Duality between $\mathcal{N} = 5$ and $\mathcal{N} = 6$
Chern-Simons matter theory

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Abstract: We provide evidences for the duality between $\mathcal{N} = 6$ $U(M)_4 \times U(N)_{-4}$
Chern-Simons matter theory and $\mathcal{N} = 5$ $O(\tilde{M})_2 \times USp(2\tilde{N})_{-1}$ theory for a suitable
$\tilde{M}, \tilde{N}$ by working out the superconformal index, which shows perfect matching. For
$\mathcal{N} = 5$ theories, we show that supersymmetry is enhanced to $\mathcal{N} = 6$ by explicitly
constructing monopole operators filling in $SO(6)_R R$-currents. Finally we work out
the large $N$ index of $O(2N)_{2k} \times USp(2N)_{-k}$ and show that it exactly matches with
the gravity index on $AdS_4 \times S^7/D_k$, which further provides additional evidence for
the duality between the $\mathcal{N} = 5$ and $\mathcal{N} = 6$ theory for $k = 1$. 
1. Introduction

There have been much progress in understanding 3-d superconformal field theories recently. Many of 3-d superconformal field theories are realized as supersymmetric Chern-Simons matter (SCSM) theories. Quite exhaustive classes of $\mathcal{N} \geq 4$ SCSM theories are constructed in \cite{1, 2, 3, 4, 5, 6, 7}. Among them, the most famous example is ABJM theory, $\mathcal{N} = 6$ $U(N)_k \times U(N)_{-k}$ Chern-Simons matter theory describing M2 branes on $\mathbb{C}^4/Z_k$. The curious fact of ABJM theory is that supersymmetry is enhanced to $\mathcal{N} = 8$ for $k = 1, 2$. This was discussed at \cite{8, 9, 10} and further clarified where the monopole operators filling $\mathcal{N} = 8$ R-currents are explicitly constructed \cite{11}.

Another interesting feature of SCSM theories is that 3-d analogue of Seiberg-duality holds for some classes of the theories. For example there are various evidences that $\mathcal{N} = 2$ $U(N)_k$ SCSM theory with $N_f$ fundamental flavors is dual to $\mathcal{N} = 2$ $U(N_f - N + |k|)_{-k}$ with $N_f$ fundamental flavors with additional meson fields \cite{12, 13, 14}. While in 4-d Seiberg dualities hold for $\mathcal{N} = 1$ supersymmetric theories, there are several evidences that such duality holds for SCSM theories with $\mathcal{N} \geq 2$ theories \cite{13, 16}.

In these respects, we had better explore further models, which would enrich our understanding of 3-d SCSM theories. In this note we are interested in another type of dual pairs of SCSM theory in 3-d. The theories we are interested in are $\mathcal{N} = 6$ $U(M)_4 \times U(N)_{-4}$ and $\mathcal{N} = 5$ $O(M)_2 \times USp(2\tilde{N})_{-1}$. There is a conjecture that this
pair is dual to each other for suitable \( \hat{M}, \hat{N} \) for given \( M, N \) as will be explicitly stated in the main text. This duality is interesting in several ways. Firstly, \( \mathcal{N} = 6 \) \( U(N + l)_{4} \times U(N)_{-4} \) Chern-Simons theory itself exhibits Seiberg-like dualities, i.e., it is dual to \( U(N)_{4} \times U(N + 4 - l)_{-4} \) with \( l \leq 4 \). And the similar holds for \( \mathcal{N} = 5 \) theory as well. In addition, there is a separate duality connecting \( \mathcal{N} = 6 \) theory and \( \mathcal{N} = 5 \) theory, whose physical origin is not clear at this point. Index computation provides evidence that \( \mathcal{N} = 6 \) \( U(M)_{4} \times U(N)_{-4} \) theory is dual to \( \mathcal{N} = 5 \) \( O(\hat{M})_{2} \times USp(2\hat{N})_{-1} \) theory. As a byproduct of the computation, we can provide evidence for Seiberg-like dualities among \( \mathcal{N} = 6 \) theories and among \( \mathcal{N} = 5 \) theories for simple cases. Secondly, since \( \mathcal{N} = 5 \) theory is dual to \( \mathcal{N} = 6 \) theory for a particular choice of Chern-Simons level, the supersymmetry of \( \mathcal{N} = 5 \) theory should be extended to \( \mathcal{N} = 6 \). Adopting the method of [11], we explicitly construct the monopole operators filling in \( Spin(6) \) \( R \)-currents, thereby showing the supersymmetry is indeed enhanced at section 3. Finally we work out the large \( N \) limit of the superconformal index of \( \mathcal{N} = 5 \) \( O(2N)_{2k} \times USp(2N)_{-k} \) theory and matches it to the gravity index of \( AdS_{4} \times S^{7}/D_{k} \) with \( D_{k} \) dihedral group of \( 4k \) elements. This not only provides evidence for the conjecture that \( \mathcal{N} = 5 \) theory has the gravitational dual of \( AdS_{4} \times S^{7}/D_{k} \) but also provides additional evidence for the equivalence between the \( \mathcal{N} = 5 \) \( O(2N)_{2} \times USp(2N)_{-1} \) theory and \( \mathcal{N} = 6 \) \( U(N)_{4} \times U(N)_{-4} \) theory, which is in turn dual to the gravity theory on \( AdS_{4} \times S^{7}/Z_{4} \) since \( D_{1} = Z_{4} \). In appendix we provide the details of the index formulae used in the main text.

