Problems of modern cosmology: How dominant is the vacuum?

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It would be hard to find a cosmologist today who does not believe that the vast bulk of the Universe (ninety-five percent or more) is hidden from our eyes. We review the evidence for this remarkable consensus, and for the latest proposal, that the mysterious dark matter consists of as many as four separate ingredients: baryons, massive neutrinos, new “exotic” dark matter particles, and vacuum energy, also known as the cosmological constant (Λ). Of these, only baryons fit within standard theoretical physics; the others, if their existence is confirmed, will mean rewriting textbooks. Fresh experimental evidence has recently appeared for and against all four components, so that the subject is in a state of turmoil and excitement. The past few years in particular have seen the fourth (vacuum) component come into new prominence, largely at the expense of the third (exotic dark matter). We conclude our review by exploring the possibility that the energy density of the vacuum is in fact so dominant as to leave little room for significant amounts of exotic dark matter.

1 The Four Elements of Modern Cosmology

Hear first the four roots of all things: bright Zeus (fire), life-giving Hera (air), Aidoneus (earth) and Nestis (water), who moistens the springs of mortals with her tears.
- Empedokles, Fragments, c. 450 B.C.

Like the philosophers of antiquity, modern cosmologists have divided the physical world into four different realms, each characterized by its own length scale and dominated by an increasingly rarefied species of invisible matter which, while not seen directly, is inferred to exist from its gravitato-

![Fig. 1. The “four elements” of modern cosmology (adapted from a figure in a 1519 edition of Aristotle’s Libri de Caelo)
density $\Omega_{\text{CDM}}$ would far exceed that of the baryons. Theoretical particle physics provides several plausible candidates, and it could even be that this component of the cosmic fluid — identified with “water” in Fig. 1 — itself has several ingredients. However, despite intensive searches, none of these candidate particles has yet been observed.

On still larger scales — those relevant to the formation of galaxy clusters in the early universe — a different form of dark matter may be operative: the neutrino. That this particle exists is unquestioned; but the extent of its contribution (to the density of the Universe is not yet clear. If the neutrino has a small (or zero) rest mass, then it is always relativistic and can be treated for dynamical purposes like the photon. (We have therefore classified it together with light as the “air” of the new world view, Fig. 1.) In this case, neutrino contributions can be combined with those of photons ($\Omega_{\gamma}$) to give the total radiation density of the Universe, $\Omega_{R} = \Omega_{\nu} + \Omega_{\gamma}$. This is known to be insignificant at present. If, on the other hand, neutrinos are sufficiently massive, then they are no longer relativistic on average, and belong instead (with baryonic and exotic dark matter) under the category of dust-like (zero-pressure) matter, with total matter density $\Omega_{M} = \Omega_{\text{BAR}} + \Omega_{\text{CDM}} + \Omega_{\nu}$. Only in the latter case do neutrinos play a significant dynamical role in the present Universe. Recent experimental evidence has been taken by many to support this second scenario, implying (in most models) a collective neutrino density well below that of baryonic matter, but (in others) a density possibly rivalling that attributed to exotic matter.

Influential only over the largest of scales — the cosmological horizon — is the outmost species of invisible matter: the vacuum energy (also known by such names as dark energy, quintessence, $x$-matter, the zero-point field, and the cosmological constant $\Lambda$). This profusion of nomenclature betrays the fact that there is at present no consensus as to where vacuum energy originates, or how to calculate its energy density ($\Omega_{\Lambda}$) from first principles. Existing theoretical estimates of this latter quantity range over some 122 orders of magnitude, prompting most cosmologists until very recently to disregard it altogether. New observations of distant supernovae, however, suggest that the vacuum not only gravitates, but that its effective density exceeds that of all the other forms of matter put together. Thus the “standard model” of cosmology has evolved (in the space of three years) from one in which $\Omega_{\Lambda} = 0$ in practice (and is as large as $\sim 10^{122}$ in theory), to one in which there is widespread agreement that $\Omega_{\Lambda}$ must be of order unity — indicating to us that cosmology, far from being a “solved problem” as reported by some authors, remains in a far-from-settled state. We have identified vacuum energy with “fire” in Fig. 1.

Four kinds of invisible matter: dark baryons, exotic particles, neutrinos and vacuum energy — three of which imply new physics, and any one of which (with the possible exception of the neutrino) outweighs the entirety of the visible Universe. Are they all necessary? Our purpose in this review will be to re-examine the steps leading up to this conclusion. We begin with the relevant cosmological equations in $\S$ 2. The “four elements” are then introduced in turn: dark baryons ($\S$ 3), exotic matter ($\S$ 4), neutrinos ($\S$ 5) and the newly-popular vacuum energy ($\S$ 6). In $\S$ 7 we consider the hypothesis (advanced by one of the authors for ten years) that vacuum energy is in fact so dominant that there is no need for an exotic component to the dark matter. Conclusions are summarized in $\S$ 8.

2 Dark Matter and the Evolution of the Universe

The relevant equations here are Einstein’s field equations of general relativity:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} \mathcal{R} g_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$  

where the terms on the left-hand side describe the geometrical structure of spacetime, while those on the right-hand side describe its matter and and radiation content. $\mathcal{R}_{\mu\nu}$ and $\mathcal{R}$ are the Ricci tensor and curvature scalar, $g_{\mu\nu}$ is the metric tensor, and $T_{\mu\nu}$ is the energy-momentum tensor; $G$, $c$ and $\Lambda$ are

Alternative theories of gravity can also be constructed in such a way to remove the need for large amounts of dark matter in our Universe; we do not consider these here.
all constants of nature. About the value of $\Lambda$, in particular, we will say more in §

The densities of matter ($\Omega_m$) and radiation ($\Omega_r$) are stored in $\mathcal{T}_{\mu\nu}$, while that of the vacuum ($\Omega_\Lambda$) is a function of $\Lambda$. Assuming that these three components are distributed homogeneously and isotropically on large scales, and that they do not exchange energy at a significant rate, Eqs. (1) reduce to a pair of differential equations in the cosmological scale factor $R$ and its time derivatives, including the Hubble parameter $H \equiv \dot{R}/R$ (the expansion rate of the Universe). The equation for $H$ is

$$[H(z)/H_0]^2 = \Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0} - (\Omega_{\text{TOT},0} - 1)(1+z)^2,$$  

(2)

where $z \equiv (R/R_0)^{-1} - 1$ is the cosmological redshift. The subscript “0” here (and throughout our review) denotes quantities measured at the present time; i.e., at redshift $z = 0$. These are constants, and must be carefully distinguished from functions of time (or redshift) such as $\Omega_m$, $\Omega_r$, and $\Omega_\Lambda$. The constant $\Omega_{\text{TOT},0} \equiv \Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0}$ is of particular interest, because it separates spatially spherical models from hyperbolic ones. If $\Omega_{\text{TOT},0} > 1$, then the Universe is closed and finite in extent. Conversely, if $\Omega_{\text{TOT},0} < 1$, then it is open and infinite in extent. And, if $\Omega_{\text{TOT},0} = 1$ exactly, then we live in an infinite Universe which is spatially flat (“Euclidean”).

To determine the shape of the homogeneous and isotropic world has been a prime goal of cosmologists since expansion was discovered.

Eq. (3) already tells us a great deal about the evolution of the Universe. The first term on the right-hand side, $\Omega_{m,0}(1+z)^3$, shows that matter acts to increase the expansion rate $H(z)$ as one goes to higher $z$ — that is, to slow down the expansion with time. This is the braking effect of matter’s gravitational self-attraction.

The second term, $\Omega_{r,0}(1+z)^4$, shows that radiation has the same effect, but with a stronger dependence on redshift. This means that, as one moves backward in time, photons (and relativistic particles) become increasingly important compared to pressureless matter. In fact, the dynamics of the early Universe (at redshifts above $z \gg 1000$) must have been completely dominated by them. At present, however, the total density $\Omega_{r,0}$ of “radiationlike matter” (including both photons and relativistic neutrinos) is several orders of magnitude below that of nonrelativistic matter. Since we are largely concerned in this review with redshifts less than ten, we will drop the radiation term in Eq. (2) from this point onward.

The third term, $\Omega_{\Lambda,0}$, is independent of redshift, which means that its influence is not diluted with time. Vacuum energy will therefore eventually come to dominate the dynamics of the Universe in any model with $\Lambda > 0$. In the limit $t \to \infty$, in fact, the other terms drop out, and we find that the density of the vacuum may be expressed as $\Omega_{\Lambda,0} = (H_\infty/H_0)^2$, or (since $\Omega_{\Lambda,0} \equiv \Lambda c^2/3H_\infty^2$)

$$\Lambda c^2 = 3H_\infty^2,$$  

(3)

where $H_\infty$ is the limiting value of the Hubble parameter as $t \to \infty$ (assuming that this latter quantity exists; i.e., that the Universe does not recollapse in the future). This provides a little-discussed connection between $\Lambda$ (an apparent constant of nature) and the asymptotic expansion rate (a dynamical parameter). If $\Lambda > 0$, and if we are living at sufficiently late times, then Eq. (3) immediately predicts that we will measure $\Omega_{\Lambda,0} \sim 1$.

The fourth term in Eq. (2), finally, shows that an excess of $\Omega_{\text{TOT},0}$ over one (i.e., a positive curvature) acts to offset the contribution of the first three terms to the expansion rate, while a deficit (i.e., a negative curvature) enhances them. Open models, in other words, expand more quickly at any given

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2Densities throughout this review will be written in the form of the dimensionless density parameter $\Omega$, which is just the ratio of physical density ($\rho$) to the critical density $\rho_{\text{crit}} = 3H_\infty^2/(8\pi G)$. This latter quantity depends on the Hubble parameter, Eq. (1), and takes the value $\rho_{\text{crit},0} = 3H_\infty^2/(8\pi G)$ at the present time. If $H_0$ lies in the range 70 – 90 km s$^{-1}$ Mpc$^{-1}$ (§2), then $\rho_{\text{crit},0}$ is equivalent to between 5.5 and 9.1 protons per cubic meter.

3Strictly speaking, it is also possible to obtain flat and hyperbolic solutions which are fine, by suitably “identifying” different points and adopting a nontrivial topology [22]; we do not pursue this possibility here.

4This is inferred, not only from measurements such as those of the COBE satellite, but also from the fact that a too-high density of relativistic matter would have slowed expansion so much that the Universe could not have lived long enough to produce the oldest stars we see.
redshift \( z \) (and therefore last longer) than closed ones. This curvature term, however, goes only as \((1 + z)^2\), which means that its importance drops off relative to the matter and radiation terms at early times, and becomes negligible compared with that of the vacuum term at late ones.

