in the Galaxy.

Key words: accretion, accretion disks – binaries: close – stars: black holes – stars: dwarf novae – stars: neutron stars: cataclysmic variables

1. INTRODUCTION

Stellar-mass black holes (BHs) are mostly detected through dramatic X-ray outbursts exhibited by transient X-ray binaries (SXTs; Tanaka & Shibazaki 1996). About 60 BH candidates have been identified in the 50-year lifetime of X-ray astronomy (Corral-Santana et al. 2016), although only 17 of these have been confirmed by dynamical studies, i.e., their possess a mass function \( f(M) = M^3 \sin^3 i/(1 + q)^2 \) in excess of 3 M\(_\odot\), where \( q = M_2/M_1 \) is the mass ratio of the companion star to the compact object. The reason for the low rate of confirmed BHs lies in the difficulty of detecting the companion star for quiescent optical magnitudes fainter than \( \sim 22 \). Time-resolved spectroscopy with signal-to-noise ratio S/N \( \gtrsim 10 \) is typically needed to detect the weak absorption features and trace their orbital motion. Determining the binary mass ratio is even more challenging, as it also requires resolving powers \( R \gtrsim 5000 \) to measure the rotational broadening of the absorption lines (e.g., Casares & Charles 1994). In addition, moderately short integration times are essential to avoiding significant orbital smearing. It should be mentioned that alternative methods based on the radial velocities of the disk emission lines are uncertain and prone to large systematic effects (see e.g., Marsh et al. 1994). A critical review on the determination of system parameters in BH SXTs can be found in Casares & Jonker (2014).

In Casares (2015) (henceforth Paper I) we showed that the full width half maximum (FWHM) of the disk H\(_\alpha\) line in quiescent BH SXTs scales with the velocity semi-amplitude of the companion star. We here present the discovery of another correlation between \( q \) and the ratio of the double-peak separation to the line width. Both relations open the door to constrain fundamental parameters and perform dynamical studies in much fainter samples of quiescent BH candidates than is currently possible.
Table 1
Database of X-ray Transients

| Object       | q              | DP/FWHM     | References          |
|--------------|----------------|-------------|---------------------|
| V404 Cyg     | 0.067 ± 0.005  | 0.5636 ± 0.0024 | 1                   |
| BW Cir       | 0.12 ± 0.03    | 0.5534 ± 0.0032 | 2                   |
| XTE J1550-564| 0.033 ± 0.008  | 0.5705 ± 0.0039 | 3                   |
| N. Oph 77    | 0.014 ± 0.016  | 0.5591 ± 0.0119 | 4                   |
| N. Mus 91    | 0.079 ± 0.007  | 0.5679 ± 0.0037 | 5                   |
| GS 2000+1    | 0.042 ± 0.012  | 0.5801 ± 0.0094 | 6                   |
| A0620-00     | 0.067 ± 0.010  | 0.5766 ± 0.0013 | 7                   |
| N Vel 93     | 0.055 ± 0.010  | 0.5689 ± 0.0038 | 8                   |
| XTE J1118+480| 0.024 ± 0.009  | 0.5873 ± 0.0010 | 9                   |

Neutron Stars

| Cen X-4      | 0.176 ± 0.003  | 0.5446 ± 0.0001 | 10                  |
|--------------|----------------|----------------|---------------------|
| XTE J2123-058| 0.37 ± 0.15    | 0.5298 ± 0.0234 | 11                  |

References.
1. Casares (1996); 2. Casares et al. (2009); 3. Orosz et al. (2011); 4. Harlaftis et al. (1997); 5. Wu et al. (2015); 6. Harlaftis et al. (1996); 7. Marsh et al. (1994); 8. Macias et al. (2011); 9. Calvelo et al. (2009); 10. Shahbaz et al. (2014); 11. Tomsk et al. (2002).

3. THE DP/FWHM–Q RELATION

Double-peak separations (DP) were obtained by fitting a symmetric two-Gaussian model to the average H$_\alpha$ profile in every SXT and CV. In the case of eclipsing CVs, we excluded those spectra obtained within ±0.05 phases from the time of the central eclipse. The fitted model consists of a constant plus two Gaussians of identical width and height. The continuum rectified spectra were fitted in a window of ±10,000 km s$^{-1}$, centered on the H$_\alpha$ line after masking the neighboring HeI line at 6678 Å. Prior to the fit, the two-Gaussian model was degraded to the resolution of the data by convolution with the instrumental profile\(^1\) We adopted 1-$\sigma$ formal errors on the fitted parameter as derived through $\chi^2$ minimization. Figure 1 displays some fit examples using our two-Gaussian model. In addition, FWHM values were extracted from single Gaussian fits to the same average H$_\alpha$ profiles following Paper I.

