Technical Report: Outage Performance of Full-Duplex MIMO DF Relaying using Zero-Forcing Beamforming

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Abstract

In this paper, we deal with the performance analysis of full-duplex relaying in decode-&-forward cooperative networks with multiple-antenna terminals. More specifically, by analyzing the end-to-end statistics, we derive the accurate closed-form expressions of the end-to-end outage probability for both transmit and receive ZFBF scheme over Rayleigh fading environments. Some selected results show some interesting observations useful for system designers. Specifically, we observe that the outage performance can be improved by adopting the joint ZF-based precoding with different antenna configurations.

Index Terms

full-duplex relay, MIMO, decode-&-forward, zero forcing beamforming, outage probability

I. INTRODUCTION

Recently, research on cooperative communications with full-duplex (FD) relaying technique to achieve more efficient spectrum resource utilization, in addition to the network coverage extension and higher throughput, has underway as one of active research area [1], [2]. FD technique is considered as an essential component of the coming 5G wireless systems. In FD

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operation, the main drawback is the performance degradation imposed by the loopback self-interference due to signal leakage from the transmitter to the receiver at the relay side [3], [4]. Thus, the priority challenge issue is that of finding the suppression/cancellation techniques of the loopback self-interference. There were considered several approaches including combinations of analogue/digital self-interference (SI) cancellation with RF domain approaches in addition to physical isolation between the transmit and receive antennas, for example, spatial-domain suppression, time-domain cancellation, zero-forcing (ZF) beamforming, power control and so on [1], [3], [5]–[9].

Especially, [7] has dealt with the deployment of FD relaying in amplify-&-forward (AF) cooperative networks with multiple-antenna terminals. More specifically, the joint precoding/decoding design with the rank-1 ZF loopback self-interference suppression which maximizes the end-to-end performance has been proposed with the closed-form precoder/decoder solutions for transmit and receive ZF schemes and related closed-form performance results by solving appropriate optimization problems. In [7], authors showed that with the proposed joint ZF-based precoding, the end-to-end performance can be significantly improved.

There are two distinguished relaying protocols in the literature, such as AF and decode-&-forward (DF). With the AF protocol, an amplified unwanted signal, such as not the interference and noise but also the loop interference in case of FD relaying, can be forwarded to the destination, leading to the noise amplification while the DF protocol provides more reliability for moderate to strong loop interferences by paying more processing compared to the AF protocol because with the DF protocol, the possible performance degradation due to the unwanted signal amplification can be mitigated by decoding the source message and then retransmitting the regenerated decoded message to the destination. Further, for a FD relay terminal, the opti-
mal/suboptimal diversity-multiplexing tradeoff for this multi-hop setup is achievable by DF relaying [10].

Based on above observations, in this paper, we consider the outage performance analysis of FD relaying cooperative networks with multiple-antenna terminals, especially in DF protocol, using ZF beamforming (ZFBF) by adopting the joint precoding/decoding design based on the AF cooperative relaying network in [7]. More specifically, by analyzing the end-to-end statistics, we derive the accurate closed-form expressions of the end-to-end outage probability for both transmit and receive ZFBF scheme over Rayleigh fading environments.

II. SYSTEM AND CHANNEL MODELS

We employ a conventional three-node FD MIMO relay network with DF protocol consisting of one source (S), one relay (R), and one destination (D), as shown in Fig. 1. We assume that S has $N_S$ antennas and D has $N_D$ antennas while R is equipped with $N_{R_2}$ transmit antennas and $N_{R_1}$ receive antennas for FD operation. S has no direct link to D, which may result from heavy path loss and high shadowing between S and D. We also assume that we adopt all known practical RF/analog domain interference cancellation approaches [2], [5] to suppress the SI through the feedback channel. Further, we assume that a single data stream is transmitted. More specifically, S applies a precoding vector $t_S$ on the data stream, while D applies a linear receive vector $t_D$ with $\|t_S\|_F^2 = 1$ and $\|t_D\|_F^2 = 1$, respectively, where $\|\cdot\|_F$ denotes the Frobenius norm.

To suppress remained SI at a FD node, we apply conventional ZF self-interference suppression approaches at R node where ZFBF is an intuitive criterion. Let, for the transmit beamforming

\[ 1 \text{This SI introduced by loop-back channel dominates over the intended reception and cannot be perfectly cancelled in practice.} \]

\[ \text{Note that according to recently published results [2], [5], the SI can be reduced up to a sufficient level (e.g., near 100 dB interference suppression).} \]
scheme, the beamformer has the relay transmit vector, $W_T$, at transmitter side of $R$ with $N_{R_2} \times 1$ and the relay receive vector, $W_R$, at receiver side of $R$ with $N_{R_1} \times 1$, then the SI can be written as $W_R^\dagger H_{RR} W_T$. Here, ZF constraint is that this matrix product of SI is forced into the all zero $\mathbb{Z}$, $\mathbb{I}$, i.e., $W_R^\dagger H_{RR} W_T = 0$, where all possible pairs $(W_R, W_T)$ satisfying the ZF constraint constitute the ZFBF solution set.

