Preface

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Helsinki, May 23, 1995

Martti Raidal
Extended electroweak models and their tests in future colliders
Martti Raidal
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Abstract

In this Thesis some possible tests of physics beyond the Standard Model in the next generation collider experiments are considered. The main emphasis is put on the processes which may be studied in the non-conventional $e^- e^-$, $e^- \gamma$ and $\gamma \gamma$ operation modes of the next linear collider (NLC).

The sensitivity of the NLC for testing the gauge boson self-interactions through the reaction $e^- \gamma \rightarrow W^- \nu$ is investigated. New bounds for precision of measurement of the $\gamma WW$ coupling parameters $\kappa_{\gamma}$ and $\lambda_{\gamma}$ are derived in the case of polarized beams, taking into account the recent developments in the NLC design.

The minimal version of the Standard Model does not allow neutrinos to have mass as some recent astrophysical observations and oscillation measurements seem to require. In this work high energy tests of a left-right symmetric extension of the Standard Model, the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, in which small neutrino masses are generated in a natural way, are investigated. The analysis is focussed on the lepton number violating interactions of Majorana neutrinos and doubly charged Higgs bosons. An interesting process, the so-called inverse double-$\beta$ decay $e^- e^- \rightarrow W^- W^-$, is investigated and its high energy behaviour is discussed. The process is found to be useful for clarifying the nature of neutrinos and also for studying the Higgs sector of the model.

In order to avoid the hierarchy problem the left-right symmetric model can be supersymmetrized. A distinctive signature of the susy left-right model is found to be provided by the decay of doubly charged higgsino $\tilde{\Delta}^{--}$. The production of $\tilde{\Delta}^{--}$ in all collision modes of the NLC is studied. The contribution of doubly charged higgsino as a virtual state to the selectron pair production is also estimated.
List of Papers

This Thesis consists of an introductory review part, followed by four research publications:

I  M. Raidal,

Tests of gauge boson couplings in polarized $e^- \gamma$ collisions,

Nucl. Phys. B 441 (1995) 49.

II  P. Helde, K. Huitu, J. Maalampi, M. Raidal,

Gauge boson pair production in electron-electron collisions with polarized beams,

Nucl. Phys. B 437 (1995) 305.

III K. Huitu, J. Maalampi, M. Raidal,

Supersymmetric left-right model and its test in linear colliders,

Nucl. Phys. B 420 (1994) 449.

IV  K. Huitu, J. Maalampi, M. Raidal,

Slepton pair production in supersymmetric left-right model,

Phys. Lett. B 328 (1994) 60.
1 Introduction

Overwhelming amount of experimental data obtained over the past years proves that the Standard Model [1] of particle interactions has been strikingly successful in terms of experimental predictions. The discovery of the massive vector bosons at CERN [2] gave a direct evidence for the correctness of the basic ideas of gauge theories applied in the Standard Model. Furthermore, the precision analysis of LEP electroweak data has enabled physicists to constrain strictly those parameters of the model which are not directly measured. The recent discovery and mass measurement of the top quark by CDF and D0 experiments [3] at TEVATRON confirms the results of LEP precision analysis and strengthens the confidence in the Standard Model. Based on these experimental results, theoretical arguments indicate that the Standard Model indeed provides a faithful characterization of the reality at energies currently used in experiments [4].

Despite of the impressive success of the Standard Model, physicists continue looking for alternative or more fundamental theories which might replace the Standard Model at higher energies. The large number of free parameters entering in the Standard Model, the maximal parity violation of the weak interaction, the vanishing neutrino masses and the hierarchy problem are among the puzzles which motivate the searches for new models. On the experimental side, there is still no direct measurement of one central feature of $SU(2)_L$ gauge symmetry, the non-Abelian self-couplings of $W, Z$ and photon, and no direct confirmation of the Higgs mechanism [5], which generates masses in the Standard Model.

A straightforward way to obtain hints for physics beyond the Standard Model could be the precision study of gauge boson self-couplings. Strongly constrained by the gauge invariance, they are particularly sensitive to the deviations from the Standard Model structure. The new generation particle colliders currently under planning [6, 7, 8] will give us a possibility to probe the gauge boson self-couplings in various processes in different collision modes.

Another possible approach is to search for models with extended gauge sym-
metries. One of the most natural extensions of the Standard Model, the left-right symmetric model, was first proposed by J.C. Pati and A. Salam in 1974 [9]. In addition to the original idea of providing an explanation to the parity violation of the weak interaction, this model turned out to be capable of explaining the lightness of neutrinos via the so-called see-saw mechanism [10] in a dynamical way. The observed solar [11] and atmospheric [12] neutrino deficit, the recent $\nu_\mu \to \nu_e$ oscillation events of LSND experiment [14], as well as the COBE satellite hints for possible existence of a hot component of dark matter [13] seem indeed to indicate that neutrinos may have small nonvanishing mass. Although the concept of the left-right model is already twenty years old, the present interest in the model can be explained by its accessibility to experimental tests in the near future. The energy range of new colliders could allow us to study the production of heavy right-handed particles, as well as the lepton number violating processes predicted by the left-right symmetric model.

However, similarly to the Standard Model, also the left-right symmetric model suffers from the hierarchy problem, i.e. the masses of scalar particles tend to diverge quadratically. This problem can be cured by supersymmetrizing the theory. It turns out that in the supersymmetric left-right symmetric model one of the doubly charged Higgs bosons should be relatively light [15] and accessible in the future collider experiments. The mass of its superpartner, the doubly charged higgsino, is a free parameter of the model and can also be light. Therefore, the search for the doubly charged particles will be of great interest in future colliders.

In this Thesis, the extensions of the Standard Model described above are considered and their phenomenological implications are studied. In particular, the main emphasis is put on the signatures of new physics in the high energy $e^+e^-, e^-e^-, e^-\gamma$ and $\gamma\gamma$ collisions [8] possible to be explored in the next linear collider (NLC).

The work is organized as follows. A brief summary of the original publications, which the Thesis is based on, is given at the end of this Introduction.

In Section 2 the general parametrization of the triple boson vertex is reviewed.
Further restrictions on the gauge boson self-interactions are discussed and their implications on the anomalous form factors of the couplings are considered.

In Section 3 the basic structure of the left-right symmetric model is described, concentrating in particular on the Higgs sector and on the symmetry breaking mechanism. The importance of the central novel features of the model, i.e. the Majorana nature of neutrinos and the existence of doubly charged Higgs bosons, is considered in various physical processes.

In Section 4 the supersymmetric left-right symmetric model, considered in the papers this Thesis is based on, is introduced. The particle contents of the model is given, and the predictions for the masses of Higgs bosons and the susy partners of the particles are discussed.

Section 5 is devoted to the experimental tests of the above mentioned models. Reactions with the most distinctive experimental signatures in all the collision modes of the future linear collider are considered.

The conclusions are given in Section 6.

The original papers are appended at the end of the Thesis.

Summary of the Original Papers

**Paper I: Tests of gauge boson couplings in polarized $e^{-\gamma}$ collisions.**

Single $W$-boson production in $e^{-\gamma}$ collisions with polarized beams is investigated. Helicity amplitudes for general couplings and masses are derived and their properties are discussed. The results are applied to study the Standard Model. Updated estimates of the measurement precision of the photon anomalous coupling parameters $\kappa_\gamma$, $\lambda_\gamma$ at the NLC with $\sqrt{s_{ee}} = 500$ GeV are obtained. Comparison with earlier studies, done without taking polarization into account, shows a factor of 3 improvement in the measurement precision.
Paper II: Gauge boson pair production in electron-electron collisions with polarized beams.

The $W$-boson pair production in $e^-e^-$ collisions with polarized beams is investigated. The helicity amplitudes are derived for general vector-axial vector couplings and the conditions for a good high-energy behaviour of the cross-section are given. The results are applied to the heavy vector boson production in the context of the left-right symmetric model. The Ward identities and the equivalence theorem are also discussed.

Paper III: Supersymmetric left-right model and its test in linear colliders.

The phenomenological implications of a supersymmetric left-right model based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry testable in the next generation linear colliders are investigated. In particular, the emphasis is put on the doubly charged $SU(2)_R$ triplet higgsino $\tilde{\Delta}$, which is found to have very distinguishing signatures for experimental search. Its production rate in $e^+e^-$, $e^-e^-$, $e^-\gamma$ and $\gamma\gamma$ collisions is estimated and its subsequent decays are considered. These processes are found to have a clear discovery signature with a very low background from other processes.

