Quantum repeaters free of polarization disturbance and phase noise

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Original quantum repeater protocols based on single-photon interference suffer from phase noise of the channel, which makes the long-distance quantum communication infeasible. Fortunately, two-photon interference type quantum repeaters can be immune to phase noise of the channel. However, this type quantum repeaters may still suffer from polarization disturbance of the channel. Here we propose a quantum repeaters protocol which is free of polarization disturbance of the channel based on the invariance of the anti-symmetric Bell state \(|\psi^-(\lambda)\rangle = (|H\rangle|V\rangle - |V\rangle|H\rangle)/\sqrt{2} \) under collective noise. Our protocol is also immune to phase noise with the Sagnac interferometer configuration. Through single-atom cavity-QED technology and linear optics, this scheme can be implemented easily.

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I. INTRODUCTION

Many applications of quantum communication, such as quantum cryptography \([1, 2]\), rely on distributing quantum states between two distant peers, called Alice and Bob. However, the real-life channel will inevitably absorb or add noises to the information carriers (flying photons), which makes the efficiency of distributing quantum states decrease exponentially with the length of channel. To implement long-distance (\(\geq 1000\text{km}\)) quantum communication, this efficiency should scale polynomially with the length of channel. With the help of quantum repeaters \([3]\), one can implement long distance quantum communication in principle. The basic ideas of quantum repeaters are: 1) dividing the transmission channel between Alice and Bob into many short segments and distributing entangled-states between the neighboring nodes, each of which can be regarded as individual quantum-memory units, 2) through entanglement swapping and purification \([4, 5]\), the range of entanglement could be extended, 3) finally entangled-states are distributed between Alice and Bob.

The physical implementations of quantum repeaters can be based on single atom trapped in cavity \([7, 8]\) or atomic ensembles with linear optics (DLCZ) \([6]\). For the experimental progresses, one can see Refs. \([14, 15, 16, 17, 18, 19, 20]\). Most of quantum repeaters protocols utilize the same topological structure as DLCZ’s. Suppose we have two neighboring but distant quantum memories \(L\) and \(R\) which can be atomic ensembles as in the original DLCZ scheme or single atom trapped in cavity or some other quantum memories used in \([9, 10]\). \(L\) and \(R\) are both simultaneously illuminated by a pumping laser, and then with a small probability \(L\) or \(R\) will jump to another quantum state and emit one photon to the channel. The path information of the single photon emitted from \(L\) or \(R\) will be erased and then this single-photon enters the detector at the middle point. Now \(L\) and \(R\) will be entangled. This heralded entanglement generation is also a built-in entanglement purification process \([6]\). The lower the probability of photon emission is, the higher the fidelity of the resulted entangled-states is. However this is also a single-photon interference process which is sensitive to the phase drift due to unknown length drift between the path from the \(L\) to the middle point and the path from \(R\) to the middle point. Since the elementary entanglement generation and entanglement swapping are all probabilistic, one must keep this phase constant over a very long time. In fact this demand is beyond modern technology. One can see Refs. \([11, 12]\) for the detailed analysis of the phase stability problem of DLCZ.

To overcome phase instability, Refs. \([11, 12]\) proposed a new quantum repeaters architecture based on two-photon Hong-Ou-Mandel-type \([21]\) interference. Unlike the original DLCZ-type quantum repeaters which need to keep the drift of arms of the long distance interferometer below the wave-length scale, this new architecture only requires us to keep this drift below the coherence-length scale of the flying photons, which greatly facilitates the implementation of the long-distance quantum repeaters. Another choice is to use the Sagnac interferometer configuration. With the help of the optical switches, the pumping pulse to the \(L\) should be reflected by a mirror in the \(R\) before the excitation process, and similarly, the pumping pulse to the \(R\) should be reflected by an mirror in the \(L\) before the excitation process. With the Sagnac interferometer configuration, the single-photon from the \(L\) and \(R\) will have the same path-length if the length drift of the channel is negligible during the travel time of the flying photons in the channel. And in the Ref \([22]\), it has been proved that one can obtain high enough interference visibility even the fiber length up to 75km with the Sagnac interferometer configuration.

