Long range neutrino forces and the lower bound on neutrino mass

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ABSTRACT

Stellar objects (including our Sun, other stars of main sequence, white dwarfs, neutron stars etc.) contain strongly degenerate low energy sea of neutrinos (in neutron stars) or antineutrinos. The presence of this sea leads due to Pauli principle and thermal effects to effective blocking of the long-range neutrino forces. This blocking can resolve the problem of the unphysically large value of the self-energy of stars stipulated by many body long-range neutrino interactions. As a consequence no lower bound on the neutrino mass is inferred, in contrast with the statement by Fischbach.
1 The self-energy paradox

The exchange of massless neutrinos leads to long range forces. The potential of interaction of two particles $f$ due to these forces equals

$$V^{(2)}(r) = \frac{G_F^2 a_f^2}{4\pi^3 r^5} = \frac{2}{\pi r} \left( \frac{G_F a_f}{2\sqrt{2}\pi r^2} \right)^2,$$

where $G_F$ is the Fermi coupling constant, $a_f$ is the weak vector charge of particle $f$, and $r$ is the distance between the particles. For neutrons, electrons and protons one has $a_n = -1/2$, $a_e = 1/2 + 2 \sin^2 \theta$, $a_p = 1/2 + 2 \sin^2 \theta$. Neutrino exchange leads also to many body interactions which can be represented in the $k$-body case as the neutrino loop interacting with $k$ currents of $f$ (ring diagrams). Inclusion of one additional particle $f$ adds the factor

$$\frac{G_F a_f}{2\sqrt{2}\pi r^2}$$

to the potential, and for $r \sim R$, the typical size of star, this factor is extremely small.

It is claimed [2], that in spite of the smallness of factor (2) the many-body neutrino forces become dominating in stars, where the number of particles, $N$, can exceed $10^{57}$. The contribution of the $k$-body interactions to the self-energy of star, $W^{(k)}$, can be represented as

$$W^{(k)} = U^{(k)} \cdot C^N_{k} \approx \frac{1}{k!} \cdot U^{(k)} N^k,$$

where $U^{(k)}$ is the $k$-body potential averaged over the volume of star and $C^N_{k} = \frac{N!}{k!(N-k)!}$ is the number of combinations of $k$ particles among $N$ particles. It is this combinatoric factor, first discussed in [3], which leads to the dominance of the many-body interactions. The second equality is valid for values of $k \ll N$, when $C^N_{k} \approx N^k/k!$. The ratio of contributions to the self energy from $k+2$ and $k$ body interactions equals

$$\frac{|W^{(k+2)}|}{|W^{(k)}|} \approx \xi^2,$$

where according to (2) and (3) [2] the series parameter $\xi$ is

$$\xi \equiv \frac{a_f}{2\sqrt{2}\pi e} \left( \frac{G_F N}{R^2} \right) = \frac{\sqrt{2}a_f}{3e} \frac{G_F n_f}{R}$$

($e = 2.718...$). Here $R$ is the radius of star and $n_f$ is the average density of particles. Notice that $\xi$ is determined essentially by the width of matter in star. For neutron star with
R \approx 10 \text{ km} \text{ and } N \approx 10^{57} \text{ one finds the series parameter } \xi \approx 10^{13}. \text{ Thus the contribution to self-energy enormously increases with multiplicity of interaction. For } k = 8 \text{ one finds } W^{(8)} \approx 3 \cdot 10^{11} M_{NS}, \text{ i.e. formally the self-energy exceeds the mass of the star } \cite{2}. \text{ There are some other aspects of the paradox: the self energy changes the sign if two more neutrons are added to star, or when microscopically small changes of radius of star occur.}

It is claimed in \cite{2} that the only way to resolve the paradox is to suggest that neutrino has non-zero mass \( m_\nu \). In this case the effective radius of interaction becomes \( r \sim 1/m_\nu \) and in the series parameter one should substitute \( R \) by \( 1/m_\nu \):

\[
\xi_m = \frac{2a_f}{3e} \frac{G_F n_f}{m_\nu}.
\]

This means that the neutrons within the radius \( 1/m_\nu \) only contribute to the many-body potential. From the condition

\[
\xi_m < O(1),
\]

(more precisely: the exchange energy inside a given volume should be smaller than the mass of that volume) one finds the lower bound on neutrino mass \( m_\nu > 0.4 \text{ eV} \) \cite{2}. If correct, this result has a number of very important consequences.

