Manifestly $\mathcal{N} = 2$ supersymmetric regularization for $\mathcal{N} = 2$ supersymmetric field theories

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Abstract

We formulate the higher covariant derivative regularization for $\mathcal{N} = 2$ supersymmetric gauge theories in $\mathcal{N} = 2$ harmonic superspace. This regularization is constructed by adding the $\mathcal{N} = 2$ supersymmetric higher derivative term to the classical action and inserting the $\mathcal{N} = 2$ supersymmetric Pauli–Villars determinants into the generating functional for removing one-loop divergencies. Unlike all other regularization schemes in $\mathcal{N} = 2$ supersymmetric quantum field theory, this regularization preserves by construction the manifest $\mathcal{N} = 2$ supersymmetry at all steps of calculating loop corrections to the effective action. Together with $\mathcal{N} = 2$ supersymmetric background field method this regularization allows to calculate quantum corrections without breaking the manifest gauge symmetry and $\mathcal{N} = 2$ supersymmetry. Thus, we justify the assumption about existence of a regularization preserving $\mathcal{N} = 2$ supersymmetry, which is a key element of the $\mathcal{N} = 2$ non-renormalization theorem. As a result, we give the prove of the $\mathcal{N} = 2$ non-renormalization theorem which does not require any additional assumptions.

Keywords: higher derivative regularization, supersymmetry, harmonic superspace.

1 Introduction

The $\mathcal{N} = 2$ non-renormalization theorem states that the global $\mathcal{N} = 2$ supersymmetric gauge theories are finite beyond the one-loop approximation. This theorem was first enunciated in [1] for the $\mathcal{N} = 2$ supersymmetric Yang–Mills (SYM) theory. The unconstrained $\mathcal{N} = 2$ superfield formulation of the hypermultiplet was constructed in [2], where it was used for proving the finiteness of the $\mathcal{N} = 4$ SYM theory. On its basis the detailed proof of the $\mathcal{N} = 2$ non-renormalization theorem was given in [3]. Using this theorem it is possible to obtain that
\( \mathcal{N} = 2 \) supersymmetric gauge theories are finite if their one-loop \( \beta \)-function vanishes \([4]\). A proof of the \( \mathcal{N} = 2 \) non-renormalization theorem based on the harmonic superspace approach was given in \([5]\). Deriving the non-renormalization theorem one implicitly assumes existence of a regularization which does not break the gauge symmetry and \( \mathcal{N} = 2 \) supersymmetry. However, a construction of a regularization which satisfies these requirements is not evident \([6]\). In particular, the standard dimensional regularization breaks supersymmetry, and supersymmetric theories are mostly regularized by using its special modification, which is called the regularization by means of dimensional reduction \([7]\). However, the dimensional reduction is inconsistent from the mathematical point of view \([8]\). In principle, it is possible to remove the inconsistencies, but the price is the loss of manifest supersymmetry \([9]\). As a consequence, supersymmetry can be broken by quantum corrections in higher loops \([10, 11]\). In particular, the explicit calculations made in \([10]\) and subsequently corrected in \([12]\) show that for the \( \mathcal{N} = 2 \) SYM theory supersymmetry is really broken by quantum corrections in the three-loop approximation if the regularization by means of dimensional reduction is used. This implies that in this case the assumptions used in the proof of the non-renormalization theorem are broken due to the loss of manifest supersymmetry. Thus, the dimensional reduction cannot be considered as a completely satisfactory regularization for supersymmetric theories and the proof of the non-renormalization theorem contains a hole. The purpose of this paper is to remove this hole and to justify finally the \( \mathcal{N} = 2 \) non-renormalization theorem.

We would like to pay attention that there exists a consistent regularization convenient for using in gauge theories. It is called the higher covariant derivative regularization \([13, 14]\). For \( \mathcal{N} = 1 \) supersymmetric gauge theories such a regularization can be formulated in terms of \( \mathcal{N} = 1 \) superfields \([15, 16]\), so that \( \mathcal{N} = 1 \) supersymmetry is a manifest symmetry at all steps of quantum calculations. This regularization appears to be very convenient for explicit computing the quantum corrections (see, e.g. \([17, 18, 19]\)) and for proving some general statements, such as deriving the Novikov-Shifman-Vainshtein-Zakharov (NSVZ) beta-function \([20, 21, 22, 23]\) and NSVZ-like relations in all orders of the perturbation theory \([24, 25, 26, 27]\) or constructing in all orders the NSVZ-scheme \([28, 29, 30]\). In particular, it turns out that the higher derivative regularization has some essential advantages comparing with the dimensional reduction.

\( \mathcal{N} = 2 \) supersymmetric theories can be certainly considered as a special case of \( \mathcal{N} = 1 \) supersymmetric theories with extra hidden on-shell \( \mathcal{N} = 1 \) supersymmetry. However, it is unclear from the very beginning that the \( \mathcal{N} = 1 \) higher covariant derivative regularization will preserve the above hidden supersymmetry. The first attempt to construct a version of the higher derivative regularization for the \( \mathcal{N} = 2 \) SYM theories was made in \([31]\), but the invariant higher derivative term was not written explicitly. The problem was again addressed in \([32]\), where the higher covariant derivative regularization was constructed for an arbitrary \( \mathcal{N} = 2 \) supersymmetric gauge theory. However, the formulation in terms of \( \mathcal{N} = 1 \) superfields which was used in \([32]\) although preserve manifest \( \mathcal{N} = 1 \) supersymmetry, does not allow to preserve the hidden \( \mathcal{N} = 1 \) supersymmetry at all stages of quantum corrections calculating, because the gauge fixing condition in terms of \( \mathcal{N} = 1 \) superfields is incompatible with hidden supersymmetry. As a result, a removal of the above hole in the proof of the \( \mathcal{N} = 2 \) non-renormalization theorem requires the additional study. It is clear that the most natural way to carry out such a study should be based on a formulation of the \( \mathcal{N} = 2 \) supersymmetric theories in terms of unconstrained \( \mathcal{N} = 2 \) superfields where \( \mathcal{N} = 2 \) supersymmetry will be manifest.

