Particle-in-cell numerical simulation of the PHall-IIc Hall thruster

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Abstract. In this work we compare the performance of two types of Hall thrusters, namely, the SPT-100 and the PHall-IIc, the latter being developed by the Plasma Physics Laboratory at the University of Brasilia. The comparison is carried out by performing particle-in-cell simulations. We compute thruster parameters such as thrust and specific impulse. Our results can contribute to the design of electric propulsion devices for future Brazilian space missions.

1. Introduction
Hall thrusters are electric propulsion devices used in deep-space missions and in station-keeping purposes due to their high exhaust speed, high specific-impulse, and high propellant efficiency. They consist of an annular chamber with a radial magnetic field generated using electromagnetic coils and an anode set to a positive potential, and an external hollow cathode. Basically, they operate as follows. Electrons are emitted by a hollow cathode towards an anode in the rear end of the thruster, are trapped due the magnetic field, and start drifting along the annular chamber. The motion of the electrons generates a Hall current, and ionizes a gas, usually xenon. The ions are then electrostatically accelerated out of the annular chamber, generating thrust.

There are basically two types of Hall thruster technology, namely, the Stationary Plasma Thruster (SPT), and the Thruster with Anode Layer (TAL). The main difference between the TAL and the SPT is that an anode layer is installed in the acceleration channel of the SPT instead of the original dielectric material. Among Hall thrusters the SPT-100 is the device with most flight heritage on spacecrafts [1]. The most important achievement of Hall thrusters was the mission SMART-1 by the European Space Agency (ESA) in 2003. In this mission the PPS-1350-G was the first electric propulsion thruster to leave the geosynchronous earth orbit.

Electric propulsion was first elucidated by Robert Goddard in 1906 and later described by Tsiolkovskiy in 1911 [1]. After that, its application was developed by Hermann Oberth in 1929 and by Sheperd and Cleaver in 1949. Since the 60’s the United States and Soviet Union made important contributions and investments in space applications for the Hall thruster. Since 2004 the Plasma Physics Laboratory of University of Brasilia (LFP-UnB) is developing a SPT-type thruster called PHall which differs from the traditional SPT-100 thruster in channel dimensions, operating parameters, and in particular the mechanism to generate the required magnetic field. While the SPT-100 employs electromagnetic coils, the PHall uses permanent magnets [14]. The most recent operational prototype at the LFP-UnB is the PHall-IIc [28]. In this work we compare the performance of the SPT-100 and the PHall-IIc with xenon as propellant via
numerical simulations. This paper is organized as follows. Section 2 describes the computational model and the operating parameters of the SPT-100 and the PHall-IIc. The numerical results are presented in section 3, and the conclusions are presented in section 4.

2. Two-dimensional model
The two-dimensional model adopted in this paper is closely based on the model presented by Ref. [16]. We adopt cylindrical coordinates and neglect variations in the azimuthal direction to define a two-dimensional \((z,r)\) representation of the upper-half of a cross-section of a Hall thruster. This allows us to reduce the computational effort. A detailed description of the simulation domain is presented in Figure 1.

![Figure 1](image)

**Figure 1.** Two-dimensional model of a Hall thruster. The limits of the numerical domain are represented by the blue dashed line. Dielectric material walls are represented by orange squares. The boundary condition of the electrostatic potential is set to a constant value at the anode that depends on the thruster being simulated. The outer boundary is set to 0 V. Particles representing electrons are inserted at the gray region, simulating the effect of an external hollow cathode. The inner region is filled with xenon neutral gas, and the density is assumed to decrease linearly from a maximum value at the anode.

