Theory of Spin Polarized Tunneling in Superconducting Sr$_2$RuO$_4$

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A theory of tunneling conductance in ferromagnetic-metal/insulator/triplet-superconductor junctions is presented for unitary and non-unitary spin triplet pairing states which are promising candidates for the superconducting pairing symmetry of Sr$_2$RuO$_4$. As the magnitude of the exchange interaction in the ferromagnetic metal is increased, the conductance for the unitary pairing state below the energy gap is reduced in contrast to that for the non-unitary pairing state. This is due to the fact that the retro-reflectivity in the Andreev reflection for unitary pairing state is broken due to the influence from the exchange interaction.

KEYWORDS: triplet superconductor, unitary pair potential, non-unitary pair potential, ferromagnetic metal, Andreev reflection

Andreev reflection, which occurs at the interfaces between normal metals and superconductors, is one of the most important elemental processes in electron transport through the superconducting junction. It is an interesting problem to clarify the expected for a tunneling effect in ferromagnet/insulator/superconductor (F/I/S) junctions, since the retro-reflectivity of the Andreev reflection is broken due to an exchange interaction in the ferromagnets. For s-wave superconductors, these properties are well understood and new aspects of the Andreev reflection have been revealed. On the other hand, stimulated by the establishment of d-wave symmetry in high-$T_C$ superconductors, theories of the tunneling conductance of anisotropic superconductor junctions have been developed, which fully take into account the anisotropy of the pair potential. In anisotropic superconductors, the change in sign of the pair potential induces remarkable effects, i.e., a zero-bias conductance peak (ZBCP) in tunneling experiments of high $T_C$ superconductors, due to the formation of zero-energy states (ZES). Recently, previous theories have been extended to F/I/S junctions with d-wave superconductors and the influences of the exchange interaction on transport properties have been clarified. However, these theories treat only spin singlet superconductors.

The recent discovery of superconductivity in Sr$_2$RuO$_4$ has attracted much theoretical and experimental attention because this material is the first example of a noncuprate layered perovskite...
superconductor. Since this compound is isostructural to cuprate superconductors, the electronic properties in both the normal and the superconducting state are highly anisotropic. Several experiments indicate the existence of a large residual density of states of quasiparticles. Furthermore, the importance of ferromagnetic spin fluctuations in this material has been suggested. These results strongly imply that the pairing states of Sr$_2$RuO$_4$ belong to two-dimensional triplet superconducting states, i.e. $E_u$ symmetry. To clarify the phase coherence of this material peculiar to anisotropic pairing, theories of tunneling conductance and Josephson effect in this material have been presented. However, no theory has been presented for transport properties of ferromagnet/insulator/triplet superconductor (F/I/T S) junctions where remarkable differences from those of singlet superconductor cases are expected.

In this paper, a formulation of the spin-polarized tunneling conductance in F/I/T S junctions is presented by extending the theory for $d$-wave superconductors. Although the superconducting state of Sr$_2$RuO$_4$ has yet to be fully identified, we will choose two types of triplet $p$-wave pair potentials: the unitary and the non-unitary pairing states with $E_u$ symmetry. For the unitary pairing state, an incoming electron and the Andreev reflected hole have antiparallel spins from each other, as in the case of the singlet pairing state. In the unitary case, the conductance obtained below the energy gap of the superconductor is reduced drastically due to the exchange interaction. On the other hand, since the Andreev reflection in the non-unitary pairing state conserves spin, the influences of the exchange interaction are not so serious. Thus, the present results serve as a more useful guide for the experimental identification of the pairing state of Sr$_2$RuO$_4$.

For the calculation, a two-dimensional F/I/T S junction with semi-infinite double-layered structures in the clean limit is assumed. A flat interface is perpendicular to the $x$-axis and is located at $x=0$. The insulator is modeled as a delta-functional form $H\delta(x)$, where $\delta(x)$ and $H$ are the delta-function and its amplitude, respectively. The Fermi energy $E_F$ and the effective mass $m$ are assumed to be equal both in the ferromagnet and in the superconductor, for simplicity. As a model of the ferromagnetic metal, we apply the Stoner model using the exchange potential $U(x) = U\Theta(-x)$, where $\Theta(x)$ is the Heaviside step function. The magnitude of Fermi momentum in the ferromagnet for up [down] spin is denoted as $k_{F,\uparrow} = \sqrt{\frac{2m}{\hbar^2}(E_F + U)}$ [ $k_{F,\downarrow} = \sqrt{\frac{2m}{\hbar^2}(E_F - U)}$]. For simplicity, we assume the spatially constant pair potentials and neglect the effects of spin-orbit scattering. The wave functions $\Psi(x)$ are obtained by solving the Bogoliubov-de Gennes (BdG) equation according to the quasiclassical approximations. In this approximation, the effective pair potentials for quasiparticles in the superconductor are given by $\Delta(\theta_S)\Theta(x)$, where $\theta_S$ denotes the direction of the motions of quasiparticles which is measured from the normal to the interface.
The pair potential matrix is expressed as

