Radiative Corrections to Moller Scattering of Polarized Particles

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Abstract
Principal contributions to QED radiative effects for Moller scattering of polarized particles are investigated both on the Born level and taking into account radiative corrections (RC). Scattering on the case of longitudinal and transversal polarized targets is also considered. In a general way the expressions for differential cross section and polarization asymmetry (PA) have been defined, and respective graphics for longitudinal and transversal polarization asymmetries and also for radiative corrections to asymmetry are presented. All quantities are presented in terms of covariant variables. The ultrarelativistic approximation was applied for calculations. The results of a computer run are presented.

1 Introduction

For a wide serie of experiments in SLAC (E-142,E-143,E-154,E-155), it is neccesary to know the polarization of electron beam. For this aim a single arm Moller polarimeter is utilized. In order to extract (with desirable accuracy) beam polarization $p_b$ we need to calculate theoretically, taking into account the experimental conditions, the quantities $\sigma^{th}$ and $A^{th}$ - theoretical meaning of cross section (CS) and polarization asymmetry. If the experimental data for polarization asymmetry ($A^{meas}$) and target polarization ($p_t$) are known, the ratio

$$A^{meas} = p_b p_t A^{th},$$

allows to find the polarization of electron beam.
2  Method of Calculation

The Moller \((e^- + e^- \rightarrow e^- + e^-)\) scattering CS of order \(O(\alpha^3)\) can be written in the form (within the QED treatment)

\[
\sigma = \sigma_o + \sigma_V + \sigma_R,
\]

where each \(\sigma_o, \sigma_V, \sigma_R\) is defined as \(d\sigma_o, d\sigma_V, d\sigma_R/ dy\), and \(y \equiv 1 - E'/E\), where \(E(E')\) is the energy of the initial (scattered) electron. \(\sigma_o\) is the Born (non radiative) contribution of order \(O(\alpha^2)\). \(\sigma_V\) is the contribution of the diagrams with an additional virtual photon (V-contribution). It is constituted by contributions of vacuum polarization (sum over all generations of leptons and quarks), vertex renormalization and two-photon exchange diagrams. \(\sigma_R\) is the contribution of nonobservable (internal bremsstrahlung) photon radiation (R-contribution). \(\sigma_R\) can be divided into three parts: \(\sigma_R^{\text{F, R}}\) is finite when \(k \rightarrow 0\) (\(k\) is the photon momentum), \(\sigma_R^{\text{H}}\) is the contribution of "hard photons", and it is also finite, and \(\sigma_S\) is the contribution of "soft photons". The latter contains infrared divergences. All \(\sigma_R^{\text{F, H}}\) and \(\sigma_S\) may be calculated in a standard way (see, for example, \([1]-[2]\)). According to the method of ref. \([1]\), the infrared divergence vanishes when the infrared divergent part of \(\sigma_V\) (denoted as \(\delta_\nu^{\text{vert}}\), see, for example, \([1]\)) is summed with the infrared divergent part of \(\sigma_R\) (denoted as \(\delta_\nu^{\text{R}}\), see, for example, \([1]\)).

As it is wellknown, the linear, exchangeable and interfering diagrams give contribution to Moller scattering. For this reason the expression for differential cross section and polarization asymmetry are presented as a sum of three parts:

\[
\frac{d\sigma}{dy} = \sum_l \frac{d\sigma_l}{dy},
\]

and

\[
A = \sum_l A_l,
\]

where \(l = l, e, i\). In our calculations the differential cross section is presented as

\[
\sigma = \sigma_o + \frac{\alpha}{\pi}(\delta_{\text{vert}} + \delta_{\text{vac}})\sigma_o + \sigma_{\text{amm}}^u + \sigma_{\text{amm}}^p + \frac{\alpha}{\pi}(\delta_k + \delta_{2\gamma}^u)\sigma_o^u + \frac{\alpha}{\pi}(\delta_k + \delta_{2\gamma}^p)\sigma_o^p + \sigma_{\text{R}}^{F, u} + \sigma_{\text{R}}^{F, p} + \frac{\alpha}{\pi}(\delta_l + \delta_{2\gamma}^l + \delta_s + \delta_{1\gamma}^H)\sigma_o + \sigma_{2\gamma}^{H, u} + \sigma_{2\gamma}^{H, p},
\]

