1. Radiation Processes and Models

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Abstract

This is a basic introduction to the physics of compact objects in the context of high time-resolution astrophysics (HTRA). The main mechanisms of energy release and the properties of relevant radiation processes are briefly reviewed. As a specific example, the top models for the multi-wavelength variability of accreting black holes are unveiled.

1.1. Introduction

Compact objects represent a prime target for HTRA. Their physics involves strong gravitational fields, matter compressed to enormous densities, very high energy particles and huge magnetic fields. Of course, such extreme conditions cannot be produced in a laboratory on Earth. In many cases, high time-resolution observations of these objects represent a unique opportunity to test fundamental theories regarding particle interactions, the properties of dense matter or gravitation. Compact objects also inform us about fundamental astrophysical processes such as accretion and ejection in their most extreme form.

Classically, there are three types of compact objects that all form mostly (but not only) through the gravitational collapse of a normal star after exhaustion of its thermonuclear fuel. White dwarfs (hereafter WDs) have masses lower than the Chandrasekhar limit \((1.4M_\odot)\) and radii comparable to that of Earth. The quantum degeneracy pressure of the electrons balances the gravitation forces to prevent collapse. Neutron stars (hereafter NSs) are instead supported by short-range repulsive neutron–neutron interactions mediated by the strong force and also by the quantum degeneracy pressure of neutrons. They can have masses up to the Tolman–Oppenheimer–Volkoff (TOV) limit, which is approximately between 2 and 3\(M_\odot\). Above the TOV limit, a star cannot support its own weight. It is completely collapsed and forms a black hole (hereafter BH). The size of a NS is about 10 km; the size of a BH is given by the size of the horizon of events. The latter depends on the spin of the BH but remains comparable to the gravitational radius:

\[ R_g = \frac{GM_{\text{BH}}}{c^2} \approx 1.5 \frac{M_{\text{BH}}}{M_\odot} \text{km}. \]

A typical stellar-mass BH concentrates about 10 times the mass of the sun within a radius of about 15 km. This corresponds to an average density within \(R_g\) of about \(10^{15}\)\(\text{g cm}^{-3}\). The average density of the matter of a NS is of the same order, i.e., \(10^{14}\) times denser than Planet Earth! The surface gravity on a NS is more than \(10^{11}\) times that on Earth. The dynamical timescale in the environment of compact objects is very short. For example, a Keplerian orbit at six times the radius of the object corresponds, on Earth, to a geosynchronous orbit has a period of exactly one day. This timescale is reduced to a few minutes around a WD and only a few milliseconds

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2. Note that objects supported by the quantum degeneracy pressure of quarks, and named quark stars, have also been theorised and would have properties very similar to that of neutron stars, but their existence is disputed.
around a NS or a BH! Compact objects can also release tremendous amounts of energy in their environment in the form of radiation and powerful outflows. This combination of power and very short dynamical timescales makes compact objects one of the main targets of HTRA. Section 1.2 presents the basic mechanisms of energy release around compact objects through dissipation of gravitational, magnetic and rotational power. Then, in Section 1.3, the main emission processes responsible for the conversion of, fraction of this energy into photons are described. Finally, Sections 1.4 and 1.5 focus on models for the multi-wavelength variability of accreting BHs in X-ray binary systems involving, respectively, the accretion flow and the jets.

1.2. Compact Objects

Energy can be extracted from the rotation of the compact object (pulsar), its magnetic field (magnetar), from the gravitational energy of material falling onto the compact object (accretion in binary systems) or even thermonuclear fusion of outer layers of accreted materials (X-ray bursters, classical novae). In this section, I briefly introduce the different mechanisms of energy release. A much more detailed exposition of compact objects physics can be found in classical textbooks, such as those by Shapiro and Teukolsky (1983), Frank et al. (1992), Longair (1994) and Kato et al. (2008). There are also many excellent reviews covering specific recent developments of the field, for instance, Patruno and Watts (2012) on millisecond X-ray pulsars, Watts (2012) on thermonuclear burst oscillations, Gilfanov and Sunyaev (2014) on the structure of the boundary layer in accreting neutron stars, Smith (2006) on cataclysmic variables or Done et al. (2007) on accreting BHs and NSs.