\[
\Delta(\theta_S) = \begin{pmatrix}
\Delta_{\uparrow\uparrow}(\theta_S) & \Delta_{\uparrow\downarrow}(\theta_S) \\
\Delta_{\downarrow\uparrow}(\theta_S) & \Delta_{\downarrow\downarrow}(\theta_S)
\end{pmatrix}.
\]

(1)

In the calculation, we consider four kinds of pair potentials with \(E_u\) symmetry. In the following, we will call \(E_u(1)\) state for time reversal symmetry state and \(E_u(2)\) state for broken time reversal symmetry state, respectively. For the unitary pairing state, these matrix elements are given by \(\Delta_{\uparrow\downarrow}(\theta_S) = \Delta_{\downarrow\uparrow}(\theta_S) = \Delta_0(\sin\theta_S + \cos\theta_S)\), \(\Delta_{\uparrow\uparrow}(\theta_S) = \Delta_{\downarrow\downarrow}(\theta_S) = 0\) for \(E_u(1)\) state and \(\Delta_{\uparrow\downarrow}(\theta_S) = \Delta_{\downarrow\uparrow}(\theta_S) = \Delta_0 \exp(i\theta_S)\), \(\Delta_{\uparrow\uparrow}(\theta_S) = \Delta_{\downarrow\downarrow}(\theta_S) = 0\) for \(E_u(2)\) state, respectively. For non-unitary pairing state, these are \(\Delta_{\uparrow\downarrow}(\theta_S) = \Delta_0(\sin\theta_S + \cos\theta_S)\), \(\Delta_{\downarrow\uparrow}(\theta_S) = \Delta_{\downarrow\downarrow}(\theta_S) = 0\) for \(E_u(1)\) state and \(\Delta_{\uparrow\downarrow}(\theta_S) = \Delta_0 \exp(i\theta_S)\), \(\Delta_{\downarrow\uparrow}(\theta_S) = \Delta_{\downarrow\downarrow}(\theta_S) = 0\) for \(E_u(2)\) state, respectively, following the discussions by Sigrist and Zhitomirsky and Machida et al.

There are four scattering processes for an electron injection from the ferromagnet with up spin and at angle \(\theta_F\) with respect to the interface normal, as shown in Figure 1. These are Andreev reflection (AR) as a hole, normal reflection (NR) as an electron, transmission as an electron like quasiparticle (ELQ), and transmission as a hole like quasiparticle (HLQ). The transmitted ELQ and HLQ feel different effective pair potentials \(\Delta_{ss'}(\theta_{S+})\) and \(\Delta_{ss'}(\theta_{S-})\), with \(\theta_{S+} = \theta_S\) and \(\theta_{S-} = \pi - \theta_S\). The wave vectors of ELQ and HLQ are approximated by \(k_S = |k_S| \approx \sqrt{\frac{2mE_F}{\hbar^2}}\) in the framework of the quasiclassical approximation. Since the translational symmetry holds for the \(y\)-axis direction, the momenta parallel to the interface are conserved at the interface, \(k_F \sin\theta_F = k_S \sin\theta_S\). In the case of a unitary state, the retro reflectivity of the Andreev reflection is broken due to the difference in the exchange interaction felt by a hole like quasiparticle. Consequently, \(\theta_F\) is not equal to \(\theta_S'\).

The wave function \(\Psi(x)\) in the ferromagnet region for the unitary state is described by

\[
\Psi(x) = \begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} e^{ik_{F,\uparrow}x} + a_\uparrow \begin{pmatrix}
0 \\
0 \\
1 \\
0
\end{pmatrix} e^{ik_{F,\downarrow}x} + b_\uparrow \begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} e^{-ik'_{F,\uparrow}x}.
\]

(2)

That for the non-unitary state is given by

\[
\Psi(x) = \begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} e^{ik_{F,\uparrow}x} + a_\uparrow \begin{pmatrix}
0 \\
0 \\
1 \\
0
\end{pmatrix} e^{ik_{F,\downarrow}x} + b_\uparrow \begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} e^{-ik'_{F,\uparrow}x}.
\]

