Spin content of Lambda and its longitudinal polarization in $e^+e^-$ annihilation at high energies

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Abstract

Longitudinal polarization of Lambda produced in $e^+e^-$ annihilation at LEP energies is calculated in a picture for the spin content of Lambda which is consistent with the polarized deep inelastic lepton-nucleon scattering data and SU(3) flavor symmetry for hyperon decay so that the spin of Lambda is not completely carried by its $s$-valence quark. A comparison with the recent ALEPH data and the results of earlier calculations based on the static quark model in which the spin of Lambda is completely determined by the $s$-quark is given. The result shows that further measurements of such polarization should provide useful information to the question of which picture is more suitable in describing the spin effects in the fragmentation processes.
There exist now in literature two completely different pictures for the spin contents of the baryons: According to the static (or constituent) quark model, spin of a baryon belonging to the $J^P = \frac{1}{2}^+$ octet is completely determined by the three valence quarks. The sum of the spins of these valence quarks is the spin of the baryon. This picture is very successful in describing the static properties of the baryons. But according to the the data from polarized deep inelastic lepton-nucleon scattering [1] and SU(3) flavor symmetry in hyperon decay, the sum of the spins of the three valence quarks is only a small fraction of the spin of the nucleon. A large part of the baryon spin originates from the orbital angular momenta of the valence quarks and/or from the sea (i.e. the sea quarks, antiquarks and gluons). Hence, it is natural to ask which picture is suitable in describing the spin effects in the quark fragmentation process. Obviously, the answer to this question is a priori unknown and should be studied in experiments. Polarization of Lambda is an ideal place to investigate this problem because of the following: First, the spin structure of Lambda in the static quark model is very special: the spin of Lambda is completely carried by the $s$ valence quark while the $u$ and $d$ have completely no contribution. This picture is completely different from that drawn from the data of deep-inelastic lepton-nucleon scattering [1] and SU(3) flavor symmetry in hyperon decay. The deep inelastic scattering data, together with the SU(3) flavor symmetry for hyperon decay, suggest that [2] the $s$ quark carries only about 60% of the Lambda spin, while the $u$ or $d$ quark each carries about $-20\%$. Second, the polarization of the produced Lambda can easily be determined in experiments by measuring the angular distribution of the decay products. Besides, striking polarization effects have been observed for hyperons produced in unpolarized hadron-hadron collisions experiments [3]. Such effects have been observed for more than two decades and remain as a puzzle for the theoretians. Clearly, the study of the above mentioned question should be able to provide some useful information of this problem; and this makes the study even more interesting and instructive.

Polarization effects for Lambda produced in high energy reactions have been studied in different connections [2,4-12]. In some of these discussions [2,4-9], current quark picture has been used thus the picture for the spin content of Lambda drawn from the polarized deep
inelastic lepton nucleon scattering data should be applicable. But in the other [10-12], it is assumed that Lambda spin is completely determined by the $s$ quark thus picture of the static quark model should be applicable. No discussion has been made yet to the question of which of them is more suitable.

It is known from the standard model of electroweak interaction that the $s$ quark produced in $e^+e^-$ annihilation at high energies is longitudinally polarized [13]. Hence it is expected [13] that the Lambda which contains this $s$ quark should also be longitudinally polarized and such Lambda polarization can be measured in experiments. Theoretically, this Lambda polarization can be calculated and the results should be quite different using the above mentioned two different pictures for the spin contents of Lambda. Hence, measurements of the polarization should be able to show which picture is more suitable in describing such spin effects. A calculation of the longitudinal Lambda polarization in $e^+e^-$ annihilation at the $Z$-pole has been made [14] using the picture of the static quark model, but no calculation has been made yet [15] using the picture drawn from the data of deep inelastic scattering.

