Solving system of integro-differential equations using discrete Adomian decomposition method

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**ABSTRACT**

In this paper, we propose a new numerical method for solving system of integro-differential equations featuring Volterra and Fredholm integrals. The proposed method depends on the successful application of the Discrete Adomian Decomposition Method (DADM) to solve highly complicated functional equations coupled with some numerical integration schemes of Trapezoidal and Simpson rules. The scheme is further simulated with the aid of symbolic computation and demonstrated on some test problems. We then carried out error analysis in comparison with some existing methods revealed that the present method is superior due to the high level of accuracy and less computational steps.

1. Introduction

Integro-differential equations arise in many areas of mathematical physics and engineering applications. Since analytical solutions of these equations are difficult to obtain, much attention has been invested in the search for effective methods for obtaining approximate or numerical solutions of both linear and nonlinear integro-differential equations, respectively. Further, the integro-differential equations featuring nonlinear terms that are more practical in reality are still difficult to solve numerically or approximately. Therefore, several numerical methods were used for the solutions of these types of equations, such as the Galerkin method \([1]\), Runge–Kutta method \([2]\), Chebyshev collocation method \([3]\), Taylor collection method \([4]\), rationalized Haar functions method \([5]\), Galerkin methods with hybrid functions \([6]\), Adomian Decomposition Method (ADM) \([7–14]\), Laplace Adomian decomposition method \([15]\) and modified homotopy perturbation method \([16]\) among others.

However, in this paper, an efficient numerical method to treat the system of nonlinear integro-differential equations (NSIDE) will be devised. The method depends on the successful application of the Discrete Adomian Decomposition Method (DADM) \([17]\) coupled with some numerical integration schemes alongside some quadrature rules used to approximate definite integrals that cannot be computed analytically; see \([18–29]\) for some related methods for both the fractional and differential equations types. Also, as an application of the proposed method, it will be applied to systems of nonlinear Volterra and Fredholm integro-differential equations to demonstrate the efficiency of the method together with some comparison illustrations.

2. ADM for system of nonlinear integro-differential equations

We consider the system of integro-differential equation of the form

\[
\begin{align*}
\begin{cases}
\begin{align*}
u''(x) &= f_1(x) + \int_a^b (k_1(x,t)N_1(u(t)))dt, \\
u''(x) &= f_2(x) + \int_a^b (k_2(x,t)N_2(u(t)))dt,
\end{align*}
\end{cases}
\end{align*}
\]

with initial conditions:

\[
\begin{align*}
u(0) &= \alpha_1, \quad \nu'(0) = \beta_1, \\
\nu(0) &= \alpha_2, \quad \nu'(0) = \beta_2.
\end{align*}
\]

Where \( u''(x), \nu''(x) \) are in Equation (1) the second derivatives of the unknown functions \( u(x) \) which will be determined; \( k_1(x,t), k_1'(x,t), k_2(x,t) \) and \( k_2'(x,t) \) are the kernels of the integro-differential equations; \( f_1(x), f_2(x) \) are analytic functions; \( N_1(u(t)), N_2(u(t)) \) and \( N_1'(u(t)) \) and \( N_2'(u(t)) \) are nonlinear functions of \( u \) and \( \nu \), respectively.

Now, let \( L = d^2/dx^2 \), so \( L^{-1}(.) = \int_0^x \int_0^x f(\cdot) \) dxdx; then applying \( L^{-1} \) to both sides of (1), and using initial conditions, we obtain
\[
\begin{align*}
    u(x) &= \alpha_1 + \beta_1 x + L^{-1} f_1(x) + L^{-1} \int_a^b (k_1(x,t)N_1(u(t)))
    + k_1(x,t)N_1'(v(t)) dt,
    \\
v(x) &= \alpha_2 + \beta_2 x + L^{-1} f_2(x) + L^{-1} \int_a^b (k_2(x,t)N_2(u(t)))
    + k_2(x,t)N_2'(v(t)) dt.
\end{align*}
\]

(3)

To use the ADM, let

\[
    u(x) = \sum_{n=0}^{\infty} u_n(x), \quad v(x) = \sum_{n=0}^{\infty} v_n(x),
\]

and the nonlinear functions \(N_1(u(t))\) and \(N_2(v(t))\) by the infinite series of polynomials

\[
    N_1(u(t)) = \sum_{n=0}^{\infty} A_n(u(t)), \quad N_2(v(t)) = \sum_{n=0}^{\infty} B_n(v(t)),
\]

