A new low-frequency backward mode in inhomogeneous plasmas

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When an electromagnetic transverse wave propagates through an inhomogeneous plasma so that its electric field has a component in the direction of the background density gradient, there appears a disbalance of charge in every plasma layer, caused by the density gradient. Due to this some additional longitudinal electric field component appears in the direction of the wave vector. This longitudinal field may couple with the usual electrostatic longitudinal perturbations like the ion acoustic, electron Langmuir, and ion plasma waves. As a result, these standard electrostatic waves are modified and in addition to this a completely new low-frequency mode appears. Some basic features of the coupling and modification of the ion acoustic wave, and properties of the new mode are discussed here, in ordinary electron-ion and in pair plasmas.

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I. INTRODUCTION

In ordinary electron-ion plasmas, both transverse electromagnetic (TEM) and longitudinal electrostatic (LES) Langmuir wave have the same cut-off at the electron plasma frequency \( \omega_{pe} \). For very small wave-numbers the frequencies of both modes are close to each other. The two modes are physically very different and within linear theory they are typically not coupled. On the other hand, ion modes like ion acoustic (IA), and ion plasma (IP) modes are well separated from the two mentioned modes and it is believed that there is no linear coupling either. However, the situation may be quite different in the presence of a density gradient in the direction perpendicular to the direction of propagation of the TEM wave. In such a case the LES modes (IA, IP, and Langmuir) become coupled with the TEM mode even within linear theory. The coupling is more profound for the IA mode, implying that these electrostatic modes may have some electromagnetic features. The coupling with the IA mode has been studied in detail in Ref. 4 for both cold and hot ions, collisional and collision-less, isothermal and adiabatic. It was shown that in a part of spectrum, for small wave-numbers \( k \), the IA mode may become backward in the sense that \( \partial \omega / \partial k < 0 \), it gets some cut-off caused by the density gradient, and in this same domain it is coupled with TEM wave.

However, in the previous work it was not realized that the general dispersion equation, which describes coupling between TEM and LES waves in the presence of a density gradient, allows for some additional peculiar low frequency hybrid (LFH) mode in the range below the IA wave frequency \( k c_s \) and below the ion thermal mode \( k v_i \), where \( c_s, v_i \) are the sound and ion thermal speeds, respectively. In the large part of the spectrum this LFH mode is backward, \( \partial \omega / \partial k < 0 \), and it appears only in the presence of a TEM wave propagating through an inhomogeneous environment. The mode is the result of linear coupling between TEM and LES modes. At frequencies close to \( k c_s \) there is an exchange of identities of this new mode and the IA mode; the latter becomes backward above \( k c_s \) for small wave-number, while the LFH mode follows the \( k c_s \) line (but remaining below it) for \( k \to 0 \). Some basic features of this new low-frequency mode, and its coupling with the IA mode are presented in this work.

II. PLASMA WITHOUT MAGNETIC FIELD

We start with a static plasma containing two general species \( a \) and \( b \), which thus may include some ion-electron or pair (pair-ion, electron-positron) plasma, and we assume small isothermal electromagnetic perturbations that propagate in \( z \)-direction. Note that much more general cases were studied in our recent work for collisional plasma with hot ions, with the Landau damping effect, and together with the energy equation. In the present case we take a simple model in order to see some basic features of the new low-frequency mode, presented here for the first time. Linear perturbations imply the momentum equation for the general species \( j \):

\[
m_j n_{j0} \frac{\partial \vec{v}_j}{\partial t} = q_j n_{j0} \vec{E}_1 - \kappa T_{j0} \nabla n_{j1} - \kappa n_{j1} \nabla T_{j0}.
\]

Here, indices 0, 1 describe the equilibrium and perturbed quantities, respectively, for the two species \( j = a, b \), where \( q_a = e, q_b = -e, n_{j0} = n_0 \), and \( T_j = T_{j0} \). We shall assume small equilibrium gradients of the temperature and density to be in \( x \)-direction, and with the characteristic inhomogeneity length far exceeding the wavelength, so that we apply the usual local approximation analysis. We allow for the presence of both longitudinal (electrostatic) and transverse (electromagnetic) perturbations propagating in the \( z \)-direction, \( \sim -i\omega t + ikz \). The speed due to both of these perturbations

\[
\vec{v}_{j1} = i q_j \vec{E}_1 |-i v_{ji} n_{j1}\nabla T_j - i v^2_{ji} n_{j1} \nabla T_j / T_j |,
\]

