Novel behavior of upper critical field due to nematic order in \textit{d}-wave superconductors

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In recent years, there have been increasing experimental evidence suggesting the existence of an electronic nematic state in a number of unconventional \textit{d}-wave superconductors. Interestingly, it is expected that the long-range nematic order can coexist with \textit{d}-wave superconductivity. We analyze the in-plane upper critical field $H_{c2}$ after taking the influence of nematic state into account, and find that the four-fold oscillation of angle-dependent $H_{c2}$ in a pure \textit{d}-wave superconducting state is turned into a novel two-fold oscillation pattern by a weak nematic order. Moreover, such effect is much more significant at higher temperatures. These behaviors are measurable and may be used to probe the predicted coexisting nematic order in \textit{d}-wave superconductors. In addition, we show that the concrete angular dependence of $H_{c2}$ and the positions of its maximum can be strongly affected by several parameters, including temperature and $T_c$.

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In the past three decades, there have been a plenty of theoretical and experimental studies of the unusual and elusive properties of unconventional superconductors \cite{1}, among which the high-$T_c$ cuprate \cite{2} and heavy fermion \cite{3,4} superconductors have attracted particular interest. It is well-known that cuprates and heavy fermion compounds all have layered structures, with the physical quantities defined along the $c$-axis being quite different from those of the basal $a$-$b$ planes. Generically, the electronic properties turn out to be homogeneous in the $a$-$b$ planes. However, the past decade has witnessed an increasing number of experimental evidence \cite{5,12} which suggests the existence of a strong anisotropy in the electronic properties within the $a$-$b$ planes in some of these superconductors. The earliest evidence of such electronic anisotropy comes from the transport measurements performed on YBa$_2$Cu$_3$O$_{6+\delta}$ \cite{6}. The subsequent experiments have uncovered more extensive evidences in the same compound \cite{7,8} and also in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ \cite{9}. Moreover, a strong anisotropy is found in the so-called hidden-order phase of heavy fermion compound URu$_2$Si$_2$ \cite{10}. It is interesting that iron-based superconductors have also joined the growing family with strong anisotropy \cite{11,12}.

The strong anisotropy observed in the unconventional superconductors can hardly be explained by the small intrinsic lattice anisotropy. In fact, it is universally attributed to the formation of a novel electronic nematic order that spontaneously breaks the $C_4$ symmetry down to a $C_2$ symmetry \cite{14,15}. In recent years, the phase transition that leads to such an electronic nematic order and the associated nematic critical behaviors have stimulated intensive research effort, especially in the context of cuprates \cite{14,15}. In particular, the interaction between the critical fluctuation of nematic order parameter and gapless fermions \cite{19,27} is found to induce a series of anomalous behaviors.

In the meantime, much theoretical work has been done to address the very interesting possibility that a nematic quantum phase transition takes place in a \textit{d}-wave superconductor \cite{14,27}. It was found that the corresponding nematic quantum critical point (QCP), dubbed $x_c$, on the schematic phase diagram Fig. 1 exhibits a series of non-trivial properties \cite{14,27}. While the nematic QCP has been investigated extensively, little attention has been paid to the broad region surrounded by $T_c$, $T_n$ and the segment between $x_c$ and superconducting QCP $x_0$, where the nematic and superconducting orders are supposed to coexist. If the nematic-SC coexisting state really exists, it is interesting to examine the influence of such state on observable quantities.

Amongst the observable quantities that can effectively reflect the influence of nematic order, here we focus on the in-plane upper critical field $H_{c2}$. Recently, in-plane $H_{c2}$ has played an important role in the determination of the precise gap symmetry of \textit{d}-wave superconductors \cite{28,31}. The main reason is that the concrete angular dependence of in-plane $H_{c2}$ is intimately related to the angular dependence of \textit{d}-wave superconducting gap. The formation of an electronic nematic order in a \textit{d}-wave superconductor is always accompanied by a change of the gap symmetry, hence the in-plane $H_{c2}$ turns out to be a suitable quantity to characterize the physical effects caused by nematic order.

As pointed out previously \cite{19}, the nematic order developed in a $d_{x^2−y^2}$-wave superconductor leads naturally to a superconducting state with a mixed $d_{x^2−y^2} + s$-wave gap, where the additional $s$-wave component reflects the modification of the superconducting order parameter in response to the nematic order. Due to this relationship, one should be able to study the influence of the nematic order by examining how the additional $s$-wave gap changes the angular dependence of in-plane $H_{c2}$.

