Double-vector multiplet and partially broken N=4, d=3 supersymmetry

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Abstract

We elaborate on a new N=2, d=3 supermultiplet (double-vector multiplet) with a non-trivial off-shell realization of the central charge. Its bosonic sector comprises two abelian gauge vector fields forming an SO(2) vector. We present a superfield formulation of this multiplet in the central-charge extended N=2, d=3 superspace and then employ it, in the framework of the nonlinear realizations approach, as the Goldstone one for the partial breaking N=4 \rightarrow N=2 in three dimensions. The covariant equations of motion for the self-interacting Goldstone superfield arise as a natural generalization of the free ones and are interpreted as the worldvolume supersymmetric form of the equations of motion of a N=4 D2-brane. For the vector fields we find a coupled nonlinear system of Born-Infeld type and demonstrate its dual equivalence to the d=5 membrane. The double-vector multiplet can be fused with some extra N=2, d=3 multiplet to form an off-shell N=4, d=3 supermultiplet.
1 Introduction

The purely geometric target space actions for extended objects possess a lot of gauge (super)symmetries which are all realized linearly. After proper gauge fixing the corresponding world-volume actions describe a very special type of highly nonlinear theories. The main property of these theories is a nonlinear realization of broken (super)symmetries while the unbroken ones are still linearly realized. Among all spontaneously broken symmetries supersymmetry plays a very special role. Nowadays, it is clear that just the nonlinearly realized broken supersymmetries single out the Nambu-Goto-Born-Infeld type actions as the actions compatible with the requirement of invariance under both the linearly as well as the nonlinearly realized supersymmetries [1]-[16].

As usual, the partial spontaneous breaking of global supersymmetry (PBGS) implies the presence of a massless Goldstone multiplet of the residual unbroken supersymmetry. The choice of such a multiplet is not unique [6]. The proper choice of the Goldstone multiplet can greatly simplify the construction of the invariant Goldstone superfield action. Employing the $N = 1, d = 4$ tensor multiplet instead of chiral multiplet gives a seminal example of such a simplification [1], [8], [9]. In the case of chiral Goldstone multiplet (and, more generally, in all cases where the number of physical scalars in the Goldstone multiplet exceeds one), a highly nonlinear redefinition of the Goldstone scalar superfields is required to bring the action to the standard form (yielding the Nambu-Goto action for scalars in the bosonic sector). In contrast, for the case of $N = 1$ tensor Goldstone multiplet, when one of the physical scalars is off-shell dualized into the tensor gauge field (“notoph”), the problem of constructing the correct Goldstone superfield action is drastically simplified because only one physical scalar field is present off shell. The action constructed by the methods of refs. [1], [8], [9] immediately yields the static-gauge Nambu-Goto action for the single scalar, no redefinition of the Goldstone superfield is needed. From this point of view, the $N = 2 \rightarrow N = 1$ supersymmetry breaking scheme with the $N = 1, d = 4$ vector multiplet as the Goldstone one should be simplest, because this version contains no physical scalar fields at all. Indeed, this is the case [6].

In contrast to the $N = 1, d = 4$ vector multiplet, its dimensionally reduced version, the $N = 2, d = 3$ vector multiplet, contains a scalar field. Yet, in three dimensions the scalar is dual to a gauge field, suggesting the existence of another $N = 2, d = 3$ supermultiplet which contains only two vector fields in its bosonic sector. In Section 2 we construct such a multiplet. Decomposed into $N = 1, d = 3$ superfields, it comprises two $N = 1$ vector multiplets, and we naturally name it the double-vector $N = 2, d = 3$ multiplet. An additional peculiarity of this multiplet is a non-trivial off-shell realization of the central charge which appears in the closure of the two $N = 1$ supersymmetries. In Section 3 we develop a superfield description of the double-vector multiplet in $N = 2, d = 3$ superspace with a central charge. In Section 4 we consider the spontaneous breaking of $N = 4, d = 3$ supersymmetry to $N = 2, d = 3$ in the framework of nonlinear realizations and choose the double-vector multiplet as the Goldstone one accompanying this breakdown. The nonlinear generalization of the basic constraints on the double-vector multiplet gives us the superfield equations of motion. The latter we interpret as the worldvolume equations of some $N = 4$ D2-brane comprising two Maxwell gauge field strengths in its worldvolume multiplet. In Section 5 we combine the double-vector multiplet with some extra $N = 2, d = 3$ multiplet to gain an off-shell $N = 4, d = 3$ supermultiplet. By its component content, it is a dimensional reduction of the $N = 2, d = 4$ vector-tensor multiplet.
2 Double-vector multiplet in $N = 1$ superspace

In this Section we perform the direct construction of the double-vector multiplet in $N = 1, d = 3$ superspace.