Tables 1 and 2 list the parameter DP/FWHM and its propagated 1-$\sigma$ error as derived from our Gaussian model fits. The evolution of DP/FWHM with q is presented in Figure 2. The figure shows that DP/FWHM varies very rapidly for small q values, with a 10% increase for q under 0.2.

To understand this behavior, we follow Paper I and start by assuming that the FWHM of the H$_\alpha$ line is determined by gas with Keplerian velocity at a characteristic radius $R_W = \alpha R_{L1}$, with $\alpha < 1$ and $R_{L1}$ the Roche lobe of the compact star, i.e.,

$$\text{FWHM} = \left(\frac{GM}{R_W}\right)^{1/2} \sin i.$$  \hspace{1cm} (1)

On the other hand, the double-peak separation is set by the velocity of the outer disk, whose radius $R_d$ is truncated by the tidal forces of the companion star (Paczynski 1977; Papaloizou & Pringle 1977). There is ample evidence for the outer disk velocities to be sub-Keplerian (e.g., North et al. 2002), and thus we decided to adopt

$$\frac{DP}{2} = \beta \left(\frac{GM}{R_d}\right)^{1/2} \sin i.$$  \hspace{1cm} (2)

where the parameter $\beta < 1$ accounts for the fraction by which the outer disk material is sub-Keplerian. For extreme mass ratios $q \lesssim 0.25$, the disk is effectively truncated at the resonance radius of the $(j = 3, k = 2)$ commensurability or 3:1 resonance radius i.e., the radius at which the disk angular velocity is three times the angular velocity of the companion star (see Hirose & Osaki 1990; Frank et al. 2002). Therefore,

$$R_d = R_{32} = 3^{-2/3}(1 + q)^{-1/3}a$$  \hspace{1cm} (3)

where $a$ the binary separation. If we now bring Equation (3) into Equation (2) and use Eggleton’s relation (Eggleton 1983) to remove $R_{L1}/a$ we find

$$\frac{DP}{\text{FWHM}} = 3^{1/3}(1 + q)^{2/3} \beta \sqrt{Q(f(q))}$$  \hspace{1cm} (4)

where $f(q)$ is the same expression as in Equation (6) of Paper I, i.e.,

$$f(q) = \frac{0.49(1 + q)^{-1}}{0.6 + q^{2/3} \ln (1 + q^{-1/3})}.$$

By computing the ratio between the double-peak separation and the line width, we managed to cancel out the dependence on compact object mass and binary inclination. Interestingly, in contrast with the FWHM–K$_2$ correlation presented in Paper I, Equation (4) is very sensitive to the mass ratio for $q \lesssim 0.25$ i.e.,

The sake of comparison, we also plot Equation (4) in Figure 2 for $\alpha = 0.42$ (adopted from Paper I) and $\beta = 0.77$. Although not intended to be a fit, the alignment of the data with the model indicates that Equation (4) provides a good description of the observations. This endorses our interpretation that the strong dependence of DP/FWHM with q is driven by the truncation of the outer disk radius caused by the 3:1 resonance tide. The bottom panel in Figure 2 also displays the evolution of DP/FWHM for the CV sample, with the model for $\alpha = 0.42$ and $\beta = 0.83$ superimposed. The comparison of the SXT and CV data with the model indicates that the velocities at the outer accretion disk are sub-Keplerian by $\approx 20\%$, in good agreement with other studies (e.g., Wade & Horne 1988). Note that for $q \gtrsim 0.25$, the 3:1 resonance lies beyond $\approx 0.9 R_d$, and the outer disk radius is then limited by the largest non-intersecting orbits allowed by three-body interactions (Paczynski 1977). Under these circumstances the dependence of the outer disk radius on q is very weak (Frank et al. 2002) and therefore a nearly constant evolution of DP/FWHM versus q is expected for $q \gtrsim 0.25$.

