On the correlation coefficient $T(E_e)$ of the neutron beta decay, caused by the correlation structure invariant under discrete P, C and T symmetries

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We analyze the correlation coefficient $T(E_e)$, which was introduced by Ebel and Feldman (Nucl. Phys. 4, 213 (1957)). The correlation coefficient $T(E_e)$ is induced by the correlation structure $(\xi_n \cdot \vec{E}_e)(\xi_e \cdot \vec{E}_e)/E_e E_e$, where $\xi_{n,e}$ are unit spin-polarization vectors of the neutron and electron, and $(E_{e,v}, \vec{E}_{e,v})$ are energies and 3-momenta of the electron and antineutrino. Such a correlation structure is invariant under discrete P, C and T symmetries. The correlation coefficient $T(E_e)$, calculated to leading order in the large nucleon mass $m_N$ expansion, is equal to $|T(E_e)| \sim 1$, where $g_A$ is the axial coupling constant. Within the Standard Model (SM) we describe the correlation coefficient $T(E_e)$ at the level of $10^{-3}$ by taking into the radiative corrections of order $O(\alpha/\pi)$ or the outer model-independent radiative corrections, where $\alpha$ is the fine-structure constant, and the corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil. We calculate also the contributions of interactions beyond the SM, including the contributions of the second class currents.

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I. INTRODUCTION

After the discovery by Chadwick in 1932 [1], neutron has started to play an unprecedentedly important role in particle and nuclear physics, astrophysics and cosmology [2–8]. The experimental analysis of correlation coefficients of the neutron beta decay allows to obtain precise information about the structure of the Standard Model (SM) interactions [9,14] and interactions beyond the SM [2–4,15,16] at low and high energies [16,24]. For the first time the most general form of the electron–energy and angular distribution of the neutron beta decay for a polarized neutron, a polarized electron and an unpolarized proton was proposed by Jackson et al. [25]. Following Jackson et al. [25] the electron–energy and angular distribution of the neutron beta decay for a polarized neutron, a polarized electron and an unpolarized proton can be represented as follows

\[
\frac{d^5 \lambda}{dE_e dE_\nu d\Omega_\nu} (E_e, \vec{E}_e, \vec{E}_\nu, \xi_n, \xi_e) = \left(1 + 3g_A^2\right) \frac{|G|}{16\pi^2} (E_0 - E_e)^2 \sqrt{E_e - m_e^2} E_e F(E_e, Z = 1) \zeta(E_e) \left\{ 1 + b(E_e) \frac{m_e}{E_e} \right\}
\]

\[
+ a(E_e) \left( \frac{\vec{E}_e \cdot \vec{\xi}_e}{E_e E_\nu} + \frac{A(E_e)}{E_e E_\nu} \frac{\vec{\xi}_n \cdot \vec{E}_e}{E_e} + B(E_e) \frac{\vec{\xi}_n \cdot \vec{\xi}_e}{E_\nu} + K_n(E_e) \frac{(\vec{\xi}_n \cdot \vec{E}_e)(\vec{\xi}_e \cdot \vec{E}_\nu)}{E_e E_\nu} + Q_n(E_e) \frac{(\vec{\xi}_n \cdot \vec{E}_e)(\vec{\xi}_e \cdot \vec{E}_\nu)}{E_e E_\nu} \right) + \left( \frac{\vec{E}_\nu \cdot \vec{E}_e}{E_e E_\nu} + G(E_e) \frac{\vec{E}_\nu \cdot \vec{\xi}_e}{E_e} + \frac{H(E_e)}{E_e E_\nu} \frac{\vec{\xi}_n \cdot \vec{E}_e}{E_e} + \frac{N(E_e)}{E_e E_\nu} \frac{\vec{\xi}_n \cdot \vec{\xi}_e}{E_e} + Q_e(E_e) \frac{(\vec{\xi}_n \cdot \vec{E}_e)(\vec{\xi}_e \cdot \vec{E}_\nu)}{E_e E_\nu} \right) \]

\[
+ K_e(E_e) \frac{(\vec{\xi}_n \cdot \vec{E}_e)(\vec{\xi}_e \cdot \vec{E}_\nu)}{E_e E_\nu} + R(E_e) \frac{\vec{\xi}_n \cdot \vec{E}_e}{E_e} + L(E_e) \frac{\vec{\xi}_n \cdot \vec{\xi}_e}{E_e} \right\},
\]

