The decays of $B$ mesons to two-body hadronic final states are analyzed within the context of broken flavor SU(3) symmetry, extending a previous analysis involving pairs of light pseudoscalars to decays involving one or two charmed quarks in the final state. A systematic program is described for learning information from decay rates regarding (i) SU(3)-violating contributions, (ii) the magnitude of exchange and annihilation diagrams (effects involving the spectator quark), and (iii) strong final-state interactions. The implication of SU(3)-breaking effects for the extraction of weak phases is also examined. The present status of data on these questions is reviewed and suggestions for further experimental study are made.
I. INTRODUCTION

Recently [1] we analyzed the decays of $B$ mesons to two light pseudoscalar mesons $P$ within the context of flavor SU(3) [2, 3, 4]. We proposed that information on phases of the Cabibbo-Kobayashi-Maskawa (CKM) matrix could be obtained from the study of time-independent measurements of decay rates, and found that the SU(3) relations were of use in interpreting and anticipating CP-violating asymmetries in these decays.

The analyses in [1] made use of an overcomplete graphical description of amplitudes involving dominant tree $T$, color suppressed $C$, and penguin $P$ contributions, and smaller exchange $E$, annihilation $A$, and penguin annihilation $PA$ terms. Particularly useful relations followed from the neglect of these last three terms. For a $B$ meson to decay via these diagrams directly the two quarks in the meson must find each other, and hence the contributions of these diagrams were expected to be suppressed by a factor of $f_B/m_B \approx 5\%$. Tests of this assumption that relied on $B$ decays to the pseudoscalar mesons were proposed in [1].

One can also test for the absence of exchange and annihilation graphs in the decays [2, 3, 4] of $B$'s to one light pseudoscalar $P$ and one charmed meson $D$. (In these processes there is no analogue of the penguin annihilation graph.) Furthermore, various SU(3)-breaking effects can be studied in a manner not possible when both final-state mesons are light. Since a single product of CKM elements is involved in such decays, relative phases between amplitudes are a signal of final-state interactions, which thus may be probed with the help of amplitude triangles [5]. When two charmed quarks occur in the final state, as in the decays $B \to D\bar{D}$ or $B \to \eta_c P$, the analysis becomes even simpler.

The strangeness-preserving processes $B \to P\bar{D}$, involving the CKM matrix element product $V_{cb}^*V_{ud}$, have typical branching ratios of several parts in $10^3$. They dominate the much rarer $B \to PP$ processes, which involve $V_{ub}$ and are expected to have branching ratios of order $10^{-5}$. The strangeness-changing processes $B \to P\bar{D}$, involving the combination $V_{cb}^*V_{us}$, as well as the rarer processes $B \to PD$, involving the combinations $V_{ub}^*V_{cs}$ or $V_{ub}^*V_{cd}$, also provide useful information, as do the decays of $B$ mesons to $D\bar{D}$ or $\eta_c P$ final states.

Several issues arose in [1] which can be addressed in part by extending the analysis to decays involving one or two charmed quarks in the final state. We address these issues in the present paper:

(1) How large are SU(3)-breaking effects in two-body $B$ meson decays?

(2) Are contributions due to exchange ($E$) and annihilation ($A$) diagrams really negligible?

(3) Can one determine final-state interactions in a manner independent of CKM phases? One such determination involves the decays $B^+ \to \pi^+\bar{D}^0$, $B^0 \to \pi^+D^-$, and $B^0 \to \pi^0\bar{D}^0$ [3].

In Refs. [3, 4] we discussed several ways in which weak CKM phases could be determined using SU(3) triangle relations involving a variety of $B \to PP$ processes. In this paper we discuss how these analyses are affected by SU(3)-breaking effects. We will see that, for the most part, SU(3) breaking can be taken into account in a systematic way. In Ref. [8], the question was raised as to the importance of electroweak penguin
diagrams in the determination of weak phases. Although this is an important point, it is somewhat orthogonal to the main thrust of the present work. We therefore discuss it in a separate paper [9].

The impatient reader may turn directly to our conclusions (Sec. VIII) for the answers (many of which will require new measurements) to the above questions. For more leisurely perusal, the following sections may be of interest as well.

In Section II we review our SU(3) analysis [1] of $B \to PP$ processes, and extend it to $B \to P\bar{D}$, $B \to PD$, $B \to D\bar{D}$, and $B \to \eta_{c}P$ decays. The SU(3) analysis will lead to many useful relations. For all except the $B \to PP$ processes, equivalent relations can be obtained by simply replacing one or both of the pseudoscalar mesons in the final state with a vector meson. Of course, when both are vector mesons, amplitude relations will hold separately for different helicity or angular momentum states, limiting their usefulness.

The language employed involves a graphical notation equivalent to decomposition into SU(3) representations. We introduce this notation and apply it to the case of first-order SU(3) breaking in Sec. III. Measurements which test these relations, both in the presence and in the absence of exchange (E) and annihilation (A) contributions, are noted in Sec. IV. In Sec. V we examine how SU(3)-breaking effects affect the extraction of weak CKM phases. We discuss amplitude triangle relations and their implications for strong final-state interactions in Sec. VI. The present status of relevant data on two-body $B$ decays, and some future experimental prospects, are reviewed in Sec. VII.

In our approach, the graphical description is used to implement flavor SU(3) symmetry and linear SU(3) breaking in the most general form. Some of our relations follow purely from this linearly broken symmetry, while others depend on an additional (testable) dynamical assumption that permits us to ignore certain contributions. This is complementary to the model-dependent studies of two-body $B$ decays carried out in the past [11]. Such model calculations are based on further assumptions of factorizable hadronic matrix elements of the effective Hamiltonian and on specific hadronic form factors. This leads to stronger predictions than in our approach – absolute branching ratios, for example. However, the model-dependent description is also expected to involve a number of different kinds of uncertainties [11], so that it is probably only sufficient for order-of-magnitude rate estimates.

II. NOTATION AND SU(3) DECOMPOSITION

A. Definitions of states

We recapitulate some results of [1]. Taking the $u$, $d$, and $s$ quarks to transform as a triplet of flavor SU(3), and the $-\bar{u}$, $\bar{d}$, and $\bar{s}$ to transform as an antitriplet, we define mesons in such a way as to form isospin multiplets without extra signs:

$$\pi^+ \equiv u\bar{d}, \quad \pi^0 \equiv (d\bar{d} - u\bar{u})/\sqrt{2}, \quad \pi^- \equiv -d\bar{u}, \quad (1)$$

$$K^+ \equiv u\bar{s}, \quad K^0 \equiv d\bar{s}, \quad (2)$$
For reasons discussed in more detail in [1], we do not consider decays involving $\eta$ or $\eta'$ in the present paper. Since these states are octet-singlet mixtures, we would have to introduce additional SU(3) reduced matrix elements or additional graphs to describe such decays.

The $B$ mesons and their charge-conjugates are defined as

$$
B^+ \equiv \bar{b}u \quad , \quad B^0 \equiv \bar{b}d \quad , \quad B_s \equiv \bar{b}s \quad ,
$$

$$
B^- \equiv -b\bar{u} \quad , \quad \bar{B}^0 \equiv b\bar{d} \quad , \quad \bar{B}s \equiv b\bar{s} \quad .
$$

(4)

Charmed mesons are taken to be

$$
D^0 \equiv -c\bar{u} \quad , \quad D^+ \equiv c\bar{d} \quad , \quad D^+_s \equiv c\bar{s} \quad ,
$$

$$
\bar{D}^0 \equiv \bar{c}u \quad , \quad D^- \equiv \bar{c}d \quad , \quad D^-_s \equiv \bar{c}s \quad .
$$

(5)

B. Decomposition in terms of SU(3) amplitudes

1. $B \rightarrow PP$ decays were discussed in [1]. The weak Hamiltonian operators associated with the transitions $\bar{b} \rightarrow \bar{u}u\bar{q}$ and $\bar{b} \rightarrow \bar{q} (q = d \text{ or } s)$ transform as a $3^*$, 6, or $15^*$ of SU(3). When combined with the triplet light quark in the $B$ meson, these operators then lead to the following representations in the direct channel:

$$
3^* \times 3 = 1 + 8_1 \quad ,
$$

(6)

$$
6 \times 3 = 8_2 + 10 \quad ,
$$

(7)

$$
15^* \times 3 = 8_3 + 10^* + 27 \quad .
$$

(8)

These representations couple to the symmetric product of two octets (the pseudoscalar mesons), containing unique singlet, octet, and 27-plet representations, so that the decays are characterized by one singlet, three octets, and one 27-plet amplitude. Separate amplitudes apply to the cases of strangeness-preserving and strangeness-changing transitions.

2. $B \rightarrow P\bar{D}$ decays, involving the quark subprocess $\bar{b} \rightarrow \bar{c}u\bar{q}$ ($q = d \text{ or } s$), are characterized by a weak Hamiltonian transforming as a flavor octet. When combined with the initial light quark (3), this leads to final states transforming as $3, 6^*$, and $15$ representations of SU(3). These are also the representations formed by the combination of the final octet light pseudoscalar meson and triplet $\bar{D}$ meson. Thus, there are three independent SU(3) amplitudes, transforming as $3, 6^*$, and $15$, for these decays.

3. $B \rightarrow PD$ decays, involving the quark subprocess $\bar{b} \rightarrow \bar{u}c\bar{q}$ ($q = d \text{ or } s$), are characterized by a weak Hamiltonian transforming as a $3$ or $6^*$ representation. When combined with the initial light quark (3), this leads to the following representations:

$$
3 \times 3 = 3^*_1 + 6 \quad ,
$$

(9)
\[ 6^* \times 3 = 3_2^* + 15^* \]

which each have unique couplings to the final light pseudoscalar (8) and charmed meson (3'), whose tensor product involves 3*, 6, and 15* representations. Thus, these processes are characterized by four invariant amplitudes.

4. \( B \to D\bar{D} \) and \( B \to \eta_c P \) decays are characterized by transitions giving rise to a single light antiquark, transforming as an antitriplet. When combined with the initial quark, this antiquark can form a singlet or an octet in the direct channel. Thus, there will be two SU(3)-invariant amplitudes \((1 + 8)\) characterizing the decays \( B \to D\bar{D} \) but only one (8) characterizing the decays \( B \to \eta_c P \), where \( P \) is an octet member.

C. Decomposition in terms of diagrams

Diagrams describing \( B \) decays are a particularly useful representation of SU(3) amplitudes. There are six distinct diagrams, shown in Fig. 1. They consist of:

- a (color-favored) “tree” amplitude \( T \),
- a “color-suppressed” amplitude \( C \),
- a “penguin” amplitude \( P \),
- an “exchange” amplitude \( E \),
- an “annihilation” amplitude \( A \), and
- a “penguin annihilation” amplitude \( PA \).

Of course, not all diagrams contribute to all classes of decays. In particular,

1. All six diagrams contribute to the decays \( B \to PP \) (see Fig. 1), but only five distinct linear combinations appear in the amplitudes.

2. Three diagrams (\( T, C, E \)) contribute to the decays \( B \to P\bar{D} \) (see Fig. 2).

3. Four diagrams (\( \hat{T}, \hat{C}, \hat{E}, \hat{A} \)) contribute to the decays \( B \to PD \) (see Fig. 2).

4. Three diagrams (\( \hat{T}, \hat{P}, \hat{E} \)) contribute to the decays \( B \to D\bar{D} \), but they only appear in two combinations (\( \hat{T} + \hat{P}, \hat{E} \)). Only one diagram (\( \hat{C} \)) contributes to the decays \( B \to \eta_c P \) (see Fig. 3).

As expected, one obtains the same number of diagrams (or combinations of diagrams) contributing to the various classes of \( B \) decays as was found previously using group theory.

In Tables 1-8, in the “SU(3) invariant” column, we present the decomposition in terms of diagrams of all the \( B \) decays in the four classes:

1. \( B \to PP \) (Tables 1 and 2).
Figure 1: Diagrams describing $B \rightarrow PP$ decays for $\Delta S = 0$ processes (unprimed amplitudes) or $|\Delta S| = 1$ processes (primed amplitudes). The $q$ quark denotes any member of the SU(3) triplet, $u, d, s$, whereas $q'$ denotes $d$ or $s$.

2. $B \rightarrow P\bar{D}$ (Tables 3 and 4),

3. $B \rightarrow PD$ (Tables 5 and 6),

4. $B \rightarrow D\bar{D}$ and $B \rightarrow \eta_c P$ (Tables 7 and 8).

