MINIMAL REALISTIC SU(5) SCENARIO

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We present phenomenological aspects of the simplest realistic SU(5) grand unified theory—the theory with the 5, 15, and 24 dimensional representations in the Higgs sector. We show that a successful gauge coupling unification sets experimentally accessible upper bound on the total proton decay lifetime. It also relates proton decay lifetime to scalar leptoquark mass in an experimentally testable manner. We also discuss an addition of gauge singlets—both fermions and bosons—to the simplest scenario and comment on relevant phenomenological consequences of such modifications.

1 Introduction

Grand unified theories (GUTs) are considered to be the most viable candidates for the physics beyond the Standard Model. Through matter unification and unification of strong and electroweak interactions they always generate two predictions: (1) gauge couplings unify and (2) proton decays. Testing the first prediction is practically impossible since unification takes place at energy scales beyond our reach. However, the second one can be experimentally investigated thereby offering a very promising way to test grand unification. It is thus important to single out and investigate grand unified theories where proton decay is both accurately predicted and experimentally reachable.

We have recently proposed\cite{1,2} the simplest realistic GUT model with both of those properties. It comes in a form of a particularly simple extension of the well-known Georgi-Glashow SU(5) model\cite{3}. In particular, the scenario contains not only the usual three generations of matter fields and the 5 and 24 dimensional Higgs representations, but also one 15 dimensional Higgs. And, it includes all possible SU(5) invariant operators.

Despite a large number of parameters in the Lagrangian the scenario is still very predictive. This predictivity is primarily attributed to the simplicity of the Higgs sector. But, two generic features of non-supersymmetric SU(5) framework also boost predictivity. Firstly, SU(5) is
the only simple group with the Standard Model (SM) embedding that has unique single step symmetry breaking. This allows for an accurate determination of the unified scale—the so-called GUT scale $M_{\text{GUT}}$. Secondly, the least model dependent and usually dominant contribution to proton decay comes from an exchange of only one set of superheavy gauge bosons with mass $M_V$. Clearly, if $M_V$ is taken to define to the GUT scale the former property also implies accurate prediction for proton decay. That is exactly what happens in our case.

In the following, we briefly present the simplest realistic $SU(5)$ theory. Again, even though the proposed scenario has uncorrelated regions in the Yukawa sector, the simplicity of its Higgs sector guarantees both its testability and refutability in near future.

2 The simplest $SU(5)$ scenario

The scenario we propose has a following particle content of the Higgs sector: $5 = \Psi = (\Psi_D, \Psi_T) = (1, 2, 1/2) + (3, 1, -1/3)$, $15 = \Phi = (\Phi_a, \Phi_b, \Phi_c) = (1, 3, 1) + (3, 2, 1/6) + (6, 1, -2/3)$ and $24 = \Sigma = (\Sigma_8, \Sigma_3, \Sigma_{(3, 2)}, \Sigma_{(3, 2)}, \Sigma_{24}) = (8, 1, 0) + (1, 3, 0) + (3, 2, -5/6) + (\overline{3}, 2, 5/6) + (1, 1, 0)$, where we use the SM ($SU(3) \times SU(2) \times U(1)$) decomposition to set our notation. $\Sigma_{(3, 2)}$ and $\Sigma_{(3, 2)}$ are fields eaten by the superheavy gauge fields $V$. We define the GUT scale through their common mass, i.e., we set $M_V = M_{\text{GUT}}$. As always, a vacuum expectation value (VEV) of $\Sigma_{24}$ breaks $SU(5)$ while VEV of $\Psi_D$ triggers the SM symmetry breaking. During the latter stage $\Phi_a$ develops an induced VEV that eventually yields neutrino mass through the type II see-saw mechanism.

On renormalizable level our scenario predicts $Y_D = Y^T_E$ at the GUT scale, where $Y_{D(E)}$ is the down quark (charged lepton) Yukawa coupling matrix. This prediction however disagrees with experiments, especially in the case of the first and second generation. In order to correct that we include higher-dimensional $SU(5)$ invariant operators. (If we demand renormalizability we must introduce 45 dimensional Higgs representation. However, falsifiability of such an extension is not guaranteed.)

Clearly, our scenario accommodates realistic fermionic mass spectrum. What remains to be investigated is the issue of gauge unification and its compatibility with proton decay constraints.

