Supplementary Materials for

Search for exotic spin-dependent interactions with a spin-based amplifier

Haowen Su, Yuanhong Wang, Min Jiang*, Wei Ji, Pavel Fadeev, Dongdong Hu, Xinhua Peng*, Dmitry Budker

*Corresponding author. Email: dxjm@ustc.edu.cn (M.J.); xhpeng@ustc.edu.cn (X.P.)

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I. EXPERIMENTAL SETUP

In this section, we describe the details of our experimental setup, which is used to search for the exotic spin-dependent interactions ($V_{4+5}$ and $V_{12+13}$). As shown in Fig. S1(a), a vapor cell contains spatially overlapping ensembles of $^{129}$Xe and $^{87}$Rb. $^{129}$Xe spins act as an amplifier for the signal from exotic oscillating magnetic fields. $^{87}$Rb atoms are not only used to polarize the $^{129}$Xe spins through spin-exchange collisions, but also function as a magnetometer to measure the effective magnetic field generated by $^{129}$Xe spins. Thus, a $^{129}$Xe-based spin amplifier and $^{87}$Rb magnetometer can be realized by using a single $^{129}$Xe-$^{87}$Rb vapor cell.

A mass rotor is used as a source of the exotic spin-dependent interactions (see Fig. S2). A commercial miniaturized atomic magnetometer (from QuSpin Inc.) is used to monitor the magnetic field noise during the search measurement. In the following, we describe these elements one by one.

![FIG. S1. Experimental setup of the exotic interaction searches.](image)

(a) Two-dimensional diagram of the spin-based amplifier and the $^{87}$Rb magnetometer. A vapor cell with $^{129}$Xe and $^{87}$Rb is used. A circularly polarized pump beam propagating along $z$ polarizes $^{87}$Rb atoms. $^{129}$Xe spins are polarized through spin-exchange collisions with polarized $^{87}$Rb atoms. Hyperpolarized $^{129}$Xe spins can greatly amplify the signal from external oscillating magnetic fields and generate an effective magnetic field $B_{\text{eff}}$ on $^{87}$Rb atoms due to Fermi-contact interactions. In this case, $^{87}$Rb atoms also act as a sensitive magnetometer to detect the effective magnetic field of $^{129}$Xe. BE, beam expander; LP, linear polarizer; $\lambda/4$, quarter-wave plate; PD, photodiode; PEM, photoelastic modulator; DAQ, data acquisition. (b) Three-dimensional diagram of the experimental setup consisting of the spin-based amplifier, the $^{87}$Rb magnetometer and the QuSpin magnetometer. The QuSpin magnetometer mounted on a plate is located in the vicinity of the vapor cell ($\approx 3$ cm).

A. Schematic of the spin-based amplifier

The spin-based amplifier was recently demonstrated in Ref. [40]. It employs hyperpolarized long-lived $^{129}$Xe nuclear spins, which amplify the external magnetic field by more than two orders of magnitude. At the same time, $^{129}$Xe spins are also sensitive to exotic interactions. The key element of the spin-based amplifier is a 0.5 cm$^3$ cubic vapor cell containing 5 torr of isotopically enriched $^{129}$Xe, 250 torr N$_2$ as buffer gas, and a droplet (several milligrams) of isotopically enriched $^{87}$Rb. $^{129}$Xe spins are polarized through spin-exchange collisions with optically polarized $^{87}$Rb atoms. A bias magnetic field $B_0^z$ along $z$ is produced with a set of solenoids coils.

The $^{129}$Xe Larmor frequency $\nu_0$ is tuned by adjusting the bias magnetic field $B_0^z$. By tuning the $^{129}$Xe Larmor frequency, the operation frequency of the spin-based amplifier can be matched to the oscillation frequency $\nu$ of the measured magnetic field. In this situation, the $^{129}$Xe spin magnetization is tilted away from the bias-field direction $z$ [see Fig. S1(a)]. Due to Fermi-contact
interaction between $^{87}\text{Rb}$ and $^{129}\text{Xe}$ spins, the $^{129}\text{Xe}$ transverse magnetization generates an effective field $B^{\text{eff}}_\rho$ on $^{87}\text{Rb}$ atoms, which is \textit{in situ} measured with the $^{87}\text{Rb}$ magnetometer. As we show below (see Sec. III), the effective field $B^{\text{eff}}_\rho$ can be two orders of magnitude larger than the external measured magnetic field. In the case of the pseudo-magnetic field experienced by the $^{129}\text{Xe}$ spins due to the exotic interactions, these spins act as a transducer converting the pseudo-magnetic field into the effective magnetic field probed with $^{87}\text{Rb}$ spins. Here, we should emphasize that although the $^{129}\text{Xe}$-$^{87}\text{Rb}$ vapor cell is similar to that used in comagnetometers, the spin-based amplifier is quite different from comagnetometers (see Sec. IV A).

We have calibrated the performance of the spin-based amplifier. The spin-based amplifier is sensitive to the external oscillating fields in the $xy$ plane (see Sec. III). The amplification factor $\eta$ is calibrated to be $\approx 116$ in our experiment (see Sec. III A). In our experiment, a bias field $B^{0}_z$ (for example, $B^{0}_z \approx 423$ nT) is applied along $z$ to tune the $^{129}\text{Xe}$ Larmor frequency to $v_0 \approx 4.995$ Hz. By adjusting the operation frequency $v_0$, the spin-based amplifier can detect external magnetic field ranging from 1 to 1000 Hz (or even higher). The full width at half maximum (FWHM) of the spin-based amplifier is calibrated as 24 mHz (see Sec. III B), making it possible to filter out far-off-resonant field.

### B. Schematic of $^{87}\text{Rb}$ magnetometer

Using a $^{129}\text{Xe}$-$^{87}\text{Rb}$ vapor cell, not only a $^{87}\text{Rb}$-based spin amplifier can be built, but also a $^{87}\text{Rb}$ magnetometer can be realized. In our experiment, $^{87}\text{Rb}$ atoms act as a magnetometer to detect the $^{129}\text{Xe}$ effective oscillating field $B^{\text{eff}}_\rho$ [40, 46, 54, 55]. As shown in Fig. S1(b), the $^{129}\text{Xe}$-$^{87}\text{Rb}$ vapor cell is shielded with a five-layer cylindrical $\mu$-metal shield and heated to 140 °C. $^{87}\text{Rb}$ spins are polarized with circularly polarized laser light tuned to the peak of the D1 transition at 795 nm. The $^{87}\text{Rb}$ electron spin polarization along $z$ is sensitive to transverse magnetic fields along $x$ and $y$ and thus function as a magnetometer. The transverse electron spin polarization of $^{87}\text{Rb}$ atoms along $x$ is measured using linearly polarized probe light blue-detuned 110 GHz from the D2 transition at 780 nm. The optical rotation of the probe beam after passing the vapor cell is (see, for example, Refs. [55-57])

$$\theta = \frac{1}{4} lr c f n P_x^e D(v), \quad (S1)$$

where $l \approx 8$ mm is the optical path length, $r_e = 2.8 \times 10^{-13}$ cm is the classical radius of the electron, $c$ is the speed of light, $f$ is the oscillator strength (about 1/3 for D1 light and 2/3 for D2 light), $D(v) = (v - v_0)/[(v - v_0)^2 + (\Delta v/2)^2]$, $v$ is the frequency of the probe laser light, $\Delta v$ is the full-width at half-maximum (FWHM) of the optical D2 transition of frequency $v_0$. $P_x^e$ is the electron spin polarization of $^{87}\text{Rb}$ atoms along $x$

$$P_x^e = \langle \sigma_x \rangle = \rho(+1/2) - \rho(-1/2); \quad (S2)$$

where $\sigma_x$ is the Pauli matrix and $\rho$ is the number densities. To suppress the influence of low-frequency noise of the probe beam, the polarization of the probe beam is modulated with a photoelastic modulator (PEM) at 50 kHz [see Fig. S1(a)]. After being demodulated with a lock-in amplifier (SRS Model 830), the signal is acquired with a 24-bit analog-to-digital card (NI 9239).

The response of $^{87}\text{Rb}$ magnetometer can be written as (see Sec. III)

$$P_x^e \propto B_x B^{0}_z - B_y \Delta B \left(\frac{B^{0}_z}{(B^{0}_z)^2 + (\Delta B)^2}\right), \quad (S3)$$

where $\Delta B \approx 2300$ nT is calibrated in our experiment. To tune the $^{129}\text{Xe}$ Larmor frequency at $v_0 \approx 4.995$ Hz, the bias field $B^{0}_z$ is set as 423 nT. Based on Eq. (S3), $^{87}\text{Rb}$ magnetometer is simultaneously sensitive to the magnetic field along $x$ and $y$. In our experiment, the measured $B_x$ and $B_y$ are not from the external environment, but from the effective field generated by $^{129}\text{Xe}$ transverse magnetization in the $xy$ plane (see Sec. III).

### C. Schematic of the rotor

The exotic spin- and velocity-dependent interactions $V_{4+5}$ and $V_{12+13}$ are generated by a single cube-shaped BGO crystal $25.0 \times 25.0 \times 25.0$ mm$^3$ total volume. As shown in Fig. S2, a 112.34-g BGO crystal is mounted in a box at one end of a symmetric aluminum rod. The center of the 48.76-cm aluminum rod is located 58.32 cm away from the center of the $^{129}\text{Xe}$ vapor cell. The center of the aluminum rod is connected to a servo motor fixed on a stable aluminum platform, through a cylindrical titanium rod. Driven by the motor, the BGO crystal and the aluminum rod rotate with a frequency $v \approx 4.995$ Hz in the $xz$ plane. In order to precisely determine the rotation frequency $v$, a bronze needle is fixed on the titanium rod to trigger pulses when passing through the optoelectronic switch.
While the rotor system is mostly symmetric, the small imbalance due to the BGO crystal can cause some mechanical vibration. However, we found that this vibration can be neglected in our experiment. We explain this as follows. As shown in Fig. S2, the apparatus used to generate the exotic spin-dependent interactions mainly consists of a single BGO mass, a symmetric aluminum rod, a servo motor, and a platform. The BGO mass is about 112.34 g weight, the aluminum rod is about 610.34 g weight, and the platform used to fix the rotor is about 100 kg weight. The asymmetric mass of the rotor system is only $\approx 0.1\%$ of the whole rotor system. Moreover, the barycenter of the whole system is about 0.02 cm away from the center of the platform, which is significantly smaller than the size of the platform ($40 \times 40$ cm$^2$). Therefore, the mechanical vibration caused by the asymmetric BGO mass can be neglected. Indeed, testing the system with the rubidium magnetometer made insensitive by far-detuning of the probe beam and the analysis of the signals from the auxiliary QuSpin magnetometer have not produced any noticeable signals at the rotation frequency and its harmonics.