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is used in the continuity equation which becomes

\[ -i\omega \frac{n_{j1}}{n_0} + \nabla \cdot \vec{\nu}_1 + \frac{q_j}{m_j\omega_j^2} \nabla E_1 \cdot \nabla n_0 = 0. \]  

(3)

In what follows we use the fact that \( \vec{k} \perp \nabla n_0, \vec{k} \perp \nabla T_j \) and without any approximation from Eq. (3) we have

\[ n_{j1} = \frac{q_jn_0}{m_j\omega_j^2} \nabla \cdot \vec{E}_1 + \frac{q_j}{m_j\omega_j^2} \vec{E}_1 \cdot \nabla n_0, \]  

(4)

\[ \omega_j^2 = \omega^2 - k^2v_{Tj}^2 + \frac{v_{Tj}^2}{T_j}. \]

Here, \( n_0', T_j' \) are the first and second derivatives of \( n_0, T_j \) in the \( x \)-direction. The electric field in principle has both longitudinal and transverse components; the one in the term with the density gradient is due to transverse plane-polarized electromagnetic perturbations. It produces the third derivative \( \nabla^3 \vec{E}_1 \) in Eq. (2) which becomes

\[ -\frac{iq_jk^2v_{Tj}^2}{m_j\omega_j^2} \nabla T_j \left( E_{1x} \frac{n_0'}{n_0} + \nabla \cdot \vec{E}_1 \right). \]  

(6)

We may omit the third order small terms, and the velocity becomes

\[ \vec{v}_{j1} = \frac{iq_j}{m_j\omega_j} \vec{E}_1 - \frac{iq_jv_{Tj}^2}{m_j\omega_j^2} \nabla (\nabla \cdot \vec{E}_1) \]

\[ -\frac{iq_jv_{Tj}^2}{m_j\omega_j^2} \frac{n_0'}{n_0} \nabla E_{1x} - \frac{iq_jk^2v_{Tj}^4}{m_j\omega_j^2} \nabla T_j \nabla \cdot \vec{E}_1. \]  

(7)

Here and further in the text the second derivative of temperature in \( \omega_j \) is neglected, in accordance with the used local approximation. Eq. (7) is used in the general wave equation

\[ c^2k^2 \vec{E}_1 - c^2\vec{k} \cdot \vec{E}_1 - \omega^2 \vec{E}_1 - \frac{i\omega \hbar}{\varepsilon_0} = 0, \]  

(8)

This yields the following wave equation expressed through the perturbed electric field only:

\[ c^2k^2 \vec{E}_1 - c^2\vec{k} \cdot \vec{E}_1 - \omega^2 \vec{E}_1 + (\omega_{pa}^2 + \omega_{pb}^2) \vec{E}_1 \]

\[ -\frac{n_0'}{n_0} \left( \frac{\omega_{pa}^2 v_{Tj}^2}{\omega_a^2} + \frac{\omega_{pb}^2 v_{Tj}^2}{\omega_b^2} \right) \nabla E_{1x} \]

\[ -k^2 \nabla \cdot \vec{E}_1 \left( \frac{\omega_{pa}^2 v_{Tj}^2}{\omega_a^2} \nabla T_a + \frac{\omega_{pb}^2 v_{Tj}^2}{\omega_b^2} \nabla T_a \right) \]

\[ -\left( \frac{\omega_{pa}^2 v_{Tj}^2}{\omega_a^2} + \frac{\omega_{pb}^2 v_{Tj}^2}{\omega_b^2} \right) \nabla \left( \nabla \cdot \vec{E}_1 \right) \]

\[ -\nabla \cdot \vec{E}_1 \left[ \frac{\omega_{pa}^2 v_{Tj}^2}{\omega_a^2} \left( \frac{\nabla n_0}{n_0} + \frac{\nabla T_a}{T_a} \right) \right. \]

\[ \left. + \frac{\omega_{pb}^2 v_{Tj}^2}{\omega_b^2} \left( \frac{\nabla n_0}{n_0} + \frac{\nabla T_b}{T_b} \right) \right] = 0. \]  

