Dynamical Model for Determination of Horizontal Forces on Crane Runway during Motion of the Crane

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Abstract. During a motion of an overhead travelling crane on the crane runway, horizontal forces between the crane and the crane runway girder occur. The reason of these forces can be the acceleration or deceleration of the crane and then most significant part of these horizontal forces is skewing of the crane. There are several methods for calculation of these forces. These methods depend mainly on the chosen computational model. Some comparison of these methods was made in a past conference by the author of this article. The most significant load caused by motion of crane is the skewing. In the past, the cause of the skewing was considered to be caused by different speeds of the end-carriages of the crane which result to the horizontal transverse loads of the crane runway. In the present, the skewing of crane is defined as the motion of the overhead bridge crane with the constant velocity, but with the angle relative to the crane runway. It means that the overhead bridge crane is angled relative to the crane runway and moves with constant velocity. During this motion, the crane wheels are angled relative to the rail of the crane runway and the transverse forces occur on crane wheels. But this situation can occur also during the acceleration or deceleration of the crane. Present standards do not take into account this situation. A dynamical model is presented in this paper, describing the motion of the overhead double bridge crane during its acceleration. The dynamical model includes things which can influence the motion of the crane, the forces acting on crane wheels. The model thus describes the behaviour of the crane during acceleration in general and enables to determinate the horizontal transverse forces. The basic assumption of this model is that there is no contact between the rim of the crane wheel and the rail of the crane runway.

1. Introduction

The transverse horizontal forces between the overhead bridge crane and the rail of crane runway girder occur during a motion of the crane. These forces are caused by various causes. They can be caused by accelerating or decelerating of the crane, acceleration or deceleration of a crab and by the skewing of the crane which results into significant transverse forces between the crane wheel and the rail of the crane runway. There are some procedures and computational models which enable to calculate these horizontal transverse forces. Some of them were presented and compared in [1]. The comparison of different models shows the big differences between models and results. These big differences are caused by different interpretations of what is skewing of crane. N. A. Lobov [2][3][4] created dynamical model describing motion of the overhead bridge crane. This model enables to determinate the horizontal transverse forces. The model considers the crane as the rigid body with the load of crab rigidly connected to the crane. It means that the swinging of the load on the crab rope was neglected.
2. Creation of the Dynamical Model of the overhead double bridge crane

The dynamical model is based on the model created by N.A Lobov [2]. But dynamical model in this paper is extended by some factors, that Lobov's model did not consider. This includes the deformation of crane bridges (so-called S-deformation), the swinging of the crab load in the direction of the crane and the inclusion of crane drive control. Designed model is designed for a double bridge crane. The model is made up of rigid bodies that are connected to each other by equivalent springs, which represent the system's deformations. The motion equations of the model are assembled using Lagrange equations II. kinds. The Figure 1 shows the created dynamical model.

![Figure 1. Dynamical model of the overhead bridge crane](image)

Legend to the Figure 1:
- $m_m$, $J_m$ - the weight of the crane bridge and the moment of inertia to its center of gravity
- $m_p$, $J_p$ - the weight of the endcarriage and the moment of inertia to its center of gravity
- $m_b$ - weight of load hanging on the crab
- $c$ - the bending stiffness of the free ends of the endcarriages
- $c_{mp}$ - an equivalent rotating spring replacing the deformation of the crane bridges due to different endcarriages speeds (so-called S-deformation)
- $c_{sb}$ - equivalent stiffness replacing the rope and simulating the load swing
- $R_1, R_2, R_3, R_4$ transverse forces between the crane wheels and the rail of the crane runway
- $W_1, W_2$ the resistance forces generated by the rolling of the wheel and acting against the movement of the crane
- $P_1, P_2$ engine driving forces (representing by torque moment in the dives)
- $x, y, y_1, y_2, \phi, x_1, x_2, x_3, x_4, y_b$ the unknown positioning of the system over time

The rigidity of the spring $c$ is given by the condition of the same flexibility as the free end of endcarriage member. The spring extension $x_1, x_2, x_3, x_4$ corresponds to the deflections at the end of the cross member. The procedure for detecting the stiffness of the free end of the crossbar on the crane structure shows Figure 2. The applied force is the unit and the detected horizontal deformation of the endcarriage end corresponds to the compliance of the end of the cross member, which is introduced in the inverted value into the model as the stiffness $c$.

