Decay Constants and Distribution Amplitudes of B Meson in the Relativistic Potential Model

Hao-Kai Sun\[ and Mao-Zhi Yang\[†

School of Physics, Nankai University, Tianjin 300073, P.R. China

(Dated: September 29, 2016)

Abstract

In this work we study the decay constants of $B$ and $B_s$ mesons based on the wave function obtained in the relativistic potential model. Our results are in good agreement with experiment data which enables us to apply this method to the investigation of $B$-meson distribution amplitudes. A very compact form of the distribution amplitudes is obtained. We also investigate the one-loop QCD corrections to the purely leptonic decays of $B$ mesons. We find that, after subtracting the infrared divergence in the one-loop corrections using the factorization method, the QCD one-loop corrections to the leptonic decay amplitude will be zero.

PACS numbers: 12.39.Pn,12.39.St,13.30.Ce,14.40.Nd

Keywords: Decay Constant, Distribution Amplitudes, Factorization
I. INTRODUCTION

The study of B-meson decays, especially the exclusive semi-leptonic and two-body non-leptonic channels, presents us rich information for testing and understanding the Standard Model (SM). In the past two decades, as the running and upgrading of B-factories, BaBar and Belle and also the CLEO, LHCb experiments, a great amount of experimental data has been accumulated. Although a lot of models and/or approaches have been developed in theoretical research, the poor knowledge of non-perturbative Quantum Chromodynamics (QCD) effects still limit theoretical predictions severely. In two-body non-leptonic decays of B-meson, QCD factorization theorems [1–4] and perturbative QCD methods [5–9] have been developed, which allows to separate the non-perturbative effects out as the universal quantities, such as, the light-cone distribution amplitudes (LCDA) and the form factors. The B-meson LCDA has been studied extensively and thus several models are proposed in the literature [10–18].

Inspired by the construction of initial bound state in Ref.[19] and based on our previous works on mass spectrum [20, 21] and wave functions of B-meson [22], we try an alternate way to study the LCDA with the help of the wave function obtained in the relativistic potential model [20, 21]. Considering the recent experimental data on the pure leptonic decays of B meson, in this paper, we focus on a careful investigation about the decay constants and the distribution amplitues (DAs) of B-meson.

In particular, the decay constants of the heavy-light mesons are related directly to the pure leptonic decay widths and thus provide a way to check different theoretical models and may give hints for the physics beyond the Standard Model (SM). During the past decades, many methods have been applied to the study of the decay constants from QCD sum rules [23–29], the Bethe-Salpeter equation [30, 31], the field correlator method [32], the soft-wall holographic approach [33], the potential models [22, 34–37], etc., to the Lattice QCD simulations [38–45]. Up to now there are still large uncertainties for the value of $|V_{ub}|$ [46] and only the $\tau$-channel in purely leptonic decay are measured in experiment [47–50] (also with large uncertainties). Our results are well located in the experimental error bars [47–51]. Further tests are needed from experiments in future with enhanced precision (most possibly comes from Belle II / SuperKEKB collaboration [52, 53]).

As we do not make the light-cone and heavy-quark approximations, our study on B-
meson distribution amplitudes achieves more general results. The analytical forms both in coordinate and momentum space are obtained. When they are transformed to the commonly used form of LCDA, the figures clearly show that our results are reasonable since they obey the model-independent limitations [13]. We also consider the pure leptonic decays of \(B\)-meson to QCD one-loop level, and we find that one-loop corrections in QCD will be zero after subtracting the infrared divergence by using the factorization method.

Our paper is organized as follows. In Sec.II, after briefly introducing how the initial state is constructed, we focus on the detailed procedures and restrictions used to calculate the decay constants of the \(B\)-meson. Through the pure leptonic decay channels, our results are verified with the experiment data and we also give a simple discussion about the possible experiment in the future. Next, we continue to the study of the matrix element of \(B\) meson which defines the distribution amplitudes (DAs) in Sec.III. The analytical forms of the matrix element and DAs are obtained and figures are shown as illustrations. We deduce a compact expression of the matrix element containing the whole Dirac structure. Sec.IV is devoted to the study of the pure leptonic decay of the \(B\)-mesons up to one-loop level in QCD and Sec.V is for the conclusion and discussion.

II. DECAY CONSTANTS OF \(B\) AND \(B_s\) MESONS

Recently, the spectra of heavy-light quark-antiquark system have been studied in the relativistic potential model in our previous works, where hyper-fine interactions are included [20, 21]. The whole spectra of \(B\) and \(D\) system are well in agreement with experimental measurements. Hence in this work, we extend our previous work [20, 21] by studying the decay properties of \(B\) meson with the wave functions obtained in the relativistic potential model. We study the decay constants of \(B\) and \(B_s\) mesons at first, and then give a compact form of distribution amplitudes of \(B\)-meson, which shall be useful for studying \(B\) decays.

The decay constant of a pseudoscalar meson is defined by the matrix element of the axial current between the meson state and the vacuum

\[
\langle 0 | \bar{q} \gamma^\mu \gamma^5 Q | P \rangle = i f_P P^\mu
\]

(1)

where the axial current is composed of a light antiquark field \(\bar{q}\) and a heavy quark field \(Q\).

We describe the pseudoscalar meson as the bound state of an antiquark and a quark [19]
\[ P(\vec{P}) = \frac{1}{N_L^{1/3}} \sum_i \int d^3 k_q d^3 k_Q \delta^{(3)}(\vec{P} - \vec{k}_q - \vec{k}_Q) \Psi_0(\vec{k}_q) \]
\[ = \frac{1}{\sqrt{2}} \left[ c^i(\vec{k}_Q, \uparrow) b^i(\vec{k}_q, \downarrow) - c^i(\vec{k}_Q, \downarrow) b^i(\vec{k}_q, \uparrow) \right] \bigg| 0 \bigg\rangle \]

where \( N_L \) is the normalization factor, and the normalization conditions will be shown explicitly below. \( i \) stands for the QCD color index and \( \frac{1}{\sqrt{3}} \) is the corresponding normalization factor. Also, the factor \( \frac{1}{\sqrt{2}} \) is the normalization factor for the spin fields which are indexed by up or down arrows. Inside the square parenthesis, \( b^i \) and \( c^i \) are the creation operators of the light antiquark \( \bar{q} \) and the heavy quark \( Q \), respectively.

The function \( \Psi_0(\vec{k}_q) \) is the normalized wave function of the pseudoscalar meson at ground state (subscript 0) in the momentum space, which describes the wave function of the two quark constituents in a meson. It is noted here that these quark constituents are the effective quarks carrying a gluon cloud and therefore the quarks have constituent masses [54].

The wave function can be solved from the Schrödinger type wave equation with relativistic dynamics
\[ (H_0 + H')\Psi(\vec{r}) = E\Psi(\vec{r}), \]
where \( H_0 + H' \) is the effective Hamiltonian (their explicit expressions can be found in reference [21]) and \( E \) is the energy of the meson. The first term \( H_0 \) contains the kinetic part and the effective potential which is taken as a combination of a Coulomb term and a linear confining term inspired by QCD [34, 55, 56].

The second term \( H' \) is the spin-dependent part of the Hamiltonian resulting from one-gluon-exchange diagram in the nonrelativistic approximation [34, 57], and new terms which accounts for contributions of nonperturbative dynamics in the bound state system and relativistic corrections for the light quark in the heavy meson [20, 21].