(9)

Here, \( \omega_{pj}^2 = c^2n_0/\varepsilon_0m_j \) and the electric field includes both transverse and longitudinal components.
A. Mechanism of transverse-longitudinal electric field coupling

The mechanism of the coupling can be understood from Fig. 1 where we have a density gradient in $x$-direction and an EM wave propagating along the $z$-axis, $\vec{k} = k\vec{e}_z$. Due to the electric field $\vec{E}_{\text{trans}} = E_{x1}\vec{e}_x$, plasma particles with opposite charges move in opposite directions. The physics is essentially the same for both electron-ion and pair-ion plasma, although in the former case the displacement of particles is more effective for electrons of course. In case of particles of the same mass, displacements of positive particles, that were initially in an arbitrary layer (which we denote as $x = 0$) with density $n_0(0)$, is represented by the sinusoidal line. Two arbitrary points $A(0, z_1)$ and $B(0, z_2)$ at two different positions in $z$-direction are displaced to $A'(x_1, z_1)$ and $B'(x_2, z_2)$. As a result, due to background density gradient, the amount of positively charged particles at $A(0, z_1)$ and $B(0, z_2)$ will no longer be the same; in the point $A(0, z_1)$ it is reduced (they are replaced by the particles of the same species which come from some other point in $x$ direction with lower density), and in the point $B(0, z_2)$ it is increased. In the same time, because of opposite motion of negatively charged particles, the amount of negative charges at $A(0, z_1)$ will be increased (they are displaced and moved from the area with a higher density) and at $B(0, z_2)$ decreased. Something similar happens at every point in the $z$ direction, and for every layer along the $x$ axis. This means that there will be a difference of charge in the chosen arbitrary points, and this further implies that there will be an additional electric field $E_{1z}$ in the $z$ direction, as indicated in Fig. 1.

![Diagram of longitudinal electric field](image)

FIG. 1. Origin of a longitudinal electric field in case of a transverse electromagnetic wave propagating through an inhomogeneous plasma.

B. The case of equilibrium with opposite density and temperature gradients

So far nothing is assumed about a possible relation between the temperature and density gradients in the equilibrium. One possible and physically plausible scenario may include a plasma with a balance of the two gradients in the equilibrium:

$$\frac{\nabla T_j}{T_j} = -\frac{\nabla n_0}{n_0}.$$  \hspace{1cm} (10)

Note that this also allows for different temperatures of the two species $T_b = \alpha T_a$. With this, the last two terms in Eq. 9 vanish and we consequently have the following wave equation:

$$c^2k^2\vec{E}_1 - c^2k^2(\vec{k} \cdot \vec{E}_1) - \omega^2\vec{E}_1 + (\omega^2pa + \omega^2pb)\vec{E}_1$$

$$- \frac{\nabla E_{1x}}{L_n} \left( \frac{\omega^2pa^2\nu_{T_a}^2}{\omega_a^2} + \frac{\omega^2pb^2\nu_{T_b}^2}{\omega_b^2} \right)$$

$$+ \frac{k^2\epsilon_0}{L_n} \left( \frac{\omega^2pa^2\nu_{T_a}^2}{\omega_a^2} + \frac{\omega^2pb^2\nu_{T_b}^2}{\omega_b^2} \right) \nabla \cdot \vec{E}_1$$

$$- \left( \frac{\omega^2pa^2\nu_{T_a}^2}{\omega_a^2} + \frac{\omega^2pb^2\nu_{T_b}^2}{\omega_b^2} \right) \nabla \cdot (\nabla \cdot \vec{E}_1) = 0.$$ \hspace{1cm} (11)

Here $L_n = \omega^2pa^2/\nu_{T_a}$ is the characteristic scale-length for the equilibrium density gradient, $n_0 = \omega^2pa^2/\nu_{T_a}$, and $L_n = -L_T$.

