$\Xi^-$ emission probabilities at $(K^-,K^+)$ reaction points and the $\Xi N$ interaction

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$\Xi^-$ escaping and scattering probabilities in the $(K^-,K^+)$ reaction with nuclear targets is investigated on the basis of the eikonal approximation. Calculations are performed using the $S=-2$ sector of the Nijmegen model-D interaction, whose hard core radii are treated phenomenologically. The $\Xi^-N$ elastic cross sections are derived from the $^9\text{Be}(K^-,K^+)$ data of the BNL-E906 experiment, which is useful to determine the hard-core radii in $P$-states. The $\Xi$ single-particle potential energy is investigated with the G-matrix calculation in nuclear matter, which is sensitive to the hard-core radii in $S$-states. The important constraint for the $\Xi N$ interaction is obtained by combining the two analyses.

§1. Introduction

It is very important to study the properties of $\Xi N$ and $\Xi$-nucleus interactions in hypernuclear phenomena, because free-space $\Xi N$ scattering experiments are difficult to perform with current experimental apparatus. Interesting information was obtained from events of simultaneous emissions of two $\Lambda$ hypernuclei (twin $\Lambda$ hypernuclei) in the KEK-E176 experiment. The obtained values of binding energies $B_\Xi$ lead to an important conjecture for the $\Xi$-nucleus potential. The BNL-E885 collaboration measured the missing mass spectra for the $^{12}\text{C}(K^-,K^+)X$ reaction. Reasonable agreement between this data and theory is realized by assuming a $\Xi$-nucleus potential $U_\Xi(\rho) = -V_0 f(r)$ with well depth $V_0 \sim 14$ MeV within the Wood-Saxon (WS) prescription.

Recently, a new experiment (BNL-E906) has been performed in order to detect charged particles emitted from $(K^-,K^+)$ reaction points with a $^9\text{Be}$ target using a cylindrical detector system. Our concern in this work is with the emission probabilities of $\Xi^-$ particles produced through the quasi-free process in the target. In recent analysis, $\Xi^-$ emission events have been separated into two categories, the $\Xi^-$ escape process without any interaction after its production, and the scattering process with a target nucleon, for which the obtained probabilities are $63.6 \pm 8.2\%$ (escaping probability) and $14.5 \pm 2.6\%$ (scattering probability), respectively. From the scattering probability, the $\Xi^-N$ elastic scattering cross section in a nuclear medium was found to be $20.9 \pm 4.5 ($stat$) ^{\pm 2.5}_{\pm 2} ($syst$) \text{mb}$ by using the eikonal approximation.

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in which the number of the nucleons was eight and the reaction cross sections of $K^-$ and $K^+$ with the nucleons were 28.0 mb and 19.5 mb, respectively. Also, the elastic scattering probability was separated into the $\Xi^-$ part (5.8 ± 3.5%) and the $\Xi^-n$ part (8.7 ± 4.4%).

Nara et al. studied the $K^+$ momentum spectrum from $(K^-, K^+)$ reactions in the framework of inter-nuclear cascade-type calculations. Here, two-step processes such as $K^-p \rightarrow M'Y$ followed by $M'N \rightarrow K^+Y$, where $M'$ represents intermediate mesons, play a significant role in $K^+$ yields, especially for heavy targets. The contributions from these processes seem to be non-negligible, even in the high-momentum region of $K^+$, where quasi-free $\Xi^-$ productions are dominant. Because the $\Xi^-$ emission probabilities are defined as the ratio of the number of emitted $\Xi^-$ particles to produced $\Xi^-$ particles, the two-step contributions to $(K^-, K^+)$ events make them larger. The ratio of the two-step processes in the $(K^-, K^+)$ events, denoted as $\alpha_{\text{two}}$, is estimated to be about 10% at most. The above values of the escaping probability, 63.6%, and the scattering probability, 14.5%, for $\alpha_{\text{two}} = 0$% change to 67.1% (71.0%) and 15.1% (15.8%), respectively, for $\alpha_{\text{two}} = 5$% ($\alpha_{\text{two}} = 10$%).

