On marginal deformation of WZNW model and PP-wave limit of deformed $AdS_3 \times S^3$ string geometry

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Abstract
We discuss the Penrose limit of the classical string geometry obtained from a truly marginal deformation of $SL(2) \otimes SU(2)$ WZNW model.

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The PP-wave geometry and exactly solvable string theory in the corresponding backgrounds have received growing attention after appearance of the article of BMN, in which the authors proposed some holographic description. This new holographic description was based on the fact that PP-waves can be obtained as a Penrose limit of different supersymmetric AdS\(_p\) × S\(_{10-\ p}\) geometries of IIB and M-theories and therefore inherit the holographic properties of the latter. On the other hand, strings in PP-waves can be quantized exactly, as in flat space at least in the light-cone gauge, and thus in this case of the string/gauge theory dualities one can work with the full string spectrum.

Another example of the exactly quantized and holographic string is the string in the AdS\(_3\) × S\(_3\) × M\(_4\) background, the exact quantization of which is connected with the well known equivalence between the string in the AdS\(_3\) × S\(_3\) background and the SL(2, R) × SU(2) Wess-Zumino-Novikov-Witten model, and the holographic conjecture allows to construct the boundary conformal algebra from the string world sheet objects. The Penrose limit of the AdS\(_3\) × S\(_3\) string, quantization in this case and some holographic properties were investigated recently in [12, 13, 14].

In this note we study a deformed AdS\(_3\) × S\(_3\) background using a truly marginal deformation of the WZNW model for the SL(2) ⊗ SU(2) group manifold given by the operator \(\int d^2 z (J_{sl} \bar{J}_{su} + J_{su} \bar{J}_{sl})\) which is bilinear in corresponding U(1) currents of the Cartan subalgebra. This type of deformation for the SU(2) and SU(2) ⊗ SU(2) WZNW models was considered in [16], [17], and in [18] for the case of SL(2) in the null direction. The main idea of this consideration at that time was to use O(d, d) transformation of the compactified string (WZNW model) to generate a new classical solution of string theory from known ones, because this transformation can be obtained from the marginally deformed WZNW model and therefore will maintain world sheet conformal invariance. Some deformation of the SL(2) × SU(2) WZNW model was used in [19] for the description of the five-dimensional non-extreme rotating black holes with large charges. We will consider a truly marginal deformation of the SL(2) ⊗ SU(2) WZNW model using the method of direct solution of the corresponding differential equation for deformed action and currents developed in [17] for the SU(2) case. Then we consider the Penrose limit of this deformed string geometry and find the corresponding PP-wave background.

There are different parameterizations of the WZNW model for the group
manifolds $SL(2,R)/AdS_3$ and $SU(2)/S^3$:

$$S(g) = \frac{k}{32\pi} \int_{\partial B} Tr (\partial_\mu g^{-1} \bar{\partial}^\mu g) + \frac{k}{48\pi} \int_B \varepsilon^{ijk} Tr (\partial_i gg^{-1} \partial_j gg^{-1} \partial_k gg^{-1})$$

(1)

First of all we can use Euler angles for both models:

$$g = e^{i\theta_1 \sigma_3} e^{ix \sigma_2} e^{i\theta_2 \sigma_3}$$

(2)

In these coordinates the action (1) for $SU(2)$ is:

$$S[x,\theta] = \frac{k}{2\pi} \int d^2 z \left( \partial x \bar{\partial} x + \partial \theta_1 \bar{\partial} \theta_1 + \partial \theta_2 \bar{\partial} \theta_2 + 2 \cos 2x \partial \theta_2 \bar{\partial} \theta_1 \right).$$

(3)

The action for $SL(2,R)$ is obtained from (3) by replacing $x$ by $iy$ and $k$ by $-k$:

$$S[y,\psi] = \frac{k}{2\pi} \int d^2 z \left( \partial y \bar{\partial} y - \partial \psi_1 \bar{\partial} \psi_1 - \partial \psi_2 \bar{\partial} \psi_2 - 2 \cosh 2y \partial \psi_2 \bar{\partial} \psi_1 \right)$$

(4)

