Asymmetric Dual-Mode Constellation and Protograph LDPC Code Design for Generalized Spatial MPPM Systems

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Abstract—To achieve reliable and efficient transmissions in free-space optical (FSO) communication, this paper designs a new protograph low-density parity-check (PLDPC) coded generalized spatial multipulse position modulation (GSMPPM) system over weak turbulence channels. Specifically, we investigate an ADM constellation with the size of a power of two can be generated. The proposed ADM constellations are capable of satisfying the high-reliability requirement for FSO applications. 

Index Terms—Generalized space shift keying, multipulse pulse-position modulation, protograph low-density parity-check codes, weak turbulence channel, free-space optical communication.

I. INTRODUCTION

FREE-SPACE optical (FSO) communication has been widely used for ground-to-satellite communication systems due to the high transmission rate and high reliability [1], [2], [3]. In practical applications, due to the effect of the atmospheric turbulence (also known as scintillation) is generated in FSO communications. The atmospheric turbulence caused random fluctuations of the amplitude and phase for the received signal at the receiver, which seriously deteriorates the system performance [6], [7]. To quantify the effect of atmospheric turbulence, several mathematical models have been proposed to describe the distribution of turbulence fading in FSO communication systems. For example, the lognormal distribution has been used to describe the weak turbulence channel in FSO communication systems [8], [9].

The weak turbulence channel is one of the most typical channel models to describe FSO communication scenarios, which have attracted significant research interest in the past two decades [10], [11], [12], [13]. In the FSO communication systems based on weak turbulence, the optical signal experiences random fluctuations of the amplitude and phase for the received signal at the receiver, which seriously deteriorates the system performance [6], [7]. As an alternative, PPM can improve the energy efficiency by increasing the order of modulation [15]. However, the improvement of energy efficiency is obtained at the cost of bandwidth efficiency, which leads to a limited transmission capacity. As a variant of PPM, multipulse position modulation (MPPM) has been proposed to improve the bandwidth efficiency by utilizing multiple pulsed slots during each symbol transmission [18]. In the past decade, researchers have conceived various designs of MPPM constellations in FSO communication systems. In [4], a Gray labeling search (GLS) algorithm which considers the Hamming distance between adjacent symbols has been proposed to construct a new type of MPPM constellations. Furthermore, an MPPM constellation subset selection (MCSS) algorithm has been proposed [19], in which the MPPM symbols are continuously added to the constellation subset by maximizing the Hamming distance between two adjacent symbols. Through this way, an MPPM constellation with the size of a power of two can be generated.
The above two MPPM constellations have been only employed in single-input-single-output (SISO) technique, which cannot attain spatial diversity. To tackle the above issue, multiple-input multiple-output (MIMO) technique has been introduced in FSO communication systems [20]. Space shift keying (SSK) [21] is a spatial modulation technique that utilizes transmit antennas with distinct path characteristics to distinguish the transmitted symbols. To be specific, in [22], a low-complexity spatial pulse position modulation (SPPM) scheme that combines SSK with PPM has been proposed. In [23], the authors propose a generalized spatial pulse position modulation (GSPPM) scheme by utilizing the pulse inversion technique. The GSPPM scheme requires a DC bias to achieve pulse inversion, which adds additional power consumption. Moreover, the GSPPM scheme employs a single pulse position modulation (PPM), which cannot be further optimized in the signal-domain constellation. In [24], the authors have considered a modulation scheme based on a generalized spatial modulation (GSM) with multiple pulse amplitude and position modulation (MPAPM) in the visible light communication (VLC) systems. This work only exploits the conventional mapping rule of GSM and MPAPM, but does not design new bit-to-symbol mapping rules. Furthermore, in the GSM-MPAPM scheme, the number of activated LED groups must be a power of two, and thus all possible activated LED groups cannot be used sufficiently. In addition, a spatial MPPM (SMPPM) scheme adopting SSK and MPPM has been considered in [25]. So far, an in-depth investigation on GSPPM is still lacking. Actually, a conventional GSPPM constellation includes a spatial-domain constellation (i.e., effective activated antenna groups) and a signal-domain constellation (i.e., MPPM constellations). The sizes of both two types of constellations should be a power of two, and thus part of antenna groups and MPPM symbols keep idle. Therefore, how to design an efficient GSPPM constellation using more antenna groups and MPPM symbols is a challenging problem. An alternative method to improve the system performance is the employment of error-correction codes (ECCs). For example, Reed-solomon (RS) codes [17], [26] and Low-density parity-check (LDPC) codes [27], [28] have been used in the FSO systems. Among all ECCs, a class of structured LDPC codes, i.e., protograph LDPC (PLDPC) codes [29], have received tremendous attention due to low complexity and close-to-capacity performance [30]. As well, the researchers have proposed a protograph extrinsic information transfer (PEXIT) [31] algorithm for predicting the decoding thresholds of PLDPC codes in specific communication scenarios. With the aid of the PEXIT algorithm, the authors have constructed a PLDPC code (i.e., code-B) for Poisson channels [32]. Nevertheless, the code-B PLDPC code may not perform well over weak turbulence channels due to the different distribution characteristics. Hence, it is crucial to design a type of PLDPC code tailored for the PLDPC-coded GSPPM system over such a scenario. Inspired by the above motivation, we make a comprehensive investigation on the PLDPC-coded GSPPM systems over weak turbulence channels. Thus, the contributions of this work are summarized as follows.

1) We propose a new GSPPM scheme based on an asymmetric dual-mode (ADM) constellation search algorithm by removing the constraint condition that the sizes of spatial-domain constellation and signal-domain constellation must be a power of two.
2) We analyze the constellation-constrained capacity of the proposed GSPPM constellations in the case of using different MPPM slots.
3) With the aid of the PEXIT algorithm, we construct an improved PLDPC code, referred to as I-PLDPC code, to achieve excellent decoding thresholds in the GSPPM system.
4) Analytical and simulation results reveal that the proposed PLDPC-coded ADM-aided GSPPM system remarkably outperforms the existing counterparts over weak turbulence channels.

