Flavorful Supersymmetry from Higher Dimensions

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Abstract

We present models of flavorful supersymmetry in higher dimensions. The Higgs fields and the supersymmetry breaking field are localized in the same place in the extra dimension(s). The Yukawa couplings and operators generating the supersymmetry breaking parameters then receive the same suppression factors from the wavefunction profiles of the matter fields, leading to a specific correlation between these two classes of interactions. The resulting phenomenology is very rich, while stringent experimental constraints from the low-energy flavor and CP violating processes can all be satisfied. We construct both unified and non-unified models in this framework, which can be either strongly or weakly coupled at the cutoff scale. We analyze one version in detail, a strongly coupled unified model, which addresses various issues of supersymmetric grand unification. The models presented here provide an explicit example in which the supersymmetry breaking spectrum can be a direct window into the physics of flavor at a very high energy scale.
1 Introduction

One of the longstanding puzzles of the standard model is the distinct pattern of masses and mixings of the quarks and leptons. While supersymmetry addresses many of the other mysteries of the standard model, including the instability of the electroweak scale and the lack of a dark matter candidate, it is not clear if and how supersymmetry helps us understand the flavor puzzle of the standard model at a deeper level. Recently, it has been pointed out that the supersymmetry breaking parameters can exhibit nontrivial flavor structure, and that measurement of these parameters at the LHC can give insight into the flavor sector of the standard model [1, 2]. In particular, it has been shown in Ref. [2] that the class of models called flavorful supersymmetry, in which the supersymmetry breaking parameters receive similar suppressions to those of the Yukawa couplings, can evade all the current experimental bounds and have very distinct signatures at the LHC. In this paper we present explicit models of flavorful supersymmetry.

In this paper we construct models in higher dimensional spacetime where supersymmetry breaking and the Higgs fields reside in the same location in the extra dimension(s). This provides a simple way to realize the necessary correlation between the structures of the supersymmetry breaking parameters and the Yukawa couplings [3, 2]. To preserve the successful prediction for supersymmetric gauge coupling unification, we take the size of the extra dimension(s) to be of order the unification scale. The hierarchical structure for the Yukawa couplings is generated by wavefunction overlaps of the matter and Higgs fields [4], and the correlation between flavor and supersymmetry breaking is obtained by relating the location of the Higgs and supersymmetry breaking fields in the extra dimension(s). Models along similar lines were considered previously in Ref. [5], where flavor violation in the supersymmetry breaking masses is induced by finite gauge loop corrections across the bulk. Here we consider models in which matter fields interact directly with the supersymmetry breaking field, giving the simplest scaling for flavorful effects in the supersymmetry breaking parameters.\footnote{Flavor violation in higher dimensional supersymmetric models was also discussed in different contexts, see [6, 7].}

While not necessary, the extra dimension(s) with size of order the unification scale can also be used to address various issues of supersymmetric grand unified theories. Grand unification in higher dimensions provides an elegant framework for constructing a simple and realistic model of unification [8, 9]. It naturally achieves doublet-triplet splitting in the Higgs sector and suppresses dangerous proton decay operators, while preserving successful gauge coupling unification. Realistic quark and lepton masses and mixings are also accommodated by placing matter fields in the bulk of higher dimensional spacetime [9, 10, 11]. We thus first construct a grand unified model of flavorful supersymmetry which can successfully address these issues. In this model
we also adopt the assumption of strong coupling at the cutoff scale motivated by the simplest understanding of gauge coupling unification in higher dimensions [12, 13], although this is not a necessity to realize flavorful supersymmetry.

There are a variety of ways to incorporate supersymmetry breaking in the present setup. An important constraint on the flavorful supersymmetry framework is that superpotential operators leading to the supersymmetry breaking scalar trilinear interactions must be somewhat suppressed, unless the superparticles are relatively heavy. While it is possible that this suppression arises accidentally or from physics above the cutoff scale, we mainly consider the case where the suppression is due to a symmetry under which the supersymmetry breaking field is charged. This symmetry can also be responsible for a complete solution to the $\mu$ problem, the problem of the supersymmetric Higgs mass term (the $\mu$ term) being of order the weak scale and not some large mass scale. This leads to a scenario similar to the one discussed in Refs. [14, 15], in which the $\mu$ term arises from a cutoff suppressed operator [16] while the gaugino and sfermion masses are generated by gauge mediation [17, 18]. The present setup, however, also leads to flavor violating squark and slepton masses that are correlated with the Yukawa couplings, characterizing flavorful supersymmetry.

We stress that only the extra dimension(s) and the field configuration therein are essential for a realization of flavorful supersymmetry. All the other ingredients, including grand unification, strong coupling, and the particular way of mediating supersymmetry breaking, are not important. While the model described above provides an explicit example of flavorful supersymmetry in which many of the issues of supersymmetric unification are addressed in a relatively simple setup, it is straightforward to eliminate some of the ingredients or to extend the model to accommodate more elaborate structures. In particular, we explicitly discuss a construction in which the theory is weakly coupled at the cutoff scale, which can be straightforwardly applied to models with various spacetime dimensions or gauge groups.

The organization of the paper is as follows. In the next section we present a unified model of flavorful supersymmetry with the assumption that the theory is strongly coupled at the cutoff scale. We explain how the relevant correlation between the Yukawa couplings and supersymmetry breaking parameters is obtained. Phenomenology of the model is studied in section 3, including constraints from low-energy processes, the superparticle spectrum, and experimental signatures. In section 4 we construct a model in warped space, which allows us to obtain a picture of realizing flavorful supersymmetry in a 4D setup, through the AdS/CFT correspondence. In section 5 we present a weakly coupled, non-unified model, which does not possess a symmetry under which the supersymmetry breaking field is charged. Extensions to larger gauge groups or higher dimensions are also discussed. Finally, conclusions are given in section 6.
2 Model

In this section we present a unified, strongly coupled model. We adopt the simplest setup, $SU(5)$ in 5D, to illustrate the basic idea. Extensions to other cases such as larger gauge groups and/or higher dimensions are straightforward. It is also easy to reduce the model to a non-unified model in which the gauge group in 5D is the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$.

2.1 $SU(5)$ grand unification in 5D

We consider a supersymmetric $SU(5)$ gauge theory in 5D flat spacetime with the extra dimension compactified on an $S^1/Z_2$ orbifold: $0 \leq y \leq \pi R$, where $y$ represents the coordinate of the extra dimension [8, 9]. Under 4D $N = 1$ supersymmetry, the 5D gauge supermultiplet is decomposed into a vector superfield $V(A_\mu, \lambda)$ and a chiral superfield $\Sigma(\sigma + iA_5, \lambda')$, where both $V$ and $\Sigma$ are in the adjoint representation of $SU(5)$. We impose the following boundary conditions on these fields:

\begin{align}
V : & \begin{pmatrix}
(+, +) & (+, +) & (+, +) & (+, -) & (+, -) \\
(+, +) & (+, +) & (+, +) & (+, -) & (+, -) \\
(+, -) & (+, -) & (+, -) & (+, +) & (+, +) \\
(-, -) & (-, -) & (-, -) & (-, +) & (-, +) \\
(-, +) & (-, +) & (-, +) & (-, -) & (-, -)
\end{pmatrix}, \\
\Sigma : & \begin{pmatrix}
(+, +) & (+, +) & (+, +) & (+, -) & (+, -) \\
(+, +) & (+, +) & (+, +) & (+, -) & (+, -) \\
(+, -) & (+, -) & (+, -) & (+, +) & (+, +) \\
(-, -) & (-, -) & (-, -) & (-, +) & (-, +) \\
(-, +) & (-, +) & (-, +) & (-, -) & (-, -)
\end{pmatrix},
\end{align}

where + and − represent Neumann and Dirichlet boundary conditions, and the first and second signs in parentheses represent boundary conditions at $y = 0$ and $y = \pi R$, respectively. This reduces the gauge symmetry at $y = \pi R$ to $SU(3) \times SU(2) \times U(1)$, which we identify with the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ (321). The zero-mode sector contains only the 321 component of $V$, $V^{321}$, which is identified with the gauge multiplet of the minimal supersymmetric standard model (MSSM).

The Higgs fields are introduced in the bulk as two hypermultiplets transforming as the fundamental representation of $SU(5)$. Using notation where a hypermultiplet is represented by two 4D $N = 1$ chiral superfields $\Phi(\phi, \psi)$ and $\Phi^c(\phi^c, \psi^c)$ with opposite gauge transformation properties, our two Higgs hypermultiplets can be written as $\{H, H^c\}$ and $\{\bar{H}, \bar{H}^c\}$, where $H$ and $\bar{H}^c$ transform as 5 and $\bar{H}$ and $H^c$ transform as 5* under $SU(5)$. The boundary conditions are
given by

\[ H(5) = H_T(3, 1)_{-1/3}^{(+,-)} \oplus H_D(1, 2)_{1/2}^{(+,+)} \],
\[ H^c(5^*) = H_T^c(3^*, 1)_{1/3}^{(-,+)} \oplus H_D^c(1, 2)_{-1/2}^{(-,-)} \]

for \{H, H^c\}, and similarly for \{\bar{H}, \bar{H}^c\}. Here, the right-hand-side shows the decomposition of \(H\) and \(H^c\) into representations of \(321\) (with \(U(1)_Y\) normalized conventionally), together with the boundary conditions imposed on each component. The zero modes consist of the \(SU(2)_L\)-doublet components of \(H\) and \(\bar{H}\), and are identified with the two Higgs doublets of the MSSM, \(H_a\) and \(H_d\).

Matter fields are also introduced in the bulk. To have a complete generation, we introduce three hypermultiplets transforming as \(10\), \(\{T, T^c\}\), \(\{T', T'^c\}\), \(\{T'', T''^c\}\), two transforming as \(5^*, \{F, F^c\}\), and \(\{F', F'^c\}\), and one transforming as \(1\), \(\{O, O^c\}\), for each generation. The boundary conditions are given by

\[ T(10) = T_Q(3, 2)_{1/6}^{(+,+)} \oplus T_U(3^*, 1)_{-2/3}^{(-,-)} \oplus T_E(1, 1)_{1}^{(+,+)} \],
\[ T'(10) = T_Q'(3, 2)_{1/6}^{(+,-)} \oplus T_U'(3^*, 1)_{-2/3}^{(-,+)} \oplus T_E'(1, 1)_{1}^{(+,-)} \],
\[ T''(10) = T_Q''(3, 2)_{1/6}^{(-,+)} \oplus T_U''(3^*, 1)_{-2/3}^{(-,-)} \oplus T_E''(1, 1)_{1}^{(+,+)} \],
\[ F(5^*) = F_D(3^*, 1)_{1/3}^{(+,+)} \oplus F_L'(1, 2)_{-1/2}^{(+,-)} \],
\[ F'(5^*) = F_D'(3^*, 1)_{1/3}^{(+,-)} \oplus F_L'(1, 2)_{-1/2}^{(+,+)} \],
\[ O(1) = O_N(1, 1)_{0}^{(+,+)} \].

The boundary conditions for the conjugated fields are given by \(+ \leftrightarrow -\), as in Eqs. (3, 4). With these boundary conditions, the zero modes arise only from \(T_Q, T_U', T_E'', F_D, F_L\) and \(O_N\), which we identify with a single generation of quark and lepton superfields of the MSSM (together with the right-handed neutrino), \(Q, U, E, D, L\) and \(N\).

There are two important scales in the theory: the cutoff scale \(M_\star\) and the compactification scale \(1/R\). We take the ratio of these scales to be \(\pi R M_\star \approx 16 \pi^2 / g^2 C \approx O(10 – 100)\), where \(g\) is the 4D gauge coupling at the unification scale, \(g = O(1)\), and \(C \approx 5\) is the group theoretical factor for \(SU(5)\). This makes the theory strongly coupled at \(M_\star\), suppressing calculable threshold corrections to gauge coupling unification [12, 13].

