Zecale: Reconciling Privacy and Scalability on Ethereum

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Abstract. In this paper, we present Zecale, a general purpose SNARK proof aggregator that uses recursive composition of SNARKs. We start by introducing the notion of recursive composition of SNARKs, before introducing Zecale as a privacy preserving scalability solution. Then, we list application types that can emerge and be built with Zecale. Finally, we argue that such scalability solutions for privacy preserving state transitions are paramount to emulate “cash” on blockchain systems.

Keywords: scalability, privacy, digital cash, crypto-economics, zero-knowledge proofs, Ethereum

1 Introduction

As blockchain systems gained in popularity, several challenges have arisen revealing some of the current limitations of the technology. While privacy is a known issue on blockchains such as Bitcoin [Nak09] and Ethereum [But14], scalability is another important concern. In fact, by their very nature of “append only ledgers”, the more users transact on a blockchain, the more data is added to the state over which validators reach consensus. This increase in blockchain data, if not controlled, can lead to fewer validator nodes (and full nodes) yielding more and more centralization in the system. In addition to that, sudden increase in transaction volumes can dramatically inflate the transaction cost, as witnessed in late 2017 after the raise in popularity of the CryptoKitties game1 that congested the Ethereum network2 and triggered a sharp increase in gas price.

Even today, the important amount of transactions on public blockchains pushes the fees up, making networks like Ethereum less attractive to certain users3.

Over the past years, several solutions have been developed in order to improve blockchain scalability. Some approaches rely on a technique called “sharding” which consists in splitting the entire state of the network into partitions denoted

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1 https://www.cryptokitties.co/

2 https://www.coindesk.com/cat-fight-ethereum-users-clash-cryptokitties-congestion

3 See these tweets for instance: https://twitter.com/intocryptoast/status/1263372756625702913?s=21 https://twitter.com/sassal0x/status/1264555874992848898?s=21 or https://twitter.com/fennie_wang/status/1266083935093583877?s=21 for instance.
“shards”. Doing so allows to move away from the “all miners verify all transactions” approach by having several network partitions validating transactions concurrently \(^4\). Other projects such as Coda \(^5\) introduce the notion of “succinct blockchain”, which is built through the use of recursive composition of SNARKs (see Section 3.2). In Coda the entire transaction history is replaced by an argument\(^4\) of computational integrity certifying that the state of the blockchain is valid. As such, any node willing to join the network only needs to verify one (small) argument (a SNARK) instead of going through and verifying the entire transaction history.

Similar techniques have been investigated to develop “layer 2” scaling solutions on Ethereum, and constructions such as “ZK-Rollups” \(^5\) have gained tremendous interest\(^5\).

Finally, as witnessed in the Bitcoin community, relay networks have been developed as a way to improve blockchains scalability through better block propagation \(^5\). In fact, in \(^5\) the authors explain that the use of a relay network along blockchain systems like Bitcoin can help improving the throughput of the system by shortening the block interval — by speeding up the block propagation while avoiding an increase in the block orphan rate. We note however, that, while improving the transaction processing time, such solutions do not allow to reduce the size of the blockchain.

Our contribution. In this paper we present Zecale — a general purpose aggregator using recursive composition of SNARKs that allows to improve the scalability of SNARK-based applications on Ethereum via aggregation of transactions off-chain. We show how the privacy solution ZETH \(^5\) is complemented by Zecale, and how both solutions can be used to implement digital cash systems.

To do so, we will consider the use of a relay network as a way to gain sender anonymity when coupled with privacy-preserving protocols such as ZETH. Additionally, we will show how scaling solutions like Zecale can be deployed on nodes constituting the relay network in order to off-load state transaction verification work — normally carried out by miners — to relays, and thus enabling to keep the transaction history and the blockchain data condensed.

We note that the focus of this study is articulated around the Ethereum \(^5\) blockchain. Nevertheless, all results presented here are directly applicable to blockchains supporting smart-contracts deployment and equipped with a Turing-complete execution environment.

\(^4\) i.e. a computationally sound proof

\(^5\) While the name “zk-rollup” is wide-spread in the community, we find this name a bit confusing. In fact, zero-knowledge is not strictly required (and generally not used at all in “zk-rollup” projects). A better definition of “rollups” may be “proofs of computational integrity of the verification of a set of transactions”, but this long description diverges from the agreed upon and wide-spread terminology. As such, we will stick to the agreed upon vocabulary and use the “zk-rollup” term in this paper. We gently warn the reader than this term can be misleading.
2 Background and Notations

Let PPT denote probabilistic polynomial time. Let $\lambda \in \mathbb{N}$ be the security parameter. We assume that all algorithms described receive as an implicit parameter the security parameter written in unary, $1^\lambda$. All algorithms are modeled as Turing machines. An efficient algorithm is a probabilistic Turing machine running in polynomial time. All adversaries are modeled as efficient algorithms. An adversary and all parties they control are denoted by letter $A$. We write $\text{negl}$ (resp. $\text{poly}$) to denote a negligible (resp. polynomial) function. Following [Sho04, BR06], we structure security proofs as sequences of games. As such, we say that the adversary wins game $\text{GAME}$ defined on $X$ if they make $\text{GAME}$ return 1. We say that $A$ wins $\text{GAME}$ with negligible advantage if $\text{Adv}_{A,X}(\lambda) \leq \text{negl}(\lambda)$.

Finally, we denote by $[]: \mathbb{S}^n \times [n] \rightarrow \mathbb{S}$ the infix operator that takes a tuple and an index as inputs and returns the element at the given index in the tuple. We use the notation $x[i]$ as syntactic sugar for $x[]i$ (e.g. $x \leftarrow (A, 7, U)$, $x[2] = U$).

Bilinear groups. Let $\mathcal{G}$ be a bilinear group generator taking $1^\lambda$ as input and returning a tuple $(r, \mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_T, e)$, where $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_T$ are cyclic groups of prime order $r$, and $e$ is a map $e: \mathcal{G}_1 \times \mathcal{G}_2 \rightarrow \mathcal{G}_T$, such that $e$ is non-degenerate and bilinear. We further assume that arithmetic in the groups and computing $e$ is efficient, that $\mathcal{G}_1 \neq \mathcal{G}_2$, and that there does not exist efficiently computable homomorphisms between the two source groups (pairing of type III [GPS08]).

We denote by $g_1, g_2, g_T$ the generators of the groups $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_T$ respectively. We denote by $+$ the group operation in the two source groups $\mathcal{G}_1, \mathcal{G}_2$. Additionally, we define the infix operator $\cdot: \mathbb{F}_r \times \mathcal{G}_{1,2} \rightarrow \mathcal{G}_{1,2}$ that represents the successive application of the group operation - e.g. $k \cdot g_1 = g_1 + \cdots + g_1$ ($k$ times). The group operation for the target group $\mathcal{G}_T$ is denoted by $\ast$, and the “exponentiation” operator $\hat{}: \mathcal{G}_T \times \mathbb{F}_r$ denotes its successive application - e.g. $g_T^k = g_T \ast \cdots \ast g_T$ ($k$ times).

3 Preliminaries

3.1 Brief overview of IP, NIZK and SNARK

In contrast with standard mathematical proofs, Goldwasser, Micali and Rackoff introduced the notion of zero-knowledge proofs. These proofs enable a prover $P$ to prove a theorem to a verifier $V$ without revealing anything other than the fact that the theorem is correct [GMR85].
In their seminal work, the authors focused on interactive protocols, where both P and V are modeled as interactive Turing machines communicating by sharing their reading and writing tapes. In brief, an interactive proof is a procedure by which a prover wants to prove a theorem to a verifier. To that end, the verifier is allowed to flip coins, and use these coin flips to ask questions (i.e. “send challenges”) to the prover. The prover answers the verifier’s questions, and after several interactions, the verifier either accepts or rejects the theorem.

Informally, the complexity class admitting interactive proofs (IP) is defined as the class of languages L with the following properties:

Completeness:

$$\exists P \text{ s.t. } \Pr[V.Ff(x, \pi) = \text{true} \mid \pi \leftarrow \text{Transcript}(P, V)] \geq \frac{2}{3} \quad x \in L$$

Soundness:

$$\forall P, \Pr[V.Ff(x, \pi) = \text{true} \mid \pi \leftarrow \text{Transcript}(P, V)] \leq \frac{1}{3} \quad x \notin L$$

where Transcript(P, V) is the set of messages exchanged between the prover and the verifier, and where Vf is the verification algorithm ran by the verifier to either accept or reject x. Note that interaction is a very efficient tool for soundness amplification as multiple executions of an interactive protocol can be used to bring the soundness error down.

Informally, we say that the protocol is zero-knowledge if the verifier only learns the validity of the theorem being proven and nothing else.

Importantly, the notion of IP as per [GMR85] does not make any computational assumption on the prover (modeled as “all powerful”), while the verifier is assumed to have limited resources (i.e. be PPT). Likewise, the computational model assumes that both P and V have a random tape, but that the verifier’s coin flips are private (i.e. private coin).

This restriction on coin tosses contrasts with the notion of Arthur-Merlin (AM) protocols introduced by Babai [Bab85, BM88], in which the verifier’s coin tosses are public, and thus accessible to the prover. In fact, in this model all the verifier’s messages can be replaced by the output of a random beacon [Rab83] for instance. Shortly after, Goldwasser and Sipser [GS89] showed however, that “private coin tossing” is no stronger than “public coin tossing” and their work led to the well acclaimed IP = AM result.

Interestingly, Shamir showed that IP = PSPACE [Sha92], demonstrating the great power of randomness and communication. While IP is demonstrated to be a wide complexity class, Goldreich and Oren showed [Ore87, GO94] that re-

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6 k repetitions allow to bound the soundness error by $\frac{1}{3^k}$

7 a simplified proof was published by Shen [She92]

8 we know that NP ⊆ PSPACE
moving interactions between prover and verifier while preserving zero-knowledge yields a collapse to BPP in the plain model.\(^9\)

As a way to compensate the removal of interactions, Blum, Feldman and Micali [BFM88], and then Blum, De Santis, Micali and Persiano [BSMP91] studied the notion of non-interactive zero-knowledge proofs, in which prover/verifier communications and randomness are substituted by a shared common random string. This setting, generalized to the common reference string (CRS) model, and declined in various flavors where, for example, the reference string is assumed to have a specific structure,\(^{10}\) has been used to design a wide range of non-interactive zero-knowledge proof systems (NIZK).

We further note that, moving to the non-interactive setting allows to obtain publicly verifiable proofs, which is of great interest for various cryptographic applications (e.g. blockchain protocols).