The remainder of this paper is organized as follows. In Section II, we propose the PLDPC-coded GSPPM system and estimate its corresponding constellation-constrained capacity. In Section III, we present the design method of the ADM constellations. In Section IV, we construct a new PLDPC code for the PLDPC-coded GSPPM system. Simulation results are presented in Section V, and the conclusion is made in Section VI.

II. SYSTEMS MODEL

A. Conventional GSPPM

Assuming that there are $N_t$ transmit antennas and $N_r$ receive antennas in the conventional GSPPM. At each transmission instant, $N_a (2 \leq N_a \leq N_t/2)$ out of $N_t$ transmit antennas are chosen as an activated antenna group. Apparently, the number of all possible activated antenna groups is $N_s = \binom{N_t}{N_a}$, where $\binom{N_t}{N_a} = N_t! / N_a! (N_t - N_a)!$, and “!” denotes the factorial. However, the number of $N_s$ is typically not a power of two, and thus all possible activated antenna groups cannot be utilized substantially. To ensure that the number of the spatial-domain constellation (i.e., effective activated antenna groups) is a power of two, one usually selects $N_e$ effective activation antenna groups to transmit an $M$-ary MPPM symbol, where $N_e = 2^{\lfloor \log_2 N_s \rfloor}$ (i.e., closest to $N_s$), and $\lfloor \cdot \rfloor$ denotes the floor function. Generally, each MPPM symbol consists of $l$ slots, with $l_a (2 \leq l_a \leq l/2)$ pulsed slots and $(l - l_a)$ non-pulsed slots. Thereby, the MPPM symbol can be characterized by a length-$l$ vector $z_p = [z_{p1}, z_{p2}, \ldots, z_{pl}]$, where $z_{pq} \in \{0, 1\}$, $q = 1, 2, \ldots, l$, $p = 1, 2, \ldots, M$, “1” represents a pulsed slot, and “0” means a non-pulsed slot.
Accordingly, there are two kinds of constellations (i.e., spatial-domain constellation and signal-domain constellation) in the conventional GSMPM. The size of the spatial-domain constellation depends on the number of effective activated antenna groups, while the size of the signal-domain constellation depends on the order of MPPM. Thus, every \( m = \lfloor \log_2 N_s + \log_2 M_{\text{max}} \rfloor \) coded bits are divided into two parts at each transmission instant, the first \( m_1 = \lfloor \log_2 N_s \rfloor \) coded bits are used to select the effective activated antenna group, while the remaining \( m_2 = \lfloor \log_2 M_{\text{max}} \rfloor \) coded bits are mapped into an \( M \)-ary MPPM symbol.

### B. PLDPC-Coded GSMPM System

The block diagram of a PLDPC-coded GSMPM system is shown in Fig. 1, which has \( N_t \) transmit antennas and \( N_r \) receive antennas. Specifically, at the transmitter, a length-\( k \)-information-bit sequence \( u = \{ u_1, u_2, \ldots, u_k \} \) is first encoded by a PLDPC encoder to generate a length-\( s \)-codeword \( c = \{ c_1, c_2, c_3, \ldots, c_s \} \). Subsequently, \( c \) is passed to a GSMPM modulator after permuted by a random interleaver. After that, \( m \) coded bits are mapped into GSMPM symbol. Thus, a length-\( l \)-sequence \( Z = \{ z_1, z_2, \ldots, z_m \} \) can be yielded. Note that we use \( (N_t, N_r, N_N, l, \alpha_s, M_s) \) to represent a GSMPM constellation with size of \( M_s = 2^m \) in this paper. Finally, the GSMPM symbol sequence \( Z \) is passed through a weak turbulence channel and each GSMPM symbol \( z_p \) is converted into a transmission vector \( x \). The channel output \( y \) can be written as

\[
y = \frac{P_t}{\sqrt{N_a}} H x + w,
\]

where \( y = \begin{bmatrix} y_1, y_2, \ldots, y_{N_r} \end{bmatrix}^T \) denotes the received signal vector with size of \( N_r \times 1 \) by all receive antennas, \( y_i = \begin{bmatrix} y_{1i}, y_{2i}, \ldots, y_{li} \end{bmatrix} \) is the received signal in the \( i \)-th receive antenna; \( x = \begin{bmatrix} x_1, x_2, \ldots, x_{N_t} \end{bmatrix}^T \) denotes the GSMPM signal vector with size of \( N_t \times 1 \) sent by transmit antennas, \( x_i = \begin{bmatrix} x_{1i}, x_{2i}, \ldots, x_{li} \end{bmatrix} \) is the GSMPM signal sent by the \( j \)-th transmit antenna; \( w \) denotes the noise matrix with size of \( N_r \times l \), in which each element is the real additive Gaussian noise with zero-mean and variance \( \sigma_w^2 = N_0 / 2 \), and \( N_0 \) is the noise power spectral density. Moreover, \( P_t = P_a \cdot \gamma \) denotes the peak transmit power of the MPPM, where \( P_a \) is the average transmit power (all modulation patterns use the same \( P_a \)). \( \gamma = 1 / \tau \) is the peak-to-average power ratio (PAPR), and \( \tau = l_a / l \) is the duty cycle of each MPPM symbol. In addition, \( H = (h_{ij}) \) denotes the channel coefficient matrix of size \( N_r \times N_t \), where \( h_{ij} \) is the channel coefficient from the \( j \)-th transmit antenna to the \( i \)-th receive antenna in the PLDPC-coded GSMPM system over a weak turbulence channel. The channel coefficient \( h_{ij} \) of the weak turbulence channel follows a lognormal distribution, and its probability density function (PDF) is given by [8] and [9]

\[
f(h_{ij}) = \frac{1}{2h_{ij} \sqrt{2\pi} \sigma_{\ln h_{ij}}} \exp \left( -\frac{(\ln(h_{ij}) - 2\mu)^2}{8\sigma_{\ln h_{ij}}^2} \right),
\]

where \( \sigma^2_{\ln h_{ij}} = 0.25 \ln (1 + \sigma^2_w) \) is the log-amplitude variance, \( \sigma_1 (\sigma_1^2 < 1) \) is the channel scintillation index, and \( \mu = -\sigma^2_w \) is the log-amplitude mean [33]. To ensure that the average power is not impacted by channel fading, the channel coefficients are normalized as \( \mathbb{E}[h_{ij}^2] = 1 \), where \( \mathbb{E}[\cdot] \) stands for the expectation function. The signal-to-noise ratio (SNR) can be expressed as [22] and [23]

\[
\text{SNR} = \frac{\left( \ln P_t^c \right)}{2 \text{R} \sigma^2_w},
\]

where \( R \) is the code rate.