Paper IV: Slepton pair production in supersymmetric left-right model.

The pair production of sleptons in electron-positron collisions is investigated in a supersymmetric left-right model. The cross section is found to be considerably larger than in the minimal supersymmetric version of the Standard Model (MSSM) because of larger number contributing graphs. A novel process is a doubly charged higgsino exchange in u-channel, which makes the angular distribution of the final state particles and the final state asymmetries different from those of the MSSM. It also allows for the flavour non-diagonal final states $\tilde{\ell}\tilde{\mu}$, $\tilde{e}\tilde{\tau}$ and $\tilde{\mu}\tilde{\tau}$, forbidden in the MSSM. These processes give indirect information about neutrino mixings since they depend on the same couplings as the Majorana mass terms of right-handed neutrinos.
2 Anomalous Triple Boson Couplings

2.1 Self-Interactions of Weak Bosons

The weak gauge boson sector of the Standard Model reflects two fundamental principles: the non-Abelian gauge symmetry of the Standard Model based on the gauge group $SU(2)_L \times U(1)_Y$ and the spontaneous breaking of the gauge symmetry which provides the $W$- and $Z$-bosons with their masses. An immediate consequence of the non-Abelian local gauge symmetry is the existence of gauge boson self-interactions which arise from the kinetic Lagrangian of the gauge fields,

$$L_{\text{gauge}} = -\frac{1}{2} \text{Tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4} \text{Tr}(B_{\mu\nu}B^{\mu\nu}),$$  

with the field strengths

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu], \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

where $W_\mu = \tau^a W^a_\mu/2$ and $W^a_\mu$ and $B_\mu$ denote the non-Abelian and Abelian field operators of gauge bosons, respectively. The gauge principle leads to the universality of the weak coupling constant, i.e. the strength of the $W$-boson coupling to fermions is given by the same parameter $g$ which appears in the three gauge boson as well as in the four gauge boson self-couplings in the Lagrangian (1).

The Lagrangian (1) describes only the transverse components of the gauge bosons. The longitudinal components of the massive gauge bosons would arise from the mass terms of the Lagrangian which, however, are not gauge invariant. The only known possibility to provide the gauge bosons with masses, so that the gauge invariance of the Lagrangian is preserved, is the spontaneous symmetry breaking mechanism [5].

In the Standard Model the linear realization of the spontaneous symmetry breaking is used. There exists a doublet of Higgs scalars in the theory, the neutral component of which obtains a non-vanishing vacuum expectation value. This gives masses to the gauge bosons, and the three unphysical Goldstone bosons associated with the broken symmetries become the longitudinal components of the vector bosons. If the Higgs boson is very heavy, the weak boson sector becomes strongly interacting at
high energies \[16\]. The latter possibility is not considered in this work, and it is assumed that the doublet Higgs boson is lighter than $\mathcal{O}(800)$ GeV which ensures that the gauge boson self-interaction processes remain perturbative.

To date, there has been no direct tests of either the non-Abelian nature nor the exact realization of spontaneous symmetry breaking in the electroweak boson system. The couplings of $W$- and $Z$-bosons to fermions are tested with a good accuracy, as these interactions can be studied at the energies of the present experiments at tree level. The triple and quartic gauge boson vertices, however, enter the low energy phenomena only through loop corrections and are therefore quite poorly tested. The existing $pp$ colliders measure the photon coupling to $W$-boson with errors larger than 100% \[17\]. The first reaction which will directly test the non-Abelian gauge structure of the weak bosons is the pair production of $W$-bosons in $e^+e^-$ collisions at LEP 200 \[6\]. This reaction probes both $\gamma WW$ and $ZWW$ interaction vertices. At the NLC one will also be able to study the reaction $e^-\gamma \rightarrow W^-\nu$, which gives information about the $\gamma WW$ coupling alone. This reaction is investigated in Paper I of this Thesis.

The possible deviations of the gauge boson self-interactions from their gauge theory form have in general quite remarkable effects, since they would spoil the delicate cancellation among the different amplitudes dictated by the gauge symmetry. This would mean that the good high energy behaviour of the total amplitude is lost and unitarity is violated at high energies. To preserve unitarity some new physical phenomena beyond the Standard Model should become effective above a certain energy scale $\Lambda \sim \mathcal{O}(1)$ TeV.

The effects of new physics below $\Lambda$ can be described by effective interaction terms of light fields in the Lagrangian. This is analogous to the situation in the Fermi theory where the weak interactions are described by the non-renormalizable pointlike higher order local operators, which lead to the violation of unitarity at higher energies. Unitarity is preserved by introducing a massive vector boson $W$ to the theory. In this particular case the scale $\Lambda$ can be identified with the mass of
the gauge boson, $M_W$. In general, the parameter $\Lambda$ should be regarded as a physical
cutoff scale where the theory has to be replaced by a more fundamental one.

There are two different approaches to parametrization of the anomalous triple
boson interactions. In the first approach, the so-called standard phenomenological
parametrization [18], one writes down the most general Lagrangian involving the
interaction of three gauge bosons which is allowed by the Lorentz invariance alone.
Being the most general, this approach can be criticized [4, 19] on the basis that it
does not respect the $SU(2)_L \times U(1)_Y$ gauge invariance. However, the phenomenologi-
cal Lagrangian can be regarded as a particular parametrization of a gauge invariant
Lagrangian written down in the unitary gauge where only the terms which describe
the triple boson coupling are presented [20]. Because of the gauge non-invariant
formulation, the standard phenomenological parametrization can be used to study
the effects of anomalous couplings at tree level only.

The gauge invariant formulation of the anomalous couplings emerges from the
effective field theory [21] - the theory where all the possible higher dimensional
operators allowed by the symmetries are added to the Standard Model Lagrangian:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\text{effective}},$$

$$\mathcal{L}_{\text{effective}} = \frac{1}{\Lambda^2} \sum_1^\infty \alpha_i^6 \mathcal{O}_i^6 + \frac{1}{\Lambda^4} \sum_1^\infty \alpha_i^8 \mathcal{O}_i^8 + ...$$

Here $\mathcal{O}^n$ denote the operators of dimension $n$ and $\alpha_i$ are the effective coupling
constants. In this theory all the appearing divergences can be absorbed in the
renormalization of the coefficients of the operators [22]. Therefore, it is applicable
also in the loop calculations. If the assumption that in the first approximation new
physics can be described by the lowest dimensional operators alone (dimension 6 in
the Standard Model) is made, then all the parameters employed by the standard
phenomenological approach are not independent any more [22]. In the standard
phenomenological parametrization this would correspond to imposing additional
global symmetries [23].
2.2 Standard Phenomenological Parametrization of Triple Boson Coupling

Let us now consider in more detail the standard phenomenological parametrization of the triple boson coupling applied in Paper I to the tree level reaction $e^- \gamma \rightarrow \nu W^-$. On general grounds, it has been shown that a charged spin-$J$ particle can have $(6J + 1)$ electromagnetic form-factors including $C_\mp$, $P_\mp$ and $CP_\mp$-violating terms [24]. There is also the equal number of invariant form-factors for the spin-1 coupling to a charged spin-$J$ particle, provided that the following conditions hold:

$$\partial_\mu V^\mu = 0, \quad \partial_\mu W^\mu = 0,$$

where $V^\mu$ denotes either the photon or $Z^0$ field and $W^\mu$ stands for the $W^-$ field. In this case the most general Lagrangian describing the $\gamma WW$ and $ZWW$ couplings in a Lorentz invariant way can be parametrized as follows [18]:

$$L_{VWWW}/g_{VWWW} = \frac{i g_1 V}{M_W^2} (W_{\mu \nu}^\dagger W^{\mu \nu} - W_{\nu}^\dagger W_{\mu}^\nu) + \frac{i \lambda_V}{M_W^2} W_{\mu \nu}^\dagger W^{\mu \nu},$$

$$+ g_5 V^\mu \partial_\mu V^\nu - W_{\mu}^\dagger W_{\nu} + \frac{1}{2} g_4 V^\mu \partial_\mu V^\nu + 2 g_5 V^\mu \partial_\mu V^\nu.$$  

Here $W_{\mu \nu} = \partial_\mu W_{\nu} - \partial_\nu W_{\mu}$ and $V_{\mu \nu} = \partial_\mu V_{\nu} - \partial_\nu V_{\mu}$ denote the Abelian field strength of $W^-$ and $\gamma/Z^0$ fields, respectively, and $g_1 V, \kappa_V, \lambda_V, g_4 V, g_5 V, \tilde{K}_V$ and $\tilde{\lambda}_V$ are independent form factors.

