Trellis channel joint equalization and demodulation of SEFDM signals

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Abstract. An algorithm of channel joint equalization and demodulation of multicarrier signals with nonorthogonal frequency spacing (spectrally efficient frequency division multiplexing, SEFDM) is proposed. It is based on DFT (Discrete Fourier Transform) and BCJR (Bahl, Cocke, Jelinek and Raviv) algorithm adapted to work with frequency samples of received signal. The performance of proposed receiver is comparable with performance of classic OFDM-signal receivers.

1. Introduction
Modern wireless networks are based on Orthogonal frequency division multiplexing (OFDM) signals. One of the possible approaches for future generations of wireless systems is spectrally efficient frequency division multiplexing (SEFDM) [1-3] which is a kind of faster-than-Nyquist (FTN) signals [4-5]. This type of signals represents the OFDM signal with reduced frequency spacing between subcarriers by 1/α times, where α is a frequency compression coefficient.

The known approaches to the problem of SEFDM signal demodulation are successive interference cancellation [3, 6], sphere decoder [7], matched filtering (MF) [6,7] and trellis demodulator based on BCJR (Bahl, Cocke, Jelinek and Raviv) algorithm [8, 9]. Sphere decoder approach provides near ML results but has extremely high complexity and thus applicable in general for SEFDM signals with small number of subcarriers. The successive interference cancellation and MF algorithms has low complexity but poor BER performance for high compression values. Trellis demodulator provides the acceptable BER performance for high number of subcarriers still suffering from the high complexity for high order mapping schemes [8].

The SEFDM signal behavior in fading channels is covered in papers [10-14]. The works [10-11] extend the sphere decoding approach to the joint equalization and demodulation of SEFDM signal. In [12] SEFDM-symbol was considered as an equivalent OFDM-symbol with reduced number of subcarriers. It allows to use for channel estimation pilot OFDM-symbols with the same duration as data SEFDM-symbols duration. The main drawback of the joint equalizer and demodulator based on the sphere decoder algorithm is its infeasibility for systems with high number of subcarriers (N > 40).

In [13] a receiver based on representation of SEFDM signal as equivalent OFDM signal and one-tap ZF-equalizer in frequency domain was introduced. The main drawback of this approach is usage of additional blocks of Fourier transforms for processing equivalent OFDM signal.

In this work we apply BCJR algorithm to the problem of joint equalization and demodulation of SEFDM signal. The analysis of SEFDM BER performance is given for pedestrian LTE fading channel under assumption of perfect channel knowledge.

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The rest of the paper is organized as follows. The system model is described in section II, proposed joint SEFDM equalizer and demodulator structure is described in section III and simulation results are presented in section IV. The conclusions are in section V.

2. System model

A symbol of baseband multicarrier signal with \( N \) subcarriers and duration \( T \) is expressed as:

\[
s(t) = \sum_{k=-N/2}^{N/2-1} C_k e^{j2\pi k t T}.
\] (1)

Here \( \omega_k = k \omega_1 = 2\pi k / T \) is \( k \)-th subcarrier, \( C_k \) is a complex modulation symbol for \( k \)-th subcarrier. The frequency spacing between adjacent subcarriers is \( \Delta f \) and \( \omega_k = 2\pi k / T \). In the case of OFDM signal \( \Delta f = \Delta f_{\text{OFDM}} = 1/T \), while for SEFDM \( \Delta f = \alpha / T \), where \( \alpha < 1 \) is the subcarrier frequency spacing normalized to \( \Delta f_{\text{OFDM}} \), (for OFDM \( \alpha = 1 \)).

The discrete version of (1) can be expressed as follows:

\[
s_n = \sum_{k=-N/2}^{N/2-1} C_k e^{j2\pi k n T} = \sum_{k=-N/2}^{N/2-1} C_k e^{j2\pi k n N / L}.
\] (2)

Here \( n = 0, \ldots, (L - 1) \), \( L \) is the number of time samples in one SEFDM-symbol, \( L = F_s T \), \( N \alpha \) if sampling frequency is chosen as \( F_s = N \Delta f \).

The equation (2) can be written in the matrix form as:

\[
\vec{s} = \vec{F}_L^{-1} \times \vec{c}.
\] (3)

Here \( \vec{c} = \{ C_k \}_{k=-N/2}^{N/2-1} \) and \( \vec{s} = \{ s_k \}_{k=0}^{L-1} \) are column vectors, \( \vec{F}_L^{-1} \) is a \( L \) by \( N \) inverse Fourier matrix with elements equal to \( e^{j2\pi k n N / L} \), where \( k \) is a column index and \( n \) is a row index of matrix. From (3) it follows that SEFDM signal can be generated using Inverse Discrete Fourier Transform (IDFT) transform with ignoring \( N-L \) samples in the output.

