Properties of the decay $H \rightarrow \gamma\gamma$ using the approximate $\alpha_s^4$-corrections and the principle of maximum conformality

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(Dated: November 26, 2018)

The Higgs boson decay channel, $H \rightarrow \gamma\gamma$, is one of the most important channels for probing the properties of the Higgs boson. In the paper, we reanalyze its decay width by using the QCD corrections up to $\alpha_s^4$-order level. The principle of maximum conformality has been adopted to achieve a precise pQCD prediction without conventional renormalization scheme-and-scale ambiguities. By taking the Higgs mass as the one given by the ATLAS and CMS collaborations, i.e. $M_H = 125.09 \pm 0.21 \pm 0.11$ GeV, we obtain $\Gamma(H \rightarrow \gamma\gamma)|_{\text{LHC}} = 9.364^{+0.075}_{-0.075}$ GeV.

PACS numbers: 12.38.Bx, 13.66.Bc, 14.40.Pq

After the discovery of the Higgs boson at the Large Hadron Collider (LHC) in year 2012 [1–4], the remaining task is to confirm and learn more of its various properties either experimentally or theoretically. Theoretically, it is important to study its various decay modes within the framework of Standard Model (SM). As an important example, the Higgs decays into two photons, $H \rightarrow \gamma\gamma$, which could be observed at the LHC or a high luminosity linear collider, shall provide a clean platform for studying the Higgs properties.

The SM Higgs couples dominantly to the massive particles, the leading-order (LO) term of $H \rightarrow \gamma\gamma$ is already at the one-loop level, which inversely makes its higher-order QCD corrections very complex. At present, the LO, the next-to-leading order (NLO), the N$^3$LO, the approximate N$^3$LO, and the approximate N$^4$LO predictions on the decay width $\Gamma(H \rightarrow \gamma\gamma)$ have been done in Refs.[5–18]. As shall be shown below, even though only the fermionic contributions which form a gauge-invariant subset have been considered in the N$^3$LO and N$^4$LO terms [18], those state-of-art terms give us the opportunity to achieve a more precise prediction on $\Gamma(H \rightarrow \gamma\gamma)$.

According to the renormalization group invariance, the perturbatively calculable physical observable, corresponding to an infinite perturbative series, should be independent to any choices of the renormalization scheme and renormalization scale. Due to the mismatching of the running coupling ($\alpha_s$) and its coefficients at the same order, there is renormalization scheme-and-scale ambiguities for the fixed-order pQCD predictions. Those ambiguities always provide key errors for pQCD predictions, which are generally assumed to be decreased when more loop terms have been included. For example, Ref.[18] shows that when going from the LO level to the approximate N$^4$LO level, the renormalization scale dependence decreases continuously. However, such decreasing of scale dependence is caused by compensation of scale dependence for all orders. Conventionally, the scale is chosen as the typical momentum flow the process, there are however many problems for such conventional treatment [19, 20]. It is thus important to find a proper scale-setting way to set the renormalization scale so as to achieve an accurate fixed-order prediction.

In the literature, the principle of maximum conformality (PMC) [21–25] has been suggested to eliminate the renormalization scheme-and-scale ambiguities. Its key idea is to fix the renormalization scale based on the renormalization group equation (RGE); and when one applies the PMC, all non-conformal terms that govern the $\alpha_s$-running behavior of the pQCD approximant, should be systematically resummed. The PMC prediction satisfies the renormalization group invariance and all the self-consistency conditions of the renormalization group [26]. Since the PMC resums all of the $\{\beta_i\}$-terms, the divergent renormalon terms like $n l_0^i a^n$ generally disappear, naturally leading to a convergent pQCD series. Furthermore, the definite PMC conformal series can also be adopted to reliably predict the contributions from uncalculated high-order terms [27]. In this paper we apply the PMC to set the renormalization scale for the decay width $\Gamma(H \rightarrow \gamma\gamma)$ up to N$^4$LO level and show that an accurate scale-independent prediction can indeed be achieved. For clarity, we shall adopt the PMC single-scale approach (PMC-s) [28] to do the scale-setting.

The decay width of the Higgs decays into two photons at the one-loop level takes the form

$$\Gamma(H \rightarrow \gamma\gamma) = \frac{M_H^3}{64\pi} |A_W + \sum_f A_f|^2,$$

where $M_H$ is the Higgs mass, $A_W$ denotes the contribution which arises from purely bosonic diagrams, and $A_f$ stands for the contribution from the amplitudes with
f = (t, b, c, τ), which corresponds to top quark, bottom quark, charm quark and τ lepton, respectively.