In the report of the E906 experiment, the most important results of the theoretical calculations are cited, where a simple model is proposed for the $\Xi^-$ emission process. The outline of this paper is as follows. First, the plausibility of this model is investigated in light of the E906 data. Second, the observed $\Xi^-$ escaping and scattering probabilities are shown to give an important constraint for the $\Xi N$ interaction, which complements the $\Xi$ well depths derived from the data of E885 experiment.

§2. $\Xi^-$ emission probabilities and the $\Xi^-N$ elastic cross section

We now formulate our model. The transition rates $w_{\Xi}^{\text{scat}}$ due to elastic scattering between a quasi-free $\Xi$ (momentum $\vec{p}_{\Xi}$) and nucleons (momenta $\vec{p}_N$) in nuclear medium are defined by

$$w_{\Xi}^{\text{scat}} = \int d^3\vec{p}_N \int d^3\vec{p}_{\Xi N} \int d^3\vec{p}_{\Xi N}' \int d^3\vec{P}_{\Xi} \rho F_N(\vec{p}_N) v_{\Xi N} \frac{d\sigma}{d\Omega} \times \delta((p_{\Xi N}' - \vec{P}_{\Xi N})/\vec{P}_{\Xi N}) \delta^3(\vec{P}_{\Xi N} - \vec{P}_{\Xi N}') \theta(p_{\Xi}' - p_F)\theta(p_{\Xi}' - q_{\Xi}),$$

(2.1)

where $\rho$ is the nuclear density, $d\sigma/d\Omega$ is the $\Xi N$ differential scattering cross section for the transition from the initial relative momentum $p_{\Xi N}$ to the final one $p'_{\Xi N}$, and $v_{\Xi N}$ is the relative velocity. Also, $\vec{P}_{\Xi N}$ and $\vec{P}_{\Xi N}'$ denote the initial and final center-of-mass momenta, respectively. The function $F_N(p_N) = \left(\frac{3}{4}\pi p_F^3\right)^{-1} \theta(p_F - p_N)$ represents a Fermi distribution of nucleons, with $p_F$ being the Fermi-momentum. The factor $\theta(p_{\Xi}' - p_F)$ is the Pauli exclusion factor for the final nucleon state of momentum $p_{\Xi}'$, and $\theta(p_{\Xi}' - q_{\Xi})$ allows transitions only to $\Xi$ states whose momenta ($p_{\Xi}'$) are larger than the critical momentum $q_{\Xi}$ determined by $\frac{q_{\Xi}^2}{2M_{\Xi}} + U_{\Xi}(\rho) = 0$, $U_{\Xi}(\rho)$ being a $\Xi$ single-particle potential in medium. This constraint for $\Xi$ states results from the fact that lower-momentum components of scattered $\Xi$ particles are considered as being absorbed into target nuclei and not detected experimentally. In this work we
We define here the $\Xi^-$ absorption rate $w_{\Xi^-}^{\text{abs}}$ phenomenologically. This is determined from the probability of $\Xi^-$ vanishing due to processes such as $\Xi^- p \to \Xi^0 n$ and $\Xi^- p \to \Lambda \Lambda$ in target nuclei. The corresponding $\Xi^-$ absorption cross section is denoted as $\sigma_{\text{abs}}^{\Xi^-}$. The $\Xi^-$ mean free path (MFP) $\lambda_{\Xi^-}$ in medium is given as follows. The MFPPs for scattering and absorption cross sections are represented by $\lambda_{\Xi^-}^{\text{scat}} = v_{\Xi^-}/w_{\Xi^-}^{\text{scat}}$ and $\lambda_{\Xi^-}^{\text{abs}} = 1/(\rho \sigma_{\text{abs}}^{\Xi^-})$, where $v_{\Xi^-}$ is the velocity of $\Xi^-$. Then we have the relation $1/\lambda_{\Xi^-} = 1/\lambda_{\Xi^-}^{\text{scat}} + 1/\lambda_{\Xi^-}^{\text{abs}}$. Using Eq. (2.1), $\lambda_{\Xi^-}^{\text{scat}}$ is obtained as a function of $p_{\Xi^-}$ and $\rho$.

