Radiative seesaw: a case for split supersymmetry

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We revive Witten’s mechanism for the radiative seesaw induced neutrino masses in SO(10) grand unified theory. We propose its extension to charged fermion masses as a possible cure for wrong tree level mass relations. We offer two simple realizations that can produce a realistic fermionic spectrum. The first one requires two 10 dimensional Higgses in the Yukawa sector and utilizes radiative effects for charged fermion masses. The second one trades one 10 for a 120 dimensional Higgs and leads to the SO(10) theory with less parameters in the Yukawa sector. The mechanism works only if supersymmetry is broken at the GUT scale while gauginos and higgsinos remain at TeV. This provides a strong rationale for the so called split supersymmetry.

I. INTRODUCTION

The simplest and the most popular approach for generating small neutrino masses is based on the seesaw mechanism \cite{1}. This scenario must be implemented in a well defined theory in order to be predictive and testable. The SO(10) grand unified theory provides a natural framework since it contains automatically righthanded neutrinos and due to unification constraints restricts the seesaw scale. With the advent of neutrino masses it can be thus argued that SO(10) is actually the minimal realistic grand unified theory. In this context SU(5), which was tailor made for massless neutrinos, becomes cumbersome and ridden by too many parameters.

Among a number of different ways of realizing the seesaw mechanism in SO(10), the one of Witten \cite{2} stands out for its simplicity and beauty. It is based on two-loop radiatively induced and calculable righthanded neutrino masses if the $B-L$ symmetry is broken by a 16 dimensional Higgs multiplet. We call it the radiative seesaw mechanism. Obviously it must fail in any low energy supersymmetric theory due to the nonrenormalization theorem of the
superpotential. Since in the last two decades most of the effort went into supersymmetric grandunification, this appealing approach unfortunately fell from grace. Still, this mechanism is too appealing to be given up. In this letter we revive this approach and, equally important, we extend it to the charged fermion masses. In the process we suggest two simple minimal realizations that could lead to realistic theories.

Regarding low-energy supersymmetry, its main motivation is the control of the gauge hierarchy in perturbation theory. If one is to accept the fine-tuning of the Higgs mass the way one does for the cosmological constant, the scale of supersymmetry breaking becomes a dynamical issue to be determined by the unification constraints. In principle in SO(10) gauge coupling unification needs no supersymmetry at all provided there is an intermediate scale [3]. Actually, even without supersymmetry there is a very appealing mechanism of understanding a gauge hierarchy based on the attractor vacua [4]. The need for low-energy supersymmetry disappears also in the landscape picture where one stops worrying about the smallness of the weak scale. This fits nicely with the anthropic arguments in the favour of the small cosmological constant [5].

On the other hand, if one abandons the need for the perturbative stability of the Higgs mass, grandunification does not tell us what the effective theory at TeV energies relevant for LHC is. This is the burning question and any guidance is badly needed. In the minimal SU(5) theory the options are limited: unification constraints require either low-energy supersymmetry or split supersymmetry with light gauginos and higgsinos and heavy sfermions [6, 7].

The trouble as we said is that SU(5) is not a good theory of neutrino masses and furthermore, it cannot decide between the supersymmetric and the split supersymmetric options above.

The motivation behind this letter is twofold. We wish to construct a simple and predictive realistic theory based on the radiative generation of neutrino masses and, at the same time, we would like to determine the effective low-energy theory. Obviously, as we said, the low-energy theory cannot be supersymmetric, since the righthanded neutrino mass would then be suppressed by the small scale of supersymmetry breaking. We will show though, that the phenomenological and unification constraints lead automatically to split supersymmetry. This provides a strong motivation for a large scale of supersymmetry breaking. The LSP dark matter is then a welcome consequence rather than an input as in the original work.
The original model of Witten utilized a single 10 dimensional Higgs and ended up predicting neither quark nor leptonic mixings and the usual bad mass relations \( m_s = m_{\mu} \) and \( m_d = m_e \) at the GUT scale. Even worse, neutrino masses scale as up quark masses. The failure does not lie in the radiative mechanism of the righthanded neutrino mass, but rather in the oversimplistic Yukawa Higgs sector. In order to get a correct mass spectrum of charged fermions one must complicate the Yukawa sector. One possibility is adding a \( 126_H \) dimensional Higgs representation, which then works successfully at the tree level. This has been worked out in detail in the context of supersymmetric SO(10), but can equally well be implemented in the nonsupersymmetric version. In the radiative mechanism case one should instead add another 10\(_H\) or 120\(_H\). In this work we discuss both versions and show how they promise to offer realistic theories of fermion masses and mixings. It may appear impossible to have a realistic theory with two 10’s due to the fact that the above bad relations apparently do not depend on the number of these multiplets. This is not true though once we go beyond the tree level. We find that the Witten’s radiative approach is readily generalized to light fermions.