The higher-order N2LO, N3LO and N4LO expressions given in Refs. [17, 18] are for the top-quark running mass (m_t). As has been argued in Ref. [29], we have to transform those terms into the ones for the top-quark pole mass (M_t) so as to avoid the entanglement of the {β_i}-terms from either the top-quark anomalous dimension or the RGE, thus avoiding the ambiguity in applying the PMC. Such transformation of mss can be done by using the relation between m_t and M_t, and the relation up to O(α_s^3) level can be read from Ref. [30].

For convenience, we rewrite the decay width as

\[ \Gamma(H \to γγ) = \frac{M_H^3}{64π} \left( A_{LO}^2 + A_{EW} \frac{α}{π} \right) + R(μ_r), \]  

(2)

where α is the fine-structure constant. The LO contribution A_{LO} and the electroweak (EW) correction A_{EW} are [17]

\[ A_{LO} = A_W^{(0)} + A_f^{(0)} + ̂A_t A_t^{(0)}, \]

(3)
\[ A_{EW} = 2A_{LO} A_{EW}^{(1)}, \]

(4)

where A_W^{(0)} is the purely bosonic contribution to the amplitude, A_f^{(0)} is the contribution from the amplitude with f = (b, c, τ), ̂A_t = 2Q_t^2 α\sqrt{2G_F}/π, G_F is the Fermi constant, and Q_t is the top-quark electric charge. All of them have been calculated in Refs. [5, 6], i.e.

\[ A_W^{(0)} = \frac{α\sqrt{2G_F}}{2π} \left[ 2 + \frac{3}{τ_W} + \frac{3}{τ_W} \left( 2 - \frac{1}{τ_W} \right) f(τ_W) \right], \]
\[ A_f^{(0)} = \sum f = b, c, τ \frac{α\sqrt{2G_F}}{πτ_f} Q_f^2 \left[ 1 + \left( 1 - \frac{1}{τ_f} \right) f(τ_f) \right], \]
\[ A_t^{(0)} = 1 + \frac{7}{30} + \frac{2}{21} r_t^2 + \frac{26}{525} r_t^4 + \frac{512}{17325} r_t^6 + \frac{1216}{63063} r_t^8 + \frac{3}{9555} r_t^{10}, \]

where

\[ f(τ) = \begin{cases} \text{Arcsin}^2(√τ) & \text{for } τ \leq 1 \\ \frac{-1}{2} \ln \frac{1+1-τ^{-1}}{1-1-τ^{-1}} - iπ^2 & \text{for } τ > 1 \end{cases} \]

Q_f denotes the electric charge for f = (b, c, τ), τ_W = M_H^2/(4M_W^2), τ_t = M_H^2/(4M_t^2) and τ_f = M_H^2/(4M_f^2), and the expression for the NLO electroweak term A_{EW}^{(1)} can be read from Refs. [30, 31].

The QCD corrections to the decay width Γ(H → γγ) are separately represented by R(μ_r), whose perturbative series up to (n + 1)-loop level can be written as

\[ R_n(μ_r) = \sum_{i=1}^{n} r_i(μ_r) a_i^*(μ_r), \]

(5)

where a_s = α_s/π, μ_r is the renormalization scale. The perturbative coefficients r_i in the \( \overline{\text{MS}} \)-scheme up to α_s^3-order level can be derived from Refs. [17, 18]. To apply the PMC, the n_f-power series (n_f being the active flavor number) in the coefficients r_i should be rewritten into conformal terms and non-conformal β_i-terms [24, 25], i.e.

\[ r_1 = r_{1,0}, \]
\[ r_2 = r_{2,0} + r_{2,1} β_0, \]
\[ r_3 = r_{3,0} + r_{3,1} β_0 + 2r_{3,1} β_0 + r_{3,2} β_0^2, \]
\[ r_4 = r_{4,0} + r_{4,1} β_2 + 2r_{4,1} β_1 + \frac{5}{2} r_{3,2} β_0 β_1, \]
\[ + 3r_{4,1} β_0 + 3r_{4,2} β_0^3 + r_{4,3} β_0^4, \]
\[ \cdots, \]

