HADRONIC B DECAYS WITH QCD FACTORIZATION

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A brief summary of outstanding theoretical issues and recent results from the QCD factorization approach to exclusive hadronic B decays is provided.

Keywords: B decays; factorization

1. INTRODUCTION

Hadronic two-body decays of B mesons play a central role in the on-going programme to clarify the sources of CP violation and rare flavour-changing processes. Where experimental information is not sufficient to determine strong interaction amplitudes from the data themselves, the matrix elements ⟨f|Oi|¯B⟩ (with Oi an operator from the effective weak interaction and f a two-body final state) must be provided by theory.

QCD factorization \(^\text{1}\) is a synthesis of the heavy quark expansion (\(m_b \gg \Lambda_{QCD}\)) with soft-collinear factorization for hard processes (particle energies \(\gg \Lambda_{QCD}\)) to compute the matrix elements ⟨\(M_1M_2|O_i|\bar{B}\)⟩ in an expansion in \(1/m_b\) and \(\alpha_s\). Only the leading term in \(1/m_b\) assumes a simple form. The basic formula is

\[
\langle M_1M_2|O_i|\bar{B}\rangle = F^{BM_1}(0) \int_0^1 du T^{I}_i(u)\Phi_{M_2}(u) + \int d\omega du dv T^{II}_i(\omega, u, v)\Phi_B(\omega)\Phi_{M_1}(v)\Phi_{M_2}(u),
\]

where \(F^{BM_1}(0)\) is a standard heavy-to-light form factor at \(q^2 = 0\), \(\Phi_{M_i}\) and \(\Phi_B\) are light-cone distribution amplitudes, and \(T^{I,II}_i\) are perturbatively calculable hard-scattering kernels. \((M_1\) and \(M_2\) are light mesons, and \(M_1\) is the meson that picks up the spectator antiquark from the \(\bar{B}\) meson.) The formula shows that there is no long-distance interaction between the constituents of the mesons \(M_2\) and the \((BM_1)\) system at leading order in \(1/m_b\). This is the precise meaning of factorization.

2. THEORETICAL ISSUES

2.1. Factorization of spectator scattering

The factorization argument for the first term on the right-hand side of (1) is very similar to the case when \(M_1\) is a heavy meson \(^\text{12}\). The second term is more interest-
ing, since it involves a hard interaction with the soft spectator antiquark in the $\bar{B}$ meson, which introduces the scale $(m_b\Lambda_{\text{QCD}})^{1/2}$. The detailed structure of this term has now been clarified with the help of soft-collinear effective theory (SCET).3,4

It is a consequence of “colour transparency” that the meson $M_2$ (which does not pick up the spectator quark) factorizes from the $(BM_1)$ system already when the scale $m_b$ is integrated out. Thus one obtains (matching to SCET$_I$)

\[
\langle M_1 M_2 | O_i | \bar{B} \rangle = \int_0^1 du \Phi_{M_2}(u) \left\{ T^I_i(u) F^{BM_1}(0) + \int d\tau C^{II}_i(\tau; u) \Xi^{BM_1}(\tau; 0) \right\}, \tag{2}
\]

where $\Xi^{BM_1}(\tau; 0)$ denotes a new bi-local form factor related to the SCET$_I$ matrix element $\langle M_1 | \xi(0) A_{\perp c}(sn_c) h_v(0) | \bar{B} \rangle$. The same function appears in the factorization formula for the heavy-to-light form factor.5 We can therefore apply the factorization argument for form factors6 to write $\Xi^{BM_1}(\tau; 0)$ as (matching to SCET$_{II}$)

\[
\Xi^{BM_1}(\tau; 0) = \int d\omega dv J(\tau; \omega, v) \Phi_B(\omega) \Phi_{M_1}(v), \tag{3}
\]

which implies (1) with $T^{II}_i(\omega, u, v) = \int d\tau C^{II}_i(\tau; u) J(\tau; \omega, v)$. The short-distance function $J(\tau; \omega, v)$ contains the scattering with the spectator antiquark at the scale $(m_b\Lambda_{\text{QCD}})^{1/2}$. Using (3) provides more predictive power, since $\Xi^{BM_1}(\tau; 0)$ is otherwise unknown, but is not mandatory, since $M_2$ factorizes at the scale $m_b$. This is exploited in what is sometimes (inappropriately) called the SCET approach to hadronic $B$ decays.5

Has factorization for $B$ decays to two light mesons been proven? SCET provides precise prescriptions for the extraction of the various short-distance functions and up to now all results are in agreement with (1). The remaining issues are more fundamental. SCET factorization “proofs” often tend to assume what should be part of the proof, namely that SCET is the correct framework. This requires an investigation of the analytic structure of Feynman integrals in QCD. The observation that soft-collinear factorization in SCET$_{II}$ is in general not maintained in loop corrections due to the absence of a regulator that respects factorization6,7 provides an example of the possible pitfalls in SCET factorization “proofs”.

