Energy harvesting from flow induced vibration; study using a distributed parameter model

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Abstract. Energy harvesting from ambient vibration gained attention in the last decade to self power devices which need very low electric input. This finds applications in powering wireless sensor networks, structural health monitoring and human implants. In this work, the possibility of harnessing vibration energy from flow induced vibration is investigated numerically. A fluid stream passing over a bluff body make the body to oscillate due to vortex shedding. Piezoelectric transduction can be used to capture this energy from the oscillator by proper design of a mechanism. The governing equation for the distributed system was obtained using extended Hamilton principle followed by modal analysis. Continuous system model with a rigid support at the joint in the fluid flow is considered. Different wake oscillator models are considered and its effectiveness is investigated. Facchinetti model gives better representative result for the wake. Optimization using genetic algorithm is performed to identify the values of parameters for maximum energy harvesting and to reduce primary system vibration.

1. Introduction
Identifying sources of energy which is clean, renewable and harmless to the environment is one of the promising research work done all over the world in the past decade. Present day technological advancements had owed to decrease in power consumption of devices along with decrease in size of the systems. The concept of localisation has seen a rapid growth in recent years where off-grid solar power plants are installed in rooftops satisfying localised energy needs and reducing the dependency on the grid making homes self-sufficient. Microelectromechanical systems (MEMS) devices consumes low power and can be utilised in remote areas. Wireless Sensor Networks uses multiple MEMS to process or collect information and it requires self energy for uninterrupted operations. Use of finite power sources in biological implants leads to repetitive replacements making like uncomfortable. Providing power to under water mechanisms installed for structural health monitoring can be critical task considering working situation. So as to meet the energy requirement for the systems located in the isolated areas local finite sources of energy like batteries were utilized early which require periodic replacement. Self energising systems have the potential to replace batteries providing uninterrupted power for devices. These self energising systems can be viewed as an effort to trap nature’s energy that commonly goes unused[1]. Electromagnetic, electrostatic and piezoelectric transduction are the basic three vibration to electric energy conversion mechanisms. In the last decade piezoelectric transduction has received the maximum attention of these three mechanisms. PZT-5A and PZT-5H are the most widely
used piezoelectric ceramics in energy harvesting applications[2]. Piezoelectric materials have the ability to exploit ambient vibrations to generate electric potential and are proposed for harvesting energy to meet local energy demands[3]. A new method on exploitation of the dynamical features of non-linear oscillators was proposed by Cottone[4] which can be applied to piezoelectric energy harvesting technology. Ambient vibrations greatly have significant predominance in low frequency components with the energy distributed over a wide spectrum of frequencies. A wide spectral response is shown by the nonlinear oscillator presented in this study and matches closely with what is available in the environment when operated in an appropriate way[5]. Large power densities, minute in size and ease of application are the main advantages of piezoelectric materials in energy harvesting[6].

In nature as well as in human made constructions flow induced vibrations of bluff bodies are very common. Circular cylinder and prism shaped bluff bodies are omnipresent in engineering systems as components are exposed to fluid flow. Self-excited oscillations in these structures gives rise to unwanted problems. Heat exchangers, offshore structures, submerged pipelines, high-rise buildings, silos, chimneys and overhead transmission lines/cables are few examples that are prone to vortex induced vibration which is considered a negative phenomenon reducing life span and causing structural damage. Anton and Sodano have reviewed the progress made in the field of energy harvesting and completely self powered devices by supporting it with experiments[7]. The involved flow could be either contained gas/liquid or generally unconfined flow due to wind and water currents. A bluff cylinder with an appreciable downstream flow can undergo vortex induced vibration. Vortices are produced in the wake of the bluff body and disengage almost periodically from either side of the bluff body and form a street of vortices called Von Karman vortex street. The afterbody or the downstream flow interacts with the fluid wake to give rise to the pressure loading on the body downstream of the separation points. In this wind speed range the frequencies measured in the wake do not contain the expected Strouhal frequency. But it contains a frequency very close to the natural frequency of the body which is the phenomenon of synchronization or lock-in. Energy harvesting from flow induced vibration using two degree of freedom and three wake oscillator models has been investigated and proved to be effective[8].

