Light quark energy loss in a soft-wall AdS/QCD model

Xiangrong Zhu\textsuperscript{1,a}, Zi-qiang Zhang\textsuperscript{2,b}

\textsuperscript{1} School of Science, Huzhou University, Huzhou 313000, China
\textsuperscript{2} School of Mathematics and Physics, China University of Geosciences, Wuhan 430074, China

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Abstract We investigate the energy loss of light quarks in a holographic QCD model with conformal invariance broken by a background dilaton. We perform the analysis within falling string and shooting string, respectively. It turns out that the two methods give the same result: the presence of confining scale and chemical potential tends to enhance the energy loss, in accord with previous findings of drag force and jet quenching parameter.

1 Introduction

One of the main purposes of the heavy-ion collisions experiments is to explore the QCD phase diagram and the properties of new state of matter produced through collisions at high energy density. It is believed that the experimental program at Relativistic Heavy-Ion Collider (RHIC) and Large Hadron Collider (LHC) have created a new state of matter so-called quark gluon plasma (QGP) \cite{1,2}. One of the striking features of such substance is jet quenching: the energy loss of high energy partons produced through collisions as they interact with the plasma before they fragment into hadrons (for recent reviews see \cite{3,4}). On the other hand, it has been observed that QGP behaves as a strongly coupled fluid \cite{5–7}, which involves nonperturbative physics suitable for the application of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence.

AdS/CFT \cite{8–10}, the duality between the type IIB superstring theory formulated on AdS\textsubscript{5} × S\textsuperscript{5} and $\mathcal{N} = 4$ super Yang-Mills theory (SYM) in four dimensions, provides a useful tool for understanding the strong interaction, i.e. quantum chromodynamics (QCD). During the last two decades, the AdS/CFT correspondence has yielded many important insights for studying various aspects of QGP (see \cite{11,12} for recent reviews with many phenomenological applications).

An interesting example of such applications is jet quenching. For instance, the drag force which describes the energy loss for heavy quarks moving through $\mathcal{N} = 4$ SYM plasma was studied in \cite{13,14}. On the other hand, the jet quenching of light quarks moving through $\mathcal{N} = 4$ SYM plasma has been addressed based on various approaches, e.g., jet quenching parameter \cite{15,16}, falling string \cite{17–21}, shooting string \cite{22,23}. Further study in this direction can be found in \cite{24–29}.

The purpose of this paper is to study the light quark energy loss (using falling string and shooting string) in a nonconformal holographic model. The motivation is that the conformal SYM plasma is different from the actual QGP produced in heavy ion collisions, which is highly nonconformal in the crossover region. To address this issue, various “holographic QCD models” are conceived, such as top-down models (derived from string theory) \cite{30–33}, bottom-up models (derived from experimental data and lattice results) \cite{34–37} and some improved holographic QCD models \cite{38–40}. Here, we will employ a type of bottom-up models, i.e. the SW\textsubscript{T,μ} model \cite{41}, which is defined by the AdS with a charged black hole to describe finite temperature and density multiplied by a warp factor to generate confinement. Another motivation for this paper is that the drag force \cite{42} and jet quenching parameter \cite{43} have been investigated in the same model and the results show that the presence of chemical potential and confining scale increases the two parameters thus enhancing the energy loss. Inspired by this, we wonder whether chemical potential and confining scale have the same effect on the energy loss of light quarks using falling string and shooting string.

The outline of the paper is as follows. In the next section, we introduce the SW\textsubscript{T,μ} model presented in \cite{41}. In Sect. 3, we study the energy loss of light quarks in this model within falling string and shooting string, in turn. The last part is devoted to conclusion and discussion.
2 Setup

First, we briefly review the SW\(_{T,\mu}\) model. One considers the following action [44-46]

\[
I = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left( R + \frac{12}{R^2} - \frac{R^2}{g^2_F} F_{\mu\nu} F^{\mu\nu} \right),
\]

where \(\kappa^2 = 8\pi G\). \(R\) is the Ricci scalar. \(R\) represents the AdS radius. \(g_F\) denotes an effective dimensionless gauge coupling constant. \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\) with \(A_\mu\) the \(U(1)\) gauge field. The 5-dimensional solution of equation of motion coming from (1) can be written as

\[
ds^2 = \frac{R^2}{z^2} \left( -f(z) dt^2 + dx^2 + dz^2 \right),
\]

with

\[
f(z) = 1 - (1 + Q^2) \left( \frac{z}{z_h} \right)^4 + Q^2 \left( \frac{z}{z_h} \right)^6,
\]

where \(Q\) is the charge of black hole. \(z\) represents the fifth coordinate with \(z = z_h\) the horizon, defined by \(f(z_h) = 0\).