The $y$-component of Eq. (11) yields one TEM wave $\omega^2 = \omega^2pa + \omega^2pb + k^2c^2$ which is decoupled from the rest. The $z$ and $x$ components are coupled and they yield

$$\left[ -\omega^2 + \omega^2pa + \omega^2pb + k^2 \left( \frac{\omega^2pa\nu_{T_a}^2}{\omega_a^2} + \frac{\omega^2pb\nu_{T_b}^2}{\omega_b^2} \right) \right] E_{1z}$$

$$- \frac{ik}{L_n} \left( \frac{\omega^2pa\nu_{T_a}^2}{\omega_a^2} + \frac{\omega^2pb\nu_{T_b}^2}{\omega_b^2} \right) E_{1x} = 0,$$ \hspace{1cm} (12)

$$\left[ -\omega^2 + \omega^2pa + \omega^2pb + k^2c^2 \right] E_{1x}$$

$$+ \frac{ik^3}{L_n} \left( \frac{\omega^2pa\nu_{T_a}^2}{\omega_a^2} + \frac{\omega^2pb\nu_{T_b}^2}{\omega_b^2} \right) E_{1z} = 0.$$ \hspace{1cm} (13)

Eqs. (12, 13) describe coupled longitudinal $E_{1z}$ and transverse (electromagnetic) $E_{1x}$ perturbations.

Obviously, the coupling described by Eqs. (12, 13) vanishes in the absence of inhomogeneity $L_n \to \infty$. In this limit Eq. (13) yields yet another EM transverse wave with
the electric field in the $x$-direction, while in e-i plasmas Eq. (12) describes the usual IA, IP, and Langmuir modes.

As Fig. 1 suggests, the high-frequency longitudinal electric field (produced by the transverse EM wave) will cause simultaneous high-frequency density oscillations in the $z$-direction. In the presence of an additional independent longitudinal (ion-acoustic, or electron Langmuir, or ion-plasma) mode propagating in the $z$-direction, obviously there may be coupling of these two longitudinal displacements of completely different origin, and this is what Eqs. (12, 13) describe, but this is not all. In fact, as it will be shown below, there appears an extraneous oscillatory longitudinal mode, which is partly backward and very low-frequency.

From all this it is self-evident that $\vec{E}_{\text{trans}}$ does not have to be polarized strictly in the $(x, z)$-plain. To have the described high-frequency longitudinal motion caused by the transverse wave, it is enough that the electric field vector of the transverse wave has a component in the $x$-direction regardless how small, as we stressed earlier.

So in the presence of inhomogeneity, the dispersion equation obtained from Eqs. (12, 13) reads:

$$\omega^2 \left( 1 - \frac{\omega_{pa}^2}{\omega_a^2} - \frac{\omega_{pb}^2}{\omega_b^2} \right) \left( \epsilon^2 k^2 + \omega_{pa}^2 + \omega_{pb}^2 - \omega^2 \right) + \frac{k^4}{L_n^2} \left( \frac{\omega_{pa}^2 v_{ta}^2}{\omega_a^2} + \frac{\omega_{pb}^2 v_{tb}^2}{\omega_b^2} \right) \left( \frac{\omega_{pa}^2 v_{ta}^2}{\omega_a^2} + \frac{\omega_{pb}^2 v_{tb}^2}{\omega_b^2} \right) = 0. \quad (14)$$

Here, $\omega_{a,b}^2 \approx \omega^2 - k^2 v_{ta,b}^2$. In the absence of inhomogeneity this equation describes the usual TEM, Langmuir, and IA modes. With equilibrium gradients it yields some extra branches, like the ion thermal and more importantly a completely new low frequency mode described below for electron-ion and pair-ion plasmas.

