Direct Estimate of the Gluon Polarization in the Nucleon

John Ellis∗
Theory Division, CERN, Geneva, Switzerland

Marek Karliner†
Cavendish Laboratory, Cambridge University, England
on leave from
School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv, Israel

Abstract

We make a first crude direct estimate of the net gluon polarization in the proton, $\Delta G$, combining data on the asymmetries in high-$p_T$ hadron production from the HERMES, SMC and COMPASS collaborations. Although these data sample a restricted range of $x$, they provide no hint that $\Delta G$ is large. Fixing the normalizations of different theoretical parametrizations using the hadron asymmetry data, we find typical central values of $\Delta G \sim 0.5$, with uncertainties of similar magnitude. Values of $\Delta G \geq 2$ are disfavoured by $\Delta \chi^2 \sim 9$ to 20, depending on the parametrization used.

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∗john.ellis@cern.ch
†marek@proton.tau.ac.il
1 Introduction

The spin structure of the proton is still uncertain, despite considerable experimental effort, theoretical ingenuity and several surprises [1]. It certainly differs from naive expectations formulated within the non-relativistic quark model, with strange quarks apparently polarized oppositely to the proton: $\Delta s < 0$ [2], and the quarks altogether apparently contributing only about 30\% of the proton spin: $\Delta \Sigma \equiv \Delta u + \Delta d + \Delta s \sim 0.3 \pm 0.1$ [2]. However, whether the remainder of the proton spin is due to gluons $\Delta G$ and/or orbital angular momentum $L_z$ remains an open question [1]. One particular theoretical interpretation of the proton spin is provided by chiral soliton models, which suggest that $\Delta \Sigma, \Delta G \sim 0$, with $L_z \sim 0.5$, up to corrections of higher order in $1/N_c$ and $m_s$ [3].

The question of the magnitude of $\Delta G$ was given a high profile by suggestions that gluons might be making a significant negative contribution to the net quark spins, [4]-[6] which might even be sufficient to explain all the negative polarization of the strange quarks: $\Delta s \sim -(\alpha/2\pi) \times \Delta G$, which would require $\Delta G \gtrsim 2 \, ^{*}$. Thus there are two conceptually interesting values for $\Delta G$: a value $\sim 1/2$ which would be comparable to the proton spin and hence make an important direct contribution to it, and a value $\sim 2$ which could ‘explain away’ the strange quark spin. It is worth keeping in mind that both of these effects require positive $\Delta G$, and that very few models suggest a negative value.

First attempts to estimate $\Delta G$ from data have been made using measurements of the polarized structure function $g_1$ at different momentum scales $Q^2$ to extract $\Delta G$ indirectly using next-to-leading order (NLO) QCD [8, 9]. These first attempts have been inconclusive, unable to exclude any of the interesting theoretical possibilities $\Delta G \sim 0, 0.5$ or 2. It is hoped that new structure function measurements, e.g., by COMPASS [10], will be able to refine this indirect extraction of $\Delta G$.

However, the main objective of COMPASS is the direct determination of $\Delta G$ via asymmetries in the production of $\bar{c}c$ and high-$p_T$ hadron pairs. The direct determination of $\Delta G$ is also a key objective of the RHIC polarized beam programme [11]. First results from both PHENIX at RHIC [12] and

\footnote{However, this suggestion is controversial, in particular because the gluon renormalization of the quark spin is scheme-dependent [7].}
COMPASS at both high \cite{13} and low $Q^2$ \cite{14} have recently been announced, the former on the asymmetry $A_{LL}$, and the latter on the asymmetry in high-$p_T$ hadron-pair production. The high-$Q^2$ COMPASS result was in fact the third result on this asymmetry, having been preceded by measurements by the HERMES \cite{16} and SMC \cite{17} collaborations.

The current PHENIX result is difficult to analyze in terms of $\Delta G$, since the error is still quite large, the central value is outside the physical region, and the relation to $\Delta G$ is not single-valued. The errors in the high-$Q^2$ measurement by COMPASS are no smaller than those of the previous HERMES and SMC measurements, but the errors in their low-$Q^2$ are significantly smaller, and COMPASS benefits in the comparison with HERMES from having more generous kinematics, which reduces potential issues related to higher-twist effects. In fact, as we discuss in more detail below, the COMPASS, SMC and HERMES data seem to be telling a consistent story.

