A Lie Group Structure Underlying the Triplectic Geometry

M. A. Grigoriev

Tamm Theory Division, Lebedev Physics Institute, Russian Academy of Sciences

We consider the pair of degenerate compatible antibrackets satisfying a generalization of the axioms imposed in the triplectic quantization of gauge theories. We show that this actually encodes a Lie group structure, with the antibrackets being related to the left- and right- invariant vector fields on the group. The standard triplectic quantization axioms then correspond to Abelian Lie groups.

1 Introduction

Triplectic geometry was introduced in [1] (see also [2, 3, 4, 5]) as the structure underlying the geometrically covariant (triplectic) generalization of the Sp(2)-symmetric Lagrangian quantization of general gauge theories [6, 2].

The most essential ingredient of the triplectic geometry is a pair of compatible and appropriately degenerate antibrackets. The triplectic quantization prescription also makes use of additional objects: the odd vector fields and a density appearing in the triplectic master equation and path-integral. However, the most interesting features characteristic of the triplectic geometry originate from the antibracket structure.

The triplectic antibrackets naturally encode two different geometrical structures [5]. The first is given by a complex structure and two transversal polarizations; these compatible structures are induced on the space of marked functions of the antibrackets. The second geometric object originating from the antibrackets is an Abelian group structure, i.e., the structure of globally defined nondegenerate commuting vector fields induced on the intersection \( L \) of the symplectic leaves of the antibrackets (i.e., on the “manifold of fields” in triplectic quantization).

Studied in [4, 5] were mutually commutative compatible antibrackets. The mutual commutativity condition states that the algebra of marked functions of the antibracket \((~,~)\) is commutative with respect to the antibracket \((~,~)\), and vice versa. This condition is imposed for the consistency with the Sp(2)-invariant quantization in the canonical coordinates [1, 2, 3]. As we are going to show, however, this condition turns out to be quite restrictive from the geometrical standpoint. We will see that with this condition relaxed, the compatible antibrackets induce a Lie group structure on the submanifold \( L \), and conversely, any Lie group admits a “triplectic bundle” endowed with a pair of compatible antibrackets. Thus, the construction of the “triplectic bundle” over a Lie group is in some sense canonical, and a natural set of axioms may be thought to be the one that does not lead to any further restrictions on this group. However, the mutual commutativity axiom implies that the group is necessarily Abelian.

This letter organized as follows: in section 2, we introduce the basic definitions and assumptions of the triplectic formalism and consider the simplest and most transparent example where all the basis marked functions are Grassmann-odd and hence the corresponding Lie group is not a
super

In the section 2.2, we allow the basis marked functions to have arbitrary Grassmann
parity and construct the corresponding submanifold that carries a Lie (super)group structure. In
section 3, we explain the inverse construction: given an arbitrary Lie group, we construct the
triplectic antibrackets on a certain bundle over it. Finally, we make some remarks regarding the
applications to the triplectic quantization and discuss possible generalizations of our construction.

2 From triplectic geometry to Lie group structure

We begin with a brief reminder of the basic structures and their properties in the geometrically
covariant approach to the $Sp(2)$-invariant quantization. In contrast to the geometry unders
lying the covariant formulation of the standard BV formalism (see [8, 9, 10, 11, 12]), the triplectic
quantization prescription requires the antibrackets to be appropriately degenerate and to satisfy
some additional constraints which we consider momentarily. We start, however, with the mutual
commutativity condition removed; the marked functions of each antibracket are thus allowed to
have nonzero commutators with respect to the other antibracket.

