Possible imprints of cosmic strings in the shadows of galactic black holes

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Abstract

We examine the shadow cast by a Kerr black hole pierced by a cosmic string. The observable images depend not only on the black hole spin parameter and the angle of inclination, but also on the deficit angle yielded by the cosmic string. The dependence of the observable characteristics of the shadow on the deficit angle is explored. The imprints in the black hole shadow left by the presence of a cosmic string can serve as a method for observational detection of such strings.

It’s well known that the shadow (or the apparent shape) of a compact relativistic object encodes information about the nature of this object [1]. That is why the apparent shapes of various black holes and other compact objects, such as wormholes and naked singularities, have been intensively studied in the last years. The shadows of the black holes and naked singularities from the Kerr-Newmann family of solutions of the Einstein-Maxwell equations have been thoroughly investigated in [2]-[8]. The shadow of a black hole with a NUT-charge has been obtained in [9]. The black hole shadows in Einstein-Maxwell-dilaton gravity, Chern-Simons modified gravity and braneworld gravity have been examined in [10], [11], [12]. The apparent shape of the Sen black hole has been studied in [13]. The wormhole shadows have been recently investigated in [14], [15].

With the advance of technology the experimental observation of the shadows of compact objects is now possible. Experiments that allow such observations include the Event Horizon Telescope [16], which is a system of earth-based telescopes measuring in the (sub)millimeter wavelength, the space-based radio telescopes RadioAstron and Millimetron [17], [18], and the space-based X-ray interferometer MAXIM [19]. In the next few years these missions are expected to reach resolution high enough to observe the shadow of the supermassive compact object at the center of our galaxy or those located at nearby galaxies [18]. The results of these experiments should be compared with the theoretical models. In this way the observations will reject some of the models or will make it possible to distinguish between different types of compact objects [20]. Even more, the mentioned observations can be used for detection of theoretically predicted objects and effects not observed so far. Such a theoretical prediction is the possible existence

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of cosmic strings. These objects are expected to have formed during phase transitions in the early universe through spontaneous symmetry breakings [21]. It has also been shown that cosmic strings generally form at the end of inflation within the framework of various supersymmetric grand unification theories [22].

A cosmic string makes the space-time around it a conical space-time with a deficit angle \( \delta = 8\pi G \mu / c^4 \) where \( \mu \) is the string tension and \( G \) and \( c \) are the gravitational constant and speed of light, respectively. The deficit angle manifests itself physically by giving rise to interesting phenomena and effects [21], [23]-[27] which can be used for detection of cosmic strings. For example, the gravitational lensing phenomena serve as direct evidence for cosmic strings although none have been detected yet. However, it is unlikely that the pure gravitational lensing of a single string could be measured in the violent astrophysical conditions. It is much more natural to search for signs of cosmic strings in the gravitational lensing by the whole system consisting of the cosmic string and the matter surrounding it. In a galactic context we can consider a model configuration consisting of a central galactic black hole pierced by a cosmic string. The gravitational lensing of such configurations has been recently investigated in [27] and it has been shown that the presence of a cosmic string leaves observable imprints. Then, in view of these results, it is very natural to expect that the presence of a cosmic string would also leave imprints in the shadow of the black hole pierced by it.

With this motivation in mind, we continue the theoretical analysis of the black hole shadows. The aim of the current paper is to investigate the apparent shape of a rotating black hole pierced by a cosmic string, and compare the results with the case of the Kerr black hole. Then we consider the deviations from the Kerr case as a possible test for the existence of cosmic strings on a galactic level.

The metric of the spacetime describing a rotating Kerr black hole pierced along the axis of symmetry by a cosmic string\(^1\) is given by [23]

\[
ds^2 = - \left( 1 - \frac{2Mr}{\rho^2} \right) dt^2 + \rho^2 \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \zeta \frac{\sin^2 \theta}{\rho^2} \left( \zeta \Sigma^2 d\varphi - 4aMr \, dt \right) \, d\varphi, \tag{1}
\]

where \( r, \theta \) and \( \varphi \) are the Boyer-Lindquist coordinates and the metric functions \( \Delta, \rho \) and \( \Sigma \) are defined as usual

\[
\Delta \equiv r^2 - 2Mr + a^2, \quad \rho^2 \equiv r^2 + a^2 \cos^2 \theta, \quad \Sigma^2 \equiv (a^2 + r^2)^2 - a^2 \Delta \sin^2 \theta, \tag{2}
\]

where \( M, a \) and \( \zeta \) are parameters. The parameter \( \zeta \) (\( 0 < \zeta \leq 1 \)) describes the influence of the string on the metric and it is related to the deficit angle of the string by \( \delta = 2\pi (1 - \zeta) \).

