Negative energy states in the Reissner-Nordström metric

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We consider electrogeodesics on which the energy $E < 0$ in the Reissner-Nordström metric. It is shown that outside the horizon there is exactly one turning point inside the ergoregion for such particles. This entails that such a particle passes through an infinite chain of black-white hole regions or terminates in the singularity. These properties are relevant for two scenarios of high energy collisions in which the presence of white holes is essential.

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I. INTRODUCTION

One of the most physically interesting processes in black hole physics is the famous Penrose process [1]. It consists in the possibility of energy extraction from black holes. This requires the existence of the negative energy states with respect to a remote observer. Then, if some particle having the energy $E_0$ decays to two fragments and particle 1 sits on the orbit with the energy $E_1 < 0$, particle 2 can return to infinity with $E_2 > E_0$. The region where such states can exist is called ergoregion (ergosphere). It can be realized in the metric of rotating black holes - the simplest example is the Kerr one. For the Schwarzschild metric such an effect is absent. Meanwhile, it was found that the counterpart of the Penrose process for static but electrically charged black holes described by the Reissner-Nordström (RN) metric, is also possible [2]. Many aspects of the Penrose process were investigated but,
strange as it may seem, the question about the nature of trajectories with $E < 0$, remained in the shade until recently. It was posed in [3] where it was shown that corresponding geodesics must originate and terminate under the horizon. This means that the situation when in the ergoregion a particle with $E < 0$ oscillates between two turning points outside the horizon or moves along the circular orbit, is forbidden. This result was extended to a rather wide class of stationary rotating axially symmetric black holes in [4].

The aim of the present work is to elucidate this issue for the RN metric. The trajectories under discussion play in the electric version of the Penrose process [2] a role similar to that played by corresponding geodesics in the standard Penrose process [1]. An additional motivation stems from the fact that (i) the collisional version of the Penrose process also exists (see [5] for a review and (ii) it can be related to the high energy processes in the centre of mass frame in the vicinity of rotating black holes [6]. The latter process is called the Bañados-Silk-West (BSW) effect under the names of its authors. The counterpart of the BSW effect as well as a version of the collisional Penrose process (even with formally unbounded efficiency) are possible not only for rotating black holes but also for the RN metric [7], [8]. Moreover, recently the analogue of the latter was considered even for the white charged holes in the extremal case [9] (that has also its counterpart in the Kerr case [10]). Now, we consider a generic (nonextremal or extremal) RN black hole.

We use the system if units in which fundamental constants $G = c = 1$.

II. BASIC EQUATIONS

Let us consider motion of particle with the mass $m$ and electric charge $q$ in the background of the RN black hole. The metric has the form

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\omega^2, \quad d\omega^2 = d\theta^2 + \sin^2 \theta d\phi^2,$$

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2},$$

where $M$ is the black hole mass, $Q$ being its electric charge. (For the definiteness we assume that $Q > 0$.) The event horizon $r_+$ lies at the largest root of the equation $f = 0$,

$$r_+ = M + \sqrt{M^2 - Q^2}.$$
Then, the equations of motion read

\[ m\dot{t} = \frac{X}{f}, \quad (4) \]
\[ X = E - q\varphi, \quad (5) \]
\[ m\dot{r} = \sigma P, \quad \sigma = \pm 1, \quad (6) \]
\[ P = \sqrt{X^2 - f(m^2 + \frac{L^2}{r^2})}. \quad (7) \]

Here, \( E \) is the energy, \( L \) being the angular momentum. They are conserved since the metric does not depend on \( t \) and \( \phi \). Dot denotes derivative with respect to the proper time \( \tau \). The electric Coloumb potential \( \varphi = \frac{Q}{r} \).

The forward-in-time condition \( \dot{t} > 0 \) gives us

\[ X \geq 0, \quad (8) \]

where equality can be reached on the horizon only, where \( f = 0 \).

III. PROOF OF THE STATEMENT

We are interested in the states with nonpositive energies, so

\[ E = -|E|. \quad (9) \]

Then, (8) gives us \( q = -|q| < 0 \), so

\[ X = \frac{Q|q|}{r} - |E|. \quad (10) \]

A particle with \( E < 0 \) cannot reach infinity since it would have there \( X < 0 \) in contradiction with (8). It means that in the region \( r > r_+ \) it has a turning point \( r_1 \), where \( P(r_1) = 0 \). Now, we will show that in this region it has only one such a point. To this end, we will demonstrate that \( (P^2)' < 0 \), prime denotes \( \frac{d}{dr} \). Then, the function \( P(r) \) is monotonically decreasing in this region, so \( P(r_+) \geq P(r) \geq P(r_1) = 0 \).

The proof is rather easy, despite some algebra. Direct calculation gives us

\[ \frac{(P^2)'}{2} = \frac{BL^2}{r^3} - \frac{m^2(Mr - Q^2)}{r^3} - \frac{Q|q|}{r^2} X, \quad (11) \]
where
\[ B = f + \frac{Q^2}{r^2} - \frac{M}{r} = 1 - \frac{3M}{r} + \frac{2Q^2}{r^2}. \] (12)

Here, \( Mr - Q^2 \geq Mr_+ - Q^2 \geq 0 \). As a result,

\[ B \leq f. \] (13)

If \( B \leq 0 \), the fact that \((P^2)' < 0\) is obvious since all terms in (11) are negative outside the horizon. We assume that \( B > 0 \) and will continue derivation.

