A three-wave mixing kinetic inductance traveling-wave amplifier with near-quantum-limited noise performance

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We present a theoretical model and experimental characterization of a microwave kinetic inductance traveling-wave amplifier (KIT), whose noise performance, measured by a shot noise thermometer, approaches the quantum limit. Biased with a dc current, the KIT operates in a three-wave mixing fashion, thereby reducing by several orders of magnitude the power of the microwave pump tone compared to conventional four-wave mixing KIT devices. It is built in an artificial transmission line intrinsically matched to 50 Ω, whose dispersion allows for a controlled amplification bandwidth. We experimentally measure 17.6±1.4 dB of gain across a 2 GHz bandwidth, with an input 1 dB compression power of -63 dBm within that bandwidth, in qualitative agreement with theory. Using the KIT as the first amplifier in an amplification chain, we measure a system-added noise of 0.61±0.08 K between 3.5 and 5.5 GHz, about one eighth the noise obtained when using only a representative classical amplifier. The KIT contribution to this added noise is estimated to be 0.2±0.1 K, consistent with the quantum limit on amplifier added noise. This device is therefore suitable to read large arrays of microwave kinetic inductance detectors or thousands of superconducting qubits.

I. INTRODUCTION

Can a microwave amplifier be quantum limited, while at the same time have enough gain, power handling, and bandwidth to read thousands of frequency-multiplexed superconducting resonators, like those in qubit systems or microwave kinetic inductance detectors (MKIDs)? When designed with resonant structures, Josephson-based parametric amplifiers have demonstrated high gain and quantum limited performances [1–7]. However, despite efforts to increase the bandwidth up to a few hundred megahertz via impedance engineering [8, 9], or to increase the power handling up to a few hundred femtowatts via Kerr engineering [10–12], they still cannot read more than a handful of resonators simultaneously. When designed with nonresonant structures, i.e. transmission lines, Josephson traveling-wave parametric amplifiers (JTWPA) have high gain over gigahertz bandwidth [13–15], but disappointingly exhibit similar power handling capabilities as their resonant counterparts. Recent studies [16–18] suggest that a three-wave mixing (3WM) JTWPA with a finely controlled and canceled Kerr nonlinearity should increase tenfold the device’s power handling. Compelling experiments have yet to prove the feasibility of this approach, for which the JTWPA’s design and fabrication increase in complexity. We propose to tackle this question from another angle: starting with the intrinsic broadband and high power handling capabilities of a kinetic inductance traveling-wave amplifier (KIT) [19], we build a near-quantum-limited amplifier.

The current limitations on microwave amplification affect many scientific endeavors. Although proof-of-principle “quantum supremacy” was demonstrated by a quantum computer containing a few tens of qubits [20], this number has to scale by at least an order of magnitude to run powerful quantum algorithms [21, 22]. In the hunt for exoplanets, cameras with tens of thousands of MKID pixels are being built [23], and proposals to search for very light warm dark matter also necessitate the use of a great number of MKID pixels [24, 25]. All these applications are either already limited by the amplifier noise, or would greatly benefit from the advent of wideband, high gain, high power handling, quantum limited amplifiers.

The KIT we present in this article is a step toward a practical, quantum-limited amplifier, whose bandwidth and power handling are compatible with high channel-count applications. Operating in a 3WM fashion, and fabricated out of a single layer of Nb-Ti-N, it consists of a weakly dispersive artificial transmission line [26, 27], for which we control the phase matched bandwidth with dispersion engineering. This limits spurious parametric conversion processes that otherwise degrade the power handling and noise performance. We measure an average gain of 17.6 dB over a 2 GHz bandwidth, and a typical 1 dB input compression power of -63 dBm in that bandwidth. Using a shot-noise thermometer (SNT) [28, 29] we measure the added noise of a readout chain with the KIT as first amplifier. We demonstrate a system-added noise temperature of 0.61 ± 0.08 K between 3.5 and 5.5 GHz, where 0.2 ± 0.1 K is estimated to originate from the KIT itself, suggesting quantum-limited operation at the chip level. It is the first time that the broadband noise properties of a KIT are fully characterized.

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II. THEORY AND DESIGN

KITs exploit the nonlinear kinetic inductance of a superconducting line to generate parametric interaction between pump, signal, and idler photons. In 3WM, a single pump photon converts into signal and idler photons, whereas four-wave mixing (4WM) converts two pump photons in this fashion. Operating a KIT with 3WM offers two key advantages over 4WM. First, as the pump frequency is far detuned from the amplification band, it is easily filtered, which is often necessary to avoid saturating the following amplifier. Second, it reduces the rf pump power, because energy is extracted from dc power to convert pump photons, which avoids undesirable heating effects from the strong rf pump, including those happening in the packaging. More precisely, when biased with a dc current $I_d$, the KIT inductance per unit length $L$ is [30]:

$$L = L_d (1 + \epsilon I + \xi I^2),$$

(1)

where $I$ is the rf current, $L_d$ is the KIT inductance under dc bias, at zero rf current, $\epsilon = 2L_d/(I_s^2 + I_i^2)$ and $\xi = 1/(I_s^2 + I_i^2)$. The current $I$ sets the scale of the nonlinearity; it is typically $\sim 10^5$ higher than that of Josephson devices, thereby conferring KITs $\sim 10^4 - 10^6$ higher power handling capabilities than their Josephson equivalents. The term $\epsilon I$ permits 3WM, while $\xi I^2$ permits 4WM.

Solving the coupled mode equations (CME) for a pump at frequency $\omega_p$, signal at $\omega_s$, and idler at $\omega_i$, such that $\omega_p = \omega_s + \omega_i$, yields the 3WM phase matching condition for exponential gain:

$$\Delta_k = -\frac{\xi I_{p0}^2}{8} (k_p - 2k_s - 2k_i),$$

(2)

see appendix A1. Here, $\Delta_k = k_p - k_s - k_i$ with $k_j$, $j \in \{p, s, i\}$ the pump, signal and idler wavenumbers, and $I_{p0}$ is the rf pump amplitude at the KIT’s input. In a non-dispersive transmission line $\Delta_k = 0$, and thus equation 2 can naturally be fulfilled over a very wide frequency range in KITs, where $I_{p0} \ll I_\circ$. Although desirable within the amplification bandwidth, it is undesirable out of that bandwidth, where multiple parametric conversion processes take place [30, 31]. They deplete the pump, thereby degrading the amplifier’s power handling, and they induce multiple conversion mechanisms at each frequency, thereby increasing the amplifier’s added noise.

