Analysis of the influence of a strong magnetic field gradient on convection process of paramagnetic fluid in the annulus between horizontal concentric cylinders

W A Wrobel, E Fornalik-Wajs, J S Szmyd
Department of Fundamental Research in Energy Engineering, Faculty of Energy and Fuels, AGH – University of Science and Technology, 30 Mickiewicza Ave., 30-059 Krakow, Poland
E-mail: wrobelwit@gmail.com

Abstract. The paper presents findings of analysis of the effect of a strong magnetic field gradient on the natural thermal convection of paramagnetic fluid. The study have been done in a 3D annulus between horizontal coaxial concentric cylinders filled with paramagnetic fluid. The inner cylindrical surface was heated and the outer cylindrical surface was cooled. Measuring vessel has been placed in a magnetic field gradient inside the superconductive magnet. The research has been divided into two parts in order to validate theoretical model. In the experimental part: an impact analysis of a strong magnetic field on value of the Nusselt number for three various configurations of the magnetic field relative to the gravitational field was carried out. The value of Rayleigh number was approximately Ra = 1.5•10^5. In the numerical part: the forces acting on electrically non-conductive, paramagnetic fluid situated in a gravitational field and a strong inhomogeneous magnetic field exposed to the temperature difference were described. The result showed that high magnetic field can be used to either enhance or suppress thermal convection of paramagnetic fluid.

1. Introduction
Natural convection has been studied from a wide range of perspectives because of its appearance in numerous industrial processes. Natural convection between isothermal horizontal concentric cylinders was first studied by Beckmann. Similar studies were conducted by Voigt and Krischer. Kraussold extended these investigations on larger Prandtl numbers. They also formulated correlation equations for the average heat transfer coefficient [1]. Itoh, Fujita, Hishiwaki and Hirata [2] proposed a new definition of characteristic length for the type of annuli. Based on that they presented the average heat transfer coefficient for a range of pseudo-conduction and laminar flows. Kuehn and Goldstein [3] extended the earlier studies on the turbulent range. At first, the numerical solution for natural convection was obtained by Crawford and Lemlich. Powe, Carley and Carruth [4] carried out the calculations for convection in the range of pseudo-conduction state to unsteady flow. They found that the results of calculations agreed with the experimental results obtained by Powe et al. [1]. Braithwaite, Beaugnon and Tourneier [5] demonstrated experimentally the possibility of heat transfer intensification during natural convection of paramagnetic fluid in a strong non-uniform magnetic field. They also proposed a mathematical description of the forces acting on a paramagnetic fluid, based on temperature difference. Huang and Edwards [6] showed that the convective heat transfer of electrically non-conducted paramagnetic fluid can be effectively controlled by a non-uniform magnetic field. Further research related to thermomagnetic convection were conducted by Ozoe [7]. The aims of this work were to validate numerical model from previous study and compare it with experimental data obtained in experimental study.
2. Experimental system and experimental procedures

2.1. Experimental setup

The experimental system is shown in Figure 1. The setup consists of: superconducting magnet, thermostat, power supply with ammeter and voltmeter, thermocouples with data acquisition system, measuring vessel and digital camera. Applied superconducting magnet was able to achieve 10 T of magnetic induction.

![Figure 1 Pictorial drawing of the experimental system.](image1)

The experimental vessel, an annular enclosure with a round rod core and a cylindrical outer wall, is presented in Figure 2. The core diameter was 0.02 m and the inner diameter of the outer cylinder 0.054 m. The outer cylinder was made of 0.003 m thick aluminum, while the core was a copper rod that could be electrically heated. The cavity aspect: inner radius to outer radius is equal to $r_i/r_o = 0.37$ and gap to length ratio is $(r_o - r_i)/l = 0.185$. The core surface could be assumed to be isothermal due to high thermal conductivity of copper. Resistance wire was wound around a ceramic element and placed in a hole drilled inside the copper bar. The electric heater was then connected to DC power supply and the voltage and electrical current were constantly controlled. The outer cylinder was cooled with water kept at constant temperature inside a thermostatting bath. A 0.006 m thick Plexiglas plate was inserted roughly half way up the cylinder. The core temperature and temperature of the aluminum side wall were measured with T-type thermocouples. The enclosure was closed with a 0.006 m thick plexiglas plate from one side and a 0.009 m thick ebonite plate from the other. The experimental fluid was injected through a small hole in Plexiglas plate used to close the system. The total height of the assembled enclosure was 0.092 m.