In this paper, we calculate in-plane $H_{c2}$ after taking into account the effects of nematic order. On the basis of these results, we will show that the well-known four-fold oscillation of angle-dependent $H_{c2}$ in pure $d_{x^2−y^2}$ superconductors is strongly modified by the nematic order and completely turned into a two-fold oscillation even when the additional $s$-wave gap induced by nematic order is still quite weak. Moreover, the effects of nematic order is more significant at higher temperatures. We also study
how $H_{c2}$ is affected by various physical parameters, such as temperature, $T_c$ and fermion velocity, and find that the concrete angular-dependence of $H_{c2}$ is very sensitive to these parameters. We will perform calculations of $H_{c2}$ in the contexts of both heavy fermion and cuprate superconductors, and then make a comparison between the behaviors of $H_{c2}$ in these two types of superconductors. Our predictions on $H_{c2}$ are unambiguous and could be directly verified by experiments.

We consider a general layered $d$-wave superconductor. The physical properties in the fundamental $a$-$b$ plane is nearly isotropic, but the coupling between $a$-$b$ planes is relatively weaker. The Fermi surface of layered superconductor usually has a complicated shape. To simplify our calculations, it is convenient to choose a rippled cylinder Fermi surface with a dispersion \[ \varepsilon(k) = \frac{k_x^2 + k_y^2}{2m} - 2t_{c} \cos(k_z c). \] Now suppose an external magnetic field $H$ is imposed to the basal plane. For type-II superconductors, any field $H$ weaker than the lower critical field $H_{c1}$ cannot penetrate the superconductor due to the Meissner effect. As $H$ exceeds $H_{c1}$ and further grows, the superconducting gap is gradually destructed by the orbital effect in many superconductors. The gap is entirely suppressed once $H$ reaches an upper critical field $H_{c2}$, which can be obtained by solving the corresponding linearized gap equation. Under certain circumstances, the Pauli paramagnetic limiting effect can also suppress the gap by breaking spin singlet pairs, and may even be more important than the orbital effect in some peculiar superconductors \[ \text{[32, 33]} \]. In order to make a general analysis, we consider both of these effects in the following.

It is useful to rewrite the magnetic field vector $\mathbf{H}$ in terms of a vector potential. We define the $a$-axis as $x$-coordinate and the $b$-axis as $y$-coordinate, and choose a vector potential
\[
\mathbf{A} = (0, 0, H(-x \sin \theta + y \cos \theta)),
\]
where $\theta$ is the angle between in-plane $H$ and $a$-axis. For conventional $s$-wave superconductors, the paring gap is isotropic and $H_{c2}$ is therefore $\theta$-independent. For $d$-wave superconductors, however, the gap is strongly anisotropic, so $H_{c2}$ becomes $\theta$-dependent. Now the field vector is of the form
\[
\mathbf{H} = \nabla \times \mathbf{A} = (H \cos \theta, H \sin \theta, 0).
\] The generalized derivative operator is given by
\[
\Pi(\mathbf{R}) = -i \nabla_{\mathbf{R}} + 2c \mathbf{A}(\mathbf{R})
= -i \partial_{e_x} e_x - i \partial_{e_y} e_y
+ (-i \partial_z + 2c H (-x \sin \theta + y \cos \theta)) \mathbf{e}_z.
\]

