Analysis and Experimental Verification of Cross-coupled 2-DOF SPM Motor with Halbach Array

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In this paper, a cross-coupled 2-DOF SPM motor with Halbach array is proposed. In the proposed 2-DOF motor, the permanent magnets (PMs) are arranged in both the linear direction and the rotational direction so that the motor generates thrust force in the linear direction and torque in the rotational direction. The Halbach array is applied to both the linear direction and the rotational direction. The proposed motor is analyzed using 3-D FEA. In the analysis, the model with Halbach array only in the linear direction, the model with Halbach array only in the rotational direction, and the model with Halbach array in both the linear direction and the rotational direction are compared. The analysis results show that the model with Halbach array in both directions generates higher force and torque than not only the model without Halbach array but also the models with Halbach array in either the linear direction or the rotational direction. In the analysis, the magnetic flux is also analyzed in addition to the force and torque. Additionally, experiments are conducted to confirm the effectiveness of the Halbach array.

Keywords: 2-DOF motor, halbach array, direct drive, multi degrees of freedom, linear motion, rotational motion

1. Introduction

Nowadays, a lot of robots are used in various fields (1). In the industrial field, they perform different tasks, such as welding, painting, and assembling, instead of humans. The robots which help humans to work have been introduced into medical and rehabilitation fields. Surgery robots which doctors control reduce doctor’s burdens. Power-assist suits support patients who walk for rehabilitation. The demand of robots is increasing more and more. Because of that, research of robots has been conducted actively so that robots can realize more complicated motion (2)–(4).

The robots have multi-degree-of-freedom (multi-DOF) structure so that they can realize the complicated motion. More DOF is necessary as they perform more complicated tasks. However, one DOF generally consists of one motor in order to control each DOF independently. Because of that, they would be bigger and heavier as more complicated tasks are required. Research of multi-DOF motors attracts attention because they would reduce the number of motors in robots while keeping the number of DOF.

Multi-DOF motors are important not only to reduce the size and weight of robots but also to realize human’s motion by robots. A shoulder joint of a human does not rotate about one axis but rotates about multiple axes. Therefore, a multi-DOF motor which rotates about multiple axes is necessary to realize the motion of human’s shoulder by robots. The multi-DOF motors which realize such motion are spherical motors (5)–(7). A spherical motor has a spherical mover and a stator which covers the mover. The spherical motors have two or three DOF and the movers rotate about multiple axes. There are a lot of robots that have not only rotary motors but also linear motors such as SCARA robots in the industrial fields. SCARA robots are used to transport products. The robot generally has two rotary motors to move the end effector of itself in the horizontal direction and one rotary motor to rotate the end effector with the position of the end effector kept. Additionally, the robot has one linear motor to move the end effector in the vertical direction. Because of that, multi-DOF motors which realize not only the rotational motion but also the linear motion are necessary to reduce the number of motors in robots further. One of such motors is the linear and rotary permanent magnet actuator (8).

The linear and rotary permanent magnet actuator generates thrust force in one axis direction and torque about the axis. The actuator realizes the linear motion by generating the thrust force and the rotational motion by generating the torque. Moreover, the actuator realizes both the linear and rotational motion simultaneously by generating both the thrust force and the torque. However, the actuator has nine kinds of windings. It is necessary to control the current of each winding independently. Because of that, the controller would become big and expensive.

The 2-DOF actuators having less kinds of windings have been proposed (9)–(10). Each 2-DOF actuator has two sets of 3-phase windings. One set of 3-phase windings generates thrust force. The other set generates torque. However, it means that each set of 3-phase windings generates either the thrust force or the torque, but not both. Because of that, it is necessary to increase the turning of both sets in order to improve both the thrust force and the torque. It is difficult to improve both the thrust force and torque volume ratio.

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In order to solve that problem, the cross-coupled 2-DOF direct drive motor has been proposed \((13)\). The 2-DOF motor is a surface permanent magnet (SPM) motor, which has a mover composed of a shaft and permanent magnets (PMs). The PMs are arranged on the surface of the shaft. The stator of the 2-DOF motor has two kinds of 3-phase windings. One kind of 3-phase windings is superposed on the other kind in the 2-DOF motor. Because of that, one PM is opposed to both kinds of 3-phase windings. If the magnetic flux of one PM is improved, both kinds of Lorentz force are improved. Therefore, it is possible to improve the thrust force and torque volume ratio.

The number of motors in the robots that need both the rotational motion and the linear motion such as SCARA robots would decrease by applying the cross-coupled 2-DOF motors to the robots. The size and weight of the robots would decrease by the decrease in motors mounted in the robots.

In the 2-DOF motor, the thrust force and torque are further improved by the improvement of the interlinkage flux across windings. There are some methods to improve interlinkage flux in motors. One of the methods is to use ferromagnetic materials as a PM yoke and a coil back yoke. Another method is the Halbach array \((15)\). For example, the force density of the tubular modular PM machine has increased by applying the Halbach array to the motor \((15)\). The increase rate of the force density is 1.09–1.13.

The helical motion PM motor with the Halbach array, which is one of 2-DOF motors, has been proposed \((17)\). The mover has two PM layers. In one PM layer, N and S poles are alternately placed to the linear direction. In the other PM layer, N and S poles are alternately placed to the rotational direction. Because of that, the Halbach array is applied to each PM layer one-dimensionally as well as 1-DOF motors.

