A radiative-convective equilibrium model to study young giant exoplanets by direct imaging

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ABSTRACT

Context. Since the end of 2013 a new generation of instruments optimized to directly image young giant planets is coming available on 8-m class telescopes, both at Very Large Telescope and Gemini, in the southern hemisphere. Beyond the achievement of high contrast and the discovery capability, these instruments are designed to obtain photometric and spectral information to characterize the atmospheres of these planets.

Aims. We aim to build a simple tool to interpret future photometric and spectral measurements from these instruments, in terms of physical parameters of the planets.

Methods. We developed a model for young giant exoplanets (Exoplanet Radiative-convective Equilibrium Model or Exo-REM). Input parameters are planet’s surface gravity (g), effective temperature (T_{eff}) and elemental composition. Under the additional assumption of thermochemical equilibrium, the model predicts the equilibrium temperature profile and mixing ratio profiles of the most important gases. Opacity sources include the H$_2$-He collision-induced absorption and molecular lines from H$_2$O, CO, CH$_4$ (updated with the Exomol linelist), NH$_3$, VO, TiO, Na and K. Absorption by iron and silicate cloud particles is added above the expected condensation levels with a fixed scale height and a given optical depth at some reference wavelength. Scattering was not included at this stage.

Results. We applied Exo-REM to photometric and spectral observations of the planet beta Pictoris b obtained in a series of near IR filters. We derived T_{eff} = 1550 \pm 150 \, \text{K}, \log(g) = 3.5 \pm 1, and a radius R = 1.76 \pm 0.24 \, R_{Jup} (2-\sigma error bars). These values are comparable to those found in the literature, although with more conservative error bars, but consistent with the model accuracy. We finally investigated the precision to which the above parameters can be constrained from SPHERE measurements using different sets of near IR filters as well as near low resolution spectroscopy.

Key words. radiative transfer, planets and satellites: atmospheres, planets and satellites: gaseous planets, stars: individual (beta Pictoris)

1. Introduction

Following the detection of 51 Peg b by Mayor & Queloz (1995), by velocimetry, almost 2000 exoplanets\textsuperscript{1} are known as of today (March 10, 2015) and many more candidates are awaiting confirmation (Rowe et al. 2015). Among them, only a few were detected by direct imaging.

The first image of a planetary mass object orbiting a star, 2M1207 b, was obtained by Chauvin et al. (2004) with NaCo at the Very Large Telescope (VLT). In this particular case, the mass ratio was highly favorable as the central star is a Brown Dwarf (BD). No specific device to attenuate the star, like a coronagraph, was needed. But the detection was enabled by the use of an adaptive optics (AO) system in the coronagraph, was needed. But the detection was enabled by the use of an adaptive optics (AO) system in the L′ band. Later, larger mass ratios became feasible with the improvement of AO systems. For now, the planet with the largest mass ratio with respect to its host star and for which we have an image is HD95086 b with a mass of 4.5 \pm 0.5 \, M_{Jup} around a star of 1.6 \, M_{Sun} (Rameau et al. 2013). Conveniently, direct imaging also allows to collect spectroscopic data if one is able to attenuate the starlight at the location of the planet. Janson et al. (2010) presented the first spatially resolved spectra of HR8799 c in the spectral range 3.88–4.10 \, \mu m, a planet that is 6.5 mag dimmer than its parent star (in the L′ band) at an angular separation of 0.96\arcsec (Marois et al. 2008). More recently, near IR low resolution spectra of the planet beta Pictoris b (Lagrange et al. 2010) were obtained with GPI (Macintosh et al. 2014) in the J and H bands (Bonnefoy et al. 2014; Chilcote et al. 2015), an instrument tailored for the search of young giant planets.

Nowadays, the instruments for direct imaging combine AO, coronography and differential imaging to detect faint planets. The Spectro-Polarimetric High-contrast Exoplanet Research, SPHERE (Beuzit et al. 2008), installed at the VLT, is designed to perform high contrast imaging for detecting young giant planets and for characterizing their atmospheres. SPHERE provides broad and narrow band photometry and spectroscopy in the near infrared (NIR) range with the InfraRed Dual-band Imager and Spectrograph (IRDIS, Langlois et al. 2010) and the Integral Field Spectrograph (IFS, Claudi et al. 2008), but also photometry and polarimetry in the visible range with the Zurich Imaging Polarimeter (ZIMPOL, Schmid et al. 2010).

To directly detect the light from a planet around another star than the sun, the following conditions have to be met:

- the star-to-planet angular separation must be larger than the angular resolution offered by an 8-m telescope in the NIR (25–50 mas for SPHERE). This reins the sample of tar-

\textsuperscript{1} exoplanet.eu
gets to less than 100 pc as well as the minimal physical separation to a few AU.

- the star-to-planet brightness ratio must be smaller than the achievable instrumental contrast, which is typically $10^3$–$10^4$ at less than 1". Only giant planets can be warm enough at young ages to produce a detectable emission (Burrows et al. 1995; Chabrier et al. 2000). These Young Extrasolar Giant Planets (YEGP) are ~10–100 millions years old.

One theoretical challenge is to understand planetary formation mechanisms. BD and YEGP are two types of objects almost impossible to differentiate but formed in different ways. The basic idea is to consider that BD are objects formed by gravitational instabilities (Boss 2001) like stars but without a sufficient mass to start to burn hydrogen, unlike planets formed by core-accretion (Lin & Ida 1997). Since 1995 (Nakajima et al. 1995) more than one thousand BD have been discovered\(^2\), which has triggered a large theoretical effort among the community. It was noticed very soon that their atmospheres must contain clouds below $T_{\text{eff}} \approx 2600$ K to account for the spectroscopic observations (Tsuji 2005).

In parallel, atmospheric models for objects with mass and temperature lower than an M dwarf, were developed since the end of the 1990’s. The basic idea is to include in a H-He atmosphere some chemistry and other physical processes to account for the range of pressure-temperature expected in such low mass stars. The models differ in particular by their treatment of dust opacity. In the AMES-Dusty model (Allard et al. 2001) the amount of dust at any level is simply calculated level-by-level from thermochemical equilibrium and conservation of elements. Ackerman & Marley (2001) and Tsuji (2002) follow a semi-empirical approach, with a few free parameters, to estimate the distribution of dust particles above the expected condensation level. More sophisticated models such as BT-Settl (Allard et al. 2003) and Drift-PHOENIX (Helling et al. 2008) aim at describing the actual cloud formation process and include microphysics of dust particles and atmospheric convection. These models were used to constrain the main parameters like effective temperature, surface gravity, atmospheric compounds or radius of the detected planets (Bonnefoy et al. 2013) but also to predict abilities of new instruments (Hanot et al. 2010; Vigan et al. 2010). Most of these models are developed for BD and applied to exoplanets. Although, the radius of exoplanets can be similar to that of BD, they have lower masses. Therefore, the range of surface gravity considered in BD atmospheric models ($\log(g) > 3.5$) do not necessarily cover the entire range expectable for YEGP ($\log(g) > 2$).