Following the general methods presented in previous papers \[ \text{[34–39]} \], one can obtain the following linearized gap equation:
\[
- \ln \left( \frac{T}{T_c} \right) \Delta(\mathbf{R}) = \int_0^{\pi} d\phi \frac{\pi T}{\sin(\pi T) \eta(T)} \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{2\pi} \frac{d\phi}{2\pi} \left\{ 1 - \cos \left[ \eta \left( h + \frac{1}{2} v_F(\mathbf{k}) \right) \right] \right\} \Delta(\mathbf{R}),
\]
where $\chi = k_z c$ and the function $\Delta(\mathbf{R})$ is
\[
\Delta(\mathbf{R}) = \left( \frac{2c H}{\pi} \right)^{\frac{1}{4}} e^{-eh(\pm x \sin \theta - y \cos \theta)^2}.
\] Here, for simplicity, we do not include the Landau level mixing \[ \text{[31, 31, 31]} \] which will not affect our conclusion. For the chosen cylinder Fermi surface, the Fermi velocity vector takes the form \[ \text{[32]} \]
\[
\mathbf{v}_F(\mathbf{k}) = v_a \mathbf{e}_x \sin \phi_x + v_a \mathbf{e}_y \sin \phi_y + v_c \mathbf{e}_z \sin \phi_z.
\] The Fermi velocity component along $c$-axis is $v_c = 2t_{c} c$. The velocity $v_a$ is defined as $v_a = v_0 \sqrt{1 + \lambda \cos(\chi)}$, where $v_0 = \frac{k_{F_0}}{m}$ with Fermi momentum being related to Fermi energy $\epsilon_F$ by $k_{F_0} = \sqrt{2m \epsilon_F}$. The ratio $\frac{v_c}{v_a} = \lambda$ with $\lambda = \frac{\gamma}{\epsilon_F}$ and $\gamma = \frac{\Delta}{2\mu_B}$. Moreover, we define $h' = -\frac{\mu_B H}{2\gamma}$, where $\mu_B$ is Bohr magneton and $g$ is the gyromagnetic ratio. The orbital effect of magnetic field is reflected in the factor $v_F(\mathbf{k}) \cdot \Pi(\mathbf{R})$, whereas the Pauli paramagnetic effect is reflected in the factor $h'$. The concrete behavior of $H_{c2}$ is determined by the complex interplay of these two effects.

We choose the direction of field $H$ as a new $z'$ axis, and then define
\[
(e_x', e_y', e_z') = (e_x \sin \theta - e_y \cos \theta, -e_z, e_x \cos \theta + e_y \sin \theta).
\]
In the coordinate frame spanned by \((e'_x, e'_y, e'_z)\), we have

\[
\mathbf{v}_F(\mathbf{k}) = v_a \sin(\theta - \varphi) e'_x - v_c \sin(\chi) e'_y + v_a \cos(\theta - \varphi) e'_z,
\]

and

\[
\Pi(\mathbf{R}) = \sqrt{v_c H} \left[(a_+ + a_-) e'_x - i (a_+ - a_-) e'_y + \sqrt{2} a_0 e'_z\right],
\]

where

\[
a_{\pm} = \frac{1}{2\sqrt{v_c H}} \left[-i \sin \theta \partial_x + i \cos \theta \partial_y \mp \partial_z \right. \\
\left. \pm 2 i e H (x \sin \theta - y \cos \theta)\right],
\]

\[
a_0 = \frac{1}{\sqrt{2} e H} \left[-i \partial_x \cos \theta - i \partial_y \sin \theta\right],
\]

which satisfy \(a_{-}, a_{+} = 1\) and \(a_{\pm}, a_{0} = 0\).

In Eq. (4), the gap symmetry is encoded in the function \(\gamma_{\alpha}(\mathbf{k})\). For isotropic s-wave pairing, \(\gamma_{\alpha}(\mathbf{k}) = 1\). For \(d_{x^2-y^2}\)-wave pairing, one can write the gap as [28]

\[
\gamma_{d}(\mathbf{k}) = \sqrt{2} \cos(2\varphi)
\]

To make a general analysis, we consider a mixed gap

\[
\gamma_{d+s}(\mathbf{k}) = \sqrt{1 - r^2} \sqrt{2} \cos(2\varphi) + r,
\]

where \(r\) is a tuning parameter. When \(r = 0\), the above mixed gap reduces to the pure \(d_{x^2-y^2}\)-wave gap, whereas \(r = \sqrt{2}/3\) corresponds to the upper limit of the influence of nematic order. For \(r \in (0, \sqrt{2}/3)\), there is a mixing between \(d_{x^2-y^2}\)-wave and s-wave gaps, and there are still four gapless nodes. Apparently, \(r\) measures the ratio of s-wave component to the mixed gap. Once a nematic order is formed in a \(d_{x^2-y^2}\)-wave superconductor, the effective superconducting gap is described by Eq. (12), with \(r\) reflecting the influence of the nematic order.