The conditions (5) imply that the scalar part of massive vector bosons can be neglected. These conditions are valid for the on-shell massive vector particles, as follows from the wave equations of $W$ and $Z$. In the case of off-shell massive vector bosons the scalar components do not contribute if the vector bosons couple to massless fermions (this fact follows from the Dirac equation). For the processes where light fermion masses are negligible compared with the energy involved, the Lagrangian (6) provides indeed the most general parametrization of the triple boson...
couplings. This is true for the processes like $e^-e^+ \rightarrow W^-W^+$ and $e^-\gamma \rightarrow W^-\nu$ which are of primary interest in the future experiments at LEP 200 and NLC.

In the pair production of the electroweak bosons in $e^-e^+$ collisions, where both $\gamma WW$ and $ZWW$ couplings play a role, there are altogether 14 independent parameters. The reaction $e^-\gamma \rightarrow W^-\nu$ studied in Paper I involves the $\gamma WW$ vertex only, and in the most general case it depends on 7 parameters.

On the other hand, we know from experiments that not all of the parameters involved in the general coupling can be treated on the same footing. There are well established symmetry principles which are known to hold with a good accuracy. Imposing these symmetries decreases the number of free parameters in the general Lagrangian (6). Obviously, electromagnetic $U(1)_{em}$ gauge invariance is such a symmetry. It fixes the electric charge of $W^-$ to be equal to $e$, implying $g_1^\gamma = 1$. Since in general the $CP$-violating effects are measured to be very small, it is safe to assume that they do not have any detectable impact on $\gamma WW$ coupling. Thus, in the first approximation the $CP$-violating terms can also be neglected.

The most general Lagrangian in agreement with these assumptions is of the form

$$L_{\gamma W W} = -ie(W^\dagger_\mu W^\nu A^\nu - W^\dagger_\mu A_\nu W^{\mu\nu} + \kappa_\gamma W^\dagger_\mu W^\nu F^{\mu\nu} + \frac{\lambda_\gamma}{M_W^2} W^\dagger_\mu W^\nu F^{\nu\mu} + \frac{\lambda_\gamma}{M_W^2} W^\dagger_\mu W^\nu F^{\nu\mu} + \frac{\lambda_\gamma}{M_W^2} W^\dagger_\mu W^\nu F^{\nu\mu}), \quad (7)$$

where $A_\mu$ and $F_{\mu\nu}$ denote the photon field and the photon field strength, respectively. The static electromagnetic properties of $W^-$, the magnetic dipole moment $\mu_W$ and the electric quadrupole moment $Q_W$, are related to the parameters appearing in (7) through

$$\mu_W = \frac{e}{2M_W}(1 + \kappa_\gamma + \lambda_\gamma), \quad (8)$$

$$Q_W = -\frac{e}{M_W^2}(\kappa_\gamma - \lambda_\gamma). \quad (9)$$

In the Standard Model at tree level one has $\kappa_\gamma = 1$ and $\lambda_\gamma = 0$.

In expressing the $\gamma WW$ vertex given by the Lagrangian (7) in the momentum space one can take into account the kinematics of the process at hand. In the case of the process $e^-\gamma \rightarrow W^-\nu$ the photon and one of the $W$’s are on mass-shell. The
Figure 1: Feynman rule for $\gamma WW$ vertex parametrized in terms of $\lambda_\gamma$ and $\kappa_\gamma$.

The corresponding vertex has the following form:

$$
\Gamma_{\nu\mu\rho}(k,k_1) = -2k_\nu g_{\mu\rho} - (1 + \kappa_\gamma - \lambda_\gamma)k_1\mu g_{\nu\rho} + (k + (\kappa_\gamma - \lambda_\gamma)k_1)\rho g_{\mu\nu} + \frac{\lambda_\gamma}{M_W^2}(k + k_1)\rho((k \cdot k_1)g_{\mu\nu} - k_\nu k_1\mu),
$$

where $k$ and $k_1$ denote the momenta of the incoming photon and of the outgoing on mass-shell $W^-$, respectively (see Fig.1).

Are there any general arguments which enable us to estimate the magnitude of the deviations from the Standard Model? If the underlying theory is a gauge theory conserving $SU(2)_L \times U(1)_Y$, then anomalous triple boson interactions can occur at the earliest at one loop level [26]. Since there is one power of a gauge coupling associated with every vertex and a loop phase space factor $1/16\pi^2$ suppressing the anomalous coupling, then even in the case of constructive contribution of several particles one would not expect any deviations from the Standard Model couplings to be larger than $\sim 1\%$. This is roughly the accuracy which one can expect to be achieved in the NLC with c.m. energy $\sqrt{s} = 500$ GeV. Hence the NLC will be the first accelerator where one would expect to achieve meaningful constraints on the underlying new physics by studying the anomalous boson couplings.
3 Minimal Left-Right Symmetric Model

One of the still unsolved problems in particle physics is the understanding of the parity violation observed in the low energy experiments. A possible solution is to assume that the interaction Lagrangian is left-right symmetric, i.e. at the level of the Lagrangian the left- and right-handed fields are treated on the same basis, but the vacuum is non-invariant under the parity transformation. That is to say, the left-right symmetry is spontaneously broken at very high energies, causing parity violation effects at low energies. This idea is realized in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge model \[9\]. At low energies this model reproduces all the features of the Standard Model, while at high energies parity is conserved and the new particle degrees of freedom predicted by the model, heavy gauge bosons, right-handed neutrinos and new Higgs bosons, start to become manifest.

In addition to a dynamical explanation of the parity violation of weak interaction there are several other hints \[27\] which may indicate that the left-right symmetry could play a fundamental role in Nature. One of the most appealing features of the left-right symmetric model is that with a suitably chosen Higgs sector, it is capable to explain the smallness of neutrino masses. The question whether neutrinos are exactly massless or have a small mass is not settled yet. As mentioned in the Introduction, there are some astrophysical observations, i.e. the deficit of solar neutrinos \[11\] and the possible indication of the COBE satellite results for a hot component of dark matter \[13\], as well as some neutrino oscillation results \[14\], which seem to support the idea of massive neutrinos. In the left-right symmetric model such situation can occur in a natural way, whereas in the Standard Model the neutrinos are exactly massless.

Another shortcoming of the Standard Model is the lack of clear physical meaning of the $U(1)$ generator. In the left-right symmetric model it becomes the $B - L$ quantum number, the only anomaly free quantum number left ungauged in the Standard Model, and all the weak interaction generators have a clear physical meaning. Thus the Gell-Mann formula for the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ theory
becomes
\[ Q = I_{3L} + I_{3R} + \frac{B - L}{2}. \]  

(11)

While the smallness of \( CP \)-violation is unexplained in the Standard Model, it is solved dynamically in the left-right symmetric model \[27\]. The suppression of \( CP \)-violating interactions arises due to the suppression of the right-handed currents and therefore originates from the spontaneous breaking of \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) symmetry.

Due to the breaking of \( B - L \) symmetry there exist lepton number violating interactions in the left-right symmetric model. At very low energies these interactions can be searched for in the neutrinoless double-\( \beta \) decay experiments \[28\]. At high energy collider experiments the \( B - L \) violating interactions would give rise to processes with very clean signatures. One such reaction, the \( W \) pair production in \( e^- e^- \) collisions, is investigated in Paper II.

Finally, the baryon number violating interactions of the Majorana neutrinos may provide us with an explanation of the matter-antimatter asymmetry in the Universe \[29\].

