Flavor SU(3) breaking effects in the chiral unitary model for meson-baryon scatterings

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We examine flavor SU(3) breaking effects on meson-baryon scattering amplitudes in the chiral unitary model. It turns out that the SU(3) breaking, which appears in the leading quark mass term in the chiral expansion, can not explain the channel dependence of the subtraction parameters of the model, which are crucial to reproduce the observed scattering amplitudes and resonance properties.

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Properties of baryonic excited states are investigated with great interest both theoretically and experimentally. Recently, the chiral unitary model has been successfully applied to this problem, especially to the first excited states of negative parity ($J^P = 1/2^-$) such as $\Lambda(1405)$ and $N(1520)$. In this method, based on the leading order interactions of the chiral Lagrangian and the unitarization of the S-matrix, the baryon resonances are dynamically generated as quasi-bound states of ground state mesons and baryons. It reveals the importance of chiral dynamics not only in the threshold but also in the resonance energy region.

In the chiral unitary model for the meson-baryon scattering, we consider the coupled channel scatterings of octet mesons and baryons. Imposing the unitarity condition on the scattering amplitudes $T_{ij}$ in the $N/D$ method, we obtain the scattering equation in the matrix form Refs. [3, 8]:

$$T_{ij} = V_{ij} + V_{ik}G_{ik}T_{kj},$$

where $V_{ij}$ denotes the elementary tree level interaction derived from the chiral Lagrangian. This equation can be solved algebraically. The loop integral $G_{i}$ is the fundamental building block in the chiral unitary model and are regularized by the dimensional regularization:

$$G_{i}(\sqrt{s}) = i \int \frac{d^4q}{(2\pi)^4} \frac{2M_i}{(P-q)^2 - M_i^2 + i\epsilon} \frac{1}{s} = \frac{2M_i}{(4\pi)^2} \left[ a_i(\mu) + \ln \frac{M_i^2}{\mu^2} + \frac{m_i^2 - M_i^2 + s}{2s} \ln \frac{m_i^2}{M_i^2} + \frac{\hat{q}_i}{\sqrt{s}} \left[ \ln n_{++} + \ln n_{+-} - \ln n_{-+} - \ln n_{--} \right] \right],$$

with $n_{\pm \pm} = \ln(\pm s \pm (M_i^2 - m_i^2) + 2\sqrt{s}\hat{q}_i)$, the masses of baryon and meson $M_i$ and $m_i$, the three-momentum of the meson $\hat{q}_i$, the total energy in the center of mass system $\sqrt{s}$ and the regularization scale $\mu$. In the present calculation, we follow the method shown in Refs. [3, 8, 4], and calculate only the $s$-wave amplitudes since the contributions from the $p$-wave (and higher partial waves) are much less important in energies considered here [10].

In actual calculations, it is necessary to determine the renormalization constants ($a_i$’s) in Eq.[3] so as to reproduce experimental data. The constants $a_i$ are equivalent to the subtraction constants in the dispersion theory formulation [5] and, in fact, are free parameters of the model. As a consequence, they have depended very strongly on scattering channels, as shown in Table I.

In this work, we investigate whether such channel dependence of the subtraction constants could be explained by the SU(3) breaking terms of the chiral perturbation theory. By doing this, we expect that the free parameters of $a_i$’s could be controlled with suitable physics ground, namely the SU(3) breaking terms, in order to extend this model to various channels with predictive power. Here we keep using just one subtraction constant $\alpha$ from the SU(3) breaking effects in all channels to regularize the loop function $G_{i}$.

The use of only one subtraction constant is justified in the SU(3) symmetric limit [4, 11]. Under the flavor SU(3) symmetry, the scattering amplitude should be expressed as a diagonal matrix in the SU(3) basis $(1, 8, \cdots)$, which is transformed from the particle basis $(\pi N, \eta N, \cdots)$ by a fixed unitary matrix given by the SU(3) Clebsch-Gordan coefficients. Each components of the amplitude $T(D)$ separately satisfies the scattering equation like Eq.[2] in each irreducible representation $D$. Therefore, on one hand, the function $G$ represented in a matrix form becomes a diagonal matrix in the SU(3) basis. On the other hand, since the $G$ function is given as a loop integral as shown in Eq. [2], it is also diagonal in the particle basis. Therefore the subtraction constants $a_i$’s are components of a diagonal matrix both in the SU(3) and particle bases. Such a matrix for the subtraction constants should be proportional to unity. Hence, it is concluded that there is only one subtraction constant $\alpha$ in the SU(3) limit.

