Design of optical system with three-dimensional image visualization using an array of microlenses

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Abstract. The algorithm of calculation of the optical imaging system of a three-dimensional image based on the method of integral photography was given in this article. The algorithm is easy to use and allows the calculate schemes with different microlens arrays, a CCD array of the camera and the projector.

1. Introduction
A person perceives the world as a three-dimensional space filled with three-dimensional objects, and has long sought to reproduce the most plausible reality, for this purpose he is constantly inventing new ways of visualizing three-dimensional objects. One of them is the creation of three-dimensional image in the space on the basis the method of the integral photography [1].

The main problem of the reconstruction is the quality of the obtained three-dimensional image. In this paper we present the reasoning and the algorithm for calculating the basic parameters of the three-dimensional optical imaging system (integrated) images using a microlens array. The calculations take into account not only the resolution of the CCD matrix of the camera and the projector, and physiological characteristics of the human eye.

2. Schematic diagram of the recording and reconstruction of the integral image
The optical recording scheme presented in figure 1, consists of three components: the array of microlenses 1, the focusing lens 3 and the CCD camera (while recording) / projector (while reconstructing) 4.

![Figure 1](image.png)

**Figure 1.** The optical scheme of the integral image recording: 1 microlens array; 2 - plane of elementary images; 3 - focusing lens; 4 - CCD sensor camera.
The microlenses array is a matrix consisting of a lens of small size. Each microlens captures a certain angle of the object and generates a two-dimensional image in a plane elementary image. The array of elemental images 2 with the help of focusing lens is recorded on the CCD camera.

The projector is used instead of the camera to obtain an integral image. At the same time, the distances between the components of the scheme remain unchanged. In this case, the light beams passing through the microlens array cross at one point, forming an image of the object. After crossing the beams follow the observer, who perceives these diverging beams as the beams propagating from the real object.

3. The calculation of the basic parameters of the optical scheme
2.1 The choice of microlens array (figure 2)

![Figure 2](image)

Figure 2. The choice of microlens array: 1 - microlens array; 2 - plane of elementary images

The main parameters of the microlens array are the number of microlenses in the array, the focal length $f_{ml}$ and the pitch $p$ - the distance between the centers of two microlenses. The pitch size corresponds to the size of the diameter of one microlens. It is necessary to take into account the number of pixels corresponding to one microlens. This parameter defines the minimum distance between two neighbouring image points $\delta_{img}$, which can be calculated using the following formula:

$$\delta_{img} = \frac{p}{N}$$

where N - the number of vertical pixels per a microlens.
According to figure 2 the minimum distance between two points of the object will be determined by the following expression:

\[ \delta_{obj} = \frac{s_{ml} - s_{ml}'}{s_{ml}'} \delta_{img}, \]  

(2)

where \( s_{ml} \) and \( s_{ml}' \) - the front (the distance from the front surface of the microlens to the subject) and the back (the distance from the back surface of the microlens to the image of the object) line segments respectively.

The lateral resolution of the object \( R_{obj} \) and its image \( R_{img} \) can be defined as the reciprocal value of the minimum distance between two points, i.e.

\[ R_{obj} = \frac{1}{\delta_{obj}}, \]  

(3)

\[ R_{img} = \frac{1}{\delta_{img}}. \]  

(4)

Angular field microlens is defined as

\[ \omega = \frac{2 \cdot \arctg \left( \frac{p}{2s_{ml}'} \right)}{2}. \]  

(5)

Then we get

\[ \omega = 2 \cdot \arctg \left( \frac{p}{2s_{ml}'} \right). \]  

(6)

The depth of the obtained image during reconstruction can be found from the following expression:

\[ \Delta z = 2 \frac{s_{ml}}{p} \delta_{obj}. \]  

(7)

2.2 The choice of focusing lens

The task of the focusing lens is to project the plane of elementary images constructed by microlens array on the CCD matrix of the camera. While choosing it is necessary to pay attention to the linear increase of lens which is defined as:

\[ V_{l} = \frac{2y_{ml}'}{2y_{ml}}. \]  

(8)

where \( 2y_{ml}' \) - the linear dimension of the plane of elementary images of the object \( 2y \) (see figure 1); \( 2y_{ml}'' \) - linear dimension of the plane of elementary images on the CCD array camera.

2.3 The choice of the camera and projector

During the selection of the camera and projector, it is necessary to pay attention to their main parameters: the resolution and the size of one pixel. These parameters must be identical, otherwise the resulting integral image will be of poor quality.
To determine the value of the front and back line segments of the optical components of the system it is necessary to use well-known formulae of geometrical optics.

