Generation of higher derivatives operators and electromagnetic wave propagation in a Lorentz-violation scenario

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We study the perturbative generation of higher-derivative Lorentz violating operators as quantum corrections to the photon effective action, originated from a specific Lorentz violation background, which has already been studied in connection with the physics of light pseudoscalars. We calculate the complete one loop effective action of the photon field through the proper-time method, using the zeta function regularization. This result can be used as a starting point to study possible effects of the Lorentz violating background we are considering in photon physics. As an example, we focus on the lowest order corrections and investigate whether they could influence the propagation of electromagnetic waves through the vacuum. We show, however, that no effects of the kind of Lorentz violation we consider can be detected in such a context, so that other aspects of photon physics have to be studied.
I. INTRODUCTION

Since the years 1990s, a systematic search for Lorentz symmetry breaking in a wide range of phenomena has been undertaken by both experimental and theoretical physicists, using as a fundamental tool the Standard Model Extension (SME) developed by Kostelecky and collaborators [1, 2]. The idea is to incorporate in the Standard Model (SM) Lagrangian new terms involving constant background tensors, thus introducing Lorentz violation (LV) while maintaining fundamental properties such as renormalizability and gauge invariance. The extension of this idea to include gravity was first worked out in [3], and it has been recently extended to include higher derivative operators involving LV [4, 5]. More on the status of this subject can be found for example in [6].

One natural question is the origin of such a multitude of LV terms added to the SM Lagrangian. They might arise from a spontaneous breaking of Lorentz symmetry in a more fundamental theory, such as String Theory [7], but they could also be derived as quantum corrections in an extended theory of electromagnetic, scalar or gravitational fields involving couplings of these fields to heavy spinor fields in a Lorentz breaking manner. This idea was developed in [8], where one of the simplest Lorentz-breaking extensions of electrodynamics, the CFJ model [9], was shown to arise in the low-energy effective action of a particular LV model where a massive fermion is integrated out. This idea was extended to the perturbative generation of other LV terms: in [10], for example, we have shown how a model including a massive fermion that couples in a LV way to the gauge field $A^\mu$ and to a pseudoscalar $\phi$ can generate the coupling between an axion-like particle and the photon, which is very relevant for contemporary experimental searches for axions or other light pseudoscalars. Our aim now is to study the physical consequences of one of the couplings introduced in [10], this time focusing on the pure Maxwell sector of the theory.

It is known that a very efficient tool for obtaining the complete low-energy one-loop effective action is the proper-time method [11] (for a review, see for example [12]), which was adapted for gauge theories in [13]. Within the study of the Lorentz violating theories, the only example we know of the application of this method is given in [14] where it was used to obtain the non-Abelian CFJ and the gravitational Chern-Simons terms as perturbative corrections in some LV models.

In this work, we adopt the version of the proper-time method based on the zeta function regularization to obtain the complete low-energy effective action of the gauge field in the Lorentz-breaking extension of QED which we considered [10]. The effective action thus calculated can be used as a starting point for the study of further physical effects of these LV couplings. With this idea in mind, we look for the influence of the LV on the propagation of electromagnetic waves in
the one-loop corrected theory. It is known that certain Lorentz-breaking extensions of QED display birefringence and rotation of the polarization plane of electromagnetic waves in the vacuum \[15–19\]. Furthermore, different issues related to wave propagation in various Lorentz-breaking extensions of QED were studied in a number of papers \[20–29\]. However, there is up to now a very small number of studies of wave propagation in higher-derivative Lorentz-breaking theories, such as \[30–32\], where Lorentz-breaking modifications up to third order in derivatives were studied, and issues related to unitarity and causality were considered. It is worth to mention that other higher-derivative contributions to the effective action, also up to the third order in derivatives, were shown to emerge as quantum corrections \[33–35\].

Considering our model in particular, the one loop corrections we take into account are the standard nonlinear corrections to the Maxwell theory, known as the Euler-Heisenberg Lagrangian, with the contributions of first order in the LV parameters. Since there are active precision experiments involving photons propagating in a strong magnetic field \[36, 37\], this study could pinpoint a window of opportunity to probe for the LV background we consider. We write the modified Maxwell’s equations in vacuum, but find out that no modification due to LV in the wave propagation appears in our model. However, we point out that an experimental bound on the mass \(M\) could in principle be derived once these experiments are able to measure the nonlinear corrections with sufficient precision.

