Dynamic wake tracking and characteristics estimation using a cost-effective LiDAR

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Abstract. Recent developments in Light Detection And Ranging (LiDAR) systems provide the possibility of remote measurement of the upcoming wind speed. Despite significant advances in remote sensing, extracting useful inflow characteristics from a limited number of line-of-sight measurements still requires assumptions of the inflow. Typically, the wind direction is derived based on the assumption of horizontal homogeneous inflow that is well satisfied in flat terrain and over sufficiently large time averages. However, such an assumption is violated if the wake from a neighbouring turbine impinges the inflow, and the velocity deficit in the wake causes a bias on the wind direction, misinterpreting as yaw misalignment by the downstream turbine. The actual yaw misalignment can be recovered by isolating the effect of the wake velocity deficit from the ambient inflow. A scanning LiDAR can easily track and characterise the wake; however, it is non-trivial for a cost-effective LiDAR with only a few fixed laser beams. Therefore, this paper presents a dynamic wake tracking and characteristics estimation algorithm for a cost-effective LiDAR. The proposed algorithm provides estimates of the wake centre location and other wake characteristics by exploiting the nature of wake meandering dynamics and state estimation theory. Assuming neutral stratification of the atmospheric boundary layer, the simulation results show that the wake position and its characteristics estimation is achievable in full and partial wake situations, thus presenting an estimation framework for potential applications, including yaw misalignment control and wake steering control.

1. Introduction
In recent years, the ability to sample an oncoming wind field has been become possible with the advent of the Light Detection And Ranging (LiDAR) systems. The knowledge of the oncoming wind inflow enables the development of advanced turbine control methods, for example, preview control (e.g. [1]), yaw control (e.g. [2]) and wake steering (e.g. [3]). These techniques can potentially lead to an increase in the annual energy production and reduction in loads of wind turbines. Thus, this has sparked recent interest in improving turbine control techniques using real-time wind measurements from the LiDAR systems.

One of the popular research areas is to use the oncoming wind measurements to reduce the turbine yaw misalignment. Maintaining the turbine alignment with the dominant wind direction can increase the power capture and at the same time reduce loads of key turbine components such as the blades and main shaft. Typically, a yaw control system uses the wind direction measurement from a wind vane that are mounted upon the rear of the nacelle. The measurement from a wind vane is inherently subjected to rotor-induced flow distortion due to...
Figure 1: Illustration of a two-beam nacelle-mounted LiDAR in a yaw misaligned situation (left), and partial waked situation (right). From LiDAR measurements $U_{los}^+$, $U_{los}^-$, both situations are not distinguishable, thus, the turbine subjected to the partial wake is then misinterpreted as yaw misaligned.
be estimated based on solely a few fixed point measurements from the LiDAR. Specifically, such an estimation algorithm is developed upon the basis of the dynamic wake meandering model (DWM) and state estimation, as illustrated in Figure 2. The dynamic wake meandering model characterises the wake as a passive tracer driven by the large-scale turbulence structures in the atmospheric boundary layer [10, 11]. Based on the generic wake dynamics, the Kalman filter is employed to estimate not only the dynamic movement of the wake structure but in addition the wake characteristics from the LiDAR measurements. The knowledge of the oncoming or emitting wake enables not only the yaw misalignment correction in a wake situation, but also makes the closed-loop wake steering possible.

2. Methodology

The design of the estimator is depicted in Figure 2. The left part of this figure illustrates a snapshot of a turbine located in the downstream and subjected to a partial wake generated from an upstream turbine. The colour scale from yellow to blue represents the wind speed from high to low. The blue colour illustrates the wake velocity deficit, whereas the yellow colour denotes the ambient turbulent flow field. The LiDAR provides two measurements of the wind speed, represented by the red dots. One of the measurements is clearly in the wake, and the other is in the ambient wind flow. This could contribute to an additional yaw error as one of the LiDAR measurements is corrupted by the wake, if the yaw controller uses the LiDAR measurements. The cross and dash line denotes the estimated location of the wake centre from the estimator and the mean wake deficit Gaussian profile. For brevity, this work only considers a two-dimensional situation, where the wind inflow, wake and LiDAR measurement are at a horizontal plane. The proposed method is self-explanatory to a three-dimension situation.

