$N = 2$ massive superparticle:
the minimality principle and the k-symmetry

D.V.Uvarov$^a$ * and A.A.Zheltukhin$^{a,b}$ †

$^a$ NSC Kharkov Institute of Physics and Technology
310108, Kharkov, Ukraine,

$^b$ Institute of Theoretical Physics , University of Stockholm
Box 6730, S-11385 Stockholm, Sweden

Abstract

The electromagnetic interaction of massive superparticles with $N = 2$ extended Maxwell supermultiplet is studied. It is proved that the minimal coupling breaks the k-symmetry. A non-minimal k-symmetric action is built and it is established that the k-symmetry uniquely fixes the value of the superparticle’s anomalous magnetic moment.

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1. Introduction

It is well known that consistency of the theories of superstrings, super-$p$ - (and $D$-) branes [1,2] may be achieved only if the k-symmetry is present in these theories [3-5]. However, the presence of the k-symmetry in a free supersymmetric theory doesn’t guarantee its presence in a theory with interaction. This fact accounts for an interest to the problem of the k-symmetry preservation when passing from free supersymmetric models to ones with interaction. The model of a charged superparticle coupled to external superpotential [7-9, 12] is one of the simplest supersymmetric schemes with interaction. In the
case of $N = 1$ massless superparticle the model with interaction possessing
the k-symmetry can be obtained by strict following to the minimality principle. An important consequence of the k-symmetry existence is presence of
the correct constraints for superfield strengths which remove unphysical fields
and single out the $N = 1$ Maxwell supermultiplet. Analogous situation takes
place in the case of $N = 1$ supergravity.

In Ref.[4] has been proposed an extended ($N > 1$) free superparticle model
which possesses the k-symmetry and, unlike the $N = 1$ case, permits the co-
variant (although reducible) division of the grassmanian constraints without
introduction of the auxiliary variables such as twistor-like ones [10]. However,
consistency of the minimality principle and the k-symmetry is violated
when passing to that superparticle model coupled to extended $N > 1$ super-
potential. As revealed by Luzanna and Milevski in Ref.[12], this breakdown requires, the modifications of the model, analogous to those of the Refs.[13,14]
for the spinning particles. These modifications, based on the mass “renorm-
alization” prescription, turned out to be equivalent to the introduction of
an anomalous magnetic moment (AMM) for the spinning particles as it was
established in Ref.[15], where a superfield description was given. Moreover,
no restrictions on the AMM magnitude of the spinning particle appeared.

In some sense a similar situation appears in the model of $N = 2$ mas-
sive superparticle which is studied below. However, here the requirement
of the k-symmetry existence severely restricts the AMM value of superparticle.
Clarification of this statement is a reason for this paper.

The paper is organised as follows: section 2 is devoted to the study of
the free $N = 2$ massive superparticle model [4]. In the section 3 we examine
this model when the minimal coupling with an external $N = 2$ superpoten-
tial is introduced. Then the problem of the k-symmetry breaking for this
coupling is discussed. In the section 4 we build k-symmetric action for the
superparticle interacting with the external $N = 2$ superpotential. We show
that the restoration of the k-symmetry of the action is provided by means
of the nonminimal terms introduction. This nonminimal extension of the
model corresponds to taking into account the electromagnetic interactions
of the superparticle caused by its AMM. Moreover, the value of this AMM
turns out to be fixed.

2. $N = 2$ massive superparticle model

Among various superparticle models of the most interest are those, which
possess the k-symmetry. In particular, these are massless superparticles in
$D = 3, 4, 6, 10$ with an arbitrary number of supersymmetries [5]. At the same
time a transition to the corresponding massive superparticle model violates the k-symmetry. However, when \( N > 1 \) there exists the possibility to avoid these difficulties by means of Wess-Zumino-Witten-like term introduction. For the \( D = 4 \) case such model was suggested in [4] and for the \( D = 6, 10 \) in [6].

We resort the \( D = 4, N > 1 \) case [4], where to the Brink-Schwarz action was added the additional term

\[
\theta_i^\alpha A^{ij} \dot{\theta}_j^\beta + \bar{\theta}_{\dot{\alpha}i} A^{ij} \dot{\theta}_j^\beta,
\]

where \( A^{ij} \) is a real antisymmetric matrix depending on the superparticle’s mass. This term is invariant under global supersymmetry transformations up to the total derivative and is a 1D analogue of the super-p-branes Wess-Zumino-Witten term [5]. In the \( N = 2 \) case this matrix is simply \( m \epsilon^{ij} \) and the superparticle action takes form

\[
S = \int d\tau \sqrt{-\omega^\mu \omega_\mu} + m \int d\tau \left( \theta_i^\alpha \dot{\theta}_i^\alpha + \bar{\theta}_{\dot{\alpha}i} \dot{\bar{\theta}}_{\dot{\alpha}i} \right),
\]

(1)

where \( \omega^\mu = \dot{x}^\mu + i \theta_i^\alpha \sigma_\mu^{\alpha\dot{\alpha}} \dot{\bar{\theta}}_{\dot{\alpha}i} - \bar{\theta}_{\dot{\alpha}i} \sigma_\mu^{\alpha\dot{\alpha}} \dot{\theta}_i^\alpha \) are the supersymmetric Cartan forms. Our notations mainly coincide with those of the Ref.[11] (see also Appendix A). Introducing the worldline einbein action (1) can be represented in the following form

\[
S = \frac{1}{2} \int d\tau \left( \frac{\omega^2}{g} - gm^2 \right) + m \int d\tau \left( \theta_i^\alpha \dot{\theta}_i^\alpha + \bar{\theta}_{\dot{\alpha}i} \dot{\bar{\theta}}_{\dot{\alpha}i} \right). \quad (1')
\]

The Hamiltonian analysis [16] we begin with the introduction of the momentum variables

\[
p_{\mu} = \frac{\partial L}{\partial \dot{x}^\mu} = \frac{\omega_\mu}{g}, \quad p_g = \frac{\partial L}{\partial \dot{g}} = 0
\]

\[
\pi_\alpha^i = \frac{\partial L}{\partial \dot{\theta}_i^\alpha} = -i \bar{\theta}_{\dot{\alpha}i} \omega_{\alpha\dot{\alpha}} \frac{g}{g} - m \theta_i^\alpha, \quad \bar{\pi}_{\dot{\alpha}i} = \frac{\partial L}{\partial \dot{\bar{\theta}}_{\dot{\alpha}i}} = -i \theta_i^\alpha \omega_{\alpha\dot{\alpha}} \frac{g}{g} - m \bar{\theta}_{\dot{\alpha}i},\]  

(2)

leading to the set of the primary constraints \( p_g \approx 0 \),

\[
V_\alpha^i = \pi_\alpha^i + ip_{\alpha\dot{\alpha}} \bar{\theta}_{\dot{\alpha}i} + m \theta_i^\alpha \approx 0; \quad \bar{V}_{\dot{\alpha}i} = \bar{\pi}_{\dot{\alpha}i} + i \theta_i^\alpha p_{\alpha\dot{\alpha}} + m \bar{\theta}_{\dot{\alpha}i} \approx 0
\]

(3)

and the standard Hamiltonian

\[
H_0 = \dot{x}^\mu p_\mu + \dot{\theta}_i^\alpha \pi_\alpha^i + \dot{\bar{\theta}}_{\dot{\alpha}i} \bar{\pi}_{\dot{\alpha}i} - L = \frac{g}{2} (p^2 + m^2).
\]

(4)

Having the constraints requires introduction of the full Hamiltonian

\[
H = H_0 + \lambda_\alpha V_\alpha^i + \bar{\lambda}_{\dot{\alpha}} \bar{V}_{\dot{\alpha}i} + \varphi p_g,
\]

(4')
which is a linear combination of the primary constraints. To proceed further we need to have the Poisson brackets definition

\[
\{ p_\mu, x^\nu \} = -i \delta_\mu^\nu, \quad \{ p_g, g \} = -i,
\]

\[
\{ \pi_i^\alpha, \theta^\beta_j \} = -i \delta_i^\alpha \delta_j^\beta, \quad \{ \bar{\pi}^\dot{\alpha}_i, \bar{\theta}^\dot{\beta}_j \} = -i \delta_i^{\dot{\alpha}} \delta_j^{\dot{\beta}}.
\]

Using (5) we can evaluate several important Poisson brackets

\[
\{ V_\alpha, V_\beta \} = -2im \epsilon_{\alpha\beta} \lambda_i^i, \quad \{ \bar{V}_{\dot{\alpha}}, \bar{V}_{\dot{\beta}} \} = -2im \epsilon_{\dot{\alpha}\dot{\beta}}, \quad \{ V_\alpha, \bar{V}_{\dot{\alpha}} \} = 2p_\alpha, \quad (\text{6})
\]

\[
\{ p^2 + m^2, V_\alpha \} = 0, \quad \{ p^2 + m^2, \bar{V}_{\dot{\alpha}} \} = 0. \quad (\text{7})
\]

