Neutralino Dark Matter with Inert Higgsinos and Singlinos

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Abstract

We discuss neutralino dark matter arising from supersymmetric models with extra inert Higgsinos and singlinos, where inert means that their scalar partners do not get vacuum expectation values. As an example, we consider the extended neutralino sector of the $E_6$ SSM, which predicts three families of Higgs doublet pairs, plus three singlets, plus a $Z'$, together with their fermionic superpartners. We show that the two families of inert doublet Higgsinos and singlinos predicted by this model provide an almost decoupled neutralino sector with a naturally light LSP which can account for the cold dark matter relic abundance independently of the rest of the model, providing that the ratio of the two usual Higgs doublets satisfies $\tan \beta < 2$. 
1 Introduction

The existence of weak scale supersymmetry (SUSY) is theoretically well motivated, because of its ability to stabilise the electro-weak symmetry breaking (EWSB) scale. One of the benefits of weak scale SUSY with conserved \( R \)-parity is that the lightest supersymmetric particle (LSP) is absolutely stable and provides a weakly interacting massive particle (WIMP) candidate capable of accounting for the observed cold dark matter (CDM) relic density \( \Omega_{\text{CDM}} h^2 \approx 0.1 \) [1, 2]. In particular, the lightest neutralino in SUSY models is an excellent such candidate, providing its mass, composition and interactions are suitably tuned to result in the correct value of \( \Omega_{\text{CDM}} h^2 \).

The Minimal Supersymmetric Standard Model (MSSM) [3] provides the simplest supersymmetric extension of the Standard Model (SM) in which the superpotential contains the bilinear term \( \mu H_d H_u \), where \( H_{d,u} \) are the two Higgs doublets whose neutral components \( H^0_{d,u} \) develop vacuum expectation values (VEVs) at the weak scale and the \( \mu \) parameter has the dimensions of mass. However, since this term respects supersymmetry, there is no reason for \( \mu \) to be of order the weak scale, leading to the so-called \( \mu \) problem [4]. Also, the MSSM suffers a fine-tuning of parameters at the per cent level [5].

To address the above shortcomings of the MSSM one may replace the \( \mu \) term of the MSSM by the low energy VEV of a singlet field \( S \) via the interaction \( \lambda SH_d H_u \). For example, such a singlet coupling can be enforced by a low energy \( U(1)' \) gauge symmetry arising from a high energy \( E_6 \) GUT group [6]. Within the class of \( E_6 \) models there is a unique choice of Abelian gauge group, referred to as \( U(1)_N \), which allows zero charges for right-handed neutrinos. This choice of \( U(1)_N \), which allows large right-handed neutrino Majorana masses, and hence a high scale see-saw mechanism, defines the so-called Exceptional Supersymmetric Standard Model (E\( _6 \)SSM) [7, 8].

In the E\( _6 \)SSM, in order to cancel gauge anomalies involving \( U(1)_N \), the low energy (TeV scale) theory must contain the matter content of three complete 27 representations of \( E_6 \) (minus the neutral right-handed neutrinos which acquire intermediate scale masses). It is clear that the E\( _6 \)SSM predicts a rich spectrum of new states at the TeV scale corresponding to the matter content of three 27 component families. Since each 27 includes a pair of Higgs doublets plus a singlet, the E\( _6 \)SSM predicts in total three families of Higgs doublets and three families of Higgs singlets\(^1\). The two Higgs doublets familiar from the MSSM are denoted as \( H_d \) and \( H_u \), while the two further replicas of these Higgs doublets predicted by the E\( _6 \)SSM are denoted as \( H^d_1, H^u_1 \) and \( H^d_2, H^u_2 \). Each 27 representation

\(^1\)Each 27 component family also includes a pair of vector-like charged \( \pm 1/3 \) coloured states \( D, \bar{D} \) which are readily produced at the LHC and provide a clear signature of the model.
also contains a separate SM singlet, namely the singlet $S$ whose VEV yields an effective $\mu$ term, plus two further copies of this singlet, $S_1$ and $S_2$. In the E$_6$SSM the extra Higgs doublets, $H_d^1$, $H_u^1$, $H_d^2$, $H_u^2$, and singlets, $S_1$, $S_2$, are not supposed to develop VEVs and the scalar components of these superfields are consequently called “inert”. From the perspective of dark matter, of particular interest are the fermionic partners of these inert Higgs doublet and singlet superfields, which we refer to as “inert Higgsinos/singlinos”. Such inert Higgsinos/singlinos will in general mix with the other neutralinos and therefore change the nature of lightest neutralino. If the LSP is the lightest neutralino, identified as a WIMP CDM candidate, then the calculation of the thermal relic density will necessarily be affected by the presence of such inert Higgsinos/singlinos.

The purpose of this paper is to study neutralino dark matter in the presence of inert Higgsinos/singlinos. As an example, we shall consider the extended neutralino sector of the E$_6$SSM, which includes three families of Higgs doublet pairs, plus three singlets, plus a $Z'$, together with their fermionic superpartners. The study here should be compared to that of the USSM [9] which, in addition to the states of the MSSM, also includes a singlet, $S$, plus plus a $Z'$, together with their fermionic superpartners, namely the singlino $\tilde{S}$ and an extra gaugino $\tilde{B}'$. In the USSM the neutralino LSP may have components of the extra gaugino $\tilde{B}'$ and singlino $\tilde{S}$ in addition to the usual MSSM neutralino states, which can have interesting consequences for the calculation of the relic density $\Omega_{\text{CDM}}h^2$. In the present study we include all the above states of the USSM, plus the extra inert Higgsino doublets predicted by the E$_6$SSM but not included in the USSM, namely $\tilde{H}^d_1$, $\tilde{H}^u_1$, $\tilde{H}^d_2$, $\tilde{H}^u_2$, and the singlinos, $\tilde{S}_1$ and $\tilde{S}_2$, but we do not include the corresponding inert scalars, which do not play a role in the heavy inert scalar limit. We also do not include any of the exotic coloured states, $D$ and $\bar{D}$, since in general we would not expect them to play a significant role in the calculation of the dark matter relic abundance.

We shall study neutralino dark matter in the E$_6$SSM, as defined above, both analytically and numerically, using MicrOMEGAs [23]. We find that results for the relic abundance in the E$_6$SSM are radically different from those of both the MSSM and the USSM. This is because the two families of inert doublet Higgsinos and singlinos predicted by the E$_6$SSM provide an almost decoupled neutralino sector with a naturally light LSP which can account for the cold dark matter relic abundance independently of the rest of the model. Typically the LSP will originate predominantly from the neutralinos contained in the inert Higgsino/singlino families and such an LSP will be able to account for the dark matter relic abundance and satisfy current experimental data.\footnote{There is a lot of interest in the excess positron signal that has been recently observed by the PAMELA, ATIC and Fermi collaboration (see e.g. [14]). It has been speculated that this could have been produced...}
through an s-channel Z-boson, via its inert Higgsino doublet components which couple to the Z-boson. This leads to a constraint that the LSP mass must exceed half the Z-boson mass, to avoid the LEP constraints on the Z-boson width, which can be satisfied providing that the ratio of the two usual Higgs doublet VEVs (\(\tan \beta\)) is less than about 2.

