Accounting casual character of the cutting tool life at the optimization of the cutting regimes

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Abstract. In the real production functioning of the cutting tool takes place in the conditions of properties dispersion of the technological system elements that defines stochastic character of the cutting process parameters. As a result casual character has a basic data-out of cutting process - the cutting tool life. It is presented the method of the cutting regimes optimization taking into account the variation coefficient and the distributing law of the cutting tool life as casual value. The analysis of the optimization criteria (productivity and prime price) as functions of casual argument - the cutting tool life is executed. There are got analytical dependences, which allow taking into account dispersion of the cutting tool life at determination of its optimum value. There are set the correction coefficient on the productivity and prime price optimum cutting speeds taking into account the coefficient of cutting tool life variation. There are set the necessary for providing of the set level of reliability the coefficient of the cutting regimes change.

1. Introduction

An increase of the productivity and decline of prime price of making of products are the actual task of machine-building production. Most effectively it decides on the basis of optimization of the modes of cutting on the criteria of burst performance and minimum prime price under various conditions treatments [1]. The basic methods of decision of tasks of optimization are based on consideration of parameters of cutting process as the determined value and to replacement of casual parameters their mean values [2]. However in the real production functioning of the cutting tool takes place in the conditions of the properties dispersion of the technological system elements that defines stochastic character of the cutting process parameters. As a result casual character has a basic data-out of cutting process - the cutting tool life. Presently picture of the cutting tool life as casual value is well enough grounded [3], however questions of account of laws of its distributing practically not examined and require further development. It is known the decision of task of determination of optimum cutting tool life period on the criterion of maximal productivity [4], and also on the criterion of minimum prime price [5]. However in these works the questions of determination of the optimum cutting regimes which providing of the assured level of cutting tool faultlessness practically are not considered that requires further development of the indicated method.

The purpose of this work is perfection of method of the cutting regimes optimization taking into account the law of distributing of cutting tool life and coefficient of its variation with providing of the assured level of cutting tool faultlessness.

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2. Problem formulation

In the presented work the task of oneself-reactance optimization of cutting speed at the set values of cutting depth and feed on the criteria of maximal productivity and minimum prime price is examined.

In this case as a parameter of optimization cutting tool life is determined at first, and then it is considered cutting speed. As the basic optimization criteria variable parts of the productivity \( K_p \) and prime price \( K_c \) depending on the cutting regimes are accepted.

\[
K_p = (t_o + t_c t_o / T)^{-1}; \quad K_c = A(t_o + t_c t_o / T) + A' t_o / T,
\]

where \( t_o \) - the basic time of treatment; \( t_c \) - the restoration time of the cutting tool; \( T \) - the cutting tool life period; \( A \) - the expenses for 1 minute of the equipment work; \( A' \) – the expenses for 1 the cutting tool life period; \( A' = C_r / KT \) (\( C_r \) - cost of the cutting tool, \( K \) - an amount of the cutting tool life period).

The objective functions expressing dependence of the optimization criteria on the cutting tool life:

\[
K_p (T) = C(T^m + t_c T^{m-1})^{-1}; \quad K_c (T) = C(T^m + c T^{m-1}),
\]

where \( C = \pi D L t_x V y_v^{-1}/1000 C_V K_V \), \( c = t_c + A' / A \) – permanent coefficients; \( D, L \) - diameter and length of treatment; \( V \) – cutting speed; \( S \) – feed; \( C_V, K_V \) – factors and \( x, y, m \) – the indexes characterizing degree of influence of the depth \( t_o \), feed \( S \) and cutting tool life \( T \) for cutting speed \( V \).

Stochastic character of functioning of the cutting tool is conditioned by the action of casual parameters of cutting process [1]. To their number take the out-of-control changes of physical-mechanical properties of purveyance and cutting tool, static and dynamic descriptions of equipment. As a result casual character has a basic data-out of cutting process - cutting tool life \( T \).