### 3.1 Basic Structure of the Model

In the left-right symmetric model the quark and lepton doublets

\[
Q_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}, \quad \psi_{L,R} = \begin{pmatrix} \nu \\ l^- \end{pmatrix}_{L,R}
\]  

(12)

are assigned to the gauge group \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) with the quantum numbers

\[
Q_R : (0, \frac{1}{2}, \frac{1}{3}), \quad Q_L : (\frac{1}{2}, 0, \frac{1}{3}), \\
\psi_R : (0, \frac{1}{2}, -1), \quad \psi_L : (\frac{1}{2}, 0, -1)
\]  

(13)

in agreement with eq.(11). There are three coupling constants \( g_R, g_L \) and \( g' \) associated with the symmetry groups \( SU(2)_R \), \( SU(2)_L \) and \( U(1)_{B-L} \), respectively. For reasons of symmetry the coupling constants \( g_L \) and \( g_R \) are usually assumed to be equal. Due to the \( SU(2)_R \) symmetry the model has two extra gauge bosons, \( W_R \) and
$Z_R$, as compared with the Standard Model. Since there is no experimental evidence for right-handed currents \[30, 31\], the new gauge bosons must be much heavier than the ordinary $W$ and $Z$ \[32\].

Regarding the Higgs sector of the left-right theories, there are several possibilities (see \[33\] and references therein). All the models contain a bidoublet field

$$
\phi(\frac{1}{2}, \frac{1}{2}, 0) = \begin{pmatrix}
\phi_1^0 \\
\phi_1^+ \\
\phi_2^- \\
\phi_2^0
\end{pmatrix}.
$$

(14)

Non-vanishing vacuum expectation values of its neutral members $\phi_1^0$ and $\phi_2^0$ are responsible for giving masses to the ordinary gauge bosons $W_L$ and $Z_L$ and contribute also to the masses of $W_R$ and $Z_R$. They do not break the $U(1)_{B-L}$ symmetry, however. In order to break the $U(1)_{B-L}$ symmetry, and to give large enough masses to the right-handed bosons, one has to add extra Higgs multiplets to the theory. These additional representations could be, for example, $SU(2)_R$ doublets with a non-vanishing $U(1)_{B-L}$ charge \[34\], but in this case neutrinos would be Dirac particles and the model fails to explain the smallness of the neutrino masses. In the case of $SU(2)_R$ triplet fields the neutrinos are Majorana particles, and the resulting neutrino mass matrix takes naturally the form suitable for the see-saw mechanism \[10\]. A model with one bidoublet $\phi(\frac{1}{2}, \frac{1}{2}, 0)$ and one right-handed Higgs triplet,

$$
\Delta_R(1, 0, 2) = \begin{pmatrix}
\Delta_R^+/\sqrt{2} \\
\Delta_R^0 \\
-\Delta_R^+/\sqrt{2}
\end{pmatrix},
$$

(15)

is the minimal scheme, and it possesses all the characteristic features of the left-right symmetric models.

In spontaneous symmetry breaking the neutral components of the Higgs multiplets acquire the vacuum expectation values

$$
< \Delta_R > = \begin{pmatrix}
0 & 0 \\
v_R & 0
\end{pmatrix},
$$

(16)

$$
< \phi > = \begin{pmatrix}
\kappa_1 & 0 \\
0 & \kappa_2
\end{pmatrix}.
$$

(17)
$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry breaks down to $U(1)_{em}$ symmetry in two stages. In the first stage the right-handed triplet $\Delta_R$ breaks the initial symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ to the Standard Model symmetry $SU(2)_L \times U(1)_Y$. At the same time the right-handed charged gauge boson acquires a mass

$$M_{WR}^2 = \frac{g_R^2 v_R^2}{2}. \quad (18)$$

By minimizing the triplet Higgs potential

$$V_{\Delta} = -\mu^2 \text{Tr} \Delta_R \Delta_R^\dagger + \rho_1 (\text{Tr} \Delta_R \Delta_R^\dagger)^2 + \rho_2 \text{Tr} \Delta_R \Delta_R \text{Tr} \Delta_R^\dagger \Delta_R^\dagger, \quad (19)$$

one finds $v_R = \mu^2 / \rho_1$.

The second stage of the spontaneous symmetry breaking, \textit{i.e.} $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$, is controlled by the vacuum expectation value of the bidoublet $<\phi>$. Apart from giving masses to $W_L$ and $Z_L$, $<\phi>$ may slightly mix $W_L$ with $W_R$. The resulting mass eigenstates are

$$W_1 = \cos \xi W_L + \sin \xi W_R,$$

$$W_2 = -\sin \xi W_L + \cos \xi W_R, \quad (20)$$

where the mixing angle $\xi$ is proportional to $\xi \sim \kappa_1 \kappa_2 / v_R^2$ \cite{33}.

During the two-stage symmetry breaking also the three neutral gauge bosons of the theory mix, resulting in a physical massless state, the photon, and two massive states $Z_1$ and $Z_2$, where $Z_1$ can be identified with the known neutral weak boson whereas $Z_2$, predominantly the state $Z_R$, is heavier.

Experimental knowledge of the suppression of the right-handed currents \cite{30, 31, 32} forces us to assume

$$v_R \gg \max(\kappa_1, \kappa_2). \quad (21)$$

If the right-handed symmetry is broken at a scale much higher than the electroweak scale, the masses of the right-handed particles, determined by the new scale, are naturally high. In the gauge boson sector the mixing between the left and right bosons is small ($\xi \rightarrow 0$), and the mass eigenstates $W_1$ and $W_2$ with $M_{W_2}^2 \gg M_{W_1}^2 = \frac{g_R^2 v_R^2}{2}$.
$g_L^2(\kappa_1^2 + \kappa_2^2)/2$ correspond predominantly to $W_L$ and $W_R$, respectively. It has been argued [33] that in a phenomenologically consistent model the gauge boson mixing angle should be exactly zero. In the following we shall always neglect the gauge boson mixing and take the weak eigenstates to be equal to the mass eigenstates. This simplification is supported by the experimental data, since the mixing angle $\xi$ is measured to be smaller than $\xi < 0.04$ [35].

Another consequence of the relation (21) concerns the neutrino masses. Before studying the neutrinos of left-right symmetric model explicitly, let us discuss possible neutrino masses in a model independent way.

### 3.2 Generation of Neutrino Masses

The most general mass term for a spin-$\frac{1}{2}$ fermion allowed by Lorentz invariance can be expressed as a sum $L_{\text{mass}} = L_D + L_M$ of the Dirac mass terms [36]

$$L_D = m_D \bar{\nu} R \nu_L + h.c.$$  \hspace{1cm} (22)

and the Majorana mass terms

$$L_M = m_L \bar{\nu}^c R \nu_L + m_R \bar{\nu}^c L \nu_R + h.c.$$  \hspace{1cm} (23)

Here $\nu^c$ denotes the charge conjugated neutrino field defined by $\nu^c = C \bar{\nu}^T$ where $C$ is the charge conjugation matrix, and $m_D$, $m_L$ and $m_R$ are constants. Under the global transformation

$$\nu \rightarrow e^{i\theta} \nu, \quad \nu^c \rightarrow e^{-i\theta} \nu^c$$  \hspace{1cm} (24)

the Dirac mass terms $L_D$ transform into themselves, whereas the Majorana mass terms $L_M$ acquire an extra phase $e^{2i\theta}$. Thus, in contrast to the ordinary Dirac mass terms, Majorana mass terms do not preserve additive quantum numbers like electric charge or lepton number. Therefore, charged fermions can admit only a Dirac mass while the neutral fermions like neutrino can have mass terms of both types. If Majorana mass terms are present, the theory will contain interactions which violate lepton number by two units, $\Delta L = 2$. 
In the case of a pure Dirac mass term \((m_L = m_R = 0)\) the physical states are Dirac particles, \(i.e.\) the neutrino and the anti-neutrino are different particles. As can be seen from the mass Lagrangian \((22)\), both left and right chirality components of a Dirac neutrino have the same mass and there is no kinematical suppression of the production of a right-handed neutrino. If both Dirac and Majorana mass terms, or Majorana mass terms alone, are present, then the physical mass eigenstate neutrinos are their own antiparticles:

\[
\nu^c = e^{i\lambda} \nu,
\]

where \(\lambda\) is a real phase.

Since the mass parameters are arbitrary, one may realize a situation where \(m_R\) is large compared with \(m_L\) and \(m_D\). This would ensure that the processes involving right-handed neutrino interactions are kinematically forbidden at low energies.

