Revisiting the Higgs sector of a 3-3-1 model in light of the 126 GeV signal at the LHC

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We explore in this paper certain phenomenological consequences regarding the scalar sector of an SU(3)_c ⊗ SU(3)_L ⊗ U(1)_X gauge model with right-handed neutrinos. Our analysis is performed in a particular theoretical approach of treating electro-weak gauge models with spontaneous symmetry breaking in which a single free parameter (a) finally remains to be tuned, once all the Standard Model phenomenology is recovered. Although it predicts a unique surviving Higgs (regardless of the involved electro-weak gauge group SU(N)_L ⊗ U(1)_X), the general method proves itself flexible enough to accommodate the traditional approach in which three scalar triplets exhibit three breaking scales for supplying masses to all the gauge bosons and fermions in the model and to allow for three surviving neutral Higgs bosons in the end. The novelty here is that, when reaching the mass basis, only two physical scalars—H_1 (specific to this model) and H_2 (the SM-like one) with m(H_1) ≫ m(H_2)—take part in various interactions (whose couplings are explicitly calculated), while H_3 comes out massless and sterile. A plausible phenomenological scenario implying the recent Higgs signal—m(H_{SM}) = 126 GeV (announced in the summer of 2012 by both the ATLAS and CMS collaborations at the CERN-LHC)—is then discussed within the framework of our method.

1. Introduction

The Standard Model (SM) [1–3]—based on the gauge group SU(3)_c ⊗ SU(2)_L ⊗ U(1)_Y undergoing a spontaneous symmetry breaking (SSB) in its electro-weak sector—has established itself as a successful theory in explaining the strong, weak, and electromagnetic forces acting among the known elementary particles. Nevertheless, some recent experimental evidence—regarding mainly the neutrino oscillation (see Ref. [4] and references therein)—definitely calls for certain extensions of the SM. In order to cover the new and richer phenomenology, any realistic theoretical model must conceive a consistent device responsible for generating the masses of both fermion and boson sectors. In the SM this role is accomplished by the so-called “Higgs mechanism” [5–9], which so far seems to be the paradigmatic procedure to give particles their appropriate masses, while the renormalizability is kept valid throughout. The Higgs mechanism enforces a suitable SSB up to the electromagnetic U(1)_{em} group, which is thus regarded as the residual symmetry of the model. Despite its success in the gauge boson sector, this procedure implies not only a great number of Yukawa coupling coefficients (undetermined on theoretical grounds) in the fermion sector, but also the existence of a still-elusive neutral scalar particle: the Higgs boson.

Among the possible extensions of the SM, the so-called “3-3-1” class of models [10–14] emerged two decades ago and has since earned a wide reputation due to a systematic and compelling study
of its phenomenology. It is essentially based on the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ gauge group that undergoes in its electro-weak sector an SSB up to the universal electromagnetic $U(1)_{em}$ symmetry, like in the SM. The discrimination among various models in this class [15–17] can be done on the particle content criterion, each model supplying in its own right some new and distinct phenomenological consequences. We deal here with a particular model [13,14] that includes (along with the left-handed charged lepton) both the left-handed and right-handed corresponding neutrinos in the triplet representations of the fermion sector. Besides recovering all the particles coming from the SM (three charged leptons, three neutrinos, six quarks, and four gauge bosons), it predicts the occurrence of three new exotic quarks and five new gauge bosons. Apart from other versions [10,11] that claim the existence of exotic electric charges (quarks with $\pm 5e/3$, $\pm 4e/3$ or bosons with $\pm 2e$), the version under consideration here implies only ordinary electric charges, even for the exotic particles!

A few words about the method employed here to “solve” this class of models. Proposed initially by Cotăescu [18], it essentially consists of a general algebraical procedure in which electro-weak gauge models with high symmetries ($SU(N)_L \otimes U(1)_X$) achieve their SSB in only one step up to $U(1)_{em}$ by means of a special Higgs mechanism. This predicts a single physical scalar remaining in the spectrum and supplies the exact expressions for the masses and neutral charges of all particles involved in the model. Here we work out the modified original version of the procedure and use its prescriptions as an efficient tool in investigating electro-weak 3-3-1 gauge models. It can successfully accommodate the traditional approach that assumes three neutral Higgs scalars. To this end a proper parametrization of the scalar sector is paired by an orthogonal restriction among scalar multiplets to warrants for only three Higgs scalars surviving the SSB, while all other degrees of freedom (Goldstone bosons) are “eaten” by the gauge bosons to become massive. The advantage of our approach resides in the fact that a realistic boson mass spectrum appears to be simply a matter of tuning a single remaining free parameter, here called $a$.

The purpose of this paper is to give an estimate of the properties of the surviving neutral Higgs bosons from a 3-3-1 model with right-handed neutrinos (331RHN) based on this particular approach of tuning a single free parameter [19,20] to get mass spectra, once the SM phenomenology is recovered. We focus on the Higgs couplings such as $HVV$ (where $V$ denotes any vector gauge boson of the model, namely $W$, $Z$, $Z'$, $X$, or $Y$) and $H\bar{f}f$ (with $f$ any fermion; let it be a lepton or a quark), in view of obtaining possible signatures at the LHC. The recent $m(H_{SM}) = 126$ GeV announced by the ATLAS [21] and CMS [22] collaborations at the CERN-LHC could be a natural outcome of our approach; therefore, special attention will be paid to its consequences within this framework.

The paper is organized as follows. In Sect. 2 we offer a brief review of the 3-3-1 gauge model at hand. The scalar sector (with its Higgses) is discussed in Sect. 3, while in Sect. 4 certain numerical estimates and plausible scenarios are considered. Section 5 deals properly with the Higgs phenomenology. Section 6 is reserved for sketching our conclusions.

2. Brief review of the model

The study of the 331RHN models has revealed a rich phenomenology [23–34], including flavor changing neutral current (FCNC) processes, proper GIM mechanisms, a restrictive $Z'$-boson phenomenology, exotic $T$-quark properties, and a strong CP problem that can be elegantly solved due to the natural Peccei–Quinn symmetry possessed by the quark sector of this class of models. Some suitable solutions for the neutrino mass issue [35–45] have been successfully proposed. With regard to the scalar sector and Higgs phenomenology, a series of papers [46–51] has been published too. However, we consider it worthwhile reviewing once more the main features of constructing a 331RHN
model in order to establish the framework in which our theoretical calculations are performed. It is supplied by the gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ undergoing an SSB in its electro-weak sector.

Fermions. The main pieces are the irreducible representations of the gauge group, which are assigned to fermion left-handed multiplets (triplets, in the case at hand). The corresponding right-handed singlets are nothing but 1D representations of the same group, since $(f_R)^c = (f^c)_L$ where the upper index $c$ denotes the charge conjugation and left-handedness (right-handedness) is achieved by the action of the well known projectors $P_{R,L} = \frac{1 \pm \gamma^5}{2}$. Briefly, the fermion content can be put as follows:

\[ f_{aL} = \begin{pmatrix} v^c_a \\ v_a \\ e_a \end{pmatrix}_L \sim (1, 3, -1/3) \quad e_{aR} \sim (1, 1, -1) \quad (1) \]

Lepton families

Quark families

\[ Q_{iL} = \begin{pmatrix} D_i \\ -d_i \\ u_i \end{pmatrix}_L \sim (3, 3^* , 0) \quad Q_{3L} = \begin{pmatrix} U_3 \\ u_3 \\ d_3 \end{pmatrix}_L \sim (3, 3, +1/3) \quad (2) \]

\[ d_{iR}, d_{3R} \sim (3, 1, -1/3) \quad u_{iR}, u_{3R} \sim (3, 1, +2/3) \quad (3) \]

\[ U_{3R} \sim (3, 1, +2/3) \quad D_{iR} \sim (3, 1, -1/3) \quad (4) \]

with $i = 1, 2$.

The electric charge operator of this particular 331RHN gauge model—when following the prescriptions of the Cotăescu method [18]—is expressed by a linear combination of generators in the manner $Q^\rho = \frac{2}{\sqrt{3}} T^\rho_8 + X$ for each representation $\rho$, as was worked out in Ref. [20].

In the representations displayed above, one has to assume that two generations of quarks transform differently from the third one in order to cancel all the axial anomalies (by an interplay between families, although each one remains anomalous by itself). In this way one prevents the model from compromising its renormalizability by triangle diagrams. The capital letters denote the exotic quarks ($D_1$, $D_2$, and $U_3$) included in each family. Many authors consider that $U_3 = T$ and $D_i = D, S$ as a possible explanation for the unusual heavy masses of the third generation of quarks (the one to transform differently from the other two). For reasons evident in the following sections, we adopt the same identification.