While in conventional traveling-wave amplifiers dispersion engineering prevents only pump harmonic generation, we suppress all unwanted parametric conversion processes by designing our KIT as a weakly dispersive artificial transmission line. Originally developed to have the KIT matched to 50 Ω [26], this line consists of a series of coplanar waveguide (CPW) sections, or cells, each with inductance $L_d$, flanked by two interdigitated capacitor (IDC) fingers that form the capacitance to ground $C$, such that $Z = \sqrt{L_d/C} = 50$ Ω, see Fig. 1a. Each IDC finger then constitutes a low-Q quarter-wave resonator, with capacitance $C_f = C/2$ and inductance $L_f$ (Fig. 1b), set by the finger’s length. In practice, we choose $\omega_f = 1/\sqrt{L_f C_f} = 2\pi \times 36$ GHz, and $Q = 1/Z \sqrt{L_f C_f} = 3.3$, to generate a slight dispersion at low frequencies, where the pump, signal and idler modes lie. The dashed line in Fig. 2a shows the dispersive part of the wavenumber $k_t = k - k_0$ with $k_0 = \omega \sqrt{L_d C}$, as a function of frequency. It is calculated by cascading the ABCD matrices of the KIT cells, see appendix B. As it slightly deviates from zero, no triplet $\{k_s, k_i, k_p\}$ can satisfy Eq. 2.

To retrieve phase-matching over a desired bandwidth, we engineer another dispersion feature by periodically increasing the line impedance. It creates a resonance in the line’s phase response (and a stopband in the line’s amplitude response), at a frequency $\omega_l$ controlled by the loading periodicity [26, 32]. Figure 2a shows $k^*$ in a line periodically loaded at 80 Ω, with $\omega_l = 2\pi \times 8.5$ GHz. Because the nonlinear wavenumber close to resonance sharply varies, there exists values of $k^*$ (above $\omega_l$) for which we can find triplets $\{k_s, k_i, k_p\}$ that satisfy Eq. 2 (examples of their nonlinear parts are shown in colored dots). A slight variation of the pump frequency $\omega_p$ significantly affects which pair of signal and idler frequencies is phase matched.

At these matched frequencies, the 3WM gain grows exponentially with $k_p$ (in radian per cell), with the KIT length, and with the relative inductance modulation amplitude $\delta L$, generated by the rf current and scaling with $I_d$. More precisely, the phase matched, small signal power gain can be written as:

$$G_s \propto \cosh^2 \left( \frac{1}{8} \delta L k_p N_c \right),$$

(3)

where $N_c$ is the total number of cells, see appendix A2. Typically, when operating our KIT, $\delta L \sim 7 \times 10^{-3}$; thus,
with $L_d \sim 50 \, \text{pH/cell}$ (see Sec. III), we need $N_c > 5 \times 10^4$ to get $G_s > 15 \, \text{dB}$ at $\omega_p \sim 2\pi \times 9 \, \text{GHz}$.

Since maximum gain is achieved with phase-matching, $\omega_p$ influences the gain profile. To calculate this profile, we insert the dispersion relation $k(\omega)$ into the CME, and solve them numerically, see appendix B. Figure 2b shows signal power gain profiles, calculated for the pump frequencies represented in Fig. 2a. As expected, the gain is maximal at the signal and idler frequencies for which the triplets $\{k_s, k_i, k_p\}$ satisfy Eq. 2. When these frequencies are far apart, there is a region in between, where phase matching is not sufficient, and the gain drops. By reducing $\omega_p$, we can lower the distance between phase-matched signal and idler frequencies, and therefore obtain a wideband, flat gain region. Further reducing $\omega_p$, we reach the value where phase-matched signal and idler frequencies are equal, beyond what phase matching is nowhere fulfilled anymore, and the gain drops. Fundamentally, the wideband nature of the gain depends on the convexity of the dispersion relation, and therefore on the fingers' length and capacitance to ground. As $\omega_f$ or Q increase, $k^*$ is less convex, and thus closer to a broadband, phase-matched situation, but at the cost of allowing extra parametric processes to arise.

### III. EXPERIMENTAL REALIZATION

Because the kinetic inductance nonlinearity is weak, in practice a KIT comprises a transmission line tens of centimeters long. To maximize this nonlinearity, and to minimize its length, the line as well as IDC fingers are made 1 $\mu$m wide, and each unit cell is 5 $\mu$m long, see Fig. 3a and b. Fabricated of a 20 nm thick Nb-Ti-N layer via optical lithography, it yields $I_s \sim 7 \, \text{mA}$, and a sheet inductance $L_K \sim 10 \, \text{pH/square}$. Thus, $L_d \sim 50 \, \text{pH}$, and in order to retrieve $Z = 50 \, \Omega$, each finger is made 102 $\mu$m long. The loading (Fig. 3a) consists of cells with shorter fingers (33.5 $\mu$m, $Z = 80 \, \Omega$), arranged periodically to generate a resonance at 8.5 GHz, thereby positioning the pump frequency in a way compatible with our filtering capabilities.

We lay out the line in a spiral topology, on a $2 \times 2$ cm chip (Fig. 3c), which contains $N_c = 6.6 \times 10^4$ cells, equivalent to 33 cm. To avoid spurious chip and microstrip modes, electrical grounding inside the chip is ensured with spring-loaded pogo pins, descending from the top copper lid of the packaging, and contacting the chip between the line traces, see appendix D. They also improve the chip-packaging thermal link, which otherwise mostly relies on wire-bonds.

![Figure 2. Influence of the phase matching on the gain profile. (a) The nonlinear wavenumber $k^*$ is calculated as a function of frequency for the transmission line represented in Fig. 1 (dashed), and for a similar line, periodically loaded at 80 $\Omega$ (black). The nonlinear part of four triplets $\{k_s, k_i, k_p\}$ that satisfy the phase matching condition (Eq. 2) are indicated with colored dots: in purple $\omega_s - \omega_i = 4 \, \text{GHz}$, green $\omega_s - \omega_i = 3 \, \text{GHz}$, red $\omega_i - \omega_s = 2 \, \text{GHz}$, and blue $\omega_i = \omega_s$. Additionally, a gray line indicates a pump frequency for which phase matching is nowhere fulfilled. (b) Solving the CME (Eqs. A5), the signal power gain profile is calculated (Eq. A9) for the related pump frequencies (the colors match with panel a). The KIT length, is $N_c = 6.6 \times 10^4$ cells.](image)

![Figure 3. Micrographs of the KIT. (a) The transmission line (false color red) is periodically loaded with shorter IDC fingers. (b) Line and fingers are 1 $\mu$m wide, and each cell is 5 $\mu$m long. (c) The overall KIT is laid out in a spiral configuration on a $2 \times 2$ cm chip, and clamped on a copper packaging.](image)

