Polaron states of electrons over the anisotropic surface of liquid helium

Sviatoslav S. Sokolov¹,², Antônio Carlos A. Ramos¹, and Nelson Studart¹

¹Departamento de Física, Universidade Federal de São Carlos, 13565-905 São Carlos, São Paulo, Brazil
²B. I. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, 61164 Kharkov, Ukraine

Abstract

The energetics and transport properties of the polaron in the anisotropic surface over liquid helium are investigated. The localization radii and the energy of the ground and excited states are calculated using the variational method within the hydrodynamic model of the polaron. In particular, we have considered maximal anisotropy which corresponds to the system of electrons in quasi-one-dimensional channels over liquid helium. The polaron binding energy is found and the temperature for the polaron formation is estimated below 0.1 K. Solving the hydrodynamic equations for fluid velocities, the polaron mobilities along and across the channel are determined. The possibility of experimental observation of polarons by measuring the frequency of spectroscopic transitions as well the mobility as functions of the holding electric field is addressed.

PACS numbers: 71.38.+i; 73.20.Dx; 73.90.+f

I. INTRODUCTION

An electron together with its self-induced polarization in a medium forms a quasiparticle which has been named polaron. Besides its importance as a standard theoretical model of a fermionic particle coupled to a boson scalar field, the polaron has been observed in some physical systems. In particular, there has been great interest in the search of polaron states for surface electrons levitated over liquid helium whose properties turn out the system as a good candidate for the formation of a polaron of reduced dimensionality. Predicted theoretically long time ago, the surface polaron over liquid helium has been the subject of great amount of experimental work during the last decades.

Theoretical approaches to investigate the surface polaron over helium are twofold. One is based on the description of the dimple state (electron plus the deformation of the isotropic helium surface due to the pressing field) through the minimization of the total energy functional of the dimple which leads to a system of coupled equations of motion. The transport properties are evaluated in terms of classical hydrodynamical equations since the polaron has a high effective mass. The other one involves the concept of a Fröhlich-like polaron (a single electron coupled to ripplons). The conductivity was calculated in terms of a force-force correlation function within the linear response theory. Despite the great difference between the methods, the final results for the structure of the ripplonic polaron in both approaches show fair qualitative and quantitative agreement.

In a seminal paper, Shikin and Monarkha have determined the ground state of the electron in an isotropic surface deformation through the solution of the Schrödinger equation for the electron trapped in the dimple and the mechanical equilibrium equation using the Fourier-Bessel transform due to the circular symmetry of the equations. The profile of the surface deformation was taken in the harmonic approximation (HA) which allows them to obtain the localization length of the electron from the Gaussian wave function, i.e. the polaron radius. Later Monarkha using a variational method (VM) was able to obtain the localization length from the minimization of the polaron energy calculated with a trial Gaussian wave function. In these works, the influence of external fields and the thickness of the helium film on the static and dynamical properties of the polaron was investigated. Marques and Studart have solved in a self-consistent way the Schrödinger equation and the mechanical equilibrium equation obtaining both the electron wave function and the profile of the dimple. The comparison of the numerical results with those from the HA and VM shows that the VM provides a more exact description of the electron wave function than the HA. More recently Farias and Peeters have also used the VM for determining both ground and excited polaron states taking into account the effect of a positive impurity charge localized in the substrate that supports the helium film.

Recently, there has been a growing interest in the effects of a corrugated helium surface in the properties of surface electrons. One motivation is to make use of suspended helium films to increase electron densities which are limited in the case of bulk helium by a surface instability and by the impossibility of obtaining high-mobility electrons on a
thin film, in which high densities could be achieved, due to surface roughness of the substrate. The other one is to confinesurface electrons in one and zero-dimensions as have been realized in semiconductor heterostructures. 

Quasi-one-dimensional (Q1D) electron systems on the surface of liquid helium have been realized by either geometric or electrostatic mechanisms which provide the distortion of the helium surface and a confining electric field holds the surface electrons along the formed liquid channels. Multi-wire systems have been created using dielectric optical gratings, substrate wrap by nylon threads, and metallic gate structures. A single wire was also performed using a sharply bent polymer film and metallic strips on printed circuit board. In such a Q1D system, the electron motion is restricted, in addition to the quantum well due to the holding electric in the direction normal to the liquid surface, by a lateral confinement and can be modelled in first order of approximation by a harmonic potential. In such conditions, the formation of an asymmetric polaron may become also possible.

In this paper, we address the question of polaron properties by using the hydrodynamic approach in the case of an anisotropic potential $U(x,y)$ due to surface corrugation and in particular for the Q1D electron system described by a parabolic confinement in the $y$ direction. The general formalism can be applied for both circularly symmetric and asymmetric polaron states. We consider the properties of both ground and excited polaron states and the anisotropic transport properties of the polaron are investigated for surface electrons on He$^4$ and He$^3$.

The paper is organized as follows. The general formalism and the main relations are described in Sec. II. In Sec. III we analyze the properties of the ground and excited polaron states. In Sec. IV we investigate the polaron mobility when a driving electric field is applied parallel to the liquid surface. In Sec. V we summarize our main results.

II. THEORETICAL FORMALISM

Electrons above the free surface ($z = 0$) of a helium film with thickness $d$ is prevented to penetrate into the liquid because of a high potential barrier ($\sim 1$ eV) at the liquid-vapor interface. If an electric field $E_z$ is applied in the $z$ direction, the electrons are trapped in a quantum well due to image forces coming from the liquid helium and substrate and determined by the potential $V(z) = eE_zz - \Delta_0/\epsilon - \Lambda_1/(z + d)$ where $\Delta_0 = e^2(\epsilon_{He} - 1)/4(\epsilon_{He} + 1)$, $\Lambda_1 = e^2\epsilon_{He}(\epsilon_s - \epsilon_{He})/2(\epsilon_{He} + \epsilon_s)$ with $\epsilon_{He}$ and $\epsilon_s$ the dielectric constants of helium and substrate respectively. If the barrier height is approximated as infinity, the condition $\Psi(x,y,z = 0) = 0$ for the electron wave function must be fulfilled for a flat surface. The situation changes drastically if we take into account the surface deformation $\xi(x,y)$. Now the boundary condition for $\Psi(x,y,z)$ has to be imposed at $z = \xi(x,y)$. Shikin and Monarkha show that the transformation to a new variable $z' = z - \xi(x,y)$ allows to avoid the perturbation in the boundary condition for $\Psi(x,y,z)$ leading, however, to modifications in the Schrödinger equation. For $\xi(0,0) \ll d$ and $\langle z \rangle \ll d$, where $\langle z \rangle$ is the mean electron distance from the surface, these modifications result, in particular, in the dependence of $V(z)$ on an effective holding electric field given by $E_z^* = E_z + \Lambda/ed$ with $\Lambda = e^2(\epsilon_s - 1)/4(\epsilon_s + 1)$ where we take $\epsilon_{He} \simeq 1$, and the appearance of an additional term $eE_z^*\xi(x,y)$.

