Research Article

Effect of Longitudinal Gradient on 3D Face Stability of Circular Tunnel in Undrained Clay

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The longitudinal gradient existed in shield-driven tunnel crossing river or channel has a longitudinal gradient, which is often ignored in most stability analyses of the tunnel face. Considering the influence of the longitudinal gradient into A(a) continuous velocity field, the present paper, conducting a limit analysis of the tunnel face in undrained clay, adopted to yield the upper-bound solutions of the limit pressure supporting on a three-dimensional tunnel face. The least upper bounds of the collapse and blow-out pressures can be obtained by conducting an optimization procedure. These upper-bound solutions are given in the design charts, which provide a simple way to assess the range of the limit pressure in practice. The influence of the longitudinal gradient becomes more significant with the increase of \( \frac{c}{D_s} \) and \( C/D \). The blow-out pressure for tunneling in a downward movement could be overestimated and the collapse pressure for tunneling in an upward movement could be conversely underestimated, with ignoring the influence of the longitudinal gradient.

1. Introduction

Shield-driven excavation is widely used in a subway or road tunnel crossing the river or channel. Generally, these tunnels have a longitudinal gradient and the river or channel bed also has a small undulation. For vehicle-driving safety, the longitudinal gradient of the tunnels should be less than 3%, but such a small gradient is often neglected in stability analyses of the tunnel face. However, there are many shield-driven excavations in construction for railway and gas-insulated line (GIL) crossing the river, and the longitudinal gradient may exceed 5% and even reach 10%. In these situations, the longitudinal gradient may affect the face stability during boring excavations, which cannot be ignored in practice.

Methods for stability analysis of the tunnel face can be categorized as limit analysis method [1–5], limit equilibrium method [6–8], and numerical analysis method [9–11]. Zhao et al. [12] conduct limit analysis of face stability of tunnels with a longitudinal gradient, and the failure mechanism with a multiple elliptical cone is used to obtain upper-bound solutions of the active and passive failure pressure for supporting the face of the shallow tunnel in frictional soils. Recently, Huang et al. [13] employed the continuous velocity
field proposed by Mollon et al. [14] to analyze the face stability of longitudinally inclined tunnels in anisotropic purely cohesive soils.

However, their studies are limited to the excavations in a horizontal ground surface, and they cannot be used to predict the supporting pressure on the face of tunnels crossing a river or channel. The purpose of this study is to investigate the influence of the longitudinal gradient on face stability of the tunnel crossing the river or channel. The continuous velocity field of Mollon et al. [14] is adopted here to obtain the upper-bound solutions of supporting pressure of circular tunnels in undrained clay.

2. Kinematic Approach of Limit Analysis in Tunnel Face Stability

2.1. Problem Definition. For a tunnel crossing the river or channel, the shield-driven excavations will be processing downward and upward, as shown in Figure 1. The angles $\beta_{TF}$ and $\beta_{G}$ represent the inclination of the tunnel face and the ground surface (i.e., the river or channel bed) from the vertical plane, respectively. When the angle $\beta_{TF}$ is negative, the movement of the shield-driven excavation is downward. $\eta$ in the longitudinal gradient of the tunnel is often used in practice and the value of $\eta$ is equal to tan ($\beta_{TF}$). When a circular tunnel with a diameter of $D$ is advancing in uniform clay, the cover depth of the tunnel crown is denoted as $C$ and the uniform pressure $\sigma_s$ is supporting on the tunnel face. The clay is considered as short-term undrained conditions during the excavations and then its undrained shear strength is $\sigma_u$. The presented three-dimensional (3D) stability analyses of tunnel face involve both collapse and blow-out failures, by which the corresponding critical supporting pressure on the tunnel face can be determined [15–17].

2.2. Three-Dimensional Failure Mechanism of the Tunnel Face. Mollon et al. [14] recently proposed a continuous velocity field for collapse and blow-out of the 3D circular face of tunnel in undrained clay and then conducted limit analysis to determine the critical supporting pressure. However, the least upper-bound solution of the pressure $\sigma_s$ is not obtained due to the numerically explicit finite-difference discretization on the 3D velocity field. Zhang et al. [4] adopted the closed-form solution for the velocity field expressed by Klar and Klein [18] to calculate the lowest upper-bound solutions. Some comparisons demonstrate the criticality of the proposed velocity field in plasticity of limit analysis. In this paper, the velocity field is extended to investigate the influence of the longitudinal gradient on face stability of tunnel crossing the river or channel. The following is just an outline of the velocity field obtained from the closed-form solution. The details explaining the velocity field and its associated equations are given in [4, 5, 14]. These equations are repeated here to provide a framework for result presentation.

