Modeling Spatially-Correlated Cellular Networks by Using Inhomogeneous Poisson Point Processes
Shanshan Wang, Marco Di Renzo, Xiaojun Xi

To cite this version:
Shanshan Wang, Marco Di Renzo, Xiaojun Xi. Modeling Spatially-Correlated Cellular Networks by Using Inhomogeneous Poisson Point Processes. 22nd International ITG Workshop on Smart Antennas WSA 2018, Mar 2018, Bochum, Germany. 10.5281/zenodo.3529633 . hal-02376548

HAL Id: hal-02376548
https://centralesupelec.hal.science/hal-02376548
Submitted on 9 Jul 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Distributed under a Creative Commons Attribution 4.0 International License
Abstract—We introduce a new methodology for modeling and analyzing downlink cellular networks, where the Base Stations (BSs) constitute a motion-invariant Point Process (PP) that exhibits some degree of interactions among the points, e.g., spatial inhibition (repulsion) or spatial aggregation (clustering). The proposed approach is based on the theory of Inhomogeneous Poisson PPs (I-PPPs) and is referred to as Inhomogeneous Double Thinning (IDT) approach. The proposed approach consists of approximating a motion-invariant PP with an equivalent PP that is made of the superposition of two conditionally independent I-PPPs. The inhomogeneities are mathematically modeled through two distance-dependent thinning functions and a tractable expression of the coverage probability is obtained.

Index Terms—Cellular Networks, Stochastic Geometry.

I. INTRODUCTION

The theory of Poisson Point Processes (PPPs) has been extensively employed for modeling emerging cellular network architectures [1], [2]-[4]. Modeling cellular networks by using PPPs has the inherent advantage of analytical tractability. Practical cellular network deployments, however, are likely to exhibit some degree of interactions among the locations of the Base Stations (BSs) [5]. Therefore, several other spatial models have been proposed for overcoming the complete spatial randomness property of PPPs [6]-[9].

In this paper, we propose a new approach for modeling cellular networks, which is based on the theory of Inhomogeneous PPs (I-PPPs) and is referred to as Inhomogeneous Double Thinning (IDT) approach. In particular: i) we introduce two distance-dependent intensity functions to create the inhomogeneities based on the spatial inhibition properties empirically observed in practical cellular networks, ii) we devise a method for approximating the network panorama of the typical user of a generic motion-invariant PP with the network panorama of a probe user located at the origin of the approximating I-PPP, and iii) we introduce a new tractable analytical expression of the coverage probability of cellular networks.

II. THE IDT APPROACH

A. Cellular Networks Modeling Using I-PPPs

The BSs constitute the points of two conditionally independent isotropic I-PPPs, denoted by \( \Phi_{BS}^{(F)} \) and \( \Phi_{BS}^{(K)} \), with intensity measures \( \Lambda_{\Phi_{BS}^{(F)}}(\cdot) \) and \( \Lambda_{\Phi_{BS}^{(K)}}(\cdot) \) [4], respectively. Since I-PPPs are non-stationary, we are interested in computing the coverage probability of a probe user that is located at the origin. The BS serving the probe user is assumed to belong to \( \Phi_{BS}^{(F)} \) and the interfering BSs are assumed to belong to \( \Phi_{BS}^{(K)} \).

The aim of the proposed IDT approach is to approximate a generic network model based on a motion-invariant PP, \( \Psi_{BS} \), by appropriately choosing the inhomogeneities of the two conditionally independent I-PPPs \( \Phi_{BS}^{(F)} \) and \( \Phi_{BS}^{(K)} \). The approximation is devised in such a way that the coverage probability, \( P_{cov} \), of the typical user under the network model corresponding to \( \Psi_{BS} \) is closely approximated with the coverage probability, \( \tilde{P}_{cov}^{(o)} \), of the probe user located at the origin under the network model corresponding to \( \Phi_{BS}^{(F)} \) and \( \Phi_{BS}^{(K)} \), i.e., \( \tilde{P}_{cov}^{(o)} \approx P_{cov} \). More precisely, the spatial inhomogeneities of \( \Phi_{BS}^{(F)} \) and \( \Phi_{BS}^{(K)} \) are parameterized by using the triplet of parameters \( (a_F, b_F, c_F) \) and \( (a_K, b_K, c_K) \), respectively, which are obtained from empirical data [5]-[9].

B. Tractable Framework of the Coverage Probability

The following theorem provides a tractable expression of \( \tilde{P}_{cov}^{(o)} \). Two case studies are considered: i) the network is infinitely large and ii) the network has a finite size whose radius is \( R_A \). The second case study is useful to compare the analytical framework against estimates obtained by using empirical data, especially for small values of the path-loss exponent. The following notation is used: \( T \) is the decoding threshold, \( P_{tx} \) is the transmit power, \( \sigma^2 \) is the noise power, \( \kappa \) and \( \gamma \) are the path-loss constant and the path-loss exponent, respectively, \( \lambda_{BS} \) is the BS density, \( I(\cdot) \) is the indicator function, \( \frac{\gamma}{\gamma + d} \) is the Gauss hypergeometric function.

