Combining the Best of Graphical Models and ConvNets for Semantic Segmentation

Michael Cogswell  
Virginia Tech  
cogswell@vt.edu

Xiao Lin  
Virginia Tech  
linxiao@vt.edu

Senthil Purushwalkam  
Virginia Tech  
senthil@vt.edu

Dhruv Batra  
Virginia Tech  
dbatra@vt.edu

Abstract

We present a two-module approach to semantic segmentation that incorporates Convolutional Networks (CNNs) and Graphical Models. Graphical models are used to generate a small (5-30) set of diverse segmentations proposals, such that this set has high recall. Since the number of required proposals is so low, we can extract fairly complex features to rank them. Our complex feature of choice is a novel CNN called SegNet, which directly outputs a (coarse) semantic segmentation. Importantly, SegNet is specifically trained to optimize the corpus-level PASCAL IOU loss function. To the best of our knowledge, this is the first CNN specifically designed for semantic segmentation. This two-module approach achieves 52.5% on the PASCAL 2012 segmentation challenge.

1. Introduction

Training deep Convolutional Neural Networks (CNNs) with large amounts of labeled data has produced impressive results for classification and detection of objects and attributes [11, 18, 23, 28]. The natural next question to ask is – can these deep models be generalized beyond simple prediction spaces (as in multi-way classification) to complex, structured prediction spaces as in semantic segmentation, keypoint/pose estimation, and coarse 3D estimation?

There are two main challenges in this generalization:

• Does vision = “lots of classification”? Most recent applications of CNNs to new tasks such as detection and segmentation have framed these tasks as “lots of classification”, either of scanning window patches [5,9,12,21,22] or region proposals [11,14,15]. While these results are encouraging, such formulations ignore the rich structure in the output space. In semantic segmentation, the goal is to label each pixel with an object class. Labels of nearby pixels tend to be correlated, and independent per-pixel predictions loose this valuable signal. These intuitions are also reflected in the choice of the evaluation metrics used by community – for instance, mean Jaccard Index (or Intersection-over-Union (IOU)) used by PASCAL segmentation, as opposed to the naïve Hamming distance.

• Limited training data. Unlike classification, which requires image-level labels, and detection, which requires bounding boxes, higher-level scene understanding tasks such as semantic segmentation, or coarse 3D estimation often require dense pixel-level annotations that are time consuming and expensive to collect. Thus, such datasets are significantly smaller in scale than classification, despite ongoing valiant efforts [19].

Goal. At a high level, the goal of this paper is to address the above two challenges – to leverage improvements in CNN-based classification for higher-level vision tasks in a manner that uses the large training corpus available for classification without “shoe-horning” the task at hand into repeated classification.

Overview. We present a novel CNN-based approach for semantic segmentation, the task of labeling each pixel in an image with an object class. Fig. 1 illustrates our two-module approach. Module 1 uses a graphical model to produce multiple semantic segmentation proposals. Module 2 uses a novel CNN called SegNet, which is used to score and re-rank these proposals, resulting in the final prediction.

Contributions. Our primary technical contribution is SegNet, a novel CNN that directly outputs a (coarse) semantic segmentation. Importantly, SegNet is task-aware, and specifically trained to optimize the corpus-level PASCAL IOU loss function. To the best of our knowledge, this is the first CNN specifically designed for semantic segmentation. While our experiments focus on this one specific application (semantic segmentation), at a high-level, our approach presents a general recipe for combining the
strengths of graphical models (modeling dependencies) and deep learning (learning rich features) for a range of applications. The recipe is simple – use graphical models to generate a small set of proposals and CNNs to score them. Formulating the problem this way has a number of advantages:

- **Wider receptive field without loss in resolution**: As CNNs get deeper, each output pixel gets to see a larger patch of input and reason about more context. Unfortunately, the output also gets coarser due to the pooling layers. Thus, practitioners are left with a dilemma – either build shallow networks that have limited performance or deeper richer networks that lose localization information. Our 2-module approach does not face this problem: SegNet gets to look at not just a patch or a segment, but the entire image to make its predictions. The loss in resolution is acceptable because the SegNet prediction simply needs to re-rank holistic proposals, which are full resolution.

- **Leveraging classification corpus while learning output structure**: The first few layers of SegNet are warm-started with Krizhevsky et al.’s classification network (AlexNet) trained on ImageNet [18]. These weights have learned the expected Gabor-like filters, and are good low-level features for natural images. We make the last few layers task-aware by optimizing corpus-level structured loss on PASCAL.

- **Graphical models encode knowledge about output structure**: Re-ranking proposals produced by graphical models allows us to reason about segmentation structure in a second way – through the large body of work tying graphical models and structured prediction.

2. Related Work

As Fig. 1 suggests, our work relates to two themes – deep learning and proposal-based vision pipelines.

**Convolutional Neural Networks.** Image classification and object detection are formulated as patch classification problems, where the patch is either the whole image or comes from a set of boxes sampled across scale and aspect ratio. Segmentation is a natural extension of this view, and a number of recent approaches have classified uniform patches sampled in a grid [5, 9, 12, 21, 22].