Direct imaging of YEGP is characterized by low flux, low signal to noise and low spectral resolution. In that respect, the models that are used to interpret these images should be representative of the level of data quality. For that purpose we specifically developed a model to analyze direct imaging of YEGP for instruments like SPHERE. It is a radiative-convective equilibrium model, assuming thermochemical equilibrium for self luminous planets without stellar heating. It allows us to explore low surface gravity, i.e. low mass YEGP.

The radiative-convective equilibrium model is described in Section 2. In Section 3, we apply Exo-REM to the well-known planet β Pictoris b, derive physical parameters from existing measurements, and compare our results to previously published investigations. In Section 4, we analyze the uncertainties on the derived physical parameters as a function of photometric errors in the context of SPHERE observations. Conclusion is drawn in Section 5.

2. Model description

2.1. Radiative-convective equilibrium model

2.1.1. Numerical method

In a one-dimensional radiative-convective equilibrium model, the net flux ($\pi F$) is assumed to be constant as a function of pressure level. This net flux $\pi F$ is equal to:

$$\pi F = \sigma T_{\text{eff}}^4$$

where $T_{\text{eff}}$ is the effective temperature of the planet. We first solve for purely radiative equilibrium and neglect heating from the parent star. This is justified as long as we are interested in hot young giant exoplanets relatively far from their parent star. Assuming a planet with $T_{\text{eff}} = 700$ K at a distance of 7 AU from the star, the stellar flux absorbed by the planet would amount to less than 0.1% of the planet’s thermal emission. Discarding scattering, the net flux at pressure level $p$, in a plane parallel geometry, is given by:

$$\pi F(p) = 2\pi \int_0^{\infty} d\sigma \int_{\tau_\sigma(p)}^{\infty} B_\sigma(\tau'_\sigma) E_2(\tau'_\sigma - \tau_\sigma) d\tau'_\sigma - \int_{\tau_\sigma(p)}^{\infty} B_\sigma(\tau'_\sigma) E_2(\tau_\sigma - \tau'_\sigma) d\tau'_\sigma,$$

where $\tau_\sigma(p)$ is the optical depth at pressure level $p$ and wavenumber $\sigma$, $B_\sigma(\tau'_\sigma)$ is the Planck function at the temperature of level of optical depth $\tau'_\sigma$ and wavenumber $\sigma$, and $E_2$ is the second-order exponential integral.

The integral over wavenumber is calculated over the range $[\sigma_{\text{min}}, \sigma_{\text{max}}]$, with $\sigma_{\text{min}} = 20$ cm$^{-1}$ and $\sigma_{\text{max}} = 16000$ cm$^{-1}$. This range is sliced into $n_{\sigma}$ intervals of width $\delta\sigma = 20$ cm$^{-1}$, over which the Planck function is taken as constant. The radiative transfer integral over each interval of width $\delta\sigma$ is calculated

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through a correlated-k distribution method with \( n_p \) quadrature points (Goody & Yung 1989). The atmospheric grid consists of \( n_p \) atmospheric levels equally spaced in \( \ln(p) \) between pressure levels \( p_{\text{max}} \) at the bottom of the grid \((j = 1)\) and \( p_{\text{min}}\) at the top \((j = n_p, \) in this paper we use \( n_p = 64)\). Assuming a linear variation of the Planck function \( B \) with optical depth \( \tau \) within any layer \([p_{j-1}, p_{j})\), i.e.

\[
B_\sigma (\tau_\sigma) = B_\sigma (\tau_{\sigma(j-1)}) \frac{\tau_\sigma - \tau_{\sigma j}}{\tau_{\sigma(j-1)} - \tau_{\sigma j}} + B_\sigma (\tau_{\sigma j}) \frac{\tau_{\sigma j} - \tau_\sigma}{\tau_{\sigma(j-1)} - \tau_{\sigma j}}
\]  

(3)

the contribution of this layer to the flux at wavenumber \( \sigma \) can be analytically calculated and expressed as a linear combination of the Planck functions at pressure levels \( p_j \) and \( p_{j-1} \). We also add contribution from below the atmospheric grid \((p > p_{\text{max}})\) assuming a semi-infinite layer with the same variation of the Planck function as in Eq. (3) for the first layer. Summing over all layers and spectral intervals, the net flux at level \( p_j \) can then be expressed as:

\[
\pi F(p_j) = 2\pi \delta \sigma \sum_{i=1}^{n_s} \sum_{j=1}^{n_p} B_{\sigma i} (T_j) \sum_{l=1}^{n_s} \sigma_l A(\sigma_l, p_j, p_j, l),
\]  

(4)

where \( B_{\sigma i} (T_j) \) is the Planck function at temperature of the \( j \)th pressure level of the grid and wavenumber \( \sigma_i \) at the middle of the \( l \)th spectral interval of width \( \delta \sigma \). \( \sigma_l \) is the weight applied to the \( l \)th set of \( k \)-correlated coefficients used in the quadrature for the spectral integration over any spectral interval \((\sum_{l=1}^{n_s} \sigma_l = 1)\).

\( A(\sigma_l, p_j, p_j, l) \) is a dimensionless factor that couples pressure levels \( p_j \) and \( p_j \) and only depends on the grid of optical depths for the \( l \)th set of \( k \)-correlated coefficients of the \( j \)th spectral interval.

We then search for the temperature profile that ensures radiative equilibrium, i.e.:

\[
\pi F(p_j) = \sigma T_{\text{eff}}^4
\]  

(5)

for \( j \) varying from 2 to \( n_p \) (we do not use the flux at the first, deepest, level as a constraint because the variation of the Planck function at deeper levels is fixed arbitrarily in the model to that of the first layer). To solve this system of \( n_p \) equations, we use a constrained linear inversion method described in Vinatier et al. (2007) and based on Conrath et al. (1998). The algorithm minimizes the quadratic difference \( (\chi^2) \) between desired \((\sigma T_{\text{eff}}^4)\) and calculated fluxes with the additional constraint that the solution temperature profile lies close to the reference one. Starting from an initial guess profile \( T_0 \), an approximate solution \( T_1 \) is derived from the equation:

\[
T_n = T_{n-1} + \alpha S K^{-1} \Delta F
\]  

(6)

with \( n = 1 \), where \( \Delta F \) is the difference vector between the desired and calculated fluxes \((\sigma T_{\text{eff}}^4 - \pi F(p_j))\), \( K \) is the kernel.
matrix with $K_{ij}$ equal to the derivative of the flux at level $p_j$ with respect to the temperature at level $p_i$, $S$ is a normalized two-point Gaussian correlation matrix that provides a vertical filtering of the solution needed to avoid numerical instabilities, and $\alpha$ a scalar parameter that controls the emphasis placed on the proximity of the solution $T_1$ to the reference profile $T_0$. We used a correlation length of 0.4 pressure scale height. The kernel matrix is calculated from Eq. (4), neglecting the dependence of $A$ with temperature which it is generally much weaker than that of the Planck function:

$$K_{ij} = 2\pi\delta\sigma \sum_{n=1}^m \frac{\partial B_n(i)(T_j)}{\partial T_j} \sum_{l=1}^n \sigma_l A(\sigma_l, p_i, p_j, l), \quad (7)$$

Matrix $C$ is equal to:

$$C = \alpha KS K^T + E \quad (8)$$

where $E$ is a diagonal matrix with $E_{jj}$ equal to the square of the flux error acceptable at the $j$th pressure level, usually set to 0.1% of $\sigma_T$.