The right part of Figure 2 illustrates the proposed estimator. Let $x_k$ denotes the actual state (location of the wake centre and other wake information) whilst $\hat{x}_k$ represent their estimates. The crucial role of the estimator is to ensure the state estimate is as close as possible to the actual state. The estimator predicts the current wake centre and its characteristics $\hat{x}_k$ by employing a simplified dynamic model of the wake $f$ and the previous state estimate $\hat{x}_{k-1}$. Based upon the estimated wake centre and its information $\hat{x}_k$, the estimated velocity deficit at the LiDAR measurement point $\hat{y}_k$ is computed by using the wake velocity deficit (Gaussian) profile $h$. The estimated wind speed $\hat{y}_k$ is then subtracted from the actual measurement $y_k$ to form an error signal $e_k$. This error is then multiplied with the filter gain $L_k$ to introduce a correction term that is used in the dynamic wake meandering model in an attempt to minimise the error signals in the subsequent iterations. Notice that the Kalman filter is a recursive filter, and the filter gain is computed that minimises the mean square error $\mathbb{E} \left( (x_k - \hat{x}_k)(x_k - \hat{x}_k)^T \right)$ for given measurements $y_k$, where $\mathbb{E}$ denotes the expectation of sequence and $(\cdot)^T$ is the transpose of a vector.
3. Wake velocity deficit and dynamic wake meandering models

This section presents the simplified models that describe the wake velocity deficit and the dynamics of the wake characteristics. These simplified models are then subsequently used in the estimator.

3.1. Wake velocity deficit model

The wake velocity deficit model is presented, and such a model can be used by the estimator to identify the wake centre and other information based on the wind speed measurement from the LiDAR. From empirical studies [10], the downstream wake velocity deficit model can be approximated as a Gaussian function in a far wake situation. The wind velocity measurement in the wake in longitudinal direction $U_w \in \mathbb{R}$ is defined as follows:

$$ U_w(t) = \bar{U}_\infty - \Delta U(t), \quad (1) $$

where $\bar{U}_\infty \in \mathbb{R}$ denotes the mean free-stream wind velocity, $\Delta U \in \mathbb{R}$ is the longitudinal velocity deficit. This wake velocity deficit can be expressed mathematically as follows [12]:

$$ \Delta U(y_m, y_w, \sigma_w, U_{peak}, t) = U_{peak}(t)e^{-\frac{1}{2}\left(\frac{y_m(t) - y_w(t)}{\sigma_w(t)}\right)^2} + v_1(t), \quad (2) $$

where $\Delta U \in \mathbb{R}$ denotes the longitudinal velocity deficit measured at the lateral distance $y_m \in \mathbb{R}$ from the hub, $y_w$ is the lateral wake centre location, whilst $U_{peak}, \sigma_w \in \mathbb{R}$ are the peak magnitude and standard deviation of the Gaussian wake profile. The measurement noise and modelling uncertainty are denoted as a white noise $v_1 \in \mathbb{R}$.

3.2. Dynamics of the wake structure and its characteristics

The motion of the wake structure is described by the dynamic wake meandering model [11]. The dynamic wake meandering model is based upon the fundamental conjecture that the transport of wakes in the atmospheric boundary layer can be modelled as a passive tracer driven by the large-scale turbulence structures. The meandering process is then described by a stochastic transport media as well as of a suitable definition of the cut-off frequency defining the large-scale turbulence structures.

The wake meandering dynamics in the lateral direction can be modelled as follows [11]:

$$ \dot{y}_w(t) = v_c(t) + n_1(t), \quad (3a) $$

where the $y_w \in \mathbb{R}$ denotes the lateral wake centre location, $\dot{\cdot} := \frac{d\cdot}{dt}$ is the time-derivative and $n_1 \in \mathbb{R}$ is the white noise that represents the modelling uncertainty. The characteristic velocity in the lateral direction is represented by $v_c \in \mathbb{R}$. This characteristic velocity $v_c$ is governed by a stochastic/coloured noise, and its dynamics is similar to a low-pass filter, defined as follows:

$$ \dot{v}_c(t) = -\omega_c v_c(t) + \omega_c n_2(t), \quad (3b) $$

where $n_2 \in \mathbb{R}$ denotes the white noise, and the cut-off frequency $f_c := \frac{\omega_c}{2\pi}$ is determined by the ambient mean wind speed $\bar{U}_\infty \in \mathbb{R}$ in the longitudinal direction and instantaneous wake velocity deficit diameter $D_w \in \mathbb{R}$ (details can be found in [11]):

$$ f_c = \frac{\bar{U}_\infty}{2D_w}. \quad (3c) $$

**Assumption 1** The mean ambient wind speed $\bar{U}_\infty$ is known in this work. The wake velocity deficit diameter $D_w$ is assumed to be constant at the measurement distance. In future work, the mean wind speed $\bar{U}_\infty$ and velocity deficit diameter $D_w$ will be estimated by the algorithm.
Besides, the dynamics of other wake information such as peak magnitude and standard deviation in (2) is much slower than the dynamics of the wake structure movement. Thus, their dynamics is assumed to be constant as follows:

\[ \dot{U}_{\text{peak}}(t) = n_3(t), \quad \dot{\sigma}_w(t) = n_4(t), \]  

where \( n_3, n_4 \in \mathbb{R} \) denote the process noises.

4. State estimation

The state estimator design is based on a celebrated Kalman filtering approach [13]. A Kalman filter provides the optimal state estimates of a linear system by minimising the mean square state error or state error covariance matrix \( P_k := E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \). Given the fact that the models discussed in Section 3 is nonlinear, an extended Kalman filter (EKF) is employed to estimate the states of the system, namely the wake characteristics and centre location. The EKF is similar to Kalman filter, except that it computes the estimates based on the nonlinear equations and determines the state covariance matrix \( P \) by linearising around the current state estimate. Notice that the model in Section 3 is in continuous-time. It is, however, more convenient if the model can be expressed in discrete-time thus a discrete-time EKF can be employed in typical computational hardware. The state-space representation of the models in (2), (3) and (4) is described as follows:

\[ x_{k+1} = f(x_k) + n_k, \quad y_k = h(x_k) + v_k, \]  

where \( f : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_x} \) denotes the dynamical model of the wake meandering dynamics (3) and the dynamics of the wake characteristics (4), whilst \( h : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_y} \) is the measurement model that describes the wake velocity deficit (2), and \( n_k, v_k \) denote the vector of the process and measurement noises. The state \( x_k \in \mathbb{R}^{n_x} \) of the system at sample time \( k \) consists of the lateral location of the wake centre \( y_w \), characteristic lateral velocity \( v_c \), the peak magnitude and standard deviation of the Gaussian profile \( U_{\text{peak}}, \sigma_w \):

\[ x_k := [y_{w,k}, v_{c,k}, U_{\text{peak},k}, \sigma_{w,k}]^T. \]

The measurement \( y_k \in \mathbb{R}^{n_y} \) to the estimator consists the velocity deficits (2):

\[ y_k := [\Delta U_{1,k}, \ldots, \Delta U_{n_y,k}]^T, \]

where \( n_y \) denotes the number of LiDAR beam measurements. The velocity deficits is calculated based on (1) using the LiDAR measurement \( U_w \) at measurement position \( y_m \) subtracting the mean ambient velocity. Notice that the true LiDAR measurement is the line-of-sight velocity \( U_{\text{los}} \) instead of its longitudinal component \( U_w \). The derivation involves the turbine yaw angle, the pan and tilt angles of the LiDAR laser beams. In addition, the mean ambient velocity \( \bar{U}_\infty \) is currently assumed to be known, which might be formulated into the system state in the future work.