![Figure 2 Cross-section of the measuring vessel.](image2)

![Figure 3 Diagram showing the setup used to visualize the flow field.](image3)
Visualization of the velocity field was conducted with a camera attached to the measuring vessel. Diagram showing the location of the cameras in the set is presented in Figure 3. In order to obtain the flow fields the tracer particles reflecting light were added to fluid. The interior of the measuring vessel was illuminated by a set of diodes (LEDs) spread in a thin ring made of Plexiglas. The calculation of the velocity field was carried by using 7.2 LaVision Davis software.

2.2. Experimental procedures

The enclosure was filled with the experimental fluid. The measurement vessel was placed in one selected position in magnetic field presented in Figure 4. Position of the enclosure in the magnetic field was a variable parameter. The water temperature in the constant temperature bath was set at 18 °C and the supplied power of the electrical heater placed in the enclosure’s core set to 27 W. Once the power was set, the apparatus was left to reach a steady state conditions. This lasted about 30–40 min, after which the measured values were recorded. The recorded temperature data was averaged and then analyzed. The data taken during the last 10 min before changing the parameters was included in the average. After temperature measurement the strength of magnetic field supplied power was set to the next level. Strength of magnetic field was changed from 1 to 10 T in the centre of coil, which corresponded to the square field gradient equal to 0.6 - 40 T^2/m in considered positions. When series of data was taken in position A, measuring vessel was moved to the position B and then to position C for subsequent set of measurements.

2.3. Experimental fluid

A 50% volume aqueous solution of glycerol with a 0.8 mol/(kg of solution) concentration of gadolinium nitrate hexahydrate (Gd(NO_3)_3·6H_2O) was used as the working fluid. The magnetic susceptibility of the obtained experimental fluid was 3.38·10^{-4} [–]. The density, kinematic viscosity, mass magnetic susceptibility, specific heat, thermal diffusivity, thermal expansion coefficient were all measured experimentally. The other, like dynamic viscosity, magnetic susceptibility, thermal conductivity, Prandtl number were calculated from previous properties. The mass magnetic susceptibility of the fluid was measured with the magnetic susceptibility balance. Dynamic viscosity is a function of temperature and the expression describing it was obtained from the experimental data:

$$
\mu(T) = 3.878 - 0.03457 \times 10^{-3} T + 1.031 \times 10^{-7} T^2 - 1.0281 \times 10^{-11} T^3
$$

(1)
where: $\mu$ is dynamic viscosity, $T$ is temperature in Kelvin. This polynomial is valid in temperature range: 290÷345 K. The properties of the experimental fluid are listed in Table 1.

| Properties                  | Value       | Unit |
|-----------------------------|-------------|------|
| Density                     | 1.41·10$^3$ | kg/m$^3$ |
| Dynamic viscosity           | 1.30·10$^{-2}$ | kg/(m·s) |
| Kinematic viscosity         | 9.25·10$^{-6}$ | m$^2$/s |
| Magnetic susceptibility      | 3.38·10$^4$ | [-] |
| Mass magnetic susceptibility | 2.69·10$^{-3}$ | m$^3$/kg |
| Specific heat               | 2.92·10$^3$ | J/(kg·K) |
| Thermal conductivity        | 3.76·10$^{-1}$ | W/(m·K) |
| Thermal diffusivity         | 9.13·10$^{-8}$ | m$^2$/s |
| Thermal exp. coefficient.   | 3.93·10$^{-4}$ | 1/K |
| Prandtl number              | 101         | [-] |

2.4. Estimation of the Nusselt number
To describe the net convection heat transfer rate, a quantified value known as the Nusselt number could be used:

$$\bar{N}u = \frac{Q_{\text{conv}}}{Q_{\text{cond}}}$$

where: $Q_{\text{conv}}$ indicates the convection heat transfer rate and $Q_{\text{cond}}$ is the theoretical heat transfer rate due to conduction [8]:

$$Q_{\text{conv}} = Q_{\text{heater}} - Q_{\text{loss}}$$

$Q_{\text{heater}}$ is the supplied heat transfer rate described by and $Q_{\text{loss}}$ is the heat loss to the environment. More details about way of calculating heat lost are given in reference [8]. Theoretical conduction heat transfer rate $Q_{\text{cond}}$ was calculated from:

$$Q_{\text{cond}} = \frac{2\pi\lambda L(T_h - T_c)}{\ln(R_i/R_o)}$$

where: $L$ is a length of cylinder, $\lambda$ is thermal conductivity of experimental fluid, $T_h, T_c$ is temperature of hot and cold walls, $R_i, R_o$ is inner and outer radius of the vessel. Due to the configuration of the experimental enclosure the heat loss was quite small in relation to the power supplied.

