Star clusters as laboratories for stellar and dynamical evolution

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Open and globular star clusters have served as benchmarks for the study of stellar evolution due to their supposed nature as simple stellar populations of the same age and metallicity. After a brief review of some of the pioneering work that established the importance of imaging stars in these systems, we focus on several recent studies that have challenged our fundamental picture of star clusters. These new studies indicate that star clusters can very well harbour multiple stellar populations, possibly formed through self-enrichment processes from the first-generation stars that evolved through post-main-sequence evolutionary phases. Correctly interpreting stellar evolution in such systems is tied to our understanding of both chemical-enrichment mechanisms, including stellar mass loss along the giant branches, and the dynamical state of the cluster. We illustrate recent imaging, spectroscopic and theoretical studies that have begun to shed new light on the evolutionary processes that occur within star clusters.

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1. An historic look at stellar evolution and star clusters

Understanding stellar evolution has been one of the most important pursuits of observational astronomy over the past century. In 1905, Ejnar Hertzsprung demonstrated that stars emitting light with a similar spectrum, and with the same parallax, could have very different luminosities. Hertzsprung referred to these nearby stars, which therefore had the same temperature and distance, as ‘giants’ (high luminosity) and ‘dwarfs’. Just a few years after this study, Hertzsprung and Henry Norris Russell obtained and analysed new observations that characterized the basic properties of hundreds of nearby field stars (Russell 1913; 1914a) and for the nearest co-moving groups (e.g., the Pleiades and Hyades; see figure 1). This pioneering work led to one of the most important diagnostic tools in astrophysics, the luminosity versus spectral type plane, rightfully named the Hertzsprung–Russell (H–R) diagram.

The first H–R diagrams illustrated clear evidence of groupings and sequences of stars, leading to speculation of the probable order of stellar evolution, as well as some interesting correlations (Russell 1914a,b). For example, based on these observations, Russell remarks that “there appears, from the rather scanty evidence at present available, to be some correlation between mass and luminosity”. Remarkably, these initial observations were rather complete, illustrating a rich main
sequence extending over 10 magnitudes from A-type stars to M dwarfs (including several eclipsing variables), an abundant population of red giants, and the first white dwarf observed (40 Eri B, seen by W. Herschel in 1783).†

Figure 1. The first published Hertzsprung–Russell (H–R) diagrams of the Pleiades and Hyades clusters (left and middle; Hertzsprung 1911), as well as for nearby field stars (right; Russell 1914a). Plotted on the vertical axis are the magnitudes (e.g., more luminous stars are at the top) and on the horizontal axis are the peak wavelengths or spectral types of the stars (e.g., hotter, bluer stars towards the left).

Star clusters were recognized as excellent laboratories to test theories of stellar evolution soon after the first H–R diagrams of the Pleiades and Hyades were published. Over the years, it was quickly realized that each individual cluster is a simple stellar population, being coeval, cospatial and isometalic (some of this knowledge came later) and, therefore, represents a controlled testbed with well-established properties that are common to all member stars (see also Bruzual 2010). Yet, the constituent stars in any given cluster span a large range in stellar mass, the most important parameter dictating stellar evolution as we now know. The present observations of an H–R diagram for any star cluster therefore represents a snapshot of how stellar evolution has shaped the population at a given age and metallicity; the other properties only have a secondary influence. As we link together the observations of many of these clusters, over a wide range in age and metallicity, a complete picture of stellar evolution begins to emerge.

Fortunately, the timing of Hertzsprung and Russell’s first studies of the H–R diagrams of nearby star clusters coincided with the construction of the first large reflecting telescopes. The Mount Wilson Observatory’s 1.5 m (1908) and 2.5 m (1917) facilities and the Dominion Astrophysical Observatory’s 1.8 m (1918) offered the exciting possibility of undertaking the first systematic (resolved) studies of many

† Given the lack of other ‘faint white stars’, Russell initially questioned the quality of the spectrum of this star, which is a member of a triple system (Russell 1914a).
of the clusters in Messier’s famous catalog (Messier 1774). In a series of papers, Harlow Shapley catalogued photometry for hundreds of individual stars in the nearest star clusters (e.g., Shapley 1917; and related articles in the 1910s and 1920s) and provided the first clues as to the properties of these systems. For example, Shapley measured the distances and distributions in space, sizes, luminosities and even proper motions of several nearby star clusters. These photometric studies led directly to the first ideas on stellar physics (e.g., the static main sequence and the mass–luminosity relation; Eddington 1926), well before the source of a star’s energy was known. It was only later, in the 1930s and 1940s, that hydrogen fusion and the proton–proton chain were detailed, leading to much of our current picture of stellar evolution (e.g., Chandrasekhar 1939; Bethe & Marshak 1939; Hayashi 1949; Henyey et al. 1959).

2. Present-day H–R diagrams of open and globular star clusters

Star clusters are now understood to be the end products of star-formation processes such as those we see in regions like the Orion OB association (e.g., Warren & Hesser 1977). The clusters form from the collapse and fragmentation of a turbulent molecular cloud (e.g., Harris & Pudritz 1994; Elmegreen et al. 2000; Bate et al. 2003; see also Clarke 2010; Lada 2010). Modern-day observations of Milky Way star clusters generally group these systems into two classes, the Population I open clusters that have lower total mass (tens to thousands of stars) and are mostly confined to the Galactic disc (Janes & Adler 1982), and Population II globular clusters (tens of thousands to hundreds of thousands or more stars) that are much more massive and make frequent excursions into the Galactic halo (Zinn & West 1984; Harris 1996).

Despite their importance for several aspects of stellar astrophysics, including an understanding of stellar evolution, many rich Milky Way star clusters have been historically neglected from observational studies given their large angular sizes, great distances and/or heavy extinction. For those systems that have been studied, the existing photometry often only extends to $V \sim 20$ mag, limiting scientific studies to just the giants and brighter main-sequence stars of the cluster. The advent of wide-field CCD cameras on 4m-class telescopes has recently provided us with a wealth of new data on the open clusters of our Galaxy. For example, projects such as the WIYN (Mathieu 2000) and CFHT (Kalirai et al. 2001a) Open Star Cluster Surveys have systematically imaged nearby northern-hemisphere clusters in multiple filters down to a depth of $V = 25$ mag, enabling a number of new studies.

