A Single-lined Spectroscopic Binary Companion to an Active and Deep Contact Binary in a Quintuple Stellar System

W.-P. Liao, S.-B. Qian, X.-D. Shi, L.-J. Li, N.-P. Liu, J.-J. He, L. Zang, and P. Li

Abstract

V410 Aur is a known deep and low-mass-ratio contact binary with a spectroscopically tertiary component and a visual companion. However, the physical and orbital properties of the tertiary are unknown. We constructed a curve with 117 new eclipse times and those collected from the literature, which shows a cyclical variation with a period of 25.44 (±1.17) yr and a projected semimajor axis of 0.348 (±0.0021) days while it undergoes a long-term period decrease at a rate of $dP/dt = -1.58 \times 10^{-7}$ days$^{-1}$. The cyclical variation is analyzed for the light-travel time effect. The minimum mass of the third body is determined as 1.39 (±0.13) $M_\odot$ that is much larger than the inferred value (0.97 $M_\odot$) of spectroscopic investigation, which indicates that the spectroscopically tertiary is a single-lined spectroscopic binary with an unseen component. The maximum orbital semimajor axis 6.19 (±0.67) au of the third body is determined. Gaia detected a visual companion to V410 Aur at practically the same distance from the Sun nicely confirming the physical bond. These results reveal that V410 Aur contains a single-lined spectroscopic binary with a visual companion in a quintuple stellar system. TESS photometric solutions confirmed that V410 Aur is a deep overcontact binary with a fill-out factor of 73.83 (88)% where the additional light contribution is about 24.80 (18)% of the massive component. The parabolic variation in the $(O-C)$ curve may be caused by the mass transfer from the massive component to the less massive one in the deep overcontact binary.

Supporting material: machine-readable tables

1. Introduction

A series of radial velocity studies of W UMa-type binary stars observed at the Dominion Astrophysical Observatory (DAO) and at the David Dunlap Observatory (DDO) had been carried out by Rucinski’s team since 1999: Lu & Rucinski (1999, Paper I); Rucinski & Lu (1999, Paper II); Rucinski et al. (2000, Paper III); Lu et al. (2001, Paper IV); Rucinski et al. (2001, Paper V); Rucinski et al. (2002, Paper VI); Rucinski et al. (2002, Paper VII); Rucinski et al. (2003, Paper VIII); Pych et al. (2004, 2011, Paper IX); Rucinski et al. (2005, 2007a, Paper X); Pribulla et al. (2006, Paper XI); Pribulla et al. (2007a, Paper XII); Rucinski et al. (2008, Paper XIII); Pribulla et al. (2009a, Paper XIV); Pribulla et al. (2009b, Paper XV). They selected close binary systems with orbital periods shorter than 1 day, brighter than 11 mag, with declinations $>-20^\circ$, and accessible to medium-resolution spectroscopic studies with 2 m class Northern Hemisphere telescopes. After this spectroscopic program of radial velocity measurements had been conducted, a series of follow-up investigations were carried out in order to study the evolution of W UMa-type contact binary stars. A series of papers presented combined spectroscopic and photometric orbital solutions for those objects in the above W UMa Project (Kreiner et al. 2003; Baran et al. 2004; Zola et al. 2004; Gazanas et al. 2005, 2006; Zola et al. 2005, 2010; Gazeas et al. 2011; papers I–VIII, respectively). Finally, a total of 138 W UMa binaries have good combined spectroscopic and photometric parameters. In addition, to study the formation of very close binary stars with periods shorter than 1 day, a series of research on “contact binaries with additional components” are presented (D’Angelo et al. 2006; Pribulla & Rucinski 2006; Rucinski et al. 2007b; Papers I–III, respectively). They found that the presence of close companions is a very common phenomenon for very close binaries with orbital periods shorter than one day, and the tertiary companions may facilitate or enable the formation of contact binaries via acquiring and/or absorbing the angular momentum during the evolution of multiple systems. The above series of work provides a good opportunity to study the physical and orbital characteristics of the multiple systems by light-travel-time effect (LTTE) analysis.

Although third components around some binaries have been detected in the spectroscopic observations and/or photometric solutions, they are not clearly seen in the LTTE. In order to study the physical and orbital characteristics of the tertiary companions, we have carried out long-term multicolor photometric observations of some well-known spectroscopic triple and higher systems (e.g., Qian et al. 2006, 2008; Zhu et al. 2011, 2013a, 2013b; Liao et al. 2019, 2021), including V410 Aur in the present paper.