Additionally, the cross-coupled 2-DOF SPM motor with the Halbach array has been proposed \((18)\). In the 2-DOF motor, the Halbach array is applied to the PM arrangement not only one-dimensionally but also two-dimensionally. The cross-coupled 2-DOF SPM motor has higher flexibility in the design of the PM arrangement with the Halbach array than the helical motion PM motor. Therefore, the cross-coupled 2-DOF SPM motor has the potential to increase the torque and force more significantly by applying the Halbach array than 1-DOF motors and the helical motion PM motor. The analysis results of the cross-coupled 2-DOF SPM motor have shown that the thrust force and torque are increased by applying the Halbach array to the 2-DOF motor.

In this paper, the cross-coupled 2-DOF SPM motor with the Halbach array is researched for two purposes. One purpose is to confirm whether the Halbach array improves the 2-DOF motor more significantly than 1-DOF motors such as the tubular modular PM machine \((15)\). Therefore, the specifications of the 2-DOF motor are defined as the increase in the thrust force and torque by 1.13 times, by which the force density of the tubular modular PM machine has increased.

In the analysis and experiments, it is confirmed whether the thrust force and torque of the 2-DOF motor increase by 1.13 times.

The other purpose is to clarify the effect of the Halbach array peculiar to the 2-DOF motor. For the purpose, not only the torque and thrust force but also the magnetic flux is analyzed. The effect of the Halbach array on not only the model without ferromagnetic materials but also the model with ferromagnetic materials is analyzed because it is expected that not only the Halbach array but also the ferromagnetic materials are applied to the 2-DOF motor to improve the torque and thrust force.

2. The Structure of the Proposed Motor

2.1 Basic Structure of the Cross-coupled 2-DOF Motor

The structure of the cross-coupled 2-DOF motor and the PM arrangement in the mover is shown in Figs. 1–3. Figure 1 indicates the 2-DOF motor without the coil back yoke. Figure 2 indicates the 2-DOF motor with the coil back yoke. The 2-DOF motor has the stator which is composed of two kinds of 3-phase windings. The windings are rolled around the bobbin which is made from a non-magnetic material such as the resin material. Therefore, the 2-DOF motor does not have cores for windings. One kind of 3-phase windings composes inner windings. The other kind of 3-phase windings composes outer windings. The inner windings and outer windings are orthogonally arranged. Because of that, these windings are called cross-coupled windings.

Figure 3 indicates the development view of the mover, outer windings, and inner windings. Figure 3(a) shows the relationship between the PMs in the movers and the outer windings. Figure 3(b) shows the relationship between the PMs and the inner windings.

The PMs are arranged not only in \(z\) direction but also in \(\theta\) direction. In the paper, \(z\) direction indicates the linear direction and \(\theta\) direction indicates the rotational direction about \(z\) direction. A surface of the PM is S pole or N pole. These poles are alternately placed to \(z\) direction and \(\theta\) direction, like squares on a chessboard.

The inner and outer windings are wired along diagonals of the PMs. When the current is flowed in the outer and inner windings, Lorentz force \(F_{\text{outer}}\) and \(F_{\text{inner}}\) shown in Fig. 3 are...
generated. The direction of $F_{\text{outer}}$ is different from the direction of $F_{\text{inner}}$. $F_{\text{outer}}$ is composed of Lorentz force in the right helical direction, which is the combination of $+z$ direction and $+\theta$ direction. $F_{\text{inner}}$ is composed of Lorentz force in the left helical direction, which is the combination of $-z$ direction and $-\theta$ direction. Therefore, the sum of $z$ directional components of $F_{\text{outer}}$ and $F_{\text{inner}}$ produces thrust force in $z$ direction $F_z$ shown in Fig. 1. The difference between $\theta$ directional components of $F_{\text{outer}}$ and $F_{\text{inner}}$ produces torque in $\theta$ direction $T_{\theta}$.

$F_z = K_{\text{Fout}} I_{q\text{out}} + K_{\text{Fin}} I_{q\text{in}}$ \hspace{1cm} (1)

$T_{\theta} = K_{\text{Fout}} I_{q\text{out}} - K_{\text{Fin}} I_{q\text{in}}$ \hspace{1cm} (2)

Subscripts $\text{out}$ and $\text{in}$ indicate parameters of the outer windings and inner windings. $K_F$, $K_F$, and $I_q$ show a thrust constant, a torque constant, and a q-axis current. Therefore, only $F_z$ is generated when $I_{q\text{in}} = \frac{K_{\text{Fout}}}{K_{\text{Fin}}} I_{q\text{out}}$. Only $T_{\theta}$ is generated when $I_{q\text{in}} = \frac{K_{\text{Fin}}}{K_{\text{Fout}}} I_{q\text{out}}$.

