The scalar $f_0(500)$ and $f_0(980)$ resonances and vector mesons in the single Cabibbo-suppressed decays $\Lambda_c \to pK^+K^-$ and $p\pi^+\pi^-$

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In the chiral unitary approach, we have studied the single Cabibbo-suppressed decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$ by taking into account the s-wave meson-meson interaction as well as the contributions of the intermediate vectors $\phi$ and $\rho$. Our theoretical results for the ratios of the branching fractions of $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$ with respect to the one of $\Lambda_c \to p\phi$ are in agreement with the experimental data. Within the picture that the scalar resonances $f_0(500)$ and $f_0(980)$ are dynamically generated from the pseudoscalar-pseudoscalar interaction, we have calculated the $K^+K^-$ and $\pi^+\pi^-$ mass distributions respectively for the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$. One can find a broad bump structure for the $f_0(500)$ and a narrow peak for the $f_0(980)$ in the $\pi^+\pi^-$ mass distribution of the decay $\Lambda_c \to p\pi^+\pi^-$, which is compatible with the BESIII measurement. For the $K^+K^-$ mass distribution, in addition to the narrow peak for the resonance $\phi$, one can see an enhancement structure near the $K^+K^-$ threshold. We encourage our experimental colleagues to measure these two decays, which would be helpful to understand the nature of the $f_0(500)$ and $f_0(980)$.

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I. INTRODUCTION

The non-leptonic decays of the lightest charmed baryon $\Lambda_c$ play an important role in the study of strong and weak interactions [1–6]. In the last decades, lots of the information about the $\Lambda_c$ decays has been accumulated [7–11], which provides a good platform to investigate the possible final state interference effects where some resonances can be dynamically generated [12–17].

Recently, the BESIII Collaboration has reported the branching fractions of the $\Lambda_c \to pK^+K^-$, $p\pi^+\pi^-$,

$$\frac{B(\Lambda_c \to p\phi)}{B(\Lambda_c \to pK^+K^-)} = (1.81 \pm 0.33 \pm 0.13)\%,$$

$$\frac{B(\Lambda_c \to pK^+K^-)_{\text{non-}\phi}}{B(\Lambda_c \to pK^+K^-)} = (9.36 \pm 2.22 \pm 0.71)\%,$$

$$\frac{B(\Lambda_c \to p\pi^+\pi^-)}{B(\Lambda_c \to pK^+K^-)} = (6.70 \pm 0.48 \pm 0.25)\%,$$

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and also measured the $\pi^+\pi^-$ and $K^+K^-$ mass distributions, respectively \cite{18}, where one can find a broad bump around 500 MeV for the scalar resonance $f_0(500)$ and a narrow sharp around 980 MeV for the scalar resonance $f_0(980)$ in the $\pi^+\pi^-$ mass distribution, in addition to the peak for the $\rho$ meson. Later, the LHCb Collaboration has also reported these ratios using the proton-proton collision data \cite{11},

$$
\frac{B(\Lambda_c \to pK^+K^-)}{B(\Lambda_c \to pK^-\pi^+)} = (1.70 \pm 0.03 \pm 0.03)\%,
$$

$$
\frac{B(\Lambda_c \to p\pi^+\pi^-)}{B(\Lambda_c \to pK^-\pi^+)} = (7.44 \pm 0.08 \pm 0.18)\%.
$$

Before the BESIII and LHCb results, the above two decay modes have also been observed by the NA32 \cite{19}, E687 \cite{20}, CLEO \cite{21}, and the Belle Collaborations \cite{7}.

Within the chiral unitary approach, the scalar resonances $f_0(500)$, $f_0(980)$, $a_0(980)$, and $\kappa(800)$ appear as composite states of meson-meson, automatically dynamically generated by the interaction of pseudoscalar-pseudoscalar where the kernel for the Bethe-Salpter equation is taken from the chiral Lagrangians \cite{22,27}. The productions of $f_0(500)$ and $f_0(980)$ have been recently studied with the chiral unitary approach and the final state interactions in the decays of the $\bar{B}$ and $\bar{B}_s \ \cite{28,31}, \chi_{c1} \ \cite{32,33}, \chi_{c2} \ \cite{32,33}, D^0 \ \cite{34}, \tau^- \ \cite{35}, \text{and } J/\psi \ \cite{36}.

