Stabilization of Compactification Volume
In a Noncommutative Mini-Super-Phase-Space

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Abstract

We consider a class of generalized FRW type metrics in the context of higher dimensional Einstein gravity in which the extra dimensions are allowed to have different scale factors. It is shown that noncommutativity between the momenta conjugate to the internal space scale factors controls the power-law behavior of the scale factors in the extra dimensions, taming it to an oscillatory behavior. Hence noncommutativity among the internal momenta of the mini-super-phase-space can be used to explain stabilization of the compactification volume of the internal space in a higher dimensional gravity theory.

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1 Introduction

It is a long standing problem that gravity (the Einstein Hilbert action) is plagued with severe non-renormalizability and string theory has appeared as the only consistent theory of quantum gravity with a reasonable low energy limit. Consistent string theories are, however, all formulated in dimensions higher than four, the best understood of them in ten dimensions. Besides the extra dimensions consistency of string theory requires supersymmetry. Neither the extra dimensions nor supersymmetry are not backed by the current observational data. Hence, to turn string theory to a viable physical model one needs to address the above issues. In this work we will concentrate on the extra dimensions and leave the supersymmetry to other works.

Theories in higher than four space-times dimensions was considered by Kaluza-Klein (KK) in the early days of Einstein gravity, with the hope of unifying gauge theories with gravity. In the KK setup the question “why the extra dimensions are not observed today?” is answered by considering the extra dimensions to be wrapping a compact manifold of a small (Planckian) size. This is essentially what is also used in modern day string theory. Namely, the six extra dimensions are compactified on a Ricci flat complex, Kähler three-fold, a Calabi-Yau three-fold ($CY_3$) whose volume is of the size of strings (or Planck) scale \[1\].

Using ($CY$) compactification idea one immediately faces the “compactification and moduli problem”, i.e. why and how among so many possibilities for the shape and size of the $CY_3$ a specific one, which has resulted in the Universe and physics we see today, has been chosen? In other words, the question is whether string theory bears a (dynamical) mechanism which lands us in a specific point in the so-called string landscape \[3\].

Recent developments in string theory compactification when various form-field fluxes are turned on, the flux compactification, has taught us that the shape moduli (complex structure moduli) can be fixed once the fluxes are chosen \[4\]. The size moduli (Kähler structure moduli) and in particular the overall volume modulus are, however, more involved and their fixing is somewhat less established and clear in the flux compactification setup, e.g. see \[5, 6, 7\] in the string theory context and \[8, 9, 10\] in the context of large extra dimension models.

It is, of course, desirable to invoke a more dynamical reasoning than flux compactification. For example looking for scenarios where the extra dimensions become small, while we see the expansion in the three space dimensions, in the early stages of the cosmic evolution, and in particular during inflation. There have also been proposals to realize this “dynamical moduli fixing” scenarios within string theory, which usually go under the name of “string gas

\[1\] There is also the Randall-Sundrum (RS) scenario (which is also called RS “compactification”) \[2\] which is not based on the idea of compact Ricci flat manifold as extra dimensions, but employs the warped metric to make the extra dimensions invisible to the “low energy” physics observed today.
cosmology” \cite{11,12} or their more recent improvements “brane gas cosmology” \cite{13}. These models are based on the fact that in string cosmology we are dealing with a gas of strings and branes rather than a gas of point particles described by standard local field theories.

In this paper we examine a different new idea for the dynamical stabilization of the volume. In our model we restrict ourselves to Einstein gravity in higher dimensions. In particular we start with a generalized FRW cosmology plus noncommutativity in the space of momenta conjugate to the metric in the extra dimensions and show that this noncommutativity tames the runaway behavior of the scale factors of the internal space. The idea of noncommutativity in the context of cosmology and its relevance to moduli stabilization, has been previously discussed in the literature. For example in \cite{14} noncommutativity in the extra directions and that it can help with stabilization of the volume of compactification has been discussed; in \cite{15} noncommutativity in the superspace (and not the real physical space) in the context of four dimensional gravity theory was considered and the noncommutative version of Wheeler-DeWitt equation discussed. Our model, in a sense, has a combination of the two. Namely, in our model we deal with a deformed superspace (and not the real physical space) in the context of a higher dimensional theory. The noncommutativity we consider is in the momentum part of the mini-super-phase-space and is only in the extra dimensions.