**SNARK.** We now focus on zk-SNARKs, a special type of NIZK. Informally, a (zero-knowledge) Succinct Non-interactive Argument of Knowledge (SNARK), is a small proof of knowledge, which is non-interactive and sound against computationally bounded adversaries. Not surprisingly, the appealing performances of NIZKs like SNARKs take their source in the shift from an “all powerful prover” (like in IP) to a “computationally restricted” prover; allowing protocol designers to rely on cryptographic assumptions to design efficient protocols.

The first known SNARK construction, proposed by Micali [Mic94] coined Computationally Sound (CS) Proofs is based on Kilian’s construction [Kil92] and proven secure in the Random Oracle model. Following, various SNARK constructions have been established over the past decades (e.g. [GGPR13,Gro10,Lip12]). While different SNARK constructions offer different trade-offs, we will be focusing here on the pairing-based SNARK construction [Gro16] “compiled from” Linear Interactive Proofs (LIP) as proposed by Bitansky et al. [BCI+13].

Remark 1. Our focus on [Gro16] is justified by the fact that this scheme admits very small arguments (3 group elements), which, when used along a blockchain system, allows to minimize the size of the transactions. We note however that our result can be used with any other “NP-complete” SNARK schemes.

Below, we provide an informal definition of a SNARK, and invite the reader to consult e.g. [GMT17] for precise definitions.

*Informal definition of SNARKs.* Let \( R \subset \{0,1\}^* \times \{0,1\}^* \) be a polynomial-time decidable binary relation. We assume that \( \lambda \) is explicitly deductible from the description of \( R \). Let \( L = \{ x \mid \exists w \text{ s.t. } R(x,w) \} \) be the NP language defined by \( R \).

\(^9\) note that the word “collapse” here reflects our current understanding of the complexity class hierarchy. In fact, the relation between P and BPP and the relation between P and NP is not yet fully understood. As of today, it is believed that \( P = BPP \) [Gol11, IW97, CRT98] and that \( P \neq NP \).

\(^{10}\) in which case we talk about “Structured Reference String” (SRS)
Here, $x$ is the instance\(^{11}\) and $w$ is the witness. Roughly speaking, $\Psi$ is a publicly verifiable zero-knowledge Succinct Non-interactive Argument of Knowledge (zk-SNARK) if $\Psi$ comports four PPT algorithms $KGen, P, V, Sim$ such that:

**CRS generator:** $KGen$ is a PPT algorithm that takes an NP-relation $R$ as input, runs a one time setup routine, and outputs a common reference string (CRS) $crs$ for this relation along with a trapdoor $td$.

**Prover:** $P$ is a PPT algorithm that given $(crs, x, w)$, such that $(x, w) \in R$, outputs an argument $\pi$.

**Verifier:** $V$ is a PPT algorithm that on input $(crs, x, \pi)$ returns either 0 (reject) or 1 (accept).

**Simulator:** $Sim$ is a PPT algorithm that on input $(crs, td, x)$ outputs an argument $\pi$.

We require a proof system $\Psi$ to have the following four properties:

**Completeness:** $\Psi$ is complete if an honest verifier accepts a proof made by an honest prover. That is, the verifier accepts a proof made for $(x, w) \in R$.

**Knowledge soundness:** $\Psi$ is knowledge-sound if from an acceptable proof $\pi$ for instance $x$ it is feasible for a specialized algorithm called extractor to extract a witness $w$ such that $(x, w) \in R$.\(^{12}\)

**Zero knowledge:** $\Psi$ is zero-knowledge if for any $x \in L$ no adversary can distinguish a proof made by an honest prover on input $(crs, x, w)$ from a proof made by the simulator on input $(crs, x, td)$ but no witness $w$.

**Succinctness:** $\Psi$ is succinct if the proof $\pi$ is sub-linear in the size of the instance and witness.

We note that despite knowledge soundness being stronger than “plain” soundness, it is sometimes still too weak of a notion to satisfy the requirements necessary to deploy a scheme in real world systems. In fact, knowledge-soundness does not protect against Man-in-The-Middle (MiTM) attacks where an adversary can forge an acceptable SNARK after seeing a set of acceptable arguments. In other words, knowledge soundness does not ensure that the SNARK is non-malleable. Recent zk-SNARKs such as [GM17, BG18] comply with a strong notion of soundness — simulation extractability (or simulation knowledge soundness) [Sah99, SCO⁺01] — which prevents MiTM attacks, and which is desirable in many real life applications. SNARKs satisfying the property below are referred to as SE-SNARKs:

**Simulation extractability:** $\Psi$ is simulation-extractable if from any proof $\pi$ for instance $x$ output by an adversary with access to an oracle producing simulated proof on given inputs, it is possible for an extractor to extract a witness $w$, such that $(x, w) \in R$.

\(^{11}\) also referred to as “public/primary input”

\(^{12}\) note that “knowledge soundness” $\Rightarrow$ “soundness”.
3.2 Recursive composition of SNARKs

As we know that SNARKs like [Gro16] can be used to generate arguments for any NP statements (i.e. $x \in L$, where $L$ is an NP language), and since we know that the verification algorithm ran by the verifier $V$ is itself in NP, one can wonder if it is possible to generate an argument certifying that another argument has correctly been verified. In other words, is it possible to generate a proof that another proof has correctly been checked?\(^{13}\)

This question was first studied by Valiant [Val08] who proposed composable proofs of knowledge as a way to achieve Incrementally Verifiable Computation (IVC). Further, Bitansky, Canetti, Chiesa and Tromer introduced the notion of Proof Carrying Data [CT10], along with a “boostrapping” technique [BCCT13] to obtain complexity-preserving SNARKs allowing to recursively compose proofs. The first practical instantiation of recursive (pairing-based) SNARK composition was achieved by Ben-Sasson et al. [BCTV17] using cycles of MNT elliptic curves [MNT01], as further studied by Chiesa et al. [CCW19]. Informally, using special tuples of elliptic curves such as cycles (see [SS11,CCW19] for formal definitions) allows to remove overhead due to finite field characteristic mismatch, and allows to achieve “infinite recursive SNARK composition”\(^{14}\).

Recent work such as [BCG+18] showed that bounded recursion was sufficient to build meaningful applications. In their construction, the authors introduced a pairing-friendly amicable chain of elliptic curve instantiated as (BLS12-377, CP), where CP is obtained via the Cocks-Pinch method [CP01] (see [FST10, Section 4.1]). A more efficient instantiation of the chain was later proposed by El Housni and Guillevic [HG20].

3.3 Ethereum

In the following, we assume familiarity with blockchain systems, and more precisely Ethereum. We refer the reader to [But14], or [Woo] for an introduction.

4 Zecale

4.1 Motivations

Despite their broad interest, blockchain systems are often criticized for their inability to “scale”. Unfortunately, multi-dimensional notions like scalability are often simplified to simple metrics such as Transactions Per Second (TPS). Such simplifications turn the “blockchain scalability problem” into a TPS maximization problem that fails to capture the true nature of the issue.

\(^{13}\) In the following we will use “proof” and “argument” interchangeably to denote computationally sound proofs.

\(^{14}\) It is possible to generate a proof $A$ that another proof $B$ was correctly verified, where $B$ is itself a proof that another proof $C$ was correctly verified, where $C$ is itself a proof that another proof $D$ was verified etc.
What does scalability mean in the context of blockchain systems? Answering this question is fundamental to frame the problem that needs to be solved. Well established payment systems can support thousands of transactions per second, and are systematically used as comparison point to assess the performances of blockchains. Importantly, however, the value proposition of blockchain systems lies in their decentralized nature. As such, keeping the network as decentralized as possible by preserving the network’s performances under addition of new nodes in the distributed system is paramount.

Hence, a scalable blockchain system is one that can support a large number of users (high throughput/TPS), as well as a large number of untrusted validator nodes (highly decentralized). This raises the following paradox:

Intuitively, maximizing TPS necessitates to produce more transactions (data) that are processed by the system. However, maximizing the number of validating nodes on the system requires to keep the hardware requirements – to run a node – (bandwidth, processing, storage) as low as possible, and thus requires to keep the amount of data, produced and processed, as small as possible.

In the following, we refer to a “scalable blockchain” as one that implements the right trade-offs allowing to maximize the combination of all scalability parameters.

The cost of privacy. Another long standing issue with blockchains is the lack of privacy. In fact, because of their very nature of “append only” distributed ledgers, all transactions need to be validated by all miners. As commonly assumed, the transaction data needs to be visible to carry out the verification, which roughly means that sender, recipient and amount of a transaction need to be “in the clear” to keep the system sound. Protocols such as Zerocash/Z-cash \cite{BCG+14,HBHW16}, however, contrast with approaches relying on transparency for security, and rather propose to rely on zk-SNARKs as a way to prove that transactions “follow the rules of the system” without disclosing their attributes.

Recently, Rondelet and Zajac \cite{RZ19} leveraged the (quasi\footnote{due to the need to pay gas for each operations on the Ethereum state, and due to the block gas limit, it is clear that one can easily come up with a state transition that cannot be executed on Ethereum (either because the cost of carrying out the state transition is bigger than the block gas limit, or because the cost of the state transition is bigger than the current supply of Ether)\textsuperscript{16}} Turing-complete smart-contract platform of Ethereum as a way to encode a privacy preserving state transition similar to Zerocash. However, and not surprisingly, privacy preserving state transitions like ZETH \cite{RZ19} are significantly more expensive (gas-wise) to carry out than plain Ethereum transactions\footnote{this is not surprising. Indistinguishability has a price.}. This is partially due to high storage requirements on the smart-contract, and the necessity to carry out multiple cryptographic checks as part of the state transition. Like most SNARK-based applications, one check carried out during the state transition
is the verification of the SNARK proof \( \pi \) for the base application statement (e.g. ZETH).

The common framework for SNARK-based layer 2 applications (i.e. smart-contract) is to:

1. Verify the SNARK proof
2. If the SNARK proof verification is successful, then, execute the state transition specified by the application logic (e.g. carry out changes in the blockchain state etc.)

Implementing complex smart-contract logic may require to pass a lot of data as input to the smart contract. This means, broadcasting big transactions and blocks on the peer-to-peer network and processing big pieces of data as part of the block mining process. This exacerbates the “scalability paradox” above-mentioned.