In section II, we shall describe the Lorentz-breaking extension of QED that we consider, which is a subset of the model considered in \[10\]. The calculation of the one-loop low-energy effective action for the photon field is presented in section III. The first order corrections generated in the Maxwell action are made explicit in section IV and there we show the LV does not modify wave propagation in the vacuum. Section V summarizes our conclusions and perspectives.

\section{II. THE MODEL}

We consider a high energy model containing the photon field \(A_\mu\) and a single massive charged fermion field \(\psi\). The Lagrangian describing our model is given by

\[
\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} \left[ i \partial - M - \gamma^\mu (g A_\mu + F_{\mu\nu} d^\nu) \right] \psi,
\]

where \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\) is the field strength, \(M\) is the fermion mass and \(g\) is the electric charge. Within our model, LV enters via the nonminimal CPT-breaking interaction \(d^\nu F_{\mu\nu} \bar{\psi} \gamma^\mu \psi\), where \(d^\mu\) is a constant background vector (not an axial vector as in \[8\]). Our main interest in this particular
interaction comes from the fact that it was shown to lead to relevant results for the search of axion-like particles \[10\], which motives us to investigate further physical consequences of this term.

Since we assume the fermion mass to be very high, in the low-energy regime we can integrate in the fermion field, expressing the one loop effective action \(S^{(1)}_{\text{eff}}[A]\) by means of a functional trace as follows,

\[
S^{(1)}_{\text{eff}}[A] = - \text{Tr} \ln \left( i \tilde{\partial} - M - \gamma^\mu \tilde{A}_\mu \right),
\]

where

\[
\tilde{A}_\mu = g A_\mu + F_{\mu \nu} d^\nu.
\]

In terms of the operator \(\tilde{D}_\mu = \partial_\mu + i \tilde{A}_\mu\), the effective action in Eq. (2) can be written as

\[
S^{(1)}_{\text{eff}}[A] = - \text{Tr} \ln \left( i \tilde{\partial} - M \right).
\]

We note that the proper-time method requires the use of an even-order differential operator \[12\], so we rewrite the trace on the previous equation as follows,

\[
\text{Tr} \ln \left( i \tilde{\partial} - M \right) = \frac{1}{2} \text{Tr} \left[ \ln \left( i \tilde{\partial} - M \right) + \ln \left( i \tilde{\partial} - M \right) \right]
= \frac{1}{2} \text{Tr} \left[ \ln \left( i \tilde{\partial} - M \right) + \ln \left( -i \tilde{\partial} - M \right) \right]
= \frac{1}{2} \text{Tr} \ln \left( \tilde{\partial}^2 + M^2 \right),
\]

where in the second line we have inserted \(\gamma_5 \gamma_5 = I\) and used of the cyclicity of the trace. Thus we can write

\[
S^{(1)}_{\text{eff}}[A] = - \frac{1}{2} \text{Tr} \ln \left( \tilde{D}^2 + i \Sigma^{\mu \nu} \tilde{F}_{\mu \nu} + M^2 \right),
\]

where

\[
\Sigma^{\mu \nu} = \frac{1}{4} \left[ \gamma^\mu, \gamma^\nu \right] ; \quad \tilde{F}_{\mu \nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu.
\]

This form of the one loop effective action is suitable for the evaluation by means of the proper-time method, as we show in the next section.