As recently as the 1980s, many cosmologists were persuaded that Eq. (3) could be substantially simplified, not only by neglecting the second (radiation) term, but also the third (vacuum) and fourth (curvature) terms on the right-hand side. This appeared reasonable at the time, for four principal reasons. First, these terms differ sharply from each other (and from the first term) in their dependence on redshift \( z \), and the probability that we should happen to find ourselves in an era when all four terms have similar values would seem a priori very remote. By this “Dicke coincidence” argument, it was felt that only one term ought to dominate at any given time \( \Omega \). Second, the vacuum term in particular was avoided for historical and theoretical reasons (to be discussed in \( \Omega \)). Third, a period of cosmic inflation was widely asserted to have driven \( \Omega_{TOT}(t) \) to exactly unity in the early universe \( \Omega \). And finally, this “standard Einstein-de Sitter” (EdS) model was favored on grounds of simplicity. These arguments are no longer valid today. We are justified in neglecting the radiation term, and only the radiation term in Eq. (2), leaving

\[
H^2(z) = H_0^2 \left[ \Omega_{M,0}(1 + z)^3 + \Omega_{\Lambda,0} - (\Omega_{TOT,0} - 1)(1 + z)^2 \right].
\]

This is the modern version of what is usually called Friedmann’s equation in cosmology. It may be integrated numerically for the cosmological scale factor \( R(t) \) as a function of time.

Several examples are plotted in Fig. 2, including closed models (1 through 5) and one flat (6) and open model (7). Model 1, with \( \Omega_{M,0} = 0.014 \) and \( \Omega_{\Lambda,0} = 1.08 \), has been proposed in \( \Omega \) and \( \Omega \) and will be discussed further in \( \Omega \). The others all have \( \Omega_{M,0} = 0.3 \), a figure widely quoted today for the total density of gravitating matter (\( \Omega \)). Model 6, with \( \Omega_{\Lambda,0} = 0.7 \) (known as the \( \Lambda \)CDM model), has been singled out as the newest “standard model” of cosmology. Two timescales are plotted (top and bottom), depending on the present value \( H_0 \) of Hubble’s parameter (\( \Omega \)).

Along each of the curves, we have marked the points where \( \Omega_{M}(z) \) takes on maximum values (\( \Delta \)), the points of inflection (\( \star \)), and the points where \( \Omega_{M}(z) \) takes on maximum values and \( H(z) \) reaches a minimum (\( \bigtriangledown \)). One finds that \( R \) takes the special value \( R_{\Lambda} \equiv 1/\sqrt{\Lambda} \) at the points (\( \Delta \)). The cosmological constant may thus be understood physically (in closed models) as the curvature of space at the time when the matter density parameter goes through its maximum (see \( \Omega \) for discussion).

Joining the points of inflection in Fig. 2 are two dashed lines marked \( \Omega_{\Lambda,E} \) (for “Einstein limit”). One must have \( \Omega_{\Lambda,0} < \Omega_{\Lambda,E} \) (a function of the matter density \( \Omega_{M,0} \)) in order for expansion to originate in a big bang singularity. Solutions with \( \Omega_{\Lambda,0} = \Omega_{\Lambda,E} \) go over to Einstein’s static model as \( t \to \infty \). When \( \Omega_{\Lambda,0} > \Omega_{\Lambda,E} \), \( R(t) \) drops to a nonzero minimum and starts to climb again in the past direction; these are usually known as Eddington-Lemaître (or “bounce”) models.

The value of \( \Omega_{\Lambda,E} \) can be computed for a given model by differentiating Eq. (\( \Omega \)) at the points of in-

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Footnote:

That this is not necessarily so has been shown by several authors \( \Omega, \Omega \). A suitable inflationary phase may be grafted onto models with \( \Omega_{TOT,0} \neq 1 \). The probability of finding oneself in such a universe depends, not just on the raw amount of inflation, but on factors such as the elapsed time and the distribution of initial conditions (such as phase transitions \( \Omega \)) which preceded the inflationary epoch.
flection (*). This leads to a cubic polynomial which must be solved parametrically in general [10, 12] but has a little-appreciated direct solution for cases in which \( \Omega_{M,0} \leq 0.5 \) [7]. With the conviction that \( \Omega_{M,0} = 1 \) now fading in the astronomical community, and most cosmologists calling for values of \( \Omega_{M,0} \approx 0.3 \), it may be worthwhile to dust off this formula again. In modern form it reads [13, 123]

\[
\Omega_{\Lambda, E} = 1 - \Omega_{M,0} + \frac{3}{2} \Omega_{M,0}^{2/3} \times \left[ \left( 1 - \Omega_{M,0} + \sqrt{1 - 2\Omega_{M,0}} \right)^{1/3} + \left( 1 - \Omega_{M,0} - \sqrt{1 - 2\Omega_{M,0}} \right)^{1/3} \right]. \quad (5)
\]

One finds that \( \Omega_{\Lambda, E} = 1.10 \) if \( \Omega_{M,0} = 0.014 \), for instance [7], while \( \Omega_{M,0} = 0.06 \) leads to \( \Omega_{\Lambda, E} = 1.25 \). Combined analysis of data on cosmic microwave background (CMB) fluctuations from the COBE, BOOMERANG and MAXIMA experiments implies that \( \Omega_{TOT,0} \leq 1.24 \) at 2σ confidence [68]. Since \( \Omega_{N,0} \) is certainly less than \( \Omega_{TOT,0} \), we infer that \( \Omega_{N,0} < \Omega_{\Lambda, E} \) in models with \( \Omega_{M,0} \geq 0.06 \), which may be regarded as a proof of the existence of the big bang in these models [3]. For higher matter densities \( 0.2 \leq \Omega_{M,0} \leq 0.5 \) (as discussed in [4]), one obtains even larger values of \( \Omega_{\Lambda, E} \), between 1.5 and 2.0 (see the Einstein limits in [14]).

The differences between the models shown in Fig. 2 become apparent when their evolution is plotted on a phase diagram, with matter density parameter along one axis and vacuum density parameter along the other. The key equations [14] are

\[
\Omega_{\Lambda}(z) = \frac{\Omega_{\Lambda,0}}{[\Omega_{M,0}(1 + z)^3 + \Omega_{\Lambda,0} - (\Omega_{M,0} + \Omega_{\Lambda,0} - 1)(1 + z)^2]} \\
\Omega_{M}(z) = (\Omega_{M,0}/\Omega_{\Lambda,0}) \Omega_{\Lambda}(z) (1 + z)^3. \quad (6)
\]

Fig. 3 depicts the same family of models as Fig. 2, with redshift factors \( [1+z = R_0/R(t)] \) labelled at intervals along the curves. Also marked are contours of constant deceleration, defined by \( q = -\dot{R}R/\dot{R}^2 = \Omega_{M}/2 - \Omega_{\Lambda}. \) This parameter takes values of 0.5 at each point of maximum matter density parameter (Δ), zero at the inflection points (*), and −1 at the points of minimum expansion rate (∇).

All positive-\( \Lambda \) models begin in Fig. 3 at the point (1,0) and evolve asymptotically toward (0,1) as \( t \to \infty \). Flat (Euclidean) models follow a straight line; any deviation from critical density produces a curved path. Those to the right of Model 6 are all closed. Models 5 through 2 are increasingly unlikely insofar as they violate the above-mentioned observational bound \( \Omega_{TOT,0} \leq 1.24 \) on total density [68]. Model 2, in particular, cannot describe the real universe. However, the model immediately adjacent to it in phase space (Model 1) is perfectly acceptable in this regard, since it has \( \Omega_{TOT,0} = 1.094 \). Very different combinations of \( \Omega_{M,0} \) and \( \Omega_{\Lambda,0} \), in other words, can produce almost identical trajectories in phase space. Indeed, from the perspective of Fig. 3, the popular Euclidean models appear implausibly fine-tuned. One would not expect the Universe to take the shortest path through phase space, any more than one would expect a star to follow a straight line from the main sequence to the red giant branch on a Hertzsprung-Russell diagram.

The slow expansion rate and high matter density parameter between the points marked (Δ) and [6]Of course, the Universe we have assumed (homogeneous and isotropic) is simpler than most stars, and might be restricted to such a path for reasons having to do with some higher symmetry of nature. While this may be so, however, any such symmetry would lie outside the context of Einstein’s theory of gravity as it stands.
single out this stage of evolution for large-scale structure formation. If $\Omega_{\Lambda,0}$ is of the same order (or less) than $\Omega_{\text{M,0}}$, however, this process must occur very quickly. Consider $\Lambda$CDM, represented by Model 6 in Fig. 2. Analysis of the Hubble and William Herschel Deep Fields suggests that the number density of galaxies at redshifts $z \approx 4 - 6$ (i.e., at scale factors $R/R_0 \approx 0.1 - 0.2$) is equal to that at $z = 0$. If so, then these objects were in place by $z = 4$, or (consulting Fig. 2 for Model 6, and using either the top or bottom scale for $H_0$) within 1.2-1.5 Gyr after the big bang. This poses a serious challenge for the model, since primordial density fluctuations must not only decouple from the Hubble expansion on short timescales, but do so at a time when the expansion rate (the slope of the curve in Fig. 2) is some six times its present value. The problem is even worse in models with lower values of $\Omega_{\Lambda,0}$ (e.g., Model 7).

The standard way to address this has been to suppose that most of the matter density is in an exotic new form (CDM) which is able to decouple from the primordial fireball before the baryons, preparing potential wells for them to fall into. This approach successfully accelerates structure formation on large scales, but is perhaps too successful on smaller ones. Galaxy-sized regions are formed with excessively peaked central masses (the “density cusp problem”) and too many low-mass fragments (the “substructure problem”). These problems may be resolved within the CDM picture by refining the properties of the exotic matter; proposals include warm, fuzzy, fluid and self-interacting dark matter.

Alternatively, difficulties with the growth of large-scale structure are substantially eased in models with larger ratios of $\Omega_{\Lambda,0}$ to $\Omega_{\text{M,0}}$. In Model 1, for instance, Fig. 2 shows that redshift $z = 4$ corresponds to between 9.3 and 11.9 Gyr after the big bang (depending on the value of $H_0$), giving the galaxies seven times longer to form. The expansion rate at this redshift is only 0.7 times its present value. Nor is the low (present) density of gravitating matter a problem in this model, because $\Omega_{\text{M}}(z)$ reaches levels as high as four times the critical density at redshifts near $z \approx 5$ (Fig. 3). It is natural to associate this redshift with the onset of galaxy formation, and it would be of great interest to test “slightly-closed” cosmologies of this kind via numerical simulations.