Following [41], the SW\(_{T,\mu}\) model can be obtained by the metric (2) multiplied by a warp factor

\[
ds^2 = \frac{R^2}{z^2} h(z) \left( -f(z) dt^2 + dx^2 + dz^2 \right),
\]

with

\[h(z) = e^{\frac{c^2 z^2}{2}},\]

where \(c\) denotes the deformation parameter which has the dimension of energy. \(h(z)\) distorts the metric and brings the confining scale.

The temperature of the black hole is

\[T = \frac{1}{\pi z_h} \left( 1 - \frac{Q^2}{2} \right), \quad 0 \leq Q \leq \sqrt{2}.
\]

The chemical potential is

\[\mu = \sqrt{3} Q/z_h.
\]

Notice that for \(Q = 0\), the SW\(_{T,\mu}\) model becomes the Andreev model [47]. For \(c = 0\), it reduces to the AdS-Reissner Nordstrom black hole [45,46]. For \(Q = c = 0\), it restores to the AdS black hole.

3 Energy loss of light quarks in the SW\(_{T,\mu}\) model

3.1 Falling string

Various authors argued [17–21] that the jet quenching of light quarks could be characterized by a (maximum) stopping distance of a massless particle falling along the null geodesic in the dual geometry in different ways, e.g., with or without the addition of fundamental-charge matter, specify different initial conditions. Though each approaches the subject from different vantage points, the analysis of the stopping distance traveled by the falling string in AdS\(_5\)-Schwarzschild are consistent. Here we will follow the argument in [20,21] to study the jet quenching of light quarks in the SW\(_{T,\mu}\) model by analyzing the stopping distance of an image jet induced by a massless source field, characterized by a massless particle falling along the null geodesic. According to this scenario, the R-charged current is generated by a massless gauge field in the gravity dual and the induced current is regarded as an energetic jet passing through the medium. When the wave packet of the massless gauge field falls into the horizon of the dual geometry, the image jet on the boundary dissipates and then thermalizes in the medium. The stopping distance is therefore defined as the distance for a jet passing through the medium before it thermalizes.

In the WKB approximation, the wave packet of the massless gauge field in the gravity dual is supposed to be localized in the momentum space such that the wave function of the gauge field could be factorized as

\[A_j(t, z) = \exp \left[ \frac{i}{\hbar} \left( q_k x_k + \int dq_k z_k \right) \right] \tilde{A}_j(t, z),\]

where \(q_k\) denotes the 4-momentum, conserved as the metric preserves the translational symmetry along the 4-dimensional spacetime. \(q_j\) represents the momentum along the bulk direction. \(A_j(t, z)\) refers to the slow-varying with respect to \(t\) and \(z\). \(j, k\) are the 4-dimensional spacetime coordinates.

In the classical limit, i.e., \(\hbar \rightarrow 0\), the equation of motion of the wave pack reduces to a null geodesic,

\[0 = (ds^2) = dx^i g_{ij} dx^j + dz g_{zz} dz,
\]

yielding

\[\frac{dz}{d\xi} = \frac{1}{\sqrt{g_{zz}}} \left[ -g_{ij} \frac{dx^i}{d\xi} \frac{dx^j}{d\xi} \right]^{1/2},\]

where \(\xi\) is an affine parameter for the trajectory. As the 4-dimensional translation invariance,

\[g_{ij} \frac{dx^j}{d\xi} = 1.
\]
is conserved and proportional to $q_i$, leading to

$$\frac{dx^i}{d\zeta} \propto g^{ij} q_j.$$  \hspace{1cm} (12)

Then, dividing (12) by (10) gives

$$\frac{dx^i}{dz} = \sqrt{g_{zz}} \frac{g^{ij} q_j}{(-q_k g^{kl} q_l)^{1/2}},$$  \hspace{1cm} (13)

one can check that the null geodesic in the above equation remains unchanged even when one uses the Einstein frame.