The probability of $\Xi^-$ reaction processes in a finite nucleus can be estimated on the basis of the eikonal approximation. Assuming forward scattering, $\theta_{K^+} = \theta_{\Xi^-} = 0^\circ$, the $(K^-, K^+)$ effective number for the reaction (scattering and absorption) of the produced quasi-free $\Xi^-$ and nucleons is given by

$$N_{\Xi^-}^{\text{reac}} = \int_0^\infty 2\pi b \, db \int_{-\infty}^\infty dz \, \rho(\sqrt{b^2 + z^2})$$
$$\times \exp \left\{ -\sigma_{K^-N} \int_z^{\infty} \rho(\sqrt{b^2 + z'^2}) dz' - \sigma_{K^+N} \int_z^\infty \rho(\sqrt{b^2 + z'^2}) dz' \right\}$$
$$\times \left\{ 1 - \exp \left( -\int_{-\infty}^z \frac{1}{\lambda_{\Xi^-}} dz' \right) \right\},$$

(2.2)

with $1/\lambda_{\Xi^-} = 1/\lambda_{\Xi^-}^{\text{scat}} + 1/\lambda_{\Xi^-}^{\text{abs}}$. Here, $\sigma_{K^-N}$ and $\sigma_{K^+N}$ are the isospin averaged total cross sections between $K^-$ and $K^+$ and a nucleon. We set $\sigma_{K^-N} = 28.0$ mb and $\sigma_{K^+N} = 19.5$ mb in this work. The total effective number $N_{\text{total}}^{\Xi^-}$ for the $(K^-, K^+)$ reaction is given by omitting the factor $\left\{ 1 - \exp \left( -\int_{-\infty}^z \frac{1}{\lambda_{\Xi^-}} dz' \right) \right\}$ in the above expression, and the reaction probability of $\Xi^-$ in medium is given by

$$P_{\Xi^-}^{\text{reac}} = N_{\Xi^-}^{\text{reac}} / N_{\text{total}}^{\Xi^-}.$$  

(2.3)

Similarly, we obtain the effective number $N_{\Xi^-}^{\text{abs}}$ for $\Xi^-$ absorption in medium by replacing $\lambda_{\Xi^-}$ in Eq. (2.2) with $\lambda_{\Xi^-}^{\text{abs}}$. Then, the scattering part of the effective number, which originates from the $\Xi^- N$ scattering in medium, is given as

$$N_{\Xi^-}^{\text{scat}} = \int_0^\infty 2\pi b \, db \int_{-\infty}^\infty dz \, \rho(\sqrt{b^2 + z^2})$$
$$\times \exp \left\{ -\sigma_{K^-N} \int_z^{\infty} \rho(\sqrt{b^2 + z'^2}) dz' - \sigma_{K^+N} \int_z^\infty \rho(\sqrt{b^2 + z'^2}) dz' \right\}$$
\[
\times \exp \left( -\int_z^\infty \frac{1}{\lambda_{\text{abs}}} \, dz' \right) \left\{ 1 - \exp \left( -\int_z^\infty \frac{1}{\lambda_{\text{scat}}} \, dz' \right) \right\}.
\] (2.4)

Then the \(\Xi^-\) scattering probability for the \(\Xi^-\) emission after \(\Xi^- N\) scattering in medium is given by
\[
P_{\text{scat}}^{\Xi^-} = \frac{N_{\Xi^- \text{scat}}}{N_{\text{total}}}.
\] (2.5)

The \(\Xi^-\) escaping probability for \(\Xi^-\) emission with no interaction after the production is given by
\[
P_{\text{esc}}^{\Xi^-} = 1 - \frac{N_{\Xi^- \text{reac}}}{N_{\text{total}}}.
\] (2.6)

