Bounding effective parameters in the chiral Lagrangian for excited heavy mesons

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Abstract

We use recent experimental data on charmed mesons to constrain three coupling constants in the effective lagrangian describing the interactions of excited heavy-light mesons with light pseudoscalar mesons at order $m_Q^{-1}$. Predictions in the beauty sector are also derived.
The coupling constants and the mass parameters in effective Lagrangians which reproduce QCD in specific limits represent important input parameters for the description of the hadron processes. Therefore their determination, either by theoretical approaches or by phenomenological analyses, is relevant for the use of the related effective theory. The case of the effective Lagrangian describing the strong interactions of heavy-light hadrons with the octet of pseudo Goldstone bosons is not an exception, and it is noticeable that data recently collected at the B factories and at the Fermilab Tevatron can constrain a few of such parameters, thus allowing to exploit this theoretical framework to make, for example, predictions that can be tested at the new experiments. This note is devoted to such a discussion.

The heavy quark chiral effective theory is constructed starting from the spin-flavour symmetry occurring in QCD for hadrons comprising a single heavy quark, in the infinite heavy quark mass limit, and from the chiral symmetry valid in the massless limit for the light quarks [1]. The heavy quark spin-flavour symmetry allows to classify heavy $Q\bar{q}$ mesons into doublets labeled by the value of the angular momentum $s_\ell$ of the light degrees of freedom:

$$s_\ell = s_\bar{q} + \ell,$$

$s_\bar{q}$ being the light antiquark spin and $\ell$ the orbital angular momentum of the light degrees of freedom relative to the heavy quark [2]. The lowest lying $Q\bar{q}$ mesons correspond to $\ell = 0$, then $s^P_\ell = \frac{1}{2}$; this doublet comprises two states with spin-parity $J^P = (0^+, 1^-)$: $P = D_{(s)}(B_{(s)})$ and $P^* = D_{(s)}^*(B_{(s)}^*)$ mesons in case of charm (beauty) heavy quark, respectively. For $\ell = 1$ it could be either $s^P_\ell = \frac{1}{2}$ or $s^P_\ell = \frac{3}{2}$. The two corresponding doublets have $J^P = (0^+, 1^+)$ and $J^P = (1^+, 2^+)$. We denote the members of the $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$ doublet as $(P_0^*, P_1^*)$ and those of the $J^P_{s_\ell} = (1^+, 2^+)_{3/2}$ doublet as $(P_1, P_2^*)$, with $P = D, D_s, B, B_s$. The negative and positive parity doublets can be respectively described by the fields $H_a, S_a$ and $T^\mu_a, a = u, d, s$ being a light flavour index:

$$H_a = \frac{1 + \gamma^\nu}{2}[P^*_{a\mu}\gamma^\mu - P_a\gamma_5],$$ (1)

$$S_a = \frac{1 + \gamma^\nu}{2}[P^\mu_{1\mu}\gamma_5 - P^*_0],$$ (2)

$$T^\mu_a = \frac{1 + \gamma^\nu}{2}\left\{P^{\mu\nu}_{2\mu}\gamma_5 - P_{1\mu}\sqrt{\frac{3}{2}}\gamma_5\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - u^\mu)\right]\right\},$$ (3)

with the various operators annihilating mesons of four-velocity $v$ which is conserved in strong interaction processes. The heavy field operators contain a factor $\sqrt{m_P}$ and have dimension 3/2.
The octet of light pseudoscalar mesons can be introduced using the representation 
\[ \xi = e^{i \frac{M}{f_\pi}} \] and \( \Sigma = \xi^2 \); the matrix \( \mathcal{M} \) contains \( \pi, K \) and \( \eta \) fields:

\[
\mathcal{M} = \begin{pmatrix}
\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & \pi^+ & K^+
\pi^- & -\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & K^0
K^- & \bar{K}^0 & -\sqrt{\frac{1}{2}} \eta
\end{pmatrix}
\]  

(4)

with \( f_\pi = 132 \) MeV.

