Impact of Variable Fluid Properties and Double Diffusive Cattaneo–Christov Model on Dissipative Non-Newtonian Fluid Flow Due to a Stretching Sheet

Khalil M. Khalil 1, A. Soleiman 1 and Ahmed M. Megahed 2, * and W. Abbas 3

1 Department of Mathematics, College of Science and Arts—Gurayat, Jouf University, Sakakah 77454, Saudi Arabia; kmshalaby@ju.edu.sa (K.M.K); asoliman@ju.edu.sa (A.S.)
2 Department of Mathematics, Faculty of Science, Benha University, Benha 13518, Egypt
3 Basic and Applied Science Department, College of Engineering and Technology, Arab Academy for Science, Technology and Maritime Transport, Cairo 11511, Egypt; wael_abass@aast.edu

* Correspondence: ahmed.abdelbaqk@fsc.bu.edu.eg

Abstract: The present work focuses on the attributes of flow, heat, and mass transfer together with double diffusive Cattaneo–Christov mechanism with regards to their applications. The aim of this study is to investigate the non-Newtonian Powell–Eyring fluid flow, taking into account the twofold impact of the heat generation mechanism and the viscous dissipation due to an extensible sheet. The chemical reaction between the fluid particles and the fluid variable properties is assumed in this study. The motive behind this study is the continuous and great interest in the utilization of non-Newtonian liquids in organic and technical disciplines. This model is administered and governed by the momentum equation, energy equation, and concentration, all of which are in the form of partial differential equations. With the help of the shooting technique, the numerical solution is obtained. Graphs show the characteristics of flow, heat, and mass transfer mechanisms for various governing parameters. Additionally, significant physical non-dimensional quantities have been presented in a tabular form. The outcomes detect that increasing the Deborah number, which is connected with the mass transfer field and the chemical reaction parameter, decreases the concentration distribution.

Keywords: Cattaneo–Christov model; Powell–Eyring fluid; chemical reaction; variable fluid properties

1. Introduction

With the rapid progress of engineering design innovations, the non-Newtonian fluid as a new material has gained substantial attention. This wide interest is owing to their importance in numerous industrial applications, such as modern technology, transportation, biomedicine, and electronics. Important applications also include the expulsion of elastic sheets, veins, water system channels and the cooling of continuous strips, etc. Because of these diverse and novel technological and innovative applications for non-Newtonian models, many researchers have looked into this topic and its various applications. For example, a non-Newtonian micropolar type was presented by Haliza et al. [1], with further subtleties on the topic of non-Newtonian Maxwell fluids able to be viewed in [2–4], while the non-Newtonian Casson fluid has been evaluated under the impact of slip phenomena by Imran et al. [5]. On the same topic, Sharma et al. [6] explored the non-Newtonian Casson–Williamson fluid flow over a stretched sheet of non-uniform thickness, whereas Muhammad and Samia [7] investigated the non-Newtonian Powell–Eyring fluid due to a stratified sheet with mixed convection. Similarly, Megahed and Abbas [8] studied non-Newtonian cross fluid under the impact of the thermal stratification phenomenon.

The investigations listed above are limited to traditional heat and mass transfer models that are controlled by Fourier laws. The characteristics of heat transfer can be determined precisely if we consider the fact that the relaxation time for velocity fields must occur during
the fluid flow. The relaxation time phenomenon that influences both the concentration
and the temperature fields is governed by the Cattaneo–Christov model [9]. Nowadays,
the Cattaneo–Christov heat motion model is a type of Fourier law amended by fusing
the relaxation time term with the spread of heat flux through the physical model [10].
Hayat et al. [11] explored the impact of variable conductivity on the non-Newtonian Jeffrey
fluid with the Cattaneo–Christov model and variable thickness. Cattaneo–Christov double
diffusion influence in the non-Newtonian Walters-B nanofluid has been investigated by
Hayat et al. [12]. The Cattaneo–Christov model was further scrutinized by Sui et al. [13] for
Maxwell nanofluids with a slip velocity phenomenon, by Shehzad et al. [14] for third-grade
fluid flow, by Li et al. [15] for a viscoelastic fluid model under the effect of the magnetic
field, by Meraj et al. [16] for a Jeffrey fluid model together with Darcy–Forchheimer
flow, and by Shehzad et al. [17] for chemically non-Newtonian Maxwell liquid with 3D
hydromagnetic flow. Subsequent studies regarding the topic of the Cattaneo–Christov
model are introduced in Refs. [18–21]. The shooting method was used in most prior studies,
as it is in this one, although there are a variety of different numerical approaches that can
be used to solve the Navier–Stokes equations, which have been studied in Refs. [22,23].