The received signal in the case of multipath propagation is the sum of convolution of SEFDM samples with channel impulse response and AWGN samples:

\[
r_j = \sum_{j=0}^{J-1} s_{j-i} h_j + w_j,
\] (4)

where, \( J \) is the number of paths, \( h_j \) represents the complex gain of the \( j \)-th path and \( w_j \) is Additive white Gaussian noise (AWGN) sample. In the case of Rayleigh fading the gains of paths are simulated as Gaussian random numbers.

Assuming that the intersymbol interference is cancelled employing cyclic prefix and the constant fading process for the duration of sending at least one SEFDM-symbol, the equation (4) can be expressed in the matrix form:

\[
r = H_{L \times L} \times \vec{s} + w = H_{L \times L} \times \vec{F}_L^{-1} \times \vec{c} + \vec{w}.
\] (5)

Here \( L \) by \( L \) matrix \( H_{L \times L} \) contains circularly shifted sequences \( \vec{h} = \{ h_j \}_{j=0}^{L-1} \).

Reception of multicarrier signals is usually done in the frequency domain.

\[
\vec{r}^{\text{DFT}} = \vec{F}_N \times \vec{H}_{L \times L} \times \vec{F}_L^{-1} \times \vec{c} + \vec{w}^{\text{DFT}}.
\] (6)

Here \( \vec{r}^{\text{DFT}} = \{ \vec{r}_m \}_{m=-N/2}^{N/2-1} \) are spectral samples which are employed for estimation of initial modulation symbols \( \vec{c} = \{ C_k \}_{k=-N/2}^{N/2-1} \).
The considered system model is presented on figure 1. It consists of serial concatenation of a mapper, IDFT-based SEFDM modulator, Rayleigh multipath channel, discrete AWGN channel and DFT-based joint SEFDM equalization and demodulation block. The latter comprises \(N\)-point DFT block and joint SEFDM equalization-demodulation block. The mapper associates groups of \(D\) channel bits with complex modulation symbols from a particular constellation (e.g., QPSK, QAM-16, QAM-64, etc.) with alphabet size \(M = 2^D\). DFT-based SEFDM modulator transforms these symbols into \(L\) time samples of SEFDM-symbol. At the receiver side the DFT block generates \(N\) spectral samples from received noisy time samples of SEFDM-symbol. At last, receiver block performs joint equalization and demodulation and provides the detected bits.

![System model](image1)

**Figure 1.** System model.

3. SEFDM receiver

As follows from (3) spectral samples \(x^{DFT} = \{R_m\}_{m=-N/2}^{N/2}\) at the FFT output (figure 1) can be represented as a sum of AWGN and linear combination of all subcarriers:

\[
R_m = \sum_{k=-N/2}^{N/2-1} C_k a_{mk} + w_m^{DFT}.
\] (7)

Where coefficients \(a_{mk}\) are the elements of matrix \(A_{N+N} = F_{N+N} \times H_{L+N} \times F_{L+N}^{-1}\) with row index \(m+N/2\) and column index \(k+N/2\). These coefficients introduce intersubcarrier interference caused by SEFDM modulation which should be eliminated by the receiver. Example of normalized absolute values of \(a_{mk}\) for SEFDM signal with number of subcarriers \(N = 200\), \(\alpha = 3/4\), pedestrian LTE channel and \(m = 10\) is represented in the figure 2.

![Normalized sequence](image2)

**Figure 2.** Normalized sequence \(|a_{mk}|, m = 10, N = 200, \alpha = 3/4.**
The major effect on the $R_m$ is provided by $k$-th subcarrier with modulation symbol $C_k$ and the nearest subcarriers with numbers $(k \pm 1)$, $(k \pm 2)$ and so on. In this way, we can consider only a small number $K$ of nearest to position $k$ subcarriers in (7):

$$R_m \approx \sum_{k=(K-1)/2}^{(K-1)/2-1} C_k a_{mk} + w^{DFT}_m. \quad (8)$$

Assuming no AWGN, we can interpret (8) as a linear filter with $K$ coefficients $a_{mk}$ which are different for all $m$. Such filter processes the sequence of $R_m$ and can be regarded as a Markov process with a finite number of states $M^{K-1}$, where $M$ is modulation order (2 for BPSK, 4 for QPSK, etc.). In Markov process the probability of a system state at step $k$ depends only on the system state at step $k-1$.

