Abstract

The electromagnetic current of bound systems in the light-front is constructed in the Breit-Frame, in the limit of momentum transfer $q^+ = (q^0 + q^3)$ vanishing. In this limit, the pair creation term survives and it is responsible for the covariance of the current. The pair creation term is computed for the $j^+$ current of a spin one composite particle in the Breit-frame. The rotational symmetry of $j^+$ is violated if the pair term is not considered.
The electromagnetic current in the light-front can be constructed from the covariant Feynman one-loop triangle diagram, which is integrated in the $k^- (= k^0 - k^3)$ component of the internal loop momentum [1–4]. The Cauchy integration just includes the residue at the pole of the forward propagating spectator particle in the photon absorption process for $q^+ = 0$. It is understood that it is possible to eliminate pairs created out or annihilating into the vacuum, which leads to a description in terms of of a two-particle light-front wave-function [1,2]. In general, this procedure keeps the covariance under kinematical boost transformations, but the current loses its physical properties under general rotations and parity transformations. As we will show, the pair terms should be evaluated for the full covariance of the current.

Long ago, the problems with the naive Cauchy integration of $k^-$ in momentum loops, which takes into account only the residue at the pole of the forward propagating particle, had been addressed in Ref. [5]. However, in that work, it was not discussed the electromagnetic current. The purpose of our work is to derive the pair creation terms. They are responsible for the covariance of the photon absorption process in the light-front. We show how the pair terms maintain the rotational symmetry of the ”good”-component of the electromagnetic current ($j^+$) in the Breit-frame for a spin one two-fermion composite system. In this case the angular condition is valid. The angular condition has been discussed in the context of the Hamiltonian light-front dynamics [7–10].

We begin our discussion with an illustrative example, where a pair term appears. We show that, the $j^- (= j^0 - j^3)$ component of the current of a two-boson bound state has contribution from the pair term in the Breit-frame with momentum transfer such that $q^+ = q^- = 0$. This completes a previous work on the subject [1], where $j^+$ and $j_\perp$ have been calculated from the triangle diagram. For these components only the contribution from the residue corresponding to the forward propagating boson survives in the Cauchy integration over $k^-$ in the momentum loop. The calculation have been done in the Breit-frame, where the momentum transfer $q^\mu$ is along the transverse direction $x$; $q^\mu = (0, q_x, 0, 0)$. No pair terms survive for $j^+$ and $j_\perp$, which is not valid for $j^-$. The current $j^-$ is known to be the
"bad"-component of the current \[\mathbf{(3)}\], thus in principle it receives contribution from the pair term. It is not apparent how such term appears for \(q^+ = 0\), and to obtain it, a careful analysis of the limit \(q^+ \to 0\) is required.

The \(j^-\)-component of the electromagnetic current is given by:

\[
   j^- = \Gamma^2 \int \frac{d^4k}{(2\pi)^4} \frac{(2k - P' - P)^-}{(k^2 - m^2 + i\epsilon)((k - P)^2 - m^2 + i\epsilon)((k - P')^2 - m^2 + i\epsilon)},
\]

where \(P\) and \(P'\) are the initial and final quadri-momenta of the composite boson; \(\Gamma\) is the vertex of the composite state and \(m\) is the boson mass.

The integral, Eq. (1), in the light-front coordinates, \(k^+\) and \(k^-\), is written as

\[
   j^- = \Gamma^2 \int \frac{d^2k_\perp dk^+dk^-}{2(2\pi)^4} \frac{(2k - P' - P)^-}{k^+(P^+ - k^+)(P'^+ - k^+)(k^- - f_1 - i\epsilon)} \left(\frac{P^- - k^- - f_2 - i\epsilon}{P'^+ - k^+}\right),
\]

where \(f_1 = k_\perp^2 + m^2\), \(f_2 = (P - k)^2_\perp + m^2\), \(f_3 = (P' - k)^2_\perp + m^2\). The Cauchy integration over \(k^-\) in Eq.(3) has two nonzero contributions to the residue calculation, one coming from the interval \(0 < k^+ < P^+\) (I) and another from \(P^+ < k^+ < P'^+\) (II). We use \(q^+\) nonzero with \(P'^+ = P^+ + q^+\), and let then \(q^+\) goes to zero, such that the Breit-frame is recovered. Surprising, the contribution from (II) is nonzero in the above limit.