Let $M$ be the “triplectic manifold,” i.e., a $3N$-dimensional supermanifold endowed with a pair
of compatible antibrackets. We assume $M$ to be connected (i.e., the body of $M$ is connected
in the standard sense). Let $\mathcal{C}_M$ be the superalgebra of smooth functions on $M$. An antibracket
on $M$ is a bilinear skew-symmetric map $\mathcal{C}_M \otimes \mathcal{C}_M \to \mathcal{C}_M$ satisfying the Leibnitz rule and the
Jacobi identity. A pair of antibrackets $(\ , \ )^a$, where $a = 1, 2$, is called compatible if every linear
combination $(\ , \ ) = \alpha(\ , \ )^1 + \beta(\ , \ )^2$ with constant $\alpha$ and $\beta$ is also an antibracket, i.e. satisfies the
Jacobi identity. The compatibility condition is equivalent to

$$(-1)^{(\varepsilon(F)+1)(\varepsilon(H)+1)}((F,G)^{(a)}H^b) + \text{cycle}(F,G,H) = 0, \quad F,G,H \in \mathcal{C}_M,$$

with the curly brackets denoting the symmetrization of indices $a, b$. This condition is often referred
to as the symmetrized Jacobi identity \[2\].

We take the antibrackets on $M$ to be everywhere of rank $2N$ and to be jointly nondegenerate \[3, 5\]; the latter means that the corresponding odd Poisson bivectors do not have common zero modes,
i.e. if a 1-form $\phi$ is such that $E^1\phi = E^2\phi = 0$, then $\phi = 0$, where $E^1$ and $E^2$ denote the bivectors
corresponding to the antibrackets.

We denote by $i_a : M_a \to M$ the foliations of $M$ into symplectic leaves of the respective
antibracket. We also assume for simplicity that the foliations $M_a \to M$ are fibrations, and let
$N_1 = M/M_2$ and $N_2 = M/M_1$ be the corresponding base manifolds. Then, we also have the
projections $\pi_a : M \to N_a$.

A powerful tool in studying degenerate brackets is provided by their marked functions (the
Casimir functions). A function $\phi \in \mathcal{C}_M$ is a marked function of the antibracket $(\ , \ )$ if $(\phi, F) = 0$
for any $F \in \mathcal{C}_M$. An important property of the marked functions of compatible antibrackets is as
follows.

**Proposition 2.1** Let $\phi, \psi \in \mathcal{C}_M$ be marked functions of the first antibracket $(\ , \ )^1$ (respectively, of $(\ , \ )^2$). Then so is $(\phi, \psi)^2$ (respectively, $(\phi, \psi)^1$).
The proof is a direct consequence of the compatibility of the antibrackets. It follows from the proposition that the algebra of marked functions of the first (the second) antibracket is closed with respect to the second (respectively, the first) bracket.

Let \( \xi_{1i} \) (respectively, \( \xi_{2\alpha} \)) be (locally) a minimal set of basis marked functions of the second (respectively, the first) antibracket. For example, \( \xi_{1i} \) can be the transversal coordinates to the symplectic leaves of the second antibracket. The conditions imposed on the compatible antibrackets naturally translate into the language of marked functions. In particular, the rank assumption imposed on the antibrackets implies that there are only \( N \) independent marked functions of each antibracket; the nondegeneracy condition implies that all the functions \( \xi_{1i} \) and \( \xi_{2\alpha} \) are independent.

Recalling the assumption that the foliations \( i_a : \mathcal{M}_a \to \mathcal{M} \) are fibrations, we see that \( \xi_{1i} \) (\( \xi_{2\alpha} \)) actually constitute a local coordinate system on \( \mathcal{N}_1 \) (respectively, \( \mathcal{N}_2 \)). Therefore, it is natural to regard each \( \mathcal{N}_a \) as the manifold of marked functions of the respective antibracket.

We now are in a position to study the noncommutative antibrackets. To avoid some technical complications, we begin with the following instructive example.

### 2.1 An instructive example

For simplicity, we now assume all the basis marked functions \( \xi_{1i} \) and \( \xi_{2\alpha} \) to be Grassmann-odd. As we have assumed the foliations of the symplectic leaves of the antibrackets to be fibrations, there exist base manifolds \( \mathcal{N}_1 \) and \( \mathcal{N}_2 \), which we further assume to be connected. Since all the basis marked functions are Grassmann-odd, \( \mathcal{N}_1 \) and \( \mathcal{N}_2 \) are odd superspaces. The algebra \( \mathcal{C}_{\mathcal{N}_1} \) (\( \mathcal{C}_{\mathcal{N}_2} \)) is actually a Grassmann algebra generated by \( \xi_{1i} \) (respectively, by \( \xi_{2\alpha} \)).