Motivated by successful gauge coupling

\(^2\)It is possible to extract both \(U\) and \(E\) from a single hypermultiplet \(\{T', T'^c\}\) by adopting the boundary conditions \(T'(10) = T_Q'(3, 2)_{1/6}^{(+,-)} \oplus T_U'(3^*, 1)_{-2/3}^{(-,+)} \oplus T_E'(1, 1)_{1}^{(+,+)}\), in which case we do not introduce the hypermultiplet \(\{T'', T''^c\}\). In fact, this is what we obtain if we naively apply the orbifolding procedure to the matter hypermultiplets. The model also works in this case, with the extra constraint of \(M_{U'} = M_E\) (see section 2.2) and \(g_Q = g_L\) (see section 2.3).

\(^3\)Our estimate on the strong coupling scale is conservative. It is possible that \(M_\star R\) can be larger by a factor of \(\approx \pi\), but it does not affect our results.
unification at about $10^{16}$ GeV in supersymmetric models, we take the cutoff scale and the scale of the extra dimension to be

$$M_* \approx 10^{17} \text{ GeV}, \quad 1/\pi R \approx 10^{15} \text{ GeV}.$$  \hfill (11)

More detailed discussions on gauge coupling unification will be given in section 3.4.

### 2.2 Quark and lepton masses and mixings

With the boundary conditions given in the previous subsection, the matter content of the theory below $1/R$ reduces to that of the MSSM and right-handed neutrinos: $V^{321}$, $H_u$, $H_d$, $Q_i$, $U_i$, $D_i$, $L_i$, $E_i$ and $N_i$, where $i = 1, 2, 3$ is the generation index. The Yukawa couplings for the quarks and leptons are introduced on the $y = 0$ and $y = \pi R$ branes. The sizes of the 4D Yukawa couplings are then determined by the wavefunction values of the matter and Higgs fields on these branes. This can be used to generate the observed hierarchy of quark and lepton masses and mixings [4, 5, 11]. Here we consider particular configurations of these fields, relevant to our framework.

A nontrivial wavefunction profile for a zero mode can be generated by a bulk mass term. A bulk hypermultiplet $\{\Phi, \Phi^c\}$ can generally have a mass term in the bulk, which is written as

$$S = \int d^4x \int_0^{\pi R} dy \int d^2\theta M_\Phi \Phi \Phi^c + \text{h.c.},$$ \hfill (12)

in the basis where the kinetic term is given by $S_{\text{kin}} = \int d^4x \int dy \left[ \int d^4\theta (\Phi^\dagger \Phi + \Phi^c \Phi^c) + \int d^2\theta \Phi^c \partial_y \Phi + \text{h.c.} \right]$ [19]. The wavefunction of a zero mode arising from $\Phi$ is proportional to $e^{-M_\Phi y}$, so that it is localized to the $y = 0$ ($y = \pi R$) brane for $M_\Phi > 0$ ($< 0$), and flat for $M_\Phi = 0$. (The $\Phi^c$ case is the same with $M_\Phi \rightarrow -M_\Phi$.) In the present model, we have a bulk mass for each of the Higgs and matter hypermultiplets. For clarity of notation, we specify these masses by the subscript representing the corresponding zero mode: $M_{H_u}$, $M_{H_d}$, $M_{Q_i}$, $M_{U_i}$, $M_{D_i}$, $M_{L_i}$, $M_{E_i}$ and $M_{N_i}$.

We mainly consider the case that the two Higgs doublets $H_u$ and $H_d$ are strongly localized to the $y = \pi R$ brane:

$$M_{H_u}, M_{H_d} \ll -\frac{1}{R}.$$ \hfill (13)

The relevant Yukawa couplings are then those on the $y = \pi R$ brane

\[
S = \int d^4x \int_0^{\pi R} dy \delta(y - \pi R) \int d^2\theta \left\{ (\lambda_u)_{ij} T_{Q_i} T'_{U_j} H_D + (\lambda_d)_{ij} T_{Q_i} T'_{D_j} \bar{H}_D + (\lambda_e)_{ij} T'_{L_i} T''_{E_j} \bar{H}_D + (\lambda_\nu)_{ij} T'_{L_i} O_{N_j} H_D \right\} + \text{h.c.},
\]  \hfill (14)
where the sizes of the couplings are naturally given by \((\lambda_u)_{ij}, (\lambda_d)_{ij}, (\lambda_e)_{ij}, (\lambda_{\nu})_{ij} \approx 4\pi/M_5^{3/2}\) using naive dimensional analysis [20, 12]. This leads to the low-energy 4D Yukawa couplings

\[
W = (y_u)_{ij}Q_iU_jH_u + (y_d)_{ij}Q_iD_jH_d + (y_e)_{ij}L_iE_jH_d + (y_{\nu})_{ij}L_iN_jH_u,
\]

with

\[
(y_u)_{ij} \approx 4\pi \epsilon_{Q_i} \epsilon_{U_j}, \quad (y_d)_{ij} \approx 4\pi \epsilon_{Q_i} \epsilon_{D_j}, \quad (y_e)_{ij} \approx 4\pi \epsilon_{L_i} \epsilon_{E_j}, \quad (y_{\nu})_{ij} \approx 4\pi \epsilon_{L_i} \epsilon_{N_j},
\]

where the factors \(\epsilon_{\Phi} (\Phi = Q_i, U_i, D_i, L_i, E_i, N_i)\) are given by

\[
\epsilon_{\Phi} = \sqrt{\frac{2M_\Phi}{(1 - e^{-2\pi R M_\Phi}) M_*}} e^{-\pi R M_\Phi} \approx \left\{ \begin{array}{ll}
\sqrt{\frac{2M_\Phi}{M_*}} e^{-\pi R M_\Phi} & \text{for } \pi R M_\Phi \gtrsim 1 \\
\sqrt{\frac{2M_\Phi}{M_*}} & \text{for } |\pi R M_\Phi| \ll 1 \\
\sqrt{\frac{2M_\Phi}{M_*}} e^{-\pi R M_\Phi} & \text{for } \pi R M_\Phi \lesssim -1
\end{array} \right.
\]

Realistic Yukawa couplings are obtained by localizing lighter generations more towards the \(y = 0\) brane so that their wavefunction overlaps with the Higgs fields are more suppressed. For example, we can take

\[
\begin{align*}
\epsilon_{Q_1} & \approx \tilde{y}^{-1/2} \epsilon^2, \\
\epsilon_{Q_2} & \approx \tilde{y}^{-1} \epsilon, \\
\epsilon_{Q_3} & \approx \tilde{y}^{-1/2}, \quad \epsilon_{E_1} \approx \tilde{y}^{-1/2} \epsilon^2, \\
\epsilon_{E_2} & \approx \tilde{y}^{-1} \epsilon, \\
\epsilon_{E_3} & \approx \tilde{y}^{-1/2},
\end{align*}
\]

and

\[
\tan \beta \equiv \frac{\langle H_u \rangle}{\langle H_d \rangle} \approx \epsilon^{-1},
\]

where \(\epsilon \sim O(0.1)\) and \(\tilde{y} \approx 4\pi \approx 1/\epsilon\), to reproduce the gross structure of the observed quark and lepton masses and mixings. The suppression factors of Eq. (18) are obtained by taking bulk masses

\[
M_{Q_3, U_3, E_3} \approx -\frac{1}{R}, \quad M_{Q_2, U_2, D_i, L_i, E_2} \approx \frac{0.5 - 1}{R}, \quad M_{Q_1, U_1, E_1} \approx \frac{1.5}{R}.
\]

Small neutrino masses are obtained through the seesaw mechanism by introducing Majorana masses for the right-handed neutrinos on the \(y = \pi R\) brane

\[
S = \int d^4x \int_0^{\pi R} dy \, \delta(y - \pi R) \int d^2 \theta \frac{(M_N)_{ij}}{2M_*} O_{N_i} O_{N_j} + \text{h.c.}
\]

The values of \(\epsilon_{N_i}\) are then not relevant to the low-energy masses and mixings (unless \(N_i\)'s are localized to the \(y = 0\) brane extremely strongly), since they cancel out in the expression for the light neutrino masses.

The localization of various fields in the extra dimension with the bulk masses of Eqs. (13, 20) is depicted schematically in Fig. 1. The quark and lepton masses and mixings are given by
Figure 1: A schematic depiction of the localization for various fields. Here, $X$ represents the supersymmetry breaking field (see section 2.3).

\[
\begin{align*}
(m_t, m_c, m_u) & \approx v (1, \epsilon^2, \epsilon^4), \\
(m_b, m_s, m_d) & \approx v (\epsilon^2, \epsilon^3, \epsilon^4), \\
(m_{\tau}, m_{\mu}, m_{\epsilon}) & \approx v (\epsilon^2, \epsilon^3, \epsilon^4), \\
(m_{\nu_{\tau}}, m_{\nu_{\mu}}, m_{\nu_{\epsilon}}) & \approx \frac{v^2}{M_N} (1, 1, 1),
\end{align*}
\]

and

\[
\begin{align*}
V_{\text{CKM}} & \approx \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \\
V_{\text{MNS}} & \approx \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},
\end{align*}
\]

where $O(1)$ factors are omitted from each element, and $V_{\text{CKM}}$ and $V_{\text{MNS}}$ are the quark and lepton mixing matrices, respectively. This reproduces the gross structure of the observed quark and lepton masses and mixings [21].

The matter configuration considered here can be extended easily to account for the more detailed pattern of the observed masses and mixings. For example, we can localize $L_1$ slightly more towards the $y = 0$ brane to explain the smallness of the $e3$ element of $V_{\text{MNS}}$, which is experimentally smaller than about 0.2. The other elements of $V_{\text{CKM}}$ and $V_{\text{MNS}}$, as well as the mass eigenvalues, can also be better fitted by choosing the bulk masses more carefully. Here we simply adopt Eq. (20) (and its variations, discussed in section 3.1) for the purpose of illustrating the general idea.

There are also variations on the location of the Higgs fields. For example, we can localize
the two Higgs doublets on the \( y = 0 \) brane, instead of the \( y = \pi R \) brane: \( M_{H_u}, M_{H_d} \gtrsim 1/\pi R \). In this case, the localization should not be very strong so that their colored-triplet partners, whose masses are given by \( \approx 2M_{H_u}e^{-\pi RM_{H_u}} \) and \( 2M_{H_d}e^{-\pi RM_{H_d}} \), do not become too light. The location of the matter fields can simply be flipped with respect to \( y = \pi R/2 \): \( M_\Phi \rightarrow -M_\Phi \) for \( \Phi = Q_i, U_i, D_i, L_i, E_i, N_i \). Another possibility is to (slightly) delocalize \( H_u \) and/or \( H_d \) from the brane. In this paper, we focus on the case of Eq. (13), where \( H_u \) and \( H_d \) are strongly localized to the \( y = \pi R \) brane.

### 2.3 \( \mu \) term, \( U(1)_H \), and flavorful supersymmetry

In order to have a complete solution to the doublet-triplet splitting problem, a possible large mass term for the Higgs doublets on the \( y = \pi R \) brane, \( \delta (y - \pi R) \int d^2 \theta H_D \bar{H}_D \), must be forbidden by some symmetry. Moreover, to understand the weak scale size of the mass term (\( \mu \) term) for the Higgs doublets, the breaking of this symmetry must be associated with supersymmetry breaking. One possibility to implement this idea is to consider a \( U(1)_R \) symmetry under which the two Higgs doublets are neutral \([9]\). Here we consider the case that the symmetry is a non-\( R \) symmetry.

