A study of the possible interactions between fermions assuming only rotational invariance has revealed 15 forms for the potential involving the fermion spins. We review the experimental constraints on unobserved macroscopic, spin–dependent interactions between electrons in the range below 1 cm. An experiment using 1 kHz mechanical oscillators as test masses has been used to constrain mass–coupled forces in this range. With suitable modifications, this experiment can be used to explore all 15 possible spin–dependent interactions between electrons in this range with unprecedented sensitivity.

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I. INTRODUCTION

The possible existence of unobserved interactions of nature with ranges of mesoscopic scale (microns to millimeters) and very weak couplings to matter has begun to attract a great deal of scientific attention. Many theories beyond the Standard Model possess extended symmetries that, when broken at high energy scales, lead to weakly coupled, light bosons such as axions, familons, and Majorons, which can generate relatively long–range interactions [1]. Several theoretical attempts to explain dark matter and dark energy also produce new weakly coupled long–range interactions. The fact that the dark energy density, of order \((1 \text{ meV})^4\), corresponds to a length scale of \(~100 \mu\text{m}\) encourages searches for new phenomena at this scale in particular [2]. Particles which might transmit such interactions are sometimes referred to generically as WISPs (Weakly-Interacting Sub-eV Particles) [3] in recent theoretical literature, or as “portals” to a hidden sector [4].

A general classification of interactions between non-relativistic fermions assuming only rotational invariance reveals 16 different operator structures [5]. Of these, 15 involve the spin of at least one of the particles and 7 their relative momentum. In general, experimental constraints on unobserved interactions that depend on the spin and/or velocity of the particles are fewer and less stringent than those for static, spin–independent interactions [2]. However, new experimental results from initial searches for the former interactions have accelerated over the last few years. In particular, the velocity–dependent interactions involving the spin of both particles have been constrained at long range using the geomagnetic field [6], and at the atomic scale from an analysis of spin–exchange interactions [7].

One approach to the search for mesoscopic forces uses planar, 1 kHz mechanical oscillators as test masses with a stiff conducting shield in between to suppress backgrounds [8]. With modifications including spin–polarized test masses, this technique can be used to create localized spin sources with non-zero relative velocity. It thus has the capability to probe essentially all of the spin and velocity–dependent interactions described in [5], with unprecedented sensitivity in the mesoscopic range.

This paper is organized as follows. Sec. II reviews the parameterization in [5], as applied to the proposed experiment. The current mesoscopic limits on polarized electron interactions are reviewed in Sec. III. The experiment is described in Sec. IV and the sensitivity calculations in Sec. V.

II. PARAMETERIZATION

In the non–relativistic, zero–momentum transfer limit, the long–range potential \(V_i\) \((i = 1,...,16)\) in the general classification in [5] for single boson exchange depends (in the enumeration in [5]) on 72 dimensionless coupling constants \(f_{i}^{1,2}\). Here, the superscripts denote the species of interacting fermions.

In the experiment described in Sec. IV the polarized particles (that is, the particles with non-zero projection of spin averaged over the volumes of the test masses) are electrons. There are nine components of the spin–spin potential between two polarized electrons. Three are static, given (in SI units, and adopting the numbering scheme in [5]) by:

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\[ V_2 = f_2^e \frac{\hbar c}{4\pi} (\hat{\sigma}_1 \cdot \hat{\sigma}_2) \left( \frac{1}{r} \right) e^{-r/\lambda} \]

\[ V_3 = f_3^e \frac{\hbar^3}{4\pi m_e^2 c} \left[ (\hat{\sigma}_1 \cdot \hat{\sigma}_2) \left( \frac{1}{\lambda r^2} + \frac{1}{r^3} \right) - (\hat{\sigma}_1 \cdot \hat{r})(\hat{\sigma}_2 \cdot \hat{r}) \left( \frac{1}{\lambda^2 r^2} + \frac{3}{\lambda r^2} + \frac{3}{r^3} \right) \right] e^{-r/\lambda} \]

\[ V_{11} = -f_{11}^e \frac{\hbar^2}{4\pi m_e} [ (\hat{\sigma}_1 \times \hat{\sigma}_2) \cdot \hat{r} ] \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda} . \quad (1) \]