In a first experiment, we measure the gain, bandwidth, and power handling of the KIT, when cooled to $\sim 30 \, \text{mK}$. Mounted as the sole amplifier in the chain, thereby ensuring characterization over its full bandwidth, two biastees at the KIT input and output ports combine dc and rf currents. Figure 4a shows KIT gain profiles, acquired at two different pump frequencies. The current amplitudes are $I_d = 1.5 \, \text{mA}$, and $I_{R\Phi} = 180 \, \mu\text{A}$ ($\sim 28 \, \text{dBm}$ in power, calibrated $\text{in situ}$ by comparing dc and rf nonlinear phase
shifts, see appendix A 3). For the higher pump frequency, the gain drops in the middle of the amplification bandwidth. For the lower one, the gain profile is flatter, with an average value of 17.6±1.4 dB between 3.5 and 5.5 GHz. Both profiles agree qualitatively with behaviors explained in Sec. II. There are gain ripples with 8 MHz characteristic frequency (see Fig. 4c), equivalent to 62.5 cm in wavelength (the phase velocity being \( v_p = 1/\sqrt{L_0 C} \sim 1000 \) cell per nanosecond), or about twice the KIT length. We thus attribute their presence to a mismatch in impedance between KIT and microwave feed structures before and after the KIT. This mismatch results in a pump standing wave pattern, which influences the signal amplification, depending on its frequency. Figure 4b shows the gain at 4.5 GHz (obtained at the lower pump power), as a function of \( P_t \), the input power of a probe tone. The gain compresses by 1 dB from its small signal value for \( P_{-1dB} = -63 \) dBm, about 7 dB lower than theoretical predictions, see appendix C 2. This discrepancy, suggesting substantial room for improvement, may be due to effects not included in our model, such as standing wave patterns, or defects in the line, which locally lower effects not included in our model, such as standing wave patterns, or defects in the line, which locally lower

\[
\frac{1}{G} = 2 \pi \lambda L \frac{N}{N_H} \frac{\hbar}{k_b} \frac{\omega}{\omega_k} - \frac{2 \pi}{\omega} \frac{I_p}{I_{p0}} \frac{L_0}{d} \frac{1}{\omega_k} \frac{1}{\omega_p}.
\]

At higher dc current bias (bound by the dc critical current of the transmission line, \( \sim 2.4 \) mA in our device), lower rf pump power can be used to obtain equivalent small signal gain, at the cost of a reduced 1 dB compression power. Conversely, reducing \( I_d \) and increasing \( I_{p0} \) improves power handling capabilities, but the gain is then limited by a superconductivity breaking phenomenon. We suspect the presence of weak links \[34\], and we are currently investigating the line breakdown mechanism.

IV. NOISE PERFORMANCE

The combined gain, bandwidth, and power handling performances are promising, provided that the KIT also presents a competitive noise performance. Measuring this noise is a topic of current interest \[27, 35\], and we execute the task using a self-calibrated shot-noise thermometer (SNT) \[28, 29\]. We perform a direct measurement of the output spectral density, which incorporates the true chain's microwave loss. The SNT acts as a dynamic variable noise source, allowing for a continuous sweep in input noise temperature, and our measurement scans the entire high-gain region of the KIT bandwidth.

As in any phase insensitive traveling-wave and resonant amplifier, the practical, usable bandwidth, is half of the presented amplification bandwidth. It is the bandwidth in which signals coming from microwave devices can be phase-insensitively amplified. The other half, barring the idler frequencies, contains a copy of signals in the first half. That is why the gain in Fig. 4a is nearly symmetric about the half pump frequency (\( \sim 4.5 \) GHz). The asymmetry - gain and ripples marginally bigger above the half pump frequency - originates from a frequency dependent saturation power. In fact, higher frequencies possess a higher saturation power, see appendix C 1. We see this effect here because we chose a signal power close to \( P_{-1dB} \) in order to maximize the signal-to-noise ratio in this measurement, where the KIT remains the sole amplifier.

Apart from the superconducting chip, any microwave component adds loss, which degrades the quoted noise performance. Because our device necessitates the use of two bias tees, we must include them as part of the amplifier; however, supplemental microwave components used here may otherwise change (for instance, qubit experiments usually employ isolators to prevent amplifier back action \[13, 20\]). As such, our measurement puts an upper bound on the amplifier’s added noise \( N \) (and to its noise temperature \( T_N = N\hbar\omega/k_b \)), because we measure directly only the system-added noise \( N_s \). In the future, putting the SNT onto a variable temperature stage will
allow us to calibrate out microwave loss up to the desired reference plane (first bias tee’s input), thereby directly accessing $N_o$. Figure 5 schematizes the amplification chain. Noise from the SNT is transmitted to the amplifier with efficiency $\lambda$, amplified with gain $G$, further transmitted to the high electron mobility transistor (HEMT) with efficiency $\eta$, and amplified with gain $G_H$. After further room temperature amplification, it finally reaches the spectrum analyzer (SA). Thus, the output noise (in units of photons) can be written as

$$N_o = G_c (N_i + N_s),$$

where $G_c$ is the net gain of the chain (including transmission efficiencies) from SNT to SA, $N_i$ is the SNT-generated noise, and where

$$N_s = \left(1 - \frac{\lambda}{\eta G}\right) N_f + \frac{N_H}{\eta G \lambda},$$

see appendix F1. Here, $N_f \simeq 1/2$ is the Johnson noise at 30 mK, and $N_H$ is the HEMT added noise. With sufficient gain $G$ (and sufficient transmission efficiency), the first term in the right hand side of Eq. 5 dominates over the second one, in which case the HEMT contribution to the system-added noise is minimal.

Figure 6c presents the input referred, system-added noise temperature $T_s = N_s \hbar \omega / k_b$, with $\hbar$ the reduced Plank constant and $k_b$ the Boltzmann constant, measured over the full KIT’s amplification bandwidth. In this experiment, we reduced the KIT gain to $G = 13.5 \pm 0.5$ dB between 3.5 and 5.5 GHz (Fig. 6a) in order to avoid having big gain ripples, and to ensure gain stability over the acquisition time ($\sim 12$ hrs). At each frequency, we measure $N_o$ over a 5 MHz resolution bandwidth (RBW), comparable to typical resonant amplifiers bandwidth, as a function of the SNT voltage bias (Fig. 6b), and fit to obtain $N_s$ (see appendix F2). It yields $T_s = 0.61 \pm 0.08$ K, about one eighth the system-added noise temperature obtained when the KIT is not pumped ($T_s' = 5.1 \pm 1.4$ K, see appendix G).

