RESONANT TUNNELING AND CHARGING EFFECTS, A PATH INTEGRAL APPROACH

JÜRGEN KÖNIC1, HERBERT SCHOELLER1,2, GERD SCHÖN1 AND ROSARIO FAZIO1,3

1 Institut für Theoretische Festkörperphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany
2 Department of Physics, Simon Fraser University, Burnaby, B.C., V5A 1S6, Canada
3 Istituto di Fisica, Facolta di Ingegneria, 95129 Catania, Italy

Abstract. Electron tunneling through small metallic islands with low capacitance is studied. The large charging energy in these systems is responsible for nonperturbative Coulomb blockade effects. We further consider the effect of electron interactions in the electrodes. In junctions with high resistance compared to the quantum resistance transport can be described by sequential tunneling. If the resistance is lower, quantum fluctuations, higher order coherent processes, and eventually resonant tunneling become important. We present a path integral real-time approach, which allows a systematic diagrammatic classification of these processes. An important process is “inelastic resonant tunneling”, where different electrons tunnel coherently between the electrodes and the island. Physical quantities like the current and the average charge on the island can be deduced. We find a strong renormalization of the system parameters and, in addition, a finite lifetime broadening. It results in a pronounced broadening and smearing of the Coulomb oscillations of the conductance. These effects are important in an experimentally accessible range of temperatures. The electron interaction in the electrodes is modeled by a Luttinger liquid. It leads to non-analytic kernels in the effective action. The diagrammatic expansions can be performed also in this case, resulting in power-law current-voltage characteristics.
1. Introduction

Electron transport through mesoscopic metallic islands coupled to electrodes has been the subject of extensive research [1-3]. The small size and capacitance of these systems imply a strong Coulomb interaction energy $E_{ch}$, which gives rise to a variety of single-electron phenomena. At low temperatures tunneling can be suppressed (Coulomb blockade). The charging energy can be tuned by gate voltages. When the energy difference between adjacent charge states $E_{ch}(n \pm 1) - E_{ch}(n)$ is lower than the temperature $kT$ or the bias voltage $eV$, a current flows through the system. As a consequence, the conductance shows a series of peaks as a function of the gate voltage $V_G$ (linear response) and further structure at larger bias voltage $V$ (nonlinear response). The classical description is sufficient as long as the resistance $R_T$ of a single barrier is much higher than the quantum resistance $R_K = h/e^2$, i.e. for $\alpha_0 \ll 1$, where $\alpha_0 \equiv R_K/(4\pi^2 R_T)$. In this regime, transport occurs in sequences of uncorrelated tunneling processes. The rates can be obtained in lowest order perturbation theory in the tunneling amplitudes. They enter a master equation [4-8], from which the probabilities for different charge states $n$ and the currents can be calculated.

For very low temperatures or when the dimensionless conductance $\alpha_0$ is not small the classical description breaks down. Quantum fluctuations and higher order coherent tunneling processes become important [9-15]. This includes cotunneling, where two electrons tunnel coherently in different junctions, thus avoiding the Coulomb blockade. Furthermore, resonant tunneling, where electrons tunnel coherently back and forth between the island and the electrodes, play a role. A description is called for, which allows a systematic classification of all these processes. In comparison to the well known phenomenon of resonant tunneling of single electrons one encounters here two complications. One lies in the fact that the metallic system contains many electrons. With overwhelming probability different electron states are involved in the different transitions of the coherent process. The second arises since the Coulomb interaction is strong and, hence, cannot be accounted for in perturbation theory.

In the present article we develop a systematic diagrammatic technique to identify the processes of sequential tunneling, inelastic cotunneling and resonant tunneling. We study the time evolution of the density matrix. In an earlier paper [16] we have formulated the problem, after a separation of charge and fermionic degrees of freedom, in a many-body expansion technique. Here we reformulate it in a real-time path-integral representation. The latter is well known from the studies of dissipation in quantum mechanics. Caldeira and Leggett [18], following methods pioneered by Feynman and Vernon [17], studied the problem of Ohmic dissipation. Dissipation associ-
Resonant Tunneling and Charging Effects, a Path Integral Approach

An essential step in the present work is a transformation of the Feynman-Vernon functional for electron tunneling from a phase (coordinate) representation to a charge (momentum) representation [5]. The path-integral method is well-suited to account for the strong correlations due to the Coulomb interaction in a perturbative or nonperturbative analysis in the tunneling. In contrast, usual Green’s function techniques cannot be used since the Coulomb interaction has to be retained in the unperturbed Hamiltonian. The same problem arises in the context of local, strongly correlated Fermi systems like the Kondo and Anderson model [20, 21]. For these systems diagrammatic techniques have been derived by Barnes [22] from a slave-boson description. Rammer [23] developed a graphical density-matrix description for the dynamics of a particle coupled to a heat bath. The problem can also be formulated in terms of Liouville operators [24].

As examples we will study the electron box and single electron transistors. The box consists of a metallic island coupled via a tunnel junction with capacitance $C_J$ to an electrode. It is further coupled capacitively ($C_G$) to a voltage source $V_G$. The charging energy is $E_{ch}(n) = (n e - Q_G)^2/(2C)$, where $C = C_J + C_G$ denotes the total capacitance of the island. It depends on the number of excess electrons $n$ on the island and on the continuously varying external charge $Q_G = C_G V_G$. The single electron transistor consists of a metallic island which is coupled by two tunnel junctions to two electrodes (see Fig. 1). Here a transport voltage $V = V_L - V_R$ drives a current. The island is further coupled capacitively to a gate voltage $V_G$. The charging energy of this system depends again on the number of electrons $n$ on the island, $E_{ch}(n) = (n e - Q_G)^2/(2C)$. Here $C = C_L + C_R + C_G$ is the sum of the two junctions and the gate capacitances and $Q_G = C_G V_G + C_L V_L + C_R V_R$. In tunneling processes we further have to account for the work done by the voltage source (see below). In both examples the total island capacitance defines the scale for the charging energy $E_C \equiv e^2/2C$.

The tunneling is described by tunneling Hamiltonians. We consider “wide” metallic junctions, which implies that there are many transverse channels. As a result “inelastic” higher order tunneling processes, involving different electron states for each step, dominate over those higher order processes which involve the same state repeatedly. Accordingly, in the effective action description presented below only simple loop diagrams are retained.

