Circumbinary discs around merging stellar–mass black holes

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\textbf{ABSTRACT}

A circumbinary disc around a pair of merging stellar–mass black holes may be shocked and heated during the recoil of the merged hole, causing a near-simultaneous electromagnetic counterpart to the gravitational wave event. The shocks occur around the recoil radius, where the disc orbital velocity is equal to the recoil velocity. The amount of mass present near this radius at the time of the merger is critical in determining how much radiation is released. We explore the evolution of a circumbinary disc in two limits. First, we consider an accretion disc that feels no torque from the binary. The disc does not survive until the merger unless there is a dead zone, a region of low turbulence. Even with the dead zone, the surface density in this case may be small. Second, we consider a disc that feels a strong binary torque that prevents accretion on to the binary. In this case there is significantly more mass in regions of interest at the time of the merger. A dead zone in this disc increases the mass close to the recoil radius. For typical binary-disc parameters we expect accretion to be significantly slowed by the resonant torque from the binary, and for a dead zone to be present. We conclude that provided significant mass orbits the binary after the formation of the black hole binary and that the radiation produced in recoil shocks can escape the flow efficiently, there is likely to be an observable electromagnetic signal from black hole binary mergers.

\textbf{Key words:} accretion, accretion discs — binaries: general — hydrodynamics — gravitational waves — black hole physics

1 INTRODUCTION

The direct detection of gravitational waves (GW) from binary black hole mergers has naturally raised the question of whether there could be a corresponding electromagnetic (EM) signal. Recently de Mink & King (2017) suggested that the gravitational effect of a BH + BH merger on a circumbinary disc might lead to an EM signal delayed by the light–travel time from the merging binary to this disc (see also Rossi et al. 2016, where this effect is applied to mergers of supermassive black holes). Given the lack of simultaneity, identifying this latter signal would require spatial coincidence, and so positional information from several GW detectors. This should be available as new detectors become operational.

At least two detailed binary evolution scenarios, the classical common–envelope and chemically homogeneous pictures, show how a massive binary can remain bound as it evolves to a BH + BH binary (see the discussion in de Mink & King 2017, Section 2, and references therein). In both cases the two stars shed large fractions of their original masses. Unless all of this mass is expelled to infinity, or somehow completely re–accreted by the black holes, a residual circumbinary disc must remain, as the only stable configuration for the shed mass.

In this paper we consider the evolution of this circumbinary disc around a merging black hole binary. Specifically, we are interested in whether there is sufficient material in the disc at the time of the merger for an electromagnetic signal to be generated by the perturbation from the gravitational wave recoil and the sudden drop in mass of the binary.

The circularisation radius of material falling back after a supernova explosion is very small (Perna et al. 2014; Perna, Lazzati & Giacomazzo 2016). This suggests that the angular momentum of the ejected material is small in the frame of the star. Thus the material is ejected with a velocity equal to that of the mass losing star plus a velocity radial from the mass losing star with magnitude $v_{\text{ej}}$. For ejection velocities greater than the escape velocity from the binary, $v_{\text{ej}} > v_{\text{esc}}$, the material is lost. For $v_{\text{ej}} \ll v_1$, where $v_1$ is the orbital velocity of the mass–losing star, the material does not escape the Roche-Lobe of the star. For $v_{\text{ej}} \sim v_1$ the material has a spread of angular momentum around the angular momentum of the mass–losing star: the material at the back of the star has less, and the material at the front of the star has more. Thus we expect approximately half of the ejected material...
to have a velocity such that it ends up in a circumbinary configuration with a spread of angular momenta and thus circularisation radii. Whether the material circularises, or passes within the binary and cancels angular momentum with gas on similar but opposed orbits, to circularise at smaller radii, requires more detailed numerical simulations than we attempt here.