In the \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) left-right symmetric model the neutrino masses arise from the following Yukawa interaction Lagrangian:

\[
L_Y = f \bar{\psi}_L \phi \psi_R + i h \bar{\psi}^c_L \tau_2 (\tau \cdot \Delta_R) \psi_R + h.c.,
\]

where \(f\) and \(h\) denote unknown Yukawa coupling constants. The first term in \((26)\) is of the Dirac type and the second of the Majorana type. After spontaneous symmetry breaking neutrinos acquire masses which in a suitable basis can be written as follows:

\[
L_{mass} = - (\bar{\nu}^c_R, \bar{\nu}_R) \begin{pmatrix}
0 & m_D \\
m_D & m_R
\end{pmatrix} \begin{pmatrix}
\nu_L \\
\nu^c_L
\end{pmatrix}.
\]

The resulting mass matrix is of the form which realizes the see-saw mechanism \([10]\). The off-diagonal terms \(m_D = f \kappa_1\) are determined by the left-handed symmetry breaking scale and are therefore expected to be of the order of a typical Dirac fermion mass. The non-zero diagonal entry \(m_R = h v_R\) is set by the breaking scale of the \(SU(2)_R\) symmetry, which justifies the assumption \(m_D \ll m_R\).

The physics content of the Lagrangian \((27)\) becomes apparent when it is diagonalized. It can be written in the canonical form

\[
L_{mass} = -(m_1 \bar{\chi}_1 \chi_1 + m_2 \bar{\chi}_2 \chi_2),
\]
where the eigenvectors $\chi_1$ and $\chi_2$ are expressed in terms of the self-conjugate fields $\chi_L = \nu_L + \nu_R^c$ and $\chi_R = \nu_R + \nu_L^c$ as follows:

$$\chi_1 \approx \chi_L - \frac{m_D}{m_R} \chi_R,$$

$$\chi_2 \approx \chi_R + \frac{m_D}{m_R} \chi_L. \quad (29)$$

The masses of these two Majorana neutrinos $\chi_1$ and $\chi_2$ are approximately

$$m_1 \approx \frac{m_D^2}{m_R}, \quad m_2 \approx m_R, \quad (30)$$

and the states have opposite $CP$-parities,

$$\eta_{CP}(\chi_1) = -i, \quad \eta_{CP}(\chi_2) = +i. \quad (31)$$

The Majorana nature of the neutrinos is obvious since $\nu_{L,R}^c = (\nu_{R,L})^c$. According to eq.(29), the heavy mass eigenstate $\chi_2$ corresponds predominantly to the right-handed neutrino, and the light mass eigenstate $\chi_1$ predominantly to the left-handed neutrino. Neglecting the small mixing of the order of $m_D/m_R$, the heavy neutrino interacts via $(V + A)$ currents, while the light, Standard-Model-like neutrino, interacts via $(V - A)$ currents. As can be seen from (30), the mass of the light neutrino is directly connected to the scale $v_R$ where the left-right symmetry is broken so that the larger this scale, the smaller the neutrino mass. In the limit $v_R \to \infty$, light neutrino mass vanishes and the left-right symmetric model becomes in all respects indistinguishable from the Standard Model.

Despite of the searches for the new gauge bosons and $(V + A)$ currents in accelerators as well as in low-energy weak interaction experiments, no indications of their existence has been found so far. An interesting possibility for the future collider experiments would be to investigate the $|\Delta L| = 2$ interactions, which in the framework of the left-right symmetric model are mediated by massive neutrinos and doubly charged Higgs bosons. Such studies will not only reveal the nature of the massive neutrinos, but also shed light on the Higgs sector of the theory. In Paper II, one analysis of this type is carried out.
4 Supersymmetric Left-Right Model

The left-right symmetric model has enabled us to explain successfully some of the questions left unanswered in the Standard Model. On the other hand, like in the Standard Model, the left-right symmetric model suffers from the hierarchy problem: the masses of the Higgs scalars diverge quadratically when loop corrections are taken into account. One can cure this problem by making the theory supersymmetric. Due to the symmetry between fermions and bosons, there exist supersymmetric partners for every ordinary particle, which cancel the quadratic divergences. While there is no evidence for the existence of the susy partners, the interest in supersymmetry was renewed when it turned out that in $SU(5)$ grand unified model the unification of coupling constants at high energies occurs only when the model is supersymmetric [37]. However, in the $SO(10)$ theory, which would be the grand unified framework for the left-right model, supersymmetry would not be necessary for this reason [38].

Apart from the existence of superpartners, the biggest difference between susy and non-susy models concerns the Higgs sector. From the phenomenological point of view a major difference is that in susy models some of the Higgs bosons must be rather light. It has been shown that in supersymmetric models with arbitrary Higgs sector the mass of the neutral Standard Model like Higgs particle cannot be heavier than about 150 GeV [40].

Susy left-right symmetric model has been previously studied by many authors [41, 42, 43, 44, 45]. Paper III of this Thesis introduces a minimal susy left-right model where the number of Higgs fields is the smallest possible. The minimal set of Higgs fields in the non-susy left-right model consists of a bidoublet $\phi_u$ and a $SU(2)_R$ triplet $\Delta_R$. After supersymmetrization, the cancellation of chiral anomalies among the fermionic partners of the triplet Higgs fields $\Delta_R$ requires introduction of a second triplet $\delta_R$ with opposite $U(1)_{B-L}$ quantum number. Due to conservation of the $B - L$ symmetry, $\delta_R$ does not couple to leptons or quarks. In order to avoid a trivial Kobayashi-Maskawa matrix for quarks, also another bidoublet $\phi_d$ should be added to the model. This is because supersymmetry forbids a Yukawa coupling
where the bidoublet appears as a conjugate.

The vacuum expectation values for the Higgses, which break the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ into the $U(1)_{em}$, can be chosen as follows:

$$
\langle \phi_u \rangle = \begin{pmatrix} \kappa_u & 0 \\ 0 & 0 \end{pmatrix}, \quad \langle \phi_d \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_d \end{pmatrix}, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad \langle \delta_R \rangle = 0.
$$

(32)

Here $\kappa_{u,d}$ are of the order of the electroweak scale $10^2$ GeV, since they give mass to $W_L$ and $Z_L$. As discussed in the previous Section, the vacuum expectation value $v_R$ of the triplet Higgs has to be large in order to have the masses of the new gauge bosons $W_R$ and $Z_R$ sufficiently high. Since in (32) only one of the neutral fields in each of the bidoublets $\phi_u$ and $\phi_d$ are assumed to acquire a non-vanishing vacuum expectation value, the charged gauge bosons do not mix, and $W_L$ corresponds to the observed weak bosons.

In Papers III and IV the supersymmetric version of the left-right symmetric model and its phenomenological implications for high energy collisions are investigated. There it is assumed that the superpotential has the following form:

$$
W = h_u^Q \hat{Q}_L^T \hat{\phi}_u \hat{Q}_R + h_d^Q \hat{Q}_L^T \hat{\phi}_d \hat{Q}_R \\
+ h_u^L \hat{L}_L^T \hat{\phi}_u \hat{L}_R + h_d^L \hat{L}_L^T \hat{\phi}_d \hat{L}_R + h_\Delta \hat{L}_R^T \tau_2 \hat{\Delta}_R \hat{L}_R \\
+ \mu_1 \text{Tr}(\tau_2 \hat{\phi}_u^T \tau_2 \hat{\phi}_d) + \mu_2 \text{Tr}(\hat{\Delta}_R \hat{\delta}_R).
$$

(33)

Here $\hat{Q}_{L(R)}$ stands for the doublet of left(right)-handed quark superfields, $\hat{L}_{L(R)}$ stands for the doublet of left(right)-handed lepton superfields, $\hat{\phi}_u$ and $\hat{\phi}_d$ are the two bidoublet Higgs superfields, and $\hat{\Delta}_R$ and $\hat{\delta}_R$ the two triplet Higgs superfields. In superpotential (33) the $R$-parity, $R = (-1)^{3(B-L)+2S}$, is preserved. This and the assumption $< \tilde{\nu} >= 0$ ensures that susy partners with $R = -1$ are produced in pairs and that the lightest supersymmetric particle is stable.

