Pseudo-three-dimensional model for hydraulic fracturing with foams

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Abstract. Pseudo-three-dimensional cell-based model for hydraulic fracturing with foams is presented. Accounting for compressibility and non-Newtonian rheology of the foam the obtained model introduces a simple and efficient method to compute the fracture growth in a layered rock formation for such complex fluids. Numerical results reveal an influence of the foam properties on the fracture propagation and predict the difference in proppant transport for the foam fracturing comparing to fracturing with ordinary fluids.

1. Introduction
Hydraulic fracturing is a well-known technology in the petroleum industry which aims to increase the productivity of reservoir by pumping pressurized liquid into a wellbore to create cracks and increase the effective permeability. Using foam as pumped fracturing fluid is very attractive due to lower water usage reducing the damage to sensitive formations and rheology properties providing good proppant-carrying capacity [1, 2].

The modeling of the fracture growth process is a difficult task and commonly employs simplifications that assume incompressibility and Newtonian viscosity of the fracturing fluid [3–5]. However, these assumptions are not valid for the foam. Foam may contain up to 95% gas phase making fluid highly compressible and changing the rheology behaviour. These factors are making the model equations more complicated. A wide range of foam fracturing models is developed accounting for different aspects of the foam properties [6–10].

This work focuses on the generalization of the existing two-dimensional foam fracturing model [11] to pseudo-three-dimensional PKN-based [3, 4] formulation. In this model foam is considered as a single phase power-law fluid with the effective density and viscosity depending on its quality — the volumetric part of the gas phase in a foam. Fracture growth in length is assumed to be greater than in its height, known model [12,13] is used to predict height increment proposing corrections for the compressible fluid case.

2. Foam fracturing model
Consider a problem of a hydraulic fracture propagation driven by the injection of compressible power-law fluid in a multi-layered infinite homogeneous linear elastic medium with minimum horizontal stress contrast.
2.1. Geometry of the problem
The geometry of the problem is presented in Fig. 1.

According to PKN-based pseudo-three-dimensional problem formulation [4], fracture is
mainly growing in $x$-direction with length $L(t)$ which is greater than varying in $x$ fracture height
$H(x,t)$ and $H \gg w$ due to $E \gg \sigma_{cp}$ condition, where $E$ is the plain strain Yong’s modulus
of the rock, $\sigma_{cp}$ is minimum horizontal stress of the perforated layer and $w(x, y, t)$ is the fracture
opening along $z$-axis. System $\{l_i, \sigma_i\}$ defines heights $l_i$ of the rock layers and corresponding
stresses $\sigma_i$. $H_{\text{high}}(x, t)$ and $H_{\text{low}}(x, t)$ are the distances between $xz$-plane and the upper and the
lower tips of the fracture, $H = H_{\text{high}} + H_{\text{low}}$.

2.2. Model equations
Basing on the model developed in [11] we should modify the model equations accounting for the
varying height of the fracture. The continuity equation then is given by

$$\frac{\partial}{\partial t}(\rho H \bar{w}) + \frac{\partial}{\partial x}(\rho Q) + \rho \bar{Q}_L = 0,$$

(1)

where $\bar{w}(x, t)$ is the average opening in the fracture cross-section, $\rho(x, t)$ is the density of the
fluid, $Q$ is the volumetric flow rate, $Q_L$ is the volume rate of the fluid leakoff per unit fracture
length. The following relation between flow rate $Q$ and the pressure gradient $\partial p/\partial x$ represents
the transformed equations of motion in this formulation,

$$Q = -\psi \frac{2^{n+1}}{(2K)^{\frac{1}{n}}} H \left( \frac{\partial p}{\partial x} \right)^{\frac{1}{n}} \text{sign} \left( \frac{\partial p}{\partial x} \right),$$

(2)
where \( K, n \) are the flow consistency index and the flow behavior index of the power-law fluid respectively which depend on the fluid pressure \( p \). \( \psi(x) \) is a function, depending on the fracture cross-section geometry,

\[
\psi(x) = \frac{n}{2n + 1} \left[ \frac{1}{H} \int_{-H_{low}}^{H_{high}} \left( \frac{w(x, y)}{\bar{w}(x)} \right)^{2n+1} \, dy \right].
\] (3)