We consider that the bare \( \mu \) term, \( \int d^2 \theta H_u H_d \), is forbidden, but the effective \( \mu \) term is generated by the operator \( \int d^4 \theta X^+ H_u H_d \) through supersymmetry breaking, where \( X \) is a supersymmetry breaking field \([16]\). We then find that the relevant symmetry is \( U(1) \) (a Peccei-Quinn symmetry) whose charge assignment can be taken, without loss of generality, as

\[
Q_i(q_Q), \quad U_i(-1 - q_Q), \quad D_i(-1 - q_Q), \quad L_i(q_L), \quad E_i(-1 - q_L), \quad N_i(-1 - q_L),
\]

\[
H_u(1), \quad H_d(1), \quad X(2),
\]

where \( q_Q \) and \( q_L \) are real numbers, and we have assumed that the Yukawa couplings are invariant under the symmetry. In the context of the 5D theory, this assignment can be implemented by considering \( U(1) \) charges for a hypermultiplet \( \{\Phi, \Phi^c\} \) (\( \Phi = H, H', T_i, T_i', T_i'', F_i, F_i', O_i \)) such that the charge of \( \Phi \) follows that of the corresponding zero mode, while the charge of \( \Phi^c \) is the opposite to that of \( \Phi \). This \( U(1) \) symmetry commutes with 5D supersymmetry. The \( X \) field is introduced on the \( y = \pi R \) brane, either as a brane field or a bulk field whose zero mode is strongly localized to the \( y = \pi R \) brane by a bulk mass term \( M_X \ll -1/R \) (see Fig. 1).

The \( U(1) \) symmetry of Eqs. (24, 25), which we call \( U(1)_H \), has several immediate virtues. First of all, the most general interactions between the Higgs and \( X \) fields, located on the \( y = \pi R \) brane, leads (up to the quadratic order in \( X \)) to the following interactions in 4D:

\[
\mathcal{L} \approx \int d^4 \theta \left[ \left( \frac{1}{\Lambda} X^+ H_u H_d + \text{h.c.} \right) + \frac{1}{\Lambda^2} X^+ X H_u^+ H_u + \frac{1}{\Lambda^2} X^+ X H_d^+ H_d \right],
\]

(26)
where we have used naive dimensional analysis to estimate the sizes of various coefficients, and omitted an $O(1)$ factor in each term. The mass scale $\Lambda$ is defined by

$$\Lambda \equiv \frac{M_*}{4\pi} \approx 10^{16} \text{ GeV},$$

(27)

where we have used Eq. (11). After supersymmetry is broken by the $F$-term vacuum expectation value (VEV), $F_X$, of the $X$ field (see the next subsection), these interactions lead to the $\mu$ term and soft supersymmetry breaking masses for the Higgs fields of order $F_X/\Lambda$ at the scale $M_*:

$$\mu \approx \frac{F_X}{\Lambda}, \quad m_{H_u}^2 \approx m_{H_d}^2 \approx \left(\frac{F_X}{\Lambda}\right)^2.$$ (28)

(Note that $O(1)$ coefficients are omitted in these equations, so that the ratio of $m_{H_u}^2$ to $m_{H_d}^2$, for example, can be an arbitrary $O(1)$ number.) An important point here is that the operator $\mathcal{L} \approx \int d^4\theta \left( X^\dagger X H_u H_d / \Lambda^2 + \text{h.c.} \right)$ is prohibited by $U(1)_H$, so that the holomorphic supersymmetry breaking mass-squared for the Higgs doublets ($B\mu$ term) is not generated at order ($F_X/\Lambda$)$^2$ at tree level.\(^4\) The low-energy value of the $B\mu$ term is then generated by contributions from the gaugino masses through renormalization group evolution. This is crucial to avoid the supersymmetric $CP$ problem, since for weak scale superparticle masses an arbitrary relative phase between the $\mu$ and $B\mu$ terms leads to an unacceptably large electric dipole moment for the electron.

Another important implication of $U(1)_H$ is that possible $y = \pi R$ brane operators $\delta(y - \pi R) \int d^4\theta \left( X T_{Q_i} T_{U_j} H_D + X T_{Q_i} F_{D_j} \tilde{H}_D + X F_{L_i} T_{E_j} H_D + X F_{L_i} O_{N_j} H_D \right) + \text{h.c.}$, which reduce in 4D to $\int d^4\theta \left( X Q_i U_j H_u + X Q_i D_j H_d + X L_i E_j H_d + X L_i N_j H_u \right) + \text{h.c.}$, are forbidden. If these operators were present, they would lead to supersymmetry breaking scalar trilinear interactions ($A$ terms) of order $(a_u)_{ij} \approx 4\pi \epsilon_{Q_i} \epsilon_{U_j} (F_X/\Lambda)$, $(a_d)_{ij} \approx 4\pi \epsilon_{Q_i} \epsilon_{D_j} (F_X/\Lambda)$, $(a_e)_{ij} \approx 4\pi \epsilon_{L_i} \epsilon_{E_j} (F_X/\Lambda)$ and $(a_\nu)_{ij} \approx 4\pi \epsilon_{L_i} \epsilon_{N_j} (F_X/\Lambda)$, which are not necessarily proportional to the corresponding Yukawa matrices in flavor space. Here, $(a_f)_{ij}$ ($f = u, d, e, \nu$) are defined by $\mathcal{L}_{\text{soft}} = -(a_u)_{ij} \tilde{q}_{ij} h_u - (a_d)_{ij} \tilde{q}_{ij} d h_d - (a_e)_{ij} \tilde{e}_{ij} h_e - (a_\nu)_{ij} \tilde{\nu}_{ij} h_\nu + \text{h.c.}$ While these terms are suppressed by $\epsilon$ factors, they still provide sizable contributions to low-energy flavor violating processes, because an $A$-term insertion flips the chirality of the sfermion and thus eliminates one factor of the Yukawa coupling from an amplitude. We then find that with $O(1)$ coefficients, the rate for $\mu \to e\gamma$ is expected to be larger than the experimental upper bound by a couple of orders of magnitude for weak scale superparticle masses \cite{2, 3}. This problem does not arise in the present model.

The interactions between the matter and $X$ fields relevant to soft supersymmetry breaking parameters take the form $\delta(y - \pi R) \int d^4\theta \left( X^\dagger X T_{Q_i} T_{U_j} + X^\dagger X T_{U_i} T_{U_j} + X^\dagger X F_{D_i} F_{D_j} + X^\dagger X F_{L_i} F_{L_j} + \right.$

\footnote{There are contributions to the $B\mu$ term of order $F_X^2 (X)/\Lambda^3$ and $F_X m_{3/2}/\Lambda$, where $m_{3/2}$ is the gravitino mass, coming from operators $\mathcal{L} \approx \int d^4\theta \left( X^\dagger X H_u H_d / \Lambda^3 + \text{h.c.} \right)$ and the supergravity effects of the first term of Eq. (26), respectively. These contributions are, however, negligibly small, since $(X)/\Lambda \approx \Lambda / M_{Pl} \ll 1$ and $m_{3/2} \approx F_X / M_{Pl} \ll F_X / \Lambda$, where $M_{Pl} \approx 2 \times 10^{18}$ GeV is the reduced Planck scale (see section 2.4).}
$X^\dagger XT^\mu_{E_i} T^\nu_{E_j} + X^\dagger XO^\dagger_{N_i} O_{N_j}$, which reduce in 4D to

$$\mathcal{L} \approx \int d^4\theta \sum_{\Phi} \sum_{i,j} \frac{\epsilon_{\Phi_i} \epsilon_{\Phi_j}}{\Lambda^2} X^\dagger X \Phi_i \Phi_j,$$

(29)

where $\Phi = Q, U, D, L, E, N$. This leads to the following supersymmetry breaking squared masses for the squarks and sleptons at the scale $M_*:

\begin{align}
(m^2_{\tilde{q}})_{ij} &\approx \epsilon_{Q_i} \epsilon_{Q_j} \left( \frac{F_X}{\Lambda} \right)^2, \\
(m^2_{\tilde{u}})_{ij} &\approx \epsilon_{U_i} \epsilon_{U_j} \left( \frac{F_X}{\Lambda} \right)^2, \\
(m^2_{\tilde{d}})_{ij} &\approx \epsilon_{D_i} \epsilon_{D_j} \left( \frac{F_X}{\Lambda} \right)^2, \\
(m^2_{\tilde{l}})_{ij} &\approx \epsilon_{L_i} \epsilon_{L_j} \left( \frac{F_X}{\Lambda} \right)^2, \\
(m^2_{\tilde{e}})_{ij} &\approx \epsilon_{E_i} \epsilon_{E_j} \left( \frac{F_X}{\Lambda} \right)^2,
\end{align}

(30)

where we have omitted supersymmetry breaking masses for the right-handed sneutrinos, which are not relevant for low-energy phenomenology. Through Eq. (16), these masses are related to the Yukawa couplings — lighter generation scalars receive only small contributions, while heavier generation scalars can receive sizable ones. This is exactly the pattern needed to realize the flavorful supersymmetry scenario, which arises here from the fact that the Higgs and supersymmetry breaking fields reside in the same location in the extra dimension.

As shown in Ref. [2], the existence of flavor non-universal contributions of Eqs. (30, 31) does not contradict the low-energy data on flavor or $CP$ violating processes for wide parameter regions. Since the masses of Eqs. (30, 31) are highly flavor non-universal, they cannot be the dominant contribution to the soft masses (except possibly for some of the third generation sfermions), and we need an extra flavor universal contribution as well as the gaugino masses. These are generated in the present model by gauge mediation, as discussed in the next subsection.

Finally, the $U(1)_H$ symmetry forbids any superpotential term involving only the $X$ field. Since breaking supersymmetry requires a linear $X$ term in the superpotential, this implies that supersymmetry is not broken unless $U(1)_H$ is broken, providing a solid relation between breaking of supersymmetry and that of $U(1)_H$.

### 2.4 Supersymmetry breaking and the low-energy spectrum

To induce supersymmetry breaking VEV $F_X$, we need a linear term of $X$ in the superpotential. This implies that $U(1)_H$ must be broken either explicitly or spontaneously. Here we simply parameterize the effect of $U(1)_H$ breaking in the $X$ potential by a dimensionless chiral spurious parameter $\eta$, which we assume to have the $U(1)_H$ charge of $-2$. The resulting physics does not depend much on the underlying origin of this breaking.

The most general low-energy 4D interactions of $X$ consistent with the (broken) $U(1)_H$ symmetry is given by the following Kähler potential and superpotential:

$$K \approx X^\dagger X - \frac{1}{4\Lambda^2} (X^\dagger X)^2 + \cdots,$$

(32)
\[ W \approx c + \mu_X^2 X + \frac{\mu_X^4}{4\pi \Lambda^3} X^2 + \frac{\mu_X^6}{(4\pi)^2 \Lambda^6} X^3 + \cdots, \] (33)

where \( c \) is a constant term in the superpotential, needed to cancel the cosmological constant, and \( \mu_X^2 \equiv 4\pi \eta \Lambda^2 \). Here, again, we have used naive dimensional analysis to estimate the sizes of various coefficients (except for the \( c \) term), and omitted an \( O(1) \) factor in each term.\(^5\) Note that the terms in Eqs. (32, 33) arise from operators localized on the \( y = \pi R \) brane, except for the \( c \) term which can have contributions from other sources as well.