3. Mathematical model
Three-dimensional laminar incompressible fluid flow motion and heat transfer by natural convection are described by the conservation equation for mass, momentum and energy. The governing equation can be presented in form [9]:

$$\frac{DP}{Dt} + \rho \vec{v} \cdot \vec{v} = 0$$

$$\rho \frac{D\vec{v}}{Dt} = -\vec{v} p + \mu(T) \vec{v}^2 + \vec{F}_{\text{buoy}}$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = a \vec{v}^2 T$$

where: $\rho$ is density, $t$ is time, $\vec{v}$ is velocity, $p$ is pressure, $\mu(T)$ is dynamic viscosity (temperature-dependent), $\vec{F}_{\text{buoy}}$ is buoyancy force, $T$ is temperature: $a$ is thermal diffusivity.

The driving force for free convection in gravitational field is usually coming from density difference between hot and cold regions of the fluid.
\[ \vec{F}_{\text{grav}} = \rho \vec{g} \]  \hspace{1cm} (8)

where: \( \rho \) is the density and \( \vec{g} \) is the gravitational acceleration. If the fluid is placed in the magnetic field gradient, and has magnetic susceptibility different than zero \( (\chi \neq 0) \), magnetizing force acting per unit volume in the magnetic field gradient could be defined as [5]:

\[ \vec{F}_{\text{mag}} = \mu_0 \vec{M} \nabla \vec{H} \]  \hspace{1cm} (9)

where: \( \mu_0 \) is vacuum permeability or magnetic constant, \( \vec{M} \) is magnetization, \( \vec{H} \) is external magnetic field. For paramagnetic fluid magnetic susceptibility \( \chi \) was much smaller than unity, then it could be assumed \( \vec{H} \approx \vec{B}/\mu_0 \), which gave:

\[ \vec{F}_{\text{mag}} \approx \frac{\chi \nabla \vec{B}^2}{2\mu_0} \]  \hspace{1cm} (10)

Therefore the body force acting on the fluid \( \vec{F}_{\text{body}} \) was presented as [8]:

\[ \vec{F}_{\text{body}} = \vec{F}_{\text{grav}} + \vec{F}_{\text{mag}} = \rho \vec{g} + \frac{\chi \nabla \vec{B}^2}{2\mu_0} \]  \hspace{1cm} (11)

When the temperature difference occurred in the system, the mass magnetic susceptibility and density changed with it. The mass magnetic susceptibility \( \chi_m \) depended on the temperature following the Curie’s law:

\[ \chi_m = \frac{C}{T} \]  \hspace{1cm} (12)

where \( T \) is the temperature and \( C \) is the Curie’s constant.

According to Braithwaite et al. [5], and applying the approximation done by Tagawa et al. [7, 10], the quantities \( \rho(T) \) and \( \rho\chi_m(T) \) could be linearized by Taylor’s expansion. The buoyancy force \( \vec{F}_{\text{buoy}} \) acting on the paramagnetic fluid took its final form [8]:

\[ \vec{F}_{\text{buoy}} = -\vec{g} \rho_0 \beta(T - T_0) \left( 1 + \frac{1}{\beta T_0} \right) \frac{\rho_0 \chi_m \beta}{2\mu_0} (T - T_0) \nabla \vec{B}^2 \]  \hspace{1cm} (13)

The magnetic field was calculated on the basis of Biot–Savart’s law.

4. Numerical procedures

ANSYS Fluent software was utilized to calculate conduction and convection phenomena under the influence of magnetic field. A special module for the magnetising buoyancy force was programmed and introduced into the software. The grid points were non-uniformly allocated and the total number of grid points was 28x90x90. The applied grid and thermal boundary condition are shown in Figure 5.

**Figure 5** Applied grid and thermal boundary condition. 
The body force was the sum of gravitational buoyancy and magnetising buoyancy forces, as shown in previous section. The calculation was done iteratively. In the first step the magnetic buoyancy force was calculated in the new magnetic module, while the gravitational buoyancy force was obtained from the standard Fluent module, both of which were based on the temperature field. The pressure term was then estimated and the velocity could be determined. In the next step the temperature was calculated. The iterations were ended when the convergence was obtained. The convergence parameter for the continuity and momentum equations was $10^{-4}$, while for the energy equation it was $10^{-6}$. Numerical calculations were performed for the configuration of the magnetic and gravitational field corresponding to items A, B, and C in experimental part.

5. Experimental and numerical results
Temperature measurement between inner and outer cylindrical surface results are shown in Figure 6. For a given heat flux on the inner wall (27 W) a clear influence of position in magnetic field and strength of magnetic field could be observed. Temperature difference for Position A was decreasing with increasing magnetic field. The relation was similar in Position C, but in this configuration decrease in temperature was smaller. For Position A and C the magnetic field always enhanced the convective motion. For Position B the temperature difference was increasing with increasing magnetic field and reached the maximum at 6.5 T. Further increase in magnetic field, caused decrease in the temperature difference. It indicated that in Position B the convective motion decreased with increasing magnetic field reaching the minimum at 6.5 T, and then increased.