In the particular case when \( \zeta = 1 \), the metric reduces to the well-known metric of Kerr. The parameter \( a \) is the angular momentum per unit mass. \( M \), however, does not coincide with the physical mass of the black hole. The physical mass \( M_{\text{phys}} \) and the physical angular momentum \( J_{\text{phys}} \) of the black hole pierced by a cosmic string are given by [23]

\[
M_{\text{phys}} = \zeta M, \quad J_{\text{ phys}} = \zeta J. \tag{3}
\]

\(^1\)For numerical solutions describing rotating black holes with cosmic string hair we refer the reader to [28].
The motion of test particles in spacetime is determined by the geodesic equations or equivalently by the Hamilton-Jacobi equation

\[ -2 \frac{\partial S}{\partial \lambda} = g_{\alpha\beta} \frac{\partial S}{\partial x^\alpha} \frac{\partial S}{\partial x^\beta}, \]

where \( S \) is the particle action, \( \lambda \) is the affine parameter along the geodesics of the metric \( g_{\alpha\beta} \). Since our spacetime described by the metric (1) is stationary and axisymmetric we have two conserved quantities - the energy of the particle \( E \) and its angular momentum \( L_z \) about the axis of symmetry. As in the case of pure Kerr spacetime, we have another conserved quantity, namely the Carter constant \( K \), leading to the separability of the Hamilton-Jacobi equation, which then has a solution of the form

\[ S = \frac{1}{2} m^2 \lambda - E t + L_z \varphi + S_r(r) + S_\theta(\theta), \]

where \( m \) is the mass of the test particle. Using (5) the Hamilton-Jacobi equation reduces to the following equations for \( S_r(r) \) and \( S_\theta(\theta) \):

\[ (S'_r)^2 = \left( \frac{a^2 + r^2}{\Delta} \right) E^2 + \frac{a^2}{\xi^2} L_z^2 - \frac{4aMr}{\Delta} E L_z - \frac{m^2r^2 + K}{\Delta} \equiv R(r), \]

\[ (S'_\theta)^2 = K - m^2a^2 \cos^2 \theta - E^2a^2 \sin^2 \theta - \frac{1}{\xi^2 \sin^2 \theta} L_z^2 \equiv \Theta(\theta). \]

Then (5) takes the form

\[ S = \frac{1}{2} m^2 \lambda - E t + L_z \varphi + \int \sqrt{R(r)} \, dr + \int \sqrt{\Theta(\theta)} \, d\theta. \]

Hence by using the standard procedure we find the null geodesics (i.e. \( m = 0 \)) in the spacetime of a rotating black hole pierced by a cosmic string, namely

\[ \rho^2 \frac{dt}{d\lambda} = \frac{1}{\Delta} \left[ (a^2 + r^2)^2 - \frac{2aMr}{\xi} \right] - a^2 \sin^2 \theta, \]

\[ \rho^2 \frac{dr}{d\lambda} = \sqrt{(a^2 + r^2)^2 + \frac{a^2}{\xi^2} \xi^2 - \frac{4aMr}{\xi} \xi} - \Delta \eta \equiv \sqrt{R}, \]

\[ \rho^2 \frac{d\theta}{d\lambda} = \sqrt{\eta - a^2 \sin^2 \theta - \frac{1}{\xi^2 \sin^2 \theta} \xi^2} \equiv \sqrt{\Theta}, \]

\[ \zeta \rho^2 \frac{d\varphi}{d\lambda} = \frac{\xi}{\sin^2 \theta} - \frac{a}{\Delta} \left( \frac{a}{\xi} \xi - 2Mr \right), \]

with \( \xi \equiv L_z/E \) and \( \eta \equiv K/E^2 \) being the impact parameters. We have also redefined the affine parameter \( E\lambda \rightarrow \lambda \).

