LESSONS FROM $\overline{B} \to X_S \gamma$ IN TWO HIGGS DOUBLET MODELS

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ABSTRACT
The next-to-leading order predictions for the branching ratio $\text{BR}(\overline{B} \to X_S \gamma)$ are given in a generalized class of two Higgs doublets models. Included are the recently calculated leading QED corrections. It is shown that the high accuracy of the Standard Model calculation is in general not shared by these models. Updated lower limits on the mass of the charged Higgs boson in Two Higgs Doublet Models of Type II are presented.

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1 Introduction

Two Higgs Doublet Models (2HDMs) are conceptually among the simplest extensions of the Standard Model (SM). They contain additional sources of flavour change due to their extended Higgs sectors. Studies of the $\overline{B} \to X_s \gamma$ decay in this class of models, therefore, can already test how unique is the accuracy of the Standard Model prediction for this branching ratio $\text{BR}(\overline{B} \to X_s \gamma)$. This generalization allows a simultaneous study of different models, including Type I and Type II, by a continuous variation of the (generally complex) charged Higgs couplings to fermions. It allows also a more complete investigation of the question whether the measurement of $\text{BR}(\overline{B} \to X_s \gamma)$ closes the possibility of a relatively light $H^\pm$ not embedded in a supersymmetric model.

This summer (1998), a new (preliminary) measurement of this decay rate was reported by the CLEO Collaboration \cite{1} $\text{BR}(\overline{B} \to X_s \gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}$, which, compared to older results, is based on 53% more data ($3.3 \times 10^6 \overline{B} \overline{B}$ events). The upper limit allowed by this measurement, reported in the same paper, is $4.5 \times 10^{-4}$ at 95% C.L.. The ALEPH Collaboration has measured $\text{BR}(\overline{B} \to X_s \gamma) = (3.11 \pm 0.80 \pm 0.72) \times 10^{-4}$ from a sample of $b$ hadrons produced at the $Z$–resonance.

The theoretical prediction for $\text{BR}(\overline{B} \to X_s \gamma)$ has a rather satisfactory level of accuracy in the SM. The main uncertainty, slightly below $\pm 10\%$, comes from the experimental error on the input parameters. The more genuinely theoretical uncertainty, due to the unknown value of the renormalization scale $\mu_b$ and the matching scale $\mu_W$, which was unsatisfactorily large at the leading–order (LO) level, was reduced to roughly $\pm 4\%$ when the NLO QCD corrections to the partonic decay width $\Gamma(b \to X_s \gamma)$ were completed \cite{12}. (See ref. \cite{12} for the milestone papers which brought to the complete LO calculation.) In addition, non–perturbative contributions to $\text{BR}(\overline{B} \to X_s \gamma)$, scaling like $1/m_b^2$ and $1/m_t^2$, were computed. Very recently, the leading QED and some classes of electroweak corrections were also calculated \cite{13,14,15,16}.

Following the procedure described in refs. \cite{12,13,14,15,16}, and including QED corrections as in ref. \cite{15}, we obtain a theoretical prediction \cite{15} in agreement with the existing data:

$$\text{BR}(\overline{B} \to X_s \gamma) = (3.32 \pm 0.14 \pm 0.26) \times 10^{-4}. \quad (1)$$
The first error in [()] is due to the $\mu_u$ and $\mu_W$ scale uncertainties; the second, comes from the experimental uncertainty in the input parameters.

A detailed study of $\overline{B} \to X_s \gamma$ at the NLO in QCD [()] in 2HDMs, on the contrary, shows that the NLO corrections and scale dependences in the Higgs contributions to $\text{BR}(\overline{B} \to X_s \gamma)$ are very large, irrespectively of the value of the charged Higgs couplings to fermions. This feature remains undetected in Type II models, where the SM contribution to $\text{BR}(\overline{B} \to X_s \gamma)$ is always larger than, and in phase with, the Higgs contributions. In this case, a comparison between theoretical and experimental results for $\text{BR}(\overline{B} \to X_s \gamma)$ allows to conclude that values of $m_{H^\pm} = O(m_W)$ can be excluded. Such values are, however, still allowed in other 2HD models.