2. Superconformal index

Let us first discuss the structure of \( \mathcal{N} = 6 \) \( U(M)_{k} \times U(N)_{-k} \) theory, and the \( \mathcal{N} = 5 \) \( O(M)_{2k} \times USp(2N)_{-k} \) theory, which we abbreviate \( U(M|N) \), \( OSp(M|2N) \) respectively.\(^1\) The \( \mathcal{N} = 6 \) \( U(M|N) \) theory contains four superfields \( C_{I} \) in the fundamental representation \( 4 \) of the \( R \)-symmetry \( SU(4)_{R} \cong Spin(6)_{R} \) and in the bifundamental representation of the gauge group. The superfields \( C_{I} \) can be written in the \( \mathcal{N} = 2 \) formalism as \( C_{I} = (A_{1}, A_{2}, \bar{B}^{1}, \bar{B}^{2}) \) where \( A_{a} \) and \( B_{\dot{b}} \) are four \( \mathcal{N} = 2 \) chiral multiplets in the representation \( (2, 1) \) and \( (1, 2) \) of \( SU(2)_{A} \times SU(2)_{B} \subset SU(4)_{R} \) respectively. The theory has the superpotential

\[
W = \frac{2\pi}{k} \epsilon^{ab} \epsilon^{\dot{a}\dot{b}} \text{tr} (A_{a}B_{a}A_{\dot{b}}\bar{B}_{\dot{b}}) = \frac{4\pi}{k} \text{tr} (A_{1}B_{2}A_{\dot{1}}\bar{B}_{\dot{2}} - A_{1}B_{1}A_{\dot{2}}\bar{B}_{\dot{2}}) .
\]  

(2.1)

The \( \mathcal{N} = 5 \) \( OSp(M|2N) \) theory also contains the superfields \( C_{I} = (A_{1}, A_{2}, \bar{B}^{1}, \bar{B}^{2}) \) with identifications \( A_{1} = B_{1}^{T}J \) and \( A_{2} = B_{2}^{T}J \) where \( J \) is the invariant antisymmetric matrix of the symplectic group. These identifications preserve \( USp(4)_{R} \subset SU(4)_{R} \) such that the \( R \)-symmetry of the theory is \( USp(4)_{R} \cong Spin(5)_{R} \). The theory has

\(^1\)In our convention, \( SO(2)_{1} \) Chern-Simons theory is equivalent to \( U(1)_{1} \) theory.
the superpotential
\[ W = \frac{4\pi}{k} \text{tr} \left( A_1 J A_2^T A_2 J A_1^T - A_1 J A_1^T A_2 J A_2^T \right). \] (2.2)

Curiously, it seems that there exist a duality between these two theories when the Chern-Simons levels are \((k, -k) = (4, -4)\) for the first theory and \((2k, -k) = (2, -1)\) for the second theory. More specifically, the following dual relations were conjectured \cite{17}:

\[
\begin{align*}
U(N)_4 \times U(N)_{-4} & \leftrightarrow O(2N)_2 \times USp(2N)_{-1} \\
U(N + 2)_4 \times U(N)_{-4} & \leftrightarrow O(2N + 2)_2 \times USp(2N)_{-1} \\
\{ U(N + 1)_4 \times U(N)_{-4} \} & \leftrightarrow \{ O(2N + 1)_2 \times USp(2N)_{-1} \} \\
\{ U(N + 3)_4 \times U(N)_{-4} \} & \leftrightarrow \{ USp(2N)_1 \times O(2N + 1)_{-2} \}
\end{align*}
\]

In fact, at \cite{17} the duality for the first two theories are not precisely stated and we improve it in our paper.\(^2\) The last pair on the left hand side are mapped to the last pair on the right hand side. Furthermore, these two theories on each side also are related by Seiberg-like duality. This is clear from the brane construction of \cite{17}, where Seiberg-like duality can be inferred from the NS-5 brane movement past the infinite coupling.\(^3\) In the \(\mathcal{N} = 2\) setting, the Seiberg-like dualities for product gauge groups are discussed in \cite{18}. See \cite{19} also related discussions on 3-d Seiberg-like dualities.

As shown at \cite{17}, all these theories have moduli space as the symmetric product of \(C^4/Z_4\). This is the first evidence for the conjectured duality. In order to provide further evidences for the claimed dualities, we resort to the superconformal index computation. Let us discuss the general structures of the index. We consider the superconformal index for 3-d \(\mathcal{N} = 2\) superconformal field theory (SCFT). The superconformal index for a higher supersymmetric theory can be defined using their \(\mathcal{N} = 2\) subalgebra. The bosonic subgroup of the 3-d \(\mathcal{N} = 2\) superconformal algebra is \(SO(2, 3) \times SO(2)\). There are three Cartan elements denoted by \(\epsilon, j\) and \(R\) which come from three factors \(SO(2)_\epsilon \times SO(3)_j \times SO(2)_R\) in the bosonic subalgebra. One can define the superconformal index for 3-d \(\mathcal{N} = 2\) SCFT as follows \cite{20}:

\[ I = \text{Tr}(-1)^F \exp(-\beta\{Q, S}\}) x^{\epsilon+j} y^{F_j} \] (2.3)

where \(Q\) is a supercharge with quantum numbers \(\epsilon = \frac{1}{2}, j = -\frac{1}{2}\) and \(R = 1\) and \(S = Q^\dagger\). They satisfy following anti-commutation relation:

\[ \{Q, S\} = \epsilon - R - j := \Delta. \] (2.4)

\(^2\)At \cite{17}, it’s claimed that the first two theories on the left hand side are mapped to the first two theories on the right hand side although which to which is not clear.

