Magnetic Wormholes and Vertex Operators

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We consider wormhole solutions in 2 + 1 Euclidean dimensions. A duality transformation is introduced to derive a new action from magnetic wormhole action of Gupta, Hughes, Preskill and Wise. The classical solution is presented. The vertex operators corresponding to the wormhole are derived. Conformally coupled scalars and spinors are considered in the wormhole background and the vertex operators are computed.
I. INTRODUCTION

Recently, considerable attention has been focused on the study of topology changing processes. The effects of wormholes are interesting and they play an important role in the quantum theory of gravity. Hawking [1] has argued that such processes might be responsible for the loss of quantum coherence. Coleman has advanced the proposition that wormholes introduce indeterminacy in the constants of Nature [2]. He has argued persuasively that the constants of Nature are randomly distributed in the ensemble of universes. Furthermore, he argued that the probability distribution is peaked for zero value of the cosmological constant [3]. There has been considerable amount of activity following the work of Coleman some of which are given in [4]. However, the mechanism proposed by Coleman has been subjected to criticism. It is worthwhile to ask how much the physics at low energies is affected due to the wormholes. Hawking [1] has introduced the concept of vertex operators to account for the effect of wormholes at energies much smaller than the Plank scale. Thus it is possible to construct effective Lagrangians at low energies.

The purpose of this note is to study wormholes in 2+1 euclidean dimensions. We consider the Lagrangian introduced by Gupta et al. [5] where abelian gauge field is responsible for the existence of classical solution of the Einstein-Maxwell equation. This is the so called the magnetic wormhole solution. Recently, it has been shown [6] that such 3-dimensional action can be derived from a 4-dimensional theory by adopting dimensional reduction technique. The 4-dimensional theory has Einstein-
Hilbert action and the action of an antisymmetric tensor field. The dimensionally reduced action, suitably choosing the compactification, gives rise to the action of Gupta et al.

We introduce a duality transformation so that a scalar field is dual to the electromagnetic field strength. The field equations are presented and explicit solutions are given for the geometry $R^1 \times S^2$.

It is observed that there is a global conserved charge in this theory which is responsible for stabilising the classical wormhole solution. This property is very useful to construct vertex operators for the wormholes. Recently, Hawking and Grinstein and Maharana have analysed the structure of these vertex operators explicitly for the axionic wormholes. The effects of wormholes of size smaller than the scale of the observation are taken care of once the gauge-invariant, bi-local effective interaction terms, vertex functions, are added to the original action. Presently, it is widely believed that the considerable knowledge about these vertex operators may provide a good understanding of the topology change in quantum gravity.

Next we consider matter fields in the background of the wormhole. These fields are not responsible for the existence of the classical wormhole solutions. However, the correlation functions for these field configurations, while they are far away from the throat of the wormholes, are expected to be different from flat space correlation functions. We compute such correlation functions by introducing the vertex operators. The vertex operators for the scalar fields and spinor fields are presented in section-IV.
II. 2+1-DIMENSIONAL MAGNETIC WORMHOLE

The magnetic wormhole solutions in three euclidean dimensions have been obtained by Gupta et al. [5]. The action is

\[ S = \int d^3x \sqrt{g} \left( -\frac{R}{16\pi G} + \frac{1}{4e^2} F^2 \right) \]  

(1)

where \( G = (16\pi)^{-1} M_p^2 \) and \( e \) are newtonian gravitation constant and electromagnetic coupling constant, respectively. \( M_p \) is the Plank mass and \( \hbar = c = 1 \). Here the vector potential is such that the field strength admits a monopole like configuration. The Einstein gravitational field equations are

\[ G_{\mu\nu} = \frac{M_p^2}{4e^2} \left( 2F_{\mu\lambda}F_{\nu}^{\lambda} - \frac{1}{2}g_{\mu\nu}F^2 \right) \]  

(2)

where

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \]  

(3)

While the matter field equation is

\[ \frac{1}{\sqrt{g}} \partial_{\mu} (F^{\mu\nu} \sqrt{g}) = 0 \]  

(4)

Metric and the field strength, which satisfy the above field equations, have the following form

\[ ds^2 = dt^2 + a^2(t) \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \]  