The angular momentum of the material forming the circumbinary disc may not necessarily be initially aligned to the binary angular momentum. For a circular black hole binary, differential precession driven by the binary torque combined with viscous dissipation causes the disc to move towards alignment or counter-alignment with the binary depending only on the initial inclination (e.g. King et al. 2005; Nixon, King & Pringle 2011; Foucart & Lai 2013, 2014). As the disc angular momentum is usually significantly smaller than the binary angular momentum (Nixon 2012), the alignment or counteralignment condition is straightforward: if the disc begins closer to alignment, it aligns, and if it begins closer to counteralignment it counteraligns. How this alignment proceeds depends upon properties of the disc. Papaloizou & Pringle (1983) showed that the propagation of warps occurs in two distinct ways, the dominant mechanism determined by the relative magnitudes of the disc viscosity and pressure. If the viscosity parameter is less than the disc aspect ratio, \( \alpha < H/R \), then the warp propagates through waves, otherwise the warp evolves diffusively (see Nixon & King 2016, for a review of warped disc physics). Since the disc aspect ratio of circumbinary discs around black holes is likely to be small, the inner parts of the disc are likely to be aligned (or counteraligned) while the outer parts of the disc may be strongly warped.

The strength of the tidal torque from the binary on the inner parts of the disc depends upon the inclination of the angular momentum of the material, being strongest when it is aligned to the binary angular momentum (e.g. Lubow, Martin & Nixon 2013; Miranda & Lai 2013). For a retrograde circumbinary disc around a circular binary the tidal torque from Lindblad resonances is zero (Nixon et al. 2011). Nixon & Lubos (2013) showed that resonances do occur in retrograde circumbinary discs around eccentric binaries, but that they are weak enough that they only modulate the accretion flow on to the binary rather than providing a torque of sufficient strength to arrest accretion.

The accretion rate from the inner edge of the circumbinary disc on to the components of a binary depends upon the strength of the tidal torque and properties of the disc. This has received significant attention as it impacts a variety of astrophysical systems (e.g. Artymowicz & Lubow 1994, 1996; Roedig et al. 2012; Shi et al. 2012; D’Orazio, Haiman & MacFadyen 2013; Farris et al. 2014). The gas is subject to two competing torques. The binary transfers angular momentum to the gas, causing the inner disc to move outwards, and viscosity redistributes that angular momentum to larger radii in the disc, allowing the inner disc to shrink. Thus discs subject to a weak torque (or strong viscosity) can move in and accrete on to the binary components. In contrast a strong torque (or weak viscosity) leads to the disc being held out. For the small disc aspect ratio expected in black hole binaries, the accretion rate is significantly suppressed (Ragusa, Lodato & Price 2012). If the tidal torque is sufficiently strong, accretion on to the binary may be completely halted. Disc solutions in this case correspond to decretion discs (Pringle 1991) rather than traditional accretion discs in which material flows freely through the inner boundary (Pringle 1981).

In Section 2 we first consider how much material is required in a circumbinary disc for an observable electromagnetic signal to be generated. In the rest of this work, we then estimate how much material is present in an evolving circumbinary disc. Because of the uncertainties in the strength of the tidal torque and the resulting accretion flow on to the binary, we consider two extreme limits. In Section 5 we consider the disc evolution in the case that the binary provides no torque on the disc and material freely flows inwards and is accreted on to the binary components. The disc is a traditional accretion disc. In Section 4 we examine the case where the binary torque is strong enough to prevent all flow on to the binary. We draw our conclusions in Section 5.

### 2 ELECTROMAGNETIC SIGNAL FROM A CIRCUMBINARY DISC

Gravitational wave emission during a binary black hole coalescence produces a sudden drop in the total central mass. The weaker potential causes circumbinary disc orbits to expand (e.g. Rosotti, Lodato & Price 2013). This excites density waves which dissipate their energy, giving an electromagnetic signal. However, as the black holes merge the merged hole suffers a momentum kick due to the asymmetry of the gravitational wave emission. As the hole is kicked, the specific angular momentum of the gas in the circumbinary disc is altered, leading the gas orbits to become eccentric and causing shocks, dissipation and circularisation on the local orbital timescale. This can give a rather stronger electromagnetic signal (Rossi et al. 2010).

