A Gaussian beam method for ultrasonic non-destructive evaluation modeling

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Abstract. The propagation of high-frequency ultrasonic body waves can be efficiently estimated with a semi-analytic Dynamic Ray Tracing approach using paraxial approximation. Although this asymptotic field estimation avoids the computational cost of numerical methods, it may encounter several limitations in reproducing identified highly interferential features. Nevertheless, some can be managed by allowing paraxial quantities to be complex-valued. This gives rise to localized solutions, known as paraxial Gaussian beams. Whereas their propagation and transmission/reflection laws are well-defined, the fact remains that the adopted complexification introduces additional initial conditions. While their choice is usually performed according to strategies specifically tailored to limited applications, a Gabor frame method has been implemented to indiscriminately initialize a reasonable number of paraxial Gaussian beams. Since this method can be applied for an usefully wide range of ultrasonic transducers, the typical case of the time-harmonic piston radiator is investigated. Compared to the commonly used Multi-Gaussian Beam model [1], a better agreement is obtained throughout the radiated field between the results of numerical integration (or analytical on-axis solution) and the resulting Gaussian beam superposition. Sparsity of the proposed solution is also discussed.

1. Introduction

Ultrasonic Non-Destructive Evaluation (NDE) uses high-frequency acoustic and elastic waves to probe components without affecting their integrity. Since the industrial materials and structures are reaching high levels of complexity, the associated ultrasonic NDE technologies and diagnosis methods require up-to-date simulation tools. It means that an end-user solution should encompass many different realistic applications while providing essential results in reasonable time. However, the studied waves can propagate over a few hundreds of wavelengths before being monitored and analyzed. Furthermore, there is a wide range of wave mechanisms potentially involved in the testing process: flaw scattering, mode conversion, dispersion, attenuation, beam forming. All these underlying physical processes do make any quantitative and efficient model challenging to develop.

In the CIVA software [2], the propagation of ultrasonic body waves is usually achieved semi-analytically with a ray-based model [3], which was initially implemented for piecewise media. This approach related to Dynamic Ray Tracing techniques [4] has also been developed for taking continuous description of welds into account [5]. In the context of high-frequency modeling, these asymptotic techniques naturally stand out in performing long time propagation efficiently...
while direct numerical methods, such as finite-difference, finite element or spectral methods are very costly in practice. In addition, it underlies a well-suited framework for tracking field contributions throughout complex media and through interactions with interfaces. Conversely, highly interferential features fields – namely caustics, shadows or postcritical incidences – are notoriously misdescribed by a ray-based formulation [6]. Fortunately, these so-called singularities can be circumvented inside such a convenient framework by generalizing to complex-valued paraxial quantities [7].

2. Context : radiating ultrasonic bulk wave planar transducer

The complex phase function that follows provides a decaying Gaussian profile away from the real ray path, giving rise to a localized solution commonly known as a paraxial Gaussian beam (GB). The ultrasonic field is then expanded into a superposition of several GBs. Such a complexification of the paraxial quantities introduces additional degrees of freedom into each GB. Their choice and their justification are the starting point of an extensive literature in different fields over the past three decades. We should reference contributions in Geophysics [8, 9], Acoustics [1], Quantum Mechanics [10] and radar engineering [11]. It appears that the various existing strategies developed to fix these GB parameters are strongly related to the nature of the problem. The method we propose shall then take some ultrasonic NDE particularities into account, leading us to formulate modeling assumptions.

Harmonic source term

An ultrasonic bulk wave transducer, used as a transmitter, converts an electrical signal to acoustical energy and radiates a sound beam. Generally, the excitation signal is a pulse which is described by a central frequency and a limited bandwidth. Common Fourier analysis allows us to recover a time domain response by considering all the harmonic responses of interest, independently. Therefore, we only consider the source term as an harmonic piston radiator, oscillating at the angular frequency $\omega$.

Propagation media

Depending on the testing protocol, the active area on the face of the bulk wave transducer is immersed in a coupling media (immersion transducer) or in contact with an elastic solid (contact transducer). For sake of simplicity, the propagation media is assumed to be a homogeneous fluid of wave speed $c$ and density $\rho$ and we shall then model the radiated pressure field. Thus, we define $k = \omega/c$.

