Multi-periodic Refinery Scheduling Based on Generalized Disjunctive Programming

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Abstract. Refinery complex production process usually involves some distinct or implied production rules and expert experiences. Representation and utilizing these heuristic rules is conducive to efficient scheduling optimization. In this paper, the heuristic rules were formulated using disjunctive form and logical proposition, and a discrete-time based multi-periodic generalized disjunctive programming (GDP) scheduling model was built. Due to the combining of logic expressions and algebra expressions, heuristic rules were utilized by the proposed model, while scheduling optimization was ensured. The formulated model was used to deal with a refinery scheduling problem. Case study shows the model’s feasibility and efficiency.

1. Introduction

Oil refining industry is vital to national economy, and production scheduling plays important role in efficient production, cost saving and enterprise profit improving. Establishing scheduling model is the key element. Mixed-integer linear programming (MILP) and Mixed-integer non-linear programming (MINLP) are major modelling approaches. Shah [1] built a comprehensive MILP optimization model based on continuous-time formulation for refinery scheduling problem. Li [2] presented a unit-specific event-based continuous-time MINLP scheduling model to optimize operations of blending various crude oils in storage tanks. To integrate production system and utility system for refinery-wide simultaneous optimization, Zhao [3] introduced a MINLP scheduling model which was then decomposed into a MILP model and a nonlinear programming for further solution., Pedro [4] formulated the gasoline blend scheduling problem as a MINLP model. So far most researches represented refinery production process by algebra expressions.

Refinery process is more complex, there are not only physical reactions but also chemical reactions. Production units are tightly connected which results in higher coupling scheduling model. Thereby not all elements of production process can be formulated precisely by common algebra model. In real-life situations solutions of scheduling model usually need to be modified by production experts to generate executable schemes for production controllers. Some worse situations maybe met that there are no feasible schemes due to the complex production conditions and corresponding scheduling models. To conquer such problem, only using common modelling method is not sufficient, it is need to incorporate expert experiences to acquire feasible and optimal schemes.

Production rules and expert experiences play important roles in actual refining process, feasible schemes can be easily find, but optimal schemes are hard to get to realize cost saving and profit improving. It is necessary to combine heuristic rules with algebra model, which will facilitate formulating complex refinery process and improve solving efficiency, therefore feasible and optimal schemes can be got. Grossmann [5] proposed a modelling framework named generalized disjunctive
programming which integrates algebraic equations, disjunctions and logic propositions. GDP can combine algebra model and heuristic rules that are represented by logical proposition and disjunctive form. Castro [6] used GDP to model logistic constraints of crude oil pooling problem to optimize blending and delivery operations. Mostafaei [7] modelled scheduling problem of transporting refined petroleum products by straight pipelines with multiple single and dual purpose nodes, GDP was utilized to reformulate convex hull to simplify model solution. Literature [8] gives an outline of GDP for process scheduling problems that are illustrated with the STN and RTN models.

It is necessary to utilize production rules and expert experiences to tackle scheduling problem of refinery production process. A multi-periodic discrete-time GDP model was proposed in this paper to optimize processing units operations. The scheduling problem of a refinery and some heuristic rules are presented firstly. Corresponding GDP scheduling model is built, in which heuristic rules are represented as disjunctions and logic propositions. The GDP model’s feasibility and efficiency are illustrated by presented case study. Concluding remarks are given to summarize the research work.

2. Problem Statement
The flow sheet of a refinery is shown in figure 1. Processing units of the refinery production process include crude distillation unit (CDU), fluid catalytic cracking unit (FCC), continuous catalytic reforming unit (CCR) and diesel hydrogenation unit (DH). The CDU convert crude oil into different outputs which are feed to downstream units for further refining. The end-products are finally produced thought the refining process network. The task of production scheduling is to generate feasible and optimal operations schemes of processing unit to maximize enterprise profit at the condition of meeting market requirements.

