Dynamic analysis of the extended space charge layer using chronopotentiometric measurements

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Abstract
In this paper, we experimentally verified the length (LESC) and the concentration (cESC) of the extended space charge (ESC) layer in front of the electrical double layer (EDL) using the chronopotentiometric measurement and the equivalent circuit model analysis. From the experimentation, the coupled-response of the EDL and the ESC layer was discriminated from the contribution of electro-osmotic flow (EOF). In addition, we derived the potential differences across the ESC (VESC) layer using the circuit model of the ICP layer under rigorous consideration of ESC and EDL. As a result, we obtained that VESC was linearly proportional to the square of the applied current (iapplied). Hence, LESC and cESC were quantitatively provided, where LESC is linear to the iapplied and cESC is constant regardless of iapplied. Thus, this experimentation could not only clarify an essential ICP theory but also guide in ESC-based applications.

Keywords: Ion concentration polarization, Chronopotentiometric measurement, Extended space charge layer

Introduction
Perm-selective ion selective transportation through an electrochemical nanoporous membrane has been widely utilized for numerous engineering applications such as desalination [1–7], preconcentration [8–17] and energy harvesting [2, 18–22]. In such systems, ion concentration gradients are formed on both sides of the nanoporous membrane, which is called an ion concentration polarization (ICP) phenomenon [23, 24]. Typically, a zone where electrolyte concentration was extremely low was formed at the anodic side of membrane, while the electrolyte concentration significantly increased at the cathodic side of membrane in the case of cation-selective membrane [25]. These zones were called the ion depletion zone and the ion enrichment zone, respectively. In order to characterize these zone, numerous theories and experimentations had been conducted such as the possible overlimiting conductance (OLC) mechanisms by instabilities [26–32], diffusioosmosis [33], electro-osmotic flow (EOF) and surface conduction (SC) [34–38], etc. Most of these studies pointed that all these nonlinear electro-kinetic phenomena were stemming from the development of extended space charge (ESC) layer in front of the electrical double layer (EDL) at an overlimiting current regime, predicted by Rubinstein and Zaltzman [39, 40]. Recently, the electrical impedance spectroscopy (EIS) have been proposed to probe the existence of the ESC layer [41–43]. However, the direct confirmation based on AC electric field analysis was lacking, because the electrical response was tightly involved with the couplings of EDL, ESC layer and EOF.

Therefore, in this study, we suggested an experiment and a circuit analysis for obtaining the potential across the ESC layer (VESC). First of all, chronopotentiometric measurement (dc bias with constant current) was used to discriminate the electrical response of both the EDL and the ESC layer out of EOF. Furthermore, we proposed the equivalent circuit model of an ion depletion zone reflecting EDL and ESC layer, where each resistor and each
capacitor are serially connected. From those analyses, we finally obtained the relationship between $V_{\text{ESC}}$ and the applied current, which has never been proposed before. Finally, we quantitatively derived the ESC layer information such as the length, the total charge and the concentration. Therefore, this study would be one of essential basis for ICP research not only in fundamental aspect but also various applications based on ICP.

Materials and methods

Device fabrication

As shown in Fig. 1a, we fabricated a micro/nano-fluidic device consisting of the main microchannel (1 cm length, 100 μm height and 15 μm depth), the buffer microchannel (1 cm length, 100 μm height, and 15 μm) and two side microchannels (40 mm length, 15 μm height and 15 μm depth). For the external hydrodynamic injection, the two side microchannels were tangentially connected to the main microchannel, which is 50 μm apart from the end of the main microchannel. The side microchannels on both sides of the main microchannel was installed for easiness of the experiment [44] and preventing ever-increasing ICP layer [45]. By injecting fresh electrolyte solution through the side microchannels, the diffusion length was reduced as an order of a hundred micron, confining the ICP layer as the triangular shape as shown in Fig. 1b, c. The main building block of device were made of a polydimethyl siloxane (PDMS, Sylgard 184 silicone elastomer kit, Dow corning). We followed the general soft-lithographical fabrication method for PDMS [46]. The Nafion nanoporous membrane was patterned on the glass substrate based on the surface patterning method [45, 47]. Simply, Nafion was patterned using a straight microchannel (200 μm width x 50 μm depth) on a glass side, and the PDMS piece of the main

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**Fig. 1** a Image of micro/nanofluidic device used in this work and the magnified view of the device. Main and buffer microchannel had the dimension of 15 μm depth x 100 μm width and the side microchannel had one of 15 μm depth x 150 μm. The ICP layer **b** without or **c** with external flow from two side microchannels.
microchannel was irreversibly bonded in the middle using a plasma bonder (CuteMP, Femto Science, Korea) to a designated position on top of the Nafion-patterned glass.