The \(\Xi^-\) emission probability is given as the sum of the above two probabilities:
\[
P_{\text{emit}}^{\Xi^-} = 1 - P_{\text{abs}}^{\Xi^-} = P_{\text{esc}}^{\Xi^-} + P_{\text{scat}}^{\Xi^-},
\] (2.7)

with the \(\Xi^-\) absorption probability \(P_{\text{abs}}^{\Xi^-} = \frac{N_{\Xi^- \text{abs}}}{N_{\text{total}}}\). The probabilities \(P_{\text{esc}}^{\Xi^-}\), \(P_{\text{scat}}^{\Xi^-}\), \(P_{\text{emit}}^{\Xi^-}\) and \(P_{\text{abs}}^{\Xi^-}\) can be compared with the E906 data. Here, it is very important in our treatment that the experimental values of \(P_{\text{esc}}^{\Xi^-}\) and \(P_{\text{scat}}^{\Xi^-}\) be obtained separately in the analysis of the E906 data. As another expression for the \(\Xi^-\) scattering probability, we define here the in-medium scattering cross section \(\tilde{\sigma}_{\text{scat}}\) by the relation \(\lambda_{\text{scat}}^{\Xi^-} = \frac{1}{\rho \tilde{\sigma}_{\text{scat}}}\). Here, the value of \(\tilde{\sigma}_{\text{scat}}\) can be obtained phenomenologically without using Eq. (2.1), by treating it as a parameter in Eq. (2.4) to reproduce the data of \(P_{\text{scat}}^{\Xi^-}\).

The proton and neutron distributions in \(^9\text{Be}\) are represented as \(\rho(r) = \rho_0 (1 + \alpha (r/1.65)^2) \exp(-(r/1.65)^2)\) with \(\alpha = 1.46\) (proton) and 2.10 (neutron) on the basis of a theoretical calculation.\(^8\)

It is necessary to use appropriate \(\Xi N\) interactions for calculations, though at present we do not have sufficient knowledge of realistic models. In this work we use mainly the \(S = -2\) sectors of \(SU(3)\)-invariant OBE models, which have been used successfully to explore properties of \(S = -2\) hypernuclear systems such as double-\(\Lambda\) nuclei and \(\Xi\)-nucleus bound states. Among the various interaction models, the Nijmegen D (ND)\(^11\) seems to be suitable at present, because this model reproduce the attractive \(\Xi^-\)-nucleus potentials found in the KEK-E885 experiment\(^2\) with a reasonable choice of the hard-core radii \(r_c\), as shown below. ND should be considered here as a useful model for our purpose to analyze the data of both the \(\Xi^- N\) cross section and the \(\Xi\) well depth in medium complementarily. Another \(\Xi N\) interaction, which we use only for comparing with ND, is of a purely phenomenological Wigner-type represented by a one-range Gaussian (ORG) form \(v(r) = v_{\text{ORG}} \exp(-(r/\beta)^2)\) with \(\beta = 1.0\) fm, for which the isospin dependence is not taken into account.

Now our concern is in determining how the E906 data can be used to determine hard-core radii \(r_c\) in ND \((v_{\text{ORG}}\) in ORG). Hereafter hard-core radii in \(S\), \(P\) and \(D\) states are denoted \(r_c(S)\), \(r_c(P)\) and \(r_c(D)\). The E906 data gives the \(\Xi N\) scattering in medium at \(\Xi\) momentum around 550 MeV/c, where the \(P\)-state contributions are more important than the \(S\)-state contributions, and the results are sensitive to the value of \(r_c(P)\), not only to that of \(r_c(S)\). Contrastingly, the \(S\)-state contributions are
Table I. In the cases of $\alpha_{\text{two}} = 0.0, 0.05$ and $0.10$, the values of $\sigma_{\text{abs}}$ (mb) and $r_c(P)$ (fm) are chosen so as to reproduce the experimental values of $P_{\text{abs}}^{\Xi^-}$, $P_{\text{esc}}^{\Xi^-}$ and $P_{\text{scat}}^{\Xi^-}$. The notation $R(\Xi^- p/\Xi^- n)$ is used to represent $P_{\text{scat}}^{\Xi^-}(p)/P_{\text{scat}}^{\Xi^-}(n)$. The values of $P_{\text{esc}}^{\Xi^-}$ are the fitted ones and agree with the experimental values. See the main text for the definitions of $\sigma_{\text{abs}}$ (mb) and $\sigma_{\text{scat}}$ (mb).