**Scalable privacy.** **Zecale** aims to lessen the impact of privacy preserving state transitions like ZETH on the overall system, by minimizing the amount of data sent and processed on-chain. To do this, **Zecale** off-loads the verification of all zk-SNARK proofs of an application to a piece of software called the *aggregator*. This software component uses recursive proof composition in order to generate a proof of computational integrity certifying that all zk-SNARK proofs have correctly been verified off-chain. This proof is then sent on-chain along with the instances associated with the aggregated proofs. This *unique* proof is then verified on-chain, and each primary inputs are processed according to the associated verification bit. This technique allows to aggregate \( N \) SNARK proofs into a single one, and allows to send a *unique* transaction on-chain, without altering the soundness of the system – *the system remains sound as long as the SNARK-scheme is secure* (see Section 4.6 for more details). This idea is summarized Fig. 1.

### 4.2 Technique

As above-mentioned, **Zecale** leverages recursive proof composition in order to delegate the individual SNARK proof verification to an off-chain software component – the *aggregator* (see Fig. 2). The proof generated by the aggregator is then sent on-chain, along with the primary inputs of all verified proofs in order to execute the state transitions on the state machine (e.g. Ethereum). Since the proof generated by the aggregator is verified on-chain by all miners, only “one level of recursion” is needed in **Zecale**. This is similar to BCG+18 in that regard.

As a consequence, we propose to use a *pairing-friendly amicable chain* as a way to implement efficient bounded recursion and bypass several open problems (see CCW19) and inefficiencies related to the use of low embedding degree cycles of elliptic curves17.

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17 if the embedding degrees of the curves are small, then dlog and pairing-based hardness assumptions need to be enforced via the use of prime fields with larger char-
**Fig. 1**: Left: transaction execution routine without Zecale. A set of transactions are processed, all transactions contain a zk-SNARK proof that needs to be verified. Right: transaction execution routine when Zecale is used. Only one transaction – and SNARK proof – is processed. \( N \) represents the maximum number of zktx that can be included in a block, where zktx is a transaction containing a SNARK.

The use of the chain of elliptic curves allows us to generate a set of “nested” zk-SNARKs over the first curve, and to generate a “wrapping” SNARK\(^{18}\) – proving correct verification of the set of nested zk-SNARKs – over the second curve. As such, it is required that all SNARK-based applications aiming to be used along with Zecale generate zk-SNARK proofs over the first curve of the pairing-friendly amicable chain used. The authors of ZEXE \( [BCG + 18] \) initially proposed a chain instantiated by a curve from the BLS12 family (BLS12-377) and a curve generated using the Cocks-Pinch method. A more efficient instantiation made of BLS12-377 and a Brezing-Weng \( [BW05] \) curve – denoted BW6-761 – was recently proposed by El Housni and Guillevic \( [HG20] \). Any such 2-chain (resp. cycle) of pairing-friendly elliptic curves can be used in Zecale. We propose characteristics which renders group and field operations less efficient since more “limbs” are needed to represent the algebraic structures’ elements.

\(^{18}\) note that this proof is not zero-knowledge
to use (BLS12-377, BW6-761) as it reflects the state of the art at the time of writing.

4.3 Zecale language

In order to provably verify each individual “nested” zk-SNARK proofs, it is necessary to encode the SNARK verification algorithm of the base application (i.e. $\Psi_{\text{app}}.V$) in the correct form: one allowing to efficiently generate a proof of computational integrity of the execution of the algorithm. This way to encode machine computation so that its correctness can easily be verified via few probabilistic algebraic checks is called arithmetization. SNARKs such as [Gro16] use a type of arithmetization called Quadratic Arithmetic Programs (QAPs) introduced in [GGPR13]. While describing the full set of algebraic constraints encoding the SNARK verification algorithm as an arithmetic circuit – defined over a finite field – is outside of the scope of this paper, we provide below a high-level description of the $\text{NP}$-relation ($R^{\text{zec}}$) characterizing the language of Zecale denoted $L^{\text{zec}}$. This is summarized Fig. 3.

Let $\text{BATCH\_SIZE}$ be a fixed protocol parameter representing the size of the batch of nested proofs.

\[
\text{ZecaleRelation}(x_{\text{zec}}, (\text{crs}_{\text{app}}, \{(\pi_{\text{app}i}, x_{\text{app}i})\}_{i \in [\text{BATCH\_SIZE}]}))
\]

\[
\text{for } i \in [\text{BATCH\_SIZE}] \text{ do}
\]
\[
b_i \leftarrow \Psi_{\text{app}}.V(\text{crs}_{\text{app}}, \pi_{\text{app}i}, x_{\text{app}i})
\]

\[
\text{endfor}
\]
\[
x_{\text{zec}} = \sum_{i \in [\text{BATCH\_SIZE}]} b_i \cdot 2^i
\]

where $\Psi_{\text{app}}.V$ is the SNARK verification algorithm associated with the SNARK scheme used in the base application. For [Gro16], this algorithm consists in carrying out the check:

\[
e(\pi.A, \pi.B) = e(\upsilon.\alpha, \upsilon.\beta) \ast e\left(\sum_{i=0}^{|x|} x_i \upsilon_{\text{vk}i}.u(x) + \upsilon.\alpha \cdot x_i(x) + w_i(x), \upsilon.\gamma\right) \ast e(\pi.C, \upsilon.\delta),
\]

where $\upsilon$ is the part of the $\text{crs}$ used by the verifier.

Fig. 3: Pseudo-code representation of the set of constraints that all pairs of inputs $(x, w)$ need to satisfy to belong to $R^{\text{zec}}$.

The attentive reader may realize that the verification check of Groth16 as described in Fig. 3 is linear with the number of public inputs. As multiple SNARK-based applications may have different number of primary inputs, one needs to overcome some challenges in order to enable the Zecale relation to be generic enough to be used along with a wide class of SNARK-based applications. In fact, it is important to remember that Groth16 can be used to prove any statement in $\text{NP}$ (due to the $\text{NP}$-completeness of QAPs), but a new setup phase needs to be ran for each new language, posing some practical challenges.
Fortunately, the authors of [GGPR13] proposed a “trick” to break the linear complexity of the SNARK verification check by simply applying an ordinary (i.e. not extractable) collision-resistant hash function to the statement. We follow the same approach and propose to extend the NP-relations of all base applications — that aim to be used with Zecale — with this additional hashing step to reduce the number of their primary inputs to the same constant.

To simplify notations in the following sections, we define, for each prime $p$ and hash function $H$, the following constant $\text{InpNb} = \left\lceil \frac{\text{lh}}{\left\lfloor \log_2(p) \right\rfloor + 1} \right\rceil$, where $\text{lh}$ is the digest length of $H$. Moreover, we introduce the two following functions:

- $\text{toField}_{H,p}: \{0,1\}^{\text{lh}} \to (\mathbb{F}_p)^{\text{InpNb}}$ which takes a hash digest and returns its “prime field representation” (the function is injective, i.e. if $\text{lh} > \left\lfloor \log_2(p) \right\rfloor + 1$, $\text{toField}_{H,p}$ returns the tuple of elements in $\mathbb{F}_p$ necessary to uniquely represent the digest “in the field”), and
- $\text{toDigest}_{H,p}: (\mathbb{F}_p)^{\text{InpNb}} \to \{0,1\}^{\text{lh}}$ defined such that for all digests $h$ of $H$, we have $\text{toDigest}_{H,p}(\text{toField}_{H,p}(h)) = h$. Finally, $\mathbb{F}_r$ (resp. $\mathbb{F}_{rw}$) is the scalar field of the first (resp. second) curve of the 2-chain we use.

Using these additional notations, we represent the “generic” Zecale relation Fig. 4.

\begin{table}[h]
\centering
\begin{tabular}{l}
\textbf{ZecaleRelation}(x_{zec}, (\text{crs}_{app}, \{ (\pi_{app,i}, x_{app,i}) \}_{i \in \text{[BATCH_SIZE]}})) \\
\textbf{for} $i \in \text{[BATCH_SIZE]}$ \textbf{do} \\
\hspace{1em} // As assumed, the relation of the base application uses \\\n\hspace{1em} // $H$ to do the “hashing trick” described in [GGPR13]. \\\n\hspace{1em} // Here $x_{app,i}$ represents the “non-hashed” instance that \\\n\hspace{1em} // we now hash to verify the nested SNARK. \\\n\hspace{1em} x_{zec}.xH_i = \text{toField}_{H,r_n}(H(x_{app,i})) \\\n\hspace{1em} b_i \leftarrow \Psi_{app}.V(\text{crs}_{app}, \pi_{app,i}, x_{zec}.xH_i) \\\n\textbf{endfor} \\
\hspace{1em} x_{zec}.xValid = \sum_{i \in \text{[BATCH_SIZE]}} b_i \cdot 2^i \\\n\hspace{1em} x_{zec}.vkHash = \text{toField}_{H,r_w}(H(\text{crs}_{app}))
\end{tabular}
\caption{Pseudo-code representation of the generic Zecale relation}
\end{table}

In the rest of the document, we use $R^{zec}$ to refer to the NP-relation described Fig. 4

19 Converting a bit string to a field element can be done by taking the sum of all $i$th bits of the string multiplied by $2^i$, e.g. $(110101)_2$ is represented by $(\sum_{i \in \text{[1101]}} \{110\}[i] \cdot 2^i, \sum_{i \in \text{[101]}} \{101\}[i] \cdot 2^i) = (6, 5) \in (\mathbb{F}_7)^2$
A note on zero-knowledge. Since the Zecale aggregator generates a “wrapping” proof to certify the correct verification of a set of “nested” proofs, it is not necessary for the “wrapping” proof to be zero-knowledge. In fact, if “nested” SNARKs already are zero-knowledge, there is no additional security gain to also enforce zero-knowledge for the “wrapping” proof. As such, the randomization steps carried out by the prover $P$, during the proof generation, to protect against malicious verifiers, and ensure zero-knowledge, can be omitted. This allows to fasten the aggregator algorithm by removing unnecessary operations.

A note on batch verification. A well known method to fasten the verification of a set of SNARKs is to use batching techniques. While it may be tempting to modify the NP-relation $R^{zec}$ to generate a proof of correct batch verification of a set of zk-proofs, this approach presents a set of practical limitations. We investigate the case of batch verification in more details in Appendix A and Appendix B.

4.4 Smart-contract logic

In addition to generate SNARK proofs for statements of the form $x \in L^{zec}$, Zecale requires some smart-contract logic to function. In fact, after aggregating all the base application’s zk-SNARKs, it is necessary to verify the “wrapping” proof on-chain, and execute the base application logic for all the primary inputs associated with valid “nested” proofs.

The smart-contract specifying the set of Zecale state transitions uses a dispatch mechanism that forwards the set of instances – associated to valid proofs – to the base application contract that will then process them by executing the base application logic on the state machine.

