WITTEN DEFORMATION OF RAY-SINGER ANALYTIC TORSION

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August, 1994

Abstract. Let $F$ be a flat vector bundle over a compact Riemannian manifold $M$ and let $f : M \rightarrow \mathbb{R}$ be a self-indexing Morse function. Let $g^F$ be a smooth Euclidean metric on $F$, let $g^F_t = e^{-2tf}g^F$ and let $\rho^{RS}(t)$ be the Ray-Singer analytic torsion of $F$ associated to the metric $g^F_t$. Assuming that $\nabla f$ satisfies the Morse-Smale transversality conditions, we provide an asymptotic expansion for $\log \rho^{RS}(t)$ for $t \rightarrow \infty$ of the form $a_0 + a_1 t + b \log (\frac{t}{\pi}) + o(1)$. We present explicit formulae for coefficients $a_0, a_1$ and $b$. In particular, we show that $b$ is a half integer.

0. Introduction

0.1. The Ray-Singer analytic torsion. Let $M$ be a compact manifold of dimension $n$ and let $F$ be a flat vector bundle on $M$. Let $g^F$ and $g^{TM}$ be smooth metrics on $F$ and $TM$ respectively.

In [RS] Ray and Singer introduced a numerical invariant of these data which is called the Ray-Singer analytic torsion of $F$ and which we shall denote by $\rho^{RS}$.

0.2. The Witten deformation. Suppose $f : M \rightarrow \mathbb{R}$ is a Morse function. For $t > 0$, we denote by $g^F_t$ the smooth metric on $F$

\begin{equation}
(0.1)
\quad g^F_t = e^{-2tf}g^F.
\end{equation}

Let $\rho^{RS}(t)$ be the Ray-Singer torsion on $F$ associated to the metrics $g^F_t$ and $g^{TM}$.

Denote by $\nabla f$ the gradient vector field of $f$ with respect to the metric $g^{TM}$. Let $B$ be the finite set of zeroes of $\nabla f$.

We shall assume that the following conditions are satisfied (cf. [BFK3, page 5]):

(1) $f : M \rightarrow \mathbb{R}$ is a self-indexing Morse function (i.e. $f(x) = \text{index}(x)$ for any critical point $x$ of $f$).

\begin{itemize}
\item 1991 Mathematics Subject Classification. Primary: 58G26.
\item Key words and phrases. Analytic torsion, Witten deformation, Asymptotic expansion, Ray-Singer metric, Milnor metric.
\end{itemize}
(2) The gradient vector field $\nabla f$ satisfies the Smale transversality conditions [Sm1, Sm2] (for any two critical points $x$ and $y$ of $f$ the stable manifold $W^s(x)$ and the unstable manifold $W^u(y)$, with respect to $\nabla f$, intersect transversally).

(3) For any $x \in B$, the metric $g^F$ is flat near $B$ and there is a system of coordinates $y = (y^1, \ldots, y^n)$ centered at $x$ such that near $x$

$$g^{TM} = \sum_{i=1}^n |dy^i|^2, \quad f(y) = f(x) - \frac{1}{2} \sum_{i=1}^{\text{index}(x)} |y^i|^2 + \frac{1}{2} \sum_{i=\text{index}(x)+1}^n |y^i|^2.$$

0.3. Asymptotic expansion of the torsion. Burghelea, Friedlander and Kappeler ([BFK3]) have shown that the function $\log \rho^{RS}(t)$ has asymptotic expansion for $t \to \infty$ of the form

$$\log \rho^{RS}(t) = \sum_{j=0}^{n+1} a_j t^j + b \log t + o(1).$$

The coefficient $a_0$ is calculated in [BFK3] in terms of the parametrix of the Laplace-Beltrami operator.

In the present paper we shall calculate all coefficients in the asymptotic expansion [BFK3]. In fact, we shall show that the coefficients $a_j = 0$ for $j > 1$ and the coefficient $b$ is a half integer.

0.4. To formulate our result, we need to introduce some notation (cf. [BZ1]).

Let $\nabla^{TM}$ be the Levi-Civita connection on $TM$ corresponding to the metric $g^{TM}$, and let $e(TM, \nabla^{TM})$ be the associated representative of the Euler class of $TM$ in Chern-Weil theory.