1.2.1 Rotation

All stars are rotating (the rotation period of the sun is about 27 d). In the case of NSs and BHs, the angular momentum is essentially conserved during the gravitational collapse. The compactification of the star then implies faster rotation. As a result, a newborn NS can have a spin period $P$ of the order of $P = 10^{-100}$ ms. On the other hand, most WDs have slow rotation due to the removal of angular momentum during the loss of the progenitor’s outer envelope in a planetary nebula. Nevertheless, a WD can later be spun up by accretion (see Section 1.2.3). The fastest spinning WD has a period of only $P = 13$ s (Mereghetti et al., 2011). The rotational energy of a star can be estimated simply as $E_{\text{rot}} = I \Omega^2 / 2 = 2I \pi^2 P^{-2}$ where $I$ is the moment of inertia, which, for a homogeneous sphere of mass $M$ and radius $R$ is $I = 2MR^2 / 5$. For a NS, this energy can represent a small but significant fraction of the rest mass energy of the star:

$$\frac{E_{\text{rot}}}{\mathcal{M}c^2} \approx 10^{-4} \left( \frac{R}{10 \, \text{km}} \right)^2 \left( \frac{P}{0.01 \, \text{s}} \right)^{-2}.$$  \hspace{1cm} (1.1)

In absolute terms the rotation energy of a young neutron star is comparable to the energy radiated by the sun during one billion years. However, this energy is released over much smaller timescales of the order of $10^3$–$10^7$ yr. Indeed, the extraction of rotational energy implies spin-down. The rotational power is related to the spin-down rate as

$$\dot{E}_{\text{rot}} = \frac{d}{dt} \left( \frac{I \Omega}{2} \right) = 4I \pi^2 \frac{\dot{P}}{P^3} \simeq 3 \times 10^{30} \frac{M}{M_\odot} \left( \frac{R}{10 \, \text{km}} \right)^2 \left( \frac{P}{0.01 \, \text{s}} \right)^{-2} \frac{\dot{P}}{P} \, \text{erg} \, \text{s}^{-1}. \hspace{1cm} (1.2)$$

The ‘braking’ time scale over which the rotational power is released can be estimated as $\dot{E}_{\text{rot}} / \dot{P} \sim P / \dot{P}$. Pulsars are NSs emitting beams of radiation leading to the highly
coherent modulation of the observed light curve at the spin period. In these sources, both the spin and spin-down rate can be measured accurately (see Chapter 5 by A. Possenti in this volume), and this allows one to estimate the released rotation power. In the case of the Crab Pulsar, the observed period is \( P \approx 33 \text{ ms} \), and the spin-down rate is \( \dot{P}/P \approx 10^{-11} \text{ s}^{-1} \), which leads to \( \dot{E}_{\text{rot}} \approx 10^{38} \text{ erg s}^{-1} \). This power is comparable to the accumulated radiative output of about \( 10^9 \) stars like the sun. This power is larger than the observed luminosity of the source but comparable to the power required to feed the surrounding pulsar wind nebula. This coincidence indicates that the pulsar wind is powered by the rotational energy of the pulsar.

The rotational power is extracted via the effects of the strong magnetic field of the neutron star. Indeed, during the gravitational collapse leading to the formation of the NS, the conservation of magnetic flux across the stellar surface implies that the magnetic field is amplified by a factor \( \sim R_e^2/R^2 \), where \( R_e \) is the initial radius of the star, and \( R \) is the radius of the compact object. For a star of the size of the sun compacted into a 10 km radius, this amplification factor is of the order of \( 5 \times 10^9 \). As a result, the typical surface magnetic field of a young NS is of the order of \( 10^{11} - 10^{12} \text{ G} \). For comparison, Earth’s magnetic field is \( \sim 0.5 \text{ G} \); a fridge magnet is \( \sim 50 \text{ G} \); the strongest continuous field yet produced in a laboratory is about \( 5 \times 10^5 \text{ G} \). Because of the high conductivity of the NS matter, its magnetic field dissipates only very slowly, on timescales much longer than the ‘braking’ timescale of the NS. In practice, it can be considered a constant. The young NS, therefore, behaves as a huge, spinning magnet. It is known from electromagnetic theory that a magnetic dipole rotating in a vacuum radiates. The radiated energy can be estimated simply as