Figure 2 illustrates the velocity field of face collapse of the tunnel with a longitudinal gradient $\eta$. The overall velocity field has the shape of a torus, and its maximal velocity flow line is a circle about the origin $O$. The radius of the circle $R_f$ is assumed as

$$R_f = R_i + C' = R_i + \frac{\sin R_G}{\sin (R_G - \beta_{TF})}C,$$

(1)

where $R_i = D/2 + L_1$ and $L_1$ is the distance of the maximum velocity flow line from the tunnel face center.

Selecting a curvilinear coordinate system $(r, \theta, \beta)$ to express the velocity field, there are three components of the velocity: the radial velocity ($v_r$), the orithoral velocity, ($v_\theta$) and the axial velocity ($v_\beta$). In any circular cross-section $\Pi_\beta$, the orithoral velocity $v_\theta$ is equal to zero. The axial velocity $v_\beta$ is perpendicular to the plane $\Pi_\beta$, and its value decreases in a parabolic way from the corresponding maximal velocity flow line to the outside boundary. The expression of $v_\beta$ is given below:

$$v_\beta (r, \theta, \beta) = v_m (\beta) \left[ 1 - \frac{r^2}{R_{\max}^2 (\theta, \beta)} \right],$$

(2)

where $v_m$ and $R_{\max}$ can be expressed as

$$v_m (\beta) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{R_{\max}(\theta, \beta)} \frac{1}{1 - \left( r^2 / R_{\max}^2 (\theta, \beta) \right)} rdr d\theta,$$

(3)

$$R_{\max} (\theta, 0) = R_i + \frac{\theta}{\beta_{TF}} \left( R_i - \beta_{TF} \right),$$

(4)

where $\beta_{TF}$ and $\beta_{G}$ represent the inclination of the tunnel face and the ground surface (i.e., the river or channel bed) from the vertical plane, respectively. The presented three-dimensional (3D) stability analyses of tunnel face involve both collapse and blow-out failures, by which the corresponding critical supporting pressure on the tunnel face can be determined [15–17].

Substitution of equations (3) and (4) into equation (2) can form the expression of the velocity $v_\beta$ as

$$v_\beta (r, \theta, \beta) = \frac{2A_2^2}{A_1^2} \left( \frac{8A_1^2\eta^2}{A_1^2A_3^2} \right),$$

(6)

where the coefficients $A_1$, $A_2$, and $A_3$ are

$$A_1 (\beta) = (1 + 2T_1) (2B_G - 2\beta) + 4R_f (\beta - \beta_{TF}),$$

(7a)

$$A_2 (\theta) = 2T_1 \cos \theta + \sqrt{1 - 2T_1^2 + 2L_1^2 \cos 2\theta},$$

(7b)

$$A_3 = (1 + 2T_1) (2B_G - 2\beta_{TF}),$$

(7c)

$\bar{[\cdot]}$ means normalized by the tunnel diameter $D$ (e.g., $T_1 = L_1/D$).

As presented by Klar and Klein [18], the undrained clay is incompressible and then the radial velocity $v_r$ can be analytically obtained, and its expression is given as

$$v_r (r, \theta, \beta) = \frac{2A_4A_3^2\eta}{(R_f - r \cos \theta)A_1} \left( 1 - \frac{4A_1^2\eta^2}{A_1^2A_3^2} \right),$$

(8)

where the coefficient $A_4$ is

$$A_4 = 4 \bar{R}_f - 2 (1 + 2T_1).$$

(9)
Figure 1: Tunneling with a longitudinal gradient in (a) downward movement; (b) upward movement.