The light variability of V410 Aur was discovered by the Hipparcos satellite (HIP 023337). It shows a typical light curve of W UMa-type binary with an orbital period of 0.3663 days, and an epoch for the photometric primary eclipse of HJD 2448500.1760 was given. The Hipparcos parallax has a large error $4.77 \pm 5.39$ mas. However, the Gaia mission will make up this lack. In reality, the Gaia mission (EDR3 and DR2) had provided parallaxes of V410 Aur and of its visual companion at $1^\prime.732$ (Gaia Collaboration et al. 2016a, 2016b, 2018, 2021), which will make us derive the precise distance of V410 Aur.
and its visual companion in the final discussion. The radial velocity study of V410 Aur was given by Rucinski et al. (2003). They determined spectroscopic orbital elements as follows: it is an A-type contact system; \( q_{ph} = 0.144(13); \) 
\((M_1 + M_2) \sin^3 i = 1.42(10); \) 
third light contribution \( l_3 = 26(1);\) 
spectral type Sp. = G0/2 V. They also found that the mean velocity of the close binary has a significant difference from the average velocities of the third component.

Photometric investigations of the system were carried out by several authors. Selam (2004) analyzed the light curves of V410 Aur based on the Hipparcos photometry with the light-curve synthesis method proposed by Rucinski. The preliminary photometric elements were given for this system: third light contribution \( l_3 = 27.7;\) 
mass ratio \( q = 0.25;\) 
degree of contact \( f = 50;\) 
and orbital inclination \( i = 87°.5.\) In the study of a series of deep, low-mass-ratio overcontact binary systems, Yang et al. (2005) model light curves of V410 Aur observed with the 1 m telescope at Yunnan Observatories by adding one spot on the primary star and a third light. Their light curves displayed asymmetries that were not obvious. They found V410 Aur is an A-subtype W UMa binary with low mass ratio and deep degree of overcontact: \( q_{ph} = 0.1428(28); i = 78°.6(0.8); f = 52.4(6.2); l_3 = 0.93(0.27); l_3R = 1.19(0.24);\) 
and \( l_31 = 0.61(0.24);\) 
The absolute physical parameters were determined and listed in their Table 9. Next year, photometric observations of the system were obtained by Gazeas et al. (2006) with the 0.40 m telescope at the University of Athens Observatory, Athens, Greece. A small negative O’Connell effect (O’Connell 1951) was displayed in their light curves. A spotted plus third light solution was given by them: \( q = 0.137; (assumed); i = 80°.6(1.0); f = 72%; l_3 = 15%.\) The absolute physical parameters were calculated and listed in their Table 6. The light curve of Oh et al. (2007), however, displayed a positive O’Connell effect, with the first light maximum at phase 0.25 being brighter than the second maximum at phase 0.75. A spotted plus third light solution was given: \( q = 0.1602(16); i = 85°.92(0.63); f = 56.5%; l_3 = 1.4%.\) The absolute physical parameters were calculated and listed in their Table 2. Until 2017, Luo et al. (2017) published new photometric observations of V410 Aur from 2014 December 19 to 2015 February 8 (about 52 day span of time); four sets of multicolor light curves were obtained. The shape of their light curves exhibit many peculiar features: the depths of two minima vary from season to season; the light intensity at minimum is variable, strongly sloping with phase; the light maxima are asymmetric and vary as well. They suggested that V410 Aur is a very active W UMa system. Their two spotted plus third light solutions suggested a moderate fill-out factor of \( f \approx 29%;\) 
an orbital inclination of 82°.2, and third light contribution of \( l_3 < 2.5%.\) The two jumps of the dominant spot distortion phase between phases 0.0 and 0.5 were explained by flip-flop activity. For an easy comparison, individual photometric elements are listed in Table 1.

In the series of “Contact Binaries with Additional Components,” Pribulla & Rucinski (2006) stated that the spectroscopic tertiary companion is possibly SB1 due to its RV variability by those extant data. In the spectroscopic search for faint tertiaries of D’Angelo et al. (2006), they fitted the spectrum of the contact binaries and checked whether adding a spectrum of a fainter tertiary improves the fit significantly by using the data set that was used in Papers I–IX of the series “Radial Velocity Studies of Close Binary Stars” carried out by Rucinski’s team. The tertiary flux ratio of 0.22 was inferred from its contribution to the broadening function (in the multiple star catalog, it was 0.38 (Tokovinin 1997)). With the tertiary flux ratio, they determined the absolute magnitude of the tertiary. Then, they thought that the fainter tertiary should be on the main sequence. With the main-sequence mass–luminosity relation (they used the Tables 15.7 and 15.8 of Cox (2000)), the tertiary mass \( M_3 \) was inferred as 0.97 \( M_\odot \) from the absolute V-band magnitude. In a search for companions using Canada–France–Hawaii Telescope adaptive optics (AO), Rucinski et al. (2007b) stated V410 Aur has a distant visual companion at \( \alpha = 10°.716(6) \) (that was confirmed by the Gaia mission at \( \alpha = 10°.732, \) and the estimated spectral type of the visual companion is K2 V.

