Four Dimensional Black Holes
and Strings with Rescaled Tension

Edi Halyo*

Department of Physics
Stanford University
Stanford, CA 94305

ABSTRACT

We give a microscopic description of extreme and near–extreme Reissner–Nordstrom black holes in four dimensions in terms of fundamental strings in a background of magnetic five branes and monopoles. The string oscillator numbers and tension are rescaled due to the Rindler space background with the string mass fixed. The entropy of the black holes is reproduced correctly by that of the string with the tension rescaling taken into account.

* e–mail address: halyo@dormouse.stanford.edu
1. Introduction

If string theory is a quantum theory of gravity it has to provide a microscopic explanation for black hole entropy given by the Bekenstein–Hawking formula

\[ S = \frac{A}{4G_N} \]  

in terms of string degrees of freedom. One would expect that the black hole entropy arises from the degeneracy of the string states which describe a black hole with given mass and charges. Naively, this expectation fails due to the fact that the black hole and string entropies scale differently with the mass; \( S_{BH} \sim M^2 \) whereas \( S_{st} \sim M \) in four dimensions. However, in ref.[1] it was shown that the scaling is the same if one considers the Rindler energy of a string in the black hole background and identifies it with the string entropy. In [2] this result was expanded and it was shown that \( S_{BH} \) and \( S_{st} \) completely agree for Schwarzschild black holes in any dimension.

During the last year, different microscopic descriptions of four dimensional Neveu–Schwarz (NS) and Ramond–Ramond (RR) charged Reissner–Nordstrom (RN) black holes appeared in the literature [3,4,5]. A common feature of these is that the essential degrees of freedom of the black hole seem to be those of long (D) strings. For extreme and near–extreme black holes these degrees of freedom are non or weakly interacting respectively. Other objects such as five (D) branes which are present do not play a dynamical role in the black hole but simply constitute backgrounds for the string. This picture also applies to rotating black holes and emission of low energy neutral, charged and fixed scalars [7,8,9,10].

Recently, another microscopic description of five dimensional NS charged RN black holes was given in ref. [11]. In this paper, we extend the results of ref. [11] to four dimensional extreme and near–extreme RN black holes. We show that these black holes can be described by a string with two charges in a background of magnetic five branes and monopoles. Due to the background the string oscillator numbers and tension are rescaled [13] whereas the mass remains fixed. The mass
of the black hole is given by the sum of the background and string masses. The black hole entropy is given only by that of the string with the tension rescaling taken into account. This is done by calculating the Rindler energy of the string in the background of magnetic five branes and monopoles and identifying it with the entropy of the string.

This paper is organized as follows. In section 2, we review the classical four dimensional RN black hole solutions. In section 3, we give the microscopic description of these black holes in terms of a string with two charges in a background of magnetic five branes and monopoles. We apply the method of ref. [11] to extreme and near–extreme four dimensional RN black holes. Section 4 contains our conclusions.

2. Classical Reissner–Nordstrom Black Holes in Four Dimensions

In this section, we review the solution for the NS charged RN black hole in four dimensions. The classical solution to the low energy equations of motion in type II string theory compactified on $T^6$ is given by the metric $g_{\mu\nu}$, the NS antisymmetric tensor $B_{\mu\nu}$ and the dilaton $g^2 = e^{2\phi}$. The RR three form, the self–dual five form and the RR scalar are set to zero. Also, the asymptotic value of the dilaton $\phi$ is taken to be zero. The classical four dimensional RN black hole metric is given by

$$ds^2 = -\chi^{-1/2} \left( 1 - \frac{r_0}{r} \right) dt^2 + \chi^{1/2} \left[ \left( 1 - \frac{r_0}{r} \right)^{-1} dr^2 + r^2 d\Omega_2^2 \right]$$  \hspace{1cm} (2)

where

$$\chi = \left( 1 + \frac{r_0^2 \sinh^2 \alpha}{r^2} \right) \left( 1 + \frac{r_0^2 \sinh^2 \beta}{r^2} \right) \left( 1 + \frac{r_0^2 \sinh^2 \gamma}{r^2} \right) \left( 1 + \frac{r_0^2 \sinh^2 \delta}{r^2} \right)$$  \hspace{1cm} (3)

The solution is parametrized by eight parameters, $\alpha, \beta, \gamma, \delta, r_0$ and the compactified one and four volumes $2\pi R, 2\pi R'$ and $(2\pi)^4 V$. The total energy of the black hole is

$$E = \frac{RR'V r_0}{2g^2 \alpha^4} (cosh 2\alpha + cosh 2\beta + cosh 2\gamma + cosh 2\delta)$$  \hspace{1cm} (4)

