Predicting precursors of slow glassy dynamics with high precision by machine learning relative motion

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The recent developments in machine learning have enabled accurate predictions of heterogeneous dynamics of glassy dynamics. However, existing machine-learning models are designed so that they learn a single dynamic quantity and relate it to the structural features of glassy liquids. In this Letter, we propose a new graph neural network model which learns relative motion between neighboring pairs of particles, in addition to self motion of particles. By relating the structural features to these two different dynamical variables, the model autonomously acquires the ability to discern how two different types of dynamic processes, strain fluctuations and particle rearrangements, affect the self motion of particles undergoing fast β relaxation. Our model affords a methodology to treat these two at the same time and realizes highly precise predictions of rearranging hotspots in glassy systems.

When a liquid is rapidly cooled, it freezes into a glass which retains a random amorphous structure. This phenomenon, referred to as the glass transition, is ubiquitous, but its origin still remains to be clarified. The elucidation of this origin has been an important research topic for decades [1, 2]. Along with the drastic slowdown of atomic motion accompanying the glass transition, localized domains of particles that rearrange more preferentially than others grow up [3–6]. Identifying structural origin for this phenomenon has long been an central problem in the field. Structural analyses of local geometric orders [7–9] and of real-space normal modes of vibrations derived from static structures [10–14] have accumulated evidences for structure–dynamics correspondence. However, the random featureless structure of glasses still prevents us from fully understanding the origin.

The recent developments in machine learning methodologies and network analyses, have enabled the accurate determination of structure–dynamics correspondence in glassy systems [15–22]. Among them, graph neural networks (GNNs) [23], a deep-learning algorithm that operates on a graph, have been found to be an suitable methodology [20]. GNNs have been proven to be useful for making predictions of materials properties by directly representing structures of molecules or atomic clusters as nodes and edges of the graph [24–27]. Bapst et al. [20] have leveraged such ability of GNNs to the task of predicting particle mobility in a glass-forming liquid from its static structure, and have shown that GNNs outperform other machine-learning methodologies. This high accuracy in the predictions of glassy dynamics has enabled development of inverse modeling as a further application [28]. It has also led to the development of a cost-effective linear-regression model that can overcome the high computational cost of GNNs but still achieves an equivalent precision in its prediction [21, 22].

In the study by Bapst et al. [20], the prediction accuracy of GNNs are found to outperform not only the that of other machine-learning methods but also of the normal mode analysis, at all the temperature accessible by simulations and across all the temporal scales involve the glassy dynamics. The normal mode analysis is a method to predict how short-time vibrations are spatially distributed. Because short-time vibrations (soft modes) are known to be intimately linked to the slow α relaxations [30–34], the normal mode analysis yields a strong baseline of physics-based prediction of glassy dynamics from static structure [10, 29]. More recently, these two dynamics are shown to exhibit identical spatial heterogeneity once the former is appropriately coarse-grained [9]. The prediction accuracy of GNNs, nevertheless, becomes relatively low in the fast β dynamics than in the slow α relaxation [20, 22]. These considerations lead us to a plausible conjecture that the dynamics is made difficult to predict due to elastic fluctuations, which are erased out after coarse-graining the fast β dynamics.

Here, we propose a GNN model, named “BOnd TAar-geting Network (BOTAN)”, which realizes highly precise predictions of fast β dynamics by resolving this problem. BOTAN can learn a target characteristic quantity assigned on edges corresponding to pairs of close-by particles. This model can be positioned as an extension of the GNN used in the previous study [20], which we term as NT-GNN (“node-targeting GNN”) because it was trained using only the particle propensity for motion at each node of the graph. BOTAN and NT-GNN are based on the interaction network [35, 36] consisting of an encoder–process–decoder architecture, where features of nodes and edges in a graph are mutually computed by exchanging messages between a pair of two-layer multi-layer perceptrons (MLPs) assigned for nodes and edges. This architecture is suitable in extracting intricate relationships from simulation data computed with high precision. Because the network treats node and edge features equivalently, regressing the network toward the target quantities on edges as well as nodes [37] is possible by decoding edge embeddings by an MLP [38].