It is known that the manifest \( \mathcal{N} = 2 \) supersymmetric formulation of the \( \mathcal{N} = 2 \) theories is given in the terms of the \( \mathcal{N} = 2 \) harmonic superspace \([33, 34, 35]\) (see also \([36]\)). In particular, using this formalism it is possible to construct the \( \mathcal{N} = 2 \) supersymmetric gauge fixing procedure. That is why in this paper we formulate the higher covariant derivative regularization for \( \mathcal{N} = 2 \) supersymmetric gauge theories in \( \mathcal{N} = 2 \) harmonic superspace. As a result, we obtain a version of the higher covariant derivative regularization which allows to calculate quantum corrections.
in a manifestly \(\mathcal{N} = 2\) supersymmetric way. Existence of such a regularization justifies the proof of the \(\mathcal{N} = 2\) non-renormalization theorem. Therefore, we present a way of calculating the quantum corrections which actually ensures absence of divergences beyond the one-loop approximation.

The paper is organized as follows: In Sect. 2 we recall basic information about the \(\mathcal{N} = 2\) supersymmetric gauge theories and \(\mathcal{N} = 2\) harmonic superspace. Sect. 3 is devoted to the formulation of the higher covariant derivative regularization in the harmonic superspace. This is done by using the background field method so that the constructed regularization does not break the background gauge invariance. This allows to justify the proof of the non-renormalization theorem, which is considered in Sect. 3.3. In Sect. 4 we present another simple proof of \(\mathcal{N} = 2\) non-renormalization theorem based on the NSVZ \(\beta\)-function. The last Sect. 5 is devoted to explicit calculating the one-loop divergences for the general \(\mathcal{N} = 2\) SYM theory with matter by the help of the regularization constructed in this paper. In particular, we demonstrate factorizations of integrals for the \(\beta\)-function into integrals of double total derivatives and vanishing of the one-loop anomalous dimensions for hypermultiplets.

2 \(\mathcal{N} = 2\) supersymmetric gauge theories in the harmonic superspace

Manifest \(\mathcal{N} = 2\) supersymmetry at all stages of calculating quantum corrections is achieved by using \(\mathcal{N} = 2\) harmonic superspace. It is obtained from the ordinary \(\mathcal{N} = 2\) superspace with the coordinates \((x^\mu, \theta_a^i, \bar{\theta}_{ia})\) by adding the complex coordinates \(u_i^+, u_i^- = (u_i^+)^*\), such that

\[
u_i^+ u_i^- = 1. \quad (1)
\]

In the language of \(\mathcal{N} = 2\) harmonic superspace the gauge field is a component of the real (with respect to a specially defined conjugation \(\gamma\)) analytic superfield \(V^{++}\). The analyticity means that it satisfies the conditions

\[
D^+_a V^{++} = 0; \quad \bar{D}^+_\bar{a} V^{++} = 0, \quad (2)
\]

where \(D^+_a\) and \(\bar{D}^+_\bar{a}\) are the supersymmetric covariant derivatives contracted with \(u_i^+\). The superfield \(V^{++}\) belongs to the Lie algebra of the gauge group so that \(V^{++} = e_0(V^{++})^A t^A\), where \(e_0\) is a bare coupling constant and the Hermitian generators \(t^A\) are normalized by the condition \(\text{tr}(t^A t^B) = \delta^{AB}/2\). In order to write the action for the \(\mathcal{N} = 2\) SYM theory we also define the superfield

\[
V^{--}(X, u) = \sum_{n=1}^{\infty} (-i)^{n+1} \int du_1 du_2 \ldots du_n \frac{V^{++}(X, u_1) V^{++}(X, u_2) \ldots V^{++}(X, u_n)}{(u^+_1 u_1^+)(u^+_2 u_2^+) \ldots (u^+_n u_n^+)}, \quad (3)
\]

where \(X\) denotes the set of the coordinates \((x^\mu, \theta^i, \bar{\theta}_i)\), which is the same in all \(V^{++}\) in the numerator, and \((u_1^+ u_2^+) \equiv u_1^+ u_2^+\). This superfield is related to the strength tensors \(W\) and \(\bar{W}\) by the equations

\[
W \equiv e^{i\nu} W e^{-i\nu} = -\frac{i}{2}(\bar{D}^+)^2 V^{--}; \quad \bar{W} \equiv e^{i\nu} \bar{W} e^{-i\nu} = \frac{i}{2}(D^+)^2 V^{--}. \quad (4)
\]

\(^{1}\)In our notation, \(\alpha\) numerates components of the left spinor, \(\dot{\alpha}\) numerates components of the right spinor, and the index \(i = 1, 2\) numerates \(\theta\)-s.