(11)

where the β-pattern at each perturbative order is a superposition of RGE, and all the coefficients r_{i,j} can be fixed from the n_f-power series at the same order by using the degeneracy relations among different orders. r_{i,0} are conformal coefficients which are exactly free of μ_r for the present channel, and r_{i,j\neq0} are non-conformal coefficients which are functions of μ_r, i.e.,

\[ r_{i,j} = \sum_{k=0}^{j} C_k^i r_{i-k, j-k} \ln^k(μ_r^2/M_H^2), \]

(12)

where r_{i,j} = r_{i,j}(μ_r = M_H). The needed {β_i}-functions also under the \( \overline{\text{MS}} \)-scheme are available in Refs. [32–40].

Following the standard procedures of the PMC single-scale approach [28], the pQCD corrections to the decay width Γ(H → γγ) can be simplified as

\[ R_n(μ_r)|_{PMC} = \sum_{i=1}^{n} r_{i,0} a_i^*(Q_+), \]

(13)

where Q_+ is the PMC scale. Using the known pQCD corrections up to N2LO level, Q_+ can be fixed up to next-to-next-to-leading-log (N^3LL) accuracy, i.e.,

\[ \ln \frac{Q_+^2}{M_H^2} = \sum_i T_i a_i^*(M_H), \]

(14)

whose first three coefficients with i = (0, 1, 2) can be determined by the known five-loop QCD corrections to the decay width Γ(H → γγ), which are

\[ T_0 = -\frac{r_{2,1}}{r_{1,0}}, \]
\[ T_1 = 2\left(\frac{r_{2,0} r_{2,1} + r_{1,0} r_{3,1}}{r_{1,0}^2} + \frac{r_{2,1}^2 - r_{1,0} r_{3,2}}{r_{1,0}^2}\right) β_0, \]
\[ T_2 = 4\left(\frac{r_{1,0} r_{2,0} r_{3,1} - r_{2,0}^2 r_{3,1}}{r_{1,0}^2} + \frac{3r_{1,0} r_{2,1} r_{3,0} - r_{1,0} r_{2,0} r_{3,2}}{r_{1,0}^2}\right) β_0, \]

(15–16)

\[ - \frac{r_{2,0} r_{2,1}^2 + 2(r_{2,0} r_{2,1}^2)}{r_{1,0}} β_0 + \frac{3(r_{2,1}^2 - r_{1,0} r_{3,2})}{2r_{1,0}^2} β_1, \]

where a_s = α_s/π, μ_r is the renormalization scale. The perturbative coefficients r_i in the \( \overline{\text{MS}} \)-scheme up to α_s^3-order level can be derived from Refs. [17, 18]. To apply the PMC, the n_f-power series (n_f being the active flavor number) in the coefficients r_i should be rewritten into conformal terms and non-conformal β_i-terms [24, 25], i.e.
Eq. (14) indicates that the scale $Q_*$ is free of $\mu_r$, together with the fact that the conformal coefficients $\hat{r}_{1,0}$ are also free of $\mu_r$, the net PMC prediction $R_n(\mu_r)|\text{PMC}$ is scale-independent. Thus the conventional scale ambiguity is eliminated. As a subtle point, due to unknown even higher-order terms in $Q_*$ perturbative series, there is residual scale dependence for $Q_*$. However such kind of residual scale dependence is different from conventional renormalization scale ambiguity, which is usually negligible due to both the $\alpha_s$-suppression and the exponential suppression. This property has been confirmed in many PMC applications done in the literature.

To do the calculation, we take the following ones as their central values [41]: the $W$-boson mass $M_W = 80.379$ GeV, the $\tau$-lepton mass $M_\tau = 1.77686$ GeV, the $b$-quark pole mass $M_b = 4.78$ GeV, the $c$-quark pole mass $M_c = 1.67$ GeV, the $t$-quark pole mass $M_t = 173.07$ GeV, and the Higgs mass $M_H = 125.9$ GeV. The Fermi constant $G_F = 1.1663787 \times 10^{-5}$ GeV$^{-2}$ and the fine structure constant $\alpha = 1/137.03599913$. We adopt the four-loop $\alpha_s$-running and $\alpha_s(M_Z = 91.1876$GeV$) = 0.1181$ to fix the $\alpha_s$-running behavior.