Existing studies like that shown in Fig. 4 by the Virgo Consortium have so far been restricted to flat and open models. Of these, $\Lambda$CDM (the new “standard CDM” model) produces a mass distribution in much better agreement with the observed distribution of light than EdS (the old “standard CDM” model, still denoted SCDM in many works). The improvement is especially pronounced at higher redshifts (top two panels). This has been taken as evidence for a significant $\Lambda$-term. More detailed analysis, however, reveals that the power spectrum of the mass distribution does not agree with that observed for galaxies in either of these two models. The discrepancies have typically been
attributed to nonlinear, scale- and morphology-dependent bias factors [16]; a complementary approach would be to consider an expanded repertoire of \( \Omega_{M,0} \) and \( \Omega_{\Lambda,0} \) values.

### 3 Baryons

Beginning from the observed luminosity density of the Universe, one can infer the total density of luminous baryonic matter (i.e., that in stars) by making various reasonable assumptions about the fraction of galaxies of different morphological type, their ratios of disk-type to bulge-type stars, and so on. The latest such estimate is \([10]\)

\[
\Omega_{\text{LUM}} = (0.0027 \pm 0.0014) h_0^{-1}, \tag{7}
\]

where \( h_0 \) is the present value of Hubble’s parameter expressed in units of 100 km \text{s}^{-1} \text{Mpc}^{-1}.

This latter parameter is unfortunately not yet fixed by observation, and we pause to discuss its value before proceeding. Using various relative-distance methods, all calibrated against Cepheid variable stars in the Large Magellanic Cloud (LMC), the Hubble Key Project (HKP) team has determined that \( h_0 = 0.71 \pm 0.06 \) \([88]\). Fundamental physics methods (e.g., the Sunyaev-Zeldovich effect, gravitational lensing time delays [GLTDs]) have higher uncertainties but are roughly consistent with this, \( h_0 = 0.65 \pm 0.08 \) \([101]\). The near convergence of these approaches has been widely hailed, with many authors asserting that precision values of \( h_0 \) are just around the corner, awaiting only a final round of experimental refinements.

On the other hand, a recalibrated LMC Cepheid period-luminosity relation based on a much larger Cepheid sample (from the OGLE collaboration) leads to a considerably higher value of \( h_0 = 0.85 \pm 0.05 \) \([126]\). And a new VLBI measurement of the transverse velocity of water masers in NGC4258 gives a purely geometric distance to this galaxy \([60]\) which also implies that the traditional calibration is off, boosting Cepheid-based estimates by \( 12 \pm 9\% \) \([84]\). This would raise, e.g., the HKP value to \( h_0 = 0.80 \pm 0.09 \). There is some additional independent support for this recalibration in observations of eclipsing binaries \([55]\) and “red clump stars” in the LMC \([119]\). Fundamental physics approaches are also not immune to systematic effects: GLTD-based values of \( h_0 \), which are routinely computed assuming EdS, rise by about 7% (on average) in \( \Lambda \)CDM, and 9% in open models.

“Hubble fatigue” may therefore have prompted cosmologists to embrace prematurely small levels of uncertainty in \( h_0 \). We attempt to allow for this by retaining two possible values for \( h_0 \) in this review; a “low value” of \( h_0 = 0.7 \) and a “high value” of \( h_0 = 0.9 \). This seemingly modest range of choices turns out to discriminate quite powerfully between the cosmological models considered here. To a large extent this is a function of their ages. Fig. 2 reveals, for example, that \( \Lambda \)CDM (represented by Model 6) is 13.5 Gyr old if \( h_0 = 0.7 \), or 10.5 Gyr if \( h_0 = 0.9 \). The oldest metal-poor halo stars seen in the Milky Way have an age of 15.6\( \pm 4.6 \) Gyr \([27]\), setting a firm lower limit of 11.0 Gyr on the age of the Universe.

This is enough to rule out \( \Lambda \)CDM with the high value of \( h_0 \), but not the low one.

Model 1, on the other hand, faces the opposite problem: Fig. 2 shows that it has a total age of 30.2 Gyr if \( h_0 = 0.9 \), or 38.8 Gyr if \( h_0 = 0.7 \). Both numbers are larger than most cosmologists are prepared to accept. However, upper limits on the age of the Universe are not as secure as lower ones. One must take into account, for instance, that the galaxy formation associated above with redshifts \( z \approx 4 \) occurred between 9 and 12 Gyr after the big bang in this model, so that galaxies would not be older than \( 24 \pm 3 \) Gyr in any case.

There are various ways to test such a hypothesis. One might expect to find a greater spread in galaxy ages (and hence colors) at \( z \approx 4 \), given their longer formation time. Galaxies in Model 1 had \( \sim 6 \) Gyr to form, or about \( \sim 25\% \) of their nominal lifetime, according to Fig. 2 (with \( h_0 = 0.9 \)). The corresponding fraction in Model 6 is \( \lesssim 10\% \). This may not necessarily translate into a difference between observed color spreads, however, since high-redshift galaxies are seen principally during (relatively brief) episodes of star formation.

Very old galaxies, if they exist, should also be present at lower redshifts. They would be inherently faint and reddened, making them difficult to find and distinguish from younger objects which are
simply obscured by dust. Nevertheless, several candidates have been noted in the past years, including a number of low surface brightness galaxies \[15\] and extremely red objects \[25, 64\] whose ages based on simple (“single-burst”) evolution models appear to be as high as \(\sim 17\) Gyr or more.

If our own Milky Way is not unusually young, we should also expect to find large numbers of dead stars in the galactic halo. These would act as microlenses, inducing variability in background stars and quasars, even if they were too dim to be seen directly. Such objects may now have been detected in the direction of the LMC (see below).

Returning now to the density of luminous matter, we find with our values for \(h_0\) that Eq. (7) gives

\[
\Omega_{\text{LUM}} = 0.0034 \pm 0.0018 .
\]  

(8)

The visible Universe, in other words, constitutes an insignificant 0.5\% or less of the critical density.

It may, however, be that many of the baryons in the Universe are not visible. How significant could these dark baryons be? The theory of cosmic nucleosynthesis provides us with an independent method for determining the density of total baryonic matter in the Universe, based on the assumption that the light elements we see today were forged in the furnace of the hot big bang. Results using different light elements agree tolerably well, which is impressive in itself. The primordial abundances of \(^4\)He (by mass) and \(^7\)Li (relative to H) imply a baryon density of \(\Omega_{\text{BAR}} = (0.011 \pm 0.005) h_0^{-2}\) \[90\], whereas new measurements based exclusively on the primordial D/H abundance give a higher value: \(\Omega_{\text{BAR}} = (0.019 \pm 0.002) h_0^{-2}\) \[118\]. These two results, both given at the 2\(\sigma\) confidence level, are superimposed on a plot of predicted light element abundances in Fig. 5. They do not overlap. In our view it is premature at present to exclude either one. We therefore adopt the same strategy here as with Hubble’s parameter, retaining a “low” baryon density of 0.01\(h_0^{-2}\) and a “high” one of 0.02\(h_0^{-2}\) throughout our review. Combining this with our high and low values of \(h_0\), we conclude that the baryonic density lies in the range

\[
\Omega_{\text{BAR}} = 0.012 - 0.041 ,
\]  

(9)

a result in very good agreement with that obtained by the entirely independent method of adding up individual contributions from all known repositories of baryonic matter via their estimated mass-to-light \((M/L)\) ratios \[46\]. If \(\Omega_{\text{TOT}}\) is close to unity, as it now seems (§6), then it follows from Eq. (4) that baryons — and everything that would have been recognized as “matter” before 1930 — make up less than 5\% (by mass) of the known Universe.

Most of these baryons, moreover, have not been seen. The baryonic dark matter fraction \(f_{\text{BDM}}(\equiv \Omega_{\text{BDM}}/\Omega_{\text{BAR}}) = 1 - \Omega_{\text{LUM}}/\Omega_{\text{BAR}}\) lies in the range

\[
f_{\text{BDM}} = 77\% - 95\% ,
\]  

(10)

where we have used Eq. (8) together with our high and low values of \(h_0\) and \(\Omega_{\text{BAR}}\). Where could these dark baryons be? One possibility is that they are smoothly distributed in a gaseous intergalactic medium, which would have to be strongly ionized in order to explain why it has not left a more obvious signature in quasar absorption spectra. Recent observations using O\(\text{VI}\) absorption lines as a tracer of ionization suggest that the contribution of such
material to \( \Omega_{\text{BAR}} \) is at least \( 0.003h_0^{-1} \) \cite{15}, comparable to \( \Omega_{\text{LUM}} \). Numerical simulations are able to reproduce many observed features of the Lyman \( \alpha \) (Ly\( \alpha \)) forest with as many as \( 80 - 90\% \) of the baryons in this form \cite{33}.

Dark baryonic matter, however, could also be bound up in clumps of matter such as substellar objects (jupiters, brown dwarfs) or stellar remnants (white, red and black dwarfs, neutron stars, black holes). The former are not thought to be numerous enough to be important, given their low mass. The latter are limited in the opposite sense; black holes cannot be more massive than about \( 10^5M_\odot \) since this would lead to dramatic tidal disruptions and lensing effects which are not seen \cite{17}. The critical mass range for dark baryon clumps is thus within a few orders of magnitude of the solar mass. Microlensing constraints based on quasar variability do not seriously limit such objects at present, setting an upper bound of 0.1 (well above \( \Omega_{\text{BAR}} \)) on their combined contributions to the density parameter of an EdS universe \cite{107}.

A likely detection of dark compact objects within our own galactic halo has recently been reported by the MACHO microlensing survey of stars in the LMC \cite{4}. The lenses, with masses in the range \((0.15 - 0.9)M_\odot\), appear to account for between 8% and 50% of the high rotation velocities seen in the outer parts of the Milky Way — depending on the halo model chosen, and extrapolating \((\text{at } 2\sigma \text{ confidence})\) from the \(~15\) events actually seen.\footnote{An alternative interpretation of the data, that most or all of these lenses are actually faint stars inside the LMC itself, now appears to be strongly disfavored \cite{59}.} The identity of these objects has been hotly debated, with some authors linking them to faint, fast-moving objects apparently detected in the Hubble Deep Field \cite{27}. It is unlikely that they could be traditional white dwarfs, since these are formed from massive progenitors whose metal-rich ejecta we do not see \cite{11}. Degenerate “beige dwarfs,” which might be able to form above the hydrogen-burning mass limit of 0.08 \( M_\odot \) without fusing \cite{58} are one possibility. Another would be a population of ancient, low-mass \((\lesssim 0.6M_\odot)\) stars which have simply cooled into invisibility.\footnote{Existing limits on the density of halo objects in this mass range \cite{52} necessarily extrapolate in a number of ways from known stellar populations, and are based on theoretical stellar evolution models \cite{24} which do not currently extend beyond 20 Gyr.}

### 4 Exotic Dark Matter

Three main reasons have been proposed for going beyond dark baryons and introducing a second species of invisible matter, the exotic cold dark matter (CDM), into the Universe: (1) a range of observational arguments imply that the density parameter of total gravitating matter \((\Omega_{m,0} = \Omega_{\text{BAR}} + \Omega_{\text{CDM}} + \Omega_{\nu})\) is higher than that provided by baryons and neutrinos alone; (2) our current understanding of large-scale structure formation requires the process to be helped along by quantities of cold (i.e., nonrelativistic) gravitating matter in the early universe, creating the potential wells for infalling baryons; and (3) theoretical physics supplies several plausible (albeit still hypothetical) candidate CDM particles with the right properties.

