Mathematical Control Model of Secondary Coextrusion Composition Preparation

V V Dyadichev¹, A V Kolesnikov², A V Dyadichev³

¹V.I. Vernadsky Crimean Federal University, Academy of Construction and Architecture, Engineering Center “Bio-positive Construction and Resource-saving”, 3 Pavlenko Str., Simferopol, Crimea, 295000, Russia
²Lugansk National University named after Vladimir Dal, Department of Automation and Computer-Integrated Technologies, 20-A Molodezhnyi Sq., Lugansk, 91034, Ukraine

E-mail: mr.dyadichev@mail.ru

Abstract. The control of polymer waste recycling process is an important modern problem. A partial solution to the problem can be got by using coextrusion technology. As a result of its application with respect to polymer waste, we can get a multilayer qualitative product, containing waste and safe for usage. Because of economic causes, the problem of using this technology is the correct preparation of secondary coextrusion composition based on the requirements to the quality of the obtained product. The scientific paper presents the mathematical control model of secondary polymer composition preparation. The developed model allows carrying out an operational control of composition components consumption, determining and correcting the consumption of the most expensive astringent component in order to guarantee the required quality of the product. The mathematical model is the basis for construction of intellectual automated systems for controlling polymer waste recycling process by coextrusion technology.

1. Introduction

Coextrusion secondary composition is intermediate raw material in the process of recycling polymer waste into a new qualitative product by coextrusion method [1-6]. The standard mathematical models of the compositions determine the required content of the principal astringent component (the primary polymer) while changing the values of the rest components of the composition [7-9], which provide the necessary durability characteristics of the designed multilayer product and rheological properties of the composition [10-11].

2. Topicality, scientific and practical significance

The usage of standard models of the composition allows optimizing the process of composition preparation control [12]. It is important to take into consideration the peculiarities of the technology function process, i.e. interaction of the components measured in the course of time. It is necessary to take into consideration the actual position of tools of measurement and metering of the composition components with respect to the extruders of the coextrusion equipment, which determines the correspondence of the measured components consumption to the real consumption of raw materials,
getting into the extruder in the time \( \tau \), which is different for each component [13]. Such a problem can be solved with the help of the mathematical model of the coextrusion composition preparation process.

3. Problem setting
Designing the mathematical control model of secondary coextrusion composition preparation which will allow carrying out an operational control of composition components consumption, determining and correcting the consumption of the most expensive astringent component in order to guarantee the required quality of the new product with the usage of secondary raw material.

4. Theoretical part
A mathematical model of any manufacturing process is the totality of interconnected equations (models), describing the work of separate technological segments [14]. With regard to the manufacturing process of preparing the coextrusion composition, such segments are: bunkers, dispensers, transporting conveyer units, grinders, an extruder and a coextrusion head [15]. The totality of their equations will present the general view of the mathematical model of the composition preparation process [16].

Designing the mathematical model of manufacturing coextrusion composition on the basis of the composition standard model allows confining ourselves to mathematical description only of dispensers and blending devices, presenting the rest of the objects, such as transporting segments, with delay [17]. If further to add to the obtained model the models of extrusion with transportation and of coextrusion, then there will appear the possibility to solve the problem of designing and researching the process model of getting a multilayer product [18].

The existing technologies of preparing the composition from polymer waste consist of separate processing lines of preparing components which are joined in the extruder. The weigh metering device for each line, in general, is a segment, formed by the bunker and the dispenser, which for loose materials is characterized by the following parameters:

- weight efficiency of the material delivery to the bunker \( Q_1 \), kg/h;
- material supply of the bunker \( M_b \), kg;
- the consumption of the material from the bunker \( Q_2 \), kg/h;
- the speed of the dispenser (material) belt movement, m/s and the length of the conveyer \( l \), m;
- material supply on the dispenser belt \( M \), kg;
- the consumption of the material at the dispenser output \( Q_3 \), kg/h.

Using the equation of the bunker’s material balance:

\[
\frac{dM_b}{dt} = Q_1(t) - Q_2(t);
\]  
(1)

and of the dispenser’s conveyer belt:

\[
Q_3(t) = Q_2\left(t - \frac{l}{\vartheta}\right);
\]  
(2)

finally we get the equation:

\[
\frac{dM_b}{dt} = -Q_3(t - \tau) + Q_1(t),
\]  
(3)

where \( \tau = l/\vartheta \) - is the constant of transportation lag.

