Computer Aided Evaluation of Fir Tree Blade Root Profile Based on Particle Swarm Algorithm

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Abstract. Aiming at the problems of inefficiency and inaccuracy in the evaluation of fir tree blade root profile, a method of computer aided profile evaluation based on particle swarm algorithm is proposed. The evaluation process is automated by this method, the theoretical profile and the coordinates of the measured points are read automatically, the measured points and the theoretical profile are aligned automatically, the profile deviation is calculated, and whether the blade root is qualified is determined. The mathematical model and calculation method of profile evaluation are given, the measured points and theoretical profile are aligned precisely according to the principle of least square by making use of the particle swarm optimization algorithm with adaptive inertia weight (APSO) after coarse alignment, and the influence of position error between design reference and measurement reference on the results of profile evaluation is eliminated. Simulation experiment and practical application manifest better accuracy and stability can be obtained by using APSO algorithm compared with other algorithms when evaluating the fir tree blade root profile. The computer aided profile evaluation method proposed in this paper could totally replace the traditional manual method.

1. Introduction

With the rapid development of economy and society, steam turbines are widely used in all walks of life. As a component of steam turbine, the blade is the key factor to ensure the reliability of steam turbine operation [1]. The three parts of the blade are blade root, blade profile and blade tip, among them the blade root is the connecting part between the blade and the rim, which can ensure that the blade can be firmly fixed on the impeller under any operating conditions [2]. For different kind of blade roots with the same size, the fir tree blade roots have the highest bearing capacity. The processing technology of this kind of blade root is complex and requires high accuracy. Therefore, it is necessary to evaluate the blade root profile in order to ensure the consistency of the root profile and consistency of the blade’s frequency.

Profile is the most widely used item in the national and international standard of part shape tolerance, but it is difficult to measure and evaluate [3]. The position error caused by the inconsistency of design reference and measurement reference will reduce the accuracy of the evaluation result when measuring profile of a line by CMM, so it is necessary to align the measured points with the theoretical profile in order to reduce this kind of error [4].

Alignment is an unconstrained nonlinear optimization problem. Since the traditional optimization algorithms, such as analytical method and numerical analysis method, take a long time to solve such optimization problems, and have low accuracy, even cannot solve them, stochastic optimization algorithm based on swarm intelligence is put forward continually [5]. Among them, the particle swarm
The algorithm has unique advantages in dealing with complex nonlinear optimization, such as simple principle, easy implementation, few parameters, no need to limit the initial value and high accuracy [3], so it’s widely used.

The existing evaluation method of fir tree blade root profile mainly has the following problems: the evaluation process is manual, and it relies on staff to read the coordinates of the measured points and to align the measured points with the theoretical profile. It takes at least 10 minutes from reading the measured points coordinates and theoretical profile of a fir tree straight blade root profile to evaluating whether the blade root is qualified. The work process is complicated, especially when aligning the measured points and the theoretical profile, so the best alignment is achieved by the staff changing the amount of rotation or translation constantly on the CAD software. Two measured points are selected on each working surface by the manual method and then aligned with the theoretical profile. The problem is that if there are overshoots in other unselected measured points on the working surface, the evaluation result obtained by the manual method is incorrect.

To solve the problems mentioned above, a method of computer aided fir tree blade root profile evaluation based on particle swarm algorithm is proposed in this paper. The evaluation process is automated by this method, the theoretical profile and the coordinates of the measured points are read automatically, the measured points and the theoretical profile are aligned automatically, the profile deviation is calculated, and whether there are any overshoots, that is, whether the blade root is qualified or not is judged, besides, all the measured points on the working surface are taken into account when aligning the measured points with the theoretical profiles and calculating the profile deviation, therefore, the evaluation process is simple, and the accuracy and the efficiency of evaluation are improved.

2. Establishment of evaluation model

2.1 Establish mathematical model of theoretical profile

The theoretical profile of fir tree blade root given by the design drawing is composed of multiple straight lines and arcs, which is difficult to be described with a mathematical expression. Since the qualifications of the blade roots are judged only by the profile of the straight line segment on the working surface, the equations of the straight line segment on all working surfaces are established as the mathematical model of the theoretical profile.