The $AdS_3 \times S^3$ string equipped with the NS-NS antisymmetric field $B_{\mu \nu}$ and constant dilaton $\varphi_0$ is just the combination of these two Lagrangians. The forms of the metric and $B$-field

$$ds^2 = k \left( -\cosh^2 y dt^2 + dy^2 + \sinh^2 y d\psi^2 + \cos^2 x dr^2 + dx^2 + \sin^2 x d\theta^2 \right),$$

$$H = dB = 2k \sinh 2y dy \wedge dt \wedge d\psi + 2k \sin 2x dx \wedge d\tau \wedge d\theta$$

(5)

(6)

can be read off from the action

$$S^{AdS_3 \times S^3}[x, y, \psi, \theta] = \frac{k}{2\pi} \int d^2 z \left( \partial x \bar{\partial} x + \partial \theta_1 \bar{\partial} \theta_1 + \partial \theta_2 \bar{\partial} \theta_2 + 2 \cos 2x \partial \theta_2 \bar{\partial} \theta_1 + \partial y \bar{\partial} y - \partial \psi_1 \bar{\partial} \psi_1 - \partial \psi_2 \bar{\partial} \psi_2 - 2 \cosh 2y \partial \psi_2 \bar{\partial} \psi_1 \right)$$

(7)

Another useful parameterization in the case of $SL(2,R)$ only is the Gauss decomposition parameterization:

$$g = \begin{pmatrix} 1 & 0 \\ \gamma & 1 \end{pmatrix} \begin{pmatrix} e^y & 0 \\ 0 & e^{-y} \end{pmatrix} \begin{pmatrix} 1 & \bar{\gamma} \\ 0 & 1 \end{pmatrix}.$$

This decomposition leads to the action

$$S[y,\gamma] = \frac{k}{2\pi} \int d^2 z (\partial y \bar{\partial} y + e^{2y} \partial \bar{\partial} \gamma).$$
using symmetrization and antisymmetrization of the fields and the following change of variables

\[ \theta_{1,2} = \frac{\tau \pm \theta}{2}, \quad \psi_{1,2} = \frac{t \pm \psi}{2} \]  

(8)

We return to this coordinate system later after deformation of our theory, but for the moment we use expression (7) for the \( AdS^3 \times S^3 \) string because this form of the WZNW action is useful in the following respect. The equations of motion corresponding to the variations of the fields \( \psi_i, \theta_i \) lead directly to the conservation of the following chiral currents

\[ J_{\theta} = \partial \theta_1 + \cos 2x \partial \theta_2, \]  

(9)

\[ \bar{J}_{\theta} = \bar{\partial} \theta_2 + \cos 2x \bar{\partial} \theta_1, \]  

(10)

\[ J_{\psi} = -\partial \psi_1 - \cosh 2y \partial \psi_2, \]  

(11)

\[ \bar{J}_{\psi} = -\bar{\partial} \psi_2 - \cosh 2y \bar{\partial} \psi_1. \]  

(12)

We now consider a minimal deformation with deformation parameter \( \lambda \) of fields and interactions, preserving chiral conserved currents in the following way:

\[ S_{\lambda}^{SL(2) \times SU(2)} = \frac{k}{2\pi} \int d^2 z \left( \partial x \bar{\partial} x + \partial \theta_1^\lambda \bar{\partial} \theta_1^\lambda + \partial \theta_2^\lambda \bar{\partial} \theta_2^\lambda + 2\Sigma_\theta^\lambda(x,y) \partial \theta_2^\lambda \bar{\partial} \theta_1^\lambda \right) \]

+ \[ \partial y \bar{\partial} y - \partial \psi_1^\lambda \bar{\partial} \psi_1^\lambda - \partial \psi_2^\lambda \bar{\partial} \psi_2^\lambda - 2\Sigma_\psi^\lambda(x,y) \partial \psi_2^\lambda \bar{\partial} \psi_1^\lambda \]

+ \[ 2\Sigma_\theta\psi^\lambda(x,y) \partial \theta_2^\lambda \bar{\partial} \psi_1^\lambda + 2\Sigma_\psi\theta^\lambda(x,y) \partial \psi_2^\lambda \bar{\partial} \theta_1^\lambda \]  

(13)

This form of deformed interactions preserves the existence of the conserved chiral currents

\[ J_{\theta}^\lambda = \partial \theta_1^\lambda + \Sigma_\theta^\lambda(x,y) \partial \theta_2^\lambda + \Sigma_\psi^\lambda(x,y) \partial \psi_2^\lambda, \]  