Now, according to the Dirac prescription [16], we study equations, obtained from the primary constraints by the temporal conservation conditions:

\[
\{ p_g, H \} = 0 \implies \chi = p^2 + m^2 \approx 0, \quad \text{(secondary constraint)}
\]

\[
\{ H, V_\alpha \} = -2im \lambda^i_\alpha + 2p_{\alpha\dot{\beta}} \bar{\lambda}^{\dot{\beta}i} = 0, \quad \{ H, \bar{V}_{\dot{\alpha}} \} = 2\lambda^i_\alpha p_\alpha - 2im \bar{\lambda}_{\dot{\alpha}i} = 0. \quad (\text{9})
\]

A system of linear equations has nontrivial solutions when and only when its determinant vanishes. In our case

\[
\det A \equiv \det\begin{pmatrix}
-im \delta^\beta_\alpha \delta^i_j & \delta^i_j p_{\alpha\dot{\beta}} \\
-\delta^i_j p_{\dot{\alpha}\dot{\beta}} & -im \delta^\dot{\alpha}_{\dot{\beta}} \delta^i_j
\end{pmatrix} = (p^2 + m^2)^4 \approx 0. \quad (\text{10})
\]

This condition however does not fix system’s rank, which in our case equals four \( \boxed{\text{4}} \), so only half of the equations (9) are linear independent and, as a consequence, four of the Lagrange multipliers \( \lambda(\bar{\lambda}) \) remain unfixed. This indicates the presence of the local fermionic symmetry of the action (1′), which is just the k-symmetry, and existence of the four spinorial first-class constraints, generating this symmetry. The rest of the spinorial constraints in (3) belong to the second-class. At this point the new problem arises: how is it possible to produce the covariant division of the first- and second-class spinorial constraints? The matrix A (10) prompts us to use the set of projectors [17], separating the first- and second-class constraints

\[
P_{I,II} = \frac{1}{2} \left(1 \pm \Pi\right), \quad (\text{11})
\]

where

\[
\Pi = \delta^i_j \begin{pmatrix}
0 & -i p_{\alpha\dot{\beta}} \\
i \sqrt{-p^2} & 0
\end{pmatrix}. \quad (\text{11′})
\]

\(^1\)Unlike the interaction case here we need no additional conditions to halve the rank to obtain the k-symmetry
and Π satisfies the strong relation Π² = 1. The multiplier (√−p²) was introduced here for a normalization. The projectors P_I and P_{II} obey the following relations: P_{I,II}² = P_{I,II}; P_IP_{II} = P_{II}P_{I} = 0. Then the first-and second-class constraints acquire the form

$$\begin{pmatrix} V^{(1)i}_{\alpha} \\ \tilde{V}^{(1)\dot{\alpha}i} \end{pmatrix} = P_I \begin{pmatrix} V^j_{\beta} \\ \tilde{V}^{\dot{\beta}j} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} V^i_{\alpha} - ip_{\alpha\beta} \tilde{V}^{\dot{\beta}i} \\ \tilde{V}^{\dot{\alpha}i} - ip_{\dot{\alpha}\dot{\beta}} \tilde{V}^{\dot{\beta}i} \end{pmatrix}$$

(12)

Although we managed to separate constraints in the manifestly covariant way we have got, however, the linearly dependent sets of the first- and second-class constraints

$$i\tilde{p}^{\dot{\beta}\alpha}V^{(1)i}_{\alpha} = V^{(1)\dot{\beta}i}, \quad i\tilde{p}^{\dot{\beta}\alpha}V^{(2)i}_{\alpha} = V^{(2)\dot{\beta}i}.$$  

(13)

One can use projector P_I for constructing the k-symmetry transformation laws for the action (1): $\delta x^\mu = -i\theta_i \sigma^\mu \delta \bar{\theta}^i + i\delta \theta_i \sigma^\mu \bar{\theta}^i$,

$$\begin{pmatrix} \delta \theta^i_{\alpha} \\ \delta \bar{\theta}^{\dot{\alpha}i} \end{pmatrix} = P_I \begin{pmatrix} \kappa^j_{\beta} \\ \kappa^{\dot{\beta}j} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \kappa^i_{\alpha} - i\omega_{\alpha\beta} \kappa^{\dot{\beta}i} \\ \kappa^{\dot{\alpha}i} + i\omega_{\dot{\alpha}\dot{\beta}} \kappa^{\dot{\beta}i} \end{pmatrix}.$$  

(14)

The bosonic constraint χ belongs to the first-class here (being a reparametrization generator), but the external superpotential coupling converts it to the second-class as will be seen below

$$\{\chi, \chi\} = 0, \quad \{\chi, V_{\alpha}\} = 0, \quad \{\chi, \tilde{V}_{\dot{\alpha}}\} = 0.$$  

(15)

Thus, our analysis explicitly shows how to covariantly separate the fermionic constraints and to construct the k-symmetry transformations. The total constraints algebra is presented in the Appendix B.

The next step in our analysis will be investigation of the consistency between the k-symmetry and the minimal coupling procedure to introduce the interaction of a superparticle with an external superpotential.

### 3. N = 2 massive superparticle coupled to external superpotential
We start from the following action of the $N = 2$ massive charged superparticle coupled to an external superpotential

$$S_{\text{min}}^{(c)} = \int d\tau \left[ \frac{1}{2} \left( \frac{\omega^\mu \omega_\mu}{g} - gm^2 \right) + m \left( \theta^a \dot{\theta}^i_a + \overline{\theta}^{\dot{\alpha}} \dot{\bar{\theta}}^{\dot{\alpha} \dot{i}} \right) \right]$$

$$+ ie \int d\tau \left( \omega^\mu A_\mu + \dot{\theta}^a A^i_a + \dot{\bar{\theta}}^{\dot{\alpha} \dot{i}} \bar{A}^{\dot{\alpha} i} \right).$$

Here we restrict ourselves by the electromagnetic U(1) group case. The gauge superfields $A_M(x^\mu, \theta, \bar{\theta}) = (A_\mu, A^i_\alpha, A^{\dot{i}}_{\dot{\alpha}})$ contain a great number of unphysical component fields, which have to be removed by imposing gauge invariant constraints on the superfield strengths. The requirement of the k-symmetry existence will restrict the admissible form of these constraints.

Now we consider the Hamiltonian treatment of the model (16) and introduce the canonical momenta

$$p_\mu = \frac{\partial L}{\partial \dot{x}^\mu} = \frac{\omega^\mu}{g} + ie A_\mu, \quad p_g = \frac{\partial L}{\partial \dot{g}} = 0$$

$$\pi^i_\alpha = \frac{\partial L}{\partial \dot{\theta}^i_\alpha} = -\frac{i \omega_\alpha \dot{\theta}^{\dot{\alpha}}_i}{g} - m \theta^i_\alpha + e A^i_\alpha \overline{\theta}^{\dot{\alpha}}_i \dot{\bar{\theta}}^{\dot{\alpha} i}$$

$$\bar{\pi}^{\dot{\alpha} i} = \frac{\partial L}{\partial \dot{\bar{\theta}}^{\dot{\alpha} i}} = -\frac{i \theta^\alpha \dot{\theta}^i_\alpha}{g} - m \bar{\theta}^{\dot{\alpha} i} + e \bar{\theta}^{\dot{\alpha} i} A_\alpha \dot{\theta}^i_\alpha + ie \bar{A}^{\dot{\alpha} i}.$$ (17)

The primary constraints following from these definitions are the following:

$$p_g \approx 0$$

$$V^i_\alpha = \pi^i_\alpha + ip_{a\dot{\alpha}} \bar{\theta}^{\dot{\alpha} i} + m \theta^i_\alpha - ie A^i_\alpha \approx 0, \quad \bar{V}^{\dot{\alpha} i} = \bar{\pi}^{\dot{\alpha} i} + i \theta^\alpha p_{a\dot{\alpha}} + m \bar{\theta}^{\dot{\alpha} i} - ie \bar{A}^{\dot{\alpha} i} \approx 0,$$ (18)

and the canonical Hamiltonian is given by

$$H_0 = \frac{q}{2} \left[ (p^\mu - ie A^\mu)^2 + m^2 \right]$$ (19)

The total Hamiltonian is

$$H = H_0 + \lambda^\alpha V^i_\alpha + \bar{\lambda}^{\dot{\alpha}} \bar{V}^{\dot{\alpha} i} + \varphi p_g.$$ (20)