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where we have used the notations in Refs. 28–33. Then, $g_A$ and $G_V$ are the axial and vector coupling constants, respectively, $\xi_n$ and $\xi_e$ are unit spin–polarization vectors of the neutron and electron 28–29, 31 (see also 33), $d\Omega_e$ and $d\rho$ are infinitesimal solid angles in the directions of electron $\vec{k}_e$ and antineutrino $\bar{\nu_e}$ 3–momenta, respectively, $E_0 = (m_n^2 - m_p^2 + m_e^2)/2m_p = 1.2926$ MeV is the end–point energy of the electron–energy spectrum 28, 31, $F(E_e, Z = 1)$ is the relativistic Fermi function, describing the electron–proton final–state Coulomb interaction, is equal to (see, for example, 40, 41 and a discussion in 28)

$$F(E_e, Z = 1) = \left(1 + \frac{1}{2}\right) \frac{4(2p_e m_e \beta)^{2\gamma}}{t^2(3 + 2\gamma)} \frac{e^{\pi\alpha/\beta}}{(1 - \beta^2)^{\gamma/2}} \Gamma\left(1 + \gamma + i \frac{\alpha}{\beta}\right)^2,$$

where $\beta = k_e/E_e = \sqrt{E_e^2 - m_e^2}/E_e$ is the electron velocity, $\gamma = \sqrt{1 - \beta^2} - 1$, $r_p$ is the electric radius of the proton 42. The correlation coefficient $b(E_e)$ is the Fierz interference term 43. The structure and the value of the Fierz interference term may depend on interactions beyond the SM 43. An information of a contemporary theoretical and experimental status of the Fierz interference term can be found in 44–48 (see also 42). The correlation coefficients of the electron-antineutrino correlations $a(E_e)$, the electron asymmetry $A(E_e)$, the antineutrino asymmetry $B(E_e)$ and others $G(E_e)$, $H(E_e)$, $Q_e(E_e)$ and $K_e(E_e)$ survive to leading order in the large nucleon mass $m_N$ expansion 28–29, 31 and depend on the axial coupling constant $g_A$ only. The correlation coefficients $a(E_e)$ and $Q_e(E_e)$ do not violate invariance under discrete symmetries, i.e. parity conservation (P-invariance), time–reversal invariance (T-invariance) and charge conjugation invariance (C-invariance) 11. This is unlike the correlation coefficients $A(E_e)$, $B(E_e)$, $G(E_e)$, $H(E_e)$ and $K_e(E_e)$, which provide a quantitative information about effects of parity violation (P-odd effects) 28. In turn, the correlation coefficients $K_e(E_e)$ and $Q_e(E_e)$ appear only to next-to-leading order in the large nucleon mass $m_N$ expansion. They are caused by the contributions of weak magnetism and proton recoil 49 (see also 41, 50, 51 and 28) and measure the strength of P-odd effects. The correlation coefficients $D(E_e)$, $R(E_e)$ and $L(E_e)$ characterize quantitatively the strength of effects of violation of time-reversal invariance (T-odd effects). In addition, the correlation coefficients $D(E_e)$ and $L(E_e)$ are responsible also for violation of charge invariance (C-odd effects), whereas the correlation coefficient $R(E_e)$ defines also quantitatively effects of violation of parity invariance (P-odd effects) 28, 29. In the SM the correlation coefficients $D(E_e)$, $R(E_e)$ and $L(E_e)$ are induced by the distortion of the Dirac wave function of the electron in the Coulomb field of the proton 28, 29, 52, 53 (see also 28, 31). The radiative corrections of order $O(\alpha/\pi)$, where $\alpha$ is the fine–structure constant 14, to the neutron lifetime and correlation coefficients in Eq. 11 were calculated in 54–62 (see also 56, 51 and 28, 29, 31). The corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil, were calculated in 49, 41, 54, 51 (see also 28, 29, 31). So one may argue that within the SM the correlation coefficients in Eq. 11 were fully investigated theoretically to order $10^{-3}$ caused by radiative corrections of order $O(\alpha/\pi)$ and the corrections of order $O(E_e/m_N)$, induced by weak magnetism and proton recoil.

The electron-energy and angular distribution in Eq. 11 was supplemented by the term with the correlation structure $(\xi_n \cdot \bar{\xi}_n)(\bar{k}_e \cdot \xi_e)/E_e E_0$ and the correlations coefficient $T(E_e)$ by Ebel and Feldman 63. Such a correlation structure is invariant under discrete P, C and T symmetries 11, 25, 29. The main peculiarity of the correlation coefficient $T(E_e)$ in comparison to other correlation coefficients, introduced by Ebel and Feldman 63, is that the correlation coefficients proposed by Jackson et al. 25 in Eq. 11, is to be finite with the absolute value of order of 1, i.e. $|T(E_e)| \sim 1$, to leading order in the large nucleon mass $m_N$ expansion. This paper is addressed to the detailed analysis of the structure of the correlation coefficient $T(E_e)$.