In summary, V410 Aur has been well analyzed by spectroscopy and photometry, an additional companion is detectable both spectroscopically and photometrically, but a measurable LTTE in orbital period changes is lacking. The first orbital period investigations were performed by Yang et al. (2005), and they suggested a long-term period increase by using only nine available eclipsing times. Since then, no studies on orbital period changes have been published. Some photometric surveys, such as SuperWASP (SWASP; Butters et al. 2010), the Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2015), and the Kamogata Wide-field Survey (KWS)\(^5\) have observed V410 Aur. Considering the fact that new high-precision data have been accumulated, we will reanalyze the orbital period changes of V410 Aur based on a total of 256 eclipse times. Expect to find a measurable LTTE in the orbital period investigations, and study the physical and orbital characteristics of the tertiary companion. In addition, we will analyze the new 26 day continuous observation TESS light curves of the system and discuss the evolutions of it.

### Table 1

| Parameter | Selam (2004) | Yang et al. (2005) | Gazeas et al. (2006) | Oh et al. (2007) | Luo et al. (2017) |
|-----------|--------------|-------------------|---------------------|-----------------|-----------------|
| \( T_1 (K) \) | ... | 6040 | 5890 | 5942 | 5950 |
| \( q (M_2/M_1) \) | 0.25 | 0.1428 | (0.0028) | 0.137 | 0.1602 | 0.144 |
| \( q_r (\mu) \) | 87.5 | 78.6(0.8) | 80.6(1.0) | 85.92 | (0.63) |
| \( l_3 (%) \) | 27.7 | ... | ... | ... | ... |
| \( l_3R (%) \) | ... | 17.6(1.0) | 1.34 | 1.9 |
| \( l_3V (%) \) | ... | 12.4(1.0) | 1.41 | 1.8 |
| \( l_3K (%) \) | ... | 13.8(1.0) | 1.43 | 1.7 |
| \( f (\%) \) | 50 | 52.4(6.2) | 72 | 56.5 | 29 |
| Spot | N | One | One | One | Two |

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\(^5\) [http://kws.cetus-net.org/](http://kws.cetus-net.org/)
National Astronomical Research Institute of Thailand (NARIT), which is located at Lijiang Gaomeigu station of YNOs (LJ-70). The Andor iKon-L 936 camera was equipped on LJ-70. Reduction of the CCD images was made with the DAOPHOT package of IRAF in a standard process including bias and flat corrections and aperture photometry. Then differential magnitudes between V410 Aur and the comparison star were derived. Based on a series of eclipse light curves, a parabolic fitting method was used to determine the times of light minimum in different bands that are nearly the same as those determined with the Kwee & van Woerden (1956) method. Sixteen new eclipse times were obtained and are listed in Table 2.

2.2. Spectra Observations and the Stellar Atmospheric Parameters of V410 Aur

In order to determine stellar atmospheric parameters of V410 Aur, the spectra observations were performed on 2021 February 7 and October 20 by using the Beijing Faint Object Spectrograph and Camera (BFOSC) mounted on the 2.16 m telescope in Xinglong station of National Astronomical Observatories, Chinese Academy of Sciences. Low-dispersion spectrometer BFOSC and grism G7 at a slit width of 1″.8 were used, the corresponding spectral resolution is about ~700, and the observation wavelength range is 400–680 nm (Fan et al. 2016). The exposure time was set as 600 s. We use IRAF to process the observation images and extract the spectra. Normalized flux was obtained, and the atmospheric absorption lines were corrected. The observed spectra are displayed in Figure 1 with black lines. The observed spectra are usually regarded as the spectral lines of the primary component due to the low resolution (Tian & Chang 2020). We use the University of Lyon Spectroscopic Analysis Software (ULySS; Koleva et al. 2009) through full spectra fitting with model spectra generated by an interpolator with the ELODIE library (Prugniel & Soubiran 2001) to obtain the stellar atmospheric parameters. The fitted spectra are shown with blue lines in Figure 1. The mean values of stellar atmospheric parameters according to two observations were derived as follows: $T_{\text{eff}} = 5760 \pm 114$ K, log $g = 3.77 \pm 0.02$ cm s$^{-2}$, and $[Fe/H] = -0.09 \pm 0.01$ dex. Because the