However, the above two solutions to the phase instability of DLCZ-type quantum repeaters still suffer from the polarization disturbance of the channel. For the two-photon Hong-Ou-Mandel-type interference quantum repeaters proposed by Refs. \([11, 12]\), the polarization disturbance of the channel may introduce error in the elementary entanglement generation step. Suppose the channel between \(L\) and the middle point maps the horizontal polarization state \(|H\rangle\) to the \(|F\rangle = (|H\rangle + |V\rangle)/\sqrt{2}\) and the vertical polarization state \(|V\rangle\) to the \(|S\rangle = (|H\rangle - |V\rangle)/\sqrt{2}\), then the type-I Bell-States-
Measurement (BSM-I) performed at the middle point cannot distinguish the error case of both the two ensembles of L emitting one photon from the correct case. This may be an extreme case, since the channel disturbance should not introduce such a big change of polarization. But we can conclude that the polarization disturbance indeed introduces errors to this quantum repeaters proposal. For the Sagnac interferometer method, the different polarization disturbances between the two channels from L and R to the middle point obviously lower the interference visibility. Of course, polarization controllers may be used to alleviate the polarization disturbance, as demonstrated in the experiment of the Ref. [22]. But for the real-life quantum repeaters the application of polarization controllers will make the system more complicated and this method cannot eliminate some polarization disturbances due to rapid vibrations of the fiber channel.

Here, in this article we propose a new quantum repeaters architecture that utilizes the invariance of the anti-symmetric Bell state $|\psi^-(\rangle = (|H)(V) - |V)(H)\rangle) / \sqrt{2}$ under collective noise. In fact, when the coherence time of photons is larger than the delay time resulting from the polarization mode dispersion, the polarization disturbance can be treated as an unitary transition only acting on polarization space $|H\rangle \rightarrow |V\rangle$. Our scheme is totally free of polarization disturbance of the channel. Through Sagnac interferometer configuration, the phase stability can be also achieved if the distance between neighboring nodes is not too long. Though topological structure of our scheme is similar to those in the Refs. [11, 12], the physical implementation should not be atomic ensembles, since one ensemble may emit two or more photons. The single photon source type quantum memory, such as single atom trapped in cavity may be appropriate for our scheme. The detailed discussion of our scheme will be given in section II and a conclusion will be given in section III.

II. ARCHITECTURE

The basic architecture for distributing entanglement for neighboring nodes of our scheme is shown in Fig. 1.

![Diagram](image)

**Fig. 1.** BS: 50:50 beam-splitter; D1 and D2 represent two single photon detectors respectively; FM: Faraday mirror; FM: phase modulator; PBS: polarization beam-splitter which transmits horizontal polarized photons and reflects vertical polarized photons

As in Fig. 1., Alice and Bob are two neighboring nodes.

The memory units $L_1, L_2, R_1$ and $R_2$ should be some single-photon source type quantum-memories, such as a single three-level atom trapped in a cavity shown in Fig. 1. Initially, the four three-level atoms, each in the $L_1, L_2, R_1$ and $R_2$ respectively, are all in the ground state $|g\rangle$. We simultaneously pump the four atoms and with a small probability $p$, one of these four atoms emits Stokes photon with frequency $\omega_{\text{St}} = \omega_L - \omega_V$. In fact the pumping laser for the quantum memories $L_1, L_2$ must be reflected by a mirror in R before the pumping process and so does the pumping laser for $R_1$ and $R_2$. For simplicity, we do not draw this Sagnac interferometer configuration in Fig. 1. We only concern the case that the two single-photon detectors have two clicks. With probability $p^2$, two of the four atoms emit one photon respectively, and this could be given in the operators form:

$$|\Psi\rangle = \left(1 + \sqrt{p} (S^\dagger_{L_1} H^\dagger_{L_1} + S^\dagger_{L_2} V^\dagger_{L_2} + S^\dagger_{R_1} H^\dagger_{R_1} + S^\dagger_{R_2} V^\dagger_{R_2})ight. + p(S^\dagger_{L_1} S^\dagger_{L_2} H^\dagger_{L_1} V^\dagger_{L_2} + S^\dagger_{R_1} S^\dagger_{R_2} H^\dagger_{R_1} V^\dagger_{R_2})