Since our ideas on structure formation may yet change, and the candidate particles may not materialize, the case for exotic CDM turns on the observational arguments. At present these agree to within no better than a factor of five, pointing to values of \( \Omega_{m,0} \) between about 0.1 and 0.5. (Not long ago, in the 1980s, there were calls for \( \Omega_{m,0} = 1 \), but these tended to come from theorists wishing to retain the EdS model, and are no longer tenable observationally \cite{125}. ) The lower limit is crucial: if \( \Omega_{m,0} > \Omega_{\text{BAR}} + \Omega_{\nu} \), then \( \Omega_{\text{CDM}} > 0 \).

The arguments can be broken into two classes: those which are purely empirical, and those which assume in addition the validity of the gravitational instability (GI) picture of structure formation. Let us begin with the empirical arguments. One has been encountered already: the galactic rotation curve. If the MACHO results are taken at face value, and if the Milky Way is typical, then it is probable that dark compact objects make up less than 50% of the halo mass in spiral galaxies. The remaining halo dark matter does not appear to consist of baryonic matter in known forms such as dust, rocks, hot or cold gas, or hydrogen snowballs \cite{59}.

The total amount of dark matter in spiral galaxies, however, is rather limited. The easiest way
to see this is to divide the total dynamical mass of the Milky Way (including its unseen halo matter) by its luminosity.$^9$ The resulting mass-to-light ratio, $M/L = (21 \pm 7) M_\odot/L_\odot$, is a mere $\sim 0.02 h_0^{-1}$ times that of a critical-density universe, $(M/L)_{\text{crit}} = (1136 \pm 138) h_0 M_\odot/L_\odot$.\footnote{The mass of which can be inferred from its $x$-ray temperature. Total cluster mass is measured by one or all of the three methods listed above (virial, $x$-ray, or lensing). At sufficiently large radii the cluster may be taken as representative of the Universe as a whole, so that $\Omega_{M,0} = \Omega_{\text{BAR}}/(M_{\text{BAR}}/M_{\text{TOT}})$, where $\Omega_{\text{BAR}}$ is fixed by nucleosynthesis.\footnote{Applied to various clusters, this procedure leads to $\Omega_{M,0} = 0.3 \pm 0.1$ \cite{10} — a result which is almost certainly an upper limit, partly because baryon enrichment is more likely to take place inside the cluster than out, and partly because dark baryonic matter (e.g., MACHOs) is not taken into account; this would raise $M_{\text{BAR}}$ and lower $\Omega_{M,0}$.}

Most of the mass of the Universe, in other words, is spread over scales larger than galaxies, and it is here that the arguments for exotic CDM are most compelling. The $M/L$-ratio method is in fact the most straightforward: one measures $M/L$ for a chosen region, corrects for the corresponding value in the “field,” and divides by $(M/L)_{\text{crit}}$ to obtain $\Omega_{M,0}$. Much, however, depends on the region. A widely respected application of this approach, that of the CNOC team, uses rich clusters of galaxies. These systems sample large volumes of the early Universe, have dynamical masses which can be measured by three independent methods (the virial theorem, $x$-ray gas temperatures and gravitational lensing), and are subject to fairly well-understood evolutionary effects. They are found to have field $M/L$-ratios of $(213 \pm 59) h_0 M_\odot/L_\odot$, giving $\Omega_{M,0} = 0.19 \pm 0.06$ (1$\sigma$ confidence) when $\Omega_{\Lambda,0} = 0$ \cite{20}. This result scales as $(1 - 0.4 \Omega_{\Lambda,0}) \cite{21}$ so that, e.g., $\Omega_{M,0}$ drops to 0.11 \pm 0.04 when $\Omega_{\Lambda,0} = 1$. The weak link in this chain of inference is that rich clusters may not be characteristic of the Universe as a whole. Only about 10\% of galaxies are found in such clusters. If individual galaxies (like the Milky Way, with $M/L \approx 21$) are substituted for clusters, then the inferred value of $\Omega_{M,0}$ drops by a factor of ten, approaching $\Omega_{\text{BAR}}$ (\cite{8}) and removing the need for exotic CDM. A recent comprehensive effort to address the impact of scale on $M/L$ arguments concludes that $\Omega_{M,0} = 0.16 \pm 0.05$ when regions of all scales (from individual galaxies to superclusters) are considered \cite{10,10}.

A second line of argument uses the cluster baryon fraction $(M_{\text{BAR}}/M_{\text{TOT}})$ of baryonic to total gravitating mass in clusters. Baryonic matter is defined as

\[ \frac{H}{\Omega} \sim 0.02 h_0^{-1} \]

\[ (M/L)_{\text{crit}} = (1136 \pm 138) h_0 M_\odot/L_\odot \\cite{21}. \]

The resulting mass-to-light ratio, $M/L = (21 \pm 7) M_\odot/L_\odot$, is a mere $\sim 0.02 h_0^{-1}$ times that of a critical-density universe, $(M/L)_{\text{crit}} = (1136 \pm 138) h_0 M_\odot/L_\odot$.\footnote{The result is based on comparisons with simulations that assume $\Omega_{M,0} = 0.37$ plus spatial flatness. It is hence subject to systematic uncertainty (in the downward direction) of as much as $-20\% \cite{10}$, possibly more if $\Omega_{\Lambda,0} > 0.63$.}

A final, recent entry into the list of purely empirical methods uses the separation of radio galaxy lobes as standard rulers, a variation on the classical angular-size distance test in cosmology. The widths, propagation velocities and magnetic field strengths of these lobes have been measured for 14 radio galaxies with the aid of long-baseline radio interferometry, leading to the (1$\sigma$) constraint $\Omega_{M,0} < 0.10$ for $\Omega_{\Lambda,0} = 0$, or $\Omega_{M,0} = 0.10^{+0.25}_{-0.10}$ for flat models ($\Omega_{\Lambda,0} = 1 - \Omega_{M,0}$) \cite{54}.

We consider next the GI-based measurements of $\Omega_{M,0}$, which are “circular” in the sense that they assume that large-scale structure formed via gravitational instability from a Gaussian spectrum of primordial density fluctuations — a process which (as we currently understand it) could not have taken place as it did unless $\Omega_{M,0}$ is considerably larger than $\Omega_{\text{BAR}}$. According to GI theory, formation of large-scale structure is more or less complete by $z \approx \Omega_{M,0}^{-1} - 2 \\cite{23}$. Therefore one can constrain $\Omega_{M,0}$ by looking for evidence of number density evolution of large-scale structures such as galaxy clusters. In a low-density universe, this would be relatively constant out to at least $z \sim 1$, whereas in a high-density universe one would expect the abundance of clusters to drop rapidly with $z$ because they are still in the process of forming. In fact, massive clusters are seen at redshifts as high as $z = 0.83$, leading to the (1$\sigma$) limits $\Omega_{M,0} = 0.17^{+0.14}_{-0.09}$ for $\Omega_{\Lambda,0} = 0$ models, and $\Omega_{M,0} = 0.22^{+0.13}_{-0.07}$ for flat ones \cite{8}.

Evolution of the mass power spectrum $P(k)$ constrains $\Omega_{M,0}$ in a similar way. Here one uses the

\[ \frac{H}{\Omega} \sim 0.02 h_0^{-1} \]

\[ (M/L)_{\text{crit}} = (1136 \pm 138) h_0 M_\odot/L_\odot \\cite{21}. \]
fact that structures of a given mass form by the collapse of large-scale regions in a low-density universe, or smaller-scale regions in a high-density one. Comparing $P(k)$ for the present-day distribution of galaxy clusters to that for the distribution of matter at some earlier epoch therefore yields an estimate of $\Omega_{M,0}$. Using the mass power spectrum of Lyα absorbers at $z \approx 2.5$, for instance, one finds that $\Omega_{M,0} = 0.46^{+0.12}_{-0.10}$ (1σ) for $\Omega_{\Lambda,0} = 0$ models \cite{2}. This result goes as approximately $(1 - 0.4\Omega_{\Lambda,0})$, so that the central value of $\Omega_{M,0}$ drops to 0.34 in a flat model, and 0.28 if $\Omega_{\Lambda,0} = 1$.

A final group of measurements comes from galaxy peculiar velocities. These are produced by the gravitational potential of locally over- (or under-) dense regions relative to $\Omega_{M,0}$, but also depend on $\Omega_{M,0}$ itself. The power spectra of the velocity and density distributions can be related within the context of GI theory. A typical bound derived from several such studies is $\Omega_{M,0} > 0.3$ \cite{2}. Dependence on $\Omega_{\Lambda,0}$ is modest since these tests probe relatively small volumes, but lower limits derived in this way can depend significantly on $h_0$ as well as the spectral index $n$ of the density distribution. In \cite{2}, where the latter is normalized to CMB fluctuations, results take the form $\Omega_{M,0} h_0^{1.3} n^2 \approx 0.33 \pm 0.07$ (2σ). The preferred value of $\Omega_{M,0}$ therefore drops from 0.53 ± 0.11 (if $h_0 = 0.7$) to 0.38 ± 0.08 (if $h_0 = 0.9$), where we have assumed $n = 1$.

To summarize, one may say that purely empirical arguments lean toward values of $\Omega_{M,0} \sim 0.3$ or lower, whereas GI (gravitational instability) theory-based results tend to come in at $\sim 0.2$ and higher. If there is flexibility in the lower limits on $\Omega_{M,0}$, it lies in the empirical methods, especially if contributions from dark baryons are near their upper limit \cite{3}. It is unlikely, however, that the limits based on GI theory can be stretched far enough to remove the need for exotic CDM. We therefore conclude that this component of the dark matter has a density parameter in the range

$$\Omega_{CDM} = \begin{cases} 0.1 - 0.5 & \text{(GI theory)} \\ 0 - 0.4 & \text{(otherwise)} \end{cases}$$

(11)

If our current understanding of structure formation via gravitational instability is correct, then exotic CDM must exist. Conversely, if exotic CDM does not exist, then our understanding of structure formation is incomplete.

