Deriving DERs VAR-Capability Curve at TSO-DSO Interface to Provide Grid Services

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Abstract—The multitudes of inverter-based distributed energy resources (DERs) can be envisioned as distributed reactive power (var) devices (mini-SVCS) that can offer var flexibility at the TSO-DSO interface. To facilitate this vision, a systematic methodology is proposed to derive an aggregated var capability curve of a distribution system with DERs at the substation level, analogous to a conventional bulk generator. Since such a capability curve will be contingent on the operating conditions and network constraints, an optimal power flow (OPF) based approach is proposed that takes inverter headroom flexibility, unbalanced nature of the system, and coupling with grid side voltage into account, along with changing operating conditions. Further, the influence of several factors such as compliance to IEEE 1547 on the capability curve is thoroughly investigated on IEEE 37 bus and 123 bus distribution test systems along with unbalanced DER proliferation. Validation with nonlinear analysis is presented along with demonstrating a scenario with T-D co-simulation.

Index Terms—Aggregated flexibility, cosimulation, distributed energy resources, DER, transmission system, var provision.

I. INTRODUCTION

REACTIVE power (var) balance plays a vital role in maintaining transmission grid resiliency, and the availability of sufficient var capability is often considered an indicator of voltage security [1], [2]. The var-related ancillary services have been mainly achieved by large synchronous generators and other strategically deployed var devices such as static synchronous compensator (STATCOM) and static var compensator (SVC). However, a growing footprint of distributed energy resources (DERs) is replacing fossil fuel-based generation, which may result in a shortage of regional var availability [3], [4]. It has initiated a discussion on utilizing DERs as alternative sources in the future grid and bulk generation plants to provide essential ancillary services to the grid, such as ramping requirements, ensuring adequate inertia, and maintaining var balance [5], [6]. This paper concerns the latter topic with DER as the focus. As voltage insecurity or voltage instability is usually load-driven and DERs are closer to the load centers than the generating stations, DERs may serve some part of the var requirement.

Most of the inverter-based DERs can control the real power and var independently. Much of the extant literature focuses on utilizing the var control potential of DERs to improve the performance of distribution systems (DS), i.e., voltage challenges [7], [8], loss minimization, [9] and other indices. However, utilization of DERs’ var potential for the benefit of the transmission systems (TS) is being explored more recently.

We present a methodology that enables thousands of inverter-based DER devices from tens of feeders at a transmission load bus that can be utilized as geographically distributed var resources (‘mini- SVCs’). These ‘mini- SVCs’ can provide enhanced flexibility options to the transmission system operators (TSOs) through aggregation by the distribution system operators (DSO). Our assertion is founded on the following reasoning: 1) The inverter-based DERs can inject/absorb var via fast local volt/var controls [7], [8], thus can provide a significant amount of fast and continuous capacitive/inductive var support, locally or at an aggregated level; 2) The proposition of DERs’ var provision is gaining strength with revised DER integration standards such as IEEE1547-2022 [10], California Rule 21, Hawaii Rule 14 [11] and Germany grid codes [4] that have made it obligatory for DERs to provide var support for grid requirements; 3) The local and distributed nature of DER makes it a suitable contestant for var provision. An assessment study for East Denmark identifies the DERs var provision scheme as economically competitive to conventional dynamic var devices. [4]; 4) The required infrastructure and protocol for DSO-TSO interaction have started gaining attention e.g. some TSOs in Europe are implementing a payment structure for voltage control where DSOs can participate in var provision based on the day-ahead reactive power plans sent out by TSO [12].

Thus, in this new environment of DERs, a consensus emerges from the literature that motivates TSOs to consider DERs var flexibility in their optimization. Previous works have aggregated the capability of asynchronous generators or DFIG for large wind farms without considering DS constraints [13], [14]. Reference [15] has attempted to approximate the DER flexibility using a geometric approach without physical network constraints. References [3], [16], [17] introduce the optimization-based approaches with a focus on the TSO-DSO interaction. A mixed integer linear program (MILP) is discussed at the transmission level in [18] that couples demand reduction with on-load tap changer (OLTC) headroom to account for distribution system
flexibility; however, it does not model the distribution systems. Reference [19] proposed an interval constrained power flow optimization technique to estimate the flexibility cost maps provided by distribution systems at TSO-DSO boundary buses by incorporating the maximum cost user is willing to pay. Similarly, [20] proposed an iterative optimization approach to estimate hourly flexibility provided by energy storage and distributed generator units. Another OPF-based method is proposed by [21] that solves non-convex formulation to estimate extreme points of a flexibility chart, with non-optimal solution. These methods do not take the unbalanced nature of distribution systems and DER penetration into account that can substantially affect the capability region. Reference [22] proposed an ellipsoidal inner approximation of the set of feasible power injection trajectories at the substation, however a fixed power factor is modeled instead of considering active and reactive power coupling through inverter constraints. Inverter constraints are important which may become critical and binding when we aggregate a large number of devices. Another OPF-based method is proposed to provide voltage support to the TN, while optimizing the DN operation and satisfying the power quality constraints by [23], however, the visualization and process of capability charts are not discussed.

Overall, we identify the following main limitations of the current work: (1) These works do not account for and analyze the impact of unbalance in distribution systems on the flexibility estimation, (2) The flexibility is not characterized as a function of inverter headroom as per the IEEE 1547 standard and, (3) these papers do not discuss how the capability curves could be impacted by various practical aspects of distribution system such as grid side voltage, unbalanced DER distribution. It is worth mentioning that the proposed methodology and framework are not limited to DERs connected to medium voltage (MV) level and can be extended to the house-level DERs as well (connected at low voltage (LV)), especially since we are able to capture the unbalanced nature of distribution system. The present paper is an extension of the conference paper, [24], where the extension discusses the implications of considering headroom while operating the DERs and its impact on the TSO’s ability to utilize the DSO’s var availability. A more comprehensive, detailed understanding of the DER var estimation at the TSO-DSO interface along with validation is presented in this extended paper. This paper also discusses the potential impact of these flexibility curves on the transmission grid.

