Low-Complexity Transmission Mode Selection in MU-MIMO Systems

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Abstract—We propose a low-complexity transmission strategy in multi-user multiple-input multiple-output downlink systems. The adaptive strategy adjusts the precoding methods, denoted as the transmission mode, to improve the system sum rates while maintaining the number of simultaneously served users. Three linear precoding transmission modes are discussed, i.e., the block diagonalization zero-forcing (BDZF), the cooperative zero-forcing (CZF), and the cooperative matched-filter (CMF). Considering both the number of data streams and the multiple-antenna configuration of users, we modify the common CZF and CMF modes by allocating data streams. Then, the transmission mode is selected between the modified ones according to the asymptotic sum rate analyses. As instantaneous channel state information is not needed for the mode selection, the computational complexity is significantly reduced. Numerical simulations confirm our analyses and demonstrate that the proposed scheme achieves substantial performance gains with very low computational complexity.

I. INTRODUCTION

Multi-user multiple-input multiple-output (MU-MIMO) systems have drawn a lot of attention in the past decades. In order to handle the exponential growth in mobile data traffic, many novel techniques such as mmWave, full-duplex transmission and large-scale antenna system (LSAS) are proposed to improve the performance of MU-MIMO systems. In particular, LSAS, which achieves huge spectral-efficiency and energy-efficiency gains [1], [2], has recently been considered as a technological breakthrough that holds great potential for the future wireless communication.

In MU-MIMO downlink systems, the capacity region can be achieved by employing Dirty Paper Coding (DPC) at the transmitter [3]. But DPC is too complicated to be used in practice. Consequently, many linear precoding schemes, which show impressive performance with much lower complexity, have been proposed for practical multi-user downlink systems, such as block diagonalization (BD) [4], zero-forcing (ZF), and matched-filter (MF). Recently, linear precoding in LSAS has become a hot research topic. [5] and [6] compared the performance of ZF and MF precoders in multi-user multiple-input single-output (MU-MISO) LSAS. [7] studied the regularized ZF (RZF) precoder of MU-MISO LSAS in-depth, such as the optimal regularization parameter for RZF. However, the existing studies on LSAS mainly focus on single-antenna users rather than multi-antenna users, and have not been further developed for transmission strategy design. Although various kinds of efficient transmission strategies have been designed in the conventional MIMO systems, they are not suitable for LSAS because most of them need online computations relating to the instantaneous channel state information (CSI). For example, [8] developed an efficient transmission scheme by adapting linear precoding and signal modulation, relying upon the instantaneous channel capacity of probability of error. With the increasing amount of system antennas, the computational complexity of the above scheme becomes unbearable.

In this paper, we propose a low-complexity transmission strategy in downlink MU-MIMO LSAS. The adaptive strategy adjusts the precoding methods, denoted as the transmission mode, to enhance the system sum rate performance. First, we get deterministic sum rate approximations for the block diagonalization zero-forcing (BDZF), the cooperative zero-forcing (CZF) and the cooperative matched-filter (CMF) modes. In particular, we obtain a good upper bound of the sum rate of full-spatial-multiplexing CZF, which has never been addressed in the aforementioned LSAS works. These deterministic approximations enable us to propose an very low-complexity transmission strategy without any instantaneous CSI. Therefore, in order to achieve better sum rate performance and maintain the number of simultaneously served users, we modify the common CZF and CMF precoding schemes by scheduling the optimal amount of data streams as far as possible. Furthermore, one of the modified CZF and CMF modes is selected for higher sum rate. Such strategy cannot be developed in MU-MISO works [5]–[7], [9] as it takes advantage of the multi-antenna configuration of users.

