The application of discrete mathematics to microeconomics

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Abstract: Mathematical logic is one of the two cornerstones of discrete mathematics course, which uses mathematical methods to study the relationship between the premise and conclusion in reasoning. Microeconomics is the study of individual economic units, how market mechanisms work and function when resources are scarce, and ways to improve their functioning. Game logic is a new direction in the field of logic game theory is mainly applied to behavioural theory in microeconomics. This paper studies the combination of logic and game theory from the propositional logic in mathematical logic.

1. Introduction

Discrete mathematics is an important branch of modern mathematics, with the study of the structure of discrete quantities and their mutual relations as the main research object. It has a wide range of applications in the field of computer science and technology, and is an essential prerequisite course for many professional courses of computer and communication engineering [1-2]. Discrete mathematics teaching content is not only involved in computer hardware, computer software research and has a close relationship, with distinct characteristics, the basis of its concept, theory and method in the computer professional courses such as operating system, compiling principle, data structure and algorithm analysis and design, database has a broad application in depth[3]. Discrete mathematics courses mainly include quantitative logic, set theory, algebraic structure, graph theory.

Logic is a science that studies the thinking forms and laws of human beings. It can be divided into formal logic, dialectical logic and mathematical logic according to the different research objects and methods. Mathematical logic is a branch of mathematics that studies reasoning. It uses mathematical methods, namely a set of symbols, to study the formal relationship between premise and conclusion in reasoning, so it is also called symbolic logic. It plays an important role in computer science and people's life, and plays an important role in theoretical guidance. Mathematical logic is divided into propositional logic and predicate logic. This paper only does some research on the application of mathematical logic in microeconomics.

2. Basic applications of propositional logic

Propositional logic is a simpler and more basic part of modern logic. It does not consider the analysis of propositions into the combination of non-propositional components such as individual words, predicates and quantifiers, but only studies the compound propositions composed of propositions and propositional connectors, especially the logical properties and inference rules of propositional connectors [4].

In the reasoning of propositional logic, let A1, A2, A3, …, Ak be propositions, if A1 ∧ A2 ∧ … ∧ Ak → B is always true, then it is called from premiseA1, A2, …, Ak infer B. They call B is A1,
A₂……Aₖ logical conclusion, denote A₁ ∧ A₂ ∧ …… ∧ Aₖ ⇒ B

For example, a public security officer is investigating a theft and the following facts are known:

1) A or B has stolen the recorder.
2) If A steals the tape recorder, the time of the crime should not be before midnight.
3) If B's testimony is correct, the lights in the house remain on at midnight.
4) If B's testimony is incorrect, the crime took place before midnight.

Who stole the tape recorder?

Assuming that p: A stole a tape recorder. q: B stole a tape recorder. r: The crime was committed before midnight. s: B's testimony is correct; t: The lights went out at midnight.

Preconditions: p ∨ q, p → ¬r, s → t, ¬s → r, ¬t

Conclusion: According to logical reasoning

Reasoning process:
① s → t   P
② ¬t   P
③ ¬s   T ①② I
④ ¬s → r   P
⑤ r   T ③④ I
⑥ p → ¬r   P
⑦ ¬p   T ⑤⑥ I
⑧ p ∨ q   P
⑨ q   T ⑦⑧ I
⑨ indicates that B stole the recorder.

3. Combination of logic and game theory

Since the second half of last century, the theory and method of game theory have been applied to logic in various forms. Logicians represented by Sintica put forward "assignment logic", which is a logical game. His main idea is to turn the actions of the players in the game into connectives in logic. For example, the decision behavior of game players can correspond to the implication expression in logic, and the choice behavior can correspond to the disjunctive expression in logic.

In recent years, with the continuous improvement of game theory, people find that there are many problems that need to be solved by logic theory in game theory, and it is the current development trend that logic is applied to game theory. Game is closely related to us and exists game phenomenon all the time in our life. How can you win the game? As a "rational person", we need to take a reasonable strategy. The choice of strategy is usually based on rigorous reasoning. Strong game thinking comes from strong logical accomplishment. Therefore, game logic arises at the historic moment, and it is indispensable to improve human's game thinking ability.

In traditional logic, we divide reasoning into deductive reasoning and inductive reasoning according to the different relationship between premise and conclusion. In game logic, the basis is different, it is to study the choice of the players in the game, how the players make the choice, and how the choice is deduced. The outcome of the choice depends heavily on two things. One is the structure of the game and the other is the strategy chosen by the players. The game structure is the payment function of the players after choosing different strategies. According to the relationship between the premise and the conclusion of the participants' reasoning, reasoning can be divided into deductive reasoning and inductive reasoning, reasoning in game logic and reasoning in traditional logic. For deductive reasoning, as long as the premise is true and the reasoning form is effective, the conclusion must be true. In inductive reasoning, the premises are true and the conclusion is not necessarily true, so we say that deductive reasoning is necessary reasoning and inductive reasoning is probable reasoning. Reasoning in game logic differs from that in traditional logic in that the assumptions used are static and invariable when people reason in traditional logic. In game logic, the
assumptions used are both static and dynamic.