\[
\dot{E}_d = \frac{B_p^2 R_e^6 \Omega^4}{6 c^3} \sin^2 \alpha \approx 10^{39} \left( \frac{B_p}{10^{12} \text{ G}} \right)^2 \left( \frac{R}{10 \text{ km}} \right)^6 \left( \frac{P}{0.01 \text{ s}} \right)^{-4} \sin^2 \alpha \text{ ergs}^{-1}, \tag{1.3}
\]

where \( B_p \) is the magnetic field amplitude at the poles of the compact star, and \( \alpha \) is the misalignment angle between the direction of the dipole and the spin of the neutron star. This radiated power can be matched to the observed rotational power in pulsars in order to estimate the pulsar magnetic field, its age, etc. But in reality, the magnet is not spinning in a vacuum. Huge electric fields are generated close to the surface of the NS. Those fields extract charged particles from the NS. These charged particles distribute themselves around the star to neutralise the electric field; an extended magnetosphere is formed. Streams of charged particles leave the star at a high latitude where the magnetic field lines are open, leading to the formation of a wind. The rotation energy is dissipated mostly through the interaction of the magnetic field of the NS with the magnetosphere and the wind. Nevertheless, the energy losses remain comparable to those predicted by the simple magnetic dipole radiation model.

### 1.2.2 Magnetic Field Dissipation

Some NSs appear to have negligible rotation energy but huge magnetic fields up to \( 10^{14} - 10^{16} \text{ G} \). These strongly magnetised NSs define the class of magnetars (see, e.g., Woods and Thompson, 2006). It has been shown (Thompson and Duncan, 1993) that such magnetic fields may have built up through dynamo processes (similar to that generating Earth or the sun’s magnetic fields) during the first moments of the life of the NS. This strong dynamo amplification occurs only if the NS is formed with a spin period \( P < 5 \text{ ms} \). After 10–30 s, the core has cooled down and is not hot enough for efficient dynamo. The magnetic braking is very efficient and the rotation energy is dissipated very quickly. After a few minutes, the rotation has slowed down to a period of a few seconds.
But the magnetic fields remain so strong that they can push and move material around in the star’s interior and crust. This leads to large amounts of magnetic dissipation during the first $10^{4}$ yr. Magnetic dissipation in the interior of the star keeps the star hot, bright and producing mostly X-ray thermal emission. Magnetic dissipation may also occur in the surrounding magnetosphere due to twisting of the magnetic field lines and reconnection (similar to the solar corona). This can lead to bursts of non-thermal X-ray and gamma-ray radiation, which are observed in sources called ‘soft gamma-ray repeaters’.

1.2.3 Accretion

Accretion is the growth of a massive object by gravitationally attracting more matter. This is an ubiquitous astrophysical process leading to the formation of planets, stars, galaxies and the growth of super-massive BHs. The gravitational energy released during accretion onto super-massive BHs also appears to regulate the joint growth of the BHs and their host galaxies. Accretion onto stellar compact objects can be observed if the compact object is in a binary system and can accrete gas from its companion star. As this type of accretion occurs in bright nearby sources evolving on human timescales it is relatively easy to observe and study, and some of the knowledge gained may be extrapolated to other accreting systems such as super-massive BHs in active galactic nuclei (AGNs). As will be discussed below, accretion onto a compact star in an accreting binary system depends on the nature of the donor star (high mass versus low mass star). It depends also on the nature of the compact star. Gas accreting onto a NS or a WD will ultimately hit the hard surface of the star. This has observable effects, such as the presence of a boundary layer in which the gas is stopped or the triggering of nuclear explosions (X-ray bursts in NSs, classical novae in WDs) when enough material has been accreted onto the surface. The structure of the accretion flow may also be affected by the strong magnetic field of the star as is the case for NSs in X-ray pulsars or WDs in polars. Such complications do not occur in accreting BHs where we can observe accretion in its ‘purest’ form; as the gas crosses the event horizon without notable effects, all the observed radiation must originate from the accretion flow.