Now let us show the Lagrangian with the flavor SU(3)
breaking terms, which we use in the present work. The SU(3) breaking appears as the quark mass terms in the chiral expansion:

\[ \mathcal{L}_{SB} = - \frac{Z_0}{2} \text{Tr} \left( d_m B \left[ \xi m \xi + \xi^i m \xi^i, B \right] \right) \\
+ f_m \overline{B} \left[ \xi m \xi + \xi^i m \xi^i, B \right] \right) \\
- \frac{Z_1}{2} \text{Tr}(\overline{B}B) \text{Tr}(mU + U^\dagger m) \]

where \( f_m + d_m = 1 \). Here we employ the standard notation \[12\]: \( \xi(\Phi) = \exp \left\{ i \Phi / \sqrt{2} f \right\} \) and \( U(\Phi) = \xi^2 \). The 3 \times 3 matrices \( B \) and \( U(\Phi) \) represent the baryon and meson fields. At this stage, we introduce one meson decay constant \( f \), which is taken as an averaged value \( f = 1.15 f_\pi \) with \( f_\pi = 93 \text{ MeV} \). The quark mass matrix is defined as \( m = \text{diag}(\bar{m}, \hat{m}, \tilde{m}) \) with isospin symmetry. \( m_u = m_d \equiv \hat{m} \). The parameters \( Z_0, Z_1, f_m/d_m \) can be determined by the baryon masses and the \( \pi N \) sigma term, and therefore we have no new free parameters. Here we have \( Z_0 = 0.528, Z_1 = 1.56 \) and \( f_m/d_m = -0.31 \) with \( m_u/\hat{m} = 26 \), which are determined in chiral perturbation theory for meson masses. The terms in Eq. \(3\) are of order \( O(p^2) \), based on the Gell-Mann-Oakes-Renner relation \[13\], which implies \( m_q \propto m^2 \). There are other chirally symmetric terms of order \( O(p^2) \). Here we do not take into account these terms, since we concentrate on the effects of the flavor SU(3) breaking.

Let us show the numerical results of the \( K \pi \) induced scatterings. We use a single subtraction constant \( a \), and compare the results with and without the SU(3) breaking terms. In each case, the subtraction constant is determined by fitting threshold branching ratios \[14, 15\]:

\[
\begin{align*}
\gamma &= \frac{\Gamma(K^- p \to \pi^+ \Sigma^-)}{\Gamma(K^- p \to \pi^- \Sigma^+)} \sim 2.36 \pm 0.04, \\
R_c &= \frac{\Gamma(K^- p \to \text{charged particles})}{\Gamma(K^- p \to \text{all})} \sim 0.664 \pm 0.011, \\
R_n &= \frac{\Gamma(K^- p \to \pi^0 \Lambda)}{\Gamma(K^- p \to \text{neutral particles})} \sim 0.189 \pm 0.015.
\end{align*}
\]

Without the symmetry breaking terms, we find the optimal value \( a = -1.96 \) (A). Now including the symmetry breaking term, the optimal value takes \( a = -1.59 \) (B). The calculated threshold values are presented in Table \(\ref{table:threshold}\). From the table, we see that the agreement with data is improved by including the symmetry breaking effect. Note that this improvement is achieved without new free parameters.

Using these optimal values, we calculate the cross sections of \( K^- p \to \) (various channels) and plot them in Fig. \(\ref{fig:cross_sections}\). Results of (A) are shown by dotted lines and those of (B) by dash-dotted lines. For (A), the agreement with data is still good, which is the well known result of the chiral unitary model \[1, 3, 8\]. Originally the subtraction constants in the \( S = -1 \) channel are not very much dependent on the channels and take values around \( a_i \sim -2 \) as shown in Table \(\ref{table:subtraction}\). Now including the symmetry breaking terms (B), we find that agreement with data becomes worse (dash-dotted lines), contrary to our expectation, although the threshold branching ratios are better reproduced.

| Experiment | \( \gamma \) | \( R_c \) | \( R_n \) |
|------------|-------------|------------|------------|
| (A)        | 2.36 \pm 0.04 | 0.664 \pm 0.011 | 0.189 \pm 0.015 |
| (B)        | 2.19        | 0.623      | 0.179      |
| (C)        | 2.35        | 0.626      | 0.172      |

**TABLE II: Threshold branching ratios calculated by using** \( a = -1.96 \) without the SU(3) breaking interaction (A), \( a = -1.59 \) with the SU(3) breaking interaction (B), \( a = -1.68 \) with the SU(3) breaking interaction and the physical \( f \) (C). Experimental values are taken from Refs. \[14, 15\].

