Analysis of the $X(1835)$ as a baryonium state with Bethe-Salpeter equation

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Abstract

In this article, we take the $X(1835)$ as a pseudoscalar baryonium state, and calculate the mass spectrum of the baryon-antibaryon bound states $p\bar{p}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$, and $\Lambda\bar{\Lambda}$ in the framework of the Bethe-Salpeter equation with a phenomenological potential. The numerical results indicate the $p\bar{p}$, $\Sigma\bar{\Sigma}$ and $\Xi\bar{\Xi}$ bound states maybe exist, and the $X(1835)$ can be tentatively identified as the $p\bar{p}$ bound state.

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1 Introduction

In 2003, the BES collaboration observed a significant narrow near-threshold enhancement in the proton-antiproton ($p\bar{p}$) invariant mass spectrum in the radiative decay $J/\psi \rightarrow \gamma p\bar{p}$ [1]. The enhancement can be fitted with either an $S$- or $P$-wave Breit-Wigner resonance function. In the case of the $S$-wave fitted form, the mass and the width are $M = 1859^{+3}_{-10}+5^{+10}_{-25}$ MeV and $\Gamma < 30$ MeV respectively. In 2005, the BES collaboration observed a resonance state $X(1835)$ in the $\eta'\pi^+\pi^-$ invariant mass spectrum in the process $J/\psi \rightarrow \gamma \pi^+\pi^-\eta'$ with the Breit-Wigner mass $M = (1833.7 \pm 6.2 \pm 2.7)$ MeV and the width $\Gamma = (67.7 \pm 20.3 \pm 7.7)$ MeV respectively [2]. Many theoretical works were stimulated to interpret the nature and the structure of the new particle, such as the $p\bar{p}$ bound state [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], the pseudoscalar glueball [16, 17, 18, 19], and the radial excitation of the $\eta'$ [20, 21, 22], etc.

In this article, we take the $X(1835)$ as a baryonium with the quantum numbers $J^{PC} = 0^{-+}$, and calculate the mass spectrum of the baryon-antibaryon bound states $p\bar{p}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$, and $\Lambda\bar{\Lambda}$ with the Bethe-Salpeter equation [23, 24]. The Bethe-Salpeter equation with phenomenological potentials is a powerful theoretical tool in studying bound states and has given many successful descriptions of the hadron properties [24, 25, 26].

The article is arranged as follows: we solve the Bethe-Salpeter equation for the baryon-antibaryon bound states in Sec.2; in Sec.3, we present the numerical results and discussions; and Sec.4 is reserved for our conclusions.

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The Bethe-Salpeter equation is a conventional approach in dealing with the two-body relativistic bound state problems. We write down the ladder Bethe-Salpeter equation for the pseudoscalar $p\bar{p}$ bound state,

\[
S^{-1}
\begin{pmatrix}
q + \frac{P}{2} \\
q - \frac{P}{2}
\end{pmatrix}
\chi(q, P)S^{-1}
\begin{pmatrix}
q - \frac{P}{2} \\
q + \frac{P}{2}
\end{pmatrix}
= \int \frac{d^4k}{(2\pi)^4} \gamma_5 \chi(k, P) \gamma_5 G(q - k), \tag{1}
\]

where the $P_\mu$ is the four-momentum of the center of mass of the $p\bar{p}$ bound state, the $q_\mu$ is the relative four-momentum between the proton and antiproton, $\gamma_5$ is the bare baryon-meson vertex, the $\chi(q, P)$ is the Bethe-Salpeter amplitude of the $p\bar{p}$ bound state, and the $G(q - k)$ is the interaction kernel. With a simple replacement of the corresponding parameters, we can obtain the Bethe-Salpeter equations for the $\Sigma \bar{\Sigma}$, $\Xi \bar{\Xi}$ and $\Lambda \bar{\Lambda}$ bound states if they exist.

In the flavor $SU(3)$ symmetry limit, the interactions among the octet baryons and the pseudoscalar mesons can be described by the lagrangian $L$,

\[
L = \sqrt{2} \left( D \text{Tr} \left( \bar{B} \{P, B\} \right) + F \text{Tr} \left( \bar{B} \{P, B\} \right) \right), \tag{2}
\]

where

\[
B = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda \\
\Sigma^- \\
\Xi^- \\
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta
\end{pmatrix}
\begin{pmatrix}
\Sigma^+ \\
\Xi^+ \\
\pi^+ \\
\eta
\end{pmatrix}
\begin{pmatrix}
p \\
n \\
K^+ \\
K^0
\end{pmatrix},
\]

\[
P = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda \\
\Sigma^- \\
\Xi^- \\
\pi^-
\end{pmatrix}
\begin{pmatrix}
\Sigma^+ \\
\Xi^+ \\
\pi^+ \\
\eta
\end{pmatrix}
\begin{pmatrix}
p \\
n \\
K^+ \\
K^0
\end{pmatrix},
\]

and the $D$ and $F$ are two parameters for the coupling constants. From the lagrangian, we can obtain