Gauge bosons. The gauge bosons of the model are connected to the generators of the electro-weak $su(3)$ Lie algebra, expressed by the usual Gell–Mann matrices $T_a = \lambda_a/2$. So, the Hermitian diagonal generators of the Cartan sub-algebra are

\[ D_1 = T_3 = \frac{1}{2} \text{Diag}(1, -1, 0), \quad D_2 = T_8 = \frac{1}{2\sqrt{3}} \text{Diag}(1, 1, -2). \quad (5) \]
In this basis the gauge fields are $A^0_\mu$ (corresponding to $U(1)_X$) and $A_\mu \in su(3)$, so that the gauge fields can be represented as

$$A_\mu = \frac{1}{2} \begin{pmatrix} A^3_\mu + A^8_\mu / \sqrt{3} & \sqrt{2} X_\mu & \sqrt{2} Y_\mu \\ \sqrt{2} X^*_\mu & -A^3_\mu + A^8_\mu / \sqrt{3} & \sqrt{2} W^*_\mu \\ \sqrt{2} Y^*_\mu & \sqrt{2} W^*_\mu & -2 A^8_\mu / \sqrt{3} \end{pmatrix}, \quad (6)$$

where $\sqrt{2} W^\pm_\mu = A^0_\mu \mp i A^7_\mu$, $\sqrt{2} Y^\pm_\mu = A^4_\mu \pm i A^5_\mu$, and $\sqrt{2} X_\mu = A^1_\mu - i A^2_\mu$, respectively. One notes that, apart from the charged Weinberg bosons ($W$) from the SM, there are two new complex boson fields, $X$ (neutral) and $Y$ (charged), as off-diagonal entries in Eq. (6).

The diagonal Hermitian generators must provide the model with the neutral gauge bosons $A^c_{\mu}$, $Z_\mu$, and $Z'_\mu$. Therefore, on the diagonal terms in Eq. (6), a generalized Weinberg transformation (gWt) must be performed in order to separate the massless electromagnetic field from the other two neutral massive fields. One of the two massive neutral fields will be identified with the $Z^0$-boson of the SM. The details of the general procedure with gWt can be found in Ref. [18] and its concrete realization in the model of interest here in Refs. [19,20].

**Scalar sector.** In the general method [18], the scalar sector of any $SU(n)_L \otimes U(1)_X$ electroweak gauge model must consist of $n$ multiplets $\phi^{(1)}, \phi^{(2)}, \ldots, \phi^{(n)}$, each consisting of $n$ complex scalar-fields. The main hypothesis of the general method assumes that the multiplets must satisfy the orthogonal condition $\phi^{(i)} + \phi^{(j)} = \eta^2 \delta_{ij}$ in order to reduce the number of real parameters from $2n^2$ to $n^2$ in the scalar sector and thus eliminate unwanted degrees of freedom (Goldstone bosons) that could eventually survive the SSB. Here $\varphi \sim (1, 1, 0)$ is a gauge-invariant neutral field variable acting as a norm in the scalar space and $n$ is the dimension of the fundamental irreducible representation of the $SU(n)_L$ group. The parameter matrix $\eta = (1 - \eta_0) \text{Diag}(\eta_1, \eta_2, \ldots, \eta_n)$ with the property $Tr \eta^2 = 1 - \eta^2_0$ is in our approach [18] a key ingredient too: it is introduced in order to obtain a suitable non-degenerate boson mass spectrum with a unique breaking scale ($\varphi$), successively the SSB. Obviously, $\eta_0, \eta_i \in [0, 1)$. Then, the Higgs Lagrangian density (Ld) reads:

$$L_H = \frac{1}{2} \eta^2_0 \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \sum_{i=1}^n \eta^2_i \left( D_\mu \phi^{(i)} \right)^2 + \left( D^\mu \phi^{(i)} \right)^2 - V(\phi^{(i)}) \quad (7)$$

where $D_\mu \phi^{(i)} = \partial_\mu \phi^{(i)} - ig A_\mu \phi^{(i)} + g' y^{(i)} A^0_\mu \phi^{(i)}$ acts as covariant derivatives of the model. $g$ and $g'$ are the coupling constants of the groups $SU(n)_L$ and $U(1)_X$ respectively. The real characters $y^{(i)}$ stand as a kind of hyper-charge of the new theory and they must fulfill the condition $Tr (Y) = 0$.

The particular Higgs mechanism with the above ingredients was designed as a proper mathematical tool to achieve the SSB and mimic the SM outcome, namely a unique physical scalar Higgs field $\varphi$ in the spectrum. The latter introduces in the theory the overall breaking scale ($\varphi$).

For the particular 331RHN model under consideration here, the most general choice of parameters is given by the matrix $\eta^2 = (1 - \eta^2_0) \text{Diag} \left[ 1 - a, \frac{1}{2} (a - b), \frac{1}{2} (a + b) \right]$. It obviously meets the trace condition required by the general method for any $a, b \in [0, 1)$. After imposing the phenomenological condition $M^2_Z = M^2_W / \cos^2 \theta_W$ (established at the SM level), the procedure of diagonalizing the neutral boson mass matrix (for details, see Refs. [19,20]) eliminates one parameter and thus the parameter matrix becomes $\eta^2 = (1 - \eta^2_0) \text{Diag} \left[ 1 - a, a \frac{(1 - \tan^2 \theta_W)}{2}, a \frac{1}{2 \cos^2 \theta_W} \right]$. 

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3. Higgs bosons of the 3-3-1 model

3.1. Spontaneous symmetry breaking

Scalar field redefinition. In the following we accommodate the non-canonical general method [18], briefly reviewed above, with the traditional approach, in which there are 3 distinct breaking scales resulting from the potential minimum condition. Each of these is determined by the vacuum expectation values (VEV) of the three scalar triplets. For this purpose we redefine the initial scalar triplets in the following way:

\[ \phi^{(1)} \rightarrow \eta_1 \phi^{(1)} \equiv \rho, \quad \phi^{(2)} \rightarrow \eta_2 \phi^{(2)} \equiv \chi, \quad \phi^{(3)} \rightarrow \eta_3 \phi^{(3)} \equiv \phi, \]

or, in equivalent notation (with the upper index showing the electric charge of the field it labels):

\[ \rho = \begin{pmatrix} \rho_1^0 \\ \rho_2^0 \\ \rho_3^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \\ \chi_3^- \end{pmatrix} \sim (1, 3, -1/3), \quad \phi = \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \phi_3^- \end{pmatrix} \sim (1, 3, +2/3). \]

Obviously, these new fields obey orthogonal relations in a new form, namely:

\[ \rho^+ \rho = \eta_1^2 \psi^2, \quad \chi^+ \chi = \eta_2^2 \psi^2, \quad \phi^+ \phi = \eta_3^2 \psi^2. \]

Scalar potential. The simplest potential that preserves renormalizability can now be put in the following general form:

\[ V = \mu_1^2 \rho^+ \rho + \mu_2^2 \chi^+ \chi + \mu_3^2 \phi^+ \phi + \lambda_1 (\rho^+ \rho)^2 + \lambda_2 (\chi^+ \chi)^2 + \lambda_3 (\phi^+ \phi)^2 \\
+ \lambda_4 (\rho^+ \rho) (\chi^+ \chi) + \lambda_5 (\rho^+ \rho) (\phi^+ \phi) + \lambda_6 (\phi^+ \phi) (\chi^+ \chi) \\
+ \lambda_7 (\rho^+ \chi) (\rho^+ \phi) + \lambda_8 (\phi^+ \phi) (\chi^+ \phi) + \lambda_9 (\phi^+ \chi) (\chi^+ \phi). \tag{11} \]

The above potential could be added with trilinear terms as \( f \varepsilon^{ijk} \rho_i \chi_j \phi_k \) h.c., allowed by the gauge symmetry of the model. This can be done in order to get the most general case (see Refs. [48,49]). However, such terms in the potential vanish if one enforces some discrete symmetries, for instance \( \rho \rightarrow -\rho, \chi \rightarrow \chi, \phi \rightarrow \phi \) (as was considered in Ref. [50]). For the sake of simplicity, we adopt here the latter restriction and deal with the simpler potential in Eq. (11). It is simplified even more by the orthogonal assumption, so that the third row in Eq. (11) erases itself too.