We start with two $N = 1, d = 3$ vector multiplets $\psi^i_a$ ($i = 1, 2$),

$$D_a \psi^{ai} = 0 \Rightarrow \left\{ \begin{array}{l} D^2 \psi^i_a = D^c D_c \psi^i_a = -i \partial_{ab} \psi^{bi} a, \\
\partial_{ab} D^a \psi^{bi} = 0 \end{array} \right. ,$$

(2.1)

where

$$D_a = \frac{\partial}{\partial \theta^a} + \frac{i}{2} \theta^b \partial_{ab} \quad \{D_a, D_b\} = i \partial_{ab} ,$$

(2.2)

$a, b = 1, 2$ being the $d = 3$ $sl(2, R)$ spinor indices.

The main question we address here is whether it is possible to realize an additional $N = 1$ supersymmetry on $\psi^i_a$ in such a way that it forms, together with the manifest one, an $N = 2, d = 3$ supersymmetry. Assuming the second supersymmetry to be linearly realized, the corresponding transformation law of the superfields $\psi^i_a$ appears to be

$$\delta \psi^i_a = \epsilon^{ij} e^b D_b \psi^j_a .$$

(2.3)

One can easily check that the standard requirement of compatibility of the transformations (2.3) with the basic constraints (2.1) puts our superfields on-shell,

$$D^a D_b \psi^i_a = 0 \Rightarrow \partial_{ab} \psi^{bi} = 0 .$$

(2.4)

To have an off-shell supermultiplet, we should modify the transformation law (2.3). The seemingly unique possibility of such a modification is to admit $\theta$-dependent terms. Requiring the standard closure of the second supersymmetry,

$$\{S_a, S_b\} = P_{ab} ,$$

(2.5)

together with the preservation of the constraints (2.1) completely fixes the additional terms and produces

$$\delta \psi^i_a = \epsilon^{ij} \left( e^b D_b \psi^j_a + i e_c \theta^c \partial_{ab} \psi^{bi} \right) .$$

(2.6)

Due to the explicit presence of $\theta$ in (2.3), the bracket of the manifest and the second $N = 1$ supersymmetry yields a central charge transformation

$$\delta \zeta \psi^i_a = i 2 \epsilon^{ij} \partial_{ab} \psi^{bj}$$

(2.7)

which commutes with both supersymmetries ($c$ is the corresponding transformation parameter). Hence, we are facing the $N = 2, d = 3$ supersymmetry algebra with a central charge in

$$\{Q_a, S_b\} = \epsilon_{ab} Z .$$

(2.8)

Thus, our double-vector multiplet requires an off-shell central charge which vanishes on-shell.

The free action for the double-vector multiplet has the very simple form

$$S = \int d^3 x d^2 \theta \left( \psi^{ai} \psi^i_a \right) .$$

(2.9)

It reveals a manifest internal $SO(2)$ symmetry which commutes with both supersymmetries and acts on the index $i$. All the component fields in $\psi^i_a$, including two Maxwell $d = 3$ field strengths, are $SO(2)$ vectors.

1 The indices are raised and lowered as follows: $V^a = \epsilon^{ab} V_b, \quad V_b = \epsilon_{bc} V^c, \quad \epsilon_{ab} \epsilon^{bc} = \delta^c_a.$
3 Double-vector multiplet in $N = 2$ superspace

Since the double-vector multiplet has a non-zero central charge, it is necessary to enlarge the usual $N = 2$ superspace $\{ x^{ab}, \theta^a, \bar{\theta}^a \}$ by one extra bosonic coordinate $z$. The algebra of covariant derivatives in this extended superspace has the following form:

$$\{ D_a, \bar{D}_b \} = i \partial_{ab} - \varepsilon_{ab} \partial_z \, , \, \{ D_a, D_b \} = 0 \, , \, \{ \bar{D}_a, \bar{D}_b \} = 0 \, ,$$