To overcome the problem of high computation time with CFD solvers to calculate lift, phenomenological models with excitation on the structure due to fluid are modelled using a self-excited nonlinear oscillator. Non-linear solutions have the potential for describing a much larger range of phenomenon and are correspondingly more difficult. The works on these was kick-started by Bishop and Hassan [9] continued by Hartlen and Currie[10], Skop and Griffin[11], Iwan and Blevins[12], Skop and Balasubramanian[13] and Facchinetti[14]. Wang and Ko have exploited the pressure difference developed during vortex shedding using diaphragm to harvest energy through experiments and finite element models[15].

In this study a PZT-5A bimorph cantilever beam is modeled as a distributed system and mounted rigidly to the cylinder which is excited by fluid structure interaction. Parametric study has been performed and its parameters are optimised using genetic algorithm.

2. Mathematical Modelling
The system is composed of an elastically mounted cylinder with the damper attached with a PZT-5A bimorph cantilever beam substructure made of aluminum and piezoceramic layer made of PZT-5A under fluid flow as shown in figure 1. The following system can be modelled as a lumped cylinder with distributed cantilever beam to extract the energy across the full length of the beam. where $M_1$ represents the mass of the cylinder, $M_2$ represents the mass of the bimorph cantilever, $K_1$, $C_1$ represents the stiffness and damping respectively. $Y_1$ represents the displacement of cylinder(Mass 1). $C_p$ represents the internal capacitance, $R_i$ is the resistance of the piezoelectric equivalent circuit.
2.1. Flow Equations

2.1.1. Hartlen and Currie Lift Oscillator Model The oscillating lift force is assumed to be proportional to the instantaneous value of fluctuating lift coefficient. The characteristic of the wake oscillator model were inferred from comparison of experimental data. It is self-excited oscillator with its natural frequency is proportional to the wind speed. It is coupled with structural oscillator by a term which is proportional to the transverse cylinder velocity. The dimensional form of the instantaneous lift coefficient, $C_L$ is given by

$$\ddot{C}_L - \delta w_s C_L + \gamma w_s \dot{C}_L^3 + w_s^3 C_L = F_C$$

(1)

Where $F_C = b w_n Y_1$ is the coupling term which gives the effect of structural variations on the vortex formation and shedding. Wake oscillator parameter like $b, \gamma, \delta$ are constants determined from experiments. The parameter values for the model are obtained from Hartlen & Currie [10] ($\delta=0.02; \gamma=0.6667; b=0.4; C_{L0}=0.2$)

2.1.2. Facchinetti wake oscillator model The dynamic nature of the lift coefficient formed by von Karman vortex street is modelled as a self-excited and self-limiting nonlinear oscillator called Van der Pol oscillator. Equation of Facchinetti model[14] is shown below,

$$\ddot{q} + \varepsilon w_s (q^2 - 1) \dot{q} + w_s^2 q = F_C$$

(2)

The fluid variable $q$ is inferred as a non-dimensionalised vortex lift coefficient given by $q = \frac{2 C_L}{C_{L0}}$, where the reference lift coefficient $C_{L0}$ which is lift coefficient observed on a fixed cylinder subjected to vortex shedding. $F_C$ is the force coupling term and is given by the relation $F_C = \frac{A Y_1}{A}$, where $A=12, \varepsilon=0.3$ ($A$ and $\varepsilon$ are experimentally determined constants).

The the type of coupling used to model differentiates between Fachinetti’s wake oscillator model and Hartlen and Currie’s lift oscillator model is. In Hartlen and Currie’s lift oscillator model, the coupling is through the velocity term whereas in Fachinetti wake oscillator model the acceleration coupling.