Notations: We use uppercase boldface letters for matrices and lowercase boldface for vectors. $(\cdot)^H$, $(\cdot)^T$, tr$(\cdot)$, $E[\cdot]$ and $[\cdot]$ denote the conjugate transpose, the pseudo-inverse, the trace, the expectation, and the round down operation, respectively. $CN(m, \Theta)$ denotes the circularly-symmetric complex Gaussian distribution with mean vector $m$ and covariance matrix $\Theta$. $\xrightarrow{a.s.}$ denotes the almost sure convergence, and $\xrightarrow{d}$ denotes convergence in distribution.
II. SYSTEM MODEL

We consider a MU-MIMO downlink system composed of an $M$-antenna base station and $K$ simultaneously served $N$-antenna users. We assume $M \geq K$, so that user scheduling is not taken into account. Perfect CSI is assumed available at the base station. The base station sends $N_k$ data streams to the $k$-th user ($1 \leq N_k \leq N$), so that the total number of data streams of the system is $L = \sum_{k=1}^{K} N_k$. The transmitted signal $x \in \mathbb{C}^{M \times 1}$ is defined as

$$x = Ws = \sum_{k=1}^{K} W_k s_k$$

with an average total power constraint $E[\text{tr}(xx^H)] \leq P$, where $P$ is the total available transmit power. $W = [W_1, \ldots, W_K] \in \mathbb{C}^{M \times L}$ is the total precoding matrix at the base station, and $s = [s_1^H, \ldots, s_L^H] \in \mathbb{C}^{L \times 1}$ is the information-bearing vector from the base station to all the $K$ users. $W_k \in \mathbb{C}^{M \times N_k}$ and $s_k \in \mathbb{C}^{N_k \times 1}$ denotes the precoding matrix and the data vector for the $k$-th user, respectively, $N_k$ antennas are pre-selected at the $k$-th user to receive signals and the $N_k \times 1$ received signal vector is

$$y_k = H_x x + n_k,$$

where $H_x \in \mathbb{C}^{N_k \times M}$ with independent $\mathcal{CN}(0, 1)$ entries is the channel matrix from the base station to the $k$-th user, $n_k \in \mathbb{C}^{N_k \times 1}$ with $\mathcal{CN}(0, \sigma^2_n)$ entries is the additive white Gaussian noise at the $k$-th user. The $i$-th received data stream of the $k$-th user (hereinafter referred to as data stream $(k,i), 1 \leq i \leq N_k, 1 \leq k \leq K$) is given by

$$y_{k,i} = h_{k,i}w_{k,i}s_{k,i} + \sum_{j=1, j \neq i}^{N_k} w_{k,j}s_{k,j} + h_{k,i} \sum_{l=1, l \neq k}^{K} W_l s_l + n_{k,i},$$

where $h_{k,i} \in \mathbb{C}^{1 \times M}$, $w_{k,i} \in \mathbb{C}^{M \times 1}$, $s_{k,i}$ and $n_{k,i}$ is the $i$-th row of $H_x$, the $i$-th column of $W_k$, the $i$-th element of $s_k$ and the $i$-th entry of $n_k$, respectively.

Denoting the signal-to-interference-and-noise ratio (SINR) of the data stream $(k,i)$ by $\text{SINR}_{k,i}$, the rate of data stream $(k,i)$ is given by

$$r_{k,i} = \log_2(1 + \text{SINR}_{k,i}).$$

Then the system sum rate can be calculated as

$$\mathcal{R} = \sum_{k=1}^{K} \sum_{i=1}^{N_k} r_{k,i}.$$  

III. TRANSMISSION MODES REVIEW

We briefly go over the three transmission modes (i.e., BDZF, CZF and CMF) of MU-MIMO in this section.