At the receiver, the received signal \( y \) is detected by the max-sum approximation of log-domain maximum a-posterior probability (Max-log-MAP) algorithm [17], [34] in the GSMPM detector. Then, the extrinsic log-likelihood ratios (LLRs) output from the GSMPM detector are sent to a deinterleaver. After that, these extrinsic LLRs will be fed to a PLDPC decoder to perform belief-propagation (BP) [35], [36] decoding.

### C. Constellation-Constrained Capacity

Given the channel state information (CSI), the maximum rate of a reliable transmission can be determined by the average mutual information (AMI) [37]. Assume that the selected GSMPM symbol from the GSMPM constellation is equiprobable, the constellation-constrained AMI of a code modulation (CM) scheme over a weak turbulence channel can be calculated as [38]

\[
C_{\text{CM}} = m - \mathbb{E}_{x, y, H} \left[ \log_2 \frac{\sum_{r \in \Omega} p(y | r, H)}{p(y | x, H)} \right],
\]

where

\[
p(y | x, H) = \prod_{t=1}^{l} \prod_{q=1}^{l} \prod_{i=1}^{N} \prod_{j=1}^{N} p(y_t^q | x_i^j, H) = \exp \left[ -\frac{\sum_{j=1}^{N} \sum_{i=1}^{N} \left( y_t^q - \sum_{j=1}^{N} \frac{P_{a}}{l_a} h_{ij} x_i^j \right)^2}{2\sigma_w^2} \right],
\]

In Eq. (3), \( \Omega \) denotes a GSMPM constellation, \( y_t^q \) denotes the received signal of the \( q \)-th slot at the \( t \)-th receive antenna, \( x_i^j \) denotes the signal transmitted by the \( q \)-th slot at the \( j \)-th transmit antenna, and \( p(y | x, H) \) is the probability density function (PDF) of the received signal vector \( y \) under the conditions of the channel coefficient matrix \( H \) and the GSMPM symbol vector \( x \). Therefore, in Eq. (4), the PDF of the elements \( h_{ij} \) in the channel coefficient matrix \( H \) follows Eq. (2). In addition, the constellation-constrained AMI of a bit-interleaved coded modulation (BICM) scheme can be calculated as [38]

\[
C_{\text{BICM}} = m - \sum_{k=1}^{m} \mathbb{E}_{b, y, H} \left[ \log_2 \frac{\sum_{x \in \mathcal{X}} p(y | x, H)}{\sum_{x \in \mathcal{X}^b} p(y | x, H)} \right],
\]

where \( \mathcal{X}^b \) denotes the subset of GSMPM constellation \( \Omega \) with the \( k \)-th bit being \( b \in \{0, 1\} \).

### III. DESIGN OF PROPOSED ADM CONSTELLATIONS FOR PLDPC-CODED GSMPM SYSTEM

#### A. Proposed ADM Constellations

In the conventional GSMPM modulation scheme, the \( m_t \) coded bits are used to select effective activated antenna groups, while the \( m_s \) coded bits are modulated by using MPPM symbols for effective activated antenna groups. Note that each effective activated antenna group shares the same MPPM
Algorithm 1 ADM Constellation Parameter Selection

1 Initialization: Given parameters $N_e, N_a, l,$ and $l_a$, calculate $N_s, M_{\text{max}}, m, M_s$, and $M = 2^\lfloor \log_2 M_{\text{max}} \rfloor$. Set $T \theta = 0$, $i = 0$, $N_{\text{add}} \leftarrow N_s - N_e$.

2 While $T \theta = 0$ do:

3 Calculate $M_A \leftarrow [M_s/N_s] + i$ and $M_B$;

4 If $(M_A > M_B)$ & & $(M_A + M_B) \leq M_{\text{max}}$ then $T \theta = 1$;

5 Continue;

6 $N_{\text{add}} \leftarrow N_{\text{add}} - 1$;

7 If $N_{\text{add}} = 0$ then

8 Reset $N_{\text{add}}$ and $i \leftarrow i + 1$;

10 Finalization: Output parameters: $N_{\text{add}}, M_{\text{max}}, M, m$, $M_A$ and $M_B$.

from the $M_{\text{max}} - M_A$ idle MPPM symbols. Then, each effective activated antenna group sends the MPPM symbols in the sub-constellation set $\Psi_A$. Also, we add the additional activated antenna groups to send the MPPM symbols in the constellation $\Psi_B$, where the number of the additional activated antenna groups is $N_{\text{add}}$, and the additional activated antenna groups are selected from the $N_e$ idle activated antenna groups. The size of the spatial-domain constellation is $N_e + N_{\text{add}}$, and the size of the signal-domain constellation is $M_A + M_B$.

As seen, neither the size of the spatial-domain constellation nor the size of the signal-domain constellation is a power of two. More importantly, we consider the mapping relationship between MPPM symbols and coded bits in the ADM constellation. Actually, we propose a novel method to relabel MPPM symbols in the ADM constellation based on the maximum Hamming distance criterion. The detailed design steps of the proposed ADM constellations in the PLDPC-coded GSMPPM system are as follows.