**FIG. 1:** Total cross sections of \( K^- p \) scatterings (\( S = -1 \)) as functions of \( P_{lab} \), the three-momentum of initial \( K^- \) in the laboratory frame. Dotted lines show the results with \( a = -1.96 \) without SU(3) breaking (A), dash-dotted lines show the results including the SU(3) breaking with \( a = -1.59 \) (B), and solid lines show the results including the SU(3) breaking and the physical \( f \) with \( a = -1.68 \) (C). Open circles with error bars are experimental data taken from Refs. \[10, 17, 16\].
sonance has been generated at around
mesons and baryons is so strong that an unexpected res-
flavor symmetry. Then the attractive force between the
without the symmetry breaking, since it reproduces the
the
interaction constant
a

ment in the total cross sections of the
meson decay constants does not make drastic improve-
lines. While the inclusion of the SU(3) breaking on the
ratios. The results are shown in Figs.1, 2 with the solid
in this case is
a

are experimental data taken from Ref. [25].
π
π
1
3
√

1380

1440

1200

1400

1600

FIG. 2: Mass distributions of the \( \pi \Sigma \) channel with \( I = 0 \).
Dotted line shows the result with \( a = -1.96 \) (A), dash-dotted
line shows the result including the SU(3) breaking with \( a =
-1.59 \) (B), and solid line shows the result including the SU(3)
breaking and the physical \( f \) with \( a = -1.68 \) (C). Histogram
are experimental data taken from Ref. 27.

In Fig. 2 we show the \( \pi \Sigma \) mass distribution, in or-
der to investigate the \( \Lambda(1405) \) resonance. For (A) we
obtained the dotted curve which agrees well with experi-
mental data. If we include the symmetry breaking terms
(B), once again, the agreement becomes worse as shown
by dash-dotted line. A sharp peak is pronounced around
\( \sqrt{s} = 1420 \) MeV, in obvious contradiction with the ob-
served spectrum.

We also perform calculations with the inclusion of an-
other source of the SU(3) flavor breaking, that is, the
meson decay constants. We use the empirical values of
the decay constants: \( f_\pi = 93 \) MeV, \( f_K = 1.22f_\pi \), \( f_\eta =
1.3f_\pi \). The optimal value of the subtraction constant \( a \)
in this case is \( a = -1.68 \) (C) to reproduce the threshold
ratios. The results are shown in Figs. 1, 2 with the solid
lines. While the inclusion of the SU(3) breaking on the
meson decay constants does not make drastic improve-
ment in the total cross sections of the \( K^-p \) scatterings
as shown in Fig. 1, the shape of the peak in the \( \pi \Sigma \) mass
distribution becomes milder. However, the improvement
is not enough to reproduce the experimental spectra.

We perform similar analyses for the \( \pi N \) scattering for
the \( S = 0 \) channel. At first we use the common subtra-
tion constant \( a = -1.96 \) obtained in the \( S = -1 \) channel
without the symmetry breaking, since it reproduces the
\( \Lambda(1405) \) property well and we want to check the SU(3)
flavor symmetry. Then the attractive force between the
mesons and baryons is so strong that an unexpected res-
onance has been generated at around \( \sqrt{s} \simeq 1250 \) MeV.
Therefore we choose the values of \( a \) for \( S = 0 \) by fit-
ting the \( S_{11} \) scattering amplitudes of the \( \pi N \) channel up
to the energy \( \sqrt{s} \sim 1400 \) MeV. We show the calculated
scattering amplitudes of the \( S_{11} \, \pi N \) channel in Fig. 3
for the following three cases: (A) \( a = 0.53 \) without SU(3)
breaking, (B) \( a = 1.33 \) with SU(3) symmetry breaking
and (C) \( a = 2.24 \) with physical meson decay constants.
In all cases, the scattering amplitudes and cross sections
(we do not show the cross sections here) are not well re-
produced. The results of (A) seems to have some struc-
ture around the \( N(1535) \) energies, but it is far from the
observed amplitude. Reasonable agreement with data is
achieved only when channel dependent subtraction con-
stants are introduced as shown in Table I.