As the new target quantity for learning on edges, we introduce pair-distance change after time t,

\[ E_{ij}(t) = r_{ij}(t) - r_{ij}(0), \]

which characterizes relative motion between neighbor pairs.
Here, $r_{ij}(t)$ denotes the pairwise distance between particles $i$ and $j$ at time $t$. Only the pairs are considered within a threshold distance ($r_c = 2.0\sigma_{AA}$) at the initial time, $t = 0$. The dataset is trajectory data of a 3D binary Kob–Andersen Lennard–Jones mixture [39] with 4,096 particles, wherein the pairwise force is smoothed at its cutoff [40, 41]. The data for training and prediction tests are generated from 400 and 100 initial configurations, respectively. The actual target quantity of machine learning is the ensemble average over the result of 32 independent simulation runs generated from each of initial configurations, so-called the isoconfigurational ensemble [10, 29, 31, 42]. In the remainder of this Letter, we use the dimensionless units based on the Lennard–Jones potential and target quantities of learning are always the average on isoconfigurational ensembles for which explicit notations are abbreviated. Machine learning hyperparameters are set similar to NT-GNN [20]. In particular, the standard $L_2$-norm loss function is used for learning and the input graph is constructed in a manner that pairs of nodes are connected by an edge when the corresponding pair distance is lower than $r_c$. For training, different epoch numbers are set for the different time points and temperatures to avoid overlearning [38].

In Fig. 1, the prediction result of the pair-distance change, $E_{ij}(t)$, for a particle configuration in the test dataset, using BOTAN trained at $t = (a)\ 130$ and (b) $4120$ under the lowest temperature considered $T = 0.44$. The data are shown in color maps projected onto segments satisfying $r_{ij}(t = 0) < 1.35$. The “actual” pair-distance change (the isoconfigurational ensemble average) from the simulation is also shown for each as the ground truth. The time points correspond to $0.03\tau_\alpha$ and $\tau_\alpha$, with $\tau_\alpha$ the $\alpha$-relaxation time. Particularly for short-time prediction (a) $t = 130$, we find that BOTAN well discerns which specific pairs become separated. Over a long-time (b) $t = 4120$, as long as $\tau_\alpha$, the prediction deviates from the ground truth diffuse but still captures the spatial contrast between mobile and immobile regions. The prediction accuracy of BOTAN over $E_{ij}(t)$ can be quantified by using the Pearson correlation coefficient [20–22] that express the proximity between the predicted and ground truth data. This coefficient should be quantified carefully to ensure that the comparison is performed over equivalent particle pairs. Therefore, among pairs of type A particles (pairs related to type B are excluded), pairs within the first neighboring distance $r_{ij}(t = 0) < 1.35$ are chosen in this comparison, because neighbor pairs in the first and second neighbor shells are expected to exhibit different distance changes on average. The prediction accuracy thus evaluated is high in the short time, and monotonically declines with time. This result marks a clear contract to that of the previous NT-GNN [20], where the prediction accuracy of particle propensity drops to the lowest in the identical time scale. This high prediction accuracy in the short time originates from a simple anti-correlation relation between pair-distance change $E_{ij}(t)$ and the initial distance $r_{ij}(t = 0)$; in Fig. 1 (d), a probability distribution function (PDF) map is shown as functions of these quantities for closest neighbor pairs of type A particles in the training dataset at $t = 130$ ($0.03\tau_\alpha$) at $T = 0.44$.

The pair-distance changes, for most of neighbor pairs, remain one order of magnitude smaller than the particle radii. This quantity, therefore, is mainly highlighting strain distribution associated with the elastic fluctuations. Such anti-correlation gradually disappears as the time increases [38].

BOTAN is modeled by adding a decoder applied to edge embeddings, whereas a decoder can be retained for node embeddings. It then becomes possible to train BOTAN with target quantities for both, that is, particle propensity for the self-displacement

$$S_i(t) = |r_i(t) - r_i(0)|$$

in addition to the pair-distance changes $E_{ij}(t)$. Here, $r_i(t)$ denotes the 3D position of particle $i$ at time $t$. The actual tar-
get quantities are again their isoconfigurational ensemble averages. The simultaneous learning of these quantities can be realized by setting the loss function as a weighted sum of $L_2$-norm losses for $S_i(t)$ and $E_{ij}(t)$. The model is then trained with other hyperparameters remaining unchanged. To avoid overlearning, we again set different epoch numbers for the different time points and temperatures [38].

