High scale perturbative gauge coupling in R-parity conserving SUSY SO(10) with longer proton lifetime

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Abstract. It is well known that in single step breaking of R-parity conserving SUSY SO(10) that needs the Higgs representations $126 \oplus 126$, the GUT-gauge coupling violates perturbative constraint at mass scales few times larger than the GUT scale. Therefore, if the SO(10) gauge coupling is to remain perturbative up to the Planck scale($\equiv 2 \times 10^{18}$ GeV), the scale $M_U$ of the GUT symmetry breaking is to be bounded from below. The bound depends upon specific Higgs representations used for SO(10) symmetry breaking but, as we find, can not be lower than $1.5 \times 10^{17}$ GeV. In order to obtain such a high unification scale we propose a two-step SO(10) breaking through $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C (g_{2L} \neq g_{2R})$ intermediate gauge symmetry. We estimate potential threshold and gravitational corrections to the running of gauge couplings and show that they can make the picture of perturbative GUT-gauge coupling running consistent at least up to the Planck scale. We also show that when SO(10) $\rightarrow G_{2213}$ by Higgs representations $210 \oplus 51$, gravitational corrections alone with negligible threshold effects may guarantee such perturbative gauge coupling. The lifetime of the proton is found to increase by nearly 6 orders over the present experimental limit for $p \rightarrow e^+ \pi^0$. For the proton decay mediated by dim.5 operator a wide range of lifetimes is possible extending from the current experimental limit up to values 2 – 3 orders longer.

1 Introduction

In spite of its astounding success the nonsupersymmetric standard model(SM) suffers from the well known gauge hierarchy problem. It fails to explain the available data on neutrino masses and mixings and also fails to exhibit unification of the three known gauge couplings at higher scales. One compelling reason to solve the gauge hierarchy problem is to go beyond the SM through weak-scale SUSY as in the minimal supersymmetric standard model(MSSM) [1]. The MSSM has the added virtues that, in addition to explaining the origin of electroweak symmetry breaking, it provides a candidate for dark matter of the Universe. If the SM fermion representations are extended by the addition of one right-handed neutrino per generation and the corresponding extension is made in MSSM, the model can account for neutrino masses and mixings through seesaw mechanisms [2,3,4].

Another amazing aspect of MSSM has been noted to be the unification of the three gauge couplings of disparate strengths and origins when extrapolated to as high a scale as $M_U = 2 \times 10^{16}$ GeV [5]. However, the meeting of the three gauge couplings can be truly termed the grand unification [6,7] of the three basic forces of nature provided the merged coupling constants evolve as a single gauge coupling at higher scales and some simple ansatz for this have been hypothesized through SUSY GUTs such as SU(5), SO(10), $E_6$, and a number of others [7,8,9].

While R-parity violation as an automatic consequence of MSSM spoils the predictive power of supersymmetric theories, an additional elegant feature of SUSY SO(10) breaking down to MSSM is its potentiality to conserve R-parity. As the minimal left-right symmetric GUT SO(10) contains the maximal subgroup $SU(2)_L \times SU(2)_R \times SU(4)_C$ of Pati-Salam [6] which in turn contains $SU(2)_L \times U(1)_R \times SU(4)_C$, $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \equiv G_{2213}$, $SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C$ and $SU(2)_L \times U(1)_{Y} \times SU(3)_C \equiv G_{213} \equiv SM$ as its subgroups [10]. Thus, subject to the consistency with the renormalization group constraints, SUSY SO(10) gauge symmetry may break to the SM gauge group directly in one-step or through an intermediate gauge symmetry to the MSSM [11,20]. In addition to other superheavy representations needed to implement the GUT symmetry breaking, two different popular choices of Higgs representations being extensively used to obtain the MSSM from SUSY SO(10) are $16 \oplus 10$ and $126 \oplus 126$. While the first choice violates R-parity, the second conserves it. The Higgs representations $126 \oplus 126$ in SUSY SO(10) have been found to solve a number of problems on fermion masses through renormalizable interactions. To cite a few, it rectifies the bad SU(5)-mass relation in the right direction in SO(10) to yield $m_\tau = 3m_\mu$. It attributes large atmospheric neutrino mixing to $b - \tau$ unification and accommodates the
masses and mixings of three neutrino flavors, in addition to the observed masses and mixings of all other fermions, via Type II seesaw mechanism [12, 13, 14, 15, 21]. However, because of large contribution to the $\beta$-function coefficient of the gauge coupling evolution, the presence of $126 \oplus 126$ in R-parity conserving SUSY SO(10), in addition to other Higgs representations, violates perturbative constraint on the GUT gauge coupling ($\alpha_G < 1$) even at mass scales few times larger than the GUT scale ($\approx 2 \times 10^{16}$ GeV). Although there might be deeper reasons to believe that the R-conserving SUSY at such scales could be nonperturbative, it is desirable to have a perturbative theory at least up to the compactification scale ($M_{GS} \approx 10^{17}$ GeV) or the Planck scale ($M_P = 2 \times 10^{18}$ GeV).

Proton decay is a necessary prediction of a number of GUTs including SU(5) and SO(10). The decay mode $p \to e^+ \pi^0$ common to both SUSY and non-SUSY GUTs is mediated by superheavy gauge bosons carrying fractional charges and the corresponding effective Lagrangian has a dim.6 operator. In SUSY GUTs superpartners of fermions and heavy colour triplets of Higgs bosons give rise to new decay modes such as $p \to K^+ \pi^0$, $p \to K^+ \tau^\pm$, and others. The mediation of the heavy superpartner leads to a dim.5 operator in the effective Lagrangian for these supersymmetric decay modes. Recent experimental measurements provide improved limits on the lifetimes for both these types of decay modes,

$$\tau (p \to e^+ \pi^0) \geq 4 \times 10^{33} \text{years} \quad (1)$$

$$\tau (p \to K^+ \tau^\pm) \geq 2.2 \times 10^{31} \text{years} \quad (2)$$

While eq.(1) gives the bound $M_U \geq 5.6 \times 10^{15}$ GeV, eq.(2) yields the limit on the superheavy colour triplet Higgsino mass as $M_{\tau^\pm} \geq 10^{17}$ GeV. Although this has been treated as a severe constraint on SUSY SU(5) [16], easier methods have been suggested to evade it [17, 18]. In R-parity conserving SUSY SO(10) another interesting suggestions have been made to increase proton lifetime of supersymmetric decay mode through specific Yukawa textures, but in this case the GUT gauge coupling remains perturbative only up to $\mu = \text{few} \times 2 \times 10^{16}$ GeV [19].

In this paper we show that with the similar choices of Higgs representations as in the single step breakings of R-parity conserving SUSY SO(10), when the GUT gauge symmetry is allowed to break down to MSSM through $G_{2213}$-intermediate gauge symmetry investigated recently [20], perturbative GUT gauge coupling is ensured at least up to the Planck scale due to threshold and gravitational corrections. Although in this paper we have addressed the issue of perturbative gauge coupling up to the reduced Planck scale ($\approx 2 \times 10^{18}$ GeV), we have checked that our method also works even if we use the Planck scale as $M_P \approx 1.2 \times 10^{19}$ GeV according to the definition of the Particle Data Group. The realization of perturbative grand unification in R-parity conserving SO(10) which has not been possible otherwise is demonstrated for the first time in this paper. Other new contributions of the present paper compared to [20] are derivations of gravitational corrections in the presence of Higgs representations $54$ and $210 \oplus 54$ which contribute to SO(10) breaking near the GUT scale. Combining perturbative criteria with R-parity conservation in SUSY SO(10) we obtain lower bounds on the unification scale in different cases. Very significant increase of proton lifetimes is obtained leading to the greater stability of the particle.