Source geometry

Mono-element transducers can take many different shapes according to the NDE (or medical imaging) application to be carried out [12, 13]. Even limiting to the planar case, those are not necessarily of the axial symmetry (e.g. individual elements in ultrasonic arrays). Furthermore, their dimension can take value from less than a wavelength to several wavelengths. Then we only focus on baffled planar radiators, but nonetheless of any 2D shape $S$ and amplitude profile.

3. Method : phase-space GB summation

The seminal idea behind the studied method is to initialize a GB set from a Gabor frame representation of the source term [14]. Once a properly tuned Gaussian window function is phase-space shifted for generating the frame representation, a paraxial approximation of the associated beam propagators gives rise to a fully defined GB set. We then obtain an explicit (but approximated) formulation of the full radiated field by summing up all the GB contributions. Before giving more details about the approach, the reader is invited Figure 1 to find out the geometry of the problem and some introduced notations.
3.1. Source representation

According to the preceding assumptions, the 2D source term is known a priori and is reduced to the harmonic amplitude $\hat{v}_0(x)$ of the radiator normal velocity. Its Gabor frame representation is given by

$$\hat{v}_0(x) = \sum_\gamma \hat{a}_\gamma \hat{\psi}_\gamma(x).$$

(1)

The set $\{\hat{\psi}_\gamma\}$ is defined as

$$\hat{\psi}_\gamma(x) = \hat{\psi}(x - x_m)e^{ik_x^\gamma \cdot (x - x_m)},$$

where

$$\left\{ \begin{array}{l}
x_m = (m_1\bar{x}, m_2\bar{x}) \\
k_x^\gamma = (n_1\bar{k}_x, n_2\bar{k}_x)
\end{array} \right.$$  

are the space and the phase shifts of the \textit{mother window} $\hat{\psi}(x)$, respectively. We use the vector index $\gamma = (m_1, m_2; n_1, n_2) = (m; n) \in \mathbb{Z}^2 \times \mathbb{Z}^2$ to denote the \textit{synthesis function} $\hat{\psi}_\gamma$ unequivocally. The parameters $\bar{x}$ and $\bar{k}_x$ are the spatial and spectral displacements units respectively. For $\{\hat{\psi}_\gamma\}$ to be a frame, $\bar{x}$ and $\bar{k}_x$ must satisfy

$$\bar{x}\bar{k}_x = 2\pi\nu,$$

(2)

where $0 < \nu \leq 1$ is the overcompleteness factor. Once this is verified, the coefficients $\hat{a}_\gamma$ are given by

$$\hat{a}_\gamma = \langle \hat{v}_0, \hat{\psi}_\gamma \rangle = \int_S \hat{v}_0(x)\hat{\psi}_\gamma^*(x)\,dx,$$

(3)

where $\hat{\psi}_\gamma^*$ is the conjugate of the \textit{analysis function} $\hat{\psi}_\gamma$. The set $\{\hat{\psi}_\gamma\}$ is the dual frame associated to the Gabor frame $\{\hat{\psi}_\gamma\}$. An explicit expression of the latter cannot be found in general, but

Figure 1: Gaussian Beam parametrization. A baffled planar radiator of arbitrary shape $S$ is considered. The coordinates of any point on the radiator surface are denoted by $x \equiv (x_1, x_2)$. The motion of the radiator surface acts as a piston with an arbitrary time-harmonic normal velocity $\hat{v}_0(x)$. A Gabor frame $\{\hat{\psi}_\gamma\}$ is introduced to decompose the source term. Each element $\hat{\psi}_\gamma$ of the frame is centered at the lattice point $(x_m, k_x^n)$ in the $(x, k_x)$ phase-space. The pressure field radiated into $z > 0$ by the synthesis window $\hat{\psi}_\gamma$ is evaluated asymptotically at the observation point $r \equiv (x, z)$ by the Gaussian beam $\hat{B}_\gamma$. 

it can be approximated for $\nu \to 0$ [14] by
$$
\hat{\varphi}_\gamma(x) \approx \left[ \frac{\nu^2}{\|\hat{\psi}_\gamma\|^2} \right] \hat{\psi}_\gamma(x).
$$
(4)

Finally, we choose the mother window $\hat{\psi}$ to be a 2D symmetric Gaussian
$$
\hat{\psi}(x) = e^{-k|x|^2/2z_R},
$$
(5)
where the significance of $z_R$ is given below.