\(^{2}\)Throughout this paper we mostly work in the \(\lambda\)-frame and omit the subscript \(\lambda\) for the superfields in the \(\lambda\)-frame. The subscript \(\tau\) points out that a superfield is written in the \(\tau\)-frame.
In our notation \((\bar{D}^+)^2 \equiv \bar{D}_a^a \bar{D}^+_a\), \((D^+)^2 \equiv D^+ a D^+_a\), and the bridge superfield \(v\) is defined as a solution of the equation
\[
V^{++} \equiv -ie^{iv} D^{++} e^{-iv},
\]
where
\[
D^{++} = u^{+i} \frac{\partial}{\partial u^{-i}}; \quad D^{--} = u^{-i} \frac{\partial}{\partial u^{+i}}.
\]
It is important that the superfields \(W^\tau\) and \(\bar{W}^\tau\) depend only on the coordinates of ordinary superspace and are independent of the harmonic variables, \(D^{\pm\pm} W^\tau = 0\).

The action of the pure \(\mathcal{N} = 2\) SYM theory in \(\mathcal{N} = 2\) harmonic superspace has the form
\[
S_{\text{SYM}} = -\frac{1}{32e^0} \text{Re} \text{tr} \int d^4x d^2\theta d^2\bar{\theta} W_r^2 = -\frac{1}{32e^0} \text{Re} \text{tr} \int d^4x d^2\theta d^2\bar{\theta} du W^2
\]
\[
= \frac{1}{16e^0} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \int d^4x d^2\theta du_1 du_2 \ldots du_n \frac{V^{++}(X, u_1) V^{++}(X, u_2) \ldots V^{++}(X, u_n)}{(u_1^+ u_2^+ u_3^+) \ldots (u_n^+ u_1^+)}.
\]
This action is invariant under the gauge transformations
\[
V^{++} \to e^{-i\lambda} V^{++} e^{i\lambda} - ie^{-i\lambda} D^{++} e^{i\lambda},
\]
where \(\lambda\) is a real (with respect to \(\bar{\lambda}\)) analytic superfield. Under these transformations
\[
V^{--} \to e^{-i\lambda} V^{--} e^{i\lambda} - ie^{-i\lambda} D^{--} e^{i\lambda}; \quad W \to e^{-i\lambda} We^{i\lambda}; \quad \bar{W} \to e^{-i\lambda} \bar{W} e^{i\lambda}.
\]

The general renormalizable \(\mathcal{N} = 2\) supersymmetric gauge model consists of the pure Yang–Mills theory and hypermultiplets in a certain representation of the gauge group. In \(\mathcal{N} = 2\) harmonic superspace the hypermultiplets are described by the analytic superfields \(\phi^+\). The action for the hypermultiplet with the mass \(m_0\) can be written as
\[
S_{\text{matter}} = -\int d^4x d^\theta du \bar{\phi}^+ \left( D^{++} + iV^{++} - m_0(\theta^+)^2 + m_0(\bar{\theta}^+)^2 \right) \phi^+.
\]
In spite of the manifest dependence on \(\theta^+\) and \(\bar{\theta}^+\) this action is \(\mathcal{N} = 2\) supersymmetric, because for the massive representation corresponding to the hypermultiplet supersymmetry algebra is modified by the central charge \(Z = m_0\) (see, e.g., [39]).

The action \((10)\) is invariant under the gauge transformations [8] complemented by the transformation of the hypermultiplet superfield
\[
\phi^+ \to e^{-i\lambda} \phi^+; \quad \bar{\phi}^+ \to \bar{\phi}^+ e^{i\lambda}.
\]

### 3 \(\mathcal{N} = 2\) higher covariant derivative regularization

#### 3.1 \(\mathcal{N} = 2\) higher derivative term

Let us considered the general \(\mathcal{N} = 2\) supersymmetric theory described by the action
\[
S = S_{\text{SYM}} + S_{\text{matter}},
\]
\[\text{In the } \lambda\text{-frame this equation can be written as } D^{\pm\pm} W + i[V^{\pm\pm}, W] = 0.\]
where $S_{\text{SYM}}$ is given by Eq. (7) and $S_{\text{matter}}$ is given by Eq. (10), assuming that the analytical superfield $\phi^+$ lies in an arbitrary representation $R$ of the gauge group. In order to introduce the higher covariant derivative regularization we add to the action the $\mathcal{N} = 2$ supersymmetric higher derivative term

$$S_\Lambda = -\frac{1}{128\xi_0^2\Lambda^2}\text{tr} \int d^4x d^8\theta \bar{W}_\tau W_\tau = -\frac{1}{128\xi_0^2\Lambda^2}\text{tr} \int d^4x d^8\theta du \bar{W}W,$$

which is evidently also invariant under the gauge transformations (5). One can show that the expression (13) (up to notation) coincides with the higher derivative term which was obtained in [32] by using the Noether method for $\mathcal{N} = 1$ superfields.

### 3.2 The background field method and the gauge fixing procedure

In the case of using $\mathcal{N} = 2$ harmonic superspace one can fix a gauge without breaking manifest $\mathcal{N} = 2$ supersymmetry. It is convenient to do this using the background field method. In the harmonic superspace it can be formulated as follows [42, 39]. First, we split the analytic gauge superfield $V^{++}$ into the background and quantum parts by making the substitution

$$V^{++} = V^{++} + v^{++}. \quad (14)$$

Then we can fix the gauge without breaking the background gauge invariance

$$V^{++} \to e^{-i\lambda}V^{++} e^{i\lambda} - i e^{-i\lambda}D^{++} e^{i\lambda} ; \quad v^{++} \to e^{-i\lambda}v^{++} e^{i\lambda} ; \quad \phi^+ \to e^{-i\lambda}\phi^+ \quad (15)$$

