Future singularity free accelerating expansion with the modified Poisson brackets

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Abstract

We show that the second accelerating expansion of the universe appears smoothly from the decelerating phase, which follows the initial inflation, in the two-dimensional soluble semi-classical dilaton gravity along with the modified Poisson brackets with noncommutativity between the relevant fields. This is contrast to the fact that the ordinary solution of the equations of motion following from the conventional Poisson algebra describes permanent accelerating universe without any phase change. In this modified model, it turns out that the noncommutative Poisson algebra is responsible for the remarkable phase transition to the second accelerating expansion.

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I. INTRODUCTION

It has been proposed that the recent intriguing accelerating expansion of the universe is of relevance to the curious behavior of dark energy described by the negative equation-of-state parameter [1]. In the well-known Friedmann equation, it is easily expected for our universe to show the decelerating expansion if the positive energy condition has been met as seen from $3\ddot{a}/a = -4\pi G(\rho + 3p)$, where $a$ is the scale factor, and $\rho$ and $p$ are the energy density and the pressure, respectively. So, the dark energy, which is responsible for the accelerating expansion of the universe, is defined by the state parameter, $\omega (\equiv p/\rho) < -1/3$ and even more $\omega < -1$ for the phantom energy to compensate ordinary matters. Note that the density of the dark energy is assumed to be positive so that the pressure should be negative, which naturally results in the negative state parameter [3].

On the other hand, cosmological problems are usually hard to solve exactly and thus often considered in some simplified models in order to get some clues and insights for solving them. One such model is the exactly soluble two-dimensional gravity [4, 5, 6, 7, 8, 9, 10, 11, 12], which has attracted much attention in the study of cosmology in various aspects [13, 14, 15, 16, 17, 18] (for a review of two or higher dimensional dilaton gravity, see Ref. [19]). From this perspective, it would be interesting to study whether the model can show accelerating expansion of the universe after the decelerating expansion or not. A recent work [20] shows that it may be possible to obtain the transition from the decelerating universe to the accelerating universe by assuming noncommutativity during the finite time. However, this model essentially encounters the future singularity in a finite proper time unless an appropriate regular geometry is patched. So, we would like to obtain cosmological solutions to describe the future-singularity-free accelerating universe without patching after the initial acceleration corresponding to the first inflation, whose whole profile is essentially similar to our universe chronologically. For this purpose, we add two local counter terms with the Polyakov action of the conformal anomaly in the semi-classical action, and impose some modified Poisson brackets with noncommutativity between the relevant fields. This process naturally yields the modified set of semi-classical equations of motion involving the noncommutative parameter, and remarkably leads us to have the desired solutions.

In the next section, we shall introduce the Callan-Giddings-Harvey-Strominger model [4, 5] coupled to massless conformal matter fields and its quantum correction expressed by
the Polyakov nonlocal action. Furthermore, two local terms with a constant \(\gamma\) are added in order to solve the model exactly. Then, the usual semi-classical equations of motion obeying the conventional Poisson algebra are solved, and the finite accelerating expansion is obtained under some conditions; however, they do not exhibit any change in phase. In sect. III, new equations of motion based on the modified Poisson brackets are shown to give the nontrivial solutions depending on the noncommutative parameter. For \(\gamma > 2\), it is shown that the second accelerating expansion of the universe appears smoothly from the decelerating phase which follows the initial accelerating expansion. The second acceleration is finite and eventually vanishes so that the infinite accelerating expansion or the curvature singularity does not exist. Finally, some comments and discussions follow in sect. IV.

II. PERMANENT ACCELERATING EXPANSION

In this section, we study the two-dimensional semi-classical dilaton gravity action composed of the conformal matter fields and its quantum correction, which are described by the nonlocal Polyakov action with two covariant local terms as follows [4],

\[
S = S_{DG} + S_{cl} + S_{qt},
\]

(1)

The first term on the right-hand-side is the well-known string inspired dilaton gravity action written as

\[
S_{DG} = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla\phi)^2 + 4\lambda^2 \right],
\]

(2)

and the classical matter action of \(N\) conformal fields \(S_{cl}\) and its one-parameter-family quantum correction \(S_{qt}\) are given by

\[
S_{cl} = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ -\frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right],
\]

(3)

\[
S_{qt} = \frac{\kappa}{2\pi} \int \sqrt{-g} \left[ -\frac{1}{4} R \frac{1}{\Box} R + (\gamma - 1)(\nabla\phi)^2 - \frac{\gamma}{2} \phi R \right],
\]