3.2. Paraxial Gaussian beams and field expansion

Now that we have the Gabor frame representation (1) of the source term, we want to formulate
the pressure field radiated at the observation point $r$. This can be represented by a plane wave
integral, in particular for a given synthesis function $\hat{\psi}_\gamma$ we obtain the beam propagator $\hat{B}_\gamma(r)$
into $z > 0$,
$$
\hat{B}_\gamma(r) = \rho c \left( \frac{k}{2\pi} \right)^2 \int \hat{\psi}_\gamma(k^x) e^{i(k^x \cdot x + k^z z)} dk^x,
$$
(6)
where $\hat{\psi}_\gamma$ is the 2D spatial Fourier transform of $\hat{\psi}_\gamma$ and
$$
k^z = +\sqrt{k^2 - |k^x|^2},
$$
(7)
is the $z-$component of the wave vector $k \equiv (k^x, k^z)$. The integral (6) cannot be explicitely
directly, but it can be evaluated asymptotically by saddle point integration [15]. We then find out
$$
\hat{B}_\gamma(r) \approx \rho c \sqrt{\frac{\det M_\gamma(z_\gamma)}{\det M_\gamma(0)}} \exp \left[ ikz_\gamma + i\frac{k}{2} x^T M_\gamma(z_\gamma) x \right],
$$
(8)
and we recognize the expression of a paraxial Gaussian beam formulated in ray-centered
coordinates $(x_\gamma, z_\gamma)$ (see [4] for example). The complex-valued curvature $2 \times 2$ matrix $M_\gamma$
describes the paraxial behaviour of the radiated field around the beam axis emerging from $x_m$
in the radiator plane, and supported by the wave vector $k_\gamma$. The matrix $M_\gamma$ only varies with $z_\gamma$ and its imaginary part is positive definite for all $z_\gamma$. This complex-valued matrix explicitly
depends on the ray emerging direction and $z_R$, introduced in (5), which is the GB collimation
length (or Rayleigh length). Finally, the pressure field $\hat{p}(r)$ radiated by the source term is
obtained by summing up all the paraxial Gaussian beams $\hat{B}_\gamma$ (8) weighted by the respective $\hat{a}_\gamma$
coefficients (3), giving
$$
\hat{p}(r) \approx \sum_{\gamma \in \mathbb{Z}^2 \times \mathbb{Z}^2} \hat{a}_\gamma \hat{B}_\gamma(r).
$$
(9)

3.3. Bounds and compression of the representation

As it is suggested in (9), there is an infinite number of GB contributing to the field synthesis.
Fortunately, the effective GB set can be drastically shrunk in many different ways.

Spectral range  Firstly, after expliciting the plane wave spectrum of $\hat{\psi}_\gamma$, we can show that $k_n^z$ is
the single non-degenerate saddle point for the phase function of the integrand in (6). Since we
only consider the propagating beams in (9) (see Figure 1) that occur only for
$$
k_n^z = +\sqrt{k^2 - |k_n^x|^2} \in \mathbb{R}_+^*,
$$
(10)
the associated vectors index $\gamma$ are then bounded to values of $n$ that verify (10). Without deriving
more explicit conditions on $n$ values, $K$ denotes the subset of the corresponding vectors index $\gamma$. 

4
Spatial range Secondly, according to (3) the amplitude of coefficient $\hat{a}_\gamma$ is non-negligible only if the associated analysis function $\hat{\phi}_\gamma$ sufficiently overlaps the source term. Since the dual frame set $\{\hat{\phi}_\gamma\}$ is approximated by (4), the analysis function $\hat{\phi}_\gamma$ amplitude is a 2D Gaussian symmetric Gaussian centered in $\mathbf{x}_m$. We can then define a threshold above which the amplitude of the analysis function is thought to be significant so we can easily estimate how close to the source term the function $\hat{\phi}_\gamma$ must be centered. Finally, given the fact that the considered radiator is baffled, the eligible vectors index $\gamma$ are also bounded to such values of $\mathbf{m}$. In the same manner as above, $\mathcal{X}$ denotes the subset of the corresponding vectors index $\gamma$.