The photon orbits are in general of two types - orbits falling into the black hole and others scattered away from the black hole to infinity. An observer far from the black hole will be able to see only the photons scattered away from the black hole, while those
captured by the black hole will form a dark region. This dark region observed on the luminous background is the shadow of the black hole.

The boundary of the black hole shadow is the critical orbit that separates the escape and plunge orbits. In order to find the shadow boundary we reformulate the problem as one-dimensional problem for a particle in an effective potential by rewriting the radial geodesic equation in the form

\[
\left( \rho^2 \frac{dr}{d\lambda} \right)^2 + U_{\text{eff}}(r) = 0,
\]

where \( U_{\text{eff}}(r) = -R(r) \). In this formulation, it is clear that the critical orbit between escape and plunge, which is obviously unstable and circular, corresponds to the highest maximum of the effective potential. The conditions therefore for the critical spherical orbit determining the boundary of the black hole shadow are

\[
U_{\text{eff}} = 0, \quad \frac{dU_{\text{eff}}}{dr} = 0, \quad \frac{d^2U_{\text{eff}}}{dr^2} \leq 0,
\]

or, equivalently, \( R = 0, \quad \frac{dR}{dr} = 0 \) and \( \frac{d^2R}{dr^2} \geq 0 \). Since the effective potential \( U_{\text{eff}} \) (or equivalently \( R \)) depends on \( r \) as well as \( \xi \) and \( \eta \), the conditions (9) give in fact a parametric relation between the impact parameters that should be satisfied on the shadow boundary. Of course, in addition to the conditions above, the impact parameters should be such that \( \Theta(\theta) \geq 0 \).

Taking into account the explicit form of the function \( R \) in our case, the solution to the conditions \( R = 0 \) and \( \frac{dR}{dr} = 0 \), that also satisfies \( \Theta(\theta) \geq 0 \), is given by

\[
\xi = -\frac{\zeta}{a(r-M)} \left[ r^2(r - 3M) + a^2(r + M) \right],
\]

\[
\eta = \frac{2}{(r-M)^2} \left[ r^2(r^2 - 3M^2) + a^2(r^2 + M^2) \right].
\]

The condition \( \frac{d^2R}{dr^2} \geq 0 \) leads to the following explicit inequality

\[
3r^2 + a^2 - \frac{1}{(r-M)^2} \left[ r^2(r^2 - 3M^2) + a^2(r^2 + M^2) \right] \geq 0.
\]

Equations (10) and (11) define the boundary of the shadow in parametric form. It is clear from the derivation that the boundary of the shadow is determined only by the spacetime metric and does not depend on the details of the emission mechanisms.

In real observations, however, what is seen, is in fact the projection of the shadow on the observer’s sky defined as the plane passing through the black hole and normal to the line of sight. Taking this into account, it is more natural to present the shadow boundary in the so-called celestial coordinates \( \alpha \) and \( \beta \). The celestial coordinates are defined by

\[
\alpha = \lim_{r \to \infty} \left( -r^2 \sin \theta_0 \frac{d\varphi}{dr} \right),
\]

\[
\beta = \lim_{r \to \infty} \left( -r^2 \cos \theta_0 \frac{d\varphi}{dr} \right).
\]
\[ \beta = \lim_{r \to \infty} \left( r^2 \frac{d\theta}{dr} \right), \]  
(13)

where the limit is taken along the null geodesics and \( \theta_0 \) is the inclination angle between the axis of rotation of the black hole and the line of sight of the observer. From the definition of the celestial coordinates and using the null geodesics equations (8), we get

\[ \alpha = -\frac{\xi}{\zeta \sin \theta_0}, \]  
\[ \beta = \sqrt{\eta - a^2 \sin^2 \theta_0 - \frac{\xi^2}{\zeta^2 \sin^2 \theta_0}}. \]  
(14)