by inserting into the generating functional

$$1 = \Delta_{\text{FP}} \delta \left( \nabla^{++}v^{++} - f^{(+4)} \right), \quad (16)$$

where the background covariant derivative is given by

$$\nabla^{++}v^{++} \equiv D^{++}v^{++} + i[V^{++}, v^{++}]. \quad (17)$$

It is well known [39] that in this case the Faddeev–Popov determinant $\Delta_{\text{FP}}$ can be presented as a functional integral over the Faddeev–Popov ghosts, which are described by the anticommuting analytical superfields $b$ (antighost) and $c$ (ghost):

$$\Delta_{\text{FP}} = \int DbDc \exp(iS_{\text{FP}}), \quad (18)$$

where the action for the Faddeev–Popov ghosts is given by

$$S_{\text{FP}} = \frac{1}{e_0^2}\text{tr} \int d^4x d^8\theta du b \nabla^{++} \left( \nabla^{++}c + i[v^{++}, c] \right). \quad (19)$$

(The ghost superfields $b$ and $c$ belong to the adjoint representation of the gauge group.) Then it is convenient to integrate over $f^{(+4)}$ taking into account the identity

$$1 = \Delta_{\text{NK}} \int Df^{(+4)} \exp \left( -\frac{i}{32\xi_0^2}\text{tr} \int d^4x d^8\theta du_1 du_2 \right. \nabla^{+}\phi_1 (u_1) e^{-\Omega}\nabla^{++}\phi_2 (u_2) e^{-\Omega}) e^{i\phi_2 (u_2) - i\phi_1 (u_1)} \right) \left( 1 + \frac{32\xi_0^2}{\Lambda^2} f^{(+4)} e^{i\phi_2 (u_2) - i\phi_1 (u_1)} \right). \quad (20)$$

$^4\mathcal{N} = 1$ superfields are defined as lowest components of $W_\tau$ by the equations [40, 41] $W_\tau \equiv 2\sqrt{2}e^{i\Omega}e^{-\Omega}$ and $(\nabla_2)^{\alpha}W_\tau \equiv -4e^{i\Omega}W_\tau e^{-\Omega}$, where the vertical line denotes the conditions $\theta^2 = 0$ and $\bar{\theta}_2 = 0$.
Here the subscripts numerates the harmonic variables, e.g., \( f_1^{(+4)} \equiv f^{(+4)}(X, u_1) \) etc. The superfield

\[
v = v \Big|_{v^{++}=0} \quad (21)
\]
is introduced in order to obtain the expression invariant under the background gauge transformations \((15)\), under which

\[
e^{i\theta} \rightarrow e^{-i\lambda} e^{i\theta} e^{i\tau},
\]

where \( \tau = \tau(x, \theta) \) is independent of the harmonic variables. The bridge superfield \( v \) is related with the background gauge superfields by the equations

\[
V^{++} = -ie^{i\theta} D^{++} e^{-i\theta}, \quad \quad V^{--} = -ie^{i\theta} D^{--} e^{-i\theta}.
\]

Also in Eq. \((20)\) we use the notation

\[
\overset{\sim}{\Box} \equiv -\frac{1}{32} \left( (D^{+})^4 (\nabla^{--})^2 \right)
\]

for the analog of the Laplace operator, which maps analytic superfields into analytic superfields, where the background covariant derivative is given by \( \nabla^{--} \equiv D^{--} + iV^{--} \).

Inserting the expression \((20)\) into the generating function corresponds to adding the gauge fixing action

\[
S_{gf} = -\frac{1}{32\xi_0} \text{tr} \int d^4x d^8\theta \, du_1 du_2 \, e^{-i\theta_1 (u_1 + u_2)} e^{i\theta_1} (u_1 + u_2)^2 e^{-i\theta_1} (1 + \frac{\overset{\sim}{\Box}}{\Lambda^2}) v^{++} (X, u_1) v^{++} (X, u_2), \quad (25)
\]

which is invariant under the background gauge transformations \((15)\) and \((22)\).

Using the equation

\[
-\frac{1}{32} (D^{+})^4 (D^{--})^2 v^{++} = \partial^2 v^{++}
\]

one can verify that the terms quadratic in the quantum superfield \( v^{++} \) (which do not contain the background superfield) can be written as

\[
S_{g1}^{(2)} + S_{g2}^{(2)} + S_{g3}^{(2)} = -\frac{1}{8\xi_0^2} \text{tr} \int d^4x d^4\theta \, du \, v^{++} (X, u) \, \partial^2 \left( 1 + \frac{\partial^2}{\Lambda^2} \right) v^{++} (X, u) + \frac{1}{32\xi_0} \left( 1 - \frac{1}{\xi_0} \right) \text{tr} \int d^4x d^4\theta \, du_1 \, du_2 \, v^{++} (X, u_1) \frac{1}{(u_1 + u_2)^2} \left( 1 + \frac{\partial^2}{\Lambda^2} \right) v^{++} (X, u_2).
\]

In the case \( \xi_0 = 1 \) these terms have the most simple form

\[
-\frac{1}{8\xi_0^2} \text{tr} \int d^4x d^4\theta \, du \, v^{++} \, \partial^2 \left( 1 + \frac{\partial^2}{\Lambda^2} \right) v^{++}.
\]

