Partial wave interference mechanism in gluonic dissociation of $J/\psi$

Binoy K. Patra$^1$ and V. J. Menon$^2$

1 Dept. of Physics, Indian Institute of Technology, Roorkee 247 667, India
2 Dept. of Physics, Banaras Hindu University, Varanasi 221 005, India

We explicitly take into account the effect of hydrodynamic expansion profile on the gluonic breakup of $J/\psi$'s produced in an equilibrating parton plasma. Attention is paid to the space-time inhomogeneities as well as Lorentz frames while deriving new expressions for the gluon number density $n_g$, average dissociation rate $\langle \Gamma \rangle$, and $\psi$ survival probability $S$. A novel type of partial wave interference mechanism is found to operate in the formula of $\langle \Gamma \rangle$. Nonrelativistic longitudinal expansion for small length of the initial cylinder is found to push the $S(pT)$ graph above the no flow case considered by us earlier. However, relativistic flow corresponding to large length of the initial cylinder pushes the curve of $S(pT)$ downwards at LHC but upwards at RHIC. This mutually different effect on $S(pT)$ may be attributed to the different initial temperatures generated at LHC and RHIC.

Introduction

Among the well known mechanisms of $J/\psi$ dissociation the one due to gluonic bombardment deserves special attention here. Recently the present authors considered the statistical mechanics of important physical observables viz. the gluon number density, thermally-averaged $g - \psi$ break-up rate, and the $\psi$ meson survival probability appropriate to RHIC/LHC initial conditions. It is a well-recognized fact that the longitudinal/transverse expansion of the medium controls the master rate equations for the time-evolution of the plasma temperature and parton fugacities. But the literature does not tell how the fluid velocity profile itself influences the Lorentz transformations connecting the rest frames of the fireball, plasma, and $\psi$ meson. In other words, since the flow velocity profile causes inhomogeneities in space-time, hence the scenario of $J/\psi$ inhomogeneities in space-time, hence the scenario of $J/\psi$ dissociation may be affected in a quite nontrivial manner and the aim of the present paper is to address this hitherto unsolved problem.

Theory and Calculations

If $K = (K^0, \vec{K})$ is the the gluon 4-momentum in the fireball rest frame and $k = (k^0, \vec{k})$, the gluon 4-momentum in the local comoving frame of the plasma is then given by Lorentz transformations

$$K : u = k^0 ; \quad K^0/k^0 = \gamma(1 + \vec{v} \cdot \vec{k}),$$

where $u = (\gamma, \gamma \vec{v})$ is the fluid 4 velocity. Then the gluon number density using the Bose-Einstein distribution can be expressed as

$$n_g(x) = \frac{16}{\pi^2} \gamma T^3 \sum_{n=1}^{\infty} \frac{\lambda_n^6}{n^2}$$

This result shows in a compact manner how the number density depends upon $\gamma$, $T$, and $\lambda_n$. Next, we turn to the question of applying statistical mechanics to gluonic break-up of the $J/\psi$ moving inside an expanding parton plasma. In the fireball frame consider a $\psi$ meson of mass $m_\psi$, four momentum $p_\psi = (p_\psi^0, \vec{p}_\psi)$, three velocity $\vec{v}_\psi$ and dilation factor $\gamma_\psi$. The invariant quantum mechanical dissociation rate $\Gamma$ for $g - \psi$ collision may be written as

$$\Gamma = v_{rel} \sigma$$

where $v_{rel}$ is the relative flux and $\sigma$ the cross section measured in any chosen frame. Its thermal average over gluon momentum in the fireball frame reads

$$\langle \Gamma(x) \rangle = \frac{16}{n_g(x)} \int \frac{d^3K}{(2\pi)^3} \Gamma f$$

Let $q = (q^0, \vec{q})$ be the gluon 4 momentum measured in $\psi$ meson rest frame. Since the relative flux becomes $v_{rel}^{\text{Rest}} = c = 1$ hence our invariant $\Gamma$ reduces to the QCD based cross section

$$\Gamma = \sigma_{\text{Rest}} = B(Q^0 - 1)^{3/2}/Q^{05} ; \quad q^0 \geq \epsilon_\psi$$

$$Q^0 = \frac{q^0}{\epsilon_\psi} \geq 1 ; \quad B = \frac{2\pi}{3} \left( \frac{32}{3} \right)^{1/2} \frac{1}{m_\psi c^2 m_\psi m_\psi}$$
where $\epsilon_\psi$ is the $J/\psi$ binding energy and $m_c$ the charmed quark mass. The energy variable for the massless gluon transforms via

$$K^0 = \gamma_\psi (q^0 + \vec{v}_\psi \cdot \vec{q}) = \gamma_\psi q^0 (1 + |\vec{v}_\psi| \cos \theta_{qw})$$ \hspace{1cm} (6)$$

with $\theta_{qw}$ being the angle between $\hat{q}$ and $\vec{v}_\psi$ unit vectors. Furthermore, the fluid 4-velocity $w = (w^0, \vec{w})$ seen in $\psi$ rest frame will be given by the Lorentz transformations

