A FORMULATION WITHOUT PARTIAL WAVE DECOMPOSITION
FOR SCATTERING OF SPIN-\(\frac{1}{2}\) AND SPIN-0 PARTICLES

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A new technique has been developed to calculate scattering of spin-\(\frac{1}{2}\) and spin-0 particles. The so called momentum-helicity basis states are constructed from the helicity and the momentum states, which are not expanded in the angular momentum states. Thus, all angular momentum states are taken into account. Compared with the partial-wave approach this technique will then give more benefit especially in calculations for higher energies. Taking as input a simple spin-orbit potential, the Lippman-Schwinger equations for the T-matrix elements are solved and some observables are calculated.

Keywords: 3D technique, momentum-helicity basis

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1. Introduction

As an alternative to the partial wave (PW) decomposition a so called three-dimensional (3D) technique has been developed for the nucleon-nucleon (NN) system. It has been successfully applied to NN scattering and the deuteron in Refs. \cite{1} and \cite{2} respectively for the Bonn-B and the AV18 realistic potential models. The basic idea in 3D approach is to take the momentum states, as part of the basis states, without expanding them into the angular momentum states. Thus, all angular momentum states are taken into account. It is then very important especially for higher energy region, while in PW calculations adding more higher angular momentum states may at some point become not feasible, for instance, due to restriction on computer resources. For lower energy regions still the 3D technique appears as a good alternative.

In this work we derived a 3D formulation for scattering of spin-\(\frac{1}{2}\) and spin-0 particles. The work will then be very useful, for example, for kaon-nucleon (KN) investigation. The formulation described in Section \cite{2} is simpler than that for NN scattering, since our system consists of a pair of nonidentical particles with the
total spin being $\frac{1}{2}$. There is no antisymmetrizing like in Ref. 1. Finally by means of
symmetry relations we get only a single Lippmann-Schwinger equation. In Section 3
we show some calculations for a simple spin-orbit interaction based on the Malfliet-
Tjon potential. We summarize in Section 4.

2. Formulation

2.1. The momentum-helicity basis

The momentum-helicity basis states are eigenstates to the parity operator $P$ and
the helicity operator $\mathbf{s} \cdot \hat{\mathbf{p}} = \frac{1}{2} \sigma \cdot \hat{\mathbf{p}}$, and are defined as

$$|p; \hat{p}\lambda\rangle = \frac{1}{\sqrt{2}} \left( |p\rangle \mp \sigma \cdot \hat{p} \right) |\hat{p}\lambda\rangle,$$

(1)

with

$$|p; \hat{p}\lambda\rangle \equiv |p\rangle |\hat{p}\lambda\rangle,$$

(2)

$p$ the relative momentum, $|\hat{p}\lambda\rangle$ and $\lambda = \pm \frac{1}{2}$ the helicity eigenstate and eigenvalue
for spin $\frac{1}{2}$, $\eta_\pi = \pm 1$ the parity eigenvalue, the subscript $\pi = \pm$ the parity eigenstate
label. The momentum-helicity basis states in Eq. (1) has the normalization as

$$\pi \langle p'; \hat{p}'\lambda' | p; \hat{p}\lambda\rangle_\pi = \delta_{\eta_\pi' \eta_\pi} \left[ \delta(p' - p) \delta_{\lambda' \lambda} - i \eta_\pi \delta(p' + p) \delta_{\lambda' - \lambda} \right]$$

(3)

and the completeness relation as

$$\frac{1}{2} \sum_{\pi \lambda} \int dp |p; \hat{p}\lambda\rangle_\pi \langle p; \hat{p}\lambda\rangle_\pi = 1.$$

(4)

2.2. The potential and T-matrix elements

The potential $V$ matrix elements and the T-matrix elements in the momentum-
helicity basis are defined as

$$V^\pi_{\lambda' \lambda} (p', p) \equiv \pi \langle p'; \hat{p}'\lambda' | V | p; \hat{p}\lambda\rangle_\pi,$$

(5)

$$T^\pi_{\lambda' \lambda} (p', p) \equiv \pi \langle p'; \hat{p}'\lambda' | T | p; \hat{p}\lambda\rangle_\pi.$$

(6)

The potential matrix elements obey the following symmetry relations:

$$V^\pi_{\lambda' \lambda} (p', p) = -i \eta_\pi V^\pi_{\lambda' \lambda} (p', -p)$$

(7)

$$V^\pi_{\lambda' \lambda} (p', p) = i \eta_\pi V^\pi_{\lambda' \lambda} (-p', p)$$

(8)

and similarly the T-matrix elements. Since the basis states are eigenstates to the he-
llicity operator, it would be convenient if the potential in momentum representation
is expressed, in general, as

$$V (p', p) \equiv \langle p'| V | p \rangle = \sum_i f_i \left( p', p, \hat{p}' \cdot \hat{p} \right) (\sigma \cdot \hat{p})^{a_i} (\sigma \cdot \hat{p})^{b_i},$$