It follows from Proposition 2.1 that the first antibracket determines a skew-symmetric mapping \( (\cdot, \cdot)^1 : \mathcal{C}_{\mathcal{N}_1} \otimes \mathcal{C}_{\mathcal{N}_1} \to \mathcal{C}_{\mathcal{N}_1} \) satisfying the Jacobi identity. In fact, the antibracket makes \( \mathcal{C}_{\mathcal{N}_1} \) considered as a linear space into a Lie superalgebra, with the antibracket being the supercommutator; here, each even element from the Grassmann algebra \( \mathcal{C}_{\mathcal{N}_1} \) is to be considered as an odd element of the Lie superalgebra, and vice versa. However, we do not discuss this in more detail and concentrate instead on some quotient of this Lie algebra.

We explicitly write the antibracket in \( \mathcal{C}_{\mathcal{N}_1} \) as

\[
(\xi_{1i}, \xi_{1j})^1 = C^{k}_{ij} \xi_{1k} + \cdots ,
\]

where the dots mean higher-order terms in \( \xi \). It follows from the skew-symmetry of the antibracket that \( C^{k}_{ij} = -C^{k}_{ji} \). Moreover, in the first order in \( \xi \) the Jacobi identity for the antibracket implies that

\[
C^{m}_{ij} C^{n}_{mk} + C^{m}_{jk} C^{n}_{mi} + C^{m}_{ki} C^{n}_{mj} = 0 ,
\]

which is the Jacobi identity for the Lie algebra whose structure constants are \( C^{k}_{ij} \).

We now explicitly construct this Lie algebra. To this end, we consider the vector fields on \( \mathcal{M} \)

\[
L_i = (\xi_{1i}, \cdot)^1,
\]

While this letter was in preparation, we received the paper [13], where the antibracket on the Grassmann algebra corresponding to a given Lie algebra is considered in a slightly different context.
which form an \( N \)-dimensional Lie algebra modulo the vector fields vanishing at \( \xi_{1i} = 0 \),
\[
[L_i, L_j] = C^k_{ij} L_k + \ldots .
\]  
(2.5)
Proceeding similarly with the algebra \( \mathcal{C}_{\mathcal{N}_2} \) of the marked functions of the first antibracket, we arrive at the vector fields
\[
R_\alpha = (\xi_{2\alpha}, \cdot)^2,
\]  
(2.6)
satisfying
\[
(\xi_{2\alpha}, \xi_{2\beta})^2 = C^\gamma_{\alpha\beta} \xi_{2\gamma} + \ldots .
\]  
(2.7)
In fact, \( L_i \) as well as \( R_\alpha \) can be restricted to the submanifold \( \mathcal{L} \) determined by the equations \( \xi_{1i} = 0, \xi_{2\alpha} = 0 \). This allows us to consider these vector fields as defined on \( \mathcal{L} \). As a consequence of the rank conditions, \( L_i \) form a basis of \( T\mathcal{L} \), and so do \( R_\alpha \). They also satisfy the relations
\[
[L_i, L_j] = C^k_{ij} L_k, \quad [R_\alpha, R_\beta] = C^\gamma_{\alpha\beta} R_\gamma.
\]  
(2.8)
It follows from the compatibility of the antibrackets that
\[
[L_i, R_\alpha] = 0 .
\]  
(2.9)
Relations (2.8) and (2.9) are precisely those of the left- and right-invariant vector fields on the Lie group corresponding to the Lie algebra determined by the structure constants \( C^k_{ij} \). We thus conclude that \( \mathcal{L} \) is diffeomorphic to \( G \) [14].

2.2 The general construction

We now turn to the general situation, where we allow the basis marked functions to have arbitrary Grassmann parities.