We denote that $H_{SR}$ and $H_{RD}$ are the S-R and R-D channels, respectively, while $H_{RR}$ denotes the loopback self-interference channel. We also assume that all the channels between nodes experience block fading. Thus, they remain constant over a long observation time (i.e., time slot), and varies independently from one slot to another. This assumption applies to networks with a low mobility and corresponds to slow fading (block) channels where coding is performed over one block. In addition, all links are subject to non-selective independent Rayleigh block fading and additive white Gaussian noise (AWGN).

Then, let the equivalent S-R and R-D channels be $h_{SR} = H_{SR} t_S$ and $h_{RD} = H_{RD}^\dagger t_D$, respectively, then the input signal at $R$ and the received signal at $R$ can be written as respectively

$$r_{IN} = h_{SR} x_S + H_{RR} W_T x_R + n_{RR}, \quad (1)$$

and

$$r_R = W_R^\dagger r = W_R^\dagger h_{SR} x_S + W_R^\dagger H_{RR} W_T x_R + W_R^\dagger n_{RR}, \quad (2)$$

where $x_S$ is the transmitted symbol at $S$ with zero-mean and average power $E \{|x_S|^2\} = P_S$, , $n_{RR}$ is $N_{R_1} \times 1$ AWGN vector with zero-mean and identity covariance matrix $E \left\{ n_{RR} n_{RR}^\dagger \right\} = I_{N_{R_1}}$, and $(\cdot)^\dagger$ denotes the conjugate transpose.

Here, we add the ZF constraint that the design of $W_T$ and $W_R$ ensure no loopback self-interference for full-duplex operation at $R$. Then, once the ZF constraint is met, the received
signal after $W_R$ at $R$ becomes as

$$r'_R = W_R^\dagger r = W_R^\dagger h_{SR}x_S + W_R^\dagger n_{RR},$$

(3)

with the covariance and the received power at $R$ as, respectively,

- **Covariance:**

$$\sum_R = E\left\{ |r'_R|^2 \right\} = E\left\{ (W_R^\dagger h_{SR}x_S + W_R^\dagger n_{RR}) (x_S^* h_{SR}^\dagger W_R + n_{RR}^* W_R) \right\}$$

$$= E\left\{ W_R^\dagger h_{SR}x_Sx_S^* h_{SR}^\dagger W_R + W_R^\dagger h_{SR}x_S n_{RR}^\dagger W_R + W_R^\dagger n_{RR}x_S^* h_{SR}^\dagger W_R + W_R^\dagger n_{RR}n_{RR}^* W_R \right\}$$

$$= W_R^\dagger h_{SR} E\left\{ |x_S|^2 \right\} h_{SR}^\dagger W_R + W_R^\dagger E\left\{ n_{RR}n_{RR}^* \right\} W_R$$

$$= P_S W_R^\dagger h_{SR} h_{SR}^\dagger W_R + W_R^\dagger n_{RR} n_{RR}^* W_R,$$

(4)

- **Received Power:**

$$\text{Tr} \left( \sum_R \right) = P_S \text{Tr} \left( W_R^\dagger h_{SR} (W_R^\dagger h_{SR})^\dagger \right) + \text{Tr} \left( W_R^\dagger W_R^\dagger \right)$$

$$= P_S \left| W_R^\dagger h_{SR} \right|^2 + \left| W_R^\dagger \right|^2_F$$

$$= P_S \left| W_R^\dagger h_{SR} \right|^2 + 1 = P_S \left| h_{SR}^\dagger W_R \right|^2 + 1,$$

(5)

where $\text{Tr} (\cdot)$ is the trace operation.

Then, for received signal at $D$, similarly, once the ZF constraint is met, the received signal at $D$ can be given as

$$r_D = h_{RD}^\dagger W_T x_R + n_{RD},$$

(6)

where $x_R$ is the relay transmit signal with zero-mean and average power $E\left\{ |x_R|^2 \right\} = P_R$ and $n_{RD}$ is AWGN with zero mean and unit-variance. Here, the covariance and the received power at $D$ are as, respectively,
- **Covariance:**

\[
\sum_{D} = E \{ |r_{D}|^2 \} = E \left\{ \left( h_{RD}^\dagger W_{TR} x_{R} + n_{RD} \right) \left( x_{R}^* W_{TR}^\dagger h_{RD} + n_{RD}^* \right) \right\} \\
= \sum_{D} = E \left\{ h_{RD}^\dagger W_{TR} x_{R} x_{R}^* W_{TR} h_{RD} + h_{RD}^\dagger W_{TR} x_{R} n_{RD}^* + n_{RD} x_{R}^* W_{TR}^\dagger h_{RD} + n_{RD} n_{RD}^* \right\} \\
= E \left\{ h_{RD}^\dagger W_{TR} x_{R} x_{R}^* h_{RD} + n_{RD} n_{RD}^* \right\} \\
= h_{RD}^\dagger W_{TR} E \{ |x_{R}|^2 \} W_{TR} h_{RD} + E \{ n_{RD} n_{RD}^* \} \\
= P_{R} h_{RD}^\dagger W_{TR} W_{TR}^\dagger h_{RD} + 1, \tag{7}
\]