Following Ref. [39], one can easily calculate the Nielsen–Kallosh determinant \( \Delta_{NK} \). It is given by a product of two contributions, one of which can be presented as an integral over the commuting analytic Nielsen–Kallosh superfield \( \beta \) in the adjoint representation of the gauge group

\[
\Delta_{NK} = \int D\beta \exp(iS_{NK}) \cdot \text{Det}^{1/2}(NK; V^{++}), \quad (29)
\]
where

\[ S_{NK} = \frac{1}{e_0^2} \text{tr} \int d^4 x \, d^4 \theta^+ du \, \nabla^{++} \beta \nabla^{++} \beta. \]  

(30)

The second determinant can be also presented as a functional integral over anticommuting analytic superfields in the adjoint representation \( \gamma^{(+4)} \) and \( \gamma \):

\[
\text{Det}(NK; V^{++}) = \int D\gamma^{(+4)} D\gamma \exp \left\{ \frac{i}{e_0^2} \text{tr} \int d^4 x \, d^4 \theta^+ du \, \gamma^{(+4)} \nabla^2 \left( 1 + \frac{\nabla^2}{\Lambda^2} \right) \gamma \right\},
\]

(31)

but the degree 1/2 does not allow to modify \( S_{NK} \) in such a way to include this contribution.

### 3.3 Degree of divergence and the non-renormalization theorem

In this subsection we will evaluate the superficial degree of divergence for an arbitrary global \( \mathcal{N} = 2 \) supersymmetric gauge theory and prove that any such theory is finite beyond the one-loop approximation. The analysis is based on two properties. First, the effective action is manifestly \( \mathcal{N} = 2 \) supersymmetric. It is stipulated by manifest \( \mathcal{N} = 2 \) supersymmetry of the theory regularized by the \( \mathcal{N} = 2 \) supersymmetric higher covariant derivative regularization\(^5\). Second, the regularized effective action is manifestly gauge invariant. It is stipulated by background field method developed in the previous subsection. Therefore, for evaluating the superficial degree of divergence \( \omega \) we can use manifest \( \mathcal{N} = 2 \) supersymmetry and the manifest gauge invariance. Also we will take into account the discussion of the degree of divergence carried out in \([5]\).

Let us study an arbitrary \( L \)-loop supergraph \((L > 1)\) and set \( m_0 = 0 \), because masses cannot increase the degree of divergence. In the beginning, we will consider the limit \( \Lambda \to \infty \) which corresponds to the non-regularized theory. In this case the momentum integrals do not contain any dimensionful parameters and the degree of divergence can be calculated using dimensional considerations. Calculating the contribution to the effective action of a certain supergraph we obtain the integral over \( d^8 \theta \) and all external momenta. It is easy to see that in the coordinate representation the dimensions of the gauge superfield, the hypermultiplet, and the Faddeev–Popov ghosts are \([V^{++}(x, \theta, u)] = m^0\), \([\phi(x, \theta, u)] = m^1\), and \([b(x, \theta, u)] = [c(x, \theta, u)] = m^1\). Therefore, in the momentum representation \([V(p, \theta, u)] = m^{-4}\) and \([\phi(p, \theta, u)] = [b(p, \theta, u)] = [c(p, \theta, u)] = m^{-3}\). As a consequence, the dimension of the integral over \( d^8 \theta \) and external lines (including the corresponding momentum integrals) is \( m^{(4+N_\phi+N_c)} \), where \( N_\phi \) and \( N_c \) are numbers of the hypermultiplet and ghost external legs, respectively. The dimension of the momentum \( \delta \)-function (which leads to the energy–momentum conservation) is \( m^{-4} \). Moreover, if there are \( N_D \) spinor derivatives acting to the external gauge lines, they give a factor of the dimension \( m^{N_D/2} \).

Taking into account that effective action is dimensionless, we obtain that the dimension of the remaining momentum integral (which is equal to the degree of divergence for the non-regularized theory) is \([5]\)

\[
\omega = -N_\phi - N_c - \frac{1}{2} N_D.
\]

(32)

Now, let us proceed to calculating the degree of divergence for the theory containing the higher derivative term \([13]\). Due to the presence of this term the degree of momentums in the denominator of the gauge propagator is increased by 2. Also the degree of momentums in the purely gauge vertices is increased by 2. Therefore, in the regularized theory the degree of divergence is given by

\[^5\text{Namely, the manifest } \mathcal{N} = 2 \text{ supersymmetry was assumed but not proved in all other regularization schemes.}\]
$$\omega = -N_\phi - N_c - \frac{1}{2} N_D - 2(P - V),$$

(33)

where $V$ is a number of the purely gauge vertices and $P$ is a number of the gauge propagators. If the regularized effective action is manifestly $\mathcal{N} = 2$ supersymmetric and formulated on the base of background field method, the quantity $N_D$, associated with external vector superfield lines, is always positive beyond the one-loop (see discussion of this point in [34]). Evidently, beyond the one-loop approximation we have $P - V > 0$ and, therefore, in this case $\omega < 0$.

In one-loop approximation ($L = 1$) the effective action is given by the functional determinants of the differential operators acting on superfields and requires a separate consideration. If the background field is included into the propagator as in Ref. [5], then the one-loop diagrams do not contain external lines and $N_D = 0$. Therefore, $\omega = 0$. This implies that the divergencies in principle can be present in the one-loop diagrams (including the one-loop subdiagrams in multiloop diagrams). As a consequence, one-loop divergencies cannot be removed by adding the regularizing higher derivative term that is a typical feature of the higher covariant derivative regularization [43]. In order to regularize them by a manifestly $\mathcal{N} = 2$ supersymmetric and gauge invariant way one should introduce into the generating functional the appropriate manifestly $\mathcal{N} = 2$ supersymmetric and gauge invariant Pauli–Villars determinants, as it was first done in [44] for conventional field theory.

