Pole Placement Based State Feedback for DC Motor Position Control

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Abstract. The paper proposes DC Motor position controller by applying state feedback control which is tuned by pole placement method. DC motor is one of most widely-used industrial and robotics actuators. Hence, DC Motor controller is supposed to have good performance with high efficiency. The research designed state feedback controller which functions to set position of DC motor with best pole parameter value. DC Motor simulation with pole placement is made in MATLAB. Based on test results, best pole values for the DC Motor is \([-10+j10 -10 -10-j10]\) with system response performance as: 0% overshoot, 0.4459 second in settling time, and 0.2558 second in rise time.

1. Introduction
Technology development and advance affects in human daily life [1]. Various electronic devices keep developing into more sophisticated and easy-to-use tools [2]. The use of electrical motors has covered many areas such as transportation system [3], robots [4][5], aerospace [6], computers [7][8], and automation [9]. Nowadays, it is beneficial for system with electrical motors to have good performance characteristics and efficiency [10][11].

One of the most used industrial actuators is DC motor [12]. DC motors are popular due to its low-power and high precise servo applications with reasonable cost and ease of control characteristics [13]. The main problem of controlling a system with DC motor is determining how much electrical power needed so that the DC motor rotates in the desired angular position or speed [14]. Hence, some controllers, both linear [15] and nonlinear controllers, have been proposed to control DC motor systems [16].

Linear state feedback control is theoretically an attractive method for controlling a linear plant [17]. It is proved that state feedback controller has better performances in terms of overshoot and settling time as a linear controller for DC motor [18]. It has the flexibility of shaping the dynamics of the closed-loop system to meet the desired specification [19]. The state feedback controller gains matrix K is not unique for a given system but depends on the desired closed-loop pole locations which determine the speed and damping ratio of a response [20].

However, this flexibility property may be a trivial issue to be solved. It is already known that different closed-loop pole locations may result in different performance specifications [21] [22]. This research will propose new position controller for DC motor by applying state feedback control with pole placement method as its tuning method [23]. The test will be done by simulation in MATLAB. Pole
locations of the system will be observed to analyze the characteristics of the pole location to the system response performances as to guarantee the best pole locations available.

2. Systems Design
   A. Modeling of DC Motor
   The DC motor system is modelled in state space representation. The system input is the DC motor voltage supply, which can be written as \( V \). Meanwhile, the system output will be angular position of DC motor which is \( \theta \). Mechanical point of view analysis of DC motor is based on The Newton Law II about rotation [24]. It says that the sum of torsion is the multiplication of weight of inertia with angular acceleration. Therefore, the equations can be written as,
   \[
   \Sigma \tau = Ia \\
   T - f_k = Ia \\
   K_m i - K_f \frac{d\theta}{dt} = J \frac{d^2 \theta}{dt^2}
   \]
   where \( K_m \) is armature constant of DC motor, \( f_k \) is the friction force happens to DC motor, and \( J \) is weight of inertia of DC motor. While in electrical point of view, Kirchhoff Law of Voltage can be applied in DC motor System. It says that the sum of potential differences in a closed circuit is equal to zero. Hence, some equations can be obtained and written as,
   \[
   v = v_R + v_L + v_{Emf} \\
   v = iR + L \frac{di}{dt} + K_b \frac{d\theta}{dt} \\
   \frac{di}{dt} = \frac{V}{L} - i \frac{R}{L} - \frac{K_b d\theta}{dt}
   \]
   where \( v_{Emf} \) is the induced electromotive force in DC motor and \( K_b \) is the electromotive force constant in DC motor. Variables in this system model are electric current \( i \), angular position \( \theta \), and angular speed \( \omega \) which is the first derivative of \( \theta \). Based on the variables and equations above, the state space representation of DC motor is able to be written as,
   \[
   \dot{x}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{K_f}{J} & \frac{K_m}{J} \\ 0 & -\frac{K_b}{J} & \frac{R}{L} \end{bmatrix} \begin{bmatrix} \theta \\ \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} v
   \]
   \[
   y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} v
   \]