We consider a black hole binary system of total mass \( M \), composed of two black holes of mass \( M_{\text{bh}} = 30 M_\odot \). Heating by shocks occurs on the dynamical timescale

\[
\tau_{\text{dyn}} = \frac{GM}{v_{\text{rec}}^2} = 2.2 \left( \frac{M}{60 M_\odot} \right) \left( \frac{v_{\text{rec}}}{1000 \text{ km s}^{-1}} \right)^{-2} \text{ hr},
\]

where \( v_{\text{rec}} \) is the recoil velocity imparted to the centre of mass. The gas in the disc orbits the central mass at Keplerian angular frequency \( \Omega = \sqrt{GM/R^3} \), where \( R \) is the radius from the binary centre of mass. The recoil radius is where the Keplerian velocity of the disc is equal to the recoil velocity

\[
R_{\text{rec}} = \frac{GM}{v_{\text{rec}}^2} = 11.5 \left( \frac{M}{60 M_\odot} \right) \left( \frac{v_{\text{rec}}}{1000 \text{ km s}^{-1}} \right)^{-2} R_\odot
\]

(de Mink & King 2017). The magnitude of the recoil velocity depends sensitively on the pre–merger black hole spin vectors (e.g. Blecha et al. 2016). If the spins are within a few degrees of alignment then the kick may be small, \( \lesssim 600 \text{ km s}^{-1} \). However, if the spins are not aligned larger kicks up to several thousand km s\(^{-1}\) can occur (Campanelli et al. 2007; Lousto & Zlochower 2011). Spin alignment is unlikely in X-ray binaries (e.g. Martin, Pringle & Tout 2007, 2009; Martin, Tout & Pringle 2008; Martin, Reis & Pringle 2008; King & Nixon 2016) or in SMBH binaries (Lodato & Gerosa 2013), but could perhaps be facilitated in close binaries where the stellar spins have been synchronised with the binary orbit. In this work, we assume that kick velocities of order 500–1000 km s\(^{-1}\) are possible, and that some ejected material settles into a circumbinary disc. We are interested in how much material may still be present at the recoil radius at the time of the merger.

The luminosity generated by the kick is calculated with rate of dissipation of kinetic energy as

\[
L = \frac{fM_{\text{rec}} v_{\text{rec}}^3}{\tau_{\text{dyn}}},
\]

where \( M_{\text{rec}} \) is an estimate of the mass of the disc within the region that feels the kick strongly and \( f \) is a scaling factor. The scaling factor is calibrated to the numerical simulations by Rossi et al. 2010.
who found $f \approx 0.1$ for a kick angle $\theta = 15^\circ$. The peak luminosity is somewhat insensitive to the kick angle. For typical parameters the luminosity is
\[ L = \frac{f M_{\text{rec}} v_{\text{rec}}^3}{t_{\text{dyn}}} \approx 1.5 \times 10^{33} \left( \frac{f}{0.1} \right) \left( \frac{M_{\text{rec}}}{0.001 M_\odot} \right) \left( \frac{v_{\text{rec}}}{1000 \text{ km s}^{-1}} \right)^3 \text{ erg s}^{-1}. \] (4)

There is a radius outside of which all particles in the disc will become unbound after the kick, this is given by
\[ R_{\text{db}} = \left( \cos \theta + \sqrt{\cos^2 \theta + 1} \right)^2 R_{\text{rec}} \] (5)
(Rossi et al. 2010), where $\theta$ is the angle between the kick velocity direction and the disc plane. This is maximal if the kick is in the plane of the disc. We approximate this radius with
\[ R_{\text{db}} = k R_{\text{rec}}. \] (6)

We find the average parameter of $k = 3.64$ by assuming an isotropic distribution for $\theta$ and integrating equation (5) over $0 < \theta < 90^\circ$. This corresponds to $\theta = 46.4^\circ$. We use this in our estimates for the mass $M_{\text{rec}}$ in the rest of this work.

A further constraint on the circumbinary disc is that it must survive until the black hole merger takes place. This depends sensitively on the initial separation of the binary at the time when the black hole binary forms. The timescale for the decay of an equal mass binary as a result of gravitational waves is
\[ t_{\text{GW}} = 1.1 \times 10^9 \left( \frac{a_b}{10 R_\odot} \right)^4 \left( \frac{M}{60 M_\odot} \right)^{-3} \text{ yr} \] (7)
(Peters 1964), where $a_b$ is the separation of the binary. The inspiral may be accelerated by the torque from a prograde circumbinary disc (e.g. Armitage & Natarajan 2003; Lodato et al. 2009) and even more significantly from a retrograde disc (Nixon et al. 2011). The separation of the binary does not affect the structure of the disc at large radii from the binary. In this work we assume that the disc remains in quasi–steady state as the disc and the binary separation evolve.