**Theorem 1:** \( \tilde{P}_{cov}^{(o)} \) can be formulated as follows:

\[
\tilde{P}_{cov}^{(o)} = \exp\left(-\xi T\sigma^2/\sigma^2_{Jx}\right) \exp\left(-I(\xi)\right) U_{IN}(\xi) d\xi \\
+ \int_{\Theta} \exp\left(-\xi T\sigma^2/\sigma^2_{Jx}\right) \exp\left(-I(\xi)\right) U_{OUT}(\xi) d\xi
\]

(1)

where \( d_F = \frac{\sigma^2_{tx}}{\sigma^2_{Jx}} d_K = \frac{\sigma^2_{tx}}{\sigma^2_{Jx}} \), \( \Theta \rightarrow \infty \) and \( I(\xi) = I_{Jx}(\xi) \) for infinite-size networks, \( \Theta \rightarrow \kappa R_A^{\lambda} \) and \( I(\xi) = I_{Jx}(\xi) \) for finite-size networks, and \( I_{Jx}(\xi), I_{RA}(\xi), U_{IN}(\xi), U_{OUT}(\xi) \) are defined in (2).
that the IDT approach is accurate, tractable and capable of
analyzing practical cellular network deployments. The numerical results are depicted in Figs. 1. In the figure, three curves are shown: i) the curve labelled “Empirical (R)” corresponds to a cellular network whose BSs are distributed according to a Ginibre Point Process (GPP) [7], ii) the curve labelled “PPP-IDT” is obtained by using the IDT approach in Theorem 1, and iii) the curve labelled “PPP-H” corresponds to the benchmark cellular network deployment where the BSs are distributed according to a homogeneous PPP of density $\lambda_{BS}$. We evince that the IDT approach is accurate, tractable and capable of reproducing the spatial interactions of a widely used non-PPP spatial model, such as the GPP model.

IV. CONCLUSION

In this paper, we have introduced a new tractable approach for modeling and analyzing cellular networks where the locations of the BSs exhibit some degree of spatial interactions. The proposed IDT approach has been shown to be analytically tractable and accurate with the aid of Monte Carlo simulations.

It can be used for analyzing and optimizing several emerging transmission technologies, e.g., [10] and [11].

ACKNOWLEDGMENT

This work is supported in part by the European Commission through the H2020-MSCA ETN-5Gwireless project under Grant Agreement 641985, the H2020-MSCA ETN-5Gaura project under Grant Agreement 675806, and the Agence Nationale de la Recherche Scientifique (ANR) through the project SpatialModulation (Société de l’Information et de la Communication – Action Plan 2015).

REFERENCES

[1] J. G. Andrews, F. Baccelli, and R. K. Ganti, “A tractable approach to coverage and rate in cellular networks”, IEEE Trans. Commun., vol. 59, no. 11, pp. 3122-3134, Nov. 2011.
[2] M. Di Renzo, A. Guidotti, and G. E. Corazza, “Average rate of downlink heterogeneous cellular networks over generalized fading channels - A stochastic geometry approach”, IEEE Trans. Commun., vol. 61, no. 7, pp. 3050-3071, July 2013.
[3] M. Di Renzo, “Stochastic geometry modeling and analysis of multi-tier millimeter wave cellular networks”, IEEE Trans. Wireless Commun., vol. 14, no. 9, pp. 5038-5057, Sep. 2015.
[4] M. Di Renzo, W. Lu, and P. Guan, “The intensity matching approach: A tractable stochastic geometry approximation to system-level analysis of cellular networks”, IEEE Trans. Wireless Commun., vol. 15, no. 9, pp. 5963-5983, Sep. 2016.
[5] A. Guo and M. Haenggi, “Spatial stochastic models and metrics for the structure of base stations in cellular networks”, IEEE Trans. Wireless Commun., vol. 12, no. 11, pp. 5800-5812, Nov. 2013.
[6] M. Haenggi, “The mean interference-to-signal ratio and its key role in cellular and amorphous networks”, IEEE Wireless Commun. Lett., vol. 3, no. 6, pp. 597-600, Mar. 2014.
[7] N. Deng, W. Zhou, and M. Haenggi, “The Ginibre point process as a model for wireless networks with repulsion”, IEEE Trans. Wireless Commun., vol. 14, no. 1, pp. 107-121, Jan. 2015.
[8] Y. Li et al., “Statistical modeling and probabilistic analysis of cellular networks with determinantal point processes”, IEEE Trans. Commun., vol. 63, no. 9, pp. 3405-3422, Sep. 2015.
[9] J. Kibilda, B. Galkin, and L. A. DaSilva, “Modelling multi-operator base station deployment patterns in cellular networks”, IEEE Trans. Mob. Comput., vol. 15, no. 12, pp. 3087-3099, Dec. 2016.
[10] M. Di Renzo et al., “Spatial modulation for generalized MIMO: Challenges, opportunities and implementation”, Proc. of the IEEE, vol. 102, no. 1, pp. 56-103, Jan. 2014.
[11] E. Basar et al., “Index modulation techniques for next-generation wireless networks”, IEEE Access, vol. 5, pp. 16693-16746, Aug. 2017.