Strong Decays of the Radial Excited States $B(2S)$ and $D(2S)$

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ABSTRACT

The strong OZI allowed decays of the first radial excited states $B(2S)$ and $D(2S)$ are studied in the instantaneous Bethe-Salpeter method, and by using these OZI allowed channels we estimate the full decay widths: $\Gamma_{B^0(2S)} = 24.4$ MeV, $\Gamma_{B^+(2S)} = 23.7$ MeV, $\Gamma_{D^0(2S)} = 11.3$ MeV and $\Gamma_{D^+(2S)} = 11.9$ MeV. We also predict the masses of them: $M_{B^0(2S)} = 5.777$ GeV, $M_{B^+(2S)} = 5.774$ GeV, $M_{D^0(2S)} = 2.390$ GeV and $M_{D^+(2S)} = 2.393$ GeV.

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In the past few years, there are many new states observed in experiments. Among them, the new states $D_{s0}^*(2317)$, $D_{s1}(2460)$ [1], $B_{s1}(5830)$ and $B_{s2}(5840)$ [2] are orbitally excited states, which are also called $P$ wave states. So far, great progress has been made on the physics of orbital excited states $D_{s0}^*(2317)$ and $D_{s1}(2460)$ [3], and there are already exist some investigations of $B_{s1}(5830)$ and $B_{s2}(5840)$ [4]. Around the energy of these hadrons, according to constitute quark model, there may be the radial excited $S$ wave states $B(2S)$ and $D(2S)$. But due to their absence, the experimental and theoretical studies for the radial excited $2S$ states $B(2S)$ and $D(2S)$ are still missing in the literature.

We know that the first radial excited $2S$ state has a node structure in its wave function, which means relativistic correction of $2S$ state is much larger than the one of corresponding basic state, even the $2S$ state is a heavy meson, so to consider the physics of radial excited state a relativistic method is needed. Bethe-Salpeter equation [5] and its instantaneous one, Salpeter equation [6], are famous relativistic methods to describe the dynamics of a bound state. In a previous letter [7], we have solved the full Salpeter equations for pseudoscalar mesons, the masses of first radial excited $2S$ states are obtained, they are $M_{B^0(2S)} = 5.777$ GeV, $M_{B^+(2S)} = 5.774$ GeV, $M_{D^0(2S)} = 2.390$ GeV and $M_{D^+(2S)} = 2.393$ GeV.

The mass of $B(2S)$ is 310 MeV higher than the threshold of mass scale of $B^*\pi$, but lower than the threshold of $B_s^*K$, and the mass of $D(2S)$ is 240 MeV higher than the threshold of mass scale of $D^*\pi$, but lower than the threshold of $D_s^*K$, so the strong decays $B(2S) \to B^* + \pi$ and $D(2S) \to D^* + \pi$ are OZI allowed strong decays, and they are dominate decay channels of $B(2S)$ and $D(2S)$, respectively. In this letter, we calculate the strong decay widths of $B(2S) \to B^* + \pi$ and $D(2S) \to D^* + \pi$ in the framework of Bethe-Salpeter method.

Since one of the final state is $\pi$ meson in the OZI allowed $B(2S)$ or $D(2S)$ strong decay, we use the reduction formula, PCAC relation and low energy theorem, so for the strong decays (considering the $B^0(2S) \to B^{*+}\pi^-$ as an example) shown in Fig. 1, the transition matrix element can be written as [8]:

$$T = \frac{P_{f_2}^\mu}{f_{P_{f_2}}} (B^{*+}(P_{f_1}) | \bar{u} \gamma_\mu \gamma_5 d | B^0(P)), \quad (1)$$

where $P$, $P_{f_1}$ and $P_{f_2}$ are the momenta of the initial state $B^0(2S)$, final states $B^{*+}$ and $\pi^-$, respectively, and $f_{P_{f_2}}$ is the decay constant of $\pi^-$ meson.