At low temperature the electron number $n(Q_G)$ in the electron box takes the value which minimizes the charging energy. It increases in steps of unity when $Q_G$ is increased. Tunneling processes in the transistor are only possible in lowest order perturbation theory if the electrochemical potential of one electrode is high enough to allow one electron to enter the island, say $eV_L > E_{ch}(n+1) - E_{ch}(n)$, while the electrochemical potential of
the other electrode allows the next tunneling process to that electrode, i.e. 
\( E_{ch}(n+1) - E_{ch}(n) > eV_R \). Within the window set by these two conditions 
the current is \( 4R_I I = V - 4(Q_G - (n + 1/2)e^2)/(C^2V) \). This implies that 
the conductance \( G(Q_G) = \partial I/\partial V \) shows, as a function of \( Q_G \), an e-periodic 
series of structures of width \( CV \) with, at \( T = 0 \), vertical steps at its limits. 
At finite temperature the average value \( \langle n(Q_G) \rangle \) and the steps in the linear 
and nonlinear conductance are washed out.

Quantum fluctuations further wash out these steps, even at zero tem-
perature. We will describe these processes diagrammatically. In the most 
interesting case we can re-sum the diagrams and obtain closed expressions 
for the stationary density matrix and the spectral density describing the 
charge excitations of the system. Our main findings are: (i) After a resum-
mation of the leading logarithmic terms in \( \alpha_0 \ln (E_C/|\omega|) \) we find a renor-
malization of the energy and dimensionless conductance. (ii) The coherent 
processes where electrons tunnel an arbitrary number of times between the 
leads and the island (resonant tunneling) give rise to a broadening of the 
charge state levels. Both effects need to be retained in a conserving theory 
which obeys sum rules and current conservation. Both effects are observable 
in a real experiment at accessible temperatures.

In order to study the effect of electron interactions we consider in the 
last section a model where the electrodes are assumed to be Luttinger 
liquids [25]. Due to the interactions the single particle density of states has 
a power-law asymptotic at low energies, and Fermi liquid theory does not 
apply anymore. For a quantum wire with an arbitrarily small barrier this 
leads to a suppression of transport at low energies [26, 27]. We find that 
the effective action in the Luttinger liquid - normal metal junction has the 
same form as in the metallic case. However, the kernels are modified, taking 
a power law form with exponents depending on the interaction strength 
in the Luttinger liquid. The diagrammatic expansions can be performed
Resonant Tunneling and Charging Effects, a Path Integral Approach

2. Real time evolution of the density matrix

The description of the single electron transistor is based on the Hamiltonian

\[ H = H_L + H_R + H_I + H_{ch} + H_t. \]  

(1)

Here \( H_L = \sum_{k, \sigma} (\epsilon_k + eV_L)c_{k\sigma}^\dagger c_{k\sigma} \) describes noninteracting electrons in the left lead, with similar expressions for the island (with states denoted by \( q \)) and the right lead. The electrodes are treated as reservoirs, i.e. they remain in thermal equilibrium. The Coulomb interaction \( H_{ch} \) is assumed to depend only on the total charge on the island, as expressed by the charging energy \( E_{ch}(n) \) introduced above. Charge transfer is described by the standard tunneling Hamiltonian, e.g. the tunneling in the left junction by

\[ H_{t,L} = \sum_{k,q,\sigma} T_{kq} c_{k\sigma}^\dagger c_{q\sigma} + h.c. \]  

(2)

The tunnel matrix elements \( T_{kq} \) are considered independent of the states \( k \) and \( q \). They can be related to the tunnel restitances \( R_{L/R} \) of the junctions, \( 1/R_{L/R} = (4\pi e^2/\hbar)N_{L/R}(0)N_I(0)|T_{L/R}|^2 \), where \( N_r(0) \) are the densities of states of the the electrodes and the island, \( r = L, R, I \). We further assume that the junctions, although small, still accommodate many transverse channels. From a comparison of Andreev reflection and single electron tunneling in small normal-superconducting junctions we have concluded that the number of channels is \( N_{ch} \geq 10^3 \).

The quantum mechanical many-body problem of electrons coupled by Coulomb interactions can be reformulated in a path integral representation. In order to handle the interaction one performs a Hubbard-Stratonovich transformation. It introduces collective variables \( \varphi_r \), which are the quantum mechanical conjugates of the charges in the electrodes or island. The capacitive interaction between electrons is replaced in this way by an interaction of the electrons with the fields \( V_r \equiv \hbar \dot{\varphi}_r/e \). After this stage the electronic degrees of freedom can be traced out. The next step involves an expansion in the electron propagators. Since we consider wide junctions only simple loops need to be retained [28]. Their iteration introduces in each order a factor \( N_{ch} \), hence they dominate over more complicated higher order loops. After the trace has been performed the system can no longer be described by a Hamiltonian. Instead we deal with a reduced density matrix \( \rho(\{\varphi_{r,1}\}, \{\varphi_{r,2}\}) \). It depends on an effective action, which is expressed in the phases \( \varphi_{r,\sigma} \) corresponding to the forward and backward propagator
The structure of the theory is familiar from Refs. [17, 18], where a quantum system coupled to a harmonic oscillator bath has been considered. The analogous model describing the effect of electron tunneling has been presented in Refs. [19, 5]. We, therefore, do not present the derivation of the steps here, rather we quote only the result.

For transparency we describe the formalism for a single junction. The time evolution of its reduced density matrix is given by ($\hbar = k = 1$)

$$\rho(t_f; \varphi_{1f}, \varphi_{2f}) = \int d\varphi_{1i} d\varphi_{2i} \int_{\varphi_{1i}}^{\varphi_{1f}} D\varphi_{\infty}(\mathbb{L}) \int_{\varphi_{c}}^{\varphi_{e}} D\varphi_{c}(\mathbb{L}) e^{iS[\varphi_{1}, \varphi_{2}]} \rho(t_i; \varphi_{1i}, \varphi_{2i}) .$$  \hspace{1cm} (3)

Here $\varphi_{1}$ and $\varphi_{2}$ refer to the forward and backward time evolution. The effective action is given by

$$S_{[\varphi_{1}, \varphi_{2}]} = S_{\text{ch}}[\varphi_{1}] - S_{\text{ch}}[\varphi_{2}] + S_{l}[\varphi_{1}, \varphi_{2}] , \hspace{1cm} (4)$$

The first term represents the charging energy $S_{\text{ch}}[\varphi] = \int_{t_i}^{t_f} dt \frac{C}{2} (\dot{\varphi}/e)^2$. Electron tunneling is described by [19, 5]

$$S_{l}[\varphi_{1}, \varphi_{2}] = 4\pi i \sum_{\sigma, \sigma'} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \alpha_{\sigma, \sigma'}(t - t') \cos[\varphi_{\sigma}(t) - \varphi_{\sigma'}(t')] , \hspace{1cm} (5)$$

where $\alpha_{\sigma, \sigma'}$ are given in Fourier space by

$$\alpha_{\sigma, 1}(\omega) = (-1)^{\sigma+1} \alpha_-^{\sigma}(\omega) \hspace{1cm}, \hspace{1cm} \alpha_{\sigma, 2}(\omega) = (-1)^{\sigma} \alpha_+^{\sigma}(\omega) \hspace{1cm} (6)$$

$$\text{and} \hspace{1cm} \alpha_+^{\sigma}(\omega) = \pm \frac{1}{\alpha_0 \omega} \exp(\pm \beta \omega) - 1 . \hspace{1cm} (7)$$

The tunneling term couples the forward and backward propagators. This arises in the step where the microscopic degrees of freedom are eliminated.