Apart from the requirement \(\tan \beta < 2\), the very stringent constraints on MSSM or USSM parameter space, which come from requiring that the model explains the relic density in terms of relic neutralinos, become completely relaxed, since in the \(E_6\)SSM neutralino dark matter depends almost exclusively on the parameters of the almost decoupled inert Higgsino sector. We expect similar results to apply to any singlet-extended SUSY model with an almost decoupled inert doublet Higgsino / Higgs singlino sector.

The remainder of the paper is organised as follows. In Section 2 we briefly review the \(E_6\)SSM which provides the motivation for including three families of Higgs doublets and singlets. In Section 3 we discuss the inert Higgsino sector of the \(E_6\)SSM and the effective model which we shall study, and highlight the most important couplings for our analysis of the LSP dark matter relic density. In Section 4 we display the complete neutralino and chargino mass matrices of the considered model. In Section 5 we present some analytical results which provide useful insight into the new inert sector physics. These results are subsequently used to understand and interpret the results of Section 6, in which the results of a full numerical dark matter relic density calculation using MicrOMEGAs [23] are presented. The paper is concluded in Section 7.

2 The \(E_6\)SSM

One of the most important issues in models with additional Abelian gauge symmetries is the cancellation of anomalies. In \(E_6\) theories, if the surviving Abelian gauge group factor is a subgroup of \(E_6\) and the low energy spectrum constitutes complete 27 representations of \(E_6\), then the anomalies are cancelled automatically. In the \(E_6\)SSM the 27 of \(E_6\) containing the three quark and lepton families decompose under the \(SU(5) \times U(1)_N\) subgroup of \(E_6\) as follows:

\[
27 \rightarrow (10, 1)_i + (5^*, 2)_i + (5^*, -3)_i + (5, -2)_i + (1, 5)_i + (1, 0)_i.
\]

by annihilating dark matter in the galactic halo [15], but it has also been suggested that the signal could be explained as coming from normal astrophysical sources such as nearby pulsars [16]. In this paper we shall not try to interpret these data as arising from neutralino dark matter, but instead we assume some astrophysical explanation of the data.
The first and second quantities in the brackets are the $SU(5)$ representation and extra $U(1)_N$ charge while $i$ is a family index that runs from 1 to 3. From Eq. (1) we see that, in order to cancel anomalies, the low energy (TeV scale) spectrum must contain three extra copies of $5^* + 5$ of $SU(5)$ in addition to the three quark and lepton families in $5^* + 10$.

To be precise, the ordinary SM families which contain the doublets of left-handed quarks $Q_i$ and leptons $L_i$, right-handed up- and down-quarks ($u^c_i$ and $d^c_i$) as well as right-handed charged leptons, are assigned to $(10, 1)_i + (5^*, 2)_i$. Right-handed neutrinos $N^c_i$ should be associated with the last term in Eq. (1), $(1, 0)_i$. The next-to-last term in Eq. (1), $(1, 5)_i$, represents SM singlet fields $S_i$ which carry non-zero $U(1)_N$ charges and therefore survive down to the EW scale. The three pairs of $SU(2)$-doublets ($H^d_i$ and $H^u_i$) that are contained in $(5^*, -3)_i$ and $(5, -2)_i$ have the quantum numbers of Higgs doublets, and we shall identify one of these pairs with the usual MSSM Higgs doublets, with the other two pairs being inert Higgs doublets which do not get VEVs. The other components of these $SU(5)$ multiplets form colour triplets of exotic quarks, $D_i$ and $\overline{D}_i$, with electric charges $-1/3$ and $+1/3$ respectively. The matter content and correctly normalised Abelian charge assignment are in Tab. 1.

| $\sqrt{\frac{5}{3}}Q^Y_i$ | $Q$ | $u^c$ | $d^c$ | $L$ | $e^c$ | $N^c$ | $S$ | $H_u$ | $H_d$ | $D$ | $\overline{D}$ | $H'$ | $\overline{H}'$ |
|--------------------------|----|------|------|----|------|------|----|-------|-------|----|------------|------|--------|
| $\sqrt{40}Q^N_i$        | 1  | 1    | 2    | 2  | 1    | 0    | 5  | $-2$  | $-3$  | $-2$| $-3$       | 2    | $-2$   |

Table 1: The $U(1)_Y$ and $U(1)_N$ charges of matter fields in the $E_6$ SSM, where $Q^N_i$ and $Q^Y_i$ are here defined with the correct $E_6$ normalisation factor required for the RG analysis.

We also require a further pair of superfields $H'$ and $\overline{H}'$ with a mass term $\mu' H'\overline{H}'$ from incomplete extra $27'$ and $\overline{27}'$ representations to survive to low energies to ensure gauge coupling unification. Because $H'$ and $\overline{H}'$ originate from $27'$ and $\overline{27}'$, these supermultiplets do not spoil anomaly cancellation in the considered model. Our analysis reveals that the unification of the gauge couplings in the $E_6$ SSM can be achieved for any phenomenologically acceptable value of $\alpha_3(M_Z)$, consistent with the measured low energy central value, unlike in the MSSM which requires significantly higher values of $\alpha_3(M_Z)$, well above the central measured value [10].

Table 3.

Since right-handed neutrinos have zero charges they can acquire very heavy Majorana masses. The heavy Majorana right-handed neutrinos may decay into final states with lepton number $L = \pm 1$, thereby creating a lepton asymmetry in the early Universe.

\[^3\]The two superfields $H'$ and $\overline{H}'$ may be removed from the spectrum, thereby avoiding the $\mu'$ problem, leading to unification at the string scale [11]. However we shall not pursue this possibility in this paper.
Because the Yukawa couplings of exotic particles are not constrained by the neutrino oscillation data, substantial values of CP-violating lepton asymmetries can be induced even for a relatively small mass of the lightest right-handed neutrino \((M_1 \sim 10^6 \text{ GeV})\) so that successful thermal leptogenesis may be achieved without encountering any gravitino problem [12].

In \(E_6\) models the renormalisable part of the superpotential arises from the \(27 \times 27 \times 27\) decomposition of the \(E_6\) fundamental representation. The most general renormalisable superpotential that is allowed by the \(E_6\) symmetry can be written in the following form:

\[
W_{E_6} = W_0 + W_1 + W_2, \quad (2)
\]

\[
W_0 = \lambda_{ijk} S_i (H_d j H_u k) + \kappa_{ijk} S_i (D_j \overline{D}_k) + h_{ijk}^N N_i^c (H_{u_j} L_k) + h_{ijk}^U u_i^c (H_{u_j} Q_k) + h_{ijk}^D d_i^c (H_{d_j} Q_k) + h_{ijk}^E e_i^c (H_{d_j} L_k), \quad (3)
\]

\[
W_1 = g_{ij}^Q D_i (Q_j Q_k) + g_{ijk}^q \overline{D}_i d_j^c u_k^c, \quad (4)
\]

\[
W_2 = g_{ij}^N N_i^c D_j d_k^c + g_{ijk}^E e_i^c D_j u_k^c + g_{ijk}^D (Q_i L_j) \overline{D}_k. \quad (5)
\]

The superpotential of the \(E_6\) SSM clearly involves a lot of new Yukawa couplings in comparison to the SM. In general these new interactions violate baryon number conservation and induce non-diagonal flavour transitions. To suppress baryon number violating and flavour changing processes one can postulate a \(Z_2^H\) symmetry under which all superfields except one pair of \(H_d^i\) and \(H_u^i\) (say \(H_d \equiv H_3^d\) and \(H_u \equiv H_3^u\)) and one SM singlet field \((S \equiv S_3)\) are odd. The \(Z_2^H\) even Higgs doublets then play the role of the conventional Higgs doublets which get VEVs and are allowed to couple to the normal SM matter. Here we have chosen the third generation to be even, so the inert superfields must therefore belong to the first and second generations. The \(Z_2^H\) symmetry then explains why the inert Higgs doublets and singlets do not get VEVs.