Let $\psi(TM, \nabla^{TM})$ be the Mathai-Quillen ([MQ]) $n - 1$ current on $TM$ (see also [BGS, Section 3] and [BZ1, Section IIId]).

Let $\nabla^F$ be the flat connection on $F$ and let $\theta(F, g^F)$ be the 1-form on $M$ defined by (cf. [BZ1, Section IVd])

$$\theta(F, g^F) = \text{Tr} \left[ (g^F)^{-1} \nabla^F g^F \right].$$

Set

$$\chi(F) = \sum_{i=0}^n (-1)^i \dim H^i(M, F),$$

$$\chi'(F) = \sum_{i=0}^n (-1)^i i \dim H^i(M, F).$$

Let $\rho^M$ be the torsion of the Thom-Smale complex (cf. Section 1).
Theorem 0.5. The function \( \log \rho^{RS}(t) \) admits an asymptotic expansion for \( t \to \infty \) of the form

\[
(0.6) \quad \log \rho^{RS}(t) = a_0 + a_1 t + b \log \left( \frac{t}{\pi} \right) + o(1),
\]

where the coefficients \( a_0, a_1 \) and \( b \) are given by the formulas

\[
(0.7) \quad a_0 = \log \rho^M - \frac{1}{2} \int_M \theta(F, g^F)(\nabla f)^*(TM, \nabla^{TM});
\]

\[
(0.8) \quad a_1 = -\text{rk}(F) \int_M f e(TM, \nabla^{TM}) + \chi'(F);
\]

and

\[
(0.9) \quad b = \frac{n}{4} \chi(F) - \frac{1}{2} \chi'(F).
\]

Remark 0.6. Note that \( \chi(F) = 0 \) if \( n \) is odd. Hence, (0.9) implies that the coefficient \( b \) is a half integer for any \( n \).

0.7. The method of the proof. Our method is completely different from that of [BFK3]. In [BFK3] the asymptotic expansion (0.3) is proved by direct analytic arguments and, then is applied to get a new proof of the Ray-Singer conjecture [RS].

In the present paper we use the Bismut-Zhang extension of this conjecture ([BZ1]) in order to obtain the Theorem 0.5.

Acknowledgments. It is a great pleasure for me to express my deep gratitude to Michael Farber for bringing the papers [BFK2, BFK3] to my attention and for valuable discussions.

1. Milnor metric and Milnor torsion

In this section we follow [BZ1, Chapter I].

1.1. The determinant line of the cohomology. Let \( H^\bullet(M, F) = \bigoplus_{i=0}^n H^i(M, F) \) be the cohomology of \( M \) with coefficients in \( F \) and let \( \det H^\bullet(M, F) \) be the line

\[
(1.1) \quad \det H^\bullet(M, F) = \bigotimes_{i=0}^n \left( \det H^i(M, F) \right)^{(-1)^i}.
\]
1.2. The Thom-Smale complex. Suppose \( f : M \rightarrow \mathbb{R} \) is a Morse function satisfying the Smale transversality conditions [Sm1, Sm2] (for any two critical points \( x \) and \( y \) of \( f \) the stable manifold \( W^s(x) \) and the unstable manifold \( W^u(y) \), with respect to \( \nabla f \), intersect transversally).

Let \( B \) be the set of critical points of \( f \). If \( x \in B \), we denote by \( F_x \) the fiber of \( F \) over \( x \) and by \([W^u(x)]\) the real line generated by \( W^u(x) \). For \( 0 \leq i \leq n \), set

\[
C^i(W^u, F) = \bigoplus_{x \in B \atop \text{index}(x) = i} [W^u(x)] \otimes \mathbb{R} F_x.
\]

By a basic result of Thom ([Th]) and Smale ([Sm2]) (see also [BZ1, pages 28–30]), there is a well defined linear operators \( \partial : C^i(W^u, F) \rightarrow C^{i+1}(W^u, F) \), such that the pair \((C^*(W^u, F), \partial)\) is a complex and there is a canonical identification of \( \mathbb{Z} \)-graded vector spaces \( H^\ast(C^*(W^u, F), \partial) \simeq H^\ast(M, F) \). By [KMu] there is a canonical isomorphism

\[
\det H^\ast(M, F) \simeq \det C^*(W^u, F).
\]

1.3. The Milnor metric. The metric \( g^F \) on \( F \) determines the structure of Euclidean vector space on \( C^*(W^u, F) \).