(3.10)

where $D_a, \bar{D}_b$ read

$$D_a = D_a + \frac{1}{2} \bar{\theta}_a \partial_z \equiv \frac{\partial}{\partial \theta^a} - i \frac{\bar{\theta}}{2} \partial_{ab} + \frac{1}{2} \bar{\theta}_a \partial_z \, ,$$

$$\bar{D}_a = \bar{D}_a + \frac{1}{2} \theta_a \partial_z \equiv -\frac{\partial}{\partial \bar{\theta}^a} + i \frac{\theta}{2} \partial_{ab} + \frac{1}{2} \theta_a \partial_z \, .$$

(3.11)

The supersymmetry transformations are

$$\delta x^{ab} = \frac{i}{4} \left( \varepsilon^{(a} \theta^{b)} + \varepsilon^{(a} \theta^{b)} \right) \, , \, \delta z = -\frac{1}{2} \left( \varepsilon^a \bar{\theta}_a - \varepsilon^a \theta_a \right) \, , \, \delta \theta^a = \varepsilon^a \, , \, \delta \bar{\theta}^a = \bar{\varepsilon}^a \, .$$

(3.12)

The double-vector multiplet in $N = 2, d = 3$ central-charge extended superspace can be described by a chiral fermionic superfield $\eta^a$:

$$D_a \bar{\eta}_b = 0 \, , \, \bar{D}_a \eta_b = 0 \, ,$$

(3.13)

subject to the additional constraints

$$D^a \eta_a = 0 \, , \, \bar{D}^a \bar{\eta}_a = 0 \, .$$

(3.14)

As a consequence of (3.13), (3.14) and their corollaries

$$D^2 \eta^a = \bar{D}^2 \bar{\eta}^a = 0 \, ,$$

(3.15)

the expansion of $\eta_a$ in the anti-commuting coordinates is “ultra-short”,

$$\eta_a = \nu_a + \theta^b A_{ab} + \frac{1}{2} \theta^b \bar{\theta}^c \left( \varepsilon_{bc} \partial_z \nu_a - i \partial_{bc} \nu_a \right) \, ,$$

(3.16)

where $A_{ab}$ is further restricted to be a Maxwell-field strength

$$A_{ab} = A_{ba} \, , \, \partial_{ab} A^{ab} = 0 \, .$$

(3.17)

It is worth emphasizing that the constraints (3.13), (3.14) entirely fix the dependence of the superfields $\eta_a, \bar{\eta}_a$ on the central charge coordinate $z$:

$$\partial_z \eta_a = -i \partial_{ab} \bar{\eta}_b \, , \, \partial_z \bar{\eta}_a = i \partial_{ab} \eta^b \quad \rightarrow \quad \partial_z \nu_a = -i \partial_{ab} \nu^b \, , \, \partial_z A_{ab} = -i \partial_{ac} A^c_{b} \, .$$

(3.18)

Thus, our complex $N = 2, d = 3$ superfield $\eta_a$ subject to the constraints (3.13), (3.14) is just the same double-vector multiplet considered in the previous Section.

The free action for the double-vector multiplet in the central-charge extended $N = 2$ superspace can be written as

$$S = \int d^3 x d^4 \theta \, (\theta \bar{\theta}) (\eta \bar{\eta}) \, .$$

(3.19)
Despite the presence of the explicit $\theta$ dependence in (3.19), the action is supersymmetric as a consequence of the above constraints. One of these consequences is the property that the Lagrangian density $L_{\text{free}} = \eta^a \bar{\eta}_a$ in (3.13) obeys the constraints

$$D^2 L_{\text{free}} = \bar{D}^2 L_{\text{free}} = 0.$$  (3.20)

We also note that in (3.19) one can use the integration measure of the standard $N = 2$ superspace without central charge,

$$d^3 x d^4 \theta = \frac{1}{16} d^3 x (D^a D_a) (\bar{D}_b \bar{D}^b),$$

because the derivative of the Lagrangian density with respect to $z$ is a total $x$-derivative:

$$\partial_z L_{\text{free}} = i \partial_{ab} \left( \eta^a \bar{\eta}^b \right).$$  (3.21)

In the components (3.16) the action (3.19) reads

$$S = \frac{1}{2} \int d^3 x \left( -i \nu^a \partial_{ab} \bar{\nu}^b + \frac{1}{2} A^{ab} \bar{A}_{ab} \right).$$  (3.22)

Hence, the free equations of motion which follow from (3.22)

$$\partial_{ab} \nu^b = 0, \quad \partial_{ac} A^c_a + \partial_{bc} A^c_a = 0$$  (3.23)

are equivalent to discarding the central charge dependence of the double-vector multiplet in the basic constraints (3.13), (3.14). Thus the latter together with

$$\partial_z \eta_a = \partial_z \bar{\eta}_a = 0,$$  (3.24)

represent the superfield form of the equations of motion.