However, the \(Z_2^H\) can only be approximate (otherwise the exotics would not be able to decay). To prevent rapid proton decay in the \(E_6\) SSM, a generalised definition of \(R\)-parity should be used. We give two examples of possible symmetries that can achieve this. If \(H_d^i, H_u^i, S_i, D_i, \overline{D}_i\) and the quark superfields \((Q_i, u_i^c, d_i^c)\) are even under a discrete \(Z_2^L\) symmetry while the lepton superfields \((L_i, e_i^c, N_i^c)\) are odd (Model I) then the allowed superpotential is invariant with respect to a \(U(1)_B\) global symmetry with the exotic \(D_i\) and \(\overline{D}_i\) identified as diquark and anti-diquark, i.e. \(B_D = -2/3\) and \(B_{\overline{D}} = +2/3\). An alternative possibility is to assume that the exotic quarks, \(D_i\) and \(\overline{D}_i\), as well as lepton superfields, are all odd under \(Z_2^B\) whereas the others remain even. In this case (Model II) the \(\overline{D}_i\) and \(D_i\) are leptoquarks [7].
3 The Inert Higgsino Couplings

The most important couplings in our analysis are the trilinear couplings between the three generations of up- and down-type Higgs doublets and Higgs SM singlets contained in the superpotential of the $E_6$ SSM in Eq. (3),

$$\lambda_{ijk}S_i H_d^j H^0_{uk} = \lambda_{ijk}(S_i H_d^j H^0_{uk} - S_i H_d^j H^0_{uk}) \tag{6}$$

The trilinear coupling tensor $\lambda_{ijk}$ in Eq. (6) consists of 27 numbers, which play various roles. The purely third family coupling $\lambda_{333} \equiv \lambda$ is very important, because it is the combination $\mu = \lambda s/\sqrt{2}$ that plays the role of an effective $\mu$ term in this theory (where $s/\sqrt{2}$ is the VEV of the third family singlet scalar $S_3 \equiv S$). Some other neutralino mass terms, such as those involving $\tilde{S}$, are also proportional to $\lambda$. The couplings of the inert (first and second generation) Higgs doublet superfields to the third generation Higgs singlet superfield $\lambda_{3a\beta} \equiv \lambda_{a\beta}$ (where $\alpha, \beta, \gamma$ index only the first and second generations) directly contribute to neutralino and chargino mass terms for the inert Higgsino doublets. $\lambda_{a3\beta} \equiv f_{d\alpha\beta}$ and $\lambda_{a3\beta} \equiv f_{u\alpha\beta}$ directly contribute to neutralino mass terms involving an inert doublet Higgsino and singlino.

The 13 Higgs trilinear couplings mentioned thus far are the only couplings that obey the proposed $Z^H_2$ symmetry. This symmetry (under which all superfields other than the third generation $H_d$, $H_u$ and $S$ are odd) is proposed in order to prevent flavour changing neutral currents in the SM matter sector by eliminating non-diagonal flavour transitions. There is, however, no specific reason to suspect that it is respected by the $\lambda_{ijk}$ couplings or by superpotential couplings involving the exotic quarks. Indeed, if $Z^H_2$ is respected by the latter then the lightest exotic quark state(s) would be stable. This would presumably lead to a relic density of heavy exotic quark states inconsistent with observation. If $\lambda_{ijk}$ obeyed $Z^H_2$ exactly then, as we will see below, the neutralino mass matrix (and also the chargino mass matrix) would be decoupled into two independent systems and the lightest from each sector would be stable. We shall refer to the $Z^H_2$ breaking couplings involving two third generation superfields as $\lambda_{3a3} \equiv x_{da}$, $\lambda_{33a} \equiv x_{ua}$ and $\lambda_{a33} \equiv z_{a}$. The notation for the $\lambda_{ijk}$ couplings used in this paper are compiled in Tab. 2.

| $\lambda_{ijk}$ | $\lambda$ | $\lambda_{a\beta}$ | $f_{d\alpha\beta}$ | $f_{u\alpha\beta}$ | $x_{da}$ | $x_{ua}$ | $z_{a}$ |
|-----------------|--------|--------------------|-----------------|-------|-------|-------|-------|
| $ijk$           | 333    | $3\alpha\beta$    | $\alpha\beta$   | $\alpha\beta$ | $\alpha\beta$ | $3\alpha$ | $\alpha\beta$ |

Table 2: The notation for the $\lambda_{ijk}$ couplings.

The remaining 8 $Z^H_2$ breaking couplings $\lambda_{a\beta\gamma}$ are of less importance. As long as only the third generation Higgs doublets and singlet acquire VEVs then these couplings do not
appear in the neutralino or chargino mass matrices. Additionally, they only appear in Feynman rules that involve the inert Higgs scalars and we assume that these are given soft SUSY breaking masses that are heavy enough such that these particles do not contribute to any processes relevant for the current study.

As a final note, one could perhaps argue that these couplings should be arranged to help ensure that only the third generation singlet scalar radiatively acquires a VEV. However, as the contributions to the running of the singlet scalar square masses could be coming mostly from the heavy exotic quarks, there is little reason to impose any constraints from such considerations on the $\lambda_{ijk}$ couplings.

4 The Neutralino and Chargino Mass Matrices

In the MSSM there are four neutralino interaction states, the neutral wino, the bino and the two Higgsinos. In the USSM, two extra states are added, the singlino and the bino'. In the conventional USSM basis

$$\tilde{\chi}_\text{int}^0 = (\tilde{B} \ W^3 \ H_d^0 \ H_u^0 \ | \ \tilde{S} \ \tilde{B}')^T$$

and neglecting bino-bino' mixing (as justified in Ref. [9]) the USSM neutralino mass matrix is then

$$M^\text{n}_\text{USSM} = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta & 0 & 0 \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta & 0 & 0 \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu & -\mu s_\beta & g_1' v c_\beta Q^N_d \\ -m_Z s_W s_\beta & m_Z c_W s_\beta & 0 & -\mu & -\mu c_\beta & g_1' v s_\beta Q^N_u \\ 0 & 0 & -\mu s_\beta & -\mu c_\beta & 0 & g_1' s Q^N_s \\ 0 & 0 & g_1' v c_\beta Q^N_d & g_1' v s_\beta Q^N_u & g_1' s Q^N_s & M'_1 \end{pmatrix},$$

where $M_1$, $M_2$ and $M'_1$ are the soft gaugino masses, $\mu_s = \lambda v / \sqrt{2}$, $\langle H_d \rangle = v \cos \beta / \sqrt{2}$ and $\langle H_u \rangle = v \sin \beta / \sqrt{2}$. In the E_6 SSM this is extended. We take the full basis of neutralino interaction states to be

$$\tilde{\chi}_\text{int}^0 = (\tilde{B} \ W^3 \ H_d^0 \ H_u^0 \ | \ \tilde{S} \ \tilde{B}')^T$$

