Fluctuation spectrum of fluid membranes coupled to an elastic meshwork:
jump of the effective surface tension at the mesh size

Jean-Baptiste Fournier,1, 2 David Lacoste,1 and Elie Raphaël3, 2
1Laboratoire de Physico-Chimie Théorique, ESPCI, 10 rue Vauquelin, F-75231 Paris cedex 05, France
2Fédération de Recherche CNRS 2438 “Matière et Systèmes Complexes”, F-75231 Paris cedex 05, France
3Laboratoire de Physique de la MATièRe Condensée, UMR CNRS 7125, Collège de France, 11 Place Marcelin Berthelot, F-75231 Paris cedex 05, France
(Dated: March 22, 2022)

We identify a class of composite membranes: fluid bilayers coupled to an elastic meshwork, that are such that the meshwork’s energy is a function $F_{el}[A_{el}]$ not of the real microscopic membrane area $A$, but of a smoothed membrane’s area $A_{\xi}$, which corresponds to the area of the membrane coarse-grained at the mesh size $\xi$. We show that the meshwork modifies the membrane tension $\sigma$ both below and above the scale $\xi$, inducing a tension-jump $\Delta \sigma = dF_{el}/dA_{\xi}$. The predictions of our model account for the fluctuation spectrum of red blood cells membranes coupled to their cytoskeleton. Our results indicate that the cytoskeleton might be under extensional stress, which would provide a means to regulate available membrane area. We also predict an observable tension jump for membranes decorated with polymer “brushes”.

PACS numbers: 87.16.Dg, 68.03.Cd, 87.16.Gj, 68.47.Pe

Random surfaces 1 play an important role in many areas of physics, spanning from biophysics 2 and chemical physics 3 to high-energy physics 4. In the last decades, there has been considerable interest in the properties of fluid bilayer membranes composed of amphiphilic molecules in water. It is well-known 3 that their fluctuations are dominated by the membrane bending rigidity, $\kappa$, which leads to a spectrum proportional to $(\kappa q^4 + \sigma q^2)^{-1}$ 4, 5, 8. Here, $\sigma$ is an effective surface tension, i.e., an adjustable thermodynamic parameter arising from the constraint of constant surface area, which is usually several orders of magnitude smaller than the surface tensions of ordinary liquid interfaces 4, 10. Additional complexity arises if the membrane interacts with other systems (e.g., a rigid substrate, another membrane, a network of polymers or filaments), 11, 12, 13, or contains inclusions (passive or active) 14, 15.

In this paper, we identify a class of composite membranes, i.e., of membranes coupled to an external system (like the cytoskeleton of red blood cells 10), for which the coupling energy can be described, in a first approximation, by a function of the membrane area coarse-grained at a characteristic length scale $\xi$ (the mesh size in the cytoskeleton case). For such systems, we show that the effective membrane tension should exhibit a jump of amplitude $\Delta \sigma$ at the scale $\xi$. In other words, the fluctuation spectrum is proportional to $(\kappa q^4 + \sigma q^2)^{-1}$ for $q \lesssim \xi^{-1}$, and to $(\kappa q^4 + \sigma^2 q^2)^{-1}$ for $q \gtrsim \xi^{-1}$. As we shall see, $\Delta \sigma = \sigma^c - \sigma^s$ is directly related to the coupling between the membrane and the external system. Understanding the scale dependence of the effective surface tension, and in particular its value at short length-scales, is important in a number of phenomena, e.g., membrane adhesion 17, 18, cell fusion 19, 20, and other microscopic biological mechanisms, such as endocytosis 21. It should also help interpret experiments on composite membranes 22, 23, 24, which attempt to determine the value of $\kappa$ by fitting the fluctuation spectrum.