After substituting (10) in (14), we find

\[ \alpha = \frac{1}{a(r-M)\sin \theta_0} \left[ r^2(r-3M) + a^2(r+M) \right], \]

\[ \beta = \sqrt{\frac{2r^2(r-3M^2) + a^2(r^2+M^2)}{(r-M)^2} - a^2 \sin^2 \theta_0 - \frac{r^2(r-3M) + a^2(r+M)^2}{a^2(r-M)^2 \sin^2 \theta_0}}. \]

(15)

These two equations give in parameteric form the boundary of the shadow in celestial coordinates. Formally (15) coincide with the celestial coordinates for the shadow of the pure Kerr black hole. However, although the string parameter \( \zeta \) does not enter explicitly the equations for the shadow boundary in celestial coordinates, it indeed influences the black hole shadow through the parameter \( M \), which is related to the physical mass of the black hole via eq. (3), i.e. \( M_{\text{phys}} = \zeta M \).

The shadow of the Kerr black hole pierced by a cosmic string is presented in Figs. 1–4 for several inclinations angles, spin and string parameters.

There are two observables that characterize the shadow, namely the radius \( R_s \) of the shadow and the so-called dent \( D_s \). The radius of the shadow is defined as the circle passing through the three points located at the top, bottom and right end of the shadow (see Fig. 5). The other important characteristic, the dent \( D_s \), is defined as the difference between the left end points of the circle and the shadow. It is also useful to define a distortion as \( \delta_s \equiv D_s/R_s \) (see Fig. 6).

For small deficit angles (i.e. for \( \zeta \) close to 1) the shadow of the black hole is very close to that of the Kerr black hole. However with the increase of the deficit angle the radius of the shadow boundary increases as it is seen in Fig. 6. The characteristic deformations of the shadow are also more pronounced for large deficit angles as one can see in Fig. 7.

In general, for a fixed mass \( M_{\text{phys}} \), the shadow radius \( R_s \) and the distortion \( \delta_s \) depend on three parameters - the spin parameter \( a_s = J_{\text{phys}}/M_{\text{phys}}^2 \), the string parameter \( \zeta \) and the inclination angle \( \theta_0 \). For fixed inclination angles, the dependence of \( R_s \) on the spin parameter \( a_s \) for different values of the string parameter \( \zeta \) is shown in Fig. 6. It is seen that the radius \( R_s \) depends weakly on the spin parameter, while the dependence on the string parameter is more strongly expressed.

For fixed \( M_{\text{phys}} \) and \( \theta_0 \) the dependence of the distortion \( \delta_s \) on the spin parameter for different values of the string parameter is shown in Fig 7.

All these results show that the presence of a cosmic string piercing the black holes leaves observable imprints in the shadows of the black holes. If we can measure the mass, the spin parameter and the inclination, then our results and more precisely the dependences shown in Fig. 6 and Fig. 7 allow for a determination of the string parameter \( \zeta \) which is equivalent to the detection of the presence of a cosmic string if \( \zeta \neq 0 \).
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References

[1] H. Falcke, F. Melia, and E. Agol, Astrophys.J. 528, L13 (2000); J.-P. Luminet, Astron. Astrophys. 75, 228 (1979).

[2] J. Bardeen, Black Holes, Edited by C. DeWitt and B.S. DeWitt; Ècole d’été de Physique Théoretique, Les Houches 1972 (Gordon and Breach Science Publishers, New York, 1973).

[3] S. Chandrasekhar, The mathematical theory of black holes (Oxford Univ. Press, 1992).

[4] V. Frolov and A. Zelnikov, Introduction to black hole physics (Oxford Univ. Press, 2011).

[5] P. J. Young, Phys. Rev. D 14, 3281 (1976).

[6] A. de Vries, Class. Quant. Grav. 17, 123 (2000).

[7] R. Takahashi, Astrophys. J. 611, 996 (2004).

[8] K. Hioki and Kei-ichi Maeda, Phys. Rev. D 80, 024042 (2009).

[9] A. Abdujabbarov, F. Atamurotov, Y. Kucukakca, B. Ahmedov and U. Camci, Astrophys. Space Sci. 344, 429 (2013).

[10] L. Amarilla and E. Eiroa, Phys.Rev. D 87, 044057 (2013).

[11] L. Amarilla, E. Eiroa and G. Giribet, Phys. Rev. D 81, 124045 (2010).

