Effect of electron collisions on steady states of plasma diodes with an electron potential barrier

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Abstract. Solutions corresponding to steady states of the Pierce diode are derived in the presence of collisions of electrons with background particles and reflection of electrons from the virtual cathode is investigated. Two families of solutions: Bursian and non-Bursian are explored. It is demonstrated that all non-Bursian solutions disappear when the ratio of the average collision frequency to the electron plasma frequency is of the order of 10%.

1. Introduction

To optimize performances of the thermionic energy converter (TIC) with a high emitter temperature, it is necessary to take into account collisions of electrons with atoms and ions, since the maximum specific power of such a device is reached at cesium vapor pressures corresponding to the transition region from collisionless to collisional mode for electrons [1]. In addition, in this mode, a potential barrier exists in the plasma, which reflects portion of electrons back to the emitter. Many necessary results for TIC can be obtained analytically using the Pierce diode model, in which a monoenergetic electron flow moves through a uniform background of immobile ions. This is due to the fact that the Pierce diode makes it possible to approximate with good accuracy the state of a collisionless TIC [2]. In our previous paper, [3], we studied the effect of electron-atom collisions on the steady states of the Pierce diode in the regime when there are no potential barriers (virtual cathode) for electrons in the plasma. In this paper, we solve a similar problem for the diode, in which some electrons are reflected from the virtual cathode.

2. Statement of the problem

We consider the planar diode with an electrode distance $d$ and a potential difference $U$ being applied across electrodes. The mono-energetic electron flow enters from the emitter ($z = 0$) with the density $n_0$ and velocity $v_0$ perpendicularly to the emitter surface. The inter-electrode space is uniformly occupied by infinitely massive immobile ions of constant density $n_i$. Ion background is taken into account through the dimensionless neutralization parameter

$$\gamma = n_i/n_0.$$  \hspace{1cm} (1)

We put $\gamma = 1$ in the paper. The electrons move with no collisions in the self-consistent electric field $E$. It can be calculated from the scalar potential $\varphi$ which depends on the coordinate $z$ only.
In the 1D time-independent case, the basic governing equations are the equation of continuity, the momentum equation and the Poisson’s one. According to Refs. [3], [4] in this case the presence of collisions can be incorporated in the momentum equation by a dissipative term which is proportional to the electron velocity with the coefficient \( \nu \) being ratio of an average collision frequency to the plasma frequency. Therefore the basic governing equations can be expressed in dimensionless form as:

\[
\begin{align*}
\nu u &= H(\zeta; \zeta_r, r), \quad u \frac{du}{d\zeta} = -\varepsilon - \nu u, \\
\varepsilon &= -\frac{d\eta}{d\zeta}, \quad \frac{d\zeta}{d\zeta} = -n + \gamma 
\end{align*}
\]

with the following boundary conditions: \( n(0) = 1, \ u(0) = 1, \ \eta(0) = 0 \) and \( \eta(\delta) = V \). In Eqs. (2) \( r \) is the reflection coefficient which stands for a portion of electrons reflected by the potential barrier [5] and \( H(\zeta; \zeta_r, r) = 1 + r \) when \( \zeta \) locates to the left of the reflection point \( \zeta_r \) and \( 1 - r \) – to the right of it. To express all relevant variables in terms of dimensionless quantities, we introduced energy and length units which are the kinetic energy of electrons at the emitter \( W_0 \) and the beam Debye length \( \lambda_D \), respectively

\[
\lambda_D = \left[ \frac{2e_0 W_0}{e^2 n_0} \right]^{1/2} \approx 0.3238 \cdot 10^{-2} \frac{V_0^{3/4}}{e^{1/2} \text{[cm]}}, \quad W_0 = m v_0^2/2. \tag{3}
\]

Here, the current density \( j_0 = e n_0 v_0 \) and accelerating voltage \( V_0 = W_0/e = m v_0^2/(2e) \) of the injected beam are taken in Amperes per square cm and Volts, respectively; the symbols \( e \) and \( m \) represent the electron charge and electron mass; the free-space permittivity \( \epsilon_0 = 8.854 \cdot 10^{-12} C^2/Nm^2 \). The dimensionless coordinate, time, velocity, potential and electric field strength are introduced as: \( \zeta = z/\lambda_D, \ t = t\omega_0, \ u = v/\sqrt{2W_0/m}, \ \eta = e\phi/(2W_0), \ \varepsilon = eE\lambda_D/(2W_0) \); here \( \omega_0 = \left[ e^2 n_0/(m\epsilon_0) \right]^{1/2} \) is the characteristic frequency. Dimensionless inter-electrode gap and voltage between collector and emitter are denoted via \( \delta \) and \( V \) respectively.

After proceeding to the Lagrangian coordinate \( \tau \) and carrying out the Lagrange transformation \( \zeta = \int_0^\tau u(\tau')d\tau' \) the system of equations (2) can be reduced to a single equation

\[
\frac{d^2u}{d\tau^2} + \nu \frac{du}{d\tau} + \gamma u = H(\zeta; \zeta_r, r) \tag{4}
\]

with initial conditions

\[
u u = H(\zeta; \zeta_r, r) \]

\[
\begin{align*}
\zeta(\tau) &= \frac{1 + r}{\gamma} \tau - \frac{1}{\gamma^2} (\gamma\varepsilon_0 + (1 + r)\nu) + \frac{1}{\gamma^2} \exp \left( -\frac{\nu}{2} \tau \right) \times \\
&\times \left\{ [\gamma\varepsilon_0 + (1 + r)\nu] \cos(\beta\tau) + \frac{1}{\beta} \left[ \nu \left( \gamma\varepsilon_0 + (1 + r)\nu \right) - \gamma(1 + r - \gamma) \right] \sin(\beta\tau) \right\}, \\
u(\tau) &= \frac{1 + r}{\gamma} - \frac{1}{\gamma} \exp \left( -\frac{\nu}{2} \tau \right) \times \\
&\times \left\{ (1 + r - \gamma) \cos(\beta\tau) + \frac{1}{\beta} \left[ \gamma(\varepsilon_0 + \nu) + \nu \left( 1 + r - \gamma \right) \right] \sin(\beta\tau) \right\}. \tag{6}
\end{align*}
\]
Here “effective frequency” $\beta = \sqrt{\gamma - \nu^2/4}$ and $\varepsilon_0$ is the emitter electric field strength.