For practical purposes, in Figure 3 we display the variation of DP/FWHM versus q in logarithmic units. A least-squares linear fit yields

$$\log q = -6.88(0.52) - 23.2(2.0)\log \left(\frac{\text{DP}}{\text{FWHM}}\right)$$

with a Pearson correlation coefficient $r = 0.95$. To estimate the error in q implied by this relation, we computed the difference with respect to the true observed values for our 11 SXTs. The distribution of differences can be approximated by a normal
Table 2
Database of Cataclysmic Variables

| Object              | q          | DP/FWHM     | References |
|---------------------|------------|-------------|------------|
| GK Per              | 0.55 ± 0.21| 0.5551 ± 0.0007 | 1          |
| SDSS J100658.40+233724 | 0.51 ± 0.08 | 0.5402 ± 0.0013 | 2          |
| U Gem               | 0.359 ± 0.013 | 0.5663 ± 0.0013 | 3, 4       |
| IP Peg              | 0.32 ± 0.08  | 0.5760 ± 0.0035 | 5          |
| CTCV J1300–3052    | 0.25 ± 0.03  | 0.5681 ± 0.0007 | 6          |
| HT Cas              | 0.150 ± 0.015 | 0.5825 ± 0.0027 | 7          |
| OY Car              | 0.102 ± 0.003 | 0.6100 ± 0.0003 | 8          |
| V2051 Oph           | 0.19 ± 0.03  | 0.5737 ± 0.0043 | 9          |
| SDSS 103533.02+055158.3 | 0.055 ± 0.002 | 0.6311 ± 0.0016 | 8          |
| WZ Sge              | 0.088 ± 0.013 | 0.6150 ± 0.0007 | 10         |
| SDSS J143317.78+101123.3 | 0.069 ± 0.003 | 0.5777 ± 0.0025 | 8          |

Note. Values for GK Per, IP Peg, and CTCV J1300–3052 have been obtained through the $V \sin i$ technique while those for U Gem and WZ Sge by measuring the radial velocity curves of the white dwarf and the donor star. The remaining $q$ values are derived by modeling the eclipses of the white dwarf and the hot spot in optical light curves.

References. (1) Morales-Rueda et al. (2002); (2) Southworth et al. (2009); (3) Friend et al. (1990); (4) Long & Gilliland (1999); (5) Beekman et al. (2000); (6) Savoury et al. (2012); (7) Wood & Horne (1990); (8) Littlefair et al. (2008); (9) Baptista et al. (1998); (10) Steeghs et al. (2007).

Figure 1. Example of double-Gaussian fits to Hα profiles in SXTs. A selection of average spectra, representing the entire range of FWHMs, is depicted.
function with $\sigma(q) = 0.015$. This indicates that mass ratios can be realistically obtained from Equation (6) with a typical $\sim 25\%$ uncertainty.

4. ORBITAL EFFECTS

The spectra that we used to produce the DP/FWHM–$q$ correlation are orbital averages. This is partly because individual spectra rarely possess enough signal-to-noise for the technique to be applicable. But also because orbital means average out possible asymmetries in individual spectra from, for example, hot spots or disk eccentricities that could potentially bias the determination of $q$.

At this point we decided to explore the impact of line asymmetries in the results of our technique. Since this can only be tested on data with sufficient signal-to-noise, we have focused on the 154 high-quality GTC spectra of XTE J1118+480 obtained along four different orbits over two years. We have performed Gaussian fits on every individual spectrum and computed mass ratios using Equation (6). The distribution of $q$ values is found to peak at $q = 0.024$ (the bottom panel in Figure 4), with 68% of the values contained between $q = 0.021$ and 0.045. This indicates that if $q$ were to be obtained from a single individual spectrum, the typical uncertainty would be about $\sim 56\%$.

In order to trace the effect of line asymmetries we also extracted the $V/R$ parameter from each spectrum, with $V/R$ defined as the ratio of equivalent widths between the blue and the red part of the H$_\alpha$ profile. The two halves of the line are set from the rest wavelength till $\pm 2500$ km s$^{-1}$. For example, $V/R = 0.8$ indicates a line with the red part stronger by 20% while $V/R = 1$ implies a symmetric profile. A plot of $q$ versus $V/R$ (see top panel in Figure 4) seems to show a trend, with asymmetric profiles preferring slightly lower $q$ values, although the large scatter prevents from drawing a firm conclusion.

