Electromagnetic Effects in Superconductors in Stationary Gravitational Field

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Received 9 September 2004

The general relativistic modifications to the resistive state in superconductors of second type in the presence of a stationary gravitational field are studied. Some superconducting devices that can measure the gravitational field by its red-shift effect on the frequency of radiation are suggested. It has been shown that by varying the orientation of a superconductor with respect to the earth gravitational field, a corresponding varying contribution to AC Josephson frequency would be added by gravity. A magnetic flux (being proportional to angular velocity of rotation $\Omega$) through a rotating hollow superconducting cylinder with the radial gradient of temperature $\nabla_r T$ is theoretically predicted. The magnetic flux is assumed to be produced by the azimuthal current arising from Coriolis force effect on radial thermoelectric current. Finally the magnetic flux through the superconducting ring with radial heat flow located at the equatorial plane interior the rotating neutron star is calculated. In particular it has been shown that nonvanishing magnetic flux will be generated due to the general relativistic effect of dragging of inertial frames on the thermoelectric current.

Keywords: relativity stars; superconductors; general relativity; electromagnetic fields

PACS numbers: 04.20.-q; 04.40.-b; 04.80.Cc

1. Introduction

Although the effects of gravitational field on the electromagnetic properties of superconductors have been widely discussed by a number of authors (see, for example, $^1-^6$) there exist unconsidered problems which are of physical and astrophysical interest. To the best of our knowledge theoretical investigation of the effects of general relativity on electrodynamics of laboratory II-type superconductors has not been done. This subject is very interesting, on one hand, from point of view to detect weak gravitational effects in the solar system since superconductors provide sensitive and accurate measurements.
On other hand it is important for astrophysics of magnetized relativistic compact objects since the core of neutron stars forms a matter in the superconducting state of II-type arising from the realistic estimation that the superconducting protons coherence length \( (2 - 6\, fm) \) for stellar matter is typically much smaller than the London screening length \( (100 - 300\, fm) \). Magnetic flux vortices are the result of the type-II superconductivity in the inner crust and core region. The magnetic field of the neutron star is confined into individual vortices of flux \( \Phi = \pi \hbar c/e = 2 \times 10^{-7} G \cdot cm^2 \). The number density of such vortices is \( n_v = B/\Phi \approx 10^{19} B_{12} \, cm^{-2} \), where \( B_{12} = B/10^{12} G \). The knowledge of the electromagnetic properties of flux vortices in gravitational field may be relevant for understanding some observable phenomena of pulsars. However, we should mention here that in recent papers (see, for example, 8) it was discussed inconsistency of the standard picture of the neutron star core, composed of a mixture of a neutron superfluid and a proton type-II superconductor with observations of long period precession in isolated pulsars.

The present work is the sequel of our previous investigation on gravitational and rotational effects on electromagnetic properties of conductors and superconductors. In our previous paper we did not consider the electromagnetic properties of type-II superconductors in the gravitational field. One of the purpose of the paper is to study gravitational effects on type-II superconductivity. The second purpose of the paper is to examine a question on generation of magnetic field in superconductors arising from the effects of gravity or inertia. Since the mechanism of the generation of neutron star magnetic fields is under big astrophysical interest, we apply the discussed mechanism to the interior of slowly rotating neutron star in the simple toy model.

The paper is organized as follows. In the section 2 we consider the general relativistic modifications to the electromagnetic properties of II-type superconductor when the Lorentz force created by transport current flowing in the superconductor exceeds the pinning force and the vortices started to move laterally to the current (This stage is called as resistive one due the appearance of the resistance from the energy dissipation.). This generalizes the resistive state in II-type superconductors when the gravitational field is present. In addition, in subsection 2.1 the weak general relativistic contribution on the frequency of radiation from II-type superconductor with transport current in an applied magnetic field is calculated. More concretely we propose here an experiment where the AC Josephson effect in a superconductor of II-type has a combined electric and gravitational origin.

The magnetic flux through superconducting hollow cylinder arising from interplay between thermally generated electric current and uniform rotation is obtained in section 3. In section 4 new possible mechanism for magnetic field production inside a rotating relativistic star is discussed. The last section 5 summarizes our results.