It follows from \( P^2 \geq 0 \) that

\[ \frac{L^2}{r^2} \leq \frac{(\frac{Q|q|}{r} - |E|)^2 - fm^2}{f}. \] (14)

Then, by substitution into (11) we obtain that

\[ \frac{(P^2)'}{2} \leq \frac{C}{rf}, \] (15)

where

\[ C = B \left( \frac{\frac{Q|q|}{r} - |E|}{f} \right)^2 - fD, \] (16)

\[ D = m^2 f + \frac{Q|q|}{r} (\frac{\frac{Q|q|}{r} - |E|}{f}). \] (17)

Taking into account (13), we can rewrite this as

\[ C \leq Hf, \] (18)

where

\[ H = X^2 - X \frac{Q|q|}{r} - m^2 = -X |E| - m^2 < 0. \] (19)

Thus \( C < 0 \), so \( P' < 0 \) that is just what we wanted to prove. Thus, no more than 1 turning point can exist outside the event horizon for particles with a negative (or zero) energy.

From another hand, one turning point in the outer region is inevitable, as is explained above. It means that a particle with \( E < 0 \) appears in the outer region from the inner region only for a finite interval of the proper time, bounces back and moves inside. There, the particle under discussion can bounce from the singularity and appear in the outer space again but in another universe \[11, 12\]. The process can continue endlessly. Alternatively, such a particle can fall in the singularity (or originate from it), provided \( L = 0 \) and either (i) \(|q| > m \) or (ii) \(|q| = m, Mm \geq Q |E|\). (There is an exceptional case \(|E| = m = |q|, M = Q \) when a particle remains in the rest in the field of the extremal black hole.)


IV. DISCUSSION AND CONCLUSIONS

Thus we showed that for the RN metric the situation with the electogeodesics (trajectories corresponding to motion under the electromagnetic force and gravitation only) is similar to that in the Kerr metric [3] or more general rotating black-white holes [4]. In the ergosphere, there is exactly one turning point outside the horizon. It follows from this that a particle with $E < 0$ emerges from the inner (white hole) region and, afterwards, returns to the inside region under the horizon. It passes through an infinite chain of different black-white hole region or ends up hitting the singularity.

The relevance of a white hole region entails, in particular, that two types of high energy collisions connected with white hole are possible. In the first type of a scenario it can appear from the white hole region and collide with another particle having $E > 0$. If collision occurs near the horizon, the energy in the center of mass frame becomes unbounded [13]. In the second type of scenario two particle with $E > 0$ collide in the black hole region. They can produce a particle with $E < 0$ that oscillates in the bounded region but passes through different black-white hole regions [9].

In our treatment, we assumed that the space-time represents the eternal black-white hole. In the case of collapse of charged matter, the properties of the trajectories under discussion can change. This requires further investigation.

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[1] R. Penrose, Riv. Nuovo Cimento, Num. Spec. I, (1969) 252.
[2] G. Denardo and R. Ruffini, On the energetics of Reissner Nordström geometries, Phys. Lett. B 45 (1974) 259.
[3] A. A. Grib, Yu. V. Pavlov, V. D. Vertogradov, Geodesics with negative energy in the ergosphere of rotating black holes, Mod. Phys. Lett. A 29 (2014) 1450110, arXiv:1304.7360 [gr-qc].
[4] O. B. Zaslavskii, On geodesics with negative energies in the ergoregions of dirty black holes. Mod. Phys. Lett. A 30 (2015) 1550055, \texttt{arXiv:1412.1725} [gr-qc].

[5] J. D. Schnittman, The Collisional Penrose Process. Gen. Relat. and Gravitation. 50 (2018) 77, \texttt{arXiv:1910.02800} [astro-ph].

[6] M. Bañados, J. Silk, S.M. West, Kerr black holes as particle accelerators to arbitrarily high energy, Phys. Rev. Lett. 103 (2009) 111102, [hep-ph].

[7] O. B. Zaslavskii, Acceleration of particles by nonrotating charged black holes, Pis'ma Zh. Eksp. Teor. Fiz. 92, 635 (2010) [JETP Lett. 92, 571 (2010)], \texttt{arXiv:1001.4598} [gr-qc].

[8] O. B. Zaslavskii, Energy extraction from extremal charged black holes due to the BSW effect. Phys. Rev. D 86 (2012) 124039, \texttt{arXiv:1207.5209} [gr-qc].

[9] Zaslavskii O. B., Super-Penrose process for extremal charged white holes. \texttt{arXiv:2005.11090} [gr-qc].

[10] M. Patil, T. Harada, Extremal Kerr white holes as a source of ultra high energy particles, \texttt{arXiv:2004.12874} [gr-qc].

[11] J. C. Graves and D. R. Brill, Oscillatory character of Reissner-Nordström metric for an ideal charged wormhole, Phys. Rev. 120 (1960) 1507.

[12] B. Carter, The complete analytic extension of the Reissner-Nordström metric in the special case \(e^2 = m^2\), Phys. Lett. 21 (1966) 423.

[13] Grib A. A., Pavlov Yu. V., Are black holes totally black? Gravit. Cosmol. 21, 13 (2015), \texttt{arXiv:1410.5736} [gr-qc].