Here, \( \hat{s}_{1,2} = \hbar \hat{\sigma}_{1,2}/2 \) are the spins of electrons (in test masses 1 and 2), \( \hat{r} = \hat{r}/r \) is the unit vector along the direction between them, \( \hbar \) is Planck’s constant, \( c \) is the speed of light in vacuum, \( m_e \) is the electron mass, and \( \lambda \) is the interaction range. The remaining six components depend on the relative velocity \( \vec{v} \) of the electrons:

\[ V_{6+7} = -f_{6+7}^{ee} \frac{\hbar^2}{4\pi m_e c} [(\hat{\sigma}_1 \cdot \vec{v})(\hat{\sigma}_2 \cdot \hat{r})] \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda} \]

\[ V_8 = f_8^e \frac{\hbar}{4\pi} [(\hat{\sigma}_1 \cdot \vec{v})(\hat{\sigma}_2 \cdot \vec{v})] \left( \frac{1}{r} \right) e^{-r/\lambda} \]

\[ V_{14} = f_{14}^e \frac{\hbar^2}{4\pi} [(\hat{\sigma}_1 \times \hat{\sigma}_2) \cdot \vec{v}] \left( \frac{1}{r} \right) e^{-r/\lambda} \]

\[ V_{15} = -f_{15}^{ee} \frac{\hbar^2}{8\pi m_e^2 c^2} \left[ (\hat{\sigma}_1 \cdot (\vec{v} \times \hat{r}) \right] (\hat{\sigma}_2 \cdot \hat{r}) + (\hat{\sigma}_1 \cdot \hat{r}) \left[ \hat{\sigma}_2 \cdot (\vec{v} \times \hat{r}) \right] \right] \left( \frac{1}{\lambda^2 r^2} + \frac{3}{\lambda r^2} + \frac{3}{r^3} \right) e^{-r/\lambda} \]

\[ V_{16} = -f_{16}^{ee} \frac{\hbar^2}{8\pi m_e^2 c^2} \left[ (\hat{\sigma}_1 \cdot (\vec{v} \times \hat{r}) \right] (\hat{\sigma}_2 \cdot \vec{v}) + (\hat{\sigma}_1 \cdot \vec{v}) \left[ \hat{\sigma}_2 \cdot (\vec{v} \times \hat{r}) \right] \right] \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda} . \quad (2) \]

There are six components in the case where only one test mass is polarized. The potentials between a polarized electron and an unpolarized atom of atomic number \( Z \) and mass number \( A \) are given by:

\[ V_{4+5} = -Z \left[ f_4^{ee} + f_5^{en} + \left( \frac{A - Z}{Z} \right) f_5^{en} \right] \frac{\hbar^2}{8\pi m_e c} [\hat{\sigma}_1 \cdot (\vec{v} \times \hat{r})] \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda} \]

\[ V_{9+10} = Z \left[ f_9^{ee} + f_1^{en} + \left( \frac{A - Z}{Z} \right) f_1^{en} \right] \frac{\hbar^2}{8\pi m_e} (\hat{\sigma}_1 \cdot \hat{r}) \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda} \]

\[ V_{12+13} = Z \left[ f_6^{ee} + f_7^{en} + \left( \frac{A - Z}{Z} \right) f_7^{en} \right] \frac{\hbar}{8\pi} (\hat{\sigma}_1 \cdot \vec{v}) \left( \frac{1}{r} \right) e^{-r/\lambda} . \quad (3) \]

where \( \hat{r} \) points from the electron to the atom and \( \vec{v} \) is their relative velocity. Following [3], only one linear combination of the separate components in Eq. 3 has been used (as in the expression for \( V_{6+7} \) in Eq. 1), and the coupling constants are given in terms of the \( f_{1,2}^{1,2} \) by:

\[ f_{1}^{1,2} = -f_{4}^{1,2} - f_{5}^{1,2} \]

\[ f_{2}^{1,2} = -f_{9}^{1,2} - f_{10}^{1,2} \]

\[ f_{3}^{1,2} = f_{12}^{1,2} + f_{13}^{1,2} \quad (4) \]

The potentials \( V_{11}, V_{12+13}, \) and \( V_{16} \) violate parity (\( P \)), \( V_{6+7} \) violates time-reversal symmetry (\( T \)), and \( V_{9+10}, V_{14} \) and \( V_{15} \) violate both \( P \) and \( T \). The potentials \( V_3 \) and \( V_{9+10} \) are the dipole–dipole and monopole–dipole interactions studied by Moody and Wilczek [9]. The remaining potential \( V_1 \) corresponds to the well-known Yukawa type between unpolarized objects, to which the sensitivity of the experiment in Sec. I][X] is discussed elsewhere [10].

For the case of spin-0 or spin-1 boson exchange, the coefficients \( f_{1}^{1,2} \) can be expressed in terms of the scalar and pseudoscalar couplings \( g_s, g_p \) or vector and axial couplings \( g_v, g_A \), respectively. The case of single massive spin-0 exchange is derived in [4], as is the case for spin-1 in the context of a massive \( Z' \) boson. The results are summarized in Table II with various simplifications, for
the experiment in Sec. IV.

III. EXPERIMENTAL LIMITS

Fig. 1 shows the experimental limits on static spin–spin interactions between electrons (Eq. 1) in the range between 1 μm and 10 cm. The best limits above 1 cm derive from the spin–polarized torsion pendulum experiment in the Eot–Wash group at the University of Washington, previously used to constrain spin–dependent forces at terrestrial and astronomical ranges 11. The “spin pendulum” consists of an array of Alnico and SmCo permanent magnets arranged so that the orbital moments in the latter cancel the spin moments in the former, resulting in a polarized test mass with negligible external field. A recent shorter–range version of this experiment 12 used a set of similarly–designed spin sources placed 15–20 cm from the pendulum, arranged in several configurations to enhance sensitivity to $V_2$, $V_3$, and $V_{11}$ in Eq. 1.