We use here a space-like signature \((-,-,+,-)\) and a system of units in which \( c = 1 \) (unless explicitly shown otherwise for convenience). Greek indices are taken to run from 0 to 3.
2. Second type superconductor in resistive state in presence of gravitational field

Suppose that superconducting sample is placed in the gravitational field being assumed to be stationary with respect to the superconductor as whole, that is there exist a timelike Killing vector \( \xi(t) \) being parallel to the four velocity of the superconductor \( u^\alpha \). On defining \( \Lambda = -\xi^\alpha(t) \xi_\alpha(t) \), Killing equation \( \xi_{\mu;\nu} + \xi_{\nu;\mu} = 0 \) implies that \( \partial_\alpha \Lambda^{1/2} = \Lambda^{1/2} w_\alpha \), where \( w_\alpha = u_{\alpha,\beta} u^\beta \) is the absolute acceleration of superconductor. The electric and magnetic fields as seen by observers who are at rest with respect to the superconductor are \( E^\alpha = F_{\alpha \beta} u^\beta \) and \( B^\alpha = \frac{1}{2} \eta_{\alpha \beta \mu \nu} F_{\beta \mu} u_{\nu} \), where \( F_{\alpha \beta} = A_{\beta,\alpha} - A_{\alpha,\beta} \) is the field tensor, \( A_\alpha \) is the four-potential of electromagnetic field, \( \eta_{\alpha \beta \mu \nu} = \sqrt{-g} \epsilon_{\alpha \beta \mu \nu} \) is the pseudo-tensorial expression for the Levi-Civita symbol \( \epsilon_{\alpha \beta \mu \nu} \), \( g = \det|g_{\alpha \beta}| \).

Electromagnetic field applied to the sample is supposed to be stationary and \( \mathcal{L}_\xi F_{\alpha \beta} = 0 \) (which is equivalent to \( \mathcal{L}_\xi A_\alpha \) in the Lorentz gauge), where \( \mathcal{L}_\xi \) denotes the Lie derivative with respect to vector field \( \xi^\alpha(t) \).

For laboratory II-type superconductor the London penetration depth \( \lambda \) is bigger than the coherence length \( \xi (\lambda \gg \xi) \) and external magnetic field \( B^e \) penetrates the superconductor as an array of vortex lines but in the space between vortices the material remains superconducting. Assume that the massive superconductor is in resistive state in an applied magnetic field. For the simplicity we will consider vortices which form quadratic lattice (see Fig. 1). Their axes are aligned along the magnetic field lines.

If the superconductor is in the mixed state and transport current, created by
the external source, flows in the direction being perpendicular to the vortices then the Lorentz force acting on the vortices appears. The vortex state has zero electrical resistivity if the flux tubes are prevented from moving in response to an external magnetic field. However due to the pinning of vortices it is necessary to create the finite electric current (called as critical one) for motion of vortices.

Transport current creates Lorentz force under which the whole vortex structure moves with relative velocity \( v^{(v)} \) defined through decomposition formula:

\[
  u^{(v)} = u^{\alpha} + \frac{v^{(v)}}{\sqrt{1 - v^{(v)^2}}} \approx u^{\alpha} + v^{(v)}.
\]  

(1)

Hereafter we consider slow motion case and neglect terms being quadratic in the relative velocity \( v^{(v)} \).

The flux tubes must migrate perpendicular to the direction of current and perpendicular to the magnetic field under the influence of Lorentz force, i.e. the normal core regions move also as the electrons drifting in an external electric field. Such a drift or current causes ohmic heat loss. Thus migration of the flux tubes through the superconductor causes the occurrence of dissipation and the sample exhibits an electrical resistance.

Now we calculate the dissipation of electromagnetic energy due to the motion of magnetic field inside the superconductor. The electromagnetic energy-momentum tensor is

\[
  T_{(em)}^{\alpha\beta} = \frac{1}{4\pi} \left( F^{\alpha\sigma} F^{\beta\sigma} - \frac{1}{4} g^{\alpha\beta} F_{\mu\nu} F^{\mu\nu} \right).
\]

(2)

According to the first pair of Maxwell equations

\[
  F_{\mu\nu;\alpha} = -F_{\nu\alpha;\mu} - F_{\alpha;\mu\nu}
\]

(3)

the divergency of energy-momentum tensor (2)

\[
  T^{\beta}_{(em)\alpha\beta} = \frac{1}{4\pi} \left( F^{\alpha\sigma;\beta} F^{\beta\sigma} + F_{\alpha\sigma} F^{\beta\sigma}_{;\beta} - \frac{1}{2} F^{\mu\sigma} F_{\mu\nu;\alpha} \right)
\]

