PQCD Analysis of Hard Scattering in Nuclei

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Abstract

We review the extension of the factorization formalism for perturbative QCD to soft initial- and final-state scattering associated with hard processes in nuclei.

1 Introduction

In this talk, we would like to review a few results from a perturbative QCD (pQCD) treatment of the scattering of hadrons and leptons in nuclei, based on factorization, work in collaboration with Ma Luo [1, 2, 3] and, more recently, Xiaofeng Guo [4]. At the outset, it may be useful to clarify the relation of this work to the recent papers of Baier et al. (BDMPS), described by Dokshitzer at this workshop [5]. We have tried to illustrate this relation schematically in Fig. 1. The BDMPS analysis begins (Fig. 1a) with the classic treatment of radiation induced when a charged particle passes through a large target, due originally to Landau, Pomeranchuk and Migdal (LPM). This analysis does not require the presence of a hard scattering, but describes the coherent results of many soft scatterings. Its primary subject has traditionally been induced energy loss. Our analysis (GLQS) begins with the perturbative QCD treatment of hard-scattering in a small target (Fig. 1b), in which the primary subject of interest is momentum transfer. A complete analysis (Fig. 1c) of hard scattering in a large target, involves both energy loss and the transverse momenta due to initial- and final-state soft scatterings. Our work is a step in this direction, attempting to stay as close as possible to the pQCD formalism, in which we may readily quantify corrections. To be specific, we consider only a single soft initial- or final-state interaction in addition to the hard scattering. Our central observation is that for suitably-defined jet and related inclusive cross sections this is the first order in an expansion in the quantity

$$\frac{A^{1/3} \times \lambda^2}{Q^2},$$

(1)

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where $\lambda$ represents a nonperturbative scale, which we shall identify with a higher-twist parton distribution below. That additional scatterings are suppressed by factors of $1/Q^2$ is perhaps surprising. Let us review why this is the case, at least for certain cross sections.

The basic analysis of hard-scattering in nuclear matter (cold or hot) is quite simple. To be specific, consider the scattering of a quark. A hard-scattering with momentum transfer $Q$ can resolve states whose lifetimes are as short as $1/Q$, for instance quarks off-shell by order $Q$, but still less that $Q$. The off-shellness of the scattered quark increases with the momentum transfer simply because the number of available states increases with increasing momentum. Similarly, the scattered quark, of momentum $p'$ is typically off-shell by order $m_J \lesssim Q$. We may think of $m_J$ as the momentum of the jet into which quark fragments. If we are to recognize the jet, we must have $m_J \ll E_J = p'_0$. On the other hand, the counting of available states ensures that $m_J \gg \Lambda_{QCD}$.

Now the scattered quark has a lifetime in its own rest frame $\Delta t(p') \sim 1/m_J$ with $m_J \ll E_J$. In the target rest frame, however, this becomes, for large enough $E_J/m_J$, $\Delta t(\text{target}) \sim 1/m_J \left( \frac{E_J}{m_J} \right) > R_A$, where $R_A$ is the (fixed) target size. Thus, at high enough energy the lifetime of the scattered quark will exceed the target size, even though the quark itself is far off the mass shell, typically by a scale that grows with the momentum transfer $Q$.

Further couplings of the off-shell quark are suppressed, first of all by the strong coupling evaluated at scale $m_J$, and, more importantly, by an overall factor of $1/m_J^2 \sim 1/Q^2$, since the effective size of the scattered quark decreases with momentum transfer in this manner.

In summary, for inclusive processes such as jet production, high-$Q$ implies that process-dependent multiple scattering is power-suppressed compared to single scattering. Initial-state interactions internal to the nucleus are leading-power, but factorize. Thus the “Cronin
effect”, $A^\alpha$-dependence with $\alpha > 1$, due to multiple scattering, is higher-twist for inclusive distributions, while the “EMC” effect for parton distributions in nuclei is (almost by definition) leading-twist.

