The Quantum Molecular Dynamics Model (IQMD) is used to investigate the origin of the collective transverse velocity observed in heavy ion experiments. We find that there are three contributions to this effect: initial-final state correlations, potential interactions and collisions. For a given nuclear equation of state (eos) the increase of the transverse velocity with increasing beam energy is caused by the potential part. For a given beam energy the collective transverse velocity is independent of the nuclear eos but the relative contributions of potential and collisions differ. In view of the importance of the potential interactions between the nucleons it is not evident that the similarity of the radial velocities measured for fragments at beam energies below 1 AGeV and that for mesons at beam energies above 2 AGeV is more than accidental.

A major goal of heavy ion collision studies is to extend our knowledge of the properties of dense and hot nuclear matter. In the past these studies were concentrated on the production of secondary particles like pions or kaons and on anisotropies in the momentum distribution like the directed in-plane flow (bounce-off) or the out of plane flow (squeeze-out) of nuclear matter. For recent reviews of the different collective flows we refer to [3,4].

Recently also the isotropic radial flow has gained large interest because at AGS and at CERN energies the existing data at mid-rapidity of most of the mesons and baryons can be well described by the assumption that the system is thermalized and expands with a collective velocity of nuclear matter. At these low energies most of the pions come from the decay of nuclear resonances and therefore cannot be used to determine a transverse velocity and kaons are too rare to be of use for such an analysis.

The radial flow velocity determined with help of the fragment kinetic energies up to 1A GeV smoothly joints that determined with help of mesons and baryons starting from 11A GeV. This raised the question whether both transverse collective flows have a common origin and whether their functional dependence on the beam energy allows for conclusions about a possible transition between hadronic matter and a quark gluon plasma.

A common origin of the transverse flow of fragments and mesons is not at all evident. Nucleons which are bound asymptotically in fragments do usually not suffer collisions with a large momentum transfer, otherwise they are not bound anymore. On the contrary, the creation of mesons requires a large momentum transfer.

At beam energies below 1AGeV the radial flow seems to be a good candidate for measuring the nuclear equation of state. Increasing compression leads to an increasing pressure which is released isotropically. The directed transverse flow in the reaction plane, which in principle also depends on the pressure, is known to be sensitive on the potential gradient. The squeeze-out of high energy particles perpendicular to the reaction plane allows for a view inside the reaction zone. However, this effect is connected to the geometry and the time scales of the system.

At CERN and AGS energies a detailed theoretical investigation of the origin of the transverse velocity is difficult because many details of the expansion of the hadronic matter like the cross sections between baryonic and mesonic resonances are unknown. Therefore calculations rely on a multitude of assumptions.

At energies around 1 A GeV the situation is much better. Here we have models which simulate the complete reaction and we find agreement between the results of these models and experiment for many observables. It is the aim of this article to take advantage of this situation and to investigate the origin of the transverse expansion of fragments in detail. Details of the Quantum molecular dynamics models, which are used for our study, can be found in the refs. where also the different equations of state, mentioned in this article, are explained. We used for this work the IQMD version of the model with a variance of the Gaussian wave function in coordinate space of $L = 4.33\text{fm}^2$ and (if not otherwise stated) with static potentials because the compressional
energy is difficult to determine with momentum dependent interactions. A small value of $L$ yields a higher number of fragments than a large one and reproduces better the experimental multiplicity. This analysis is strongly based on the behavior of fragments. Therefore we have chosen this small $L$ value. The choice of this value of $L$ yield only a slight enhancement of the asymptotic value of $\beta_t$ for the 1 GeV case, where most of the analysis is done. For lower energies there is a visible reduction of the asymptotic $\beta_t$.

In the QMD simulations there are three different processes which contribute to the transverse velocity:

- The nucleons which finally form a fragment $A$ may have already initially a finite transverse velocity

$$< v_t(A) > = \left\langle \frac{1}{A} \sum_{j=1}^{A} v_i^j \hat{e}_r \right\rangle,$$

where $v_i^j$ equals $v_i^j'(t=0)$ and the sum runs over all nucleons which are finally entrained in the fragment of size $A$. $<>$ means averaging over all available nuclei of size $A$ observed at mid-rapidity ($y_{CM}/y_{proj} \leq 0.3$). This velocity may depend on the size of the fragment. This type of initial-final state correlations have been investigated by Gossiaux et al. [17].

- The potential between the nucleons may contribute to the transverse velocity

$$v_F(A) = \left\langle \frac{1}{mA} \int dt \sum_{j=1}^{A} -\nabla U^j(t) \hat{e}_r \right\rangle.$$

To investigate its influence we sum up all contributions from the nuclear force to the transverse velocity.