- **Received Power:**

\[
\text{Tr} \left( \sum_{D} \right) = P_{R} \text{Tr} \left( \left( h_{RD}^\dagger W_{TR} \right) \left( h_{RD}^\dagger W_{TR} \right)^\dagger \right) + 1 \\
= P_{R} \left| h_{RD}^\dagger W_{TR} \right|^2 + 1 \overset{\text{or}}{=} P_{R} \left| W_{TR}^\dagger h_{RD} \right|^2 + 1. \tag{8}
\]

Note that as results with (5) and (6), we can adopt the design/analytical approaches used in [7] because our problem is eventually similar to what solve the problem of the joint precoding/decoding design in [7].

### III. Statistical Analysis with Receive ZFBF

Based on [7], [12], we first assume \( W_{T} = h_{RD} \), then \( W_{R} \) should be aligned to the direction of \( h_{SR} \) projected to the orthogonal direction of \( H_{RR} h_{RD} \). Therefore, by applying the similar approach used in [7], \( W_{R} \) can be found from the projection onto the orthogonal space of \( H_{RR} h_{RD} \), with the orthogonal projector onto the left null space of \( H_{RR} h_{RD} \), \( \hat{D} \), such that

\[
\hat{D} \triangleq I_{N_{R_{1}}} - \frac{H_{RR} h_{RD} (H_{RR} h_{RD})^\dagger}{(H_{RR} h_{RD})^\dagger H_{RR} h_{RD}} = I_{N_{R_{1}}} - \frac{H_{RR} h_{RD} h_{RD}^\dagger H_{RR}^\dagger}{\| H_{RR} h_{RD} \|_{F}^2}, \tag{9}
\]

where \( \hat{D} \) is idempotent and \( \frac{H_{RR} h_{RD} (H_{RR} h_{RD})^\dagger}{(H_{RR} h_{RD})^\dagger H_{RR} h_{RD}} \) is the orthogonal projector with rank one (isolate the signal in a single direction \( H_{RR} h_{RD} \)), and \( I_{N_{R_{1}}} - \frac{H_{RR} h_{RD} (H_{RR} h_{RD})^\dagger}{(H_{RR} h_{RD})^\dagger H_{RR} h_{RD}} \) is the orthogonal projector...
with rank \((N_{R_1} - 1)\) (eliminate the signal in the direction \(H_{RR}h_{RD}\)) or it is also called the complementary projector to \(\frac{H_{RR}h_{RD}H_{RR}h_{RD}}{H_{RR}h_{RD}H_{RR}h_{RD}}\).

Once \(W_R\) is determined, the SNR monotonically depends on the quantity \(\|\hat{D}h_{SR}\|_F^2\). Here, \(t_S\) is embedded in \(\|\hat{D}h_{SR}\|_F^2\). Then, by adopting similar approach used in [7], we can also consider that the SNR monotonically depends on the quantity \(\|\hat{D}H_{SR}\|_F^2\) where \(\|\hat{D}H_{SR}\|_F^2 = \lambda_{\text{max}} \left( \left( \hat{D}H_{SR}\right)^\dagger \left( \hat{D}H_{SR} \right) \right) = \lambda_{\text{max}} \left( H_{SR}^\dagger \hat{D} \hat{D} H_{SR} \right) \). Here, because \(\hat{D}\) is idempotent, \(\|\hat{D}H_{SR}\|_F^2\) can be simplified as

\[
\|\hat{D}H_{SR}\|_F^2 = \lambda_{\text{max}} \left( H_{SR}^\dagger \hat{D} H_{SR} \right). \tag{10}
\]

Therefore, we can re-write \(\|\hat{D}H_{SR}\|_F^2\) as

\[
\|\hat{D}H_{SR}\|_F^2 = \lambda_{\text{max}} \left( H_{SR}^\dagger \left( I_{N_{R_1}} - \frac{H_{RR}h_{RD}(H_{RR}h_{RD})^\dagger}{H_{RR}h_{RD}H_{RR}h_{RD}} \right) H_{SR} \right). \tag{11}
\]