The electron motion along the plane $(x,y)$ in the presence of a magnetic field $B$ in the $z$ direction is described by the Schrödinger equation

$$\frac{1}{2m}[(\hat{p}_x + \frac{eB}{2c}y)^2 + (\hat{p}_y - \frac{eB}{2c}x)^2] \psi(x,y) + [eE_z^*\xi(x,y) + U(x,y)] \psi(x,y) = \varepsilon \psi(x,y).$$

Here $\hat{p}_x$ and $\hat{p}_y$ are the $x$ and $y$ components of the momentum operator and we have chosen the symmetric gauge of the vector potential $\vec{A} = (-By/2, Bx/2, 0)$. Note that due to the explicit $(x,y)$-dependence of the potential term in Eq. (1), $\hat{p}_x$ and $\hat{p}_y$ are not conserved. The confinement potential $U(x,y)$ will be considered as a general anisotropic parabolic well given by

$$U(x,y) = \frac{m\omega_0^2}{2}(\alpha x^2 + y^2).$$

where $\alpha$ is the anisotropic parameter. For $\alpha = 1$, we restore the circular symmetry and $\alpha = 0$ corresponds the case of a Q1D electron considered in Refs. [11,12]. Then Eq. (1) can be rewritten as

$$-\frac{\hbar^2}{2m}\nabla^2 \psi - \frac{\hbar\omega_c}{2}\hat{L}_z \psi + \left[\frac{m}{8}(\omega_x^2 x^2 + \omega_y^2 y^2) + eE_z^*\xi\right] \psi = \varepsilon \psi,$$

where $\nabla$ is the 2D-Laplacian, $\omega_x^2 = \omega_0^2 + 4\alpha\omega_0^2$, $\omega_y^2 = \omega_0^2 + 4\alpha\omega_0^2$ with $\omega_c = eB/mc$ the cyclotron frequency, and $\hat{L}_z$ is the angular momentum operator along $z$ which is also not conserved for $\alpha \neq 1$ when the axial symmetry is lost. As a consequence, the second term in Eq. (2) does not contribute to energy eigenvalues $\varepsilon$ if the $\psi(x,y)$ is taken real. If we assume that $\psi$ is real and vanishes at infinity the electron energy can be calculated from

$$\varepsilon = \frac{\hbar^2}{2m}\left[n^2 + \frac{1}{2}(n^2 + n + 1)\right].$$

where $n = \langle \omega_x x^2 + \omega_y y^2 \rangle / \omega_c^2$. On the other hand, in the case of a Q1D system, the electron motion is described by the Schrödinger equation in the $y$ direction.

$$\frac{1}{2m}[\hat{p}_y^2 + eE_z^*\xi] \psi(x,y) = \varepsilon \psi(x,y).$$

where $\hat{p}_y$ is the momentum operator in the $y$ direction. The general solution is given by

$$\psi(x,y) = \sum_{n} \psi_n(x) \exp(i\alpha_n y) \exp(-i\varepsilon_n t),$$

where $\alpha_n = n\pi/d$ is a quantized parameter and the energy levels $\varepsilon_n$ are determined by the eigenvalue problem $\hat{L}_z \psi_n(x) = \varepsilon_n \psi_n(x)$. The wave function $\psi_n(x)$ is a normalized Gaussian function of $x$ and $y$.

The wave function $\psi(x,y)$ can be expanded in the form of the superposition of the states $\psi_n(x)$ as

$$\psi(x,y) = \sum_n \psi_n(x) \exp(i\alpha_n y) \exp(-i\varepsilon_n t).$$

where $\alpha_n = n\pi/d$ is a quantized parameter and the energy levels $\varepsilon_n$ are determined by the eigenvalue problem $\hat{L}_z \psi_n(x) = \varepsilon_n \psi_n(x)$. The wave function $\psi_n(x)$ is a normalized Gaussian function of $x$ and $y$.

The wave function $\psi(x,y)$ can be expanded in the form of the superposition of the states $\psi_n(x)$ as

$$\psi(x,y) = \sum_n \psi_n(x) \exp(i\alpha_n y) \exp(-i\varepsilon_n t).$$

where $\alpha_n = n\pi/d$ is a quantized parameter and the energy levels $\varepsilon_n$ are determined by the eigenvalue problem $\hat{L}_z \psi_n(x) = \varepsilon_n \psi_n(x)$. The wave function $\psi_n(x)$ is a normalized Gaussian function of $x$ and $y$.

The wave function $\psi(x,y)$ can be expanded in the form of the superposition of the states $\psi_n(x)$ as

$$\psi(x,y) = \sum_n \psi_n(x) \exp(i\alpha_n y) \exp(-i\varepsilon_n t).$$

where $\alpha_n = n\pi/d$ is a quantized parameter and the energy levels $\varepsilon_n$ are determined by the eigenvalue problem $\hat{L}_z \psi_n(x) = \varepsilon_n \psi_n(x)$. The wave function $\psi_n(x)$ is a normalized Gaussian function of $x$ and $y$.

