Coupling of Double Tearing Instability with Shear Flow and Solar Flare

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The nonlinear behavior of the double tearing mode with shear flow parallel to the magnetic field is investigated numerically in cylindrical geometry. The double tearing instability is found to be unstable with two peak current density profile, and under the influence of shear flow, it exhibits a complicated behavior in the physical and parameter space. With a Sech form shear flow, it is found that when the velocity shear near the resonant surface is small, the shear flow has a stabilizing effect, but for larger velocity shear, the double tearing modes are destabilized. The possible physical implications of the coupling of double tearing instability with shear flow for the triggering of solar flares are discussed.

1. Introduction

Magnetic field line reconnection plays an important role in many aspects of plasma phenomena in the universe, such as the evolution of solar flares (Priest, 1982), the development of the Earth's magnetosphere (Shi et al., 1988), and the relaxation in laboratory plasma for nuclear fusion research (Furth, 1985).

On the sun, the magnetic field is mostly concentrated in the magnetic flux tube structure, one of the manifestations of such a magnetic structure is coronal loops with large longitudinal currents which are excited due to the complicated motion of the sunspots. Recent observation and studies shown that the interaction of current loops is an essential ingredient in the trigger of solar flares (Machado et al., 1988; Sakai, 1989). Further more, large shear flow motions are observed between the footpoints of solar flares and along the magnetic field line, and flares are closely connected with shear flow. It has been shown both analytically (Holfmann, 1975; Chen and Morrison, 1990) and numerically (Einaudi and Rubini, 1986; Ofman et al., 1991; Deng and Wang, 1992) that shear flows could have significant effect on the resistive tearing instability. The double tearing mode can exit whenever the plasma has multiple rational surface \((K \cdot B = 0 \text{ at more than one location})\) and islands are formed on adjacent rational surfaces. In this paper the double tearing instability with shear flow parallel to the magnetic field line is investigated numerically in cylindrical geometry.

2. Magnetohydrodynamic Equations

We start from the resistive magnetohydrodynamic equations. For simplicity, we consider a long and thin standard cylinder with a constant plasma density. It has been proved that the incompressibility with constant isotropic resistivity is valid for the double tearing instability (Ofman et al., 1991). We also assume that the plasma pressure is small than the magnetic pressure, and the axial component of magnetic field is large than other component (Parker, 1972). The magnetic field \(B\) and the plasma velocity \(V\) may be taken:

\[
B = \nabla \psi \times e_z + B_{z0} e_z, \tag{1}
\]

\[
V = \nabla \phi \times e_z, \tag{2}
\]
where \( \psi \) and \( \phi \) are \( z \)-components of the potential function of \( B \) and the stream function of \( V \) respectively. All physical quantities are written in the normalized form, and resulting equations can be written as (Strauss, 1976): where

\[
J_z = -\nabla^2 \psi, \quad u = -\nabla^2 \phi, \quad \nabla_\perp = \nabla - e_z \frac{\partial}{\partial z},
\]

\( u \) and \( J_z \) are the toroidal components of vorticity and current density, the subscript \( \perp \) represents the component which are perpendicular to toroidal magnetic field respectively.

3. Computational Procedures and Numerical Results

As the toroidal magnetic field is set to be \( B_{\phi}(r) = 1 \), the equilibrium is completely determined through the specification of the safety factor \( q = (rB_\phi)/(RB_\theta) \) profile, where \( B_\theta \) and \( B_\phi \) are poloidal and toroidal components of the magnetic field. The exact equilibrium solution can be written as (Waddell \textit{et al.}, 1976)

\[ \frac{\partial \psi}{\partial t} = B \cdot \nabla \phi - \eta J_z + E_z, \]
\[ \psi_{eq}(r) = \int_{r}^{r'} \frac{r'dr'}{q(r')} \quad (5) \]

We choose the hollow \( q \) profile to model the skin current phase:

\[ q(r) = q(0) \beta \left[ \frac{1 + \left( \frac{r}{r_0} \right)^2}{\beta + \left( \frac{r}{r_0} \right)^2} \right]^{\frac{1+p}{p}} \quad (6) \]

where \( q(0), \beta, p \) and \( r_0 \) are free parameters.