The size of the patch in consideration (or “receptive field”) determines the amount of context available – [9] and [22] use multi-scale CNNs to increase the receptive field while limiting increase in model complexity; [22] simply makes their convolution filters larger. Most notably, [21] uses a recurrent CNN to gain depth and a larger receptive field, while limiting the parameters that need be learned. Our network is deeper, so our receptive field is naturally large.

Interestingly, a number of these approaches find that the structure in natural images isn’t well respected by their CNN predictions. Thus, the CNN predictions are post-processed using graphical models [9] to insert structural knowledge back into the pipeline. To contrast, our proposal re-ranking step can be thought of as a sophisticated form of post-processing.

**Graphical Models and Proposals.** Modern approaches for object detection and semantic segmentation increasingly rely on category independent bounding-box and segment proposals [2, 4, 25]. In both cases, the search space (#boxes, #segments) is overwhelmingly large, and the goal is to reduce the search space to enable expensive processing, without throwing out good solutions. [11] and [14] achieved...
state of the art performance on detection and segmentation respectively by classifying bounding-box and region proposals using a CNN.

Most proposal methods need to produce on the order of 200-5000 proposals to get sufficiently high recall. Our approach may be viewed as an instantiation of the same philosophy, only operating a step “downstream”. Specifically, we produce entire image labelings, not category-independent box/segment proposals. Interestingly, this allows us to use significantly fewer proposals – on the order of 10-30 per image. We are motivated by the observation made in recent work [26] – a set of just 10 image labelings has the potential to improve PASCAL segmentation by 15%-points (33% relative gain). Using fewer proposals allows even more complex scoring of those proposals by sophisticated secondary modules, as we do in this work.

In a manner similar to us, the most successful detection and segmentation methods, do not use their CNNs for localization; rather the CNNs are used to score proposals [11,14]. On the other hand, CNNs generate dense features in [9,23], but are outperformed by proposal-based methods.

3. Approach

We begin by describing the SegNet module in our approach, and then explain how it is used to score semantic segmentation proposals.

3.1. SegNet: Predicting Coarse Segments

Architecture. As shown in Fig. 1, our architecture contains 8 convolutional layers and each is fed through rectified linear non-linearities except the last, which is fed through a pixel-wise C-way softmax to label an image with C classes. There are no fully connected layers.

Comparison with a classification net. The first 5 layers in SegNet are convolutional layers (conv), with 96, 256, 384, 384, then 256 filters, Max pooling (pool) and local response normalization follow the first two layers, similar to AlexNet. The 5th conv layer produces 256 feature maps of size 13 × 13, but we do not pool after this layer, and this is where the architectures diverge. In standard classification CNN, conv layers are typically followed by fully-connected (fc) layers. We do not have fc layers. Instead, we add two more conv layers with 128 feature maps each and a third conv layer with as many feature maps as the number of classes (including the ‘background’ class). These C final feature maps can be thought of as ‘semantic feature maps’ since they give pixel-wise probabilities for each class and those are interpretable.

Crucially, we initialize the first 5 conv layers with weights from CaffeNet trained on ImageNet, thus utilizing the large classification corpus. During training, we keep these conv-layer weights fixed and only learn the weights of the newly added layers. We apply dropout [16] before each of the added feature maps during training. By feeding each pixel at the output through a softmax activation function (normalized over classes) we can output ‘semantic feature maps’ which collectively give a distribution over classes at each output pixel in the 13 × 13 grid. An example which shows 21 feature maps for the 21 PASCAL classes is shown in Fig. 4.

Since SegNet contains no fully-connected layers, the only weights are the filters. This is greatly beneficial since a majority of the parameters in standard classification nets lie in the fully-connected layers. Indeed, SegNet contains less than 10% of CaffeNet parameters. Interestingly, in previous work, Zeiler and Fergus [27] have observed that weights in convolutional filters provide more information per weight because removing fully connected layers (containing most parameters) does not lead to a proportional decline in classification performance.

Since SegNet predictions are coarse (low-resolution), we need to down-sample high-resolution segmentation ground truths to derive the annotation for training SegNet parameters. Each pixel in a down-sampled segmentation corresponds to a patch in the high-res version, so we compute distributions over classes in this patch, yielding a soft segmentation ground-truth, similar to our predictions.

Our baseline loss for training SegNet is the standard cross-entropy computed between a pixel’s predicted class distribution and the ground truth’s distribution. Notice that this loss function is “decomposable” over pixels – it treats segmentation as independent classification problems at each pixel.

3.2. Optimizing a Segmentation-Specific Loss

Recall that the standard evaluation criteria used in segmentation tasks is Intersection-over-Union (IOU) averaged across classes. Although imperfect (in the sense that it does not reward boundary alignment), it does captures some notions of a good segmentation better than decomposable metrics such as Hamming. Unfortunately, this metric does not decompose over pixels or even images. In fact, it is a corpus-level metric, and can only be computed for an entire dataset, not individual images. Fortunately, we only need a loss’s gradient to train a CNN, so we can directly optimize such a metric. The supplementary material shows our our derivation of IOU’s gradient. Before going further, it’s
kinds of mistakes (Demonstrated)

For the sake of building an understanding, consider the gra-

Optimizing IOU.

