Analysis of Students’ Mathematical Imagination Ability in Solving Problems

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Abstract. The purpose of this study is to determine the ability of mathematical imagination to solve mathematical problems. Mathematical problems in this study use fractional content. This research is descriptive qualitative research. In this study subjects included 3 junior high school students, where the subject was deliberately taken by sample technique, i.e. considering the ability to communicate and an indication of the ability to bring up mathematical imagination. Data collection is done by observation, mathematical imagination test, and interview. Data analysis techniques include data reduction, data performance, and drawing conclusion. Conclusion drawing used triangulation method. The outcome of this study is that the ideas raised by the students in solving problems are not yet another form of content, but rather in the form of technical calculations. Students do not understand the concept of set in solving fractional content problems. Thus, students are only able to bring sensory imagery.

1. Introduction
Mathematics contains a collection of abstract ideas, so learning requires a higher thinking. To understand math, high mental activity consists of logic, thinking and imagination [1]. Mathematical discovery does not come from logic but imagination [2]. There is a need for imagination to create different ideas in geometry and prove the theory [3]. Thus, it is suspected that there is an important relationship between ideas formed from experiences and ideas formed using imagination.

Some types of imagination that can be developed in mathematical learning are called mathematical imagination [4]. Mathematical imagination is related to the mental activity of students in an effort to solve mathematical problems by calculating, measuring size, and shape. Imagination is a form of transition to mathematical thinking, from the real world to the world of symbols [5]. With the ability of mathematical imagination, problems in the real world can be expressed in symbols and can be done mathematically to find solutions.

Imagination should be involved in learning activities [6, 7, 8]. The use of imagination is the key which allows the development of new methods of learning mathematics where it is one of the practice of using imagery to find new ideas to solve problems [9]. Thus, teachers should focus on the mathematical imagination of students in the learning process. However, in reality this mathematical imagination does not have special attention from the teacher during learning class.

Based on the result of observation and interview, that mathematics learning in schools is still inductive. That is, the teacher explains the content, then gives an example and how it is resolved. In the
meantime, students are done with recording and understanding what the teacher has said. In this process, students use the left brain more. The left brain plays a role in logical, analytical, verbal, sequential and statistical matters. With such learning, students will be able to do mathematical calculation but are weak in analysing a problem that can be solved mathematically. This can be seen when the teacher gives a different problem than the given example, for example by replacing the variable being asked. It will also be difficult for the students to solve the problems given, especially if the given problems are non-routine. Students will find it difficult to analyse these different or non-routine problems to solve their problem.

One of the possible causes is because there has never been a study that discusses mathematical imagination in the school. So that the teacher does not have the data and recommendations to develop learning, that can accommodate students’ imagination development. Apart from this, the resources for learning how to adapt the right brain of students in solving mathematical problems in Indonesia are still limited. Unlike Taiwan, who have emphasized their learning activities to develop students’ creativity and imagination. Creativity and imagination in education are increasingly emphasizing in the whole world, for example in Taiwan with the development of policies and action plans and in the educational programs with the implementation of creativity and imagination education [10].

Imagination is the work of the right brain. The right brain is the centre of human creativity. The right brain becomes the centre of the human mind’s power because in this place where everything is related to space and time such as intuition, imagination, and creativity [11]. These things are very important in determining the problems, especially non-routine ones. Thus learning needs to be done that not only can explore the potential of the student’s left brain, but also his right brain, because the human brain has the privilege of linking sensed reality with information (associations). There are three types of mathematical imagination, i.e. sensory imagination, creative imagination, and recreational imagination [12, 13].

The sensory imagination is basically perception, which is the experience gained from a stimulus. The creative imagination is the emergence of ideas in unusual and unpredictable ways. Meanwhile, the recreational imagination is the ability to construct ideas from a different perspective of an experience.