An important step for a systematic description of tunneling processes is the change from the phase to a charge representation, accomplished by

$$\rho(t_f; n_{1f}, n_{2f}) = \sum_{n_i} \rho(t_i; n_i, n_i) \int d\varphi_{1f} d\varphi_{2f} d\varphi_{1i} d\varphi_{2i} \int_{\varphi_{1i}}^{\varphi_{1f}} D\varphi_{\infty}(\mathbb{L}) \int_{\varphi_{c}}^{\varphi_{e}} D\varphi_{c}(\mathbb{L}) \int D\varphi_{\infty}(\mathbb{L}) \int D\varphi_{c}(\mathbb{L}) \times \exp \left( -in_1 \dot{\varphi}_{1i} + in_1 \dot{\varphi}_{1f} - iS_{\text{ch}}[n_1] + i \int dt n_1 \dot{\varphi}_{1i} \right) \times \exp \left( +in_2 \dot{\varphi}_{2i} - in_2 \dot{\varphi}_{2f} + iS_{\text{ch}}[n_2] - i \int dt n_2 \dot{\varphi}_{2i} \right) \times \exp \left\{ 4\pi i \sum_{\sigma, \sigma' = 1, 2} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \alpha_{\sigma, \sigma'}^{\sigma}(t - t') \cos[\varphi_{\sigma}(t) - \varphi_{\sigma'}(t')] \right\} . \hspace{1cm} (8)$$
Here $S_{ch}[n] = \int_{t_i}^{t_f} dt \frac{1}{2} (ne)^2$. In systems with discrete charges the integrations over $\varphi$ include a summation over winding numbers [5]. We further assume that the initial density matrix is diagonal in the charges. In the absence of tunneling the bare forward and backward propagators involve only the charging energy $\exp(\pm i S_{ch}[n])$. We expand the tunneling part of the action $\exp(iS_t)$ and integrate over $\varphi$. Each of the exponentials $\exp[\pm i \varphi_\sigma(t)]$ causes a change of the number of electrons on the island by $\pm e$ at time $t$ on the forward or backward branch, $\sigma = 1$ or 2, respectively. These changes occur in pairs and are connected by a line representing $\alpha^{\sigma,\sigma'}(t-t')$. The two correlated transitions can occur on the same branch $\sigma = \sigma'$, or on different branches.

The latter are of particular interest. Imagine we started in a state with $n$ charges $\rho(t_i) = |n\rangle\langle n|$. Then each transition, described by $\exp(i[\varphi_1(t) - \varphi_2(t')])$ changes the charge on both branches by $+e$ and the density matrix acquires a finite value also for states $|(n+1)e\rangle$. After integrating over the two times $t$ and $t'$, limited by $t_i \leq t' \leq t \leq t_f$, we find

$$\langle n+1\rho(t_f)|n+1 \rangle = (t_f - t_i)2\pi \alpha^+(\Delta E_{ch}(n)),$$

where $\Delta E_{ch}(n) = E_{ch}(n+1) - E_{ch}(n)$. Obviously we can interpret the coefficient of the time difference as transition rate, and indeed we reproduce the well-known single electron tunneling rate.

In the following we consider the single electron transistor. Each capacity $C_r$ introduces a charging term $S_{ch}$, where $r = L, R, G$ denotes the reservoirs on the left, right and the gate, and each tunnel junction introduces a term $S_t$, all depending on the appropriate phase difference $\varphi_I - \varphi_r$. The phases in the reservoirs $r$ are assumed to be well-defined quantities $\varphi_r = eV_r t$ without fluctuations. The contribution of the charging energy which is related to the work done by the transport voltage can be accounted for in the tunneling lines by $\alpha^+(\omega - eV_r)$, and the charging energy becomes $E_{ch}(n) = (ne - Q_G)^2/(2C)$.

Each term of the expansion can be visualized by a diagram. In Fig. (2) we show several important processes. There is a closed time-path consisting of two horizontal lines joined at $t_f$. They correspond to the forward propagator from $t_i$ to $t_f$ (upper line) and the backward propagator from $t_f$ to $t_i$ (lower line). Along the time-path we arrange vertices. They are connected in pairs by (dashed) tunneling lines, either within one propagator or between the two propagators. We draw all topological different diagrams with directed tunneling line and evaluate the diagrams according to the following rules:

1. Assign charge states $n$ and the corresponding charging energy to each element of the propagators. Elements of the forward (backward) propagator between $t$ and $t'<t$ carry factors $\exp[\mp i E_{ch}(n)(t-t')]$. 


Figure 2. Example of a diagram showing various tunneling processes: on the left sequential tunneling in the left and right junctions, then a term which preserves the norm, next a cotunneling process, and on the right resonant tunneling in the left junction.

2. Each vertex represents an exponential \( \exp[\pm i \varphi_\sigma(t)] \) of the tunneling contribution to the action. It changes the charge from \( n \) to \( n \pm 1 \).

3. Pairs of vertices are connected by a directed tunneling line \( \alpha_+(t - t') [\alpha_-(t - t')] \) for the electrodes \( r = L, R \), if the line of is running backward [forward] with respect to the closed time-path. The charge increases (decreases) along the time-path by 1 if a tunneling line comes in (goes out).

4. Each diagram carries an prefactor \((-i)^M(-1)^m\), where \( M \) is the total number of vertices and \( m \) their number on the backward propagator.

5. Integrate over the internal times and sum over the reservoirs.

In order to calculate stationary transport properties it is convenient to change to an energy representation. This is achieved by the following transformation. In each diagram we order the times from left to right and label them by \( t_j \), irrespective on which branch they are. We further set \( t_i = -\infty \) and \( t_f = 0 \). We then encounter integrals of the type

\[
\int_{-\infty}^{0} dt_1 \int_{t_1}^{0} dt_2 \ldots \int_{t_{M-1}}^{0} dt_M e^{\eta t_1} e^{-i \Delta E_1(t_2 - t_1)} e^{-i \Delta E_2(t_3 - t_2)} \ldots e^{-i \Delta E_M(-t_M)}
\]

\[
= (-i)^M \frac{1}{\Delta E_1 - i\eta} \cdot \frac{1}{\Delta E_2 - i\eta} \ldots \frac{1}{\Delta E_M - i\eta}.
\]

Here \( \Delta E_j \) is the difference of the energies of the upper and lower propagator and – if present – the frequency of the tunneling line within the segment limited by \( t_j \) and \( t_{j+1} \). The convergence factor \( e^{\eta t_1} (\eta \to 0^+) \) is related to an adiabatic switching of the tunneling term \( H_t \). The rules in energy representation read:

1. Draw all topological different diagrams. These are the same as in time space. In addition to the charging energy assigned to the propagators we assign a frequency \( \omega \) to each tunneling line.

