Best median values for cosmological parameters

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Our procedure to obtain best values for cosmological parameters from five recent multiparameter fits is as follows. We first study the values quoted for $r$, $\alpha_v$, $w+1(\omega_0+1)$, $w_1$ and $\Omega_k$, arriving at the conclusion that they do not differ significantly from zero, and their correlations to other parameters are insignificant. In what follows they can be therefore ignored. The neutrino mass sum $\Sigma m_v$ also does not differ significantly from zero, but since neutrinos are massive their sum must be included as a free parameter. We then compare the values obtained in five large flat-space determinations of the parameters $\Sigma m_v$, $\omega_b$, $\omega_m$, $h$, $\tau$, $n_s$, $A_s$ and $\sigma_8$. For these we compute the medians and the 17-percentile and 83-percentile errors by a described procedure.

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I. INTRODUCTION

Since July 2004 five large analyses [1, 2, 3, 4, 5] of cosmological parameters have appeared (we call them papers P1, P2, P3, P4 and P5) based on partly overlapping data. They represent a wealth of new information which, however, may appear contradictory to the outsider and difficult to digest. Since each analysis determines parameter values separately for different subsets of data and for several choices of parameter spaces, the results are legio. For instance, these five papers quote altogether 56 different values for the density of matter in the Universe, $\omega_m$. The parameter values exhibit notable differences due to differing choices of priors, methods of analysis, simulation variance and statistical fluctuations. The purpose of this review is to compare these five analyses, and to try to extract as reliable values as possible. Although the authors of the analyses may be unlikely to believe in the results of anyone else, theoreticians do need recommended values.

II. DATA SETS

All analyses quote results from fits to different subsets and combinations of data sets. This is of course of importance in order to test different sets for consistency or for correlations. However, since the purpose of the present analysis is to obtain as accurate information about parameter values as possible, we consider only fits to maximal data sets.

All the five papers P1–P5 use the CMB $\langle TT \rangle$ and $\langle TE \rangle$ power spectra on large angular scales from the first year Wilkinson Microwave Anisotropy Probe (WMAP) observations. P1 [1] combines this with the constraints from the Sloan Digital Sky Survey (SDSS) galaxy clustering analysis [3], the SDSS galaxy bias analysis [8], and the SNIa constraints [10]. A distinctive feature of the P1 analysis is the inclusion of Ly-\(\alpha\) forest data [11]. P1 mentions explicitly that inclusion of power spectra from the CMB observations on small angular scales by the Cosmic Background Imager (CBI) in

2004 [12], the Very Small Array (VSA) [13], and the Arcminute Cosmology Bolometer Array Receiver (ACBAR) [14] do not affect their results much, nor does the inclusion of the 2 degree Field Galaxy Redshift Survey (2dFGRS) [15] galaxy clustering power spectrum.

P2 [2] uses an SDSS "Main sample" [16] of galaxy clustering data which is of earlier date than the sample used in P1. The distinctive feature of the P2 analysis is the inclusion of spectroscopic data for a recent sample of 46 748 luminous red galaxies (LRG) in the range $0.16 < z < 0.47$ which show a significant acoustic peak in the redshift-space two-point correlation function (related to the galaxy power spectrum). P2–P5 do not include Ly-\(\alpha\) forest data, some of them mentioning that the information is still susceptible to systematic errors.

P3 [3] uses in addition to WMAP [3, 7] the following sets of CMB data: the BOOMERANG $\langle TT \rangle$ power spectra, the BOOMERANG $\langle TE \rangle + \langle EE \rangle + \langle BB \rangle$ polarization spectra, and the $\langle TT \rangle$ power spectra of DASI [17], VSA [13], ACBAR [14], MAXIMA [18], and CBI 2004 data [12]. The shapes of the SDSS and 2dFGRS power spectra [3, 15] are used as constraints, but not the amplitudes. In addition the SNIa "gold" set [10] is included.

P4 [4] relies on either the 2dFGRS [15] or the SDSS [8] power spectrum of galaxy clustering, the WMAP data [3, 7], but in contrast to P1 they also use the $\langle TT \rangle$ power spectra of VSA [13], ACBAR [14], and CBI 2004 data [12]. We use the parameter values derived from 2dFGRS data rather than SDSS, because P4 represents 2dFGRS.