(4)

takes form

\[
  T^{\beta}_{(em)\alpha\beta} = \frac{1}{4\pi} F_{\alpha\sigma} F^{\sigma\beta}.
\]

(5)

Now use of the second pair of Maxwell equations

\[
  F^{\alpha\beta}_{;\beta} = 4\pi J^{\alpha}
\]

(6)

gives

\[
  T^{\alpha\beta}_{(em);\beta} = F^{\sigma\alpha} J_{\sigma},
\]

(7)

where the four current \( J_\alpha = \rho_e u_\alpha + j_\alpha \), \( \rho_e \) is the electric charge density, \( j_\alpha \) is the conduction current.
Now if we scalarly multiply the equation (7) to 4-velocity of the superconductor $u_\alpha$ we get the formula
\[ u_\alpha T^{\alpha\beta}_{(em)};\beta = u_\alpha F^{\sigma\alpha} J_\sigma = E^{\sigma} j_\sigma \]  
(8)
for the electromagnetic energy change along the world lines of superconducting medium.

The conversion of electromagnetic energy to heat as a result of the migration of flux vortices will basically take place as the occurrence of local electric field $E_\alpha$. The migration of flux vortices causes the magnetic field to change with time which results in an electric field
\[ E^\alpha = \eta^{\alpha\beta\mu\nu} u_\beta v^\mu(v_\nu) B_\nu . \]  
(9)

The electric field (9) then accelerates the unpaired electrons which can pass their energy taken from the electric field, on to the lattice and, hence, produce a heat. The proper resistance of superconductor $\rho_f$, which appears due to the lateral motion of magnetic flux with respect to the transport current, is called as flux-flow resistance and differs from the ohmic resistance $\sigma^{-1}$ for normal electrons.

2.1. The gravitational red-shift correction to the frequency of radiation from II-type superconductor in resistive state

Now we will show a similarity between Josephson junction in general relativistic AC generation state\textsuperscript{3,6} and superconductor of II-type in the resistive state in the presence of stationary gravitational field.

Left hand side of equation (9) can be written as
\[ E^\alpha = -\Lambda^{-1/2} A^\alpha_{\beta\xi(t)} + \Lambda^{-1/2} A_{\beta\alpha} \xi^\beta_{(t)} . \]  
(10)

Taking into account that 4-potential $A_\alpha$ must satisfy the equation $\mathcal{L}_\xi A_\alpha = 0$, one can get the following expression
\[ E^\alpha = \Lambda^{-1/2} \varphi_{,\alpha} , \quad \varphi \equiv A_{\rho\xi(t)} . \]  
(11)
Substituting (11) into (9)\textsuperscript{a}
\[ \Lambda^{-1/2} \varphi_{,\alpha} = \eta_{\alpha\beta\mu\nu} u_\beta v^\mu(v_\nu) B_\nu . \]  
(12)

Suppose that the type-II superconductor is in the Schwarzschild space-time
\[ ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \]  
(13)
where the timelike Killing vector can be chosen so that $\Lambda = 1 - 2M/r$. If the curvature effects are negligible, then the apparatus may be regarded as having an

\textsuperscript{a}We neglect here the effect of dynamical part of gravitational field on motion of flux vortices since we make the concrete analysis in the Schwarzschild space-time.
acceleration, \( g \), relative to a local inertial frame, and thus, \( \Lambda = \left(1 + 2gz/c^2\right)^2 \), where \( z \) is the height above some fixed point.

Consider a rectangular superconducting plate carrying an electric current being parallel to its plane. It is in vertical gravitational field \( g \) and held in an applied perpendicular magnetic field \( B \). If \( \nabla V \) is a potential difference on the length of period \( a \) in \( z \) direction then equation (12) takes form

\[
\Lambda^{-1/2} \nabla V = a \left( Bv \right).
\] (14)

It is clear that the displacement of vortices is translationally symmetric with respect to the period \( a \). Thus one could expect there is an alternating component of voltage \( \nabla V \) with the frequency

\[
\omega = 2\pi \frac{\nu_0}{a}.
\] (15)

Inserting (15) into (14) gives

\[
\Lambda^{-1/2} \nabla V = \frac{\omega (Ba^2)}{2\pi c}.
\] (16)

Taking into account that \( Ba^2 \) is the magnetic flux connected with the single vortex, i.e. flux quantum is \( \Phi_0 = \pi hc/e \) we can rewrite (16) in the form

\[
\Lambda^{-1/2} \nabla V = \frac{\hbar \omega}{2e}.
\] (17)

As a consequence of (12) and (17), an alternating current of frequency

\[
\omega = \frac{2e}{\hbar} \Lambda^{-1/2} \nabla V
\] (18)

is produced. Consequently \( \omega = \omega_0 \left(1 - gz/c^2\right) \), where \( \omega_0 \) is a constant.

