Adversarial training in communication constrained federated learning

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Abstract

Federated learning enables model training over a distributed corpus of agent data. However, the trained model is vulnerable to adversarial examples – designed to elicit misclassification. We study the feasibility of using adversarial training (AT) in the federated learning setting. Furthermore, we do so assuming a fixed communication budget and non-iid data distribution between participating agents. We observe a significant drop in both natural and adversarial accuracies when AT is used in the federated setting as opposed to centralized training. We attribute this to the number of epochs of AT performed locally at the agents, which in turn effects (i) drift between local models; and (ii) convergence time (measured in number of communication rounds). Towards this end, we propose \textit{FedDynAT} – a novel algorithm for performing AT in federated setting. Through extensive experimentation we show that \textit{FedDynAT} significantly improves both natural and adversarial accuracy, as well as model convergence time by reducing the model drift.

1 Introduction

Federated learning is a paradigm for multi-round model training over a distributed dataset \cite{19, 12}. At the beginning of every round, the participating clients obtain the most recent model from a central server. Clients use the model to perform multiple epochs of local training and share only the parameter updates with the server. The server, in turn, employs a fusion algorithm to aggregate the updates from the various clients and generate a new model for the next round. While federation allows the clients to retain control and privacy of their raw data \cite{2, 26}, the final model continues to remain vulnerable to carefully crafted adversarial examples \cite{25, 3, 8} designed to elicit misclassification at inference time. Complementary to our research, there exists a significant body of work on defenses (both empirical \cite{18} and certifiable \cite{21, 6}) against adversarial attacks for centralized model training. Among them, adversarial training \cite{18} (AT), formulated as an optimization of the saddle point problem, has emerged as the method of choice for training robust models. In this paper, we study the feasibility of adopting AT to the federated learning setting for training robust models, under realistic constraints on both the client data distribution (non-iid) and the maximum number of possible communication rounds.

To optimize the minimax objective, AT typically uses multiple iterations of a two-step sequential process: (i) Use Projected Gradient Descent (PGD) to maximize the loss and generate adversarial counterparts for every sample in the training data; (ii) Use Stochastic Gradient Descent (SGD) to optimize the model parameters and minimize the expected loss over the adversarial examples. From an optimization perspective, an immediate consequence of adopting AT to the federated setting is a distributed solution to the minimax objective. In other words, the above two steps, which were earlier performed over the entire data, are now distributed across multiple clients and performed on their local data. Our goal is to design a fusion algorithm, which over multiple rounds ensures that the aggregate model has comparable performance in terms of both natural accuracy (measured on benign test samples) and adversarial accuracy (measured on adversarial test samples) w.r.t the model trained in the centralized setting. As a first step towards feasibility analysis, we reproduce the observation in \cite{19} for adversarial training. We perform weighted averaging of adversarially trained models from two clients. The models are trained on an iid distributed subset of
CIFAR10 data, starting from a common initialization. The result shown in Fig. 1 provides initial evidence that the aggregate model leads to a significant reduction in the loss on the entire training set for both natural and adversarial samples compared to either client models (extreme values of $w$). This implies that fusion can also improve adversarial accuracy ($\text{FedAvg}$ had established that fusion helps for natural training).

Figure 1: Loss on CIFAR-10 training set (natural and adversarial samples) using aggregate models. The aggregate model parameters $\theta = w\theta_1 + (1-w)\theta_2$, where $\theta_1$ and $\theta_2$ are client model parameters, with common initialization but updated independently using local AT at the clients. Mixing weights $w \in [-0.2, 1.2]$.

**Challenges:** However, there are several challenges that need to be addressed. Using centralized clean training as baseline, we observed a higher drop in both natural and adversarial accuracies when AT is used in the federated setting as opposed to centralized training. We attribute this to number of epochs of local AT, denoted by $E$, performed at each client, which in turn causes the local models at the clients to drift further from each other. This is over and above the drift that is already introduced by the non-iid data distribution at the clients. We use SVCCA [20] scores to quantify model drift across clients. In addition, the convergence time (in terms of the number of communication rounds) is also increased. This is of particular concern as it increases the communication cost significantly.

**Contributions:** Towards this end, we propose $\text{FedDynAT}$, which builds on the techniques proposed for preventing catastrophic forgetting in federated learning on non-iid data [22, 13] by augmenting them with a dynamic local AT schedule ($E$-schedule). We make the following contributions.

- We study the performance of adversarial training in a federated setting with non-iid data distribution and discuss the challenges involved as compared to normal training.
- We present a novel algorithm for improving the performance of adversarial training in federated setting with non-iid data distribution, called $\text{FedDynAT}$, and evaluate its performance over multiple datasets and models. We compare the performance with other state-of-the art methods used in normal federated learning.
- We analyze the performance of different algorithms in the federated setting using model drift across various clients and provide insights on the relative performance.