| HJD 2400000+ | Error (days) | Method | Reference | HJD 2400000+ | Error (days) | Method | Reference | HJD 2400000+ | Error (days) | Method | Reference |
|-------------|--------------|--------|-----------|-------------|--------------|--------|-----------|-------------|--------------|--------|-----------|
| 48500.1760  | 0.0010       | CCD    | (1)       | 54140.37224 | 0.00059      | CCD    | SWASP     | 56632.74250 | 0.00050      | CCD    | (29)      |
| 51284.4460  | 0.0010       | CCD    | (2)       | 54141.4713  | 0.00070      | CCD    | SWASP     | 56645.38491 | 0.00263      | CCD    | KWS       |
| 51467.8112  | 0.0060       | CCD    | (3)       | 54142.38531 | 0.00069      | CCD    | SWASP     | 56646.29919 | 0.00296      | CCD    | KWS       |
| 51630.6444  | 0.0060       | CCD    | (3)       | 54168.3972  | 0.0002       | Pe     | (19)      | 56646.66291 | 0.00271      | CCD    | KWS       |

Note. This is a sample of the full table, which is available in its entirety in machine-readable format.

(This table is available in its entirety in machine-readable form.)

Figure 1. Spectra of V410 Aur observed on 2021 February 07 and October 20. The black and blue lines represent the observed and fitted spectra, respectively.
secondary component is quite faint and the primary component contributes the most light to the total system, the stellar atmospheric parameters obtained by ULYSS can be used to describe the atmospheric characteristic of the primary component. The temperature of the primary star is taken as 5760 K in the latter process of solving the TESS light-curve solutions.

3. Orbital Period Changes

Orbital period changes of V410 Aur have been given by Yang et al. (2005), and a simple continuous period increase was displayed by using just several data points. The physical and orbital characteristics of the tertiary component were unclear. A detailed analysis of the orbital period changes for the system is urgently required.

By using the methods mentioned by Shi et al. (2021a) and Liu et al. (2015), 27 times of light minima from the SWASP data, 44 times of light minima from the TESS data, and 30 times of light minima from the KWS data were obtained in the present paper. It is shown that the SWASP and TESS data have high precision. Combined with 16 new times of light minima determined by our observations and those collected from the literature, a total of 256 times of light minima spanning about 30 yr are listed in Table 2, and they are published in the machine-readable format. Those shown in the second column are errors of times of light minima, in the third column are the observational methods, where “CCD” refers to charge-coupled device photometry and “Pe” to photoelectric photometry.

The \((O-C)\) curves were computed with the following linear ephemeris given in \((O-C)\) gateway created by Drs. Anton Paschke (database administrator) and Bc.Luboš Bráň (WEB/SQL programator) on the referenced website.\(^6\)

\[ Min1 = 2448500.176 + 0^d.3663562 \times E. \] (1)

In \((O-C)\) curves, the magenta open circles refer to data from the literature, red open circles to the TESS data, dark yellow ones to the SWASP data, and wine red ones to the KWS data; while blue, green, and black dots to 60 cm, 1 m, and 70 cm telescopes data, respectively. As one can see from the \((O-C)\) diagram, there is a periodic variation that exists in \((O-C)\) diagram, which was not discovered by the previous \((O-C)\) trend (Yang et al. 2005); we use the following general equation to fit the \((O-C)\) diagram (Irwin 1952):

\[
O - C = \Delta T_0 + \Delta P_0 E + \frac{\beta}{2} E^2 \\
+ A \left[ \left( 1 - e_3^2 \right) \sin (\nu + \omega) \right. \\
\left. + e_3 \cos \nu \right] + e_3 \sin \omega \\
\]

\[
= \Delta T_0 + \Delta P_0 E + \frac{\beta}{2} E^2 \\
+ A \left[ \sqrt{1 - e_3^2} \sin E \cos \omega + \cos E \sin \omega \right] \\
\]

The Kepler equation is as follows:

\[
M = E^* - e_3 \sin E^* = \frac{2\pi}{P_3} (t - T_3) , \] (2)

where the meaning of each parameter is described in a recently published paper of Liao et al. (2021).

The corresponding results of Equations (2) and (3) are exhibited in Table 3. There are two cases of analysis results, i.e., (1) Case A, linear term plus cyclical variation \((\beta = 0, \) without quadratic fit); (2) Case B, quadratic term plus cyclical variation \((\beta \neq 0, \) with quadratic fit). The fitting results are displayed in Figures 2 and 3, respectively. It can be seen from these two figures that the data can be fitted satisfactorily in both cases. Further, in order to compare the goodness between the linear (Case A) and quadratic (Case B) fits of the \((O-C)\) diagrams, the Bayes information criterion (BIC) method (e.g., Burnham & Anderson 2002) was adopted. The BIC method was defined as

\[
BIC = n \log (\text{RSS}/n) + k \log n , \] (4)

where RSS is the residual sum of squares, \(n\) is the data number, and \(k\) is the number of the fitted parameters. The values of RSS and BIC of the Case A and Case B fits are shown in Table 3. Statistically, the quadratic fit (Case B) is better than the linear fit (Case A) for V410 Aur.