3. The proposed method

We will consider a general decision for determination of parameters of function of one casual argument [6]. A casual value \( Y \) is nonlinear function of casual argument \( X \) with the mathematical expectation \( M \), and dispersion \( D_X: Y = \phi(X).\) Decomposing a function \( y = \phi(x) \) in the Taylora row in the vicinity of point \( m_i \) (saving the first three members in decomposition), have approximately:

\[
Y = \phi(x) \approx \phi(M_x) + \phi'(M_x)(X - M_x) + \frac{1}{2} \phi''(M_x)(X - M_x)^2.
\]

Mathematical expectation \( M_y \) of casual value \( Y \) is determined as follows:

\[
M_y = \phi(M_x) + \frac{1}{2} \phi''(M_x)D_x.
\]

Taking into account the executed analysis the mathematical expectations \( M_{KP} \) and \( M_{KC} \) of objective functions of the productivity \( K_p(T) \) and prime price \( K_c(T) \) as functions of casual argument \( T \) are appear as follows dependences:

\[
M_{KC} = C(T^m + c T^{m-1}) + \frac{C}{2} \frac{\partial^2}{\partial T^2} (T^m + c T^{m-1}) (V_T T)^2,
\]

\[
M_{KP} = C(T^m + t_c T^{m-1})^{-1} + \frac{C}{2} \frac{\partial^2}{\partial T^2} (T^m + t_c T^{m-1})^{-1} (V_T T)^2.
\]

Conformities to law of change of the the mathematical expectations of objective functions as functions of casual argument it is expedient to examine on the basis of relative function \( M_{dKP} =
$M_{KP}/K_P(T)$, $M_{KC} = M_{KC}/K_C(T)$, which in a kind of two-parameter dependences from the cutting tool life $T$ and coefficient of its variation $V_T$ are presented on a figure 1 and figure 2. For calculations there are accepted the following parameters: $t_c = 5\text{min}$; $A'/A = 2,5\text{min}$; $c = 7,5\text{min}$.

![Figure 1](image1.png)  
**Figure 1.** Graphs of change of the mathematical expectation objective function of the productivity $M_{oKP}$, depending on the cutting tool life $T$ and coefficient of its variation $V_T$.

![Figure 2](image2.png)  
**Figure 2.** Graphs of change of the mathematical expectation objective function of the prime price $M_{oKC}$ depending on the cutting tool life $T$ and coefficient of its variation $V_T$.

Charts testify to extreme character of considered objective functions from the parameter of optimization $T$. The got dependences confirm that the mathematical expectations of objective functions as functions of casual argument are differed from a value of objective functions, which are accounted on the mathematical expectation of its argument $\phi(M_x) = \phi(T)$. This difference is quantitative characterized in size the second element in the formula (5) and formula (6), which with the sufficient degree of exactness can serve as the estimation of error of close approximation of casual models by the determined models. Thus there are changed not only the value of objective function but also optimum value of casual argument. At the value of the coefficient of variation $V_T = 0$ the optimum on the productivity the cutting tool life periods $T_{oP} = 20\text{min}$ and the optimum on the prime price the cutting tool life periods $T_{oC} = 30\text{min}$. With the increase of coefficient of variation the optimum cutting tool life period increases. At the value of the coefficient of variation $V_T = 1$ the optimum on the productivity the cutting tool life periods $T_{oP1} = 42\text{min}$ and the optimum on the prime price the cutting tool life periods $T_{oC1} = 56\text{min}$.