(5)

\[ F = f(t) \varepsilon = f(t)a^2(t)d\theta \wedge \sin \theta d\phi \]  

(6)

Integral over \( \varepsilon \) gives the surface area of the two sphere,

\[ \int_{S^2} \varepsilon = 4\pi a^2(t) \]
The magnetic flux is given by

\[ \Phi = \int_{S^2} F = \int_{S^2} F_{\mu\nu}d\Sigma^{\mu\nu} = n\Phi_0 \quad (7) \]

and thus

\[ f(t) = \frac{n}{2a^2(t)} . \quad (8) \]

The magnetic flux, \( \Phi \), is required to be the integer multiple of unit quantum flux \( \Phi_0 \) (equal to \( 2\pi \)) if particles of the unit charge are to be introduced on the wormhole background. The scale factor, \( a(t) \), satisfies the equation (from \( o-o \) component of the Einstein equation)

\[ (\partial_t a)^2 = 1 - \frac{a_o^2}{a^2} , \quad (9) \]

where

\[ a_o^2 = \frac{\pi G n^2}{e^2} \quad (10) \]

and solution to \( a(t) \) is

\[ a^2 = a_o^2 + t^2 . \quad (11) \]

Thus the wormhole configuration is such that there are two asymptotically flat regions (corresponding to \( t = \pm\infty \)) connected by a tube of minimum throat size \( a_o \).

It is convenient to introduce a dual field of electromagnetic field strength \( F_{\mu\nu} \). We may recall in this context the four dimensional axionic wormhole solutions described by Giddings and Strominger [9] and Myers [10]. The effective action of string theory contains the square of the field strength of the antisymmetric field strength tensor \( B_{\mu\nu} \). The effective action considered in the ref. [8] is

\[ \int \left( -\frac{1}{16\pi G} R + \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right) \sqrt{g} d^4 x . \quad (12) \]
The field strength can be locally expressed as

$$H_{\mu\nu\lambda} = \varepsilon_{\mu\nu\lambda\rho} \partial^\rho A$$  \hspace{1cm} (13)$$

where $A$ is pseudoscalar field, so that $d^* H = 0 = dH$. Here asterisk denotes the Hodge dual. In three euclidean dimensions, we can write

$$F_{\mu\nu} = \varepsilon_{\mu\nu\lambda} \nabla^\lambda \phi$$  \hspace{1cm} (14)$$
or $^* F = d\phi$ is a closed one-form; $d^* F = 0 = dF$. The dual action is obtained by substituting eq.(14) in the eq.(1),

$$S = \int d^3 x \sqrt{g} \left( -M_P^2 R + \frac{1}{2e^2} (\nabla \phi)^2 \right) .$$  \hspace{1cm} (15)$$

It describes a theory of the scalar field coupled to gravity. This scalar field is directly responsible for the stability of the wormhole and providing it with negative stress energy. For the spherically symmetric ansatz, eq.(5), of the metric the Einstein and matter field equations are satisfied for

$$\phi = \frac{n}{2a_o} \arctan \frac{l}{a_o} .$$  \hspace{1cm} (16)$$

We note that the field equation for the dual scalar field, $\phi$, has the form of a current conservation law and consequently

$$\int_{S^2} (\sqrt{\bar{g}} g^{oo} \partial_o \phi) = 2\pi n ,$$  \hspace{1cm} (17)$$

where the integration is taken over 2-sphere around $t=0$. Here $n$ is required to be integer after quantisation.

**III. VERTEX OPERATOR**

The concept of vertex operator, in the context of wormholes, was first introduced by Hawking \[1\]. In order to
account for the effects of wormholes on the physical phenomena, at length scales much larger than the wormhole size, one can construct effective actions following the proposal of Hawking. We can envisage the scenario where the initial configuration is characterized by the geometry $\Sigma_i$ with charge $Q_i$, which evolves to a geometry $\Sigma_f$ and charge $Q_f$. The amplitude can be represented in the Feynman’s path integral approach. However, it must be emphasized that the wormhole is stabilized by a conserved global charge,

$$Q = \int_{\Sigma} d\Sigma^\mu J_\mu \quad (18)$$

which follows from the current conservation equation, $J^\mu;_\mu = 0$. An elegant formulation of the topology changing processes, in Hamiltonian path integral approach, was given by Grinstein [11]. In what follows we adopt the ref. [11].