(14)

\[ \bar{J}_{\theta}^\lambda = \bar{\partial} \theta_2^\lambda + \Sigma_\theta^\lambda(x,y) \bar{\partial} \theta_1^\lambda + \Sigma_\psi^\lambda(x,y) \bar{\partial} \psi_1^\lambda, \]  

(15)

\[ J_{\psi}^\lambda = -\partial \psi_1^\lambda - \Sigma_\psi^\lambda(x,y) \partial \psi_2^\lambda + \Sigma_\theta\psi^\lambda(x,y) \partial \theta_2^\lambda, \]  

(16)

\[ \bar{J}_{\psi}^\lambda = -\bar{\partial} \psi_2^\lambda - \Sigma_\psi^\lambda(x,y) \bar{\partial} \psi_1^\lambda + \Sigma_\psi\theta^\lambda(x,y) \bar{\partial} \theta_1^\lambda. \]  

(17)

Here all \( \Sigma \) s are unknown functions of \( \lambda \) and radial parameters \( x, y \), and \( \theta_i^\lambda \) and \( \psi_i^\lambda \) are initial fields \( \theta_i \) and \( \psi_i \) mixed with some general \( \lambda \)-dependent 4 × 4 matrices \( A(\lambda) \):

\[ \begin{pmatrix} \psi_1^\lambda \\ \psi_2^\lambda \\ \theta_1^\lambda \\ \theta_2^\lambda \end{pmatrix} = A(\lambda) \begin{pmatrix} \psi_1 \\ \psi_2 \\ \theta_1 \\ \theta_2 \end{pmatrix} \]  

(18)
\[
\partial_{\lambda} \begin{pmatrix}
\psi_1^\lambda \\
\psi_2^\lambda \\
\theta_1^\lambda \\
\theta_2^\lambda
\end{pmatrix} = \partial_{\lambda} A(\lambda) A^{-1}(\lambda) \begin{pmatrix}
\psi_1^\lambda \\
\psi_2^\lambda \\
\theta_1^\lambda \\
\theta_2^\lambda
\end{pmatrix},
\]  

(19)

We will be interested in the mixed deformation preserving zero value of the beta function. This corresponds to a truly marginal deformation which is bilinear in chiral conserved currents from the Cartan subalgebra of the \(SL(2)\) and \(SU(2)\) algebras. For obtaining the truly marginal deformation we have to solve the equation

\[
\partial_{\lambda} S_{\lambda}^{SL(2)\times SU(2)} = \frac{2k}{2\pi} \int d^2 z \left( J_\theta^\lambda \bar{J}_\psi^\lambda + J_\psi^\lambda \bar{J}_\theta^\lambda \right)
\]

(20)

with the corresponding initial conditions

\[
\begin{align*}
\Sigma_{\theta\theta}^{\lambda=0}(x, y) &= \cos 2x, \\
\Sigma_{\psi\psi}^{\lambda=0}(x, y) &= \cosh 2y, \\
\Sigma_{\theta\psi}^{\lambda=0}(x, y) &= \Sigma_{\psi\theta}^{\lambda=0}(x, y) = 0, \\
A(0) &= \mathbb{I}.
\end{align*}
\]

(21)

Solving (20) we have to fix all unknown \(\Sigma\) s and \(A, B\) matrices also. The substitution of (13) in (20) leads to the following set of differential equations for unknown \(\Sigma\) terms

\[
\begin{align*}
\partial_{\lambda} \Sigma_{\theta\theta} &= \Sigma_{\theta\theta} (\Sigma_{\psi\theta} + \Sigma_{\theta\psi}), \\
\partial_{\lambda} \Sigma_{\psi\psi} &= \Sigma_{\psi\psi} (\Sigma_{\psi\theta} + \Sigma_{\theta\psi}), \\
\partial_{\lambda} \Sigma_{\theta\psi} &= \Sigma_{\theta\psi}^2 - \Sigma_{\theta\theta} \Sigma_{\psi\psi} + 1, \\
\partial_{\lambda} \Sigma_{\psi\theta} &= \Sigma_{\psi\theta}^2 - \Sigma_{\theta\theta} \Sigma_{\psi\psi} + 1,
\end{align*}
\]

(23-26)

with the initial conditions (21) and the following equations for \(\theta_i, \psi_i\) with the initial condition (24):

\[
\begin{align*}
\partial_{\lambda} \psi_1^\lambda &= \theta_2^\lambda, & \partial_{\lambda} \psi_2^\lambda &= \theta_1^\lambda, \\
\partial_{\lambda} \theta_1^\lambda &= -\psi_2^\lambda, & \partial_{\lambda} \theta_2^\lambda &= -\psi_1^\lambda, \\
\psi_1^{\lambda=0} &= \psi_1, & \theta_1^{\lambda=0} &= \theta_1.
\end{align*}
\]