The scalar potential arising from Eqs. (32, 33) can be minimized in supergravity. Assuming that the coefficient of the \((X^\dagger X)^2/\Lambda^2\) term in the Kähler potential is negative, the minimum of \( X \) is given by the competition between the \( X \) mass term arising from \( V \approx (\mu_X^2/\Lambda^2)|X|^2 \subset |\partial W/\partial X|^2(\partial^2 K/\partial X \partial X)^{-1} \) and the linear term \( V \approx -2\mu_X^2 c(X + X^\dagger)/M_{Pl}^2 \) arising in supergravity. The constant \( c \) is determined to cancel the vacuum energy \( V \approx |\partial W/\partial X|^2 - 3|W|^2/M_{Pl}^2 \) as \( c \approx \mu_X^2 M_{Pl}/\sqrt{3} \). This, therefore, leads to the following supersymmetry breaking minimum

\[ \langle X \rangle \approx \frac{2\Lambda^2}{\sqrt{3} M_{Pl}} \approx 10^{14} \text{ GeV}, \quad F_X \approx \mu_X^2, \quad (34) \]

with the mass-squared for the \( X \) excitation given by \( m_X^2 \approx \mu_X^2/\Lambda^2 \). Note that the \( X \) VEV, \( \langle X \rangle \approx 10^{14} \text{ GeV} \), is smaller than the compactification scale, \( 1/\pi R \approx 10^{16} \text{ GeV} \), so that the 4D analysis of the potential minimization is justified. In fact, with \( \mu_X \) much smaller than \( \langle X \rangle \) to reproduce the weak scale superparticle masses (see Eqs. (28, 30, 31) and below), the only relevant terms in the potential minimization are the first two terms of Eqs. (32) and (33).

The supersymmetry breaking of Eq. (34) can be transmitted to the MSSM gauginos and scalars by gauge mediation by coupling \( X \) to the messenger fields \( f \) and \( \bar{f} \): \( W = \lambda X f \bar{f} \) [22]. The minimum of \( X \) in Eq. (34) is not destabilized as long as the coupling \( \lambda \) is sufficiently small, \( \lambda^2 n_f/16\pi^2 \lesssim (\Lambda/M_{Pl})^2 \), where \( n_f \) is the number of components for the messenger fields. We introduce the messenger fields in the bulk as hypermultiplets: \( \{ f, f^c \} \) and \( \{ \bar{f}, \bar{f}^c \} \). The boundary conditions are given by

\[ f(5) = f_D(3, 1)_{-1/3}^{(+1)} \oplus f_L(1, 2)_{1/2}^{(+1)}, \quad (35) \]

\[ f^c(5^*) = f_D^c(3^*, 1)_{1/3}^{(-1)} \oplus f_L^c(1, 2)_{-1/2}^{(-1)}, \quad (36) \]

and similarly for \( \{ \bar{f}, \bar{f}^c \} \), leading to the zero modes from \( f_D, f_L, \bar{f}_D \) and \( \bar{f}_L \). Here, we have chosen the messenger fields to be a pair of \( 5 + 5^* \), for simplicity, but they can in general be an arbitrary number of pairs of arbitrary \( SU(5) \) representations (as long as they do not make the standard

\(^5\)The most general insertions of the spurious parameter \( \eta \) allows us to write down the tree-level \( \mu \) term in the superpotential, with \( \mu \approx 4\pi \eta \Lambda \approx \mu_X^2/\Lambda \). This contribution is the same order as the one in Eq. (28); see Eq. (34).
model gauge couplings strong at or below $\sim 1/R$). The messenger fields have interactions to $X$ on the $y = \pi R$ brane:

$$S = \int d^4x \int_0^{\pi R} dy \, \delta(y - \pi R) \int d^2\theta (\eta_D X f_D \bar{f}_D + \eta_L X f_L \bar{f}_L) + \text{h.c.},$$

(37)

where the couplings $\eta_D$ and $\eta_L$ are of order $4\pi/M_\ast (4\pi/M_\ast^{3/2})$ from naive dimensional analysis if $X$ is a $y = \pi R$ brane (bulk) field. This determines the $U(1)_H$ charges of the $f = f_D + f_L$ and $\bar{f} = \bar{f}_D + \bar{f}_L$ fields such that the sum of the $f$ and $\bar{f}$ charges is $-2$. (The $f^c$ and $\bar{f}^c$ fields have the opposite charges to $f$ and $\bar{f}$, respectively.)

The messenger multiplets in general have the bulk mass terms of the form of Eq. (12), $M_f$ and $M_{\bar{f}}$. The interactions of Eq. (37) then lead to the 4D superpotential

$$W = \lambda_D X f_D \bar{f}_D + \lambda_L X f_L \bar{f}_L,$$

(38)

where $f_D, f_L, \bar{f}_D$ and $\bar{f}_L$ represent the zero-mode chiral superfields, and

$$\lambda_D \approx \lambda_L \approx 4\pi \epsilon_f \epsilon_f,$$

(39)

where $\epsilon_f, \epsilon_\bar{f}$ are given by Eq. (17) with $\Phi = f, \bar{f}$. The stability condition for the potential is $\lambda_D^2 n_f/16\pi^2 \lesssim (\Lambda/M_\ast)^2 \approx 10^{-4}$, which can be easily satisfied, for example, by taking $M_f, M_\bar{f} \gtrsim 1/\pi R$, i.e., $f_D, f_L, \bar{f}_D$ and $\bar{f}_L$ localized towards the $y = 0$ brane. At the scale

$$M_\text{mess} \approx \lambda_D (X) \approx \frac{\lambda_D \Lambda^2}{M_\text{Pl}},$$

(40)

the messenger fields are integrated out, generating the gauge-mediated contributions to the MSSM gaugino and scalar masses [17, 18]:

$$M_a = N_\text{mess} \frac{g_a^2}{16\pi^2} \frac{F_X}{\langle X \rangle}, \quad m_{\bar{f}}^2 = 2N_\text{mess} \sum_a C_a^\bar{f} \left( \frac{g_a^2}{16\pi^2} \right)^2 \left| \frac{F_X}{\langle X \rangle} \right|^2,$$

(41)

where $a = 1, 2, 3$ represents the standard model gauge group factors, $g_a$ are the standard model gauge couplings at $M_\text{mess}$, $\bar{f} = q, u, d, \tilde{e}, H_u, H_d$, and $C_a^\bar{f}$ are the quadratic Casimir coefficients.

The supersymmetry breaking parameters and the $\mu$ parameter in our theory receive contributions of Eqs. (28, 30, 31) generated at the scale $M_\ast$ and those of Eq. (41) generated at the scale $M_\text{mess}$. The low-energy superparticle masses are then obtained by evolving the parameters of Eqs. (28, 30, 31) from $M_\ast$ to $M_\text{mess}$, adding the contributions of Eq. (41) at $M_\text{mess}$, and then evolving the resulting parameters from $M_\text{mess}$ down to the weak scale. Because of the wavefunction suppression factors $\epsilon_f, \epsilon_\bar{f}$, which are exponentially sensitive to the bulk masses $M_f, \bar{f}$, the value of $M_\text{mess}$ can in general be anywhere between $\approx 100$ TeV and $O(0.1)\langle X \rangle \approx 10^{13}$ GeV. Here,
the upper bound comes from the stability condition on \( \lambda_{D,L} \), while the lower bound from the messenger stability. Note that since the gauge-mediated contributions of Eq. (41) have the size

\[
M_a \approx (m_f^2)^{1/2} \approx \frac{F_X}{\Lambda} \left( \frac{g^2}{16\pi^2} \frac{M_{Pl}}{\Lambda} \right) \approx \frac{F_X}{\Lambda},
\]

where \( g \) represents the standard model gauge couplings, they are comparable to the tree-level contributions to the Higgs-sector parameters of Eq. (28).\(^6\) On the other hand, the flavor non-universal contributions of Eqs. (30, 31) are suppressed due to the \( \epsilon \) factors associated with the quark and lepton superfields (except possibly for the third generation). This therefore reproduces precisely the pattern for the low-energy supersymmetry breaking masses in flavorful supersymmetry.

The model also has other flavor violating contributions to the supersymmetry breaking parameters, but they are all small. For example, loops of the higher dimensional gauge and messenger fields produce flavor violating scalar squared masses at \( 1/R \), but they are of order

\[
N_{\text{mess}}(g^2/16\pi^2)^2 |F_X/\Lambda|^2 \approx (\langle X \rangle/\Lambda)^2 m_f^2
\]

and thus small. The \( y = 0 \) brane Kähler potential operators connecting the matter (and messenger) fields, e.g. \( \delta(y) \int d^4 \theta T_i^T T_j T_l f^T f \), also generate flavor violating scalar squared masses through loops of the matter (or messenger) fields. Using naive dimensional analysis to estimate the coefficients of the operators, we find that this contribution is at most of order

\[
|F_X/\Lambda|^2 / (\pi R M_*)^5
\]

and negligible. Possible contributions from bulk higher dimension operators are also expected to be small based on similar dimensional arguments. Finally, \( y = 0 \) brane localized kinetic terms, e.g. \( \delta(y) \int d^4 \theta T_i^T T_j \), can introduce flavor violation by giving corrections of order \( 1/M_* R \approx 1/16\pi^2 \) to the kinetic terms of the low energy 4D fields. After canonically normalizing the 4D fields, these corrections affect both the Yukawa couplings and the supersymmetry breaking parameters. Interestingly, however, this does not affect the mass insertion parameters used in section 3.1 at the order of magnitude level. In other words, we can always take the basis for the low energy 4D fields such that the Yukawa couplings and supersymmetry breaking parameters are given by Eqs. (16, 30, 31) at \( M_* \) even in the presence of the general brane kinetic terms.\(^7\) Below, we assume that this basis is taken.

Setting the size of the dominant contributions to the supersymmetry breaking and \( \mu \) parameters to be the weak scale, we obtain

\[
F_X/\Lambda \approx (100 \text{ GeV} - 1 \text{ TeV})\text{ from Eq. (42).}
\]

The value of \( F_X \) is then determined as

\[
\sqrt{F_X} \approx (10^{8.5} - 10^{9.5}) \text{ GeV using Eq. (27).}
\]

This leads to the gravitino mass

\[
m_{3/2} \approx \frac{F_X}{\sqrt{3} M_{Pl}} \approx (0.1 - 10) \text{ GeV},
\]

\( ^6 \)In contrast with the situation discussed in Ref. [14], there is no reason in the present theory that the \( \mu \) term must be suppressed compared with the gauge-mediated contributions. In fact, they are naturally expected to be comparable.

\( ^7 \)In fact, this property persists even if the corrections to the 4D kinetic terms are of order unity.
implying that the gravitino is the lightest supersymmetric particle (LSP). Together with the flavor non-universal contributions of Eqs. (30, 31), this can lead to spectacular signatures at the LHC [2], some of which will be discussed in section 3.5.

2.5 Neutrino masses, R parity, and dimension five proton decay

The $U(1)_H$ charge assignment of Eqs. (24, 25) contains two free parameters $q_Q$ and $q_L$. These parameters can be restricted by imposing various phenomenological requirements [15]. For example, if we require that dangerous dimension-five proton decay operators $W \sim Q_i Q_j Q_k L_l$ and $U_i U_j D_k E_l$ are prohibited by $U(1)_H$, then we obtain the conditions $3q_Q + q_L \neq 0$ and $3q_Q + q_L \neq -4$, respectively. Similarly, if we require that $U(1)_H$ forbids dimension-four $R$-parity violating operators $W \sim L_i H_u$, $Q_i D_j L_k$, $L_i J_j E_l$, and $K \sim L_i H_d$, we obtain $q_Q \neq -1$, $q_L \neq 1$, and $q_L \neq 1$.