Let us first write the transition amplitude from an arbitrary initial state $|\Sigma_i, \phi_i\rangle$ to some final state $|\Sigma_f, \phi_f\rangle$ which in path integral representation is given by [12],

$$\langle \Sigma_f, \phi_f | \Sigma_i, \phi_i \rangle = \int_{b.c.} D[g, \phi] \exp \left( -S(g, \phi) \right) \quad (19)$$

where $D[g, \phi]$ is a measure on the space of all field configurations $g$ and $\phi$, $S(g, \phi)$ is the action of the fields, and the integral is taken over all fields which have given boundary conditions $\langle b.c. \rangle$; $\phi(x, t = t_f) = \phi_f$ and $\phi(x, t = t_i) = \phi_i$ on surfaces $\Sigma_f$ and $\Sigma_i$ respectively. We are interested in initial and final configurations with definite charges. Such states are defined by geometries $\Sigma_i$, $\Sigma_f$ and charge densities $J_i^\phi$, $J_f^\phi$ respectively. Here $J_\phi$, the charge density, is the canonically conjugate momentum of the scalar field $\phi$. After some lengthy but
straight forward calculations, we arrive at the following expression for the transition amplitude,

$$\langle \Sigma_f, J_f^\phi | \Sigma_i, J_i^\phi \rangle = \int d\phi_i d\phi_f e^{i\int_{\Sigma_i} d\Sigma \cdot J_i^\phi - \int_{\Sigma_f} d\Sigma \cdot J_f^\phi} \times \langle \Sigma_f, \phi_f | \Sigma_i, \phi_i \rangle$$

$$= \int d\phi_i d\phi_f e^{i(Q_f^\phi - Q_i^\phi)} \int D[g, \phi] \exp (-S[g, \phi])$$

(20)

where $Q_i = \int_{\Sigma_i} d\Sigma \cdot J_i^\phi$ and $Q_f = \int_{\Sigma_f} d\Sigma \cdot J_f^\phi$ are the conserved charges on respective surfaces. Using the identity

$$\phi_f = \int_{t_i}^{t_f} dt \partial_t \phi + \phi_i$$

we can write

$$[Q_f^\phi - Q_i^\phi] = i(Q_f - Q_i)\phi_i + i \int_{t_i}^{t_f} d\Sigma_t J_t^\phi \partial_t \phi_i$$

Finally the integration over $\phi_i$ and $\phi_f$ will give

$$\langle \Sigma_f, J_f^\phi | \Sigma_i, J_i^\phi \rangle = \delta(Q_f - Q_i) \int D[g, \phi] e^{-S+i \int_{t_i}^{t_f} d^3x \sqrt{g} J_{\phi}^\mu \partial_{\mu} \phi}$$

(21)

where we have used the fact that $\partial_i \phi = 0$, i.e., $\phi$ does not vary over space like surface $\Sigma$, see eq.(16). The above expression tells us that charge of initial and final configurations should be conserved. Now we are ready to do integration over $\phi$. The result of the integration of $\phi$ is reproduced if we set into the integrand on the right hand side of the above equation

$$\phi_{b, \mu}^i = i J_{\mu}$$

(22)

and remove the integration over $\phi$. The value of the field, $\phi_{b}^i$, in above equation corresponds to the effective background or saddle point configuration. Note that we
have dropped suffix $\phi$ of $J^\mu_\phi$ and here onward we shall write only $J^\mu$.

The path integral in eq.(21) does include the wormhole fluctuations. It is not known so far how to handle these fluctuations in a spacetime theory explicitly. However, it does not stop one to formulate a low energy effective theory. Here one can get rid of the fluctuations of wavelengths shorter than some scale $L \gg a_o$ by integrating them out. In this approach the effects of wormhole fluctuations of size smaller than $L$ can be summarized through inserting \textit{vertex functions} in the integrand [1].