The previously mentioned examinations reveal that a lot of investigations might be
cited regarding the matter of Cattaneo–Christov through different types of non-Newtonian
fluid. In any case, fewer studies are considered through the literature review that imply the
combination of the Cattaneo–Christov model and the non-Newtonian Powell–Eyring fluid.
Yet, no review has talked about the Cattaneo–Christov heat flux model on a dissipative
non-Newtonian Powell–Eyring fluid with variable fluid properties and a chemical reaction
mechanism. The creativity of the displayed problem is enhanced with the extra effects of
the heat generation mechanism and the viscous dissipation phenomenon. The current study
is performed with the assumption of the fluid flow due to the linear stretching of an elastic
sheet. Finally, we adopted a shooting procedure to deal with such a physical problem.

2. Fundamental Governing Equations

Herein, we will consider consistent two-dimensional laminar Powell–Eyring fluid
flow and heat mass transfer together with the Cattaneo–Christov phenomenon. This type
of non-Newtonian fluid can be evermore characterized by the parameters $\beta$ and $c$. The fluid
velocity components are assumed to be symbolized by $u$ and $v$ in $x-$ and $y-$ directions,
respectively (Figure 1). The fluid particles are likewise assumed to chemically react together
with their gaining warmth due to the presence of the heat generation source. It is also
presumed that the sheet has a constant temperature $T_w$ and it is extended with speed
$u_w = ax$.  

![Figure 1. Physical configuration.](image-url)
Furthermore, the impact of the viscous dissipation on the heat transfer mechanism through the thermal boundary layer is taken into account. Considering the above physical assumptions, the administering equations for the Powell–Eyring model can be introduced as follows [24]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{u \partial T}{\partial x} + v \frac{\partial T}{\partial y} + \lambda_1 \Phi_E = \frac{1}{\rho_{\infty} \epsilon \rho_p} \left( \kappa \frac{\partial T}{\partial y} \right) + \frac{Q}{\rho_{\infty} \epsilon \rho_p} (T - T_\infty) \tag{2}
\]

\[
\frac{u \partial C}{\partial x} + \frac{v \partial C}{\partial y} + \lambda_2 \Phi_C = D \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_\infty). \tag{3}
\]

The fluid flow is exposed to the following boundary conditions:

\[
u = u_w = ax, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0, \tag{5}
\]

\[
u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{at} \quad y \to \infty, \tag{6}
\]

where

\[
\Phi_E = u \frac{\partial u}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} + u \frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + v \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + 2uv \frac{\partial^2 T}{\partial x \partial y} + \frac{\partial u}{\partial x} \frac{\partial T}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial C}{\partial x}, \tag{7}
\]

\[
\Phi_C = u \frac{\partial u}{\partial x} \frac{\partial C}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial C}{\partial y} + u \frac{\partial C}{\partial x} \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \frac{\partial C}{\partial y} + 2uv \frac{\partial^2 C}{\partial x \partial y} + \frac{\partial u}{\partial x} \frac{\partial C}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial C}{\partial x}, \tag{8}
\]

where \( \kappa \) is the fluid thermal conductivity, \( \rho_{\infty} \) is the ambient density, \( \mu \) is the Powell–Eyring viscosity, \( T \) is the fluid temperature, \( C \) is the fluid concentration, \( T_\infty \) is the sheet temperature, \( C_w \) is the sheet concentration, \( T_\infty \) is the encompassing fluid temperature, \( C_\infty \) is the encompassing fluid concentration, \( k_0 \) is the rate of chemical reaction, \( \lambda_1 \) is the relaxation time for the heat flux, \( D \) is the coefficient of diffusion, \( \lambda_2 \) is the relaxation time for the mass flux and \( Q \) is the coefficient of heat generation (absorption).