We provide Fig. 6 a pseudo-code description of the processAggrTx function representing the logic of the smart-contract Zecale$C$ specifying Zecale state transitions. Furthermore, we represent Fig. 5 the difference between a stand-alone base-application and one used along with Zecale.
Fig. 5: Left: Pseudo-code of the state transition of a stand-alone SNARK-based application. Right: Pseudo-code of the state transition of a SNARK-based application implementing the “dispatch mechanism” to be used with Zecale.
ZecaleC.processAggrTx(\(\pi_{\text{zec}}, x_{\text{zec}}, \{x_{\text{app}, i}\}_{i \in [\text{BATCH\_SIZE]}}, \text{ZbaseAppAddr}\))

1: // 1. Check the inputs before carrying out any expensive computation
2: if \(x_{\text{zec}}.x\text{Valid} > \sum_{i \in [\text{BATCH\_SIZE}]} 2^i\) then
3: abort
4: endif
5: // Make sure that the application inputs have not been maliciously tampered with by the Zecale aggregator
6: for \(i \in [\text{BATCH\_SIZE}]\) do
7: if \(-[x_{\text{zec}}.x\text{H}_i = \text{toField}_{H, r_n}(H(x_{\text{app}_i}))]\) then
8: abort
9: endif
10: endfor
11: // 2. Check the aggregation/wrapping SNARK
12: if \(-\Psi_{\text{zec}}.V(\text{crs}_{\text{zec}}, \pi_{\text{zec}}, x_{\text{zec}}) = \text{true}\) then:
13: abort
14: endif
15: // 3. Execute the base application state transitions
16: // 3.1 If none of the nested proofs are correct, there is nothing to dispatch
17: if \(x_{\text{zec}}.x\text{Valid} = 0\)
18: abort
19: endif
20: // 3.2 Otherwise, dispatch the instances associated with the valid nested proofs
21: dispatchData \(\leftarrow \{0\}^{\text{BATCH\_SIZE}}\)
22: for \(i \in [\text{BATCH\_SIZE}]\) do
23: if \(x_{\text{zec}}.x\text{Valid} \& 0x1\) then
24: dispatchData[i] \(\leftarrow x_{\text{app}_i}\)
25: endif
26: endfor
27: // Right shift by 1 position
28: shr(x_{\text{zec}}.x\text{Valid}, 1)
29: endfor
30: \(\text{ZbaseAppC} \leftarrow \text{createContractInstance}(\text{ZbaseAppAddr})\)
31: byteData \(\leftarrow \text{encodeToBytes}(\text{dispatchData})\)
32: \(\text{ZbaseAppC}.\text{dispatch}(x_{\text{zec}}.vkHash, \text{byteData})\)
33: return 1

Fig. 6: Pseudo-code specifying the Zecale state transition

In brief, the Zecale state transition can be decomposed into 3 steps:
Check the inputs: As nested proofs are verified off-chain, it is important to make sure that the instances of the nested proofs, forwarded on-chain by the aggregator to execute the base application logic, are the same as the one used during the off-chain “nested” proof verification. In other words, it is key to make sure that the base application state transition processes the “right instances” (instances that haven’t been tampered with by the aggregator). Likewise, we check that $x_{\text{zec}}.x_{\text{Valid}} \in \left[\sum_{i \in [\text{BATCH SIZE}]} 2^i\right]$ in order to abort the state transition as soon as possible if not the case.

Verify the aggregation SNARK proof: Once the inputs are checked to be of the right form, the wrapping proof is verified. If the proof verifies correctly, then network participants have overwhelming confidence that the set of nested proofs have correctly been verified off-chain.

Forward the application data to the base application: The base application verifies that:

- The calling contract is the genuine Zecale contract — to be sure that all necessary checks have successfully been carried out.
- The received data is made of instances for this application.

Finally, the base application logic is executed. Note however, that we have explicitly represented the check that consists in verifying that all instances $(x_{\text{app}})$ associated to valid nested SNARKs lie in the right finite field. In fact, failure to do such a check can expose the underlying application to modular arithmetic-based attacks like [Sem19].

Example 1. Let $\text{BATCH SIZE} = 3$, $x_{\text{zec}}.x_{\text{Valid}} = 5$, and let $(x_{\text{app}0}, x_{\text{app}1}, x_{\text{app}2})$ be the application instances. Since the size of the proofs batch is 3, at most 3 zk-SNARKs are valid. As such $x_{\text{zec}}.x_{\text{Valid}}$ is bounded by $(111)_2 = 7$. We check that $x_{\text{zec}}.x_{\text{Valid}} < 7$, which is satisfied here. The binary representation of $x_{\text{zec}}.x_{\text{Valid}}$ is $(101)_2$. As such, the nested zk-SNARKs at indices 0 and 2 in the batch verified correctly, while the proof at index 1 was deemed incorrect by the verification algorithm. This means that, out of the tuple of instances $(x_{\text{app}0}, x_{\text{app}1}, x_{\text{app}2})$, only $\text{dispatchData} \leftarrow (x_{\text{app}0}, x_{\text{app}2})$ will be forwarded to the base application contract to be processed (e.g. added to the Merkle tree of commitments etc. if the base application is ZETH).

4.5 Instantiation of the SNARK scheme

Despite the fact that Groth16 is not universal [GKM+18] and only weakly simulation extractable [KV20], it remains of broad interest in applied settings because of its nearly optimal argument size and efficiency.

As our main focus is to minimize the size of data exchanged and processed by miners while keeping the cost of the Zecale state transition as small as possible,
we believe that Groth16 is a good candidate to instantiate the SNARK scheme used in Zecale.

It is important to note, however, that recent proof systems such as, e.g. [GWC19, CHM+20] could also be used to instantiate the SNARK scheme used in Zecale. Likewise, there are no requirements to use the same proof system in Zecale and in the base application. For instance, it is feasible for ZETH to use Groth16, while generating Zecale wrapping proofs can be done using another proof system, such as [GM17] for instance.

4.6 Security

In the following, we prove that Zecale is secure. Namely, we show that the protocol preserves the soundness of the underlying blockchain system.

To study the soundness of Zecale we want to show that the probability that an adversary $A$ can use Zecale in order to break the soundness of a ledger $L$ is negligible in $\lambda$. We do so by defining the soundness game $ZCL$-$SND$ below, and argue that the probability of winning this game (also referred to as $Adv_{A,L}^{ZCL}$-$SND$(\$\lambda\$)) is negligible.

We denote by APPS a set of applications deployed on $L$ during the execution of the Setup algorithm. In the following, an application app will be represented by a tuple $(\text{crs}_{\text{app}}, \text{baseAppAddr}, \text{ZbaseAppAddr})$. Moreover, we denote by $I = \Pi \diamond \chi$, $|I| = |\chi| = |\Pi| = \text{BATCH SIZE}$ the set of pairs of SNARKs proofs generated with $\Psi_{\text{app}}(\Pi)$, and associated set of primary inputs ($\chi$). We denote by $\diamond$ the operator that takes two ordered sets of same cardinality as input and builds a resulting set which $i$th element is the pair of the $i$th elements of the input sets. Furthermore, $S \subset \chi$.

\[
\text{ZCL-SND}(\lambda)
\]

$\text{(crs}_{\text{zecl}}, L, \text{APPS}) \leftarrow \text{Setup}(\lambda)$

$\text{state} \leftarrow A_{\text{G,C,ωapp,ωzecl}}(\text{crs}_{\text{zecl}}, \text{APPS})$

$capp \leftarrow s \text{APPS}$

$(\pi, x_{\text{zecl}}, I, S) \leftarrow A_{\text{O,C,ωapp,ωzecl}}(\text{crs}_{\text{zecl}}, \text{APPS}, \text{state}, capp)$

$b \leftarrow \text{ZecaleP(cr}\text{s}_{\text{zecl}}, \pi_{\text{zecl}}, x_{\text{zecl}}, \chi, S, capp.ZbaseAppAddr) \land \exists_{\exists}$

$\exists_{x_i} \in S, (\pi_i, x_i) \in I, \neg \text{BaseAppP(π}_i, x_i, \text{capp.baseAppAddr)}$

return $b$

In the game above, $A$ can do 2 types of oracle queries to the ledger $L$:

- processTx which takes $(\pi, x, \text{baseAppAddr})$, and which execute the function processTx of baseAppC at address baseAppAddr on input $(\pi, x)$.
- processAggrTx which takes $(\pi_{\text{zecl}}, x_{\text{zecl}}, \{x_i\}_{i \in \text{[BATCH SIZE]}}$, ZbaseAppAddr) as inputs and executes the function processAggrTx of the contract ZecaleC on the same inputs.
Likewise $\mathcal{A}$ is allowed to do oracle queries to generate “nested” (resp. “wrapping”) proofs — i.e. call $\Psi_{\text{app}}.P$ (resp. $\Psi_{\text{zec}}.P$) — and verify them — i.e. execute $\Psi_{\text{app}}.V$ (resp. $\Psi_{\text{zec}}.V$):

- genNestedProof: takes $(\text{crs}, x, w)$ as input, where $\text{crs}$ is the CRS of one of the applications in APPS, and returns the output of $\Psi_{\text{app}}.P(\text{crs}, x, w)$.
- verifNestedProof: takes $(\text{crs}, \pi, x)$ as input, where $\text{crs}$ is the CRS of one of the applications in APPS, and returns the output of $\Psi_{\text{app}}.V(\text{crs}, \pi, x)$.
- genWrappingProof: takes $(x_{\text{zec}}, \text{crs}, \{\pi_i, x_i\}_{i \in \text{BATCH}\_\text{SIZE}})$ as input, where $\text{crs}$ is the CRS of one of the applications in APPS, and returns the output of $\Psi_{\text{zec}}.P(\text{crs}_{\text{zec}}, x_{\text{zec}}, \{\pi_i, x_i\}_{i \in \text{BATCH}\_\text{SIZE}})$.
- verifWrappingProof: takes $(\text{crs}_{\text{zec}}, \pi_{\text{zec}}, x_{\text{zec}})$ as input, and returns the output of $\Psi_{\text{zec}}.V(\text{crs}_{\text{zec}}, \pi_{\text{zec}}, x_{\text{zec}})$.

In the game $\text{ZCL-SND}$, $\text{ZecaleP}(\text{crs}_{\text{zec}}, \pi_{\text{zec}}, x_{\text{zec}}, \chi, S, \text{ZbaseAppAddr})$ is the predicate that returns true if the value of decodedData equals $S$ when $\text{ZecaleC}.\text{processAggrTx}$ is called on $(\pi_{\text{zec}}, x_{\text{zec}}, \chi, \text{ZbaseAppAddr})$, and returns false otherwise.