In this work, motivated by the channel dependence
of the parameters and symmetry consideration, we have
tried to reproduce the observed cross sections and the
resonance properties using a single subtraction constant.
In the \( S = -1 \) channel, without the symmetry breaking
terms, \( a = -1.96 \) is determined by the threshold branch-
ing ratios of the \( K^-p \) scatterings. With this parameter
(A), the total cross sections of the \( K^-p \) scatterings are
reproduced well, as well as the mass distribution for
\( \Lambda(1405) \) is. (See Figs. 1 and 2.) This value is closed
to the \( a \sim -2 \) corresponds to \( \Lambda = 630 \) MeV in the three-
momentum cut-off reguralization. The elementary in-
teraction of the \( KN \) system is sufficiently attractive, and
a resummation of the coupled channel interactions gen-
erates the \( \Lambda(1405) \) resonance at the correct position, by
imposing the unitarity condition with the natural value
for the cut-off parameter. Hence the wave function of
\( \Lambda(1405) \) is largely dominated by the \( KN \) component.

On the other hand, in the \( S = 0 \) channel, if one uses
the natural value for the subtraction constant as in the
\( S = -1 \) channel, the attraction of the meson-baryon
interaction becomes so strong that an unexpected res-
onance is generated at around \( \sqrt{s} \simeq 1250 \) MeV. Therefore,
repulsive component is necessary to reproduce the
observed \( \pi N \) scattering. However, with the fitted value
\( a \sim 0.5 \), the \( N(1535) \) resonance is not generated.

From the above observation, we see that the unitarized
amplitudes are very sensitive to the attractive component
of the interaction. Even including the SU(3) breaking
terms, the interaction derived from the chiral Lagrangian
alone do not describe all scattering amplitudes simulta-
neously. Both the fundamental interaction and the sub-
traction constants are important in order to reproduce
proper results. For smaller \( a \), the interaction becomes
more attractive, and for larger \( a \), less attractive. For
\( S = 0 \), we need to choose \( a \sim 0.5 \) in order to suppress

FIG. 3: Real and imaginary parts of the \( S_{11} \) T-matrix am-
plitudes of \( \pi N \rightarrow \pi N \). Dotted lines show the results with
\( a = 0.53 \) (A), dash-dotted lines show the results including
the SU(3) breaking interaction with \( a = 1.33 \) (B), and solid
lines show the results including the SU(3) breaking and the
physical \( f \) with \( a = 2.24 \) (C). Open circles with error bars are
experimental data taken from Refs. 26.
the attraction from the \(\pi N\) interaction in contrast with the natural value \(a \sim -2\) in the \(S = -1\) channel. Therefore, it is not possible to reproduce both the \(\Lambda(1405)\) resonance properties and the low energy \(\pi N\) scattering with a common subtraction constant.

At this point, it is useful to discuss slightly in detail the structure of the \(\Lambda(1405)\) resonance. Although the properties of \(\Lambda(1405)\) have not been reproduced well with the SU(3) breaking terms as shown in the \(\pi \Sigma\) mass distribution (Fig. 2), we still have found the two poles for \(\Lambda(1405)\) in the scattering amplitudes in the second Riemann sheet. The property of the two poles are investigated recently in detail and is related to the SU(3) structure of the meson and baryon states \([11, 27, 28]\). In the present study, we find \(z_1(B) = 1424 + 1.6i\) and \(z_2(B) = 1389 + 135i\) for the parameter (B). The pole \(z_1\), which is located very close to the real axis, is responsible for the sharp peak. When the SU(3) breaking of the meson decay constants is introduced (C), the poles are \(z_1(C) = 1424 + 2.6i\) and \(z_2(C) = 1363 + 87i\), where \(z_1\) is still close to the real axis, while \(z_2\) moves significantly.

