Coherent $\pi^0\pi^0$ photoproduction on lightest nuclei

M. Egorov and A. Fix

Laboratory of Mathematical Physics, Tomsk Polytechnic University, 634050 Tomsk, Russia

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Coherent photoproduction of $\pi^0\pi^0$ on the deuteron and $^3$He is calculated. The isoscalar and isovector parts of the elementary photoproduction amplitude was determined by fitting the measured total cross section on protons and neutrons. The dependence of the cross section on the isospin of the target nucleus is discussed.

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I. INTRODUCTION

Coherent photoproduction of neutral pions on nuclei, when the target nucleus remains in its ground state, are widely used in meson-nuclear physics. Firstly, these reactions allow obtaining nuclear structure information. Since the incident photon probes the entire nuclear volume, the cross section directly depends on the nuclear density distribution. An important advantage of reactions involving neutral pions is that they are not complicated by the coulomb interaction, so that the description of the final state becomes more simple. Among the recent works one can mention Ref. [1], in which the dependence of the cross section for $\gamma^7$Li $\rightarrow \pi^0^7$Li on the $^7$Li parameters was explored. Generally, the results obtained from the photoproduction reactions seem to be of comparable quality to that provided by other processes, for example, by elastic particle-nuclear scattering.

The second important aspect, related to the coherent meson photoproduction, is the investigation of the photoproduction mechanism. The case in point are the spin-isospin selection rules provided by the quantum numbers of the target nucleus. These rules make it possible to identify the contributions of individual components of the photoproduction amplitude. Recently, experiments were carried out in order to study partial transitions in nuclei proceeding when the meson is produced [2]. Such experiments became possible due to significant improvements in detection of the mesons. This allowed one to overcome difficulties in identifying individual nuclear transitions, which were the main obstacle of using nuclear reactions to study meson photoproduction dynamics.

In the present work we consider coherent photoproduction of two neutral pions

$$\gamma + A \rightarrow \pi^0 + \pi^0 + A.$$  \hspace{1cm} (1)

The corresponding single nucleon process

$$\gamma + N \rightarrow \pi^0 + \pi^0 + N$$ \hspace{1cm} (2)

is still not very well understood. There are significant deviations between the analyses provided by different groups (see, for example, the discussion in Ref. [3]). As a rule they give an acceptable description of the available data, although the partial wave content of the corresponding amplitudes may be rather different. The complete experiment, which may resolve this ambiguity, appears to be extremely difficult when two mesons are produced [4]. Some results for photoproduction of $\pi^0\pi^0$ on protons are obtained only in the low energy region [3] where one can effectively use a truncated expansion of the amplitude over partial waves.

The information which may be obtained on the elementary amplitude is usually considered as a main motivation for studying coherent photoproduction on nuclei. There are, however, difficulties arising from a rather strong sensitivity of the coherent cross section to different model ingredients, like off-shell behavior of the elementary amplitude, effects of the final pion-nuclear interaction, role of two-nucleon production mechanisms, etc. These problems seem to be unavoidable at least within the models in which the impulse approximation is used as a basic zero-order approximation. Hence, more or less firm quantitative conclusion can be drawn only as long as sophisticated microscopic models are adopted. At the same time, in the $\pi^0\pi^0$ case where even the general partial wave structure is rather poorly known, already qualitative results may provide very useful information.

Here we are mainly focused on the isospin structure of the $\pi^0\pi^0$ photoproduction amplitude. The corresponding selection rules provided by the charge of the nucleus enable us to get information about the role of isoscalar and

*Electronic address: fix@tpu.ru
FIG. 1: Left panel: total cross section for $\pi^0\pi^0$ photoproduction on protons (solid line) and neutrons (dashed line). The data are taken from Ref. [3] (triangles) and [7] (circles). Right panel: solid line: total cross section for $\pi^0\pi^0$ photoproduction on protons (same as on the left panel), dash-dotted curves: the proton cross section calculated with only the spin-flip $\vec{L}$ and the spin independent part $K$ in Eq. (5).