The magnetic field \(B(z,r)\) is assumed to be static and purely radial (i.e., the axial component is neglected) and is modeled by the following piecewise-defined function [16]

\[
B(z,r) = \begin{cases} 
B_0 \exp \left( -16 \left( \frac{z-z_c}{z_c} \right)^2 \right) \hat{r}, & r_i < r < r_f, \\
0, & \text{otherwise}.
\end{cases}
\]

where \(B_0\) represents the maximum value of the modulus of the magnetic field, \(\hat{r}\) is the radial unit vector, \(r_i\) is the radial position where the inner dielectric wall is located (i.e., the beginning of the channel) and \(r_f\) is the radial position of the outer dielectric wall (i.e., the end of the acceleration channel in the radial direction). The resulting vector field is shown in Fig. 2. We note that a realistic model should include the axial component of \(B\), nevertheless, this simplified model captures the essential features of the magnetic field, for example, the nearly-radial field...
that becomes more intense near the exit of the acceleration region. The modulus of the magnetic field has a maximum value near the exit of the acceleration region that depends on the device. The value of $B_0$ for the SPT-100 and the PHall-IIc is given in Table 1.

![Figure 2. The magnetic vector field $\mathbf{B}(r, z)$ in the simulation domain of the SPT-100. The size of the vectors is proportional to the vector modulus, and is also represented using a color scale.](image)

The xenon neutral gas density distribution $n_N$ inside the acceleration region is assumed to be constant in the radial direction, and decreases linearly in the axial direction from a maximum value $n_0$ at the anode to zero at the end of the channel, therefore

$$n_N(z) = \begin{cases} n_0(1 - \frac{z}{L}), & 0 < z \leq L, \\ 0, & \text{otherwise}. \end{cases}$$

where $L$ is the length of the acceleration channel.

The boundary conditions of the electrostatic potential $\phi$ are implemented using Dirichlet boundary conditions. The potential is set to zero at all boundaries except for the anode position, where $\phi$ is set to a constant positive value depending on the device (see Table 1).

The simulation is solved using the particle-in-cell approach [17, 18], in which the position and velocities of particles are obtained from the equations of motion due to the electric and magnetic fields

$$m_\alpha \frac{d\mathbf{v}_\alpha}{dt} = q_\alpha \left( \mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B} \right),$$

(1)

where $m_\alpha$ is the mass of the particle ($\alpha = i, e$), $\mathbf{v}_\alpha$ represents the velocity of the particle, $q_\alpha$ is the charge of the particle and $\mathbf{E}$ is the electric field at the position of the particle. From the distribution of the positions of particles the charge density can be obtained. The Poisson equation is solved from the charge density to obtain new values of the electrostatic potential

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

(2)

where $\rho$ is the charge density and $\epsilon_0$ is the permittivity of free space. From Eq. (2) the electric field can be obtained

$$\mathbf{E} = -\nabla \phi$$

(3)
The electric field obtained from Eq. (3) and the static magnetic field $\mathbf{B}$ are then used to obtain the velocities and positions of the particles from Eq. (1), and the cycle is restarted. Table 1 shows the operating parameters of the SPT-100 and the PHall-IIc.

| Thruster          | SPT-100 | PHall-IIc |
|-------------------|---------|-----------|
| Channel length (m)| 0.025   | 0.0235    |
| Channel width (m) | 0.015   | 0.025125  |
| Exit length (m)   | 0.01    | 0.01      |
| Anode (V)         | 300     | 105.5     |
| Cathode (V)       | 0       | 0         |
| Electron emission current (A) | 4.5 | 4.46 |
| Initial electron energy (eV)     | 25  | 5.61   |
| Maximum magnetic field (T)        | $120 \times 10^{-4}$ | $100 \times 10^{-4}$ |
| Maximum xenon density (m$^{-3}$)  | $10^{21}$ | $10^{21}$ |
| Shrink factor $\zeta$  | 0.02    | 0.02      |

Each particle in the simulation represents a cloud of real particles in order to reduce the number of particles in the simulation and the execution time. The position of the “superparticle” representing a cloud of real particles is given by the center of mass of the cloud, and the velocity is given by the mean velocity of the cloud [18]. In our simulations each superparticle represents $10^5$ real particles. In the following we refer to the superparticles as simply particles.