$I_{q\text{in}} = \frac{K_{\text{Fout}}}{K_{\text{Fin}}} I_{q\text{out}}$, and d-axis current $I_{d\text{out}}$ and $I_{d\text{in}}$ are calculated as (3) and (4).

\[
\begin{bmatrix}
I_{d\text{out}} \\
I_{q\text{out}}
\end{bmatrix} = C(\alpha_{\text{out}}) \begin{bmatrix}
I_{U\text{out}} \\
I_{V\text{out}}
\end{bmatrix} \\
\begin{bmatrix}
I_{d\text{in}} \\
I_{q\text{in}}
\end{bmatrix} = C(\alpha_{\text{in}}) \begin{bmatrix}
I_{U\text{in}} \\
I_{V\text{in}}
\end{bmatrix}
\]

(3)

(4)

The combination of $I_U$, $I_V$, and $I_W$ means 3-phase current. $\alpha$ indicates electrical angle. $\alpha_{\text{out}}$ and $\alpha_{\text{in}}$ are calculated as (5) and (6).

\[
\alpha_{\text{out}} = \pi \left( \frac{z}{L_m} + \frac{2\theta}{\pi} \right)
\]

(5)

\[
\alpha_{\text{in}} = \pi \left( \frac{z}{L_m} - \frac{2\theta}{\pi} \right)
\]

(6)

$L_m$ indicates the length of a pole in z direction.

In (3) and (4), $C$ means the transformation matrix for dq axis. $C$ is indicated as (7).

\[
C(\alpha) = \begin{bmatrix}
\cos(\alpha) & \cos(\alpha - \frac{\pi}{2}) & \cos(\alpha + \frac{\pi}{2}) \\
\sin(\alpha) & \sin(\alpha - \frac{\pi}{2}) & \sin(\alpha + \frac{\pi}{2})
\end{bmatrix}
\]

(7)

### 2.2 2-DOF Motor with Halbach Array

The Halbach array is applied to the cross-coupled 2-DOF motor. N and S poles are alternately placed to z direction and $\theta$ direction. Therefore, there are two kinds of PM arrangements with the Halbach array as shown in Fig. 4.

In one arrangement, the Halbach array is applied to the arrangement in $z$ direction as shown in Fig. 4(a). In the other arrangement, the Halbach array is applied to the arrangement in $\theta$ direction as shown in Fig. 4(b). In the paper, one arrangement is called $z$ arrangement. The other arrangement is called $\theta$ arrangement.

In the figures, $L_m$ and $L_p$ indicate the length of a pole and a pole pair. $L_{\text{unc}}$ and $L_{\text{odd}}$ indicate the length of a PM magnetized in the radial direction. Especially, $L_{\text{unc}}$ and $L_{\text{odd}}$ are the length in z direction and $\theta$ direction. $r_m$ and $\theta_m$ indicate the radius of the mover and the angle of a PM magnetized in the radial direction. Therefore, there is the following relationship between $L_{\text{odd}}$, $r_m$, and $\theta_m$ as shown in Fig. 4.

\[
L_{\text{odd}} = r_m \theta_m
\]

(8)

In Fig. 4(a), $L_{\text{unc}}$ indicates the length of a PM magnetized in $z$ direction for the Halbach array. Therefore, there is the relationship between $L_{\text{unc}}$, $L_{\text{unc}}$, and $L_m$ as (9).

\[
L_m = L_{\text{unc}} + L_{\text{unc}}
\]

(9)
In Fig 4(b), \( L_{dh} \) and \( \theta_h \) indicate the length and the angle of a PM magnetized in \( \theta \) direction for the Halbach array. In the motor, the number of poles in \( \theta \) direction is defined as 4. Therefore, there is the relationship between \( L_{dh} \), \( \theta_h \), \( L_{mh} \), and \( L_{mz} \) as (10)–(12)

\[
L_{dh} = \frac{r_m \theta_h}{L_m} \quad \cdots \quad (10)
\]

\[
L_m = L_{mh} + L_{dh} \quad \cdots \quad (11)
\]

\[
\theta_m + \theta_h = 90 \text{deg}. \quad \cdots \quad (12)
\]

Additionally, it is possible to apply the Halbach array to the arrangements in both \( z \) direction and \( \theta \) direction as shown in Fig. 5. In the paper, the arrangement shown in Fig. 5 is called \( z\theta \) arrangement.

### 3. Analysis of the Proposed Motor

#### 3.1 Analysis Setup

Some kinds of the analysis of the 2-DOF motor with the Halbach array are conducted with 3-D FEA in order to confirm the effectiveness of the Halbach array. The analysis parameters are shown in Table 1.

At first, \( T_o \) and \( F_z \) are analyzed. \( T_o \) and \( F_z \) are expected to be changed by changing the PM arrangement. In the analysis, the PM length of the Halbach array in \( \theta \) direction \( L_{zh} \) and the PM angle of the Halbach array in \( \theta \) direction \( \theta_h \) are changed. \( L_{zh} \) is converted by 2.50 mm and \( \theta_h \) is converted by 10.0 deg with \( \theta_m \) kept constant. Therefore, when \( L_{zh} = 0.00 \text{ mm} \) and \( \theta_h = 0.00 \text{ deg} \), \( L_{mz} \), \( L_{mh} \), and \( L_{dh} \) are defined as following

\[
L_{dh} = 0.00 \text{ mm} \quad \cdots \quad (13)
\]

\[
L_{mz} = L_{mh} = L_{mz} \quad \cdots \quad (14)
\]

The PM arrangement at \( L_{zh} = 0.00 \text{ mm} \) and \( \theta_h = 0.00 \text{ deg} \) indicates the PM arrangement shown in Fig. 3.

In the analysis, not only the models without ferromagnetic materials but also the models with ferromagnetic materials are analyzed, because \( T_o \) and \( F_z \) increase by applying ferromagnetic materials to the 2-DOF motor.

