Description of the $D_s^*(2320)$ resonance as the $D\pi$ atom

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(Dated: March 25, 2022)

We discuss the possibility that the recently reported resonance in the $D_s\pi^0$ spectrum can be described in terms of residual $D\pi$ interactions.

The BaBar collaboration has recently reported a narrow resonance in the $D_s^+(1968)\pi^0$ spectrum \(^1\). The mass of the resonance $M_c = 2.32$ GeV is significantly below the $DK$ threshold, and the width $\Gamma \sim 9$ MeV is of the order of a typical hadronic decay width for a light meson emission from a charmed resonance.

In the charmed sector there are three, stable under hadronic decays, light-flavored, $c\bar{q}$, $q = u, d, s$, $D$-mesons, the $D^0(1870)$, $D^+(1870)$ and $D_s(1968)$ together with their spin excitations with $J^P = 1^-$, the $D^*(2010)$ and the $D_s^*(2110)$ in the $u, d$ and strange sector respectively \(^2\). Other well established resonances have $J^P = 1^+$, the $D_1(2420)$ and the $D_{s1}(2536)$. In terms of the quark model classification the ground states with $J^P = 0^-$ correspond to $2S+1L_J = 1^S_0\ c\bar{q}$ states, the $J^P = 1^-$ natural parity states are identified as $3S_1$ states and the $J^P = 1^+$ unnatural parity resonances are the $J = 1$ members of the $L_{c\bar{q}} = 1$ multiplet containing states with the following quantum numbers, $\bar{P}_0$, $\bar{P}_1$, $\bar{P}_2$. The predicted $1^P_2$ states could be assigned to $D_2^*(2460)$ and $D_{s2}^*(2573)$ resonances and two more states, the $\bar{P}_0$ and a linear combination of the $\bar{P}_1$ and $\bar{P}_2$ are still to be found.

As pointed out by Barnes et al. the identification of the BaBar state with the $\bar{P}_0$ member of the $L_{c\bar{q}} = 1$ multiplet is unlikely \(^3\). Its mass is 230 MeV below the average of the $D_{s1}$ ($\bar{P}_1$) and $D_{sJ}$ masses. Furthermore from the heavy quark symmetry it is expected that two out of the four $L_{c\bar{q}} = 1$ states, corresponding to the $j\bar{q} = L_{c\bar{q}} + 1/2\bar{q} = 3/2$ doublet are narrower then the other two from the $j\bar{q} = 1/2$ doublet. The former can be identified with the narrow $D_{s1}$ and $D_{sJ}$ states, while the latter would include the $\bar{P}_0$ state, which in a quark model is predicted to have width of the order of hundreds of MeV \(^4\).

To summarize, the measured charmed mesons resonances, with the exception of the latest BaBar state seem to agree well with the quark mode. From the point of view of this classification, two states, one with $J^P = 0^+$ and one with $J^P = 1^+$ are missing; however, they may well be much broader than those observed.

Since the $J^P = 0^+$ BaBar state is not expected to belong to a $c\bar{q}$ family we investigate the possibility it is molecular in nature. This could happen if there is a strong flavor-singlet attraction between the pion and the $c\bar{s}$ mesons. Since $m_c/m_s < 10\%$ one could consider the BaBar state as a result of a pion being captured by a non-relativistic (even static) charmed meson. Since the width of the resonance measured by BaBar, $(\Gamma \lesssim 10$ MeV) is small compared to the energy difference between nearby coupled channels, e.g. $|m_{D_s^*(2320)} - m_{D_{sK}}| = 40$ MeV, channels other than the measured $D_s\pi$ should be unimportant.

Even though it is expected that there are residual flavor-neutral interactions mediated by glueball (pomeron) exchanges, the details of such processes are presently unknown. It is possible, however to formulate the problem using effective interactions once the relevant energy-momentum scales have been identified. In particular, the $D_s\pi$ interactions are mediated via multi-gluon exchange and its spectral properties at low mass can be saturated by $\eta'$ exchanges thus correlated with matter fields \(^5\). The virtual light quark matter fields coupled to $\pi$ or $D$ mesons probe the light quark distribution in these particles up to momentum scales of the order of the QCD scale $\Lambda \sim 0.5 - 1$ GeV, thus momenta in virtual meson propagators should be truncated at $p \lesssim \Lambda$. The effective $D\pi$ interaction obtained this way could then be used to calculate the $D\pi$ scattering amplitude \(^6\). This requires iteration of the real part of virtual $D\pi$ exchanges. Since we are not explicitly including contributions from other channels the energies of the intermediate states have to be truncated at $E(p) \leq E_{th}$. For example for the $DK$ threshold, $E_{th} = 2.36$ GeV, which implies the relative momentum in the $D\pi$ system $p \leq 340$ MeV. Thus the cutoff, $\mu$ on the loop integrals over $D\pi$ states should be of the order of a few hundred MeV. Of course if all coupled channels were explicitly included, it would be possible able to set $\mu \rightarrow \infty$.