Similarly, the Advanced Camera For Surveys (ACS) Survey of Galactic Globular Clusters has provided homogeneous imaging of a large fraction of the Milky Way’s globulars that lacked previous high-resolution observations (Sarajedini et al. 2007). We illustrate the quality of these new data in figure 2 for a sample of five very rich

† Available online at http://messier.obspm.fr/xtra/history/m-cat?1.html
† A database of Galactic open cluster H–R diagrams is available at http://www.univie.ac.at/webda; Mermilliod (1995).
‡ A database of Galactic globular clusters is available at http://venus.mporzio.astro.it/~marco/gc/
Figure 2. (top) H–R diagrams of five rich open star clusters observed as a part of the Canada–France–Hawaii Telescope Open Star Cluster Survey (Kalirai et al. 2001a). The clusters are arranged from the oldest in the top left-hand corner (8 Gyr) to the youngest in the top right-hand corner (200 Myr). (bottom) H–R diagrams of five relatively unexplored globular clusters from the ACS Survey of Galactic Globular Clusters (Sarajedini et al. 2007). Note that the photometry for some of the brightest red giants in these H–R diagrams suffers from saturation effects.

open star clusters (Kalirai et al. 2001b,c, 2003, 2007a, 2008) and five poorly studied globular clusters (Sarajedini et al. 2007).

(a) Inverting H–R diagrams to constrain stellar evolution theory

Interpreting stellar evolutionary processes from observations of star clusters ideally involves comparing the observed H–R diagrams of these systems to synthetic diagrams produced from theoretical stellar evolution models (e.g., Aparicio et al. 1990; Tosi et al. 1991; Skillman & Gallart 2002). The synthetic H–R diagram is constructed based on Monte Carlo extractions of (mass, age) pairs, given a stellar initial mass function, star-formation law and time interval for the star-formation activity. Each extracted synthetic star is placed in the observed H–R diagram by interpolation of the adopted stellar evolution tracks, and is given a photometric error based on the actual measured errors of stars at that brightness. Of course, any star is only retained in the analysis if it actually would have been detected in the observations, as calculated from incompleteness corrections based on artificial-star tests.

For a given set of models, the comparison above can yield tight constraints on
the fundamental properties of the star cluster, such as its age, metallicity, binary fraction and mass. By extending these comparisons to multiple sets of models, e.g., the Padova (Girardi et al. 2002), Yale–Yonsei (Demarque et al. 2004), VandenBerg (VandenBerg et al. 2006) or Dartmouth (Dotter et al. 2008) groups, the different assumptions that are invoked to explain processes that do not come from first principles can be tested. For example, depending on the treatment of core rotation, diffusion, gravitational settling and core overshooting, the location of points in the synthetic H–R diagram will vary and the distribution of points along various evolutionary stages will be unique (see Kalirai & Tosi 2004 for some comparisons).

(b) The present study

There exist excellent reviews on the constraints that evolutionary sequences in H–R diagrams have provided for the theory of how stars evolve. For example, Renzini & Fusi Pecci (1988) highlighted our understanding 20 years ago of the hydrogen-burning main sequence (both for low- and higher-mass stars), subgiant branch, red-giant branch, horizontal branch and asymptotic giant branch, and subsequent evolution on to the white-dwarf cooling sequence (see also Iben 1965, 1967).

In this article, we shift focus to new inferences on stellar evolution that have emerged from some of the most accurate and deepest H–R diagrams constructed to date (of any stellar population). For example, very accurate photometric observations of several globular clusters have now revealed that these stellar systems may not be the simple populations that we have assumed for so many years and, in fact, may contain sets of stars with different initial conditions. Ultradeep imaging of the nearest systems has, for the first time, provided us with a complete inventory of the stellar species in these clusters, extending from the hydrogen-burning limit, through the turnoff, to the brightest giants and down to the faintest cluster remnants. By combining these imaging data with new constraints from spectroscopy, links have been made to directly connect the properties of the main-sequence stars to their eventual white-dwarf state, and therefore to probe stellar mass-loss rates over a wide range of environments. We also discuss the importance of coupling our basic knowledge of stellar evolution, as gauged from observations of star clusters, to the dynamical evolution of these clusters. This includes a look at how the death of stars in these clusters can affect the global properties of the system and the evolution of stars within. Finally, we draw some links on how the knowledge that we gain from these local calibrators can directly influence the interpretation of photometry of galaxies in the distant Universe. We conclude with an encouraging discussion of the exciting possibilities for advancing our understanding of stellar evolution from new surveys that are envisioned in the coming decade.

3. Multiple stellar populations in individual star clusters

For a coeval system, the present-day stars on the main sequence and in post-main-sequence evolutionary phases may be affected by the first-generation evolving stars. For example, the ejecta from the evolution of the most massive stars in the cluster, i.e., the first supernovae, may pollute the atmospheres of neighbouring stars. As we will see in §5 even stars only a few times more massive than the Sun will lose 75% of their mass in just a few hundred million years after they evolve off the main se-
quence. This expelled material, e.g., from the asymptotic giant branch, was subject to nucleosynthesis at the base of the convective envelope and can therefore act as another source of pollution for less-evolved stars in a cluster environment. Indeed, evidence of variations in chemical species among the red giants of nearby globular clusters has existed for over 30 years (e.g., Cohen 1978; Peterson 1980; Norris et al. 1981; Gratton 1982). More recently, high-resolution spectroscopic studies have been extended to unevolved stars with similar results, as summarized in excellent review papers by Kraft (1994) and Gratton et al. (2004). This work suggests that self-enrichment of the intracluster medium and of unevolved stars in globular clusters can produce light elements (e.g., C, N, Na, O, Mg and Al) that are altered in their ratios due to proton-capture reactions, a scenario that is not seen among field halo stars.