The debate, of course, becomes moot if exotic CDM (with $\Omega_{CDM} \sim 0.3$) is actually discovered in the laboratory. Theorists have proposed a colorful list of particle candidates, with varying degrees of testability. Two have emerged as most plausible: the axion and the weakly interacting massive particle (WIMP)\footnote{Other contenders include WIMPzillas, primordial black holes, magnetic monopoles, solitons (in various dimensions), cosmic string loops, shadow matter and mirror matter. We refer the interested reader to the latest conference reports \cite{8,9} or semipopular works such as \cite{10,11}.}. Either one of these could in principle make up the CDM because each, if it exists, is (1) cold (i.e., nonrelativistic in the early Universe), and (2) expected on theoretical grounds to have a collective density within a few orders of magnitude of the critical density (see Appendix). Ambitious experimental detection efforts around the world are now directed at both particles. While they have not turned up anything so far, most of the theoretical parameter space remains unexplored.

## 5 Neutrinos

Since neutrinos indisputably exist, and in great numbers (their number density $n_\nu$ is $3/11$ times that of the CMB photons, or $112$ cm$^{-3}$ per species), they have been leading particle dark matter candidates for longer than either the axion or the WIMP. They gained prominence in 1980 when teams in the U.S.A. and Soviet Union both claimed to have evidence of nonzero neutrino rest masses. While these claims did not stand up, a new round of more sophisticated experiments is once again suggesting that $m_\nu$ (and hence $\Omega_\nu$) $> 0$.

Dividing $n_\nu m_\nu$ by the critical density of the Universe, one obtains immediately \cite{12}

$$\Omega_\nu = \left(\sum m_\nu c^2 / 94 \text{ eV}\right) h_0^{-2},$$

(12)

where the sum is over the three neutrino species.\footnote{The calculations in this section are strictly valid only for $m_\nu c^2 \lesssim \text{MeV}$. More massive neutrinos with $m_\nu c^2 \sim \text{GeV}$ were once considered as CDM candidates but are no longer viable since LEP experiments rule out additional neutrino species with $m_\nu c^2 < 46 \text{ GeV}$ (i.e., half the $Z_0$-mass).}
masses are 3 eV ($\nu_e c^2$), 0.19 MeV ($\nu_\mu c^2$) and 18 MeV ($\nu_\tau c^2$), so it would appear feasible in principle for these particles to close the Universe. In fact $m_{\nu_e}$ and $m_{\nu_\mu}$ are limited far more stringently by Eq. (12) than by laboratory bounds.

Perhaps the best-known theory along these lines in recent years is that of Sciama [108], who postulated a population of 29 eV $\tau$-neutrinos, decaying via $\nu_\tau \rightarrow \nu_\mu + \gamma$ into much lighter $\mu$-neutrinos and 15 eV photons on timescales of $\tau_\nu \lesssim 3 \times 10^{23} \text{s}$. Decay photons with these properties would solve a number of astrophysical puzzles, such as the high degree of ionization in the interstellar medium and the large intergalactic flux of hydrogen-ionizing photons. The proposed neutrinos would moreover provide exactly the critical density if $\Omega_\nu < 0.56$. This model, however, has now been ruled out by the absence of a strong 15 eV emission line in the extragalactic background light [92].

More generally, the strongest upper limits on $\Omega_\nu$ come from our current understanding of structure formation. Neutrinos are hot (i.e., relativistic at the time of decoupling from the primordial fireball) and therefore able to stream freely out of density perturbations in the early Universe, erasing them before they have a chance to grow. Good agreement with observations of large-scale structure can be achieved in models with $\Omega_\nu$ as high as 0.2, but only if $\Omega_{\text{bar}} + \Omega_{\text{CDM}} = 0.8$ and $h_0 = 0.5$ [19]. A more realistic upper limit follows from a statistical exploration of the entire parameter space and leads to the conclusion that $m_{\nu_e} c^2 \lesssim (9.2 \text{ eV}) \Omega_{\text{CDM}} (2\sigma)$ over $0 \leq \Omega_{\text{CDM}} \leq 0.6$ for flat models [28]. Eq. (12) then implies that $\Omega_\nu < 0.12 \Omega_{\text{CDM}}$ (if $h_0 = 0.9$) or $0.20 \Omega_{\text{CDM}}$ (if $h_0 = 0.7$) — a neutrino density below that attributed to exotic CDM (11) and still well above that of the baryons (3). [7]

Unexpected new lower limits on $\Omega_\nu$ have come from atmospheric (Super-Kamiokande [14]), solar (SAGE [1]), Homestake [24], GALLEX [56], and accelerator-based (LSND [4]) neutrino experiments.

In each case it appears that two neutrino species are oscillating into each other, a process which can only take place if $m_\nu > 0$. The strongest evidence comes from Super-Kamiokande, which has reported oscillations between $\tau$- and $\mu$-neutrinos with $5 \times 10^{-4} \text{ eV}^2 < \Delta m^2_{\tau\mu} c^4 < 6 \times 10^{-3} \text{ eV}^2 (2\sigma)$, where $\Delta m^2_{\tau\mu} \equiv |m_{\nu_\tau}^2 - m_{\nu_\mu}^2|$ [14]. If neutrino masses are hierarchical, like the masses of other fermions, then $m_{\nu_\tau} \gg m_{\nu_\mu}$ and $m_{\nu_\tau} c^2 > 0.02 \text{ eV}$. In this case it follows from Eq. (12) that $\Omega_\nu > 0.0003$ (if $h_0 = 0.9$) or 0.0005 (if $h_0 = 0.7$). If, instead, neutrino masses are nearly degenerate, then $\Omega_\nu$ cannot be determined from this result, but will in any case still lie below the upper bound imposed by structure formation above. We conclude that the possible range of values for this parameter is

$$\Omega_\nu = 0.0003 - 0.2\Omega_{\text{CDM}}.$$ (13)

The neutrino contribution to $\Omega_{\text{tot,b}}$ is anywhere from an order of magnitude below that of the visible stars and galaxies (3) up to as much as half that attributed to exotic CDM (11). If $\Omega_{\text{CDM}}$ is small (or zero), however, then $\Omega_\nu$ must lie near the lower, rather than the upper end of this range; since a large density of neutrinos will interfere with structure formation (as noted above) unless something like exotic CDM is present to help hold primordial density perturbations together.

6 Vacuum Energy

There are at least four good arguments for the cosmological constant. The first is mathematical: $\Lambda$ plays a role in Eqs. (1) similar to that of the additive constant in an indefinite integral [103]. The second is dimensional: $\Lambda$ specifies the curvature radius $R_\Lambda \equiv 1/\sqrt{\Lambda}$ of a (closed) Universe at the moment when the matter density parameter $\Omega_M(z)$ passes through its maximum (2), thereby providing a fundamental length scale for cosmology (see [18] for discussion). The third is dynamical: $\Lambda$ determines the asymptotic expansion rate of the Universe according to Eq. (4), $\Delta c^2 = 3H^2_\infty$. And the

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13These (possibly quite significant) values of $\Omega_\nu$ do not invalidate our earlier assumption (3) that the present density of radiationlike matter is negligible. While neutrinos are relativistic at decoupling, they lose energy and become nonrelativistic on timescales $t_{\text{SN}} \approx 190,000(m_{\nu_e} c^2/eV)^{-1} \text{yr}$ [7] — well before the present epoch for neutrinos which are massive enough to be of interest.
fourth is material: $\Lambda$ is related to the energy density of the vacuum via $\rho_\Lambda c^2 = \Lambda c^4/8\pi G$.

With all these reasons to take this term seriously, why have most cosmologists since Einstein set $\Lambda = 0$? Computational convenience is one explanation. Another is the smallness of most effects associated with the $\Lambda$-term. Einstein himself set $\Lambda = 0$ in 1931 “aus Gründen der logischen Ökonomie” — for reasons of logical economy — because he saw no hope of measuring this quantity experimentally at the time. He is often quoted as adding that its introduction in 1915 was the “biggest blunder” of his life (“die größte Eselei in meinem Leben”). This statement, which does not appear anywhere in Einstein’s writings but was rather attributed to him by Gamow [47], is sometimes interpreted as a rejection of the very idea of a cosmological constant. It more likely represents Einstein’s rueful recognition that, by invoking the $\Lambda$-term solely to obtain a static solution of the field equations, he had narrowly missed what would surely have been one of the greatest triumphs of his life: the prediction of cosmic expansion.

The relation between $\Lambda$ and the energy density of the vacuum has led to a new quandary in more recent times: the fact that $\rho_\Lambda$ as estimated in the context of quantum field theories such as quantum chromodynamics (QCD), electroweak (EW) and grand unified theories (GUTs) implies impossibly large values of $\Omega_{\Lambda,0}$ (Table 1). These theories have been successful in the microscopic realm. Here, however, they are in gross disagreement with the observed facts of the macroscopic world, which tell us that $\Omega_{\Lambda,0}$ cannot be much larger than order unity. This “cosmological constant problem,” is undoubtedly another reason why many cosmologists have preferred to set $\Lambda = 0$, rather than deal with a parameter whose microphysical origins are still unclear (see [22] for review).

This, however, is no longer an appropriate response because observations now indicate that $\Omega_{\Lambda,0}$, while nowhere near the size suggested by Table 1, is nevertheless greater than zero. The cosmological constant problem has therefore become more baffling, in that any quantum field-theoretic account of this parameter must apparently contain a cancellation mechanism which is not only good to some 44 (or 122) decimal places, but which begins to fail at precisely the 45th (or 123rd). One possibility is to treat $\Lambda$ as a dynamical quantity rather than a constant of nature, in which case the observed smallness of $\Omega_{\Lambda,0}$ might be attributed to the age of the Universe [77]. It is equivalent to introduce a fifth element (known as quintessence) into cosmology [78]. In general, however, this means extending Einstein’s equations (1) to incorporate new (and so far unobserved) phenomena such as scalar fields, which in turn introduce new terms (intermediate to the $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$-terms) into Eq. (3). While these ideas hold theoretical promise, it is likely that they too must involve fine-tuning if they are to reproduce exactly the values of $\Omega_{\Lambda,0}$ observed. As an alternative explanation, it has been proposed that a universe in which $\Omega_{\Lambda,0}$ was too large (or small) might be incapable of giving rise to intelligent observers, so that the fact of our own existence already “requires” $\Omega_{\Lambda,0} \sim 1$ [122].

Let us pass now to the observational arguments for $\Omega_{\Lambda,0}$. Some have been mentioned already. It was noted in [38] (Fig. 4) that numerical simulations of large-scale structure formation match models with $\Omega_{\Lambda,0} = 0.7$ better than those with $\Omega_{\Lambda,0} = 0$. Additional simulations would allow us to explore the parameter space with higher resolution.

Some of the arguments for exotic CDM in [88] also show a (more modest) dependence on $\Omega_{\Lambda,0}$. The trend in most cases is toward higher values of $\Omega_{\Lambda,0}$ in conjunction with lower values of $\Omega_{M,0}$. In the arguments from cluster $M/L$-ratios and mass power spectra, for example, we have seen that raising $\Omega_{\Lambda,0}$ from zero to one corresponds to a drop of $\sim 40\%$ in the preferred value of $\Omega_{M,0}$.