The entropy of the black hole is found from the area of the horizon using the
Bekenstein–Hawking formula

\[ S = \frac{A_H}{4G_4} = \frac{8\pi RR'V r_0^2}{g^2\alpha'^4} \cosh\alpha \cosh\beta \cosh\gamma \cosh\delta \]  

(5)

where the ten and four dimensional Newton constants are given by \( G_{10} = 8\pi^2 g^2 \alpha'^4 \) and \( G_4 = G_{10}/(2\pi)^6 RR'V \). The RN black hole carries four NS charges

\[ Q_5 = \frac{R'r_0}{2\alpha'} \sinh(2\alpha) \]  

(6a)

\[ Q_1 = \frac{V R'r_0}{2g^2\alpha'^3} \sinh(2\beta) \]  

(6b)

\[ n = \frac{R^2 R'V r_0}{2g^2\alpha'^4} \sinh(2\gamma) \]  

(6c)

\[ Q_m = \frac{r_0}{2} \sinh(2\delta) \]  

(6d)

These are the charges of the black hole under the NS three form \( H_3 \), its dual \( H_7 \), Kaluza–Klein two form coming from the metric and a magnetic monopole charge* . The magnetic monopole is the only object which can be added to the five dimensional solution to stabilize the new compactified coordinate with radius \( R' \). Note that due to the fact that we have a RN black hole the scalars in the solution, i.e. the dilaton \( \phi \) and the compactification volumes \( R, R', V \) (which are a priori fields) are constant in space. The RN condition is \( \alpha = \beta = \gamma = \delta \) so that the four contributions to the mass of the black hole are equal. The extreme limit is obtained by \( r_0 \to 0 \) and \( \alpha, \beta, \gamma, \delta \to \infty \) with the charges \( Q_5, Q_1, n, Q_m \) fixed. The cases with less than four charges are obtained by setting the corresponding angles to zero.

The properties of the black hole can be written in a suggestive way if we trade the eight parameters \( \alpha, \beta, \gamma, \delta, r_0, R, R', V \) for \( N_5, \bar{N}_5, N_1, \bar{N}_1, n_L, n_R, N_m, \bar{N}_m \) defined by

\[ N_5 = \frac{r_0 R'}{4\alpha'} e^{2\alpha} \]  

(7a)

* I thank Arvind Rajaraman for proposing the magnetic monopole as the fourth charge.
In terms of the above numbers, the charges of the black hole are
\[ Q_5 = N_5 - \bar{N}_5, \]
\[ Q_1 = N_1 - \bar{N}_1, \]
\[ n = n_L - n_R, \]
\[ Q_m = N_m - \bar{N}_m. \]
The black hole mass is
\[ M_{BH} = RV \frac{r_0^2 R'}{4g^2 \alpha'^3} e^{-2\alpha} + R \frac{V r_0}{4g^2 \alpha'^3} e^{-2\beta} + \frac{1}{R} (n_L + n_R) + \frac{RR'V}{g^2 \alpha'^4} \]
(8)
The entropy can be written as
\[ S = 2\pi (\sqrt{N_1} + \sqrt{\bar{N}_1}) (\sqrt{N_5} + \sqrt{\bar{N}_5}) (\sqrt{n_L} + \sqrt{\bar{n}_R})(\sqrt{N_m} + \sqrt{\bar{N}_m}) \]
(9)
The extreme limit is given by \( \bar{N}_5 = \bar{N}_1 = n_R = \bar{N}_m = 0 \). For small deviations beyond extremality \( \bar{N}_5 \sim \bar{N}_1 \sim n_R \sim \bar{N}_m << N_5 \sim N_1 \sim n_L \sim N_m \). Note that all three anti-charges must become nonzero in order to satisfy the RN nature of the nonextreme black hole.

Our aim in the next section will be to show that the RN black hole mass and entropy are given by that of a fundamental closed string with a rescaled tension in the background of magnetic five branes and monopoles. We will identify \( Q_1 \) and \( n \) with the net winding and momentum number of the string whereas \( Q_5 \) and \( Q_m \) will be the net five brane (winding) and magnetic monopole numbers respectively.
3. Black Holes and Strings with Rescaled Tension

In this section, we show that four dimensional RN black holes can be described by strings with two charges in a background of magnetic five branes and monopoles. The mass of the black hole is given by the sum of the masses of the background and the string. The black hole entropy is given only by that of the string which is related to the Rindler energy of the string in the background. The background of $Q_5$ magnetic five branes and $Q_m$ monopoles is Rindler space near the horizon. We calculate the Rindler energy of the string in this background and relate it to its entropy. We find that this equals the entropy of the black hole; therefore we conclude that the string carries all of the black hole entropy. We show that this method works for the extreme and near-extreme four dimensional black holes.

