Hadrosynthesis at SPS and RHIC and the statistical model

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Abstract. An analysis of hadron abundances in heavy ion collisions from SPS to RHIC energy within the statistical-thermal model is presented. Pb–Pb collisions at 40 A GeV are analysed for the first time here. Unlike stated in similar recent studies, the data analysis rules out a complete strange chemical equilibrium in full phase space. In fact, the use of multiplicities integrated in full phase space or in a limited rapidity window at SPS energy gives rise to different results for the extra-strangeness suppression parameter $\gamma_S$ while the extracted values of temperature and baryon-chemical potential do not vary significantly. This behaviour raises the question whether the observed hadronic strangeness phase space saturation at RHIC within a small mid-rapidity window would hold in a possible full phase space analysis.

1. Introduction

The statistical model of hadronisation, in different versions, is being used extensively in heavy ion collisions as a tool to determine the global conditions of the matter at the stage where interactions among hadrons cease (freeze-out). [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The agreement of the model with the data is somehow impressive [11], and its validity has been unexpectedly proved in elementary collisions as well [12, 13, 14], where inelastic rescattering between final hadrons is believed to be negligible. This latter finding has given rise to the interpretation that the hadronisation process itself generates hadrons according to the equiprobability of multi-hadronic phase space states (maximum entropy) [13, 14] and the apparent chemical equilibrium among different species is driven by hadronisation itself rather than inelastic rescattering [10].

The far most used observables in the statistical model analyses are hadronic multiplicities and ratios of them. The main reason of this choice resides in their Lorentz invariance, implying their independence on the momenta of cluster (or fireballs) and thence on dynamical effects such as flow or collective momentum inherited by the pre-hadronisation dynamical evolution [11, 14]. On the other hand, both transverse and longitudinal momentum spectra do depend on dynamics besides the local properties of hadronising matter and the extraction of statistical-thermodynamical parameters based

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on them is more complicated. Moreover, they can only probe the state of hadronic matter at the later stage of kinetic freeze-out - i.e. when also elastic collisions cease - which does not necessarily coincide with the chemical freeze-out - i.e. when inelastic collisions cease - in heavy ion collisions.

The detailed description of the statistical model which has been used for the present analysis can be found elsewhere [14, 7]. As far as heavy ion collisions, in the examined energy range, are concerned, the $4\pi$ integrated multiplicities can be described by means of a grand-canonical formula depending on four free parameters (temperature $T$, baryon-chemical potential $\mu_B$, volume $V$ and extra strangeness suppression parameter $\gamma_S$) provided that the actual fluctuations of cluster masses and charges for fixed volumes meet a special requirement: they have to be the same relevant to the splitting of one equivalent global cluster (EGC) into an arbitrary number of sub-clusters [14]. If this is the case, the volume $V$ is meant to be the sum of the volumes of the clusters, measured in the EGC rest frame.

The formulae and the methods used in this analysis are described in detail in ref. [7]. In this paper, we will mainly discuss the results obtained in the analysis of three different systems: Pb–Pb collisions at $\sqrt{s_{NN}} = 8.7$ and 17.2 GeV, and Au–Au collisions at $\sqrt{s_{NN}} = 130$ GeV.

2. Analysis and results for SPS

The basic analysis of Pb–Pb collisions at SPS has been performed with NA49 data measured at a beam momentum of 40 (preliminary) and 158 $A$ GeV corresponding to nucleon-nucleon centre-of-mass energies of 8.7 and 17.2 GeV. The multiplicities or their ratios are measured or extrapolated to full phase space. The results are shown in tables 1 and 2 along with relevant references. The fit quality is satisfactory in both cases, taking into account that some measurements are still preliminary and that the only sizeable discrepancy at 158 $A$ GeV, the $\bar{\Lambda}/\Lambda$ ratio [4] is probably going to disappear [17]. The $\gamma_S$ fitted values turns out to be in agreement with each other and less than 1 at both energies. This clearly indicates that strangeness is not fully equilibrated in these collisions. The value of temperature and baryon-chemical potential in Pb–Pb collisions at 40 $A$ GeV, determined here for the first time, lie on the expected curve drawn by the values determined in previous analyses (see fig. [4]). It is worth pointing out that recent measurements of short-lived strongly decaying particles in NA49, such as $\Lambda(1520)$ [18, 19] have found a large discrepancy with respect to the prediction of the statistical model in this particular analysis ($1.45 \pm 0.7$ measured [18] against 3.516 expected). Within the statistical model, this can only be accounted for by admitting that the decay products of $\Lambda(1520)$ undergo significant elastic rescattering in the hadronic medium, thus losing memory of their invariant mass correlation.