![Figure 1. Simplified flow sheet of a refinery](image)

Heuristic rules such as production rules and expert experiences play important roles in efficient refinery operations scheduling. There are some production rules in the refinery shown in figure 1. CDU and FCC have two running modes respectively that are catalytic mode and oil mode of CDU and petrol mode and diesel mode of FCC. In actual production, if CDU catalytic mode runs, then FCC runs. If CDU oil mode runs, then CCR runs. If CDU oil mode and FCC petrol mode run, then DH runs. If FCC diesel mode runs, then DH runs. These heuristic rules contribute to solve scheduling problem, and no suitable schemes can be got without considering them.

3. GDP Scheduling Model
A multi-periodic discrete-time GDP model for refinery scheduling optimization is proposed in this section. The presented formulation integrates production rules mentioned above using disjunctive form and logical proposition, therefore binary variables and Boolean variables both exist. Model nomenclature of sets, parameters and decision variables is as follows:

**Sets**
- $P$: set of time periods
- $UM$: set of processing units
- $MS$: set of materials
RM: set of raw materials
FP: set of final products
M(i): set of processing modes of unit i
PM(i): feed materials set of unit i
OM(i): output materials set of unit i
CM(j): set of units consuming material j
PM(j): set of units producing material j

Parameters
V_i^U, V_i^L: maximum and minimum processing capacity of unit i
C_j^U, C_j^L: maximum and minimum storage of material j
B_j^U, B_j^L: maximum and minimum supply of raw material j
mpjp: price of raw material j during time period p
S_j^U, S_j^L: maximum and minimum demand of product j
spjp: price of product j during time period p;
β_jg: output ratio of material j of unit i in processing mode g

Decision Variables
rip: binary variables denoting whether unit i runs during time period p
Rip: Boolean variable corresponding to rip
rigp: binary variables denoting whether unit i runs processing mode g during time period p
Rigp: Boolean variable corresponding to rigp
Vigp: throughputs of processing mode g of unit i during time period p
Bj: purchasing amount of raw material j
Sj: selling amount of product j

If unit i is on during time period p, then one processing mode must run, and unit throughputs is limited by processing capacity. If unit i shut down, then no processing mode should be taken, and the throughputs are zero. The above two heuristic rules are expressed by disjunctive form (1).

\[
\begin{bmatrix}
R_{ip} \\
\sum_{g \in M(i)} r_{igp} = 1 \\
V_i^L \leq V_{ip} \leq V_i^U \\
\sum_{g \in M(i)} V_{igp} = V_{ip}
\end{bmatrix}
\lor
\begin{bmatrix}
-N_{ip} \\
\sum_{g \in M(i)} r_{igp} = 0 \\
V_i^L \leq V_{ip} \leq V_i^U \\
\sum_{g \in M(i)} V_{igp} = 0
\end{bmatrix}, \forall i \in UM, \forall p
\quad (1)
\]

Algebraic constraint (2) and (3) calculate the amounts of input materials and output materials. The consuming quantity equals producing quantity as algebraic equation (4) shown.

\[
V_{igp} = \sum_{g \in M(i)} \sum_{j \in PM(i)} v_{igjp}, i \in UM, p \in P
\quad (2)
\]

\[
vo_{igp} = \beta_{igp} V_{igp}, i \in UM, g \in M(i), p \in P
\quad (3)
\]

\[
\sum_{j \in PM(i)} v_{igjp} = \sum_{j \in OM(i)} v_{igjp}, i \in UM, g \in M(i), p \in P
\quad (4)
\]

Units operation heuristic rules illustrated in section 2 were formulated by expression (5) and (6). Disjunctive form (1) represents production rules that if specific unit processing mode is on, then corresponding unit runs. Similar heuristic rules involve processing modes of different units were described by logical proposition (6).

\[
\begin{bmatrix}
R_{ip} \\
\sum_{i \in UM} r_{ip} = 1
\end{bmatrix}, \forall i' \in UM, i' \neq i, p \in P
\quad (5)
\]

\[
R_{igp} \land R_{i'g'p} \rightarrow R_{i'p}, i, i', i'' \in UM, i \neq i' \neq i'', p \in P
\quad (6)
\]
Constraint (7) is material balance constraint. The storage material j is limited as constraint (8) presented. The purchase or sell volume of material j should satisfy market requirements.