Tentative lower limits on $\Omega_{\Lambda,0}$ have come from galaxy number counts. The comoving volume is enhanced at large redshifts ($z \gtrsim 2$) for high-$\Lambda$ models, leading to greater (projected) number densities at faint magnitudes. In practice, it has proven difficult to disentangle this effect from galaxy evolution.
Early claims of a best fit at $\Omega_{\Lambda,0} \approx 0.9$ [45] have been disputed on the basis that the steep increase seen in numbers of blue galaxies is not matched in the $K$-band [48]. Attempts to account for evolution in a comprehensive way have recently produced the $2\sigma$ lower limit $\Omega_{\Lambda,0} > 0.53$ [115], with a good fit (for flat models) at $\Omega_{\Lambda,0} \approx 0.8$ [105]. A preliminary plot of constraints across the $\Omega_{M,0} - \Omega_{\Lambda,0}$ plane [105] is consistent with these results.

Measurements of $\Omega_{\Lambda,0}$ from gravitational lens statistics are based on a similar premise: the increase in path length to a given redshift in high-$\Lambda$ models should mean that more lensed sources are seen. Somewhat surprisingly, comparison of the observed frequency of lensed quasars to that expected produces results in poor agreement with those inferred from galaxy counts; leading in fact to the strongest current $2\sigma$ upper limit on vacuum density: $\Omega_{\Lambda,0} < 0.66$ for flat models [76]. Dust could obscure the distant lenses and allow for a larger value of $\Omega_{\Lambda,0}$ [3]. This objection can however be met by moving to radio lenses, which give an only slightly weaker $2\sigma$ bound: $\Omega_{\Lambda,0} < 0.73$ for flat models or $\Omega_{\Lambda,0} < 0.4 + 1.5\Omega_{M,0}$ for nonflat ones [8]. Other sources of potential systematic error remain, including source redshift distributions, survey incompleteness, lens modelling and evolution in the mass profiles of the lensing objects. A recent $2\sigma$ limit from radio lenses is $\Omega_{\Lambda,0} < 0.95$ [26].

Lensing provides a second constraint on $\Omega_{\Lambda,0}$ in closed models, based on the redshift of the antipodes (the set of points at radial coordinate distance $\pi$). It has been shown [3] that the lensing cross section blows up for sources at this distance, implying that the antipodes cannot be nearer than the farthest normally lensed object, currently a galaxy at $z = 4.92$ [42]. It is straightforward to compute the antipodal distance in terms of $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$; one finds in this way that $\Omega_{\Lambda,0} < 1.57$ if $\Omega_{M,0} = 0.3$, an upper limit that drops to $< 1.30$ if $\Omega_{M,0} = 0.1$ [7]. The upper limit is $\Omega_{\Lambda,0} < 1.10$ if $\Omega_{M,0} = 0.014$, essentially the same as the Einstein limit discussed in §2.

Fig. 6 shows a two-dimensional slice of this model, along with the EdS model drawn to the same scale. The antipodes (in the closed case) are at $z \approx 12$, well beyond the highest-redshift objects seen to date. Fig. 6 also reveals nearly ten times as much linear distance between redshifts $z = 3$ and $z = 4$ in the spherical model as there is in the flat one; this is why one expects to see more lensed sources (and faint galaxies) in a high-$\Lambda$ universe.

\[\Omega_{M,0} = 0.014, \Omega_{\Lambda,0} = 1.08, h_0 = 0.9\]

\[\Omega_{M,0} = 1, \Omega_{\Lambda,0} = 0, h_0 = 0.5\]
The best constraints on $\Omega_{\Lambda,0}$ come from Type Ia supernovae (SNe Ia), used as standard candles in the classical magnitude-redshift relation, can measure $\Omega_{\Lambda,0}$ more robustly than either galaxy counts or lensing statistics because evolution is less of a concern for these objects. The Supernova Cosmology Project [77] and High-z Supernova Search teams [102] have lately caused a great stir with their reports that a systematic dimming of SNe Ia at $z \sim 0.5$ by about 0.25 mag relative to that expected in an EdS model is best explained by

$$\Omega_{\Lambda,0} \approx \frac{4}{3} \Omega_{M,0} + \frac{1}{3} \pm \frac{1}{2} \quad (2\sigma). \quad (14)$$

This leads at once to a conclusion that most cosmologists [8] have long been reluctant to consider: that we live in a vacuum-dominated universe. Several words of caution are in order, however. Observations must reach $z \sim 2$ before one can truly be certain of discriminating between models like those shown in Fig. 2. Intergalactic dust could mimic the effects of a cosmological constant, if it were “sifted” during the process of ejection from galaxies [8]. The neglect of evolution may be more serious than claimed [81]. And the physics of SNe Ia explosions needs to be better understood [64].

The best constraints on $\Omega_{\Lambda,0}$ come from CMB fluctuations; and in particular, from the new detection of the first peak in their angular power spectrum by the BOOMERANG [29] and MAXIMA [57] experiments. The location of this peak is a direct measure of the largest size of fluctuations in the primordial plasma at the moment of last scattering, as seen through the “lens” of a curved Universe. The two detections, combined with earlier data from the COBE satellite [14], imply that [68]

$$\Omega_{\Lambda,0} = 1.11^{+0.13}_{-0.12} - \Omega_{M,0} \quad (2\sigma). \quad (15)$$

This result is more reliable than all the others discussed so far because it bypasses “local” phenomena such as supernovae, galaxies, and even lensed quasars; taking us directly back to the radiation-dominated era when physics was very simple.

Let us therefore use Eq. (15) to calculate the energy density of the vacuum. Summing the baryon, exotic CDM, and neutrino densities — Eqs. (8), (11) and (13) respectively — gives the total matter density $\Omega_{M,0}$. Substituting this into Eq. (15), we find the following ranges of values:

$$\Omega_{\Lambda,0} = \begin{cases} 0.3 - 1.1 & \text{(GI theory)} \\ 0.4 - 1.2 & \text{(otherwise)} \end{cases} \quad (16)$$

Vacuum energy, the new and invisible “fire” of modern cosmology, is thus indeed dominant, making up the bulk of a dark Universe in which light and baryons — the constituents of our familiar world — appear almost incidental.

7 How much exotic dark matter?

The basis for $\Lambda$CDM as the new favorite among cosmological models lies in the approximate orthogonality of the CMB and supernova bounds, Eqs. (14) and (15). Indeed, if we take both of these results at face value, we can substitute one into the other and solve to find $\Omega_{\Lambda,0} = 0.78 \pm 0.23$ and $\Omega_{M,0} = 0.33 \pm 0.22$. The fact that this latter number is very near the center of the range of allowed values for $\Omega_{M,0}$ in Eq. (11) has then been taken as a further sign of the basic correctness of both the $\Lambda$CDM model in particular and the GI theory of structure formation in general.

While this is a self-consistent account, and one that agrees with most observations, it suffers from one flaw: it is inherently improbable. The densities of baryonic matter (and exotic CDM, should it exist) evolve at a very different rate from neutrinos; and both of these components evolve at very different rates from vacuum energy. So one has three kinds of matter which should not have anything like the same density parameters at any given time — and yet two of them (at least) do. In the $\Lambda$CDM picture, in particular, it seems that we happen to live at a time when $\Omega_{\Lambda,0}$ and $\Omega_{M,0}$ are separated by a mere factor of two. To illustrate the unlikelihood of this “preposterous universe,” Carroll [22] has plotted the evolution of $\Omega_{M}$ and $\Omega_{\Lambda}$ for 35 powers of ten in scale factor in the past and future directions, showing that the probability of finding oneself at the moment when they should be within even one order of magnitude of each other is exceedingly remote.

\[10\] With some exceptions [32, 77, 80]; see [5].
It may be misleading to characterize this problem in logarithmic terms. Several Gyr were necessary to form the first galaxies and stars; and they will all be gone after a hundred — so it is natural that we should find ourselves within this span of cosmic history, at least. In Fig. 7, we have plotted the evolution of $\Omega_\Lambda(t)$ and $\Omega_M(t)$ on a linear scale for the first $\sim 80$ (100) Gyr after the big bang in the $\Lambda$CDM model with $h_0 = 0.9$ (0.7). This plot confirms that $\Lambda$CDM is improbable, in the sense that the values of $\Omega_M$ and $\Omega_\Lambda$ observed at the present epoch (0.3 and 0.7 respectively) are atypical.

Fig. 7 also shows the evolution of $\Omega_\Lambda$ and $\Omega_M$ in the vacuum-dominated, $\Lambda$+baryon model discussed in several places above (Model 1 in Figs. 2 and 3), with $\Omega_{\Lambda,0} = 1.08$ and no exotic CDM ($\Omega_{M,0} = 0.014 \approx \Omega_{\text{BAR}}$). This universe, which was originally proposed in [79, 80] and which we shall term here the “$\Lambda$bar” model, is a good deal less preposterous than $\Lambda$CDM in the sense that a factor of $\sim 80$ (rather than two) separates the presently observed values of vacuum and energy density. Indeed these parameters are much closer to their “cosmological average” values of one and zero respectively. While this in itself does not constitute a case for the model, it prompts us to wonder whether $\Lambda$ might not be more important than most cosmologists have been willing to consider. Could vacuum energy be not just the dominant, but the only significant component of the dark matter?

Such an idea would have been unthinkable only a few years ago, when it was still routine to set $\Lambda = 0$ and cosmologists had two main choices: the “one true faith” ($\Omega_{M,0} \equiv 1$), or the “reformed” (with each believer being free to choose his or her own value near $\Omega_{M,0} \approx 0.3$). All this has been irrevocably altered by the CMB experiments [2]. If there is a single guiding principle in choosing models now, it is no longer $\Omega_{M,0} \approx 0.3$, and certainly not $\Omega_{\Lambda,0} = 0$; it is $\Omega_{M,0} + \Omega_{\Lambda,0} \approx 1$ from the power spectrum of the CMB. With this in mind, we will devote this final section of our review to exploring the feasibility of the $\Lambda$bar model, which has $\Omega_{M,0} + \Omega_{\Lambda,0} = 1.094$, in excellent agreement with Eq. (15).

To begin with, any model of this kind, with a small $\Omega_{M,0}$ and a high $\Omega_{\Lambda,0}$, must face three main objections: (1) the lower limits on $\Omega_{M,0}$ in §3; (2) the upper limits on $\Omega_{\Lambda,0}$ in §3; and (3) the age problem in §3. Let us briefly review these points before moving on to what are, to us, the strongest arguments in favor of the $\Lambda$bar idea.