(5)

where \(A_n, B_n\) are the so-called Adomian polynomials that can be constructed for all forms of nonlinearity according to specific algorithms set by Adomian [21–24] given by

\[
    \frac{1}{n!} \frac{d^n}{dt^n} \left( \sum_{\lambda=0}^{\infty} \lambda^i u(x(t)) \right)_{\lambda=0}, \quad n = 0, 1, 2, \ldots
\]

(6)

Substituting (4)–(6) in (3) we get the recursive relation

\[
    \begin{align*}
    u_0 &= \alpha_1 + \beta_1 x + L^{-1} f_1(x), \\
v_0 &= \alpha_2 + \beta_2 x + L^{-1} f_2(x), \\
    u_{k+1} &= L^{-1} \int_a^b (k_1(x,t)A_k(t) + k_1(x,t)A_k'(v(t))) dt, \quad k \geq 0, \\
v_{k+1} &= L^{-1} \int_a^b (k_2(x,t)B_k(t) + k_2(x,t)B_k'(v(t))) dt, \quad k \geq 0.
\end{align*}
\]

(7)

Applying formula (8) on Equation (7) to obtain

\[
    \begin{align*}
    u_0 &= \alpha_1 + \beta_1 x + L^{-1} f_1(x), \\
v_0 &= \alpha_2 + \beta_2 x + L^{-1} f_2(x), \\
    u_{k+1} &= L^{-1} \left( \sum_{i=0}^{n} w_n,k(x,t_{n+i}) \cdot (A_k(t_{n+i}) + A_k'(t_{n+i})) \right), \quad k \geq 0, \\
v_{k+1} &= L^{-1} \left( \sum_{i=0}^{n} w_n,k(x,t_{n+i}) \cdot (B_k(t_{n+i}) + B_k'(t_{n+i})) \right), \quad k \geq 0.
\end{align*}
\]

(9)

Thus, the approximate solution of the equations using DADM can be obtained by summing the approximate values of the component \(u_k(x), v_k(x), k \geq 0\) given in Equation (9) at nodes \(x_{i}, i = 0, 1, 2, \ldots, n\) which are the same points of quadrature rule. The solutions \(u(x_i), v(x_i), k \geq 0\) at these nodes using DADM of Equation (9) can be written as

\[
    u(x_i) = \sum_{k=0}^{\infty} u_k(x_i), \quad v(x_i) = \sum_{k=0}^{\infty} v_k(x_i).
\]

(10)

In practice, all the terms of the series in (10) cannot be determined, so the solution will be approximated by the series:

\[
    \psi_1(x_i) = \sum_{k=0}^{n} u_k(x_i), \quad \psi_2(x_i) = \sum_{k=0}^{n} v_k(x_i).
\]

(11)

4. Computational results and analysis

Example 1

Consider the system of nonlinear Volterra integro-differential equation

\[
    \begin{align*}
    u''(x) &= 1 - \frac{1}{2} x^3 - \frac{1}{2} v^2(x) + \int_{0}^{x} (u^2(t) + v^2(t)) dt, \\
u(0) &= 1, \quad u'(0) = 2, \\
v''(x) &= -1 + x^2 - xu(x) + \frac{1}{2} x \int_{0}^{x} (u^2(t) - v^2(t)) dt, \\
v(0) &= -1, \quad v'(0) = 0.
\end{align*}
\]

(12)

with the exact solution

\[
(u(x), v(x)) = (x + e^x, x - e^x).
\]

In order to use the quadrature rule for Equation (12), let \(t = x_i\), so we get

\[
    \begin{align*}
    u''(x) &= 1 - \frac{1}{2} x^3 - \frac{1}{2} v^2(x) + \frac{1}{2} x \int_{0}^{x} (u^2(x_i) + v^2(x_i)) dt, \\
u(0) &= 1, \quad u'(0) = 2, \\
v''(x) &= -1 + x^2 - xu(x) + \frac{1}{2} x \int_{0}^{x} (u^2(x_i) - v^2(x_i)) dt, \\
v(0) &= -1, \quad v'(0) = 0.
\end{align*}
\]
Applying \( L^{-1}(.) = \int \frac{f(.)}{x} \, dx \) to both sides of the above equation, we get
\[
\begin{align*}
  u(x) &= 2x + 1 + \frac{1}{2}x^2 - \frac{1}{2}L^{-1}v^2(x) \\
  v(x) &= -1 - \frac{1}{2}x^2 + \frac{1}{2}x^4 - L^{-1}xu(x) \\
  &+ \frac{1}{4}L^{-1}x \int (u^2(x,v) - v^2(x,v)) \, dt.
\end{align*}
\] (13)

Thus, to evaluate the above system of equations in (13), we go by the following numerical integration schemes:

**i) Trapezoidal Rule**

We divide the interval \((0, 1)\) into subinterval of equal lengths \(h = 0.2, n = 5\) and denote \(x_i = a + ih, 0 \leq i \leq 5\).