(10)
The relation in Eq. (11) applies also to the T-matrix elements, lead to the following uncoupled integral equation:

where \( f \), \( V \) and \( \mu \) can be separated as

\[
V_{N_{\lambda}}^\pi (p', p, x') = e^{i\lambda \phi'} V_{N_{\lambda}}^\pi (p', p, x'), \quad (x' \equiv \cos \theta').
\]

(11)

The relation in Eq. (11) applies also to \( T_{N_{\lambda}}^\pi (p', p, x') \). The symmetry relations in Eqs. (7) & (8) together with the relation in Eq. (11), all for both the potential and the T-matrix elements, lead to the following uncoupled integral equation:

\[
T_{N_{\lambda}}^\pi (p', p, x') = V_{N_{\lambda}}^\pi (p', p, x') + \lim_{\epsilon \to 0} \int_0^\infty dp'' \int_{-1}^1 dx'' \frac{V_{N_{\lambda}}^\pi (p', p'', x''') T_{N_{\lambda}}^\pi (p'', p, x''')}{\frac{\mu^2}{2\epsilon} + i \epsilon - \frac{\mu'^2}{2\epsilon}} ,
\]

which we just call the Lippmann-Schwinger (LS) equation for \( T_{N_{\lambda}}^\pi (p', p, x') \), with \( \mu \) being the reduced mass of the system and

\[
V_{N_{\lambda}}^\pi (p', p'', x''') = \int_0^{2\pi} d\phi'' e^{i\lambda (\phi'' - \phi')} V_{N_{\lambda}}^\pi (p', p'').
\]

(13)

For \( T_{N_{\lambda}}^\pi (p', p, x') \) there are symmetry relations given as

\[
T_{N_{\lambda}}^\pi (p', p, x') = -2\lambda \eta_{\pi} T_{N_{\lambda}}^\pi (p', p, -x')
\]

(14)

\[
T_{N_{\lambda}}^\pi (p', p, x') = 2\lambda \eta_{\pi} T_{N_{\lambda}}^\pi (p', p, -x')
\]

(15)

\[
T_{N_{\lambda}}^\pi (p', p, x') = 4\lambda \eta_{\pi} T_{N_{\lambda}}^\pi (p', p, -x').
\]

(16)

These symmetry relations in (14), (15), (16) reduce the number of equations to be solved. For each parity state we need to solve the LS equation given in Eq. (12) only for, say, \( T_{\frac{\pi}{2}, \frac{\pi}{2}}^\pi (p', p, x') \).

3. Example of Calculations

We show here just as an example some calculations for a simple spin-orbit potential given as

\[
V(r) = V_c(r) + V_s(r) = \hat{1} \cdot s,
\]

(17)

with \( \hat{1} \) being the angular momentum operator, \( s = \frac{1}{2} \sigma \), the radial functions \( V_c(r) \) and \( V_s(r) \) of the type of the Malfliet-Tjon potential:

\[
V_c(r) = -V_{ca} e^{-\mu_c r} + V_{cb} e^{-2\mu_c r}, \quad V_s(r) = -V_{sa} e^{-\mu_s r} + V_{sb} e^{-2\mu_s r},
\]

(18)

\( V_{ca} = 3.22, V_{cb} = 7.39, \mu_c = 1.55, V_{sa} = 2.64, V_{sb} = 7.39, \mu_s = 0.63 \). The mass of particle 1 acting as the projectile is of nucleon mass and that of particle 2 acting as the target is of kaon mass. It would certainly be much more interesting if we
could test our work on KN scattering. But to do that we need a KN interaction in operator form, which is unfortunately not yet available.

Figure 1 shows the spin averaged differential cross section for various projectiles laboratory energies from a few MeV up to 1 GeV. We want to stress out that the algebraic and numerical effort in 3D approach is the same for all different energies.

4. Summary

We have developed a 3D technique for scattering of spin-$\frac{1}{2}$ and spin-0 particles. It will be very useful for investigation on, for example, KN system, given a KN interaction in operator form. The technique shows to be a good alternative to the PW technique, especially for higher energy region, where PW calculations may become not feasible. As an example some calculations based on a simple spin-orbit potential are performed.

References

1. I. Fachruddin, Ch. Elster, and W. Glöckle, Phys. Rev. C62, 044002 (2000).
2. I. Fachruddin, Ch. Elster, and W. Glöckle, Phys. Rev. C63, 054003 (2001).
3. R. Machleidt, Adv. Nucl. Phys. 19, 189 (1989).
4. R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C51, 38 (1995).
5. R. A. Malfliet and J. A. Tjon, Nucl. Phys. A127, 161 (1969).