**Definition 1.4.** The Milnor metric \( \| \cdot \|_{\det H^\ast(M, F)}^M \) on the line \( \det H^\ast(M, F) \) is the metric corresponding to the obvious metric on \( \det C^*(W^u, F) \) via the canonical isomorphism (1.3).

**Remark 1.5.** By Milnor [Mi1, Theorem 9.3], if \( g^F \) is a flat metric on \( F \) then the Milnor metric coincides with the Reidemeister metric defined through a smooth triangulation of \( M \). In this case \( \| \cdot \|_{\det H^\ast(M, F)}^M \) does not depend upon \( F \) and \( g^TM \) and, hence, is a topological invariant of the flat Euclidean vector bundle \( F \).

1.6. The Milnor torsion. Let \( \partial^* \) be the adjoint of \( \partial \) with respect to the Euclidean structure on \( C^*(W^u, F) \). Using the finite dimensional Hodge theory, we have the canonical identification

\[
H^i(C^*(W^u, F), \partial) \simeq \{ v \in C^i(W^u, F) : \partial v = 0, \partial^* v = 0 \}, \quad 0 \leq i \leq n.
\]

As a vector subspace of \( C^i(W^u, F) \), the vector space in the right-hand side of (1.4) inherits the Euclidean metric. We denote by \( | \cdot |_{\det H^\ast(M, F)}^M \) the corresponding metric on \( \det H^\ast(M, F) \).

The metrics \( \| \cdot \|_{\det H^\ast(M, F)}^M \) and \( | \cdot |_{\det H^\ast(M, F)}^M \) do not coincide in general. We shall describe the discrepancy.

Set \( \Delta = \partial \partial^* + \partial^* \partial \) and let \( P : C^*(W^u, F) \rightarrow \text{Ker} \Delta \) be the orthogonal projection. Set \( \Pi^\perp = 1 - \Pi \).
Let $N$ and $\tau$ be the operators on $C^\bullet(W^u, F)$ acting on $C^i(W^u, F)$ ($0 \leq i \leq n$) by multiplication by $i$ and $(-1)^i$ respectively. If $A \in \text{End}(C^\bullet(W^u, F))$, we define the supertrace $\text{Tr}_s[A]$ by the formula

$$\text{Tr}_s[A] = \text{Tr}[\tau A].$$

For $s \in \mathbb{C}$, set

$$\eta^M(s) = -\text{Tr}_s \left[ N(\Delta)^{-s} \Pi^\perp \right].$$

**Definition 1.7.** The Milnor torsion is the number

$$\rho^M = \exp \left( \frac{1}{2} \frac{d\eta^M(0)}{ds} \right).$$

The following result is proved in [BGS, Proposition 1.5]

$$\| \cdot \|_{\det H^\bullet(M, F)}^M = \| \cdot \|_{\det H^\bullet(M, F), t}^M \cdot \rho^M.$$  

**1.8. Deformation of Milnor metric.** The metric $\| \cdot \|_{\det H^\bullet(M, F)}^M$ depends on the metric $g^F$. Let $g^F_t = e^{-2tf}g^F$ and let $\| \cdot \|_{\det H^\bullet(M, F), t}^M$ be the corresponding Milnor metric. Set

$$\tilde{\chi}'(F) = \text{rk}(F) \sum_{x \in B} (-1)^{\text{index}(x)} \text{index}(x).$$

As $f$ is a self-indexing Morse function

$$\text{rk}(F) \sum_{x \in B} (-1)^{\text{index}(x)} f(x) = \tilde{\chi}'(F).$$

Obviously,

$$\| \cdot \|_{\det H^\bullet(M, F), t}^M = e^{-i\tilde{\chi}'(F)} \cdot \| \cdot \|_{\det H^\bullet(M, F)}^M.$$  

**2. Ray-Singer metric and Ray-Singer torsion**

**2.1. $L_2$ metric on the determinant line.** Let $(\Omega^\bullet(M, F), d^F)$ be the de Rahm complex of the smooth sections of $\Lambda(T^*M) \otimes F$ equipped with the coboundary operator $d^F$. The cohomology of this complex is canonically isomorphic to $H^\bullet(M, F)$.