2.2. Mode shape
Equations of motion are derived using the Euler-Bernoulli beam assumption. The partial differential equation is solved using the variable separable method. Free vibration equation of motion is given by,

$$c^2 \frac{\partial^4 w(x,t)}{\partial x^4} + \frac{\partial^2 w(x,t)}{\partial t^2} = 0; \text{ where } c = \sqrt{\frac{EI}{\rho A}}$$

(3)
Free vibration solution can be obtained using the method of separation of variables and is of the form

$$W(x) = C_1 \cos \beta x + C_2 \sin \beta x + C_3 \cosh \beta x + C_4 \sinh \beta x$$

The natural frequencies of the beam can be obtained from

$$w = (\beta l)^2 \sqrt{\frac{EI}{\rho Al^4}}$$

2.2.1. Boundary conditions

(i) At the left end $$x = 0$$ the resting force on the mass is balanced by shear force at beam

$$\frac{\partial}{\partial x} \left( EI \frac{\partial^2 w(0,t)}{\partial x^2} \right) = - \left( M_1 \frac{\partial^2 w(0,t)}{\partial t^2} + k_1 w(0,t) \right)$$

(ii) The bending moment at $$x = 0$$ is zero ; $$E I W''(0) = 0$$

(iii) Bending Moment at $$x = l$$ is zero ; $$W''(l) = 0$$

(iv) Shear force is zero at $$x = l$$ ; $$W'''(l) = 0$$

2.2.2. Solution for mode shapes

Apply the boundary condition we get the transcendental equation

$$1 - \frac{1}{\cos \beta l \cosh \beta l} + \left( \frac{1}{r(\beta l)^3} - \frac{\beta l}{\mu} \right) (\tan \beta l - \tanh \beta l) = 0$$

Solve the above equation using Newton Raphson method to get the equation for mode shape

$$W_n(x) = C_4 n \left( \chi_n \cos \beta_n x + \cosh \beta_n x \right) + \psi_n \sin \beta_n x$$

Optimal solution to frequency equation is found at $$\mu = 0.3$$ and $$r = 5$$ with $$\beta_1 l = 0.1861$$, $$\beta_2 l = 3.965$$, $$\beta_3 l = 7.089$$, $$\beta_4 l = 10.22$$

Applying the orthogonality and normalization condition for natural mode yields

$$\int_0^l \rho A W_i^2(x) \, dx + M_1 W_i^2(0) = 1 ; i = 1, 2, 3...$$

Above equation is first simplified into algebraic equation MATLAB symbolic toolbox. Solve the resulting algebraic equation to get $$C_4 n$$. Mode shapes are shown in figure 3

| Modes | $$\beta_n l$$ | $$\psi_n$$ | $$\chi_n$$ | $$C_4 n$$ |
|-------|---------------|------------|------------|----------|
| Mode 1 | 16.203 | 0.925 | -0.311 | 0 |
| Mode 2 | 132.167 | 94.382 | -3.534 | 0.992 |
| Mode 3 | 236.300 | 45.709 | -0.946 | 2.0358 |

Table 1: Parameters of mode shape with $$\mu = 0.3$$, $$r = 5$$

Figure 3: Mode Shape Plot
2.3. Modal equations

2.3.1. Extended Hamilton principle

Modal equations are derived using the extended hamilton principle. The total Kinetic energy
\[ T(t) = \frac{1}{2} \int_{V_s} \rho_s \left( \frac{\partial w(x,t)}{\partial t} \right)^2 \, dV_s + \frac{1}{2} \int_{V_p} \rho_p \left( \frac{\partial w(x,t)}{\partial t} \right)^2 \, dV_p + \frac{1}{2} M_1 \left( \frac{\partial w(x,t)}{\partial t} \right)^2 \] (10)