### 3 ACCRETION DISC MODEL

We now consider the evolution of a circumbinary disc freely accreting on to the binary components. The total mass of the disc is taken as $M_d = 0.001 M_\odot$. This otherwise arbitrary choice is designed to illustrate that a detectable electromagnetic signal is possible even from a residual disc of far lower mass than that lost ($\sim M$) in the earlier evolution of the binary (see de Mink & King 2017, Section 2). Since $M_d \approx M_{\text{rec}}$, equation (4) shows the effect of different choices. We take a simple model in which we assume that the disc aspect ratio $H/R$ is constant over the radial extent of the disc.

Angular momentum transport in accretion discs is thought to be driven by turbulence. In many cases, turbulence can be driven by the magnetorotational instability (MRI, Balbus & Hawley 1991). A critical level of ionisation in the disc is required for this mechanism to operate: if the disc is sufficiently hot, it may be sufficiently thermally ionised. In parts of the disc with a lower temperature, it may be that the surface layers of the disc are MRI–active because they are ionised by external sources such as cosmic rays (e.g. Glassgold, Najita & Isella 2004). A “dead zone” forms at the disc midplane where the MRI does not operate (Gammie 1996; Gammie & Menou 1998; Perna, Lazzati & Giacomazzo 2011). In this Section we first consider a disc model in which the MRI operates throughout, and then we consider a disc model with a dead zone.

#### 3.1 Fully turbulent disc

If the disc is fully MRI active, the lifetime of the disc is approximated by the viscous timescale at the circularisation radius of the gas. The viscous timescale is
\[ t_v = \frac{R^2}{v}, \] (8)
where the viscosity is parameterised with
\[ v = \alpha c_s H, \] (9)
with $\alpha$ is the dimensionless Shakura & Sunyaev (1973) viscosity parameter. The scale height of the disc is $H = c_s/\Omega$, where $c_s$ is the sound speed, and so
\[ t_v = \alpha \left( \frac{H}{R} \right)^2 \frac{R^2}{\Omega}. \] (10)
And for typical values we find the viscous timescale to be
\[ t_v = 6.5 \times 10^5 \left( \frac{\alpha}{0.1} \right)^{-1} \left( \frac{H/R}{0.01} \right)^{-2} \left( \frac{M}{60 M_\odot} \right)^{1/2} \left( \frac{R}{10 R_\odot} \right)^2 \text{ yr}. \] (11)

The lifetime of a fully MRI active disc with no binary torque is relatively short, and in this case there may be no circumbinary material left at the time of the merger unless the binary black holes are formed with a separation $a_b \leq 1 R_\odot$.

#### 3.2 Disc with a dead zone

A dead zone forms at the disc midplane if two conditions are satisfied. First, the temperature must be lower than the critical required for the MRI to operate, $T_{\text{crit}} \approx 800 \text{ K}$ (Umebasyahi & Nakano 1988). Above this temperature, the disc is thermally ionised and MRI active. Second, the surface density must exceed the critical that can be sufficiently ionised by external sources, $\Sigma_{\text{crit}}$. For a central black hole binary, the only source of external ionisation is cosmic rays. These may ionise a maximum surface density of $100 \text{ g cm}^{-2}$ (e.g. Gammie 1996; Matsubara, Padriz & Thommes 2009; Zhu, Hartmann & Gammie 2009). Effects such as recombination may lead to a much smaller active layer (e.g. Wardle 1999; Balbus & Terquem 2001; Fleming, Stone & Hawley 2000; Martin et al. 2012). In this work we take a fiducial value of $\Sigma_{\text{crit}} = 10 \text{ g cm}^{-2}$.