The experiment WA97 [20] has measured multiplicities of many strange particles in a limited phase space region ($\Delta y = 1$) around mid-rapidity in Pb–Pb collisions at 158 $A$ GeV in several centrality bins. In general, a cut on phase space like this,
Table 1. Fitted parameters in Pb–Pb collisions at a beam energy of 40 A GeV and 158 A GeV \(^{[7]}\) with full phase space multiplicities and in Au–Au collisions at \(\sqrt{s}_{NN} = 130\) GeV with multiplicity ratios measured at mid-rapidity.

| \(\sqrt{s}_{NN}\) (GeV) | \(T\) (MeV) | \(\mu_B/T\) | \(\chi^2/dof\) |
|--------------------------|------------|-------------|--------------|
| 8.7                      | 149.3 ± 2.4| 2.637 ± 0.068| 0.822 ± 0.058| 13.5/3     |
| 17.2                     | 158.1 ± 3.2| 1.509 ± 0.075| 0.789 ± 0.052| 14.4/6     |
| 130                      | 167.0 ± 7.2| 0.274 ± 0.030| 1.04 ± 0.10  | 10.3/13    |

Figure 1. Temperatures and baryon-chemical potentials fitted with the statistical-thermal model analysis of hadron abundances in full phase space \(^{[7]}\). For the RHIC point, hadronic multiplicity ratios at mid-rapidity have been used.

introduces a non-trivial dependence of the yields on the distribution of cluster charges (baryon number, electric charge and strangeness) as a function of rapidity, which can be otherwise disregarded in the 4\(\pi\) analysis \(^{[3, 14]}\). The statistical model is, by construction, not able to predict such distributions and, therefore, a dynamical model is needed. However, although an appropriate fit of abundances within the statistical model requires integration over 4\(\pi\), we have nonetheless performed a similar analysis especially in order to study the effect of the phase space cut upon the extracted parameters and compare our results with previous analyses where particle ratios measured in both full and limited phase space have been fitted at the same time \(^{[3]}\). It must be pointed out that, in order to keep the same number of free parameters of the full phase space fit, this analysis has been performed by enforcing the net vanishing strangeness constraint, which might not
Table 2. Comparison between fitted and preliminary measured particle multiplicities in Pb–Pb collisions at a beam energy of 40 $A$ GeV. Results for 158 $A$ GeV can be found in ref. [7].

| Particle | Fitted  | Measured  | Reference |
|----------|---------|-----------|-----------|
| $\pi^+$  | 264.6   | 282 ± 15  | [21]      |
| $\pi^-$  | 293.3   | 312 ± 15  | [21]      |
| $K^+$    | 51.91   | 56 ± 3    | [21]      |
| $K^-$    | 19.53   | 17.8 ± 0.9| [22]      |
| $\Lambda$| 38.32   | 45.6 ± 3.4| [23]      |
| $\bar{\Lambda}$| 0.7179 | 0.71 ± 0.07| [23] |
| $N_p$    | 352.5   | 349 ± 5   | [21]      |

Figure 2. Statistical-thermal model parameters fitted by using hadronic multiplicities measured by WA97 experiment in Pb–Pb collisions at a beam energy of 158 $A$ GeV in various centrality bins (centered at 120, 204, 289 and 350 participant nucleons from left to right) compared with NA49 full phase space data fit.

The results of the fits to WA97 multiplicities are shown in fig. 2 along with those relevant to the fit to $4\pi$ multiplicities measured by NA49 in the most central collisions. It can be clearly seen that the thermal-statistical parameters fitted by using WA97 data do not essentially change going from central to peripheral collisions. Moreover, the extra strangeness suppression parameter is about 1, unlike in the $4\pi$ NA49 analysis. The different value of $\gamma_S$ fitted by using the limited and full phase space multiplicity samples apply in a limited kinematic region.
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proves quite definitely that ratios of hadronic yields measured in different kinematical regions cannot by any means be mixed in a single statistical model analysis, as has been done in ref. [6] where $\gamma_S = 1$ was claimed. It is also worth emphasizing that the fact that $\gamma_S < 1$ in full phase space does not depend on the inclusion or not in the fit of particles carrying two strange quarks such as $\Xi$ and $\phi$ [7].

3. Analysis and results at RHIC

We have used the same model to fit ratios of hadronic multiplicities measured at RHIC in Au–Au collisions at $\sqrt{s_{NN}} = 130$ GeV in a small window around mid-rapidity ($\Delta y \simeq 1$), which is the probably the widest kinematical region accessible to RHIC experiments. As a limited region of phase space is involved, all the caveats discussed in the previous section concerning the use of such data hold. The data sample is the same as in ref. [8] with one more measurement of $\phi/h^-$ ratio [24]. In this analysis, the net vanishing strangeness constraint has been used and hadronic weak decays products have been included in the multiplicities. The results are shown in table 1 and are in very good agreement with those in ref. [9]. They are quite consistent with ref. [8] (taking into account that only 50% of weak decay products were included) while the temperature is somewhat lower than that quoted in ref. [25].
4. Discussion and conclusions