**III. EVALUATION OF QUANTUM CORRECTIONS**

We calculate now the explicit form of the one-loop contribution to the effective action in Eq. (6) using the zeta function regularization (for a review on this approach, see \[38\]). The zeta function
\( \zeta(s) \) is an important tool used to represent functional determinants in quantum field theory, and it is written in terms of the integral over proper time \( \tau \) as follows

\[
\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \tau^{s-1} \text{Tr} \left( e^{-O \tau} \right) \, d\tau,
\]

where \( O \) represents a dimensionless differential operator. It follows from this expression that

\[
\left. \frac{d\zeta}{ds} \right|_{s=0} = \text{Tr} \left( \ln O \right),
\]

and therefore

\[
S_{\text{eff}}^{(1)} [A] = -\frac{1}{2} \left. \frac{d\zeta}{ds} \right|_{s=0},
\]

where, for the sake of our work,

\[
O \sim \tilde{D}^2 + i \Sigma^{\mu\nu} \tilde{F}_{\mu\nu} + M^2.
\]

Equation (9) summarizes the use of the zeta function method for obtaining the one-loop quantum corrections. Up to our knowledge, although it has been applied in many physically interesting models including gravity [38] and supersymmetric field theories [13], this method was never used for the study of Lorentz-breaking theories.

Now, let us briefly describe how we apply the zeta function approach in our model, following [13].

Substituting Eq. (10) in Eq. (8), we have

\[
\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty d\tau \tau^{s-1} e^{-\frac{m^2}{\mu^2} \tilde{K} \left( \frac{\tau}{\mu^2} \right)},
\]

where we identify the kernel trace

\[
\tilde{K} \left( \frac{\tau}{\mu^2} \right) = e^{-\frac{m^2}{\mu^2} \tilde{D}^2} \text{tr} \left\{ e^{-i \frac{\tau}{\mu^2} \Sigma^{\mu\nu} \tilde{F}_{\mu\nu}} \right\}.
\]

Here the trace is taken over spin indices, and the \( \mu \) parameter has mass dimension one and is introduced to make the differential operator in Eq. (10) dimensionless. The kernel trace can be related with the local kernel through the expression

\[
\tilde{K} (\tau) = \int d^4x \text{ tr} \left\{ e^{-i \tau \Sigma^{\mu\nu} \tilde{F}_{\mu\nu}} \right\} K (\tau),
\]

with

\[
K (\tau) = \lim_{x \to x'} e^{-\tau \tilde{D}^2} \delta^4 (x - x').
\]

Substituting Eq. (13) in Eq. (11), we obtain

\[
\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty d\tau \tau^{s-1} e^{-\frac{m^2}{\mu^2} \tilde{K} \left( \frac{\tau}{\mu^2} \right)} \int d^4x \text{ tr} \left\{ e^{-i \frac{\tau}{\mu^2} \Sigma^{\mu\nu} \tilde{F}_{\mu\nu}} \right\} K \left( \frac{\tau}{\mu^2} \right).
\]
To calculate the trace and the determinant in the right-hand side of the Eq. (17), we carry out the determinant. Therefore,

$$K(\tau) = \frac{1}{16\pi^2\tau^2} \det \left[ \frac{-\tau i \tilde{F}}{\sinh(-\tau i \tilde{F})} \right]^{1/2}, \quad (16)$$

where in this expression the tensor $\tilde{F}$ is considered as a four by four matrix in calculating the determinant. Therefore,

$$\zeta(s) = \frac{\mu^4}{16\pi^2} \frac{1}{\Gamma(s)} \int_0^\infty d\tau \tau^{s-3} e^{-\frac{m^2}{\mu^2}} \int d^4x \operatorname{tr} \left\{ e^{-i \frac{\tau}{\mu^2} \Sigma_{\mu\nu} \tilde{F}_{\mu\nu}} \right\} \det \left[ \frac{-\tau i \tilde{F}}{\mu^2 \sinh(-\frac{\tau}{\mu^2} i \tilde{F})} \right]^{1/2}. \quad (17)$$