Moreover, $\text{BaseAppP}(\pi_i, x_i, \text{baseAppAddr})$ is the predicate that returns true if $\text{baseAppC}.\text{processTx}(\pi_i, x_i)$, where $\text{baseAppC}$ is the application contract at address $\text{baseAppAddr}$, returns 1. This predicate returns false in all other cases.

All in all, winning the game above means that the adversary used $\text{Zecale}$ as a way to carry out changes in the ledger $\mathcal{L}$’s state that should not have been done by only using the stand-alone base application contract (i.e. without $\text{Zecale}$).

**Theorem 1.** Let $\mathcal{A}$ be a PPT adversary. If $H$ is a collision-resistant hash function (i.e. $\text{Adv}^{\text{coll-res}}_{\mathcal{A}, H}(\lambda) \leq f(\lambda)$, $f$ negligible), and $\Psi_{\text{zec}}$ is a sound SNARK scheme (i.e. $\text{Adv}^{\text{snark-snd}}_{\mathcal{A}, \Psi_{\text{zec}}}(\lambda) \leq g(\lambda)$, $g$ negligible), then $\text{Adv}^{\text{ZCL-SND}}_{\mathcal{A}, \mathcal{L}}(\lambda) \leq \text{negl}(\lambda)$.

**Proof.** Let $X = (\text{crs}_{\text{zec}}, \pi_{\text{zec}}, x_{\text{zec}}, \chi, S, \text{capp.ZbaseAppAddr})$ be such that $\text{ZecaleP}(X) = \text{true}$. Additionally, let $\mathcal{I}, S \subset X$ be such that $\exists x_i \in S, (\pi_i, x_i) \in \mathcal{I}$, $\neg \text{BaseAppP}(\pi_i, x_i, \text{capp.baseAppAddr})$.

By looking at $\text{baseAppC}$ (see Fig. 5), we see that for $\text{BaseAppP}(\pi_i, x_i, \text{capp.baseAppAddr})$ to return false, at least one of the two predicates below needs to be not satisfied:

\begin{align*}
(E_1) & \quad x_{\text{app}i} \in (F_{\pi_i})|^{|a_{\text{app}i}|} \\
(E_2) & \quad \Psi_{\text{app}}.V(\text{crs}_{\text{app}}, \pi_i, x_i)
\end{align*}

We study the probability of these two events below.

$(E_1)$ Since the $\text{ZecaleP}$ predicate was satisfied (by assumption), this means that the check $\text{decodedData} \in (F_{\pi_i})^{\text{decodedData}}$ was satisfied. The probability to satisfy this check on $\text{Zecale}$ while not satisfying it on the base application contract is 0 (the same check is done on both $\text{ZbaseAppC}$ and $\text{baseAppC}$).

\footnote{Note that the requirement on toField being an injective map is important, as it allows to be sure that the function is not a “source of collisions.”}
We distinguish three cases for which ZecaleP can return true, yet one of the BaseAppP predicates returns false because the SNARK verification check $\Psi_{app} \triangleright V(crs_{app}, \pi_i, x_i)$ is not satisfied:

A. $\pi_{zec} \leftarrow \Psi_{zec} \triangleright P(crs_{zec}, x_{zec}, crs_{app}, I)$, where $I$ is a set containing proof/instance pairs for $capp$, but where at least one proof $(\pi_i)$ in the set is not valid (i.e. $x_i \notin L^{capp}$), and its associated instance is in $S$. For ZecaleP to be true, and since the invalid instance was included in $S$, this means that the associated invalid proof was deemed valid since the check on $x_{Valid}$ in $R^{zec}$ was successful. Since the proof was correctly rejected on BaseAppP, this means that the SNARK $\Psi_{zec}$ is not sound since the statement $x_{zec} \in L^{zec}$ was deemed correct while in reality $x_{zec} \notin L^{zec}$. We denote the probability of this event by $Adv_{A, \Psi_{zec}}(\lambda)$.

B. $\pi_{zec} \leftarrow \Psi_{zec} \triangleright P(crs_{zec}, x_{zec}, crs_{app}, I)$, where $I$ is a set containing only valid proof/instance pairs for $app$, where $app \neq capp$. In this case, it is also clear that BaseAppP will return false on all input pairs in $I$ as they have been generated for another application (under another CRS which is different from $capp.crs_{app}$, the one used in BaseAppP in the game). However, for ZecaleP to be true, this means that the check $toDigest_{H, rw}(vkHash)$ = $storage["app.crs"]$ (line 8 in the dispatch function in Fig. 5) was successful. However, by looking at Fig. 4 we know that the value of $x_{zec}.vkHash$ is constrained to equal to $toField_{H, rw}(H(\overline{app}))$ for $x_{zec}$ to be in $L^{zec}$. Hence, assuming that $\Psi_{zec}$ is sound, for ZecaleP to accept, it is necessary to have $toDigest_{H, rw}(toField_{H, rw}(H(\overline{app}))) = H(capp)$, which by the definition of $toDigest_{H, rw}$ and $toField_{H, rw}$ means that $H(\overline{app}) = H(capp)$, where $capp \neq app$. This implies that $A$ needs to find a collision in $H$. We denote the probability of this event by $Adv_{\lambda, H}(\lambda)$.

C. $\pi_{zec} \leftarrow \Psi_{zec} \triangleright P(crs_{zec}, x_{zec}, crs_{app}, \{\pi_i, x_i\}_{i \in \text{[BATCH SIZE]}})$, where $\{\pi_i, x_i\}_{i \in \text{[BATCH SIZE]}}$ is a set containing only valid proof/instance pairs for $capp$. Furthermore, we set $I \leftarrow \{\pi_i, x_i\}_{i \in \text{[BATCH SIZE]}}$, where $\exists j \in \text{[BATCH SIZE]}$, $x_j \neq x_j$ (i.e. one of the “nested” instance has been tampered with by the aggregator after generating $\pi_{zec}$). For ZecaleP to accept such input (which is rejected by BaseAppP), the check $x_{zec}.x_{Hj} = toField_{H, rw}(H(\overline{x_j}))$ (lines 7-11 Fig. 6) needs to be satisfied. However, since $x_{zec}.x_{Hj}$ is constrained to be equal to $toField_{H, rw}(H(x_j))$ (see Fig. 4), since $toField_{H, rw}$ is injective and since $\overline{x_j} \neq x_j$, then $A$ broke the collision resistance of $H$.

Since both events $E_1$ and $E_2$ above are independent and since $E_2$ is a conjunction of independent events, then the probability $Pr[E_1 \lor E_2] = Pr[E_1] + Pr[E_2]$ which means that $Adv_{\lambda, L}(\lambda) = 0 + \left(Adv_{\lambda, \Psi_{zec}}(\lambda) + 2 \cdot Adv_{\lambda, H}(\lambda)\right)$ is negligible in $\lambda$.

Remark 2. We note that the hash function $H$ used to hash the instances associated to the “nested proofs” does not necessarily need to be the same hash function as the one used the hash the verification key. It is perfectly feasible —
and (maybe) in some contexts even desirable — to instantiate these hash functions differently as long as both functions comply with the security requirements mentioned above.

Remark 3. As observed by Duncan Tebbs [Teb20] it is necessary to check on \( \text{ZbaseAppC} \) that the calling Zecale contract (i.e. \( \text{ZecaleC} \)) is genuine in order to be sure, on the application contract, that all the checks necessary to make the protocol sound are properly carried out. Failure to check the address of the calling contract would render the protocol vulnerable to a range of malicious calling contracts.

5 Applications

In this section we present a few types of applications of Zecale, and conclude by arguing that Zecale and privacy preserving solutions like ZETH can be used as building blocks to implement blockchain-based digital cash systems.

We note that none of the applications discussed below are mutually exclusive nor that they comprehensively represent the landscape of potential applications of Zecale.

5.1 Application types 1: Blockchain users run the Zecale aggregator locally

In this setting, the Zecale aggregator piece of software is ran on the network users’ machines in order to aggregate their own batches of transactions locally. Doing so allows users to save gas since only one transaction is sent on-chain, and only one proof needs to be verified as part of the smart-contract execution. Running Zecale locally to batch one’s transactions allows to save at most \( g_{\text{Saved}} \) gas:

\[
g_{\text{Saved}} = \text{DGAS} \cdot (\text{BATCH\_SIZE} - 1) + \text{BATCH\_SIZE} \cdot \text{VNProofGas} - \text{VWProofGas}
\]

where \( \text{DGAS} \) is the intrinsic gas of a transaction, \( \text{VNProofGas} \) the gas necessary to verify one nested proof, and \( \text{VWProofGas} \) is the gas necessary to verify the wrapping proof.

Would batch verification make sense in this setting? In the context where Zecale only aims to be used in this setting (i.e. as a way to equip network users with a way to batch their transactions locally), it may be tempting to modify \( R^{\text{Ze}} \) in order to support batch verification of proofs. In fact, many of the practical security issues discussed in Appendix [B] do not apply here anymore since all proofs in the batch are generated by the user directly. However, since all nested proofs are both generated and batch-verified (as part of the wrapping proof generation) by the user, it is important, for the security of the system, to make sure that a malicious user cannot craft a set of scalars in the batch verification
equation Eq. (9) that could violate the soundness of the system. In other words, it is necessary to make sure that the set of scalars used in the batch verification equation are not under the prover’s control. While picking a set of random field elements cannot be enforced using arithmetic gates, one may want to leverage Pseudo-Random Functions (PRFs) as a way to deterministically derive a set of random scalars from the set of proofs in the batch and add this derivation process to $R^{zec}$. Further discussion on that is provided in Appendix A.

5.2 Application type 2: Blockchain miners run the Zecale aggregator

Another application of Zecale consists in adding the aggregator software logic as part of the block production on the blockchain. Doing so requires to make some changes to the blockchain protocol – which in many cases, may not be desired – but could however, lead to interesting scenarios and platform economics in which miners are incentivized to aggregate transactions in the blocks they produce. One such protocol modification may, for instance, consist in extending the block production reward with an aggregation reward, and design a penalty that would diminish the block production reward in the event where a block proposed by a miner contains transactions that could have been aggregated. More drastically, one may want to change the validity rules of a block [Woo, Section 4.3], by enforcing that all blocks containing data that could have been aggregated via Zecale, are deemed invalid on the network and thus rejected.

Such protocols would minimize data redundancy on-chain and would allow to minimize the growth of data on the blockchain, at the cost of modifying the base protocol.

5.3 Application type 3: Creation of an aggregation market

Finally, in this 3rd category of applications we argue that solutions such as Zecale could be used to enrich the blockchain ecosystem by creating new economic opportunities.