The distance from the back surface of the microlenses to the object image

\[ s_{ml}' = (1 - V_{ml}) f_{ml}', \]  

(9)

where \( V_{ml} = \frac{2y'}{2y} \) is the linear increase of the microlenses; \( 2y \) and \( 2y' = p \) are the linear dimension of the object and the elementary image respectively.

The distance from the front surface of the microlens to an object is defined as

\[ s_{ml} = \frac{s_{ml}'}{V_{ml}}, \]  

(10)

The distance from the front surface of the focusing lens to the plane of elementary images

\[ s_{f}' = (1 - V_{f}) f_{f}', \]  

(11)

where \( f_{f}' \) is the focal length of the lens.

The distance from the back surface of the focusing lens to the CCD array:

\[ s_{f} = \frac{s_{f}'}{V_{f}}. \]  

(12)

The distance between the microlens array and the focusing lens will be determined from the following equation

\[ d = s_{ml}' + s_{f}. \]  

(13)

For the high-quality reconstructed image it is necessary to take into account the physiological characteristics of the eye. The resolution of the eye is one minute. From this we can determine the maximum distance from which a person can observe the obtained three-dimensional image. In accordance with figure 2, it will match the value of the expression (14):

\[ s_{main} = \frac{\delta_{obj}}{0.0175} + \frac{\Delta z}{2}. \]  

(14)

4. Calculation example of the optical scheme for recording of the integral image.

This section presents the calculation of the optimal parameters of the optical system. CCD array of the camera and the projector have the following basic parameters: resolution 640 × 480, and the size of one pixel, which is 5.6 \( \mu \)m. Microlens array has a pitch \( p = 0.25 \)mm and focal length \( f_{ml}' = 0.57 \)mm (the array was taken as an example offered by the company Flexible Optical BV [2]).

The number of pixels of the CCD per one microlens is important [3]. We assume that one microlens has 40x40 pixels. Then, the minimum distance between two neighboring points of the object and the elementary image according to the formulae (1) and (2) will be \( \delta_{mg} = 0.02 \)mm and \( \delta_{obj} = 0.5 \)mm.

For example we take the object, that size is 20mm. Then, linear increase of the microlens is \( V_{ml} = 0.0125 \). The plane of the elementary images will be located at a distance equal
Next, we define linear dimensions of plane of the elementary images formed by the microlenses and the CCD array camera. Let us calculate the microlens in the array. Considering the size of the CCD matrix in pixels and the fact, that it is necessary 40x40 pixels per one microlens, we receive an array of microlenses 16x12. From this, we find linear dimension of the plane of elementary images. Multiplying the number of microlens on the pitch \( p \) we get the size of the microlens array equal \( 4 \times 3 \) mm. We similarly define the linear dimension of the CCD array camera. Multiply the number of pixels on the size of one pixel. We find, that the linear dimension of the CCD array is \( 3.6 \times 2.7 \) mm. Thus, for the further calculations we got the parameters \( 2y_{ml} = 4 \) mm and \( 2y''_{ml} = 3.6 \) mm.

Now we define the parameters of the focusing lens with a focal length equal to, for example, \( f''_l = 100 \) mm. According to the formula (8) the increase of the lens is \( V_l = 0.896 \times \). The back and the front line segments are equal \( s'_l = 10.4 \) mm; \( s_l = 11.6 \) mm according to the formulae (11) and (12).

The distance between microlens arrays and the focusing lens in accordance with formula (13) is \( d = 12.2 \) mm. The CCD array of the camera is located at a distance \( s'_l = 10.4 \) mm. The angular field is 25 degrees, and the depth of the image in accordance with (7) is \( \Delta z = 180,12 \) mm.

The maximum distance that a person can observe the obtained three-dimensional image in accordance with formula (14) is \( s_{main} = 1724 \) mm.

Thus in this paper we have described a method of creating a three-dimensional image of the object in real space, based on the method of integral photography, developed by G. Lipman in the early XX century, but has not lost its relevance nowadays.

The algorithm for calculating of the optical scheme is easy to use and allows the calculate schemes with different microlens arrays, a CCD array of the camera and the projector.

5. References.
[1] P P Sokolov, Journal of the Society of Naturalists 1911, p. 23-29
[2] Xiao X, Javidi B, Martinez-Corral M, Stern A, Journal of Applied Optics 2013, 52, №4, p. 546–560