New Solutions for Scalar–Isoscalar $\pi\pi$ Phase Shifts

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Abstract. The scalar–isoscalar $\pi\pi$ phase shifts are calculated in the $\pi\pi$ energy range from 600 MeV to 1600 MeV. We use results of the CERN–Cracow–Munich collaboration for the reaction $\pi^- p_\uparrow \to \pi^+ \pi^- n$ on a transversely polarized target at 17.2 GeV/c $\pi^-$ momentum. Energy–independent separation of the $S$–wave pseudoscalar amplitude ($\pi$ exchange) from the pseudovector amplitude ($a_1$ exchange) is carried out. Below the $K\bar{K}$ threshold we find two solutions for the $\pi\pi$ phase shifts, for which the phases increase slower with the effective $\pi\pi$ mass than the P-wave phases ("flat" solutions) and two solutions for which the phases increase faster than the P-wave phases ("steep" solutions). Above 1420 MeV both sets of phase shifts increase with energy faster than in the experiment on an unpolarized target. This fact can be related to a presence of the scalar resonance $f_0(1500)$.

One of the main sources of information on the scalar-isoscalar $\pi\pi$ interactions is the $\pi^+\pi^-$ partial wave analysis (PWA) yielding the $S$–wave. Virtually all PWA’s were based on the old CERN–Munich experiment [1] for the reaction $\pi^- p \to \pi^+ \pi^- n$ on nonpolarized target at 17.2 GeV/c. The number of observables provided by such experiment is much smaller than the number of real parameters needed to describe the partial waves. Consequently, the dominance of pseudoscalar exchange, equivalent to the absence of pseudovector exchange and several other physical assumptions have been made in previous studies [1–6].

In this analysis we use results of PWA performed by the CERN–Cracow–Munich collaboration for reaction $\pi^- p_\uparrow \to \pi^+ \pi^- n$ on polarized target at 17.2 GeV/c [7]. The data cover the $\pi\pi$ effective mass range from 600 MeV to 1600 MeV and the $t$–momentum transfer range from $-0.2$ (GeV/c)$^2$ to $-0.005$ (GeV/c)$^2$.

Combination of results of experiments on polarized and nonpolarized target yields a number of observables sufficient for performing a quasi–complete and energy independent PWA without any model assumptions. This analysis is only quasi–complete because of an unknown phase between two sets of transversity amplitudes. Nevertheless, intensities of partial waves could be determined in a completely model–independent way. This removed ambiguities appearing in earlier studies, except for the old "up–down" ambiguity [2]. General belief (see e.g. [8,9]) was that the "up–down" ambiguity had been resolved definitely in favour of the "down" solution. We disagree with this belief since all the studies of the full (i.e. including polarized-target) data are consistent with both the "up" and "down"
Let us denote by $f_0$ a system of two pions in a relative $S$–wave isospin 0 state. Transition amplitude for the $f_0$ production process $\pi^-p \rightarrow f_0 n$ can be written as the following matrix element

$$T_{sp=sn} = < u_{sp}^s | A \gamma_5 + \frac{1}{2} B \gamma_5 \gamma_\mu(p_\pi + p_f)^\mu | u_{p_1}^{sp} >,$$

(1)

where $p_1, p_2, p_\pi$ and $p_f$ are proton, neutron, incoming pion, and final $f_0$ four-momenta, $s_p$ and $s_n$ are proton and neutron spin projections, $u_{sp}$ and $u_{p_1}^s$ are the corresponding four-spinors, A and B are functions of the Mandelstam variables $s = (p_1 + p_\pi)^2$ and $t = (p_1 - p_2)^2$ at fixed $f_0$ mass $m_{\pi\pi}$. Part A of the amplitude corresponds to the pseudoscalar (or one pion) exchange while part B describes the pseudovector exchange which we shall briefly call $a_1$ exchange. Functions A and B have to be determined from experiment.

In this paper we use two transversity amplitudes $g$ and $h$, adequate for describing $f_0$ production on a transversely polarized target: $g \equiv < n \downarrow | T | p \uparrow >$ and $h \equiv < n \uparrow | T | p \downarrow >$. Separation of the amplitudes $g$ and $h$ into two components ($g = g_A + g_B$ and $h = h_A + h_B$) proportional to the invariant amplitudes A and B has been described in [10]. The amplitudes $g_i$ and $h_i$ ($i = A, B$) have been averaged over $t$.