![Figure 2. Bending stiffness of the free ends of the endcarriages](image)

The stiffness of the cmp springs between the bridges and endcarriages is defined on the same compliance of the deformation of the actual crane and model construction. Determination of the longitudinal deformation at the selected force $F$ (so-called S-deformation) shows Figure 3.

![Figure 3. S deformation of the crane](image)

If we load the crane structure by a known force $F$ and subtract the corresponding longitudinal deformation of the crane, we can according the Figure 4 calculate the stiffness of the cmp springs.
Figure 4. Scheme for calculating the stiffness of cmp

Legend to the Figure 4

- F chosen force
- y deformation in the real structure calculated for example by FEM method
- L span of the crane

From the Figure 4 can be derived:

$$c_{mp} = \frac{F \cdot L^2}{4 \cdot y}$$  \hspace{1cm} (1)

The cb can be derive from the equality of motion differential equations for forced oscillation of matter suspended on the rope and the mass attached to the spring, or simply from Figure 5.
Figure 5. Forces on the load hanging on the rope

Legend to the Figure 5

- m weight of the load
- l length of the rope
- φ angle of the rope
- x horizontal displacement of the load
- F force caused the horizontal displacement of the load
- cb stiffness of the equivalent spring

From Figure 5 can be derived the following formula

\[ c_b = \frac{m \cdot g}{l} \]  \hspace{1cm} (2)

The forces R1, R2, R3 and R4 are the forces that occur as a result of the lateral slip of the wheel when it is rolled along the rail, and are for this model also the forces that loads the crane runway and the crane structure. The forces R1 through R4 are determined according to the linear sliding theory as follows

\[ R_1 = K_1 \cdot \sigma_1 \]  \hspace{1cm} (3)
\[ R_2 = K_2 \cdot \sigma_2 \]  \hspace{1cm} (4)
\[ R_3 = K_3 \cdot \sigma_3 \]  \hspace{1cm} (5)
\[ R_4 = K_4 \cdot \sigma_4 \]  \hspace{1cm} (6)
where:
- \( K_1 \) až \( K_4 \) are slip constants
- \( \sigma_1 \) až \( \sigma_4 \) are transverse slides on individual wheels during crane driving

Determination of slip constants and transverse slides can be found for example in [5].

The driving torques \( M_1, M_2 \) are moments occurring on the motor shaft and are defined in accordance with the drive control.

3. Motion equations of the dynamic model

For deriving of the motion equations of the dynamical model were used Lagrange equations II. Kinds:

\[
\frac{d}{dt}(\frac{dE_k}{dq_j}) - \frac{dE_k}{dq_j} = Q_j
\]

where:
- \( E_k \) is the kinetic energy of the system
- \( q_j \) are unknown coordinates \((x, y_1, y_1, \varphi, y_b, x_1 \text{ až } x_4)\)
- \( Q_j \) is a generalized force

When we express the kinetic energy of the system and substitute it into equation (7) we get the motion equations of the system:

\[
(m + m_k) \cdot \ddot{x} = -R_1 - R_2 - R_3 - R_4
\]