To summarize, the effective $D\pi$ flavor-singlet interaction should have a natural strength if the scale in the interaction is of the order of the QCD scale ($\Lambda$) and the $D\pi$ amplitude is truncated at momenta of the order of a few hundred MeV ($\mu$).

The effective interaction can be deduced from an effective QCD Lagrangian which includes anomalous $U_A(1)$ symmetry breaking \(^7\). For the system under study, the relevant part of such a Lagrangian is given by \(^8\).
The first two terms represent the lowest order terms of a nonlinear chiral Lagrangian, with \( U = \exp(i\pi^a T_\alpha / f_\pi + i\sqrt{3} \eta_0 / f_\pi) \), \( \pi^a \) and \( \eta_0 \) being the octet and singlet meson fields, respectively. We have neglected small terms which differentiate between the flavor octet, \( f_\pi \) and the flavor singlet meson decay constants. The \( Q = (\alpha / 4\pi) F \) represents the gluon field and the term linear in \( Q \) is responsible for the anomalous coupling of the gluon to matter fields and for the \( U_A(1) \) symmetry breaking. The first \( Q^2 \)-dependent term can be interpreted as the kinetic term of the gluon field. Finally the last two terms represent flavor-singlet, lowest dimension gluon coupling to the light meson octet and the charmed, \( D \) meson field. The coupling constant \( \beta = -0.63 \) can be determined from the \( \eta' \rightarrow \pi\pi\eta \) decay and, as discussed above, the unknown coupling \( c \) is expected to be of the order of \( \Lambda^{-2} \). Using the equations of motion, the \( Q \)-field can be replaced by the matter field which among others leads to the following interactions,

\[
\mathcal{L}_{\pi\eta\eta_0} = \frac{3}{2} \frac{\beta}{f_\pi^2} \eta_0^2 \partial_\mu \pi^a \partial^\mu \pi^a, \quad \mathcal{L}_{\pi\eta_0\eta_0} = \frac{c}{6 \, f_\pi^2} \eta_0^2 \partial^2.
\]

These result in an effective \( D\pi \) Lagrangian given by

\[
L_{D\pi} = \frac{\beta}{4f_\pi^2} \int dx dy (\partial_\mu \pi^a(x))^2 (\eta_0^2(x)\eta_0^2(y)) D^2(y).
\]

The expectation value of the \( \eta \) (gluon) field is replaced by an instantaneous contact term, smeared over the QCD scale, \( \Lambda \), \( \langle \eta_0^2(x)\eta_0^2(y) \rangle \sim -\delta(x-y)\Lambda^4 \delta^2(x-y)/m_\eta^4 \), resulting in a final \( D\pi \) effective potential,

\[
V_{D\pi} = \frac{\beta c}{4f_\pi^2} \int dx (\partial_\mu \pi^a(x))^2 D^2(x),
\]

with \( c = O(\Lambda^2) \sim O(1 \, \text{GeV}^2) \). Due to absence of multiparticle, relativistic effects and the low momentum approximation (\( p \lesssim \mu \)), the scattering amplitude can be determined from \((1 - VG)^{-1} V\) with \( G = (E - \sqrt{m_D^2 + p^2 - \sqrt{m_\pi^2 + p^2 + i\epsilon}}) \) being the free \( D\pi \) propagator. The scattering \( S \)-wave phase shift can then be easily calculated for the potential of Eq. (4) and is given by,

\[
\tan \delta(E) = \frac{E_\pi^2(p)pc\beta f(p/\mu)}{32\pi f_\pi^2 E(p)(1 - J[E(p)])},
\]

with \( E(p) = E_\pi(p) + E_D(p) = \sqrt{m_\pi^2 + p^2} + \sqrt{m_D^2 + p^2} \) and \( J(E) \) being the contribution from the real part of \( D\pi \) loop cutoff by a form factor \( f(p/\mu) \) with \( \mu = O(\text{few 100 MeV}) \),

\[
J(E) = \frac{c\beta}{32\pi^2 f_\pi^4} \int_0^\infty dk \frac{k^2 E^2(k)f(k/\mu)}{E_{\pi}(k)E_D(k)[E(p) - E(k) + i\epsilon]}.
\]

