An Analytical Stochastic Dynamic Programming Algorithm for Energy Storage Price Arbitrage

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Abstract—This paper presents a computation-efficient stochastic dynamic programming algorithm for solving energy storage price arbitrage. We formulate the price arbitrage problem using stochastic dynamic programming and model real-time prices as a Markov process. Then we propose an analytical solution algorithm using a piecewise linear approximation of the value-to-go function. Our method does not discretize control and state of the energy storage. Our solution algorithm achieves extreme computation performance and solves the proposed arbitrage problem for one operating day in less than one second on a personal computer. We demonstrate our approach using historical price data from four price zones in New York Independent System Operator, with case studies comparing the performance of different stochastic models and storage settings. Our results show that the proposed method captures 50% to 90% of arbitrage profit compared to perfect price forecasts. In particular, our method captures more than 80% of arbitrage profit in three out of the four price zones when considering batteries with more than two-hour duration and realistic degradation cost.

Index Terms—Energy storage; Power system economics; Stochastic optimal control.

I. INTRODUCTION

Decarbonization in the electric power sector is pivotal for sustainable developments as other sectors become electrified. The renewable energy share of the US electricity generation mix has reached more than 20% by 2020 and is projected will double by 2050 [1]. Electrical energy storage resources provide reliable and affordable solutions to many challenges in a power system with a high renewable share, such as peak shaving [2], voltage support [3], and frequency regulation [4]. Pilot programs have shown storage is a cost-effective and profitable solution in these applications. In a competitive market, storage that maximizes profit will also help to maximize social welfare [5]. The Federal Energy Regulatory Commission (FERC) issued Order 841 [6] to further remove barriers for energy storage to participate in wholesale electricity markets and urge independent system operators and regional transmission organizations to develop bidding and dispatch models for energy storage.

Yet, the participation of energy storage in wholesale energy markets has been limited compared to other applications, even with the fast dropping cost of energy storage [7]. As the most significant market and foundation of deregulated power systems, it is critical for storage to participate in wholesale energy markets efficiently. However, although all system operators have allowed energy storage to participate, current market designs have difficulties in dispatching storage accurately, restricted by the capability of forecasting and the limitation of computation power. Therefore, practical and efficient bid and control designs are heavily reliant on energy storage to better participate in energy markets.

This paper proposes an analytical stochastic dynamic programming (SDP) algorithm for optimizing energy storage price arbitrage in real-time energy markets with extreme computation efficiency. Our method targets a generic energy storage model with constant efficiency and discharge cost. Compared to optimization-based storage bidding and control methods such as bi-level optimization [8]–[10], our method is lightweight and easy to implement. While compared to other dynamic programming or reinforcement learning methods [11], [12], our method does not discretize energy storage controls and states. Our method is efficient to train and can explicitly incorporate physical storage parameters such as efficiency and degradation. The main contribution of this paper is as follows.

1) We develop an analytical algorithm for solving storage SDP for real-time energy price arbitrage without using third-party solvers. The algorithm uses less than one second to solve a real-time arbitrage problem for one operating day, including 288 stages (5-minute market clearing) and 22 price nodes per stage.

2) We model real-time price uncertainties as a Markov process. The Markov models are trained using historical price data, and are then incorporated into SDP to model price node transition probabilities.

3) We compare different Markov process models for real-time price arbitrage, including stage and season dependency. The result shows that considering stage-wise dependency in price prediction improves model performance significantly.

4) We demonstrate our algorithm by performing price arbitrage in different price zones in New York Independent System Operator (NYISO). Results show that the proposed method captures 50% to 90% of arbitrage profit compared to perfect price forecasts depending on the storage setting and price zones.

The remainder of this paper is organized as follows. Section II presents the background and discusses previous works in this topic. Sections III formulates the problem, and Sections IV provides the solution method to proposed models. Section V includes simulation results and discusses the result compare to different benchmarks. Section VI concludes the paper and proposes a future course of research.