P5 [5] analyzes exclusively CMB data: $\langle TT \rangle$ from [3, 7, 12, 17] and in particular all the $\langle TE \rangle + \langle EE \rangle$ polarization data now available [3, 8, 10, 17]. They do not use LSS constraints.

It would have been interesting to include also the recent analysis of cosmic shear [19]. However this would not have been consistent because the parameter space analyzed is quite different from that of papers P1 – P5.
III. PARAMETER SPACES

The most general parameter space explored in the five papers is 13-dimensional

\[ p = (\omega_b, \omega_m, \Omega_k, \Sigma m_\nu, \tau, h, \sigma_8, b, w, n_s, A_s, \alpha_s, r) . \]

The normalized matter density parameter \( \Omega_m \) does not appear explicitly in this list because it is given by \( \Omega_m = \omega_m / h^2 \) where \( h \) is the Hubble parameter in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). Similarly, \( \Omega_b = \omega_b / h^2 \) is the normalized baryon density parameter. One has \( \omega_m = \omega_{dm} + \omega_b \) and hence \( \Omega_m = \Omega_{dm} + \Omega_b \), where \( \Omega_{dm} = \omega_{dm} / h^2 = \Omega_{cdm} + \Omega_\nu \) is the density parameter of dark matter, sometimes denoted \( \Omega_c \) for CDM. The density parameter of dark energy is then \( \Omega_\Lambda = 1 - \Omega_m - \Omega_k \), where \( \Omega_k \) is the curvature or vacuum density. The ratio of pressure to energy density for dark energy is \( w = w_0 + w_1 (1 - a) \), where \( a \) is the scale parameter of the Universe. \( \sigma_8 \) is the rms linear mass perturbation in \( 8 h^{-1} \) Mpc spheres.

The fraction of dark matter that amounts to neutrinos is \( \frac{\Omega_\nu}{\Omega_{dm}} \). Assuming that the neutrinos are Majorana particles with standard freeze-out, the sum of the three neutrino masses is \( \Sigma m_\nu = 94.4 \Omega_\nu f_\nu \text{ eV} \).

The parameters \( \tau, n_s, \alpha_s, A_s, r, b \) describe fluctuation properties: the scalar spectral index is \( n_s \), the scalar amplitude is \( A_s \), the tensor spectral index \( n_t \) is not included because there is no information, the ratio of tensor to scalar amplitudes is \( r = A_t / A_s \), and the running of \( n_s \) with \( k \) is \( \alpha_s \). The parameter \( \tau \) measures the Thomson scattering optical depth to decoupling, and \( b \) is the bias factor describing the difference in amplitude between the galaxy power spectrum and that of the underlying dark matter. For \( b \) no value is quoted.

IV. METHOD OF ANALYSIS

Since the data sets are to a large extent overlapping they are not independent. Moreover, parts of the reported errors are systematic and not stochastic, so different data sets cannot be summarized using statistical methods, either frequentist or Bayesian. The breakdown into data subsets differing in methods of data sampling, statistical analysis, and simulation demonstrate notable variations in the parameter values (see for example Chu et al. \[20\]). We clearly want to use results from as large data compilations as possible, not from subsets.

Many parameters are determined with quite low precision, the published 1\( \sigma \) errors are large, the correlations likewise insignificant, and the dependences on the choices of data subsets, priors, and parameter spaces are statistically weak. The tendency in the cosmological literature is to draw optimistic conclusions from 1\( \sigma \) and 2\( \sigma \) confidence regions obtained by marginalization in many-parameter spaces. We think that parameter estimates from the five data sets, at their present state of accuracy, can well be combined without sophisticated methods. We then proceed as follows.