In this work, we perform the calculations of the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$ using the chiral unitary approach and the final state interactions of the meson-meson interaction in coupled channels. The two pions in the final states of the decay $\Lambda_c \to p\pi^+\pi^-$ can propagate in s-wave, which will generate the $f_0(500)$ and $f_0(980)$ resonances, and for the decay $\Lambda_c \to pK^+K^-$, the $f_0(980)$ resonance dynamically generated from the s-wave $K^+K^-$ final state interaction will result in an enhancement structure close to the $K^+K^-$ threshold.

The paper is organized as follows. In Section II, we present the formalism and ingredients for the decay amplitudes of the $\Lambda_c \to pK^+K^-$ and $p\pi^+\pi^-$ decays. Numerical results for invariant mass distributions of the $K^+K^-$ and $\pi^+\pi^-$ and discussions are given in Section III, followed by a short summary in the last section.

II. FORMALISM

In this section, we will present the formalism for the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$. The three-body decays of $\Lambda_c$ can preform in s-wave, where the final state interactions of $\pi^+\pi^-$ or $K^+K^-$ will dynamically generate the scalar resonances $f_0(500)$ and $f_0(980)$.

In addition, the three-body decays can happen via the intermediate vector mesons $\rho^0$ or $\phi$. We first introduce the formalism for the mechanism of final state interactions of $\pi^+\pi^-$ or $K^+K^-$ in s-wave in Subsect. II A, then we show the details for the mechanism of the $\Lambda_c$ decay via the intermediate vector mesons $\rho^0$ and $\phi$ in Subsect. II B

A. s-wave final state interactions of $K^+K^-$ and $\pi^+\pi^-$

Following Refs. \cite{37,40}, we take the decay mechanism of the internal $W$ emission mechanism for the decays $\Lambda_c \to pK^+K^-$ and $\Lambda_c \to p\pi^+\pi^-$ as depicted in Figs. I(a) and (b). For the weak decays of $\Lambda_c$, the $c$ quark decays into a $W^+$ boson and a $s$ (or $d$) quark, then the $W^+$ boson decays into a $\bar{s}u$ (or $\bar{d}u$) pair. In order to give rise to the final states of $pK^+K^-$ (or $p\pi^+\pi^-$), the $ss$ (or $dd$) quark pair need to hadronize together with the $\bar{q}q$ ($= \bar{u}u + \bar{d}d + \bar{s}s$)
produced in the vacuum, $H^{(a)}$ or $H^{(b)}$, which are given by,

$$H^{(a)} = V^{(a)} s(\bar{u}u + \bar{d}d + \bar{s}s)\frac{1}{\sqrt{2}}(ud - du) = V^{(a)} (M^2)_{33} p,$$

$$H^{(b)} = V^{(b)} d(\bar{u}u + \bar{d}d + \bar{s}s)\frac{1}{\sqrt{2}}(ud - du) = V^{(b)} (M^2)_{22} p,$$

where $V^{(a)}$ and $V^{(b)}$ are the weak interaction strengths. We use $|p| = \frac{1}{\sqrt{2}} |u(ud - du)|$, and $|\Lambda_c| = \frac{1}{\sqrt{2}} |c(ud - du)|$. $M$ is the $q\bar{q}$ matrix,

$$M = \begin{pmatrix} uu & ud & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}.$$

The matrix $M$ in terms of pseudoscalar mesons can be written as,

$$M \Rightarrow P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{6}} \\ \pi^- \\ K^- \end{pmatrix} - \frac{1}{\sqrt{2}} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{6}} \begin{pmatrix} K^+ \\ K^0 \\ -\eta \sqrt{3} + \frac{2\eta'}{\sqrt{6}} \end{pmatrix}.$$  \hspace{1cm} (8)