The normalization conditions for wave function are
\[ \int d^3k \left| \Psi_0(\vec{k}) \right|^2 = 1, \]
\[ \left\{ c(\vec{k}, s), c^\dagger(\vec{k}', s') \right\} = \delta_{ss'} \delta^{(3)}(\vec{k} - \vec{k}'), \]
\[ \langle P(\vec{P}) | P(\vec{P}') \rangle = (2\pi)^3 2E \delta^{(3)}(\vec{P} - \vec{P}'). \]

Note that we omit the color index of the operator \( c \) and use \( s, s' \) to denote the spin states. Substituting Eq.(2) into Eq.(4c) and considering Eq.(4a) and Eq.(4b), we can obtain the
normalization factor

\[ N_L = \frac{1}{(2\pi)^{3/2}E}. \]  

(5)

The wave function has been solved numerically in our previous work [21]. For \( B \) meson, the wave function can be expressed by

\[ \Psi_0(\vec{k}) = \frac{\varphi_0(|\vec{k}|)}{|\vec{k}|}Y_{00}(\theta, \phi) \]  

(6)

where \( \varphi_0(|\vec{k}|) \) is the reduced wave function. The numerical result of \( \varphi_0(|\vec{k}|) \) can be shown in Fig.1.

Since it is convenient to have an analytical form of the wave function \( \Psi_0(\vec{k}) \) for the numerical calculation, we fit the wave function obtained in our previous work [21] with an exponential function and finally got the fitted form for the \( B(S) \) meson wave function as

\[ \Psi_0(\vec{k}) = a_1e^{a_2|\vec{k}|^2+a_3|\vec{k}|+a_4}, \]  

(7)

where the parameters for \( B \) meson: \( a_1 = 4.55, a_2 = -0.39, a_3 = -1.55, a_4 = -1.10 \) and for \( B_s \) meson: \( a_1 = 1.60, a_2 = -0.43, a_3 = -1.28, a_4 = -0.22 \). The illustrations are shown in Fig.2.

In the calculation of the decay constants, the four-momentum conservation law should hold

\[ k_q + k_Q = P, \]  

(8)

where \( k_{q,Q} \) and \( P \) are the momenta of the quark constituents and the meson respectively.
FIG. 2. The wave functions (W.F.) of $B$-meson.

With the restriction above, we take the ACCMM scenario [58, 59], where the light quark is kept on-shell, while the heavy quark off-shell,

\begin{align}
E_q + E_Q &= m_P, \\
E_q^2 &= m_q^2 + |\vec{k}|^2, \\
m_Q^2(\vec{k}) &= E_Q^2 - |\vec{k}|^2.
\end{align}

Eq.(9a) is the energy conservation in the meson rest frame. We also assume that the running mass of the heavy quark must be positive $m_Q(\vec{k}) \geq 0$. Thus the actual range of the momentum $|\vec{k}|$ is limited, which is shown as the cut lines in Figs.1 and 2.

Substituting Eq.(2) into Eq.(1) in the rest frame and contracting the quark (antiquark) creation operators with the quark (antiquark) annihilation operators in the quark field of the axial current $\bar{q}\gamma^\mu\gamma^5Q$, we obtain

\begin{equation}
 f_P = \sqrt{\frac{3}{(2\pi)^3 m_P}} \int d^3k \Psi_0(\vec{k}) \frac{(E_q + m_q)(E_Q + m_Q) - |\vec{k}|^2}{\sqrt{E_q E_Q (E_q + m_q) (E_Q + m_Q)}}.
\end{equation}

where the integral over the variable $\vec{k}$ shall be limited in the finite range according to Eqs.(9a)–(9c).

The parameters used in this work are [21]

\begin{equation}
 m_s = 0.32 \text{ GeV}, \quad m_u = m_d = 0.06 \text{ GeV},
\end{equation}

and the mesons’ masses are taken from PDG [46]

\begin{equation}
 m_B = 5.28 \text{ GeV}, \quad m_{B_s} = 5.37 \text{ GeV}.
\end{equation}
TABLE I. Theoretical results of the decay constants of $B$-mesons.

The errors are estimated by varying the parameters in the allowed ranges. The total errors are around 7% for the decay constants of $B$ and $B_s$ mesons. We also calculate the ratio of the decay constants of $B$ and $B_s$ mesons $f_{B_s}/f_B$. The final results obtained are

$$f_B = 219 \pm 15 \text{ MeV}, \quad f_{B_s} = 266 \pm 19 \text{ MeV}, \quad f_{B_s}/f_B = 1.21 \pm 0.09.$$  \hfill (13)

During past decades, many theoretical methods or models have been developed on the calculation of the $B$-meson decay constants. Here we list some of the results for comparison in Table I. From the table, one can see that our results are consistent with most of the theoretical predictions.

The branching ratio of pure leptonic decay of $B$ meson can be calculated by the following
TABLE II. Experimental results for $B(B^+ \to \tau^+ \nu_\tau)$. 

| Experiment | Tag       | $B$(units of $10^{-4}$) |
|------------|-----------|------------------------|
| Belle      | Hadronic  | $0.72^{+0.27}_{-0.25} \pm 0.11$ |
| Belle      | Semi-leptonic | $1.25 \pm 0.28 \pm 0.27$ |
| BABAR      | Hadronic  | $1.83^{+0.53}_{-0.49} \pm 0.24$ |
| BABAR      | Semi-leptonic | $1.7 \pm 0.8 \pm 0.2$ |

The branching ratio of $B \to \tau^+ \nu_\tau$ channel is measured by Belle and BABAR collaborations [47–50], which is shown in Table II.

$B(B^\pm \to l^\pm \nu_l) = \frac{G_F^2 m_l^2 m_B}{8\pi} \left(1 - \frac{m_l^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$, \hspace{1cm} (14)

where $G_F$ is the Fermi constant, $V_{ub}$ the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, $m_B$ and $m_l$ the $B^\pm$ meson and lepton masses respectively, and $\tau_B$ is the life time of $B^\pm$ meson.

In this work, we obtain

$B(B^+ \to e^+ \nu_e) = (1.17 \pm 0.18) \times 10^{-11}$, \hspace{1cm} (15a)

$B(B^+ \to \mu^+ \nu_\mu) = (5.01 \pm 0.78) \times 10^{-7}$, \hspace{1cm} (15b)

$B(B^+ \to \tau^+ \nu_\tau) = (1.41 \pm 0.22) \times 10^{-4}$, \hspace{1cm} (15c)

where the errors mostly result from the uncertainties of the decay constants $f_B$ and the CKM matrix element $|V_{ub}|$ [46]

$|V_{ub}| = (4.09 \pm 0.39) \times 10^{-3}$. \hspace{1cm} (16)

The branching ratio of $B \to \tau^+ \nu_\tau$ channel is measured by Belle and BABAR collaborations [47–50], which is shown in Table II.

Taking the large uncertainties of the experimental data into consideration, our predicted branching ratio of the decay channel $B^+ \to \tau^+ \nu_\tau$ (Eq.(15c)) is consistent with the experimental results.

As an upgrade of the Belle / KEKB experiment, the Belle II / SuperKEKB will start taking data from 2018. With a designed luminosity $8 \times 10^{35} \text{cm}^{-2}\text{s}^{-1}$, which is about 40 times larger than its predecessor, data sample corresponding to 50 ab$^{-1}$ will be accumulated within five years of operation [53]. It is expected to reduce both the statistical and systematic errors of the $B^+ \to \tau^+ \nu_\tau$ decay mode by a factor about 7 [65].
III. B-MESONS DISTRIBUTION AMPLITUDES

Based on the success of our predictions on the mass spectrum [20, 21] and the decay constants of B-mesons, we continue to study of the matrix element of B meson which defines the DAs. The matrix element and DAs are generally used in studying hadronic decays of B meson.