We take the shear velocity in the form: with

\[ V(r) = rG(r), \quad (7) \]

\[ G(r) = \alpha \text{Sech} \left( \frac{r - r_0}{R_v} \right) \quad (8) \]

where \( \alpha \) is constant, \( R_v \) is shear parameter, \( r_0 \) is the position of the resonant surface.

The equations are solved with our nonlinear, initial value computer code, which uses a partial-implicit algorithm and the predict-correct method. The time-step satisfies the numerical stability condition (Richtmyer and Morton, 1967).

\[ \Delta t \leq \frac{2R}{aS \max \left| n - (m/q) \right|} \quad (9) \]

Fig. 2. The helical flux function \( \psi_c \) contour for \( m/n = 2/1 \) modes.

Fig. 3. The corresponding distribution of disturbed plasma velocity for \( m/n = 2/1 \) modes.
where $S = T_r/T_A$, $T_r = (\mu_0 a^2)/\eta_0$, $T_A = R_0(\mu_0 \rho)^{1/2}/B_{0\phi}$, $a$ and $R$ are the minor and major radii of the torus, $m$ and $n$ denote the poloidal and toroidal mode number, respectively.

Figure 1 gives the double-peak current density profile with $q(0) = 7.0$, $q(1) = 6.0$, $p = 1$.

In the presence of shear flow, the solution of the basic equations can no longer be symmetric and the $\psi$ and $\phi$ must have full Fourier series expansion in the poloidal and toroidal directions.

When the hollow $q$ profile with double-peak current density has the same rational value at two values of the minor radius. This leads to the possibility of a pair of magnetic islands with the same helicity, developing in a sequence of flux contours. Figures 2 and 3 give the helical flux function $\psi_r$ contour and the distribution of disturbed plasma velocity for $m/n = 2/1$ modes respectively.

When $G(r) = 0$, the results of double tearing modes without shear flow are recovered in agreement with Pritchett (Pritchett et al., 1980). If the separation of the rational surface is sufficiently small, the growth rate is predict to scale as $S^{-1/3}$ and the structure of the mode proves to be essentially identical with that of the $m = 1$ tearing mode. The perturbed flux function $\psi$ varies considerably across the singular layer, the constant-$\psi$ approximation is badly violated in this case. The main reason for this is that in a plasma with multiple rational surface, magnetic islands form on adjacent rational surface and drive each other, therefore enhancing the growth rate over the standard tearing mode; With increasing separation, the mode makes a transition to the $S^{-3/5}$ scaling and the structure of the standard tearing mode, and the constant-$\psi$ approximation is reasonably well satisfied. In Fig. 4, the normalized growth rate $\gamma T_r$ of the $2/1$ mode is given as a function of $\beta$ with $q(0) = 7.0$, $q(1) = 6.0$, $p = 1$ kept fixed.

When considering the interaction of double tearing modes with shear flow ($G(r) \neq 0$), the shear flow in the vicinity of the resonant surface determines the mode structure and growth rate. In Figs. 5 and 6 we give out the evolution of normalized growth rate $\gamma T_r$ verse time and the dependance of the normalized growth rate $\gamma T_r$ on the shear parameter $R_r$ respectively.

It is clearly seen from Fig. 6 that the Sech form shear flow has great effect on the growth rate of the double tearing modes. When the $\alpha$ is small or $R_r$ is large, the flow has a stabilizing effect, but for larger

![Fig. 4. The normalized growth rate $\gamma T_r$ of the $2/1$ mode as a function of $\beta$ with $q(0) = 7.0$, $q(1) = 6.0$, $p = 1$, $S = 10^4$ kept fixed.](image-url)
Fig. 5. The normalized growth rate $\gamma T_s$ versus time $t/T_s$ with $q(0) = 6.5$, $q(1) = 5.6$, $p = 1$, $S = 1 \times 10^4$, $\alpha = 5 \times 10^3$, $R_s = 0.1$.