The non-linearity of the problem requires an iterative process in which $T_{n+1}$ is obtained from Eq. (6) after updating the reference profile to $T_{n+1}$ and recalculating the kernel matrix $K$ for profile $T_{n+1}$. The iteration process is pursued till $\chi^2$ is less than 1 and no longer significantly decreases. The $\alpha$ parameter in Eq. (7) is chosen small enough to ensure convergence and large enough to reduce the number of iterations needed. Typically 10 iterations are needed. Note that the final solution does not depend on the initial profile $T_0$ or on the choice of $\alpha$. For $T_{n+1}$, we used one of the three temperature profiles calculated by Allard et al. (2003) for $T_{eff} = 900, 1300$ and 1700 K. We choose the one having $T_{eff}$ closest to the input value to ensure rapid convergence.

![Fig. 3. Temperature profiles calculated for the set of $T_{eff}$ and log$(g)$ parameters used to generate the spectra in Fig. 2. Triangles and squares on the log$(g)$ = 4 profiles indicate the bottom of the iron and silicate clouds respectively.](image)

In a second step, the solution profile is checked against convective instability by comparing the model lapse rate $\nabla = \frac{\ln(T_{i+1})}{\ln(p_{i+1})/p_i}$ with the adiabatic value $\nabla_{ad} = R/C_P$, where $R$ is the gas constant and $C_P$ the temperature-dependent molar heat capacity for the H$_2$-He atmosphere. Regions where the lapse rate exceeds the adiabatic value cannot be convective. They are found in the bottom of the pressure grid $p > p_{ad}$, with $p_{ad}$ being the level where the lapse rate equals the adiabatic value. In that case, convective heat transfer occurs setting back the lapse rate to the adiabatic value. In Exo-REM, we do not solve explicitly for convection. We add a convective flux to the radiative flux in Eq. (4) through an analytical function that is essentially zero when $\nabla \leq \nabla_{ad}$ and rapidly gets very large when $\nabla > \nabla_{ad}$. We found that the following function:

$$F_{conv}(p_j) = 10^{-3}\sigma T_{eff}^4 e^{200(\nabla_{ad} - \nabla)}$$

is adequate to ensure negligible superadiabaticity in the final solution profile. We then set the lapse rate of the purely radiative solution to the adiabatic value, plus a small amount (0.015), at levels $p > p_{ad}$. The kernel matrix $K$ is calculated adding

$$\frac{\partial F_{conv}(p_j)}{\partial T_j} = 0.2\sigma T_{eff}^4 e^{200(\nabla_{ad} - \nabla)/(T_{j+1}T_{j}^{\nabla \sigma_{ff}})} \quad (a)$$

and

$$\frac{\partial F_{conv}(p_j)}{\partial T_{j-1}} = -0.2\sigma T_{eff}^4 e^{200(\nabla_{ad} - \nabla)/(T_{j}T_{j-1}^{\nabla \sigma_{ff}})} \quad (b), \quad (10)$$

to $K_{ij}$ in Eq.(7). The iterative process is finally restarted with the modified flux and kernel till convergence is achieved. Typically another set of 10 iterations is needed. Our model in this paper has 64 pressure levels equally spaced in $\ln(p)$ between 50 bar and 0.01 mbar.

### 2.1.2. Spectroscopic data

As mentioned above, the spectral flux was calculated over 20 cm$^{-1}$ intervals using a $k$-correlated distribution method. For each molecule and each interval, we calculated a set of $n_k = 16$ $k$-coefficients ($l=1$, $n_k$) for the interval [0.95:1.00] of the normalized frequency $f^*$ and 8 for the interval [0.95:1.00]. The values of $g^*$ and associated weights $\sigma_l$ are those of the eight-point Gaussian-Legendre quadrature for each of the two $g^*$ intervals. The $k$-coefficients were calculated for a set of 15 pressures between 100 bar and 0.01 mbar (2 values per decade) and, for each pressure, a set of 6 temperatures, increasing with pressure to encompass model temperature profiles encountered in the literature for exoplanets with 500 K < $T_{eff}$ < 2000 K. Absorptivity spectra for a given pressure and temperature were calculated using a line-by-line radiative transfer program with a frequency step equal to the Doppler half-width of the lines.

We considered the eight most important molecules and atoms in terms of opacity for relatively cool exoplanets (500 K < $T_{eff}$ < 2000 K): H$_2$O, CO, CH$_4$, NH$_3$, TiO, VO, Na and K. The origin of the line lists and the intensity cutoff used to calculate

| Table 1. Atom and molecular opacity sources |
|------------------------------------------|
| **Opacities** | **Intensity cutoff** (cm molecule$^{-1}$) | **References** |
| H$_2$O | 10$^{-22}$ at 2500 K | HITEMP line list (Rothman et al. 2010) |
| CO | 10$^{-22}$ at 3000 K | |
| CH$_4$ | 10$^{-27}$ at 1500 K | Yurchenko & Tennyson (2014) |
| NH$_3$ | 5 × 10$^{-27}$ at 4000 K | Yurchenko et al. (2011) |
| TiO, VO | 10$^{-22}$ at 4000 K | Plez (1998) (with update) |
| Na, K | | Kramida et al. (2014) |
| H$_2$-H$_2$, H$_2$-He | | Burrows & Volobuyev (2003) |
| Borysow et al. (1988, 1989) | | |
| Borysow & Fromhold (1989) | | |
| Borysow et al. (2001) | | |
| Borysow (2002) | | |
the absorptivity spectra are given in Table 1. Note that, in the previous version of the model used by Baudino et al. (2013, 2014a,b), Galicher et al. (2014) and Bonnefoy et al. (2014), the methane linelist originated from Albert et al. (2009), Boudon et al. (2006), Daumont et al. (2013) and Campargue et al. (2012) for CH\(_4\), and from Nikišin et al. (2002, 2006, 2013) for CH\(_3\)D. Our new methane linelist now comes from the Exomol database (Yurchenko & Tennyson 2014).