In order to make the phenomenon of gap mixing more transparent, we plot the mixed gap in Fig. 2. When \(r = 0\), the pure \(d_{x^2-y^2}\)-wave gap exhibits a discrete \(C_4\) symmetry. Once \(r\) becomes finite, the \(C_4\) symmetry is broken down to a \(C_2\) symmetry, which is obviously driven by the formation of nematic order. It is interesting to examine how \(r\), and other physical parameters as well, influence the angular dependence of \(H_{c2}\). At first glance, it seems that \(H_{c2}\) should always exhibit its maximum along the antinodal direction \((\theta = 0, \pi)\) since the mixed gap is maximal in this direction. As will be shown below, this is actually not the case. For certain parameters, \(H_{c2}\) could exhibit its maximum along the nodal direction where the mixed gap vanishes. In addition, although the mixed gap has four nodes for \(r < \sqrt{2}/3\), the four-fold oscillation pattern of \(H_{c2}\) can be completely changed by the nematic order.

Substituting Eq. (7), Eq. (8) and Eq. (12) into the linearized gap equation (4) and making average over the ground state \(\Delta(\mathbf{R})\), following the procedure presented in Ref. [34], we finally obtain an integral equation of \(H_{c2}\),

\[
\ln \frac{1}{t} = \int_0^{+\infty} du \frac{1 - \cos(hu)}{\sinh(u)} \left\{ \int_0^{\pi} \frac{1}{2\pi} \int_0^{2\pi} \frac{d\varphi}{2\pi} \times \left[ (1 - r^2) (1 + \cos(4\varphi) \cos(4\theta)) \right. \right.
\]

\[
\left. \left. + 2 \sqrt{2} r \sqrt{1 - r^2} \cos(2\varphi) \cos(2\theta) + r^2 \right] \right. \times \exp \left[ -pu^2 \left[ \sin^2(\varphi) + \frac{u^2}{v_a^2} \sin^2(\chi) \right] \right] \right\}.
\]

where

\[
t = \frac{T}{T_c}, \quad h = \frac{g \mu_B e H_{c2}}{2\pi k_B T} e H_{c2}, \quad \rho = \frac{\hbar v_s^2}{8\pi^2 k_B T^2} e H_{c2}.
\]

The angular dependence of in-plane \(H_{c2}\) can be obtained by solving this equation.

The equation of \(H_{c2}\) contains a number of parameters, and cannot be really solved without choosing concrete magnitudes for these parameters. Although our analysis is actually material independent, we need to consider some realistic samples so as to obtain a detailed angular dependence of \(H_{c2}\). In this paper we consider two types of d-wave superconductors: cuprates and heavy fermion compounds. It is fair to say that the experimental evidence for nematic order in cuprates is far more extensive and compelling than that in heavy fermion compounds. To date, there are only limited reports suggesting the existence of nematic state in one single heavy fermion compounds. However, it is still useful to make a general analysis and compare the unusual behaviors of \(H_{c2}\) obtained in different sorts of compounds. As will be shown below, this can help us to gain valuable insights on the difference between orbital effect and Pauli paramagnetic effect. Moreover, \(H_{c2}\) manifests unexpected properties even in the absence of nematic order, which makes it not only interesting but necessary to perform a detailed analysis in the contexts of heavy fermion compounds.
Here, $g = 1$, $v_0 = 2000\text{m/s}$, $\lambda = 0.5$ and $\gamma = 1$. We suppose $T_c = 1K$, which is a typical value for heavy fermion superconductors. (a) $t = 0.1$; (b) $t = 0.9$.

Fig. 3 shows the $\theta$-dependence of $H_{c2}$ for a number of different values of $r$. We choose $T_c = 1K$, $v_0 = 2000\text{m/s}$ and $g = 1$, which are suitable quantities for heavy fermion compounds. The results are obtained at two representative temperatures, $t = 0.1$ and $t = 0.9$, respectively. Though heavy fermion compounds have layered structures, the coupling between different layers are not very small, so we choose $\lambda = 0.5$. In addition, we take $\gamma = 1$ throughout the whole paper.