The spectroscopic work on chemical abundances discussed above hints that globular clusters may not be as simple and clean as we naturally assume. A remarkable observation by D’Antona et al. (2005) changed the paradigm completely. By using the Wide Field Planetary Camera 2 (WFPC2) aboard the Hubble Space Telescope (HST), D’Antona et al. measured the thickness of the main sequence of the massive cluster NGC 2808 and, for the first time, found photometric evidence of multiple stellar populations on the main sequence. Using techniques similar to those described above, D’Antona et al. carefully characterized their photometric errors on the main sequence and found that 20% of the cluster stars are bluer than expected from synthetic H–R diagrams. They explained the results by invoking a scenario where the helium abundance of these stars was much larger than for the bulk of the main-sequence stars. D’Antona et al. go on to suggest three phases of star formation in the cluster, with different helium enhancement in each phase, from the winds of previous-generation, massive, asymptotic-giant-branch stars (note that the helium yield is a robust prediction from asymptotic-giant-branch evolutionary models; Ventura et al. 2001). Interestingly, enhanced helium abundances will also lead to bluer tails in a cluster’s horizontal-branch morphology, suggesting a possible connection between the main-sequence observations and the already observed dichotomy of NGC 2808’s horizontal branch (see also D’Antona & Caloi 2004). Of course, as has been outlined extensively in the literature, the morphology of a cluster’s horizontal branch may also (or entirely) depend on several other factors such as age, metallicity, mass loss, late flashers and rotation (see Sweigart & Catelan 1998).

Verification of the picture above came from extremely precise HST/ACS imaging observations of NGC 2808 by Piotto et al. (2007), who resolve three main sequences in the cluster for a single turnoff (see figure 3). This remarkable observation is consistent with multiple stellar populations of approximately the same age with varying helium abundances (see inset panel), as outlined initially by D’Antona et al. (2005). Such multiple sequences, including those along the subgiant branch which may imply age variations, have now been observed in the H–R diagrams of several massive globular clusters such as NGC 1851, NGC 6388, NGC 6656, NGC 6715 and 47 Tuc (Milone et al. 2008; Anderson et al. 2009; Piotto 2009). For the most massive Milky Way globular cluster, ω Cen, Bedin et al. (2004) demonstrated multiple turnoffs, subgiant branches and a bifurcation in the main sequence (see also Piotto et al. 2005; Villanova et al. 2007). At younger ages, several LMC clusters have been shown to contain two distinct turnoff branches separated by an age difference of 300 Myr (Mackey et al. 2008). When combined with internal spreads in metal
4. Pushing H–R diagrams to the limit: the complete stellar inventory of a star cluster

Even the most accurate photometric studies of the nearest star clusters have been unable to catalogue stars along the entire main sequence, and rarely probe to the photometric depths required to detect even the brightest of the remnants of stellar evolution for the bulk of all stars, white dwarfs. At very low masses, the mass–luminosity relation for hydrogen-burning stars is very steep, and therefore a small change in stellar mass yields a large change in luminosity. At solar metallicity, a 0.1 $M_\odot$ star has $M_V = 15.9$ mag (Chabrier et al. 2000), giving it a faint optical apparent magnitude of $V > 27$ mag at a distance of just 2 kpc. Compounding the faintness of these stars, the veil of foreground and background objects also increases with photometric depth, making it difficult to isolate a clean lower main sequence in most studies. For the first time, these obstacles have been crossed, leading to the most accurate and complete H–R diagram ever published for a star cluster. This, a 126-orbit HST/ACS integration of the globular cluster NGC 6397 (Richer et al. 2006, 2008), has led to photometry of the lowest-mass hydrogen-burning stars that...
exist in such systems. We describe this remarkable data set here, as it represents a backdrop for several of the studies in the subsequent sections.

We present the H–R diagram of NGC 6397 in figure 4. In the left panel, a tight sequence representing the cluster is clearly delineated from the general field contamination down to $m_{F606W} = 26$ mag. At this point on the main sequence, the Dotter et al. (2008) mass–luminosity relation for the metallicity of NGC 6397, $[\text{Fe/H}] = -2.03 \pm 0.05$ dex (Gratton et al. 2003), indicates a mass of 0.092 M$_\odot$, 10% higher than the expected hydrogen-burning limit at 0.083 M$_\odot$ (see also King et al. 1998). Lower-mass stars on the stellar sequence of NGC 6397 are isolated from the line-of-sight foreground and background stars by proper-motion selection. In the top panel, the astrometry of all stars in the field is measured by comparing these ACS observations to archival WFPC2 data over 60% of the field of view (King et al. 1998). The tight clump in this diagram at $\mu_l \cos(b) = -13.27 \pm 0.04$ mas yr$^{-1}$ and $\mu_b = -11.71 \pm 0.04$ mas yr$^{-1}$ (Kalirai et al. 2007b) represents the NGC 6397 stars, and the corresponding H–R diagram for just these stars is shown in the right-hand panel. The stellar main sequence of the cluster extends another four magnitudes down to $m_{F606W} = 30$ mag. The data are still 75% complete at this limit, below which no stars are detected (see Richer et al. 2006, 2008). Therefore, this represents a detection of the hydrogen-burning limit, a fundamental prediction of stellar evolution theory, at a mass of $M \sim 0.083$ M$_\odot$, based on the Dotter et al. (2008) models, and at a similar mass based on the Baraffe et al. (1998) models.

Above the hydrogen-burning limit, the main sequence illustrated in figure 4 extends 14 magnitudes to the present-day turnoff of the cluster at $m_{F606W} = 16$ mag. In Richer et al. (2008), these data are compared to two independent stellar evolution models to yield valuable insight into the parameter space (e.g., mass ranges) over which the models match the observations versus regions where there is a mismatch. Specifically, these data allow tests of the detailed shape of the main sequence itself, through comparison of observed slope changes along the main sequence to those predicted by the models. For this work, Dotter et al. (2007) recently constructed a new grid of stellar evolution tracks, isochrones, luminosity functions and synthetic horizontal-branch models to compare to such H–R diagrams as this and those in the ACS Globular Cluster Survey (e.g., figure 2). Finally, we note that comparison of stellar models to these clean sequences also yields an estimate of the stellar initial mass function in these environments by relating the present-day observed mass function in specific fields to the global primordial function through dynamical simulations of the cluster (see Richer et al. 2008).