4.1. Operations of the EKF

Typically, an EKF consists of two steps, prediction and measurement update. Notice that superscripts \( x^+ \) := \( x_{k|k}, x^- := x_{k|k-1} \) are used and they are the \textit{a posteriori} and \textit{a priori} state estimate respectively, that denote the estimated state variables \( x \) at sample time \( k \) based on the measurement up to \( k \) \( (x^+_k) \) and \( k - 1 \) \( (x^-_k) \), respectively.
Table 1: Root mean square error (RMSE) and variance accounted for (VAF) for the estimated wake centre by the EKF with various tunings and number of LiDAR beams measuring at different lateral locations from the hub.

|           | RMSE [D]       | Weights | $Q_{U_{peak}} = Q_{\sigma_w}$ |
|-----------|----------------|---------|--------------------------------|
|           | $10^{-3}$      | $10^{-4}$| $10^{-5}$ | $10^{-6}$ | $10^{-7}$ |
| Two beams | 0.434 0.338 0.208 0.212 0.251 |
| Four beams| 0.258 0.190 0.159 0.150 0.210 |
| Six beams | 0.139 0.123 0.117 0.122 0.160 |

In the prediction step, the EKF predicts the estimate and error covariance based on the system models and information from the previous step. The predicted/a priori estimates for the state $\hat{x}_k$ and error covariance matrix $P_k$ can be calculated as follows:

$$\hat{x}_k = f(\hat{x}_{k-1}), \quad P_k^- = F_k P_{k-1}^- F_k^T + Q_k, \quad F_k := \frac{\partial f(x_{k-1}^+)}{\partial x}.$$  \hspace{1cm} (6)

where the $\hat{x}_{k-1}^+$ and $P_{k-1}^+$ are the updated/a posteriori state estimate and covariance estimate from the previous step, whilst $Q_k \in \mathbb{R}^{n_x \times n_x}$ is the process noise covariance matrices to the dynamic model, chosen by the designer.

Next, in the measurement update step, the prediction is corrected by exploiting the measurement, yielding the updated estimates $\hat{x}_k^+$ and error covariance $P_k^+$, defined as follows:

$$\hat{y}_k = h(\hat{x}_k), \quad \hat{x}_k^+ = \hat{x}_k + L_k (y_k - \hat{y}_k), \quad P_k^+ = (I - L_k H_k) P_k^-, \quad H_k := \frac{\partial h(x_k^-)}{\partial x},$$ \hspace{1cm} (7)

where $L_k \in \mathbb{R}^{n_y \times n_x}$ is the filter gain. The filter gain is computed as follows:

$$L_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}.$$ \hspace{1cm} (8)

where $R_k \in \mathbb{R}^{n_y \times n_y}$ is the measurement noise covariance matrix that is also a parameter tuned by the designers. Subsequently, the estimate of system states (wake characteristics and wake centre location) $\hat{x}_k^+$, and the velocity deficits $\hat{y}_k$ are the best estimates for measurements up to and including sample time $k$. At the next sample time, the estimator repeats the prediction and measurement update steps.

5. Simulation results
This section demonstrates the performance of the proposed estimation scheme by conducting simulations of the estimator under full and partial wake situations.

5.1. Simulation set-up
The turbine wake in this study was generated using the DWM module within the HAWC2 aeroelastic code [14]. This DWM module has been validated against the experimental data [8,15,16]. In this study, the turbulent wind field was generated by a Mann turbulence model with the mean ambient wind speed of $U_\infty = 16$ m/s and the ambient and meandering turbulence

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intensity of 10%. The turbine used to generate the wake source is a DTU 10MW reference turbine with rotor diameters $D$ of 178.3m [17]. The LiDAR measurements were assumed to be recorded at 3.3 rotor diameters from the wake source, equivalently 0.7D upstream the affected rotor, because the spacing between turbines is chosen based on the Lillgrund wind farm.

5.2. Tuning parameters of the estimator

The tuning parameters of the proposed EKF estimator are the weights $Q$ and $R$ in (6) and (8), that are known as the covariances of the process and measurement noises, respectively. Such covariances are typically unknown; thus, the weights $Q$ and $R$ are often tuned based on empirical experiences. The weight $R$ tends to be chosen small if the measurement noise is believed to be small. Similarly, the weight $Q$ is chosen small if the confidence in the dynamical model is high. In addition, the effect of these matrices $Q$ and $R$ are relative to each other must be considered; thus, the control designers need to find the right balance. The matrix $Q$ is chosen as a diagonal matrix $\text{diag}(Q_{y_w}, Q_{v_c}, Q_{U_{\text{peak}}}, Q_{\sigma_w})$. In this work, the weights for the measurement and for the wake centre dynamics $y_w$ and $v_c$ were fixed as $R = 1$ and $Q_{y_w} = Q_{v_c} = 10^{-2}$. The weights for the peak magnitude $U_{\text{peak}}$ and the standard deviation $\sigma_w$ were investigated.