2.1 Unification vs. proton decay

There are four masses—$M_{\text{GUT}}$, $M_{\Sigma_3}$, $M_{\Phi_a}$ and $M_{\Phi_b}$—and two equations that govern gauge coupling unification at the one-loop level in our case. Actually, there are three renormalization group equations—one for each gauge coupling of the SM. However, elimination of the unified coupling constant $\alpha_{\text{GUT}}$ leaves only two relevant equations. These are

$$\frac{B_{23}}{B_{12}} = 0.719 \pm 0.005, \quad \text{and} \quad \ln \frac{M_{\text{GUT}}}{M_Z} = \frac{184.9 \pm 0.2}{B_{12}},$$  \hspace{1cm} (1)$$

where the right-hand sides reflect the latest experimental measurements of the SM parameters. The left-hand sides depend on particular mass spectrum of the particle content of the theory at hand. More precisely, $B_{ij} = B_i - B_j$, where $B_i$ coefficients are given by:

$$B_i = b_i + \sum_I b_{iI} r_I, \quad r_I = \frac{\ln M_{\text{GUT}}/M_I}{\ln M_{\text{GUT}}/M_Z}.$$  \hspace{1cm} (2)$$

$b_i$ are the SM coefficients while $b_{iI}$ are the one-loop coefficients of any additional particle $I$ of mass $M_I$ ($M_Z \leq M_I \leq M_{\text{GUT}}$). (Recall, for the case of $n$ light Higgs doublet fields $b_1 = 40/10 + n/10$, $b_2 = -20/6 + n/6$ and $b_3 = -7$.) The $B_{ij}$-coefficient contributions in our scenario are listed in Table. Note that the SM case yields $B_{23}/B_{12} = 0.53$. This means that additional particles
Table 1: $B_{ij}$ coefficients.

| $B_{23}$ | $B_{12}$ |
|----------|----------|
| $\frac{11}{25}$ | $\frac{1}{3}$ |

with intermediate masses $M_I$ are required for successful unification. In our case these particles are clearly $\Sigma_3$, $\Phi_a$ and $\Phi_b$.

In this paper we present the outcome of the two-loop level unification analysis in terms of $M_{\Sigma_3}$ and $M_{\Phi_a}$ contours in the $M_{\text{GUT}}$–$M_{\Phi_b}$ plane in Fig. 1. Stars represent points that correspond to exact numerical unification while lines represent linear interpolation.

![Figure 1: Unification of the gauge couplings at the two-loop level. Stars correspond to exact numerical two-loop unification solutions. There are two sets of lines of constant value. The steeper set is associated with $M_{\Phi_a}$ and the other one represents the lines of constant $M_{\Sigma_3}$. All masses are in GeV units. The region to the left of the vertical dashed line is excluded by the proton decay experiments.](image-url)

The sail-like region in Fig. 1 represents the viable parameter space under the assumption that $\Psi_T$, $\Sigma_8$ and $\Phi_c$ are at or above the GUT scale. It is bounded from the left and below by experimental limits on $M_{\Phi_a}$ and $M_{\Phi_b}$, respectively. The right bound stems from the requirement that $M_{\Sigma_3} \geq M_Z$. Note that $\Phi_b$ is a scalar leptoquark and hence very interesting generator of new physics. For example, Large Hadron Collider (LHC) aims to place more stringent lower limits on the mass of $\Phi_b$ at 1 TeV.

The region to the left of the vertical thick dashed line in Fig. 1 is excluded by the present limits on the proton decay lifetime. In order to generate this bound we appropriately take the case of maximal flavor suppression of the gauge $d = 6$ proton decay operators and use $\alpha = 0.015 \text{GeV}^3$ for the value of nucleon matrix element. Experimental limit we take as input reads $\tau_p(p \to \pi^0 e^+) > 5.0 \times 10^{33}$ years.

What happens if we relax the $M_{\Psi_T}, M_{\Sigma_8}, M_{\Phi_c} \geq M_{\text{GUT}}$ assumption? As one lowers $M_{\Psi_T}$ and $M_{\Phi_c}$ the $M_{\Phi_a} = 130 \text{GeV}$ line in Fig. 1 moves slowly to the left while, at the same time, the $M_{\Sigma_3} = M_Z$ line moves very rapidly in the same direction until the allowed region shrinks to a point. Hence, any scenario in which $M_{\Psi_T}$ or $M_{\Phi_c}$ or both are below $M_{\text{GUT}}$ would be more exposed to the tests through the proton decay lifetime measurements and accelerator searches than the scenario shown in Fig. 1.

If, on the other hand, one lowers the mass of $\Sigma_8$, the $M_{\Phi_a} = 130 \text{GeV}$ line moves to the
right more rapidly than the $M_{\Sigma_3} = M_Z$ line until the allowed region becomes a point when $M_{\Sigma_3}$ reaches $M_Z$. At that point $M_{\text{GUT}}$ is $4.6 \times 10^{13}$ GeV for $M_{\Sigma_3} = M_Z$, $M_{\Phi_a} = 242$ GeV and $\alpha_{\text{GUT}}^{-1} = 37.06$. We use these particular values to derive an accurate upper bound on the proton decay lifetime:

$$\tau_p^{(\text{two-loop})} \leq 1.4 \times 10^{36} \text{ years}. \quad (3)$$

This bound follows from a more general model independent inequality:

$$\tau_p \leq 6 \times 10^{39} \alpha_{\text{GUT}}^{-2} (M_V/10^{10}\text{GeV})^4 (0.003\text{GeV}^5/\alpha)^2 \text{ years}, \quad (4)$$

which is applicable to any simple group with the SM embedding. (Note, in the case of scenarios with partial gauge coupling unification such as the Pati-Salam or flipped $SU(5)$ scenario the proton can be stable.)