According to (18) the frequency of the alternating current depends on the altitude in the gravitational field and hence frequencies \( \omega_1 \) and \( \omega_2 \) for the heights above some points \( z_1 \) and \( z_2 \) (\( z = z_2 - z_1; z_1 < z_2 \)) are connected through

\[
\omega(z_2) = \omega(z_1)(1 - gz/c^2).
\] (19)

Using typical numbers for laboratory experiments in the Earth’s gravitational field as voltage \( \nabla V = 100V \), height \( z = 10cm \) and gravitational acceleration \( g = 9.8 \times 10^2 cm \cdot s^{-2} \) one can get the general relativistic red-shift for the frequency

\[
\omega = \frac{2e}{\hbar} \frac{gz}{c^2} \nabla V \approx 3.3 Hz.
\] (20)

This is the gravitational redshift of vortices, and superconductors of II-type in principle could provide a test of gravitational field with help of the Josephson effect.
3. Effect of uniform rotation on a hollow superconducting cylinder with radial heat flow

It was obtained in \(^6\) that an azimuthal temperature gradient applied to a hollow bimetallic superconducting cylinder (ring) furnishes a small magnetic field consisting of a contribution due to the trapped flux \(n\Phi_0\) (\(n\) is the number of initially trapped quanta) plus the field induced by the thermal current \(j_{(s)\nu} = \Lambda^{-1/2}\sigma\beta\partial_\nu \tilde{T}\) (\(\beta\) is the thermoelectric coefficient, \(\sigma\) is the conductivity of the normal component).

Here taking into account thermoelectric effects in superconducting state \(^6\) we extend to the superconducting case our result \(^10\) on the rotational effects on the radial conduction current of thermoelectric origin. Consider now bimetallic hollow cylinder of outer radius \(r_2\) and internal radius \(r_1\) in the presence of radial heat flow (see Fig. 2).

Fig. 2. Superconducting cylinder of outer radius \(r_2\) and internal radius \(r_1\) rotating with angular velocity \(\Omega\) along its axis. \(\Delta T = T_2 - T_1\), where \(T_2(T_1)\) is the hot (cold) layer temperature.
As was shown in 6, in the bulk of a superconductor the total thermoelectric current \( j^{\alpha} = j^{\alpha}_{(s)} + j^{\alpha}_{(n)} \) vanishes, i.e. the normal current density \( j^{\alpha}_{(n)} \) is cancelled locally by a counterflow of supercurrent density \( j^{\alpha}_{(s)} \), so that

\[
\begin{align*}
  j^{\alpha}_{(n)} &= \Lambda^{-1/2} \sigma \beta \partial_{\alpha} \tilde{T} - \sigma R_H (F_{\nu \alpha} + u_{\alpha} u^\sigma F_{\nu \sigma}) j^{\nu}_{(n)} + \sigma R_{gg} j^{\beta}_{(n)} A_{\alpha \beta} = \\
  &- j^{\alpha}_{(s)} = 2 n_s e + \frac{m_s c}{m_s} \left[ \hbar \partial_{\alpha} \vartheta - \frac{2 e}{c} A_{\alpha} \right],
\end{align*}
\]

(21)

where \( A_{\alpha \beta} = u_{[\alpha, \beta]} + u_{[\beta] w_{\alpha]} \) is the relativistic rate of rotation, \( n_s \) and \( m_s \) represent the density and mass of Cooper pairs, \( \vartheta \) is the phase of superconducting wavefunction. \( R_H \) is the Hall constant and \( R_{gg} \) is the galvanogravitomagnetic coefficient (\( R_{gg} \approx 0.8 \times 10^{-22} \text{s}^2 \), see for details 11). Thus the initial electric current (initiating a real radial thermoelectric current) could be assumed to be \( j^r_{(n)} = \sigma \beta \partial_r T \) (the orthonormal components are hatted).