The wave function $\psi(x,y)$ can be expanded in the form of the superposition of the states $\psi_n(x)$ as

$$\psi(x,y) = \sum_n \psi_n(x) \exp(i\alpha_n y) \exp(-i\varepsilon_n t).$$

where $\alpha_n = n\pi/d$ is a quantized parameter and the energy levels $\varepsilon_n$ are determined by the eigenvalue problem $\hat{L}_z \psi_n(x) = \varepsilon_n \psi_n(x)$. The wave function $\psi_n(x)$ is a normalized Gaussian function of $x$ and $y$.
The total energy of the complex “electron + dimple” can be defined as

\[ W = \varepsilon + \frac{\sigma}{2} \int \int \left[ (\nabla \xi)^2 + k_c^2 \xi^2 \right] \, dx \, dy, \]  

where \( k_c^2 = \rho g / \sigma \) is the capillary constant, \( g' = g (1 + 3 f / \rho g d^4) \), \( f \) is the van der Waals coupling constant of the helium to the substrate, \( \sigma \) and \( \rho \) are the surface tension coefficient and the mass density of helium, respectively, and \( g \) is the gravitational acceleration. As is seen from Eq. (3), the inequality \( \varepsilon > \varepsilon_0 \) is always satisfied. Minimizing the Eq. (3), we obtain the mechanical equilibrium equation

\[ \sigma (\nabla^2 \xi - k_c^2 \xi) = e E_\perp^* \psi^2. \]  

Note that the quantity of \( e E_\perp^* \psi^2 \) plays the role of an electron pressure on the liquid surface.

For the sake of completeness, we can rewrite the resulting Schrödinger equation after removing \( \xi(x,y) \) and \( \psi(x,y) \), as:

\[ -\frac{\hbar^2}{2m} \nabla^2 \psi - \frac{\hbar \omega_c}{2} \mathcal{L}_x \psi + \frac{m}{8} (\omega_\perp^2 x^2 + \omega_\parallel^2 y^2) \psi = e E_\parallel^* \psi + W \psi = \varepsilon \psi. \]  

Equation (10) is quite general and can be solved self-consistently and the results are used to evaluate the total polaron energy \( W \) and surface profile \( \xi(x,y) \). However this procedure is totally numerical and cumbersome even in the symmetric case \( \alpha = 1 \). [11] We prefer to make reasonable guesses about the structure of \( \psi(x,y) \) and obtain analytical results for the energetics and transport properties of the asymmetric polaron.

We also hereafter limit ourselves to the most studied case of the Q1D system model [21,22], where the surface electrons are confined by a lateral potential well corresponding to the completely anisotropic limit, \( \alpha = 0 \), i.e. \( U(y) = m \omega_\perp^2 y^2 / 2 \) with the characteristic frequency defined as \( \omega_\perp = \sqrt{e E_\perp^* / m R} \), where \( R \sim 10^{-4} \sim 10^{-3} \) cm is the curvature radius of the liquid in the channel. In the absence of a magnetic field the spectrum for the free electron motion is given by \( E(k_z, n) = \hbar^2 k_z^2 / 2m + \hbar \omega_\perp (n + 1/2) \) and the electron localization in the \( y \) direction estimated by the parameter \( L_0 = (\hbar / m \omega_\perp)^{1/2} \ll R \) and for this reason, curvature effects play no significant role because \( E_\perp^* \) pushes the electron to the bottom of the channel. This parameter somewhat changes in the presence of a magnetic field. [23] Typical values of \( L_0 \) are of the order of \( 10^{-9} \) cm [21,22] and the condition \( L_0 \ll R \) is satisfied with great accuracy. Obviously for the polaron state the localization parameter along the \( y \) direction will be significantly modified.
III. POLARON ENERGETICS

A. Ground-state

Based on the general structure of Eq. (11) we choose the trial function to describe the ground state of the polaron as

$$\psi_0(x, y) = \frac{1}{\pi^{1/2}(\ell_x \ell_y)^{1/2}} \exp \left[ -\frac{1}{2} \left( \frac{x^2}{\ell_x^2} + \frac{y^2}{\ell_y^2} \right) \right],$$

(11)

where $\ell_x$ and $\ell_y$ are the electron localization lengths in $x$ and $y$ directions, respectively. Substituting Eq. (11) into Eqs. (6)-(9), one obtains the surface deformation $\xi_0(x, y)$, and the total energy $W_0$ of the polaron ground state as

$$\xi_0(x, y) = \frac{eE^*_x}{4\pi^2\sigma} \int dx \int dy \exp \left[ -\left( \frac{k_x^2 \ell_x^2 + k_y^2 \ell_y^2}{2} \right) / 4 \cos(k_x x) \cos(k_y y) \right]$$

$$W_0 = -\frac{(eE^*_x)^2}{8\pi^2\sigma} \int dx \int dy \exp \left[ -\left( \frac{k_x^2 \ell_x^2 + k_y^2 \ell_y^2}{2} \right) / 2 \right] \frac{\hbar^2}{4m} \left( \frac{1}{\ell_x^2} + \frac{1}{\ell_y^2} \right)$$

$$+ \frac{m}{16} [\omega_c^2 \ell_x^2 + (\omega^2_c + 4\omega_0^2) \ell_y^2].$$

(12)

The ground state energy $\varepsilon^{(0)}$ of electron has the same form except by the coefficient of the integral which in this case is $(eE^*_x)^2/4\pi^2\sigma$. In the limit $k_x^2 (\ell_x^2 + \ell_y^2) \ll 1$, the integral in Eq. (12) can be evaluated analytically resulting in

$$W_0 \simeq -\frac{(eE^*_x)^2}{4\pi\sigma} \ln \left[ \frac{2\sqrt{2}}{\sqrt{\gamma}k_c(\ell_x + \ell_y)} \right] + \frac{\hbar^2}{4m} \left( \frac{1}{\ell_x^2} + \frac{1}{\ell_y^2} \right) + \frac{m}{16} [\omega_c^2 \ell_x^2 + (\omega_c^2 + 4\omega_0^2) \ell_y^2],$$

(13)

where $\gamma = \exp C$, and $C = 0.5772...$ is the Euler-Mascheroni constant. The value of $\varepsilon^{(0)}$ is near the same obtaining the coefficient 2 instead of 4 in the denominator of first term of Eq. (13).