As anticipated, $T_s$ follows the symmetric nature of $G$ with respect to the half pump frequency $\omega_p/2 \sim 4.4$ GHz. It increases at frequencies where the gain decreases, except for a noticeable, noisier region close to $\omega_p/2$. It may come from a narrow-band 4WM conversion process occurring around $\omega_p$, creating photons which in turn, convert via 3WM.

Assuming transmissions efficiencies $\lambda = \eta = 0.6 \pm 0.1$, consistent with the value of $T_s'$, we can estimate a lower bound for $T_N$, as usually reported (see appendix F3). In fact, removing the effect of microwave loss (including that of the two bias tees), we obtain $T_N \gtrsim 0.2 \pm 0.1$ K, consistent with the quantum limit on amplifier added noise. It also shows that the HEMT contribution to the

FIG. 6. System-added noise measurement of a microwave amplification chain with a KIT as first amplifier. (a) The gain’s frequency dependence is measured with a VNA, at a 5MHz intermediate frequency (IF) bandwidth. (b) The output noise temperature $T_o$ is measured with a SA (5 MHz RBW), while varying the SNT dc voltage bias $V$. We divide $T_o$ by the chain’s gain $G_c$ to refer it to the KIT input; we also subtract the zero voltage SNT temperature $\hbar \omega/(2k_b)$ so that $T_o$ visually matches the zero bias value of $T_o$. In practice, $T_s$ is extracted from fitting the whole output noise response. Three colored curves indicate output noises at 4, 5, and 6 GHz, with fits superimposed in black lines. (c) Data from the output noise spectra are compiled to form the frequency dependent system-added noise $T_s$. Uncertainties are indicated by the gray area surrounding the black line.
system-added noise is non-negligible, because $T_s$ is significantly higher than 0.2 K. This contribution increases as the KIT gain drops, and as the HEMT noise itself increases. Thus, improvements in the gain of the amplifier will undoubtedly improve $T_s$.

There are three strategies to achieve higher gain, which directly follow from Eq. 3: higher pump power (i.e. increase $\delta t$), longer line (increase $N_c$), or higher inductance per unit cell (increase $k_p$). All face non-trivial challenges, starting with better understanding of the line breakdown mechanism [34]. If it comes from imperfections in the line (weak links), a higher resolution fabrication process, like electron beam lithography, may improve the performance of the device. Also, running the amplifier at higher gain will require better damping of the gain ripples, whose amplitude grows with gain.

V. CONCLUSION

A microwave amplifier can be near-quantum-limited over a wide frequency range, without sacrificing power handling capabilities. Engineering the phase-matched bandwidth is key, because it suppresses spurious parametric conversion processes. We demonstrate this idea on a KIT, whose combined gain, bandwidth, power handling, and noise performances are fully characterized. This KIT has the potential to initiate a qualitative shift in the way arrays of superconducting detectors, such as MKIDs, process information, moving them to a quantum-limited regime. Straightforward improvements, such as on-chip bias tees, or achieving higher gain should lower the noise further.

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Appendix A: COUPLED-MODE THEORY OF A DC-BIASED KIT

1. Coupled-mode equations

The phase matching condition for exponential gain, Eq. 2 is obtained by solving the CME while pumping in a 3WM fashion, i.e. such that $\omega_p = \omega_s + \omega_i$, and in the presence of 3WM and 4WM terms see, eq. 1. The CME relate the current amplitudes $I_j$, $j \in \{p, s, i\}$ at the frequencies $\omega_j$, $j \in \{p, s, i\}$; they are obtained by injecting equation 1 into the telegrapher’s equations, and by operating the harmonic balance (HB) with only these three frequencies.

More precisely, the telegrapher’s equations in a lossless transmission line relate current $I$ and voltage $V$ as:

$$\frac{\partial I}{\partial x} = -C \frac{\partial V}{\partial t}$$

$$\frac{-\partial V}{\partial x} = L \frac{\partial I}{\partial t}$$

with $x$ a length per unit cell. Injecting Eq. 1 into Eqs. A1, we obtain:

$$v_p^2 \frac{d^2 I}{dx^2} - \frac{\partial I}{\partial t} = \frac{1}{2} (\frac{1}{2} I^2 + \frac{1}{2} \xi I^3)$$

with $v_p = 1/\sqrt{CL_d}$ the phase velocity. To solve Eq. A2 we perform the HB: we assume that the current in the transmission line is a sum of three terms at three different frequencies:

$$I = \frac{1}{2} (I_p(x)e^{i(k_j x - \omega_j t)} + I_s(x)e^{i(k_s x - \omega_s t)} + I_i(x)e^{i(k_i x - \omega_i t)} + c.c.)$$

and we then keep only the mixing terms from Eq. A2 that emerge at these frequencies. This approach is valid in our case, because the phase matching bandwidth is limited by dispersion engineering (see appendix B), and thus mostly these three frequencies are able to mix together. Under the slow-varying envelope approximation, $|d^2I_j/dx^2| \ll |k_j dI_j/dx|$ for $j \in \{p, s, i\}$, the left hand side of Eq. A2 yields:

$$v_p^2 \frac{d^2 I}{dx^2} - \frac{\partial I}{\partial t} = \sum_{j=p, s, i} ik_j \frac{dI_j}{dx} e^{i(k_j x - \omega_j t)} + c.c.$$  

Using $\omega_p = \omega_s + \omega_i$, we collect terms at $\omega_j$, $j \in \{p, s, i\}$ in the right hand side (rhs) and form the CME:

$$\frac{dI_p}{dx} = \frac{ik_p}{4} I_s I_i e^{-i\Delta k x} + \frac{ik_p}{8} I_p(|I_p|^2 + 2|I_s|^2 + 2|I_i|^2)$$

$$\frac{dI_s}{dx} = \frac{ik_s}{8} I_p I_i e^{i\Delta k x} + \frac{ik_s}{8} I_s (2|I_p|^2 + |I_s|^2 + 2|I_i|^2)$$

$$\frac{dI_i}{dx} = \frac{ik_i}{8} I_p I_s e^{i\Delta k x} + \frac{ik_i}{8} I_i (2|I_p|^2 + |I_s|^2 + 2|I_i|^2),$$

with $\Delta k = k_p - k_s - k_i$. The phase matching condition, Eq. 2, is found for a strong pump, where $|I_p|, |I_i| \ll |I_p|$. Assuming the pump undepleted, $|I_p(x)| = I_{p0}$, Eqs. A5 rewrite:

