Application of elastic stability of flat and spatial shapes of rods in agriculture

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Abstract. In various elements of building structures, pipelines and tanks, ships and submarines, aircraft and spacecraft, in the oil and gas industry, in agriculture, elastic rods, shells and plates are widely used, for which the stability of the forms of equilibrium determines the conditions for their trouble-free operation. Today, the literature contains a huge number of works devoted to both the solution of individual particular problems and the development of general methods for analyzing the stability of equilibrium forms of elastic systems. Despite this, many problems still need to be studied. In particular, in the well-known works on the theory of stability of elastic systems, the most difficult problems include the problems of bending of heavy elastic rods, experiencing the joint action of inhomogeneous force factors. These tasks remain poorly understood, despite their importance for many agricultural and industrial processes. The article investigates the stability of elastic rods (stems), loaded at the ends by concentrated moments, and investigates the influence of the characteristics of distributed stiffness on the value of critical moments, which is a mathematical model of maintaining the stability of the stem of a cereal plant during lodging. For stems with the same bending stiffness in different planes, within the framework of the static approach, the influence of the variability of the distributions along the rods of bending stiffness on the critical values of the torques at which the loss of stability occurs is studied.

1. Introduction

The need to develop scientifically based recommendations, technologies and techniques to increase the resistance of cereals to unfavorable cultivation factors, including lodging, due to which a significant proportion of the crop is lost, was noted in all fundamental works on the technology of their cultivation [1]. The article proposes to use the methods of mathematics and technical mechanics to solve the problems of crop losses [2].

Distinguish between basal lodging of plants, due to the weak adhesion of the root system of plants to the soil, which is especially evident after heavy rains and winds. Stem lodging of plants, caused by a significant bend of the stem from the weight of the above-ground plant mass on the lower part of the stem [3]. Naturally, both processes (basal and stem lodging) are interrelated, affect each other, and are complex and complex in nature for various plants [4].

It is necessary to study the architectonics of plants, the physical and mechanical properties of the tissue of their stems, the conditions for the stability of their rectilinear form of equilibrium and bending under the influence of various natural factors, in particular, rain and wind load [5].
This article deals with the worst case of bending - a freestanding wet plant. In field conditions, the effect of the wind weakens due to its screening by other plants.

Mathematical study of "large" stem bends (especially outside the elastic deformations) is associated with the integration of complex and cumbersome nonlinear integro-differential equations [3]; an approximate solution to this problem in order to estimate the values of bending stresses under the action of a wind load is given below.

2. Materials and methods

The object of the research was the varieties of winter wheat, spring wheat, rye, triticale, winter barley, spring barley, rice, sorghum, corn hybrids and bamboo plants of the genus leaf grate, zoned in the Krasnodar Territory and Adygea.

There are two main methods and the corresponding methods for determining the elastic and strength (elasticity and yield limits) properties of the tissue of cereal stalks - by testing them for tension and bending.

Stem compression tests, as well as torsion tests (to determine the modulus of elasticity, elasticity and yield strength during shear) are not of great practical importance, since the occurrence of significant axial compressive or torsional stresses in plant stems due to external natural factors and leading to irreversible deformation of the stems, unlikely.

Testing stems for tension, compression, torsion is associated with serious technical difficulties: the ends of the stem samples must be clamped into special cartridges, while the tissue of the stems at the ends of the samples is deformed from compression, which introduces significant errors in the measurement results [6, 7].

In the stems of cereals during lodging of plants, various types of stresses are encountered, but bending stresses are the main ones. As is known from technical mechanics, bending is a dangerous type of deformation, since minor influences on the bar can lead to significant deflections. Cereals lodge not because of axial compression and stretching, but precisely because of the bending of the stem and, to a greater extent, at the root. For this reason, it is important to experimentally investigate the elastic properties of plant tissues by artificially creating bending stresses. With the help of a special setup [2], we recorded the deflections of plant stem samples at various loads and loading steps depending on the type of crop (from 1.5 N for rice stems and up to 1000 N for bamboo stems). The experiment was terminated in the event of a break in the stem of the plant.