Let us first concentrate on Fig. 3(a). We can see that the $\theta$-dependence of $H_{c2}$ exhibits a four-fold oscillation at $r = 0$, which is well consistent with previous results 28, 29. As $\theta$ changes from 0 to $\pi$, there are two peaks for $H_{c2}$, locating at $\pi/4$ and $3\pi/4$ respectively. As $r$ is growing continuously, these two peaks move away from their original positions and approach each other. Once $r$ exceeds certain critical value $r_c$, they converge to one single peak located at $\theta = \pi/2$. As shown in Fig. 3(a), $H_{c2}$ exhibits a two-fold oscillation at $r = 0.2$, reflecting the strong influence of the nematic order. After careful calculations, we find the critical value $r_c \approx 0.149$ for $t = 0.1$. For $r$ greater than $r_c$ (remaining smaller than $\sqrt{2/3}$), the mixed gap still has four nodes as $\theta$ grows from 0 to $2\pi$, but $H_{c2}$ exhibits only one peak from 0 to $\pi$. Apparently, a small amount of nematic order-induced $s$-wave gap is strong enough to trigger the four-to-two fold transition of $H_{c2}$.

Similar to Fig. 3(a), $H_{c2}$ presented in Fig. 3(b) also exhibits a four-fold oscillation at small $r$ and a two-fold oscillation when $r$ is greater than a critical value $r_c$. We find the critical value $r_c \approx 0.013$ for $t = 0.9$. This result indicates that the effect of nematic order is much more significant at high temperatures.

In realistic experiments, the existence of nematic state is usually identified by measuring the spatial dependence of certain observable quantities 10, 11, such as specific heat and thermal conductivity. The in-plane $H_{c2}$ provides a new route to probe the nematic state. One advantage of this route is that the nematic state can be determined by measuring the angular dependence of $H_{c2}$ even when the nematic order is rather weak, because the two-fold oscillation can be easily distinguished from the four-fold oscillation. Another interesting result is that the critical value $r_c$ obtained at high temperatures is much smaller than that of low temperatures. If $r$ is fixed at a given value, such as $r = 0.05$, there should be a four-to-two fold transition of $H_{c2}$ as $T$ increases from $T = 0$ to $T_c$.

One can notice a very important difference between Fig. 3(a) and Fig. 3(b): $H_{c2}(\theta)$ exhibits a minimum at $\theta = 0$ for $t = 0.1$, but exhibits a maximum at $\theta = 0$ for $t = 0.9$. Two main conclusions can be drawn from these results. First, the maximum of $H_{c2}$ is not necessarily along the anti-nodal directions where the $d_{2g}$-wave gap is maximal. Second, $H_{c2}$ exhibits apparently different $\theta$-dependencies at low and high temperatures, and there is a transition of the oscillation pattern of $H_{c2}$ at certain critical temperature $T_0$. 

![Diagram](image-url)
oscillation transition. A nematic order is responsible for this four- to two-fold oscillation once $H$ becomes sufficiently large. This property clearly demonstrates the existence of a temperature-driven transition between two different oscillation patterns of $\theta$-dependent $H_{c2}$.

We feel it necessary to examine the effects of different temperatures on the behaviors of $H_{c2}$ in more details. We show the $t$-dependence of $H_{c2}(\theta = 0^\circ)$ in Fig. 4(a), that of $\Delta H_{c2} = H_{c2}(\theta = 45^\circ) - H_{c2}(\theta = 0^\circ)$ in Fig. 4(b) and that of $\Delta H'_{c2} = H_{c2}(\theta = 90^\circ) - H_{c2}(\theta = 0^\circ)$ in Fig. 4(c). First of all, notice that $H_{c2}$ shown in Fig. 4(a) is not monotonic at lower temperatures, which agrees well with the results of Ref. [31]. Such non-monotonicity is presumably due to the Pauli paramagnetic effect [31]. Furthermore, notice that the differences $\Delta H_{c2}$ and $\Delta H'_{c2}$, shown in Fig. 4(b) and Fig. 4(c) respectively, are positive at lower temperatures but negative at higher temperatures. This property clearly demonstrates the existence of a temperature-driven transition between two different oscillation patterns of $\theta$-dependent $H_{c2}$.

The above theoretical analysis can be readily applied to study the behaviors of $H_{c2}$ in the contexts of cuprates. We show the $\theta$-dependence of $H_{c2}$ in Fig. 5 for several values of $r$ after assuming $T_c = 40K$, $v_0 = 10^3 m/s$ and $r = 0.1$, which are typical values for cuprates. Since the coupling between different layers in cuprates is quite small, we take $\gamma = 0.01$. Fig. 5(a) is obtained at $t = 0.1$, and Fig. 5(b) at $t = 0.95$. It is easy to see that $H_{c2}$ exhibits a four-fold oscillation at $r = 0$ and a two-fold oscillation once $r$ becomes sufficiently large. This feature is qualitatively the same as that in the case of heavy fermion compounds. Once again, the formation of a nematic order is responsible for this four- to two-fold oscillation transition.