In Sec.2, we discuss the origin of high-scale violation of perturbation theory in SUSY SO(10). In Sec.3 we discuss analytically threshold and gravitational corrections. In Sec.4 we show how these corrections elevate the unification scale so as to satify perturbative constraint on the GUT gauge coupling at least up to the Planck scale. In Sec.5 we discuss increase in proton lifetimes in different cases. Summary and conclusions are stated in Sec.6.

### 2 Perturbative constraint and lower bounds on unification scale

With R-parity conserving a minimal SO(10) model having 26 parameters has been identified to be the one with Higgs representations: $210 \oplus 126 \oplus 126 \oplus 10$ [31] for which a very interesting method of proton lifetime increase has been suggested [19]. In order to account for neutrino masses and mixings in SUSY SO(10) through Type II seesaw dominance, the realistic symmetry breaking pattern has been shown to require $210 \oplus 54 \oplus 126 \oplus 126 \oplus 10$ [21] where both $210$ and $54$ are present. We will show that in this case with $G_{2213}$ intermediate breaking gravitational corrections alone may be sufficient to guarantee perturbative gauge coupling at higher scales. But, in the single step breaking scenario above the GUT scale, not only these two models but also other variants of R-parity conserving SUSY SO(10) violate perturbation theory even at mass scales $\mu = \text{few} \times 2 \times 10^{16}$ GeV whenever the Higgs representations $126 \oplus 126$ are present in the model.

Above the GUT scale ($\mu > M_U$) the GUT fine structure constant $\alpha_G (\mu) = \frac{g^2 (\mu)}{4 \pi}$, where $g_G =$GUT coupling, evolves at one-loop level as

$$\frac{1}{\alpha_G (\mu)} = \frac{1}{\alpha_G (M_U)} - \frac{a}{2 \pi} \ln \frac{\mu}{M_U} \quad (3)$$

The $\beta$-function coefficient in eq.(3) consists of gauge, matter and Higgs contributions,

$$a = a_{\text{gauge}} + a_{\text{matter}} + a_{\text{Higgs}} \quad (4)$$

The gauge bosons of SO(10) in the adjoint representation $45$, three generations of matter in the spinorial representations $16$, and their superpartners contribute as
Table 1. Contribution of Higgs representations to SUSY SO(10) \(\beta\)-function coefficient for the GUT gauge coupling evolution

| Rep. | \(a_{Higgs}\) | Rep. | \(a_{Higgs}\) |
|------|---------------|------|---------------|
| 10   | 1             | 45 ⊕ 16 ⊕ 16 ⊕ 10 | 13  |
| 54   | 12            | 54 ⊕ 45 ⊕ 16 ⊕ 16 ⊕ 10 | 25  |
| 120  | 28            | 210 ⊕ 16 ⊕ 16 ⊕ 10   | 61  |
| 16   | 2             | 45 ⊕ 126 ⊕ 126 ⊕ 10 | 79  |
| 45   | 8             | 210 ⊕ 126 ⊕ 126 ⊕ 10 | 127 |
| 126  | 35            | 54 ⊕ 45 ⊕ 126 ⊕ 126 ⊕ 10 | 91  |
| 210  | 56            | 210 ⊕ 54 ⊕ 126 ⊕ 126 ⊕ 10 | 139 |

\[ a_{gauge} = -24, \quad a_{matter} = 6 \]  \(\text{(5)}\)

The Higgs contributions of different SO(10) irreducible representations are shown in Table 1.

Noting that \(a_{gauge} + a_{matter} = -18\), use of eq.(5) in eqs.(3)-(4) gives at \(\mu = A > M_U\),

\[ \frac{1}{\alpha_G(A)} = \frac{1}{\alpha_G(M_U)} + \frac{18}{2\pi} \ln \frac{A}{M_U} - \frac{a_{Higgs}}{2\pi} \ln \frac{A}{M_U} \]  \(\text{(6)}\)

If the gauge coupling constant encounters a Landau pole at \(A\), \(\alpha_G(A) \rightarrow \infty\) and eq.(6) leads to

\[ a_{Higgs} \leq 18 + \frac{2\pi}{\ln \frac{A}{M_U}} \times \frac{1}{\alpha_G(M_U)} \]  \(\text{(7)}\)

On the other hand the perturbative condition

\[ \alpha_G(A) \leq 1 \]  \(\text{(8)}\)

leads to the constraint

\[ a_{Higgs} \leq 18 + \frac{2\pi}{\ln \frac{1}{\alpha_G(M_U)}} \left[ \frac{1}{\alpha_G(M_U)} - 1 \right] \]  \(\text{(9)}\)

In the single step breakings of all SUSY GUTs \(M_U \approx 2 \times 10^{16} \text{ GeV}\), \(\alpha_G(M_U)^{-1} \approx 25\), and the upper bound defined by inequality (9) has been estimated [22].

For SUSY SO(10) with \(\{45 \oplus 16 \oplus 10\}\), \(a_{Higgs} = 13\) and the perturbative constraint remains valid for higher scales and perturbative grand unification is guaranteed at least up to the Planck scale [22]. However, for minimal SO(10) with \(\{210 \oplus 126 \oplus 126 \oplus 10\}\), \(a_{Higgs} = 127\) and the perturbation theory can not be guaranteed to hold up to the Planck scale in the grand desert model. Thus, in the single step breaking of SUSY SO(10) to MSSM, whenever larger Higgs representations like \(126 \oplus 126\) are used to break the \(SU(2)_R \times U(1)_B-L \subset SO(10)\) or \(SU(2)_R \times SU(4)_C \subset SO(10)\), leading to the seesaw mechanism and Majorana neutrino masses, the large contribution to the Dynkin indices violates perturbation theory at \(A = \text{few} \times 2 \times 10^{16} \text{ GeV}\). This has led to the investigations of perturbative grand unification of SO(10) through the use of Higgs representations \(16 \oplus 10\) instead of \(126 \oplus 126\) in the supergrand-desert scenario [22].

It is clear that in R-parity conserving SUSY SO(10) the Higgs contribution to the \(\beta\)-function coefficient for the gauge coupling evolution satisfies \(a_{Higgs} < 71\). Noting that \(\alpha_G(M_U) \approx 0.043\) and demanding that perturbative condition is satisfied up to \(A = M_{pl} = 2 \times 10^{18} \text{ GeV}\), then the inequality (9) gives the lower bound,

\[ M_U > 10^{17} \text{GeV} \]

This lower bound on the unification scale has to be satisfied in any R-parity conserving SUSY SO(10) if the GUT gauge coupling is to remain perturbative up to the Planck scale. It is interesting to note that this lower bound accidentally matches the Higgsino mass limit obtained from the current experimental limit of the proton lifetime for \(p \rightarrow K^+ \mu^+ \nu\).