Unitary gauge. One can easily observe that the SSB is accomplished in the unitary gauge by three VEVs, as follows:

\[ \begin{pmatrix} \eta_1 \langle \varphi \rangle + H_{\rho} \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \eta_2 \langle \varphi \rangle + H_{\chi} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ \eta_3 \langle \varphi \rangle + H_{\phi} \end{pmatrix}, \]

with the overall VEV

\[ \langle \varphi \rangle = \frac{\sqrt{\mu_1^2 \eta_1^2 + \mu_2^2 \eta_2^2 + \mu_3^2 \eta_3^2}}{\sqrt{2 (\lambda_1 \eta_1^4 + \lambda_2 \eta_2^4 + \lambda_3 \eta_3^4) + \lambda_4 \eta_1^2 \eta_2^2 + \lambda_5 \eta_1^2 \eta_3^2 + \lambda_6 \eta_2^2 \eta_3^2}} \]

resulting from the minimum condition applied to the potential (11). We note that, in the original method of Cotăescu, \( \varphi \) would have been the unique Higgs boson in the model and its VEV \( \langle \varphi \rangle \) the unique breaking scale. By redefining the scalar mutiplets, the canonical outcome with three surviving
Scalars in the spectrum naturally occurs. They are, in order, \( H_\rho, H_\chi, H_\phi \) in the weak basis. We now look for their couplings. To this end, one can write explicitly the terms in the potential \( V \) after SSB has taken place:

\[
V = -\left[ \mu_1^2 (\eta_1 \langle \varphi \rangle + H_\rho)^2 + \mu_2^2 (\eta_2 \langle \varphi \rangle + H_\chi)^2 + \mu_3^2 (\eta_3 \langle \varphi \rangle + H_\phi)^2 \right] \\
+ \left[ \lambda_1 (\eta_1 \langle \varphi \rangle + H_\rho)^4 + \lambda_2 (\eta_2 \langle \varphi \rangle + H_\chi)^4 + \lambda_3 (\eta_3 \langle \varphi \rangle + H_\phi)^4 \right] \\
+ \left[ \lambda_4 (\eta_1 \langle \varphi \rangle + H_\rho)^2 (\eta_2 \langle \varphi \rangle + H_\chi)^2 + \lambda_5 (\eta_1 \langle \varphi \rangle + H_\rho)^2 (\eta_3 \langle \varphi \rangle + H_\phi)^2 \right] \\
+ \lambda_6 (\eta_2 \langle \varphi \rangle + H_\chi)^2 (\eta_3 \langle \varphi \rangle + H_\phi)^2. \tag{14}
\]

### 3.2. Scalar field couplings

The next step is to identify for each Higgs its own coupling terms. These are, in order:

(i) **linear terms (must be absent—as in the SM—so one gets three constraints on the parameters):**

\[
H_\rho : -\mu_1^2 + \left( 2\lambda_1 \eta_1^2 + \lambda_4 \eta_2^2 + \lambda_5 \eta_3^2 \right) \langle \varphi \rangle^2 = 0 \\
H_\chi : -\mu_2^2 + \left( 2\lambda_2 \eta_2^2 + \lambda_4 \eta_1^2 + \lambda_6 \eta_3^2 \right) \langle \varphi \rangle^2 = 0 \\
H_\phi : -\mu_3^2 + \left( 2\lambda_3 \eta_3^2 + \lambda_5 \eta_1^2 + \lambda_6 \eta_2^2 \right) \langle \varphi \rangle^2 = 0; \tag{15}
\]

(ii) **HH quadratic terms:**

\[
H_\rho H_\rho : -\mu_1^2 + \left( 6\lambda_1 \eta_1^2 + \lambda_4 \eta_2^2 + \lambda_5 \eta_3^2 \right) \langle \varphi \rangle^2 = 4\lambda_1 \eta_1^2 \langle \varphi \rangle^2 \\
H_\chi H_\chi : -\mu_2^2 + \left( 6\lambda_2 \eta_2^2 + \lambda_4 \eta_1^2 + \lambda_6 \eta_3^2 \right) \langle \varphi \rangle^2 = 4\lambda_2 \eta_2^2 \langle \varphi \rangle^2 \\
H_\phi H_\phi : -\mu_3^2 + \left( 6\lambda_3 \eta_3^2 + \lambda_5 \eta_1^2 + \lambda_6 \eta_2^2 \right) \langle \varphi \rangle^2 = 4\lambda_3 \eta_3^2 \langle \varphi \rangle^2 \tag{16}
\]

(iii) **HHH trilinear terms:**

\[
H_\rho H_\rho H_\rho : \lambda_1 \eta_1 \langle \varphi \rangle, \quad H_\rho H_\chi H_\chi : 2\lambda_4 \eta_1 \langle \varphi \rangle, \quad H_\rho H_\phi H_\phi : 2\lambda_5 \eta_1 \langle \varphi \rangle, \\
H_\chi H_\rho H_\rho : 2\lambda_4 \eta_2 \langle \varphi \rangle, \quad H_\chi H_\chi H_\chi : 4\lambda_2 \eta_2 \langle \varphi \rangle, \quad H_\chi H_\phi H_\phi : 2\lambda_6 \eta_2 \langle \varphi \rangle, \tag{18}
\]

\[
H_\phi H_\rho H_\rho : 2\lambda_5 \eta_3 \langle \varphi \rangle, \quad H_\phi H_\chi H_\chi : 2\lambda_6 \eta_3 \langle \varphi \rangle, \quad H_\phi H_\phi H_\phi : 4\lambda_3 \eta_3 \langle \varphi \rangle;
\]

(iv) **HHHH quartic terms:**

\[
H_\rho H_\rho H_\rho H_\rho : \lambda_1, \quad H_\rho H_\chi H_\chi H_\chi : \lambda_2, \quad H_\rho H_\phi H_\phi H_\phi : \lambda_3. \tag{19}
\]

The above couplings are slightly shifted if cubic terms are considered in the potential and the discrete symmetry relaxed, but their concrete formulas will be worked out in a future paper.
3.3. Higgs masses

From the above displayed couplings, by inspecting the quadratic terms, one can identify the Higgs mass matrix as:

\[ M_H^2 = 8 \begin{pmatrix} \lambda_1 \eta_1^2 & \lambda_4 \eta_1 \eta_2 & \lambda_5 \eta_1 \eta_3 \\ \lambda_4 \eta_1 \eta_2 & \lambda_2 \eta_2^2 & \lambda_6 \eta_2 \eta_3 \\ \lambda_5 \eta_1 \eta_3 & \lambda_6 \eta_2 \eta_3 & \lambda_3 \eta_3^2 \end{pmatrix} \langle \phi \rangle^2. \quad (20) \]

In the phenomenological case of interest in the following (as one can see in Sect. 4), there is a VEV hierarchy \( \langle \rho \rangle \gg \langle \chi \rangle, \langle \phi \rangle \) that is \( \eta_1 \to 1 \) and \( \eta_2, \eta_3 \to 0 \) in our parametrization; in other words, the parameter \( a \) is very small \( a \to 0 \). This also ensures a correct boson mass spectrum \([19,20]\). In addition, one can set \( \eta_0 \equiv 0 \), for vanishing the kinetic term of the real scalar \( \phi \) in the Cot\( \tilde{a}\)escu method, since we have adapted his general method to the canonical approach and there is no need for such an extra physical field.

For the sake of simplicity, we state here certain assumptions with regard to the \( \lambda \). We can safely consider \( \lambda_1 \simeq \lambda_2 \simeq \lambda_3 \equiv \lambda \) for the diagonal entries and \( \lambda_4 \simeq \lambda_5 \simeq \lambda_6 \equiv \lambda' \) for the off-diagonal ones, so that Eq. (20) becomes:

\[ M_H^2 = 8 \begin{pmatrix} \lambda \eta_1^2 & \lambda' \eta_1 \eta_2 & \lambda' \eta_1 \eta_3 \\ \lambda' \eta_1 \eta_2 & \lambda' \eta_2^2 & \lambda' \eta_2 \eta_3 \\ \lambda' \eta_1 \eta_3 & \lambda' \eta_2 \eta_3 & \lambda \eta_3^2 \end{pmatrix} \langle \phi \rangle^2. \quad (21) \]

Consequently, the Higgs mass matrix can be computed by decoupling the \( H_1 \) from its partners \( H_2 \) and \( H_3 \), so that \( H_1 \cong H_\rho \) and its mass yields:

\[ m_1^2 \simeq 8 \lambda \eta_1^2 \langle \phi \rangle^2. \quad (22) \]

Assuming that \( H_1 \) does not mix with the two remaining Higgses, the physical basis can be reached by a simple \( 2 \times 2 \) rotation constructed just as

\[ R = \frac{1}{\sqrt{\eta_2^2 + \eta_3^2}} \begin{pmatrix} \eta_2 & \eta_3 \\ -\eta_3 & \eta_2 \end{pmatrix} \quad (23) \]

and applied on the “small” part (rows and columns 2, 3) of Eq. (20). This leads straightforwardly to:

\[ H_2 \cong \frac{\eta_2 H_\chi + \eta_3 H_\phi}{\sqrt{\eta_2^2 + \eta_3^2}} \quad (24) \]

\[ H_3 \cong \frac{-\eta_3 H_\chi + \eta_2 H_\phi}{\sqrt{\eta_2^2 + \eta_3^2}}. \quad (25) \]