However, as we have mentioned above, the technique outlined in this paper is most useful on phase averaged spectra because individual spectra typically have very limited signal-to-noise. Consequently, we have extracted $q$ values by fitting
Gaussians to the four orbital averages of the 154 individual spectra. The distribution of $q$ values has a mean at 0.026 and a standard deviation of 0.005, indicating that the typical error on phase averaged spectra is reduced to $\sim$19% i.e., smaller than the 25% uncertainty drawn from the correlation. We therefore conclude that the uncertainty expected from the application of our technique to phase averaged spectra is about 32%.

5. DISCUSSION: APPLICATION TO THREE FAINT BHS

In Paper I we showed that the FWHM of the H$_\alpha$ line in quiescent SXTs and CVs is formed at $\approx$42% of $R_{L1}$. Furthermore, it is tightly correlated with the projected velocity of the donor star, $K_2$, and thus the quantity FWHM/$K_2$ can be used to extract dynamical information from single epoch low-resolution ($R \approx$ 500) spectroscopy. In addition, we showed that FWHM/$K_2$ is weakly dependent on $q$, resulting in a $\sim$27% flatter slope for (long-period) CVs.

We here now present a new method to estimate the binary mass ratio in quiescent BH SXTs from the properties of the H$_\alpha$ line. We have proved that the quantity DP/FWHM is strongly dependent on $q$, with a 10% variation for $q \lesssim 0.25$. The reason behind this is the efficient truncation of the outer disk radius by the 3:1 tidal resonance of the donor star. The correlation of DP/FWHM with $q$, therefore, opens a new avenue with which to measure mass ratios in quiescent BH SXTs. The double-peak separation can be solidly measured by fitting a symmetric double-Gaussian model to phase averaged H$_\alpha$ profiles. We estimate that instrumental resolution better than 25% of the double-peak separation is required to resolve the latter. This typically demands resolving powers of only $R \gtrsim 1000$ i.e., a factor $\sim$5 lower than required to measure $q$ using the $V \sin i$ technique. More significantly for observational feasibility, this method makes use of the disk H$_\alpha$ line which, with a typical EW $\sim 50$ Å, is much stronger than the weak atmospheric features of the donor star.

The relations presented in this paper and in Paper I thus allow for a reasonably accurate estimation of the system parameters in very faint SXTs which otherwise cannot be tackled with current instrumentation and standard techniques. As an example, we have applied our method to the BH SXTs XTE J1650–500, XTE J1859+226 and Swift J1357–0933. They all have $R \approx 22–23$ and none of them has yet a mass ratio determination. We have produced averaged spectra for XTE J1650–500 and XTE J1859+226 using the data presented in Table 1 of Paper I. Regarding Swift J1357–0933, we have used a more recent and extended database reported in Mata Sánchez et al. (2015). Figure 5 displays the averaged spectra of the three BHs together with the best double-Gaussian model fits, which result in DP/FWHM = 0.5828 ± 0.0150, 0.5741 ± 0.0072, and 0.5805 ± 0.0027 for XTE J1650–500, XTE J1859+226, and Swift J1357–0933, respectively. The mass ratios implied by Equation (6) are $q = 0.026 \pm 0.003$, 0.049 ± 0.012 and 0.040 ± 0.005, respectively. The quoted uncertainties correspond to 68% confidence regions and have been computed using a Monte Carlo simulation with $10^5$ realizations. We note in passing that our mass ratio for Swift J1357–0933 is in excellent agreement with an independent estimate based on the radial velocity curve of the wings of the H$_\alpha$ line (Mata Sánchez et al. 2015).

Regarding Swift J1357–0933, we are in the position to provide a credible BH mass using our scaling H$_\alpha$ relations. By combining our mass ratio with the mass function obtained by means of the FWHM/$K_2$ correlation (Mata Sánchez et al. 2015) and a conservative value of the inclination $i = 80^\circ \pm 10^\circ$ (Corral-Santana et al. 2013; Mata Sánchez et al. 2015), we find a $M_1 = 12.4 \pm 3.6 \, M_\odot$. This confirms Swift J1357–0933 as one of the most massive BH low-mass X-ray binaries in the Galaxy, rivaled only by GRS 1915+105 (Reid et al. 2014).

To conclude, we would like to stress that the relations presented here and in Paper I will help deepen the search for new BH transients to substantially fainter limits. They will also prove very useful in extracting fundamental parameters from large numbers of data to be delivered by new spectroscopic surveys such as GAIA or WEAVE.