For search of the optimum value of the cutting tool life period $T$ taking into account casual character of the criterion and parameter of optimization there are decided differential equalizations:

$\frac{\partial}{\partial T} \left[ \left( T^m + t_c T^{m-1} \right)^{-1} + \frac{1}{2} \frac{\partial^2}{\partial T^2} \left( T^m + t_c T^{m-1} \right)^{-1} \left( V_T T \right)^2 \right] = 0. \quad (7)$

$\frac{\partial}{\partial T} \left[ \left( T^m + c T^{m-1} \right) + \frac{1}{2} \frac{\partial^2}{\partial T^2} \left( T^m + c T^{m-1} \right) \left( V_T T \right)^2 \right] = 0. \quad (8)$

4. **Analysis of the receive results**

Without the account of casual character of the cutting tool life (coefficient of variation $V_T = 0$) optimum on the productivity and prime price the cutting tool life periods $T_{oP}$ and $T_{oC}$ are determined by the known dependences:
\[ T_{oP} = (1/m-1)\bar{t}_c ; \quad T_{oC} = (1/m-1)(t_\bar{r} + A'/A). \]  \hfill (9)

With the increase of coefficient of variation \( V_T \) the optimum tool life periods are increased. Correction coefficient on the optimum tool life periods \( \mu_P(V_T) = T_{oP}(V_T)/T_{oP}(0), \mu_C(V_T) = T_{oC}(V_T)/T_{oC}(0) \) can be certain, coming from decisions for the different values of the coefficient of tool life variation of the following differential equalizations:

\[ 1 - \frac{\partial}{\partial T} \left[ \frac{\partial^2}{\partial T^2} \left( T^m + t_c T^{m-1} \right)^{-1} (V_T T)^2 \right] = 0 ; \quad 1 - \frac{\partial}{\partial T} \left[ \frac{\partial^2}{\partial T^2} \left( T^m + c T^{m-1} \right)(V_T T)^2 \right] = 0. \] \hfill (10)

The optimum on the productivity and prime price tool life periods with the account of casual character of the cutting tool life are determined by the dependences:

\[ T_{oP}(V_T) = \mu_P(V_T)(1/m-1)\bar{t}_r ; \quad T_{oC}(V_T) = \mu_C(V_T)(1/m-1)(t_\bar{r} + A'/A). \] \hfill (11)

The correction coefficients \( \mu_P(V_T) \) and \( \mu_C(V_T) \) can be certain with the use of graphs, which are presented on a figure 3. At the value of the coefficient of variation \( V_T = 0 \) dispersion absents and \( \mu_P(V_T) = \mu_C(V_T) = 1 \). At the value of the coefficient of variation \( V_T<0,3 \) the degree of its influence on the optimum tool life periods is very insignificant, and it is possible to ignore this coefficient.

![Figure 3. Graphs of dependence of the correction coefficients \( \mu_P(V_T) \) and \( \mu_C(V_T) \) on the coefficient of variation \( V_T \).](image1)

![Figure 4. Graphs of dependence of the approximated correction coefficients \( \mu_{Pa}(V_T) \) and \( \mu_{Ca}(V_T) \) on the coefficient of variation \( V_T \).](image2)

Substantial influence on the optimum tool life periods the coefficient of variation has at \( V_T > 0,4 \). In this case a possible error in the estimation of the optimum tool life periods can arrive at 80% that testifies to the necessity of account of casual character of the criterion and parameter of optimization. For the close estimation of optimum tool life period at \( V_T > 0,3 \) the correction coefficients \( \mu_P(V_T) \) and \( \mu_C(V_T) \) can be approximated by the followings dependences (figure 4):

\[ \mu_{Pa}(V_T) = e^{0.9(V_T - 0.3)} ; \quad \mu_{Ca}(V_T) = e^{0.8(V_T - 0.3)}. \] \hfill (12)

At the analysis of the cutting tool life as a casual size the law of Veybulla [3] is most widespread for which integral \( P(t) \) and differential to the \( f(t) \) function of distributing:

\[ P(t) = e^{-(t/a)^p} ; \quad f(t) = (b/a)(t/a)^{(b-1)}e^{-(t/a)^p}, \] \hfill (13)
where \( a = T_C / \Gamma (1 + 1/b) \); \( b(V_T) = \exp (-1.092 \ln V_T) \) - the parameters of the law; \( \Gamma (1 + 1/b) \) - gamma-function.