In this paper we make a preliminary combination of the HERMES \cite{16}, SMC \cite{17} and COMPASS \cite{13,14} data, seeking a first direct indication of the possible magnitude of $\Delta G$ based on hadron-asymmetry data. Despite their limited precision, these data are already precise enough for such an analysis to be carried out within the framework of existing parametrizations of the possible polarized gluon distribution. On the other hand, they are not yet sufficiently precise to merit a fully-fledged NLO fit.

We find that $\Delta G$ is unlikely to be as large as was desired in attempts to ‘explain away’ the negative value of $\Delta s$, though a substantial gluonic contribution to the spin of the proton can certainly not be excluded. Using three different parametrizations of the polarized gluon distribution, we find central values of $\Delta G \sim 0.5$, with errors of similar magnitude. The suggestion that $\Delta G \geq 2$ is disfavoured by $\Delta \chi^2 \sim 9$ to 20, depending on the parametrization adopted, assuming that the polarized gluon distribution does not exhibit unexpected behaviour outside the limited $x$ range covered by the current experiments.

2 Available Experimental Information

We first review the relevant experimental information that is currently available. Three recent attempts to extract $\Delta G$ from NLO analyses of deep-
inelastic structure function data yield the following estimates:

\[
\begin{align*}
\Delta G &= 1.026 \pm 0.549 & \text{Set 3, Ref. [8]} & \equiv BB3 \\
\Delta G &= 0.931 \pm 0.669 & \text{Set 4, Ref. [8]} & \equiv BB4 \\
\Delta G &= 0.533 \pm 1.931 & \text{Ref. [9]} & \equiv AAC
\end{align*}
\]

The first two estimates were made by the same group, and the difference between their central values may be indicative of the systematic errors in this indirect approach to \(\Delta G\) that are associated with the choice of parametrization of the polarized gluon distribution. The larger error in the third estimate may mark a more realistic assessment of the systematic errors in this indirect approach. According to these analyses, each of the parametrizations used in [8, 9] would be compatible with \(\Delta G = 2\) at the 2-\(\sigma\) level.

The measurements of the high-\(p_T\) hadron production asymmetry that we use here are the following:

\[
\begin{align*}
\text{HERMES [16]} & : \quad \Delta G/G = 0.41 \pm 0.18 \pm 0.03 & 0.06 < x_G < 0.28, \\
\text{SMC [17]} & : \quad \Delta G/G = -0.20 \pm 0.28 \pm 0.10 & \langle x_G \rangle = 0.07, \\
\text{COMPASS [13]} & : \quad \Delta G/G = 0.06 \pm 0.31 \pm 0.06 & \langle x_G \rangle = 0.13, \\
& & Q^2 > 1 \text{ GeV}^2 \\
\text{COMPASS [14]} & : \quad \Delta G/G = 0.024 \pm 0.089 \pm 0.057 & \langle x_G \rangle = 0.095, \\
& & Q^2 < 1 \text{ GeV}^2
\end{align*}
\]

where we have indicated in each case the available information on the kinematic range of the measurement.

In addition, COMPASS has recently released preliminary results [15] for gluon polarization from open charm, based on the 2002-2003 data:

\[
\Delta G/G = -1.08 \pm 0.73 \text{ at } \langle x_G = 0.15 \rangle, \text{ RMS}=0.08.
\]

This channel has very little background, but very low statistics, resulting in large errors. We therefore do not use these preliminary open charm data in our fits, but note that that the central value of \(\Delta G\) is negative, providing additional qualitative evidence against a large positive \(\Delta G\).

In order to convert the measurements into estimates of \(\Delta G\), one needs to assume a suitable form for the unpolarized gluon distribution \(G(x, Q^2)\) and
specify the relevant momentum transfer scale $Q^2$. As our defaults, we use a recent MRST [18] gluon distribution and assume that $Q^2 \sim 5 \text{ GeV}^2$, and we discuss later the sensitivity to the assumed value of $Q^2$.

3 Fits to Asymmetry Data

In making our direct estimates of $Q^2$, we assume three trial forms for the polarized gluon distribution $\Delta G(x, Q^2)$, proposed by the groups mentioned earlier [8, 9]. The explicit expressions for the three parametrizations are available as FORTRAN codes that can be downloaded from the HEPDATA site [19]. Schematically, they can be written in the following form which emphasizes the overall normalization:

\begin{align}
\text{BB3 : } \Delta G(x, Q^2) &= A_{BB3} \cdot f_{BB3}(x, Q^2) \tag{BB3}, \\
\text{BB4 : } \Delta G(x, Q^2) &= A_{BB4} \cdot f_{BB4}(x, Q^2) \tag{BB4}, \\
\text{AAC : } \Delta G(x, Q^2) &= A_{AAC} \cdot f_{AAC}(x, Q^2) \tag{AAC}.
\end{align}

We treat the overall normalizations $A_{ijk}$ as free parameters, but retain as defaults the values of the other parameters chosen in [8] and the central fit values found in [9]. We then fit the overall normalizations to the three asymmetries (2), and hence determine the integrated gluon polarization $\Delta G$.