The first four states are the MSSM interaction states, the $\tilde{S}$ and $\tilde{B}'$ are the extra states added in the USSM and the final six states are the extra inert doublet Higgsinos and Higgs singlinos that come with the full E_6 SSM model. Under the assumption that only the third generation Higgs doublets and singlet acquire VEVs the full Majorana mass
matrix is then

\[
M_{E_{6}SSM}^{n} = \begin{pmatrix} M_{_{USSM}}^{n} & B_{2} & B_{1} \\ B_{2}^{T} & A_{22} & A_{21} \\ B_{1}^{T} & A_{12}^{T} & A_{11} \end{pmatrix}, \tag{10}
\]

where the sub-matrices involving the inert interaction states are given by

\[
A_{\alpha\beta} = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \lambda_{\alpha\beta} s & f_{u_{\alpha\beta}} v \sin \beta \\ \lambda_{\beta\alpha} s & 0 & f_{d_{\beta\alpha}} v \cos \beta \\ f_{u_{\alpha\beta}} v \sin \beta & f_{d_{\beta\alpha}} v \cos \beta & 0 \end{pmatrix}, \tag{11}
\]

and the \(Z_{2}^{H}\) breaking sub-matrices by

\[
B_{\alpha} = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x_{\alpha\alpha} s & z_{\alpha} v \sin \beta & x_{\alpha} v \cos \beta \\ x_{u_{\alpha\alpha}} s & z_{\alpha} v \cos \beta & 0 \\ x_{\alpha\alpha} v \sin \beta & x_{\alpha} v \cos \beta & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{12}
\]

Similarly we take our basis of chargino interaction states to be

\[
\tilde{\chi}_{\text{int}}^{\pm} = \begin{pmatrix} \tilde{\chi}_{\text{int}}^{+} \\ \tilde{\chi}_{\text{int}}^{-} \end{pmatrix},
\]

where

\[
\tilde{\chi}_{\text{int}}^{+} = \begin{pmatrix} \tilde{W}^{+} \\ \tilde{H}_{u}^{+} \\ \tilde{H}_{u_{2}}^{+} \\ \tilde{H}_{u_{1}}^{+} \end{pmatrix} \quad \text{and} \quad \tilde{\chi}_{\text{int}}^{-} = \begin{pmatrix} \tilde{W}^{-} \\ \tilde{H}_{d}^{-} \\ \tilde{H}_{d_{2}}^{-} \\ \tilde{H}_{d_{1}}^{-} \end{pmatrix}. \tag{13}
\]

The corresponding mass matrix is then

\[
M_{E_{6}SSM}^{c} = \begin{pmatrix} C^{T} \end{pmatrix},
\]

where

\[
C = \begin{pmatrix} M_{2} & \sqrt{2} m_{W} \sin \beta & 0 & 0 \\ \sqrt{2} m_{W} \cos \beta & \mu & \frac{1}{\sqrt{2}} x_{d_{2}} s & \frac{1}{\sqrt{2}} x_{d_{1}} s \\ 0 & \frac{1}{\sqrt{2}} x_{u_{2}} s & \frac{1}{\sqrt{2}} \lambda_{22} s & \frac{1}{\sqrt{2}} \lambda_{21} s \\ 0 & \frac{1}{\sqrt{2}} x_{u_{1}} s & \frac{1}{\sqrt{2}} \lambda_{12} s & \frac{1}{\sqrt{2}} \lambda_{11} s \end{pmatrix}. \tag{14}
\]

It is clear that a generic feature of the \(E_{6}\)SSM is that the LSP is usually (naturally) composed mainly of inert singlino and ends up being typically very light. One can see this
by inspecting the new sector blocks of the extended neutralino mass matrix in Eq. (10), such as $A_{11}$, and assuming a hierarchy of the form $\lambda_{\alpha\beta} s \gg f_{(u,d)\alpha\beta} v$. This is a natural assumption since we already require that $s \gg v$ in order to satisfy the current experimental limit on the $Z'$ mass of around 1 TeV [13], as discussed in Ref. [8].

For both the neutralinos and the charginos we see that if the $Z^H_2$ breaking couplings are exactly zero then the new part of the $E_6$ SSM mass matrix becomes decoupled from the USSM mass matrix. However, although approximate decoupling is expected, exact decoupling is not, and will therefore not be considered.

5 Analytical Discussion

According to standard cosmology, at some time in the past, before Big Bang Nucleosynthesis (BBN), the LSP would have decoupled from equilibrium with other species still in equilibrium with the photon. This decoupling from chemical equilibrium would have happened roughly when the particle’s inelastic interaction rate (maintaining chemical equilibrium) became less than the expansion rate of the universe $H = \dot{a}/a$, where $a$ is the scale factor of the universe. When such a chemical “freeze-out” occurs the number density of the frozen out species (the LSP here) typically remains much larger than it would have been if the species had remained in chemical equilibrium as the universe cooled. From this point onwards it is approximately just the number density at freeze-out that determines the relic density of the stable particle today. Generally the larger a stable relic’s annihilation and co-annihilation cross-sections would have been before freeze-out, the lower its relic density in the universe would be today [17].

In order for such a relic particle to be “cold” (as in “cold dark matter”) the freeze-out temperature must be much less than the mass of the particle, such that the particle was non-relativistic at freeze-out. The measured value used for the total present day cold dark matter relic density is $\Omega_{\text{CDM}}h^2 = 0.1099 \pm 0.0062$ [21]. If a theory predicts a greater relic density of dark matter than this then it is ruled out, assuming standard pre-BBN cosmology. A theory that predicts less dark matter cannot in the same way be ruled out, but if the theory is supposed to be the low energy effective theory of the complete theory that describes the universe then it should account for all of the observed dark matter. The LSP relic density calculation has already been widely studied in the MSSM [18] and especially in the constrained MSSM [19].

It will be useful to get some analytical understanding of the calculation of the relic abundance coming from the new neutralino/chargino physics of the $E_6$ SSM before looking
at the results of the full numerical simulation. To this end, in this section, we consider just one inert Higgs family consisting of two inert Higgs doublets and one inert Higgs singlet, which we shall label as the first generation. We shall assume that the $Z_2^H$ breaking couplings of the first (inert) Higgs generation to the third (conventional) Higgs generation are large enough to allow the heavier states of the USSM to decay into the LSP, formed mostly from inert states, but also small enough such that we can consider the inert Higgsinos to be approximately decoupled from the rest of the neutralino mass matrix for the purposes of obtaining an analytical estimate of the mass eigenstates. This amounts to considering the single block $A_{11}$ of the extended neutralino mass matrix in Eq. (10) and ignoring the rest. We emphasise that this is for the purposes of the simple analytical estimates in this section only and that in the next section we shall perform a full numerical analysis without any approximation.