Deductive reasoning is a process of drawing conclusions based on inference rules and information known to all participants, which is applicable to complete information games. We divide deductive reasoning into deductive reasoning in a dynamic game with complete information and deductive reasoning in a static game with complete information.

In a game, there is not necessarily public knowledge for the players. There are two situations: one is that both players know some knowledge, but they do not know whether the other party knows it or not. Similarly, the other party does not know whether it knows it or not. But only one side of the game knows, one side knows more, the other side knows less. That knowledge is asymmetric. So you have to use inductive reasoning, so you can divide it into inductive reasoning in a static game with incomplete information and inductive reasoning in a dynamic game with incomplete information.

4. Deductive reasoning in the prisoner's dilemma

In the complete information game, players can derive the strategy choices of other players based on public knowledge. If the public knowledge is the only knowledge possessed by all participants, then they can use the public knowledge to reason, and the conclusions they get must be true. This process of reasoning is called deductive reasoning [5].

A brief introduction to the classic case in game theory - the prisoner's dilemma. Suppose two thieves A and B are caught by the police for breaking into a house together. Police put the two men in separate cells for questioning, the two thieves cannot exchange information between each other, each of the actions of the complete understanding of the other. For each suspect, the police promised that if both men confessed to the crime and handed over the stolen goods, and the evidence proved strong, they would both be sentenced to six years in prison. If the other suspect refuses to confess rather than make a confession, he is charged with obstructing public office for an additional two years, while the confessor is given a reduced sentence of six years for meritorious performance and released immediately. If they both refuse to admit the theft, police can only put them each in jail for a year for breaking and entering.

The game table of the two thieves is shown in Table 1:

| The thief a     | The thief b | Refused to confess |
|----------------|-------------|-------------------|
| confession     | -6, -6      | 0, 8              |
| Refused to confess | -6, 0     | 1, 1              |

In this case, the best strategy between the two thieves was that both sides refused to confess, and the result was that the two thieves were only sentenced to one year. Assume that every thief is a "rational economic man" who makes a choice in his favor. Thief A made such reasoning: "If B refused to admit the fact of stealing, and I confessed at the same time, then I have performed meritorious service, and will probably be released without guilt. If B confessed to the theft, and I refused, then B would be acquitted and I would go to prison for eight years. If I and B both confessed to the theft, we could both get six years in prison, which would be far better than my eight years in prison. In all of these cases, confessing to the theft would have saved me time in prison, and I should not have considered B's choice out of self-interest."

Thief A's reasoning is correct from the point of view of personal interest. No matter what B chooses, A's optimal strategy is to confess. Obviously, this truth B is also very clear, B will choose to confess. The Nash equilibrium of this game is confession. The two thieves could have co-operated by refusing to admit the theft and each would have been jailed for a year. But since information cannot be exchanged, both thieves suspect the other of selling out to protect themselves from a psychological point of view. Using formal reasoning, P represents refusal to confess, and Q represents two more years in prison. From the perspective of rational man's self-interest, every player is not willing to spend more time in prison. The above statement can be transformed into the following reasoning form:
\((p \rightarrow q) \land \neg q \rightarrow \neg p\)

The two thieves were fully aware of the relevant legal policies, i.e., the strategic choices between each other and the corresponding legal consequences. Since the two thieves choose the strategy simultaneously (that is, confess and refuse to confess), the game behavior between them is a process of complete information static game logic. The form of reasoning used by the two thieves in this process is deductive reasoning. The basis of this form of reasoning is the thief's common-understanding of legal sentencing. In fact, this is a paradox presented in the prisoner's dilemma: the two decision-making agents could gain the common benefit through cooperation, but as rational people, they choose to betray the strategy at the expense of their own interests. This paradox reflects a profound problem when individual rationality conflicts with collective rationality.

5. Conclusion
Mathematical logic is an important part of discrete mathematics, which has important theoretical and application value. The game theory is also an important knowledge point in microeconomics. Due to the limited cognitive ability of human beings, there are many problems worth further studying in the reasoning of game logic.

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References:
[1] Guo Jinglei, Jin Cong. Microchip learning model of discrete mathematics course [J]. Computer Education, 2018(08): 62-65.
[2] Tang Jianfeng, Gao Zhen, Huang Jie. Discrete Mathematics Teaching Reform Exploration [J]. Computer Education, 2019(06): 65-67.
[3] Chen Guangxi, Gu Tianlong. "Discrete Mathematics" Excellent Course Teaching Reform Practice [J]. Journal of Guilin University of Electronic Science and Technology, 2018(04): 300-302.
[4] Alizadeh Majid, Ardestir Mohammad. Basic propositional logic and the weak excluded middle [J]. Logic Journal of the IGPL, 2019, 27(3): 70-75.
[5] Tianqun Pan. Deductive and Inductive Reasoning in Game Behavior and Its Problems [J]. Studies in Dialectics of Nature, 2003, (03): 39-43+53.