Lattice analysis of semi-leptonic form factors*

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We present preliminary results from simulations done on 170 $32^3 \times 64$ lattices at $\beta = 6.0$ using quenched Wilson fermions. This talk focuses on the $Q^2$ behavior of the form-factors, extrapolation in quark masses, dependence on renormalization scheme, and comparison with heavy-quark effective theory (HQET). Even though we cannot estimate errors due to quenching and discretization, our results are consistent with experimental results for $D$ decays. We present results for the Isgur-Wise function and estimate $\xi'(w = 1) = 0.97(6)$.

1. TECHNICAL DETAILS

We briefly mention some details of our analysis to extract the form-factors at $Q^2 = 0$ relevant to phenomenology. A full analysis will be presented elsewhere. The details of the lattices are contained in the papers on the hadron spectrum and decay constants. Preliminary results on a sub-set of lattices have been presented at LATTICE 94 and DPF 94. We do not have data to extrapolate to $m_b$ or to $a = 0$, thus our results are relevant for $D$ decays, i.e. $D \to K\ell\nu$, $D \to \pi\ell\nu$, $D \to K^*\ell\nu$, and $D \to \rho\ell\nu$, calculated at $a^{-1} = 2.33(4)$ GeV.

The decaying $D$ meson is created at rest by using a $\vec{p} = 0$ source. On each lattice we make two measurements by creating the $D$ meson at $t = 7$ and 57, for a total statistical sample size of 340. The sink for the final state meson is at $t = 32$ in both cases. The time-slice of the weak operator is taken to vary between $10 \leq t \leq 30$ and $35 \leq t \leq 55$ respectively, and the insertion is at 5 lowest values of lattice momenta. The quark propagators are created using a Wuppertal smeared source as described in [2].

In order to isolate the desired matrix element ($ME$) we construct a ratio of 3-point to 2-point correlation functions. We have a choice of using either smeared-smeared (SS) or smeared-local (SL) 2-point correlation functions. We calculate the $ME$ both ways and take the average as our best estimate. Figure 1 shows a typical example of signal for the ratio of correlators in the SL case: the quality in the SS case is very similar. We have seen a steady improvement in the consistency between these two estimates of $ME$ with statistics. With the current sample they are within $1\sigma$ in all cases.

**Pole dominance hypothesis (PDH):** It states that all form-factors, $f(Q^2)$, have the structure

$$f(Q^2) = f(0)/(1 - Q^2/M^2),$$

where $M$ is the mass of the nearest resonance.
with the right quantum numbers. To test PDH we make two kinds of fits: (i) single parameter “pole” fit where $M$ is the lattice measured value of the resonance mass, (ii) two parameter “best” fit where $M$ and $f(0)$ are free parameters. An example of these fits is shown in figure 2. Overall we find that only $f_0$ and $f_V$ are well described by the “pole” form. $f_+$ and $f_{A_0}$ are consistent with “pole” form with $M < M_{pole}$, while for $f_{A_1}$ and $f_{A_2}$ we find $M > M_{pole}$. $f_{A_1}$ and $f_{A_2}$ show a much smaller $Q^2$ dependence than expected from pole dominance, however the data are too noisy to make a definite statement. We use results from “best” fit for our final estimates.

**HQET:** At leading order in $1/m_c$, HQET predicts that the Isgur-Wise function $\xi$ describes all form-factors relevant to $D \to Kl\nu$ and $D \to K^*l\nu$ decays. Neglecting $O(\alpha_s)$ corrections, one gets

$$\xi(w) = Rf_+(q^2) = R \left(1 - \frac{q^2}{(M_f + M_i)^2}\right)^{-1} f_0(q^2)$$

$$= R^* V(q^2) = R^* A_0(q^2) = R^* A_2(q^2)$$

$$= R^* \left(1 - \frac{q^2}{(M_f + M_i)^2}\right)^{-1} A_1(q^2),$$

where $M_i$ and $M_f$ are the initial and final meson masses and

$$w = v_i \cdot v_f = \frac{M_i^2 + M_f^2 - q^2}{2M_i M_f}; \; R = \frac{2\sqrt{M_i M_f}}{M_i + M_f}.$$
Figure 4. Extrapolation of \( f_+(Q^2 = 0) \) to \( m_u \). \( f_+(D \to K\ell\nu) \) is extracted from points labeled by squares and octagons, while \( f_+(D \to \pi\ell\nu) \) is from data with degenerate \( q\bar{q} \) points (crosses).