In the rotating frame of reference (with angular velocity \( \Omega \))

\[
d s^2 = - \left( c^2 - \Omega^2 r^2 \right) dt^2 + 2 \Omega r^2 d\phi dt + dr^2 + r^2 d\phi^2 + dz^2
\]

(22)

the radial electric current will be affected by the Coriolis force and therefore an azimuthal current

\[
j^{\phi}_{(n)} = - j^{\phi}_{(s)} = R_{gg} \sigma^2 \beta (\nabla_r T) \Omega
\]

(23)

in the bulk of the superconductor is generated.

Then integrating the equation (21) along the contour, which is in the bulk of the superconductor one can easily get

\[
\Phi_b = n \Phi_0 + \frac{m_s c}{4 n_s e^2} R_{gg} \sigma^2 \beta (\nabla_r T) 2 \pi \Omega.
\]

(24)

Thus the magnetic flux due to the Coriolis force acting on superconducting current is expected to be \( \Phi_c = \left( \tau m_s c / 2 n_s c^2 \right) R_{gg} \sigma^2 \beta (\nabla_r T) \Omega \). Using the typical numbers for conductivity of normal element \( \sigma = 9 \times 10^{16} \text{s}^{-1} \), thermoelectric coefficient \( \beta = 10^{-5} \text{V} \cdot \text{K}^{-1} \), angular velocity \( \Omega = 2 \pi \times 10^4 \text{s}^{-1} \), concentration of superconducting electrons \( n_s = 10^{23} \text{cm}^{-3} \) and \( \nabla_r T = 10 \text{K} \cdot \text{cm}^{-1} \) one can evaluate the additional magnetic flux as \( \Phi_c = 5 \times 10^{-13} \text{G} \cdot \text{cm}^2 \).

4. Magnetic flux through superconducting ring inside a rotating neutron star

Finally, as we noted the mechanism of magnetic field production discussed above may be relevant to the problem of origin of magnetic fields in rotating neutron stars whose substance is expected to be superconducting at high densities. Assume that the fields interior star are stationary and axisymmetric, and ignore the self-gravity of the electromagnetic field. This is realistic assumption since even for the magnetized star with high magnetic field the electromagnetic field energy is negligible with compare to the rest mass energy (see, for example, 12).
Since the angular momentum $a$ of the neutron star will not be zero in general, the interior background geometry is given by the metric at the first order in the angular momentum in a coordinate system ($ct, r, \theta, \phi$) (see, for example, $^{13}$) as follows
\[ ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 - 2\omega(r)r^2 \sin^2 \theta dt d\phi + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 , \] (25)
where $\omega(r)$ can be interpreted as the angular velocity of a free falling (inertial) frame and is also known as the Lense-Thirring angular velocity. The radial dependence of $\omega$ in the region of spacetime internal to the star has to be found as the solution of the differential equation
\[ \frac{1}{r^3} \frac{d}{dr} \left( r^4 j \frac{d\omega}{dr} \right) + 4 \frac{d}{dr} j \omega = 0 , \] (26)
where we have defined
\[ \bar{j} \equiv e^{-\left(\Phi + \Lambda\right)} , \] (27)
and where
\[ \bar{\omega} \equiv \Omega - \omega , \] (28)
is the angular velocity of the fluid as measured from the local free falling (inertial) frame.

In the vacuum region of spacetime external to the star, on the other hand, $\omega(r)$ is given by the simple algebraic expression
\[ \omega(r) \equiv \frac{d\phi}{dt} = -\frac{g_{\phi\phi}}{g_{\phi\phi}} = \frac{2J}{r} , \] (29)
where $J = I(M, R)\Omega$ is the total angular momentum of metric source as measured from infinity and $I(M, R)$ its momentum of inertia. Outside the star, the metric (25) is completely known and explicit expressions for the other metric functions are given by
\[ e^{2\Phi(r)} \equiv \left( 1 - \frac{2M}{r} \right) = e^{-2\Lambda(r)} , \] (30)
where $M$ and $R$ are the mass and radius of the star as measured from infinity.