The localization lengths $\ell_x$ and $\ell_y$, which appear in Eqs. (11)-(14), have been evaluated by a few methods. In particular, the HA similar to that used in Ref. [1] can be applied and is based on the following approximate expression for the surface deformation

$$\xi_0(x, y) \simeq \xi(0, 0) + \frac{1}{2} \left[ \xi_x^\prime(0, 0)x^2 + \xi_y^\prime(0, 0)y^2 \right].$$

(15)

The expansion is similar to that of the potential energy near its minimum value in the 2D oscillatory problem and reduces the problem of the electron motion in the dimple to the motion in a parabolic confinement potential, by defining $\ell_x$ and $\ell_y$ as localization parameters of the 2D harmonic oscillator. We prefer however to use the VM which allows to obtain $\ell_x$ and $\ell_y$ from the minimization conditions of the energy $W_0$, i.e. $\partial W_0 / \partial \ell_x = \partial W_0 / \partial \ell_y = 0$ of the minimum. Note that in the VM the key point for consideration is the expression for the polaron energy, given by the Eq. (13), whereas the structure of the dimple potential is less important and is taken into account through Eq. (11). From the experimental point of view, using the VM seems more convenient because the characteristic value of the helium surface depression in the centre of the dimple is about $10^{-8}$ cm for $E^*_x \sim 3$ kV/cm which is impossible to detect directly. Hereafter this estimative is based on actual holding fields in experiments on electrons along Q1D channels on bulk helium, where the effects of the film thickness are neglected. [47]. On the other hand, as will see, the energy gap between the ground and excited polaron states, calculated using the VM, may in principle be accessible experimentally.

In the VM, $\ell_x$ and $\ell_y$ can be find from the roots of the system of equations

$$\frac{1}{\ell_x^2} - \frac{1}{L_B^2 \ell_x} = \frac{1}{L_B^2 \ell_y} = 0 \quad \text{and} \quad \frac{1}{\ell_y^2} - \frac{1}{L_B^2 \ell_y} = \frac{1}{L_B^2 \ell_x} - \frac{1}{L_B^2} = 0,$$

(16)

where $L_B^2 = 2\pi e^2 / m(eE^*_x)^2$ and $L_B^2 = 2\hbar / m\omega_c$. Eqs. (16) have been solved numerically. The results are presented in Fig. 1 as a function of the holding field for some values of the magnetic field and for $^4$He as the liquid substrate. The analytical solution of Eqs. (16) is possible in some limiting cases. For $B = 0$ and very high holding fields $E^*_x \gg 4$...
kV/cm satisfying the condition $L_F \ll L_0$, one obtains $\ell_x \simeq \ell_y \simeq \sqrt{2}L_F$ which is the localization parameter for the symmetric polaron where the effects of the lateral confinement along $y$ are negligible. For holding fields in the range $1 < E_\perp^* < 3$ kV/cm, in the opposite limit $L_F \gg L_0$, one has $\ell_x \simeq L_F$ and $\ell_y \simeq L_0$ which correspond to the numerical results shown in Fig. 1. Hence the localization length $\ell_y$ is almost the same as that for the electron moving freely along the Q1D channel. Numerical estimates at $B = 0$ are $\ell_x \sim 10^{-5}$ cm and $\ell_y \sim 10^{-6}$ cm for $1 < E_\perp^* < 3$ kV/cm. These values are significantly smaller than the curvature radius $R$. It means that not only the condition $\ell_y \ll R$, which supports the validity of our Q1D confinement model, is fulfilled but also the condition $\ell_x \ll R$ is satisfied. The application of the magnetic field leads to a decrease of localization lengths in comparison with those for $B = 0$. For high $B$ ($\omega_c \gg \omega_0$), one obtains from Eqs. (14) $\ell_x \simeq \ell_y \simeq 2L_F$ for $L_F \ll L_B$ and $\ell_x \simeq \ell_y \simeq L_F$ for $L_F \gg L_B$. We point out that $\ell_x$ and $\ell_y$, calculated using the HA, give different asymptotic values: $\ell_x \simeq \ell_y \simeq L_F$ for $L_F \ll L_0, L_B$ and for $B = 0$, the results are $\ell_x \simeq L_F/\sqrt{2}$ and $\ell_y \simeq L_0$ at $L_F \gg L_0$. For high magnetic fields and $L_F \gg L_B$, one has $\ell_x \simeq \ell_y \simeq L_B$. It is interesting also to note that if the VM is used for the energy of the electron trapped in the dimple $\varepsilon^{(0)}$, instead of $W_0$, the results are the same as those obtained in HA. One can conclude the solutions of Eqs. (14) yielding $\ell_x = \ell_y$, which is the case of a symmetric polaron, appear either in the limit of very high holding field (small $L_F$) or for high magnetic fields ($\omega_c \gg \omega_0, L_F \gg L_B$).

Using the localization lengths from Eq. (14), we depicted in Fig. 2 the polaron energy as a function of the holding field for some values of the magnetic field. For $\ell_x \simeq L_F$ and $\ell_y \simeq L_0$ and $B = 0$, Eq. (14) is rewritten as

$$W_0 \simeq -\frac{(eE_\perp^*)^2}{4\pi\sigma} \ln \frac{2\sqrt{2}}{\sqrt{\gamma_k_c L_F}} + \frac{\hbar^2}{4mL_F^2} + \frac{\hbar\omega_0}{2}.$$  (17)

Defining the binding energy $E_b$ as the energy of the polaron state minus the electron energy $\hbar\omega_0/2$ of the free motion in the lowest subband of the lateral potential, one obtains

$$E_b \simeq -\frac{(eE_\perp^*)^2}{4\pi\sigma} \ln \frac{2\sqrt{2}}{\sqrt{\gamma_k_c L_F}} + \frac{\hbar^2}{4mL_F^2}.$$  (18)

Note that we have considered very low temperatures $T \ll \hbar\omega_0$ where only the lowest subband $n = 0$ is occupied (for the classical SE system $k_x \sim \sqrt{2mT/\hbar}$). The polaron state is preferable energetically at $T < |E_0|$, otherwise thermal motion can liberate electron from the dimple. We estimate $E_b \simeq -0.03$ K and $E_\ast \simeq -0.3$ K for holding electric fields of $1$ V/cm and $3$ kV/cm, respectively, in the case of a $^4$He substrate. One can conclude that the energetic conditions for the formation of the polaron in the Q1D electron system on the liquid helium surface are almost the same as in the case of the 2D symmetric polaron.