A. IEEE 1547 and Headroom Correlation

The IEEE 1547-2022 standard mandates DERs to have var capability to absorb/inject at least 44% of the inverter apparent power (kVA) rating [10]. This requires that a solar photovoltaic (PV) inverter could only produce real power up to 90% of it’s kVA rating as depicted in Fig. 1. It can be either achieved by over-sizing inverter or by creating a real-power headroom. Implication of over-sizing is that for most of the time, when the solar insolation is not peaking, full inverter capacity will not be utilized which means a higher investment than needed. Therefore, it is expected that most inverters will not be oversized and will need 10% of real power headroom during the peak solar PV generation (minimum headroom) to allow for 44% of the inverter rating for reactive power modulation. An appropriate headroom is essential to provides real and reactive power ancillary services to the bulk grid, therefore modeled in the proposed capability curve in this work.

B. Key Contributions

The present work provides the following unique contribution to DER capability/flexibility aggregation: 1) The proposed methodology provides an aggregated net Q-capability curve as a function of real power headroom resembling a virtual conventional generator capability curve. This will enable TSO to model both P and Q flexibility as resources from DS in their planning. 2) The proposed approach captures the unbalanced nature of the distribution system, which is shown to be a crucial factor affecting flexibility. (3) To provide useful and comprehensive insight, the influence of several factors on aggregated capability is investigated, such as grid side voltage, daily load profile, revised integration standard 1547, inverter sizing, etc. 4) Few simple applications of the derived DER flexibility for the transmission grid have been demonstrated using T-D co-simulation. T-D cosimulation allows to observe the impact of flexibility on TS while ensuring that DS operational constraints are not violated.

Section II sets up the conceptual framework for var provision. Section III builds capability curve characterization for which an OPF-based process is outlined in Section IV. Section V presents DS case studies with a discussion on the IEEE1547 standard compliance and unbalanced DER proliferation in DS. Section VI demonstrates the impact of DER var provision on the grid via T-D cosimulation.

II. OVERALL CONCEPTUAL VAR SUPPORT FRAMEWORK

Fig. 2 depicts the overall framework of providing DERs’ var support to the grid in an integrated T-D system proposed in this work. Consider a transmission grid connected to multiple DS with high penetration of inverter-based DERs. In this study, distributed solar PV are considered as DERs. The whole physical system can be seen in three parts, i.e., the transmission grid,
boundary buses (substation), and the distribution buses with DERs. In this framework, we envision an aggregator entity DSO at the substation level, which exchanges information with the TSO and the DER devices. As shown in the Fig. 2, the framework consists of two major functions the DSO performs. However, in this paper, we only focus on the first function, which is to dynamically aggregate the net var capability curve of the DS at the substation level every 10–15 minutes time scale based on a short-term forecast and send it to the TSO to include it in their planning and operational activities. Here we assume that the TSO has its own planning and control methods to request var support from the DSO in case of emergency. The second function of DSO is to dispatch optimal inverter var set-points to individual DER devices to meet the var support requested by the grid; however, in this work, we do not provide details of this functionality. The scope of this paper is to focus on developing a general framework to aggregate DER var capability.

III. CAPABILITY CURVE CHARACTERIZATION

A typical DS connected to a substation bus with solar PV penetration is shown in Fig. 3. Load and PV generation at ith node are denoted by \( p_i \) and \( q_i \), and \( p_i^g \) and \( q_i^g \) respectively, where \( p \) and \( q \) denote real and reactive power component respectively. The distribution loads and DERs can be aggregated separately as \( p_{sub}^g \) and \( q_{sub}^g \) at the substation, as shown in the Fig. 3.

Consequently, the whole DS can further be aggregated as the net power demand at substation which includes actual loads, DERs, and losses as shown in the same Fig. 3. In this section, we will systematically build the characterization of aggregated var capability curve.

A. Var Capability of Individual Solar PV

For each individual PV inverter, the device flexibility domain \( C_i \) can be characterized as follows:

\[
C_i = \begin{cases} 
(p_i^g, q_i^g) & p_i^g + q_i^g \leq S_i^g \\
0 & p_i^g \leq 0 \\
|p_i^g| & p_i^g \leq p_i^g_{rated} 
\end{cases}
\]

where \( S_i^g \) and \( p_i^g_{rated} \) are the hardware capacity of the inverter and solar panel, respectively, whereas \( p_i^g \) is the maximum solar generation possible at a given point of time in a day. \( C_i \) represents the available flexibility in var generation or absorption by the inverter for all possible amounts of real power generation. We consider the following sign convention: the positive value represents the consumption/absorption, and the negative value means the generation/injection of real/reactive powers. A typical flexibility domain of a solar PV inverter can be graphically drawn as shown in Fig. 4. The outer envelope of the domain \( C_i \) can be defined as a function \( q_i^{g,cap} = f(p_i^g) \) which can be termed as the device Q-capability curve. This curve collects maximum reactive power values that an individual DER inverter can inject or absorb for a given real power generation. The domain \( C_i \) shrinks or increases as the operating point \( p_i^g \) moves along the horizontal axis throughout the day.

B. Net DER Aggregation

Before developing the net capability of the whole network, let’s understand the aggregation of DERs. An aggregated DER flexibility domain, \( C_{sub} \), can be defined as the total flexibility provided by all the DERs combined at the substation as follows:

\[
C_{sub} = \begin{cases} 
(p_{sub}^g, q_{sub}^g) & p_{sub}^g = \sum_{i=1}^{N} p_i^g \\
q_{sub}^g = \sum_{i=1}^{N} q_i^g \\
(p_i^g, q_i^g) & (p_i^g, q_i^g) \in C_i 
\end{cases}
\]
consider is that the inverter var injection or absorption affects the voltage profile of the DS, and consideration of voltage limits may shrink the flexibility domain in certain operating conditions, as visible in the Fig. 6.