**Chemical preparation**

Potassium chloride 1 mM solution were used for the experimentation. For tracking the electrokinetic flows and visualizing the ion concentration profile around the ion concentration polarization (ICP) layer, the negatively charged particle ($d = 0.2 \mu m$, Invitrogen) and the fluorescent dye (Alexa488, Sigma Aldrich) were mixed in the prepared solution [33, 37, 48].

**Experimental setup**

From the two side microchannels, we pumped the prepared solution with the volume rate, 20 nL/min using a syringe pump (PHD2000, Harvard apparatus) for 30 min until the injected flows were stabilized at the main microchannel. Then, we applied the external current source through the reservoir of the main microchannel utilizing the source measure unit (SMU 236, Keithley) while the two reservoirs of the buffer microchannel were grounded. Note that the reservoir of the two side microchannels were electrically floated during ICP. With a customized LabView program, we performed four experimentations as followed: (1) the chronopotentiometric measurement ($V$–$t$) from 1 to 30 nA with an 1 nA interval for each 3 min, (2) the chronoamperometric measurement ($I$–$t$) from 0.3 to 9.9 V with an 0.3 V interval for each 3 min, (3) the voltage–current ($V$–$I$) responses from 0 to 30 nA with a step current 1 nA for every 60 s per step and (4) the current–voltage ($I$–$V$) responses from 0 to 9.9 V with a step voltage 0.3 V for every 60 s per step. In order to capture the optical image of an ICP layer, we used a CCD camera (DP73, Olympus) and the image was obtained through the commercial software program (CellSens, Olympus).

**Results and discussions**

**Chronopotentiometric measurement**

Figure 2 showed the representing chronopotentiometric measurement of the ICP system, where the red line and the blue line indicated the electrical response at both the overlimiting current regime and the ohmic current regime, respectively. Previous studies neglected the voltage behavior at the ohmic current regime, while they described the voltage behavior at the overlimiting current regime as: (1) The initial voltage value was ohmic voltage which was subject to the electrodialysis system. (2) The sharp voltage hop (1st hop) appeared and the voltage value depended on the type of membrane. (3) A linear voltage growth (2nd hop) regime was followed, where the electroconvection initiated at this time, and then (4) the voltage value was saturated as the microvortices saturated both the size and speed [49, 50]. However, the aforementioned steps were insufficient to explain the voltage behavior in chronopotentiometry since the ICP layer model was missing. Furthermore, the internal structures inside ion depletion zone has never been suggested as an electrokinetic circuit model. Thus, we would introduce a unified equivalent circuit model including EDL and ESC as well as 2nd EOF in the following section.

Especially at the OLC regime, the voltage responses during the chronopotentiometric measurement showed the two voltage hops ($V_{1st}$ and $V_{2nd}$) as shown in Fig. 3a. When the current was applied at $t=0$ (sec) from the main microchannel, the $V_{1st}$ was followed due to the capacitance of both the ESC and the EDL, which the corresponding image and the circuit was shown in image i) in Fig. 3a and inset of in Fig. 3b, respectively. When the EOF was generated at $t=15$ s, the voltage was increasing until the EOF size saturated at $t>50$ s with the value $V_{2nd}$~$i_{applied}$ as shown in image ii) in Fig. 3a [50]. In this experimentation, we applied the various current values from 12 nA to 30 nA so that we can obtain the $V_{1st}$–$i_{applied}$ relations as shown in Fig. 3b. Note that the $V_{1st}$ is not linear to the $i_{applied}$ indicating that the ohm’s law is not valid due to the appearance of the ESC layer as expected by Rubinstein and Zaltzman [40].