Nowadays at the complexes of polymer waste recycling the weight dispensers with belt conveyers are used. These weight dispensers also have efficiency control by the alteration of the belt movement speed (as distinct from the dispensers with the change of the bunker’s output section) [19]. That is, the
bunker of the weight dispenser serves as the feeder of the material supply $M_b$. That is why to solve the problem of the dispenser’s control we will limit ourselves by the research of the correlation between the material consumption at the input $Q_3$ and the output $Q_i$ of the measuring conveyer and the task $Q'_i$ for i-dispenser:

$$ Q'_i = m_i P_i, $$

where $m_i$ is the mass of the i-component in 1 m$^3$ of the composition, kg/m$^3$;

$P_i$ is the efficiency of the complex, m$^3$/h.

Such a statement of question is competent as the usage of nonstandard dispensers for the materials, which are difficult to meter on the basis of the conveyer scales and of simple schemes of the conveyer’s speed regulation, has become widespread [20]. In this case the conveyer, onto which the platform scales are placed, is the metering one, and the preceding conveyer is the feeder. One of the conveyers has to be obligatory regulated [21].

In such a statement the equations of the material balance of the metering devices will be presented by the following formulas in case of change:

- the consumption of the material at the input:

$$ \frac{dM}{dt} = Q_3(t) - Q_i(t) = Q_3(t) - Q_3(t - \tau) = Q_3(t + \tau) - Q_3(t); $$

- the tasks for the material consumption:

$$ \frac{dM}{dt} = Q'_i - Q_i(t). $$

By analogy the equations of the material balance for the segment “the accumulating capacity – the filler of the filler” can be obtained, they have a similar form.

Hereby, from the equation (6) it is evident that the real consumption of the material at the output of each dispenser is determined by the task $Q'_i$ and the changed speed of the material flow on the measuring conveyer of the dispenser $\frac{dM}{dt}$. Using the results of the mathematical description of the process of the components’ weight metering and the equations of the composition model’ regression, we get a general form of the mathematical model of the composition preparation:

$$ Q_{pp}^3 = a_{0pp}P_x + a_{1pp}Q'_p - \frac{dM_{sp}}{dt} + a_{2pp}Q_p^3 - \frac{dM_f}{dt} + a_{3pp}Q_p^3 - \frac{dM_{fw}}{dt}, $$

$$ R = a_{0R} + a_{1R}Q_p^3 - \frac{dM_{sp}}{dt} + a_{2R}Q_{pp}^3 - \frac{dM_{pp}}{dt} + a_{3R}Q_p^3 - \frac{dM_f}{dt} + a_{4R}Q_{fw}^3 - \frac{dM_{fw}}{dt}, $$

$$ \tau_0 = a_{0\tau} + a_{1\tau}Q_p^3 - \frac{dM_{sp}}{dt} + a_{2\tau}Q_{pp}^3 - \frac{dM_{pp}}{dt} + a_{3\tau}Q_p^3 - \frac{dM_f}{dt} + a_{4\tau}Q_{fw}^3 - \frac{dM_{fw}}{dt}, $$

where $Q_{pp}^3$ is the task for the consumption of the primary polymer, kg/h;

$P_c$ is the set efficiency of the coextrusion complex, m$^3$/h;
\[ Q^3_{sp}, \quad Q^3_1, \quad Q^3_{fw} \] are the tasks for the consumption of the secondary polymer, the filler and the filled waste respectively, calculated by the formula (4), kg/h;

\[ a_{un}, \quad a_{fr}, \quad a_s \] are the coefficients of the regression equations for the compositions of different normative durability;

\[ \frac{dM_i}{dt} \] is the increase of the material supply, depending on the constructive peculiarities of the dispensers’ elements and their regulation methods:

\[
\begin{align*}
  a_{fr} &= a_{fr} / P_c, \\
  a_{t} &= a_{t} / P_c.
\end{align*}
\]

As a rule, the placing of dispensers in the coextrusion complexes is done due to the condition of their work in the mode of local regulation or components’ stabilization with regard to the device type, assembly and maintenance facilities, the possibility to regulate the material flow and other factors. That is why while implementing the model in the real time mode it is necessary to take into consideration the delay of components’ measured values arrival to the extruder’s input (figure 1). Let us present all the elements of the production string from the dispenser to the extruder as the segments with transportation lag.