Isovector transitions in this reaction. The numerical calculations are performed on the deuteron and $^3$He. Our main purpose is to study the presumably strong sensitivity of the cross section to the isospin of the target. The results are expected to be important as a firm testing ground for our knowledge on the two-pion photoproduction dynamics.

II. FORMALISM

The formalism, which we used to calculate the cross section (1), was partially considered in our previous work [5] on coherent photoproduction of $\pi^0\eta$ pairs. The cross section for coherent photoproduction of two mesons on a nucleus $A$ with spin $J$ is given in the center-of-mass frame by

$$\frac{d\sigma}{d\Omega_p d\Omega_q' d\omega_{\pi\pi}} = \frac{1}{(2\pi)^5} \frac{E_A E'_A q^* p}{8 E_{\gamma}(E_{\gamma} + E_A)^2} \frac{1}{2(2J+1)} \sum_{\lambda M_i M_f} |T_{\lambda M_i M_f}|^2,$$

where the matrix element $T_{\lambda M_i M_f} = \langle M_f | \hat{T}_{\gamma A} | M_i \rangle$ determines the transition between the nuclear states with definite spin projection. The energy of the incident photon and the initial and the final energies of the target nucleus are denoted respectively by $E_{\gamma}$, $E_A$ and $E_A'$. As independent kinematical variables we took the invariant $\pi\pi$ mass $\omega_{\pi\pi}$, the spherical angle $\Omega_p$ of the final nucleus momentum $\vec{p} = (p, \Omega_p)$ and the spherical angle of the momentum $\vec{q}^* = (q^*, \Omega_{q^*})$ of one of the two pions in their center-of-mass frame. The photon polarization index $\lambda = \pm 1$ will be omitted in subsequent expressions.

In order to calculate the amplitude $T_{M_i M_f}$ we used the impulse approximation taking the operator $\hat{T}_{\gamma A}$ as a superposition of operators $\hat{t}_{\gamma N}$ for photoproduction on free nucleons

$$\hat{T}_{\gamma A} = \sum_i \hat{t}_{\gamma N}(i),$$

where the summation is performed over all nucleons in the target. The spin structure of the single nucleon operator has the well known form

$$\hat{t}_{\gamma N} = K + i \vec{L} \cdot \vec{\sigma},$$

where $\vec{\sigma}$ is the Pauli spin operator, and the amplitudes $\vec{L}$ and $K$ describe transitions with and without flip of the nucleon spin. Both amplitudes depend on the photon polarization index $\lambda$ and on the kinematical variables as described above.
For \( t_{\gamma N} \) we adopted the phenomenological analysis \( 3 \) for \( \gamma p \to \pi^0\pi^0p \). The analysis contains the resonance \( D_{13}(1520) \) whose parameters were taken from the Particle Data Group listing (PDG) \( 6 \) and were not varied. Other partial amplitudes having the spin-parity \( J^\pi = 1/2^- \), \( 1/2^+ \) and \( 3/2^+ \), as well as a possible contribution in \( J^\pi = 3/2^- \) (in addition to \( D_{13}(1520) \)) were taken in the form

\[
t_{J^\pi} = [t_B + t_R(W_{\gamma N})] G_\Delta F_{\Delta \to \pi N}.
\]

Here the first term in the brackets corresponds to the background, which is assumed to depend smoothly on the energy, whereas the second term containing possible contributions from baryon resonances in the \( s \)-channel, can undergo rapid variation with energy. The factors \( G_\Delta \) and \( F_{\Delta \to \pi N} \) are the \( \Delta \)-isobar propagator and the \( \Delta \to \pi N \) decay vertex function. The details of parametrization of the terms \( t_B \) and \( t_R \) are described in Ref. \( 3 \). The parameters were adjusted to the angular distributions of the incident photons in the frame, in which the \( z \)-axis was taken along the normal to the final state plane. The latter contains the momenta of all three final particles in the center-of-mass system. The predictions of this fit for the total cross section are presented in Fig.\( 4 \) (the solid curve). In the same figure we show the contributions coming from only the spin independent part \( K \) and the spin-flip amplitude \( \tilde{L} \).