Generalizing the current in the definition of the decay constant in Eq.(1) from local to non-local operators and making use of the Fierz identity, we obtain the matrix element between the B meson state and the vacuum in coordinate space

\[ \tilde{\Phi}_{\alpha\beta}(z) = \langle 0 | \bar{q}(z) | Q_\alpha(0) \rangle | \bar{B}(P) \rangle \]

\[ = \frac{1}{4} \langle 0 | \bar{q}(z)Q(0) \rangle | \bar{B} \rangle \left( \frac{1}{4} \langle 0 | \bar{q}(z)\gamma^5 Q(0) \rangle | \bar{B} \rangle (\gamma^5)_{\alpha\beta} \right. \\
\left. + \frac{1}{8} \langle 0 | \bar{q}(z)\sigma^{\mu\nu}\gamma^5 Q(0) \rangle | \bar{B} \rangle (\sigma_{\mu\nu} \gamma^5)_{\alpha\beta} + \frac{1}{4} \langle 0 | \bar{q}(z)\gamma^\mu Q(0) \rangle | \bar{B} \rangle (\gamma^5)_{\mu\alpha}\beta \\
\right. - \frac{1}{4} \langle 0 | \bar{q}(z)\gamma^\mu \gamma^5 Q(0) \rangle | \bar{B} \rangle (\gamma^5)_{\alpha\beta} \] (17a)

for the remainder of the paper. Its definition is

\[ [z, 0] \equiv \text{Pexp}\left( i \int_0^z dx^\mu A_\mu(x) \right). \] (18)

According to the discrete symmetries of C, P, T and the properties of the Dirac matrices, the five matrix elements in the right-hand side of Eq.(17b) are related to four DAs \( \tilde{\phi}_i \) (\( i = P, T, A1, A2 \) and the tilde means they are in coordinate space) as defined in Ref.[10]

\[ \langle 0 | \bar{q}(z)Q(0) | \bar{B} \rangle = 0, \] (19a)

\[ \langle 0 | \bar{q}(z)\gamma^5 Q(0) | \bar{B} \rangle = -if_B m_B \tilde{\phi}_P, \] (19b)

\[ \langle 0 | \bar{q}(z)\sigma^{\mu\nu}\gamma^5 Q(0) | \bar{B} \rangle = -if_B \tilde{\phi}_T(P^\mu z^\nu - P^\nu z^\mu), \] (19c)

\[ \langle 0 | \bar{q}(z)\gamma^\mu Q(0) | \bar{B} \rangle = 0, \] (19d)

\[ \langle 0 | \bar{q}(z)\gamma^\mu \gamma^5 Q(0) | \bar{B} \rangle = f_B \left( i \tilde{\phi}_{A1} P^\mu - m_B \tilde{\phi}_{A2} z^\mu \right). \] (19e)

In our scenario, we calculate these five matrix elements in the B-meson rest frame \( P^\mu = (m_B, \vec{0}) \) by using the B meson state defined in Eq.(2). We confirmed that the matrix elements in Eq.(19a) and Eq.(19c) are indeed zero

\[ \langle 0 | \bar{q}(z)Q(0) | \bar{B} \rangle = \langle 0 | \bar{q}(z)\gamma^\mu Q(0) | \bar{B} \rangle = 0. \] (20)
For the pseudoscalar DA in Eq.(19b), we obtain (let $N_B \equiv \frac{i}{f_B} \sqrt{\frac{3}{(2\pi)^4 m_B}}$ for simplicity.)

$$\tilde{\phi}_P(z) = N_B \int d^3k \Psi_0(k) - \frac{(E_q + m_q)(E_Q + m_Q) + |k|^2}{\sqrt{E_q E_Q (E_q + m_q)(E_Q + m_Q)}} e^{-i k \cdot z},$$

(21)

where $k^\mu_q = (E_q, \vec{k})$ is the four-momentum of the light quark in the meson rest frame. It should be understood that the wave function $\Psi_0(k)$ has an arbitrary phase angle and it can be fixed in the calculation to obtain the positive decay constant by the definition in Eq.(1).

For the other DAs in Eqs.(19c) and (19e) (the detailed derivation can be found in Appendix A), we define two functions $A_T$ and $A$ at first

$$A_T(k^1, k^2, k^3) \equiv \Psi_0(k) \frac{E_q + m_q + E_Q + m_Q}{\sqrt{E_q E_Q (E_q + m_q)(E_Q + m_Q)}},$$

(22a)

$$A(k^1, k^2, k^3) \equiv \Psi_0(k) \frac{E_Q + m_Q - E_q - m_q}{\sqrt{E_q E_Q (E_q + m_q)(E_Q + m_Q)}},$$

(22b)

where $k^1, k^2, k^3$ are the components of the light quark momentum $\vec{k}$, and $\vec{k} = (k^1, k^2, k^3)$. Then we obtain the expressions

$$\tilde{\phi}_T(z) = N_B \int d^3k \left[ \frac{1}{3} \sum_i \int_0^{k_i} A_T(\eta_i, \ldots) \eta d\eta \right] e^{-i k \cdot z},$$

(23a)

$$\tilde{\phi}_{A2}(z) = N_B \int d^3k \left[ \frac{1}{3} \sum_i \int_0^{k_i} A(\eta_i, \ldots) \eta d\eta \right] e^{-i k \cdot z},$$

(23b)

$$\tilde{\phi}_{A1}(z) = -N_B \int d^3k e^{-i k \cdot z}$$

$$\left[ \Psi_0(k) \frac{(E_q + m_q)(E_Q + m_Q) - |k|^2}{\sqrt{E_q E_Q (E_q + m_q)(E_Q + m_Q)}} + E_q A(k^1, k^2, k^3) \right].$$

(23c)

For the details of the summation in the square parentheses containing the ellipsis, see Eq.(A3).

Now, with the Eqs.(21) and (23a)–(23c), the matrix element of $B$-meson in coordinate space can be rewritten as

$$\tilde{\Phi}_{\alpha\beta}(z) = \frac{-if_B}{4} \left\{ \left[ m_B \tilde{\phi}_P + \frac{1}{2} \tilde{\phi}_T (P^\mu z^\nu - P^\nu z^\mu) \sigma_{\mu\nu} \right. \right.$$

$$\left. + \left( \tilde{\phi}_{A1} P^\mu + i m_B \tilde{\phi}_{A2} z^\mu \right) \gamma_5 \right\} \alpha\beta.$$

(24)

In order to obtain the expressions of the DAs in momentum space, we make use of the amplitude of a decay process which can be expressed as a convolution

$$F = \int d^4z \tilde{\Phi}_{\alpha\beta}(z) \tilde{T}_{\beta\alpha}(z).$$

(25)
Substituting Eq.\((24)\) into Eq.\((25)\) and making some reductions (see Appendix B for details), we obtain

\[
\Phi_{\alpha\beta}(l) = \begin{cases} 
- & \text{if } B_m B_4 \\
\frac{i}{2} \phi_P(l) \sigma_{\mu\nu} \left( v^\mu \frac{\partial}{\partial l^\nu} - v^\nu \frac{\partial}{\partial l^\mu} \right) \\
\phi_{A1}(l) \gamma_5 & \phi_{A2}(l) \gamma_\mu \\
\end{cases} \end{eqnarray} 
\]

(26)

and

\[
F = \int d^3 l \Phi_{\alpha\beta}(l) T_{\beta\alpha}(l) \bigg|_{t^2 = m_q^2} .
\]

(27)

It is understood that the derivative \(\frac{\partial}{\partial l^\mu, \nu}\) in Eq.\((26)\) (which is called the momentum space projector \([11, 66]\)) act on the hard scattering kernel \(T_{\beta\alpha}(l)\) before \(l = k_q\) is set.