If supersymmetry were an exact symmetry, the masses of particles and their susy partners would be the same. Since none of the superpartners has been observed, one has to break supersymmetry. In Papers III and IV breaking has been assumed to
happen softly, *i.e.* the superpotential contains the most general non-supersymmetric mass terms for scalars and gauginos, which do not give rise to quadratic divergences:

\[
\mathcal{L}_{\text{soft}} = -\frac{1}{2} \sum_i m_i^2 |\varphi_i|^2 - \frac{1}{2} \sum_\alpha M_\alpha \lambda_\alpha \lambda_\alpha + B \varphi^2 + A \varphi^3 + h.c. \tag{34}
\]

Here \(\varphi_i\) denote scalar fields and \(\lambda_\alpha\) stands for gaugino fields. The soft masses should not be much heavier than \(\mathcal{O}(1)\) TeV \[46, 47\] because their contribution to the lightest Higgs mass would become too large otherwise. In order to preserve the naturalness of the theory the supersymmetric mass parameters \(\mu_i\) in the superpotential (33) should be close to the scale of soft supersymmetry breaking. In order to avoid an unnatural hierarchy of the mass parameters at the Lagrangian level, one can assume that the parameters \(|\mu_i|\) are also of the order of the electroweak scale.

Particularly interesting objects from the phenomenological point of view are the doubly charged fermions, the superpartners of the triplet scalars \(\tilde{\Delta}^{++}\) and \(\tilde{\delta}^{++}\). Their mass matrix is particularly simple, since doubly charged higgsinos do not mix with gauginos. The susy mass terms for triplet higgsinos are given by

\[
\mathcal{L}_{\text{triplet mass}} = -\mu_2 (\tilde{\Delta}^+\tilde{\delta}^- + \tilde{\Delta}^{++}\tilde{\delta}^{--} + \tilde{\Delta}^0\tilde{\delta}^0) + h.c. \tag{35}
\]

which implies that the mass of the doubly charged higgsino is set by the parameter \(\mu_2 = M_{\Delta^{++}}\). Thus the doubly charged higgsino mass is a free parameter of the model which by naturality arguments should be close to the weak scale. The triplet higgsinos, like the triplet Higgses, carry two units of lepton number, and therefore the final state of their decay must also have even lepton number in the case of \(R\)-parity conservation. This follows from the Lagrangian

\[
\mathcal{L}_{\tilde{\Delta}l} = -2 h_{\Delta} \bar{\tilde{\Delta}} \tilde{l}, \tag{36}
\]

which describes an interaction between the doubly charged higgsino, lepton and the slepton.

There are five charginos \(\psi_j^\pm\) and nine neutralinos \(\psi_i^0\) in this model. The physical particles \(\tilde{\chi}_i^\pm\) and \(\tilde{\chi}_i^0\) are found by diagonalizing the mass Lagrangian:

\[
\tilde{\chi}^\pm_i = \sum_j C_{ij}^\pm \psi_j^\pm, \tag{37}
\]
\[ \chi_i^0 = \sum_j N_{ij} \psi_j^0, \]  

where \( C_{ij}^\pm \) and \( N_{ij} \) denote the diagonalizing matrices of charginos and neutralinos, respectively. The neutralinos are Majorana particles, whereas the charginos combine to form Dirac fermions.

The masses of susy particles depend on the following parameters: the soft gaugino masses, the supersymmetric Higgs masses \( \mu_1 \) and \( \mu_2 \), the vacuum expectation values \( \kappa_u, \kappa_d, \) and \( v_R \), and the gauge coupling constants. It has been shown \[15\] that one of the doubly charged Higgses \( \Delta^{++} \) in the supersymmetric left-right model must be lighter than a few hundred GeV. In Papers III and IV we have carried out the analysis of the particle contents of the model for different values of the mass parameters. For large soft gaugino masses (around 1 TeV) the neutralinos are predominantly higgsinos, whereas for smaller soft masses (200 GeV) they are mainly gauginos. The difference in the neutralino interactions in these two cases should manifest itself in collider experiments. For simplicity, in Paper IV the mixing of the left and right selectrons is assumed to be negligible, and their masses \( m_{\tilde{e}_L} \) and \( m_{\tilde{e}_R} \) are taken to be equal.
5 Tests of the Extended Models

The main emphasis in the Papers of this Thesis is on the high energy signatures of the beyond-the-Standard-Model schemes described in the foregoing Sections. The experimental environment one has in mind is mainly the NLC. According to present plans \([8]\), in the first stage the NLC will operate at the \(e^+e^-\) c.m. energy of \(\sqrt{s} = 0.5\) TeV, and later the energy will be increased up to \(\sqrt{s} = 2\) TeV. The anticipated luminosity of the \(e^+e^-\) collision mode is \(\mathcal{L} = 10^{34}\) cm\(^{-2}\)s\(^{-1}\).

While the \(e^-e^+\) option will be the main operation mode of the next linear collider, also \(e^-e^-, e^-\gamma\) and \(\gamma\gamma\) collisions are technically feasible \([8]\). The \(e^-e^-\) collider design is, in principle, simpler than the \(e^-e^+\) one since there is no need to create positrons. For the photon colliders the photon beam can be obtained by scattering linearly polarized laser light off of the electron beam. The result is a polarized photon beam with very hard spectrum strongly peaked at the maximum energy which is about 84% of the electron beam energy \([48]\). The growing interest in the non-conventional collision modes arises not only from the need of having complementary tests of physical quantities but also from the fact that for many purposes the new options provide more useful reactions to study than the conventional \(e^-e^+\) mode. For example, the \(e^-e^-\) option is particularly suitable for the study of lepton number violating interactions, since its initial state carries lepton number two. The \(e^-\gamma\) and \(\gamma\gamma\) modes give us a direct access to processes which in the other collision modes appear only as subprocesses of higher order reactions. Hence the new collision options of the NLC are considered to be very useful for studying various extensions of the Standard Model.

5.1 Measurements of Anomalous Triple Boson Coupling

At present the most stringent experimental bounds for the \(\gamma WW\) vertex parameters \(\kappa_\gamma\) and \(\lambda_\gamma\) defined in the Lagrangian \([7]\) are obtained by the TEVATRON CDF
experiment from a direct measurement of photon-$W$ interaction \[17\]:

\[-2.3 \leq 1 - \kappa_\gamma \leq 2.2,\]

\[-0.7 \leq \lambda_\gamma \leq 0.7,\]  \hspace{1cm} (39)

given at 95\% CL. More stringent, but less direct limits have been obtained from studies of electroweak radiative corrections of low energy data by applying the effective Lagrangian formalism of dimension-6 operators using, however, some additional assumptions \[4\]. It has been shown in ref.\[22\] that there is no rigorous model-independent bounds for the anomalous triple boson coupling from radiative corrections. Therefore, the precise direct measurements are needed in the future colliders in order to improve the limits (39).

The first electron collider which will probe the $\gamma WW$ coupling is LEP 200. Because of the low energy and low luminosity, LEP 200 will be able to measure the coupling only with a precision not better than about 10\% \[49\]. Theoretically, one should not expect anomalous couplings to be larger than 1\% \[26\], and hence the first stage of the NLC is likely to be the first place to produce relevant new constraints.

The $W$-boson pair production \[18\]

\[
e^-e^+ \rightarrow W^-W^+\]  \hspace{1cm} (40)
is a particularly clean process to study the triple boson couplings and to find constraints on the anomalous terms. It will be an important reaction to be experimentally studied in the NLC. Taking the collision energy to be $\sqrt{s} = 500$ GeV, integrated luminosity 50 fb$^{-1}$ and beam polarization 90\%, the sensitivity of the NLC with respect to the parameters $\kappa_\gamma$ and $\lambda_\gamma$ at 95\% CL is anticipated to be \[19\]

\[-0.0052 \leq 1 - \kappa_\gamma \leq 0.0057,\]

\[-0.012 \leq \lambda_\gamma \leq 0.021.\]  \hspace{1cm} (41)

A disadvantage of the processes \[41\] is that it does not allow separate tests of the anomalous photon and $Z^0$ couplings, since both $\gamma WW$ and $ZWW$ vertices are
involved in the reaction. The $e^-\gamma$ collision option of the NLC will be an ideal place for studying the photon anomalous couplings separately.