   B. State Feedback Controller Design Based on Pole Placement Method
   In general, the controller design can be illustrated as in Figure 1. The controller can be divided into two parts: Integral Control (Integral gain block) and the state feedback controller (state feedback block) [25]. The plant block contains the model of DC motor in state space representation. The system output, which is also the feedback of the augmented system is the angular position of DC motor \( \theta \). It will be compared to the input reference and will result in error value. The control objective of the augmented system is to
set the angular position of DC motor system output $\theta$ in accordance to the desired reference input. The plant is modeled as state space representation with

$$\dot{x} = Ax + Bu,$$  \hspace{1cm} (10)

where $u$ is the control input from state feedback controller and is defined as

$$u = -Kx.$$  \hspace{1cm} (11)

Hence, the augmented system with state feedback controller applied to the system can be written as

$$\dot{x} = \tilde{A}x = (A - BK)x.$$  \hspace{1cm} (12)

![Diagram](https://via.placeholder.com/150)

**Figure 1.** DC motor open-loop control block diagram

System response in this research can be observed from some output parameters of Simulink MATLAB, and will become the reference in controller parameter tuning process. If the system response already matched with the objective of the research, then tuning process can be stopped and the parameter values will be tested with some variants of reference input values.

It must be noted that, in order to be able to apply state feedback controller with pole placement method, DC Motor system model needs to be in controllable state, especially for $A$ and $B$ matrices. To check whether it is already in controllable canonical form or not, a rank of controllability matrix $M$ needs to be checked. The controllability matrix $M$ can be written as follow,

$$M = [B : AB : \cdots : A^{n-1}B]$$  \hspace{1cm} (13)

If the model is already in controllable canonical form, then the values of $K$ can be found. It is designed and derived from the combination of desired closed-loop pole locations using a method called Ackermann formula.

### C. Pole Placement with Ackermann Formula

After the desired closed loop poles have been decided, the next step is to define $K$. The DC motor model used in the research is categorized as low order system, with $n = 3$ so that the determination of $K$ can be made by using Ackermann formula.

Determination of $K$ by using Ackermann formula can be done in MATLAB Simulink by using `acker` function. The Ackermann formula to determine state feedback gain matrix $K$ for an arbitrary positive integer $n$ can be written as follows,

$$K = [0 \ 0 \ \cdots \ 1][B : AB : \cdots : A^{n-1}B]^{-1}\phi(A)$$  \hspace{1cm} (14)

Where

$$\phi(A) = \alpha_n I + \alpha_{n-1}A + \cdots + \alpha_1A^{n-1} + A^n$$  \hspace{1cm} (15)

The constants of $\alpha$ can be acquired with $\mu$ as desired poles from the desired characteristic equation as follows,

$$|sI - A + BK| = (s - \mu_1)(s - \mu_2)\cdots(s - \mu_n)$$  \hspace{1cm} (16)

$$|sI - A + BK| = s^n + \alpha_1s^{n-1} + \cdots + \alpha_{n-1}s + \alpha_n$$  \hspace{1cm} (17)
3. Result and Discussion

A. Open-loop test DC Motor system

Open-loop test is done to observe the dynamic characteristics of DC motor system model which has been designed. The response then will be compared to the real system response. Besides, the open-loop test response is held to analyze how DC motor system reacts to the input voltage. List of parameters with their respective values is as follow. The resistance is 100Ω, the Inductance is 0.5H, Friction force constant 0.1Nms, the armature constant is 0.01Nm/A, the moment of inertia is 0.01kgm².

The open-loop test is held by giving 12-volt supply voltage to the DC motor system model. Angular position of DC motor then will be observed as system response performances. DC motor control block diagram in Simulink MATLAB is shown as in Figure 2 and the system model in state space is,

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -10 & 7.5 \\ 0 & -0.15 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \quad C = [1 \ 0 \ 0] \quad \text{(18)}
\]

**Figure 2.** DC motor open-loop control block diagram

The result of open-loop test of DC motor system model is shown in Figure 3. Based on the figure, DC motor system can achieve certain position when it was given 12-volt voltage supply. However, it will continue to rotate if the simulation still runs. The test results show that the system model can be used to represent real system. The real system shows similar result in open-loop test, which has unstable characteristics.