(4)

and \(\kappa = (N - 24)\hbar/12\). The higher order quantum correction beyond the one-loop is negligible in the large \(N\) approximation where \(N \rightarrow \infty\) and \(\hbar \rightarrow 0\), so that \(\kappa\) is assumed to be positive finite constant, while the cosmological constant \(\lambda^2\) is set to zero for simplicity. Note that the local ambiguity terms in Eq. (4) correspond to those of the Russo-Susskind-Thorlacius (RST) model [6] for \(\gamma = 1\), and to the Bose-Parker-Peleg model [8] for \(\gamma = 2\). In our work, \(\gamma\) is assumed to be \(\gamma > 2\) for our goal.
In the conformal gauge, \( ds^2 = -e^{2\rho} dx^+ dx^- \), if we define new fields as follows

\[
\chi = e^{-2\phi} + \kappa \left( \rho - \frac{\gamma}{2} \phi \right), \quad (5)
\]
\[
\Omega = e^{-2\phi} - \frac{\kappa}{2} (\gamma - 2) \phi, \quad (6)
\]

the total action \((1)\) is written as

\[
S = \frac{1}{\pi} \int d^2 x \left[ -\frac{1}{\kappa} \partial_+ \chi \partial_- \chi + \frac{1}{\kappa} \partial_+ \Omega \partial_- \Omega + \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i \right], \quad (7)
\]

and constraints are given by

\[
\kappa t_\pm = -\frac{1}{\kappa} (\partial_\pm \chi)^2 + \frac{1}{\kappa} (\partial_\pm \Omega)^2 + \frac{1}{2} \sum_{i=1}^{N} (\partial_\pm f_i)^2, \quad (8)
\]

where \( t_\pm \) reflects the nonlocality of the anomaly term in the Polyakov action. This integration function from the nonlocality should be determined by the choice of the boundary condition for the geometrical vacuum and matter state, which may be constant or time-dependent depending on model. The quantum energy-momentum tensors from Eq. \((11)\) can be written in the form of

\[
T^+_{\pm} = \frac{\kappa (\gamma - 2)}{2 \Omega'} \left[ \partial_+ \partial_- \Omega - \frac{\Omega''}{(\Omega')^2} (\partial_\pm \Omega)^2 \right] - \partial_+ \partial_- (\chi - \Omega), \quad (9)
\]
\[
T^\pm_{\pm} = \frac{\kappa}{\Omega'} \left[ \partial^2 \Omega - \frac{\Omega''}{(\Omega')^2} (\partial_\pm \Omega)^2 \right] + \partial^2 (\chi - \Omega) - \kappa \left[ \frac{\partial_\pm \Omega}{\Omega'} + \frac{1}{\kappa} \partial_\pm (\chi - \Omega) \right]^2 - \kappa t_\pm
\]
\[
-\frac{\kappa (\gamma - 1)}{(\Omega')^2} (\partial_\pm \Omega)^2 - \frac{\kappa}{2 \Omega'} \left[ \partial^2 \Omega - \frac{\Omega''}{(\Omega')^2} (\partial_\pm \Omega)^2 \right], \quad (10)
\]

where \( \Omega' = d\Omega/d\phi \) and \( \Omega'' = 4e^{-2\phi} \). Assuming the homogeneous spacetime, the Lagrangian and the constraints are reduced to

\[
L = -\frac{1}{2\kappa} \dot{\chi}^2 + \frac{1}{2\kappa} \dot{\Omega}^2 + \frac{1}{4} \sum_{i=1}^{N} \dot{f}_i^2, \quad (11)
\]
\[
\kappa t_\pm = -\frac{1}{4\kappa} \dot{\chi}^2 + \frac{1}{4} \dot{\Omega}^2 + \frac{1}{8} \sum_{i=1}^{N} \dot{f}_i^2, \quad (12)
\]

where the Lagrangian is defined by \( S/L_0 = \frac{1}{4} \int dt L \) with \( L_0 = \int dx \). The overdot denotes the derivative with respect to the coordinate time \( t \) defined by \( dx^\pm = dt \pm dx \). Then, the Hamiltonian is

\[
H = -\frac{\kappa}{2} P_\chi^2 + \frac{\kappa}{2} P_\Omega^2 + \sum_{i=1}^{N} P_{f_i}^2, \quad (13)
\]
where the canonical momenta are given by \( P_\chi = -\frac{1}{\kappa} \dot{\chi} \), \( P_\Omega = \frac{1}{\kappa} \dot{\Omega} \), \( P_{f_i} = \frac{1}{2} \dot{f}_i \).