Read the straight line segments on all working surfaces in the design drawing, as shown in the bold line segment in figure 1, and then calculate the slope $k_n$ and the intercept $b_n$ of each line segment. The equation of the theoretical straight line segments on each working surface is shown in formula (3).
2.2 Determine the minimum distance from the measured points to the theoretical profile

The profile deviation of each measured point is the minimum distance from the point to the theoretical profile [6]. Since the same reference is selected as far as possible in the design drawing when measuring parts with CMM, the deviation between the measured and theoretical profiles will not be very large [7]. Here are the concrete steps to determine the minimum distance from the measured point to the theoretical profile:

1. Read coordinates of measured points: when measuring blade roots with CMM, the intersection of symmetrical line of blade roots profile and profile is taken as the origin of coordinate system, and the measured points and theoretical profiles are displayed in the same coordinate system, as shown in figure 2;

2. Coarse alignment: coarse alignment makes all measured points and theoretical profiles basically adapt, and the deviation is controlled within a certain range [8]. As shown in figure 2, all measured points are moved to the intersection of the symmetry line of the theoretical profile and the theoretical profile based on the origin of the coordinate system.

3. Extract all measured points on the working surface: since judging whether the blade roots are qualified only depends on the evaluation of the profile of the straight line segment on the working surface, just all measured points on the working surface can be considered in aligning measured points with theoretical profile and calculating profile deviation. On the premise of small deviation, the measured points on the working surface can be selected according to the X and Y coordinates of the measured points, respectively, between the X and Y coordinates of the starting point and the end point of the theoretical line segment on the working surface.

4. Calculate the minimum distance from all measured points on the working surface to the theoretical profile: calculate the minimum distance from the measured points \((x_i, y_i)\) on each working surface to the theoretical profile, as shown in the formula (4):

\[
d_i = \frac{y_i - k_n x_i - b_n}{\sqrt{k_n^2 + 1}}
\]

Among them, the slope \(k_n\) and the intercept \(b_n\) of each working surface are calculated respectively by formula (1) and formula (2). If the calculated distance is greater than 0, which indicates that the measured point is above the theoretical profile, and if it is less than 0, which indicates that the measured point is below the theoretical profile.

2.3 Establish the mathematical model of evaluation

All measured points after coarse alignment are regarded as points with three degrees of freedom on the plane. The purpose of precise alignment is to complete the containment process of all measured points to the theoretical profile by fine adjustment of all measured points [8]. The best alignment between the theoretical profile and the measured points can be achieved only by translating and rotating coordinate transformation [7]. Its coordinate transformation matrix is:

\[
T = \begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
u & v & 1
\end{bmatrix}
\]
That is to say, the relationship between the new coordinates and the old coordinates of the measured points after translation and rotation transformation is as follows:

\[
\begin{align*}
    x'_i &= x_i \times \cos \theta - y_i \times \sin \theta + u \\
    y'_i &= x_i \times \sin \theta + y_i \times \cos \theta + v
\end{align*}
\]

In the formula, \( u \) and \( v \) are coordinate translations of X axis and Y axis respectively, and \( \theta \) is coordinate rotation.

After coordinate transformation, formula (4) is used to calculate the minimum distance from each measured point on the working surface to the theoretical profile, and the calculation formula is shown in formula (7).

\[
d^i_n = \frac{y'_i - b_k \times x'_i - b_k}{\sqrt{k_n^2 + 1}} = \frac{(x_i \times \sin \theta + y_i \times \cos \theta + v) - k_n \times (x_i \times \cos \theta - y_i \times \sin \theta + u) - b_k}{\sqrt{k_n^2 + 1}}
\]

The principle of line profile evaluation is usually the minimum area [9], but the blade root profile is composed of many straight lines and arcs, so it is difficult to describe by a mathematical expression, and it’s complex and difficult to achieve the minimum condition. Therefore, the least square principle is used to evaluate the line profile, that is, to minimize the sum of the squares of the distance from each measured point on the working surface to the theoretical profile. The objective function established by the least squares principle is as follows:

\[
F = \min \sum_{i=1}^{N} d^2_i
\]

The evaluation model is an unconstrained and non-linear three-dimensional optimization with optimization variables \((u, v, \theta)\).

3. Evaluation method based on particle swarm algorithm

3.1 Basic principles
KENNEDY et al. [10] proposed a swarm intelligence optimization algorithm in 1995, called particle swarm algorithm, which simulates bird population by using the concepts of "population" and "evolution". In the solution space, particle swarm algorithm firstly generates the position and velocity of each particle randomly, and then searches for the optimal solution and optimizes the particle swarm by iterating and updating continuously according to the current global optimum and its own optimum direction [11]. Assuming that the position of the ith particle in the D-dimensional search space is \( X^t = (x^t_1, x^t_2, ..., x^t_D) \), and the velocity is \( V^t = (v^t_1, v^t_2, ..., v^t_D) \), the velocity and position of the particle can be updated in the optimization process in the following way:

\[
\begin{align*}
    v^t(n+1) &= w \times v^t(n) + c_1 \times r_1 \times [p^t \times x^t(n)] + c_2 \times r_2 \times [g^t \times x^t(n)] \\
    x^t(n+1) &= x^t(n) + v^t(n+1)
\end{align*}
\]

Among them, both \( r_1 \) and \( r_2 \) are random number uniformly distributed between 0 and 1. \( w \) is the coefficient to maintain the original speed, so it is called inertia weight factor. \( c_1 \) and \( c_2 \) are learning factor.

