Research on Underwater Target Tracking Based on Multiple Fading Factors Strong Tracking SCKF

Zhang Xun¹, Xu Yue-qian¹, Zhang Hong-han¹, Yang Hui-ying², Chen Bin³,⁴
¹College of Automation, Harbin Engineering University, Harbin 150001, China
²China Xi’an Satellite Control Center, Xi’an 710043, China
³Shandong Luneng Intelligence Technology Co., LTD, Jinan, China
⁴State Grid Shandong Electric Power Research Institute, Jinan, China
xyqxdz2009@yeah.net

Abstract. In the research on complex maneuvering underwater target tracking, the single fading factor adjusts the estimation of all the state variables with the same value. However, the uncertainty of different state variable is different. Only one single fading factor can not get each state variable optimal estimation. Aiming at this problem, multiple fading factors strong tracking SCKF algorithm is proposed. Simulation results show that the multiple fading factors strong tracking SCKF algorithm has higher estimation accuracy than the CKF algorithm, SCKF algorithm and STSCKF algorithm when the underwater target system model is uncertain.

1. Introduction
Underwater target tracking technology has developed for decades[1]. An important topic in underwater target tracking research is the filtering algorithm for estimating the motion state of underwater target[2].

Cubature Kalman Filter(CKF) is a new nonlinear filtering algorithm proposed by Haykin in 2009[3]. Compared to EKF, CKF does not linearize nonlinear functions, which avoids linearization errors. Compared to UKF, the weight of CKF sampling point is always positive and the filtering accuracy is higher when the system state vector is high-dimensional[4-5]. In order to keep the covariance matrix positive definite in the CKF filtering process, square-root CKF(SCKF) is proposed in [6]. Aiming at the problem caused by uncertainty of the target motion model, [7] proposes the idea of strong tracking filter based on EKF. [8] combines the idea of strong tracking filter with the CKF algorithm and proposes the STCKF algorithm. [9] combines the idea of strong tracking filter with the SCKF algorithm and proposes the STSCKF algorithm. The idea of strong tracking filter proposed in [7] uses one single fading factor to adjust the estimation of all the state variables. However, the uncertainty of different state variable is different. Only one single fading factor can not make optimal estimation for each state variable. Hence, [10] proposes multiple fading factors strong tracking filter algorithm based on EKF. By introducing multiple fading factors into the state predicted error covariance matrix, the estimation of each state variable is adjusted to be optimal based on corresponding uncertainty when the system model is uncertain.

In this paper, the solution formula of multiple fading factors for CKF algorithm is derived. Combined with SCKF algorithm, multiple fading factors STSCKF (MSTSCKF) algorithm is proposed. In the simulation, the MSTSCKF algorithm is applied to underwater target tracking field.
Simulation results show that the estimation accuracy of MSTSCKF algorithm is better than STSCKF algorithm, SCKF algorithm and CKF algorithm when the system model is uncertain, which proves that MSTSCKF algorithm can play an important role in underwater target tracking field.

2. Multiple Fading Factors Solution Formula for MSTSCKF Algorithm

We consider the following nonlinear system. It is assumed that the process noise and measurement noise in the system are additive noise:

\[ x_k = f(x_{k-1}) + w_{k-1} \]  \hspace{1cm} (1)

\[ z_k = h(x_k) + v_k \]  \hspace{1cm} (2)

where \( x_k \) denotes state vector and \( z_k \) denotes measurement vector at discrete time \( k \); nonlinear function \( f(\cdot) \) and \( h(\cdot) \) are state function and measurement function, respectively; \( w_{k-1} \) and \( v_k \) are process and measurement Gaussian white noise with covariances \( Q_{k-1} \) and \( R_k \), respectively. For the nonlinear system above, [10] introduces a multiple fading factors solution method based on EKF when the system model is uncertain as follows.

