Clustering Aspects of Highly Excited States and Neutron-Rich Nuclei

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Recent researches of clustering aspects in stable and unstable nuclei based on antisymmetrized molecular dynamics (AMD) are reported. We will focus on the clustering of highly excited states of ²⁴Mg, the low-lying spectroscopy and novel exotic clustering in the vicinity of the island of inversion.

§1. Introduction

Clustering aspect is one of the most important ingredients of the nuclear many-body problem. It embodies the dynamical aspects of nuclear system, namely assembling and disassembling of nucleons, which make nuclear physics challenging and fascinating problem. Owing to the theoretical and experimental developments, the domain of the cluster studies is rapidly expanding toward highly excited and neutron-rich nuclei. For example, the studies of unstable nuclei have revealed novel types of clustering represented by Be isotopes,¹–³ and analogous structures have been discussed in heavier system.⁴,⁵ The intensive discussions on the na gas-like state in ¹²C⁶ and ¹⁶O⁷ renewed our interest in the clustering of the highly excited states. Theoretical and experimental studies are rapidly developing toward heavier systems.

AMD has been one of powerful theoretical approaches to investigate those clustering aspects in the highly excited states and in the neutron-rich nuclei. In this article, we report some of recent activities of AMD studies for clustering of stable and unstable nuclei. The dominance of the cluster states around the threshold energies despite of the mean-field nature of the low-lying states is remarked using ²⁴Mg⁸ as an illustrative example (§3). AMD has achieved sufficient accuracy and predictive power for the study of the low-lying mean-field dynamics in heavier unstable nuclei. It enabled us to discuss and predict the coexistence of the spherical and deformed states⁹ and neutron-halo structure¹⁰,¹¹ in the island of inversion (§4). Clustering will also play an important role in the vicinity of the island of inversion. We predict the coexistence of the mean-field states and the novel types of clustering which are composed of clusters and many valence neutrons at small excitation energies in F isotopes far from stability.¹²
§2. Theoretical framework of AMD

The Hamiltonian used in this study is

$$H = \sum_{i=1}^{A} t_i - t_{cm} + \sum_{i<j}^{A} v_{ij}^N + \sum_{i<j}^{Z} v_{ij}^C. \quad (2.1)$$

The Gogny D1S parameter set is used as the effective interaction $v_{ij}^N$. The Coulomb interaction $v_{ij}^C$ is approximated by the sum of twelve Gaussians, and the center-of-mass kinetic energy $t_{cm}$ is removed exactly. The variational wave function of AMD is represented by a parity-projected Slater determinant of deformed Gaussian wave packets,\(^{13,14}\)

$$\Phi^\pi = P^\pi A\{\varphi_1, \varphi_2, \ldots, \varphi_A\}, \quad \varphi_i(r) = \phi_i(r) \chi_i \xi_i, \quad (2.2)$$

$$\phi_i(r) = \exp \left\{ - \sum_{\sigma=x,y,z}^{\nu_\sigma} \frac{Z_{i\sigma}}{\sqrt{\nu_\sigma}} \right\}, \quad \chi_i = a_i \chi_1 + b_i \chi_1, \quad \xi_i = \{p \text{ or } n\}, \quad (2.3)$$

where $P^\pi$ is the parity projector. $Z_i$, $a_i$, $b_i$ and $\nu_\sigma$ are the parameters determined by the variational calculation with constraints. The constraints applied in this study are explained for each topic discussed below. After the variational calculation, we project out eigenstates of the total angular momentum, and the calculation is completed by performing GCM,

$$\Phi_{\alpha}^{J\pi} = \sum_{i}^{J} \sum_{K=-J}^{J} c_{K\alpha} \hat{P}_{MK}^{J} \Phi_{i}^{\pi}. \quad (2.4)$$

The coefficients $c_{K\alpha}$ are determined by solving the Hill-Wheeler equation.

§3. Clusters of highly excited states — the case of $^{24}$Mg —

Fig. 1. (a) Calculated and observed partial level scheme and $B(E2)$ of $^{24}$Mg for the positive-parity states. (b) Overlaps of the wave function between each state and GCM basis wave function that indicate the distribution of the wave function in the $\beta-\gamma$ plane. Figures are taken from Ref. 8).
As an example to show the universality of clustering dynamics in stable nuclei, we focus on $^{24}\text{Mg}$. It is well known that triaxial deformation of mean-field is essential to describe low-lying spectrum. Indeed, AMD+GCM calculation$^8$ with constraints on quadrupole deformations ($\beta$ and $\gamma$) successfully describes the low-lying rotational bands and shows the importance of triaxial deformation for the description of the $K^\pi = 2^+$ band as shown in Fig. 1. As we see in the density distribution of the intrinsic wave function, quadrupole deformed mean-field structure dominates the low-lying states and no prominent clustering appears on the $\beta\gamma$ energy surface.