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It might seem intuitive to aim to maximize IOU, but in

Consider high-resolution predictions and ground truth. Let

TPk denote the number of true positive for class k across

the dataset, i.e. the number of pixels across all images that

are annotated and predicted as class k. Analogously, let

FPk denote the false positives, FNk the false negatives,

and GTk the sum of ground truth pixels for class k. Then,

the Jaccard Index for class k can be defined as:

\[ IOU_k = \frac{TP_k}{TP_k + FP_k + FN_k}, \]

which is averaged across categories to yield the final metric:

\[ IOU = \frac{1}{K} \sum_{k=1}^{K} IOU_k \]

(2)

It might seem intuitive to aim to maximize IOU, but in

our experiments we have found the optimization to be easier

when we minimized the Union-over-Intersection (UOI)

instead. In our preliminary experiments with random ini-

tialisation, the IOU objective function always led to an all-

background prediction whereas UOI minimization leads to

a much better solution. We explain why this might be the

case.

Optimizing IOU.

First, let us use the fact that \( TP_k + FN_k = GT_k \) to rewrite the gain function as:

\[ IOU_k = \frac{GT_k - FN_k}{GT_k + FP_k} \]

(3)

For the sake of building an understanding, consider the gra-

dients of the \( IOU_k \) gain function with respect to the two

kinds of mistakes \( (FP_k \) and \( FN_k \)):

\[ \frac{\partial (IOU_k)}{\partial (FP_k)} = \frac{- (GT_k - FN_k)}{(GT_k + FP_k)^2} \]

(4)

\[ \frac{\partial (IOU_k)}{\partial (FN_k)} = \frac{-1}{GT_k + FP_k} \]

(5)

Notice that there are two things non-ideal about these gra-

dients. First, as the number of mistakes \( (FP_k \) or \( FN_k \)) in-

crease the gradients diminish. Second, as the number of

mistakes reduce, the gradients increase. Such a behavior

hammers convergence of first-order methods.

Note that we only analyzed the effect of \( FP_k, FN_k \) on

\( IOU_k \), and but not on \( IOU_{k'} \) for other categories \( k' \).

Each pixel can be assigned only one category, and thus the

mistakes \( \{FP_k, FN_k\}_{k=1}^{K} \) are not independent of each

other. Thus, we need to also analyze the other terms

\( \partial (IOU_k) / \partial (FP_k) \). Fig. 2 shows a simulation where we

computed the behavior of IOU as a function of increasing

\( FP_k \) and \( FN_k \). Our illustration of the gradients of \( IOU_k \)

provides an intuition for the behavior of the IOU function.

Optimizing UOI.

Now we show that Union-over-Intersection (UOI) is a smoother optimization function

based on the behavior of its gradient and that it shares a

natural relation with IOU. In a manner similar to IOU, the

UOI function can be written as:

\[ UOI = \frac{1}{K} \sum_{k=1}^{K} UOI_k \]

\[ = \frac{1}{K} \sum_{k=1}^{K} \frac{TP_k + FP_k + FN_k}{TP_k} \]

\[ = \frac{1}{K} \sum_{k=1}^{K} \frac{GT_k + FP_k}{GT_k - FN_k} \]

(6)

(7)

(8)

Consider the gradient of \( UOI_k \) w.r.t. the number of mis-

takes \( FP_k \) and \( FN_k \):

\[ \frac{\partial UOI_k}{\partial FP_k} = \frac{1}{GT_k - FN_k} \]

\[ \frac{\partial UOI_k}{\partial FN_k} = \frac{GT_k + FP_k}{(GT_k - FN_k)^2} \]

(9)

(10)

We can see that \( UOI_k \) has more desirable properties com-

pared to \( IOU_k \), as illustrated in Fig. 2. When the number of

mistakes are large, the gradient is large. As the number of

mistakes decrease, the gradient decreases as well.

Does UOI optimise IOU?

Since we have now established that the UOI function has more desirable traits, we should

understand whether the two objectives are related. Does minimization of UOI lead to the maximization of IOU?

We show that IOU can be lower-bounded by a decreasing function of UOI:

\[ \sum_k IOU_k \geq f \left( \sum_k UOI_k \right) \]

(11)

Since \( f(x) \) is a decreasing function in \( x \), we can see that decreasing UOI leads to increasing the lower-bound on IOU.
Importantly, we can show that the bound is tight— that the maximum possible value of \( \sum_k IOU_k \) (=K) is achievable, although it requires that \( TP_k \neq 0 \) for all classes \( k \). In our coarse segmentation setting this can always be achieved by not allowing any soft outputs to be 0.