To proceed, we calculate the stooping distance. Supposing that the three momentum $q$ to point in one of $x$ directions, e.g., the $x_3$ direction, implying $q_i = (-\omega, 0, 0, |q|)$, where $\omega$ and $q$ are the energy and spacial momentum of the light quark, respectively. Then plugging (4) into (13), the stopping distance for the $SW_{T,\mu}$ model can be obtained,

$$x = \int_0^{z_0} \frac{dz}{e^{c^2 z^2} \sqrt{\frac{\omega^2}{|q|^2} - \frac{1-\left(1+\frac{a^2 z^2}{c^4} \right)^2}{c^4 \frac{a^2}{z^4}}}},$$  \hspace{1cm} (14)

note that when one turns off the chemical potential and confining scale effects by setting $\mu = c = 0$, the above equation recovers the result of SYM [20,21].

Before going further, we need to turn to numerics. First, we determine the value range of $c$. Here we tend to study the light quark energy loss in a class of models parametrized by $c$ rather than in a specific model with fixed $c$. To that end, we make $c$ dimensionless by normalizing it at fixed $T$ and express $\mu$ in unit of it as well. The knowledge of lattice calculations suggests [48] that the range of $0 \leq c/T \leq 2.5$ is most relevant for a comparison with QCD. We take that range here.

In Fig. 1, we compare the stopping distance of a light quark moving in the $SW_{T,\mu}$ model with the same one moving in $N = 4$ SYM plasma. The left panel corresponds to $x/x_{SYM}$ versus $\mu/T$ with fixed $c/T$ while the right one represents $x/x_{SYM}$ versus $c/T$ with fixed $\mu/T$, where $x_{SYM}$ denotes the stopping distance of SYM. From these figures, one finds that the stopping distance in the $SW_{T,\mu}$ model is smaller than that of SYM. In particular, the left panel tells us that with fixed $c/T$, increasing $\mu/T$ leads to decreasing $x/x_{SYM}$. Namely, the inclusion of chemical potential decreases the stopping distance thus enhancing the energy loss, in accord with that found in [49]. On the other hand, one can see from the right panel that the confining scale has similar effect. Given the above, one could reach the following conclusion: the presence of chemical potential and confining scale both decrease the stopping distance thus enhancing the energy loss, consistently with the findings of the drag force [42] and jet quenching parameter [43].

3.2 Shooting string

This subsection is devoted to the analysis of the light quarks energy loss using shooting string. According to [22,23], one considers a particular type of classical string motion: the string endpoint is close to horizon initially and move towards the boundary, carrying some energy and momentum which are gradually bled off into the rest of the string during its rise, hence, this motion is called finite-endpoint-momentum shooting string, or shooting string for short.

Next, we will follow the approach in [22,23] to study the light quark energy loss within shooting string for the background metric (4). The instantaneous energy loss of light quarks takes the form

$$\frac{dE}{dx} = -\frac{|L|}{2\pi a' z^2},$$  \hspace{1cm} (15)

where $L$ represents the null geodesics that the endpoint follows. From the above equation, it appears that small $z$ (mean-
ing the endpoint starts near the boundary) will result in large energy loss, indicating the jets will be quenched quickly and cannot be seen. So to hedge against this, one assumes the energy loss, indicating the jets will be quenched quickly and (since the endpoint starts near the boundary) will result in large energy loss for the SW model as

\[ \frac{dE}{dx} = \frac{e^{z^2} \sqrt{1 - \left(1 + \frac{\mu^2 z^2}{3}ight) \frac{z^4}{z_h^4} + \frac{\mu^2 \eta^4}{3 z_h^4}}}{2 \pi \alpha'} \],

(22)

Finally, using (4), (15) and (20), one obtains the energy loss for the SW model as

\[ \frac{dE}{dx} = \frac{e^{z^2} \sqrt{1 - \left(1 + \frac{\mu^2 z^2}{3}ight) \frac{z^4}{z_h^4} + \frac{\mu^2 \eta^4}{3 z_h^4}}}{2 \pi \alpha'} \],