\(^3\)More precisely, for example, \(U(N + 1)_4 \times U(N)_{-4}\) is equivalent to \(U(N + 3)_4 \times U(N)_{-4}\) if we combine Seiberg-like duality and parity transformation.
In the index formula, the trace is taken over gauge-invariant local operators in the SCFT defined on $\mathbb{R}^{1,2}$ or over states in the SCFT on $\mathbb{R} \times S^2$. As is usual for Witten index, only BPS states satisfying the bound $\Delta = 0$ contributes to the index and the index is independent of $\beta'$. If we have additional conserved charges commuting with chosen supercharges $(Q, S)$, we can turn on the associated chemical potentials and the index counts the number of BPS states with the specified quantum number of the conserved charges denoted by $F_j$ in eq. (2.3). We simply set $y_j = 1$ in the subsequent computation.

The index can be exactly calculated using the localization technique, deforming the action by a $Q$-exact term and making the Gaussian approximation exact [21, 22]. The deformation we adopt breaks the $R$-symmetry from Spin$(N)_R$ to Spin$(N - 2) \times SO(2)_R$. For the details, readers may refer to the appendix and [14, 21]. In the subsequent computation for confirming dualities, it is crucial that we consider $O(N)$ theory instead of $SO(N)$ for the $OSp(N|2M)$ theory.

Let us first consider the simplest case $U(1|1)$ theory. The index for the $U(1|1)$ theory is given by

$$I_{U(1|1)}(x) = 1 + 4x + 11x^2 + 25x^3 + 12x^5 + 44x^6 + 42x^7 + 32x^8 + O(x^9).$$  \hspace{1cm} (2.5)

On the other hand, the indices for the $OSp(2|2)$ theory and the $OSp(4|2)$ theory are given by

$$I(x)_{OSp(2|2)} = 1 + 4x + 11x^2 + 25x^3 + 12x^5 + 44x^6 + 24x^7 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ (2.6)$$

$$I(x)_{OSp(4|2)} = 1 + 4x + 4x^2 + 12x^3 + 8x^4 + 27x^5 + 27x^6 + 36x^7 + 36x^8 + O(x^9). \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ (2.7)$$

The $U(1|1)$ theory and the $OSp(2|2)$ theory have the same index while the $OSp(4|2)$ theory doesn’t. Therefore, we provide the evidence that the $U(1|1)$ theory and the $OSp(2|2)$ theory are a dual pair while one can expect that the $U(3|1)$ theory and the $OSp(4|2)$ theory are another dual pair. The index for the $U(3|1)$ theory is given by

$$I_{U(3|1)}(x) = 1 + 4x + 4x^2 + 12x^3 + 8x^4 + 27x^5 + O(x^6), \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ (2.8)$$

which is the same as that for the $OSp(4|2)$ theory as expected, up to the order of $x^4$. In a similar manner, we tested the duality for some low rank cases. The result is given in Table [1].

One can explicitly examine the computation result of the index in detail. For example, the result indicates that every dual pair has four gauge-invariant operators with $\epsilon + j = 1$. By state-operator correspondence of the conformal field theory, gauge invariant operators on $\mathbb{R}^3$ have corresponding states in $\mathbb{R} \times S^2$. These states can be easily found and are given as follows:

$$\bar{A}^a \bar{B}^b |0, \ldots; 0, \ldots\rangle$$  \hspace{1cm} (2.9)
The superconformal index

| $U_4 \times U_{-4}$ | $O_2 \times USp_{-1}$ | The superconformal index |
|---------------------|------------------------|--------------------------|
| $U(1|1)$            | $OSp(2|2)$              | $1 + 4x + 11x^2 + 12x^3 + 25x^4 + 12x^5 + 44x^6 + 24x^7 + 32x^8 + O(x^9)$ |
| $OSp(2|2)$          |                        | $1 + 4x + 11x^2 + 12x^3 + 25x^4 + 12x^5 + 44x^6 + 24x^7 + 32x^8 + O(x^9)$ |
| $U(3|1)$            | $OSp(4|2)$              | $1 + 4x + 12x^2 + 8x^3 + 27x^4 + O(x^5)$ |
| $OSp(4|2)$          |                        | $1 + 4x + 12x^2 + 8x^3 + 27x^4 + 36x^5 - 36x^6 + O(x^7)$ |
| $U(2|2)$            | $OSp(4|4)$              | $1 + 4x + 22x^2 + 56x^3 + 131x^4 + 252x^5 + 516x^6 + O(x^7)$ |
| $OSp(4|4)$          |                        | $1 + 4x + 22x^2 + 56x^3 + 131x^4 + O(x^5)$ |
| $U(3|2)$            | $OSp(5|4)$              | $1 + 4x + 22x^2 + 60x^3 + 134x^4 + 200x^5 + 556x^6 + O(x^7)$ |
| $OSp(5|4)$          |                        | $1 + 4x + 22x^2 + 60x^3 + 134x^4 + 200x^5 + 556x^6 + O(x^7)$ |
| $U(2|1)$            | $OSp(3|2)/USp(2)_1 \times O(3)_{-2}$ | $1 + 4x + 12x^2 + 8x^3 + 27x^4 + 32x^5 - 20x^6 + 128x^7 - 65x^8 + O(x^9)$ |
| $U(4|1)$            |                        | $1 + 4x + 12x^2 + O(x^3)$ |

Table 1: The superconformal indices for some low rank dual pairs.

for the $U(M|N)$ theory and

$$\tilde{A}^{[a}\tilde{A}^{b]}|0,\cdots;0,\cdots\rangle,$$ \hspace{1cm} (2.10)

for the $OSp(M|2N)$ theory where $|m,n,\cdots;m,n,\cdots\rangle$ is a bare monopole state.