Note that, for $B \rightarrow PP$, we include only the contributions from the $T$, $C$ and $P$ diagrams. As discussed in [1], and reiterated in the introduction, we expect the $E$, $A$ and $PA$ diagrams to be suppressed by $f_B/m_B \approx 5\%$. We will be testing the validity of this assumption with the $B \rightarrow P\bar{D}$ system. The decomposition of $B \rightarrow PP$ decays in terms of all six diagrams can be found in [1].
Figure 2: Diagrams describing decays $B \to P\bar{D}$ or $B \to PD$ governed by CKM factors $V_{cb}^*V_{ud}$ or $V_{cb}^*V_{us}$ ($\approx \lambda V_{cb}^*V_{ud}$) (barred amplitudes), and $V_{ub}^*V_{cs}$ or $V_{ub}^*V_{cd}$ ($\approx -\lambda V_{ub}^*V_{cs}$) (tilded amplitudes). The $q$ quark denotes any member of the SU(3) triplet, $u,d,s$, whereas $q'$ denotes $d$ or $s$.

Figure 3: Diagrams describing $B \to D\bar{D}$ (a,c,d), or $B \to \eta_c P$ (b) decays with $|\Delta S| = 1$ (unprimed amplitudes, $q' = s$) or with $\Delta S = 0$ (primed amplitudes, $q' = d$). The $q$ quark denotes any member $u,d,s$ of the SU(3) triplet.
Table 1: Decomposition of $B \to PP$ amplitudes for $\Delta C = \Delta S = 0$ transitions in terms of graphical contributions shown in Fig. 1. Amplitudes $E$, $A$, and $PA$ (and the corresponding SU(3)-breaking terms) are neglected.

| Final state | SU(3) invariant | SU(3) breaking |
|-------------|----------------|---------------|
| $B^+ \to \pi^+ \pi^0$ | $-(T + C)/\sqrt{2}$ | $P$ |
| $K^+ \bar{K}^0$ | | $P_3$ |
| $B^0 \to \pi^+ \pi^-$ | $-(T + P)$ | |
| $\pi^0 \pi^0$ | $-(C - P)/\sqrt{2}$ | |
| $K^0 \bar{K}^0$ | $P$ | $P_3$ |
| $B_s \to \pi^+ K^-$ | $-(T + P)$ | $-(T_2 + P_2)$ |
| $\pi^0 \bar{K}^0$ | $-(C - P)/\sqrt{2}$ | $-(C_2 - P_2)/\sqrt{2}$ |

Table 2: Decomposition of $B \to PP$ amplitudes for $\Delta C = 0$, $|\Delta S| = 1$ transitions in terms of graphical contributions shown in Fig. 1. Amplitudes $E'$, $A'$, and $PA'$ (and the corresponding SU(3)-breaking terms) are neglected.

| Final state | SU(3) invariant | SU(3) breaking |
|-------------|----------------|---------------|
| $B^+ \to \pi^+ K^0$ | $P'$ | $P'_1$ |
| $\pi^0 K^+$ | $-(T' + C' + P')/\sqrt{2}$ | $-(T'_1 + C'_1 + P'_1)/\sqrt{2}$ |
| $B^0 \to \pi^- K^+$ | $-(T' + P')$ | $-(T'_1 + P'_1)$ |
| $\pi^0 K^0$ | $-(C' - P')/\sqrt{2}$ | $-(C'_1 - P'_1)/\sqrt{2}$ |
| $B_s \to K^+ K^-$ | $-(T' + P')$ | $-(T'_1 + T'_2 + P'_1 + P'_2)$ |
| $K^0 \bar{K}^0$ | $P'$ | $P'_1 + P'_2$ |
Table 3: Decomposition of amplitudes for processes governed by $V_{cb}^*V_{ud} \sim \mathcal{O}(\lambda^2)$ in terms of graphical contributions shown in Fig. 2.

| Final state | $SU(3)$ invariant | $SU(3)$ breaking |
|-------------|-------------------|------------------|
| $B^+ \to \pi^+ D^0$ | $T + \bar{C}$ | |
| $B^0 \to \pi^+ D^-$ | $T + E$ | $(\bar{C} - \bar{E})/\sqrt{2}$ |
| $K^+ D_s^-$ | $\bar{E}$ | $\bar{E}_2$ |
| $B_s \to K^0 D^0$ | $C$ | $C_2$ |
| $\pi^+ D_s^-$ | $\bar{T}$ | $\bar{T}_2$ |

Table 4: Decomposition of amplitudes for processes governed by $V_{cb}^*V_{us} \sim \mathcal{O}(\lambda^3)$ in terms of graphical contributions shown in Fig. 2.

| Final state | $SU(3)$ invariant | $SU(3)$ breaking |
|-------------|-------------------|------------------|
| $B^+ \to K^+ D^0$ | $\lambda(T + \bar{C})$ | $\lambda(T_1 + \bar{C}_1)$ |
| $B^0 \to K^+ D^-$ | $\lambda T$ | $\lambda T_1$ |
| $K^0 D^0$ | $\lambda \bar{C}$ | $\lambda \bar{C}_1$ |
| $B_s \to \pi^+ D^-$ | $\lambda \bar{E}$ | $\lambda \bar{E}_1$ |
| $\pi^0 D^0$ | $-\lambda \bar{E}/\sqrt{2}$ | $-\lambda \bar{E}_1/\sqrt{2}$ |
| $K^+ D_s^-$ | $\lambda(T + E)$ | $\lambda(T_1 + T_2 + E_1 + E_2)$ |
Table 5: Decomposition of amplitudes for processes governed by $V_{ub}V_{cs} \sim \mathcal{O}(\lambda^3)$ in terms of graphical contributions shown in Fig. 2.

| Final state | SU(3) invariant | SU(3) breaking |
|-------------|----------------|----------------|
| $B^+ \rightarrow K^+D^0$ | $-\bar{C} - \bar{A}$ | $-\bar{C}_1 - \bar{A}_1$ |
| $K^0D^+$ | $\bar{A}$ | $\bar{A}_1$ |
| $\pi^0D^+_s$ | $-\tilde{T}/\sqrt{2}$ | $-\tilde{T}_1/\sqrt{2}$ |
| $B^0 \rightarrow K^0D^0$ | $-\bar{C}$ | $-\bar{C}_1$ |
| $\pi^-D^+_s$ | $-\bar{T}$ | $-\bar{T}_1$ |
| $B_s \rightarrow K^-D^+_s$ | $-\bar{T} - \bar{E}$ | $-\bar{T}_1 - \bar{T}_2 - \bar{E}_1 - \bar{E}_2$ |
| $\pi^-D^+$ | $-\bar{E}$ | $-\bar{E}_1$ |
| $\pi^0D^0$ | $\bar{E}/\sqrt{2}$ | $\bar{E}_1/\sqrt{2}$ |

Table 6: Decomposition of amplitudes for processes governed by $V_{ub}V_{cd} \sim \mathcal{O}(\lambda^4)$ in terms of graphical contributions shown in Fig. 2.

| Final state | SU(3) invariant | SU(3) breaking |
|-------------|----------------|----------------|
| $B^+ \rightarrow \pi^+D^0$ | $\lambda(\bar{C} + \bar{A})$ | $-\lambda\bar{A}$ |
| $\pi^0D^+$ | $\lambda(\bar{T} - \bar{A})/\sqrt{2}$ | $-\lambda\bar{A}_2$ |
| $K^0D^+_s$ | $-\lambda\bar{A}$ | $-\lambda\bar{A}_2$ |
| $B^0 \rightarrow \pi^-D^+$ | $\lambda(\bar{T} + \bar{E})$ | $\lambda\bar{E}_2$ |
| $\pi^0D^0$ | $\lambda(\bar{C} - \bar{E})/\sqrt{2}$ | $\lambda\bar{E}_2$ |
| $K^-D^+_s$ | $\lambda\bar{E}$ | $\lambda\bar{E}_2$ |
| $B_s \rightarrow K^-D^+$ | $\lambda\bar{T}$ | $\lambda\bar{T}_2$ |
| $K^0D^0$ | $\lambda\bar{C}$ | $\lambda\bar{C}_2$ |
Table 7: Decomposition of amplitudes for $|\Delta S| = 1$ processes involving a $c\bar{c}$ pair in the final state [leading behavior $\sim \mathcal{O}(\lambda^2)$] in terms of graphical contributions shown in Fig. 3.

| Final state invariant breaking | Final state invariant breaking |
|-------------------------------|-------------------------------|
| $B^+ \to D_s^+ D^0$          | $T + P$                      |
| $\eta_c K^+$                  | $\hat{T} + \hat{P}$         |
| $B^0 \to D_s^+ D^0$          | $T + P$                      |
| $\eta_c K^0$                  | $\hat{T} + \hat{P}$         |
| $B_s \to D_s^+ D_s^-$         | $T + P + \hat{E}$           |
| $\eta_c \pi^0$                | $\hat{T} + \hat{P} + \hat{E}$ |
| $D^+ D^-$                     | $\hat{E}$                   |
| $D^0 \bar{D}^0$               | $-\hat{E}$                  |

Table 8: Decomposition of amplitudes for $\Delta S = 0$ processes involving a $c\bar{c}$ pair in the final state [leading behavior $\sim \mathcal{O}(\lambda^3)$] in terms of graphical contributions shown in Fig. 3.

| Final state invariant breaking | Final state invariant breaking |
|-------------------------------|-------------------------------|
| $B^+ \to D^+ \bar{D}^0$      | $T' + P'$                    |
| $\eta_c \pi^+$                | $\hat{C}'$                   |
| $B^0 \to D^+ \bar{D}^- + D^0 \bar{D}^0$ | $T' + P' + \hat{E}'$ |
| $\eta_c \pi^0$                | $\hat{C}' / \sqrt{2}$       |
| $B_s \to D_s^+ D_s^- + \eta_c \bar{K}^0$ | $T'' + P''$ |
| $\eta_c \bar{K}^0$            | $T_2'' + \hat{P}_{2''}$     |

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III. SU(3)-BREAKING EFFECTS

In the previous section, we discussed the decomposition of the various $B$ decays in terms of SU(3)-invariant amplitudes. We now turn to a discussion of SU(3)-breaking effects.

A. SU(3)-breaking diagrams

Flavor SU(3) is broken by the difference in the $u$, $d$ and $s$ quark masses. Since the mass matrix transforms as a $3 \times 3^* = 1 + 8$ of SU(3), we use the octet piece to break SU(3) (the singlet is, by definition, SU(3)-invariant). This breaking is first order (i.e. linear) in the quark masses. In operator language, this corresponds to the introduction of an operator $M$ into the SU(3)-invariant amplitudes, in which $M$ is a linear combination of $\lambda_3$ and $\lambda_8$ (the $\lambda_i$ are the usual Gell-Mann matrices). The $\lambda_3$ piece can be neglected, since it corresponds to isospin breaking, which is expected to be very small. We therefore have $M \sim \lambda_8$. It is now possible to construct all SU(3)-breaking operators à la Savage and Wise [3], and to examine their effects on $B$ decays.

It is simpler, however, to think of the above in terms of a diagrammatic decomposition of SU(3) amplitudes. It is the $s$-quark mass (or, more precisely, the difference of the $s$-quark and the $d$-quark masses) which breaks SU(3). The Gell-Mann matrix $\lambda_8 \sim \text{diag}[1,1,-2]$ can be written as the identity (which is SU(3)-invariant) plus the matrix $\text{diag}[0,0,-3]$. Thus, SU(3)-breaking operators will be nonzero only when an $s$-quark is involved and an SU(3)-breaking diagram can be obtained from the SU(3)-preserving diagrams of Fig. 1 by putting an “X” on any $s$-quark line. The “X” corresponds to a mass-difference insertion $(m_s - m_d)/\Lambda$, where $\Lambda$ is the scale of SU(3) breaking. The SU(3)-breaking diagrams are shown in Fig. 4:

- There are two SU(3)-breaking diagrams which can be obtained from a $T$ diagram:
  (1) in the $T_1$ diagram, the $s$-quark is among the decay products of the $W$; (2) in the $T_2$ diagram, the $s$-quark is the spectator quark.

- There are two SU(3)-breaking diagrams which can be obtained from a $C$ diagram:
  (1) in the $C_1$ diagram, the $s$-quark is among the decay products of the $W$; (2) in the $C_2$ diagram, the $s$-quark is the spectator quark.

- There are three SU(3)-breaking diagrams which can be obtained from a $P$ diagram:
  (1) in the $P_1$ diagram, there is a $b \to s$ transition; (2) in the $P_2$ diagram, the $s$-quark is the spectator quark; (3) in the $P_3$ diagram, an $s\bar{s}$ quark pair is created.

- There are two SU(3)-breaking diagrams which can be obtained from a $E$ diagram:
  (1) in the $E_1$ diagram, the $s$-quark is in the decaying ($B_s$) meson; (2) in the $E_2$ diagram, an $s\bar{s}$ quark pair is created.

- There are two SU(3)-breaking diagrams which can be obtained from a $A$ diagram:
  (1) in the $A_1$ diagram, the $s$-quark is among the decay products of the $W$; (2) in the $A_2$ diagram, an $s\bar{s}$ quark pair is created.
There are two SU(3)-breaking diagrams which can be obtained from a $PA_1$ diagram. They are not shown in Fig. 4 since we will never make use of them. However we list them here for completeness: (1) in the $PA_1$ diagram, the $s$-quark is in the decaying $(B_s)$ meson; (2) in the $PA_2$ diagram, an $s\bar{s}$ quark pair is created.