At an age of 12 Gyr, the present-day turnoff mass of a globular cluster such as NGC 6397 is $\sim 0.8$ M$_\odot$. All stars that were more massive than this limit have exhausted their core-hydrogen supply and evolved to post-main-sequence evolutionary phases (e.g., the subgiant and red-giant phases, which can be seen in figure 2). The final state of this evolution for 98% of all stars is white dwarfs. These stars are the direct products of helium burning and so their core composition is C and O. The stars contain no nuclear fuel to sustain fusion reactions. As a white dwarf, a star will simply emit light as it cools and becomes dimmer as time passes.

Star clusters are privileged sites for studying the evolution of main-sequence stars into white dwarfs (see, e.g., the review by Moehler & Bono 2009). Although all stars formed at the same time, the most massive objects exhausted hydrogen rapidly and formed white dwarfs earlier, and thus have now cooled to fainter magnitudes.
Figure 4. The proper-motion (top) and H–R diagram (bottom) of the nearby globular cluster NGC 6397 from a 126-orbit exposure with HST/ACS (Richer et al. 2008). On the main sequence, the proper-motion-cleaned data set (right) extends to the hydrogen-burning limit at $m_{F606W} = 30$ mag (these are the stars within the red circle in the proper-motion diagram). The complete white-dwarf cooling sequence is also characterized at the faint blue end of H–R diagram, extending almost six magnitudes down to $m_{F606W} = 29$ mag (left). These results are discussed in §4.

Given the continuous stream of stars evolving off the main sequence, a white-dwarf cooling sequence will form at the faint blue end of the H–R diagram with hotter, newly formed white dwarfs at the top of the sequence. Just as discussed above for the faintest main-sequence stars, the HST/ACS observations of NGC 6397 shown in figure unveil the complete white-dwarf cooling sequence of the cluster, stretching almost six magnitudes in the H–R diagram. The detection of these stars provides a complete stellar picture of this globular cluster and leads to several interesting findings. For example, modelling of the cooling sequence of the white dwarfs in the cluster yields an extremely accurate age measurement for this globular cluster, $t = 11.47 \pm 0.47$ Gyr (Hansen et al. 2007)

† The faintest white dwarfs are missing from the proper-motion-cleaned H–R diagram in figure as these stars were not detected in the shallower, earlier-epoch WFPC2 observations.

‡ This was recently updated to an age of $\sim 12.21 \pm 0.35$ Gyr with the inclusion of a new
verifies theoretical predictions of the colours of very cool white dwarfs (e.g., the blue hook; see Bergeron et al. 1995; Hansen 1999; Saumon & Jacobson 1999), and the connection between the main sequence and the white-dwarf cooling sequence allows the first probe of the stellar mass function above the present-day turnoff of a globular cluster (e.g., through the initial–final-mass relation; see below and Richer et al. 2008).

5. Post-main-sequence evolution and stellar mass loss

Correctly characterizing the amount of mass loss that stars suffer through post-main-sequence evolution represents one of the most important and fundamental goals of stellar astrophysics. Unlike life on the main sequence, the total post-main-sequence evolutionary lifetime of most stars up to the tip of the asymptotic giant branch constitutes only 5% of the total stellar lifetime, and therefore this represents a very dynamic stage in a star’s life. One very important implication of accurately constraining chemical yields from stars during post-main-sequence evolution has already been discussed, namely, the pollution of helium from winds of massive asymptotic giant branches into the intracluster medium and on to unevolved stars. Even prior to this evolutionary stage, the mass-loss rate for low-mass stars on the first-ascent red-giant branch can drastically affect the eventual fate of an evolving star. For example, the total integrated red-giant-branch mass loss can affect the location from which a star leaves the red-giant branch (e.g., Castellani & Castellani 1993; D’Cruz et al. 1996; Kalirai et al. 2007a) and therefore alters the upper red-giant-branch luminosity function. Equally important, higher rates of mass loss lead to hotter exposed stars, which will occupy a bluer position on the subsequent core-helium-burning horizontal branch (Rood 1973). Extremely high levels of mass loss on the red-giant branch can also lead to stars with very thin hydrogen envelopes that bypass both of these phases and evolve directly to the white-dwarf cooling sequence with helium cores (e.g., Hansen 2005; Kalirai et al. 2007a), or those that experience late flashes (see, e.g., D’Cruz et al. 1996).

Unfortunately, theoretical predictions of post-main-sequence stellar mass loss are difficult to calculate due to insufficient understanding of mass-loss mechanisms on the red-giant branch and at the helium flash, and also to the unknown number of thermal pulses on the asymptotic giant branch (see, e.g., Habing 1996; Weidemann 2000). The mass-loss rates depend on the assumed composition of the dust grains, the dust-to-gas ratio (which likely correlates with metallicity), the expansion velocity of the stellar envelope and the temperature/luminosity of the star (see, e.g., Groenewegen 2006). Given our lack of knowledge of these basic stellar properties, variations in different mass-loss recipes are large (see figure 5), leading to large uncertainties in our basic predictions from stellar evolution models and, therefore, our interpretation of the properties of stars in these phases today.

The eventual fate of most stars after suffering through mass-loss stages on the red- and asymptotic giant branches is the white-dwarf cooling sequence. Therefore, imprinted within the properties of white dwarfs, such as their masses, are the integrated events that their main-sequence progenitors evolved through. A connection
Figure 5. Behaviour of several red-giant-branch mass-loss prescriptions as a function of metallicity for stars with a fixed age of 12 Gyr (see Catelan 2009 for more information and full references). The total mass loss on the branch has been normalized in each model to 0.10 M⊙ at [Fe/H] = −1.54 dex to highlight the different behaviours of these relations with respect to metallicity.

between the final remnant mass and the initial progenitor mass represents a powerful probe of mass loss, and is therefore one of the most important relations in all of stellar astrophysics. In addition, such a relation can only be constructed for white dwarfs that are members of star clusters, as the total stellar lifetime (e.g., the age of the cluster) is known for each remnant. Building this fundamental link requires a step beyond photometric observations. As most white dwarfs are degenerate A-type stars, their spectra exhibit the Balmer sequence of absorption lines, heavily broadened due to the intense pressure on the surface (i.e., a white dwarf is one million times denser than a typical rock on Earth). Measuring these Balmer lines through spectroscopic observations and reproducing them with atmosphere models (e.g., Bergeron et al. 1992) yields the masses and cooling ages of the remnants (i.e., the time since the star left the tip of the asymptotic giant branch), and the difference between this age and the cluster age represents the main-sequence lifetime of the progenitor star up to the tip of the asymptotic giant branch. Therefore, the initial mass of the star becomes known through the mass–main-sequence-lifetime relation for stars of the metallicity of the cluster.