Once we take into account the spectral and the spatial ranges, (9) is reduced to

$$\hat{p}(\mathbf{r}) \approx \sum_{\gamma \in \mathcal{K} \cap \mathcal{X}} \hat{a}_\gamma \hat{B}_\gamma(\mathbf{r}).$$

Compression Furthermore, it is possible to take advantage of the overcompleteness of the Gabor frame representation (1) of the source term. Indeed, if $\nu = 1$, the frame $\{\hat{\psi}_\gamma\}$ is critically complete and it becomes a basis where there is a single possible representation for $\hat{v}_0$. In other words, the frame overcompleteness produces a redundancy in the representation that makes its compression possible without degrading the result of (1). In practice, this phase-space representation reveals a strong localization of the coefficients $\hat{a}_\gamma$ around clearly defined region in the $(\mathbf{x}_m, \mathbf{k}_n^x)$ domain. It appears that the coefficients $\hat{a}_\gamma$ are non-negligible for phase-space points near the Lagrange Manifold [14]. Such a rather high localization is an additional argument for compression.

The compression for seeking sparse representation is a recent topic of high interest. For example, Andersson et al [16] apply iterative soft thresholding to a Gaussian wave packet representation to obtain sparse approximation of oscillatory functions. Instead of using such an iterative algorithm, we propose a compression strategy which is simply based on a single thresholding applied to coefficients $\hat{a}_\gamma$, followed by the reassignment of the discarded energy. Formally, we define a relative threshold $\epsilon < 1$ and $\Gamma_\epsilon$ denotes the subset of the vectors index $\gamma$ corresponding to the selected coefficients $\hat{a}_\gamma$, given by

$$\Gamma_\epsilon = \left\{ \gamma \in \mathcal{K} \cap \mathcal{X}, |\hat{a}_\gamma| \geq \epsilon \max_{\gamma \in \mathcal{K} \cap \mathcal{X}} |\hat{a}_\gamma| \right\}.$$  

Since the thresholding step cuts a potentially non-negligible energy from the initial representation, the selected coefficients $\hat{a}_\gamma$ are uniformly weighted by a reassignment ratio $\alpha$ given by

$$\alpha = \sqrt{\frac{\sum_{\gamma \in \mathcal{K} \cap \mathcal{X}} |\hat{a}_\gamma|^2}{\sum_{\gamma \in \Gamma_\epsilon} |\hat{a}_\gamma|^2}} > 1.$$  

The energy conservation by the weighting operation is straightforward. Finally, the phase-space bounded GB solution (11) of the radiated field $\hat{p}(\mathbf{r})$ is reduced to

$$\hat{p}(\mathbf{r}) \approx \sum_{\gamma \in \Gamma_\epsilon} \alpha \hat{a}_\gamma \hat{B}_\gamma(\mathbf{r}).$$

In the following section, we use (11) to evaluate the approximated radiated field, and (14) to assess the influence of the compression.
4. Simulation results and discussion
As an example, we consider the typical case of a 5 MHz, 6 mm diameter circular uniform piston transducer in water, giving
\[ \hat{v}_0(x) = \hat{v}_0, \quad |x| \leq \frac{L}{2}. \]
The corresponding wavelength in the water is \( \lambda = 0.3 \) mm, and we can introduce the near field distance \( N = L^2/4\lambda \) associated to a circular uniform piston radiator [17]. This quantity is well-suited for localizing the nulls and the maxima of the on-axis field. For all the following simulation results, we use an highly overcomplete frame where \( \nu = 0.09 \). The Rayleigh length \( z_R \) that characterizes the mother window \( \hat{\psi} \) is chosen in such a way that the analysis functions \( \hat{\phi}_\gamma \) ensure a balanced representation with sufficient localization in both phase and space domains. Here, we arbitrary choose the width of the analysis function to be fourth the transducer diameter. In order to complete the frame definition, the spatial and spectral displacements units, \( \bar{x} \) and \( \bar{k} \) respectively, are choosen according to \( \nu \) and \( z_R \) values in order to provide a full coverage of the phase-space. Thereafter, subsets \( K \) and \( \mathcal{X} \) are easily identified. Then, noting that (3) can be expressed as windowed Fourier transforms, the coefficient set \( \{ \hat{a}_\gamma, \gamma \in K \cap \mathcal{X} \} \) is computed quickly by performing Fast Fourier Transforms (FFTs).

**Main beam region**  As an overview, we calculate the radiated pressure field via (11) in the main beam region that contains the transducer axis. The 2D cross-sectional result is shown Figure 2.

Figure 2: Cross-sectional image of the normalized magnitude of the pressure field of a 6 mm diameter circular piston transducer radiating into water at 5 MHz as calculated with the GB solution given by (11)

Qualitatively, the field structure is familiar. Even in the near field region for \( z < N \), it is able to exhibit an highly interferential regime where there is a well-known alternance between nulls and maxima throughout the on-axis field, and expected side lobes throughout the cross-axis field. This calls for quantitative comparisons.

