X-RAY GAS DENSITY PROFILE OF CLUSTERS OF GALAXIES FROM THE UNIVERSAL DARK MATTER HALO

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ABSTRACT

The X-ray cluster gas density distribution in hydrostatic equilibrium is computed from the universal density profile of the dark matter halo proposed by Navarro, Frenk, & White in two recent studies. If one assumes the isothermality, the resulting distribution is well approximated by the conventional $\beta$-model. We predict the core radius $r_c$, the $\beta$-parameter, and the X-ray luminosity of clusters as a function of the temperature $T_X$ of clusters in some representative cosmological models and compare them with observations and results of numerical simulations. The predicted size of $r_c$ is a factor of 3–10 smaller than the average of observed values. If both the universal density profile and the hydrostatic equilibrium are reasonable approximation to the truth, then this suggests either that the previous X-ray observations systematically overestimated the core radius of gas densities in clusters of galaxies or that some important physical mechanisms, which significantly increase the core radius, are still missing.

Subject headings: cosmology: theory — dark matter — galaxies: clusters: general — X-rays: galaxies

1. INTRODUCTION

While it has been recognized for a long time that the gas density profile of X-ray clusters is well approximated by the isothermal $\beta$-model,

$$n_g(r) = \frac{n_{g0}}{[1 + (r/r_s)^2]^{3/2}} ,$$

the origin of this functional form is not yet accounted for. Navarro, Frenk, & White (1996, 1997, hereafter NFW96, NFW97) found that the virialized halos in numerical simulations are well fitted by the following universal profile:

$$\rho_{DM}(r) = \frac{\delta_c \rho_c}{(r/r_s)(1 + r/r_s)^2} ,$$

where $\rho_c$ is the critical density of the universe at $z = 0$, and

$$\delta_c(M) \approx 3 \times 10^3 \Omega_m [1 + z_f(M)]^3 ,$$

$$r_c(M) = \frac{r_{vir}(M)}{c(M)} = \frac{1}{c(M)} \left( \frac{3M}{4\pi \Delta_c \rho_c} \right)^{1/3} .$$

In the above expressions, $\Omega_m$ is the density parameter at $z = 0$, $\Delta_c$ is the collapse factor in a spherical nonlinear model, $z_f(M)$ is the average formation redshift of objects of mass $M$, and the concentration parameter $c(M)$ is related to $\delta_c(M)$ as

$$\delta_c = \frac{\Delta_s}{\frac{3}{2} \ln (1 + c) - c/(1 + c)} .$$

NFW96 also mentioned that under the gravitational potential of the dark halo (eq. [2]), the gas in hydrostatic equilibrium reaches a profile quite similar to equation (1). Using a hydrodynamical simulation, Eke, Navarro, & Frenk (1997, hereafter ENF) examined in more detail the properties of X-ray clusters in a cold dark matter model with $\Omega_m = 0.3$, $\Delta_s = 0.7$, $h = 0.7$, and $\sigma_8 = 1.05$, where $\lambda_0$ is the dimensionless cosmological constant, $h$ is the Hubble constant in units of $100$ km s$^{-1}$ Mpc$^{-1}$, and $\sigma_8$ is the top-hat mass fluctuation amplitude at $8 h^{-1}$ Mpc. They also found that the isothermal $\beta$-model describes well the simulated cluster gas distribution.

In the present paper we derive an analytical expression for the gas density profile embedded in the universal dark matter halo (eq. [2]) assuming hydrostatic equilibrium and isothermal distribution (§ 2). The resulting distribution is remarkably well fitted by equation (1). With this we are able to predict the core radius, the $\beta$-parameter, and the X-ray luminosity of clusters as a function of the mass $M$ or temperature $T_X$ of clusters once the cosmological model is specified. Our predictions are compared with the results of ENF simulations and observations in § 3. Finally, § 4 is devoted to discussion of further implications.