The effective QCD Lagrangian is constructed imposing invariance under heavy quark
spin-flavour transformations and chiral transformations. The kinetic term

\[
\mathcal{L} = i \text{Tr} \{ \bar{H}_b \gamma^\mu D_{\mu ba} H_a \} + \frac{f_\pi^2}{8} \text{Tr} \{ \partial^\mu \Sigma \partial^\mu \Sigma^\dagger \}
+ \text{Tr} \{ \bar{S}_b (i \gamma^\mu D_{\mu ba} - \delta_{ba} \Delta_S) S_a \} + \text{Tr} \{ \bar{T}_b^\mu (i \gamma^\mu D_{\mu ba} - \delta_{ba} \Delta_T) T_{a\mu} \}
\]  

(5)

involves the operators \( D \) and \( A \):

\[
D_{\mu ba} = -\delta_{ba} \partial_\mu + V_{\mu ba} = -\delta_{ba} \partial_\mu + \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right)_{ba}
\]  

(6)

\[
A_{\mu ba} = i \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right)_{ba}
\]  

(7)

and the mass parameters \( \Delta_S \) and \( \Delta_T \) which represent the mass splittings between positive and negative parity doublets. They can be expressed in terms of the spin-averaged masses:

\[ \Delta_S = \overline{M}_S - \overline{M}_H \] and \( \Delta_T = \overline{M}_T - \overline{M}_H \) with

\[
\overline{M}_H = \frac{3M_{P^*} + M_P}{4}
\]

\[
\overline{M}_S = \frac{3M_{P^0} + M_{P^*}}{4}
\]

\[
\overline{M}_T = \frac{5M_{P^2} + 3M_{P^1}}{8}
\]

(8)

At the leading order in the heavy quark expansion the decays \( H \to H'M \), \( S \to H'M \) and \( T \to H'M \) (\( M \) a light pseudoscalar meson) are described by the lagrangian terms:

\[
\mathcal{L}_H = g \text{Tr} \{ \bar{H}_a H_b \gamma_\mu \gamma_5 A_{\mu ba}^a \}
\]

\[
\mathcal{L}_S = h \text{Tr} \{ \bar{H}_a S_b \gamma_\mu \gamma_5 A_{\mu ba}^a \} + h.c.
\]

\[
\mathcal{L}_T = \frac{h'}{\Lambda_\chi} \text{Tr} \{ \bar{H}_a T_b^\mu (iD_\mu A + iD A_\mu)_{ba} \gamma_5 \} + h.c.
\]

(9)

where \( \Lambda_\chi \) is a chiral symmetry-breaking scale; we use \( \Lambda_\chi = 1 \) GeV. \( \mathcal{L}_S \) and \( \mathcal{L}_T \) describe transitions of positive parity heavy mesons with the emission of light pseudoscalars in \( S \)
and \(D\) wave, respectively. The coupling constants \(h\) and \(h'\) weight the interactions of \(S\) and \(T\) heavy-light mesons with the light pseudoscalar mesons.

Corrections to the heavy quark limit induce symmetry breaking terms suppressed by increasing powers of \(m_Q^{-1}\) [3]. Mass degeneracy between the members of the meson doublets is broken by the terms:

\[
\mathcal{L}_{1/m_Q} = \frac{1}{2m_Q} \left\{ \lambda_H Tr[\tilde{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}] - \lambda_S Tr[\tilde{S}_a \sigma^{\mu\nu} S_a \sigma_{\mu\nu}] + \lambda_T Tr[\tilde{T}_a^\alpha \sigma^{\mu\nu} T_a^\alpha \sigma_{\mu\nu}] \right\}
\]

(10)

where the constants \(\lambda_H\), \(\lambda_S\) and \(\lambda_T\) are related to the hyperfine mass splittings:

\[
\lambda_H = \frac{1}{8} \left( M_{P^*}^2 - M_{P^0}^2 \right)
\]

\[
\lambda_S = \frac{1}{8} \left( M_{P_1^1}^2 - M_{P_0^1}^2 \right)
\]

\[
\lambda_T = \frac{3}{8} \left( M_{P_2^2}^2 - M_{P_1^1}^2 \right).
\]

(11)

Other two effects stemming from spin symmetry-breaking terms concern the possibility that the members of the \(s_\ell = \frac{3}{2}^+\) doublet can also decay in \(S\) wave into the lowest lying heavy mesons and pseudoscalars, and that a mixing may be induced between the two \(1^+\) states belonging to the two positive parity doublets with different \(s_\ell\). The corresponding terms in the effective Lagrangian are:

\[
\mathcal{L}_{D_1} = \frac{f}{2m_Q \Lambda} Tr[H_a \sigma^{\mu\nu} T_b^\alpha \sigma_{\mu\nu} \gamma_5 (iD_\alpha A_\theta + iD_\theta A_\alpha)_{ba}] + h.c.
\]