**Proof.** Assume \( 0 < IOU_i \). Now consider the decreasing function \( f(x) = \frac{1}{x} \). We need to show that:

1. \( \sum_k IOU_k \geq \sum_k \frac{1}{IOU_k} \)
2. There exists a value of \( UOI \) for which \( IOU = 1 \)

Let \( x_i = IOU_i \). Notice that \( 0 < x_i \leq 1 \). This means that:

\[
\sum_{k=1}^{K} x_k \geq x_i \Rightarrow \frac{1}{x_i} \geq \frac{1}{\sum_k x_k}
\] (12)

Now, if we notice

\[
\frac{1}{x_i} + \sum_{k \in \{1, \ldots, K\} \setminus \{i\}} \frac{1}{x_k} = \sum_{k=1}^{K} \frac{1}{x_k}
\] (13)

then we can see

\[
\sum_{k} \frac{1}{x_k} \geq \frac{1}{\sum_k x_k} \Rightarrow \sum_{k} x_k \geq \frac{1}{\sum_k \frac{1}{x_k}}
\] (14)

\[
\Rightarrow \sum_{k} IOU_k \geq \frac{1}{\sum_k UOI_k}
\] (15)

Hence, the lower-bound of IOU has been shown as a decreasing function of UOI. If \( UOI_i = 1 \) \( \forall i \), clearly \( \sum_{k=1}^{K} IOU_k = K \) and \( IOU = 1 \). This implies that UOI acts as a good surrogate objective for optimizing IOU. □

### 3.3. Semantic Segmentation Proposals

Our second module is a pipeline which uses a graphical model to generate semantic segmentation proposals. We directly use the O2P+DivMBest approach of [26]. They use the O2P model [3] which generates approximately 150 CPMC segments [4] for each image, then scores them using Support Vector Regressors trained over second-order pooled features [3]. These segmentations are greedily pasted to form a semantic segmentation. Finally, DivMBest is used to generate multiple diverse semantic segmentation proposals. [26] showed that one of these proposals tends to be significantly more accurate that the 1-best semantic segmentation. Let the oracle segmentation be the most accurate segmentation in the set. The oracle accuracy at just 10 proposals is 15%-points higher than the 1-best segmentation.

### 4. Post-Processing: Coarse to Full

To evaluate segmentations from SegNet we need to post-process these coarse segmentations to produce “full-sized” semantic segmentation. We’ll refer to the up-sampled segmentation as \( \hat{P}_{jk} \) where each index \( j \) corresponds to a pixel in the original image and \( k \) indexes classes. Let \( \hat{p}_{jk} \) denote the probability of class \( k \) predicted at coarse pixel \( i \) (which corresponds to a patch in the full resolution image) by the last layer of a SegNet.

We propose four post-processing strategies that are arranged by increasing sophistication; the first two methods are simple heuristics while the last two methods try to pick good proposals.

**(Naïve)** A simple way to do this, which we’ll call naïve upsampling just copies the argmax of a coarse pixel into all pixels in the patch it came from:

\[
\hat{P}_{jk} = \text{argmax}_{k} \hat{p}_{jk}
\] (16)

**(Superpixel)** The next step is to try and respect object boundaries using small superpixels (using SLIC [1]), which are labeled by coarse segmentations. For each superpixel, we aggregate distributions over categories from patches overlapping with this superpixel. Each distribution is weighted by the percentage of pixels in the superpixel that are also in the patch. This gives a distribution for superpixels. We take the argmax for each patch distribution. Neither this smart upsampling, nor the previous naïve upsampling are competitive.

**(SegNet)** Next, we use SegNet outputs to pick proposals. We down-sample each DivMBest segmentation to \( 13 \times 13 \) soft segmentations, similar to how ground truth was downsampled for training. Call \( \hat{q}_{ik}^m \) the probability of the \( m^{th} \) DivMBest downsampled segmentation at patch \( i \). We score the consistency of \( \hat{p} \) and \( \hat{q}_k^m \) with the symmetric-KL augmented by a background penalty term:

\[
S(m) = \sum_i \left[ D_{KL}(\hat{p}_i || \hat{q}_k^m) + D_{KL}(\hat{q}_k^m || \hat{p}_i) + 0.02 \hat{q}_k^m \right].
\] (17)

where \( D_{KL} \) is the Kullback-Leibler divergence and \( 0.02 \hat{q}_k^m \) is a regularizer that penalizes background prediction. The background penalty comes from observing that background (class 0) is frequently overpredicted; adding this term consistently improved validation performance.

**(SegNet+SVM)** The is our final approach, which works best, and uses SegNet segmentations as a feature, and training a re-ranker to pick the best proposal from DivMBest. Specifically, similar to [26], we train a ranking Support Vector Machine to choose the best proposal according to a variety of features which describe proposals. We use both sophisticated hand-engineered features taken from [26], and
simple features based on SegNet outputs. This is similar to R-CNN [11] and SDS [14], which each use an SVM trained on CNN features to evaluate proposals.

**Segmentation features:**

(SegNet) As in the previous section, we calculate KL divergence between proposals and CNN segmentations, and consider each direction the divergence can be computed separately (2 dimensions).

We also considered expected intersection, expected union, expected intersection over expected union, and expected union over expected intersection. Each of these statistics is computed for all PASCAL classes plus background (84 dimensions).

(CNN Classification) We use class-wise scores from an SVM trained on DeCAF features [6] for PASCAL classification (20 dimensions). In addition, we extend image classification to predict not just the existene of objects, but also whether a category in an object is greater than a certain size or not. To train an SVM for class C and threshold t ∈ [0, 1], we set the ground truth label for class C to 0 if the percent of C pixels in an image is below t. The thresholds are chosen per-class by sorting images by percentage of C pixels then using the C-percent of the image at the 20th, 40th, 60th, and 80th percentile. (80 dimensions).