Clearly, an improvement in the proton lifetime measurements by a $4^4$ factor is called for to completely rule out this GUT scenario. The situation is actually even more promising; even a mild improvement in the proton lifetime bounds (by a factor of fifteen) would make our scenario incompatible with exact unification unless either $\Phi_b$ or $\Sigma_3$ resides below $10^{10}$ GeV. (This certainly makes them accessible in accelerator experiments.) The next generation of proton decay experiments aims at improving lower bounds on partial lifetimes by a few orders of magnitude. For instance, the goal of Hyper-Kamiokande is to explore the proton lifetime at least up to $\tau_p/B(p \rightarrow e^+\pi^0) > 10^{35}$ years and $\tau_p/B(p \rightarrow K^+\bar{\nu}) > 10^{34}$ years in about 10 years. Thus, our minimal GUT scenario will be tested and/or ruled out at the next generation of proton decay and accelerator experiments.

3 Adding singlets

Let us comment on possible additions of $SU(5)$ singlets—both spin 1/2 and spin 0 particles—to our scenario.

As we argued, our scenario is tailor made to generate neutrino masses through the Type II seesaw mechanism. But, one can also introduce right-handed neutrinos—singlets of $SU(5)$—and generate additional neutrino mass contribution through the so-called Type I seesaw mechanism. That sort of addition obviously cannot change our discussion on the gauge coupling unification and related constraints—at least at the one-loop level. Hence, there is no need to modify our previous discussion on proton decay. However, the addition of right-handed neutrinos has potential to accommodate viable mechanism that explains the origin of the baryon asymmetry observed in the universe.

Among viable mechanisms to explain primordial matter-antimatter asymmetry, leptogenesis has undoubtedly become one of the most compelling scenarios. In our case, leptogenesis could proceed through the decay of the lightest triplet scalar $\Phi_a$. As has been shown the scalar triplet with a mass $M_{\Phi_a} \geq 10^{9-10}$ GeV constitutes a natural candidate for a successful triplet leptogenesis if our minimal $SU(5)$ scenario is extended with at least one singlet fermion field. Combining the above leptogenesis bounds with the ones shown in Fig. 1 we observe that successful triplet leptogenesis excludes the region of the parameter space where the mass of the triplet $M_{\Phi_a}$ is below $10^9-10^{10}$ GeV. This in turn would imply that leptoquark $\Phi_b$ could be light enough ($M_{\Phi_b} < 10^{6-7}$ GeV) to open the possibility of direct production at the next generation of collider experiments. It should be noted however that in general if both Type I and Type II seesaws are present leptogenesis could proceed in a way that avoids generation of any bounds on the mass spectrum of relevant particles.

Let us also comment on a possible addition of scalar $SU(5)$ singlet fields from point of view of a successful inflationary scenario. Namely, in view of the latest three-year WMAP
data there has been a renewed interest in a class of inflationary models based on a quartic Coleman-Weinberg potential where a gauge singlet scalar field plays a flaton role. It has been recently shown that such a class of models can be in good agreement with WMAP data. This conclusion also applies to our scenario if one additional $SU(5)$ singlet scalar field is included.

Finally, another $SU(5)$ scalar singlet can also be introduced to address the existence of Dark Matter. This approach however would require additional discrete symmetries in order to forbid certain couplings.

4 Additional comments

- Higher-dimensional operators help us generate realistic mass spectrum of the SM fermions. And, these same operators also play a role in deriving a correct upper bound on the total proton lifetime. Clearly, there is a prospect to use this connection to establish an upper bound on scale of the ultraviolet completion of our theory. We will present our findings on this issue in near future.

- It is often argued that if $\Sigma_{24}$ breaks $SU(5)$ down to the SM and $\Psi_D$ gets electroweak VEV $v$ then $\Sigma_3$ must get an induced VEV at the tree-level. We find this to be model dependent statement. Namely, if only $SU(5)$ invariance is imposed then one can always arrange that $\Sigma_3$ does not get a VEV at the tree-level by fine-tuning.

5 Summary

We have presented phenomenological aspects of the simplest realistic $SU(5)$ model, where the Higgs sector is composed of the $5$, $15$ and $24$ dimensional representations. The scenario accommodates realistic fermion masses and gauge coupling unification. Moreover, it predicts experimentally accessible proton decay. More specifically, there exists an upper bound on the total proton decay lifetime $\tau_p \leq 1.4 \times 10^{36}$ years. And, the bound correlates with the leptoquark mass. Since the next generation of proton decay experiments is expected to improve current bounds by a few orders of magnitude, our simplest non-supersymmetric $SU(5)$ model will be certainly tested or ruled out. The proposed scenario, if further extended with singlet fermions and bosons, could accommodate leptogenesis mechanism and be brought in agreement with other cosmological observations.

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