The rest of the paper is organized as follows. In section 2, we present a preliminary set up for the model starting with a $D$ dimensional gravity theory and use the $(D-1)+1$ ADM decomposition to work out the Hamiltonian constraint, through which using Hamilton-Jacobi dynamics, one can obtain the gravitational equations of motion. In section 3, we add the other crucial ingredient of our model, namely turning on the noncommutativity in the momentum space. We then re-solve the gravitational equations of motion for the noncommutative case and show that while the scale factor in the four space-time dimensions has a power-law growth with respect to the co-moving time, the scale factor in the extra dimensions exhibits an oscillatory behavior, thus stabilizing the volume modulus. We end with discussions and remarks and discuss a possible setting to realize our model within string theory flux compactifications.

\section{Preliminary set up of the model}

Consider a $D$ dimensional universe defined as

$$\mathcal{M} = R \times S \times M \times N,$$

where $\times$ are wrapped products, $S$ is the spatial part of our four dimensional space-time and $M, N$ are the internal (extra) dimensions. We choose $M$ and $N$ to be compact $m, n$
dimensional manifolds, hence $D = m + n + 4$. We parameterize the line element as
\[ ds^2 = -e^{2\rho(t)} dt \otimes dt + e^{2\alpha(t)} dx^i \otimes dx^i + e^{2\alpha(t)} dy^j \otimes dy^j + e^{2\nu(t)} dz^k \otimes dz^k, \] (2.2)
where $i = 1, 2, 3, j = 1, \ldots, m$, $k = 1, \ldots, n$ and $\alpha, \nu$ are the scale factors we have associated with $S, M$ and $N$ respectively. Dynamics of the metric is governed by the $D$ dimensional action
\[ S = \frac{1}{2\kappa_0^2} \int d^D x \sqrt{g} R[g] \] (2.3)
where $R[g]$ is the scalar curvature of the metric (2.2), $\kappa_0^2$ is the $D$-dimensional gravitational (Newton) constant. Assuming that all the fields (the metric components) are only functions of $t$, plugging (2.2) into the action (2.3) we obtain
\[ S = \frac{1}{2\kappa^2} \int dt d^3 x \mathcal{L}, \] (2.4)
where $\kappa$ is the four dimensional Newton constant,
\[ \kappa^2 = \frac{\kappa_0^2}{V_M V_N}, \]
with $V_M, V_N$ being the volumes of $M, N$ internal manifolds, and
\[ \mathcal{L} = \frac{1}{2} e^{-\rho + \rho_0} \left[ (3 - 9) \dot{a}^2 + (m - m^2) \dot{u}^2 + (n - n^2) \dot{v}^2 - 2 (3m \dot{a} \dot{u} + 3n \dot{a} \dot{v} + nm \dot{u} \dot{v}) \right]. \] (2.5)
Here, dot represents differentiation with respect to $t$ and
\[ \rho_0(t) = 3a(t) + mu(t) + nv(t). \] (2.6)
For the above Lagrangian one may write the corresponding Hamiltonian as
\[ \mathcal{H} = \frac{e^{\rho + \rho_0}}{2 + n + m} \left[ \alpha p_a^2 + \beta p_u^2 + \gamma p_v^2 - p_a p_u - p_a p_v - p_u p_v \right], \] (2.7)
where $p_a, p_u$ and $p_v$ are the momenta conjugate to $a, u$ and $v$ respectively and
\[ \alpha = \frac{n + m - 4}{6}, \quad \beta = \frac{n + 2}{2m}, \quad \gamma = \frac{m + 2}{2n}. \] (2.8)
The equations of motion corresponding to this Hamiltonian, noting that the overall constant factor $\frac{1}{2 + n + m}$ has been dropped, become
\begin{align*}
\dot{a} &= \{a, \mathcal{H}\}_P = e^{-\rho + \rho_0} (2\alpha p_a - p_u - p_v), \quad \dot{p}_a = \{p_a, \mathcal{H}\}_P = \mathcal{H} e^{\rho - \rho_0} \{p_a, e^{-\rho + \rho_0}\}_P, \\
\dot{u} &= \{u, \mathcal{H}\}_P = e^{-\rho + \rho_0} (2\beta p_u - p_a - p_v), \quad \dot{p}_u = \{p_u, \mathcal{H}\}_P = \mathcal{H} e^{\rho - \rho_0} \{p_u, e^{-\rho + \rho_0}\}_P, \\
\dot{v} &= \{v, \mathcal{H}\}_P = e^{-\rho + \rho_0} (2\gamma p_v - p_a - p_u), \quad \dot{p}_v = \{p_v, \mathcal{H}\}_P = \mathcal{H} e^{\rho - \rho_0} \{p_v, e^{-\rho + \rho_0}\}_P.
\end{align*} (2.9)
Since the momentum conjugate to $\rho(t)$ does not appear in Hamiltonian (2.7), the equation of motion for $\rho$,