The results appear to be the first short–range limits for electrons interpreted directly in terms of these potentials. They are reported in 12 as limits on the couplings $(g_2^e)^2$, $(g_3^e)^2$, and $g_1^eg_0^e$, respectively, and are shown in Fig. 1 according to those parameterizations and the $f_{11}^{ee}$. The limits on $f_2^{ee}$ and $f_3^{ee}$ are 1–4 orders of magnitude more sensitive than previous results in the range near 1 cm, and the limit on $f_{11}^{ee}$ appears to be the first such constraint in the range of interest.

Fig. 1 also shows the limits on $f_3^{ee}$ that can be derived from the spin–polarized torsion pendulum at the University of Virginia 13. The spin sources in this experiment consisted of compensated rare earth ferrimagnets, which inspired the proposed experiment in Sec. IV in the form of powder pressed into high–permeability cylinders and polarized along their symmetry axes. The results of the original experiment are reported in terms of a fraction $\alpha$ of the strength of the (infinite–ranged) magnetic dipole–dipole interaction between electrons:

$$\alpha = (1.6 \pm 6.9) \times 10^{-12}. \quad (5)$$

The test cylinders were oriented side-by-side with their axes parallel, a configuration which strongly suppressed the $\hat{r} \cdot \hat{r}$ terms in the dipole–dipole potential and in which the finite–sized test masses could be approximated by point dipoles up to correction factors of order unity. The curve in Fig. 1 is thus obtained by converting the limit on $\alpha$ to a magnetic dipole–dipole energy and equating it to the expression for $V_2$ in Eq. 1, where $r$ is fixed at the 3.4 cm test mass separation reported in 13. The long–range limit of the curve corresponds to the result reported for this experiment in 11.

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1 The exact long-range limit is stronger than the result in 10, on account of an apparent error in Eq. 4.12 of that reference, at

FIG. 1. Projected sensitivity of proposed experiment to static spin–spin interactions (Eq. 1), with current limits and theoretical prediction. Interaction strength according to all parameterizations in Table I is plotted versus the range $\lambda$ (lower axes) and the mass of an unobserved boson (upper axes). Included regions are above the curves. For $V_2$, solid curve is the 1 σ direct limit on $(g_2^e)^2$ 12, also expressed as $f_2^{ee}$. Dashed curve is the limit from 13 re-interpreted in terms of Eq. 1. For $V_3$, lower projected curve corresponds to the case of polarization normal to the test mass surfaces; upper curve to polarization in the test mass planes. Bold solid curve is the direct limit on $(g_3^e)^2$ 12, also re-scaled to $f_3^{ee}$ in Eq. 1. Dashed curves are the limit from 14 and the anomaly from Refs. 15–16 re-interpreted in terms of Eq. 1. Thin solid curve is the prediction for the axion 5. For $V_{11}$, solid curve is the direct limit on $g_1^eg_2^e$ 12, also expressed as $f_{11}^{ee}$. 

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TABLE I. Coefficients $f_s^{\pm}$ in terms of scalar, pseudoscalar, vector, and axial coupling constants for the case of single massive spin-0 and spin-1 boson exchange, following \[9\], as applied to the experiment in Sec. III. The approximation $A = 2Z$ is used in Eq. 3 for couplings to unpolarized masses, which for the case of the proposed experiment (which uses silicon masses) is accurate to within 1%. The results for $f_s^{\pm} + f_s^0 + f_s^{\pm}$ ($s = 1$) and $f_s^{\pm} + f_s^0 + f_s^{\pm}$ ignore additional terms scaled by $m_{e}/m_{p,n}$ and $m_{e}/M$, where $M$ is explained in \[3\].

| Parameter | $s = 0$ | $s = 1$ |
|-----------|---------|---------|
| $f_s^0$   | $g_A^2$ | $g_A^2$ |
| $f_s^0$   | $-\frac{1}{2}(g_A^0)^2$ | $\frac{1}{2}(g_A^0)^2 + (g_A^0)^2$ |
| $f_s^{\pm}$ | $0$ | $g_A^2g_V^2$ |
| $f_s^{\pm}$ | $0$ | $-\frac{1}{2}(g_A^2)^2$ |
| $f_s^{\pm}$ | $0$ | $(g_A^2)^2$ |
| $f_s^{\pm}$ | $0$ | $2g_A^2g_V^2$ |
| $f_s^{\pm} + f_s^0 + f_s^{\pm}$ | $\frac{1}{2}2g_A^0(g_A^0 + g_A^0 + g_A^0)$ | $\frac{1}{2}2(g_A^0)^2 + (g_A^0)^2 + g_A^0g_V^2 + g_A^0g_V^2$ |
| $f_s^{\pm} + f_s^0 + f_s^{\pm}$ | $g_A^0g_A^0 + g_A^0g_A^0 + g_A^0g_A^0$ | $g_A^0g_A^0 + g_A^0g_A^0 + g_A^0g_A^0$ |
| $f_s^{\pm} + f_s^0 + f_s^{\pm}$ | $0$ | $2g_A^2g_V^2$ |

$^a$ This is the more generic notation used or implied in \[9\].