Figure 2: Continuous velocity field for the undrained failure of the tunnel face with a longitudinal gradient.
Figure 3: Stability numbers for the face stability of the circular tunnel with different longitudinal gradients.
2.3. Formula and Upper-Bound Solutions. Within the upper-bound theorem of limit analysis, equating the external rate of work to the internal rate of dissipation in the assumed velocity field can determine the critical pressure supporting on the tunnel face. The external rate of work includes the work rate done by the soil weight, the possible uniform surcharge ($\sigma_s$) acting on the ground surface, and the supporting pressure acting on the tunnel face. The work rate of the soil weight can be expressed as
Figure 5: Influence of the parameter $L_1/D$ on the stability number $N$ for blow-out.

\[
W_y = \gamma \int_0^{\pi/2} \int_0^{2\pi} \int_0^{R_{\text{max}}(\theta, \beta)} (v_r \sin \beta + v_r \cos \theta \cos \beta) dr \cdot r d\theta (R_t - r \cos \theta) d\beta.
\]

(10)

The rate of work done by the surcharge $\sigma_s$ and pressure $\sigma_t$ is, respectively, given as

\[
W_{\sigma_s} = \sigma_s \int_0^{2\pi} \int_0^{R_{\text{max}}(\theta, \beta = \beta_C)} v_r(\beta = \beta_C) dr \cdot r d\theta,
\]

(11)
\[
W_{si} = -\int_0^{2\pi} \int_0^{R_{max}(\theta; \beta=0)} \sigma_i v_i(\beta = \beta_{TF}) dr \cdot r d\theta.
\]

The total internal dissipation of energy is obtained by integration within the whole volume \( V \) as
\[
\dot{D} = \int_V s_u \cdot 2 \max (\{ |\dot{e}_i| \}) dV
= s_u \int_0^{\pi/2} \int_0^{2\pi} \int_0^{R_{max}(\theta; \beta)} 2 \max (\{ |\dot{e}_i| \}) dr \cdot r d\theta (R_f - r \cos \theta) d\beta,
\]
where \( \dot{e}_i \) is the principal strain rate component. The expressions of the components can be found in Klar and Klein [18] or Zhang et al. [4].

Equating the external rate of work (equations 10–12) to the rate of internal energy dissipation (equation 13) can give the expression of the supporting pressure as
\[
\sigma_t = \sigma_s N_s + \gamma DN_f - s_u N_c,
\]
where the dimensionless coefficients are given by
\[ N_s = \frac{\int_0^{2\pi} \int_0^{R_{\text{max}}(0,\theta,\beta)} v_\beta (\beta = \beta_C) d\tau \cdot \tau d\theta}{\int_0^{2\pi} \int_0^{R_{\text{max}}(0,\theta,\beta)} v_\beta (\beta = \beta_{\text{Tf}}) d\tau \cdot \tau d\theta} \quad (15a) \]

\[ N_f = \frac{\int_0^{\pi/2} \int_0^{2\pi} \int_0^{R_{\text{max}}(0,\theta,\beta)} (v_y \sin \beta + v_r \cos \theta \cos \beta) d\tau \cdot \tau d\theta (R_f - r \cos \theta) d\beta}{\int_0^{2\pi} \int_0^{R_{\text{max}}(0,\theta,\beta)} v_\beta (\beta = \beta_{\text{Tf}}) d\tau \cdot \tau d\theta} \quad (15b) \]

Figure 7: Layout of the derived continuous velocity field for shallow tunnel blow-out \((C/D = 0.5, y/D/s_u = 10, \text{and } \eta = 6\%): (a) tunneling in downward movement; (b) tunneling in upward movement.
It can be found that the coefficient $N_s$ is equal to 1. In addition, the coefficients $N_c$ and $N_c'$ can be approximately calculated by using the numerical integration methods (e.g., trapezoidal rule and Simpson’s rule). For the tunnel face subjected to blow-out, the upper-bound of the limit pressure can be expressed as

$$\sigma_t = \sigma_s + yDN_y + \varepsilon_sN_c. \quad (16)$$

Figure 8: Layout of the derived continuous velocity field for deep tunnel collapse ($C/D = 4$, $yD/s_y = 10$, and $\eta = 6\%$): (a) tunneling in downward movement; (b) tunneling in upward movement.
The dimensionless coefficients $N_y$ and $N_c$ are the same as those given in the case of collapse.

