Study of the angular coefficients and corresponding helicity cross sections of the $W$ boson in hadron collisions

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We present the Standard Model prediction for the eight angular coefficients of the $W$ boson, which completely describe its differential cross section in hadron collisions. These coefficients are ratios of the $W$ helicity cross sections and the total unpolarized cross section. We also suggest a technique to experimentally extract the coefficients.

I. INTRODUCTION

The analytical study of the $W$ boson is essential for the understanding of many open questions related to the electroweak physics, like the origin of the electroweak symmetry breaking and the source of the $CP$ violation. Since its discovery the $W$ hadronic cross section, mass, and width have been measured with great precision [1]. On the other hand complete physical information is contained in the boson’s angular distribution in three dimensions, given by its differential cross section, which can be written as a sum of helicity cross sections. These quantities are related to the nature of the electroweak processes, the $W$ polarization, and the presence of QCD effects. In this paper we address these issues in the case of $W$ produced in hadron collisions.

The total differential cross section of the $W$ production in a hadron collider [2] is given by the equation:

$$
\frac{d\sigma}{dq_T^2 dy d\cos\theta d\phi} = \frac{3}{16\pi} \frac{d\sigma^u}{dq_T^2 dy} [1 + \cos^2 \theta]
$$

$$
+ \frac{1}{2} A_0 (1 - 3 \cos^2 \theta) + A_1 \sin 2\theta \cos \phi
$$

$$
+ \frac{1}{2} A_2 \sin^2 \theta \cos 2\phi + A_3 \sin \theta \cos \phi
$$

$$
+ A_4 \cos \theta + A_5 \sin^2 \theta \sin 2\phi
$$

$$
+ A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi 
$$

(1)

where $q_T$ and $y$ are the transverse momentum and the rapidity of the $W$ in the lab frame and $\theta$ and $\phi$ are the polar and azimuthal angles of the charged lepton from the $W$ decay in the Collins-Soper (CS) frame [3]. The CS frame is used because in this frame we can experimentally reconstruct the azimuthal angle $\phi$ and the polar quantity $|\cos \theta|$. Our ignorance of the $W$ longitudinal momentum, which is due to our inability to measure the longitudinal momentum of the neutrino in a hadron collider, introduces a two-fold ambiguity on the sign of $\cos \theta$. The quantity $d\sigma^u/dq_T^2 dy$ is the angles-integrated unpolarized cross section.

The dependence of the cross section on the leptonic variables $\theta$ and $\phi$ is completely manifest and the dependence on the hadronic variables $q_T^2$ and $y$ is completely hidden in the angular coefficients $A_i(q_T, y)$. This allows us to treat the problem in a model independent manner since all the hadronic physics is described implicitly by the angular coefficients and it is decoupled from the well understood leptonic physics. The angular coefficients are ratios of the helicity cross sections of the $W$ and $d\sigma^u/dq_T^2 dy$. In order to explicitly separate the hadronic from the leptonic variables, the helicity amplitudes were used to describe the hadronic tensor associated with the hadronic production of the $W$ [4]. The leptonic tensor on the other hand is analytically known, leading to analytic functions of the angles of the charged lepton in Equation (1). It is common to integrate Equation (1) over $y$ and study the variation of the angular coefficients as a function of $q_T$.

If the $W$ is produced with no transverse momentum, it is polarized along the beam axis because of the $V$-$A$ nature of the weak interactions and helicity conservation. In that case $A_4$ is the only non-zero coefficient. If only valence quarks contributed to the $W^+$ production, $A_4$ would equal 2, and the angular distribution (1) would be $\sim (1 \pm \cos \theta)^2$, a result that was first verified by the UA1 experiment [5].

If the $W$ is produced with non-negligible transverse momentum, balanced by the associated production of jets, the rest of the angular coefficients are present and the cross section depends on the azimuthal angle $\phi$ as well. The last three angular coefficients – $A_5$, $A_6$, and $A_7$ – are non-zero only if gluon loops are present in the production of the $W$ [6]. Hence, in order to study all the angular coefficients and associated helicity cross sections of the $W$ in a hadron collider, we have to consider the production of the $W$ with QCD effects at least up to order $\alpha_s^2$.

