The proving skill profile of prospective math teacher with high math ability and high math anxiety

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Abstract. Math anxiety often affects students in performing mathematical tasks including the tasks of proving. This qualitative research was to explore the ability of prospective math teacher who have high mathematical ability but with high level of math anxiety in solving math proving problems. The subject of this research was one student of prospective math teacher. Data were collected through tests, assignments, questionnaires and task-based interviews with the researcher himself as the main instrument. The results of this study indicate that: the subject can modify the problem in the form of the theorem appropriately, the subject is less precise in establishing the hypothesis and conclusion, the method used is suitable and can be used to prove the theorem given; the subject can choose the strategy or the means used to implement the method appropriately, the subject is less accurate in formulating the assumptions, the ideas put forward are always equipped with adequate arguments, but the arguments presented are less systematic and logic.

1. Introduction
Proving is one powerful way to build a correct mathematical understanding based on logical and systematic reasoning. Nevertheless, to prove mathematical statements is not an easy matter. Proving processes is an endeavor that demands high-order thinking skills to construct ideas and express them logically and systematically [1]. Proving in mathematics is indeed the most difficult topic to study [2, 3] because to prove mathematical statements require comprehensive understanding and mastery of concepts, facts, procedures, and skills in mathematics. In proving, one must be able to use all his/her mathematical abilities to, in a convincing manner, indicate whether a particular statement is clearly declared right or wrong [4].

Another factor that contributes to proving skill is mathematical anxiety. Math anxiety is the feeling of discomfort and disturbance that some people experience when facing mathematical problems [5]. Many students who suffer from mathematics anxiety lack of a confidence in their ability to do mathematics [6, 7] including in the case of proving. In addition, memory performance is disrupted by mathematical anxiety [8]. Under these conditions, the brain has difficulty constructing logical and systematic arguments to prove mathematical statements conclusively.

Many study results showed that the proving and development of understanding of mathematical proving was still a challenge for many students [9]. Other findings also showed that prospective mathematics teachers have many difficulties in understanding and writing proof [10]. In fact, these skills are very important in mathematics and need to be developed, as a means of logical reasoning. In mathematics learning, students are expected to produce a logical argument and present formal proving
that effectively explains their reasoning [11]. This can be achieved if students master mathematics material well, including proof [12,13]. Through proving the student can verify, explain, communicate, and systematize the statement into the deductive system [14] so that it can be accepted and believed to be true. Therefore, the study of proving becomes urgent to be examined, especially for prospective mathematics teacher, because they will play an important role in learning [15].

2. Method
This research was qualitative research. Data were collected through tests, assignments, and interviews. The instruments used were the Proving Task, Math Capability Test, and Math Anxiety Test. Prior to use, each of these instruments was validated by an expert and tested to ensure that all of these instruments were applicable. Mathematical ability test was given to 48 students of mathematics education program. A student is said to have a high mathematical ability if the score is obtained more than 65, on a scale of 100. Based on the results of the test, only six students have high mathematical ability. All of them are female students. Meanwhile, the mathematical anxiety test consists of 30 questions with four different answers. The answer score range from 1 to 4, so the minimum score is 30 and the maximum is 120. A student is said to have high mathematical anxiety when the score is over 75. Math anxiety test results showed that, only one of those six high math ability students have a high level of math anxiety. Thus, the student was selected as the subject of this research. Subsequently, the selected subject was given the first proving task which contained two questions of proving. After examining the results of her work, the researcher interviewed the subject to clarify the unclear things found in the worksheet. By applying the time triangulation method, two weeks later, the subject was given another second proving task which also contains two questions similar to the two previous questions. Based on the results of her work, researchers interviewed the subject again to clarify the things that was not clear. Furthermore based on the results of the work and the interview, the researcher reduced, displayed, and drew conclusions about the proving skill profile of the subject.

3. Results and discussion

3.1. The Proving Skill in Geometry
To explore the profile of the proving skill in geometry, the subject was asked to do two questions of proving. The two questions are: (1) Prove that if $ABCDEF$ is a regular hexagonal with side length is $a$ unit then the area of $BCEF$ is $\frac{3}{2}a^2\sqrt{3}$ unit; and (2) Prove that the regular hexagonal area with the side length is $a$ is $\frac{3}{2}a^2\sqrt{3}$ unit.