We compare the prediction ability of BOTAN with that of NT-GNN by Bapst et al. In Fig. 2, predictions results for $t = 130\,(0.03\tau_d)$ for $T = 0.44$ of (a) NT-GNN and of (b) BOTAN, in addition to (c) the “actual” propensity directly evaluated from the trajectory data in the test dataset. As all the snapshots are shown in the same cross section as in Fig. 1 (a), the spatial correspondence can be seen between the predictions of $S_i(t)$ and $E_{ij}(t)$. BOTAN clearly makes better prediction of spatially heterogeneous patterns of particle propensity than BOTAN. Contrarily, prediction over a longer time $t = \tau_d$ remains at the same level of accuracy as in the previous model, indicating that the addition of edge targets in BOTAN has not improved its ability over the long-time structural relaxation [38].

Two kinds of additional plots created in Fig. 2 provide us with a qualitative insights into the reason for the improvement in the prediction. One is (d) displacement field locally averaged over nearest neighbors [38], computed from the particle trajectory. This plot demonstrates the existence of collective motion at a length scale far exceeding the particle size, highlighting the cooperative nature of fast $\beta$ dynamics [43]. Although the detailed nature of this collective motion is beyond the scope of this Letter, this fast $\beta$ dynamics is expected to be intimately related to strain fluctuations arising from quasilocalized modes and also from phonons [44–46]; the other is error maps of predictions as the difference between the predicted and ground truth propensity map, plotted for (e) BOTAN and (f) NT-GNN. In the latter, (f), the large scale spatial pattern in the error map represents its inability to capture the full heterogeneity in the particle motion. However, the locally-averaged displacement field in (d), plotted with 3D vectors, demonstrates the existence of collective motion, whose length scale considerably exceeds the particle size. On comparing these two, it is observed that the locally-averaged displacements are spatially correlated with the prediction errors, being parallel with the contour lines of the prediction error map. The errors are not perfectly but clearly suppressed in the prediction of BOTAN in (e). The clear spatial correlation suggests that the collective fluctuations, which is less intimately related to the local structures, is the possible reason for the decline in prediction accuracy of fast $\beta$ dynamics.

We assess the prediction accuracy of BOTAN in comparison with the previous model and further investigate the short-time collective fluctuations affecting the dynamics by computing the Pearson correlation coefficients. Because two types of particles (A and B) of a 3D KALJ liquid are expected to exhibit different propensities over a long period, we evaluate the prediction accuracy by the Pearson correlation coefficient using only the data for type A particles. Figure 3 shows how accurately BOTAN and of NT-GNN predict the particle propensity $S_i(t)$, for three different temperatures, $T = (a)\, 0.44$, (b) 0.50, and (c) 0.64. The trend of time dependence is in agreement with the previous results [20]: in the shorter time before reaching the plateau region ($t < 0.2$), the Pearson correlation coefficients assume high values, then they falls below 0.5, and afterwards gradually increases to reach its peak at around the $\alpha$-relaxation time. Conversely, BOTAN exhibits a remarkable improvement in the prediction accuracy in the time region of $t < 100$ for all the temperatures under investigation.

To this end, we note the pair-distance changes $E_{ij}(t)$ have its own significance in spite of extreme simplicity. Because the pair-distance changes characterize relative motion, they are not directly affected by collective fluctuations, which makes a clear difference from the quantities defined on self-displacement. The pair-distance changes characterize how each pair tends to deform due to the local shear deformation around rearranging cores; such an aspect of structural changes.
has long been addressed in the concept of “Shear Transformation Zones” [45, 47–49]. By leveraging information learned via this quantity, BOTAN internally fixes the prediction errors over particle propensity for self motion \( S_i(t) \). Notwithstanding such remarkable improvement, predictions of long-time dynamics (as long as \( \tau_\alpha \)) remain at the same level of accuracy as in the NT-GNN. The improvement in the prediction of short-time dynamics clearly points toward the importance of considering a simple pairwise correlation, in addition to the many-body structural correlations, in predicting the fast \( \beta \) dynamic process.