An interesting possibility arises if $q_L = 0$. In this case we can have the following superpotential on the $y = \pi R$ brane:

$$S = \int d^4 x \int_0^{\pi R} dy \, \delta(y - \pi R) \int d^2 \theta \frac{\kappa_{ij}}{2} X O_{N_i} O_{N_j} + \text{h.c.},$$

(44)

which, together with the last term of Eq. (14), leads to

$$W = \frac{\kappa_{ij}}{2} X N_i N_j + (y_{\nu})_{ij} L_i L_j H_u,$$

(45)

in the low-energy 4D theory. Using naive dimensional analysis, the couplings $\kappa_{ij}$ and $(y_{\nu})_{ij}$ are given by $\kappa_{ij} \approx 4\pi \epsilon_{N_i} \epsilon_{N_j}$ and $(y_{\nu})_{ij} \approx 4\pi \epsilon_{L_i} \epsilon_{N_j}$. The vacuum of Eq. (34) is not destabilized as long as $\kappa_{ij} < \mathcal{O}(0.1)$, which can be easily satisfied by taking $\epsilon_{N_i}$ to be somewhat small, i.e., by taking $M_{N_i} \gtrsim -1/\pi R$. Small neutrino masses are then generated by the seesaw mechanism through the $X$ VEV of Eq. (34). Note that the $\epsilon_{N_i}$ factors cancel out from the generated neutrino masses:

$$(m_{\nu})_{ij} \approx 4\pi \epsilon_{L_i} \epsilon_{L_j} \frac{\langle H_u \rangle^2}{\langle X \rangle}.$$

(46)

It is interesting that with $\langle X \rangle \approx 10^{14}$ GeV, this is in the right ballpark to explain the experimental data on neutrino oscillations.\(^8\)

It is not necessary to impose all the requirements above for the $U(1)_H$ charge assignment. For example, $R$-parity violating operators can be forbidden simply by imposing matter parity

\(^8\)The interactions of Eq. (45) also generate supersymmetry breaking masses of order $(y_{\nu}^2/16\pi^2) F_X / \langle X \rangle$ for $L_i$ and $H_u$ through loops of $N_i$ ($A$ terms at one loop and non-holomorphic supersymmetry breaking masses at two loops [23]). This effect, however, is small for $y_{\nu} \ll 1$, compared with the contributions of Eqs. (28, 41).
in addition to $U(1)_H$. Nevertheless, it is interesting that one can consider the $U(1)_H$ assignment that satisfies all these requirements. For example, one can adopt

$$ q_Q = \frac{4}{3} + 2n, \quad q_L = 0, \quad (47) $$

where $n$ is an integer. The $U(1)_H$ symmetry is spontaneously broken by the VEV of $X$, but the charge assignment of Eq. (47) leaves a discrete $Z_6$ symmetry after the breaking. The product of $Z_6$ and $U(1)_Y$ contains the (anomalous) $Z_3$ baryon number and (anomaly-free) $Z_2$ matter parity ($R$ parity) as subgroups. This symmetry, therefore, strictly forbids the $R$-parity violating operators, and the lightest supersymmetric particle is absolutely stable.

In the rest of the paper, we assume that the LSP is absolutely stable (although it is not necessarily required by the model). This can be achieved either by choosing the $U(1)_H$ charges so that all the $R$-parity violating operators are forbidden even after the $U(1)_H$ breaking, as is the case for Eq. (47), or simply by imposing matter (or $R$) parity.

### 2.6 Origin of $U(1)_H$ breaking

In section 2.4, we have simply parameterized the effect of (small) $U(1)_H$ breaking by a spurious parameter $\eta \ll 1$. This breaking controls the size of the coefficient $\mu_X^2$ for the $X$ linear term in the superpotential, and thus the size of supersymmetry breaking. There are a variety of possibilities for the origin of the required small breaking. For example, it may simply arise as a result of string theory dynamics at the cutoff scale $M_*$. Here, we discuss two explicit examples for the origin of $U(1)_H$ breaking. The validity of the model as well as its basic phenomenological consequences discussed in section 3 have little dependence on this physics.

The first possibility is that the $U(1)_H$ breaking effect arises from the mixed $U(1)_H$ anomaly with respect to the hidden sector gauge group. The scale $\mu_X$ then arises from dimensional transmutation associated with the hidden sector gauge group. This scenario can be implemented in our higher dimensional framework simply by promoting the model discussed in Refs. [15, 24] to higher dimensions. Specifically, we consider a supersymmetric $SU(5)_{\text{hid}} \times SU(5)$ gauge theory on 5D flat spacetime, where the latter $SU(5)$ factor is identified with the unified gauge group, whose gauge multiplet obeys the boundary conditions of Eqs. (1, 2). The Higgs and matter fields are singlet under $SU(5)_{\text{hid}}$, and have the same $SU(5)$ gauge quantum numbers and boundary conditions as in section 2.1. The location for the Higgs, matter and $X$ fields, as well as their $U(1)_H$ charges, are also the same as before.

The messenger fields $\{f, f^c\}$ and $\{\bar{f}, \bar{f}^c\}$ are also introduced in the bulk as before, with the interactions to the $X$ field given by Eq. (37). Instead of introducing arbitrary explicit $U(1)_H$ breaking, however, here we assign the gauge quantum numbers $(5^*, 5)$ to $f$ and $\bar{f}^c$, and
(5, 5′) to $\bar{f}$ and $f^c$, where the numbers in parentheses represent the quantum numbers under $SU(5)_{\text{hid}} \times SU(5)$. Below the compactification scale $\approx 1/\pi R$, this reduces to the model discussed in [15, 24]. In particular, the required $X$ linear term in the superpotential is generated:

$$W_{\text{eff}} = \lambda \Lambda_{\text{hid}}^2 X,$$

(48)

where $\Lambda_{\text{hid}}$ is the dynamical scale of $SU(5)_{\text{hid}}$, and we have taken $\lambda_D \approx \lambda_L \approx \lambda$. Note that this superpotential is “exact,” i.e., no higher order terms in $X$ are generated.

A virtue of the higher dimensional setup in the context of $SU(5)_{\text{hid}} \times SU(5)$ is that the nontrivial wavefunction profiles of $f$ and $\bar{f}$ needed to suppress $\lambda_{D,L}$ (to satisfy the stability condition $\lambda_{D,L}^2 \lesssim 10^{-3}$) also suppress the superpotential coupling $W = \zeta f \bar{f} H_u H_d / \Lambda$ in the low-energy 4D theory, which can arise from the $y = \pi R$ brane localized operator and leads to an unwanted large $\mu$ term unless $\zeta \lesssim \lambda_{D,L}$. Using naive dimensional analysis, we find $\lambda_{D,L} \approx \zeta \approx 4\pi \epsilon_f \epsilon_{\bar{f}}$, so that we do not have a large $\mu$ term from the superpotential operator.

Another possibility for the $U(1)_H$ breaking is that $U(1)_H$ is spontaneously broken. Since $U(1)_H$ has a mixed anomaly with respect to $SU(3)_C$, this provides a solution to the strong CP problem [25]. We do not attempt here to construct a complete model of this kind. It is, however, straightforward to realize this possibility at the level of a non-linear sigma model, i.e. the axion field being realized nonlinearly.

## 3 Phenomenology

In this section we study phenomenology of the model presented in the previous section. We study constraints from flavor and CP violation and the variation of the superparticle spectrum allowed by these constraints. We find that there are a variety of possibilities for the next-to-lightest supersymmetric particle (NLSP), which decays into the LSP gravitino with the lifetime of $O(10^2 - 10^6 \text{ sec})$. We also discuss proton decay, precision gauge coupling unification, and possible experimental signatures.

### 3.1 Constraints from flavor violation and the variety of the spectrum

Phenomenology of the model depends on the wavefunction profiles for the quark and lepton zero modes, which are controlled by the bulk masses for these fields. In the low-energy 4D theory, these affect the Yukawa matrices, Eq. (16), and the flavor violating contribution to the squark and slepton masses generated at $M_*$, Eqs. (30, 31). This effect is parameterized by the factors $\epsilon_{\Phi} (\Phi = Q_i, U_i, D_i, L_i, E_i, N_i)$ in Eq. (17).

The values for the $\epsilon_{\Phi}$ factors are restricted by requiring that the gross structure of the observed quark and lepton masses and mixings are reproduced by these factors. This, however,
still leaves some freedoms for the choice of the $\epsilon_\Phi$ factors. For example, scaling $\{\epsilon_{Q_i}, \epsilon_{U_i}, \epsilon_{D_i}\} \rightarrow \{\alpha \epsilon_{Q_i}, \alpha^{-1} \epsilon_{U_i}, \alpha^{-1} \epsilon_{D_i}\}$ does not change the quark masses and mixings. Taking these freedoms into account, here we consider

$$\begin{align*}
\epsilon_{Q_1} &\approx \tilde{y}_q^{-\frac{1}{2}} \alpha_q \epsilon^2, & \epsilon_{U_1} &\approx \tilde{y}_q^{-\frac{1}{2}} \alpha_q^{-1} \epsilon^2, & \epsilon_{D_1} &\approx \tilde{y}_q^{-\frac{1}{2}} \alpha_q^{-1} \alpha_\beta \epsilon, \\
\epsilon_{Q_2} &\approx \tilde{y}_q^{-\frac{1}{2}} \alpha_q \epsilon, & \epsilon_{U_2} &\approx \tilde{y}_q^{-\frac{1}{2}} \alpha_q^{-1} \epsilon, & \epsilon_{D_2} &\approx \tilde{y}_q^{-\frac{1}{2}} \alpha_q^{-1} \alpha_\beta \epsilon, \\
\epsilon_{Q_3} &\approx \tilde{y}_q^{-\frac{1}{2}} \alpha_q, & \epsilon_{U_3} &\approx \tilde{y}_q^{-\frac{1}{2}} \alpha_q^{-1}, & \epsilon_{D_3} &\approx \tilde{y}_q^{-\frac{1}{2}} \alpha_q^{-1} \alpha_\beta \epsilon,
\end{align*}$$

(49)

where $\epsilon = O(0.1)$ and $\alpha_q$, $\alpha_l$ and $\alpha_\beta$ are numbers parameterizing the freedoms unfixed by the data of the quark and lepton masses and mixings. Note that the range of $\alpha_{q,l,\beta}$ is restricted such that the $\epsilon$ parameters, $\epsilon_{Q_i,U_i,D_i,L_i,E_i}$, do not exceed $\approx 1$; see Eq. (17). (The value of $\alpha_\beta$ is also restricted so that $\tan \beta$ stays within the regime in which none of the Yukawa couplings blow up below the cutoff scale.) The pattern of Eqs. (49, 50) is a straightforward generalization of Eq. (18), and the resulting quark and lepton masses and mixings are still given by Eqs. (22, 23).

The parameters $\alpha_q$, $\alpha_l$ and $\alpha_\beta$, however, alter the size of the flavor violating contribution to the squark and slepton masses, and are thus constrained by low-energy flavor and $CP$ violating processes. We use the mass insertion method [26] to derive constraints on these parameters. The experimental bounds on the mass insertion parameters can be found, e.g., in Ref. [27], and are summarized in Ref. [2]. In the quark sector, the most stringest bounds come from $K-\bar{K}$, $D-\bar{D}$ and $B-\bar{B}$ mixings and $\sin 2\beta$, while in the lepton sector the most stringent one comes from the $\mu \rightarrow e\gamma$ process, giving

$$\begin{align*}
\sqrt{\text{Re}(\delta^d_{12})_{LL/RR}} &\approx (10^{-2} - 10^{-1}), & \sqrt{\text{Re}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}} &\approx 10^{-3}, \\
\sqrt{\text{Im}(\delta^d_{12})_{LL/RR}} &\approx (10^{-3} - 10^{-2}), & \sqrt{\text{Im}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}} &\approx 10^{-4}, \\
|\delta^d_{12}|_L \approx (10^{-2} - 10^{-1}), & |\delta^d_{12}|_L = |\delta^u_{12}|_L \approx (10^{-3} - 10^{-2}), \\
|\delta^d_{13}|_L \approx (0.1 - 1), & |\delta^d_{13}|_L = |\delta^u_{13}|_L \approx 10^{-2},
\end{align*}$$

(52)

$$\begin{align*}
|\delta^e_{12}|_L \approx (10^{-4} - 10^{-3}),
\end{align*}$$

(54)
where we have kept only the bounds relevant to our model. In deriving the above bounds, we have taken the gluino and squark masses to be the same order of magnitude with $m_{\tilde{q}} \simeq 500$ GeV, and the same for the weak gaugino and slepton masses with $m_{\tilde{f}} \simeq 200$ GeV. For heavier superparticles, the bounds become weaker linearly with increasing superparticle masses, except for that on $|\delta_{12}^{e}|_{LL}$, which scales quadratically with $m_{\tilde{f}}$.