Below we remind some useful Poisson brackets relations following from Eqs.(18-19)

$$\{ V^i_\alpha, V^{j}_\beta \} = -2ie \epsilon^{i\beta}_{\alpha\beta} - e F_{\alpha\beta}; \quad \{ \bar{V}^{\dot{i}}_{\dot{\alpha}}, \bar{V}^{\dot{j}}_{\dot{\beta}} \} = -2ie \epsilon^{\dot{j}\beta}_{\dot{\alpha}\dot{\beta}} - e F_{\dot{\alpha}\dot{\beta}};$$

$$\{ V^i_\alpha, \bar{V}^{\dot{i}}_{\dot{\alpha}} \} = 2 \mathcal{P}_{a\dot{\alpha}} - e F_{a\dot{\alpha}} \text{, where } \mathcal{P}_{a\dot{\alpha}} = p_{a\dot{\alpha}} - ie A_{a\dot{\alpha}};$$

$$\{ H_0, V^i_\alpha \} = \frac{eg}{2} \mathcal{P}^{\beta\dot{\beta}}_{\beta\dot{\beta}a} F_{\beta\dot{\beta}a}; \quad \{ H_0, \bar{V}^{\dot{i}}_{\dot{\alpha}} \} = \frac{eg}{2} \mathcal{P}^{\beta\dot{\beta}}_{\beta\dot{\beta}a} F_{\beta\dot{\beta}a}. \quad (21)$$
The field strengths are defined in [18]. Temporal conservation of the primary constraints leads to the secondary one \( \chi = \mathcal{P} + m^2 \approx 0 \) and to the system of the linear equations for the Lagrange multipliers

\[
E_\alpha = Q_\alpha + \lambda^\beta M_{\alpha \beta} + \bar{\lambda}^{\dot{\beta}} N_{\alpha \dot{\beta}} = 0
\]
\[
\bar{E}_{\dot{\alpha}} = \bar{Q}_{\dot{\alpha}} + \lambda^\beta M_{\beta \dot{\alpha}} + \bar{\lambda}^{\dot{\beta}} \bar{M}_{\dot{\beta} \dot{\alpha}} = 0
\]
\[
Q_\alpha \lambda^\alpha + \bar{Q}_{\dot{\alpha}} \bar{\lambda}^\dot{\alpha} = 0,
\]
where we have used the following notations:

\[
Q_\alpha = \frac{eg}{2} \mathcal{P}^{\beta \dot{\beta}} F_{\beta \dot{\beta} \alpha}, \quad \bar{Q}_{\dot{\alpha}} = \frac{eg}{2} \mathcal{P}^{\dot{\beta} \beta} \bar{F}_{\dot{\beta} \beta \dot{\alpha}};
\]
\[
M_{\beta \alpha} = -2i\text{m} \varepsilon_{\beta \alpha} - eF_{\beta \alpha}, \quad \bar{M}_{\dot{\beta} \dot{\alpha}} = -2i\text{m} \varepsilon_{\dot{\beta} \dot{\alpha}} - e\bar{F}_{\dot{\beta} \dot{\alpha}};
\]
\[
N_{\alpha \dot{\alpha}} = 2\mathcal{P}_{\alpha \dot{\alpha}} - eF_{\alpha \dot{\alpha}}.
\]

It is easy to show that the last equation in (22) is a consequence of the others. Similarly to the free case, the existence of the four spinorial first-class constraints imposes certain restrictions on the rank of the system (22). Namely, this rank should be equal to four. The matrix of the system (22) equals

\[
A = \begin{pmatrix} M & N \\ N^T & \bar{M} \end{pmatrix}
\]

Respectively, the matrix of the extended system can be written in the form

\[
A_{\text{ex}} = \begin{pmatrix} M & N & -Q \\ N^T & \bar{M} & -\bar{Q} \end{pmatrix}
\]

Using well-known properties of the rank invariance one can persuade that rank \( A = \text{rank} \, R \) and det \( A = \text{det} \, R \), where

\[
R = \begin{pmatrix} M & N \\ 0 & \bar{M} - N^T M^{-1} N \end{pmatrix}.
\]

We suppose that \( \text{det} \, M \neq 0 \), so rank \( M = 4 \). Indeed, \( \text{det} \, M \neq 0 \) when the interaction is turned off, so it is quite reasonable to assume its conservation when interaction is turned on. Then we find that rank \( A = \text{rank} \, M = \text{rank} \, R = 4 \) and

\[
Y_{\beta \dot{\alpha}} = M_{\beta \dot{\alpha}} - N_{\alpha \beta} M^{-1} \bar{M}^{-1} N_{\beta \dot{\alpha}} = 0.
\]
The system (22) is compatible, when and only when rank $A = \text{rank } A_{ex} = 4$ or, equivalently,

$$\text{rank } A_{ex} = \text{rank} \begin{pmatrix} M & N & -Q \\ 0 & 0 & -\bar{Q} + NT^{-1}M \bar{Q} \end{pmatrix} = 4,$$  \hspace{1cm} (28)

The latter equation implies the constraint

$$\bar{Q}_{\hat{\alpha}} - N_{\beta\hat{\alpha}}M^{-1\alpha\beta}Q_\alpha = 0$$  \hspace{1cm} (29)

provided that $\det N \neq 0$.

Thus, we conclude that the system’s rank is halved, if and only if the new constraints (27,29) are satisfied. In view of this observation the total Hamiltonian (20) should be extended to the form

$$H = \frac{g}{2} \chi - \frac{1}{2} Q^\alpha M_{\alpha\beta} V_\beta - \frac{1}{2} \bar{Q}^\hat{\alpha} \bar{M}_{\hat{\alpha}\hat{\beta}} \bar{V}^\hat{\beta} + \frac{1}{2} \lambda^\alpha \left( V^\alpha - N_{\alpha\hat{\beta}} \bar{M}^{-1\hat{\alpha}\hat{\beta}} \bar{V}^\hat{\alpha} \right)$$

$$\frac{1}{2} \bar{\lambda}^{\hat{\alpha}} \left( \bar{V}^\hat{\alpha} - N_{\beta\hat{\alpha}} M^{-1\beta\alpha} V_\alpha \right) \approx 0,$$

(30)

where $\lambda^\alpha$ and $\bar{\lambda}^{\hat{\alpha}}$ are connected via either $E_{\alpha}^1, \bar{E}_{\hat{\alpha}}^1$ or $E_{\alpha}^2, \bar{E}_{\hat{\alpha}}^2$. The new Lagrange multipliers $\lambda$ define the first-class constraints. It is well known that the first-class constraints form a closed algebra. To prove that we are dealing with just this case, we are to calculate the Poisson brackets of the two second-class constraints

$$\{ V^{(1)}_\alpha, V^{(1)}_\beta \} = Y_{\alpha\beta} + (\text{linear and quadratic terms in } V(V)) \approx 0,$$  \hspace{1cm} (31)

where $Y_{\alpha\beta} = M_{\alpha\beta} - N_{\alpha\hat{\beta}} \bar{M}^{-1\hat{\alpha}\hat{\beta}} N_{\beta\hat{\alpha}} \approx 0$. $Y_{\alpha\beta}$ is the complex conjugate constraint to $\bar{Y}_{\hat{\alpha}\hat{\beta}}$. Note that it has the polynomial structure in $P_{\alpha\hat{\beta}}$ of the second power with the coefficients constructed from the spinorial components of the superfield strengths. This essential feature will play the crucial role in our further analysis. Now we are going to study the Poisson brackets for $V^{(1)}_\alpha$ and $Y_{\beta\gamma}$

$$\{ V^{(1)}_\alpha, Y_{\beta\gamma} \} = a_{\alpha\beta\gamma} + \sum_{\text{cycl}(\alpha\beta\gamma)} \left( b_{\alpha\beta\gamma} \bar{P}_{\gamma\lambda} + c_{\alpha\beta\gamma} \bar{P}_{\beta\lambda} \bar{P}_{\gamma\hat{\rho}} \right) + d_{\alpha\beta\gamma} \bar{P}_{\alpha\lambda} \bar{P}_{\beta\rho} \bar{P}_{\gamma\hat{\delta}} +$$

$$+ (\text{linear, quadratic and cubic terms in } V(V)),$$  \hspace{1cm} (32)

\footnote{As system’s rank is halved now we can consider any of these complex conjugate pairs as independent equations.}