5.1 Inert Neutralino Masses and Mixing for One Family

Within the first generation we use the basis

$$\tilde{\chi}_0^{\text{int}} = (\tilde{H}_{d1}^0 \tilde{H}_{u1}^0 \tilde{S}_1)^T$$

(15)

and the neutralino mass matrix is then, from Eq. (11),

$$A_{11} \equiv A = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \lambda's & f_u v \sin \beta \\ \lambda's & 0 & f_d v \cos \beta \\ f_u v \sin \beta & f_d v \cos \beta & 0 \end{pmatrix},$$

(16)

where $\lambda' = \lambda_{11} \equiv \lambda_{311}$, $f_d = f_{d11} \equiv \lambda_{131}$ and $f_u = f_{u11} \equiv \lambda_{113}$. As discussed earlier, it is natural to assume that $\lambda's \gg f v$ and this will lead to a light, mostly first-generation-singlino lightest neutralino.

Finding the mass eigenvalues of the matrix $A$ involves solving a reduced cubic equation. Doing an expansion in $f v/\lambda's$ the three neutralino masses from the first generation are

$$m_1 = \frac{1}{\sqrt{2}} \frac{f_d f_u v^2}{\lambda' s} \sin(2\beta) + \cdots,$$

(17)

$$m_2 = \frac{\lambda's}{\sqrt{2}} - \frac{m_1}{2} + \cdots,$$

(18)

$$m_3 = -\frac{\lambda's}{\sqrt{2}} - \frac{m_1}{2} + \cdots.$$  

(19)

The lightest state is mostly singlino (as we will confirm below) and the two heavier states are nearly mass degenerate, split by the LSP mass. At $\beta = 0$ or $\pi/2$ the lightest
neutralino becomes massless. This is when only one of the third generation conventional Higgs doublets has a VEV. The LSP, even if very weakly interacting, must be heavier than a few MeV so that it would not contribute to the expansion rate prior to nucleosynthesis, changing nuclear abundances [7].

We shall define the neutralino mixing matrix \( N \) by

\[
N_i^aM^{ab}N_j^b = m_i \delta_{ij} \quad \text{no sum on } i. 
\]  

(20)

The lightest state is then made up of the following superposition of interaction states:

\[
\tilde{\chi}_1^0 = N_1^1 \tilde{H}_d^0 + N_1^2 \tilde{H}_u^0 + N_1^3 \tilde{S}_1. 
\]  

(21)

Again expanding in \( f v/\lambda' \)s we have

\[
N_1 = \begin{pmatrix}
-\frac{f_u^w}{\lambda's} \cos \beta + \cdots \\
-\frac{f_u^w}{\lambda's} \sin \beta + \cdots \\
1 - \frac{1}{2} \left( \frac{v}{\lambda'} \right)^2 \left[ f_d^2 \cos^2(\beta) + f_u^2 \sin^2(\beta) \right] + \cdots
\end{pmatrix}, 
\]  

(22)

confirming that the LSP is mostly singlino in this limit. The other eigenvectors, which determine the composition of neutralinos 2 and 3, are

\[
N_i = \sqrt{\frac{1}{a_i^2 + b_i^2 + \cdots}} \begin{pmatrix}
a_i \\
b_i \\
1
\end{pmatrix}, 
\]  

(23)

where

\[
-b_2 = a_2 = \frac{\lambda's}{v} [f_d \cos \beta - f_u \sin \beta]^{-1} + \cdots, 
\]  

(24)

\[
b_3 = a_3 = \frac{\lambda's}{v} [f_d \cos \beta + f_u \sin \beta]^{-1} + \cdots. 
\]  

(25)

Note that \( a, b \gg 1 \) and that \( a_2 \) and \( b_2 \) flip sign at \( f_d \cos \beta = f_u \sin \beta \) whereas \( a_3 \) and \( b_3 \) are always positive. Very approximately these eigenvectors are then

\[
N_2 = \frac{1}{\sqrt{2}} \begin{pmatrix}
-1 \\
1 \\
0
\end{pmatrix} \text{sign}(f_u s_\beta - f_d c_\beta), 
\]  

(26)

\[
N_3 = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 \\
1 \\
0
\end{pmatrix}. 
\]  

(27)
Under the assumptions of this section the lightest chargino is simply the first generation charged Higgsino with a mass $m_c = \lambda' s / \sqrt{2}$.

### 5.2 Annihilation Channels

From Eq. (17) it is seen that the LSP mass $m_1$ is proportional to $v^2 / s$ and so is naturally small since $v \ll s$. To understand this, recall that $Z-Z'$ mixing leads to two mass eigenstates, $Z_2 \sim Z'$ and $Z_1 \sim Z$, and limits on $Z-Z'$ mixing and on the $Z_2$ mass place lower limits on $s$, with $v \ll s$ being always satisfied. For example, when $s = 3000$ GeV the $Z_2$ mass is about 1100 GeV and $v^2 / s \approx 20$ GeV. The LSP mass further decreases as $s$ becomes larger in the considered limit. In practice, it is quite difficult to arrange the LSP mass to exceed about 100 GeV.

![Figure 1: s-channel LSP annihilation diagrams.](image)

In view of the above discussion the LSP is expected to be relatively light, and so we begin by looking at $s$-channel annihilation, which can result in lighter mass final states. The most important diagrams are shown in Fig. 1 and it will turn out that the most important of these annihilations have a $Z$-boson in the $s$-channel (or strictly speaking $Z_1 \sim Z$). The $\tilde{\chi}_1^0 \tilde{\chi}_1^0 Z$ gauge coupling in this diagram is suppressed by a factor of

$$\frac{1}{2} \left( \frac{v}{\lambda'} \right)^2 \left[ f_u^2 \sin^2(\beta) - f_d^2 \cos^2(\beta) \right] + \cdots$$

under the assumptions of this section, since the LSP only couples through its small Higgsino components. This coupling vanishes completely at $f_d \cos \beta = f_u \sin \beta$, which is when the LSP contains a completely symmetric combination of $\tilde{H}_{d1}^0$ and $\tilde{H}_{u1}^0$. In the MSSM a Higgsino-like LSP is typically such a symmetric combination of up- and down-type Higgsino and therefore does not couple very strongly to the $Z$-boson. In this model, however, the LSP is unlikely to have very similar admixtures of $\tilde{H}_{d1}^0$ and $\tilde{H}_{u1}^0$.

Full gauge coupling strength $s$-channel $Z$-boson annihilations tend to leave a relic density that is too low to account for the observed dark matter, but in this model the coupling of the mostly singlino LSP to the $Z$-boson is typically suppressed, as it only couples through its doublet Higgsino admixture, leading to an increased relic density if
this is the dominant annihilation channel. As $\lambda'$ decreases, the proportion of the LSP that is made up of inert doublet Higgsino, rather than inert singlino, increases. This can be seen in Eq. (22). This then increases the strength of the overall $\tilde{\chi}_1^0 \tilde{\chi}_1^0 Z$ coupling. The inclusive cross-section for s-channel annihilation through a $Z$-boson is therefore highly dependent on $\lambda'$, which affects both the coupling and the LSP mass $m_1$. The effect of independently increasing the coupling is always to increase the cross-section, but the effect of independently increasing the LSP mass can be to either increase or decrease the cross-section, depending on which side of the $Z$-boson resonance it is on. In the considered limit both the mass and coupling are proportional to $1/(\lambda')^2$, and the annihilation cross-section is given by,

$$\sigma(\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow Z* \rightarrow \text{anything}) \propto \left(\frac{1}{\lambda'}\right)^4 \left(\frac{1}{m_Z^2 - (2m_1)^2}\right)^2 \left(f_u^2 s_{\beta}^2 - f_d^2 c_{\beta}^2\right)^2. \quad (28)$$

The s-channel annihilation through the lightest Higgs boson will also become important if the LSPs are on resonance in the relic density calculation.