2. For each segment derived from \( t_j \leq t \leq t_{j+1} \) with \( j \geq 1 \) we assign a resolvent \( \frac{1}{\Delta E_j - i\eta} \) where \( \Delta E_j \) is the difference of the energies of the
Resonant Tunneling and Charging Effects, a Path Integral Approach

forward and backward propagator, plus the sum of the frequencies of the tunneling lines in the given segment. The latter have to be taken positive for lines from the left to the right and negative for lines from the right to the left.

3. The prefactor is given by \((-1)^{m+l}\), where \(m\) is the total number of vertices on the backward propagator and \(l\) the total number of resolvents.

4. For each coupling of vertices we write \(\alpha^+ (\omega) [\alpha^- (\omega)]\), if the tunneling line of reservoir \(r\) is going backward (forward) with respect to the closed time-path.

5. Integrate over the frequencies of tunneling lines and sum over the reservoirs.

3. Master equation and stationary probabilities

We denote the sum of all diagrams by \(\Pi_{n_1,n_1'}^{n_2,n_2'}\), where the indices indicate the left and right charge states on the two branches, \(\sigma = 1,2\). They can be expressed by an irreducible self-energy part \(\Sigma_{n_2,n_2'}^{n_1,n_1'}\), defined as the sum of all diagrams where any vertical line cutting through them crosses at least one tunneling line. The time evolution of the density matrix can be expressed as an iteration in the style of a Dyson equation, illustrated in Fig.(3)

\[
\Pi_{n_2,n_2'}^{n_1,n_1'} = \Pi^{(0)}_{n_2,n_2} \delta_{n_1,n_1'} \delta_{n_2,n_2'} + \sum_{n_2',n_2''} \Pi^{(0)}_{n_2,n_2'} \Sigma_{n_2',n_2''}^{n_1,n_1'} \Pi^{(0)}_{n_2'',n_2'} \Pi^{(0)}_{n_1,n_1'}. 
\] (10)

Here \(\Pi^{(0)}\) is the free propagator.

We start from an density matrix which is diagonal, \(\rho(t_i,n_1,n_2) = P^{(0)}(n_1) \delta_{n_1,n_2}\). This means that the density matrix remains diagonal except during transitions described by \(\Sigma\). Hence, we consider \(\Sigma_{n,n'} = \Sigma_{n,n'}^{n,n'}\). We identify the solution of Eq. (10) – after multiplication with \(P^{(0)}(n)\) from the left – as the stationary distribution function \(\sum_n P^{(0)}(n) \Pi_{n,n'}^{n,n'} = P^{st}(n')\).

Figure 3. The iteration of processes for \(\Pi\), describing the time evolution of the density matrix.
Hence Eq. (10) reduces to

\[
P_{st}(n') = P^{(0)}(n') + \sum_{n''} P_{st}(n'') \Pi_{n'',n'}^{(0)}.
\]  

(11)

The last term in Eq. (11), \(\Pi^{(0)}\), describes a propagation in a diagonal state (i.e. equal energies on the forward and backward propagator) which does not contain a tunneling line. According to the rules in the energy representation this introduces a factor \(1/i\eta\), and in the limit \(\eta \to 0^+\) we find \(\sum_{n'} P_{st}(n') \Pi_{n',n}^{(0)} = 0\). By attaching the rightmost vertex of each diagram \(\Sigma\) to the upper and lower propagator we can show that \(\sum_{n'} \Sigma_{n,n'} = 0\), which allows us to rewrite Eq.(3) also in the form

\[
0 = -P_{st}(n) \sum_{n' \neq n} \Sigma_{n,n'} + \sum_{n' \neq n} P_{st}(n') \Sigma_{n',n}.
\]

(12)

We recover the structure of a stationary master equation with transition rates given by \(\Sigma_{n',n}\). In general the irreducible self-energy \(\Sigma\) yields the rate of all possible correlated tunneling processes. We, furthermore, see that the stationary solution \(P_{st}(n)\) is independent of the initial distribution \(P^{(0)}(n)\).

For illustration we evaluate now all diagrams which contain no overlapping tunneling lines. After each transition the system is in a diagonal state of the density matrix and a probability can be assigned. Examples are visualized on the left hand side of Fig. (2). This means we include those processes which are also described by the master equation with rates obtained in lowest order perturbation theory. In the limit considered the irreducible part \(\Sigma^{(1)}\) consists of two classes of diagrams, those where a tunneling line connects two vertices on one propagator (which implies that \(n'_i = n_i\)) and those where it connects the two propagators (\(n'_i = n_i \pm 1\), with \(n_1 - n_2 = n'_1 - n'_2\)) (see Fig.(4)). According to the rules the rates are

\[
\Sigma_{n,n}^{(1)} = 2\pi i \alpha^\pm (\pm \Delta E_{ch}^\pm) , \quad \Sigma_{n,n}^{(1)} = -2\pi i \sum_{\pm} \alpha^\pm (\pm \Delta E_{ch}^\pm)
\]

(13)

where \(\Delta E_{ch}^\pm = E_{ch}(n \pm 1) - E_{ch}(n)\).

4. Higher order tunneling processes

4.1. COTUNNELING

In situations where single electron tunneling is suppressed by Coulomb blockade the lowest order contribution to the current arises due to cotunneling. It is described by a diagram with tunneling processes in the left and in the right junction, where the corresponding lines \(\alpha_L(t_L - t'_L)\) and
Figure 4. Representation of the self-energy $\Sigma^{(1)}$, defined to contain no overlapping tunneling lines. Only one representative of each class is shown, the remaining ones are obtained by changing the direction of the arrows and exchanging the position on the forward and backward propagator.

$\alpha_R(t_R - t'_R)$ overlap in time. This means there is no intermediate time where the density matrix is diagonal, which would describe sequential tunneling. An example is shown in Fig. (2). Performing the integrations we find for the rate

$$
\Sigma_{\text{cot}} = \frac{\hbar}{2\pi e^4 R_L R_R} \int d\epsilon_1 d\epsilon_2 d\epsilon_3 d\epsilon_4 f(\epsilon_1)[1 - f(\epsilon_2)]f(\epsilon_3)[1 - f(\epsilon_4)]
$$

$$
\times \left( \frac{1}{\epsilon_2 - \epsilon_1 + E_1} + \frac{1}{\epsilon_4 - \epsilon_3 + E_2} \right)^2 \delta(eV + \epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4), \quad (14)
$$

plus a divergent term. This divergence is canceled if we take into account consistently all processes of the same order. This includes a process where the island has already the charge $(n \pm 1)e$ and one tunneling event brings it back to $ne$. The corresponding terms are products of a single electron tunneling rate of one junction and the stationary probability $P_{n \pm 1}^{st}$ in first order in $\alpha_0$. Adding all these terms we obtain the result given above.