To show that collective elastic fluctuations are one of the reasons for the loss of prediction accuracy of NT-GNN, we also let NT-GNN learn two additional quantities less affected by elastic vibrations than the self displacement \( S_i(t) \). These quantities are both defined to quantify difference between the actual displacement and the uniform strain displacement of the neighbors

\[
\mathcal{R}_i(t) = \frac{1}{N_{n.n.}} \sum_{j \in n.n.} \mathbf{Y}^\alpha_{ij}(t), \quad \mathcal{D}_i(t) = \sqrt{\frac{1}{N_{n.n.}} \sum_{j \in n.n.} \mathbf{Y}^\beta_{ij}(t)},
\]

\[
\mathbf{Y}^\alpha_{ij}(t) = [r_i^\alpha(t) - r_j^\alpha(t)] - [r_i^\alpha(0) - r_j^\alpha(0)] \cdot (\delta_{ij} + \epsilon_{ij})
\]

The vector and tensor components \( \alpha, \beta \in \{x,y,z\} \) are explicitly denoted for clarity. Here, \( N_{n.n.} \) denotes the number of nearest-neighbor particles and \( \epsilon_{ij} \) represents the local strain tensor that minimizes \( \mathcal{D}_i(t) \) with respect to local particle displacements, with \( \alpha, \beta \in \{x,y,z\} \) the spatial components [38]. In the short-time limit \( t \to 0 \) where this local strain reduces to zero, the former quantity \( \mathcal{R}_i(t) \) reduces to relative displacement with respect to neighbor environments [38, 50]. The latter quantity \( \mathcal{D}_i(t) \) is a variant of \( \mathcal{R}_i(t) \) having qualitatively similar meaning. This \( \mathcal{D}_i(t) \) is widely used in the literature [14, 47, 48, 51, 52], as it is susceptible to hot spots where particle rearrangement preferably occurs. In Fig. 3, Pearson correlation coefficients for type-A particles are also shown for predictions of \( \mathcal{R}_i(t) \) and \( \mathcal{D}_i(t) \) by NT-GNN. The predictions of \( \mathcal{R}_i(t) \) and \( \mathcal{D}_i(t) \) improve especially in the time regions \( t < 0.1 \tau_\alpha \), for these quantities less affected by collective thermal fluctuations, inversely implying their own relevance to the decline in the prediction accuracy of \( S_i(t) \) by NT-GNN in the short time.

In conclusion, we have introduced a new GNN model that realizes faithful predictions of the \( \beta \) relaxation of glassy dynamics, with far more improved accuracy than ever. The key for the improvement is that the model learns relative motion between neighbor pairs to related it to two-body structural correlation, in addition to the self motion of the particles under the influence of vibrations. The GNN model then acquires ability to autonomously “interpret” how particle motion is affected by different dynamical effects, transparently reconciling the fast \( \beta \) and slow \( \alpha \) relaxation dynamics. As a consequence, the model affords a method, more reliable than ever, to predict which inherent structure the system will move onto, namely, rearranging hotspot as a precursor of slow glassy relaxation.

![FIG. 3](image_url)  
**FIG. 3.** Prediction accuracies of BOTAN and NT-GNN are compared by showing the Pearson correlation coefficients \( \rho \) between predicted and actual particle propensities \( S_i(t) \) as functions of time \( t \) for \( T = (a) 0.44, (b) 0.50, \) and \( (c) 0.64 \). Additionally, those of \( \mathcal{R}_i(t) \) and \( \mathcal{D}_i(t) \) by NT-GNN are also considered with the same quantity. Error-bars show the median, best, and worst of five independently trained models.

In a broader context, the key design principle of our GNN model lies in incorporating and resolving interplay of different types fluctuations on different spacial scales. As an immediate application, similar machine-learning method can be applied for studying various phenomena where collective flow takes place – deformations under external strain or more diverse problems including e.g. dense active matter [53]. Moreover, the high fidelity of prediction by our GNN model over thousands of molecular dynamics simulations steps may potentially be used for finding the “reaction coordinates” along which the fluctuations should be enhanced for efficient sampling of molecular trajectory. Extensions to the recently discussed machine-learning-aided sampling technique may also be interesting [54, 55].