The calculation of the stiffness of the stem in bending was determined by the formula (1):

\[ D = \left[0.5(D_1^4 + D_2^4)\right]^{1/4} \]

\[ d = \left[0.5(d_1^4 + d_2^4)\right]^{1/4}, \]

where \( D \) and \( d \) are diameters of the stem sample at its ends.

The results of measurements showed that the ratio \( k = d / D \) is quite stable and on average is a value of \( k \) from 0.53-0.65 (wheat, rye, triticale, barley, rice) to 0.71-0.86 (bamboo, sorghum, corn) - with the exception of the upper internode of the stem about 0.15–0.20 m long (directly at the panicle (ear)), where this ratio is about 15–30% less.

3. Results and discussion

In fig. 1 schematically depicts a curved stem and the loads acting on it. The architectonics of plants is created in such a way that their stem in any of its sections equally resists the action of external forces and moments of forces [8].
Figure 1. Scheme for determining bending stresses in the plant stem under the action of wind load: 1 - wind direction, 2 - ear, panicle; 3 - angle; 4 - stem; 5 – root.

The equation of moments of forces in the parametric form $X = X(S)$ and $Y = Y(S)$ along the stem axis $S$ is written as follows:

$$EI(S) \cdot \alpha'(S) = M + F \cdot S + q(\xi) \int_0^S (S - \xi) \cos \alpha(\xi) d\xi + \int_0^S (S - \xi) q(\xi) \sin \alpha(\xi) d\xi,$$

where $EI(S)$, $q(\xi)$ are the bending stiffness and weight of a running meter of the stem with leaves varying along the length of the stem;

$q_c$ - wind load distributed along the length;

$S$ is the current length of the stem (with its total length $L$);

$\xi$ - local coordinate (0 ≤ $\xi$ ≤ $S$); $M$, $F$ are still unknown constants,

$\alpha(S)$ - current angle of deviation of the stem axis from the vertical;

$\alpha'(S)$ - current, in parametric form, curvature of the stem axis, equal to

$$K = \frac{Y'(S) \cdot X''(S) - Y''(S) \cdot X'(S)}{[Y'^2(S) + X'^2(S)]^{1/2}}.$$ 

The character of all functions from $S$ and from $\xi$ is exactly the same.

Since the main purpose of the research is to determine the forces acting on the stem and the moments of forces corresponding to the elastic limit of the stem tissue $\sigma_y$, the radius of curvature $\rho_o = 1/\alpha'(0)$ at the root in the first approximation can be taken from the condition

$$\rho_o = \frac{EI_o}{\sigma_y W_o},$$

where $EI_o$, $W_o$ - bending stiffness and moment of resistance of the section at the root.

$$\alpha'(0) = \frac{\sigma_y W_o}{EI_o}$$
Accordingly, the curvature of the axis is. The equation is solved by the method of successive approximations, for which the first approximation of the functions is taken in the form:

\[ \alpha(\xi) = a_0 + a_1 \xi; \quad \alpha'(\xi) = a_1; \]  

From completely admissible initial conditions, \( \alpha(0) = 0 \) and \( \alpha'(0) = \frac{\sigma_y W_o}{EI_o} \) we will obtain, \( a_0 = 0 \),

\[ \alpha(\xi) = a_1 \xi \]

consequently

\[ \alpha(\xi) = k \xi; \quad \sin \alpha(\xi) = \sin k \xi; \quad \cos \alpha(\xi) = \cos k \xi, \]

where the initial curvature

\[ k = \frac{\sigma_y W_o}{EI_o \cdot n}. \]

By varying the coefficient \( n \) (usually in the range of 0.5 - 3.5) during the calculations, one should ensure that the initial curvature approximately corresponds to the average curvature of the stem axis after the second approximation.