4.2. RESONANT TUNNELING

The perturbative approach breaks down at low temperatures or for large values for the dimensionless conductance $\alpha_0$. Specifically we will show that the classical master equation is valid only for $\alpha_0 \ln \left( \frac{E_C}{kT} \right) \ll 1$, whereas for larger values resonant tunneling processes become important.

To proceed we have to find a systematic criterion which diagrams should be retained and summed. For this we note that during a tunneling process the reservoirs contain an electron excitation. Two parallel tunneling lines imply two such excitations. Our criterion is that we take into account only those matrix elements of the total density matrix, i.e. reservoirs plus charge states, which differ at most by two excitations in the leads or (equivalently) in the island. Graphically this means that any vertical line will cut at most two tunneling lines.
We will concentrate here on situations where only two charge states with \( n = 0, 1 \) need to be considered. This is sufficient when the energy difference of the two charge states \( \Delta_0 = E_{ch}(1) - E_{ch}(0) \) and the bias voltage \( eV = eV_L - eV_R \) are small compared to \( E_C \), which is the energy associated with transitions to higher states. The combination of the two restrictions, diagrams with at most two parallel tunneling lines in the two-state problem, implies that no diagram contains crossing tunneling lines. In this case we can evaluate the irreducible self-energy analytically.

In order to sum all diagrams which may contain up to two parallel tunneling lines we introduce a diagram labeled by \( \phi_{n_1, n_2}^{n_1, n_2}(r, \omega) \) (see Fig.(5)). It has an open tunneling line associated with tunneling in the junction \( r \).

![Figure 5](image)

**Figure 5.** a) Self-consistent equation for \( \phi_n^\tau(\omega) \). A summation over the electrodes \( r \) and the direction of the tunneling lines is implied. b) Representation of the self-energy \( \Sigma_{n_1} \) within our approximation.

Carrying the frequency \( \omega \). Consequently it remains in an off-diagonal state at one side. For the two state problem we need only

\[
\phi_n^\tau(\omega) \equiv \phi_{n, 0}^{n, 1}(r, \omega)
\]

with \( n = 0, 1 \), for which we can formulate the iteration depicted diagrammatically in Fig.(5). It yields

\[
\phi_n^\tau(\omega) = \pi(\omega) \left[ \alpha_r^+(\omega) \delta_{n, 0} - \alpha_r^-(\omega) \delta_{n, 1} + \alpha_r(\omega) \int d\omega' \sum_{r'} \frac{\phi_{n'}^r(\omega')^*}{\omega' - \omega - i\eta} \right].
\]

Here we encounter the propagator \( \pi(\omega) \equiv \Pi^{(1), 1}_{\delta, 0}(\omega) \). It is given by the propagator \( \Pi \), restricted to first order in the tunneling, and we start and end in an off-diagonal state. Furthermore, since the parallel tunneling line carries a frequency \( \omega \) the energies of the upper and lower lines are effectively
shifted relative to one another. Notice that due to the restriction to a two state problem there are no diagrams contained in \( \Pi_{1,0,0}^{(1,1)}(\omega) \) where a tunneling line connects the upper and lower propagator. We can express it by the first order self-energy \( \sigma(\omega) \equiv \Sigma_{1,0,0}^{(1,1)}(\omega) \), which is the off-diagonal version of the expression known from the first order discussion, with the added complication of a parallel tunneling line with frequency \( \omega \).

The irreducible self-energy \( \Sigma \) is obtained from \( \phi(\omega) \) by connecting the tunneling line with the appropriate direction to the upper and the lower propagator and adding both contributions (see Fig. (5)). We get for \( n = 0, 1 \)

\[
\Sigma_{n,1} = -2i \text{Im} \int d\omega \sum_r \phi_n^*(\omega),
\]

while \( \Sigma_{n,0} \) follows from the rule \( \Sigma_{n,0} + \Sigma_{n,1} = 0 \).

Applying our rules for the calculation of the diagrams we find

\[
\pi(\omega) = \frac{1}{\omega - \Delta_0 - \sigma(\omega)}, \quad \sigma(\omega) = -\int d\omega' \frac{\alpha(\omega')}{\omega' - \omega - i\eta}.
\]

Here and for the following we introduce the notations \( \alpha(\omega) = \alpha^+(\omega) + \alpha^-(\omega) \), \( \alpha^\pm(\omega) = \sum_r \alpha^\pm_r(\omega) \), \( \alpha_r(\omega) = \alpha^+_r(\omega) + \alpha^-_r(\omega) \) and \( \alpha_0 = \sum_r \alpha^r_0 \).

Notice that \( \alpha^\pm_r(\omega) = \alpha_r(\omega) f(\pm(\omega - eV_r)) \) where \( f(\omega - eV_r) = f_r(\omega) = 1/\left[ \exp(\beta(\omega - eV_r)) + 1 \right] \) is the Fermi function of reservoir \( r \).

We solve the integral equations (16) and obtain

\[
\Sigma_{0,1} = -\Sigma_{0,0} = 2\pi i \frac{\lambda_+}{\lambda}, \quad \Sigma_{1,0} = -\Sigma_{1,1} = 2\pi i \frac{\lambda_-}{\lambda}.
\]

with

\[
\lambda_+ = \int d\omega |\alpha^+(\omega)|^2 \pi(\omega)|^2, \quad \lambda = \int d\omega \pi(\omega)|^2.
\]

Inserting these quantities in the kinetic equation (12) we arrive at the stationary probabilities \( P^n_0 = \lambda_- \) and \( P^1_1 = \lambda_+ \). Both probabilities are positive and normalized \( \lambda_+ + \lambda_- = 1 \).