$$\mathcal{H} = \mathcal{L} = 0,$$  \hspace{1cm} (2.10)

appears as a constraint which should be imposed on the solutions of (2.9). Therefore, as long as (2.10) is satisfied $\rho$ could be taken any function of time. To solve (2.9) it is convenient to adopt the harmonic time gauge

$$\rho(t) = \rho_0(t) = 3a(t) + mu(t) + nv(t).$$  \hspace{1cm} (2.11)

The equations then simplify to

$$\ddot{a} = 0, \quad \ddot{u} = 0, \quad \ddot{v} = 0,$$  \hspace{1cm} (2.12)

whose solutions are of the form

$$a(t) = a_1t + b_1, \quad u(t) = a_2t + b_2, \quad v(t) = a_3t + b_3,$$  \hspace{1cm} (2.13)

where $a_i, b_i$ are arbitrary constants. We should now make sure that the (Hamiltonian) constraint (2.10)

$$\mathcal{M}_{ij}a_ia_j \equiv (3 - 9)a_1^2 + (m - m^2)a_2^2 + (n - n^2)a_3^2 - 2(3ma_1a_2 + 3na_1a_2 + nma_2a_3) = 0$$  \hspace{1cm} (2.14)

can be satisfied. This equation does have solutions for real $a_i$, because the $3 \times 3$ matrix $\mathcal{M}_{ij}$ has negative eigenvalues. More concretely it has two positive and one negative eigenvalues. One may solve (2.14) and obtain e.g. $a_3$ as a function of $a_1$ and $a_2$.

The above result is obtained in the harmonic time gauge. To see the significance of this result for cosmological purposes it is better to use the comoving frame. This could be done introducing the comoving time $\tau$:

$$\frac{d\tau}{dt} = e^{\rho(t)},$$  \hspace{1cm} (2.15)

leading to

$$\tau = \frac{1}{K} e^{Kt}, \quad K = 3a_1 + ma_2 + na_3.$$  \hspace{1cm} (2.16)

In terms of $\tau$, the scale factors $e^a$ becomes

$$e^{\rho(\tau)} \sim \tau^{a_1 + ma_2 + na_3}.$$  \hspace{1cm} (2.17)

Therefore, for generic values of the initial conditions $a_i$, we find a power-law behavior in all of the spatial directions and hence there is no stabilization for the internal dimensions. This

\[2\] In fact the Hamiltonian (2.7) is not positive definite. This is, however, a well known fact in gravity e.g. see [16]. To see this recall that gravity can be viewed as a gauge theory with the local Lorentz symmetry.
is somewhat expected because the internal scale factors in terms of the lower dimensional theory, after Kaluza-Klein compactification, appear as scalar fields with run away potentials driving the power-law behavior. In the next section we demonstrate how noncommutativity among the internal conjugate momenta \( p_u, p_v \) can lead to a mechanism which dynamically stabilizes the volume of the extra dimensions by modifying the run away behavior.

### 3 Noncommutative mini-super phase space

The Hamiltonian (2.7) describes a classical (or quantum) mechanical system on a three dimensional mini-super-space or a six dimensional mini-super-phase-space. In the previous section we used standard canonical phase space with the usual Poisson brackets. We would now like to modify the canonical brackets on the mini-super-phase-space. We do this by changing the Poisson brackets of conjugate momenta to be non-vanishing. Since \( a, u, v \) do not appear in the Hamiltonian explicitly, making them noncommuting will not modify the equations of motion.

We impose noncommutativity between the internal momenta:

\[
\{P_v, P_u\}_P = \xi, \\
\{v, u\}_P = 0.
\]

We’ll, however, keep the Hamiltonian to have the same functional form in terms of \( P \)'s as before. Such a noncommutativity can be motivated by string theory corrections to the Einstein gravity. In the next section we discuss this point further.