For the stiffness and linear weight of the stem (with leaves), we use very accurate (with a coefficient of determination not less than 0.97) exponential representations of the dependences \( EI(S) \), \( q(\xi) \), and \( W(S) \):

\[ EI(S) = EI_o e^{-\frac{S}{L} \beta}; \quad q(\xi) = q_o e^{-\frac{\xi}{L} \gamma}; \quad W(S) = W_o e^{-\frac{S}{L} \phi}, \]

where \( W_o \) - moment of resistance of the section of the stem at the root;
\( \phi \) – decrement of decreasing value.

The values \( W_o, \phi \) are also determined by the least squares method using the "exact" formulas for calculating \( W(S) \). At the ear (panicle) the moment of resistance of the section is denoted by \( W_k \).

\[ W(S) = \frac{\pi}{32 \cdot D(S)} \left[ D(S)^4 - d(S)^4 \right], \]

where

\[ D(S) = D_o - (D_o - D_o) \frac{S}{L} \]

and

\[ d(S) = d_o - (d_o - d_o) \frac{S}{L}. \]

Taking into account the assumptions made, equation (6) is rewritten as:

\[ EI_o e^{-\frac{S}{L} \beta} \cdot \alpha'(S) = M + F \cdot S + q_o \int_0^S (S - \xi) \cos k \xi d\xi + q_o \int_0^S (S - \xi) e^{-\frac{\xi}{L} \gamma} \sin k \xi d\xi \]

and, after integration on the right-hand side and obvious transformations,

\[ \alpha'(S) = e^{\frac{S}{L} \beta} \left\{ M + F \cdot S + \frac{q_o}{\kappa^2} (1 - \cos kS) + \frac{q_o}{(A_o \kappa)^2} \left[ A_o kS - \frac{2 \gamma}{\kappa L} + e^{-\frac{S}{L} \gamma} \left( \frac{2 \gamma}{\kappa L} \cos kS + B_o \sin kS \right) \right] \right\}, \]

where

\[ A_o = \left( \frac{\gamma}{\kappa L} \right)^2 + 1; \quad B_o = \left( \frac{\gamma}{\kappa L} \right)^2 - 1. \]
Integration and differentiation of the last equation gives expressions for determining the current angle of deviation of the stem axis from the vertical and the intensity of the change in curvature (for calculating shear forces):

\[
\alpha(S) = \frac{e^{\frac{S}{L}}}{EI_o} \left\{ \frac{ML}{\beta} + F \left( \frac{SL}{\beta} - \frac{L^2}{\beta^2} \right) + \frac{q_o}{\kappa \lambda} \left[ \frac{L}{\beta} - \frac{1}{C_o \kappa L} \left( \beta \cos \kappa S + \sin \kappa S \right) \right] \right. \\
\left. + \frac{q_o}{(A_i \kappa)^2} \left[ A_i \kappa \left( \frac{SL}{\beta^2} - \frac{L^2}{\beta^2} \right) - 2\gamma \frac{\beta}{\kappa L} + 2\epsilon e^{-\frac{S}{L}} \frac{\beta - \gamma}{\kappa L} \left( \beta - \gamma \cos \kappa S + \sin \kappa S \right) \right] \\
+ \frac{B_o e^{-\frac{S}{L}}}{D_o \kappa} \left( \frac{\beta - \gamma}{\kappa L} \sin \kappa S - \cos \kappa S \right) \right\} + A; \\
\alpha^*(S) = \frac{e^{\frac{S}{L}}}{EI_o} \left\{ M \frac{\beta}{L} + F \left( \frac{1}{L} + \frac{\beta}{L} \right) S + \frac{q_o}{\kappa \lambda} \left( 1 - \cos \kappa S + \kappa \sin \kappa S \right) \right. \\
\left. + \frac{q_o}{(A_i \kappa)^2} \left( A_i \kappa \left( 1 + \frac{\beta}{L} \right) S - \frac{2\gamma \beta}{\kappa L^2} + 2\epsilon e^{-\frac{S}{L}} \frac{\beta - \gamma}{\kappa L} \left( \beta - \gamma \cos \kappa S - \kappa \sin \kappa S \right) \right] + \\
+ \frac{B_o e^{-\frac{S}{L}}}{D_o \kappa} \left[ \frac{\beta - \gamma}{L} \sin \kappa S + \kappa \sin \kappa S \right] \right\},
\]