If we now define non-vanishing Poisson brackets between elementary fields as follows

\[
\{ \Omega, P_\Omega \}_{\text{PB}} = \{ \chi, P_\chi \}_{\text{PB}} = \{ f_i, P_{f_i} \}_{\text{PB}} = 1, \tag{14}
\]

then Hamiltonian equations of motion \[ \dot{\mathcal{O}} = \{ \mathcal{O}, H \}_{\text{PB}} \] for canonical fields are derived as

\[
\begin{align*}
\dot{\chi} &= -\kappa P_\chi, \\
\dot{\Omega} &= \kappa P_\Omega, \\
\dot{f}_i &= 2 P_{f_i}, \tag{15}
\end{align*}
\]

which are solved as

\[
\begin{align*}
\chi &= -\kappa P_{\chi_0} t + \chi_0, \tag{17} \\
\Omega &= \kappa P_{\Omega_0} t + \Omega_0, \tag{18}
\end{align*}
\]

where \( P_{\chi_0}, P_{\Omega_0}, \chi_0, \text{ and } \Omega_0 \) are arbitrary constants, and \( P_{f_i} = 0 \) for the sake of simplicity. Note that these semiclassical solutions (17) and (18) from the Hamiltonian equations of motion (15) and (16) are essentially equivalent to those of Euler-Lagrangian equations of motion from Eq. (11) because the fields \( \Omega \) and \( \chi \) are not the quantum operators. If they are operators, they should be decomposed into the positive and the negative frequency parts along with the creation and annihilation operators. So, this is not the quantization of the quantization for the model (3). In the next section, we will modify the conventional Poisson brackets in this semiclassical regime, in order to obtain the modified semiclassical equations of motion.

We now turn to the issue of the expanding universe. We first consider the expansion condition for the scale factor,

\[
\frac{da}{d\tau} = \frac{d\rho}{dt} = \frac{\kappa P_{\Omega_0}}{-2e^{-2\phi} - \kappa(\gamma - 2)/2} - (P_{\chi_0} + P_{\Omega_0}) \geq 0, \tag{19}
\]

where the scale factor \( a(\tau) \) is a function of the comoving time \( \tau \) and is defined by \( ds^2 = -d\tau^2 + a^2(\tau)d\mathbf{x}^2 \), that is, \( d\tau = e^{\rho(t)} dt \) and \( a(\tau) = e^{\rho(t)} \). Since the denominator in Eq. (19) is less than \(-\kappa(\gamma - 2)/2\), we get the following condition,

\[
P_{\chi_0} + \frac{\gamma}{\gamma - 2} P_{\Omega_0} \leq 0, \tag{20}
\]
where $P_{\Omega_0} > 0$ is assumed. As for the constraints, substituting the solutions (17) and (18) into the constraint equations (12) give

$$\kappa t_\pm = -\frac{\kappa}{4}(P_{\chi_0}^2 - P_{\Omega_0}^2),$$

(21)

where $t_\pm$ is constant determined by the matter state.

The curvature scalar is calculated as

$$R = \kappa^2 P_{\Omega_0}^2 e^{-4\phi} \left[\frac{e^{-2\phi} + \kappa(\gamma - 2)/4}{4}\right]^3 \exp\left[2(P_{\chi_0} + P_{\Omega_0})t - \frac{2}{\kappa}(\chi_0 - \Omega_0)\right],$$

(22)

which can not be negative, since we have assumed that $\kappa$ is positive and $\gamma > 2$. Because the curvature scalar in two dimensions is directly proportional to the second derivative of the scale factor, Eq. (22) shows that the universe exhibits permanent accelerating expansion without any decelerating expansion. Note that the curvature scalar (22) converges to zero at both ends if the following condition is met,

$$P_{\chi_0} + \frac{\gamma + 2}{\gamma - 2}P_{\Omega_0} > 0.$$

(23)

Then, from the expansion condition (20) and convergence of the scalar curvature (23), we get

$$-\frac{\gamma + 2}{\gamma - 2}P_{\Omega_0} < P_{\chi_0} \leq -\frac{\gamma}{\gamma - 2}P_{\Omega_0}.$$  \hspace{1cm} (24)

Thus, we have the singularity-free accelerating solution under this condition. For other choices of the constants, what we get is the decelerating solution which unfortunately does not show any phase change of the universe. In the next section, by modifying the Poisson brackets (14), we will find another solution showing the desired phase change to the accelerating expansion from decelerating expansion following the initial exponentially accelerating expansion.