1. Electron-ion plasma

   a. Ion acoustic range. In electron-ion plasmas we may discuss Eq. (14) in the IA frequency range, and for simplicity we may assume $T_a \ll T_b$, where $a$ denotes ions $a = i$, and therefore $b = e$. Assuming only that $\omega^2 \ll \omega_{pi}^2, \omega_{pe}^2, k^2 v_{Te}^2$ (all well-justified for the IA frequency range), this yields the modified IA mode$^{[40]}

$$\omega^2 = k^2 c_s^2 \left( 1 + \frac{m_i}{m_e} \frac{1}{k^2 L_n^2 + 1 + k^2 \lambda_e^2} \right) \frac{1}{1 + k^2 \lambda_{de}^2}, \quad (15)$$

$$\lambda_e = \frac{c}{\omega_{pe}}, \quad \lambda_{de} = \frac{v_{Te}}{\omega_{pe}}, \quad c_s^2 = \frac{k T_e}{m_i}.$$

The IA mode is partly backward, $\partial \omega / \partial k < 0$, due to the second term within brackets in Eq. (15), and this is in the range of small $k$, i.e., for a strong enough inhomogeneity:

$$L_n^2 < \frac{m_i}{m_e} \frac{\lambda_e^2}{(1 + k^2 \lambda_e^2)^2}.$$
direct mode which goes to zero following the line $kc_s$ for $k \to 0$.

These all features can be seen in Fig. 2 which shows the actual ion acoustic wave (line $b$) in an arbitrary inhomogeneous ($L_n = 0.5$ m) argon-electron plasma with $n_e = 10^{16}$ m$^{-3}$, $T_e = 10^4$ K, $T_i = 2 \cdot 10^3$ K. This line is nicely described by the approximate analytical expression (13). The usual IA mode $kc_s$ in a homogenous plasma is presented by line $a$. Clearly, in the small $k$ range it is very different from the actual IA wave in inhomogeneous plasma, line $b$. The new, gradient-driven low frequency hybrid mode (LFH) is presented by line $c$; its maximum frequency in the graph is about 44 kHz and it is achieved at $\lambda = 0.06$ m, i.e., $k = 90$ (note that the corresponding ion acoustic wave minimum in the same $k$-range is about 208 kHz). For large $k$ (i.e., in the range where it is backward), the approximate analytical expression (10) describes the LFH mode rather accurately.

2. Pair-ion plasma

In pair-ion plasma[9,12] without density gradient Eq. (14) yields the following dispersion equation:

$$1 = \frac{\omega_p^2}{\omega^2 - k^2 v_T^2} + \frac{\omega_p^2}{\omega^2 - k^2 v_T^2}, \quad \omega_p^2 = \frac{e^2 n_0}{\varepsilon_0 m}. \quad (17)$$

Eq. (17) is discussed in our earlier work[13] It gives a longitudinal electrostatic Langmuir mode, and the ion sound mode in the pair plasma (the latter only on condition $T_n \neq T_b$).

In the presence of the density/temperature gradients, the LFH mode with mixed transverse-longitudinal features can be found in the pair-ion plasma as well. In the limit $\omega^2 \ll \omega_p^2, k^2 v_T^2$, Eq. (14) reduces to

$$\omega^2 = \frac{v_T^2}{L_n^2 (1 + k^2 r_d^2/2)(1 + k^2 \lambda_i^2/2)}. \quad (18)$$

Here, $\lambda_i = c/\omega_p$ is the inertial length for the two species, and $r_d = v_T/\omega_p$ is the plasma Debye radius. Eq. (18) is an approximate solution valid only for relatively large wave-numbers; in this range of $k$ it is also backward, it should have similar features as the mode in e-i plasma, in particular in the limit $k \to 0$, but this cannot be checked analytically because the condition $\omega^2 \ll k^2 v_T^2$ becomes violated.

Eq. (14) can be solved numerically for any pair plasma (electron-positron, hydrogen pair plasma $H^\pm$, or fullerene pair plasma). In cylindrical configuration in pair-plasma experiments the radial scale was very small, so now we choose $L_n = 0.03$ m. In Fig. 3 we present only the new hybrid mode for an arbitrary pair-proton plasma with the temperature $T_e = T_b = 5 \cdot 10^3$ K, and for two densities $n_0 = 10^{14}$ m$^{-3}$, and $n_0 = 10^{16}$ m$^{-3}$. The approximate solution given by Eq. (18) is practically identical to the numerical one presented in Fig. 3 which is obtained from Eq. (14).