| TABLE I: Comparison of empirically defined errors |
|-------------------------------------------------|
| \( a_1 \pm \delta a_1 \) | \( a_2 \pm \delta a_2 \) | Our method | \( |a_2 - a_1|/2 \) |
| \( 1 \pm 5.0 \) | \( 2 \pm 5.0 \) | 1.5 \( \pm 4.80 \) | 1.5 \( \pm 5.5 \) |
| \( 1 \pm 1.0 \) | \( 2 \pm 1.0 \) | 1.5 \( \pm 1.08 \) | 1.5 \( \pm 1.5 \) |
| \( 1 \pm 0.5 \) | \( 2 \pm 0.5 \) | 1.5 \( \pm 0.71 \) | 1.5 \( \pm 1.0 \) |
| \( 1 \pm 0.2 \) | \( 2 \pm 0.2 \) | 1.5 \( \pm 0.58 \) | 1.5 \( \pm 0.7 \) |
| \( 1 \pm 0.5 \) | \( 2 \pm 1.0 \) | 1.33 \( ^{+1.09}_{-0.67} \) | |
within the quoted 1σ errors, thus \( r \) is not significantly correlated to the basic parameters. The largest correlation is with \( n_s \), still an increase of only about 1σ (this is discussed in detail in P1). When any of the parameters \( \alpha_s \), \( \Sigma m_\nu \), \( w \) is allowed to vary, the parameter space becomes 8- or 9-dimensional resulting in a further deterioration of the information on the basic parameters as well as on \( r \).

P3 quotes \( r < 0.36 \) and P4 quotes \( r < 0.41 \) from fits in the same 7-parameter space. The inclusion of \( r \) as a free parameter degrades slightly the information on some basic parameters. As in the case of P1, \( r \) is not significantly correlated to any other parameter except slightly to \( n_s \), at the level of about 1σ.

P2 and P5 do not quote any value for \( r \). Thus there is no significant evidence for \( r \neq 0 \), only for a commonly acceptable limit \( r < 0.4 \).

The parameter \( \alpha_s \) is determined by P1 and P3. P1 quotes fits in two different parameter spaces:
\[
\begin{align*}
& r < 0.45, \quad \alpha_s = -0.006^{+0.012}_{-0.011}, \quad w = -1, \quad \text{and} \\
& r < 0.45, \quad \alpha_s = -0.011 \pm 0.012, \quad w = -0.91 \pm 0.09.
\end{align*}
\]
In neither case is \( \alpha_s \) significantly different from zero.

P3 quotes \( \alpha_s = -0.051^{+0.027}_{-0.026} \) when \( w = -1 \) in rather marked disagreement with P1. Because of this conflict we choose not to combine it with P1. The effect of including \( \alpha_s \) in the P1 and P3 fits increases the errors of the basic parameters considerably. In P1 no significant correlations are found, in P3 there are some correlations at the 1σ level. That the running of \( n_s \) is poorly determined is not surprising since we shall see later that also \( n_s \) is poorly determined.

To proceed with \( w \) we take \( r = 0 \) and \( \alpha_s = 0 \). With this assumption four values of \( w \) have been determined;
\[
\begin{align*}
P1: & \quad -0.99 \pm 0.09, \quad P2: \quad -0.80 \pm 0.18, \quad P3: \quad -0.94^{+0.09}_{-0.10}, \\
P4: & \quad -0.85^{+0.18}_{-0.17}.
\end{align*}
\]

The inclusion of \( w \) as a seventh parameter in P1 and P3 has very little effect on the errors and central values of the basic six, more so in P4, but the correlations are still insignificant. One notices in P1, however, that keeping \( r \) and \( \alpha_s \) free improves the \( w \) errors and reduces its central value by slightly more than 1σ.

Including \( w \) as a fifth parameter in P2, degrades the errors of their basic four parameters, and changes their central values by less than 1σ.

The above values can then be used to define the median and errors
\[
w = -0.93^{+0.13}_{-0.10}.
\]

P1 also determines the parameters \( w_0, \ w_1 \) in the combination \( w = w_0 + w_1 (1 - a) \), with the result
\[
w_0 = -0.98 \pm 0.19, \quad w_1 = 0.05^{+0.83}_{-0.65}.
\]

Thus there is no significant information indicating \( w \neq -1 \) or \( w_1 \neq 0 \) at present. In a Bayesian study of the need for either of the parameters \( w \) or \( n_s \) (using a modified P1 data set) Mukherjee et al. [22] conclude that theories with more parameters are not disfavored, but that the improvement does not warrant the additional complexity in the theory. Thus we choose results from 7-parameter fits which assume \( w = -1 \).

A fortiori \( w_1 \) is useless.

The available determinations of the curvature parameter \( \Omega_k \) for the case \( w = -1 \) are
\[
P2: \quad -0.010 \pm 0.009, \quad P3: \quad -0.027 \pm 0.016, \quad P4: \quad -0.074^{+0.049}_{-0.052}.
\]

The general tendency is confirmed that the inclusion of \( \Omega_k \) degrades the information on the basic parameters, also the correlations are insignificant with one exception: there is a positive correlation between \( \Omega_k \) and \( h \), most visible in P3. The above set of data define the median and errors
\[
\Omega_k = 1 - \Omega_0 = -0.023^{+0.017}_{-0.050}.
\]

We conclude that there is very little information on curvature at present, and that \( h \) to some extent takes its rôle.