(12)

\[
\mathcal{L}_{mix} = \frac{g_1}{2m_Q} Tr[S_a \sigma^{\mu\nu} T_{ia} \sigma_{\mu\nu} v^\alpha] + h.c.
\]

(13)

Notice that \(\mathcal{L}_{D_1}\) describes both \(S\) and \(D\) wave decays. The mixing angle between the two \(1^+\) states:

\[
|P_1^{phys}\rangle = \cos \theta |P_1\rangle + \sin \theta |P_1'\rangle
\]

\[
|P_1^{phys}\rangle = -\sin \theta |P_1\rangle + \cos \theta |P_1'\rangle
\]

(14)

(15)

can be related to the coupling constant \(g_1\) and to the mass splitting:

\[
\tan \theta = \sqrt{\frac{\delta^2 + \delta_g^2 - \delta}{\delta_g}}
\]

(16)

where \(\delta = \Delta_T - \Delta_S\) and \(\delta_g = -\sqrt{\frac{2}{3}} \frac{g_1}{m_Q}.\)

The parameters in the various terms of the effective Lagrangian are universal and their determination is important in the definition of the effective theory and in the applications.
to the hadron phenomenology. Data recently collected on charmed and charmed-strange mesons, together with information on previously known positive parity charmed states, allow us to determine some of them by an analysis previously impossible due to the lack of enough experimental input.

A new result is the observation of charmed mesons which can be accommodated in the $s^P_\ell = \frac{1}{2}^+$ doublet. Two broad states which could be identified as the $D^*_0$ and $D'_1$ mesons have been observed by Belle [4], FOCUS [5] and CLEO [6] Collaborations. The masses and widths measured by Belle (the only experiment which separately observes the two states) are: $M_{D^*_0} = 2308 \pm 17 \pm 15 \pm 28$ MeV, $\Gamma(D^*_0) = 276 \pm 21 \pm 18 \pm 60$ MeV and $M_{D'_1} = 2427 \pm 26 \pm 10 \pm 15$ MeV, $\Gamma(D'_1) = 384^{+107}_{-75} \pm 24 \pm 70$ MeV, while the average values from the various experiments are: $M_{D^*_0} = 2351 \pm 27$ MeV, $\Gamma(D^*_0) = 262 \pm 51$ MeV (from Belle and FOCUS), and $M_{D'_1} = 2438 \pm 30$ MeV, $\Gamma(D'_1) = 329 \pm 84$ MeV (from Belle and CLEO). In the charm-strange sector the two mesons $D^*_{sJ}(2317)$ and $D'_{sJ}(2460)$ [7] naturally fit in the doublet ($D^*_{s0}$, $D'_{s1}$). Being below the $DK$ and $D^*K$ decay thresholds, respectively, they are narrow [8].

The two sets of measurements allow to determine a few parameters in eqs.(5), (10), (12) and (13). In Table 1 we collect the values of $\lambda_H$, $\lambda_S$ and $\lambda_T$ obtained using the masses of the charmed and beauty states reported by PDG [9] with two exceptions. The first one is $\lambda_S$ in case of non-strange charmed mesons, which we derive from the Belle measurement. Had we used mass values averaged over Belle, CLEO and Focus measurements we would have obtained a smaller value for $\lambda_S$, compatible within the uncertainties with the value in Table 1. The second exception concerns $B^*_{s}$, reported by PDG in the list of particles needing confirmation with $m_{B^*_{s}} = 5416 \pm 3.5$ MeV; we use the mass recently measured by the CLEO Collaboration: $m_{B^*_{s}} = 5414 \pm 1 \pm 3$ MeV [10]. In Table 1 we also report the spin averaged masses and the mass splitting between positive and negative doublets.

A few considerations are in order. First, observable SU(3) effects appear in charm determinations of $\lambda_H$, while analogous effects are not evident in $\lambda_{S,T}$ due to the larger experimental uncertainties. Second, a sizeable heavy quark mass effect remains in $\lambda_H$ when it is determined from charm and beauty data, meaning that further terms in the heavy quark mass expansion should be considered for describing data at the present level of accuracy. Finally, $\lambda_H$ and $\lambda_S$ are not equal, at odds with the relation $\lambda_H \simeq \lambda_S$ suggested by a description of negative and positive $s_\ell = \frac{1}{2}$ states as chiral partners [11].