The flavor indices $a,b$ and $\tilde{b}$ run over 1, 2. The omitted gauge indices for (2.9) and (2.10) are properly contracted to form gauge invariant states. The expression for the gauge indices of (2.11) is schematic. The expression means that the matter fields are excited satisfying the Gauss law constraint such that the states having nonvanishing GNO charges are gauge invariant. These states exist regardless of the rank of the gauge group. The existence of such states is crucial to examine the supersymmetry enhancement in the next section.

### 3. Supersymmetry enhancement

The relevant facts about the $\mathcal{N} \geq 4$ superconformal algebra are explained at \cite{23}.

We are interested in the stress tensor multiplet. The lowest component of the stress tensor multiplet is an $SO(3)$ scalar and an antisymmetric rank 4 tensor of $Spin(\mathcal{N})_R$ with the conformal dimension $\epsilon = 1$, where $SO(3)$ denotes the rotation group of $\mathbb{R}^3$. Another component of our interest is $R$-current, which is antisymmetric rank 2 $Spin(\mathcal{N})_R$ tensor with the conformal dimension 2. Starting from one component,

\footnote{See also \cite{24} for related but incomplete discussion of the enhanced supersymmetry for $\mathcal{N} = 5$ theory.}
one obtains another component of the different conformal dimension by acting supercharges and its conjugate. For example, $R$-currents can be obtained from the scalar components by acting supercharges twice so that the $R$-current components have spin 1 with respect to $SO(3)$.

If there is a conserved global current, it belongs to a supermultiplet. The lowest component of this supermultiplet is an $SO(3)$ scalar which is an antisymmetric 2nd rank tensor of $Spin(N)_{R}$ with $\epsilon = 1$. Note that for $N = 6$, the lowest component of the stress tensor multiplet and that of the global current multiplet are on the same representation of $Spin(6)_{R}$ with the same conformal dimension. It is indeed argued that this global part is a part of the stress tensor multiplet so that every $N = 6$ has the global $U(1)$ symmetry.

Now we would like to argue that $R$-symmetry $Spin(5)_{R}$ is enhanced to $Spin(6)_{R}$ for the special case of $N = 5$ theory we discussed before so that it has $N = 6$. The strategy is to look for the lowest scalar component of the stress tensor multiplet then obtain the needed $R$-currents by acting superconformal generators on the scalar component.

Let us consider the lowest scalar component. For $N = 6$ this is the rank 4 antisymmetric tensor representation of $Spin(6)_{R}$, $15$. It decomposes under $Spin(5)_{R} \subset Spin(6)_{R}$ as $15 = 5 \oplus 10$ where 5 and 10 are respectively rank 4 and rank 3 antisymmetric tensor representations of $Spin(5)_{R}$. 5 is the representation of the lowest scalar component of the $Spin(5)_{R}$ stress tensor multiplet. As explained at the previous section, we adopted the deformation breaking the $Spin(5)_{R}$ $R$-symmetry down to $Spin(3) \times SO(2)_{R} \simeq SU(2) \times U(1)_{R}$, under which the 3rd rank tensor representation 10 of $Spin(5)_{R}$ decomposes as $10 = 1_{0} \oplus 3_{0} \oplus 3_{1} \oplus 3_{-1}$.

If a scalar state in the representation $3_{1}$ has energy 1, it is a BPS state such that it must appear in the superconformal index. Indeed, in the previous section we found four scalar BPS states with energy 1 for the $OSp(M|2N)$ theory: $\tilde{A}^{[a}_{\hat{1}_1} \tilde{A}^{b]}_{\hat{1}_1} |0, \cdots ; 0, \cdots \rangle$ and $\tilde{A}^{[a}_{\hat{1}_1} \tilde{A}^{b]}_{\hat{1}_1} |1, \cdots ; 1, 0, \cdots \rangle$. According to [11], as one varies a parameter, cohomology classes appear and disappear in pairs so that members of the pair have $R$-charge differing by 1 and energy, angular momentum differing by 1/2. Since there is no spinor BPS state that has energy 1/2 and $R$-charge 0, or energy 3/2 and $R$-charge 2, the above four scalar states are protected from the deformation; i.e., they still exist in the undeformed theory. The first state with vanishing GNO charges is in the representation $1_{1}$ of $SU(2) \times U(1)_{R}$ and would be in the representation 5 of $Spin(5)_{R}$. What we are really interested in are the next three. The next three states are in the representation $3_{1}$ of $SU(2) \times U(1)_{R}$. Furthermore, they must be in a representation of $Spin(5)_{R}$, the $R$-symmetry of the undeformed theory. The multiplet $3_{1}$ must lie in

\footnote{This is why we examine the lowest component of the stress tensor multiplet at first instead of the $R$-currents themselves. By looking at the scalar component, it’s easier to argue that wanted states exist in the strong coupling region.}
a representation of $\text{Spin}(5)_R$ having the highest weight $(1,1)$ because the multiplet $3_1$ contains BPS states with a weight $(1,1)$. Representation of $\text{Spin}(5)_R$ containing $(1,1)$ as the highest weight is the rank 3 antisymmetric tensor representation $\mathbf{10}$. Therefore, we conclude that three scalar BPS states $\vec{A}_1^{(a_1 b_1)} |1,0,\cdots;1,0,\cdots\rangle$ are in the representation $\mathbf{10}$ of $\text{Spin}(5)_R$ and that the undeformed $\text{OSp}(M|2N)$ theory has the 3rd rank antisymmetric tensor multiplet $\mathbf{10}$. Combined with $\mathbf{5}$ of the lowest scalar component of the $\text{Spin}(5)_R$, these provide the lowest scalar component of the $\text{Spin}(6)_R$.