2. HYDROSTATIC EQUILIBRIUM OF GAS AND DARK MATTER

Consider an isothermal spherical gas cloud with temperature $T_X$; then its density distribution $\rho_g$ in hydrostatic equilibrium satisfies

$$\frac{kT_X}{\mu m_p} \frac{d\ln \rho_g}{dr} = - \frac{GM(r)}{r^2} ,$$

where $\mu$ and $m_p$ denote the mean molecular weight (we adopt 0.59 below) and the proton mass. If one neglects the gas and galaxy contributions to the gravitational mass on the right-hand side, then equation (2) yields

$$M(r) = 4\pi \delta_c \rho_c r_s^3 \ln \left( 1 + \frac{r}{r_s} - \frac{r_s}{r + r_s} \right) ,$$

and equation (6) can be analytically integrated to give

$$\rho_g(r) = \rho_{g0} \exp \left\{ - \frac{27}{2} \left[ 1 - \ln \left( 1 + \frac{r}{r_s} \right) \right] \right\}$$

$$= \rho_{g0} e^{-27h_0} \left( 1 + \frac{r_s}{r} \right)^{27h_0/(2r/r_s)} ,$$

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The cluster gas temperature $T_b$ is expected to be close to the virial temperature $T_{\text{vir}}(M)$ of the dark halo. In the profile (2), the latter is in fact dependent on the radius $r$:

$$kT_{\text{vir}}(r) = \gamma \frac{G\mu m_p M(r)}{3r},$$

where $\gamma$ is a fudge factor of order unity that should be determined by the efficiency of the shock heating of the gas; ENF adopted $\gamma = 1.5$ while Kitayama & Suto (1997) adopted 1.2 as their canonical value in the analysis of X-ray cluster number counts. If one substitutes equation (10) into equation (9), one finds that

$$b(r) = \frac{2}{9\gamma} \left[ \ln \left( 1 + \frac{r}{r_s} \right) - \frac{r}{r + r_s} \right]^{-1}.$$  

Throughout the present paper we assume that the cluster gas is isothermal; this is observationally true for most clusters and theoretically expected because of the effect of the thermal conduction. In that case it is reasonable to adopt either $T_{\text{vir}}(r)$ or $T_{\text{vir}}(r_s)$ as the gas temperature $T_b$. This corresponds to $b(r_s) \approx 1.15/\gamma$, or $b(T_{\text{vir}}) = 2c/(9\gamma)[\ln(1 + c) - c/(1 + c)]^{-1} \approx (1.3 \sim 2)/\gamma$.

The derived functional shape (eq. [8]) may appear completely different from equation (1) at a first glance. As is shown in Figure 1, however, it is surprisingly well approximated by the isothermal $\beta$-model profile:

$$\rho_b(r) = \frac{\rho_{b0} A(b)}{[1 + (r/r_{\text{core}})^2]^{\frac{3}{2}+\beta/2}},$$

with $A(b) = -0.178b + 0.982$, $r_{\text{core}} = 0.22r_s$, and $\beta = 0.9b$ over $0.01r_s < r < 10r_s$. Interestingly, this implies that $\beta_{\text{core}}(r_s) \approx 1/\gamma$ and $\beta_{\text{eff}}(r_{\text{vir}}) \approx (1-2)/\gamma$ if we take $T_b = T_{\text{vir}}(r)$ and $T_{\text{vir}}(r_{\text{vir}})$, respectively (also see Fig. 2b). With $\gamma = 1 – 1.5$, those values agree very well with typical observed ones, i.e., $\beta = 1.2 – 0.6$. This is why Waxman & Miralda-Escudé (1995), NFW96, and ENF were able to reproduce a gas distribution very similar to the $\beta$-model in their specific models, and in fact the agreement should be quite generic and almost independent of the adopted parameters.