For magnetic fields satisfying the conditions $\omega_c \gg \omega_0, L_F \ll L_B$, the binding energy, defined by extracting the cyclotron energy $\hbar\omega_\theta/c$ of the lowest Landau level from $W_0$ in Eq. (14) can be written as

$$E_b \simeq -\frac{(eE_\perp^*)^2}{4\pi\sigma} \ln \frac{\sqrt{2}}{\sqrt{\gamma_k_c L_B}},$$  (19a)

which agrees with the asymptotic value of $E_\ast$ for the symmetric polaron ($\omega_\theta = 0$) for very high $B$. For $B = 5.5$ T ($\omega_c = 10^{12}$ s$^{-1}$), we estimate $L_B = 1.52 \times 10^{-6}$ cm and $E_\ast \simeq -0.36$ K for $E_\perp^* = 3$ kV/cm, which is very close to that for $B = 0$.

We have also estimated the profile of the surface from Eq. (13). For large distances such that $k_c x$ and $k_c y \gg 1$, $\xi(x,y)$ decreases exponentially as a function of the distance $r = \sqrt{x^2 + y^2}$ in the same manner as for symmetric polaron. (11) The value of the dimple depth in its centre can be written as

$$\xi_0(0,0) \simeq -\frac{eE_\perp^*}{2\pi\sigma} \ln \frac{4}{\sqrt{\gamma_k_c (\ell_x + \ell_y)}}.$$  (20)

For $B = 0$ and $\ell_x \simeq L_F$, $\ell_y \simeq L_0$, we obtain $\xi_0(0,0) \simeq -(eE_\perp^*/2\pi\sigma) \ln(4/\sqrt{\gamma_k_c L_F}) \simeq -1.9 \times 10^{-8}$ cm for $E_\perp^* = 3$ kV/cm if the liquid substrate is $^4$He. For high $B$, where $L_F \gg L_B$ and $\omega_c \gg \omega_0$, one has $\xi_0(0,0) \simeq -(eE_\perp^*/2\pi\sigma) \ln(2/\sqrt{\gamma_k_c L_B}) \simeq -2.2 \times 10^{-8}$ cm.

It is interesting how to recover from our results those for the symmetric polaron where $\omega_\theta = 0$. In this case $x$ and $y$ directions are equivalent and the $z$ component of the angular moment conserves. Due to this one can replace the operator $\hat{L}_z$ of angular moment in Eqs. (3) and (10) by its eigenvalue $l_z$ which leads to the new contribution $\hbar\omega_{l_z}/2$ to the electron energy $\varepsilon$ in the case of non-zero magnetic field. We should put $l_z = 0$ in the ground state of the polaron with lowest angular momentum. Moreover this contribution is zero for real electron wave functions.
in the polaron state even for nonzero angular momentum. If \( \omega_0 = 0 \), the solution of Eqs. (16) is \( \ell = \ell_x = \ell_y \) with 
\[ 1/\ell^2 = 1/4L_B^2 + \sqrt{1/16L_F^2 + 1/4L_B^2} \]
and the electron wave function for the ground state is written as
\[ \psi_0(x, y) = \frac{1}{\pi^{1/4} \ell} e^{-r^2/2\ell^2} \]
which leads to the same results for \( W_0, \xi_0(r) \) as previously found in the case of a symmetric polaron. \[ \text{In particular, the polaron energy is} \]
\[ W_0 \approx -\frac{(eE_L)^2}{4\pi\sigma} \left[ \ln \frac{1}{\sqrt{\gamma}k_F L_F} - 1 \right]. \]
One can easily also obtain that the second-order derivative of \( \xi(r) \) at the centre of the dimple \( [\xi''(r,0)]_0 = eE_L^2 / 2\pi\sigma\ell^2 \)
which shows the existence of the minimum at \( r = 0 \).

### B. Excited states

We must choose a trial excited-state wave function orthogonal to ground state wave function given by Eq. (11), which as we have seen is the same as wave function for the 2D asymmetric harmonic oscillator in Cartesian coordinates. Hence it is natural to propose the wave function of excited states of the 2D harmonic oscillator as the trial functions for the excited polaron states:
\[ \psi_{10}(x, y) = \frac{\sqrt{\pi}}{\pi^{1/2} (\delta \delta_y)_{1/2}} \exp \left[ \frac{1}{2} \left( \frac{x^2}{\delta_x^2} + \frac{y^2}{\delta_y^2} \right) \right]. \]

Evidently another excited state \( |0,1 \rangle \) can be considered with wave function \( \psi_{01}(x, y) \). The wave function \( \psi_{01} \) has the same form as \( \psi_{10} \) by replacing \( x \) by \( y \) in Eq. (21). We find however that the state \( |1,0 \rangle \) has energy smaller than \( |0,1 \rangle \). For this reason the state \( |1,0 \rangle \) should be considered the first excited polaron state and we are looking for results for this state. The results for the state \( |0,1 \rangle \) can be easily obtained in a straightforward way.

Following the same procedure as for the ground state we arrive to following expressions of the polaron energy and the profile of the dimple in excited state:
\[ W_{10} = -\frac{(eE_L)^2}{8\pi^2\sigma} \int dk_x \int dk_y \left( 1 - k_x^2 \delta_x^2 / 2 \right) \exp \left[ -\left( k_x^2 \delta_x^2 + k_y^2 \delta_y^2 \right) / 2 \right] + \frac{\hbar^2}{4m} \left( \frac{3}{\delta_x^2} + \frac{1}{\delta_y^2} \right) \]
\[ + \frac{m}{16} \left[ 3\omega_0^2 \delta_x^2 + (\omega_0^2 + 4\omega_c^2) \delta_y^2 \right] \]
and
\[ \xi_{10}(x, y) = -\frac{eE_L}{4\pi^2\sigma} \int dk_x \int dk_y \left( 1 - k_x^2 \delta_x^2 / 2 \right) \exp \left[ -\left( k_x^2 \delta_x^2 + k_y^2 \delta_y^2 \right) / 4 \right] \cos(k_x x) \cos(k_y y). \]

As before the electron energy \( e^{(10)} \) has the coefficient \( -(eE_L)^2 / 4\pi^2\sigma \) in first term in Eq. (22).