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[38] See Supplemental Material [URL will be inserted by publisher] for the following details: dataset generation, GNN model and training procedures, dependence of pair-distance changes on the initial distances with varying time, comparison of predictions of BOTAN and NT-GNN at $\tau_\alpha$, and the definition of locally-averaged displacement field.

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Supplemental Material for “Predicting precursors of glassy structural relaxation with high precision by machine learning relative motion”

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I. MACHINE LEARNING MODEL ARCHITECTURE

In this section, we explain the neural network architecture of BOTAN and NT-GNN, and also provide information that was not provided in the previous study conducted by Bapst et al. [S2]. Let $G(V, E)$ be a graph that includes a set of nodes $V$ and edges $E$. For a pair of nodes $v, u \in V$, an edge between $v$ and $u$ is represented as $e_{v,u}$. The neighboring edges of a vertex $v \in V$ are represented as $N(v) \subset E$. Given input feature vectors $h^0_v$ and $h^0_e$ for a node $v \in V$ and an edge $e \in E$ respectively, the goal is to compute the corresponding output features $h^{out}_v$ and $h^{out}_e$. As shown in a block diagram in FIG. S1, and by denoting an encoder, a multi-layer perceptron (MLP) and a decoder as $EN(\cdot)$, $MLP(\cdot)$ and $DE(\cdot)$, each layer to compute feature vectors for node $v$ is formulated as follows. First, input feature vectors are encoded by respective encoders

$$h^0_v = EN(h^0_v), \quad h^0_e = EN(h^0_e).$$  \hspace{1cm} \text{(S1)}$$

Then, update of the edge and node features via MLPs are repeated for $n$ times

$$h^m_{e,v,u} = MLP(h^{m-1}_{e,v,u} \oplus h^{m-1}_v \oplus h^0_v),$$ \hspace{1cm} \text{(S2)}

$$h^m_v = MLP\left(\sum_{e \in N(v)} h^m_e \oplus h^0_v\right),$$ \hspace{1cm} \text{(S3)}

where $m$ stands for the iteration index of $n$ repeat cycle ($0 < m \leq n$). In each cycle, messages are passed between nodes and edges, wherein encoded feature vectors are concatenated together before every update. The decoders placed as the final layer yields the output features on each node and edge

$$h^{out}_v = DE(h^m_v), \quad h^{out}_e = DE(h^m_e).$$ \hspace{1cm} \text{(S4)}$$

In practice, $EN(\cdot)$, $MLP(\cdot)$ and $DE(\cdot)$ are all implemented by two hidden layers of 64 units with rectified linear units (ReLU) non-linearity. At every repeat cycle of update on edges and nodes in Eqs. (S2) and (S3), information of the neighbor shell (inside the distance of $2.0r_{AA}$) is propagated. We set the number of this repetition as $n = 7$ so that the information propagates over the whole system (with a box length of 15.06) in a single epoch of learning.

Figure S1. Block diagram of the GNN indicating the steps of computation in an epoch. “EN” and “DE” are abbreviations of “encoder” and “decoder”, respectively. The GNN has an encoder-process-decoder structure consisting of the connected layers of MLPs with two hidden layers of 64 units with ReLU activation.
In addition to the node output feature $h^\text{out}_v$ which have been exploited to learn particle propensity in previous studies [S2, S7, S8], we introduced the decoder for the edge feature $h^\text{out}_e$ in this model, which is an essential innovation in the present study. It is essential that edges receive messages from both neighbor nodes $v, u$ in each repetitive cycle of message passing in Eq. (S2), though this detail does not affect the results and was not explicitly addressed in the study by Bapst et al. [S2].

In this study, a graph data for learning is constructed based on the particle configuration wherein the particles are represented by its nodes. Edges are assigned to pairs of particles $i$ and $j$ within a distance threshold value $r_c(t = 0) \leq r_c$, so that the neighborhood relation can be represented. We chose a threshold length $r_c = 2.0\sigma_{AA}$ that has the best predictive performance on particle propensity [S2]. Then these graphs are fed into the GNN by encoding particle types (A and B) and 3D relative positions between particles independently into the nodes and edges as single floating-point numbers, wherein the types (A and B) are converted into 0 and 1.