In light of Equations (1)–(4), pick the following proper dimensionless transformations as:

\[
u = ax f'(\eta), \quad v = -\sqrt{\frac{a}{\nu_\infty}} f'(\eta), \quad \eta = \frac{a}{\nu_\infty}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \tag{9}
\]

Considering that the liquid conductivity and the liquid viscosity modifies with temperature as indicated by the accompanying relations [25]:

\[
\mu = \mu_\infty e^{-\gamma \theta}, \tag{10}
\]

\[
\kappa = \kappa_\infty (1 + \epsilon \theta), \tag{11}
\]

where \( \mu_\infty \) is the viscosity of the Powell–Eyring fluid at the ambient, \( \gamma \) is the viscosity parameter, \( \kappa_\infty \) is the liquid thermal conductivity at the surrounding and \( \epsilon \) is the thermal conductivity parameter. Now, considering the last supposition for both the liquid viscosity and the liquid conductivity, Equation (1) is consequentially fulfilled while the remaining equations reduce to the following system:

\[
f''(e^{-\gamma \theta} + \alpha (1 - (\delta f')^2)) - \gamma f' e^{-\gamma \theta} - f'' + f'' = 0, \tag{12}
\]
\[
\frac{1}{Pr} \left( (1 + \epsilon \theta) \theta'' + \epsilon \theta'^2 \right) + f \theta' - De_1 \left( f f' \theta' + f^2 \theta'' \right) + \beta_1 \theta + Ec f'' \left( e^{-\gamma \theta} + \kappa \right) - \frac{\alpha \delta f'^2}{3} = 0 \quad (13)
\]

\[
\phi'' + Sc f \phi' - Sc De_2 \left( f f' \phi' + f^2 \phi'' \right) - Sc \xi \phi = 0. \quad (14)
\]

Additionally, after invoking the previous dimensionless variables, the related physical boundary conditions are transformed as

\[
f = 0, \quad f' = 1, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0, \quad (15)
\]

\[
f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{at} \quad \eta \to \infty, \quad (16)
\]

The arisen dimensionless governing parameters are the Powell–Eyring parameters \( \alpha, \delta \), the thermal Deborah number \( De_1 \), the Eckert number \( Ec \), the Prandtl number \( Pr \), the heat generation (absorption) parameter \( \beta_1 \), the Deborah number which related to the mass transfer field \( De_2 \) and the chemical reaction parameter \( \xi \) which can be characterized as follows:

\[
\alpha = \frac{1}{\mu_\infty \rho c}, \quad \delta = \frac{\alpha_\mu_\infty^2 \rho_\infty}{2c^2 \mu_\infty}, \quad De_1 = \lambda_1 a, \quad Ec = \frac{\mu_\infty}{c_p(T_w - T_\infty)}, \quad (17)
\]

\[
Pr = \frac{\mu_\infty c_p}{\kappa_\infty}, \quad \beta_1 = \frac{Q}{a \rho c_p}, \quad De_2 = \lambda_2 a, \quad \xi = \frac{k_0}{\alpha}. \quad (18)
\]

Moreover, as indicated by the previous dimensionless variables in Equation (9), the local skin friction \( Cf_x \), local Nusselt number \( Nu_x \) and local Sherwood number \( Sh_x \) are introduced as follows:

\[
Cf_x Re^{\frac{1}{2}} = - \left( (e^{-\gamma \theta(0)} + \alpha) f''(0) - \frac{\alpha \delta}{3} f'^3(0) \right), \quad Nu_x Re^{\frac{1}{2}} = -\theta'(0), \quad Sh_x Re^{\frac{1}{2}} = -\phi'(0) \quad (19)
\]

where \( Re = \frac{\mu_\infty x}{\nu_\infty} \) is the local Reynolds number.