The most important of the potential t-channel processes are shown in Fig. 2. In practice these channels will not play a significant role compared to the s-channel annihilations considered previously, but we discuss them for completeness. The t-channel particle for these processes is one of the neutralinos or the chargino of the first generation (for producing neutral Higgs scalars / $Z$-bosons or charged Higgs scalars / $W$-bosons respectively). In the first diagram, t-channel annihilation to conventional third generation Higgs scalars, the couplings are just $f$ couplings of the first generation and appropriate mixing matrix elements. With the chargino or with neutralino 2 or 3 in the t-channel the diagram is approximately inert singlinos annihilating with an inert doublet Higgsino in the t-channel and the couplings are approximately just $f_d$ and $f_u$ for producing $H_d$ and $H_u$ interaction states respectively. The LSP mass is smaller than the other masses by a factor of order $v^2/s^2$. With another LSP in the t-channel the first diagram therefore receives an enhancement of order $s^2/v^2$ for the t-channel propagator at low momentum, but has a suppression of order $v^2/s^2$ in the couplings due to the LSP only containing doublet type first generation Higgsinos with amplitudes of order $v/s$. 

![Figure 2: t-channel LSP annihilation diagrams.](image-url)
The second diagram in Fig. 2 represents annihilation to massive gauge bosons. To very good approximation these bosons only couple to weak isospin doublets and not to SM singlets (since $Z-Z'$ mixing must be small). These diagrams are therefore suppressed by order $v^2/s^2$ in the couplings even with a chargino or with neutralino 2 or 3 in the t-channel. On top of this suppression these diagrams also receive an additional suppression of order $v^2/s^2$ in the couplings, but an enhancement of order $s^2/v^2$ in the propagator when the t-channel contains the LSP. Although this second type of diagram is suppressed relative to the first (assuming $v^2/s^2 \ll f$) it has a greater chance of being kinematically allowed. As previously stated, inert scalar Higgs-bosons are assumed heavy and annihilation to these particles is not considered.

6 Numerical Analysis

We now turn to the full model, in which the LSP is determined from the neutralino mass matrix in Eq. (10) where there are two copies of the family considered in the previous section as well as 6 unknown mixing parameters between the two families. In general, after rotation to the mass eigenstates, we expect that two states are much lighter than the rest, both inert-singlino-like in the $\lambda's \gg fv$ limit. In this section we use numerical methods to predict the relic density. We first diagonalize the neutralino, chargino and Higgs scalar mass matrices numerically. The 1-loop USSM Higgs mass corrections from top and stop loops are implemented from Ref. [20]. Corrections from exotic quark and squark loops are not included in our analysis, as these have been shown to be small [7], and CP violation is not considered. Having done this MicrOMEGAs 2.2 [23] is then used to numerically compute the present day relic density, including the relevant (co-)annihilation channel cross-sections and the LSP freeze-out temperature. MicrOMEGAs achieves this by calculating all of the relevant tree-level Feynman diagrams using CalcHEP. The CalcHEP model files for the considered model are generated using LanHEP [24]. The MicrOMEGAs relic density calculation assumes standard cosmology in which the LSP was in equilibrium with the photon at some time in the past.

6.1 The Parameter Space of the Model

Motivated by the running of the gauge couplings from the GUT scale, we assume that the GUT normalised couplings of the two $U(1)$ gauge groups, $U(1)_Y$ and $U(1)_N$, are

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4An exception to this is in the large $M'_1$ limit in which the LSP could originate from the lower block of the USSM neutralino mass matrix in Eq. (8) due to a mini see-saw mechanism as discussed in Ref. [9].
equal and that the mixing between the two groups is negligible, giving \( g_1' \approx 0.46 \). The free parameters are then the trilinear Higgs couplings \( \lambda_{ijk} \), the singlet VEV \( s \), \( \tan \beta \), the soft \( \lambda_{333} \) coupling \( A_\lambda \) and the soft gaugino masses. It will turn out that the soft gaugino masses usually have little effect on the dark matter physics. One can see this by observing the neutralino mass matrix, Eq. (10), where the USSM terms coming from the soft gaugino masses do not directly mix with terms from the new \( E_6 \) SSM inert sector. The scalar Higgs doublet and singlet soft SUSY breaking masses are determined from the scalar potential minimalisation conditions given \( s, v, \tan \beta \) and \( A_\lambda \). The regular squark and slepton sectors as well as the potential issue of mixing between the two \( U(1) \) gauge groups are the same as in the USSM [9].

In the following analysis we shall choose \( s = 3000 \) GeV and \( \mu = 400 \) GeV which gives \( \lambda = 2\sqrt{2}/15 \approx 0.19 \) and makes the \( Z_2 \) (i.e. \( Z' \)) mass about 1100 GeV. Although much of the physics is highly dependent on \( s \), this specific choice of \( s \) does not limit the generality of the results obtained. This is explained below. We also choose \( M_1 = M'_1 = M_2/2 = 250 \) GeV. These relations between the soft gaugino masses are motivated by their running from high scale, but the value is not. In this analysis the squarks and sleptons will not play a significant role in the calculation of dark matter relic abundance since the LSP will always be much lighter. We choose equal soft sfermion masses to be \( M_S = 800 \) GeV and the stop mixing parameter, \( X_t = A_t - \mu \cot(\beta) \), to be \( X_t = \sqrt{6} M_S \) as in Ref. [7]. This results in a lightest CP-even Higgs mass in excess of 114 GeV for all parameter space considered below. The soft \( \lambda \) coupling \( A_\lambda \) is set by choosing the pseudo-scalar Higgs mass \( m_A \). We choose \( m_A = 500 \) GeV.

We initially assume the \( Z_2^H \) breaking \( \lambda_{ijk} \) couplings to be small (0.01) for the following analysis. The main properties of the physics can then be seen by varying three parameters \( \lambda' = \lambda_{22} = \lambda_{11}, f = f_{d22} = f_{u22} = f_{d11} = f_{u11} \) and \( \tan \beta \). The first and second generation mixing couplings are set to \( \lambda_{21,12} = \epsilon \lambda' \) and \( f_{(d,u)(21,12)} = \epsilon f \). Assuming this parameter choice the sub-matrices of the neutralino mass matrix Eq. (11) become

\[
A_{22} = A_{11} = -\frac{1}{\sqrt{2}} \begin{pmatrix}
0 & \lambda' s & f v \sin \beta \\
\lambda' s & 0 & f v \cos \beta \\
fv \sin \beta & f v \cos \beta & 0
\end{pmatrix}
\]  
\[
A_{21} = \epsilon A_{22}.
\]  

This simple parametrisation is sufficient for illustrating the generic properties of the physics. Deviations from this parametrisation are discussed afterwards.

Note that the analytical results of the previous section provide an essential context in which to understand the numerical results of this section. According to the above
parametrization, the two generations are approximately degenerate and the mixing terms are not too large. In this case the LSP and the second lightest neutralino will each contain approximately equal contributions from each generation.