The amplitude \( 6 \) was fitted to the proton data. To calculate the photoproduction on nucleon one needs information about the reaction on a neutron \( \gamma n \to \pi^0\pi^0n \). The contribution of \( D_{13}(1520) \) to \( \gamma n \to \pi^0\pi^0n \) was determined by the PDG values of the corresponding helicity \( D_{13} \to n\gamma \) amplitudes \( A_{1/2} \) and \( A_{3/2} \). To fix the isotopic structure of other contributions we used the following simple prescription. In the isotopic spin space of the nucleon the operator \( t_{\gamma N} \) is given by

\[
t_{\gamma N} = t_0 + t_1 \tau_3,
\]

where \( \tau_3 \) is the \( z \)-component of the nucleon isospin operator \( \vec{\tau} \). The coefficients \( t_0 \) and \( t_1 \) in Eq.\( 7 \) determine transitions to the final \( \pi\pi N \) state when the initial nucleon absorbs a photon having isospin \( I = 0 \) and \( I = 1 \), respectively. Therefore, the amplitudes of \( \pi^0\pi^0 \) photoproduction on the proton and on the neutron read

\[
t_{\gamma p} = t_0 + t_1, \quad t_{\gamma n} = t_0 - t_1.
\]

A consequence of Eq.\( 8 \) is that proton and neutron amplitudes are related to each other as

\[
t_{\gamma n} = (2\alpha - 1) t_{\gamma p},
\]

where the parameter \( \alpha \) is the ratio of the isoscalar part \( t_0 \) to the proton amplitude

\[
\alpha = \frac{t_0}{t_{\gamma p}}.
\]

It determines the relative contribution of the isoscalar \( (I = 0) \) and the isovector \( (I = 1) \) photons to the reaction. In the general case its value is energy dependent. However for our qualitative estimations we took it as constant. To obtain \( \alpha \) we used experimental data for the total cross section of \( \gamma n \to \pi^0\pi^0n \), presented in Ref. \( 5 \). Since the cross section is proportional to the square of the modulus of \( t_{\gamma n} \), the value of \( \alpha \) can not be unambiguously determined. In particular, the data \( 7 \) for \( \gamma n \to \pi^0\pi^0n \) in Fig.\( 1 \) may be described both with \( \alpha = 0.08 \) and \( \alpha = 0.75 \). However, since the major part of the resonances in the energy region considered are excited by the isovector photons and, therefore, the larger value 0.75 seems to be rather unlikely, we used

\[
\alpha = 0.08.
\]

As is noted above, to calculate photoproduction of \( \pi^0\pi^0 \) on nuclei we used impulse approximation. The kinematic in the \( \gamma N \) subsystem was fixed using the prescription of Ref. \( 3 \) in which the elementary amplitude \( t_{\gamma N} \) is frozen at some mean value of the initial nucleon momentum \( \langle \vec{p}_i \rangle \) determined in the overall center-of-mass frame as

\[
\langle \vec{p}_i \rangle = -\frac{1}{A} \vec{k} - \frac{A - 1}{2A} \vec{Q},
\]

where \( A \) is the nuclear mass number, \( \vec{k} \) is the photon momentum, and \( \vec{Q} \) is the three-momentum transferred to the nucleus. The energy \( E_i \) of the nucleon is fixed by the on-mass-shell condition \( E_i^2 = M_N^2 + \vec{p}_i^2 \). The four-momentum \( (E_f, \vec{p}_f) \) of the final nucleon was then determined by the energy-momentum conservation at the single-nucleon vertex

\[
E_f = E_i + E_i - \omega_1 - \omega_2,
\]

\[
\vec{p}_f = \vec{k} + \vec{p}_i - \vec{q}_1 - \vec{q}_2,
\]
FIG. 2: Total cross section of the incoherent reaction $\gamma d \rightarrow \pi^0\pi^0np$. The filled and open circles are taken from Ref. [9] and [10] respectively. The dashed and the solid curves are obtained without and with inclusion of the neutron-proton interaction in the final state. The dash-dotted curve represents the value of the third term on the r.h.s. of Eq. (24) (see discussion in the text). On the right panel the low-energy region from threshold up to $E_\gamma = 500$ MeV is shown separately.

