Discrete-Time Accuracy Analysis of the Time-Domain Regular Perturbation Model for Unamplified Links

Citation for published version (APA):
Barreiro, A., Liga, G., Fehenberger, T., & Alvarado, A. (2021). Discrete-Time Accuracy Analysis of the Time-Domain Regular Perturbation Model for Unamplified Links. arXiv, 2021, [2106.05088].
https://arxiv.org/abs/2106.05088

Document license:
CC BY

Document status and date:
Published: 10/06/2021

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain.
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.
Discrete-Time Accuracy Analysis of the Time-Domain Regular Perturbation Model for Unamplified Links

Astrid Barreiro*, Student Member, IEEE Gabriele Liga*, Member, IEEE Tobias Fehenberger† Member, IEEE, and Alex Alvarado* Member, IEEE

*Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands
†ADVA, 82152 Martinsried/Munich, Germany

Abstract—The accuracy of a discrete-time channel model based on regular perturbation is numerically studied for unamplified links. We analyse the distance between discrete nonlinear interference points and show that such distance can be used to estimate the effective channel memory.

I. INTRODUCTION

G OOD channel models are key to establish the mathematical framework for a better understanding of nonlinear phenomena and to effectively design fibre-optic communication systems [1]. The nonlinear Schrödinger equation (NLSE), which mathematically describes the propagation of signals in an optical fibre, is the origin for the derivation of most analytical models of nonlinear interference (NLI). In particular, perturbation theory has been broadly employed as the main mathematical method to approximate the NLSE solution and find relatively simplified analytical expressions for the NLI [2], [5], [6].

One of the most popular approximations is based on a regular perturbation (RP) expansion on the nonlinear parameter $\gamma$ and truncated to the first-order term. Considerable amount of validation work has been carried out on a waveform level [4], [2], [5], [6] for this first-order RP. However, the question of how this accuracy translates to the discrete-time domain has been, to the best of our knowledge, not well addressed in the literature.

In this paper, we numerically study the accuracy of the first-order RP on $\gamma$ with respect to received symbols observed after chromatic dispersion compensation, matched filtering, and sampling. We consider unamplified links whose study is important for its application to island-to-island communication systems, intra and inter-data center interconnect, and the fact such links are building block for more complex (multi-span) systems.

We assess the convergence of the first-order RP on $\gamma$ in terms of the model’s memory, i.e., on the number of interfering symbols that need to be considered around the symbol of interest to achieve the highest model’s accuracy. The model’s accuracy is first estimated in terms of predicted NLI variance, a metric often used in the literature. The accuracy of the model is also estimated using the pairwise distance for each NLI point. This pairwise metric enables a comprehensive assessment of the model’s ability to capture the true NLI distribution.

II. SYSTEM MODEL

To assess the accuracy of a perturbative discrete-time channel model, we consider a single channel, single polarization system shown in Fig. 1. The complex transmitted symbols $a = \ldots, a_{-1}, a_0, a_1, \ldots$ are converted into an optical field $A(t, 0)$ using linear modulation. Propagation through the optical fiber is performed via the split step Fourier method (SSFM), which we consider the true channel. At the receiver, chromatic dispersion compensation (CDC) is applied to the received field $A(t, L)$. Matched filter and sampling at the symbol rate is used to obtain the vector of symbols $\hat{y} = \ldots, y_{-1}, y_0, y_1, \ldots$, where $y_0$ represents the received symbol at time instant 0.

The first-order RP on the nonlinear parameter $\gamma$ approximates the exact solution of the NLSE and leads to a discrete-time channel model given by $\hat{y}_0 \approx y_0$, where

$$\hat{y}_0 \approx a_0 + \Delta a_0, \quad (1)$$

with

$$\Delta a_0 = \gamma G \sum_{k_1=-N}^{N} \sum_{k_2=-N}^{N} \sum_{k_3=-N}^{N} a_{k_1} a_{k_2} a_{k_3} S_{k_1, k_2, k_3} \quad (2)$$

and where $k_1, k_2, k_3$ are time indices, and $S_{k_1, k_2, k_3}$ are complex coefficients that model self-phase modulation [6]. In (2), $G$ is a scaling factor to control the transmit power $P$. As shown in [2], we focus on a finite memory model, which converges to the first order perturbative model when $N \to \infty$. The value of $N$ is referred as the model memory, where $2N + 1$ is the number of interfering symbols.

The methodology we propose to evaluate the accuracy of the model is shown in Fig. 1. The channel model in (2) is...
Fig. 2: NLI variance vs. model memory $N$ for QPSK (a) and 1-3-APSK (b). The filled markers represent the model memory estimated via the NLI variance of the RP model on $\gamma$.