The \( x \) dependence of \( \psi_{10}(x, y) \) contributes strongly to decrease the electronic pressure on the liquid surface at \( x = 0 \), at the centre of the dimple. According to Eq. (13), the pressure is proportional to \( \psi_{10}^2(x, y) \). As a consequence, in the limit \( k_x^2 (\delta_x^2 + \delta_y^2) \ll 1 \), the second-order partial derivative \( [\xi''_{xy}]_{10}(0,0) \) calculated by Eq. (23)
\[ [\xi''_{xx}]_{10}(0,0) = -[\xi''_{yy}]_{10}(0,0) \approx -eE_L^2 / \pi \sigma (\delta_x + \delta_y)^2 \]
This result is a direct consequence of the structure of the trial wave function \( \psi_{10}(x, y) \) and means that the function \( \xi_{10}(x, y) \) has a maximum at \( x = 0 \) for fixed \( y \) whereas a minimum at \( y = 0 \) at fixed \( x \) and one can not conclude definitely about the nature of the dimple potential as a function of two variables \( x \) and \( y \) at \( x = y = 0 \). In such condition the use of the HA to excited states is inappropriate because it is based on the expansion given by Eq. (13) near the minimum of the dimple potential at \( x = y = 0 \). For this reason the VM is the only consistent way to calculate the polaron properties in the excited state.

As before the localization lengths \( \delta_x \) and \( \delta_y \) can be obtained, in VM, from the conditions of a minimum of energy \( W_{10} \) given by Eq. (22). For \( k_x^2 (\delta_x^2 + \delta_y^2) \ll 1 \), we obtain
\[ W_{10} \approx -\frac{(eE_L)^2}{4\pi\sigma} \left[ \ln \frac{2 \sqrt{2 \delta_x \delta_y}}{\sqrt{\gamma}k_F (\delta_x + \delta_y)} - \delta_x (2\delta_x + 3\delta_y) \right] + \frac{\hbar^2}{4m} \left( \frac{3}{\delta_x^2} + \frac{1}{\delta_y^2} \right) \]
\[ + \frac{m}{16} \left[ 3\omega_0^2 \delta_x^2 + (\omega_0^2 + 4\omega_c^2) \delta_y^2 \right]. \]
Imposing that $\partial W_{10}/\partial x = \partial W_{10}/\partial y = 0$ we arrive to the system of equations

$$
\begin{align*}
\frac{3}{\delta_x^2} - \frac{1}{L_F^2 \delta_x (\delta_x + \delta_y)} \left[ 1 + \frac{\delta_y (\delta_x + 3 \delta_y)}{4(\delta_x + \delta_y)^2} \right] - \frac{3}{L_B^2} &= 0 \\
\frac{1}{\delta_y^2} - \frac{1}{L_F^2 \delta_y (\delta_x + \delta_y)} \left[ 1 - \frac{\delta_x (\delta_x + 3 \delta_y)}{4(\delta_x + \delta_y)^2} \right] - \frac{1}{L_B^2} &= 0.
\end{align*}
$$

In the limit of very high holding fields where $L_F \ll L_0, L_B$, we found $\delta_x \approx 2.12L_F$; $\delta_y \approx 1.73L_F$. The localization parameters obtained from the minimization of $\varepsilon^{(10)}$ are $\sqrt{2}$ times smaller. If $B = 0$, the analytical solution of Eqs. (23) can be found in the limit $L_0 \ll L_F$: $\delta_x \approx \sqrt{3}L_F$; $\delta_y \approx L_0$. Finally for high $B$ ($\omega_c \gg \omega_0$) one obtains $\delta_x \approx \delta_y \approx L_B$.

We now discuss the energy gap $\varepsilon^{(10)} - \varepsilon^{(0)}$ between the ground and excited states of the electron trapped in the dipole. The numerical results of the spectroscopic transition frequency $\omega_{10-0} = \varepsilon^{(10)} - \varepsilon^{(0)}/\hbar$ are presented in Fig. 3 as a function of the holding field for zero magnetic field and two substrates $^3$He and $^4$He. In the range $1 < E_z^* < 3$ kV/cm, the curves can be described by the analytical expression $\omega_{10-0} = (eE_z^*)^2/(2\pi\delta \hbar)$, and changes from $9.8 \times 10^8$ s$^{-1}$ to $8 \times 10^9$ s$^{-1}$ for the polaron over $^3$He which corresponds to increasing electron energy from $7.5 \times 10^{-3}$ K to $6.2 \times 10^{-2}$ K. This increase is significantly smaller than $|E_b|$ calculated by Eq. (18), and the electron transition from ground to excited states does not destroy the polaron state. It is interesting to note that $W_{10} - W_0 \approx (\varepsilon^{(10)} - \varepsilon^{(0)})/2$ in such conditions is significantly smaller than $|E_b|$ and that also $\omega_{10-0} \ll \omega_0$, which characteristic values vary from $5.9 \times 10^{10}$ s$^{-1}$ to $1.0 \times 10^{11}$ s$^{-1}$ when $E_z^*$ increases from 1 to $3$ kV/cm. One should emphasize that the spectroscopic frequencies $\omega_{01-0}$ for the transition from the ground to the excited state $|0, 1\rangle$ are significantly higher than $\omega_{10-1}$ and are the same as $\omega_0$. It means that after such a spectroscopic transition the electron energy increases from 0.4 K to 0.7 K when $E_z^*$ lies in the range of $(1 - 3)$ kV/cm. These energies are significantly higher than $|E_b|$ and this transition should destroy the polaron state. For this reason only the state $|1, 0\rangle$ can be considered as the excited polaron state.

The surface deformation for this state is given by

$$
\xi_{10}(0,0) = -\frac{eE_z^*}{2\pi\sigma} \ln \frac{4}{\sqrt{\pi}k_c(\delta_x + \delta_y)} - \frac{\delta_x}{\delta_x + \delta_y},
$$

which follows from Eq. (22) in the limit of $k_c \sqrt{\delta_x^2 + \delta_y^2} \ll 1$. The absolute values of $\xi_{10}(0,0)$ are smaller than those of $\xi_{00}(0,0)$. For $E_z^* = 3$ kV/cm, $\xi_{10}(0,0) \approx -1.67 \times 10^{-8}$ cm for $B = 0$.