Then, we have,

$$H^{(a)} = V^{(a)} (M^2)_{33} p = V_p V_{cs} V_{us} \left( K^- K^+ + K^0 K^0 + \frac{1}{3} \eta \eta \right) p,$$

$$H^{(b)} = V^{(b)} (M^2)_{22} p = V_p V_{cd} V_{ud} \left( \pi^+ \pi^- + \frac{1}{2} \pi^0 \pi^0 + \frac{1}{3} \eta \eta - \frac{2}{\sqrt{6}} \pi^0 \eta + K^0 K^0 \right) p,$$  \hspace{1cm} (9) \hspace{1cm} (10)

where we neglect the $\eta'$ because of its large mass. $V_p$ is the meson-meson production vertex which contains all dynamical factors. In this work we take $V_{cs} = V_{ud} = -\sin\theta_c = 0.22534$, $V_{cd} = V_{us} = \cos\theta_c = 0.97427$ [41].

After the production of a meson-meson pair, the final state interaction in the s-wave between the mesons takes place, which can be parameterized by the re-scattering in the hadron level, as show in Figs. [2] and [3] where we will take into account both contributions from the diagrams of Fig. [1].

On the other hand, the decays $\Lambda_c \rightarrow pK^+K^-$ and $\Lambda_c \rightarrow p\pi^+\pi^-$ can also proceed with the following steps: i) the charmed quark turns into $W^+$ and the $s$ or $d$ quark, with the $K^+$ or $\pi^+$ emission from the $W^+$; ii) the remaining quarks $s$ or $d$ and $ud$ in the $\Lambda_c$ hadronize to the $K^- p$ or $\pi^- p$. Although this mechanism of the external $W$ emission is color favored, we do not know any information about the relative weight and phase between the external $W$ emission and internal $W$ emission. Furthermore, the external $W$ emission process provides the contributions for the $pK^+K^-$

FIG. 1: The diagrams of the internal $W$ emission for the $\Lambda_c$ decays, (a) $\Lambda_c \rightarrow pK^+K^-$, (b) $\Lambda_c \rightarrow p\pi^+\pi^-$. 

![Diagram](image-url)
and $p\pi^+\pi^-$ as background, and do not affect much the invariant mass distributions of the final state $K^+K^-$ and $\pi^+\pi^-$. Since the purpose of this work is to study the scalar mesons, dynamically generated from the $s$-wave meson-meson interactions, we will leave the contributions from the external $W$ diagrams in future studies, when more accurate experimental data are available.

Finally, the amplitudes of the decays $\Lambda_c \rightarrow pK^+K^-$ and $\Lambda_c \rightarrow p\pi^+\pi^-$ in $s$-wave can be expressed as,

$$t_{s-wave}^{\Lambda_c \rightarrow pK^+K^-} = V_p V_c V_{us} \left[ 1 + G_{\pi^0K^0} t_{\pi^0K^0 \rightarrow K+K^-} + 2G_{K^0\bar{K}^0} t_{K^0\bar{K}^0 \rightarrow K+K^-} + G_{\pi^+\pi^-} t_{\pi^+\pi^- \rightarrow K+K^-} + \frac{1}{2} G_{\eta\eta} \hat{t}_{\eta\eta \rightarrow K+K^-} \right],$$

$$t_{s-wave}^{\Lambda_c \rightarrow p\pi^+\pi^-} = V_p V_c V_{us} \left[ 1 + G_{\pi^0K^0} t_{\pi^0K^0 \rightarrow \pi^+\pi^-} + 2G_{K^0\bar{K}^0} t_{K^0\bar{K}^0 \rightarrow \pi^+\pi^-} + G_{\pi^+\pi^-} t_{\pi^+\pi^- \rightarrow \pi^+\pi^-} + \frac{1}{2} G_{\eta\eta} \hat{t}_{\eta\eta \rightarrow \pi^+\pi^-} \right],$$

where we include the factor 1/2 in the intermediate loops involving a pair of identical mesons. The scattering matrix $t_{i \rightarrow j}$ has been calculated within the chiral unitary approach in Refs. 22, 29, 34, 42, 43, and we take $\hat{t}_{\eta\eta \rightarrow K+K^-} = \sqrt{2} t_{\eta\eta \rightarrow K+K^-}$ and $\hat{t}_{\eta\eta \rightarrow \pi^+\pi^-} = \sqrt{2} t_{\eta\eta \rightarrow \pi^+\pi^-}$ as Ref. 42. $G_i$ is the loop function for the two mesons propagator in the $i$th channel, as given by,