In the sequel we shall only use parameter values determined under the assumptions of flat space and \( r = \alpha_s = w + 1 = 0 \). This maximizes the information on the basic parameters, and it does not entirely neglect the effects of possibly non-vanishing \( \Omega_k \) or \( r \) because of the presence of the slight (\( \Omega_k, h \)) and (\( r, n_s \)) correlations.

**B. Neutrinos**

Although there are only upper limits determined for \( \Sigma m_\nu \) there is clear evidence from neutrino oscillations that the neutrinos have mass. Therefore it is more motivated to include \( f_\nu \) or \( \Sigma m_\nu \) in the fits than any of the parameters discussed in the previous subsection. There is neutrino information in all papers except P5 and P2; the latter notes that \( n_s \) has the same effect as massive neutrinos. P3 and P4 obtain relatively small values for \( n_s \) and higher neutrino masses, but this is not a very significant correlation.

The Ly-\( \alpha \) forest data present more difficulties than the other data sets in the form of nuisance parameters which have to be marginalized over and systematic errors which have to be estimated. The Ly-\( \alpha \) forest data are orthogonal to the other data in the sense that they are responsible for much of the improvement on the primordial spectrum shape and amplitude: \( \sigma_s \) is more accurately determined and the neutrino mass limit is much tighter. P1 admits that more work is needed, reflecting the sceptical attitude in the other papers.

The 95% C.L. results for \( \Sigma m_\nu \) in units of eV are
\[
P1: \quad < 0.42 \ (< 1.54 \text{ without Ly-}\alpha), \quad P3: \quad < 1.07, \quad P4: \quad < 1.16.
\]

Our recommendation is cautiously
\[
\Sigma m_\nu < 1.1 \text{ eV}.
\]

In the following we shall use those parameter values in P1, P3 and P4 that were obtained with \( \Sigma m_\nu \) as the seventh free parameter. The P2 data, obtained with \( \Sigma m_\nu = 0 \), shall be used in a special way to be described
shortly. One notes in the P1, P3 and P4 fits that some of the parameters are quite insensitive to whether \(\Sigma m_\nu\) is free or not: this is true for \(\omega_{\text{b}}, \omega_{\text{m}}, n_\tau\) and \(A_s\). For these parameters we consider the values obtained in P5 to be sufficiently unbiased to be used here.

C. \(\omega_{\text{b}}, \Omega_{\text{b}}, \omega_{\text{m}}, \Omega_{\text{m}}, \ h\)

Let us compare P1 and P2. The SDSS "Main sample" in P2 is not exactly the same as what P1 calls "all". The effects of the spectroscopic LRG data on the Main sample are quoted, generally they are small. For flat \(w = -1\) cosmology the LRG data change the parameters obtained from the Main sample by the amounts \(\Delta \omega_{\text{m}} = -0.004, \Delta \Omega_{\text{m}} = -0.007, \Delta h = -0.004, \Delta n_\tau = -0.017\).

Assuming that the LRG data would cause the same corrections to the P1 results, we concoct a set of LRG-corrected results that we call P1+LRG.

The \(\omega_{\text{b}}\) values found in the five analyses are very consistent, uncorrelated with other parameters and robust against different choices of parameter sets and priors, as pointed out by P4. They are P1: 2.36 \(\pm\) 0.09, P3: 2.24 \(\pm\) 0.12, P4: 2.24 \(\pm\) 0.11, P5: 2.31 \(\pm\) 0.13.

We can combine these with the BBN value \(\omega_{\text{b}} = 2.2 \pm 0.2\), to obtain values of the median and errors

\[
10^2 \omega_{\text{b}} = 2.28^{+0.12}_{-0.13}. \tag{5}
\]

The parameter \(\omega_{\text{m}}\) is determined better than \(\Omega_{\text{m}} = \frac{\omega_{\text{m}}}{h^2}\), thus the route to \(\Omega_{\text{m}}\) is to use \(\omega_{\text{m}}\) and \(h\). The values for \(\omega_{\text{m}}\) have often been obtained in this way. Also, since WMAP constrains \(\omega_{\text{m}}\) rather than \(\Omega_{\text{m}}\), one then avoids a large \((\Omega_{\text{m}}, h^2)\) correlation. We turn first to the Hubble parameter \(h\) which is measured to be P1+LRG: 0.706 \(\pm\) 0.022, P3: 0.648 \(\pm\) 0.038, P4: 0.691 \(\pm\) 0.038.

