Baryon Number Violation in Particle Decays

RATHIN ADHIKARI\((a)\)* and RAGHAVAN RANGARAJAN\((b)\)†

\((a)\) Jagadis Bose National Science Talent Search
716, Jogendra Gardens, Kasba, Kolkata (Calcutta) 700 078, India

\((b)\) Theoretical Physics Division, Physical Research Laboratory
Navrangpura, Ahmedabad 380 009, India

Abstract

It has been argued in the past that in baryogenesis via out-of-equilibrium decays one must consider loop diagrams that contain more than one baryon number violating coupling. In this note we argue that the requirement with regard to baryon number violating couplings in loop diagrams is that the interaction between the intermediate on-shell particles and the final particles should correspond to a net change in baryon number and that this can be satisfied even if the loop diagram contains only one baryon number violating coupling. Put simply, we show that to create a baryon asymmetry there should be net B violation to the right of the ‘cut’ in the loop diagram. This is of relevance to some works involving the out-of-equilibrium decay scenario.

* E-mail: rathin\_adhikari@yahoo.com
† E-mail: raghavan@prl.ernet.in
It is well known that to obtain a baryon asymmetry in the out-of-equilibrium baryon number violating decays of heavy particles one must consider the interference between tree level diagrams and higher order loop diagrams. Furthermore, some particles in the loop must be able to go on shell for the net asymmetry to be non-zero. This is typically illustrated by drawing a ‘cut’ through the lines representing particles that have gone on shell. In the Appendix of Ref. [1], the authors had argued that a further requirement is that one must consider loop diagrams that contain more than one B violating coupling. In this brief note we argue that the requirement with regards to B violating couplings is that the interactions on the right of the ‘cut’ should correspond to a non-zero change in the baryon number. Furthermore, this can be satisfied even if the loop diagram contains only one B violating coupling and we refer the reader to such an example.

Consider a particle $X$ and its antiparticle $\bar{X}$ each of which can decay to final states with different baryon number. Let $f$ be a specific final state with baryon number $B_F$ that $X$ decays to. Assuming CP violation, the partial decay rates for $X$ going to specific final states and for $\bar{X}$ going to the corresponding final states can be different. Therefore we now consider the amplitude $A(\bar{X} \rightarrow \bar{f})$ for the decay of $\bar{X}$ to $\bar{f}$. By the CPT theorem,

$$A(\bar{X} \rightarrow \bar{f}) = A(f \rightarrow X).$$

(1)

Therefore $^1$

$$\langle \bar{f}|\bar{X}\rangle_{\text{in}} = \langle \bar{X}|f\rangle_{\text{in}}.$$

(2)

Inserting a complete set of in states $^2$

$$\langle X|f\rangle_{\text{in}} = \sum_g \langle X|g\rangle_{\text{out}} \langle g|f\rangle_{\text{in}}$$

(3)

The sum over states above includes integration over momenta. Then

$$\sum_{f_{B_F}} |\langle \bar{f}|\bar{X}\rangle|^2 = \sum_{f_{B_F}} \sum_g \langle X|g\rangle_{\text{out}} \langle g|f\rangle_{\text{in}} \langle X|f\rangle_{\text{in}}^*$$

$$= \sum_{f_{B_F}} \sum_g \langle X|g\rangle_{\text{out}} \langle g|f\rangle_{\text{in}} \langle f|X\rangle$$

$^1$ The issue of whether it is possible to have unstable particles in asymptotic states $^3$ is ignored in Ref. [1] and by us.

$^2$As in Ref. [1], we shall henceforth drop the subscript for the $|X\rangle$ states as $|X\rangle_{\text{in}} = |X\rangle_{\text{out}}$ for one particle states.
\[
= \sum_{f} \sum_{g} \langle X|g\rangle_{out} \langle g|f\rangle_{in} \langle f|X\rangle - \sum_{f_{B}\neq B_{F}} \sum_{g} \langle X|g\rangle_{out} \langle g|f\rangle_{in} \langle f|X\rangle
\]
\[
= \sum_{f_{B}=B_{F}} \sum_{g} \langle X|g\rangle_{out} \langle g|f\rangle_{in} \langle f|X\rangle
\]
\[
+ \sum_{f} \sum_{g_{B}\neq B_{F}} \langle X|g\rangle_{out} \langle g|f\rangle_{in} \langle f|X\rangle
\]
\[
= \sum_{\bar{B}_{F}} \langle g|X\rangle_{out} \langle X|g\rangle_{out} \langle g|f\rangle_{in} \langle f|X\rangle
\]
\[
+ \sum_{f_{B}=B_{F}} \sum_{g_{B}\neq B_{F}} \langle X|g\rangle_{out} \langle g|f\rangle_{in} \langle f|X\rangle
\]
\[
= \sum_{\bar{B}_{F}} |\langle g|X\rangle_{out}|^{2}
\]
\[
+ \sum_{f_{B}=B_{F}} \sum_{g_{B}\neq B_{F}} \left[ \langle X|g_{out}\rangle \langle g|f\rangle_{in} \langle f|X\rangle - \langle X|f_{out}\rangle \langle f|g\rangle_{in} \langle g|X\rangle \right]
\]
\[
= \sum_{\bar{B}_{F}} |\langle g|X\rangle_{out}|^{2}
\]
\[
+ \sum_{f_{B}=B_{F}} \sum_{g_{B}\neq B_{F}} \left[ A^{*}(X \rightarrow g)A(f \rightarrow g)A^{*}(f \rightarrow X)
\right.
\]
\[
- A^{*}(X \rightarrow f)A(g \rightarrow f)A^{*}(g \rightarrow X) \right]
\]
\[
(4)
\]
where the sum over \( f_{B}(\bar{f}_{B}) \) is over all final states with a fixed baryon number, \( B_{F}(-B_{F}) \), and includes integration over momenta of the \( f(\bar{f}) \) states.

