Known Bits Puncturing for Systematic Polar Coded OFDM Systems

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Abstract: This article proposes a known bits puncturing scheme for systematic polar code. The puncturing deteriorates BER performance because the removed bits are generally unknown at the receiving side. Polar code supports systematic encoding which generate the codeword that has the same value of information bits. Exploiting this characteristic, known bits, which are pre-shared between the transmitter and the receiver, are punctured from the codeword. The receiving side then set punctured bits to the known values and it can assist the decoding capability. Computer simulation verifies its effectiveness in terms of BER as well as throughput performance.

Keywords: Polar codes, systematic polar codes, puncturing

Classification: Wireless communication technologies

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1 Introduction

Polar code [1] is one of the error correction codes and classified as the linear block code. It uses a phenomenon called channel polarization for encoding, and can attain the capacity of a binary-input discrete memoryless channel (B-DMC). Moreover, polar code can be implemented with a simple encoder and a simple successive cancellation (SC) decoder. Meanwhile, polar codes have some limitations; one of them is that the code length becomes a power of two due to the encoder structure. Although the puncturing can realize an arbitrary code rate to enhance the transmission efficiency, the BER performance is deteriorated.

Lots of approaches have been proposed to prevent the BER performance degradation from the puncturing. First, [2] conceived to determine the puncturing position according to the channel capacities. Second work in [3] investigated the puncturing pattern from the generator matrix $G$. $G$ is characterized by the fact that there are some columns whose column weight is 1. Column bits are punctured wherein the column has weight of 1. Then, the column and row are deleted, column weight are calculated again. The above operation is repeated until the desired code length is attained. Third, [4] employed bit-reversal permutation. This method considers a vector having the same size as the codeword. Let $N_p$ denote the number of punctured bits, the first $N_p$ bits in the vector are set to zeros and others are ones. Bit-reversal permutation is then performed and the indices having zeros are punctured.

The above approaches are based on the non-systematic polar coding. Meanwhile, polar codes support a systematic encoding where the original input data appears as it is in the encoder output [5]. Systematic polar codes can outperform the original non-systematic ones in terms of BER [6]. In this case, puncturing is generally applied to the parity bits part. Exploiting this feature, we proposed a known bits puncturing for systematic polar codes [7]. It embeds known bits in the codeword of encoder output and punctures these bits. At the receiver side, these known bits are inserted and input to the decoder. Known bits are expected to assist the decoding capability and thus BER performance can be improved. This letter presents new results other than [7] with various numbers of punctured bits and we compared our approach with existing ones [2][3][4] to strengthen our contribution.

The rest of this letter is organized as follows. Section 2 describes the systematic polar coding and puncturing. Sections 3 and 4 presents the proposed scheme and computer simulation results. Section 5 concludes the letter.

2 System Model

2.1 Systematic Polar Coding

Let $M$ and $N$ ($M < N$, $N = 2^n$) be the codeword and the data length of $(N, M)$ polar codes, where $n$ is a positive integer. An $N \times N$ generator matrix $G_n$ for the $(N, M)$
polar codes is obtained by

\[ G_n = F_2^\otimes n, \]  

where \( F_2^\otimes n \) are the \( n \)th Kronecker power of \( F_2 \). We define the source word \( x \), the codeword \( z \) and the set of indices containing information bits \( \mathcal{A} \). Therefore, we define information bits \( x_\mathcal{A} \) in which channel capacity are large and insert the data bits. Frozen bits \( x_{\bar{\mathcal{A}}} \) in which channel capacity are small and insert 0. The non-systematic codeword can be written as

\[ z = x_\mathcal{A} G_\mathcal{A} + x_{\bar{\mathcal{A}}} G_{\bar{\mathcal{A}}}, \]  

where \( G_\mathcal{A} \) and \( G_{\bar{\mathcal{A}}} \) are the submatrices of \( G \) consisting of rows with indices in \( \mathcal{A} \) and \( \bar{\mathcal{A}} \), respectively. The systematic polar code is constructed by specifying a set of indices of the codeword \( z \) as the indices of the information bits. For this explanation, we prepare similar set \( \mathcal{B} \) to split the codeword as \( z = (z_\mathcal{B}, z_{\bar{\mathcal{B}}}) \), and rewrite Eq. (3) as