The limited precision of the presently available asymmetry data is insufficient to merit a fully-fledged multi-parameter NLO fit. Even with the much more precise deep-inelastic scattering data set, the authors of Ref. [8] found it necessary to give up on a fully-fledged fit to the parameters of the form they proposed for $\Delta G(x, Q^2)$, imposing by hand several constraints and leaving the overall normalization as the main free parameter. On the other hand, Ref. [9] did not impose supplementary constraints on their parametrization of the polarized gluon distribution, which is why the errors for $\Delta G$ that they quote are considerably larger.

Our procedure is clearly hostage to unforeseen properties of the polarized gluon distribution $\Delta G(x, Q^2)$ at values of $x$ outside the experimental ranges given in (2). This possibility would introduce a systematic error that we are unable to quantify.

Before fitting the values of the normalizations $A_{ijk}$ in the different parametrizations, we have first assumed values that correspond to an integral
The experimental results for $\Delta G/G$ compared with the theoretical parametrizations (3) using normalizations adjusted to yield $\Delta G = 2$. As seen in Fig. 1, each of the parametrizations (3) reproduces the general trend of the values of $\Delta G$ indicated by the data from the SMC, COMPASS and HERMES. However, each of the parametrizations also violates the unitarity bound $\Delta G(x) \leq G(x)$ at larger $x$. Ignoring this problem for the time being, we find $\chi^2 = 16.4, 10.5$ and 22.5 for the BB3, BB4 and AAC functions, respectively. If one caps each polarized distribution by the unitarity bound, keeping the same normalization at lower $x$ where there are measurements, the integrals are reduced to $\Delta G = 1.63, 1.99$ and 1.80, respectively. If one were to attempt to compensate for these reductions in $\Delta G$ by increasing the normalization factors $A_{BB3, BB4, AAC}$, the $\chi^2$ values would each be increased.

Much better fits to the parametrizations (3) are obtained when the overall normalizations are allowed to float: $\chi^2 = 1.1, 1.5$ and 2.5 and, as seen in Fig. 2 each parametrization now respects the unitarity bound $\Delta G(x) \leq G(x)$.

*We repeat that such a value was compatible with the NLO analyses of [3] [4], though not the central value suggested by their fits.
Figure 2: Experimental results for $\Delta G/G$ [10, 17, 13, 14] compared with the theoretical parametrizations [8, 9], with normalizations adjusted to yield best fits to the data.

for all $x$. In view of all the assumptions and uncertainties, we are reluctant to quote the differences in $\chi^2$, $\Delta \chi^2 = 15.3$, 9.0 and 20.0, as numbers of standard deviations by which $\Delta G \geq 2$ is disfavoured. In particular, the $\Delta \chi^2$ found for the AAC fit would change significantly if the full freedom of the parametrization were explored. However, it is clear that the current asymmetry data offer no hint in favour of the option that $\Delta G \geq 2$. The following are the best fit values of $\Delta G$ that we find for each of the parametrizations [3] and the formal errors:

$$
\begin{align*}
BB3 : \Delta G & = 0.31 \pm 0.43, \quad \chi^2 = 1.1, \\
BB4 : \Delta G & = 0.39 \pm 0.54, \quad \chi^2 = 1.5, \\
AAC : \Delta G & = 0.57 \pm 0.32 \quad \chi^2 = 2.5.
\end{align*}
$$

The best fits are compared with the data in Fig. 2.