4. 26 day Continuous TESS Light-curve Solutions

The 26 day continuous time-series photometry data of TESS for V410 Aur, from BJD 2458816 to 2458841 with a 30 minutes cadence, could provide a good opportunity to study the continuous variations in the light curve of the system. We obtained data from TESSCut (Brasseur et al. 2019) and extracted the object star and background by applying a percentile threshold method (Liu N. P. et al. 2021, in preparation). The

| Parameter | Value | Unit |
|-----------|-------|------|
| Case A: linear term plus cyclical variation \((\beta = 0)\) | | |
| Revised epoch, \(T_0\) | 2448500.1504(±0.0011) | HJD |
| Revised period, \(P_0\) | 0.3663566(±0.0000001) | days |
| Eccentricity, \(e_3\) | 0.18(±0.01) | \(\ldots\) |
| Long-term change of the orbital period, \(\beta\) | (0 assumed) | \(\text{day}^{-1}\) |
| Projected semimajor axis, \(A\) | 0.0300(±0.0004) | days |
| Orbital period, \(P_3\) | 22.59(±0.20) | yr |
| Orbital phase, \(\varphi\) | 96.5(±5.47) | degree |
| Time of periastron passage, \(T_3\) | 2457331.4(±0.01) | HJD |
| RSS | 3.49 \times 10^{-6} | |
| BIC | –1997 | |
| Mass function, \(f(m)\) | 0.28(±0.01) | \(M_\odot\) |
| Mass, \(M_{\text{min}}\) | 1.26(±0.02) | \(M_\odot\) |
| Orbital semimajor axis, \(a_{\text{max}}\) | 5.91(±0.14) | au |
| Case B: quadratic term plus cyclical variation \((\beta \neq 0)\) | | |
| Revised epoch, \(T_0\) | 2448500.1420(±0.0022) | HJD |
| Revised period, \(P_0\) | 0.3663586(±0.0000004) | days |
| Eccentricity, \(e_3\) | 0.17(±0.02) | \(\ldots\) |
| Long-term change of the orbital period, \(\beta\) | –1.58(±0.41) \times 10^{-10} | \(\text{day}^{-1}\) |
| Projected semimajor axis, \(A\) | 0.0348(±0.0021) | days |
| Orbital period, \(P_3\) | 25.44(±1.17) | yr |
| Longitude of the periastron passage, \(\omega_3\) | 164.5(±12.10) | degree |
| Time of periastron passage, \(T_3\) | 2456568.5(±0.02) | HJD |
| RSS | 0.929 \times 10^{-6} | |
| BIC | –2141 | |
| Mass function, \(f(m)\) | 0.34(±0.06) | \(M_\odot\) |
| Mass, \(M_{\text{min}}\) | 1.39(±0.13) | \(M_\odot\) |
| Orbital semimajor axis, \(a_{\text{max}}\) | 6.19(±0.67) | au |

\(^6\) http://var2.astro.cz/ocgate/
TESS data of V410 Aur were shown in the upper panel of Figure 4. We divided the TESS data into four segments, i.e., 58816-58822, 58823-58827, 58828-58834, and 58835-58841. Then, using the equation of $\text{BJD} = \text{BJD}_0 + P \times E$ (here BJD is the observing time, BJD$_0$ is the reference time, $P$ is the orbital period, and $E$ is the cycle number), we calculated the phase of observations and plotted phase light curves in the lower panel of Figure 4. The light curves corresponding to the four segments are displayed in black rectangle (□), magenta open circles (○), red plus (+), and blue cross (×), respectively.

As one can see from the lower panels of Figures 4 and 5, it has a typical light curve of W UMa-type binary with total-eclipse characteristics, which will enable us to determine reliable photometric solutions of this system (Liao et al. 2013, 2017; Li...
Several peculiarities are found in V410 Aur: (1) The light curves of V410 Aur are always asymmetric during 26 day continuous observations, exhibiting a noticeable negative O’Connell effect, where the first light maximum at phase 0.25 (Max I) is fainter than the second maximum at phase 0.75 (Max II). It is a common and complex phenomenon in eclipsing binaries that Max I and Max II are not equal in height. Any mode that can cause local brightness change of the surface of the binary components, such as spots, mass transfer, and so on, could cause this phenomenon (Shi et al. 2021b). Moreover, it is found that the height of the Max II in the light curves went lower from the first to the fourth light curves, while those of the Max I were unchanged. (2) and the depth of the primary minimum went shallower from the first to the fourth light curves, while the depth of the secondary minimum went deeper. In the first two sets of light curves (∼12 days), the light minima at phase 0.5 were brighter than the light minima at phase 1.0. In the third light curve, however, two minima depths gradually became close. Until the last light curve, the shape of the light curve began to show two minima of almost the same depths. This changing behavior was also discovered by Luo et al. (2017) in their span.
of 52 day observations; (3) near the minima at phase 0.5 in our last three sets of light curves, a sloping behavior existed, which was also found in Figure 1 of Luo et al. (2017).