However, ferromagnetic materials are heavier than non-magnetic materials. It is difficult to apply ferromagnetic materials to the mover if the acceleration is required more strongly than the force. It is necessary to consider the motor with only the coil back yoke.

Additionally, it is possible to switch a mover and a stator although the paper defines the shaft and the PMs as the mover and the windings as the stator. It is necessary to consider the mover without ferromagnetic materials for the high acceleration. Therefore, it is necessary to consider the motor with only the ferromagnetic shaft. In the analysis, four cases are analyzed.

**Case 1** With a non-magnetic shaft, without a coil back yoke

**Case 2** With a non-magnetic shaft, with a coil back yoke

**Case 3** With a ferromagnetic shaft, without a coil back yoke

**Case 4** With a ferromagnetic shaft, with a coil back yoke

In each case, \( T_o \) and \( F_z \) generated by the inner windings and outer windings are analyzed. When \( T_o \) and \( F_z \) by the inner windings are analyzed, the mover is moved in the left helical direction and the current is applied to only the inner windings. When \( T_o \) and \( F_z \) by the outer windings are analyzed, the mover is moved in the right helical direction and the current is applied to only the outer windings. The analysis is conducted not in the dynamic condition but in the static condition. Therefore, \( T_o \) and \( F_z \) do not include the disturbance caused by iron loss.

Next, the magnetic flux density generated by the PMs is analyzed. The magnetic flux density is expected to be changed by changing the PM arrangement. In the analysis, four cases are analyzed in the same way as the first analysis.

At last, the effect of the coil back yoke, such as magnetic attraction force and iron loss, is analyzed. The analysis is conducted under the condition that the mover rotates at 20 Hz, which is the maximum angular velocity of human joints when a human throws a ball.

In the analysis, the turning of the inner windings is equal to the outer windings. Generally, the turning of the inner windings and outer windings are defined so that the thrust constant of the inner windings is equal to the outer windings. Because of that, the calculation is simple when the reference value of the current is calculated from the desired force and torque in the 2-DOF motor. The thrust constant depends on the inductance of windings.

However, the inductance of the inner windings to the outer windings is changed depending on whether the motor has the coil back yoke or not. The optimal turning in the motor is changed by the coil back yoke.

Additionally, it is recommended that the turning should be
kept in order to compare the model without the coil back yoke and the model with the coil back yoke. Therefore, the turning of the inner windings is equal to the outer windings in the analysis and the experiments which are conducted after the analysis.

### 3.2 Analysis of Torque and Thrust Force

$T_\theta$ and $F_z$ generated by the inner windings in Case 1 are shown in Fig. 6 and Fig. 7. These figures show the change of $T_\theta$ and $F_z$ depending on the change of $L_{hz}$ and $\theta_h$. In the figures, the points at $L_{hz} = 0.00 \text{ mm}$ and $\theta_h = 0.00 \text{ deg}$ mean $T_\theta$ and $F_z$ generated by the model without the Halbach array. Therefore, the points mean $T_\theta$ and $F_z$ generated by the PM arrangement shown in Fig. 3. The points at $L_{hz} > 0.00 \text{ mm}$ and $\theta_h = 0.00 \text{ deg}$ mean $z$ arrangement shown in Fig. 4(a). The points at $L_{hz} = 0.00 \text{ mm}$ and $\theta_h > 0.00 \text{ deg}$ mean $\theta$ arrangement shown in Fig. 4(b).

From the figures, it turns out that $T_\theta$ and $F_z$ are increased by applying the Halbach array to the 2-DOF motor. Additionally, $T_\theta$ and $F_z$ increase as $L_{hz}$ and $\theta_h$ increase. $T_\theta$ and $F_z$ by the outer windings are increased by applying the Halbach array as well as the inner windings. When $L_{hz} = 10.00 \text{ mm}$ and $\theta_h = 30.00 \text{ deg}$, the highest $T_\theta$ and $F_z$ are generated by the inner windings and outer windings. Therefore, the highest $T_\theta$ and $F_z$ are generated when $L_{hz} = 21.50 \text{ mm}$ and $\theta_h = 60.00 \text{ deg}$, which are calculated from (9) and (12).

#### Table 2. $L_{hz}$ and $\theta_h$ to generate the highest torque and thrust force

| Case | Inner windings | Outer windings | Inner windings | Outer windings |
|------|----------------|----------------|----------------|----------------|
| 1    | 10.0           | 10.0           | 10.0           | 10.0           |
| 2    | 10.0           | 10.0           | 10.0           | 10.0           |
| 3    | 5.00           | 5.00           | 10.0           | 10.0           |
| 4    | 5.00           | 5.00           | 10.0           | 10.0           |

Table 2 shows $L_{hz}$ and $\theta_h$ which cause the highest $T_\theta$ and $F_z$. $L_{hz}$ and $\theta_h$ are not different between Case 1 and Case 2 and between Case 3 and Case 4. However, they are different between Case 1, 2 and Case 3, 4. Therefore, it turns out that the optimal PM arrangement depends on the materials of the shaft.

