Responses of transmission-line networks to electrostatic discharge electromagnetic pulses

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Abstract. A modelling method of transmission-line networks based on Baum–Liu–Tesche (BLT) equation is investigated, and a use of BLT equation for calculating transmission-line load responses in the time domain is discussed. From the frequency-domain BLT equation, the expressions for the propagation function at each terminal load are derived. Using the propagation function and time-domain convolution calculation, the transient load responses in the time domain are gained. For special cases of a dendriform network and an annular network, the transient load responses excited by electrostatic discharge (ESD) pulse are calculated. Compared with test results, and the agreement of the calculation is excellent. The modelling and calculating method could be used to investigate the load responses of transmission-line networks, and estimate the influence of electromagnetic pulses on networks.

1. Introduction
Electrostatic discharge (ESD) is a natural phenomenon in dry environment. ESD current is a transient pulse with short duration, high peak power and concomitant electromagnetic emission. ESD electromagnetic pulse (EMP) can influence the performance of circuits and devices, and transmission line and its network are the main coupling ways of EMP energy.

In recent years, increasing attention is drawn to the area of ESD effects and ESD protection. It mainly focuses on direct coupling of ESD energy to devices or equipment. This work emphasizes indirect coupling by transmission-line networks, and investigate loads responses of networks when ESD occurred at any place. Different loads, structures and line length of network are also considered.

2. BLT equation
In 1970s, there was an interest in developing transmission-line models that could be used for analyzing large multi-conductor cable harnesses in aircraft. This research resulted in the Baum–Liu–Tesche (BLT) equation, which is a frequency domain matrix equation describing the behavior of voltages and currents at all of the junctions (interconnections) of the multiconductor cables in the network [1–2]. Recently, an extension of the BLT-equation concept was developed to include the effects of electromagnetic (EM) field propagation in space (along trajectories that can be thought of as being an imaginary field propagation “tube”) [3-4].

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The multi-conductor transmission-line equations in frequency domain can be expressed as:

\[
\begin{align*}
\frac{d}{dx}(V_n(x)) &= -(Z_{n,m}) \cdot (I_n(x)) + (V_n^{(s)}(x)) \\
\frac{d}{dx}(I_n(x)) &= -(Y_{n,m}) \cdot (V_n(x)) + (I_n^{(s)}(x))
\end{align*}
\] (1)

where \((Z_{n,m})\) and \((Y_{n,m})\) are the per-unit-length impedance matrix and admittance matrix respectively; \((V_n^{(s)}(x))\) and \((I_n^{(s)}(x))\) are the per-unit-length voltage matrix and current matrix respectively. The subscript \(m\) and \(n\) express the conductor sequences.

The two conductor transmission line models are shown in figure 1. The wave \(W_1\) and \(W_2\) can be expressed in matrix form as:

\[
W_1(x) = (V_n(x))_+ = (V_n(x)) + (Z_{C_n,m}) \cdot (I_n(x)) \tag{2}
\]

\[
W_2(x) = (V_n(x))_- = (V_n(x)) - (Z_{C_n,m}) \cdot (I_n(x)) \tag{3}
\]

where, \(Z_{C_n,m}\) is the impedance matrix, \((V_n(x))_+\) is the incidence wave and \((V_n(x))_-\) is the reflection wave. The total voltage and current are as follows:

\[
(V_n(x)) = \frac{1}{2} ((V_n(x))_+ + (V_n(x))_-) \tag{4}
\]

\[
(Z_{C_n,m}) \cdot (I_n(x)) = \frac{1}{2} ((V_n(x))_- - (V_n(x))_+). \tag{5}
\]

![Figure 1. Transmission-line model.](image)

So, the equation (1) can be expressed as:

\[
\left(\frac{d}{dx} + q \gamma_{C,n}\right) (V_n(x))_q = (V_n^{(s)}(x))_q \tag{6}
\]

with

\[
[\gamma_{C,n}^2] = \begin{bmatrix}
\gamma_1^2 & 0 & \cdots & 0 \\
0 & \gamma_2^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \gamma_n^2
\end{bmatrix}, \quad q = \begin{cases}
1 & \text{for incidence wave} \\
-1 & \text{for reflection wave}
\end{cases}
\]

According to the foregoing equations, the solution can be obtained as:
\[ W_i(x) = e^{-(\gamma_{ia})x} \cdot (V_a(0)) + \int_0^x e^{-(\gamma_{ia})|x-s|} \cdot (V_a^{(S)}(s)) \, ds \]  
\( i = 1, 2 \) 