The Cauchy integration in \(k^-\) for \(0 < k^+ < P^+\) (I), receives contribution from the residue at the forward propagating on-mass-shell pole of the spectator particle in the photon absorption process. This pole is placed in the lower semi-plane of complex-\(k^-\) at \(k^- = (f_1 - i\epsilon)/k^+\). It is the only pole which contributes to the Cauchy integration in \(j^+\) and \(j_\perp\) in the limit of vanishing \(q^+\) [1]. The Cauchy integration of \(j^-\) in region (I) results

\[
   j^- = -i\Gamma^2 \int \frac{d^2k_\perp dk^+}{2(2\pi)^3} \left[ \frac{2f_1 - k^+(P'^+ - P^-)}{k^2(P^+ - k^+)(P'^+ - k^+)(P'^- - f_1 - i\epsilon)} \right] \theta(P^+ - k^+) \theta(k^+) \left(\frac{P^- - f_2 - i\epsilon}{P'^+ - k^+} - \frac{f_3}{P'^+ - k^+}\right).
\]

The second contribution to the Cauchy integration of \(k^-\) in Eq.(3) comes from the region (II) where \(P^+ < k^+ < P'^+\). The integration can be performed in the upper half of the complex-\(k^-\) plane. Only the residue at the pole \(k^- = P'^- - (f_3 - i\epsilon)/(P'^+ - k^+)\) contributes to the Cauchy integration,
\[ j^{-II} = -i \Gamma^2 \int \frac{d^2k_+ dk^+}{(2\pi)^3} \left[ \frac{-2f_3 f_3 + P^+ - P^-}{k^+(P^+ + k^+)(P^+ - k^+)(P^+ - f_3 f_2 - f_1 k^+)} \right] \theta(P^+ - k^+) \theta(k^+ - P^+) \]

\[
\left( \frac{f_3}{P^+ - k^+} - \frac{f_2}{P^+ - k^+} + P^- - P'\right) .
\]

The physical process represented by Eq. (5) is the pair creation by the photon. The denominator \( \left[ \frac{f_3}{P^+ - k^+} - \frac{f_2}{P^+ - k^+} + P^- - P'\right]^{-1} \) corresponds to the forward propagator of the virtual three-particle system composed by the initial bound state, the antiparticle and the particle produced by the incoming photon. The denominator \( [P^+ - \frac{f_3}{P^+ - k^+} - \frac{f_1}{k^+}]^{-1} \) is the forward propagator of the virtual two-particle system, which composes the bound light-front wave-function in the final state.

Before performing the limit of \( q^+ \to 0_+ \), we make the following variable transformation, \( x = (k^+ - P^+)/q^+ \). Eq. (5) becomes

\[ j^{-II} = i \Gamma^2 \int \frac{d^2k_+ dx}{(2\pi)^3} \left[ \frac{-2f_3 q^+(1 - x)(P^+ - P^-)}{(P^+ + q^+ x)(1 - x)^2} \right] \theta(1 - x) \theta(x) \]

\[
\left( \frac{\frac{f_3}{1 - x} + \frac{f_2}{x} + q^+(P^- - P^-)}\right) .
\]

In Eq. (6) the limit \( q^+ \to 0_+ \) can be done, resulting

\[ j^{-II} = i \frac{\Gamma^2}{P^+} \int \frac{d^2k_+ dx}{(2\pi)^3} \frac{\theta(1 - x) \theta(x)}{x f_3 + (1 - x) f_2} = i \frac{\Gamma^2}{P^+} \int \frac{d^2k_+ \ln(f_3) - \ln(f_2)}{(2\pi)^3} \frac{f_3 - f_2}{f_3 - f_2} .
\]

The sum of \( j^{-I} \) and \( j^{-II} \) is equal to the covariant expression Eq. (4), \( j^- = j^{-I} + j^{-II} \).