The inner edge of the dead zone is fixed by the radius at which the temperature of the fully turbulent steady state disc model is equal to the critical temperature required for the MRI. The sound speed in the disc is given by
\[ c_s = \sqrt{\frac{k T}{\mu m_{\text{H}}}}. \] (12)
where $k$ is the Boltzmann constant, $T$ is the temperature, $\mu = 2.3$ is the mean molecular weight and $m_{\text{H}}$ is the mass of a hydrogen atom. Thus the disc temperature is
\[ T = 3.3 \times 10^4 \left( \frac{H/R}{0.01} \right)^{1/2} \left( \frac{\mathcal{M}}{60 M_\odot} \right)^{1/2} \left( \frac{R}{10 R_\odot} \right)^{-1} \text{ K}. \] (13)
Setting $T = T_{\text{crit}}$ we find the radius of the inner edge of the dead zone as

$$R_{\text{out}} = 417 \left( \frac{H/R}{0.01} \right)^2 \left( \frac{M}{60 \, M_\odot} \right) \left( \frac{T_{\text{crit}}}{800 \, \text{K}} \right)^{-1} \, R_\odot. \quad (14)$$

Note that both the temperature and the inner radius of the dead zone depend sensitively upon the disc aspect ratio.

The surface density of the disc in the dead zone is higher than that of a steady accretion disc since material builds up there over time (e.g. Martin & Lubow 2011, 2013; Martin et al. 2012b). The radius of interest here, $R_{\text{out}}$, is likely much larger than a dead zone, unless the disc aspect ratio is smaller than 0.01. The dead zone acts as a bottleneck in the accretion flow. We assume that the viscosity in the dead zone is zero, although in practice there may be a small amount of turbulence there (e.g. Martin & Lubow 2014). The surface density at radii inside the dead zone radius is smaller than that of a steady accretion disc since material builds up there over time (e.g. Martin & Lubow 2011, 2013; Martin et al. 2012b). The surface density at radii inside the dead zone is smaller than that of a disc without a dead zone, and the reduced accretion on to the binary means that the disc survives longer. The bottleneck occurs at the innermost dead zone radius, $R_{\text{d}}$. The accretion rate through the active surface layer is approximately given by

$$M_{\text{d}} = 3 \pi \Sigma \nu \sqrt{M_{\text{crit}}}.$$  \quad (15)

so

$$M_{\text{d}} = 7.2 \times 10^{-8} \left( \frac{\alpha}{0.1} \right)^{1/2} \left( \frac{H/R}{0.01} \right)^3 \left( \frac{M}{60 \, M_\odot} \right) \times \left( \frac{\Sigma_{\text{d}}}{10 \, \text{g cm}^{-2}} \right)^{1/2} \left( \frac{T_{\text{crit}}}{800 \, \text{K}} \right)^{-1/2} \, M_\odot \, \text{yr}^{-1}. \quad (16)$$

We estimate the lifetime of the disc as

$$t_{\text{lifetime}} = \frac{M_\odot}{M_{\text{d}}}, \quad (17)$$

where $M_\odot$ is the total mass of the disc. For our typical parameters this is

$$t_{\text{lifetime}} = 8.4 \times 10^6 \left( \frac{M_{\text{d}}}{0.001 \, M_\odot} \right)^{1/2} \left( \frac{\Sigma_{\text{d}}}{10 \, \text{g cm}^{-2}} \right)^{-1} \left( \frac{H/R}{0.01} \right)^3 \times \left( \frac{T_{\text{crit}}}{800 \, \text{K}} \right)^{1/2} \, \text{yr}. \quad (18)$$

Again, this depends very sensitively on the disc aspect ratio. The lifetime of this disc may be sufficiently long for material to remain around until the merger.

Inside the inner dead zone radius, $R_{\text{d}}$, the disc behaves as a steady accretion disc with accretion rate $M_{\text{d}}$. Away from the inner boundary, the steady state surface density of the disc is approximately $\Sigma = M_{\text{d}}/(3 \pi \nu)$. So if the recoil radius is smaller than than the radius of the inner edge of the dead zone, the surface density at the recoil radius is

$$\Sigma(R_{\text{rec}}) \approx 66.4 \left( \frac{M}{60 \, M_\odot} \right)^{1/2} \left( \frac{\Sigma_{\text{d}}}{10 \, \text{g cm}^{-2}} \right) \left( \frac{T_{\text{crit}}}{800 \, \text{K}} \right)^{-1/2} \times \left( \frac{H/R}{0.01} \right) \left( \frac{R_{\text{rec}}}{10 \, R_\odot} \right)^{-1/2} \, \text{g cm}^{-2}. \quad (19)$$