### 3. IMPLICATIONS OF THE DERIVED GAS DENSITY PROFILE

Given the apparent success in describing the observed shape of the X-ray cluster gas profiles in the universal dark matter halo potential, it is reasonable to examine the predicted scales of the physical variables in further detail. In fact, all the relevant quantities can be computed using Appendix A of NFW97 once the cosmological model is fully specified.

Figure 2 plots the ratio of $r_{\text{vir}}$ and $r_{\text{core}}(0.22r_s)$ (panel a), the ratio of the virial temperatures at $r$ and at $r_{\text{vir}}$ (panel b), and $r_{\text{core}}(0.22r_s)$ against the temperature $T_b = T_{\text{vir}}(r)$. In what follows we set $\gamma = 1.5$ for definiteness, but it does not change the conclusions below. We adopt the shape parameter $\Gamma$ to characterize the power spectrum, which is equivalent to $\Omega_0 h \exp \{-\Omega_0 [1 + (2b)^{1/2}/\Omega_0] \}$ in cold dark matter (CDM) models with the baryon density parameter $\Omega_0$ (Sugiyama 1995). The representative cosmological models that we consider include standard CDM (SCDM) ($\Omega_0 = 1$,
\( \lambda_0 = 0, \Gamma = 0.5, \sigma_g = 1.2 \), open CDM (OCDM) \((\Omega_0 = 0.3, \lambda_0 = 0, \Gamma = 0.25, \sigma_g = 1.0)\), and lambda CDM (LCDM) \((\Omega_0 = 0.3, \lambda_0 = 0.7, \Gamma = 0.2, \sigma_g = 1.0)\).

Although \( r_{\text{vir}}/r_c \) is around 50 on scales of rich clusters (Fig. 2a), the corresponding temperatures \( T_{\text{vir}}(r_{\text{vir}}) \) and \( T_{\text{vir}}(r_c) \) agree within a factor of 2 (Fig. 2b). Therefore, the assumption of isothermal clusters is acceptable, and one obtains reasonable values for \( \beta_{\text{eff}} \) comparable to the observed one (§ 2) even if we adopt either \( T_{\text{vir}}(r_{\text{vir}}) \) or \( T_{\text{vir}}(r_c) \) as the gas temperature. In Figure 2c, we also plot the \( r_s \) versus \( T_X \) relations of observed clusters from the Einstein (Jones & Forman 1984; David et al. 1993), and EXOSAT (Edge & Stewart 1991a, 1991b; see also Kitayama & Suto 1996) observations. Clearly the predicted \( r_{s,\text{eff}} \) is a factor of 3–10 smaller than the average of observed values.

Equation (6) does not involve the amplitude of gas density; the central density \( \rho_{\text{cen}} \) should be determined by some other consideration. For this purpose, we use

\[
\frac{M_{\text{gas}}(r_{\text{vir}})}{M(r_{\text{vir}})} = f_{\text{gas}} \Omega_b \frac{\Omega_0}{\Omega_0},
\]

where \( f_{\text{gas}} \) is the gas mass fraction of the total baryon in the cluster. Observationally, \( f_{\text{gas}} \) is typically 0.7–0.9. With equations (7) and (8), equation (13) reduces to

\[
\rho_{\text{cen}} = f_{\text{gas}} \Omega_b \rho_c \frac{\Omega_0}{\Omega_0} e^{27b/2} \left[ \ln (1 + c) - \frac{c}{1 + c} \right]
\times \left[ \int_0^c x^2(1 + x)^{27b/2} dx \right]^{-1}. \tag{14}
\]

This can be translated to the central number density of electron \( n_{\text{e,\text{cen}}} \) assuming the primordial abundance of hydrogen and helium \((X = 0.76)\), which is plotted in Figure 3a. For definiteness, we set \( b = 0.7/0.9 \) so as to reproduce a typical shape of observed X-ray clusters and adopt \( \Omega_b = 0.015 \ h^{-2} \) and \( f_{\text{gas}} = 0.8 \). As is clear from Figure 3a, the predicted \( n_{\text{e,\text{cen}}} \) is larger than typical observed values. This is a direct consequence of the small \( r_c \) in this model given the total gas mass ratio in the cluster at \( r_{\text{vir}} \) (Fig. 2c).