2.2. Radiative corrections

The short-distance functions $T^I_i$, $T^{II}_i$ are currently computed up to order $\alpha_s$. This means that the strong-interaction phases (hence, direct CP asymmetries) and spectator-scattering effects are known only at leading order (LO). A next-to-leading order (NLO) calculation of these quantities is particularly important in view of phenomenological evidence (mainly from the $\pi\pi$ and $\pi K$ final states) that the strong-interaction phases and the ratio of the colour-suppressed to colour-allowed tree amplitude, $C/T$ (or $\alpha_2/\alpha_1$), might be larger than the (LO) factorization result.

A complete NLO calculation of phases and spectator-scattering is not yet available. The NLO correction to the function $J(\tau; \omega, v)$ has been computed5,10,19 and
has been found to increase $C/T$. A partial calculation of the NLO penguin contributions to $T_{11}^{11}$ has been performed. The completion of these calculations will answer the question whether the apparent discrepancies between LO factorization and data have a short-distance explanation.

2.3. **Power corrections**

2.3.1. **Scalar penguins**

The only $1/m_b$ power correction for which there is unambiguous evidence is the (pseudo)scalar penguin contribution to the QCD penguin amplitude ($a_6$). The scalar penguins arise from the Fierz transformation of the $V+A$ penguin operators in the effective weak Hamiltonian and their interference with the $V-A$ penguin contribution strongly discriminates between final states of two pseudoscalar mesons (PP), one pseudoscalar and one vector meson (PV or VP), and two vector mesons (VV). Fortunately, the leading scalar penguin contribution is calculable despite being a power correction, and its inclusion into the factorization formula (1) is mandatory for a successful phenomenology. An understanding of the factorization properties of this power correction would provide a better justification for this procedure, but has not yet been completed. A preliminary investigation indicates that factorization is not preserved, but the factorization-breaking terms appear to be numerically suppressed. An enumeration of power-suppressed diagram topologies related to this question can also be found in Ref. 20.

2.3.2. **Weak annihilation**

Weak annihilation constitutes a class of $1/m_b$ power corrections which does not factorize as in (1). A phenomenological model in terms of a single parameter, which also allows for a long-distance strong-interaction phase, is often used to account for this contribution. In this model the single most important annihilation amplitude comes from penguin operators, and it is indistinguishable from the QCD penguin amplitude. There is evidence from data that an additional contribution to the QCD penguin amplitude is required that could be ascribed to weak annihilation, and which is compatible with the assumed size of this contribution (up to 25% of the penguin amplitude for final states of two pseudoscalar mesons and significantly larger for vector meson final states) in the standard parameterization. An estimate of the annihilation amplitude with QCD sum rules leads to numbers in a similar range. In the absence of higher-order calculations of the QCD penguin amplitude the data does not allow to draw the conclusion that the weak annihilation mechanism is significant at all. Evidence for this comes from the annihilation-dominated decay $\bar{B}_d \rightarrow D_s^+ K^-$ with an observed branching fraction in agreement with the estimate from the phenomenological model. A better quantitative control of weak annihilation is nevertheless one of the key issues in the theory of hadronic $B$ decays, in particular for final states containing one or two vector mesons.
2.3.3. 3-particle contributions

Some power corrections related to $q\bar{q}g$ light-cone distribution amplitudes have been evaluated.\textsuperscript{23,24}

2.3.4. Charm penguins

Charm penguins, that is, contractions that contain a charm quark loop, have been discussed for some time as a source of potentially large corrections.\textsuperscript{25} While these discussions centered on the issue whether charm penguins could induce numerically large power corrections, it has recently been claimed\textsuperscript{5} that charm penguins may not factorize at all even at leading power in contradiction to (1). To be precise, the short-distance part of the charm penguin loops is part of the standard calculation of the penguin amplitudes. The issue is whether charm quark loops have (incalculable) long-distance contributions that do not vanish as $m_b$ goes to infinity. The argument says that charm loops have a significant contribution from the charm threshold region, in which non-relativistic power counting must be applied and factorization is not guaranteed.