The net decay rate, \( \bar{\Gamma}(B_{F}) \), for \( \bar{X} \) to all final states with baryon number \( -B_{F} \) is proportional to the expression on the left-hand side above, while the net decay rate, \( \Gamma(B_{F}) \), for \( X \) to all final states with baryon number \( B_{F} \) is proportional to the first term on the right-hand side above.

For a 2-body decay scenario, the difference term, i.e., the second term on the right-hand side above, is 0 to \( O(\lambda^{2}) \), where \( \lambda \) is any coupling in the theory. At \( O(\lambda^{4}) \), the difference term can be rewritten as
\[
\sum_{f_{B}=B_{F}} \sum_{g_{B}\neq B_{F}} \left[ A^{*}_{c}(X \rightarrow g)A_{c}(g \rightarrow f)A_{c}(X \rightarrow f) + A^{*}_{c}(X \rightarrow f)A_{c}(g \rightarrow f)A_{c}(X \rightarrow g) \right]
\]

3
\[ 2\text{Re} \sum_{f \neq g_0} \sum_{g \neq B_F} \left[ A_c(X \to g)A_c(g \to f)A^*_c(X \to f) \right], \]  

(5)

where \( A_c \) is the connected (tree level) amplitude and we have used \( A^*_c(a \to b) = -A_c(b \to a) \) which is valid for diagrams in which no internal particles go on shell.

Thus to obtain a difference in \( \Gamma(B_F) \) and \( \bar{\Gamma}(-B_F) \) one requires that there exist intermediate states \( g \) such that the transition between the states \( g \) and the final states \( f \) involves a net change in baryon number. In other words, the requirement for an asymmetry is that there must exist diagrams such that the process to the right of the ‘cut’ should violate baryon number and the net asymmetry is then proportional to the amplitudes associated with these diagrams.

If the process \( X \to g \), where \( B_g \neq B_F \), involves a \( B \) violating coupling then the total one loop amplitude for \( X \to f \) must involve more than one \( B \) violating coupling for it to contribute to the net asymmetry. In this case, the statement that the one loop diagram must involve more than one \( B \) violating coupling to obtain an asymmetry, as argued in Ref. [1], holds. This is shown in fig. 1. In fact, the insertion of internal states \( g \) in Ref. [1] implicitly assumes that the process \( X \to g \) involves \( B \) violation. However, an asymmetry between \( \Gamma(B_F) \) and \( \bar{\Gamma}(-B_F) \) may be achieved even if the one loop amplitude for \( X \to f \) involves only one \( B \) violating coupling. If, for example, \( X \) carries no baryon number and \( B_F = 1 \) then the one loop amplitude for the process \( X \to f \) in which the states \( g \) carry no baryon number can involve only one \( B \) violating coupling (see fig. 2) and yet satisfy the requirement for an asymmetry discussed above. This is the case in Ref. [3], albeit for a 3-body decay scenario. (Our conclusions can be extended to the asymmetry from 3 body decays.)

If the processes \( g \to f \) involve one or more than one \( B \) violating coupling, as in fig. 3, the total contribution of the corresponding loop diagrams to the net asymmetry will still be 0 if \( B_g = B_F \). Note that the asymmetry is not 0 for individual decay channels when \( B_g = B_F \). But summing over all intermediate and final states gives zero net asymmetry for processes in which the intermediate states and the final states have the same baryon number. This result is relevant for the calculations of the asymmetry in, for example, Ref. [1,4].

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FIG. 1. Tree level and one loop diagrams for $X \rightarrow f$ and net $\Delta B = 1$. The loop diagram has $B_g \neq B_F$ and involves more than one $B$-violating coupling.

FIG. 2. One loop diagram with $B_g \neq B_F$ and only one $B$-violating coupling.
FIG. 3. One loop diagram with $B_g = B_F$ and with one and more than one $B$-violating couplings.