$$G_i = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(p-q)^2-m_i^2+i\epsilon} \frac{1}{q^2-m_i^2+i\epsilon} = i \int \frac{d^4q}{(2\pi)^4} \frac{\omega_1 + \omega_2}{2\omega_1\omega_2} \frac{1}{\sqrt{s} + \omega_1 + \omega_2}(\sqrt{s} - \omega_1 - \omega_2 + i\epsilon),$$

where $\sqrt{s}$ is the invariant mass of the meson-meson pair, and the meson energies $\omega_i = \sqrt{(q^2)^2+m_i^2}$ ($i = 1, 2$). The integral on $\vec{q}$ in Eq. (13) is performed with a cutoff $|\vec{q}_{\text{max}}| = 600$ MeV, as used in Refs. 29, 34, 42. The transition amplitude $t_{ij}$ is obtained by solving the Bethe-Salpeter equation in coupled channels,

$$T = \frac{V}{1 - VG},$$

where five channels $\pi^+\pi^-$, $\pi^0\pi^0$, $K^+K^-$, $K^0\bar{K}^0$, and $\eta\eta$ are included. The elements of the diagonal matrix $G$ are given by the loop function of Eq. (13), and $V$ is the matrix of the interaction kernel corresponding to the tree level.
transition amplitudes obtained from phenomenological Lagrangians \[22\] and can be expressed as \[42\],

\[
\begin{align*}
V_{11} &= -\frac{1}{2f^2}s, \quad V_{12} = -\frac{1}{\sqrt{2}f^2}(s - m^2), \quad V_{13} = -\frac{1}{4f^2}s, \quad V_{14} = -\frac{1}{4f^2}s, \\
V_{15} &= -\frac{1}{3\sqrt{2}f^2}m^2, \quad V_{22} = -\frac{1}{2f^2}m^2, \quad V_{23} = -\frac{1}{4\sqrt{2}f^2}s, \quad V_{24} = -\frac{1}{4\sqrt{2}f^2}s, \\
V_{25} &= -\frac{1}{6f^2}m^2, \quad V_{33} = -\frac{1}{2f^2}s, \quad V_{34} = -\frac{1}{4f^2}s, \quad V_{35} = -\frac{1}{12\sqrt{2}f^2}(9s - 6m^2 - 2m^2), \\
V_{44} &= -\frac{1}{2f^2}s, \quad V_{45} = -\frac{1}{12\sqrt{2}f^2}(9s - 6m^2 - 2m^2), \quad V_{55} = -\frac{1}{18f^2}(16m^2_K - 7m^2),
\end{align*}
\]

(15)

where \(f\) is the pion decay constant, \(f = f_\pi = 93\) MeV, and \(m_\pi, m_K, \) and \(m_\eta\) are the averaged masses of the pion, kaon, and \(\eta\) mesons, respectively \[41\].

With the amplitudes of Eqs. (11) and (12), we can write the differential decay width for the decays \(\Lambda_c \rightarrow pK^+K^-\) and \(\Lambda_c \rightarrow p\pi^+\pi^-\) in s-wave,

\[
\frac{d\Gamma_{s-wave}}{dM_{inv}} = \frac{1}{(2\pi)^3} \frac{p_p \bar{k}}{4M_{\Lambda_c}} \left| t_{s-wave}^{\Lambda_c \rightarrow pK^+K^-} \right|^2,
\]

(16)

where \(M_{inv}\) is the invariant mass of the \(K^+K^-\) or \(\pi^+\pi^-\) system, \(p_p\) is the momentum of the proton in the \(\Lambda_c\) rest frame, and \(\bar{k}\) is the momentum of the \(K^+\) (or \(\pi^+\)) in the rest frame of the \(K^+K^-\) (or \(\pi^+\pi^-\)) system,

\[
p_p = \frac{\lambda^{1/2}}{2M_{\Lambda_c}} \left( M_{\Lambda_c}, M_{p}^2, M_{inv}^2 \right), \quad \bar{k} = \frac{\lambda^{1/2}}{2M_{inv}} \left( M_{inv}^2, m_{K^+}^2, m_{K^-}^2 \right),
\]

(17)

with the Källén function \(\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx\). The masses of the baryons and mesons involved in our calculations are taken from PDG \[41\].