It is now straightforward to establish which SU(3)-breaking diagrams contribute to the various $B$ decays:

1. $B \to PP$: all six diagrams contribute to these decays (albeit in only five distinct linear combinations), so all 13 SU(3)-breaking diagrams will contribute, though only in 10 distinct linear combinations.

2. $B \to P\bar{D}$: The $\tilde{T}$, $\tilde{C}$ and $\tilde{E}$ diagrams contribute to these decays, so there are six possible SU(3)-breaking contributions ($\tilde{T}_1$, $\tilde{T}_2$, $\tilde{C}_1$, $\tilde{C}_2$, $\tilde{E}_1$, $\tilde{E}_2$).

3. $B \to PD$: The $\tilde{T}$, $\tilde{C}$, $\tilde{E}$ and $\tilde{A}$ diagrams contribute to these decays, so there are eight possible SU(3)-breaking contributions ($\tilde{T}_1$, $\tilde{T}_2$, $\tilde{C}_1$, $\tilde{C}_2$, $\tilde{E}_1$, $\tilde{E}_2$, $\tilde{A}_1$, $\tilde{A}_2$).

4. $B \to D\bar{D}$: Two combinations ($\hat{T} + \hat{P}$, $\hat{E}$) of the three diagrams $\hat{T}$, $\hat{P}$ and $\hat{E}$ contribute to these decays, so there are four possible SU(3)-breaking contributions.
\[ (\hat{T}_1 + \hat{P}_1, \hat{T}_2 + \hat{P}_2, \hat{E}_1, \hat{E}_2). \] (\(\hat{P}_3\) never appears since in these decays a \(c\bar{c}\) quark pair is created, not an \(s\bar{s}\) pair.)

\(B \to \eta_c P\): The \(\hat{C}\) diagram contributes to these decays, so there are two possible SU(3)-breaking contributions (\(\hat{C}_1, \hat{C}_2\)).

In Tables [4-8] in the “SU(3) breaking” column, we present the SU(3)-breaking contributions to all the \(B\) decays in the four classes:

1. \(B \to PP\) (Tables [1] and [2]).
2. \(B \to PD\) (Tables [3] and [4]),
3. \(B \to PD\) (Tables [5] and [6]),
4. \(B \to DD\) and \(B \to \eta_c P\) (Tables [7] and [8]),

For \(B \to PP\), we include only the SU(3)-breaking contributions which are derived from the \(T, C\) and \(P\) diagrams and their primed counterparts. Those SU(3)-breaking diagrams which are related to the \(E, A\) and \(PA\) diagrams are expected to be much smaller (see below).

Note that, in \(T\)-type diagrams, the weak current is coupled directly to a final-state meson (see Figs. [1-3]). Therefore, assuming factorization, SU(3)-breaking effects in the decay of the \(W\) can be directly related to meson decay constants. Specifically, for \(B \to PP\) and \(B \to PD\),

\[
\left| \frac{T + T_1}{T} \right| = \frac{f_K}{f_\pi} ,
\]

while for \(B \to PD\) and \(B \to DD\) we have

\[
\left| \frac{T + T_1}{T} \right| = \frac{f_{D_s}}{f_D} .
\]

(In the above, the symbol “\(T\)” represents any \(T\)-type diagram in Figs. [1-3].)

Since the \(T_2, C_2\) and \(P_2\) corrections involve the spectator quark, these can be interpreted as form-factor corrections. The remaining SU(3)-breaking corrections are related to the difference in the production amplitudes for \(s\bar{s}\) and \(u\bar{u}\) (\(d\bar{d}\)).

In all cases, the SU(3)-breaking diagrams may have different strong phases than the parent diagrams, so that final-state phases can be affected. In particular, in Eqs. (11) and (12) above, the quantity \(1 + T_1/T\) is in general equal to the ratio of decay constants times an unknown phase.

B. Expected sizes of the various diagrams

Not all of the SU(3)-invariant contributions are expected to be equally large – we expect there to be a range of magnitudes. The SU(3)-violating contributions should obey a similar hierarchy.

For example, the \(T, C, E\) and \(A\) contributions to a particular decay all have the same CKM matrix elements. However, for dynamical reasons, the \(T\) diagram is expected to
dominate. The $C$ diagram is color-suppressed, so naively its magnitude should be smaller than that of the $T$ diagram by a factor $1/3$. Model calculations suggest that the ratio $|C/T|$ is in fact somewhat smaller, about $0.2 \ [12, 13]$. For the purposes of comparison, we will take $|C/T| \sim \lambda$. (Note that the use of the parameter $\lambda$ here is not related in any way to the CKM matrix elements of $C$ and $T$ – it is simply used to keep track of the relative size of the two diagrams.) As previously mentioned, the $E$ and $A$ diagrams are expected to be suppressed relative to the $T$ diagrams by a factor $f_B/m_B \approx 5\% \sim \lambda^2$. (Again, the parameter $\lambda$ is used here only as an approximate measure of the relative size.) Thus, the approximate relative sizes of these four SU(3)-invariant contributions are $|T| : |C| : |E|, |A| = 1 : \lambda : \lambda^2$.

We do not know how large the SU(3)-breaking effects are. Our one clue comes from the ratio $f_K/f_\pi = 1.2$, which appears naturally if factorization is assumed, i.e. $(f_K - f_\pi)/f_\pi \approx 0.2 \sim \lambda$. Assuming all SU(3)-breaking effects are of this order, we expect $|T_i/T| \sim \lambda$, $|C_i/C| \sim \lambda$, etc. (If the SU(3)-breaking effects should turn out to be significantly larger, then our lowest-order parametrization of SU(3) breaking would probably be suspect.)

The $P$ and $PA$ contributions have to be considered separately, since they have different CKM matrix elements than the $T$, $C$, $E$ and $A$ diagrams. We will discuss them as they arise in the various $B$ decays below.

A word of caution to the reader: In what follows, we estimate the relative sizes (in powers of $\lambda$) of the SU(3)-invariant and SU(3)-breaking diagrams which contribute to all two-body hadronic $B$ decays. In later sections we often use this estimated hierarchy to isolate the largest effect in a particular decay (including appropriate explanations, of course). However, one must be careful not to take this hierarchy too literally. Not only are these only educated guesses, but $\lambda$ is not that small a number – a factor of 4 enhancement or suppression can easily turn an effect of $\mathcal{O}(\lambda^n)$ into an effect of $\mathcal{O}(\lambda^{n\pm 1})$.

1. $B \to PP$ decays: For these decays, the $\bar{b} \to \bar{u}u\bar{d}$ and $\bar{b} \to \bar{u}u\bar{s}$ transitions must be analysed separately, since the penguin contributions play a different role in the two cases.

The dominant diagram in $\bar{b} \to \bar{u}u\bar{d}$ decays is $T$, whose CKM matrix elements are $V_{ub}^*V_{ud}$. Based on the above discussion, relative to $|T|$ we expect that $|C|, |E|, |A|$, and the SU(3) corrections to $T$, $C$, $E$ and $A$ are suppressed by various powers of $\lambda$. The $P$ diagram is also smaller than the $T$ diagram, but its suppression factor is more uncertain. The CKM matrix elements for $P$ are $V_{tb}^*V_{td} \ [14]$. Although $|V_{td}| > |V_{ub}|$, there are suppressions due to the loop and to $\alpha_s(m_b) \simeq 0.2$. Allowing for the possibility that the $P$ matrix elements are enhanced relative to the $T$ matrix elements, a conservative estimate is $|P/T| \sim \mathcal{O}(\lambda)$ (although this is likely to be somewhat smaller $\ [15]$). The $PA$ diagram should be suppressed relative to the $P$ diagram by a factor $f_B/m_B \sim \lambda^2$. Thus, for $\Delta C = \Delta S = 0$ transitions, relative to the dominant $|T|$ contribution we expect the following approximate hierarchy to hold:
\[ \mathcal{O}(\lambda^0) : |T| \]
\[ \mathcal{O}(\lambda) : |C|, |P|, \text{SU}(3) \text{ corrections to } T, \]
\[ \mathcal{O}(\lambda^2) : |E|, |A|, \text{SU}(3) \text{ corrections to } C \text{ and } P, \]
\[ \mathcal{O}(\lambda^3) : |PA|, \text{SU}(3) \text{ corrections to } E \text{ and } A, \]
\[ \mathcal{O}(\lambda^4) : \text{SU}(3) \text{ corrections to } PA. \]

This implies that, if one neglects the \( E \) and \( A \) contributions to such decays, it is consistent to also ignore all \( \text{SU}(3) \)-breaking effects except the corrections to \( T \).

For \( \bar{b} \to \bar{u}u\bar{s} \) transitions, the relevant CKM matrix elements in a \( T' \) diagram are \( V_{ub}^*V_{us} \sim \mathcal{O}(\lambda^4) \), while those for the \( P' \) diagram (which corresponds to a \( \bar{b} \to \bar{s} \) transition) are \( V_{tb}^*V_{ts} \sim \mathcal{O}(\lambda^2) \). There is a suppression for the \( P' \) diagram due to the loop and to \( \alpha_s(m_b) \), and we estimate this as above to be \( \mathcal{O}(\lambda) \). The conclusion is that, in these decays, it is the \( P' \) diagram which dominates. Thus, for \( \Delta C = \Delta S = 0 \) transitions, relative to \( |P'| \) we expect the following approximate hierarchy of contributions:

\[ \mathcal{O}(\lambda^0) : |P'| \]
\[ \mathcal{O}(\lambda) : |T'|, \text{SU}(3) \text{ corrections to } P', \]
\[ \mathcal{O}(\lambda^2) : |C'|, |PA'|, \text{SU}(3) \text{ corrections to } T' \]
\[ \mathcal{O}(\lambda^3) : |E'|, |A'|, \text{SU}(3) \text{ corrections to } C' \text{ and } PA', \]
\[ \mathcal{O}(\lambda^4) : \text{SU}(3) \text{ corrections to } E' \text{ and } A'. \]

It should be stressed, however, that this estimated hierarchy is on less solid ground than that for \( \bar{b} \to \bar{u}u\bar{d} \) transitions, since our knowledge of penguin contributions to hadronic \( B \) decays is rather sketchy at the moment. However, if this hierarchy holds, then it is probably consistent to ignore the \( C' \) contribution in Table 2 as well as all \( \text{SU}(3) \)-breaking effects except the \( P'_s \).

2. \( B \to P\bar{D} \) decays: The largest contribution to these decays is \( \bar{T} \). Relative to \( |\bar{T}| \) we expect that \( |\bar{C}|, |\bar{T}_1| \) and \( |\bar{T}_2| \) are \( \mathcal{O}(\lambda) \); \( |\bar{E}|, |\bar{C}_1| \) and \( |\bar{C}_2| \) are \( \mathcal{O}(\lambda^2) \); and \( |\bar{E}_1| \) and \( |\bar{E}_2| \) are \( \mathcal{O}(\lambda^3) \).

3. \( B \to P\bar{D} \) decays: The largest contribution to these decays is \( \bar{T} \). Relative to \( |\bar{T}| \) we expect that \( |\bar{C}|, |\bar{T}_1| \) and \( |\bar{T}_2| \) are \( \mathcal{O}(\lambda) \); \( |\bar{E}|, |\bar{A}|, |\bar{C}_1| \) and \( |\bar{C}_2| \) are \( \mathcal{O}(\lambda^2) \); and \( |\bar{E}_1|, |\bar{E}_2|, |\bar{A}_1| \) and \( |\bar{A}_2| \) are \( \mathcal{O}(\lambda^3) \).

4. \( B \to D\bar{D} \) and \( B \to \eta_c P \) decays: For the \( B \to D\bar{D} \) decays, the \( \bar{T} \) diagram dominates, and the \( \bar{E} \) diagram is suppressed relative to it by a factor of \( \mathcal{O}(\lambda^2) \). As for the \( \bar{P} \) diagram, its CKM matrix elements are about the same size as those of \( \bar{T} \), but there are suppressions due to the loop, to \( \alpha_s(m_b) \sim 0.2 \), and to the fact that a \( c\bar{c} \) pair must be created. Taking all factors into account, the total suppression is probably of \( \mathcal{O}(\lambda^2) \), stronger than that in \( B \to PP \) decays. With this assumption, relative to \( |\bar{T}| \) we expect that \( |\bar{T}_1| \) and \( |\bar{T}_2| \) are \( \mathcal{O}(\lambda) \), \( |\bar{E}| \) and \( |\bar{P}| \) are \( \mathcal{O}(\lambda^2) \), and \( |\bar{E}_1|, |\bar{E}_2|, |\bar{P}_1| \) and \( |\bar{P}_2| \) are \( \mathcal{O}(\lambda^3) \). For \( B \to \eta_c P \) decays, the \( \bar{C} \) diagram dominates, and the \( |\bar{C}_1| \) and \( |\bar{C}_2| \) corrections are suppressed relative to it by a factor of \( \mathcal{O}(\lambda) \).