Now, computing explicitly the \( 2 \times 2 \) diagonal mass matrix \( \hat{M} = RMR^T \), one gets:

\[ \hat{M} = \left( \frac{\lambda}{\eta_2^2 + \eta_3^2} \right) + \left( \frac{2 \lambda'}{\eta_2^2 + \eta_3^2} \right) \eta_2 \eta_3 \left( \lambda - \lambda' \right) \left( \eta_2^2 - \eta_3^2 \right) \langle \phi \rangle^2, \quad (26) \]

which, in order to be truly diagonal, must satisfy the condition \( \lambda = \lambda' \), since evidently \( \eta_2 \neq \eta_3 \) in our approach. Apparently, after decoupling we get a spectrum similar to that of the well known two-Higgs-doublet models.
Under these circumstances, the corresponding masses are:

\[ m_2^2 = 8\lambda a \langle \varphi \rangle^2 \quad (27) \]
\[ m_3^2 = 0. \quad (28) \]

This result is an important outcome, since there are three VEVs in the weak basis of the model but only two massive surviving physical Higgs scalars in the end. Moreover, the massless Higgs proves itself sterile. Indeed, its resulting couplings vanish and thus ensure a safe behavior with respect to the experimental data that forbid an active massless Higgs in the spectrum. By roughly inspecting the Higgs spectrum (Eqs. (22) and (27)), one notes that:

(a) \( H_1 \) (the heavier Higgs, specific to the 331RHN model) acquires its mass \( m_1 \approx 2\sqrt{2\lambda (1 - a)} \langle \varphi \rangle \), which can be approximated as \( m_1 \approx 2\sqrt{2\lambda} \langle \varphi \rangle \) when the parameter \( a \to 0 \), and

(b) \( H_2 \) (the SM-like Higgs) exhibits \( m_2 \approx 2\sqrt{2\lambda a} \langle \varphi \rangle \) approximated as \( m_2 \approx 2\sqrt{\lambda} \langle \varphi \rangle_{SM} \) under the same assumption. Here, we have anticipated and exploited the splitting relation between \( \langle \varphi \rangle \) and \( \langle \varphi \rangle_{SM} \) (see Sect. 3.4.1, Eq. (32)).

Consequently, one can express the heavier Higgs in terms of the SM-like one, namely:

\[ m(H_1) = \frac{\langle \varphi \rangle}{\langle \varphi \rangle_{SM}} m(H_{SM}). \quad (29) \]

Some recent observations at LHC [21,22] indicate that \( m(H_{SM}) \approx 126 \) GeV. This state of affairs leads to a heavy Higgs at around \( m(H_1) \approx 0.52 \langle \varphi \rangle \) TeV, if we take into consideration the established \( \langle \varphi \rangle_{SM} \approx 246 \) GeV.

Now, the overall breaking scale \( \langle \varphi \rangle \) can be tuned. If it lies above 1.2 TeV, the last result is in good agreement with the limit for Higgs masses that excludes other Higgses below 600 GeV. At the same time, Eq. (27) allows for calculating \( \lambda \approx 0.03 \).

### 3.4. Higgs interactions

In order to analyze the possible phenomenological consequences regarding the Higgs sector and its processes (Higgs production, Higgs decays) one has to observe the terms that provide us with the couplings of the Higgs bosons to the gauge bosons of the model (\( HVV \)) and to the fermions (\( H \bar{f} f \)) respectively. They can be read from the resulting Ld in unitary gauge after SSB:

\[
\mathcal{L}_H = \frac{g^2}{4} \left[ (\eta_1 \langle \varphi \rangle + H_\rho)^2 + (\eta_2 \langle \varphi \rangle + H_\chi)^2 \right] X^+ X^\mu \\
+ \frac{g^2}{4} \left[ (\eta_1 \langle \varphi \rangle + H_\rho)^2 + (\eta_3 \langle \varphi \rangle + H_\phi)^2 \right] Y^+ Y^\mu \\
+ \frac{g^2}{4} \left[ (\eta_2 \langle \varphi \rangle + H_\chi)^2 + (\eta_3 \langle \varphi \rangle + H_\phi)^2 \right] W^+ W^\mu \\
+ \frac{g^2}{8 \cos^2 \theta_W} \left[ (\eta_2 \langle \varphi \rangle + H_\chi)^2 + (\eta_3 \langle \varphi \rangle + H_\phi)^2 \right] Z Z^\mu \\
+ \frac{g^2}{8} \left( \frac{4 \cos^2 \theta_W}{3 - 4 \sin^2 \theta_W} \right) (\eta_1 \langle \varphi \rangle + H_\rho)^2 Z^\prime Z^\prime \mu
\]
The seemingly unusual couplings are precisely those that lead to the boson mass spectrum previously obtained by the author [19,20] within the framework of the general Cotăescu method with a unique surviving scalar field and a unique breaking scale.

### 3.4.1. Boson mass spectrum.

From the above expression (30) the boson mass spectrum can be inferred, simply by identifying the proper terms as the mass $L_d$:

$$\mathcal{L}_{\text{mass}} = m_W^2 W_\mu^+ W_\mu + \frac{1}{2} M_Z^2 Z^\mu Z_\mu + M_X^2 X_\mu^+ X_\mu + M_Y^2 Y_\mu^+ Y_\mu + \frac{1}{2} M_{Z'}^2 Z'^\mu Z'^\mu. \quad (31)$$

Some rapid calculus drives straightforwardly to the boson mass spectrum obtained by using the Cotăescu method in Refs. [19,20], namely:

- $M_W^2 = m^2 a$,
- $M_Z^2 = m^2 a / \cos^2 \theta_W$,
- $M_X^2 = m^2 (1 - a / 2 \cos^2 \theta_W)$,
- $M_Y^2 = m^2 [1 - a (1 - \tan^2 \theta_W) / 2]$,
- $M_{Z'}^2 = m^2 [4 \cos^2 \theta_W - a (3 - 4 \sin^2 \theta_W + \tan^2 \theta_W)] / (3 - 4 \sin^2 \theta_W)$.

We have used the notation $m^2 = g^2 \langle \phi \rangle^2 (1 - \eta_\phi^2) / 4$. The mass spectrum is now just a matter of tuning the parameter $a$ in accordance with the possible values for $\langle \phi \rangle$. As we have already mentioned, one can set parameter $\eta_\phi^2$ (of the general method) to be equal to zero, so that for our purpose here $m^2 \simeq g^2 \langle \phi \rangle^2 / 4$. The connection between the two scales is now evident by exploiting the concrete expressions for the mass of the $W$ boson: $M_W^2 = \frac{1}{4} g^2 \langle \phi \rangle^2_{\text{SM}} = \frac{1}{4} g^2 a \langle \phi \rangle^2$ (in the SM and in our approach respectively). This identity yields:

$$\sqrt{a} \langle \phi \rangle \equiv \langle \phi \rangle_{\text{SM}}. \quad (32)$$

One can remark for the neutral boson sector that diagonalizing the resulting mass matrix [19] has been performed by imposing a specific relation between $M_W$ and $M_Z$, namely $M_Z^2 = M_W^2 / \cos^2 \theta_W$. This is why one is finally left with a single free parameter to be tuned, $a$. Moreover, the rotation matrix doing the diagonalization job has established the mixing angle $\sin \phi = 1/2 \sqrt{1 - \sin^2 \theta_W}$. The traditional approach in the literature assumes $\phi$ as a free parameter restricted on experimental grounds. Here it is fixed, the role of ensuring the experimentally observed gap between $m(Z')$ and $m(Z)$ being realized exclusively by the free parameter $a$. In addition, we mention that the correct coupling match is recovered by means of our method, namely it stands $g' = g \sqrt{3} \sin \theta_W / \sqrt{3 - 4 \sin^2 \theta_W}$ (for details, see Ref. [20]). All the couplings in the neutral currents of the model (or, in other words, the neutral charges of the fermions) are exactly obtained and need no approximation. They also reproduce for the SM fermions their established values (for a detailed list, the reader is referred to the table in Ref. [20]).
### Table 1. Higgs couplings to gauge bosons.