## 2 Adversarial Training in Federated Setting

Following [18], we define centralized adversarially trained model parameters, $\hat{\theta}$, as a solution to the following min-max problem:

$$
\min_{\theta} \rho(\theta); \quad \rho(\theta) = \mathbb{E}_{(x, y) \sim F} \left[ \max_{\theta} \mathcal{L}(f_\theta(\tilde{x}), y) \right]
$$

where $f_\theta$ is the model parameterized by $\theta$ and $\tilde{x}$ is an $l_\infty$ adversarial sample, within $\epsilon$-ball of the benign sample $x$, with class label $y$.

**Problem Formulation:** We consider adversarial training between $K$ clients, each having their share of the training data $D_k \sim F_k$ and $D(= \bigcup_{k=1}^K D_k) \sim F$. Our objective is to train a joint machine learning model
without any exchange of training data between clients. Motivated by our initial observations in (Figure 1) we investigate adversarial training in federated environment as a combination of local adversarial training at individual clients followed by periodic synchronization and aggregation of model parameters at a trusted server. The aggregation, achieved by fusing the model weights of individual clients, results in a global model. The expectation is that after a sufficiently large number of aggregation rounds $t_0$, this global model will be an approximate solution to (1), i.e., $\rho(\theta_{t_0}) \approx \rho(\hat{\theta})$, providing robustness to adversarial attacks at all clients.

Let $\theta^t_k \in \mathbb{R}^n$ be the model parameters at client $k = 1, \ldots, K$ at the beginning of round $t$. The aggregation of model weights is achieved using a fusion operator $F : \mathbb{R}^{K \times n} \to \mathbb{R}^n$. We denote the fusion output at the end of round $t$ as $\theta^{t+1}$.

A training round $t$ involves $E_t$ epochs of adversarial training using local data at each agent. Thus, $\theta^t_k$ is a solution to the following min-max problem obtained over $E_t$ epochs with initial value $\theta^t_k = \theta^t_k$:

$$
\min_{\theta} \rho_k(\theta); \quad \rho_k(\theta) = \mathbb{E}_{(x, y) \in D_k} \left[ \max_{\|\tilde{x} - x\|_\infty \leq \epsilon} L(f_\theta(\tilde{x}), y) \right]
$$

and

$$
\theta^{t+1} = F(\theta^t_1, \theta^t_2, \ldots, \theta^t_K) \quad (2)
$$

The general optimization framework for solving Eqn. (2) is given in Algorithm 1. We consider two specific implementations of the fusion algorithm: Federated Averaging Adversarial Training (FedAvgAT) and Federated Curvature Adversarial Training (FedCurvAT). These fusion algorithms differ in the choice of their loss function $L$ and fusion operator $F$.

**Algorithm 1 Federated adversarial training framework**

| Input: number of clients: $K$, number of communication rounds: $R$, PGD Attack $A_{T, \epsilon, \alpha}$ with 3 parameters: $T, \epsilon, \alpha$ (number of PGD steps, perturbation ball size, step size), number of local training epochs per round: $E$, Initial model weights: $\theta^0$ |
|---|
| for $t = 0$ to $R - 1$ do |
| Server sends $\theta^t$ to all devices |
| for client $k = 0$ to $K - 1$ do |
| $\theta^t_k = AT(E, \theta^t, A_{T, \epsilon, \alpha})$ |
| Send $\theta^t_k$ back to server |
| end for |
| (Server performs aggregation over agent updates) |
| $\theta^{t+1} = F(\theta^t_1, \theta^t_2, \ldots, \theta^t_K)$ |
| end for |

**2.1 FedAvgAT Algorithm**

The first fusion algorithm we consider is Federated Averaging [19] which is a widely used algorithm in federated learning.

At the end of round $t$, each client sends its model parameters, $\theta^t_k$, back to server. The server performs a weighted average of the model parameters to obtain the aggregate model parameters $\theta^{t+1}$ for the next round as:

$$
\theta^{t+1} = \frac{1}{|D|} \sum_k |D_k| \theta^t_k \quad (3)
$$

**2.2 FedCurvAT Algorithm**

With non-iid data distribution between clients, local models tend to drift apart and inhibit learning using FedAvg. In FedCurv [22], a penalty term is added to the local training loss, compelling all local models to converge to a shared optimum.