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IV. TESTS FOR SU(3) BREAKING AND NEGLECT OF E, A DIAGRAMS

We now inspect Tables 1-8 for relations which test for the magnitude of SU(3)-breaking terms and for the absence of E and A diagrams. We consider pairs of tables together, since they are generally related by a factor \( \lambda \). We first discuss relations which are expected to hold in the presence of SU(3) breaking, usually as a consequence of the isospin properties of the weak Hamiltonian. We then discuss general tests for SU(3) breaking, keeping E and A contributions, and finally note the additional relations which follow if such terms are neglected. In what follows we shall always work to first order in SU(3) breaking. We remind the reader that, aside from the decays \( B \to PP \), one is free to change one or both final-state pseudoscalar mesons to a vector meson in all the relations to be quoted below.

A. \( B \to PP \) decays

We refer the reader to [1] for our conventions regarding identical particles. Amplitudes are defined in such a way that their squares always yield decay rates with the same constant of proportionality.

1. Relations following merely from isospin consist of the equality

\[
A(B_s \to \pi^+ \pi^-) = -\sqrt{2}A(B_s \to \pi^0 \pi^0) \quad ,
\]

the triangle relation

\[
\sqrt{2}A(B^+ \to \pi^+ \pi^0) = A(B^0 \to \pi^+ \pi^-) + \sqrt{2}A(B^0 \to \pi^0 \pi^0) \quad ,
\]

and the quadrangle relation [16]

\[
A(B^+ \to \pi^+ K^0) + \sqrt{2}A(B^+ \to \pi^0 K^+) = A(B^0 \to \pi^- K^+) + \sqrt{2}A(B^0 \to \pi^0 K^0) \quad .
\]

2. Within SU(3) symmetry, \( B^+ \) decays to \( \pi \pi \) and \( \pi K \) are related [1, 3, 7]:

\[
A(B^+ \to \pi^+ K^0) + \sqrt{2}A(B^+ \to \pi^0 K^+) = \lambda \sqrt{2}A(B^+ \to \pi^+ \pi^0) \quad .
\]

The general treatment of SU(3) breaking for \( B \to PP \) decays (including E, A, and PA terms) involves a large number of contributions, since all the quarks in the final state transform as flavor triplets or antitriplets. In the remaining relations, based on Tables 1 and 2 we ignore the effects of E, A, and PA and the corresponding SU(3)-breaking terms. Numerous tests for the presence of E, A, and PA were suggested in [1]. Even with this simplification, we find that SU(3)-breaking effects are harder to separate from one another than in the cases involving one or more charmed quarks in the final state. We find the following relations:

3. One amplitude relation is preserved in the presence of SU(3) breaking:

\[
A(B^+ \to K^+ K^0) = A(B^0 \to K^0 K^0) \quad .
\]
Both amplitudes are $P + P_3$. This relation would not necessarily hold in the presence of unequal $A$ and $PA$ contributions, since the left-hand side receives a contribution $A$ while the right-hand side has an additional $PA$ term \[1\].

4. Several combinations of SU(3)-breaking terms can be extracted from the data:

\[
\begin{align*}
\Gamma(B_s \rightarrow \pi^+K^-)/\Gamma(B^0 \rightarrow \pi^+\pi^-) &= 1 + 2 \text{Re}[(T_2 + P_2)/(T + P)] , \\
\Gamma(B_s \rightarrow \pi^0K^0)/\Gamma(B^0 \rightarrow \pi^0\pi^0) &= 1 + 2 \text{Re}[(C_2 - P_2)/(C - P)] , \\
\Gamma(B_s \rightarrow K^0\bar{K}^0)/\Gamma(B^+ \rightarrow \pi^+K^0) &= 1 + 2 \text{Re}(P'_2/P') , \\
\Gamma(B_s \rightarrow K^+\bar{K}^-)/\Gamma(B^0 \rightarrow \pi^-K^+) &= 1 + 2 \text{Re}[(T'_2 + P'_2)/(T' + P')] .
\end{align*}
\]

(Only the real parts of the SU(3)-breaking terms appear here and below, since we are working only to linear order in these terms.) Our program of ignoring $E$, $A$ and $PA$ terms is equivalent to keeping only the lowest-order corrections to the dominant term in any decay. If our estimates (see Sec. III B) of the approximate sizes of the various SU(3)-breaking terms are correct, $C_2$, $P_2$ and $T'_2$ are negligible to the order at which we are working. Furthermore, in the SU(3) corrections on the right-hand sides of the above equations, we need only keep the largest terms in both the numerator and denominator. The other contributions are subdominant and can be ignored. Thus, at this level of approximation the SU(3)-breaking quantity that is measured in Eq. (20) above is $\text{Re}(T_2/T)$, while the quantity $\text{Re}(P'_2/P')$ is measured in both Eqs. (22) and (23). To this order, since we have neglected $E$ and $PA$ terms in the denominator of the left-hand side of Eq. (21) which are of the same order as SU(3)-breaking terms, the SU(3)-breaking factor on the right-hand side should be ignored; we cannot say anything about $O(\lambda)$ corrections in this case.

SU(3)-breaking terms modify the triangle relation \[18\]:

\[
A(B^+ \rightarrow \pi^+K^0) + \sqrt{2}A(B^+ \rightarrow \pi^0K^+) = \left(1 + \frac{T'_1 + C'_1}{T' + C'}\right)\lambda\sqrt{2}A(B^+ \rightarrow \pi^+\pi^0) .
\]

We have argued above that the $C'_1$ and $C'$ terms in the above expression give subdominant SU(3) corrections, and are therefore negligible. Thus, using Eq. \[1\]), we see that the SU(3)-breaking effect which enters the relation between the $I = 3/2$ $B \rightarrow \pi K$ amplitude and $\lambda$ times the $I = 2$ $B \rightarrow \pi\pi$ amplitudes is just $f_K/f_\pi$ (times a possible strong phase). This is, in fact, what we estimated previously \[1\].

To relate various contributions in $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$ decays to one another, Silva and Wolfenstein \[17\] neglected $E$ and $PA$ in $B^0 \rightarrow \pi^+\pi^-$ and assumed that $T'_1/T' = P'_0/P'$ in $B^0 \rightarrow \pi^-K^+$. We find that this assumption is difficult to test using the decays of Tables \[\pi\] and \[\pi\].

B. $B \rightarrow PD$ decays

1. An isospin amplitude relation connects the amplitudes for $B_s \rightarrow \pi \bar{D}:

\[
A(B_s \rightarrow \pi^+D^-) = -\sqrt{2}A(B_s \rightarrow \pi^0D^0) .
\]
2. Isospin triangle relations connect the amplitudes for $B \to \pi \bar{D}$:

$$ A(B^+ \to \pi^+ \bar{D}^0) = A(B^0 \to \pi^+ D^-) + \sqrt{2} A(B^0 \to \pi^0 \bar{D}^0) \ .$$

and the amplitudes for $B \to K \bar{D}$:

$$ A(B^+ \to K^+ \bar{D}^0) = A(B^0 \to K^+ D^-) + A(B^0 \to K^0 \bar{D}^0) \ .$$

3. One relation among six amplitudes holds in the presence of first-order SU(3) breaking when $E$ terms are retained:

$$ A(B_s \to K^+ D_s^-) - A(B_s \to \pi^+ D^-) - A(B^0 \to K^+ D^-)$$

$$ = \lambda [A(B_s \to \pi^+ D_s^-) + A(B^0 \to K^+ D_s^-) - A(B^0 \to \pi^+ D^-)] \ .$$

4. The real part of the ratio $(\bar{T}_1 + \bar{C}_1)/(\bar{T} + \bar{C})$ may be learned from the ratio

$$ \Gamma(B^+ \to K^+ \bar{D}^0)/\lambda^2 \Gamma(B^+ \to \pi^+ \bar{D}^0) = 1 + 2 \text{ Re}[(\bar{T}_1 + \bar{C}_1)/(\bar{T} + \bar{C})] \ .$$

One must write this relation in terms of the real part of the ratio of the SU(3) breaking and SU(3) invariant terms since strong final-state phases may not be the same in the $K^+ \bar{D}^0$ and $\pi^+ \bar{D}^0$ channels. Once again, if our estimates of the approximate sizes of the SU(3)-breaking terms are correct, the $\bar{C}_1$ and $\bar{C}$ terms in the above expression are negligible since they are simply higher-order corrections. In this case the above rate ratio is simply equal to $(f_K/f_{\pi})^2$ [see Eq. (13)].

5. Other rate ratios provide information on combinations of parameters:

$$ \Gamma(B^0 \to K^0 \bar{D}^0)/\lambda^2 \Gamma(B_s \to K^0 \bar{D}^0) = 1 + 2 \text{ Re}[(\bar{C}_1 - \bar{C}_2)/\bar{C}] \ .$$

$$ \Gamma(B_s \to \pi^+ D^-)/\lambda^2 \Gamma(B^0 \to K^+ D_s^-) = 1 + 2 \text{ Re}[(\bar{E}_1 - \bar{E}_2)/\bar{E}] \ .$$

If we now neglect all $\bar{E}$ contributions (there are no $\bar{A}$ terms in $B \to P \bar{D}$ decays),

6. Three decay rates vanish:

$$ \Gamma(B^0 \to K^+ D_s^-) = \Gamma(B_s \to \pi^+ D^-) = \Gamma(B_s \to \pi^0 \bar{D}^0) = 0 \ .$$

Upper limits on the size of $\bar{E}$ terms can already be obtained from the data: $B(B^0 \to K^+ D_s^-)/B(B^0 \to \pi^+ D^-) < 1/12$ and $B(B^0 \to K^+ D_s^-)/B(B^+ \to \pi^+ \bar{D}^0) < 1/20$ [18]. Of course, the measurement of these ratios will have to improve by more than an order of magnitude in order to detect $\bar{E}$ effects at the expected level, but it is interesting that we already have significant experimental evidence regarding the suppression of the $\bar{E}$ terms. We will discuss the experimental data further in Sec. VII.

In addition we learn more about the SU(3)-breaking terms:

7. The real parts of $\bar{T}_{1,2}/\bar{T}$ and $\bar{C}_{1,2}/\bar{C}$ can be learned separately from the ratios

$$ \Gamma(B^0 \to K^+ D^-)/\lambda^2 \Gamma(B^0 \to \pi^+ D^-) = 1 + 2 \text{ Re}(\bar{T}_1/\bar{T}) \ .$$
One isospin quadrangle holds:
\[ \Gamma(B_s \to \pi^+ D_s^-)/\Gamma(B^0 \to \pi^+ D^-) = 1 + 2 \text{Re}(\bar{T}_2/T) \quad , \]
\[ \Gamma(B_s \to K^+ D_s^-)/\Gamma(B^0 \to K^+ D^-) = 1 + 2 \text{Re}(\bar{T}_2/T) \quad , \]
\[ \Gamma(B^0 \to K^0 \bar{D}^0)/2\lambda^2 \Gamma(B^0 \to \pi^0 \bar{D}^0) = 1 + 2 \text{Re}(\bar{C}_1/C) \quad , \]
\[ \Gamma(B_s \to \bar{K}^0 \bar{D}^0)/2\Gamma(B^0 \to \pi^0 \bar{D}^0) = 1 + 2 \text{Re}(\bar{C}_2/C) \quad . \]

Using Eq. (34), the first relation above is in fact equal to \((f_K/f_\pi)^2\). Furthermore, if our estimated hierarchy is correct, the \(\bar{C}_i\) terms are about the same size as the \(\bar{E}\) terms which we have neglected. Therefore, to this order, the last two relations are reliable only up to \(\mathcal{O}(1)\), not to \(\mathcal{O}(\lambda)\).

8. A consistency check may be performed by comparing the results of Eqs. (34) and (35).

C. \(B \to PD\) decays

Since these processes are at most of order \(\lambda^3\), they will be less valuable for testing SU(3) breaking and neglect of \(E\) than the \(B \to P\bar{D}\) decays mentioned above. These processes do provide a unique testing ground for the presence of \(A\) contributions, however. Moreover, the ratio \(\bar{C}/\bar{T}\) of color-suppressed to color-non-suppressed amplitudes (which may differ from the corresponding ratio \(\bar{C}/\bar{T}\) for \(B \to P\bar{D}\) decays) is important for the measurement of the weak phase \(\gamma\) using \(B \to KD_{CP}\) decays.