The obtained $\lambda_S$ values in several heavy ion collisions are shown in fig. 3 while the ratio between newly produced valence s quarks and u, d quarks for direct hadrons (the so-called Wroblewski factor) is shown in fig. 4 for both heavy ion and elementary collisions. This plot updates previous studies \[7\]. It is clear that the behaviour of strange quark production in heavy ion is very different from that in elementary collisions where $\lambda_S$ is seemingly constant $\simeq 0.2$. The relative strangeness production seems to attain a maximum around a beam energy of 30 $A$ GeV \[26\] and decrease then to an asymptotic value of $\simeq 0.4$. However, the largest energy point for heavy ion collisions in fig. 4, the RHIC one, has been calculated on the basis of a fit to mid-rapidity yields and, thus, this $\lambda_S$ is not likely to be the same as in full-phase space on the basis of what has been discussed about Pb–Pb data at SPS in Sect. 2, rather an upper limit of it.

A major issue in this kind of studies is related to the achievement or not of full hadron chemical equilibrium in heavy ion collisions. The analysis of $4\pi$ multiplicities in heavy ion collisions clearly indicate that strangeness chemical equilibrium is \textit{globally} not achieved in any of the examined systems (see fig. 3) \[3\]. Recent claims that $\gamma_S = 1$ || It has been proposed that even light flavours could be out of equilibrium inside the hadronising volume \[10\] but, in our opinion, the present degree of accuracy in hadronic yield measurements does

\[\begin{align*}
\lambda_S & \mid \text{SIS} & \text{AGS AuAu} & \text{SPS PuPb 40 (prel.)} & \text{RHIC (midrap)} \\
\mid \text{SIS} & \text{AGS SiAu} & \text{SPS PbPb SS SAg} & & \\
\end{align*}\]

\[\begin{align*}
\sqrt{s} \text{ (GeV)} & & 1 & 10 & 10^2 & 10^3 \\
\lambda_S & & 0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1 \\
\end{align*}\]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Wroblewski factor $\lambda_S$ determined within the statistical model in several elementary \[13, 14\] and heavy ion collisions \[3, 7\] as a function of (nucleon-nucleon) centre-of-mass energy. Unlike all other points, the RHIC value has been obtained by using mid-rapidity hadron yields.}
\end{figure}
in Pb–Pb collisions \cite{3} depend on a mixture of hadronic ratios measured in different rapidity windows. The question now arises whether the full chemical equilibrium is at least achieved locally. The fact that $\gamma_S$ turns out to be about 1 by using hadronic yield ratios at mid-rapidity might be taken as a clearcut indication of the formation of a completely equilibrated hadron gas in the central region. However, it must be pointed out that this could not be the case.

The usual global statistical-thermal analysis of hadron multiplicities relies on a particular distribution of cluster charges and masses once their volumes are fixed \cite{13, 14} but with no requirement on cluster charge distribution as a function of rapidity, provided that one deals with full phase space multiplicities. Otherwise stated, the same $4\pi$ yields can be obtained with many different distributions of cluster charges (e.g. strangeness) as a function of rapidity and the ultimate reason of this special property is the Lorentz invariance of hadron multiplicities \cite{3, 14}. It must be emphasized that this kind of requirement justifying the use of one global statistical-thermal formula for hadron abundances and based on charges and masses fluctuations of clusters for fixed volumes, is more general than the usual one based on the hydrodynamical picture. Indeed, in the latter argument, each cluster is described by grand-canonical parameters such as temperature and baryon-chemical potential and this implies that all of them have to be large enough; on the other hand, fluctuations of cluster masses and charges are anyway fixed by the grand-canonical framework, so the involved assumptions are altogether stronger than in the previous case. Furthermore, it might happen that clusters endowed with non-vanishing strangeness, or with a larger number of strange quarks to be coalesced in the final hadrons, tend to lie in the mid-rapidity region (see fig. 5), thus creating an apparent enhancement of strange particles thereabout even though the underlying $\gamma_S$ was less than 1 throughout. Therefore, one could obtain a $\gamma_S$ about 1 by fitting mid-rapidity ratios even if this was not the case. Indeed, the dependence of fitted $\gamma_S$, as well as the other parameters, on the rapidity window has been studied in ref. \cite{27} on the basis of RQMD simulated data along with strangeness neutrality and a clear dependence of $\gamma_S$ on the rapidity cut has been found.

In summary, the issues of local full chemical equilibrium and of the soundness of the use of mid-rapidity multiplicities are not settled yet. More and more detailed studies are necessary, particularly in view of the experiments at RHIC and LHC where only a limited window in rapidity is accessible.

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Central region

Or $S=2$ $S=-2$

Figure 5. The distribution of cluster charges as a function of rapidity may favour configurations with non-vanishing strangeness in the central region, creating an apparent enhancement of strange particle production at mid-rapidity and faking $\gamma_S \simeq 1$.

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