The Nordtvedt effect in the Trojan asteroids

R.B. ORELLANA$^{1,*}$ and H. VUCETICH$^{2,*}$

1 Observatorio Astronómico de La Plata, Paseo del Bosque, (1900)La Plata, Argentina
2 Departamento de Física, Universidad Nacional de La Plata, C.C. 67, (1900)La Plata, Argentina

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Abstract. Bounds to the Nordtvedt parameter are obtained from the motion of the first twelve Trojan asteroids in the period 1906-1990. From the analysis performed, we derive a value for the inverse of the Saturn mass $3497.80 \pm 0.81$ and the Nordtvedt parameter $-0.56 \pm 0.48$, from a simultaneous solution for all asteroids.

Key words: relativity - gravitation - asteroids - astronomical constants - celestial mechanics

1. Introduction

The asteroids located in the vicinity of the equilateral triangle solutions of Lagrange ($L_4$ and $L_5$), known as the Trojan asteroids, are particularly sensitive to a possible violation of the Principle of Equivalence (Nordtvedt, 1968) because they act as a resonator selecting long period perturbations. Theories of gravitation alternatives to General Relativity predict a difference between inertial ($m_i$) and passive gravitational ($m_g$) masses of a planetary-sized body (the so-called Nordtvedt effect) equal to:

$$\frac{m_g}{m_i} = 1 + \Delta,$$  \hspace{1cm} (1)

For the sun, the correction term $\Delta$ is equal to:

$$\Delta_\odot = \frac{15GM_\odot}{2R_\odot c^2} \eta,$$  \hspace{1cm} (2)

*Member of C.O.N.I.C.E.T. (Argentina)
$M_{\odot}$ is the solar mass, $R_{\odot}$ is the solar radius and $\eta$, the Nordtvedt parameter, is a linear function of the PPN parameters (Will, 1981). It can be shown that the null result that General Relativity predicts comes from the cancellation of several large contributions to $\Delta$; and thus a null result for the Nordtvedt effect becomes a strong test of General Relativity.

The standard value of $\Delta_{\odot}$ is

$$\Delta_{\odot} = 1.59 \times 10^{-5} \eta.$$  \hspace{1cm} (3)

For $\eta \approx 1$, this term produces a shift of the Lagrange points, $L_4$ and $L_5$, toward Jupiter by an amount of approximately one arcsecond. This effect is very large and easily discerned by standard astrometric techniques if systematic errors can be controlled in the photographic observations.

In 1988, we realized a first determination of the Nordtvedt parameter using the first six Trojan asteroids (Orellana-Vucetich, 1988), and we obtained a value of $\eta = 0.0 \pm 0.5$, in agreement with the General Relativity prediction. In this paper, we incorporate six new asteroids and we increase the span of observations. The enlarged set permits a rigorous analysis of systematic errors, specially biases in the star catalogue reference system.

In the following sections, we shall discuss the new results.

2. Data set and Ephemeris

As discussed in the previous work, only those asteroids with a minimum observation time of 60 years were selected, which is required to determine the mass correction of Saturn. An error in the Saturn mass introduces a spurious shift in the Lagrange equilibrium point. At present, six new Trojan asteroids can be incorporated giving rise to twelve asteroids satisfying that condition.

The observations were obtained from the MPC (after 1950) and from several publications (before 1950) from 1906 to 1990. A total amount of 1383 observations were employed (see Table 1), which were reduced to astrometric position and the observation time to ephemeris time. This observational material will be separately published.

The equations of motion of each Trojan asteroid were numerically integrated, together with those of the outer planets, using a heliocentric coordinate system referred to the equinox and equator of 1950.0. A standard predictor-corrector of fifth order with a step of five days was used for the numerical integration. The positions of the outer planets were tested with the standard ephemeris (Eckert, 1951). Besides, the variational equations for each asteroid and Jupiter were simultaneously integrated.
Table 1. Observational set parameters. The columns show the number and name of the asteroid, time span covered by observations, accepted number and the number of the libration point.