Firstly, we present the total decay width $\Gamma(H \rightarrow \gamma \gamma)$ up to N$^4$LO level under conventional scale-setting in Figs. (1, 2). Fig. (1) shows that under conventional scale-setting, the scale dependence becomes smaller when more loop terms are included. This agrees with the conventional wisdom that by finishing enough higher-order calculation, one may achieve a desirable scale-independent prediction. The N$^4$LO total decay width under conventional scale-setting gives

$$\Gamma(H \rightarrow \gamma \gamma)|_{\text{Conv.}} = 9.626^{+0.002}_{-0.002}\text{ KeV},$$

where central value is for $\mu_r = M_H$, and the renormalization scale error is for $\mu_r \in [M_H/2,2M_H]$.

It is noted that such nearly scale-independent for the N$^4$LO total decay width $\Gamma(H \rightarrow \gamma \gamma)$ under conventional scale-setting is caused by large cancellations of the scale dependence among different orders. This can be explicitly seen from Table I, in which the individual decay widths at LO+EW, N$^2$LO, N$^3$LO and N$^4$LO levels are presented. We define a parameter $\kappa_i$ to measure the scale dependence of the separate decay widths at different orders, i.e.

$$\kappa_i = \left| \frac{\Gamma_i|_{\mu_r=M_H/2} - \Gamma_i|_{\mu_r=2M_H}}{\Gamma_i|_{\mu_r=M_H}} \right|,$$

where the subscript $i$ stands for NLO, N$^2$LO, N$^3$LO, N$^4$LO and Total decay widths, respectively. Table I shows that under conventional scale-setting,

$$\kappa_{\text{NLO}} = 20\%, \quad \kappa_{\text{N}^2\text{LO}} = 1.2 \times 10^{3}\%, \quad \kappa_{\text{N}^3\text{LO}} = 19\%, \quad \kappa_{\text{N}^4\text{LO}} = 1.6 \times 10^{3}\%.$$  

Large magnitude of $\kappa$ indicates that under conventional scale-setting, there are large scale errors for each orders, and such kind of scale errors cannot be affected by the high-order terms.

On the other hand, as shown by Fig. (2), the PMC prediction is almost scale-independent for each order, and the PMC prediction on $\Gamma(H \rightarrow \gamma \gamma)$ quickly approaches its “physical” value due to a faster convergence than conventional pQCD series. Because the magnitude of the newly added N$^3$LO and N$^4$LO terms are only about 28% and 4% of that of the N$^2$LO terms whose magnitude is small, our previous N$^2$LO PMC prediction agrees with the present prediction [42] with high precision. Table I shows that after applying PMC scale-setting, both the separate decay widths and the total decay width are nearly unchanged for $\mu_r \in [M_H/2,2M_H]$. The N$^4$LO total decay width under PMC scale-setting is

$$\Gamma(H \rightarrow \gamma \gamma)|_{\text{PMC}} = 9.626\text{ KeV}.$$  

The four-loop and five-loop fermionic contributions are helpful to set an accurate PMC scale. The nearly scale-independence for each order under PMC scale-setting is caused by the fact that the effective PMC scale $Q_*$ can be
fixed up to \(N^2\)LL-accuracy by using the known five-loop pQCD corrections, i.e.

\[
\ln \left( \frac{Q^2}{M_H^2} \right) = 1.321 - 4.271 \alpha_s(M_H) + 21.029 \alpha_s^2(M_H). 
\]  

(23)

We present \(Q^*_s\) in Fig. (3), in which \(Q^{(1)}_s\) is computed at the LL accuracy, \(Q^{(2)}_s\) is at the NLL accuracy and \(Q^{(3)}_s\) is at the \(N^3\)LL accuracy.

Secondly, the Padé approximation approach (PAA) provides a practical way for promoting a finite series to an analytic function [43-45], which has recently been suggested to give a reliable prediction of uncalculated high-order terms by using the PMC conformal series [27].

As an attempt, following the same method described in detail in Ref.[27], we give a PAA+PMC prediction for \(R_n(M_H)\) by using the preferable \(\left[0/(n-1)\right]\)-type Padé series, which is in Fig. (4).

Then the total decay width

\[
\Gamma_5(H \rightarrow \gamma\gamma)|_{\text{PMC}} = \left[9.626 \pm 5.354 \times 10^{-5}\right] \text{KeV}, 
\]  

(25)

where the error is the PAA+PMC prediction of uncalculated high-order pQCD contributions, which is negligible.