Since we do not make the light-cone approximation \(z^2 = 0\), the “new” convolution formula of Eq.\((27)\) is reduced to a 3D-integral which reserves more information about the transverse-momentum-dependence of the DAs and the hard scattering kernel. As expected, we obtain the four DAs in the momentum space,

\[
\begin{align*}
\phi_P(k_q^\mu) &= -N_B \left( \frac{E_q + m_q}{E_Q + m_Q} \right) \sqrt{k_3^2} \Psi_0(k), \\
\phi_T(k_q^\mu) &= \frac{N_B}{3} \sum_i \int_0^{k_3^i} A_T(\eta, \ldots) \eta d\eta, \\
\phi_{A2}(k_q^\mu) &= \frac{N_B}{3} \sum_i \int_0^{k_3^i} A(\eta, \ldots) \eta d\eta, \\
\phi_{A1}(k_q^\mu) &= -N_B \left[ \Psi_0(k) \left( \frac{E_q + m_q}{E_Q + m_Q} \right) \sqrt{k^2} + E_q A(k_1, k_2, k_3) \right],
\end{align*}
\]

(28a-d)

In general, the functional form of these DAs above plays an important role in the study of the \(B\)-meson decays \([13]\). Thus it is necessary and useful to give an illustration of them.

For simplicity, let \(k = (0, 0, k_3^i)\) and in Fig.3 we show how these four DAs vary with respect to \(k_3^i\). In the heavy-quark limit, one can obtain that one of the axial-vector DA \(\phi_{A2}\) is equal to the axial-tensor DA \(\phi_T\) \([10]\). For our results, as shown in Figs.3(c) and (d), these two DAs are indeed very close indicating that our scenario is reasonable and their difference reflects the influence of the finite heavy-quark mass.

One can also see that the figures for \(B\) and \(B_s\) mesons are very similar in the shape but for the same \(k_3^i\), the absolute value of the \(B\) meson is a bit larger than that of the \(B_s\) meson.
which is consistent with the fact that the DAs is inversely proportional to square root of the decay constants and masses.

In addition, the light-cone coordinate is widely used in the study of the DAs (for examples, see [11, 13, 18, 66–69] and references therein.) in which the DAs depend on a single variable \( \omega \) which is the light-cone projection \( k_+ \) of the light antiquark momentum in the meson rest frame. Through the definitions \( k_\pm = \frac{E_{q, \pm} k^3}{\sqrt{2}} \) and \( k^\mu_{\perp} = (0, k^1, k^2, 0) \), performing the
integration over the transverse momentum \(k_\perp\), we can obtain the light-cone distribution amplitudes (LCDAs) in our scenario. Usually, the \(k_\perp\)-integral is restricted by a scale \(\mu\) i.e. \(|k_\perp| < \mu\) from the general analysis \([2, 70]\). In our model, the wave function is spherical with respect to \(k^1, k^2,\) and \(k^3\). The integral region of the \(k_\perp\) has an upper bound which is determined by equations Eqs.(9a)–(9c) and this is shown clearly as the cut lines in Fig.1.

As shown in Fig.4 the distribution amplitude \(\phi_{A1}\) is taken as an example here since it equals to the most generally used LCDA \(\phi_B^+\) in the framework of heavy quark effective theory (HQET) and our results do not conflict with the general analysis \([13]\) and has a similar shape with some guessed models.

Next we try to get a compact form of the matrix element \(\tilde{\Phi}_{\alpha\beta}(z) = \langle 0|\bar{q}_\beta(z)[z,0]|Q_\alpha(0)|\bar{B}(P)\rangle\).

We substitute Eq.(21) and Eqs.(23a–23c) into Eq.(17b) and after a few steps of simplification, we obtain

\[
\tilde{\Phi}_{\alpha\beta}(z) = -\frac{1}{4} \sqrt{\frac{3m_B}{(2\pi)^3}} \int d^3k \frac{\Psi_0(\kappa)e^{-ik_\perp z}}{\sqrt{E_qE_Q(E_q + m_q)(E_Q + m_Q)}} \begin{bmatrix} b \\ c \end{bmatrix} \begin{bmatrix} c \\ a \end{bmatrix} \Bigg\} \quad \alpha\beta \\
\quad \text{(D.R.)} (29a)
\]

\[
= -\frac{1}{4} \sqrt{\frac{3m_B}{(2\pi)^3}} \int d^3k \cdot \frac{\Psi_0(\kappa)e^{-ik_\perp z}}{\sqrt{E_qE_Q(E_q + m_q)(E_Q + m_Q)}} \begin{bmatrix} b - c \\ b + c \end{bmatrix} \begin{bmatrix} c - a \\ c + a \end{bmatrix} \Bigg\} \quad \alpha\beta \\
\quad \text{(W.R.).} \quad (29b)
\]

where we define three \(2 \times 2\) matrices

\[
a = (E_q + m_q)I_{2 \times 2}, \quad b = (E_Q + m_Q)I_{2 \times 2}, \quad c = \kappa \cdot \vec{\sigma}
\]

(30)
\[ K(\vec{k}) \equiv \frac{-N_B\Psi_0(\vec{k})}{\sqrt{E_qE_Q(E_q + m_q)(E_Q + m_Q)}}. \] (31)

Then the convolution formula of Eq.(27) can be rewritten as

\[ F = \int d^3k \frac{-ifBmB}{4}K(\vec{k}) \left\{ \begin{pmatrix} b \\ c \end{pmatrix} \begin{pmatrix} c & a \end{pmatrix} \right\}_{\alpha\beta} T_{\beta\alpha}(k_q) \bigg|_{k_\perp^2 = m_q^2} \] (32)

where the spinor matrices are given in Dirac Representation (D.R.).

Next we introduce two light-like vectors \( n_{\perp}^\pm = (1, 0, 0, \mp 1) \) and define \( \Psi^\pm \equiv n_{\perp}^\mu \gamma_\mu = (\begin{pmatrix} 1 \\ \sigma^3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -\sigma^3 \\ -1 \end{pmatrix}) \), with which another form of Eq.(26) can be derived

\[ \Phi_{\alpha\beta}(k_q^\mu) = \frac{-ifBmB}{4}K(\vec{k}) \left\{ \begin{pmatrix} b \\ c \end{pmatrix} \begin{pmatrix} c & a \end{pmatrix} \right\}_{\alpha\beta} \]

\[ = \frac{-ifBmB}{4}K(\vec{k}). \]

\[ \left\{ \begin{pmatrix} (E_Q + m_Q) \frac{1 + \Psi}{2} \left( \begin{pmatrix} k_+ + m_q \\ \sqrt{2} \end{pmatrix} \phi_+ + \begin{pmatrix} k_+ - m_q \\ \sqrt{2} \end{pmatrix} \phi_- - k_{\perp}^\mu \gamma_\mu \right) \gamma^5 \\
- (E_q + m_q) \frac{1 - \Psi}{2} \left( \begin{pmatrix} k_- + m_q \\ \sqrt{2} \end{pmatrix} \phi_+ + \begin{pmatrix} k_- - m_q \\ \sqrt{2} \end{pmatrix} \phi_- - k_{\perp}^\mu \gamma_\mu \right) \gamma^5 \right\}_{\alpha\beta}. \] (33)

Compared with the commonly used results (for instance, see Eq.(109) in Ref.[11] and Eq.(2.48) in Ref.[18]), this new form reveals the whole Dirac structure of the momentum projector. The part containing \( \frac{1 + \Psi}{2} \) is proportional to the heavy quark’s mass and is the only term in the framework of HQET. Since when the heavy-quark mass \( m_Q \) goes infinity, the contribution of other part in Eq.(33) will be very small and can be ignored. Therefore, as we take the finite heavy-quark mass, the \( (E_q + m_q) \) part will give extra contribution and may be an important correction in the study of \( B \)-meson decays.