Then, taking as t the moment of the time at the extruder’s input and taking into consideration the formula (6), we transform the equations (7) – (9) of the process model to the following form:

\[
Q^3_{PP}(t) = a_{0,PP} + a_{1,PP} Q^3_{sp}(t - \tau_{sp}) + a_{2,PP} Q^3_{sp}(t - \tau_{sp}) + Q^3_{PP}(t - \tau_{f1}) + Q^3_{PP}(t - \tau_{f2}) + Q^3_{fw}(t) + + a_{3,PP} Q^3_{fw}(t - \tau_{fw})
\]

\[
R = a_{0R} + a_{1R} Q^3_{sp}(t - \tau_{sp}) + a_{2R} Q^3_{PP}(t) + a_{3R} Q^3_{PP}(t - \tau_{f1}) + Q^3_{PP}(t - \tau_{f2}) + Q^3_{fw}(t) + + a_{4R} Q^3_{fw}(t - \tau_{fw}) \geq R_{add}
\]

\[
\tau_a = a_{0a} + a_{1a} Q^3_{sp}(t - \tau_{sp}) + a_{2a} Q^3_{PP}(t) + a_{3a} Q^3_{PP}(t - \tau_{f1}) + Q^3_{PP}(t - \tau_{f2}) + Q^3_{fw}(t) + + a_{4a} Q^3_{fw}(t - \tau_{fw}) \leq \tau_{0a}\text{add}
\]

where \( a_{0,PP} = a_{0,PP} P_c \);

\[
\begin{align*}
  \tau_{sp} &= \tau_1 + \tau_2 + \tau_3; \\
  \tau_{f1} &= \tau_2 + \tau_7; \\
  \tau_{f2} &= \tau_5 + \tau_8; \\
  \tau_{fw} &= \tau_4 + \tau_5 + \tau_6; \\
  \vec{\tau}(\tau_1, ..., \tau_6) &= \text{the vector of the components’ movement lag};
\end{align*}
\]

\( Q^3_{sp}, \quad Q^3_{PP}, \quad Q^3_{PP}, \quad Q^3_{fw} \) are the real consumption of the components at the dispensers’ output, which are brought to the extruder’s input:

\[
Q^3_{Exp}(t) = Q^3_{sp}(t - \tau_1 - \tau_2 - \tau_3);
\]

\[
Q^3_{Exp}(t) = Q^3_{PP}(t - \tau_2 - \tau_7) + Q^3_{PP}(t - \tau_5 - \tau_8);
\]

\[
Q^3_{Exp}(t) = Q^3_{PP}(t);
\]
Figure 1. The scheme of placing metering devices for recording the components’ consumption at the polymer material waste recycling complex.

5. Conclusions
Thus, the developed in this research model of the composition preparation process allows carrying out an operational control of composition components consumption, determining and correcting the consumption of the most expensive astringent component in order to guarantee the required quality of the composition.

The successful implementation of the manufacturing process of the polymer composition preparation mostly depends on the quality of composition components mixing, providing durable adhesive compositions from polymers, the choice of design factors of extruders, augers, heads, and other factors.