The four-velocity components of stellar medium are given by
\[ u^\alpha \equiv e^{-\Phi(r)} \left( 1, 0, 0, 0 \right) ; \quad u_\alpha \equiv e^{\Phi(r)} \left( -1, 0, 0, -\omega r^2 \sin^2 \theta e^{-2\Phi(r)} \right) . \] (31)

In the coordinate system (25) and with the electromagnetic fields referred to the medium (31), the Ohm’s law (21) for the normal component of electric current takes form
\[ j_{(n)r} = \sigma E_r + \sigma R_{\phi\phi}^\phi j_{(n)\phi}^\phi r^2 \sin^2 \theta e^{-\Phi} \left\{ -\omega + \omega r \Phi_{,r} - \frac{\omega^r_{,r}}{2} \right\} + \sigma \beta e^{-\Phi} \left( e^\Phi T \right) ,r \] (32)
\[ j_{(n)\theta} = \sigma E_\theta + \sigma R_{\phi\phi}^\phi j_{(n)\phi}^\phi \omega r^2 \sin \theta \cos \theta e^{-\Phi} + \sigma \beta T_{,\theta} , \] (33)
Assume for simplicity of calculations that a superconducting ring (thin disk of finite size with the inner radius \( r_{in} \) and outer one \( r_{ex} \)) is located in the equatorial plane (\( \theta = \pi/2 \)), symmetrically around the axis of rotation. Then the nonvanishing radial heat flow through superconducting ring may create electric current of thermoelectric origin:

\[
j_{(n)r} = \sigma \beta e \Phi \left( e^\Phi T \right)_{r}.
\]

Then as a consequence of azimuthal component of Ohm’s law (34) the azimuthal electric current will be created interior the superconductor

\[
\dot{j}_{(s)\alpha} = -\frac{2n_{s}e}{m_{s}} \left[ \hbar \partial_{\alpha} \theta - \frac{2e}{c} A_{\alpha} \right] = -R_{gg} \sigma^{2} \beta e^{-2(\Phi+\Lambda)} \left( e^\Phi T \right)_{r} \left\{ \omega r - \omega r^{2} \Phi_{r,r} + \frac{\omega^{'r^{2}}}{2} \right\}.
\]

By integrating the equation (35) along the contour \( r = R \), which is in the bulk of the superconducting ring one can get

\[
\Phi_{b} = n\Phi_{0} - \frac{\pi m_{s}c}{2n_{s}e^{2}} R_{gg} \sigma^{2} \beta e^{-2(\Phi+\Lambda)} \left( e^\Phi T \right)_{r} \left\{ \omega r - \omega r^{2} \Phi_{r,r} + \frac{\omega^{'r^{2}}}{2} \right\} \bigg|_{r=R}.
\]

Thus we considered the mechanism of production of stellar magnetic field in superconductor with the heat flow. The arguments on generation of magnetic field by the thermoelectric effects in the conducting crust of the neutron star with the immense gradient of temperature were extensively studied in papers 14, 15, 16. Since there is uncertainty in microphysics of neutron stars we decided do not provide here any evaluation for magnetic flux (36) for this toy model. The realistic model with the numerical evaluation will be developed in our future work.

5. Conclusion

Here we investigated the effect of the gravitation on the electromagnetic properties of II-type superconductors and the effects of uniform rotation on superconductors with a heat flow. In particular we have shown that gravity can affect the alternating voltage in II-type superconductor. The proposal is presented which uses gravity to create an additional emf and consequently may change the superconducting Josephson effect frequency. The frequency is becoming a function of the gravitational field. Thus the results on the electromagnetic properties of II-type superconductor embedded in gravitational field could lead to a way towards measuring of the tiny general relativistic effects in the earth conditions.

We have also derived the expression for magnetic flux through rotating hollow type-I superconductor of cylindrical shape when a temperature gradient is applied...
in radial direction between the inner and outer boundaries of the sample. In principle, the magnetic field due to the rotational effects on the electric current in the superconducting state should be observable in this type of laboratory experiment, although its magnitude is less than flux quantum for many orders.

The mechanism of the generation of magnetic field inside rotating neutron star due to the frame dragging effect has been considered. A possibility of magnetic field production has been shown in the toy model, where for the simplicity superconducting matter has been considered as the disc in the equatorial plane.

Acknowledgments

BA and VK greatly thank TWAS and ICTP for financial support towards their visit to India and acknowledge the warm hospitality at the IUCAA where the work has been done. BA acknowledges the partial financial support from NATO through the reintegration grant EAP.RIG.981259. Financial support for this research is partly provided by the UzFFR (project 1-06) and projects F2.1.09 and F2.2.06 of the UzCST.

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