[12] L. Amarilla and E. F. Eiroa, Phys. Rev. D 85, 064019 (2012).

[13] K. Hioki and U. Miyamoto, Phys. Rev. D 78, 044007 (2008).

[14] C. Bambi and K. Freese, Phys. Rev. D 79, 043002 (2009).

[15] P. Nedkova, V. Tinchev and S. Yazadjiev, The Shadow of a Rotating Traversable Wormhole (2013) [arXiv:1307.7647[gr-qc]].

[16] http://www.eventhorizontelescope.org

[17] http://www.asc.rssi.ru/radioastron/

[18] T. Johannsen, D. Psaltis, S. Gillessen, D.P. Marrone, F. EOzel, S.S. Doeleman, and V.L. Fish, Astrophys. J. 758, 30 (2012).

[19] http://bhi.gsfc.nasa.gov/maxim-home.html
[20] Z. Li and C. Bambi, Measuring the Kerr spin parameter of regular black holes from their shadow (2013)[arXiv:1309.1606[gr-qc]]

[21] A. Vilenkin and P. Shellard, Cosmic strings and other topological defects, (Cambridge Univ. Press, 1994).

[22] R. Jeannerot, J. Rocher and M. Sakellariadou, Phys. Rev. D 68, 103514 (2003).

[23] D. Galtsov and E. Masar, Class. Quant Grav., 6, 1313 (1989).

[24] E. Hackmann, B. Hartmann, C Lämmerzahl and P. Sirimachan, Phys. Rev. D82, 044024 (2010).

[25] B. Hartmann, C Lämmerzahl and P. Sirimachan, Phys. Rev. D83, 045027 (2011).

[26] T. Clifton and J. Barrow, Phys. Rev. D 81, 063006 (2010).

[27] S.-W. Wei and Y.-X. Liu, Phys. Rev. D85, 064044 (2012).

[28] R. Gregory, D. Kubiznak and D. Wills, JHEP 1306, 023 (2013).

[29] S. Vásquez and E. Esteban, Nuovo Cim., 119B, 489 (2004)
Figure 1: The shadow of the Kerr black hole pierced by a cosmic string (solid line) and the Kerr black hole (dashed line) with inclination angle $\theta_0 = \pi/2$ rad for different rotation and string parameters. The physical mass of both solutions is set equal to 1. The celestial coordinates ($\alpha, \beta$) are measured in the units of physical mass.
Figure 2: The shadow of the Kerr black hole pierced by a cosmic string (solid line) and the Kerr black hole (dashed line) with inclination angle $\theta_0 = \pi/3 \, \text{rad}$ for different rotation and string parameters. The physical mass of both solutions is set equal to 1. The celestial coordinates $(\alpha, \beta)$ are measured in the units of physical mass.
Figure 3: The shadow of the Kerr black hole pierced by a cosmic string (solid line) and the Kerr black hole (dashed line) with inclination angle $\theta_0 = \pi/4$ rad for different rotation and string parameters. The physical mass of both solutions is set equal to 1. The celestial coordinates $(\alpha, \beta)$ are measured in the units of physical mass.
Figure 4: The shadow of the Kerr black hole pierced by a cosmic string (solid line) and the Kerr black hole (dashed line) with inclination angle $\theta_0 = \pi/6$ rad for different rotation and string parameters. The physical mass of both solutions is set equal to 1. The celestial coordinates $(\alpha, \beta)$ are measured in the units of physical mass.
Figure 5: The shadow of Kerr black hole (solid line) and the circle (dashed line) passing through the three points located at the top, bottom and rightmost end of the shadow. The radius of this circle is $R_s$. The difference between the left end points of the circle and the black hole’s shadow is $D_s$. The definition of the distortion parameter is $\delta_s \equiv D_s / R_s$.

Figure 6: The spin parameter $a_*$ of the Kerr black hole pierced by a cosmic string as a function of the circle radius $R_s$ for a few different inclination angles $\theta_0$ and string parameters $\zeta$. 
Figure 7: The spin parameter $a_*$ of the Kerr black hole pierced by a cosmic string as a function of the distortion parameter $\delta_s$ for a few different inclination angles $\theta_0$ and string parameters $\zeta$. 