The recursive relation is therefore expressed as
\[
\begin{align*}
  u_0 &= 2x + 1 + \frac{1}{2}x^2 - \frac{1}{2}L^{-1}v^2(x) \\
  v_0 &= -1 - \frac{1}{2}x^2 + \frac{1}{2}x^4 - \int (u^2(x,v) - v^2(x,v)) \, dt,
\end{align*}
\]

\[
\begin{align*}
  u_{k+1} &= -\frac{1}{4}L^{-1}A_k(x) + \frac{1}{4}L^{-1}(x(B_k(t_0) + C_k(t_0))) \\
  &+ \sum_{j=1}^{4} x(B_k(t_j) + C_k(t_j)) + B(A_k(t_5)) \\
  &+ C_k(t_5), k \geq 0, \\
  v_{k+1} &= -\frac{1}{4}L^{-1}xu_k(x) + \frac{1}{4}L^{-1}(x(B_k(t_0) - C_k(t_0))) \\
  &+ \sum_{j=1}^{4} x(B_k(t_j) - C_k(t_j)) + B(A_k(t_5)) \\
  &- C_k(t_5), k \geq 0,
\end{align*}
\]

or
\[
\begin{align*}
  u_1 &= 0.16666667x^3 + 0.04166667x^4 + \ldots, \\
  v_1 &= -\frac{1}{2}x^2 - 0.125x^4 + \ldots, \\
  u_2 &= 0.025x^5 - 0.01377777x^5 + \ldots, \\
  v_2 &= -0.00555556x^6 - 0.00035730x^7 + \ldots, \\
  \vdots
\end{align*}
\]

The series solution is then obtained by summing the above iterations,
\[
\begin{align*}
  u(x) &= u_0 + u_1 + u_2 + \ldots, \\
  v(x) &= v_0 + v_1 + v_2 + \ldots.
\end{align*}
\] (14)

Table 1 and Figure 1 compare the exact solution \(u(x)\) and its approximate solution using DADM based on Trapezoidal rule. Only five components were used from \(x = 0\) to \(x = 1\) at an interval of 0.2 and the respective absolute errors are presented.

Here, the results produced by our method with only few components (\(m = 5\)) are in a very good agreement with the exact solution results. Figure 1 clearly shows that all the values of DADM overlapped the values of the exact solutions which give our method an edge over other reported methods of solving system of integro-differential equations.

Moreover, Table 2 and Figure 2 compare the exact solution \(v(x)\) and its approximate solution using DADM based on Trapezoidal rule. Only five components were
used from \( x = 0 \) to \( x = 1 \) at an interval of 0.2 and the respective absolute errors are presented.

In a similar manner, it can be deduced from Table 2 and Figure 2 that the results produced by our method with only a few components (\( m = 5 \)) are in a very good agreement with the exact solution results. In fact, Figure 2 clearly shows that all the values of DADM overlapped the values of the exact solutions which give our method an edge over other reported methods of solving system of integro-differential equations.

**(ii) Simpson’s Method**

We divide the interval \((0, 1)\) into subinterval of equal lengths \( h = \frac{0.5}{n} \), \( n = 5 \) and denote \( x_i = a + ih, 0 \leq i \leq 20 \), the recursive relation is given by

\[
\begin{align*}
    u_0 &= 2x + 1 + \frac{1}{2}x^2 - \frac{1}{60}x^5, \\
    v_0 &= -1 - \frac{1}{2}x^2 + \frac{1}{12}x^4,
\end{align*}
\]