4.3. THE CURRENT

The expression for the current at time \( t \) in the junction \( r \) can be written as

\[
I_r(t) = -2ie\langle \int_{-\infty}^{t} dt' \sum_\sigma \alpha_1^{1,\sigma}(t - t') \sin[\varphi_{r,1}(t) - \varphi_{r,\sigma}(t')] \rangle,
\]

where the expectation value is taken with the reduced density matrix discussed above, where the larger of the times is \( t = t_f \). If we express the expectation value in the charge representation we see that the sin-functions
in Eq.(21) describe the transfer of charges at times $t_f$ and $t'$. We, therefore, have to evaluate the two real-time correlation functions describing charge transfer processes at times $t$ and $t'$

$$C^>(t, t') = -i(e^{-i\varphi(t)} e^{i\varphi(t')}) , \quad C^<(t, t') = i(e^{i\varphi(t')} e^{-i\varphi(t)}).$$

(22)

In the stationary limit the current can be expressed by

$$I_{st} = -ie \int d\omega \left\{ \alpha_r^{+}(\omega)C^>(\omega) + \alpha_r^-C^<(\omega) \right\}.$$  

(23)

It is further convenient to introduce a spectral density for charge excitations

$$A(\omega) = \frac{1}{2\pi i} [C^<(\omega) - C^>(\omega)].$$

(24)

The correlation functions and spectral density can be evaluated in the approximation which we have used before,

$$C^{(>)}(\omega) = (+(-)2\pi i \sum_r \alpha_r(\omega)f(+(-)[\omega - eV_r])|\varpi(\omega)|^2$$  

(25)

and $$A(\omega) = \alpha(\omega)|\varpi(\omega)|^2.$$  

(26)

The current can then be written as

$$I_{st} = \frac{e}{\hbar}4\pi^2 \int d\omega \sum_{r'} \frac{\alpha_{r'}(\omega)\alpha_r(\omega)}{\alpha(\omega)} A(\omega)[f_{r'}(\omega) - f_r(\omega)].$$

(27)

These results satisfy the conservation laws and sum rules. The current is conserved, $\sum_r I_{st} = 0$, and vanishes in equilibrium when $V_r = 0$. The spectral density is normalized $\int d\omega A(\omega) = 1$, and the usual relationships between the correlation functions and the spectral density in equilibrium are reproduced. The classical result can be recovered if we use the lowest order approximation in $\alpha_0$ for the spectral density $A^{(0)}(\omega) = \delta(\omega - \Delta_0)$. We conclude with the observation that quantum fluctuations yield energy renormalization and broadening effects, which manifest themselves in the spectral density via the real and imaginary part of the self-energy $\sigma(\omega)$ given in Eq.(18). It will be evaluated in the next section.

5. Results and applications

5.1. CHARGE FLUCTUATIONS IN THE SINGLE ELECTRON BOX

In equilibrium when $V_R = V_L$, the SET-transistor is equivalent to the single electron box. The average excess particle number can be obtained from (20) and (26) $\langle n \rangle = \int d\omega f(\omega)A(\omega)$. In the classical limit $A^{(0)}(\omega) = \delta(\omega -$
\( \Delta_0 \) one obtains \( \langle n^{cl} \rangle = f(\Delta_0) \) where the energy difference \( \Delta_0 = E_C(1 - 2C_GV_G) \) depends on the gate voltage. Thus, \( \langle n^{cl}(Q_G) \rangle \) shows a step at \( Q_G = C_GV_G = 1/2 \), which is smeared by temperature.

At larger values of \( \alpha_0 \) or lower temperature we have to include the self-energy \( \sigma(\omega) \) (18) in the spectral density (26). The two limits, \( T = 0 \) and \( |\omega| \leq T \), can be analyzed analytically. In the first case, the spectral density has the form

\[
A(\omega) \approx \frac{|\omega|}{\Delta_0} \cdot \frac{\tilde{\Delta}(\omega)\tilde{\alpha}(\omega)}{|\omega - \Delta(\omega)|^2 + [\pi\Delta(\omega)\tilde{\alpha}(\omega)]^2}, \quad T = 0
\]

where

\[
\tilde{\Delta}(\omega) = \frac{\Delta_0}{1 + 2\alpha_0 \ln(\frac{E_C}{|\omega|})} \frac{1}{1 + \pi^2 \tilde{\alpha}(\omega)^2}, \quad \tilde{\alpha}(\omega) = \frac{\alpha_0}{1 + 2\alpha_0 \ln(\frac{E_C}{|\omega|})}.
\]

The spectral density \( A(\omega) \) has a maximum at the renormalized energy difference \( \Delta \), which is obtained from the self-consistent solution of

\[
\Delta = \tilde{\Delta}(\Delta).
\]

It further has a broadening of order \( \pi \Delta \alpha \) where \( \alpha = \tilde{\alpha}(\Delta) \). For \( \pi \alpha \ll 1 \) the broadening can be neglected. In this case our results coincide with the RG-analysis of Refs. [14, 11], which shows that the leading logarithmic terms are included in our diagram series.

At finite temperatures \( |\omega| \leq T \), we get

\[
A(\omega) \approx \frac{\Delta}{\Delta_0} \cdot \frac{\alpha \omega \coth(\frac{\omega T}{2})}{|\omega - \Delta|^2 + [\pi \alpha \omega \coth(\frac{\omega T}{2})]^2}, \quad |\omega| \leq T
\]

where

\[
\Delta = \frac{\Delta_0}{1 + 2\alpha_0 \ln(\frac{E_C}{|\omega|})} \frac{\alpha_0}{1 + 2\alpha_0 \ln(\frac{E_C}{2\pi T})}.
\]

The broadening is of order \( \pi \alpha T \). It can only be neglected if for \( \pi \alpha \ll 1 \).

As a consequence of quantum fluctuations the step of the average charge in the electron box near the degeneracy points is washed out. We neglect broadening effects (\( \pi \alpha \ll 1 \)) and assume that the energies of the ground state and the first excited state near the degeneracy point depend symmetrically on the distance, \( \frac{E_C}{4} \pm \frac{\Delta}{2} \). In this case the partition function is \( Z \approx 2e^{\frac{E_C}{4}} \cosh(\frac{\Delta}{2T}) \) and the average excess charge, \( \langle n \rangle = Q_G/e - T \frac{\partial}{\partial \Delta_0} \ln Z \), becomes

\[
\langle n \rangle \approx \frac{\Delta_0 - \Delta}{2\Delta_0} + \frac{\Delta}{\Delta_0} f(\Delta).
\]
Depending on the temperature we have to choose the appropriate limiting form for $\Delta$. At finite temperature the slope at $\Delta_0 = 0$ is

$$\frac{\partial \langle n \rangle}{\partial \Delta_0} \bigg|_{\Delta_0=0} = -\{4T[1 + 2\alpha_0 \ln \left(\frac{E_C}{2\pi T}\right)]^2\}^{-1}.$$  

It is reduced compared to the classical result $-1/4T$. This anomalous temperature dependence can serve as an indication for coherent higher order tunneling processes. Below we will demonstrate that the linear and nonlinear conductance of a transistor shows more pronounced deviations from the classical result, making it more suitable for experimental verification.