II. DATASET DETAILS

We employ a Kob–Andersen-type binary Lennard–Jones mixture in three dimensions [S1, S2] which is usually given by an interatomic pairwise potential

$$u(r) = 4\epsilon_{\mu\nu} \left[\left(\frac{\sigma_{\mu\nu}}{r}\right)^{12} - \left(\frac{\sigma_{\mu\nu}}{r}\right)^{6}\right].$$

$\mu, \nu \in \{A, B\}$ denotes the particle types, where the number of particles belonging to type A (denoted as $N_A$) is 80% of the total number $N = 4096$. The interaction energy and particle size are defined as $\epsilon_{AA} = 1.0$, $\epsilon_{AB} = 1.5$, $\epsilon_{BB} = 0.5$, $\sigma_{AA} = 1.0$, $\sigma_{AB} = 0.8$, and $\sigma_{BB} = 0.88$. In this study, distances, time, and temperature are denoted in units of $\sigma_{AA}$, $\tau = \sqrt{m\sigma_{AA}/\epsilon_{AA}}$, and $\epsilon_{AA}/k_B$, with $k_B$ the Boltzmann constant.

For improving the accuracy of numerical integration over a large number of steps, the pairwise potential is modified in the track of previous studies as [S3, S4]

$$U(r) = u(r) - u(r_c) - (r - r_c) \frac{du(r)}{dr} \bigg|_{r=r_c}, \quad (S5)$$

so that the force and the potential vanish at the cutoff length $r = r_c$. The cutoff lengths are set differently for different combinations of pair species depending on the combination of pair species as $r_c = 2.5\sigma_{ab}$.

The procedure of dataset generation is similar to that in the previous study [S2] except that the system size is fixed. For all the simulations, the number density of the system is fixed at $N/V = 1.2$. Time step of numerical integration is kept at $\Delta t = 10^{-3}$ throughout. We set three target temperatures, $T = 0.44$, 0.50, and 0.64. In Fig. S2, the self-part of the intermediate scattering functions for particles of type A

$$F_s(k, t) = \frac{1}{N_A} \sum_{j \in A} \exp\left[ik \cdot (r_j(t + t_0) - r_j(t_0))\right], \quad (S6)$$

Figure S2. Intermediate scattering function $F_s(k, t)$ in Eq. (S6) is shown for temperatures $T = 0.44, 0.47, 0.50, 0.56$, and $0.64$ as functions of time. Vertical dotted lines indicate time points corresponding to $0.03\tau_u$ and $\tau_u$ for $T = 0.44$, which are mainly investigated in the Letter.
are shown for these temperatures (and additionally $T = 0.47$ and 0.56), where the wavenumber $k$ is set at $2\pi/\sigma_{\text{AA}}$. The $\alpha$-relaxation times $\tau_\alpha$ are evaluated as 4120, 217, and 16.5, for $T = 0.44$, 0.50, and 0.64, respectively. In the Letter, time points $t = 130 (0.03\tau_\alpha)$ and 4120 ($\tau_\alpha$) are mainly investigated at the lowest temperature $T = 0.44$, which is indicated in the same Fig. S2 with vertical dotted lines.

For each of these three target temperatures, 500 independent particle configurations are generated as follows. After keeping the temperature at $T = 5.0$ for the time lapse of $t_h = 10^4$, the system is cooled rapidly (in $t_{\text{cool}} = 20$) to the target temperature. The temperature is subsequently kept constant until the system reaches the steady state up to time scale of $40\tau_\alpha$. The final particle configuration thus obtained is used as the initial one at $t = 0$ in the production run for generating the dataset.

From the 500 initial configurations, we have introduced the isoconfigurational ensemble [S5, S6] by running 32 separate microcanonical (NVE) simulations all starting from the same configuration; however, the initial velocities are randomly given from the Maxwell-Boltzmann distribution at the target temperature. These simulations are conducted approximately up to $30\tau_\alpha$, where the particle configurations are sampled at logarithmically increasing time points (see Table I in Sec. I). Among the 500 initial configurations, 400 configurations (12,800 runs) are used for training and the other 100 (3,200 runs) are for testing.