The first efforts to construct this mapping, called the initial–final-mass relation, date back to Weidemann (1977). This was followed by a two-decade-long effort, primarily by D. Reimers and D. Koester (see Koester & Reimers 1981, 1985, 1993, 1996; Reimers & Koester 1982, 1989, 1994; Weidemann & Koester 1983), also including studies by Weidemann (1987, 1997) and Jeffries (1997). A review of the earlier work is provided in Weidemann (2000) and a compilation of more recent results is presented in Ferrario et al. (2005), excluding very recent studies by Dob-
bie et al. (2006, 2009), Williams & Bolte (2007), Williams et al. (2009) and Kalirai et al. (2007a, 2008, 2009a). The synthesis of all of these studies is presented in figure 6, which displays the total integrated mass loss as a function of the initial mass of stars (note, most of the host star clusters of these white dwarfs have solar metallicity). Clearly, the trend indicates that more massive stars will lose a higher fraction of their total mass in post-main-sequence evolution, yet still form more massive white dwarfs. To quantify stellar mass-loss rates, the white-dwarf data suggest that, for the most massive main-sequence stars that will form white dwarfs, the post-main-sequence stellar yield is about 85%, and this decreases smoothly to $\sim 75\%$ for intermediate-mass stars with $3 < M_{\text{initial}} < 4 M_\odot$. A more rapid decline is seen for stars with $M \lesssim 2 M_\odot$. At this mass, stars will lose $\sim 70\%$ of their total mass. However, this decreases down to just $\sim 55\%$ for stars approximately the mass of the Sun.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure6.png}
\caption{Total integrated mass loss through stellar evolution, constrained from connecting white-dwarf mass measurements in open clusters to their progenitor masses (see §5). The solid line represents a fit to the data points.}
\end{figure}

Although the dependence of mass loss on initial mass is well constrained from figure 6, there are as yet few constraints on the dependence on metallicity (e.g., to constrain the theoretical predictions illustrated in figure 5). From measuring infrared excesses from dust in red giants (wind-driven mass loss), Origlia et al. (2007; 2009, personal communication) find no correlation of increased mass loss from $[\text{Fe}/\text{H}] = -2.0$ dex systems to $[\text{Fe}/\text{H}] = -0.7$ dex systems. Kalirai et al. (2009a) also verify this with a comparison of the masses of white dwarfs in the metal-poor globular cluster M4 to a (small) extrapolation of the solar-metallicity initial–final-mass relation to $M_{\text{initial}} = 0.8 M_\odot$. Formally, they find the masses of white dwarfs forming in Population II systems to be $M = 0.53 \pm 0.01 M_\odot$. This suggests that
the processes of stellar mass loss in post-main-sequence evolution will drive away one third of the mass of stars, in excellent agreement with theoretical predictions, which have long estimated that \(0.8 \, M_\odot\) stars should produce \(0.51 < M < 0.55 \, M_\odot\) white dwarfs (Renzini & Fusi Pecci 1988; Renzini et al. 1996). Interestingly, for the \([\text{Fe/H}] = +0.40\) dex metallicity cluster NGC 6791, Kalirai et al. (2007a) measure a mass distribution of white dwarfs peaked at significantly lower masses than expected, suggesting evidence of enhanced mass loss at supersolar metallicities.

The characterization of white-dwarf populations in star clusters with different properties (e.g., metallicity) can clearly influence our understanding of mass-loss rates and the trends of those rates with environment. As we discuss in §7, several new imaging and spectroscopic projects are likely to yield abundant data in this field in the coming decade.

6. Stellar death and the dynamical evolution of clusters

Stars within clusters are born, evolve and then die in some fashion. How they die turns out to be important for the entire cluster. Massive stars may end their lives as black holes and if the cluster is able to retain these objects, the black hole can accrete matter, grow and eventually dominate the cluster dynamics in the core. A more modest star will terminate its existence as a neutron star, which may make its presence known as an X-ray source. A collection of these in the cluster core can also eventually provide an important source of heating for the cluster. As we have already discussed, lower-mass stars terminate their lives as white dwarfs and other than the dynamical effect of them losing up to 90% of their mass (this mass is likely lost from the cluster) and decreasing the overall cluster potential, they have not been suspected of playing a major role in the ongoing cluster dynamics. Recent observations and simulations have changed this picture, however, so that it now appears that white dwarfs are significant players in the dynamical evolution of star clusters. There are a number of important players in this story. Therefore, let us see how each of these contribute to the overall picture. The cast of characters include the cluster binary frequency, the perceived need for intermediate-mass black holes (IMBHs) in some globular clusters and a new scenario which incorporates the discovery that white dwarfs are apparently given a kick during their formation.

(a) The cluster binary frequency

There is a general misconception that the binary frequency in the disc of the Galaxy is high. While it is true that it is elevated (> 50%) among high-mass disc stars, for low-mass stars (like those currently on the main sequence in globular clusters) the frequency is low, i.e., just 30% of M dwarfs in the disc are in binary systems (Lada 2006) and the fraction is even lower in the Galactic halo. This incorrect belief of a high binary frequency among low-mass field stars may have been carried across to globular clusters, where it has been the accepted paradigm for many years that the binary fraction is 20–100%.