3.1. Extreme Reissner–Nordstrom Black Holes

In this section we consider extreme RN black holes with four charges. From eqs. (4) and (5) they have mass (since $N_5 = N_1 = n_R = N_m = 0$)

$$M_{BH} = \frac{RV}{g^2 \alpha'^3} N_5 + \frac{R}{\alpha'} N_1 + \frac{n_L}{R} + \frac{RR^2 V}{g^2 \alpha'^4} N_m$$

and entropy

$$S = 2\pi \sqrt{N_1 N_5 n_L N_m}$$

For the RN black hole the four contributions to the mass are equal. Following the arguments of ref. [11], we will describe this black hole as a string with two charges in a background of $N_5$ magnetic five branes and $N_m$ magnetic monopoles. Two of the charges of the black hole, $N_1$ and $n_L$ are the winding and momentum numbers of the string so that the mass of the string is

$$M_{st}^2 = Q^2_R + \frac{4N_R}{\alpha'} = Q^2_L + \frac{4N_L}{\alpha'}$$

with

$$Q_{R,L} = \left( (N_1 - \bar{N}_1) \frac{R}{\alpha'} \pm \frac{(n_L - n_R)}{R} \right)$$
where the oscillator numbers are given by

\[ N_{L,R} = \frac{1}{4} \left[ (\sqrt{N_1} + \sqrt{\bar{N}_1})(\sqrt{n_L} + \sqrt{n_R}) \pm (\sqrt{N_1} - \sqrt{\bar{N}_1})(\sqrt{n_L} - \sqrt{n_R}) \right]^2 \]  (14)

Note that \( N_{L,R} \) above satisfy the level matching condition for any \( N_1, \bar{N}_1, n_L, n_R \). Using the definitions of \( N_{L,R} \) we find that for the extreme black hole with \( n_R = \bar{N}_1 = 0, N_R = 0 \) and \( N_L = N_1 n_L \). Therefore the string is in a BPS state. This is not surprising since we are considering an extreme RN black hole which is described by a BPS string state on a BPS background. The mass of the background with the string is

\[ M = \frac{RV}{g^2 \alpha'^3} N_5 + \frac{RR'^2 V}{g^2 \alpha'^4} N_m + \sqrt{\frac{4N_1 n_L}{\alpha'}} \]  (15)

which is equal to that of the black hole given by eq. (10). It is easy to show that the lowest excitations of the string with rescaled tension have energy \( \sim 1/(RN_5 N_1 N_m) \) which is precisely that expected for the four dimensional black hole.

The black hole entropy is more difficult to obtain however. Naively, the entropy of the black hole is given by that of the string

\[ S = 2\pi \sqrt{\frac{c}{6}} \left( \sqrt{N_L} + \sqrt{N_R} \right) \]  (16)

Using the expressions for \( N_{L,R} \), we see that we do not get the correct black hole entropy in eq. (11). This is not surprising since the above formula for entropy holds for a free string whereas we have a string in a background of five branes and monopoles. In the curved background of a black hole the Rindler energy of an object is given by \( dE_R = dM/2\pi T_R \). Comparing this with \( dM = T dS \) for the string we find that

\[ S = 2\pi \left( \frac{T_R}{T} \right) E_R \]  (17)

The Rindler energy is a dimensionless quantity which should be identified with the (square root of the) oscillator number of the string and not with its mass. This is also apparent from eq. (17) and the expression for the string entropy.
We now compute the Rindler energy of the above string in the background of $Q_5$ five branes and $Q_m$ monopoles. The metric for this background is obtained from eq. (2) by taking $\alpha = \gamma = 0$ and is given by

$$ds^2 = -\chi^{-1/2} \left(1 - \frac{r_0}{r}\right) dt^2 + \chi^{1/2} \left[\left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\Omega_3^2\right]$$

(18)

where

$$\chi = \left(1 + \frac{r_0 \sinh^2 \alpha}{r}\right) \left(1 + \frac{r_0 \sinh^2 \delta}{r}\right)$$

(19)

Near horizon, the metric in eq. (18) gives the Rindler space–time. In this limit

$$r \to r_0, \quad \chi \to (1 + \sinh^2 \alpha)(1 + \sinh^2 \delta) = \cosh^2 \alpha \cosh^2 \delta$$

(20)

We rescale the metric by

$$r' = r\chi^{1/4}, \quad r'_0 = r_0\chi^{1/4}$$

(21)