5.2. CONDUCTANCE OSCILLATIONS IN THE SET-TRANSISTOR

The linear conductance $G$ of a transistor follows from (27)

$$G = -\frac{e^2}{h} 4\pi^2 \int d\omega \frac{\alpha_R(\omega)\alpha_L(\omega)}{\alpha(\omega)} A(\omega)f'(\omega). \quad (34)$$

Since the derivative of the Fermi function restricts the integration variable to the regime $|\omega| \leq T$ we can use the form (31) and obtain

$$G = \frac{e^2}{h} 2\pi^2 \frac{\alpha_R\alpha_L}{\alpha} \int d\omega \frac{\omega/T}{\sinh(\omega/T)} \cdot \frac{\alpha \omega \coth \left(\frac{\omega}{2T}\right)}{\omega - \Delta^2 + \left[\pi\alpha \coth \left(\frac{\omega}{2T}\right)\right]^2} \quad (35)$$

where $\alpha = \alpha_R + \alpha_L$ and we have to use the finite temperature form for $\Delta$ and $\alpha$. The conductance has a maximum at $\Delta_0 = 0$, given by

$$G_{\text{max}} = \frac{e^2}{h} 2\pi^2 \frac{\alpha_R\alpha_L}{\alpha} \left[\frac{\pi}{2} - \arctan \left(\frac{(\pi\alpha)^2 - 1}{2\alpha\pi}\right)\right], \quad (36)$$

with a broadening given by

$$\gamma \approx \frac{\int d\Delta_0 G(\Delta_0)}{G_{\text{max}}} = \frac{\pi^3 T[1 + 2\alpha_0 \ln \left(\frac{E_C}{2\pi T}\right)]}{\pi - 2 \arctan\left[\frac{(\pi\alpha)^2 - 1}{2\alpha\pi}\right]} \quad (37)$$

In the regime $2\alpha_0 \ln \left(\frac{E_C}{2\pi T}\right) \ll 1$ the peak height $G_{\text{max}}$ is constant and $\gamma$ proportional to $T$, which is the classical result for sequential tunneling. For $2\alpha_0 \ln \left(\frac{E_C}{2\pi T}\right) \sim 1$ and $\pi\alpha \ll 1$, $G_{\text{max}}$ and $\gamma$ contain logarithmic terms in the temperature indicating energy renormalization effects due to higher-order tunneling processes. For $T \to 0$ the maximum value as well as the broadening vanish, $G_{\text{max}} \sim \frac{1}{\ln T}$ and $\gamma \sim T \ln T$. For $2\alpha_0 \ln \left(\frac{E_C}{2\pi T}\right) \sim 1$ and $\pi\alpha \sim 1$, we have to retain the finite lifetime effects contained in (36) and (37).

The most pronounced signature of quantum fluctuations is contained in the differential conductance $G(V) = \frac{\partial I_{\text{st}}(V)}{\partial V}$. In this case the finite voltage

$$G(V) = -\frac{e^2}{h} \sum_{\nu} \frac{\alpha_R(\nu)\alpha_L(\nu)}{\alpha(\nu)} A(\nu)f'(\nu) \cdot \frac{\omega}{\sinh(\omega/T)} \cdot \frac{\alpha \omega \coth \left(\frac{\omega}{2T}\right)}{\omega - \Delta^2 + \left[\pi\alpha \coth \left(\frac{\omega}{2T}\right)\right]^2}$$.

In the regime $2\alpha_0 \ln \left(\frac{E_C}{2\pi T}\right) \ll 1$ the peak height $G_{\text{max}}$ is constant and $\gamma$ proportional to $T$, which is the classical result for sequential tunneling. For $2\alpha_0 \ln \left(\frac{E_C}{2\pi T}\right) \sim 1$ and $\pi\alpha \ll 1$, $G_{\text{max}}$ and $\gamma$ contain logarithmic terms in the temperature indicating energy renormalization effects due to higher-order tunneling processes. For $T \to 0$ the maximum value as well as the broadening vanish, $G_{\text{max}} \sim \frac{1}{\ln T}$ and $\gamma \sim T \ln T$. For $2\alpha_0 \ln \left(\frac{E_C}{2\pi T}\right) \sim 1$ and $\pi\alpha \sim 1$, we have to retain the finite lifetime effects contained in (36) and (37).

The most pronounced signature of quantum fluctuations is contained in the differential conductance $G(V) = \frac{\partial I_{\text{st}}(V)}{\partial V}$. In this case the finite voltage
Figure 6. The differential conductance in the nonlinear response regime as function of the gap energy normalized to the transport voltage $V$, for $T = 0$ and $\alpha_0^L = \alpha_0^R = 0.05$ ($R_L = R_R = R_T$), we consider a symmetric bias and chose (1) $V/E_C = 0.1$, (2) $V/E_C = 0.01$, (3) $V/E_C = 0.001$. For comparison, (0) shows the result for the classical case obtained from lowest order perturbation theory and the master equation.

$eV$ provides another energy scale and the renormalization and life-time effects are probed over a wider energy range even at zero temperature. The $T = 0$ result obtained from Eq. (27) is plotted in Fig.(6). For comparison we also show the result obtained in perturbation theory. (We had quoted the expression for the current in this limit in the Introduction.) In the classical limit the conductance is nonzero only in the range $|\Delta_0| \leq \frac{eV}{2}$ with vertical steps at the edges. The result of Fig.(6) displays clearly the renormalization effects and, moreover, the finite life-time broadening. It is clear that at finite temperature the conductance is washed out further. These effects should be observable in an experiment with realistic parameters.

6. Interactions in the leads

We will now address the problem of interactions in the leads by modeling the leads as 1-dimensional Luttinger liquids. In contrast to Kane and Fisher
we assume that the interactions in the island are sufficiently described by the charging energy. We derive a path integral description for this case and find modifications of the effective action as compared to the metallic case. However, the physics associated with the Coulomb blockade remains the same.

The Hamiltonian of the system is still given by Eq. (1), where the Hamiltonians of the leads describe Luttinger liquids. They are given by [26]

$$H_L = \int \frac{dx}{2\pi} \sum_j v_j \left[ \frac{g_j}{2} (\nabla \phi_j)^2 + \frac{1}{2g_j} (\nabla \theta_j)^2 \right], \quad (38)$$

i.e. they contain a sum of spin ($j = \sigma$) and charge ($j = \rho$) degrees of freedom. The parameters $g_j$ describe the interaction strength ($g_j = 2$ in the noninteracting limit), and $v_j$ are the velocities of spin and charge excitations. We also introduce the bosonic fields $\phi_s = \phi_\rho + s\phi_\sigma$ and $\theta_s = (1/2) [\theta_\rho + s\theta_\sigma]$ for spin up ($s = 1$) and down ($s = -1$) fermions. These fields obey the commutation relation $[\phi_s(x), \theta_{s'}(x')] = (i\pi) \Theta(x - x') \delta_{s,s'}$.