Privacy preserving solutions like ZETH allow to keep the value and the recipient of a payment secret. Nevertheless, because of the need to pay for the gas of the state transition, network users can observe when another user sends a ZETH transaction (see [RZ19] for more discussions on this).

Despite this information leakage (i.e. Ethereum balances going “up and down”), the transaction graph remains blurred (see [RZ19, Figure 2]). Users emitting ZETH transactions cannot be associated to specific payments. Likewise, while transaction unobservability is not achievable on a blockchain system (miners and other network users can see the transactions in the system), we note that the possibility to emit “dummy ZETH payments” – at the cost of paying the price of the state transition – allows to create additional noise in the system in order to achieve payment unobservability.

Nevertheless, this ability to detect when a user triggers a privacy preserving state transition on the distributed state machine (whether it is a dummy payment
or not) may prevent the deployment and adoption of solutions like **ZETH** in countries where the use of Privacy Enhancing Technologies (PETs) is prohibited. Making sure to remove this information leakage is of tremendous importance for the wide deployment of such solutions.

Below, we show how one could leverage solutions such as **Zecale** along with the use of Anonymous Communication (AC) protocols like mix-networks in order to *bypass* gas-related information leakages and hide the sender of transactions.

**Anonymous communication protocols and sender anonymity.** As above-mentioned, solutions like **ZETH** allow to achieve *recipient anonymity* and *relationship anonymity* (as defined in [PH09, BKM+16]). Unfortunately, *sender anonymity* is not ensured by the protocol because of the need to pay gas to execute state transition on the distributed state machine.

While removing the need to pay gas for each transaction on the blockchain would undoubtedly remove some types of leakages and could ultimately be used to allow transactions to be emitted from newly created accounts (with no funds) as a way to gain sender anonymity; it is clear that such solution would expose the system to a wide class of attacks – such a Denial of Services (DoS). Instead, another way to obtain sender anonymity would be to leverage a network of *relay nodes* that would listen to incoming messages/transactions and emit/relay them on-chain. Such service could be rewarded by a *relay fee*.

Resorting to relay nodes as a way to gain sender anonymity by moving the need to pay gas onto another entity is not without risk however. In addition to the need to design a sound crypto-economic protocol and implementing the right incentive structure to reward relays and keep the system secure; it is also necessary to make sure that the *system remains censorship resistant* to make sure that no malicious relay node can prevent transactions from being sent and executed on-chain – even if deemed irrational by the incentive structure.

Interestingly, if a SNARK-based state transition *only* processes a zero-knowledge SNARK proof along with the associated set of public inputs (that do not leak the sender of the transaction) during its execution, it is possible to use anonymous communication protocols to simply route the zero-knowledge proof and the instance to a relay. Routing such information using an anonymous network based on onion routing [GRS99] (e.g. **TOR** [DMS10]) or mix-networks [Cha03, Adi06, SP06] along with cryptographic packet format [DG09] (e.g. **Loopix** [PHE+17] or **Nym** [NYM19]) would protect the transaction originator from any malicious relays, and would render any targeted censorship strategy inefficient (see Fig. 7).

**Zecale as an aggregation proxy to scale SNARK-based blockchain applications.** In addition to simply relay received zk-SNARK proofs on-chain, we envision scenarios where *relay nodes* could also run the **Zecale** aggregator as a way to compress the set of received proofs, into a single SNARK-proof to be sent on-chain (see Fig. 5).

While relaying transactions allows transaction originators to become anonymous, aggregating received zk-SNARKs before relaying them on-chain allows
blockchain validators/miners to exchange and process less data. Like “honest relaying”, “honest aggregation” of transactions should also be worth a fee that could, for instance, be paid by blockchain miners to reward the aggregators for their “data compression” work, which ultimately renders the blockchain system more efficient.

**Remark 4.** We note that for blockchains based on Nakamoto-style consensus [Nak09], block production work is intrinsically different from zk-SNARK proof production. As such, we believe that the distinct roles of miners and aggregators are very complementary and reflect the different nature of the computational tasks.
Delegating the wrapping proof generation (i.e. the aggregation of zk-SNARK proofs) to a set of aggregators seems to be a very natural model.

**Toward the formation of “aggregation pools”?** As we know that carrying out Groth16 SNARK verification as part of proving that $x \in \mathbf{L}^{\text{zec}}$ is expensive (i.e. pairings are expressed by large R1CS), we believe that new form of “pools” (similar to mining pools) could emerge on the aggregation network. In fact, Wu et al. [WZC+18] proposed DIZK, a system that distributes the generation of a zero knowledge proof across machines in a compute cluster. Some blockchain systems leveraging Zecale may be willing to maximize the size of the batch of aggregated zk-SNARK proofs – i.e. maximize $\text{BATCH\_SIZE}$ – in order to maximize data compression and flatten the growth of the blockchain data. However, doing so would significantly complicate the set of algebraic constraints representing $\mathbf{R}^{\text{zec}}$ and increase the degrees of the interpolated polynomials manipulated during the proof generation process. This could be compensated by the formation of “aggregation pools” where a set of contributors could allocate some compute resources to the generation of a wrapping proof, and be rewarded pro-rata the resources allocated. We believe that further investigation of the formation of aggregation pools is of great interest. However, we defer such study for later work.

6 **ZETH and Zecale as building blocks of digital cash systems**

Using ZETH as base application of Zecale as in Section 5.3 allows to gain sender anonymity and diminish the amount of data sent and processed on-chain. As mentioned in [CR20, Remark A.2.2], ZETH has been designed to be used in various setting, and the separate addressing scheme of the protocol [CR20 Section 1.4] allows to distinguish between users having an Ethereum account and those with only a ZETH address. While Ethereum account holders can trigger arbitrary state transitions, using Zecale along with ZETH is a way to expose the ZETH state transition – to users holding only a ZETH address – through relay and aggregator nodes. We believe that such mechanism is a step toward implementing digital cash systems on blockchains like Ethereum, but stress that sound crypto-economics are fundamental to keep the system secure. The possibility to retrieve funds on the ZETH mixer in the form of a public output value is certainly a mechanism that can be used to define such incentive structures.

A note on latency. An obvious drawback of composing protocols such as ZETH, Zecale and Ethereum is the increased settlement latency (i.e. the time between the emission of the transaction by the user and the processing of the transaction on-chain). In fact, it is clear that sending zk-SNARK proofs to a relay node over an anonymous network in order to relay them on the blockchain adds latency to execution of the state transition on the distributed state machine
(i.e. blockchain). Any extra computation carried out on the relay node – like the proof aggregation – increases this latency even more.

Importantly, high latency in the system may discourage users from using it. As such, and in addition to aggressively optimizing their infrastructure and software to generate wrapping proofs (see Section 5.3), operators of aggregation nodes may be willing to pay high gas price to fasten the inclusion of their aggregation transactions in the blockchain. In a way that echoes and resembles [CST20], operators of Zecale aggregator nodes would bid to maximize their chances to see their aggregate transactions mined first on the chain. Such high gas price may decrease the aggregation profits but could allow to capture more market shares by offering lower latency access to the ZETH contract to carry out “fast” cash payments.

6.1 Towards better cash

Today, cash still plays a fundamental role in the economy. Despite a decline in the number of cash payments due to alternative and friction-less digital payment methods, cash remains hugely important, especially for the numerous “un-banked” people [Cla18].

While transitioning to digital cash seems unavoidable, this transition presents interesting opportunities but also numerous challenges ans risks [Tan96,DH16].

A digital cash system as described in Section 6 is secure, untraceable, distributed but does not, however, allow to carry out payments at no fees (which is the case for cash). Nevertheless, not only exposing the ZETH state transition via an anonymous aggregation network can grant restricted and secure access to the distributed ledger to people that do not hold an account on the ledger (but only a ZETH address), but it also makes it possible to implement Anti-Money Laundering (AML) policies for cash payments by modifying the ZETH language in order to (additionally) prove – in zero-knowledge – that a payment satisfies a compliance predicate (without disclosing the payment details).

We believe that being able to securely grant wide access to the ZETH state transition as well as cryptographically enforce financial regulatory policies is a step toward solving some of the drawbacks of cash and could pave the way to build a more efficient and secure but also more inclusive and stable economy.

7 Implementation

In this section we provide an overview of the software architecture of Zecale. We invite the reader to consult the open-source project [23] for more details on the implementation.

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22 the central counter-party processing the payments is replaced by a distributed ledger

23 https://github.com/clearmatics/zecale
7.1 Ethereum client

As mentioned in Section 3.2, any pairing friendly cycle/amicable chain of elliptic curves can be used in Zecale to generate the “wrapping” SNARK proof (over one curve) certifying the correct verification of a set of “nested” zk-SNARK proofs (generated over another curve). Unfortunately, the current version of Ethereum does not offer the possibility to carry out arithmetic over a wide class of elliptic curves. As of today, implementing Zecale requires to fork from Ethereum mainnet, and add new precompiled contracts to support the BW6-761 pairing group operations. Nevertheless, Ethereum Improvement Proposals such as [Vla19] may change this situation and expose a wide class of pairing groups via precompiled contracts.

Remark 5. Additional EIPs such as [ASB19] have recently been incorporated to the Ethereum protocol, making layer 2 scalability solutions like Zecale more efficient.

7.2 Zecale aggregator

Following a similar software architecture as in ZETH [RZ19, Section 6], we decided to implement the Zecale aggregator as a self-contained software component written in C++ and using a modified version of the libsnark and libff libraries. This software component exposes a Remote Procedure Call (RPC) interface allowing to receive zk-SNARK proofs to aggregate. The received zk-SNARK proofs and corresponding instances are then added in an “application pool” which represents the set of “nested” zk-SNARK proofs to aggregate for a given SNARK-based application.

In addition to receive zk-SNARK proofs to aggregate, other endpoints have been added to the Application Programming Interface (API) allowing to register applications (i.e. deposit the crs associated to a given application), along with the back-end logic to enable a Zecale aggregator to function across several applications. A high-level representation of the software components composing Zecale is provided Fig. 9.

7.3 Zecale contract

A pseudo-code of the smart-contract logic is provided Fig. 6. To be deployed on Ethereum, such contract can be implemented using the Solidity programming language for instance.