Accretion Power

Accreting matter falls into the potential wells of a compact object and loses gravitational energy. For accretion to occur, gravitational energy (and angular momentum) must be dissipated away, mostly in the form of radiation. If accretion occurs at a mass accretion rate $\dot{M}$ onto an object of size $R$ and mass $M$, the gravitational power that is dissipated is given by the gravitational potential on the surface

$$\dot{E}_{ac} = \dot{M} \frac{GM}{R} = \eta \dot{M} c^2,$$

where $\eta = GM/Rc^2$ is called the accretion efficiency. It represents the amount of energy released per unit of mass energy accreted. The accretion efficiency onto a WD is of the order of $10^{-4}$ while accretion onto a neutron star reaches 0.1. Due to the absence of a hard surface during accretion onto a BH, the efficiency depends on the structure of the accretion flow. For thin accretion discs (see below), the accretion efficiency is in the range 0.057–0.42, depending on the BH spin. For comparison, accretion onto Planet Earth has $\eta \sim 10^{-9}$, while the sun has $\eta \sim 10^{-6}$. The efficiency of thermonuclear fusion of hydrogen is $7 \times 10^{-3}$. Accretion onto a compact object is, therefore, much more efficient than fusion at releasing energy.
Eddington Limit

The accretion power is, however, limited by the amount of matter that we are able to accrete. There is a fundamental limit on the mass accretion rate that is set by the Eddington luminosity. The Eddington luminosity is the maximum luminosity for which the gravitational force on a fluid element exceeds the radiation pressure (i.e., the maximum luminosity at which matter can be accreted). Let us consider a fluid element of mass $m$ located at distance $d$ from the center of the compact object, which radiates isotropically at luminosity $L$. The amplitude of the force of gravity is $F_{\text{grav}} = GMm/d^2$. The radiation pressure force is proportional to the local radiation flux and is directed in opposite direction. It is given by $F_{\text{rad}} = L\kappa m/(4\pi d^2 c)$, where the opacity $\kappa$ is a measure of the effectiveness of the transfer of momentum from radiation to the fluid element. Since both forces are proportional to $m/d^2$, the condition for equilibrium $F_{\text{grav}} = F_{\text{rad}}$ is independent of both the mass and distance of the fluid element. This condition defines the Eddington luminosity

$$L_E = \frac{4\pi GMc}{\kappa} \approx 1.46 \times 10^{38} \frac{M}{M_\odot} \text{ ergs}^{-1}. \quad (1.5)$$

The accreting gas is usually hot and ionised, so the opacity is dominated by electron scattering. In this case, $\kappa = \kappa_{es} = 0.34 \text{ cm}^2 \text{ g}^{-1}$ for standard abundances. This is the value of $\kappa$ that was assumed in the numerical estimate given in Equation 1.5. This immediately implies a maximum accretion rate

$$\dot{M}_E = \frac{L_E}{\eta c^2} = 2.6 \times 10^{-9} (M/M_\odot) \eta^{-1} \frac{M_\odot}{\text{yr}}. \quad (1.6)$$

For a compact object with a hard surface (NS or WD), the Eddington mass accretion rate depends only on its size

$$\dot{M}_E = \frac{4\pi cR}{\kappa} = 1.8 \times 10^{-8} \left( \frac{R}{10 \text{ km}} \right) \frac{M_\odot}{\text{yr}}. \quad (1.7)$$

The maximum luminosity and accretion rate are estimated for a spherical accretion geometry and radiation field, and deviations from spherical are of course expected to occur and may allow to exceed somewhat the Eddington limit (which indeed appears to be violated in some sources). Nevertheless, this gives a good estimate of the maximum power that can be extracted through accretion onto a compact object. This power is huge. The Eddington luminosity of a 10$M_\odot$ BH is comparable to the combined luminosity of 10$^6$ stars like the sun.

Mass Transfer

However, there is another limitation related to the capacity of the compact object to attract and capture gas from the donor star. This is the so-called mass transfer problem. Before falling onto the compact object, the gas must escape the pull of the donor star. The effective gravitational potential in a binary system is determined by the masses of the stars and the centrifugal force arising from the motion of the two stars around one another. One may write this potential as a function of $r_1$ and $r_2$, the distance to the centre of each star, and $r_3$, the distance to the rotation axis

$$\phi = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{\Omega_{\text{orb}}^2 r_3^2}{2}, \quad (1.8)$$

where $\Omega_{\text{orb}}$ is the orbital angular velocity. The equipotential surfaces form two lobes surrounding each of the stars that are called the Roche lobes. The two lobes are connected...
through a point called the first Lagrangian point (or L1), where the sum of the centrifugal and gravitational forces vanishes. This is a saddle point in the potential that forms a pass that the gas from the donor has to climb before being able to fall into the influence of the compact object.