The paper is organized as follows. In section II within the standard effective $V - A$ theory of weak interactions 64 we calculate the correlation coefficient $T(E_e)$ to leading order in the large nucleon mass $m_N$ expansion. Such a correlation coefficient was introduced by Ebel and Feldman 63, who calculated the correlation coefficient by using the effective phenomenological interactions by Jackson et al. 25, 26 only. We show that the absolute value of $T(E_e)$ is of order of 1, i.e. $|T(E_e)| \sim 1$. According to 25, 26, the correlation structure responsible for $T(E_e)$ does not violate invariance under discrete P, C and T symmetries 11. In section III within the SM we give a detailed description of the correlation coefficient $T(E_e)$ at the level of $10^{-3}$ including the radiative corrections of order $O(\alpha/\pi)$, calculated to leading order in the large nucleon mass $m_N$ expansion, and the corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil. In section IV we calculate the contribution of interactions beyond the SM expressed in terms of the phenomenological coupling constants of the effective low-energy weak interactions proposed by Jackson et al. 25. In section V we calculate the contribution of the second class currents or the $G$–odd correlations (regarding $G$–parity invariance of strong interactions, we refer to the paper by Lee and Yang 52) by Weinberg 62. In section VI we discuss the obtained results and perspectives of 1) an experimental analysis of the correlation coefficient $T(E_e)$ and ii) an improvement of the SM description of $T(E_e)$ at the level of $10^{-5}$. In section VII we give a detailed calculation of the contribution of the neutron radiative decay $n \rightarrow p + e^- + \bar{\nu}_e + \gamma$ to the correlation coefficient $T(E_e)$ providing a removal of the dependence of the radiative corrections on the infrared cut–off.
II. THE STRUCTURE OF THE CORRELATION COEFFICIENT $T(E_e)$ TO LEADING ORDER IN THE LARGE NUCLEON MASS EXPANSION IN THE EFFECTIVE $V - A$ THEORY

The analysis of the electron–energy and angular distribution of the neutron beta decay for a polarized neutron, a polarized electron and an unpolarized proton within the standard effective $V - A$ theory of weak low-energy interactions, carried out even to leading order in the large nucleon mass expansion, shows that the set of correlation coefficients in Eq. (1), which survive in such a limit, is not complete. In order to show this we use the standard effective Lagrangian of $V - A$ low–energy weak interactions \[ \mathcal{L}_W(x) = -G_V [\bar{\psi}_p(x)\gamma_\mu(1 - g_A\gamma^5)\psi_n(x)]e(x)\gamma^\mu(1 - \gamma^5)\psi_e(x), \] (3) where $G_V$ and $g_A$ are the vector and axial coupling constants \[2, 3, 11\]. $\psi_p(x)$, $\psi_n(x)$, $\psi_e(x)$ and $\psi_\nu(x)$ are the field operators of the proton, neutron, electron and antineutrino, respectively. $\gamma^\mu = (\gamma^\mu, \gamma^\nu)$ and $\gamma^5$ are the Dirac matrices \[39\]. In the non-relativistic approximation for the neutron and proton the amplitude of the neutron beta decay, calculated with the effective Lagrangian Eq. (3), is \[28\]

\[ M(n \to p e^- \bar{\nu}_e) = -2m_nG_V \left\{ [\bar{\psi}_p^\dagger \phi_n] [\bar{\psi}_e] \gamma_0 (1 - \gamma^5) \psi_\nu + g_A [\bar{\psi}_p^\dagger \phi_n] [\bar{\psi}_e] \gamma_0 (1 - \gamma^5) \psi_\nu \right\}, \] (4)

where $\phi_n$ and $\phi_p$ are Pauli wave functions of the neutron and proton, whereas $\psi_n$ and $\psi_p$ are Dirac wave functions of the electron and antineutrino, $\bar{\sigma}$ are Pauli $2 \times 2$ matrices of the neutron spin \[39\]. According to \[28\], the electron–energy and angular distribution of the neutron beta decay for a polarized neutron, a polarized electron and an unpolarized proton, calculated with the effective Lagrangian Eq. (3), is \[28\]