4  

$N = 4 \rightarrow N = 2$ PBGS with double-vector multiplet

As one of the possible applications of the double-vector multiplet, we can try to utilize it as the Goldstone superfield describing the partial breaking $N = 4 \rightarrow N = 2$ in $d = 3$. Indeed, it contains two fermionic fields which could be identified with the Goldstone fermions of two broken $N = 1, d = 3$ supersymmetries. Then $\eta_a, \bar{\eta}_a$ can be treated as the superfield Goldstone fermions associated with the spontaneously broken supersymmetry generators.

Leaving aside, for the time being, the search for the corresponding Goldstone off-shell superfield action generalizing (3.19), we pursue here a different goal. Namely, we shall get the corresponding superfield equations of motion (together with the Bianchi identities) starting from the nonlinear realization of the global supersymmetry group along the lines of [13].

As far as we are interested in the equations of motion, we can discard the central charge dependences of our superfields. Hence, our starting point will be the $N = 4, d = 3$ Poincaré superalgebra {f without} central charges:

$$\{Q_a, \bar{Q}_b\} = -P_{ab}, \quad \{S_a, \bar{S}_b\} = -P_{ab}. \quad (4.1)$$
Assuming the $S_a, \bar{S}_a$ supersymmetries to be spontaneously broken, we introduce the Goldstone superfields $\xi^a(x, \theta, \bar{\theta}), \bar{\xi}^a(x, \theta, \bar{\theta})$ as the parameters of the following coset

$$
g = e^{ix^{ab} P_{ab} e^{i q^a Q_a - i \theta^a \bar{Q}_a} e^{i \xi^a S_a - i \bar{\xi}^a \bar{S}_a}}. \quad (4.2)$$

Using the Cartan forms

$$
g^{-1}dg = i \omega^{ab} P_{ab} + i \omega^a \partial_a Q_a - i \bar{\omega}^a \partial_a \bar{Q}_a + i \omega_a S_a - i \bar{\omega}_a \bar{S}_a,
\omega^{ab} = dx^{ab} + i \frac{1}{4} \left( d\theta^a \bar{\theta}^b + d\bar{\theta}^a \theta^b + d\xi^a \bar{\xi}^b + d\bar{\xi}^a \xi^b \right),
\omega^a = d\theta^a, \quad \bar{\omega}^a = d\bar{\theta}^a, \quad \omega_a = d\xi^a, \quad \bar{\omega}_a = d\bar{\xi}^a, \quad (4.3)$$

one can define the covariant derivatives

$$
\nabla_{ab} = \left( E^{-1} \right)_{cd} \partial_{cd},
\nabla_a = D_a - \frac{i}{2} \left( \xi^b D_a \bar{\xi}^c + \bar{\xi}^b D_a \xi^c \right) \nabla_{bc},
\nabla_{\bar{a}} = \bar{D}_a - \frac{i}{2} \left( \bar{\xi}^b \bar{D}_a \bar{\xi}^c + \bar{\xi}^b \bar{D}_a \xi^c \right) \nabla_{bc}, \quad (4.4)
$$

where

$$F^{cd} = \frac{1}{2} \left( \delta^c_d \delta^d_b + \delta^d_a \delta^c_b - \frac{i}{2} \xi^{(c} \partial_{ab} \bar{\xi}^{d)} - \frac{i}{2} \bar{\xi}^{(c} \partial_{ab} \xi^{d)} \right) \quad (4.5)$$

and $D_a, \bar{D}_a$ are “flat” covariant derivatives (8.11) without central charge. The covariant derivatives obey the following algebra

$$
\{ \nabla_a, \nabla_b \} = i \nabla_{ab} - i \left( \nabla_a \xi^m \nabla_b \bar{\xi}^m + \nabla_a \bar{\xi}^m \nabla_b \xi^m \right) \nabla_{mb},
\{ \nabla_a, \nabla_{\bar{b}} \} = -i \left( \nabla_a \xi^m \nabla_{\bar{b}} \bar{\xi}^m + \nabla_a \bar{\xi}^m \nabla_{\bar{b}} \xi^m \right) \nabla_{mb},
\{ \nabla_{\bar{a}}, \nabla_{\bar{b}} \} = -i \left( \nabla_{\bar{a}} \xi^m \nabla_{\bar{b}} \bar{\xi}^m + \nabla_{\bar{a}} \bar{\xi}^m \nabla_{\bar{b}} \xi^m \right) \nabla_{mb}. \quad (4.6)
$$