**Roche lobe overflow.** Mass transfer may occur simply if the donor fills its Roche lobe. This may result from an increase of the stellar radius during the evolution of the star. This can also be driven by changes in the orbital parameters that make the Roche lobe smaller. This can also occur through loss of angular momentum (by emission of gravitational waves, magnetic braking, tidal torques, mass loss in a stellar wind, etc.). Also, mass transfer, over time, will change the mass ratio of the two components and affect the orbital parameters. So even if the donor fills its Roche lobe, accretion may be unstable and may not be sustained. It can be shown that stable lobe overflow can occur only if the mass of the donor is smaller than the mass of the accretor. If these two conditions are fulfilled, then steady mass accretion occurs at rates of the order of $\dot{M} \sim 10^{-10} - 10^{-9}$. Such systems are called low-mass X-ray binaries (hereafter LMXBs).

**Wind accretion.** A massive early type companion (O or B) can lose mass in a wind at a rate $10^{-6} - 10^{-5} \, M_\odot \, yr^{-1}$ and supersonic velocity comparable to the escape velocity of the star

$$v_w \sim v_{esc} = \sqrt{\frac{2GM_*}{R_*}} \sim 10^3 \, \text{km s}^{-1}.$$  

(1.9)

The compact star will gravitationally capture matter from a roughly cylindrical region with an axis along the relative wind direction. This cylinder represents the volume where the wind particle kinetic energy is less than the gravitational potential (e.g., Bondi and Hoyle, 1944; Bondi, 1952; Davidson and Ostriker, 1973; Lamb et al., 1973; Frank et al., 1992). The radius of the cylinder, called the accretion radius or gravitational capture radius, is given by

$$r_{acc} \simeq \frac{2GM}{v_{rel}^2 + c_s^2},$$

(1.10)

where $c_s$ is the sound speed in wind, $v_{rel}^2 = v_{orb}^2 + v_w^2$, and $v_{orb}$ is the orbital velocity of the gas.

The net amount of gas captured and accreted by the compact object can be obtained by combining this relationship with Kepler’s third law and the continuity equation (assuming spherically symmetric and steady mass loss)

$$\dot{M} = \pi r_{acc}^2 n_{rel} (10^{-5} - 10^{-4}) \dot{M}_w \sim 10^{-11} - 10^{-9} M_\odot \, yr^{-1}.$$  

(1.11)

We can see that both wind accretion in HMXB and Roche lobe overflow in LMXB allow mass transfer at a rate that is smaller than, and yet a significant fraction of, the Eddington limit.

**Keplerian Accretion Discs**

Once the material is captured, it would orbit the compact object indefinitely unless it can get rid of its angular momentum and be accreted. It is viscosity, and the associated viscous torques between annuli in the disc, that allows angular momentum to be transferred outwards and mass to spiral inwards. Any original ring of particles will thus spread out into a disc, with the outer radius being determined by tidal torques from the
companion. However, ordinary molecular viscosity is completely inadequate to account for the observed properties of discs, and some kind of turbulent viscosity is invoked. The best candidate as the origin of this turbulent viscosity is the magneto-rotational instability that develops in differentially rotating flows and can generate fully developed magnetohydrodynamic (MHD) turbulence that provides efficient angular momentum transport (Balbus and Hawley, 1991, 1998). The characteristic scale of the turbulence must be less than the disc thickness, $H$, and the characteristic turbulent speed is expected to be less than the speed of sound, $c_s$, since there is no strong evidence for turbulent shocks. The viscosity, $\nu$, is therefore often parametrised by writing it as $\nu = \alpha c_s H$, placing all the uncertainties in the unknown parameter $\alpha$, taken to be less than 1. This is the standard alpha-disc model of accretion discs (Shakura and Sunyaev, 1973). This model also assumes that radiation cooling in the disc is very efficient. All the locally dissipated accretion power is radiated away on the spot in the form of blackbody radiation. As a consequence, the disc is cold and geometrically thin. From these two assumptions, it is possible to determine analytically the full disc structure as a function of the distance $r$ from the compact object. Although there is still no reliable way of calculating $\alpha$, it turns out that the observable properties of a steady accretion disc are largely independent of $\alpha$. In particular, the total luminosity of the disc is simply half of the gravitational power at the disc inner radius $R_i$, $L_{\text{disc}} = 2GM \dot{M}/R_i$. Most of the luminosity comes from the inner part of the accretion flow; 80 per cent of the power is radiated within $10R_i$. The temperature profile in the disc is