The total headroom, $p_{sub}^{hr}$, aggregated at the substation level, can be defined such that

$$p_{sub}^{hr} = \sum_{i} S_{i}^{q} \left( 1 - p_{sub}^{g,hr} \right)$$

(3)

$p_{sub}^{g,hr}$ ranges from $p_{min}^{hr}$ to 1, where $p_{min}^{hr}$ is the minimum possible aggregated headroom at a given time in a day. $p_{min}^{hr}$ corresponds to the maximum generation possible at a given time.

$$\sum_{i} S_{i}^{q} \left( 1 - p_{min}^{hr} \right) = \sum_{i} p_{i}^{q}$$

(4)

Thus, $p_{min}^{hr}$ can be obtained as,

$$p_{min}^{hr} = 1 - \frac{\sum_{i} p_{i}^{q}}{\sum_{i} S_{i}^{q}}$$

(5)

Now, for each DER, we can write,

$$p_{i}^{q} = S_{i}^{q} \left( 1 - p_{i}^{g,hr} \right)$$

(6)

where $p_{i}^{g,hr}$ is headroom for $i$th DER. Finally, $p_{sub}^{g,hr}$ can be written in form of $S_{i}^{q}$ and $p_{i}^{g,hr}$ as

$$p_{sub}^{g,hr} = \left( \sum_{i=1}^{N} S_{i}^{q} p_{i}^{g,hr} \right) / \sum_{i=1}^{N} S_{i}^{q}$$

(7)

The real power headroom for DER can be achieved via inverter oversize, storage, or curtailment. The real power headroom also reflects possible uncertainties in solar generation. For instance, the conceptual capability chart shown in Fig. 6 represents the flexibility region at a given time $t$. In this chart, a particular point on the x-axis corresponds to the headroom associated with the solar generation forecast. All other points correspond to the deviation from the forecast solar output (that can be seen as either uncertainty or deliberate curtailment). Thus, the proposed chart reflects the higher flexibility of the system and provides more options to TSO to handle var-related grid events. Nonetheless, utilizing this flexibility involves a greater discussion on policy, customer comfort, and related cost-benefit analysis, which is beyond the scope of this paper.

IV. PROCESS OF CAPABILITY ESTIMATION

A. System Modeling

In this section, we will use linearized ‘LinDist3Flow’ equations [25] for an unbalanced three-phase DS to develop a graph-representation model [8]. Consider a radial DS with $N + 1$ nodes represented by a tree graph $T = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{0, 1, \ldots, N\}$ is a set of DS nodes, indexed by $i$ and $j$. For simplicity, let’s assume each $i \in \mathcal{N}$ has all three phases $a, b$ and $c$. The set $\mathcal{E} = \{(i, j)\}$ contains all line segments with $i$ as the upstream and $j$ as the downstream node. Each line element $(i, j) \in \mathcal{E}$ will also have three phases. The subset $\mathcal{N}_j$ is a collection of all immediate downstream neighboring buses of node $j$. The secondary side of the substation is denoted by node

where $p_{sub}^{q}$ and $q_{sub}^{q}$ are the total real power and var generation from DERs. The outer envelope of the domain $C_{sub}^{q}$ can be defined as a function $q_{sub}^{cap} = f (p_{sub}^{q})$ that we call an aggregated DER capability curve as shown in Fig. 5. The horizontal axis can also be seen as varying aggregated real power headroom where point A and origin denote minimum and maximum possible aggregated headroom, respectively. A point $H (p_{sub}^{g,hr}, q_{sub}^{g,hr})$ on the curve implies that for a given value of $p_{sub}^{g,hr}$, the maximum possible var absorption is $q_{sub}^{g,hr}$. Note that a given $p_{sub}^{g,hr}$ can be achieved in more than one way by different headroom combinations of individual PV generations. In other words, H also denotes the operating point to achieve $q_{sub}^{g,hr}$ var absorption with minimum possible aggregated headroom. All other possibilities of achieving $q_{sub}^{g,hr}$ which fall inside the domain will require higher aggregated headroom than necessary.

However, the more useful information for TSO is the net available var at the substation, which includes aggregated load, DER, and network losses. Therefore, we define the aggregated net var capability curve that provides the information of the maximum net var injection/absorption possible at the substation, which is seen by the transmission system as net var demand. Henceforth, we will refer to it as aggregated capability curve for brevity. We have seen in Fig. 5 that DER headroom provides real power flexibility that can further enhance the var flexibility region. Therefore, we define aggregated capability curve as function of aggregated DER headroom at the substation, $p_{sub}^{g,hr}$, i.e. $q_{net}^{cap} = f (p_{sub}^{g,hr})$. A conceptual curve at a given operating condition is shown in Fig. 6 that depicts the capacitive and inductive var flexibility domain. Another important point to
Let $M$ be an $3N \times 3N$ graph incidence matrix of $T$. The $i$th column of matrix $M$ corresponds to the line segment $(i, j) \in E$ with entries $M(i, k) = e$ and $M(j, k) = -e$, where $e$ is a $3 \times 3$ identity matrix. All other entries of $M$ are zero. Now, according to the LinDistFlow model, the voltages at nodes $i$ and $j$ can be written as

$$V_i V_j^* = V_j V_i^* - Z_{ij} P_j - Z_{ij}^* Q_j \quad (8)$$

where $Z_{ij} = [V_a V_b V_c]^T$ represents the vector of voltage phasors at node $j$. Similarly, $P_j = [P_a P_b P_c]^T$ and $Q_j = [Q_a Q_b Q_c]^T$ denote the real and reactive power entering at node $j$. $Z_{ij}^P$ and $Z_{ij}^Q$ are the constant three-phase impedance matrices for line segment $(i, j)$ as follows [25]:

$$Z_{ij}^P = \begin{bmatrix} -2r_{aa} & r_{ab} - \sqrt{3}r_{ab} & r_{ac} + \sqrt{3}r_{ac} \\ r_{ba} + \sqrt{3}r_{ba} & -2r_{bb} & r_{bc} - \sqrt{3}r_{bc} \\ r_{ca} - \sqrt{3}r_{ca} & r_{cb} + \sqrt{3}r_{cb} & -2r_{cc} \end{bmatrix}_{ij} \quad (9)$$

$$Z_{ij}^Q = \begin{bmatrix} -2x_{aa} & x_{ab} + \sqrt{3}x_{ab} & x_{ac} - \sqrt{3}x_{ac} \\ x_{ba} - \sqrt{3}x_{ba} & -2x_{bb} & x_{bc} - \sqrt{3}x_{bc} \\ x_{ca} + \sqrt{3}x_{ca} & x_{cb} - \sqrt{3}x_{cb} & -2x_{cc} \end{bmatrix}_{ij} \quad (10)$$