An examination of the distribution of core to half-mass radii \((r_c/r_h)\) in globular star clusters indicates that many clusters have large core radii relative to their

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half-mass radii: fully 50% have this ratio $> 0.3$ (see figure 4). So, most clusters do not currently appear to be in a core-collapsed (or binary-burning) phase of evolution. They may still be in the standard initial contraction phase or they may have had their core sizes enhanced through some excess energy source in the cluster. N-body or Monte Carlo models (e.g., Heggie et al. 2006) demonstrate that, with zero binaries, a cluster rapidly goes into core collapse and only with an appreciable primordial binary fraction ($> 10\%$) could core collapse be avoided or delayed for a time comparable to the age of the Universe. So, putting an appreciable binary fraction into the models helps bring the theoretical structural picture of globular clusters closer to the real data, although even with 100% binaries, very large core radii could not be achieved in dynamical simulations. This was demonstrated early on, for example by Vesperini & Chernoff (1994), who studied the dependence of $r_c/r_h$ on binary frequency and showed that this ratio was always $< 0.05 - 0.08$. So, the structure of the bulk of the globular star clusters cannot be explained simply by assuming a large binary fraction: the cluster must either be dynamically young or have some new energy source.

![Figure 7](image-url)

Figure 7. Distribution of the ratio of the core to half-mass radii for globular star clusters. All data were taken from the Harris (1996) catalogue. Note that the majority of clusters have large $r_c/r_h$ ratios, with fully 55% having a ratio in excess of 0.3.

In a recent paper, Davis et al. (2008a) show that beyond the half-mass radius in NGC 6397, for which we have superb HST/ACS data as discussed earlier, the binary frequency is very low, only 1–2%. In fact, a literature search yields that virtually all determinations of binary fractions at large clustercentric radii suggest a low value. This result may be unrelated to the primordial binary frequency, as it is expected that the binaries will sink to the cluster centre as they are generally more massive objects. There is some justification for this as determinations of the
binary frequency in globular cluster cores suggest values at least 3–5 times higher than at large radii.

The situation was recently clarified by Hurley et al. (2007) who analysed a 100,000-object (single stars and binaries) $N$-body simulation of a globular cluster. They show convincingly that the primordial cluster binary frequency was basically preserved outside the cluster half-mass radius and that new binaries were produced in the core, resulting in a higher frequency there. This is an important result, as it strongly suggests that the primordial frequency is very low and that binaries cannot play an important role in delaying or preventing core collapse in most clusters. Further information on the importance of the binary fraction of star clusters is provided in Goodwin (2010).

(b) Intermediate-mass black holes in globular clusters?

With binaries clearly not the solution to large cluster cores, the paradigm shifts to IMBHs. These are black holes in the mass range from roughly 100 to a few times 10,000 $M_\odot$. Such objects could form from a stellar-mass black hole that subsequently accreted material from stars shredded by its tidal field or from runaway mergers of massive stars in the early cluster history. An extension of the supermassive black hole–velocity dispersion relation for galaxy bulges predicts IMHBs of several thousand solar masses for the velocity dispersions typically encountered in the cores of massive globular clusters (a few tens of km s$^{-1}$; e.g., Valluri et al. 2005).

IMBHs act in a way similar to binaries in heating a cluster. For a binary system, an interaction with a third star causes the binary to harden and its total energy to become more negative, while the interloper goes off with higher kinetic energy to conserve energy. An IMBH acts in much the same way, capturing a star in a tight orbit while interactions with other stars eventually harden the orbit and provide excess kinetic energy to the interloping star. $N$-body calculations by numerous authors (e.g., Gill et al. 2008; Trenti et al. 2007) show that large cluster cores could easily be obtained with the presence of an IMBH with a mass of a few percent of that of the total cluster. Some authors have even made statements to the effect that an IMBH was required in the cores of those clusters exhibiting very large core radii.

An observational signature of the presence of an IMBH in a star cluster is an increase in the cluster’s velocity dispersion near the core (e.g., Baumgardt et al. 2005). Observations to search for such a signature are very demanding, as the cluster core has a very high stellar density. This is particularly difficult if the observations are radial-velocity measurements, where there may be enormous amounts of scattered light in the slit of the spectrograph and often multiple stars on the same slit. Several heroic attempts have been made to search for this effect in a few clusters such as M15 (Gerssen et al. 2002; van der Marel et al. 2002), ω Cen (Noyola et al. 2008) and G1 in M31 (Gebhardt et al. 2002, 2005). While none of these cases was rock-solid, they were certainly suggestive of the presence of IMBHs in some clusters.

A few years ago, we began a programme with several students (Andres Ruberg, Ronald Gagne and Saul Davis) to explore proper motions in the cores of globular clusters, mainly as a tool to determine a geometric distance to these systems but also to search for the signature of an IMBH. This is a more efficient approach.
than radial-velocity measurements as the stellar light does not have to be dispersed (avoiding overlap as much as possible), observation time is much less than for spectroscopic measurements and high-spatial-resolution instruments such as \textit{HST} or adaptive-optics imagers on large ground-based telescopes could be used to image the cluster cores. As a trial case, we used the \textit{Gemini North} adaptive-optics near-infrared system ALTAIR/NIRI to image the northern metal-rich globular cluster M71, which has a relatively fluffy core, $r_c/r_h = 0.38$. The observations were of very high quality and we resolved the proper-motion dispersion in the cluster core $(250 \pm 20 \mu\text{arcsec yr}^{-1})$. As we demonstrate in figure 8 the proper motions of stars as a function of their distance from the centre of the cluster (a new determination of the cluster centre was made with recent \textit{HST} data) is flat with no hint of a rise towards the centre (indicating that an IMBH is not present). In a recent superb study of $\omega$ Cen, Anderson & van der Marel (2009) find a similar result using proper motions from \textit{HST} imaging and claim that their observations do not support the conclusions of Noyola \textit{et al.} regarding the presence of an IMBH in this cluster. These two detailed examples certainly weaken the case for IMBHs in globular clusters.

![Figure 8. Proper-motion distribution of stars within about 15'' of the centre of the globular cluster M71. These data are from Gemini North ALTAIR/NIRI images with just under a two-year time baseline between the observations. The vertical line is the sphere of influence $(GM_{BH}/\sigma^2)$ of a 100 M$_\odot$ black hole in the centre of M71 (where $\sigma$ is the stellar velocity dispersion, $M_{BH}$ the black-hole mass and $G$ the gravitational constant). Inside this radius, we would expect to see an increase in the velocities of the stars if there were a black hole of this mass present. Clearly, no such upturn is observed. (These data were part of the MSc thesis of A. Ruberg at the University of British Columbia.)](image)

There are other potential heating mechanisms in globular star clusters which may be operating and are generally not included in the current generation of mod-
els. Among these are the presence of stellar-mass black-hole binaries (Hurley 2007), evaporation of the stellar-mass black-hole population (Mackey et al. 2007), stellar collisions (Chatterjee et al. 2009) and core oscillations (Giersz & Heggie 2009; Heggie & Giersz 2009). However, the observations discussed in the next section suggest a mechanism that selectively puffs up the young white-dwarf distribution (Davis et al. 2008b), implying something specific happening to them when they are born.