\footnote{Here and further we omit explicit expressions for the terms proportional to the constraints $V(V)$ because they are irrelevant for the definition of the constraint’s class.}
where

\[ d\dot{\lambda}_\alpha^\delta = -8i\bar{M}^{-1}i^{\dot{\lambda}_\alpha^\delta} \bar{D}_\lambda^\beta \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma}; \]

\[ c^\gamma_{\alpha^\beta} = 4iD_\alpha^\beta \bar{M}^{-1}i^{\dot{\lambda}_\alpha^\delta} \]

\[ + 4ieF_\alpha^\beta \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} \bar{D}_\lambda^\delta \bar{M}^{-1}i^{\dot{\lambda}_\delta^\alpha}; \]

\[ \dot{c}^\dot{\alpha}_{\alpha^\beta} = -8e\bar{M}^{-1}i^{\dot{\lambda}_\alpha^\delta} \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} F_{\alpha^\beta\dot{\alpha}}^\gamma - 4ie\bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} \bar{D}_\lambda^\delta \left( \bar{M}^{-1}i^{\dot{\lambda}_\delta^\alpha} F_{\alpha^\beta\dot{\alpha}}^\gamma \right); \]

\[ \dot{b}_{\alpha^\beta} = 2i\bar{M}^{-1}i^{\dot{\lambda}_\alpha^\delta} \bar{D}_\lambda^\beta M_{\alpha^\beta} - 4e^2\bar{M}^{-1}i^{\dot{\lambda}_\alpha^\delta} \bar{F}_{\alpha^\dot{\beta}}^\gamma \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} M_{\alpha^\dot{\beta}}^\dot{\lambda} \bar{F}_{\dot{\beta}}^\delta \bar{M}^{-1}i^{\dot{\lambda}_\delta^\alpha} \]

\[ F_{\beta^\alpha} = 2ie^2\bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} \bar{D}_\lambda^\delta \left( \bar{M}^{-1}i^{\dot{\lambda}_\delta^\alpha} F_{\alpha^\beta}^\gamma \right); \]

\[ \dot{b}_{\alpha^\beta} = -4e\bar{M}^{-1}i^{\dot{\lambda}_\alpha^\delta} F_{\alpha^\beta\dot{\alpha}}^\gamma - 2ieD_\alpha^\beta \left( \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} F_{\alpha^\beta}^\gamma \right) - 4e^2F_{\gamma\beta}^\delta \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} \bar{M}^{-1}i^{\dot{\lambda}_\gamma^\alpha} \]

\[ F_{\alpha^\beta\dot{\gamma}} = 2ie^2F_{\gamma\beta}^\delta \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} \bar{D}_\lambda^\delta \left( \bar{M}^{-1}i^{\dot{\lambda}_\delta^\alpha} F_{\alpha^\beta\dot{\alpha}}^\gamma \right); \]

\[ a_{\alpha^\beta\gamma} = -iD_\alpha^\beta M_{\beta^\gamma} + 2e^2F_{\alpha^\beta^\gamma}^\delta \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} \bar{F}_{\dot{\beta}}^\delta \bar{M}_{\dot{\beta}}^\gamma + ie^2D_\alpha^\beta \left( F_{\dot{\beta}}^\delta \bar{M}^{-1}i^{\dot{\lambda}_\delta^\alpha} F_{\dot{\alpha}}^\gamma \right) + \]

\[ 2e^2F_{\alpha^\beta^\gamma}^\delta \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} \bar{F}_{\dot{\beta}}^\delta M_{\beta^\gamma} + 2e^3F_{\alpha^\beta}^\delta \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} F_{\beta^\delta}^\gamma \bar{M}^{-1}i^{\dot{\lambda}_\gamma^\alpha} F_{\alpha^\beta\dot{\alpha}}^\gamma + \]

\[ 2e^3F_{\alpha^\beta}^\delta \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} \bar{F}_{\beta^\delta}^\gamma \bar{M}^{-1}i^{\dot{\lambda}_\gamma^\alpha} F_{\alpha^\beta\dot{\alpha}}^\gamma + ie^3F_{\alpha^\beta}^\delta \bar{M}^{-1}i^{\dot{\lambda}_\beta^\gamma} \bar{D}_\lambda^\delta \left( F_{\beta^\delta}^\gamma \bar{M}^{-1}i^{\dot{\lambda}_\gamma^\alpha} F_{\dot{\alpha}}^\gamma \right). \]

\( Y_{\alpha^\beta\gamma} \) possesses the polynomial structure of the third power with respect to \( P_{\alpha^\beta} \) with the coefficient functions depending on the superfield strengths. It is not a function of the present constraints, so we are forced to consider it as a new constraint. Again, we calculate the Poisson brackets for \( Y_{\alpha^\beta\gamma} \) and \( V_2^{(1)} \) to obtain a new fourth power polynomial constraint, which should be then considered as a new constraint. In the limit, the above-described procedure leads to an infinite sequence of polynomial constraints of arbitrary high power with respect to \( P_{\alpha^\beta} \) with the coefficient functions constructed from the superfield strengths. A controlled analysis of the exact form of these infinite constraints chain is rather difficult, since we have not found any recursion procedure for their presentation as functions of \( Y_{\alpha^\beta} \).

The only reason, why we have got infinite set of the constraints, is that the object \( Y_{\gamma_1...\gamma_n\alpha^\beta} \) appearing on every stage was considered as the creation of a new constraint. A possibility to avoid such uncontrolled multiplication of the constraints supposes their identical fulfilment (for the total set of \( P_{\alpha^\beta} \)) starting from a certain stage. Taking into account the structure mentioned above, the identical fulfilment actually signifies some restriction for the superfield configurations. The identical fulfilment of the n-th stage constraint yields the identical fulfilment of the next stage constraints. Although we can’t realize this procedure for an arbitrary stage (because of the sophisticated structure of the appearing expressions), here we present the explicit consideration of the first and the second stages.
At the first stage we deal with $Y_{\alpha\beta}$. For our purpose it will be convenient to introduce the SU(2)-decomposition of the superfield strengths

$$ F_{\alpha\beta} = -\epsilon_{\alpha\beta\dot{\alpha}\dot{\beta}} W + \tau_{aij} \tilde{F}^a_{\alpha\beta}; \quad F_{\dot{\alpha}\dot{\beta}} = \epsilon_{\alpha\dot{\beta}} \bar{W} + \tau_{aij} \tilde{F}^a_{\alpha\dot{\beta}}, $$

and then the matrix inverse to $M$ takes the form

$$ \bar{M}^{-1}_{\dot{\alpha}\dot{\beta}} = \frac{i}{2(m - ieW/2)} \left\{ \epsilon_{\dot{\alpha}\dot{\beta}} + \sum_{n=1}^{\infty} \left[ \frac{ie}{2(m - ieW/2)} \right]^n \tilde{F}_{\dot{\alpha}_n\dot{\beta}_n} \cdots \tilde{F}_{\dot{\alpha}_1\dot{\beta}_1} \right\}. $$

After substitution of the explicit expressions for the $M$, $\bar{M}$ and $N$ matrices and using the mass shell constraint $\chi = p^2 + m^2 \approx 0$, one finds:

- the quadratic term
  $$ \sum_{n=1}^{\infty} \left[ \frac{ie}{2(m - ieW/2)} \right]^n \tilde{F}_{\dot{\alpha}_n\dot{\beta}_n} \cdots \tilde{F}_{\dot{\alpha}_1\dot{\beta}_1} = 0 \implies \tilde{F}_{\dot{\alpha}\dot{\beta}} = 0; $$

- the linear term
  $$ F_{\alpha\beta} = 0; $$

- the free term
  $$ -2i(m + ie\bar{W}/2)\epsilon_{\alpha\beta} + \frac{2im^2\epsilon_{\alpha\beta}}{(m - ieW/2)} - e\tilde{F}_{\alpha\beta} = 0. $$

Taking into account the linear independence of the $\tau$-matrices leads to the constraints

$$ \tilde{F}_{\alpha\beta} = 0 $$

and

$$ (m + ie\bar{W}/2)(m - ieW/2) = m^2. $$

After the substitution of the constraints (35-37) into the superfield Bianchi identities, we find the following restrictions for the physical superfields $W$ and $\bar{W}$: $D_\alpha W = \bar{D}_\dot{\alpha} W = 0$ and $D^{ij} W - \bar{D}^{ij} \bar{W} = 0$, which isolate $N = 2$ Maxwell supermultiplet $(z, \lambda^i_\alpha, C^{ij}, v_\mu)$. The last constraint (37') is preserved. Its differentiation imposes additional restrictions on the chiral superfields $W$ and $\bar{W}$ which eliminate some physical degrees of freedom. Thus, it is impossible to consider $Y_{\alpha\beta}$ as an identical constraint.

At the second stage the nullification of the cubic term in $\mathcal{P}_{\alpha\dot{\alpha}}$ from Eqs.(32) yields the two possibilities:

- a) $\bar{D}_\dot{\alpha} \tilde{F}_{\dot{\alpha}\dot{\beta}} = 0$, $\bar{D}_\dot{\alpha} W = 0$;
- b) $\tilde{F}_{\dot{\alpha}\dot{\beta}} = 0$. 