1) ADM Constellation Parameter Selection: In order to utilize all activated antenna groups as much as possible, how to determine the number $N_{\text{add}}$ of additional activated antenna groups is one of the most critical issues. Given system parameters $N_e, N_a, l, l_a$, and $M_e$, all possible activated antenna groups can be calculated as $N_e$, and the number of full-MPPM symbols is $M_{\text{max}}$. First, we randomly select $N_e$ as effective activated antenna groups, thus the number of the remaining activated antenna groups is $N_i$. The additional activated antenna groups are selected from the remaining activated antenna groups. We define two MPPM sub-constellation sets $\Psi_A$ and $\Psi_B$. As such, each effective activated antenna group sends GSMPPM symbols by using constellation set $\Psi_A$ with size of $M_A$, while each additional activated antenna group sends GSMPPM symbols by using constellation set $\Psi_B$ with size of $M_B$. Finally, the relationship between the $M_A$, $M_B$, and $N_{\text{add}}$ can be represented as

$$M_B = [(M_s - N_e \cdot M_A)/N_{\text{add}}],$$

subject to: $M_A > M_B$, $M_A + M_B \leq M_{\text{max}}$.  

(6)

where the initial value of $M_A$ is set to $[M_s/N_s]$ and the initial value of $N_{\text{add}}$ is set to $N_s - N_e$. To elaborate further, the ADM constellation parameter selection is summarized in Algorithm 1.

2) ADM Constellation Formulation: (i) The parameters $N_{\text{add}}$, $M_A$ and $M_B$ of an ADM constellation are determined by Algorithm 1. Considering an ADM GSMPPM constellation scheme with $M_e$ MPPM symbols, there exist $M_e$ labels in the corresponding constellations mapper, each label $\xi_\alpha = [\xi_1^\alpha, \xi_2^\alpha, \ldots, \xi_n^\alpha]$ consists of $m$ labeling bits, where $\alpha = 1, 2, \ldots, M_e$, $\xi_n^\alpha \in [0, 1]$, and $\xi_\alpha$ is the binary representation of the index value ($\alpha - 1$). To conveniently determine the MPPM symbols in sub-constellation set $\Psi_A$, we first divide the $N_e M_A$ labels into $N_a$ label subsets. The index values corresponding to the labels in the $l$th label subset $\xi_\lambda$ within $\Psi_A$ belong to the interval $[lM, (M_A - 1) + lM]$ (see Table I), and the remaining labels correspond to the MPPM symbols in sub-constellation set $\Psi_B$, where $\lambda = 0, 1, \ldots, N_a - 1$. 

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Here, we take the $\lambda$th label subset $\xi_\lambda$ as an example and select the corresponding sub-constellation set $\Psi_A$. Especially, $M_A$ MPPM symbols must be selected from a full MPPM symbol set $\Phi$ of size $M_{\text{max}}$ to constitute an MPPM sub-constellation set $\Psi_{A,p}$, where $p = 1, 2, \ldots, (M_A)$.

The selected sub-constellation set $\Psi_A$ is divided evenly from the $i$th label (i.e., $\xi_i = [\xi_i^1, \xi_i^2, \ldots, \xi_i^{c_m}]$) in $\xi$, where $i = 1, 2, \ldots, M_A$. Further, the Hamming distance of arbitrary two different MPPM symbols (i.e., $\Psi_{A,p_i}$ and $\Psi_{A,p_j}$) is defined as $D^{ij}$. One can calculate the Hamming distance $D_{A,p_i} = \frac{1}{M_A-1}\sum_{j=1,i\neq j}^M D^{ij}$ between the $i$th MPPM symbol $\Psi_{A,p_i}$ and the remaining $M_A - 1$ MPPM symbols. Then, the average Hamming distance of the sub-constellation $\Psi_A$ can be obtained by $D_{A,p} = \frac{1}{M_A}\sum_{i=1}^{M_A} D_{A,p_i}$. Assume that $D_{A,p}$ is the maximum average Hamming distance $D_a$, we select the sub-constellation $\Psi_{A,p}$ for the next operation. For the sub-constellation set $\Psi_{A,p}$, we relabel each MPPM symbol based on the maximum Hamming distance criterion. To be specific, when $D^{ij} = 2a$ (i.e., the largest Hamming distance), we maximize the Hamming distance $d$ of arbitrary two different labels corresponding to two different MPPM symbols $\Psi_{A,p_i}$ and $\Psi_{A,p_j}$. If the case of $d = m$ does not exist, we consider $d = m-1, m-2, \ldots, 1$ and so on. Based on the above operations, we can obtain a sub-constellation set $\Psi_A$.

(ii) In order to decrease the interference between the GSMPMM symbols in two different activated antenna groups, the same MPPM symbol cannot exist in both $\Psi_A$ and $\Psi_B$. Thus, we should remove all MPPM symbols belonging to $\Psi_A$ from $\Phi$ (i.e., $\tilde{\Phi}_A = \Phi/\Psi_A$). Given a spectral efficiency $\rho$, the additional activated antenna groups will cause the truncation of the effective activated antenna groups corresponding to signal-domain constellations. For example, when $N_1 = 4$, $N_2 = 2$, and $M_A = 6$, two MPPM symbols corresponding to the labels [00110] and [00111] cannot combine with the effective activated antenna groups to form two GSMPMM symbols, it can only form two GSMPPM symbols with the additional activated antenna groups (1, 3) and (2, 4), respectively. Therefore, we reclassify the remaining $M$ labels into $N_{\text{add}}$ label subsets in a sequential order, where $M = 2^m(M - M_A)$. When $M - M_A \leq N_{\text{add}}$, the index value corresponding to the $\mu$th label in the $\mu$th label subset $\zeta_{\mu}$ is $(\beta - M_A + \mu)$, where $\mu = 0, 1, \ldots, N_{\text{add}} - 1, \beta = 1, 2, \ldots, M_B$ (see Table I). Otherwise (i.e., $M - M_A > N_{\text{add}}$), the index values corresponding to the labels are divided evenly into $N_{\text{add}}$ subsets in a sequential order (see Table II). Especially, each label in a label subset $\zeta_{\mu}$ corresponds to an MPPM symbol in sub-constellation $\Psi_B$. Taking the label subset $\zeta_{\mu}$ as an example, we select $M_B$ MPPM symbols from set $\tilde{\Phi}_B$ to form a sub-constellation $\Psi_{B,p}$, where $p = 1, 2, \ldots, (M_{\text{max}} - M_A)$. Likewise, we calculate the average Hamming distance of the sub-constellation $\Psi_{B,p}$, which can be obtained by $D_{B,p} = \frac{1}{M_{B}(M_{B}-1)}\sum_{i=1}^{M_{B}}\sum_{j=1,i\neq j}^{M_{B}} D^{ij}$.