1. Two isospin relations between amplitudes exist:
\[ \sqrt{2}A(B^+ \to \pi^0 D^+_s) = A(B^0 \to \pi^- D^+_s) \quad , \]
\[ A(B_s \to \pi^- D^+) = -\sqrt{2}A(B_s \to \pi^0 D^0) \quad . \]

2. One isospin triangle can be found:
\[ A(B^+ \to K^+ D^0) + A(B^+ \to K^0 D^+) = A(B^0 \to K^0 D^0) \quad . \]

3. One isospin quadrangle holds:
\[ A(B^+ \to \pi^+ D^0) + \sqrt{2}A(B^+ \to \pi^0 D^+) = A(B^0 \to \pi^- D^+) + \sqrt{2}A(B^0 \to \pi^0 D^0) \quad . \]

4. One relation among six amplitudes is valid in the presence of all first-order terms:
\[ A(B^0 \to \pi^- D^+) - A(B^0 \to K^- D^+_s) - A(B_s \to K^- D^+) \]
\[ = \lambda [A(B_s \to K^- D^+_s) - A(B_s \to \pi^- D^+) - A(B^0 \to \pi^- D^+_s)] \quad . \]

We also obtain a number of additional results:

5. The following SU(3)-breaking terms can be extracted from ratios of rates:
\[ \lambda^2 \Gamma(B^+ \to K^+ D^0)/\Gamma(B^+ \to \pi^+ D^0) = 1 + 2 \text{Re}[(\bar{C}_1 + \bar{A}_1)/(\bar{C} + \bar{A})] \quad , \]
\[ \lambda^2 \Gamma(B^+ \to K^0 D^+)/\Gamma(B^+ \to \bar{K}^0 D^+_s) = 1 + 2 \text{Re}[(\bar{A}_1 - \bar{A}_2)/\bar{A}] \quad , \]
\[ \lambda^2 \frac{\Gamma(B^0 \rightarrow K^0 D^0)}{\Gamma(B_s \rightarrow \bar{K}^0 D^0)} = 1 + 2 \text{Re}[\hat{C}_1 \hat{C}_2]/\hat{C} ] \quad (45) \]
\[ \lambda^2 \frac{\Gamma(B^0 \rightarrow \pi^- D_s^+)}{\Gamma(B_s \rightarrow K^- D^+)} = 1 + 2 \text{Re}[\hat{T}_1 \hat{T}_2]/\hat{T} ] \quad (46) \]
\[ \lambda^2 \frac{\Gamma(B_s \rightarrow \pi^- D^+)}{\Gamma(B^0 \rightarrow K^- D_s^+)} = 1 + 2 \text{Re}[\hat{E}_1 \hat{E}_2]/\hat{E} ] \quad . (47) \]

Now we examine the consequence of neglecting \( \hat{E} \) and \( \hat{A} \) contributions.

6. The following 5 rates vanish:

\[ \Gamma(B^+ \rightarrow K^0 D^+ ) = \Gamma(B_s \rightarrow \pi^- D^+) = \Gamma(B_s \rightarrow \pi^0 D^0) \]
\[ = \Gamma(B^+ \rightarrow \bar{K}^0 D_s^+) = \Gamma(B^0 \rightarrow K^- D_s^+) = 0 \quad . (48) \]

The vanishing of the rate for \( B^+ \rightarrow K^0 D^+ \) implies, through the isospin triangle \( \Box \), a relation between amplitudes with isospins 0 and 1 in the direct channel, and the equality of the amplitudes for \( B^+ \rightarrow K^+ D^0 \) and \( B^0 \rightarrow K^0 D^0 \). Since these processes are color-suppressed, the violation of the rate relation \( \Gamma(B^+ \rightarrow K^+ D^0) = \Gamma(B^0 \rightarrow K^0 D^0) \) would probably be the most stringent test we could devise for the presence of annihilation (\( \hat{A} \)) contributions.

7. The one quadrangle relation \( \Box \) becomes two amplitude relations:

\[ A(B^+ \rightarrow \pi^+ D^0) = \sqrt{2} A(B^0 \rightarrow \pi^0 D^0) \quad , \quad (49) \]
\[ \sqrt{2} A(B^+ \rightarrow \pi^0 D^+) = A(B^0 \rightarrow \pi^- D^+) \quad . \quad (50) \]

8. In addition the following \( SU(3) \)-breaking terms can be extracted:

\[ \lambda^2 \frac{\Gamma(B^+ \rightarrow K^+ D^0)}{\Gamma(B^+ \rightarrow \pi^+ D^0)} = 1 + 2 \text{Re}(\hat{C}_1/\hat{C} ) \quad , \quad (51) \]
\[ \Gamma(B_s \rightarrow \bar{K}^0 D^0)/\Gamma(B^+ \rightarrow \pi^+ D^0) = 1 + 2 \text{Re}(\hat{C}_2/\hat{C} ) \quad , \quad (52) \]
\[ \lambda^2 \frac{\Gamma(B^0 \rightarrow \pi^- D_s^+)}{\Gamma(B^0 \rightarrow \pi^- D^+)} = 1 + 2 \text{Re}(\hat{T}_1/\hat{T} ) \quad , \quad (53) \]
\[ \Gamma(B_s \rightarrow K^- D_s^+)/\Gamma(B^0 \rightarrow \pi^- D_s^+ ) = 1 + 2 \text{Re}(\hat{T}_2/\hat{T} ) \quad . \quad (54) \]
\[ \Gamma(B_s \rightarrow K^- D^+)/\Gamma(B^0 \rightarrow \pi^- D^+) = 1 + 2 \text{Re}(\hat{T}_2/\hat{T} ) \quad . \quad (55) \]

If the \( \hat{C}_i \) terms are of the same order as the \( \hat{E} \) and \( \hat{A} \) terms, as we expect, the first two of the above rate relations should be reliable only up to \( O(1) \).

9. A consistency check may be performed by comparing the left-hand sides of the last two equations.

D. \( B \rightarrow D \bar{D} \) and \( B \rightarrow \eta_c P \) decays

Here we must discuss the relations implied by Tables \( \Box \) and \( \Box \) separately, since a single factor of \( \lambda \) no longer relates the two. Although \( \hat{T}'/\hat{T} \simeq \hat{C}'/\hat{C} \simeq \hat{E}'/\hat{E} \simeq -\lambda \) (and similarly for the corresponding \( SU(3) \)-breaking terms), the ratio \( \hat{P}'/\hat{P} \) is expected to be only of order \( \lambda \), but not to the same accuracy.

By now our methods should have become clear to the reader, but we enumerate the consequences of the tables explicitly for the sake of completeness.
1. Numerous isospin relations may be written. These consist of the amplitude relations

\[ A(B^+ \to D_s^+ D^0) = A(B^0 \to D_s^+ D^-) \] \hspace{1cm} (56)

\[ A(B^+ \to \eta_c K^+) = A(B^0 \to \eta_c K^0) \] \hspace{1cm} (57)

\[ A(B_s \to D^+ D^-) = -A(B_s \to D^0 \bar{D}^0) \] \hspace{1cm} (58)

\[ A(B^+ \to \eta_c \pi^+) = \sqrt{2} A(B^0 \to \eta_c \pi^0) \] \hspace{1cm} (59)

and the triangle relation

\[ A(B^+ \to D^+ \bar{D}^0) = A(B^0 \to D^+ D^-) + A(B^0 \to D^0 \bar{D}^0) \] \hspace{1cm} (60)

The consequences of the \( I = 0 \) nature of the \( \bar{b} \to c\bar{c}s \) transition for \( B \to KJ/\psi \) decays were pointed out some time ago [20].

2. The effects of color-suppressed and exchange-type \( SU(3) \)-breaking amplitudes can be extracted from the data:

\[ \lambda^2 \Gamma(B^+ \to \eta_c K^+)/\Gamma(B^+ \to \eta_c \pi^+) = 1 + 2 \text{ Re}(\hat{C}_1/\hat{C}) \] \hspace{1cm} (61)

\[ \Gamma(B_s \to \eta_c K^0)/\Gamma(B^+ \to \eta_c \pi^+) = 1 + 2 \text{ Re}(\hat{C}_2/\hat{C}) \] \hspace{1cm} (62)

\[ \Gamma(B^0 \to D_s^+ D_s^-)/\Gamma(B^0 \to D^0 \bar{D}^0) = 1 + 2 \text{ Re}(\hat{E}_2/\hat{E}) \] \hspace{1cm} (63)

\[ \lambda^2 \Gamma(B_s \to D^+ D^-)/\Gamma(B^0 \to D^0 \bar{D}^0) = 1 + 2 \text{ Re}(\hat{E}_1/\hat{E}) \] \hspace{1cm} (64)

3. The effects of the combination \( \hat{T}_2' + \hat{P}_2' \) can be extracted from the ratio

\[ \Gamma(B_s \to D^+ D^-)/\Gamma(B^+ \to D^+ \bar{D}^0) = 1 + 2 \text{ Re}[(\hat{T}_2' + \hat{P}_2')/(\hat{T}' + \hat{P}')] \] \hspace{1cm} (65)

If we now ignore exchange-type diagrams, we find several more relations:

4. Several amplitudes vanish. Thus,

\[ A(B_s \to D^+ D^-) = A(B_s \to D^0 \bar{D}^0) = A(B^0 \to D_s^+ D_s^-) = A(B^0 \to D^0 \bar{D}^0) = 0 \] \hspace{1cm} (66)

As one consequence, the triangle relation (60) becomes an amplitude equality:

\[ A(B^+ \to D^+ \bar{D}^0) = A(B^0 \to D^+ D^-) \] \hspace{1cm} (67)

5. The effects of the combination \( \hat{T}_2 + \hat{P}_2 \) can be extracted from the ratio

\[ \Gamma(B_s \to D_s^+ D_s^-)/\Gamma(B^+ \to D_s^+ \bar{D}^0) = 1 + 2 \text{ Re}[(\hat{T}_2 + \hat{P}_2)/(\hat{T} + \hat{P})] \] \hspace{1cm} (68)

where we recall that we are working only to first order in \( SU(3) \) breaking.

The tree contributions \( \hat{T} \) and \( \hat{T}' \) always occur in combination with the corresponding penguin terms \( \hat{P} \) and \( \hat{P}' \). A number of additional consequences would follow if we were to assume that \( \hat{P}'/\hat{P} \simeq \hat{T}'/\hat{T} \), or that the penguin terms (which must produce a \( c\bar{c} \) pair) are negligible. In the latter case one could determine the ratio \( (f_{D_s}/f_D)^2 \) [see Eq. (12)] by comparing \( \Gamma(B^+ \to D^+ \bar{D}^0) \) with \( \lambda^2 \Gamma(B^+ \to D_s^+ \bar{D}^0) \). Other rate ratios

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which can be used to obtain $(f_{D_s}/f_D)^2$ are $\Gamma(B^0 \to D^+D^-)/\lambda^2 \Gamma(B^0 \to D_{s}^+D^-)$ and $\Gamma(B_s \to D^+D_s^-)/\lambda^2 \Gamma(B_s \to D_{s}^+D_s^-)$.

V. SU(3) BREAKING AND THE EXTRACTION OF CKM PHASES

In Refs. [6, 7] we presented a number of ways to extract CKM phases, strong phases, and the sizes of individual diagrams from $B \to PP$ decays. All these analyses made use of unbroken SU(3) symmetry (as well as the neglect of $E$, $A$ and $PA$ diagrams) to relate $B \to \pi\pi$, $B \to \pi K$ and $B \to K\bar{K}$ decays. In this section we discuss the implications of SU(3)-breaking effects for such analyses. (Note that electroweak penguins [8, 9], which we neglect here, may be of equal, or greater, importance than SU(3)-breaking effects. If such contributions are large, they may well invalidate the analyses of Refs. [6, 7]. However, if they are small, then SU(3) breaking is the important factor, which is why it is useful to consider it separately, as we do here.)

Ref. [6] makes use of the SU(3) triangle relation of Eq. (18), rewritten below for convenience:

$$A(B^+ \to \pi^+ K^0) + \sqrt{2}A(B^+ \to \pi^0 K^+) = \lambda \sqrt{2}A(B^+ \to \pi^+\pi^0) .$$

If $A$-type diagrams are neglected, these three amplitudes have the following graphical decomposition:

$$A(B^+ \to \pi^+\pi^0) = -\frac{1}{\sqrt{2}}(T + C) ,$$
$$A(B^+ \to \pi^0 K^0) = P' ,$$
$$A(B^+ \to \pi^0 K^+) = -\frac{1}{\sqrt{2}}(T' + C' + P') .$$

(69)

Now consider the triangle formed from the three CP-conjugate processes:

$$A(B^- \to \pi^-\bar{K}^0) + \sqrt{2}A(B^- \to \pi^0 K^-) = \lambda \sqrt{2}A(B^- \to \pi^-\pi^0) .$$

(70)

The $P'$ amplitude is dominated by the CKM matrix elements $V_{tb}^*V_{ts}$, whose phase is $\pi$. Thus, this amplitude is common to both triangles:

$$A(B^+ \to \pi^+ K^0) = A(B^- \to \pi^-\bar{K}^0).$$

(71)

The weak phase of the $T + C$ amplitude is $\gamma$. Thus we have

$$|A(B^+ \to \pi^+\pi^0)| = |A(B^- \to \pi^-\pi^0)| .$$

(72)

The third amplitude, $T' + C' + P'$, has two contributions ($T' + C'$ and $P'$) with different weak and strong phases. Hence there can be CP violation in the decay $B^\pm \to \pi^0 K^\pm$.