In our experiment, we search for the spin- and velocity-dependent interactions $V_{4+5}$ and $V_{12+13}$ generated by the rotating BGO crystal. In principle, there is also a contribution due to the exotic interactions generated with the rotating aluminum rod. However, the exotic pseudo-magnetic fields generated by the BGO crystal and the aluminum rod are spectrally separated (see Sec. II). Indeed, the pseudo-magnetic field generated by the rotating BGO crystal has the harmonics of $v$ (i.e., $v, 2v, 3v, ...$). In contrast, the pseudo-magnetic field generated by the aluminum rod has the harmonics of $2v$ (i.e., $2v, 4v, 6v, ...$). In our experiment, the operation frequency of the spin-based amplifier is set at $v$. Due to the relatively narrow bandwidth $\approx 24$ mHz of the spin-based amplifier (see Sec. III B), it only amplifies the signal of the first harmonic $v$ and filters out the signal of higher harmonics (for example, the second harmonic $2v$). Therefore, the pseudo-magnetic fields generated by the rotating aluminum rod can be neglected.

**D. Schematic of the QuSpin magnetometer**

In our experiment, we use a commercial miniaturized atomic magnetometer (from QuSpin Inc.) to monitor the spurious magnetic field, for example, from the rotor. The QuSpin magnetometer is a centimeter-scale atomic spin-exchange-relaxation-free (SERF) magnetometer. The $12.4 \times 16.6 \times 24.4$ mm$^3$ QuSpin magnetometer contains a $3 \times 3 \times 3$ mm$^3$ $^{87}\text{Rb}$ vapor cell with $\approx 10^{13}$ Rb atoms and a single laser for both optical pumping and probing. In contrast to our spin-based amplifier, the QuSpin magnetometer uses electron spins to detect and thus measure the real magnetic field (and not the pseudo-magnetic field measured by the spin-based amplifier). As shown in Fig. S1(a), the QuSpin magnetometer is placed on a plate made of polyether ether ketone (PEEK) and is located about 3 cm away from the vapor cell of the spin-based amplifier. The QuSpin magnetometer is sensitive to magnetic fields along $x$ and $y$, similar to the spin-based amplifier. This configuration guarantees that the spurious magnetic field experienced by the QuSpin magnetometer is almost the same as that seen by the spin-based amplifier. The
magnetic sensitivity of the magnetometer is about 20 fT/√Hz at frequencies between 3 and 100 Hz.

There are two reasons to use the QuSpin magnetometer. (a) The QuSpin magnetometer can be used to find the source of magnetic impurities. We use the QuSpin magnetometer to measure the field changes inside the magnetic shields when the rotor rotates at frequency $\nu$. The data-processing method of the QuSpin magnetometer is the same as that of spin-based amplifier (see Sec. V C). In initial experiments, a weak signal was detected at first harmonic with the QuSpin magnetometer. It was eventually traced to slightly magnetic screws used to mount the BGO crystal that were subsequently replaced. (b) The QuSpin magnetometer can be used to reduce the influence of magnetic field noise. If there is excess magnetic field noise detected with the QuSpin magnetometer (for example, due to incompletely shielded magnetic activity in the building), the data of the spin-based amplifier acquired at same time are vetoed.

II. NUMERICAL SIMULATIONS OF THE EXOTIC PSEUDO-MAGNETIC FIELDS

This section presents numerical simulations of the pseudo-magnetic fields generated by the exotic spin- and velocity-dependent interactions. The exotic interactions studied here are\[ V_{4+5} = -f_{4+5} \frac{\hbar^2}{8\pi mc} (\hat{\sigma} \cdot (\vec{\nu} \times \hat{r})) \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda}, \quad (S4) \]
\[ V_{12+13} = f_{12+13} \frac{\hbar}{8\pi} (\hat{\sigma} \cdot \vec{v}) \left( \frac{1}{r} \right) e^{-r/\lambda}, \quad (S5) \]
where $f_{4+5}, f_{12+13}$ are dimensionless coupling constant, $c$ is the speed of light in vacuum, $\hat{\sigma}$ is the spin vector and $m$ is the mass of the polarized fermion, $\vec{v}$ is the relative velocity between two interacting fermions, $\hat{r}$ is the unit vector in the direction between them, and $\lambda = \hbar (m_b c)^{-1}$ is the force range (or the boson Compton wavelength) with $m_b$ being the light boson mass. Figure S3 shows the relative position $\hat{r}$ and velocity $\vec{v}$ between the vapor cell of the spin-based amplifier and the rotating BGO crystal.

![FIG. S3. Schematic for the numerical simulation.](image)

Based on Eqs. (S4) and (S5), the exotic spin- and velocity-dependent interactions are predicted to be produced by moving unpolarized mass, such as a rotating bismuth germanate insulator [Bi$_4$Ge$_3$O$_{12}$ (BGO)] crystal (see Fig. S2). The exotic interactions induce energy shift of $^{129}$Xe spins in the vapor cell [29]

\[ -\mu_{Xe} \cdot \vec{B}_{j}^{exo} = V_j, \quad (S6) \]

where $\mu_{Xe}$ is the magnetic moment of $^{129}$Xe spins, $V_j$ represents the potential that we measure ($V_{4+5}$ or $V_{12+13}$) and $\vec{B}_{j}^{exo}$ is the exotic pseudo-magnetic field produced by the interaction $V_j$. In the following, we numerically simulate the exotic pseudo-magnetic fields $\vec{B}_{4+5}^{exo}$ and $\vec{B}_{12+13}^{exo}$ generated by the BGO crystal and the aluminum rod in our experiment.

A. Simulation of the pseudo-magnetic fields generated by the rotating BGO crystal

Based on Eq. (S6), the pseudo-magnetic fields $\vec{B}_{j}^{exo}$ produced by the spin-dependent interactions are generated by the rotating BGO crystal. Here, taking experimental parameters into consideration (see Fig. S3), we perform the numerical simulation of the
pseudo-magnetic fields $B_{4+5}^{\text{exo}}$.

1. Simulation of $V_{4+5}$

Here we consider the pseudo-magnetic field $B_{4+5}^{\text{exo}}$ generated by a single rotating BGO crystal. The pseudo-magnetic field can be obtained by integrating over the volume of the single BGO crystal (25.0 × 25.0 × 25.0 mm$^3$),

$$B_{4+5}^{\text{exo}} = f_{4+5} \frac{\hbar^2}{8\pi mc|\vec{\mu}_{\text{Xe}}|} \iiint \rho(\vec{r}) (\vec{\nabla} \times \vec{r}) \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda} dr,$$

(S7)

where $\rho(\vec{r})$ is the density of BGO crystal’s nucleons at location $\vec{r}$ and $m$ is the mass of a neutron. Compared to the distance between the spin-based amplifier and the rotor, the length of BGO crystal is sufficiently small. The “volume effect” of the BGO crystal can be neglected

$$B_{4+5}^{\text{exo}} \approx f_{4+5} \frac{\hbar^2}{8\pi mc|\vec{\mu}_{\text{Xe}}|} N (\vec{\nabla} \times \vec{r}) \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda},$$

(S8)

where $N$ is the total number of BGO crystal’s nucleons.

Based on Eq. (S7), the pseudo-magnetic field $B_{4+5}^{\text{exo}}$ is proportional to $f_{4+5}$. To clearly show the pseudo-magnetic field strength, we take $f_{4+5}^{\text{sim}} = 10^{-18}$ and $\lambda = 1.0$ m as an example to simulate the spectrum of $B_{4+5}^{\text{exo}}$. The pseudo-magnetic field should be along $y$ because vector $\hat{r}$ and $\vec{v}$ are in the $xz$ plane and their cross-product $\vec{v} \times \hat{r}$ is along $y$. Figure S4(a) shows that the pseudo-magnetic field $B_{4+5}^{\text{exo}} = B_{4+5}^{\text{sim}}$ is not a pure trigonometric function. Moreover, as shown in Fig. S4(b), $B_{4+5}^{\text{sim}}$ contains harmonic frequencies at $n, 2n, 3n, \ldots$. The ratios of the field strengths at harmonic frequencies are $B_{00}^{(1)} : B_{00}^{(2)} : B_{00}^{(3)} \approx 5.1 : 2.9 : 1.4$.

![Image](https://example.com/image1)

**FIG. S4. Simulations of $V_{4+5}$ produced by the rotating BGO crystal.** Signal from the simulated pseudo-magnetic field $B_{4+5}^{\text{exo}}$ is generated by the rotating BGO [bismuth germanate insulator Bi$_4$Ge$_5$O$_{12}$] crystal. We take $f_{4+5}^{\text{sim}} = 10^{-18}$ and $\lambda = 1.0$ m as an example to numerically simulate the pseudo-magnetic field $B_{4+5}^{\text{exo}}$. (a) Time-domain signal of $B_{4+5}^{\text{sim}}$. (b) Fourier transformation spectrum of $B_{4+5}^{\text{sim}}$.