$$w^0 = \gamma_\psi \gamma (1 - \vec{v} \cdot \vec{v}_\psi)$$ $$\vec{w} = \gamma [\vec{v} - \gamma_\psi \vec{v}_\psi + (\gamma_\psi - 1)(\vec{v} \cdot \vec{v}_\psi)\vec{v}_\psi]$$ \hspace{1cm} (7)$$

and the scalar product

$$K \cdot u = q \cdot w = q^0 w^0 - q^0 |\vec{w}| \cos \theta_{qw}$$ \hspace{1cm} (8)$$

where $\theta_{qw}$ is the angle between $\hat{q}$ and $\hat{w}$. Finally, the thermally-averaged rate of (4) can be calculated as \[9\]

$$\langle \Gamma(x) \rangle = \frac{8\epsilon_\psi^3 \gamma_\psi}{\pi^2 n_g} \sum_{n=1}^{\infty} \lambda_g \int_1^\infty dQ^0 Q^0 (Q^0)^2 \sigma_{\text{Rest}}(Q^0) e^{-C_n Q^0}$$

$$\times \left[ I_0(\rho_n) + I_1(\rho_n)|\vec{v}_\psi| \cos \theta_{qw} \right]$$ \hspace{1cm} (9)$$

where $C_n = n \epsilon_\psi w^0 / T$, and $\rho_n = n \epsilon_\psi Q^0 |\vec{w}| / T$. This demonstrate how the mean dissociation rate $\langle \Gamma(x) \rangle$ depends on the hydrodynamic flow through $|\vec{w}|$ (or $w^0$) as well as the angle $\theta_{qw}$.

From the analytical viewpoint it is much more advisable to work with the modified rate

$$\langle \tilde{\Gamma}(x) \rangle = n_g(x) \langle \Gamma(x) \rangle$$ $$\approx \frac{8\epsilon_\psi^3 \gamma_\psi}{\pi^2} \lambda_g \int_1^\infty dQ^0 Q^0 (Q^0)^2 \sigma_{\text{Rest}} H$$ $$\propto \lambda_g \gamma_\psi H$$ \hspace{1cm} (10)$$

Here the entire dependence on the flow velocity $w$ is contained in the function

$$H \equiv e^{-C_1 Q^0} \left[ I_0(D_1 Q^0_\rho) + I_1(D_1 Q^0_\rho) |\vec{v}_\psi| \cos \theta_{qw} \right], D_1 \equiv \frac{\epsilon_\psi |\vec{w}|}{T}$$ \hspace{1cm} (11)$$

We are now ready to discuss some consequences of (10) in three cases viz. static medium in the fireball frame, no flow in the $J/\psi$ rest frame, and ultrarelativistic flow in either frame.

First, we consider static medium in fireball frame where $\vec{v} = \vec{0}$, $\gamma = 1$, $w^0 = \gamma_\psi$; $\vec{w} = -\gamma_\psi \vec{v}_\psi$. This is precisely the case treated in our earlier paper \[4\]. Due to the assumed absence of flow there is no inhomogeneity with respect to $x$. At fixed $p_T$ the steady increase of $\langle \tilde{\Gamma} \rangle$ with $T$ in Fig.1 is caused by the growing exp $(-C_1 Q^0)$ factors of the estimate(11) whereas at fixed $T$ the monotonic decrease of $\langle \tilde{\Gamma} \rangle$ with $p_T$ in Fig.1 has a very interesting explanation.
FIG. 2: $\langle \tilde{\Gamma}(x) \rangle$ using Eqs.(9,10) as a function of temperature (transverse momenta) at different transverse momenta (temperature) for the ultrarelativistic longitudinal flow velocity $v = 0.9 \ c$.

For the case under study $\vec{w} = -\gamma_\psi \vec{v}_\psi$ is antiparallel to $\vec{v}_\psi$ so that $\cos \theta_{\psi w} = -1$. Hence partial wave terms $I_0$ and $I_1$ of $H \ interfere$ destructively in Fig. 4 making $\langle \tilde{\Gamma} \rangle$ small as $|\vec{v}_\psi|$ grows.

Secondly, we consider no flow in $J/\psi$ rest frame where 3 velocities of the plasma and $\psi$ meson coincide at some $x$ in the fireball frame, i.e., $\vec{v} = \vec{v}_\psi$. Here the values are consistently higher than the above no-flow case.