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II. BACKGROUNDS AND LITERATURE REVIEW

Increasing penetration of renewable energy in power systems increases the fluctuation of electricity prices, and real-time market price arbitrage will become more profitable [13]. Electrochemical battery energy storage can switch between full charging and discharging power in less than a second, providing it with a unique advantage to arbitrage real-time price differences [14]. FERC Order 841 further ensures energy storage resources have equal access to wholesale electricity markets, and a growing number of researchers are focusing on battery energy storage arbitrage in the real-time markets.

Model predictive control (MPC) is one of the most widely used control methods in energy storage applications [15]. MPC is an optimization-based control strategy. It solves an optimization problem over a finite future horizon with predictions, but only applies the first control step. MPC repeatedly optimizes the best-predicted system performance over a finite horizon with new system states update at each time step [16]. Although MPC relies on some heuristic and expert knowledge in specific tasks, it achieves good performance in practice [17].

Due to MPC’s excellent flexibility to incorporate different objectives and constraints, previous works utilized MPC based controllers in energy storage applications. Shan et al. investigated energy storage for smoothing wind and solar power generation with MPC controllers [18]. Khalid et al. [19] presented a controller based on MPC for the efficient operation of the BESS for frequency regulation. Meng et al. [20] proposed a distributed MPC scheme, which coordinates BESS for voltage regulation. The University of Queensland used an MPC controller for performing price arbitrage in electricity spot markets and achieved 70% profit compared to the perfect forecast [21]. However, MPC must use a separate module to generate future scenarios and can be inefficient to incorporate uncertainty considerations. MPC is also computationally expensive, especially in edge implementations, because it has to repetitively solve optimization problems.

Stochastic dynamic programming (SDP) is a systematic approach to optimize price arbitrage in highly stochastic scenarios compared to MPC. SDP has been a classic approach to optimize hydro power and pump hydro scheduling but uses relatively simple stochastic models focused on weekly to seasonal scheduling [22]–[24]. The major challenge of applying SDP in more sophisticated scenarios like price arbitrage is the computational cost, which grows exponentially as the number of decision stages and scenarios increases. Therefore, SDP has been less attractive in BESS real-time arbitrage due to the high frequent decision making nature (usually 5-minute) and prolonged decision-making horizon.

Nevertheless, few exploratory attempts have been made to apply SDP in price arbitrage. One approach is to solve SDP using a Markov decision process (MDP), in which the storage control pairs with system and uncertainty states. It discretizes energy storage state of charge (SoC) and power, and has been applied in applications such as price arbitrage and frequency regulation [25], [26]. MPD has also been combined with reinforcement learning in energy storage price arbitrage [27], or used to investigate the welfare optimization considering consumer and producer surplus [28]. However, these studies have to discretizes states to map control to observed states. It induces a trade-off problem: the problem is too big solve in reasonable times if states are discretized to fine granularity, while coarse state granularity may not be accurate enough for storage arbitrage because of charging and discharging efficiency.

Stochastic dual dynamic programming (SDDP) is an alternative approach to solve SDP by approximating the value-to-go function using linear cuts based on the dual variable [29]. SDDP does not discretize storage control or states, and the computation performance is better than solving SDP using MDP [30]. However, SDDP uses forward-backward iterations to generate dual cuts, and the number of iterations increases significantly with the number of uncertainty stages and nodes [31]. Comparably, the method proposed in this paper follows a similar theory foundation with SDDP, but we use an analytical algorithm to generate complete piecewise linear value function approximations. Our method does not discretize actions or states, and solves the arbitrage SDP problem in a backward path without iterating. As a result, our method provides improved accuracy and computation speed.