Once we obtain the needed scalar component, we can obtain $R$-currents by acting superconformal generators. By acting $Q^{[a}Q^{b]}$ on the scalar component $\mathbf{10}$, we obtain vector states of spin 1 which are in the representation $\mathbf{5}$ of $\text{Spin}(5)_R$. Since the vector states in $\mathbf{5}$ have energy 2, operators corresponding to those vector states are conserved currents by unitarity. We now have additional conserved currents in the representation $\mathbf{5}$ of $\text{Spin}(5)_R$ along with the $R$-currents in the adjoint representation $\mathbf{10}$ of $\text{Spin}(5)_R$. They must fit into the adjoint representation of some Lie group. In this case, it should be $\text{Spin}(6)$. The adjoint representation of $\text{Spin}(6)$ decomposes under its subgroup $\text{Spin}(5)$ as $\mathbf{15} = \mathbf{10} \oplus \mathbf{5}$. Thus, $\mathcal{N} = 5$ supersymmetry of the $\text{OSp}(M|2N)$ theory is enhanced to $\mathcal{N} = 6$. Note that this enhancement occurs only for $O(M)_{2k} \times USp(2N)_{-k}$ with $k = 1$. The BPS states transforming as $3_1$ having energy 1 at $(2,1)$ exist only for $k = 1$. For higher $k > 1$ one cannot have such states with energy 1 due to the Gauss constraints which require higher energy states.

By slightly modifying the above argument, one can show that $\mathcal{N} = 5$ superconformal theory with $U(1)$ global symmetry leads to $\mathcal{N} = 6$ superconformal theory. Note that $U(1)$ global current belongs to a supermultiplet, whose lowest scalar component has conformal dimension or energy $\epsilon = 1$ and 2nd rank antisymmetric tensor of $\text{Spin}(5)_R$, $\mathbf{10}$. We already have the scalar component of rank 4 tensor of $\text{Spin}(5)_R$ in the stress tensor multiplet, which transforms as $\mathbf{5}$. By applying $Q^{[a}Q^{b]}$ on the multiplet $\mathbf{10}$ again, we obtain vector states $\mathbf{5}$ of $\epsilon = 2$ while acting on $\mathbf{5}$ we have vector states $\mathbf{10}$. Together they transform as the adjoint representation of $\text{SO}(6)_R$ and we have $\mathcal{N} = 6$ superconformal theory. Note that the conserved current for the $U(1)$ global symmetry exists apart from such $\mathbf{15}$ $R$-currents, which means that

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6We also have a $\text{Spin}(5)_R$ singlet $\mathbf{1}$ vector state of spin 1 due to the equivalence of the 2nd rank and 3rd rank antisymmetric representations of $\text{Spin}(5)_R$. This is consistent with the conclusion for $\mathcal{N} = 6$ theories of the fact that a $\mathcal{N} = 6$ theory always has the $U(1)$ global symmetry, which must be also true for our theories if they indeed have $\mathcal{N} = 6$ symmetry. One can check that norms of vector states $\mathbf{1}$ and $\mathbf{5}$ do not vanish, for example, by decomposing the $\mathcal{N} = 6$ stress tensor multiplet with respect to the $\mathcal{N} = 5$ subalgebra because the norms are completely determined by the superconformal algebra alone. The $\mathcal{N} = 6$ stress tensor multiplet contains the scalar component $\mathbf{15}$ of $\text{Spin}(6)_R$, which decomposes as $\mathbf{15} = \mathbf{5} \oplus \mathbf{10}$ under $\text{Spin}(5) \subset \text{Spin}(6)_R$, and the vector components $\mathbf{15} \oplus \mathbf{1}$ of $\text{Spin}(6)_R$, which again decomposes as $\mathbf{15} + 1 = \mathbf{10} \oplus 5 \oplus 1$ under the $\text{Spin}(5)$. Thus, any $\mathcal{N} = 5$ theory having a scalar component $\mathbf{10}$ of $\text{Spin}(5)_R$ indeed has vector components $\mathbf{5} \oplus \mathbf{1}$ of $\text{Spin}(5)_R$ with nonvanishing norm.
\( \mathcal{N} = 6 \) theory still has that \( U(1) \) symmetry as a global symmetry.

4. Large \( N \) index for \( \mathcal{N} = 5 \) theories and gravity index on \( AdS_4 \times S^7 / D_k \)

In this section, we will argue that the superconformal index for \( \mathcal{N} = 5 \) theory at large \( N \) exactly matches the gravity index on \( AdS_4 \times S^7 / D_k \). Especially for \( k = 1, D_1 = Z_4 \) and the exact match gives an additional evidence for the equivalence between \( OSp \) type \( \mathcal{N} = 5 \) theory with \( k = 1 \) and \( U \) type \( \mathcal{N} = 6 \) theory with \( k = 4 \) at large \( N \). At large \( N \), the difference between \( OSp(2N|2N) \) and \( OSp(2N+1|2N) \) is negligible and two theories give the same index. The subtlety between \( SO(N) \) and \( O(N) \) is also negligible at large \( N \).