### 3.4 Removing one-loop divergences by the Pauli–Villars determinants

In this section we develop the harmonic superspace Pauli-Villars regularization for the one-loop divergences which remain after adding the higher derivative term (13) to the classical action.

In $\mathcal{N} = 2$ harmonic superspace the Pauli–Villars determinants are constructed using the expression for the action of the massive hypermultiplet. Following Ref. [32], for this purpose we introduce the (commuting) analytic Pauli–Villars superfields $\phi^+ (\text{in the adjoint representation of the gauge group})$ and $\phi^+_I (\text{which lies in the same representation as the superfield } \phi^+)$ and construct the Pauli–Villars determinants

$$\text{Det}(PV, M_0; V^{++})^{-1} = \int D\phi^+ D\bar{\phi}^+ \exp(iS_\phi);$$

$$\text{Det}(PV, M_I; V^{++})^{-1} = \int D\phi^+_I D\bar{\phi}^+_I \exp(iS_I),$$

(34)

where the actions for the Pauli–Villars fields are now written as

$$S_\phi = -\frac{2}{\kappa^2} \int d^4x d^4\theta^+ du \bar{\phi}^+ \left( D^{++} \phi^+ + i[V^{++}, \phi^+] - M_0(\theta^+)^2 \phi^+ + M_0(\bar{\theta}^+)^2 \phi^+ \right)$$

$$S_I = -\int d^4x d^4\theta^+ du \bar{\phi}^+_I \left( D^{++} + iV^{++} - M_I(\theta^+)^2 + M_I(\bar{\theta}^+)^2 \right) \phi^+_I.$$

(35)

(It is assumed that the superfield $V^{++}$ is split into the background and quantum parts according to Eq. (14).) The masses of the Pauli–Villars superfields $M_0$ and $M_I$ are proportional to the parameter $\Lambda$ in the higher derivative term, the coefficient of the proportionality being independent of the (bare) coupling constant.

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\*Existence of this property was assumed in [5], however the regularization scheme which provided such a property was not proposed. Here we eliminate this hole in proof of the $\mathcal{N} = 2$ non-renormalization theorem.
In the next section we demonstrate by explicit calculation that inserting the Pauli–Villars determinants (34) leads to regularizing all one-loop divergencies.

Using the Pauli–Villars determinants (34) it is possible to construct the regularized generating functional as

\[
Z = \int Dv^{++} D\bar{\phi}^{+} D\phi^{+} Db Dc D\beta \text{Det}(PV, M_0; V^{++})^{-1} \prod_{I=1}^{\eta} \text{Det}(PV, M_I; V^{++})^{c_I} \\
\times \text{Det}^{1/2}(NK; V^{++}) \exp \left( iS + iS_{\Lambda} + iS_{gf} + iS_{\text{ghosts}} + iS_{\text{sources}} \right),
\]

where \(c_I\) are the coefficients which satisfy the conditions \(\sum_{I=1}^{\eta} c_I = 1\) and \(\sum_{I=1}^{\eta} c_I M_I^2 = 0\). The action \(S\) is a sum of Eqs. (7) and (10), \(S_{\Lambda}\) is the higher derivative term (13), \(S_{gf}\) is the gauge fixing term (25), and \(S_{\text{ghosts}} = S_{FP} + S_{NK}\). The source term \(S_{\text{sources}}\) includes all necessary sources. The effective action is defined by the standard way on the base of \(Z\).

Thus, we obtain the \(\mathcal{N} = 2\) supersymmetric regularization which has never been considered before and hope that it will be useful for various concrete calculations.

4 The exact NSVZ \(\beta\)-function and \(\mathcal{N} = 2\) non-renormalization theorem

The higher derivative regularization constructed in this paper allows to reformulate a statement of the non-renormalization theorem in terms of the NSVZ \(\beta\)-function [45, 32]. The matter is that there are strong evidences that the NSVZ relation is satisfied by the renormalization group functions defined in terms of the bare coupling constant if the higher covariant derivatives are used for the regularization [17, 24, 25, 26, 27]. The \(\mathcal{N} = 2\) supersymmetric theories can be considered as a special case of \(\mathcal{N} = 1\) supersymmetric theories. In particular, for \(\mathcal{N} = 2\) gauge theories the NSVZ \(\beta\)-function gives (see, e.g., [32]).

\[
\beta(\alpha_0) = -\frac{\alpha_0^2}{\pi} \left( C_2 - T(R_0) \right) \left( 1 - \gamma_\phi(\alpha_0) \right),
\]

where \(\gamma_\phi(\alpha_0)\) is the anomalous dimension of the hypermultiplet. If the theory is formulated in terms of \(\mathcal{N} = 1\) superfields it is at least very difficult (if possible) to prove that \(\gamma_\phi = 0\). However, this can be easily done using the regularization constructed in this paper. Really, the diagrams contributing to the anomalous dimension of the hypermultiplet \(\gamma_\phi(\alpha_0)\) have \(\mathcal{N}_\phi = 2\) (see Eq. (32)), and, therefore, are finite. Thus, the anomalous dimension vanishes and the \(\beta\)-function is given by the purely one-loop expression

\[
\beta(\alpha_0) = -\frac{\alpha_0^2}{\pi} \left( C_2 - T(R_0) \right).
\]

Eq. (38) and vanishing of the anomalous dimension \(\gamma_\phi\) imply that \(\mathcal{N} = 2\) supersymmetric gauge theories are finite beyond the one-loop approximation, and the hypermultiplets are not renormalized. For the \(\mathcal{N} = 4\) SYM theory \(R_0 = \text{Adj}\) and \(T(R_0) = T(\text{Adj}) = C_2\). As a consequence, we obtain the known results that the \(\beta\)-function vanishes and the theory is finite in all orders.