| Asteroid   | Span       | No. Obs. | L |
|------------|------------|----------|---|
| (588)Achilles | 1906–1990 | 155      | 5 |
| (617)Patroclus | 1906–1989 | 114      | 4 |
| (624)Hector  | 1906–1990 | 237      | 5 |
| (659)Nestor  | 1908–1990 | 91       | 5 |
| (884)Priamus | 1917–1989 | 95       | 4 |
| (911)Agamenon| 1919–1984 | 124      | 5 |
| (1143)Odysses| 1930–1990 | 164      | 5 |
| (1172)Aneas  | 1930–1987 | 112      | 4 |
| (1173)Anchises| 1930–1989 | 76       | 4 |
| (1208)Troilus| 1931–1989 | 45       | 4 |
| (1404)Ajax   | 1936–1989 | 60       | 5 |
| (1437)Diomedes| 1937–1989| 110      | 5 |

3. The new results

The initial conditions for each asteroid were obtained from the Ephemeris for Minor Planets 1991, for the epoch 2448600.5JD, and were adjusted through a differential correction in order to obtain a reference orbit. Table 2 shows the initial conditions for the reference orbit of each asteroid and Table 1 shows the remaining observations once those with residuals larger than 3 were eliminated (Arley, 1950).

By using this reference orbit, several adjustments were carried out for each asteroid. The obtained results are illustrated in Table 3 and Table 4. Table 3 shows the separate and the joint solutions of the inverse of the Saturn mass and the Nordtvedt parameter with the orbital elements of each asteroid.

The values of the inverse of the Saturn mass are in rough agreement with those
Table 2. Reference orbit initial conditions. The columns show the asteroid number, coordinate and velocity of the asteroid, referred to the equator and equinox of 1950.0 at the epoch 2448600.5JD.

| Ast | $x$       | $y$       | $z$       | $\dot{x}$ | $\dot{y}$ | $\dot{z}$ |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| 588 | -3.9025983497742 | -3.2736728834505 | -2.5316108761226 | 0.0043537492285 | -0.0047928164771 | -0.0022300129791 |
| 617 | -1.0627928190858 | 3.9020964430745  | 3.6159794283693  | -0.0069836057153 | -0.0015890388164 | 0.001178570651 |
| 624 | -3.0501202747117 | 2.9280029114512  | 3.6158794283693  | -0.0039228257827 | 0.0011148785811 |
| 659 | -5.4934657772575 | -1.0458211881904 | -0.6352066069516 | -0.0026266504514 | 0.0002951259888 | -0.0012659129792 |
| 884 | 0.1672107459144  | 5.0820722624050  | 2.7350374790094  | 0.0066515083585 | 0.0006701997987 | -0.006529235805 |
| 911 | -3.5738399893703 | -2.6975381380999 | -3.2968414383167 | -0.0038870724565 | -0.0026647074728 | 0.0018697093635 |
| 1143| -4.0433167628393 | -3.4378827005772 | -1.4783781326443 | -0.0052935067994 | -0.0012659129792 | 0.0018697093635 |
| 1172| -0.6758109101219 | 5.4416321288771  | 1.4386284702578  | -0.0038870724565 | -0.0026647074728 | 0.0018697093635 |
| 1173| -1.5913219103482 | 5.1937286624775  | 2.2182587621419  | -0.0052935067994 | -0.0012659129792 | 0.0018697093635 |
| 1208| -1.2576214476810 | 3.1558146702659  | 4.2762472615874  | -0.0064452466474 | -0.0029968982314 | 0.0010308113922 |
| 1404| -4.3891545802031 | -2.5604362090287 | -2.9392856669528 | 0.004423909857 | -0.002204981599 | -0.0014436102720 |
| 1437| -2.8517339598160 | -3.0129925076262 | -3.313758997109 | 0.0058058535762 | -0.002204981599 | -0.0014436102720 |
Table 3. Mass correction and the Nordtvedt parameter: preliminary results. The columns show the asteroid number, the value of Saturn inverse mass, the Nordtvedt parameter (obtained from separate fits to each asteroid) and the values obtained fitting both parameters to each asteroid.