\[ \frac{d^2\lambda_n(E_e, \vec{k}_e, \vec{\kappa}_e, \vec{\zeta}_e, \vec{\zeta}_e)}{dE_e d\Omega_e d\Omega_\nu} = (1 + 3\lambda^2) \frac{|G_V|^2}{16\pi^2} (E_0 - E_e)^2 \sqrt{E_0^2 - m_n^2} E_e F(E_0, Z = 1) \sum_{\text{pol}} \frac{|M(n \to p e^- \bar{\nu}_e)|^2}{(1 + 3g_A^2)|G_V|^264m_n^2E_eE_\nu}, \] (5)

where we sum over polarizations of the massive fermions. The sum over polarizations of the massive fermions is defined by \[28, 29, 31\]

\[ \sum_{\text{pol}} \frac{|M(n \to p e^- \bar{\nu}_e)|^2}{(1 + 3g_A^2)|G_V|^264m_n^2E_eE_\nu} = \frac{1}{(1 + 3g_A^2)8E_eE_\nu} \{ \text{tr}((1 + \xi_0 \cdot \bar{\sigma}) \cdot \text{tr}((k_e + m_e\gamma^5\zeta_e)\gamma_0\gamma^5\nu_e(1 - \gamma^5)) + g_A \text{tr}((1 + \xi_0 \cdot \bar{\sigma}) \cdot \text{tr}((k_e + m_e\gamma^5\zeta_e)\gamma_0\gamma^5\nu_e(1 - \gamma^5)))) \}

+ g_A^2 \text{tr}((1 + \xi_0 \cdot \bar{\sigma}) \cdot \text{tr}((k_e + m_e\gamma^5\zeta_e)\gamma_0\gamma^5\nu_e(1 - \gamma^5)))) \}, \] (6)

where $\zeta_e$ is the 4-vector of the spin–polarization of the electron. It is defined by \[33\]

\[ \zeta_e = (\xi_e^0, \vec{\xi}_e) = \left( \frac{\vec{k}_e \cdot \vec{\xi}_e}{m_e}, \frac{\vec{k}_e \cdot \vec{\zeta}_e}{m_e(E_e + m_e)} \right). \] (7)

The 4-vector of the spin–polarization of the electron $\zeta_e$ is normalized by $\xi_e^2 = -1$. It obeys also the constraint $k_e \cdot \zeta_e = 0 \ [33]$. Calculating the traces over the nucleon degrees of freedom and using the properties of the Dirac matrices \[34\], the metric of the Minkowski space–time, $\gamma^\mu\gamma^\nu$ is the metric tensor of the Minkowski space–time, $\varepsilon^{\alpha\mu\nu\beta}$ is the Levi–Civita tensor defined by $\varepsilon^{0123} = 1$ and $\varepsilon_{\alpha\mu\nu\beta} = -\varepsilon^{\alpha\mu\nu\beta}$ \[39\], we transcribe the right-hand-side (r.h.s.) of Eq. (6) into the form \[28, 29, 31\]

\[ \sum_{\text{pol}} \frac{|M(n \to p e^- \bar{\nu}_e)|^2}{(1 + 3g_A^2)|G_V|^264m_n^2E_eE_\nu} = \frac{1}{4E_e} \left\{ \left(1 + B_0 \frac{\vec{k}_e \cdot \vec{\kappa}_e}{E_\nu} \right) \text{tr}((k_e + m_e\gamma^5\zeta_e)\gamma_0(1 - \gamma^5)) + \left(A_0 \xi_0 + a_0 \frac{\vec{k}_e}{E_\nu} \right) \cdot \text{tr}((k_e + m_e\gamma^5\zeta_e)\gamma_0(1 - \gamma^5)))) \right\}, \] (9)

where $a_0$, $A_0$ and $B_0$ are defined in terms of the axial coupling constant $g_A$ only \[2, 3\]

\[ a_0 = \frac{1 - g_A^2}{1 + 3g_A^2}, \quad A_0 = \frac{2g_A(1 - g_A)}{1 + 3g_A^2}, \quad B_0 = \frac{2g_A(1 + g_A)}{1 + 3g_A^2}. \] (10)
Having calculated the traces over lepton degrees of freedom, we arrive at the expression

\[ \sum \frac{|M(n \rightarrow pe^-\bar{\nu}_e)|^2}{(1 + 3g_A^2)|G_F|^2 |\bar{m}_e| |E_E| |\bar{E}_0|} = (1 + B_0 \bar{\xi}_e \cdot \bar{\vec{k}}_e |E_E|) \left( 1 - \frac{\bar{\xi}_e \cdot \bar{\vec{k}}_e}{|E_E|} \right) + A_0 \bar{\xi}_e + a_0 \bar{\vec{k}}_e \cdot \frac{\bar{\vec{k}}_e}{|E_E|} \cdot \frac{m_e}{|E_E|} \right) \frac{\bar{\xi}_e}{|E_E|} \left( e \cdot \bar{\vec{k}}_e \cdot \bar{\vec{k}}_e \right) \]