In the appendix we derive the superconformal index for \( SO(2N)_{2k} \times USp(2N)_{-k} \), which is given by

\[
I^{SO(2N)_{2k} \times USp(2N)_{-k}} = \sum_{\{n_i, \tilde{n}_i\}} \frac{x^0}{(2\pi)^2} \exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} I_{sp}(x^n, e^{in\lambda_i}, e^{in\tilde{\lambda}_i}) \right].
\]

with the single letter index \( I_{sp} \) is

\[
I_{sp}(x, e^{i\lambda_i}, e^{i\tilde{\lambda}_i}) = f(x) \sum \sum_{i,j} \left( e^{\pm i(\lambda_i - \tilde{\lambda}_j)x|n_i - \tilde{n}_j|} + e^{\pm i(\lambda_i + \tilde{\lambda}_j)x|n_i + \tilde{n}_j|} \right) - \sum \sum_{i<j} \left( e^{\pm i(\lambda_i - \tilde{\lambda}_j)x|n_i - n_j|} + e^{\pm i(\lambda_i + \tilde{\lambda}_j)x|n_i + n_j|} \right) - \sum e^{\pm 2i\lambda_i x|2\tilde{n}_i|} - \sum e^{\pm 2i\tilde{\lambda}_i x|2n_i|}
\]

where \( f(x) := \frac{2x\frac{1}{x}}{1 + x} \) (4.2)

Now we will take the large \( N \) limit on the superconformal index. From eq. (4.2), we will use a similar large \( N \) analysis technique used in [21] (see also [26]). To take the large \( N \) limit, we first introduce \( (n = 0, 1, \ldots) \)

\[
\rho_n = \sum_{j=N_1+1}^{N} e^{i n \lambda_j} + e^{-i n \lambda_j}, \quad \chi_n = \sum_{j=N_2+1}^{N} e^{i n \tilde{\lambda}_j} + e^{-i n \tilde{\lambda}_j}.
\]

We assume that first \( N_1 \) (\( N_2 \)) monopole fluxes for \( SO(2N) \) (\( USp(2N) \)) are non-zero
and the rest are all zero. In terms of \((\rho_n, \chi_n)\) variables,

\[
\exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} I_{sp}(x^n, e^{i n \lambda_i}, e^{i n \tilde{\lambda}_i}) \right] \\
= \exp \left[ - \sum_{p \text{ odd}}^{\infty} \frac{1}{p} \rho_p^2 - \sum_{p \text{ even}}^{\infty} \frac{1}{2p} (\rho_p^2 - 2\rho_p) - \sum_{p \text{ odd}}^{\infty} \frac{1}{2p} \chi_p^2 - \sum_{p \text{ even}}^{\infty} \frac{1}{2p} (\chi_p^2 + 2\chi_p) + \sum_{p} \frac{1}{p} f(x^p) \rho_p \chi_p \right] \\
\times \exp \left[ \sum_{p=1}^{\infty} \frac{1}{p} \rho_p \left( \sum_{i=1}^{N_2} x^{p|\tilde{n}_i|}(e^{ip\lambda_i} + e^{-ip\lambda_i}) f(x^p) - \sum_{i=1}^{N_1} x^{p|n_i|}(e^{ip\lambda_i} + e^{-ip\lambda_i}) \right) \right] \\
\times \exp \left[ \sum_{p=1}^{\infty} \frac{1}{p} \chi_p \left( \sum_{i=1}^{N_1} x^{p|n_i|}(e^{ip\lambda_i} + e^{-ip\lambda_i}) f(x^p) - \sum_{i=1}^{N_2} x^{p|\tilde{n}_i|}(e^{ip\lambda_i} + e^{-ip\lambda_i}) \right) \right] \\
\times \exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} I_{sp}^{OSp(2N_1|2N_2)}(x^n, e^{i n \lambda_i}, e^{i n \tilde{\lambda}_i}) \right]. \tag{4.4}
\]

where \(I_{sp}^{OSp(2N_1|2N_2)}\) denotes the single letter index for \(SO(2N_1) \times USp(2N_2)\) theory, which is the same as \([4.2]\) except that the index \(i\) in \((\lambda_i, n_i)\) (or \((\tilde{\lambda}_i, \tilde{n}_i)\)) runs from 1 to \(N_1\) (or \(N_2\)). In the large \(N\) limit, the holonomy variable integrations can be replace by integration of \((\rho_n, \chi_n)\) variables

\[
\int \prod_{i=1}^{\infty} \frac{d\lambda_i d\tilde{\lambda}_i}{(2\pi)^2} \rightarrow \int \prod_{i} d\rho_i d\chi_i. \tag{4.5}
\]

The infinite dimensional integral for \((\rho_n, \chi_n)\) is gaussian and can be easily performed. Doing the gaussian integration and simplifying the formula, we finally get

\[
I_{N \to \infty}^{O(2N)_{2k} \times Sp(2N)_{-k}}(x) = I^{(0)}(x) I'(x). \tag{4.6}
\]

\(I^{(0)}(x)\) comes from zero monopole fluxes.

\[
I^{(0)}(x) = \prod_{n=1}^{\infty} \frac{1}{\sqrt{1 - f^2(x^n)}} \exp \left[ - \frac{f(x^{2n})}{2n(1 + f(x^{2n}))} \right]. \tag{4.7}
\]