where, $Z_{\phi, i,j} = r_{\phi, i,j} + jx_{\phi, i,j}$ is impedance between phase $\phi$ and $\psi$ of line segment $(i, j)$. Now, let’s define the vector of squared voltage magnitude as a new variable $Y_j = V_j V_j^* = [v_a v_b v_c]^T$ for $j \in N \backslash \{0\}$. Assuming the reference node 0 voltage as $V_0$, when voltages at each node can be written in compact form as follows:

$$[M_0 M]^T [Y_0^T Y^T]^T = -Z_D^P P - Z_D^Q Q \quad (11)$$

where $M_0$ is a matrix of size $3N \times 3$ with the first entry as $e$ and the rest as zero. $Z_D^P$ and $Z_D^Q$ are square matrices of size $3 N$ and are constructed by putting $Z_{ij}^P$ and $Z_{ij}^Q$ as the diagonal entries respectively at the row corresponding to the line segment $(i, j)$. Vectors $Y, \Psi, P, Q$ follow the definition of a vector $X = [X_1^T \ldots X_N^T]^T$. The line flows $S_{\phi, j} = P_{\phi, j} + Q_{\phi, j}$ can be written in terms of net injections (assuming negligible losses) as following:

$$S_{\phi, j} \approx -s_{\phi, j} + \sum_{k \in N_j} S_{\phi, k} \quad (12)$$

where $s_{\phi, j} = p_{\phi, j} + q_{\phi, j}$ is the net injection at node $j$ and phase $\phi$ where, $\phi = a, b, c$. Further, $p_{\phi, j} = q_{\phi, j}^l$ and $q_{\phi, j} = q_{\phi, j}^l - q_{\phi, j}^g$ where superscript $g$ and $l$ denote power generated by DER and consumed by loads, respectively. It is assumed that the DERs are located at the nodes collected in a subset $\mathcal{G} \subseteq N$. For all other $j \in N - G$, $p_{\phi, j}^l$ and $q_{\phi, j}^g$ are considered zero. Note that usually, in the LinDistFlow model, line losses are neglected, which introduces a small error in the voltage drop calculation as indicated by [26]. However, to increase accuracy, we will later incorporate a loss factor in voltage drop calculation.

In this formulation, voltage dependency of loads is modeled as

$$p_{\phi, j}^l (V_{\phi, j}) = p_{\phi, j}^l (a_{\phi, j}^0 + a_{\phi, j}^l |V_{\phi, j}|^2) \quad (13)$$

where $a_{\phi, j}^0$ and $a_{\phi, j}^l$ denote constant power and constant impedance portion of the load with $a_{\phi, j}^0 + a_{\phi, j}^l = 1$. Thus, (12) can be re-written in compact form as

$$M P = p^q - (p^l a^0 + diag(p^l)) Y a^1 \quad (15)$$

$$M Q = q^q - (q^l a^0 + diag(q^l)) Y a^1 \quad (16)$$

where, $diag(x)$ returns a square diagonal matrix with vector $x$ on the diagonal. Using (11), (15), and (16), voltages in the form of power injections can be written as follows:

$$Y = K^{-1} [R^q (p^q - p^l a^0) + X^q (q^q - q^l a^0) - M^{-T} M_0 Y_0] \quad (17)$$

where,

$$R^q = -M^{-T} Z_D^P M^{-1} \quad (18)$$

$$X^q = -M^{-T} Z_D^Q M^{-1} \quad (19)$$

$$K = I_{3N} + R^q diag (p^l) a^1 + X^q diag (q^l) a^1 \quad (20)$$

where $I_{3N}$ is an identity matrix of size $3 N$. $K$ models the effect of voltage-dependent load on the voltage profile and becomes identity when $a^l = 0$. The substation voltage is assumed to be balanced with magnitude $v_0$. Further, due to the radial structure of the network, $-M^{-T} M_0 Y_0$ is the same as $v_0^0$, where $1$ is a column vector of size $3 N$ with all entries as $1$. Note that (17) does not account for the voltage drop due to line losses. Therefore, following the lossy LinDistFlow from [27], we incorporate a diagonal matrix $L$ of size $3 N$ with $L_j$ as $l$th entry that denotes the loss factor for line segment $l$ as $R^q = -M^{-T} Z_D^P M^{-1} L$ and $X^q = -M^{-T} Z_D^Q M^{-1} L$. $L$ is estimated based on the procedure outlined in [27].

Substation secondary voltage $v_0$ can be controlled via an OLTC within a range as $v_0 = v^{\text{tol}, r}$, where $v^{\text{tol}}$ is primary side transmission voltage and $r$ is tap ratio of OLTC. Usually, each tap provides $\pm 0.0063$ pu voltage regulation with maximum $\pm 16$ taps. Therefore the maximum possible values of $r$ are $1 \pm 0.1$.

**B. Estimating System Losses and Net Var Demand**

In a multi-phase unbalanced system, losses also include the impact of mutual coupling impedance with other phases. In a 3-phase unbalanced system, losses incurred in each phase of the line ending at node $j$, $L_j$, can be written in vector form as

$$L_j = d \left[ [I_a I_b I_c]^T \right] = \begin{bmatrix} Z_{aa} I_a^2 + Z_{ab} I_b I_a + Z_{ac} I_c I_a \\ Z_{ba} I_b^2 + Z_{bb} I_b I_a + Z_{bc} I_c I_b \\ Z_{ca} I_c^2 + Z_{cb} I_b I_c + Z_{cc} I_c I_c \end{bmatrix}_j \quad (21)$$

where, $[I_a I_b I_c]_j$ and $Z_j$ are current and impedance of line ending at node $j$. $Z_{aa}$ and $Z_{ab}$ represent self impedance of phase $a$ and mutual impedance between phase $a$ and $b$, and likewise. $d(X)$ returns the diagonals of a square matrix $X$ as a vector. Following [27], [28], voltage phasors are assumed to be balanced, i.e., $\Phi_1 \approx \Phi_2 \approx \Phi_3 \approx \Phi_4$, and are used in this approximation, currents in each phase can be written as
\[ I_a = S_a / V_a = \alpha^2 S_a / V_c = \alpha S_a / V_c \]
\[ I_b = \alpha S_b / V_b = S_b / V_c = \alpha^2 S_a / V_c \]
\[ I_c = \alpha S_c / V_a = \alpha S_c / V_b = S_c / V_c \]

By replacing (22) in (21) such that losses in phase \( \phi \) is a function of \( V_{\phi} \), the loss term can be written as

\[ L_j = \begin{bmatrix} S_a/y_a \ Z_{aa} \ Z_{ab}\alpha \ Z_{ac}\alpha^2 \ S_a^* \ Z_{ba} \ Z_{bb} \ Z_{bc}\alpha \ S_b^* \ Z_{ca}\alpha \ Z_{cb}\alpha^2 \ Z_{cc} \ S_c^* \end{bmatrix} \begin{bmatrix} S_a^* \ S_b^* \ S_c^* \end{bmatrix} \]

(23)

where, \( y_{\phi,j} = V_{\phi,j}^2 \). Henceforth, we drop the subscript \( \phi \) for the convenience of the notations. (23) is able to capture the impact of the mutual impedance of other phases as well, which may be significant in unbalanced distribution systems.