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24 Only arithmetic over BN-254 and BLS12-381 is exposed via precompiled contracts at the time of writing: https://github.com/ethereum/go-ethereum/blob/0c82928981028e8b32b5852c38b095d2e0d26b04/core/vm/contracts.go#L83
25 see https://github.com/clearmatics/libsnark and https://github.com/clearmatics/libff
26 https://solidity.readthedocs.io/en/v0.7.0/
We note that, as previously mentioned, an explicit check — verifying that all received nested instances lie in the right finite field — is done by the application contract in Zecale. This check is kept on the smart-contract rather than “moved” as an extra constraint in $R_{\text{Zeale}}$ for efficiency reasons. In fact, $\text{ZecaleRelation}$ is defined over the scalar field $F_{r_w}$ of BW6-761 which is also the base field of BLS12-377. That means that all variables/wires manipulated in the algebraic representation of the Zecale NP-relation are defined over $F_{r_w}$, while the “nested instances” are defined over the scalar field $F_{r_n}$ of BLS12-377. As such, for a given $x$, checking that $x \in F_{r_n}$ in $\text{ZecaleRelation}$ incurs a non-negligible overhead due to the characteristic mismatch of the two scalar field, i.e. $r_w \neq r_n$. Hence, to bypass the necessity to provably carry out an expensive range-check on the instance, we decided to keep this check on the application smart-contract.

Importantly, since any entry in a set of “nested instances” lying outside of the field $F_{r_n}$ will cause the Zecale state transition to abort, implementers of Zecale aggregator services may — at their will — decide to carry out this membership test on “nested instances” before accepting the incoming $(\pi, x)$ pair from a user. As such, all requests containing “nested instances” that are not elements of $F_{r_n}$ will be deemed invalid and end up being rejected by the aggregation service. Alternatively, the smart-contract logic (as illustrated Fig. [6] and Fig. [5]) can be modified to ignore/skip instances failing to satisfy the field membership test, without aborting.

Finally, we note that elements of the base field of BLS12-377 and of the base and scalar field of BW6-761 have a binary representation that exceeds the 256-bit word length of the Ethereum (stack-based) Virtual Machine (EVM). As such, representing elements of such fields requires multiple entries on the stack, which makes manipulating such elements relatively expensive gas-wise. As such,
implementers may want to allocate some time to optimize the implementation of costly operations of the Zecale smart-contract\textsuperscript{27}, or may want to implement additional precompiled contracts to carry out costly operations natively on the client.

8 Conclusion

In this paper we presented Zecale, a general purpose SNARK proof aggregator allowing to scale SNARK-based applications on Ethereum. We explained how Zecale works and showed that multiple applications could benefit from it. Likewise, various new types of applications and new market opportunities could emerge from this work, and we believe that Zecale and blockchain-based privacy preserving protocols constitute valuable building blocks for digital cash systems.

Importantly, while Zecale diminishes the data sent and processed on-chain, and thus, makes SNARK-based applications more scalable, we emphasize that this work is not "the" solution to on-chain scalability, but is rather what we believe to be a step toward building more scalable systems.

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\textsuperscript{27} Using inline assembly for instance (see \url{https://solidity.readthedocs.io/en/v0.7.0/assembly.html?highlight=assembly})
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A A note on batch verification

We start by defining the notion of batch SNARK verification by adapting the definition of batch verification of signatures from Camenisch, Hohenberger and Pedersen [CHP12].

Definition 1 (Batch verification of SNARKs). Let \( \lambda \) be the security parameter and \( R \) be an NP-relation which associated language is \( L \). Suppose \( \Psi = (\text{KGen}, P, V, \text{Sim}) \) is a SNARK scheme, \( n \in \text{poly}(\lambda) \), and let \( \text{crs} \leftarrow \Psi.\text{KGen}(R) \). We call probabilistic \( \text{VBATCH} \), a batch verification algorithm where the following conditions hold:

Batch-Completeness:

\[
\Psi. V(\text{crs}, x_j, w_j) = \text{true}, \forall j \in [n] \\
\Rightarrow \text{VBATCH}(\text{crs}, (x_0, w_0), \ldots, (x_{n-1}, w_{n-1})) = \text{true}
\]

Batch-Soundness:

\[
\Pr\left[ \text{VBATCH}(\text{crs}, (x_0, w_0), \ldots, (x_{n-1}, w_{n-1})) = \text{true} \mid \exists j \in [n] \text{ s.t. } V(\text{crs}, x_j, w_j) = \text{false} \right] \leq \text{negl}(\lambda)
\]

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A.1 Intuition behind the soundness of batching.

In [BGR98] Bellare et al. present a set of probabilistic checks to perform fast batch verification for modular exponentiation and digital signatures. Later, Ferrara et al. [FGHP09] and Blazy et al. [BFI+10] proposed batching methods in pairing-based settings.

Similar methods can be used to batch verify a set SNARKs generated for the same language (i.e. a set of SNARKs generated under the same crs). As such, instead of checking a set of $N$ proofs separately by running $N$ times the verification algorithm, a single equation can be used to decide whether all SNARKs in the set are valid. If the batch verification equation is not satisfied, this means that at least one proof in the set is not valid.

As such it is possible to transform system of verification equations, as below, into a single probabilistic check (we further assume here that the algorithm $\Psi \cdot V()$ can be fully represented by a single check that tests that an equation - $V_{eq}$ - equals 0 if the proof is valid).

\[
\begin{align*}
\Psi \cdot V_{eq}(\text{crs}, x_0, \pi_0)(\doteq 0) \\
\Psi \cdot V_{eq}(\text{crs}, x_1, \pi_1)(\doteq 0) \\
\vdots \\
\Psi \cdot V_{eq}(\text{crs}, x_{N-1}, \pi_{N-1})(\doteq 0)
\end{align*}
\] (2)

Indeed, representing the system of equations in Eq. (2) as a vector defined over a vector space, informally allows to “isolate” each proof verification into “its own dimension”. In fact, $\Pi$ can be obtained by decomposition in term of the standard basis of the vector space, i.e. :

\[
\Pi = f_0 \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \ldots + f_{N-1} \cdot \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}
\] (3)

where $f_i = \Psi \cdot V_{eq}(\text{crs}, x_i, \pi_i), \in \mathbb{F}_r$.

Since $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ldots , \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis of the vector space $\mathcal{V}$, and by definition of a basis, we know that by the linear independence property of the vectors forming a vector space basis, $\Pi = 0 \Rightarrow f_i = 0, \forall i \in [N]$, i.e. all proofs are correct.

While checking if $\Pi = 0$ can be done by checking all coordinates of $\Pi$, which corresponds to checking each individual proofs one by one, it is possible to fasten this check by applying a linear transformation $T : (\mathbb{F}_r)^N \rightarrow \mathbb{F}_r$ defined as: $Mx$, where $M = [m_0, \ldots, m_{N-1}]$ is a $1 \times N$ matrix defined over $\mathbb{F}_r$ such that $\exists i \in [N]$ s.t. $m_i \neq 0$, and $x = [x_0, \ldots, x_{N-1}]^\top \in \mathcal{V}$. 
Applying this linear transformation on \( \Pi \) allows to “project the vector to the number line” and simply check that the transformation input is mapped to 0 by doing a single check. Trivially, \( x = 0 = [0, 0, \ldots, 0]^T \in (\mathbb{F}_r)^N \) is mapped to 0 by \( T \). However, \( \text{Ker}(T) = \{ (x_0, \ldots, x_{N-1}) \in \mathcal{V} \mid m_0x_0 + \ldots + m_{N-1}x_{N-1} = 0 \mod r \} \neq \{0\} \) (i.e. \( \text{Ker}(T) \supset \{0\} \), but \( \text{Ker}(T) \not\subset \{0\} \)). In fact, reducing the dimensions of the space via the transform \( T \) breaks the “one check ↔ one dimension” idea above-mentioned, and it is now important to make sure that no adversary can forge a vector \( \Pi \) such that \( \Pi \in \text{Ker}(T) \setminus \{0\} \), i.e. its projection by the linear transformation above yields the origin of the number line. This would violate the batch-soundness property (see Definition 1).

To measure the likelihood for an adversary to hold a vector of SNARK proof that violates the soundness of the batch equation, it is necessary to estimate the number of solutions of the equation:

\[
m_0x_0 + \ldots + m_{N-1}x_{N-1} = 0 \mod r
\]  

(4)

where \( m_i \)'s and \( x_i \)'s \( \in \mathbb{F}_r \). As the linear functional \( T \) is assumed to be non-trivial, we know that \( \text{Ker}(T) \subset \mathcal{V} \). However, knowing how smaller the kernel of the linear functional is compared to the source vector space is crucial to prove the soundness of batching.

Intuitively, given \( T \), the equation Eq. (4) holds if either all terms are equal to 0, or if 2 terms - out of the \( N \) terms - “cancel each other”, i.e. are additive inverse in the field, or if 3 terms “cancel each other” etc. This means that the number of solutions of Eq. (4) is bounded by the sum of binomial coefficients,

\[
\sum_{k=0}^{N} \binom{N}{k} = 2^N
\]

As we now know that \( |\text{Ker}(T)| < 2^N, \forall T \leftrightarrow \mathfrak{V} \), where \( \mathfrak{V} \) is the set of elements in the algebraic dual space of \( \mathcal{V} \) minus the trivial map \( T_t \) (where \( \text{Ker}(T_t) = \mathcal{V} \)), on challenge a random - non-trivial - linear functional, the probability for an adversary \( A \) (who previously committed to his vector of proofs - to “kill” adaptivity) to hold a vector of proofs such that it falls in \( \text{Ker}(T) \) is:

\[
\frac{|\text{Ker}(T)|}{|\mathfrak{V}|} < \frac{2^N}{(|\mathbb{F}_r|)^N} < \left(\frac{2}{r}\right)^N
\]

If \( r \) is a large prime such that \( r \) is encoded on \( \lambda \) bits (i.e. \( r > 2^\lambda \)), the probability that the vector of proofs held by the prover lies in the kernel of the challenged linear functional is bounded by \( \left(\frac{2}{2^\lambda}\right)^N = \frac{1}{2^{N(\lambda - 1)}} \), which is negligible in \( \lambda \).\(^{28}\)

\(^{28}\) We can also use \([\text{Sch74, Lemma 1}]\) to bound \( |\text{Ker}(T)| \) (described as the set of solutions of a multivariate polynomial of degree 1) by \( r^{n-1} \), and show that the probability to have a vector that lies in \( \text{Ker}(T) \) is bounded by \( r^{n-1} = \frac{1}{r} < \frac{1}{2^\lambda} \).
Modeling proof-batching as an interactive protocol. Below, we model proofs batching as an interactive protocol between a prover and a verifier. Informally, the prover has a vector of SNARKs and wants to show the verifier that all proofs are valid.