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The treatment of the quadratic terms leaves the only possibility $\tilde{F}_{\dot{\alpha}\dot{\beta}} = 0$ and $\bar{D}_{\dot{\gamma}} W = 0$ together with the constraint

$$2\delta_j^i F'_{\alpha\beta,\dot{\gamma}k} + i \bar{D}_{\dot{\gamma}k} F^i_{\alpha\dot{\beta}j} = 0$$

(38)

and its complex conjugate, where the spinor-vector superfield strength components were decomposed on spin $\frac{1}{2}$ and spin $\frac{3}{2}$ parts

$$F_{\alpha\beta,\dot{\gamma}k} = \varepsilon_{\dot{\beta}\dot{\gamma}} V_{\alpha k} + F'_{\alpha\dot{\beta},\dot{\gamma}k}.$$

(39)

The substitution of the expressions (38,39) together with their complex conjugate into the Bianchi identities allows to determine the superfields $V$ and $\bar{V}$

$$V_{\dot{\gamma}} = \frac{i}{2} D_{\dot{\gamma}} W, \quad \bar{V}_{\dot{\gamma}} = -\frac{i}{2} \bar{D}_{\dot{\gamma}} \bar{W}. $$

Now, by analogy with the first stage, the linear terms impose too strong constraints on the chiral superfields $W$ and $\bar{W}$: $D_{\dot{\beta}} W = \bar{D}_{\dot{\beta}} \bar{W} = 0$. And, again, we conclude that the identical fulfilment of $Y_{\alpha\beta\dot{\gamma}}$ eliminates physical degrees of freedom.

Although we could not prove it evidently, it is quite reasonable to conjecture that the identical fulfilment of the next stage constraints will also eliminate the physical degrees of freedom. Then the necessity for the introduction of nonminimal terms to preserve the k-symmetry becomes evident.

4. A nonminimal coupling of $N = 2$ massive superparticle

There exists a possibility to introduce some nonminimal terms for a superparticle possessing not only the electrical charge $e$, but also an AMM $\mu$, in such a way that the minimal structure of the interactions caused by the electric charge, will be preserved. Taking into account the dimensional reasons ($[\mu] = L$ in the system $c = \hbar = 1$) we can construct the dimensionless gauge invariant scalars $\mu F_{\mu\dot{\alpha}}$ and $\mu \bar{F}_{\dot{\alpha}\dot{\mu}}$ linear in the field strengths. Analogous considerations were used in [19] for the introduction of nonminimal terms by means of the extension of the superconnection $1$–form. Then the superparticle action can be written in the form

$$S^{(c,\mu)} = -m \int d\tau \sqrt{-F} \omega^\mu \omega_\mu + m \int d\tau \left( \hat{\theta}^{\dot{\alpha}} \hat{\theta}_{\dot{\alpha}} + \hat{\Omega}_{\dot{\alpha}} \hat{\dot{\Omega}}_{\dot{\alpha}} \right)
+ ie \int d\tau \left( \omega^\mu A_\mu + \hat{\theta}^{\dot{\alpha}} A_{\dot{\alpha}} + \hat{\dot{\theta}}_{\dot{\alpha}} \bar{A}_{\dot{\alpha}} \right)$$

(40)

where $F = \left( 1 - \frac{i\mu}{4} F_{\mu\dot{\alpha}} \right) \left( 1 - \frac{i\mu}{4} F_{\dot{\alpha}\dot{\mu}} \right)$. The rescaled mass $m^* = (m F)^{1/2}$ in the first term is supersymmetric and gauge invariant. Similar procedure in
the second term would violate global supersymmetry. Introduction of the world-line einbein gives

\[ L^{(e,\mu)} = \frac{1}{2} \left( \frac{F \omega^2}{g} - g m^2 \right) + m \left( \theta^\alpha \dot{\theta}_\alpha + \bar{\theta}_\dot{\alpha} \dot{\bar{\theta}}^\dot{\alpha} \right) \]  

\[ + i e \left( \omega^\mu A_\mu + \dot{\theta}^\alpha A_\alpha + \dot{\bar{\theta}}^\dot{\alpha} A^\dot{\alpha} \right). \]  

(41)

Note that after the redefinition: \( g \rightarrow \frac{g}{F} \) the contribution of AMM can be presented in the form of the potential term \( gF m^2 \) which disappears when \( m = 0 \).

The canonical momentum variables are

\[ p_\mu = \frac{\partial L}{\partial \dot{x}^\mu} = \frac{F}{g} \omega_\mu + i e A_\mu, p_g = \frac{\partial L}{\partial \dot{g}} = 0, \]

\[ \pi^i_\alpha = \frac{\partial L}{\partial \dot{\theta}^i_\alpha} = -i \frac{F}{g} \omega^i_\alpha \bar{\theta}^{\dot{\alpha}i} - m \theta^i_\alpha + e A_\alpha \bar{\theta}^{\dot{\alpha}i} + ie A_i^i, \]  

\[ \bar{\pi}_{\dot{\alpha}i} = \frac{\partial L}{\partial \dot{\bar{\theta}}^i_{\dot{\alpha}}} = -i \frac{F}{g} \theta^i_\alpha \omega^i_\alpha - m \bar{\theta}^{\dot{\alpha}i} + e \theta^i_\alpha A_\alpha + i e \bar{A}_{\dot{\alpha}i} \]  

(42)

and the corresponding canonical Hamiltonian is given by

\[ H_0 = \frac{g}{2F} \left[ (p^\mu - ie A^\mu)^2 + m^2 \right]. \]  

(43)

The definition (42) yields the primary constraints \( p_g \approx 0 \) and

\[ V^i_\alpha = \pi^i_\alpha + ip_{a\dot{\alpha}} \bar{\theta}^{\dot{\alpha}i} + m \theta^i_\alpha - ie A^i_\alpha \approx 0, \]

\[ \bar{V}_{\dot{\alpha}i} = \bar{\pi}_{\dot{\alpha}i} + i \theta^0_i p_{a\dot{\alpha}} + m \bar{\theta}^{\dot{\alpha}i} - ie \bar{A}_{\dot{\alpha}i} \approx 0. \]  

(44)

The total Hamiltonian reads

\[ H = H_0 + \lambda^\alpha V^i_\alpha + \bar{\lambda}_{\dot{\alpha}} \bar{V}_{\dot{\alpha}i} + \varphi p_g. \]  

(43’)

Note that, unlike \( H \) and \( H_0 \), the primary constraints don’t ”feel” nonminimal terms. As in Section 3, the temporal conservation of the primary constraints leads to the secondary constraint \( \chi = \mathcal{P}^2 + m^2 \approx 0 \) and to the linear system of equations for Lagrange multipliers coinciding with (23), if the following redefinitions are taken into account

\[ Q^{\alpha} = \frac{e g}{2F} \mathcal{P}^{\dot{\beta}\dot{\gamma}} F_{\beta\dot{\gamma}^\alpha} + \frac{igm^2}{2F} D_{\dot{\alpha}F}, \]

\[ \bar{Q}_{\dot{\alpha}}^{\dot{\alpha}} = \frac{e g}{2F} \mathcal{P}^{\beta\dot{\beta}} F_{\beta\dot{\beta}^{\dot{\alpha}}} + \frac{igm^2}{2F} D_{\dot{\alpha}F}. \]

(45)

Again, to have the first-class spinorial constraints, generating the k-symmetry, we need to halve the system rank. The necessary and sufficient condition of the rank halving is the presence of the constraints (27,29), as in the section 3. Again the spinorial first-class constraints read

\[ V^{(1)}_{\alpha} = V_\alpha - N_{\alpha\dot{\beta}} \bar{M}^{-1} \bar{V}_{\dot{\alpha}}. \]  

(46)
So, the analysis performed in the previous section is also valid here, and we can formulate the conclusion: the possibility to avoid a sequence of infinite constraints is the identical fulfillment of the constraint (27). Substituting explicit expressions for $M$ and $N$ matrices and using the mass-shell condition we get:

- the quadratic term

$$\sum_{n=1}^{\infty} \frac{i e}{2(m - i e W/2)} \tilde{F}^{\hat{\beta}\hat{\alpha}_1} \ldots \tilde{F}^{\hat{\alpha}_{n-1}\hat{\alpha}_n} = 0 \implies \tilde{F}^{\hat{\beta}\hat{\alpha}} = 0; \quad (47)$$

- the linear term

$$F_{\alpha\hat{\beta}} = 0; \quad (48)$$

- the free term

$$-2i(m + i e W/2)\varepsilon_{\alpha\beta} + \frac{2im^2 F_{\alpha\beta}}{(m - i e W/2)} - e\tilde{F}_{\alpha\beta} = 0.$$ 

in each order. Taking into account $\tau$-matrices linear independence one has:

$$\tilde{F}_{\alpha\beta} = 0 \quad (49)$$

and

$$(m + i e \tilde{W}/2)(m - i e W/2) = (m + i m \tilde{W})(m - i m W). \quad (49')$$

Substituting the constraints (47-49) into the Bianchi identities for the superfield strengths leads to the standard constraints on $N = 2$ physical superfields

$$\bar{D}_\alpha W = D_\alpha \tilde{W} = D^{ij} W - \bar{D}^{ij} \tilde{W} = 0. \quad (50)$$