Finally, it is worth remarking that, assuming the above parametrisation, the effect on the neutralino and chargino inert sectors of changing $s$ is simply equivalent to that of changing $\lambda'$ (although the $Z'$ mass will depend on $s$). This means that the following results are also applicable for other experimentally consistent values of $s$, scaled by $\lambda'$.

### 6.2 Neutralino and Chargino Spectra

Fig. 3 shows how the spectrum of chargino masses varies with $\lambda'$. Although the plot is for $\tan \beta = 1.5$, as one can see from the chargino mass matrix, Eq. (14), the inert sector has no dependence on $\tan \beta$, with the mass terms just being proportional to the singlet VEV. The almost constant masses are those mass eigenvalues coming mostly from the USSM sector, the third generation charged Higgsino and wino. The charginos coming mostly from the inert sector vary with $\lambda'$ as expected and drop below the 94 GeV experimental lower limit at some value of $\lambda'$, depending on the value of $s$. The effect of the $\epsilon = 0.1$ mixing between generations can be seen in the splitting between the two inert sector
charginos. Where lines cross in Fig. 3 the chargino masses are of opposite sign. When chargino mass lines of the same sign approach each other, they veer away from each other at the would-be crossing point due to the effect of interference.

Fig. 4 shows how the spectrum of neutralino masses varies with $\lambda'$. The inert neutralino spectrum is dependent on $\tan \beta$, but each of the qualitative features can be understood. We see the two light neutralino states that become heavier as $\lambda'$ decreases from unity until the approximation $\lambda's \gg f v$ breaks down. At this point $f v \sin \beta$ begins to dominate and the LSP mass decreases with decreasing $\lambda'$ as the dominance of $f v \sin \beta$ becomes greater. In this low $\lambda'$ region the LSP is no longer mostly inert singlino, but mostly inert up-type Higgsino. The six almost unvarying neutralino masses are those mostly from the USSM sector, which is not mixing very much with the new sector in this figure. We have already seen that the inert sector chargino masses continue to be set by $\lambda'$ as we go down into the low $\lambda'$ region, resulting in light charginos in this region. By contrast, the four inert sector neutralinos begin to be governed by the $f v$ terms rather than the $\lambda'$s terms in the low $\lambda'$ region and therefore approach a constant value in this region.

As in the case of the charginos, the effect of the $\epsilon = 0.1$ mixing can be seen in the splitting between the two light neutralinos and the four heavier inert neutralinos which are both split by this mixing and further split by the light neutralino mass as predicted.

Figure 4: Neutralino masses (magnitude only) against $\lambda'$ with $f = 1$, $\epsilon = 0.1$, $\tan \beta = 1.5$, $s = 3000$ GeV and $Z^H_2$ breaking $\lambda_{ijk}$ couplings set to 0.01.
Figure 5: Component structure of the LSP in terms of the inert interaction states against \( \lambda' \) with \( f = 1, \epsilon = 0.1, \tan \beta = 1.5, s = 3000 \text{ GeV} \) and \( Z_2^H \) breaking \( \lambda_{ijk} \) couplings set to 0.01.

in the previous section.

Fig. 5 shows how the make-up of the LSP in terms of the inert interaction states varies with \( \lambda' \). The behaviour in the \( \lambda' \)'s \( \gg f v \) limit is as predicted in Eq. (22). We also see how the dominant component of the LSP changes from inert singlino to inert up-type Higgsino in the low \( \lambda' \) region.

6.3 Dark Matter Relic Density Predictions

Using the parametrization in Eqs. (29,30) we use MicroMEGAs 2.2 to numerically compute the present day relic density. Fig. 6 shows a contour plot of the LSP mass and relic density \( \Omega_\chi h^2 \) regions in the \((\lambda', \tan \beta)\)-plane, with \( s = 3000 \text{ GeV}, \epsilon = 0.1 \) and \( f = 1 \). We focus on small values of \( \lambda' < 0.4 \) since for large \( \lambda' \) the LSP would be very light state, predominantly inert singlino, which would not annihilate very efficiently through any channel, leading to a too high relic density \( \Omega_\chi h^2 \gtrsim \Omega_{\text{CDM}} h^2 \) (such regions are shaded dark green). As \( \lambda' \) is decreased below 0.3 the LSP mass increases and approaches about half of the Z-boson mass and there is a region where the prediction for \( \Omega_\chi h^2 \) is consistent with the measured 1-sigma range of \( \Omega_{\text{CDM}} h^2 \) (such regions are shaded red). When the LSP mass is around 40 GeV it contains enough inert doublet Higgsino component such that s-channel Z-boson...
annihilation becomes strong enough to account for the observed relic density. As the LSP mass is increased further from 40 GeV and approaches 45 GeV, the annihilations before freeze-out become on resonance with a $Z$-boson in the s-channel and the predicted relic density becomes too low (such regions are shaded light green).

However the regions where the LSP mass is less than half of the $Z$-boson mass are excluded by LEP limits on the $Z$-boson width. The point is that the same couplings which lead to successful relic density, via annihilation through an s-channel $Z$-boson, will also violate the LEP collider limits on the $Z$-pole arising from $Z$-boson decay two LSPs. Such a $Z$-boson decay channel would contribute to the invisible $Z$ width $^5$. The measurement of the invisible $Z$ width at LEP is used to give strong bounds on the number of light neutrino species [22]. The PDG average for the effective number of light neutrinos as inferred from the invisible $Z$ width is $2.92 \pm 0.05$ [22]. Because of the coupling suppression of the LSP due to its inert singlino component amplitudes, helicity suppression and also significant phase-space suppression, the branching ratio to two LSPs would have a contribution to

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$^5$By contrast $Z$ decays involving the second lightest neutralino would contribute to the total width, because the second neutralino would decay to the LSP before reaching the detector.
the invisible width significantly less than that of a neutrino, but still large enough to violate the LEP limit. Note that in the MSSM this limit does not arise since either the LSP is bino-like and so does not couple to the $Z$ or is Higgsino or Wino like in which case it would have accompanying almost degenerate charginos and therefore must have a mass greater than about 100 GeV in any case. Here we can have an inert Higgsino/singlino LSP with a mass lower than half of the $Z$-boson mass while still having experimentally consistent inert-doublet-Higgsino-like charginos. The regions in Fig. 6 where the LSP is lower than half of the $Z$-boson mass, namely to the right of the hatched line, are therefore ruled out by the $Z$ decay width measurements at LEP. Fortunately there are successful regions indicated in red to the left of the hatched line in Fig. 6, where the LSP mass is greater than 45 GeV thereby avoiding the LEP limit, as we discuss below.

We note at this point that the requirement that the LSP mass exceeds 45 GeV implies low $\tan \beta$, and this is the reason for the restricted range of $\tan \beta$ in Fig. 6. This can be seen from Eq. (17) where we found that the LSP mass should be approximately proportional to $\sin 2\beta$, i.e. to the product of the two doublet Higgs VEVs, which is maximized for $\sin 2\beta = 1$ corresponding to $\tan \beta = 1$. In the $E_6$SSM an experimentally acceptable
lightest Higgs mass can be achieved even with tan $\beta$ as low as about 1.2 \[7\], so having low tan $\beta$ is not a problem in such models.