A. BDZF

In the BDZF mode, BD technique is utilized to precancel inter-user interference followed by ZF precoders to remove the inter-stream interference of each user. Hence, $W_k$ is defined as a cascade of two matrices, i.e.,

$$W_k = \alpha_k B_k D_k,$$

where $\alpha_k$ is the power control parameter. In this paper, uniform power allocation among data streams is adopted, so we get $\alpha_k = \sqrt{PN_k/(\text{tr}(D_k B_k H_k^H B_k^H D_k^H H_k^H K N_k L))}$. $B_k \in \mathbb{C}^{M \times T}$ is designed with the general method introduced in [4] to remove the inter-user interference in (3). If we denote $H_k = H_x B_k$, the ZF precoding matrix $D_k \in \mathbb{C}^{T \times N_k}$ is $D_k = H_k^{-1}$. To ensure the support of $N_k$ data streams for the $k$-th user, $T_k$ should satisfy the constraint $N_k \leq T_k \leq M + N_k - L$. The SINR of data stream $(k,i)$ is given by

$$\text{SINR}_{k,i}^{\text{BDZF}} = \frac{PN_k}{\sigma^2_n \text{tr}(D_k B_k D_k^H B_k^H K N_k L)} = \frac{PN_k}{\sigma^2_n \text{tr}(H_k H_k^H)^{-1}}.$$  

B. CZF

For CZF, the MU-MIMO system is treated as an equivalent single-user MIMO (SU-MIMO) system. The equivalent channel $H \in \mathbb{C}^{L \times M}$ from the base station to all the $K$ users is $H = [H_1^H, H_2^H, \ldots, H_K^H]^H$. The CZF precoding matrix is

$$W_{\text{CZF}} = \beta(H)^H = \beta H^H (HH^H)^{-1},$$

where $\beta = \sqrt{P/\text{tr}(H H^H)}$ is utilized to normalize the transmit power. The SINR of data stream $(k,i)$ is

$$\text{SINR}_{k,i}^{\text{CZF}} = \frac{P}{\sigma^2_n \text{tr}(H H^H)^{-1}}.$$  

C. CMF

For CMF, the MU-MIMO system is also treated as a SU-MIMO system. MF instead of ZF precoding is utilized. The CMF precoding matrix is

$$W_{\text{CMF}} = \gamma H^H,$$

where $\gamma = \sqrt{P/\text{tr}(H H^H)}$. For data stream $(k,i)$, we get

$$y_{k,i} = \gamma \|h_{k,i}\|^2 s_{k,i} + \gamma h_{k,i} \sum_{l=(m) \neq (k,i)} h^H_{l,m} s_{l,m} + n_{k,i}.$$  

The corresponding SINR is

$$\text{SINR}_{k,i}^{\text{CMF}} = \frac{\gamma^2 \|h_{k,i}\|^4}{\sigma^2_n + \gamma^2 |h_{k,i}|^2 \sum_{l=(m) \neq (k,i)} h^H_{l,m} h_{l,m} + h_{k,i}^H h_{k,i}}.$$  

IV. PROPOSED TRANSMISSION MODE SELECTION SCHEME

In this section, we propose a simple and effective transmission mode selection scheme based on the analysis of the asymptotic sum rate performance of the three aforementioned transmission modes.
A. Sum Rate Analysis of Large-Scale MU-MIMO Systems

In BDZF mode, in order to figure out the sum rate, we need to focus on \( \text{tr}(HH^H)^{-1} \). Since the elements of \( H_k \) are i.i.d. \( \mathcal{CN}(0, 1) \) random variables and \( B_k \) is \( \mathcal{CN}(0, 1) \) random variables. Using the results of [10], we get \( \text{tr}(HH^H)^{-1} \to_{a.s.} N_k/(T_k - N_k) \) as \( T_k \) and \( N_k \) go to infinity with keeping a finite ratio \( T_k/N_k \). The sum rate is given by

\[
R_{\text{BDZF}} \to_{a.s.} \sum_{k=1}^{K} N_k \log_2 \left( 1 + \frac{P(T_k - N_k)}{\sigma_n^2 L} \right). \tag{13}
\]

For CMF, similar to [5], we have

\[
R_{\text{CMF}} \overset{d}{\to} L \log_2 \left( 1 + \frac{P(M + 1)}{\sigma_n^2 L + P(L - 1)} \right). \tag{14}
\]