To evaluate Eq. (1), we need to calculate the hadron matrix element $\langle B^{*+}(P_{f_1}) | \bar{u} \gamma_\mu \gamma_5 d | B^0(P) \rangle$. It is well known that the Mandelstam formalism [9] is one of proper approaches to compute the hadron matrix elements sandwiched by the Bethe-Salpeter or Salpeter wave functions of two bound-state. With
the help of this method, in leading order, the hadron matrix elements in the center of mass system of initial meson can be written as [8, 10]:

\[
\langle B^{++}(P_f)|\bar{u}\gamma_\mu\gamma_5|B^0_{2S}(P)\rangle = \int \frac{d\vec{q}}{(2\pi)^3} Tr \left[ \tilde{\varphi}^{++}(\vec{q}) \gamma_\mu \gamma_5 \varphi^{++}(\vec{q}) \frac{P}{M} \right].
\]  

where \(\vec{q}\) is the relative three-momentum of the quark-anti-quark in the initial meson \(B^0(2S)\) and \(\vec{q}' = \vec{q} + \frac{m'_i}{m_1 + m_2} \vec{r}\), \(M\) is the mass of \(B^0(2S)\), \(\vec{r}\) is the three dimensional momentum of the final meson \(B^{++}\), \(\varphi^{++}_p\) is the positive energy B.S. wave function for the relevant mesons and \(\varphi^{++}_{p_{f_1}} = \gamma_0(\varphi^{++}_{p_{f_1}})^{\dagger}\).

For the initial state pseudoscalar meson \(B^0(2S)\) \((J^P = 0^-)\), the positive energy wave function takes the general form [7]:

\[
\varphi^{++}_p(\vec{q}) = \frac{M}{2} \left\{ \left[ f_1(\vec{q}) + f_2(\vec{q}) \right] m_1 + m_2 \right\} \frac{\omega_1 + \omega_2}{m_1 + m_2} \frac{P}{M} - \frac{\vec{q}}{m_2 \omega_1 + m_1 \omega_2} \left[ \frac{\vec{q}}{m_2 \omega_1 + m_1 \omega_2} \right] \gamma_5.
\]  

where \(q_\perp = (0, \vec{q})\) and \(\omega_i = \sqrt{m_i^2 + \vec{q}^2}\), \(f_i(\vec{q})\) are eigenvalue wave functions which can be obtained by solving the full \(0^-\) state Salpeter equations. For the final state vector meson \(B^{++}\) \((J^P = 1^-)\), the positive energy wave function takes the general form [11]:

\[
\varphi^{++}_{1+}(\vec{q}) = \frac{1}{2} \left[ A \, \varphi^{\lambda}_1 + B \, \varphi^{\lambda}_1 \right] P_{f_1} + C(\varphi^{\lambda}_2 \varphi^{\lambda}_2 - \varphi^{\lambda}_2 \cdot \epsilon^{\lambda}_2) + D(P_{f_1} \varphi^{\lambda}_2 - P_{f_1} \varphi^{\lambda}_2 \cdot \epsilon^{\lambda}_2)
+ q_\perp \epsilon^{\lambda}_1 (E + F \, P_{f_1} + G \, q_\perp + H \, P_{f_1} q_\perp),
\]  

where \(\epsilon\) is the polarization vector of meson, and \(A, B, C, D, E, F, G, H\) are defined as:

\[
A = M' \left[ f_5(\vec{q}) - f_6(\vec{q}) \right] \frac{\omega_1 + \omega_2}{m_1' + m_2'},
B = \left[ f_6(\vec{q}) - f_5(\vec{q}) \right] \frac{m_1' + m_2'}{\omega_1' + \omega_2'},
\]
where $M'$ is the mass of $B^{*+}$, eigenvalue wave functions $f_i(q')$ can be obtained by solving the full 1$^-$ state Salpeter equations.

In calculation of transition matrix element and solving the full Salpeter equation, there are some parameters have to be fixed, the input parameters are chosen as follows [7]: $m_b = 5.224$ GeV, $m_c = 1.7553$ GeV, $m_d = 0.311$ GeV, $m_u = 0.307$ GeV. The values of the decay constants we use in this letter are $f_{\pi\pm} = 0.1307$ GeV, $f_{\pi^0} = 0.13$ GeV [12]. With the parameters, the masses of the radial excited 2$S$ states are present: $M_{B^{0}(2S)} = 5.777$ GeV, $M_{B^{+}(2S)} = 5.774$ GeV, $M_{D^{0}(2S)} = 2.390$ GeV and $M_{D^{+}(2S)} = 2.393$ GeV. The numerical strong decay widths of $B(2S)$ and $D(2S)$ mesons are shown in Table 1.

