Coulomb drag in double quantum wells with a perpendicular magnetic field

M.W. Wu, H.L. Cui, and N.J.M. Horing

Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, NJ

07030

Abstract

Momentum transfer due to electron-electron interaction (Coulomb drag) between two quantum wells, separated by a distance $d$, in the presence of a perpendicular magnetic field, is studied at low temperatures. We find besides the well known Shubnikov-de Haas oscillations, which also appear in the drag effect, the momentum transfer is markedly enhanced by the magnetic field.

PACS number(s): 73.20.Dx, 73.50.Jt, 73.40.Ty
Coulomb drag effect of double quantum wells has recently attracted much experimental and theoretical attention. This is because, notwithstanding the fact that the effects of electron-electron collisions only have indirect consequences for transport properties of single isolated quantum wells as they conserve total momentum and cannot transfer or relax it, the drag effect of double quantum wells is unique in that it provides an opportunity to directly measure electron-electron interaction through a transport measurement where momentum is transferred from one well to the other. Consider two quantum wells containing electrons so close to each other that the electrons in the two respective wells experience the Coulomb forces originating from the other well and yet far enough away from each other that direct charge transfer between the two wells is not possible. Thus, if a current $J$ is driven through one of the wells, then an induced current is dragged in the other well. Alternatively, if no current is allowed to flow in the other well, an electric field $E$ is induced. The so called transresistivity $\rho$, which describes the momentum transfer between the two wells, is hereupon defined as

$$\rho = \frac{E}{J}.$$  

To our knowledge, almost all the works on this subject are focused on screening effects associated with plasmon modes (see, eg. [17–19]), or on deviations of the transresistivity from $T^2$ law at low temperature (from zero to several Kelvin degrees) (see, eg. [4,9,16]). Magnetic field effects on Coulomb drag in planar quantum wells have, to a large extent, escaped attention. It is known that in the classical limit, the magnetic field does not affect the Coulomb drag effect. Nevertheless, for higher magnetic fields, when Landau quantization becomes important, the situation is quite different. Since the mechanism for Coulomb drag is carrier-carrier scattering, the transresistivity is proportional to the square of the effective interaction between the two wells. The available phase space for electron-electron scattering is modified drastically by the magnetic field due to the formation of Landau levels, consequently affecting Coulomb drag.

The purpose of this letter is to elucidate the significance of the magnetic field, which is
applied perpendicular to the quantum wells, in regard to the Coulomb drag effect in double quantum wells at low temperature. We also include the role of dynamic screening of the Coulomb interactions between two quantum wells. As may be expected, we find the well known Shubnikov-de Haas oscillations, which feature prominently in the drag effect when temperature is sufficiently low. Our analysis further shows that the Coulomb drag effect is remarkably enhanced by high magnetic field at low temperature.

The transresistivity, which can be calculated from either the Boltzmann equation \([1,12]\), the momentum balance equation method \([9,14]\), or, very recently, the Kubo linear-response formula \([18,19]\), is given by

\[
\rho = \frac{1}{4\pi^2 n_1 n_2 e^2} \int_0^\infty dq q^3 \int_{-\infty}^\infty d\omega \left| \frac{v(q)e^{-\beta q}}{\varepsilon(q, \omega)} \right|^2 \left[ -n'(\frac{\omega}{T}) \right] \Pi^{(1)}(q, \omega) \Pi^{(2)}(q, \omega). \tag{2}
\]

In this equation, \(n_1, n_2\) stands for the sheet density of first (second) quantum well. \(n(x) = [\exp(x) - 1]^{-1}\) is the Bose distribution function and \(n'(x) = \frac{d}{dx} n(x)\). \(v(q) = 2\pi e^2/\kappa q\) is the bare 2D Coulomb interaction, with \(\kappa\) being the background dielectric constant. The dynamic screening of the Coulomb interaction, in random phase approximation, reads

\[
\varepsilon(q, \omega) = [1 - v(q)\Pi^{(1)}(q, \omega)][1 - v(q)\Pi^{(2)}(q, \omega)] - v(q)^2 e^{-2\beta q}\Pi^{(1)}(q, \omega)\Pi^{(2)}(q, \omega). \tag{3}
\]