The transition amplitude can be written in the following factorized form by introducing the vertex operators,

\[
\langle \Sigma_f, J^f_\mu | \Sigma_i, J^i_\mu \rangle = \langle \Sigma_f, J^f_\mu | V \rangle \times \langle V | \Sigma_i, J^i_\mu \rangle \quad (23)
\]

where

\[
\langle \Sigma, J_\mu | V \rangle \alpha = \int D[\varphi, \phi] e^{-S+S+ \int d^3x \sqrt{-g} J^\mu_\phi \cdot \phi} \quad (24)
\]

Note that the amplitudes on the right hand side of (23) are computed in flat space and integral over configurations does no more include wormhole fluctuations. Their sole effect is accounted by the introduction of the vertex operator, $V$. Further wormhole carries charge and one should no more insist on $J^\mu_\mu = 0$ rather following is true,

\[
J^\mu_\mu = Q \frac{\delta^3(x-x_o)}{\sqrt{g}} , \quad (25)
\]

and

\[
\int d^3x \sqrt{-g} (J^\mu_\phi)_{,\mu} = Q \phi(x_o) + \int d^3x \sqrt{-g} J^\mu_\phi_{,\mu}. \quad (26)
\]

It determines the vertex operator, $V$, on the r.h.s. of the eq.(24) to be $e^{-iQ\phi(x_o)}$ for compensation.
We present below an alternative method of extracting vertex operators \[8\]. Here expectation value of field operators in wormhole background is approximated by their saddle point values in semiclassical limit (\(\hbar = 0\)),

\[
\langle \phi(x_1) \cdots \phi(x_n) \rangle_w = \langle 1 \rangle_w \phi^b(x_1) \cdots \phi^b(x_n) + O(\hbar)
\]

(26)

where \(\langle 1 \rangle_w\) is the normalisation and \(\phi^b(x)\) is the saddle point value of the scalar field. We can read from equations (22) and (25) that \(\phi^b\) satisfies the green function equation,

\[
\Box \phi^b(x) = iQ \frac{\delta^3(x - x_o)}{\sqrt{g}}.
\]

(27)

Eq. (27) is the same as the one satisfied by the Feynman propagator, \(\triangle_F(x, x_o)\). Thus we can write the product of fields on right hand side of eq. (26) as

\[
\langle \phi(x_1) \cdots \phi(x_n) \rangle_w = \langle 1 \rangle_w \left( -iQ \right)^n \prod_{i=1}^{n} \triangle_F(x_i, x_o).
\]

(28)

In order to reproduce the above results, we must chose an appropriate form for the vertex operator. It is evident that \(V(\phi(x_o))\) should have the following form

\[
V(x_o) = \langle 1 \rangle_w \frac{1}{n!} (-iQ)^n \phi^n(x_o)
\]

(29)

so that relation

\[
\langle \phi(x_1) \cdots \phi(x_n) \phi(y_1) \cdots \phi(y_n) \rangle_w = \langle \phi(x_1) \cdots \phi(x_n) V(x_o) \rangle_o \langle V(x_o) \phi(y_1) \cdots \phi(y_n) \rangle_o
\]

(30)

can be checked to be consistent with eq. (26) when we choose \(V\) as given by (29). Note that points \(\{x_1, \cdots, x_n\}\) belong to one asymptotically flat region whereas \(\{y_1, \cdots, y_n\}\) belong to the another asymptotically flat region. The tilde in \(V(\tilde{x}_o)\) is used to distinguish the vertex operators in two asymptotic regions of
the wormhole. We have obtained the form of the vertex operators for product of a string of $n$ fields at different spacetime points. The general expression for $V$ now can be written

$$V(x_o) = \langle 1 \rangle_w e^{-iQ\phi(x_o)}.$$  \hspace{1cm} (31)

Now we turn our attention towards the emission and absorption of gravitons by the wormholes. It is expected that one might be able to compute correlation functions involving the metrics $g_{\mu\nu}(x_i)$. Such correlation function will be gauge dependent. One can sideline this difficulty by calculating correlations involving the object like $\prod_i R(x_i)$, where $R(x)$ is the scalar curvature. The result is given by the background configuration,

$$\langle R(x_1) \cdots R(x_n) \rangle_w = \langle 1 \rangle_w R^b(x_1) \cdots R^b(x_n) + O(h)$$  \hspace{1cm} (32)

where background curvature is

$$R^b(x) = -\frac{8r_o^2}{|x - x_o|^4(1 + \frac{r_o^2}{|x - x_o|^2})^4}.$$  \hspace{1cm} (33)