$$W_{\gamma N} = \sqrt{(E_\gamma + E_i)^2 - (\vec{k} + \vec{p}_i)^2}.$$  \hspace{1cm} (15)

If the prescription (12) together with the mass-shell condition for the initial nucleon are used, the final nucleon turns out to be very close to its mass-shell. The difference $|M_N - \sqrt{E_f^2 - \vec{p}_f^2}|$, where $E_f$ and $\vec{p}_f$ are defined according to (13) and (14), does not surpass 4 MeV in the whole kinematic region considered. This is quite important in our case since our amplitude (6) is of pure phenomenological nature and is determined in a relatively narrow energy region $W_{\gamma N} - (M_N + 2M_\pi) \leq 325$ MeV. In Sect. III we also discuss sensitivity of the results to the prescription for fixing the single nucleon kinematic.

III. RESULTS AND DISCUSSION

Using for $\alpha$ the value in (11) we firstly calculated the cross section for quasifree photoproduction of $\pi^0\pi^0$ on the deuteron. The results shown in Fig. 2 demonstrate rather good agreement with the data which is achieved within only the impulse approximation, without much need for rescattering corrections. A direct calculation of the nucleon-nucleon rescattering shows that this effect leads to a slight increase of the total cross section in the whole energy region (compare dashed and solid lines in Fig. 2). To include the neutron-proton interaction in the final state we used the standard formalism described, for example, in Ref. [11] and we would like to refer the reader to this paper for more details.

One important point is in order. As is noted above, the incoherent cross section appears to be rather insensitive to the model ingredients. For example, the dominant contribution to incoherent photoproduction on a deuteron, provided by the impulse approximation, is proportional to the incoherent sum $|t_{\gamma p}|^2 + |t_{\gamma n}|^2$, folded with the momentum distribution of the nucleons in the deuteron. As a consequence, if a realistic nuclear wave function is used in conjunction with the elementary photoproduction operator adjusted to the single nucleon data then already the impulse approximation provides a rather good description of the unpolarized cross section on a nucleus. The mechanisms involving more than one nucleon, like final state rescatterings, meson exchange currents, nuclear isobar configurations etc, are important in rather narrow kinematical region where the cross section is, as a rule, rather small. At the same time, in the coherent channel the reaction amplitude is proportional to the absolute square of $t_{\gamma p} + t_{\gamma n}$

$$\sigma_{\gamma d} \sim |t_{\gamma p} + t_{\gamma n}|^2,$$ \hspace{2cm} (16)
FIG. 3: Left panel: total cross section for coherent $\pi^0\pi^0$ photoproduction on the deuteron (dashed line) and $^3\text{He}$ (solid line). The dash-dotted line is the cross section on $^3\text{He}$, obtained with Blankenbecler-Sugar choice of the $\gamma N$ invariant energy $W_{\gamma N}$ (see Eq. (29) and the corresponding discussion in the text). Right panel: total cross section for $\gamma^7\text{Li} \rightarrow \pi^0\pi^0^7\text{Li}$, calculated with $^7\text{Li}$ parameters from Ref. [15].

so that not only the moduli but also the relative phase between $t_{\gamma p}$ and $t_{\gamma n}$ play a role. The nature of this phase may be rather complicated. Even if the reaction is dominated by the single resonance excitation, the nontrivial phase may appear due to large background contributions in those channels to which the $\pi^0\pi^0$ channel is coupled through the unitary relations. This should primarily be the single pion channel $\pi N$ which will generate a dressing of the bare vertex for resonance photoexcitation $\gamma N \rightarrow R$, thus leading to an energy dependent electromagnetic coupling $g_{\gamma N \rightarrow R}$ whose phase may be different for neutrons and protons. In the case of $\eta$ photoproduction this question was addressed rather detailed in Ref. [12]. In the present work we do not calculate these effects, since, as we believe, their impact on our qualitative results is not crucial.