Here, \(\frac{H_{RR}h_{RD}(H_{RR}h_{RD})^\dagger}{H_{RR}h_{RD}H_{RR}h_{RD}}\) has rank one. Thus, it can be re-written as

\[
\|\hat{D}H_{SR}\|_F^2 = \lambda_{\text{max}} \left( H_{SR}^\dagger U^\dagger \left( I_{N_{R_1}} - \text{diag}(1,0,\cdots,0) \right) U H_{SR} \right) \tag{12}
\]

\[
= \lambda_{\text{max}} \left( \hat{H}_{SR}^\dagger \text{diag}(0,1,\cdots,1) \hat{H}_{SR} \right),
\]

where \(\hat{H}_{SR} = U H_{SR}\) and \(U\) is unitary matrix. Thus, \(\|\hat{D}H_{SR}\|_F^2\) can be finally re-written as

\[
\|\hat{D}H_{SR}\|_F^2 = \lambda_{\text{max}} \left( \tilde{H}_{SR}^\dagger \tilde{H}_{SR} \right), \tag{13}
\]

where \(\tilde{H}_{SR}\) is \((N_{R_1} - 1) \times N_S\) matrix. Here, based on our channel model assumptions, \(\hat{H}_{SR}^\dagger \tilde{H}_{SR}\) follows the Wishart distribution. As a result, \(\|\hat{D}H_{SR}\|_F^2\) is the maximum eigenvalue of a Wishart matrix \(\tilde{H}_{SR}^\dagger \tilde{H}_{SR}\) with dimension \((N_{R_1} - 1) \times N_S\).

Then, the SNRs, \(\gamma_{SR,1}\) and \(\gamma_{RD,1}\), can be finally written as

\[
\gamma_{SR,1} = \gamma_{SR_{\text{max}},1} = P_S \|\hat{D}H_{SR}\|_F^2 = P_S \lambda_{\text{max}} \left( \tilde{H}_{SR}^\dagger \tilde{H}_{SR} \right) = P_S \lambda_{SR_{\text{max}},1}, \tag{14}
\]
and

\[ \gamma_{RD,1} = \gamma_{RD,max,1} = P_R \| H_{RD} \|_F^2 = P_R \Lambda_{RD,max,1}. \]  \hspace{1cm} (15) \]

Here, based on channel assumptions in Sec. II, the links are subject to i.i.d. Rayleigh block fading with the average SNRs, \( \gamma_{SR} \) and \( \gamma_{RD} \). It means that all the channels are stationary during a single transmission and identical.

Therefore, with the help of [13], the PDF expression of \( \Lambda_{SR,max,1} \) and \( \Lambda_{RD,max,1} \) can be written as

\[ f_{\Lambda_{SR,max,1}}(\gamma) = \sum_{n=1}^{\min(N_S, N_{R_1} - 1)} \sum_{m=|N_S - (N_{R_1} - 1)|} \frac{D_{n,m}^1}{m!} \left( \frac{n}{\gamma_{SR}} \right)^{m+1} \gamma^m \exp \left( -\frac{n}{\gamma_{SR}} \cdot \gamma \right), \]  \hspace{1cm} (16) \]

and

\[ f_{\Lambda_{RD,max,1}}(\gamma) = \sum_{k=1}^{\min(N_{R_2}, N_D)} \sum_{l=|N_{R_2} - N_D|} \frac{C_{k,l}^1}{l!} \left( \frac{k}{\gamma_{RD}} \right)^{l+1} \gamma^l \exp \left( -\frac{k}{\gamma_{RD}} \cdot \gamma \right). \]  \hspace{1cm} (17) \]

Then, by transforming density functions of (16) and (17) with (14) and (15), the PDF expressions of \( \gamma_{SR,1} \) and \( \gamma_{RD,1} \) can be finally obtained as

\[ f_{\gamma_{SR,1}}(x) = f_{\Lambda_{SR,max,1}} \left( \frac{x}{P_S} \right) \cdot \frac{1}{P_S} \]

\[ = \sum_{n=1}^{\min(N_S, N_{R_1} - 1)} \sum_{m=|N_S - (N_{R_1} - 1)|} \frac{D_{n,m}^1}{m!} \left( \frac{n}{P_S \gamma_{SR}} \right)^{m+1} x^m \exp \left( -\frac{n}{P_S \gamma_{SR}} \cdot x \right), \]  \hspace{1cm} (18) \]

and

\[ f_{\gamma_{RD,1}}(x) = f_{\Lambda_{RD,max,1}} \left( \frac{x}{P_R} \right) \cdot \frac{1}{P_R} \]

\[ = \sum_{k=1}^{\min(N_{R_2}, N_D)} \sum_{l=|N_{R_2} - N_D|} \frac{C_{k,l}^1}{l!} \left( \frac{k}{P_R \gamma_{RD}} \right)^{l+1} x^l \exp \left( -\frac{k}{P_R \gamma_{RD}} \cdot x \right). \]  \hspace{1cm} (19) \]