Here \(\Pi^{(j)}(q, \omega)\) is the electron density-density correlation function of \(j\)th quantum well, with \(\Pi^{(1)}(q, \omega) (\Pi^{(2)}(q, \omega))\) denoting the real (imaginary) part of it. In the presence of magnetic field, it is given by \([21]\)

\[
\Pi^{(j)}(q, \omega) = \frac{1}{\pi \alpha^2} \sum_{nn'} C_{nn'}(x) \Pi^{(j)}(n, n', \omega), \tag{4}
\]

where

\[
C_{nn'}(x) = m_2! m_1! x^{m_1 - m_2} e^{-x}[L_{m_2}(x)]^2 \tag{5}
\]

with \(m_1 = \max(n, n')\) and \(m_2 = \min(n, n')\). \(L_{m_2}(x)\) is the associated Laguerre polynomial. \(\alpha\) is the radius of the ground cyclotron orbit, given by \(\alpha = (eB)^{-1/2}\) with \(B\) denoting the magnetic field. \(x = \alpha^2 q^2/2\). The quantity \(\Pi^{(j)}(n, n', \omega)\) can be expressed as
\[
\begin{align*}
\text{Re}\Pi^{(j)}(n, n', \omega) &= -\frac{1}{\pi} \int_{-\infty}^{\infty} dz [f_j(z) - f_j(z + \omega)] \text{Re} G_n^{(j)}(z + \omega) \text{Im} G_{n'}^{(j)}(z), \\
\text{Im}\Pi^{(j)}(n, n', \omega) &= -\frac{1}{\pi} \int_{-\infty}^{\infty} dz [f_j(z) - f_j(z + \omega)] \text{Im} G_n^{(j)}(z + \omega) \text{Im} G_{n'}^{(j)}(z).
\end{align*}
\]

In these equations, \(f_j(z)\) denotes the Fermi distribution function of \(j\)th well, given by \(\left\{\exp[\frac{z - \mu_j}{T}] + 1\right\}^{-1}\) with \(\mu_j\) denoting chemical potential, which is determined, for total electron sheet density \(n_j\), by the relation
\[
\frac{1}{\pi^2} \sum_{n=0}^{\infty} \int d\omega f_j(\omega) \text{Im} G_n^{(j)}(\omega) = n_j.
\]

\(G_n^{(j)}(z)\), the Green function for the \(n\)th Landau level of \(j\)th well, can be expressed, in the self-consistent Born approximation \(\text{[22,23]}\), as
\[
G_n^{(j)}(\omega) = \frac{2}{\Gamma_j^2} \left[\omega - \varepsilon_n - \sqrt{(\omega - \varepsilon_n)^2 - \Gamma_j^2}\right],
\]
with \(\Gamma_j^2 = 2\omega_c/\pi \tau_0^{(j)}\) and \(\tau_0^{(j)}\) is the electron transport lifetime in the absence of a magnetic field at zero temperature, related to the mobility \(\mu_0^{(j)}\) under the same conditions by \(\tau_0^{(j)} = m\mu_0^{(j)}/e\). \(\varepsilon_n = (n + 1/2)\omega_c\) is the Landau level energy with \(\omega_c = eB/m\).

Based on Eqs. (2) and (8) we can calculate the transresistivity, as a function of magnetic field at low temperatures, to examine the influence of Landau quantization of magnetic field on the Coulomb drag effect. The magnetic field considered in our calculation is taken large enough so that the Landau level energy \(\hbar\omega_c\) is larger or at least comparable to \(k_BT\), and therefore quantum effects are important. Parameters pertaining to a GaAs-AlGaAs-GaAs double quantum well structure are used, with electron effective mass \(m = 0.07m_e\) (\(m_e\) is the free electron mass), barrier thickness \(b = 200\text{Å}\) and the background dielectric constant \(\kappa = 12.9\). We suppose that both quantum wells share the same sheet density \(n_1 = n_2 = 10^{15}\text{ m}^{-2}\), and the mobility at zero temperature is \(\mu_0^{(1)} = \mu_0^{(2)} \equiv \mu_0 = 25\text{ m/Vs}\).