![FIG. 3. The determined effective scale \(Q_s\). \(Q^{(1)}_s\) is at the LL accuracy, \(Q^{(2)}_s\) is at the NLL accuracy and \(Q^{(3)}_s\) is at the \(N^3\)LL accuracy.](image)

![FIG. 4. Comparison of the exact ("EC") and the predicted \([0/(n-1)]\)-type "PAA" pQCD approximant \(R_n(M_H)\) under PMC scale-setting. It shows how the PAA predictions change when more loop-terms are included.](image)

![FIG. 5. The PMC prediction of the decay width \(\Gamma(H \rightarrow \gamma\gamma)\) versus the Higgs mass \(M_H\).](image)
the Higgs mass as the one given by the ATLAS and CMS collaborations [46, 47], i.e. \( M_H = 125.09 \pm 0.21 \pm 0.11 \) GeV, we obtain

\[
\Gamma(H \rightarrow \gamma\gamma)|_{\text{LHC}} = 9.364^{+0.076}_{-0.075} \text{ KeV}, \tag{26}
\]

FIG. 6. The fiducial cross section \( \sigma_{\text{fid}}(pp \rightarrow H \rightarrow \gamma\gamma) \) using the \( \Gamma(H \rightarrow \gamma\gamma) \) up to N^3LO level. The LHC-XS prediction [48], the ATLAS measurements [49–51] and the CMS measurement [52] are presented as a comparison.

Thirdly, as an application of the \( H \rightarrow \gamma\gamma \) decay width, we predict the “fiducial cross section” of the process \( pp \rightarrow H \rightarrow \gamma\gamma \), which has been predicted by the LHC-XS group under the conventional scale-setting [48] and has been measured by ATLAS and CMS collaborations with increasing integrated luminosities [49–52]. A PMC prediction has previously been given in Ref. [53] by using \( \Gamma(H \rightarrow \gamma\gamma) \) up to N^2LO level. Taking the same parameters as those of Refs. [48, 53, 54], e.g. \( M_H = 125 \) GeV and \( \Gamma_{H} = 173.3 \) GeV, and by using the present \( \Gamma(H \rightarrow \gamma\gamma) \) up to N^4LO level, we obtain \( \sigma_{\text{fid}}(pp \rightarrow H \rightarrow \gamma\gamma) = 30.1^{+2.3}_{-2.2} \text{ fb} \), \( 38.3^{+2.9}_{-2.8} \text{ fb} \), and \( 85.8^{+5.7}_{-5.3} \text{ fb} \) for the proton-proton center-of-mass collision energy \( \sqrt{s} = 7, 8 \) and 13 TeV, respectively. Here the errors are dominated by the error of the Higgs inclusive cross section. A comparison of the recent experimental data is put in Fig. (6). A better agreement with the data at \( \sqrt{s} = 7, 8 \) TeV can be achieved by applying the PMC. The ATLAS and CMS measurements at \( \sqrt{s} = 13 \) TeV are still of large errors and are in disagreement, and the PMC prediction prefers the CMS data [52].

As a summary, the PMC uses the basic RGE to set the \( \alpha_s \)-running behavior; the resultant conformal series is independent to the initial choice of renormalization scale and renormalization scheme, and thus eliminates the conventional scheme-and-scale ambiguities. By using the known QCD corrections up to the known approximate five-loop level, we can fix the effective PMC scale up to N^2LL level and an accurate scheme-and-scale independent prediction for the decay width \( \Gamma(H \rightarrow \gamma\gamma) \) can be achieved. The residual scale dependence due to unknown high-order terms are negligible, and the PMC prediction is almost scale-independent for each order. The PAA+PMC treatment indicates that our present N^4LO decay width \( \Gamma(H \rightarrow \gamma\gamma)|_{\text{PMC}} \) is already approaches its “physical” value, since the contribution of the uncalculated even higher-order is negligible due to a more convergent renormalon-free conformal series.

Acknowledgments: This work was supported in part by Natural Science Foundation of China under Grant No.11625520, No.11547010 and No.11705033; by the Project of Guizhou Provincial Department of Science and Technology under Grant No.2016GZ42963 and the Key Project for Innovation Research Groups of Guizhou Provincial Department of Education under Grant No.KY[2016]028 and No.KY[2017]067.

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