In this paper we describe another mechanism of suppression of the long-range interactions. It can systematically lead to \( \xi < 1 \), so that many body forces do not dominate in the self-energy, and no lower bound on the neutrino mass is inferred. The suppression effect originates from the neutrino sea which exists in stars, in contrast with a statement done in reference \cite{2}.

## 2 Neutrino sea

The incorrect statement about neutrino sea in \cite{2} follows from the observation that the neutrino cross-section at low energies are extremely small, so that the neutrinos freely escape from the star. However, as it has been shown by Loeb \cite{4}, the low energy neutrinos are trapped in the stars due to refraction. Refraction effect is stipulated by coherent neutrino interaction with particles of medium. For low energies the refraction index, \( G_F n_f / E \), becomes of the order 1 and the effect of the complete inner reflection takes
Neutrinos have circular orbits inside the star \[4\]. Equivalently, one can describe the effect in terms of potential created by particles of medium.

For definiteness let us consider the neutron star. For neutrinos, the star can be viewed as the potential well with the depth:

\[
V \approx \frac{1}{2} G_F n_n = 10 \text{ eV} \left( \frac{n_n}{10^{37} \text{cm}^{-3}} \right).
\]

(8)

The potential is attractive for neutrinos which have the positive weak charge and repulsive for antineutrinos. Therefore produced antineutrinos leave the star, whereas the neutrinos turn out to be trapped. Neutrinos fill in the levels of the potential well of the star. As the result the sea is strongly degenerate \[4\]. The energy distribution of neutrinos can be approximated by the thermal Fermi-Dirac distribution

\[
n_\nu(E) = \frac{1}{1 + \exp \frac{E - \mu}{T}}
\]

(9)

with chemical potential

\[
\mu \approx V = a_f G_F n_f
\]

and the effective temperature \(T \ll \mu\). (The effective temperature describes the degree of filling in the levels of the well.) For neutron star we get numerically \(\mu \approx 50 \text{ eV}\).

The total density of the neutrinos \[4\] equals:

\[
\int \frac{d^3 p}{(2\pi)^3} n(p) \approx \frac{V^3}{6\pi^2} \approx 3 \cdot 10^{17} \text{ cm}^{-3}.
\]

(10)

Notice, this number is much smaller than that of neutrons: \(n_\nu/n_N \sim 10^{-20}\), and the energy stored in these neutrinos is too small to influence the dynamics of star.

The degenerate sea exists starting from early stages of evolution in all stars. In the Sun the potential \(V\) is created by the interactions of neutrinos with protons, neutrons and electrons \(V = \sum a_f G_F n_f \sim 10^{-13} \text{ eV}\). It has an opposite sign to that in the neutron star so that the Sun has the antineutrino sea. (Also white dwarfs have antineutrino sea, and only in the process of neutronization in the protoneutron star, when \(n_n\) becomes bigger than \(2n_p\), the potential changes the sign). According to (10) the concentration of neutrinos in the sea of the Sun equals \(n_\nu \sim 10^{-28} \text{ cm}^{-3}\) and the total number of these neutrinos is \(10^5\). However even this small number can be sufficient to resolve the energy paradox.
There is a number of processes which can contribute to creation of the neutrino sea. For example, in the neutron stars the neutronisation itself can lead to strongly degenerate spectrum of neutrinos. One can consider also the neutrino pair bremsstrahlung in the collisions of neutrons, URCA processes, neutrino production by the weak field etc. However the most efficient mechanism of production of very low energy neutrinos is the one related to the long-range many-body forces themselves [5]. Indeed, as we described before, many body forces are induced by the neutrino loops which couple with k-currents of particles of medium. If in such a diagram one of the propagators of neutrino between points $x$ and $y$ is substituted by emission of neutrino in $x$ and absorption of neutrino in $y$ (and vice versa) one gets the diagram for neutrino pair production. Obviously, this process has the same combinatoric enhancement as the self-energy contributions: $\sigma^{(k)} \propto k U^{(k)} C^{(k)}_N$. So that the efficiency of the neutrino pair production increases together with the self-energy.