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1The idea of chiral doubling as a consequence of the restoration of the chiral symmetry has been recently challenged in [12], where it is argued that when the symmetry is broken in the Nambu-Goldstone mode through the appearance of pions, it cannot be manifested somewhere in the spectrum.
Table 1: $\lambda_i$ parameters obtained using data in PDG [9]. For the determination of $\lambda_S$ in case of $c\bar{q}$ see text. The spin-averaged masses for the various doublets and the mass splittings $\Delta_S$ and $\Delta_T$ are also reported.

|        | $c\bar{q}$                  | $c\bar{s}$                  | $b\bar{q}$                  | $b\bar{s}$                  |
|--------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $\lambda_H$ | $(261.1 \pm 0.7 \text{ MeV})^2$ | $(270.8 \pm 0.8 \text{ MeV})^2$ | $(247 \pm 2 \text{ MeV})^2$ | $(252 \pm 10 \text{ MeV})^2$ |
| $\lambda_S$ | $(265 \pm 57 \text{ MeV})^2$  | $(291 \pm 2 \text{ MeV})^2$  |                             |                             |
| $\lambda_T$ | $(259 \pm 10 \text{ MeV})^2$  | $(266 \pm 6 \text{ MeV})^2$  |                             |                             |
| $M_H$    | $1974.8 \pm 0.4 \text{ MeV}$  | $2076.1 \pm 0.5 \text{ MeV}$ | $5313.5 \pm 0.5 \text{ MeV}$ | $5404 \pm 3 \text{ MeV}$    |
| $M_S$    | $2397 \pm 28 \text{ MeV}$    | $2424 \pm 1 \text{ MeV}$    |                             |                             |
| $M_T$    | $2445.1 \pm 1.4 \text{ MeV}$  | $2558 \pm 1 \text{ MeV}$    |                             |                             |
| $\Delta_S$ | $422 \pm 28 \text{ MeV}$    | $348 \pm 1 \text{ MeV}$    |                             |                             |
| $\Delta_T$ | $470.3 \pm 1.5 \text{ MeV}$  | $482 \pm 1 \text{ MeV}$    |                             |                             |

Since the parameters $\lambda_H$, $\lambda_S$, $\lambda_T$ are independent of the heavy quark mass, they can be used to constrain the masses of excited beauty mesons. We impose that the splittings $\Delta_S$ and $\Delta_T$ are the same for charm and beauty: this is true in the rigorous heavy quark limit, while at $O(1/m_Q)$ such an assertion corresponds to assuming that the matrix element of the kinetic energy operator is the same for the three doublets. Furthermore, SU(3)$_F$ effects are included in the determinations based on the charm-strange sector. The results are reported in Table 2. It is worth noticing that the $B_{s0}^*, B_{s1}^*$ masses are below the $BK$ and $B^*K$ thresholds, and therefore they are expected to be narrow (as also argued in [13, 8]). The search for such narrow resonances is in the physics programmes of collider experiments, the Tevatron and the LHC.

**Table 2: Predicted masses of excited beauty mesons.**

|        | $B_{(s)0}^*$ $(0^+)$ | $B_{(s)1}^*$ $(1^+)$ | $B_{(s)1}^*$ $(1^+)$ | $B_{(s)2}^*$ $(2^+)$ |
|--------|----------------------|----------------------|----------------------|----------------------|
| $b\bar{q}$ | $5.70 \pm 0.025 \text{ GeV}$ | $5.75 \pm 0.03 \text{ GeV}$ | $5.774 \pm 0.002 \text{ GeV}$ | $5.790 \pm 0.002 \text{ GeV}$ |
| $b\bar{s}$ | $5.71 \pm 0.03 \text{ GeV}$  | $5.77 \pm 0.03 \text{ GeV}$  | $5.877 \pm 0.003 \text{ GeV}$  | $5.893 \pm 0.003 \text{ GeV}$  |

In the above determinations we have neglected the mixing angle between the two $1^+$ states $D_1$ and $D'_1$. Considering, instead, the result $\theta_c = -0.10 \pm 0.03 \pm 0.02 \pm 0.02 \text{ rad}$ [4] and using $\Delta_T$ and $\Delta_S$ in Table 1 together with eq.(16) and $m_c = 1.35 \text{ GeV}$, we can compute the coupling $g_1$ in (13): $g_1 = 0.008 \pm 0.006 \text{ GeV}^2$, therefore compatible with
zero. In the beauty system, for $m_b = 4.8$ GeV, one obtains: $\theta_b \simeq -0.028 \pm 0.012$ rad.