More recently, longitudinal Lambda polarization in $e^+e^-$ annihilation at the $Z$-pole (which is therefore dominated by those from $Z$ decay) has been measured [16] by the ALEPH Collaboration at CERN. A comparison of the data [16] with the calculated results of [14] has been made [16], and they are in good agreement with each other. This means the above mentioned static quark model picture for Lambda spin structure is consistent with the data [16]. Does this mean that the static (or constituent) quark model but not that from deep inelastic lepton nucleon scattering should be used in the fragmentation process? To answer this question, calculations have to be carried out using a picture which is consistent with the deep inelastic scattering data so that a comparison with the ALEPH data [16] can be made.

In this note, we present the results of such a calculation and compare them with those obtained in [14] and the ALEPH data [16]. The calculations have been carried out using the same method as that in [14]. Here, we first consider the contribution of the Lambdas which are directly produced in the hadronization process. Such hyperons are divided into
two groups: those which contain the leading $u$, $d$ or $s$ quark and those which do not. The latter kind of Lambdas, i.e. those which do not contain the initial $u$, $d$ or $s$ quark from $e^+e^-$ annihilation, are assumed [14] not to be polarized [17] but the former kind can be polarized since the initial $u$, $d$ or $s$ quark is longitudinally polarized. The polarization of such Lambda is different in different pictures for the spin structure of Lambda. More precisely, the polarization of such Lambda is equal to the fraction of spin carried by the quark which has the flavor of the initial quark multiplied by the polarization of this initial quark. The polarizations of the initial quarks from $e^+e^-$ annihilations are determined by the standard model for electroweak interactions, and given by [13],

$$
P_f = -\frac{A_f(1 + \cos^2 \theta) + B_f \cos \theta}{C_f(1 + \cos^2 \theta) + D_f \cos \theta}
$$

where $\theta$ is the angle between the outgoing quark and the incoming electron, the subscript $f$ denotes the flavor of the quark and

$$
A_f = 2a_f b_f (a_f^2 + b_f^2) - 2(1 - \frac{m_Z^2}{s})Q_f a b_f,
$$

$$
B_f = 4ab(a_f^2 + b_f^2) - 2(1 - \frac{m_Z^2}{s})Q_f a b_f,
$$

$$
C_f = \frac{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}{s^2}Q_f^2 + (a^2 + b^2)(a_f^2 + b_f^2) - 2(1 - \frac{m_Z^2}{s})Q_f a b_f,
$$

$$
D_f = 8ab a_f b_f - 4(1 - \frac{m_Z^2}{s})Q_f b b_f,
$$

where $m_Z$ and $\Gamma_Z$ are the mass and decay width of $Z$; $a$, $b$, $a_f$ and $b_f$ are the axial and vector coupling constants of electron and quark to $Z$ boson, which are functions of the Weinberg angle $\theta_W$. (See table 1 in [13]). Averaging over $\theta$, we obtain $P_f = -0.67$ for $f = u, c, t$ and $P_f = -0.94$ for $f = d, s, b$.

The fractional contributions ($\Delta U_\Lambda$, $\Delta D_\Lambda$, and $\Delta S_\Lambda$) of different flavors ($u$, $d$ and $s$) to Lambda spin are calculated using the deep inelastic lepton-nucleon scattering data on $\Gamma_1 \equiv \int_0^1 g_1(x)dx$ [where $g_1(x)$ is the spin-dependent structure] and those for the constants
The detailed procedure of extracting the $\Delta U_\Lambda$, $\Delta D_\Lambda$, and $\Delta S_\Lambda$ from the data for $\Gamma^p_1$ for proton, and those for $F$ and $D$ is summarized in the Appendix. The obtained results are given in Table 1.