(8)

where \( C_o = \left( \frac{\beta}{\kappa L} \right)^2 + 1 \), \( D_o = \left( \frac{\beta - \gamma}{\kappa L} \right)^2 + 1 \); \( A \) – constant of integration.

To determine the three constants \( M, F \) and \( A \) in equations (7) - (9), there are three boundary conditions.

1. The root has an elastic seal, so you can write

\[ EI_o \alpha'(0) - \theta_o \sin \alpha(0) = 0 \]

or, due to the smallness \( \alpha(0) \), \( EI_o \alpha'(0) - \theta_o \alpha(0) = 0 \)

Thus, when solving the problem of stem bending, the specific moment \( \theta_o \) is the moment of forces at the place where the stem is embedded in the root, referred to the value of the angular displacement of the stem in it.

2. The shearing force at the end of the stem is zero (the displacement of the stem is not limited), i.e. \( EI_o \alpha'(L) = 0 \)

3. A bending moment acts at the end of the stem due to the weight of the ear (panicle) and the wind load on it, i.e. \( EI_o \alpha'(L) - \theta_o \sin \alpha(L) - \theta_o \cos \alpha(L) = 0 \), where is the specific moment \( \theta_o \) – moment of forces from the weight of the ear, referred to the sine of the angular displacement of the upper end of the stem from the vertical; \( \theta_o = 0.5 q_o h^2 \) – specific moment from the action of wind load on the panicle, \( \text{Nm}/\cos \alpha(L) \).

Substitution of the written boundary conditions into equations (7) - (9) gives:

\[ M \left( \frac{EI_o}{\theta_o} \right) - \frac{L}{\beta} + \frac{FL^2}{\beta^2} - A \cdot EI_o = N_1 \]

(1)

\[ M + FL + N_2 - \theta_o \sin \left\{ \frac{1}{EI_o} \left[ \frac{ML}{\beta} + \frac{F L^2}{\beta^2} (\beta - 1) + A \cdot EI_o + N_1 \right] \right. - \\
\left. \theta_o \cos \left\{ \frac{1}{EI_o} \left[ \frac{ML}{\beta} + \frac{F L^2}{\beta^2} (\beta - 1) + A \cdot EI_o + N_1 \right] \right\} = 0 \]

(2)
\[ M \frac{\beta}{L} + F(1 + \beta) + N_4 = 0 \]

where abbreviations \( N_1, N_2, N_3, N_4 \) are determined by the formulas:

\[
N_1 = \frac{q_o}{\kappa L^2} \left( \frac{L}{\beta} - \frac{\beta}{C_o \kappa^2 L} \right) - \frac{q_o}{(A_o \kappa^3)^2} \left( A_o \kappa L^2 \beta^2 + 2\gamma \cdot \frac{2\gamma}{\kappa \beta} + \frac{2\gamma}{D_o \kappa^2 L} \left( \frac{\beta - \gamma}{\kappa L} \right) + B_o \right); \\
N_2 = \frac{q_o}{\kappa^2} \left( 1 - \cos \kappa L \right) + \frac{q_o}{(A_o \kappa^3)^2} \left( A_o \kappa L \frac{2\gamma}{\kappa L} + e^{-\gamma} \left( \frac{2\gamma}{\kappa L} \cos \kappa L + B_o \sin \kappa L \right) \right); \\
N_3 = \frac{q_o}{\kappa^2} \left( \frac{L}{\beta} - \frac{1}{C_o \kappa L} \cos \kappa L + \sin \kappa L \right) + \\
+ \frac{q_o}{(A_o \kappa^3)^2} \left( A_o \kappa L \left( \beta - 1 \right) \frac{2\gamma}{\kappa \beta} + 2\gamma e^{-\gamma} \frac{2\gamma}{D_o \kappa^2 L} \left( \frac{\beta - \gamma}{\kappa L} \cos \kappa L + \sin \kappa L \right) + \\
+ B_o e^{-\gamma} \left( \frac{\beta - \gamma}{\kappa L} \sin \kappa L - \cos \kappa L \right) \right); \\
N_4 = \frac{q_o}{\kappa L} \left( 1 - \cos \kappa L \right) + \sin \kappa L \right) + \\
+ \frac{q_o}{(A_o \kappa^3)^2} \left( A_o \kappa L \left( 1 + \beta \right) \frac{2\gamma}{\kappa \beta} + 2\gamma e^{-\gamma} \frac{2\gamma}{D_o \kappa^2 L} \left[ \left( \frac{\beta - \gamma}{\kappa L} \cos \kappa L - \sin \kappa L \right) \right] + \\
+ B_o \kappa e^{-\gamma} \left( \frac{\beta - \gamma}{\kappa L} \sin \kappa L + \cos \kappa L \right) \right). \\
\]

From the third and first equations, \( M \) and \( A \) is expressed through \( F \):

\[
M = -FL \left( \frac{1}{\beta} + 1 \right) - N_4 \frac{L}{\beta}; \\
A = F \left( \frac{2L^3}{EI_o \beta^2} + \frac{L^2}{EI_o \beta} - \frac{L}{\theta_o \beta} - \frac{L}{\theta_o} \right) - \frac{N_1}{EI_o} + \frac{N_4 L}{\beta} \frac{L}{EI_o \beta} - \frac{1}{\theta_o}. \\
\]

Substituting these formulas into the second equation, we get:

\[
\theta_x \sin(Z_1 + FZ_2) + \theta_u \cos(Z_1 + FZ_2) + \frac{FL}{\beta} + Z_3 = 0, \\
\]

where

\[
Z_1 = \frac{N_4 L}{\beta} \left( \frac{L}{EI_o \beta} - \frac{L}{\theta_o} \frac{1}{\theta_o} \right) + \frac{N_1}{EI_o} - \frac{N_1}{EI_o} \right); \\
Z_2 = 2\frac{L^2}{\beta^2} \left( \frac{1}{EI_o} - \frac{1}{EI_o} \right) + \frac{L^2}{EI_o \beta} - \frac{L}{\theta_o \beta} - \frac{L}{\theta_o} \right); \\
Z_3 = \frac{N_4 L}{\beta} - N_2 \right). \\
\]

The solution of the transcendental equation (13) with respect to \( F \) in the algorithm for the general solution of the stem bending problem is performed by Newton’s method of successive approximations:

\[
F_{i+1} = F_i - \frac{f(F)}{f'(F)}, \text{ } i = 1, 2, 3, \ldots \\
\]

where

\[
f(F) = \theta_x \sin(Z_1 + FZ_2) + \theta_u \cos(Z_1 + FZ_2) + \frac{FL}{\beta} + Z_3 \right); \\
\]

where
Calculations have shown that up to 5 - 10 iterations are usually sufficient to obtain the zero of the function $f(F)$. After finding $F$, $M$ and $A$ are calculated by formulas (11) - (12).

To calculate the bending of the stems of the studied plants (bending stresses $\sigma$ and $S$), the angles of deflection from the vertical $\alpha$ ($S$), shearing forces $U$ ($S$), the shape of the curve in XY coordinates, the found optimal diameters and lengths of plant stems and other initial data (physical and mechanical properties of stem tissue, wind load area, etc.) and formulas:

$$\sigma(S) = \frac{EI(S)\alpha'(S)}{W(S)}; \quad U(S) = EI(S)\alpha''(S).$$

$\alpha(S)$ – calculated by the formula (8).