These issues are illustrated in Sec. 4, after defining in Sec. 2 the class of 2HDMs considered, and presenting the NLO corrections at the amplitude level in Sec. 3. A brief discussion on the existing lower bounds on $m_{H^\pm}$ coming from direct searches at LEP is included in Sec. 3. This rests on observations brought forward in refs. [()] contributed to this conference.

2 Couplings of Higgs bosons to fermions

Models with $n$ Higgs doublets have generically a Yukawa Lagrangian (for the quarks) of the form:

$$-\mathcal{L} = h^d_{ij} \overline{q}^i_L \phi_1 d^c_R j + h^u_{ij} \overline{q}^i_L \phi_2 u^c_R j + h.c. ,$$

(2)

where $q^i_L, \phi_i, (i = 1, 2)$ are SU(2) doublets ($\overline{\phi}_i = i\sigma^2 \phi^*_i$); $u^c_R, d^c_R$ are SU(2) singlets and $h^d, h^u$ denote 3 × 3 Yukawa matrices. To avoid flavour changing neutral couplings at the tree–level, it is sufficient to impose that no more than one Higgs doublet couples to the same right–handed field, as in eq. [()].

After a rotation of the quark fields from the current eigenstate to the mass eigenstate basis, and an analogous rotation of the charged Higgs fields through a unitary $n \times n$ matrix $U$, we assume that only one of the $n - 1$ physical Higgs bosons is light enough to lead to observable effects in low energy processes. The $n$–Higgs doublet model then reduces to a generalized 2HDM, with the following Yukawa interaction for this charged physical Higgs boson denoted by $H^+$:

$$\mathcal{L} = \frac{g}{\sqrt{2}} \left\{ \frac{m_d}{m_W} X_{ij} \overline{V}_{li} V_{lj} d^c_R i + \frac{m_u}{m_W} Y_{ij} \overline{V}_{ri} V_{ij} u^c_R i \right\} H^+. $$

(3)

In [()], $V$ is the Cabibbo–Kobayashi–Maskawa matrix and the symbols $X$ and $Y$ are defined in terms of elements of the matrix $U$ (see citations in ref. [()]). Notice that $X$ and $Y$ are in general complex numbers and therefore potential sources of CP violating effects. The ordinary Type I and Type II 2HDMs (with $n = 2$), are special cases of this generalized class, with $(X, Y) = (-\cot \beta, \cot \beta)$ and $(X, Y) = (\tan \beta, \cot \beta)$, respectively.

We do not attempt to list here the generic couplings of fermions to neutral Higgs fields. In a 2HDM of Type II, the neutral physical states are two CP–even states $h$ and $H (m_h < m_H)$ and a CP–odd state $A$. In this case, only one additional rotation matrix is needed, parametrized by the angle $\alpha$, which is independent of the rotation angle $\beta$ of the charged sector. This independence stops to be true when this model is supersymmetrized since supersymmetry induces a relation between $\tan 2\alpha$ and $\tan 2\beta$.

3 NLO corrections at the amplitude level

The NLO corrections are calculated using the framework of an effective low–energy theory with five quarks, obtained by integrating out the the $t$–quark, the $W$–boson and the charged Higgs boson. The relevant effective Hamiltonian for radiative $B$–decays

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V^*_{ts} V_{tb} \sum_{i=1}^{8} C_i(\mu)O_i(\mu)$$

(4)

consists precisely of the same operators $O_i(\mu)$ used in the SM case, weighted by the Wilson coefficients $C_i(\mu)$. The explicit form of the operators can be seen elsewhere [()].

