The structure of $^{12}$C and $^{16}$O

Martin Freer

School of Physics and Astronomy, University of Birmingham, Birmingham, B15 2TT, UK

M.Freer@bham.ac.uk

Abstract. The recent advances in the understanding of the two nuclei $^{12}$C and $^{16}$O are discussed in the light of recent measurements and developments in nuclear theory. The focus is the collective excitations, which may be related to dynamical symmetries which in turn reveal the nature of the underlying nuclear structure.

1. Nuclear Correlations

The complexity revealed in the binding of light nuclei is revealing. It exposes in sharp relief details of the strong interaction in a manner that is akin to the grazing incidence light on the moon’s surface exposes detail of historical impacts. Figure 1 illustrates, for nuclei up to Neon isotopes, bound and unbound nuclei. At the neutron drip-line the effects of neutron-neutron correlations are manifest in the Borromean nature of $^6$He, $^8$He, $^{11}$Li and $^{14}$Be, with their $A$-1$n$ counterparts $^5$He, $^7$He, $^{10}$Li and $^{13}$Be all being unbound to neutron decay. Similarly, on the proton-rich side $^{10}$C may be thought of as super-Borromean (or perhaps more accurately Brunnian [1]), composed of the constituents $^4$He+$^4$He+p+p.

The lack of the existence of bound $^9$B and $^6$Be means that the removal of any of the 4 constituents, results in the dissociation of $^{10}$C; and indeed, the lack of bound $^8$Be and $^5$Li nuclei also implies that removal of any two components results in the spontaneous decay of the remaining ones. This rather delicate balance of bound and unbound substructures again may be linked to correlations. In this particular instance it is the very high binding energy of the alpha-particle which is playing a driving role.

This high binding owes its origins to the maximal correlations between pairs of protons and neutrons and indeed between the n-p pairs. This yields one of the highest binding energies of light nuclei and is the origin of the unbound nature of $^8$Be, which decays into two alpha-particles with a Q-value of 92 keV. Moreover, the correlations are manifest in the very high lying (>20 MeV) first excited state. It is these
properties that make the nucleus $^4$He such a good nuclear cluster. The evidence for the influence of the alpha-cluster on the properties of light nuclei has deep historical roots, extending back to the 1930s with the work of Hafstad and Teller [2] through the 1960s with the contribution of Ikeda [3] to more recent developments involving molecular type structures where neutrons are exchanged between alpha-particle clusters [4].

The treatment of the phenomenon of clustering from a theoretical perspective had, until recently, been rather rudimentary, where the cluster and core were considered to be pre-existing entities and collective excitations are generated through orbital motion of the cluster about the core. Great advances have been made in theory in which $A$-body calculations may be performed that reveal the single-particle and cluster-like states within the same framework [5]. Similarly, chiral effective field theory (cEFT) calculations performed on the lattice are beginning to provide a real insight into the structure of light nuclear systems from a perspective in which the interaction is grown from the QCD degrees of freedom. In particular, the focus for such calculations have been the structure of $^{12}$C and in particular the Hoyle-state [6,7].

2. Carbon-12

Given the underlying influence of the correlations described above, the structure of $^{12}$C naturally may be thought of, in zeroth order, as an assembly of three alpha-particles. The Ikeda picture [3] reveals that the full realization of clustering does not occur until the appropriate decay threshold is reached. In the case of $^{12}$C the 3 alpha-particle decay threshold is just above 7 MeV, and the Hoyle-state at 7.65 MeV famously has not only been associated with a well-developed alpha-cluster structure, but also plays a key role in nucleosynthesis [8]. Below this threshold, it is expected that the cluster-structure is reduced and eventually the shell-model like states are arrived at.