Algorithm 2 ADM Constellation Formulation

1. **Initialization:** Given parameters $N_{\text{add}}, M_A, M_B, M_{\text{max}}, m, l$ and $l_\lambda$, generate a full MPPM symbol set $\Phi$ of size $M_{\text{max}}$. Set $D_a = 0$, $D_b = 0$, Label = 0.

2. for $p = 1, 2, \ldots, (M_A)$ do

3. Generate a sub-constellation $\Psi_{A,p}$ and calculate $D_{A,p}$;

4. if $D_{A,p} > D_a$ then

5. Label $\leftarrow p$; $D_a \leftarrow D_{A,p}$;

6. for $i = 1, 2, \ldots, M_A$ do

7. for $j = i + 1, i + 2, \ldots, M_A$ do

8. if $D^{ij} = 2a$ then

9. Maximize the Hamming distance $d$ between two different labels in $\xi_\lambda$;

10. $\Psi_A \leftarrow \Psi_A, \text{Label}$;

11. Remove MPPM symbols in $\Psi_A$ from $\Phi$, i.e., $\Phi = \Phi/\Psi_A$.

12. for $\eta = 1, 2, \ldots, (M_{\text{max}} - M_B)$ do

13. Generate a sub-constellation $\Psi_{B,\eta}$ and calculate $D_{B,\eta}$;

14. if $D_{B,\eta} > D_b$ then

15. Label $\leftarrow \eta$; $D_b \leftarrow D_{B,\eta}$;

16. for $i = 1, 2, \ldots, M_B$ do

17. for $j = i + 1, i + 2, \ldots, M_B$ do

18. if $D^{ij} = 2a$ then

19. Maximize the Hamming distance $d$ between two different labels in $\zeta_{\mu}$;

20. $\Psi_B \leftarrow \Psi_B, \text{Label}$.

21. **Finalization:** Output $\Psi_A, \Psi_B$.

Assume that $D_{B,\eta}$ is the maximum average Hamming distance $D_b$, we select the sub-constellation $\Psi_{B,\eta}$ for the next operation. Then, when $D^{ij} = 2a$ (i.e., the largest Hamming distance), we maximize the Hamming distance $d$ of arbitrary two different labels corresponding to two MPPM symbols $\Psi_{B,\eta}, i$ and $\Psi_{B,\eta}, j$. Finally, the sub-constellation set $\Psi_B$ can be obtained by the above operation.

Based on the above two steps, a type of ADM constellations can be formulated. To illustrate further, the design method for ADM constellations is summarized in Algorithm 2. For example, with the aid of Algorithm 2, the ADM constellations with spectral efficiencies $\rho = 5$ bits per channel use (bpcu) and $\rho = 6$ bpcu are shown in Tables I and II, respectively.

Remark: The proposed ADM constellation can be directly applicable to other turbulence channels (e.g., the Malaga turbulence channel and the gamma-gamma turbulence channel), since the design criterion is independent of the fading distribution.

B. Performance Analysis

To verify the effectiveness of the proposed ADM constellations, the CM and BICM capacities of different GSMPMM
constellations over the weak turbulence channel can be estimated by using the constellation-constrained capacity analysis method in Section II-C. In the PLDPC-coded GSMPPM systems, we calculate the constellation-constrained capacities of the proposed ADM constellations, the optimized constellations, natural constellation [39], MCSS constellation [19], and GLS constellation [4] with $\rho = 5$ bpcu and $\rho = 6$ bpcu are illustrated in Fig. 2 and Fig. 3, respectively, where $N_t = 4$, $N_r = 4$, $N_a = 2$, and $\sigma_x = 0.3$. Note that the quantization of turbulence in the weak turbulence channel is implemented by setting the value of the log-amplitude variance $\sigma_x$ [17]. For the case of $(4, 4, 2, 5, 2, 32)$ GSMPPM scheme with $\rho = 5$ bpcu, in Fig. 2(a), one can see that the proposed ADM constellation is closest to the CM capacity. In particular, the proposed ADM constellation is closest to the CM capacity.

In Fig. 2(b), we can observe similar results for $(4, 4, 2, 6, 2, 32)$ GSMPPM scheme. Likewise, the same phenomenon can be observed when the spectral efficiency $\rho = 6$ bpcu (see Fig. 3). Therefore, it can be concluded that the proposed ADM constellations are able to obtain better performance with respect to the existing counterparts in the PLDPC-coded GSMPPM systems.

Moreover, we analyze the energy efficiency of the proposed GSMPPM scheme, the existing GSPPM scheme [23] and GSM-MPAPM scheme [24]. Note that the energy efficiency can be defined as the number of transmitted information bits per unit energy [40]. For a fair comparison, the average transmit power $P_a$ of all modulation constellations and the optimized constellation can obtain much larger capacities than the other three constellations when the code rate $R > 0.35$ (i.e., $R = C_{BICM}/m$). In particular, the proposed ADM constellation is closest to the CM capacity. In Fig. 2(b), we can observe similar results for $(4, 4, 2, 6, 2, 32)$ GSMPPM scheme. Likewise, the same phenomenon can be observed when the spectral efficiency $\rho = 6$ bpcu (see Fig. 3). Therefore, it can be concluded that the proposed ADM constellations are able to obtain better performance with respect to the existing counterparts in the PLDPC-coded GSMPPM systems.

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the higher energy efficiency than the GSPPM scheme. On the other hand, the energy efficiency of the GSM-MPAPM scheme is dependent on the level number of pulse amplitude of signal $A$ and the peak transmit power $P_t$. When $A = 2$, $P_t_1 = 1$, and $P_t_2 = 4$, the energy efficiency of the GSM-MPAPM scheme is 0.18 (i.e., less than 0.2). Thus, the energy efficiency of the proposed GSMPPM scheme is also higher than that of the GSM-MPAPM scheme.