$$\frac{dI_p}{dx} = \frac{ik_p}{8} I_p I_p^2$$

$$\frac{dI_s}{dx} = \frac{ik_s}{8} I_p I_i e^{i\Delta k x} + \frac{ik_s}{8} I_s I_p^2$$

$$\frac{dI_i}{dx} = \frac{ik_i}{8} I_p I_s e^{i\Delta k x} + \frac{ik_i}{8} I_i I_p^2,$$
which results in \( I_p(x) = I_{p0} \exp(i \xi k_p I_{p0}^2 x/8) \). Signal and idler amplitudes are then searched with the form: \( I_j(x) = \tilde{I}_j(x) \exp(i \xi I_{j0}^2 k_j x/4), j \in \{s, i\} \). Equations A6 then yield:

\[
\frac{d\tilde{I}_s}{dx} = \frac{i k_0 c \xi}{4} I_{p0} \tilde{I}_s e^{i \Delta_\beta x},
\]

\[
\frac{d\tilde{I}_i}{dx} = \frac{i k_0 c \xi}{4} I_{p0} \tilde{I}_s e^{i \Delta_\beta x},
\]

(A7)

with \( \Delta_\beta = \Delta_k + \frac{\xi I_{j0}^2}{2} (k_p - 2 k_s - 2 k_i) \) and \( \Delta_k = k_p - k_s - k_i \). The system of equations A7 has known solutions [36]. In particular, when phase matching is achieved, i.e. \( \Delta_\beta = 0 \), we obtain:

\[
\tilde{I}_s = \cosh (g_3 x) \tilde{I}_{s0},
\]

\[
\tilde{I}_i = i \sqrt{\frac{C}{k_s}} \sinh (g_3 x) \tilde{I}_{s0},
\]

(A8)

with \( g_3 = \frac{c I_{p0}}{\sqrt{k_s k_x}} \), and with initial conditions \( I_s(0) = I_{s0} \) and \( I_i(0) = 0 \). The signal power gain

\[
G_s(x) = \left| \frac{I_s(x)}{I_{s0}} \right|^2
\]

(A9)

is then exponential with \( x \): \( G_s = \cosh^2(g_3 x) \).

3. Pump phase shift

From the phase matching condition, Eq. 2, it is clear that only the 4WM term \( \xi \) creates a dispersive phase shift of pump, signal and idler. In other words, in a pure 3WM case, \( \xi = 0 \) and the phase matching condition becomes \( \Delta_k = 0 \), naturally fulfilled in a dispersion-less transmission line. While detrimental for noise properties (see Sec. IV), we can use the continued presence of 4WM to our advantage, because it allows us to calibrate the pump power, down to the KIT input.

In fact, in such a situation \( I_d \) and \( I_{p0} \) influence the pump tone phase shift, which we can measure unambiguously (i.e. not \( \mod 2\pi \)) with a VNA. More precisely, although the phase \( \phi = \arg(S_{21}) \) read by a VNA is \( 2\pi \)-wrapped, its shift \( \delta_\phi = \phi - \phi_0 \) from an initial value \( \phi_0 \) can be continuously monitored when \( I_d \) and \( I_{p0} \) vary, and thus unambiguously determined. This phase shift in turn translates into a wavenumber variation \( \delta_k = -\delta_\phi/N_c \). If initially at zero dc bias and small rf pump amplitude, then \( \delta_k = \beta_P - k_p \), with \( \beta_P \) the pump wavenumber, dependent on \( I_d \) and \( I_{p0} \), and \( k_p \) the linear wavenumber. When a single pump tone travels along the line (no input signal), we are by default under the strong pump approximation, and the first equation of Eqs. A6 gives \( I_p(x) = I_{p0} \exp(i \xi k_p I_{p0}^2 x/8) \). In addition, the current \( I \) in the line then writes as \( I = 1/2 \{I_p(x) \exp[i(k_p x - \omega t)] + c.c. \} \), and thus the pump wavenumber is \( \beta_p = \xi k_p I_{p0}^2/8 + k_p \), which leads to \( \delta_k = \xi k_p I_{p0}^2/8 \). Because \( k_p = \omega_p \sqrt{L_d C} \), we can re-write:

\[
\delta_k = \frac{1}{8} \frac{I_{p0}^2}{I_d} \omega_p \sqrt{L_d C} \frac{1 + \frac{I_d^2}{I_d^2}}{1 + \frac{I_d^2}{I_d^2}},
\]

(A13)

therefore \( I_{p0} \) and \( I_d \) induce similar phase shift in the line. Knowing \( I_d \) (room temperature parameter), we thus get \( I_{p0} \) at the KIT input.

Appendix B: ABCD TRANSFER MATRICES

The dispersion relations, Fig. 2a are calculated by cascading the ABCD matrices of the KIT cells, a method suitable for any periodic loading pattern. We then compute the KIT \( S_{21} \) scattering parameter as \( S_{21} = 2/(A + B/Z_0 + C Z_0 + D) \) [32], where \( Z_0 = 50 \Omega \) is the input and output ports impedance, and finally get

\[
k = -\text{unwrap} [\arg (S_{21})]/N_c.
\]

In the unloaded case, represented in Fig. 1, the ABCD matrix cell is:

\[
T_c = \begin{bmatrix}
1 & \frac{i L_d \omega}{2 - L_d C \omega^2} \\
\frac{1}{2 - L_d C \omega^2} & 1 - \frac{i L_d \omega}{2 - L_d C \omega^2}
\end{bmatrix}.
\]

(B1)

All the cells being identical, the KIT’s ABCD matrix is simply \( T_K = T_c^{N_c} \). In Fig. 2a we used \( N_c = 6.6 \times 10^4 \) to match the length of our fabricated KIT, and \( L_d = 45.2 \)
pH, $C = 18.8$ fF, and $L_f = 1.02$ nH, values that match our design (fingers are 102 µm long and 1 µm wide).