Standard particle swarm optimization (PSO) algorithm is prone to fall into local optimum in the early stage of optimization, and presents "premature" phenomenon, and it also reduces the search efficiency due to the loss of population diversity in the later stage of search [12]. \( w \) plays an important role in the convergence of particle swarm algorithm because it can maintain particle motion inertia and determines the trend of enlarging search space. In order to balance the ability of global exploration and local development of particle swarm algorithm and find the optimal solution quickly and accurately, the standard particle swarm optimization algorithm can be changed into the adaptive inertia weight particle swarm optimization (APSO) algorithm by using the adaptive dynamic inertia weight coefficient of formula (10).
Among them, \( w_{\text{min}} \) and \( w_{\text{max}} \) respectively are the minimum and maximum inertia weight, \( f \) is the fitness value of the current particle, \( f_{\text{avg}} \) and \( f_{\text{min}} \) respectively are the average fitness value and the minimum fitness value of all the current particles.

When the fitness value of the particle is better than the average fitness value, the inertia weight factor of the particle is reduced to protect the particle and enhance the local search ability. When the fitness value of the particle is worse than the average fitness value, the inertia weight factor of the particle is increased to make the particle have a better search area and accelerate the global search, which is conducive to approaching the optimal solution [13].

### 3.2 Algorithm steps

The specific steps of adaptive inertia weight particle swarm optimization (APSO) algorithm are as follows:

1. Set parameters: considering the collocation of each parameter, the size of particle swarm is chosen to be 30, \( w_{\text{max}} = 1.2, w_{\text{min}} = 0.2, c_1 = c_2 = 2 \), and the maximum search algebra is 100 on the basis of many experiments.

2. Initialize the velocity and position of particle swarm: velocity and position of initial particle swarm are generated by random number, \( X_i^0 = (u_i^0, v_i^0, \theta_i^0) \) and \( V_i^0 = (V_{u_i}^0, V_{v_i}^0, V_{\theta_i}^0) \), where \( i \) is particle number.

3. Determine the fitness function: the objective function established according to the least square principle is the minimum sum of squares of the minimum distances from each measuring point on the working surface to the theoretical profile corresponding to the optimal particle \((u^*, v^*, \theta^*)\), i.e. formula (8).

4. Store the optimal fitness \( g_{\text{best}} \) of the population and the optimal fitness \( p_{\text{best}} \) of the individual.

5. Update the inertia weight coefficient \( w \) according to formula (10).

6. Update the position and velocity of particles according to formula (9).

7. Update the optimal fitness of individual and population.

8. Judge whether the termination condition is satisfied: the termination condition is that the maximum number of cycles is 100 or the difference between the optimal fitness values of adjacent populations is less than the given threshold value of 0.000001. The search is stopped and the results are output when the termination condition is met, and the search is returned (4) to continue when the termination condition is not met.

Coordinate transformation parameters \((u, v, \theta)\) are obtained after precise alignment, and the minimum distances from each measured point on each working surface to the theoretical profile, i.e. profile deviations, are calculated by formula (4). The profile deviations of all working surfaces whether exceed the specification are compared, and the qualified blade root is determined when not exceeding, and the unqualified blade root is determined when exceeding.

### 3.3 Analysis of simulated experimental results

In order to test the effectiveness of the algorithm, the adaptive inertia weight particle swarm optimization (APSO) algorithm, the linear variable inertia weight particle swarm optimization (LDIW) algorithm [14], the random inertia weight particle swarm optimization (RIW) algorithm [15] and the non-linear decreasing inertia weight particle swarm optimization (NLDIW) algorithm [16] are simulated and compared. In the experiment, different PSO algorithms are set the same parameters. The theoretical profile of the experiment is the blade root profile design drawing. The coordinates of the points on the theoretical profile obtained from the design drawing are the coordinates of the theoretical
points. The coordinates of the points obtained after setting a set of random position errors \((u, v, \theta)\) to the coordinates of the theoretical points are the coordinates of a simulated set of measuring points. The test function is established according to formula (8). In order to reduce the experimental error, 10 groups of random position errors are given to obtain 10 groups of simulated coordinates of measurement points. Finally, 10 test functions \(f_1\)–\(f_{10}\) are obtained, and their theoretical optimum values are all 0. In order to reduce statistical errors, each group of experiments was repeated 30 times, that is, four algorithms were run 30 times respectively for each test function. The comparison results of each algorithm on mean (MEAN) and standard deviation (STD) were shown in table 1, and the best results of each test function were expressed in bold.