There are two different possible collision schemes for the photon colliders [50]. First, the photon conversion region is very close to the interaction point and the entire photon spectrum interacts with the electron beam. From the physics point of view this realization of the $e^-\gamma$ collisions is undesired because of the high rate of background processes initiated by the electrons which have been used for creating the photon beam, and also because of the low monochromaticity of the photon beam.

In the second collision scheme the distance between the conversion and interaction points is larger. The electrons used for producing the photon beam are removed by applying a strong magnetic field, and therefore the $e^-\gamma$ collisions are clean and highly monochromatic. The achievable luminosities in this case are found to vary from 30 fb$^{-1}$ at VLEPP to 200 fb$^{-1}$ at TESLA per year [50] depending on the linear collider design.

The most sensitive process to the photon anomalous coupling in $e^-\gamma$ collision mode is

$$e^-\gamma \rightarrow W^-\nu.$$  \hspace{1cm} (42)

The process has been previously investigated in ref.[51]. In Paper I the updated analysis of the reaction (42), taking into account beam polarization as well as recent developments in the linear collider design, is carried out. The center of mass energy is assumed to be $\sqrt{s_{e\gamma}} = 420$ GeV, corresponding to the peak value of the photon energy spectrum. The integrated luminosity is estimated to be $L_{int} = 50$ fb$^{-1}$. A $\chi^2$ analysis of the differential cross section of the process, which turns out to be the most sensitive observable with regards to the parameters $\kappa_\gamma$ and $\lambda_\gamma$, yields for the measurement sensitivity of the $e^-\gamma$ collider at 90% CL the following bounds:

$$-0.01 \leq 1 - \kappa_\gamma \leq 0.01,$$

$$-0.012 \leq \lambda_\gamma \leq 0.007.$$  \hspace{1cm} (43)

The estimate of Paper I shows that the beam polarization together with the monochromaticity of the photon beam improves the sensitivity by a factor of 3. Comparison
with the bounds (11) reveals that the process (12) allows to constrain the parameter \( \lambda \) more strictly than the process (10). Let us emphasize again, the bounds (13) are independent of \( ZWW \) coupling.

In order to further increase the measurement precision of the anomalous triple boson coupling one needs higher collision energies. It has been estimated in ref.[49] that the NLC with a center of mass energy \( \sqrt{s} = 1 \) TeV will allow measurements with the precision of about 0.1%.

5.2 Searches for Left-Right Symmetry

Up to date no new heavy gauge bosons, heavy neutrinos or new types of interactions going beyond the Standard Model have been discovered. In order to make the left-right symmetric model consistent with this fact one must constrain the parameters of the model, such as the masses of the right-handed gauge bosons \( W_R \) and \( Z_R \) and the Lorentz structure of the interactions.

The experimental constraints are obtained mainly from the low-energy weak interaction processes, where one searches for possible manifestations of \((V + A)\) currents, and from high energy collider experiments, which give direct mass limits for the new particles. The present lower limit for the mass of \( W_R \), obtained from a direct search in TEVATRON, is \( M_{W_R} \geq 652 \) GeV [32]. This is comparable with the limits from the low energy processes \( K \rightarrow \pi \mu \bar{\nu}, \mu \rightarrow e \nu \bar{\nu} \) [30] where \( W_R \) appears as a virtual intermediate state. The most stringent limit quoted in the literature is derived from the \( K_L - K_S \) mass difference [31]: \( M_{W_R} \geq 1.6 \) TeV.

All these constraints, however, are subject to various assumptions about the details of the left-right symmetric model. The gauge couplings of \( SU(2)_L \) and \( SU(2)_R \) gauge groups are usually taken to be equal \( g_L = g_R \), the Cabibbo-Kobayashi-Maskawa matrix for the right-handed quarks is assumed to be the same as for the left-handed quarks, \( V_R = V_L \), and the right-handed neutrinos are assumed to be light. All the limits discussed will be considerably weakened if these simplifying assumptions are relaxed [52]. For example, the muon decay \( \mu \rightarrow e \nu \bar{\nu} \) bounds do not
hold if the mass of the right-handed neutrino exceeds 50 MeV. The TEVATRON bounds on $M_{W_R}$ are degraded if the right-handed neutrinos decay in the detector, or if $(V_R)_{ud} \ll (V_L)_{ud}$. Also the $K_L - K_S$ limit can be weakened if $V_R \neq V_L$ and $g_R < g_L$.

Experimentally the most intriguing prediction of the left-right symmetric model is the existence of lepton number violating processes associated with the Majorana nature of neutrinos. One of the processes studied at low energies is the neutrinoless double-$\beta$ decay \[28\],

\[(A, Z) \to (A, Z + 2) + 2e^-, \quad (44)\]

where a nucleus decays into another nucleus by emitting of two electrons. In the left-right symmetric model this reaction can arise from the diagram depicted in Fig. 2 where the gauge bosons can be either an ordinary $W_L$ or a right-handed $W_R$.

The important feature of the process is that it takes place only when the mediated neutrino is a massive Majorana particle. In the Standard Model the lepton number is conserved and the reaction (44) is forbidden. So far, there exists no evidence for the neutrinoless double-$\beta$ decay. This can be used to derive constraints for the light neutrino mass which at the moment is $\langle m_\nu \rangle < 0.68 \text{ eV} \quad [28]$. 

The collision energies of the future collider experiments would possibly allow for the study of the interactions of the particles of the right-handed sector directly.
One interesting reaction which may be possible to investigate in the NLC is

\[ e^-e^- \rightarrow W^-W^- . \]  

This is the inverse process to the neutrinoless double-\( \beta \) decay. While the reaction \( e^+e^- \rightarrow W^+W^- \), which will be soon explored at LEP 200, is possible irrespectively of whether neutrinos are Dirac or Majorana particles, the reaction (45) can occur only in the Majorana case and is thus forbidden in the Standard Model.

The reaction (45) proceeds via a neutrino exchange in the t- and u-channels and via a doubly charged triplet Higgs \( \Delta^{--} \) exchange in the s-channel (see Fig. 3). There is a strong cancellation between the contributions from the different channels in the amplitude, which guarantees the good high energy behaviour of the cross section. The final state in the process (45) can be either \( W_LW_L \), \( W_RW_R \) or \( W_LW_R \). Because the mixing between \( W_L \) and \( W_R \) is known to be small, the last channel is suppressed. The \( W_LW_L \) final state is, in turn, suppressed by the smallness of the light neutrino mass and the triplet Higgs Yukawa coupling to the \( W_L \). Thus the final state \( W_RW_R \) is the most relevant one, provided that the collision energy exceeds the kinematical threshold.

The phenomenological aspects of the reaction (45) have been previously studied in ref.\[54\]. In Paper II it is investigated in a more general approach by assuming a general form for the couplings involved and taking into account the polarization of the initial and final state particles. General conditions which the couplings should satisfy to ensure the good high energy behaviour of the process are derived there. These conditions concern the case where the final state \( W \)'s are longitudinally polarized, since the possible divergences would occur in this channel. In gauge theories the longitudinal components of massive gauge bosons play a special role since they are created by the Higgs mechanism. In the original Lagrangian they correspond to the Goldstone bosons. At high energies the gauge boson interactions are indistinguishable from the corresponding Goldstone boson interactions. This result is known as the equivalence theorem \[55\]. Since the singly charged Goldstone boson interaction with \( \Delta^{--} \) follows from the potential (19), studies of the process (45)
Figure 3: Feynman diagrams for the inverse neutrinoless double-β decay.

could give valuable information about the triplet Higgs potential.

If $W_R$'s were too heavy to be pair produced in the NLC , one could still see signals of doubly charged triplet Higgses in $e^- e^-$ collisions through other processes. For example, one can look for a s-channel $\Delta^{--}$ resonance in the reaction

$$e^- e^- \rightarrow l^- l^-,$$

(46)

where $l^-$ can be $e^-$, $\mu^-$ or $\tau^-$. The uncertainty associated with this process is that it depends on the unknown $\Delta^{++} l^- l^-$ Yukawa coupling constant $h$. Assuming $h$ to be of the order of the gauge coupling constant one will be able to see at the NLC the doubly charged Higgs bosons with masses as high as $M_\Delta = 10$ TeV [56]. Since the mass $M_\Delta$ is set by the right-handed symmetry breaking scale, it may be very large. In the case of the susy left-right model, however, where $\Delta^{--}$ should be light [15], the NLC with $\sqrt{s} = 1$ TeV should suffice to discover it.