III. FORMULATION

We formulate energy storage arbitrage using stochastic dynamic programming as a price response problem assuming the storage can update its control over time period \( t \) after observing the newest real-time market price \( \lambda_t \), which the system operator will announce before the dispatch period starts. We assume the distribution if \( \lambda_{t+1} \) is dependent on the price realization from the previous period \( \lambda_t \). The arbitrage problem is formulated as

\[
Q_{t-1}(e_{t-1} \mid \lambda_t) = \underset{b_t, p_t}{\max} \lambda_t(p_t - b_t) - c_p e_t + V_t(e_t \mid \lambda_t)
\]

\[
V_t(e_t \mid \lambda_t) = E_{\lambda_{t+1}}[Q_t(e_t \mid \lambda_{t+1}) \mid \lambda_t]
\]

where \( Q_{t-1} \) is the maximized energy storage arbitrage revenue from time period \( t \) till the end of the operating horizon \( T \). \( Q_{t-1} \) is dependent on the energy storage SoC at the end of the previous time period \( e_{t-1} \), and the real-time market price \( \lambda_t \). The first term in the objective function is the market income over period \( t \) as the product of \( \lambda_t \) and the battery output power \( (p_t - b_t) \), where \( p_t \) is the discharge power and \( b_t \) is the charge power. The second term of the objective function models the discharge cost, in which \( c_p \) is the marginal discharge cost. The last term is the value-to-go function that models the opportunity value of the energy \( e_t \) left in the battery. We consider a stage-dependent price model such that the future price realization \( \lambda_{t+1} \) is dependent on the current price, such that \( V_t \) is dependent on \( \lambda_t \). (1b) shows \( V_t \) is the conditional expectation of the future arbitrage revenue \( Q_{t-1} \) given \( \lambda_t \).

The arbitrage problem subjects to the following constraints

\[
0 \leq b_t \leq P, \quad 0 \leq p_t \leq P
\]

\[
p_t = 0 \text{ if } \lambda_t < 0
\]

\[
e_t - e_{t-1} = -p_t/\eta + b_t\eta
\]

\[
0 \leq e_t \leq E
\]
where (1c) models the storage power rating $P$ over charge and discharge power. (1d) enforces the battery to not discharge when prices are negative, a sufficient condition for avoiding simultaneous charge and discharge [32]. (1e) models the energy storage SoC evolution constraint with efficiency $\eta$. (1f) models the energy limit $E$ over the storage SoC.

IV. SOLUTION METHOD

We introduce an analytical algorithm for solving the arbitrage problem. First, we model the stochastic price process using an order-1 Markov process and summarize how we train the Markov process using historical data. Then we reformulate the arbitrage problem based on the Markov process and present a SoC valuation method and control policy. In the end, we introduce the complete solution algorithm.

A. Markov Process Model

We model real-time price uncertainty using an order-1 Markov Process model. The Markov model includes $T$ stages, and each stage has $N$ nodes. We discretize real-time prices into a set of time-dependent nodes $\pi_{t,i}$, each node clusters prices within the range $[\pi_{\text{low}}, \pi_{\text{high}}]$ where $\pi_{\text{low}}$ is the lower range bound and $\pi_{\text{high}}$ is the upper range bound. Note that the smallest price node does not have a lower bound, and the highest price node does not have an upper bound. Each period has the same number of nodes $N = \{1, 2, \ldots, N\}$, but the node value may be different. Price nodes are associated with the following time-varying transition probability matrix $M_t = \{\rho_{i,j,t} | i \in N, j \in N\}$ indicating the probability that the price will transit from $\pi_{t,i}$ to $\pi_{t+1,j}$, $(i, j) \in N$.

We generate the probability transition matrix $M_t$ from historical real-time price (RTP). All transition matrices in this paper are on an hourly resolution, i.e., $M_t$ is identical in same hour $H_h$, where $H_h = \{1, 2, \ldots, 24\}$. This approximation is motivated by the fact that generators are scheduled hourly in day-ahead, and also to reduce the effect of limited training historical data size. We calculate $M_{H_h}$ base on the sum of state transit incidents happens in $H_h$ for all days in training historical data. Given a set of training price data set $\lambda_t$, we calculate the hourly transition probability as $\forall t \in H_h$

$$\rho_{i,j,H_h} = \sum_{t \in H_h} I\{\lambda_{t+1} \in \pi_{t+1,j}\} I\{\lambda_t \in \pi_{t,i}\} \sum_{t \in H_h} I\{\lambda_t \in \pi_{t,i}\}$$