In the four specific examples of Higgs representations shown in Table 1 which correspond to R-parity conserving, the Higgs contributions to the \(\beta\)-function coefficients in the respective cases and the inequality (9) give different values of lower bounds on the unification scale. In particular for the choices of the Higgs representations (I), \(\{210 \oplus 126 \oplus 126 \oplus 10\}\), (II), \(\{54 \oplus 45 \oplus 126 \oplus 126 \oplus 10\}\), (III), \(\{210 \oplus 54 \oplus 126 \oplus 126 \oplus 10\}\), and (IV), \(\{45 \oplus 126 \oplus 126 \oplus 10\}\) the lower bounds on the unification scale turn out to be \(M_U = 5.8 \times 10^{17} \text{ GeV}, M_U = 3 \times 10^{17} \text{ GeV}, M_U = 6.25 \times 10^{17} \text{ GeV}, \) and \(M_U = 1.5 \times 10^{17} \text{ GeV}\), respectively. Thus the smallest lower bound corresponds to the one for the Higgs representation \(\{45 \oplus 126 \oplus 126 \oplus 10\}\) as expected with minimal contribution \(a_{Higgs} = 79\). These lower bounds suggest that if the perturbative criteria on the GUT gauge coupling is to be satisfied, the unification scale has to be elevated by at least one order compared to the conventional value. Further, the perturbative constraint has the implication that, in R-conserving SUSY SO(10), the larger is the Higgs contribution to the \(\beta\)-function coefficient, the greater must be the unification scale. The lower bounds are to be satisfied irrespective of the SO(10) breaking to MSSM through a single step or through an intermediate gauge symmetry.

In the next section we show how the presence of \(SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C\) intermediate gauge symmetry...
symmetry at higher scales yields perturbative $\text{SO}(10)$ up to the Planck scale even if we use the Higgs representations $\{126 \oplus 126\}$ with or without $210$ or other Higgs representations such as $54$ and $45$ for high-scale breaking of SUSY $\text{SO}(10)$.

3 Threshold and gravitational corrections on mass scales

It is clear from eq. (9) that, if in a specific GUT scenario the unification scale $M_U$ can be closer to the Planck or the complication scale than in the single step breaking case, the contribution of the Higgs representation to the RHS of (9) can be larger without violating the inequality. In eqs. (11) the intermediate $\text{SU}(2)\times\text{SU}(3)$ breaking in SUSY $\text{SO}(10)$ was investigated,

$$\text{SO}(10) \times \text{SUSY} \frac{\Phi_U}{M_U} = \text{SU}(2)\times\text{SU}(3) \times \text{SUSY} \frac{M_2 \Phi_{213}}{M_2 \times \text{SUSY} \frac{M_Z}{M_Z}} \times \text{SU}(3)_C \times \text{SU}(3)_C$$

where the Higgs representations responsible for the GUT symmetry breaking were chosen as $\Phi_U \equiv 210$, or $54 \oplus 45$, which also break D-Parity at the GUT scale while permitting the left-right asymmetric gauge group $G_{2213} (2213 \not\equiv g_{2R})$ to survive down to the intermediate scale [23]. In such an R-parity conserving symmetry breaking chain quite significant threshold corrections arising out of spreading of masses around the intermediate scale and the GUT scale and gravitational corrections arising out of 5 – dim. operators induced by the Planck or the complication scales [24, 25, 26, 27] were noted. In this section we estimate these effects in detail to explore the possibility of increasing $M_U$ which is necessary for the existence of perturbative gauge coupling at higher scales. While the gravitational corrections originating from the 5-dim. operator due to $210$ was investigated in [20], in this work we investigate the corresponding effects due to $54$ and $210 \oplus 54$ while studying the threshold effects of the latter. The evolution of gauge couplings in the two different mass ranges is expressed as,

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(M_U)} + \frac{a_i}{2\pi} \ln \frac{M_U}{M_Z} + \theta_i - \Delta_i, \quad i = 1Y, 2L, 3C,$$

$$\frac{1}{\alpha_i(M_U)} = \frac{1}{\alpha_i(M_U)} + \frac{a'_i}{2\pi} \ln \frac{M_U}{M_U} + \theta'_i - \Delta'_i - \Delta'_i^{(gr)}, \quad i = 2L, 2R, BL, 3C. \quad (12)$$

where the second, third, and the fourth terms in the RHS of eqs. (11)-(12) represent one-loop, two-loop, threshold, and gravitational corrections, respectively [20]. In eqs. (11)-(12) $\alpha_Y = 33/5, a_{2L} = 1, a_{3C} = a''_{3C} = -3, a'_{2L} = 1, a'_{2R} = 5$ and $a'_{BL} = 15$. The two-loop coefficients ($b_{ij}$) below the intermediate scale and ($b'_{ij}$) above the intermediate scale have been obtained in [20]. Below $M_T$ the presence of $G_{2213}$ in MSSM gives,

$$b_{ij} = \left( \begin{array}{ccc} 199 & 27 & 88 \\ 25 & 24 & 9 \\ 14 & 1 & 3 \end{array} \right), \quad i, j = 1Y, 2L, 3C. \quad (13)$$

Above the intermediate scale, the two-loop beta-function coefficients in the presence of SUSY $G_{2213}$ symmetry are

$$b'_{ij} = \left( \begin{array}{ccc} 25 & 3 & 3 \\ 24 & 9 & 1 \\ 14 & 8 & 1 \end{array} \right), \quad i, j = 2L, 2R, BL, 3C. \quad (14)$$

These coefficients occur in two-loop contributions represented by $\theta_i$ and $\theta'_i$ in the two mass ranges,

$$\theta_i = \frac{1}{4\pi} \sum_j B_{ij} \ln \frac{\alpha_j(M_U)}{\alpha_j(M_Z)}$$

$$\theta'_i = \frac{1}{4\pi} \sum_j B'_{ij} \ln \frac{\alpha_j(M_U)}{\alpha_j(M_I)}$$

$$B_{ij} = \frac{b_{ij}}{a_{ij}}, \quad B'_{ij} = \frac{b'_{ij}}{a'_{ij}}. \quad (15)$$

For the sake of simplicity we have neglected the Yukawa contributions to two-loop effects on gauge couplings. While the functions $\Delta_i$ include threshold effects at $M_Z$ and $M_T$ with

$$\Delta_i = \Delta_i^{(Z)} + \Delta_i^{(f)}$$

$\Delta'_i$ include threshold effects at $M_U$.

It may be recalled that although in nonsupersymmetric gauge theories threshold effects contain both constant terms as well as logarithmic terms, it was noted in [32] that the constant terms are absent in supersymmetric threshold corrections.

In the presence of $G_{2213}$ intermediate symmetry the particle spectra of Higgs scalars, fermions, gauge bosons, and their superpartners with masses lighter than $M_U$ are the same in all four cases being considered in this paper. Then, under the assumption that all superheavy particles with masses larger than $M_U$ decouple from the Lagrangian, the contributions to two-loop effects on gauge couplings. While the functions $\Delta_i$ include threshold effects at $M_Z$ and $M_T$ with

$$\Delta_i = \Delta_i^{(Z)} + \Delta_i^{(f)}$$

$\Delta'_i$ include threshold effects at $M_U$.

