An Improved Research on Frequency-Fixed Cascaded SOGI Single-Phase PLL

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Abstract. The second-order generalized integrator (SOGI) with band-pass filtering characteristics is widely used in the design of advanced single-phase phase-locked loops (PLLs). But this type of PLL has insufficient ability to restrain dc biases, which limits their applications under certain conditions. Recently, a PLL adopting two cascade SOGI structures is called the cascaded generalized integrator PLL (CGI-PLL), which has perfect dc offsets blocking capability. However, the two output signals of the quadrature signal generator (QSG) formed by frequency-fixed CGI have different amplitudes, which makes the second harmonic oscillations appear in the estimated parameters of the PLL while the frequency deviates from the nominal frequency. Similarly, the in-phase output signal of CGI-based QSG is not synchronized with the fundamental frequency component of the grid voltage under frequency drifts, which is the reason for the phase offset error in the PLL outputs in the steady state. Some compensation strategies are added to correct the error that exists in the frequency-fixed CGI-PLL when the frequency deviates from the nominal value.

1. Introduction

A large number of researches have been carried out on single-phase locked loops (PLLs) in recent years. The single-phase PLL based on the second-order generalized integrator (SOGI) is undoubtedly one of the attractive PLLs. The construction of SOGI-PLL is uncomplicated and easy to realize. It consists of a basic SOGI block to generate two quadrature signals from the input signal [1]. To implement the synchronization function, these quadrature signals are sent to an embedded synchronous reference frame-based PLL (SRF-PLL) [2]. However, the single-phase PLL based on the SOGI is influenced by the dc biases in the input signal, and the dc component will exist in the quadrature signal, thus introducing the PLL and affecting the phase-locked results.

In [3], a frequency-fixed PLL adopting two cascade SOGI structures is called the cascaded generalized integrator PLL (CGI-PLL), which has perfect dc offset blocking capability. The self-adaptive parameters are avoided simultaneously, thus ensuring stability and practicability. It is well known that frequency-fixed SOGI-PLLs cause second harmonic ripples in the estimated parameters when the input frequency deviates from the nominal value [1]. Certainly, the output of CGI-PLL will...
be contaminated by double-frequency problem. Moreover, there is a small phase difference between the in-phase output signal of the CGI-based QSG and the input signal under frequency deviations. This phase difference will cause a slight deviation in the output of the PLL, resulting in inaccurate phase-locked results. In [3], considering that the input signal is under the most serious frequency deviations, a satisfactory output outcome is achieved through reasonable design of system parameters. The main content of this paper is to introduce some compensation strategies into the CGI-PLL to deal with the problems under frequency deviations.

2. Analysis of fixed-frequency CGI-based QSG

For the convenience of narration, the schematic diagram of [3] is shown in figure 1, where $V_{in}$ represents the input voltage, $\omega_n$ is the nominal grid frequency which is $2\pi 50$ rad/s in this work, $\hat{\omega}$ and $\hat{\theta}$ are the estimated frequency and phase angle, respectively. It is worth noting that $k$ represents the damping coefficient. It can affect the closed-loop system bandwidth [1].

According to this, the transfer function of CSI-QSG can be obtained as follows,

$$G_\alpha(s) = \frac{V_\alpha}{V_{in}}(s) = \frac{(k\omega_n s)^2}{(s^2 + k\omega_n s + \omega_n^2)^2}$$  \hspace{1cm} (1)

$$G_\beta(s) = \frac{V_\beta}{V_{in}}(s) = \frac{k^2\omega_n^2 s}{(s^2 + k\omega_n s + \omega_n^2)^2}$$  \hspace{1cm} (2)

The bode plots of the transfer functions $G_\alpha$ and $G_\beta$ are depicted in figure 2. The following conclusions can be drawn from figure 2 and transfer functions,

(1) The $\alpha$-axis output signal of the CGI-based QSG has the same amplitude and phase as the fundamental frequency component of the input signal at the nominal frequency, indicating that $\alpha$-axis
can accurately estimate the fundamental frequency component of the input. There will be phase and amplitude errors in \( \alpha \)-axis output in the case of frequency drift;

(2) The \( \beta \)-axis output signal of the CGI-based QSG has the same amplitude as the fundamental frequency component of the input with a phase lag of 90° at the nominal frequency. There will be amplitude errors between \( \beta \)-axis output and the fundamental component of input and this signal is no longer orthogonal under frequency deviations;

(3) the \( \alpha \)-axis and \( \beta \)-axis outputs of the CGI-based QSG always maintain a phase difference of 90°, indicating that they are always orthogonal.

3. Proposed PLL structure

It can be seen from above conclusion that the CGI-PLL works effectively at nominal frequency. However, under frequency drift, dual-frequency oscillations appear in the estimated phase, frequency, and amplitude. Moreover, the estimated amplitude and phase have oscillation and offset errors respectively at non-nominal frequencies. This phenomenon can be evidently captured in figure 3.