P3 and P4 exhibit clearly the effect of making the neutrino mass variable, a change of \(-3.1\%\) and \(-6.4\%\) respectively. This is a measure of neutrino mass bias in P2 and P5 that we therefore do not include. The chosen input yields the median and errors

\[
h = 0.687^{+0.034}_{-0.047}. \tag{6}
\]

in flat space, in remarkably good agreement with our constrained fit in 1998 \(h = 0.68 \pm 0.05\), based on Cepheid distances only.

From the above values of \(\omega_{\text{b}}\) and \(h\) we derive the fraction of baryonic energy in the Universe

\[
\Omega_{\text{b}} = 0.048^{+0.005}_{-0.004}. \tag{7}
\]

Turning now to the \(m\) and \(dm\) parameters, the available input data are the P1+LRG value \(\Omega_{\text{m}} = 0.277 \pm 0.025\), the P2 value \(\omega_{\text{m}} = \omega_{\text{dm}} + \omega_{\text{b}} = 0.142 \pm 0.005\), and the \(\omega_{\text{dm}}\) values

\[
P3: 0.126 \pm 0.007, \ P4: 0.110 \pm 0.006, \ P5: 0.112 \pm 0.011.
\]

Combining these values with \(h\) and \(\omega_{\text{b}}\) in the most efficient way, we find the median and errors

\[
\omega_{\text{m}} = 0.139 \pm 0.011, \quad \Omega_{\text{m}} = 0.286^{+0.030}_{-0.028}. \tag{8}
\]

This is in excellent agreement with what WMAP quoted in 2003, \(\Omega_{\text{m}} = 0.27 \pm 0.04\). It follows from this that the dark energy content in flat space is

\[
\Omega_{\Lambda} = 0.714^{+0.026}_{-0.030}, \tag{9}
\]

and the fraction of dark matter

\[
\Omega_{dm} = 0.238^{+0.030}_{-0.028}. \tag{10}
\]

D. \(\tau, n_\tau, A_s, \sigma_8\)

The parameters \(\tau\) and \(\Omega_k\) are significantly correlated \(4\), so if \(\Omega_k\) is taken to be zero, also \(\tau\) will be small. P1 and P2 use the prior \(\tau < 0.3\) which appears a bit tight; even so P1 obtains the highest value for \(\tau\). Since all papers use WMAP data, they inherit the problems with \(\tau\) appearing different in the Northern and Southern hemispheres \(21, 22\). Thus this parameter is neither reliably nor precisely determined. The values that can be quoted are

\[
P1: 0.185^{+0.052}_{-0.046}, \ P3: 0.108^{+0.049}_{-0.047}, \ P4: 0.143^{+0.076}_{-0.071}, \ P5: 0.147 \pm 0.085. \text{ twocolonm From this, the values of the median and errors are}
\]

\[
\tau = 0.147^{+0.068}_{-0.064}. \tag{11}
\]

The parameter \(n_\tau\) is measured by P1, P3, P4 and P5. The LRG data in P2 do not yield independent information on \(n_\tau\) and \(\omega_{\text{m}}\), so P2 adds them in the form of a constraint \(\omega_{\text{m}} = 0.130(n_\tau/0.98)^{-1.2} \pm 0.011\) that we do not use. The available data are then

\[
P1+LRG: 0.972^{+0.026}_{-0.023}, \ P3: 0.95 \pm 0.02, \ P4: 0.957^{+0.033}_{-0.029}, \ P5: 0.975 \pm 0.038.
\]

From this, the values of the median and errors are

\[
n_\tau = 0.962^{+0.030}_{-0.027}, \tag{12}
\]

thus \(n_\tau\) is marginally less than 1.0.