The Wilson–Devinney (W–D) program (Van Hamme & Wilson 2007; Wilson 2012; Wilson & Devinney 1971) was employed to model the TESS light curves of V410 Aur. Thanks to Professor Van Hamme, who is one of the authors of the W–D program, we were kindly provided with the band files of TESS. Some observations taken in the long cadence mode (30 minutes cadence) can cause a change in the light variation amplitude and the shape of the minima for short-period eclipsing binaries, perhaps even enough to lose their resemblance to the original features. In the process of solving these light-curve solutions, the phase smearing effect should

Figure 5. Left panel: the light curves (black open circles) of the four segments and their theoretical fitting light curves (red solid lines) with third light \( l_3 \) and spot computed using the W–D code, and residuals (observed minus calculated light curves) from the solutions are shown in black dots. Right panel: the theoretical geometrical structure at phase 0.25.
be considered, the control parameter NGA (Number of Gaussian quadrature abscissas in phase or time smearing simulation) should be set as 2–10, and NGA = 3 should easily suffice for most applications (WD program manual, Zola et al. 2017). Due to the TESS data of V410 Aur having a 30 minutes cadence, the phase smearing effect was considered, and the control parameter NGA was set to 3. The temperature of the primary star (Star 1) was determined to be 5760 K by the present authors, which is the mean of the two observations. The primary star is a late type, so its gravity-darkening and bolometric albedos coefficients are set to $g_{1,2} = 0.32$ and $A_{1,2} = 0.5$. The bolometric and bandpass limb-darkening coefficients were obtained by using an internal computation with the logarithmic law. According to the radial velocity studies of Rucinski et al. (2003), the mass ratio was fixed at $q_{\text{SIP}} = 0.144$, and the spectroscopic mass ratio will make light-curve solutions gain in quality.

Because of the fact that the TESS light curves of V410 Aur are asymmetric and the massive tertiary in the system could contribute light to the total system, we decided to calculate a photometric solution with the spot and third light. Therefore, in the process of fitting the light curves, the free parameters include the phase shift, the orbital inclination ($i$), the effective surface temperature of the secondary ($T_s$), the monochromatic luminosity of the primary ($L_1$), the modified dimensionless surface potential ($\Omega_1 = \Omega_2$ for contact configuration), the third light ($L_3$), and the spot parameters (the latitude $\theta$, the longitude $\psi$, the radius $r$, and the temperature factor $T_s/T_\odot$). (See note in Table 4 for details.) Iterations were performed until convergence. The final solutions for four sets of light curves were derived and listed in column 2 to column 5 of Table 4, and the mean values of the four solutions are shown in column 6. The theoretical light curves described with a cool spot and large third light were displayed as red solid lines in Figure 5, in which residuals (observed minus calculated light curves) from the solutions are shown in black dots. Those in the right panel are the theoretical geometrical structures at phase 0.25.

### 5. Discussions and Conclusions

V410 Aur is a known triple system in which the spectroscopic signature of a fainter tertiary companion is obviously found by Rucinski et al. (2003); they obtained $q_{\text{SIP}} = 0.144(13)$ and $(M_1 + M_2)\sin^3i = 1.42(10) M_\odot$. In the DDO observations of Rucinski et al. (2003), a relatively wide spectrograph slit of 300 $\mu$m corresponded to the angular size on the sky of 1″.8 and the projected width of 4 pixels was used (see their Paper II). It is suggested that the slit edge to the center is ~0″.9; thus, the visual companion at a separation of ~0″.7 detected in the AO observations (Rucinski et al. 2007b) and Gaia mission may not have been completely in the slit. Moreover, from the three-Gaussian-model fits to the broadening profiles of D’Angelo et al. (2006), the visual companion may contribute to the spectra, but some of its light fell outside the slit. Based on the tertiary flux ratio, a spectroscopic tertiary mass of 0.97 $M_\odot$ was inferred by them, which would not correspond to the visual companion; furthermore, the mass of a closer tertiary was more or less overestimated due to some of the visual companion’s light that fell into the slit. In AO and Gaia detection, the