However, it is hasty to conclude the absence of the clustering in $^{24}\text{Mg}$. When we turn to the highly excited states, according to Ikeda’s diagram, a couple of the threshold energies that decompose the system into two or three clusters are located around 10 to 20 MeV, and we can expect a variety of cluster states. Indeed, a recent $\alpha$ inelastic scattering experiment by Kawabata et al.$^{15}$ has shown the presence of excited $0^+$ states with large isoscalar (IS) monopole transition strength in the vicinities of the threshold energies. As pointed out in the cases of $^{12}\text{C}$, $^{11}\text{B}$ and $^{16}\text{O}$,$^{16,17}$ the large fraction of the isoscalar (IS) monopole strengths below the GMR can be associated with the cluster structure. Therefore, the highly excited $0^+$ states reported in Ref. 15) shed light on cluster states of $^{24}\text{Mg}$.

To investigate nature of the highly excited $0^+$ states, we have applied AMD with a constraint on the principal quantum number $N$ which is defined as

$$\hat{N} = \sum_{i=1}^{A} a_i^\dagger \cdot a_i = \sum_{i=1}^{A} \left[ \frac{p_i^2}{4\hbar^2\nu_0} + \nu_0r_i^2 - \frac{3}{2} \right], \quad \nu_0 = (\nu_x\nu_y\nu_z)^{1/3}. \quad (3.1)$$

The lowest Pauli allowed value for $^{24}\text{Mg}$ is $N = 28$, and increase of $\langle N \rangle$ excites nucleons across major shells. The obtained results shown in Fig. 2(a) are rather surprising. Since the low-energy spectrum is dominated by the triaxial deformed mean-field, one expects the increase of $N$ will populate the particle-hole excitations within the same mean-field as the low-lying state. Contradict to this naive assump-
tion, we have found that once we promote nucleons into higher major shell, $^{24}$Mg shows a variety of clustering. For example, the constraint to $N = 30$ ($2\hbar\omega$ excitation) populates the $\alpha + ^{20}$Ne cluster structure and $N = 32$ ($4\hbar\omega$ excitation) does the $^{12}$C+$^{12}$C and $2\alpha + ^{16}$O cluster structures. This result is very impressive to show that the cluster structures dominate the states around threshold energies, even though the low-lying states are dominated by the mean-field dynamics.

By performing the GCM using the basis wave functions with cluster structures, we have obtained five excited $0^+$ states, while we only have two $0^+$ states with the wave functions obtained by the quadrupole constraint (Fig. 2(b)). All of them have prominent cluster structure except for the $0^+_2$ state. The $0^+_3$ and $0^+_5$ states have $^{12}$C+$^{12}$C, and $2\alpha + ^{16}$O and $\alpha + ^{20}$Ne cluster structures appear as the $0^+_4$ and $0^+_6$ states, respectively. These states have considerable magnitudes of the IS monopole strengths. Detailed comparison with the experiments and investigations for other $N = Z$ nuclei are ongoing now.

§4. Shape coexistence and neutron-halo in the island of inversion

4.1. Coexistence of spherical and deformed states, and neutron orbitals

The nuclear spectra in the island of inversion have been one of most fascinating topics to investigate nuclear shell erosion far from stability. Recently thanks to the development of the experiments, the information of the non-yrast states are rapidly increasing. For example, the coexistence of spherical and deformed shapes has been discussed on the basis of the presence of the low-lying $0^+_2$ states in $^{30}$Mg$^{18}$ and $^{32}$Mg.$^{19}$ The low-lying yrast and non-yrast spectra of odd-mass isotopes are the most sensitive probes to study the neutron orbits far from stability and the coexistence of spherical and deformed states, since the last neutron’s orbit is quite sensitive to the nuclear deformation and it determines the spin-parity of the system.