### III. CORRELATION BETWEEN PAIR-DISTANCE CHANGE AND INITIAL DISTANCE WITH VARYING TIME

BOTAN, the GNN model we propose in this study, learns pair-distance changes $E_{ij}(t)$, defined in Eq. (1) in the Letter. In the short time region corresponding to the fast $\beta$ relaxation, the high prediction accuracy of BOTAN, at least partly, originates from the simple anti-correlations between $E_{ij}(t)$ and the initial distances $r_{ij}(t = 0)$. In Fig. S3, the relation between these quantities are shown in the form of probability distribution functions (PDF) over all particle pairs of type A (within the distance $r_{ij}(t = 0)$), over all the training dataset for the temperature $T = 0.44$ (400 initial configurations, namely, 12800 runs), for different time points $t = 13.0, 130, 412, 1300, 4120, \text{and } 13000$. BOTAN leverages the anti-correlation appearing in the short time region for improving the predictions of particle propensity for motion.

![Figure S3](image_url)

Figure S3. The PDF showing the relationship between the initial pair distance $r_{ij}(t = 0)$ at $T = 0.44$, for time points $t = (a) 13.0, (b) 130 \text{ (the same as Fig. 1 (d) in the Letter), (c) 412, (d) 1300, (e) 4120, and (f) 13000.}
IV. TRAINING PROCEDURE

The learning setups and hyperparameters in the present study are similar to those used by Bapst et al. [S2]. The training dataset is learned to minimize the loss function, wherein the neural network is optimized by Adam algorithm without norm regularization with the learning rate of $10^{-4}$ with a standard implementation in PyTorch. The data loaded for training are augmented by applying random rotation of the simulation box to the particle positions, as all the target quantities for learning are scalar variables that are rotationally invariant.

The loss functions for learning are defined by

$$L_M = pL_S + (1 - p)L_E,$$

(S7)

where $L_S$ and $L_E$ are standard $L_2$-norm loss functions respectively defined for deviations of particle propensity $S_i(t)$ and pair-distance changes $E_{ij}(t)$. These may be written as

$$L_S = \sum_{j=1}^{N} \left[ \hat{S}_j(t;r_i) - \langle S_j(t) \rangle_{IC} \right]^2$$

(S8)

$$L_E = \sum_{j=1}^{N} \sum_{k \neq n} \left[ \hat{E}_{jk}(t;r_i) - \langle E_{jk}(t) \rangle_{IC} \right]^2,$$

(S9)

where $\langle \cdot \rangle_{IC}$ represents the average over the isoconfigurational ensemble and $\hat{S}_j(t;r_i)$ and $\hat{E}_{jk}(t;r_i)$ denote the values of particle propensity and pair-distance change as the output from the GNN. By using the loss function $L_M$, the model can be trained simultaneously via both $S_i(t)$ and $E_{ij}(t)$, and further, these quantities can be inferred simultaneously as well. $p$ is treated as a hyperparameter ($0 \leq p \leq 1$) as it decides the weight of learning between nodes and edges. Two limiting cases are $p = 0$ wherein the model only learns pair-distance changes $E_{ij}(t)$ (corresponding to Fig. 1 in the Letter), and $p = 1$ which reduces to NT-GNN [S2]. For the simultaneous learning on nodes and edges ($0 < p < 1$), we make the choice of $p = 0.4$ in this study, which is justified by the ablation experiment in which Pearson correlation coefficients are calculated with changing $p$ for $t = 130$ and $T = 0.44$, as shown in Fig. S4.

The training with BOTAN (with edge targets, $p = 0$ and $p = 0.4$) is performed without minibatching of the graph input. The learning requires a larger number of epochs under the present hyperparameter setting than with NT-GNN ($p = 1$). Consequently, we need to stop training at an epoch before overlearning occurs. The time $t$ used as the time point under investigation and the number of epochs used for this training procedure is shown in Table I. In our training with NT-GNN ($p = 1$), on the other hand, the minibatch sizes of the graph input are fixed at 5 and the number of epochs for learning are fixed at 1000. No overlearning was confirmed in all the cases. Although Bapst et al. applied no minibatching in their previous study [S2], this difference does not bring much differences in the results presented in this Letter.