In our experiment, by tuning the bias field $B_{4+5}^{\text{sim}}$, the operation frequency of the spin-based amplifier is matched to the first harmonic at $\nu$ to obtain the maximal signal from the pseudo-magnetic field. Due to the relatively narrow bandwidth $\approx 24$ mHz of the spin-based amplifier (see Sec. III B), only the first harmonic (i.e. $B_{00}^{(1)} \cos(2\pi \nu t) \hat{y}$) can be resonantly amplified and other harmonics can be neglected.

2. Simulation of $V_{12+13}$

The pseudo-magnetic field of $V_{12+13}$ can be similarly derived as

$$B_{12+13}^{\text{exo}} = -f_{12+13} \frac{\hbar}{8\pi |\vec{\mu}_{\text{Xe}}|} \iiint \rho(\vec{r}) (\vec{\nabla} \times \vec{r}) e^{-r/\lambda} dr,$$

(S9)

Compared to the distance between the spin-based amplifier and the rotor, the length of BGO crystal is sufficiently small. The “volume effect” of the BGO crystal can be neglected

$$B_{12+13}^{\text{exo}} \approx -f_{12+13} \frac{\hbar}{8\pi |\vec{\mu}_{\text{Xe}}|} \frac{\vec{v}}{r} e^{-r/\lambda}.$$

(S10)
In contrast to $\mathbf{B}_{\text{exo}}^{\text{ac}}$, due to vector $\mathbf{v}$ in $xz$ plane, the pseudo-magnetic field $\mathbf{B}_{12+13}^{\text{ac}} = B_{12+13}^{x}\hat{x} + B_{12+13}^{z}\hat{z}$ contains two components along $x$ and $z$. We note that the spin-based amplifier is insensitive to the oscillating field along $z$, because the oscillating field along $z$ can not induce $^{129}\text{Xe}$ transverse magnetization. Therefore, only $x$-component of the pseudo-magnetic field $B_{12+13}^{x}\hat{x}$ should be considered.

Based on Eq. (S9), the pseudo-magnetic field $B_{12+13}^{x}\hat{x}$ is proportional to $f_{12+13}$. To clearly show the pseudo-magnetic field strength, we take $f_{12+13}^{\text{sim}} = 10^{-33}$ and $\lambda = 1.0$ m as an example to numerically simulate the pseudo-magnetic field $B_{12+13}^{x}\hat{x}$. Figure S5(a) shows that the numerically simulated pseudo-magnetic field $B_{12+13}^{x}\hat{x}$ is not a pure trigonometric function. Moreover, as shown in Fig. S5(b), $B_{12+13}^{x}\hat{x}$ contains harmonic frequencies at $v, 2v, 3v, ...$. The ratios of the field strengths at harmonic frequencies are $B_{\text{ac}}^{(1)} : B_{\text{ac}}^{(2)} : B_{\text{ac}}^{(3)} \approx 5.5 : 1.7 : 0.5$.

Similarly, by tuning the bias field $B_{z}^{0}$, the operation frequency of the spin-based amplifier is matched to the first harmonic at $v$ to obtain the maximal signal from the pseudo-magnetic field. Due to the relatively narrow bandwidth $\approx 24$ mHz of the spin-based amplifier (see Sec. III B), only the first harmonic (i.e. $B_{\text{ac}}^{(1)} \cos(2\pi vt)\hat{x}$) can be resonantly amplified and other harmonics can be neglected.

B. Simulation of the pseudo-magnetic fields generated by the rotating aluminum rod

In above discussion, we do not consider the pseudo-magnetic fields generated by the rotating aluminum rod. The reason is that the pseudo-magnetic fields generated by the rotating BGO crystal and the rotating aluminum rod are spectrally separated. We explain the details as follows.

The 610.34-g aluminum rod $(487.6 \times 30.5 \times 15.2$ mm$^3$) contains $3.64 \times 10^{26}$ unpolarized nucleons, which can also induce the energy shift of $^{129}\text{Xe}$ spins. Therefore, we now analyze the pseudo-magnetic field generated by the rotating aluminum rod. We take $f_{4+5}^{\text{sim}} = 10^{-18}$, $f_{12+13}^{\text{sim}} = 10^{-33}$ and $\lambda = 1.0$ m as an example. Figure S6(a) and figure S6(c) shows the time-domain signal of the oscillating pseudo-magnetic fields $B_{4+5}^{\text{Al}}\hat{y}$ and $B_{12+13}^{\text{Al}}\hat{x}$ by our numerical simulation. Figure S6(b) and figure S6(d) shows the Fourier transformation spectrum of them. It is important to find that the pseudo-magnetic fields produced by the aluminum rod do not contain odd harmonics at $v, 3v, 5v, ...$ and only contain even harmonics at $2v, 4v, 6v, ...$. The reason is that the configuration of the rotating aluminum is central-symmetric around the rotation axis and axial-symmetric along $z$, as shown in Fig. S2. In one period, one end of the aluminum rod comes to the nearest position to the vapor cell twice, the pseudo-magnetic field reaches maximum twice, and the dominant frequency becomes $2v$.

In our experiment, the operation frequency of the spin-based amplifier is tuned to $v_0 \approx v$. Thus, it is not necessary to consider the pseudo-magnetic fields produced by the rotating aluminum, because it contains no harmonics at $v$.
The response is described by the coupled Bloch equations [40], the exotic spin- and velocity-dependent interactions $V_{4+5}$ and $V_{12+13}$ can produce pseudo-magnetic fields. The Fermi-contact interaction between $^{87}$Rb electron spins; and $^{129}$Xe nuclear spin-spin interaction [40, 46, 47] introduces an effective magnetic field $B_{\text{ac}}^{\text{eff}}$. Here, we only consider the case of a single-frequency magnetic field due to the narrow bandwidth of the spin-based amplifier ($\approx 24$ mHz). The response to the oscillating $x$ magnetic field is the same with the case of oscillating $y$ magnetic field.

Fig. S6. Simulations of $V_{4+5}$ and $V_{12+13}$ produced by the rotating aluminum rod. Signal of the pseudo-magnetic field is generated by the rotating aluminum rod. We take $f_{4+5} = 10^{-18}$, $f_{12+13} = 10^{-33}$ and $\lambda = 1.0$ m as an example to numerically simulate the pseudo-magnetic fields. (a) Time-domain signal of $B_{4+5}^{\text{Al}}\hat{y}$. (b) Fourier transformation spectrum of $B_{4+5}^{\text{Al}}\hat{y}$. (c) Time-domain signal of $B_{12+13}^{\text{Al}}\hat{x}$. (d) Fourier transformation spectrum of $B_{12+13}^{\text{Al}}\hat{x}$.

III. THE RESPONSE OF SPIN-BASED AMPLIFIER TO THE EXOTIC SPIN- AND VELOCITY-DEPENDENT INTERACTIONS

Based on Sec. II, the exotic spin- and velocity-dependent interactions $V_{4+5}$ and $V_{12+13}$ can produce pseudo-magnetic fields. In this section, we present the analysis of the response of our spin-based amplifier to such pseudo-magnetic fields.

We first derive the resonant response of the spin-based amplifier to the pseudo-magnetic field, for example, $B_{\text{ac}}^{\text{exo}} = B_{\text{ac}}^{\text{eq}} \cos(2\pi v t) \hat{y}$. Here, we only consider the case of a single-frequency magnetic field due to the narrow bandwidth of the spin-based amplifier ($\approx 24$ mHz). The response to the oscillating $x$ magnetic field is the same with the case of the oscillating $y$ magnetic field.

The spin-based amplifier employs $^{129}$Xe noble gas, which spatially overlaps with $^{87}$Rb gas in the same vapor cell, as shown in Fig. S7. The response is described by the coupled Bloch equations [40]

$$\frac{\partial \mathbf{P}^e}{\partial t} = \frac{\gamma_e}{Q}(B_{\text{ac}}^{\text{eq}} + B_{\text{ac}}^{\text{exo}} + \beta M_0^0 \mathbf{P}^e) \times \mathbf{P}^e + \frac{P_0^{\text{eq}} - P_e^e}{T_e Q},$$

$$\frac{\partial \mathbf{P}^n}{\partial t} = \gamma_n(B_{\text{ac}}^{\text{eq}} + B_{\text{ac}}^{\text{exo}} + \beta M_0^0 \mathbf{P}^e) \times \mathbf{P}^n + \frac{P_0^{\text{eq}} - P^n}{T_{2n}, T_{2n}, T_{1n}},$$

where $\mathbf{P}^e$ ($\mathbf{P}^n$) is the polarization of $^{87}$Rb electron ($^{129}$Xe nucleus); $\gamma_e$ ($\gamma_n$) is the gyromagnetic ratio of the $^{87}$Rb electron ($^{129}$Xe nucleus); $Q$ is the electron slowing-down factor originated from hyperfine interaction and spin-exchange collisions; $B_{\text{ac}}^{\text{eq}}$ is the applied bias field; $M_0^0$ ($M_0^n$) is the maximum magnetization of $^{87}$Rb electron ($^{129}$Xe nucleus) associated with full spin polarizations; $P_0^{\text{eq}}$ ($P_0^n$) is the equilibrium polarization of the $^{87}$Rb electron ($^{129}$Xe nucleus); $T_e$ is the common relaxation time of $^{87}$Rb electron spins; and $T_{1n}$ ($T_{2n}$) is the longitudinal (transverse) relaxation time of $^{129}$Xe spins.

The Fermi-contact interaction between $^{87}$Rb and $^{129}$Xe spins introduces an effective magnetic field [40, 46, 47]

$$B_{\text{eff}}^{\text{ex}} = \beta M_0^e \mathbf{P}^e,$$
Spin dynamics of the spin-based amplifier

In the operation of the spin-based amplifier, the effective magnetic field $B_{\text{eff}} \approx B_{ex}^0$ can be approximated as a constant, much larger than the effective field $B_{\text{ex}}^n$ and $P_{\text{ex}}^n$. Because the applied $B_{\text{ex}}^0$ is much larger than the effective field $\lambda M_{0z}^0 P_{\text{ex}}^0$ (on the order of nT), we can further neglect $\lambda M_{0z}^0 P_{\text{ex}}^0$ in Eq. (S12). Therefore, based on Eq. (S12), the $^{129}\text{Xe}$ spins evolve under the bias field $B_{\text{ex}}^0$. As a result, Eq. (S12) can be independently solved at first and $P^n$ can be evaluated. (2) Then, we can take the solution of $P^n$ into Eq. (S11) and solve for $P^n$.