TABLE I: Fiber parameters

| Parameter                  | Value                      |
|----------------------------|----------------------------|
| Nonlinearity parameter $\gamma$ | $1.2 \text{ W}^{-1} \text{km}^{-1}$ |
| Fiber attenuation $\alpha$     | $0.2 \text{ dB/km}$        |
| Group velocity dispersion $\beta_2$ | $-21.7 \text{ ps}^2/\text{km}$ |

used as a parallel response function that based on the channel input $a$, estimates symbol-by-symbol the real channel output $y$. The vectors $\mathbf{y}$ and $\hat{\mathbf{y}}$ are used to compute the NLI variances $\text{Var}[Y_0 - A_0]$ and $\text{Var}[\hat{Y}_0 - A_0] = \text{Var}[\Delta A_0]$, which follows from (1) and (2).

In addition, we propose to study the random variable

$$W(a) \triangleq |Y_0(a) - \hat{Y}_0(a)|, \quad (3)$$

where the dependency on the transmitted symbol sequence $a$ is made explicit to emphasize that $Y_0$ and $\hat{Y}_0$ are obtained using the exact transmitted sequence $a$. In the next section we study the distribution of the random variable $W(a)$ as well as its root mean square error.

III. NUMERICAL RESULTS

The system under study is a 32 Gbaud single-channel transmission with root-raised cosine pulse shaping (10% roll-off) over a single span of standard single-mode fiber, with parameters summarized in Table I.

Fig. 2(a) shows the NLI variances for $L = 200$ km as a function of the model’s memory size $N$ for three different transmitted powers ($P = 2, 5, 8$ dBm, with $P = 5$ dBm being optimum launch power) and for a quaternary phase shift keying (QPSK) modulation format. We observe that the variance predicted by the model in (1)–(2) saturates at $N = 5$ symbols for all transmitted powers (considering a convergence > 95% of the asymptotic variance value). This indicates no significant power dependence of the channel memory in the considered power interval, which combines both linear and low non-linear regimes. The variance predicted by the model for large values of $N$ converges to the variance estimated using SSFM simulations (dashed lines). In Fig. 2(b), the NLI variance is analysed for a 1-3 amplitude-phase shift keying (APSK) format on a per-ring basis and for the three above mentioned transmitted powers. In 1-3-APSK, one point is at the origin (i.e., the first ring $R_1 = 0$) and the other 3 points are equally spaced on a ring $R_2$. Fig. 2(b) shows that the model predicts well different values of the variance for the two constellation rings, where the variance of $R_1$ is considerably lower than the variance for $R_2$. This is particularly evident from the insets in Fig. 2(a), specially for $P = 8$ dBm. Moreover, the saturation of the variance for the two rings is achieved at different memory values (see filled circles). These results indicate that the channel memory (in a NLI variance sense) is input-dependent.

To extend the picture given by the NLI variance, we analyse the random variable in (3). In particular, we study the mean value, which corresponds to the root mean squared error (RMSE) between the received symbols in (1) and the corresponding SSFM estimates, i.e., $\text{RMSE} = \mathbb{E}[W]$. The RMSE is able to capture the proximity between the NLI deviations predicted by the model and the corresponding values obtained via the SSFM, thus providing a more comprehensive picture of the model’s convergence to the true NLI distribution. Fig. 3 shows the RMSE as a function of the model’s memory $N$, for QPSK (a) and 1-3 APSK (b). Fig. 3(a) shows that for all powers the RMSE saturates at a value of $N = 5$, confirming the results obtained via a variance-based memory estimation. This is also confirmed by the histograms of $W$ (insets in Fig. 3) which are similar for all powers (up to a rescaling of their $x$-axis). The asymptotic RMSE decreases as the transmitted power is decreased, which indicates that the
The work of A. Barreiro and A. Alvarado has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (grant agreement No 757791). The work of G. Liga is funded by the EuroTechPostdoc programme under the European Union’s Horizon 2020 research and innovation programme (Marie Skłodowska-Curie grant agreement No. 754462).

REFERENCES

[1] A. Bononi, R. Dar, M. Secondini, P. Serena, and P. Poggiolini, “Fiber Nonlinearity and Optical System Performance,” 2020, pp. 287–351. [Online]. Available: http://link.springer.com/10.1007/978-3-030-16250-4_19
[2] A. Vannucci, P. Serena, and A. Bononi, “The RP method: A new tool for the iterative solution of the linear Schrödinger equation,” Journal of Lightwave Technology, 2002.
[3] E. Forestieri and M. Secondini, “Solving the nonlinear Schrödinger equation,” Optical Communication Theory and Techniques, no. 1, pp. 3–11, 2005.
[4] V. Oliari, E. Agrell, and A. Alvarado, “Regular perturbation on the group-velocity dispersion parameter for nonlinear fibre-optical communications,” Nature Communications, vol. 11, no. 1, pp. 1–11, 2020. [Online]. Available: [http://dx.doi.org/10.1038/s41467-020-14503-w]
[5] M. Secondini and E. Forestieri, “Analytical fiber-optic channel model in the presence of cross-phase modulation,” IEEE Photonics Technology Letters, vol. 24, no. 22, pp. 2016–2019, 2012.
[6] R. Dar, M. Feder, A. Mecozzi, and M. Shitaif, “Properties of nonlinear noise in long, dispersion-uncompensated fiber links,” Opt. Express, vol. 21, no. 22, pp. 25685–25699, nov 2013.