Figure 2 shows the Antisymmetrised Molecular Dynamics (AMD) calculations [5] for the structure of the ground-state (a1) and Hoyle-state (a2). The ground-state is compact, though still reveals the 3alpha structure, which is also strongly influenced by the $^8$Be+alpha substructure. The “collective” excitations of this state (b1) and (c1) – 2+ and 4+ states - reveal a very similar intrinsic structure. The Hoyle-state is a loose assembly of alpha-particles, again influenced by the $^8$Be+alpha partition. However, the excitations do not strongly resemble the intrinsic structure of the Hoyle-state and hence in this model a rotational description might not be appropriate.

The condensate (Bose Einstein Condensate (BEC)) description of the 3alpha system was also presented at the present meeting by Funaki-san. This reveals that the Hoyle-state may be described in terms of a wave-function that captures the bosonic properties of the alpha-particles, whilst recognizing their internal fermionic structure which is revealed as the alpha-particles overlap through an antisymmetrisation operator. The description of the Hoyle-state in this model lies outside the low energy shell model basis, and can reproduce many of the experimental characteristics; most notably the angular distribution for electron inelastic scattering from the ground state to the Hoyle-state [9]. Here, the underpinning wave-function is one which has the three alpha-particles with a common s-state wave function, in a potential formed from their mutual
interaction. The antisymmetrisation operator then serves to suppress the amplitudes at small distances and the cluster structure then emerges. Collective excitations then, in large part, correspond to rotations. The contribution to the present meeting from Funaki-san indicates that the 2⁺ excitation is dominated by an alpha-particle rotating around an $^8\text{Be}_{\text{gs}}$ with two units of angular momentum, with a smaller component of $^8\text{Be}(2^+)\text{+alpha}$. A 4⁺ excitation has dominant components from the rotation of the alpha-particle around the $^8\text{Be}_{\text{gs}}$ core with $L=4$ and $^8\text{Be}(2^+)\text{ core with } L=2$ and only a small contribution from $^3\text{Be}(4^+)$. Understanding these collective excitations is rather important, inasmuch that they are fundamentally linked to the nature of the Hoyle-state itself. If, as indicated by the AMD calculations, the dominant component is governed by $^8\text{Be+alpha}$, then the collective modes would be linked to either rotations of the $^8\text{Be}$ component, the alpha-particle motion around the $^8\text{Be}$, or combinations of these collective degrees of freedom. This is a similar description to that of the condensate picture. Guidance may be derived from the collective excitations of the $^8\text{Be}$ nucleus; the excitation energies of the 2⁺ and 4⁺ excitations are 3.03 and 11.35 MeV. If these are mirrored in $^{12}\text{C}$ then the 2⁺ and 4⁺ excitations of the Hoyle-state should lie at 7.65+3.03 = 10.68 MeV and 7.65+11.35 = 19.00 MeV. The experimental situation has over recent years become clearer. A 2⁺ excitation at ~9.8 MeV [10-13] with a width of 700-800 keV (perhaps even larger [13]) has been firmly identified. There is also an indication of a broad 4⁺ state close to 13.3 MeV [14]. The experimental 2⁺ state lies in close proximity to that extrapolated from $^8\text{Be}$, but the 4⁺ lies somewhat lower in energy by about 6.5 MeV. The lowering in energy with respect to the collective excitations of $^8\text{Be}$ may be interpreted as either the 2 correlated alpha-particles possess a larger separation than that in the $^8\text{Be}_{\text{gs}}$ band, or that the states are generated in a different manner. If as indicated by both the AMD and BEC approaches the motion of the alpha-particle contributes strongly to the collectivity, then it is possible that the energy of the collective excitations may be lowered such that they lie closer to the experimental equivalents. The BEC approach indeed does predict lower energy 2⁺ and 4⁺ states, but actually lower in energy than the experimental counterparts. This, thus, remains an open question, which could perhaps be resolved if it might be possible to measure the B(E2) transition rates between these states.