For the given $k$-range the thermal mode $kv_T$ is with the frequency in the range above $10^6$ Hz and the plasma frequency is $\sim 10^7$ Hz, so the new mode presented in Fig. 3 is far separated from these frequencies, and it is truly a different branch of oscillations. Note that $r_d \ll \lambda \ll \lambda_i$ so the backward features are mainly due to $k^2 \lambda_i^2$ term in Eq. (18).

For a larger laboratory configuration the mode may be observable by naked eyes. Taking for example $L_n = 0.2$ m and $\lambda = 0.01$ m for the same temperature as above and $n_0 = 10^{14}$ m$^{-3}$ yields $\omega = 3$ Hz.

The same behavior can be shown by taking fullerene pair plasma, but frequencies in that case are far below Hz.

Taking parameters which would correspond to experiments[10,11] is not appropriate because of the following two major problems: particle gyro radius becomes comparable with the plasma column radius, and the density scale length $L_n$ (in radial direction) is much shorter than the parallel wavelength. In such a case an eigenmode analysis is needed[13] and the local approximation is not applicable, so this is avoided here in order to show that the new mode and its backward features are not the result of the finite geometry and boundary effects.

III. PLASMA IN MAGNETIC FIELD

In the laboratory environment, instead of the equilibrium condition[10] we may have a more realistic situation of a plasma confined by an external magnetic field. Hence, we shall assume the presence of a background magnetic field directed along the $z$-axis, and with perturbations propagating in the same direction. In such a case the equilibrium implies the presence of the diamag-
The equilibrium may obviously be satisfied even for a homogeneous temperature and this is a frequent situation in lab-plasmas. Also, in pair-ion experiments no temperature gradient is mentioned, so we can omit it in (19) and keep only the density gradient. In such a case, and for other parameters as above for electron ion plasma we have \( v_{c0} = 1.7/B_0 \text{ m/s} \), and for the pair-ion plasma parameters \( v_{j0} = 14.3/B_0 \text{ m/s} \), so an eventual magnetic field shear inhomogeneity caused by this diamagnetic current is negligible. On the other hand, eventual equilibrium magnetic field gradient, which may in principle appear in the equilibrium condition \( \nabla [p_0 + B_0^2/(2\mu_0)] = 0 \), can also be made negligible if plasma-\( \beta \) is small and this is the case for the parameters used so far in the text. Note also that \( \vec{v}_{j0} \) will not contribute to the previously used continuity equation through the term \( n_1 \nabla \cdot \vec{v}_{j0} \) because \( \nabla \cdot \vec{v}_{j0} \equiv 0 \).

Another issue are the terms \( \vec{v}_1 \times \vec{B}_0 \equiv \vec{v}_{\perp1} \times \vec{B}_0 = \vec{v}_{\parallel 1} \times \vec{B}_1 \) that should appear in the momentum equation \( [1] \) in the presence of the magnetic field. Regarding their role with respect to the TEM wave, the first can be made negligible because the Lorentz force due to this term acts on the particle within time interval that is far shorter than the gyro-rotation time (particle changes direction within very short time intervals corresponding to the TEM wave). Note that for the pair-ion case discussed in the previous section and assuming \( B_0 = 0.3 \text{ T} \) (like in the experiment) for the two densities we have \( \omega_p/\Omega_i \approx 2, 22 \), where \( \Omega_i = q_i B_0/m_i \). The frequency of the electromagnetic wave is in fact much higher than \( \omega_p \approx 10^6 \text{ Hz} \); for the smallest \( k \) in Fig. 3 it is of the order of \( 10^{11} \text{ Hz} \). So it would be surely justified to omit \( \vec{v}_1 \times \vec{B}_0 \) term even for the parameters used so far for plasma without the magnetic field. The second term \( \vec{v}_{\parallel 0} \times \vec{B}_1 \) is even smaller.

For the electron-ion plasma the Lorentz force due to TEM wave can make difference for electrons only, but it can also be negligible if the magnetic field is not too strong. For example, for the parameters used in Fig. 2 the physics related to the TEM mode alone will not change if the introduced magnetic field is kept below 0.01 T, but the field can be allowed to be much stronger if the number density is assumed higher.

However, in the case of the LFH mode studied in the present text, there appears a slow transverse particle dynamics associated with this mode as well, and this can further be affected by the background field. For this motion there are several possibilities which can be discussed separately.