(c) A potential new paradigm: kicked white dwarfs

It seems that we require a new paradigm to explain the core dynamics in globular star clusters. Fortunately, one has come forward based on detailed studies of white dwarfs in a few clusters. Davis et al. (2008b) showed that in our single field in NGC 6397 at about 2 $r_h$, the young white dwarfs have a more extended radial distribution than older white dwarfs. ‘Young’ here means younger than about three relaxation times (at the location of the field in the cluster, the relaxation time is $\sim 0.3$ Gyr) while ‘old’ is between 5 and 12 relaxation times. A similar result is also seen in M4 (Davis 2008c). The difference in mass between these two populations is negligible, so the result cannot be due to mass segregation. In any case, the effect actually goes the wrong way for mass segregation. Just before the young white dwarfs were born, they were the most massive visible stars in the cluster, at about 0.8 $M_\odot$, and hence should be centrally concentrated. The fact that they were not implies that something happened to these stars when they were born to fluff up their distribution. Our interpretation is that they were given a small natal kick (of about 3–5 km s$^{-1}$) and they then took several relaxation times to exhibit the radial distribution appropriate to their mass. Monte Carlo simulations seem to back up these ideas (Fregeau et al. 2009). It should be clear here that the only direct evidence for the kick at this time is the more extended distribution of the younger white dwarfs. A more definitive result would be a measurement of the proper-motion dispersion of the two populations (the young white dwarfs are predicted to have a smaller velocity dispersion if they were recently kicked; D. Lynden–Bell 2009, personal communication), but the data currently are not good enough to detect this in these faint stars (the measurement will be made in HST Cycle 17).

How does this explain the large cluster cores? A kicked white dwarf will lose energy to its surrounding stars as it is moving too fast for its mass in equipartition. In this sense, it supplies heat to the cluster, puffing up its core (Heyl 2008b). Monte Carlo models were calculated for clusters with and without white-dwarf kicks (Fregeau et al. 2009) and those where the kick was present typically had larger cores by about a factor of ten and generally did not reach core collapse in a Hubble time. The kick will not, however, be important in all clusters. In the models calculated thus far (Fregeau et al. 2009), those that have velocity dispersions approximately equal to the size of the kick speed can have their cores expanded by the kicks at late times by about an order of magnitude. For those clusters where the velocity dispersion is significantly larger than the kick, no effect should be observed. The younger white dwarfs are, in fact, more centrally concentrated, as expected in the high-velocity-dispersion cluster $\omega$ Cen (Calamida et al. 2008). We have extensive HST/ACS imaging of 47 Tuc due to arrive in Cycle 17 and the prediction is that the young white dwarfs in this high-velocity-dispersion cluster should also be more

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centrally concentrated than the older ones, in sharp contrast to what is seen in NGC 6397 and M4.

A physical mechanism for a white-dwarf kick is not immediately apparent. Some sort of asymmetric mass loss in the late asymptotic-giant-branch or planetary-nebula phases seems most obvious, but a kick at the core-helium flash, if it is off centre, may also be possible (Ivanova 2009, personal communication). To provide a white dwarf with a kick of 4 km s\(^{-1}\) would necessitate an asymmetry in the total amount of mass lost, with typical wind velocities in all post-main-sequence phases of evolution of about 10%. We are currently investigating this observationally by exploring the radial distributions of stars in all evolutionary phases beyond the turnoff, but a theoretical effort here could prove to be quite rewarding.

This kick paradigm is not without its own difficulties. In a cluster where the kicks are likely to be important (velocity dispersion \(\sim\) 5 km s\(^{-1}\), the approximate size of the expected kick), the crossing time in the cluster will be \(\sim\) 1 pc / 5 km s\(^{-1}\) \(\sim\) 2 \(\times\) 10\(^5\) years. If the kick is truly an impulsive event, it should occur on this or shorter timescales. Only the core flash, the second ascent of the asymptotic giant branch or the planetary-nebula phase are this brief, but most of the stellar mass loss after the main-sequence turnoff occurs on the red-giant branch, which lasts about 10\(^8\) years. Hansen (2009) also pointed out that wide binaries containing a white dwarf should have been disrupted by almost any size kick, yet some wide common-proper-motion pairs containing a white dwarf do exist. What is not clear is whether these are true binaries or the remnants of a disrupted cluster, but further investigation of this point is important.

(i) **Confirmation and future directions of white-dwarf-kick research**

There are several tests of the kick hypothesis that can be made in addition to the aforementioned tests of the radial-density and proper-motion distribution of young versus old white dwarfs in clusters with high velocity dispersions. In clusters with very low velocity dispersions, one expects to observe a dearth of white dwarfs as a kick of 3–5 km s\(^{-1}\) can exceed the escape velocity from the system. There are already some claims in the literature that open clusters possess too few white dwarfs (Weidemann 1977; Williams 2002; Fellhauer et al. 2003), but caution should be exercised here, as it is not difficult to hide white dwarfs in binary systems in open clusters (Hurley & Shara 2003). There are a number of globular clusters that possess very low velocity dispersions (e.g., \(\sigma < 1\) km s\(^{-1}\): Pal 5, Pal 13 and Pal 14). However, most are too distant for white-dwarf studies. The nearest systems with low dispersions in which this test is possible are NGC 6366 (\(\sigma \sim 1.3\) km s\(^{-1}\)), NGC 5053 (\(\sigma \sim 1.4\) km s\(^{-1}\)) and NGC 5466 (\(\sigma \sim 1.7\) km s\(^{-1}\)), the nearest of which will have the tip of the white-dwarf cooling sequence at \(V \sim 26\) mag.