The fermionic field operators $\hat{\Psi}$ can be expressed in terms of the spin and charge degrees of freedom [30]

$$\hat{\Psi}_{L,s}^\dagger(x, \tau) = \sqrt{\rho_{0,s}} \sum_{m = \pm 1} e^{i m k_F x} e^{i \frac{\pi}{2} \frac{|q|}{v_j \tau} (e^{iqx} \hat{b}_{j,q}^\dagger + e^{-|q|v_j \tau} \hat{b}_{j,-q})} \hat{\Psi}_{R,s} (x, \tau) = \sqrt{\frac{\pi}{2g_j}} \sum_{q \neq 0} \frac{1}{2qL} \left| e^{iqx} \hat{b}_{j,q}^\dagger - e^{-|q|v_j \tau} \hat{b}_{j,-q} \right|.$$ \quad (39)

The tunneling occurs in the junctions at the points $x = 0$ and $x = d$. We obtain an effective action, as in the metallic case, by introducing the auxiliary field related to the voltage drop across the junction $\phi(\tau)$ and performing a gauge transformation which transfers the phase dependence into the tunneling matrix elements. After a cumulant expansion we express the partition function as a path integral involving the Euclidean action

$$S[\phi] = \int_0^\beta d\tau \frac{C}{2e^2} \left( \frac{\partial \phi}{\partial \tau} \right)^2 - \int_0^\beta d\tau \, d\tau' \, 2\alpha(\tau - \tau') \cos[\phi(\tau) - \phi(\tau')]. \quad (41)$$
It has the same form as for noninteracting electrons. However, the kernel
\[ \alpha(\tau) = \frac{k_F^2}{2\pi} |T|^2 N(0) \left( \frac{\pi/v_0 \beta k_F}{\sin(\pi \tau/\beta)} \right)^\eta \] (42)
carries an exponent \( \eta \), which is given by the parameters of the Luttinger liquid \( \eta = 1 + \frac{1}{8} [g_{\rho} + g_{\sigma} + \frac{4}{g_{\rho}} + \frac{4}{g_{\sigma}}] \). In Fourier space the kernels are proportional to \( \omega^{-\eta} \). As a result the \( I-V \) characteristics of a normal metal-Luttinger liquid junction will also show a power law dependence proportional to \( \text{sign}(V) V^{\eta-1} \).

We point out that the effective action differs from the one obtained in Ref. [31] for the model of a Luttinger liquid with two constriction. The physics associated with Coulomb blockade is very similar whether we describe the central island as a Luttinger liquid or as a metal island. We finally point out that a similar action was discussed in Ref. [29] in connection with shake-up effects close to the Fermi energy due to tunneling, and in Ref. [32] for junctions coupled to Ohmic baths.

Acknowledgements

We would like to acknowledge stimulating discussions with G. Falci, A.D. Zaikin and G. Zimanyi. This work was supported by the Swiss National Science Foundation (H.S.) and by the ‘Deutsche Forschungsgemeinschaft’ as part of ‘Sonderforschungsbereich 195’.

References

1. D.V. Averin and K.K. Likharev, in Mesoscopic Phenomena in Solids, ed. B.L. Altshuler, P.A. Lee and R.A. Webb (Elsevier, Amsterdam, 1991), p. 173
2. Several review articles are contained in Single Charge Tunneling, NATO ASI Series, Vol. 294, H. Grabert and M.H. Devoret, eds., New York, Plenum Press (1992)
3. For further articles we refer to Z.Phys.B-Condensed Matter 85 (1991)
4. I.O. Kulik and R.I. Schekhter, Sov. Phys. JETP 41, 308 (1975)
5. G. Schön and A.D. Zaikin, Phys. Rep. 198, 237 (1990)
6. C.W.J. Beenakker, Phys. Rev. B 44, 1646 (1991)
7. D.V. Averin, A.N. Korotkov and K.K. Likharev, Phys. Rev. B 44, 6199 (1991)
8. Y. Meir, N.S. Wingreen and P.A. Lee, Phys. Rev. Lett. 66, 3048 (1991)
9. P. Lafarge, H. Pothier, E.R. Williams, D. Esteve, C. Urbina and M.H. Devoret, Z. Phys. B - Condensed Matter 85, 327 (1991)
10. L.I. Glazman and K.A. Matveev, Sov. Phys. JETP 71, 1031 (1990)
11. K.A. Matveev, Sov. Phys. JETP 72, 892 (1991)
12. D.S. Golubev and A.D. Zaikin, Physica B 203 (1994)
13. H. Grabert, Physica B 194-196, 1011 (1994); Physica B 203 (1994)
14. G. Falci, J. Heinz, G. Schön and G.T. Zimanyi, Physica B 203 (1994)
15. W. Zwerger, preprint
16. H. Schoeller and G. Schön, to be publ. in Phys. Rev. B; Physica B 203 (1994)
17. R.P. Feynman and F.L. Vernon, Ann. Phys. (N.Y.) 24, 118 (1963)
18. A.O. Caldeira and A.J. Leggett, Ann. Phys. (NY) 149, 374 (1983)
19. U. Eckern, G. Schön and V. Ambegaokar, Phys. Rev. B 30, 6419 (1984)
20. N.E. Bickers, Rev. Mod. Phys. 59, 845 (1987)
21. A.C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge Univ. Press, 1993).
22. S.E. Barnes, J. Phys. F: Metal Phys. 7, 2637 (1977); Phys. Rev. B 33, 3209 (1986)
23. J. Rammer, Rev. Mod. Phys. 63, 781 (1991)
24. D. Loss and H. Schoeller, Physica 150A, 199 (1988); J. Stat. Phys. 54, 765 (1989);
    J. Stat. Phys. 56, 175 (1989)
25. F.D.M. Haldane, J. Phys. C 14, 2585 (1981); J. Sólyom, Adv. Phys. 28, 201 (1979)
26. C.L. Kane and M.P.A. Fisher, Phys. Rev. Lett. 68, 1220 (1992); Phys. Rev. B 46, 15233 (1992)
27. K.A. Matveev and L.I. Glazman, Phys. Rev. Lett. 70, 990 (1993)
28. For a systematic discussion of higher order terms, which describe physical processes
    such as Andreev reflection in normal-superconductor junctions or elastic cotunneling, we refer to C. Bruder, R. Fazio and G. Schön, Physica B 203 (1994)
29. M. Ueda and F. Guinea, Z.Phys. 85, 413 (1991)
30. F.D.M. Haldane, Phys. Rev. Lett. 47, 1840 (1981)
31. C.L. Kane and M.P.A. Fisher, Phys. Rev. B 46, 15233 (1992)
32. M. Sassetti and U. Weiss, this volume