Similarly, mesoscopic limits on $f_s^{\pm}$ can be derived from the experiment by Ni and co-workers at the National Tsing Hua University in Taiwan \[14, 18, 19\]. This experiment used a SQUID magnetometer to monitor the interaction between spin-polarized test masses (also consisting of compensated rare-earth ferrimagnets) and a sample of paramagnetic salt, as the test masses were rotated around the sample at a distance of about 5 cm. The results of this experiment are also reported in terms of the electron magnetic dipole–dipole interaction. The most sensitive result \[14\] is: $\alpha = (1.2 \pm 2.0) \times 10^{-14}$. The test mass polarization was oriented either directly toward or away from the salt, maximizing the contribution from the $\hat{\sigma} \cdot \hat{r}$ terms. The curve in Fig. 4 is thus obtained by converting $\alpha_s$ to a magnetic dipole–dipole energy and equating it to the expression for $V_3$ in Eq. 4.12, with $r$ fixed at 5 cm. Again, the long–range limit of the curve corresponds to the result reported for this experiment in \[9\].

Below about 2 mm, stronger limits on $f_s^{\pm}$ can be inferred from precision measurements of the hyperfine splitting in the ground state of positronium \[15, 21, 22\]. There is currently a $\sim 4\pi$ difference between these measurements and QED theory \[16, 23, 24\]. The horizontal line in the middle plot in Fig. 4 results from equating the energy discrepancy to the expression for $V_3$ in Eq. 4.12, with $r$ fixed at the positronium Bohr radius (0.1 nm). An analogous analysis of the same system has been used to constrain unparticles \[25\].

The $V_3$ plot also shows the prediction for the axion (for the case of a spin–0 interaction), for which there exists an explicit relationship between the coupling strength and the range. The value is derived from \[9\], and also re-scaled to $f_s^{\pm}$ according to Table I.

Fig. 2 shows the limits on velocity–dependent spin–spin interactions (Eq. 2) in the range of interest. For the case of electrons, these interactions appear to be unconstrained in this range. At $\lambda = 1$ km, the lower limit of the range analyzed in \[6\], the constraints on electron interactions range from $10^{-32}–10^{-22}$ for the case of $V_{14}$ and $V_8$, to $10^{-7}–10^{-1}$ for the case of $V_{16}$ and $V_{15}$, with the remaining interactions constrained at $10^{-17}–10^{-12}$.

For comparison, the solid line in the $V_4$ plot is the limit calculated for the nucleon coupling $f_s^{np}$ by a California State University-East Bay collaboration, based on the analysis of atomic spin exchange interaction cross sections \[7\]. The analysis compared the theoretical cross sections, calculated with the usual spin–dependent electromagnetic potentials responsible for spin exchange replaced with potentials of the form in Eqs. 1 and 2 with data from He–Na collisions. The result for $V_8$ is reported in \[7\] as a limit on the coupling $g_A^0g_A^0$, and has been re-scaled in Fig. 2 according to Table I with the additional substitution $\tilde{\sigma}_2 = 3h\tilde{\sigma}_2/2$ in the equation for $V_8$ to account for the Na nuclei which carried the proton spin. The limit has also been extended beyond the micron range reported in \[7\].

Mesoscopic limits on the interactions in Eq. 3 are at least partially confirmed by the authors. The term containing the fine structure constant in that equation is mis-scaled by a factor of $4\pi$ \[9\]. This is compensated somewhat by the larger value of $r$ (10 cm) assumed in \[9\] for the experiment in \[18\] and \[19\], scales as $1/r$, and results in the expected $1/r^3$. The authors of \[9\] suspect this to be a typographical error, which has been confirmed \[20\].

As noted in \[9\], the explicit potential, which appears in \[18\] and \[19\], scales as $1/r$, as opposed to the expected $1/r^3$. The authors of \[9\] suspect this to be a typographical error, which has been confirmed \[20\].
FIG. 2. Projected sensitivity of proposed experiment to velocity–dependent spin–spin interactions (Eq. 2). For comparison, the solid curve in the $V_8$ plot is an extension of the 2 $\sigma$ limit on $g_V^u g_V^p$ for nucleons \[7\], also re-scaled to $f_{np}$ in Eq. 2.