- The collisions between the baryons may increase the transverse velocity as well

$$v_C(A) = \left\langle \frac{1}{A} \sum_{j=1}^{A} \sum_{coll} (\vec{v}_j + \vec{v}_2 - \vec{v}_j - \vec{v}_2) \hat{e}_r \right\rangle.$$

To calculate this contribution we add the transverse velocity change in all individual collisions $(v_j, v_2) \rightarrow (v'_j, v'_2)$ of the nucleons $j$ of a fragment of the size $A$.

Before we start with a detailed investigation of the origin of the transverse flow $\beta_t$ we compare our calculations with the experimental results to make sure that the gross features of the transverse flow are well reproduced in our simulations. As it was shown in IQMD the transverse expansion of the nuclear matter shows a linear velocity profile for central collisions at intermediate energies. This linear profile is used as input to expansion models which assume the expansion to be due to a thermal and a collective part. Assuming $E_t(A) = 1/2 mA\beta_t^2 + 3/2T$ we disentangle these two parts by fitting the transverse velocities at mid-rapidity ($y_{CM}/y_{proj} \leq 0.3$) using two parameters $\beta_t$ and $T$:

$$v_t(A) = \sqrt{\frac{2E_t(A)}{Am}} = \sqrt{\beta_t^2 + \frac{3T}{Am}},$$

where $\beta_t$ is the collective transverse velocity, $T$ is the temperature, $m$ is the mass of the nucleon and $A$ the mass number of the fragments. $v_t(A)$ is the mean transverse velocity of fragments with the mass number $A$. We made two fits in order to be comparable with the two sets of experiments [7,8]: one which includes all masses up to ten and a second where only masses smaller than five are included.

### Au+Au, very central collisions

![Transverse velocity vs. E(inc)](image_url)

**Fig. 1.** Transverse velocity $\beta_t$ obtained in IQMD simulations using a hard momentum dependent interaction as compared to the different experiments.

Fig. 1 shows the results of the simulations for an impact parameter of $b = 1$ fm using a hard equation of state with momentum dependent interactions. These calculations are compared with the available experimental data. The $\beta_t$ value of the different experiments have been determined differently and in addition the impact parameter cut is different. The difference between the data of [10] and of [8] is not understood yet. We observe that the simulations follow very closely the experimental data.

We see as well in fig.1 that even at the lowest beam energies a finite collective transverse velocity is observed. In this energy range Nebauer et al. [10] discussed on correlations between the radial expansion and the directed
transverse flow. However, they studied semicentral collisions where the radial flow is small and the transverse flow is strong. In this article we will only regard very central collisions where the transverse flow is vanishing and the radial flow is strongest and analyze the influence of potential and collisions.

Fig. 2 shows the transverse velocities taken at mid-rapidity for fragments produced in the reaction Au(250 A MeV)+Au at b=1fm. The circles denote the final velocities of the fragments. The dotted curve shows the result of a fit of the form of eq.1 and illustrates the method of determining the asymptotic radial velocity as described above.

The triangles mark the initial total transverse velocity of the fragment nucleons. Clusters of nucleons which have a velocity pointing away from the interaction zone have a higher chance to survive the reaction without being destroyed than those clusters which have a momentum which points into the reaction zone. For the diamonds we have added to the initial transverse velocity the transverse velocity caused by the potential. The difference between the diamonds and the circles, the total transverse velocity, is the transverse velocity caused by collisions. We see a continuous decrease of $v_t(A)$ with the fragment mass up to some asymptotic value. Thus, at 250 A MeV the transverse velocity has essentially two components: an initial transverse velocity and that caused by collisions. The potential plays only a minor role.

It has now to be analyzed whether the vanishing influence of the potential is a genuine effect or only valid for a beam energy of 250A MeV. Figure 3 shows the asymptotic velocity and its decomposition into the different contributions. We display the initial transverse velocity of those nucleons which form finally a fragment as well as the sum of the initial transverse velocity and that caused by the potential. The difference between this curve and the asymptotic value is the transverse velocity caused by collisions. For energies larger than 500A MeV we see an almost constant contribution from the collisions (of about $\beta = 0.1$) and from the initial velocity distribution of the fragments (of about $\beta = 0.04$). The increase of $\beta_t$ is almost exclusively caused by the potential. Higher beam energies yield a higher compression and therefore stronger forces. Only at energies well above 2 GeV (where one may question the validity of our model) we observe saturation. The low energy sector is more complicated. We see that below 250A MeV the forces become attractive (the reaction develops towards a scenario expected for deep inelastic collisions) and therefore they lower the contribution from the initial velocity distribution. Collisions become more and more Pauli blocked and do not contribute anymore. At high beam energies the nucleons which are finally entrained in fragments have been (in central collisions) initially close to the surface at the back end of the nuclei, because there the number of collisions they suffer is lowest [17]. Also their initial momentum points away from the reaction zone. These initial-final state correlations change at lower energies. There, due to the increased Pauli blocking, projectile matter can traverse the target and vice versa and hence the nucleons which are finally found in fragments have initially a quite uniform distri-