Working to NLO precision means that one is resuming all the terms of the form $\alpha_s^2(m_b) \ln^n(m_b/M)$, as well as $\alpha_s(m_b) (\alpha_s^2(m_b) \ln^n(m_b/M))$. The symbol $M$ stands for one of the heavy masses $m_W$, $m_t$ or $m_H$ which sets the order of magnitude of the matching scale $\mu_W$. This resummation is achieved through the following 3 steps: 1) matching the full standard model theory with the effective theory at the scale $\mu_W$. The Wilson coefficients are needed at the $O(\alpha_s)$ level; 2) evolving the Wilson coefficients from $\mu = \mu_W$ down to $\mu = m_b$, where $\mu_b$ is of the order of $m_b$, by solving the appropriate renormalization group equations. The anomalous dimension matrix has to be calculated up to order $\alpha_s^2$; 3) including corrections to the matrix elements of the operators $\langle s|O_i(\mu)|b \rangle$ at the scale $\mu = \mu_b$ up to order $\alpha_s$.

Only step 1) gets modified when including the charged Higgs boson contribution to the SM one. The new contributions to the matching conditions have been worked out independently by several groups [()] by simultaneously integrating out all heavy particles, $W$, $t$, and $H^+$ at the scale $\mu_W$. This is a reasonable approximation provided $m_{H^\pm}$ is of the same order of magnitude as $m_W$ or $m_t$.

Indeed, the lower limit on $m_{H^\pm}$ coming from LEP I, of 45 GeV, guarantees already $m_{H^\pm} = O(m_W)$. There exists a higher lower bound from LEP II of 55 GeV for any value of $\tan \beta$ [()] for Type I and Type II models,
which has been recently criticized in ref.\textsuperscript{1}. This criticism is based on the fact that there is no lower bound on $m_A$ and/or $m_h$ coming from LEP\textsuperscript{4}. As it was already mentioned, in 2HDMs (unlike in the MSSM), the two rotation angles of the neutral and charged Higgs sector, $\alpha$ and $\beta$, are independent parameters. Therefore, the pair–production process $e^+e^- \rightarrow Z^* \rightarrow hA$ and the Bjorken process $e^+e^- \rightarrow Z^* \rightarrow Zh$, which are sufficient in the MSSM to put lower limits on $m_h$ and $m_A$ separately, imply in this case only that $m_h + m_A \gtrsim 100$ GeV\textsuperscript{5}.

The other two production mechanisms possible at LEP I (they require larger numbers of events than LEP II can provide) are the decay $Z \rightarrow h/A \gamma$ and the radiation of $A$ off $t$ and $\tau^+\tau^-$ pairs. The latter, allows for sizable rates only for large values of $\tan\beta$ and it yields the constraint $\tan\beta \lesssim 40$ for $m_A = 15$ GeV, in a 2HDM of Type II\textsuperscript{6}.

The former one, limits weakly $\tan\beta$ to be in the range \{0.2, 100\} for $m_{h/A} \approx 10$ GeV\textsuperscript{6}. Indirect searches from the anomalous magnetic moment of the muon also lead to constraints on $\tan\beta$ for a light $h$ and $A$. For a pseudoscalar this limit is stronger than that obtained from the radiation mechanism at LEP I only for $m_A < 2$ GeV\textsuperscript{6}.

Therefore, one of the neutral Higgs bosons can still be light. Charged Higgs bosons pair–produced at LEP II can then decay as $H^+ \rightarrow h/A \gamma$, and the quark level matrix element of the operator $O_{X}$, can be written in the compact form

$$A(b \rightarrow s\gamma) = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \mathcal{D} \langle s\gamma | O_{X} | b \rangle_{\text{tree}}.$$  \hspace{1cm} (5)

(\text{It should be noticed that a subset of Bremsstrahlung contributions was transferred to} \mathcal{D}, \text{as described in ref.\textsuperscript{1}}.) \text{For the following discussion it is useful to decompose the reduced amplitude} \mathcal{D} \text{in such a way that the dependence on the couplings} X \text{and} Y \text{(see eq. (3)) becomes manifest:}

$$\mathcal{D} = \mathcal{D}_{SM} + XY^* \mathcal{D}_{XY} + |Y|^2 \mathcal{D}_{YY}.$$  \hspace{1cm} (6)