\[
\left( m_p + \frac{1}{2} \cdot m_m + \frac{1}{2} \cdot s^2 \cdot J_m + m_k \cdot \left( \frac{s - e}{2 \cdot s} \right)^2 + m_b \cdot \left( \frac{s - e}{2 \cdot s} \right)^2 + I_m \cdot \frac{i_p^2}{R_k} \right) \cdot \ddot{y}_b + \frac{1}{2} \cdot m_m - \frac{1}{2} \cdot s^2 \cdot J_m + m_b \cdot \left( \frac{s - e}{2 \cdot s} \right) \cdot \left( 1 - \frac{s - e}{2 \cdot s} \right) + m_b \cdot \left( \frac{s - e}{2 \cdot s} \right) \cdot \left( 1 - \frac{s - e}{2 \cdot s} \right) + I_m \cdot \frac{i_p^2}{R_k} \cdot \ddot{y}_1 - \frac{m_b \cdot s \cdot e}{2 \cdot s} \cdot \ddot{y}_b = M_1 \cdot \frac{i_p}{R_k} \cdot \eta_p - W_1 \cdot \frac{2}{s} \cdot c_{np} \cdot \left( \frac{v_1 - v_2}{2 \cdot s} - \varphi \right)
\]

\[
\left( \frac{1}{2} \cdot m_m - \frac{1}{2} \cdot s^2 \cdot J_m + m_k \cdot \left( \frac{s - e}{2 \cdot s} \right) \cdot \left( 1 - \frac{s - e}{2 \cdot s} \right) + m_b \cdot \left( \frac{s - e}{2 \cdot s} \right) \cdot \left( 1 - \frac{s - e}{2 \cdot s} \right) + I_m \cdot \frac{i_p^2}{R_k} \right) \cdot \ddot{y}_1 + \frac{m_p + \frac{1}{2} \cdot m_m + \frac{1}{2} \cdot s^2 \cdot J_m + m_k \cdot \left( \frac{s - e}{2 \cdot s} \right)^2 + m_b \cdot \left( \frac{s - e}{2 \cdot s} \right)^2 + I_m \cdot \frac{i_p^2}{R_k} \right) \cdot \ddot{y}_2 - m_b \cdot \left( 1 - \frac{s - e}{2 \cdot s} \right) \cdot \ddot{y}_b = M_2 \cdot \frac{i_p}{R_k} \cdot \eta_p - W_2 \cdot \frac{2}{s} \cdot c_{np} \cdot \left( \frac{v_1 - v_2}{2 \cdot s} - \varphi \right) - m_b \cdot \frac{s - e}{2 \cdot s} \cdot \ddot{y}_1 - m_b \cdot \left( 1 - \frac{s - e}{2 \cdot s} \right) \cdot \ddot{y}_2 + m_b \cdot \ddot{y}_b = -c_b \cdot y_b
\]
\[ 2 \cdot J_p + 2 \cdot m_s \cdot b^2 = (R_1 + R_2 - R_3 - R_4) \cdot a + 4 \cdot c_{hp} \cdot \left( \frac{y_1 - y_2}{2 \cdot s} - \varphi \right) \] (12)

\[ c \cdot x_1 = R_1 \] (13)
\[ c \cdot x_2 = R_2 \] (14)
\[ c \cdot x_3 = R_3 \] (15)
\[ c \cdot x_4 = R_4 \] (16)

where
- \( M_1, M_2 \) the driving moments of the crane's motors
- \( R_k \) radius of crane whell
- \( \text{ip} \) gear ratio
- \( \eta_p \) efficiency of the gear

4. Results and discussions
The dynamical model described above was created for several assumptions. The main assumption is no contact wheels rim with the rail. No damping of the system is considered, because the damping is not possible to determine practically. Although the crane's steel structure has a small relative attenuation, a number of other damping elements are present in the real structure, for example, in the form of friction etc. It is not possible to define the initial preload before starting of the movement of the crane. This preload for real cranes arises as a result of a previous ride and lasts after the crane stops. A rigid crane track is considered in the model. Ideal crane geometry, crane wheels and crane tracks are assumed. Small crane rotation is assumed. This assumption is fulfilled easily because of the relatively small gap between wheel rim and the rail. Ideal geometry of crane, crane wheels and crane tracks is assumed.

5. Conclusions
The dynamical model describing the motion of the overhead bridge crane on the crane runway was created. The construction of the model was explained and the motion equations were derived. For creation of the model several assumptions were made for the derivation. It is planned to use this dynamical model for further research of the determination of horizontal transverse forces, especially caused by skewing of the overhead bridge crane.

References
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