To further verify the advantage of our proposed ADM constellations, the decoding thresholds of a code rate-1/2 accumulate-repeat-by-4-jagged-accumulate (AR4JA) code [31] in the PLDPC-coded GSMPPM systems are analyzed by utilizing the PEXIT algorithm [37], [41]. In addition, the optimized, natural, GLS and MCSS constellations are used as benchmarks. Note that the transmitted codeword length is assumed to be 4500 and the maximum number of BP iterations $t_{BP}$ is set to be 100. As can be seen from Table III, in the cases of $(4, 4, 2, 5, 2, 32)$ and $(4, 4, 2, 6, 2, 32)$ GSMPPM schemes (i.e., $\rho = 5$ bpcu), the decoding thresholds of the AR4JA code with the proposed ADM constellation and the optimized constellation are smaller than those with other three existing constellations. When the spectral efficiency $\rho = 6$ bpcu (i.e., $4$ in the PEXIT algorithm, when the a-posteriori MIs of all variable nodes (VNs) in a protograph converge to 1, one can obtain the minimum SNR (i.e., decoding threshold). In other words, the decoding threshold represents the minimum SNR that allows a PLDPC code to achieve error-free transmission [37], [41]. Interested readers are referred to the above articles as well as the references therein for more details of the PEXIT algorithm.
TABLE III
Decoding Thresholds (i.e., dB) of the AR4JA Code in the GSMPM System With the Proposed ADM Constellation, the Optimized Constellation, Natural Constellation, MCSS Constellation, and GLS Constellation Over a Weak Turbulence Channel, Where $N_t = 4$, $N_r = 4$, $N_p = 2$, and the Modulation Patterns Are (4, 4, 2, 5, 2, 32), (4, 4, 2, 6, 2, 32), (4, 4, 2, 7, 2, 64) and (4, 4, 2, 8, 2, 64)

| Constellation | Modulation Pattern | (4, 4, 2, 5, 2, 32) | (4, 4, 2, 6, 2, 32) | (4, 4, 2, 7, 2, 64) | (4, 4, 2, 8, 2, 64) |
|---------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| Natural [39]  | -3.0734             | -3.1873             | -3.6949             | -3.7976             |
| MCSS [19]     | -2.9364             | -3.1692             | -3.6861             | -3.8987             |
| GLS [4]       | -3.2972             | -3.9282             | -4.0969             | -4.1966             |
| Optimized     | -3.4875             | -3.7466             | -4.2324             | -4.3361             |
| ADM           | -3.6942             | -3.8542             | -4.3475             | -4.4732             |

(4, 4, 2, 7, 2, 64) and (4, 4, 2, 8, 2, 64) GSMPM schemes, the proposed ADM constellations still have excellent error performance. Importantly, the decoding thresholds of the proposed ADM constellations are the lowest, which indicates that our proposed ADM constellations are the best scheme for the PLDPC-coded GSMPM systems.

IV. Design and Analysis of PLDPC Codes for ADM-Aided GSMPM System

A. Proposed I-PLDPC Code

A PLDPC code is represented by a Tanner graph, which consists of several small sets of check nodes (CNs), variable nodes (VNs), and edges [29], [31]. The VNs and CNs are connected by their associated edges. In a protograph, parallel edges are allowed. In addition, the protograph with a code rate $R = (p_c - p_v)/p_c$ can be represented by a base matrix $B_o = (b_{i,j})$ of size $p_c \times p_v$, where $b_{i,j}$ denotes the number of edges connecting CN $c_i$ and VN $v_j$. A large protograph (resp. parity-check matrix) of size $P_c \times P_v$ corresponding to the PLDPC code, can be constructed by performing a lifting operation on a given protograph (resp. base matrix), where $P_c = T p_c$, $P_v = T p_v$ and $T$ denotes a lifting factor. Typically, the lifting operation (i.e., copy and permute) can be implemented by a modified progressive-edge-growth (PEG) algorithm [42].

It is well known that the BER performance of a PLDPC code may show different performance in different communication scenarios. For specific communication scenarios, it is essential to design PLDPC codes with outstanding performance. In the PLDPC code design, a pre-coding structure and a certain proportion of degree-2 VNs can improve the decoding thresholds [31], [43]. Nevertheless, as the typical minimum distance ratio (TMDR) [44] is extremely susceptible to degree-2 VNs, a finite-length code with excessive degree-2 VNs always has error floor in the high SNR region. For instance, the accumulate-repeat-3-accumulate (AR3A) code [31] and the AR4JA code possess two degree-2 VNs and one degree-2 VN, respectively. According to the analyses, the AR3A code does not possess TMDR and has an error floor in the high SNR region. Conversely, the AR4JA codes benefits from TMDR and does not have any error floor in the high SNR region [31]. As such, some constraints should be imposed on the protograph at the initial stage of PLDPC-code design, as follows.

1) A pre-coding structure: the pre-coding structure includes a CN and two VNs. Especially, the CN connects only a degree-1 VN and a highest-degree punctured VN.

2) Appropriate proportion of degree-2 VNs: If the number of CNs is $p_c$, the number of degree-2 VNs must satisfy $1 \leq p_{c,2} \leq p_c/2$. Otherwise, TMDR cannot be guaranteed.

3) Low complexity: To ensure the low encoding/decoding complexity, the number of parallel edges connecting the CN $c_i$ and the VN $v_j$ is limited up to 3 (i.e., $b_{i,j} \in \{0, 1, 2, 3\}$). To further reduce the search space, an additional constraint is imposed, i.e., $b_{i,1} + b_{i,2} + b_{i,3} + b_{i,4} = b_{i,1} + b_{2,6} + b_{3,6} + b_{4,7} = b_{1,2} + b_{2,7} + b_{3,7} + b_{4,7} > 2$.