As mentioned above, we identify the marked functions of the first (the second) antibracket with functions on \( \mathcal{N}_2 \) (respectively, on \( \mathcal{N}_1 \)). Proposition 2.1 tells us that the antibracket \((\cdot,\cdot)^1\) (respectively, \((\cdot,\cdot)^2\)) induces an antibracket on \( \mathcal{N}_1 \) (on \( \mathcal{N}_2 \)), and therefore \( \mathcal{N}_a \), \( a = 1, 2 \), become odd Poisson manifolds. Let \( p_1 \in \mathcal{N}_1 \) be a point where the antibracket \((\cdot,\cdot)^1\) vanishes\(^3\), i.e., \((\phi, \psi)^1|_{p_1} = 0\) for all \( \phi, \psi \in \mathcal{C}_{\mathcal{N}_1} \). This point is evidently a symplectic leaf of the antibracket \((\cdot,\cdot)^1\) considered as an antibracket on \( \mathcal{N}_1 \). Let \( \xi_{1i} \) be some coordinates in a neighborhood \( U_{p_1} \subset \mathcal{N}_1 \) of \( p_1 \) such that \( \xi_{1i} = 0 \) at \( p_1 \). Then we have
\[
(\xi_{1i}, \xi_{1j})^1 = C^k_{ij}(\xi_1) \xi_{1k}.
\]  
(2.10)
with some functions \( C^k_{ij}(\xi_1) \). We will view the tensor \( C^k_{ij} = C^k_{ij}(0) \) as the structure constants of some Lie (super) algebra \( \mathfrak{g}_1 \).

Let \( p_2 \in \mathcal{N}_2 \) be a vanishing point of the second antibracket on \( \mathcal{N}_2 \). The coordinate system \( \xi_{2\alpha} \) on \( U_{p_2} \subset \mathcal{N}_2 \) is chosen such that \( \xi_{2\alpha}|_{p_2} = 0 \). Similarly to (2.10), we have the equation
\[
(\xi_{2\alpha}, \xi_{2\beta})^2 = C^\gamma_{\alpha\beta}(\xi_2) \xi_{2\gamma}.
\]  
(2.11)
\(^3\)we assume that the set of zeroes (i.e., the points where the corresponding odd Poisson bivector vanishes) of the first (the second) antibracket on \( \mathcal{N}_1 \) (respectively, on \( \mathcal{N}_2 \)) is non-empty.
The structure constants $C_{\alpha\beta}^\gamma = C_{\alpha\beta}^\gamma(0)$ also give rise to a Lie algebra, which we denote by $g_2$. Thus, we have associated a pair of Lie algebras $g_1$ and $g_2$ to a pair of compatible antibrackets. Note that the Lie algebras $g_1$ and $g_2$ are of the same dimensions.

The vanishing points $p_a \in N_a$ correspond to the submanifold $\mathcal{L} \subset \mathcal{M}$ determined by the equations $\xi_{1i} = \xi_{2\alpha} = 0$, where $\xi_{1i}$ and $\xi_{2\alpha}$ are considered as functions on $\mathcal{M}$. In different words, $\mathcal{L} = \pi_1^{-1}p_1 \cap \pi_2^{-1}p_2$. The functions $\xi_a$ are well-defined in some neighborhood $U_{\mathcal{L}}$ of $\mathcal{L}$ in $\mathcal{M}$. Indeed, $\xi_a$ are functions on $\pi_1^{-1}U_{p_1}$; we then choose $U_{\mathcal{L}} = \pi_1^{-1}U_{p_1} \cap \pi_2^{-1}U_{p_2}$, which is evidently a neighborhood containing $\mathcal{L}$. Thus the vector fields

$$L_i = (\xi_{1i}, \cdot)^1, \quad R_\alpha = (\xi_{2\alpha}, \cdot)^2.$$ (2.12)

are defined on the entire neighborhood $U_{\mathcal{L}}$. We now observe that the vector fields restrict to $\mathcal{L}$. Indeed, it follows from Eqs. (2.10)–(2.11) that

$$(L_i \xi_{1j})|_{\mathcal{L}} = (R_\alpha \xi_{2\beta})|_{\mathcal{L}} = 0, \quad L_i \xi_{2\alpha} = R_\alpha \xi_{1j} = 0,$$ (2.13)

which are precisely the conditions for $L_i$ and $R_\alpha$ to restrict to $\mathcal{L}$.