| Couplings $HVV$ | $\times (m^2/\langle \phi \rangle)$ | $\times (2M_W^2/\langle \phi \rangle_{SM})$ |
|-----------------|---------------------------------|---------------------------------|
| $H_1 X_\mu^+ X_{\mu}^-$ | $2\eta_1$ | $\sqrt{\frac{1 - \alpha}{\alpha}}$ |
| $H_2 Y_\mu^+ Y_{\mu}^-$ | $2\eta_1$ | $\sqrt{\frac{1 - \alpha}{\alpha}}$ |
| $H_1 Z_\mu^+ Z_{\mu}^-$ | $\frac{4 \cos^2 \theta_W}{3 - 4 \sin^2 \theta_W} \eta_1$ | $\frac{2 \cos^2 \theta_W}{3 - 4 \sin^2 \theta_W} \sqrt{\frac{1 - \alpha}{\alpha}} = 0.74 \sqrt{\frac{1 - \alpha}{\alpha}}$ |
| $H_2 W_\mu^+ W_{\mu}^-$ | $2\sqrt{a}$ | $1$ |
| $H_2 Z_\mu^+ Z_{\mu}^-$ | $\frac{\sqrt{a}}{\cos^4 \theta_W}$ | $\frac{1}{2 \cos^2 \theta_W} = 0.65$ |
| $H_2 X_\mu^+ X_{\mu}^-$ | $(1 - \tan^2 \theta_W)\sqrt{a}$ | $\frac{1 - \tan^2 \theta_W}{2} = 0.35$ |
| $H_2 Y_\mu^+ Y_{\mu}^-$ | $\frac{\sqrt{a}}{\cos^4 \theta_W}$ | $\frac{1}{2 \cos^2 \theta_W} = 0.65$ |
| $H_2 Z_\mu^+ Z_{\mu}^-$ | $\frac{1 + (1 - 2 \sin^2 \theta_W)^3}{2 \cos^4 \theta_W (3 - 4 \sin^2 \theta_W)} \sqrt{a}$ | $\frac{1 + (1 - 2 \sin^2 \theta_W)^3}{4 \cos^4 \theta_W (3 - 4 \sin^2 \theta_W)} = 0.23$ |

#### 3.4.2. Higgs couplings to gauge bosons.

From Eq. (30) one infers the general expressions for the Higgs couplings to gauge bosons in the 331RHN model. They are:

$$g(H_1 VV) \simeq g(H_\rho VV)$$

$$g(H_2 VV) \simeq \frac{1}{\sqrt{2}} \left[ g(H_x VV) \sqrt{1 - \tan^2 \theta_W} + g(H_\phi VV) \frac{1}{\cos \theta_W} \right]$$

$$g(H_3 VV) \simeq \frac{1}{\sqrt{2}} \left[ g(H_\phi VV) \sqrt{1 - \tan^2 \theta_W} - g(H_x VV) \frac{1}{\cos \theta_W} \right].$$

An amazing (but welcome) result finally occurs, namely $H_3$ is sterile since all its couplings are equal to zero, according to Eq. (35). The numerical values in our scenario for $H_1$ and $H_2$ couplings appear in the right-hand column of Table 1. The couplings of the form $HHVV$ can be obtained from the ones in Eqs. (33)–(35) by simply dividing by $2 \langle \phi \rangle$.

#### 3.4.3. Yukawa sector.

There is one more sector in the 3-3-1 model of interest here—the so-called Yukawa sector—that contains all interactions of the type fermion–Higgs–fermion allowed by the gauge invariance of the theory. Successively the SSB, these terms play the role of supplying masses in the fermion sector, once some Yukawa complex matrices are introduced. Based on these Yukawa couplings, the interaction terms can be identified as well.

We must note from the very beginning that these complex matrices introduce a great amount of arbitrariness, as the masses of the heavy quarks ($U, D_1, D_2$) have still not been determined experimentally. However, under some speculative assumptions, rough estimates of these heavy quarks can be made. For the charged leptons, the Yukawa couplings are straightforwardly accessible, since there are no other charged leptons in the 331RHN model, except for those known from the SM, and no mixing occurs among them. At the same time, though we postpone the issue for a future work, we must mention that there could be some new states in the lepton sector to generate neutrino masses.
The procedure can work with this scalar content at the expense of adding a right-handed neutrino \( v_{\alpha R} \sim (1, 1, 0) \) to each fermion family without altering the renormalizability of the model. This addition can supply the canonical see-saw terms, but the details will be worked out elsewhere, since this exceeds the scope of the present paper.

Generically, the Yukawa sector is divided in two distinct parts, corresponding to leptons and quarks respectively:

\[
\mathcal{L}_Y = \mathcal{L}_Y^{\text{lepton}} + \mathcal{L}_Y^{\text{quark}}.
\]  

(36)

It reads explicitly:

\[
-\mathcal{L}_Y = Y^e \bar{f}_{\alpha L} \phi e_R + Y^\mu \bar{f}_{\mu L} \phi \mu_R + Y^\tau \bar{f}_{\tau L} \phi \tau_R
+ (Y^d_\chi)_{ij} \bar{Q}_{iL} \chi^* d_{jR} + (Y^d_\chi)_{ij} \bar{Q}_{iL} \chi^* d_{3R} + (Y^d_\chi)_{ik} \bar{Q}_{iL} \chi^* d_{kR}
+ (Y^d_\phi)_{3j} \bar{Q}_{3L} \phi d_{jR} + (Y^d_\phi)_{33} \bar{Q}_{3L} \phi d_{3R} + (Y^d_\phi)_{3k} \bar{Q}_{3L} \phi D_{kR}
+ (Y^u_\phi)_{ij} \bar{Q}_{iL} \phi^* u_{jR} + (Y^u_\phi)_{ij} \bar{Q}_{iL} \phi^* u_{3R} + (Y^u_\phi)_{ik} \bar{Q}_{iL} \phi^* u_{kR}
+ (Y^u_\chi)_{3j} \bar{Q}_{3L} \chi^* u_{jR} + (Y^u_\chi)_{33} \bar{Q}_{3L} \chi^* u_{3R} + (Y^u_\chi)_{3k} \bar{Q}_{3L} \chi^* u_{kR}
+ (Y^D_\phi)_{ik} \bar{Q}_{iL} \rho^* D_{kR} + (Y^D_\phi)_{ik} \bar{Q}_{iL} \rho^* d_{jR} + (Y^D_\phi)_{ik} \bar{Q}_{iL} \rho^* d_{3R}
+ (Y^U_\phi)_{3j} \bar{Q}_{3L} \rho^* u_{jR} + (Y^U_\phi)_{33} \bar{Q}_{3L} \rho^* u_{3R} + (Y^U_\phi)_{3k} \bar{Q}_{3L} \rho^* u_{kR} + H.c.
\]  

After SSB and assuming the family identification anticipated in Sect. 2, Eq. (36) splits into three distinct groups of terms: (i) lepton Yukawa terms, (ii) up-type Yukawa terms, and (iii) down-type Yukawa terms. In order to get the true couplings in the scalar sector one has to inspect Eq. (36) and take into consideration the expressions of the physical scalar fields, especially Eq. (24).

(1) Lepton Yukawa terms. The lepton masses are straightforward, since they need no diagonalizing.

\[
m(e) = Y^e \eta_3 \langle \varphi \rangle, \quad m(\mu) = Y^\mu \eta_3 \langle \varphi \rangle, \quad m(\tau) = Y^\tau \eta_3 \langle \varphi \rangle.
\]  

(38)

In the SM these masses are in order:

\[
m(e) = Y^e_{SM} \frac{\langle \varphi \rangle_{SM}}{\sqrt{2}}, \quad m(\mu) = Y^\mu_{SM} \frac{\langle \varphi \rangle_{SM}}{\sqrt{2}}, \quad m(\tau) = Y^\tau_{SM} \frac{\langle \varphi \rangle_{SM}}{\sqrt{2}}.
\]  

(39)

By assuming the VEV ratio provided by Eq. (32), one can get the Yukawa couplings for charged leptons in the 331RHN model:

\[
Y^e = Y^e_{SM} \cos \theta_W, \quad Y^\mu = Y^\mu_{SM} \cos \theta_W, \quad Y^\tau = Y^\tau_{SM} \cos \theta_W.
\]  

(40)

Hence, the couplings \( g \ (H_2 e \bar{e}), \ g \ (H_2 \mu \bar{\mu}), \) and \( g \ (H_2 \tau \bar{\tau}) \) can be estimated against the corresponding SM ones, namely \( g_{SM} \ (H_2 e \bar{e}), \ g_{SM} \ (H_2 \mu \bar{\mu}), \) and \( g_{SM} \ (H_2 \tau \bar{\tau}) \) respectively, by simply taking into consideration Eq. (24), which tells us which is the SM-like boson in our approach.