At any client $k$, the loss function used for training $L(f_\theta(\tilde{x}_k), y_k)$ comprises of two components:

1. Training loss on local data, $L_k(\theta)$. 

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2. Regularization loss, $R_k(\theta)$, given by:

$$R_k(\theta) = \sum_{j \in K \setminus k} (\theta - \hat{\theta}_{t-1,j})\text{diag}((\hat{I}_{t-1,j})(\theta - \hat{\theta}_{t-1,j}))$$

At the beginning of round $t$, starting from the initial point $\theta^t$ given by Eqn.(3), each node optimizes $L_k(\theta) + \lambda R_k(\theta)$. At end of round $t$, client $j$ sends its local model weights $\hat{\theta}_{t,j}$ and the diagonal of Fisher Information Matrix $\hat{I}_{t,j}$, evaluated on its local dataset and local model weights. $\lambda$ is the regularization hyperparameter which balances the tradeoff between local training, and model drift from the local client weights at the end of previous round.

3  Impact of adversarial training on federated model performance

In this section, we empirically outline the primary challenges in adopting AT to federated setting. These challenges provide useful insights and motivate our proposed solution. We use FedAvg as our baseline fusion algorithm and compare the relative impact on performance due to FedAvgAT.

Datasets and model architecture for evaluation

We conduct our evaluation with two datasets: CIFAR-10 and EMNIST-Balanced. We use Network-in-Network (NIN) and VGG-9 network for training on the CIFAR-10 dataset. For experiments with EMNIST-Balanced, we use a network with two convolution layers followed by two fully connected layers. We refer to this model as EMNIST-M in subsequent sections.

3.1 Increased drop in natural and adversarial accuracy with federated AT and non-iid data

In federated learning deployments, data partitions across clients invariably exhibit non-iid behavior. Thus, we begin by investigating the effect of adversarial training with non-iid data. Details for the data partition strategy for non-iid data can be found in Section 5.1. Table 1 reports the performance of FedAvgAT for both natural training and adversarial training for all the three models. The number of local training epochs, $E = 1$. We also report results for centralized training (entire dataset available at the training site) for better comparison.

First, we observe that adversarial training with iid data distribution leads to performance that is close to that of centralized setting for all the three models. Second, as has been indicated in prior work, we validate that using FedAvg with non-iid data, leads to a drop in accuracy even for natural training. However, as the main focus of our paper, we note a much larger drop in both natural and adversarial accuracies when performing adversarial training on non-iid data when compared to iid data. It is most severe for EMNIST-M model, where the adversarial accuracy drops from 72% in the centralized setting to 58%.

|                | NIN         | VGG         | EMNIST-M   |
|----------------|-------------|-------------|------------|
| Natural        |             |             |            |
| iid            | (88%, 0%)   | (91%, 0%)   | (87%, 0%)  |
| non-iid        | (78.5%, 0%) | (86%, 0%)   | (82.5%, 0%)|
| Adversarial    |             |             |            |
| iid            | (75%, 42%)  | (75%, 44%)  | (84.5%, 72%)|
| non-iid        | (67%, 36%)  | (68.5%, 36%)| (84%, 58%) |

Table 1: Performance for natural and adversarial training for centralized and FedAvgAT with K=5 clients. The performance is reported as a tuple: (natural accuracy, adversarial accuracy).
We hypothesize increased model drift with non-iid data as the reason for the significant reduction in model performance with FedAvgAT. Model drift refers to models learning very different representations of the data. At the start of a round, every client begins local adversarial training with the same model weights. However, the generated adversarial examples depend on both the local samples and the local model. As the local training progresses, the adversarial examples at the clients can become highly tuned towards the local data and model and may differ a lot from adversarial examples generated at other clients. Hence the models which are trained on these samples can exhibit significant drift. We validate our hypothesis and evaluate model drift in detail using SVCCA [20] in Section 5.3.

3.2 Increased communication overhead

In this section, we study the effect of adversarial training on the convergence rate of the global model. The convergence plots for FedAvg and FedAvgAT on non-iid data between $K = 5$ clients, with $E = 1$ are shown in Fig. 2. While FedAvg converges within 300 communication rounds, FedAvgAT takes about 800 rounds to converge. This highlights that adversarial training requires more communication rounds compared to natural training in a federated setup. With faster compute infrastructure at the clients, increased communication cost is often the primary bottleneck for training of the models in a federated setting.

4 FedDynAT Algorithm

To address the twin challenges described in Section 3, we propose a new algorithm, Federated Dynamic Adversarial Training (FedDynAT). While prior work [28, 24] have studied the tradeoff between communication overhead, and the convergence accuracy for natural training with iid data distribution. In this paper, we present this trade-off for adversarial training with non-iid data distribution.

4.1 Influence of $E$ on convergence

To motivate FedDynAT, we first discuss the sensitivity of AT in a federated setting with respect to the number of local adversarial training epochs, $E$, between successive model aggregation. We vary the value of $E$ and study its impact on convergence speed as well as accuracy achieved.

The convergence plots for FedAvgAT for $E = 1, 5$ and 20 are presented in Fig. 3. Setting $E = 1$ leads to the best performance (for both natural and adversarial accuracy) but takes 800 communication rounds to converge. The adversarial accuracy plots for $E = 5$ and $E = 20$ converge much faster to almost the same accuracy as $E = 1$ (after 800 rounds), however, as training progresses the accuracy drops significantly.