3.1 Threshold effects with effective mass parameters

We follow the method of effective mass parameters due to Carena, Pokorski, and Wagner [28] to estimate threshold effects which have been also utilised to study such effects.
in SUSY SU(5) by introducing two sets of effective mass parameters, one set for the SUSY threshold and the other set for the GUT threshold [29]. In [20] their effects have been examined on SUSY SO(10) with $G_{2123}$ intermediate symmetry by defining one set of effective mass parameters for each threshold. Although these parameters at the weak-scale SUSY threshold have been approximately estimated [28, 29], no such estimations are available for higher thresholds and they would be assumed to deviate at most by a factor 6(1/6) from the corresponding scales. Following the standard procedure, the effective mass parameters are defined through the following relations,

$$
\Delta_i^2 = \sum_\alpha \frac{b_\alpha}{2\pi} \ln \frac{M_\alpha}{M_i} = \frac{b_i}{2\pi} \ln \frac{M_i}{M_Z},
$$

$$
i = 1Y, 2L, 3C; \mu = M_Z; \tag{16}
$$

$$
\Delta_i^l = \sum_\alpha \frac{b'^\alpha}{2\pi} \ln \frac{M'_\alpha}{M_l} = \frac{b'_l}{2\pi} \ln \frac{M'_l}{M_l},
$$

$$
i = 1Y, 2L, 3C; \mu = M_I; \tag{17}
$$

$$
\Delta' = \Delta'_U = \sum_\alpha \frac{b''_\alpha}{2\pi} \ln \frac{M''_\alpha}{M_U}
$$

$$
= \frac{b''_I}{2\pi} \ln \frac{M''_I}{M_U},
$$

$$
i = 2L, 2R, BL, 3C; \mu = M_U; \tag{18}
$$

where $b_i$ refers to the actual $G_{2123}$ submultiplet near $\mu = M_Z$, $M_I$ or $G_{2123}$ submultiplet near $\mu = M_U$ and $M_\alpha, M'_\alpha, M''_\alpha$ refer to the actual component masses. The three sets of effective mass parameters are $M_I, M'_I,$ and $M''_I$. The coefficients $b_i = \sum b^\alpha_i$ and $b'_l = \sum b'^\alpha_l$ have been defined in eqs.(16)-(18) following [20, 28]. The numbers $b_i$ and $b'^\alpha_l$ refer to the contributions of the multiplet $\alpha$ to the $\beta$-functions of $U(1)_Y, SU(2)_L,$ and $SU(3)_C$ gauge couplings. Similarly $b''_I$ refers to the contributions of the multiplet $\alpha$ to the $\beta$-functions of $U(1)_Y, SU(2)_L, SU(2)_R, SU(3)_C,$ and $U(1)_{B-L}$ gauge couplings [20].

The threshold effects on the mass scales $M_I$ and $M_U$ are then expressed in the form

$$
\Delta \ln \frac{M_I}{M_Z} = a \ln \frac{M''_R}{M_U} + b \ln \frac{M''_{BR}}{M_U} + c \ln \frac{M''_{BL}}{M_U} + d \ln \frac{M''_{CI}}{M_U} + e \ln \frac{M''_{CI}}{M_I} - 1.56,
$$

$$
\Delta \ln \frac{M_U}{M_Z} = a' \ln \frac{M''_R}{M_U} + b' \ln \frac{M''_{CR}}{M_U} + 0.105 \tag{19}
$$

where the numerical values are due to the weak-scale SUSY threshold effects. The values of the parameters computed for the four different cases are,

**Case(I): $210 \oplus 126 \oplus 126 \oplus 10$**

$$(a, b, c, d, e) = (-25, -57/4, 130, -355/4, -9/4),$$

$$(a', b') = (26, -213/8) \tag{20}$$

**Case(II): $54 \oplus 45 \oplus 126 \oplus 126 \oplus 10$**

$$(a, b, c, d, e) = (-77/4, -45/4, 405/4, -135/2, -9/4),$$

$$(a', b') = (81/4, -81/4) \tag{21}$$

**Case(III): $210 \oplus 54 \oplus 126 \oplus 126 \oplus 10$**

$$(a, b, c, d, e) = (-109/4, -61/4, 565/4, -95, -9/4),$$

$$(a', b') = (113/4, -57/2). \tag{22}$$

**Case(IV): $45 \oplus 126 \oplus 126 \oplus 10$**

$$(a, b, c, d, e) = (-35/4, -31/6, 185/4, -385/12, -9/4),$$

$$(a', b') = (37/4, -77/8). \tag{23}$$

Although Cases (I)-(II) were derived in Ref.[20] some numerical and typographical errors have been corrected here while Cases (III)-(IV) are new.

### 3.2 Gravitational corrections from dim.5 operators

In this subsection we derive gravitational corrections in Case(II) and Case(III) while such corrections in Case(I) were discussed in [20]. In addition to the renormalizable part of the Lagrangian of SUSY GUT, a 5-dim operator can be induced either in 4 - dim. gravity at the Planck scale ($M_C = M_{Pl} = 2 \times 10^{18}$ GeV) or due to compactification of extra dimension(s) at scales $M_C = M_{CS} \sim 10^{17}$ GeV [25].

$$
\mathcal{L}_{gr} = -\eta \frac{1}{2M_C} \text{Tr} (F_{\mu\nu} \Sigma F^{\mu\nu}) \tag{24}
$$

where, for example, $\Sigma = 210, 54 \subset SO(10)$ that contribute to the GUT symmetry breaking near $M_U$ and $M_C = \text{compactification scale}(M_{CS})$ of extra dimension(s), or the Planck scale ($M_{Pl}$) in 4 - dim. gauge theory. When $\Sigma = 45 \subset SO(10)$ the contribution of the 5 - dim. operator in eq.(24) identically vanishes. We will confine to the Cases (I)-(III) for gravitational corrections.

Although there are no exact theoretical constraint on $\eta$ it could be positive or negative with plausible values up to $|\eta| \approx O(10)$. Whereas 210 and 54 are present in cases I and II, respectively, both are present in case III. In [20] gravitational effects were derived only for the Case (I) corresponding to $\Sigma = 210$ with a normalization factor 1/8 instead of 1/2 as given in eq.(24) [26]. In order to compare with gravitational corrections resulting from eq.(24) with $\Sigma = 54$ we evaluate them for the Case (I) with the common normalization factor of 1/2. In a number of earlier investigations the effects of such operators on GUT predictions have been found to be quite significant [18, 20, 25, 26, 27]. In the presence of $SO(10) \rightarrow G_{2123}$ such operators modify the GUT boundary condition on the coupling.
constants which has the general form at $\mu = M_U$,

$$\alpha_{2L}(M_U)(1 + \epsilon_{2L}) = \alpha_{2R}(M_U)(1 + \epsilon_{2R})$$

$$\alpha_{BL}(M_U)(1 + \epsilon_{BL}) = \alpha_{3C}(M_U)(1 + \epsilon_{3C})$$

$$\alpha_G(M_U)$$  \hspace{1cm} (25)