(3)

The reflection probabilities of AR(\(a_{\uparrow}(\downarrow)\)) and NR(\(b_{\uparrow}(\downarrow)\)) are determined by solving the BdG equations under the boundary conditions. The reflection probabilities for down spin injection are also obtained...
in a similar way. The normalized tunneling conductance is expressed as

\[ \sigma_T(eV) = \frac{\int_{-\pi/2}^{\pi/2} d\theta_S \cos \theta_S (\sigma_{S,\uparrow} + \sigma_{S,\downarrow})}{\int_{-\pi/2}^{\pi/2} d\theta_S \cos \theta_S (\sigma_{N,\uparrow} + \sigma_{N,\downarrow})} \]

where \( \sigma_{N,\uparrow}[\downarrow] \) denotes the tunneling conductance for the up[down] spin quasiparticle injection in the normal state and is given by

\[ \sigma_{N,\uparrow} = \frac{4\lambda_+}{(1 + \lambda_+)^2 + Z_{\theta_S}^2}, \quad \lambda_+ = \sqrt{1 \pm \frac{U}{E_F \cos^2 \theta_S}} \]

\[ \sigma_{N,\downarrow} = \frac{4\lambda_-}{(1 + \lambda_-)^2 + Z_{\theta_S}^2} \theta(\sigma_0 \pm |\theta_S|) \]

with \( Z_{\theta_S} = \frac{Z}{\cos \theta_S} \) and \( Z = \frac{2mH}{\hbar^2 k_F} \).

The quantity \( \sigma_{S,\uparrow}[\downarrow] \) is the tunneling conductance for the up[down] spin electron injection in the superconducting state. In the unitary pairing state, we note that the Fermi surface effect largely influences the reflection process. The retro-reflectivity of AR is broken due to the influence of the exchange interaction in the ferromagnet. In the following, we will consider the situation where \( k_{F,\downarrow} < k_S < k_{F,\uparrow} \) is satisfied [Fig. 2]. For \( |\theta_S| > \sqrt{\cos^{-1}(U/E_F)} \equiv \theta_C \), the Andreev reflection does not exist as a propagating wave. This novel property in the F/I/S junction is caused by the fact that the Andreev reflected hole has an antiparallel spin to that of the injection electron.

Based on the calculation of singlet superconductors, the tunneling conductance \( \sigma_{S,\uparrow}[\downarrow] \) is given by

\[ \sigma_{S,\uparrow} = \sigma_{N,\uparrow}\frac{1 - |\Gamma_+\Gamma_-|^2 (1 - \sigma_{N,\downarrow}) + \sigma_{S,\downarrow} |\Gamma_+|^2}{1 - \Gamma_+\Gamma_- \sqrt{1 - \sigma_{N,\downarrow}} \sqrt{1 - \sigma_{S,\uparrow}} \exp[i(\varphi_\uparrow - \varphi_\downarrow)]} \Theta(\theta_C - |\theta_S|) \]

\[ + [1 - \Theta(\theta_C - |\theta_S|)]\sigma_{N,\uparrow}\frac{1 - |\Gamma_+\Gamma_-|^2}{1 - \Gamma_+\Gamma_- \sqrt{1 - \sigma_{N,\downarrow}} \sqrt{1 - \sigma_{S,\uparrow}} \exp[i(\varphi_\uparrow - \varphi_\downarrow)]} \]

\[ \sigma_{S,\downarrow} = \sigma_{N,\downarrow}\frac{1 - |\Gamma_+\Gamma_-|^2 (1 - \sigma_{N,\uparrow}) + \sigma_{S,\uparrow} |\Gamma_+|^2}{1 - \Gamma_+\Gamma_- \sqrt{1 - \sigma_{N,\uparrow}} \sqrt{1 - \sigma_{S,\downarrow}} \exp[i(\varphi_\uparrow - \varphi_\downarrow)]} \Theta(\theta_C - |\theta_S|) \]

\[ \exp(i\varphi_\downarrow) = \frac{1 - \lambda_- + iZ_{\theta_S}}{\sqrt{1 - \sigma_{S,\downarrow}(1 + \lambda_- + iZ_{\theta_S})}} \exp(-i\varphi_\uparrow) = \frac{1 - \lambda_+ - iZ_{\theta_S}}{\sqrt{1 - \sigma_{S,\uparrow}(1 + \lambda_+ + iZ_{\theta_S})}} \]