6. References
[1] Dyadichev V V, Kolesnikov A V, Menyuk S G and Dyadichev A V 2018 Improvement of extrusion equipment and technologies for processing secondary combined polymer materials and mixtures IOP Conf. Series: J. of Physics: Conf. Series "Applied Mechanics and Systems Dynamics" (Moscow) 1210 DOI: 10.1088/1742-6596/1210/1/012035
[2] Kim V S 1988 Dispersion and mixing in plastics production processes (Moscow: Chemistry) 293
[3] Davis B et al 1998 Grooved feed single screw extruders – improving productivity and reducing viscous heating effects Polym. Eng. Sci. 7 38 1199
[4] Han C D 1981 Multiphase flow in polymer processing (New York: Academic Press)
[5] Steller R T 1990 Theoretical model for flow of polymer melts in the screw channel Polym. Eng. Sci.. 7 30 400-7
[6] Rauwendaal C 2001 Polymer extrusion (Munich, Hauser Garduer) 777
[7] Pape J, Potente H and Obermann C 1999 Influence of model simplifications on the accuracy of simulation results in single screw extruders 15th Annual Meeting of the Polymer Processing Society (the Netherlands: Den Bosch)
[8] Elemans P 2000 Enhancing dry-color efficiency in starve-fed injection molding 58th SPE ANTEC 2582-6
[9] Elemans P and van Wunnik J M 2000 The effect of feeding mode on the dispersive mixing efficiency in single-screw extrusion 58th SPE ANTEC 265-7
[10] Shvetsov P A 1988 Plastics processing technology (Moscow: Chemistry) 462
[11] Dyadichev V, Kolesnikov A, Dyadichev A, Menyuk S and Dyadicheva E 2018 Model of creation a multilayer structure for production of building pipes MATEC Web Conf.: VI Int. Scient. Conf. “Integration, Partnership and Innovation in Construction Science and Education” (IPICSE-2018) (Moscow) 251 DOI: 10.1051/matecconf/201825101006
[12] Gutsalenko Yu, Bratan S, Roshchupkin S, Dyadichev V and Menyuk S 2018 Investigation of the structure and properties of copper-tin bonding M2-01 in diamond grinding wheel introducing additional energy in the form of electric discharges into the processing zone Materials Today: Proc.: Int. Conf. on Modern Trends in Manufacturing Techn. and Equipment 2018 (ICMTMTE_2018): Materials Science (Sevastopol) 11 586-90.
[13] Lyubomirskiy N V, Fedorkin S I, Bakhtin A S and Bakhtina T A 2017 Structuring of composite systems based on lime harden through carbonation and secondary limestone raw materials Malaysian Constr. Res. J. (MCRJ) 3 23 15-26
[14] Fic S, Lyubomirskiy N V and Barnat-Hunek D The Influence of the natural aggregate roughness on the ITZ adhesion in concrete Mat. Sci. Forum: “Materials and Technologies in Construction and Architecture” 931 564-7
[15] Gusev A, Shul’gin V, Braga E, Zamnius E, Starova G, Lyssenko K, Eremenko I and Linert W 2018 Luminescent properties of zinc complexes of 4-formylpyrazolone based azomethineligands: excitation-dependent emission in solution J. of Luminescence 202 370-6
[16] Gusev A N, Shul’gin V F, Braga E V, Nemec I, Minaev B F, Baryshnikov G V, Trávníček Z, Ágren H, Eremenko I L, Lyssenko K A and Linert W 2018 Synthesis and photophysical properties of Zn(II) Schiff base complexes possessing strong solvent-dependent solid-state fluorescence Polyhedron DOI: 10.1016/j.poly.2018.08.019
[17] Shul’gin V, Pevzner N, Gusev A, Sokolov M, Panyushkin V, Devterova J, Kirillov K, Martynenko I and Linert W 2018 Tb(III) complexes with 1-phenyl-3-methyl-4stearoyl-pyrazol-5-one as a material for luminescence Langmuir–Blodgett films J. of Coordination Chemistry DOI: 10.1080/00958972.2018.1536783
[18] Yankovskaya V S, Dovhyi I I, Bezhin N A, Milyutin V V, Nekrasova N A, Kapranov S V and Shulgin V F 2018 Sorption of cobalt by extraction chromatographic resin on the base of di-(tert-butylbenzo)-18-crown-6 J. of Radioanalytical and Nuclear Chemistry 2 318 1085–97
[19] Dyadichev V V, Kolesnikov A V, Dyadichev A V and Dyadicheva E A 2018 Model of the formation of a multilayer structure of a given quality for the recycling process of secondary construction and polymeric materials Coll. of sci. papers “Construction and technological safety”: Scientific and technical journal on construction and architecture 10 62 97-102
[20] Malitskova E A and Potapov I I 1997 Recycling of waste plastics (Moscow: Avis Original) 159
[21] Fogarty J, Rauwendaal C J, Fogarty D and Rios A 2001 Turbo-screw, new screw design for foam extrusion SPE ANTEC Techn. Papers

Acknowledgements
The study was carried out with the financial support of the Ministry of Education and Science of the Russian Federation within the framework of the scientific project 10.1622.2017/4.6.