We shall consider two different systems for which our model predicts a jump in surface tension: (i) membranes decorated with polymer “brushes” 25, and (ii) red blood cell membranes interacting with their attached cytoskeleton 10. Polymer decorated membranes are useful in providing a sterical stabilization for liposome drug carriers 26, 27; we expect that better understanding their membrane tension should yield new insight in their stability and mechanical properties. Here, we shall mainly focus, however, on the composite membrane of the red blood cell (or erythrocyte). The cytoskeleton of red blood cells is a roughly triangular flexible protein meshwork, composed of spectrins attached to the bilayer by association with integral membrane proteins 10. The resulting mechanical strength enables red blood cells to undergo large extensional deformations as they are forced through narrow capillaries. In the last two decades, the red blood cell membrane has been thoroughly studied and a rich description of its attached cytoskeleton has been obtained 28, 29, 30. Very recently, the spectrum of the short wavelength membrane fluctuations of red blood cells 22 has been successfully fitted by including the effect of confinement from the cytoskeleton 24; however, this approach leads to an unexplained, rather abrupt change in the value of the membrane tension. We shall show that our model explains quantitatively this change, provided that the cytoskeleton is under (extensional) stress, which is an important issue currently debated in cell biology 31, 32. Specific implications concerning red blood cells as well as possible experiments on
polymer decorated membranes will be discussed at the end of the paper.

Let us introduce our model by considering a membrane (embedded in the Euclidian three-dimensional space) connected to an elastic meshwork. The meshwork is composed of \( N \) Hookean springs (spring constant \( \lambda \), free length \( \ell_0 \)) and is attached to the membrane through its nodes. The shape of the membrane is specified by giving the position \( \mathbf{R}(s^1, s^2) \) of a membrane element as a function of two intrinsic coordinates. The total free energy of the system may be written as \( F[\mathbf{R}] = F_{\text{memb}}[\mathbf{R}] + F_{\text{el}}[\mathbf{R}] \), where \( F_{\text{memb}}[\mathbf{R}] \) is the membrane free energy and

\[
F_{\text{el}}[\mathbf{R}] = N \frac{1}{2} \lambda (\xi[\mathbf{R}] - \ell_0)^2
\]

is the meshwork elastic energy. Since the meshwork cannot follow the wiggles of the membranes of wavelength shorter than the mesh size (see Fig. 1), the spring elongation \( \xi[\mathbf{R}] \) actually depends on the membrane conformation. To simplify, we have assumed that \( \xi[\mathbf{R}] \) is identical for all the springs, which is reasonable since the membrane is fluid. The mesh size, i.e., the spring length, can thus be expressed as

\[
\xi^2[\mathbf{R}] = g \frac{A_\xi[\mathbf{R}]}{N},
\]

where \( A_\xi[\mathbf{R}] \) is the area of the membrane coarse-grained at the mesh size, and \( g \) is a constant depending on the meshwork topology (e.g., \( g = 2\sqrt{3} \) for triangular meshwork). Determining \( \xi[\mathbf{R}] \) is the two-dimensional analogous of measuring the length of a fractal coast (e.g., the coast of Brittany) with a straight ruler, and determining the ruler length that allows to go through the whole coast in \( N \) steps.

Solving Eq. (2) for \( \xi \) in the general case of an arbitrary membrane shape is a rather difficult task. In order to simplify the problem, let us consider the case of a quasi-spherical membrane of fixed volume \( V \) and fixed area \( A \). If \( V \) is expressed as \( V = \frac{4}{3} \pi R_0^3 \), the area can then be written in the form \( A = \bar{A} + \delta A \) where \( \bar{A} = 4\pi R_0^2 \gg \delta A \) is the projected area. We now consider a very small, almost planar portion \( \bar{a} = \epsilon \bar{A} \) of the projected area, that still contains a rather large number \( n = \epsilon N \) of springs. We represent the local membrane shape by its fluctuation modes \( u_q \) above the local projected area. Locally, Eq. (2) can be expressed as

\[
\frac{n}{g} \xi^2[u] = a_\xi[u] = \bar{a} + \sum_{q < \xi^{-1}[u]} \frac{1}{2} q^2 |u_q|^2 + \mathcal{O}(u_q^2),
\]

where \( a_\xi \approx \epsilon A_\xi \) is the local coarse-grained area. Consistently with the second-order expansion, the upper limit \( \xi^{-1}[u] \) in the above sum can be replaced by its zeroth order approximation:

\[
\xi_0^{-1} = \sqrt{\frac{n}{g a}} = \sqrt{\frac{N}{g A}}.
\]
bending rigidity originating from the meshwork, which may also be scale-dependent.