The importance of the determination of the $W$ angular coefficients is discussed in [7], and is summarized here. This study allows us to measure for the first time the differential cross section of the $W$ and study its polarization, since the angular coefficients are related to the helicity cross sections. It also helps us verify the QCD effects in the production of the $W$ up to order $\alpha_s^2$. In addition, $A_3$ is only affected by the gluon-quark interaction and its measurement could constrain the gluon parton dis-
distribution functions. Moreover, the next-to-leading order (NLO) coefficients $A_5$, $A_6$, and $A_7$ are $P$-odd and $T$-odd and may play an important role in direct $CP$ violation effects in $W$ production and decay [6]. Finally, a quantitative understanding of the $W$ angular distribution could be used to test new theoretical models and to facilitate new discoveries.

In this paper we present the Standard Model prediction for the angular coefficients $A_i$ as a function of $q_T$, for proton-antiproton collisions at 1.8 TeV, using the DYRAD Monte Carlo program [8], an event generator of $W$+jet up to order $\alpha_s^2$. Three of the coefficients, including $A_3$ which is the dominant one up to $q_T = 100$ GeV, are presented for the first time. We also suggest a new method for the extraction of the angular coefficients.

II. STANDARD MODEL PREDICTION FOR THE $W$ ANGULAR COEFFICIENTS

In order to study the angular distribution of the $W$ we have to choose a particular charge for the boson. In this paper we present the results for the $W^-$. The angular coefficients for the $W^+$ can be extracted by $CP$ transformation. In the Collins-Soper frame, the $CP$ transformation leaves $\phi$ unchanged and takes $\cos \theta$ to $\pi - \cos \theta$. If we assume that Equation (1) describes $W^-$ bosons, we have to change the sign of coefficients $A_1$, $A_4$, and $A_5$, in order to describe $W^+$ bosons, without changing the definition of the Collins-Soper frame.

We generate Monte Carlo $W$+jet events up to $\alpha_s^2$, including up to one gluon loop, from proton-antiproton collisions at $\sqrt{s} = 1.8$ TeV, using the DYRAD generator. We run with minimal kinematic and acceptance cuts, with a minimum transverse energy for the jet of 10 GeV and jet-jet angular separation of $\Delta R = \sqrt{(\Delta\eta_{lab})^2 + (\Delta\phi_{lab})^2} = 0.7$, in the pseudorapidity-phi space in the lab frame. The CTEQ4M parton distribution functions were used.

We measure the $\theta$ and $\phi$ angles of the charged lepton in the Collins-Soper $W$ rest-frame. At the Monte Carlo event-generator level, we know the momentum of the neutrino, so there is no two-fold ambiguity on the sign of $\cos \theta$. The CS frame is the rest-frame of the $W$ where the $z$-axis bisects the angle between the proton momentum ($\vec{p}_{CS}$) and the opposite of the antiproton momentum ($-\vec{p}_{CS}$) in the CS frame. The signs of the angular coefficients depend on the way the Collins-Soper $x$-axis and $y$-axis are defined. In this paper, we define them so that the $x$-$z$ plane coincides with the $p_{CS} - \vec{p}_{CS}$ plane and the positive $y$-axis has the same direction as $\vec{p}_{CS} \times \vec{p}_{CS}$. The SM distribution of the $\phi$ and $\cos \theta$ for four $q_T$ bins (15-25, 25-35, 35-65 and 65-105 GeV) is shown in Figures 1 and 2 respectively. We note that at low transverse momentum of the $W$, the $\phi$ distribution is almost flat, whereas the $\cos \theta$ distribution almost follows the $(1 + \cos \theta)^2$ law. In the fourth $q_T$ bin, there is a strong $\phi$-dependence of the cross section and the $\cos \theta$ distribution is almost a straight line ($|\cos \theta|$ is flat). There is a correlation between $\cos \theta$ and $M_T^W$, with low $\cos \theta$ corresponding to low $M_T^W$ events.