**Figure 3.** Open-loop test result

B. Proposed controller test

Simulation test of the controller is done by implementing state feedback controller to DC motor system model. The input of the simulation system is the reference signal and the output is the DC motor angular position. The reference signal that is used in the simulation test result is step unit with amplitude as 1 rad.
The simulation test is grouped into two categories are: 1) closed-loop real poles; 2) complex conjugates with real poles. Simulation model design is shown on Figure 4. It illustrates the augmented system of a DC motor with state feedback controller based on pole placement method.

![Figure 4. Augmented system of DC Motor with State Feedback Controller](image)

**a. Real Poles**

The test is done to see the response of poles which are located in real axis. The test result is shown in Table 1 and Figure 5. Analysis result of Table 2 and Figure 5(a) shows that negative real poles will make the system have no overshoot in response. Moreover, more negative the values of the poles are, the faster the rise time of the system response, or it can be said that the system responds faster to achieve steady state.

| Desired Poles | Gains | Systems Response Performance Specifications |
|---------------|-------|---------------------------------------------|
|               | $k_1$ | $k_2$ | $k_3$ | Rise Time (s) | Settling time (s) | Overshoot (%) |
| $[-5 -5 -5]$  | 8.3333| 1.5167| 0.5   | 0.8498        | 1.5070           | 0             |
| $[-5 -10 -5]$ | 16.6667| 1.5167| 3     | 0.7259        | 1.2975           | 0             |
| $[-5 -7 -7]$  | 16.3333| 1.7833| 2.5   | 0.6938        | 1.2324           | 0             |
| $[-7 -10 -5]$ | 23.3333| 2.1833| 4     | 0.6440        | 1.1489           | 0             |
| $[-7 -7 -7]$  | 22.8667| 2.3167| 3.5   | 0.6062        | 1.0741           | 0             |
| $[-10 -5 -10]$ | 33.3333| 3.1833| 5.5   | 0.5894        | 1.0617           | 0             |
| $[-10 -7 -10]$ | 46.6667| 4.5167| 6.5   | 0.4940        | 0.8774           | 0             |
| $[-10 -10 -10]$ | 66.6667| 6.5167| 8     | 0.4313        | 0.7588           | 0             |

**b. Complex Poles**

The test is done to investigate the response of poles formed by combination of real poles and complex conjugates poles. The test results are shown in Table 3, Figure 5(b). Based on Table 3 and Figure 6, the location of poles affects the system response in the settling time specifications. One dominant pole, such as $\mu = 10$ will result an overshoot in the system performances.

**4. Conclusions**

Based on the research, some conclusions can be made. Closed-loop pole locations affect the system’s stability and performances. Real poles make the system response has no overshoot. Moreover, more negative the values of the poles, the faster the rise time and the settling time are. Overall, the best result is given by the closed-loop poles as $[-10 + j10 -10 -10 -10]$. Those poles make the system responds in no overshoot, 0.4459 seconds in settling time, and 0.2558 seconds in rise time.
Table 2. Closed-loop real and complex poles chosen controller gains and simulation test result

| Desired Poles | Gain | Systems Response Performance Specifications |
|---------------|------|---------------------------------------------|
|               | $k_1$ | $k_2$ | $k_3$ | Rise Time (s) | Setting Time (s) | Overshoot (%) |
| $[-5 - 5j -5 + 5j -10]$ | 33.3333 | 3.1833 | 3.0 | 0.3789 | 0.9158 | 2.6204 |
| $[-7 - 7j -7 + 7j -5]$ | 32.6667 | 5.0500 | 2.5 | 0.4626 | 0.8944 | 0 |
| $[-10 - 10j -10 + 10j -5 - 5j]$ | 66.6667 | 9.8500 | 5.5 | 0.4512 | 0.8821 | 0 |
| $[-10 - 10j -10 + 10j -7]$ | 93.3333 | 11.1833 | 6.5 | 0.3314 | 0.6413 | 0 |
| $[-7 + 7j -7 - 7j]$ | 45.7333 | 5.5833 | 3.5 | 0.3646 | 0.6226 | 0 |
| $[-7 - 7j -7 + 7j -10 - 10j]$ | 65.3333 | 6.3833 | 5.0 | 0.3064 | 0.5013 | 1.4463 |
| $[-10 + 10j -10 - 10j -10]$ | 133.3333 | 13.1833 | 8.0 | 0.2558 | 0.4459 | 0 |

Figure 5. (a) Step response of closed-loop real poles (b) Step responses of closed-loop real and complex poles

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