A higher value of $E$ is found to benefit the convergence speed for natural training as shown in [19]. However, the fall in adversarial accuracy with higher $E$ in the later phase of training can be explained with the findings in earlier studies [29, 9]. They show that the adversaries generated only in the later phase of the training are potent to the final model. A higher value of $E$ results in models drifting too much from each
other due to non-iid data distribution (and hence the locally generated adversaries) but it starts to impact only in the later phase of the training.

![Graphs showing accuracy vs. communication round plots with FedAvgAT for E = 1, 20, 50 on NIN. The bottom row is a zoomed version of the convergence plot, for first 100 communication rounds.]

**Figure 3:** The top row shows the accuracy vs. communication round plots with FedAvgAT for $E = 1, 20, 50$ on NIN. The bottom row is a zoomed version of the convergence plot, for first 100 communication rounds.

### 4.2 FedCurvAT improves performance over FedAvgAT with higher $E$

FedCurv, as described in Section 2.2, has been shown to result in better performance as compared to FedAvg in the context of non-iid data distribution [22]. We evaluate performance of FedCurvAT on non-iid data and its dependency on $E$. Table 2 compares the performance of FedCurvAT and FedAvgAT for different values of $E$ for NIN, VGG and EMNIST-M models.

We see a significant improvement in adversarial accuracy with FedCurvAT compared to FedAvgAT. Further, the improvement increases as we increase $E$. We explain the dependence of model drift on $E$ in Section 5.3. We attribute this improvement to FedCurvAT’s regularized loss which helps in reducing model drift. Increased drift of local models affects the adversarial accuracy of the global model, reflected in FedAvgAT results for $E = 20$ and $E = 50$.

### 4.3 Our algorithm: FedDynAT

The experiments (in Sections 4.1 and 4.2) lead to two primary observations: (i) A high value of $E$ leads to large model drift affecting the adversarial accuracy of the global model. A low value of $E$ results in a higher adversarial accuracy at convergence but converges slowly, resulting in high communication cost. (ii) FedCurvAT outperforms FedAvgAT for high values of $E$, and the improvement increases with $E$.

We combine the insights in our algorithm FedDynAT, which follows a dynamic schedule for the number of local training epochs at each round. The training starts with a high value of $E = E_0$, and decays its value every $F_E$ rounds by a decay factor of $\gamma_E$. Further, we use FedCurv as the fusion algorithm (but it does not preclude the use of FedAvg for fusion). Details of the algorithm are outlined in Algorithm 2.

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### Table 2: FedAvgAT vs FedCurvAT for various $E$ values after 50 communication rounds, for NIN, VGG and EMNIST-M models.

| Model     | $E$ | $K$  | FedAvgAT         | FedCurvAT         |
|-----------|-----|------|------------------|-------------------|
| **NIN**   |     |      |                  |                   |
|           | 20  | 2    | (62.1%, 27.5%)   | (66.6%, 33.5%)    |
|           | 5   | 5    | (66.6%, 31%)     | (66.6%, 30.1%)    |
|           | 10  | 5    | (63.8%, 24.5%)   | (65.2%, 32%)      |
|           | 50  | 5    | (60.5%, 21.7%)   | (60.9%, 30.7%)    |
| **VGG**   |     |      |                  |                   |
|           | 20  | 2    | (60.2%, 26.7%)   | (67.1%, 30.5%)    |
|           | 5   | 5    | (67.7%, 31.3%)   | (68.3%, 31.3%)    |
|           | 10  | 5    | (60.5%, 21.7%)   | (64%, 28.5%)      |
|           | 50  | 5    | (64%, 30.6%)     | (64%, 28.7%)      |
| **EMNIST-M** |   |      |                  |                   |
|           | 5   | 2    | (80%, 53.3%)     | (81%, 55%)        |
|           | 10  | 2    | (80.3%, 52.6%)   | (83.1%, 56%)      |
|           | 5   | 5    | (74.1%, 47%)     | (78.6%, 50.4%)    |
|           | 10  | 5    | (72.8%, 42.5%)   | (76%, 45.7%)      |

### Algorithm 2 FedDynAT

**Input:** number of clients: $K$, number of communication rounds: $R$, PGD Attack: $A_{T,\epsilon,\alpha}$, Initial model weights: $\theta^0$, Local training epochs at start: $E_0$, Decay rate: $\gamma_E$, Decay frequency: $F_E$

for $t = 0$ to $R - 1$ do

\[
E_t = \left\lfloor E_0 + \frac{t}{F_E} \gamma_E \right\rfloor
\]

Server sends $\theta^t$ to all devices

for client $k = 0$ to $K - 1$ do

$\theta^t_k = AT(E_t, \theta^t, A_{T,\epsilon,\alpha})$

Send $\theta^t_k$ back to server

end for

(Server performs aggregation over agent updates)

$\theta^{t+1} = FedCurv(\theta^t_1, ..., \theta^t_K)$

end for

## 5 Experiments and Results

### 5.1 Experimental set-up

We list here the implementation details, and the non-iid data partition strategy across clients. All experiments were conducted on a VM with eight Intel(R) Xeon(R) CPU E5-2690 v4 @ 2.60GHz CPUs and one NVIDIA V100 GPU.