where \(\sigma_o = \sigma_o^u + \sigma_o^p\), and \(\sigma_{a, b}^{u, p}\) are the spin-independent and spin-dependent Born contributions. \(\delta_{\text{vert}}\) and \(\delta_{\text{vac}}\) are the spin-independent finite factorized contributions of vertex and vacuum polarization corrections. \(\sigma_{\text{amm}}^{u, p}\)
are the spin-independent and spin-dependent contributions attributed to anomalous magnetic momentum. $\delta_{2,\gamma}^{u,p}$ are the spin-independent and spin-dependent corrections due to two-photon exchange. $\delta_k$ can be written in a standard way. For references see [3]. $\sigma_{\gamma}^{F,u,p}$ are the spin-independent and spin-dependent contributions of infrared free part of R-contribution. $\delta_l, \delta_s, \delta_{1}^{H}$ and $\delta_{\lambda}^{H}$ are spin-independent factorized corrections derived from the infrared divergent vertex contribution, finite "soft-photon" contribution, finite "hard-photon" contribution and the sum of infrared divergent parts in R and V contributions ($\delta_{\lambda}^{H}$). $\sigma_{2,u}^{H}$ and $\sigma_{2,p}^{H}$ are the spin-independent and spin-dependent corrections of infrared free part of hard-photon contribution. All contributions are defined in a standard way, (see, for example, references [1, 3]).

The longitudinal and transversal polarization asymmetries are, usually, defined as

$$A_l = \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}},$$

and

$$A_t = \frac{\sigma_{\uparrow\rightarrow} - \sigma_{\uparrow\leftarrow}}{\sigma_{\uparrow\rightarrow} + \sigma_{\uparrow\leftarrow}}.$$  (6), (7)

The arrows show the polarization of the beam and the target, respectively.

The corrected polarization asymmetry is presented as

$$A_{QED}^{QED} = A_o (1 + \frac{\alpha}{\pi} (\delta_{2,\gamma}^{p} - \delta_{2,\gamma}^{u}) + \sigma_{amn}^{p} - \sigma_{amn}^{u} + \frac{\sigma_{n}^{u} \sigma_{R}^{p} - \sigma_{n}^{p} \sigma_{R}^{u}}{(\sigma_{n}^{u})^{2}} + O(\alpha^2),$$  (8)

where $A_o$ is the Born asymmetry, and $A_{o} \equiv A_{lo}$ or $A_{o} \equiv A_{to}$. $\sigma_{amn}^{p}$ and $\sigma_{amn}^{u}$ are the spin-dependent and spin-independent contributions due to anomalous magnetic momentum. $(\sigma_{n}^{u} \sigma_{R}^{p} - \sigma_{n}^{p} \sigma_{R}^{u})/(\sigma_{n}^{u})^2$ is due to R-contribution. Where $\sigma_{R}^{u} = \sigma_{R}^{F,u} + \sigma_{2,u}^{H}$ and $\sigma_{R}^{p} = \sigma_{R}^{F,p} + \sigma_{2,p}^{H}$.