To calculate the angular coefficients from the angles of the charged lepton, we use the method of moments. We
first define the moment of a function \( m(\theta, \phi) \) as

\[
< m(\theta, \phi) > = \frac{\int \int d\sigma(q_T, y, \theta, \phi)m(\theta, \phi)d\cos \theta d\phi}{\int \int d\sigma(q_T, y, \theta, \phi)d\cos \theta d\phi}
\]  
(2)

We can easily prove that:

\[
< m_0 > \equiv < \frac{1}{2} (1 - 3 \cos^2 \theta) > = \frac{3}{20} (A_0 - \frac{2}{3})
\]

\[
< m_1 > \equiv < \sin 2\theta \cos \phi > = \frac{1}{5} A_1
\]

\[
< m_2 > \equiv < \sin^2 \theta \cos 2\phi > = \frac{1}{10} A_2
\]

\[
< m_3 > \equiv < \sin \theta \cos \phi > = \frac{1}{4} A_3
\]

\[
< m_4 > \equiv < \cos \theta > = \frac{1}{4} A_4
\]

\[
< m_5 > \equiv < \sin^2 \theta \sin 2\phi > = \frac{1}{5} A_5
\]

\[
< m_6 > \equiv < \sin 2\theta \sin \phi > = \frac{1}{5} A_6
\]

\[
< m_7 > \equiv < \sin \theta \sin \phi > = \frac{1}{4} A_7
\]

(3)

For a set of discrete generator (or experimental) data, we substitute the integrals of Equation (2) by sums and the cross section values by the weights \( w_i \) of the Monte Carlo events.

\[
< m(\theta, \phi) > = \frac{\sum_{i=1}^{N} m(\theta_i, \phi_i)w_i}{\sum_{i=1}^{N} w_i}
\]

(4)

By solving Equations (3) for the angular coefficients and substituting the moments by the discrete expressions (4), we extract the Standard Model prediction. By ignoring the \( W \) rapidity, we actually calculate the y-integrated angular coefficients, which are now functions of just \( q_T \). The results are shown in Figure 3. The angular coefficients \( A_1, A_4, \) and \( A_6 \) are presented for the first time. The DYRAD Monte Carlo generator is more reliable for \( q_T > 10 \) GeV, which is also the transverse energy cut for our jets, and this value determines the minimum of our \( q_T \)-axis. The maximum is determined by the Monte Carlo statistics.

We notice that indeed \( A_1 \) is the only surviving major coefficient at low \( q_T \) values. It is also the only angular coefficient that decreases as \( q_T \) increases. The angular coefficient \( A_1 \), although it is a leading order (LO) coefficient, is much smaller than the leading order coefficients \( (A_0, A_2, A_3, \) and \( A_4) \) and comparable to the next-to-leading order ones \( (A_5, A_6, \) and \( A_7) \). The coefficients \( A_0 \) and \( A_2 \) would be exactly equal if gluon loops were not included \cite{2}. At order \( \alpha_s^2 \), \( A_0 \) is consistently greater than \( A_2 \). To better determine \( A_1, A_5, A_6, \) and \( A_7 \) we use the four \( q_T \) bins, to improve the statistics. The result is shown in Figure 4. There are relations that directly connect the angular coefficients with the helicity cross sections of the \( W \) \cite{2}. We first extract the unpolarized cross section of the \( W \) as a function of \( q_T \) and using our prediction for the angular coefficients, we arrive at the Standard Model prediction for the \( W \) helicity cross sections at \( \sqrt{s} = 1.8 \) TeV, shown in Figure 5. Here \( d\sigma_i \) is the helicity cross...
section that corresponds to the angular coefficient $A_i$.

III. EXPERIMENTAL DETERMINATION OF THE $W$ ANGULAR DISTRIBUTION

E. Mirkes [2] first realized the problem of directly measuring the angular coefficients. The angular distributions of Figures 1 and 2 are seriously distorted after the effects of the detector are considered and quality cuts are imposed on the data sample. To study the effect, we treat the generator leptons as electrons and we pass them through a detector simulator [9]. The new $\phi$ and $\cos \theta$ distributions are shown in Figures 6 and 7 respectively. The shapes of the muon distributions are the same as that for electron $W$+jet events, however less events are detected, because of the lower muon acceptance of a typical hadron collider detector. The main reason for the difference between Figures 1, 2 and Figures 6, 7 is the leptons transverse momentum cuts ($p_{T,\ell} > 20$ GeV and $p_{T,\nu} > 20$ GeV) and the charged lepton rapidity cut (central leptons are considered, $|y| < 1$).