\[
g_{\pi^0 pp} = -g_{\pi^0 nn} = D + F, \quad g_{\pi^0 \Sigma^+ \Sigma^+} = -g_{\pi^0 \Sigma^- \Sigma^-} = 2F,
\]

\[
g_{\pi^0 \Xi^- \Xi^-} = -g_{\pi^0 \Xi^0 \Xi^0} = D - F, \quad g_{\eta pp} = g_{\eta nn} = -\frac{D - 3F}{\sqrt{3}},
\]

\[
g_{\eta \Sigma^+ \Sigma^+} = g_{\eta \Sigma^- \Sigma^-} = g_{\eta \Sigma^0 \Sigma^0} = -g_{\eta \Lambda \Lambda} = \frac{2D}{\sqrt{3}},
\]

\[
g_{\eta \Xi^- \Xi^-} = g_{\eta \Xi^0 \Xi^0} = -\frac{D + 3F}{\sqrt{3}}, \tag{4}
\]

and write down the kernel $G(q - k)$ explicitly,

\[
G(q - k) = \frac{g^2(q - k)C_\pi}{(q - k)^2 + m_\pi^2} + \frac{g^2(q - k)C_\eta}{(q - k)^2 + m_\eta^2}, \tag{5}
\]
where the coefficients $C_\pi = (1 + \alpha)^2$, $4\alpha^2$, $(1 - \alpha)^2$, 0 and $C_\eta = \frac{(1 - 3\alpha)^2}{3}$, $\frac{4}{3}$, $\frac{(1 + 3\alpha)^2}{3}$, $\frac{4}{3}$ for the $p\bar{p}$, $\Sigma\Sigma$, $\Xi\Xi$, $\Lambda\Lambda$ bound states respectively; $g^2(k) = D^2$ and $\alpha = \frac{E}{\lambda}$. In this article, we choose the value $\alpha = 0.6$ from the analysis of the hyperon semileptonic decays \cite{27}, and take the coupling constant $g^2(k)$ as a modified Gaussian distribution, $g^2(k) = A \left(\frac{k^2}{\mu^2}\right)^2 \exp\left(-\frac{k^2}{\mu^2}\right)$, where the strength $A$ and the distribution width $\mu^2$ are two free parameters. The ultraviolet behavior of the modified Gaussian distribution warrants the integral in the Bethe-Salpeter equation is convergent.

We can perform the Wick rotation analytically and continue $q$, $k$ into the Euclidean region. The Euclidean Bethe-Salpeter amplitude of the pseudoscalar $p\bar{p}$ bound state can be decomposed as

$$\chi(q, P) = \gamma_5 \{ F(q, P) + i P F_1(q, P) + i q F_2(q, P) + [q, P] F_3(q, P) \} ,$$

due to Lorentz covariance \cite{25}. In the coordinate space, the Bethe-Salpeter amplitude is defined by $\chi_{\alpha\beta}(x, y) = \langle 0 | T[q_{\alpha}(y)\bar{q}_{\beta}(x)] | X \rangle$, where the $q(x)$ is the interpolating current of the proton, $q(x) = e^{ik\cdot x} \mathcal{C}\gamma_\mu u_j(x)\gamma_5\gamma_\mu d_k(x)$.

We can perform the Fierz re-ordering in the Dirac spinor space to obtain the following identity,

$$q_\alpha(y)\bar{q}_{\beta}(x) = -\frac{1}{4} \delta_{\alpha\beta} q(x)y(y) - \frac{1}{4} (\gamma^\mu)_{\alpha\beta} \bar{q}(x)\gamma_\mu q(0) - \frac{1}{8} (\sigma^{\mu\nu})_{\alpha\beta} \bar{q}(x)\sigma_{\mu\nu} q(y) + \frac{1}{4} (\gamma^\mu \gamma_5)_{\alpha\beta} \bar{q}(x)\gamma_\mu \gamma_5 q(y) + \frac{1}{4} (i\gamma_5)_{\alpha\beta} \bar{q}(x)i\gamma_5 q(y) .$$

The QCD sum rules indicate that the couplings of the axial-vector currents and the pseudoscalar currents to the octet pseudoscalar mesons are much stronger than other interpolating quark currents \cite{28}. In this article, we can take the approximation

$$\chi(q, P) = \gamma_5 \{ F(q, P) + i P F_1(q, P) \} ,$$

for simplicity.

Multiplying both sides of the Bethe-Salpeter equation by $\gamma_5 [q, P]$ and doing the trace in the Dirac spinor space, we can obtain an simple relation $F = 2M_p F_1$, the amplitudes $F(q, P)$ and $F_1(q, P)$ are not independent.