Next we use the standard trick [17] to re-introduce the canonical variables

\[
p_u = P_u + \frac{\xi}{2} v, \\
p_v = P_v - \frac{\xi}{2} u.
\]

It is evident that \( \{p_v, p_u\}_P = 0 \). The rest of the analysis reduces to the standard ones once we rewrite Hamiltonian in terms of the canonical variables \( u, v, p_u, p_v \)

\[
\mathcal{H}_{nc} = \frac{e^{-\rho + \rho_0}}{2 + n + m} \left[ \alpha p_a^2 + \beta (p_a - \frac{\xi}{2} v)^2 + \gamma (p_v + \frac{\xi}{2} u)^2 - p_a (p_u - \frac{\xi}{2} v) - p_a (p_v + \frac{\xi}{2} u) - (p_u - \frac{\xi}{2} v) (p_v + \frac{\xi}{2} u) \right]
\]

where \( \alpha, \beta \) and \( \gamma \) are defined in (2.8). As we see the Hamiltonian (3.3) is very similarly to that of a charged particle moving in an external magnetic field proportional to \( \xi \) in the \( u, v \) directions and the flat potential in the \( u \) and \( v \) directions is lifted to a harmonic oscillator potential due to noncommutativity.
The equations of motion can now be easily worked out as
\[ \eta \dot{a} = e^{-\rho + \rho_0} \left(2 \alpha p_a - p_u - p_v - \frac{\xi}{2}(u - v) \right), \]
\[ \eta \dot{p}_a = \eta H_{nc} e^{\rho - \rho_0} \{ p_a, e^{-\rho + \rho_0} \}_P, \tag{3.4} \]
\[ \eta \dot{u} = e^{-\rho + \rho_0} \left(2 \beta p_a - p_u - p_v - \frac{\xi}{2}(2 \beta v + u) \right), \]
\[ \eta \dot{p}_u = \eta H_{nc} e^{\rho - \rho_0} \{ p_u, e^{-\rho + \rho_0} \}_P + e^{-\rho + \rho_0} \left(-\gamma \xi p_v + \frac{\xi}{2} p_u + \frac{\xi}{2} p_v - \frac{\gamma \xi^2}{4} u - \frac{\xi^2}{4} v \right), \tag{3.5} \]
\[ \eta \dot{v} = e^{-\rho + \rho_0} \left(2 \gamma p_v - p_u - p_v + \frac{\xi}{2}(2 \gamma u + v) \right), \]
\[ \eta \dot{p}_v = \eta H_{nc} e^{\rho - \rho_0} \{ p_v, e^{-\rho + \rho_0} \}_P + e^{-\rho + \rho_0} \left(\beta \xi p_u - \frac{\xi}{2} p_v - \frac{\beta \xi^2}{2} v - \frac{\xi^2}{4} u \right), \tag{3.6} \]

where
\[ \eta = 2 + m + n = D - 2, \]

and equation of motion for $\rho$ again appears as the Hamiltonian constraint
\[ H_{nc} = 0. \tag{3.7} \]

Physically one should expect this, because the Hamiltonian constraint is the result of time re-parametrization invariance which remains even when the noncommutativity is turned on.

In the harmonic time gauge (2.11) the above equations simplify as
\[ \eta \ddot{a} = -\xi (\dot{u} - \dot{v}), \]
\[ \eta \ddot{u} = -\xi (\dot{u} + 2\beta \dot{v}), \]
\[ \eta \ddot{v} = \xi (\dot{v} + 2\gamma \dot{u}), \tag{3.8} \]

whose solutions are
\[ a(t) = \frac{b}{\tan \theta} \left[ \sqrt{\gamma} \cos(\omega t + \phi + \theta) + \sqrt{\beta} \cos(\omega t + \phi) \right] + H_0 t + C_1, \]
\[ u(t) = b \sqrt{\beta} \sin(\omega t + \phi) + C_2, \]
\[ v(t) = -b \sqrt{\gamma} \sin(\omega t + \phi + \theta) + C_3, \tag{3.9} \]

where $b, C_1, C_2, C_3, H_0$ and $\phi$ are integration constants and
\[ \theta = \arccos \left( \frac{1}{\sqrt{4 \beta \gamma}} \right), \tag{3.10} \]
\[ \eta \omega = \xi \sqrt{4 \beta \gamma} - 1 = +\xi \tan \theta. \tag{3.11} \]