The two 10\(_H\)’s version is appealing since the charged fermion masses are corrected radiatively, whereas the version with 120 is attractive due to the smaller number of parameters.

II. THE MODEL

The natural theory to start with is the one with 16\(_H\) (and, normally in supersymmetry one takes also 16\(_H\)) and 45\(_H\) Higgses. This however is not enough, since it can be shown that it leaves SU(5) unbroken. One can simply add a 54\(_H\) or use 210\(_H\), which works by itself. The choice is not so important for \( \nu_R \); what is crucial is to use the 16\(_H\). It may be relevant though for radiatively induced corrections to light fermion masses (see model B below). Either choice leaves the rank unbroken, i.e. at least a \( B-L \) symmetry remains intact (usually also SU(2)\(_R\) remains a good symmetry). The next stage of symmetry breaking is achieved by \( \langle 16_H \rangle = M_R \). Whether or not \( M_R \) lies at \( M_{\text{GUT}} \) is determined by the unification and phenomenological constraints. In this theory one ends up with a single step breaking, i.e. \( M_R = M_{\text{GUT}} \approx 10^{16} \) GeV, due to the neutrino mass considerations. This is discussed below.

On top of that we need a “light” Higgs responsible for the electroweak scale. The simplest
and the most common choice is a $10_H$ dimensional multiplet with the Yukawa interaction schematically

$$\mathcal{L}_Y = 16_F Y_{10} 10_H 16_F.$$  \hspace{1cm} (1)

As is well known, righthanded neutrino masses, being SU(5) singlets, can only arise from a five index antisymmetric $\overline{126}$ representation, missing in this approach. In the language of the SU(2)$_L \times$SU(2)$_R \times$SU(4)$_C$ Pati-Salam symmetry (hereafter denoted as PS) one needs a nonzero vev in the $(1, 3, 10)$ direction. Thus it must be generated radiatively and it can only appear at the two loop level shown in Fig. 1.

![Fig. 1: A contribution to the radiatively generated fermion mass.](image)

One obtains

$$M_{\nu_R} \approx \left(\frac{\alpha}{\pi}\right)^2 Y_{10} \frac{M_R^2}{M_{GUT}}.$$  \hspace{1cm} (2)

Notice that we write $M_R^2/M_{GUT}$ instead of $M_{GUT}$ in (2) in order to be as general as possible. Of course this was a nonsupersymmetric theory. Today we know that this must fail as mentioned in the introduction. The failure of gauge coupling unification in the standard model forces the SU(2)$_R$ breaking scale $M_R$ responsible for righthanded neutrino mass to lie much below $M_{GUT}$: $M_R \approx 10^{13}$ GeV. This in turn leads to too small righthanded
neutrino masses: $\max (m_{\nu_R}) \leq 10^8$ GeV, since from $d = 6$ proton decay constraints $M_{GUT}$ must definitely lie above $10^{15}$ GeV.

This won’t do: light neutrino masses will become generically too large. A possible way out is to give up the predictability and simply fine-tune the Dirac neutrino masses through a complicated enough Yukawa sector. This would be against the the original motivation of calculating and predicting fermion masses and mixings. Furthermore, so light righthanded neutrinos seem to be in contradiction with leptogenesis constraints [11]. Instead it is much more natural to look for a theory with $M_R \approx M_{GUT}$, since the scope of our program is the implementation of the Witten’s mechanism in the minimal and predictive scenario.

Unification constraints then apparently imply low energy supersymmetry, which would kill the radiative effect. The way out of this impasse is quite unique: for the sake of the one-step GUT symmetry breaking one should have light gauginos and higgsinos and at the same time the supersymmetry breaking scale close to $M_{GUT}$ in order to be in accord with neutrino masses.

Thus we need to extend the original radiative mechanism to a (strongly broken) supersymmetric theory. In Fig. 2 we give a typical contribution due to supersymmetric partners in the loops; the others are easily obtained.