We write the total magnetic field experienced by $^{87}\text{Rb}$ spins as $B = B_{\text{ex}}^0 + B_{\text{ex}}^n P^n$ and obtain the simplified Bloch equations,

$$\frac{\partial B^e}{\partial t} = \frac{\gamma_e B^e}{Q} \times \dot{P} + \frac{P^n_0 \hat{\beta} \times \dot{P}^e}{T_{\text{eQ}}},$$

$$\frac{\partial P^n}{\partial t} = \frac{\gamma_n}{T_2 \text{Rb}} \left( B_{z}^{0 \hat{\beta}} + B_{\text{ex}}^{0 \hat{\beta}} \right) \times \dot{P}^e + \frac{P^n_0 \hat{\beta} \times \dot{P}^e}{T_{2n} T_{2n} T_{1n}}.$$ (S14)

(S15)

We first solve for the evolution of $^{129}\text{Xe}$ spins under a bias field $B_{\text{ex}}^0$. The Larmor frequency of $^{129}\text{Xe}$ spins is $\nu_0 = \gamma_n B_{\text{ex}}^0 / (2\pi)$. We derive the steady-state solution of three components of $\dot{P}^e$ by solving Eq. (S16) in rotating frame and transforming back to the laboratory frame,

$$P_{\text{e}}^x = \frac{1}{2} P_{0}^n \gamma_n B_{\text{ex}}^{0 \hat{\beta}} T_{2n} \cos(2\pi v t) + \frac{2\pi(v - v_0) T_{2n}^2 \sin(2\pi v t)}{1 + (\gamma_n B_{\text{ex}}^{0 \hat{\beta}}/2)^2 T_{1n} T_{2n} + [2\pi(v - v_0)]^2 T_{2n}^2},$$

$$P_{\text{e}}^y = \frac{1}{2} P_{0}^n \gamma_n B_{\text{ex}}^{0 \hat{\beta}} T_{2n} \sin(2\pi v t) - \frac{2\pi(v - v_0) T_{2n}^2 \cos(2\pi v t)}{1 + (\gamma_n B_{\text{ex}}^{0 \hat{\beta}}/2)^2 T_{1n} T_{2n} + [2\pi(v - v_0)]^2 T_{2n}^2},$$

$$P_{\text{e}}^z = P_{0}^n \frac{\gamma_n}{1 + (\gamma_n B_{\text{ex}}^{0 \hat{\beta}}/2)^2 T_{1n} T_{2n} + [2\pi(v - v_0)]^2 T_{2n}^2}.$$ (S17)

(S18)

(S19)
According to \( \mathbf{B}_\text{eff}^n = \beta M_0^n \mathbf{P}^n \), we can derive the effective field experienced by \(^{87}\text{Rb}\) spins as

\[
\begin{align*}
\mathbf{B}_\text{eff}^n &= \frac{1}{2} \beta M_0^n P_0^n \gamma n B_{\text{ac}}^{\text{exo}} T_{2n} \cos(2\pi \nu T) + 2\pi (v - v_0) T_{2n}^2 \sin(2\pi \nu T) \times \\
&\quad \left( 1 + (\gamma n B_{\text{ac}}^{\text{exo}} / 2)^2 T_{1n} T_{2n} + [2\pi (v - v_0)]^2 T_{2n}^2 \right)^{1/2} \\
&\quad \left( \text{effective field generated by } ^{129}\text{Xe} x \text{ magnetization} \right) \\
&\quad + \frac{1}{2} \beta M_0^n P_0^n \gamma n B_{\text{ac}}^{\text{exo}} T_{2n} \sin(2\pi \nu T) - 2\pi (v - v_0) T_{2n}^2 \cos(2\pi \nu T) \times \\
&\quad \left( 1 + (\gamma n B_{\text{ac}}^{\text{exo}} / 2)^2 T_{1n} T_{2n} + [2\pi (v - v_0)]^2 T_{2n}^2 \right)^{1/2} \\
&\quad \left( \text{effective field generated by } ^{129}\text{Xe} y \text{ magnetization} \right).
\end{align*}
\tag{S20}
\]

A similar expression can be obtained for an oscillating pseudo-magnetic field \( \mathbf{B}_{\text{ac}}^{\text{exo}} = B_{\text{ac}}^{\text{exo}} \cos(2\pi \nu T) \hat{x} \) along \( x \).

Based on Eq. (S20), the effective field \( \mathbf{B}_\text{eff}^n \) consisting of two components along \( x \) and \( y \) can be measured in situ by the \(^{87}\text{Rb}\) magnetometer [40, 46]. We now derive the magnetic response of \(^{87}\text{Rb}\) magnetometer [see Eq. (S14)]. In our experiment, the \( x \) component of \(^{87}\text{Rb}\) polarization \( P_x^r \) is detected with a probe beam [see Eq. (S1)]. Thus, we only need to obtain the explicit expression of \( P_x^r \). Under a quasi-static field, the steady-state solution is

\[
P_x^r \propto \frac{B_x B_0^0 - B_0 \Delta B}{(B_0^0)^2 + (\Delta B)^2},
\]

where \( \Delta B = 1/(\gamma n T_x) \approx 2300 \) nT. In our experiment, a bias field \( B_0^0 = 423 \) nT is applied along \( z \). In this situation, the \(^{87}\text{Rb}\) magnetometer is simultaneously sensitive to the \( x, y \) magnetic fields generated by the effective \(^{129}\text{Xe}\) magnetic field \( \mathbf{B}_\text{eff}^n \), as shown in Fig. S7.

In the following, we derive the amplification factor and bandwidth of the spin-based amplifier through theoretical analysis and calibration experiments.

### A. Amplification factor

As seen in Eq. (S20), the pseudo-magnetic field \( \mathbf{B}_{\text{ac}}^{\text{exo}} \) can induce an oscillating \(^{129}\text{Xe}\) nuclear magnetization, generating a considerable effective magnetic field \( \mathbf{B}_\text{eff}^n \) on \(^{87}\text{Rb}\) spins. Surprisingly, the signal from \( \mathbf{B}_\text{eff}^n \) can be greatly larger than that from the oscillating pseudo-magnetic field \( \mathbf{B}_{\text{ac}}^{\text{exo}} \). To quantify the amplification effect, we define an amplification factor:

\[
\eta = |\mathbf{B}_\text{eff}^n|/|\mathbf{B}_{\text{ac}}^{\text{exo}}|.
\tag{S21}
\]

To derive the amplification factor \( \eta \) on resonance, we assume that the pseudo-magnetic field strength is small and thus the term \( (\gamma n B_{\text{ac}}^{\text{exo}} / 2)^2 T_{1n} T_{2n} \) can be neglected in Eq. (S20) (see Ref. [40]). In this situation, \( \mathbf{B}_\text{eff}^n \) can be written as

\[
\mathbf{B}_\text{eff}^n (v \approx v_0) = \frac{4\pi}{3} \kappa_0 M_0^n P_0^n \gamma n T_{2n} |\cos(2\pi \nu T) \hat{x} + \sin(2\pi \nu T) \hat{y}| B_{\text{ac}}^{\text{exo}}.
\tag{S22}
\]

Based on Eq. (S22), the effective field \( \mathbf{B}_\text{eff}^n \) is a circularly polarized field and its amplitude is equal to \( \frac{4\pi}{3} \kappa_0 M_0^n P_0^n \gamma n T_{2n} \cdot B_{\text{ac}}^{\text{exo}} \). Lastly, the amplification factor is

\[
\eta = \frac{4\pi}{3} \kappa_0 M_0^n P_0^n \gamma n T_{2n}.
\tag{S23}
\]

As seen in Eq. (S23), to achieve a large amplification factor, many experimental parameters \( \{ \kappa_0, M_0^n, P_0^n, T_{2n} \} \) are important. The \( \kappa_0 \) enhancement is only a part of the amplification factor, and is not enough to guarantee a significant amplification effect. Here \( P_0^n \) is the polarization of \(^{129}\text{Xe}\) nucleus; \( \gamma n \) is the gyromagnetic ratio of the \(^{129}\text{Xe}\) nucleus; \( M_0^n \) is the maximum magnetization of \(^{129}\text{Xe}\) nucleus associated with full spin polarizations; \( P_0^n \) is the equilibrium polarization of the \(^{129}\text{Xe}\) nucleus; \( T_{2n} \) is the transverse relaxation time of \(^{129}\text{Xe}\) spins.

To realize considerable amplification effect, long transverse relaxation time of nuclear spins needs to be achieved, high nuclear vapor density need to be required, and high polarization of nuclear spins needs to be realized. If these prerequisites are not satisfied, there would be no considerable amplification factor \( \eta \) to realize amplification effect of spin-based amplifiers. Therefore, the key new ingredient of the spin-based amplifier is the use of hyperpolarized long-lived nuclear spins, for example, \(^{129}\text{Xe}\), as a pre-amplifier to significantly amplify the signal from spin-dependent interactions. Specifically, the relaxation time of \(^{129}\text{Xe}\) nuclear spins are about \( > 20 \) s, the polarization of \(^{129}\text{Xe}\) nuclear spins can achieve \( > 30\% \), and the amplification factor \( \eta \approx 116 \).
can be achieved. On the other hand, for a small $\kappa_0 \approx 5.9$ enhancement of $^3$He-K system, the amplification factor can achieve $10^9$ (see Sec. VI A) due to the long relaxation time of $^3$He spins (on the order of 1000 s.)