3 Blocking effect

Let us show now that presence of the degenerate neutrino sea can lead to the blocking of the long range forces stipulated by the neutrino exchange. For this let us consider the propagator of massless neutrino in the degenerate neutrino sea:

$$S(x - y) = \int \frac{dk^4}{(2\pi)^4} e^{-ik(x-y)} \hat{k} \times \left[ \frac{i}{k^2 + i\epsilon} - 2\pi \delta(k^2) \theta(k_0) n_\nu(k_0) - 2\pi \delta(k^2) \theta(-k_0) n_{\bar{\nu}}(-k_0) \right].$$

(11)

The thermal distribution $n_\nu(E)$ is given in (9); for $n_{\bar{\nu}}(E)$ one should substitute $\mu \rightarrow -\mu$ in $n_\nu(E)$. The first term is the vacuum propagator. The second term corresponds to real neutrinos of the sea. This term describes the process, in which instead of the exchange of a virtual particle between two points $x$, $y$, one has the absorption of neutrino in the point $x$ and the emission of neutrino in the point $y$ and vice-versa; similar consideration is applied to the last term in (11).

Let us find the potential due to neutrino exchange using Schwinger formalism [6]. According to this formalism the neutrons can be treated as static classical sources of quantized neutrino field. Then the multi-body potential due to neutrino loops, $V(\bar{x}_1, \bar{x}_2, \bar{x}_3, ...)$,
can be found by calculation of the integral over the energy of
\[ tr[\gamma_0 P_L S(E, \bar{x}_1 - \bar{x}_2) \gamma_0 P_L S(E, \bar{x}_2 - \bar{x}_3) ...]. \]

Here \( S(E, \bar{x}) \) is the neutrino propagator being a function of the energy and of the position, \( P_L \) is the chirality projector, and \( tr \) is the trace over the spinorial indices. Using equation (11) we find the propagator:
\[
S(E, \bar{x}) = (-E\gamma_0 + i\partial_i\gamma_i)\Delta(E, \bar{x}),
\]
\[
\Delta(E, \bar{x}) = -\frac{i}{4\pi r} \left[ (1 - n_\nu(E)) \exp(iEr) + n_\nu(E) \exp(-iEr) \right].
\]

Substituting the propagators of this type in Schwinger formula [6] we obtain the two-body potential:
\[
V^{(2)}(r) = \frac{(G_F a_f)^2}{4\pi^3 r^5} \left[ \cos(2\mu r) + \mu r \sin(2\mu r) + \frac{\cos(2\mu r)}{2} F(2\pi Tr) \right] \frac{2\pi Tr}{\sinh 2\pi Tr}
\]

instead of (1). In (13) \( F(x) = x \coth(x) - 1 \); this function behaves like \( x \) for large \( x \), whereas for small \( x \) one has \( F(x) = x^2/3 + \mathcal{O}(x^4) \). For zero temperatures the potential (13) reduces to
\[
V^{(2)}(r) = \frac{(G_F a_f)^2}{4\pi^3 r^5} \left[ \cos(2\mu r) + \mu r \sin(2\mu r) \right].
\]

This result coincides with the one which can be obtained from the calculations by Horowitz and Pantaleone [7]. The potential (13) shows the blocking effect: the presence of the neutrino sea results in fastly oscillating factors \( \cos(2\mu r) \) and \( \sin(2\mu r) \) as well as in an exponential damping of the long range neutrino forces.