The amplitudes $t_{\gamma p}$ and $t_{\gamma n}$, obtained as described above, were then used to calculate the cross sections of the coherent reactions

$$\gamma + d \rightarrow \pi^0 + \pi^0 + d,$$

(17)

$$\gamma + ^3\text{He} \rightarrow \pi^0 + \pi^0 + ^3\text{He}.$$  

(18)

For the ground states of deuteron and $^3\text{He}$ we adopted the phenomenological wave functions from Refs. [13] and [14], respectively. These functions describe the corresponding form factors in the whole momentum transfer region, as covered by the energy region considered here. Since the deuteron has zero isospin, only the isoscalar part $t_0$ in Eq. (7) contributes to the reaction (17). Using in Eq. (9) the deuteron quantum numbers and taking into account only the spherically symmetric part in its wave function one obtains the approximate relation

$$\sigma_{\gamma d} \sim 4 \left[ \langle |K_0|^2 \rangle + \frac{2}{3} \langle |L_0|^2 \rangle \right].$$

(19)

Here the notation $\langle ... \rangle$ means averaging over the nuclear momentum distribution, available for a given energy. The lower index “0” in $K_0$ and $L_0$ relates to the isoscalar part of these amplitudes in (5).

According to the analysis outlined above the isoscalar part $t_0$ in Eq. (7) amounts only 8% of the absolute value of the proton amplitude $t_{\gamma p}$. Therefore, the cross section of the reaction (17) is expected to be extremely small. From the same considerations one can conclude that the cross section for $^3\text{He}$ should be at least one order of magnitude larger than that on the deuteron. Indeed, since the isospin of $^3\text{He}$ is 1/2, the corresponding cross section within the same approximation as in Eq. (19) reads

$$\sigma_{\gamma ^3\text{He}} \sim 9 \left[ \langle |K_0 + \frac{1}{3} K_1|^2 \rangle + \frac{1}{9} \langle |L_0 - L_1|^2 \rangle \right].$$

(20)
Taking

\[ \left| \frac{K_0}{K_1} \right| \approx \left| \frac{\bar{L}_0}{\bar{L}_1} \right| \approx 8.7 \cdot 10^{-2}, \]  
(21)

according to (11) and taking into account the approximate relation

\[ |K| \approx 1/\sqrt{2}|\bar{L}|, \]  
(22)

as follows from the calculations presented in Fig. 1 we obtain

\[ \frac{\sigma_{\gamma^3\text{He}}}{\sigma_{\gamma d}} \approx 45. \]  
(23)

Our results for the cross sections (17) and (18) are plotted in Fig. 3. As one can see, their ratio is in rough agreement with the estimation (23). At the energy \( E_\gamma = 450 \text{ MeV} \) the value of \( \sigma_{\gamma^3\text{He}}/\sigma_{\gamma d} \) comprises about 46, at \( E_\gamma = 550 \text{ MeV} \) it reaches 31 and at \( E_\gamma = 700 \text{ MeV} \) it decreases to 13. Thus, the cross section for coherent \( \pi^0\pi^0 \) photoproduction should demonstrate a strong dependence on the isospin of the target. This is primarily due to the smallness of the isoscalar part \( t \), as seen in Eq. (20). Namely, within the approximation, in which only the spherically symmetric spatial part of the deuteron scalar matter form factor \( F_\gamma(Q) \) is a kinematical factor whose exact form is insignificant for the present discussion. The last term, proportional to the isoscalar part \( t_0 \) of the \( \pi^0\pi^0 \) photoproduction amplitude, as is predicted by our calculation.