The most important point here is that the scattered particle remains off-shell for its entire transit of the target. Thus, its interactions with the target may be treated by the formalism of perturbative QCD, which, however, must be extended to include corrections that decrease with extra powers of momentum transfer. Up to the first such “higher-twist” contribution, a general cross section has the representation \[ \sigma(Q) = H^0 \otimes f_2 \otimes f_2 + \left( \frac{1}{Q^2} \right) H^1 \otimes f_2 \otimes f_4 + O \left( \frac{1}{Q^4} \right), \] (2)

where $\otimes$ represents covolutions in fractional momenta carried by partons, and $f_n$ represents a parton distribution of twist $n$. Target-size dependence due to multiple scattering can only appear in the second term in this expansion.

2 Parton-Nucleus Scattering in Perturbative QCD

2.1 Factorization at Leading and Nonleading Powers

Let us review some of the details of a factorized cross section like (2). The first term, consisting of only twist-two matrix elements has the detailed form,

\[ \omega d\sigma^2 = \sum_{ij} \int dy f_{j/p_2}(y, Q) \int dx f_{i/p_1}(x, Q) \hat{\sigma}_{ij}(xp_1, yp_2, p') , \] (3)

where we may take $p'$ as the momentum of an observed jet. The fragmentation of a jet, suitably defined, is calculable in perturbation theory, and may be absorbed into the “hard scattering function” $\hat{\sigma}$. The $f_{a/p}$ are distributions of parton type $a$ in hadron $p$. They have the interpretation of expectation values in the hadronic state of products of fields on the light cone, for instance, for a quark distribution

\[ f_{q/p}(x, Q) = \int \frac{dy}{2\pi} e^{ixp^+y} \langle p|\bar{q}(0)\frac{\gamma^+}{2} q(y^-)|p\rangle , \] (4)

where for simplicity we choose the $A^+ = 0$ gauge, assuming $\vec{p}$ is in the plus direction. Eq. (3) is illustrated by Fig. 2a. As shown, the convolution in eq. (3) is in terms of the momentum fractions $x$ and $y$ carried by partons $i$ and $j$, from hadrons $p_1$ and $p_2$, respectively, into the hard scattering.

Fig. 2b is the corresponding picture for a higher-twist contribution to hard scattering. In this case two partons $i$ and $i'$ with momenta $x_1 p_1$ and $x_2 p_1$ from the target (the “nucleus”) collide with a single parton $j$ of momentum $yp_2$ (from the “projectile”),

\[ \omega d\sigma^4 = \sum_{(ii')j} \int dy f_{j/p_2}(y, Q) \int dx_1 dx_2 dx_3 T_{(ii')/p_1}(x_1, x_2, x_3, Q) \hat{\sigma}^{(4)}_{(ii')j}(xp_1, yp_2, p'). \] (5)
The expectation value $T$ corresponding to this multiparton contribution from the target is typically of the form \[7\],

\[
T_{(ii')/p}(x_1, x_2, x_3, Q) = \int \frac{dy^-_1 dy^-_2 dy^-_3}{(2\pi)^3} e^{ip^+(x_1 y^-_1 + x_2 y^-_2 + x_3 y^-_3)} \langle p| B_i(0) B_{i'}^\dagger(y^-_3) B_{i'}^\dagger(y^-_2) B_i(0) |p \rangle ,
\]

where $B_i$ is the field corresponding to a parton of type $i = q, \bar{q}, G$. In eq. (5), the hard part $\hat{\sigma}^{(4)}_{(ii')}_{+j}$ depends on the identities and momentum fractions of the incoming partons, but is otherwise independent of the structure – in particular the size – of the target (and projectile).

To find $A$-enhancement due to multiple scattering, we must look elsewhere.

### 2.2 $A$-Enhancement from Matrix Elements

For definiteness, we consider photoproduction or deeply inelastic scattering on a nucleus \[1,3\]. In this case, the additional soft scattering is always a final-state interaction. The structure of the target is manifest only in the matrix element $T$ in eq. (3). Each pair of fields in the matrix element (4) represents a parton that participates in the hard scattering. The $y^-_i$ integrals parameterize the distance between the positions of these particles along the path of the outgoing scattered quark. In eq. (6), integrals over the distances $y^-_i$ generally cannot grow with the size of the target because of oscillations of the exponential factors $e^{ip^+ x_i y^-_i}$. Poles from $\hat{\sigma}$ in the $x_i$ integrals, associated with the scattered particle, however, can result in finite contributions from points where two of the $x_i$ vanish [1-3]. An example is shown in Fig. 3. It is important to emphasize that using a pole in the complex $x_i$ (longitudinal momentum) space to do the integral does not correspond to assuming on-shell propagation for the scattered quark. Indeed, the $x_i$ integrals are not pinched between coalescing singularities at that point, and the same results could be derived by performing the $x_i$ integrals without ever going through the $x_i = 0$ points.