Here, \( D_{n,m}^1 = c_{n,m} K_{a,b} \frac{m!}{m+n-1} \) and \( C_{k,l}^1 = c_{k,l} K_{a,b} \frac{n!}{k+l} \) in [13] where \( K_{a,b} = \frac{1}{\prod_{i=1}^{n} (a-i)! (b-i)!} \), \( a = \min(N_S, N_{R_1} - 1) \), and \( b = \max(N_S, N_{R_1} - 1) \) and the coefficients \( c_{n,m} \) and \( c_{k,l} \) are determined by applying a curve fitting on the plot of \( \frac{d}{da} a \times a \) Hankel matrix in [13]. Note that \( D_{n,m}^1 \) and
C_{k,l}^1$ can be computed with the help of [13], [14] by using most symbolic softwares such as Maple, Mathematica, or Matlab. In Appendix A for users’ convenience, we provide the Matlab based code based on the proposed algorithm in [14]. With this code, we can directly compute the exact values of the coefficients instead of any curve fitting or approximation.

### IV. Statistical Analysis with Transmit ZFBF

In this case, similar to receive ZF case in Sec. III, by fixing $W_R = h_{SR}$, $W_T$ should be aligned to the direction of $h_{RD}$ projected to the orthogonal direction of $H_{RR}^\dagger h_{SR}$. Therefore, $W_T$ can be found from the projection onto the orthogonal space of $H_{RR}^\dagger h_{SR}$ with the orthogonal projector onto the left null space of $H_{RR}^\dagger h_{SR}$, $\hat{B}$, such that

$$\hat{B} \triangleq I_{N_{R_2}} - \frac{H_{RR}^\dagger h_{SR} (H_{RR}^\dagger h_{SR})^\dagger}{H_{RR}^\dagger h_{SR} H_{RR}^\dagger} = I_{N_{R_2}} - \frac{H_{RR}^\dagger h_{SR} H_{RR}^\dagger}{\|H_{RR}^\dagger h_{SR}\|_F^2},$$  \hspace{1cm} (20)

where $\hat{B}$ is idempotent, $\frac{H_{RR}^\dagger h_{SR} (H_{RR}^\dagger h_{SR})^\dagger}{H_{RR}^\dagger h_{SR} H_{RR}^\dagger}$ is the orthogonal projector with rank one, and $I_{N_{R_2}} - \frac{H_{RR}^\dagger h_{SR} (H_{RR}^\dagger h_{SR})^\dagger}{H_{RR}^\dagger h_{SR} H_{RR}^\dagger}$ is the orthogonal projector with rank $(N_{R_2} - 1)$. Then, similarly, as a result, $W_T$ is determined, we can also consider that the SNR monotonically depends on the quantity $\|\hat{B} h_{RD}\|_F^2$, where $\|\hat{B} h_{RD}\|_F^2 = \lambda_{\text{max}} \left( H_{RD}^\dagger \hat{B} h_{RD} \right)$. Then, $\|\hat{B} h_{RD}\|_F^2$ becomes

$$\|\hat{B} h_{RD}\|_F^2 = \lambda_{\text{max}} \left( H_{RD}^\dagger U (I_{N_{R_2}} - \text{diag}(1,0,\cdots,0)) U h_{RD} \right)$$

$$= \lambda_{\text{max}} \left( \tilde{H}_{RD}^\dagger \text{diag}(0,1,\cdots,1) \tilde{H}_{RD} \right),$$  \hspace{1cm} (21)

where $\tilde{H}_{RD} = U h_{RD}$ and $U$ is unitary matrix. Then, $\|\hat{B} h_{RD}\|_F^2$ can be finally re-written as

$$\|\hat{B} h_{RD}\|_F^2 = \lambda_{\text{max}} \left( \tilde{H}_{RD}^\dagger \tilde{H}_{RD} \right),$$  \hspace{1cm} (22)

where $\tilde{H}_{RD}$ is $(N_{R_2} - 1) \times N_D$ matrix. As a result, $\|\hat{B} h_{RD}\|_F^2$ is the maximum eigenvalue of a Wishart matrix $\left( \tilde{H}_{RD}^\dagger \tilde{H}_{RD} \right)$ with dimension $(N_{R_2} - 1) \times N_D$. Then, similar to receive ZF
case, the SNRs, $\gamma_{SR,1}$ and $\gamma_{RD,1}$, can be finally written as

$$\gamma_{SR,2} = \gamma_{SR,\max,2} = P_S \|\mathbf{H}_{SR}\|_F^2 = P_S \Lambda_{SR,\max,2}$$

with dimension $(N_R \times N_S)$, \(23\)

$$\gamma_{RD,2} = \gamma_{RD,\max,2} = P_R \|\mathbf{\hat{H}}_{RD}\|_F^2 = P_R \Lambda_{\max} \left(\mathbf{\hat{H}}_{RD} \mathbf{\hat{H}}_{RD}^H\right) = P_R \Lambda_{RD,\max,2}.$$ \(24\)