After theoretical analysis, we introduce the experimental procedures to calibrate the amplification factor. The bias field $B_z^0$ is set as $\approx 423$ nT to tune the $^{129}$Xe Larmor frequency to $v_0 \approx 4.995$ Hz. A resonant oscillating field, here, $v \approx 4.995$ Hz is applied along $y$ with an arbitrary waveform generator (Keysight Model 33210A); the output signal of $^{87}$Rb magnetometer is an oscillating voltage signal and we record its amplitude as $A_{\text{Reson}}$. A far-off-resonant oscillating field is applied along $y$ at a frequency of $320$ Hz (the response is independent of frequency in a broad range) and we record the amplitude $A_{\text{FORes}}$ of the $^{87}$Rb magnetometer output signal. In this situation, we only need to consider $B_{ac,y}^0$ without considering $B_{\text{eff}}^n$. By comparing the above two amplitudes, we obtain the amplification factor

$$\eta \approx \frac{A_{\text{Reson}}}{A_{\text{FORes}}}.$$ (S24)

As described in the main text [see Fig. 2(a)], $\eta$ is calibrated to be $\approx 116$ in our $^{129}$Xe spin-based amplifier.

In the near future, further improvement of the search sensitivity can be anticipated using a $^3$He-K system (see Sec. VI A). Based on theoretical calculations [40], the amplification factor could be as large as $10^4$ in a $^3$He-K system. The sensitivity to the signal from $B_{ac}^{\text{xox}}$ can potentially reach a few aT/$\sqrt{\text{Hz}}$. Accordingly, it is possible to establish more stringent constraints on $f_{4,5}$, $f_{12,13}$ (see Sec. VI B) and other spin-dependent interactions, for example, the spin-spin-velocity interactions (see Sec. VI B 1) through the $^3$He-K system.

### B. Bandwidth

Up to now, we have only considered the amplification factor in the resonant case. Next, we analyse the amplification performance in the near-resonant case. Based on Eq. (S20), the amplification factor reaches maximum on resonance, and rapidly decreases when the oscillation frequency of applied magnetic field $v$ is far off-resonant. Therefore, there is a bandwidth for our spin-based amplifier. Based on Eq. (S20), we now determine the bandwidth. The effective magnetic field $|B_{\text{eff}}^n|$ can be rewritten in the amplitude spectrum,

$$|B_{\text{eff}}^n(v)| \propto \frac{\Lambda/2}{\sqrt{(v-v_0)^2 + (\Lambda/2)^2}}.$$ (S25)

We assume that the pseudo-magnetic field amplitude is weak and thus the term $(\gamma_nB_{ac}^{\text{xox}}/2)^2T_{1n}T_{2n}$ can be neglected in Eq. (S20). The full width at half maximum (FWHM) is $\sqrt{3}\Lambda$. Thus, the spin-based amplifier can enhance the signal in a corresponding frequency range, which is well suited for resonantly searching for exotic spin- and velocity-dependent interactions.

As a demonstration, we describe the bandwidth-calibration experiment. We set the bias field as $B_z^0 \approx 423$ nT. In a practical measurement, we usually do not know the exact Larmor frequency $v_0$ without prior calibration. In this situation, to find $v_0$, we scan the oscillating field frequency over a frequency range, corresponding to the narrow bandwidth of the spin-based amplifier. An oscillating field with strength $\approx 13$ pT is applied along $y$. The oscillating field frequency is automatically changed through an AWG (keysight 33210A). We fit the experimental data, find the maximum as the resonant signal amplitude $A_{\text{Reson}}$ and obtain the bandwidth $\sqrt{3}\Lambda \approx 24$ mHz of the spin-based amplifier based on Eq. (S25), as shown in Fig. 2(a) in the main text.

### IV. COMPARISON BETWEEN OUR WORK AND EXISTING TECHNIQUES

In this section, we would like to emphasize the differences between the spin-based amplifier and other existing techniques. The spin-based amplifier has been demonstrated by the recent work [40]. We show that it is quite different from, for example, comagnetometers and other resonant detection methods.

#### A. Comparison to the comagnetometer

Although the vapor cell (e.g., $^{87}$Rb-$^{129}$Xe cell) of the spin-based amplifier is similar to that of “self-compensating” comagnetometers [20, 21], our technique is quite different from them. We explain the detailed difference as follows.

In the operation of the spin-based amplifier, a bias magnetic field $B_z^0$ is applied to tune the $^{129}$Xe Larmor frequency to match the oscillation frequency of the measured magnetic field. The bias field $B_z^0$ typically ranges from $10^2$ to $10^5$ nT, corresponding to the measured frequency from 1 to 1000 Hz. In contrast, a “self-compensating” comagnetometer should operate at a specific
near-zero bias field $B_0^0$, which is equal and opposite to the sum of the effective magnetic fields of $^{87}\text{Rb}$ and $^{129}\text{Xe}$. In this situation, the $^{87}\text{Rb}$ Larmor frequency is equal to $^{129}\text{Xe}$ Larmor frequency, which is typically around 1 Hz.

Due to the different operation conditions between two techniques, the explored frequency range is different. The comagnetometer is usually used to search for low-frequency exotic signals (for example, 0.18 Hz [49]). Whereas the spin-based amplifier can be applied for searching signals ranging from 1 to 1000 Hz and thus is well suited to exploring the parameters of new physics in the high-frequency range. For example, the spin-based amplifier can be applied to search for the dark photon [6], axion-like particles [40], spin-dependent interactions [19, 49] and other exotic signals. In this work, we have demonstrated the application to a search for exotic velocity-dependent interactions, where the strength of the pseudo-magnetic field generated by the velocity-dependent interactions is proportional to the velocity.

In comagnetometers, $^{87}\text{Rb}$ and $^{129}\text{Xe}$ are strongly coupled to each other via Fermi-contact interactions, because their Larmor frequencies are equal. Due to the strong coupling, a “self-compensating” effect occurs in comagnetometers, leading to the comagnetometer being insensitive to the normal magnetic field. In contrast, the $^{87}\text{Rb}$ and $^{129}\text{Xe}$ are weakly coupled in the spin-based amplifier, which remains sensitive to the normal magnetic field. Specifically, the spin-based amplifier employs the hyperpolarized long-lived nuclear spins as an amplifier for the resonant signal from the external fields by a gain factor of $>100$. The spin-based amplifier can be further improved to attotesla-level sensitivity (see Sec. VI A).

B. Comparison to other resonant detection methods

Although both our work and other NMR resonant searches are aimed to measure the NMR magnetization generated by exotic spin-dependent interactions, the readout scheme of the NMR magnetization between our work and other searches is different. As shown in Fig. S8(a), previous works all consider the “remote” readout scheme, where the NMR magnetization is measured from a distance with atomic or SQUID magnetometers. For example, as proposed in Refs. [33, 35], the NMR magnetization induced by exotic interactions generates a dipole magnetic field and is measured with external atomic magnetometers. In contrast, our work uses a “contact” readout scheme in which the polarized nucleons and the detector are spatially overlapping in the same vapor cell. Due to the Fermi-contact interactions, the induced magnetization of $^{129}\text{Xe}$ generates a considerable effective magnetic field that can be measured in situ by atomic magnetometer [see Fig. S8(b)].

In the above two cases, the magnetic field at position $\mathbf{r}$ generated by NMR magnetization is

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{3(\hat{\mathbf{r}} \cdot \mathbf{m}) - \mathbf{m}}{r^3} + \frac{2\mu_0}{3} \mathbf{m} \delta(\mathbf{r}),$$  \hspace{1cm} (S26)

where $\mathbf{m}$ is the magnetic dipole moment. In our experiment, the “contact” term in Eq. (S26) is the $^{129}\text{Xe}$ effective magnetic field $\mathbf{B}_{\text{eff}}^{129}\text{Xe}$ described by the Fermi-contact interactions. This “contact” readout scheme offers a significant advantage: nuclear spin signals can be enhanced due to large Fermi-contact enhancement factor (on the order of 600), measured in situ with an atomic magnetometer (see Sec. III). In contrast, it is experimentally challenging to prepare high nuclear-spin polarization and maintain readout sensitivity for “remote” scheme.

FIG. S8. Two readout schemes of NMR magnetization. (a) “Remote” readout scheme. The nuclear sample and the atomic magnetometer are spatially separated. The NMR magnetization induced by the spin-dependent interactions is proposed to be read out by external atomic magnetometers. The magnetic dipole field measured is proportional to $r^{-3}$. (b) “Contact” readout scheme. The nuclear sample and the atomic magnetometer are spatially overlapping in the same vapor cell. The NMR magnetization induced by the spin-dependent interactions generates an effective magnetic field $\mathbf{B}_{\text{eff}}^{129}\text{Xe}$ read out in situ by atomic magnetometer.
V. DATA ANALYSIS

This section presents the detailed information of our data analysis to determine the constraints (95% C.L.) on \( f_{4+5}, f_{12+13} \) from search experiments. The data processing procedure consists of five parts, i.e., calibration experiments, search experiments, analysis of mean values and statistical errors, analysis of systematic errors, and extraction of the constraints. We explain the details as follows.

TABLE S1. Summary of experimental parameters and their corresponding systematic errors. The corrections to \( f_{4+5} \) at \( \lambda = 1.0 \) m are listed.

| Parameter                      | Value                      | \( \Delta f_{4+5} \left( \times 10^{-20} \right) \) |
|-------------------------------|-----------------------------|----------------------------------------------------|
| Mass of BGO (g)               | 112.34 ± 0.02 g            | ±0.0002                                            |
| Position of pivot x (mm)      | −3.4 ± 0.5                 | −0.0114 +0.0108                                    |
| Position of pivot y (mm)      | 6.0 ± 0.3                  | ±0.0001                                            |
| Position of pivot z (mm)      | 583.2 ± 1.1                | ±0.0051                                            |
| Length of aluminum rod (mm)   | 487.6 ± 0.7                | ±0.0025                                            |
| Phase delay \( \phi \) (deg)  | 79.1 ± 6.0                 | −0.6648                                           |
| Calib. const. \( \alpha \) (V/nT) | 1.91 ±0.06               | +0.6333 +0.9274                                    |
| Final \( f_{4+5} \left( \times 10^{-20} \right) \) | 1.08                      | ±2.48 (statistical)                                |
| \( (\lambda = 1.0 \) m)       |                            | ±0.69*                                             |

* The origin of coordinates was at the center of the vapor cell.