(22)

note that for \( \mu = c = 0 \), the above equation recovers the result of SYM [22,23],

\[ \left( \frac{dE}{dx} \right)_{SYM} = -\frac{\pi \sqrt{T}}{2} T^2 (1 + \pi T x)^2, \]

(23)

where \( T = 1/\pi z_h \) and \( \lambda = \frac{\mu^2}{2} N_c = \frac{1}{\alpha'} \). One finds that for SYM case, \( dE/dx \) looks like \( T^2, x T^3 \) and \( x^2 T^4 \), for small, intermediate and large \( x \), respectively. We tried to derive the analytic expression for (22) to see how \( dE/dx \) depends on the powers of \( x \) and \( T \) but have not succeeded yet, thus, we need to turn to numerics. The procedures of the numerical evaluation are as follows: First, sending \( z_s \to 0 \), then for a given value of \( \mu/T \), one can integrate (21) and invert to get \( z(x) \). Next, substituting \( z(x) \) into (22) one can obtain \( dE/dx \) as a function of \( c/T, \mu/T \) and \( x \).

In Fig. 2, we compare the instantaneous energy loss of a light quark moving in the SW model with its counterpart in \( N = 4 \) SYM plasma (for ease of numerical calculation, we take \( T = 170 MeV \) here, but we have checked that by varying the value of \( T \), the ratio \( (dE/dx)/(dE/dx)_{SYM} \) barely changes). The left panel corresponds to \( (dE/dx)/(dE/dx)_{SYM} \) versus \( \mu/T \) with fixed \( c/T \) while the right one denotes \( (dE/dx)/(dE/dx)_{SYM} \) versus \( c/T \) with fixed \( \mu/T \), where \( (dE/dx)_{SYM} \) represents the energy loss of SYM case. For both panels, one finds the energy loss in the SW model is larger than that of SYM. In particular, increasing \( \mu/T \) or \( c/T \) both increase \( (dE/dx)/(dE/dx)_{SYM} \). Namely, the inclusion of chemical potential and confining scale both increase the energy loss,
4 Conclusion and discussion

Jet quenching in high-energy heavy-ion collisions can be used to probe properties of QGP. Studying the jet quenching in strongly coupled nonconformal plasma with finite quark density may shed qualitative insights into analogous questions in QCD. In this paper, we investigated the energy loss of light quarks in a nonconformal holographic model i.e. the SW_{T,\mu} model using falling string and shooting string, in turn. We discussed how chemical potential and confining scale modify the energy loss for both cases, respectively. For the former, we calculated the stopping distance of a massless particle moving along the null geodesic and found the presence of chemical potential and confining scale both decrease the stopping distance thus increasing the energy loss. For the latter, we computed the instantaneous energy loss based on the finite endpoint momentum framework and found the same result: increase chemical potential and confining scale both enhance the energy loss. These results agree with those obtained by the drag force and jet quenching parameter in the same model [42,43]. Taking all of this together, one may draw a conclusion that the effects of chemical potential and confining scale on the energy loss of heavy quarks and light quarks are consistent.

However, it must be admitted that there are some limitations to the present study. The major disadvantage is that the SW_{T,\mu} model is not a bona fide solution of the classical equations of motion. Considering jet quenching in some consistent models, e.g. [50,51] would be instructive. We hope to report our progress in this regard in the near future.