The total potential energy \( U(t) \) is given by:
\[ U(t) = \frac{1}{2} \int_{V_s} T_{xx} S_{xx} \, dV_s + \frac{1}{2} \int_{V_p} T_{pp} S_{xx} \, dV_p + \frac{1}{2} K_1 w^2(0,t) \] (11)

The total internal electrical energy, \( W_{ie}(t) \), in the piezo-electric layer is given by
\[ W_{ie}(t) = \frac{1}{2} \int_{V_p} ED \, dV_p = \frac{1}{2} \int_{V_p} \left( -\frac{v(t)}{h_p} \right) + e_{31} \left[ \frac{\partial^2 w(x,t)}{\partial x^2} - e_{33} \frac{v(t)}{h_p} \right] \, dV_p \] (12)

where \( D \) is the electric Displacement and \( E \) is the electric field. The total non-conservative work is due to the external forces and also due to electric charge output.
\[ W_{nc}(t) = \int_0^l f(x,t) w(x,t) \, dx + Q(t)v(t) \] (13)

where \( f(x,t) \) is the distributed force along the length of the beam, \( Q \) is the charge.

Application of the generalized Hamilton’s principle gives
\[ \int_{t_1}^{t_2} (\delta T - \delta U + \delta W_{ie} + \delta W_{nc}) \, dt = 0 \] (14)
\[ m \frac{\partial^2 w(x,t)}{\partial t^2} + G \frac{\partial^4 w(x,t)}{\partial x^4} - J_p v(t) \left[ \frac{d\delta(x)}{dx} - \frac{d\delta(x-l)}{dx} \right] = f(x,t) \] (15)

where \( G = (E_s I_s + e_{11}^p I_p) \) is the bending stiffness term of the composite cross section for the constant electric field condition and \( J_p = e_{11} b_h \left( \frac{h_p}{b_h} + \frac{h_s}{2} \right)^2 - \left( \frac{h_s}{2} \right)^2 \)

2.3.2. Modal analysis

According to modal analysis, we assume the solution for the equation to be a linear combination of the normal modes of the beam as
\[ w(x,t) = \sum_{i=1}^{\infty} W_i(x) \eta_i(t) \] (16)

where \( W_i(x) \) represents the normal modes which were earlier derived for the equation of the form
\[ G \frac{d^4 W_i(x)}{dx^4} - mw_i^2 W_i(x) = 0 \] (17)

and \( \eta_i(t) \) are the generalised coordinates. Solving above equations, we get
\[ \ddot{\eta}_i(t) + 2\zeta_i w_i \dot{\eta}_i(t) - \theta_i v(l) = f_i(l) \; \text{where} \; \theta_i = J_p \left[ \frac{dW_i(x)}{dx} \right]_{x=l} - \frac{dW_i(x)}{dx} \bigg|_{x=0} \] (18)
\[ f_i(t) = \int_0^l f(x,t) W_i(x) \, dx = \int_0^l F_{ii,ft} \delta(x-0) W_i(x) \, dx + F_{ii,ft} W_i(0) \] (19)

\( F_{ii,ft} = \frac{1}{2} \rho_f u^2 C_L \) is the fluctuating lift force per uit length of the cylinder, is acting on the cylinder at the point \( w(0,t) \) where \( \rho_f \) is the fluid density, \( u \) the velocity of flow, and \( C_L \) the instantaneous coefficient of lift. Assume that the lift force is mostly acting on the cylinder.
2.4. Governing equations

\[ \ddot{\eta}_i(t) + 2\zeta_i w_i \dot{\eta}_i(l) + w_i^2 \eta_i(l) - \theta_i v(l) = f_{\text{lift}} W_i(0); \text{ where } i = 1, 2, 3 \] (20)

\[ C_p \dot{v}(l) + \frac{v(l)}{R_l} + \sum_{i=1}^{\infty} \theta_i \dot{\eta}_i(t) = 0 \] (21)

\[ \ddot{C}_L + \delta w_s \dot{C}_L + \frac{\gamma}{w_s} C_L^3 + w_s^2 C_L = bw_n \sum_{i=1}^{\infty} \dot{\eta}_i(t) W_i(x) \bigg|_{x=0} \] (Hartlen and Currie model) (22)

\[ \ddot{C}_L + \epsilon w_s C_L \left( \frac{2C_L}{C_{L0}} - 1 \right) + w_s^2 C_L = aC_{L0} \sum_{i=1}^{\infty} \dot{\eta}_i(t) W_i(x) \bigg|_{x=0} \] (Facchinetti model) (23)