Figures 8–11 show the analysis results of $T_\theta$ and $F_z$. Fig. 8 and Fig. 9 show $T_\theta$ and $F_z$ generated by the inner windings. In the figures, data of Case 1 indicate the summarization of the analysis results in Fig. 6 and Fig. 7. Figure 10 and Fig. 11 show $T_\theta$ and $F_z$ generated by the outer windings. In the figures, the data of the models with $z\theta$ arrangement in Case 1 and Case 2 indicate the analysis results when $L_{hz} = 10.00 \text{ mm}$ and $\theta_h = 30.00 \text{ deg}$. Ones in Case 3 and Case 4 indicate the analysis results when $L_{hz} = 5.00 \text{ mm}$ and $\theta_h = 10.00 \text{ deg}$. Therefore, the data of the models with $z\theta$ arrangement in each case indicate the highest $T_\theta$ and $F_z$ generated by the inner windings and outer windings in each case.

The data of the models with $z$ arrangement in Case 1 and Case 2 show the analysis results when $L_{hz} = 10.00 \text{ mm}$ and $\theta_h = 0.00 \text{ deg}$. Ones in Case 3 and Case 4 show the analysis...
results when \( L_{hc} = 5.00 \text{ mm} \) and \( \theta_h = 0.00 \text{ deg} \).

The data of the models with \( \theta \) arrangement in Case 1 and Case 2 show the analysis results when \( L_{hc} = 0.00 \text{ mm} \) and \( \theta_h = 30.0 \text{ deg} \). Ones in Case 3 and Case 4 show the analysis results when \( L_{hc} = 0.00 \text{ mm} \) and \( \theta_h = 10.0 \text{ deg} \).

Figures 12–15 show the increase rates of \( T_\theta \) and \( F_z \) shown in Figs. 8–11. The figures show the torque rates and thrust force rates of the models with the Halbach array to the model without the Halbach array.

There is little difference of the increase rates between Case 1 and Case 2 and little difference of the increase rates between Case 3 and Case 4. Therefore, it turns out that the effect of the Halbach array does not depend on the coil back yoke but depends on the materials of shafts so much.

The model with the Halbach array in both \( z \) direction and \( \theta \) direction generates higher torque and force than the models with the Halbach array in either \( z \) direction or \( \theta \) direction in each case. Synergistic effect, which is the combination of the Halbach array applied to the linear direction and the Halbach array applied to the rotational direction, is caused in the 2-DOF motor.

There is difference of the increase rates between Case 1 and Case 3 and difference of the increase rates between Case 2 and Case 4. The Halbach array has a more potent effect on an aluminum shaft than an iron shaft.

Rotary motors basically have PMs arranged in \( \theta \) direction. From the analysis results of the proposed motor, it turns out that the increase of \( T_\theta \) by applying the Halbach array to the PM arrangement in \( \theta \) direction is small. Because of that, the increase of \( T_\theta \) is small if the Halbach array is applied to rotary motors. Even if the Halbach array is applied to the 2-DOF system which is composed of a linear motor and a rotary motor, only \( F_z \) increases drastically. However, the synergistic effect increases not only \( F_z \) but also \( T_\theta \) by the increase rate of \( F_z \) in the linear motor if the Halbach array is applied to the 2-DOF system which is composed of the cross-coupled 2-DOF SPM motor. The proposed motor contributes miniaturization of multi-DOF systems.

Table 3 and Table 4 show the averages of the increase rates
of $T_p$ and $F_z$ shown in Figs. 12–15. In the paper, the specifications of the 2-DOF motor are defined as the increase in the thrust force and torque by 1.13 times as described in Section 1. In Case 1 and Case 2, the average increase rates of the models with $z$ arrangement are close to 1.13. The average increase rates of the models with $z\theta$ arrangement are higher than the specifications of the 2-DOF motor. It turns out that the models with $z\theta$ arrangement in Case 1 and Case 2 meet the required performance.

### 3.3 Analysis of Magnetic Flux

Before the analysis of magnetic flux, it is confirmed whether there is the effect of magnetic saturation or not. The relative permeability of Case 4, where the analysis model has the ferromagnetic shaft and the coil back yoke, is analyzed because it is expected that higher magnetic density is generated in Case 4 than Case 1–3. Figure 16 shows the contour plots of the relative permeability in Case 4. Relative permeability is a ratio of permeability to air. For example, the relative permeability of air is 1.00. Therefore, relative permeability does not have a unit. Figure 16(a) indicates the plot when $L_{hc} = 0.00$ mm and $\theta_h = 0.00$ deg. Figure 16(b) indicates the plot when $L_{hc} = 5.00$ mm and $\theta_h = 10.00$ deg.

From figures, it turns out that the relative permeability of the coil back yoke is much higher than air. The minimum relative permeability of the ferromagnetic shaft and coil back yoke is more than 260. Therefore, it turns out that magnetic saturation does not occur in the 2-DOF motor.

In the analysis of magnetic flux, the magnetic flux density in the detection area shown in Fig. 17 is analyzed. In Fig. 17, $\theta_p$ indicates the angle of a pole pair. The number of poles is defined as 4 in the motor. Therefore, $\theta_p$ is defined as 180 deg in the motor. The detection area is located at the level of the inner windings. Therefore, the area is located 50.5 mm from the central axis.

Figure 18 shows the magnetic flux density in Case 1, where the model with the aluminum shaft and without the coil back yoke. Figure 18(a) shows the result of the model without the Halbach array. Figure 18(b) shows the result of the model with $z\theta$ arrangement. The magnetic flux density shown in Fig. 18(b) is analyzed when $L_{hc} = 10.00$ mm and $\theta_h = 30.00$ deg.