This surface density is constant in time as long as the dead zone has not been accreted. The mass of the disc up to a radius $R_{\text{d}}$ is

$$M_{\text{d}}(R_{\text{rec}}) \approx 5.6 \times 10^{-7} \left( \frac{\Sigma_{\text{d}}}{10 \, \text{g cm}^{-2}} \right) \left( \frac{T_{\text{crit}}}{800 \, \text{K}} \right)^{1/2} \left( \frac{M}{60 \, M_\odot} \right)^2 \times \left( \frac{H/R}{0.01} \right) \left( \frac{\nu_{\text{rec}}}{1000 \, \text{km s}^{-1}} \right)^3 \left( \frac{k}{3.64} \right)^{3/2} \, M_\odot. \quad (20)$$

This is quite small, but this is a lower limit since if the inner edge of the dead zone is smaller than $R_{\text{d}}$, the surface density must be higher than this.

In summary, a disc around a black hole that provides no tidal torque on the disc is unlikely to survive until the time of the black hole merger, unless it contains a dead zone. The dead zone significantly extends the lifetime of the disc since material accretes on to the binary at a much lower rate. However, the surface density in the inner parts of the disc that feel the kick is low, and may not produce a detectable signal at the expected source distances. Comparing (20) with (4) suggests a peak luminosity of $\sim 2 \times 10^{38} \, \text{erg s}^{-1}$.

### 4 DECRESSION DISC MODEL

We now consider the case of a binary torque strong enough to prevent all accretion on to the binary. Angular momentum is added to the inner parts of the disc. The disc spreads outwards while the surface density decreases in time. We assume that the disc is fully MRI active in this case.

#### 4.1 Quasi–steady state disc

The disc evolves as a quasi–steady state decretion disc. The steady state decretion disc solution satisfies

$$\nu \Sigma = \frac{(-M)}{3 \pi} \left( \frac{R_{\text{out}}}{R} \right)^{1/2} - 1,$$

where $R_{\text{out}}$ is the outer disc edge and $-M$ is the outward accretion rate through the disc. Since we are interested in the inner parts of the disc, we use the approximation $\nu \Sigma \approx R^{1/2}$ that is valid for $R \ll R_{\text{out}}$. We find the constant of proportionality from the total mass of the disc that remains constant in time. We note that this approximation leads us to underestimate the mass in the inner parts of the disc so our calculated masses can be taken as a lower limit.

We find the surface density

$$\Sigma = 3.94 \times 10^{4} \left( \frac{M}{60 \, M_\odot} \right) \left( \frac{M_{\text{d}}}{0.001 \, M_\odot} \right) \left( \frac{R}{10 \, R_\odot} \right)^{-1} \left( \frac{R_{\text{out}}}{10^{4} \, R_\odot} \right)^{-1} \, \text{g cm}^{-2}. \quad (22)$$

We note that the initial outer radius of the disc is determined by the angular momentum of the material that forms the disc. We show in Section 4.2 that for late times, the initial outer radius does not significantly affect the properties of the disc. The mass within radius $R_{\text{d}}$ is

$$M_{\text{d}}(R_{\text{rec}}) = 2.5 \times 10^{-4} \left( \frac{M}{60 \, M_\odot} \right)^2 \left( \frac{M_{\text{d}}}{0.001 \, M_\odot} \right) \left( \frac{R_{\text{rec}}}{1000 \, \text{km s}^{-1}} \right)^2 \, M_\odot. \quad (23)$$

For our typical parameters, this is more than two orders of magnitude larger than the accretion disc mass given in equation (20). Comparing equation (23) with (4) suggests a peak luminosity of $\sim 10^{41} \, \text{erg s}^{-1}$. However, since the decretion disc spreads outwards in time, we must consider how this mass changes in time.