Since $4 \beta \gamma = 1 + 2 \left( \frac{m+n+2}{mn} \right) > 1$, $\omega$ and $\theta$ are both real valued. It can be easily checked that the above noncommutative solutions in the $\xi \rightarrow 0$ go over to the commutative ones (2.13).
As we see now, $u$ and $v$ show an oscillatory behavior. They have a phase difference $\theta$ which depends only on the number of the dimensions of $M$ and $N$. For example for $m = n$ case,

$$
\cos \theta = \frac{\text{#extra dimensions}}{\text{#total spacetime dimensions}} = \frac{n}{n + 2}, \quad \omega = \frac{1}{n\sqrt{n + 1}} \xi.
$$

For the ten dimensional case, $\cos \theta = 3/5$, $\omega = \xi/6$.

We must now impose the Hamiltonian constraint (3.7). It is basically an equation for $H_0$. Plugging the solutions (3.9) into $\mathcal{H}_{nc}$ we obtain

$$
H_0^2 = \frac{1}{2 \times 3mn} b^2 \xi^2
$$

(3.12)

As we expect, $H_0$ turns out to be proportional to the only dimensionful parameter in our problem, $\xi$. Moreover, as one would physically expect the constants $C_i$ and $\phi$ which could be absorbed in the scaling of coordinates and choice of origin of time, does not appear in the final expression for the Hamiltonian constraint and hence in $H_0$. For the special ten dimensional case and when $m = n = 3$, $H_0 = b\xi/3\sqrt{6}$.

Dropping the $C_i$ and $\phi$ the scale factors of the internal dimensions can be written as

$$
\begin{align*}
    e^{2u(t)} &= \exp \left[2b\sqrt{\beta} \sin \omega t\right], \\
    e^{2v(t)} &= \exp \left[-2b\sqrt{\gamma} \sin(\omega t + \theta)\right], \\
    e^{2a(t)} &= \exp \left[\frac{2b}{\tan \theta} \left(\sqrt{\gamma} \cos(\omega t + \theta) + \sqrt{\beta} \cos \omega t\right)\right] e^{2H_0 t},
\end{align*}
$$

(3.13)

where $H_0$ is given by (3.12). A quick look at the above equations reveals that as the time $t$ progresses the behavior of the internal scale factors, the first two equations, is oscillatory while that of the visible spacetime has a growing behavior.

To gain a better understanding of this behavior it is useful to work in the more standard comoving FRW frame. As discussed in the previous section for this we need to redefine the time coordinate. To simplify the computations let us consider the special case which is of particular interest to string theory,

$$
m = n = 3,
$$

corresponding to a ten dimensional space-time. For this case (2.11) becomes,

$$
\rho = \frac{b\xi}{\sqrt{6}} t + \frac{3b}{\sqrt{6}} \cos \left(\frac{\xi}{6} t + \frac{\theta}{2}\right)
$$

(3.14)

and the co-moving time $\tau$ is given by $d\tau = e^\rho dt$. The form of the scale factors and the volume ratio with respect to the co-moving time $\tau$ are drawn in the Figures. For the present epoch, that is for $t \gg \frac{1}{K}$ in (3.14), the first term has the dominant contribution and one can write $\tau = \frac{1}{K} e^{Kt}$, $K = 3H_0 = b\xi/\sqrt{6}$, similarly to the commutative case (2.16).

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Figure 1: The left plot shows the scale factors with respect to the harmonic time $t$ measured in units of $H_0^{-1}$. The harmonic time coordinate $t$ is essentially logarithm of the comoving time $\tau$ (for large $t$). These figures are drawn to demonstrate the oscillatory behavior. The solid line represents the scale factor for the extra dimensions using equation (3.13). The dashed line represents the scale factor of the ordinary universe. The plot on the right shows the ratio of the volume of the internal space to that of the ordinary space. These figures are drawn for $n = m = 3$, $b = 1$ and $\xi \sim H_0$.