For all species except alkali, we calculated line absorption up to 120 cm\(^{-1}\) from line center using a Voigt profile multiplied by a \(\chi\) factor to account for sub-Lorentzian far wings. For \(\chi\), we used the profile derived by Hartmann et al. (2002) for H\(_2\)-broadened lines of methane. The far wing absorption of Na and K has been shown to strongly affect the near-infrared spectra of brown dwarfs and extra-solar giant planets (Burrows et al. 2000). For Na and K, we used a Voigt profile \(V(\sigma)\) in the impact region, up to a detuning frequency \((\delta \sigma)\) of 30(T/500)\(^{-0.6}\) cm\(^{-1}\) for Na and 50(T/500)\(^{-0.6}\) cm\(^{-1}\) for K, following Burrows et al. (2000). The Lorentz half-widths, calculated by the impact theory, are 0.27(T/296)\(^{-0.70}\) cm\(^{-1}\) atm\(^{-1}\) for Na and 0.53(T/296)\(^{-0.70}\) cm\(^{-1}\) atm\(^{-1}\) for K. Beyond the detuning frequency, we used a profile in the form:

\[
F(\sigma - \sigma_0) = V(\delta \sigma)[(\delta \sigma / (\sigma - \sigma_0))]^{3/2} \exp[-(h c (\sigma - \sigma_0)/kT)(\sigma - \sigma_0)/\sigma_F] \tag{11}
\]

where \(\sigma - \sigma_0\) is distance from line center, \(V(\delta \sigma)\) is the Voigt profile at the detuning frequency \(\delta \sigma\) and \(\sigma_F\) is a parameter that we adjusted to best reproduce the absorption cross sections calculated by Burrows & Volobuyev (2003) for the red wings of the Na/K + H\(_2\) systems as shown in their Fig. 6. We derived \(\sigma_F = 5000\) cm\(^{-1}\) for Na and 1600 cm\(^{-1}\) for K from best fitting of the 0.6-0.9 \(\mu\)m and 0.8-1.0 \(\mu\)m regions for Na and K respectively. Profiles were calculated up to 9000 cm\(^{-1}\) of line center.

Besides line opacity, we added the collision-induced absorption from H\(_2\)-H\(_2\) and H\(_2\)-He using data files and subroutines provided by A. Borysow\(^3\). These are based on publications by Borysow et al. (2001) and Borysow (2002) for H\(_2\)-H\(_2\), and Borysow et al. (1988, 1989) and Borysow & Frommhold (1989) for H\(_2\)-He.

We finally added absorption by cloud particles, discarding scattering. We considered condensates from Si and Fe, the two most abundant condensing elements in exoplanets with \(T_{\text{eff}}\) in the range 500-2000 K (Lunine et al. 1989). For silicates, we used the optical constants of crystalline forsterite Mg\(_2\)SiO\(_4\) published by Jäger et al. (2003) and for Fe liquid particles those from Ordal et al. (1988).

### 2.2. Atmospheric model

#### 2.2.1. Gas composition

The vertical profiles of H\(_2\)O, CO, CH\(_4\), NH\(_3\), TiO, VO, Na and K are calculated, at each iteration, from thermochemical equilibrium assuming a 0.83/0.17 H\(_2\)/He volume mixing ratio and solar system elemental abundances from Table 3 of Lodders (2010). Note that, in the model, it is also possible to use enrichment factors over the solar values, independently for C, O, N and heavier elements. We considered only the species that significantly affect the profiles of the above mentioned molecules in cool giant exoplanets according to Burrows & Sharp (1999) and Lodders & Fegley (2006). These species are given in Table 2 (Column 2). We also included species involved in the formation of silicate and iron clouds to determine their condensation levels. Equilibrium abundances are derived from the equations of conservation for each element and using the standard molar free energies \(\Delta G^0(T)\) listed in Chase (1998) to calculate equilibrium constants involving the species in Table 2 (Column 2). Calculation is done level-by-level, starting from the deepest level of our grid, at highest pressure and temperature, and moving upwards in the grid. When a condensate appears in a given layer, its constituent elements are partly removed from the gas phase and the new elemental abundances in the gas phase are used to calculate equilibrium abundances in the overlying layer. If no condensation occurs, the same elemental abundances are used in the overlying layer. As in Lodders & Fegley (2006), we take into account dissolution of VO in perovskite (CaTiO\(_3\)) assuming an ideal solid solution and Henry’s law.

#### 2.2.2. Cloud model

Absorption by silicate and iron clouds is included above their respective condensation level \(p_c\) up to one hundredth of this pressure level. A particle-to-gas scale height ratio of 1 is assumed. We assumed spherical particles and used the Mie theory to calculate the absorption Mie efficiency \(Q_{\text{abs}}\) as a function of wavelength. The particle size distribution follows a gamma distribution with a mean radius \(r\) and an effective variance of 0.05.

As discussed by Ackerman & Marley (2001) and Marley et al. (2012), the cloud opacity is expected to be proportional to the pressure \(p_c\) at the condensation level, proportional to the total concentration of the condensing element (Si or Fe) embedded in various molecules at level \(p_c\), and inversely proportional to the gravity \(g\). This relates to the available column density of condensing material at the condensation level. We write the optical depth of the cloud as:

\[
\tau_{\text{cloud}} = \tau_{\text{ref}} \frac{p_c}{p_{\text{ref}}} \tag{12}
\]

where \(p_{\text{ref}} = 1\) bar. Because the solar elemental ratios Si/H and Fe/H are about the same, we assumed the particle column densities of the silicate and iron clouds are in the ratios of the pressure of their condensation levels, and thus that their \(\tau_{\text{ref}}\) at any wavelength are in the ratios of their \(Q_{\text{abs}}\) at this wavelength.

We then keep one free parameter in this cloud model, which is \(\tau_{\text{ref}}\) for the Fe cloud at some reference wavelength.

### Table 2. Compounds considered in thermochemical equilibrium calculations

| Compounds of interest | Species included in chemical equilibrium calculations |
|-----------------------|------------------------------------------------------|
| H\(_2\)O, CO, CH\(_4\) | H\(_2\)O, H\(_2\)O\(^n\), CO, CH\(_4\), NH\(_3\), N\(_2\), NH\(_3\)SH\(^n\), H\(_2\)S, Na, Na\(_2\)S\(^n\), H\(_2\)S, HCl, NaCl, K, KCl, KCl\(^n\), NH\(_3\), NH\(_3\)Cl\(^n\) |
| TiO, VO                | Ti, TiO, TiO\(_2\), V, VO, VO\(_2\), Ca, CaTiO\(_3\), VO\(_2\), H\(_2\)O |
| Mg\(_2\)SiO\(_4\)*, MgSiO\(_3\)*, SiO\(_2\)* | Mg, SiO, H\(_2\)O, Mg\(_2\)SiO\(_4\)*, MgSiO\(_3\)*, SiO\(_2\)*, Fe, Fe* |

1: Species marked with asterisks are condensates

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In running Exo-REM, we found that in some cases, when \( \tau_{\text{ref}} \)

is large and the condensation curve close to the solution temperature profile, the model is unstable through the iteration process and does not converge towards a radiative equilibrium solution. This is because adding particulate opacity increases significantly the temperature just above the condensation level. The temperature in this region may then become larger than the condensation temperature and no self-consistent solution can be found. This instability was also seen by Morley et al. (2012) who advocated a patchy atmosphere to solve this problem and reach a radiative equilibrium state.

### 2.3. Input and output parameters

The input parameters of the model are the effective temperature \( T_{\text{eff}} \), the acceleration of gravity \( g \) at 1 bar, which affects the atmospheric scale height and thus the optical depth profiles, and the oversolar enrichment factors \( \alpha \) for C/H, N/H, O/H and heavier elements X/H (\( \alpha = 1 \) for solar system values). The other set of free parameters are the optical depth of the iron cloud \( \tau_{\text{eff}} \) at 1.2 \( \mu \)m and a reference condensation level of 1.0 bar, and the mean radius \( r \) of the cloud particles.