The results obtained above are based on the fact that the meshwork energy is actually a function of \( A_\xi \left[ R \right] \) through Eqs. (1) and (2). The same conclusions concerning the effective tension therefore hold for any composite membrane for which the coupling may be approximated by a function \( F_{el}(A_\xi) \) of the membrane area coarse-grained at a characteristic scale \( \xi \). The general expression giving the tension jump occurring at \( \xi_0 \approx \xi \) is simply

\[
\Delta \sigma = \frac{\partial F_{el}}{\partial A_\xi} \bigg|_{A_\xi = \xi_0}.
\]  

(9)

Before making predictions concerning other systems, such as polymer decorated membranes, let us discuss in more detail the cytoskeleton case.

Inspired by the recent work of Gov et al. [24], we have re-analyzed the data of Zilker et al. [22] on the fluctuation spectrum of normal red blood cells in the wavelength range 300 nm < \( \lambda < 6000 \) nm. By fitting these data, the authors of Ref. [24] have demonstrated the necessity to introduce a confining potential, arising from the presence of the cytoskeleton. They acknowledge, however, that the data at very short scales are better fitted by a value of the membrane tension much smaller than the one obtained at large scales. Since the cytoskeleton is well described by a meshwork of elastic springs [16, 32], we believe that the theory put forward in this paper provides an explanation to this jump in surface tension. Fluctuation spectra are usually fitted via an effective wavevector-dependent bending rigidity \( \kappa_q \) defined by

\[
\langle |q| \rangle^2 = \frac{k_B T}{\kappa_q q^2}.
\]  

(10)

According to Eq. (3), this quantity should roughly exhibit a discontinuity at the mesh wavevector \( q_0 = \xi_0^{-1} \). This is indeed the case, as evidenced by the solid line fit in Fig. 2. The position of the discontinuity being fairly marked at \( \xi_0^{-1} \approx 125 \) nm, and the value \( \kappa \approx 2 \times 10^{-20} \) Nm being already well documented [16, 32, 22], we have fitted the data to Eq. (3) for the parameters \( \gamma \), \( \sigma^< \) and \( \sigma^> \), only, obtaining \( \gamma = (5.2 \pm 0.6) \times 10^6 \) Jm\(^{-4}\), \( \sigma^< = (1.5 \pm 0.2) \times 10^{-6} \) Jm\(^{-2}\) and \( \sigma^> = (-0.8 \pm 1.2) \times 10^{-7} \) Jm\(^{-2}\). According to our model, \( \Delta \sigma = \sigma^< - \sigma^> = 1.6 \times 10^{-6} \) Jm\(^{-2}\) should be given by Eq. (4). Indeed, assuming \( \xi_0 > t_0 \) (and yet \( \xi \approx t_0 \)) we deduce \( \lambda \approx 10^{-6} \) Jm\(^{-2}\), which agrees well with the measured cytoskeleton elastic constants [31]. From the positive sign of \( \Delta \sigma \), our analysis predicts that the cytoskeleton is under extensional stress: this important issue would require more experiments to test. Finding \( \sigma^> \) essentially zero, or even negative, is also quite remarkable. It suggests that one important role of the cytoskeleton might be to regulate, via its elastic stress, the membrane tension at length-scales shorter than the cytoskeleton mesh, in order to produce either large fluctuations, or even a short-scale buckling. This new type of area regulation (see Ref. [32]) might prove useful to allow for large deformations.

