Contact force control of an active pantograph for high speed trains

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Abstract. In this paper, a mathematical model of the pantograph with flexibility is developed based on experiments, and then an optimal controller together with a sliding observer is proposed to regulate the contact force in the presence of variation with respect to the equivalent stiffness of the catenary system. Furthermore, some physical interpretations of the closed-loop dynamics and pole-zero cancelations are given by analysis from a viewpoint of the output zeroing.

1. Introduction

For high-speed trains, active control of the pantograph is crucial technology to collect electrical current from the overhead contact wire supported by vertical droppers, hangers and cantilevers (catenary system, as shown in Fig.1). The contact force variation can cause contact losses, electric arc formations and sparking. This deteriorates the quality of current collection and increases the electrical related wear, and is one of the limiting factors for the maximum train speed. The increase of the static contact force, which might be considered as a possible solution for this problem, is not an efficient way, because it increases mechanical abrasive wear and produces an excessive uplift of the contact wire. Therefore, maintaining the contact force in an admissible region is crucial for high speed trains and thus modeling and control of active pantograph-catenary systems have been taken much attention from many researchers. So far, some models and controllers for the systems have been proposed. For example, Arnold and Simeon developed a rather rigorous model with PDEs and DAEs and then proposed a numerical solution method [1], Makino et al. developed a wing-shaped low-noise collector and proposed an $H_\infty$ controller with a disturbance observer [2], Yamashita and some of the authors of the present paper et al. developed a low-noise active pantograph, and then applied a PID controller or an impedance control method [3], Chartter et al. proposed a controller based on the back-stepping method together with a high-gain observer [4], Allota, Pisano, et al. proposed higher order sliding mode controllers [5]-[7], Sanchez-Rebollo et al. proposed a hardware-in-the-loop strategy with a PID controller [8].

In order to regulate the contact force, the authors have been developing an active pantograph with a pneumatic actuator, and proposed a sliding mode controller with a rigid frame model [9]. However, it was found through our experiments that the frame had flexibility which could not be ignored to control the contact force. In this paper, a mathematical model of the pantograph with flexibility is developed based on the experiments, and then an optimal controller together with a sliding mode observer is proposed to regulate the contact force in the presence of variation with respect to the equivalent...
stiffness of the catenary system, which can be caused by cantilevers. Furthermore, some physical interpretations of the closed-loop dynamics and pole-zero cancelations are given by analysis from a viewpoint of the output zeroing.

![Catenary](image1.png)

**Figure 1.** Catenary

![Active pantograph](image2.png)

**Figure 2.** Active pantograph

![Pantograph model with flexibility](image3.png)

**Figure 3.** Pantograph model with flexibility
2. Mathematical model and analysis

A schematic of the active pantograph we have been developing is shown in figure 2. Taking account of the flexibility in the frame, a three degree of freedom model is used as shown in figure 3, where $m_s$, $m_t$, $m_{f1}$ and $m_{f2}$ are masses of the overhead wire, the pantograph head, the upper frame and the lower frame, respectively, $f_0$ is static uplift force generated by the main spring in figure 2 (the other variables are defined as shown in table 1). Assuming that the overhead line and the shoe on the pantograph head are connected all the time, and that the origin in the coordinates is the equilibrium point, the equations of motion of the masses are given by

\begin{align}
(m_t + m_s)\ddot{x}_t &= -(c_s + c_x)\dot{x}_t - (k_s + k_x)x_t + c_s\dot{x}_{f1} + k_x x_{f1} + \xi_t \quad (1) \\
m_{f1}\ddot{x}_{f1} &= c_s\dot{x}_t + k_s x_t - (c_s + c_{f1})\dot{x}_{f1} - (k_s + k_{f1})x_{f1} + c_{f1}\dot{x}_{f2} + k_{f1}x_{f2} \quad (2) \\
m_{f2}\ddot{x}_{f2} &= c_{f1}\dot{x}_{f1} + k_{f1}x_{f1} - (c_{f1} + c_{f2})\dot{x}_{f2} - k_{f1}x_{f2} + f_a \quad (3)
\end{align}

where $\xi_t \equiv \Delta k_t x_t$ is the uncertainty/disturbance due to the change of the equivalent stiffness $k_t$ of the catenary system. Taking $\mathbf{x} \equiv \begin{bmatrix} x_t & \dot{x}_t & x_{f1} & \dot{x}_{f1} & x_{f2} & \dot{x}_{f2} \end{bmatrix}^T$ as the state vector, it follows the state equation