+ S^\dagger_{L_1} S^\dagger_{R_1} H^\dagger_{L_1} R^\dagger_{R_1} + S^\dagger_{L_2} S^\dagger_{R_2} H^\dagger_{L_2} R^\dagger_{R_2}

+ S^\dagger_{L_1} S^\dagger_{R_2} H^\dagger_{L_1} R^\dagger_{R_2} + S^\dagger_{L_2} S^\dagger_{R_1} H^\dagger_{L_2} R^\dagger_{R_1} + O(p))(g\rangle_{LR}|0\rangle_p$$

(1)

in which, $S^\dagger = |\psi\rangle\langle g|$ means the atomic transition from the ground level $|g\rangle$ to the meta-stable level $|\psi\rangle$, meanwhile $H^\dagger$ and $V^\dagger$ represent the creation operators for the horizontal and vertical polarized photons respectively. Before the photons enter the channel, they will pass through the unbalanced interferometer consisting of two Faraday mirrors and one polarization-dependent phase modulator in the long arm of the interferometer. The function of Faraday mirrors is to eliminate the possible polarization disturbance of the interferometer [23, 24]. The phase modulator in the long arm of the unbalanced interferometer only adds $\pi$ phase to the horizontal polarized photons does not change the vertical polarized photons. Only concerning the events which may introduce two clicks to the detector 1 and detector 2 and assuming the beam-splitter will add $\pi/2$ phase to the reflected photons and won’t change the transmitting photons, we can deduce that:

$$H^\dagger_{L_1} V^\dagger_{L_1} = \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} (H^\dagger_{L_1} V^\dagger_{L_1} - H^\dagger_{L_2} V^\dagger_{L_2}) + \frac{1}{\sqrt{2}} (H^\dagger_{L_1} V^\dagger_{L_2} - V^\dagger_{L_1} H^\dagger_{L_2}) \right)

H^\dagger_{R_1} V^\dagger_{R_1} = \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} (H^\dagger_{R_1} V^\dagger_{R_1} - H^\dagger_{R_2} V^\dagger_{R_2}) + \frac{1}{\sqrt{2}} (H^\dagger_{R_1} V^\dagger_{R_2} - V^\dagger_{R_1} H^\dagger_{R_2}) \right)$$

(2)

in which, the subscript 1(2) represents the time-bin of the photons passing through the long arm and short arm of interferometer respectively, which result in the two different time-bins, and subscript $L(R)$ represents the left(right) channel as in Fig. 1. The second term of the right side of the equation (2) is just the anti-symmetric $|\psi^-(\rangle$ state. From this equation we know if the two photons occupy the two different time-bins, they will be in the $|\psi^-(\rangle$ state. Conversely, if the two photons occupy the same time-bin 1 or 2, it will not be $|\psi^-(\rangle$ state. According to Ref. [23], polarization disturbance can be well approximated by a collective unitary transformation as long as the delay between the two time-bins is small com-
pared to the variation of disturbance, which has been verified by experiment [27]. Therefore through the invariance of \(|\psi^-\rangle\) state and erasing the path information, the entanglement can be well established. Now we can conclude that if the detector 1 and detector 2 get one click in each of the two time-bins, we can get the entangled-states given by

\[ S_{L_1}^+ S_{L_2}^+ H_{L_1}^+ V_{L_1}^+ + S_{R_1}^+ S_{R_2}^+ H_{R_1}^+ V_{R_1}^+ \rightarrow \]

\[ \frac{1}{2\sqrt{2}}(S_{L_1}^+ S_{L_2}^+ - S_{R_1}^+ S_{R_2}^+)(-H_{1D_1}^+ V_{2D_1}^+ + V_{1D_1}^+ H_{2D_1}^+ + H_{1D_2}^+ V_{2D_2}^+) \]

\[ - V_{1D_1}^+ H_{2D_2}^+) \]

\[ + \frac{1}{2\sqrt{2}}(S_{L_1}^+ S_{L_2}^+ + S_{R_1}^+ S_{R_2}^+)(H_{1D_1}^+ V_{2D_2}^+ + H_{1D_2}^+ V_{2D_2}^+) - V_{1D_2}^+ H_{2D_1}^+ ) \]