subject to the same constraints (1c)-(1f). Note that the major change of the discretized formulation compared to the original one in (1) is that the maximized revenue $Q_{t-1,i}(e_t)$ and the value-to-go function $V_{t,i}(e_t)$ now are uniquely associated with the price nodes $\pi_{t,i}$. $Q_{t-1,i}(e_{t-1})$ is the maximized arbitrage profit when the storage initial SoC is $e_{t-1}$ at price node $\pi_{t,i}$. $V_{t,i}(e_t)$ is the opportunity value of the storage SoC $e_t$ at the end of the time period $t$ at price node $\pi_{t,i}$. The conditional expectation calculation of the value function in (1b) is now replaced with (3b) using the price node transition probability $\rho_{i,j,t}$, which models the probability of transitioning to price node $\pi_{t+1,j}$ when the current price is $\pi_{t,i}$. Hence $V_{t,i}(e_t)$ is the expected SoC value function counting in all transition possibilities associated with the current price node $\pi_{t,i}$.

C. Analytical Value Function Update

Our proposed algorithm is based on the following result that updates $q_{t-1,i}$ from $v_{t,i}$, where $q_{t-1,i}$ is the derivative of
$Q_{t-1,i}$ and $v_{t,i}$ is the derivative of $V_{t,i}$

\[
q_{t-1,i}(e) = \begin{cases} 
\pi_{t,i} \left( e + P \right) & \text{if } \pi_{t,i} \leq v_{t,i}(e + P) \eta \\
\pi_{t,i} \eta & \text{if } v_{t,i}(e + P) \eta < \pi_{t,i} \leq v_{t,i}(e) \eta \\
v_{t,i}(e) & \text{if } v_{t,i}(e) \eta < \pi_{t,i} \leq v_{t,i}(e) / \eta + c]^+
\end{cases}
\]

The derivation of (4) is summarized in the Appendix, while an intuitive explanation is given as follows. The five listed cases correspond to the five control cases in the same order:

- Storage charges at full power $P$ when price is very low;
- Storage charges at partial power between 0 and $P$ when price is moderately low;
- Storage does not charge or discharge when price is similar to the current marginal opportunity value;
- Storage discharges at partial power between 0 and $P$ when price is moderately high;
- Storage discharges at $P$ when price is very high.

while exact conditions are as listed in (4).

To explain the principle of (4), we use first-order optimality condition to solve (3a) by taking the derivative with respect to the final SoC $e_t$ as

\[
q_{t-1,i}(e_{t-1}) = \pi_{t,i} \frac{\partial p_t}{\partial e_t} - \frac{\partial b_t}{\partial e_t} + v_{t,i}(e_t) = 0
\]

and according to (1e) and the Karush–Kuhn–Tucker (KKT) conditions we can conclude the following conditions

\[
\frac{\partial p_t}{\partial e_t} = \begin{cases} 
-1 / \eta & \text{if (1e) is not binding} \\
0 & \text{if (1e) is binding}
\end{cases}
\]

\[
\frac{\partial b_t}{\partial e_t} = \begin{cases} 
\eta & \text{if (1e) is not binding} \\
0 & \text{if (1e) is binding}
\end{cases}
\]

\[
\frac{\partial e_{t-1}}{\partial e_t} = \begin{cases} 
1 & \text{if (1e) is binding} \\
0 & \text{if (1e) is not binding}
\end{cases}
\]

now it is intuitive to see that in the second and fourth case of (4) the storage charge or discharge at partial power so that (1c) is not binding, hence $q_{t-1,i}$ equals to either $\pi_{t,i}/\eta$ or $(\pi_{t,i} - c)/\eta$ depending on the charge or discharge status; in the first and the fifth case the storage charge or discharge at full power, so the power derivatives are zero hence $q_{t-1,i}(e_{t-1}) = v_{t,i}(e_t)$, in which $e_t$ can be replaced with $e_{t-1} + P \eta$ or $e_{t-1} - P / \eta$ depending on the charge or discharge condition. In the third case both the charge and discharge power are bounded by non-negative value so $q_{t-1,i}(e_{t-1}) = v_{t,i}(e_t)$ directly holds.