3. Results and Discussion

Frequency response has been generated for the model using two wake oscillator models. Material which is used for the structure is PZT-5A and the primary structure material is taken as Aluminium. Electromechanical constants considered for numerical simulation of distributed parameter model are Length of the beam, \( l = 30\text{mm} \); Width of the beam, \( b = 5\text{mm} \); Thickness of the beam, \( h = 0.05\text{mm} \); Thickness of the PZT, \( h_p = 0.05\text{mm} \); Spring Stiffness, \( k_1 = 100\text{N/m} \); Elastic modulus of PZT, \( C_{11} = 61\text{GPa} \); Elastic modulus of aluminium, \( E_s = 70\text{GPa} \); Mass density of Piezo, \( \rho_p = 7750\text{kg/m}^3 \); Piezo-electric constant, \( E_{31} = -10.4\text{C/m}^2 \); Permitivity constant, \( E_{33} = 13.3\text{nF/m} \); \( C_p = 39.9\text{nF} \); \( R = 2\Omega \).

3.1. Hartlen and Currie Oscillator Model

![Cylinder Displacement vs Time](image1)

![Coefficient of Lift vs Time](image2)

Figure 4: System response curve with Hartlen Currie for the constant flow velocity of 4.08m/s
(a) Displacement (b) Co-efficient of lift (\( C_p = 39.9\text{nF}, R=2\Omega \))

The initial conditions used for simulation are \( x_1 = 10; x_2 = 0; x_3 = 0; x_4 = 0; x_5 = 10^{-9}; x_6 = 0; x_7 = 0; x_8 = 10^{-3}; x_9 = 0 \). Figures 4 (a) and (b) shows the time history plot of the system response at 4.08m/s which corresponds to the first resonance point. The displacement of the cylinder settles down at 0.1848 nm. The fluctuating lift coefficient of 0.2 from figure 4 (c). Figures 4 (a) and (b) is plotted for the system response with 2\( \Omega \). Phase-plane plot with closed curve is obtained which represents a periodic solution for the cylinder.
From figure 5 (a) the displacement of the cylinder at the first natural frequency is minimum, but at the second and third natural frequencies significant cylinder displacement is observed. Amplitude seems to be maximum at the second resonance of about 25.8µm. Around flow velocity of 134.1 m/s there is a occurrence of minimum displacement followed by third resonance. Maximum generated voltage of 4.303V is obtained at third resonance. Figure 5 (b) shows the maximum harvested power of 9.259 mW at third resonance.

3.2. Facchinetti’s Wake Oscillator Model

Figures 6 (a) and (b) shows the time history plot of displacement of cylinder and coefficient of lift respectively at first resonance point. Displacement is found to settle to 35µm at a flow velocity of 4.087m/s which corresponds to first resonance condition. Coefficient of lift attains a steady state value of 0.3001. Figure Phase-plane plot with closed curve is obtained which represents a periodic solution for the cylinder.
Figure 7: Facchinetti wake oscillator model with a variation in flow velocity (a) Displacement (b) Harvested Power (\( C_p = 39.9\text{nF}, R=2\text{k}\Omega \))

Figures 7 (a) shows the variation of displacement as a function of flow velocity. From figure 7 (a) first resonance is shown with a peak which was not observed in the Hartlen and Currie model. Second resonance shows a maximum displacement of 0.42mm. Around flow velocity of 126 m/s there is a occurrence of minimum displacement followed by third resonance. Maximum generated voltage of 6V is obtained at third resonance. Figure 7 (b) shows the maximum harvested power of 5 mW at third resonance. But the displacement of the primary system is very high at third resonance, hence this generated voltage at third resonance is not feasible practically.