Therefore, similar to receive ZF case, the PDF expressions of $\Lambda_{SR,\max,2}$ and $\Lambda_{RD,\max,2}$ can be also written as

$$f_{\Lambda_{SR,\max,2}}(\gamma) = \sum_{n=1}^{\min(N_S,N_{R_1})} \sum_{m=\lceil N_S - N_R \rceil} \frac{D_{n,m}}{m!} \left(\frac{n}{\gamma_{SR}}\right)^{m+1} \gamma^m \exp \left(-\frac{n}{\gamma_{SR}}\right),$$ \(25\)

and

$$f_{\Lambda_{RD,\max,2}}(\gamma) = \sum_{k=1}^{\min(N_D,N_{R_2} - 1)} \sum_{l=\lceil N_D - (N_R - 1) \rceil} \frac{C_{k,l}^2}{l!} \left(\frac{k}{\gamma_{RD}}\right)^{l+1} \gamma^l \exp \left(-\frac{k}{\gamma_{RD}}\right).$$ \(26\)

Then, similarly, with $\gamma_{SR,2} = P_S \Lambda_{SR,\max,2}$ and $\gamma_{RD,2} = P_R \Lambda_{RD,\max,2}$, the closed-form expressions of PDF of $\gamma_{SR,2}$ and $\gamma_{RD,2}$ can be finally obtained as

$$f_{\gamma_{SR,2}}(x) = f_{\Lambda_{SR,\max,1}} \left(\frac{x}{P_S}\right) \cdot \frac{1}{P_S}$$

$$= \sum_{n=1}^{\min(N_S,N_{R_1})} \sum_{m=\lceil N_S - N_R \rceil} \frac{D_{n,m}}{m!} \left(\frac{n}{P_S \gamma_{SR}}\right)^{m+1} x^m \exp \left(-\frac{n}{P_S \gamma_{SR}} \cdot x\right),$$ \(27\)

and

$$f_{\gamma_{RD,2}}(x) = f_{\Lambda_{RD,\max,1}} \left(\frac{x}{P_R}\right) \cdot \frac{1}{P_R}$$

$$= \sum_{k=1}^{\min(N_D,N_{R_2} - 1)} \sum_{l=\lceil N_D - (N_R - 1) \rceil} \frac{C_{k,l}^2}{l!} \left(\frac{k}{P_R \gamma_{RD}}\right)^{l+1} x^l \exp \left(-\frac{k}{P_R \gamma_{RD}} \cdot x\right).$$ \(28\)

V. OUTAGE PERFORMANCE

The outage probability, $P_{OUT}$, is defined as the probability that the instantaneous end-to-end SNR falls below a target SNR. Here, based on the mode of operation in Sec. II, the overall system outage occurs a communication failure in one of two links (i.e., from S to R or from R
Therefore, the overall channel outage probability can be expressed in terms of individual link outage probabilities as

$$P_{\text{OUT}} = P_{\text{OUT,SR}} + (1 - P_{\text{OUT,SR}}) P_{\text{OUT,RD}},$$

(29)

where

$$P_{\text{OUT,SR}} = \Pr [\gamma_{SR,i} < \gamma_T] = \int_0^{\gamma_T} f_{\gamma_{SR,i}}(x) \, dx \quad \text{for} \quad i = 1, 2$$

(30)

and

$$P_{\text{OUT,RD}} = \Pr [\gamma_{RD,i} < \gamma_T] = \int_0^{\gamma_T} f_{\gamma_{RD,i}}(x) \, dx \quad \text{for} \quad i = 1, 2.$$  

(31)

Similarly, in terms of the mutual-information rate, the event of an information-outage occurs when the received date rate falls below some fixed threshold, $R_0$, and each outage probability terms can be expressed for $i = 1, 2$ as

$$P_{\text{OUT,SR}} = \Pr [\log_2 (1 + \gamma_{SR,i}) < R_0] = \Pr [\gamma_{SR,i} < 2^{R_0} - 1]$$

$$= \int_0^{2^{R_0} - 1} f_{\gamma_{SR,i}}(x) \, dx,$$

(32)

$$P_{\text{OUT,RD}} = \Pr [\log_2 (1 + \gamma_{RD,i}) < R_0] = \Pr [\gamma_{RD,i} < 2^{R_0} - 1]$$

$$= \int_0^{2^{R_0} - 1} f_{\gamma_{RD,i}}(x) \, dx.$$  

(33)

Note that in this case, the closed-form expression of the outage probability can be directly obtained by replacing $\gamma_T$ in the closed-form result of (29) with $2^{R_0} - 1$. 