Next, we consider $\mathcal{L}$ as a submanifold in $\mathcal{M}_1 = \pi_1^{-1}p_1 \subset \mathcal{M}$. We have

**Proposition 2.2** $\mathcal{L}$ is Lagrangian submanifold of $\mathcal{M}_1$. In particular, $\mathcal{L}$ is $N$-dimensional.

We consider the functions $\xi_{1i}$ and the vector fields $L_i$ as being defined on $\mathcal{M}_1$. The proof of the proposition follows immediately from relations (2.10) and the fact that $\xi_{1i}$ are independent functions on $\mathcal{M}_1$. Since $\mathcal{M}_1$ is odd symplectic (i.e., the antibracket $(\cdot, \cdot)^1$ is nondegenerate on $\mathcal{M}_1$) we see that $L_i$ are linearly independent on $\mathcal{L}$, and therefore form a basis of $T\mathcal{L}$. We can treat the vector fields $R_\alpha$ similarly, which gives us another set of globally defined vector fields on $\mathcal{L}$. To summarize, we have

**Theorem 2.3** $\mathcal{L}$ is endowed with globally defined vector fields $L_i$ and $R_\alpha$ that satisfy

$$[L_i, L_j] = C_{ij}^k(0)L_k, \quad [R_\alpha, R_\beta] = C_{\alpha\beta}^\gamma(0)R_\gamma,$$

$$[L_i, R_\alpha] = 0.$$ (2.14)

Therefore, $\mathcal{L}$ is diffeomorphic to some Lie (super) group $G$.

As a corollary of the theorem, we observe that the Lie algebras $g_1$ and $g_2$ are isomorphic.

Finally, we note that in the case considered in the previous subsection, there is only one zero $p_1$ ($p_2$) of the first (the second) antibracket on $N_1$ (respectively, on $N_2$). Thus, there is only one submanifold $\mathcal{L} = \pi_1^{-1} \cap \pi_2^{-1} \subset \mathcal{M}$ and the Lie group associated to the antibrackets is uniquely determined.

In the general case, however, we have families $Z_a \subset N_a$ of the vanishing points and hence a family of super Lie groups. This raises the question of whether different points from $Z_a$ actually correspond to different Lie groups. We do not address this question here, however, and turn instead to the converse problem of constructing triplectic antibrackets corresponding to a given Lie group.
3 The inverse construction

The above considerations provide us with the construction of triplectic antibrackets on the duplicated odd cotangent bundle over any Lie group.

Let $G$ be a Lie group and $\mathfrak{g}^*$ the corresponding Lie coalgebra. It is well known that the cotangent bundle $T^*G$ is trivial. There exist two natural ways to identify $G \times \mathfrak{g}^*$ with $T^*G$; the first one is to view $\mathfrak{g}^*$ as the Lie coalgebra of left-invariant 1-forms on $G$ and the second is to view $\mathfrak{g}^*$ as the Lie coalgebra of right-invariant 1-forms.

Since $T^*G$ has a canonical symplectic structure, we can also equip $G \times \mathfrak{g}^*$ with a symplectic structure and hence with the Poisson bracket. We thus have canonical Poisson brackets on $G \times \mathfrak{g}^r_l$ and on $G \times \mathfrak{g}^r_r$, with the subscripts $r$ or $l$ indicating the way in which we identify $G \times \mathfrak{g}^*$ with $T^*G$.

Recall that by changing the parity of the fibres, we can associate a supermanifold to every vector bundle. In the present case we associate to $T^*G$ the supermanifold $\Pi T^*G$ (with $\Pi$ denoting the parity reversing functor), which has the canonical antibracket (the one corresponding to the canonical odd-symplectic structure). Thus $G \times \Pi \mathfrak{g}^*_r$ (respectively, $G \times \Pi \mathfrak{g}^*_l$) is also endowed with a natural antibracket. Let $x^A$ be a local coordinate system on $G$ and $L_i = L_i^A \frac{\partial}{\partial x^A}$ a basis of $\mathfrak{g}^*_l$ considered as the left-invariant vector fields. Let also $\xi_{1i}$ be the coordinates on $\Pi \mathfrak{g}^*_l$ corresponding to the basis dual to $L_i$. Then the canonical antibrackets on $G \times \Pi \mathfrak{g}^*_r$ are

$$
(\xi_{1i}, x^A)^1 = L_i^A, \quad (\xi_{1i}, \xi_{1j})^1 = C_{ij}^k \xi_{1k},
$$

(3.1)

with all the other antibrackets vanishing. Here we have introduced the structure constants via $[L_i, L_j] = C_{ij}^k L_k$. In this way, we can also obtain an explicit form of the canonical antibracket on $G \times \Pi \mathfrak{g}^*_r$.