IV. QCD ONE-LOOP CORRECTIONS TO LEPTONIC DECAYS OF \( B \)-MESON

In Sec.II, we study the leptonic decays of \( B \) meson in tree level. In this section, we shall extend this study by including QCD one-loop corrections. In considering one-loop
corrections in QCD, if one naively calculate the loop diagrams, one will encounter not only ultraviolet divergence, but also infrared divergence. Factorization method can be applied to obtain the infrared safe amplitude on the quark level. Now that we have to consider the transition amplitude at quark level as the first step to study the process of meson decay, let us take the free quark state $|\bar{u}(k) b^s(p - k)\rangle$ as the initial state at first. Factorization means that the matrix element of a physics transition process $F^\mu$ can be expressed as the convolution of the wave function of the initial state and the hard transition amplitude $T$

$$F^\mu = \Phi \otimes T \quad (34)$$

where the circle-time $\otimes$ denotes the convolution like Eq. (25), and $\mu$ denotes the Lorentz index that may appear in the physical transition matrix element. All the infrared contributions are absorbed into the wave function $\Phi$, while the hard amplitude $T$ is infrared safe.

In the perturbation theory, the matrix element $F^\mu$, which relevant to the quark transition process, the wave function $\Phi$ and the hard-scattering kernel $T$ can all be expanded by the power of $\alpha_s$. Therefore the factorization formula takes the form

$$F^\mu = F^{(0)\mu} + F^{(1)\mu} + \cdots = \Phi \otimes T \quad (35)$$

where the superscripts $(n)$ indicate the perturbation levels. After calculating both the matrix element $F^{(1)\mu}$ and the wave function $\Phi^{(1)}$ at one-loop order, one can extract the hard amplitude $T^{(1)}$ by using Eq. (35), that is

$$\Phi^{(0)} \otimes T^{(1)} = F^{(1)\mu} - \Phi^{(1)} \otimes T^{(0)} \quad (36)$$

At one-loop level, both the matrix element $F^{(1)\mu}$ and the wave function $\Phi^{(1)}$ are infrared divergent. Through the subtraction in the right-hand side of Eq. (36), the infrared divergence may be cancelled. Then we can get the hard amplitude $T^{(1)}$ which is infrared safe.

At tree level, the factorization is straightforward and we show the results briefly. The matrix element $F^\mu$, as shown in Fig. 5, is obtained

$$F^{(0)\mu}_{ba} = \langle 0 | \bar{u(r)} \gamma^\mu_L b^s(p - k) \rangle$$

$$= \frac{1}{(2\pi)^3} \sqrt{\frac{m_a m_b}{k^0 (p - k)^0}} \bar{v}^r(k) \gamma^\mu_L u^s(p - k) \quad (37)$$
where the coefficient \( \frac{1}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0(p-k)^0}} \) results from our convention, and \( \bar{v} \) and \( u \) are the spinor wave functions of the quarks \( \bar{u} \) and \( b \) respectively. The superscripts \( r \) and \( s \) are the spin labels.

The definition and result of the wave function based on the free quark state \(|\bar{u}^r(k)b^s(p-k)\rangle\) are

\[
\Phi^{(0)\bar{u}b}_\alpha\beta(\tilde{k}) = \int d^4z e^{i\tilde{k}\cdot z} \langle 0|\bar{u}_\beta(z)[z,0]b_\alpha(0)|\bar{u}^r(k)b^s(p-k)\rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0(p-k)^0}} (2\pi)^4 \delta^{(4)}(\tilde{k} - k) \bar{v}^r_\beta(k) u^s_\alpha(p-k). \tag{38}
\]

Matching the matrix element in Eq.(37) and the wave function in Eq.(38) into the factorization formula

\[
F^{(0)\mu}_{\bar{u}b} = \int \frac{d^4\tilde{k}}{(2\pi)^4} \Phi^{(0)\bar{u}b}_\alpha\beta(\tilde{k}) T^{(0)\beta\alpha}(\tilde{k}) = \frac{1}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0(p-k)^0}} \bar{v}^r_\beta(k) T^{(0)\beta\alpha}_\beta(\tilde{k}) u^s_\alpha(p-k), \tag{39}
\]

we can deduce the hard-scattering kernel

\[
T^{(0)\beta\alpha}_\beta(\tilde{k}) = (\gamma^\mu_L)_\beta\alpha. \tag{40}
\]

This tree-level result is independent of the momentum \( k \) and is a constant matrix which plays an important role in the reduction at one-loop level.

Next, we establish the factorization at one-loop level and evaluate the hard-scattering kernel at this level. Fig.6 (a)–(c) show the three Feynman diagrams for the matrix element \( F^{(1)\mu} \): (a) denotes the vertex correction, (b) and (c) correspond to the renormalization of the external quark fields.
FIG. 6. Feynman diagrams at one-loop level for $F$.

In Fig. 6 (a), all the momenta of quark and gluon lines are shown. The contribution of Fig. 6 (a) is

$$F_V^{(1)} = \frac{1}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0 (p-k)^0}} (-ig_s^2) C_F \bar{v}^r(k).$$

$$\int \frac{d^4l}{(2\pi)^4} \frac{\gamma^\rho}{m_u - (l-k)} \frac{1}{m_b - (p-k + l)} \gamma^\rho \frac{1}{l^2} \cdot u^a(p - k). \quad (41)$$

where $g_s$ is the strong coupling constant.

The results for Fig 6 (b) and (c) corresponding to the renormalization of the light anti-quark field $\bar{u}$ and the heavy quark field $b$ are

$$F_{bR}^{(1)} = \frac{1}{2}(Z_b^2 - 1) F_{b\bar{u}}^{(0)}, \quad F_{\bar{u}R}^{(1)} = \frac{1}{2}(Z_{\bar{u}}^2 - 1) F_{b\bar{u}}^{(0)}. \quad (42)$$

The renormalization constants are defined in terms of the one-particle irreducible (1PI) diagrams $\Sigma$ by

$$Z_{b, \bar{u}}^2 = 1 + \frac{d\Sigma}{dp^4} \bigg|_{p=m}. \quad (43)$$

The corrections for the wave functions at one-loop order contain 6 Feynman diagrams which have been divided into two groups. They are shown in Figs. 7 and 8. It will be shown later that, when the contribution of the diagrams in Fig. 8 convolutes with the hard-scattering kernel at tree level, the result will be zero.

The contribution to the wave function from vertex correction (Fig. 7(a)) is

$$\Phi_{\alpha\beta}^{(1)V}(k) = (2\pi)^4 \delta^4(l + k(k+\bar{k}) \frac{1}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0 (p-k)^0}} (-ig_s^2) C_F \int \frac{d^4l}{(2\pi)^4} \left[ \bar{v}^r(k) \gamma^\rho \frac{1}{m_u - (l-k)} \right]_{\beta} \left[ \frac{1}{m_b - (p-k + l)} \gamma^\rho \frac{1}{l^2} \cdot u^a(p - k) \right]_{\alpha}. \quad (44)$$
and the wave function renormalization of the heavy quark field (Fig. 7 (b)) and the light quark field (Fig. 7 (c)) are
\[
\Phi^{(1)}_{ab\bar{c}} = \frac{1}{2}(Z^b_2 - 1)\Phi^{(0)ab\bar{c}}, \quad \Phi^{(1)}_{a\bar{c}b} = \frac{1}{2}(Z^b_2 - 1)\Phi^{(0)a\bar{c}b}.
\]
(45)

Then it is straightforward to obtain the results after the convolution with the hard-scattering kernel at tree level \( T^{(0)}_{\beta\alpha} = (\gamma^\mu_2)_{\beta\alpha} \) and we find that
\[
F^{(1)}_{V} = \Phi^{(1)V}_{\alpha\beta} \otimes T^{(0)}_{\beta\alpha},
\]
(46a)
\[
F^{(1)}_{bR} = \Phi^{(1)b}_{\alpha\beta} \otimes T^{(0)}_{\beta\alpha},
\]
(46b)
\[
F^{(1)}_{\bar{u}R} = \Phi^{(1)\bar{u}}_{\alpha\beta} \otimes T^{(0)}_{\beta\alpha}.
\]
(46c)