Giving the parameters (e.g., cover depth $C$, tunnel diameter $D$, angles $\beta_{TF}$ and $\beta_{G}$, unit weight of soil $\gamma$, undrained strength $s_u$, and ground surcharge $\sigma_s$), independent variable describing the profile of the velocity fields is the parameter $L_1$. Performing an optimization procedure can search the least upper-bound solution of the supporting collapse or blow-out pressure. As defined by Davis et al. [1], stability number $N$ is also used here to produce the design chart for prediction of the tunnel pressures and its expression is given below:

$$N = \frac{\sigma_s + \gamma H - \sigma_i}{s_u}, \quad (17)$$

where $H = C + D/2$. Substitution of equations (14) and (16) into equation (17) will result in

$$N = N_c + \frac{\gamma D}{s_u} \left( \frac{C}{D} + \frac{1}{2} - N_y \right), \quad \text{for collapse,} \quad (18a)$$

$$N = -N_c + \frac{\gamma D}{s_u} \left( \frac{C}{D} + \frac{1}{2} - N_y \right), \quad \text{for blow-out.} \quad (18b)$$

3. Results and Discussion

Computations are carried out for both of collapse and blow-out failures, with the ratio of $C/D$ ranging from 0 to 5. Various longitudinal gradients of the tunnels, $\eta = 3\%, 6\%$,
and 9%, are considered in the calculations. The negative value of $\eta$ represents downward movements of the shield-driven tunneling, otherwise it is upward movements. Figure 3 illustrates the design charts for both of collapse and blow-out cases. The stability number is plotted against the ratio of $C/D$ for $\gamma DS_{u} = 1$, 5, 10, and 20. As expected, the stability number increases with the ratio $C/D$ increasing. To demonstrate the influence of the longitudinal gradient, the results for horizontal excavations $\eta = 0$ are included in these charts. As the value of $\gamma DS_{u}$, $C/D$, and $\eta$ increases, the influence is more significant for both of the collapse and blow-out failures. For tunneling in greater undrained strength of the clay, the longitudinal gradient can be ignored in face stability. However, if the clay has small value of the undrained strength, the influence of the longitudinal gradient should be considered in assessment of face stability.

For upward tunneling, both of the collapse and blow-out pressures increase with longitudinal gradient increasing, and they decrease with $\eta$ increasing. It means that the upward tunneling requires greater supporting pressure against the collapse failures and conversely the downward tunneling requires smaller supporting pressure against the blow-out failures. Neglecting the influence of the longitudinal gradient in stability analysis of the tunnel face could yield the collapse failures in upward excavations or blow-out failures in downward excavations.

To obtain the least upper-bound solutions, the independent variable $L_1/D$ should be determined for the critical failure surface of the tunnel face. Figures 4 and 5 show the value $L_1/D$ with the ratio of $C/D$ for collapse and blow-out, respectively. As the ratio of $C/D$ increases, the maximum velocity flow line of collapse failure tends to be the center of the torus shaped mechanism. For the shallow tunnel with large value of $\gamma DS_{u}$ when the $L_1/D$ approaches to 0.5, i.e., the boundary of the failure mechanism will be lower. In this situation, the failure is rigidly rotating with the origin O. When the shallow tunnel face ($C/D < 1$) is subjected to blow-out failures, the value $L_1/D$ increases obviously with the increasing $C/D$. For downward tunneling, the value $L_1/D$ can reduce as the $C/D$ increases.

Figures 6 and 7 illustrate the profile of the collapse and blow-out failure surface for the shallow tunnel face, respectively. Both of the downward and upward excavations with a longitudinal gradient $\eta = 6\%$ are involved. The maximum velocity flow line of collapse failures in downward and upward movements is located on the lower boundary of the mechanism. For the blow-out failures, the corresponding maximum velocity flow line is closer to the center of the mechanism. In addition, the deep tunnel is also considered here, as shown in Figures 8 and 9. The continuous velocity field becomes larger but the movements are almost happened next to the tunnel face.

4. Conclusions

Based on the continuous velocity field of the tunnel face, an analytical approach is derived for the face stability of the shield-driven tunnel with a longitudinal gradient. Both of the collapse and blow-out failures are involved to predict the range of the limit pressure supporting on the tunnel face. Design charts are produced for the inclined tunnel in undrained clay. They are convenient to assess the limit pressure on the face of tunnels crossing a river or channel. The longitudinal gradient of tunneling has significant effects on the face stability. Its influence is more obvious as the values of $\gamma DS_{u}$, $C/D$, and $\eta$ increase. Neglecting the longitudinal gradient in stability analysis of the tunnel face can yield an underestimation of the collapse pressure for tunneling in upward movement, and an overestimation of the blow-out pressure for tunneling in downward movement. Such an underestimation or overestimation is more significant as the longitudinal gradient increases.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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