When one compares the triangle to the CP-conjugate triangle, the angle between the amplitudes $\lambda \sqrt{2}A(B^+ \to \pi^+\pi^0)$ and $\lambda \sqrt{2}A(B^- \to \pi^-\pi^0)$ is just $2\gamma$ (see Fig. 5). There is a twofold ambiguity corresponding to the interchanging of $\gamma$ and $\delta_{TC'} - \delta_{P'}$, where $\delta_{TC'}$ and $\delta_{P'}$ are the strong phases of the $(T' + C')$ and $P'$ amplitudes, respectively.
Figure 5: Triangle relating amplitudes $T' + C' + P' = -\sqrt{2}A(B^+ \to \pi^0 K^+)$, $P' = A(B^+ \to \pi^+ K^0)$, and $T' + C' = \lambda(f_K/f_\pi)A(B^+ \to \pi^+ \pi^0)$, as well as the corresponding charge-conjugate processes (denoted by bars over symbols for amplitudes). The angle between $T' + C'$ and $T^\prime + C'$ is $2\gamma$.

How does this analysis hold up when we consider SU(3) breaking? From Tables 1 and 2, the decomposition of the amplitudes in terms of SU(3)-invariant and SU(3)-breaking contributions is

$$A(B^+ \to \pi^+ \pi^0) = -\frac{1}{\sqrt{2}}(T + C),$$
$$A(B^+ \to \pi^+ K^0) = P' + P'_1,$$
$$A(B^+ \to \pi^0 K^+) = -\frac{1}{\sqrt{2}}(T' + C' + P' + T'_1 + C'_1 + P'_1).$$

(73)

In other words, the third side of the triangle is really $-(T' + C' + T'_1 + C'_1)$, whereas above we assumed it was $-\lambda(T + C) = -(T' + C')$. The error incurred is simply

$$1 + \frac{T'_1 + C'_1}{T' + C'} \simeq 1 + \frac{T'_1}{T'},$$

(74)

where, on the right-hand side, we have neglected the $C'$ and $C'_1$ terms as being subdominant (see Sec. III B). (This approximation is at the same level as the neglect of $A$-type diagrams.) However, from Eq. (11), assuming factorization, we have

$$1 + \frac{T'_1}{T'} = \frac{f_K}{f_\pi} e^{i\delta_{SU(3)}},$$

(75)

where we have included a possible additional strong phase. This simply reflects the fact that, in the presence of SU(3) breaking, the $T$ and $T'$ amplitudes no longer have the same strong phase: $\delta_{T'} = \delta_T + \delta_{SU(3)}$. Therefore, taking into account SU(3)-breaking effects, Eq. (18) should read

$$A(B^+ \to \pi^+ K^0) + \sqrt{2}A(B^+ \to \pi^0 K^+) = \frac{f_K}{f_\pi} e^{i\delta_{SU(3)} \lambda} \sqrt{2}A(B^+ \to \pi^+ \pi^0).$$

(76)
This does not change things substantively. \( \delta_{TC'} \) and \( \delta_{P'} \) are now the strong phases of \((T' + C' + T_1' + C_1')\) and \(P' + P_1'\), respectively. Apart from this, the analysis of Ref. [7] still holds, as long as the factor \( f_K/f_\pi \) is included. The weak phase \( \gamma \) can be obtained, up to a twofold ambiguity which interchanges it and the strong phase \( \delta_{TC'} - \delta_{P'} \).

Ref. [7] describes two types of analyses. The first is essentially an extension of the analysis described above, except that it also allows one to extract the strong phases and sizes of the individual \( T' \), \( C' \) and \( P' \) diagrams. It makes use of the isospin \( \pi K \) quadrangle [Eq. (17)], rewritten below for convenience:

\[
A(B^+ \to \pi^+K^0) + \sqrt{2}A(B^+ \to \pi^0K^+) = A(B^0 \to \pi^-K^0) + \sqrt{2}A(B^0 \to \pi^0K^0) .
\]

The key point is that, within SU(3) symmetry, one diagonal of the \( \pi K \) quadrangle is related to the amplitude for \( B^+ \to \pi^+\pi^0 \) [Eq. (18)]: \((T' + C') = \lambda(T + C)\). Thus, by measuring the four \( \pi K \) rates, as well as the rate for \( B^+ \to \pi^+\pi^0 \), one can construct the diagram in Fig. 6. This allows the extraction of \(|P'|, |T'|, |C'|, \Delta_{C'}^\prime \) and \(\Delta_T \), as well as the quantity \(\Delta_{P'} - \Delta_T - \gamma\), where \(\Delta_i \equiv \delta_i - \delta_{TC'}\). If one also measures the CP-conjugate processes, \( \gamma \) can be disentangled from \(\Delta_{P'} - \Delta_T\), as in [8]. (It should be pointed that there is some uncertainty in the determination of \(|C'|\) and \(\Delta_{C'}^\prime \), although the amplitude \(\lambda'\) is negligible compared to \(P'\) and \(T'\), it is not so small when compared to \(C'\) — we estimate \(|\lambda'/C'| \sim \lambda\). Thus, the precision in our determinations of \(|C'|\) and \(\Delta_{C'}^\prime \) is limited by the neglect of \(\lambda'\).)

If one considers SU(3) corrections, Fig. 6 still holds, except that (i) \(P', T'\) and \(C'\) now include their SU(3) corrections \(P_1', T_1'\) and \(C_1'\), respectively, and (ii) the diagonal is no longer \((T' + C')\), which is directly related to \((T + C)\), but rather \((T' + C' + T_1' + C_1')\). However, we showed above how to relate this SU(3)-corrected diagonal to \((T + C)\): up to small corrections, \(|T' + C' + T_1' + C_1'| = (f_K/f_\pi)\lambda|T + C|\). Thus, the analysis still holds, except that the strong phases that are measured include SU(3)-breaking effects. (To be precise, the SU(3) correction \(C_1'\) should be neglected everywhere. It is expected to be of the same order as \(E\) and \(A\)-type diagrams, which have been ignored. This means that, just as in the SU(3)-invariant case, the determination of \(|C'|\) is accurate to only about 25\%, and \(\Delta_{C'}^\prime \) is similarly affected.)

The second analysis of Ref. [7] is a bit more complicated. If one assumes unbroken SU(3) symmetry, one has two triangles with a common base [see Eqs. (16), (17) and (18)]:

\[
\lambda \sqrt{2}A(B^+ \to \pi^+\pi^0) = \lambda A(B^0 \to \pi^+\pi^-) + \lambda \sqrt{2}A(B^0 \to \pi^0\pi^0) , \\
\lambda \sqrt{2}A(B^+ \to \pi^+\pi^0) = A(B^0 \to \pi^-K^+) + \sqrt{2}A(B^0 \to \pi^0K^0) .
\]

In terms of diagrams, these two triangles can be written

\[
\lambda(T + C) = \lambda(T + P) + \lambda(C - P) , \\
\lambda(T + C) = (T' + P') + (C' - P') .
\]

By measuring the rates for \(B^+ \to \pi^+K^0\) and \(B^0 \to K^0K^0\), one can obtain the magnitudes of \(P'\) and \(P\), respectively. With these 7 rate measurements, one can construct
Figure 6: Amplitude triangles based on Eqs. (17) and (18) permitting the extraction of strong phases and the weak phase $\gamma$ in the SU(3) symmetry limit and with linear SU(3) symmetry breaking.
the diagram of Fig. 4, in which the apex of the subtriangle $T' + C' = (T' + C')$ is determined, up to a twofold ambiguity, from the intersection of the two circles. The key point here is that this fixes the orientation of the vectors $P$ and $P'$. Thus we can obtain their phases, relative to the $(T' + C')$ amplitude (the horizontal line). These relative phases are $\Delta_P + \alpha$ and $\Delta_{P'} - \gamma$, respectively (we have assumed that the weak phase of the $P$ diagram is given approximately by $\text{Arg}(V^*_b V_d) = -\beta$ [14], and we have used $\alpha = \pi - \beta - \gamma$). However, within SU(3) symmetry, $\Delta_P = \Delta_{P'}$, so that one can combine these two phase measurements to obtain the weak CKM phase $\beta$. In addition, one can also obtain the strong phases and magnitudes of the various diagrams. If one also measures the rates for the CP-conjugate processes, it is possible to obtain $\gamma$, $\alpha$, and $\Delta_P$ separately. (Note that the precision with which the magnitude and phase of $C' = \lambda C$ can be determined is limited as in the first construction by the neglect of $A$- and $PA$-type diagrams.)

Unfortunately, in the presence of SU(3)-breaking effects, this analysis does not stand up as well as the previous two. We will be able to extract $\gamma$ and certain strong phases in a way independent of the previous constructions, but we will not be able to obtain the other CKM phases.

There are two places where SU(3)-breaking effects effects are important. First, $P$ and $P'$ get different SU(3) corrections: the amplitude $P'$ in the decay $B^+ \rightarrow \pi^+ K^0$ gets an SU(3) correction $P'_1$, while the amplitude $P$ in the decay $B^0 \rightarrow K^0 \bar{K}^0$ has a $P_3$ correction. Thus, the equality $\Delta_P = \Delta_{P'}$ is likely to be broken, so that the CKM angle $\beta$ cannot be extracted as described above. Furthermore, the $P_3$ correction to $P$ is not present in the isospin triangle [Eq. (16)]. This means that there is some uncertainty as to the position of the apex of the subtriangle. Thus, the orientation of the $P + P_3$ vector is poorly determined even if we somehow knew that $\Delta_P = \Delta_{P'}$, we still could not obtain $\beta$ precisely. (Note that the orientation of $P' + P'_1$ can still be fairly accurately obtained since $|P| \ll |P'|$, a small correction to $P$ has very little effect on the orientation of $P' + P'_1$ as determined from the intersection of the $P$ and $P'$ circles in Fig. 4.)

Second, there are really two subtriangles: $T + C = (T + C)$ and $T' + C' = (T' + C')$. Assuming a perfect SU(3) symmetry, these subtriangles are congruent, and simply scale by $\lambda$. However, this is no longer true in the presence of SU(3) breaking. We know how to take certain SU(3)-breaking effects effects into account. For example, assuming factorization, $T'$ and $T$ are related by $(f_K/f_\pi) \exp(i\delta_{\text{SU}(3)})$, as are $(T' + C')$ and $(T + C)$ to a good approximation (the error is at the level of $\sim \lambda^2$ relative to the dominant $T$ and $T'$ diagrams.) However, $C'$ and $C$ are not so clearly related. A priori, we do not know the relation between these two amplitudes. In this case, the small difference between $(T' + C')$ and $(T + C)$ can have a significant effect. Since the $C$ diagram is smaller than the $T$ diagram by a factor of $\lambda$, the small error one makes in relating $(T' + C')$ to $(T + C)$ can be a large error in the determination of the magnitudes and phases of $C$ and $C'$ (in addition to the error incurred by neglecting $A$- and $PA$-type diagrams). This in turn leads to a further uncertainty in the position of the apex of the subtriangle.

On the other hand, since $|T|$ and $|T'|$ are much larger than $|C|$ and $|C'|$, respectively, the small uncertainty in the position of the apex of the subtriangle has little effect on
Figure 7: Amplitude triangle based on Eqs. (77) permitting the extraction of strong phase shift differences and the weak phases $\beta, \gamma$ in the SU(3) symmetry limit. With linearly broken SU(3), only the extraction of $\gamma$ and certain strong phases is possible (see text).
the determination of \(|T'\) and \(\Delta_T\). Thus, the quantity \(\Delta_P' - \Delta_T' - \gamma\) can be extracted in the same way as in the first analysis of Ref. \[7\]. If one measures the CP-conjugate processes, one can similarly disentangle \(\gamma\) and \(\Delta_P' - \Delta_T'\). One of the advantages of this method over the previous one was that the weak phase \(\beta\) could be obtained. In the presence of SU(3) breaking this is no longer the case. Furthermore, the determinations of \(|C'\) and \(\Delta_C'\) remain imprecise. However even in the presence of SU(3) breaking, this method can still be used to independently determine \(\gamma\) and some of the strong phases.

VI. FINAL-STATE INTERACTIONS

A. \(B \to \pi \bar{D}\) decays

The decays \(B^+ \to \pi^+ \bar{D}^0\), \(B^0 \to \pi^+ \bar{D}^-\), and \(B^0 \to \pi^0 \bar{D}^0\) involve one amplitude leading to a final state with \(I = 1/2\) and one amplitude leading to a final state with \(I = 3/2\). Specifically, the weak Hamiltonian for the transition \(\bar{b} \to \bar{c}ud\bar{d}\) transforms as \(I = I_3 = 1\), permitting the following decomposition of the amplitudes in terms of the \(\pi \bar{D}\) isospins:

\[
A(B^+ \to \pi^+ \bar{D}^0) = A_{3/2},
\]

\[
A(B^0 \to \pi^+ D^-) = (2/3)A_{1/2} + (1/3)A_{3/2},
\]

\[
\sqrt{2}A(B^0 \to \pi^0 \bar{D}^0) = -(2/3)A_{1/2} + (2/3)A_{3/2}.
\]

These amplitudes clearly satisfy a triangle relation, as already written in (26). Since a single CKM element dominates the decays, a non-zero area for this triangle would signify differences in final-state phases between the \(I = 1/2\) and \(I = 3/2\) amplitudes. This circumstance has been used by H. Yamamoto \[4\] to place upper limits on such phase differences, not only in the decays \(B \to \pi \bar{D}\), but also in \(B \to \pi \bar{D}^\ast\) and \(B \to \rho \bar{D}\). A similar method has already been used in the decays \(D \to \pi K\) and related processes to conclude that there are important final-state phase differences between the \(I = 1/2\) and \(I = 3/2\) \(\pi K\) and \(\pi K^\ast\) states \[24\].