Now we are ready to place additional, manifestly covariant constraints on our Goldstone superfields $\xi_a, \bar{\xi}_a$. Following [13], we should covariantize the “flat” equations of motion. We have seen that the basic constraints for the double-vector multiplet (3.13), (3.14) augmented with the condition of indepedence on the central charge coordinate, eq. (3.24), give us the on-shell multiplet. Thus the sought-for covariantized system is

$$
\nabla_a \bar{\xi}_b = 0 \ , \ \nabla_a \xi_b = 0 \ , \ \nabla^a \xi_a = 0 \ , \ \nabla^a \bar{\xi}_a = 0. \quad (4.7)
$$

As it usually happens in the nonlinear realizations approach, the equations (4.7) are, despite their simple form, rather complicated, due to the highly nonlinear structure of the covariant derivatives. The complete analysis of these equations is beyond the scope of the present paper, but in order to understand which kind of dynamics they encode it is instructive to consider their bosonic limit. We find that they amount to the following equations for the vectors $F^{ab} \equiv \nabla^a \xi^b |_{\theta = 0}, \bar{F}^{ab} \equiv -\nabla^a \bar{\xi}^b |_{\theta = 0}$:

$$
\partial_{ab} F^{cb} + F^{cm}_b \partial_{mn} F^{cb}_c = 0 \ , \ \partial_{ab} \bar{F}^{cb} + F^{cm}_b \partial_{mn} \bar{F}^{cb}_c = 0. \quad (4.8)
$$
Like the previously treated cases [13], the system (4.8) is a disguised form of the Born-Infeld equations for two Maxwell gauge field strengths, augmented with the Bianchi identities. Indeed, after rewriting eqs. (4.8) as

\[
\left(1 - \frac{1}{4} F^2 \bar{F}^2\right) \partial_{ab} F_c^b - \frac{1}{2} F_a^c \partial_{bc} (F^2) - \frac{1}{4} \left(\bar{F}^2\right) F_b^c \partial_{ac} (F^2) = 0,
\]

one can bring them into the following equivalent form

\[
\partial_{ab} V^{ab} = 0, \quad \partial_{ab} \bar{V}^{ab} = 0,
\]

where

\[
V^{ab} = \frac{4 F^{ab} + 2 F^2 \bar{F}^{ab}}{4 - F^2 \bar{F}^2}, \quad \bar{V}^{ab} = \frac{4 \bar{F}^{ab} + 2 \bar{F}^2 F^{ab}}{4 - F^2 \bar{F}^2},
\]

\[
G^{ab} = \frac{4 F^{ab} - 2 F^2 \bar{F}^{ab}}{4 - F^2 \bar{F}^2}, \quad \bar{G}^{ab} = \frac{4 \bar{F}^{ab} - 2 \bar{F}^2 F^{ab}}{4 - F^2 \bar{F}^2}.
\]

After introducing the “genuine” field strengths \(V^{ab}, \bar{V}^{ab}\) eqs. (4.10) are recognized as the Bianchi identities, while eqs. (4.11) acquire the familiar form with

\[
G^{ab} = \frac{1 + V \bar{V}}{\sqrt{1 + V \bar{V}^2}} V^{ab} - V^2 \bar{V}^{ab}, \quad \bar{G}^{ab} = \frac{1 + V \bar{V}}{\sqrt{1 + V \bar{V}^2}} \bar{V}^{ab} - \bar{V}^2 V^{ab}.
\]

Thus, in this new basis the action for the bosonic core is the Born-Infeld type action for the \(SO(2)\) invariant system of two \(d = 3\) gauge field strengths \(V, \bar{V}\)

\[
S = \int d^3 x \left\{ \sqrt{\left(1 + V \bar{V}\right)^2 - V^2 \bar{V}^2} - 1 \right\}
= \int d^3 x \left\{ \sqrt{\det \left[ \eta_{mn} + (V_m \bar{V}_n + \bar{V}_m V_n) \right]} - 1 \right\}.
\]

(up to an overall coupling constant). Note that in the limit \(V_m = \bar{V}_m\) it does not immediately yield the standard Born-Infeld action with Lagrangian density

\[
\sim \sqrt{\det \left[ \eta_{mn} + \sqrt{2} \epsilon_{mnk} V^k \right]},
\]

but it is easy to show that these two representations are equivalent.