We now take the direct sum of the bundles, $\mathcal{M} = (G \times \Pi \mathfrak{g}^*_r) \oplus (G \times \Pi \mathfrak{g}^*_l)$. It can be equipped with the canonical triplectic antibrackets.

**Proposition 3.1** The canonical antibrackets on $G \times \Pi \mathfrak{g}^*_r$ and $G \times \Pi \mathfrak{g}^*_l$ naturally induce a pair of compatible antibrackets on $\mathcal{M} = (G \times \Pi \mathfrak{g}^*_r) \oplus (G \times \Pi \mathfrak{g}^*_l)$. The antibrackets are compatible, jointly nondegenerate and satisfy the rank condition formulated in section 2.2. Thus, every Lie (super)group admits a canonical triplectic bundle.

Indeed, we can identify $\mathcal{M}$ with $G \times \Pi \mathfrak{g}^*_r \times \Pi \mathfrak{g}^*_l$; we then consider the canonical antibrackets on $G \times \Pi \mathfrak{g}^*_r$, which we denote by $(\ , \ )^1$, and the trivial (vanishing) antibracket on $\Pi \mathfrak{g}^*_l$. We thus obtain the product antibracket $(\ , \ )^1$ on the product $G \times \Pi \mathfrak{g}^*_r \times \Pi \mathfrak{g}^*_l$. The second antibracket $(\ , \ )^2$ on $G \times \Pi \mathfrak{g}^*_r \times \Pi \mathfrak{g}^*_l$ is constructed similarly.

It should be noted that in contrast to the group case, there does not exist, in general, a natural “triplectic” bundle over an arbitrary manifold $\mathcal{L}$. In fact, the construction of the “triplectic bundle” essentially requires the base $\mathcal{L}$ to be parallelizable. This fact may be viewed as a serious limitation of the triplectic quantization in its present form. Indeed, the triplectic antibrackets cannot be

\[ A \text{ similar construction in the case of ordinary Poisson bracket on } G \times \mathfrak{g}^* \text{ is well known. Its generalization to the case of antibracket has been considered in F. } \]
constructed (even formally) in a covariant way in the model whose target space is a non-flat manifold. Over a curved manifold, the tripectic structure would be defined only locally and would depend on the choice of coordinates. This is in contrast with the standard BV quantization [7], where the canonical antibracket exists on the odd cotangent bundle over any field space.

A remarkable feature of our construction is that it can be repeated for the standard (non-super) geometry. In particular, there a exist a pair of compatible Poisson brackets on $\mathcal{M}' = (G \times g_l^*) \oplus (G \times g_r^*)$ for every Lie group. Moreover, there exists a direct analogue of Theorem 2.3 for a pair of appropriately degenerate compatible Poisson brackets.

4 Conclusions

We have shown that the tripectic structure (consisting of a pair of appropriately degenerate and compatible antibrackets) induces the structure of a Lie group on the intersection of certain symplectic leaves of the antibrackets. The interest in the pairs of degenerate antibrackets originates from the tripectic quantization of general gauge theories. In the tripectic scheme, however, one needs mutually commutative antibrackets, and therefore the group has to be Abelian, which appears to be a strong geometrical constraint on the applicability of the tripectic quantization.

Our approach to the Lie group structures is somewhat reminiscent of the well-known fact that one can naturally associate a Lie algebra to every symplectic leaf of a Poisson bracket [16]. In the tripectic geometry, it is not only a Lie algebra but also the corresponding Lie group that is associated with the zero-rank symplectic leaf of the antibracket induced on the algebra of marked functions. An important point of our construction is that it has a direct analogue for the ordinary (non-super) differential geometry based on the standard Poisson bracket.