![Figure 6](image_url)  
Figure 6 The dependence of the temperature difference on inner and outer wall of the magnetic field inside the coil for experimental data.

![Figure 7](image_url)  
Figure 7 Comparison of Nusselt number dependence of the magnetic field in the center of the coil for numerical and experimental data.

Various temperature difference between the inner and outer wall at constant heat flux on the inner wall, caused variations of the Nusselt number value. Nusselt number dependence on the intensity of the magnetic field is presented in Figure 7. The values obtained experimentally were compared with those obtained numerically. Very good agreement between numerical and experimental results could be observed.

In order to show the fluid flow and convection motion the flow visualization was carried out. Visualized cross-section of the vessel in three positions for the field strength inside the coil equal to 8 T is shown in Figure 8. For position A stream of liquid over the heated inner wall was lifted up. This flow resembled the flow due to the gravitational force, but judging from the values of Nusselt number, convective flow was enhanced. This is because the directions of gravitational and magnetic forces are the same and they strengthened each other. Completely different flow was observed at location B. At magnetic induction of 8 T the warm fluid close to the inner wall was pushed down. Estimation of uncertainty and measurements error was presented in [8].
Figure 8 The velocity field obtained experimentally and numerically for the magnetic field strength at the centre of the coil equal to 8 T. Cross section of the measuring vessel. Where $Ra^{GM}$ is a modification of Rayleigh number include both gravity and magnetic buoyancy force [8].
This shows that the magnetizing force was stronger than gravitational one and the magnetic field determined the nature of the flow. Flow profile at position C was the result of the sum of magnetic and gravitational forces. In this position, the magnetic force was directed horizontally. Because of that the warm stream of liquid above the core was pushed to the side. Comparison of flow streams results obtained numerically and experimentally shows satisfactory agreement.

6. Conclusion
The results show that magnetising force affects the heat transfer rate and that a strong magnetic field can control the thermal convection of paramagnetic fluid. Depending on the direction and strength of the magnetic field, the enhancement or suppression of convective motion was variously observed. Numerical code have been validate and very good agreement between the experimental and numerical data was found. Applied software with an additional module can be used to analyse more complex systems for example heat exchangers. Detailed numerical and experimental investigation was presented in the PhD thesis [11].

Acknowledgments
The results were supported by the Polish Ministry of Science and Higher Education (Grant AGH 11.11.210.198) and ACK Cyfronet (Grant: MNiSW/IBM_BC_HS21/AGH/014/2011).

References
[1] Kuehn T H and Goldstein R J 1976 An experimental and theoretical study of natural convection in the annulus between horizontal concentric cylinders Journal of Fluid Mechanics vol. 74 no. 4 pp 695-719
[2] Itoh M, Fujita T, Nishiwaki N, and Hirata M 1970 A new method of correlating heat-transfer coefficients for natural convection in horizontal cylindrical annuli Journal of Heat and Mass Transfer vol. 13 pp 1364-1368
[3] Kuehn T H and Goldstein R J 1976 Correlating equations for natural convection heat transfer between horizontal circular cylinders International Journal of Heat and Mass Transfer vol. 19 no. 10 pp 1127–1134
[4] Powe R E, Carley C T, and Carruth S L 1971 Numerical simulation of natural convection in cylindrical annuli Journal of Heat Transfer vol. 93 no. 2 pp 210-220
[5] Braithwaite D, Beaugnon E, and Tournier R 1991 Magnetically controlled convection in a paramagnetic fluid Nature vol. 354 pp 134 - 136
[6] Huang J, Edwards B F, and Gray D D 1998 Magnetic control of convection in nonconducting paramagnetic fluids Physical Review E vol. 57 no. 1 pp 29–31
[7] Ozoe H 2005 Magnetic Convection (Imperial College Press)
[8] Wrobel W, Fornalik-Wajs E, and Szym J S 2010 Experimental and numerical analysis of thermo-magnetic convection in a vertical annular enclosure International Journal of Heat and Fluid Flow vol. 31 no. 6 pp 1019–1031
[9] Bejan A 2004 Convection Heat Transfer (John Wiley & Sons, Inc.)
[10] Tagawa T, Shigemitsu R, and Ozoe H 2002 Magnetizing force modeled and numerically solved for natural convection of air in a cubic enclosure: effect of the direction of the magnetic field International Journal of Heat and Mass Transfer vol. 45 no. 2 pp 267-277
[11] Wrobel W 2012 An Analysis of Influence of High Magnetic Field on Convection Process of Paramagnetic Fluids in the Annulus Between Concentric Cylinders (in polish) PhD Thesis, AGH-University of Science and Technology, Krakow, Poland