5 One-loop quantum corrections
According to the non-renormalization theorem considered in the previous section, the divergences can appear only in the one-loop approximation. Due to the background gauge invariance and renormalizability these divergences are encoded in the renormalization constants, so that the counterterms $\Delta S \equiv S - S_{\text{ren}}$ can be presented in the form

$$\Delta S = -\frac{1}{32\pi^2}\text{Re}\text{ tr} \int d^4x d^2\theta_1 d^2\theta_2 du \left(Z_\alpha \mathcal{W}^2[V^{++} + Z_c v_R^{++}] - \mathcal{W}^2[V^{++} + v_R^{++}]\right)$$

$$\quad + \frac{1}{e^2} \text{tr} \int d^4x d^4\theta^+ du \left((Z_\alpha Z_\alpha - 1)b_R \nabla^{++}\nabla^{++} c_R + i(Z_\alpha Z_\alpha Z_v - 1)b_R [v_R^{++}, c_R]\right)$$

$$\quad - \int d^4x d^4\theta^+ du \left((Z_\phi - 1)\tilde{\phi}_R^{++}\phi_R^{++} + i(Z_\phi Z_v - 1)\tilde{\phi}_R^{++} v_R^{++} \phi_R^{++}\right), \quad (39)$$

where the subscript $\scriptstyle R$ denotes the renormalized fields and $e$ is the renormalized coupling constant. By definition, the sum of $\Delta S$ and the divergent part of the effective action $\Gamma_\infty$ is finite. Thus, the renormalization constants $Z_\alpha, Z_\phi, Z_v, \text{ and } Z_c$ completely define the divergent part of the effective action. In order to find these renormalization constants we can consider only two-point Green functions of the various superfields using the above constructed version of the higher covariant derivative regularization in the harmonic superspace. (For simplicity, here we will consider the massless case $m_0 = 0$ and the gauge $\xi = 1$.)

First, we consider the two-point Green functions of the matter superfields and the Faddeev–Popov ghosts (which are given by the diagrams presented in Fig. 1). We obtained that these diagrams give the vanishing contributions similar to the calculation made in [35]. The only difference is the presence of higher derivatives in the propagator of the quantum gauge superfield. For example, the one-loop contribution to the two-point function of the hypermultiplet superfields is proportional to

$$\int \frac{d^4p}{(2\pi)^4} d^8\theta du \tilde{\phi}_R^{++}(p, \theta, u)C(R)_{ij}D^{-\gamma}\phi_R^{++}(-p, \theta, u) \int \frac{d^4k}{(2\pi)^4} \frac{e^2}{k^4(1 + k^2/\Lambda^2)(k + p)^2} = 0. \quad (40)$$

(In order to derive the last equality we note that the integration measure contains $(D^+)^4$ and $(D^+)^4 D^{-\gamma} \phi^+ = 0$ due to analyticity of $\phi^+$.)

As a consequence, $Z_\phi = 1 + O(\alpha_0^2)$ and $Z_v, Z_\alpha = 1 + O(\alpha_0^2)$. This implies that in the considered approximation the anomalous dimension of the hypermultiplet vanishes, $\gamma_\phi(\alpha_0) = O(\alpha_0^2)$.

![Figure 1: One-loop diagrams which contribute to the two-point Green functions of the matter superfields and the Faddeev–Popov ghosts.](image)

Next, we consider the diagrams which give the one-loop renormalization of the coupling constant $Z_\alpha$. This renormalization constant can be found by calculating the two-point Green function of the background superfield $V^{++}$ (which corresponds to the bold wavy external lines). The corresponding one-loop diagrams are presented in Fig. 2. The result can be written as

$$\frac{d\Gamma^{(2)}_V}{d\ln \Lambda}_{\Lambda \to \infty} = \frac{1}{128\pi^2} \text{tr} \int d^8\theta du_1 du_2 \frac{1}{(u_1^+ u_2^+)^2} \int \frac{d^4p}{(2\pi)^4} V^{++}(-p, \theta, u_1) V^{++}(p, \theta, u_2)$$

$$\times \left(I_{\text{gauge}} + I_{\text{FP}} + I_{\text{NK}} + I_\phi + I_\theta + O(\alpha_0)\right). \quad (41)$$
where the derivative with respect to \( \ln \Lambda \) is calculated at a fixed value of the renormalized coupling constant \( \alpha \).

\( I_{\text{gauge}} \) denotes the contribution of the diagrams containing a loop of the quantum gauge superfield presented in the first column of Fig. 2. We have obtained

\[
I_{\text{gauge}} = 0. \tag{42}
\]

(In order to obtain this result it is necessary to take into account vertices containing higher derivatives which (in the one-loop approximation) cancel higher derivatives in the propagators. Thus, although the result is same as in the case in which the higher derivatives are absent, its derivation is essentially different.)