The problem of distortion of the $\phi$ and $\cos \theta$ distributions due the detector effects and quality cuts is not the only one. A more fundamental problem is the actual measurement of these angles. To measure them, we need to reconstruct the $W$ in the three dimensional momentum space, in order to boost to its center of mass. The longitudinal momentum of the neutrino is not measured, but it is constrained by the mass of the $W$, based on the equation:

$$p_{\nu}^z = \frac{1}{(2p_{T}^\ell)^2} \left( \frac{Ap_{T}^\ell \pm E\sqrt{A^2 - 4(p_{T}^\ell)^2(p_{T}^\nu)^2}}{2} \right)$$

(5)

where

$$A = M_{W}^2 + q_{T}^2 - (p_{T}^\ell)^2 - (p_{T}^\nu)^2,$$
E^l is the energy of the charged lepton, p_T^l and p_T^\nu are the transverse momentum of the charged lepton and the neutrino and p_z^l is the longitudinal momentum of the charged lepton. The two solutions for the longitudinal momentum of the neutrino lead to two solutions for the W longitudinal momentum. Both solutions correspond to the same \phi but to opposite \cos \theta values.

Moreover, according to Equation (5), for each event, we have to input a mass for the W to get p_T^\nu and eventually \phi and \vert \cos \theta \vert. The mass of W is not known on event-by-event basis, we just know its pole mass and its Breit-Wigner width. Based on these two established values, we can plot the uncertainty in the measurement of \phi and \vert \cos \theta \vert introduced by the uncertainty in the mass of the W. For each Monte Carlo event we generate W masses that are greater than the transverse mass for the particular event and follow the Breit-Wigner distribution. In Figure 8 we see that the systematic error on the measurement of \phi is very small, but the \cos \theta systematic error is significant, especially at low \vert \cos \theta \vert and at big values of the transverse mass of the W. This makes the direct measurement of the \vert \cos \theta \vert distribution more challenging.

IV. EXPERIMENTAL EXTRACTION OF THE ANGULAR COEFFICIENTS

In [7] it is suggested that the experimental distributions of Figures 6 and 7 should be divided by the Monte Carlo distributions obtained using isotropic W decays. This method results in distributions similar to those shown in Figures 1 and 2 and the extraction of the angular coefficients is easier. Here we present a method that does not bias the experimental data by Monte Carlo data. Instead, it uses the knowledge of the detector and its effect on the theoretical distributions. We will demonstrate the method for the \phi analysis.

If we integrate Equation (1) over \cos \theta and y, we get:

\[
\frac{d\sigma}{dq_T^2 d\phi} = C'(1 + \beta_1 \cos \phi + \beta_2 \cos 2\phi + \beta_3 \sin \phi + \beta_4 \sin 2\phi)
\]

where

\[
C' = \frac{1}{2\pi} \frac{d\sigma}{dq_T^2}, \quad \beta_1 = \frac{3\pi}{16} A_3, \quad \beta_2 = \frac{A_2}{4}
\]

\[
\beta_3 = \frac{3\pi}{16} A_4, \quad \beta_4 = \frac{A_5}{2}
\]

The observed \phi distribution is given by Equation (6), only if we ignore the effects of the detector and kinematic cuts. In any other case, there is an acceptance and efficiency function \text{ae}(q_T, \cos \theta, \phi) which multiplies (1) before it is integrated over \cos \theta and as a result, no angular coefficient is completely integrated out. In the actual data, what we measure is the number of events, which is:

\[
N(q_T, \phi) = \int \frac{d\sigma}{dq_T^2 d\cos \theta} \cdot \text{ae}(q_T, \cos \theta, \phi) d\cos \theta \int L dt + N_{bg}(q_T, \phi)
\]

where \mathcal{L} is the luminosity and \text{ae}(q_T, \cos \theta, \phi) is the acceptances and efficiencies for the particular W transverse momentum and pixel in the (\cos \theta, \phi) phase space. \text{N}_{bg}(q_T, \phi) is the background for the given \phi bin and q_T. If we combine Equations (8) and (1), then the measured distribution is:

\[
N(q_T, \phi) = C'(f_{-1} + \sum_{i=0}^{7} A_i f_i) + N_{bg}(q_T, \phi)
\]

where \[ C' = \mathcal{L} \int dt, \quad \text{and} \quad f_i \text{ are the fitting functions,}
\]

integrals of the product of the explicit functions of \cos \theta and \phi and the \text{ae}(\cos \theta, \phi):