The remaining constraint (49') either fixes the AMM magnitude $\mu = \frac{e}{2m}$, imposing no further constraints on $W$ and $\tilde{W}$, or eliminates the physical degrees of freedom. So, unlike the minimal case, nonminimal one gives rise to the first-class constraints (and the k-symmetry), but at certain field configurations (50) and $\mu = \frac{e}{2m}$. Then the superparticle Lagrangian is presented as

$$L^{(e,\mu(e))} = \frac{1}{2} \left[ \frac{(m - i e W/2)(m + i e \tilde{W}/2)\omega^2}{g} - g m^2 \right] + m \left( \theta^\alpha \dot{\theta}_\alpha + \bar{\theta}_\alpha \dot{\bar{\theta}}_\alpha \right) + i e \left( \omega^\mu A_\mu + \dot{\theta}^\alpha A_\alpha + \dot{\bar{\theta}}_{\dot{\alpha}} \bar{A}_{\dot{\alpha}} \right)$$

and the first-class spinorial constraints are written in the form

$$V^{(1)}_\alpha = V_\alpha - \frac{i P_{\alpha\hat{\beta}} \tilde{V}^{\hat{\beta}}}{(m - i e W/2)}; \quad \bar{V}^{(1)}_{\hat{\alpha}} = \bar{V}_{\hat{\alpha}} - \frac{i P_{\hat{\beta}\hat{\alpha}} \tilde{V}^{\hat{\beta}}}{(m + i e W/2)}, \quad (52)$$

only half of them being independent:

$$\frac{i P^{\hat{\alpha}\alpha} V^{(1)}_\alpha}{(m + i e W/2)} \approx \bar{V}^{(1)}_{\hat{\alpha}}. \quad (53)$$
To obtain the explicit expressions for the total Hamiltonian (43') we have to solve equations (23) subjected to the constraints (47-50) and substitute the solution into (43'). Equations (23) can be written in the form

\[-ieg \frac{1}{4F} \mathcal{P}_{\alpha \beta} \bar{D}^{\dot{\alpha} \dot{\beta}} \bar{W} + \frac{eg}{4F} (m + ie\bar{W}/2) \bar{D}^{\dot{i}}_{\dot{\alpha}} W - 2i(m + ie\bar{W}/2) \lambda^{\dot{i}}_{\dot{\alpha}} + \bar{\lambda}^{\dot{i}}_{\dot{\beta}} \mathcal{P}_{\alpha \dot{\beta}} = 0\]

\[ieg \frac{1}{4F} \mathcal{P}_{\beta \dot{\alpha}} \bar{D}^{\dot{\iota}} W - \frac{eg}{4F} (m - ieW/2) \bar{D}_{\dot{\alpha} \dot{\iota}} W + 2\lambda^{\dot{i}}_{\dot{\alpha}} \mathcal{P}_{\beta \dot{\alpha}} - 2i(m - ieW/2) \bar{\lambda}^{\dot{i}}_{\dot{\alpha}} = 0,\]  

(54)

where we used the spinor-vector superfield strength components following from the Bianchi identities solution

\[F_{\mu \dot{\alpha}} = i \left( \sigma_{\mu} \bar{D}^{\dot{\alpha}} \right) \bar{W}, \quad F_{\mu \dot{\iota}} = -i \left( D_{\dot{\iota}} \sigma_{\mu} \right) \bar{W}.\]  

(55)

Solving (54) with respect to \(\lambda_{\alpha}^{\dot{\alpha}}\) and \(\bar{\lambda}_{\dot{\alpha}}^{\dot{\iota}}\), and substituting the solution in (43') we obtain

\[H = \frac{g}{2F} T + \lambda^{\alpha}_{\alpha} \bar{V}^{(1)1} + \bar{\lambda}^{\dot{\alpha}}_{\dot{\alpha}} \bar{V}^{(1)\dot{\alpha} 2} = \]

\[\frac{g}{2F} \left[ \chi - \frac{eV^{\alpha 2}_{\alpha} \mathcal{P}_{\alpha \dot{\alpha}} \bar{D}^{\dot{\alpha} \dot{\beta}} \bar{W}}{4(m + ieW/2)} + \frac{ie}{4} D_{\alpha 2} W V^{\alpha 2} + \frac{eD^{\dot{1}}_{\dot{\alpha}} W \mathcal{P}_{\dot{\beta} \dot{\alpha}} \bar{V}^{\dot{\alpha} 1}}{4(m - ieW/2)} + \frac{ie}{4} \bar{D}^{\dot{1}}_{\dot{\alpha}} \bar{W} \bar{V}^{\dot{\alpha} 1} \right] + \lambda^{\alpha}_{\alpha} \bar{V}^{(1)1} + \bar{\lambda}^{\dot{\alpha}}_{\dot{\alpha}} \bar{V}^{(1)\dot{\alpha} 2} \approx 0.\]  

(56)

Both the spinorial first-class constraints and the bosonic reparametrization generator \(T\) are not SU(2)-invariants, however, the latter may be written in completely invariant fashion by means of the shift

\[\lambda_{\alpha} \rightarrow \lambda_{\alpha} + \frac{ieg}{8F} D_{\alpha} \bar{W}, \quad \bar{\lambda}^{\dot{\alpha}} \rightarrow \bar{\lambda}^{\dot{\alpha}} - \frac{ieg}{8F} \bar{D}^{\dot{\alpha}} \bar{W}.\]  

(57)

Then the full Hamiltonian takes form

\[H = \frac{g}{2F} T + \lambda^{\alpha}_{\alpha} \bar{V}^{(1)1} + \bar{\lambda}^{\dot{\alpha}}_{\dot{\alpha}} \bar{V}^{(1)\dot{\alpha} 2} = \]

\[\frac{g}{2F} \left[ (P^2 + m^2) - \frac{ie}{4} D^{\alpha} W \bar{V}^{\alpha} + \frac{ie}{4} \bar{D}^{\dot{\alpha}} \bar{V}^{\dot{\alpha}} \right] + \lambda^{\alpha}_{\alpha} \bar{V}^{(1)1} + \bar{\lambda}^{\dot{\alpha}}_{\dot{\alpha}} \bar{V}^{(1)\dot{\alpha} 2}.\]  

(58)

It contains only half of the spinorial Lagrange multipliers and the same number of the spinorial first-class constraints.

\[
\text{As the system rank equals four, we can choose either } \lambda_{\alpha 1} \text{ and } \bar{\lambda}^{\dot{\alpha} 2} \text{ or } (\lambda_{\alpha 2} \text{ and } \bar{\lambda}^{\dot{\alpha} 1}) \text{ as independent variables.}
\]
It is straightforward to construct the k-symmetry transformation laws implied by the action (40): \( \delta x^\mu = -i \theta_i \sigma^\mu \delta \bar{\theta}^i + \delta \theta_i \sigma^\mu \bar{\theta}^i \),

\[
\begin{pmatrix}
\delta \theta^i_k \\
\delta \bar{\theta}^k_{\bar{a}}
\end{pmatrix} = P_I \begin{pmatrix}
\kappa^i_k(\tau) \\
\bar{\kappa}_{\bar{a}}^k(\tau)
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
\kappa^i_k - \frac{im^* \omega_{\alpha \beta} \bar{\kappa}_{\bar{b}}^i}{\sqrt{-\omega^2(m + i\epsilon W/2)}} \\
\bar{\kappa}_{\bar{a}}^i + \frac{im^* \omega_{\bar{a} \bar{b}} \kappa^i_k}{\sqrt{-\omega^2(m - i\epsilon W/2)}}
\end{pmatrix},
\]

where \((\kappa^i_k(\tau), \bar{\kappa}_{\bar{a}}^i(\tau))\) are arbitrary functions of \(\tau\). Here we used the projector \([17]\), which satisfies the well-known conditions \(P^2_I = P_I\) and \(\Gamma^2 = 1\), to halve the bispinor components

\[
P_I = \frac{1}{2} \delta^i_j (1 + \Gamma) = \frac{1}{2} \delta^i_j \begin{pmatrix}
\delta^\beta_k & \frac{-im^* \omega_{\alpha \beta}}{\sqrt{-\omega^2(m + i\epsilon W/2)}} \\
\frac{-im^* \omega_{\bar{a} \bar{b}}}{\sqrt{-\omega^2(m - i\epsilon W/2)}} & \delta_{\bar{a}}^\beta
\end{pmatrix},
\]