The net reactive power demand at the substation, \( q_{\text{net}}^{\text{net}} \) can be written as the sum of the net injection of var at each node due to load, DER inverter, and reactive power losses incurred at each line across all three phases as follows.

\[ q_{\text{net}}^{\text{net}}(y_j, q_j^g) = \sum_{\phi \in \{a,b,c\}} \left( \sum_{j \in \mathcal{G}} q_{\phi,j}^q - \sum_{j \in \mathcal{N}} q_{\phi,j}^f + \sum_{j \in \mathcal{L}} L_{\phi,j} \right) \]

(24)

C. DER-OPF Formulation

Our objective here is to construct the net capability curve \( q_{\text{net}}^{\text{cap}} \) as shown in Fig. 6. To achieve it, we need to estimate both the capacitive \( (q_{\text{net}}^{\text{cap}}) \) and inductive \( (q_{\text{net}}^{\text{cap}}) \) var capabilities of the network, which is same as minimizing and maximizing the net var flow at the substation. Based on the already defined preliminaries, the following DER-OPF can be written to estimate \( q_{\text{net}}^{\text{cap}} \):

\[ \text{minimize } q_{\text{net}}^{\text{cap}}(y_j, q_j^g) \]

(25a)

subject to

\[ \mathcal{Y} = K^{-1} \left[ R^a \left( p^a - p_j^a \right) + X^q \left( q^q - q_j^q \right) + 1_{y_0} \right] \]

(25b)

\[ p_j = S_j^g \left( 1 - p_j^{g,hr} \right) - p_j^g \left( a_0 + a_1 y_j \right) \quad \forall j \in \mathcal{N} \]

(25c)

\[ q_j = q_j^g - q_j^f \left( a_0 + a_1 y_j \right) \quad \forall j \in \mathcal{N} \]

(25d)

\[ y \leq y_j \leq \bar{y} \quad \forall j \in \mathcal{N} \]

(25e)

\[ L_j \leq I_j \leq T_j \quad \forall j \in \mathcal{N} \]

(25f)

\[ \left| q_j^g \right| \leq \sqrt{S_j^2 - S_j^g \left( 1 - p_j^{g,hr} \right)^2} \quad \forall j \in \mathcal{G} \]

(25g)

\[ p_j^{g,hr} \leq p_j^{g,hr} \leq 1 \quad \forall j \in \mathcal{G} \]

(25h)

\[ \sum_{j \in \mathcal{G}} S_j^g p_j^{g,hr} = P_{\text{sub}} \cdot \sum_{j \in \mathcal{G}} S_j^g \]

(25i)

\[ v_{tm} \leq v_0 \leq v_{tm} \]

(25j)

Constraints (25b–25d) denote the power flow and (25e) ensures the voltages are within the ANSI limits [29]. \( y \) and \( \bar{y} \) are upper and lower allowable voltage limits, and are usually taken as 1.05 and 0.95, respectively. Thermal violation constraints of distribution feeder lines are represented by (25f) where \( L_j \) and \( T_j \) denote upper and lower thermal limits of line ending in node \( j \). Constraint (25g) manifest the hardware capacity limit of an inverter. Constraint (25h) provides the range of OLTC tap. Note that the aggregated headroom at the substation level, \( p_{\text{sub}}^{g,hr} \), is an input parameter in this optimization, whereas individual DER headroom, \( p_j^{g,hr} \), is an optimization variable. The relation between these two is given by (25i). To avoid integer programming, \( r \) is taken as a continuous variable in the OLTC constraint in (25j). Upper and lower saturation limits on OLTC tap ratios are denoted by \( \tau \) and \( \xi \) respectively. Note that the \( v_{tm} \) is an input parameter to the formulation whereas \( v_0 \) is an optimization variable. This allows us to perform a parametric study of the capability curve with respect to grid side voltage. The solution of the optimization (25a) provides the optimal var set dispatch \( (q_j^{g}) \) and optimal headroom \( (p_j^{g,hr}) \) for each DER and optimal secondary side voltage set-point \( (v_0) \).

Similar to (25a), the upper part of the net capability curve \( \left( q_{\text{net}}^{\text{cap}} \right) \) can be estimated by maximizing the net var demand at the substation, which is the same as the following.

\[ \text{minimize } -q_{\text{net}}^{\text{net}}(y_j, q_j^g) \]

subject to

\[ (25b) - (25j) \quad \forall j \in \mathcal{N} \]

(26a)

Unfortunately, the objective function in (26a) is not convex due to losses term \( L_j \) in (24) being quadratic, as shown in (23). However, it can be converted to a convex expression by removing the \( L_j \) term as follows:

\[ \text{minimize } -q_{\text{net}}^{\text{net}}(y_j, q_j^g) \]

subject to

\[ (25b) - (25j) \quad \forall j \in \mathcal{N} \]

(27a)

(27b)

Usually, the var losses are a much smaller component of \( q_{\text{net}}^{\text{net}} \) compared to combined var consumption by the loads and the inverters. Therefore, it doesn’t affect the optimal point significantly. In fact, in most cases, the optimal point of (27a) is also optimal for (26a) except when lower voltage boundary constraints of (25e) at all nodes are not active. In those cases, (26a) tries to further reduce voltage to its minimum to increase losses which adds a negligible error in optimal net var flow \( q_{\text{net}}^{\text{cap}} \) calculated by (27a). Therefore, \( q_{\text{net}}^{\text{cap}} \) is estimated via (24) using optimal \( q_j^{g} \) resulting from (27a).