An important part of a Hall thruster is the hollow cathode, which provides electrons to the device. Representing the cathode in the simulation domain would break the cylindrical symmetry of the model, for this reason we insert electrons directly in the narrow region indicated in Fig. 1. This approach has been used in previous works [19, 16, 23]. Particles leaving the domain are removed from the simulation.

The simulation time can also be reduced by applying a set of self-similar geometric scalings [19]. The scaling is applied by reducing the thruster dimensions linearly, while the plasma parameters are kept constant. The value of the scaling parameter $\zeta$, sometimes called “shrink factor”, is indicated in Table 1, and the scaling laws are shown in Table 2. The unscaled parameter values can be recovered easily by applying the inverse scaling.

### 3. Numerical results

We start our simulations by defining a initial xenon gas distribution and introducing electrons through the exit region shown in Fig. 1. The electrons are initially trapped in the acceleration

| Parameter                  | Scaling law         |
|----------------------------|---------------------|
| Length                     | $L^* = \zeta L$     |
| Electric potential         | $\phi^* = \phi$     |
| Magnetic field             | $\mathbf{B}^* = \zeta^{-1} \mathbf{B}$ |
| Mass flow rate             | $\dot{m}^* = \zeta \dot{m}$ |
| Current                    | $I^* = \zeta I$     |
| Electric field             | $\mathbf{E}^* = \zeta^{-1} \mathbf{E}$ |
| Particle number density    | $n^* = \zeta^{-1} n$ |
| Temperature                | $T^* = T$           |
channel due to the magnetic field and ionize the xenon neutral gas via MonteCarlo collisions, producing xenon ions. The cloud of electrons expands towards the anode, and the production of xenon ions \( \text{Xe}^+ \) increases due to the higher gas density near the anode. The \( \text{Xe}^+ \) particles are accelerated toward the exit region due to the electrostatic potential. These processes describe the initial behavior of the SPT-100 and PHall-Iic simulations, and reproduce the transient start-up of a Hall thruster [24]. Figure 3 shows the steady-state distribution of \( \text{Xe}^+ \) particles over the simulation domain for the SPT-100 and the PHall-Iic simulations. For both thrusters most of \( \text{Xe}^+ \) particles are located inside the acceleration region, except near the walls due to the formation of plasma sheaths.

![Figure 3. Distribution of particles representing \( \text{Xe}^+ \) ions (red points) at the end of the SPT-100 thruster simulation (left panel) and at the end of the PHall-Iic simulation (right panel). The grey rectangles represent dielectric walls.](image)

The particle-in-cell approach allows to compute the electrostatic potential \( \phi \) and the associated electric field self-consistently from the charge density, which is obtained from the distribution of charged particles over the simulation domain. Figure 4 shows the steady-state \( \phi \) for the SPT-100 and the PHall-Iic, represented by a color scale. The spatial profiles of the electrostatic potential of the SPT-100 and the PHall-Iic are similar, however the maximum value of \( \phi \) of the SPT-100 is higher than that of the PHall-Iic. This is due to the higher potential applied to the anode (see Table 1).

The ion flow can be examined in detail by plotting the axial \( z \) component of the velocity \( v_z \) of the \( \text{Xe}^+ \) particles as a function of \( z \). The resulting scatter plot shown in Fig. 5 allows us to identify three different regions of the ion flow. Near the anode (\( z \sim 0 \)) a number of particles have a negative velocity, which indicates that there is a backflow of ions towards the anode which is a common characteristic of Hall thrusters [19]. This region is called the diffusion region due to the weak magnetic field [19]. At the interior of the channel (\( 0.005 \leq z \leq 0.03 \)) the values of the axial component of the ion velocity fluctuate around \( v_z = 0 \), which characterizes the ionization region. At the exit region of the SPT-100 and the PHall-Iic there is a clear change of regime in which \( v_z \) displays a trend with a positive slope, due to the ions being accelerated by the electrostatic potential. These results confirm that the internal structure of the ion flow of the PHall-Iic is similar to the SPT-100, however the resulting ion acceleration is lower.