To determine the couplings $h'$ and $f$ in eqs.(9-12) we consider the widths of the two members of the $s^0_T = \frac{3}{2}^+$ doublet, $D_1$ and $D_2^*$, identifying the observed meson $D_1(2420)$ with the physical state (14):

$$\Gamma(D_2^* \rightarrow D^+\pi^-) = \frac{4 m_D}{15\pi m_{D_2^*}} \frac{|\vec{p}_\pi|^5}{f_\pi^2} \frac{1}{\Lambda^2} \left( h' - \frac{f}{m_c} \right)^2$$  \hspace{1cm} (17)

$$\Gamma(D_2^0 \rightarrow D^{*+}\pi^-) = \frac{2 m_{D^*}}{5\pi m_{D_2^*}} \frac{|\vec{p}_\pi|^5}{f_\pi^2} \frac{1}{\Lambda^2} \left( h' - \frac{f}{m_c} \right)^2$$  \hspace{1cm} (18)

$$\Gamma(D_1^0 \rightarrow D^{*+}\pi^-) = \frac{2 m_{D^*}}{3\pi m_{D_1}} \frac{|\vec{p}_\pi|^5}{f_\pi^2} \frac{1}{\Lambda^2} \left[ \left( h' + \frac{5}{3m_c} \right)^2 + \frac{32 f^2}{9 m_c^2} \right]$$  \hspace{1cm} (19)

and

$$\Gamma(D_1^0 \rightarrow D^{*+}\pi^-) = \frac{1}{2\pi m_{D_1'}} \frac{|\vec{p}_\pi|^2}{f_\pi^2} (m_c^2 + |\vec{p}_\pi|^2) \ .$$  \hspace{1cm} (20)

Analogous equations hold for charmed-strange meson transitions: $D_{s2}^{*+} \rightarrow D^{(s)+}K^0$, $D^{(*)0}K^+$ and $D_{s1}^{+} \rightarrow D^{*+}K^0$, $D^{*0}K^+$. Assuming that the full widths are saturated by two-body decay modes with single pion (kaon) emission, we can determine $h'$ and $f$ using recent results from Belle Collaboration [4]:

$$\Gamma(D_2^0) = 45.6 \pm 4.4 \pm 6.5 \pm 1.6 \text{ MeV}$$

$$\Gamma(D_1^0) = 23.7 \pm 2.7 \pm 0.2 \pm 4.0 \text{ MeV} \ .$$  \hspace{1cm} (21)

Notice that, while $\Gamma(D_1^0)$ is compatible with the value reported by PDG ($\Gamma(D_1^0) = 18.9^{+4.6}_{-4.5}$ MeV) and with a recent measurement by CDF Collaboration [14], the width of $D_2^0$ is larger than the PDG value ($\Gamma(D_1^0) = 23 \pm 5$ MeV), while it is consistent with a Focus measurement: $\Gamma(D_2^0) = 38.7 \pm 5.3 \pm 2.9$ MeV [5] and a CDF measurement: $\Gamma(D_2^0) = 49.2 \pm 2.1 \pm 1.2$ MeV [14]. We use $h = -0.56$ [15]. In the plane $(h', f)$ four regions are allowed by data which, due to symmetry $(h', f) \rightarrow (-h', -f)$, reduce to the two inequivalent regions depicted in fig.1.

Notice that the two terms in square brackets in eq.(19) correspond to D-wave and S-wave pion emission, respectively. A further constraint is the Belle measurement of the helicity angle distribution in the decay $D_{s1}(2536) \rightarrow D^{*+}K^0_S$, with the determination of the ratio

$$R = \frac{\Gamma_S}{\Gamma_S + \Gamma_D} \ ,$$  \hspace{1cm} (22)
Figure 1: Regions in the \((h', f)\) plane constrained by the widths of \(D_2^{*0}\) and \(D_1^0\). Only the region A is also compatible with the constraints on the parameter \(R\) in eq.(22).

\(\Gamma_{S,D}\) being the S and D wave partial widths, respectively [16]: \(0.277 \leq R \leq 0.955\) (a measurement of the ratio \(R\) versus the phase difference between \(S\) and \(D\) was obtained by CLEO Collaboration for non-strange mesons [17]). Although the range of \(R\) is wide, it allows to exclude the region \(B\) in fig.1, leaving only the region \(A\) that can be represented as

\[
    h' = 0.45 \pm 0.05 \quad f = 0.044 \pm 0.044 \text{ GeV}.
\]

(23)

The coupling constant \(f\) is compatible with zero, indicating that the contribution of the lagrangian term (12) is small. Since also the coupling \(g_1\) turns out to be small, the two \(1^+\) states corresponding to the \(s'_1 = \frac{1}{2}^+, \frac{3}{2}^+\) practically coincide with the physical states.