For output, the model provides the radiative-convective equilibrium temperature profile \( T(\rho) \), the corresponding vertical profiles of the absorbers at chemical equilibrium, and the spectrum at the resolution of the \( k \)-correlated coefficient distribution, i.e. 20 cm\(^{-1}\).

### 2.4. Examples of model outputs

This section shows examples of model outputs (spectra, temperature and abundance profiles) for various input parameters, allowing us to investigate the effect of surface gravity, effective temperature and clouds. All models here, unless specified, assume a solar metallicity. For models with clouds (silicates and iron), we used \( \tau_{\text{ref}} = 1 \) and a mean particle size of 30 \( \mu \)m (Ackerman & Marley 2001). We do not consider water vapor (H\(_2\)O) condensation and thus formation of ice clouds, which would occur in the upper atmospheres of planets with \( T_{\text{eff}} \) less than \( \sim 600 \) K.

Figure 1 presents spectra calculated for a cloud-free atmosphere, \( \log(g) = 4 \) and two values of \( T_{\text{eff}} \), 700 and 2000 K. Water vapor absorbs all over the spectral range for both effective temperature, with strongest bands centered at 0.94, 1.14, 1.38, 1.87 and 2.7 \( \mu \)m. Other absorption features are due to other compounds as indicated in the figure. Besides H\(_2\)O, K and Na absorption are visible both in low- and high-temperature spectra through their resonant lines at 767 and 589 nm respectively. On the other hand, TiO, VO and CO absorptions are important only for large \( T_{\text{eff}} \) while CH\(_4\) bands play a significant role at low temperatures. NH\(_3\) has a weaker effect, only visible in planets with a low \( T_{\text{eff}} \), and provides additional absorption around 3.0, 2.0 and 1.5 \( \mu \)m.

Figure 2 shows spectra calculated for \( T_{\text{eff}} \) varying from 700 to 1700 K and \( \log(g) \) varying from 3 to 5. The fluxes correspond to a planet having a Jupiter radius and located at 10 pc. For each set of parameters, a cloud-free model and a model with clouds are shown. For the case with \( \log(g) = 5 \) and \( T_{\text{eff}} = 700 \) K, only the cloud-free case is shown because cloud condensation occurs below our pressure grid and cannot be taken into account. The temperature profiles corresponding to these sets of parameters and to a cloudy atmosphere are shown in Fig. 3. The locations of the iron and silicate clouds are also indicated.

For a given \( T_{\text{eff}} \), adding cloud absorption yields a smoother spectrum, decreasing the contrast between absorption bands and spectral windows. This is because cloud opacity, concentrated near the cloud base in the 1600–2100 K range, (depending on \( \log(g) \) and \( T_{\text{eff}} \), as shown in Fig. 3), reduces more strongly the flux in the windows, which originate from deeper levels than the flux in the absorption bands. Cloud opacity also affects the relative fluxes in the various photometric bands. Essentially, for a given \( T_{\text{eff}} \), the flux is reduced below \( \sim 1.7 \) \( \mu \)m and increased longwards. Therefore, the flux is significantly lower in the Y and J bands, at which the atmosphere is the most transparent, and higher in the K, L, and M bands where atmospheric opacity is larger. In the set of examples we show in Fig. 2, the strongest cloud effects are seen for \( \log(g) = 5 \) and \( T_{\text{eff}} = 1000, 1400, \) or 1700 K. In these cases, the emission resembles that of a blackbody at temperature \( T_{\text{eff}} \). The pressure levels where \( T = T_{\text{eff}} \), representative of the mean emission level, are the deepest of the set in Fig. 3, \( \sim 0.4 \) bar, and thus the cloud opacity at this level is the largest, according to Eq. (12).

Note that we only show here the effect of clouds for a single set of parameters: \( \tau_{\text{ref}} = 1 \) and a particle scale height equal to the pressure scale height. For this parametrization, the cloud optical depths at a given pressure level are the same for any location of the condensation level (see Eq. 12) and any value of \( T_{\text{eff}} \) or \( \log(g) \). Therefore, we cannot draw any conclusion on the relative effect of cloud opacity as a function of \( T_{\text{eff}} \) or \( \log(g) \) from the calculations shown in Fig. 2. For example, it could be reasonable to assume that \( \tau_{\text{ref}} \) varies as \( 1/\rho \), as does the pressure scale height, following the parametrization of Ackerman & Marley (2001) (e.g. their Eq. 18). This would reduce the increasing effect of clouds with increasing gravity. Also, if the cloud is more confined near the condensation level than assumed here, i.e. a particle-to-gas scale height ratio lower than 1, the effect of clouds would be much reduced for cases with \( T_{\text{eff}} = 700 \) or even
1000 K since particles would be confined to levels well below the mean emission level.

Obviously, the main effect of increasing the effective temperature is to increase the emitted flux but, in addition, changes in the spectral shape between 1 and 4 µm can be noted. As Teff increases, CH₄ bands, mostly visible at 1.7, 2.3 and 3.3 µm, become less and less intense while the CO band at 4.7 µm as well as TiO and VO absorption below 1.3 µm become visible. Such large variations may easily be detected from narrow-band photometry or low-resolution spectroscopy; TiO and VO signatures occur in the J-band, CO affects the M-band at high Teff while, at low Teff, CH₄ has a strong effect in the H, K and L-bands, and NH₃ a marginal effect in the K-band.

The spectral variations with effective temperature are, of course, due to changes in composition as illustrated in Fig. 4. Carbon is partitioned between CO and CH₄, with a CO/CH₄ ratio depending on temperature and, to a lesser extent, on pressure. In the 700-K planet, methane dominates over carbon monoxide above the 1-bar level whereas in the 1700-K planet, CO dominates over the whole pressure grid. Similarly, nitrogen is partitioned between N₂ and NH₃, the latter being abundant in the observable atmosphere only for relatively low effective temperatures (≤ 800 K). We also note that, as effective temperature decreases, TiO and VO get confined to deeper levels and have thus less influence on the outgoing flux. The depletion of TiO and VO in the upper (colder) atmosphere is due to perovskite (CaTiO₃) formation and VO condensation respectively. Alkali Na and K affect all spectra in our grid but are confined at deeper levels in the case of low Teff atmospheres. They are removed from the upper atmosphere by formation of Na₂S condensate and KCl condensation respectively.

Fig. 5. Effect of varying the metallicity Z in a cloudy model with Teff = 1400 K and log(g) = 3.9

Fig. 5 shows the effect of varying the metallicity for given Teff and log(g) assuming no clouds. As expected, increasing the metallicity increases the depth of all absorption bands. For example, considering the water vapor band at 2.7 µm, a metallicity of Z = +0.5 produces a band depth (2.2 / 2.7 µm) twice larger than in the case with Z = -0.5. In principle, the metallicity of an observed exoplanet could thus be deduced from low-resolution spectroscopy provided that the temperature profile modeled from radiative-convective equilibrium is reliable, which also requires that Teff and log(g) can be accurately derived from the spectra. The unknown effect of clouds may be a stronger limitation in some cases since cloud absorption reduces the band depths and may mimic some decrease in metallicity.