To calculate the trace and the determinant in the right-hand side of the Eq. (17), we carry out the expansions

$$\det \left[ \frac{-\tau i \tilde{F}}{\mu^2 \sinh(-\frac{\tau}{\mu^2} i \tilde{F})} \right]^{1/2} = 1 - \frac{\tau^2}{12\mu^2} \tilde{F}^2 + \frac{\tau^4}{144\mu^8} \left[ \frac{2}{5} \operatorname{Tr}(\tilde{F}^4) + \frac{1}{2} \tilde{F}^4 \right] + \cdots, \quad (18)$$

where $\operatorname{Tr}$ means a trace over the spacetime indices of $\tilde{F}$, i.e.,

$$\operatorname{Tr}(\tilde{F}^2) = \tilde{F}^{\mu\nu} \tilde{F}_{\mu\nu} = -\tilde{F}^2, \quad \operatorname{Tr}(\tilde{F}^4) = \tilde{F}^{\mu\nu} \tilde{F}_{\mu\nu} \tilde{F}^{\alpha\beta} \tilde{F}_{\alpha\beta}, \quad (19)$$

and

$$\operatorname{tr}\left\{ e^{-i \frac{\tau}{\mu^2} \Sigma_{\mu\nu} \tilde{F}_{\mu\nu}} \right\} = \operatorname{tr}(I) - i \frac{\tau}{\mu^2} \operatorname{tr}(\Sigma_{\mu\nu} \tilde{F}_{\mu\nu}) - \frac{\tau^2}{2\mu^4} \operatorname{tr}(\Sigma_{\mu\nu} \tilde{F}_{\mu\nu})^2$$

$$+ i \frac{\tau^3}{6\mu^6} \operatorname{tr}(\Sigma_{\mu\nu} \tilde{F}_{\mu\nu})^3 + \frac{\tau^4}{24\mu^8} \operatorname{tr}(\Sigma_{\mu\nu} \tilde{F}_{\mu\nu})^4 + \cdots. \quad (20)$$

It can be shown that

$$\operatorname{tr}(\Sigma_{\mu\nu} \tilde{F}_{\mu\nu})^2 = -2\tilde{F}^2; \quad \operatorname{tr}(\Sigma_{\mu\nu} \tilde{F}_{\mu\nu})^4 = \tilde{F}^4 - (\star \tilde{F}^{\mu\nu} \tilde{F}_{\mu\nu})^2, \quad (21)$$

whereas traces of an odd number of $\Sigma$ matrices vanish, and the dual electromagnetic tensor is defined as $\star \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \tilde{F}_{\alpha\beta}$. Putting all this together, we obtain the one-loop quantum corrections to the effective action of the photon field $A_\mu$ in the form

$$S_{eff}^{(1)}[A] = -\frac{1}{48\pi^2} \int d^4x \left\{ -\ln \left( \frac{M^2}{\mu^2} \right) \tilde{F}^2 + \frac{1}{8M^4} \left[ \frac{2}{15} \operatorname{Tr}(\tilde{F}^4) - \frac{1}{3} \tilde{F}^4 \right.$$

$$- \frac{1}{2} (\star \tilde{F}^{\mu\nu} \tilde{F}_{\mu\nu})^2 \left. \right] + \frac{1}{16M^8} \tilde{F}^2 \left[ \frac{2}{5} \operatorname{Tr}(\tilde{F}^4) + \frac{1}{2} (\star \tilde{F}^{\mu\nu} \tilde{F}_{\mu\nu})^2 \right] \right.$$  

$$+ \frac{5}{96M^{12}} \left[ \tilde{F}^4 - (\star \tilde{F}^{\mu\nu} \tilde{F}_{\mu\nu})^2 \right] \left[ \frac{2}{5} \operatorname{Tr}(\tilde{F}^4) + \frac{1}{2} \tilde{F}^4 \right] + \cdots \right\}. \quad (22)$$
The Lorentz violation is implicitly taken into account inside the $\tilde{F}_{\mu\nu}$, since from (3) and (7), we have

$$\tilde{F}_{\mu\nu} = gF_{\mu\nu} + d^\lambda (\partial_\mu F_{\nu\lambda} - \partial_\nu F_{\mu\lambda}) \ ,$$

and, for example

$$\tilde{F}^2 = \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = g^2 F_{\mu\nu}F^{\mu\nu} + 4gd_\alpha F_{\mu\nu}(\partial^\mu F^{\nu\alpha})$$

$$+ 2d^\lambda d_\alpha [ (\partial_\mu F_{\nu\lambda})(\partial^\mu F^{\nu\alpha}) - (\partial_\mu F_{\nu\lambda})(\partial^\nu F^{\mu\alpha}) ] \ .$$