We first discuss the collective modes of the coupled electron gas. They are given by the zeros of the real part of the dielectric function Eq. (3), \(\text{Re}[\varepsilon(q, \omega(q))] = 0\) at finite temperature, with the imaginary part describing the damping. As in the nonmagnetic field case \(\text{[24]}\), there are two collective modes, as shown in Fig. 1 in which we plot the integrand
of Eq. (2) versus $\omega/T$ for different values of $q$ normalized by $\alpha$, when $T = 3$ K (Fig. 1(a)) and 10 K (Fig. 1(b)). The magnetic field in these figures is 1 T. From the figure we can see that as $q$ increases, the two modes both increase in frequency $\omega(q)$ and become closer. On the other hand, as $q$ goes to zero, one mode $\omega(q)$ also tends to zero, which is the so called acoustic mode, and the other is in the nature of an optical mode with $\omega(q) \neq 0$. We point out here that this feature of the optical mode in the presence of the magnetic field is quite different from the nonmagnetic case, as in that circumstance, the optical mode also tends to zero as $q \to 0$, with its small $q$ limiting behavior at zero temperature being $q^{1/2}$ [19,24]. This is because, in the absence of magnetic field, the collective excitation reflects the nature of 2D electron gases, whereas in the presence of the field, this 2D freedom is further restricted by Landau quantization.

In Fig. 2 we plot the numerically calculated transresistivity $\rho$, which is normalized by $\rho_0$ [25] as a function of magnetic field $B$ at different temperatures $T = 3, 4, 6.5, 10, \text{ and } 15$ K. We include ten Landau levels in our computation. In order to exhibit the screening effect, we plot the transresistivity both with and without screening in Fig. 2(a) and (b) respectively. From both of these figures we can see that the magnetic field has a strong effect on the transresistivity, especially at low temperatures. We note, from the figures for $T$ below about 4 K, the transresistivity exhibits distinctive Shubnikov-de Haas oscillations as the magnetic field increases from 0.5 T to 2.5 T (with the filling factor $\nu$ decreasing from 8.3 to 1.7). It is also clear from the figures that the minima for the curve of $T = 3$ K from the right to the left correspond to the filling factors $\nu=2, 4, 6, 8$, respectively, which means the Fermi level is between the first and the second, second and third, $\cdots$ etc., Landau levels. Hence the available phase space is greatly reduced, thus reducing the transresistivity. As the magnetic field continues to increase, eventually all the electrons are accommodated in the first Landau level, the transresistivity is markedly enhanced, becoming an order of magnitude larger than that in the low field case. However, when temperature becomes higher, the oscillations are washed out due to thermal fluctuation and the magnetic enhancement of the drag effect becomes moderate as $B$ increases.
We also note from the figures that screening is more effective for low temperatures than for higher ones. This can be well illustrated from the fact that in Fig. 2(a) one can see that when electron screening effect is included, for any magnetic field, the drag effect always increases as temperature rises. However in Fig. 2(b) one can see, especially in the high magnetic field regime ($B > 2.5$ T), that the lower the temperature, the higher the transresistivity. This implies that screening reduces the drag effect at low temperature faster than at high temperature. This is readily understood since the degree of degeneracy is higher at low temperatures, so that the screening of electrons is more effective than at high temperature case.

In summary, we have studied the effects of a quantizing magnetic field on the momentum transfer due to electron-electron Coulomb interaction between two spatially separated quantum wells. We find that, at low temperatures, the magnetic field not only induces Shubnikov-de Haas oscillations, but it also markedly enhances momentum transfer due to electron-electron interactions.