\[ z_\mathcal{B} = x_\mathcal{A} G_{\mathcal{A}\mathcal{B}} + x_{\bar{\mathcal{A}}} G_{\bar{\mathcal{A}}\mathcal{B}}, \]  

\[ z_{\bar{\mathcal{B}}} = x_\mathcal{A} G_{\mathcal{A}\bar{\mathcal{B}}} + x_{\bar{\mathcal{A}}} G_{\bar{\mathcal{A}}\bar{\mathcal{B}}}, \]  

where \( G_{\mathcal{A}\mathcal{B}} \) is the submatrix of \( G \) consisting of the array of elements \((G_{i,j})\) with \( i \in \mathcal{A} \) and \( j \in \mathcal{B} \). It is apparent that the set \( \mathcal{B} \) is the same as \( \bar{\mathcal{A}} \). Therefore we can rewrite Eqs. (4) and (5) as

\[ z_\mathcal{A} = x_\mathcal{A} G_\mathcal{A} + x_{\bar{\mathcal{A}}} G_{\bar{\mathcal{A}}}, \]  

\[ z_{\bar{\mathcal{A}}} = x_\mathcal{A} G_{\mathcal{A}\bar{\mathcal{A}}} + x_{\bar{\mathcal{A}}} G_{\bar{\mathcal{A}}\bar{\mathcal{A}}}. \]  

Referring to Fig. 1, Eq. (6) can be simplified as

\[ z_\mathcal{A} = x_\mathcal{A} G_\mathcal{A}. \]  

Fig. 1 shows systematic polar coding when \( N = 2^3 \), \( \mathcal{A} = \{4, 6, 7, 8\} \). In this case, the generator matrix of systematic polar codes is given by

\[ G_3 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 
\end{bmatrix}. \]  

Eqs. (8) and (9) show that \( x_\mathcal{A} \) and \( z_\mathcal{A} \) have the same values in systematic polar code. Systematic polar codes are decoded by the SC same as non-systematic ones. Since the decoded bits are at the stage where zeros are inserted into the same indices as the frozen bits in Fig. 1, it is encoded again to obtain the information data.

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2.2 Puncturing

As an illustration of puncturing, we assume \((S, T)\) polar codes of rate \(R = T/S\), where \(S\) and \(T\) denote the codeword and the data length, respectively. If the desired codeword length is \(U \ (U < S)\), \(|\mathcal{P}|\) bits are thinned out. \(|\mathcal{P}|\) and \(\mathcal{P}\) are the number of punctured bits and the puncturing pattern, respectively. \(|\mathcal{P}| = S - U\) is satisfied and there exists some puncturing strategies to determine \(\mathcal{P}\) such as [2], [3] and [4].

In this case, the coding rate \(R\) is improved from \(T/S\) to \(T/(S - |\mathcal{P}|) = T/U\). Therefore, puncturing improves transmission efficiency in exchange for error correction capability. The number of punctured bits and puncturing pattern can be flexibly determined according to the desired codeword length as well as the desired coding rate requirements. At the receiving side, the received signal is decoded by setting log-likelihood ratios (LLRs) of the corresponding punctured bits to zero.

3 Proposed Scheme

Conventional puncturing methods have a problem that decoding performance is deteriorated due to insertion of the incorrect value to the punctured part. To solve this problem, this paper proposes a known bits puncturing for systematic polar codes.

In systematic polar codes, information bits appear as they are in the codeword. It is possible to embed known bits fixedly in codeword by exploiting this property. Suppose the position and pattern of fixed bits are pre-shared between transmitter and receiver, LLRs of the punctured bits can be set to infinity at the receiving side.

For example, Fig. 2 shows a schematic of the proposed system when block length \(N=2^3\), frozen bits \(X_F = \{1, 2, 3, 5\}\) and information bits \(X = \{4, 6, 7, 8\}\). In this paper, puncturing positions are determined in descending order of the channel capacity. Therefore, the shared known bits \(X_P\) are allocated to the frozen bits having the high channel rank, i.e. \(X_P = \{3, 5\}\), as shown in Fig. 2. \(Z = \{Z_1, Z_2, X_3, X_6, X_7, X_8\}\) is the transmission data, which becomes \(Y = \{Y_1, Y_2, Y_4, Y_6, Y_7, Y_8\}\) through the channel \(W\) and arrives at the receiver. We then insert the known bits \(X_3\) and \(X_5\) to the third and fifth positions to \(Y\), and hence \(Y\) is obtained as \(\{Y_1, Y_2, X_3, Y_4, X_5, Y_6, Y_7, Y_8\}\). The proposed scheme can be constructed by the following procedures.