We next consider the contribution of those Lambda’s from the decay of other hyperons in the same octet as Lambda. These hyperons can also be polarized if they contain the initial $u$, $d$ or $s$ quark, and the polarization can be transferred to Lambda’s in the decay processes. The polarization of such Lambda is thus equal to the polarization of the hyperon multiplied by the probability for the polarization to be transferred to Lambda. Hence, to calculate such contribution, we need to calculate the polarization of the such hyperon before it decays and the probability for the polarization to be transferred to Lambda in the decay process. The polarization of hyperon in the same octet as Lambda can easily be calculated using exactly the same method as that for Lambda. There are three such hyperons, i.e. $\Sigma^0$, $\Xi^0$ and $\Xi^-$ which may decay into $\Lambda$. We calculated the fractional contributions of different flavors of quarks to their spins in the way described in the Appendix and obtained the results shown in Table 1. These results are as reliable as those for Lambda, and are therefore [2] as reliable as those for the nucleons. $\Sigma^0$ decay into $\Lambda$ by emitting a photon, i.e., $\Sigma^0 \rightarrow \Lambda \gamma$. Whether the polarization of $\Sigma^0$ is transferred to the produced Lambda in this decay process has been discussed in [18]. It has been shown that, on the average, the produced $\Lambda$ is also polarized (in the opposite direction as $\Sigma^0$) if $\Sigma^0$ was polarized before its decay, and the polarization is $-1/3$ of that of the $\Sigma^0$. The hyperon $\Xi$ decays into $\Lambda$ through $\Xi \rightarrow \Lambda \pi$, which is a parity non-conserving decay and is dominated by S-wave. The polarization of the produced $\Lambda$ is equal to that of the $\Xi$ multiplied by a factor $(1 + 2\gamma)/3$, where $\gamma$ can be found in review of particle properties [19] as $\gamma = 0.87$.

It is now still impossible to calculate the polarizations of the produced hyperons that belong to the baryon decuplet in a way consistent with that for those in the octet. This is because no deep-inelastic scattering data on any one of such baryons is available. It is therefore impossible to calculate the fractional contributions of different flavors to the spin of such hyperon. Hence, it is impossible to estimate the contributions of decays of
such hyperons which contain the initial $u$, $d$ or $s$ quark to the polarization of Lambda in the final state of $e^+e^-$ annihilations in the same way as that for the octet hyperons. Qualitative analysis suggests that the influences of such hyperons should not be very large. This is because, first, their production rates are relatively small, and second, since the mass differences between such hyperons and Lambda are relative large, their decays contribute mainly to Lambda’s in the central region of the $e^+e^-$ annihilation (i.e. those with relatively small momenta). This region is dominated by those Lambda’s which do not contain the initial quark and are unpolarized.

To make a quantitative estimation, we need a hadronization model to calculate all the different contributions to the Lambda’s from all the different sources discussed above. For this purpose, we used, as in [14], the LUND model [20] as implemented by JETSET [21]. We explicitly calculated the different contributions, and obtained the results shown in Fig.1. We see in particular that the contribution from the decay of the decuplet hyperons is indeed relatively small. We calculated Lambda polarization $P_\Lambda$ for the following two cases: In the first case, we completely neglect the contribution from decuplet hyperon decay to $P_\Lambda$ and obtained the results shown by the solid line in Fig.2. In the second case, we used the results for the polarization of the decuplet hyperons obtained from the static quark model as an approximation to estimate the contribution of such hyperon decay to $P_\Lambda$. We added the results to $P_\Lambda$ and obtained the dashed line in Fig.2. For comparison, we included in the figure also the results from the static quark model without (dotted line) or with (dash-dotted line) the contributions from decuplet hyperon decay.

From these results, we see that there is indeed a significant difference between those obtained in [14] based on the picture of the static quark model and those obtained in the present estimation using a picture based on the polarized deep-inelastic lepton-nucleon scattering data [1] and SU(3) flavor symmetry for hyperon decay. It seems that the ALEPH data [16] favors the former but cannot exclude the latter since the error bars are still too large. We see also that, although the influence from the decuplet is indeed relative small, but it is not negligible in particular for moderate $z$. We can also see that further measurements
of \( P_\Lambda \) with higher accuracy are needed to distinguish between these two kinds of models. The large \( z \) region is most suitable for such a study since in this region not only the magnitude of \( P_\Lambda \) itself is large but also the difference between the prediction of the two different models is large. It will be also particularly helpful to measure the polarization only for those Lambda’s which are not decay products of decuplet hyperons.