The results (12), (13), (14) correspond to thermal equilibrium. Actually the neutrino sea distribution is non-thermal already due to the fact that the antineutrinos freely escape from the star. In this connection let us consider extreme situation of the complete antineutrino stripping. It corresponds to the omission of the last term in propagator (11). In this case the potential acquires an additional contribution:
\[
V^{(2)}_{\text{stripped}} = V^{(2)}(r) + \Delta V^{(2)}(r),
\]

where
\[
\Delta V^{(2)}(r) = -\frac{(G_F a_f)^2}{8\pi^3 r^5} \sum_{n=1}^{\infty} \left\{ \exp(-\mu/T) \right\}^n \frac{n/(2Tr)^2}{1 + \left[ n/(2Tr) \right]^2}.
\]

For strongly degenerate sea \( (T \ll \mu) \) it is enough to use the first term in the series (13), so that
\[
\Delta V^{(2)}(r) \propto \frac{(G_F a_f)^2}{8\pi^3 r^5} \exp(-\mu/T).
\]
The contribution $\Delta V^{(2)}$ has no oscillatory behaviour; it reflects the incompleteness of filling in the low energy levels $E \to 0$ (in accordance with (9)). Strong blocking effects requires strong degeneracy of the sea: $T << \mu$. For example, the many body interactions in the neutron star are suppressed (so that $\xi < 1$), if $\exp(-\mu/T) < 10^{-13}$, or $\mu/T > 30$.

Thus, the effect of the term (15) is suppressed, if the sea is strongly enough degenerate. The contribution of the term (13) to the self-energy is suppressed due to fast oscillations. Indeed, the integration over the volume of star, the motion of particles as well as macroscopic motion of medium, fluctuations of density and radius of the star etc., lead to strong cancellation of contributions from distances $r > 1/\mu$. Effectively this gives the cut of the long-range forces at $r \sim 1/\mu$.

Similar oscillating factors appear in many body potentials [5].

Thus, if in the series parameter $\xi$ of Eq. (5) the radius $R$ is substituted by $1/\mu$, one gets for the parameter in presence of the degenerate sea:

$$\xi_{\mu} = \frac{\sqrt{2}}{3e} \frac{a_f G_F n_f}{\mu} = \frac{\sqrt{2}}{3e} < 1. \quad (16)$$

The series parameter becomes smaller than one. Moreover, it does not depend on properties of the star: density, radius, chemical composition. This shows the universality of the mechanism; it works for any stars. Due to $\xi_{\mu} < 1$ and presence of additional factor $1/k!$ in $W^{(k)}$ the many body forces never exceed the lowest contributions, and this solves the paradox.

In conclusion, let us outline the dynamics of blocking. Let us start by very diluted stellar object — star at early stages of evolution, for which even without blocking one has $\xi < 1$. With increase of density the self-energy increases. However, simultaneously the potential of medium increases, and in the same degree the efficiency of low energy neutrino pair production increases. The neutrinos (or antineutrinos ) freely escape from the star with velocity of light and antineutrinos (or neutrinos) fill in the potential well. The degenerate neutrino sea leads to Pauli blocking and thermal damping of the small energy neutrino exchanges and therefore to blocking of the long-range forces. Moreover due to Pauli principle the sea will block further production of neutrino pairs.
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References

[1] G. Feinberg and J. Sucher, Phys. Rev., 166 (1968) 1638 G. Feinberg, J. Sucher and C.-K. Au, Phys. Rep., 180 (1989) 83; S. D. H. Hsu and P. Sikivie, Phys. Rev. D49 (1994) 4951.

[2] H. Kloor, E. Fischbach, C. Talmadge, and G. L. Greene, Phys. Rev. D49 (1994) 2098, E. Fischbach, hep-ph 9603396.

[3] H. Primakoff and T. Holstein, Phys. Rev. 55 (1939) 1218.

[4] A. Loeb, Phys. Rev. Lett. 64 (1990) 115.

[5] A. Yu. Smirnov and F. Vissani, (in preparation)

[6] J. Schwinger, Phys. Rev. 94 (1954), 1362; J. B. Hartle, Phys. Rev. D 1 (1970), 394; see also ref. [2].

[7] C. J. Horowitz and J. Pantaleone, Phys. Lett. B319 (1993) 186.