If we get two clicks in the same detector we obtain \((S_{L_1}^+ S_{L_2}^+ - S_{R_1}^+ S_{R_2}^+)/\sqrt{2}\), and if one click in each detector \((S_{L_1}^+ S_{L_2}^+ + S_{R_1}^+ S_{R_2}^+)/\sqrt{2}\) is prepared.

Meanwhile, events described by \(S_{L_1}^+ S_{R_1}^+ H_{L_1}^+ H_{R_1}^+\), \(S_{L_2}^+ S_{R_2}^+ V_{L_1}^+ V_{R_1}^+\), \(S_{L_1}^+ S_{R_1}^+ H_{L_2}^+ V_{R_2}^+\), \(S_{L_2}^+ S_{R_2}^+ V_{L_2}^+ H_{R_2}^+\) in the equation (1) are also probable to click the detectors twice, e.g. both \(L_1\) and \(R_1\) both emit horizontal polarized photons. Obviously, photons as described by \(H_{L_1}^+ H_{R_1}^+\), \(V_{L_1}^+ V_{R_1}^+\), \(H_{L_1}^+ V_{R_1}^+\) or \(V_{L_1}^+ H_{R_1}^+\) cannot constitute the \(|\psi^-\rangle\) state. Hence these photon-states suffer from random polarization disturbance, and we cannot predict the states of these photons after traveling along the two different channels. Therefore, in this elementary entanglement distributing step, we cannot eliminate these errors. Assuming we get the first click in detector 1 and the second click in detector 2 and neglect dark counts of the detectors, we will get be a mixture given by:

\[ \rho_{LR} = \frac{1}{\sqrt{6}}(\sqrt{2}\rho_{ENGLR} + |sg\rangle_{L_1}L_2\langle sg| \otimes |sg\rangle_{R_1}R_2\langle sg| \]

\[ + |gs\rangle_{L_1}L_2\langle gs| \otimes |gs\rangle_{R_1}R_2\langle gs| \]

\[ + |gs\rangle_{L_1}L_2\langle gs| \otimes |sg\rangle_{R_1}R_2\langle sg|) \]

in which, \(\rho_{ENGLR}^L = |\Psi^L\rangle_{LR}\langle \Psi^L|\) and \(|\Psi^L\rangle_{LR} = (S_{L_1}^+ S_{L_2}^+ + S_{R_1}^+ S_{R_2}^+)|\otimes|_{L_1L_2R_1R_2}/\sqrt{2}\). Note that in the above equation we have assumed that all error events do not result in any entanglement between \(L\) and \(R\). Obviously this assumption is just for simplicity and does not spoil the preciseness in this paper, since all error events can be eliminated by the entanglement swapping step. The first item of the above equation represents the correct case while the others represent the error cases. From the above equation, we know there are many errors after the first entanglement distributing step. Fortunately, all these possible errors can be eliminated with a simple entanglement swapping step, which is shown in Fig. 2.

Suppose that we have successfully performed entanglement distributing between nodes \(L\), \(A\) and \(B\), \(R\), which means that the \(L\), \(A\) and \(B\), \(R\) are in the mixture \(\rho_{LA}\) and \(\rho_{BR}\) respectively. The entanglement swapping process is same as those in Refs. [6, 11, 12], in which we pump the atoms in \(A\) and \(B\) to excite the transition from \(|g\rangle\) to \(|e\rangle\). From the equation (4), the correct case that the \(L\), \(A\) are in the state \(|\Psi^L\rangle_{LA} = (S_{L_1}^+ S_{L_2}^+ + S_{R_1}^+ S_{R_2}^+)|\otimes|_{L_1L_2A_1A_2}/\sqrt{2}\) and the \(B\), \(R\) are in the state \(|\Psi^L\rangle_{BR} = (S_{B_1}^+ S_{B_2}^+ + S_{R_1}^+ S_{R_2}^+)|\otimes|_{B_1B_2R_1R_2}/\sqrt{2}\), has a probability of 1/9. Note that the \(A\) and \(B\) are in the same location and thus any polarization or phase noise is negligible. For this correct case and while only components that will induce detectors double clicks are considered, we deduce that the successful entanglement swapping process is given by:

\[ |\Psi^L\rangle_{LA}|\Psi^L\rangle_{BR} \rightarrow \frac{1}{8}(\sqrt{2}\rho_{ENGLR} + |sg\rangle_{L_1}L_2\langle sg| \otimes |sg\rangle_{R_1}R_2\langle sg| \]

\[ + |gs\rangle_{L_1}L_2\langle gs| \otimes |gs\rangle_{R_1}R_2\langle gs| \]

\[ + |gs\rangle_{L_1}L_2\langle gs| \otimes |sg\rangle_{R_1}R_2\langle sg|) \]

in which the subscript 1, 2, 3 and 4 in the photon creations operators represent the corresponding modes for the photon detectors. According to the above equation, if we have two clicks in detector 1 and 2 or 3 and 4, the entanglement swapping is successful and thus entanglement between \(L\) and \(R\) is established with probability of 1/2. Here we abandon the \(H_{1}^2, V_{2}^2, H_{4}^2\) and \(V_{3}^2\), since these events may be introduced by error cases in equation (4), which will be discussed in the followings.

Concretely speaking, if one of \(L\), \(A\) and \(B\), \(R\) is in the correct case and the other is in the wrong case, obviously the detectors will have only 1 click or 3 clicks. Thus we can distinguish this case from the correct case. If both the \(L\), \(A\) and \(B\), \(R\) are in the wrong case, the entanglement swapping process will result in the following photons creations given by:
According to equation (6), obviously neither of these items results in $H^1_B V^3_V$ and $H^1_B V^1_V$. Therefore, the two clicks in both detectors 1 and 2 or 3 and 4 will project the required entangled state $|\Psi^+\rangle$. This new setup can be totally free of polarization disturbance of channel. With Sagnac interferometer configuration, phase noise can be also alleviated. Single photon source quantum memory should be adopted for physical implementation of this proposal, e.g., single three-level atom trapped in high-finesse cavity is a good candidate. With an unbalanced Faraday-Michelson interferometer we can project the two-photon $|H\rangle|V\rangle$ wave-packet into the $|\Psi^+\rangle$. Thus neither of these solutions can overcome the polarization disturbance.

III. CONCLUSION

The original DLCZ-type heralded quantum repeaters suffer from the phase noise and polarization disturbance of the channel. The Sagnac interferometer configuration may overcome the phase noise when the drift of the channel length is not too fast, but still needs feedback polarization controllers. The two-photon Hong-Ou-Mandel interference type quantum repeaters may be free of phase noise but also vulnerable to the polarization disturbance. Thus neither of these solutions can overcome the polarization disturbance.

In this paper, we propose a new quantum repeaters architecture, which is based on the invariance of anti-symmetric Bell state $|\psi^-\rangle = (|H\rangle|V\rangle - |V\rangle|H\rangle)/\sqrt{2}$ under collective unitary transformation. This new setup can be totally free of polarization disturbance of channel. With Sagnac interferometer configuration, phase noise can be also alleviated. Single photon source quantum memory should be adopted for physical implementation of this proposal, e.g., single three-level atom trapped in high-finesse cavity is a good candidate. With an unbalanced Faraday-Michelson interferometer we can project the two-photon $|H\rangle|V\rangle$ wave-packet into the $|\psi^-\rangle$ state distinguishable by two very near time-bins. Although the elementary entanglement distributing may introduce some errors, the following entanglement swapping step can eliminate all possible errors. In a word, the final entanglement state can be of high fidelity and immune to both polarization and phase disturbance of the channels.

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