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A. Closed-form results for Receive ZFBF case

For S-R link, by substituting the PDF in (18) into the outage probability in (30), the outage probability of S-R link can be written as

\[
P_{OUT,SR} = \int_0^{\gamma_T} f_{\gamma_{SR,1}}(x) \, dx
= \sum_{n=1}^{\min(N_S,N_{R1} - 1)} \sum_{m=\lfloor N_S - (N_{R1} - 1) \rfloor} D_{n,m}^1 \left( \frac{n}{P_{S\!\!\!/SR}} \right)^{m+1} \int_0^{\gamma_T} x^m \exp \left( -\frac{n}{P_{S\!\!\!/SR}} \cdot x \right) \, dx.
\]

(34)

Then, by applying [15, Eq. (3.381-1)] and then mathematical simplification, the closed-form result can be obtained as the incomplete Gamma function

\[
P_{OUT,SR} = \sum_{n=1}^{\min(N_S,N_{R1} - 1)} \sum_{m=\lfloor N_S - (N_{R1} - 1) \rfloor} D_{n,m}^1 \left( \frac{n}{P_{S\!\!\!/SR}} \right)^{m+1} \gamma \left( m + 1, \frac{n\gamma_T}{P_{S\!\!\!/SR}} \right),
\]

(35)

where \( \gamma(\cdot, \cdot) \) is the lower incomplete Gamma function.

For R-D link, similarly, the outage probability of R-D link (31) can be written with the PDF (19) as

\[
P_{OUT,RD} = \int_0^{\gamma_T} f_{\gamma_{RD,1}}(x) \, dx
= \sum_{k=1}^{\min(N_{R2},N_D)} \sum_{l=\lfloor N_{R2} - N_D \rfloor} C_{k,l}^1 \left( \frac{k}{P_{R\!\!\!/RD}} \right)^{l+1} \int_0^{\gamma_T} x^l \exp \left( -\frac{k}{P_{R\!\!\!/RD}} \cdot x \right) \, dx.
\]

(36)

Then, with (3.381.1), the closed-form result can be also obtained as the incomplete Gamma function

\[
P_{OUT,RD} = \sum_{k=1}^{\min(N_{R2},N_D)} \sum_{l=\lfloor N_{R2} - N_D \rfloor} C_{k,l}^1 \left( \frac{k}{P_{R\!\!\!/RD}} \right)^{l+1} \gamma \left( l + 1, \frac{k\gamma_T}{P_{R\!\!\!/RD}} \right).
\]

(37)

Note that based on [16], we can see that a diversity order of \( N_S (N_{R1} - 1) \) and \( N_{R2} N_D \) at the receiver side and the transmitter side, respectively. Thus, the full-duplex receive ZF design can achieve a diversity order of \( \min(N_S (N_{R1} - 1), N_{R2} N_D) \).
B. Closed-form results for Transmit ZFBF case

In this case, the outage probability formulas have the similar integral form. Therefore, with the closed-form results of PDFs in (27) and (28), by adopting the same definite integral table used in (35) and (37), both closed-form results can be obtained, respectively as

\[ P_{\text{OUT,SR}} = \int_0^{\gamma_T} f_{\gamma_{SR,2}}(x) \, dx = \sum_{n=1}^{\min(N_{S},N_{R_1})} \sum_{m=\lfloor N_{S}-N_{R_1} \rfloor}^{(N_{S}+N_{R_1})} \frac{D_{n,m}^2}{m!} \cdot \gamma \left( m + 1, \frac{n \gamma_T}{P_{S} \gamma_{SR}} \right), \]  

(38)

\[ P_{\text{OUT,RD}} = \int_0^{\gamma_T} f_{\gamma_{RD,2}}(x) \, dx = \sum_{k=1}^{\min(N_{D},N_{R_2}-1)} \sum_{l=\lfloor N_{D}-(N_{R_2}-1) \rfloor}^{(N_{D}+N_{R_2}-1)} \frac{C_{k,l}^2}{l!} \cdot \gamma \left( l + 1, \frac{k \gamma_T}{P_{R} \gamma_{RD}} \right), \]  

(39)

Note also that, similarly, we can see that a diversity order of \( N_{D}(N_{R_2}-1) \) and \( N_{S}N_{R_1} \) at the transmitter side and the receiver side, respectively. Thus, the full-duplex transmit ZF design can achieve a diversity order of \( \min(N_{D}(N_{R_2}-1),N_{S}N_{R_1}) \).