1) Gyro-effects will not appear for unmagnetized particles, which corresponds to the frequency limit:

\[ \Omega_j^2 < \omega^2 \ll \omega_{pi}^2, k^2 v_{Tj}^2 . \]  \hspace{1cm} (20)

In pair plasmas (e.g., electron-positron, \( H^+ \)) the condition \( \omega_{pe} \) is satisfied in a weak field \( B_0^{-5} \text{ T} \), and for \( T_0 = 10^3 \text{ K}, n_0 = 10^{18} \text{ m}^{-3}, L_n = 0.01 \text{ m} \). Hence, in this case the magnetic field effects can be omitted from the terms describing the perpendicular perturbed speed, and the analysis can be done similar as before with neglected temperature gradient only. The wave Eq. \( [9] \) now becomes

\[ c^2 k^2 \vec{E}_1 - c^2 k^2 (\vec{k} \cdot \vec{E}_1) - \omega^2 \vec{E}_1 + (\omega_{pa}^2 + \omega_{pb}^2) \vec{E}_1 \]

\[ - \left( \frac{\omega_{pa}^2 v_{Ta}^2}{\omega_b^2} + \frac{\omega_{pb}^2 v_{Tb}^2}{\omega_b^2} \right) \left[ \frac{p'_0}{n_0} \nabla E_{1z} + \nabla (\vec{k} \cdot \vec{E}_1) \right] + (\nabla \cdot \vec{E}_1) \frac{\nabla n_0}{n_0} = 0, \quad \omega_j^2 = \omega^2 - k^2 v_{Tj}^2 . \]  \hspace{1cm} (21)

The \( y \)-component yields again a separate TEM mode and from the other two components we obtain the dispersion equation

\[ \omega^2 \left( 1 - \frac{\omega_{pa}^2}{\omega_b^2} - \frac{\omega_{pb}^2}{\omega_b^2} \right) (c^2 k^2 + \omega_{pa}^2 + \omega_{pb}^2 - \omega^2) - \frac{k^2}{L_n^2} \left( \frac{\omega_{pa}^2 v_{Ta}^2}{\omega_b^2} + \frac{\omega_{pb}^2 v_{Tb}^2}{\omega_b^2} \right)^2 = 0 . \]  \hspace{1cm} (22)

In the pair plasma, for \( v_{Ta} = v_{Tb} = v_T \), dispersion equation \( [22] \) yields the same expression \( [18] \) for the frequency.

2) For electron-ion plasma and for parameters and geometry used in Fig. 2 and assuming \( B_0 = 0.01 \text{ T} \) or smaller, the frequency of the LFH mode (line c) in the given \( k \)-range is always above the ion gyro-frequency. So the frequency range \( [20] \) applies for ions and their dynamics would remain the same. But electrons are magnetized for the same parameters (their gyro-frequency is \( 10^9 \text{ Hz} \)) and their dynamics would become much more complex. Within some reasonable approximations their perpendicular speed can be written as:

\[ \vec{v}_{e\perp 1} = - \frac{1}{B_0} \frac{\vec{E}_1 \times \vec{E}_{1z}}{\Omega_e} - \frac{v_{Te}^2}{\Omega_e} \vec{e}_z \times \nabla_{\perp} n_{e\perp} \frac{1}{n_0} \frac{\Omega_e B_0}{\partial t} \frac{\partial E_{1\perp}}{\partial t} . \]  \hspace{1cm} (23)

Here, only the last term (the polarization drift) describes the motion in the direction of the TEM wave electric field, and only this electron motion can contribute to the appearance of the longitudinal electric field (see Fig. 1), but it is typically negligible. The other two terms describe electron drift motion in the \( y \)-direction. Electron parallel dynamics (along the magnetic field) will be the same as before. From Fig. 1 it is clear that the longitudinal electric field will appear whenever any of the two species move along the density gradient. In the present case this will be mainly due to ion direct motion in the TEM wave.
field. So some sort of the LFH mode is expected to develop again, but ions are less mobile and the mode may be considerably modified.