FIG. 2: A supersymmetric contribution to the radiatively generated fermion mass. In our notation the tilde stands for the supersymmetric partners, i.e. $\tilde{45}_V$ denotes gauginos, $\tilde{16}_F$ squarks and sleptons and $\tilde{10}_H$ and $\tilde{16}_H$ higgsinos.
In the exact supersymmetric limit of course all the diagrams cancel against each other. Eq. (2) gets simply traded for

\[ M_{\nu_R} \approx \left( \frac{\alpha}{\pi} \right)^2 Y_{10} \frac{M^2_{GUT}}{M_{GUT}} f \left( \frac{\tilde{m}}{M_{GUT}} \right), \]  

where \( \tilde{m} \) is the scale of supersymmetry breaking, or in other words the difference between the scalar and fermion masses of the same supermultiplet. This is valid only for \( \tilde{m} \) not above \( M_{GUT} \). The function \( f(x) \to 0 \) when \( x \to 0 \) and \( f(x) = \mathcal{O}(1) \) if \( x = \mathcal{O}(1) \).

Due to the two loops suppression the only way to have large enough righthanded neutrino masses is through single step symmetry breaking \( M_R \approx M_{GUT} \) and the large \( \tilde{m} \approx M_{GUT} \). Thus, independently of the details of the realistic Yukawa sector, one is forced to the split supersymmetry picture.

If we keep only one \( 10_H \), we will of course have \( m_D = m_L \) and \( m_U = m_{\nu_D} \) for all three generations and the vanishing mixing angles. This is due to the well known quark-lepton symmetry of the \( 10_H \) vev being in the (2,2,1) of the PS symmetry. As a remedy we offer two simple possibilities. The first one uses another \( 10_H \) and the second one interchanges it for \( 120_H \). We describe them now in more detail.

A. Model A

Add another \( 10_H \); this allows for nonvanishing mixings since up and down fermion mass matrices are not anymore proportional to each other. At first glance, though, the above problem of equal down quark and charged lepton masses persists. There is a nice way out however: a radiatively induced (2,2,15) component of the effective \( 126 \) through the two loop diagrams as before, but with light fermions as external states and a small (order electroweak scale) vev of the \( 16_H \):

\[ M_f \approx \left( \frac{\alpha}{\pi} \right)^2 \left( c_1 Y_{10}^{(1)} + c_2 Y_{10}^{(2)} \right) \frac{M_{R}M_{Z}}{M_{GUT}} g \left( \frac{\tilde{m}}{M_{GUT}} \right), \]  

where \( c_i \) contain various numerical factors from the above diagrams and the mixings between between the SU(2)$_L$ doublets in \( 10_H \), and \( 16_H \), while \( g(x) \) has similar properties as \( f(x) \) for \( x \) close to zero and of order 1. These mixings arise from the interactions in the superpotential

\[ W_H = \alpha_i 16_H 10^i_H 16_H. \]  

(5)
The contribution \( m_\mu = -3m_s \) by itself would imply \( M_{\text{GUT}} \), which works very well after being run down to \( M_Z \). We thus propose this radiative mechanism as a possible natural way to obtain correct mass relations for charged fermions. There is more to it: unless there is low-energy supersymmetry such effects should be taken into account even in models that apparently work at the tree level (for example, see below the discussion of model B).

Admittedly, a conspiracy between the tree level and the two loop contributions is needed in order to achieve correct relations for the first two generations. At the same time the gauge coupling at the GUT scale must be large enough: \( (\alpha/\pi)^2 > 10^{-3} \) or so, in order for the muon and the strange quark to weigh enough. This requires the existence of complete SU(5) multiplets at an intermediate scale and is naturally present in many models of the mediation of supersymmetry breaking. The appealing feature of this is an enhancement of the \( d = 6 \) proton decay rate which can make proton decay observable in the near future; see the last reference in [6]. Recall that \( d = 5 \) proton decay is negligible in this version of the split supersymmetry with sfermion masses at the GUT scale. In view of this a detailed analysis of different channel branching ratios of \( d = 6 \) proton decay along the lines of [12] is called for.

On top of that, the righthanded neutrino mass matrix must be presumably rather hierarchical in order to compensate for a tree level hierarchy in \( M_{\nu_D} \). Obviously a careful numerical analysis is needed at this point, but the challenge is highly nontrivial and is beyond the scope of this letter. This is similar (and even more constrained) to the situation encountered in the type I seesaw case in the minimal SO(10) with \( 10_H \) and \( 126_H \) case. There the type II seesaw [13] works very well and offers a natural connection between \( b - \tau \) unification and the large atmospheric mixing angle [14, 15]. Here the type II contribution, although present, is strongly suppressed. It originates from the same type of diagrams as \( M_{\nu_R} \), when the vevs of \( 16_H \) point in the SU(2)_L rather than SU(2)_R direction. While the two loop suppression of \( M_{\nu_R} \) enhances the type I contribution to the seesaw formula, the same loop effect basically kills the type II effect.