The Lorentz violation causes a space-time anisotropy that leads us to a preferred frame, in other words, we expect a dependence of physical measurements on the directions of the Lorentz violation parameters. To make evident such dependence, one may rewrite Eq. (24) explicitly in terms of the electric and magnetic fields $E$ and $B$ respectively, up to the first order in the LV parameter, as follows

$$\tilde{F}^2 = 2g^2 (B^2 - E^2) + 4g \{ (d^0) [B \cdot (\nabla \times E) + E \cdot (\partial_0 E)]$$

$$- d \cdot [E \times (\partial_0 B)] + E \cdot [\nabla (d \cdot E)] + (\nabla \cdot B) (d \cdot B)$$

$$- B \cdot [\nabla (d \cdot B)] + d \cdot [(\nabla \times B) \times B] \}$$

thus unveiling the dependence of the action on the orientation of the the fields $E$ and $B$ with respect to the Lorentz violation parameter $d' = (d^0, \ d)$.

Summing up the results of this section, the full effective action $S_{\text{eff}} [A]$ can be represented in the following way

$$S_{\text{eff}} [A] = \int d^4x \left[ - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g^2}{48\pi^2} \ln \left( \frac{M^2}{\mu^2} \right) F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_{F^4}$$

$$+ \frac{g}{12\pi^2} \ln \left( \frac{M^2}{\mu^2} \right) d_\alpha F_{\mu\nu}(\partial^\mu F^{\nu\alpha}) + \cdots \right] \ .$$

The second term in (26) is similar to the standard vacuum polarization correction to the Maxwell theory. The third is the lowest order non-linear correction to the Maxwell theory, which in the case of QED is known in literature by the name of Euler-Heisenberg term (for a review, see f.e. [39]).

More explicitly, from Eq. (22), we obtain

$$\mathcal{L}_{F^4} = - \frac{1}{8\pi} \cdot \frac{1}{48\pi M^4} \left[ \frac{2}{15} \text{Tr} \left( \tilde{F}^4 \right) - \frac{1}{3} \tilde{F}^4 - \frac{1}{2} \left( \star \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} \right)^2 \right] \bigg|_{d' = 0} \ ,$$

(27)
which, by using that
\[ \text{Tr} \left( \tilde{F}^4 \right) \bigg|_{d' = 0} = g^4 \left[ 2 \left( \mathbf{E}^2 - \mathbf{B}^2 \right)^2 + 4 \left( \mathbf{E} \cdot \mathbf{B} \right)^2 \right] \], \tag{28a} \]
\[ \tilde{F}^4 \bigg|_{d' = 0} = 4g^4 \left( \mathbf{E}^2 - \mathbf{B}^2 \right)^2, \left( \ast \tilde{F}^{\mu \nu} \tilde{F}_{\mu \nu} \right)^2 \bigg|_{d' = 0} = 16g^4 \left( \mathbf{E} \cdot \mathbf{B} \right)^2, \tag{28b} \]
can be cast as,
\[ \mathcal{L}_{F^4} = \frac{R}{8\pi} \left( \mathbf{E}^2 - \mathbf{B}^2 \right)^2 + \frac{S}{8\pi} \left( \mathbf{E} \cdot \mathbf{B} \right)^2, \tag{29} \]
where
\[ R = \frac{g^4}{45\pi M^4}, S = 7R. \tag{30} \]
This reproduces the well known Euler-Heisenberg Lagrangian, the first nonlinear corrections due to QED, if the charge \( g \) and the mass \( M \) are taken as the electron charge \( e \) and mass \( m \), respectively \[40\]. The fourth term in (26) represents the correction due to LV in the model. The ellipsis in Eq. (26) denotes higher orders corrections both in the power of fields, and of the LV parameters, which will not be considered in this work.