|        | LDIW | RIW | NLDIW | APSO |
|--------|------|-----|-------|------|
| \(f_1\) | MEAN | 1.846921 | 0.421235 | 0.027417 | 3.03E-08 |
|        | STD  | 6.477843 | 0.555528 | 0.128993 | 1.35E-23 |
| \(f_6\) | MEAN | 0.800084 | 3.247817 | 0.198267 | 2.66E-08 |
|        | STD  | 1.988631 | 8.714020 | 0.719797 | 6.73E-24 |
| \(f_2\) | MEAN | 3.375443 | 1.659608 | 2.37E-07 | 2.33E-08 |
|        | STD  | 11.56951 | 1.012498 | 7.93E-07 | 1.35E-23 |
| \(f_7\) | MEAN | 2.457351 | 0.725759 | 0.178154 | 2.96E-08 |
|        | STD  | 8.801943 | 1.909158 | 0.972897 | 2.02E-23 |
| \(f_3\) | MEAN | 2.033052 | 2.456432 | 0.066531 | 2.32E-08 |
|        | STD  | 4.406666 | 3.806849 | 0.361025 | 2.65E-10 |
| \(f_8\) | MEAN | 0.819157 | 0.858997 | 0.287741 | 2.87E-08 |
|        | STD  | 1.760255 | 1.029441 | 1.021959 | 1.68E-23 |
| \(f_5\) | MEAN | 2.242007 | 0.656291 | 2.459556 | 2.74E-08 |
|        | STD  | 5.503789 | 0.644185 | 13.13641 | 1.68E-23 |
| \(f_{10}\) | MEAN | 1.862712 | 1.498907 | 0.018181 | 2.82E-08 |
|        | STD  | 5.952021 | 5.073484 | 0.071767 | 0 |

The experimental results in table 1 show that the averages of the APSO algorithm are closer to the theoretical optimal value than those of the other three algorithms in 10 test functions and the standard deviations are smaller than those of the other three algorithms, which manifests better accuracy and stability can be obtained by using APSO algorithm compared with other three algorithms when evaluating the fir tree blade root profile.

4. Application example

In this paper, the fir tree straight blade root profile design drawing of an enterprise is taken as the theoretical profile, and the points coordinates of the blade root section measured by the CMM are taken as the measured points coordinates to evaluate the fir tree blade root profile. Firstly, the theoretical profile and the measured points coordinates are read automatically, and the mathematical model of the theoretical profile is established. Then, all the measured points are coarsely aligned. The maximum profile deviation of all measured points on the working surface is 0.116641mm after coarsely alignment. Finally, the APSO algorithm is used to achieve the fine alignment of the measured points and the theoretical profiles according to the principle of least squares (minimize the sum of the squares of the distances from each measured point to the theoretical profile on the working surface). The maximum profile deviation of all measured points on the working surface obtained after precise alignment is 0.008003mm, which does not exceed specification 0.013mm, and the blade root is judged to be qualified. The position error between the design reference and the measurement reference obtained by this method is (144.154592, 104.415621, 0.000505), and the time from reading the theoretical profile and the measured point coordinates to the end of the evaluation is 10 seconds, which is much less than this of the manual method. In addition, the evaluation method based on particle swarm algorithm takes into account all the measured points on the working surface in the precise alignment, which is more accurate than the manual method only considering two measured points on each working surface. Therefore, the computer aided profile evaluation method proposed in this paper could totally replace the traditional manual method and has high efficiency and accuracy.

5. Conclusion
In this paper, a method for evaluating fir tree blade root profile based on particle swarm algorithm is proposed. The mathematical model and calculation method of profile evaluation are given. Firstly, the theoretical profile and the measured point coordinates are read automatically. Then, coarse alignment and precise alignment are used to align the measured points with the theoretical profile accurately, so as to eliminate the position errors by reference misalignment in the measurement. Finally, the maximum profile deviation of all measured points on the working surface is calculated, and whether the blade root is qualified is determined. Simulation experiment and practical application manifest better accuracy and stability can be obtained by using APSO algorithm compared with other algorithms when evaluating the fir tree blade root profile. The computer aided profile evaluation method proposed in this paper could totally replace the traditional manual method.

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