Since eqs. (4.7) represent an \(N = 4\) supersymmetrization of the Born-Infeld equations (4.10), (4.11), they can be interpreted from the brane’s viewpoint as manifestly worldvolume supersymmetric equations for some space-time filling \(N = 4\) supersymmetric D2-brane. This brane possesses an internal rigid \(SO(2)\) symmetry and incorporates two abelian gauge fields in its worldvolume multiplet, such that they form an \(SO(2)\)

\[\text{We use the } d = 3 \text{ Minkowski signature } (+ - -) \text{ and normalize the vectors so that } A_m B^m = A_{ab} B^{ab}.\]
vector. Note that this $SO(2)$ naturally arises in the nonlinear realization setting as the automorphism group of the spontaneously broken part of $N = 4, d = 3$ supersymmetry realized on the generators $S_a, \bar{S}_b$ in (1.1). It is easy to show (see below) that the action (4.14) is dual to the static-gauge form of the Nambu-Goto action for the membrane in $D = 5$, with an $SO(2)$ doublet of scalar fields. So it is plausible that the full superfield system is dual to the $N = 1, D = 5$ scalar supermembrane which in the static gauge has just two transverse bosonic directions. In this picture, the above $SO(2)$ reappears as the rotation group of the transverse directions, a remnant of the full $D = 5$ Lorentz group $SO(1, 4)$ corresponding to the breakdown pattern $SO(1, 4) \to SO(1, 2) \times SO(2)$. This supermembrane can be also obtained via the double dimensional reduction from the $N = 1, D = 6$ scalar 3-brane which, from the $d = 4$ world-volume point of view, is the theory of partial breaking $N = 2 \to N = 1$ with a chiral Goldstone superfield \[^5\]. Note that the “genuine” $N = 4$ D2-brane built with the worldvolume $N = 2, d = 3$ vector multiplet seems not to admit a straightforward off-shell dualization to the considered system. Indeed, this D2-brane clearly lacks the internal off-shell $SO(2)$ symmetry which is manifest in the system described here. The duality equivalence is expected to be restored on-shell, with the $SO(2)$ becoming a kind of duality rotation. Of course, all these statements can be made more precise after constructing an action for the above system and rederiving eqs. (4.7) just from it.

As the last topic of this Section let us sketch the proof of the duality equivalence of the BI action (4.14) to the static-gauge Nambu-Goto action for two scalar bosons.

One adds the Bianchi identities (4.10) with Lagrange multipliers $\phi, \bar{\phi}$ to (4.14), passing to the equivalent action

$$\tilde{S} = \int d^3x \left\{ \sqrt{\left(1 + \mathcal{V}\bar{\mathcal{V}}\right)^2 - V^2\bar{V}^2} - 1 + \partial \phi \bar{V} + \bar{\partial} \bar{\phi} V \right\}. \quad (4.15)$$

Then one varies with respect to $\bar{V}^{ab}, V^{ab}$ which are unconstrained in (4.15) and gets for them the following algebraic equation

$$\frac{1}{\sqrt{\left(1 + \mathcal{V}\bar{\mathcal{V}}\right)^2 - V^2\bar{V}^2}} \left[ \left(1 + \mathcal{V}\bar{\mathcal{V}}\right) V^{ab} - V^2\bar{V}^{ab} \right] + \partial_{ab} \phi = 0 \quad (4.16)$$

and its conjugate. These equations can be solved for $V, \bar{V}$, expressing the latter in terms of $\partial_{ab} \phi, \partial_{ab} \bar{\phi}$. This is rather straightforward. The simplifying observation is that these equations immediately imply

$$V \partial \bar{\phi} = \bar{V} \partial \phi$$

which, after substituting into (4.15), allows one to represent the dual action in terms of the two invariants $V\bar{V}$ and $V^2\bar{V}^2$, namely

$$\tilde{S} = \int d^3x \left\{ \frac{1 + V^2\bar{V}^2 - (V\bar{V})^2}{\sqrt{\left(1 + V\bar{V}\right)^2 - V^2\bar{V}^2}} - 1 \right\}. \quad (4.17)$$