The shape of the \(\pi \Sigma\) mass distribution is strongly influenced by the location of the poles. In this case, the poles \(z_2\) is sensitive to the pion decay constant. Since the resonance of \(z_2(B)\) has a strong coupling to the \(\pi \Sigma\) channel \([11]\), the resonance properties are very much affected by the \(\pi \Sigma\) interaction. In the chiral Lagrangian, the interaction is attractive as in the Weinberg-Tomozawa term, which contains a coupling strength proportional to the inverse square of the pion decay constant. Therefore, by changing the decay constant from the SU(3) averaged value (107 MeV, case B) to the physical value (93 MeV, case C), the strength of the attractive \(\pi \Sigma\) interaction is enhanced by \(\sim 30\%\). This shifts the real part of \(z_2\) to the lower side. At the same time this reduces the phase space and hence the imaginary part decreases.

To summarize shortly, we have studied the flavor SU(3) symmetry breaking effect in the meson-baryon scatterings in the chiral unitary model. A reasonable prescription from symmetry consideration by including the SU(3) breaking mass terms, which appear in the next-to-leading order of the chiral expansion, make theoretical predictions worse. So far, except for the use of channel dependent subtraction constants, we do not know what would resolve this problem. In the present framework, the role of the subtraction constants is very important.

A better understanding may be provided by introducing genuine resonance components. Very naively such states could be quark originated as expected from the success of the quark model for baryon resonances. Full coupled channel studies of meson-baryon and quark degrees of freedom would be useful in order to resolve the problem discussed in the present study. Such an analysis will provide more microscopic understanding for the resonance structure.

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[1] N. Kaiser, P. B. Siegel, and W. Weise, Phys. Lett. B362, 23 (1995).
[2] B. Krippa, Phys. Rev. C58, 1333 (1998).
[3] E. Oset and A. Ramos, Nucl. Phys. A635, 99 (1998).
[4] M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002).
[5] E. Oset, A. Ramos, and C. Bennhold, Phys. Lett. B527, 99 (2002).
[6] T. Inoue, E. Oset, and M. J. Vicente Vacas, Phys. Rev. C65, 035204 (2002).
[7] A. Ramos, E. Oset and C. Bennhold, Phys. Rev. Lett. 89, 252001 (2002).
[8] J. A. Oller and U. G. Meissner, Phys. Lett. B500, 263 (2001).
[9] T. Hyodo, S. I. Nam, D. Jido, and A. Hosaka, in preparation.
[10] D. Jido, E. Oset and A. Ramos, Phys. Rev. C 66, 055203 (2002).
[11] D. Jido, J. A. Oller, E. Oset, A. Ramos, and U. G. Meissner, nucl-th/0303062.
[12] For example, J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the standard model* (Cambridge University Press, London, 1992).
[13] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).
[14] R. J. Nowak et al., Nucl. Phys. B139, 61 (1978).
[15] D. N. Tovee et al., Nucl. Phys. B33, 493 (1971).
[16] T. S. Mast, M. Alston-Garnjost, R. O. Bangerter, A. S. Barbaro-Galtieri, F. T. Solmitz, and R. D. Tripp, Phys. Rev. D14, 13 (1976).
[17] J. Ciborowski et al., J. Phys. G8, 13 (1982).
[18] R. O. Bangerter, M. Alston-Garnjost, A. S. Barbaro-Galtieri, T. S. Mast, F. T. Solmitz, and R. D. Tripp, Phys. Rev. D23, 1484 (1981).
[19] T. S. Mast, M. Alston-Garnjost, R. O. Bangerter, A. S. Barbaro-Galtieri, F. T. Solmitz, and R. D. Tripp, Phys. Rev. D11, 3078 (1975).
[20] M. Sakitt, T. B. Day, R. G. Glasser, N. Seeman, J. H. Friedman, W. E. Humphrey, and R. R. Ross, Phys. Rev. 139, B719 (1965).
[21] M. Csejthey-Barth et al., Phys. Lett. 16, 89 (1965).
[22] J. K. Kim, Phys. Rev. Lett. 14, 29 (1965); Columbia University Report, Nevis 149 (1966).
[23] W. Kittel, G. Pitter, and I. Wacek, Phys. Lett. 21, 349 (1966).
[24] H. Göing, Nuovo Cim. 16, 848 (1960).
[25] R. J. Hemingway, Nucl. Phys. B253, 742 (1985).
[26] Center of Nuclear Study, http://gwdac.phys.gwu.edu.
[27] D. Jido, A. Hosaka, J. C. Nacher, E. Oset, and A. Ramos, Phys. Rev. C66, 025203 (2002).
[28] C. Garcia-Recio, J. Nieves, E. Ruiz Arriola, and M. J. Vicente Vacas, hep-ph/0210311.