Let $*$ be the Hodge operator associated to the metric $g^{TM}$. We equip $\Omega^\bullet(M, F)$ with the inner product

$$\langle \alpha, \alpha' \rangle_{\Omega^\bullet(M, F)} = \int_M \langle \alpha \wedge \ast \alpha' \rangle_{g^F}.$$  

By Hodge theory, we can identify $H^\bullet(M, F)$ to the corresponding harmonic forms in $\Omega^\bullet(M, F)$. These forms inherit the Euclidean product (2.1). Thus the line $\det H^\bullet(M, F)$ inherits a metric $\| \cdot \|_{\det H^\bullet(M, F)}^{\text{RS}}$, which is also called the $L_2$ metric.
2.2. The Ray-Singer torsion. Let $dF^*$ be the formal adjoint of $dF$ with respect to the metrics $g^{TM}$ and $g^F$.

Set $\Delta = dF dF^* + dF^* dF$ and let $\Pi : \Omega^\bullet(M, F) \to \text{Ker } \Delta$ be the orthogonal projection. Set $\Pi^\perp = 1 - \Pi$.

Let $N$ be the operator defining the $\mathbb{Z}$-grading of $\Omega^\bullet(M, F)$, i.e. $N$ acts on $\Omega^i(M, F)$ by multiplication by $i$.

If an operator $A : \Omega^\bullet(M, F) \to \Omega^\bullet(M, F)$ is trace class, we define its supertrace $\text{Tr}^s[A]$ as in (1.5).

For $s \in \mathbb{C}$, set
$$
\eta^{RS}(s) = - \text{Tr}_s \left[ N(\Delta)^{-s} \Pi^\perp \right].
$$

By a result of Seeley [Se], $\eta^{RS}(s)$ extends to a meromorphic function of $s \in \mathbb{C}$, which is holomorphic at $s = 0$.

**Definition 2.3.** The Ray-Singer torsion is the number

$$
\rho^{RS} = \exp \left( \frac{1}{2} \frac{d\eta^{RS}(0)}{ds} \right).
$$

2.4. The Ray-Singer metric. We now remind the following definition (cf. [BZ1, Definition 2.2]):

**Definition 2.5.** The Ray-Singer metric $\| \cdot \|^{RS}_{\text{det } H^\bullet(M, F)}$ on the line $\text{det } H^\bullet(M, F)$ is the product

$$
\| \cdot \|^{RS}_{\text{det } H^\bullet(M, F)} = | \cdot |^{RS}_{\text{det } H^\bullet(M, F)} \cdot \rho^{RS}.
$$

**Remark 2.6.** When $M$ is odd dimensional, Ray and Singer [RS, Theorem 2.1] proved that the metric $\| \cdot \|^{RS}_{\text{det } H^\bullet(M, F)}$ is a topological invariant, i.e. does not depend on the metrics $g^{TM}$ or $g^F$. Bismut and Zhang [BZ1, Theorem 0.1] described explicitly the dependents of $\| \cdot \|^{RS}_{\text{det } H^\bullet(M, F)}$ on $g^{TM}$ and $g^F$ in the case when $\text{dim } M$ is odd.

2.7. Bismut-Zhang theorem. Let $\nabla^{TM}$ be the Levi-Civita connection on $TM$ corresponding to the metric $g^{TM}$, and let $e(TM, \nabla^{TM})$ be the associated representative of the Euler class of $TM$ in Chern-Weil theory.

Let $\psi(TM, \nabla^{TM})$ be the Mathai-Quillen ([MQ]) $n - 1$ current on $TM$ (see also [BGS, Section3] and [BZ1, Section IIId]).

Let $\nabla^F$ be the flat connection on $F$ and let $\theta(F, g^F)$ be the 1-form on $M$ defined by (cf. [BZ1, Section IVd])

$$
\theta(F, g^F) = \text{Tr} \left[ (g^F)^{-1} \nabla^F g^F \right].
$$

Now we remind the following theorem by Bismut and Zhang [BZ1, Theorem 0.2].
Theorem 2.8 (Bismut-Zhang). The following identity holds

$$\log \left( \frac{\| \cdot \|_{RS}^{H^\bullet(M,F)} \|}{\| \cdot \|_{H^\bullet(M,F)}^{M}} \right)^2 = -\int_M \theta(F, g^F) (\nabla f)^* \psi(TM, \nabla TM).$$

2.9. Dependence on the metric. The metrics $\| \cdot \|_{RS}^{H^\bullet(M,F)}$ and $\| \cdot \|_{H^\bullet(M,F)}^{M}$ depend, in general, on the metric $g^F$. Let $g_t^F = e^{-2t} g^F$ and let $\| \cdot \|_{RS}^{H^\bullet(M,F),t}$ and $\| \cdot \|_{M_H^\bullet(M,F),t}$ be the Ray-Singer and Milnor metrics on $H^\bullet(M, F)$ associated to the metrics $g_t^F$ and $g^{TM}$.