In order to compare our model with the above bounds, we need to obtain the structure of the squark and slepton mass matrices at low energies. We first consider the flavor universal contribution. It comes from two different sources. The first is gauge mediation, generated at the scale $M_{\text{mess}}$, while the other is a $U(1)$ Fayet-Iliopoulos $D$-term piece, $\text{Tr}(Y_{f}m_{f}^{2}) \neq 0$, of the soft masses generated at $M_{s}$, Eqs. (28, 30, 31). The sfermion masses at a low energy, $\mu_{R}$, can then be written as

$$m_{f}^{2}(\mu_{R}) \simeq 2N_{\text{mess}} \sum_{a=1}^{3} C_{a}^{f} \frac{g_{a}^{4}(M_{\text{mess}})}{(16\pi^{2})^{2}} \left[ 1 + \frac{N_{\text{mess}}}{b_{a}} \left( 1 - \frac{g_{a}^{4}(\mu_{R})}{g_{3}^{4}(M_{\text{mess}})} \right) \right] \frac{F_{X}^{2}}{\langle X \rangle^{2}}$$

$$- \frac{6Y_{f}}{5} \frac{g_{1}^{2}(\mu_{R})}{16\pi^{2}} \left( x_{H_{u}} - x_{H_{d}} + \frac{x_{Q_{3}}\alpha_{3}}{\tilde{y}} - 2 \frac{x_{U_{3}}}{\tilde{y}_{U}^{2}} + \frac{x_{E_{3}}\alpha_{3}}{\tilde{y}_{E}^{2}} \right) \frac{F_{X}^{2}}{\Lambda^{2}} \ln \frac{M_{s}}{\mu_{R}},$$

where $(b_{1}, b_{2}, b_{3}) = (33/5, 1, -3)$ are the 321 beta-function coefficients, $Y_{f}$ represents hypercharges in the normalization that $Q$ has $Y_{f} = 1/6$, and $x_{H_{u}, H_{d}, Q_{3}, U_{3}, E_{3}}$ are the $O(1)$ factors in front of the corresponding soft masses generated at $M_{s}$. (Here, we have kept only the leading terms in $\epsilon$.) As we will see in section 3.2, the $U(1)$ $D$-term piece can considerably affect the superparticle spectrum, leading to interesting phenomenology.

The flavor violating elements of the sfermion mass matrices are renormalized among themselves, and are also generated from the flavor universal piece through the Yukawa couplings. These effects, however, do not significantly modify the values of these elements in most of the parameter space. We therefore take the approximation that the flavor non-universal part of the sfermion masses is parameterized by Eqs. (30, 31) with Eqs. (49, 50) at low energies. The chirality-preserving mass insertion parameters are then obtained by dividing these flavor violating elements by the (average) diagonal elements in the super-CKM basis.

With the low-energy mass parameters described above, one can study the constraints from flavor and $CP$ violation. The scalar trilinear interactions in our model are generated only by renormalization group evolution, so that they are proportional to the corresponding Yukawa couplings with real proportionality constants, in the basis where the gaugino masses are real. They, therefore, do not contribute to flavor or $CP$ violating processes. The constraints on the $\alpha$ parameters are then obtained from Eqs. (52 – 54). We find that for $\tilde{y} = 4\pi$ and $\epsilon = 0.05$, all

---

A possible contribution to $m_{\tilde{l}}^{2}$ from loops of the right-handed neutrinos is also not important as long as $(y_{\nu})_{ij} \lesssim O(1)$, which is the case for the $\epsilon$ factor assignment of Eq. (50) with $\alpha_{l} \approx O(1)$.  

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constraints from the quark sector are satisfied, while $\mu \rightarrow e\gamma$ gives

$$\alpha_l \lesssim 1.8,$$

(56)

with no further constraints on $\alpha_q$ or $\alpha_\beta$. Taking $\epsilon = 0.1$, the constraints become stronger with both $\mu \rightarrow e\gamma$ and $K-\bar{K}$ mixing, giving

$$\alpha_l \lesssim 0.9, \quad \alpha_\beta \lesssim 1.4.$$  

(57)

These bounds are obtained for the superparticle mass scale of $m_\tilde{q} \sim 400$ GeV, with $F_X/M_\ast \sim 1$ TeV. (This corresponds to $m_\tilde{q} \sim 1.2$ TeV, which is sufficient to avoid the LEP II bound on the physical Higgs boson mass.) While these bounds are rough ones, they show that there exists a consistent parameter region. For heavier superparticles, the bounds become weaker and the region expands.

### 3.2 The NLSP

As we have seen in section 2.4, the LSP is the gravitino with mass $\approx (0.1 - 10)$ GeV. In order to study phenomenology, it is important to determine which particles can be the NLSP. Since the dominant contribution to the masses of most supersymmetric particles comes from gauge mediation, we first consider the spectrum without the corrections from tree-level pieces generated at $M_\ast$. Since the masses are determined by the gauge charge, the lightest particles will be those neutral under $SU(3)_C$ and $SU(2)_L$. Therefore, the lightest gaugino is a neutralino, $\chi_0^1$ which is mostly bino, and the lightest sfermions are the right-handed sleptons. The mass of the bino at low energy is given by

$$m_{\tilde{B}(\mu_R)} \approx N_{\text{mess}} \frac{g_1^2(\mu_R) F_X}{16\pi^2 \langle X \rangle},$$

(58)

while the mass of the sleptons can be derived from Eq. (55). From these two equations we see that with increasing $N_{\text{mess}}$ the sleptons become lighter than the bino, while increasing $M_{\text{mess}}$ makes the sleptons heavier because of renormalization group effects. Calculations show that for $N_{\text{mess}} = 1$ the bino is always the NLSP, while for larger $N_{\text{mess}}$ the sleptons can be lighter. In the case of $N_{\text{mess}} = 3 \ (5)$, for example, the sleptons are lighter than the bino for $M_{\text{mess}} \lesssim 10^{10} \ (10^{12})$ GeV.

The bino mass in the present model is the same as in gauge mediation, but the slepton masses can deviate. As discussed in section 3.1, the sleptons receive the contribution from the $U(1)_Y$ $D$-term, indicated by the second line of Eq. (55). This contribution is flavor universal so it does not affect the splitting among sleptons, but it affects the relation between the sleptons and the bino. The other correction to gauge mediation comes from the tree-level masses in Eqs. (30, 31). From Eqs. (49, 50), we see that these mass terms are $\epsilon$ suppressed for most fields, but the effect
can be $O(1)$ for $\tilde{\tau}_R$, and the unknown coefficient could even be negative as long as the sum of the tree-level and gauge mediated pieces bring the physical mass above direct detection bounds. This means that $\tilde{\tau}_R$ could lie anywhere in the spectrum of $\tilde{e}_R$, $\tilde{\mu}_R$ and $\tilde{B}$.

The splitting between $\tilde{e}_R$ and $\tilde{\mu}_R$ is controlled almost entirely by the splitting at $M_s$ because the renormalization group running is universal up to small effects from the muon Yukawa coupling. Phenomenology is governed by the splitting between mass eigenstates which is given by

$$m_{\tilde{\mu}_R} - m_{\tilde{e}_R} \approx \frac{m_{\tilde{\mu}_R}^2 - m_{\tilde{e}_R}^2}{2\sqrt{m_{\tilde{e}_R,\tilde{\mu}_R}^2}} \approx O(0.01) \left( \frac{\alpha_\beta}{\alpha_l} \right)^2 \left( \frac{\Lambda}{M_{Pl}} \right)^2 \frac{m_{\tilde{B}}^2}{\sqrt{m_{\tilde{e}_R,\tilde{\mu}_R}^2}}.$$  (59)

The splitting between light generation sfermions is much larger than in the usual gauge mediation scenario. It can be large enough that the heavier one can decay to the lighter by emission of an electron and a muon.

There are corners of parameter space where the NLSP is not a right-handed slepton or bino. Since the contribution from the $U(1)_Y$ $D$-term in Eq. (55) has opposite signs for the left-handed and right-handed sleptons, it could invert the usual order between these two species. The lighter stop could also be the NLSP because, like $\tilde{\tau}_R$, it has an $O(1)$ tree-level contribution to its mass. The stops also have a contribution from the large top Yukawa coupling, which decreases the masses through renormalization group evolution. While the tree-level piece is expected to be smaller than the $SU(3)_C$ gauge mediation piece, negative tree-level and Yukawa effects could combine to give a strongly interacting NLSP. We do not consider these exotic NLSPs in the rest of this paper because they require large cancellation between independent effects.

### 3.3 Proton decay

Dimension four proton decay in the present model can be forbidden by the $U(1)_H$ symmetry or matter parity. Dimension five proton decay caused by colored Higgsino exchange is also absent because of the form of the Higgsino mass matrix determined by higher dimensional spacetime symmetry [9]. Proton decay in the present model can thus arise only from dimension six operators and cutoff suppressed dimension five operators.

As discussed in section 2.5, we can take the charge assignment of $U(1)_H$ such that the operators $W \sim Q_iQ_jQ_kL_l$ and $U_iU_jD_kE_l$ are forbidden: $3q_Q + q_L \neq 0, -4$. In this case, dimension five proton decay arises only from operators on the $y = \pi R$ brane which involve the X VEV. The relevant interactions are $W \sim X^mQ_iQ_jQ_kL_l$ and $X^mU_iU_jD_kE_l$, which can be written for $3q_Q + q_L = -2m$ and $3q_Q + q_L = 2m - 4$ ($m \in \mathbb{Z} > 0$), respectively. In the low-energy 4D
effective theory, these interactions lead to dimension five operators

\[ W \approx 4\pi \epsilon_{Q_i} \epsilon_{Q_j} \epsilon_{L_l} \frac{\Lambda^{m-1}}{M_{Pl}^m} Q_i Q_j Q_k L_l \quad \text{and} \quad 4\pi \epsilon_{U_i} \epsilon_{U_j} \epsilon_{D_k} \epsilon_{E_l} \frac{\Lambda^{m-1}}{M_{Pl}^m} U_i U_j D_k E_l , \]

where the coefficients are evaluated using naive dimensional analysis, and we have used \( \langle X \rangle \approx \frac{\Lambda^2}{M_{Pl}} \). We find that the approximate sizes of these operators are obtained by replacing the colored Higgsino mass by \( 4\pi M_{Pl}^m/\Lambda^{m-1} \) in the corresponding expressions in the minimal supersymmetric \( SU(5) \) grand unified theory. The resulting proton decay rate is thus much smaller than the current experimental bound for all the values of \( 3q_Q + q_L \neq 0, -4 \).