It is worth noting, that the smallness of the cross section for \( \gamma d \to \pi^0\pi^0 d \) makes it possible to 'measure' the \( \pi^0\pi^0 \) production on a neutron as a difference of the total inclusive cross sections on a deuteron and the free proton. Indeed, using closure for the inclusive (coherent plus incoherent) cross section on a deuteron, one obtains

\[ \sigma_{\gamma d}^{\text{incl}} \approx \langle \sigma_{\gamma p} \rangle + \langle \sigma_{\gamma n} \rangle + 2K F_d(Q) \Re \left\{ \frac{1}{3} \langle \bar{F}_p \cdot \bar{L}_n \rangle + \langle K^*_p K_n \rangle \right\}, \]  
(24)

where, as in the previous expressions, the notation \( \langle ... \rangle \) means averaging over the Fermi momentum distribution. \( K \) is a kinematical factor whose exact form is insignificant for the present discussion. The last term, proportional to the deuteron scalar matter form factor \( F_d(Q) \), depending on the transferred three-momentum \( Q \), is generally rather small as may be seen from Fig. 2 where it is plotted by the dash-dotted line.

It is also interesting to note one special property of photoproduction of two neutral pions on \( ^3\text{He} \) which is well seen in Eq. (20). Namely, within the approximation, in which only the spherically symmetric spatial part of the \( ^3\text{He} \) nuclear wave function is taken into account, the protons form a correlated pair with total spin \( S = 0 \). Therefore, the production of mesons with a nucleon spin-flip can proceed on the neutron only. Otherwise we will have in the final state two protons with the same orbital and spin momentum, contradicting the Pauli exclusion principle. This property is clearly manifest in Eq. (20), where the spin dependent part is proportional to \( |\bar{L}_0 - \bar{L}_1| = |\bar{L}_{\gamma n}| \). This means, that one can use the ratio of helium to proton cross sections as a measure of the nucleon spin-change probability in \( \pi^0\pi^0 \) photoproduction.

Using the results presented in Fig. 3 one can make a rough estimate of the cross sections for \( \pi^0\pi^0 \) photoproduction on \( ^7\text{Li} \). Within a simple shell model this nucleus is described as a system, containing a core, which is built up from four nucleons with the total isospin \( T = 0 \), and three valent nucleons. Because of smallness of the isoscalar part \( t_0 \) of the amplitude \( t_{\gamma N} \) the contribution of the core should be small, and the major part of the cross section is expected to come from the valent nucleons. In this respect the cross section for \( \gamma ^7\text{Li} \to \pi^0\pi^0 ^7\text{Li} \) may be estimated according to

\[ \sigma_{\gamma^7\text{Li}} \approx R \sigma_{\gamma^3\text{He}} \]  
(25)

with

\[ R = \frac{\int \left| F_{\gamma^7\text{Li}}^{(1p)}(Q) \right|^2 d\Omega}{\int \left| F_{\gamma^3\text{He}}(Q) \right|^2 d\Omega}, \]  
(26)

where \( Q \) is the momentum transfer, and \( F_A(Q) \) are the corresponding nuclear matter form factors. The notation \((1p)\) means that for \(^7\text{Li}\) only the contribution of the \(1p\)-orbit is taken into account. The integration is performed over the phase space available for each reaction. To estimate the ratio \( R \) we used for a ground state \(^7\text{Li}\) wave function a simple oscillator shell model, which gives for the form factors the well known expressions:

\[ F_{\gamma^3\text{He}}(Q) = \exp^{-{(Q r_0)^2}/4}, \]  
(27)
Taking $r_0 = 1.8 \text{ fm}$ and $r_0 = 1.67 \text{ fm}$ for $^3\text{He}$ and $^7\text{Li}$, respectively, one obtains for $R$ the value close to 0.41 at $E_{\gamma} = 400 \text{ MeV}$ which decreases to $R = 0.25$ at $E_{\gamma} = 750 \text{ MeV}$. In Fig. 2 we present the cross section calculated with the double-well $^7\text{Li}$ wave function from Ref. [15]. The latter gives rather satisfying description of the $^7\text{Li}$ charge form factor in a wide range of momentum transfer. As one can see, our results are in general agreement with the estimation 0.41 $\leq R \leq 0.25$, obtained as discussed above.