**B. \(\Lambda_c\) decays via the intermediate vector mesons \(\phi\) and \(\rho^0\)**

In this section, we will present the formalism for the decays \(\Lambda_c \rightarrow pK^+K^-\) and \(\Lambda_c \rightarrow p\pi^+\pi^-\) via the intermediate mesons \(\phi\) and \(\rho^0\). The quark level diagrams for the two-body decays of \(\Lambda_c\) into a proton and a vector meson are shown in Fig. \[3\].

At the quark level, the quark components of the vector mesons are,

\[
\rho^0 = \frac{1}{\sqrt{2}}(u\bar{d} - d\bar{u}), \quad \phi = s\bar{s}, \quad \omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \quad \bar{K}^*\rho^0 = s\bar{d}.
\]

(18)

The amplitudes can be written as,

\[
\begin{align*}
t_{\Lambda_c \rightarrow pp\rho^0} &= -\frac{1}{\sqrt{2}}V_p^V V_{cd} V_{ud}, \quad t_{\Lambda_c \rightarrow p\phi} = V_p^V V_{cs} V_{us}, \\
t_{\Lambda_c \rightarrow pp\omega} &= \frac{1}{\sqrt{2}}V_p^V V_{cd} V_{ud}, \quad t_{\Lambda_c \rightarrow p\bar{K}^*\rho^0} = V_p^V V_{cs} V_{ud},
\end{align*}
\]

(19)

(20)

where \(V_p^V\) is a normalization factor for the \(\Lambda_c\) decay into proton and a vector meson. The factor of \(1/\sqrt{2}\) in the above amplitudes comes from the quark component of the \(\rho^0\) and \(\omega\). With those amplitudes, the decay width for the two-body decay of \(\Lambda_c\) into proton and a vector meson in s-wave is,

\[
\Gamma_{\Lambda_c \rightarrow pV} = \frac{\lambda^{1/2}}{16\pi M_{\Lambda_c}^3} \left| t_{\Lambda_c \rightarrow pV} \right|^2,
\]

(21)
where $V$ stands for the vector mesons $\rho^0$, $\phi$, $\omega$, and $\bar{K}^{*0}$.

The $K^+K^-$ and $\pi^+\pi^-$ mass distributions respectively for the $\phi$ and $\rho^0$ mesons can be obtain by converting the total rate for vector production into a mass distribution as Refs. [28, 44],

\[
\frac{d\Gamma_{\Lambda_c \to p\rho^0, \rho^0 \to \pi^+\pi^-}}{dM_{\text{inv}}} = \frac{2m_{\rho}^2}{\pi} \frac{\tilde{\Gamma}_\rho \tilde{\Gamma}_{\Lambda_c \to p\rho^0}}{(M_{\text{inv}}^2 - m_{\rho}^2)^2 + m_{\rho}^2 \tilde{\Gamma}_\rho^2},
\]
\[
\frac{d\Gamma_{\Lambda_c \to p\phi, \phi \to K^+K^-}}{dM_{\text{inv}}} = \frac{m_{\phi}^2}{\pi} \frac{\tilde{\Gamma}_{\phi} \tilde{\Gamma}_{\Lambda_c \to p\phi}}{(M_{\text{inv}}^2 - m_{\phi}^2)^2 + m_{\phi}^2 \tilde{\Gamma}_{\phi}^2},
\]

where we have considered that the $K^+K^-$ decay accounts for 1/2 of the $K\bar{K}$ decay width of the $\phi$ meson. Since $\rho^0 \to \pi^+\pi^-$ and $\phi \to K^+K^-$ are in $p$-wave, we take

\[
\tilde{\Gamma}_\rho = \Gamma_{\rho^0} \left(\frac{\sqrt{M_{\text{inv}}^2 - 4m_{\rho}^2}}{m_{\rho}^2 - 4m_{\pi}^2}\right)^3, \quad \tilde{\Gamma}_{\phi} = \Gamma_{\phi} \left(\frac{\sqrt{M_{\text{inv}}^2 - 4m_{K}^2}}{m_{\phi}^2 - 4m_{K}^2}\right)^3,
\]