These couplings are:

\[
g^{H_2 e \bar{e}} = g_{SM}, \quad g^{H_2 \mu \bar{\mu}} = g_{SM}, \quad g^{H_2 \tau \bar{\tau}} = g_{SM}.
\]  

(41)

Independently of the parameter \( a \), one can affirm that in the 331RHN model each lepton couples to an SM-like Higgs identically to how it does in the SM.
(II) Quark Yukawa terms. In the quark sectors, all $Y$ are Yukawa complex coefficients, their upper indices indicating the type of sector they label (up or down respectively) and their lower indices denoting the particular Higgs they interact with.

A considerable amount of arbitrariness occurs due to the large number of unknown coefficients in the Yukawa matrices. This state of affairs complicates our job to estimate the production rates and decay widths for the SM-like Higgs boson. There are evidently two complex mass matrices—a $5 \times 5$ one for the “down” sector and a $4 \times 4$ one for the “up” sector—that remain to be diagonalized in order to get the true couplings of the quarks to the physical Higgs bosons. A thorough analysis of the quark sector in the model at hand will be performed in detail elsewhere, once the masses of the exotic quarks are experimentally available. However, for our purposes here, certain approximations will be considered in order to rapidly get the mass basis (physical states) in the quark sector and hence the Higgs–quark couplings of interest. Here is the point where the family choice we made in Sect. 2 evidently proves helpful and quite natural, since the third generation of quarks seems the heaviest of all and its decoupling will be essential in our further estimates.

From Eq. (37), these general mass matrices yield:

$$
M^u = \begin{pmatrix}
Y_{11} \eta_3 & Y_{12} \eta_3 & Y_{13} \eta_3 + Y_{31} \eta_2 & Y_{14} \eta_3 + Y_{41} \eta_1 \\
Y_{21} \eta_3 & Y_{22} \eta_3 & Y_{23} \eta_3 + Y_{32} \eta_2 & Y_{24} \eta_3 + Y_{42} \eta_1 \\
Y_{31} \eta_2 + Y_{13} \eta_3 & Y_{32} \eta_2 + Y_{23} \eta_3 & Y_{33} \eta_2 + Y_{34} \eta_1 & Y_{34} \eta_2 + Y_{43} \eta_1 \\
Y_{41} \eta_1 + Y_{14} \eta_3 & Y_{42} \eta_1 + Y_{24} \eta_3 & Y_{43} \eta_1 + Y_{34} \eta_2 & Y_{44} \eta_1
\end{pmatrix} \langle \phi \rangle \quad (42)
$$

for the “up” sector, and

$$
M^d = \begin{pmatrix}
Y_{11} \eta_2 & Y_{12} \eta_2 & Y_{13} \eta_2 + Y_{31} \eta_3 & Y_{14} \eta_2 + Y_{41} \eta_1 \\
Y_{21} \eta_2 & Y_{22} \eta_2 & Y_{23} \eta_2 + Y_{32} \eta_3 & Y_{24} \eta_2 + Y_{42} \eta_1 \\
Y_{31} \eta_3 + Y_{13} \eta_2 & Y_{32} \eta_3 + Y_{23} \eta_2 & Y_{33} \eta_3 & Y_{34} \eta_1 + Y_{43} \eta_1 \\
Y_{41} \eta_1 + Y_{14} \eta_2 & Y_{42} \eta_1 + Y_{24} \eta_2 & Y_{43} \eta_1 + Y_{34} \eta_3 & Y_{44} \eta_1
\end{pmatrix} \langle \phi \rangle \quad (43)
$$

for the “down” sector respectively.

In the above matrices, of course, $Y^u_{ij} \neq Y^d_{ij}$, but in order to simplify the notations we have omitted the upper indices and take each diagonalization in its own right.

In order to get the physical basis, some approximations can be enforced. They must be natural in terms of decoupling the heavier states from the lighter ones, once we have considered it in the scalar sector as well. The first step means that the exotic quarks of the 331RHN model decouple from the SM-like ones. That assumes the vanishing of certain entries in the matrices (42)–(43) so they become:

$$
M^u = \begin{pmatrix}
\frac{1}{2 \cos^2 \theta_W} & \frac{1}{2 \cos^2 \theta_W} & \frac{1}{2 \cos^2 \theta_W} & 0 \\
\frac{1}{2 \cos^2 \theta_W} & \frac{1}{2 \cos^2 \theta_W} & \frac{1}{2 \cos^2 \theta_W} & 0 \\
\frac{1}{2 \cos^2 \theta_W} & \frac{1}{2 \cos^2 \theta_W} & \frac{1}{2 \cos^2 \theta_W} & 0 \\
0 & 0 & 0 & \sqrt{a} \langle \phi \rangle
\end{pmatrix}
$$

(44)
where \( Y_{14}\eta_3 + Y_{41}^*\eta_1 \simeq 0, Y_{24}\eta_3 + Y_{42}^*\eta_1 \simeq 0, Y_{34}\eta_2 + Y_{43}^*\eta_1 \simeq 0 \) could be safely set. Furthermore, in the spirit of the SM, one can claim that off-diagonal entries in the above matrix are equal to zero since there is always a basis in which no mixing in the “up” sector occurs, while it must be performed only in the “down” sector.

Under these circumstances, the mass states of the up quarks are, in order:

\[
\begin{align*}
\circ m(u) &= Y_u \sqrt{\frac{1}{2\cos^2\theta_W}} \langle \varphi \rangle_{\text{SM}} \\
\circ m(c) &= Y_c \sqrt{\frac{1}{2\cos^2\theta_W}} \langle \varphi \rangle_{\text{SM}} \\
\circ m(t) &= Y_t \sqrt{\frac{1}{2(1-\tan^2\theta_W)}} \langle \varphi \rangle_{\text{SM}} \\
\circ m(T) &= Y_T \sqrt{\frac{(1-a)}{a}} \langle \varphi \rangle_{\text{SM}}.
\end{align*}
\]

In the “down” sector things can be solved similarly, by setting \( Y_{14}\eta_2 + Y_{41}^*\eta_1 \simeq 0, Y_{24}\eta_3 + Y_{42}^*\eta_1 \simeq 0, Y_{34}\eta_2 + Y_{43}^*\eta_1 \simeq 0, Y_{15}\eta_3 + Y_{51}^*\eta_1 \simeq 0, Y_{25}\eta_2 + Y_{52}^*\eta_1 \simeq 0, Y_{35}\eta_3 + Y_{53}^*\eta_1 \simeq 0 \).

The resulting mass matrix yields:

\[
M_d = \begin{pmatrix}
\begin{array}{ccc}
Y_{d1}^d & Y_{d1}^d & 0 \\
Y_{d2}^d & Y_{d2}^d & 0 \\
Y_{d3}^d & Y_{d3}^d & \frac{1}{2\cos^2\theta_W} & 0 & 0 \\
0 & 0 & 0 & Y_{d4}^D & Y_{d5}^D \\
0 & 0 & 0 & Y_{d4}^D & Y_{d5}^D
\end{array}
\end{pmatrix}
\sqrt{a} \langle \varphi \rangle.
\]

At this point, the CKM-like matrix must enter the stage and do its job of diagonalizing the SM-like part of the above matrix. However, a rough way to get the mass states is to consider that the \( b \) quark decouples from its companions \( d \) and \( s \) (their masses lie in a strong hierarchy). Then, the Cabibbo matrix is sufficient to mix \( d \) and \( s \) so that finally one gets the following masses:

\[
\begin{align*}
\circ m(d) &= Y_d \sqrt{\frac{1-\tan^2\theta_W}{2}} \langle \varphi \rangle_{\text{SM}} \\
\circ m(s) &= Y_s \sqrt{\frac{1-\tan^2\theta_W}{2}} \langle \varphi \rangle_{\text{SM}} \\
\circ m(b) &= Y_b \sqrt{\frac{1}{2\cos^2\theta_W}} \langle \varphi \rangle_{\text{SM}} \\
\circ m(D) &= Y_D \sqrt{\frac{(1-a)}{a}} \langle \varphi \rangle_{\text{SM}} \\
\circ m(S) &= Y_D \sqrt{\frac{(1-a)}{a}} \langle \varphi \rangle_{\text{SM}}.
\end{align*}
\]