In the preceding discussion we have compared the experimental data with polarized gluon parametrizations $\Delta G(x, Q^2)$ evaluated at a canonical value
$Q^2 = 5 \text{ GeV}^2$. However, each of the measured values of $\Delta G/G$ in eq. (2) results from averaging by the relevant experiments over a range of values of $x$ and $Q^2$. The limited information in the experimental papers and the low precision of the currently available data make it impossible to provide an accurate estimate of systematic errors due to this averaging. We can, however, get a semi-quantitative estimate of the relevant error by repeating the fitting procedure at several values of $Q^2$, as shown in Table I.

| $Q^2$ (GeV$^2$) | BB4 | BB3 | AAC |
|-----------------|-----|-----|-----|
| $\Delta G$     | $\chi^2$ (best fit) | $\chi^2$ ($\Delta G = 2$) | $\Delta G$ | $\chi^2$ (best fit) | $\chi^2$ ($\Delta G = 2$) | $\Delta G$ | $\chi^2$ (best fit) | $\chi^2$ ($\Delta G = 2$) |
| 1.5             | 0.31$\pm$0.40 | 1.6  | 0.46$\pm$0.44 | 10.7 | 13.4 | 0.58$\pm$0.31 | 2.2  | 22.8 |
| 2.0             | 0.33$\pm$0.43 | 1.5  | 0.33$\pm$0.37 | 1.2  | 21.2 | 0.57$\pm$0.31 | 2.3  | 23.4 |
| 5.0             | 0.39$\pm$0.54 | 1.5  | 0.31$\pm$0.43 | 1.1  | 16.4 | 0.57$\pm$0.32 | 2.5  | 22.5 |
| 10.0            | 0.43$\pm$0.62 | 1.5  | 0.30$\pm$0.48 | 1.1  | 13.5 | 0.58$\pm$0.33 | 2.6  | 20.6 |

*Fits to the HERMES, SMC and COMPASS $\Delta G/G$ data [4], for $Q^2 = 1.5$, 2, 5 and 10 GeV$^2$, using the parametrizations [3] of $G(x, Q^2)$. For each parametrization we list the best-fit value of $\Delta G$ and its $\chi^2$, as well as the $\chi^2$ value corresponding to $\Delta G = 2$. The trend shown by these fits is clear: for $1.5 \leq Q^2 \leq 10 \text{ GeV}^2$ (which includes the preferred value $Q^2 = 3 \text{ GeV}^2$ quoted in [14] and the three parametrizations [3], the best fit values $\Delta G$ range between 0.30$\pm$0.48 and 0.58$\pm$0.33. In all cases the $\Delta G = 2$ value is significantly disfavoured.

The current discussion is based on the COMPASS 2002 - 2003 data with $Q^2 > 1 \text{ GeV}^2$ [13] and approximately 10 times more data with $Q^2 < 1 \text{ GeV}^2$ [14]. The present statistics will be approximately doubled with the 2004 data, when the full 2004 set of all $Q^2$ COMPASS data is analyzed, the statistical error on $\Delta G/G$ is expected to go down to $\pm 0.05$ [13], compared with $\pm 0.31$ in the currently available 2002-03 data set. However, the inclusion of $Q^2 < 1 \text{ GeV}^2$ data introduces additional theoretical uncertainties.
Still, it is clear that the inclusion of the full 2004 data set will significantly increase the precision with which the first moment $\Delta G$ can be estimated. To see the effect of the increased precision of the future COMPASS data, we have repeated the current fits, setting the COMPASS $\Delta G/G$ error at $\pm 0.06$, while keeping the HERMES and SMC data unchanged. The expected error on $\Delta G$ shrinks down to $\pm 0.44$, $\pm 0.55$ and $\pm 0.16$, for the BB3, BB4 and AAC parametrizations, respectively. In the cases of the two BB parametrizations, the expected error reduction is rather modest, but we attribute this to the fact that in this exercise the fit includes only one $x$ value with increased precision. This underlines the importance of providing the high-precision values of $\Delta G/G$ over a range of $x$.

4 Summary and Prospects

We have made in this paper a first direct estimate of the net gluon polarization in the nucleon, based on hadron-asymmetry data in deep-inelastic scattering [13, 14, 16, 17]. Despite being very crude and incomplete in its kinematic coverage, this direct estimate has an error that is comparable with that provided indirectly by NLO analyses of deep-inelastic structure functions [8, 9]. We find a favoured value of $\Delta G \sim 0.5$, with a formal error of similar magnitude. Values of $\Delta G \geq 2$ are disfavoured by $\Delta \chi^2 \sim 9$ to 20, depending on the parametrization of the polarized-gluon distributions that is used.