Plugging Eq. (11) into Eq. (5) we obtain the electron–energy and angular distribution of the neutron beta decay, calculated to leading order in the large nucleon mass \( m_N \) expansion. We get

\[ \frac{d^3 \lambda_n(E_e, \bar{\vec{k}}_e, \bar{\vec{\mu}_p}, \bar{\vec{\nu}_n}, \bar{\vec{\xi}}_e)}{dE_e d\Omega_e d\Omega_p} = (1 + 3\lambda^2) \frac{|G_F|^2}{10\pi^{7/2}} (E_0 - E_e)^2 \sqrt{E_L^2 - m_e^2} E_e F(E_e, Z = 1) \]

From the comparison of Eq. (12) with Eq. (11) we determine the following correlation coefficients in the electron–energy and angular distribution, calculated to leading order in the large nucleon mass \( m_N \) expansion:

\[ a(E_e) = a_0 \quad , \quad A(E_e) = A_0 \quad , \quad B(E_e) = B_0 \quad , \quad G(E_e) = -1 , \]

\[ H(E_e) = -a_0 m_e \frac{|E_e|}{|E_E|} \quad , \quad N(E_e) = -a_0 \frac{m_e}{|E_E|} \quad , \quad Q_e(E_e) = -a_0 \quad , \quad K_e(E_e) = -a_0 . \]

(13)

The correlation structure of the last term in Eq. (12) was introduced by Ebel and Feldman [3] with the correlation coefficient \( T(E_e) \). According to our analysis carried out above to leading order in the large nucleon mass expansion, it is equal to \( T(E_e) = -B_0 \). The correlation structure responsible for the correlation coefficient \( T(E_e) \) is invariant under discrete P, C and T symmetries [1].

Having calculated the correlation coefficient \( T(E_e) \) to leading order in the large nucleon mass expansion and without radiative corrections, we may proceed to the analysis of the structure of this correlation coefficient by taking into account the radiative corrections of order \( O(\alpha/\pi) \) or so-called outer model-independent radiative corrections [6] and corrections of order \( O(E_e/m_N) \), caused by weak magnetism and proton recoil. This should give within the SM the theoretical description of the correlation coefficient \( T(E_e) \) at the level of \( 10^{-3} \).

III. THE CORRELATION COEFFICIENT \( T(E_e) \) TO ORDER \( 10^{-3} \)

The procedure of the calculation of the radiative corrections of order \( O(\alpha/\pi) \) and the corrections of order \( O(E_e/m_N) \), caused by weak magnetism and proton recoil, has been expounded well in [28, 29, 31]. For the calculation of these corrections we use the following effective Lagrangian [28, 29, 31]

\[ \mathcal{L}_{\text{W}3}(x) = -G_V \left\{ \bar{\psi}_p(x) \gamma_\mu (1 - g_A \gamma_5) \gamma_\nu \psi_n(x) \right\} + \frac{\kappa}{2m_N} g^\nu [\bar{\psi}_p(x) \sigma_{\mu \nu} \psi_n(x)] \right\} \left\{ \bar{\psi}_e(x) \gamma_\mu (1 - \gamma^5) \gamma_\nu (1 - \gamma^5) \psi_e(x) \right\} \]

\[ e \left\{ \bar{\psi}_p(x) \gamma_\mu \gamma_\nu \psi_n(x) - \bar{\psi}_e(x) \gamma_\mu \gamma_\nu \psi_e(x) \right\} A_\mu (x) . \]

(14)

In comparison to the effective Lagrangian Eq. (35), we have added i) the hadronic current an additional term with the Lorentz structure, defined the Dirac matrix \( \sigma^{\mu \nu} = \frac{i}{2}(\gamma^{\mu} \gamma^{\nu} - \gamma^{\nu} \gamma^{\mu}) \) [20], where \( \kappa = \kappa_p - \kappa_n = 3.7059 \) is the isovector anomalous magnetic moment of the nucleon, induced by the anomalous magnetic moments of the proton \( \kappa_p = 1.7929 \) and the neutron \( \kappa_n = -1.9130 \) and measured in nuclear magneton [14], and ii) the electromagnetic interactions for the proton and electron, where \( e \) is the electric charge of the proton and \( A_\mu (x) \) is the 4-vector potential of the electromagnetic field.