Another system to which our model can be applied is a membrane bearing an anchored polymer brush. For a fixed number \( p \) of polymers in good solvent conditions, we have [23]

\[
\frac{F_{el}}{p k_B T} \approx N b^{5/3}(1 - 2\chi)^{1/3} \Sigma^{5/6},
\]  

(11)

where \( N \) denotes now the index of polymerization, \( b \) the monomer length, \( \chi \) the Flory parameter, and \( \Sigma \) the grafting density. As illustrated in Fig. 3 the relevant area defining the grafting density is not the total area \( A \), but the area \( A_\xi \) coarse-grained at the blob scale. Hence,

\[
\Sigma \approx \frac{p}{A_\xi} \approx \frac{1}{\xi^2}.
\]  

(12)

Expressing as in Eq. (5) the brush energy in terms of the local coarse-grained area, or equivalently, using Eq. (9), we obtain

\[
\Delta \sigma \approx -N k_B T b^{5/3}(1 - 2\chi)^{1/3} \left( \frac{p}{A} \right)^{11/6}.
\]  

(13)

This actually corresponds to the brush osmotic pressure integrated over the brush height: \( \Delta \sigma \approx -\Pi_{osm} H = -k_B T(p/A)^{3/2} \times b N (b^2 p/A)^{1/3} \). Note that \( \Delta \sigma < 0 \), i.e., the brush reduces the large-scale tension with respect to the short-scale one. It actually draws some of the membrane area stored at length-scales shorter than the blob size in order to lower its grafting density. Let us estimate \( \Delta \sigma \) in typical experiments [12]: with \( N = 100 \), \( b = 1 \) nm, and a (projected) area per chain of 100 nm\(^2\), we obtain \( |\Delta \sigma| \approx 10^{-4} \) Jm\(^{-2}\), which compares with ordinary membrane tensions. At length-scales larger than the
brush height $H$, the brush elasticity also yields a renormalization of $\kappa$ [36]. Note that for distortions wavevectors in the interval $H^{-1} < q < \xi^{-1}$, the brush will actually heal over $\sim q^{-1}$, implying that the tension jump will probably be spread over this interval [37]. Our result agrees with a recent direct calculation of Bickel et al. [38], showing that membranes bearing polymers in the "mushrooms" regime should possess an excess tension $\sim 2 k_B T \Sigma$ at scales shorter than the gyration radius $R_g$. Indeed, at the grafting density $\Sigma \sim 1/R_g^2$ which crosses-over to the brush regime, this gives $\Delta \sigma \sim - \Pi_{\text{osm}} H$, with $\Pi_{\text{oosm}} \sim k_B T/R_g^2$ and $H \sim R_g$. A possible way to test our prediction Eq. (13) would be to measure, on a given vesicle decorated with a polymer brush, the variation of the large-scale membrane tension induced either by a modification of the solvent quality or by a chemical alteration of the index of polymerization.

To conclude, the jump of the effective surface tension evidenced in this paper arises from a competition between the free energy of the membrane, which requires a certain amount of excess area to be stored in the short wavelength modes, and the free energy of the coupled elastic system, which is lowered if either less or more excess area (depending on the system) is transferred to the large-scale modes. Very generally, a wavevector-dependent membrane tension should therefore occur whenever an external system coupled to the membrane produces a scale-dependent area transfer.

We acknowledge fruitful discussions with A. Ajdari, F. Gallet, and P.-G. de Gennes.

References

[1] Statistical Mechanics of Membranes and Interfaces, edited by D. R. Nelson, T. Piran, S. Weinberg (World Scientific, Singapore, 1989). Fluctuating geometries in statistical mechanics and field theory, edited by F. David, P. Ginsparg, and J. Zinn-Justin (Elsevier, Amsterdam, 1996).

[2] Structure and Dynamics of Membranes, edited by R. Lipowsky, E. Sackman (Elsevier, Amsterdam, 1995).