We have argued in §3 that the lower limits on $\Omega_{M,0}$ are of two kinds: those which are framed in the context of gravitational instability theory (thus tacitly assuming the existence of large amounts of CDM) and those which are not. Of these, the former are certainly incompatible with $\Omega_{M,0} = 0.014$. The latter, however, are more flexible, especially when their $\Omega_{\Lambda,0}$- and $h_0$-dependence is taken into account (as it often is not). The distinction is important, because CDM may not be needed in the $\Lambda$bar model: structures have far longer to form, and they do so at a time when expansion is slower and densities are higher.

The observational constraints on $\Omega_{\Lambda,0}$ in §3 are not much more secure than those on $\Omega_{M,0}$. Supernovae favor a value of $\Omega_{\Lambda,0}$ closer to $\sim 0.8$, but have large (and possibly underestimated) uncertainty factors. The antipodal redshift argument restricts us to $\Omega_{\Lambda,0} < 1.10$, which is just above the value we have adopted — suggesting that this could become a useful test of the theory when ob-

![Fig. 7. Evolution of $\Omega_M$ and $\Omega_\Lambda$ in the $\Lambda$bar and $\Lambda$CDM models (Models 1 and 6 in Figs. 2 and 3). Time is set to zero at the present epoch; $t_{BB}$, the time of the big bang, is calculated using $h_0 = 0.9$ for $\Lambda$bar (bottom scale) and $h_0 = 0.7$ for $\Lambda$CDM (top scale). Compare Fig. 3.](image)
servations are eventually pushed to $z \approx 12$ (Fig. 6). Gravitational lensing statistics remain the strongest argument against values of $\Omega_{\Lambda,0}$ as large as that considered here. It would be a more compelling one, however, if we understood why galaxy counts (which rely on essentially the same reasoning) lead to such different conclusions.

Finally, as we have argued in §§2 and 3, the age of the Abar universe should be seen as an asset rather than a liability, particularly in the area of structure formation. One also expects to find a faint population of very old, dead stars; these may be the halo objects whose existence is implied by the MACHO survey. The most direct way to rule out the Abar model based solely on age considerations would be to prove that $h_0$ is less than or equal to what we have referred to as the “low” value ($0.7$) above. Alternatively, if evidence continues to mount for the “high” value ($0.9$), then the “age problem” will begin to put pressure on the ΛCDM, rather than the the Abar model.

Let us turn now to the observational case for the Abar idea, which rests on two main lines of evidence: Lyα absorption spectra, and the lack of a second peak in the power spectrum of CMB fluctuations. We consider these in turn.

The forest of Lyα absorption lines was first used to measure $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$ in [79, 80]; the most recent review of this method appears in [120]. The idea is extremely simple: one supposes that Lyα absorbers, like galaxies, are distributed with a cell-like structure, and that absorption lines are produced when the line of sight to a distant quasar cuts through the cell walls (Fig. 8). The crucial assumption is then made that the cells expand with the Hubble flow, and that evolution within them is secondary and can be neglected. This is not unreasonable, given that the expansion velocity of a typical cell would be of order $\sim 3000$ km/s, whereas peculiar motions inside the cell walls might be no more than about $\sim 300$ km/s. This picture is also in accord with the latest observational work on Lyα absorbers which, although still tentative, suggests that they are indeed distributed in structures which expand with decreasing redshift [30], and have comoving size $\sim 26h_0^{-1}$ Mpc at $z = 2.6$ [27] [4].

In fact, neither assumption may be strictly necessary; a similar analysis has been performed under the assumption that Lyα absorbers make up a homogeneous population of clouds with constant comoving size, and it leads to results consistent with those presented here [43].

One then simply counts the absorption lines and measures the mean spacing $\Delta \lambda$ between them. This gives the redshift spacing $\Delta z(z)$ of the cells as a function of redshift, which may in turn be related to their comoving coordinate size $\Delta \chi$ by

$$[\Delta z(z)]^2 = \left( \frac{R_0 \Delta \chi}{c} \right)^2 H^2(z) ,$$

where $\chi$ is the radial coordinate distance and $H^2(z)$ is given as a cubic in $(1 + z)$ by Eq. (4). Since $\Delta z(z)$ is an observable, Eq. (17) becomes a third-order polynomial regression formula for $[\Delta z(z)]^2$; and one, moreover, with no linear term.

Fig. 9 shows a plot of $[\Delta z(z)]^2$ versus $(1 + z)$, based on published spectra of 21 different quasars with a total of 1320 Lyα absorption lines, weighted according to resolution (see [80] for details). At first sight it does not seem that the regression curve is strongly constrained by the data. However, the fit is in fact remarkably robust. The reason for this is that the curve can consist of only three components: a constant, a downward-opening quadratic, and a cubic originating at $(0,0)$. The regression coefficients that meet these conditions span only a very narrow range of values, and lead directly to the $2\sigma$ (statistical) results [4]

$$\begin{align*}
\Omega_{M,0} &= 0.014 \pm 0.006 \\
\Omega_{\Lambda,0} &= 1.08 \pm 0.03 .
\end{align*}$$

Fig. 8. Schematic illustration of a network of cell-like structures, with Lyα absorption lines seen by an observer in the spectrum of a distant quasar due to the fact that the line of sight passes through the cell walls.
Eqs. (18) define what we have referred to as the Λbar model; this term could equally be applied to other vacuum-dominated models without significant amounts of CDM (e.g., Fig. 10).

The Λbar model presented in Eqs. (18) passes several basic consistency tests. Firstly, the sum of $\Omega_{M,0} + \Omega_{\Lambda,0}$ matches that seen in the CMB experiments ($\Omega_{M,0} + \Omega_{\Lambda,0} = 1$). Secondly, the value of $\Omega_{M,0}$ is within the bounds imposed by cosmic nucleosynthesis ($\Omega_{M,0} \approx 0.16h_0^{-2}$ for $h_0 = 0.9$, or $h_0 \geq 0.71$ for $\Omega_{M,0} = 0.01h_0^{-2}$). And thirdly, the regression curve (heavy solid line in Fig. 9) passes through $z = 0$ at $\Delta z \approx 0.009$, in excellent accord with the distribution of galaxy structure seen in our own cosmic neighborhood (the empty rectangle in the upper left-hand corner of Fig. 9). These phenomena involve independent physics on widely different scales, and we would regard it as remarkable for a simple procedure like the one described above to agree with all three by chance alone.

The third point however deserves some clarification. The fact that the scale of the local galaxy distribution matches that of Lyα absorbers agrees with our simple picture, in which both types of matter cluster predominantly along the cell walls (Fig. 8). However, there is still significant debate in the literature over the extent of correlations between Lyα absorbers and other structures. Why not “calibrate” the regression curve at $z = 0$ using absorbers rather than visible galaxies? Ultraviolet Lyα spectra are now available from the Hubble Space Telescope (HST) over the redshift range $0 \leq z \leq 1.3$. The absorber material, however, is not spread uniformly over the cell walls, but rather occurs in filaments and knots. Lines of sight to a few quasars, therefore, are far less likely to “notice” the cell walls at low redshifts, where cell size is enormous. Indeed, the HST spectra show a mean redshift spacing $\langle \Delta z \rangle \approx 0.04$, four times larger than that seen in the galaxy distribution. In the local universe, in other words, galaxy surveys are better guides to large-scale structure than Lyα absorption spectra.

A different restriction comes into play at high redshifts, where one might expect from the above argument that Lyα spectra would be increasingly reliable. It is certainly true that lines of sight to quasars at $z \gtrsim 4$ will “notice” most of the cell walls they intersect, since the cells themselves are smaller (by a factor $1+z$) at these redshifts. Another effect, however, also goes as $1+z$: Doppler broadening of lines due to peculiar motions of the Lyα absorbers in the cell walls. At $z \approx 4$ the latter will already be of order $\sim 1500 \text{ km/s}$ rather than $\sim 300 \text{ km/s}$, introducing spurious lines which would masquerade as small-scale structure. Recent Keck/HIRES spectra of a $z = 4.1$ quasar confirm this suspicion, yielding a mean redshift spacing $\langle \Delta z \rangle \approx 0.002$, three times smaller than the fit to the regression curve at $z \approx 4$ (Fig. 9). The Lyα method outlined here is most sensitive to the cosmological parameters for quasars in the redshift range $2 < z < 4$. The most recent Keck/HIRES spectra of a quasar at $z = 3.1$, for example, shows $\langle \Delta z \rangle \approx 0.006$, in excellent agreement with Fig. 9 at $z \approx 3$.

We have not addressed the possibility of bias due to inherent evolution within the cell structure. There is still debate about the mean comoving scale of the Lyα distribution in the literature, to say nothing of its possible time rate of change. To obtain higher values of $\Omega_{M,0}$ from the above analysis, however, evolution must be in the right direction.

An appeal to ionization, for instance, should not ionize high-redshift clouds more than local ones;
this would raise \( \Delta z \) at high \( z \) and lower the inferred value of \( \Omega_{\text{M},0} \). Higher matter densities can in principle be obtained if evolution is such that substructure increases with redshift. Observations and numerical simulations do suggest the possibility of such a trend in the Ly\( \alpha \) forest, as sheet-like structures give way to filaments and knots with time \[101\]. It has been estimated that allowance for an effect of this kind might raise the value of \( \Omega_{\text{M},0} \) from that in Eq. \(18\) to as much as \( \sim 0.05 \) \[124\].

Questions of a more mathematical nature may be raised by the smallness of the uncertainties in Eqs. \(18\). The possible impact of spectral resolution, equivalent width, and line blending on the line-counting procedure have been considered in \[24\], with the conclusion that corrections arising from these factors will be minor. The statistical robustness of the Ly\( \alpha \) method can be attributed to the absence of a linear term in Eq. \(1\) for Hubble’s parameter. More work, however, could be done to reduce the possibility of unmodelled systematic errors. Ultimately one would like to see \( \Delta z(z) \) extracted from a power spectrum analysis using high signal-to-noise spectra (like those now coming from Keck/HIRES) together with automated line-fitting and counting procedures \[71\].

We move now to the second argument for a \( \Lambda \)-type universe, one which relies on a new analysis of the angular power spectrum of the CMB (Fig. 10). While the angular positions of the peaks in this spectrum fix the sum of matter and vacuum densities \( (1) \), their relative heights are largely a function of the matter density alone. The odd-numbered peaks are produced by regions of the primordial plasma which have been maximally compressed by infalling material, while the even ones correspond to maximally rarefied regions which have rebounded due to photon pressure. A high baryon-to-photon ratio enhances the compressions and retards the rarefractions, thus suppressing the size of, e.g., the second peak relative to the first. The strength of this effect depends on the fraction of baryons (relative to the more weakly-bound neutrinos and CDM particles) in the overdense regions.