These boundary conditions lead to the corresponding gravitational corrections on the four gauge couplings,

$$\Delta^{gr}_r = -\frac{\epsilon_i}{\alpha_G}, \quad i = 2L, 2R, BL, 3C$$  \hspace{1cm} (26)

Then using the procedure of [20], analytic formulas for the gravitational corrections of the two mass scales are derived,

$$\left(\ln \frac{M_I}{M_Z}\right)^{gr} = \frac{2\pi(A' \epsilon' - A \epsilon'')}{\alpha_G(AB' - A'B)}$$

$$\left(\ln \frac{M_U}{M_Z}\right)^{gr} = \frac{2\pi(B \epsilon'' - B' \epsilon')}{\alpha_G(AB' - A'B)}$$  \hspace{1cm} (27)

where

$$B = B' = \frac{5}{3}a_Y - \frac{2}{3}a_{BL} - a_{2R}$$

$$A = a_{2R} + \frac{5}{3}a_{BL}$$

$$A' = a_{2R} + \frac{2}{3}a_{BL} + a_{3C} - \frac{8}{3}a_{3C}$$

$$\epsilon'' = \epsilon_{2L} + \epsilon_{2R} + 2\frac{\epsilon_{BL}}{3} - \frac{8}{3}\epsilon_{3C}$$

$$\epsilon' = \epsilon_{2R} + 2\frac{\epsilon_{BL}}{3} - \frac{5}{3}\epsilon_{2L}$$  \hspace{1cm} (28)

We will need the numerical values of $A, A', B, B'$ defined through eq.(28) which are the same in all R-parity conserving cases with $G_{2213}$ intermediate gauge symmetry,

$$A = 40/3, \quad A' = 24,$$

$$B = B' = -4$$  \hspace{1cm} (29)

With the generalized formulas given by eqs.(25)-(28) and the numerical values given in eq.(29) we discuss specific gravitational corrections in three different cases as given below.

Case (I): $2(10) \oplus 126 \oplus \overline{126} \oplus 10$

In this case $\Sigma = 2\overline{10}$ and we denote the unknown parameter in eq.(24) as $\eta = \eta_1$. After taking into account a factor 4 in the normalization of the gauge kinetic term [26, 27] and using an approximate relation between the GUT-scale VEV $\phi_0$ and the degenerate masses of superheavy gauge bosons, $M_U \approx (2/9)^{1/2} g_G \phi_0$, we have

$$\epsilon_{2L} = -\epsilon_{2R} = -\epsilon_{3C} = \frac{1}{2} \epsilon_{BL} = \epsilon_1,$$

$$\epsilon' = \epsilon_{2R} + 2\frac{\epsilon_{BL}}{3} - \frac{5}{3}\epsilon_{2L} = 4\epsilon_1,$$

$$\epsilon'' = \epsilon_{2L} + \epsilon_{2R} + 2\frac{\epsilon_{BL}}{3} - \frac{8}{3}\epsilon_{3C} = 4\epsilon_1$$  \hspace{1cm} (30)

where

$$\epsilon_1 = \frac{3\eta_1 M_U}{4 M_C \sqrt{4\pi\alpha_G}}$$  \hspace{1cm} (31)

Using eqs.(29)-(30) in eq.(27) gives

$$\left(\ln \frac{M_I}{M_Z}\right)^{gr} = \frac{2\pi\epsilon_1}{\alpha_G},$$

$$\left(\ln \frac{M_U}{M_Z}\right)^{gr} = 0$$  \hspace{1cm} (32)

which were derived in [20] but with a different normalization factor for $\epsilon_1$.

Case (II): $\overline{54} \oplus 45 \oplus 126 \oplus \overline{126} \oplus 10$

In this case $\pmatrix{45 \oplus \overline{10} \subset \text{SO}(10)}$ does not contribute to the dim.5 operator of eq.(24). Using $\Sigma = \overline{54}$ and denoting $\eta = \eta_2$ in eq.(24), we derive

$$\epsilon_{3C} = \epsilon_{BL} = \epsilon_2,$$

$$\epsilon_{2L} = \epsilon_{2R} = -\frac{3}{2}\epsilon_2,$$

$$\epsilon' = \epsilon_{2R} + 2\frac{\epsilon_{BL}}{3} - \frac{5}{3}\epsilon_{2L} = \frac{5}{3}\epsilon_2,$$

$$\epsilon'' = \epsilon_{2L} + \epsilon_{2R} + 2\frac{\epsilon_{BL}}{3} - \frac{8}{3}\epsilon_{3C} = -5\epsilon_2$$  \hspace{1cm} (33)

where

$$\epsilon_2 = \frac{3\eta_2 M_U}{4 M_C \sqrt{15\pi\alpha_G}}$$  \hspace{1cm} (34)

Using eq.(29) and eqs.(33)-(34) in eq.(27), we get,

$$\left(\ln \frac{M_I}{M_Z}\right)^{gr} = \frac{5\pi\epsilon_2}{\alpha_G},$$

$$\left(\ln \frac{M_U}{M_Z}\right)^{gr} = \frac{5\pi\epsilon_2}{4\alpha_G}$$  \hspace{1cm} (35)

Eq.(35) has the implication that if we attempt to change the unification mass by one order purely by gravitational corrections, then the intermediate scale would change by approximately four orders.
Case (III): $210 \oplus 54 \oplus 126 \oplus 126 \oplus 10$

The importance of this case emphasizing the presence of $54$ in addition to $210$ for realistic SUSY SO(10) breaking leading to Type II seesaw dominance for neutrino masses has been elucidated in [21] in the single step breaking case. In our case with $G_{2213}$ intermediate symmetry both $54$ and $210$ contribute separately to the dim.5 operator with

$$L_{gr} = \frac{-\eta_2}{2M_G} \text{Tr}(F_{\mu \nu} \phi_{210} F^{\mu \nu})$$

$$L_{gr} = \frac{-\eta_4}{2M_G} \text{Tr}(F_{\mu \nu} \phi_{54} F^{\mu \nu})$$

(36)

Then

$$\Delta_i^{gr} = - (\epsilon^2_i + \epsilon_{210}) / \alpha_G$$

$$i = 2L, 2R, BL, 3C$$

The relations (32) and (35) hold separately leading to

$$\ln \left( \frac{M_U}{M_Z} \right)^{gr} = \frac{5 \pi \epsilon_2}{4 \alpha_G}$$

$$\ln \left( \frac{M_I}{M_Z} \right)^{gr} = \frac{5 \pi \epsilon_2}{4 \alpha_G} + \frac{2 \pi \epsilon_1}{\alpha_G}.$$ (37)

Comparing eqs. (31) and (34) gives $\epsilon_1 / \epsilon_2 = (15/4)^{1/2} \eta_1 / \eta_2$. In the next section we use these results to study the effects of gravitational corrections on SO(10) gauge coupling.