For the width of \(D_{s1}(2536)\) we predict

\[
    \Gamma(D_{s1}(2536)) = 2.5 \pm 1.6 \text{ MeV}
\]

(24)

compatible with the present bound: \(\Gamma(D_{s1}(2536)) < 2.3\) MeV [9].

It is possible to predict the widths of excited \(B_{(s)}\) mesons, the results are collected in Table 3. Moreover, for \(B_1\) and \(B_{s1}\) the ratios (22) turn out to be compatible with zero: \(R = 0.01 \pm 0.01\) (for \(B_1\)) and \(R = 0.1 \pm 0.1\) (for \(B_{s1}\)). A word of caveat is needed here, since these predictions are obtained only considering the heavy quark spin-symmetry breaking terms in the effective Lagrangian; corrections due to spin-symmetric but heavy
Table 3: Predictions for decay widths and branching fractions of $J_{s_1}^P = (1^+, 2^+)_{±}$ beauty mesons. The full widths are obtained assuming saturation of the two-body modes.

| Mode          | $\Gamma$(MeV) | BR  | Mode          | $\Gamma$(MeV) | BR  |
|---------------|---------------|-----|---------------|---------------|-----|
| $B_{2s}^{0} \to B^{+}\pi^-$ | $20 \pm 5$ | 0.34 | $B_{s2}^{0} \to B^{+}K^-$ | $4 \pm 1$ | 0.37 |
| $B_{2}^{0} \to B^{0}\pi^0$ | $10.0 \pm 2.3$ | 0.17 | $B_{2}^{0} \to B^{0}K^0$ | $4 \pm 1$ | 0.34 |
| $B_{2}^{0} \to B^{*+}\pi^-$ | $18 \pm 4$ | 0.32 | $B_{2}^{0} \to B^{*+}K^-$ | $1.7 \pm 0.4$ | 0.15 |
| $B_{2s}^{0} \to B^{*0}\pi^0$ | $9.3 \pm 2.2$ | 0.16 | $B_{s2}^{0} \to B^{*0}K^0$ | $1.5 \pm 0.4$ | 0.13 |
| $B_{2}^{0}$ | $57.3 \pm 13.5$ | | | | |
| $B_{1}^{0} \to B^{*+}\pi^-$ | $28 \pm 6$ | 0.66 | $B_{s1}^{0} \to B^{*+}K^-$ | $1.9 \pm 0.5$ | 0.54 |
| $B_{1}^{0} \to B^{*0}\pi^0$ | $14.5 \pm 3.2$ | 0.34 | $B_{s1}^{0} \to B^{*0}K^0$ | $1.6 \pm 0.4$ | 0.46 |
| $B_{1}^{0}$ | $43 \pm 10$ | | | | |
| $B_{1}^{0}$ | | | $B_{s1}^{0}$ | $3.5 \pm 1.0$ | |

Flavour breaking terms involve additional couplings for which no sensible phenomenological information is currently available, so that they cannot be reliably bounded. Keeping this in mind, we notice that the full widths of $B_{2(s)2}^{0}$ and $B_{(s)1}^{0}$ are determined with remarkable accuracy. As for the present experimental information concerning these states, PDG reports in the listing of states needing confirmation a $B_{s1}^{*}(5732)$ signal, which could be considered as stemming from several narrow and broad resonances, with (average) mass $5698 \pm 8$ MeV and (average) width of $128 \pm 18$ MeV [9]. The separation of this signal in its components could be done using our predictions. A $B_{s1}^{*}(5850)$ signal is also reported with mass $5853 \pm 16$ MeV and width of $47 \pm 22$ MeV [9], within our predictions.

In conclusion, we have exploited recent observations and measurements concerning excited charm mesons to determine two coupling constants governing their strong decays to light pseudoscalar mesons, as well as the mixing parameter between the two $J^P = 1^+$ states. Furthermore, we have estimated masses and decay rates of corresponding beauty states, and these predictions will be checked at the future experimental environments, such as the LHC, where such states could be observed.
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