The standard Euler-Heisenberg corrections of QED are quite small effects, which are nevertheless expected to be probed in the near future, investigating modifications in the propagation of photons in a region with a strong magnetic field, as the vacuum magnetic birefringence \[36, 37\]. The nonlinear effects in our model are actually smaller, since they involve the mass \( M \) which is much larger than the electron mass. On general, LV effects are also expected to be very tiny, since no evidence for them has been observed so far. We assume that, in principle, nonlinear and LV effects in our model are comparable, so that none will be discarded, and we perform a more robust analysis of the possible effects of the LV background we are considering. Therefore, we will not study wave propagation considering only the terms quadratic in \( F \) appearing in Eq. (26), which would amount to study modifications of wave propagation originated only from the LV, but we will include also the effects of the non-linearities in the fields, thus including all the terms presented in Eq. (26) in our analysis.

**IV. ELECTROMAGNETIC WAVE PROPAGATION**

In this section we investigate the possible influence of the Lorentz-violation background described by Eq. (11) in the electromagnetic wave propagation in the vacuum. It is convenient to
rewrite Eq. (26) as follows,

\[ S_{\text{eff}} [A] = \int d^4x \left[ -\frac{1}{16\pi} (1 + C g^2) F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_{F^2} - \frac{C g}{4\pi} d_\alpha F_{\mu\nu} (\partial^\mu F^{\nu\alpha}) + \cdots \right], \tag{31} \]

where \( C = -\frac{1}{3\pi} \ln \left( \frac{M^2}{\mu^2} \right) \). In this equation, we have changed the normalization of the fields to coincide with the ones used in [40]. The covariant field equations derived from Eq. (31) read

\[ \partial_\mu \left[ (1 + C g^2) F^{\mu\nu} - R \left( F^{\alpha\beta} F_{\alpha\beta} \right) F^{\mu\nu} - \frac{S}{4} \left( * F^{\alpha\beta} F_{\alpha\beta} \right) F^{\mu\nu} + C g d_\alpha \left( \partial^\alpha F^{\mu\nu} + \partial^\mu F^{\nu\alpha} - \partial^\nu F^{\mu\alpha} \right) \right] = 0. \tag{32} \]

The modified Maxwell’s equations derived from this action are more conveniently written in terms of the vectors \( \mathbf{D} \) and \( \mathbf{H} \) defined as

\[ D_i = 4\pi \frac{\partial}{\partial E_i} \left[ -\frac{1}{16\pi} (1 + C g^2) F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_{F^4} \right], \tag{33a} \]
\[ H_i = -4\pi \frac{\partial}{\partial B_i} \left[ -\frac{1}{16\pi} (1 + C g^2) F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_{F^4} \right], \tag{33b} \]

or, more explicitly, as,

\[ \mathbf{D} = (1 + C g^2) \mathbf{E} + 2 R \left( \mathbf{E}^2 - \mathbf{B}^2 \right) \mathbf{E} + S (\mathbf{E} \cdot \mathbf{B}) \mathbf{B}, \tag{34a} \]
\[ \mathbf{H} = (1 + C g^2) \mathbf{B} + 2 R \left( \mathbf{E}^2 - \mathbf{B}^2 \right) \mathbf{B} + S (\mathbf{E} \cdot \mathbf{B}) \mathbf{E}. \tag{34b} \]

With these definitions, the equations of motion derived from Eq. (31) are given by

\[ \nabla \cdot \mathbf{D} = -C g d \cdot \left\{ -\nabla \times (\nabla \times \mathbf{E} + \partial_0 \mathbf{B}) \right\}, \tag{35} \]

and

\[ -\partial_0 \mathbf{D} + (\nabla \times \mathbf{H}) = -d^0 \nabla \times [\nabla \times \mathbf{E} + \partial_0 \mathbf{B}] + d \cdot \nabla \left[ \partial_0 \mathbf{E} - \nabla \times \mathbf{B} \right] - d \times \left[ \partial_0^2 \mathbf{B} - \nabla^2 \mathbf{B} \right] - \nabla \left( d \cdot \left[ \partial_0 \mathbf{E} - \nabla \times \mathbf{B} \right] \right), \tag{36} \]

together with the unmodified equations

\[ \nabla \cdot \mathbf{B} = 0, \ \nabla \times \mathbf{E} = -\partial_0 \mathbf{B}. \tag{37} \]