In the real work as indexes of faultlessness of cutting tool is examined middle cutting tool life period and gamma-percent cutting tool life period \( T \):

\[
T_C = a \Gamma (1 + 1/b) ; \quad T_\gamma = a (- \ln (\gamma / 100))^{1/b} .
\]  

(14)

A coefficient, characterizing correlation gamma-percent cutting tool life period \( T_\gamma \) with middle cutting tool life period \( T_C \) can be expected on a formula:

\[
K_\gamma = T_\gamma / T_C = [- \ln (\gamma / 100)]^{1/b} / \Gamma (1 + 1/b) .
\]  

(15)

Conformities to law of change of correction coefficient \( K_\gamma \) taking into account correlation of gamma-percent cutting tool life period \( T_\gamma \) with middle cutting tool life period \( T_C \) depending on the coefficient of the cutting tool life variation are presented on a figure 5. Coefficients are presented for different values gamut – percent: \( \gamma = 90\% \) (\( K_{90} \)); \( \gamma = 80\% \) (\( K_{80} \)).

For the close estimation of optimum tool life period at \( V_T > 0.3 \) the correction coefficients \( K(V_T) \) can be approximated by the followings dependences (figure 6):

\[
K_{90} (V_T) = 0.62 e^{-2.5(V_T - 0.3)} ; \quad K_{80} (V_T) = 0.75 e^{-1.7(V_T - 0.3)} .
\]  

(16)

Optimum on the productivity and prime price cutting speeds taking into account the coefficient of the cutting tool life variation are determined on the proper the optimum cutting tool life periods \( T_{oP} \) and \( T_{oC} \) (9) by the dependences:

\[
V_{oP}(V_T) = \frac{K_{VP} K_{V\gamma} (V_T) C_V K_V}{T_{oP}^m (V_T) S_y v t_p^x_v} ; \quad V_{oC}(V_T) = \frac{K_{VC} K_{V\gamma} (V_T) C_V K_V}{T_{oC}^m (V_T) S_y v t_p^x_v} ,
\]  

(17)

were \( K_{VP}(V_T) \) - the correction coefficients on the optimum cutting speeds taking into account the coefficient of the cutting tool life variation; \( K_{V\gamma}(V_T) \) - correction coefficients on the optimum cutting speeds taking into account the
For the close estimation of correction coefficients on the optimum cutting speeds at coefficient of cutting tool life variation \( V_T > 0.3 \) on foundation of correction coefficients \( \mu_{\text{pa}}(V_T) \) и \( \mu_{\text{ca}}(V_T) \) can be set the followings dependences (\( m = 0.2 \)):

\[
K_{VP}(V_T) = \left[ \mu_{P}(V_T) \right]^{-m} ; \quad K_{VC}(V_T) = \left[ \mu_{C}(V_T) \right]^{-m} ; \quad K_{V\gamma}(V_T) = \left[ K_{\gamma}(V_T) \right]^{m}.
\] (18)

The correction coefficients on the productivity and prime price optimum cutting speeds \( K_{VP}(V_T) \) and \( K_{VC}(V_T) \) can be certain with the use of the graphs which are presented on a figure 7. The coefficients taking into account different values gamma-percent \( \gamma \) are presented on a figure 8.

5. Conclusions
The conducted researches testify that for providing of the maximal productivity and minimum prime price of the mechanical treatment in the conditions of the considerable cutting tool life dispersion it is necessary to rise the optimum cutting tool life period and according to reduce the optimum cutting speed. On the basis of the developed method of the cutting regimes optimization taking into accounts the casual character of the cutting tool life there are set the correction coefficient on the productivity and prime price optimum cutting speeds taking into account the coefficient of cutting tool life variation.

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