$$T(R) = T_0 \left( \frac{R_i}{r} \right)^{3/4} \left( 1 - \frac{R_i}{r} \right)^{1/4},$$

where

$$T_0 = \left( \frac{3GM \dot{M}}{8\pi \sigma R_i^4} \right)^{1/4} \approx 6 \times 10^7 \left( \frac{R_i}{R_G} \right)^{-3/4} \left( \frac{M^2}{L_E} \right)^{1/4} \left( \frac{M}{M_\odot} \right)^{-1/4} \text{ K.}$$

The temperature has a maximum around $3R_i$ and then decreases with a distance like $r^{-3/4}$. This maximum temperature can reach a few keVs in NSs and BHs. The innermost and most luminous part of the accretion disc will therefore produce mostly X-ray radiation. The spectral energy distribution (SED) of the whole accretion disc is constituted of the sum of all the black bodies emitted at different radii in the disc with a different temperatures. Longer wavelengths allow one to probe more distant regions of the accretion flow. We note the standard Shakura-Sunyaev disc model assumes pure Newtonian physics, the general relativistic version of the thin disc model was introduced by Novikov and Thorne (1973).

**Hot Accretion Flows**

The thin accretion disc model assumes full energy thermalisation, and this implies a low temperature, a high density and a small disc scale height of $H/R \sim 10^{-3}$. $10^{-4}$. If, instead, the disc is hot, then the gas pressure makes the scale height larger, $\simeq 0.1-0.5$, and this reduces the density. Consequently, the radiation cooling is less efficient and a high temperature can be maintained. Indeed, the density becomes so small that the collision time-scales between electrons and ions of the plasma can be long compared to the accretion timescale. Then the protons acquire most of the gravitational energy through viscous heating, but this energy can only be radiated efficiently by the electrons. Since electrons and protons are decoupled, the
two populations end up having very different temperatures. The protons can reach \( \sim 10^{12} \) K while the electrons are much colder (and yet very hot), \( \sim 10^9 \) K in the innermost part of the accretion flow. In this kind of hot accretion flow, the accretion energy is not radiated away locally; it can be carried inward with the flow until it is swallowed by the BH or hits the surface of the compact star. These advection-dominated accretion flows (Ichimaru, 1977; Narayan and Yi, 1994) produce mostly non-thermal Comptonisation radiation (see Section 1.3.4). Hot accretion flows have been extensively studied. Analytic solutions have been found also taking into account the effects of convection; convection-dominated accretion flows (CDAF, Narayan et al., 2000) and outflows; advection-dominated inflow–outflow solution (ADIOS, Blandford and Begelman, 1999). Hot accretion flow have also been found in numerical simulations (Stone et al., 1999).

**Jet Launching from Accretion Flows**

Accreting NSs and BHs (an perhaps also WDs) can launch highly collimated jets of magnetised plasma that travel at near light speed, carrying away a significant fraction of the accretion power (Mirabel and Rodríguez, 1994; Fender and Gallo, 2014). These jets propagate over large distances and can have an enormous impact on the surrounding medium over distance scales that are far out of the gravitational reach of the BH itself. The details of the formation of those jets are unclear; none of the models and simulations take into account all the physics. But all models require magnetic fields. The mechanism proposed by Blandford and Payne (1982) involves magnetic field lines threading the accretion disc. Provided that the magnetic field lines are sufficiently inclined with respect to the disc, the centrifugal force can throw away some of the accreting material, which remains tied to the rotating magnetic field lines like a bead on a wire. Then the jet needs to somewhat collimate, and this requires large-scale coherent magnetic field structures. Another mechanism involves electromagnetic extraction of energy from a Kerr BH. Indeed, Penrose (1969) and Christodoulou (1970) have shown that up to 30 per cent of the mass energy of a maximally spinning BH can be theoretically extracted. Then, Blandford and Znajek (1977) have shown, that an accretion disc can allow this energy to be extracted and drive powerful jets. The magnetic field carried by the accreted gas remains threaded through the BH horizon. The frame of the field lines is dragged along with the rotation of the BH. These rotating field lines induce an electromagnetic force that accelerates charged plasma at relativistic speeds along the axis of rotation. Due to the radial component of the field, the particles spiral as they leave.