D. Decoupling Range of Capability Curve With TN Voltage

It is pertinent to discuss that OLTC tap ratio provides a limited decoupling between the primary and secondary sides of the substation within the range of \( r \). Due to this, the desired optimal secondary voltage \( v_0 \) can be achieved by adjusting the tap ratio for any value of \( v_{tm} \), which lies in the decoupling range \( D \) defined...
This decoupling is lost when \( v_{tm} \notin D \) i.e. the OLTC tap gets saturated. To address this concern, DSO estimates the var capability \( q_{\text{cap}} = [q_{\text{cap}}^{\text{net}}, q_{\text{cap}}^{\text{net}}] \) and \( D \) for nominal value of \( v_{tm} = 1 \) and send this information to TSO to be included in their optimizations. It is advised that the TSO use both \( q_{\text{cap}}^{\text{net}} \) and \( D \) as constraints while estimating the var requirement service from DSO. However, there might arise situations when expected \( v_{tm} \notin D \). In such cases, DSO can not guarantee the \( q_{\text{cap}}^{\text{net}} \) but can provide an estimated bound on \( q_{\text{cap}}^{\text{net}} \) for worst-case value of \( v_{tm} \) i.e. \( q_{\text{cap}}^{\text{net}} = [q_{\text{cap}}^{\text{net}} - \epsilon q, q_{\text{cap}}^{\text{net}} - \epsilon q] \) by performing parametric study of the capability curve estimation with respect to \( v_{tm} \) as also shown in the results section later.

Flow chart of the overall process of the var capability curve estimation for a given operating condition is shown in Fig. 7. Note that the while one capability estimation process will only include 100 s of DERs of 1 feeder, multiple feeders are assumed to be coupled at the substation transformer, and thus, the TSO gets the capability information from thousands of devices using the proposed framework.

V. TEST CASE STUDY

A. Reactive Power Flexibility Region (RPFR)

To numerically evaluate the var flexibility provided by DER, we define reactive power flexibility region (RPFR) as the range

\[
\mathcal{D} = \left[ v_0^*/1, v_0^*/1 \right]
\]

(a) IEEE 37 node test system

Fig. 7. Flow chart of the process of estimating var capability curve as a function of DER Headroom.

(b) IEEE 123 node test system

Where \( q_{\text{base}}^{\text{net}} \) is the net var demand at the substation when all DER inverters operate in unity power factor mode. The \( a \) and \( b \) denote the maximum available capacitive and inductive var support in MVar, respectively. A higher magnitude of both \( a \) and \( b \) with negative and positive signs respectively represent a larger flexibility region. A zero value of both \( a \) and \( b \) denotes no available var flexibility.

B. Test System Description

An unbalanced 3-phase IEEE distribution 37 bus test system is considered with around 4 MW as peak load and around 90% solar PV penetration as shown in Fig. 8(a). Here, we define the penetration level as a ratio of peak solar generation to peak load demand. Around 100 Single phase DER (solar PV) units are equally distributed throughout the DS nodes in all three phases. Inverter ratings are considered 1.1 times the peak solar generation. Maximum and minimum values of \( v_{tm} \) are considered as 0.9 and 1.1. The results are also validated on a larger IEEE 123 node test systems shown in Fig. 8(b).

C. Modeling Validation

The successful implementation of the proposed var support framework is highly dependent on the accuracy of the
To verify, the voltages profiles from this model are compared with the full nonlinear DS (GridlabD) model for both IEEE 37 and 123 test systems. Fig. 9 compares the voltages of all three phases of the 123 bus system at all the nodes at peak load scenario without any DERs. It can be seen that the voltage profile from the linearized model is very close to the non-linear solver. The Table I shows that the mean and maximum voltage errors are within 0.02% and 0.07% respectively. The high accuracy of the LinDist3Flow model in the base case is due to an appropriate selection of loss factors based on the offline study. We will keep the same loss factor for capability curve estimations and report the validation error in the next subsections. Note that the error in substation net var for both systems is within 0.2% and 1%, respectively.

**D. Aggregated Net Capability Curves**

Let’s consider two cases with different loading conditions to compare the aggregated net capability curves, i.e., high loading case 1 with peak load and low loading case 2 with half of the peak load. The capability curve for case 1 is shown in Fig. 10 as a function of DER headroom with solid black lines. The region enveloped by the solid black lines is the P-Q flexibility or capability domain; the dashed black line is base var demand, $q_{\text{base}}^{\text{net}} = 2.12 \text{ Mvar}$. The region above and below the base var demand line can be seen as inductive and capacitive var support regions, respectively. Essentially, any point in the flexibility domain can be achieved by appropriate headroom. The RPFR ($a, b$) values for both case 1 and case 2 are computed using (29) and are compared in Table II for different DER real power headroom levels (e.g., for Case 1, 10%: $a = 0.34 - 2.12 = -1.78$ and $b = 4.01 - 2.12 = 1.89$). It can be seen that the capacitive support region (magnitude of $a$) increases with increasing the headroom. For both cases 1 and 2 as increasing real power headroom frees the inverter capacity as well as reduces the voltages due to an increase in net load. This provides more scope for DERs to supply var, leading to a higher magnitude of $a$. However, the inductive var support region (magnitude of $b$) first increases with the headroom but starts decreasing towards the end for case 1 while for case 2, it continuously increases. This is because increasing the headroom and inductive var support cause low voltages, and after a certain headroom level, the voltage of at least one node reaches its minimum limit. Whereas, in case 2, the voltages do not reach the minimum limit due to low load conditions for almost all the headroom percentages, as shown in Fig. 11.