The nonlinearity in the electromagnetic fields is encoded in the left hand side of Eqs. (35) and (36). Due to the nonlinearity, one cannot simply state that the right hand side of these equations vanishes in the vacuum, as they would in the linear electrodynamics. It is true, however,
that the nonlinear corrections to the usual Maxwell equations are of first order in the small coefficients $R$ and $S$, and since the right hand side of Eqs. (35) and (36) is already of first order in the LV parameter $d^\mu$, this nonlinear corrections would amount to a second order effect, that can be disregarded. The outcome is that the LV parameter disappears from (35) and (36), which reduce to

\[ \nabla \cdot \mathbf{D} = 0, \quad (38a) \]
\[ -\partial_0 \mathbf{D} + (\nabla \times \mathbf{H}) = 0. \quad (38b) \]

Now these equations are identical in form to the ones used to find the usual Euler-Heisenberg modifications to wave propagation (see for example [41]), so they will predict the same kind of effects, except in our case we expected them to be smaller than in the usual QED, since they involve inverse powers of the large mass $M$.

As an example, one may consider weak electromagnetic fields $\mathbf{E}_P$ and $\mathbf{B}_P$ propagating in the presence of a constant, strong magnetic field $\mathbf{B}_0$. More specifically, one assumes

\[ \mathbf{E} = \mathbf{E}_P ; \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_P, \quad (39) \]

with the conditions

\[ |\mathbf{E}_P| \ll (R)^{-1/2}, (S)^{-1/2}, \quad (40) \]
\[ |\mathbf{B}_P| \ll \mathbf{B}_0 \ll (R)^{-1/2}, (S)^{-1/2}, \quad (41) \]

which allows one to linearize the equations, thus finding plane waves solutions. In this case, one finds different dispersion relations depending whether the electric field $\mathbf{E}_P$ is perpendicular or parallel to the constant magnetic field $\mathbf{B}_0$,

\[ \omega_\perp = k \left( 1 - \frac{7RB_0^2}{2} \right), \quad \omega_\parallel = k \left( 1 - 2RB_0^2 \right), \quad (42) \]

that is to say, one finds birefringence in wave propagation. These results are of the same form as in the usual QED, except they are much smaller since Eq. (30) involves the fermion mass $M$, much larger than the electron mass.

In conclusion, even if wave propagation in vacuum cannot yield any experimental constrains on the LV parameter $d^\mu$, the model we consider in this work, involving a very massive fermion field which intermediates the LV appearing in very high energy to the low energy effective action of the photon field, still generates a correction in the standard Euler-Heisenberg Lagrangian. Therefore, in principle a bound on the mass $M$ could be inferred from experimental studies, insofar these become able to measure the effects described in (42) with sufficient precision.
V. CONCLUSIONS

The extension of the minimal SME to include higher derivative Lorentz violating couplings is a new undertaking in a very active area in theoretical and experimental physics. Even if, by dimensional reasons, the nonminimal couplings are expected to be smaller than the minimal ones, they might be relevant in new physical contexts yet under experimental exploration, such as the interaction between photons and light pseudoscalars [10]. In this work, we paved the way for further explorations of the physical consequences of one of the nonminimal couplings discussed in [10], by calculating the corrections generated by it in the low energy effective action of the Maxwell field. We used the proper-time method, together with the zeta function regularization, to integrate out the heavy fermion responsible for the introduction of LV in our model. With this result at hand, one may study the consequences of this class of LV in photon phenomenology. This is an approach that has shown itself to be rather fertile in the context of the minimal SME, providing strong phenomenological bounds on several of the SME coefficients. In our case, we were led to consider the case of photons propagating in a region with a strong, constant magnetic field, investigating whether LV effects could compete with the effects of nonlinear effects induced by the quantum corrections, as it was done in [40] for a specific minimal LV coupling. The end result was that no effects of the Lorentz violating parameter $d_\mu$ can be detected in wave propagation in vacuum, and the only remnant of the high energy LV background we considered is actually the correction due to the presence of the very massive fermion in the Euler-Heisenberg Lagrangian. This allows in principle to find bounds on the mass of this fermion, assuming these effects can be measured with sufficient precision.