\[
\begin{align*}
    u_{k+1} &= -\frac{1}{2}L^{-1}A_k(x) + \frac{0.5}{6}L^{-1}(x(B_k(t_0) + C_k(t_0))) \\
            &+ 4\sum_{i=1}^{10} x_i(B_k(t_{2i-1}) + C_k(t_{2i-1})) + 2\sum_{i=1}^{9} x_i(B_k(t_{2i})) \\
            &+ C_k(t_0) + x(B_k(t_{20}) + C_k(t_{20})), k \geq 0, \\
    v_{k+1} &= -L^{-1}u_k(x) + \frac{0.5}{12}L^{-1}(x(B_k(t_0) - C_k(t_0))) \\
            &+ 4\sum_{i=1}^{10} x_i(B_k(t_{2i-1}) - C_k(t_{2i-1})) + 2\sum_{i=1}^{9} x_i(B_k(t_{2i})) \\
            &- C_k(t_20) + x(B_k(t_{20}) - C_k(t_{20})), k \geq 0,
\end{align*}
\]

or

\[
\begin{align*}
    u_1 &= 0.16666667x^3 + 0.04166667x^4 + \ldots, \\
    u_2 &= -\frac{1}{6}x^2 - 0.125x^4 + \ldots, \\
    u_3 &= -0.02555555x^6 - 0.00039682x^7 + \ldots, \\
    &\vdots
\end{align*}
\]

The series solution is then obtained by summing the above iterations as in Equation (14). Table 3 and Figure 3 present the comparison between the exact solution \( u(x) \) and its approximate solution using DADM based on Simpson’s rule. Only five components were used from \( x = 0 \) to \( x = 0.5 \) at an interval of 0.1 and the respective absolute errors are presented.

Here, the results produced by our method with only a few components (\( m = 5 \)) are in a very good agreement with the exact solution results. Besides, Figure 3 clearly shows that all the values of DADM overlapped the values of the exact solutions which give our method an edge over other reported methods of solving system of integro-differential equations.

Moreover, Table 4 and Figure 4 compare the exact solution \( v(x) \) and its approximate solution using DADM based on Simpson’s rule. Only five components were used from \( x = 0 \) to \( x = 0.5 \) at an interval of 0.1 and the respective absolute errors are as well presented.

Similarly, it can be deduced from Table 4 and Figure 4 that the results produced by our method with only few components (\( m = 5 \)) are in a very good agreement with the exact solution results. In fact, Figure 4 clearly shows that all the values of DADM overlapped the values of

**Table 3.** The comparison between the exact solution \( u(x) \) and the approximate solution using DADM based on the Simpson’s rule.

| x    | Exact     | DADM       | Absolute Error |
|------|-----------|------------|----------------|
| 0    | 1.00000000 | 1.00000000 |               |
| 0.10 | 1.20517092 | 1.20517092 | 1.42545776e-15|
| 0.20 | 1.42140276 | 1.42140276 | 1.86990969e-13|
| 0.30 | 1.64985881 | 1.64985881 | 2.00290725e-12|
| 0.40 | 1.89182470 | 1.89182470 | 3.2300992e-11  |
| 0.50 | 2.14872127 | 2.14872127 | 9.09490688e-10 |

**Figure 3.** Curves of the exact solution \( u(x) \) and the approximate solution using DADM based on the Simpson’s rule.

**Figure 4.** Curves of the exact solution \( v(x) \) and the approximate solution using DADM based on the Simpson’s rule.
the exact solutions which give our method an edge over other reported methods of solving system of integro-differential equations.

**Example 2**

Consider the system of nonlinear Fredholm integro-differential equation

\[
\begin{aligned}
\begin{cases}
\frac{d}{dx}u(x) &= 2 + \frac{12}{3}x - \int_0^x (u^2 + v^2)\, dx, \\
u(0) &= 1, \quad u'(0) = 0,
\end{cases}
\end{aligned}
\]

\[
\begin{aligned}
\begin{cases}
\frac{d}{dx}v(x) &= -2 + \frac{4}{3}x - \int_0^x (u^2 - v^2)\, dx, \\
u(0) &= 1, \quad u'(0) = 0,
\end{cases}
\end{aligned}
\]

with the exact solution

\[ (u(x), v(x)) = (1 + x^2, 1 - x^2) \]

Applying \( L^{-1}(.) = \int_0^x (. )\, dx \) to both sides of Equation (15), we get,

\[
\begin{aligned}
\begin{cases}
u(x) &= 1 + x^2 + \frac{12}{30}x^3 - \int_0^x (u^2 + v^2)\, dx, \\
v(x) &= 1 - x^2 + \frac{4}{18}x^3 - \int_0^x (u^2 - v^2)\, dx.
\end{cases}
\end{aligned}
\]