A. Calibration experiments

We first perform calibration experiments to determine all required parameters including mass of the BGO \( M \), parameters of the rotor \( x, y, z, L \), phase delay \( \phi \), calibration constant \( \alpha \), as shown in Table S1. We introduce them as follows.

1. Calibration of the mass of the BGO crystal. We used a commercial electronic scale to measure the mass \( (M) \) of the BGO crystal. The mass of the BGO crystal is measured to be

\[
M \approx 112.34 \pm 0.02 \text{ g.} \quad (S27)
\]

Accordingly, the total number \( N \) of the nucleons is estimated to be \( N \approx 6.707 \times 10^{25} \).

2. Calibration of the parameters of the rotor. We use a precise ruler to measure the position of the rotation center (the center of the aluminum rod). The origin of coordinates is at the center of the vapor cell. Figure S3 shows the position of the rotor in our coordinates. The position \( (x, y, z) \) of the rotation center is

\[
x \approx −3.4 \pm 0.5 \text{ mm}, \\
y \approx 6.0 \pm 0.3 \text{ mm}, \\
z \approx 583.2 \pm 1.1 \text{ mm}. \quad (S28)
\]

The length \( L \) of the aluminum rod is

\[
L \approx 487.6 \pm 0.7 \text{ mm}. \quad (S29)
\]

The position \( (x, y, z) \) and the length \( L \) are used to calculate the relative direction \( \hat{r} \) and the distance \( r \), which are used to calculate the pseudo-magnetic field based on Eqs. (S8) and (S10).

3. Calibration of the phase delay \( \phi \). The phase delay \( \phi \) represents the phase difference between the input oscillating magnetic field and the corresponding output signal of the spin-based amplifier. The phase delay \( \phi \) is important to determine the reference signal for the “lock-in” detection scheme (see Sec. VC and Eq. (S35)). Figure S9(a) shows the schematics of measuring \( \phi \). Specifically, a function generator (AWG) is connected to a set of saddle coils along \( y \) and applies an oscillating voltage to generate an oscillating magnetic field whose frequency is matched to the \( ^{129}\text{Xe} \) Larmor frequency. Then we simultaneously acquire the input oscillating voltage signal and the corresponding output signal [see Fig. S9(b)].
We fit two signals with sine waves to obtain the phase delay $\phi$. The experiment is repeated many times to determine the mean value and the standard deviation of $\phi$. The phase delay $\phi$ is finally determined as

$$\phi \approx 79.1 \pm 6.0 \text{ deg.}$$  \hspace{1cm} (S30)

(4) Determination of the calibration constant $\alpha$. The calibration constant is the coefficient that transfers the output signal (V) of the spin-based amplifier to the unknown pseudo-magnetic field (nT). To determine the calibration constant $\alpha$, a resonant oscillating field $B_{y} \cos(2\pi \nu t) \hat{y}$ is applied; we record the amplitude of the output signal as $A_{\text{Reson}}$. The calibration constant on resonance can be obtained

$$\alpha = \frac{A_{\text{Reson}}}{B_{y}^{v}} \approx 1.91 \text{ V/nT.}$$  \hspace{1cm} (S31)

We now determine the statistical error of $\alpha$. In our experiments, there are two factors leading to the fluctuation of the calibration constant $\delta \alpha$: (a) the intrinsic instability of the spin-based amplifier $\delta \alpha_{\text{int}}$; (b) the external instability of the rotor rotation frequency $\delta \nu$. We first determine the standard deviation $\delta \alpha_{\text{int}}$ caused by the intrinsic instability of the spin-based amplifier. Specifically, we apply an ac magnetic field along $y$ to the spin-based amplifier, and initially match the frequency of the applied oscillating field and $^{129}$Xe Larmor frequency. Then we continuously take data for one hour and compute the response over every 20 s interval. The fluctuation of the response signal strength (the peak-to-peak value) is about 2.9% for one hour. The corresponding fluctuation of the calibration factor is $\pm 2.9\% \times 1.91 \approx \pm 0.06 \text{ V/nT}$. Accordingly, the statistical error of the intrinsic instability is $\delta \alpha_{\text{int}} \approx 1.91 \pm 0.06 \text{ V/nT}$.

Second, we determine the standard deviation $\delta \alpha_{\text{ext}}$ caused by the external instability $\delta \nu$ of the rotor rotation frequency

$$\delta \alpha_{\text{ext}} = \alpha \left( \frac{\Lambda/2}{\sqrt{(\delta v)^2 + (\Lambda/2)^2}} - 1 \right).$$  \hspace{1cm} (S32)

To calibrate the instability of the rotation frequency $\nu$, a bronze needle is fixed on the titanium rod to trigger the optoelectronic pulses. Triggered optoelectronic pulses are acquired simultaneously with a 24-bit acquisition card. As shown in Fig. S10, the interval between the rising edges of neighbouring pulses are determined as a single period. We average all periods and obtain the corresponding rotation frequency

$$\nu \approx 4.995 \pm 0.004 \text{ Hz.}$$  \hspace{1cm} (S33)
FIG. S10. **The triggered optoelectronic pulses.** Time-domain signal of the triggered optoelectronic pulses measured by the optoelectronic switch. (a) Sequences of the triggered optoelectronic pulses with a frequency 4.995 Hz. (b) Interval for a single optoelectronic pulse is about 1.5 ms, which is negligible compared to the interval between two rising edges of two pulses.

Due to the narrow bandwidth (24 mHz) of the spin-based amplifier (see Sec. III B), the instability of the rotation frequency is the dominant fluctuation of the calibration constant, which reduces the amplification factor and the corresponding calibration constant $\alpha$. Based on Eq. (S32), the frequency difference $\delta \nu \approx 0.004$ Hz results in 14.6% of the fluctuation of the calibration constant $-14.6\% \times 1.91 \approx -0.28$ V/nT. Accordingly, the statistical error of the external instability $\delta \alpha_{\text{ext}} \approx 1.91^{0.06 - 0.28}$ V/nT.

Combining these two statistical errors in quadrature, we obtain the final calibration constant $\alpha$

$$\alpha \approx (1.91^{+0.06 - 0.28}) \text{ V/nT.}$$

Overall, we obtain all experimental parameters in second column of Table S1 by performing calibration experiments. Based on the experimental parameters, the pseudo-magnetic field can be obtained for the following data processing in Sec. V C and Sec. V D.

### B. Search experiments

After calibration experiments, we use the spin-based amplifier to search for the exotic spin-dependent interactions. The detailed procedure of search experiments consists of the following steps.

1. Set the rotation frequency of the rotor as $\nu \approx 4.995$ Hz for counterclockwise rotation. By adjusting the bias field $B_0$, the operation frequency of the spin-based amplifier is matched to the rotation frequency $\nu_0 \approx \nu \approx 4.995$ Hz. By optimizing the performance of the spin-based amplifier, the calibration factor $\alpha$ reaches a maximum $\approx 1.91$ V/nT for our experiments. The signal of triggered optoelectronic pulses and the output of the spin-based amplifier are simultaneously acquired for one hour.

2. Stop data acquisition and re-optimize the performance of the spin-based amplifier to the maximum calibration factor $\alpha \approx 1.91$ V/nT. Acquire search data for one hour by repeating step (1) and subsequently re-optimizing the spin-based amplifier. Finally, the search data is totally acquired for 5 hours for counterclockwise rotation at $\nu \approx 4.995$ Hz.

3. Change the rotation frequency to $\nu \approx \{4.114, 4.552\}$ Hz and repeat steps (1) and (2).

4. Reverse the rotation direction to clockwise and repeat the steps (1)-(3).

After the search experiments, we acquire five-hours data of counterclockwise cycles and five-hours data of clockwise cycles at each frequency $\nu \approx \{4.114, 4.552, 4.995\}$ Hz.
C. Analysis of mean values and statistical errors

Here we describe the procedure to extract the constraints on the coupling strength $f_{4+5}$ and $f_{12+13}$ from the experimental data. In our data processing, “lock-in” analysis detection scheme is used to extract the weak signal from the pseudo-magnetic fields with a known carrier frequency from noisy signal. Consider $f_{4+5}$ as an example. The procedure of extracting $f_{12+13}$ is the same. The procedure can be divided into three steps, as shown in Fig. S11.