The velocity–dependent interactions $V_{4+5}$ and $V_{12+13}$ appear to be unconstrained for the case of polarized electrons. For comparison, the solid line in the $V_{4+5}$ plot is the limit on the corresponding coupling for polarized nucleons from an experiment at the Paul Scherrer Institute \[20\]. This experiment used Ramsey’s technique of separated oscillatory fields to compare the precession rate of polarized cold neutrons in a beam passing in close proximity to a polished copper plate with the precession of neutrons in a reference beam. The result in \[20\], which assumes no coupling to electrons ($f^{ee}_\perp = 0$) and $f^{np}_\perp = f^{nn}_\perp = f^{nnN}_\perp$, is interpreted as a limit on the coupling $(g_A)^2$; the contribution from any $g_V$ term is assumed negligible given the much stronger short–range constraints on this parameter from torsion pendulum experiments with unpolarized test masses. The limit in Fig. 3 ($\approx (g_A)^2/4$) has been re-scaled in accordance with these assumptions.

Similarly, the solid line in the $V_{12+13}$ plot is the limit on the corresponding coupling for polarized neutrons derived from the neutron spin rotation experiment at NIST \[27\]. This experiment is designed to be sensitive
to the rotation $\phi$ of the polarization of a transversely polarized beam of neutrons passing through a liquid $^4$He target. The rotation $\phi$ arises from a $P$–violating $\phi \bar{\phi}$ term in the forward scattering cross section, whether induced by an interaction such as $V_{12+13}$ or the Standard Model weak interaction to which the experiment is ultimately designed to be sensitive. The analysis in $[27]$ uses the result on $\phi$, currently an upper limit, to constrain $V_{12+13}$. The limit is reported in terms of $g_N g_A^\phi$, where $g_N$ contains a factor $Z = 2$ for $^4$He. Equating the expression for $V_{12+13}$ in $[27]$ to Eq. $3$ for polarized neutrons, and using $Z = 2$, $A = 4$ yields the result $(f_\text{e+} + f_\text{e-}^\text{np} + f_\text{e-}^\text{en} = g_N g_A^\phi)$ in Fig. $3$.

The best mesoscopic limit on the $V_{9+10}$ interaction for electrons derives from the Axion-Like Particle (ALP) torsion pendulum in the Eot-Wash group, which consists of a thin silicon wafer suspended between the two halves of a split toroidal magnet $[28]$. The magnet provides the polarized electrons, and the wafer a source of unpolarized nucleons highly insensitive to the classical magnetic field present. The limit in $[28]$ is reported in terms of $g_N^* g_F$, where $g_N^* = g_N^x = g_N^y = g_N^z / A$ and it is assumed $g_F = 0$. Since the unpolarized mass consists of silicon, the limits in Fig. $3$ ($f_\text{e+} + f_\text{e-}^\text{np} + f_\text{e-}^\text{en} \approx 2g_N^{*} g_F$) are scaled according to Table $1$ with these assumptions, where the dashed line is the projected thermal limit from $[28]$. The same scaling applies to the prediction for the axion, shown in the $V_{9+10}$ plot for the case of an $s = 0$ interaction. The prediction is again from $[30]$, updated to account for the value of $\theta_{QCD}$ $[30]$ inferred from the current best limit on the electric dipole moment of the neutron $[31]$.

Finally, the $V_{9+10}$ plot in Fig. $3$ also shows the indirect limits derived from a combination of data from laboratory experiments and astrophysical arguments $[29]$. These are limits on the coupling $g_N^{*} g_F^S$, i.e., for the case of an $s = 0$ interaction, in which the constraints on $g_N^{*}$ come from short–range gravity experiments with unpolarized test masses $[32]$ $[33]$, and the limit on $g_F^S$ comes from stellar cooling. They have been scaled by the same factor in Fig. $3$ as the limit in $[28]$ to maintain consistency with the results in $[29]$. As noted in $[5]$, analogous constraints on $g_N^{*} g_F^{S}$ can be inferred by combining the same results for $g_N^{*}$ with the stellar cooling limit on $g_F^{S}$. Using $g_F^S \leq 1.3 \times 10^{-14}$ from $[29]$, the resulting limits are shown in the $V_{4+5}$ plot for the case of an $s = 0$ interaction.