\(I'(x)\) is given by

\[
I'(x) = \sum_{\{n_i\}, \{\tilde{n}_i\}} \frac{x^{\theta_0}}{(\text{sym})} \int \left( \prod \frac{d\lambda_i}{2\pi} \right) e^{2iK \sum_{i,j} (n_i \lambda_i - \tilde{n}_i \tilde{\lambda}_i)} \exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} I_{sp}(x^n, e^{i n \lambda_i}, e^{i n \tilde{\lambda}_i}) \right],
\]

where

\[
I_{sp}'(x, e^{i \lambda_i}, e^{i \tilde{\lambda}_i}) = f(x) \sum_{i,j} e^{\pm i (\lambda_i - \tilde{\lambda}_j)} (x^{n_i - \tilde{n}_j} - x^{n_i + n_j}) \\
- \sum_{i<j} e^{\pm i (\lambda_i - \tilde{\lambda}_j)} (x^{n_i - n_j} - x^{n_i + n_j}) - \sum_{i<j} e^{\pm i (\tilde{\lambda}_i - \lambda_j)} (x^{\tilde{n}_i - n_j} - x^{\tilde{n}_i + \tilde{n}_j}).
\]
By comparing the above formulae with the large $N$ index formulae for $U(N)_k \times U(N)_{-k}$ theory in [21], one can see that

$$I'(x) = I_{N \to \infty:(+)}^{U(N)_{2k} \times U(N)_{-2k}}(x)$$

(4.8)

except the Casimir energy $\epsilon_0$. The difference between Casimir energies in two formulae is $\sum n_i - \sum \tilde{n}_i$. However, as already noticed in [21], only monopole fluxes satisfying $\sum n_i = \sum \tilde{n}_i$ contribute to the large $N$ index and thus the difference in $\epsilon_0$ vanishes at large $N$. In [21] the large $N$ index for $U(N)_k \times U(N)_{-k}$ theory is shown to be factorized into three factors, contribution from zero monopole flux and contributions from positive/negative monopole fluxes only. $I_{N \to \infty:(+)}^{U(N)_k \times U(N)_{-k}}$ denote the factor from positive fluxes only which is actually the same as the factor from negative fluxes only. Note that for $\mathcal{N} = 5$ theories we only need to consider contributions from positive monopole fluxes only, thanks to Weyl symmetries. Thus we found following relation between large index for $\mathcal{N} = 5$ and $\mathcal{N} = 6$ theories.

$$I_{N \to \infty}^{O(2N)_{2k} \times USp(2N)_{-k}}(x) = I^{(0)}(x) I_{N \to \infty:(+)}^{U(N)_{2k} \times U(N)_{-2k}}(x).$$

(4.9)

To show the equality between the large $N$ index and gravity index on $S^7/D_k$, we will assume that the large $N$ index for $U(N)_k \times U(N)_{-k}$ is the same as gravity index on $S^7/Z_k$. This is not yet proved but checked in various sectors [21] and believed to be true. Two generators $\alpha, \beta$ of the dihedral group $D_k$ act on $S^7$ as (see section 3 in [23])

$$\alpha := \exp\left(\frac{2\pi i}{k} J_3\right), \quad \beta := \exp(\pi i J_2).$$

(4.10)

$J_{1,2,3}$ are three generators (with normalization $[J_i, J_j] = i\epsilon_{ijk} J_k$) of $SU(2) \simeq SO(3)$ in $SO(5) \times SO(3) \subset SO(8)$, isometry group on $S^7$. $J_3$ can be identified with the baryonic $U(1)_b$ symmetry in $U(N|N)$ theories. Graviton spectrum on $S^7/D_k$ can obtained by keeping only $D_k$ invariant states in graviton spectrum on $S^7$. In terms of the $SU(2)$ charges, the $D_k$ invariant states can be divided by two types

- type $I : |\ell, m = 0\rangle, \quad \ell \in 2\mathbb{Z},$
- type $II : (|\ell, m\rangle + |\ell, -m\rangle), \quad \ell \in \mathbb{Z}, \quad m \in k\mathbb{Z}_+.$

(4.11)

$\mathbb{Z}$ and $\mathbb{Z}_+$ denote the set of integers and of positive integers respectively. States $|\ell, m\rangle$ are represented by its total angular momentum $\ell$ and angular momentum in the 3rd direction, $m = J_3$. In the second line, we used the fact that $\{J_3, \beta\} = 0$ and $\beta^2 = 1$ in integer-spin representations of $SU(2)$. Gravity index from gravitons of type $I$ are analyzed in [23] and it gives exactly the same factor $I^{(0)}(x)$ in (4.7). Gravitons of type $II$ can be thought as $Z_2$ invariant gravitons on $S^7/Z_{2k}$ with non-zero $U(1)_b$ charge where the $Z_2$ flips the sign of $U(1)_b$ charge. Thus, the gravity index from gravitons...
of type II is $I_{S^7/Z_{2k}}^{(\pm)}$, gravity index from single graviton with positive $U(1)_b$ charge on $S^7/Z_{2k}$. By assuming the equality between large $N$ index for $U(N|N)$ theory and gravity index on $S^7/Z_k$, $I_{S^7/Z_{2k}}^{(\pm)}$ is nothing but $I_{N \rightarrow \infty}^{U(N)_{2k} \times U(N)_{-2k}}$. In summary, we found that

$$I_{S^7/D_k} = I^{(0)}(\text{from gravitons of type I}) \times I_{N \rightarrow \infty}^{U(N)_{2k} \times U(N)_{-2k}}(\text{from gravitons of type II}) . \quad (4.12)$$

This perfectly matches the large $N$ index in eq. (4.9) for $O(2N)_{2k} \times USp(2N)_{-k}$ theory. Note that for $k = 1$, $D_1 = Z_4$ so that the large $N$ index for $O(2N)_2 \times USp(2N)_{-1}$ theory is the same as the gravity index on $AdS_{S^7/Z_4}$, which in turn is the same as the large $N$ index for $U(N)_4 \times U(N)_{-4}$ theory.