The second and the third columns in Fig. 2 contain diagrams with a loop of the Faddeev–Popov ghosts and Nielsen–Kallosh ghosts, respectively. Because the Faddeev–Popov ghosts are anticommuting, while the Nielsen–Kallosh ghosts commute, we obtain \( I_{\text{FP}} = -2I_{\text{NK}} \), where we also take into account that the determinant \( \langle 31 \rangle \) gave the vanishing contribution. Both \( I_{\text{FP}} \) and \( I_{\text{NK}} \) are not well-defined, but the well-defined result is obtained after adding the loop of the Pauli–Villars superfield \( \phi^+ \). This contribution is given by the diagram in the fourth column in Fig. 2. Also this diagram gives a contribution of the matter superfield \( \phi^+ \). After calculating the diagrams in Fig. 2 we have obtained

\[
I_{\text{FP}} + I_{\text{NK}} + I_{\phi} = -8\pi C_2 \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln \Lambda} \left( \frac{1}{q^2} - \frac{1}{q^2 + M_0^2} \right) \tag{43}
\]

\[
= 2\pi C_2 \int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q^\nu} \frac{d}{d\ln \Lambda} \left[ \frac{1}{q^2} \left( \ln q^2 - \ln(q^2 + M_0^2) \right) \right] = -\frac{C_2}{\pi};
\]

\[
I_{\phi} = 8\pi T(R) \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln \Lambda} \left( \frac{1}{q^2} - \sum_{I=1}^n c_I \frac{1}{(q^2 + M_I^2)^2} \right)
\]

\[
= -2\pi T(R) \int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q^\nu} \frac{d}{d\ln \Lambda} \left[ \frac{1}{q^2} \left( \ln q^2 - \sum_{I=1}^n c_I \ln(q^2 + M_I^2) \right) \right] = \frac{T(R)}{\pi}. \tag{44}
\]

(Calculating these integrals we take into account that the masses of the Pauli–Villars superfields \( M_0 \) and \( M_I \) are proportional to the parameter \( \Lambda \).) Thus, both these integrals are well-defined integrals of double total derivatives. (This is a typical feature obtained if supersymmetric theories are regularized by higher covariant derivatives, which was first noted in [46, 47].) Substituting the results for these integrals into Eq. \( (41) \) we obtain

\[
Z_\alpha = 1 + \frac{\alpha_0}{\pi} \left( C_2 - T(R_0) \right) \ln \frac{\Lambda}{\mu} + O(\alpha_0^3). \tag{45}
\]
As a consequence, in the considered approximation
\[
\frac{\beta(\alpha_0)}{\alpha_0^2} = -\frac{1}{\alpha_0} \frac{d \ln Z_0}{d \ln \Lambda} = -\frac{1}{\pi} \left( C_2 - T(R_0) \right) + O(\alpha_0). \tag{46}
\]

The renormalization constant $Z_v$ in the one-loop approximation can be found by calculating the diagrams presented in Fig. 3.

Figure 3: One-loop diagrams which give the two-point Green function of the quantum gauge superfield $v^{++}$.

The corresponding contribution to the effective action has the form
\[
\frac{d\Gamma^{(2)}_v}{d \ln \Lambda} \bigg|_{\Lambda \to \infty} = \frac{1}{128\pi} \text{tr} \int d^8\theta \, du_1 du_2 \frac{1}{(u_1^+ u_2^+)^2} \int \frac{d^4 p}{(2\pi)^4} \left. v^{++}(-p, \theta, u_1) v^{++}(p, \theta, u_2) \right| \left. (\tilde{I}_{\text{gauge+FP}} + \tilde{I}_\phi + \tilde{I}_\varphi + O(\alpha_0)) \right. 
\tag{47}
\]

The contributions of the matter and Pauli–Villars superfields coincide with the corresponding contributions to Eq. (41), $\tilde{I}_\phi = I_\phi$ and $\tilde{I}_\varphi = I_\varphi$. The remaining part of the result can be presented in the form
\[
\tilde{I}_{\text{gauge+FP}} + \tilde{I}_\varphi = -8\pi C_2 \int \frac{d^4 q}{(2\pi)^4} \frac{d}{d \ln \Lambda} \left( \frac{1}{q^2} - \frac{1}{(q^2 + M_0^2)^2} \right) = -\frac{C_2}{\pi}. \tag{48}
\]

(Again, the terms containing higher derivatives are present at the intermediate steps of the calculation, but cancel each other in the final result.) Therefore, the overall contributions to the two-point Green functions of the background and quantum gauge superfields are given by the same integrals. This implies that all divergencies are absorbed into the renormalization of the coupling constant and the quantum gauge superfield is not renormalized, $Z_v = 1 + O(\alpha_0^2)$.

Thus, we see that the version of the higher covariant derivative regularization proposed in this paper allows regularizing all one-loop divergencies and subdivergencies. Moreover, using this regularization we have calculated all renormalization constants which encode all divergences of the considered theory.

6 Summary

In this paper we formulate the higher covariant derivative regularization and corresponding background field method for $\mathcal{N} = 2$ supersymmetric gauge theories in the harmonic superspace. This regularization is completely mathematically consistent and does not break the $\mathcal{N} = 2$ supersymmetry and gauge invariance of the theory in calculating the effective action. Using of $\mathcal{N} = 2$ harmonic superspace allows to make the gauge fixing procedure in a manifestly $\mathcal{N} = 2$ supersymmetric way. Due to the background field method the quantum corrections are also invariant under the background gauge transformations. Thus, we construct the procedure which allows to calculate loop quantum contributions to the effective action without loss of manifest $\mathcal{N} = 2$ supersymmetry and gauge invariance. As a result, we justify an assumption in proof of
the $\mathcal{N} = 2$ non-renormalization theorem implied in the previous proof of this theorem. Also we illustrate application of the constructed regularization by the explicit calculation of the one-loop renormalization constants for the general renormalizable $\mathcal{N} = 2$ SYM theory.

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