We have considered the geometric structures induced on the intersection of symplectic leaves of compatible antibrackets. This is only a half of the full tripectic geometry. The other part is concentrated on the manifold of marked functions of the antibrackets; the corresponding geometry was studied in the case of mutually commutative antibrackets in [3]. This is also interesting to generalize to the case of non-commutative antibrackets.

Another interesting aspect of the tripectic geometry is related to an additional structure, the odd vector fields $V^a$ originating from the one-form $F$ on $\mathcal{M}$ [3] that is compatible with the antibrackets. In particular, $F$ gives rise to a Kähler structure on the manifold of marked functions provided the appropriate nondegeneracy conditions are satisfied by $F$. This also endows $\mathcal{L}$ with an even Poisson bracket. In the case considered in this paper, it can be checked that $F$ determines a left-right-invariant Poisson bracket on $\mathcal{L}$. It seems very probable that the present construction can be generalized to include the Poisson–Lie group structures.

Acknowledgements I am grateful to A. M. Semikhatov for his attention to this work and fruitful discussions and suggestions. I also wish to thank I. A. Batalin, M. A. Soloviev, and I. V. Tyutin and especially, O. M. Khudaverdian and I. Yu. Tipunin for illuminating discussions. This work was supported in part by the RFBR Grant 98-01-01155 and by the INTAS-RFBR Grant 95-0829.
References

[1] I. A. Batalin, R. Marnelius, and A. M. Semikhatov, Nucl. Phys. B446 (1995) 249.
[2] I. A. Batalin and R. Marnelius, Phys. Lett. B446 (1995) 44.
[3] I. A. Batalin and R. Marnelius, Nucl. Phys. B465 (1996) 521.
[4] M. A. Grigoriev and A. M. Semikhatov, Phys. Lett. B417 (1998) 259–268.
[5] M. A. Grigoriev and A. M. Semikhatov, “A Kähler Structure of the Triplectic Geometry”, hep-th/9807023
[6] I. A. Batalin, P. M. Lavrov, and I. V. Tyutin, J. Math. Phys. 31 (1990) 1487.
    I. A. Batalin, P. M. Lavrov, and I. V. Tyutin, J. Math. Phys.32 (1990) 532.
    I. A. Batalin, P. M. Lavrov, and I. V. Tyutin, J. Math. Phys.32 (1991) 2513.
[7] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B Vol.102 (1981) 27.
    I. A. Batalin and G. A. Vilkovisky, Phys. Rev. D Vol.28 (1983) 2567.
[8] I. A. Batalin and I. V. Tyutin, Int. J. Mod. Phys. A8 (1993) 2333.
    I. A. Batalin and I. V. Tyutin, Mod. Phys. Lett. A8 (1993) 3673.
    I. A. Batalin and I. V Tyutin, Mod. Phys. Lett. A9 (1994) 1707.
[9] A. Schwarz, Commun. Math. Phys.155 (1993) 249–260.
[10] H. Hata and B. Zwiebach, Ann. Phys. 229 (1994) 177–216.
    A. Sen and B. Zwiebach, Commun. Math. Phys.177 (1996) 305.
[11] O. M. Khudaverdian and A. Nersessian, Mod. Phys. Lett. A8 (1993) 2377.
[12] K. Bering, “Almost Parity Structure, Connections and Vielbeins in BV Geometry”, MIT-CTP-2682, physics/9711011.
[13] V. A. Soroka, “Linear Odd Poisson Bracket on Grassmann Variables”, hep-th/9811252
[14] S. Sternberg, “Lectures on Differential Geometry”, Prentice Hall, Inc. Englewood Cliffs, N.J. 1964.
[15] J. Alfaro, P. H. Damgaard, Phys. Lett. B369 (1996) 289–294.
[16] Karasev M. V. and Maslov V. P., “Nonlinear Poisson Brackets: Geometry and Quantization”, Amer. Math. Soc., Providence, RI (1991)