Fig. 3. (a) Energy surfaces of the positive- and negative-parity states of $^{31}$Mg as function of quadrupole deformation parameter $\beta$ calculated by AMD. The AMD results are taken from Ref. 9. (b) Low-lying spectrum of $^{31}$Mg calculated by AMD+GCM compared with the experimental assignment suggested in Ref. 29.)
AMD combined with GCM is one of the powerful theoretical approaches to investigate the non-yrast states of island of inversion nuclei as well as the yrast states.\textsuperscript{9),20),21) As one of such examples, Fig. 3 shows the calculated energy surfaces, predicted\textsuperscript{9} and observed spectrum of $^{31}$Mg.\textsuperscript{22)–29) The energy surfaces of $^{31}$Mg (Fig. 3(a)) have the local minima with different nuclear deformations that correspond to the 0$p1h$, 1$p2h$, 2$p3h$ and 3$p4h$ neutron configurations in ascending order of deformation. AMD+GCM calculation (Fig. 3(b)) shows that the ground state is strongly deformed, which is comparable with neighboring nuclei such as $^{32}$Mg, and has an almost pure 2$p3h$ configuration in which two neutrons are promoted into $pf$ shell across $N = 20$ shell gap. The calculated magnetic moment of the ground state is $-0.91 \mu_N$, while the observed value is $-0.88 \mu_N$.\textsuperscript{22) Since the spherical 0$p1h$ states in which the $N = 20$ magicity is retained give positive magnetic moment, the breakdown on the $N = 20$ magic number is confirmed without ambiguousness. AMD has predicted that the ground state is followed by 3/2$^+$, 5/2$^+$ and 7/2$^+$ states with 2$p3h$ configuration to constitute the rotational ground band due to the strong deformation. Furthermore, because of the quenching of the $N = 20$ shell gap, coexistence of three different configurations at small excitation energy was also predicted. The 1$p2h$ configuration appears as the 3/2$^-$ and 7/2$^-$ states at very small excitation energies, the strongly deformed 3$p4h$ configuration constitutes $K^\pi = 3/2^-$ rotational band starting from 720 keV, and the almost spherical 0$p1h$ configuration appears as the 5/2$^+$ state at 1.6 MeV. By the measurements of $\beta$-decays,\textsuperscript{23)–25) one proton or neutron knockout reactions,\textsuperscript{26),27) and Coulomb excitation,\textsuperscript{28) most of these excited states have been observed and it has been shown that many observables good agreement with AMD predictions. Thus, the coexistence of various neutron configurations and deformed states is now establishing. Further comprehensive study of non-yrast states in the island of inversion will be reported in a forthcoming paper.

4.1.1. Neutron-halo with a deformed core in the island of inversion

Recent experiments at RIBF have revealed the possible neutron-halo structure of $^{31}$Ne. The large Coulomb breakup cross section\textsuperscript{30) and interaction cross section\textsuperscript{31)} are reported. In the case of $^{31}$Ne, since the core nucleus $^{30}$Ne is located in the middle of the island of inversion, the assumption of the spherical inert core is inadequate. The last neutron of $^{31}$Ne may be coupled to the strongly deformed and magic number broken core. Therefore, the analysis based on the full microscopic theory is necessary. Since a single-particle wave function is represented by a Gaussian wave packet, AMD cannot describe properly the tail part of halo nucleus which shows the exponential damping. This shortage is overcome by combining AMD with resonating group method (AMD+RGM). We use the RGM-type wave function for $^{31}$Ne,

\begin{equation}
\Psi_{JM} = \sum_{\alpha l} c_{\alpha l} A [\chi_{\alpha l}(r)Y_{lm}(\hat{r})\phi_n\phi_{30\text{Ne}}(\alpha)]_{JM},
\end{equation}