**On-axis field**  In the case of a circular uniform piston, the on-axis pressure field is known exactly by solving the Rayleigh-Sommerfeld integral, giving
\[ \hat{p}_{\text{ref}}(z) = \rho c \hat{v}_0 \left[ e^{ikz} - e^{ik\sqrt{z^2+(L/2)^2}} \right]. \] (15)

An other interesting point of quantitative comparison is provided by the Multi-Gaussian Beam (MGB) model. Commonly used in NDE modeling, this model expands the pressure field radiated by axisymmetric transducers into a superposition of a few Gaussian beams. In this approach, the GB parameters result from an offline greedy non-linear minimization of an error function. As those parameters are wrote down in a referenced table [1], they can be re-used for convenience of implementation as they are. However, this model notoriously suffers from poor near field
description and a lack of genericity and it fails in simulating steering beam fields of phased array transducers [18]. The on-axis field is computed in the near field region for \( z \in [0, N/2] \) with our GB solution (11). The latter is compared to the exact solution (15) and the MGB output (see the results in Figure 3a). As expected, the MGB model does not match with the near field interferences for \( z \lesssim N/4 \). Conversely, our solution can keep oscillating for \( z \to 0 \) and reaches a better agreement than the MGB solution.

\[
\hat{p}(z) = \frac{1}{\rho c} \hat{v}_0 |\hat{p}|_{\text{ref}}(z)
\]

(a) Normalized magnitude of the pressure field calculated with (–) the exact solution (15), (△) the MGB model [1] and (•) the GB solution given by (11).

\[
\varepsilon(z) = \frac{\max_{r \in \mathbb{R}^3} |\hat{p}_\text{ref}(r)|}{|\hat{p}(r)| - |\hat{p}_\text{ref}(r)|}
\]

(b) Normalized absolute error level calculated for (△) the MGB model and (•) the GB solution given by (11).

Figure 3: The on-axis pressure field of a 5 MHz, 6 mm diameter circular piston transducer radiating into water is calculated in the near field region for \( z \in [0, N/2] \).

Let us take a closer look at the discrepancies by calculating the normalized absolute error level \( \varepsilon \), defined by

\[
\varepsilon(r) = \frac{\max_{r \in \mathbb{R}^3} |\hat{p}_\text{ref}(r)|}{|\hat{p}(r)| - |\hat{p}_\text{ref}(r)|}
\]

where \( \hat{p} \) is any approximated solution of the radiated pressure field and \( \hat{p}_\text{ref} \) the adopted reference solution. As it is defined, the error level \( \varepsilon \) locally depicts the deviation from the reference solution in relation to the meaningful magnitude. The error levels \( \varepsilon \) related to the above comparison are shown Figure 3b. Throughout the near field region, the error curve associated to our GB solution stays below the one associated to the MGB model and decreases continuously with \( z \). The latter aspect can be explained by the choice of discarding the evanescent modes (10) whose the contribution should get significant in the very near field region.

Cross-axis field For off-axis observation points, the harmonic response cannot be derived analytically. A natural approach consists in the numerical evaluation of the Rayleigh-Sommerfeld integral using a method for which the associated error is controlled [20]. Then, the chosen off-axis reference solution is the integral formulation given by Hutchins et al [19] and computed by a Gaussian quadrature. Radiated cross-axis pressure field is now considered at the near field distance \( (z = N) \) and in the very near field region \( (z = N/4) \), for \( x \in [-L, L] \). The results are shown Figure 4 and Figure 5, respectively.
Figure 4: The cross-axis pressure field of a 5 MHz, 6 mm diameter circular piston transducer radiating into water is calculated at the near field distance $z = N$ for $x \in [-L, L]$.

For $z = N$, the MGB solution and our GB solution are indistinguishable from the reference (see Figure 4a), so that the main beam and side lobes are very well described. For the studied region of interest, the obtained error $\varepsilon$ is less than 1% in both cases (see Figure 4b). Furthermore,
our GB solution is even more precise than the MGB model as its associated error is strictly inferior to the one calculated with the MGB model. For \( z = N/4 \), the MGB solution appears to be less accurate than our GB solution. It fails in reproducing the side oscillations and it underestimates the field magnitude in the vicinity of the transducer axis (see Figure 5a). A closer look at the error levels attests those differences, where the calculated error \( \varepsilon \) for our GB solution is still reasonable (less than 5%) and substantially below the one calculated for the MGB model (see Figure 5b).