\[
f_i(q_T, \phi) = \int_{0}^{\pi} g_i(\theta, \phi) \text{ae}(q_T, \cos \theta, \phi) d\cos \theta,
\]

\[ i = -1, \ldots, 7 \]
where
\[
\begin{align*}
g_{-1}(\theta, \phi) &= 1 + \cos^2 \theta, \\
g_0(\theta, \phi) &= 1/2(1 - 3\cos^2 \theta) \\
g_1(\theta, \phi) &= \sin 2\theta \cos \phi, \\
g_2(\theta, \phi) &= 1/2 \sin^2 \theta \cos 2\phi \\
g_3(\theta, \phi) &= \sin \theta \cos \phi, \\
g_4(\theta, \phi) &= \cos \theta \\
g_5(\theta, \phi) &= \sin^2 \theta \sin 2\phi, \\
g_6(\theta, \phi) &= \sin \theta \sin \phi \\
g_7(\theta, \phi) &= \sin \theta \sin \phi
\end{align*}
\]

The \(f_i\) functions can be calculated explicitly if we know the acceptance and the efficiency of the detector. Because we multiply by \(ae(q_T, \cos \theta, \phi)\) before integrating over \(\cos \theta\), no \(f_i\) is exactly zero. As a result, all coefficients are in principle measurable with the \(\phi\) analysis and not just \(A_2\) and \(A_3\), as Equation (6) suggests. In practice, the \(A_2\) and \(A_3\) are measurable with a greater statistical significance, because the terms \(A_i f_i(\phi)\) are much smaller, for \(i \neq 2, 3\). As a result, these terms affect less the \(\phi\) distribution. Figures 9 and 10 show the \(f_i\) functions for electron acceptance and efficiencies. The shape of the \(f_i\) functions is almost identical for the muons. For perfect acceptance and no kinematic cuts (\(ae = 1\)), the only surviving \(f_i\) functions would be \(f_{-1}, f_2, f_3, f_5,\) and \(f_7\), and they would be equal to \(8/3, 2/3 \cos 2\phi, \pi/2 \cos \phi, 4/3 \sin 2\phi\) and \(\pi/2 \sin 2\phi,\) in accordance to Equation (6). Figure 11 shows the \(\phi\) distribution (9) for the low \(q_T\) bin, with the background neglected and with only one coefficient varying at a time. We see that the \(\phi\) distribution is primarily sensitive to \(A_2\) and \(A_3\), and these coefficients are the easier measurable ones with the \(\phi\) analysis. Figure 12 shows the Monte Carlo expected experimental \(\phi\) distributions for a data sample of the size of the Tevatron Run I.

The final step is to extract the angular coefficients using the data of Figure 12. We keep the \(A_{i \neq 2, 3}\) coefficients frozen at their Standard Model values we determined above and we fit the distributions to the \(f_i\) varying \(A_2\)
and $A_3$ simultaneously. The result of the fit can be seen in Figure 12 and the extracted coefficients in Figure 13. We conclude that the measured angular coefficients are close to the values we extracted in section II, verifying that the method is self-consistent and could be used for an experimental measurement of the $W$ angular coefficients. The same technique can be applied in $Z$ boson experimental studies – which do not demonstrate any problems in the kinematic reconstruction of the boson – using the future statistically significant datasets of the Tevatron and the LHC.

V. SUMMARY

The Standard Model prediction for the angular coefficients and the associated helicity cross sections of the $W$ production in a hadron collider up to order $\alpha_s^2$ in QCD and at $\sqrt{s} = 1.8$ TeV was presented. The experimental measurement of the angular distributions is distorted due to the acceptances and efficiencies of the detector and the application of quality cuts to reduce backgrounds. Two additional issues are the $W$ mass width effect and the resolution of the two-fold ambiguity in the longitudinal momentum of the neutrino. We presented the effect of these factors on the angular distributions and noted that both problems do not affect the azimuthal angle of the charged lepton in the CS frame. Finally, we suggested a method of extracting the angular coefficients without having to divide the experimental data by Monte Carlo distributions of isotropic $W$ decays. Passing the generator data through a detector simulator and analyzing the resulting data, we were able to get back the angular coefficients we determined from the direct analysis of the generator data, demonstrating that this procedure is reliable for the experimental measurement of the angular coefficients.

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