It is noted that there are two scales in the above equations, i.e., the factorization scale \( \mu_F \) in the wave functions \( \Phi^{(1)b,a\bar{c}}_{\alpha\beta} \) and the renormalization scale \( \mu_R \) in the matrix element \( F^{(1)\mu}_{b,aR} \). Here we set \( \mu_F = \mu_R \).
At last, we turn to the contributions of Feynman diagrams in Figs. 8. The contribution of Fig. 8(a) contains a gluon propagator both starting from and ending at the Wilson-line. In the light-cone approximation and working in the Feynman gauge, this propagator vanishes since \( z \) is a null vector on the light-cone \((z^2 = 0)\). As for our case, the result is still zero. First, we obtain

\[
\Phi^{(1)0a}_{\alpha\beta}(k) = \frac{1}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0 (p - k)^0}} \langle i g_s^2 \rangle C_F \bar{\psi}^r(k) \beta \int \frac{d^4 l}{(2\pi)^4} \int d^4 z e^{i k \cdot z}.
\]

Next, substitute Eq.(47b) into the convolution formula, and perform the partial integration. By noting that the hard-scattering kernel is a constant Dirac matrix, we can demonstrate

\[
\Phi^{(1)0a}(k) \otimes T^{(0)}_{\beta\alpha} = \int \frac{d^4 \tilde{k}}{(2\pi)^4} \frac{2}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0 (p - k)^0}} \bar{\psi}^r(k) \beta \int \frac{d^4 l}{(2\pi)^4} \int d^4 z \int_0^x dx dy \frac{e^{i(xl - k \cdot y) \cdot z}}{l^2} \frac{e^{i k \cdot z}}{l^2} \left( \frac{\partial^2}{\partial k_\mu \partial k_\nu} T^{(0)}_{\beta\alpha} \right) u^s(p - k)_\alpha = 0.
\]

For the other two diagrams in Figs. 8, the corresponding contribution of Fig. 8(b) is

\[
\Phi^{(1)0b}_{\alpha\beta}(k) = \frac{i g_s^2 C_F}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0 (p - k)^0}} \int \frac{d^4 l}{(2\pi)^4} \int d^4 z \int_0^x dx \frac{e^{i(xl - k \cdot z)}}{l^2} \bar{\psi}^r(k) \beta \left( \frac{\partial}{\partial k_\rho} e^{i k \cdot z} \right) \left[ \frac{1}{m_b - (\not{\bar{p}} - \not{k} + \not{l})} \gamma_\rho u^s(p - k) \right]_\alpha.
\]

and then the convolution is

\[
\Phi^{(1)0b}_{\alpha\beta}(k) \otimes T^{(0)}_{\beta\alpha} = \int \frac{d^4 \tilde{k}}{(2\pi)^4} \frac{-i g_s^2 C_F}{(2\pi)^3} \sqrt{\frac{m_u m_b}{k^0 (p - k)^0}} \int \frac{d^4 l}{(2\pi)^4} \int d^4 z \int_0^x dx \frac{e^{i(xl - k \cdot z)}}{l^2} \bar{\psi}^r(k) \beta e^{i k \cdot z} \left( \frac{\partial}{\partial k_\rho} T^{(0)}_{\beta\alpha} \right) \left[ \frac{1}{m_b - (\not{\bar{p}} - \not{k} + \not{l})} \gamma_\rho u^s(p - k) \right]_\alpha = 0.
\]
Similarly, we can obtain that the contribution of Fig.8 (c) is also zero.

Finally, combining the Eqs. (46a)–(46c), Eq. (48) and Eq. (50) together, we can demonstrate that $T^{(1)\mu}_{\bar{b}b} = \Phi^{(1)\bar{b}b}_{\alpha\beta} \otimes T^{(0)}_{\beta\alpha}$ and thus considering Eq. (36), the total contribution to the hard-scattering kernel at one-loop level $T^{(1)}_{\beta\alpha}$ is zero.

Therefore the QCD one-loop corrections to the leptonic decay of $B$ meson are zero in the factorization scheme.

V. DISCUSSION AND CONCLUSION

Using the wave function that is obtained in the relativistic potential model in our previous work [21], where the hyperfine interactions are included, the decay constants and pure leptonic decays of the $B$-mesons are studied in this work. To keep the four-momentum conservation between the quark-antiquark pair and the meson, we use the ACCMM scenario [58, 59] to treat the constituent quarks, where the heavy quark is taken to be off-shell, while the light antiquark is kept on shell. Compared with our earlier work [22], the difference is that the wave function used here is obtained by considering the hyperfine interactions in the wave equation, and the heavy quark is treated off-shell in the decay process. The off-shellness of the heavy quark can be explained as absorbing the effective effects of the gluon cloud around the heavy quark. With such a treatment, the branching ratios of leptonic decays of $B$ meson obtained in this work are consistent with experimental data.

Based on the success of studying the leptonic decays of the $B$ meson, we further obtain the distribution amplitudes for $B$ meson both in coordinate and momentum space. The distribution amplitudes of $B$ meson are widely used in the study of $B$-meson decays. In addition, we obtain another form of the non-local matrix element in Eq. (33). Considering the success of the ACCMM scenario in studying the leptonic decays, the heavy quark in the distribution amplitude needs to be treated to be off-shell to maintain the momentum and energy conservation. The new form of the non-local matrix element obtained in this work, Eqs. (29a) or (29b) and Eq. (33) shall be useful in the study of the semi-leptonic and non-leptonic $B$ decays, where the longitudinal and transverse components are automatically included.

We finally studied the QCD one-loop corrections within the framework of the factorization approach. We find that, after suntracting the infrared divergence, the QCD one-loop
corrections will be zero.

**Appendix A: Derivation of the distribution amplitudes \( \tilde{\phi}_T(z) \), \( \tilde{\phi}_{A1}(z) \), and \( \tilde{\phi}_{A2}(z) \)**

In this appendix, we give a brief derivation of the three distribution amplitudes stated in Eqs.(23a)-(23c). The direct result about \( \tilde{\phi}_T(z) \) in Eq.(19c) is

\[
\tilde{\phi}_T(z) z^i = i N_B \int d^3 k \psi_0(\vec{k}) \frac{E_q + m_q + E_Q + m_Q}{\sqrt{E_q E_Q (E_q + m_q) (E_Q + m_Q)}} k^i e^{-ik_q z} \tag{A1}
\]

where \( k^i \) stands for any of the components of momentum \( \vec{k} \) and \( N_B = \frac{i}{f_B} \sqrt{\frac{3}{(2\pi)^3 m_B}} \). Note that \( z_i e^{-ik_q z} = \frac{\partial}{\partial k^i} e^{-ik_q z} \) and make use of \( A_T(k^1, k^2, k^3) \) defined in Eq.(22a),

\[
A_T(k^1, k^2, k^3) k^1 = \frac{\partial}{\partial k^1} \int_0^{k^1} A_T(\eta, k^2, k^3) \eta \, d\eta \tag{A2a}
\]

\[
\Rightarrow \tilde{\phi}_T(z) z^1 = N_B \int d^3 k \int_0^{k^1} A_T(\eta, k^2, k^3) \eta \, d\eta (-z_1) e^{-ik_q z} \tag{A2b}
\]

\[
\Rightarrow \tilde{\phi}_T(z) = N_B \int d^3 k \left[ \frac{1}{3} \sum_i \int_0^{k_i} A_T(\eta, \ldots) \eta \, d\eta \right] e^{-ik_q z} \tag{A2c}
\]

where Eq.(A2b) is derived from Eq.(A2a) by partial integration. The summation in the square parentheses is short for the following form

\[
\sum_i \int_0^{k_i} A_T(\eta, \ldots) \eta \, d\eta = \int_0^{k^1} A_T(\eta, k^2, k^3) \eta \, d\eta + \int_0^{k^2} A_T(k^1, \eta, k^3) \eta \, d\eta + \int_0^{k^3} A_T(k^1, k^2, \eta) \eta \, d\eta. \tag{A3}
\]

The situation is similar for the derivation of \( \tilde{\phi}_{A2}(z) \).