1. Determine the code length and information data length.
2. Determine frozen and information bit positions by using ether density evolution [8] or the Gaussian approximation [9].
3. Calculate channel capacity of frozen bit positions

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4. Define $|X_P|$ channels having large channel capacity.
5. Treat these channels as information bits and encode systematically.
6. Transmit the remaining signal after puncturing these known bits.
7. The receiver set the LLRs of $X_P$ to infinity.
8. Perform the polar encoding again to obtain the original data.

4 Simulation Result

4.1 Parameters

Based on Orthogonal Frequency Division Multiplexing (OFDM) transmission with 20 MHz bandwidth, the numbers of subcarriers, data symbols and pilot symbols, $N_{sc}$, $N_{data}$, $N_{pilot}$ are 64, 16, and 2, respectively. With the non-punctured codeword, code length is 2048 and coding rate is 1/2. The number of punctured bits, $B_{punc}$, is set to 128, 256, and 384. Random bit interleaving is also applied to the codeword. The codeword is mapped to symbols by quadrature phase-shift keying (QPSK), i.e. Modulation order, $N_{mod}$, is 2. The modulated signals are serial-to-parallel transformed and then converted to time-domain signal via inverse fast Fourier transform (IFFT). The channel model follows 15 path Rayleigh fading with exponential decay, its interval is 50 ns and the maximum Doppler frequency is 10 Hz. The guard interval is 16 samples. This evaluation defines the conventional schemes as; a scheme without puncturing (Conv.1), a scheme with puncturing (Conv.2). Furthermore, schemes proposed in [2], [3] and [4] are referred to as Conv.3, Conv.4 and Conv.5, respectively.

4.2 Results

As shown in Fig. 3(a), it can be seen that the Prop. can improve the BER performance more than the Conv.2, and this gain is about 2 dB at BER $= 10^{-4}$. Comparing the Prop. with the Conv.1, the BER of the Prop. almost coincides to that of the Conv.1.

Observing Fig. 3(b), the proposed scheme improves the BER performance as the number of punctured bits increases compared to the conventional scheme. In particular, referring to the case where 384 bits are punctured, the proposed scheme...
suppresses the degradation of the BER performance. In Fig. 3(c), the BER performance of the proposed scheme is compared with those of Conv.3, Conv.4 and Conv.5. The BER performance of the proposed scheme is better than existing puncturing schemes, and this gain is about 1 dB at BER $= 10^{-5}$.

The improvement of achievable throughput, $\beta$, can be calculated as

$$\beta = \frac{N_{\text{data}} + N_{\text{pilot}}}{N_{\text{data}} + N_{\text{pilot}} - \frac{B_{\text{punc}}}{N_{\text{mod}}N_{\text{sc}}}}. \quad (10)$$

It becomes 5.9%, 12.5%, and 20% when $B_{\text{punc}}$ is set to 128, 256, and 384, respectively. The proposed scheme can improve the transmission efficiency while suppressing the BER performance degradation.

As a result, the above evaluations clarified that the proposed known bits puncturing is effective for systematic polar coded OFDM systems.

5 Conclusion

This article proposed the known bits puncturing by exploiting the property of systematic polar coding. It allows the decoder to assist the error correction capability thanks to fixed known bits while maintaining the advantage of improving the transmission efficiency. Computer simulation results verified its effectiveness that can enhance achievable throughput as well as the good BER performance better than existing puncturing schemes. We can conclude that the proposed scheme could be the most valuable approach to enhance systematic polar codes capability in a simple and practical manner.

(a) BER performance comparison with conventional scheme (vs. Conv.1 and Conv.2, $B_{\text{punc}} = 128$)

(b) BER performance with various numbers of punctured bits

(c) BER performance comparison with existing schemes (vs. Conv.3, Conv.4 and Conv.5, $B_{\text{punc}} = 128$)

Fig. 3. Simulation results.