Introduction of the projector \(P_{II}\), with the standard properties \(P_{I,II} = P_{I,II}\), \(P_I P_{II} = P_{II} P_I = 0\), gives us the way to separate the first- and the second-class constraints

\[
\begin{pmatrix}
V^{(1)i}_\alpha \\
\bar{V}^{(1)i}_{\bar{a}}
\end{pmatrix} = P_I \begin{pmatrix}
V^{(1)i}_\beta \\
\bar{V}^{(1)i}_{\bar{b}}
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
V^{i}_\alpha - \frac{i P_{\alpha \beta} \bar{V}^{\beta i}}{(m - i\epsilon W/2)} \sqrt{m^2} \\
\bar{V}^{\alpha i}_{\bar{b}} + \frac{i P_{\beta \bar{a}} V^{i}_\beta}{(m + i\epsilon W/2)} \sqrt{m^2}
\end{pmatrix},
\]

\[
\begin{pmatrix}
V^{(2)i}_\alpha \\
\bar{V}^{(2)i}_{\bar{a}}
\end{pmatrix} = P_{II} \begin{pmatrix}
V^{(2)i}_\beta \\
\bar{V}^{(2)i}_{\bar{b}}
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
V^{i}_\alpha + \frac{i P_{\alpha \beta} \bar{V}^{\beta i}}{(m - i\epsilon W/2)} \sqrt{m^2} \\
\bar{V}^{\alpha i}_{\bar{b}} - \frac{i P_{\beta \bar{a}} V^{i}_\beta}{(m + i\epsilon W/2)} \sqrt{m^2}
\end{pmatrix}
\]

These projectors can be presented in the form

\[
P_{I,II} = \frac{1}{2} \delta^i_j (1 \pm \Gamma),
\]

where

\[
\Gamma = \begin{pmatrix}
0 & -i P_{\alpha \beta} \sqrt{m^2} \\
i P_{\bar{a} \bar{b}} \sqrt{m^2} & 0
\end{pmatrix},
\]
The first- and the second-class constraints are (separately) linear dependent:

\[
V^{(1) i}_{\alpha} \frac{i\bar{P}^{\beta\alpha}}{(m + ieW/2)} \sqrt{\frac{m^*}{-\bar{p}^2}} = \bar{V}^{(1) \bar{i}}, \quad V^{(2) i}_{\alpha} \frac{-i\bar{P}^{\beta\alpha}}{(m + ieW/2)} \sqrt{\frac{m^*}{-\bar{p}^2}} = \bar{V}^{(2) \bar{i}}. \tag{63}
\]

In the conclusion let us show that the introduced coupling constant \(\mu\) actually describes the superparticle AMM. For this purpose we are to consider the term \(\frac{i\mu}{2g} \omega^2(\bar{W} - W)\) in (41). Separating the photon part in W superfield component decomposition we obtain

\[
W = \cdots - 2i\theta_{i}^{\mu} \sigma^{\mu\nu} \theta^{\nu i} + \cdots \tag{64}
\]

While substituting this expression back into (41) and passing to the pseudo-classical bispinor variables [19]

\[
\Psi^{i} = \begin{pmatrix}
(-i\omega^{2}/g)^{\frac{1}{2}} \theta^{i}_{\alpha} \\
(i\omega^{2}/g)^{\frac{1}{2}} \bar{\theta}^{\bar{i}\alpha}
\end{pmatrix}
\]

and introducing the spin operator \(\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^{\mu}, \gamma^{\nu}]\) (\(\gamma\)-matrices are taken in the Weyl basis) we find

\[
\left. \frac{i \mu}{2g} \omega^{\mu} \omega_{\mu}(\bar{W} - W) \right|_{\text{photon}} = \mu \Psi^{i} \Sigma^{\mu\nu} \Psi^{j} v_{\mu\nu}(x) + \text{(higher corrections)} \tag{65}
\]

As is seen, the expression (65) is just the ordinary Pauli term.

### 5. Conclusions

We have examined the \(N = 2\) extended massive superparticle model [4] (with and without interactions) following to the Dirac prescription and evolving the results [12]. In the free case this model possesses the k-symmetry which allows to gauge away a half of fermionic degrees of freedom. Including the minimal interaction with the Abelian gauge superfield \(A_{M}(x, \theta, \bar{\theta})\) and demanding the k-symmetry existence, as in the free case, imposes too strong constraints on the superfield strengths \(F_{MN}\), which eliminate the component fields of the \(N = 2\) Maxwell multiplet. So we present an explicit proof for the conclusion that the model with minimal coupling actually does not permit k-invariant terms of interaction. To restore the k-invariance we introduced the nonminimal terms into the Lagrangian of the superparticle, which describes the AMM caused interactions, as we’ve shown. Thus the idea of the nonminimal terms introduction, earlier advanced in [12], gets here its explicit realization by means of constructing the nonminimal Lagrangian. This Lagrangian is invariant under the k-symmetry transformations, only if the AMM value of the massive superparticle is rigorously fixed.
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Appendix A

In the present paper we use following metric signature: $\eta_{\mu\nu} = \text{diag}(-, +, +, +)$. SU(2) and SO(1,3) spinor indices can be raised or lowered in completely covariant way:

$$\theta_\alpha^i = \varepsilon_{\alpha\beta}\varepsilon^{ij}\theta_\beta^j,$$
$$\bar{\theta}_{\dot{\alpha}}^i = \varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon^{ij}\bar{\theta}_{\dot{\beta}}^j,$$

where $\varepsilon^{12} = \varepsilon_{12} = 1$. $\sigma$-matrices have the following properties:

$$\bar{\sigma}^{\mu\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\sigma^{\mu}_{\dot{\beta}}, \quad \sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\nu\dot{\alpha}} = -2\eta^{\mu\nu},$$
$$\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\mu\dot{\beta}} = -2\delta^{\beta}_{\dot{\alpha}}\delta^{\dot{\alpha}}_{\beta}, \quad \sigma^{(\mu}_{\alpha\dot{\alpha}}\sigma^{\nu)\dot{\beta}}_{\beta} = -2\eta^{\mu\nu}\delta^{\dot{\beta}}_{\beta},$$
$$\bar{\sigma}^{(\mu\dot{\alpha}}\sigma^{\nu)\dot{\beta}}_{\beta} = -2\eta^{\mu\nu}\delta^{\dot{\beta}}_{\beta}, \quad \sigma^{[\mu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\nu]_{\beta} = 4\sigma^{\mu\nu\dot{\beta}}_{\alpha},$$
$$\bar{\sigma}^{[\mu\dot{\alpha}}\sigma^{\nu]_{\beta} = 4\sigma^{\mu\nu\dot{\beta}}_{\alpha}.$$

The above formulae can be used to define the scalars:

$$\theta\zeta \equiv \theta_i^\alpha s^i_{\alpha} = -\zeta\theta, \quad \bar{\theta}\bar{\zeta} \equiv \bar{\theta}_{\dot{\alpha}}\bar{\zeta}_{\dot{\alpha}} = -\bar{\zeta}\bar{\theta},$$
$$\theta\sigma^{\mu\nu}\zeta \equiv \theta_i^\alpha s^{\mu}_{\alpha\dot{\alpha}} s^i_{\dot{\alpha}}, \quad \theta\sigma^{\mu\nu}\zeta \equiv \theta_i^\alpha s^{\mu\nu\dot{\beta}}_{\alpha\dot{\beta}} s^i_{\dot{\beta}}.$$

The conjugation rules for spinors, derivatives, $\varepsilon$-matrixes and potentials are the following

$$(\theta_i^\alpha)^\dagger = \bar{\theta}_{\dot{\alpha}}^i, \quad (\theta_{\dot{\alpha}}^i)^\dagger = -\bar{\theta}_{\dot{\alpha}}^i,$$
$$(D_i^\alpha)^\dagger = \bar{D}_{\dot{\alpha}}^i, \quad (D^i_{\alpha})^\dagger = -\bar{D}_{\dot{\alpha}}^i,$$
$$(A_i^\alpha)^\dagger = \bar{A}_{\dot{\alpha}}, \quad (A_{\alpha}^i)^\dagger = -\bar{A}_{\dot{\alpha}},$$
$$(A^\mu)^\dagger = -A^\mu, \quad (\bar{D}F)^\dagger = (-)^F\bar{D}(F)^\dagger,$$
$$\varepsilon_{\alpha\beta}^\dagger = \varepsilon_{\dot{\alpha}\dot{\beta}}, \quad (\varepsilon^{\alpha\beta})^\dagger = \varepsilon^{\dot{\alpha}\dot{\beta}},$$
$$\varepsilon_{ij}^\dagger = -\varepsilon^{ij}, \quad (\varepsilon^{ij})^\dagger = -\varepsilon^{ij}.$$
Bispinor formalism formulae are written as:

$$\Psi^i = \begin{pmatrix} \xi^i \bar{\alpha} \\ \bar{\chi}^i \alpha \end{pmatrix} \implies \bar{\Psi} \equiv (\Psi)^\dagger \gamma^0 = (-\chi^\alpha, \bar{\chi}^\dot{\alpha}).$$

Dirac matrices in the Weyl basis take form:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}; \quad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}.$$

Any vector can be transformed to a bispinor and vice versa:

$$v_{\alpha\dot{\alpha}} = \sigma^\mu_{\alpha\dot{\alpha}} v_\mu, \quad v_\mu = -\frac{1}{2} \bar{\sigma}^{\mu\dot{\alpha}} v_{\alpha\dot{\alpha}}.$$