The non-equilibrium growth model \([4, 12, 13]\) is used to predict an increment of the fracture height \( H_{low} \) and \( H_{high} \),

\[
\frac{\partial H_{high}}{\partial t} = (K_{up}K_{dyn}) \frac{n+2}{n} H^{\frac{n-2}{2n}},
\] (4)

\[
\frac{\partial H_{low}}{\partial t} = (K_{down}K_{dyn}) \frac{n+2}{n} H^{\frac{n-2}{2n}},
\] (5)

where \( K_{up} \) and \( K_{down} \) are the stress intensity coefficients at the upper and the lower fracture tips,

\[
K_{up, down} = \sqrt{\frac{\pi H}{2}} \left[ p_{net} + p_{static} \left( H_{low} \pm \frac{H}{4} \right) - \Delta \sigma \right] + \sqrt{2} \frac{n-1}{\pi H} \sum_{i=1}^{n-1} \left( \sigma_{i+1} - \sigma_{i} \right) \frac{H}{2} \arccos \left( \frac{H - 2h_{i}}{H} \right) \pm \sqrt{h_{i}(H - h_{i})} \right] - K_{IC},
\] (6)

\( \Delta \sigma = \sigma_{i} - \sigma_{cp} \), \( p_{net} = p - \sigma_{cp} \), \( K_{IC} \) is the constant stress intensity factor, \( n_{i} \) is the number of layers for the \( x \)-cross-section of the fracture laying in with heights \( h_{i} \) (see Fig. 1). \( p_{static}(y) \) is the hydrostatic pressure term, for incompressible fluid \( p_{static}(y) = \rho g y \) \((g \) is the gravity constant). \( K_{dyn} \) is expressed as follows,

\[
K_{dyn} = \left[ \frac{E}{2n + 2} \left( \frac{2(n + 2)}{\sqrt{\pi(2 - n)}} \right) \left( \frac{K}{E} \left[ \cos \left( \frac{(1 - \frac{2}{n+2})\pi}{\sin \left( \frac{2\pi}{n+2} \right)} \right) \right]^{n+1} \left( \frac{2n+1}{n} \right) \frac{1}{n(n+2)^{n}} \right)^{\frac{1}{n+2}} \right]^{-1},
\] (7)

The relation between the fracture opening and the pressure is

\[
w(x, y) = H \left[ \frac{4}{E} \left[ p_{net} + p_{static} \left( H_{low} - z_{n}H \right) - \Delta \sigma \right] \sqrt{z_{n}(1 - z_{n})} + \frac{4}{\pi E} S_{w} \right],
\] (8)

\[
S_{w} = \sum_{i=1}^{n_{i}} (\sigma_{i+1} - \sigma_{i}) \left( q_{i} - z_{i} \right) \arccosh \left( \frac{z_{n}(1 - 2q_{i})}{|z_{n} - q_{i}|} \right) + \sqrt{z_{n}(1 - z_{n}) \arccos (1 - q_{i})},
\] (9)

where \( z_{n}(x) = y/H(x) \) and \( q_{i}(x) = \sum_{k=1}^{i} \frac{h_{k}(x)}{H(x)} \).

For the leakoff term \( Q_{L} \) the Carter’s equation \([14]\) is assumed to be applicable,

\[
Q_{L}(x, t) = \frac{2HC_{L}}{\sqrt{1 - t_{0}(x)}},
\] (10)

where \( t_{0}(x) = t |_{x=L} \) and \( C_{L} \) is the fluid leakoff coefficient. Fracture length is determined as

\[
L(t) = \min \left( x |_{\psi(x, t) = 0} \right).
\]
2.3. Compressibility and rheology
The compressibility of the foam is caused by the presence of the gas phase in the fluid and may be taken into account by introducing the following relations [11,15],
\[
\rho(p) = \frac{(1 - \Gamma_0) \rho_l + \Gamma_0 \rho_g}{(1 - \Gamma_0) p + p_0 \Gamma_0}, \quad \Gamma(p) = \frac{p_0 \Gamma_0}{(1 - \Gamma_0) p + p_0 \Gamma_0}
\] (11)
where \(\Gamma\) is introduced as the volumetric part of the gas phase in the foam, \(\Gamma_0\) is the quality of the foam at the pressure \(p_0\) with density \(\rho_l\) of the liquid phase and \(\rho_g\) of the gas phase. These equations follow from the Boyle’s law approach and allow to consider foam as a single-phase fluid.