Since the right hand side of eq.(32) involves products of curvature scalars at their saddle point values thus we are entitled to use the field equation to facilitate semiclassical computations. One can check that the vertex operators derived for the fields, $\phi$, correctly reproduce the desired results when we use the relations. Thus

$$\langle R(x_1) \cdots R(x_n) \rangle_w = (8\pi G)^n \langle \phi^{\mu} \phi_{,\mu}(x_1) \cdots \phi^{\mu} \phi_{,\mu}(x_n) V(x_o) \rangle_o \langle V(\tilde{x}_o) \rangle_o + O(h).$$

In the following section we shall also include matter fields in wormhole background. Correspondingly the structure of vertex operators will get modified.
IV. MATTER FIELD SECTOR

First let us write the metric of Gupta et al. \([5](\text{eq. } 3)\), in the following conformally flat form

\[
d s^2 = \Omega^2(x) \delta_{\mu\nu} dx^\mu \, dx^\nu
\]  
(34)

where

\[
\Omega^2(x) = \left(1 + \frac{a_o^2}{|x - x_o|^2}\right)^2
\]  
(35)

and we find

\[
|x - x_o| = \frac{a_o}{2} \exp(\sinh^{-1} \frac{t}{a_o}).
\]  
(36)

We note that two asymptotically flat regions correspond to \(x \to \infty\) (or \(t \to \infty\)) and \(x \to x_o\) (or \(t \to -\infty\)). The singularity at \(x = x_o\) is not a curvature singularity. It can be seen from the inversion of the coordinates in the sphere of radius \(a_o/2\) as follows

\[
(y - y_o)_\mu (x - x_o)_\mu = \frac{a_o^2}{4} = r_o^2 \text{(say)}.
\]  
(37)

This brings the point at infinity and the null infinity surface in the x-space to the origin and the light cone at the origin in y-space, respectively. The origin of the original spacetime(x-space) and its light cone are sent to infinity. Metric in y-space becomes conformal to that of x-space

\[
d y^\mu \, d y_\mu = \left(\frac{r_o^2}{|x - x_o|^2}\right)^2 \, d x^\mu \, d x_\mu,
\]

whence eq. (34) becomes

\[
ds^2 = \left(1 + \frac{|x - x_o|^2}{r_o^2}\right)^2 \, d y^\mu \, d y_\mu.
\]

Now we are ready to find correlation functions for matter fields in wormhole background. Though this, generally, is a difficult task. But the calculation is much simplified for conformally coupled fields in conformal backgrounds. These fields have zero vacuum expectation values.
We divide the spacetime region of the wormhole into two subspaces, viz., $|x_i - x_o| \gg r_o$ and $|x_i - x_o| \ll r_o$. We shall denote later ones by a tilde. Let us first consider the propagation of the scalar fields, $S(x)$, from one side of the wormhole to the other side. Since scalar field is conformally coupled in three dimensions, it immediately gives for two-point correlation function in wormhole background

$$\langle S(x) S(\tilde{x}) \rangle_w = \langle 1 \rangle_w \Omega^{-1/2}(x) \Omega^{-1/2}(\tilde{x}) \triangle_F (x, \tilde{x}) \quad (38)$$

where the flat space propagator is

$$\triangle_F (x, \tilde{x}) = \langle S(x) S(\tilde{x}) \rangle_o = \frac{1}{4\pi} \frac{1}{|x - \tilde{x}|} . \quad (39)$$

In the limit $|x - x_o| \gg r_o \gg |\tilde{x} - x_o|$

$$\Omega(x) \approx 1, \quad \Omega(\tilde{x}) \approx \frac{r_o^2}{|\tilde{x} - x_o|^2} \frac{1}{|x - \tilde{x}|} \approx |x - x_o| . \quad (40)$$