\begin{table}
\centering
\caption{Physical parameter}
\begin{tabular}{|l|c|l|c|}
\hline
parameter & value & parameter & value \\
\hline
\text{catenary stiffness} & $k_t$ & 1100N/m & \text{cantenary damping} & $c_t$ & 100Ns/m \\
\text{mass of catenary} & $m_t$ & 100kg & \text{shoe-upper frame stiffness} & $k_s$ & 38000N/m \\
\text{damping} & $c_s$ & 60Ns/m & \text{mass of shoe} & $m_s$ & 2.13kg \\
\text{upper frame-lower frame stiffness} & $k_{f1}$ & 19218N/m & \text{upper frame-lower frame damping} & $c_{f1}$ & 0Ns/m \\
\text{mass of lower frame} & $m_{f2}$ & 10kg & \text{contact force} & $f_a$ & [N] \\
\text{contact force} & $x_{f1}$ & [m] & \text{axis force} & $x_{f1}$ & [m] \\
\text{displacement of contact wire/shoe} & $x_{f2}$ & [m] & \text{displacement of upper frame} & $x_{f2}$ & [m] \\
\text{contact force} & $\xi_t$ & [N] & \text{uncertainty/disturbance due to catenary’s stiffness variation} & $\hat{x}_{f2}$ & [N] \\
\hline
\end{tabular}
\end{table}
Since the contact force includes inertial force of the overhead wire and pantograph head, in order to obtain an expression of the contact force we need the following equations of motion with respect to each mass independently.

\[
m_i \ddot{x}_i = -c_i \dot{x}_i - k_i x_i + f_c - \Delta k x_i
\]

(5)

\[
m_i \ddot{x}_i = -c_i (x_i - \dot{x}_{f1}) - k_i (x_i - x_{f1}) - f_c
\]

(6)

From these equations, it follows that the contact force can be represented by

\[
f_c = \frac{1}{m_i + m_f} [k_i m_i - m_k k_i, c_i m_i - m_k c_i, m_k k_i, m_i c_i, 0, 0] x - \frac{m_i}{m_i + m_f} \xi_i
\]

(7)

\[\triangleq c x + d \xi_i\]

It should be noted that the relative degree between the uncertainty \( \xi_i \) and the contact force \( f_c \) is zero because the uncertainty \( \xi_i \) appears in the output equation (7). With the state equation (4) and output equation (7), the numerator polynomial of the transfer function from the control force to the contact force is given by

\[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-k_i c_i & -c_i & k_i & c_i & 0 & 0 \\
-m_i & m_i & m_i & m_i & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & k_f1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_f1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
x + f_c \\
0 \\
0 \\
1 \\
-m_f2
\end{bmatrix}
\]

(4)

\[
A - sI - b = (m_i s^2 + c_i s + k_i)(c_{f1} s + k_{f1})
\]

(8)

where \( c_{f1} \approx 0 \) from our identification experiments, and thus the zeros of the transfer function are obtained as follows:

\[
s = \frac{-k_i}{c_i}, \quad -c_i \pm \sqrt{c_i^2 - 4m_i k_i}
\]

(9)

It should be noted that two complex zeros of the transfer function are the same as the poles of the nominal catenary subsystem given by (5), and that the relative degree is three. If \( c_{f1} \neq 0 \), the relative degree would be four. On the other hand, the relative degree of the transfer function from the disturbance \( \xi_i \) to the contact force is zero as mentioned above. In general output regulation or disturbance rejection problems, the relative degree and pole-zero cancelation play an important role in
controller design. That is, from the above observations, we can see that it is impossible to reject the disturbance completely in our system, because the relative degree of the transfer function from the control input to the contact force is less than that of the transfer function from the disturbance. Furthermore, in order to reduce the effect of the disturbance on the contact force, some of the closed-loop poles should be assigned in exactly the same location as the catenary poles, yielding pole-zero cancellation. This will be discussed in the simulation section again.

3. Controller design
First, we design an optimal type 1 servo system by full state feedback where the controlled variable is the contact force, and then design a sliding mode observer. Figure 4 shows the block diagram of the whole system.