Figure 19 shows the spacial frequency components extracted from the magnetic flux density by 2-D Discrete
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Fig. 20. Comparison of $\Phi(1,1)$

Table 5. Increase rates of $\Phi(1,1)$ by $z\theta$ arrangement

| Case  | Increase rate |
|-------|---------------|
| 1     | 1.171         |
| 2     | 1.174         |
| 3     | 1.030         |
| 4     | 1.028         |

Fig. 21. An detection area for magnetic flux

Fourier Transformation (DFT). Each spacial frequency component $\Phi(k_z,k_\theta)$ is calculated as shown in (15).

$$\Phi(k_z,k_\theta) = \sum_{p_z=0}^{N_z-1} \sum_{p_\theta=0}^{N_\theta-1} \phi(D_zp_z, D_\theta p_\theta) e^{-2\pi j \left( \frac{k_z p_z}{N_z} + \frac{k_\theta p_\theta}{N_\theta} \right)}$$

(15)

$N_z$ and $N_\theta$ indicate the numbers of sampling in $z$ direction and $\theta$ direction. $D_z$ and $D_\theta$ indicate the sampling intervals in $z$ direction and $\theta$ direction. $\phi(D_zp_z, D_\theta p_\theta)$ means the magnetic flux density at the sampling point where $z$ is $D_zp_z$ and $\theta$ is $D_\theta p_\theta$. $k_z$ and $k_\theta$ indicate the harmonic order in $z$ direction and $\theta$ direction.

Figure 19(a) shows $\Phi(k_z,k_\theta)$ of Fig. 18(a). Figure 19(b) shows $\Phi(k_z,k_\theta)$ of Fig. 18(b). The figures have peak at $k_z = 1$ and $k_\theta = 1$. Therefore, the fundamental harmonic component is the highest component in the spacial frequency components.

Figure 20 shows the fundamental harmonic components in the models without the Halbach array and the models with $z\theta$ arrangement. Table 5 shows the increase rates of $\Phi(1,1)$ shown in Fig. 20. There is little difference between the average of the increase rates of $T_\theta$, $F_z$ shown in Table s 3, 4 and the increase rates of $\Phi(1,1)$ shown in Table 5. It turns out that $T_\theta$ and $F_z$ increase because $\Phi(1,1)$ is improved.

Additionally, the magnetic flux in the detection area shown in Fig. 21 is analyzed. The length of the detection area in $z$ direction is 31.5 mm and angle in $\theta$ direction is 90 deg. Therefore, the detection area which is equal to one pole is analyzed. In the analysis, the magnetic flux is analyzed at two points. At first, the magnetic flux on the surface of PMs is analyzed. Next, the magnetic flux located at the level of the inner windings is analyzed.

Figure 22 and Fig. 23 show the analysis results of the magnetic flux. Although the magnetic flux on the surface of PMs is decreased by applying the Halbach array, the magnetic flux at the level of the inner windings increases. Therefore, it turns out that the rate of the interlinkage flux across the inner windings to the magnetic flux generated by the PMs is increased by applying the Halbach array although the magnetic flux generated by the PMs does not increase.

Figure 24 and Fig. 25 show the vector plots of the magnetic flux density in Case 1. The magnetic flux in Fig. 24(b) and Fig. 25(b) is focused on the center of poles in comparison with Fig. 24(a) and Fig. 25(a). Owing to that, the magnetic flux density enclosed in the red circle in Fig. 24(b) and Fig. 25(b) is lower than Fig. 24(a) and Fig. 25(a). Therefore, it turns out that the leakage flux, which does not cross the
inner windings, is decreased by applying the Halbach array.

However, from the comparison with Fig. 20 and Fig. 23, it turns out that the magnetic flux at the level of the inner windings does not increase by applying the Halbach array as significantly as \( \Phi(1, 1) \) does. The rates of the fundamental harmonic component to total magnetic flux is increased by applying the Halbach array.

In comparison with Fig. 25(a), more magnetic flux in Fig. 24(a) is focused on the center axis. There is less difference between Fig. 24(a) and Fig. 24(b) than difference between Fig. 25(a) and Fig. 25(b). Owing to that, the model with the Halbach array in \( z \) direction generates higher torque and force than not only the model without the Halbach array but also the models with the Halbach array in \( \theta \) direction.

In comparison with Fig. 24(a) and Fig. 25(a), there is less magnetic flux flowing in the shaft in Fig. 24(b) and Fig. 25(b). Instead of that, there is more magnetic flux flowing in the PMs in Fig. 24(b) and Fig. 25(b). The PMs have almost the same magnetic permeability as aluminum. However, the magnetic permeability of the PMs is lower than iron. Because of that, the increase rates of \( \Phi(1, 1) \) in Case 3 and Case 4 is lower if the Halbach array is applied to the 2-DOF motor.

### 3.4 Analysis of Magnetic Attraction Force and Iron Loss

\( T_\theta \) of Case 2 and Case 4, which have the model with the coil back yoke, is analyzed in the condition that the mover rotates in \( \theta \) direction. Figure 26 shows the analysis result. The analysis is conducted in the static condition that current is not input. Therefore, \( T_\theta \) does not include the torque caused by iron loss and Lorentz force but includes only the cogging torque caused by magnetic attraction.