For CZF, in the case of \( M > L \), as proposed in many previous works [5, 7, 10, 11], the sum rate is

\[
R_{\text{CZF}} \to_{a.s.} L \log_2 \left( 1 + \frac{P(M - L)}{\sigma_n^2 L} \right). \tag{15}
\]

In the case of \( M = L \), we propose a good upper bound for the sum rate of CZF here. To the best of our knowledge, this has never been addressed in the existing works. The sum rate can be rewritten as

\[
R_{\text{CZF}} = L \log_2 \left( 1 + \frac{P}{\sigma_n^2 \text{tr}(HH^H)} \right) = L \log_2 \left( 1 + \frac{P}{\sigma_n^2 \sum_{l=1}^{L} \lambda_l^{-1}} \right), \tag{16}
\]

where \( \lambda_l \) indicates the \( l \)-th eigenvalue of the matrix \( HH^H \). At low signal-to-noise ratio (SNR), we have

\[
R_{\text{CZF}} \approx \frac{PL}{\sigma_n^2 \sum_{l=1}^{L} \lambda_l^{-1}} \leq \sum_{l=1}^{L} \rho \lambda_l \approx \sum_{l=1}^{L} \log_2 (1 + \rho \lambda_l), \tag{17}
\]

where \( \rho = P/(L \sigma_n^2) \), “\( \approx \)” derived from \( \log_2 (1 + x) \approx x \) for sufficiently small \( x \), and “\( \leq \)” obtained by the Arithmetic-Harmonic Mean inequality. At high SNR, we have

\[
R_{\text{CZF}} \approx \log_2 \left( \frac{P}{\sigma_n^2 \sum_{l=1}^{L} \lambda_l^{-1}} \right) \leq \log_2 \left( \prod_{l=1}^{L} \rho \lambda_l \right) \approx \sum_{l=1}^{L} \log_2 (1 + \rho \lambda_l), \tag{18}
\]

as a result of \( \log_2 (1 + x) \approx \log_2 x \) for sufficiently large \( x \), and the less or equal relation is obtained by the Geometric-Harmonic Mean inequality. Therefore, we obtain

\[
R_{\text{CZF}} \leq \sum_{l=1}^{L} \log_2 (1 + \rho \lambda_l). \tag{19}
\]

Similar to [10], the upper bound of \( R_{\text{CZF}} \) for \( M = L \) is (we omit details here due to space limitations)

\[
R_{\text{CZF}}^{\text{II}} \overset{a.s.}{\to} 2M \log_2 \left( \frac{1 + \sqrt{1 + 4\rho}}{2} \right) - M \log_2 e (\sqrt{1 + 4\rho} + 1)^2. \tag{20}
\]

B. Optimal Number of Data Streams in CZF

When \( M > L \), if \( M \) and \( \sigma_n^2 \) are fixed, we can easily get the explicit solution of the optimal number of data streams by setting \( dR_{\text{CZF}}/dL = 0 \), i.e.,

\[
L_{\text{CZF}}^* = \begin{cases} \frac{MP \omega}{(P - \sigma_n^2)(1 + \omega)}, & P \neq \sigma_n^2 \\ \frac{M}{e \ln 2}, & P = \sigma_n^2 \end{cases}, \tag{21}
\]

where \( \omega = W \left( (P - \sigma_n^2)/(\sigma_n^2 e) \right) \) and \( W(\cdot) \) is the Lambert W function.

Remark 1. Similar results about \( L_{\text{CZF}}^* \) appear in [11, 12]. However, the case of \( M = L \) should not be ignored. It is necessary to discuss whether \( M \) is the optimal number of data streams.