| Ast | $M_\odot/M_S$ (Separate fits) | $\eta$ (Separate fits) | $M_\odot/M_S$ (Joint fit) | $\eta$ (Joint fit) |
|-----|-------------------------------|------------------------|---------------------------|--------------------|
| 588 | 3499.83 ± 0.88                | −0.37 ± 0.18           | 3499.43 ± 0.89            | −0.45 ± 0.18       |
| 617 | 3489.83 ± 1.26                | 0.60 ± 0.15            | 3488.57 ± 1.57            | −0.22 ± 0.17       |
| 624 | 3502.26 ± 1.59                | −0.80 ± 0.14           | 3497.64 ± 1.71            | −0.97 ± 0.16       |
| 659 | 3499.72 ± 1.01                | 0.31 ± 0.16            | 3498.97 ± 1.24            | −0.20 ± 0.19       |
| 884 | 3499.38 ± 1.10                | 0.26 ± 0.24            | 3499.35 ± 1.33            | −0.01 ± 0.28       |
| 911 | 3510.63 ± 4.13                | −0.61 ± 0.33           | 3508.43 ± 5.15            | −0.29 ± 0.40       |
| 1143| 3495.86 ± 1.17                | −0.82 ± 0.18           | 3496.38 ± 2.69            | −0.09 ± 0.42       |
| 1172| 3496.84 ± 0.63                | 1.05 ± 0.16            | 3498.19 ± 0.77            | 0.57 ± 0.19        |
| 1173| 3494.77 ± 1.06                | 1.45 ± 0.34            | 3493.93 ± 1.66            | −0.32 ± 0.50       |
| 1208| 3487.86 ± 5.39                | −1.39 ± 1.01           | 3474.18 ± 10.3            | 2.97 ± 1.92        |
| 1404| 3497.29 ± 2.06                | −0.01 ± 0.26           | 3491.20 ± 3.15            | 1.07 ± 0.43        |
| 1437| 3489.73 ± 2.30                | −0.14 ± 0.29           | 3489.69 ± 2.32            | 0.03 ± 0.28        |

obtained by other researches, although their dispersion is greater than the formal errors. On the other hand, the values of the Nordtvedt parameter $\eta$ show a bias toward negative values and is strongly correlated with the L4, L5 positions.

This fact suggests the existence of systematic errors in the reference system of the star catalogues. In order to analyze this effect, a simultaneous adjustment of all 12 asteroids was made. A total of 77 parameters were adjusted, including the correction to the Saturn mass, the Nordtvedt parameter and three parameters that describe biases of the star
Table 4. Final values of the physical parameters, as obtained from a simultaneous fit of all asteroids. The columns show the suppressed asteroid number, and the adjusted values of the inverse mass of Saturn, the Nordtvedt parameter and the three astrometric bias parameters. The last line contains the “jackknifed” values of the parameters.

| Ast  | $M_\odot/M_S$ | $\eta$  | $\Delta\psi$ | $\Delta\phi$ | $\Delta b$ |
|------|--------------|---------|--------------|--------------|-----------|
| —    | 3497.78 ± 0.40 | -0.17 ± 0.07 | 0.84 ± 0.12 | -0.19 ± 0.14 | -0.08 ± 0.05 |
| 588  | 3497.20 ± 0.46 | -0.25 ± 0.07 | 0.94 ± 0.12 | -0.39 ± 0.15 | -0.04 ± 0.05 |
| 617  | 3498.42 ± 0.42 | -0.18 ± 0.09 | 0.81 ± 0.14 | -0.21 ± 0.16 | -0.06 ± 0.05 |
| 624  | 3497.76 ± 0.41 | -0.11 ± 0.07 | 0.67 ± 0.13 | -0.19 ± 0.16 | -0.13 ± 0.05 |
| 659  | 3497.72 ± 0.44 | -0.21 ± 0.07 | 0.88 ± 0.12 | -0.22 ± 0.15 | -0.05 ± 0.05 |
| 884  | 3497.55 ± 0.43 | -0.16 ± 0.07 | 0.85 ± 0.12 | -0.18 ± 0.15 | -0.04 ± 0.05 |
| Q911 | 3497.66 ± 0.39 | -0.15 ± 0.07 | 0.77 ± 0.11 | -0.10 ± 0.15 | -0.12 ± 0.05 |
| 1143 | 3497.70 ± 0.45 | -0.19 ± 0.07 | 0.85 ± 0.12 | -0.22 ± 0.15 | -0.06 ± 0.05 |
| 1172 | 3497.79 ± 0.44 | -0.19 ± 0.07 | 0.82 ± 0.12 | -0.19 ± 0.15 | -0.10 ± 0.05 |
| 1173 | 3497.84 ± 0.42 | -0.13 ± 0.07 | 1.03 ± 0.13 | -0.29 ± 0.15 | -0.10 ± 0.05 |
| 1208 | 3497.87 ± 0.41 | -0.15 ± 0.07 | 0.81 ± 0.12 | -0.11 ± 0.15 | -0.06 ± 0.05 |
| 1404 | 3497.71 ± 0.42 | -0.19 ± 0.07 | 0.80 ± 0.12 | -0.12 ± 0.15 | -0.08 ± 0.05 |
| 1437 | 3497.85 ± 0.42 | -0.18 ± 0.07 | 0.85 ± 0.12 | -0.16 ± 0.15 | -0.09 ± 0.05 |
| Jack | 3497.80 ± 0.81 | -0.14 ± 0.12 | 0.75 ± 0.31 | -0.29 ± 0.32 | -0.16 ± 0.10 |

catalogue reference system.