It is worth mentioning that \( b - \tau \) unification is natural in this approach due to the tree level dominance of the \( 10_H \) Higgses. Furthermore, the model has the same small number of Yukawa couplings as the minimal renormalizable model with \( 10_H \) and \( 126_H \): \( 15 = 3 + 6 \times 2 \) real parameters [16].
B. Model B

Instead of another $10_H$ one can add a $120_H$ representation. Although a larger representation, it has even less Yukawa couplings, due to its antisymmetric nature in generation space: $9 = 3 + 3 \times 2$ real parameters. The charged fermion masses with $10_H$ and $120_H$ have been studied both analytically and numerically in [17] for even more restrictive choice of parameters. The preliminary study indicates that the theory can work, but we believe that more detailed study is needed, especially since the neutrinos were not included. Some of the effects of $120_H$ were also studied in a model with $10_H$ and $126_H$ Higgses as a subleading effect [18] and for the choice of type II seesaw. Here thus there is an interesting double challenge of less parameters and no choice for the type of seesaw: it must be type I as we stressed above.

At first glance in this case loops seem irrelevant for the charged fermion masses, since there is $(2,2,15)$ effect already at the tree level. However its contribution is antisymmetric in generation space since it originates from $120_H$. Thus the same two loop effects as in the model A that generate a symmetric $(2,2,15)$ in the effective $126_H$ must be included when a careful numerical analysis is performed. In this case there are additional diagrams where the external $16_H$ are traded for say $120_H$ or $120_H$.

C. Some phenomenological issues

Obviously with scalar masses at $M_{GUT}$ the $d = 5$ proton decay operators become completely negligible (it is amusing that even the possible $d = 4$ operators in this case become harmless). In model A the usual $d = 6$ gauge boson induced proton decay is necessarily enhanced by a larger gauge coupling and thus likely observable in the next generation of proton decay experiments [21]. In model B this depends on whether or not there are extra complete multiplets at some intermediate scale. Model A is further characterized by symmetric Yukawa couplings. In this way one can obtain interesting relations among different decay channels [12].

The main characteristic of the split supersymmetry is the cosmologically stable lightest neutralino as the dark matter candidate and a long lived gluino. Gluino lifetime is given by
\[ \tau(\text{gluino}) = 3.10^{-2} s \left( \frac{\tilde{m}}{10^9 \text{GeV}} \right)^4 \left( \frac{1 \text{TeV}}{m_{\text{gluino}}} \right)^5. \] (6)

With \( \tilde{m} \) bigger than \( 10^{15} \text{ GeV} \) as in this theory gluinos lighter than 10 TeV would be cosmologically stable. If gluinos form heavy nuclei, which seems plausible (for a recent analysis see [19] and references therein), such nuclei should have been discovered by now. The lack of such evidence is normally attributed to gluino decay. In our case, this would imply gluino mass above 10 TeV, completely out of LHC reach. Furthermore, one must make sure that gauge couplings still unify, not impossible due to possible GUT scale threshold effects. Another possibility is to appeal to a low reheat temperature after inflation so that gluinos are not produced (or they are washed out).

One of the appealing aspects of low energy or split supersymmetry is the possibility of a neutralino being a dark matter candidate. If gluinos are really stable and thus need to be washed out, one must make sure that the LSP neutralino is not washed out at the same time. This would put an interesting constraint on inflation model building.

### III. CONCLUSIONS AND OUTLOOK

In this letter we made a strong case for the radiative seesaw mechanism. The simplicity and the elegance of this approach makes it definitely worth reviving. We find that the price that needs to be paid to make it work is actually very low: it may be possible to add just another Higgs multiplet, either \( 10_H \) (model A) or \( 120_H \) (model B). Admittedly more work is needed to be sure that either of these models actually fits all the low-energy data; otherwise it may be necessary to complicate further the Yukawa sector.

We also argued that similar radiative effects play an important role for light charged fermion masses. Such effects are necessarily present in the theories with radiative seesaw and they may even provide a cure for the wrong GUT scale relations in the minimal theory. In particular, if the Yukawa sector contains only \( 10_H \)'s, these effects may be sufficient for having correct strange quark and muon masses (and certainly for down quark and electron masses).

This paves way for new class of highly predictive and simple SO(10) models. The immediate important consequence is that supersymmetry must be broken at the GUT scale, but with light gauginos and higgsinos. Our work provides simultaneously a strong rationale
for both radiative seesaw mechanism and split supersymmetry. What makes it particularly appealing is that both scenarios are potentially testable in the near future.

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**Note added** After this work was completed a new paper [20] appeared which discusses radiative generation of fermion masses, but in a quite different approach (utilizing singlet fermions). This paper also contains references to earlier works in the field.

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