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Appendix

The way of extracting the fractional contributions of quarks of different flavors to the spin of a baryon in the \( J^P = \frac{1}{2}^+ \) octet from \( \Gamma_1^p \equiv \int_0^1 g_1^p(x)dx \) obtained in deep-inelastic lepton-proton scattering experiments and the constants \( F \) and \( D \) obtained from hyperon decay experiments is now in fact quite standard [1]. We present in this appendix the main ingredients used in this procedure in order to remind the readers of the assumptions one uses here and to show how one extends it to obtain the results for other baryons in the same octet as Lambda.

According to the quark parton model [22], we have

\[
g_1(x) = \frac{1}{2} \sum_q e_q^2 [\Delta q(x) + \Delta \bar{q}(x)],
\]

where \( \Delta q(x) = q^+(x) - q^-(x), \Delta \bar{q}(x) = \bar{q}^+(x) - \bar{q}^-(x) \) is the difference between the number density of quarks (antiquarks) of flavor \( q \) polarized in the same, and that of those polarized in the opposite, longitudinal direction as the nucleon; \( e_q \) is the electric charge of the quark in unit of electron charge. Denoting \( \Delta Q \equiv \int_0^1 [\Delta q(x) + \Delta \bar{q}(x)]dx \), we obtain,

\[
\Gamma_1 = \int_0^1 g_1(x)dx = 2 \sqrt{\frac{2}{3}} \Delta Q_0 + \frac{1}{6} \Delta Q_3 + \frac{1}{6\sqrt{3}} \Delta Q_8,
\]

where \( \Delta Q_0 \equiv \frac{1}{12} \sqrt{\frac{2}{3}}(\Delta U + \Delta D + \Delta S) \); \( \Delta Q_3 \equiv \frac{1}{2}(\Delta U - \Delta D) \), and \( \Delta Q_8 \equiv \frac{\sqrt{6}}{6}(\Delta U + \Delta D - 2\Delta S) \) are the singlet, triplet, and octet terms. The singlet term \( \Delta Q_0 \) is proportional to the fraction
\[ \Sigma = \Delta U + \Delta D + \Delta S \] of spin of the nucleon carried by the light quarks. If SU(3) flavor symmetry is hold, \( \Sigma \) should be the same for baryons in the same SU(3) multiplet. Using the method of operator product expansion, one relates the \( \Delta Q_a \)'s to the matrix elements of local operators \( A^\mu_a = \bar{q}\gamma_\mu \gamma_5 \frac{\lambda^a}{2} q \) by

\[ 2MS^\mu \Delta Q_a = \langle P, S | A^\mu_a | P, S \rangle \]  \hspace{1cm} (8)

where \( a = 0, 3, \) or 8; \( S, P \) and \( M \) are spin, momentum and mass of the baryon \( B \) and \( \lambda^a \) are the usual SU(3) matrices acting in the flavor space. The matrix elements of the local operators are at \( Q^2 = 0 \) and can be measured in hyperon decays. Under the assumption that SU(3) flavor symmetry is valid among the baryons, one can use the Wigner-Eckart theorem and obtains [23],

\[ \langle \psi_b | A^\mu_a | \psi_c \rangle = 2MS^\mu (if_{abc}F + d_{abc}D) \]  \hspace{1cm} (9)

where \( \psi_b \) and \( \psi_c \) are the basis of the eight dimensional representation of SU(3) with spin \( S \); \( f_{abc} \) and \( d_{abc} \) are the totally antisymmetric and symmetric structure constants for SU(3) group; and the quantities \( F \) and \( D \) are constants which are independent of the particular states in the same multiplet.