However, $H_{c2}$ shown in Fig. 5 differs from that in Fig. 4 in several aspects. First, it is widely believed that the Pauli paramagnetic effect plays little role in cuprates. We have re-calculated $H_{c2}$ under the same parameters given in the last paragraph after removing the Pauli effect, and found a nearly negligible change of the behavior of $H_{c2}$. This indicates that the Pauli paramagnetic effect can be guaranteed to be unimportant once an appropriate set of parameters are chosen. Second, as shown in Fig. 5(a), the maximal value of $H_{c2}$ is well beyond 100 Tesla at low temperature $t = 0.1$. This result is actually expectable since it has been known for a long time that $H_{c2}$ of cuprates is roughly $100 \sim 200$ Tesla. Such a strong magnetic field is apparently too large to be realized in laboratories, so the predicted low-temperature behavior of $H_{c2}$ can hardly be tested by experiments. Moreover, we find the critical value $r_c = 0.513$ for $t = 0.1$, which means the four-fold oscillation of $H_{c2}$ will not be converted to two-fold as long as the nematic order is not extremely strong.

We next consider the results obtained at some finite temperature, shown in Fig. 5(b). If the temperature is fixed at $t = 0.95$, the magnitude of $H_{c2}$ is always smaller than 10 Tesla, which is not very strong and can be achieved in laboratories. In addition, we find the critical value of $r$ at $t = 0.95$ is quite small: $r_c = 0.043$. Therefore, the effects of nematic order on $H_{c2}$ is much more significant at higher temperatures, and can be most easily measured at temperatures near $T_c$. 

FIG. 5: $\theta$-dependence of $H_{c2}$ for different value of $r$. Here, $g = 0.1$, $v_0 = 10^3 m/s$, $\lambda = 0.01$, and $\gamma = 1$. We suppose $T_c = 40K$, which is a suitable value for cuprates. (a) $t = 0.1$; (b) $t = 0.95$.

FIG. 6: Relation between $\Delta H_{c2}$, $\Delta H'_{c2}$ and $t$ with $g = 0.1$, $v_0 = 10^3 m/s$, $\lambda = 0.01$, $\gamma = 1$, $T_c = 40K$ for different values of $r$. (a) $H_{c2}$ for $\theta = 0^\circ$; (b) $\Delta H_{c2} = H_{c2}(\theta = 45^\circ) - H_{c2}(\theta = 0^\circ)$; (c) $\Delta H'_{c2} = H_{c2}(\theta = 90^\circ) - H_{c2}(\theta = 0^\circ)$. 

FIG. 6: Relation between $\Delta H_{c2}$, $\Delta H'_{c2}$ and $t$ with $g = 0.1$, $v_0 = 10^3 m/s$, $\lambda = 0.01$, $\gamma = 1$, $T_c = 40K$ for different values of $r$. (a) $H_{c2}$ for $\theta = 0^\circ$; (b) $\Delta H_{c2} = H_{c2}(\theta = 45^\circ) - H_{c2}(\theta = 0^\circ)$; (c) $\Delta H'_{c2} = H_{c2}(\theta = 90^\circ) - H_{c2}(\theta = 0^\circ)$. 

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FIG. 6: Relation between $\Delta H_{c2}$, $\Delta H'_{c2}$ and $t$ with $g = 0.1$, $v_0 = 10^3 m/s$, $\lambda = 0.01$, $\gamma = 1$, $T_c = 40K$ for different values of $r$. (a) $H_{c2}$ for $\theta = 0^\circ$; (b) $\Delta H_{c2} = H_{c2}(\theta = 45^\circ) - H_{c2}(\theta = 0^\circ)$; (c) $\Delta H'_{c2} = H_{c2}(\theta = 90^\circ) - H_{c2}(\theta = 0^\circ)$. 

FIG. 6: Relation between $\Delta H_{c2}$, $\Delta H'_{c2}$ and $t$ with $g = 0.1$, $v_0 = 10^3 m/s$, $\lambda = 0.01$, $\gamma = 1$, $T_c = 40K$ for different values of $r$. (a) $H_{c2}$ for $\theta = 0^\circ$; (b) $\Delta H_{c2} = H_{c2}(\theta = 45^\circ) - H_{c2}(\theta = 0^\circ)$; (c) $\Delta H'_{c2} = H_{c2}(\theta = 90^\circ) - H_{c2}(\theta = 0^\circ)$. 