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**Table 4**

Photometric Solutions of the Four TESS Average Light Curves Using the W–D Code

| Parameter | 58816–58822 | 58823–58827 | 58828–58834 | 58835–58841 | Mean Value |
|-----------|--------------|--------------|--------------|--------------|-------------|
| $T_1$ (K) | 5760 (fixed) | 5760 (fixed) | 5760 (fixed) | 5760 (fixed) | 5760 (fixed) |
| Phase shift | 0.9932(1) | 0.9934(2) | 0.9926(2) | 0.9914(1) | 0.9927(1) |
| $q$ (M$_2$/M$_1$) | 0.144 (fixed) | 0.144 (fixed) | 0.144 (fixed) | 0.144 (fixed) | 0.144 (fixed) |
| $T_s/T_1$ | 1.0288(18) | 1.0408(22) | 1.0418(21) | 1.0543(20) | 1.0414(10) |
| $\Omega_1 = \Omega_2$ | 2.0132(17) | 2.0188(17) | 2.0229(15) | 2.017(18) | 2.0180(8) |
| $L_3/(L_1 + L_2 + L_3)_\text{TESS}$ | 0.8209(11) | 0.8168(12) | 0.8175(11) | 0.8096(12) | 0.8162(6) |
| $L_3/(L_1 + L_2 + L_3)_\text{TESS}$ | 0.2385(35) | 0.2432(36) | 0.2473(34) | 0.2631(35) | 0.2480(18) |
| $r/(pole)$ | 0.5303(26) | 0.52859(47) | 0.52745(42) | 0.52912(50) | 0.52885(23) |
| $r_1$(side) | 0.59217(75) | 0.58953(75) | 0.58769(67) | 0.59037(80) | 0.58994(37) |
| $r_1$(back) | 0.61876(92) | 0.61552(92) | 0.61328(82) | 0.61655(98) | 0.61603(46) |
| $r_2$(pole) | 0.23539(59) | 0.23339(59) | 0.23197(53) | 0.23399(64) | 0.23369(29) |
| $r_2$(side) | 0.24857(33) | 0.24610(73) | 0.24435(65) | 0.24683(78) | 0.24646(36) |
| $r_2$(back) | 0.3216(28) | 0.3129(24) | 0.3072(20) | 0.3154(27) | 0.3143(12) |
| $f$ (%) | 79.9(1.8) | 72.9(1.8) | 68.6(1.6) | 74.8(1.9) | 73.83(88) |
| $r_1$ | 0.58026(41) | 0.57787(41) | 0.57616(37) | 0.57860(44) | 0.57822(20) |
| $r_2$ | 0.26373(84) | 0.26077(76) | 0.25866(64) | 0.26166(84) | 0.26212(39) |
| $R_3/R_1$ | 0.4545(15) | 0.4513(14) | 0.4489(12) | 0.4522(15) | 0.4517(07) |
| Mean residual | 0.00637 | 0.00790 | 0.00595 | 0.00615 | 0.00639 |

**Note.** The errors are expressed in parenthesis, in units of last decimal places quoted. $\theta$ is the latitude of a starspot center, measured from 0 to π radians at the south pole. $\psi$ is the longitude of a starspot center, measured counterclockwise (as viewed from above the +z axis) from the line of the star centers from 0° to 360°. $r$ is the radius of a starspot, in radian. $T_s/T_\odot$ is the temperature factor of a spot, which specifies the ratio of the local spot temperature to the local temperature that would be obtained without the spot.

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https://faculty.fiu.edu/~vanhamme/kdc2015/
magnitude differences between the visual companion and the binary are close to 1: $\Delta K = 1.091$ mag (AO) and $\Delta G = 0.95$ mag (Gaia), respectively. They are almost consistent with an estimated spectral type of K2 V for the visual companion.

The contribution of the third light was inferred to be about 22%–38%; in a spectroscopic study, these inferred values are likely lower than the true contribution because of the fact that the visual companion’s all light did not fall into the spectrograph slit. In previous photometric solutions, some, except for two (Gazeas et al. 2006, about 15%; Selam 2004, about 28%) of those, are only about 1% or 2%. By considering an orbital inclination of $i = 83^\circ.22(14)$ determined by us, the masses of binary components were determined as $M_1 = 1.25 M_\odot$, and $M_2 = 0.18 M_\odot$. The physical parameters of V410 Aur can be calculated by applying the Equations (1)–(3) given by Brancewicz & Dworak (1980): $R_1 = 1.40 \pm 0.02$ $R_\odot$, $L_1 = 1.91 \pm 0.16$ $L_\odot$, $R_2 = 0.64 \pm 0.01$ $R_\odot$, $L_2 = 0.47 \pm 0.03$ $L_\odot$.