Similar to the 37 node system, the capability curve for IEEE 123 system is shown in Fig. 12. The dotted line shows the total inverter capacity to provide reactive power if voltage constraints are not considered. Whereas a proper consideration of voltage constraints in the distribution system leads to a reduced capability region in the 123 node system. This also shows the applicability of the proposed method to distribution systems with hundreds of nodes.
E. Validation of Capability Curves

To validate the capability charts obtained from the LinDist3Flow model, we compare it against the full non-linear distribution system solver, GridLAB-D. Fig. 13 compares the capability curves obtained from the linearized model and GridLAB-D. The errors in var estimation for each point on both $q^{cap}_{net}$ and $q^{cap}_{net}$ curves are shown in Fig. 14 (top) for 37 bus system. The maximum error in var estimation is around 1.5%, whereas the average error is less than 1% for both test systems, as tabulated in Table III. Further, it is important to check the accuracy of voltages while operating in the capability region. We check the voltage errors across all nodes and phases for various points on the upper and lower capability curves. The maximum and average errors in voltages are within 0.5% and 0.3%, respectively. Note that the voltage errors are more than the base case since we are operating on extreme points on the capability curves. Fig. 14 (middle and bottom) show a box plot of the distribution of errors in voltage across all nodes for each phase. It can be observed that phase a has a relatively smaller error in voltages.

F. Impact of Unbalanced DER on Capability Curves

DER distribution in all 3 phases of a typical distribution system may be significantly unbalanced, which in turn affects its aggregated var capability. To demonstrate, we create various scenarios with increasing levels of unbalance in DER distribution at all phases while keeping the total DER capacity (penetration level) the same as shown in Table IV. The balanced scenario has equal distribution of DERs in all 3 phases. Unbalanced I, II, and III scenario have an increasing level of unbalance due to higher DER allocation at one of the phases. These scenarios
have been created for each phase one at a time. Fig. 15(top) shows the capability charts for increasing level of unbalance in phase A. It can be seen that the higher unbalance significantly shrinks the capability region. This happens because the unbalanced distribution of DERs increases the voltage unbalance in the system, making it more voltage constrained in one of the phases. Therefore, the full inverter capacity can not be utilized to maintain voltages within the operating bounds. Fig. 15 (bottom) shows that even a similar level of unbalance in different phases affects the capability curves differently owing to the unbalanced nature (impedance and load) of distribution systems. This observation is in line with the discussion in [30], where increasing unbalance in DER penetration is shown to impact the voltage stability margin of the system. This emphasizes the importance of capturing the unbalance in the capability estimation process as proposed in this work.

G. Impact of Grid Side Voltages

From the transmission side, the primary substation voltage is a crucial factor that can affect the capability domain significantly. Though usually, we expect the $v_{tm}$ to be around 1, in case of contingencies and other events, it can significantly deviate from the nominal value. Fig. 18 shows how the capability region varies with change in $v_{tm}$ at no curtailment. Note that the flexibility region shrinks as $v_{tm}$ moves away from nominal 1 pu on either side beyond the decoupling range $D$. It can be seen that the DER-OPF becomes infeasible for $v_{tm}$ greater than 1.19 pu and less than 0.88 pu, which means no flexibility is available.

H. Day-Ahead Capability Curve

In the last section, capability curve charts were constructed for a given operating condition at time $t$, however, the day-ahead capability curves can also be estimated to be utilized by TSO for day-ahead planning. Various day ahead capability curves for the next 24 hours can be constructed for different headroom conditions associated with different amounts of uncertainties or curtailment possibilities. A normalized daily load curve and solar PV generation profile is applied to each load and PV unit, respectively, as shown in Fig. 16. Fig. 17 shows two capability curves (solid black lines) and var support regions (grey and blue shaded areas), each associated with a particular headroom. The day-ahead curve provides more visual information on how the aggregated capability varies with changing operating conditions throughout the day. It can be seen that the flexibility range is minimum at noon when the least inverter capacity is available for var support (between 0800 hrs to 1700 hrs). However, a cloud cover results in 40% additional headroom that frees the inverter capacity and expands its flexibility area, shown by the blue shaded portion during peak solar hours (0800 hrs to 1700 hrs) as shown in Fig. 17.

I. Impact of Voltage-Dependent Loads

The impact of voltage-dependent loads on the capability region is assessed by simulating various scenarios with decreasing portion of constant power load, denoted by $a_0$, and consequently increasing portion of constant impedance loads. Fig. 19 shows that the higher proportion of voltage-dependent loads increases the flexibility region in both directions. This happens because
Fig. 18. Impact of grid side voltage on the capability region.

Fig. 19. Impact of voltage-dependent load on the capability region. Lower $a_0$ denotes loads with a lower constant power portion and higher constant impedance portion.

![Table V](#)

**TABLE V**

| Scenario | $a_0$ | 0.25 | 0.5 | 0.75 | 1 |
|----------|-------|------|-----|------|---|
| RPFR change | 0% | 1.5% | 3.2% | 4.8% | 6.4% |

Fig. 20. Two different day-ahead aggregated capability curves and domains due to compliance to IEEE1547 var capability requirements.

As expected, both nominal capacitive and induction flexibility regions increase with higher DER penetration.

### K. Impact of Inverter Size

Inverter size plays a crucial role in available DER capability. Table VII compares the RPFR for different inverter sizes during peak solar generation—inverters with no oversize at peak generation results in no headroom leading to zero flexibility region. However, the headroom can be increased by freeing up the inverter capacity via curtailment or storage, as shown in Table VII (row 2). To realize headroom, Table VII shows the trade-off between oversizing and curtailment to achieve desired flexibility during peak solar generation. This trade-off is also relevant to complying with the integration standard 1547–2022 discussed in the next section.

### L. Integration Standard IEEE1547 Compliance

The recently revised DER integration standard IEEE1547-2022 has made it compulsory for each inverter-based DER unit to provide var capability of 44% of its kW rating at all operating conditions [10]. To comply with it, there are two possible options, i.e., either oversize the inverter by 1.113 times the kW rating with no DER headroom or 10.2% DER headroom during peak hours with no inverter oversize. Here, we have compared the impact of IEEE1547 compliance on the aggregated RPFR by choosing both the options in the form of two cases as shown in Fig. 20. Case 1 is shown by solid black lines with 10.2% headroom at each DER with no oversized inverters, and case 2 is shown by orange solid lines where each inverter is oversized by 1.113 times. Note that both the cases comply with IEEE1547 standard; however, case 2 has broader aggregated RPFR ($[-1.89, 2.03]$) compared to case 1 ($[-1.71, 1.82]$) at noon. At the same time, case 1 might be more viable as the real power headroom is provided occasionally when the var is needed by TSO, whereas the inverter oversize cost.
TABLE VIII
DER V AR IMPACT ON TRANSMISSION VAR GENERATION

| Scenario                        | \(Q_{\text{Gen1}}\) (Mvar) | \(Q_{\text{Gen2}}\) (Mvar) | \(Q_{\text{Gen3}}\) (Mvar) |
|---------------------------------|------------------------------|-----------------------------|-----------------------------|
| 50% DER (No var, No Contingency)| 30.28                        | 14.92                       | -3.52                       |
| 50% DER (With var, No Contingency)| 26.4                        | 3.75                        | -12.95                      |
| 50% DER (No var, Contingency)   | 85.62                        | 45.36                       | 10.21                       |
| 50% DER (With var, Contingency) | 82.30                        | 33.37                       | -0.31                       |

is permanent. Nonetheless, it depends on many other factors such as policy, incentive structure, ancillary service market, etc. Further cost-benefit analysis is needed of specific cases to arrive at any decision.