### 4 Perturbative SO(10) gauge coupling at higher scales

In all the three cases the same lighter components contained in $126 \oplus 126 \oplus 10$ contribute to the one-loop and two-loop $\beta$-function coefficients below the GUT scale and none of the components in $210, 54$, or $45$ contribute to large runnings of the gauge couplings. Thus, ignoring threshold and gravitational corrections, the two-loop solution of RGEs is the same for all the four cases with

$$M^0_U = 10^{15.2} \text{ GeV}, M^0_I = 10^{16.11} \text{ GeV},$$

$$\alpha_G = 0.043$$

(38)

Then adding threshold and gravitational corrections to two-loop solutions the mass scales are expressed as

$$\ln \left( \frac{M_U}{M_Z} \right) = \ln \left( \frac{M^0_U}{M_Z} \right) + \Delta \ln \left( \frac{M_U}{M_Z} \right) + (\ln \left( \frac{M_U}{M_Z} \right))_{gr},$$

$$\ln \left( \frac{M_I}{M_Z} \right) = \ln \left( \frac{M^0_I}{M_Z} \right) + \Delta \ln \left( \frac{M_I}{M_Z} \right) + (\ln \left( \frac{M_I}{M_Z} \right))_{gr}. $$ (39)

When we include corrections mentioned in Sec.3 through eq.(39), the resulting mass scales are modified in each case. The increased value of $M_U$ then extends the range of perturbative SO(10) gauge coupling up to the Planck scale. In what follows we discuss some examples of such solutions in each case.

The mass scales obtained including threshold corrections are denoted as $M_i^{(1)}$ and those obtained including both the threshold and gravitational corrections are denoted as $M_i^{(2)} (i=I, U)$

Case (I): As shown in Sec.2 the lower bound on the unification mass in this case is $5.8 \times 10^{17} \text{ GeV}$. Using threshold and gravitational corrections we examine how far this constraint can be satisfied. Using the effective mass parameters

$$M''_U = M_U, M''_C = 0.87 M_U,$$

$$M''_{2R} = 1.5 M_U, M''_{BL} = 1.8 M_U,$$

$$M''_{1Y} = M_I$$

we obtain including only threshold effects,

$$M^{(1)}_U = 6.54 \times 10^{17} \text{GeV}, \quad M^{(1)}_I = 7 \times 10^{11} \text{GeV}$$

Using this modified value, $M_U = M^{(1)}_U = 6.54 \times 10^{17} \text{ GeV}$ eq.(6) gives the perturbative value of the GUT-gauge coupling at $\Lambda = M_P$ with $\alpha_G (M_P) = 0.587$. The effects are more(less) prominent if the mass gap of the effective mass parameters is increased(decreased) for which the values of the corresponding gauge coupling will be smaller(larger). It is easily checked that the inequality (9) is satisfied. Since the gravitational corrections do not affect the GUT scale, but affect only the intermediate scale which is of the same order as the right-handed neutrino mass, in this case any desired value of the intermediate scale matching the scale of leptogenesis, or the Pecei-Quinn symmetry breaking scale, or even a value close to the minimal GUT scale can be obtained. Thus the model is potentially interesting from the point of view of neutrino physics, leptogenesis and strong CP-violation. Other examples of solutions for this case are shown in Table.2.

Case (II): As shown in Sec.2 the value of $a_{Higgs} = 91$ in this case gives the lower bound $M_U > 3 \times 10^{17} \text{ GeV}$. To examine how far threshold and gravitational corrections may allow such high unification scales, at first we consider only threshold corrections. Using the effective mass parameters

$$M''_U = M_U, M''_{2R} = 1.7 M_U, M''_{BL} = 2 M_U,$$

$$M''_C = 0.87 M_U, M''_{1Y} = M_I$$

gives including threshold corrections but ignoring gravitational corrections,

$$M^{(1)}_I = M_R = 2.91 \times 10^{11} \text{GeV}, \quad M^{(1)}_U = 3.85 \times 10^{17} \text{GeV}$$
Clearly the mass gaps near the GUT scale are reasonably small and are confined between 0.87M_U and 2M_U. Now adding gravitational corrections with η_2 = 3.0 gives,

\[ M_{I}^{(2)} = 9.31 \times 10^{12} \text{ GeV}, \quad M_{U}^{(2)} = 8.95 \times 10^{17} \text{ GeV} \]

Using the value of \( M_U = M_{I}^{(2)} = 8.95 \times 10^{17} \) GeV we obtain from eq.(6) the perturbative value of the gauge coupling, \( \alpha_G(M_{Pl}) = 0.084 \). Evaluating the RHS of inequality (9) gives

\[ a_{Higgs} < 180 \]

Noting from Table.1 that for this case \( a_{Higgs} = 91 \) it is clear that inequality (9) is satisfied ensuring perturbativity of SO(10) gauge coupling up to the Planck scale. Another example of solution including gravitational correction is given in Table.2.

Case (III): As shown in Sec.2 for this case \( a_{Higgs} = 139 \) and (9) gives the lower bound \( M_U \geq 6.25 \times 10^{17} \) GeV to ensure perturbative gauge coupling up to the Planck scale. The necessity of both 210 and 54 for realistic SUSY SO(10) breaking directly to MSSM has been emphasized in [21]. With \( G_{2213} \) intermediate breaking this case appears to be interesting as it shows the possibility that dominant gravitational corrections with marginal or negligible threshold effects can elevate the GUT scale closer to the Planck scale [25]. Although, in principle, threshold effects are somewhat larger in this case compared to the Cases (I)-(II) and (IV) because of the presence of extended size of Higgs representations, their actual values are controlled by the choice of the mass gap in the effective mass parameters. For example, using the effective mass parameters,

\[ M_{2L} = M_U, \quad M_{3e} = 0.87M_U, \]
\[ M_{2R} = 1.6M_U, M_{BL} = 1.6M_U, \]
\[ M_{I} = M_T \]

we obtain including only threshold effects,

\[ M_{U}^{(1)} = 7.57 \times 10^{17} \text{ GeV}, \quad M_{I}^{(1)} = 3.92 \times 10^{11} \text{ GeV} \]

Then eq.(6) gives the perturbative gauge coupling \( \alpha_G(M_{Pl}) \approx 0.25 \).

Further addition of gravitational corrections with \( \eta_1 = -3.0 \) and \( \eta_2 = 5.0 \) gives higher values of the unification scale closer to \( M_{Pl} \),

\[ M_{U}^{(2)} = 2.95 \times 10^{18} \text{ GeV}, \quad M_{I}^{(2)} = 6.96 \times 10^{12} \text{ GeV} \]

Using this high value of the unification scale \( M_U = M_{I}^{(2)} = 2.95 \times 10^{18} \) GeV we obtain from eq.(6) the perturbative value of the gauge coupling \( \alpha_G(M_{Pl}) \approx 0.049 \). We also note that the perturbative inequality (9) is easily satisfied with \( \Lambda = M_{Pl} \). Another example of such solution for this case is shown in Table.2 where both threshold and gravitational corrections have been included.