(DivMBest+ReRank) Finally, we use all features used by [26] (1966 dimensions).

5. Experiments and Results

**Setup.** We report our results on the PASCAL VOC 2012 segmentation dataset. We used the trainval data provided by the challenge, and the additional annotations collected by Berkeley [13]. We trained SegNet in a cross-val manner – we split the entire dataset into 10 folds, trained SegNet on 9 folds and computed SegNet outputs in the 10th fold. Finally, we trained the SSVM re-ranker on all training data other than val. All results and analyses reported in this paper are on val. We picked our best performing approach and uploaded to the PASCAL evaluation server to report results on test.

Table 2 shows the results of all approaches on PASCAL 2012 val set. Naïve upsampling performs worst at 31.3%. Superpixel upsampling gives a small improvement at 31.9%. Neither of these are competitive, which we suspect is due of the coarseness of the segmentations. It’s possible that more sophisticated up-sampling strategies from [5,9,21] would result in more competitive segmentations directly from the CNN. On the other hand, even our simple re-ranking of DivMBest proposals is competitive, at 48.6%. Our final method of SegNet features with the SVM re-ranker yielded best results at 53.1%. We uploaded our best performing method on the PASCAL evaluation server.
Naïve Upsampling & Superpixel Upsampling & SegNet-CE & SegNet-UIO & SegNet-UIO-CE-combination & +SVM re-ranker \\
31.3 & 31.9 & 47.4 & 48.5 & 48.7 & 53.1 \\

**Table 1:** PASCAL VOC 2012 segmentation test results. [3] is the same as picking the highest scoring DivMBest solution.

**Table 2:** PASCAL VOC 2012 segmentation val results.

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Contrary to most recent CNN results, our setting allows SegNet to perform well with relatively little data. The PASCAL segmentation dataset [8] augmented with extra annotations from [13] only has about 12000 images.

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**Ablation Studies.** We tried to tease apart the influence of different components in our pipeline. First, if we train an SVM re-ranker with SegNet features alone, it performs about the same as simple KL divergence based ranking (47.4%). Adding (CNN classification) features and (DivMBest+ReRank) features from [26] increases this performance by about 3.9% and 4.4% respectively. Using both yields an extra percent of performance.

To get a better idea of which features are considered important by SVM re-ranker, we considered various subsets of the features. Recall that the three types of features are (1) SegNet features, (2) Classification features, and (3) D-
vMBest+ReRank features from [26]. For reference, using all three resulted in 53% on val. If we only use (1) then we get 49.0%, only (2) gives 47.8%, and only (3) gives 48.1%. Thus we see that the learned SegNet features outperform non-segmentation CNN features (classification) and 1000s of dimensions of hand crafted features from [26]. However, doing both works best. Using all features but SegNet features, i.e., (2)+(3) gives 50.5%, which shows that SegNet features are important, even in the presence of other CNN-based features. Using just DivMBest+ReRank features and SegNet features, i.e. (1)+(3) performs at 52.0%, so SegNets again appear to be more important than simple CNN classification features.

Some qualitative results were also interesting. In Fig. 5 we note that (probabilistic) softness of our segmentations helps alleviate some problems with coarseness, but we point out how such problems still manifest in Fig. 4 and Fig. 3.

6. Conclusion and Future Work

To summarize, we present a two-module approach for semantic segmentation. Module 1 uses a graphical model to produce multiple semantic segmentation proposals. Module 2 uses SegNet, a novel CNN which is specifically trained for semantic segmentation task-loss, and is used to to score then re-rank these proposals, resulting in a final segmentation. Our approach achieves 52.5% on the PASCAL 2012 segmentation challenge.

Our experiments with and without proposals reach findings that are consistent with those observed in previous work – that methods which use proposals consistently outperform those which don’t, even among methods which use CNNs. In object detection non-proposals methods (sliding window) still are efficient enough to be viable for CNNs [23]. However, for segmentation, proposals are vital.

Perhaps unsurprisingly, it is clear that more work needs to be done with CNNs. Our results suggest that some information is not captured by our CNN. Achieving peak performance in our pipeline requires hand crafted features from [26], though our SegNet-based features are significant contributors.

Another path of future experiments involves Microsoft’s Common Objects in Context dataset [19], which contains an order of magnitude more segmentation ground truth than has existed before. Following AlexNet, it seems reasonable to expect better performance from a larger net trained on a bigger dataset. Since our net predicts entire segmentations instead of single labels it could have more to learn from this dataset than classification nets.

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1. Introduction and Notation

This document is meant to accompany the paper “Combining the Best of Graphical Models and ConvNets for Semantic Segmentation”. Herein we compute the derivatives of Intersection-over-Union (IOU) and Union-over-Intersection (UOI) with respect to an input feature map $z$. Two forms of the IOU derivative can be seen in (57) and (59). Corresponding forms of the UOI derivative are in (65) and (67).

