An analysis of mathematical reflective thinking skills of senior high school students

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Abstract. The present study aims to describe mathematical reflective thinking skills of senior high school students according to the problems which examine mathematical reflective thinking skills of integrals subject matter. The method in this study was qualitative with phenomenological approach. Six students of grade XI in a senior high school in Banda Aceh City were selected as the subject. The data were collected through a test consisting of mathematical reflective thinking skills problems, interviews, and documentation. The indicators of mathematical reflective thinking skills include identifying problems, interpreting problems, evaluating problems, predicting problem-solving, and drawing conclusions. The results show that there are students who have not been able to identify problems clearly and correctly, all students have not been able to interpret the integral-related problems clearly and correctly, some students have not been able to evaluate the integral-related problems clearly and correctly, some students have not been able to predict the integral-related problems clearly and correctly, and some students have not been able to draw conclusions regarding the integral-related problems clearly and correctly.

1. Introduction

Education is one of the crucial factors in preparing students to face their life challenges and issues in the future. What can be prepared is to improve the quality of human resources, knowledge and mind-set in order to develop well and, therefore, influence the development of science and technology in general. Today’s people are required to keep developing their skills to be able to solve every single problem of life.

One of the efforts to be made in developing the ability and power of thought is by mathematics. Mathematics plays an important role in the field of education. In order to overcome the problems in everyday life, mathematics can be used. Mathematics is said to be both the queen and the servant of science, meaning that it is the source of other kinds of knowledge [1]. Therefore, mathematics plays a very significant role in the development of science and technology. Mathematics becomes the part of school curriculum and underlies the development of students’ thinking skills.

In learning mathematics, certain ways of thinking is required to be able to understand the concepts as well as to apply them when solving a variety of mathematical problems. Bigot states that thinking is basically putting the relationship between the parts of knowledge owned by human beings [2]. Knowledge is defined as “the information, understanding, and skills that you gain through experience.
or education” [3]. Therefore, thinking constitutes an act of connecting some of the experiences that we previously encountered or the ones after we have taken education in the forms of facts, information or skills.

In each of the lessons, it is important that students be required to reflect on their own selves with regard to what they have learned and for what they learned it. This is supposed to be applied especially in mathematics learning because there are a considerable number of mathematical concepts requiring sufficient prerequisite knowledge for learning them. Thus, involving the process of reflection in learning mathematics is significant. As mentioned “reflective is essential to both teachers’ and students’ learning” [4].

Chee states that reflective thinking is an awareness of what is known and what is needed, this is very important to bridge the gap in the learning situation [5]. Besides, Gurol defines reflective thinking as a process of directed and precise activities in which individuals realize to follow, analyze, evaluate, motivate, obtain deep meaning, and use appropriate learning strategies [6]. Boody, Hamilton and Schon revealed that the characteristic of reflective thinking is reflection as a retrospective analysis, namely the ability to provide self-assessment; reflection as a problem solving process, namely awareness in learning; critical reflection on the self, which is to improve themselves continuously [7,8,9]. Other characteristics revealed by Primrose, Leung and Kember are selecting/linking knowledge gained to solve problems and actively making judgments about what has been done [10,11].

One’s learning outcomes and problem solving will be relevant to his/her reflective thinking skills. This is supported by the statement of Demirel, Derman and Karagedik that problem solving and reflective thinking are directly related [12]. Therefore, it is necessary that students’ reflective thinking skills be developed. Process of learning, researching and solving problems will run maximally if one’s reflective thinking skills are good enough because the reflection process is related to the selection of relevant previously owned concepts or knowledge as well as decision making in solving problems [13]. Rudd state that the important role of reflective thinking is to provide encouragement to thinkers during the problem solving process, this is because the thinkers are given the opportunity to step back and choose the best strategy to achieve the goals [14].