For simplicity, the coordinates of the curve are calculated numerically. Considering that $X'(S) = \cos \alpha(S)$ and $Y'(S) = \sin \alpha(S)$ these coordinates are calculated using the recurrent formulas:

$$X_{j+1}(S) = X_j(S) + (S_{j+1} - S_j) \cos[(\alpha_j + \alpha_{j+1})/2],$$

$$Y_{j+1}(S) = Y_j(S) + (S_{j+1} - S_j) \sin[(\alpha_j + \alpha_{j+1})/2],$$

where $j = 0, 1, 2...; \quad X_0(S) = Y_0(S) = S_0 = 0$.

The stem in these and in other calculations is divided into 15 parts, which is quite enough from the point of view of accuracy and clarity.

4. Conclusions
The calculations were carried out for all plants in the phase of milky-wax ripeness, with the initial data on the varieties that most affect the bending, including taking into account the moisture of the plants.

The initial data for calculating winter wheat with optimal architectonic parameters are shown in Table 1.

As an illustration, Figure 2 shows the shapes of the curves of the bending of the stem of short-stemmed (with optimal parameters) winter wheat depending on the wind speeds characteristic of Krasnodar.

**Table 1. Initial data for the calculation of winter wheat stems.**

| Do, mm | Dк, mm | L, m | \(\theta_o\), Nm | EIо, Nm^2 | \(\beta\), d.u. | qо, N/m | \(\gamma\), d.u. | Wо, m^3 | \(\phi\), d.u. |
|--------|--------|------|-----------------|-------------|----------|--------|----------|--------|----------|
| 4.2    | 2.2    | 0.65 | 1.0             | 0.0331      | 2.5080   | 0.1087 | 0.8418   | 7.2*10^-9 | 1.8684   |
| 4.4    | 2.3    | 0.70 | 1.1             | 0.0399      | 2.5278   | 0.1180 | 0.8855   | 8.3*10^-9 | 1.8863   |
| 5.3    | 2.8    | 0.90 | 1.5             | 0.0836      | 2.4793   | 0.1632 | 0.9322   | 1.4*10^-8 | 1.8480   |

**General initial data**

| B, N   | h, m | \(\theta_o\), Nm | \(S_x\), sm^2 | \(S_{2\gamma}\), m^2/m | \(\sigma_y\), MPa | \(\sigma_r\), MPa |
|--------|------|-----------------|---------------|------------------------|----------------|----------------|
Figure 2 shows that an increase in the wind from 8.9 to 19.7 m / s significantly increases the deflection of the stem (flesh to a gentle position of the stem axis at the ear). A similar shape of the stem bending is characteristic for other cereals as well.

Figure 2. The shape of the axis of short-stemmed winter wheat in the phase of milky-waxy ripeness at different wind speeds (Doopt = 4.2 mm; Dk = 2.2 mm; Loopt = 0.65 m).

Figure 3 shows the distribution of bending stresses in a winter wheat stalk. The greatest stresses are at the root, with the highest stresses occurring in medium-sized wheat. In general, the level of these stresses (11.1 - 14.3 MPa) is less than the elastic limit (18.5 MPa); this means that winter wheat with the calculated optimal parameters of architectonics is resistant to lodging.

Figure 3. Bending stresses along the axis of the winter wheat stem with optimal parameters of architectonics in the phase of milky-wax ripeness (wind 19.7 m / s).
As an objective function of optimizing the architectonics of cereals, the weight of an ear was taken, and the task of selecting an "ideal variety" is to comprehend such parameters of the architectonics of a plant (length, diameters, the degree of root embedment in the soil) so that at a given, desired weight of an ear (panicle), the stem would remain vertical. For corn and bamboo, the panicle or caryopsis crown is of no industrial value; therefore, the aim of breeding for food hybrids of corn is the maximum mass of grain on the cob, and for bamboo - straightness of the trunk and high quality wood.

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