**(i) Trapezoidal rule**

We divide the interval \((0,1)\) into subinterval of equal lengths \(h = 0.2, n = 5\) and denote \(x_i = a + ih, 0 \leq i \leq 5\), the recursive relation is given by

\[
\begin{aligned}
\begin{cases}
u_0 &= 1 + x^2 + \frac{12}{30}x^3, \\
v_0 &= 1 - x^2 + \frac{4}{18}x^3.
\end{cases}
\end{aligned}
\]

\[
\begin{aligned}
\begin{cases}
u_{k+1} &= -0.1L^{-1}(x(A_k(t_0) + B_k(t_0))) + \frac{4}{4} \sum_{i=1}^4 x(A_k(t_i)) \\
+ B_k(t_i)) + x(A_k(t_0) + B_k(t_0)), \\
v_{k+1} &= -0.1L^{-1}(x(A_k(t_0) - B_k(t_0))) + \frac{4}{4} \sum_{i=1}^4 x(A_k(t_i)) \\
- B_k(t_i)) + x(A_k(t_0) - B_k(t_0)),
\end{cases}
\end{aligned}
\]

or

\[
\begin{aligned}
\begin{cases}
u_1 &= -0.47488288x^3, \\
v_1 &= -0.28306689x^3, \\
u_2 &= 0.99110678x^3, \\
v_2 &= 0.0979518x^3, \\
\vdots 
\end{cases}
\end{aligned}
\]

The series solution is obtained by summation as in Equation (14). We present the comparison of the exact solution \(u(x)\) and its approximate solution using DADM based on Trapezoidal rule in Table 5 and Figure 5. Only five components were used from \(x = 0\) to \(x = 1\) at an interval of 0.2 and the respective absolute errors are presented.

**Table 5.** The comparison between the exact solution \(u(x)\) and the approximate solution using DADM based on the Trapezoidal rule.

| x   | Exact       | DADM       | Absolute Error |
|-----|-------------|------------|----------------|
| 0   | 1.000000000| 1.00000000| 0              |
| 0.20| 1.040000000| 1.03996370| 3.62969667e-05|
| 0.40| 1.160000000| 1.15970962| 2.93075374e-04|
| 0.60| 1.360000000| 1.35901998| 9.80018101e-04|
| 0.80| 1.640000000| 1.63767699| 2.32300587e-03|

**Figure 5.** Curves of the exact solution \(u(x)\) and the approximate solution using DADM based on the Trapezoidal rule.

**Table 6.** The comparison between the exact solution \(v(x)\) and the approximate solution using DADM based on the Trapezoidal rule.

| x   | Exact       | DADM       | Absolute Error |
|-----|-------------|------------|----------------|
| 0   | 1.000000000| 1.00000000| 0              |
| 0.20| 0.960000000| 0.95996436| 3.56392238e-05|
| 0.40| 0.840000000| 0.83971489| 2.85113790e-04|
| 0.60| 0.640000000| 0.63963774| 9.62259042e-04|
| 0.80| 0.360000000| 0.35771909| 2.2891032e-03  |

Here, the results produced by our method with only few components \((m = 5)\) are in a very good agreement with the exact solution results. Figure 5 clearly shows that all the values of DADM overlapped the values of the exact solutions which give our method an edge over other reported methods of solving system of integro-differential equations.

Moreover, Table 6 and Figure 6 compare the exact solution \(v(x)\) and its approximate solution using DADM based on Trapezoidal rule. Only five components were used from \(x = 0\) to \(x = 1\) at an interval of 0.2 and the respective absolute errors are presented.

In similar manner, it can be deduced from Table 6 and Figure 6 that the results produced by our method with only a few components \((m = 5)\) are in a very good agreement with the exact solution results. In fact, Figure 6 clearly shows that all the values of DADM overlapped the values of the exact solutions which give our method an edge over other reported methods of solving system of integro-differential equations.
(ii) Simpson’s Method