**Figure 3.** (left) The $^{12}\text{C}$ excitation energy spectrum from the $^{12}\text{C}(\alpha,3\alpha)\alpha$ reaction, for decays to the $^8\text{Be}$ ground-state. The 5⁻ state is observed at 22.5 MeV. (right) The angular distribution for the 5⁻ state showing the close agreement with $J^p=5^-$ [15].

The nature of the ground state of $^{12}\text{C}$ as revealed in the AMD calculations needs to be verified. The equation describing the rotations of a spinning top, predicted by Hafstad and Teller, and applied to $^{12}\text{C}$ is
Here $J$ and $K$ are the total angular momentum and the projection of the total angular momentum onto the symmetry axis, respectively, of the three centered object, where in a simple model the alpha-particles reside at the vertices of an equilateral triangle. For $K = 0$ then the rotations are about a symmetry axis which passes through the plane of the triangle – passing through one vertex and bisecting the other two. The gives rise to $0^+$, $2^+$ and $4^+$ states. For the next possible value of $K$, $K = 3$, then the rotation is about an axis which is perpendicular to, and passes through the center of, the triangle. In this instance each alpha-particle carries one unit of angular momentum and would produce $3^-$, $4^-$, $5^-$… excitations.

The ground state rotational band is known for the $2^+$ and $4^+$ members at 4.44 and 14.08 MeV, as is the $3^-$ excitation at 9.64 MeV. Recent measurements of the width of the $3^-$ state have indicated that it has a well-developed alpha-cluster structure [16] and the $4^-$ member of the band has been identified at 13.2 MeV [17,18]. A key measurement has been the determination of the $5^-$ member of the $K=3$ band at 22.5 MeV – as shown in Fig. 3. The observation of this state would appear to confirm the dynamical symmetry associated with the ground state structure.

Returning to the structure of the Hoyle-state; an alternative approach to establishing its nature would be to examine its’ decay properties. The possible charged particle decay modes are either sequential via $^8\text{Be}_{gs}+\alpha$-alpha, followed by the decay of $^8\text{Be}$ into two alpha-particles, or direct 3alpha decay. The Fermi breakup model allows the two and three-body decay phase space to be calculated. Integrating over the energies of the decay products gives

$$W_2 = \frac{1}{2} K_2 \left( \frac{1}{\sum_{b=1}^{2} m_b} \prod_{b=1}^{2} m_b \right)^{3/2} \frac{(2\pi)^{1.5}}{\Gamma(3/2)} Q_2^{0.5}$$

$$W_3 = \frac{1}{6} K_3 \left( \frac{1}{\sum_{b=1}^{3} m_b} \prod_{b=1}^{3} m_b \right)^{3/2} \frac{(2\pi)^3}{\Gamma(3)} Q_3^2$$

where

$$K_n = \left( \frac{V}{(2\pi\hbar)^3} \right)^{n-1}$$

and

$$V = \frac{4}{3} \pi \left( 1.4 \times 12^{1/3} \times 10^{-15} \right)^3.$$  Also $Q_2$ and $Q_3$ are the two and three body Q-values, respectively, and $m_b$ are the masses of the decay products.

For low decay energies the three-body phase-space is highly constrained, and the precise calculation depends on the magnitude of the of volume term $V$. Nevertheless, ratio of $W_3/W_2$ is close to 0.2 to 0.3%. An enhancement of the three-body decay probability over that predicted by phase space alone would be interesting and might signal a BEC type structure. The recent measurements of the ratio of the three- to two-body decay give an upper limit of 0.2%. The experimental precision is thus very close to the region where probing the structure of the Hoyle-state through this approach becomes interesting [19].