3. Solution Approach

The goal of this part is to use the numerical method known as the shooting method to come up with numerical solutions to our problem. In terms of ordinary differential equations, this method can be used to solve the initial value problem. This method’s technique entails converting the governing system of ordinary differential Equations (12)−(14) into a new first-order system. The resulting system is numerically integrated with the updated boundary conditions using the Runge–Kutta (RK) technique, which incorporates a shooting scheme. The most notable characteristic of the present method is the selection of appropriate finite values of \( \eta \to \infty \). To obtain \( f''(0), \theta'(0) \) and \( \phi'(0) \) for the mentioned boundary value problem, we use some initial guessed values on a specified set of miscellaneous parameters. This approach instantly turns the previously governing equations into the following equations:

\[
W_1 = f(\eta), \quad W'_1 = W_2, \quad W'_2 = W_3, \quad (20)
\]

\[
W'_3 = \frac{\gamma W_5 W_6 e^{-\gamma W_4} + W_2^2 - W_1 W_4}{e^{-\gamma W_4} + \alpha (1 - \delta W_3^2)}, \quad (21)
\]

\[
W_4 = \theta(\eta), \quad W'_4 = W_5, \quad (22)
\]

\[
W'_5 = \frac{De_1 \left( W_1 W_2 W_5 + W_1^2 W_5 \right) - W_5 \left( W_1 + \frac{\xi}{Pr} \right) - \beta_1 W_4 - Ec \left( \left( e^{-\gamma W_4} + \alpha \right) W_3^2 - \frac{\alpha \delta}{3} W_3^3 \right)}{\frac{1}{Pr} (1 + \epsilon W_4)}, \quad (23)
\]
\[ W_6 = \phi(\eta), \quad W'_6 = W_7, \quad \text{(24)} \]
\[ W'_7 = ScDe_2(W_1W_2W_7 + W_2^2W'_7) - ScW_1W_7 + Sc\xi W_6. \quad \text{(25)} \]

The boundary conditions are now as follows:

\[ W_1(0) = 0, \quad W_2(0) = 1, \quad W_4(0) = 1, \quad W_6(0) = 1, \quad \text{(26)} \]
\[ W_3(0) = \epsilon_1, \quad W_5(0) = \epsilon_2, \quad W_7(0) = \epsilon_3, \quad \text{(27)} \]

where \( \epsilon_1, \epsilon_2 \) and \( \epsilon_3 \) are determined using Newton’s methodology so that the outer boundary conditions \( W_2(\infty), W_4(\infty) \) and \( W_6(\infty) \) are fulfilled. To obtain the required solution, we employ the traditional RK technique to integrate the generated ODEs with the specified set of parameters. Finally, to meet the convergence condition, the process is repeated until the results are precise to the anticipated level of \( 10^{-7} \) accuracy.

4. Verification of Numerical Methodology

The motivation behind this segment is to prove the effectiveness and trustworthiness of the numerical results obtained via the shooting technique. For this reason, Table 1 is set up below as an examination between our results and the recently published work by Javed et al. [26]. The viscosity parameter was disregarded \( \gamma = 0 \), but both Powell–Eyring parameters \( \alpha \) and \( \delta \) were changed to see how they affected the local skin friction coefficient \( C_f \). By alluding to this table, the validity and unwavering quality of all the numerical results of this exploration are confirmed.

| \( \alpha \) | \( \delta \) | Javed et al. [26] | Present Work |
|-----------|-----------|-----------------|--------------|
| 0.0       | 0.0       | 1.0000          | 1.0000000000 |
| 0.2       | 0.0       | 1.0954          | 1.0953998521 |
| 0.4       | 0.0       | 1.1832          | 1.1831852914 |
| 0.2       | 0.0       | 1.0954          | 1.0953998521 |
| 0.2       | 0.1       | 1.0940          | 1.0940001201 |
| 0.2       | 0.2       | 1.0924          | 1.0923889767 |