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### A. Superconformal index formulae for the 3-d $O(M) \times USp(2N)$ gauge theory

The index for $SO(2N)_{2k} \times USp(2M)_{-k}$ theory can be written in the following form.

$$I_{SO(2N)_{2k} \times USp(2N)_{-k}}^{SO(2N)_{2k} \times USp(2N)_{-k}} = \sum_{\{n_i\}, \{\tilde{n}_i\}} \frac{x^{\epsilon_0}}{(\text{sym})} \int \prod d\lambda_i d\tilde{\lambda}_i (2\pi)^2 e^{2ik \sum_i (n_i \lambda_i - \tilde{n}_i \tilde{\lambda}_i)} \exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} I_{sp}(x^n, e^{i n \lambda_i}, e^{i n \tilde{\lambda}_i}) \right] . \quad (A.1)$$

Here $(\lambda_i, n_i)_{i=1,...,N}$ and $(\tilde{\lambda}_i, \tilde{n}_i)_{i=1,...,M}$ are (holonomy variables, monopole fluxes) for $SO(2N)$ and $USp(2N)$ respectively. Using an Weyl action of gauge group, one can take all the monopole fluxes be non-negative, $n_i, \tilde{n}_i \geq 0$. (sym) denote a symmetry factor, order (number of elements) of Weyl group for the unbroken gauge group by monopole fluxes. The Casimir energy $\epsilon_0$ is given by

$$\epsilon_0 = \sum_{i,j} |n_i - \tilde{n}_j| - \sum_{i<j} |n_i - n_j| - \sum_{i<j} |\tilde{n}_i - \tilde{n}_j| + \sum_i n_i - \sum_i \tilde{n}_i . \quad (A.2)$$
The single letter index $I_{sp}$ is
\[
I_{sp}(x, e^{i\lambda_i}, e^{i\tilde{\lambda}_i}) = f(x) \sum_{\pm} \sum_{i,j} (e^{\pm i(\lambda_i - \tilde{\lambda}_i)} x^{n_i - \tilde{n}_j} + e^{\pm i(\lambda_i + \tilde{\lambda}_i)} x^{n_i + \tilde{n}_j})
- \sum_{\pm} \sum_{i<j}^M (e^{\pm i(\lambda_i - \tilde{\lambda}_j)} x^{\tilde{n}_i - n_j} + e^{\pm i(\lambda_i + \tilde{\lambda}_j)} x^{\tilde{n}_i + n_j}) - \sum_{\pm}^M \sum_{i} e^{\pm 2i\tilde{\lambda}_i} x^{2\tilde{n}_i}
- \sum_{\pm}^N \sum_{i<j} (e^{\pm i(\lambda_i - \lambda_j)} x^{n_i - n_j} + e^{\pm i(\lambda_i + \lambda_j)} x^{n_i + n_j}) - \sum_{\pm}^N \sum_{i} e^{\pm i\lambda_i} x^{n_i}
\]
, where $f(x) := \frac{2x^{\frac{1}{2}}}{1 + x}$. (A.3)

We should consider the additional projection for $Z_2$ element of $O(2N)$ not belonging to $SO(2N)$ group. We choose the specific $Z_2$ action,
\[
Z_2 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}
\] (A.4)

Under this $Z_2$ action, the eigenvalues of the holonomy and the monopole are projected into
\[
e^{\pm i\lambda_i} \to \pm 1, \quad \pm n_i \to 0.
\] (A.5)

The other variables are not affected.

The single letter index for $SO(2N + 1)_{2k} \times USp(2M)_{-k}$ is given by
\[
I_{sp}(x, e^{i\lambda_i}, e^{i\tilde{\lambda}_i}) = f(x) \sum_{\pm} \sum_{i,j} (e^{\pm i(\lambda_i - \tilde{\lambda}_j)} x^{n_i - \tilde{n}_j} + e^{\pm i(\lambda_i + \tilde{\lambda}_j)} x^{n_i + \tilde{n}_j} + e^{\pm i\tilde{\lambda}_j} x^{\tilde{n}_j})
- \sum_{\pm} \sum_{i<j}^M (e^{\pm i(\lambda_i - \tilde{\lambda}_j)} x^{\tilde{n}_i - n_j} + e^{\pm i(\lambda_i + \tilde{\lambda}_j)} x^{\tilde{n}_i + n_j}) - \sum_{\pm}^M \sum_{i} e^{\pm 2i\tilde{\lambda}_i} x^{2\tilde{n}_i}
- \sum_{\pm}^N \sum_{i<j} (e^{\pm i(\lambda_i - \lambda_j)} x^{n_i - n_j} + e^{\pm i(\lambda_i + \lambda_j)} x^{n_i + n_j}) - \sum_{\pm}^N \sum_{i} e^{\pm i\lambda_i} x^{n_i}
\]
, where $f(x) := \frac{2x^{\frac{1}{2}}}{1 + x}$. (A.6)

Let us turn to $O(2N + 1)$ theory. In this case, we choose $Z_2$ action,
\[
Z_2 = \begin{pmatrix} 1 & \cdots & 0 \\ \cdots & \ddots & \cdots \\ 0 & \cdots & 1 \end{pmatrix}
\] (A.6)

an eigenvalue 1 of the holonomy in the fundamental representation is projected by
\[
1 \to -1
\] (A.7)
while the others are not influenced. Furthermore, eigenvalues $e^{\pm i\lambda_i}$ of the holonomy in the adjoint representation are projected by

$$e^{\pm i\lambda_i} = e^{\pm i\lambda_i} \cdot 1 \rightarrow e^{\pm i\lambda_i} \cdot (-1) \quad (A.8)$$

while the others, which are in the form of $e^{i(\pm \lambda_i \pm \lambda_j)} = e^{\pm i\lambda_i} \cdot e^{\pm i\lambda_j}$, are not affected, either.

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