Next, we estimate the thrust and the specific impulse from our numerical results. Since the
Figure 4. The electrostatic potential $\phi$ represented by a color scale at the end of the SPT-100 simulation (left panel) and the PHall-IIc simulation (right panel).

Figure 5. The axial velocity $V_z$ as a function of axial position $z$ of Xe$^+$ for the SPT-100 (left panel) and the PHall-IIc (right panel).

The velocity of the ions greatly exceeds the velocity of the xenon neutral particles in the exit region, the thrust $T$ can be written as [1]

$$T = \dot{m}_i \langle v_i \rangle$$  \hspace{1cm} (4)

where $\dot{m}_i$ is the ion mass flow and $\langle v_i \rangle$ is the average ion velocity. A discretized form of Eq. (4) can be written as [27]

$$T = N_p \frac{m_i}{\Delta t} \langle v_i \rangle$$  \hspace{1cm} (5)

where $N_p$ is the number of particles represented by each superparticle, and $\Delta t$ is the time step.

The specific impulse $I_{sp}$ is related to the thrust by

$$I_{sp} = \frac{T}{mg}$$  \hspace{1cm} (6)
where $\dot{m}$ is the neutral mass flow rate, and $g = 9.8 \text{ m/s}^2$ is the Earth’s acceleration of gravity. If the propellant is xenon, Eq. (6) can be written as [1]

$$I_{sp} = 1.02 \times 10^5 \frac{T}{Q}$$

(7)

where $Q$ is the xenon mass flow rate in mg/s.

We computed $T$ and $I_{sp}$ using Eqs. (5) and (7), respectively, from the numerical simulations of the SPT-100 and the PHall-IIc. The average velocity of ions $\langle v_i \rangle$ is computed by selecting particles crossing the end of the exit region (marked as outer boundary in Fig. 1), and taking into account the axial component of the ion velocity (i.e., $v_i = v_z$). The results are shown in Table 3. The computed value of the thrust of the SPT-100 is lower than the expected value of $\sim 0.080$ N from experimental results [1, 16]. Since the exit region in the computational domain has a relatively short length of 1 cm, the value of $T$ computed by this method can be underestimated [23]. The computed value of the $I_{sp}$ of the SPT-100 agrees with experimental results [1]. Table 3 also indicates that the thrust and the specific impulse of the PHall-IIc are lower than those of the SPT-100. These results are expected since the PHall-IIc thruster operates at a lower power than the SPT-100.

| Table 3. Computed thruster parameters |
|---------------------------------------|
|                                      |
| SPT-100                              |
| Thrust (N)                           |
| 0.048 ± 0.00083                      |
| Specific impulse (s)                 |
| 1636.24 ± 28.42                     |
| PHall-IIc                            |
| Thrust (N)                           |
| 0.019 ± 0.00031                      |
| Specific impulse (s)                 |
| 650.92 ± 10.65                       |

4. Conclusion
in this work we performed numerical simulations of two types of Hall thrusters, namely, the SPT-100 and the PHall-IIc being developed at the Plasma Physics Laboratory at the University of Brasilia. We used a 2D axisymmetric model to represent the thruster geometry, and the particle-in-cell method to model charged particles and solve the field equations self-consistently. Our results indicate that the thrust and the specific impulse of the PHall-IIc attain lower values than the SPT-100. These results are expected since the PHall-IIc operates at a lower power in comparison with the SPT-100. We are currently working to implement a more realistic model of the magnetic field profile, an accurate method to compute $T$ and $I_{sp}$ [23] and a model of wall erosion to estimate the operational lifetime of the PHall-IIc. Our results can be useful for designing new Hall thruster prototypes for future Brazilian space missions.

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