\[
C_{j(p-1)} - C_{jp} + \sum_{i \in iM(j)} \sum_{g \in gM(i)} v_{i^{-}g^{-}} + B_j - \sum_{i \in PM(j)} \sum_{g \in gM(i)} v_{i^{+}g^{+}} \cdot S_j = 0, \quad j \in MS, \quad p \in P
\]  

(7)

\[
C_j^L \leq C_{jp} \leq C_j^U, \quad j \in MS, \quad p \in P
\]  

(8)

\[
B_j^L \leq B_{jp} \leq B_j^U, \quad j \in RJ, \quad t \in T
\]  

(9)

\[
S_j^L \leq S_{jp} \leq S_j^U, \quad j \in PJ, \quad t \in T
\]  

(10)

The model objective is to maximize the overall profit which is computed by sell income, purchase fee and storage cost, as illustrated by expression (11).

\[
\text{max } Z = \sum_{p \in P} \sum_{j \in BM} sp_{jp} \cdot S_{jp} - \sum_{p \in P} \sum_{j \in FP} mp_{jp} \cdot B_{jp} - \sum_{p \in P} \sum_{j \in MS} sc_j \cdot C_{jp}
\]  

(11)

Refining heuristic rules were directly represented and utilized to optimize refinery production scheduling by the built GDP model. The GDP model can be solved by common optimization algorithm such as B&B algorithm after model relaxation proposed by literature [5].

4. Case Study

The formulated GDP model was used to deal with the scheduling problem of a refinery illustrated in Fig. 1. Unit annual production capacity is shown in Table 1. Each unit output material has different output ratio that is presented in Table 2.

**Table 1.** Unit annual production capacity (million tons per year)

| Unit   | CDU | FCC | CCR | DH |
|--------|-----|-----|-----|----|
| Production capacity | 4.8 | 2.5 | 0.7 | 0.9 |

**Table 2.** Output ratios of processing units (%)

|       | CDU Mode | FCC Mode | Petrol Mode | Diesel Mode |
|-------|----------|----------|-------------|-------------|
| Reforming material | 14 | 18 | — | — |
| Diesel oil | 20 | 30 | — | — |
| Catalytic material | 60 | 40 | — | — |
| Kerosene | 5 | 11 | — | — |
| Petrol oil | — | — | 48 | 23 |
| Light diesel oil | — | — | 20 | 50 |
| LPG | — | — | 12 | 11.5 |
| Petroleum coke | — | — | 15 | 12 |
| Topped oil | — | — | — | 2 |
| Refined diesel | — | — | — | 96 |
| Dry gas | 0.7 | 0.7 | 4.5 | 3 |
| Lost | 0.3 | 0.3 | 0.5 | 0.5 |

The scheduling time horizon was divided into ten discrete time periods. By incorporating heuristic rules represented as disjunctive form and logical proposition, twenty binary variables were
dynamically determined ahead during model solution. Since expressions of heuristic rules serve as logic cuts to accelerate computing, global optimal solution of the built model was obtained in five seconds. The optimal object value is 213.307 million Yuan, and the unit operations were shown in Figure 2. FCC Petrol mode was on during scheduling time–domain. Unit processing mode running scheme of CDU was presented in Figure 3 where value ‘1’ means running the processing mode.

5. Conclusion
The refinery scheduling problem based on generalized disjunctive programming is studied in this work. Refinery heuristic rules were represented as disjunctive forms and logical propositions, by which a discrete-time multi-periodic GDP scheduling model was formulated. The model integrates logic expressions and algebra expressions that could improve solution efficiency. Calculated results of a refinery scheduling problem show the model’s feasibility and efficiency. More heuristic rules can be mining by big data technology to advance scheduling optimization theory in further study.

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7. References
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