III. PHASE CHANGING UNIVERSE

We now extend the conventional (commutative) Poisson brackets to the modified (noncommutative) Poisson brackets characterized by two noncommutative constants, $\Theta$ and $\theta$, which are reminiscent of the one appearing in the noncommutative algebra of the D-brane on the constant tensor field or a point particle moving very slowly on the constant magnetic field \[21, 22, 23\]. In our case, we are trying to obtain the modified semiclassical solution...
involving the noncommutative constants from the modified semiclassical equations of motion based on the noncommutative algebra. We now consider noncommutative case of the Poisson algebra as follows \[24\]

\[
\{\Omega, P_\Omega\}_{\text{MPB}} = \{\chi, P_\chi\}_{\text{MPB}} = \{f_i, P_{f_i}\}_{\text{MPB}} = 1, \\
\{\chi, \Omega\}_{\text{MPB}} = \Theta, \quad \{P_\chi, P_\Omega\}_{\text{MPB}} = \theta, \quad \text{others} = 0,
\]

(25)

where $\Theta$ and $\theta$ are positive independent constants. Then, modified semiclassical equations of motion are given by

\[
\dot{\chi} = \{\chi, H\}_{\text{MPB}} = -\kappa P_\chi, \quad \dot{\Omega} = \{\Omega, H\}_{\text{MPB}} = \kappa P_\Omega,
\]

(26)

\[
\dot{P}_\chi = \{P_\chi, H\}_{\text{MPB}} = \kappa \theta P_\Omega, \quad \dot{P}_\Omega = \{P_\Omega, H\}_{\text{MPB}} = \kappa \theta P_\chi.
\]

(27)

Note that the original semiclassical equations of motion (15) and (16) are recovered in the limit, $\theta \to 0$. These modified semiclassical equations of motion depend only on $\theta$, since the Hamiltonian (13) is described by the pure momentum variables. Combining Eqs. (26) and (27), we get

\[
\ddot{P}_\chi = \kappa^2 \theta^2 P_\chi, \quad \ddot{P}_\Omega = \kappa^2 \theta^2 P_\Omega,
\]

(28)

and their solutions are obtained as

\[
\chi = -\alpha \sinh \kappa \theta t - \beta \cosh \kappa \theta t + C_\chi, \\
\Omega = \alpha \cosh \kappa \theta t + \beta \sinh \kappa \theta t + C_\Omega,
\]

(29)

(30)

where $\alpha$, $\beta$ and $C_\chi$, $C_\Omega$ are integration constants.

From the solutions (29) and (30), we can obtain the expansion condition at the asymptotic region, which comes from

\[
\frac{da}{d\tau} = \frac{-\kappa \theta / 2}{e^{-2\phi} + \kappa(\gamma - 2)/4} (\alpha \sinh \kappa \theta t + \beta \cosh \kappa \theta t) - \theta (\alpha + \beta)e^{\kappa \theta t} > 0.
\]

(31)

At the asymptotic future infinity and the past infinity, $t \to \pm \infty$, the following inequalities can be derived,

\[
\alpha + \beta < 0, \quad \alpha - \beta > 0,
\]

(32)

where these conditions imply that the constant $\beta$ is negative. In the intermediate region it is not easy to write down the condition in a simplified form; however, it can be shown
FIG. 1: The dotted and dashed lines show the behavior of the scale factor \( a(\tau) \) and the expansion rate \( da/d\tau \), respectively. The solid line importantly means the profile for the acceleration, \( d^2a/d\tau^2 \). The area of the scale factor in the figure represents the comoving time \( \tau \), so that the scale will not blow up in a finite comoving time \( \tau \). It is shown that the first acceleration starts from the comoving time \( \tau = 0 \) corresponding to the past infinity of \( t \rightarrow -\infty \), and then the deceleration of the universe corresponding to FRW phase appears in a finite time. Subsequently, the second acceleration turns up. In this figure, the parameters and constants are set as \( \kappa = 1 \), \( \gamma = 18 \), \( \theta = 1 \), \( \alpha = 0 \), \( \beta = -0.1 \), \( C_x = C_\Omega = 1 \), and \( P_{f_i} = 0 \).

in Fig. 1 that the positive expansion rate depicted in terms of the dashed line is possible without contraction of the universe.