We get to leading order in $\frac{r_o}{|x - x_o|}$ and $\frac{|\tilde{x} - x_o|}{r_o}$,

$$\langle S(x) S(\tilde{x}) \rangle_w \approx \langle 1 \rangle_w \left( \frac{r_o}{|\tilde{x} - x_o|} \right)^{-1} \frac{1}{4\pi} \frac{1}{|x - x_o|} . \quad (41)$$

Now, we can evaluate the vertex operators once we write the first factor in the inverted coordinates defined in eq. (37). In these coordinates the distance from the wormhole throat at $x_o$ to the point $\tilde{x}$, $\approx \frac{r_o^2}{|\tilde{x} - x_o|}$, becomes $|\tilde{y} - \tilde{y}_o|$. Eq. (41) can be expressed as

$$\langle S(x) S(\tilde{y}) \rangle_w \approx \langle 1 \rangle_w r_o \frac{1}{|\tilde{y} - \tilde{y}_o|} \frac{1}{4\pi} \frac{1}{|x - x_o|} . \quad (42)$$

If one looks carefully, it is clear that the above correlation function is in the factorised form. Let us write $\langle S(x) S(\tilde{y}) \rangle_w$ in the factorised form by introducing appropriate vertex operators

13
\[ \langle S(x) S(\tilde{y}) \rangle_w = \langle S(x) V(x_o) \rangle_o \langle S(\tilde{y}) V(\tilde{y}_o) \rangle_o \]  

(43)

Thus it follows from equations (31), (42) and (43) that the vertex operators associated with the two point function of conformally coupled scalar field is

\[ V(x_o) V(\tilde{y}_o) = c^2 e^{-i Q \phi(x_o) + i Q \phi(\tilde{y}_o)} 4\pi r_o S(x_o) S(\tilde{y}_o). \]

(44)

Note that this factorization of the correlation function in the wormhole background is crucial for the determination of the vertex operators. The normalisation constant \( c^2 = \langle 1 \rangle_w^2 \) corresponds to the ratio of unit operators evaluated in wormhole background and in flat space.

The above results are easily extendible to the case when \( n \) fields are inserted on each side of the wormhole, i.e.,

\[ \langle \prod_{i=1}^n S(x_i) S(\tilde{y}_i) \rangle_w = \langle 1 \rangle_w (4\pi r_o)^n n! \prod_{i=1}^n \frac{1}{4\pi} \frac{1}{|x_i - x_o|} \frac{1}{4\pi} \frac{1}{|\tilde{y}_i - \tilde{y}_o|}. \]

(45)

This gives vertex function to be

\[ V(x_o) V(\tilde{y}_o) = \frac{c^2 (4\pi r_o)^n}{n!} e^{-iQ[\phi(x_o) - \phi(\tilde{y}_o)]} S^n(x_o) S^n(\tilde{y}_o). \]

(46)

A factor of \( (\frac{1}{\hbar})^n \) is hidden on the right hand side of the above expression for the vertex operator in order to get correct order \( \hbar^{2n} \) for \( 2n \) propagators. Whence together one can write a complete expression for the vertex operator

\[ V(x_o) V(\tilde{y}_o) = c^2 e^{-i Q[\phi(x_o) - \phi(\tilde{y}_o)]} \exp[4\pi r_o S(x_o) S(\tilde{y}_o)]. \]

(47)

Massless Fermi fields, too, are conformally scaled with the conformal scaling of the metric, e.g., for metric given in eq. (34), we will have
\[ \psi(x) \to \Omega^{-1}(x) \psi(x). \] (48)

The propagator in wormhole background can be written as

\[ \langle \psi(x) \bar{\psi}(\tilde{x}) \rangle_w = \langle 1 \rangle_w \Omega^{-1}(x)\Omega^{-1}(\tilde{x}) S_o(x, \tilde{x}) \] (49)

where \( S_o(x, \tilde{x}) = \langle \psi(x) \bar{\psi}(\tilde{x}) \rangle_o = -\frac{i}{4\pi} \gamma^a(x-\tilde{x})a |x-\tilde{x}| \) which is the flat space propagator for fermi fields. Here \( \gamma^a \)'s are 2 \times 2 matrices with \( \{\gamma^a, \gamma^b\} = 2 \delta^{ab} I, a = 1, 2, 3, \) where \( I \) is the identity matrix. We shall find out vertex operators for Lorentz scalar operators, like \( \bar{\psi}(x) \psi(x) \).