The result of this reasoning is that matrix elements that depend on three fractional momenta, as in (6) above, simplify to a form like

\[
T_i(x, A) = \int \frac{dy^-_1}{2\pi} e^{ip^+ x_1 y^-_1} \int \frac{dy^-_2}{2\pi} \theta(y^- - y^-_1) \theta(y^-_2 - y^-_1)
\]
where $|p_A\rangle$ is the relevant nuclear state. In this form, integrals over the $y_i$ can grow with the nuclear radius as fast as $A^{1/3}$, once local color confinement is taken into account. The variable $x$ here is the fractional momentum associated with the hard parton from the target that initiates the process. The soft scattering contributes a negligible longitudinal fractional momentum. Details of the reasoning and calculation for deeply inelastic scattering are given in Ref. [3].

3 Applications

In Refs. [1] and [3], we have applied the formalism sketched above to single-particle inclusive and single-jet production for deeply inelastic scattering and photoproduction. These cases involve final-state interactions only. In each case, the leading $1/Q^2$ correction is proportional to the matrix element in eq. (6), or to a corresponding matrix element $T_G$ with four gluon fields. Of course, the value of the correction cannot be estimated without an idea of the magnitudes of the $T$’s. Since these magnitudes are nonperturbative they must be taken from experiment. At the same time, we expect the $x$-dependence of the probability to detect the hard parton to be essentially unaffected by the presence or absence of an additional soft scattering. Thus, we choose ansatz

$$T_q(x, Q) = \lambda^2 A^{1/3} f_{q/p_A}(x, Q)$$

for $T_q$, in terms of the corresponding twist-two parton distribution $f$, with $\lambda$ a constant with dimensions of mass (see eq. (1)). This assumption facilitates the comparison to data.

A quantity that is sensitive to final-state rescattering in a particularly direct way is the momentum imbalance of di-jets in photoproduction in nuclei. The $A^{4/3}$ dependence of this quantity is related to the matrix elements $T_q$ and $T_G$ by the simple formula [3]

$$\langle k_T^2 E_\ell \frac{d\sigma}{d^3\ell} \rangle_{4/3} = \sum_{a=q,g} \int dx T_a(x, A) H^{\gamma a}(xp, p_\gamma, \ell) = \lambda^2 A^{4/3} \sum_{a=q,g} \int dx f_a(x, A) H^{\gamma a}(xp, p_\gamma, \ell),$$

for $T_q$. In this form, integrals over the $y_i$ can grow with the nuclear radius as fast as $A^{1/3}$, once local color confinement is taken into account. The variable $x$ here is the fractional momentum associated with the hard parton from the target that initiates the process. The soft scattering contributes a negligible longitudinal fractional momentum. Details of the reasoning and calculation for deeply inelastic scattering are given in Ref. [3].
where $H^{\gamma a}$ is a hard-scattering function that we have computed to lowest order and where in the second equality we have used (5). The momentum $\ell$ may be identified as the momentum of the more energetic jet. By comparing eq. (7) to data, (8) we found $\lambda^2 \sim 0.05 - 0.1 \text{ GeV}^2$. This value may be used to predict anomalous $A$-enhancement for other processes.

One such process is direct photon production at measured transverse momentum, whose very moderate $A$-dependence has been measured by the E706 experiment at Fermilab. In Ref. [4], it was found that the value of $\lambda^2$ above, which produces a relatively large enhancement in dijet momentum imbalance, due to final-state interactions, produces a quite small $A$-enhancement in photoproduction, due to initial-state interactions, consistent with experiment. This may shed some light on the long-standing observation that (initial-state) transverse momentum effects in Drell-Yan cross sections are also surprisingly small [4]. Clearly, further study of this and related questions is in order.

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