Let us remind that some of dark-energy-dominant accelerating models have a defect of so-called big rip singularity that the scale blows up in a finite time [25] (for a recent review, see Ref. [26]). Since the present model is different from the previous models, we investigate whether this kind of singularity appears or not. If we rewrite the scale factor as

\[
a(\tau) = e^\rho = e^\phi \exp \left[ -\frac{1}{\kappa}(\alpha + \beta)e^{\kappa \theta t} + \frac{1}{\kappa}(C_x - C_\Omega) \right], \tag{33}
\]

then we see that it is definitely finite except it becomes infinite, \( e^\rho \sim \exp[-\frac{\tau}{\kappa(\gamma - 2)}(\alpha + \beta)e^{\kappa \theta t}] \), only for the coordinate time \( t \rightarrow \infty \) by using Eqs. (6), (30), and (32). The infinite coordinate time is related to the infinite comoving time as \( \tau(t \rightarrow \infty) \sim \infty \) and \( \tau(t \rightarrow -\infty) \sim 0 \), and
the scale factor is finite at a finite comoving time. Note that as $t \to -\infty$, the dilaton field is approximated as $e^{-2\phi} \sim e^{-\kappa \theta t}$ from Eqs. (30) and (31), and then the scale is given by $a(\tau) \sim e^{\kappa \theta t/2}$, which is reminiscent of the initial inflation.

Next, we investigate the behavior of the curvature scalar,

$$R = \frac{2d^2a/d\tau^2}{a(\tau)} = e^{-2\phi} \exp \left[ \frac{2}{\kappa}(\alpha + \beta)e^{\kappa \theta t} - \frac{2}{\kappa}(C_\chi - C_\Omega) \right] \times$$

$$\times \left\{ - \frac{\kappa^2 \theta^2}{e^{-2\phi} + \kappa(\gamma - 2)/4} \left[ (\alpha \cosh \kappa \theta t + \beta \sinh \kappa \theta t) - e^{-2\phi} \left[ \frac{\kappa^2 \theta^2}{e^{-2\phi} + \kappa(\gamma - 2)/4} \right]^2 (\alpha \sinh \kappa \theta t + \beta \cosh \kappa \theta t)^2 \right] - 2\kappa \theta^2 (\alpha + \beta) e^{\kappa \theta t} \right\}. \quad (34)$$

Note that $R \approx -\kappa^4 \theta^3 (\gamma - 2)t/4 \to +\infty$ for the limit of $t \to -\infty$, and $R \sim e^{\kappa \theta t} \exp \left[ \frac{2}{\kappa}(\gamma - 1)(\alpha + \beta)e^{\kappa \theta t} \right] \to 0$ for $t \to \infty$, where we have employed $\alpha + \beta < 0$ in Eq. (32). This model is singularity-free everywhere except the initial singularity at $\tau = 0(t \to -\infty)$. Therefore, there is no (big rip) singularity in a finite future comoving time.

Let us now study the most intriguing issue of the late acceleration. The acceleration is calculated formally as

$$\frac{d^2a}{d\tau^2} = \ddot{a} = \frac{1}{2}e^{-\phi} \exp \left[ \frac{1}{\kappa}(\alpha + \beta)e^{\kappa \theta t} - \frac{1}{\kappa}(C_\chi - C_\Omega) \right] \times$$

$$\times \left\{ - \frac{\kappa^2 \theta^2}{e^{-2\phi} + \kappa(\gamma - 2)/4} \left[ (\alpha \cosh \kappa \theta t + \beta \sinh \kappa \theta t) - e^{-2\phi} \left[ \frac{\kappa^2 \theta^2}{e^{-2\phi} + \kappa(\gamma - 2)/4} \right]^2 (\alpha \sinh \kappa \theta t + \beta \cosh \kappa \theta t)^2 \right] - 2\kappa \theta^2 (\alpha + \beta) e^{\kappa \theta t} \right\}. \quad (35)$$