Another test of the kick hypothesis is to look for radial anisotropy in the directions of the proper-motion vectors of the young and old white dwarfs (Heyl 2008a). If the young white dwarfs have recently been kicked, we would expect them to be on more radial orbits than their more relaxed elders. Unfortunately, this test was not possible with our current NGC 6397 data, as we lack a second epoch of this field with ACS (WFPC2 was used for the proper-motion work). This will be remedied in Cycle 17 when we reobserve this field with ACS, potentially yielding a clean result.

In the end, the effect of a white-dwarf kick is to convert nuclear energy from
the star into dynamical energy for the cluster. This illustrates the extreme synergy between stellar and dynamical evolution and that to obtain a complete picture of the evolution of a star cluster, one must consider them in detail together.

7. Conclusions and future outlook

The discussion in this paper focuses on the knowledge we have recently gained on the evolution of stars, primarily based on deep imaging and faint spectroscopic studies of star clusters in our Galaxy. Among the topics we have discussed are multiple stellar sequences in clusters, extremely low-mass hydrogen-burning stars, post-main-sequence evolution and stellar mass loss, white dwarfs and the interplay of stellar and dynamical evolution within clusters. As an extension of these primary studies, it is important to keep in mind a key implication of these findings as they relate to general astrophysics.

As early as the first resolved imaging studies of Milky Way clusters, it was understood that these systems are very important tracers of Milky Way structure and formation processes. For example, Harlow Shapley used the distribution of globular clusters on the sky to infer the location of the centre of the Galaxy (Shapley 1918). Today, the comparison of H–R diagrams of Milky Way open and globular clusters to stellar evolution models produces the calibration that commonly feeds population synthesis models, which, in turn, are used to interpret the properties of distant galaxies (e.g., Bruzual & Charlot 2003). At high redshifts, the light that we measure from these unresolved sources comes from the distribution of stars along post-main-sequence evolutionary phases, such as the red-giant branch, horizontal branch and asymptotic giant branch. Constraining stellar evolutionary processes, such as mass loss along the red-giant branch, therefore directly impacts measurements of star-formation histories, metallicities and mass-to-light ratios of these galaxies. As we continue to refine stellar evolution theory with resolved studies of nearby systems, we must re-define our knowledge of galactic astronomy.

Even the most ambitious projects discussed above, such as the ACS Survey of Galactic Globular Clusters, represent a pencil-beam study with respect to surveys that are on the horizon for 2010–2019. For example, Pan-STARRS, SkyMapper and LSST will provide multi-epoch, multifilter, deep homogeneous resolved photometry of most star clusters in our Galaxy. It is especially important that these surveys, unlike the Sloan Digital Sky Survey (sdss), target regions of the Galactic disc to systematically sample open star clusters. In this regard, the first wide-field imaging surveys of the southern hemisphere will image unexplored clusters that surely will fill empty regions of parameter space (e.g., age and metallicity). Kalirai et al. (2009b) present a summary of the type of science that these surveys may enable as related to star clusters.

A survey such as LSST will provide an unimaginable wealth of observational data to test stellar evolution models. With a detection limit of 24–25th magnitude in multiple optical bandpasses in a single visit, and a coadded 5σ depth in the r band of 27.8 mag, LSST will yield accurate turnoff photometry for all star clusters in its survey area out to beyond the edge of the Galaxy. For a 12 Gyr globular cluster, this photometry will extend to over three magnitudes below the main-sequence turnoff at this distance. For low-mass stars, previous surveys such as sdss and 2mass have yielded accurate photometry of faint M dwarfs out to distances of ~ 2 kpc. LSST

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will enable the first detection of such stars to beyond 10 kpc. At this distance, the colour–magnitude relation of hundreds of star clusters will be established and permit a detailed and systematic investigation of variations in the relation with age and metallicity. The present-day mass functions of the youngest clusters will be dynamically unevolved and, therefore, provide for new tests of the variation in the initial mass function as a function of environment, down to very-low-mass stars.

Pushing to the hydrogen-burning limit will also greatly benefit from new infrared observations, for example as proposed in missions such as SASIR (Bloom et al. 2009) and NIRSS (Stern et al. 2009). These projects aim to image the entire sky in the JHK (and possibly L) bands down to 24th magnitude. At an age of 1 Gyr, a $M = 0.08 M_\odot$ star has $M_V = 19$ mag (Baraffe et al. 1998) and will therefore be seen by a survey such as LSST to 500 pc. However, this star has a $V - H$ colour of 8, and is therefore much brighter in the near-infrared. With SASIR or NIRSS, the complete mass function of all star clusters, down to the hydrogen-burning limit, can be characterized out to 2.5 kpc. With the Wide Field Camera 3 (WFC3) imager on HST, and the James Webb Space Telescope, such stars can be easily probed in star clusters at distances of several tens of kpc. These observations will also extend beyond the stellar and substellar threshold and characterize L and T dwarfs. This will make feasible a new age indicator for star clusters: the magnitude difference between the onset of brown dwarfs and the hydrogen-burning limit.

Similar to the wealth of new data that is expected for low-mass hydrogen-burning stars from these surveys, future observations of white dwarfs will allow unprecedented studies. For the field population, a total of 20 000 white dwarfs are currently known (e.g., sdss; Eisenstein et al. 2006). LSST alone will increase the total sample size of white dwarfs in the Milky Way to over 50 million (Kalirai et al. 2009c). The bright tip of the white-dwarf cooling sequence is located at $M_V \sim 11$ mag and will be seen in clusters out to 20 kpc. For a 1 Gyr cluster, the faintest white dwarfs have cooled to $M_V = 13$ mag and will be detected in clusters out to 8 kpc. These white-dwarf cooling sequences not only provide direct age measurements (e.g., Hansen et al. 2007) for the clusters (and therefore fix the primary parameter in the theoretical isochrone fitting, allowing secondary effects to be measured), but can also be followed up with current (Keck, Gemini, VLT and Subaru) and future (e.g., TMT, GMT and/or E–ELT) multi-object spectroscopic instruments to yield the connection between initial and final masses over a wide range of environments. Finally, the synoptic nature of several future missions will permit proper-motion separation of cluster sequences from field-star contamination, permitting cleaner studies of stellar evolution in large samples of these systems.

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