3.1. Full state feedback control law
In order to apply the LQR technique to the tracking problem, we consider the deviation system from the steady state to the constant reference signal. Defining \( x_e \) and \( u_e \) as the deviation of the state and the control, respectively, and taking \( x_e \triangleq [x_e^T \ u_e]^T \) as the augmented state vector, the augmented deviation system can be represented by

\[
\begin{bmatrix}
\dot{x}_e \\
u_e
\end{bmatrix} = \begin{bmatrix} A & b \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_e \\
 u_e
\end{bmatrix} + \begin{bmatrix} 0 \\
 1 \end{bmatrix} v
\]

(10)

where \( e(t) \) is the tracking error, and \( v \triangleq \dot{u}_e \) which can be seen as the control input. Thus, for this plant, the following state feedback is employed.

\[
v = -[F \ K]E \begin{bmatrix} x_e \\
u_e
\end{bmatrix} \triangleq -F_e \begin{bmatrix} x_e \\
u_e
\end{bmatrix}
\]

(12)

where

\[
E = \begin{bmatrix} A & b \\ c & 0 \end{bmatrix}
\]

(13)

To obtain the optimal regulator gain \( F_e \), the following performance index is used.

\[
J = \int_0^T \{ W e^2(t) + v^2(t) \} dt
\]

(14)

where \( W \) is a weighting factor on the tracking error. Finally, the actual optimal gain is given by

\[
[F \ K] = F_e E^{-1}
\]

(15)
3.2. Sliding mode observer

Taking account of robustness against the uncertainty and the existence of sliding mode, we use two measurements, the velocity of the pantograph head and the displacement of the lower frame, for the observer. To simplify the design procedure, the state vector is redefined as

$$\tilde{x} = \begin{bmatrix} x_t & x_{f_1} & \dot{x}_{f_1} & x_{f_2} & \dot{x}_{f_2} & \dot{x}_t \end{bmatrix}^T$$

(16)

where the last two variables are available for the observer (only change of the order). With this new state vector, the system equation (4) can be rewritten by

$$\dot{\tilde{x}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{k_s}{m_{f_1}} & -\frac{k_s + k_{f_1}}{m_{f_1}} & -\frac{c_s + c_{f_1}}{m_{f_1}} & \frac{k_{f_1}}{m_{f_1}} & \frac{k_{f_1}}{m_{f_1}} & \frac{c_{f_1}}{m_{f_1}} \\ 0 & \frac{k_{f_1}}{m_{f_2}} & \frac{c_{f_1}}{m_{f_2}} & \frac{k_{f_1} + c_{f_2}}{m_{f_2}} & -\frac{k_{f_1}}{m_{f_2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tilde{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} f_a + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \xi_t$$

(17)

$$= \bar{A}\tilde{x} + \bar{b}u + \bar{d}\xi,$$

The output equation with respect to the measurements for the observer design is given by

$$\tilde{y} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\tilde{x} = \bar{C}\tilde{x}$$

(18)
Furthermore, the system matrix is partitioned as follows:

\[
\tilde{A} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{211} & A_{22}
\end{bmatrix}, \quad A_{11} \in \mathbb{R}^{4 \times 4}, \ A_{12} \in \mathbb{R}^{4 \times 2}, \ A_{211} \in \mathbb{R}^{4 \times 4}, \ A_{212} \in \mathbb{R}^{4 \times 4}, \ A_{22} \in \mathbb{R}^{2 \times 2}
\] (19)

The sliding mode observer is designed as [10],[11]

\[
\dot{\hat{x}} = \tilde{A}\hat{x} + \tilde{b}\mu - G_f e_y + G_n v \\
\hat{y} = \tilde{C}\hat{x}
\] (20)

where \( e_y \triangleq \hat{y} - \hat{y} \), \( v \triangleq -\rho \frac{e_y}{\|e_y\|} \), \( G_f \in \mathbb{R}^{6 \times 2} \), \( G_n \in \mathbb{R}^{6 \times 2} \)

The design parameter, \( G_n \), can be parameterized as

\[
G_n = \begin{bmatrix}
\begin{bmatrix}
0 & L_0 \\
L_0 & 0
\end{bmatrix}
\end{bmatrix}^{-1}, \text{ where } L = \begin{bmatrix}
L_0 & 0
\end{bmatrix}, \quad L_0 \in \mathbb{R}^{4 \times 1}, \quad P_0 \in \mathbb{R}^{2 \times 2}
\] (21)

During sliding mode, the estimation error dynamics is governed by

\[
\dot{e}_1 = (A_{11} + L_0 A_{211})e_1, \quad \text{where } e \triangleq \hat{x} - \hat{x} = \begin{bmatrix}
e_1 \\
e_y
\end{bmatrix}
\] (22)

which is insensitive to the uncertainty, \( \xi_j \). As seen from this equation, \( L_0 \) is determined to the sliding mode. The other design parameters should be determined to guarantee the existence of the sliding mode.