**Equivalent electrokinetic circuit model of the ICP layer**

At the charged membrane surface, the EDL was composed of both resistor ($R_{EDL}$) and capacitor ($C_{EDL}$) in parallel and they were connected in series to the diffuse layer resistor ($R_{bulk}$) as in Fig. 4a. This simple circuit coincided with the voltage–time behavior in the ohmic regime, which showed the gentle slope and the slight voltage hop as in Fig. 2. Once the current was applied exceeding
Fig. 3  
(a) The result of chronopotentiometric measurement at 29 nA (in the regime of OLC regime) and the corresponding images at i) $t = 10$ s and ii) $t = 60$ s, respectively. 
(b) The $V_{1st}$-$i_{applied}$ relations and the equivalent circuit model of the ICP layer at inset of the graph.
limiting current, the ESC layer grew between the EDL and the diffuse layer, where both resistor \((R_{ESC})\) and capacitor \((C_{ESC})\) should be additionally employed as in Fig. 4b. This electrical circuit model affected the total RC delay time, converting the gentle slope at ohmic current regime into the sharp one at overlimiting current regime.

Normally, time-varying voltage responses existed where the resistance and the capacitance are parallel in the circuit model. Considering that bulk solution was regarded to the quasi-neutral regions, one can ignore the resistance and the capacitance of the EDL as well as the resistance of one. This means that the voltage responses should be divided into the constant term (for diffusion layer) and the time-varying one (for EDL) as follows:

\[
V(t) = V_0 + V_{EDL} \exp \left(-\frac{t}{\tau_{EDL}}\right) \tag{1}
\]

where \(V_0\) is the potential of diffuse layer, \(V_{EDL}\) the potential of EDL and the \(\tau_{EDL}\) is the RC delay time \((\tau_{EDL} = R_{EDL} C_{EDL}\) in the circuit model). As shown in Additional file 1: Figure S4, the collapsed data of the \(V_{EDL}\) has the linear relations to the applied current density, which lead to the constant resistance values \((R_{EDL} = V_{EDL}/I)\) as 3 MΩ. Each component has the value 240±42 MΩ (for \(R_{bulk}\)), 3±0.7 MΩ (for \(R_{EDL}\)), 6±1.2 μF (for \(C_{EDL}\)), 1.09(\(I/I_{lim}\)) MΩ (for \(R_{ESC}\)) and 2.23(\(I/I_{lim}\))−1 μF (for \(C_{ESC}\), respectively. The simple calculation result and the derivations was introduced in supporting materials (Additional file 1: Table S1, Figure S4).

Valenca and co-workers reported that the microvortices by ICP induced the potential difference at \(V_{2nd}\) in the EC dominant regime [50]. This indicated that, in a certain overlimiting current value \(I_{OLC} > I_{lim}\), one can estimate the point conductance at \(I_{OLC}\) with a simple calculation as \(\sigma_{OLC} = I/V_{2nd}\). We also confirmed the conductivities in EOF regime, where the applied current is ranging from 12 nA to 29 nA, leading to OLC by EOF as the constant value of 0.21 nS in our system. Note that the experimental results and the set of data were provided in supporting materials (Additional file 1: Figure S2). In addition, critical time \((T_c)\) that initiates the EOF has the relation of the OLC conductance \((\sigma_{OLC})\) and its time-derivative one \((\dot{\sigma}_{OLC}/\dot{t})\). This means that \(T_c\) is also subject to the \(V_{2nd}\) and its time-derivative one \((\dot{V}_{2nd}/\dot{t})\). The scaling was developed and quantified in supporting materials (Additional file 1: Figure S3).

The length \((L_{ESC})\) and the concentration \((c_{ESC})\) of the ESC layer

At the ESC layer, the dimensionless length \((\tilde{L}_{ESC}\) normalized by diffusion length) should be 0.5(3ε\(V_{ESC}\))\((2/3)\) \((\hat{j})\)\((1/3)\), the total space charge density \((\Sigma_{ESC})\) should be \(\varepsilon(4/3)(2j_{VESC})\)\((1/3)\) and the concentration \((c_{ESC})\) should be 0.69(\(\varepsilon^2\psi_{ESC}^{-1}j_{j}^2\))\((1/3)\) [51, 52]. Here \(\varepsilon\) is the dimensionless Debye length; \(\psi_{ESC}\) is the dimensionless electric potential (normalized by the thermal potential \(RT/F\) and the \(j\) is the dimensionless applied current density (normalized by the cross-sectional area of microchannel). In our system, the diffusion length was 100 μm, \(\varepsilon\) was 4.26 × 10\(^{-4}\) and \(\psi_{ESC} \approx 7.89 \times 10^{-2} \times j^2\), leading to \(L_{ESC} \approx 132 \times j\) (nm), \(\Sigma_{ESC} \approx 0.225 \times j\) (μq/m\(^3\)) and \(c_{ESC} \approx 2.30\) (μM), respectively, leading to the conclusion of \(L_{ESC} \sim j_{applied}, \Sigma_{ESC} \sim j_{applied}\) and \(c_{ESC} \sim constant\) inside the ESC layer as shown in Fig. 5.