Based on the results of an interview with mathematics teacher, students still have difficulty developing their ability to solve mathematical problems, especially those that are non-routine. This indicates that students are only able to calculate but are not able to generate ideas to solve the problems. Meanwhile, there are three aspects of evaluation in mathematics learning, i.e. understanding concepts, logic and communication, and solving problems [14]. So it is necessary to know the extent of mathematical imagination ability of students to solve mathematical problems.

2. Methods
This research was conducted in April 2018 at junior high school in Sukabumi district. This kind of research is descriptive qualitative research. The data described is the ability of students’ mathematical imagination in solving mathematical problems. Participants in this study were 3 students. Participants were deliberately chosen using sample techniques. Students who become participants have the ability to communicate well and indicate that it can bring mathematical imagery to work. This signal is based on the recommendations of the teachers of mathematics teaching in the school. Data collection is done by observation method, mathematical imagination test, and interview. Analysis of the data used is data reduction, data performance and drawing conclusions [15]. Conclusion drawing used triangulation method.

3. Result and Discussion
The data obtained in this study are based on the result of mathematical imagination capability test (TKIM), observation which is equipped with recording of students’ behaviour in working on questions and interviews. The behaviour and results of student work in solving mathematical problems reflect the process that how students bring mathematical imagination capability. In this paper, the observation
analysis is not completely included. The results of observation were included in the finding of the observations. Testing tool is a problem solving a mathematical problem at a fraction of an item. In formulating the devices as questions, the question is made according to basic competencies and selected indicators. Then, evaluate the mathematics subject in the school by consulting the teacher. The following are the test instrument used in this study.

**Question:**
If A and B are natural numbers, then — — —. Determine the A and B values that meet the equation!

**Answer:**

If two segments are multiplied by 22, then obtained:
so the values of A and B are as follows:
- If A = 1 then B = 3
- If A = 3 then B = 2
- If A = 5 then B = 1

Based on the analysis done on each students’ answer sheet, the following results were obtained.

### 3.1. Participant 1 (P1)

![Figure 1. Answer Sheet P1](image)

Figure 1 shows that student has been able to imagine the completion of the problem given. This can be seen from answer P1, which contains the information asked. That is, P1 understands what is expected of the problem. P1 writes solutions by trial and error. This indicates that P1 can bring up the problem solving idea. P1 also searches for more than one solution, although there is an error in determining the value of B that B is not a natural number. In addition, P1 is not able to determine the general pattern of completion. To get more deep information, an interview was conducted between researcher (P) and participant 1 (P1).

P : when you first read about it, what was in your mind?
P1 : find the value A and B according to the known equation.
P : are you sure in this answer? P1 : no sir.
P : why do you write long answers like this?
P1 : because I'm confused about how to do it sir. Here I just try it by entering the natural number to A, then look for the B value.
P : why is it that you are looking for value of A from 1 to 5 not up to 6 and so on?
P1 : no sir, because the natural number starts from 1, 2, 3, and so on. If I enter the value A = 6, it will give a value B below 1. So I only enter the A to 5 values, sir.
P : here you write value of B are 3; 2,5; 2; 1,5; 1. Are 2,5 and 1,5 included in the natural number?
P1 : Oh, I forgot sir. I only look the value of A, it does not seem that the value of B should also be a natural number.
P : what is the correct A and B value?
P1 : If A = 1 then B = 3
   If A = 3 then B = 2
   If A = 5 then B = 3
P : is there any other way to solve this problem?
P1 : may be sir but I don’t know.

Based on the results of the P1 answers and the interviews conducted, P1 has understood the purpose of the problem presented. P1 can also imagine the resolution of the problem. This shows that P1 can bring sensory imagination. P1 solves these problems by trial and error. This means that P1 does not stop at the correct answer first. P1 still tries to solve by substituting numbers that might be in accordance with the given equation. This shows that P1 can bring creative imagination. However, P1 does not understand how to get the general form of the settlement it. This shows that P1 cannot bring forth a recreational imagination.