Taking into account the three constraints discussed above, we design a new PLDPC code with excellent performance in the PLDPC-coded GSMPM system. Specifically, in order to reduce the computational complexity for search, we consider a rate-1/2 PLDPC code and a $4 \times 7$ base matrix containing 28 elements. The corresponding initial base matrix $B_o$ can be expressed as

$$B_o = \begin{bmatrix} 1 & 0 & 0 & b_{1,4} & b_{1,5} & b_{1,6} & b_{1,7} \\ 0 & 1 & 1 & b_{2,4} & b_{2,5} & b_{2,6} & b_{2,7} \\ 0 & 0 & 1 & b_{3,4} & b_{3,5} & b_{3,6} & b_{3,7} \\ 0 & 1 & 0 & b_{4,4} & b_{4,5} & b_{4,6} & b_{4,7} \end{bmatrix}.$$ \hspace{1cm} (7)

After a simple search with a PEXIT algorithm [37], [41], one can obtain the rate-1/2 improved PLDPC code, referred to as I-PLDPC code, which enables the lowest decoding threshold and effective TMDR. The base matrix $B_1$ is represented as

$$B_1 = \begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 2 & 2 & 1 \\ 0 & 1 & 0 & 2 & 0 & 0 & 2 \end{bmatrix},$$ \hspace{1cm} (8)

where the fourth column denotes a punctured VN and the total number of edges is 22.

B. Performance Analysis

Referring to Table IV, we compare the decoding thresholds of the rate-1/2 I-PLDPC code with those of five existing codes (i.e., the AR4JA code [31], regular-(3, 6) code [45], the optimized code-B [32], enhanced (3, 6) code [37], and enhanced RJA code [37]) in the GSMPM system over a weak turbulence channel. It is revealed that the I-PLDPC code achieves gains about 0.1 dB, 0.45 dB, 0.99 dB, 0.41 dB, and 0.32 dB compared to the AR4JA code, regular-(3, 6) code, code-B, enhanced (3, 6) code, and enhanced RJA code in (4, 4, 2, 5, 2, 32) GSMPM, respectively. Similar results can be obtained from the (4, 4, 2, 6, 2, 32) GSMPM. Moreover, when the spectral efficiency $\rho = 6$ bpcu (i.e., (4, 4, 2, 7, 2, 64) and (4, 4, 2, 8, 2, 64) GSMPM schemes), the decoding thresholds of the I-PLDPC code are better than the other counterparts.
Furthermore, we measure the TMDRs of the I-PLDPC code, AR-JA code, regular-(3, 6) code, code-B, enhanced (3, 6) code, and enhanced RJA code by utilizing the AWD function [44]. Referring to Table V, we observe that the I-PLDPC code, AR-JA code, regular-(3, 6) code, enhanced (3, 6) code, and enhanced RJA code possess effective TMDRs, while code-B does not have a TMDR. It implies that the I-PLDPC code enjoys the linear-minimum-distance-growth property and does not suffer from an error floor in the high SNR region.

Based on the above analyses, it can be derived that the I-PLDPC code has desirable error performance in both the low and high SNR regions in the GSMPPM systems.

V. Simulation Results

In this section, we provide simulations of the PLDPC-coded GSMPPM systems with the proposed ADM constellations, the optimized constellations and three existing constellations (i.e., natural [39], GLS [4], MCSS [19] constellations) over weak turbulence channels. We also compare the bit-error-rates (BERs) of the proposed I-PLDPC code, AR-JA code [31], the optimized code-B [32], regular-(3, 6) code [45], enhanced (3, 6) code [37], and enhanced RJA code [37] in such scenarios. Unless otherwise mentioned, we assume that transmitted information-bit length $k = 1800$, the maximum number of BP iterations $t_{BP}$ is 100, and log-magnitude variance $\sigma_\alpha$ is 0.3 (i.e., the scintillation index $\sigma_1$ is about 0.66).

A. BER Performance of Different GSMPPM Constellations

In Fig. 4, we consider the AR-JA-coded GSMPPM systems with the spectral efficiency $\rho = 5$ bpcu. As seen from Fig. 4(a), the AR-JA code with the proposed ADM constellation exhibits better performance compared to the other four constellations in (4, 4, 2, 5, 2, 32) GSMPPM scheme. Specifically, to achieve a BER of $1 \times 10^{-5}$, the proposed ADM constellation, the optimized constellation, GLS constellation, MCSS constellation, and natural constellation require $-2.02$ dB, $-1.43$ dB, $-1.29$ dB, $-1.06$ dB, and $-1.09$ dB, respectively. Thereby, the proposed ADM constellation has about $0.59$ dB, $0.73$ dB, $0.96$ dB, and $0.93$ dB gains over the optimized constellation, GLS constellation, MCSS constellation, and natural constellation, respectively. As illustrated in Fig. 4(b), similar observations can be obtained for (4, 4, 2, 6, 2, 32) GSMPPM scheme. When the spectral efficiency $\rho = 6$ bpcu, Fig. 5 shows the BER curves of the AR-JA-coded GSMPPM systems with five different constellations. In Fig. 5(a), the proposed ADM constellation requires $-2.62$ dB to obtain a BER of $1 \times 10^{-5}$, while the natural, GLS, MCSS, and the optimized constellations require $-1.85$ dB, $-2.15$ dB, $-1.81$ dB, and $-2.36$ dB, respectively. Likewise, Fig. 5(b) also shows that the proposed ADM constellation requires the smallest SNR to achieve a BER of $1 \times 10^{-5}$ in (4, 4, 2, 8, 2, 64) GSMPPM scheme.

Subsequently, in Table III, among the proposed ADM constellation, the optimized constellation, GLS constellation, MCSS constellation, and natural constellation in
TABLE V
TMDRs of the Rate-1/2 AR4JA Code, Regular-(3, 6) Code, Code-B, Enhanced (3, 6) Code, Enhanced RJA Code, and the Proposed I-PLDPC Code

| Code Type      | I-PLDPC | AR4JA [31] | Code-B [32] | Regular [45] | Enhanced (3, 6) [37] | Enhanced RJA [37] |
|----------------|---------|------------|-------------|--------------|---------------------|------------------|
| TMDR           | 0.007   | 0.014      | N.A.        | 0.023        | 0.003               | 0.003            |

Fig. 5. BER curves of the AR4JA-coded GSMPPM systems with the proposed ADM constellation, the optimized constellation, natural constellation, MCSS constellation, and GLS constellation: (a) (4, 4, 2, 7, 2, 64) GSMPPM with \( \rho = 6 \) bpcu; (b) (4, 4, 2, 8, 2, 64) GSMPPM with \( \rho = 6 \) bpcu.