![Diagram showing the procedure of analysis of mean values and statistical errors](image)

**FIG. S11. The procedures of analysis of mean values and statistical errors.**

(1) Step 1: Obtain the coupling strength $f_{n}$ at rotation frequency $v$ for counterclockwise cycles. Here, we take $v \approx 4.995$ Hz as an example to explain the detailed data processing procedure for one-hour data. (a) After performing Fourier transformation, the frequency spectrum of the experimental signal $S(t)$ are obtained. A filter around the resonance frequency is applied. After performing inverse Fourier transformation on the filtered spectrum, we obtain the filtered time-domain reference signal and separate them into segments of one period $S_i(t)$. (b) The normalized reference signal $\cos(2 \pi v t + \phi)$ of every individual period is simulated based on Eq. (S8) corresponding to $f_{4+5}^{\text{sim}} = 1$ for the force range $\lambda = 1.0$ m based on experimental parameters in Table S1. A filter around the resonance frequency is applied. (c) Based on “lock-in” analysis scheme, we can extract every experimental coupling strength $f_{4+5}^i$ for every period ($T \approx 200.2$ ms)

$$f_{4+5}^i = \frac{1}{\alpha B_{ac}^{(1)}} \frac{\int_0^T \cos(2 \pi v t + \phi) S(t) dt}{\int_0^T \cos^2(2 \pi v t + \phi) dt}, \quad \text{Eq. (S35)}$$

where $B_{ac}^{(1)}$ is the first harmonic corresponding to $f_{4+5}^{\text{sim}} = 1$ (see Sec. II A). Figure S12 shows the histogram of all the experimental coupling strengths obtained. The fit with Gaussian distribution to the histogram gives the mean value and the standard error of the coupling strength in the case of $v \approx 4.995$ Hz

$$f_{4+5}^{1 \text{h}}(4.995 \text{ Hz}) \approx (9.98 \pm 0.83 \text{ stat}) \times 10^{-19}, \quad \text{Eq. (S36)}$$

where 1 h represents one-hour data and CCW represents counterclockwise cycles. By repeating steps (a)-(c), five coupling strengths can be obtained using five one-hour data. By averaging the five coupling strengths and considering the


uncertainty propagation, we can obtain the coupling strength for five-hour data at \( \nu \approx 4.995 \text{ Hz} \) for counterclockwise cycles,

\[
f_{\text{CCW}}^\text{exp}(4.995 \text{ Hz}) \approx (8.25 \pm 0.35_{\text{stat}}) \times 10^{-19}.
\] (S37)

FIG. S12. **The histogram of the potential experimental coupling strength** \( f_{4+5}^\text{CCW} \). Distribution of the experimental coupling strength \( f_{4+5}^\text{CCW} \) of one-hour data at \( \nu \approx 4.995 \text{ Hz} \) for counterclockwise rotation. The red solid line is a fit to a Gaussian distribution. The average and the standard error of the coupling strength is \( f_{\text{CCW}}^\text{1 h}(4.995 \text{ Hz}) \approx (9.98 \pm 0.83_{\text{stat}}) \times 10^{-19} \). The \( \chi^2 \approx 1.07 \) represents a valid fitting.

(2) Step 2: We note that the mean value of the coupling strength \( f_{\text{CCW}}^\text{exp} \) is about \( 20\sigma \) from 0, as shown in Fig. S12. To separate the signal from pseudo-magnetic fields from the spurious signal, we perform the velocity-dependent experiments. Because the velocity \( \mathbf{v} \) is proportional to the rotation frequency \( \nu \) (i.e., \( |\mathbf{v}| = \pi L \nu \)), see Fig. S3, for simplicity, we use the rotation frequency \( \nu \) instead of the velocity \( \mathbf{v} \) in the following.

According to Eqs. (S8) and (S10), the signal from the pseudo-magnetic field generated by the exotic interactions should be proportional to the rotation frequency \( \nu \). Accordingly, the output signal \( S_{\text{exp}} \) of the spin-based amplifier should be

\[
S_{\text{exp}} = k \cdot f_{\text{CCW}}^\text{exp} \cdot \nu,
\] (S38)

where the constant \( k \) can be calculated based on Eq. (S8) for \( f_{4+5} \) and Eq. (S10) for \( f_{12+13} \). Although \( k \) is shown in Eq. (S38), there is no necessity to obtain the specific value of \( k \) in our following data processing. In actual experiment, the output signal of the spin-based amplifier \( S_{\text{exp}} \) should contain two parts: the exotic signal from the pseudo-magnetic field \( S_{\text{exo}} \) and spurious signal \( S_{\text{spu}} \),

\[
S_{\text{exp}} = S_{\text{exo}} + S_{\text{spu}}.
\] (S39)

The spurious signal can be modeled as

\[
S_{\text{spu}} \approx S_0 + S_1 \nu + O(\nu^2),
\] (S40)

where \( S_0 \) characterizes the velocity-independent spurious signal and \( S_1 \nu \) characterizes the first order of the velocity-dependent spurious signal. The velocity-independent spurious signal can exist due to, for example, the intrinsic fluctuation of the spin-based amplifier (e.g., the fluctuation of the laser or temperature variations). The velocity-dependent spurious signal can result from the rotation of the rotor, for example, the residual signal from mechanical vibration or the air-vibration. In our experiment, the velocity-independent spurious signal is dominant. Based on Eq. (S39), we can further obtain the expression of the coupling strength

\[
f_{\text{CCW}}^\text{exp} = \frac{S_{\text{exp}}}{k \nu} = \frac{S_{\text{exo}} + S_{\text{spu}}}{k \nu} = f_{\text{CCW}} + \frac{S_0}{k \nu},
\] (S41)

where \( f_{\text{CCW}} = f_{4+5} + S_1 / k \) is the coupling strength to be extracted. Here, we use Eqs. (S8) and (S10) to simplify Eq. (S41) and obtain explicit expression of \( f_{\text{CCW}} \). As seen in Eq. (S41), the deviation of the mean value of the coupling strength \( f_{\text{CCW}}^\text{exp} \) from \( 1 \sigma \) can be caused by the spurious signal.
Based on Eq. (S41), we show that the velocity-independent spurious signal can be greatly eliminated by fitting the couplings strengths at different rotation frequencies. Specifically, we perform velocity-dependent experiments (see Sec. V B) by changing the rotation frequency \( \nu \). The experimental data are acquired at different rotation frequencies \( \nu_0 \approx \nu \approx \{4.114, 4.552, 4.995\} \) Hz. The corresponding coupling strengths \( f_{\text{CCW}}^{\text{exp}} \) are presented as black circles with error bars in Fig. S13. The red line is the theoretical fit of the experimental data based on Eq. (S41),

\[
f_{\text{CCW}}^{\text{exp}} \approx (-2.19 + \frac{52.44 \text{ Hz}}{\nu}) \times 10^{-19}.
\]

(S42)

Based on the fit, we can extract the coupling strength \( f_{\text{CCW}} \approx (-2.19 \pm 0.35_{\text{stat}}) \times 10^{-19} \) for counterclockwise rotation.

![Graph showing velocity dependence of the coupling strength](image)

FIG. S13. Velocity dependence of the coupling strength \( f_{\text{CCW}}^{\text{exp}} \) for \( V_{4+5} \). The extracted coupling strength \( f_{\text{CCW}}^{\text{exp}} \) for \( V_{4+5} \) at three rotation frequencies \( \{4.114, 4.552, 4.995\} \) Hz. The error bars are due to the statistical errors, as seen in Eq. (S41).

(3) Step 3: To further eliminate the velocity-dependent spurious signal, we compare the results \( f_{\text{CW}} \) and \( f_{\text{CCW}} \) between clockwise and counterclockwise mass rotations. Based on Eq. (S41), the basic idea is that the sign of the coupling strength \( f_{4+5} \) is the same for clockwise and counterclockwise rotations, while the sign of the spurious signal \( S_1/k \) is reversed. In above data processing, the “lock-in” detection scheme is used to extract the signal from spin-dependent interactions with the reference frequency. The coupling strength \( f_{4+5} \) is actually consistent with the reference signal. The reference signal of clockwise and counterclockwise cycles are different for the direction of the velocity \( \bar{\nu} \) based on Eqs. (S8) and (S10). As seen in Eq. (S41), if the velocity-dependent spurious signal \( S_1/k \) dominates, we should observe that \( f_{\text{CCW}} \) and \( f_{\text{CW}} \) have reversed sign. Indeed, we experimentally observe that there is a reversed sign for the coupling strength of clockwise cycles \( f_{\text{CW}} \approx (2.41 \pm 0.34_{\text{stat}}) \times 10^{-19} \) and counterclockwise cycles \( f_{\text{CCW}} \approx (-2.19 \pm 0.35_{\text{stat}}) \times 10^{-19} \). To eliminate the spurious signal, the final coupling strength is obtained by averaging clockwise and counterclockwise cycles

\[
f_{4+5} = \frac{f_{\text{CW}} + f_{\text{CCW}}}{2},
\]

(S43)

where CW represents the clockwise rotation. Based on Eq. (S43), the final coupling strength is \( f_{4+5} \approx (1.08 \pm 2.48_{\text{stat}}) \times 10^{-20} \) for the force range \( \lambda = 1.0 \) m, as shown in Table S1. In the same way, we obtain the final \( f_{12+13} \approx (1.13 \pm 2.24_{\text{stat}}) \times 10^{-35} \) for the force range \( \lambda = 1.0 \) m.

D. Analysis of systematic errors

In the data processing above, we have determined the mean value and the standard deviation of the parameters and thus obtain the mean value and statistical errors of the coupling strength. Subsequently, we determine the error of the coupling strength caused by the systematic errors of those experimental parameters (see Table S1) Here, we take the calibration constant \( \kappa \) as an example. First, we obtain the reference signal for the “lock-in” analysis scheme, as seen in Eq. (S35). Instead of using the mean values of the experimental parameters (including the calibration constant 1.91 V/nT in Sec. V C, the reference signal is obtained by using the upper limit of the calibration constant 1.97 V/nT and mean values of other parameters based on Eqs. (S8) and (S10). Second, using that reference signal, we employ the “lock-in” analysis scheme to extract the corresponding coupling strength \( f_{4+5}^{\text{cw}} \). Finally, the difference \( \Delta f_{4+5}^{\text{cw}} = f_{4+5}^{\text{cw}} - f_{4+5} \approx -0.027 \times 10^{-20} \) is determined as the systematic error caused by the upper limit of the calibration constant. In the same way, we can determine the systematic error \( \Delta f_{4+5}^{\text{cw}} = f_{4+5}^{\text{cw}} - f_{4+5} \approx 0.18 \times 10^{-20} \).
caused by the lower limit of the calibration constant 1.63 V/nT. The systematic errors caused by other calibration parameters in Table S1 are determined based on the same procedure. The overall systematic error ±0.69\text{sys} × 10^{−20} is derived by combining all the systematic errors in quadrature. The final coupling strength is \( f_{4+5} \approx (1.08 ± 2.48\text{stat} ± 0.69\text{sys}) \times 10^{-20}. \) In the same way, we obtain \( f_{12+13} \approx (1.13 ± 2.24\text{stat} ± 0.80\text{sys}) \times 10^{-35}. \) In our data processing, the lower terms of amplitude modulation at frequency \( \nu \) is considered and the spurious signal from them is eliminated.