In the present paper, the orbital period changes are studied in detail for the first time by using a total of 256 times of light minima. Finally, two cases of linear term plus cyclical variation and quadratic term plus cyclical variation with an eccentric orbit of a tertiary companion were selected to analyze the orbital period changes. As a result, both cases can fit the $(O - C)$ curve well, while the latter one shows larger BIC values. Thus, the results of Case B were used for the following discussions. It is discovered that the $(O - C)$ curve of V410 Aur shows a cyclical variation with a period of 25.44 ($\pm 1.17$) yr and a projected semimajor axis of 0.0348 ($\pm 0.0021$) days while it undergoes a long-term period decrease at a rate of $dP/dt = -1.58 \times 10^{-7}$ days yr$^{-1}$. The cyclical variation was analyzed for the LTTE via the presence of a third body (Liao & Qian 2010). Using the following equation and the masses of binary components, $f(m) = \frac{(M_1 \sin i_3)^3}{(M_1 + M_2 + M_3)^2} = \frac{4\pi^2}{GP^2} \times (a'_3 \sin i_3)^3$, the LTTE solutions for V410 Aur were derived and listed in the Table 1. The minimum mass of the third body at $i_3 = 90^\circ$ was estimated to be $M_{3 \text{min}} = 1.39 \pm 0.13$ $M_\odot$, and the maximum orbital semimajor axis was 6.19 ($\pm 0.67$) au. The third body is orbiting the central eclipsing binary in an eccentric orbit $(e_3 = 0.17)$. According to the relationship given by Kippenhahn et al. (2012) $L \sim M^{3.35}$, for such a massive third body, if it is a single 1.39 solar mass main-sequence star, it should be brighter than the binary components and would contribute more than 50% light to the total system. The fact that the lower contribution of this large third body may be explained that the third body might itself be a binary consisting of a visible component and an unseen component, which is like that in some analyses (e.g., Liao & Qian 2009; Liao et al. 2012, 2017). This result indicates that the companion detected by the timing variability, identical to the spectroscopically tertiary, is a single-lined spectroscopic binary (SB1), and it agrees with the description of Pribulla & Rucinski (2006) that the tertiary companion is possibly a SB1.

EDR3 and DR2 of Gaia mission released parallaxes $\pi$ of V410 Aur and its visual companion at 1".732 (Gaia Collaboration et al. 2016a, 2016b, 2018, 2021), which are listed in Table 5. The corresponding Gaia distances were calculated and listed in column 4. Mean distances of V410 Aur and its visual companion were derived as 175.7 ($\pm 0.9$) pc and 171.7 ($\pm 1.2$) pc. The visual companion to V410 Aur detected by Gaia IS at practically the same distance from the Sun, nicely confirming the physical bond. Considering $A = a_12 \sin i_3 / c$, the orbital semimajor axis of V410 Aur was calculated as $a_{12} = 6.02 \pm 0.36$ au. Then the distance between the LTTE companion and V410 Aur is 12.21 au. Based on Gaia parameters, the distance between the visual companion and V410 Aur was calculated to be 297.30 ($\pm 1.72$) au. The angular separation between the LTTE companion and V410 Aur was computed to be $0''$.0696 ($\pm 0''$.003), which is closer than that between the visual companion and V410 Aur (1''$.732$). It is clear that the observed LTTE cannot be caused by the visual companion. These results reveal that the deep-contact binary V410 Aur contains a SB1 with a distant visual companion in a quintuple stellar system (2 + 2 + 1).

By analyzing two low-dispersion spectra obtained with the 2.16 m telescope at Xinglong Station, the stellar atmospheric parameters of V410 Aur were determined. The temperature of primary star was fixed at $5760$ K during fitting the 26 day continuous observation TESS light curves for V410 Aur. The spotted plus third light solutions were derived. It is confirmed that V410 Aur is a deep overcontact binary with a large third light contribution: $f = 73.83$ (88%), $L_3 = 24.80$ (18%). This third light contribution found in the TESS photometric band is larger than most of the earlier photometric solutions, and it should contain the light contribution of closer SB1 ($L_3$) and the light contribution of the visual companion ($L_4$). Considering $L_1 + L_2 = 2.38 L_\odot$, then $L_1 + L_2$ was inferred to be 0.785 $L_\odot$. Since the estimated spectral type of the visual companion is K2 V (Rucinski et al. 2007b), $L_1$ was estimated as 0.512 $L_\odot$ by using the above relationship of $L \sim M^{3.37}$; thus, $L_3 = 0.27$ $L_\odot$. Therefore, the third body might be a SB1 with a compact component. The physical properties of this tertiary companion need to be further studied. The shapes of light curves are those of typical totally eclipsing contact binaries with asymmetric maxima. There are several variations and peculiar features, including changing the height of the Max II, the depths of minima, the light intensity at minimum, which were also discovered by Luo et al. (2017). The continuous light-curve variations of V410 Aur indicate that it is a very active W UMa system, and the variations should be caused by the evolution of...
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