Data taken by the \textsc{Boomerang} and \textsc{Maxima} experiments appear to show an almost total suppression of the second peak relative to the first, inconsistent (at the 99% level) with expectations based on the \( \Lambda \text{CDM} \) model (Fig. 10, left-hand side). The ratio of baryons to CDM in the primordial plasma, therefore, appears to be higher than predicted. A first reaction might be to keep the CDM and raise the baryon density; this however brings the theory into immediate conflict with nucleosynthesis limits on \( \Omega_{\text{B}} \) \[113\]. Other remedies (within the framework of gravitational instability theory) include tilting the spectrum of initial perturbations to disfavor smaller-scale (higher-order) peaks \[18\], erasing these peaks outright with processes such as delayed recombination \[16\] or decoherence, and varying one or more constants of nature \[124\].

The alternative is to take the apparent lack of CDM at face value. This can either be done in a half-hearted or whole-hearted way. The half-hearted way is to retain a minimum density of CDM with a statistical “prior.” Thus, requiring that \( \Omega_{\text{CDM}} > 0.1 \) but otherwise fitting the combined \textsc{Boomerang}, \textsc{Maxima} and \textsc{Cobe} data, one obtains a model with best-fit parameters \( \Omega_{\text{B}} = 0.032h_0^{-2} \) and \( \Omega_{\text{CDM}} = 0.14h_0^{-2} \) \[18\].

The whole-hearted approach, which may however require extending the standard picture of structure formation, is to drop the requirement of CDM
altogether. Results are shown in Fig. 10 (right-hand side), which is a statistical fit to $\Omega_{\text{BAR}}$ with $\Omega_{\text{CDM}} = 0$ and $h_0 = 0.75$. The values of $\Omega_{\Lambda,0}$ and $\Delta T$ (amplitude) are fixed by the position and height of the first peak. The best-fit model passes neatly through both peaks ($\chi^2 = 0.85$) and has

$$
\Omega_{M,0} = \Omega_{\text{BAR}} = 0.034
$$

$$
\Omega_{\Lambda,0} = 1.006 \, ,
$$

in good agreement with primordial nucleosynthesis ([3]) as well as Eqs. ([3]). This model has an age of 22.2 Gyr (with $h_0 = 0.75$), and a total density parameter slightly above one, in agreement with suggestions from other new analyses of the BOOMERANG and MAXIMA data ([2]. The shape of the CMB power spectrum, along with the analysis of Ly$\alpha$ absorption lines presented above, thus favors an old, closed Abar model, dominated by vacuum energy, with no significant contributions from CDM or neutrinos — a universe composed almost exclusively of “fire” and “earth.”

8 Conclusions

We have reviewed the evidence for the “four elements” of modern cosmology: baryons (“earth”), exotic cold dark matter (“water”), neutrinos and photons (“air”) and vacuum energy (“fire”). Recalling Eqs. ([1], ([11], ([13], ([15] and ([16]), we may summarize the present contributions of each element to the total density of the Universe with the following ranges of values:

$$
\Omega_{\text{BAR}} = 0.012 - 0.041
$$

$$
\Omega_{\text{CDM}} = \{ 0.1 - 0.5 \ (\text{GI theory}) \}
\Omega_{\nu} = 0.0003 - 0.2\Omega_{\text{CDM}}
\Omega_{\Lambda,0} = \{ 0.3 - 1.1 \ (\text{GI theory}) \}
\Omega_{\text{TOT,0}} = 0.99 - 1.24 \ [68] \, ,
$$

where “GI” refers to gravitational instability theory. Baryons, the stuff of which we are made, are apparently little more than a cosmic afterthought. Neutrinos and the elusive cold dark matter may both play far more significant roles in determining the past and future evolution of the Universe, though this is not certain. What is clear is that all three forms of matter are dwarfed in importance by a newcomer whose physical origin remains shrouded in obscurity: the energy of the vacuum.

We have devoted the last part of our review to the hypothesis that vacuum energy dominates so completely that there is no room for significant amounts of neutrino or exotic dark matter at all. This would considerably simplify our picture of the Universe, ease problems with the “preposterously” fine-tuned values of the observed cosmological parameters, and allow more time for galaxies and other structures to form. It would also, however, require that we modify the GI paradigm. We have reviewed the various lines of observational argument, both for and against such an idea. It appears to us quite possible that the vacuum density $\Omega_{\Lambda,0}$ is close to one, that the sole contributions to the matter density $\Omega_{M,0}$ come from a small amount of baryons and neutrinos, and that $\Omega_{\Lambda,0}$ and $\Omega_{M,0}$ together are enough to “just close” the Universe.

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Appendix: Axions and WIMPs

This appendix is a brief introduction to axions and supersymmetric weakly interacting particles (WIMPs); and in particular, to their main claim as compelling cold dark matter (CDM) candidates: the theoretical expectation that either one, if it exists, naturally has a collective density $\Omega_{\text{CDM}}$ close to the critical density.

Axions come about as part of the solution to the “strong CP problem” in quantum chromodynamics (QCD). This theory has had some enormous successes, notably in accounting for quark confinement and asymptotic freedom. However, it contains a dimensionless free parameter $\Theta$ which must be less than $\sim 10^{-10}$ in order to explain why strong interactions do not violate parity (P) or charge-parity (CP). (Upper bounds on the
electric dipole moment of the neutron tightly constrain any such violations.) To remove this unnaturally small number from the theory, particle theorists suppose that Θ is driven toward zero by the spontaneous breaking of a new symmetry of nature at energy scales fₐ. The axion (a) is the new boson which appears as a result of this phase transition, eventually acquiring a rest mass \( m_a c^2 \propto 1/f_a \). Unfortunately neither parameter is constrained \textit{a priori} by theory (although one hopes that \( f_a \) is less than \( 10^{10} \) times the QCD phase transition energy, or little would be gained from the whole mechanism). Observation, however, comes to the rescue.

Most of the axions with \( m_a c^2 \gtrsim 10^{-2} \) eV are produced thermally, and one can show by solving the Boltzmann equation that their combined present mass density would be \( \Omega_a \approx (m_a c^2/130 \text{ eV}) h_0^{-2} \). Those in the range \( 19 \text{ eV} \lesssim m_a c^2 \lesssim 32 \text{ eV} \) would therefore provide \( \Omega_a \approx 0.3 \), making them potential CDM candidates. However, axions decay into photons on timescales proportional to \( m_a^{-5} \). With masses as large as this, they would decay rapidly enough to flood the night sky with ultraviolet light. Consistency with the observed intensity of the extragalactic background light then leads to the upper bound \( m_a c^2 \lesssim 3 \) eV.

Thermal axions with \( 10^{-3} \) eV \( \lesssim m_a c^2 \lesssim 3 \) eV can be eliminated on different astrophysical grounds: they couple so weakly to ordinary matter that they could stream more or less freely out of the cores of red giants and supernovae, taking energy with them. Observations of the neutrino flux from supernova SN1987a show no evidence for such an effect.

Axions with \( m_a c^2 \lesssim 10^{-3} \) eV, finally, are largely nonthermal, with a collective density given by \( \Omega_a \approx (m_a c^2/4 \text{ eV}) h_0^{-2} \). Rest masses \( m_a c^2 \lesssim 10 \text{ eV} \) are then disqualified because they would provide too much CDM (\( \Omega_a \gtrsim 0.5 \)). The axion is thereby restricted to a relatively small window of potential rest masses, 10 \( \mu \text{eV} \lesssim m_a c^2 \lesssim 1000 \mu \text{eV} \). Its plausibility as a CDM candidate rests on the fact that this allowed window encompasses the range of values (15 \( \mu \text{eV} \lesssim m_a c^2 \lesssim 25 \mu \text{eV} \)) corresponding to \( \Omega_a \approx 0.3 \). The slow decay rate of axions in this mass range can, moreover, be enhanced by trapping them in strong magnetic fields; this is the basis for a number of ongoing experimental detection efforts in Japan and the U.S.A.

Like axions, supersymmetric WIMPs have their origin in a new symmetry of nature, spontaneously broken in the early Universe. This is supersymmetry (SUSY), which pairs each boson with a fermion superpartner, and vice versa. Because no such pairs can be formed with the known bosons and fermions of the standard model of particle physics, the number of fundamental particles must be doubled. The new SUSY partners are presumably so massive that they have not been discovered yet; the lightest SUSY particle (LSP) in particular must have a rest mass \( m_{\chi} c^2 \gtrsim 50 \text{ GeV} \).

This LSP plays the role of the WIMP in SUSY theories. It is stable, decaying solely by pair-annihilation with itself (a very slow process). This is due to an additional new symmetry of nature, known as \textit{R}-parity, which is necessary (in “minimal SUSY” models) to keep the proton from decaying via intermediate SUSY states. (There are also nonminimal SUSY theories in which this symmetry too is spontaneously broken, and the proton can decay.) \textit{R}-parity requires the number of SUSY (and non-SUSY) partners to be conserved in any reaction, so that, as the Universe cools, heavier SUSY particles can break down into lighter ones, but not into ordinary particles. Eventually, most of the SUSY mass in the Universe thus ends up in the form of LSPs.

Using the Boltzmann equation, one can calculate the collective density \( \Omega_\chi \) of these particles in terms of a number of free parameters such as the SUSY-breaking energy scale, and the composition of the LSP. Because its annihilation cross-section goes as \( m_\chi^{-2} \), one finds that an LSP more massive than \( \sim 3 \) TeV would annihilate so slowly as to overclose the Universe. This leaves an available SUSY WIMP mass window of 50 GeV \( \lesssim m_\chi c^2 \lesssim 3 \) TeV. The collective LSP density turns out to lie within three orders of magnitude of the critical density over most of this range, which makes the SUSY WIMP, like the axion, a plausible CDM candidate.

Many experiments around the world are currently searching for WIMPs in this mass range, assuming for example that they are gravitationally bound in the galactic halo and will occasionally be responsible for scattering events in target nuclei as the Earth follows the Sun around the Milky Way. One team (DAMA) has claimed evidence for a \( m_\chi c^2 = 59^{+17}_{-14} \) GeV WIMP signature at 3σ confidence, but this is disputed by a second (CDMS) which has searched a larger region of parameter space and found nothing. Strong constraints have also been placed on WIMPs in nonminimal versions of SUSY, where the LSP can decay into photons and contribute excessively to the intensity of the diffuse \( x \)- and \( \gamma \)-ray backgrounds. We watch these developments with interest.

\[^{18}\text{This is under debate, and may be up to an order of magnitude larger if string effects play an important role.}\]

\[^{19}\text{This is most likely to be the neutralino, a linear superposition the photino, the zino and two higgsinos (the spin-1/2 SUSY partners of the photon, Z and Higgs bosons). A less favored candidate, because it annihilates too slowly, is the gravitino, the spin-3/2 SUSY partner of the graviton.}\]
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