We’d like to use a supervised learning algorithm to train a model for labeling image pixels using $K$ classes. Let $\hat{Y}$ be the predicted segmentation and $Y$ the ground truth so that $\hat{Y}_i$ and $Y_i$ are random variables indicating the class of pixel $i$; each takes values in $\{1, \ldots, K\}$ and pixels labeled with indices $\{1, \ldots, N\}$. The set of pixels can either be all pixels in an image or all pixels in a minibatch, though we take it to be the set of all pixels in a minibatch. Thus, $P(\hat{Y}_i = k)$ is the probability of predicting pixel $i$ is class $k$. However, we’ll introduce the following short hand to condense the notation:

$$\hat{p}_{i,k} = P(\hat{Y}_i = k) \tag{18}$$

and

$$p_{i,k} = P(Y_i = k) \tag{19}$$

Here, the model produces a score $z_{i,k}$ for each pixel $i$ and class $k$ then predicts $\hat{Y}$ using a softmax. It assigns

$$\hat{p}_{i,k} := \frac{\exp(z_{i,k})}{\sum_{\tilde{k}} \exp(z_{i,\tilde{k}})} \tag{20}$$

First we’ll build up some machinery useful for both IOU and UOI, then we’ll compute their derivatives.

2. Intersection and Union

The intersection function captures the notion of agreement between ground truth and prediction. It’s defined for class $k \in \{1, \ldots, K\}$ as

$$I_k(\hat{Y}, Y) = \sum_i [\hat{Y}_i = k \land Y_i = k] \tag{21}$$

where $[\cdot]$ is the indicator function. We’re interested in the expected intersection,

$$E \left[ I_k(\hat{Y}, Y) \right] = E \left[ \sum_i [\hat{Y}_i = k \land Y_i = k] \right] \tag{22}$$

$$= \sum_i E \left[ [\hat{Y}_i = k \land Y_i = k] \right] \tag{23}$$

which can be reduced to the following, because the expectation of an indicator function is the probability of the event inside:

$$E \left[ I_k(\hat{Y}, Y) \right] = \sum_i \hat{p}_{i,k} p_{i,k} \tag{24}$$

The union function is about the total "footprint" in pixels of ground truth and prediction; it is defined as

$$U_k(\hat{Y}, Y) = \sum_i [\hat{Y}_i = k \lor Y_i = k] \tag{25}$$
Expected union is analogous to expected intersection:

\[
E \left[ U_k(\hat{Y}, Y) \right] = E \left[ \sum_i^{N} [\hat{Y}_i = k \lor Y_i = k] \right] = \sum_i^{N} E \left[ [\hat{Y}_i = k \lor Y_i = k] \right] = \sum_i^{N} (\hat{p}_{i,k} + p_{i,k} - \hat{p}_{i,k} \hat{p}_{i,k})
\]

(26)

(27)

(28)

In fact, it can be expressed using Intersection (inclusion-exclusion principle)

\[
E \left[ U_k(\hat{Y}, Y) \right] = \sum_i^{N} (\hat{p}_{i,k} + p_{i,k}) - E \left[ I_k(\hat{Y}, Y) \right]
\]

(29)

3. Softmax Gradient

A softmax function takes a bunch of scores and outputs probabilities. In the next section we’ll need the softmax’s gradient at any output with respect to any input. Normally we care about the gradient of an output with respect its corresponding input, but this case is a bit more general. Here \( k' \in \{1, \ldots, K\} \).

\[
\frac{\partial \hat{p}_{i,k'}}{\partial z_{i,k}} = \frac{\partial}{\partial z_{i,k}} \frac{\exp(z_{i,k'})}{\sum_k \exp(z_{i,k})} = \frac{\partial}{\partial z_{i,k}} \frac{\exp(z_{i,k'}) \sum_k \exp(z_{i,k}) - \exp(z_{i,k'}) \sum_k \exp(z_{i,k})}{\left( \sum_k \exp(z_{i,k}) \right)^2}
\]

(30)

(31)

(32)

If \( k = k' \) then

\[
\frac{\partial \hat{p}_{i,k'}}{\partial z_{i,k}} = \exp(z_{i,k}) \frac{\sum_k \exp(z_{i,k}) - \exp(z_{i,k}) \exp(z_{i,k})}{\left( \sum_k \exp(z_{i,k}) \right)^2} = \hat{p}_{i,k} - \hat{p}_{i,k}^2
\]

(33)

(34)

When \( k \neq k' \),

\[
\frac{\partial \hat{p}_{i,k'}}{\partial z_{i,k}} = -\frac{\exp(z_{i,k'}) \exp(z_{i,k})}{\left( \sum_k \exp(z_{i,k}) \right)^2} = -\hat{p}_{i,k'} \hat{p}_{i,k}
\]

(35)

(36)

Now the whole derivative can be written as one expression and simplified a bit.

\[
\frac{\partial \hat{p}_{i,k'}}{\partial z_{i,k}} = [k = k'] (\hat{p}_{i,k} - \hat{p}_{i,k}^2) - [k \neq k'] \hat{p}_{i,k'} \hat{p}_{i,k}
\]

(37)