Note that it vanishes at both ends, $t \to \pm \infty$, which is asymptotically given in the form of $d^2a/d\tau^2 \sim -te^{\kappa \theta t}$ for $t \to -\infty$ and $d^2a/d\tau^2 \sim \exp \left[ \frac{2}{\kappa}(\gamma - 1)(\alpha + \beta)e^{\kappa \theta t} \right]$ for $t \to \infty$. According to the assumption $\gamma > 2$ and $\alpha + \beta < 0$ in Eq. (32), there does not exist any divergent acceleration at both ends even though the initial curvature singularity appears. Apart from the ends, in the intermediate region, the whole profile of the acceleration (35) is plotted in Fig. 1. It shows that the universe starts with the inflation era and changes its state to Friedmann-Robertson-Walker (FRW) phase, and then it remarkably ends up with the desired second acceleration.

In order to discuss the equation of state parameter, we first set the energy-momentum
FIG. 2: The solid line shows the behavior of the equation-of-state parameter $\omega$, where the various parameters in Fig. 1 are used.

tensors as a source by using the constraint equations (12),

$$T_{\pm \pm} = -\kappa t_{\pm}$$

$$= \frac{1}{4\kappa} \left( \alpha^2 - \beta^2 \right) + \frac{\kappa^2 \theta^2}{4} \left( \alpha \sinh \kappa \theta t + \beta \cosh \kappa \theta t \right).$$

Then the energy density $\varepsilon$ and the pressure $p$ in the comoving coordinates are

$$\varepsilon = T_{\tau \tau} = e^{-2\rho} \left[ T_{++} + 2T_{+-} + T_{--} \right],$$

$$p = T_{xx} = \left[ T_{++} - 2T_{+-} + T \right].$$

Because of $T_{+-} = 0$, the equation-of-state parameter is simply $e^{2\rho}$, which is explicitly given by

$$\omega = p/\varepsilon$$

$$= e^{2\phi} \exp \left[ -\frac{2}{\kappa}(\alpha + \beta)e^{\kappa \theta t} + \frac{2}{\kappa}(C_{\chi} - C_{\Omega}) \right].$$

It is monotonically increasing and then eventually diverges because of the vanishing energy density. Note that both the energy density and the pressure are always negative so that the state parameter is positive, which is different from the previous models. In the past,
the state parameter is finite because the energy density and the pressure are all negative infinity. As time goes on, the energy density becomes finite while the pressure goes to the negative infinity. The whole profile for the state parameter is shown in Fig. 2 along with the same fixed parameter used in Fig. 1.

The energy-momentum tensor taken here is a little bit different from the conventional treatment of the energy-momentum tensors in that the geometrical part and the source part are not distinguished, since our energy-momentum tensors are induced from the vacuum polarization which is expressed by the metric and the dilaton without the classical matter. The induced source terms are combined with the original metric part and hence result in the back-reacted metric. The remnant, that is $t_\pm$, is interpreted as a source of this model, which is seen in Eq. (8). The left is a source while the right-hand-side is the back-reacted metric part.

IV. DISCUSSION

In our model, the induced stress-tensors from the one-loop effective action (4) play an important role. It consists of the integration function $t_\pm$ and the bulk part in Eqs. (10), where the former function is interpreted as a source contribution, while the latter part was solved in cooperation with the metric and dilaton coupled manner for the quantum back reaction as seen in Eq. (5). This means that the scale factor from $\Omega$ and $\chi$ reflects the back reaction of the geometry. So, the classical dilaton-gravity part and the quantum-mechanically induced part have not been separated; instead, they have been solved in terms of the redefined fields $\Omega$ and $\chi$.

The modified Poisson brackets (25) look like noncommutative commutator if we define the relevant fields as operators [21]. However, in our case, we have just defined Poisson brackets in order to obtain modified semiclassical equations of motion which recover the original semiclassical equations of motion when the noncommutative parameter $\theta$ vanishes. In the quantum mechanics, this nontrivial Poisson brackets may appear in a slowly moving electron on the constant magnetic field [23]; unfortunately, we have no idea what the counterpart is in this cosmological model.

In fact, we have addressed the same issue in the RST model in Ref. [20]. In that case, it shows just a single phase transition from the decelerating to the accelerating universe and,
even worse, the future singularity appears. However, in this improved model, we obtained the future-singularity-free second accelerating expansion along with some local ambiguity terms for the Polyakov non-local action. And, we skipped the critical case of $\gamma = 2$ which is similar to our model of $\gamma > 2$.

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