\[ \lambda_k = \text{diag} \left[ \lambda_1^0, \lambda_2^0, \ldots, \lambda_n^0 \right] \]  \hspace{1cm} (3)

\[ \lambda_i^0 = \begin{cases} \alpha c_i & \lambda_i^0 > 1 \\ 1, & \lambda_i^0 \leq 1 \end{cases} \]  \hspace{1cm} (4)

\[ c_i = \frac{\text{tr}(N_i)}{\sum_j \alpha_j M_j^0} \]  \hspace{1cm} (5)

\[ N_i = V_i - H_iQ_{k-1}H_i^T - R_k \]  \hspace{1cm} (7)

\[ M_i = F_iP_{k-1}F_i^T - R_k \]  \hspace{1cm} (8)

\[ F_i = \frac{\partial f(x)}{\partial x} \bigg|_{x=k_{i-1}} \]  \hspace{1cm} (9)

\[ H_i = \frac{\partial h(x)}{\partial x} \bigg|_{x=k_{i-1}} \]  \hspace{1cm} (10)

\[ V_i = \begin{cases} \varepsilon_i^T, & k = 1 \\ \rho V_{i+1} + \varepsilon_i^T, & k \geq 2 \end{cases} \]  \hspace{1cm} (11)

where \( \lambda_k \) denotes multiple fading factors matrix; \( \lambda_i^0 \) denotes fading factor corresponding to each state variable; \( \alpha_i \) is obtained based on prior knowledge; \( V_k \) denotes covariance of residual \( e_k \); \( 0 < \rho \leq 1 \) denotes forgetting factor, usually \( \rho = 0.95 \).

According to the EKF-based multiple fading factors solution method, the MSTSCKF-based multiple fading factors solution formula can be derived. In the EKF-based multiple fading factors solution method, before multiple fading factors matrix is introduced we have

\[ P_{x_k|k-1}^{(i)} = F_iP_{x_{k-1}|k-1}F_i^T + Q_{x_{k-1|k-1}} \]  \hspace{1cm} (12)

\[ P_{z_k|k-1}^{(i)} = E \left[ \begin{bmatrix} x_k - \hat{x}_{k|k-1} \\ z_k - \hat{z}_{k|k-1} \end{bmatrix} \right] = P_{x_k|k-1}^{(i)}H_i^T \]  \hspace{1cm} (13)

where \( P_{x_k|k-1}^{(i)} \) denotes predicted error covariance matrix before multiple fading factors matrix is introduced; \( P_{z_k|k-1}^{(i)} \) denotes cross covariance matrix before multiple fading factors matrix is introduced. According to (12) and (13), we have

\[ P_{x_k|k-1}^{(i)} - Q_{x_{k-1|k-1}} = F_iP_{x_{k-1|k-1}}F_i^T \]  \hspace{1cm} (14)
\[ H_k = \left[ P_{\text{est}, k|k-1}^{(i)} \right] \left[ \left( P_{\text{est}, k|k-1}^{(i)} \right)^{-1} \right]^T \]  

(15)

Substituting (14) and (15) into (7) and (8) yields

\[ N_k = V_k - \left[ P_{\text{est}, k|k-1}^{(i)} \right] \left[ \left( P_{\text{est}, k|k-1}^{(i)} \right)^{-1} \right]^T Q_{k-1} \left[ \left( P_{\text{est}, k|k-1}^{(i)} \right)^{-1} \right]^T - R_k \]  

(16)

\[ M_k = \left( P_{\text{est}, k|k-1}^{(i)} - Q_{k-1} \right) \left[ \left( P_{\text{est}, k|k-1}^{(i)} \right)^{-1} \right]^T \left[ P_{\text{est}, k|k-1}^{(i)} \right] \left[ P_{\text{est}, k|k-1}^{(i)} \right]^T \left[ \left( P_{\text{est}, k|k-1}^{(i)} \right)^{-1} \right]^T \]  

(17)

Hence, using (16) and (17) to replace (7) and (8) respectively, multiple fading factors solution formula based on MSTSCKF algorithm is derived. The specific steps of the MSTSCKF algorithm are described below.

3. MSTSCKF Algorithm Steps

The SCKF algorithm can keep the state error covariance matrix non-negative, suppressing the filtering divergence and improving the numerical stability. The STCKF algorithm can suppress the filtering divergence when the system model is uncertain, thus improving the estimation accuracy. The multiple fading factors algorithm provides corresponding strong tracking intensity based on the uncertainty of different state variable. If the multiple fading factors algorithm and the SCKF algorithm are combined, not only the state error covariance matrix can be kept non-negative but also the filtering divergence is well suppressed when the system model is uncertain. Hence, the MSTSCKF algorithm is proposed.

At first, the operation symbol \( \text{Tria}(A) = \left[ \text{qr}(A^T) \right]^T \) is defined. The specific algorithm steps are described as follows.