Dimension six operators are generated in the present model only through brane localized terms, since without them exchange of bulk gauge bosons does not transform a quark into a lepton or vice versa. (Note that different 321 multiplets arise from different \( SU(5) \) multiplets, see Eqs. (5 – 10).) The relevant terms are kinetic mixing operators \( K \sim T^{\dagger}T', T^{\dagger}T'', F^{\dagger}F' \) and cutoff suppressed dimension six operators \( K \sim T^{\dagger}T'T^{\dagger}T'', T^{\dagger}T'F^{\dagger}F' \) on the \( y = 0 \) brane. Here, we have omitted factors involving the gauge multiplet needed to make operators gauge invariant, and the existence of Hermitian conjugates is implied. The kinetic mixing terms lead, through unified gauge boson exchange, to dimension six operators at low energies, whose coefficients have approximately the size obtained by replacing the unified gauge boson mass by \( 1/\pi R \) in the corresponding minimal supersymmetric \( SU(5) \) expressions. For \( 1/\pi R \approx 10^{15} \) GeV, this leads to a proton decay rate somewhat larger than the current experimental bound \cite{28}. This implies that the compactification scale should be somewhat larger (by a factor of a few) or the coefficients of the original kinetic mixing operators should be suppressed (by an order of magnitude or so). This potential difficulty does not arise in weakly coupled models, an example of which will be discussed in section 5. The coefficients of low-energy dimension six operators arising from the cutoff suppressed operators are similar in size to those in the minimal supersymmetric \( SU(5) \) model, so that they do not lead to proton decay at a dangerous level.

In summary, proton decay in the present model is caused by dimension six operators, originating from terms on the \( y = 0 \) brane. Since the wavefunction values for the first and second generation fields on this brane are typically of the same order, the proton can decay into final states containing \( \mu^+ \) with a similar rate to those containing \( e^+ \). This provides interesting signatures for future proton decay experiments.

3.4 Precision gauge coupling unification

Strongly coupled grand unification in higher dimensions allows a precise calculation for gauge coupling unification \cite{12, 13}. Incalculable corrections arising from the cutoff scale physics are
suppressed, and the corrections from higher dimensional fields between the energy interval between $M_*$ and $1/\pi R$ are precisely calculated. Here we study this issue in the model of section 2.

We phrase the degree of the success of gauge coupling unification in terms of the prediction of $\alpha_s(M_Z) = g_3^2(M_Z)/4\pi$ obtained from $g_{1,2}(M_Z)$, where $g_{1,2,3}$ represent the standard model gauge couplings. In particular, we consider the deviation of the prediction in the present model, $\alpha_s^{5D}$, from that obtained by assuming the exact unification in the MSSM, $\alpha_s^{SGUT,0}$.

$$\delta\alpha_s \equiv \alpha_s^{5D} - \alpha_s^{SGUT,0} \simeq -\frac{1}{2\pi} \alpha_s^2 \Delta. \quad (61)$$

Here, $\Delta$ parameterizes corrections from higher dimensional fields, which can be calculated within higher dimensional effective field theory. Using the result of Ref. [29], we find that in the present model

$$\Delta = -3\ln(\pi RM_*) - 3\ln(\epsilon_{Q_1}\epsilon_{Q_2}\epsilon_{Q_3}) + \frac{15}{7}\ln(\epsilon_{U_1}\epsilon_{U_2}\epsilon_{U_3})$$
$$+ \frac{9}{7}\ln(\epsilon_{D_1}\epsilon_{D_2}\epsilon_{D_3}) - \frac{9}{7}\ln(\epsilon_{L_1}\epsilon_{L_2}\epsilon_{L_3}) + \frac{6}{7}\ln(\epsilon_{E_1}\epsilon_{E_2}\epsilon_{E_3}), \quad (62)$$

where we have used the approximation that the Higgs doublets are strictly localized to the $y = \pi R$ brane. (The term $-(9/7)\ln(\epsilon_{H_u}\epsilon_{H_d})$ should be added to the right-hand-side if the Higgs fields are delocalized.) Inserting Eqs. (49, 50) into this equation, we obtain

$$\Delta = -\ln(\pi RM_*) - \frac{135}{7}\ln\alpha_q - \frac{45}{7}\ln\alpha_l + \frac{45}{7}\ln\alpha_\beta. \quad (63)$$

Considering that the logarithms are expected to be of order unity, we find that $\Delta$ is typically of $O(10)$, with the sign depending on the values of $\alpha_{q,l,\beta}$. For typical superparticle spectra, including the one considered here, a good fit to the experimental values of $g_{1,2,3}(M_Z)$ is obtained for

$$\Delta^{\exp} \approx 5 \pm O(1). \quad (64)$$

The expression in our model, Eq. (63), can easily accommodate this value.

### 3.5 Collider signatures

Phenomenology of the general flavorful supersymmetry scenario has been discussed in Ref. [2]. Here we summarize some of the basic features in the context of the present model. As we saw in section 3.2, this model has a large portion of parameter space where there is a charged NLSP which is stable for the purposes of collider studies. Unlike the conventional scenarios, the NLSP in flavorful supersymmetry could be a $\tilde{T}_R$ or a right-handed slepton of a different flavor. Heavy stable charged particles are relatively easy to see at colliders. By measuring their velocity and
momentum, their mass can be deduced. The mass of the charged NLSP be can measured to better than 1% at the LHC by measuring only a few hundred NLSPs with $0.6 < \beta < 0.91$ [30].

Once the NLSP mass is known, it is possible to fully reconstruct events even in the hadronic environment of the LHC. Therefore we can determine the flavor content of the NLSP by taking its invariant mass with other leptons in the event. If the NLSP is found to be mostly selectron or smuon, this is definitive evidence for nontrivial flavor structure in the supersymmetry breaking sector, and possibly for flavorful supersymmetry. In addition, once we learn the dominant flavor of the NLSP, we can look for NLSP production in association with leptons of other flavors to measure the mixing angles of the NLSP.

Because the lifetime of the NLSP is quite long, it can be studied in a cleaner environment. One proposal involves using the muon tracker to determine where in the surrounding rock an NLSP went, and extracting pieces of rock that likely contain NLSPs to study them elsewhere [31]. Another possibility is to build a large stopper detector outside of one of the main detectors which can stop the NLSPs and then measure the decay products [32]. This would allow precise measurements of the lifetime of the NLSP as well as the masses of the decay products. As pointed out in Ref. [2], a particularly distinct signature of flavorful supersymmetry is monochromatic electrons or muons in the decay of the NLSP, indicating a two body decay of a selectron or smuon. This is not a possibility in the conventional scenarios because the $\tilde{\tau}_R$ is the NLSP, and it decays to a $\tau$ which further decays, so the many body decay causes the leptons to have a broad spectrum. Even if the NLSP is a $\tilde{\tau}_R$, a stopper detector will allow us to look for rare decays into other flavors and precisely measure the flavor content of the NLSP. The stopper detector can also check to see if the LSP is the gravitino. From the kinematics, the mass of the LSP can be measured, which can then be tested against the supergravity prediction which relates the lifetime of the NLSP to the mass of the gravitino [33].

While the signatures are much more spectacular if there is a slepton NLSP, evidence for flavorful supersymmetry can still be found with a neutralino NLSP. One possibility is to look for direct slepton production from Drell-Yan processes and measure the spectrum through kinematic variables such as $M_{T2}$ [34]. This is difficult because it requires high statistics and the Drell-Yan cross section falls rapidly with increasing slepton mass. Another possibility is to look for multiple edges in flavor-tagged dilepton invariant mass distributions as in Ref. [35]. This will allow us to find different flavors of sleptons if they are separated by more than a few GeV, which we would expect in flavorful supersymmetry. Finally, we could also study the spectrum of left-handed sleptons or even squarks to look for flavor non-universality. While these measurements are more difficult than those with stable sleptons, they could still provide information on the flavor structure of the supersymmetry breaking sector.
4 4D Realization — Model in Warped Space

The model in section 2 has been formulated in flat space, but we can also consider a similar model in warped space, along the lines of Ref. [36]. An interesting feature of this model is that it allows for a 4D interpretation through the AdS/CFT correspondence, providing a picture of realizing flavorful supersymmetry in a 4D setup.

Specifically, we take the metric

\[ ds^2 = e^{-2ky_\mu dx^\mu dx^\nu + dy^2} \tag{65} \]

where \( k \) denotes the inverse curvature radius of the warped spacetime. The two branes are located at \( y = 0 \) (the UV brane) and \( y = \pi R \) (the IR brane). The scales of these branes are chosen to be \( k \approx 10^{17} \text{ GeV} \) and \( k' \equiv k e^{-\pi k R} \approx 10^{16} \text{ GeV} \), respectively. The cutoff scale of the 5D theory is taken to be \( M_* \approx 10^{18} \text{ GeV} \). The gauge symmetry structure is as described in section 2; the bulk \( SU(5) \) symmetry is broken to \( 321 \) on the IR brane at \( y = \pi R \). The IR brane thus serves the role of breaking the unified symmetry.

The configuration of the matter and Higgs fields is as described in section 2. The locations of these fields are controlled by the bulk masses, and the resulting Yukawa couplings are given by Eq. (16), where the \( \epsilon \) factors are given by Eq. (17) with \( M_\Phi \rightarrow M_\Phi - k/2 \). The analysis of \( U(1)_H \) and supersymmetry breaking is as in sections 2.3 – 2.6. (Note that the cutoff scale on the IR brane is warped down to \( M_*' \equiv M_* e^{-\pi k R} \approx 10^{17} \text{ GeV} \).) This leads to phenomenology discussed in sections 3.1, 3.2 and 3.5. Dimension four and five proton decay is negligible for the reasons described in section 3.3. Dimension six proton decay is also not dangerous as the unified gauge boson mass is now of order \( \pi k' \approx 10^{16} \text{ GeV} \). For gauge coupling unification, we can show, using the results of [37], that the threshold correction is still given by the formula Eq. (62). (Note that the contribution from the Higgs doublets to differential running shuts off above \( M_*' \), since these fields are localized on the IR brane.) The experimental values of the low-energy gauge couplings are thus successfully reproduced, as seen in section 3.4.

The model described here has the following 4D interpretation through the AdS/CFT correspondence. At very high energies above \( k' \approx 10^{16} \text{ GeV} \), the theory is a 4D supersymmetric \( SU(5) \times G \) gauge theory, where \( SU(5) \) is the unified gauge group and \( G \) some quasi-conformal gauge group. There are three generations of matter fields, \( 3 \times (10 + 5^*) \) of \( SU(5) \) (and possibly three right-handed neutrinos), but not the Higgs fields. There are also fields charged under \( G \), some of which are charged under \( SU(5) \) as well. At the scale \( k' \approx 10^{16} \text{ GeV} \), the \( G \) sector deviates from the conformal fixed point, breaking the unified \( SU(5) \) symmetry to \( 321 \) by the gauge dynamics. It also produces the MSSM Higgs doublets and the supersymmetry breaking sector containing \( X \) as composite states. The effective theory below \( k' \) is thus the MSSM (and possibly three right-handed neutrinos) together with the supersymmetry breaking sector.
An important point is that the interaction strengths of the matter fields to the \( G \) sector are controlled by the dimensions of operators coupling matter to fields charged under \( G \). In general, these dimensions are generation dependent. Moreover, since \( G \) is strongly interacting above \( k' \), the anomalous dimensions for these operators can be large. As a result, the interaction strengths of matter to the \( G \) sector strongly vary between different generations, and since the Higgs doublets and \( X \) arise as composite states of \( G \), the interactions of matter to these states show strong generation dependence. Since the origin of this generation dependence is common for the matter couplings to the Higgs fields (the Yukawa couplings) and to the \( X \) field (supersymmetry breaking couplings), the patterns of these two classes of couplings are correlated. The correlation is exactly the one given in Eqs. (16, 29), realizing flavorful supersymmetry.