and

\[
\tilde{\Gamma}_{\Lambda_c \to pV} = \Gamma_{\Lambda_c \to pV} \frac{\lambda^{1/2} \left(M_{\Lambda_c}^2, M_{V}^2, M_{p}^2\right) m_{V}}{\lambda^{1/2} \left(M_{\Lambda_c}^2, m_{V}^2, M_{p}^2\right) M_{\text{inv}}}.
\]

### III. RESULTS AND DISCUSSION

We first extract the factors $V_p$ and $V'_p$ from the branching fractions of the $\Lambda_c$ decays. Our results for the ratios of the branching fractions of the decays $\Lambda_c \to p\bar{K}^{*0}$, $\Lambda_c \to p\omega$, $\Lambda_c \to p\rho^0$ with respect to the decay $\Lambda_c \to p\phi$ are,

\[
R_{1}^{\text{th}} = \frac{B(\Lambda_c \to p\bar{K}^{*0})}{B(\Lambda_c \to p\phi)} = 21.6,
\]
\[
R_{2}^{\text{th}} = \frac{B(\Lambda_c \to p\omega)}{B(\Lambda_c \to p\phi)} = 0.640,
\]
\[
R_{3}^{\text{th}} = \frac{B(\Lambda_c \to p\rho^0)}{B(\Lambda_c \to p\phi)} = 0.636,
\]
where $R_1^{\text{th}}$ and $R_2^{\text{th}}$ are consistent with the experimental results [41],

$$R_1^{\text{exp}} = \frac{B(\Lambda_c \to p\bar{K}^0)}{B(\Lambda_c \to p\phi)} = \frac{(1.94 \pm 0.27)\%}{(1.06 \pm 0.14) \times 10^{-3}} = 18.3 \pm 3.5, \quad (29)$$

$$R_2^{\text{exp}} = \frac{B(\Lambda_c \to p\omega)}{B(\Lambda_c \to p\phi)} = \frac{(9 \pm 4) \times 10^{-4}}{(1.06 \pm 0.14) \times 10^{-3}} = 0.85 \pm 0.39. \quad (30)$$

By fitting to the branching fractions of the decays $\Lambda_c \to p\bar{K}^0$, $\Lambda_c \to p\phi$, and $\Lambda_c \to p\omega$, we can obtain the $(V_p)^2/\Gamma_{\Lambda_c} = (4.5 \pm 0.4) \times 10^3$ MeV. With this value, the branching fraction of the decay $\Lambda_c \to p\rho^0$ is estimated to be $B(\Lambda_c \to p\rho^0) = (6.3 \pm 0.6) \times 10^{-4}$.

On the other hand, in order to extract the value of the $V_p$, we calculate the branching fraction for the decay $\Lambda_c \to pK^+K^-$ in s-wave with Eq. (16),

$$B(\Lambda_c \to pK^+K^-)_{\text{th}} = \frac{f^{M_{\Lambda_c}-M_p}}{m_{\Lambda_c}^2} \frac{d\Gamma_{\Lambda_c \to pK^+K^-}}{dM_{K^+K^-}} = \frac{(V_p)^2}{\Gamma_{\Lambda_c}} \times 5.41 \times 10^{-4}. \quad (31)$$

Based on the measured branching fraction of the $B(\Lambda_c \to pK^+K^-)_{\text{non-}\phi} = (5.3 \pm 1.2) \times 10^{-4}$ [41], we can obtain $(V_p)^2/\Gamma_{\Lambda_c} = 0.980 \pm 0.222$. Then the branching fraction of the decay $\Lambda_c \to p\pi^+\pi^-$ in s-wave can be given as,

$$B(\Lambda_c \to p(\pi^+\pi^-))_{\text{th}} = \frac{f^{M_{\Lambda_c}-M_p}}{m_{\Lambda_c}^2} \frac{d\Gamma_{\Lambda_c \to p\pi^+\pi^-}}{dM_{\pi^+\pi^-}} = \frac{(V_p)^2}{\Gamma_{\Lambda_c}} \times 2.066 \times 10^{-3} = (2.02 \pm 0.46) \times 10^{-3}. \quad (32)$$