Based on the preceding description, the importance of mathematical reflective thinking skills is obvious. However, in reality mathematical reflective thinking skills are known to remain low. This can be seen from the results of an international survey conducted by PISA (Programme for International Assessment) that Indonesia was ranked 64th out of 65 participating countries in 2012 [15] and 63rd out of 69 in 2015 [16]. The PISA Framework includes some assessments on the mathematical contents, processes and contexts. The assessment of the mathematical processes consists of formulating, using, and interpreting [16].

The categories mentioned above are related to the reflective thinking skills. First, formulating represents that students are able to identify mathematical problems in the real world. Second, employing indicates the ability of evaluating and interpreting the results of mathematical work into real world contexts. Third, interpreting shows how students reflect mathematical solutions and interpret them in the context of real problems. From the description above, it can be seen that it is important to have mathematical reflective thinking skills. Students who own this kind of skills will be certainly aware of what is needed through the learning process.

2. Methods
The method in this research was qualitative with phenomenological approach. Six students of grade XI-MIPA 2 at the even semester of the academic year of 2018/2019 in a senior high school in Banda Aceh City were selected as the subject research. The data were collected through a test consisting of mathematical reflective thinking skills problems, interviews and documentation. These data will be analyzed regarding the students’ mathematical reflective thinking skills in solving the integral problems.

The main instrument of the present study was the researcher herself. The auxiliary instrument was a set of questions of mathematical reflective thinking skills test, consisting of 5 items related to integral subject matter. The indicators of mathematical reflective thinking skills tested include identifying
problems, interpreting problems, evaluating problems, predicting problem-solving, and drawing conclusions.

3. Results and Discussion

The study aims to describe the mathematical reflective thinking skills of senior high school students regarding the integral subject matter. According to some consideration between the researcher and mathematics teacher, 6 students were selected as the subject, consisting of two student with high cognitive skills, two moderate, and two low.

The analysis results of the mathematical reflective thinking skills test of senior high school students in solving integral problems are presented as follows:

Table 1. Recapitulation of the mathematical reflective thinking skills test of senior high school students in solving integral problems

| Cognitive category | Item 1  | Item 2  | Item 3  | Item 4  | Item 5  |
|--------------------|--------|--------|--------|--------|--------|
|                    | Student | Identifying problems | Interpreting problems | Evaluating problems | Predicting problem-solving | Drawing conclusions |
| High               | A1     | √      | -      | √      | √      | √      |
|                    | A2     | √      | -      | -      | √      | √      |
| Moderate           | A3     | √      | -      | -      | -      | -      |
|                    | A4     | √      | -      | -      | √      | √      |
| Low                | A5     | -      | -      | -      | -      | -      |
|                    | A6     | -      | -      | -      | -      | -      |
| Total students     | 5      | 0      | 1      | 1      | 4      |

According to table 1, it can be seen that 5 students meet the ‘identifying the problem’ indicator; none of them meets the ‘interpreting the problem’ indicator; one student meets the ‘evaluating the problem’ indicator, 4 students meets the ‘predicting the problem solving’ indicator, and 4 students meet the ‘making conclusion’ indicator.

Based on the results of interviews, it was found that A1 is able to identify the integral-related problems, in this case the student can express what was known and what was asked and solve the problems clearly and correctly. A1 has not been able to interpret the integral-related problems, this is because the students cannot interpret the links between the questions of item 2. A1 is able to evaluate the integral-related problems, it can be seen from that the student can detect the error found in item 3 and can the error. A1 is also able to predict the resolution of the integral-related problems, in item 4 the student can use the integral concept as an alternative solution. Also, A1 is able to draw conclusions out of the integral-related problems, it is seen from that the student can draw general decisions by solving the problem of item 5.