One will get

\[ \langle \bar{\psi}(x)\psi(x) \bar{\psi}(\tilde{x})\psi(\tilde{x}) \rangle_w = \langle 1 \rangle_w \text{Tr} S_w(x, \tilde{x}) S_w(\tilde{x}, x) \] (50)

where \( \text{Tr} \) represents the trace over spinor indices. On simplification we get

\[ \langle \bar{\psi}(x)\psi(x) \bar{\psi}(\tilde{x})\psi(\tilde{x}) \rangle_w = -\frac{1}{(4\pi)^2} \Omega^{-2}(x) \Omega^{-2}(\tilde{x}) \frac{2}{|x-\tilde{x}|^4}. \] (51)

Since \(|x-x_o| \gg r_o \gg |\tilde{x}-x_o|\), using eq.(40) we obtain

\[ \langle \bar{\psi}(x)\psi(x) \bar{\psi}(\tilde{x})\psi(\tilde{x}) \rangle_w \cong -\frac{1}{(4\pi)^2} \frac{|\tilde{x}-x_o|^4}{r_o^4} \frac{2}{|x-x_o|^4} \] (52)

which indeed gets factorized if one notices eq.(37). This result is reproduced if we insert the vertex operator

\[ V(x_o) V(\tilde{x}_o) = e^{2} e^{-i Q [\phi(x_o)-\phi(\tilde{x}_o)] (-8\pi^2 r_o^4) \bar{\psi}(x_o)\psi(x_o) \bar{\psi}(\tilde{x}_o)\psi(\tilde{x}_o)} \] (53)

in the following

\[ \langle \bar{\psi}(x)\psi(x) \bar{\psi}(\tilde{x})\psi(\tilde{x}) \rangle_w = \langle \bar{\psi}(x)\psi(x) V(x_o) \rangle_o \langle \bar{\psi}(\tilde{x})\psi(\tilde{x}) V(\tilde{x}_o) \rangle_o. \] (54)
The above calculation can be done for arbitrary number of insertions of operators $\bar{\psi}\psi$ on either side of wormhole. Whence we obtain a complete expression for the vertex operator in case of fermion propagation,

$$V(x_o) V(\tilde{x}_o) = c^2 e^{-i Q \left| \phi(x_o) - \phi(\tilde{x}_o) \right|} \exp \left[ (-8\pi^2 r_o^4) \bar{\psi}(x_o) \psi(x_o) \bar{\psi}(\tilde{x}_o) \psi(\tilde{x}_o) \right]$$

(55)

Equations (47) and (55) are the complete vertex operators for the scalar and spinor fields evaluated in the background of the wormhole. We note that the matter field parts of these vertex operators are not factorized in the fields $S(x)$ and $\psi(x)$ respectively. If we were introduced $\alpha$-parameters following the formalism due to Coleman [3], it is evident that the $\alpha$-parameters will now be labeled by the global $Q$ and the attributes of the corresponding matter fields.

V. CONCLUSION AND DISCUSSION

We have worked out the dual theory for magnetic wormhole of Gupta et al. [5] in three euclidean dimensions in section-II. In the section-III, we explicitly obtained the vertex operator $V \sim e^{-i Q \phi}$ for the correlation function of scalar field operators, $\phi$ (dual to electromagnetic field strength). In the semiclassical approximation, it is shown that at low energies these correlations are reproduced in a flat background with the insertion of these vertex operators $V$. One can interpretate that the insertion of vertex operator in asymptotic flat space is equivalent to the effect of wormhole background on the propagation of fields far away from the throat.
Next, in section-IV we have considered the propagation of matter fields in the wormholes geometry. We find that vertex functions are appropriately modified. The modification accounts for introduction of field operators located at the throat of the wormhole. These vertex functions, when inserted in the low energy effective action, will reproduce correctly the low energy Green functions for the Lorentz invariant composite field operators made of scalar and fermi fields.

So far it has not been known to us how the above programme could be extended for massive \( m \ll a_o^{-1} \), or massless but not conformaly coupled fields.

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