Figure 2: The left plot shows the scale factors with respect to the co-moving time $\tau$ measured in units of $H_0^{-1}$. The solid line represents the scale factor for the extra dimensions using equation (5.13). The dashed line represents the scale factor of the ordinary universe. The plot on the right shows the ratio of the volume of the internal space to that of the ordinary space. These figures are drawn for $n = m = 3$, $b = 1$ and $\xi \sim H_0$. Note that the expected oscillatory behavior, not visible here, appears in later times. The same is true for the power law behavior shown by the scale factor of the ordinary universe, the dashed line.
4 Discussion and concluding remarks

Here we discussed a simple model for stabilizing compactification volume in a higher dimensional gravity theory. Starting from a $D = m + n + 4$ dimensional spacetime with all the spatial directions compactified, say on a torus of Planckian size, we showed that the assumption that the momenta conjugate to scale factors of the internal degrees are noncommuting plus the usual Einstein equations, lead to the very interesting result that the scale factor corresponding to three visible spatial dimensions obey a power law growth (see the figures), while the internal scale factors just oscillate. The solutions to our model involves two time scales, one is the noncommutativity scale $\xi$ and the other is $H_0$, whose ratio $b$ is an integration constant (an initial condition). The natural choice is of course $b \sim 1$ or $\xi \sim H_0$ in which case as is seen from the figures, the ratios of the internal space to the visible space volumes drops by a power law in the comoving cosmological time $\tau$.

In short, introduction of noncommutativity among the momenta in the internal part of mini-super-phase-space modifies the gravitational dynamics in such a way that the average of the volume of the internal compact directions is stable under the time evolution of the system. In a sense, as is also seen from (3.3), introduction of noncommutativity among the momenta for the internal scale factors is like the addition of a harmonic oscillator type potential for the internal directions. In other wording, the scale factors of the internal dimensions appear as scalars in the $3+1$ dimensional theory and the noncommutativity we have added removes the run away behavior of these scalars. This is essentially the same phenomenon that particles in an external magnetic field (the Landau problem) have effectively noncommuting momenta.

In our model we have assumed noncommuting momenta (3.1) which needs to be motivated. As mentioned earlier and is well-known noncommuting momenta arises when we have a charged particle in a magnetic field and the Landau problem. However, here we have a gravitational system and it is not clear what this “magnetic field” can be. Although we do not have a robust setup in a gravitational context which leads to (3.1), one might hope that turning on fluxes in the internal compact space in a string theory setting and within flux compactification models, can lead to noncommuting momenta in the internal part of the mini-super-phase-space. (Recall that flux compactifications was proposed as a way to stabilize some of the compactification moduli.) This is in the same spirit as the noncommutativity appearing on a D-brane in a Kalb-Ramond two from flux background [18]. Here for concreteness and just to demonstrate the idea we limited ourselves to a constant noncommutativity among the momenta. We, however, believe that this effect is not particular to this special case and is a generic behavior of any similar noncommutativity among momenta. It is desirable to check this explicitly and if indeed our model can be fit within string theory
settings.

In this work we considered a higher dimensional pure gravity with no other additional fields. It is of course very interesting to examine the noncommutativity idea in an inflationary setup and check if our volume stabilization method still remains effective in an inflation model.

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References

[1] M. Green, J. H. Schwarz, E. Witten, “Superstring Theory,” Vol.1& 2, Cambridge University Press, Cambridge (1987).
J. Polchinski, “String Theory,” Vol.1& 2, Cambridge University Press, Cambridge (1998).

[2] L. Randall and R. Sundrum, “A large mass hierarchy from a small extra dimension,” Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-ph/9905221]; “An alternative to compactification,” Phys. Rev. Lett. 83, 4690 (1999) [arXiv:hep-th/9906064].

[3] L. Susskind, “The anthropic landscape of string theory,” arXiv:hep-th/0302219

[4] There are large number of papers on the flux compactification and moduli fixing. One of the original papers in this direction is S. B. Giddings, S. Kachru and J. Polchinski, “Hierarchies from fluxes in string compactifications,” Phys. Rev. D 66, 106006 (2002) [arXiv:hep-th/0105097]. For more recent reviews see: M. Grana, “Flux compactifications in string theory: A comprehensive review,” Phys. Rept. 423, 91 (2006) [arXiv:hep-th/0509003]; M. R. Douglas and S. Kachru, “Flux compactification,” [arXiv:hep-th/0610102] and references therein.

[5] J. M. Maldacena and C. Nunez, “Supergravity description of field theories on curved manifolds and a no go theorem,” Int. J. Mod. Phys. A 16, 822 (2001) [arXiv:hep-th/0007018].
[6] K. Becker, M. Becker, K. Dasgupta and P. S. Green, “Compactifications of heterotic theory on non-Kaehler complex manifolds. I,” JHEP 0304, 007 (2003) \texttt{arXiv:hep-th/0301161}.