IV. PROPOSED EXPERIMENT

The proposed experiment is illustrated in Fig. $4$. It has been used previously to set limits on mass–coupled forces in the mesoscopic range and is described in detail elsewhere $[8]$. The experimental test masses consist of 1 kHz, planar mechanical oscillators with a thin shield between them to suppress backgrounds. The planar geometry is especially efficient for concentrating as much mass as possible at the range of interest. It is nominally null with respect to $1/r^2$ forces and thus effective in sup-

FIG. 3. Projected sensitivity of proposed experiment to interactions between polarized and unpolarized particles (Eq. 3), with current limits and theoretical prediction. For comparison, solid curves in the $V_{4+5}$ and $V_{12+13}$ plots are the direct limits (2 $\sigma$ and 1 $\sigma$, respectively) for the case of polarized neutrons $[26]$ $[27]$. For $V_{9+10}$, the bold solid curve is the 2 $\sigma$ direct limit on $g_N^{*} g_F^S$ $[28]$, also rescaled to $f_\text{e+} + f_\text{e-}^\text{np} + f_\text{e-}^\text{en}$ in Eq. 5. Bold dashed curve is the projected thermal limit. Thin solid curve is the prediction for the axion $[5]$. Lower solid curves in the $V_{4+5}$ and $V_{9+10}$ plots are the indirect limits inferred from stellar cooling arguments $[23]$, the additional projected curves show expected improvements from the proposed experiment with unpolarized test masses $[10]$.
pressing Newtonian backgrounds. The (active) source mass is driven at a resonance frequency of the (passive) detector mass to maximize the signal. Resonant operation places a heavy burden on vibration isolation. The 1 kHz operational frequency is chosen since in this frequency range it is possible to construct a simple, passive vibration isolation system with high dimensional stability \cite{35}, permitting the test mass surfaces to be maintained within a few microns of each other for indefinite periods.

The source mass is a nodally–mounted cantilever driven by a piezoelectric wafer attached in a region of high modal curvature. The detector is a planar double–torsional oscillator originally developed for cryogenic condensed matter physics experiments \cite{37,38}. It consists of 2 coplanar rectangles, joined along their central axes by a short segment. The resonant mode of interest is the first anti-symmetric torsion mode, in which the rectangles half of the small forward rectangle of the detector mass. In other configurations, a similar sample is attached to the underside of the forward part of the source mass. The thin, stiff, conducting shield between the test masses is not shown.

One such material, Dy$_6$Fe$_{23}$, has been used in previous experiments \cite{13,14,18,19,39}. Dy$_6$Fe$_{23}$ is a ferromagnet in which the magnetic moments of the Fe$^{3+}$ and Dy$^{3+}$ ions are aligned opposite to each other, except for a small canting angle. The contributions $M_{Fe}$ and $M_{Dy}$ of the sub-lattices to the magnetization of a sample depend on temperature in such a way that there is a compensation temperature, $T_{cp}$, at which their magnitudes are equal (Fig. 3) but opposite in sign, with a remaining magnetization due to the canting angle. A molecular field parameterization \cite{40} predicts $T_{cp} = 250$ K, close to room temperature. This is a strong reason to consider Dy$_6$Fe$_{23}$, but other materials exhibit compensation temperatures near room temperature, including ferrites \cite{41} and rare-earth iron garnets \cite{42}.

In Dy$_6$Fe$_{23}$, to a first approximation, all of $M_{Fe}$ is caused by the spin of electrons, while half of $M_{Dy}$ is due to orbital motion. Thus, there is an excess of polarized electrons, given by $n_{pol} = M_{Fe}/g_s \mu_B$ with $g_s = 2$ and $\mu_B$ the Bohr magneton. From Fig. 5 $M_{Fe} = 48 \mu_B/\text{Dy}_6\text{Fe}_{23} \sim 10^8$ A/m, which yields an ideal value of $n_{pol} = 5 \times 10^{28}$ m$^{-3}$. The actual electron polarization is expected to be lower because of the effect of non-central inter-lattice fields and because of variations of the sample magnetization; a value of $n_{pol} = 2 \times 10^{27}$ m$^{-3}$ is assumed for the sensitivity calculations (Sec. IV), which is about 10% of the maximum reported for Dy$_6$Fe$_{23}$ \cite{39}.

Spin–polarized materials can be pressed into thin wafers in a precision die, then magnetized in a field of several Tesla and affixed to the test masses. For sensitivity to some of the interactions (Eqs. 1–3), it will be necessary to magnetize the samples in the direction normal to the planes of the test masses. As this may prove difficult in the presence of demagnetizing fields, sensitivity is calculated for the cases of both normal and in-plane polarization when sensitivity to a particular interaction with either polarization is possible.

The effect on the detector $Q$ of attaching the polarized sample slab is unknown. However, silicon test mass prototypes, which are particularly attractive as low–susceptibility substrates for the spin–dependent experiments, have been measured to have $Q$’s as high as $2 \times 10^6$ between 77 K and room temperature. For the purpose of the sensitivity estimates, a very conservative value of $Q = 5000$ is assumed.