There appears to be a confusion about how the non-relativistic treatment of charm is combined with the heavy quark expansion in $m_b$.\textsuperscript{26} If $m_b \rightarrow \infty$ at fixed $m_c$, (for instance $m_c v^2 \sim \Lambda_{\text{QCD}}$), the charm quark is like a light quark compared to the scale $m_b$, and there is no principal difference between light and charm quark loops, which both factorize. If $m_b \rightarrow \infty$ at fixed $m_c$, then also the smallest non-relativistic scale $m_c v^2 \gg \Lambda_{\text{QCD}}$, the threshold region becomes perturbative, and standard perturbative non-relativistic resummation applies.\textsuperscript{27} The non-perturbative remainder is related to a power correction parameterized by the matrix element of a higher-dimensional operator. It follows that the charm penguins factorize since the long-distance contribution always carries a factor $1/m_b$.\textsuperscript{11}

3. EVIDENCE FOR (AND PROBLEMS WITH) FACTORIZATION

3.1. Evidence

The good overall agreement of the calculated branching fractions with observations, in particular with the parameter set S4 defined in Ref.\textsuperscript{17}, provides clear evidence that the leading amplitudes (colour-allowed tree and QCD penguin) are approximately correctly obtained with factorization. Perhaps the most important evidence of factorization comes from the observed non-universality of the QCD penguin amplitude between pseudoscalar and vector meson final states. In factorization there is a strong correlation with the $J^{PC}$ quantum numbers of the primary weak current, which leads to $M_1 M_2$ QCD penguin amplitudes roughly as follows:

$$PP \sim a_4 + r_x a_6, \quad PV \sim a_4 \approx \frac{PP}{3}, \quad VP \sim a_4 - r_x a_6 \sim -PV.$$  \hspace{1cm} (4)
The suppression of the PV and VP penguin amplitudes relative to PP is responsible for the smaller PV branching fractions, and the interference of penguin amplitudes explains the surprisingly different branching fractions of \( B \to \eta^{(')}K^{(*)} \)\cite{13, 28}. The relevance of factorization is further corroborated by the non-observation of large direct CP asymmetries, since it is a feature of the heavy-quark limit that the strong interaction phases are suppressed.

### 3.2. Problems

A more detailed comparison also reveals difficulties with factorization. The infamous wrong-sign prediction\cite{11} of the direct CP asymmetry in \( B \to \pi^{\mp}K^{\pm} \) is an example. This discrepancy is actually quantitatively a small effect, which may disappear due to higher-order radiative corrections or a strong phase in the weak annihilation amplitude. There also exist indications of a large colour-suppressed tree amplitude. It remains to be seen whether this can be explained by spectator-scattering alone\cite{9, 17}.

A general observation is that the prediction of strong phases can be rather uncertain not only because there is currently no NLO prediction, but also because a power correction \( \Lambda_{QCD}/m_b \) is parametrically not much smaller than \( \alpha_s(m_b) \). Nevertheless, it is difficult to see how factorization could explain strong phases around \( 90^\circ \) of a QCD penguin or colour-suppressed tree amplitude (relative to the colour-allowed tree) as are sometimes reported. Obviously, before giving up a wonderful theory one would like to see the experimental data improve to the point that the case for very large phases can be made with certainty.

### 4. THREE (EXEMPLARY) ROUTES TO CKM PARAMETERS

#### 4.1. \((\bar{\rho}, \bar{\eta})\) from \( B \to \pi\pi, \pi K\)

A global fit of \((\bar{\rho}, \bar{\eta})\) to the \( B \to \pi\pi, \pi K\) branching fractions in QCD factorization has already been performed in Ref.\cite{11} and later updates gave results in good agreement with the standard unitarity triangle fit. The most recent fit\cite{29}, which includes CP asymmetries, gives \( \gamma = (62^{+6}_{-9})^\circ \) to be compared to \( \gamma = (62^{+10}_{-12})^\circ \) from the standard fit. It is prudent to regard this result with caution, since it arises from averaging measurements that individually give rather different values for \( \gamma \).