Again, we divide the interval (0, 1) into subinterval of equal lengths $h = .05$, $n = 5$ and denote $x_i = a + ih$, $0 \leq i \leq 20$, the recursive relation is expressed by

$$\begin{align*}
\frac{u_{k+1}}{v_{k+1}} &= -\frac{0.05}{2}L^{-1}(x(A_k(t_0) + B_k(t_0)) + 4 \sum_{i=1}^{10} x_i(A_k(t_{2i-1})) \\
&+ B_k(t_{2i-1})) + 2 \sum_{i=1}^{10} x_i(A_k(t_{2i}) + B_k(t_{2i})) \\
&+ x(A_k(t_{20}) + B_k(t_{20})), k \geq 0,
\end{align*}$$

or

$$\begin{align*}
\frac{u_{k+1}}{v_{k+1}} &= -\frac{0.05}{2}L^{-1}(x(A_k(t_0) - B_k(t_0)) + 4 \sum_{i=1}^{10} x_i(A_k(t_{2i-1})) \\
&- B_k(t_{2i-1})) + 2 \sum_{i=1}^{10} x_i(A_k(t_{2i}) - B_k(t_{2i})) \\
&+ x(A_k(t_{20}) - B_k(t_{20})), k \geq 0,
\end{align*}$$

The series solution then follows from Equation (14). We give the comparison of the exact solution $u(x)$ and its approximate solution using DADM based on Simpson’s rule in Table 7 and Figure 7. Only five components were used from $x = 0$ to $x = 0.5$ at an interval of 0.1 and the respective absolute errors are presented.

Here, the results produced by our method with only few components ($m = 5$) are in a very good agreement with the exact solution results. Figure 7 clearly shows that all the values of DADM overlapped the values of the exact solutions which give our method an edge over other reported methods of solving system of integro-differential equations.

Moreover, Table 8 and Figure 8 compare the exact solution $v(x)$ and its approximate solution using DADM based on Simpson’s rule. Only five components were

![Figure 6. Curves of the exact solution $v(x)$ and the approximate solution using DADM based on the Trapezoidal rule.](image)

![Figure 7. Curves of the exact solution $u(x)$ and the approximate solution using DADM based on the Simpson’s rule.](image)

![Figure 8. Curves of the exact solution $v(x)$ and the approximate solution using DADM based on the Simpson’s rule.](image)

| Table 7. The comparison between the exact solution $u(x)$ and the approximate solution using DADM based on the Simpson’s rule. |
|---|---|---|
| $x$ | Exact | DADM | Absolute Error |
| 0 | 1.00000000 | 1.00000000 | 0 |
| 0.1 | 1.00000000 | 1.00000000 | 0 |
| 0.2 | 1.00000000 | 1.009999945 | 5.47032873e−07 |
| 0.3 | 1.00000000 | 1.03999562 | 4.37626299e−06 |
| 0.4 | 1.00000000 | 1.08998523 | 1.4769876e−05 |
| 0.5 | 1.00000000 | 1.24993162 | 6.83791091e−05 |

| Table 8. The comparison between the exact solution $v(x)$ and the approximate solution using DADM based on the Simpson’s rule. |
|---|---|---|
| $x$ | Exact | DADM | Absolute Error |
| 0 | 1.00000000 | 1.00000000 | 0 |
| 0.1 | 0.99000000 | 0.98999967 | 3.28891242e−07 |
| 0.2 | 0.96000000 | 0.95999737 | 2.63112993e−06 |
| 0.3 | 0.91000000 | 0.90999112 | 8.88006352e−06 |
| 0.4 | 0.84000000 | 0.83997895 | 2.10403959e−05 |
| 0.5 | 0.75000000 | 0.74995889 | 4.11114052e−05 |

![Table 7. The comparison between the exact solution $u(x)$ and the approximate solution using DADM based on the Simpson’s rule.](image)
used from $x = 0$ to $x = 0.5$ at an interval of 0.1 and the respective absolute errors are presented.

In a similar manner, it can be deduced from Table 8 and Figure 8 that the results produced by our method with only a few components ($m = 5$) are in a very good agreement with the exact solution results. In fact, Figure 8 clearly shows that all the values of DADM overlapped the values of the exact solutions which give our method an edge over other reported methods of solving system of integro-differential equations.

5. Conclusion

In conclusion, a numerical method for solving system of integro-differential equations is proposed. The method is based on the Discrete Adomian Decomposition Method (DADM) coupled to some numerical integration schemes. The method was further applied to solve the system of nonlinear Volterra and Fredholm integro-differential equations. The proposed method is easy to apply apart being superior to some existing methods in reaching out to approximate solutions. Further, the method is capable of reducing the huge computational steps compared to some classical methods thereby obtaining better accuracy with small number of iterations. Thus, the method can be applied to many integro-differential equations arising in real-life applications.

Disclosure statement

No potential conflict of interest was reported by the authors.

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