(27)

The solution of the latter is:

\[
\psi_1^\lambda = \psi_1 \cos \lambda + \theta_2 \sin \lambda,
\]

(29)
\[ \psi_2^\lambda = \psi_2 \cos \lambda + \theta_1 \sin \lambda, \quad (30) \]
\[ \theta_1^\lambda = \theta_1 \cos \lambda - \psi_2 \sin \lambda, \quad (31) \]
\[ \theta_2^\lambda = \theta_2 \cos \lambda - \psi_1 \sin \lambda, \quad (32) \]

which is just a rotation in the space of \((\psi_1, \psi_2, \theta_1, \theta_2)\) coordinates.

Then from the system \((23)\) to \((29)\) we can easily derive the condition \(\Sigma_{\psi \theta}^\lambda = \Sigma_{\theta \psi}^\lambda\) and obtain a solvable equation,

\[ \Sigma_{\psi\psi}^\lambda = C \Sigma_{\theta\psi}^\lambda, \quad C = \frac{\cos 2x}{\cosh 2y}, \quad (33) \]
\[ \partial_\lambda M(\lambda) = 2 \Sigma_{\psi \theta}^\lambda, \quad M(\lambda) = \ln \Sigma_{\psi \psi}^\lambda, \quad (34) \]
\[ \frac{1}{2} \partial_\lambda^2 M = \frac{1}{4} (\partial_\lambda M)^2 - C \exp(2M) - 1 = 0. \quad (35) \]

For solving \((35)\) we can define the new system

\[ F(M) = \partial_\lambda M(\lambda), \quad \partial_\lambda^2 M = \frac{1}{2} \partial_M (F^2), \quad (36) \]
\[ \frac{1}{4} \partial_M (F^2 \exp(-M)) = \exp(-M) - C \exp(M). \quad (37) \]

With solutions for \(F\) and \(M\)

\[ F(M) = 2 \sqrt{B \exp(M) - C \exp(2M) - 1}, \quad (38) \]
\[ \lambda + D = \frac{1}{2} \arctan \left[ \frac{B \exp M - 2}{\sqrt{B \exp M - C \exp 2M - 1}} \right]. \quad (39) \]

Here \(B, D\) are the arbitrary \(\lambda\)-independent functions.

Finally using the initial conditions \((21)\) we obtain the solutions

\[ \Sigma_{\psi \psi}^\lambda(x, y) = \frac{\cosh 2y}{\cos^2 \lambda + \sin^2 \lambda \cosh 2y \cos 2x}, \quad (40) \]
\[ \Sigma_{\theta \theta}^\lambda(x, y) = \frac{\cos 2x}{\cos^2 \lambda + \sin^2 \lambda \cosh 2y \cos 2x}, \quad (41) \]
\[ \Sigma_{\psi \phi}^\lambda(x, y) = \frac{\sin \lambda \cos \lambda (1 - \cos 2x \cosh 2y)}{\cos^2 \lambda + \sin^2 \lambda \cosh 2y \cos 2x}. \quad (42) \]

From this solution and from the action \((13)\), comparing with the sigma model expressions

\[ S = \frac{k}{2\pi} \int dz^2 (G_{\mu\nu}(\lambda) + B_{\mu\nu}(\lambda)) \bar{\partial}X^i \partial X^j, \quad (43) \]
\[ \phi(\lambda) = \phi_0 + \frac{1}{2} \ln \frac{\det G(0)}{\det G(\lambda)}, \quad (44) \]
we can find the metric of the new deformed string solution (in coordinates rotated with the angle $\lambda$), $H = dB$ and dilaton fields:

$$
\begin{align*}
    ds^2 &= k \left( dx^2 + dy^2 - d\psi_1^2 - d\psi_2^2 + d\theta_1^2 + d\theta_2^2 + \frac{2}{\Delta} [\cos 2xd\theta_1 d\theta_2 \\
    &\quad - \cosh 2yd\psi_1 d\psi_2 + \sin \lambda \cos \lambda (1 - \cos 2x \cosh 2y) (d\psi_2 d\theta_1 + d\psi_1 d\theta_2)] \right),
    \\
    H &= \frac{2k}{\Delta^2} \left[ 2 \cos^2 \lambda \sinh 2yd\psi_1 \wedge d\psi_2 + 2 \sin^2 \lambda \cosh^2 2y \sin 2xdx \wedge d\psi_1 \wedge d\psi_2 \\
    &\quad + 2 \cos^2 \lambda \sin 2xdx \wedge d\theta_1 \wedge d\theta_2 + 2 \sin^2 \lambda \cos^2 2x \sinh 2yd\psi_1 \wedge d\theta_1 \wedge d\theta_2 \\
    &\quad \cos 2x \sinh 2y \sin 2\lambda dx \wedge (d\psi_1 \wedge d\theta_2 + d\theta_1 \wedge d\psi_2) \\
    &\quad - \cosh 2y \sin 2x \sin 2\lambda dy \wedge (d\psi_1 \wedge d\theta_2 + d\theta_1 \wedge d\psi_2) \right],
    \\
    \phi(\lambda) &= \phi_0 + \ln \Delta, \quad \Delta = \cos^2 \lambda + \sin^2 \lambda \cosh 2y \cos 2x.
\end{align*}
$$

We will be interested in the Penrose limit of this solution. For obtaining this we have to find the appropriate coordinates near the null geodesic. We can find this limit using the following change of the coordinate system:

$$
\begin{align*}
    x &= \frac{r}{\sqrt{k}}, \\
    y &= \frac{\rho}{\sqrt{k}}, \\
    \psi_1 &= \frac{\mu x^+ + \psi \cos \lambda - \theta \sin \lambda}{2}, \\
    \psi_2 &= \frac{\mu x^+ - \psi \cos \lambda + \theta \sin \lambda}{2}, \\
    \theta_1 &= \frac{\mu x^+ - (k\mu)^{-1}x^- + \theta \cos \lambda + \psi \sin \lambda}{2}, \\
    \theta_2 &= \frac{\mu x^+ - (k\mu)^{-1}x^- - \theta \cos \lambda - \psi \sin \lambda}{2}.
\end{align*}
$$

and take $k \to \infty$. As a result we obtain the following PP-wave solutions for supergravity fields:

$$
\begin{align*}
    ds^2 &= -2dx^+ dx^- - \mu^2 \left( r^2 + \rho^2 - \sin 2\lambda \left[ r^2 - \rho^2 \right] \right) dx^{+2} + d\rho^2 + d\tau^2, \\
    H &= 4\mu (\cos \lambda + \sin \lambda) \rho d\rho \wedge dx^+ \wedge d\psi + 4\mu (\cos \lambda - \sin \lambda) r dr \wedge dx^+ \wedge d\theta, \\
    \phi(\lambda) &= \phi_0, \quad d\rho^2 = d\rho^2 + \rho^2 d\psi^2, \quad d\tau^2 = dr^2 + r^2 d\theta^2.
\end{align*}
$$

Therefore we obtain in the Penrose limit again the quadratic metric and constant fluxes but now we have some additional deformation parameter $\lambda$ leading to different masses and fluxes

$$
\mu^2_{\pm} = \mu^2 (1 \pm \sin 2\lambda) = \mu^2 (\cos \lambda \pm \sin \lambda)^2
$$
for transversal modes of the corresponding light-cone string lying in two dimensional planes coming from $AdS_3$ and from $S^3$ parts of the initial manifold. Indeed in the point $\lambda = 0$ we obtain the usual PP-wave Penrose limit of $AdS_3 \times S^3$ but for the $\lambda = \pm \frac{\pi}{4}$ we have separation of the flat 2d Euclidian space, i.e.:

$$ds^2 = -2dx^+dx^- - 2\mu^2 \rho^2 dx^{+2} + d\bar{\rho}^2 + d\vec{r}^2,$$

$$H = 4\sqrt{2}\mu d\rho \wedge dx^+ \wedge d\psi.$$ (55)

Thus we can deduce that the Penrose limit of our deformed string geometry connects the two PP-wave plane times flat plane points with the usual 6d PP-wave coming from the $AdS_3 \times S^3$ geometry. It is clear that this deformation leads to the deformation of the spectrum of the light-cone string oscillators due to the different masses of different modes and different fluxes of the NS-NS field in different directions. The dual description of this solution from the point of view of deformed CFT states of holographic theory needs further investigation and will be considered in a forthcoming publication.

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