(1) Initialization

Step 1  Assuming that initial condition \( \hat{x}_0, P_0, Q_0, R_0 \) are known. Factorize

\[ P_0 = S_0 S_0^T \]  

(18)

(2) Time Update

Step 2  Evaluate cubature points

\[ \chi_{i,k-1} = \hat{x}_{i,k-1} + S_{i,k-1} \xi_j \]  

(19)

Step 3  Evaluate the propagated cubature points

\[ \chi'_{i,k-1} = f(\chi_{i,k-1}), \; i = 1, 2, \ldots, m \]  

(20)

Step 4  Estimate the predicted state

\[ \hat{x}_{k|k-1} = \frac{1}{m} \sum_{i=1}^{m} \chi'_{i,k-1} = \mathbf{S}_{k|k-1} S_{0,k-1} \]  

(21)

where

\[ \chi'_{i,k-1} = \frac{1}{m} \left[ \chi_{1,k|k-1} - \hat{x}_{k|k-1} \right] \left[ \chi_{2,k|k-1} - \hat{x}_{k|k-1} \right] \ldots \left[ \chi_{m,k|k-1} - \hat{x}_{k|k-1} \right] \]  

(23)

\[ Q_{k|k-1} = S_{0,k|k-1} S_{0,k|k-1}^T \]  

(24)

Step 6  Evaluate multiple fading factors

\[ \chi_{i,k|k-1} = \hat{x}_{i,k|k-1} + S_{i,k|k-1} \xi_j \]  

(25)

\[ Z_{i,k|k-1} = h(\chi_{i,k|k-1}), \; i = 1, 2, \ldots, m \]  

(26)

\[ \hat{z}_{i,k|k-1} = \frac{1}{m} \sum_{i=1}^{m} Z_{i,k|k-1} \]  

(27)

\[ Z_{i,k|k-1} = \frac{1}{m} \left[ Z_{i,k|k-1} - \hat{z}_{i,k|k-1} \right] \left[ Z_{i,k|k-1} - \hat{z}_{i,k|k-1} \right] \ldots \left[ Z_{i,k|k-1} - \hat{z}_{i,k|k-1} \right] \]  

(28)
\[ \chi_{k|k-1}^{(t)} = \frac{1}{\sqrt{m}} \left[ \chi_{1,k-1}^{(t)} - \hat{x}_{1,k-1} \quad \chi_{2,k-1}^{(t)} - \hat{x}_{2,k-1} \quad \cdots \quad \chi_{m,k-1}^{(t)} - \hat{x}_{m,k-1} \right] \]  

(29)

\[ P_{k|k-1}^{(t)} = \chi_{k|k-1}^{(t)\top} \chi_{k|k-1}^{(t)} \]  

(30)

\[ P_{w,k|k-1}^{(t)} = \chi_{w,k|k-1}^{(t)\top} \chi_{w,k|k-1}^{(t)} \]  

(31)

\[ \lambda_k = \text{diag}[\lambda_1^k, \lambda_2^k, \ldots, \lambda_n^k] \]  

(32)

\[ \lambda_j^i \in \left\{ \begin{array}{ll} \lambda_j^i > 1, & j = 1, \ldots, n, \\ 1, & j = 1, \ldots, n \end{array} \right. \]  

(33)

\[ \lambda_j^0 = \alpha c_j \]  

(34)

\[ c_j = \frac{\alpha}{\sum_{j=1}^{n} M_j} \]  

(35)

\[ N_k = V_k - R_k \]  

(36)

\[ M_k = (P_{w,k|k-1}^{(t)} - Q_{w,k|k-1}) \]  

(37)

\[ V_k = \begin{bmatrix} e_k e_k^\top & \vdots \\ \vdots & \ddots & \vdots \\ e_k e_k^\top & \vdots & \vdots \end{bmatrix} + \rho V_{k-1} + e_k e_k^\top \]  

(38)

Step 7: Estimate the square-root of the predicted error covariance matrix with multiple fading factors

\[ S_{k|k-1} = \text{Tria} \left[ \sqrt{\lambda_k} \chi_{k|k-1} \quad S_{Q,k-1} \right] \]  

(39)

Step 8: Evaluate the cubature points

\[ Z_{i,k|k-1} = \hat{x}_{i,k-1} + S_{k|k-1} \xi_j \]  

(40)

Step 9: Evaluate the propagated cubature points

\[ Z_{i,k|k-1} = h(x_{i,k-1}), \quad i = 1, 2, \ldots, m \]  

(41)

Step 10: Estimate the predicted measurement

\[ \hat{z}_{i,k|k-1} = \frac{1}{m} \sum_{i=1}^{m} Z_{i,k|k-1} \]  