#### 5.1.1 non-iid data split

We conduct all our experiments with $K$ clients, $K \in \{2, 5, 10\}$. To achieve non-iid data distribution, we introduce skew in the data split. Each client gets data corresponding to all the classes, but a majority of the data samples are from a subset of classes. We denote the set of majority classes at client $k$ as $M_k$ with $M$ being the set of all classes in the dataset (e.g., for CIFAR-10 with 10 classes $|M| = 10$). $M \setminus M_k$ will be the set of minority classes. We keep $M_k$ as mutually exclusive across the clients. We ensure that each client has roughly the same number of data samples.

We characterize the data split with a skew parameter $s$ which is a percentage of the aggregate data size.
corresponding to a class in $\mathcal{M}$. We first create $\{M_k\}$ by dividing all class labels in $\mathcal{M}$ equally among $K$ clients, i.e. $|M_k| = |\mathcal{M}|/K$. We then divide the total data across clients such that a client has $s\%$ of data for any class in $\mathcal{M}\setminus M_k$ and $(100 - (K - 1) * s)\%$ of data for a class in $M_k$. Table 3 shows the skew values we have used for our non-iid data partitions.

| Dataset   | $K = 2$ | $K = 5$ | $K = 10$ |
|-----------|---------|---------|----------|
| CIFAR-10  | 1%      | 2%      | 2%       |
| EMNIST    | 0.1%    | 0.1%    | 0.1%     |

Table 3: Skew parameter $s$ value used in our experiments.

5.1.2 Implementation details

All our models (NIN, VGG-9, and EMNIST-M) are implemented in Keras [31]. We use the PGD adversarial attack $A_{T,\epsilon,\alpha}$ which has 3 parameters: $T, \epsilon, \alpha$ (number of steps, ball size, step size). For CIFAR-10, we use $T = 10, \epsilon = \frac{2}{255}, \alpha = \frac{2}{255}$. For EMNIST, we use $T = 10, \epsilon = \frac{1}{10}, \alpha = \frac{1}{20}$. AT at each client is done with a batch size of 32. For CIFAR-10, we use the SGD optimizer with a constant learning rate = $10^{-2}$ and momentum = 0.9. For EMNIST, we use the Adam optimizer with a constant learning rate = $10^{-4}$.

To train the VGG network, we use data augmentation (random horizontal/vertical shift by 0.1 and horizontal flip). We do not use data augmentation for NIN and EMNIST-M networks. We ignore all batch normalization [11] layers in the VGG architecture.

5.2 FedDynAT results

We compare FedDynAT to FedAvgAT and FedCurvAT baselines, both of which are run with a fixed value of $E$. For a given communication budget, we evaluate FedAvgAT and FedCurvAT with various values of $E$ and report performance with $E$ resulting in best natural accuracy for the global model. We report two observations in this section

- FedDynAT outperforms FedAvgAT, FedCurvAT for a fixed communication budget.
- FedDynAT achieves similar performance as FedAvgAT ($E=1$), albeit in significantly lower number of communication rounds.

Results with varying $E$-schedule

The dynamic $E$-schedule is defined by 3 parameters: the initial value $E_0$, decay rate $\gamma_E$ and decay frequency $F_E$ as described in Section 4.3. We report the results for various combination of these parameters in Fig. 4. The bottom row shows FedCurvAT results for 3 different fixed values of $E$. The top rows show the results for various configurations of the dynamic $E$ schedule. We notice an improved performance in both natural and adversarial accuracy with FedDynAT with a smooth drop in $E$ compared to FedCurvAT with fixed $E$.

5.2.1 Accuracy with different communication budgets

We set out to solve the adversarial training problem with non-iid data split in a communication constrained setting. We vary the communication budget i.e maximum number of communication rounds and demonstrate how FedDynAT performs as compared to FedAvgAT and FedCurvAT. We evaluate FedAvgAT and FedCurvAT with multiple values of $E$ and report performance corresponding to the best natural accuracy of the global model. Table 4 shows the results for Network-in-network model for CIFAR10 dataset and Table 5 shows that of EMNIST-M model for EMNIST dataset. We defer results with VGG to the Supplement.