for the unitary pairing state and

\[ \sigma_{S,\uparrow} = \sigma_{N,\uparrow}\frac{1 - |\Gamma_+\Gamma_-|^2 (1 - \sigma_{N,\downarrow}) + \sigma_{S,\downarrow} |\Gamma_+|^2}{1 - \Gamma_+\Gamma_- (1 - \sigma_{N,\downarrow})^2} \]

\[ \sigma_{S,\downarrow} = \sigma_{N,\downarrow} \]

for the non-unitary pairing state. Here, \( \Gamma_\pm \), defined for both unitary and non-unitary pairing states, is denoted by

\[ \Gamma_\pm = \frac{\Delta_0(\sin \theta_S \pm \cos \theta_S)}{eV + \sqrt{(eV)^2 - |\Delta_0(\sin \theta_S \pm \cos \theta_S)|^2}}, \quad E_u(1) \]
\[
\Gamma_\pm = \pm \frac{eV - \sqrt{(eV)^2 - |\Delta_0|^2}}{|\Delta_0|} e^{-i\theta_S}, \quad E_u(2)
\]

In the above formulations, when the ferromagnet is a normal metal (i.e., \( U = 0 \)), \( \sigma_T(eV) \) in refs. 18 and 19 are reproduced completely. On the other hand, the magnitude of the Fermi momentum becomes zero for down spin electrons for the half-metallic ferromagnet limit (i.e., \( U = E_F \)), and \( \sigma_{N,\downarrow} = 0 \). Then, the wave function of the reflected hole by AR becomes an evanescent wave, and hence does not contribute to the net current. In this case, since the numerator of the conductance formula in eq. (3) vanishes at \( eV = 0 \), the ZBCP disappears both for the \( E_u(1) \) and \( E_u(2) \) pair potentials. In particular for the \( E_u(2) \) pair potential, since the \( |\Gamma_+\Gamma_-|^2 = 1 \) is satisfied independent of \( \theta_S \), for \( eV < \Delta_0 \), the numerator is zero and the resulting conductance \( \sigma_{S,\uparrow}(eV) = 0 \).

Figure 3 shows the calculated conductance spectra of the \( E_u(1) \) pair potential for both unitary and non-unitary paring states with various \( X = U/E_F \) for \( Z = 0 \). The results in refs. 18 and 19 are reproduced for \( X = 0 \). In the unitary pairing state [see Fig. 3(a)], since the probability of the Andreev reflection is suppressed due to finite \( U \), the tunneling conductance inside the gap (\( eV < \Delta_0 \)) is drastically reduced with the increase of \( X \), similar to the case of \( d \)-wave superconductor. On the other hand for the non-unitary pairing state [see Fig. 3(b)], the line shape of \( \sigma_T(eV) \) is insensitive with increasing \( X \). Figure 4 shows the \( \sigma_T(eV) \) of the \( E_u(2) \) pair potential in the high barrier case (\( Z = 5 \)) for various \( X \). In this case, \( \sigma_T(eV) \) has a ZBCP due to the sign change of the pair potential. The height of this peak for the unitary pairing state [see Fig. 4(a)] is reduced with increasing \( X \) in contrast to that for the non-unitary case [see Fig. 4(b)]. Since \( \sigma_{S,\downarrow} = 0 \) for \( |\theta_S| > \theta_C \) for the unitary pairing state with \( eV < \Delta_0 \), \( \sigma_T(eV) \) disappears with the increase of \( X \). When the magnetization axis of the ferromagnet is antiparallel to the spin axis of the non-unitary pair potential, the height of the ZBCP is reduced and \( \sigma_T(eV) \) converges to unity with increasing \( X \) [see Fig. 4(c)]. Similar to the \( d \)-wave superconductor case, we can estimate the magnitude of the spin polarization using the height of the ZBCP.