The Bethe-Salpeter amplitude can be written as

$$\chi(q, P) = \gamma_5 \left( 1 + \frac{i P}{2M_p} \right) F(q, P) ,$$

and the Bethe-Salpeter equation can be projected into the following form,

$$\left( q^2 + M_p^2 + \frac{P^2}{4} \right) F(q, P) = \int \frac{d^4 k}{(2\pi)^4} F(k, P) G(q - k) .$$

We can introduce a parameter $\lambda(P^2)$ and solve above equation as an eigenvalue problem. If there really exists a bound state in the pseudoscalar channel, the mass of the $X(1835)$ can be determined by the condition $\lambda(P^2 = -M_X^2) = 1$,

$$\left( q^2 + M_p^2 + \frac{P^2}{4} \right) F(q, P) = \lambda(P^2) \int \frac{d^4 k}{(2\pi)^4} F(k, P) G(q - k) .$$


\[ \begin{array}{|c|c|c|c|c|}
\hline
& p\bar{p} & \Sigma\Sigma & \Xi\Xi & \Lambda\bar{\Lambda} \\
M_X [\text{MeV}] & 1833.7 & 2317.8 & 2612.4 & 2201.4 \\
E_X [\text{MeV}] & -42.9 & -61.0 & -31.0 & -30 \\
\hline
\end{array} \]

Table 1: The masses \( M_X \) and the bound energies \( E_X \) for the baryon-antibaryon bound states.

In the limit \( q^2 = 0 \), we can obtain an simple relation for the \( p\bar{p} \) bound state,

\[ M_X^2 < 4M_p^2, \quad (12) \]

i.e. the bound energy should be negative, \( E_X = 2M_p - M_X < 0 \). The Bethe-Salpeter equations for other bound states are treated with the same routine.

3 Numerical results and discussions

The input parameters are taken as \( m_\pi = 135 \text{ MeV}, m_\eta = 548 \text{ MeV}, M_p = 938.3 \text{ MeV}, M_{\Sigma^+} = 1189.4 \text{ MeV}, M_{\Sigma^-} = 1321.7 \text{ MeV}, M_\Lambda = 1115.7 \text{ MeV}, \) and \( M_X(1835) = 1833.7 \text{ MeV} \) from the Particle Data Group [29]. The strength \( A \) and the distribution width \( \mu^2 \) are free parameters, we take the values \( A = 215 \) and \( \mu = 200 \text{ MeV} \) for the \( p\bar{p} \) bound state. Furthermore, we take the simple replacements \( \mu^2 \rightarrow \mu^2 \frac{M_\Sigma^2}{M_p^2}, \mu^2 \rightarrow \mu^2 \frac{M_\Xi^2}{M_p^2} \) and \( \mu^2 \rightarrow \mu^2 \frac{M_\Lambda^2}{M_p^2} \) to take into account the flavor \( SU(3) \) breaking effects for the \( \Sigma\Sigma, \Xi\Xi \) and \( \Lambda\bar{\Lambda} \) bound states respectively.

We solve the Bethe-Salpeter equations as an eigen-problem numerically by direct iterations, and observe the convergent behaviors are very good. For the \( p\bar{p}, \Sigma\Sigma \) and \( \Xi\Xi \) bound states, there exists a solution with \( \lambda(P^2 = -M_X^2) = 1 \) and \( E_X < 0 \). On the other hand, we cannot obtain a solution to satisfy the condition \( \lambda(P^2 = -M_X^2) = 1 \) for the \( \Lambda\bar{\Lambda} \) bound state, and have to resort to the fine-tune mechanism by introducing the coupling \( g_{\eta'\Lambda\Lambda} \) between the \( \eta' \) meson and the \( \Lambda \) baryon. For example, we can obtain a \( \Lambda\bar{\Lambda} \) bound state with the bound energy \( E = -30 \text{ MeV} \) with the value \( g_{\eta'\Lambda\Lambda}^2 = 5.3g_{\eta\Lambda\Lambda}^2 \), such as a fine-tune solution should not be taken seriously. The numerical results for the Bethe-Salpeter amplitudes are shown in Fig.1 and the values of the bound states are presented in Table 1.

If those bound states exist indeed, they can be produced in the radiative \( J/\psi \) decays, i.e. \( J/\psi \rightarrow \gamma gg, gg + q\bar{q} \rightarrow p\bar{p}, \Sigma\Sigma, \Xi\Xi, \Lambda\bar{\Lambda} \), those bound states can decay to the \( \eta\pi\pi, \eta K\bar{K}, \eta'\pi\pi, \eta' K\bar{K}, \eta'\eta'\eta, \eta\eta\eta \) final states. We can search for those bound states in the \( \eta\pi\pi, \eta K\bar{K}, \eta'\pi\pi, \eta' K\bar{K}, \eta'\eta'\eta, \eta\eta\eta \) invariant mass distributions in the radiative decays of the \( J/\psi \) at the BESIII [30].
Figure 1: The Bethe-Salpeter amplitudes of the bound states, $A$, $B$, $C$ and $D$ denote the $p\bar{p}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$ and $\Lambda\bar{\Lambda}$ respectively.

4 Conclusion

In this article, we take the $X(1835)$ as a pseudoscalar baryonium state and calculate the mass spectrum of the baryon-antibaryon bound states $p\bar{p}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$, $\Lambda\bar{\Lambda}$ in the framework of the Bethe-Salpeter equation with a phenomenological potential. The numerical results indicate that the $p\bar{p}$, $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$ bound states maybe exist, and the $X(1835)$ can be tentatively identified as the $p\bar{p}$ bound state. The other bound states maybe observed in the future at the BESIII.

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