The problem of recovering the symmetric case from the previous results in the case of excited states is more complicated than for the ground state. If $\omega_0 = 0$, the excited state can be described by the trial wave function

$$
\psi_1(r, \varphi) = \left( \frac{1}{\sqrt{\pi}\Delta^2} \right) r \exp(-r^2/2\Delta^2) e^{i\xi L_z} \hat{L}_z = \pm 1,
$$

which is eigenfunction of the angular moment $\hat{L}_z$ corresponding to the excited state of the 2D harmonic oscillator in polar coordinates and $\Delta$ is the single localization parameter. However $\psi_{10}(x, y)$ is not an eigenfunction of $\hat{L}_z$ and hence cannot be reduced to $\psi_1(r, \varphi)$. For this reason the formal dependence of the excited polaron state on two localization lengths $\delta_x$ and $\delta_y$ must be kept for $\omega_0 = 0$. In particular $\delta_x \neq \delta_y$ even at very high $E_z^*$ ($L_F \ll L_0$) where the effects of the potential confinement are negligible and the properties of the symmetric excited polaron state should be reproduced. For $\delta_x \approx 2.12L_F$, and $\delta_y \approx 1.73L_F$ ($B = 0$), the symmetric polaron energy in the excited state of symmetric can be written as

$$
W_1 = -\frac{(eE_z^*)^2}{4\pi\sigma} \ln \frac{0.550}{k_c L_F} \frac{0.837}{L_B}.
$$

Only at high magnetic fields ($L_F \ll L_B$ and $\omega_c \gg \omega_0$) one has $\delta_x \approx \delta_y$, which means that the influence of $B$ on the electron localization is dominant over confinement effects in the limit when the scale of electron localization is determined by the magnetic length $L_B$.

It is interesting to know about the structure of the dipole at $r = 0$ in the limit of $\omega_0 = 0$. To make the problem more clear, we consider the second-order derivative $[\xi_{10}''']_{10}(0)$ in the point of $r = 0$ starting from wave function given by Eq. (21) with $\delta_x \approx 2.12L_F$ and $\delta_y \approx 1.73L_F$. However for the trial wave function $\psi_{10}(x, y)$, $[\xi_{10}''']_{10}$ depends explicitly on $\varphi$ after conversion to polar coordinates. This dependence has no physical sense for the symmetric polaron state and results from the inadequacy of $\psi_{10}$ to describe correctly the excited state in the limit of $\omega_0 = 0$. Since in
the necessary adjustments of the expressions of \(W\) in the liquid, which is accompanied by energy dissipation, and leads to a finite value of the polaron mobility. \[1,2,10,11\]

\(\rho_{xy}\) plane Peeters in a fully numerical calculation. \[12\]

\(m\bar{\eta}\) the energy gap between the ground and excited electron states over a helium film when a localization potential is

In order to evaluate the polaron mobility, we employ the energy balance equation in comparison with the weak coupling limit where the electron is simply scattered by ripplons at

\(\ell\) Eqs. (29)-(31) and vice versa. If

\(k\) in the case of \(E\) \(\eta\) by \[1,11\]

\(\langle\rangle\) is the step function. The general Eq. (29) is quite simplified in the following limiting cases:

\[\mu_x \simeq \frac{\pi^{3/2} \sigma^2 \varepsilon_x}{\sqrt{2e(E_\perp^*)^2 \eta}} \text{ if } \varepsilon_x \gg \varepsilon_y\]  

(30)

and

\[\mu_x \simeq \frac{\pi^{3/2} \sigma^2 \varepsilon_y}{\sqrt{2e(E_\perp^*)^2 \eta} \ln(\varepsilon_y/\varepsilon_x)} \text{ if } \varepsilon_x \ll \varepsilon_y.\]  

(31)

The polaron mobility along the \(y\) direction, \(\mu_y\), is the same as for \(\mu_x\) but the index “\(x\)” must be replaced by “\(y\)” in Eqs. (29)-(31) and vice versa. If \(\ell_x = \ell_y = \ell\), both \(\mu_x\) and \(\mu_y\) reproduce the mobility of the symmetric polaron given by \[1,11\]

---

**IV. POLARON TRANSPORT**

We now investigate the transport properties of the Q1D polaron. When a driving electric field \(E_\parallel\) is applied along the plane \(xy\), the surface deformation moves together with the trapped electron inducing a field of hydrodynamic velocities in the liquid, which is accompanied by energy dissipation, and leads to a finite value of the polaron mobility. \[1,2,10,11\] This approach is valid only in the strong coupling limit (high \(E_\perp^*\)) where the self-trapped state is energetically favored in comparison with the weak coupling limit where the electron is simply scattered by ripplons at \(T < 1\) K. \[22\] In order to evaluate the polaron mobility, we employ the energy balance equation \(eE_\parallel v_0 = d\rho_E/dt\), where \(v_0\) is the liquid velocity at infinity, and \(\rho_E\) is the energy density dissipated. The function \(d\rho_E/dt\) is obtained in a straightforward way by finding the normal velocity field induced by the polaron from the solution of the Navier-Stokes equation. The calculation procedure in the asymmetric case is similar to that used in Refs. \[1,2\] for the symmetric polaron and are based in the same system of equations and boundary conditions. However the asymmetry of the surface deformation in \(x\) and \(y\) directions should be taken into account and 2D Fourier transforms like Eqs. (7) should be employed. If the driving field is applied along the \(x\) (\(y\)) direction, the mobility is given by

\[\mu_{x(y)} = \frac{e}{2\eta S} \left[ \sum_k k k_x^2 |k|^2 \right]^{-1}, \]  

(28)

where \(\eta\) is the helium viscosity. The summation in Eq. (28) can be performed analytically and the polaron mobility in the case of \(E_\parallel\) along the \(x\) direction can be written in terms of the complete elliptic integrals \(K(t)\) and \(E(t)\) as

\[\mu_x = \frac{\pi^{3/2} \sigma^2}{\sqrt{2e(E_\perp^*)^2 \eta}} \left[ \frac{(\ell_y^2 - \ell_x^2) \Theta(\ell_x - \ell_y)}{\ell_x E \left(1 - \ell_x^2/\ell_y^2\right) - (\ell_y/\ell_x)^2 K \left(1 - \ell_x^2/\ell_y^2\right)} \right. \]

\[+ \left. \frac{(\ell_y^2 - \ell_x^2) \Theta(\ell_y - \ell_x)}{\ell_y E \left(1 - \ell_x^2/\ell_y^2\right) - K \left(1 - \ell_x^2/\ell_y^2\right)} \right] \}

(29)

Here \(\Theta(t)\) is the step function. The general Eq. (29) is quite simplified in the following limiting cases:

\[\mu_x \simeq \frac{\pi^{3/2} \sigma^2 \ell_x}{\sqrt{2e(E_\perp^*)^2 \eta}} \text{ if } \ell_x \gg \ell_y\]  

(30)
the results in the case of finite electron densities is limited by the condition that the scale of electron localization is restricted if we have a single Q1D channel. One possibility should be to use weakly coupled electron system with low level. This signal is proportional to the electron density which is especially like in multiple quantum wells in semiconductor heterostructures. If the average distance between the channels $b \gg a$, one can disregard the correlation between electrons in different channels and the total response should be enhanced by the factor $N_{ch}$. This turns more appropriate the conditions for experimental investigation of Q1D electron systems over liquid helium in the low-density limit.