The charges of the background are

$$Q_5 = N_5 = \frac{r_0 R'}{\alpha'} \sinh(2\alpha) \simeq \frac{2r_0 R'}{\alpha'} \cosh^2 \alpha$$

(22)

and

$$Q_m = N_m = \frac{r_0}{R'} \sinh(2\delta) \simeq \frac{2r_0}{R'} \cosh^2 \delta$$

(23)

where the second equalities hold in the near extreme case when $r_0 \to 0, \alpha, \delta \to \infty$, i.e. $\bar{N}_5 = \bar{N}_m = 0$. Then

$$\chi^{1/4} r'_0 = r_0 \cosh \alpha \cosh \delta = \frac{1}{2} \sqrt{N_5 N_m \alpha'}$$

(24)

Expanding near the horizon, $r' = r'_0 + y$ the metric becomes

$$ds^2 = -\chi^{-1/2} \frac{y}{r_0'} dt^2 + \frac{r_0'}{y} dy^2 + r_0'^2 d\Omega_3^2$$

(25)
The proper distance $\rho$ to the horizon is

$$\rho = \int \sqrt{\frac{r'_0}{y}} \, dy = 2 \sqrt{r'_0 \sqrt{y}}$$

(26)

Then the coefficient of $dt^2$ becomes

$$g_{00} = -\frac{\rho^2}{4 \chi^{1/2} r'_0^2}$$

(27)

One can bring the metric to Rindler form by the rescaling

$$\tau = \frac{t}{2 \chi^{1/4} r'_0}$$

(28)

where $\tau$ is the Rindler time conjugate to Rindler energy $E_R$ which is given by

$$E_R = 2 \chi^{1/4} r'_0 M = M \sqrt{N_5 N_m \alpha'}$$

(29)

As a result of eq. (17) we identify in the Rindler space–time $T = 2T_R$ and

$$E_R = 2 \sqrt{\frac{c}{6}} \left( \sqrt{N_L'} + \sqrt{N_R'} \right)$$

(30)

Now for a BPS state of free string with no charge $M^2 = 4N_L/\alpha'$. We see that, in the background of five branes and monopoles, the string oscillator number $N_L$ is rescaled by a factor of $\sqrt{N_5 N_m}$ compared to the free string (for $c = 6$). On the other hand, the mass of the free string (added to that of the background) gives the correct black hole mass. This means that we need to keep the string mass fixed but rescale $N_{L,R}$. This can be done if we assume that $\alpha'$ (or the string tension) is rescaled simultaneously with $N_{L,R}$ so that the mass remains the same, i.e. $\alpha' \to \alpha'_{\text{eff}} = N_5 N_m \alpha'$. The entropy of the string with the rescaled tension is obtained from eqs. (17) and (30) for $c = 6$

$$S = 2 \pi \sqrt{N_5 N_1 n_L N_m}$$

(31)

which is exactly the entropy of the black hole given by eq. (11). Note that for a string whose tension is rescaled as above $T = 2T_R$. 

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We found that due to the presence of the background, the string tension is rescaled together with $N_{L,R}$ so that the mass remains the same. Taking this rescaling into account we find that the string entropy is precisely the black hole entropy. Here we assume that all gravitational effects which are present in the black hole are summed up in the rescaling of the string tension. In addition, we assume that the masses of the background and the string are additive. This makes sense since both the background and the string are BPS states. Note also that the formula holds only for $c = 6$ whereas for a physical type II string $c = 12$. This means that the string is effectively confined to the the five brane world–volume and therefore oscillates only in the four transverse directions.

3.2 Near–extreme Reissner–Nordström Black Holes

We now consider slightly nonextreme RN black holes with mass

$$M_{BH} = \frac{RV}{g^2\alpha'^3}(N_5 + \bar{N}_5) + \frac{R}{\alpha'}(N_1 + \bar{N}_1) + \frac{1}{R}(n_L + n_R) + \frac{RR'V}{g^2\alpha'^4}(N_m + \bar{N}_m) \quad (32)$$

and entropy

$$S = 2\pi(\sqrt{N_1} + \sqrt{\bar{N}_1})(\sqrt{N_5} + \sqrt{\bar{N}_5})(\sqrt{n_L} + \sqrt{n_R})(\sqrt{N_m} + \sqrt{\bar{N}_m}) \quad (33)$$

where $N_1 \sim N_5 \sim n_L \sim N_m >> \bar{N}_1 \sim \bar{N}_5 \sim n_R \sim \bar{N}_m$. Note that the deviation from extremality is for the four charges simultaneously due to the RN nature of the black hole. Since the black hole is nonextreme it is described by a non BPS string state ($N_R \neq 0$) in a nonextreme background (five branes and anti five branes plus monopoles and anti monopoles). The string mass is given by