In the figure, the horizontal axis indicates the angle of the mover in \( \theta \) direction. “w/o in Case 2” means \( T_\theta \) of the model without the Halbach array in Case 2. “w in Case 2” means \( T_\theta \) of the model with \( z\theta \) arrangement in Case 2. “w/o in Case 4” means \( T_\theta \) of the model without the Halbach array in Case 4. “w in Case 4” means \( T_\theta \) of the model with \( z\theta \) arrangement in Case 4.

Coggging torque is generated. However, the cogging torque is even smaller than \( T_\theta \) shown in Fig. 8 and Fig. 10. The reason is that the stator does not have cores for windings although the stator has the coil back yoke. It turns out that the influence of the magnetic attraction force is small enough to ignore the influence.

Iron loss is composed of hysteresis loss and joule loss by eddy current. Each loss is analyzed in Case 2 and Case 4, which have the coil back yoke. The hysteresis loss is calculated based on Steinmetz equation [20] and the magnetic flux density analyzed with 3-D FEA. The joule loss is analyzed with 3-D dynamic FEA.

Table 6 indicates the hysteresis loss and the joule loss in the condition that the mover rotates at 20 Hz. From the figures, it turns out that the hysteresis loss is much smaller than the joule loss. The joule loss is the prevailing iron loss. Therefore, the stack structure should be applied to the coil back yoke to decrease the iron loss.

### 4. Experiment

#### 4.1 Experimental Setup

Some experiments are conducted to confirm effectiveness of the Halbach array. In the experiments, four cases are considered.

- **Case 1** With a non-magnetic shaft, without a coil back yoke
- **Case 2** With a non-magnetic shaft, with a coil back yoke
- **Case 3** With a ferromagnetic shaft, without a coil back yoke
- **Case 4** With a ferromagnetic shaft, with a coil back yoke

In each case, torque constants \( K_T \) and thrust constants \( K_F \) of the inner windings and outer windings are measured. These constants of four models, which are the model without the Halbach array and the models with the Halbach array in \( z \) direction, \( \theta \) direction, and both directions, are measured. At last, the experiments are conducted to confirm that only \( T_\theta \) or only \( F_z \) is generated by the combination of the inner windings and outer windings.

Figure 27 shows the 2-DOF motor for the experiments. The experimental system has an optical linear encoder to detect the position \( z \) and an optical rotary encoder to detect the angle \( \theta \). Resolution of the linear encoder and the rotary encoder is 0.1 \( \mu \)m and 0.0002 deg. \( \alpha_{\text{aux}} \) and \( \alpha_{\text{rot}} \) are calculated based on \( z \) and \( \theta \) detected by the encoders, (5), and (6).

In the 2-DOF motor, the shaft and the coil back yoke are removable parts. These experiments are conducted with the block diagram shown in Fig. 28. The transformation of UVW phases to dq axes and transformation of dq axes to UVW phases are conducted with transformation matrix \( C \), which is...
calculated based on (7), $\alpha_{out}$, and $\alpha_{in}$.

$I_{dim}$, $I_{qin}$, $I_{dout}$, and $I_{qout}$ are controlled with PI controller. The gain of the PI controller are shown in Table 7.

### 4.2 Experimental Results

Figures 29–32 show the experimental results. These figures are plotted with $I_q$ input into the windings and $F_z$ and $T_{\theta}$ generated by $I_q$. Therefore, the gradients of the lines made from plot data in the figures indicate $K_F$ and $K_T$. In each experiment, $F_z$ and $T_{\theta}$ are measured three times. Figures 29–32 are plotted with the average of $F_z$ and $T_{\theta}$ measured by three times. $F_z$ and $T_{\theta}$ linearly increase as $I_q$ increases. Therefore, it turns out the magnetic flux generated by the current does not cause magnetic saturation in the experiments.

Figure 33 and Fig. 34 show $K_T$ and $K_F$ calculated based on Figs. 29–32. $K_T$ and $K_F$ are calculated with the least squares method. Figure 35 and Fig. 36 show the increase rates of $K_T$ and $K_F$ shown in Fig. 33 and Fig. 34. The figures show the rates of the models with the Halbach array to the model without the Halbach array. Table 8 shows the average of the increase rates of $K_T$ and $K_F$ shown in Fig. 33 and Fig. 34. In the experimental results of Case 1 and Case 2, the increase rates of the models with $z\theta$ arrangement are basically higher than ones with either $z$ arrangement or $\theta$ arrangement as well as the analysis results. The increase rates of the models with $z\theta$ arrangement in Case 1 and Case 2 is higher than 1.13. Therefore, the models with $z\theta$ arrangement in Case 1 and Case 2 meet the required performance as well as the analysis results.

However, in Case 3 and Case 4, the increase rates of the models with $z\theta$ arrangement are not higher than ones with either $z$ arrangement or $\theta$ arrangement. The PM arrangement with the Halbach array in both directions is more complicated than ones with the Halbach array in one direction. The increase rates of the models with $z\theta$ arrangement might be decreased by error between ideal models and real models. Because of that, in Case 3 and Case 4, $T_{\theta}$ and $F_z$ in the experiments are lower than the analysis.