These results leave open the question of whether other physical effects that could be drawn from the modified gauge action we derived in this work could lead to observable effects, thus providing bounds on these nonminimal LV couplings. This is not a trivial question since one should not disregard the standard nonlinear effects that are also induced from the integration of the massive fermion, and we believe it is an interesting line to pursue in future works.

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[1] D. Colladay, V. A. Kostelecky, Phys. Rev. D 55, 6770 (1997), hep-ph/9703464.
[2] D. Colladay, V. A. Kostelecky, Phys. Rev. D 58, 116002 (1998), hep-ph/9809521.
[3] V. A. Kostelecky, Phys. Rev. D69, 105009 (2004), hep-th/0312310.
[4] V. A. Kostelecky, M. Mewes, Phys. Rev. D 80, (2009) 015020, arXiv:0905.0031.
[5] V. A. Kostelecky, M. Mewes, Phys. Rev. D 88, (2013) 096006, arXiv:1308.4973.
[6] V. A. Kostelecky, “Proceedings of the 6th Meeting on CPT and Lorentz Symmetry (CPT 13)”, World Scientific, 2014.
[7] V.A. Kostelecky, S. Samuel, Phys. Rev. D 39, (1989) 683.
[8] R. Jackiw, V. A. Kostelecky, Phys. Rev. Lett. 82, 3572 (1999), hep-ph/9901358.
[9] S. Carroll, G. B. Field, R. Jackiw, Phys. Rev. Lett. 41, 1231 (1990).
[10] L. H. C. Borges, A. G. Dias, A. F. Ferrari, J. R. Nascimento, A. Yu. Petrov, Phys. Rev. D 89, 045005 (2014), arXiv: 1304.5484.
[11] J. Schwinger, Phys. Rev. 82, 664 (1951).
[12] B. S. DeWitt, Dynamical Theory of Groups and Fields, N.Y. Gordon and Breach, 1965.
[13] I. N. McArthur, T. D. Gargett, Nucl. Phys. B497, 525 (1997), hep-th/9705200.
[14] T. Mariz, J. R. Nascimento, A. Yu. Petrov, L. Y. Santos, A. J. da Silva, Phys. Lett. B661, 312 (2008), arXiv: 0708.3348.
[15] V. A. Kostelecky, R. Lehnert, Phys. Rev. D63, 065008 (2001), hep-th/0012600.
[16] Z. Guralnik, R. Jackiw, S. Y. Pi, A. P. Polychronakos, Phys. Lett. B517, 450 (2001), hep-th/0106044
[17] R. Jackiw, Nucl. Phys. Proc. Suppl. 108, 30 (2002), hep-th/0110057
[18] R. Myers, M. Pospelov, Phys. Rev. Lett. 90, 211601 (2004), hep-ph/0301124
[19] T. Mariz, J. R. Nascimento, V. O. Rivelles, Phys. Rev. D75, 025020 (2007), hep-th/0609132.
[20] H. Belich, T. Costa-Soares, M. M. Ferreira Jr., J. A. Helayel-Neto, Eur. Phys. J. C41, 421 (2005), hep-th/0410104; Eur. Phys. J. C42, 127 (2005), hep-th/0411151.
[21] H. Belich, T. Costa-Soares, M. M. Ferreira Jr., J. A. Helayel-Neto, F. M. O. Moucherek, Phys. Rev. D74, 065009 (2006), hep-th/0604149.
[22] H. Belich, L. P. Colatto, T. Costa-Soares, J. A. Helayel-Neto, M. T. D. Orlando, Eur. Phys. J. C62, 425 (2009), arXiv: 0806.1253.
[23] R. Casana, M. M. Ferreira, C. Santos, Phys. Rev. D78, 105014 (2008), arXiv: 0810.2817.
[24] R. Casana, M. M. Ferreira, J. S. Rodrigues, M. R. O. Silva, Phys. Rev. D80, 085026 (2009), arXiv: 0907.1924.
[25] R. Casana, A. R. Gomes, M. M. Ferreira, P. R. D. Pinheiro, Phys. Rev. D80, 125040 (2009), arXiv: 0909.0544