There are three measurements of the amplitude \(A_s\), quoted in the form \(\ln(10^{10}A_s)\):

\[
P3: 3.1 \pm 0.1, \ P4: 3.11^{+0.15}_{-0.14}, \ P5: 3.17 \pm 0.16.
\]

From this, the values of the median and errors are

\[
\ln(10^{10}A_s) = 3.12^{+0.14}_{-0.12}. \tag{13}
\]

The final parameter in this survey is \(\sigma_8\). P2 has fixed its value at 0.85; the other experiments quote values exhibiting a rather large variance,

\[
P1: 0.890^{+0.035}_{-0.033}, \ P3: 0.74 \pm 0.08, \ P4: 0.678^{+0.073}_{-0.072}, \ P5: 0.849 \pm 0.062.
\]

From this, the values of the median and errors are

\[
\sigma_8 = 0.81^{+0.09}_{-0.14}. \tag{14}
\]

Since \(\sigma_8\) is rather strongly correlated with \(\tau\) and \(\Omega_{\text{m}}\), a more accurate determination of \(\Omega_{\text{m}}\) could have been obtained if all experiments had agreed to fix \(\sigma_8\) and \(\tau\) at some common values.
TABLE II: Recommended values of parameters

| Parameter | Definition | Median value | 83-/17-percentile, or 95% CL limit |
|-----------|------------|--------------|----------------------------------|
| $10^2\omega_b$ | $10^2 \times$ baryon density | 2.28 $^a$ | +0.12/−0.13 |
| $\Omega_b = \omega_b/h^2$ | Normalized baryon density | 0.048 $^c$ | +0.005/−0.004 |
| $\omega_m$ | Total matter density | 0.139 $^a$ | ±0.011 |
| $\Omega_m = \omega_m/h^2$ | Normalized matter density | 0.286 $^c$ | +0.030/−0.028 |
| $\Omega_{dm} = \Omega_m - \Omega_b$ | Normalized dark matter density | 0.238 $^c$ | +0.030/−0.028 |
| $\Omega_{k} = 1 - \Omega_m - \Omega_b$ | Normalized dark energy density | 0.714 $^c$ | +0.028/−0.030 |
| $h$ | Hubble parameter [100 km/s Mpc] | 0.687 $^a$ | +0.034/−0.047 |
| $\Sigma m_\nu$ | Neutrino mass sum | < 1.1 eV |
| $\tau$ | Thomson scattering optical depth to decoupling | 0.147 $^a$ | +0.068/−0.064 |
| $n_s$ | Scalar spectral index | 0.962 $^a$ | +0.030/−0.027 |
| $\ln(10^{10}A_s)$ | Scalar fluctuation amplitude | 3.12 $^a$ | +0.14/−0.12 |
| $\sigma_8$ | RMS linear mass perturbation in $8h^{-1}$ Mpc spheres | 0.81 $^a$ | +0.09/−0.14 |
| $r = A_s/A_b$ | Ratio of tensor to scalar amplitude fluctuations | < 0.4 $^b$ |
| $\alpha_s = dn_s/dk$ | Running scalar index | −0.011 $^b$ | ±0.012 |
| $w$ | Dark energy EOS | −0.92 $^b$ | +0.17/−0.12 |
| $\Omega_k$ | Normalized vacuum density | −0.023 $^b$ | +0.017/−0.050 |

$^a$ From 7-parameter fits including $\Sigma m_\nu$.
$^b$ From alternative 7-parameter fits which exclude $\Sigma m_\nu$.
$^c$ Alternative or derived parameter.

VI. CONCLUSIONS

By combining results from five large data analyses [1, 2, 3, 4, 5], we find that no significant values can be obtained at present for $r$, $\alpha_s$, $w$, $w_0$, $w_1$, and that the values for $\Omega_k$ and $n_s$ are only marginally significant, consistent with flat space and no tilt. This agrees well with a Bayesian study [22] (using much of the P1 data) which concludes that $w$ and $n_s$ do not improve any fits.

We consider it important to include $\Sigma m_\nu$ among the varied parameters since neutrino oscillation results have proven that neutrinos have mass. Yet the fits we quote yield only an upper limit for $\Sigma m_\nu$. We have shown that it is possible to recommend best values for $\omega_b$, $\omega_m$, $h$, $\tau$, $n_s$, $A_s$, $\sigma_8$, $\Omega_k$ and $n_s$, rather regardless of whether $\tau$ and $\sigma_8$ are taken to be fixed or not; fixing would improve the accuracy of several parameters. Our results are collected in Table II.

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