(42)

Step 11: Estimate the square-root of the innovation covariance matrix

\[ S_{z,k|k-1} = \text{Tria} \left[ \begin{bmatrix} Z_{k|k-1} \quad S_{R,k} \end{bmatrix} \right] \]  

(43)

where

\[ Z_{k|k-1} = \frac{1}{\sqrt{m}} \left[ Z_{1,k|k-1} - \hat{z}_{1,k-1} \quad Z_{2,k|k-1} - \hat{z}_{2,k-1} \quad \cdots \quad Z_{m,k|k-1} - \hat{z}_{m,k-1} \right] \]  

(44)

\[ R_k = S_{R,k} S_{R,k}^\top \]  

(45)

Step 12: Estimate the cross covariance matrix

\[ P_{z,k|k-1} = X_{k|k-1}^\top Z_{k|k-1} \]  

(46)

where

\[ X_{k|k-1} = \frac{1}{\sqrt{m}} \left[ X_{1,k|k-1} - \hat{x}_{1,k-1} \quad X_{2,k|k-1} - \hat{x}_{2,k-1} \quad \cdots \quad X_{m,k|k-1} - \hat{x}_{m,k-1} \right] \]  

(47)

Step 13: Estimate the Kalman gain

\[ K_k = (P_{z,k|k-1}/S_{z,k|k-1}) \]  

(48)
Step 14  Estimate the updated state and the square-root factor of the corresponding error covariance

\[
\hat{x}_k = \hat{x}_{k|k-1} + K_k (z_k - \hat{z}_{k|k-1}) \\
S_k = \text{Tria} \left( \begin{bmatrix} \hat{x}_{k|k-1} - K_k Z_{k|k-1} \end{bmatrix} K_k S_k \right)
\]

(49)  
(50)

4. Simulation

Scenario: Assuming that unmanned underwater vehicle (UUV) moves in a two-dimensional plane Cartesian coordinate system, initial position is on the point (10m, 50m). Initial velocities in the X and Y directions are 0.1m/s and 0.5m/s, respectively. UUV moves along X and Y directions at constant accelerations. Acceleration in the X direction is 0.02m/s². Acceleration in the Y direction is −0.04m/s². 

The observation point is origin of coordinate. The observation time is 50s. The observation time interval between two consecutive measurement is \( T = 0.5s \).

The motion model of UUV is CA model. Hence, we have the state equation and the measurement equation

\[
\begin{bmatrix}
T & \frac{T^2}{2} & 0 & 0 & 0 \\
0 & 1 & T & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & T \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_k \\
x_{k+1}
\end{bmatrix}
= \begin{bmatrix}
x_k + w_k
\end{bmatrix}
\]

(51)

\[
\begin{bmatrix}
r_k \\
\theta_k
\end{bmatrix} = \begin{bmatrix}
\sqrt{x_k^2 + y_k^2} \\
\tan^{-1} \frac{y_k}{x_k}
\end{bmatrix} + v_k
\]

(52)

where \( x_k = x_k \hat{x}_k \hat{x}_k \hat{y}_k \hat{y}_k \hat{y}_k \) is state vector; \( x_k \) and \( y_k \) denote positions, \( \hat{x}_k \) and \( \hat{y}_k \) denote velocities, \( \hat{x}_k \) and \( \hat{y}_k \) denote accelerations in the X and Y directions, respectively; In the first 30s, the covariance of \( w_k \) is \( Q = \text{diag} \begin{bmatrix} 0.01 & 10^{-6} & 0.01 & 10^{-6} & 10^{-6} \end{bmatrix} \); At the time 30s, the covariance of \( w_k \) mutates to \( 9Q \); \( r_k \) and \( \theta_k \) denote range and bearing, respectively; The standard deviation of measurement noise \( v_k \) is \( \text{diag} \begin{bmatrix} 0.1 \text{m} & 0.001 \text{rad} \end{bmatrix} \); The initial value of state estimation is \( \hat{x}_0 = \begin{bmatrix} 10m & 0.1 \text{m/s} & 0.02 \text{m/s}^2 & 50m & 0.5 \text{m/s} & -0.04 \text{m/s}^2 \end{bmatrix} \); The initial value of error covariance is \( P_0 = \text{diag} \begin{bmatrix} 0.1 & 10^{-2} & 10^{-3} & 0.1 & 10^{-2} & 10^{-3} \end{bmatrix} \); The prior proportionality is \( \alpha = [1.2 \ 1.2 \ 1.2 \ 1.5 \ 1.5 \ 1.5] \). Executing 100 times Monte Carlo runs, the results are shown as follows.