With the obtained values of $(V_p)^2/\Gamma_{\Lambda_c}$ and $(V_p)^2/\Gamma_{\Lambda_c}$, we show the $K^+K^-$ mass distribution for the decay $\Lambda_c \to pK^+K^-$ in Fig. 5 where we can see that the peak of the $\phi$ is clear. In addition, there is an enhancement structure close to the $K^+K^-$ threshold, which is the reflection of the resonance $f_0(980)$. Although the BESIII Collaboration has reported the $K^+K^-$ mass distribution, it is difficult to confirm this enhancement structure because of the large uncertainties of the experimental data [18]. It is worth to mention that, in the $K^+K^-$ mass distribution of the decay $\chi_{c1} \to p\bar{p}K^+K^-$ measured by the BESIII Collaboration [45], one can find an enhancement structure close to the threshold, which can be associated to the resonance $f_0(980)$. The similar structure can also be found in the decay $D_s^+ \to K^+K^-\pi^+$ measured by the BABAR Collaboration [46].

![FIG. 5: The $K^+K$ invariant mass distribution of the $\Lambda_c \to pK^+K^-$ decay. The green dotted curve stands for the contribution from the meson-meson interaction in s-wave, the blue dashed curve corresponds to the results for the intermediate vector $\phi$, and the red solid line shows the total contributions.](image-url)

The theoretical results for the $\pi^+\pi^-$ invariant mass distributions of the decay $\Lambda_c \to p\pi^+\pi^-$ are shown in Fig. 6 from where one can see a clear peak around 770 MeV, corresponding to the vector meson $\rho$, and a broad peak.
around 500 MeV, which can be associated to the scalar meson $f_0(500)$, dynamically generated from the meson-meson interactions in $s$-wave. In addition, there is a narrow sharp around 980 MeV, which can be associated to the scalar state $f_0(980)$. For comparison, the experimental data [18] has been adjusted to the strength of our theoretical calculations at the peak of $\rho^0$. We can see that the broad peak for $f_0(500)$, the peak for $\rho^0$, and a narrow sharp for $f_0(980)$ of our results are compatible with the BESIII measurement. Note that the BESIII data include also the background in the sideband region [18].

![Graph](image)

**FIG. 6:** The $\pi^+\pi^-$ invariant mass distributions of the $\Lambda_c \rightarrow p\pi^+\pi^-$ decay compared with the experimental data from Ref. [18]. The green dotted curve stands for the contribution from the meson-meson interaction in $s$-wave, the blue dashed curve corresponds to the results for the intermediate vector $\rho$, and the red solid line shows the total contributions.

**IV. CONCLUSIONS**

In this work, we have studied the decays $\Lambda_c \rightarrow pK^+K^-$ and $\Lambda_c \rightarrow p\pi^+\pi^-$, by taking into account contributions of the intermediate vector mesons, and the $s$-wave meson-meson interactions within the chiral unitary approach, where the $f_0(500)$ and $f_0(980)$ resonances are dynamically generated.

The $K^+K^-$ and $\pi^+\pi^-$ invariant mass distributions for these two decays are calculated. In the $K^+K^-$ mass distribution, one can find a narrow peak for the $\phi$, and an enhancement structure close to the $K^+K^-$ threshold, which should be the reflection of the $f_0(980)$ resonance. Although there is a hint of the enhancement structure in the Belle measurement, the signal of the $f_0(980)$ is still needed to be confirmed with more accurate measurements in future. For the $\Lambda_c \rightarrow p\pi^+\pi^-$ mass distribution, in addition to the broad peak of the $\rho^0$, one can find a bump structure around 500 MeV for the $f_0(500)$, and a narrow sharp around 980 MeV for the $f_0(980)$, in agreement with the BESIII measurement. We encourage our experimental colleagues to measure these two decays, which can be used to test the molecular nature of the scalar resonances $f_0(500)$ and $f_0(980)$.

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