The convergence plot for FedAvgAT, FedCurvAT and FedDynAT with NIN with a budget of 50 rounds is presented in Fig. 5. Notice how the natural accuracy curves for all 3 algorithms converge to similar numbers but the adversarial accuracy with FedDynAT is significantly better than FedAvgAT and FedCurvAT.
FedAvg fusion with dynamic $E$: FedDynAT uses the FedCurvature fusion algorithm. We consider another baseline where we replace the fusion algorithm with FedAvg and refer to it as FedAvgDynAT. The results with NIN in Table 4 demonstrate that FedDynAT outperforms this baseline. In Section 4.2 we established that FedCurvAT results in improvement over FedAvgAT for higher $E$. A high $E_0$ for our E-schedule contributes to better eventual performance with FedDynAT over FedAvgDynAT. We find FedAvgDynAT performs similar to FedDynAT for EMNIST dataset for the particular communication budget.

| $K$ | Budget | FedAvgAT | FedCurvAT | FedAvgDynAT | FedDynAT |
|-----|--------|----------|-----------|-------------|----------|
| 2   | 50     | (68.2%, 38.5%) | (68.7%, 37.8%) | (67.2%, 34.3%) | (68.4%, 38%) |
|     | 100    | (70.0%, 38%)  | (71%, 37.5%)  | (67.2%, 35.3%) | (70.4%, 38%) |
| 5   | 50     | (67.1%, 27.8%) | (66.6%, 31%)  | (66.5%, 34.8%) | (66.6%, 35.3%) |
|     | 100    | (64.9%, 35%)   | (65.8%, 35.2%) | (66.8%, 35%)  | (66.8%, 35.9%) |
| 10  | 50     | (61.6%, 25.4%) | (61.9%, 30.7%) | (61.8%, 30%)  | (62.3%, 32.5%) |
|     | 100    | (61.6%, 25.4%) | (61.9%, 30.7%) | (61.9%, 31.4%) | (62.6%, 33.3%) |

Table 4: FedDynAT performance on Network-in-network model for communication budgets of 50 and 100 rounds. We evaluate FedAvgAT and FedCurvAT with $E = \{1, 5, 20\}$ and report the best individual performance.

5.2.2 Convergence speed for FedDynAT

Figure 6 shows that convergence is much faster with FedDynAT compared to FedAvgAT ($E=1$) for both NIN and EMNIST-M. For reaching the same accuracy, with NIN, FedDynAT converges in 100 communication rounds while FedAvgAT takes 800 rounds to converge. Similarly, with EMNIST-M, FedDynAT converges in 50 communication rounds while FedAvgAT takes 150 rounds to converge. Results for other values of $K$ are included in the appendix.
Table 5: FedDynAT performance on EMNIST-M compared to FedAvgAT, FedCurvAT with a communication budget of 20 and 50 rounds. We evaluate FedAvgAT and FedCurvAT with $E = \{1, 5, 20\}$ and report performance with $E$ resulting in best natural accuracy.

| K Budget | FedAvgAT       | FedCurvAT      | FedAvgDynAT     | FedDynAT     |
|----------|----------------|----------------|-----------------|--------------|
| 2        | (83.5%, 57.5%) | (83.8%, 60%)   | (84%, 64%)      | (84.5%, 64.2%) |
| 50       | (84.1%, 65.5%) | (84.1%, 65.5%) | (84%, 64%)      | (84.5%, 64.5%) |
| 5        | (78.4%, 50.6%) | (81%, 55%)     | (82.1%, 56.5%)  | (82.3%, 56.2%) |
| 50       | (80.7%, 54.3%) | (81%, 55%)     | (83.4%, 59%)    | (83.6%, 58.3%) |
| 10       | (75.8%, 45.4%) | (76.8%, 45.8%) | (78.9%, 51.5%)  | (79.9%, 51.3%) |
| 50       | (78.3%, 47.5%) | (79.9%, 50%)   | (81.9%, 54.8%)  | (81.7%, 54.7%) |

5.3 Understanding performance through model drift: A SVCCA-based Analysis

Singular Vector Canonical Correlation Analysis (SVCCA) \cite{20} is a technique to compare two network representations that is invariant to affine transforms. SVCCA interprets each neuron as a vector (formed by considering the neuron activations on the input data points) and a layer as subspace spanned by that layer’s neurons.

We measure the SVCCA scores (in range [0-1]) between the corresponding layer of two models. A high score implies similar representations of two layers (or lower drift), and low score implies that the two layers have learnt different representations (indicating a higher drift). We show results for (i) dependence of model drift on $E$ (ii) comparison of model drift with and without AT in federated learning; and (iii) improvement in accuracy.
in model drift due to FedDynAT over FedCurvAT and FedAvgAT.

We report results for two of the five client NIN model\(^1\). The plots show SVCCA scores for convolution layers 5–8 where the drift is more pronounced. Layers 1–4, act as feature extractor layers and show relatively low drift.