Since the emphasis of our calculation for $\gamma^7\text{Li} \to \pi^0\pi^0^7\text{Li}$ plotted in Fig. 2 was on demonstrating insignificance of the four-nucleon core, we do not touch here upon some complications like, e.g., the role of the first excited $^7\text{Li}$ level $^1_2^-$ with $E^* = 0.478 \text{ MeV}$. Since in the data, to which the $^7\text{Li}$ wave function in Ref. [15] was fitted, its contribution was subtracted, it also does not enter our results. At the same time, due to obvious difficulty of separating this level in $\pi^0\pi^0$ photoproduction, it should be taken into account if comparison of the theoretical results with the data is intended.

Expression (7) together with the smallness of the isoscalar part $t_0$ indicates that the isotopic structure of $\pi^0\pi^0$ photoproduction is similar to that for $\eta$ photoproduction in the $S_{11}(1535)$ region. In the latter case, according to the results of Refs. [16–19], the value of $\alpha$ is close to 0.1. Due to the same reason the cross section for coherent $\pi^0\pi^0$ photoproduction on nuclei should have much in common with that for $\eta$ photoproduction. In particular, the process $A(\gamma, \eta)A_i$ is known to be suppressed on the nuclei with zero isospin. For example, direct calculations within impulse approximation as described in Refs. [18] and [20], give for the ratio $\sigma_{\gamma^3\text{He} \to \eta^3\text{He}}/\sigma_{\gamma^d \to \eta^d}$ in the region of $S_{11}(1535)$ the value about 20. The experimental results for the $\gamma^d \to \eta^d$ and $\gamma^3\text{He} \to \eta^3\text{He}$ cross sections [16, 21] indicate, that this ratio is likely to be essentially smaller, about 4. This difference is primarily due to larger value of the deuteron cross section in comparison to that predicted by the model of [18] in which the possible nontrivial phase between the neutron and the proton amplitudes is neglected (see discussion after Eq. (16) and in Ref. [12]).

Since our $\pi^0\pi^0$ amplitude has a relatively large resonant part $t_R$, as defined in Eq. (6), it should be sensitive to the variation of the $\gamma N$ invariant energy $W_{\gamma N}$. As a consequence, the nuclear cross sections should strongly depend on the prescription of choosing the $W_{\gamma N}$ value when the single-nucleon operator is embedded into the nucleus. Quite apparently, this dependence brings uncertainty into our calculations. To demonstrate the extent to which the results are affected by this uncertainty we present in Fig. 3 the cross section for $\gamma^3\text{He} \to \pi^0\pi^3\text{He}$ which was calculated directly via integration over the nucleon momentum $p_i$ without resorting to the prescription (12). The value of $W_{\gamma N}$ was fixed using the so-called Blankenbecler-Sugar choice (see also [22] and [23]), corresponding to the assumption that all three nucleons in the initial state of $^3\text{He}$ share equally its energy, that is

$$E_{i}^{lab} = \frac{1}{3}M^3\text{He},$$

where $E_{i}^{lab}$ is the energy of the active nucleon in the $^3\text{He}$ rest frame. The corresponding result is shown on the left panel of Fig. 3 by the dash-dotted line. One readily notes that for coherent $\pi^0\pi^0$ photoproduction it is rather important how the single nucleon kinematic is fixed. This is obviously a trivial consequence of rather strong energy dependence of the elementary amplitude (6).

Additional uncertainty comes from the assumption that $\alpha$ (10) is energy independent in a rather wide energy region. For the estimations presented in this paper this point is perhaps not very crucial. However, better quantitative level of the calculations requires more detailed knowledge of the isotopic structure of the elementary amplitude. Obviously, for this task we need new data for $\pi^0\pi^0$ photoproduction on a quasi-free neutron with better than in Ref. [3] statistics, wherever possible.