Based on the results of the study it was found that the subject always began the proving by restating what would be proven in the form of the theorem. The subject could formulate the theorem correctly in accordance to the rules of mathematics with reference to the given problem. Furthermore, the subject established the hypothesis (the known element; abbreviated as “Dik”, see Fig. 1) and the conclusion (the element to be proved, abbreviated as “adit”, see Fig. 1) based on the theorem that have been formulated. The subject could determine the hypotheses correctly, but she tended to add unimportant information, so the statement formulation becomes less systematic. For example, in the first question, according to the subject "it is known that the hexagon $ABCDEF$ is formed by six congruent parallel triangles, with the side length is $a$ unit" (see Fig. 1). Actually, the subject simply writes: it is known that the side length of the regular hexagonal is $a$ unit. Nevertheless, the subject added explanations about the properties of the hexagons which could in fact be explained elsewhere.
Figure 1. The subject determine hypotesis and conclusion in first question

Furthermore, based on the predefined hypothesis, the subject chose the strategy to be used. In the field of geometry, according to the given problem, subjects tended to use strategy by drawing. Based on the results of the interview, according to the subject, this image was made to clarify the hypothesized elements and saw the relationship of each other. This strategy is suitable and can be used to solve both problems. Although it seems less symmetrical, the subject could figure her ideas correctly. The drawings were equipped with sufficient explanations so readers could easily understand them (see Figure 2). Furthermore, with reference to the drawing, the subject constructed her arguments to show that conclusions are acceptable. Subjects could show it correctly, but the argument put forward was less systematic. Formulated sentences could not be understood immediately.

(a)  
(b)

Figure 2. (a) Figure made by subject to answer the first question; (b) Figure made by subject to answer the second question.

3.2. The Proving Skill in Algebra
To explore the profile of the proving skill in algebra, the subject was asked to do two questions of proving. The two questions are: (1) Prove that for every integer \( n \), if \( n^2 \) is odd then \( n \) is odd; and (2) Prove that for every integer \( n \), if \( n^2 \) is even then \( n \) is even.

Based on the results of the analysis, the subject always began the proof by restating what is proved in the form of a theorem. The subject could formulate the theorem correctly in the form of an implication with reference to the given problem. The subject argued that these problems could not be proved directly. This argument is true in accordance to the facts, but the argument is not put forward systematically so it is difficult to understand. Because the problem is difficult to prove directly, then the subject used indirect method of proving by articulating the contraposition proof of the given statement. The subject could formulate the contraposition correctly according to the rules in mathematics and with reference to the initial statement. Then on the basis of the contraposition statement, the subject established a hypothesis (known element, abbreviated as "dik") and conclusion (the element to be proved, which is abbreviated as "adit"). The subject could correctly determine hypotheses and conclusions based on the formulated statement of contraposition. Furthermore, the
subject set the assumption on which the proving was based, but the assumption specified was less precise and less systematic. In the first question, the subject assumed \( n = 2k \), which is actually the result of \( n \) is even, not an assumption (see Fig. 3a). Similarly in the second question, the subject assumed that \( n = 2k + 1 \), which is actually a result of \( n \) is odd, nor an assumption (see Fig. 3b). Under prescribed assumption, the subject constructed arguments to show that conclusions are acceptable. Subjects could show it correctly, but the arguments presented are less systematic.

\[ n^2 = (2k)^2 \]

\[ n^2 = (2k + 1)^2 \]

\( \text{Figure 3. (a) The subject set the assumption in the first question; (b) The subject set the assumption in the second question.} \)