We follow the same structure as a sigma protocol [Dam10], where the prover wants to convince the verifier that he holds a vector of valid SNARKs (and associated public inputs) for a given language - represented by its crs - which we assume to be available by all parties in the protocol. First, the prover commits to his set of pairs of proofs and associated public inputs, by sending a commitment to the verifier, then the verifier samples a challenge at random from the challenge space, and the prover answers this challenge. At the end of the protocol, the verifier either accepts or rejects.

The protocol is summarized below - where \( \Pi_i = (\pi_i, x_i) \forall i \in [N] \):

| Interactive protocol for SNARK proof batching |
|----------------------------------------------|
| **Prover**                                   |
| \( c \leftarrow \text{Commit}(\Pi) \)       |
| **crs** \( \leftarrow \Psi.KGen(R) \)        |
| **Verifier**                                 |
| \( c \)                                      |
| \( T \leftarrow \Psi \)                     |
| \( res \leftarrow \Psi.V_{eq}(\text{crs}, \Pi_0, \ldots)^T \) |
| **Decide** (\( c, T, res \))                |

Applying Fiat-Shamir to remove interactions. In most cases, the vector \( \Pi \) of proofs and primary inputs is held by an aggregator or verifier, and is built as the set of received proof/inputs from a set of provers. As such, the (batch) verifier is not one of the initial prover who generated one of the proofs in the batch. We are interested here, in the case where this distinction does not apply anymore, and where the batch verifier is one of the provers who initially generated a proof in the batch.

We consider the interactive protocol above, and make it non-interactive by using the Fiat-Shamir transform [FS86,PS96,BPW12], which yields the following - non-interactive - protocol:
Here the verifier’s challenge is derived by the prover from the first message sent in the interactive protocol. We note that the derivation of $\mathcal{T}$ can be done by calling a Random Oracle (RO) $N$ times to obtain each entry of the line matrix $M$, representing $\mathcal{T}$, where each $m_i$ of $M$ is obtained as $m_i \leftarrow \text{RO}(c | i)$. However, doing so, requires to call the RO $N$ times, which may not be desired. As such, one may want to derive $M$ as $m_0 \leftarrow \text{RO}(c)$, and derive the other $m_i, i \in [N] \setminus 0$ as $m_i \leftarrow m_0^{i+1}$. Now, the equation that determines the kernel of the linear transformation becomes a univariate polynomial of degree at most $N$, which, as we know by the (DeMillo-Lipton-)Schwartz-Zippel lemma [DL78,Zip79,Sch80], has at most $N$ roots. In fact, we would have:

$$\text{Ker}(\mathcal{T}) = \{(x_0, \ldots, x_{N-1}) \in \mathcal{V} \mid x_0 m_0 + x_1 m_0^2 + \ldots + x_{N-1} m_0^N = 0 \mod r\}$$

which can be modeled as the set of all roots of the polynomial of degree $N$ which coefficients are $(x_0, \ldots, x_{N-1})$. Schwartz-Zippel tell us that such polynomial has at most $N$ roots, as such, the only way for the equation to be satisfied (and the batch verification check to be accepted) is either,

1. to sample a challenge $\text{RO}(c)$ that lends in the set of roots of the polynomial determined by the prover’s input vector. The probability of this event is $\frac{N}{|\mathbb{F}_r|}$ which is negligible for low degree polynomials - $N \ll r$ (we note that $N \ll r$ will always be satisfied in applied settings, as carrying out a batch SNARK verification on $N \approx r$ SNARKs would be prohibitively expensive. Alternative protocols such as [BMMV19] would be more efficient to generate the “wrapping proof” at the expense of having a log-sized proof with a log-time verifier), or
2. for the prover’s polynomial to be the 0 polynomial (i.e. for all his proofs to be valid - all checks pass and all entries in the vector defining the polynomial are 0)

Importantly, the malicious prover who tries to violate the soundness of the batch verification, by continuously sampling a new vector of proofs and the corresponding challenge until the batch verification pass while the batch is invalid, does not have a probability higher than $\frac{N}{|\mathbb{F}_r|}$ to succeed since all these events are independent. In fact, by re-running the protocol in the hope to violate the
**batch-soundness**, the prover generates a new set of proofs and thus a new polynomial to evaluate after receiving the challenge from the random oracle (which output is unpredictable by the prover). Hence, the prover cannot evaluate the same polynomial at multiple points.

All the operations carried out by the prover in the non-interactive protocol above may be added to $R^{\text{zec}}$ in order to be “carried out in the SNARK”.

### A.2 Batch verification equation for Groth16.

We know that the verification routine run by $V$ in [Gro16] consists in checking if Eq. (5) holds.

$$e(\pi.A, \pi.B) = e(vk.\alpha, vk.\beta) * e(\Gamma, vk.\gamma) * e(\pi.C, vk.\delta)$$  \hspace{1cm} (5)

where

$$\Gamma = \sum_{i \in |x|} x_i \frac{vk.\beta \cdot u_i(x) + vk.\alpha \cdot v_i(x) + w_i(x)}{vk.\gamma}$$

Carrying out this check requires the verifier to compute 4 pairings along with $|x|$ scalar multiplications (i.e. a scalar multiplication for each primary input in $x$).

Using batch verification allows to save a few pairing operations for the verification of multiple SNARKs.

Applying the reasoning described in the first section of Appendix A.1 we can represent a set of Groth16 proof verification equations

$$\begin{cases}
A_1B_1 = \alpha \beta + \gamma \Gamma_1 + \delta C_1 \\
A_2B_2 = \alpha \beta + \gamma \Gamma_2 + \delta C_2 \\
\cdots \\
A_NB_N = \alpha \beta + \gamma \Gamma_N + \delta C_N
\end{cases} \Leftrightarrow \begin{cases}
A_1B_1 - \alpha \beta - \gamma \Gamma_1 - \delta C_1 = 0 \\
A_2B_2 - \alpha \beta - \gamma \Gamma_1 - \delta C_2 = 0 \\
\cdots \\
A_NB_N - \alpha \beta - \gamma \Gamma_N - \delta C_N = 0
\end{cases}$$  \hspace{1cm} (6)

as a vector $\Pi$ defined over the vector space $V = (\mathbb{F}_r)^N$, where $r$ is the prime order of $G_1, G_2, \text{and } G_T$:

$$\Pi = \begin{bmatrix}
A_1B_1 - \alpha \beta - \gamma \Gamma_1 - \delta C_1 \\
A_2B_2 - \alpha \beta - \gamma \Gamma_1 - \delta C_2 \\
\vdots \\
A_NB_N - \alpha \beta - \gamma \Gamma_N - \delta C_N
\end{bmatrix}$$  \hspace{1cm} (7)

and batch verify the set of underlying proofs. The goal of the verifier is to check that $\Pi$ is equal to $0$. 
**Groth16 batched - unrolled.** We can transform this set of equations into a single equation, by applying a random linear functional $T$, represented by a line matrix of random scalars $M = [m_0, \ldots, m_{N-1}]$, and checking that the result is $0 \in \mathbb{F}_r$, i.e. checking that $T(\Pi) = M \Pi = 0 \in \mathbb{F}_r$. After applying the group encoding on field elements and denoting by $\text{vk}$ (“verification key”) the part of the crs used by the verifier, the batch verification check becomes:

$$\prod_{i \in [N]} e(\pi_i.A, \pi_i.B)^{m_i} \geq \prod_{i \in [N]} [e(\text{vk}.\alpha, \text{vk}.\beta) * e(\Gamma_i, \text{vk}.\gamma) * e(\pi_i.C, \text{vk}.\delta)]^{m_i} \quad (8)$$

Which gives:

$$\prod_{i \in [N]} e(m_i \cdot \pi_i.A, \pi_i.B) \geq e \left( \sum_{i \in [N]} m_i \cdot \text{vk}.\alpha, \text{vk}.\beta \right) * e \left( \tilde{\Gamma}, \text{vk}.\gamma \right) * e \left( \sum_{i \in [N]} m_i \cdot \pi_i.C, \text{vk}.\delta \right) \quad (9)$$

where:

$$\tilde{\Gamma} = \sum_{j \in [|x|]} \frac{[\text{vk}.\beta \cdot u_j(x) + \text{vk}.\alpha \cdot v_j(x) + w_j(x)] (\sum_{i \in [N]} m_i \cdot x_j)}{\text{vk}.\gamma}$$

In contrast with Eq. (5), the batch verification equation Eq. (9) requires $N + 3$ pairings (instead of $4 \times N$) along with $1 + |x| + 2N$ scalar multiplications (instead of $N \times |x|$) to verify $N$ SNARKs. As such, a few pairing computations are removed at the expense of more scalar multiplications, which as we know, are significantly cheaper to carry out.

**B Practical considerations of SNARK batching**

While it is possible to batch verify a set of nested proofs in Zecale, one needs to understand the limitations of such technique. In fact, batch verification presents some challenges related forgeries detection in batches, which can ultimately, significantly slow the system down.

**B.1 Forgery detection.**

As above-mentioned, the batch verification equation for Groth16 can be used to efficiently check that all entries in a set of SNARKs are valid. In other words, the batch verification method detects the presence of forgeries in a set of SNARKs. Nevertheless, while a failing batch verification exposes the presence of forgeries in the set of inputs, it does not allow to identify which input - in the set - is a forgery. While a trivial way to identify forgeries consists in verifying all inputs individually, being able to identify forgeries in a batch efficiently is non-trivial and has lead to various work.
While “divide-and-conquer” approaches such as the one proposed by Pas-tuszak et al. \cite{Pas00} can be used to converge toward forgeries in the set of inputs, we note that this method introduces an important overhead on the system. Later, Law and Matt \cite{LM07,Mat09} proposed an improved forgery identification protocol for pairing-based signatures, improving on the initial design of binary-tree search methods.

While studying SNARK forgery identification is out of scope of this work, we note that carrying out forgery identification upon invalid SNARK batch verification gives a leverage to an adversary \( A \) who can send a low volume of SNARK forgeries to the aggregator, enough to have one forgery in each batch, in order to cause a severe slow down in the system \cite{BDLO12}.

As such, some care needs to be taken before considering using batch SNARK verification as an optimization in protocols like Zecale.

\section*{B.2 Processing and verifying invalid proofs.}

For systems like Zecale, where an aggregator piece of software receives a set of proofs and relays the result of their aggregation on-chain, it may be desired to process invalid proofs. In fact, settling a transaction containing an invalid proof on a blockchain system acts as a proof that the transaction has been processed - and not censored. As such, even if a SNARK does not verify successfully on an aggregator, one may still want to send the result of this erroneous verification on-chain to show the progress of the system, and to inform the transaction originator that the transaction has been processed and added on-chain.