| $\alpha_{\text{two}}$ | $\sigma_{\text{abs}}$ (mb) | $\sigma_{\text{scat}}$ (mb) | $r_c(P)$ (fm) | $P_{\text{esc}}^{\Xi^-}$ (fitted) | $P_{\text{scat}}^{\Xi^-}$ (fitted) | $R(\Xi^- p/\Xi^- n)$ |
|----------------------|-----------------------------|-----------------------------|--------------|----------------------------------|----------------------------------|---------------------|
| 0.00                 | 21.1                        | 21.6                        | 0.480        | 0.636                            | 0.145                            | 0.706               |
| 0.05                 | 16.4                        | 20.3                        | 0.491        | 0.671                            | 0.151                            | 0.692               |
| 0.10                 | 11.7                        | 19.1                        | 0.502        | 0.709                            | 0.158                            | 0.680               |

far more important than the $P$-state contributions for $\Xi$ well depths in nuclei. In the case of ND, therefore, the two data can be used, so that the hard-core radii of $S$- and $P$-states are determined separately. First, the value of $r_c(S) = 0.50$ fm is chosen so as to result in an appropriate attraction of the calculated $\Xi$ single-particle potentials, as shown in the following section. Next, the value of $r_c(P)$ is determined so as to reproduce the E906 data reasonably. Additionally, we assume $r_c(D) = r_c(S) = 0.50$ fm for simplicity. The isospin dependence of the hard-core radii of ND is not taken into account. It should be noted, however, that the isospin dependence in the OBE model is used to represent the experimental value, $\sim 0.10$.

We now give our results. There are essentially two free parameters in our formalism, $\sigma_{\text{abs}}$ and $r_c(P)$ in ND ($v_{\text{ORG}}$ in ORG). The former is chosen so as to reproduce the experimental value of $P_{\text{abs}}^{\Xi^-}$, namely the probability of vanishing of $\Xi$ in the $(K^-, K^+)$ reaction. We have $P_{\text{abs}}^{\Xi^-} = 0.219, 0.178, 0.133$ (for $\alpha_{\text{two}} = 0.0, 0.05, 0.10$, respectively). Next $r_c(P)$ in ND ($v_{\text{ORG}}$ in ORG) is determined so as to reproduce the experimental values of $P_{\text{esc}}^{\Xi^-}$ and $P_{\text{scat}}^{\Xi^-}$. Table I gives the calculated values with ND at $p_{\Xi^-} = 550$ MeV/c, around which the peak value of the $\Xi^-$ production strength is obtained. Here, the experimental values of $P_{\text{esc}}^{\Xi^-}$, $P_{\text{scat}}^{\Xi^-}$ and $P_{\text{scat}}^{\Xi^-}$ turn out to be reproduced precisely with the values of $\sigma_{\text{abs}}$ and $r_c(P)$ given in the table. The in-medium scattering cross sections $\sigma_{\text{scat}}$ are determined here so as to reproduce the data of $P_{\text{scat}}^{\Xi^-}$ phenomenologically by using the relation $\lambda_{\text{scat}}^{\Xi^-} = 1/(\rho_{\text{scat}} \sigma_{\text{scat}})$, instead of deriving $\lambda_{\text{scat}}^{\Xi^-}$ from the $\Xi N$ interactions.

In the E906 experiment, the elastic scatterings between the produced $\Xi^-$ particles and nucleons in the target were separated into the $\Xi^- p$ part ($5.8 \pm 3.5\%)$ and the $\Xi^- n$ part ($8.7 \pm 4.4\%)$. The ratio of the probabilities of $\Xi^- p$ and $\Xi^- n$ scatterings are regarded as corresponding to $R(\Xi^- p/\Xi^- n) = P_{\text{scat}}^{\Xi^-}(p)/P_{\text{scat}}^{\Xi^-}(n)$ in our model. The calculated values are given in Table I. These are found to be similar to the experimental value, $\sim 0.67$. It is more reasonable, of course, to also consider the isospin dependence of $r_c(S)$ simultaneously. There is no information, however, allowing us to determine them independently at present.

Similar results are obtained with ORG. For instance, the above values of $P_{\text{esc}}^{\Xi^-}$ and $P_{\text{scat}}^{\Xi^-}$ for $\alpha_{\text{two}} = 0.0$ can be reproduced with $v_{\text{ORG}} = -54.5$ MeV. Because no isospin dependence is taken into account in ORG, the obtained value of $R(\Xi^- p/\Xi^- n) = 0.603$ is significantly different from that with ND.