The comparison study has been done to verify the system response obtained from two wake oscillator modes and is plotted in figure 8. Hartlen and Currie model shows a less displacement with slight variations in amplitude. Energy harvesting is found to be maximum for Facchinetti model when compared to Hartlen and Currie model with minimum difference. As there is not much drastic variations in the system response both the models are considered for further optimisation.

Figure 8: Comparison between Hartlen Currie and Facchinetti Model (a) Cylinder Displacement (b) Harvested Power
4. Parametric Study
The electrical parameters like $c_p$ and $R$ are varied to see the effect of the parameters in the power harvested. There is a shift in flow velocities at which the peak occurs for the variation in load resistance. The power started to increase with respect to the load resistance and it reaches a maximum of about 72.78 mW at load resistance of about 100 KΩ. Further increase in load resistance decreases the power harvested so the optimum parameter of $R$ should lie in between the 1KΩ to MΩ which is shown in figure 10. The $C_p$ internal capacitance doesn’t show much variation in power harvested which is shown in figure 9.

![Harvested Power vs Flow Velocity](image1)

Figure 9: Parametric study on power with respect to $C_p$

![Harvested Power vs Flow Velocity](image2)

Figure 10: Parametric study on power with respect to $R$

5. Optimisation
Genetic algorithm[16] helps finding the optimised parameter values for maximum energy harvesting. The parameters considered for optimisation are model damping, capacitance of piezo and internal resistance of the piezo. Flow velocity and stiffness has been fixed at 5m/s and 100N/m respectively as stiffness. The range of constraints has been selected with the practically available dampers and piezo harvester. The objective function for the optimisation is defined as:

$$f = \max \left[ \text{Power}(C_p, R, \zeta_1, \zeta_2, \zeta_3) \right]$$

(24)

The the bounds taken for the parameters are:

$0.1 < \zeta_1, \zeta_2, \zeta_3 < 1, 10^{-8} < C_p < 10^{-7}, 10^3 < R < 10^6$

| Parameter | $\zeta_1$ | $\zeta_2$ | $\zeta_3$ | $C_p$ | $R$ |
|-----------|-----------|-----------|-----------|-------|-----|
| Value     | 0.7660    | 0.0483    | 0.0148    | 10nF  | 1.8906MΩ |

Table 2: Optimum parameter values obtained using genetic algorithm

Figures 11 and 12 shows an optimised and initial parameter value for harvested power and cylinder displacement respectively. Though flow velocity has been fixed for the optimisation from figure 11, it can be inferred that the optimised parameter value provides maximum harvested voltage for full range of flow velocity. Maximum harvested voltage after optimisation is found to be 0.109 W with a primary system displacement of 49.05μm at a flow velocity of 49m/s. Maximum harvested voltage after optimisation is found to be 0.4119 W with a primary system displacement of 48.69μm at a flow velocity of 156m/s.
6. Conclusion
In this work, a continuous system model for energy harvesting from vortex induced vibration is considered. First three modes are considered for the analysis. Facchinetti and Hartlen & Currie wake oscillator models were considered to model fluid structure interaction. Piezoelectric energy harvester has been utilised to capture the energy of the flow induced vibration. From the parameter study appropriate ranges for parameters are understood which was utilised for optimisation. The energy harvested has been maximised by optimising the parameter values. Facchinetti model was found to be a more accurate than Hartlen and Currie as later predicts slightly less displacement of cylinder for full range of flow velocity. Power of 0.4119 W was harnessed from the primary structure vibration with a displacement of only 48.69 µm at 156 m/s.

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