In Fig. 1 the individual $\mathcal{D}$ quantities are shown in LO (dashed) and NLO (solid) order, for $m_{H^\pm} = 100$ GeV, as a function of $\mu_b$; all the other input parameters are taken at their central values, as specified in ref.\textsuperscript{4}. To explain the situation, one can concentrate on the curves for $\mathcal{D}_{XY}$. Starting from the LO curve (dashed), the final NLO prediction is due to the change of the Wilson coefficient $C_7$,

shown by the dotted curve, and by the inclusion of the virtual QCD corrections to the matrix elements. This results into a further shift from the dotted curve to the solid curve. Both effects contribute with the same sign and with similar magnitude, as it can be seen in Fig. 1. The size of the NLO corrections in the term $\mathcal{D}_{XY}$ in (6) is

$$\frac{\Delta \mathcal{D}_{XY}}{\mathcal{D}^{LO}_{XY}} = \frac{\mathcal{D}^{NLO}_{XY} - \mathcal{D}^{LO}_{XY}}{\mathcal{D}^{LO}_{XY}} \sim -40\%.$$  \hspace{1cm} (7)

A similarly large correction is also obtained for $\mathcal{D}_{YY}$. For the SM contribution $\mathcal{D}_{SM}$, the situation is different: the corrections to the Wilson coefficient $C_7$ and the corrections due to the virtual corrections in the matrix elements are smaller individually, and furthermore tend to cancel when combined, as shown in Fig. 1.

The size of the corrections in $\mathcal{D}$ strongly depends on the couplings $X$ and $Y$ (see eq. (3)): $\Delta \mathcal{D}/\mathcal{D}$ is small, if the SM dominates, but it can reach values such as $-50\%$ or even worse, if the SM and the charged Higgs contributions are similar in size but opposite in sign.

\section{Results and Conclusions}

The branching ratio $\text{BR}(\mathcal{D} \rightarrow X_s\gamma)$ can be schematically written as

$$\text{BR}(\mathcal{D} \rightarrow X_s\gamma) \propto |\mathcal{D}|^2 + \cdots,$$  \hspace{1cm} (8)

where the ellipses stand for Bremsstrahlung contributions, electroweak corrections and non–perturbative effects. As required by perturbation theory, $|\mathcal{D}|^2$ in eq. (8) should be understood as

$$|\mathcal{D}|^2 = |\mathcal{D}^{LO}|^2 \left[ 1 + 2 \text{Re}\left( \frac{\Delta \mathcal{D}}{\mathcal{D}} \right) \right],$$  \hspace{1cm} (9)

and
Figure 2: $\text{BR}(\bar{B} \to X_s \gamma)$ for $Y = 1$, $m_{H^\pm} = 100$ GeV as a function of $X$, for $\mu_b = 4.8$ GeV (solid), $\mu_b = 2.4$ GeV (dashed) and $\mu_b = 9.6$ GeV (dash-dotted). Superimposed is the range of values allowed by the CLEO measurement.

i.e., the term $|\Delta D/D^{LO}|^2$ is omitted. If $\text{Re}(\Delta D/D^{LO})$ is larger than 50% in magnitude and negative, the NLO branching ratio becomes negative, i.e. the truncation of the perturbative series at the NLO level is not adequate for the corresponding couplings $X$ and $Y$. This happens also for modest values of $X$ and $Y$, as it is illustrated in Fig. 2, where only real couplings are considered. The values $X = 1$ and $X = −1$ in this figure, correspond respectively to the predictions of a the Type II and a Type I 2HDM with $\tan \beta = 1$.