In Fig. 1, the calculated values of the $\Xi^-$ escaping probabilities $P_{\text{esc}}^{\Xi^-}$ and the
$\Xi^-$ scattering probabilities $P_{\Xi^-}^{\text{esc}}$ are plotted as functions of $p_{\Xi^-}$ in (a) and (b), respectively. Here, the solid and dashed curves correspond to ND and ORG. The experimental values of $P_{\Xi^-}^{\text{esc}} = 63.6 \pm 8.2\%$ and $P_{\Xi^-}^{\text{esc}} = 14.5 \pm 2.6\%$ at $p_{\Xi^-} = 550$ MeV/c are shown for comparison, where the dotted horizontal bars indicate the width of the $\Xi^-$ spectrum (FWHM). The values of $P_{\Xi^-}^{\text{esc}}$ and $P_{\Xi^-}^{\text{esc}}$ for ND and ORG turn out to be adjusted so as to reproduce the experimental values at $p_{\Xi^-} = 550$ MeV/c, though their $p_{\Xi^-}$ dependences differ significantly. Thus, it is found that our two $\Xi N$ interactions result in fits of equal quality by adjusting their interaction parameters.

In Fig. 1, the calculated values of $P_{\Xi^-}^{\text{esc}}$ and $P_{\Xi^-}^{\text{esc}}$ in the case $\alpha_{\text{two}}=0$ as functions of $p_{\Xi^-}$ in (a) and (b).

The experimental values of $P_{\Xi^-}^{\text{esc}} = 63.6 \pm 8.2\%$ and $P_{\Xi^-}^{\text{esc}} = 14.5 \pm 2.6\%$ at $p_{\Xi^-} = 550$ MeV/c are shown for comparison, where the dotted horizontal bars indicate the width of the $\Xi^-$ spectrum (FWHM). The values of $P_{\Xi^-}^{\text{esc}}$ and $P_{\Xi^-}^{\text{esc}}$ for ND and ORG turn out to be adjusted so as to reproduce the experimental values at $p_{\Xi^-} = 550$ MeV/c, though their $p_{\Xi^-}$ dependences differ significantly. Thus, it is found that our two $\Xi N$ interactions result in fits of equal quality by adjusting their interaction parameters.

As demonstrated in the next section, we need to complementarily analyze the other data, such as those for the $\Xi$ well depths, in order to make clear the features of the respective $\Xi N$ interactions.

In Fig. 2, the calculated values of the $\Xi^- N$ elastic cross sections are plotted as functions of the $\Xi^-$ momentum $p_{\Xi^-}$ (MeV/c) for ND with $r_c(P) = 0.480$ fm, 0.491 fm and 0.502 fm, by solid, dashed and dotted curves, respectively. The $\Xi^- N$ cross sections were obtained by averaging the $\Xi^-p$ and $\Xi^-n$ cross sections with the weight of 3 protons and 5 neutrons. The ambiguity concerning the two-step processes is not serious for the resulting $\Xi^- N$ cross sections. This is essentially because $\Xi^- N$ scattering events were observed explicitly in the E906 experiment, in spite of the unknown two-step contributions.

There is an experimental error of 2.6\% for $P_{\Xi^-}^{\text{esc}} = 14.5\%$ in the case that we assume $\alpha_{\text{two}}=0$; explicitly, the upper and lower values of $P_{\Xi^-}^{\text{esc}}$ are 17.1\% and 11.9\%. Here, we attempt to reproduce the deviation of 2.6\% by changing only the value of
$\Xi^-$ emission probabilities at $(K^-, K^+)$ reaction points

Fig. 2. Calculated values of $\Xi^-N$ elastic cross sections in mb are plotted as functions of the $\Xi^-$ momentum $p_\Xi$ (MeV/c) for ND with $r_c(P) = 0.480$ fm, 0.491 fm and 0.502 fm, by solid, dashed and dotted curves, respectively. The $\Xi^-N$ cross sections are obtained by averaging the $\Xi^-p$ and $\Xi^-n$ cross sections with the weight of 3 protons and 5 neutrons.