Conclusions

Recent experiments have been conducted for probing the space charge at the micro- and nano-channel interface device using electrical impedance spectroscopy (EIS),

![Fig. 4 Schematics of equivalent circuit of ICP layer at a ohmic current regime and b overlimiting current regime. c A unified equivalent electrokinetic circuit model of ICP layer considering EDL, ESC and 2nd EOF.](image-url)
employing a conventional equivalent circuit model. However, those literatures revealed out that the EIS method hardly determined the ESC layer response since the multiple electrokinetic responses were tightly coupled during ICP. For example, Yossifon and co-workers probed the diffusion layer(DL) and the electrical double layer(EDL) using EIS at the micro- and nano-channel systems [43]. They found out the detailed components of the EDL by separating the electrode-fluidic interface and microchannel–nanochannel interface. From this experiment, they clearly captured the resistances and the capacitances at both EDL for satisfying the theoretical calculations. However, this demonstration fails to present ESC layer responses at the higher voltage because of the coupling effect where electroconvective flows were involved, thereby arousing another issue for differentiating them, individually. Thus, we emphasized that this equivalent circuit model, for the first time, reflected EDL and ESC individually. Thus, we emphasized that this equivalent circuit model, for the first time, reflected EDL and ESC individually.

In this paper, we experimentally investigated the ESC layer using chronopotentiometric measurement and the unified equivalent electrokinetic circuit model of internal ICP structure with the consideration of EDL, ESC and 2nd EOF. Each electrical component such as two resistors, two capacitors and dependent current source were included in the new model, confirming the voltage responses in chronopotentiometric measurement. From our rigorous experimentation, we obtained the relationship between the potential across the ESC layer and the applied current, \( V_{E_{\text{ESC}}-\text{applied}} \). Furthermore, we quantitatively provided the \( L_{E_{\text{ESC}}-\text{applied}} \) and the \( c_{E_{\text{ESC}}-\text{applied}} \). Therefore, all this experimental verification of the ESC layer could lead to the further development of ICP theory as well as the ESC/ICP layer related applications.

**Supplementary information**

**Supplementary information** accompanies this paper at https://doi.org/10.1186/s40486-020-00112-1.

**Additional file 1:** Figure S1. In order to obtain the limiting current values, we conducted the voltage-sweeping method in our systems. Under the 20nL/min flows was applied near the Nafion membrane, the limiting current value reaches 12 (nA). Figure S2. The \( V_{\text{ESC}} \) from the measurement has been obtained with the applied current, \( i \). This result showed that the slope of \( V_{\text{ESC}}^{-1} \), which is the overlimiting conductance (OLC) by electro-osmotic flows (EOF) have the constant values as 0.21 nS. Figure S3. The onset time (\( \tau_c \)) of electro-convective flows was obtained from the chronopotentiometric measurement. The \( \tau_c \) values are between 10 and 30, which result is coincided our scaling theory, \( \tau_c \sim 10^3 \). Figure S4. The \( V_{\text{lim}} \), which is the time-varying potential reflected by the electrical double layer, was obtained from the chronopotentiometric measurement. From this result, the resistance can be calculated by Ohm’s law (\( R_{\text{ESC}} \sim 1/V_{\text{lim}} \)). Table S1. The electrical components of the equivalent circuit model were calculated by simple calculation. Note that \( R_{\text{ESC}} \) and \( C_{\text{ESC}} \) remains same regardless of the applied current, \( i \), while \( R_{\text{REDL}} \) and \( C_{\text{REDL}} \) are linearly proportional to the current values (\( R 

**Abbreviations**

ICP: Ion concentration polarization; EDL: Electrical double layer; ESC: Extended space charge; EOF: Electroosmotic flow; SC: Surface conduction; OLC: Over-limiting conductance; POMS: Poly-dimethyl siloxane; EIS: Electrical impedance spectroscopy.

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**Authors’ contributions**

IC conducted the main experiment. HL advised circuit modeling. SJK supervised the project. All authors read and approved the final manuscript.

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**Availability of data and materials**

All data generated or analyzed during this study are included in this published article.

**Competing interests**

The authors declare no competing interests (both financial and non-financial).

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