\section{D. Control Policy}

After finishing the valuation calculation and obtained all value function derivatives $v_{t,i}$, we can execute the control using realized market prices $\lambda_t$. Similar to fitting the Markov price process, we first look up the corresponding value function $v_{t,i}$ associated with $\lambda_t$ by looking for the closest price node $\pi_{t,i}$ such that $\pi_{t,i} < \lambda_t < \pi_{t,i+1}$, then the storage control decision is updated as

\[
p_t = \min\{\hat{p}_t, e_{t-1}/\eta\} \quad (7a)
\]

\[
b_t = \min\{\hat{b}_t, (E - e_{t-1})/\eta\} \quad (7b)
\]

where $\hat{p}_t$ and $\hat{b}_t$ are calculated as

\[
\begin{cases} 
(0, P) & \text{if } \lambda_t < v_{t,i}(e + P) \eta \\text{or } \lambda_t \geq v_{t,i}(e) \eta \\
(0, 0) & \text{if } v_{t,i}(e + P) \eta < \lambda_t < v_{t,i}(e) \eta \\
\beta, 0 & \text{if } v_{t,i}(e/\eta + c) + \lambda_t < \lambda_t < v_{t,i}(e/\eta + c) + \lambda_t \\
(P, 0) & \text{if } \lambda_t > [v_{t,i}(e - P/\eta)/\eta + c] \quad (7c)
\end{cases}
\]

in which $\alpha$ and $\beta$ are given as follows

\[
\alpha = (v_{t,i}^{-1}(\lambda_t/\eta - e_{t-1})/\eta) \quad (7d)
\]

\[
\beta = (e_{t-1} - v_{t,i}^{-1}((\lambda_t - c)/\eta))/\eta \quad (7e)
\]

where $v_{t,i}^{-1}$ is the inverse function of $v_{t,i}$.

(7a) and (7b) enforce the battery SoC constraints over the discharge $\hat{p}_t$ and charge $\hat{b}_t$ decisions. (7c) calculates control decisions and following the same principle as (4) but use the observed price $\lambda_t$ instead of the price nodes $\pi_{t,i}$. In the first, third, and fifth cases the control decisions are clear as the storage is either at full charge power, idle, or full discharge power. In the second and fourth cases, the marginal value function $v_{t,i}$ should equal the marginal objective value $q_{t-1,i}$, therefore we can use the inverse function of $v_{t,i}$ and the price to update the control decision.

\section{E. Solution Algorithm}

We list the complete solution algorithm while enforcing the final SoC value is higher than a given threshold $e^f$. In this algorithm implementation we discretize $v_{t,i}$ into a set of $\mathcal{M}$ segments with value and SoC pairs

\[
\mathcal{V}_{t,i} = \{v_{t,i,m} | m \in \mathcal{M}\} \quad (8)
\]

associated with equally spaced SoC segments $e_{t,m}$. In our implementation, we discretized the SoC into 1000 segments. The valuation algorithm is listed as following

1) Set $T \rightarrow t$ to start from the last time period; initialize the final value-to-go function segments $\nu_{T,m}$ to zeros for $e_{t,m} > e^f$ and to a very high value (we use $1000/MWh$) for $e_{t,m} \leq e^f$. Note that the final value function does not depends on price nodes.

2) Go to the earlier time step by setting $t \rightarrow t-1$.

3) During period $t$, go through each price node for $i \in \mathcal{N}$ and value function segment $m \in \mathcal{M}$, solve (4) and store $q_{t-1,i}(e)$; note that here $q_{t-1,i}(e_{t-1,m})$ is also discretized with the same granularity as the value function.