[7] S. Gukov, S. Kachru, X. Liu and L. McAllister, “Heterotic moduli stabilization with fractional Chern-Simons invariants,” Phys. Rev. D 69, 086008 (2004) \texttt{arXiv:hep-th/0310159}.
F. Denef, M. R. Douglas, B. Florea, A. Grassi and S. Kachru, “Fixing all moduli in a simple F-theory compactification,” \texttt{arXiv:hep-th/0503124}.
O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, “Type IIA Moduli Stabilization,” JHEP 0507, 066 (2005) \texttt{arXiv:hep-th/0505160}.

[8] N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, “Stabilization of sub-millimeter dimensions: The new guise of the hierarchy problem,” Phys. Rev. D 63, 064020 (2001) \texttt{arXiv:hep-th/9809124}.

[9] U. Gunther and A. Zhuk, “Stabilization of internal spaces in multidimensional cosmology,” Phys. Rev. D 61, 124001 (2000) \texttt{arXiv:hep-ph/0002009}; “A note on dynamical stabilization of internal spaces in multidimensional cosmology,” Class. Quant. Grav. 18, 1441 (2001) \texttt{arXiv:hep-ph/0006283}.

[10] S. M. Carroll, J. Geddes, M. B. Hoffman and R. M. Wald, “Classical stabilization of homogeneous extra dimensions,” Phys. Rev. D 66, 024036 (2002) \texttt{arXiv:hep-th/0110149}.
S. Jalalzadeh, F. Ahmadi and H. R. Sepangi, “Multi-dimensional classical and quantum cosmology: Exact solutions, signature transition and stabilization,” JHEP 0308, 012 (2003) \texttt{arXiv:hep-th/0308067}.

[11] R. H. Brandenberger and C. Vafa, “Superstrings In The Early Universe,” Nucl. Phys. B 316, 391 (1989).
T. Battefeld and S. Watson, “String gas cosmology,” Rev. Mod. Phys. 78, 435 (2006) \texttt{arXiv:hep-th/0510022}.

[12] S. Watson and R. Brandenberger, “Stabilization of extra dimensions at tree level,” JCAP 0311, 008 (2003) \texttt{arXiv:hep-th/0307044}.
S. P. Patil and R. H. Brandenberger, “The cosmology of massless string modes,” JCAP 0601, 005 (2006) \texttt{arXiv:hep-th/0502069}.
R. Brandenberger, “Moduli stabilization in string gas cosmology,” \texttt{arXiv:hep-th/0509159}.
[13] S. Alexander, R. H. Brandenberger and D. Easson, “Brane gases in the early universe,” Phys. Rev. D 62, 103509 (2000) [arXiv:hep-th/0005212].

N. Shuhmaher and R. Brandenberger, “Heterotic brane gas inflation,” arXiv:hep-th/0512056.

D. A. Easson and M. Trodden, Phys. Rev. D 72, 026002 (2005) [arXiv:hep-th/0505098].

[14] F. Ardalan, “Large extra dimensions and noncommutative geometry in string theory,” arXiv:hep-th/9910064.

[15] H. Garcia-Compean, O. Obregon and C. Ramirez, “Noncommutative quantum cosmology,” Phys. Rev. Lett. 88, 161301 (2002) [arXiv:hep-th/0107250].

J. C. Lopez-Dominguez, O. Obregon, M. Sabido and C. Ramirez, “Towards noncommutative quantum black holes,” Phys. Rev. D 74, 084024 (2006) [arXiv:hep-th/0607002].

[16] S. Weinberg, “Gravitation and Cosmology – Principles and Applications of the General Theory of Relativity,” John Weily and Sons Inc. (1972).

[17] M. Chaichian, M. M. Sheikh-Jabbari and A. Tureanu, “Hydrogen atom spectrum and the Lamb shift in noncommutative QED,” Phys. Rev. Lett. 86, 2716 (2001) [arXiv:hep-th/0010175].

[18] For reviews see, e.g. R. J. Szabo, “Quantum field theory on noncommutative spaces,” Phys. Rept. 378, 207 (2003) [arXiv:hep-th/0109162].

M. R. Douglas and N. A. Nekrasov, “Noncommutative field theory,” Rev. Mod. Phys. 73, 977 (2001) [arXiv:hep-th/0106048].