5. Outcomes with Discussion

This section intends to investigate the impact of the current problem’s parameters on the momentum, temperature, and concentration of the non-Newtonian Powell–Eyring fluid through the following graphical representations. To this end, some figures are created by varying the value of a parameter within a specified range, while the others are rigidly assigned, such as \( \delta = 0.3, \gamma = 0.2, \alpha = 0.3, De_1 = 0.2, \beta_1 = 0.1, \epsilon = 0.2, Sc = 0.7, De_2 = 0.2, Ec = 0.2, \xi = 0.2 \) and \( Pr = 1.5 \). Figure 2 shows the impact of the \( \alpha \) parameter on the velocity field \( f'(\eta) \), temperature field \( \theta(\eta) \) and the concentration field \( \phi(\eta) \). The ascent in the value of this parameter causes an upgrade in the speed field \( f'(\eta) \), although the converse pattern is noticed for both the temperature \( \theta(\eta) \) and the concentration field \( \phi(\eta) \). Physically, the stimulation effect occurs as the \( \alpha \) parameter increases, causing the fluid velocity to increase while the concentration and temperature distributions decrease.
The impact of the $\delta$ parameter on the velocity profile, temperature profile, and concentration profile is shown in Figure 3. The heightening values of the $\delta$ parameter slightly improve both the temperature $\theta(\eta)$ and the concentration fields $\phi(\eta)$, whereas the opposite with the same marginal effect is noticed for the velocity field $f'(\eta)$. The Powell–Eyring parameter $\delta$ is physically dependent on the fluid density. As a result, as the Powell–Eyring parameter $\delta$ is increased, the density of the fluid particle increases, and the velocity profile decreases.

Figure 4 is plotted to examine the velocity profiles $f'(\eta)$, the temperature profiles $\theta(\eta)$ and the concentration fields $\phi(\eta)$, for expanding values of the viscosity parameter $\gamma$. The dependence of fluid viscosity on the temperature through the presence of $\gamma$ invigorates both the fluid temperature $\theta(\eta)$ and the fluid concentration $\phi(\eta)$ distribution, while the presence of the $\gamma$ parameter hinders the liquid velocity $f'(\eta)$ through the boundary layer. Physically, increasing the magnitude of the viscosity parameter acts as a retarding factor, causing the fluid velocity to slow down and, as a result, the friction between the fluid layers increases, causing the fluid temperature to enhance.

Figure 5 has been portrayed to delineate the impact of the thermal conductivity parameter $\varepsilon$. The larger values of the thermal conductivity parameter $\varepsilon$ relate to the larger temperature distribution and marginal increments in the concentration field, while the growth in a same parameter makes insignificant decreases in the speed profiles. Physically, increasing the value of $\varepsilon$ enhances the fluid thermal conductivity, which increases both the fluid temperature and the thickness of the thermal layer.
Figure 4. (a) \( f'(\eta) \) for picked \( \gamma \). (b) \( \theta(\eta) \) and \( \phi(\eta) \) for picked \( \gamma \).

Figure 5. (a) \( f'(\eta) \) and \( \phi(\eta) \) for picked \( \epsilon \). (b) \( \theta(\eta) \) for picked \( \epsilon \).

The actions of the velocity profiles \( f'(\eta) \), the temperature profiles \( \theta(\eta) \) and the concentration fields \( \phi(\eta) \) for escalating values of the thermal Deborah number \( De_1 \) are depicted in Figure 6. The velocity distribution \( f'(\eta) \) barely rises for the larger \( De_1 \) parameter, while the concentration profiles \( \phi(\eta) \) are somewhat marginally decreased with the expansion of the same parameter \( De_1 \). In addition, expanded values of the same parameter \( De_1 \) lower both the temperature profiles and the thermal boundary thickness. In terms of physics, when the thermal Deborah number rises, the relaxation time for the heat flux also increases, meaning that both the thermal boundary layer thickness and the mechanism of heat diffusion become low.

Figure 6. (a) \( f'(\eta) \) and \( \phi(\eta) \) for picked \( De_1 \). (b) \( \theta(\eta) \) for picked \( De_1 \).
Figure 7 demonstrates the impact of the Eckert number $Ec$ on the velocity $f' (\eta)$, thermal $\theta (\eta)$ and concentration $\phi (\eta)$ profiles. The upsurge in Eckert number $Ec$ clearly works by improving both the thermal boundary thickness and the temperature profile. Further, a barely noticeable upgrade for the concentration profiles $\phi(\eta)$ is made due to the Eckert number $Ec$, while the inverse marginally noticeable pattern is noticed for the speed profile $f' (\eta)$. In physical terms, due to the dissipation phenomena, enhancing the value of $Ec$ causes more heat energy to be created. This could explain why the temperature distribution within the thermal boundary layer is improving.