**Effects of the Magnetic Field of a Compact Star on the Accretion Flow**

Magnetic fields become dynamically important close to the compact object at a distance called the Alfvén radius, \( R_a \). At \( R_a \), the magnetic energy is comparable to the kinetic energy of the infalling gas: \( \rho v^2 / 2 \approx B^2 / 8 \pi \). For the purpose of simple estimates, we can assume that the accreting gas is in free fall in a spherical geometry, so that \( v \approx v_{\text{ff}} = \sqrt{2GM/R_a} \) and \( \rho = \dot{M} / (4\pi R_a^2 v_{\text{ff}}) \). Assuming a dipole magnetic field around a WD or NS of radius \( R_* \), \( B \sim B_p R_3/R_a^3 \), we then obtain

\[
R_a \approx 30 \text{ km} \left( \frac{B_p}{10^9 \text{ G}} \right)^4 \left( \frac{\dot{M}}{2 \times 10^{-8} \text{ M}_\odot \text{ yr}^{-1}} \right)^{-\frac{3}{2}} \left( \frac{M}{\text{M}_\odot} \right)^{-\frac{1}{2}} \left( \frac{R_*}{10 \text{ km}} \right)^{\frac{12}{7}}.
\]  

(1.14)
The effects of the magnetic field are important only if $R_a > R_\star$, which translates into the following condition for the surface magnetic field:

$$B_p > 1.5 \times 10^8 \left( \frac{\dot{M}}{2 \times 10^{-8} \, M_\odot \, \text{yr}^{-1}} \right)^{\frac{1}{2}} \left( \frac{\dot{M}}{M_\odot} \right)^{\frac{1}{2}} \left( \frac{R_\star}{10 \, \text{km}} \right)^{-\frac{3}{2}} \text{G}. \quad (1.15)$$

As discussed in Sections 1.2.1 and 1.2.2, in the case of NSs, this condition will be easily verified, and the magnetic fields of the star will interfere with the accretion process. At $R_a$, the magnetic field can force accreting material in corotation with the compact star. If the spin period of the compact star is longer than the orbital period at $R_a$, the centrifugal forces cannot balance gravity anymore, and the material flows along magnetic field lines onto the magnetic poles of the compact star. This forms an X-ray pulsar if the compact star is a NS and a polar in the case of a WD. The magnetic field transfers angular momentum from the accretion flow to the compact star, exerting an effective spin-up torque on the star. If the spin period of the compact star is shorter than the orbital period at $R_a$, the corotation implies that the centrifugal forces overcome gravity; accretion is stopped. This is the so-called propeller regime in which angular momentum is transported from the compact star to the ‘accretion’ flow. In this case, the magnetic field exerts a spin-down torque on the star. The spin of the compact star therefore evolves through propeller and accreting regimes to reach an equilibrium in which the spin period of the star is equal to the orbital period at $R_a$.

$$P_{\text{eq}} \approx 3 \, \text{ms} \left( \frac{B_p}{10^9 \, \text{G}} \right)^{\frac{1}{2}} \left( \frac{\dot{M}}{2 \times 10^{-8} \, M_\odot \, \text{yr}^{-1}} \right)^{\frac{1}{2}} \left( \frac{M}{M_\odot} \right)^{-\frac{1}{2}} \left( \frac{R_\star}{10 \, \text{km}} \right)^{\frac{18}{7}}. \quad (1.16)$$

This equation shows that accretion will spin up a NS up to spins of a few ms. Such sources are observed; these are the millisecond X-ray pulsars (Wijnands and van der Klis, 1998; Harding, 2013; Tauris, 2015).