3.2. Participant 2 (P2)

Figure 2 shows that P2 understands what is expected of the problem given. P2 resolves the problem by taking one of the natural numbers, then substituting it to B and obtaining A. However, in that answer there are still some errors such as number 2 (circle in figure 2) and the value A and B which are fractions. P2 is just trying to solve the problem by one answer without thinking about the other possibilities of the proper answer, so the answers are written incorrectly. To get more deep information, an interview was conducted between researcher (P) and participant 2 (P2).
P : are you sure that the answers here are correct?
P2 : no sir.
P : why not?
P2 : I don’t really understand with this content.
P : what do you think is content?
P2 : fractional sir
Participant 2 (P2)

P: well, how do you complete the addition of fractions?
P2: first, the denominator is equal, if it’s the same then multiply the numerator. Then it can be added.
P: try looking at your answer here (circle in figure 2). Where does the number 2 appear?
P2: 22 divided by 11 sir. Then I substituted the result to B
P: here is an explanation that A and B are natural numbers. Do you know natural numbers?
P2: yes I know. Numbers starting from 1, 2, 3 and so on until infinite. Oh yeah, I’m not careful sir, I’m not reading this.
P: are there any other possible answers than B = ?
P2: B = 1 and B = 3 sir.
P: yes, after B is known, you can determine the value of A. if you see your answer sheet, you are wrong in writing the values of A and B. A = 3, B = 2 not A = —, B = —
P2: yes sir my answer is wrong, but I mean that.

Based on the result of P2’s answer and interviews, P2 understood what was asked of the problem. But P2 is not looking for a possible answer to the problems given. That’s because P2 cannot imagine the possibilities of other answer that arise. This means that P2 can only bring sensory imagination to solve the problem.

3.3. Participant 3 (P3)

Figure 3 shows that P3 understands the purpose of the problem given. P3 resolves the problem by substituting one of the natural numbers, without looking for other possible numbers, so that the answer given by P3 is not correct. To get more deep information, an interview was conducted between researcher (P) and participant 3 (P3).
P: what is in your mind when you get this question?
P3: at first I was confuse about how to do it. Then I just try to input B = 2. Then I look for the value of A. I find A = 3. Therefore the answers I get A = 3 and B = 2.
P: do you have more answer than what you wrote here?
P3: no sir.
P: try this equation (point to the problem), you multiply all segments by 22.
P3: it has become A + 2B = 7 sir
P: so you try to substitute the natural numbers to A and B. What is the result?
P3: Oh yes, I just remembered Sir, that means there are many answer? I doubt it is impossible for the problem to be solved easily like this.
Based on the result of the answer and interviews, P3 understood what was asked of the problem. But P3 does not seek as many solutions as possible from solving the problem. This shows that P3 can only create sensory imagination. However, when P3 is stimulated, P3 can bring up the imagination of others.

The results of the observations made during learning and testing work show that all three participant can create mathematical imaginations. This can be seen when participants start and complete test and interviews. Thus, observation results are directly proportional to the results of tests and interviews.

Based on the results of the answer of each participant in solving the problems given, interviews and observations, it was received:

1. P1 can solve the problem correctly. In addition, P1 can also determine the possibilities of the correct answer. This shows that P1 can bring sensory imagination and creative imagination. However P1 is not able to form a general form of settlement. It can be seen by the lack of P1 in determining the general form of the settlement received, since P1 only understands that the solution given is in the form of numbers. In other word, P1 cannot bring a recreational imagination.

2. Basically P2 and P3 can solve the problem correctly. However P2 and P3 determine the possibilities of other values that are appropriate. P2 and P3 only show one settlement. In other word, P2 and P3 can create sensory imagination.

4. Conclusion

Based on the results and discussion, it can be seen that P1 can bring sensory imagery and creative imagery, whereas P2 and P3 can only bring sensory imagery. So all participants have been able to create imagination, that is sensory imagination.

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