4.1. Compressed representation

This section is devoted to assess the impact of the compression introduced in (12) and (13) to the deduced GB solution in (14). This is achieved by evaluating the on-axis pressure for \( z \in [0, 10N] \) with (14) for different values of the threshold coefficient \( \epsilon > 0 \). Then we determine the peak error level, \( \max_{z} \varepsilon(z) \), between these evaluated fields and the uncompressed result (\( \epsilon = 0 \)). In addition, we calculate the associated compression rate \( \tau_\epsilon \) that measures the part of GBs rejected following the thresholding operation. The latter is given by

\[
\tau_\epsilon = 1 - \frac{|\Gamma_\epsilon|}{|K \cap X|}.
\]

The results are listed in Table 1.

Table 1: Compression rate \( \tau_\epsilon \) and peak error \( \varepsilon \) due to compression for different threshold coefficient \( \epsilon \) values.

| \( \epsilon \) | \( \tau_\epsilon \) | \( \max_{z \in [0,10N]} \varepsilon(z) \) |
|---|---|---|
| 0 | 0% | 0% |
| \( 1.1 \times 10^{-3} \) | 71.5% | 0.05% |
| \( 5.1 \times 10^{-3} \) | 88.9% | 0.97% |
| \( 1.1 \times 10^{-2} \) | 92.2% | 3.89% |
| \( 2.1 \times 10^{-2} \) | 95.3% | 15.9% |
| \( 3.1 \times 10^{-2} \) | 96.9% | 29.5% |

For \( \epsilon = 1.1 \times 10^{-3} \), one notes that there’s already a meaningful shrinkage of the GB set. It remains less than on-third of the initial number of GBs. Such a resulting compression ratio \( \tau_\epsilon = 71.5\% \) for this low threshold value supports the idea that the coefficients \( \hat{a}_\gamma \) are strongly localized in the \((x_m, k_n)\) domain, as mentioned above. Yet one observes that the consecutive error \( \varepsilon(z) \) calculated throughout the \( z \)-axis does not exceed a marginal level of 0.05%. Further compression given by \( \epsilon = 1.1 \times 10^{-2} \) leads to 92.2% of the initial GBs to be rejected while the generated error \( \varepsilon \) is still under 4%. Reaching such a shrinkage while the final solution is weakly affected would not have been possible without the reassignment (13) of the discarded energy.

5. Conclusion and perspectives

The purpose of this paper was to derive a rather generic approach to expand an ultrasonic field into a superposition of several paraxial Gaussian beams for NDE modeling applications. Since it is well-known that any strategy devoted to initialize Gaussian beam depends heavily on the typology of the source term and the nature of the propagation media, different modeling
hypothesis have been formulated in keeping with the ultrasonic NDE requirements. Without being too restrictive, we have considered the field radiated into a fluid by baffled planar transducers of any 2D shape, vibrating as an harmonic piston radiator. Such a framework allows us to initialize a paraxial Gaussian beam set from a Gabor frame representation of the source term. The radiated field is then obtained analytically by summing up all the Gaussian beam contributions.

This method has been tested and validated on the typical case of a circular piston transducer. We noted that our approach gives better results than the commonly used Multi-Gaussian Beam model [1], in particular in the near field region. It then provides a relevant alternative for modeling unconventional transducers of non-symmetric shapes and/or of non-uniform amplitude profile. However, it remains that evanescent modes have been discarded by necessity, leading to an underestimated field magnitude in the vicinity of the transducer active area. An other feature which has not been presented here is that for a given far field region of interest, our GB solution can be improved by choosing an higher GB collimation distance \( z_R \). In contrast, the resulting field will be less accurate in the near field region because of a poorer space localization. Some technical considerations about frame tuning and re-expansion in reflective media can be found in [21].

Moreover, we assessed the influence of a compressed–reassigned representation of the source term to the approximated radiated field. This shed light on sparsity capabilities in setting up a modeling approach that relies on any redundant frame representation [16]. Such a feature is encouraging for potentially more complex media since it greatly reduces the number of Gaussian beams to be tracked.

Acknowledgments

O Jacquet would like to thank A Aubertin for his technical contribution and for his interest in this work.

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