\[ \text{Au+Au } b=1\text{fm hard no mdi} \]
The physical origin of the energy dependence of $\beta_t$ can also be studied by comparing it with the radial flow expected if the total compressional energy is converted into a radial flow. For this purpose we compare the energy difference $\Delta U = U_{\text{max}} - U_{\text{ini}}$ where $U_{\text{max}}$ is the total potential energy at the moment of the highest compression whereas $U_{\text{ini}}$ is the initial potential energy of projectile and target. The difference corresponds to the compressional energy needed to reach the state maximum compression.

$\Delta U$ is related to $\beta_t$ by

$$E = m + \Delta U = m \cdot \gamma = m \cdot (1 - \beta_t^2)^{-1/2} \tag{2}$$

The values are marked by diamonds. We see that its excitation function agrees to that of the contribution of initial $\beta_t$ plus potentials (triangles), which is the total radial flow minus the collisional part. For higher energies this curve is parallel to that of the total radial flow (circles) what verifies that the increase of the radial flow is caused by the increasing compression, whereas the collisional contribution stays rather constant.

All calculations discussed up to now have been performed using a hard equation of state with momentum dependent interactions (mdi). We now compare different equations of state at an energy of 1A GeV. In the Skyrme ansatz the repulsion of the nuclear equation of state at high densities is connected to the nuclear compressibility. Therefore we characterized in fig. 4 and 5 the equation of state by their corresponding compressibility. Soft and hard eos correspond to $\kappa = 200$ and $\kappa = 376$ MeV, respectively. Fig. 4 shows the time evolution of the density in the center of the reaction zone of a central Au+Au collision using different equations of state without momentum dependent interaction. We see that due to less repulsion the highest density is reached with the softest equation of state. Also the pressure is lower for this eos. Consequently the system will stay for a longer time at maximum compression. A higher density causes a smaller mean free path and therefore an enhanced number of collision. Both yield together a nearly linear dependence of the number of high energy collisions on the compressibility. Independent of the eos each high energetic nn collision contributes about the same amount to the radial velocity. The softer the eos the more collisions contribute to the radial velocity and the higher is the kinetic pressure caused by the collisions. On the other hand the repulsion of the potentials is weaker and causes a weaker pressure from the potential part. Therefore it has to be checked which contribution to the pressure is dominant and whether the dependence of the different contributions to the total pressure counterbalance.

Figure 5 shows the asymptotic transverse velocities $\beta_t$ on the compressibility for the system Au(1A GeV)+Au and the contribution of the potential part.
the same $\beta_t$ despite the contributions from the potential part are rather different. As already stated the potential part and the collisional part show different dependences on the nuclear eos. Seemingly the differences counterbalance.

In fig.5 we show as well as open triangles the radial velocity one would obtain if the compressional energy per nucleon is entirely converted into a radial velocity applying eq.2.: We see that this line parallels that with the actual final transverse velocities caused by the potential. Thus the increasing compressional energy is responsible for the increase of the radial velocity. The different absolute values are due to the fact that the system is not in complete equilibrium and hence the compression is lowered due to the not completely decelerated projectile and target matter.

Using the IQMD model to investigate the collective transverse expansion velocity observed in heavy ion reactions we find, as in the experiments, a continuous rise with the beam energy up to energies of 1600A MeV. Whereas the in-plane flow measures the potential gradient and the out of plane squeeze the mean free path it was hoped that the radial flow is directly related to the compressibility. This conjecture can only partially be substantiated. Indeed, the increase of the radial flow as a function of the bombarding energy is - for a given nuclear equation of state - due to the higher compression of the nuclear matter and not due to the rescattering of the fragment nucleons. The relative fraction of the potential and of the collisional contribution at a given energy depends on the other hand on the compressibility of the equation of state. Even more, the sum of the potential and collisional contribution is almost constant. Thus it is impossible to extract the compressibility from the observation of the radial flow.

The collisional contribution to the radial flow should be rather similar for the mesons as well as for the baryons. At energies around 1 AGeV, however, the potential contributes in between 30% and 60% to the total radial velocity, depending on the equation of state. This contribution is absent for the mesons if created by string decay and not by baryonic interactions like $NN \rightarrow \Delta N$. Because no potentials are at hand to continue our investigations to higher energies the doubts remain whether the one should expect a smooth transition between the collective radial velocity observed 1 AGeV for fragments and that measured for mesons at higher (AGS and CERN) beam energies.

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