With (21), we can get the largest sum rate when \( M > L \) as

\[
R_{\text{CZF}}^1 \overset{a.s.}{\to} \begin{cases} \frac{MP \omega}{(P - \sigma_n^2) \ln 2}, & P \neq \sigma_n^2 \\ \frac{M}{e \ln 2}, & P = \sigma_n^2 \end{cases}. \tag{22}
\]

Hence, by comparing the value of (20) and (22), the optimal number of data streams in CZF mode is given by

\[
L_{\text{CZF}}^* = \begin{cases} L_{\text{CZF}}^1, & R_{\text{CZF}}^1 \geq R_{\text{CZF}}^{\text{II}} \\ M, & R_{\text{CZF}}^1 < R_{\text{CZF}}^{\text{II}} \end{cases}. \tag{23}
\]

Clearly, given \( M, P \) and \( \sigma_n^2 \), we can easily get the optimal number of data streams in terms of the sum rate.

Remark 2. \( L_{\text{CZF}}^* \) is usually smaller than \( M \). If the transmit power is large enough, we have \( L_{\text{CZF}}^* = M \).

C. Modified CZF and CMF Schemes

In the common CZF mode, we have \( N_k = N \) (i.e., \( L = NK \)), which cannot ensure the optimal sum rate performance as discussed above. Moreover, \( M/N \) limits the number of simultaneously served users in both CZF and CMF modes.

We modify the common CZF mode through selecting the number of data streams and allocating the data streams to each user. The configuration \( L = L_{\text{CZF}}^* \) is ensured as far as possible to enhance the system sum rate performance.

Algorithm 1 shows the details of the modified CZF scheme.
Algorithm 1 Modified CZF Scheme
1) Calculate the optimal number of data streams \( L^*_{\text{CZF}} \) according to (23) with given \( M, P \) and \( \sigma_n^2 \).
2) Decide the number of data streams depending on \( K, N \) and \( L^*_{\text{CZF}} \).
   - A: \( L^*_{\text{CZF}} \geq NK \), each user utilizes all its \( N \) antennas to receive data, i.e., \( N_k = N \).
   - B: \( K \leq L^*_{\text{CZF}} \leq NK \), set \( N_k = \lceil L^*_{\text{CZF}} / K \rceil + \Delta_k \), where \( \Delta_k = 0 \) or 1 is randomly chosen to satisfy \( L = L^*_{\text{CZF}} \).
   - C: \( K > L^*_{\text{CZF}} \), set \( N_k = 1 \) for all \( K \).
3) The base station informs each user \( N_k \) and transmits data streams to users with CZF precoding.

Given the system parameters \( M, N, K, P \) and \( \sigma_n^2 \), we select the transmission mode which provides higher system sum rate. The modified CZF mode is selected for data transmission in the case of \( \mathcal{R}_{\text{CZF}} \geq \mathcal{R}_{\text{CMF}} \). In another case (i.e., \( \mathcal{R}_{\text{CZF}} < \mathcal{R}_{\text{CMF}} \)) the modified CMF mode is selected.

Furthermore, as the antenna configuration of the base station and users, the total available transmit power and the thermal noise are almost unchanged in practical cellular systems, i.e., \( M, N, P \) and \( \sigma_n^2 \) are fixed, we can pre-calculate sum rates with (24) and (25) for various \( K \), and then find the intervals of \( K \) for CZF and CMF modes in advance. Hence, the proposed transmission mode selection only depends on \( K \). The details are presented in Algorithm 2.

Algorithm 2 Proposed Transmission Mode Selection Scheme
1) Find \( L^*_{\text{CZF}} \) according to (21), (22), (24) and (25) with given \( M, N, P \) and \( \sigma_n^2 \).
2) Find the intervals of \( K \) for different transmission modes as follows:
   \[ \pi_{\text{CZF}} = \{ K | K \in \mathbb{N}^+, \mathcal{R}_{\text{CZF}} \geq \mathcal{R}_{\text{CMF}} \}, \]
   \[ \pi_{\text{CMF}} = \{ K | K \in \mathbb{N}^+, \mathcal{R}_{\text{CZF}} < \mathcal{R}_{\text{CMF}} \}, \]

   where \( \mathcal{R}_{\text{CZF}} \) and \( \mathcal{R}_{\text{CMF}} \) are calculated as (24) and (25), respectively.
3) Select the modified CZF or CMF mode for data transmission according to \( K \).