Decreasing $\lambda'$ further results in LSP masses above 45 GeV, and to the left of the hatched line in Fig. 6, other successful relic density regions (shaded in red) appear. These regions are punctuated by the light Higgs resonance, leading to the interesting double loop shape of the successful red regions to the left of the hatched line in Fig. 6. In these regions the LSP can have a mass significantly larger than half of the $Z$ mass, moving far enough off the Higgs and $Z$ resonances that annihilation is weakened just enough to give the correct relic density.

However another effect comes into play as $\lambda'$ decreases, namely the composition of the LSP changes from being singlino dominated to being Higgsino dominated, the cross-over point being close to $\lambda' = 0.07$, according to Fig. 5. Within a successful region one would normally expect the relic density to increase as the LSP mass goes up (corresponding to decreasing tan $\beta$ or $\lambda'$) because annihilation moves further away from the particular resonance (either Higgs or $Z$). However, for lower $\lambda'$ the cross-section actually increases with decreasing $\lambda'$, leading to a lower relic density, because the inert doublet Higgsino components in the LSP rapidly grow, as can be seen in Fig. 5. This implies that for $\lambda' < 0.07$, when the LSP is largely inert doublet Higgsino, annihilation is too strong leading to the relic density being too low (as indicated by the light green shading in Fig. 6. Note also that here the analytic approximations based on $\lambda's \gg f v$ break down, leading to the turning over of the LSP mass contours for $\lambda' < 0.08$. The effects of the t-channel $W$ and then $Z$ pair production channels can also be seen as they each become relevant.

According to the above discussion the successful regions to the left of the hatched line in Fig. 6 are not ruled out by $Z$ decay data, as the LSP is sufficiently heavy. Furthermore, for the entire successful region, the lightest chargino is heavy enough to be consistent with experiment, as can be seen on Fig. 3. This result will be recreated for all high enough values of $s$. For larger values of $s$ the successful regions and corresponding inert chargino masses are shifted down by the same amount in $\lambda'$.

When $\lambda's \gg f v$ lowering $f$ results in a lower LSP mass, as in Eq. (17). It also extends the range of $\lambda'$ in which this approximation is valid, i.e. it moves the boundary of the previously discussed low $\lambda'$ region to be further down in $\lambda'$. Fig. 7 shows the LSP mass and predicted present day relic density for different values of $\lambda'$ and $f$ with $\epsilon = 0.1$ and tan $\beta = 1.5$. The shifting of the successful region, where the LSP mass is above $m_Z/2$, down in $\lambda'$ at lower values of $f$ is apparent. At lower values of tan $\beta$ this successful region
extends further down in $f$. It should be noted that in order to predict the correct dark matter relic density, $\lambda'$ should be smaller than $f$ and that this disparity gets greater if $s$ is increased. Increasing $s$ effectively just shifts all of the features on Fig. 6 and Fig. 7 to the left.

### 6.4 Deviations from the Considered Parametrisation

Breaking the relation $f_u(22,11) = f_d(22,11)$ can have similar effects to those of changing $\tan \beta$. However, because these parameters cannot be too high (in order to be consistent with running from the grand unification scale) and because lowering them to much less than unity makes the LSP too light, $\tan \beta$ can be varied much more freely than the $f_u/f_d$ ratio.

The effect of increasing the generation mixing parameter $\epsilon$ is to increase the various mass splittings between similar inert mass eigenstates. Increasing the mixing between the first and second generations thus results in a lighter LSP, shrinking the successful region, and a lighter lightest chargino, potentially inconsistent with current chargino non-observation.

Increasing the $Z_2^H$ breaking $\lambda_{ijk}$ couplings from 0.01, it is possible to give the LSP significant components of the conventional, non-inert doublet Higgsinos and third generation singlino. However, turning up these parameters does not allow for the result of a very light LSP, usually singlino dominated, to be avoided, simply because of the non-diagonal structure of the non-gaugino part of the neutralino mass matrix. Turning up these parameters would, however, mean that the LSP could have significant couplings to regular quarks and leptons.

Other parameters only change the neutralinos and charginos mostly from the USSM sector. As long as the LSP is still mostly from the inert sector, as considered here (gaugino masses cannot be too light or else the LSP can become bino/bino' dominated), these parameters are effectively free. Squark and slepton parameters do not affect the dark matter physics of the considered model. Top and stop loops can have a significant effect on the lightest Higgs mass, but as long as this mass is experimentally allowed then these parameters are also effectively free.

### 7 Summary and Conclusions

In this paper we have studied neutralino dark matter arising from supersymmetric models with extra inert Higgsinos and singlinos. As an example, we have considered the extended neutralino sector of the $E_6$ SSM, which predicts three families of Higgs doublet pairs, plus
three singlets, plus a $Z'$, together with their fermionic superpartners which include two families of inert Higgsinos and singlinos. In our study we have considered neutralino dark matter arising from such a model both analytically and numerically, using MicrOMEGAs.

We have found that the results for the relic abundance in the $E_6$SSM are radically different from those for both the MSSM and the USSM. This is because the two families of inert Higgsinos and singlinos predicted by the $E_6$SSM provide an almost decoupled neutralino sector with a naturally light LSP which can account for the cold dark matter relic abundance independently of the rest of the model. Although the $E_6$SSM has two inert families, the presence of the second inert family is not crucial for achieving successful dark matter relic abundance.

In the successful regions where the observed dark matter relic density is reproduced the neutralino mass spectrum is well described by the analytical results of Section 5. In this region the LSP is mostly inert singlino and has a mass approximately proportional to $v^2/s$, as in Eq (17), and, as $\lambda's$ is decreased, the LSP becomes heavier and also less inert singlino dominated, picking up significant inert doublet Higgsino contributions. To avoid conflict with high precision LEP data on the $Z$-pole, the LSP, which necessarily must couple significantly to the $Z$-boson in order to achieve a successful relic abundance, should have a mass which exceeds half the $Z$-boson mass. Since the LSP mass in Eq. (17) is proportional to $f_d f_u \sin(2\beta)$, we find that regions of parameter space in which the dark matter relic density prediction is consistent with observation require low values of $\tan\beta$, less than about 2. Depending on the value of the singlet VEV $s$, the $f_{u,d}$ trilinear Higgs coupling parameters should also be reasonably large compared to the $\lambda_{\alpha\beta}$ ones. In general it is difficult for the true neutralino LSP to be heavier than about 100 GeV. In the successful regions we find the lightest chargino mass could be as low as the experimental lower limit of 94 GeV, although it could also be as high as about 300 GeV.

One of the main messages of this paper is that neutralino dark matter could arise from an almost decoupled sector of inert Higgsinos and singlinos, and if it does then the parameter space of the rest of the model is completely opened up. For example if such a model is regarded as an extension of the MSSM, then the lightest MSSM-like SUSY particle is not even required to be a neutralino, and could even be a sfermion which would be able to decay into the true LSP coming from the almost decoupled inert Higgsino/singlino sector. This is because the mostly inert-Higgsino/singlino LSP would have admixtures of MSSM neutralino states. The size of these components are set by $Z_2^H$ breaking $\lambda_{ijk}$ couplings and need not be extremely small.

Finally we remark that, although we have focussed on the $E_6$SSM, similar results
should apply to any singlet-extended SUSY model with an almost decoupled inert Higgsino
sector with a trilinear Higgs coupling as in Eq. (6).

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