The results of interview on A2 indicate that the student is able to identify the integral-related problems, proven by the fact that A2 can mention what was known and what was asked in question item 1 in a clear and correct manner. A2 has not been able to interpret the integral-related problems yet because A2 cannot relate one problem to the others existing in item 2. A2 has not been able to evaluate the integral-related problems either. This is detected from that the student cannot detect the error existing in item 3. A2 is able to predict the problem solving of the integral-related problems since the student can employ the integral concept as the alternative solving. A2 is able to draw conclusion out of the integral-related problems, seen from the fact that A2 can make general decision by solving item 5.

Similarly, A3 is able to identify the integral-related problems, in this case the student can mention and explain what was known and asked in item 1. A3 is also able to predict the integral-related problem solving, in item 4 the student can use the integral concept as an alternative solution. Moreover, A3 is able to draw conclusions from the integral-related problems, it is seen from that the student can draw general decisions by solving the problem of item 5. However, A3 has not been able to interpret the integral-related problems, this is because the student do not comprehend the relations between what was
known and what was asked. A3 has not been able to evaluate the integral-related problems either, it can be seen from the student’s mistake in error-detecting in item 3 due to inability to remember the particular concept related to the problem.

A4 is able to identify the integral-related problems, detected from that the student can mention and explain what was known and asked in the question item. A4 has not been able yet to interpret the integral-related problems since the student cannot relate what was known to what was asked within the question. A4 is able to predict the problem solving of the integral-related problems, this is seen from that the student can employ the integral concept as the alternative solving. A4 has not been able to evaluate the integral-related problems as the student cannot detect the errors existing in item 3, due to the inability to master the concept related to the problem provided. A4 is able to draw conclusion out of the integral-related problems as seen from the students’ ability in making general decision by solving item 5.

A5 is only able to identify the integral-related problems, in this case the student can describe what was known and what was asked as well as complete the question in item 1 clearly and correctly. However, A5 has not been able to interpret the integral-related problems, in this case the student did not answer the question of item 2 because the student do not remember the concept of integrals used in resolving the problem of item 3. This leads to that A5 has not been able to interpret the problem provided in item 2. A5 has not been able to evaluate the integral-related problems either, it can be seen from the student's inability to detect error in item 3 since the student do not remember the related integral concept. A5 has not able to predict the integral-related problem solving either, for item 4 the student cannot employ the integral concept as an alternative solution. Further, A5 has not been able to draw conclusions from the integral-related problems, it is seen from the student's inability to draw general decisions from solving the problem of item 5.

A6 has not been able yet to identify the integral-related problems, seen from the mistakes that the student make in mentioning what was known and what was asked as well as in answering question item 1. A6 has not been able yet to interpret the integral-related problems as well since the student does not answer question 2 at all. A6 has not been able to evaluate the integral-related problems, proven by the fact that the student cannot detect the errors existing in item 3. A6 has neither been able to predict the problem solving of the integral-related problems, as the student cannot employ the integral concept as the alternative solving of problem 4. Similarly, A6 has not been able to draw any conclusion out of the integral-related problems since no conclusion is made for problems 5.

Based on the results of interviews of 6 students obtained information that there are students who have not been able to solve problems related to indicators identifying problems, all students do not understand the relationship of problems in the problem with the indicator category interpreting the problem, some students cannot detect errors in the questions with the indicator category evaluating problem, some students cannot use the integral concept as an alternative to solving problems with indicator categories predicting problem solving, and some students have not been able to make conclusions on problems with indicator categories making conclusions.

4. Conclusion
Based on the results of research and data analysis, the authors conclude that (1) Students with high cognitive skills, some are only unable to interpret the problem because they cannot interpret the relationship between questions and some have not been able to interpret the problem and have not been able to evaluate the problem carefully and correct. (2) Students with moderate cognitive skills, have not been able to interpret the problem because they do not understand the relationship between what is known and what is asked, also have not been able to evaluate the problem because they are not careful in detecting errors in the questions. (3) Students with low cognitive skills, some students are only able to identify problems clearly and correctly and there are students who have not been able to interpret problems, evaluate problems, predict problem solving and make conclusions, this is because they do not remember the integral concepts involved.
5. References

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