We illustrate in Fig. 8 an amplitude triangle for \(B \to \pi \bar{D}\) decays, where we define \(r \equiv A_{1/2}/A_{3/2}\). The base of the triangle has unit length, while the two other sides have lengths

\[
\frac{A(B^0 \to \pi^+ D^-)}{A(B^+ \to \pi^+ \bar{D}^0)} = 1 + \frac{2r}{3},
\]

and

\[
\frac{\sqrt{2}A(B^0 \to \pi^0 \bar{D}^0)}{A(B^+ \to \pi^+ \bar{D}^0)} = \frac{2 - 2r}{3}.
\]

A line drawn from a point \(1/3\) of the way along the base to the apex then has the phase \(\phi \equiv \text{Arg}(r)\).
Figure 8: Amplitude triangle for determining the phase of \( r \equiv A_{1/2}/A_{3/2} \) in \( B \to \pi \bar{D} \) decays.

B. \( B \to K \bar{D} \) decays

A single CKM matrix element, governing the transition \( \bar{b} \to \bar{c}u\bar{s} \), also dominates the decays \( B^+ \to K^+ \bar{D}^0 \), \( B^0 \to K^+D^- \), and \( B^0 \to K^0\bar{D}^0 \). The weak Hamiltonian transforms as \( I = I_3 = 1/2 \). The decay amplitudes may be decomposed into contributions with final-state isospins \( I = 0 \) and \( I = 1 \):

\[
A(B^+ \to K^+ \bar{D}^0) = A_1', \\
A(B^0 \to K^+D^-) = (1/2)A_1' + (1/2)A_0', \\
A(B^0 \to K^0\bar{D}^0) = (1/2)A_1' - (1/2)A_0'.
\]  

Thus, they will satisfy a triangular relation \((27)\). If the triangle has non-zero area, final-state interactions are important. Similar results apply, for example, to \( B \to K^* \bar{D} \) and \( B \to K \bar{D}^* \) decays.

C. SU(3) relations between \( B \to \pi \bar{D} \) and \( B \to K \bar{D} \) decays

The results of Tables 3 and 4 imply relations among the amplitudes for \( B \to \pi \bar{D} \) and \( B \to K \bar{D} \) decays. In the absence of SU(3) breaking and exchange diagram \((E)\) contributions, we would expect \( A_1' = \lambda A_{3/2} \) and \( A_0' = (1/3)\lambda(4A_{1/2} - A_{3/2}) \). By comparing the expressions for the respective decays in terms of amplitudes \( T \) and \( C \) or \( T' \) and \( C' \), we see that if the triangles \((26) \) and \((27) \) have different shapes, one must conclude that (i) SU(3) is broken, (ii) exchange contributions are important, or (iii) both.

D. Other tests for final-state interactions

The decays \( B \to \pi \bar{D} \) and \( B \to K \bar{D} \) offer the best hope of providing clean tests for final-state interactions with reasonable decay rates and triangles whose sides are all expected to be non-vanishing. However, two additional amplitude triangles and one amplitude quadrangle may be of use in testing for final-state interactions. These are the relations \((40) \), \((41) \), and \((42) \) involving the decays \( B \to KD, B \to D\bar{D}, \) and \( B \to \pi D, \) respectively.
Since the decay $B^+ \rightarrow K^0 D^+$ is expected to proceed purely through an annihilation diagram (see Table 5), the triangle containing this amplitude should have one very short side. It may be very difficult to tell that such a triangle has non-zero area. Similarly, the decay $B^0 \rightarrow D^0 \bar{D}^0$ should proceed purely via an exchange diagram (Table 8), so its triangle may have a short side. The amplitude quadrangle (41) applies to the decays $B \rightarrow \pi D$ whose amplitudes are of order $\lambda^4$, and hence not likely to be detected soon. One could tell if such a quadrangle had non-zero area by constructing its sides as the square roots of observed rates and checking that no two or three sides added up to any other two or one side.

E. Comments on rescattering effects

In Ref. [1, 6, 7] the neglect of $E$, $A$, and $PA$ contributions in comparison with $T$, $C$, and $P$ contributions was noted explicitly to be equivalent to the assumption that certain rescattering effects are unimportant. For example, a final state which can be reached through the annihilation diagram can also be reached through a tree diagram followed by a rescattering. Several tests of this hypothesis were proposed in Ref. [1, 6, 7]. It is no surprise that this assumption leads to relations between final-state phases in different decay channels. Indeed one such phase relation was noted to exist between $B \rightarrow \pi \pi$ and $B \rightarrow \pi K$ [22].

It was subsequently pointed out [23] that a relation among final-state phases was implicit in assuming that the decay the decay $B^+ \rightarrow \pi^+ K^0$ is pure penguin (here we have neglected the annihilation diagram). This is indeed so. The point raised in Ref. [23] is that the $I = 3/2$ combination

$$A(\pi^+ K^0) + \sqrt{2}A(\pi^0 K^+) = -(T' + C')$$ (83)

and the $I = 1/2$ tree contribution to the combination

$$[2A(\pi^+ K^0) - \sqrt{2}A(\pi^0 K^+)]_{\text{tree}} = +(T' + C')$$ (84)

should have the same strong final-state phases if their sum is to vanish. In the graphical description of Ref. [1, 6, 7], this is automatically the case, since the amplitude in Eq. (83) and the tree contribution to the combination in Eq. (84) are both proportional to $T' + C'$. Thus, the equivalence of the strong final-state phases is a direct consequence of our assumption that the annihilation diagrams are negligible.

We stress that our general treatment of linearly broken flavor SU(3) in two-body $B$ decays does not forbid final-state interaction phases. Although OZI-forbidden scattering from one $q\bar{q}$ pair to another is not permitted at the quark level by our decomposition [24], each of the hadronic decay amplitudes, $T$, $C$, $P$, etc. may carry a nonzero CP-conserving phase. For example, part of the phase of $P$ can be calculated perturbatively [25].
VII. EXPERIMENTAL DATA AND PROSPECTS

In this section we give a snapshot of the present status of data. We include results which are anticipated soon with the events in hand. We then discuss briefly the improvements which would be needed to test various sectors of the theory. Our treatment of $B_s$ decays is rather sketchy since it is premature to assess experimental possibilities until more final states have been reconstructed.

A. Decays to two light mesons

Here we concentrate mainly on expected hierarchies of the dominant amplitudes $T, T', C, C', P, P'$, and the potential for confirming them. We have already mentioned in Ref. [1] the (rather demanding) levels of statistics required to test for the presence of the diagrams $E, E', A, A', P A, P A'$. Some combination of the decays $B_0 \to \pi^- \pi^+$ and $B_0 \to \pi^- K^+$ has been observed [26], with a combined branching ratio of about $2 \times 10^{-5}$. Equal mixtures of the two modes are likely, though a decisive separation awaits better particle identification. It then appears [1, 17] that the amplitude $T$ dominates the $B_0 \to \pi^- \pi^+$ decay, while $P'$ dominates $B \to \pi K^-$ (see Tables 1 and 2), as we estimated in Sec. III B.

Other $B \to \pi K$ decays which should be visible at branching ratios of $1/2$ to $1 \times 10^{-5}$ (depending on whether they involve a neutral or charged pion) are $B^+ \to \pi^+ \pi^0$, $B_s \to \pi^+ K^-$, and all the remaining processes in Table 2. For example, if the $C$ amplitude is small, one expects

$$\Gamma(B^+ \to \pi^+ \pi^0) \approx \Gamma(B^0 \to \pi^- \pi^+)/2. \quad (85)$$

If the $P'$ amplitude is the only one present in $B \to \pi K$, one expects

$$\Gamma(B^+ \to \pi^+ K^0) \approx 2\Gamma(B^0 \to \pi^0 K^0)$$

$$\approx \Gamma(B^0 \to \pi^- K^+) \approx 2\Gamma(B^+ \to \pi^0 K^+). \quad (86)$$

Present upper limits on branching ratios (at the 90% confidence level) for such processes include [27] $B(B^+ \to \pi^+ \pi^0) < 2.3 \times 10^{-5}$, $B(B^0 \to \pi^0 K^0) < 6.3 \times 10^{-5}$, $B(B^+ \to \pi^0 K^+) < 3.2 \times 10^{-5}$, and $B(B^+ \to \pi^+ K^0) < 6.8 \times 10^{-5}$, with no information available for $B_s$ decays. Updated results for some of these modes are forthcoming [28].

The $\Delta S = 0$ processes in Table 1 containing only color-suppressed and/or penguin contributions, such as $B^0 \to \pi^0 \pi^0$, should be characterized by branching ratios of order $10^{-6}$ or smaller (see also [12]). In this class of processes, Ref. [27] quotes only the limit $B(B^0 \to \pi^0 \pi^0) < 1.0 \times 10^{-5}$. Thus, one must wait for an improvement of about a factor of ten in present data before expecting to see such processes consistently. At this level, one will be able to construct a meaningful triangle based on the three distinct decay rates for $B \to \pi \pi$, and one should expect deviations from the relation (85).

A factor of ten increase in data will also permit the observation of rate differences in the various $B \to \pi K$ channels, as a consequence of interference of the term $T'$ in Table 2 with the dominant $P'$ term. If $C'$ is sufficiently small in comparison with $T'$, these
rate differences should violate the middle equality in (86) while preserving the other two equalities:

$$\Gamma(B^+ \to \pi^+ K^0) \approx 2\Gamma(B^0 \to \pi^0 K^0)$$  \hspace{1cm} (87)

and

$$\Gamma(B^0 \to \pi^- K^+) \approx 2\Gamma(B^+ \to \pi^0 K^+)$$  \hspace{1cm} (88)

Electroweak penguin contributions \[3\] \[4\] could disturb these relations, making them of particular interest for early testing.

In \(B\) decays to one light vector meson and one light pseudoscalar, there are hints of signals in several \(B \to \pi K^*\) channels \[27\]. However, as noted (e.g.) in \[3\] and \[4\], the SU(3) analysis in these channels is more involved, so we have not undertaken a general treatment of SU(3)-breaking effects. Some partial results on the role of electroweak penguins have been obtained \[3\].

**B. \(B \to PD\) decays**

We begin by discussing the \(\mathcal{O}(\lambda^2)\) processes in Table 4.

The color-favored decays of nonstrange \(B\) mesons, involving the amplitude \(T\), have been seen at branching ratio levels of 1/4 to nearly 2% \[13\], in the \(\pi \bar{D}, \pi \bar{D}^*, \rho \bar{D}\), and \(\rho \bar{D}^*\) channels. Typical upper limits on the color-suppressed \(B^0\) decays to these channels are an order of magnitude lower. As noted in \[5\], one can already construct meaningful amplitude triangles for several of these channels, placing upper limits on the relative phase shifts between \(I = 1/2\) and \(I = 3/2\) channels which are typically tens of degrees.

What level of data would be required to see effects of the \(\bar{E}\) contribution? The amplitude for such a process is expected to be only a few percent of the dominant \(\bar{T}\) contribution. The equality of \(\Gamma(B^0 \to \pi^+ D^-) \sim |\bar{T} + \bar{E}|^2\) with \(\Gamma(B_s \to \pi^+ D_s^-) \sim |\bar{T} + \bar{T_2}|^2\) is more likely to be upset by the SU(3)-breaking term \(\bar{T_2}\) than by the term \(\bar{E}\). So far one candidate for \(B_s \to \pi^+ D_s^-\) has been seen \[29\].

In order to see the effect of \(\bar{E}\) alone, one would have to detect the decay \(B^0 \to K^+ D_s^-\) (or a related process involving one or more vector mesons). The present limits \[13\] of \(B(B^0 \to K^+ D_s^-) < 2.3 \times 10^{-4}, B(B^0 \to K^+ D_s^{*-}) < 1.7 \times 10^{-4}, B(B^0 \to K^+ D_s^-) < 9.7 \times 10^{-4}, B(B^0 \to K^+ D_s^{*-}) < 1.1 \times 10^{-3}\) are not adequate to detect the presence of the \(\bar{E}\) contribution at the predicted level. The present upper limits on \(|\bar{E}/(\bar{T} + \bar{E})| < 1/\sqrt{12}\) from \(B(B^0 \to K^+ D_s^-)/B(B^0 \to \pi^+ D^*)\) and on \(|\bar{E}/(\bar{T} + \bar{C})| < 1/\sqrt{20}\) from \(B(B^0 \to K^+ D_s^-)/B(B^+ \to \pi^+ \bar{D}^0)\) must be improved considerably for an observation of decay modes dominated by \(E\) and \(A\) amplitudes, if these terms are indeed suppressed by \(f_B/m_B \sim \lambda^2\).