4) Calculate the value function of the previous time step as

\[
\nu_{t-1,i,m} = \sum_{j \in \mathcal{N}} \rho_{t,j,i} q_{t-1,i}(e_{t,m}) \quad (9)
\]
which is the derivative of (3b).

5) Go to step 2) until reaching the first time step.

Then after finishing the valuation, we perform the actual arbitrage simulation by updating the storage control decision using the actual system price \( \lambda_t \) based on procedures described in (IV-D) from the first time period until the last time period, we update the storage SoC according to (1f).

**Remark 2. Value function interpolation.** It is possible to encounter price nodes with zero transition probability in specific hours on value function calculations. Therefore, we implemented a simple value interpolation with the expected value function in (9). When the transition probability is all-zero, we interpolated the value function from the closest time period within the same price node.

**V. RESULTS**

We first compare different Markov process models to evaluate their performances. Then, we perform arbitrage in different price zones to examine the profitability robustness of the proposed method. Finally, we trade off battery revenue with battery degradation by showing how different marginal discharge cost settings and power-to-energy ratios can impact storage arbitrage profit and the total discharged energy.

**A. Data and Experiment Design**

We test the proposed arbitrage algorithm using historical price data from New York ISO (NYISO) [33]. We test the arbitrage over the 2019 data, while use data before 2019 to train the Markov process. We include prices from four zones to demonstrate arbitrage results in different price behaviors: WEST, NORTH, NYC, and LONGIL. Table I shows the average price and price standard deviations for the four zones in 2019. Fig. 2 shows the simulation procedure.

| Zone       | NYC  | LONGIL | NORTH | WEST |
|------------|------|--------|-------|------|
| Average price | 27.5 | 32.6   | 17.8  | 24.7 |
| Standard deviation | 28.8 | 50.1   | 40.2  | 37.5 |

We consider ten arbitrage settings in this study to demonstrate the impact of different stochastic models and compare them against benchmarks. The two benchmark arbitrage settings are

1) **BEN-DA.** We use DAP as predictions of RTP and execute arbitrage using model predictive control. This is the first benchmark case and represents a simple arbitrage implementation.

2) **BEN-PF.** We perform a deterministic arbitrage optimization using the actual real-time price assuming perfect forecasts. This is the second benchmark case and represents the maximum possible revenue.

and our proposed arbitrage algorithms

1) **RT-Idp.** We perform SDP arbitrage assume there is no stage-dependency over price nodes, i.e., the transition probability does not depend on the current price node, i.e., the term \( I\{\lambda_{t+1} \in \pi_{t+1,j}\} \) is removed from the Markov process training model (2).

2) **RT-Dep.** We perform SDP arbitrage with a Markov process trained using historical RTP data. Each stage in the Markov process has 22 price nodes, one represents price spikes, one represents negative prices, and the rest 20 are evenly spaced between $0/MWh to $200/MWh.

3) **RT-Dep-S.** Same as RT-Dep except we use different Markov processes for summer and non-summer days to represent seasonal patterns.

4) **RT-Dep-W.** Same as RT-Dep except we use different Markov processes for weekdays and weekends to represent weekly patterns.

5) **DB-Idp.** Similar to the RT-Idp, but the stochastic price model is trained using the DAP-RTP differences instead of using RTP directly.

6) **DB-Dep.** Similar to RT-Dep, but with a Markov process trained using historical DAP-RTP bias data. Each stage in the Markov process has 12 price nodes, with 10 nodes evenly spaced between $-50/MWh to $50/MWh and 2 nodes for negative and positive spikes. We use less price node than the RTP case because the DAP-RTP bias have a denser distribution than the RTP price.

7) **DB-Dep-S.** Same as DB-Dep except we use different Markov processes for summer and non-summer days to represent seasonal patterns.

8) **DB-Dep-W.** Same as DB-Dep except we use different Markov processes for weekdays and weekends to represent weekly patterns.

note that in all DB cases we update the price node value in each day with the corresponding DAP results.