In the loaded case, some cells have shorter fingers, see Fig. 3a. In these cells, the capacitance to ground is $C_l = L_d/Z_l^2$, where $Z_l$ is the load impedance, and a finger’s inductance is $L_l$. To compute the KIT scattering parameter, we form the ABCD matrix of the repetition pattern comprised with unloaded and loaded cells:

$$\mathbf{T}_s = \begin{bmatrix} 1 & iL_d\omega \\ \frac{\gamma^2 C_l\omega}{2 - L_f C_l \omega} & 1 - \frac{2L_d C_l \omega^2}{2 - L_f C_l \omega} \end{bmatrix}^{N_u/2} \times \begin{bmatrix} 1 & iL_d\omega \\ \frac{\gamma^2 C_l\omega}{2 - L_f C_l \omega} & 1 - \frac{2L_d C_l \omega^2}{2 - L_f C_l \omega} \end{bmatrix}^{N_l}$$

where $N_u$ is the number of unloaded cells and $N_l$ is the number of loaded cells in the pattern, which we call a supercell. As before, to get the KIT’s ABCD matrix, we simply form $\mathbf{T}_K = \mathbf{T}_s^{N_c}$, where $N_c = N_l/(N_u + N_l)$ is the number of supercells in the KIT. Here, $N_u = 60$, $N_l = 6$, $N_c = 66000$, therefore $N_c = 1000$. The plain line in Fig. 2a shows the wavenumber $k^*$ thus found, with $Z_l = 80$ Ω, and $L_f = 335$ µH, as the finger length in a loaded cell is 33.5 µm.

To compute the signal power gain, Fig. 2b, we inject the expression of $k(\omega)$ for a periodically loaded KIT (from $\mathbf{T}_K$) in the CME, Eqs. A5. We solve them numerically for different pump frequencies. For 8.8812 GHz (blue curve), 8.8992 GHz (red), 8.9256 GHz (green) and 8.9736 GHz (purple), phase matched signal and idler are detuned from the half pump frequency by 0, 1, 1.5 and 2 GHz respectively. For 8.855 GHz (gray curve), phase matching is nowhere achieved. We used $N_c = 6.6 \times 10^4$, $I_s = 7$ mA, $I_d = 1.5$ mA, and the initial conditions $I_{p0} = I_s/60$, $I_{s0} = I_{p0}/100$, and $I_{i0} = 0$, close to experimental values.

**Appendix C: KIT SATURATION**

1. Strong signal gain profile asymmetry

When the input signal power amounts to a significant fraction of the pump power, parametric amplification depletes the pump. It surprisingly generates asymmetry in the signal gain profile, with respect to the half pump frequency. Figure 7a shows signal gain profiles, calculated when phase matching is achieved at exactly half the pump frequency, i.e. for $\omega_s = \omega_l$ (corresponding to $\omega_p = 8.8812$ GHz), at various initial signal powers $P_{s0}$. They are obtained by solving the CME A5, which incorporate pump depletion effects. As $P_{s0}$ increases, the gain diminishes, and the originally flat profile tilts, with higher frequencies presenting higher gain.

Fundamentally, this asymmetry stems from the fact that when solving the CME in the case where $I_p$ varies along the KIT transmission line, the initial conditions (ICs) vary as a function of frequency. More precisely, the second and third equations in Eqs. A5, govern the evolution of $I_s$ and $I_i$ respectively; in a simplified version, they write as

$$\frac{dI_s}{dx} = \frac{i k_s \epsilon}{4} I_p I_s^*$$

$$\frac{dI_i}{dx} = \frac{i k_i \epsilon}{4} I_p I_i^*$$

We dropped the second terms in the rhs of Eqs. A5, representing the 4WM conversion processes, as the asymmetry still holds when $\xi = 0$, and we assumed perfect phase matching in 3WM, $\Delta k = 0$, i.e. a dispersionless line.

In the undepleted pump regime, $I_p(x) = I_{p0}$, and we can decouple the equations on $I_s$ and $I_i$. Deriving with respect to $x$ Eqs. C1, we get

$$\frac{d^2 I_j}{dx^2} = g_3^2 I_j,$$

with $j \in \{s, i\}$, and $g_3 = \frac{\epsilon I_{p0}}{4 \sqrt{k_s k_i}}$, as defined in appendix A1. Using the ICs $I_j(0) = I_{j0}$ and $dI_j/dx(0) = 0$ (because $I_j^*(0) = 0$), $I_s = \cosh(g_3 x) I_{s0}$, as seen in

![FIG. 7. Theory of KIT saturation, calculated by solving the full CME A5. (a) Distortion of the gain profile, at four different input signals: $I_{s0} = I_{p0}/100$ (blue curve), corresponding to a small signal regime, $I_{s0} = I_{p0}/12$ (red), $I_{s0} = I_{p0}/8$ (green), and $I_{s0} = I_{p0}/6$ (purple). The KIT length, is $N_c = 6.6 \times 10^4$ cells. Vertical lines indicate three frequencies: 3.4406 GHz (long dashed), 4.4406 GHz (plain), equal to the half pump frequency, and 5.4406 GHz (short dashed). (b) The gain of a probe tone is calculated at these three frequencies, as a function of the probe tone power. (c) The 1 dB compression power is shown as a function of frequency.](image-url)
Eqs. A8. Signal and idler wavenumbers appear as a product in this solution, hence for any \( x \) the signal amplitude \( I_s \) is symmetric with respect to the half pump frequency.

In the soft pump regime, where \( I_p(x) \) is not constant, Eqs. C1 cannot be decoupled. We can however write them in a canonical form, deriving with respect to \( x \):

\[
\frac{d^2 I_j}{dx^2} = \frac{1}{I_p} \frac{d I_j}{dx} - \frac{k_s k_i \epsilon^2}{16} |I_p|^2 I_j = 0,
\]

with \( j \in \{s, i\} \). Here, the interplay between \( I_s \) and \( I_i \) comes from \( I_p \), which contains the product \( I_s I_i \) (see Eqs. A5). Although signal and idler wavenumbers also appear only as a product in Eqs. C3, these CME lead to an asymmetric signal amplitude profile, because out of the five ICs required to solve them, one changes with frequency: \( I_p(0) = I_p \), \( I_s(0) = I_s \), \( I_i(0) = 0 \), \( dI_s/dx(0) = 0 \), and \( dI_i/dx(0) = i \epsilon I_p I_s k_i/4 \). This last IC depends on \( k_i \), which depends on the signal frequency. In the small signal limit, \( dI_i/dx(0) \rightarrow 0 \), and we recover a symmetric gain profile with respect to the half pump frequency.