To locate the compensation temperature, the experiment can be cooled radiatively with a high-emissivity shield surrounding the central apparatus. The test mass temperatures can be further adjusted with thermoelectric elements. The absolute magnetization of the samples away from the compensation temperature, from which
FIG. 5. **Left:** Magnetization of Dy$_6$Fe$_{23}$ versus temperature, in units of Bohr magnetons per formula unit (one such unit is equivalent to a sample magnetization of about $2 \times 10^4$ A/m). The figure is from [40] and the details explained therein. Here, the magnetization is measured in the presence of an applied field at all temperatures. **Right:** Magnetization versus temperature of a sample magnetized at a single temperature (away from the compensation point, $T_{cp}$), from [13]. The magnetizing field is removed prior to the measurements. The magnetization decreases and crosses zero at $T_{cp}$.

the degree of spin–polarization can be deduced, can be measured using external coils to produce a resonant, calibrated, quasi–uniform magnetic gradient to drive the test masses.

V. PROJECTED SENSITIVITIES

The sensitivity of the experiment is based on the expectation that essentially all experimental backgrounds can be suppressed below the detector thermal noise and amplifier noise. This represents an ultimate practical sensitivity; results with reduced but competitive sensitivity in the presence of other backgrounds are expected to be realized sooner.

Experimental signals are estimated by converting Eqs. 1–3 to forces and integrating them numerically over the test mass geometry, assuming values of 1 for the coupling constants. For simplicity, it is assumed that each of the interactions in Eqs. 1–3 acts independently, as is the case for the limits in Sec. III (Additional limits on the interactions in Eq. 4 are presented in [12], in which this assumption is relaxed.) The thermal noise force due to dissipation in the detector is found from the mechanical Nyquist formula,

$$F_T = \frac{4k_BTm\omega_0}{Q\tau},$$

where $k_B$ is Boltzmann’s constant, $T$ is the temperature, $m$ is the mass of the detector oscillator, $\omega_0$ is the resonance frequency, $Q$ is the mechanical quality factor, and $\tau$ is the experimental integration time. The ratio of this force to the result of the integration of Eqs. 1–3 at each value of $\lambda$ used (that is, a signal–to–noise ratio of 1) yields the sensitivity curves for the coupling constants. Since the experiment is sensitive to changes in the signal as the test mass separation is varied, the integration models the sinusoidal modulation of the source mass and calculates the Fourier amplitudes of the integrated signal. In the thermal noise limit, the amplitude of the oscillations of the detector (mass = $m$) is of order $\sqrt{\frac{k_BT}{(m\omega_0^2)}} \sim 1$ pm (Table III), thus the relative velocity term $\vec{v}$ in Eqs. 1–3 is very well approximated by the source velocity.

To maximize sensitivity at mesoscopic range, a small but reasonable minimum test mass gap (that is, distance of closest approach) is assumed. This is fixed at 120 $\mu$m. This allows for a 100 $\mu$m thick shield between the test masses (40 $\mu$m thicker than the shield used successfully in previous experiments [8], thus reserving space for additional magnetic shielding if needed). For each value of $\lambda$ investigated, the source mass amplitude is optimized for maximum signal. For the static interactions in Eqs. 1–3, this results in values of order $\lambda$. For all $\lambda$ above 1 mm, an amplitude of 1 mm is used, which is taken to represent a practical maximum with the piezoelectric drive technique. The optimization is the same for the velocity-dependent interactions. The exceptions are $V_8$ and $V_{16}$, which, on account of the $v^2$ dependence, increase monotonically with source amplitude at any $\lambda$ over the range of interest. For these interactions, the maximum practical source amplitude of 1 mm is used at each value of $\lambda$.

Sensitivity to all interactions in Eqs. 1–3 is possible

4 On account of the $v^2$ dependence, the principal signals for $V_8$ and $V_{16}$ are at twice the source frequency, for source amplitudes below $\lambda$. Given the narrow detector resonance at $\omega_0$, sensitivity to these interactions is maximized by driving the source at $\omega_0/2$. Since the corresponding reduction in source velocity leads to a reduction of the signal by a factor of 4, this is practical only for the case when the second harmonic exceeds the fundamental
in principle with simple modifications to the test mass geometry and polarization. The different configurations are illustrated in Fig. [3]. For the purposes of the sensitivity calculations, pure vertical translation of the source mass (along the z-axis in Fig. [3]) is assumed, with instantaneous velocity \( v \). This is a good approximation for the planar geometry but there will be small corrections for the actual mode shape of a practical source mass. The six configurations include four in which the spin–polarized material covers only half of the detector mass and the source mass is positioned over that half, so that the resulting force is optimized to excite the sensitive torsional mode of the detector:

C1: Detector and source polarization in–plane and parallel. Presumably the easiest configuration to attain for spin–spin interactions and sensitive to potentials proportional to \( \sigma_1 \cdot \sigma_2 \).

C2: Polarization normal to the test mass planes and parallel to \( \vec{v} \), for optimum sensitivity to \( \sigma_1 \cdot \sigma_2 \) and spin–spin interactions proportional to \( \vec{\sigma} \cdot \vec{r} \) and \( \vec{\sigma} \cdot \vec{v} \).