VI. IMPACT OF AGGREGATED DER V AR SUPPORT ON THE TRANSMISSION GRID

A. Impact on Transmission Var Generation

Let us consider the IEEE 9-bus transmission system with the distributed inverters connected at load bus 7 (T7) (100 MW, 35 Mvar). We consider different power flow scenarios for the IEEE 9-bus system with and without the var support available at the transmission system. We have considered 50% DER penetration with 10% headroom (minimum headroom) and 44% of its rating available for reactive power modulation for the cases with var support. The resultant power injection from the inverters at load bus 7 is (45 MW, 22 Mvar) (with var support). The case without var support assumes the inverters operate in unity power factor mode, which means there is no reactive power injection from inverters. The individual var generation from the three synchronous generators in IEEE 9-bus system is shown in Table VIII. In this system, for the contingency of line 5–6 outage, we demonstrate that with the var capability available to the transmission system, the total var generation from synchronous generators is reduced significantly. In cases of no contingency also, var support reduces transmission var generation.

The present framework discusses the availability of another var resource through DSO flexibility. Nonetheless, an economic analysis with reactive pricing can provide further inputs that enable the TSO to make the optimal choice.

B. Utilizing var Capability for TN Voltages

The proposed framework estimates the aggregated var capability curve for the transmission grid. However, the grid might not need the maximum var support all the time; rather, it can ask for the var support in specific needs, e.g., in case of voltage dips due to line contingencies. In this section, we will demonstrate on an integrated T-D test system, how the proposed aggregated var capability can potentially enhance the options for TSO. A T-D cosimulation platform is developed based on reference [31], [32] to accurately model the T-D interactions. An integrated T-D test system is modeled by coupling aggregated multiple IEEE 37 bus DS feeders (22,32,25) at three load buses (T5, T7, T9) of the IEEE 9 bus TS as shown in Fig. 21.

Let’s consider an operating point with peak solar generation to demonstrate the impact of minimum available var flexibility. We will compare the effects of DER var support under line T5-6 contingency for the following cases: a) No DER var support provided by any DSO; b) DSO at bus T9 provides just enough DER var support to comply with 1547; c) DSO at both bus T9 and T5 provide just enough DER var support to comply with integration standard 1547; d) DSO at bus T9 provide more DER var support than case (b) by 20% headroom. Fig. 22 compares the voltages at transmission buses for all cases. At \(t = 5\), line 5-6 is removed, which leads to a dip in voltages, and bus T5 and T9 suffer under voltage violation. In case (b), the support by only T9 is not enough to recover voltages above 0.95. In such cases, TSO either can request var flexibility from both T9 and T5 as recommended by 1547 standard, i.e., case (c) or it can request...
extra support from T9 that can be provided by some headroom, i.e., case (d). It can be seen that both case (c) and (d) recover voltages above the limit. However, the amount of voltage boosts at T9 and T5 differ based on the cases.

From the results shown in Fig. 22, we can see that in the case of no DERs, under a contingency, the voltages at the load buses are violated with respect to the lower limit of 0.95 pu. With minimum headroom to comply with the IEEE 1547-2022, var capability at T9 is used, but the voltages are not able to recover to more than 0.95 pu. If the DSO does not have provisions for changing the headroom, then the neighboring DSO can utilize its var capability to help the TSO, as shown in results in (c). However, if the DSO can adjust the DER headroom, then the DSO at T9 can increase the headroom to further enhance the amount of var support to TSO, as shown in (d).

Note that the estimation of optimal var support request profile depends on the various factors such as objective of TSO, availability of DER flexibility, economic compensation policies, etc., and needs to be achieved via an optimization process which is beyond the scope of this paper. The 4 cases here demonstrate the potential of the proposed framework that provides higher flexibility to TSO. The var capability curve can be used as an input by the TSO to optimize its operation by varying the var within the var capability curve. The var capability curve allows for TSO to request a DSO to dispatch a var at the TSO-DSO interface. The dispatch of the corresponding set-points to the individual DERs will use the same infrastructure as any other grid edge device in smart grids like smart thermostats, electric vehicle (EV) chargers, etc.

VII. CONCLUSION

This work is one of the components in the pursuit of utilizing the increasing DER penetration for the benefit of the future grid, driven by the proposition that a multitude of geographically distributed DERs with var control capability can be seen as flexible var resources (mini SVCs) for the grid. To facilitate this vision, a systematic OPF-based methodology is proposed to construct an aggregated net var capability curve of a DS with high DER penetration, analogous to a conventional bulk generator. The proposed capability curve also accounts for DER headroom that enables TSO to utilize both P and Q flexibility provided by DERs in their planning and operational activities. The impact of DS unbalance, voltage constraints, inverter sizing, and, T-D coupling on the flexibility region are discussed via results on an unbalanced IEEE 37 and IEEE 123 bus DS. Finally, it is shown how aggregated DER var flexibility can affect the transmission system performance on an integrated T-D test system in a cosimulation environment.

A. Future Work

Certainly, the formulation and details of the inclusion of the proposed flexibility in grid planning and operations remain an exciting challenge for future studies that require a larger discussion on policy, payment structure, etc. Further, The proposed framework and methodology can be extended with probabilistic approaches to consider various uncertainties in loads, model parameters, network topology, the status of DERs, etc., present in power system operation. Nonetheless, the results are encouraging and indicate that the proposed aggregated capability indeed has the potential to improve grid optimality by providing enhanced flexibility services to TSO.

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