(38)
In the first case, substitute $k'$ for $k$ to get

$$\|k = k'\| (\hat{p}_{i,k} - \hat{p}_{i,k}^2) - \|k \neq k'\| \hat{p}_{i,k}^2 = \|k = k'\| \hat{p}_{i,k} (1 - \hat{p}_{i,k}) - \|k \neq k'\| \hat{p}_{i,k} \hat{p}_{i,k}$$  \hspace{1cm} (39)

$$= \hat{p}_{i,k} \left( \|k = k'\| (1 - \hat{p}_{i,k}) + \|k \neq k'\| (-\hat{p}_{i,k}) \right)$$  \hspace{1cm} (40)

$$= \hat{p}_{i,k} \left( \|k = k'\| (1 - \hat{p}_{i,k}) + \|k \neq k'\| (0 - \hat{p}_{i,k}) \right)$$  \hspace{1cm} (41)

$$= \hat{p}_{i,k} \left( \|k = k'\| - \hat{p}_{i,k} \right)$$  \hspace{1cm} (42)

4. Gradient of Expected Intersection and Expected Union

Next, compute the derivatives of expected intersection and expected union. For intersection,

$$\frac{\partial}{\partial \hat{p}_{i,k}} E[I_{k'}(\hat{Y}, Y)] = \frac{\partial}{\partial \hat{p}_{i,k}} \sum_{j}^{N} \hat{p}_{j,k'} p_{j,k'}$$  \hspace{1cm} (43)

$$= p_{i,k'} \frac{\partial \hat{p}_{i,k'}}{\partial \hat{p}_{i,k}}$$  \hspace{1cm} (44)

In the case of union,

$$\frac{\partial}{\partial \hat{p}_{i,k}} E[U_{k'}(\hat{Y}, Y)] = \frac{\partial}{\partial \hat{p}_{i,k}} \sum_{j}^{N} (\hat{p}_{j,k'} + p_{j,k'} - \hat{p}_{j,k'} p_{j,k'})$$  \hspace{1cm} (45)

$$= (1 - p_{i,k'}) \frac{\partial \hat{p}_{i,k'}}{\partial \hat{p}_{i,k}}$$  \hspace{1cm} (46)

5. IOU Loss and its Gradient

Now, write out the IOU loss function

$$L_{IOU}(\hat{Y}, Y) = \sum_{k'}^{K} \frac{E[I_{k'}(\hat{Y}, Y)]}{E[U_{k'}(\hat{Y}, Y)]}$$  \hspace{1cm} (47)

and compute the gradient of IOU

$$\frac{\partial L_{IOU}(\hat{Y}, Y)}{\partial z_{i,k}} = \left( \sum_{k'}^{K} \frac{\partial}{\partial z_{i,k}} \frac{E[I_{k'}(\hat{Y}, Y)]}{E[U_{k'}(\hat{Y}, Y)]} \right)$$  \hspace{1cm} (48)

$$= \sum_{k'}^{K} \left( \frac{\partial}{\partial \hat{p}_{i,k}} \frac{E[I_{k'}(\hat{Y}, Y)]}{E[U_{k'}(\hat{Y}, Y)]} \right) \frac{\partial \hat{p}_{i,k}}{\partial z_{i,k}}$$  \hspace{1cm} (49)

First we’ll focus on

$$\frac{\partial}{\partial \hat{p}_{i,k}} \frac{E[I_{k'}(\hat{Y}, Y)]}{E[U_{k'}(\hat{Y}, Y)]}$$  \hspace{1cm} (50)

By substituting these into the following we get

$$\frac{\partial}{\partial \hat{p}_{i,k}} E[I_{k'}(\hat{Y}, Y)] = \frac{E[U_{k'}(\hat{Y}, Y)] \frac{\partial}{\partial \hat{p}_{i,k}} E[I_{k'}(\hat{Y}, Y)] - E[I_{k'}(\hat{Y}, Y)] \frac{\partial}{\partial \hat{p}_{i,k}} E[U_{k'}(\hat{Y}, Y)]}{E[U_{k'}(\hat{Y}, Y)]^2}$$  \hspace{1cm} (51)

$$= \frac{E[U_{k'}(\hat{Y}, Y)] p_{i,k'} \frac{\partial \hat{p}_{i,k'}}{\partial \hat{p}_{i,k}} - E[I_{k'}(\hat{Y}, Y)] (1 - p_{i,k'}) \frac{\partial \hat{p}_{i,k'}}{\partial \hat{p}_{i,k}}}{E[U_{k'}(\hat{Y}, Y)]^2}$$  \hspace{1cm} (52)

$$= \frac{E[U_{k'}(\hat{Y}, Y)] p_{i,k'} - E[I_{k'}(\hat{Y}, Y)] (1 - p_{i,k'}) \frac{\partial \hat{p}_{i,k'}}{\partial \hat{p}_{i,k}}}{E[U_{k'}(\hat{Y}, Y)]^2}$$  \hspace{1cm} (53)
Finally, substituting (53) into (49) gives

$$\frac{\partial \mathcal{L}_{\text{IOU}}(\hat{Y}, Y)}{\partial z_{i,k}} = \sum_{k'}^K \left( \frac{\partial}{\partial \hat{p}_{i,k}} E[I_{k'}(\hat{Y}, Y)] \right) \frac{\partial \hat{p}_{i,k}}{\partial z_{i,k}}$$