Thus, to get the dual action of $\phi, \bar{\phi}$, it is sufficient to find the expressions for these invariants. They are obtained from (4.10) in a rather simple form which we, however,
omit to present here. After substituting these expressions into (4.17), one arrives at the action

\[ \tilde{S} = \int d^3x \left\{ \sqrt{1 - \partial\phi \partial\bar{\phi}} - (\partial\phi)^2(\partial\bar{\phi})^2 - 1 \right\} \]  

which is recognized as the static-gauge form of the standard Nambu-Goto action for the membrane in 5-dimensional Minkowski space:

\[ \tilde{S} = \int d^3x \left\{ \sqrt{\det \left[ \eta_{mn} - (\partial_m\phi\partial_n\bar{\phi} + \partial_m\bar{\phi}\partial_n\phi) \right]} - 1 \right\} . \] (4.19)

5 A new \( N = 4, d = 3 \) supermultiplet

The double-vector multiplet we considered in the previous Sections is very similar to the \( N = 2, d = 4 \) vector-tensor multiplet [17]. It is known [17] that the vector-tensor multiplet can be combined with Fayet-Sohnius hypermultiplet to form an off-shell \( N = 4 \) SYM supermultiplet in the central charge extended superspace. Therefore, a natural question arises in our case: is it possible to add some additional \( N = 2, d = 3 \) supermultiplet to our double-vector multiplet so as to end up with an \( N = 4, d = 3 \) off-shell supermultiplet?

Some hints how to construct such \( N = 4 \) supermultiplet come from the previous Section. Namely, we showed that the double-vector multiplet can be utilized as the Goldstone superfield for a nonlinear realization of \( N = 4, d = 3 \) supersymmetry partially broken down to \( N = 2 \). This gives us the theory of a self-interacting double-vector multiplet, whose Lagrangian density, a nonlinear generalization of (3.19), could be in principle constructed in terms of \( \xi^a, \bar{\xi}^a \). We also know that the PBGS theories often allow one to combine the Lagrangian density together with the basic Goldstone superfields into a linear supermultiplet of higher supersymmetry. These facts suggest a combination of our double-vector supermultiplet with some scalar bosonic superfield \( \Phi \) to form a linear \( N = 4 \) multiplet. Of course, this additional superfield \( \Phi \) should also be properly constrained.

Let us now try to find a linear realization of an extra \( N = 2, d = 3 \) supersymmetry on the double-vector multiplet \( \eta^a, \bar{\eta}^a \) with (3.13), (3.14) together with some, yet unconstrained, scalar superfield \( \Phi(x, z, \theta, \bar{\theta}) \). Assuming the second supersymmetry to be spontaneously broken and requiring its standard closure, one can find the following realization

\[ \delta \eta^a = \epsilon^a - \epsilon^b \mathcal{D}_b \Phi, \quad \delta \bar{\eta}^a = \bar{\epsilon}^a - \bar{\epsilon}^b \bar{\mathcal{D}}_b \Phi, \quad \delta \Phi = \epsilon^a \bar{\eta}_a + \bar{\epsilon}^a \eta_a . \] (5.1)

Now we have to put the proper constraints on the superfield \( \Phi \).

First, if we wish to consider \( \Phi \) as a candidate for the off-shell Lagrangian density corresponding to the PBGS system of the previous Section, we should impose the constraints

\[ \mathcal{D}^2 \Phi = \bar{\mathcal{D}}^2 \Phi = 0 \] (5.2)

(recall (3.20)). The constraints (5.2) are in agreement with the transformation properties of \( \Phi \) (p.1).

The rest of constraints follow from the condition that the r.h.s. of \( \delta \eta^a, \delta \bar{\eta}^a \) in (5.1) obeys the same constraints as \( \eta^a, \bar{\eta}^a \) in eqs. (3.13), (3.14). This implies

\[ (i\partial_{ab} + \varepsilon_{ab}\partial_z) \mathcal{D}^b \Phi = 0 , \quad (i\partial_{ab} - \varepsilon_{ab}\partial_z) \bar{\mathcal{D}}^b \Phi = 0 . \] (5.3)
One may check that these constraints are also compatible with the transformation properties of $\Phi$ (5.1). Thus, the $N = 2$ superfields $\{\eta^a, \bar{\eta}^a, \Phi\}$, defined in the central charge extended superspace and respecting the constraints (3.13), (3.14), (5.2), (5.3), constitute indeed a linear off-shell $N = 4$ supermultiplet. One $N = 2$ supersymmetry is manifest, while another one is realized according to (5.1).