By [BZ1, Theorem 6.3]

$$\int_M \theta(F, g_t^F) (\nabla f)^* \psi(TM, \nabla TM) = \int_M \theta(F, g^F) (\nabla f)^* \psi(TM, \nabla TM) + 2t \text{rk}(F) \int_M f e(TM, \nabla TM) - 2t \chi'(F).$$

From (1.9), (2.5) and (2.6), we get

$$\log \left( \frac{\| \cdot \|_{RS}^{H^\bullet(M,F),t}}{\| \cdot \|_{H^\bullet(M,F),t}^{M}} \right)^2 = -\int_M \theta(F, g^F) (\nabla f)^* \psi(TM, \nabla TM) - 2t \text{rk}(F) \int_M f e(TM, \nabla TM).$$

3. The main result

In this section we prove Theorem 0.5, which we restate for convenience.

Theorem 3.1. The function $\log \rho^{RS}(t)$ admits an asymptotic expansion for $t \to \infty$ of the form

$$\log \rho^{RS}(t) = a_0 + a_1 t + b \log \left( \frac{t}{\pi} \right) + o(1),$$

where the coefficients $a_0, a_1$ and $b$ are given by the formulas

$$a_0 = \log \rho^{M} - \frac{1}{2} \int_M \theta(F, g^F) (\nabla f)^* \psi(TM, \nabla TM);$$

$$a_1 = -\text{rk}(F) \int_M f e(TM, \nabla TM) + \chi'(F);$$

and

$$b = \frac{n}{4} \chi(F) - \frac{1}{2} \chi'(F).$$
Proof. For each \( t > 0 \) we equip \( \Omega^\bullet(M, F) \) with the inner product
\[
\langle \alpha, \alpha' \rangle_{\Omega^\bullet(M, F), t} = \int_M \langle \alpha \wedge \star \alpha' \rangle_{g^F}.
\]
and we denote by \( | \cdot |_{RS |_{\det H^\bullet(M,F),t}} \) the \( L_2 \) metric on \( \det H^\bullet(M, F) \) (cf. Section 2.1) associated to this inner product.

From (1.7), (2.3) and (2.7), we get
\[
\log \rho_{RS}^\bullet(t) = -\frac{1}{2} \int_M \theta(F, g^F) (\nabla f)^\star \psi(TM, \nabla TM) \hspace{1cm} (3.6)
\]
and
\[
\int_M f e(TM, \nabla TM) + \log \rho^M + \log \left( \frac{| \cdot |_{\det H^\bullet(M,F),t}}{| \cdot |_{RS |_{\det H^\bullet(M,F),t}}} \right) \hspace{1cm} (3.7)
\]
Let \( dF^\star \) be the formal adjoint of \( dF \) with respect to the inner product (3.5). Set
\[
\Delta_t = dF dF^\star + dF^\star dF.
\]
Let \( \Omega^\bullet_t^{[0,1]}(M, F) \) be the direct sum of the eigenspaces of \( \Delta_t \) associated to eigenvalues \( \lambda \in [0, 1] \). The pair \( (\Omega^\bullet_t^{[0,1]}(M, F), dF) \) is a subcomplex of \( (\Omega^\bullet(M, F), dF) \).

We denote by \( \| \cdot \|_{\Omega^\bullet(M,F)} \) the norm on \( \Omega^\bullet(M, F) \) determined by inner product (3.5), and by \( \| \cdot \|_{C^\bullet(W^u,F)} \) the norm on \( C^\bullet(W^u, F) \) determined by \( g^F \) (cf. Section 1.3).

In the sequel, \( o(1) \) denotes an element of End \( (C^\bullet(W^u, F)) \) which preserves the \( \mathbb{Z} \)-grading and is \( o(1) \) as \( t \to \infty \).