In Case 1 and Case 2, the increase rates of $K_T$ and $K_F$ in the experimental results are higher than the increase rates of $T_{\theta}$ and $F_z$ in the analysis results. The difference of the increase rates between the analysis results and the experimental results in Case 1 and Case 2 are larger than Case 3 and Case 4. The Halbach array causes the neighboring PMs to repel to each other. The repelling makes diameter of the mover, which is composed of the PMs, larger than the model without the Halbach array. The PMs get close to the windings in Case 1 and Case 2. However, the shaft is composed of iron in Case 3 and Case 4. The PMs do not get so close to the windings because the PMs are attracted to the shaft by the magnetic force. Because of that, the increase rates of $K_T$ and $K_F$ in the

### Table 7. Experimental parameters

| Propotion gain  | $K_p$  | 10000.0 |
|-----------------|--------|---------|
| Integral gain   | $K_T$  | 200.0   |
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Fig. 29. Experimental results in Case 1

Fig. 30. Experimental results in Case 2

Fig. 31. Experimental results in Case 3

Fig. 32. Experimental results in Case 4

Experimental results are higher than the increase rates of $T_\theta$ and $F_z$ in the analysis results in Case 1 and Case 2.

At last, some experiments are conducted to confirm that the models with the Halbach array in both directions generate only $T_\theta$ as well as the motor without the Halbach array. Four cases are conducted as well as the first experiment. In the experiments, both $I_{qin}$ and $I_{qout}$ is applied to the motor.

$I_{qin}$ and $I_{qout}$ is defined as following.

\[ I_{qin} = \frac{K_{out}}{K_{in}} I_{qout} \] \hspace{0.5cm} (16)

\[ \frac{K_{out}}{K_{in}} = \frac{1}{2} \left( \frac{K_{Tout}}{K_{Tin}} + \frac{K_{Fout}}{K_{Fin}} \right) \] \hspace{0.5cm} (17)

Additionally, both $I_{qin}$ and $I_{qout}$ as shown in (18) is applied.
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Fig. 33. Experimental results of torque constants

Fig. 34. Experimental results of thrust constants

Fig. 35. Experimental results of increase rates of torque constants

Fig. 36. Experimental results of increase rates of thrust constants

Table 8. Average increase rates of torque constants and thrust constants

| Inner windings | Outer windings |
|---------------|---------------|
| \(z\) arrangement | \(\theta\) arrangement | \(\phi\) arrangement | \(z\) arrangement | \(\theta\) arrangement | \(\phi\) arrangement |
| Case 1 | 1.22 | 1.05 | 1.23 | 1.09 | 1.00 | 1.23 |
| Case 2 | 1.23 | 1.07 | 1.25 | 1.20 | 1.06 | 1.27 |
| Case 3 | 1.03 | 1.01 | 0.98 | 1.01 | 0.98 | 1.00 |
| Case 4 | 1.07 | 1.08 | 1.07 | 0.99 | 1.01 | 1.01 |

The experimental results are shown in Fig. 37. \(T_\theta^i\) and \(F_\phi^i\) indicate \(T_\theta\) and \(F_\phi\) generated when \(I_{gin} = \frac{K_{out}}{K_{in}}I_{qout}\). \(T_\phi^i\) and \(F_\theta^i\) indicate \(T_\phi\) and \(F_\theta\) generated when \(I_{gin} = \frac{K_{in}}{K_{out}}I_{qout}\). Therefore, it turns out that the 2-DOF motor with the Halbach array in both directions generates \(T_\theta\) and \(F_\phi\) independently.

In the experiments, 1.00 A is applied to the 2-DOF motor as \(I_{qout}\).

\[
I_{gin} = \frac{K_{out}}{K_{in}}I_{qout} \tag{18}
\]
5. Conclusion

This paper has proposed the cross-coupled 2-DOF SPM motor with the Halbach Array. The analysis of the motor has been conducted to confirm the effectiveness of the Halbach array. The magnetic flux is focused on the center of poles. Owing to that, $T_\theta$ and $F_z$ are increased by applying the Halbach array. Both $T_\theta$ and $F_z$ are increased by applying the Halbach array in $z$ direction or $\theta$ direction.

From the analysis results, it has turned out that the increase rates of $T_\theta$ and $F_z$ by applying the Halbach array in $z$ direction is higher than ones by applying the Halbach array in $\theta$ direction. Additionally, the increase rates of $T_\theta$ and $F_z$ by applying the Halbach array in both of $z$ direction and $\theta$ direction is higher than ones by applying the Halbach array in $z$ direction.

The analysis results have indicated that the increase rates of $T_\theta$ and $F_z$ in the model with non-magnetic shafts such as aluminum is higher than ferromagnetic shafts such as iron. The models with the non-magnetic shafts and $z \theta$ arrangement meet the required performance.

The analysis results have indicated that the Halbach array has a more remarkable effect on the cross-coupled 2-DOF motor than 1-DOF motors because the Halbach array is applied two-dimensionally. The proposed motor contributes miniaturization of multi-DOF systems.

In the experiment, the increase rates of $T_\theta$ and $F_z$ have been confirmed based on the increase in $K_T$ and $K_F$. The experimental results have shown that the models with the non-magnetic shafts meet the required performance as well as the analysis results. Additionally, the experimental results have shown that the 2-DOF motor with the Halbach array in both directions generates $T_\theta$ and $F_z$ independently.

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Cross-Coupled 2-DOF SPM Motor with Halbach Array

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