![Figure 7](image.png)

Figure 7. (a) $f' (\eta)$ and $\phi(\eta)$ for picked $Ec$. (b) $\theta(\eta)$ for picked $Ec$.

Finally, the endeavor to evoke the impact of both the Deborah number, which connected with the mass transfer $De_2$, and the chemical reaction parameter $\xi$ on the mass transfer mechanism $\phi(\eta)$ has been made by drawing Figure 8. It is obvious that the increment in both parameters $De_2$ and $\xi$ may help in creating an obstruction for the behavior of the fluid concentration mechanism $\phi(\eta)$. Physically, a higher $De_2$ indicates a longer period for the mass flux to relax. As a result, with greater $De_2$, the minimum mass diffusion rate occurs, lowering the fluid concentration.

![Figure 8](image.png)

Figure 8. (a) $\phi(\eta)$ for picked $De_2$. (b) $\phi(\eta)$ for picked $\xi$.

The numerical outcomes for the local Nusselt number, the local Sherwood number, and the local skin-friction coefficient are displayed in Table 2 for different values of the arising parameters. From Table 2, it may be seen that because of an increment in the Powell–Eyring parameter $a$ there is an expansion in the local skin-friction coefficient, the local Nusselt number, and the local Sherwood number, whereas the inverse pattern is noticed for the Powell–Eyring parameter $\delta$. Additionally, from the same table, we note that the viscosity parameter decreases the local skin-friction coefficient, the local Nusselt
number, and the local Sherwood number. Further, Table 2 shows that the local skin-friction coefficient increments with expanding values of the thermal Deborah number, whereas it diminishes with the expanding values of the thermal conductivity parameter and the Eckert number. Furthermore, it is discernible that both the Eckert number and the chemical reaction parameter contribute to strengthening the local Sherwood number.

| $\alpha$ | $\delta$ | $\gamma$ | $\epsilon$ | $De_1$ | $Ec$ | $De_2$ | $\xi$ | $C_f(Re_x)^{1/2}$ | $Nu_x(Re_x)^{1/2}$ | $Sh_x(Re_x)^{1/2}$ |
|---------|---------|---------|---------|-------|-----|-------|-----|----------------|----------------|----------------|
| 0.0     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 0.920887      | 0.468142       | 0.599312       |
| 0.4     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.112051      | 0.502697       | 0.622836       |
| 0.6     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.197817      | 0.513244       | 0.631776       |
| 1.0     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.354441      | 0.527314       | 0.646012       |
| 0.3     | 0.0     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.074721      | 0.497792       | 0.618709       |
| 0.3     | 0.4     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.064221      | 0.495515       | 0.617431       |
| 0.3     | 0.6     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.058640      | 0.494284       | 0.616745       |
| 0.3     | 1.0     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.046602      | 0.491591       | 0.615261       |
| 0.3     | 0.3     | 0.0     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.334321      | 0.508799       | 0.625702       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.064230      | 0.496106       | 0.617670       |
| 0.3     | 0.3     | 0.5     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 0.973111      | 0.476591       | 0.606035       |
| 0.3     | 0.3     | 0.8     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 0.888698      | 0.456668       | 0.594712       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.0   | 0.2 | 0.2   | 0.2 | 1.068531      | 0.567536       | 0.617941       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.066933      | 0.496106       | 0.617670       |
| 0.3     | 0.3     | 0.2     | 0.5     | 0.2   | 0.2 | 0.2   | 0.2 | 1.064971      | 0.420572       | 0.617541       |
| 0.3     | 0.3     | 0.2     | 0.8     | 0.2   | 0.2 | 0.2   | 0.2 | 1.063412      | 0.366929       | 0.617372       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.0   | 0.2 | 0.2   | 0.2 | 1.065840      | 0.465177       | 0.617651       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.2 | 1.066933      | 0.496106       | 0.617670       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.4   | 0.2 | 0.2   | 0.2 | 1.068043      | 0.528122       | 0.617811       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 0.0 | 0.2   | 0.2 | 1.068581      | 0.609484       | 0.617951       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.0   | 0.2 | 1.066933      | 0.496106       | 0.617670       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.5   | 0.2 | 0.2   | 0.2 | 1.064741      | 0.326569       | 0.617477       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.0 | 1.066930      | 0.496106       | 0.609967       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.5   | 0.2 | 1.066930      | 0.496106       | 0.631066       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 1.0 | 0.2   | 0.2 | 1.066930      | 0.496106       | 0.658297       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.0 | 1.066930      | 0.496106       | 0.481006       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.5 | 1.066930      | 0.496106       | 0.617670       |
| 0.3     | 0.3     | 0.2     | 0.2     | 0.2   | 0.2 | 0.2   | 0.5 | 1.066930      | 0.496106       | 0.775697       |