For the DA \( \tilde{\phi}_{A1}(z) \), after substituting Eq.(23b) and Eq.(22b) into the equation Eq.(19c), we obtain

\[
\tilde{\phi}_{A1}(z) = -N_B \int d^3 k e^{-ik_q z} \left[ \psi_0(\vec{k}) \frac{(E_q + m_q)(E_Q + m_Q) - |\vec{k}|^2}{\sqrt{E_q E_Q (E_q + m_q) (E_Q + m_Q)}} + \frac{i z_0}{3} \sum_i \int_0^{k_i} A(\eta, \ldots) \eta \, d\eta \right]. \tag{A4}
\]

Using the same trick \( iz_0 e^{-ik_q z} = \frac{\partial}{\partial E_q} e^{-ik_q z} \) and partial integration and noting that in our scenario \( E_q^2 - |\vec{k}|^2 = m_q^2 \), we get the final expression

\[
\tilde{\phi}_{A1}(z) = -N_B \int d^3 k e^{-ik_q z} \left[ \psi_0(\vec{k}) \frac{(E_q + m_q)(E_Q + m_Q) - |\vec{k}|^2}{\sqrt{E_q E_Q (E_q + m_q) (E_Q + m_Q)}} + E_q A(k^1, k^2, k^3) \right] \tag{A5}
\]
Appendix B: Deduction of the amplitude $F$

In this appendix, we show explicitly how to derive Eq. (27) from Eq. (25). Firstly, we perform the Fourier transformation on the hard scattering kernel $\tilde{T}_{\beta\alpha}(z)$ and obtain the following identity

$$F = \int d^4z \tilde{\Phi}_{\alpha\beta}(z) \int \frac{d^4l}{(2\pi)^4} e^{il\cdot z} T_{\beta\alpha}(l) = \int d^4z e^{il\cdot z} \tilde{\Phi}_{\alpha\beta}(z) \left[ \int \frac{d^4l}{(2\pi)^4} e^{il\cdot z} T_{\beta\alpha}(l) \right] (B1)$$

then substitute Eq. (24) into it, using the same trick $z^\mu e^{il\cdot z} = -i \frac{\partial}{\partial l^\mu} e^{il\cdot z}$ as in Appendix A, we obtain

$$\int d^4z e^{il\cdot z} \tilde{\Phi}_{\alpha\beta}(z) = -if_B \int d^4z \left\{ \left[ m_B \tilde{\phi}_P + \frac{-i}{2} \tilde{\phi}_T \left( P^\mu \frac{\partial}{\partial l^\nu} - P^\nu \frac{\partial}{\partial l^\mu} \right) \sigma_{\mu\nu} \right. \right.$$  

$$\left. + \left( \tilde{\phi}_A P^\mu + m_B \tilde{\phi}_A \frac{\partial}{\partial l^\mu} \right) \gamma_\mu \right\} e^{il\cdot z} \gamma_5 \right\} \right\}_\alpha\beta (B2)$$

Considering the hard scattering kernel $T_{\beta\alpha}(l)$ at the end of Eq. (B1), the derivative $\frac{\partial}{\partial l^\mu}$ can be moved to act on it through partial integration. In addition, we observe that in the four distribution amplitudes in Eq. (21) and Eqs. (23a)–(23c), only the exponential part $e^{-ik_q\cdot z}$ depends on the variable $z$. Therefore the integration over $z$ can be easily worked out and the result is a delta function $(2\pi)^4 \delta^{(4)}(l - k_q)$.

$$F = \int \frac{d^4l}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(l - k_q) \left[ \ldots \right] T_{\beta\alpha}(l) (B3)$$

After taking $P^\mu = m_B v^\mu$ and $k_q^2 = m_q^2$ (Eq. (9b)) into consideration, we obtain the final expression Eq. (27). The coefficient $-if_B m_B$ follows the convention of Refs. [10, 11].

ACKNOWLEDGMENTS

This work is supported in part by the National Natural Science Foundation of China under Contract No.11375088.

[1] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999).  
   arXiv:hep-ph/9905312 [hep-ph]

[2] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. B591, 313 (2000).  
   arXiv:hep-ph/0006124 [hep-ph]
[3] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. B606, 245 (2001), arXiv:hep-ph/0104110 [hep-ph].

[4] M. Beneke and M. Neubert, Nucl. Phys. B675, 333 (2003), arXiv:hep-ph/0308039 [hep-ph].

[5] Y.-Y. Keum, H.-n. Li, and A. I. Sanda, Phys. Lett. B504, 6 (2001), arXiv:hep-ph/0004004 [hep-ph].

[6] Y.-Y. Keum, H.-N. Li, and A. I. Sanda, Phys. Rev. D63, 054008 (2001), arXiv:hep-ph/0004173 [hep-ph].

[7] C.-D. Lu, K. Ukai, and M.-Z. Yang, Phys. Rev. D63, 074009 (2001), arXiv:hep-ph/0004213 [hep-ph].

[8] Y.-Y. Keum, H.-n. Li, and A. I. Sanda, Proceedings, 9th International Symposium on Heavy Flavor Physics, AIP Conf. Proc. 618, 229 (2002), arXiv:hep-ph/0201103 [hep-ph].

[9] Y.-Y. Keum, T. Kurimoto, H. N. Li, C.-D. Lu, and A. I. Sanda, Phys. Rev. D69, 094018 (2004), arXiv:hep-ph/0305335 [hep-ph].

[10] A. G. Grozin and M. Neubert, Phys. Rev. D55, 272 (1997), arXiv:hep-ph/9607366 [hep-ph].

[11] M. Beneke and T. Feldmann, Nucl. Phys. B592, 3 (2001), arXiv:hep-ph/0008255 [hep-ph].

[12] B. O. Lange and M. Neubert, Phys. Rev. Lett. 91, 102001 (2003), arXiv:hep-ph/0303082 [hep-ph].

[13] S. J. Lee and M. Neubert, Phys. Rev. D72, 094028 (2005), arXiv:hep-ph/0509350 [hep-ph].

[14] H. Kawamura, J. Kodaira, C.-F. Qiao, and K. Tanaka, Phys. Lett. B523, 111 (2001), arXiv:hep-ph/0109181 [hep-ph].

[15] T. Huang, X.-G. Wu, and M.-Z. Zhou, Phys. Lett. B611, 260 (2005), arXiv:hep-ph/0412225 [hep-ph].

[16] T. Huang, C.-F. Qiao, and X.-G. Wu, Phys. Rev. D73, 074004 (2006), arXiv:hep-ph/0507270 [hep-ph].

[17] C.-W. Hwang, Phys. Rev. D81, 114024 (2010), arXiv:1003.0972 [hep-ph].

[18] G. Bell, T. Feldmann, Y.-M. Wang, and M. W. Y. Yip, JHEP 11, 191 (2013), arXiv:1308.6114 [hep-ph].

[19] H. Leutwyler and M. Roos, Z. Phys. C25, 91 (1984).

[20] J.-B. Liu and M.-Z. Yang, JHEP 07, 106 (2014), arXiv:1307.4636 [hep-ph].