The wave functions of the baryons \( B \)'s in the baryon octet can be expressed in terms of the basis vectors \( \psi_a \) of the eight dimensional representation of SU(3). For example, \( |p\rangle = \frac{1}{\sqrt{2}}(\psi_4 - i\psi_5), |\Lambda\rangle = \psi_8, |\Sigma^0\rangle = \psi_3, |\Xi^0\rangle = \frac{1}{\sqrt{2}}(\psi_6 + i\psi_7), |\Xi^-\rangle = -\frac{1}{\sqrt{2}}(\psi_4 + i\psi_5). \) Using these wave-functions and Eq.(9) we can calculate the matrix elements of the axial currents between the different baryons in the octet and obtain the \( \Delta Q_3 \) and \( \Delta Q_8 \) as functions of \( F \) and \( D \). In this way, we obtain \( \Delta Q_3^p = \frac{1}{2}(D + F) \) and \( \Delta Q_8^p = \frac{1}{2\sqrt{3}}(3F - D) \). Using the experimental data for \( F \) and \( D \), and that for \( \Gamma_1^p \), we obtain from Eq.(11) the \( \Delta Q_0 \) thus \( \Sigma \) for proton, which should be the same for all the baryons in the octet. We then use this \( \Sigma \) and data for \( F \) and \( D \) to calculate the \( \Delta U, \Delta D \) and \( \Delta S \) for other baryons. The expressions of \( \Delta U, \Delta D \) and \( \Delta S \) in terms of \( \Sigma, F \) and \( D \), and their numerical results obtained using the data are listed in Table 1.
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Table 1. Fractional contributions $\Delta U$, $\Delta D$ and $\Delta S$ of the light flavors to the spin of baryons in the $J^P = \frac{1}{2}^+$ octet calculated using the static quark model (static QM) and those obtained using the data for deep inelastic lepton-nucleon scattering and those for hyperon decay under the assumption that SU(3) flavor symmetry is valid. The first column shows the obtained expressions in terms of $\Sigma$, $F$ and $D$. The static QM results are obtained by inserting $\Sigma = 1$, $F = \frac{2}{3}$ and $D = 1$ into these expressions and those in the third column are obtained by inserting $\Sigma = 0.28$, obtained from deep inelastic lepton-nucleon scattering experiments [1], and $F + D = g_A/g_V = 1.2573$, $F/D = 0.575$ obtained [19,24] from the hyperon decay experiments.

|       | $\Lambda$ |       | $\Sigma^0$ |
|-------|-----------|-------|------------|
|       | static QM | DIS data | static QM | DIS data |
| $\Delta U$ | $\frac{1}{3}(\Sigma - D)$ | 0 | -0.17 | $\frac{1}{3}(\Sigma + D)$ | 2/3 | 0.36 |
| $\Delta D$ | $\frac{1}{3}(\Sigma - D)$ | 0 | -0.17 | $\frac{1}{3}(\Sigma + D)$ | 2/3 | 0.36 |
| $\Delta S$ | $\frac{1}{3}(\Sigma + 2D)$ | 1 | 0.62 | $\frac{1}{3}(\Sigma - 2D)$ | -1/3 | -0.44 |

$\Xi^0$

|       | static QM | DIS data | static QM | DIS data |
|-------|-----------|-------|------------|
| $\Delta U$ | $\frac{1}{3}(\Sigma - 2D)$ | -1/3 | -0.44 | $\frac{1}{3}(\Sigma + D) - F$ | 0 | -0.10 |
| $\Delta D$ | $\frac{1}{3}(\Sigma + D) - F$ | 0 | -0.10 | $\frac{1}{3}(\Sigma - 2D)$ | -1/3 | -0.44 |
| $\Delta S$ | $\frac{1}{3}(\Sigma + D) + F$ | 4/3 | 0.82 | $\frac{1}{3}(\Sigma + D) + F$ | 4/3 | 0.82 |
Figures

Fig.1: Fractional contributions to Lambdas produced in $e^+e^-$ annihilation at LEP energy from different sources: The solid line denotes those Lambdas which are produced directly and contain the initial $u$, $d$ or $s$ quark; the dash-dotted and dashed lines are those from decay of octet ($\Sigma^0$, $\Xi$) and decuplet hyperons ($\Sigma^*$, $\Xi^*$) which contain the initial quarks.

$z \equiv 2p/\sqrt{s}$, where $p$ is the momentum of the produced Lambda and $\sqrt{s}$ is the total center of mass energy of the $e^+e^-$ system.

Fig.2: Longitudinal polarization of Lambda, $P_\Lambda$, from $e^+e^-$ annihilation at LEP energy as a function of $z$. (See text for more details).