In our results only the 1$^-$0$^-$ final states are calculated ($B^*\pi$ and $D^*\pi$), in our estimate of mass spectra, there are no other OZI allowed strong decay channels. For example, from the analysis of quantum number, there may be the decay channels with $P$ wave in the final states, for example, the final state can be 0$^+$0$^-$ $B(1P)\pi$ states, but due to our estimate the mass of lightest $P$ wave 0$^+$ state $m_{B(1P)} = 5.665$ GeV [13], which is larger than the threshold of $B(2S)$ (the same results for $D(2S)$ cases), so there is no phase space for this channel, if later experimental discovery of mass of this state is lower than theoretical estimate like happened to $D_{s0}(2317)$, which has been hoped much higher than 2317 MeV, this channel become a OZI allowed one, but because the phase space is very small, and it is a $P$ wave, the transition decay width should be smaller than the case when it is $S$ wave, so we can ignore the contributions of these channels and other electroweak channels, and we use these OZI allowed decay widths to estimate the full decay width of this $2S$ state.

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Table 1: The strong decay widths of the $2S$ state $B$ and $D$ mesons.

| Mode                  | $\Gamma$ (MeV) | Mode                  | $\Gamma$ (MeV) |
|-----------------------|----------------|-----------------------|----------------|
| $B^0(2S) \rightarrow B^{++}\pi^-$ | 12.3           | $D^{0}(2S) \rightarrow D^{*+}\pi^-$ | 5.48           |
| $B^0(2S) \rightarrow B^{0}\pi^0$   | 12.1           | $D^{0}(2S) \rightarrow D^{*0}\pi^0$ | 5.85           |
| $B^+(2S) \rightarrow B^{0}\pi^+$    | 11.7           | $D^+(2S) \rightarrow D^{*0}\pi^+$   | 6.05           |
| $B^+(2S) \rightarrow B^{++}\pi^0$   | 12.0           | $D^+(2S) \rightarrow D^{++}\pi^0$   | 5.80           |

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References

[1] BABAR Collaboration, B. Aubert et al, Phys. Rev. Lett. 90 (2003) 242001; Belle Collaboration, P. Krokovny et al, Phys. Rev. Lett. 91 (2003) 262002.

[2] CDF Collaboration, T. Aaltonen et al, Phys. Rev. Lett. 100 (2008) 082001; D0 Collaboration, V. M. Abazov et al, arXiv: 0711.0319.

[3] R.N. Cahn, J.D. Jackson, Phys. Rev. D68 (2003) 037502; T. Barnes, F.E. Close, H.J. Lipkin, Phys. Rev. D68 (2003) 054006; E. Beveren, G. Rupp, Phys. Rev. Lett. 91 (2003) 012003; H.-Y. Cheng, W.-S. Hou, Phys. Lett. B566 (2003) 193; W.A. Bardeen, E.J. Eichten, C.T. Hill, Phys. Rev. D68 (2003) 054024; A.P. Szczepaniak, Phys. Lett. B567 (2003) 23; S. Godfrey, Phys. Lett. B568 (2003) 254; P. Clangelo, F. De Fazio, Phys. Lett. B570 (2003) 180; Y.-B. Dai, C.-S. Huang, C. Liu, S.-L. Zhu, Phys. Rev. D68 (2003) 114011; T.E. Browder, S. Pakvasa, A.A. Petrov, Phys. Lett. B578 (2004) 365; E.E. Kolomeitsev, M.F.M. Lutz, Phys. Lett. B582 (2004) 39.

[4] Z.-G. Luo, X.-L. Chen, X. Liu, S.-L. Zhu, Eur. Phys. J. C60 (2009) 403; Z.-G. Luo, X.-L. Chen, X. Liu, Phys. Rev. D79 (2009) 074020; Z.-G. Wang, Chin. Phys. Lett. 25 (2008) 3908; Z.-G. Wang, Phys. Rev. D77 (2008) 054024; X.-H. Zhong, Q. Zhao, Phys. Rev. D78 (2008) 014029.

[5] E.E. Salpeter, H.A. Bethe, Phys. Rev. 84 (1951) 1232.

[6] E.E. Salpeter, Phys. Rev. 87 (1952) 328.

[7] C.S. Kim, G.-L. Wang, Phys. Lett. B584 (2004) 285.

[8] C.-H. Chang, C.S. Kim, G.-L. Wang, Phys. Lett. B623 (2005) 218.

[9] S. Mandelstam, Proc. R. Soc. London 233 (1955) 248.
[10] C.-H. Chang, J.-K. Chen, G.-L. Wang, Commun. Theor. Phys. 46 (2006) 467.

[11] G.-L. Wang, Phys. Lett. B633 (2006) 492.

[12] Particle Data Group, S. Eidelman et al., Phys. Lett. B592 (2004) 1.

[13] G.-L. Wang, Phys. Lett. B650 (2007) 15.