FIG. 6: Relation between $\Delta H_{c2}$, $\Delta H'_{c2}$ and $t$ with $g = 0.1$, $v_0 = 10^3 m/s$, $\lambda = 0.01$, $\gamma = 1$, $T_c = 40K$ for different values of $r$. (a) $H_{c2}$ for $\theta = 0^\circ$; (b) $\Delta H_{c2} = H_{c2}(\theta = 45^\circ) - H_{c2}(\theta = 0^\circ)$; (c) $\Delta H'_{c2} = H_{c2}(\theta = 90^\circ) - H_{c2}(\theta = 0^\circ)$.
Similar to the case of heavy fermion compounds, there might be a temperature-driven transition from four-fold oscillation of $H_{c2}$ to two-fold oscillation if $r$ is fixed at a suitable value, such as $r = 0.1$. However, from Fig. 4 we see that $H_{c2}$ always displays its maximum at $\theta = 0$ and $\theta = \pi$ in the whole range of $0 \leq t < 1$. This property is quite different from that found in the case of heavy fermion compounds. We attribute this difference to the Pauli paramagnetic effect, which plays no role in cuprates but could be very important in heavy fermion compounds.

It is helpful to plot the $t$-dependence of $H_{c2}(\theta = 0^\circ)$ in Fig. 5(a), that of $\Delta H_{c2} = H_{c2}(\theta = 45^\circ) - H_{c2}(\theta = 0^\circ)$ in Fig. 5(b) and that of $\Delta H_{c2}' = H_{c2}(\theta = 90^\circ) - H_{c2}(\theta = 0^\circ)$ in Fig. 5(c). $H_{c2}$ shown in Fig. 5(a) is monotonic as temperature is varying, which is understandable since the Pauli effect is believed to be negligible. The differences $\Delta H_{c2}$ and $\Delta H_{c2}'$, shown in Fig. 5(b) and Fig. 5(c) respectively, are always negative in the whole temperature range, which are in sharp contrast with those shown in Fig. 5(b) and Fig. 5(c).

In summary, we have performed a detailed analysis of the in-plane upper critical field $H_{c2}$ in layered $d_{x^2-y^2}$-wave superconductors after including the influence of a long-range nematic order that is supposed to coexist with superconductivity. On the basis of our results, we have reached two main conclusions. First, the well-known superconductivity. On the basis of our results, we have long-range nematic order that is supposed to coexist with a number of physical parameters including $T$ dependence of strength of nematic order. Second, the concrete angle-transition of temperatures. Moreover, there may be a four-to-two fold oscillation by a weak nematic order. The strength of the wave superconductors can be turned into a novel two-fold superconductors.

We believe that the predicted two-fold oscillation of in-plane $H_{c2}$ driven by nematic order is particularly interesting. This novel oscillation pattern is unambiguous and could be clearly identified in experiments provided it really occurs. More importantly, such two-fold oscillation can happen even when the nematic order is quite weak and the spatial anisotropy of other physical quantities is not significant enough to be observed. Therefore, this behavior may serve as an experimental signature for the existence of a long-range nematic order in $d_{x^2-y^2}$-wave superconductors.

Another interesting result is that the angle-dependence of $H_{c2}$ is actually very sensitive to a number of physical parameters. A more extensive and systematical analysis is required to fully reveal the influence of all the relevant parameters on $H_{c2}$. Such analysis is necessary since the concrete angle-dependence of $H_{c2}$ proves to be a powerful tool in the determination of precise gap symmetry of $d$-wave superconductors. This issue will be addressed in more details in a separate work.

In the above analysis, we have considered solely the superconductors with a $d_{x^2-y^2}$-wave gap symmetry. It is proposed that some superconductors might have a $d_{xy}$ gap. We have applied the same formalism to the superconductors with a $d_{xy}$-wave gap, and found that the effects of nematic order on in-plane $H_{c2}$ are very similar to those in $d_{x^2-y^2}$-wave superconductors. There is only one difference: the maximum of $H_{c2}$ rotates by $\pi/4$ compared to the case of $d_{x^2-y^2}$-wave gap.

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