Such signals can be defined on trellises and we can use the BCJR algorithm to demodulate them as in [13]. The algorithm includes four main steps: calculation of branch metrics $\gamma$, forward metrics $\alpha$, backward metrics $\beta$ and a posteriori LLRs of the symbols (figure 3). The branch metric $\gamma$ is associated with a transition between states $s_f$ and $s_l$ for trellis step $i$. It is a function of received frequency sample and available a priori information for it. The branch metric in logarithmic domain is evaluated in the following way:

$$\gamma(s_{st}, s_{st}) = -\frac{1}{\sigma^2} \left( R_{st} - \sum_{k=(K-1)/2}^{(K-1)/2-1} C_{p,i}^{p,i} a_{ik} \right)^2 + \ln P_a(C_i). \quad (9)$$

The symbols $C_{p,i}^{p,i}$ are associated with the pair of states $s_f$ and $s_l$. The second term in (9) is the logarithm of a priori probability of $i$-th modulation symbol.

The all necessary math for computation of forward metrics $\alpha$, backward metrics $\beta$ and a posteriori LLRs of the symbols is the same as in [8].

Based on (8) the proposed joint trellis SEFDM equalizer and demodulator tries to eliminate the main part of intersubcarrier interference in SEFDM-symbol but not all of it. This part of intersubcarrier interference grows with $K$ producing lower BER at the cost of higher complexity (the number of states increases with $K$). Herewith, computational complexity has linear dependency on the number of subcarriers, so algorithm is applicable to the SEFDM signals with high number of subcarrier in contrast to maximum likelihood and sphere decoder algorithms.

4. Simulation results

Simulation of proposed SEFDM receiver was done for QPSK modulation and pedestrian LTE fading channel (EPA). In all cases, the AWGN channel was considered with perfect knowledge of noise power and the signal-to-noise ratio per bit $E_b/N_0$ at the receiver. The confidence interval for all simulations is 0.1, the confidence probability is 0.95. The number of used subcarriers $N = 200$, $N_{eff} = 256$, $\alpha = 5/8$, $3/4$, $7/8$, $15/16$, 1 (classic OFDM), duration of SEFDM-symbol is 66.7 microsecond. Channel estimation is done using pilot OFDM-symbols without AWGN noise (perfect channel estimation).
Figure 4 shows BER performance of SEFDM signal receiver for number of subcarriers taken into account by receiver $K = 3$ and various $\alpha$. It is seen that BER performance becomes better with increasing of parameter $\alpha$, however even for SEFDM signal with $\alpha = 15/16$ curve saturates for BER $= 10^{-3}$ due to high interference from subcarriers not considered by the algorithm. Herewith, the level of intersubcarrier interference grows as $\alpha$ decreases, and accordingly the point of saturation also grows.

Figure 5 shows performance of SEFDM signal receiver with parameter $K = 5$. In contrast to receiver with parameter $K = 3$, BER performance for SEFDM signals with parameter $\alpha = 3/4$ becomes comparable to classic OFDM which corresponds to a SEFDM system with $\alpha = 1$, the energy loss is approximately equal to 5 dB for $10^{-3} < \text{BER} < 10^{-2}$. For SEFDM signal with parameter $\alpha = 5/8$ the effect of BER curve saturation is still observed.

![Figure 4. Performance of SEFDM BCJR-based receiver, LTE fading channel, $K=3$.](image-url)

![Figure 5. Performance of SEFDM BCJR-based receiver, LTE fading channel, $K=5$.](image-url)
5. Conclusions
BCJR based algorithm is applied to the problem of joint equalization and demodulation of SEFDM signal. The BER performance of SEFDM signals with various parameter \( \alpha \) in LTE pedestrian fading channel was analyzed using the proposed receiver structure. With parameter of receiver \( K = 3 \) (algorithm considers only 1 neighbor subcarrier in each side) the saturation effect of BER curve is observed. For parameter \( K = 5 \) the BER performance is comparable to OFDM: energy loss is less than 5 dB for \( 10^{-3} < \text{BER} < 10^{-2} \) for SEFDM signal with \( \alpha = 3/4 \). Algorithm presented in work [13] has higher energy efficiency: the energy loss of proposed algorithm is 2 dB for \( 10^{-3} < \text{BER} < 10^{-2} \). However, proposed algorithm doesn’t require the separate equalization procedure (auxiliary DFT and IDFT blocks), so it is more computationally efficient.

As BCJR algorithm provides soft decisions on its output, it can be adopted to iterative schemes in future works. The main drawback of the proposed receiver structure is its’ high computation complexity for high order mapping schemes (QAM-64, QAM-256, etc.).

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