$$M_{st}^2 = Q_R^2 + \frac{4N_R}{\alpha'} = Q_L^2 + \frac{4N_L}{\alpha'} \quad (34)$$

with

$$Q_{R,L} = \left( (N_1 - \bar{N}_1) \frac{R}{\alpha'} \pm \frac{(n_L - n_R)}{R} \right) \quad (35)$$

with $N_{L,R}$ given by eq. (14). Using the condition for the extreme RN case (which
holds approximately for the nonextreme case)

\[ \frac{N_1 R}{\alpha'} \simeq \frac{n_L}{R} \quad (36) \]

we find that the string mass is

\[ M_{st} = \sqrt{\frac{4N_1 n_L}{\alpha'} + \frac{8N_1 n_R}{\alpha'}} \quad (37) \]

For anti-charges much smaller than the charges we find that

\[ M_{st} \simeq \frac{2N_1 R}{\alpha'} + \frac{2n_R}{R} \quad (38) \]

Adding this to the mass of the background we obtain

\[ M = \frac{RV}{g^2\alpha'^3}(N_5 + \bar{N}_5) + \frac{RR^2V}{g^2\alpha'^4}(N_m + \bar{N}_m) + \frac{2N_1 R}{\alpha'} + \frac{2n_R}{R} \quad (39) \]

which is the mass of the near-extreme black hole given by eq. (32).

In order to find the entropy, we have to calculate the Rindler energy of the above string \( E_R \) in the background of \( N_5 \) five branes, \( \bar{N}_5 \) anti five branes, \( N_m \) monopoles and \( \bar{N}_m \) anti monopoles with \( N_5, N_m >> \bar{N}_5, \bar{N}_m \). Once again we find

\[ E_R = 2M^{1/4}r_0' \sim Mr_0\cosh\alpha \cosh\delta \quad (40) \]

in the near extreme limit. But now,

\[ Q_5 = N_5 - \bar{N}_5 = \frac{r_0}{\alpha'}\sinh^2(2\alpha) \quad (41) \]

and

\[ Q_m = N_m - \bar{N}_m = \frac{r_0}{\alpha'}\sinh^2(2\delta) \quad (42) \]

so that the oscillator numbers and the string tension are rescaled by the factor.
The entropy of the string is

\[ S = 2\pi (\sqrt{N'_L} + \sqrt{N'_R}) \] (43)

where \( N'_{L,R} = (\sqrt{N_5} + \sqrt{\overline{N_5}})(\sqrt{N_m} + \sqrt{\overline{N_m}})N_{L,R} \). Using the definition of \( N_{L,R} \) in eq. (14) we find the entropy of the string in the background

\[ S = 2\pi (\sqrt{N_5} + \sqrt{\overline{N_5}})(\sqrt{N_1} + \sqrt{\overline{N_1}})(\sqrt{n_L} + \sqrt{n_R})(\sqrt{N_m} + \sqrt{\overline{N_m}}) \] (44)

which is the entropy of the near–extreme black hole. The rescaling factor of the tension \( (\sqrt{N_5} + \sqrt{\overline{N_5}})(\sqrt{N_m} + \sqrt{\overline{N_m}}) \) is exactly the factor needed to convert the string entropy into the black hole entropy. We see that even in the near–extreme case the masses of the background and the string are additive. This does not make sense since these are not BPS states.

4. Conclusions

We have shown that extreme and near–extreme RN black holes in four dimensions can be described by strings with two charges in a background of magnetic five branes and monopoles. The extreme (nonextreme) black holes correspond to BPS (non BPS) states of the string in a BPS (non BPS) background. The mass of the black hole is given by the sum of the string and background masses. Due to the background, the string oscillator numbers and tension are rescaled whereas the mass remains fixed. The black hole entropy is reproduced only by that of the string with tension rescaling taken into account. The string entropy is given by its Rindler energy in the background five branes and monopoles.

In this paper as in ref. [11] we made a number of assumptions which we are not able justify. The main assumption is that all gravitational effects in the black hole can be summed up in the rescaling of the string oscillator numbers and tension. In addition, for both BPS and non BPS states of the string (which correspond to extreme and near–extreme black holes), the mass remains fixed in the Rindler
background i.e. the masses of the constituents are additive. Contrary to the BPS case this does not make sense for non BPS states. Finally, we neglected all possible effects due to the string mass on the string itself. Since for the RN black holes the mass of the string equals that of the background, this is difficult to understand.

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