C3: Polarization normal (detector only) and parallel to \( \vec{v} \), for optimum sensitivity to spin–mass interactions proportional to \( \vec{\sigma} \cdot \vec{r} \) and \( \vec{\sigma} \cdot \vec{v} \).

C4: Polarization in–plane and crossed, for sensitivity to spin–spin interactions proportional to \( (\sigma_1 \times \sigma_2) \cdot \vec{r} \) and \( (\sigma_1 \times \sigma_2) \cdot \vec{v} \).

In the two remaining configurations, the polarized material covers the entire detector surface and the source is centered over the detector, for sensitivity to interactions proportional to \( \vec{v} \times \vec{r} \). The \( \vec{v} \times \vec{r} \) term averages to zero over the surface of the detector in this configuration, however, the associated vector field has the profile of a vortex centered in the detector plane. Thus, for \( \vec{\sigma} \) parallel to the detector torsion axis, a force proportional to \( \vec{\sigma}_1 \cdot (\vec{v} \times \vec{r}) \), while averaging to zero over the entire detector plane, averages to a non-zero value on one side of the torsion axis and the negative of this value on the other, efficiently driving the torsional mode of interest:

C5: Polarization in–plane, parallel to detector torsion axis, for sensitivity to spin–mass interactions proportional to \( \vec{\sigma} \cdot (\vec{v} \times \vec{r}) \).

C6: Polarization mixed, with one parallel to detector torsion axis, for sensitivity to spin–spin interactions proportional to \( [\vec{\sigma}_{1,2} \cdot (\vec{v} \times \vec{r})] [\vec{\sigma}_{2,1} \cdot \vec{r}] \) and \( [\vec{\sigma}_{1,2} \cdot (\vec{v} \times \vec{r})] [\vec{\sigma}_{2,1} \cdot \vec{v}] \).

FIG. 6. Test mass and spin polarization configurations used to search for interactions \( V_2 \)–\( V_{16} \) as assumed in the sensitivity calculations. Here, \( \sigma_1 \) is the net polarization direction of the spins in the detector mass and \( \sigma_2 \) in the source mass. The relative velocity of the spins in each test mass is strongly dominated by the velocity of the source, \( v \). The detector torsion axis is along \( x \). C1: polarization in–plane, parallel \( (V_2, V_3) \). C2: polarization normal \( (V_2, V_3, V_{6+7}, V_6) \). C3: polarization normal, detector only \( (V_{9+10}, V_{12+13}) \). C4: polarization in–plane, crossed \( (V_{11}, V_{14}) \). C5: polarization in–plane, detector only \( (V_{4+5}) \). C6: polarization mixed \( (V_{15}, V_{16}) \). Note that in C1–C4, the source subtends half the detector area and the polarized material \( \sigma_1 \) covers only the detector area subtended. In C5 and C6, the source mass is centered over the detector and \( \sigma_1 \) covers the entire detector area.

Parameters used in the sensitivity calculations are listed in Table [III]. In the event that normal polarization is difficult due to demagnetizing fields, sensitivity will be compromised for interactions \( V_{6+7}, V_6, V_{9+10}, V_{12+13}, V_{15}, \) and \( V_{16} \).

Results for sensitivity to the static spin–spin interactions (Eq. [1]) are shown in Fig. [1]. Assuming the test mass polarization is uniform, the sensitivity to the \( V_2 \) interaction is independent of whether the spins are polarized normal or parallel to the test mass planes. The sensitivity to the \( V_2 \) interaction is comparable to the Eot–Wash and UVA experiments in the range near 1 cm, but many orders more so only a few millimeters below on the account of the small test mass separation.
The projected limit on $V_2$ is the most sensitive relative to the others, by at least 4 orders of magnitude, in the range of interest. The remaining projections can be roughly grouped into three regions of successively decreasing sensitivity, determined by the number of additional factors of $1/r$ or $v/c$ in the expressions for the corresponding interactions (Eqs. [13]) relative to $V_2$.

The sensitivity to the $V_3$ dipole-dipole interaction is about eight orders of magnitude greater than the limit inferred from positronium spectroscopy at 20 μm. The sensitivity is reduced by about 3 orders of magnitude at this range for the case of in-plane polarization. Results for sensitivity to the velocity–dependent spin–spin interactions (Eq. [2]) are shown in Fig. 2. The proposed technique would appear to have unique sensitivity in this range.

Results for sensitivity to interactions between polarized electrons and unpolarized atoms (Eq. [5]) are shown in Fig. [3]. The sensitivity to the $V_{9+10}$ monopole–dipole interaction is about eight orders of magnitude greater than the current experimental limits at 20 μm. The lower dashed curves in the $V_{4+5}$ and $V_{9+10}$ plots are the projected limit on $g_S^N g_S^N$ and $g_S^N g_P^P$, respectively, using the value for $g_S^N$ and $g_P^P$ from stellar cooling [29] and the projected limit on $g_S^N$ from the version of the proposed experiment using dense, unpolarized test masses [10].

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