Fig. 6. The normalized growth rate $\gamma T_s$ of the 2/1 mode as a function of shear parameter $R_s$ with $q(0) = 6.5$, $q(1) = 5.6$, $p = 1$, $S = 1 \times 10^4$, $\alpha = 5 \times 10^3$. 
\( \alpha \) or smaller \( R_e \), the double-tearing modes are destabilized, and the magnetic island and velocity pattern are also distorted by the shear flow. For more larger shear flow, the double-tearing modes is further destabilized by the K-H instability.

4. Summary and Discussion

In actual reconnection phenomena, the reconnection process is determined by both the global plasma characteristics as well as local plasma properties. Three dimensional MHD modes are often involved and particle acceleration in all three dimensions are possible. Space observations show both transient, small-scale, fast flows (up to 150 km s\(^{-1}\)) and persistent, large-scale slow flows (2 to 10 km s\(^{-1}\)), and more and more observations have shown that flares have been closely related with sheared flow and they tend to appear at the place where there is complicated configuration of magnetic and velocity fields (Zirin, 1988). The observation (Machado et al., 1988; Tsuneta et al., 1991) have also shown that the interaction of current loops is an essential ingredient in the triggering of the solar flare energy release. The interaction of coronal loops with large current may form a system that has a double-peak current density structure. A principal characteristic of the skin current phase is that the profile of safety factor is parabolic, and the double-tearing modes in this configuration can be considerably complicated, it can exit whenever the plasma has multiple rational surface ( Kataoka = 0 at more than one location). With Sech form shear flow, it is demonstrated that when the \( \alpha \) is small or \( R_e \) is large, the shear flow has a stabilizing effect, but for larger \( \alpha \) or smaller \( R_e \), the sheared flows destabilizes of the double tearing mode, speeds up the formation of magnetic islands in a nonlinear development and ultimately gives rise to a rapid release of magnetic energy. It is suggested that the coupling of doubling tearing modes with shear flow may play a important role in the heating of corona as well as the triggering of flares.

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REFERENCES

Chen, X. L. and P. J. Morrison, Phys. Fluids, B2, 495, 1990.
Deng, X. H. and S. Wang, The magnetic and velocity fields of solar active regions, in Proceedings of IAU Colloquium, No. 141, edited by H. Zirin, G. X. Ai, and H. M. Wang, p. 433, Astronomical Society of the Pacific, 1992.
Einaudi, G. and F. Rubini, Phys. Fluids, 29, 2563, 1986.
Furth, H. P., Phys. Fluids, 28, 1595, 1985.
Hofmann, I., Plasma Phys., 17, 143, 1975.
Machado, M. E., R. L. Moore, A. M. Hernandez, and M. G. Rovira, Astrophys. J., 326, 425, 1988.
Ofman, L., X. L. Chen, and P. J. Morrison, Phys. Fluids, 6, 1364, 1991.
Parker, E. N., Astrophys. J., 174, 499, 1972.
Pritchett, P. L., Y. C. Lee, and J. F. Drake, Phys. Fluids, 23, 1368, 1980.
Richtmeyer, R. D. and K. W. Morton, Different Methods for Initial Value Problems, Ch. 1 and 2, Interscience Pub., 1967.
Sakai, J.-I., Solar Phys., 120, 117, 1989.
Shi, Y., C. C. Wu, and L. C. Lee, Geophys. Res. Lett., 15, 295, 1988.
Strauss, H. R., Phys. Fluids, 19, 134, 1976.
Tsuneta, S. et al., Solar Phys., 136, 37, 1991.
Waddell, B. V., M. N. Rusenbluth, D. A. Monticello, and R. B. White, Nucl. Fusion, 16, 528, 1976.
Zirin, H., Astrophysics of the Sun, Ch. 10 and 11, Cambridge University Press, 1988.