It is shown in [BZ2, Theorem 6.9] that if \( t > 0 \) is large enough, there exists an isomorphism
\[
e_t : C^\bullet(W^u, F) \to \Omega^\bullet_t^{[0,1]}(M, F)
\]
of \( \mathbb{Z} \)-graded Euclidean vector spaces such that
\[
e_t^\star e_t = 1 + o(1). \hspace{1cm} (3.7)
\]
By [BZ2, Theorem 6.11], for any \( t > 0 \) there is a quasi-isomorphism of complexes
\[
P_t : \left( \Omega^\bullet_t^{[0,1]}(M, F), dF \right) \to \left( C^\bullet(W^u, F), \partial \right),
\]
which induces the canonical isomorphism
\[
H^\bullet(M, F) \simeq H^\bullet(\Omega^\bullet_t^{[0,1]}(M, F), dF) \simeq H^\bullet(C^\bullet(W^u, F), \partial) \hspace{1cm} (3.8)
\]
and such that
\[
P_t e_t = e_t^N \left( \frac{t}{\pi} \right)^{n/4-N/2} \left( 1 + o(1) \right) \hspace{1cm} (3.9)
\]
Here \( e_t^N \left( \frac{t}{\pi} \right)^{n/4-N/2} \) denotes the operator on \( C^\bullet(W^u, F) \) acting on \( C^i(W^u, F) \) by multiplication by \( e^{ti} \left( \frac{t}{\pi} \right)^{n/4-i/2} \).

It follows from (3.9), that, for \( t > 0 \) large enough, \( P_t \) is one to one.
From (3.7), (3.9) we get

\[(3.10)\quad P_t P_t^* = e^{2t N} \left( \frac{t}{\pi} \right)^{n/2-N} \left( 1 + o(1) \right).\]

Fix \( \sigma \in H^i(M, F) \) (0 \( \leq i \leq n \)) and let \( \omega_t \in \text{Ker} \Delta_t \) be the harmonic form representing \( \sigma \).

Let \( \Pi : C^\bullet(W^u, F) \rightarrow \text{Ker} \partial \) be the orthogonal projection. Then \( \Pi P_t \omega_t \in C^i(W^u, F) \) corresponds to \( \sigma \) via the canonical isomorphisms (1.4), (3.8).

Obviously,

\[(3.11)\quad P_t \omega_t \in \text{Ker} \partial, \quad e^{2t \left( \frac{t}{\pi} \right)^{n/2-i}} \left( P_t^* \right)^{-1} \omega_t \in \text{Ker} \partial^*.\]

By (3.10), we get

\[(3.12)\quad \left\| \Pi P_t \omega_t \right\|_{C^\bullet(W^u, F)} = \left\| P_t \omega_t \right\|_{C^\bullet(W^u, F)} \left( 1 + o(1) \right).\]

From (3.10), (3.12) we obtain

\[(3.13)\quad \left\| \Pi P_t \omega_t \right\|_{C^\bullet(W^u, F)} = e^{t \left( \frac{t}{\pi} \right)^{n/4-i/2}} \left\| \omega_t \right\|_{\Omega^\bullet(M, F), t} \left( 1 + o(t) \right).\]

It follows from (3.13) and from the definitions of the metrics \( \cdot \left| \cdot \right|_{\det H^\bullet(M, F), t}, \cdot \left| \cdot \right|_{\det H^\bullet(M, F), t}^{RS} \) that

\[(3.14)\quad \log \left( \frac{\left| \cdot \right|_{\det H^\bullet(M, F), t}}{\left| \cdot \right|_{\det H^\bullet(M, F), t}^{RS}(t)} \right) = t \chi'(F) + \left( \frac{n}{4} \chi(F) - \frac{1}{2} \chi'(F) \right) \log \left( \frac{t}{\pi} \right) + o(1).\]

From (3.6), (3.14) we get

\[(3.15)\quad \log \rho^{RS}(t) = -\frac{1}{2} \int_M \theta(F, g F)(\nabla f)^* \psi(TM, \nabla^{TM}) - t \text{rk}(F) \int_M f e(TM, \nabla^{TM}) + \log \rho^M + t \chi'(F) + \left( \frac{n}{4} \chi(F) - \frac{1}{2} \chi'(F) \right) \log \left( \frac{t}{\pi} \right) + o(1).\]

The proof of Theorem 3.1 is completed.

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