2. Compression power calculation

This asymmetry produces higher power handling capabilities at frequencies above \( \omega_p/2 \), compared to below \( \omega_p/2 \). Thus, considering that only half the bandwidth is usable for resonators readout, it is more advantageous to have these lie above \( \omega_p/2 \). Figure 7b shows the gain as a function of a probe tone power \( P_t \), calculated from the CME A5 at three frequencies: one at \( \omega_p/2 \), and two at \( \omega_p/2 \pm 1 \text{ GHz} \). Because phase matching is set to be optimal at \( \omega_s = \omega_i = \omega_p/2 \), gain is maximal at this frequency. The small signal gain is identical for \( \omega_p/2 \pm 1 \text{ GHz} \), however it visibly compresses at higher tone power for \( \omega_p/2 + 1 \text{ GHz} \). Figure 7c presents the 1 dB compression power \( P_{-1dB} \), calculated in the interval \([\omega_p/2 - 1, \omega_p/2 + 1]\) gigahertz. As expected, \( P_{-1dB} \) is a few dB higher when \( \omega > \omega_p/2 \). This phenomenon is reminiscent of gain distortion, seen in JPA [7]. Effects not included in the CME A5, such as standing wave patterns, or defects in the line, which locally lower \( I_s \), may cause the discrepancy between these theoretical calculations and the measurements (see Sec. III)

Appendix D: KIT PACKAGING

There are three main concerns when packaging a KIT: first, the package should be matched to 50 \( \Omega \). Any mismatch will result in reflections, creating gain ripples (see Sec. III). Second, the package should ensure good electrical grounding of the transmission line. Otherwise, given the fairly big chip size, spurious chip modes can appear within the frequency range of interest. Third, the package should ensure good thermalization inside the chip.

Because the pump power remains high for millikelvin operations, any inhomogeneous rise in temperature may trigger a hot spot near a weak-link, and possibly break superconductivity. We implemented a series of technologies to address these concerns.

Figure 8a presents the chip, clamped onto the bottom part of the copper packaging. The chip is wire-bonded on both sides to printed circuit boards (PCBs). They convert the on-chip CPW layout to microstrip, and then the central pin of sub-miniature version A (SMA) connectors are soldered onto the microstrip. We suspect imperfect PCBs, with impedance close to 52 \( \Omega \), to play a role in creating gain ripples. When designing the spiral, we carefully adjusted the radius of the turns, in conjunction with the unit cell length, to have these turns match the straight sections inductance and capacitance per length.

Electrical grounding inside the chip is ensured with pogo pins inserted in the top lid of the packaging, see Fig. 8c and d. When closing the box, these pins contact the chip between the line traces. If absent, we have measured spurious resonant modes with harmonics at gigahertz frequencies. Pins are 140 \( \mu \text{m} \) in diameter, and each applies a 20 g force to the chip.

These pins also act as thermal links to the packaging. In addition, we deposited gold strips onto the Nb-Ti-N layer, inside the spiral, between the KIT line traces, and near the chip edges, see Fig. 8b. These strips contact the pins. Absent the pogo pins, superconductivity breaks down before high gain can be reached. Finally, we gold-
bond the chip ground plane (from the deposited gold) to the copper box (instead of using standard aluminum bonding): gold remains a normal metal at millikelvin temperatures, thereby better thermalizing the chip.

Appendix E: NOISE MEASUREMENT EXPERIMENTAL SETUP

Figure 9 presents a schematic of the full experimental setup used to measure the system-added noise of a readout chain using a KIT as first amplifier. In total, noise generated by the SNT travels through three amplifiers: the KIT, a HEMT at 4K, and a room temperature amplifier (miteq), before finally being recorded with a SA.

The SNT consists of a tunnel junction (TJ), a magnet suppressing the Josephson effect, and a magnetic shield protecting other elements from its magnetic field. In addition, a bias tee routes SNT-generated rf noise to the microwave readout chain, while at the same time allowing for dc bias. In fact, the SNT is biased with an arbitrary waveform generator (AWG). It outputs a low frequency (50 Hz) triangular voltage wave on a 10 kΩ current limiting resistor, thereby creating a current $I_{\text{AWG}}$, varying between $\pm 12 \mu$A, which sweeps the SNT-generated noise value.

An oscilloscope reads the TJ voltage in situ while the AWG outputs a known current, allowing the computation of the TJ impedance $Z_{\text{TJ}} = 54 \pm 4 \Omega$, and with it, the SNT voltage bias $V = Z_{\text{TJ}} I_{\text{AWG}}$. The system-added noise temperature uncertainties are computed from those on $Z_{\text{TJ}}$, because these predominate.

Noise from the SNT is combined with rf tones (pump, and probe from a VNA to measure the gain profile) via a 20 dB directional coupled (DC) connected to the KIT input. Additionally, a 7 GHz low-pass filter placed between SNT and DC prevents the rf pump to leak back to the SNT. Because the KIT requires a fairly high pump power ($-28 \text{ dBm}$), we attenuate only the pump line by 10 dB at 4K. Then, an 8 GHz high-pass filter at 30 mK rejects noise at frequencies within the KIT amplification band, while allowing the pump to pass. A bias tee at the KIT input port combines rf signals (including noise from the SNT) with the KIT dc current bias, and a second bias tee at the KIT output separates dc from rf. The rf component then pass through 4–12 GHz isolators, and a 7 GHz low-pass filter (PE87FL1015, 0.3 dB typical insertion loss between 3.5 and 5.5 GHz, and $\sim 45 \text{ dB}$ rejection at 9 GHz), preventing the pump tone to saturate the HEMT.

The SA is operated in a zero-span mode, its acquisition triggered by the AWG. That way, it measures the output noise at a single frequency, over a 5 MHz RBW, and traces out directly curves of Fig. 6b. Varying the SA central frequency, we obtain the system-added noise over the full 3-6.5 GHz bandwidth, Fig. 6c.
Appendix F: SHOT NOISE THERMOMETRY

1. System-added noise

When propagating through the experimental setup presented in Fig. 9, noise generated by the SNT undergoes loss and amplification. Both affect the effective noise at each amplifier input, and therefore also, the noise reaching the SA. While the overall system-added noise $N_s$ encompasses microwave loss, it is informative to express it in terms of amplifiers added noise, gain, and transmission efficiencies to estimate the added noise $N_i$ intrinsic to the KIT.