where $\phi_n$ and $\chi_{\alpha l}(r)Y_{lm}(\hat{r})$ are the wave function of the valence neutron and the relative motion between the valence neutron and $^{30}$Ne, respectively. $\phi_{30\text{Ne}}(\alpha)$ is the internal wave function of $^{30}$Ne solved by AMD+GCM,\textsuperscript{20) and $\alpha$ labels the ground and excited states of $^{30}$Ne.
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Figure 4(a) shows the predicted spectrum of $^{31}$Ne by AMD+RGM. The ground state is the $3/2^-$ state and the calculated one neutron separation energy is $S_n = 0.45$ MeV, while the observed value is $0.29 \pm 1.64$ MeV. Because of this small separation energy, the density distribution of $^{31}$Ne calculated by AMD+RGM shows the long tail at large distance indicating the $p$-wave halo ($1p_{3/2}$Neutron coupled to $^{30}$Ne) as shown in Fig. 4(b). The effect of the deformed core appears as the core excitation. From the coefficients $c_{al}$, it is found that the $p_{3/2} \otimes ^{30}$Ne$(2^+)$ configuration amounts to 41%, that is larger than the $p_{3/2} \otimes ^{30}$Ne$(0^+)$ configuration which amounts to 37%. These values suggest that the $p_{3/2}$ neutron is coupled to the deformed and rotating ground band of $^{30}$Ne. Using AMD and AMD+RGM wave functions, the reaction cross sections of $^{31}$Ne and other Ne isotopes are analyzed based on the double-folding model. The results are taken from Refs. 10) and 21). Figure 4(c) compares the calculated and observed reaction cross section. We can see that AMD wave function shows overall agreement with the observation except for $^{31}$Ne, and anomalous large cross section of $^{31}$Ne is reasonably described by employing the AMD+RGM wave function. Thus, AMD combined with RGM and reaction theory is a promising method to investigate neutron-halo nuclei and their reactions in heavier mass region. Furthermore, using AMD+RGM method, it is possible to derive a core-neutron potential microscopically. This potential will enable us to study the candidates of two neutron-halo systems.

§5. Molecule-like states in the island of inversion

As a possible example of exotic phenomena driven by the shell erosion in the island of inversion, we discuss the molecule-like states of F isotopes analogous to those of Be isotopes. As well known, the stable nucleus $^{19}$F has the negative-parity state ($1/2_-$) at 0.11 MeV, which is known to have $\alpha ^{15}$N cluster structure today. Its extraordinary small excitation energy is due to the strong $\alpha$ correlation or $\alpha$ clustering between four nucleons in $sd$ shell triggered by a proton excitation into...
and proton excitation is not enough to trigger the states shown by green and blue lines, and they have a pronounced gradual increase and the states in lighter F isotopes. As neutron number increases, their excitation energies kept within the configurations. Red lines show the states with a proton excitation but all neutrons are kept within the sd shell, that are the representative of the low-lying negative-parity states in lighter F isotopes. As neutron number increases, their excitation energies gradually increase and the α clustering of these states diminishes since the increasing number of neutrons in sd shell reduces the deformation of the system. Therefore, a proton excitation is not enough to trigger the α clustering of neutron-rich F isotopes. On the contrary, a simultaneous excitation of a proton and neutrons across Z = 8 and N = 20 shell gap brings about the α clustering. These configurations are shown by green and blue lines, and they have a pronounced α cluster structure. For example, as shown in the density distribution of 21F (Fig. 5), the last neutron orbit is elongated along the symmetry axis and orbiting the entire system, while other nucleons manifest the α + N cluster-like core structure. These configurations are analogous to the molecular structure of Be isotopes in which the 2α cluster core is surrounded by valence neutrons occupying the molecular orbitals around it.

It is notable that the drastic reduction of their excitation energies toward neutron drip line is predicted. To understand this reduction, readers are reminded following points. (1) The last neutron’s orbital originates in a pf shell, and its energy is lowered in the island of inversion due to the quenching of the N = 20 shell gap. (2) If the core has cluster structure, it induces the deformation of the system and it further lowers the energy of the last neutron’s orbital as in the case of Be isotopes.

Fig. 5. (Color online) Left: Band-head energies of proton hole states of F isotopes. Red lines show the states with a proton excitation from p to sd shell, while green (blue) lines show those with one (two) neutrons in pf shell that have molecule-like structure. Right: Density distribution of the core (19F, contour lines) and two valence neutrons (color plot) of 21F.
Therefore, the neutron excitation into $pf$-orbital and $\alpha$ clustering of the core work in a cooperative way to reduce the excitation energies of molecular states in the island of inversion. Up to now, several candidates for the lighter F isotopes are experimentally known$^{32,33}$ as shown in Fig. 5 with dashed lines. More data for F isotopes near the drip line will be experimentally available in near future.

§6. Summary

In this article we have reported our recent researches of clustering aspects in stable and unstable nuclei. We have discussed the dominance of cluster states such as $^{12}\text{C} + ^{12}\text{C}$, $2\alpha + ^{12}\text{O}$ and $\alpha + ^{20}\text{Ne}$ around threshold energies. It shows that the dynamical assembling and disassembling of nucleons take place even though the low-lying states are dominated by the mean-field dynamics. The successful description of the low-lying states such as the shape coexistence and neutron-halo structure in the vicinity of the island of inversion is also demonstrated. Furthermore, AMD predicts the presence of the novel types of cluster states at very small excitation energy in F isotopes far from stability.

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