3.3. Discussion

From the results of the research described above, it appears that either in the field of geometry or in algebra, the subject always began the proof by reformulate what will be proven in the form of the theorem. The subject could formulate all the theorems correctly based on the given problem. This is a good thing to do to construct mathematical proof. This is consistent with the theory which asserts that the proof must begin with the exact statement of the theorem to be proved [16]. In addition, the subject could correctly identify hypothesis and conclusion elements, but add other non-essential information occasionally so that the explanations given are not systematic. Viewing from the steps taken by the subject in presenting the proof, it seems that the subject understands the methods of proof in mathematics, such as direct proof method and indirect proof method using contraposition. Subjects could apply these methods correctly, but the construction of the resulting argument is less systematic and logic. In addition, the subject was also less precise in formulating the assumptions that become the basis in combining her arguments. The results of these findings are in line with previous findings which suggested that highly math's anxious individuals would be less fluent in mathematical thinking and attitudes [6]. Anxiety is a natural, but if it stays excessive, it will adversely affect the performance of the brain. High math anxiety disturbs students' minds so they cannot think systematically and logically, even though they have a quite good math skills. This claim is clearly demonstrated in this study. The interview reveals that the subject understands what she has to write, but fails to share it properly in communicative written form. For instance, the subject is able to identify the hypothesis and the conclusion of the given question. However, found in her writing, the subject adds another unimportant statement which actually obscures the true meaning. As to show more evidence, the interview also discloses the fact that the subject knows what to assume but still with the absence of her ability to present the assumption systematically. Thus, though holding a well-established argument, the subject finds problem to convey it systematically that leads to difficulties in understanding her writing.

4. Conclusion

High math anxiety is very disturbing one's thinking in doing mathematics tasks included in the case of proving. Therefore, even with good mathematical abilities, students cannot combine their arguments logically and systematically if their minds are haunted by excessive anxiety. This has been demonstrated by the results of this study supported by the results of previous studies. The results of this study indicate that: the subject can modify the problem in the form of the theorem appropriately,
the subject is less precise in establishing the hypothesis and conclusion, the method used is suitable and can be used to prove the theorem given; the subject can choose the strategy or the means used to implement the method appropriately, the subject is less accurate in formulating the assumptions, the ideas put forward are always equipped with adequate arguments, but the arguments presented are less systematic and logic. In general, the subject can do the proof task properly because of her good math skills. However, the ideas put forward are less systematic for her high math anxiety as one of many influencing factors.

5. References
[1] Pantaleon K V, Juniati D and Lukito A 2018 Journal of Physics: Conference Series 947 012070
[2] Mejia-Ramos J P and Inglis M “Argumentative and proving activities in mathematics education research” in Proof and Proving in Mathematics Proceedings of the ICMI Study 19 Conference edited by F. Lin et al. (The Department of Mathematics, National Taiwan Normal University, Taipei, Taiwan, 2009), pp. 88 – 93.
[3] Heinze A 2004 The proving process in mathematics classroom – method and results of a video study Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education Vol 3 pp. 41 – 48.
[4] Polya G 1973 How To Solve It; A New Aspect of Mathematical Method (New Jersey: Princeton University Press) p. 154.
[5] Mutawah M A A 2015 The Influence of Mathematics Anxiety in Middle and High School Students Math Achievement in International Education Studies (Canadian Center of Science and Education, Vol. 8, No. 11 pp. 239 – 252.
[6] Kargar M, Tarmizi R A, and Bayat S, 2010 Procedia Social and Behavioral Sciences 8 pp. 537–542.
[7] Pantaleon K V, Kurnila V S, Tamur M, and Nendi F, 2017 Prosiding Seminar Nasional Matematika dan Aplikasinya pp. 134 – 141.
[8] Ashcraft M H and Kirk E P 2001 Journal of Experimental Psychology: General Vol.130 No.2 224 – 237.
[9] Yackel E and Hanna G 2003 A Research Companion to Principles and Standards for School Mathematics edited by Kilpatrick J, Martin W G, and Schiffer D (Reston: NCTM) p. 231.
[10] Ovez F T Dand Ozdemir E 2014 Procedia - Social and Behavioral Sciences 116 pp. 4075 – 4079.
[11] NCTM Principles and Standards for School Mathematics. (Reston: The National Council of Teacher of Mathematics, Inc.), p. 344.
[12] Pattimukay N, Juniati D, and Budiarto , Journal of Physics: Conference Series, 974(2018)012026.
[13] Ma’rufi, Budayas I K, and Juniati D 2018 Journal of Physics: Conference Series, 954 012002.
[14] Stavrou S G 2014 IUMPST: The Journal. Vol 1, pp. 1-8.
[15] Lestari N D S, Juniati D, and Suwarsono St, 2018 Journal of Physics: Conference Series, 1008 012074.
[16] J M Lee, “Some Remarks on Writing Mathematical Proofs” (University of Washington Mathematics Department, 2012) (https://www.math.washington.edu/~lee/Writing/writing-proofs.pdf).