#### 4.2 Spreading of the disc

The timescale for outward spreading of the disc is the viscous timescale at its outer edge. We model this by assuming

$$\frac{dR_{\text{out}}}{dt} = \frac{R_{\text{out}}}{t_{\text{vis}}(R_{\text{out}})}. \quad (24)$$

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We write the viscous timescale as
\[ t_v = t_0 \left( \frac{R_{\text{out}}}{R_0} \right)^{3/2}, \]  
where \( R_0 \) is the initial outer disc radius and \( t_0 \) is the initial viscous timescale at that radius, given by equation (8). Solving equation (25) with equation (11) we find that the outer radius of the disc evolves as
\[ R_{\text{out}} = R_0 \left( \frac{3}{2} \frac{t}{t_0} + 1 \right)^{2/3}. \]  
For an initial disc outer radius of \( R_0 = 10^3 R_\odot \), the viscous timescale there is \( t_0 = 6.5 \times 10^7 \) yr and for \( R_0 = 10^4 R_\odot \), the viscous timescale there is \( t_0 = 2 \times 10^7 \) yr. The left panel of figure 1 shows the surface density of the disc at the recoil radius as a function of time for initial outer disc radii \( R_{\text{out}} = 10^4 R_\odot \) and \( R_{\text{out}} = 10^5 R_\odot \), for our fiducial parameters \( M = 60 M_\odot \), \( M_d = 0.001 M \) and \( R_{\text{rec}} = 10 R_\odot \). The surface density in the decretion disc is significantly higher than that of the accretion disc given in equation (20). Further, the mass of the disc within \( R_{\text{ab}} \) is approximately
\[ M_{\text{d,rec}} = 2.5 \times 10^{-3} \left( \frac{M}{60 M_\odot} \right)^{5/3} \left( \frac{M_d}{0.001 M} \right) \left( \frac{\alpha}{0.1} \right)^{-2/3} \left( \frac{H/R}{0.01} \right)^{-4/3} \left( \frac{k}{3.64} \right) \left( \frac{t}{10^7 \text{ yr}} \right)^{-2/3} M_\odot. \]  
The dotted lines in Fig. 1 confirm that these fits to the surface density and the mass are good for times greater than about a viscous timescale at the outer edge of the disc.

The mass in this decretion disc, for our fiducial parameters, is larger than that of the accretion disc given in equation (20). Further, if there is a dead zone in the decretion disc, the surface density may be higher than that predicted here for the fully turbulent decretion disc. The structure would depend sensitively on the initial conditions. We will model this scenario with time–dependent simulations in a future paper in order to understand the disc evolution.

Fig. 2 shows the luminosity of the decretion disc as a function of the angle between the velocity kick and the disc plane, \( \theta \), as calculated by equation (4). We include the \( \theta \) dependence from equation (5). The luminosity is maximal for a kick that is in the plane and decreases by almost an order of magnitude for a kick that is perpendicular to the disc.

### 5 Conclusions

We have explored the evolution of a circumbinary disc around a merging black hole system in the two extreme limits of an accretion disc and a decretion disc. The accretion disc feels no torque from the binary. The viscous timescale of such a disc is short and it is unlikely that significant circumbinary material remains at the time of the merger unless there is a dead zone in the disc. The dead zone restricts the accretion flow through the disc and the disc life time is significantly extended. However, the surface density of the disc around the recoil radius is low, unless the dead zone extends into this region.
Figure 2. The luminosity of the decretion disc. The initial outer edge of the disc is at $R_0 = 10^6 R_\odot$. The times shown are $t = 0$ (solid line), $t = 10^7$ yr (dashed line) and $t = 10^9$ yr (dotted line).

In the opposite limit, where a strong binary torque prevents accretion, the disc behaves as a decretion disc. The surface density at the recoil radius is much larger than in the accretion disc, even without a dead zone.

The physical conditions in a disc around a BH–BH binary probably conform to the case of a decretion disc, held out by tidal torques from the binary, and containing dead zones. As we have seen, there is significant mass close to the recoil radius in this case. We conclude that dynamical readjustment of the disc after the BH merger is likely to release significant energy in electromagnetic form. Since all of the disc matter, including its outer skin, is shocked simultaneously, it appears unlikely that this energy is significantly trapped within the shocked disc. The prompt appearance of an electromagnetic counterpart, delayed by the light–travel time to the recoil radius (a few hours) therefore seems promising (cf. de Mink & King 2017).

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