The gas scale height is inversely proportional to the acceleration of gravity g. As a result, a given optical depth at a given wavelength is found at deeper pressure levels when g increases. This explains the general behavior of the temperature profiles as a function of log(g) for a given effective temperature as seen in Fig. 3. As log(g) increases, the temperature profile generally moves downwards along with the cloud condensation levels. The situation is however more complicated due to the presence of clouds and to the dependence of molecular absorptivity with pressure. The effect of gravity on the calculated spectral shape is more subtle than that of effective temperature. It is best seen in spectra of Fig. 6 having no cloud opacity. Because thermochemical equilibrium at a given temperature depends on pressure, the gas abundances at a given temperature level depend on the pressure at this level and thus indirectly on the gravity. For example, the CH₄/CO ratio at a given temperature varies as the square of pressure so that the methane mixing ratio at and above the atmospheric level where T = Teff, representative of the mean emission level, is larger for larger g. This explains the large increase in the depth of the methane bands for the Teff = 1700 K (and to a lesser extent 1400 K) profiles when log(g) increases from 3 to 5. In this case, the CH₄ mixing ratio is two orders of magnitude larger at the T = 1700 K level for log(g) = 5 than for log(g) = 3. These calculations suggest that, among objects with Teff ~ 1600-1800 K, methane absorption would be detectable at 2.3 or 3.3 µm in brown dwarfs but probably not in Jupiter-mass planets. On the other hand, for the Teff = 700 K profiles, the CH₄/H₂ mixing ratio is similar for all log(g) at and above the T = Teff level, being at its maximum value, which is twice the C/H elemental ratio. In conclusion, the effect of gravity on the spectra is significant but may be difficult to disentangle from compositional variations.

3. Application of the model to actual observations

3.1. Method

We now describe how we exploit existing data to derive characteristics of planets. As a first step, the model generates a grid of spectra for a range of physical parameters, log(g) between 2.1 and 5.5 with a step of 0.1 and Teff between 700 and 2000 K with a step of 100 K. Importantly, the explored parameter space must be large enough to encompass all acceptable solutions. For simplicity, we fixed the planet radius to one Jupiter radius (R_Jup), and leave the determination of the planet radius to the minimization part (see below).

Besides the direct geometrical effect on the observed flux, the radius also affects the variation of the acceleration of gravity with altitude, and thus the scale height at a given pressure level. We tested this effect in a few test cases in our grid by solving
for radiative equilibrium for two different radii and comparing the corresponding spectra. We observed only very small modifications of the shape of the spectrum, negligible compared with other error bars. Hence the radius may be considered as an independent scaling parameter, only affecting the observed flux through the area $\pi R^2$ seen from Earth. Physical parameters $T_{\text{eff}}$ and $g$ are derived with associated 1- or 2-$\sigma$ error bars (68% and 95% confidence level respectively) from a $\chi^2$ analysis with $n-1$ degrees of freedom (Bevington & Robinson 2003), where $n$ is the number of independent observation points (one degree of freedom is removed by the determination of $R$, see below).

Five types of clouds were considered with characteristics given in Table 3: one without cloud, three with a mean particle radius of 30 $\mu$m and $\tau_{\text{ref}} = 1$, 1 and 3, and one with a mean particle radius of 3 $\mu$m and $\tau_{\text{ref}} = 1$. The differences in the absorption efficiency $Q_{\text{abs}}$ between iron and forsterite explain the differences in the optical depth calculated for each compound. Note that, as already mentioned in section 2.2.2, the model may be unstable for $\tau_{\text{ref}} > 3$.

The data consist of a series of either photometric points (broad bands and/or narrow bands) expressed in magnitudes, or a normalized spectrum, or both. We compute the $\chi^2$ between the data $X_{\text{obs}}$ and each synthetic spectrum in our grid, once integrated over the photometric filters or convolved to the spectrograph resolution ($X_{\text{Model}}$), with the following relation:

$$\chi^2 = \sum \left( \frac{X_{\text{obs}} - X_{\text{Model}}}{\Delta X_{\text{obs}}} \right)^2$$ (13)
Table 3. Clouds parameters used in the five test grids

| < r > (µm) | τ ref | T_MgSiO_3 (λ=1.2 µm; p = 1 bar) | T_fe (λ=1.2 µm; p = 1 bar) |
|------------|-------|-------------------------------|----------------------------|
| 0         | 0     | 0                             | 0                          |
| 30         | 0.1   | 0.015                         | 0.1                        |
| 30         | 1     | 0.15                          | 1                          |
| 30         | 3     | 0.45                          | 3                          |
| 3          | 1     | 0.018                         | 1                          |

Table 4. Photometric measurements of Planet β Pictoris b.

| Filter | Apparent Magnitude | References |
|--------|--------------------|------------|
| Ys     | 15.53 ± 0.34       | Males et al. (2014) |
| J      | 14.0 ± 0.3         | Bonnefoy et al. (2013) |
| CH₄ 5.1% | 13.18 ± 0.15     | Males et al. (2014) |
| H      | 13.5 ± 0.2         | Bonnefoy et al. (2013) |
| Ks     | 12.6 ± 0.1         | Bonnefoy et al. (2011) |
| L'     | 11.02 ± 0.2        | Bonnefoy et al. (2011, 2013) |
| NB 4.05 | 11.20 ± 0.23     | Quanz et al. (2010) |
| M'     | 11.0 ± 0.3         | Bonnefoy et al. (2013) |

where ΔX_{Observed} are the uncertainties on the planet photometry. Then, in the case of photometric measurements, we derived a radius that minimizes the χ² metric:

\[ 5 \log_{10}(R) = - \log \left( \frac{\sum (X_{model} - X_{observed})^2}{\sum (\Delta X_{observed})^2} \right) \]

Therefore, the radius R (given in R_{Jup} unit) is considered as a global scaling parameter that does not influence the shape of the synthetic spectra. Finally, additional constraints based on models and measurements can be introduced in the analysis. Considering the core-accretion model (Mordasini et al. 2012) and the hot-start model (Spiegel & Burrows 2012), assuming a given age of the star, the radius range can be restrained to lower and upper boundaries. Since mass and radius are related to gravity through the relation:

\[ g = \frac{GM}{r^2} \]

radial velocity measurements when available can also be used to put additional constraints on R and g.

3.2. Pictoris b

For illustration, we now apply Exo-REM to the case of the planet β Pictoris b located at 19.44±0.05 pc (van Leeuwen 2007). Discovered back in 2008 (Lagrange et al. 2009), this object is a case study. As it orbits relatively close to its young (21±4 Myr Binks & Jeffries 2014) parent star, a precise follow-up allows a careful determination of the semi-major axis which is 8.9^{+0.4}_{-0.6} AU (Bonnefoy et al. 2014; Lagrange et al. 2013, 2014). The planet resides inside the circumstellar disk detected in 1987 (Smith & Terrile 1987). Importantly, Lagrange et al. (2012a) demonstrated that its orbital plane is in fact aligned with the warp observed by Mouillet et al. (1997) instead of the main disk plane, providing an unambiguous evidence for the disk/planet interaction. A photometric event reported in 1981 could have been produced by the transit of this planet in front of the star (Lecavelier Des Etangs et al. 1997).