Table 1. Estimates of form factors in 3 commonly used renormalization schemes defined in [4].

|       | TAD1 | TADπ | TADU₀ |
|-------|------|------|-------|
| \( f_+ \) | 0.71(4) | 0.75(4) | 0.66(4) |
| \( f_0 \) | 0.73(3) | 0.77(3) | 0.68(2) |
| \( f_V \) | 1.28(7) | 1.36(7) | 1.19(6) |
| \( f_{A_0} \) | 0.84(3) | 0.85(3) | 0.79(3) |
| \( f_{A_1} \) | 0.72(3) | 0.74(3) | 0.68(3) |
| \( f_{A_2} \) | 0.49(9) | 0.50(9) | 0.46(8) |
| \( f_{A_3} \) | 0.85(3) | 0.87(3) | 0.80(3) |
| \( f_V/f_{A_1} \) | 1.78(7) | 1.84(8) | 1.76(7) |

sin² \( p/2 + \sinh² M/2 \). We show variation of form-factors with \( M \) in Table 2.

Renormalization Constants: To relate lattice results to experimental data we need the renormalization constants \( Z_A \) and \( Z_V \). We use three Lepage-Mackenzie tadpole improved schemes described in Ref. [4]. Our preferred scheme is TAD1, and the variation with the schemes is illustrated in Table 1.

2. RESULTS at \( \beta = 6.0 \)

Our final results for \( D \to K\ell\nu \) and \( D \to K^*\ell\nu \) are shown in tables 1 and 2 along with variation with \( m_s \), type of fit, heavy-light meson mass, and the renormalization scheme. Our preferred estimates are with \( m_s(M_0) \), “best” fit, \( M_1 \), and TAD1 scheme. We also get that \( f_+^+/f_+^0 = 0.87(4). \)

3. \( d\Gamma(Q^2) \)

As explained in section 2 there are considerable uncertainties involved in extrapolating the form factors to \( Q^2 = 0 \). Therefore, we also calculate \( d\Gamma(Q^2)/dQ^2 \) by linearly extrapolating in \( m_q \) the form-factors at fixed 3-momentum transfer. In figure 5 we show the results for \( D \to Ke\bar{\nu}_e \). In figure 6 we show the longitudinal, transverse and the total decay widths for the process \( D \to K^*e\bar{\nu}_e \).

4. Isgur-Wise function

As mentioned in section 2 at the leading order in the heavy quark mass, there is one universal ‘Isgur-Wise’ function \( \xi \) which controls all the form factors.
Figure 5. $(1/V_{cs}^2)d\Gamma(q^2)/dq^2$ versus $q^2$ in GeV$^2$.
The shape is in qualitative agreement with experimental data.

Figure 6. $(1/V_{cs}^2)d\Gamma(q^2)/dq^2$ versus $q^2$ in GeV$^2$.

Figure 7. $\xi(w)/\xi(1)$ at various quark masses and momenta. The symbols label the flavor that the $C$ quark decays to, and variation of $\xi$ with $w$ is a kinematic effect. Dependence on $m_{\text{spectator}}$ is shown most clearly by the clusters of 3 points at $w \approx 1.2$. The largest value in each cluster corresponds to the lightest spectator ($U_3$). The data are fit to $\xi(w) = e^{-2}\exp\left(-\frac{2(2p^2-1)w}{w+p}\right)$, from which we estimate the slope $\xi'(1)$.

factors. In figure 7 we present data for $\xi_{\text{ren}}(w)$,

$$h_+(w) = \left[\hat{C}_1(w) + \frac{1}{2}(\hat{C}_2(w) + \hat{C}_3(w))\right] \xi_{\text{ren}}(w)$$

$$= f_+(q^2)\frac{M_i + M_f}{2\sqrt{M_i M_f}} + f_-(q^2)\frac{M_i - M_f}{2\sqrt{M_i M_f}},$$

where $\hat{C}_1$, $\hat{C}_2$, $\hat{C}_3$ are HQET renormalization constants including the leading $O(\alpha_s)$ corrections \cite{7}. The data show a slight dependence on the spectator quark mass indicative of $m_c^{-1}$ corrections. Analysis from the vector form factors is in progress. From this data we estimate the slope to be $\xi'(w = 1) = 0.97(6)$.

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