There are good prospects for a significant improvement soon in the accuracy with which $\Delta G$ is known, thanks to new data from COMPASS and RHIC. The present data are insufficient to exclude strongly the hypothesis that all the apparent negative value of $\Delta s$ might be induced by gluons via renormalization in one particular scheme. The forthcoming data should be able to resolve this issue. However, they might not be able to determine whether gluons carry a large part of the nucleon spin, $\Delta G \sim 1/2$, or whether their contribution is as small as that due to the quarks, as expected qualitatively in chiral soliton models [3]. There are surely still many interesting twists and turns still to come in our understanding of the nucleon spin, but direct determinations of the gluon spin now seem poised to make an important step forward.
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References

[1] For reviews, see e.g., J. R. Ellis and M. Karliner, arXiv:hep-ph/9601280. G. Altarelli, R. D. Ball, S. Forte and G. Ridolfi, Acta Phys. Polon. B 29 (1998) 1145 [arXiv:hep-ph/9803237].

[2] G. K. Mallot, Proc. of the 19th Intl. Symp. on Photon and Lepton Interactions at High Energy LP99, eds. J. A. Jaros and M. E. Peskin, Int. J. Mod. Phys. A 15S1 (2000) 521 [eConf C990809 (2000) 521] [arXiv:hep-ex/9912040].

[3] S. J. Brodsky, J. R. Ellis and M. Karliner, Phys. Lett. B 206 (1988) 309; J. R. Ellis and M. Karliner, Phys. Lett. B 213 (1988) 73.

[4] A. V. Efremov and O. V. Teryaev, JINR-E2-88-287, Proc. Symposium on Hadron Interactions - Theory and Phenomenology, Bechyne, Czechoslovakia, Jun. 26 - Jul. 1, 1988; eds. J. Fischer et al., publ. by Inst. Phys., Czech. Acad. Sci.

[5] G. Altarelli and G. G. Ross, Phys. Lett. B 212 (1988) 391.

[6] R. D. Carlitz, J. C. Collins and A. H. Mueller, Phys. Lett. B 214 (1988) 229.

[7] R. L. Jaffe and A. Manohar, Nucl. Phys. B 337 (1990) 509.

[8] J. Blümlein and H. Böttcher, Nucl. Phys. B 636 (2002) 225 [arXiv:hep-ph/0203155].

[9] M. Hirai, S. Kumano and N. Saito [Asymmetry Analysis Collaboration], Phys. Rev. D 69, 054021 (2004) arXiv:hep-ph/0312112.
[10] G. Baum et al. [COMPASS Collaboration], COMPASS: A Proposal for a Common Muon and Proton Apparatus for Structure and Spectroscopy, CERN-SPSLC-96-14, http://wwwcompass.cern.ch/.

[11] G. Bunce, N. Saito, J. Soffer and W. Vogelsang, Ann. Rev. Nucl. Part. Sci. 50 (2000) 525 arXiv:hep-ph/0007218.

[12] S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett. 93 (2004) 202002 arXiv:hep-ex/0404027.

[13] C. Bernet [COMPASS Collaboration], arXiv:hep-ex/0405073; C. Schill, Measurement of the gluon polarisation $\Delta G/G$ at the COMPASS experiment, talk at SPIN 2004 - 16th Int. Spin Symposium, Trieste, Italy, Oct. 10 - 16, 2004 and hep-ex/0501056; Mario Lebrig, CERN PH seminar on behalf of the COMPASS Collaboration, Oct. 25, 2004, http://agenda.cern.ch/fullAgenda.php?ida=a044463.

[14] C. Bernet, hep-ex/0507049 talk at DIS05, Medison, Wisconsin, April 27 - May 1, 2005, http://agenda.hep.wisc.edu/fullAgenda.php?ida=a056.

[15] C. Quintas, talk at QCD 05, 4-8 July 2005, Montpellier, France.

[16] A. Airapetian et al. [HERMES Collaboration], Phys. Rev. Lett. 84 (2000) 2584 arXiv:hep-ex/9907020.

[17] B. Adeva et al. [Spin Muon Collaboration (SMC)], Phys. Rev. D 70 (2004) 012002 arXiv:hep-ex/0402010.

[18] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Phys. Lett. B 604, 61 (2004) arXiv:hep-ph/0410230.

[19] M. R. Whalley et al., Durham HEP Database Group, http://www-spires.dur.ac.uk/hepdata/pdf.html This site also gives access to earlier parametrizations of the polarized-gluon distribution: M. Glueck, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D 53 (1996) 4775 and Phys. Rev. D 63 (2001) 094005; T. Gehrmann and W. J. Stirling, Phys. Rev. D 53 (1996) 6100;
D. de Florian and R. Sassot, Phys. Rev. D 62 (2000) 094025;
E. Leader, A. V. Sidorov and D. B. Stamenov, hep-ph/0111267.