4. Numerical simulation

In order to investigate the performance of the proposed controller, numerical simulation has been carried out. The parameter values of the pantograph-catenary system are shown in Table 1, where the parameter values of the pantograph are the estimates by identification experiments, and those of the catenary system are determined based on some references, e.g. Kobayashi et al. [12].

When the train speed is 360 km/h and the span length of each cantilever is about 50 m, the catenary equivalent stiffness can be assumed to change periodically with 2 Hz as shown in figure 5 [13]. The design parameters in the controller were determined taking account of this situation. For example, when the weighting factor \( W = 10^7 \), the poles and zeros in the transfer function from the reference signal to the contact force are obtained as follows:

\[
\text{poles} = \begin{bmatrix}
-34.29 \pm 142.25i \\
-90.79 \pm 85.56i \\
-0.50 \pm 3.28i \\
-113.10
\end{bmatrix} \\
\text{zeros} = \begin{bmatrix}
-7.53 \pm 182.11i \\
-107.20 \pm 140.63i \\
-0.50 \pm 3.28i \\
-160.40
\end{bmatrix}
\]
It should be noted that a pair of poles approximately cancel out a pair of zeros at $-0.50 \pm 3.28i$ which is the mode of the catenary subsystem given by equation (5). As mentioned in section 2, perfect output zeroing can be achieved by exact pole-zero cancellations which make an unobservable subspace in the state space. Therefore, it is natural that the approximate pole-zero cancellation is brought about in the present study, although the LQR technique has been employed. From a physical point of view, in addition, these closed-loop poles will make it possible that the pantograph head is following the catenary wire motion not to prevent its free motion, yielding a good regulation of the contact force.

Assuming that the uncertainty $\xi$ is an independent external disturbance, the frequency response from the disturbance $\xi$ to the contact force $f_c$ can be obtained as shown in figure 6. It can be expected from this figure that the active pantograph probably achieves the regulation of the contact force better than the passive one, because the active gain is lower than the passive one in almost all frequencies especially at the disturbance frequency ($4\pi\text{rad/s}$) without any resonances (much more stable).

In the rest of simulation, the reference signal is given as follows:

$$r(t) = \begin{cases} 
54 & \text{for } 0 \leq t < 11 \\
64 & \text{thereafter}
\end{cases}$$

to make sure of the transient response.

**Figure 5.** Perturbation with respect to the equivalent stiffness of catenary due to cantilevers

**Figure 6.** Bode diagram of the transfer function from the disturbance to the contact force
Figure 7 shows the estimation errors, $e_y$, used in the observer (20), where the true state variables are used for control to evaluate only the observer performance. It can be seen from the chattering around the origin as shown in figure 7(b) that the quasi sliding mode takes place, and that the estimation accuracy is better than that of the linear optimal observer shown in figure 7(a).

Although we neglected the actuator dynamics when designing the controller, the rest of the simulation results were obtained using the actuator model as follows:

$$G(s) = \frac{1}{0.013s + 1} e^{-0.002s}$$

which was obtained by some experiments. Furthermore, it is assumed that the measurement of the contact force with a load cell is corrupted by a band limited white noise. It can be seen from figure 8 that the quasi sliding mode takes place in spite of the existence of the actuator dynamics. Figure 9 shows the contact force in the steady state in comparison with the passive case where the static uplift force is provided. It is clear that the active pantograph achieves much better regulation performance than the passive one.

Figure 10 shows the step response of the contact force, from which it can be seen that the transient response is also good.
Figure 10. Step response of the contact force (Active)

5. Conclusions

We have developed the mathematical model and the robust feedback controller based on linear optimal control and sliding mode observer, taking account of the flexibility of the articulated frame in our pantograph. It has been emphasized that one of the key points to regulate the contact force is pole-zero cancellation in the nominal model without perturbation. A physical interpretation of this pole-zero cancellation is also given, that is, the pantograph head can follow the catenary motion not to prevent its free motion by assigning some of the closed loop poles on the catenary mode.

Because it has also been found out through many simulation results that the observer plays a key role in the control performance, we are still under investigation on which physical variables to use for the observer and how to measure them precisely in severe environments.

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