The comparison between our theoretical prediction and the BaBar result is shown in Fig. 1. Instead of plotting the data, for simplicity we plot a phase shift resulting from a Breit-Wigner (BW) parameterization of a resonance with mass \( M_r = 2.32 \, \text{GeV} \) and width, \( \Gamma_r = 10 \, \text{MeV} \) (equal to the experimental resolution of the BaBar measurement). We recall that a resonance phase shift, parametrized by a simple (without energy dependence in the width) Breit-Wigner formula gives,

\[
\sin^2 \delta_{BW}(E) = \frac{(\Gamma_r M_r)^2}{((E^2 - M_r^2)^2 + (\Gamma_r M_r)^2)} 
\]

where \( M_r \) and \( \Gamma_r \) are the mass and width of the resonance, respectively. In Fig. 1 this is shown by the shaded region, whose size was fixed to \( \Delta \sin^2 \delta_{BW} = 0.1 \) roughly corresponding to size of the errorbars in the mass distribution of the \( D\pi \) events shown in Ref. [1]. The prediction for \( D\pi \) phase shift from Eq. (5) is shown with the solid line and it was calculated using \( c = 1 \, \text{GeV}^2 \) and \( \mu = 341 \, \text{MeV} \).

Since the resonance is narrow it is clear that the position and width will be sensitive to these parameters. For example with \( c \) fixed changing \( \mu \) by \( \pm 20\% \) shifts the position of the resonance between 2.257 and 2.393 GeV and

\[
\mathcal{L} = \frac{f_\pi^2}{4} Tr (\partial^\mu U \partial_\mu U) + \frac{f_\pi^2}{4} Tr M (U + U^\dagger) + \frac{1}{2} i Q Tr [\log U - \log U^\dagger] + \frac{3}{m_\eta^2 f_\pi^2} Q^2 + Q^2 \left[ \frac{9\beta}{2f_\pi^4 m_\eta^2} Tr (\partial^\mu U \partial_\mu U) + \frac{c}{f_\pi^2} D^2 \right].
\]
TABLE I: Predictions for the $J^P = 0^+ \ c\bar{u}(\bar{d}) \ (D_0)$ and charmed-strange $c\bar{s} \ (D_{s0})$ meson masses and widths obtained with $c = 1$ and $\mu = 340 \pm 68$ MeV.

| $M_r$ [GeV] | $\Gamma_r$ [MeV] |
|-------------|----------------|
| $D_0$       | 2.15 - 2.30    |
| $D_{s0}$    | 2.44 - 2.55    |

$\Gamma$ decreases for low $M_r$ to 7 MeV, as the resonance mass approaches the $D\pi$ threshold, and increases to 22 MeV at the high mass. However, by changing both $c$ and $\mu$ within their natural ranges it is possible to restore the original resonance parameters. The increasing discrepancy between the BW parameterization and the solid line at higher mass is due to absence of phase space factors (demanded by unitarity) in the BW parameterization.

In summary we have found that using reasonable assumptions regarding flavor-independent interactions between the pion and the charmed-strange mesons, with natural parameters it is possible to reproduce a narrow resonance in the $D\pi$ spectrum. Such states should also be present in other charge modes, e.g. $D_{s0}\pi^{\pm}$. We have also checked that our findings are insensitive to the details of a formulation, e.g. we studied the nonrelativistic approximation and used the $N/D$ method [6].

A similar analysis applied to the $J^P = 0^+ \ D\pi$ and $DK$ systems produces scalar resonances with masses and widths listed in Table 1. These predictions should be easily tested by experiment because of the narrow width of the states involved. These masses are comparable with the quark model predictions of $M_{D_0} = 2.4$ GeV and $M_{D_{s0}} = 2.48$ GeV respectively [11], however none of these states have been observed yet.

It is a pleasure to acknowledge Alex Dzierba, Don Lichtenberg, Dan Krop and Scott Teige for several discussions. This work was supported in part by US DOE contract, DE-FG02-87ER40365.

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