Now we show that with negligible GUT threshold corrections but with the inclusion of gravitational corrections alone in this case it is also possible to obtain high values of the unification scale and perturbative gauge coupling up to the Planck scale. For the sake of simplicity ignoring all high scale threshold corrections by choosing \( M_{IY} = M_I \) and \( M''_U = M_U \) (i=2L, 2R, BL, 3C) and using \( \eta_1 = -20.0 \) and \( \eta_2 = 14.2 \) leads to \( \epsilon_1 = -0.128 \) and \( \epsilon_2 = 0.047 \). Then eq.(36) gives \( (\ln \frac{M_{Pl}}{M_{U}})_{gr} = 4.33 \) and \( (\ln \frac{M_{Pl}}{M_{I}})_{gr} = -1.49 \). When added to two-loop solutions including the weak-scale SUSY threshold corrections we obtain,

\[ M_{U}^{(2)} = 8.61 \times 10^{17} \text{ GeV}, \quad M_{I}^{(2)} = 1.09 \times 10^{14} \text{ GeV} \]

Using \( M_U = M_{I}^{(2)} = 8.61 \times 10^{17} \) GeV in eq.(6) gives the perturbative value of the gauge coupling at \( \Lambda = M_{Pl} \) with \( \alpha_G(M_{Pl}) = 0.175 \). We find that the RHS of the (9) is \( \approx 190 \) as compared to the value \( a_{Higgs} = 139 \) for this case and the perturbative inequality is satisfied. Thus, including gravitational corrections along the SO(10) model with such choice of Higgs representation guarantees perturbative SUSY SO(10) gauge coupling up to the Planck scale.

Case (IV): As shown in Sec.2, \( a_{Higgs} = 79 \) through (9) gives the lower bound \( M_U \geq 1.5 \times 10^{17} \) GeV in this case to ensure perturbative gauge coupling up to Planck scale. As there is no gravitational corrections due to the 5 − dim. operator for this case we will consider only threshold corrections. Using

\[ M''_U = 1.5M_U, M_{2L} = M_{BL} = 3.5M_U, \]
\[ M_{3e} = M_U, M_{IY} = M_I \]

we obtain

\[ M_{I}^{(1)} = 1.2 \times 10^{15} \text{ GeV}, \quad M_{U}^{(1)} = 4.7 \times 10^{17} \text{ GeV} \]

Using \( M_U = M_{I}^{(1)} = 4.7 \times 10^{17} \) GeV in eq. (6) gives the perturbative gauge coupling at the Planck scale with \( \alpha_G(M_{Pl}) \approx 0.10 \). The RHS of (9) is found to be \( \approx 119 \) and the inequality is satisfied.

## 5 Proton lifetime predictions

As pointed out in Sec.1, the experimental lower limit on the proton lifetime for the decay mode \( p \rightarrow e^+\pi^0 \) mediated by superheavy gauge bosons or equivalently through the effective dim.6 operator sets a lower limit on the GUT scale, \( M_U \geq 5.6 \times 10^{15} \) GeV which is easily satisfied in the supergrand desert scenario for which, excluding threshold or gravitational corrections, \( M_U = 2 \times 10^{16} \) GeV. The lower bounds on \( M_U \) obtained in Sec.2 for the Cases (I)-(IV), purely from the requirement of perturbativity of the SO(10) gauge coupling up to the Planck scale, are found to be satisfied by the RG solutions for the mass scales when
threshold corrections, or gravitational corrections, or both are included in the intermediate scale models. In the Case (IV) for which the Higgs representations $\mathbf{45} \oplus \mathbf{126} \oplus \mathbf{126} \oplus \mathbf{10}$ have the smallest size among all the four cases, the solutions of RGEs for the mass scales are consistent with the lower bound $M_U \geq 1.5 \times 10^{17}$ GeV when threshold effects are included. In each of the four cases the corresponding lower bound on the unification scale translates into a lower bound on proton lifetime. The shortest of these lower bounds on the proton lifetime occurs in the Case (IV),

$$\tau (p \rightarrow e^+\pi^0) \geq 2.1 \times 10^{39} \text{years} \quad (40)$$

In the Cases (I)-(III) the lifetimes are longer than this value as can be approximately estimated using Table 2. These analyses suggest that the decay mode $p \rightarrow e^+\pi^0$ which has lifetime at least 6 orders longer than the current limit is inaccessible to experimental observation.

Supersymmetric decay modes of the proton such as $p \rightarrow K^+\nu_\mu$, $p \rightarrow K^+\bar{\nu}_\tau$ and others are characteristic predictions in SUSY GUTs [30]. These decays are mediated by Higgsinos ($T_C$) which are superpartners of colour triplet Higgs scalars ($T_C$) having superheavy masses near the GUT scale. As pointed out the experimental lower limit on the proton lifetime given in eq.(2) sets the lower bound on the superheavy colour triplet Higgsino mass, $M_{T_C} \geq 10^{17}$ GeV.

In SUSY SU(5) there is one such pair of Higgsinos which are superpartners of Higgs colour triplets contained in $\mathbf{5} \oplus \mathbf{\overline{5}} \subset \text{SU}(5)$; in SUSY SO(10) models the colour triplet Higgs may be treated as linear combination of the triplets contained in $\mathbf{10} \oplus \mathbf{126} \oplus \mathbf{126}$ and $\mathbf{45}$, or $\mathbf{54}$, or $\mathbf{210}$ depending upon the choice of specific Higgs representations used to break the GUT symmetry to $G_{2213}$ [19]. For the sake of simplicity we ignore finer details of calculations and give plausibility arguments to show that for these decays governed by the effective dim.5 operators proton lifetimes ranging from the present experimental limit to several orders longer can be a natural prediction of the intermediate breaking scenario.

In a supergrand desert model like SUSY SU(5), the constraint on the colour triplet Higgsino mass is obtained using the unification condition including threshold corrections: $g_G(A_U) = g_{1Y}(A_U) + \Delta_{1Y}(A_U) = g_{2L}(A_U) + \Delta_{2L}(A_U)$ where $g_G = \text{GUT gauge coupling}$ and $A_U = \text{GUT scale}$. This leads to the constraint $g_G^{-2}(A_U) - g_{2L}^{-2}(A_U) = \left(3/20\pi^2\right) \ln(M_{T_C}/A_U)$ and $M_{T_C} \simeq \text{few } \times 10^{15}$ GeV [33]. However, including gravitational corrections large increase of the Higgsino mass even up to four orders of magnitude has been suggested in SUSY SU(5) [18].

But in the presence of $G_{2213}$ intermediate symmetry in the mass range $\mu = M_I - M_U$, the GUT scale constraint equating $g_{1Y}$ and $g_{2L}$ is absent since the $U(1)_Y$ gauge coupling splits above the scale $M_I$ into two separate unconstrained gauge couplings,

$$\frac{1}{g_{1Y}^2(\mu)} = \frac{2}{5} \frac{1}{g_{2L}^2(\mu)} + \frac{3}{5} \frac{1}{g_{2R}^2(\mu)}, \quad \mu = M_I - M_U$$

As the gauge symmetry near $A_U$ is no longer the SM, but it is $G_{2213}$ the simple SU(5) relation among $g_G$, $g_{3C}$ and $M_{T_C}$ is no longer valid. Further, unlike SU(5) where the Higgs colour triplet and anti-triplet are confined to its Higgs representations, $\mathbf{5} \oplus \mathbf{\overline{5}}$ in SO(10) their number is much more as they can originate from Higgs representations like $\mathbf{10} \oplus \mathbf{126} \oplus \mathbf{126}$. In view of these there is no similar precision constraint on $M_{T_C}$, as in SUSY SU(5) originating from gauge coupling unification. In the presence of such two-step breaking through $G_{2213}$ intermediate gauge symmetry the value of $M_{T_C}$ can easily exceed $10^{17}$ GeV.