**Compact Stars with a Weak Magnetic Field: Boundary Layer**

Many compact stars appear to have a magnetic field below the threshold defined by Equation 1.15. Indeed, accretion seems to cause dissipation of the magnetic field, which may be considerably reduced in the course of the history of the accreting compact star. In this case, the accretion disc can extend undisturbed very close to the surface of the star. There is, however, a region between the accretion flow and the star where the accreting gas has to decelerate from the orbital velocity to the rotation velocity of the star, spinning up the compact star in the process. This region is called the boundary layer (BL). Since most of the kinetic energy of the gas is dissipated in the BL, the luminosity of the BL is comparable to the total luminosity of the accretion disc: $L_{\text{BL}} \sim \dot{M} v_K^2/2 = G M \dot{M} / (2 R_a) \sim L_{\text{disc}}$. Matter has a significant latitude velocity component in the boundary layer, spreading above the compact star surface and decelerating due to friction like a wind above the sea (Inogamov and Sunyaev, 1999). For sources with luminosity greater than 5 per cent Eddington, the local radiation flux is Eddington. In this regime, the spreading layer temperature is independent of luminosity. Emission models predict a temperature of the order of 2.5 keV, which was confirmed by observations. The size of the belt must increase with accretion rate/luminosity. For $L \sim L_{\text{Edd}}$, the spreading layer covers the whole surface of the compact star (Gilfanov and Sunyaev, 2014).
1.3. Radiation Processes

From radio waves to gamma-rays, compact objects radiate over the whole electromagnetic spectrum. This radiation can be emitted through thermal radiation (i.e. blackbody emission related to the temperature of the object) or non-thermal processes like synchrotron or inverse Compton. Synchrotron radiation is produced by very energetic charged particles spiralling around magnetic field lines. Relativistic particles accelerated in shocks in the jets produce synchrotron emission mostly in the radio, sub-mm and infrared (IR) bands. A similar process, called curvature radiation, is related to the propagation of particles along very curved field lines. It can be important in the magnetosphere of neutron stars. An accretion flow can be very hot and contains energetic electrons of temperatures up to a billion K. It also contains many low-energy photons produced by synchrotron radiation or coming from the outer regions of the accretion flow. These photons collide with the hot electrons and gain energy at each interaction. This process is called inverse Compton scattering. If the photon can make several interactions before escaping the hot gas, the process is called Comptonisation. It leads to the production of hard X-ray radiation. Inverse Compton from relativistic particles in the jets may also produce gamma-rays. Understanding the radiation processes allows one to extract the information that is encoded in the radiation received from these objects. It enables to measure many physical parameters of the system, such as temperatures, velocities, magnetic fields, the energy of accelerated particles and their distribution. It also helps to determine the size and geometry of the emitting regions and test the predictions of theoretical models. By simultaneously observing in different bands, we learn about the joint evolution of the different components of the systems (e.g. accretion flow and jets). In this section, I summarise the main properties of the cyclo-synchrotron, curvature radiation, bremsstrahlung, inverse Compton and photon–photon pair production. A more detailed discussion of these radiation processes can be found in classical text books such as those by Rybicki and Lightman (1986); Longair (1992), and Jackson (1999).

1.3.1 Cyclo-Synchrotron

Cyclo-synchrotron radiation is the radiation produced by a charged particle accelerated by the magnetic field. In the following, I consider an $e^−$ which gyrates around the magnetic field lines with a velocity $v$ (see Figure 1.1). The component of its velocity that is parallel to the magnetic field is a constant. The transverse component of the velocity has a constant modulus and a direction that is rotating uniformly around the magnetic field line. The constant angle $\alpha$ between the velocity and the magnetic field is called the pitch angle. The rotation frequency $\nu_B = \nu_L/\gamma$, where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor of the particle, and $\nu_L = qB/(2\pi m_e c) = 2.8 \times 10^9$ B Hz is the Larmor frequency. The magnetic field $B$ is expressed in G. The radius of the ‘orbit’ around the magnetic field is given by the Larmor radius: $r_L = p_\perp/(2\pi \nu_L) = 1.7 \times 10^3 p_\perp/B$ cm, where $p_\perp = p \sin \alpha = \gamma v \sin \alpha/c$. Because the particle has a transverse acceleration, it radiates.

![Figure 1.1. Emission of an electron spiralling in a magnetic field.](image_url)