Assuming \(\rho_l \gg \rho_g\) and due to \(p \gg p_{\text{static}}\) we can define the function \(p_{\text{static}}(y)\) using Eqs. (11),
\[
p_{\text{static}}(y) = \frac{\rho_l gy p_0 \Gamma_0}{p(1 - \Gamma_0) + 1}.
\] (12)
\(\Gamma(p)\) expression allows us to use known experimental data or empirical relationships \(K(\Gamma)\) and \(n(\Gamma)\) [16–18] in order to define \(K(p)\) and \(n(p)\) for the model equations.

3. Numerical simulation
Following empirical \(K(\Gamma), n(\Gamma)\) relationships were used for the simulations in order to model the 20-lbm/Mgal guar foam rheology [17],
\[
n = n_0 (1 - 2.1006t^{7.3003}), \quad K = K_0 \exp (-1.9913 \Gamma + 8.9722 \Gamma^2),
\] (13)
where \(K_0, n_0\) are the flow consistency index and the flow behavior index of the liquid phase.

Equations (1), (2), (3), (8), (10) and relations (11), (12), (13) together form a closed system of equations for \(\bar{w}\) which can be iteratively solved using implicit finite difference methods for known \(H_{\text{low}}, H_{\text{high}}\). The calculation domain is divided into \(N_x\) cells with \(\{H_{\text{low},1}, H_{\text{high},1}\}, \{H_{\text{low},2}, H_{\text{high},2}\}, \ldots, \{H_{\text{low},N_x}, H_{\text{high},N_x}\}\) heights, which are recalculated at each time step solving (4), (5), (6), (7).

The initial condition is a closed fracture,
\[
\bar{w}(x, 0) = 0,
\] (14)
\[
H_{\text{low}}(x, 0) = H_{\text{high}}(x, 0) = \frac{l_{cp}}{2},
\] (15)
where \(l_{cp}\) is the height of the perforated layer \(\sigma_{cp}\).

The boundary conditions are
\[
\bar{w}(L', t) = 0,
\] (16)
\[
(\rho Q) \bigg|_{x=0} = q_{\text{in}},
\] (17)
where \(L'\) is the size of the calculation domain and \(q_{\text{in}}\) is an inlet mass flow rate.

Following hydraulic fracturing parameters were chosen for the numerical simulations,
\[
K_0 = 0.009 \text{ Pa} \cdot \text{s}^{n_0}, \quad n_0 = 0.9, \quad \sigma_1 = 16 \text{ MPa}, \quad \sigma_2 = 10 \text{ MPa}, \quad \sigma_3 = 13 \text{ Mpa}, \quad cp = 2, \quad l_{cp} = l_2 = 10 \text{ m}, \quad P_0 = 10 \text{ MPa}, \quad \Gamma_0 = 80 \%, \quad C_L = 0, \quad E = 25 \text{ Gpa}, \quad K_{IC} = 0, \quad \rho_l = 1000 \text{ kg/m}^3, \quad q_{\text{in}} = 200 \text{ kg/min}.
\]
\(K_0\) and \(n_0\) were chosen according to the measurements of the base fluid made in [17].
Figure 2. Fracture geometry at $t = 240$ s ($xy$-plane, $w(x,y)$ is colored using a rainbow palette, $\sigma(y)$ – brown-to-white palette), the length is 90 m, aspect ratio value $L/H(0)$ is 4.641.

Figure 3. Foam quality distribution at $t = 240$ s ($xy$-plane, $\Gamma(x)$ is colored using a rainbow palette, $\sigma(y)$ – brown-to-white palette).

The numerical results are shown in Fig. 2 and Fig. 3. The domain length is $L' = 100$ m, it is splitted into $N_x = 20$ cells, the time step is 0.24 s, the number of iterations for each time step is 15. It is necessary to pay attention to the foam quality distribution shown in Fig. 3. The model predicts 5% difference in the foam quality between the area near the wellbore and the tip of the fracture. Then the difference for the $K^{1/n}$ is about 10% and the value will increase as the minimum horizontal stresses of the reservoir decreases. This factor can significantly affect the transport properties of the proppant during the fracture propagation for the reservoirs with low $\sigma_i$. To investigate this effect the foam transport model should be developed and coupled with the foam fracturing model considered. In order to validate the numerical results and verify the model an experimental data of the foam fracturing in real wells will be used.

4. Conclusion and future work
Pseudo-three-dimensional cell-based model for hydraulic fracturing with foams is developed. The numerical simulations with the real foam parameters were performed using the suggested
rheology and compressibility models. The numerical results are showing that the effect of the presence of the gas phase in the fracturing fluid on the proppant transport properties may be taken into account by coupling the developed foam fracturing model with the proppant transport model. This will be done in future work.

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