The pair term assures the correct parity transformation properties of the current in the light-front. The parity operator for the transformation of \( z \) into \( -z \) is of non-kinematical nature, consequently one expects that parity properties coming from this operation are destroyed, if pair terms are neglected. Below, we illustrate this point. Consider the integral

\[ Z = \int \frac{dk^+ dk^-}{(2\pi)^2} \frac{k^+ - k^-}{(k^2 - m^2 + i\epsilon)((k - P)^2 - m^2 + i\epsilon)((k - P')^2 - m^2 + i\epsilon)^2} \]

that vanishes in the Breit-frame where \( q^+ = q^- = 0 \). The argument of the integral is odd under the transformation of \( k^3 \to -k^3 \) \( (k^3 = (k^+ - k^-)/2) \). However, if the Cauchy integration is performed in \( k^- \) taking into account only the residue at the pole \( k^- = (f_1 - i\epsilon)/k^+ \) for \( 0 < k^+ < P^+ \), it results nonzero.
\[ Z = i \int_{0}^{P^+} \frac{dk^+}{4\pi} \frac{k^+ - \frac{f_1}{k^+}}{k^+(P^+ - k^+)(P^+ - k^+ - \frac{f_1}{k^+} - \frac{f_2}{p^+ - k^+})(p^+ - \frac{f_1}{k^+} - \frac{f_3}{p^+ - k^+})} \]

\[
\frac{2f_3}{(2\pi)^2} \left( \frac{2f_3}{p^+ - k^+} \right) \prod_{j=2,j\neq 3}^{N} \left( \frac{f_3}{p^+ - k^+} \right) 
\]

\[
= \frac{i}{2\pi} \ln \left( \frac{f_2}{f_3} \right) \prod_{j=2,j\neq 3}^{N} \left( \frac{f_3}{p^+ - k^+} \right).
\]

This example shows that it is dangerous to commute the limit \( q^+ \to 0_+ \) with the Cauchy integration in \( k^- \). However, the last term in Eq.\((8)\) is exactly the opposite value of the residue that comes from region (II) \((P^+ < k^+ < P')\) in the limit of \( P' \to P^+ \). This contribution is obtained from the same arguments used in deriving Eq.\((3)\). By summing both residues the above integration exactly cancels out, as it should be from parity considerations. The integral which has \( k^- \) in the numerator can be classified as ”bad”, since it has contribution from the pair creation process.

The Eq.\((3)\) can be generalized to include any number of denominators. This will be important for the study of the \( j^+\)-component of the electromagnetic current for the composed vector particle. The integral presented below corresponds to the general form of the pair terms

\[
\lim_{q^+ \to 0} i \int_{P^+}^{P'} \frac{dk^+}{(2\pi)^2} \frac{(P^+ - k^+)^{n-1}}{(P^+ - k^+)^2(P' - k^+)(P' - k^+ - \frac{f_2}{p^+ - k^+})} \left( \frac{2f_3}{(2\pi)^2} \right) \prod_{j=2,j\neq 3}^{N} \left( \frac{f_3}{p^+ - k^+} \right) 
\]

\[
= \frac{i}{2\pi} \sum_{i=2}^{N} \ln \left( f_i \right) \left( (P^+)^{n-1} \right),
\]

in particular for \( N = 3 \) and \( n = 0 \), Eq.\((3)\) reduces to the integrand of Eq.\((3)\).

Now we discuss the ”good” component of the current of the composite vector particle. For our purpose of presenting the main points on how the pair terms maintain the rotational properties of \( j^+ \) in the light-front, we have chosen a \( \bar{\psi} \epsilon^\mu_i \gamma_{\mu} \psi \) coupling. The four-vector \( \epsilon^\mu_i \) is the polarization of the vector particle, in the \( i (= x, y, z) \) direction. The covariant form of \( j^+ \) is given by

\[
 j^+_{ji} = \frac{i}{2\pi} \int \frac{d^4k}{(2\pi)^4} \Lambda(k, P') \Lambda(k, P) \frac{Tr[\epsilon^\mu_j(k - P' + m)\gamma^+(k - P + m)\epsilon_i(k + m)]}{((k - P)^2 - m^2 + i\epsilon)(k^2 - m^2 + i\epsilon)((k - P')^2 - m^2 + i\epsilon)},
\]

where \( j^+_{ji} \) is written in the Cartesian instant-form spin basis, and \( \epsilon^\mu_j \) is the final polarization four-vector and \( \epsilon^\beta_i \) is the initial four-vector polarization. The vector particle four-momentum
in the Breit-frame are \( P^\mu = (P^0, -q_x/2, 0, 0) \) for the initial state, and \( P'^\mu = (P^0, q_x/2, 0, 0) \) for the final state; \( P^0 = m_v \sqrt{1 + \eta}, \) where \( \eta = -q^2/4m_v^2. \)

The regularization function, \( \Lambda(k, P) = C \left[ (k - P)^2 - m_R^2 + i\epsilon \right]^{-1} \), was chosen to turn Eq.(10) finite. The special form of the regulator, allows to identify a null-plane wave-function similar to the one proposed for the pion in Ref. [3]. The factor \( C \) is fixed by the charge normalization.