Using the amplitude of the neutron beta decay (see Eq. (D-52) in Appendix D of [28], with a replacement \( \tilde{\lambda} = -\tilde{g}_A \), where \( \tilde{g}_A = g_A(1 - E_0/2m_N) \)), following [28, 31] and skipping intermediate calculations we obtain

\[ \zeta(E_e)T(E_e) = -B_0 \left( 1 + \frac{\alpha}{\pi} f_{\beta e}(E_e, \mu) \right) + \frac{1}{1 + 3g_A^2 m_N E_0} \left( g_A(2g_A + (\kappa + 1)) - (g_A^2 + g_A(3\kappa + 2) + (\kappa + 1)) \right) \frac{E_e}{E_0} . \]

(15)
where the function \( f_{\beta \gamma}(E, \mu) \) was calculated by Sirlin [54] (for the calculation of the function \( f_{\beta \gamma}(E, \mu) \) in details we refer to [28]), \( \mu \) is a covariant infrared cut-off having a meaning of the photon mass [54]. For the definition of the function \( \zeta(E) \) we refer to [25, 26], where the function \( \zeta(E) \) was calculated to leading order in the large nucleon mass \( m_N \) expansion. However, below we shall use the function \( \zeta(E) \), calculated in [28] to order \( 10^{-3} \) by taking into account the contributions of the radiative corrections of order \( O(\alpha/\pi) \) and the corrections of order \( O(E_e/m_N) \), caused by weak magnetism and proton recoil (see also [51]). According to [67], the function \( f_{\beta \gamma}(E, \mu) \) defines the contribution of model–independent or outer radiative corrections. Following [28] the function \( f_{\beta \gamma}(E, \mu) \) can be rewritten as follows

\[
 f_{\beta \gamma}(E, \mu) = g_0(E) + \frac{1 - \beta^2}{2\beta} \ln \frac{1 + \beta}{1 - \beta} - g^{(1)}_{\beta \gamma}(E, \mu),
\]

where \( 2g_0(E) \) is Sirlin’s function, defining the outer radiative corrections of order \( O(\alpha/\pi) \) to the neutron lifetime [54].

The function \( g^{(1)}_{\beta \gamma}(E, \mu) \) corresponds to the contribution of the neutron radiative beta decay \( n \to p + e^- + \bar{\nu} + \gamma \) with a real photon \( \gamma \), which should be added, according to Berman [68] and Kinoshita and Sirlin [69] (see also Sirlin [51]), for the removal of the dependence of the neutron lifetime on the infrared cut–off. Following [28] the function \( g^{(1)}_{\beta \gamma}(E, \mu) \) can be rewritten as follows

\[
 g^{(1)}_{\beta \gamma}(E, \mu) = \int \frac{d\Omega}{4\pi} \int_{E_0}^{E_0 - E} \frac{d\omega}{\omega} \left[ 1 + \frac{\omega^2}{2E_e^2} \right] \left[ \frac{1}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 2 + \frac{\omega^2}{E_e^2} \right]
\]

where \( \omega = \sqrt{q^2 + \mu^2}, \quad q = |\vec{q}| \) and \( \vec{v} = \vec{q}/q_0 \). The analytical expression of the function \( g^{(1)}_{\beta \gamma}(E, \mu) \), which was calculated in [28], we adduce in section [VIII] for completeness. Then, in Eq. (17) we integrate over directions of the 3–momentum \( q \) of the photon [28]. As has been shown in [28, 29, 31] for the calculation of the radiative corrections to the correlation coefficients the function \( g^{(1)}_{\beta \gamma}(E, \mu) \) can be also regularized by a non-covariant infrared cut-off \( \omega_{\text{min}} \).

Setting \( \mu = 0 \) in Eq. (17) and integrating over directions of the 3–momentum of the photon we arrive at the integral [28] (see Eq.(B-15) in Appendix B in Ref. [28])

\[
 g^{(1)}_{\beta \gamma}(E, \omega_{\text{min}}) = \int_{\omega_{\text{min}}}^{E_0 - E} \frac{d\omega}{\omega} \left( \frac{E_0 - E - \omega}{E_0 - E} \right)^2 \left[ 1 + \frac{\omega^2}{2E_e^2} \right] \left[ \frac{1}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 2 + \frac{\omega^2}{E_e^2} \right],
\]

where \( \omega_{\text{min}} \) is a non-covariant infrared cut-off, which can be also treated as the photon–energy threshold of the detector [28].