With these results, one can estimate the couplings of all the quarks in the model to Higgses. As in the lepton case, they will result from a comparison with their homologue in the SM. By means of the
same procedure, the Yukawa couplings to $H_2$ are:

$$Y^u = Y^u_{SM} \cos \theta_W, \quad Y^c = Y^c_{SM} \cos \theta_W, \quad Y^t = \frac{Y^t_{SM}}{\sqrt{1 - \tan^2 \theta_W}} \quad (46)$$

$$Y^d = \frac{Y^d_{SM}}{\sqrt{1 - \tan^2 \theta_W}}, \quad Y^s = \frac{Y^s_{SM}}{\sqrt{1 - \tan^2 \theta_W}}, \quad Y^b = Y^b_{SM} \cos \theta_W. \quad (47)$$

Consequently, the couplings of the SM-like boson to SM-like quarks yield in our model:

$$g_{H_2 \bar{u}u} = g_{H_2 \bar{d}d} = g_{H_2 \bar{c}c}, \quad g_{H_2 \bar{d}d} = g_{H_2 \bar{s}s} = g_{H_2 \bar{b}b}. \quad (48)$$

$$g_{H_2 \bar{d}d} = g_{H_2 \bar{s}s} = g_{H_2 \bar{b}b} \quad (49)$$

The above identities occur due to both our parametrization and the expression of $H_2$ (Eq. (24)), so that $H_2 \tilde{f} \tilde{f}$ couplings are either $\sqrt{\frac{1 - \tan^2 \theta_W}{2}} g(H_2 \tilde{f} \tilde{f})$ or $\sqrt{\frac{1}{2 \cos^2 \theta_W}} g(H_2 \tilde{f} \tilde{f})$ depending on the weak triplet scalar that a particular fermion couples to.

### 3.4.4. Higgs couplings to bosons and fermions: summary

The couplings of the Higgses to bosons and fermions in the model at hand are summarized in Tables 1 and 2. One can easily observe that all the couplings are identical to the corresponding ones in the SM. This result confirms that the general method of treating gauge models with high symmetries, conceived by Cotăescu [18], does not violate the well known Appelquist–Carazzone [52] decoupling theorem. Since the Higgs boson seems to appear at $m(H_2) = 126$ GeV, all the predicted phenomenology in the SM is exactly valid.

In this mass region, processes such as $H_2 \to VV$ or $H_2 \to \tilde{t} \tilde{t}$ are still forbidden, while the dominant one $H_2 \to \tilde{b} \tilde{b}$ remains as such. Regarding the exotic quarks ($D$, $S$, $T$), their couplings to the heavier Higgs in the model seem to be strongly suppressed, unless their corresponding masses are very heavy (comparable to the overall breaking scale), while their couplings to the lighter Higgs are absent. A more accurate identification of the CKM-like matrix can be performed, if one observes that if $\sin^2 \theta_W \simeq 1/4$ (which is very close to the real value) then $\sqrt{\frac{1 - \tan^2 \theta_W}{2}} \simeq \sqrt{\frac{1}{2 \cos^2 \theta_W}} \simeq \sqrt{\frac{1}{\pi}}$. Now, in the decoupling limit, even the CKM matrix of the SM can be recovered in our model.

With these results, the theoretical part of our work devoted to the particular Higgs sector of the 3-3-1 model with right-handed neutrinos (but stemming from the fertile soil of the SM) is accomplished. Our goal of validating the theoretical method of Cotăescu, in which a one-parameter mass...
Table 3. Boson masses in the 331RHN model.

| Mass   | $\langle \phi \rangle = 1$ TeV | $\langle \phi \rangle = 2$ TeV | $\langle \phi \rangle = 5$ TeV | $\langle \phi \rangle = 7$ TeV | $\langle \phi \rangle = 10$ TeV |
|--------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $m(Y)$ | 321.8 GeV                     | 653.3 GeV                     | 1.64 TeV                      | 2.32 TeV                      | 3.28 TeV                      |
| $m(X)$ | 324.7 GeV                     | 654.7 GeV                     | 1.64 TeV                      | 2.32 TeV                      | 3.28 TeV                      |
| $m(Z')$| 389.9 GeV                     | 793.9 GeV                     | 1.99 TeV                      | 2.82 TeV                      | 3.99 TeV                      |
| $m(H_1)$| 512.2 GeV                     | 653.3 GeV                     | 1.64 TeV                      | 2.32 TeV                      | 3.28 TeV                      |
| $m(H_2)$| 126 GeV                       | 126 GeV                       | 126 GeV                       | 126 GeV                       | 126 GeV                       |

The spectrum is obtained, successfully accommodated the Higgs phenomenology in the decoupling limit and predicted the heaviest Higgs mass along with the overall breaking scale of the model. From now on, some plausible phenomenological scenarios can be developed. Detailed numerical results of the new physics beyond the SM energies can be obtained, once the breaking scale of the model is firmly established.

4. Plausible scenarios

At this moment one can test some plausible scenarios beyond the SM by choosing certain orders of magnitude for the overall VEV $\langle \phi \rangle$ specific to this 331RHN model. Hence, some rough estimates are obtained for the resulting phenomenology. We work out here some cases of interest—accessible at current colliders, especially CERN-LHC—in which $\langle \phi \rangle \in (1–10)$ TeV with the three VEVs aligned as:

- $<\rho> \in \left(\sqrt{1-a} \div 10 \sqrt{1-a}\right)$ TeV
- $<\chi> \simeq \left(\sqrt{\frac{1-\tan^2\theta_W}{2}}\right) \langle \phi \rangle_{SM} = 145.5$ GeV
- $<\phi> \simeq \left(\frac{1}{2 \cos \theta_W}\right) \langle \phi \rangle_{SM} = 198.3$ GeV

This implies $a \in (0.0006–0.06)$, corresponding to $\langle \phi \rangle \in (1–10)$ TeV, as results from $\sqrt{a} \langle \phi \rangle = \langle \phi \rangle_{SM}$ in order to ensure $m(W) = 80.4$ GeV and $m(Z) = 91.1$ GeV. With the allowed range of the parameter $a$, one can compute the allowed domain for boson masses. These are, at the presumed breaking scales, the ones presented in Table 3.

Before entering into a discussion of the Higgs phenomenology and its restrictions, we now estimate the implications of some verified phenomenological aspects [53]. For instance, the “wrong muon decay” gives at a 98% confidence level (CL) the result

$$R = \frac{\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)} = \left(\frac{M_W}{M_Y}\right)^4 \leq 1.2\%.$$ 

Hence $M_Y \geq 240$ GeV or equivalently—in our approach—to $a \leq 0.123$, which is already fulfilled.

A more severe constraint on the non-universal $Z'$ boson can be inferred [32] from the FCNC processes due to mixing in neutral meson systems $K^0, D^0, B_{s,d}^0, B_{s,d}^0$. The experimental measurements [53] claim, for the 331RHN model of interest, that $m(Z') \geq 2.1$ TeV [32] in order to maintain consistency with the phenomenology, as was shown [54] by calculating these limits using the precision parameters of the SM. This condition simply rules out some scenarios from Table 3. Now, the first two columns—if not even the third one as well—are excluded. The parameter range decreased to $a \in (0.0006–0.002)$. Roughly speaking, the free parameter of the model must not exceed $O(10^{-5})$, so that the overall breaking scale of such models must lie above 5 TeV.
5. Higgs phenomenology

Since the Higgs boson(s) can be regarded as the Holy Grail of particle physics, the experimental search for it (them) has become a must. Recent collaborations, such as ATLAS and CMS at the CERN-LHC or CDF and D0 at the Tevatron, seem to signal a possible appearance of this elusive boson at around $126 \text{ GeV}$. This experimental observation is naturally consistent with our scenarios considered above. In light of this state of affairs, our work supplies the specific couplings of the SM-like Higgs boson to all the particles in the 331RHN model. The Yukawa couplings in Eq. (37) represent the most general case. If we work on the hypothesis that the cubic terms are absent from the scalar potential due to the discrete symmetry $\rho \rightarrow -\rho$, then all the Yukawa couplings to this scalar triplet vanish from the quark sector $L_d$. Hence exotic quarks would remain massless at the tree level. To avoid this, one can supplement $\rho \rightarrow -\rho$ with the discrete symmetry $D_R \rightarrow -D_R$, $S_R \rightarrow -S_R$, and $T_R \rightarrow -T_R$. Under these circumstances, a complete decoupling between the two scales can occur and the whole phenomenology of the 331RHN model splits into low energy processes and high energy ones, with tree-level heavy masses for all the exotic quarks. Since all the couplings of the SM-like Higgs identically mimic the SM corresponding couplings, the low energy phenomenology comes out unchanged when compared against the SM predictions (for a comprehensive synthesis on SM-Higgs and non SM-Higgs phenomenology, see Refs. [54,55]). We call this scenario 1. A different possibility can be conceived as to keep only $\rho \rightarrow -\rho$ without any other supplemental discrete symmetry; we call this scenario 2. In the latter case, the exotic quarks come out massless at the tree level, but they can mix with ordinary ones due to couplings with $\chi$ and $\phi$, thus altering the SM-like quarks’ couplings.