6. Conclusions

In this research, we have examined the impact of the double diffusive Cattaneo–Christov phenomenon on the non-Newtonian Powell–Eyring fluid flow and heat mass transfer, together with viscous dissipation due to a stretching sheet. The controlling nonlinear equations were formed and tackled numerically by means of shooting technique. The acquired outcomes suggest the accompanying pronouncements.

1. The temperature distribution improves as the thermal conductivity parameter and the Eckert number improve, whereas the concentration distribution is affected by the chemical reaction parameter, which has a decreasing effect.
2. The higher the viscosity parameter, the slower the fluid velocity becomes, and the lower the thermal Deborah number, the higher the thermal distribution becomes.
3. The Nusselt number value diminishes for both higher values of thermal conductivity parameter and viscosity parameter.
4. There is an increase in the Sherwood number with an increment in the chemical reaction parameter or with a decline in the viscosity parameter.
5. The impact of diminishing the thermal Deborah number or expanding the thermal conductivity parameter reduces the speed profiles.

6. In the future, we intend to expand on this research by looking at chemically reactive mixed convective non-Newtonian fluid flows, as well as heat and mass fluxes, in order to control the cooling process.

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**Nomenclature**

\( a \) velocity coefficient
\( c \) Powell–Eyring parameter
\( c_p \) specific heat at constant pressure
\( C \) nanoparticles concentration
\( C_{f_x} \) skin friction coefficient
\( C_w \) surface nanoparticles concentration
\( C_\infty \) ambient nanoparticles concentration
\( D_{E1} \) thermal Deborah number
\( D_{E2} \) the mass Deborah number
\( Ec \) Eckert number
\( f \) dimensionless stream function
\( k_0 \) the rate of chemical reaction
\( Nu_x \) local Nusselt number
\( Pr \) Prandtl number
\( Q \) heat generation (absorption) coefficient
\( Re_x \) local Reynolds number
\( Sc \) Schmidt number
\( T \) fluid temperature
\( T_w \) surface temperature
\( T_\infty \) ambient temperature
\( u \) velocity component in the \( x \)– direction
\( v \) velocity component in the \( y \)– direction
\( x, y \) Cartesian coordinates

**Greek symbols**

\( \rho \) density of the fluid
\( \mu \) coefficient of viscosity
\( \nu \) kinematic viscosity
\( \theta \) dimensionless temperature
\( \phi \) dimensionless concentration
\( \lambda_1 \) relaxation time for heat flux
\( \lambda_2 \) relaxation time for mass flux
\[ \eta \] similarity variable
\[ \gamma \] viscosity parameter
\[ \epsilon \] thermal conductivity parameter
\[ \zeta \] chemical reaction parameter
\[ \beta \] Powell–Eyring parameter
\[ \beta_1 \] the heat generation (absorption) parameter
\[ \kappa \] thermal conductivity

Superscripts
\[ \prime \] differentiation with respect to \( \eta \)
\[ \infty \] free stream condition
\[ \omega \] wall condition

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