A Sensitivity Study for a MICE Liquid Hydrogen Absorber

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Abstract

The International Muon Ionization Cooling Experiment (MICE) is devoted to a study of a muon cooling channel capable of giving the desired performance for a Neutrino Factory. One of the goals is achieving an absolute accuracy of measurements of emittance reduction as high as \( \pm 0.1\% \). The paper describes results of a Monte Carlo study on allowed density variations of liquid hydrogen corresponding to the desired accuracy of the measurements.

1 Introduction

An experiment which allows the investigation of the performance of a muon cooling channel at different conditions of interest has been designed by MICE collaboration \textsuperscript{11}. In particular, high precision measurements of emittance reduction are planned to be performed with an accuracy of \( \pm 0.1\% \). Various factors should be taken into account to achieve the desired accuracy. One of the essential parameters is material density of the liquid hydrogen in the absorber. The density is influenced by temperature and pressure of the hydrogen kept within pre-determined tolerances. The Monte Carlo study with the MARS14 code \textsuperscript{2} was undertaken to calculate the dependence of the emittance reduction on hydrogen density at realistic conditions. The results can be used to determine the above-mentioned tolerances corresponding to the desired accuracy of measurements.

2 Basic Formulae

In a four-dimensional phase space with coordinates \( x, p_x, y, p_y \), the normalized emittance of a muon beam, \( \varepsilon_n \), can be calculated according to the expression \textsuperscript{3}
\[ \varepsilon_n = \pi \sqrt{\det \Sigma}, \tag{1} \]

where \( \Sigma \) is a \( 4 \times 4 \) correlation matrix,

\[
\Sigma = \begin{pmatrix}
aa & ab & ac & ad \\
ba & bb & bc & bd \\
cd & cb & cc & cd \\
da & db & dc & dd
\end{pmatrix},
\tag{2}
\]

and \( a \) is \( x - \bar{x} \), \( b \) is \( (p_x - \bar{p}_x)/m \mu c \), \( x \) and \( p_x \) are muon coordinate and momentum, respectively, along \( x \)-axis. The entries \( c \) and \( d \) are for \( y \)-axis and analogous to \( a \) and \( b \), respectively. In the expressions the bar over the symbols means statistical averaging over an ensemble of simulated muon trajectories.

To generate a proper muon beam distribution in the system, symmetry considerations are taken into account \[4\]. Firstly, Gaussian distributions are modeled for \( x, p_x, y, p_y \) in the geometrical center of the absorber, where magnetic field flips, using information on the \( \beta \)-function distribution in the channel \[1\]. Secondly, backward muon transport is modeled in the magnetic field and without the material (hydrogen and aluminum) to get a reflected incoming muon distribution in front of the absorber. And thirdly, regular muon transport through the absorber is modeled.

For a given ensemble of trajectories, the matrix \( \Sigma \) is calculated both in front of and behind the liquid hydrogen absorber. After that the emittance reduction, \( \Delta \varepsilon_n \), is calculated according to expression (1).

### 3 Geometry Model

Realistic geometry of the 35-cm liquid hydrogen absorber along with 'inflected' window design \[1, 5\] was implemented in the model (see Fig. 1). The absorber is inside a solenoid which is taken into account via its magnetic field only. Thickness of the aluminum windows of the absorber is variable with the thinnest parts being on the \( z \)-axis.

Realistic three-dimensional distributions of magnetic field over the cooling channel were taken from Ref. \[6\]. The longitudinal and radial solenoidal field distributions as implemented in the MARS model are shown in Fig. 2.

### 4 Calculated Results

The Monte Carlo calculations were performed for 200-MeV/c incident muons at various densities of the liquid hydrogen. Results of the calculations are shown in Fig. 3. Statistical uncertainty (1σ) of the calculations was about 0.02% which is less than the linear size of the symbols used in the Figure.
Figure 1: A cross section of the MARS model of a MICE liquid hydrogen absorber along with 100 sampled muon tracks.

Figure 2: The longitudinal (left) and radial (right) distributions of magnetic field around the liquid hydrogen absorber. The arrows indicate the field direction only, not magnitude. The field flips at the geometrical center of the absorber.
Figure 3: Calculated emittance reduction vs liquid hydrogen density in the central absorber of the cooling channel for 200-MeV/c muons [1]. Here 100% corresponds to 0.0708 g/cm$^3$.

One can see that a variation as high as 2% in hydrogen density gives rise to a 0.1% variation in $\Delta\varepsilon_n$. Therefore, to ensure the desired accuracy of emittance measurements, tolerance for the density of the liquid hydrogen should not exceed 2%.

5 Conclusions

Monte Carlo calculations were performed on emittance reduction of a muon beam vs hydrogen density for a MICE liquid hydrogen absorber within realistic absorber geometry and with detailed three-dimensional distribution of magnetic field. It was shown that, within the range of interest, the density dependence is clearly linear. To ensure the accuracy of emittance measurements as high as 0.1%, tolerance for the hydrogen density should be less than 2%.

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