In conclusion, we have studied the properties of tunneling conductance spectra \( \sigma_T(eV) \) in \( F/I/TS \) junctions. In the case of a unitary pair potential with \( E_u \) symmetry where the Cooper pair is formed between up and down spins, the height of the ZBCP is reduced drastically as the magnitude of the exchange interaction is increased. This is due to the breakdown of the retro-reflectivity of the Andreev reflection. For the non-unitary case, the conductance depends on the direction of the magnetization axis of the ferromagnet. When the magnetization axis is parallel (antiparallel) to the spin axis of the non-unitary pair potential, the height of the ZBCP is enhanced (suppressed) with the increase of the exchange interaction. Since a Cooper pair is formed between quasiparticles with equal spins, the exchange interaction does not significantly influence on the Andreev reflection. Based on these properties, we expect that the symmetry of the pair potential in \( \text{Sr}_2\text{RuO}_4 \) and the magnitude of the exchange interaction can be identified in future experiments by the presence of \( F/I/TS \) junction.
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[1] A. F. Andreev: Zh. Eksp. Teor. Fiz. 46 (1964) 1823 [Sov. Phys. JETP 19 (1964) 1228].
[2] M. J. M. de Jong and C. W. J. Beenakker: Phys. Rev. Lett. 74 (1995) 1657.
[3] Y. Tanaka and S. Kashiwaya: Phys. Rev. Lett. 74 (1995) 3451.
[4] Y. Tanaka and S. Kashiwaya: Phys. Rev. B 53 (1996) 9371.
[5] S. Kashiwaya, Y. Tanaka, M. Koyanagi, H. Takashima and K. Kajimura: Phys. Rev. B 51 (1995) 1350.
[6] S. Kashiwaya, Y. Tanaka, M. Koyanagi and K. Kajimura: Phys. Rev. B 53 (1996) 2667.
[7] S. Kashiwaya, Y. Tanaka, N. Terada, M. Koyanagi, S. Ueno, L. Alff, H. Takashima, Y. Tanuma and K. Kajimura: J. Phys. Chem. Solids 59, (1998) 2034.
[8] C. R. Hu: Phys. Rev. Lett. 72 (1994) 1526.
[9] I. Zutic and O. T. Valls: cond-mat/9808285.
[10] J-X. Zhu, B. Friedman, and C. S. Ting: preprint.
[11] S. Kashiwaya, Y. Tanaka, N. Yoshida and M. R. Beasley: cond-mat/9812160.
[12] Y. Maeno, H. Hashimoto, K. Yoshida, T. Fujita, J. G. Bednorz, and F. Lichtenberg: Nature (London) 372 (1994) 532.
[13] Y. Maeno and K. Yoshida: in Proceedings of the 21st International Conference on Low Temperature Physics, Prague, 1996 [Czech. J. Phys. 46 (1996) 3097, Suppl. S6].
[14] T. M. Rice and M. Sigrist: J. Phys. Condens. Matter 7 (1995) 643.
[15] M. Sigrist and M. E. Zhitomirsky: J. Phys. Soc. Jpn. 65 (1996) 3452.
[16] K. Machida, M. Ozaki, and T. Ohmi: J. Phys. Soc. Jpn. 65 (1996) 3720.
[17] M. Yamashiro, Y. Tanaka and S. Kashiwaya: Phys. Rev. B 56 (1997) 7847.
[18] M. Yamashiro, Y. Tanaka and S. Kashiwaya: J. Phys. Soc. Jpn. 67 (1998) 3224.
[19] C. Honerkamp and M. Sigrist: J. Low. Temp. Phys. 111 (1998) 898.
[20] C. Honerkamp and M. Sigrist: Prog. Theor. Phys. 100 (1998) 53.
[21] M. Yamashiro, Y. Tanaka and S. Kashiwaya: J. Phys. Soc. Jpn. 67 (1998) 3364.
[22] C. Bruder: Phys. Rev. B 41 (1990) 4017.
[23] In contrast to ref. 7, we neglect the polarization factor in eq. (4). This point is not essential because the inclusion of the factor depends on the model.
Fig. 1. Schematic illustration of the scattering process of the quasiparticle at the interface of the F/I/T S junction. For the Andreev reflected hole with down spin, the retro-reflectivity is broken due to the exchange interaction.

Fig. 2. Fermi surface effect for Andreev reflection for the unitary pairing state. $\theta_{FC}$ and $\theta_C$ are the angles in the ferromagnet and in the triplet superconductor, respectively. For $\theta_{FC} < |\theta_F|$, i.e., $\theta_C < |\theta_S|$, the Andreev reflection does not exist as a propagating wave.
Fig. 3. Normalized conductance spectra for $E_u(1)$ symmetry as a function of $X$ with $Z = 0$. Here (a) and (b) are results for the unitary pairing state and the non-unitary pairing state, respectively. $X$ is a: $X = 0.0$, b: $X = 0.5$ and c: $X = 0.9$ for the unitary pairing state and $X = 0.999$ for the non-unitary pairing state.
Fig. 4. Normalized conductance spectra for $E_u(2)$ symmetry as a function of $X$ with $Z = 5$. Here, (a) and (b) are results for the unitary pairing state and the non-unitary pairing state, respectively. (c) is the result of the magnetization axis of ferromagnet antiparallel to the spin axis of the non-unitary pair potential. $X$ is a: $X = 0.0$, b: $X = 0.5$ and c: $X = 0.999$. 