**Increased model drift with higher \(E\):** With a higher value of \(E\), local models train more on local data, consequently the model drift is higher. We illustrate the model drift phenomenon in Fig. [7] by running FedAvgAT with \(E = 20\) and \(E = 50\). The SVCCA scores with \(E = 20\) are much higher than those for \(E = 50\), indicating a lower model drift for \(E = 20\).

**Comparing model drift with FedAvg and FedAvgAT on non-iid data:** In Fig. [8], we compute SVCCA scores for FedAvg (natural training) and FedAvgAT with \(E = 50\). We observe that the similarity scores for FedAvgAT are generally lower than FedAvg indicating higher drift between the layers. This shows that the drift is higher for adversarial training. Furthermore, this drift is more prominent in the higher layers of the model (close to the output).

**Comparing model drift between FedDynAT, FedAvgAT, FedCurvAT for non-iid data:** In Fig. [9], we present a comparison of SVCCA scores for FedAvgAT, FedCurvAT and FedDynAT, all the three techniques we used for adversarial training. We observe that the similarity score for FedCurvAT is significantly higher than FedAvgAT, which validates the proposition that FedCurvAT arrests the local model drift.

However, FedDynAT is able to beat both FedCurvAT and FedAvgAT in SVCCA scores. use a constant \(E = 50\) for local training. FedDynAT uses \(E_0 = 50\), \(\gamma = 0.5\), \(F_E = 5\) for the \(E\)-schedule. As training progresses, and the value of \(E\) decreases, less epochs of local model training contributes to a lower model drift. The layers closer to the input have not been shown in the plots as they show low model drift for all 3 algorithms.

6 Related Work

To address communication overhead in distributed learning, algorithms perform multiple local epochs of training before synchronization and thus reduce the communication overhead [14][19][7]. The convergence

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\(^1\)NIN model: \(x \rightarrow \text{conv1} \rightarrow ... \rightarrow \text{conv9} \rightarrow \text{fc} \rightarrow y\)
analysis of these algorithms connects the rate of convergence with number of clients and local training epochs \[31, 32\]. These results are mostly obtained under strong assumptions on the degree of dissimilarity (i.e., non-iidness) in the data distribution between clients which may not hold in practice. Of particular relevance to our work is \[28\] which analyses tradeoff between decreased wall clock time for convergence and increase in convergence error with increasing \(E\). Based on the analysis for iid distribution, an adaptive algorithm is proposed that dynamically changes the number of local training rounds.

Several algorithms have been proposed to achieve faster convergence while reducing communication overhead with non-iid data. \[16, 22\] use additional regularization terms in the client loss function to minimize divergence between local and global models. Going beyond simple averaging of weights during fusion, \[33, 27\] perform layer-wise matching of neurons from different clients, before averaging the model.
weights leading to improved communication efficiency. To prevent interference between incompatible tasks across clients, [30] selectively transfers the knowledge between clients. All the above fusion algorithms have been studied for natural training only.

In centralized setting AT by [18] is shown to provide some of the strongest defense against PGD attacks [1] and is a widely accepted benchmark [10, 1]. In concurrent work, [34] has also implemented traditional AT in federated setting. Different from our work, their focus was on evaluating the vulnerability of both Byzantine resilient defenses and federated AT, rather than mitigating the performance impact of AT in federated setting.

7 Conclusion

We proposed a novel algorithm for performing adversarial training of deep neural networks in a federated setting. We showed that federated setting specially with non-iid data distribution significantly affects the training performance due to model drift and takes much longer to converge as compared to natural training. We showed that the proposed algorithm is able to address the above challenges effectively and is able to get comparable or better accuracy in significantly lower number of rounds. Incorporating dynamic scheduling with more advanced model fusion techniques remains a topic of future research.

Figure 9: SVCCA scores evaluated on 2 out of 5 clients for FedAvgAT, FedCurvAT and FedDynAT with NIN. FedCurvAT and FedAvgAT use a constant $E = 50$ for local training. FedDynAT uses $E_0 = 50, \gamma = 0.5, F_E = 5$ for the dynamic E-schedule.
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8 Supplementary material

8.1 Tuning hyperparameter lambda for FedCurvAT

$\lambda$ is the regularization hyperparameter in the loss function for FedCurvAT. It balances the tradeoff between local training, and model drift from the local client weights at the end of previous round. A very high value of $\lambda$ inhibits local training while a very low value of $\lambda$ is akin to FedAvgAT.

We do a gridsearch to tune $\lambda$ over a range varying from $\lambda = 0$ upto $\lambda = 100$. For $K = 2$ clients non-iid data, with $E = 20$, we use $\lambda = 5$ for NIN, $\lambda = 5$ for VGG, and $\lambda = e^{-2}$ for EMNIST-M.