![Fig.1 Comparison of tracking trajectory](image)
The results show that in the first 30s the four filtering algorithms have no significant difference in the estimation of UUV motion state. After the process noise is abruptly changed at the time 30s, the four filtering algorithms output significantly different estimation of UUV motion state: the estimation errors of CKF algorithm and SCKF algorithm increase rapidly; the estimation error of STSCKF algorithm also increases, but less than the CKF algorithm and the SCKF algorithm; the estimation error of the MSTSCKF algorithm does not change significantly.

It is seen from the results that when the system model is uncertain, CKF algorithm and SCKF algorithm can not adapt to the uncertainty of the model and the estimation error of UUV motion state
is very large; STSCKF algorithm has some adaptability to the uncertainty of the model, because the single fading factor in the algorithm has a corrective effect on the estimation of state variables. Although the single fading factor can reduce the error, it is not ensured that the estimation errors of all the state variables can be reduced to a lower normal range; the MSTSCKF algorithm adjusts the estimation of each state variable with different intensity by the multiple fading factors, ensuring the optimal estimation of each state variable and improving the accuracy and stability of the filtering result, which has good adaptability to the model uncertainty.

5. Conclusion
In this paper, MSTSCKF algorithm is proposed to overcome the shortcoming of strong tracking filter algorithm with single fading factor. The single fading factor can reduce the filtering divergence caused by the model uncertainty to a certain extent. Because different state variable has different uncertainty, it is not ensured that each state variable gets the optimal estimation adjusted by the same single fading factor. The multiple fading factors matrix adjusts each state variable based on corresponding uncertainty, so each state variable obtains the optimal estimation when system is uncertain. Combining multiple fading factors with SCKF algorithm, MSTSCKF algorithm is proposed. MSTSCKF algorithm not only keeps the covariance matrix non-negative in the recursion process but also gets each state variable the optimal estimation when system is uncertain, improving the accuracy and numerical stability of filtering result. The simulation results show that the tracking performance of MSTSCKF algorithm is better than STSCKF algorithm, SCKF algorithm and CKF algorithm. The MSTSCKF algorithm can play an important role in the underwater target tracking field.

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References
[1] Zhang Linlin, Yang Rijie, et al. Research on Underwater Maneouvre Target Tracking Technology[J]. Technical Acoustics, 2011,30(01):68-73(in Chinese).
[2] Junhai L, Ying H, Liying F. Underwater Acoustic Target Tracking: A Review[J]. Sensors, 2018, 18(2):112.
[3] Wang Xiaoxu, Pan Quan, et al. Overview of Deterministic Sampling Filtering Algorithms for Nonlinear System[J]. Control and Decision, 2012,27(06):801-812(in Chinese).
[4] Arasaratnam I, Haykin S. Cubature Kalman smoothers[J]. Automatica, 2011, 47(10):2245-2250.
[5] Arasaratnam I, Haykin S, Hurd T R. Cubature Kalman Filtering for Continuous-Discrete Systems: Theory and Simulations[J]. IEEE Transactions on Signal Processing, 2010, 58(10):4977-4993.
[6] Arasaratnam I. Haykin S. Cubature Kalman Filters[J]. IEEE Transactions on Automatic Control, 2009,54(6):1254-1269.
[7] Zhou Donghua, Xi Yugeng, Zhang Zhongjun. Suboptimal Fading Extended Kalman Filter for Nonlinear Systems[J]. Control and Decision,1990:1-6(in Chinese).
[8] Wang Hongjian, Fu guixia, et al. Strong Tracking CKF Based SLAM Method for Unmanned Underwater Vehicle[J]. Chinese Journal of Scientific Instrument, 2013,34(11):2542-2550(in Chinese).
[9] Du Zhanlong, Li Xiaomin, et al. Fault Prediction with Combination of Strong Tracking Square-root Cubature Kalman Filter and Autoregressive Model[J]. Control Theory & Applications, 2014,31(08):1047-1052(in Chinese).
[10] Zhou Donghua, Xi Yugeng, Zhang Zhongjun. A Suboptimal Multiple Fading Extended Kalman Filter[J]. ACTA AUTOMATICA SINICA, 1991.17(6):689-695(in Chinese).