V. CONCLUDING REMARKS

In this work, we have discussed the possibility for the formation of the polaron in the context of anisotropic liquid helium surface. In particular, we have evaluated the energetics and transport properties of the polaron in a Q1D channel. We have considered the properties of both ground and excited states of the asymmetric polaron. The localization lengths and the ground and excited state energies have been determined as functions of the holding electric field within the hydrodynamical model and using a variational approach. We have obtained the frequency of spectroscopic transitions between the ground and first levels in the asymmetric polaron states. The transport properties of the polaron along the corrugated helium surface have been also studied.

We think that the measurement of the frequency of spectroscopic transition $\omega_{10-0}$ between ground and excited states (see Fig. 4) should be favorable in experimental attempts to observe the polaron state in Q1D channel over liquid helium surface. Indeed, our estimates indicate that $\omega_{10-0}$ differs significantly of $\omega_0$ of spectroscopic transitions between subbands of free electron state in Q1D channel which allows to separate the experimental signals for these two different types of spectroscopy transitions.

Another possibility is measuring directly the mobility as in Ref. [1]. Our results point out that the mobility of the Q1D polaron is 40% smaller than the mobility of the 2D polaron for the same holding field in the range $E_{1}^* < 3$ kV/cm. Our value of the Q1D polaron mobility $\sim 10^6$ cm$^2$/Vs for $T = 0.1$ K and $E_{1}^* = 3$ kV/cm, is three orders of magnitude smaller than the mobility of 2D electrons over a flat helium surface and that for electrons moving freely along the channel [2], which allow us to easily distinguish them easily.

While the above estimates show that our proposed mechanism for the polaron formation in Q1D systems on helium should be compatible with accessible experiments, we add other comments to corroborate or refute our predictions. Of particular interest is the possibility to use liquid $^3$He instead $^4$He as the substrate for the confined Q1D electron system. [12] Recall that for $^3$He, whose surface tension coefficient is more than twice smaller than that of $^4$He, the binding energy $|E_b|$ can be substantially larger. Also the spectroscopic transition frequencies are twice larger for $^3$He in comparison with $^4$He. Furthermore, the description of the polaron in terms of a hydrodynamic viscous model may be doubtful for pure $^4$He for temperatures below 1 K. However, we must emphasize that experimental results [1] have been successfully interpreted by theoretical calculations from the hydrodynamic approach for $T < 1$ K, even though this interpretation has been seriously questioned. [5] Furthermore the hydrodynamic approach of viscous phenomena is well satisfactory in the case of $^3$He for $T$ down to $\sim 0.1$ K. Other attractive substrate should be also the low $T$ mixture of $^3$He and $^4$He. [22] Recent experiments on Q1D-channels have provided a higher effective holding field as for electrons on helium films which could greatly enlarging the absolute value of the binding energy $E_b$. [20]

The electron states have been described in the present work in the one-electron approximation. The validity of the results in the case of finite electron densities is limited by the condition that the scale of electron localization is small in comparison with the mean interelectronic distance $a \simeq n_i^{-1}$ along the channel axis where $n_i$ is linear electron density. For $B = 0$ and $E_{\perp} < 3$ kV/cm where $\ell_x \simeq L_F \gg \ell_y \simeq L_0$ the characteristic values of $\ell_x \sim 10^{-5}$ cm which conditions $n_i \ll 10^5$ cm$^{-1}$. At higher electron densities, correlation effects could strongly influence the polaron properties and the applicability of our theory developed for the low-density limit becomes doubtful. For low electron densities, the experimental difficulties may appear in providing the measurements of experimental signal of the response of electron system with low level. This signal is proportional to the electron density which is especially restricted if we have a single Q1D channel. One possibility should be to use weakly coupled $N_{ch}$ multi-channels [4][16] like in multiple quantum wells in semiconductor heterostructures. If the average distance between the channels $b \gg a$ one can disregard the correlation between electrons in different channels and the total response should be enhanced by the factor $N_{ch}$. This turns more appropriate the conditions for experimental investigation of Q1D electron systems over liquid helium in the low-density limit.
One hope that the asymmetric polaron can be detected at low temperatures, probably at $T < 0.1$ K, and modern experimental methods for achieving low and ultralow temperatures would offer the possibility to observe the polaron in the Q1D channel on the liquid helium surface.

VI. ACKNOWLEDGMENT

This work was partially sponsored by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Brazil.
FIGURE CAPTIONS

Fig. 1. Localization parameters $\ell_x$ (a) and $\ell_y$ (b) in units of $L_F = (2\pi \hbar^2 \sigma / m)^{1/2} / (eE_0^*)$ for the polaron ground-state on the surface of $^4$He as a function of the holding electric field $E_0^*$ for some values of the magnetic field.

Fig. 2. Total energy of the polaron ground-state as a function of the holding electric field on the surface of $^4$He as a function of the holding electric field.

Fig. 3. Frequency of spectroscopic transition from the ground to excited electronic states in the asymmetric dimple for some values of the magnetic field.

Fig. 4. Longitudinal ($\mu_1$) and transversal ($\mu_2$) polaron mobilities over $^4$He as a function of the holding electric field calculated using by Eq. (29) (solid line) and by the approximate expression [Eqs. (30) and (31)] (dashed line) for zero magnetic field.

Fig. 5. Longitudinal polaron mobility versus the holding electric field for some values of the magnetic field.
Fig. 1 (a):
Sokolov et al.
"Polaron states of ..."
Fig. 1 (b):
Sokolov et al.
"Polaron states of ..."
Fig. 3:
Sokolov et al.
"Polaron states of ..."
Fig. 4:
Sokolov et al.
"Polaron states of ..."
Fig. 5:
Sokolov et al.
"Polaron states of ..."