Since our lower bounds needed for perturbative gauge coupling up to the Planck scale as shown in Sec.4 are in the range,

$$M_U \geq (1.5 - 6.2) \times 10^{17} \text{GeV}$$

and the lifetime for the supersymmetric decay modes are proportional to $M_{T_C}^2$, the lower bound on lifetimes are expected to be longer by factors ranging between 2.2 and 38 compared to the single-step breaking scenario. This is due to the natural expectation that without additional fine tuning all superheavy components including the colour triplets would have masses close to $M_U$. Thus, the criteria of perturbative gauge coupling up to the Planck scale which are easily met by threshold or gravitational corrections in the four cases of R-parity conserving SUSY SO(10), constrain the unification scales with $M_U \geq (1.5 - 6.2) \times 10^{17}$ GeV which in turn predict for the supersymmetric decay modes of the proton,

$$\tau (p \rightarrow K^+\nu_\tau) \geq (2 - 9) \times 10^{34} \text{years} \quad (41)$$

But it is well known that even without additional fine tuning the superheavy components could be easily few times lighter or heavier than $M_U$. Stretching this factor to the value of $\simeq 1/6$ or 6 the lower limit on the proton lifetime has a wider range starting from the current experimental limit up to a value which is 2-3 orders longer. It is interesting to note that high-scale perturbative renormalization group relations (6) or (9) and the R-parity conservation in SUSY SO(10) predict these lower bounds on the unification scales, the smallest one being $M_U \simeq 1.5 \times 10^{17}$ GeV. The resulting longer values of proton lifetime predictions are consequences of generalized perturbative criteria in R-parity conserving SUSY SO(10) which are also solutions to perturbative renormalization group equations including threshold or gravitational corrections.

6 Summary and conclusion

SUSY SO(10) with $\mathbf{126} \oplus \mathbf{126}$ and other Higgs representations in the case of single-step breaking to MSSM has many attractive features for all fermion masses and mixings while ensuring R-parity conservation. But the popular argument raised against the model is that it violates perturbative gauge theory as the GUT coupling blows off
even at mass scales few times larger than the conventional GUT scale. In this paper we have shown that the requirement that the GUT gauge coupling remains perturbative up to the Planck scale imposes lower bounds on the unification scale which are at least one order larger than the conventional GUT scale. We have shown that the solutions to RGEs respecting these lower bounds are in fact possible if the threshold and/or gravitational corrections are included. The four different models discussed here ensure perturbative gauge coupling at least up to the Planck scale. The proton lifetime for $p \rightarrow e^+\pi^0$ becomes longer at least by nearly 6 orders of magnitude compared to the current experimental limit. For the supersymmetric de-creases of the proton stability in single step breaking of the proton. A different scenario for the increase at least up to the Planck scale and increases the additional advantages of high unification scale is that it is expected to be similar to single step breaking case, but the stability of the proton. A different scenario for the increase of the proton stability in single step breaking of SUSY SO(10) with R-parity conservation has been suggested recently by introducing specific textures [19] where perturbative condition on SUSY SO(10) gauge coupling holds up to $\mu = 2 \times 10^{16}$ GeV.

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Table 2. Perturbative SO(10) gauge coupling at higher scales including threshold and gravitational corrections to two-loop solutions for which $M_1^0 = 10^{15.20}$ GeV and $M_2^0 = 10^{16.11}$ GeV. The mass scales $M_i^{(i)} (i = I, U)$ have been obtained including threshold corrections and $M_i^{(2)} (i = I, U)$ including both threshold and gravitational corrections.

| Higgs Rep. | Mass parameters | $M_1^{(1)}$(GeV) | $M_2^{(1)}$(GeV) | $\eta_1$ | $\eta_2$ | $M_1^{(2)}$(GeV) | $M_2^{(2)}$(GeV) | $\alpha_S(M_2I)$ |
|------------|----------------|------------------|------------------|----------|----------|------------------|------------------|-----------------|
| $210\oplus$ | $M'_1, M'_2 L = M_U$ | $M'_1 = M_1, M_2 L = M_U$ | $M'_1, M'_2 L = M_U$ | $7 \times 10^{11}$ | $6.54 \times 10^{17}$ | $3.85 \times 10^{17}$ | $3.85 \times 10^{17}$ | 0.587 |
| $126 \oplus 126$ | $M'_2 = 1.5M_U, M''_L = 1.8M_U$ | $M'_2 = 1.5M_U, M''_L = 1.8M_U$ | $7 \times 10^{11}$ | $6.54 \times 10^{17}$ | $3.85 \times 10^{17}$ | 5.0 | $5.49 \times 10^{13}$ | $6.54 \times 10^{17}$ |
| $\oplus 10$ | $M''_C = 0.87M_U$ | $M''_C = 0.87M_U$ | $6.5$ | $5.3 \times 10^{14}$ | $2.1 \times 10^{18}$ | 0.047 |
| $54 \oplus 45\oplus$ | $M'_1 = M_1, M''_2 L = M_U$ | $M''_C = M_U$ | $9.31 \times 10^{12}$ | $8.95 \times 10^{17}$ | 0.084 |
| $126 \oplus 126$ | $M'_2 = 1.7M_U, M''_L = 2M_U$ | $M'_2 = 1.7M_U, M''_L = 2M_U$ | $2.91 \times 10^{11}$ | $3.85 \times 10^{17}$ | 0.09 |
| $\oplus 10$ | $M''_C = M_U$ | $M''_C = M_U$ | $6.5$ | $5.3 \times 10^{14}$ | $2.1 \times 10^{18}$ | 0.049 |
| $210 \oplus 54\oplus$ | $M'_1 = M_1, M''_2 L = M_U$ | $M''_C = M_U$ | $1.5$ | $2.5 \times 10^{12}$ | $1.5 \times 10^{18}$ | 0.09 |
| $126 \oplus 126$ | $M'_2 = 1.6M_U, M''_L = 1.6M_U$ | $M'_2 = 1.6M_U, M''_L = 1.6M_U$ | $3.92 \times 10^{11}$ | $7.57 \times 10^{17}$ | 0.09 |
| $\oplus 10$ | $M''_C = 0.87M_U$ | $M''_C = 0.87M_U$ | $-3.0$ | $5.0 \times 10^{12}$ | $2.95 \times 10^{18}$ | 0.125 |
| $210 \oplus 54\oplus$ | $M'_1 = M_1, M''_2 L = M_U$ | $M''_C = M_U$ | $3.3 \times 10^{14}$ | $1.43 \times 10^{16}$ | $2.0 \times 10^{14}$ | $0.175$ |
| $126 \oplus 126$ | $M'_2 = 1.5M_U, i=2L, 2R, BL, 3C$ | $M'_2 = 1.5M_U, i=2L, 2R, BL, 3C$ | $1.2 \times 10^{15}$ | $4.73 \times 10^{17}$ | $-2.0$ | $14.2$ | $1.09 \times 10^{14}$ | $8.61 \times 10^{17}$ |
| $\oplus 10$ | $M''_C = M_U$ | $M''_C = M_U$ | $-2.0$ | $14.2$ | $1.09 \times 10^{14}$ | 0.10 |

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