We have considered here a 4D theory in which the \( G \) sector is quasi-conformal and has a large 't Hooft coupling above the dynamical scale, motivated by the warped space construction. The dynamics described above, however, are independent of these assumptions. The same dynamics can also be incorporated, in principle, in a purely 4D theory whose 't Hooft coupling is not necessarily large above \( k' \). The quasi-conformal nature of the dynamics is also not essential. It will be interesting to construct an explicit example of purely 4D theory in which the \( G \) sector exhibits different renormalization group behavior, e.g. asymptotic freedom, above the dynamical scale \( \Lambda_G \approx 10^{16} \text{ GeV} \).

5 Weakly Coupled (Non-Unified) Models

In this section we present a non-unified model of flavorful supersymmetry in higher dimensions. Here we do not require that the theory is strongly coupled at the cutoff scale, nor that it possesses the \( U(1)_H \) symmetry. Rather, we assume that certain operators are small at the cutoff scale due to ultraviolet physics.

We consider a supersymmetric \( SU(3)_C \times SU(2)_L \times U(1)_Y \) gauge theory in 5D flat spacetime, compactified on an \( S^1/Z_2 \) orbifold: \( 0 \leq y \leq \pi R \). As in the model of section 2, the two Higgs doublets are localized towards the \( y = \pi R \) brane, where supersymmetry is broken by the \( F \)-term VEV of a chiral superfield \( X \). The matter fields are introduced in the bulk as hypermultiplets, whose zero modes \( Q_i, U_i, D_i, L_i, E_i \) (and \( N_i \)) are identified with the MSSM matter fields. The wavefunction profiles of the zero modes are controlled by the bulk masses \( M_\Phi \) (\( \Phi = Q_i, U_i, D_i, L_i, E_i, N_i \)), as seen in section 2.2.

We do not require that the theory is strongly coupled at the cutoff scale \( M_\ast \), which is taken to be a factor of a few above \( 1/R \). We then naturally expect that the operators located on branes have \( O(1) \) coefficients in units of \( M_\ast \). This leads to the 4D Yukawa couplings of Eq. (15)
with
\[
(y_u)_{ij} \approx \epsilon_Q \epsilon_{U_j}, \quad (y_d)_{ij} \approx \epsilon_Q \epsilon_{D_j}, \quad (y_e)_{ij} \approx \epsilon_L \epsilon_{E_j}, \quad (y_\nu)_{ij} \approx \epsilon_L \epsilon_{N_j},
\]
at low energies, where \(\epsilon_{\Phi}\) are given by Eq. (17). By choosing \(\epsilon_{\Phi}\) and \(\tan \beta\) to be as given in Eqs. (49 – 51) with \(\tilde{y} = 1\), this reproduces the gross structure of the observed quark and lepton masses and mixings, Eqs. (22, 23). The configuration of the matter fields, as well as those of the Higgs and supersymmetry breaking fields, are depicted schematically in Fig. 2.

The supersymmetry breaking parameters are generated through the interactions of the MSSM states to the \(X\) field on the \(y = \pi R\) brane. In the absence of the \(U(1)_H\) symmetry, the superpotential operators \(W \sim XQ_iU_jH_u + XQ_iD_jH_d + XL_iE_jH_d + XL_iN_jH_u\) are not forbidden in general. These operators generate flavor non-universal left-right mixing terms for the squarks and sleptons that require relatively heavy superparticles to avoid the constraints from low-energy flavor and \(CP\) violating processes. Here we assume that these operators are somehow suppressed. We also assume that the direct \(\mu\) term, \(W \sim H_uH_d\), is absent. Note that these assumptions are technically natural because of the nonrenormalization theorem. The supersymmetry breaking parameters are then generated by the Kähler potential operators and \(\mathcal{L} \sim \int d^2 \theta X \mathcal{W}_a \mathcal{W}_a + \text{h.c.},\)

*Here we have assumed that the Majorana masses for \(N_i\) are on the \(y = \pi R\) brane, but not on the \(y = 0\) brane. This can be realized, for example, by introducing the \(U(1)_{B-L}\) symmetry broken on the \(y = \pi R\) brane.*
where $W^a_i (a = 1, 2, 3)$ are the 321 gauge field strength superfields, giving

$$M_a \approx \mu \approx \frac{F_X}{M_*}, \quad m_{H_u}^2 \approx m_{H_d}^2 \approx B\mu \approx \left(\frac{F_X}{M_*}\right)^2,$$

$$m_{\tilde{d}}^2_{ij} \approx \epsilon_{Q_i} \epsilon_{Q_j} \left(\frac{F_X}{M_*}\right)^2, \quad m_{\tilde{u}}^2_{ij} \approx \epsilon_{U_i} \epsilon_{U_j} \left(\frac{F_X}{M_*}\right)^2, \quad m_{\tilde{q}}^2_{ij} \approx \epsilon_{D_i} \epsilon_{D_j} \left(\frac{F_X}{M_*}\right)^2,$$

$$m_{\tilde{E}}^2_{ij} \approx \epsilon_{L_i} \epsilon_{L_j} \left(\frac{F_X}{M_*}\right)^2, \quad m_{\tilde{e}}^2_{ij} \approx \epsilon_{E_i} \epsilon_{E_j} \left(\frac{F_X}{M_*}\right)^2,$$

$$a_{\tilde{u}}_{ij} \approx \{(y_u)_{kj} (\eta_Q)_{ki} + (y_u)_{ik} (\eta_U)_{kj} + (y_u)_{ij}\} \frac{F_X}{M_*},$$

$$a_{\tilde{d}}_{ij} \approx \{(y_d)_{kj} (\eta_Q)_{ki} + (y_d)_{ik} (\eta_D)_{kj} + (y_d)_{ij}\} \frac{F_X}{M_*},$$

$$a_{\tilde{e}}_{ij} \approx \{(y_e)_{kj} (\eta_L)_{ki} + (y_e)_{ik} (\eta_E)_{kj} + (y_e)_{ij}\} \frac{F_X}{M_*}.$$

Here, we have omitted $O(1)$ coefficients in each term, and $(\eta_{\Phi})_{ij} \approx \epsilon_{\Phi_i} \epsilon_{\Phi_j}$ ($\Phi = Q, U, D, L, E$) are general complex $3 \times 3$ matrices. This gives a correlation between the Yukawa couplings Eq. (66), and the supersymmetry breaking parameters Eqs. (67 – 72), realizing flavorful supersymmetry.

Note that because of the absence of a factor $4\pi$ in Eq. (66), the mass splittings between different generation sfermions in Eqs. (68, 69) can be larger than those in the strongly coupled case.

The model has other flavor violating contributions to the supersymmetry breaking parameters, but they can be controlled. For example, loops of the higher dimensional gauge fields produce flavor violating supersymmetry breaking masses at $1/R$, but they are not much larger than the tree-level masses in the parameter region considered, as long as the coefficients of the matter brane kinetic operators at $y = 0$ are of order $1/16\pi^2 M_*$ or smaller. Note that this size of the coefficients is technically natural. The matter 4-point Kähler potential operators on the $y = 0$ brane also give flavor violating contributions at loop level. They are, however, suppressed by a factor of $1/(\pi R M^5)$ and negligible. Possible contributions from bulk higher dimension operators are also expected to be small.

The compactification scale $1/R$ in the present model is naturally of order the unification scale $M_U \approx 10^{16}$ GeV to preserve the successful supersymmetric prediction for the low-energy gauge couplings. In this case, the gaugino and sfermion masses are of order $\tilde{m} \approx F_X/M_U$, while the gravitino mass is $m_{3/2} \approx F_X/M_{Pl}$, so that $m_{3/2} \approx (M_U/M_{Pl}) \tilde{m} \approx (1 - 10)$ GeV, leading to signatures discussed in section 3.5 with the NLSP being one of the right-handed sleptons. The compactification scale, however, can in principle take any value larger than of order a few TeV, in which case the gravitino may be (much) lighter. Note that the supersymmetry breaking parameters of Eqs. (67 – 72) are running parameters evaluated at the scale $1/R$. The
low-energy superparticle spectrum is obtained by evolving them down to the weak scale using renormalization group equations.

Here we have presented a non-unified model of flavorful supersymmetry in 5D. It is, however, straightforward to make it a unified model, e.g., based on $SU(5)$. We simply have to adopt the field content and boundary conditions of section 2.1 and follow the analysis above. To understand gauge coupling unification, we need to assume that incaulculable brane-localized gauge kinetic terms on the $y = \pi R$ brane are somehow suppressed (or universal), but dangerous proton decay can be easily suppressed, possibly by $U(1)_R$ symmetry [9]. It is also straightforward to extend the model to higher dimensions. The only requirement is that the Higgs fields and the supersymmetry breaking field $X$ are localized in the same place in the extra dimensions. An advantage of such a setup is that we can suppress cutoff scale dimension-five proton decay operators by localizing the $Q, U, E$ and $D, L$ fields in different subspaces in higher dimensions. These extensions allow us to realize flavorful supersymmetry in a wide variety of higher dimensional models, with varying spacetime dimensions, compact space geometries, and gauge groups.

6 Conclusions

In this paper we have presented explicit models of flavorful supersymmetry in higher dimensions. The basic idea is to localize the Higgs fields and the supersymmetry breaking field in the same location in the extra dimension(s). The interactions of matter fields to the Higgs fields (the Yukawa couplings) and to the supersymmetry breaking field (operators generating the supersymmetry breaking parameters) then receive the same suppression factors from the wavefunction profiles of the matter fields. This leads to a specific correlation between these two classes of interactions, realizing flavorful supersymmetry. The resulting phenomenology at future colliders is very rich, while stringent experimental constraints from the low-energy flavor and $CP$ violating processes can all be satisfied.

We have constructed a unified model of flavorful supersymmetry in 5D, in which the theory is strongly coupled at the cutoff scale. Supersymmetry breaking is mediated to the supersymmetric standard model sector by a combination of cutoff suppressed operators and gauge mediation. This model addresses various issues in supersymmetric unification. We have also presented a model in warped space, which allows us to obtain a picture of realizing flavorful supersymmetry in a 4D setup, through the AdS/CFT correspondence. Finally, we have discussed models which

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11It is interesting to note that the 321 gaugino masses do not have to be unified at the unification scale even if the model is unified because the gaugino mass operators reside on the $y = \pi R$ brane, where the active gauge group is only 321 [9].

12To be more precise, it is sufficient to require that the matter interactions to the Higgs and $X$ fields are suppressed by common wavefunction factors, allowing the Higgs and $X$ to propagate in different subspaces.
do not require that the theory is strongly coupled at the cutoff scale. This construction can be easily extended to a wide variety of higher dimensional theories, with varying spacetime geometries and gauge groups.

It is interesting to note that the present setup is very generic in the context of a single extra dimension. If we want to explain the observed hierarchical structure of the Yukawa couplings by wavefunction overlaps between the matter and Higgs fields, the simplest way is to localize the Higgs fields to one of the branes and lighter generation matter more towards the other brane. Now, if the supersymmetry breaking field $X$ is not localized to the same brane as the Higgs fields, interactions of lighter generation matter to $X$ are not suppressed, leading to large flavor violating supersymmetry breaking masses. To avoid this problem, we need to localize $X$ to the same brane as the Higgs fields (unless some other flavor universal mediation mechanism dominates). This gives the spectrum of flavorful supersymmetry.

As the LHC will turn on this year, it is important to explore possible theoretical constructions and experimental signatures of supersymmetric theories. The models presented here provide an example in which the supersymmetry breaking spectrum can be a window into the physics of flavor in the standard model. If supersymmetry is discovered at the LHC, it will be interesting to see if the longstanding assumption of flavor universality holds, or if there is a richer flavor structure within the supersymmetry breaking sector. This structure could give us information about the physics of flavor which could lie at energy scales as high as the unification or Planck scale.

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