(7)

\[ W_2(x) = e^{-(\gamma_{ia})|x-L|} \cdot (V_a(L)) + \int_L^x e^{-(\gamma_{ia})|x-s|} \cdot (V_a^{(S)}(s)) \, ds. \] 

(8)

Assumed \( x \) as \( u \), \( -x \) as \( v \), \( (\gamma_{nu}) \) and \( (\gamma_{uv}) \) could be expressed in new coordinate as:

\[ W_u(x_u) = (V_u(x_u)) = e^{-(\gamma_{nu})x_u} \cdot (V_u(0)) + \int_0^{x_u} e^{-(\gamma_{nu})|x_u-s_u|} \cdot (V_u^{(S)}(s_u)) \, ds_u, \] 

(9)

\[ W_v(x_v) = (V_v(x_v)) = e^{-(\gamma_{uv})x_v} \cdot (V_v(0)) + \int_0^{x_v} e^{-(\gamma_{uv})|x_v-s_v|} \cdot (V_v^{(S)}(s_v)) \, ds_v. \] 

(10)

\[ W_{u,v}(x_{u,v}) = (V_{u,v}(x_{u,v})) = e^{-(\gamma_{nu})x_u} \cdot (V_u(0)) + \int_0^{x_u} e^{-(\gamma_{nu})|x_u-s_u|} \cdot (V_u^{(S)}(s_u)) \, ds_u, \] 

(11)

\[ W_{u,v}(x_{u,v}) = (V_{u,v}(x_{u,v})) = e^{-(\gamma_{uv})x_v} \cdot (V_v(0)) + \int_0^{x_v} e^{-(\gamma_{uv})|x_v-s_v|} \cdot (V_v^{(S)}(s_v)) \, ds_v. \] 

(12)

Transmission-line network can be derived by the two conductor transmission-line as:

\[ (W_u(L_u)) = (V_u(L_u)) = (\Gamma_{n,m}) \cdot ((V_u(0))) + ((V_u^{(S)})). \] 

(13)

where

\[ (\Gamma_{n,m}) = 1_{u,v} \cdot e^{-(\gamma_{nu})x_u} = \begin{cases} 1 \quad & u = v \\ 0 \quad & u \neq v \end{cases} \]

and

\[ (V_u^{(S)}) = \int_0^{x_u} e^{-(\gamma_{nu})|x_u-s_u|} \cdot (V_u^{(S)}(s_u)) \, ds_u. \]

Scattering matrix equation is defined as:

\[ ((V_u(0))) = (S_{n,m}) \cdot ((V_u(0))) + ((V_u^{(S)})). \] 

(14)

Substituting (14) for (13), we obtain the following equation:

\[ (V_u(0)) = (\Gamma_{n,m}) \cdot ((V_u(0))) + ((V_u^{(S)})). \] 

(15)

Through matrix inverse, equation (15) can be transformed as:

\[ (V_u(0)) = [((1_{n,m}) - (S_{n,m}) \cdot (\Gamma_{n,m})))^{-1} \cdot ((S_{n,m}) \cdot (V_u^{(S)}))). \] 

(16)

The total voltage is the sum of incidence voltage and reflection voltage:

\[ (V_u(0)) = \frac{1}{2} [(V_u(0)) + (V_u(L_u))]. \] 

(17)

The relation of incidence voltage and reflection voltage is:

\[ ((V_u(L_u))) = (\Gamma_{n,m}) \cdot ((V_u(0))) + ((V_u^{(S)})). \]

(18)

Combining (16), (17) and (18), we can get the total voltage of node:

\[ (V_u(0)) = \frac{1}{2} [(S_{n,m}) \cdot ((1_{n,m}) - (S_{n,m}) \cdot (\Gamma_{n,m}) \cdot )^{-1} (V_u^{(S)})). \] 

(19)
3. Modelling and simulation of networks

The models of dendriform network and annular network are established, and transient responses of loads are analyzed.

3.1 Dendriform network

The dendriform network electromagnetic topology model is shown in figure 2. The lengths of tubes are: $T_1=1$ m, $T_2=3$ m, $T_3=2$ m and $T_4=5$ m, respectively. The characteristic impedances of all tubes (lines) are 50 Ohm. Using the BLT equation and Fourier transform techniques, transient responses of network to ESD EMP can be analyzed. Figure 3 shows the exciting source, which is the body-metal ESD wave and generated by NS61000-2A ESD simulator. The peak voltage is 18.6 V. The injection place is node 0. The loads of node 2, node 4 and node 5 are all 50 Ohm. The transient responses of node 2, node 4 and node 5 are shown in figure 4.