We would like to emphasize that the function \( g^{(1)}_{\beta \gamma}(E, \mu) \) removes the dependence on the infrared cut–off \( \mu \) only in the neutron lifetime or in the function \( \zeta(E) \) [28]. The dependence of the radiative corrections on the infrared cut–off in the correlation coefficients of the neutron beta decay should be removed by the contributions of the corresponding correlation coefficients in the neutron radiative beta decay [28, 29, 31]. The correlation coefficient of the neutron radiative beta decay, caused by the correlation structure \( \langle \xi_n \cdot \vec{k}_e \langle E_e \rangle \rangle / E_e \), we have calculated in section [VIII] It is given by the function \( g^{(2)}_{\beta \gamma}(E, \mu) \) (see Eqs. [A-11] and [A-12]). Adding the contribution of the neutron radiative beta decay to the correlation coefficient \( \zeta(E)T(E_e) \) in Eq. (15) with the definition of the function \( f_{\beta \gamma}(E, \mu) \) in Eq. (10) and taking the limit \( \mu \to 0 \) we transcribe the correlation coefficient \( \zeta(E)T(E_e) \) into the form

\[
 \zeta(E)T(E_e) = -B_0 \left\{ 1 + \frac{\alpha}{\pi} g_0(E) + \lim_{\mu \to 0} \left( g^{(2)}_{\beta \gamma}(E, \mu) - g^{(1)}_{\beta \gamma}(E, \mu) \right) + \frac{1 - \beta^2}{2\beta} \left( \frac{1 + \beta}{1 - \beta} \right) \right\}
\]

\[
 + \frac{1}{1 + 3g^2 A \frac{E_0}{m_N}} \left( g_0(a + (\kappa + 1)) - \left( g_0 + g_A(3\kappa + 2) + (\kappa + 1) \right) \frac{E_e}{E_0} \right).
\]

Since, according to [28, 29, 31], the difference \( \lim_{\mu \to 0} \left( g^{(2)}_{\beta \gamma}(E, \mu) - g^{(1)}_{\beta \gamma}(E, \mu) \right) \) is equal to

\[
 \lim_{\omega_{\text{min}} \to 0} \left( g^{(2)}_{\beta \gamma}(E, \omega_{\text{min}}) - g^{(1)}_{\beta \gamma}(E, \omega_{\text{min}}) \right) = \lim_{\omega_{\text{min}} \to 0} \left( g^{(2)}_{\beta \gamma}(E, \omega_{\text{min}}) - g^{(1)}_{\beta \gamma}(E, \omega_{\text{min}}) \right),
\]

we may rewrite Eq. (19) as follows

\[
 \zeta(E)T(E_e) = -B_0 \left\{ 1 + \frac{\alpha}{\pi} g_0(E) + \lim_{\omega_{\text{min}} \to 0} \left( g^{(2)}_{\beta \gamma}(E, \omega_{\text{min}}) - g^{(1)}_{\beta \gamma}(E, \omega_{\text{min}}) \right) + \frac{1 - \beta^2}{2\beta} \left( \frac{1 + \beta}{1 - \beta} \right) \right\}
\]

\[
 + \frac{1}{1 + 3g^2 A \frac{E_0}{m_N}} \left( g_0(a + (\kappa + 1)) - \left( g_0 + g_A(3\kappa + 2) + (\kappa + 1) \right) \frac{E_e}{E_0} \right).
\]
Using Eqs. (18) and (A.11), taking the limit $\omega_{\text{min}} \to 0$ and integrating over the photon energy $\omega$ in the region $0 \leq \omega \leq E_0 - E_e$ (see, for example, [28]) we define the correlation coefficient $\zeta(E_e)T(E_e)$ by taking into account the outer radiative corrections of order $O(\alpha/\pi)$, calculated to leading order in the large nucleon mass $m_N$ expansion, and the corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil

$$\zeta(E_e)T(E_e) = -B_0 \left( 1 + \frac{\alpha}{\pi} g_\alpha(E_e) + \frac{\alpha}{\pi} f_n(E_e) \right) + \frac{1}{1 + 3g_A^2 m_N} \left( 2g_A (g_A + (\kappa + 1)) \right. $$