It is instructive to consider the component structure of $\Phi$. Straightforward computation shows that the $\theta, \bar{\theta}$ expansion of $\Phi$ is also extremely short:

$$\Phi = \phi + \theta^a \psi_a + \bar{\theta}^a \bar{\psi}_a + \theta^a \bar{\theta}_a \rho + \theta^a \bar{\theta}^b \partial_{ab} v,$$

(5.4)

where the $z$-dependence of the components is restricted by

$$i \partial_{ab} \bar{\psi}^b + \partial_z \bar{\psi}_a = 0, \quad i \partial_{ab} \psi^b - \partial_z \psi_a = 0, \quad \Box \phi = 2 \partial_z^2 \phi, \quad \partial_z \rho = \frac{i}{2} \Box v, \quad \partial_z v = -i \rho.$$

(5.5)

Thus, $\Phi$ contains four scalars $\{\phi, \partial_z \phi, \rho, v\}$ and four fermions $\{\psi_a, \bar{\psi}_a\}$. This content coincides with that of $N = 2, d = 4$ vector-tensor multiplet [17], suggesting that the supermultiplet $\{\eta^a, \bar{\eta}^a, \Phi\}$ could be re-obtained from the vector-tensor one via $d = 4 \rightarrow d = 3$ dimensional reduction.

Finally, we write the $N = 4$ invariant free action for our multiplet as

$$S = \int d^3 x d^4 \theta \left( \bar{\theta} \right) \left( \eta^a \bar{\eta}_a + \mathcal{D}^a \Phi \mathcal{D}_a \Phi \right).$$

(5.6)

Like in the case of the action (3.19), the invariance of (5.6) is checked with the heavy use of the basic constraints on the superfields involved.

6 Concluding remarks

In this paper we have constructed, for the first time, a new off-shell $N = 2, d = 3$ supermultiplet which contains only vector fields among its bosonic components. This “double-vector” multiplet possesses a non-trivial realization of the central charge and, therefore, can be formulated only in the central-charge extended superspace. This multiplet fills a last gap in the variety of all possible dualizations of the $N = 2, d = 3$ chiral supermultiplet and corresponds to the dualization of both scalar fields into vectors. Using the double-vector multiplet as the Goldstone multiplet for $N = 4 \rightarrow N = 2$ partial breaking, we were able to derive the superfield equations of motion. In the bosonic sector they yield the Born-Infeld interaction of two vector fields forming an $SO(2)$ doublet. Finally, in the $N = 2, d = 3$ central-charge extended superspace we discover a scalar multiplet which, together with the double-vector multiplet, forms an off-shell $N = 4$ supermultiplet.

One may ask why we should consider the rather complicated double-vector multiplet? After all, the $N = 4 \rightarrow N = 2$ partial breaking of supersymmetry in $d = 3$ can be perfectly described by chiral as well as by vector multiplets (the straightforward dimensional reduction of the results of [3, 4]), while the dualization of the scalar in the vector $N = 2, d = 3$ supermultiplet corresponds to the reduction from an on-shell $N = 1, d = 4$ supermultiplet which cannot even be coupled to supergravity [18]. However, the main
goal of our investigation was to show that the supermultiplets in the central charge extended superspace can equally be used to describe the partial breaking of supersymmetry. Moreover, as follows from the consideration in Sect 4, one may introduce self-interactions of such multiplets not only through a non-linear deformation of the basic constraints, like in the case of the VT supermultiplet in $D = 4$ \cite{19}, but also through their “covariantization” with respect to nonlinear realizations of some higher-order supersymmetries. It is this possibility to which we would like to draw attention. One can hope to utilize other known (and still unknown) off-shell linear supermultiplets with non-trivially realized central charges for describing PBGS patterns of extended supersymmetries. For instance, given $N = 4$, $d = 4$ supersymmetry, they provide the only presently known off-shell realizations of it, and one can expect \cite{20} that they will be suitable for the construction of the appropriate PBGS actions along the lines of refs. \cite{5}-\cite{12}.

Let us finally note that in this paper we did not attempt to construct the full action for the $N = 4$, $d = 3$ D2-brane in the nonlinear or linear realization approaches. This problem will be addressed elsewhere.

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