VI. RESULTS AND CONCLUSIONS

In this section, some selected results for the outage probability are given. The simulation set-up follows the system model provided in Section II, especially \( P_{S} = P_{R} \) and \( \gamma_{SR} = \gamma_{RD} \). Although we have mainly considered a symmetric setup, we additionally consider the effect of asymmetric setups. More specifically, we consider both cases; i) when the first hop dominates over the second hop, e.g., \( P_{S}'(=\alpha_{SR}^2 P_{S}) > P_{R}'(=\alpha_{RD}^2 P_{R}) \), and ii) when the second hop dominates over the first hop, e.g., \( P_{R}'(=\alpha_{RD}^2 P_{R}) > P_{S}'(=\alpha_{SR}^2 P_{S}) \), where \( \alpha_{SR} \) and \( \alpha_{RD} \) are the path-loss factor for the S-R link and the R-D link, respectively. In addition, \( (N_{S},N_{R_1},N_{R_2},N_{D}) \) represents the each antenna configuration, where \( N_{S} \), \( N_{D} \), \( N_{R_1} \), and \( N_{R_2} \) are the number of antennas at S, D, receiver side of R, and transmitter side of R, respectively. Further, in the following figures, the lines and the markers represent the simulation and the analytical results, respectively. Note that the simulation results match the derived analytical results well.
Fig. 2 shows the results based on the receive ZF with different antenna configurations. We observe some interesting results which are useful for system designers. Specifically, although there is only single antenna mounted on D, the performance can be improved by applying appropriate design parameters at S and R, especially for receive ZF case. Here, for symmetric case, (2, 3, 2, 1) slightly outperforms (2, 2, 3, 1). Note that for asymmetric case, especially for link_{RD} > link_{SR} (or \( P'_R : P'_S = 3 : 2 \)) case, this performance gap increases due to the increased possibility of successful decoding at D and the relatively higher diversity order of S-R link for (2, 3, 2, 1) case compared to (2, 2, 3, 1) case. Contrary, for link_{SR} > link_{RD} (or \( P'_S : P'_R = 3 : 2 \)) case, the possibility of successful decoding at R increases. As a result, (2, 2, 3, 1) with the higher diversity order of R-D link can provide the better performance.

In Fig. 3 we observe that an additional performance gain can be obtained via increasing the number of antenna at R. More specifically, for receive ZF case, increasing \( N_{R_1} \) at R can obtain the additional performance gain while for transmit ZF case, the additional performance gain can be obtained via increasing \( N_{R_2} \).

In Fig. 4 for both (2, 3, 2, 2) and (2, 3, 2, 3) cases, the diversity order of these cases is the same. However, the latter case provides the better performance because the latter case has the relatively higher possibility of successful decoding at D. Additionally, for (3, 2, 2, 2) and (2, 3, 2, 2) cases, they have the same number of total antenna. However, we observe that, for receive ZF case, swapping \( N_S \) with \( N_{R_1} \) can improve the outage performance.

**APPENDIX A**

**A MATLAB Code for Evaluation of Coefficients, \( D_{n,m} \) or \( C_{n,m} \).**

```matlab
% This program is for coefficient calculation. % initialize the values
N=input('Input the value of N1 =');
```

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L=input('Input the value of N2 = '); m = min(N1,N2); n = max(N1,N2);
% For K_mn Calculation
a = prod(factorial(m - [1:m])); b = prod(factorial(n - [1:m])); K_mn = 1/(a*b);
% For G matrix (Symbolic)
syms lamda;
[i_val, j_val] = meshgrid([1:m],[1:m]); G = symfun(factorial(n-m+i_val+j_val-2) - igamma(n-m+i_val+j_val-1,lamda),lamda);
% For p_lamda_max (Symbolic)
p_lamda_max = symfun(K_mn * diff(det(G(lamda)),lamda),lamda);

% Step 1
f_eq = p_lamda_max;

% Step 2
k = 1; l = n+m-2; cont_flag = true; A=[];

while cont_flag
    % Step 3
    a_kl = double(1/K_mn * limit(f_eq/(lamda^l*exp(-k*lamda)),lamda,+inf));
    a_kl_prime = a_kl * K_mn * factorial(l)/(k^(l+1));
    format rational
    A_tmp=[k l a_kl_prime];
    A=[A;A_tmp];
    % Step 4
    g_eq = K_mn*a_kl*lamda^l*exp(-k*lamda);
    f_eq = f_eq - g_eq;
    % Step 5
    l = l-1;
    if(l >= n-m)
        cont_flag = true;
    else
        % Step 6
        k = k+1;
        if(k <= m)
            l = (n+m-2*k)*k;
        else
            cont_flag = false;
        end;
    end;
end;

fileID = fopen('Coefficient_Data.dat','w');
fprintf(fileID,'Estimated coefficient values of d_im for N1=%d and N2=%d \n\n', N,L);
fprintf(fileID,'%s %s %s \n\n','i','m','d_im');
fclose(fileID);

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Fig. 1. System Model
Fig. 2. Outage probability as the function of average SNR based on single antenna at D with different antenna configurations ($\gamma_T = 10$dB and 5dB for symmetric and asymmetric cases, respectively).
Fig. 3. Outage probability as the function of average SNR with $\gamma_T = 10$dB for both receive and transmit ZFBF cases.
Fig. 4. Outage probability as the function of average SNR with $\gamma_T = 10$dB for receive ZFBF case.