IV. CONCLUSION

In this paper we present our predictions for coherent photoproduction of $\pi^0\pi^0$ pairs on the two lightest nuclei, $d$ and $^3\text{He}$. The calculations are based on the available information about photoproduction on the nucleon, as well as on the physically reasonable assumption, that in the second resonance region the $\pi^0\pi^0$ pairs are photoproduced primarily by isovector photons. As our model shows, if these assumptions are correct, the cross sections on protons and neutrons should be of an almost equal size. In this case one should observe a relatively strong $\gamma \to \pi^0\pi^0$ transition on $^3\text{He}$ and a rather weak transition on the deuteron. This situation is very similar to that in $\eta$ photoproduction in the $S_{11}(1535)$ region, where the isovector photons make the dominant contribution. One of the motivations for the present paper was to encourage experimentalists to study the coherent $\pi^0\pi^0$ photoproduction on nuclei. The cross section of the process [18] has appreciable value, about 0.4 $\mu \text{b}$ and in principle can be measured with reasonable statistics. Such measurements could be a good test of our knowledge about the spin-isospin structure of the amplitude $\gamma N \to \pi^0\pi^0 N$. 

$$F_{\text{Li}}^{(1p)}(Q) = \left(1 - \frac{1}{6}(Qr_0)^2\right) \exp^{-(Qr_0)^2/4}.$$
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[1] Y. Maghrbi et al. [Crystal Ball Collaboration], Eur. Phys. J. A 49 (2013) 38.
[2] C. M. Tarbert et al. [Crystal Ball and A2 Collaborations], Phys. Rev. Lett. 100 (2008) 132301.
[3] V. L. Kashevarov, A. Fix, S. Prakhov, P. Aguaz-Bartolome, J. R. M. Annand, H. J. Arends, K. Bantawa and R. Beck et al., Phys. Rev. C 85 (2012) 064610.
[4] A. Fix and H. Arenhövel, Phys. Rev. C 85 (2012) 035502.
[5] M. Egorov and A. Fix, Phys. Rev. C 88 (2013) 5, 054611.
[6] K. Nakamura et al. (Particle Data Group), J. Phys. G 37 (2010) 075021.
[7] J. Ajaka et al., Phys. Lett. B 651 (2007) 108.
[8] C. Lazard and Z. Maric, Nuovo Cim. A 16 (1973) 605.
[9] V. Kleber, P. Achenbach, J. Ahrens, R. Beck, V. Hejny, J. D. Kellie, M. Kotulla and B. Krusche et al., Eur. Phys. J. A 9 (2000) 11.
[10] B. Krusche, M. Fuchs, V. Metag, M. Robig-Landau, H. Stroher, R. Beck, F. Harter and S. J. Hall et al., Eur. Phys. J. A 6 (1999) 309.
[11] A. Fix and H. Arenhövel, Eur. Phys. J. A 25 (2005) 115.
[12] F. Ritz and H. Arenhövel, Phys. Rev. C 64 (2001) 034005.
[13] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. 149 (1987) 1.
[14] V. Baru, J. Haidenbauer, C. Hanhart, and J. A. Niskanen, Eur. Phys. J. A 16 (2003) 437.
[15] L. R. Suelzle, M. R. Yeanian, and H. Crannell, Phys. Rev. 162 (1967) 992.
[16] B. Krusche, J. Ahrens, J. R. M. Annand, G. Anton, R. Beck, M. Fuchs, A. R. Gabler and F. Haerter et al., Phys. Lett. B 358 (1995) 40.
[17] C. Deutsch-Sauermann, B. Fritman and W. Norenberg, Phys. Lett. B 409 (1997) 51.
[18] A. Fix and H. Arenhövel, Z. Phys. A 359 (1997) 427.
[19] A. Fix and H. Arenhövel, Eur. Phys. J. A 19 (2004) 275.
[20] A. Fix and H. Arenhövel, Phys. Rev. C 68 (2003) 044002.
[21] F. Pheron, J. Ahrens, J. R. M. Annand, H. J. Arends, K. Bantawa, P. A. Bartolome, R. Beck, V. Bekrenev et al., Phys. Lett. B 709 (2012) 21.
[22] E. Breitmoser and H. Arenhövel, Nucl. Phys. A 612 (1997) 321.
[23] A. Fix and H. Arenhövel, Nucl. Phys. A 620 (1997) 457.