We use the following Poisson brackets definition:

$$\{C, D\} \equiv i \left( \frac{\partial C}{\partial x^\mu} \frac{\partial D}{\partial p_\mu} + (-)^C \frac{\partial C}{\partial \theta^\alpha} \frac{\partial D}{\partial \pi^\dot{\alpha}} + (-)^C \frac{\partial C}{\partial \bar{\theta}^\dot{\alpha}} \frac{\partial D}{\partial \bar{\pi}^\alpha} - (-)^{CD+D} \frac{\partial D}{\partial x^\mu} \frac{\partial C}{\partial p_\mu} - (-)^{CD+D} \frac{\partial D}{\partial \theta^\alpha} \frac{\partial C}{\partial \pi^\dot{\alpha}} - (-)^{CD+D} \frac{\partial D}{\partial \bar{\theta}^\dot{\alpha}} \frac{\partial C}{\partial \bar{\pi}^\alpha} \right),$$

where C(D) grassmannian parity equals to zero, when C(D) carries an even number of the spinor indices and equals to one otherwise.

$$\{C, D\}^\dagger \equiv \{D^\dagger, C^\dagger\}.$$

**Appendix B**

This appendix is devoted to the constraints algebra.

a) free superparticle case:

$$\{V_\alpha^{(1)}, V_\beta^{(1)}\} = \frac{-4i\varepsilon_{\alpha\beta}}{(m + \sqrt{-p^2})} \chi, \quad \{\bar{V}_\dot{\alpha}^{(1)}, \bar{V}_\dot{\beta}^{(1)}\} = \frac{-4i\varepsilon_{\dot{\alpha}\dot{\beta}}}{(m + \sqrt{-p^2})} \chi,$$

$$\{V_\alpha^{(1)}, \bar{V}_\dot{\alpha}^{(1)}\} = \frac{-4p_{\alpha\dot{\alpha}}}{(m + \sqrt{-p^2}) \sqrt{-p^2}} \chi;$$

$$\{V_\alpha^{(2)}, V_\beta^{(2)}\} = 4i(m + \sqrt{-p^2})\varepsilon_{\alpha\beta}, \quad \{\bar{V}_\dot{\alpha}^{(2)}, \bar{V}_\dot{\beta}^{(2)}\} = -4i(m + \sqrt{-p^2})\varepsilon_{\dot{\alpha}\dot{\beta}},$$

$$\{V_\alpha^{(2)}, \bar{V}_\dot{\alpha}^{(2)}\} = 4 \left( 1 + \frac{-m^2}{p^2} \right) p_{\alpha\dot{\alpha}}.$$

Each first-class constraint commutes with each second-class one and bosonic first-class constraint $\chi$ commutes with all of them.
b) Superparticle interacting with $N = 2$ Maxwell supermultiplet with the fixed AMM value. Here the constraints algebra has the following form:

$$\{G_A, G_B\} = \Omega_{AB} + \omega_{1AB\dot{\alpha}}V_{\dot{\alpha}} + \omega_{2AB\alpha}\bar{V}^{\dot{\alpha}} + \omega_{3AB\dot{\alpha}\dot{\beta}}V_{\dot{\alpha}}\bar{V}_{\dot{\beta}} + \omega_{4AB\dot{\alpha}}V_{\dot{\alpha}}\bar{V}_{\dot{\beta}} + \omega_{5AB\dot{\alpha}\dot{\beta}}\bar{V}_{\dot{\alpha}}\bar{V}_{\dot{\beta}},$$  \hfill (B.1)

where $G_A = (V^{(1),(2)}_{\dot{\alpha}}, \bar{V}^{(1),(2)}_{\dot{\alpha}}, T)$. In (B.1) only the first term determines which of the constraints belongs to the first-class and which to the second. That is why we omit below complicated expressions or other terms and use weak equalities

$$\{V^{(1)}_{\dot{\alpha}}, V^{(1)}_{\dot{\beta}}\} \approx -\frac{4i\varepsilon_{\alpha\beta}}{(m^* + \sqrt{-P^2})}\left| \frac{m + ie\bar{W}/2}{m - ieW/2} \chi \right|,$$

$$\{\bar{V}^{(1)}_{\dot{\alpha}}, \bar{V}^{(1)}_{\dot{\beta}}\} \approx -\frac{4i\varepsilon_{\dot{\alpha}\dot{\beta}}}{(m^* + \sqrt{-P^2})}\left| \frac{m - ieW/2}{m + ieW/2} \chi \right|,$$

$$\{V^{(1)}_{\dot{\alpha}}, \bar{V}^{(1)}_{\dot{\alpha}}\} \approx -\frac{4P_{\dot{\alpha}\dot{\beta}}}{(m^* + \sqrt{-P^2})\sqrt{-P^2}}\chi;$$

$$\{V^{(2)}_{\dot{\alpha}}, V^{(2)}_{\dot{\beta}}\} \approx -4i(m^* + \sqrt{-P^2})\left| \frac{m + ie\bar{W}/2}{m - ieW/2} \varepsilon_{\alpha\beta} \right|,$$

$$\{\bar{V}^{(2)}_{\dot{\alpha}}, \bar{V}^{(2)}_{\dot{\beta}}\} \approx -4i(m^* + \sqrt{-P^2})\left| \frac{m - ieW/2}{m + ieW/2} \varepsilon_{\dot{\alpha}\dot{\beta}} \right|,$$

$$\{V^{(2)}_{\dot{\alpha}}, \bar{V}^{(2)}_{\dot{\alpha}}\} \approx 4P_{\dot{\alpha}\dot{\beta}}\left( 1 + \sqrt{\frac{-m^*}{P^2}} \right);$$

$$\{V^{(1)}_{\dot{\alpha}}, V^{(2)}_{\dot{\beta}}(\bar{V}^{(2)}_{\dot{\beta}})\} \approx 0, \{\bar{V}^{(1)}_{\dot{\alpha}}, V^{(2)}_{\dot{\beta}}(\bar{V}^{(2)}_{\dot{\beta}})\} \approx 0, \{T, V^{(1),(2)}_{\dot{\alpha}}(\bar{V}^{(1),(2)}_{\dot{\alpha}})\} \approx 0.$$

**References**

1. J. Hughes, J. Liu and J. Polchinski, Phys.Lett. 180B (1986) 370; E. Bergshoeff, E. Sezgin and P.K. Townsend, Phys.Lett. 189B (1987) 75; Ann.Phys. 185 (1988) 330.
2. J. Polchinski, T.A.S.I. lectures on D-branes, hep-th/9611050; M. Duff, Supermembranes, hep-th/9611203; P.K. Townsend, Four lectures on M-theory, hep-th/9612121.
3. W. Siegel, Phys.Lett.128B (1983) 397.
4. J.A. de Azcárraga and J. Lukierski, Phis.Lett. 113B (1982) 170; Phys.Rev D28 (1983) 1337.
5. M.B. Green, J.H. Schwarz and E. Witten, Superstring theory (C.U.P. Cambridge, 1987);
   L. Brink and J.H. Schwarz, Phys. Lett. 100B (1981) 310;
6. J.A. de Azcárraga and J. Lukierski, Phys. Rev. D38 (1988) 509.
7. J.A. Shapiro and C.C. Taylor, Phys. Rep. 191 (1990) 221;
   G.V. Grigoryan, R.P. Grigoryan and I.V. Tyutin, Teor. Mat. Fiz. 111 (1997) 389.
8. A.Yu. Nurmagambetov, J.J. Rosales and V.I. Tkach, JETP Lett. 60 (1994) 145;
   I.A. Bandos and A.Yu. Nurmagambetov, Class. Quant. Grav. 14 (1997) 1597.
9. A.A. Zheltukhin and D.V. Uvarov, JETP Lett. 67 (1998) 888.
10. I.A. Bandos and A.A. Zheltukhin, Fortschr. der Phys. 41 (1993) 619.
11. J. Wess and J. Bagger, Supersymmetry and supergravity (P.U.P., Princeton, 1983).
12. L. Lusanna and B. Milevski, Nucl. Phys. B247 (1984) 396.
13. A. Barducci, Phys. Lett. 118B (1982) 112.
14. A. Barducci, R. Casalbuoni and L. Lusanna, Nuovo Cim. 35A (1976) 377.
15. A.A. Zheltukhin, Teor. Mat. Fiz. 65 (1985) 151.
16. P.A.M. Dirac, Lectures on quantum mechanics (Academic, New York, 1964).
17. J.M. Evans, Nucl. Phys. B331 (1990) 711.
18. P. West, Introduction to supersymmetry and supergravity (World Scientific, 1986).
19. A.A. Zheltukhin and V.V. Tugay, JETP Lett. 61 (1995) 532;
   Yad. Fiz. 61 (1998) 325; hep-th/9706114.