Figure 3 shows the performance of the proposed wake tracking algorithm under various tunings of $Q_{U_{\text{peak}}}$ and $Q_{\sigma_w}$. The two laser beam measurements were recorded at $y_m = \pm 0.22D$ laterally from the hub. The weights $Q_{U_{\text{peak}}}$ and $Q_{\sigma_w}$ both decreased from $10^{-4}$ to $10^{-6}$. That
implies the dynamics of $U_{\text{peak}}$ and $\sigma_w$ in (4) tend to be more constant, as reflected in second and third rows of the figure. To further the investigation, two metrics - root mean square error (RMSE) and variance accounted for (VAF) of the wake center $y_{w,k}$ - were employed and defined as follows:

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} (\hat{y}_{w,k} - y_{w,k})^2}, \quad \text{VAF} = \left(1 - \frac{\text{var}(y_{w,k} - \hat{y}_{w,k})}{\text{var}(y_{w,k})}\right) \times 100\% \quad (9)$$

where var(.) denotes variance, and $N \in \mathbb{Z}$ denotes the total number of samples. Table 1 summarises the performance of the proposed estimation algorithm applied on 12 different turbulent wind cases including full and partial wakes. The best weights $Q_{U_{\text{peak}}}$ and $Q_{\sigma_w}$ were $10^{-5}$, and the performance deteriorates as the weights reduce further (or equivalently, fixing the value for $U_{\text{peak}}$ and $\sigma_w$). That implies penalising or relaxing the dynamics of the wake characteristics heavily could provide a sub-optimal Gaussian model that might not fit the true wake structure.

5.3. Number of LiDAR measurement beams
This subsection investigates how additional LiDAR measurements could improve the performance of the proposed estimation algorithm. Cases of two, four and six measurement beams were investigated. The lateral measurement locations were at $\pm 0.22\text{D}$ for two beams case, $\pm 0.22, 0.44\text{D}$ for four beam case and $\pm 0.22, 0.44, 0.66\text{D}$ for six beams case. Figure 4 shows...
that with an increasing number of beam measurements, the estimation algorithm performed better as expected. The benefit of adding extra beams became small, for example, from the four beams to six beams case. The numerical results were summarised in Table 1, that also reflects a similar trend.

5.4. Partial wake and measurement locations
This subsection investigates the estimator performance in a partial wake situation, and how the beam measurement locations can improve or deteriorate the tracking performance. Tracking the partial wake is typically more challenging than the full wake. Figure 5 illustrates the performance of the wake tracking algorithm in a partial wake situation using two beams with various measurement locations. The measurement locations were chosen as $y_m = \pm 0.22D$, that is the same configuration as the previous investigation, $y_m = \pm 0.28D$ and $y_m = \pm 0.36D$. As the beams were moving away from the hub, the estimator performance deteriorated. The main reason is that the estimated velocity deficit $\Delta U(y_m)$ was computed based on the measurement locations. A large lateral distance between two measurement points led to a smaller estimated velocity deficit $\Delta U(y_m)$ based on the Gaussian profile. If the size of the estimated velocity deficit $\Delta U(y_m)$ is similar to the turbulent wind speed, the estimator might misinterpret the turbulence in the inflow as wake velocity deficit, thus, providing incorrect state estimates. Therefore, placing the measurement beams closer to the hub could improve the robustness of the estimator to the turbulence.
6. Conclusions

This paper demonstrated a wake tracking algorithm using a cost-effective fixed beam LiDAR with only a few beams. The proposed state estimation strategy computed the best estimates of the wake centre and characteristics using the dynamic wake meandering models and some measurements points. Simulation results were presented that showed the wake tracking algorithm achieved good performance with solely a few fixed measurement points from a LiDAR. Future work will look to investigate the tracking performance of using different dynamic wake meandering model in various wind conditions and validate the framework using large edge simulation (LES) data and experimental data.

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