The trace in the numerator for the different matrix elements of the current, are written for the possible polarization states. The ”good” ones, denoted by \( Tr_{ji}^g \), correspond to the traces that have dependence only on \( k^+ \), \( k_\perp \) and \( k^- (P^+ - k^+)^m \) with \( m = 1, 2, 3, \ldots \) The last ones do not contribute in the interval of \( P^+ < k^+ < P'^+ \) for \( q^+ \to 0_+ \), because the difference \( (P^+ - k^+)^m \) is of the order of \( (q^+)^m \) which vanishes in this limit. For the ”good” terms, only the residue at the spectator particle on-mass-shell pole contributes to the Cauchy integration over \( k^- \) in the limit of \( q^+ \to 0_+ \). This pole in Eq.(10) is placed on the lower half of the complex \( k^- \) plane at \( k^- = (k_\perp^2 + m^2 - i\epsilon)/k^+ \) with \( 0 < k^+ < P^+ \).

The traces in the numerator of Eq.(10), can be written in the following form

\[
Tr_{xx} = Tr_{xx}^g - k^- \left( k_\perp^2 + m^2 - \frac{q_x^2}{4} \right) \frac{q_x^2}{m_v^2} ; \quad Tr_{zx} = Tr_{zx}^g - 2k^- \left( k_\perp^2 + m^2 - \frac{q_x^2}{4} \right) \frac{q_x}{m_v} ; \\
Tr_{yy} = Tr_{yy}^g \quad ; \quad Tr_{zz} = Tr_{zz}^g + 4k^- \left( k_\perp^2 + m^2 - \frac{q_x^2}{4} \right) .
\]

The terms corresponding to the pair production by the photon in the Breit-frame, are constructed from

\[
B_v(q^2) = i \int \frac{d^4 k}{(2\pi)^4} \Lambda(k, P) \Lambda(k, P') \frac{k^- \left( k_\perp^2 + m^2 - \frac{q_x^2}{4} \right)}{(k^2 - m^2 + i\epsilon)((k - P)^2 - m^2 + i\epsilon)((k - P')^2 - m^2 + i\epsilon)} . \tag{12}
\]

The Cauchy integration in \( k^- \), has two pieces \( B_v^I(q^2) \) and \( B_v^{II}(q^2) \). In part (I), the residue is evaluated at the spectator on-mass-shell pole, which corresponds to the integration region of \( 0 < k^+ < P^+ \). In part (II), where \( P^+ < k^+ < P'^+ \), the pair term survives the limit of \( P'^+ \to P^+ \). It is obtained from the pair term given in Eq.(10), as

\[
B_v^{II}(q^2) = \frac{C^2}{P^+} \int \frac{d^2 k_\perp}{(2\pi)^3} \left( k_\perp^2 + m^2 - \frac{q_x^2}{4} \right) \sum_{i=2}^5 \frac{\ln(f_i)}{\prod_{j=2, i\neq j}^5 (-f_i + f_j)} ; \tag{13}
\]
where, $f_1 = k^2 + m^2$; $f_2 = (P-k)^2 + m^2$; $f_3 = (P-k)^2 + m_1^2$; $f_4 = (P-k)^2 + m_2^2$; $f_5 = (P-k)^2 + m_3^2$.

The matrix elements of $j^+$ are given by the sum of two terms, one comes from the contribution of the light-front wave-function for $0 < k^+ < P^+$ in Eq.(10) and the other comes from the pair term obtained with Eq.(13)

$$j^+_{xx} = j^+_{xx} - \frac{q^2}{m_v^2} B^{II}(q^2); \quad j^+_{zx} = j^+_{zx} - 2 \frac{q_x}{m_v} B^{II}(q^2); \quad j^+_{yy} = j^+_{yy}; \quad j^+_{zz} = j^+_{zz} + 4 B^{II}(q^2). \quad (14)$$

Observe that each matrix element of the current acquires contribution from the pair term unless $j^+_{yy}$. The angular condition in the Cartesian spin basis is, $\Delta(q^2) = j^+_{yy} - j^+_{zz} = j^+_y - j^+_z - 4 B^{II}(q^2) = 0 \quad (11)$. It is zero since the matrix elements of the current $j^+_{ij}$ of Eq.(10) have the correct transformation properties for rotation around the $x$-direction. Note that the pair terms not only affect the angular condition but each of the matrix elements of the current.