**ACKNOWLEDGMENTS**

One of the authors (MWW) would like to thank Mr. X.G. Feng, for providing information about his pertinent experimental work in progress. This research is supported by U.S. Office Naval Research (Contract No. N66001-95-M-3472), and the U.S. Army Research Office (Contract NO. DAAH04-94-G-0413).
REFERENCES

[1] P.M. Solomon, P.J. Price, D.J. Frank, and D.C.L. Tulipe, Phys. Rev. Lett. 63, 2508, (1989).

[2] T.J. Gramila et al., Phys. Rev. Lett. 66, 1216 (1991).

[3] U. Sivan, P.M. Solomon, and H. Shtrikman, Phys. Rev. Lett. 68, 1196 (1992).

[4] T.J. Gramila, J.P. Eisentein, A.H. MacDonald, L.N. Pfeiffer, and K.W. West, Phys. Rev. B 47, 12957 (1993). Physica 197B, 442 (1994).

[5] B. Laikhtman and P.M. Solomon, Phys. Rev. B 41, 9921 (1990).

[6] I.I. Boiko and Yu.M. Sirenko, Phys. Status Solidi 159, 805 (1990).

[7] P.M. Solomon and B. Laikhtman, Superlatt. Microstruct. 10, 89 (1991).

[8] A.G. Rojo and G.D. Mahan, Phys. Rev. Lett. 68, 2074 (1992).

[9] H.C. Tso, P. Vasilopoulos, and F.M. Peeters, Phys. Rev. Lett. 68 2516 (1992); ibid. 70, 2146 (1993).

[10] D.I. Maslov, Phys. Rev. B 45, 1911 (1992).

[11] J.M. Duan and S. Yip, Phys. Rev. Lett. 70, 3647 (1993).

[12] A.P. Jauho and H. Smith, Phys. Rev. B 47, 4420 (1993).

[13] L. Zheng and A.H. MacDonald, Phys. Rev. B 48, 8203 (1993).

[14] H.L. Cui, X.L. Lei, and N.J.M. Horing, Superlatt. Microstruct. 13, 221 (1993).

[15] E. Shimshoni and S.L. Sondhi, Phys. Rev. B 49, 11484 (1994).

[16] K. Flensberg and B.Y.K. Hu, Phys. Rev. Lett. 73, 3572 (1994).

[17] L. Swierkowski, J. Szymański, and Z.W. Gortel, Phys. Rev. Lett. 74, 3245 (1995).

[18] K. Flensberg, B.Y.K. Hu, A.P. Jauho, and J. Kinaret, Phys. Rev. B 52, 14761 (1995).
[19] K. Flensberg and B.Y.K. Hu, Phys. Rev. B 52, 14796 (1995).

[20] H.C. Tso and Vasilopoulos have discussed the Coulomb drag effect between quantum wires in the presence of a magnetic field [Phys. Rev. B 45, 1333 (1992)]. However their results in quantum wires do not show the same pronounced effect as those presented here.

[21] C.S. Ting, S.C. Ying, and J.J. Quinn, Phys. Rev. B 16, 5394 (1977).

[22] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. 36, 959 (1974).

[23] T. Ando, A.B. Fowler, and F. Stern, Rev. Mod. Phys. 54, 438 (1982).

[24] S. Das Sarma and A. Madhukar, Phys. Rev. B 23 805 (1981).

[25] $\rho_0$ is defined from the overall factor of Eq. (2): $e^2T^2/(\pi^6n_1^2\alpha^6\epsilon_0^2\kappa^2\Gamma^4)$ with $T = 1$ K and $B = 1$ T.

Fig. 1. Transresistivity (integrand of Eq. (2)) as a function of energy transfer $\omega$ scaled by temperature $T$, for various wave vectors $q$ normalized by $\alpha$. (a): $T = 3$ K; (b): $T = 10$ K.

Fig. 2. Transresistivity $\rho$ normalized by $\rho_0$ is plotted as a function of magnetic field $B$ for various temperatures $T = 3, 4, 6.5, 10, \text{and } 15$ K. (a): with the interlayer screening effects by $\epsilon(q, \omega)$ in Eq. (3); (b): without screening effects i.e. $\epsilon(q, \omega) = 1$. 