The correction for the Saturn mass is in good agreement with the value recommended by the IAU with the corrections suggested by the JPL results. The corrections to the star catalogue reference system are in agreement with other determinations. The value of the Nordtvedt parameter $\eta$ is not zero at the 95% confidence level.
4. Conclusions

On the data shown in Tables 3 and 4 are based the main results of this paper. Let us discuss them in some detail. The dispersion of the inverse of the Saturn mass and the Nordtvedt parameter in Table 3 suggest the existence of unmodelled systematic errors and, in particular, the strong correlation of $\eta$ with the preceding or receding position of the asteroid with respect to Jupiter, shows the influence of the equinox shift in the star catalogue reference system on our results. This latter quantity, indeed, must introduce spurious shifts $\delta \eta$ in the Lagrange points $L_4$ and $L_5$ but with opposite signs. The same should be true for an error in the Saturn mass.

We can confirm the above comments with a simple reduction of the $\eta$ and $M_S$ values in Table 3. We use the following equation to model the mass and equinox errors:

$$M_S = M_{S0} + \tau M_{S1},$$

$$\eta = \eta_0 + (M_S - 3500)\eta_1 + \tau \eta_2,$$

where $\tau = \pm 1$ according to the preceding or receding position of the asteroid with respect to Jupiter. We obtain

$M_{S0} = 3497.67 \pm 0.63$

$\eta_0 = -0.08 \pm 0.20$

and these values are in very good agreement with other determinations. The very simple model (Eqs. (4) and (5)) takes into account very well the main systematic errors in our determinations.

On the other hand, the results in the first line of Table 4 come from a similar but more complex (and more realistic) model for systematic errors. The non zero value of $\eta$ is at variance with the LLR results, and one should attempt a more rigorous estimate of the errors.

A robust procedure for error estimation is the so called jackknife process (Kinsella, 1986; Miller, 1974). It is a rigorous generalization of the well known process of discarding part of a data set to test the sensibility of a result computed from it. Let $\eta_0$ be the value of $\eta$ computed from the full data set of size $N$, and let $\eta_i$ be the values obtained deleting the set of $n_i$ observations corresponding to asteroid $i$ from the data set. The Jackknife process consists in forming the pseudovalues:

$$\eta_i^* = \frac{N \eta_0 - (N - n_i)\eta_i}{n_i}$$

(8)
which are treated as independent identically distributed random variables and the value and error of $\eta$ are computed from them. As a general rule, jackknife error estimates are larger than least squares estimates, since the process of forming pseudovalues enhances nongaussian contributions to the dispersion, such as introduced by nonlinearities and local distortions in star catalogues.

The jackknife procedure, applied to the value of Table 4, yields the results quoted in the last line of the table. All the jackknife errors estimates are about twice as big as the least squares error estimates, probably due to the existence of unmodelled systematic errors in the data set. The jackknifed estimate of $\eta$ is now consistent with zero. As we have mentioned in (Orellana-Vucetich, 1988) the gravitational energy of the Sun is overestimated in Eq. (1) by a factor 4. The final result for the Nordtvedt parameter in this paper is:

$$\eta = -0.56 \pm 0.48,$$

consistent with zero and not very different from our former result (Orellana-Vucetich, 1988). Our Eq. (9), however, includes a rigorous jackknife estimate of external errors.

Our results confirm the possibility of obtaining good estimates of the Nordtvedt parameter from the motion of the Trojan asteroids. Both the simple model of Eqs. (4) and (4) and our more elaborate jackknife analysis show that most of the errors come from the catalog reference system biases. A new reduction of the available plates with respect to the forthcoming Hipparcos catalog should yield an estimate of $\eta$ ten times as accurate as Eq. (3), of the order of magnitude of LLR determinations. Since the Nordtvedt effect is a strong test of General Relativity (Will, 1981), we think it is worth the efforts to test for its existence.

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