At the reflection point $\zeta_r = \zeta(\tau_r)$ we have $u(\tau_r) = 0, \ d/du(\tau_r) = 0$. Relations between $\varepsilon_0$ and $\tau_r$ are given as following

$$
\begin{align*}
\cos(\beta \tau_r) &= \frac{\nu(\varepsilon_0 + \nu) + (1 + r - \gamma)}{1 + r} \exp\left(-\frac{\nu}{2 \tau_r}\right), \\
\sin(\beta \tau_r) &= \frac{\beta(\varepsilon_0 + \nu)}{1 + r} \exp\left(-\frac{\nu}{2 \tau_r}\right), \\
\gamma(\varepsilon_0 + \nu)^2 + \nu(1 + r - \gamma)(\varepsilon_0 + \nu) + (1 + r - \gamma)^2 &= (1 + r)^2 \exp(\nu \tau_r).
\end{align*}
$$

(7)

For the parameters of the reflection point we obtain

$$
\begin{align*}
\zeta_r &= \frac{1}{\gamma} [(1 + r)\tau_r - \varepsilon_0], \\
\eta_r &= \frac{1}{\gamma} \left[(1 + r)(\nu \zeta_r - 1) + \frac{\varepsilon_0^2}{2}\right].
\end{align*}
$$

(8)

To the right of the reflection point $\zeta_r$ the solutions are

$$
\begin{align*}
\zeta(\tau) &= \zeta_r + \frac{1 - r}{\gamma} \left[\frac{1 - r}{\gamma^2} - \frac{1 - r}{\gamma^2} \nu + \frac{1 - r}{\gamma^2} \exp\left(-\frac{\nu}{2 (\tau - \tau_r)}\right)\right] \\
&\quad \times \left[\nu \cos \beta(\tau - \tau_r) - \frac{1}{\beta} \left(\gamma - \frac{\nu^2}{2}\right) \sin \beta(\tau - \tau_r)\right], \\
\eta &= \eta_r - \frac{\gamma}{2} (\zeta - \zeta_r)^2 + (1 - r)(\tau - \tau_r)(\zeta - \zeta_r) - \frac{(1 - r)^2}{2 \gamma} (\tau - \tau_r)^2 \\
&\quad + \frac{(1 - r)^2}{\gamma^2} \nu(\tau - \tau_r) + \frac{(1 - r)^2}{\gamma^3} (\gamma - \nu^2) - \frac{(1 - r)^2}{\gamma^3} \exp\left(-\frac{\nu}{2 (\tau - \tau_r)}\right) \\
&\quad \times \left[(\gamma - \nu^2) \cos \beta(\tau - \tau_r) + \frac{\nu}{2 \beta} (3 \gamma - \nu^2) \sin \beta(\tau - \tau_r)\right].
\end{align*}
$$

(9)

Figure 1 demonstrates an evolution of potential distributions (PD) with varying in the parameter

![Figure 1](image-url)

**Figure 1.** (a) – Potential distributions drawn for various values of $\nu$ and $\varepsilon_0$: $\nu=0.05, \varepsilon_0=0.98$ (curve 1), 0.06, 0.98 (2), 0.1, 0.97 (3) and 0.2, 0.95 (4); (b) – Time-independent states presented in $\varepsilon_0$ vs $\delta$ parametric curves drawn for $V = 0$ and various values of $\nu$: $\nu=0$ (curve 1), 0.02 (2), 0.05 (3) and 0.1 (4); $\gamma = 1$.

$\nu$. One can see that oscillatory-type solutions disappears with increase of $\nu$ and one minimum
potential solutions survive only. Mathematically, this phenomenon can be explained on the basis of the Eqs. (9). It follows from these formulas that the PD includes a term that grows linearly with $\tau$, as well as an oscillating term with a multiplier that decays according to the exponential law. As the parameter $\nu$ grows, the last term decreases strongly, and the curve $\eta(\zeta)$ goes above the line $\eta = V$ which corresponds to the boundary condition at the collector. For this reason, in the mode with no electron reflection, all solutions with $\nu > 0.06$ disappear as $\delta > 2\pi$.

Figure 2 presents the solutions in $\{\varepsilon_0, \delta\}$ parametric plane. It is seen that in the mode with no electron reflection the oscillatory-type solutions disappear at $0.05 < \nu < 0.10$. In the electron reflection mode, solutions of such type also disappear, and only solutions with one virtual cathode remain. We call such solutions the Bursian ones.

4. Conclusion
In the present paper, we studied time-independent solutions for the Pierce diode in the presence of electron-atom collisions. These solutions are obtained by the Lagrange method. All solutions can be assigned either to the Bursian or to non-Bursian (oscillating along the coordinate) families in the representation on the $\{\varepsilon_0, \delta\}$ plane. It is shown that the states corresponding to the non-Bursian family can exist only at a relatively small collision frequency.

Our study of the effect of electron collisions on the characteristics of a plasma diode gives us an opportunity to optimize the operation modes of TIC.

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