Thirdly, we illustrate the case of both the $J/\psi$ and plasma moving ultrarelativistically (in the transverse and longitudinal directions, respectively) with $\vec{v} = 0.9 \ ˆ{\vec{v}}_z$. The rough estimate (10) becomes

$$\langle \Gamma(x) \rangle \propto \frac{\lambda_0 T}{\gamma} \exp \left( -\frac{\epsilon_\psi Q_0^2}{2 T \gamma_\psi} \right) \left[ 1 - \left| \vec{v}_\psi \right|^2 \right] \tag{12}$$

At fixed $p_T$, $v$ the exponential in (11) tend to 0 as $T \to 0$ and tends to 1 as $T \to \infty$. Therefore, the growing trend of $\langle \tilde{\Gamma}(x) \rangle$ with $T$ in Fig. 2 is understandable whereas at fixed $T$, $v$ the rich behaviour of $\langle \tilde{\Gamma}(x) \rangle$ with $p_T$ in Fig. 2 arises from a sensitive competition between the bracketed factors of (11). In fact, at lower temperatures $T \leq 0.4 \ \text{GeV}$ the exponential factor increases dominantly with $p_T$ causing $\langle \tilde{\Gamma}(x) \rangle$ to grow, but at higher temperatures $T \geq 0.8 \ \text{GeV}$ the third bracket in (12) decreases prominently with $p_T$ causing $\langle \tilde{\Gamma} \rangle$ to drop. In the case of pure transverse expansion of the plasma $\cos \theta_{\psi w}$ can even become $+1$, implying constructive interference between $I_0$ and $I_1$ in (11) [10].

**$J/\psi$ SURVIVAL PROBABILITY**

Suppose at general instant $t$ in the fireball frame the plasma is contained inside a cylinder of radius $R$, and expanding longitudinally with a velocity

$$\vec{v} = z \hat{e}_z / t \ ; \ -L/2 \leq z \leq +L/2. \tag{13}$$

Then the effective survival chance of a chosen $\psi$ meson will be given by the exponential $e^{-W}$ with

$$W = \int_{t_I}^{t_{II}} dt \ \langle \tilde{\Gamma}[t] \rangle \tag{14}$$

where $t_I = t_i + \gamma_\psi \tau_F$ and $t_{II} = \min(t_I + t_{RI}, t_{life})$ with $t_{RI}$ time-taking by $J/\psi$ to traverse the system. The time dependence of gluon-number density have been provided by the solution of the master-rate equation of a chemically evolving plasma [2].

Upon averaging $e^{-W}$ over the production configuration of the $\psi$’s we arrive at the final expression for the net survival probability

$$S(p_T) = \int_{V_I} d^3x_\psi \ (R_I^2 - r_I^2) e^{-W} / \int_{V_I} d^3x_\psi \ (R_I^2 - r_I^2)$$

$$d^3x_\psi = d\phi_\psi \ r_I^4 \ d\phi_\psi \ dz_\psi \tag{15}$$

For chosen creation configuration (label I) of the $\psi$ meson the function $W$ was first computed from (14) and then averaged over the production configuration of the $\psi$’s. Here the values are consistently higher than the above no-flow case.
FIG. 3: The solid curve is the result of Ref. [4], i.e., in the absence of flow while the dotted and dashed curves represent when the plasma is undergoing longitudinal expansion with the initial values of the length of the cylinder $L_i = 0.1$ fm and 1 fm, respectively.

the survival probability was numerically evaluated using (15). Fig.3 show the dependence of $S(p_T)$ on the transverse momentum corresponding to the LHC(1) and RHIC(1) initial conditions. The dotted ($L_i = 0.1$ fm) and dashed ($L_i = 1$ fm) curves are computed in the presence of longitudinal flow while the solid curve is borrowed from [4] in the absence of hydrodynamic flow profile.

Discussions and Summary

Nonrelativistic longitudinal expansion from small length of the initial cylinder is found to push the $S(p_T)$ graph above the no flow case considered by us earlier [4]. However, relativistic flow corresponding to large length of the initial cylinder pushes the curve of $S(p_T)$ downwards at LHC but upwards at RHIC. This mutually different effect on $S(p_T)$ may be attributed to the constructive/destructive interference in (11) due to different initial conditions at LHC and RHIC.

[1] B. K. Patra, D. K. Srivastava, Phys. Lett. B 505, 113 (2001).
[2] B. K. Patra and V. J. Menon, Nucl. Phys. A 708, 353 (2002).
[3] Xiao-Ming Xu, D. Kharzeev, H. Satz, and Xin-Nian Wang, Phys. Rev. C 53, 3051 (1996).
[4] B. K. Patra and V. J. Menon, Eur. Phys. J. C 37, 115 (2004).
[5] T. S. Biro, E. van Doorn, M. H. Thoma, B. Müller, and X.-N. Wang, Phys. Rev. C 48, 1275 (1993).
[6] D. Pal, B. K. Patra, and D. K. Srivastava, Eur. Phys. Jour. C 17, 179 (2000).
[7] X.-N Wang and M. Gyulassy, Phys. Rev. D 44, 3501 (1991).
[8] M. E. Peskin, Nucl. Phys. B 156, 365 (1979); G. Bhanot and M. E. Peskin, Nucl. Phys. B 156, 391 (1979).
[9] B. K. Patra and V.J.Menon, Eur. Phys. J. C 44, 567 (2005).
[10] B.K.Patra and V.J.Menon, Eur. Phys. J. C 48, 207 (2006).