![Dendriform network topology model](image2.png)

**Figure 2.** Dendriform network topology model.

![ESD EMP waveform](image3.png)

**Figure 3.** ESD EMP waveform.

![Voltage responses](image4.png)

**Figure 4.** Voltage responses of node 2(a), node 4(b) and node 5(c).
Figure 4 shows the starting times of different loads responses are different. The results imply transmission times are different, which caused by different transmission distances.

Changing the load of node 2, the responses of node 4 are shown in figure 5. When the load doesn’t match the transmission line, the mismatching will cause reflection. The second peak of response waves is arisen from the reflection of node 2, when the load of node 2 is 30 Ohm or 1 mega-Ohm. The reflection coefficient of node is defined as

$$\Gamma = \frac{Z_{\text{load}} - Z_{\text{line}}}{Z_{\text{load}} + Z_{\text{line}}}.$$  \hspace{1cm} (20)

The reflection coefficient approximates 1 when the load is 1 mega-Ohm, so the total voltage of node 4 is doubled (blue dot line in figure 5). The reflection coefficient is -0.25 when the load is 30 Ohm. Therefore, the total voltage of node 4 decreased (red dash line in figure 5), due to the negative reflection coefficient.

The responses of node 4 are shown in figure 6 when substitute the length of tube 2 (defined as $L$). The tube length determines transmission time, so the delays of responses are different at different tube length.

![Figure 5. Voltage response at node 4 with different loads of node 2.](image1)

![Figure 6. Voltage response at node 4 with different line length of tube 2.](image2)

### 3.2 Annular network

The annular network electromagnetic topology model is shown in figure 7. The lengths of tubes are: $T_1=2$ m, $T_2=1$ m, $T_3=3$ m, $T_4=1$ m and $T_5=1$ m, respectively. The characteristic impedances of all tubes (lines) are 50 Ohm. The exciting source is shown in figure 3. The injection place is node 0. The loads of node 4 and node 5 are 50 Ohm. The transient response of node 5 is shown in figure 8.

![Figure 7. Annular network topology model.](image3)

![Figure 8. Voltage response at node 5.](image4)
The responses of annular network are different from those of dendriform network, by comparing figure 4 with figure 8. In annular network, there are two paths of node 0 (injection node) to node 5 (terminal node), one is $T_0 \rightarrow T_1 \rightarrow T_4$, the other is $T_0 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4$. The lengths of two paths are different, thus transmission times are different. The response of node 5 is the sum of the two paths, so the total response of node 5 has two peaks.

Changing the load of node 4, the responses of node 5 are shown in figure 9. Changing the length of tube 2 (defined as $L$), the responses of node 4 are shown in figure 10. Different loads could give rise to different reflection (shown in figure 9); different tube length could cause different delay (shown in figure 10), which is similar to the dendriform network.

\begin{figure}[h]
\centering
\includegraphics[width=1\textwidth]{figure9.png}
\caption{Voltage response at node 5 with different loads of node 4.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=1\textwidth]{figure10.png}
\caption{Voltage response at node 5 with different line length of tube 2.}
\end{figure}

\subsection*{3.3 Experimental validation}
To verify the validity of calculating results, the experimental method was designed. The experimental networks are composed of coaxial cables, and the structure and lines length are the same as calculating models (shown in the figure 2 and 7). The parameters of coaxial cable are: $\sigma = 5.8 \times 10^{-7}$ S/m, $r=1.955$ mm, $R=2.375$ mm, $\varepsilon_{\text{rel}}=2.1$ and $Z_0 = 50$ $\Omega$. The comparison of experimental results and computer-simulated results are shown in figure 11.

\begin{figure}[h]
\centering
\includegraphics[width=1\textwidth]{figure11.png}
\caption{Comparing of computer-simulated and experimental responses of node 2 (a) dendriform network; (b) annular network.}
\end{figure}

The agreement of computing results and experimental results are excellent for cable networks. These results show that this modelling method is effective for transmission-line network analysis.
4. Conclusion

This work verified the modelling and calculating method based on BLT equation can be used for analyzing transient responses of transmission-line network, by means of comparing the results of simulation and testing. Simulation results showed that the node responses to ESD EMP are related with network structure, lines length, and nodes load. The method and results of this work are useful for design and protection of networks against EMP.

Acknowledgments

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