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REFERENCES

Baptista, R., Catalan, M. S., Horne, K., & Zilli, D. 1998, MNRAS, 300, 233
Beekman, G., Somers, M., Naylor, T., & Hellier, C. 2000, MNRAS, 318, 9
Calvelo, D. E., Vrtilek, S. D., Steeghs, D., et al. 2009, MNRAS, 399, 539
Casares, J. 1996, in Proc. 158th Coll. of IAU 208, Astrophysics and Space Science Library, ed. A. Evans, & H. Janet Wood (Dordrecht: Kluwer), 395
Casares, J. 2015, ApJ, 808, 80
Casares, J., & Charles, P. A. 1994, MNRAS, 271, L5
Casares, J., Charles, P. A., Jones, D. H. P., Rutten, R. G. M., & Callanan, P. J. 1991, MNRAS, 250, 712
Casares, J., & Jonker, P. G. 2014, SSRv, 183, 223
Casares, J., Orosz, J. A., Zurita, C., et al. 2009, ApJS, 181, 238
Copperwheat, C. M., Marsh, T. R., Dhillon, V. S., et al. 2010, MNRAS, 402, 1824
Corral-Santana, J. M., Casares, J., Muñoz-Darias, et al. 2013, Sci, 339, 1048
Corral-Santana, J. M., Casares, J., Muñoz-Darias, T., et al. 2016, A&A, 587, A61
Eggleton, P. 1983, ApJ, 268, 368
Frank, J., King, A. R., & Raine, D. J. 2002, Accretion Power in Astrophysics (3rd ed.; Cambridge: Cambridge Univ. Press)
Friend, M. T., Martin, J. S., Connon Smith, R., & Jones, D. H. P. 1990, MNRAS, 246, 637
González Hernández, J. I., Rebolo, R., & Casares, J. 2014, MNRAS, 438, L21
Harlaftis, E. T., Horne, K., & Filippenko, A. V. 1996, PASP, 108, 762
Harlaftis, E. T., Steeghs, D., Horne, K., & Filippenko, A. V. 1997, AJ, 114, 1170
Hirose, M., & Osaki, Y. 1990, PASJ, 42, 135
Littlefair, S. P., Dhillon, V. S., Marsh, T. R., et al. 2008, MNRAS, 388, 1582
Long, K. S., & Gilliland, R. L. 1999, ApJL, 511, L916
Macias, P., Orosz, J. A., Bailyn, C. D., et al. 2011, BAAS, 43
Marsh, T. R., Robinson, E. L., & Wood, J. H. 1994, MNRAS, 266, 137
Mata Sánchez, D., Muñoz-Darias, T., Casares, J., Corral-Santana, J. M., & Shahbaz, T. 2015, MNRAS, 454, 2199
Moraes-Rueda, L., Still, M. D., Roche, P., Wood, J. H., & Lockley, J. J. 2002, MNRAS, 329, 597
North, R. C., Marsh, T. R., Kolb, U., Dhillon, V. S., & Moran, C. K. J. 2002, MNRAS, 333, 383
Orosz, J. A., Steiner, J. F., McClintock, J. E., et al. 2011, ApJ, 730, 75
Paczynski, B. 1977, ApJ, 216, 822
Papaloizou, J., & Pringle, J. E. 1977, MNRAS, 181, 441
Reid, M. J., McClintock, J. E., Steiner, J. F., et al. 2014, ApJ, 796, 2
Savoury, C. D. J., Littlefair, S. P., & Marsh, T. R. 2012, MNRAS, 422, 469
Shahbaz, T., Watson, C. A., & Dhillon, V. S. 2014, MNRAS, 440, 504
Southworth, J., Hickman, R. D. G., Marsh, T. R., et al. 2009, A&A, 507, 929
Steeghs, D., Howell, S. B., Knigge, C., et al. 2007, ApJ, 667, 412
Tanaka, Y., & Shibazaki, N. 1996, ARA&A, 34, 607
Tomsick, J. A., Heindl, W. A., Chakrabarty, D., & Kaaret, P. 2002, ApJ, 581, 570
Wade, R. A., & Horne, K. 1988, MNRAS, 324, 411
Wood, J., & Horne, K. 1990, MNRAS, 242, 606
Wu, J., Orosz, J. A., McClintock, J. E., et al. 2015, ApJ, 806, 92