Once \( n_{\text{e,\text{cen}}} \) is specified, it is straightforward to compute the X-ray bolometric luminosity:

\[
L_{\text{X,\text{bol}}} = 4\pi \int_0^{r_{\text{vir}}} r^2 dr \alpha(T_X) n_{\text{e,\text{cen}}}^2 (r)
= 4\pi \alpha(T_X) n_{\text{e,\text{cen}}}^2 e^{-27b} \int_0^c x^2(1 + x)^{27b/2} dx,
\]

where \( \alpha(T_X) \) denotes the bolometric emissivity for which we include line emissions in addition to the bremsstrahlung (Masai 1984; Kitayama, Sasaki, & Suto 1998). The results are plotted in Figure 3b, together with the observed \( L_{\text{X,\text{bol}}} \)–\( T_X \) relation (David et al. 1993; Ebeling et al. 1996; Ponman et al. 1996). Since \( L_{\text{X,\text{bol}}} \) predicted from equation (15) scales as \( h^{-3} \), unlike the observed ones \((\propto h^{-2})\), we divide our predictions by \( h \) assumed in each model in Figure 3b. A dash-dotted line indicates the best fit to the observed relation

\[
L_{\text{X,\text{bol}}} = 2.9 \times 10^{44} \ h^{-2} \left( \frac{T}{6 \ \text{keV}} \right)^{3.4} \ \text{ergs s}^{-1}, \tag{16}
\]

(Kitayama & Suto 1997). The shallower slope of the predicted \( L_{\text{X,\text{bol}}} \)–\( T_X \) relation \((\approx 2)\) than the observed one \((\approx 3.4)\) reflects a self-similar nature of the evolution in this model.

(Kaiser 1986). In fact, our model predictions reproduce the results of Kitayama & Suto (1997, their Fig. 1) and of ENF (their Fig. 15), apart from the small difference due to the different choice of adopted parameters.

4. DISCUSSION AND CONCLUSIONS

One of the most important consequences of the universal density profile is that all the physical quantities characterizing the gas density profile \((r_c, r_{\text{eff}}, \beta_{\text{eff}}, L_{\text{X,\text{bol}}}, \text{and } n_{\text{e,\text{cen}}})\) are computed as functions of the total halo mass \( M \), or almost equivalently of the corresponding gas temperature \( T_X \), once underlying cosmological parameters \((\Omega_0, \lambda_0, \Gamma, h \text{ and } \sigma_g)\) are fully specified. This can be regarded as a significant theoretical improvement of the understanding of the physical origin of the conventional \( \beta \)-model.

On the other hand, if both the universal density profile and the hydrostatic equilibrium are reasonable approximations to the truth, then our result indicates either that the previous X-ray observations systematically overestimated the core radius of clusters of galaxies or that we neglected some unknown important physical mechanisms that significantly increases the core radius. Incidentally, \( r_{\text{vir}} \) in equation (4) would be further divided by \( 1 + z_f \) if \( z_f \) really corresponds to the formation redshift of the entire cluster, and this would make \( r_c \) even smaller.

Figure 4 shows the mass profile of the universal dark matter halo. Also plotted are the estimates on the basis of the \( \beta \)-model fitting (eqs. [12] and [6]). The mass inferred from the gas density in this way underestimates the true value for \( r < r_c \). In turn, if the X-ray observations overesti-
Fukushige & Makino (1997) and more recently Moore et al. (1997) argued that their simulations with much higher spatial resolution show the steeper inner density profile like $\rho \propto r^{-1.4}$, which should predict the core radius much smaller than what we discussed above.

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