Fig. 6. BER curves of the PLDPC-coded GSMPPM systems with the proposed ADM constellation, the optimized constellation, GLS constellation, MCSS constellation, and natural constellation over weak turbulence channels with \( \sigma_x = 0.2 \) and \( \sigma_x = 0.4 \): (a) (4, 4, 2, 5, 2, 32) GSMPPM with \( \rho = 5 \) bpcu; (b) (4, 4, 2, 7, 2, 64) GSMPPM with \( \rho = 6 \) bpcu.

When the spectral efficiency \( \rho = 5 \) bpcu, it can be seen from Fig. 6(a) that the proposed ADM constellation shows better BER performance than the other four constellations in (4, 4, 2, 5, 2, 32) GSMPPM scheme with \( \sigma_x = 0.2 \). To be specific, to achieve a BER of \( 1 \times 10^{-5} \), the proposed ADM constellation, the optimized constellation, GLS constellation, MCSS constellation, and natural constellation require about \(-2.75\) dB, \(-2.35\) dB, \(-2.16\) dB, \(-1.94\) dB, and \(-2.07\) dB, respectively. Therefore, at a BER of \( 1 \times 10^{-5} \), the proposed ADM constellation has about \( 0.4 \) dB, \( 0.59 \) dB, \( 0.81 \) dB, and \( 0.68 \) dB gains compared with the optimized constellation, GLS constellation, MCSS constellation, and natural constellation, respectively. Likewise, the proposed ADM constellation also
shows better BER performance than the other four constellations in (4, 4, 2, 5, 2, 32) GSMPPM scheme with \(\sigma_x = 0.4\). Specifically, the proposed ADM constellation, the optimized constellation, GLS constellation, MCSS constellation, and natural constellation require about \(-1.68\) dB, \(-1.38\) dB, \(-1.21\) dB, \(-0.89\) dB, and \(-1.01\) dB to obtain a BER of \(1 \times 10^{-5}\), respectively. The proposed ADM constellation obtains 0.3 dB, 0.47 dB, 0.79 dB, and 0.67 dB compared with the optimized constellation, GLS constellation, MCSS constellation, and natural constellation in (4, 4, 2, 5, 2, 32) GSMPPM scheme with \(\sigma_x = 0.4\), respectively. In Fig. 6(b), similar results can be obtained from the weak turbulence channel in (4, 4, 2, 7, 2, 64) GSMPPM scheme with the spectral efficiency \(\rho = 6\) bpcu. Based on the above discussion, the PLDPC code with the proposed ADM constellation exhibits better BER performance compared with its benchmarks when \(\sigma_x = 0.2\) and \(\sigma_x = 0.4\). Moreover, the channel model considered in this paper is a weak turbulence channel (i.e., the channel scintillation index is \(\sigma_I^2 < 1\)), and hence the maximum log-amplitude variance of the weak turbulence channel is about \(\sigma_x = 0.4\). One can observe that the proposed ADM constellation is able to provide a performance gain larger than 0.3 dB at a BER of \(1 \times 10^{-5}\) in (4, 4, 2, 5, 2, 32) GSMPPM scheme with \(\sigma_x = 0.4\). Similar results can be observed from Fig. 6(b).

**B. BER Performance of Different PLDPC Codes**

Fig. 7 shows the BER curves of the proposed I-PLDPC code, AR4JA code, regular-(3, 6) code, code-B, enhanced (3, 6) code, and enhanced RAJA code with the proposed ADM constellations in GSMPPM systems: (a) (4, 4, 2, 5, 2, 32) GSMPPM with \(\rho = 5\) bpcu; (b) (4, 4, 2, 6, 2, 32) GSMPPM with \(\rho = 5\) bpcu.

Fig. 8 shows the BER curves of the proposed I-PLDPC code, AR4JA code, regular-(3, 6) code, code-B, enhanced (3, 6) code, and enhanced RAJA code with the proposed ADM constellations in GSMPPM systems: (a) (4, 4, 2, 7, 2, 64) GSMPPM with \(\rho = 6\) bpcu; (b) (4, 4, 2, 8, 2, 64) GSMPPM with \(\rho = 6\) bpcu.
code needs about \(-2.95\) dB to obtain a BER of \(1 \times 10^{-5}\) in \((4,4,2,7,2,64)\) GSMPPM, while the regular-\((3,6)\) code, the AR-\(JA\) code, code-B, enhanced \((3,6)\) code, and enhanced RJA code require \(-2.52\) dB, \(-2.62\) dB, \(-2.15\) dB, \(-2.71\) dB, and \(-2.77\) dB, respectively, to do so. In \((4,4,2,8,2,64)\) GSMPPM scheme, the I-PLDPC code also requires the lowest SNR to achieve a BER of \(1 \times 10^{-5}\), as shown in Fig. 8(b).

Based on the above discussion, the I-PLDPC code is promising with other parameter settings (i.e., different values of \(N_t, N_r, \sigma_0, l, I_p\), and \(\sigma_0\)) and obtained similar observations, which substantially demonstrate the superiority of our proposed constellations and code design.

VI. CONCLUSION

This paper investigated the performance of PLDPC-coded GSMPPM systems over weak turbulence channels. We proposed a type of novel GSMPPM constellations, called ADM constellations, which can achieve desirable capacities and convergence performance in this scenario. Furthermore, we constructed an improved PLDPC code using the PEXIT-aided computer search method, which possesses desirable decoding threshold and effective TMDR. Theoretical analyses and simulated results indicated that the PLDPC-coded GSMPPM system using the proposed ADM constellations and I-PLDPC code can exhibit noticeable performance gains with respect to the state-of-the-art counterparts. Based on the appealing advantages, the proposed PLDPC-coded GSMPPM transmission scheme stands out as a competitive alternative for high-reliability FSO applications.

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