But in a different geometry, with the TEM wave electric field making an angle $\theta$ with respect to the density gradient, the electron drift speed $\vec{v}_{\parallel}$ will have one component $v_{\parallel} \sin(\theta)$ along the density gradient. Electrons will then considerably contribute to the longitudinal electric field making an angle $\theta$ with respect to the density gradient. So some sort of the LFH mode is expected to develop again, but ions are less mobile and the mode may be considerably modified.

3) Finally, if both species are magnetized $\omega^2 < \Omega_i^2$, only drift motion can develop in direction perpendicular to the magnetic field. From Fig. 1 we have learned that, to have the effects studied here, there must be displacement (of any kind) of particles (of any species) along the density gradient, and obviously this can be either a direct motion (as presented in the figure) or a drift in the presence of the magnetic field. However, in the present case, the drift of particles along the density gradient will develop only if the electric field of the incident TEM wave is not strictly in the same direction. So some sort of LFH mode is expected again but geometry should be assumed different from the cases studied above.

In both cases 2) and 3), derivations are lengthy, particle dynamics in two perpendicular directions become coupled, and several additional modes appear (like electron and ion cyclotron, lower and upper hybrid, etc.). Yet no essential new physics is expected to emerge and such derivations will be omitted here.

IV. SUMMARY AND CONCLUSIONS

The mechanism of coupling between TEM and LES waves propagating in inhomogeneous plasmas, discovered in Ref. 4, is shown here to contain some crucial extra physics. A completely new mode is shown to exist, which was not noticed in Ref. 4. This new low frequency hybrid (LFH) mode is the result of coupling between transverse and longitudinal electric fields of electromagnetic and electrostatic waves in the presence of density gradients. So the backward features of the ion acoustic mode, discussed in Ref. 4, are a a part of a more profound and complex phenomenon: for relatively small wave-numbers the backward LFH mode exchanges identity with the IA wave, the latter becomes backward for $k \rightarrow 0$ and eventually gets some cut-off or its frequency continues to grow in this $k$-range, while the LFH mode goes towards $\omega = 0$ following the usual IA line $k_{c\alpha}$ in the same $k$-limit. These features are partly seen in Fig. 2. The properties of the LFH mode do not necessarily change in the presence of the magnetic field because transverse motion of particles (with respect to the magnetic field vector) is essentially due to the TEM wave which is of very high-frequency so that gyro-motion of particles is usually negligible. In the text, the features of the LFH mode are presented for two possible equilibria and for both electron-ion and pair plasmas.

The presented TEM-LES coupling is linear, and it happens only in the presence of a density gradient. However, for some nonlinear phenomena the LFH mode may take over the role of the usual IA wave. The obvious possibilities are the following: electron decay instability (LHF mode interacting with two Langmuir waves propagating in opposite directions), parametric backscattering (LFH mode interacting with two light waves propagating in opposite directions), and parametric decay instability (incident light wave interacting with Langmuir wave and LFH mode moving in opposite direction).

The two different equilibria discussed in Secs. [11, 12, 13] have effect on the dispersion equation in general [see the coupling term containing $L_{n}$ in dispersion equations [14, 22]], although in the case of the LFH mode this effect vanishes in approximate expressions. This approximate absence of the effect on the LFH mode may be explained in the following way. As pointed out earlier, in the case $\nabla p_{j0} = 0$ [no magnetic field, the condition [10] the last two terms in equation (9) vanish, and the remaining temperature gradient terms can be traced in Eqs. [11, 14] through the terms which contain $v_{\parallel}^{\perp}$. On the other hand, in the presence of magnetic field, the temperature gradient is assumed absent because it is not essential or required for the equilibrium, so these terms are set to zero in Eq. (9). Now the remaining density gradient terms yield the term \( \alpha^2 \) in the dispersion equation (22). But in the LFH mode limit all the terms $\omega_j^2 - k^2 v_b^2$ reduce to $-k^2 v_b^2$, and then there is an obvious cancelation of the remaining thermal terms in the coupling term with $L_n$, so these equilibrium differences vanish. Though strictly speaking this is so only for relatively large values of $k$, as can be deduced from Fig. 2 where neglecting $\omega^2$ in the small $k$ limit is not justified any longer.

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