5.3 Searches for Supersymmetric Left-Right Model

The success of supersymmetric models in explaining the theoretical ambiguities of the Standard Model has not been followed by an experimental discovery of su-
persymmetric particles. Most of the mass limits for susy particles are obtained assuming the minimal supersymmetric standard model and depend on various assumptions on the model parameters. The current best limit for the mass of lightest neutralino, which is supposed to be also the lightest supersymmetric particle obtained by ALEPH experiment at LEP is \[ m_{\tilde{\chi}_0} \gtrsim 18 \text{ GeV}. \] The mass limits for charged susy particles, the charginos and selectrons, obtained from LEP, are \[ m_{\tilde{\chi}^+, \tilde{e}^+} \gtrsim 45 \text{ GeV}. \] In these analyses the mass of the left-handed selectron is usually assumed to be larger than the mass of the right-handed selectron as follows from the minimal supersymmetric standard model with the unification assumption \[ 39 \]. The coloured states are also supposed to be heavier than the uncoloured states \[ 47 \], and therefore the squark production in colliders is disfavoured compared with the slepton production. The corresponding mass bounds for the susy left-right model particles are expected to be similar for the charged particles, since the bounds are rather model independent and are set mainly by the collision energy of LEP. The experimental bounds for neutralinos, however, are model dependent and should be considerably weakened when the model dependent assumptions are relaxed.

There is an upper limit of \( \mathcal{O}(1) \) TeV for the susy breaking scale \[ 46, 47 \], above which the mass differences between particles and their supersymmetric partners become too large to cancel the contributions to the scalar self-masses to a sufficient level. Since the masses of susy particles are set by soft mass terms, they cannot exceed this limit. This implies that susy models in the present context can be discovered or excluded after realization of the NLC and the Large Hadron Collider projects.

Tests of the minimal supersymmetric standard model in \( e^- e^-, e^-\gamma \) and \( \gamma\gamma \) collisions at the NLC have been considered in ref.\[ 58 \]. In Paper III and Paper IV of this Thesis the possible tests of the susy left-right model using the same collision modes have been investigated.

One very promising test of the susy left-right symmetric model in the NLC is the production of doubly charged higgsinos \( \tilde{\Delta}^{++} \), discussed in Paper III. They carry
two units of lepton number and decay to two leptons with equal charge and large missing energy, giving a very clean signature in experimental search. Their mass is a free parameter in the model and, as argued in Section 4, should not differ too much from the electroweak symmetry breaking scale. In Paper III we have studied the triplet higgsino production in $e^-e^-$, $e^+e^-$, $e^-\gamma$ and $\gamma\gamma$ options of the NLC via the processes

$$e^+e^- \rightarrow \tilde{\Delta}^{++}\tilde{\Delta}^{--}, \quad (47)$$
$$e^-e^- \rightarrow \tilde{\chi}^0\tilde{\Delta}^{--}, \quad (48)$$
$$\gamma e^- \rightarrow \tilde{l}^+\tilde{\Delta}^{--}, \quad (49)$$
$$\gamma\gamma \rightarrow \tilde{\Delta}^{++}\tilde{\Delta}^{--}. \quad (50)$$

With the collision energy of $\sqrt{s} = 1$ TeV the cross section of the process (47) is found to be of the order of $O(1)$ pb for the large range of particle masses. This would allow for the discovery of $\tilde{\Delta}^{--}$ with a mass close to the beam energy. The cross sections of the processes (48) and (49) are in general about an order of magnitude smaller, but since the single $\tilde{\Delta}^{--}$ production is associated with the production of another, presumably lighter, particle, then the study of the processes (48) and (49) will enlarge the $\tilde{\Delta}^{--}$ mass range testable in the NLC. Despite of the smaller collision energy of $\gamma\gamma$ compared with $e^+e^-$ collisions, the virtue of the reaction (50) is that it depends only on the unknown doubly charged higgsino mass.

If the doubly charged higgsinos are too heavy to be produced in the future colliders, their extra contribution as the virtual intermediate states in the selectron pair production,

$$e^+e^- \rightarrow \tilde{e}^+\tilde{e}^-, \quad (51)$$

in LEP 200 and NLC might reveal their existence. This possibility has been studied in Paper IV. Because of the larger number of neutralinos, compared with the minimal supersymmetric standard model, and the doubly charged higgsino involved in the process, the cross section of the reaction (51) is found in the susy left-right symmetric model to be about 5 times larger than in the minimal supersymmetric standard
model. The effects of the doubly charged higgsino can be found by studying the
distribution of final state electrons. Since $\tilde{\Delta}^{--}$ contribution to the process (51) compared with the neutralino contribution is with the opposite angular distribution, for a large range of model parameters the doubly charged higgsino contribution should be observable. This result, however, assumes that at least the right selectrons are light enough to be pair produced in the colliders.
6 Summary

While the Standard Model of electroweak interactions is known to be in a good agreement with the data collected so far, it is not excluded that new phenomena beyond it could start to manifest themselves when the experimental search moves up to the TeV-scale. The aim of this Thesis is to study the potential of the non-conventional $e^-e^-$, $e^-\gamma$ and $\gamma\gamma$ collision modes of the NLC for testing phenomena beyond the Standard Model. These collision modes, which previously have been available at much lower energy scales than the NLC will offer, have several advantages over the $e^+e^-$ mode since they allow to study several aspects of the underlying physics which are not accessible in the conventional operation mode.

The $e^-\gamma$ collision mode is particularly suitable for studying the three boson coupling $\gamma WW$ through the reaction $e^-\gamma \rightarrow W^-\nu$. In this reaction the $\gamma WW$ coupling can be measured independently from the $ZWW$ coupling, in contrast with, for example, the reaction $e^+e^- \rightarrow W^+W^-$. Assuming Lorentz and $CP$ invariance and imposing $U(1)_{em}$ symmetry, the general $\gamma WW$ coupling can be parametrized with two independent form factors $\kappa_\gamma$ and $\lambda_\gamma$. We have analysed the sensitivity of the reaction $e^-\gamma \rightarrow W^-\nu$ to these parameters, assuming beam polarization and taking into account the anticipated developments in the NLC design. With c.m. energy $\sqrt{s} = 420$ GeV and data of $50 \text{ fb}^{-1}$, at 90% CL the parameters may be constrained to the range $-0.01 \leq 1 - \kappa_\gamma \leq 0.01, -0.012 \leq \lambda_\gamma \leq 0.007$.

Motivated by its capability to explain the smallness of neutrino masses we have studied the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, in which the left-right symmetry is spontaneously broken by a right-handed triplet Higgs field. We have investigated a particularly interesting process, the so-called inverse double-$\beta$ decay $e^-e^- \rightarrow W^-W^-$, mediated by the Majorana neutrinos in t- and u-channel and by the doubly charged Higgs bosons, which carry lepton number two, in s-channel. In Paper II we derive helicity amplitudes for this reaction and give the conditions for the couplings to ensure its good high energy behaviour. The process is useful not only for clarifying the nature of neutrinos but also for studying the Higgs sector of
the theory.

In order to solve the hierarchy problem associated with the quadratic self-energy divergences of Higgs bosons the left-right symmetric model is supersymmetrized and we have studied the consequences of this model in the experiments at the NLC. The most distinctive signature of the model, two same charge leptons with missing energy, would be provided by the decay of the doubly charged higgsino. Using all the collision modes of the NLC one will be able to study doubly charged higgsino masses almost up to the c.m. energy of the collider. If the doubly charged higgsino is too heavy to be produced in experiments, it can possibly be discovered due to its contribution to the angular distribution of the selectron pair production.

The present experimental results give upper bounds for the masses of the new particles predicted by the left-right symmetric model. They do not, however, exclude the possibility considered in this work that some indications of the left-right symmetry would be discovered in the NLC. On the other hand, if such evidences were not found, it would not mean that the left-right symmetric model is excluded. Actually, according to some SO(10) grand unified scenarios, the right-handed symmetry breaking scale is $10^{10-12}$ GeV [38], much above the scale achievable in future accelerators.
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