[21] J.-B. Liu and M.-Z. Yang, Phys. Rev. D91, 094004 (2015), arXiv:1501.04266 [hep-ph].
[22] M.-Z. Yang, Eur. Phys. J. **C72**, 1880 (2012), arXiv:1104.3819 [hep-ph].

[23] A. A. Penin and M. Steinhauser, Phys. Rev. **D65**, 054006 (2002), arXiv:hep-ph/0108110 [hep-ph].

[24] J. Bordes, J. Penarrocha, and K. Schilcher, JHEP **12**, 064 (2004), arXiv:hep-ph/0410328 [hep-ph].

[25] J. Bordes, J. Penarrocha, and K. Schilcher, JHEP **11**, 014 (2005), arXiv:hep-ph/0507241 [hep-ph].

[26] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. **B701**, 82 (2011), arXiv:1101.5986 [hep-ph].

[27] S. Narison, Phys. Lett. **B718**, 1321 (2013), arXiv:1209.2023 [hep-ph].

[28] P. Gelhausen, A. Khodjamirian, A. A. Pivovarov, and D. Rosenthal, Phys. Rev. **D88**, 014015 (2013), [Erratum: Phys. Rev.D91,099901(2015)], arXiv:1305.5432 [hep-ph].

[29] S. Narison, *Proceedings, 18th High-Energy Physics International Conference in Quantum Chromodynamics (QCD 15)*, Nucl. Part. Phys. Proc. **270-272**, 143 (2016), arXiv:1511.05903 [hep-ph].

[30] Z.-G. Wang, W.-M. Yang, and S.-L. Wan, Nucl. Phys. **A744**, 156 (2004), arXiv:hep-ph/0403259 [hep-ph].

[31] G. Cvetic, C. S. Kim, G.-L. Wang, and W. Namgung, Phys. Lett. **B596**, 84 (2004), arXiv:hep-ph/0405112 [hep-ph].

[32] A. M. Badalian, B. L. G. Bakker, and Yu. A. Simonov, Phys. Rev. **D75**, 116001 (2007), arXiv:hep-ph/0702157 [HEP-PH].

[33] T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, Phys. Rev. **D82**, 074022 (2010) arXiv:1008.0268 [hep-ph].

[34] S. Godfrey and N. Isgur, Phys. Rev. **D32**, 189 (1985).

[35] P. Colangelo, G. Nardulli, and M. Pietroni, Phys. Rev. **D43**, 3002 (1991).

[36] M. Di Pierro and E. Eichten, Phys. Rev. **D64**, 114004 (2001), arXiv:hep-ph/0104208 [hep-ph].

[37] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Lett. **B635**, 93 (2006), arXiv:hep-ph/0602110 [hep-ph].

[38] C. T. H. Davies et al. (Fermilab Lattice, HPQCD, UKQCD, MILC), Phys. Rev. Lett. **92**, 022001 (2004), arXiv:hep-lat/0304004 [hep-lat].
[40] C. T. H. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na, and J. Shigemitsu, Phys. Rev. D82, 114504 (2010), arXiv:1008.4018 [hep-lat].

[41] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel, and G. P. Lepage, Phys. Rev. D85, 031503 (2012), arXiv:1110.4510 [hep-lat].

[42] A. Bazavov et al. (Fermilab Lattice, MILC), Phys. Rev. D85, 114506 (2012), arXiv:1112.3051 [hep-lat].

[43] D. Becirevic, V. Lubicz, F. Sanfilippo, S. Simula, and C. Tarantino, JHEP 02, 042 (2012), arXiv:1201.4039 [hep-lat].

[44] H. Na, C. J. Monahan, C. T. H. Davies, R. Horgan, G. P. Lepage, and J. Shigemitsu, Phys. Rev. D86, 034506 (2012), arXiv:1202.4914 [hep-lat].

[45] R. J. Dowdall, C. T. H. Davies, R. R. Horgan, C. J. Monahan, and J. Shigemitsu (HPQCD), Phys. Rev. Lett. 110, 222003 (2013), arXiv:1302.2644 [hep-lat].

[46] K. A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014).

[47] I. Adachi et al. (Belle), Phys. Rev. Lett. 110, 131801 (2013), arXiv:1208.4678 [hep-ex].

[48] B. Kronenbitter et al. (Belle), Phys. Rev. D92, 051102 (2015), arXiv:1503.05613 [hep-ex].

[49] J. P. Lees et al. (BaBar), Phys. Rev. D88, 031102 (2013), arXiv:1207.0698 [hep-ex].

[50] B. Aubert et al. (BaBar), Phys. Rev. D81, 051101 (2010), arXiv:0912.2453 [hep-ex].

[51] J. L. Rosner, S. Stone, and R. S. Van de Water, (2015), arXiv:1509.02220 [hep-ph].

[52] M. Barrett (Belle II), Proceedings, 13th Conference on Flavor Physics and CP Violation (FPCP 2015), PoS FPCP2015, 049 (2015).

[53] B. Wang (Belle-II), in 10th International Workshop on e+e- collisions from Phi to Psi (PHIPS15) Hefei, Anhui, China, September 23-26, 2015 (2015) arXiv:1511.09434 [physics.ins-det].

[54] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C29, 637 (1985).

[55] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, Phys. Rev. D17, 3090 (1978) [Erratum: Phys. Rev.D21,313(1980)].

[56] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, Phys. Rev. D21, 203 (1980).

[57] A. De Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. D12, 3589 (1975).
[58] G. Altarelli, N. Cabibbo, G. Corbo, L. Maiani, and G. Martinelli, Nucl. Phys. B208, 365 (1982).
[59] P. Colangelo, F. De Fazio, M. Ladisa, G. Nardulli, P. Santorelli, and A. Tricarico, Eur. Phys. J. C8, 81 (1999), arXiv:hep-ph/9809372 [hep-ph].
[60] C.-W. Hwang, Phys. Rev. D81, 054022 (2010), arXiv:0910.0145 [hep-ph].
[61] N. Carrasco et al., Proceedings, 31st International Symposium on Lattice Field Theory (Lattice 2013), PoS LATTICE2013, 313 (2014), arXiv:1311.2837 [hep-lat].
[62] Y. Aoki, T. Ishikawa, T. Izubuchi, C. Lehner, and A. Soni, Phys. Rev. D91, 114505 (2015), arXiv:1406.192 [hep-lat].
[63] N. H. Christ, J. M. Flynn, T. Izubuchi, T. Kawanai, C. Lehner, A. Soni, R. S. Van de Water, and O. Witzel, Phys. Rev. D91, 054502 (2015), arXiv:1404.4670 [hep-lat].
[64] Z.-G. Wang, Eur. Phys. J. C75, 427 (2015), arXiv:1506.01993 [hep-ph].
[65] P. Pakhlov and T. Uglov, International Conference on Particle Physics and Astrophysics (ICPPA 2015) Moscow, Russia, October 5-10, 2015, J. Phys. Conf. Ser. 675, 022009 (2016).
[66] Z.-T. Wei and M.-Z. Yang, Nucl. Phys. B642, 263 (2002), arXiv:hep-ph/0202018 [hep-ph].
[67] A. G. Grozin, Helmholtz International Summer School on Heavy Quark Physics Moscow, Dubna, Russia, June 6-16, 2005, Int. J. Mod. Phys. A20, 7451 (2005), arXiv:hep-ph/0506226 [hep-ph].
[68] G. Bell and T. Feldmann, JHEP 04, 061 (2008), arXiv:0802.2221 [hep-ph].
[69] T. Feldmann, B. O. Lange, and Y.-M. Wang, Phys. Rev. D89, 114001 (2014), arXiv:1404.1343 [hep-ph].
[70] P. Ball, JHEP 01, 010 (1999), arXiv:hep-ph/9812375 [hep-ph].
[71] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B650, 356 (2003), arXiv:hep-ph/0209216 [hep-ph].