3. Oxygen-16

The fingerprint of the $D_{5h}$ dynamical symmetries found in the spectroscopy of $^{12}\text{C}$ are remarkable and although have been explored recently, may be traced back to the work of Hafstad and Teller [2]. Similarly, the structure of $^{16}\text{O}$ may be interpreted in terms of a tetrahedral arrangement of 4 alpha-particles. The recent work of Bijker and Iachello [20] shows the spectrum that would be expected for such a tetrahedral structure; the sequence $0^-$, $3^-$, $4^-$, $6^-$ would be expected. The $0^-$ state corresponds to the ground state, the $3^-$ state would be the observed 6.13 MeV excitation, there is a $4^-$ state at 10.36 MeV and several $6^-$ state at 16.23, 21.05 and 21.65 MeV. The $3^-$ state would be the obvious analogue.
of the 9.64 MeV 3$^+$ state in $^{12}$C, with the tetrahedron rotating such that the axis of rotation passes through the center of the base and the vertex of the upper alpha-particle. This is a compelling picture, however, would require some reinterpretation of the nature of the rotational bands which have been “established” for $^{16}$O. In order to distinguish between the competing interpretations, precision measurements are required. These involve both a study of electromagnetic decay strengths coupled with precision measurements of the charged particle decay widths. For example, the measurements presented in [21] provide characterization of the high spin members of the $K^\pi = 0^{+/-}$ cluster bands in $^{16}$O which appear to confirm a common structure. The spectroscopy of $^{16}$O is much more complex than $^{12}$C and hence a concerted experimental effort is required to pin down the exact details of the underlying structure.

The analogue of the Hoyle-state in $^{16}$O has yet to be confirmed, however, as indicated by Funaki-san it is most likely the 15.2 MeV 0$^+$ state. The 4alpha decay threshold is at 14.4 MeV and consequently the 15.2 MeV state cannot decay strongly into this channel. There are, however, a number of resonances that decay to the $^8$Be+$^8$Be or $^{12}$C(Hoyle)+alpha final states. The pioneering measurements of Chevalier et al., [22] revealed both the energy and dominant angular momenta of the $^{16}$O resonances that decayed to $^8$Be+$^8$Be as populated in the $^{12}$C(alpha,$^8$Be)$^8$Be reaction. The energy-spin systematics of selected, narrow, resonances remarkably fell onto a $J(J+1)$ trajectory with a moment of inertia commensurate with a structure in which the a alpha-particles were arranged in a linear fashion; an alpha-particle chain. This work was published in 1967 and until the present has been held up as an example of extreme alpha-clustering.

Confirmation of such an exotic structure is clearly vital. There are a number of possible approaches; one of which is to confirm the details of the excitation function, the second is to search for higher spin members of the 4 alpha-particle chain band – the band was only observed up to spin 6. The highly detailed excitation functions for the $^{12}$C(alpha,$^8$Be)$^8$Be and $^{12}$C(alpha,$^{12}$C[7.65 MeV])alpha reactions presented in [23] show that both the original structure that was interpreted as resonances in the earlier work [22] was more complex and no evidence for an 8$^+$ state could be identified (Figure 4). This most recent study contained over 400 measurements at different energies, with significant coverage of the angular distributions which should permit the components from resonances and transfer-like processes to be disentangled.

As with $^{12}$C the field is considerably closer to being able to unpick some of the questions that surround the spectroscopy of $^{16}$O and address the underlying structure. However, comprehensive, precision, data are the key.

Figure 4. Excitation functions for reactions populating resonances in $^{16}$O decay to 4 alpha-particle final states, from [23].
4. Summary
Understanding the structure of light nuclei lies on the critical path to comprehending the nature of the strong interaction on a nuclear scale. The development of the ability to apply realistic interactions to nuclei, with the number of constituents beyond a few, transforms the potential to better understand how the correlations which a manifest in systems such as $^9$Be, $^{12}$C and $^{16}$O. Advances in experimental techniques are permitting a clearer understanding of the spectroscopy of light nuclei to constrain better the state-of-the-art models. The next steps in experimental characterization is to measure electromagnetic transition rates for states above the cluster decay threshold where branching ratios are $10^{-6}$ or less! Such studies have the potential to discriminate to the extent that it is possible to arrive at a detailed understanding of the role of cluster correlations in nuclei.

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