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References

1. E.V. Shuryak, Nucl. Phys. A 750, 64 (2005)
2. M. Gyulassy, L. McLerran, Nucl. Phys. A 750, 30 (2005)
3. M. Connors, N. Nattrass, R. Reed, S. Salur, Rev. Mod. Phys. 90, 025005 (2018)
4. G.Y. Qin, X.-N. Wang, Int. J. Mod. Phys. E 24(11), 1530014 (2015)
5. E. Shuryak, Prog. Part. Nucl. Phys. 53, 273 (2004)
6. K. Adcox et al., (PHENIX Collaboration), Nucl. Phys. A 757, 184 (2005)
7. J. Adams et al., (STAR Collaboration), Nucl. Phys. A 757, 102 (2005)
8. J.M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998)
9. S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 428, 105 (1998)
10. A. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri, Y. Oz, Phys. Rept. 323, 183 (2000)
11. J.C. Solana, H. Liu, D. Mateos, K. Rajagopal, U.A. Wiedemann, arXiv:1101.0618
12. O. DeWolfe, S.S. Gubser, C. Rosen, D. Teaney, Prog. Part. Nucl. Phys. 75, 86 (2014)
13. C.P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, L.G. Yaffe, JHEP 07, 013 (2006)
14. S.S. Gubser, Phys. Rev. D 74, 126005 (2006)
15. H. Liu, K. Rajagopal, U.A. Wiedemann, Phys. Rev. Lett. 97, 182301 (2006)
16. H. Liu, K. Rajagopal, U.A. Wiedemann, JHEP 03, 066 (2007)
17. S.S. Gubser, D.R. Gulotta, S.S. Pufu, F.D. Rocha, JHEP 10, 052 (2008)
18. P.M. Chesler, K. Jensen, A. Karch, Phys. Rev. D 79, 025021 (2009)
19. P.M. Chesler, K. Jensen, A. Karch, L.G. Yaffe, Phys. Rev. D 79, 125015 (2009)
20. P. Arnold, D. Vaman, JHEP 10, 099 (2010)
21. P. Arnold, D. Vaman, JHEP 04, 027 (2011)
22. A. Ficnar, S.S. Gubser, Phys. Rev. D 89, 026002 (2014)
23. A. Ficnar, S.S. Gubser, M. Gyulassy, Phys. Rev. Lett. B 738, 464 (2014)
24. K.B. Fadafan, R. Morad, Eur. Phys. J. C 78, 16 (2018)
25. B. Muller, D.-L. Yang, Phys. Rev. D 87, 046004 (2013)
26. R. Morad, W.A. Horowitz, JHEP 11, 017 (2014)
27. R. Rougemont, A. Ficnar, S. Finazzo, J. Noronha, JHEP 04, 102 (2016)
28. Z.-Q. Zhang, Phys. Lett. B 793, 308 (2019)
29. S. Heshmatian, R. Morad, M. Akbari, JHEP 03, 045 (2019)
30. J. Babington, J. Erdmenger, N.J. Evans, Z. Guralnik, I. Kirsch, Phys. Rev. D 69, 066007 (2004)
31. M. Kruczenski, D. Mateos, R.C. Myers, D.J. Winters, JHEP 0405, 041 (2004)
32. T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)
33. T. Sakai, S. Sugimoto, Prog. Theor. Phys. 114, 1083 (2005)
34. J. Erlich, E. Katz, D.T. Son, M.A. Stephanov, Phys. Rev. Lett. 95, 261602 (2005)
35. J. Polchinski, M.J. Strassler, JHEP 05, 012 (2003)
36. A. Karch, E. Katz, D.T. Son, M.A. Stephanov, Phys. Rev. D 74, 015005 (2006)
37. C. Csaki, M. Reece, JHEP 0705, 062 (2007)
38. U. Gursoy, E. Kiritsis, JHEP 0902, 032 (2008)
39. U. Gursoy, E. Kiritsis, F. Nitti, JHEP 0802, 019 (2008)
40. U. Gursoy, E. Kiritsis, L. Mazzanti, F. Nitti, Nucl. Phys. B 820, 148 (2009)
41. P. Colangelo, F. Giannuzzi, S. Nicotri, Phys. Rev. D 83, 035015 (2011)
42. Y. Xiong, X. Tang, Z. Luo, Chin. Phys. C 43, 113103 (2019)
43. J.G. Zhou, Z.-Q. Zhang, Chin. Phys. C 44, 105105 (2020)
44. A. Cambilin, R. Emparan, C.V. Johnson, R.C. Myers, Phys. Rev. D 60, 064018 (1999)
45. M. Cvetic et al., Nucl. Phys. B 558, 96 (1999)
46. D.T. Son, A.O. Starinets, JHEP 03, 052 (2006)
47. O. Andreev, V.I. Zakharov, Phys. Rev. D 74, 025023 (2006)
48. H. Liu, K. Rajagopal, Y. Shi, JHEP 08, 048 (2008)
49. E. Caceres, A. Kundu, D.L. Yang, JHEP 03, 073 (2014)
50. D. Li, M. Huang, JHEP 11, 088 (2013)
51. S. He, M. Huang, Q.S. Yan, Phys. Rev. D 83, 045034 (2011)