The violation of the angular condition is a consequence of taking into account only the contribution of the residue of the pole of the spectator particle in the Cauchy integration of the $k^-$ momentum, as has also been shown in a recent numerical study in this model [11]. The violation of the angular condition is given by, $\Delta^I(q^2) = j^+_{yy} - j^+_{zz} = 4 B^{II}(q^2)$. The rotational symmetry of the ”good” component of the current for spin one particle is valid if the pair creation process that survives in the Breit-frame is included in the computation of the matrix elements. The good component of the current receives contribution from the pair term for $q^+ = 0$.

In summary, we have calculated the pair production terms in the electromagnetic current of a composite particle, as a result of the integration of $k^-$ in the loop-momentum of the triangle diagram for the photon absorption process, in the limit of $q^+$ vanishing. The $j^-$ component of the electromagnetic current of a composite boson in the light-front coordinates is calculated. The residue associated with the virtual pair creation process by the photon in $j^-$, survives in the above limit and it is responsible for keeping the covariance properties of the current. In one example, we also derive the pair terms which are important for
maintaining the rotational symmetry of $j^+$, for a composite spin one particle, showing that it is not possible to leave out such contribution without violating the angular condition in the Breit frame.

ACKNOWLEDGMENTS

We thank to Prof. S.J. Brodsky for helpful discussions. This work was supported in part by the Brazilian agencies CNPq, CAPES and FAPESP. It also was supported by Deutscher Akademischer Austauschdienst and Fundação Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (Probral/CAPES/DAAD project 015/95).
REFERENCES

[1] M. Sawicki, Phys.Rev. D44 (1991)433; Phys. Rev. D46 (1992)474.

[2] T. Frederico and G.A. Miller, Phys. Rev. D45 (1992)4207; Phys. Rev. D50(1994)210.

[3] C. M. Shakin and Wei-Dong Sun, Phys. Rev C51 (1995)2171.

[4] W.Jaus, Phys. Rev. D41(1990)3394. See Appendix B, for detailed discussion of the connection of the covariant Feynman amplitudes and Hamiltonian Light-Front dynamics.

[5] S.-J. Chang and T.-M. Yan, Phys. Rev.D7(1973)1147; T.-M. Yan, Phys. Rev.D7(1973)1780.

[6] R. Dashen and M. Gell-Mann, 1966, in Proceedings of the 3rd Coral Gables Conference on Symmetry Principles at High-Energy,1966, edited by A.Perlmutter(Freeman,San Francisco,1967); S.Fubini, G.Segré and D.Walecka, Ann. Phys. 39(1966)381; V. de Alfara, S.Fubini, G.Furlan, C.Rossetti, ”Currents in Hadron Physics” (North Holland Amsterdam 1973).

[7] L.L.Frankfurt and M.I.Strikman, Nucl. Phys. B148 (1979)107; Phys. Rep. 76(1981)215.

[8] I.L.Grach and L.A. Kondratyuk, Sov. J. Nucl. Phys. 39(1984)198, L.L. Frankfurt, I.L.Grach, L.A. Kondratyuk and M.Strikman, Phys. Rev. Lett. 62 (1989) 387; P.L.Chung, F. Coester, B. D. Keister and W.N. Polizou, Phys. Rev. C37 (1988)2000; T. Frederico, E.M. Henley and G.A. Miller, Nucl. Phys. A533 (1991)617.

[9] L. L. Frankfurt, T. Frederico and M. I. Strikman, Phys. Rev. C48 (1993)2182.

[10] B.D. Keister, Phys. Rev. D49 (1994)1500; F. Cardarelli, I.L.Grach, I.M. Narodetskii, E. Pace, G. Salmé and S. Simula, Phys. Lett. B349 (1995)393.

[11] J.P.B.C. de Melo and T.Frederico, Phys. Rev. C55(1997)2043.