#### 4.2. \( \gamma \) from time-dependent CP asymmetries in \( b \to d \) transitions

Given a calculation of the penguin-to-tree amplitude ratio, \( \gamma \) (or \( \alpha \)) can be determined directly from the sin-oscillation of the time-dependent CP asymmetry in \( B \to \pi^+\pi^-\), \( S_{\pi\pi} \). The determination is rather accurate, since the strong phases \( \delta \) enter only at second order in \( \delta \). The analogous quantity \( S_{\pi\rho} = (S_{\pi^+\rho^-} + S_{\pi^-\rho^+})/2 \) in \( B \to \pi^\pm\rho^\mp \) decays is particularly clean, since the penguin amplitude is significantly smaller (see above). The experimental values \( S_{\pi\pi} = 0.13 \pm 0.13 \) and \( S_{\pi\rho} = -0.50 \pm 0.12 \) provide clear evidence for the penguin contribution, since otherwise \( S_{\pi\pi} = S_{\pi\rho} = -\sin 2(\beta + \gamma) \). Given \( \beta \) and the calculation of the penguin
amplitude, one finds $\gamma = (70^{+8}_{-8})^{\circ} \text{ (from } S_{\pi\rho})$ and $\gamma = (66^{+13}_{-12})^{\circ} \text{ (from } S_{\pi\pi})$ in nice mutual agreement and with the standard fit.

### 4.3. CKM parameters from $B \to \rho\gamma$

QCD factorization also applies to exclusive radiative decays. Recent analyses of branching fractions, CP- and isospin asymmetries in $B \to \rho\gamma$ decays have shown that $(\bar{\rho}, \bar{\eta})$ can be extracted from $b \to d\gamma$ transitions alone. Although not competitive in accuracy with the standard fit, this provides another consistency check for the CKM mechanism. In particular $\text{Br}(B^0 \to \rho^0\gamma) < 0.4 \cdot 10^{-6}$ implies $|V_{td}/V_{ts}| < 0.21$, which already cuts into the range allowed by the standard fit.

### 5. PUZZLES, NOW AND THEN

#### 5.1. The $B \to \pi K$ puzzles

Certain unexpected features of ratios of $B \to \pi K$ branching fractions and CP asymmetries have become preeminent in 2003 and have since then been extensively discussed. The most straightforward interpretation seems to be an enhanced electroweak $b \to s$ penguin amplitude with a large CP-violating phase, and an enhancement of the colour-suppressed tree amplitude. See the talk of R. Fleischer at this conference for a detailed discussion.

#### 5.2. $\sin(2\beta)$ from $b \to s$ transitions

The time-dependent CP asymmetry in $b \to s$ penguin transitions is expected to be close to $\pm \sin(2\beta)$, since $b \to c\bar{c}s$ and $b \to s\bar{s}s$ have (nearly) the same weak phase, and subleading amplitudes are CKM-suppressed. Calculations of $\Delta S_f \equiv S_f(b \to s) - \sin(2\beta)J_{\phi K_S}$ in QCD factorization confirm this expectation and yield a positive $\Delta S_f$ for $f = \pi^0K_S, \eta'K_S, \phi K_S, \omega K_S$, which is very small ($\approx 0.02$) for $f = \eta'K_S, \phi K_S$. This is in contrast to data which upon averaging over final states give $\Delta S_f = -0.19 \pm 0.07$.

#### 5.3. $B \to \eta'K$

The decay $B \to \eta'K$ has the largest branching fraction among all charmless $B$ decays. Moreover, there is an interesting pattern in related final states: $\text{Br}(\eta'K) \approx 20 \text{Br}(\eta K)$ but $\text{Br}(\eta'K^*) < \text{Br}(\eta K^*)$. The QCD factorization analysis of these decays reveals several new decay mechanisms for final state mesons with flavour-singlet components, and explains this pattern as an interference of QCD $b \to s$ penguin amplitudes, which are different for PP and PV final states and can have different signs for $\eta$ and $\eta'$ owing to their different strange content. In particular, $\text{Br}(B \to \eta'K) \approx 70 \cdot 10^{-6}$ is in the central range of theoretical results, which however carry a large uncertainty.
5.4. Polarization in $B \to VV$

Decays to two vector mesons offer additional dynamical information due to the possibility of polarization studies. In the heavy-quark limit both vector mesons are longitudinally polarized, and one expects the hierarchy $A_0 \gg A_- \gg A_+$ of helicity amplitudes. Transverse polarization is a power correction. This expectation is not always borne out by the data, since for instance the longitudinal polarization fraction is only about 0.5 for $B \to \phi K^*$. No anomaly is observed for tree-dominated decays. The resolution of this “polarization puzzle” in the context of QCD factorization is due to the observation [10] that weak annihilation makes a very large contribution to the $VV$ penguin amplitude. A large depolarization in penguin-dominated decays is therefore not in contradiction with theoretical estimations. Unfortunately, the theoretical predictions are also very uncertain due to the lack of a sensible theory for annihilation effects.

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