FIG. S14. The procedures of determining the systematic errors. The left part represents the average value and standard deviation of the parameters. The middle part represents the data processing (see Fig. S11). The right part represents the determined systematic errors.

E. Extraction of the constraints

After the data processing above, we obtain the coupling strength for the force range, for example, \( \lambda = 1.0 \text{ m}. \) According to 95% confidence level (corresponding to 1.96\( \sigma \)), the constraints on \( f_{4+5} \) can be determined as

\[
(1.08 + 1.96 \times \sqrt{2.48^2\text{stat} + 0.69^2\text{sys}}) \times 10^{-20} \approx 6.13 \times 10^{-20}.
\]

In the same way, the constraints on \( f_{12+13} \) for the force range \( \lambda = 1.0 \text{ m} \) can be determined as \( 5.79 \times 10^{-35}. \) By changing force range \( \lambda \) and repeating data processing in Secs. V C and V D, we obtain the constraints in the entire explored region, as shown in Fig. 4 in main text. Using the spin-based amplifier, we establish constraints on the spin- and velocity-dependent interactions between polarized and unpolarized nucleons in the force range of 0.03-100 m. For \( f_{4+5}, \) our work sets the most stringent constraints on \( f_{4+5} \) for the force range from 0.04 m to 100 m. For \( \lambda = 1.0 \text{ m}, \) our work improves over existing constraints by about four orders of magnitude [41, 42]. Moreover, our work sets the most stringent constraints on \( f_{12+13} \) for the force range from 0.05 m to 6 m. For \( \lambda = 0.45 \text{ m}, \) our result improves over previous laboratory limits by more than two orders of magnitude [43, 44].

VI. PROJECTED CONSTRAINTS ON THE SPIN-DEPENDENT INTERACTIONS

In this section, we discuss the further improvement of the experimental sensitivity to a wide range of exotic spin-dependent interactions. To do this, we first show that the magnetic sensitivity of the spin-based amplifier can be further improved. We then discuss the search for the spin-spin-velocity interactions. Using the spin-based amplifier, the constraints on such spin-dependent interactions could reach well beyond state-of-the-art ones.

A. Improved sensitivity of the spin-based amplifier

A further improvement of the experimental sensitivity to exotic spin-dependent interactions is anticipated. There are two main factors that limit the sensitivity of our searches for the spin-dependent interactions: (1) the amplification factor of the spin-based amplifier, and (2) the magnetic sensitivity of the alkali-metal magnetometer itself. Using experimentally realistic
tools, we anticipate that the experimental sensitivity to the spin-dependent interactions can be further improved by several orders of magnitude. For example, $^3$He-K system is a well-motivated candidate as a spin-based amplifier, because $^3$He spins have much longer coherence time ($T_2 \sim 1000$ s) and larger gyromagnetic ratio than those of $^{129}$Xe. In the following, we estimate the corresponding amplification factor and the spin-dependent interactions search sensitivity.

The use of $^3$He-$^{39}$K to the spin-based amplifier can promisingly achieve larger amplification factor as discussed below. To calculate the amplification factor, we need to calculate the term $\beta M'_6 P'_0$. Using the SI units, $\beta M'_6 P'_0$ can be expressed as [45, 52]

$$\frac{2\mu_0}{3} \kappa_0 M'_6 P'_0 = \frac{2\mu_0}{3} \kappa_0 g \mu B'_0 [\text{N}], \quad (S45)$$

where $\kappa_0$ is the enhancement factor for the given alkali metal and noble gas pairs (for example, for $^3$He and K, $\kappa_0 \approx 5.9$), $g$ is the Landé factor for noble gas nucleus, $P'_0$ is the polarization of noble gas nucleus, $[\text{N}]$ is the number density of the noble gas, and $\mu = 1.07 \times 10^{-26}$ J/T is the magnetic moment of $^3$He nucleus. We use Eq. (S45) to numerically calculate the $^3$He effective magnetic field and the corresponding amplification factor. Based on the demonstrated experimental parameters in Ref. [48], $P'_0 \approx 0.02$ is the typical polarization of the $^3$He gas, $[\text{He}] \approx 1.88 \times 10^{26}$ m$^{-3}$ corresponds to the number density with 7 amg $^3$He. We obtain $\beta M'_6 P'_0 \approx 2.0$ mG, which is in good agreement with 2.2 mG demonstrated in ref. [48]. According to Eq. (S23), the amplification factor of $^3$He-K spin-based amplifier is estimated to be $\eta \approx 10^4$. Compared to $^{129}$Xe-$^{87}$Rb systems, the amplification factor of $^3$He-K systems yields about two orders of magnitude improvement. In addition to the amplification factor, K magnetometer has already demonstrated higher magnetic sensitivity than that of Rb magnetometers [20, 21]. Overall, the projected magnetic sensitivity using $^3$He systems can be improved by four orders of magnitude and achieve $\approx 1 \text{ aT}/\sqrt{\text{Hz}}$.

B. Projected constraints on the velocity-dependent interactions

1. Projected constraints on the spin- and velocity-dependent interactions

In the main text, we present new constraints on $f_{4+5}$ and $f_{12+13}$ using a $^{129}$Xe-$^{87}$Rb amplifier. Based on a $^3$He-K system and similar setup of the rotor, the projected constraints on spin- and velocity-dependent interactions can be improved by four orders of magnitude compared to this work

$$|f_{4+5}| < 10^{-24}; |f_{12+13}| < 10^{-35}. \quad (S46)$$

These projected constraints show the potential of spin-based amplifiers for future searches for spin- and velocity-dependent interactions.

2. Projected constraints on the spin-spin-velocity interactions

In the future, it would be possible to investigate the spin-spin-velocity interactions between neutrons and electrons by using our spin-based amplifier and SmCo$_5$ electron-spin sources. We show that the projected constraints on spin-spin-velocity interactions could set the most stringent limits than previous work.

With a non-zero relative speed between the electron spin source and our spin-based amplifier, the pseudo-magnetic field is generated by the spin-spin-velocity interactions [19, 37],

$$V_{6+7} = -f_{6+7} \frac{\hbar^2}{4\pi \mu_0 c} \times \left[ (\hat{\sigma}_1 \cdot \hat{\nu}) (\hat{\sigma}_2 \cdot \hat{\nu}) \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda} \right], \quad (S47)$$

$$V_8 = f_8 \frac{\hbar}{4\pi c} (\hat{\sigma}_1 \cdot \hat{\nu}) (\hat{\sigma}_2 \cdot \hat{\nu}) \left( \frac{1}{r} \right) e^{-r/\lambda}, \quad (S48)$$

$$V_{14} = f_{14} \frac{\hbar}{4\pi} (\hat{\sigma}_1 \times \hat{\sigma}_2) \cdot \hat{\nu} \left( \frac{1}{r} \right) e^{-r/\lambda}, \quad (S49)$$

$$V_{15} = -f_{15} \frac{\hbar^3}{8\pi \mu_0 m_2 c^2} \times \left[ (\hat{\sigma}_1 \cdot (\hat{\nu} \times \hat{p})) (\hat{\sigma}_2 \cdot \hat{p}) + (\hat{\sigma}_1 \cdot \hat{p}) (\hat{\sigma}_2 \cdot (\hat{\nu} \times \hat{p})) \right] \times \left( \frac{1}{\lambda^2 r} + \frac{3}{\lambda r^2} + \frac{3}{r^3} \right) e^{-r/\lambda}; \quad (S50)$$

$$V_{16} = -f_{16} \frac{\hbar^2}{8\pi \mu_0 c^2} \times \left[ (\hat{\sigma}_1 \cdot (\hat{\nu} \times \hat{p})) (\hat{\sigma}_2 \cdot \hat{p}) + (\hat{\sigma}_1 \cdot \hat{p}) (\hat{\sigma}_2 \cdot (\hat{\nu} \times \hat{p})) \right] \times \left( \frac{1}{\lambda^2 r} + \frac{1}{\lambda r^2} + \frac{1}{r^3} \right) e^{-r/\lambda}. \quad (S51)$$

One can use the iron-shielded SmCo$_5$ magnets as the electron spin sources. Assuming the same cylindrical iron-shielded SmCo$_5$ magnets with characteristic dimensions of 4 cm as used in Ref. [19], the number of the net electron spins is about $\approx 1.8 \times 10^{24}$. 
Two spin sources will be used to generate the spin-dependent interactions. We envision the center of two spin sources located 0.3 m below and 1 m away from the center of the spin-based amplifier. The spin sources will rotate around the axis with 2 Hz frequency in $yz$ plane with the rotation diameter about 0.3 m. Under these conditions, we can estimate the projected constrains on these spin-spin-velocity interactions

$$|f_{6+7}| < 10^{-20}, |f_8| < 10^{-23}, |f_{14}| < 10^{-31}, |f_{15}| < 10^{-4}, |f_{16}| < 10^{-4}. \quad (S52)$$

The sensitivity to $|f_{6+7}|$, $|f_{14}|$, $|f_{15}|$ and $|f_{16}|$ can potentially reach into unexplored parameter space for the force range from 0.01 m to 100 m, while for $|f_8|$, this is the case for the force range from 0.01 m to 1 m. As for the $^3$He-K system, due to the improved sensitivity, the projected constraints can achieve

$$|f_{6+7}| < 10^{-24}, |f_8| < 10^{-27}, |f_{14}| < 10^{-35}, |f_{15}| < 10^{-8}, |f_{16}| < 10^{-8}. \quad (S53)$$

Therefore, our spin-based amplifier can potentially be applied into searches for spin-spin-velocity interactions.

In conclusion, although demonstrated for the spin- and velocity-dependent interactions, the spin-based amplifier is also well suited to searching for other exotic interactions.