We now briefly present the main processes that could occur at the present facilities and estimate their order of magnitude.

5.1. Higgs decays

Decays into fermions. These decays (Fig. 1) will not be affected at all in scenario 1, since the involved couplings are identical to those in the SM (see Table 2). Thus, the width of each two-body decay channel (first diagram in Fig. 1) is identical to the one resulting in the SM. That is, $\Gamma(H_2 \rightarrow j \bar{f}) = \Gamma_{SM}(H_2 \rightarrow j \bar{f})$ for every $f(= e, \mu, \tau, u, d, c, s, b, t)$. For the mass region of interest here, $m_2 = 126 \text{ GeV}$, the dominant process remains by far $H_2 \rightarrow \bar{b}b$, since $H_2$ lies far below the threshold $m(H_2) \sim 2m(t)$. Therefore, the process $H_2 \rightarrow t \bar{t}$ remains forbidden. In scenario 2, due to proper mixing in the quark sector, the Yukawa couplings of the SM-like quarks change (most likely decrease). Hence, the widths change, becoming smaller than those predicted by the SM.

The general analysis in the SM usually takes into consideration possible processes, such as three-body (second and third diagrams in Fig. 1) and four-body decays (third diagram in Fig. 2), especially because they involve significantly sizable couplings of the top quark. The second process in Fig. 1 is not kinematically allowed. Nevertheless, if we assume a replacement—bottom quark instead of top quark—in the same diagram, the process becomes likely, except for the resulting gauge boson,
Decays into gauge bosons. Due to unchanged couplings with SM-like gauge bosons, in either scenario, decays such as those in Fig. 2 are identical to those in the SM. So, $\Gamma(H_2 \rightarrow WW) = \Gamma_{\text{SM}}(H_2 \rightarrow WW)$, $\Gamma(H_2 \rightarrow ZZ) = \Gamma_{\text{SM}}(H_2 \rightarrow ZZ)$, and $\Gamma(H_2 \rightarrow Zj \bar{j}) = \Gamma_{\text{SM}}(H_2 \rightarrow Zj \bar{j})$.

Loop-induced decays. Of great importance at hadron colliders are the loop-induced decays (Fig. 3) of the SM-like Higgs.

The 331RHN model widths for these processes are, in order, $\Gamma(H_2 \rightarrow \gamma\gamma) = \Gamma_{\text{SM}}(H_2 \rightarrow \gamma\gamma)$, $\Gamma(H_2 \rightarrow \gamma Z) = \Gamma_{\text{SM}}(H_2 \rightarrow \gamma Z)$ in the first diagram of Fig. 3, where decays are induced by a triangle of an off-shell SM-like gauge boson, and $\Gamma(H_2 \rightarrow \gamma\gamma) = \Gamma_{\text{SM}}(H_2 \rightarrow \gamma\gamma)$, $\Gamma(H_2 \rightarrow \gamma Z) = \Gamma_{\text{SM}}(H_2 \rightarrow \gamma Z)$ in the second process of Fig. 3, where these loop-induced decays are due to a triangle of heavy $t$ quarks. Decays into two gluons, $\Gamma(H_2 \rightarrow gg) = \Gamma_{\text{SM}}(H_2 \rightarrow gg)$ (third diagram in Fig. 3), appear with sizable order of magnitude. Among the loop-induced decays, $H_2 \rightarrow gg$ is the dominant—competing even with some tree-level decays—while $H_2 \rightarrow \gamma\gamma$ is rather tiny. Notwithstanding, the decay into gluons is almost useless at hadron colliders due to the hadron background, whereas the photon signals are visible over a large hadronic background.

Some new contributions could introduce into the width estimates the occurrence of the heavy $Z'$ in the loop of the first diagram in Fig. 3, but its small coupling to $H_2$ makes a dramatic alteration of the total contribution from such diagrams less likely, as its own contribution seems not to overcome the $W$ and $Z$ ones. In the other two diagrams, the exotic quarks could also have played a significant role if they were coupling to SM-like Higgs, but in scenario 1 they are completely decoupled from the ordinary quarks. Scenario 2 could reveal some sizable contributions from these diagrams. A future
Fig. 4. Higgs production processes.

analysis must take into consideration concrete values for the mixing between ordinary quarks and exotic ones, which allows one to compute these quark triangle contributions in greater detail.

5.2. Higgs production

Besides the decay events in the Higgs sector, production processes are also of particular importance. At lepton colliders (LEP1, LEP2 experiments), Higgs production phenomena established strong limits on the parameter space. Those results are not altered at all by our approach, since processes such as \( e^- e^+ \rightarrow V^* \rightarrow VH \) can be mediated only by \( W \) and \( Z \), whose couplings are identical to those in the SM.

The predictions for hadron colliders seem more interesting. Here the main processes to be sought at the LHC and Tevatron are those in Fig. 4, occurring in \( pp \rightarrow V^* \rightarrow VH \) and \( pp \rightarrow H + \) hadronic jets (first two diagrams). These processes are the same as in the SM, if the decoupling scenario 1 is taken into consideration. Interesting effects with massless exotic quarks can, however, occur in scenario 2 for the processes \( uu \rightarrow UUH_2 \) and \( dd \rightarrow DDH_2 \), mediated by the off-shell heavy \( X \) boson. In such processes, the SM-like Higgs signal could appear even at lower center-of-mass (CM) energies. However, in the TeV region for CM energies, \( H_1 \) production is dominant and eclipses \( H_2 \) production, due to the greater coupling of \( H_1 \) to heavy gauge bosons.

The production rates of the third and fourth processes in Fig. 4 will also be the same as in the SM, but, despite their large cross sections, such processes are in competition with the hadronic background at the LHC. Regarding the \( H_1 \) Higgs boson specific to this model, we consider that it could appear dominantly in gluon fusion events at CM energies around 7–14 TeV associated with exotic \( T^- \)-quark mediators (third and fourth diagrams in Fig. 4). This seems the most likely, since the couplings of the \( H_1 \) are restricted only to exotic quarks \( D, S \), and the heaviest \( T \). The last one, however, exhibits the greatest coupling to \( H_1 \).

All the results presented in this section must be taken as rough estimates, a more accurate analysis for each particular process having to be performed in detail in future work on this topic. This could reveal some worthy phenomenology, but it cannot be performed unless the Yukawa couplings are established, and this exceeds the scope of this work.

6. Conclusions

In this work we have revisited the Higgs sector of a 331RHN gauge model, by combining the non-canonical Cotăescu approach, consisting of tuning a single free parameter in the scalar sector, with
the traditional three surviving Higgs scalars usually explored in the literature. This enables us to investigate some plausible scenarios supplied by an overall breaking scale $\langle \phi \rangle \in 1$–10 TeV.

Our work primarily proves that the two apparently opposite approaches are perfectly compatible. The particular one-parameter method can be successfully accommodated with the canonical approach, by simply redefining the scalar multiplets, so that instead of one surviving Higgs field there are two massive physical fields in the end, plus a massless scalar field. The latter is sterile, as its couplings to all fermions and bosons are zero. So, it escapes any interaction and has finally no contribution to any effect at LEP or LHC to contradict the available data.

This outcome of our approach suitably fits the present experimental data predicting realistic scenarios of the new physics with a breaking scale of a few TeV. At the same time, the decoupling theorem of Appelquist and Carazzone is proved to be valid in our model, so that the SM is entirely recovered in the low energy region. Our previously obtained results regarding the boson mass spectrum can be preserved. The one-parameter approach allows one to elegantly discuss the lower bounds imposed by the experimental data in the boson sector. Consequently, its resulting mass spectrum can be tuned: $M_X = M_X(a)$, $M_Y = M_Y(a)$, and $M_Z = M_Z'(a)$, and, for Higgses, $m_1 \cong 0.52 \langle \phi \rangle$ TeV, $m_2 = 126$ GeV, $m_3 = 0$.

With respect to the scalar sector, our approach allows one to express all the Higgs phenomenology (decay widths, production cross sections) in terms of the corresponding results obtained within the framework of the SM. A further development will be to consider trilinear terms in the potential and relax the discrete symmetries that vanished them here. Accurate numerical estimates remain be performed in the quark sector too, where the mixing between ordinary and exotic quarks in different scenarios could reveal certain shifts in the couplings and hence some corrections to decay rates and production cross sections. Since our work here is merely a pleading for the theoretical tool that the Cotăescu method is and for proving that it can predict realistic phenomenological consequences when applied to the 3-3-1 models, numerically accurate calculations are tasks for future work.

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