Remark 3. With the modified CZF and CMF schemes, besides better sum rate performance (especially for a large number \( K \) of users), the proposed scheme supports up to \( K = M \) users simultaneously compared with \( K = M/N \) in the common schemes.

E. Complexity Analysis
For the system model that we discuss in this paper, if we select the transmission by the brute force searching, about \( \left( \binom{K}{N} + \cdots + \binom{K}{N} \right)^K = (2^K - 1)^K \) data stream allocations need to be checked for finding the best mode, which is unacceptable in practical implementations. In contrast, in the proposed scheme, we only need to determine the interval \( \pi_{\text{CZF}} \) or \( \pi_{\text{CMF}} \) that \( K \) belongs to during the selection. The intervals can be solved by any standard numerical methods, and need to be updated only when \( M, N, P \) or \( \sigma_n^2 \) changes. If the system configuration is unchanged, only Step 3) in Algorithm 2 is required for the selection.

V. NUMERICAL SIMULATIONS
This section illustrates the performance of the proposed transmission mode selection scheme by Monte Carlo simulations. The transmit SNR is defined as \( \text{SNR} = P/\sigma_n^2 \).

In Fig. 1 we discuss an \( M = 64, N = 2 \) MU-MIMO system and set \( T_k = M - N_k + L \) in BDZF mode. The sum rate approximations of the BDZF, CZF and CMF modes are compared with the ergodic sum rates averaged over 10000 independent channel realizations. It can be observed that
the approximations in (13), (15) and (14) are accurate for $M/L > 1$. (20) is also an effective upper bound of $R_{CZF}$ in the case of $M/L = 1$. As mentioned in Section IV-D, the BDZF mode has almost the same sum rate performance as the CZF mode. Furthermore, it is clear the CZF and BDZF curves for $K = 11$ are above those for $K = 5$ and $K = 32$, which confirms that the sum rates of CZF and BDZF are not monotonically increasing with the increasing $K$ under certain SNR conditions. However, the sum rate of CMF increases with the growth of $K$. In addition, the CMF mode achieves better sum rate performance than the CZF and the BDZF modes at low SNR while the opposite is true at high SNR.

The sum rates of various transmission modes with $M = 16, N = 2$ are presented in Fig. 2. The transmit SNR is 0 dB and the power is uniformly allocated on each data stream. The legend “Proposed TS” indicates the proposed transmission mode selection scheme and the blue curves show the performance of the common CZF scheme. As expected, the proposed scheme combines the advantages of CZF and CMF, and supports up to 16 users at the same time, while the common modes only support 8 users. The brute force search is utilized as a benchmark to evaluate the sum rate performance. It can be seen that about 90% sum rates of the brute force search scheme are achieved by the proposed scheme. We also show the performance of the transmission selection scheme proposed in [6] for $M = 16, N = 1$. It is obviously that for the same number $K$ ($K < M$) of served users, our proposed scheme with multi-antenna users obtains better sum rate performance.

VI. CONCLUSIONS

We have proposed a low-complexity transmission scheme in MU-MIMO systems in the paper. Based on the comprehensive discussions on the sum rate performance of the BDZF, CZF and CMF precoding schemes, we have modified the common CZF and CMF schemes and developed a novel transmission mode selection approach to enhance the system sum rate performance while maintaining the number of simultaneously served users. The simulations show that our proposed transmission mode selection scheme achieves near optimal sum rate performance with extremely low computational complexity. In the future, we will focus on the low-complexity transmission mode selection scheme in multi-cell large-scale downlink systems.

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