Evaluation of the amplitude $A$ allows us to describe the $S$–wave $\pi^+\pi^- \rightarrow \pi^+\pi^-$ elastic scattering amplitude $a_S$ in the following way:

$$a_S = - \frac{p_\pi \sqrt{q_\pi} f}{m_{\pi\pi} \sqrt{2 \cdot \frac{f^2}{4 \pi}}} A(m_{\pi\pi}),$$

(2)

where $p_\pi$ is the incoming $\pi^-$ momentum in the $\pi^-p$ c.m. frame, $q_\pi$ is the final pion momentum in the $f_0$ decay frame, $g^2/4\pi = 14.6$ is the pion-nucleon coupling constant, and $f$ is the correction factor (see [10]).

Scalar-isoscalar pion-pion amplitude $a_0$ can be expressed as a function of $a_S$ and the $I=2$ $S$–wave amplitude $a_2$ and normalized to Argand’s form:

$$a_0 = 3 a_S - \frac{1}{2} a_2 = \frac{\eta e^{2i\delta} - 1}{2i},$$

(3)

where $\delta$ is the $I=0, S=0 \pi\pi$ phase shift and $\eta$ is the inelasticity coefficient.

In our analysis we have assumed that phases of the $P$, $D$ and $F$–waves follow phases of $\rho(770)$, $f_2(1270)$ and $\rho_3(1690)$ decay amplitudes into $\pi^+\pi^-$. In the region 600 MeV – 980 MeV we assume that only the $S$ and $P$–waves contribute. It is only in this region that fully analytical solutions of the PWA equations are possible [11]. The PWA analysis however, yields two solutions ("up" and "down") which are distinctly different in the $m_{\pi\pi}$ region from 800 MeV to 980 MeV. In addition to the "up–down" ambiguity in the moduli of the $g$ and $h$ transversity amplitudes, there is also a phase ambiguity in each $m_{\pi\pi}$ bin. This ambiguity comes from the mathematical structure of the PWA equations from which only cosines of
the relative phases of the partial waves can be obtained. In our analysis we present two arbitrary choices of the $S$–wave phases. In the first set, called "steep", $S$–wave phases grow faster than $P$–wave phases. In the other set, called "flat", increase in $S$–wave phases is slower than that for $P$–waves. Thus two sets of possible phases ("flat" or "steep") combined with two branches of moduli ("up" and "down") lead us to four solutions. Let us remark that in [2] the words "up" and "down" serve to distinguish the sets of phases while in our case they serve to distinguish the sets of moduli.

**FIGURE 1.** a) Scalar–isoscalar $\pi^+\pi^-$ phase shifts $\delta_0^0$ as a function of the effective $\pi^+\pi^-$ mass for the "up–flat" solution (open circles) and for data [1] (triangles). b) Same as in a) for the "down–flat" solution (full circles). c) Scalar–isoscalar $\pi^+\pi^-$ inelasticity coefficient $\eta$ versus the effective $\pi^+\pi^-$ mass for the "up–flat" (open circles) and "down–flat" (full circles) solutions.
Phase shifts $\delta$ shown in Fig. 1 a,b have been compared with those obtained from the analysis of the $\pi^-p \rightarrow \pi^+\pi^-n$ reaction on the unpolarized target [1] (solution B), where separation of the $\pi$ and $a_1$ exchanges was impossible. In Fig. 1a in the mass region from 600 MeV to about 1400 MeV we do not see any systematic differences between phase shifts corresponding to the ”down–flat” solution and the results of [1]. In the ”up–flat” solution around 900 MeV, however, the values of $\delta$ are higher than those measured in [1] by several tens of degrees (Fig. 1b). As it can be seen in Fig. 1c, inelasticity is close to 1 up to about 1000 MeV. A sudden drop in $\eta$ for the effective mass near 1000 MeV along with a characteristic jump of phase shift $\delta$ is caused by opening of a new $\bar{K}K$ channel and presence of the narrow $f_0(980)$ resonance [12,13]. Another decrease of $\eta$ near 1500 MeV – 1600 MeV can be related to the opening of further channels like $4\pi$ ( $\sigma\sigma$ or $\rho\rho$), $\eta\eta'$ or $\omega\omega$.

For two solutions ”up–steep” and ”down–steep” (not shown in Fig. 1c) we observe a characteristic fast increase of phase shifts near 780 MeV (see [10]). We see also rapid changes of $\eta$ which exceeds substantially 1 for two regions around 600 MeV – 700 MeV and 830 MeV – 980 MeV (especially in the “down-steep” solution). We conclude that this solution is unphysical and can be ignored, however the ”up-steep” solution, although a bit peculiar, cannot be completely excluded.

We would like to point out that for $m_{\pi\pi}$ larger than 1470 MeV, in all our solutions (“flat” and “steep”) the phase shifts show a systematically steeper increase than the phase shifts corresponding to data obtained on the unpolarized target. This increase is related to a presence of the relatively narrow (90 MeV - 170 MeV) resonance with mass 1400 MeV - 1460 MeV (see refs. [14,15]).

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