$r_c(P)$: The values of 17.1% and 11.9% are reproduced by choosing $r_c(P) = 0.525$ and 0.447, respectively. This choice is expressed as $r_c(P) = 0.480^{+0.045}_{-0.033}$ fm for convenience. In the above analysis, we assumed that the values of $\alpha_{\text{two}}$ is between 0 and 10%, giving rise to $r_c(P) = 0.480 - 0.502$ fm. Thus, it turns out that the ambiguity due to the two-step process is less than the experimental error bar.

§3. $\Xi N$ G-matrix calculation and $U_\Xi$

In order to demonstrate the properties of our interaction, let us perform the $\Xi N-\Lambda \Lambda$ channel coupled G-matrix calculations in symmetric nuclear matter with ND whose hard-core radii are taken as the above values. Calculations here are done in the framework used in a previous work [4]: We adopt the simple QTQ prescription for the intermediate-state spectrum, which means that no potential term is taken into account in the off-shell propagation.

In the previous analysis with ND, $r_c(S)$ and $r_c(D)$ are fixed at 0.5 fm, and $r_c(P)$ is adjusted so as to reproduce the E906 data. Because our determined values of $r_c(P)$ are around 0.5 fm, it is reasonable here to use the simple choice $r_c(S) = r_c(P) = r_c(D) = 0.5$ fm. In this case, we calculate the single-particle potential energies $U_\Xi$ for a zero-momentum $\Xi$ at normal density. The result is $U_\Xi = -14.3$ MeV, in which the $S$-state, $P$-state and $D$-state contributions are $U_\Xi(S) = 2.8$ MeV, $U_\Xi(P) = -15.6$ MeV and $U_\Xi(D) = -1.6$ MeV, respectively. It should be noted here that the negative value of $U_\Xi$ is due to the strongly attractive $P$-state contribution. As shown in the
previous section, $P_{\Xi^{-}}^{\text{scat}} = 14.5 \pm 2.6\%$ corresponds to $r_c(P) = 0.480^{+0.045}_{-0.033}$, which leads to $U_{\Xi} = -15.1_{-1.8}^{+1.5}$ MeV. Thus, the ambiguity of $U_{\Xi}$ due to the experimental error of the E906 data is found to be small.

We now demonstrate that our obtained values of $U_{\Xi}$ are consistent with the experimental results. In the E885 experiment, the missing mass spectra for $^{12}C(K^{-}, K^{+})X$ reaction was measured. Reasonable agreement between the data and theory is achieved by using a Wood-Saxon (WS) potential between $\Xi^{-}$ and the $^{11}$B core whose well depth is about 14 MeV. In order to compare our result with this WS potential, we construct the $\Xi$-nucleus potential from the calculated values of $U_{\Xi}(\rho)$ with a local density approximation (LDA) as

$$U_{\Xi}(r) = (t\sqrt{\pi})^{-3} \int d\rho U_{\Xi}(\rho(\rho')) \exp\left(-|\rho' - \rho|^2/\lambda^2\right)\rho(\rho') ,$$

(3.1)

where the finite-range effect of $\Xi N$ interactions is taken into account by the parameter $t$. The nuclear density distributions $\rho(\rho)$ are obtained from the Skyrme-Hartree-Fock calculations. We compare our obtained $\Xi^{-}\text{B}$ potential with the above WS potential by calculating the volume integral per nucleon, $J_{V}/A = -A^{-1} \int U(r)d^3r$, the root mean square radius, $\langle R_{V}^2 \rangle^{1/2} = \left[ \int U(r)d^3r/\int U(r)d^3r \right]^{1/2}$, and the binding energy of the ground $1S$ state, $B_{\Xi}(1S)$, obtained without taking the Coulomb interaction into account.