**Remark 3. Hour-ahead bidding.** Although our method targets a real-time price response problem, note that in stage-independent cases (RT-Idp and DB-Idp) we can generate all storage value functions hours before the real-time operation starts. Therefore, these two cases approximate the storage revenue potential if they participate in hour-ahead real-time market bidding. The storage bids are designed based on the value-to-go function.

For the battery setting, we assume 81% round-trip storage efficiency. In base cases, presumed marginal discharge cost and power to energy ratio (P/E) are $10/MWh and 0.5, respectively. Note that all marginal discharge costs in this paper are not real marginal costs of specific energy storage,
just presumed numbers for numerical experiments. The effect of marginal discharge cost and power-to-energy ratios will be discussed later. The Markov process training method is implemented in Python. All arbitrage algorithms are implemented in Matlab except the BEN-PF case is implemented using Julia and Gurobi. Although our algorithm also gives similar results in BEN-PF, we want to provide a benchmark independent of our algorithm for a fair comparison. On the computation speed, our code takes about one second on a personal computer to finish the valuation over 288 stages with 20 price nodes per stage. The Matlab implementation is available on GitHub.

### B. Model Comparison

We show the performance of different models by comparing profits from different cases to benchmark cases (BEN-PF and BEN-DA). Fig. 3 shows arbitrage profits of different models compared to BEN-PF and BEN-DA in NYC. BEN-PF is represented by 100% in this figure as the maximum possible profit. If we consider the real-time model (RT-Dep) and the DAP-RTP bias model (DB-Dep) together, stage-dependency improves arbitrage profit significantly. Profits from all stage-dependent cases exceed BEN-DA. Profits captured by DB cases are essentially higher than BEN-DA.

Seasonal pattern slightly improves the model performance with value interpolation, but it still suffers from zero transition probability. The weekly pattern has a smaller subset of weekend days, consider the number of weekend days is small. Thus, it has more zero transition probabilities, and it is even detrimental to model performance. The seasonal and weekly patterns have a negligible effect on the DAP-RTP bias model, because we predict real-time prices based on day-ahead price forecast, which already captured partial seasonal and weekly patterns through different price node values in SoC valuation.

In NYC, DAP-RTP bias model is superior to real-time model, but numerical result in one zone is not sufficient to generalize conclusion. In the following subsection, we test RT-Dep, DB-Idp, and DB-Dep in arbitrage simulations over different price zones, and different sizes of training data sets.

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1. [https://github.com/niklauskun/MarkovESValuation](https://github.com/niklauskun/MarkovESValuation)

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### C. Locations and Training Data Size

We compared RT-Dep, DB-Dep and DB-Idp cases in four different zones in NYISO, and demonstrate the impact from training data sizes. Fig. 4 shows percentage of captured profit compare to BEN-PF, the size of training datasets has no significant effect on model performance. The results show that one year’s of historical RTP data is sufficient, showing our proposed approach is also data efficient. However, suppose one wishes to capture seasonal and weekly patterns, a larger size of training data is then required to reduce incidents of zero transition probability, but we should be aware of the transformation of power systems. The training data close to testing data have more homogeneity, because the system
demand, supply, and market regulation is changing over time. Yet, a larger training dataset will lead to a more stable state transition probability. Therefore, we need a trade-off between different training datasets.

Profits from all stage-dependent cases are higher than BEN-DA. Notice that in Fig. 4(b), the model performance of RT-Dep is better than DB-Dep. The reason of this exception is LONGIL have more DAP-RTP bias spike compare to other location. As shown in Fig. 5, LONGIL and WEST have significantly more bias spike compare to other zones. However, WEST has little bias spikes in hours with low spike probability correlations, resulting in only LONGIL’s DB-Dep performance being affected by the bias spike issue. We also compare DB-Dep and DB-Idp, as DB-Idp approximates to hour-ahead real-time market bidding because the valuation result only depends on day-ahead forecasts. What stands out in all four locations is the profit loss in stage-wise dependency ignorance.