None of the strangeness-changing \(B \to PD\) decays listed in Table 4 has been reported yet. The observation of the decay \(B^+ \to K^+ D^0\) probably offers the best prospects. If SU(3) breaking can be accounted for by the ratio \(f_K/f_\pi\), as one expects to be true for the dominant \(\bar{T}\) contribution, one expects

$$\frac{\Gamma(B^+ \to K^+ \bar{D}^0)}{\Gamma(B^+ \to \pi^+ D^0)} = \frac{|f_K V_{us}|^2}{|f_\pi V_{ud}|^2} \approx 0.075$$  \hspace{1cm} (89)
while this ratio would be only about 0.051 in the absence of SU(3) breaking.

Since about 300 $B^+ \rightarrow \pi^+ \bar{D}^0$ events have already been reported by the CLEO II Collaboration [13], there should be nearly two dozen events of $B^+ \rightarrow K^+ \bar{D}^0$ in the same sample. An observed sample of some hundred $B^+ \rightarrow K^+ \bar{D}^0$ events would be able to test conclusively for the SU(3) breaking mentioned above.

In the absence of appreciable $\bar{E}$ contributions, one should expect

$$\frac{\Gamma(B^0 \rightarrow K^+ D^-)}{\Gamma(B^0 \rightarrow \pi^+ D^-)} = \frac{|f_K V_{us}|^2}{|f_\pi V_{ud}|^2} \approx 0.075$$

as well. About 80 events of $B^0 \rightarrow \pi^+ D^-$ have been reported by CLEO II so far [13].

C. $B \rightarrow PD$ decays

Here one is dealing with amplitudes which, though nominally of order $\lambda^3$ (Table 5) or $\lambda^4$ (Table 6), may be further suppressed by the smallness of $V_{ub}$ and the effects of form factors. Nonetheless, it is important to detect modes such as the color-suppressed decay $B^+ \rightarrow K^+ D^0$ if the program of Ref. [19] for determining the weak phase $\gamma$ is to be implemented.

The process with the best prospect of being seen first is the decay $B^0 \rightarrow \pi^- D^+_s$, for which there exists only the upper limit of $2.7 \times 10^{-4}$ on the branching ratio [18]. A crude estimate based on factorization in which we neglect form factor differences and color-suppressed diagrams would predict

$$\frac{\Gamma(B^0 \rightarrow \pi^- D^+_s)}{\Gamma(B^0 \rightarrow \pi^- \pi^+)} \approx \frac{f_{D_s}^3}{f_\pi^3} \approx 5$$

where we have taken $f_{D_s} \approx 300$ MeV. Thus, we expect a branching ratio for $B^0 \rightarrow \pi^- D^+_s$ of several parts in $10^5$. The decay should begin to show up with several times the present data sample. At precisely half the rate of $B^0 \rightarrow \pi^- D^+_s$, (as a consequence of isospin), one should see the decay $B^+ \rightarrow \pi^0 D^+_s$.

Observation of the color-suppressed $B \rightarrow KD$ decays will require a further increase of about tenfold in the data. At this level one may test SU(3) by comparing the processes involving $\bar{T} + \bar{T}$ or $\bar{T} + \bar{T} + \bar{T}$ in Table 5 with those involving $\lambda \bar{T}$ or $\lambda(\bar{T} + \bar{T})$ in Table 4.

D. $B \rightarrow D \bar{D}$ decays

Decays such as $B \rightarrow D^+_s \bar{D}$ (see Table 7) (and the corresponding processes involving one or two vector mesons) have been observed with branching ratios of $1 - 2 \%$ [30]. Somewhat over 100 events have been observed in the sum of all channels. Isospin invariance predicts pairwise equalities for charged and neutral $B$ decay modes.

The color-suppressed decays $B \rightarrow J/\psi K^{(*)}$ have been observed with branching ratios which are about an order of magnitude smaller than those of $B \rightarrow D^+_s \bar{D}^{(*)}$. This provides information about the ratio $|\hat{C}/\hat{T}|$ which is somewhat larger than $\lambda$. Similar branching ratios, of about $10^{-3}$, are expected for $B \rightarrow \eta_c K^{(*)}$ which should soon be observed through the hadronic decay modes of the $\eta_c$. 
The decays $B^+ \rightarrow D^+ \bar{D}^0$ and $B^0 \rightarrow D^+ D^-$ (see Table 8) should occur at several percent of the rates for $B^+ \rightarrow D_s^+ \bar{D}^0$ and $B^0 \rightarrow D_s^+ D^-$, with precise ratios dictated by ratios of heavy meson decay constants if a factorization hypothesis is adequate to describe these decays and if penguin amplitudes are negligible.

The presence of $\hat{E}$ contributions would be most cleanly illustrated by observing decays of the form $B_s \rightarrow D \bar{D}$. With $f_B/m_B \approx 5\%$, we estimate the corresponding branching ratio to be at most a few parts in $10^5$. Present fragmentary information on $B_s$ meson production does not allow us to estimate the size of the data sample that would permit such a test.

E. Overall prospects

The present sample of $B$ decays is based in large part on the 2 million nonstrange $B \bar{B}$ pairs collected so far by CLEO, with impressive reconstructions of some decay modes (including those of $B_s$) by groups at LEP and by the CDF Collaboration at Fermilab. A foreseen upgrade of the luminosity of CESR to $L = 10^{33}$ cm$^{-2}$s$^{-1}$ should provide 10 million such pairs in a year ($10^7$ s) of operation. Asymmetric $B$ factories at SLAC and KEK should provide comparable (or eventually larger) samples. Nonetheless, it seems hard to escape the conclusion that many of the tests proposed here will require larger data sets than can be achieved at electron-positron colliders. The ability of hadron colliders to produce large numbers of $B$ mesons is unquestioned; it remains to be seen whether a large enough fraction of these can be detected.

VIII. CONCLUSIONS

We have discussed prospects for experimental tests of several aspects of two-body hadronic $B$ decays, including SU(3)-breaking, the neglect of certain SU(3) amplitudes corresponding to disfavored graphs, and the elucidation of strong final-state-interaction phase differences. While decays to pairs of light pseudoscalar mesons typically involve more than one product of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, decays in which one or two of the final quarks are charmed typically have a simpler CKM structure. Consequently, the effects of interest to us can be more readily isolated.

We have discussed a staged set of measurements, starting with the present sample of nonstrange $B$ decays (dominated by CLEO II data) and progressing through the multiplication of this sample by successive factors. Results which may be testable in the near future include the following:

1) We have presented a diagrammatic description of the various SU(3)-breaking effects. Assuming factorization for $T$-type diagrams, one SU(3)-breaking diagram corresponds to the ratio of decay constants. Using this description, we expect that $\Gamma(B^+ \rightarrow K^+ \bar{D}^{(*)0})/\Gamma(B^+ \rightarrow \pi^+ \bar{D}^{(*)0}) = \left| f_K V_{us} \right|^2 / \left| f_\pi V_{ud} \right|^2 \approx 0.075$, while this ratio would be only about 0.051 in the absence of SU(3) breaking. Similar comments apply to the ratio $\Gamma(B^0 \rightarrow K^+ D^{(*)-})/\Gamma(B^+ \rightarrow \pi^+ D^{(*)-})$.

2) The study of $B \rightarrow D \bar{D}$ decays (Tables 7 and 8) can provide information on the ratio $f_D/f_{D_s}$ if factorization is assumed: $\Gamma(B^+ \rightarrow D^+ \bar{D}^0)/\Gamma(B^+ \rightarrow D_s^+ \bar{D}^0)$, $\Gamma(B^0 \rightarrow D^+ \bar{D}^0)/\Gamma(B^0 \rightarrow D_s^+ \bar{D}^0)$, $\Gamma(B^0 \rightarrow D_s^+ \bar{D}^0)/\Gamma(B^+ \rightarrow D_s^+ \bar{D}^0)$.
\[ \frac{D^+D^-}{\Gamma(B^0 \rightarrow D_s^+D^-)} \text{ and } \frac{\Gamma(B_s \rightarrow D^+D^-)}{\Gamma(B_s \rightarrow D_s^+D_s^-)} \text{ are all expected to equal } \frac{|f_{D^0V_{ud}}|^2}{|f_{D_s^0V_{us}}|^2}. \] This same ratio of CKM matrix elements and decay constants can also be obtained from \[ \Gamma(B^0 \rightarrow \pi^-D^+) \text{ and } \Gamma(B^0 \rightarrow \pi^-D_s^+), \] but this is likely to be less useful experimentally, since a small \[ O(\lambda^4) \] amplitude is involved.

3) Other SU(3)-breaking effects, associated with form factors and quark pair creation, can also be isolated by ratios of rate measurements. The list of such measurements is very long, so we refer the reader to Sec. IV for a complete discussion.

4) A search for decays such as \( B^0 \rightarrow K^{(*)}D_s^{(*)-} \) at an order of magnitude better sensitivity than present levels will start to shed light on the presence or absence of weak \( B \) meson decays involving the light spectator quark. Other processes of order \( \lambda^2 \) in the amplitude which are of this type are the decays \( B_s \rightarrow D^+D^- \) and \( B_s \rightarrow D^0D^0 \) (Table 7).

5) The processes in 4) are all of the “exchange” type. In order to look for purely “annihilation” amplitudes one must turn to the process \( B^+ \rightarrow K^0D^+ \) (Table 5), of order \( \lambda^3 \). This process is involved in an isospin triangle relation together with the decays \( B^+ \rightarrow K^+D^0 \) and \( B^0 \rightarrow K^0D^0 \). Unequal rates for these last two decays also would be evidence for the annihilation contribution.

6) Other \( O(\lambda^3) \) processes of the purely “exchange” variety include \( B_s \rightarrow \pi^+D^- \) and \( B_s \rightarrow \pi^0D^0 \) (Table 4), \( B_s \rightarrow \pi^-D^+ \) and \( B_s \rightarrow \pi^0D^0 \) (Table 5), and \( B^0 \rightarrow D^0\bar{D}^0 \) and \( B^0 \rightarrow D_s^+D_s^- \) (Table 8). These should also be suppressed.

7) Some SU(3) relations which should hold even in the presence of SU(3) breaking (but whose validity depends on the neglect of exchange and annihilation contributions) have been obtained, including the amplitude relation \[ A(B^+ \rightarrow K^+\bar{K}^0) = A(B^0 \rightarrow K^0\bar{K}^0) \] (see Sec. IV).

8) We find that the program for obtaining the weak phase \( \gamma \), in several independent ways, from \( B \rightarrow PP \) decays described in Refs. [6] and [7] is not substantially affected by a more careful consideration of SU(3) breaking (see Sec. V). Some strong phase information can also still be extracted. On the other hand, the determination of \( \beta \) proposed in Ref. [7] is much more vulnerable to such effects. The role of electroweak penguins in such determinations has been discussed in a separate paper [9].

9) Triangle relations involving the decays \( B \rightarrow \pi\bar{D} \) and \( B \rightarrow K\bar{D} \) (and related states involving vector mesons) will provide useful information on strong final-state phase shifts, since these decays are dominated by a single CKM matrix element.

10) A hierarchy of contributions to various decays has been discussed (Sec. III B), whereby one can estimate the expected rates for rare processes without reference to specific models. Rates of color-suppressed decays are expected to be intermediate between rates of color-favored processes and processes dominated by “annihilation” or “exchange” amplitudes.

To sum up, a rich set of questions may be addressed by measurements of rates for two-body \( B \) decays, from the present levels which include branching ratios of more than a percent down to levels of \( 10^{-7} \) or lower. Eventually, one will want to detect large numbers of \( B_s \) decays in order to fully implement this program.

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ACKNOWLEDGMENTS

We thank J. Cline, A. Dighe, I. Dunietz, G. Eilam, A. Grant, K. Lingel, H. Lipkin, R. Mendel, S. Stone, L. Wolfenstein, and M. Worah for fruitful discussions. J. Rosner wishes to acknowledge the hospitality of the Fermilab theory group and the Cornell Laboratory for Nuclear Studies during parts of this investigation. M. Gronau, O. Hernández and D. London are grateful for the hospitality of the University of Chicago, where part of this work was done. This work was supported in part by the United States – Israel Binational Science Foundation under Research Grant Agreement 90-00483/3, by the German-Israeli Foundation for Scientific Research and Development, by the Fund for Promotion of Research at the Technion, by the NSERC of Canada and les Fonds FCAR du Québec, and by the United States Department of Energy under Contract No. DE FG02 90ER40560.

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