In Fig. S5 and S6, the training and test loss functions are plotted as a function of epochs (so called the learning curves), for $p = 0$ (corresponding to Fig. 1 of the letter) and $p = 0.4$. Also, Pearson correlation coefficients $\rho$, the proximity between the ground truth and the predictions of the GNN model at each epoch, are also shown.

![Figure S4](image_url)  

Figure S4. Dependence of Pearson correlation coefficients (see the Letter for the definitions) between prediction result of BOTAN and the ground truth and the predictions of the GNN model at each epoch, are also shown.
In all these training scenarios, all the particles (particle pairs) are learned simultaneously, regardless of the particle types. The training with NT-GNN over 1000 epochs required approximately 2 h when using one NVIDIA A100 Tensor Core GPU (40 GB SXM). It is essentially the same for BOTAN, except that there may exist a substantial overhead arising from computation of interparticle distances, of which the actual elapsed time depends on its implementation.

V. PREDICTION OF BOTAN AND NT-GNN AT THE \( \alpha \)-RELAXATION TIME

In Fig. S7, predicted particle propensity for motion \( S_i(t) \) of (a) BOTAN and (b) NT-GNN, and also (c) the actual propensity for motion obtained as the isoconfigurational ensemble (ground truth), are plotted as a color map, for the same 2D cross section of the same particle configuration as in Fig. 2 of the Letter. Unlike in the short-time prediction shown in Fig. 2 of the Letter, the prediction of BOTAN does not largely differ from NT-GNN, meaning that data of the pair-distance changes does not help improving the prediction. The Pearson correlation coefficients for these predictions are shown in Fig. 3 of the Letter.

VI. DEFINITION OF LOCALLY-AVERAGED DISPLACEMENT

The locally-averaged displacement in Fig. 2 (d) in the Letter is defined as an isoconfigurational ensemble average inside the first neighbor shell

\[
\langle \Delta r_{ave}^i(t) \rangle_{IC} = \left( \frac{\sum_j \left[ r_j(t) - r_j(0) \right] \Theta \left( r_c - r_{ij}(0) \right)}{\sum_j \Theta \left( r_c - r_{ij}(0) \right)} \right)_{IC},
\]

where \( \Theta(x) \) denotes Heaviside’s step function and \( r_c \) is the cutoff length of coarse-graining \( r_c \) set at 1.35. Each of them is plotted in a cross-section with a thickness of 1.2, where the 3D vectors are enlarged for better visibility.

VII. CODE AVAILABILITY

The source code used for training and evaluation is available at GitHub repository https://github.com/h3-open-BDEC/pyg_botan. This code is an implementation of BOTAN using PyTorch Geometric with extensions from the original code for TensorFlow provided by Bapst et al. [S2]. Data for evaluations will also be made available via the hyperlink in the same GitHub repository for three years after the publication of the Letter.
Figure S5. Training and test losses (left axis) for $t = (a) 130$ and (b) $4120$ at $T = 0.44$ for the learning of pair-distance changes $E_{ij}(t)$ as an edge feature ($p = 0$, corresponding to Fig. 1 of the Letter), in addition to the resultant Pearson correlation coefficient (right axis) between the predicted and the ground truth data.

Figure S6. Training and test losses (left axis) for $t = (a) 130$ and (b) $4120$ at $T = 0.44$ for the simultaneous learning of particle propensity $S_i(t)$ and pair-distance changes $E_{ij}(t)$ (with $p = 0.4$, corresponding to Fig. 2 (a) and (c), of the Letter), in addition to the resultant Pearson correlation coefficients (right axis) between the predicted and the ground truth data for both $S_i(t)$ and $E_{ij}(t)$.

Figure S7. Color maps of particle propensity $S_i(t)$ at a temperature $T = 0.44$ at the $\alpha$-relaxation time $t = 4120$, predicted by (a) BOTAN and (b) NT-GNN, respectively. In (c), $S_i(t)$ computed from the dataset is also shown. The same cross section ($11.1 < z < 11.9$) of same particle configuration is used as in Fig. 1 and 2 of the letter.

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