[3] S. A. Safran, Statistical Thermodynamics of Surfaces, Interfaces, and Membranes (Addison-Wesley Publishing Company, 1994).

[4] J. F. Wheater, J. Phys. A 27, 3323 (1994).

[5] V. Helfrich, Naturforsch. 28C, 693 (1973).

[6] F. Brochard and J.-F. Lennon, J. Physique 36, 1035 (1975).

[7] H. Engelhardt, H. P. Duwe and E. Sackmann, J. Physique Lett. 46, 395 (1985).

[8] P. Méléard, J. F. Faucon, M. D. Mitov, and P. Bothorel, Europhys. Lett. 19, 267 (1992).

[9] W. Helfrich and R.-M. Servuss, Nuovo cimento D3, 137 (1984).

[10] E. Evans and W. Rawicz, Phys. Rev. Lett. 64, 2094 (1990).

[11] E. Evans and W. Rawicz, Phys. Rev. Lett. 79, 2379 (1997).

[12] R. Joannic, L. Auvray, and D. D. Lasic, Phys. Rev. Lett. 78, 3402 (1997).

[13] W. Häckl, M. Bärmann, and E. Sackmann, Phys. Rev. Lett. 80, 1786 (1998).

[14] M. Goulain, R. Bruinsma, and P. Pincus, Europhys. Lett. 22, 145 (1993).

[15] J.-B. Manneville, P. Bassereau, D. Lévy, and J. Prost, Phys. Rev. Lett. 82, 4356 (1999).

[16] B. Alberts, A. Johnson, J. Lewis, M. Raff, K. Roberts, P. Walter, Molecular Biology of the Cell 4th edition (Garland, New York, 2002).

[17] R. M. Servuss and W. Helfrich, J. Phys. France 50, 809 (1989).

[18] J. O. Radler, T. J. Feder, H. H. Strey, and E. Sackmann, Phys. Rev. E 51, 4526 (1995).

[19] R. Jahn and H. Grubmuller, Cur. Opin. in Cell Biol. 14, 488 (2002).

[20] L. Yang and H. W. Huang, Science 297, 5588 (2002).

[21] M. Marsh and H. T. McMahon, Science 285, 215 (1999).

[22] A. Zilker, H. Engelhardt and E. Sackmann, J. Physique 48, 2139 (1987).

[23] H. Strey, M. Peterson, and E. Sackmann, Biophys. J. 69, 475 (1995).

[24] N. Gov, A. G. Zilman and S. Safran, Phys. Rev. Lett. 90, 228101 (2003).

[25] P.-G. de Gennes, Adv. Coll. Int. Sci. 27, 189 (1987).

[26] T. M. Allen and A. Chonn, FEBS Lett. 223, 42 (1987).

[27] D. D. Lasic and D. Papahadjopoulos, Science 267, 1597 (1995).

[28] T. J. Byers and D. Branton, Proc. Natl. Acad. Sci. USA 82, 6153 (1985).

[29] V. Bennett, Biochim. Biophys. Acta 988, 107 (1989).

[30] N. Mohandas and E. Evans, Annu. Rev. Biophys. Biomol. Struct. 23, 787 (1994).

[31] S. K. Boey, D. H. Boal, and D. E. Discher, Biophys. J. 75, 1573 (1998), Biophys. J. 75, 1584 (1998).

[32] D. H. Boal, Biol. Ball. 194, 331 (1998).

[33] B. B. Mandelbrot, The fractal geometry of nature (W. H. Freeman, San Francisco, 1982).

[34] G. Lenormand, S. Hénon, A. Richert, J. Siméon, and F. Gallet, Biophys. J. 81, 43 (2001).

[35] C. E. Morris and U. Homann, J. Membrane Biol. 179, 79 (2001).

[36] C. Hiergeist and R. Lipowsky, J. Phys. II 6, 1465 (1996).

[37] A. Ajdari, private communication.

[38] T. Bickel and C. M. Marques, to appear in Eur. Phys. J. E (2002).