(54)

$$= \sum_{k'}^K \left( \frac{E[U_{k'}(\hat{Y}, Y)]p_{i,k'} - E[I_{k'}(\hat{Y}, Y)](1 - p_{i,k'}) \frac{\partial \hat{p}_{i,k'}}{\partial \hat{p}_{i,k}}}{E[U_{k'}(\hat{Y}, Y)]^2} \frac{\partial \hat{p}_{i,k'}}{\partial z_{i,k}} \right)$$

(55)

$$= \sum_{k'}^K \left( \frac{E[U_{k'}(\hat{Y}, Y)]p_{i,k'} - E[I_{k'}(\hat{Y}, Y)](1 - p_{i,k'}) \frac{\partial \hat{p}_{i,k'}}{\partial \hat{p}_{i,k}}}{E[U_{k'}(\hat{Y}, Y)]^2} \right)$$

(56)

$$= \sum_{k'}^K \left( \frac{E[U_{k'}(\hat{Y}, Y)]p_{i,k'} - E[I_{k'}(\hat{Y}, Y)](1 - p_{i,k'}) \frac{\partial \hat{p}_{i,k'}}{\partial \hat{p}_{i,k}}}{E[U_{k'}(\hat{Y}, Y)]^2} \right)$$

(57)

$$= \sum_{k'}^K \left( p_{i,k'} \sum_j^N (\hat{p}_{j,k'} - \hat{p}_{j,k'}p_{j,k'}) - (1 - p_{i,k'}) \frac{\partial \hat{p}_{i,k'}}{\partial \hat{p}_{i,k}} \frac{\partial \hat{p}_{i,k'}}{\partial z_{i,k}} \right)$$

(58)

$$= \sum_{k'}^K \left( p_{i,k'} \sum_j^N (\hat{p}_{j,k'} - \hat{p}_{j,k'}p_{j,k'}) - (1 - p_{i,k'}) \frac{\partial \hat{p}_{i,k'}}{\partial \hat{p}_{i,k}} \frac{\partial \hat{p}_{i,k'}}{\partial z_{i,k}} \right)$$

(59)

6. Union over Intersection

The gradient of Union over Intersection can be computed in a similar fashion. UOI loss is defined as

$$\mathcal{L}_{\text{UOI}}(\hat{Y}, Y) = \sum_{k'}^K \frac{E[U_{k'}(\hat{Y}, Y)]}{E[I_{k'}(\hat{Y}, Y)]}$$

(60)

Given (44) and (46), we can compute the derivative of UOI:

$$\frac{\partial \mathcal{L}_{\text{UOI}}(\hat{Y}, Y)}{\partial z_{i,k}} = \sum_{k'}^K \left( \frac{\partial}{\partial \hat{p}_{i,k}} E[I_{k'}(\hat{Y}, Y)] \right) \frac{\partial \hat{p}_{i,k}}{\partial z_{i,k}}$$

(61)

$$= \sum_{k'}^K \left( \frac{E[I_{k'}(\hat{Y}, Y)]}{E[I_{k'}(\hat{Y}, Y)]} \frac{\partial \hat{p}_{i,k}}{\partial \hat{p}_{i,k}} \frac{\partial \hat{p}_{i,k}}{\partial z_{i,k}} \right)$$

(62)

$$= \sum_{k'}^K \left( \frac{E[I_{k'}(\hat{Y}, Y)]}{E[I_{k'}(\hat{Y}, Y)]} \frac{\partial \hat{p}_{i,k}}{\partial \hat{p}_{i,k}} \frac{\partial \hat{p}_{i,k}}{\partial z_{i,k}} \right)$$

(63)

$$= \sum_{k'}^K \left( \frac{E[I_{k'}(\hat{Y}, Y)]}{E[I_{k'}(\hat{Y}, Y)]} \frac{\partial \hat{p}_{i,k}}{\partial \hat{p}_{i,k}} \frac{\partial \hat{p}_{i,k}}{\partial z_{i,k}} \right)$$

(64)

$$= \sum_{k'}^K \left( \frac{E[I_{k'}(\hat{Y}, Y)]}{E[I_{k'}(\hat{Y}, Y)]} \frac{\partial \hat{p}_{i,k}}{\partial \hat{p}_{i,k}} \frac{\partial \hat{p}_{i,k}}{\partial z_{i,k}} \right)$$

(65)

$$= \sum_{k'}^K \left( (1 - p_{i,k'}) \sum_j^N (\hat{p}_{j,k'}p_{j,k'}) - p_{i,k'} \sum_j^N (\hat{p}_{j,k'}p_{j,k'}) \hat{p}_{i,k'}([k = k'] - \hat{p}_{i,k}) \right)$$

(66)

$$= \sum_{k'}^K \left( (1 - p_{i,k'}) \sum_j^N (\hat{p}_{j,k'}p_{j,k'}) - p_{i,k'} \sum_j^N (\hat{p}_{j,k'}p_{j,k'}) \hat{p}_{i,k'}([k = k'] - \hat{p}_{i,k}) \right)$$

(67)