In the case of the above WS potential, the obtained values are $J_{V}/A = 133 \text{ MeV fm}^3$, $\langle R_{V}^2 \rangle^{1/2} = 3.1 \text{ fm}$ and $B_{\Xi}(1S) = 2.2 \text{ MeV}$. By setting $t = 1.0 \text{ fm}$ in the case of our G-matrix interaction derived from ND with $r_c(S) = r_c(P) = r_c(D) = 0.5 \text{ fm}$, we obtain $J_{V}/A = 126 \text{ MeV fm}^3$, $\langle R_{V}^2 \rangle^{1/2} = 2.9 \text{ fm}$, and $B_{\Xi}(1S) = 2.2 \text{ MeV}$. Thus, it is confirmed that the two $\Xi^{-}$-nucleus potentials are very similar. This results from our choice of $r_c(S) = 0.5 \text{ fm}$, because changes of $r_c(P)$ and $r_c(D)$ have far weaker effects on the $U_{\Xi}$ values.

The important information regarding the $\Xi^{-}$-nucleus potential can be obtained from the events of simultaneous emissions of two $\Lambda$ hypernuclei (twin $\Lambda$ hypernuclei) in the KEK-E176 experiment. These data give the energy difference between the initial $\Xi^{-}$ bound state and the final twin $\Lambda$ state, namely the binding energy $B_{\Xi^{-}}$ between $\Xi^{-}$ and the nucleus. For instance, the event studied in Ref. indicates the $2P$ $\Xi^{-}$ bound state in $^{12}$C with $B_{\Xi}(2P) = 0.57 \pm 0.19 \text{ MeV}$. With the above procedure we can derive the $\Xi^{-}\text{C}$ potential straightforwardly. Including the Coulomb interaction, we obtain the calculated value of 0.40 MeV for $B_{\Xi}(2P)$. This is slightly smaller than the experimental value, but it is still within the error bar.

Finally, let us point out the important contributions of the $\Xi N$ $P$-state interaction to the single-particle potential energy of $\Xi$ in dense nuclear matter, which is an interesting feature of ND. Considering the important roles of negatively-charged baryons in neutron stars, we calculate here the s.p. potential $U_{\Xi^{-}}$ in neutron matter. In Fig. 3, the $U_{\Xi^{-}}$ values for ND with $r_c(S) = r_c(P) = r_c(D) = 0.5 \text{ fm}$ are plotted as a function of $\rho/\rho_0$ by the solid curve, where $\rho_0 = 0.17 \text{ fm}^{-3}$ is the normal density. The $S$-, $P$- and $D$-state contributions are plotted by the dashed, dotted and dot-dashed curves, respectively. It should be noted here that the $S$-state contribution is strongly repulsive and the negative values of $U_{\Xi^{-}}$ are due to the strongly
§4. Conclusion

The BNL-E906 experiment was performed in order to detect charged particles emitted from \((K^-, K^+)\) reaction points on \(^9\)Be target. In recent analysis of the \(\Xi^-\) emission events, the \(\Xi^-\) escape process without any interaction after its production and the scattering process with a target nucleon have been extracted separately. These are specified by escaping and scattering probabilities, respectively. A simple model was proposed for these processes, which makes it possible to relate these probabilities with underlying \(\Xi N\) interactions.

Calculations were performed using the \(S = -2\) sector of ND, where the hard-core radii are treated as free parameters. The obtained \(\Xi^-\) escaping and scattering probabilities are consistent with the E906 data, where the choice of hard-core radii in the \(P\)-states is sensitive to the results.

Another important piece of information is obtained from the \(\Xi\) single-particle potential energy, which is derived from the \(\Xi N\) G-matrix calculations with ND in nuclear matter. The resulting value is very sensitive to the hard-core radii in the \(S\)-states. Our obtained \(\Xi\)-nucleus potential, constructed with LDA, is similar to the
WS potential obtained from the E885 experiment,

Thus, data from the E906 and E885 experiments can be used complementarily, so that the hard-core radii of ND in $S$- and $P$-states can be determined separately. The determined values of hard-core radii are not so different from those in $S = 0$ and $S = -1$ channels. This fact indicates that ND is a useful interaction model also in $S = -2$ channels, and we expect that it will be helpful to explore the effect of $\Xi$ mixing in neutron star matter. Of course, there still remain some problems regarding modeling of ND. Similar calculations are needed with use of more sophisticated interaction models. The present study given will be an important test for such $\Xi N$ interaction models.

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