We considered the whole set of available photometric measurements covering the near IR wavelengths, all the way to the mid IR. Observations (Table 4) were collected with NaCo (Lenzen et al. 2003; Rousset et al. 2003) at the VLT in the J, H, Ks, L', NB_4.05, M' bands (Bonnefoy et al. 2013, 2011; Quanz et al. 2010) and with MagAO (Close et al. 2012) in the Ys and CH₄ bands as well (Males et al. 2014). Recently, J-band (between 1.12 and 1.35 µm for a resolution of 35-39) and H-band (between 1.51 and 1.79 µm for a resolution of 44-49) spectra were obtained during the GPI (Macintosh et al. 2014) commissioning (Bonnefoy et al. 2014; Chilcote et al. 2015).

First, the grids of models were generated as explained here above without any constraint on radius and mass. We started with the analysis of the photometric data points alone. The models with no cloud and thin clouds (Fig. 7, top left and right panels) do not allow us to achieve a decent minimization, the regions of minima being located at the boundaries of the parameter space. If thicker clouds are introduced (Fig. 7 bottom), the model is able to reproduce the data points (reduced χ² is lower) and the region limited by the 1-σ contour falls within the grid boundaries.

We can constrain the effective temperature to 1500-1700K, while only a lower limit is derived for the gravity (log(g) > 4). Calculations with < r > = 3 µm do not yield a minimum χ² value as low as in the case of < r > = 30 µm and do not provide acceptable solutions at the 1-σ uncertainty level (Fig. 7 bottom right).

The same work was carried out using the J-band spectrum. In that case, the reduced χ² values are lower across the grid as a result of larger flux uncertainties (more models can fit the data). As previously, models without or with thin clouds are rejected. For atmospheres with clouds, the derived values for T_{eff} and log(g) are identical on average but have larger error bars than in the case of photometric data points (Fig. 8).

Assuming an age for the β Pictoris system of 15-25 Myr, evolutionary models predict a radius between 0.6 and 2 R_{Jup}. Including this constraint in the χ² minimization, implies a lower limit for the effective temperature of about 1400 K.

Radial velocity measurements presented in Lagrange et al. (2012b) yield constraints on the planet mass. For separations of 8, 9, 10, 11 and 12 AU, the detection limit corresponds to a mass of 10, 12, 15.5, 20, 25 M_{Jup}, while the model-dependent mass derived from photometry compared to evolutionary models is ≥ 6 M_{Jup} (Bonnefoy et al. 2013). Recently Bonnefoy et al. (2014) used an up-to-date compilation of radial velocity measurements and constrained the mass limit to 20 M_{Jup} (at 96% confidence level). For the purpose of being conservative, we retain a maximum mass of 25 M_{Jup}. These new constraints remove the models with highest gravities in the χ² maps.

The mass and radius constraints allow a more accurate determination of the physical parameters T_{eff} and log(g) as presented in Table 5. The same constraints applied to the case of the J-band spectrum lead to similar results. Therefore, we propose a new determination of T_{eff} = 1550 ± 150 K and log(g) = 3.5 ± 1.0 at the 2-σ confidence level.

These values are in good agreement with the former analyses by Bonnefoy et al. (2013) and Currie et al. (2013) who used the PHOENIX models like BT-Settl, Drift-PHOENIX or Ames-Dusty. The AMES-Cond and AMES-Dusty models represent extreme cases. Both solve for thermochemical equilibrium assuming level-by-level element conservation. In the AMES-Cond model all the dust is removed for opacity calculation while in the AMES-Dusty model the amount of dust is that derived from thermochemical equilibrium with no depletion process. In BT-Settl, the dust particle properties (number density and mean ra-
Table 5. Derived $T_{\text{eff}}$ and log($g$) of $\beta$ Pictoris b in each step of this analysis and shown by other studies.

| Reference                  | data type | constraints     | error size | $T_{\text{eff}}$ [K] | log($g$) | radius [$R_{\text{Jup}}$] | Best fit model          |
|----------------------------|-----------|-----------------|------------|-----------------------|----------|---------------------------|------------------------|
| This work                  | SED       | no              | 1 $\sigma$ | 1600 ± 100            | 4.0 ± 0.5| 1.68 ± 0.22               | clouds, 30 µm          |
| This work                  | SED       | no              | 2 $\sigma$ | 1550 ± 250            | >3.2     | 1.82 ± 0.44               | clouds                |
| This work                  | Spectrum  | no              | 1 $\sigma$ | 1350 ± 450            | >3.4     | 1.73 ± 0.12               | clouds, 30 µm          |
| This work                  | Spectrum  | no              | 2 $\sigma$ | >800                  | >2.6     | 1.76 ± 0.24               | clouds                |
| This work                  | SED       | radius and mass | 1 $\sigma$ | 1550 ± 50             | 3.8 ± 0.6| 1.50 ± 0.06               | clouds                |
| This work                  | SED       | radius and mass | 2 $\sigma$ | 1550 ± 150            | 3.5 ± 1.0| 1.63 ± 0.02               | clouds                |
| This work                  | Spectrum  | radius and mass | 1 $\sigma$ | 1600 ± 200            | 4.0 ± 0.6| 3.6 ± 1.0                 | clouds                |
| Bonnefoy et al. (2013)     | SED       | best $\chi^2$   | 1700 ± 100 |                     | 4.0 ± 0.5| 1.23 ± 1.76               | clouds, Drift Phoenix |
| Currie et al. (2013)       | SED       | no              | 1 $\sigma$ | 1575-1650             | 3.8 ± 0.2| 1.65 ± 0.06               | clouds, Modified AMES-Dusty |

Recently, Chilcote et al. (2015) published a spectrum of $\beta$ Pictoris b in the H band (Fig. 11). We did not perform the same $\chi^2$ analysis on this spectrum as for the J-spectrum because the published error bars only account for random errors (at the 1-2% level) and do not incorporate systematic uncertainties such as an estimated 10% uncertainty on the overall spectral slope. Our best fit, in a least square sense, is obtained for $T_{\text{eff}} = 1200$ K, log($g$) = 4.0 and $\tau_{\text{ref}} = 3$ (Fig. 11). However, this spectrum shows small-scale structure that is not seen in the observations. In addition, this case would imply a too large radius of 3.15 $R_{\text{Jup}}$ to reproduce the absolute flux, which is based on the photometric measurement of Males et al. (2014). We also show a case with $T_{\text{eff}} = 1600$ K, log($g$) = 4.3 and $\tau_{\text{ref}} = 3$, which agrees within error bars with the parameters derived from photometric measurements (Table 5). The agreement with the GPl H-spectrum is satisfactory, the remaining discrepancy being a ~10% spectral slope, which is considered as a possibility by (Chilcote et al. 2015, their Fig. 3). The fits we obtain are closer to the observations than models presented in Chilcote et al. (2015), possibly thanks to the use of the ExoMo database for methane opacity. To fit the shape of the observed spectrum, models with thick clouds are needed. Models with no or thin clouds produce too much contrast between the peak and both ends of the observed spectrum. In Fig. 11, we also show cases where log($g$) is varied by ±0.2 dex around the previous case to illustrate the strong sensitivity of the shape of the spectrum to this parameter. Spectral observations in this band thus provide a way to constrain the gravity of the planet, although one would need to investigate to what extent it can be disentangled from cloud opacity and metallicity.
4. SPHERE expected observations