### D. Power to Energy Ratio and Degradation Cost

We investigate the effect on model performance from power-to-energy (P2E) ratios and presumed marginal discharge cost. We model three different power ratings while keeping the energy rating normalized to 1 MWh, and include four cases with the presumed marginal discharge cost \( c \) of 0, 10, 30, 50 (\$/MWh).

Table II compares profit ratios, revenue, and annual total discharged energy with different P2E ratios and presumed marginal discharge costs. The profit gap between our model and BEN-PF narrows down as the storage duration and marginal discharge cost increase. The results show that our proposed method can capture up to 90% profit compared to perfect information arbitrage, with a low P2E ratio and high presumed marginal discharge cost. Table II also shows that arbitrage revenue per MWh increases with increment in P2E ratio and presumed marginal cost. While revenue per MWh is more sensitive to presumed marginal cost, energy storage is prone to discharge at price spike and ignores low profitable discharge timing.

Another takeaway from the result is the comparison between revenue and the total discharged energy. Battery lifetime critically depends on energy throughput, and reducing the total energy discharged saves battery life significantly. For reference, currently most battery warranties require the battery to be cycled less than once per day. In our case this corresponds to keeping the annual discharged energy to less than 0.365 GWh. Our result shows that adding a $10/MWh marginal cost can significantly reduce the energy discharged but only impact the revenue slightly. In some cases, the revenue increased after adding the $10/MWh marginal cost as it reduced the arbitrage sensitivity to minor price fluctuations. By increasing the presumed marginal discharged cost from zero to $50/MWh, we reduce the energy discharged by more than 90%, but the revenue reduction is only 30% to 40%. On the other hand, the revenue per MWh discharged increased by more than ten folds.

### VI. Conclusion

We proposed an analytical stochastic dynamic programming algorithm considering real-time price uncertainty by integrating Markov processes. Our proposed arbitrage algorithm achieves more than 70% profit compared to the perfect information case in the base. When considering batteries with larger energy capacity and higher degradation costs, our algorithm can reach up to 90% profit ratio. Our study also shows taking price auto-correlation into account can improve the profit ratio by around 10%, while incorporating day-ahead prices into the real-time price model will provide another 10% increase. On the other hand, seasonal and weekly patterns will not improve the performance much and require more training data.

In the future, we plan to apply the proposed stochastic dynamic programming framework to other grid storage applications such as renewable integration, peak shaving, and stacked services. We also plan to improve the Markov process accuracy by integrating it with state-of-art price prediction models.
APPENDIX

We start by restating result from our prior work [32]

\[ v_{t-1}(e) = \mathbb{E}[q_{t-1}(e)] = v_t(e + P \eta) F_t(v_t(e + P \eta) \eta) \]

\[ + \int_0^{v_t(e + P \eta)} w_f(u) \, du \]

\[ + v_t(e) \left[ F_t(v_t(e + \eta + c^+) - F_t(v_t(e) \eta) \right] \]

\[ + \eta \int_0^{v_t(e \eta + c^+)} w_f(u) \, du \]

\[ - c \eta \left[ F_t(v_t(e - P/\eta) \eta + c^+) - F_t(v_t(e) \eta + c^+) \right] \]

\[ + v_t(e - P/\eta) \left[ 1 - F_t(v_t(e - P/\eta) \eta + c^+) \right]. \]

(10)

which calculates the value function assuming the price follows a probability distribution function \( f_t \) and a cumulative distribution function \( F_t \). To apply this result to the Markov process case in this paper, note that at each valuation step we are essentially solving a deterministic arbitrage at a given price node, hence \( f_t \) becomes an indicator function and \( F_t \) becomes a step function. For example, the first term \( v_t(e + P \eta) F_t(v_t(e + P \eta) \eta) \) becomes \( v_t(e + P \eta) \) if price is lower than \( v_t(e + P \eta) \eta \) because \( F_t(v_t(e + P \eta) \eta) \) is one only when \( v_t(e + P \eta) \eta \) is larger than the price node value. Hence, repeat the derivation for all the terms in the above equation and we obtain the deterministic version used for the Markov process as shown in (4).

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