Figure 5 represents the lossy amplification chain. Noise generated from the SNT is transmitted with efficiency $\lambda$ to the KIT: it experiences a beam-splitter interaction with the Johnson noise $N_f \simeq 0.5$ of the fridge at 30 mK, such that

$$N_0 = \lambda N_i + (1 - \lambda)N_f$$  \hspace{1cm} (F1)

is the effective noise reaching the KIT. At the KIT output, we obtain

$$N_1 = G\lambda \left( N_i + \frac{(1 - \lambda)N_f + N}{\lambda} \right)$$  \hspace{1cm} (F2)

where $G$ is the KIT gain, and $N$ its (input referred) added noise. Transmitted to the HEMT input with efficiency $\eta$, the HEMT input noise is then

$$N_2 = \eta G\lambda \left( N_i + \frac{(1 - \lambda)N_f + N}{\lambda} \right) + (1 - \eta)N_f.$$  \hspace{1cm} (F3)

Following, the HEMT output noise reads

$$N_3 = G_H\eta G\lambda \left( N_i + \frac{(1 - \lambda)N_f + N}{\lambda} + \frac{(1 - \eta)N_f + N_H}{\eta G\lambda} \right),$$  \hspace{1cm} (F4)

where $G_H$ is the HEMT gain, and $N_H$ its added noise. The HEMT gain $G_H \sim 40$ dB is sufficient to overcome the following loss and amplifier noise. Thus, we can neglect their effect on the noise $N_o$ reaching the SA, and write $N_o = G_m N_a$, with $G_m$ the net room temperature amplification (including losses). We call $G_c = G_m G_H\eta G\lambda$ the net gain of the amplification chain. Furthermore, $(1 - \eta)N_f \ll N_H$, therefore the noise reaching the SA is

$$N_o = G_c \left( N_i + \frac{(1 - \lambda)N_f + N}{\lambda} + \frac{N_H}{\eta G\lambda} \right),$$  \hspace{1cm} (F5)

with

$$N_s = \frac{(1 - \lambda)N_f + N}{\lambda} + \frac{N_H}{\eta G\lambda}$$  \hspace{1cm} (F6)

the system-added noise.

When the KIT is not pumped, it is a lossless, noiseless, passive element. We therefore have $G = 1$ and $N = 0$. In that situation

$$N_o' = G_c' (N_i + N_o'),$$  \hspace{1cm} (F7)

where $G_c' = G_m G_H\eta \lambda$, and using Eq. F4 we get

$$N_s' = \frac{(1 - \eta)N_f + N_H}{\eta \lambda}$$  \hspace{1cm} (F8)

as the chain’s system-added noise with the HEMT as first amplifier.

2. Shot noise

The SNT relies on the fact that a voltage biased TJ generates a known shot noise [29]. The amount of noise $N_i$ delivered to the 50 Ω transmission line can be written as [6]

$$N_i = \frac{k_B T}{2\hbar \omega} \left[ \frac{eV + h\omega}{2k_B T} \coth \left( \frac{eV + h\omega}{2k_B T} \right) + \frac{eV - h\omega}{2k_B T} \coth \left( \frac{eV - h\omega}{2k_B T} \right) \right],$$  \hspace{1cm} (F9)

where $T \sim 30$ mK is the physical temperature of the SNT, and $V$ the SNT voltage bias. In practice, the AWG has a slight voltage offset, which we include as a fit parameter: we write $V - V_{\text{off}}$ instead of $V$ in Eq. F9. We then inject Eq. F9 into Eq. F5, and fit the output noise $N_o$ to obtain $G_c$, $N_s$, and $V_{\text{off}}$.

3. Amplifier-added noise measured with a wideband noise source

The amplifier added noise $N$ and system-added noise $N_s$ quoted by this method can be directly compared to most published results [1–7, 13–15, 27]. But the actual amplifier-added noise, the one that a MKID or a qubit would see, is subject to a small correcting term. This term originates from the fact that the SNT is wideband, and therefore generates noise at the signal and idler frequency. In that case, for a phase insensitive amplifier, the input referred amplifier-added noise is not only $N$, but it also includes a contribution from the input noise at the idler port $N'$. More precisely, the total added noise can be written as: [37]:

$$N_i = \frac{G - 1}{\lambda_s} \frac{1}{\lambda_i} N_s' + N,$$  \hspace{1cm} (F10)

where $\lambda_s$ ($\lambda_i$) is the transmission efficiency between SNT and KIT at the signal (idler) frequency. Here, $N$ represents the technical noise of the amplifier, and also encompasses the transmission efficiencies between the SNT and the amplifier. Thus, for a quantum-limited amplifier and lossless setup, $N = 0$. The idler input noise $N'$ depends on the port’s temperature. At 30 mK, and if the port is well thermalized, $N' = 0.5$. In general, $\lambda_s$ and $\lambda_i$ are
FIG. 10. System-added noise measurement of a microwave amplification chain with the HEMT as first amplifier. (a) The chain’s net gain $G'_{c}$ is obtained from fitting the output noise response $T'_{o}$, at each frequency. (b) Examples of $T'_{o}$ are shown for the SA centered at 4, 5 and 6 GHz, with fits superimposed in black lines. The SA RBW is 5 MHz. (c) From the fits, we also extract the system-added noise temperature $T'_{s}$ as a function of frequency. Uncertainties are indicated by the gray area surrounding the black line.

difficult to assess experimentally, and their difference increases as signal and idler frequencies are further apart. But in the limit case, where $\lambda_s = \lambda_i$, and for high gains, such that $G \simeq G - 1$, we see that $N'_{t} \geq 0.5$, in agreement with the quantum limit on amplifier added noise. Note also that Eq. F10 ensures the total added noise temperature to be always higher than the physical temperature of the device.

Assuming such a limit case, in our situation it implies that the total added noise temperature of the KIT is $T'_N = N'_t \hbar \omega / k_b \geq 0.3 \pm 0.1$ K. It is slightly higher than $T_N$, obtained by the more naive calculation. Similarly, replacing $N$ by $N'_t$ in Eq. 5, the total system-added noise temperature of the amplification chain with the KIT as first amplifier changes to $T'_s = 0.72 \pm 0.08$ K. We see that for our case, the correction relatively is small, because only half a photon is added, and the near-quantum-limited claim remains valid. However, as the amplifier is made quieter and approaches the quantum limit, this correction becomes more prominent.

Appendix G: SYSTEM-ADDED NOISE TEMPERATURE WITH UN-PUMPED KIT

When the KIT is off, we can measure the system-added noise temperature of the amplification chain $T'_{s} = N'_t \hbar \omega / k_b$, with the HEMT as first amplifier. In that case, $N'_t$ is given by Eq. F8. Figure 10c shows $T'_s$ as a function of frequency, obtained from fitting curves like those presented in Fig. 10b. The chain’s net gain $G'_{c}$ is shown in Fig. 10a.

Between 3.5 and 5.5 GHz, $T'_s = 5.1 \pm 1.4$ K. There are ripples with 130 MHz characteristic frequency, likely due to reflections in coaxial cables between SNT and HEMT. From $T'_s$ we can estimate the transmission efficiencies $\lambda$ and $\eta$ from Eq. F8, assuming $\lambda = \eta$. In fact, according to the HEMT specifications, $T_H = N_H \hbar \omega / k_b \sim 1.65 \pm 0.4$ K between 3.5 and 5.5 GHz, hence $\lambda = \eta \sim 0.6 \pm 0.1$, at $\omega = 2\pi \times 4.4$ GHz.

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