From Section 5.3, we observed that there is an increased model drift with a higher $E$. To compensate for the drift, we increase $\lambda$ on increasing $E$.

Similarly, we use a higher value of $\lambda$ as the number of clients $K$ increases. As mentioned in Section 5.1.1, our approach to generate non-iid distribution for a given dataset results in a decrease in the number of majority classes per client, on increasing $K$. This increases the degree of non-iidness in the data distribution on increasing $K$, which explains the need for higher $\lambda$.

8.2 Additional results

8.2.1 FedDynAT performance with VGG network

In Table 6, we vary the communication budget and demonstrate the improvement FedDynAT offers compared to FedAvgAT and FedCurvAT for a communication budget of 20 rounds as well as 50 rounds.

| K  | Budget | FedAvgAT       | FedCurvAT       | FedAvgDynAT      | FedDynAT       |
|----|--------|----------------|-----------------|------------------|----------------|
| 2  | 20     | (67.6%, 39.3%) | (67.6%, 39.3%)  | (67.3%, 35.5%)   | (68%, 38%)     |
|    | 50     | (70.1%, 35.4%) | (70.1%, 35.4%)  | (69.3%, 35.9%)   | (70.4%, 39%)   |
| 5  | 20     | (67.3%, 31%)   | (67.9%, 31.3%)  | (67.5%, 35.7%)   | (68.5%, 35.8%) |
|    | 50     | (68.5%, 35%)   | (68.5%, 35%)    | (69.1%, 35%)     | (69.1%, 35.6%) |
| 10 | 20     | (64%, 30.6%)   | (64%, 29.7%)    | (64.3%, 32.2%)   | (64.3%, 32.3%) |
|    | 50     | (65.1%, 29%)   | (65.1%, 28.3%)  | (65.2%, 32.2%)   | (65%, 32.7%)   |

Table 6: FedDynAT performance on VGG model for communication budgets of 20 and 50 rounds. We evaluate FedAvgAT and FedCurvAT with $E = \{1, 5, 20\}$ and report the best individual performance.

8.2.2 Convergence Speed for FedDynAT

We present the number of rounds needed for convergence with FedDynAT compared to FedAvgAT evaluated with $E = 1$. The results for NIN model are in Table 7, for the EMNIST-M model in Table 8 and for VGG in Table 9.

The corresponding convergence plots for NIN and EMNIST-M networks are discussed in Section 5.2.2 and for VGG network in Figure 10.

Observe that FedDynAT converges in significantly lower number of communication rounds as compared to FedAvgAT ($E=1$), without any substantial drop in performance.
FedDynAT FedAvgAT (E=1)

| K  | Test Accuracy | Communication Rounds |
|----|---------------|----------------------|
| 2  | (70.4%, 38%)  | 100                  |
|    | (70.5%, 39.5%)| 300                  |
| 5  | (66.8%, 35.9%)| 100                  |
|    | (67.5%, 36%)  | 800                  |
| 10 | (62.6%, 33.3%)| 100                  |
|    | (52.7%, 31.2%)| 1400                 |

Table 7: The convergence accuracy and the number of communication rounds to convergence are listed here for the NIN model using FedDynAT, and FedAvgAT evaluated with $E = 1$. FedAvgAT (E=1) for 10 clients has not converged till 1400 rounds, hence the low accuracy.

| K  | Test Accuracy | Communication Rounds |
|----|---------------|----------------------|
| 2  | (84.5%, 64.5%)| 20                   |
|    | (84.5%, 64.5%)| 40                   |
| 5  | (83.5%, 58%)  | 50                   |
|    | (83.5%, 58%)  | 150                  |
| 10 | (82.5%, 56.5%)| 100                  |
|    | (82.5%, 56.5%)| 250                  |

Table 8: The convergence accuracy and the number of communication rounds to convergence are listed here for EMNIST-M using FedDynAT, and FedAvgAT evaluated with $E = 1$.

| K  | Test Accuracy | Communication Rounds |
|----|---------------|----------------------|
| 2  | (70.4%, 39%)  | 50                   |
|    | (70.5%, 39%)  | 200                  |
| 5  | (69.1%, 35.6%)| 50                   |
|    | (69%, 35.7%)  | 300                  |
| 10 | (65%, 32.7%)  | 50                   |
|    | (57.6%, 33.9%)| 900                  |

Table 9: The convergence accuracy and the number of communication rounds to convergence are listed here for the NIN model using FedDynAT, and FedAvgAT evaluated with $E = 1$. FedAvgAT (E=1) for 10 clients has not converged till 900 rounds, hence the low accuracy.

Figure 10: Accuracy vs communication round plot for FedDynAT and FedAvgAT evaluated with $E=1$, on $K = 5$ client non-iid data split with VGG.