Theoretical predictions for the branching ratio in Type II models stand, in general, on a rather solid ground. Fig. 3 shows the low–scale dependence of $\text{BR}(\bar{B} \to X_s \gamma)$ for matching scale $\mu_W = m_{H^\pm}$, for $m_{H^\pm} > 100$ GeV. It is less than $±10%$ for any value of $m_{H^\pm}$ above the LEP I lower bound of 45 GeV. Such a small scale uncertainty is a generic feature of Type II models and remains true for values of $\tan \beta$ as small as 0.5. In this, as in the following figures where reliable NLO predictions are presented, the recently calculated leading QED corrections are included in the way discussed in the addendum of ref. 1. They are not contained in the result shown in Figs. 1 and 2, which have only an illustrative aim.

In Type II models, the theoretical estimate of $\text{BR}(\bar{B} \to X_s \gamma)$ can be well above the experimental upper bound of $4.5 \times 10^{-4}$ (95% C.L.), reported by the CLEO Collaboration at this conference, leading to constraints in the $(\tan \beta, m_{H^\pm})$ plane. The region excluded by the CLEO bound, as well as by other hypothetical experimental bounds, is given in Fig. 4. These contours are obtained minimizing the ratio $\text{BR}(\bar{B} \to X_s \gamma)/\text{BR}(b \to cl\nu)$, when varying simultaneously the input parameters within their errors as well as the two scales $\mu_b$ and $\mu_W$. For $\tan \beta = 0.5, 1, 5$, we exclude respectively $m_{H^\pm} \leq 280, 200, 170$ GeV, using the present upper bound from CLEO. Notice that the flatness of the curves shown in Fig. 3 towards the higher end of $m_{H^\pm}$, causes a high sensitivity of these bounds on all details of the calculation (see ref. 1). These details can only alter the branching ratio at the 1% level, i.e. well within the estimated theoretical uncertainty, but they may produce shifts of several tens of GeV, in either direction, in the lower bounds quoted above.

Also in the case of complex couplings, the results for $\text{BR}(\bar{B} \to X_s \gamma)$ range from ill–defined, to uncertain, up to reliable. One particularly interesting case in which the perturbative expansion can be safely truncated at the NLO level, is identified by: $Y = 1/2$, $X = 2 \exp(i\phi)$, and $m_{H^\pm} = 100$ GeV. The corresponding branching ratios, shown in Fig. 5, are consistent with the CLEO measurement, even for a relatively small value of $m_{H^\pm}$ in a large
range of $\phi$. Such a light charged Higgs can contribute to the decays of the $t$-quark, through the mode $t \to H^+ b$.

The imaginary parts in the X and Y couplings induce together with the absorptive parts of the NLO loop-functions CP rate asymmetries in $B \to X_s \gamma$. A priori, these can be expected to be large. We find, however, that choices of the couplings X and Y which render the branching ratio stable, induce in general small asymmetries, not much larger than the modest value of 1% obtained in the SM.

We conclude with the most important lessons which can be extracted out of the calculation presented here. The high accuracy reached in the theoretical prediction of $\text{BR}(B \to X_s \gamma)$ at the NLO for the SM, is not a general feature. In spite of its potential sensitivity to new sources of chiral flavour violation, the $B \to X_s \gamma$ decay may turn out to be unconstraining for many models, because of the instability of NLO calculations. This is not only a temporary situation, since it is highly unlikely that a higher order QCD improvement is carried out. Nevertheless, there are scenarios in which $\text{BR}(B \to X_s \gamma)$ can be reliably predicted at the NLO level as in 2HDMs of Type II. In these models, $m_{H^\pm}$ can then be safely excluded up to values which depend on the experimentally maximal allowed value of $\text{BR}(B \to X_s \gamma)$. Today, we find $m_{H^\pm} \gtrsim 165$ GeV. It has to be stressed, however, that in the more general 2HDMs discussed in this article, $H^+$ can be much lighter and may still be detected as a decay product of the $t$-quark.

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