In this section we discuss the ability of SPHERE to put useful constraints on gravity and effective temperature according to the quality of the data. The purpose is to link the photometric errors to the uncertainties on the physical parameters of planetary atmospheres. A related analysis to derive log(g) and $T_{\text{eff}}$ was performed by Vigan et al. (2010) but using the narrow band differential filters of SPHERE combined to AMES-Cond/Dusty (Allard et al. 2001, 2003), BT-Settl (Allard et al. 2007) and Burrows models (Burrows et al. 2006).

In the following, we consider twelve test-cases for which the model has a robust convergence, and which cover a representative range of log(g) = 2.5, 3.5, 4.5, $T_{\text{eff}}$ = 800, 1100, 1400, 1700 K, and cloud properties ($\tau_{\text{ref}}$ = 1 for 30 $\mu$m particles). For this preliminary analysis, we focus on the near IR broad band filters Y, J, H, Ks, offered in IRDIS, the SPHERE camera, as well as the Y-H mode of IFS (39 wavelengths, with 0.014-0.020 $\mu$m between adjacent pixels), the near IR spectrograph (Table 6).

The spectra of test-cases were integrated over IRDIS filters and a photometric error was added to the integrated flux to mimic an actual photometric measurement. We considered several error amplitudes (in magnitude), $\Delta$ = 0.01, 0.05, 0.1, 0.5 and 1.0, corresponding to very good to very poor data. The same error amplitude was applied to all filters, and we did not consider data with various qualities. As in the previous section, the photometry of test-cases were compared to the grid of models using the $\chi^2$ minimization (no mass or radius constraint). The uncertainties on physical parameters, $\Delta T_{\text{eff}}$ and $\Delta$ log(g), were derived from the 2-$\sigma$ contours. Results are displayed in Fig. 12. We observe that $\Delta T_{\text{eff}}$ decreases as log(g) increases and conversely, $\Delta$ log(g) increases as $T_{\text{eff}}$ increases.

When photometric errors are small, say 0.01 mag, the errors are often smaller or similar to the step of the grid. At the other extreme, when the photometric error is as large as 1 mag, all models contained in the grid match the observation, hence the errors on the physical parameters exceed the range of the grid. We conclude that the effective temperature, respectively the gravity, can be constrained to 200 K, respectively 0.5 dex, if an accuracy of 0.2 mag is met.

We also considered flux spectra, normalized to unity at the peak, with an error of 0.01, 0.05, or 0.1, constant for all wavelengths. The same exercise performed with our set of synthetic spectra (Fig. 13) indicates accuracies of 200 K for $T_{\text{eff}}$ and 0.5 dex for g, assuming a precision of 0.1.

The number of available photometric data points, as well as the covered spectral range, also have an impact on the accuracy of the retrieved physical parameters. For instance, consid-
Fig. 13. Effect of uncertainties on the normalized spectra upon uncertainties on derived \(T_{\text{eff}}\) (top) and \(\log(g)\) (bottom). Cases with 2-\(\sigma\) error bars exceeding our test grid are not plotted.

Table 6. Characteristics of SPHERE IRDIS filters and IFS spectroscopic mode

| Name   | Central wavelength [\(\mu\)m] | FWHM* [\(\mu\)m] |
|--------|-------------------------------|-------------------|
| BB Y   | 1.0425                        | 0.139             |
| BB J   | 1.2575                        | 0.197             |
| BB H   | 1.6255                        | 0.291             |
| BB Ks  | 2.1813                        | 0.3135            |
| Y-H    | 0.957-1.636                   | R = 30            |

*: Full Width at Half Maximum

SPHERE filters with in addition two NaCo filters (L’, M’). We assume an accuracy of 0.1 mag on SPHERE data. To achieve the same accuracy on physical parameters than previously (200 K for \(T_{\text{eff}}\) and 0.5 dex for \(g\)), we conclude that at least three data points are required. In addition, a significant improvement is achieved if the SPHERE photometry is complemented with the NaCo MIR filters. With L’ and M’ filters we can expect uncertainties on \(T_{\text{eff}}\) lower than 100 K and also a smaller error on \(\log(g)\).

5. Conclusions

We developed Exo-REM, a radiative convective equilibrium model to simulate the atmosphere of young Jupiters, which are privileged targets for direct imaging of exoplanets by new instruments like SPHERE mounted at the VLT. The model incorporates opacity from the molecules and atoms that are relevant to observable levels for giant exoplanets having \(T_{\text{eff}} < 2000\) K. It assumes that vertical profiles of these species are governed by thermochemical equilibrium. Cloud absorption by iron and silicate clouds is included through a simplified formalism and a limited number of free parameters.

We used Exo-REM to analyze data available for \(\beta\) Pictoris \(b\) and derive physical parameters of the planet. We inferred an effective temperature \(T_{\text{eff}} = 1550 \pm 150\) K, \(\log(g) = 3.5 \pm 1\), and a radius \(R = 1.76 \pm 0.24 R_{\text{Jup}}\) (2-\(\sigma\) error bars). These results are similar to those previously derived by other authors using different atmospheric models. The difference is that we considered 2-\(\sigma\) error bars (rather than 1-\(\sigma\)) and explored a wider range of parameters, in particular with lower values of \(g\), than in previous studies. Our 2-\(\sigma\) uncertainties include measurement error as well as model dependence on our limited set of cloud parameters (optical depth, particle radius).

We investigated the ability of SPHERE to characterize exoplanets with the IRDIS broadband filters and the Y-H spectroscopic mode of IFS. The couple of filters (H, Ks) appears best suited to constrain \(T_{\text{eff}}\), while the couple (J, Ks) is more appropriate to constrain \(\log(g)\). Combining MIR NaCo L’ and M’ observations with SPHERE photometry enables to obtain good constraints on both \(T_{\text{eff}}\) and \(\log(g)\).

We plan to explore the set of free parameters of Exo-REM more systematically than shown in this paper. In particular, we will study more extensively the effect of metallicity and of cloud parameters (scale height, reference optical depth, particle size). In future works, we may consider constraints coming from \(\text{ab-initio}\) models like BT-Settl or Drift-Phoenix, which provide guidelines for a range of realistic and physical cloud parameters. We will also add absorption by water ice particles that are expected to form in giant exoplanets having lower effective temperatures than studied here.
is open to the community. Known planets are prime targets for a thorough characterization (Lagrange et al. in prep, Zurlo et al in prep).

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