The radiative correction to asymmetry is defined as

$$\delta_{QED} = \frac{A_{QED}^{QED}}{A_o} - 1.$$  (9)

### 3 Results

In this section the results of calculations of polarization asymmetry both for longitudinally and transversally polarized target and radiative corrections to asymmetry are presented. Numerical calculations have been carried out with the help of FORTRAN code created by the authors. For analytical calculations REDUCE code has been utilized. Graphics for asymmetries and radiative corrections can be seen in figures 1-4. Obtained results are presented in table 1.
| $y$ | $A_{ol}$ | $A_{ot}$ | $A_l$ | $A_t$ | $\delta_l(\%)$ | $\delta_t(\%)$ |
|-----|---------|---------|------|------|----------------|----------------|
| .10 | -.41282 | .13334E-02 | -.48454 | .11093E-02 | 17.374 | -16.807 |
| .20 | -.66387 | .29711E-02 | -.72840 | .27305E-02 | 09.720 | -08.943 |
| .30 | -.81058 | .43023E-02 | -.86688 | .40466E-02 | 06.946 | -05.147 |
| .40 | -.88520 | .51717E-02 | -.93473 | .49056E-02 | 05.595 | -05.194 |
| .50 | -.90438 | .55191E-02 | -.95268 | .52324E-02 | 05.340 | -05.823 |
| .60 | -.87293 | .53009E-02 | -.92673 | .49923E-02 | 06.224 | -05.233 |
| .70 | -.78220 | .44700E-02 | -.84584 | .41523E-02 | 08.136 | -07.233 |
| .80 | -.61358 | .30365E-02 | -.68504 | .26956E-02 | 11.647 | -11.224 |
| .90 | -.32977 | .11483E-02 | -.38357 | .79126E-02 | 16.315 | -31.092 |
| .99 | -.04891 | .63291E-02 | -.05877 | .15068E-02 | 20.148 | -338.07 |

Table 1: Results of calculations of Born asymmetries $A_{ol,ot}$, corrected asymmetries $A_l,t$ and radiative corrections to asymmetry $\delta_{l,t}$ for both longitudinally and transversally polarized targets correspondingly, for SLAC kinematics. $E=50$ Gev.

4 Conclusions

The calculation of the differential cross section and polarization asymmetry to Moller scattering will in the future allow to find beam polarization. For this aim it is neccesary to measure polarization asymmetry with high accuracy. Iteration procedure for theoretical calculation of polarization asymmetry can be improved, and that will lead to the increase of the accuracy when beam polarization is measured. As we can see, the corrections without cuts are very large (in any ocasion about 20%), although they will be considerably reduced when the experimental cuts will be taken into account. Analysis of the corrected polarization asymmetries in the case of longitudinally and transversally polarized targets show that in a wide kinematical range corrections are considerable. In this paper the results of calculations of differencial cross section and polarization asymmetry have been presented taking into account only the contribution of linear diagrams. Results, in which the contribution of exchangable and interfering diagrams are considered will be presented in the future. For a more accurate calculation of polarization asymmetry it is neccesary to utilize a Monte Carlo generator for Moller scattering, and the contribution of correction due to external radiation of beam particles within the target must be taken into account. This is an object for future investigations.

References
[1] Bardin D.Yu., Shumeiko N.M. Nucl. Phys. 1977. v.B127. p.242.
   Bardin D.Yu., Shumeiko N.M. 1976. Dubna Preprint p2-10113
[2] Bardin D.Yu., Shumeiko N.M. 1976. Dubna Preprint p2-10114
[3] Kahane J. Phys. Rev. 1964. 135. B.975.
[4] Kukhto T.V., Shumeiko N.M., Timoshin S.I. Nucl. Phys. 1987. G.13.
   p.725
Figure 1: $y$-dependence of the born asymmetry (dashed line) and corrected asymmetry (solid line) in the polarization Møller scattering for SLAC kinematics. $E=50$ GeV. Longitudinally polarized target.

Figure 2: The QED radiative corrections to asymmetry without experimental cuts for longitudinally polarized target.
Figure 3: $y$-dependence of the born asymmetry (dashed line) and corrected asymmetry (solid line) in the polarization Møller scattering for SLAC kinematics. $E=50$ GeV, transversally polarized target.

Figure 4: The QED radiative corrections to asymmetry without experimental cuts for transversally polarized target.