3.1. Compensation second harmonic ripples

In CGI-PLL structure, the second harmonic oscillation problem in the estimated parameters is caused by the unequal amplitude of \( \alpha \)-axis and \( \beta \)-axis output signals. The amplitude correction will be performed firstly.

\[
\left| G_\alpha (j\omega) \right| = \frac{(k\omega_\alpha \omega_x)^2}{(\omega_\alpha^2 - \omega_x^2)^2 + (k\omega_x \omega_\alpha)^2} \quad (3)
\]

\[
\left| G_\beta (j\omega) \right| = \frac{k^2 \omega_\beta \omega_x}{(\omega_\beta^2 - \omega_x^2)^2 + (k\omega_x \omega_\beta)^2} \quad (4)
\]

The distinct difference between (3) and (4) lies in their molecular terms. Consequently, the double-frequency oscillations can be effectively eliminated by multiplying the \( \beta \)-axis output of the CGI-based QSG by \( \omega_\beta \) as an input signal of the Park transform instead of \( \beta \)-axis output, where \( \omega_\beta \) is an estimation frequency of CGI-PLL.

3.2. Adjusting the small phase error

The in-phase output signal of CGI-based QSG is not synchronized with the fundamental frequency component of the grid voltage under frequency drifts, which is the reason for the small phase error in the PLL outputs in the steady state. A succinct and valid way to solve this problem is to add the calculated value of the phase error to the phase estimated by the CGI-PLL. Based on (1), the phase difference between the \( \alpha \)-axis output of the CGI-based QSG and the input is equal to

\[
\delta = 2 \arctan \left( \frac{\omega_x^2 - \omega_\alpha^2}{k\omega_\alpha \omega_x} \right) \approx \frac{2(\omega_x^2 - \omega_\alpha^2)}{k\omega_\alpha \omega_x} \approx \frac{4\omega_\alpha (\omega_x - \omega_\alpha)}{k\omega_\alpha^2} = \frac{4(\omega_x - \omega_\alpha)}{k\omega_\alpha} \quad (5)
\]

Correspondingly, this result can also be derived in the time domain. In order to more intuitively evaluate the accuracy of the approximate processing, figure 4 shows the actual value and estimated value curve when the grid frequency changes. Figure 4 can basically prove the accuracy of the approximation, even with a certain frequency drift.

3.3. Processing of amplitude error

Supposing that \( V_n \) is the amplitude of the input fundamental frequency component, it can be seen from equation (2) that \( V_n \cdot |G_\beta (j\omega) | \) is the output amplitude of the CGI-based QSG. Correspondingly,
$V_n \cdot |G_\beta (j\omega)\|$ is the estimated amplitude after Park transformation and is recorded as $V_q$. Therefore, the actual amplitude can be expressed as $v_q \left( |G_\beta (j\omega)\| \right)^{-1}$, especially when frequency deviation occurs. Because of $|G_\beta (j\omega)\| \neq 1$, it means that $V_n \neq v_q$. It is necessary to make concise and accurate approximation of $|G_\beta (j\omega)\|$ in order to achieve the aforementioned purpose.

However, the expression of $|G_\beta (j\omega)\|$ is not easy to satisfy the above requirements. Furthermore, because $(\omega_q^2 - \omega_n^2)^2$ is much smaller than $(k\omega_q\omega_n)^2$ in equation (2) when the frequency of the input voltage is near the nominal value, the estimated amplitude may not be processed. So it is approximated that $|G_\beta (j\omega)\|\approx 1$. This can also be confirmed from the estimated amplitude curve in figure 3. This can also be confirmed from the estimated amplitude curve in figure 3, but the difference is that the problem of the second harmonic ripple in the curve has been properly resolved at this time.

![Figure 4. Accuracy evaluation curve.](image)

### 3.4. Proposed PLL structure
Figure 5 shows a block diagram of the proposed PLL, which is slightly improved on the basis of the structure shown in figure 1 and introduces the above-mentioned measures for correcting phase error and dual-frequency oscillations. Notice that the structure of the modified PI controller is shown in figure 5, and the integrator output is regarded as the estimated value of the frequency [4]. With the same values of $k_p$ and $k_i$, this modification results in a higher filtering capability and a more attenuated transient response in frequency estimation [6].

![Figure 5. Block diagram of the proposed PLL.](image)

### 4. Simulation results
The simulation results are shown below. The value of $k$ is taken as 1.63 in this paper. It can be seen that the CGI-PLL does not have dual-frequency ripple under frequency drifts from figure 6, and there
is no steady-state offset error under frequency drifts from figure 7. In order to achieve better stability, it is promising to apply small signal analysis to model the proposed PLL.

![Figure 6. The effect of introducing dual-frequency oscillation correction.](image1)

![Figure 7. The effect of introducing phase compensation.](image2)

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