$$\left. - (g_A^2 + g_A (3\kappa + 2) + (\kappa + 1)) \frac{E_e}{E_0} \right),$$

(22)

where the function $f_n(E_e)$ is equal to (see also [28, 29])

$$f_n(E_e) = \lim_{\omega_{\text{min}} \to 0} \left( g_{\beta\gamma}(E_e, \omega_{\text{min}}) - g_{\beta\gamma}^{(1)}(E_e, \omega_{\text{min}}) \right) + \frac{1 - \beta^2}{2\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) =$$

$$= \frac{1}{3} \frac{E_0 - E_e}{E_e} \left( 1 + \frac{1}{8} \frac{E_0 - E_e}{E_e} \right) \frac{1 - \beta^2}{\beta^2} \left[ \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 2 \right] - \frac{1}{12} \frac{E_0 - E_e}{E_e^2} + \frac{1 - \beta^2}{2\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right).$$

For the first time the function $f_n(E_e)$, defining the radiative corrections $(\alpha/\pi)$ of the electron–antineutrino correlations $a(E_e)$ and of the electron asymmetry $A(E_e)$, was calculated by Shann [55] (see also Eq.(D-58) in Appendix D in Ref. [28]).

Using the correlation function $\zeta(E_e)$, calculated in [28] (see Eq.(6) with the replacement $\lambda = -g_A$), we define the correlation coefficient $T(E_e)$

$$T(E_e) = -B_0 \left( 1 + \frac{\alpha}{\pi} f_n(E_e) \right) + \frac{1}{1 + 3g_A^2 m_N} \left( 2g_A (g_A + (\kappa + 1)) \right. $$

$$\left. - (g_A^2 + g_A (3\kappa + 2) + (\kappa + 1)) \frac{E_e}{E_0} \right) +$$

$$+ \frac{B_0}{1 + 3g_A^2 m_N} \left( - 2g_A (g_A + (\kappa + 1)) \right. $$

$$\left. + (10g_A^2 + 4g_A (\kappa + 1) + 2) \right. $$

$$\left. \frac{E_e}{E_0} - 2g_A (g_A + (\kappa + 1)) \frac{m_N^2}{E_0} \frac{E_0}{E_e} \right).$$

(24)

This is a complete description of the correlation coefficient $T(E_e)$ at the level of $10^{-3}$ including the outer radiative corrections of order $O(\alpha/\pi)$, calculated to leading order in the large nucleon mass $m_N$ expansion, and the corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil.

IV. CONTRIBUTIONS OF INTERACTIONS BEYOND THE STANDARD MODEL [25]

For the calculation of the contributions of interactions beyond the SM we use the effective phenomenological Lagrangian proposed in [25] (see also [62, 71, 72]). It reads

$$\mathcal{L}_{\text{BSM}}(x) = -G_V \left\{ [\bar{\psi}_p(x)\gamma_\mu \psi_n(x)] [\bar{\psi}_e(x)\gamma^\mu (C_V + \bar{C}_V \gamma^5) \psi_{\nu_e}(x)] + [\bar{\psi}_p(x)\gamma_\mu \gamma^5 \psi_n(x)] [\bar{\psi}_e(x)\gamma^\mu (C_A + C_A \gamma^5) \psi_{\nu_e}(x)] \right\}$$

$$+ [\bar{\psi}_p(x)\psi_n(x)] [\bar{\psi}_e(x)(C_S + C_S \gamma^5) \psi_{\nu_e}(x)] + [\bar{\psi}_p(x)\gamma^5 \psi_n(x)] [\bar{\psi}_e(x)(C_P + \bar{C}_P \gamma^5) \psi_{\nu_e}(x)] + \frac{1}{2} [\bar{\psi}_p(x)\sigma_{\mu\nu} \gamma^5 \psi_n(x)]$$

$$\times [\bar{\psi}_e(x)\sigma_{\mu\nu} (\bar{C}_T + C_T \gamma^5) \psi_{\nu_e}(x)],$$

(25)

where we have followed the notations in [28]. The effective phenomenological Lagrangian $\mathcal{L}_{\text{BSM}}(x)$ reduces to the standard effective Lagrangian $\mathcal{L}_V(x)$ of $V-A$ weak low–energy interactions Eq. (8) by the replacement $C_V = -\bar{C}_V = 1, C_A = -\bar{C}_A = g_A$ and $C_S = C_S = C_P = C_P = 0$. Following [25] (see Appendix G in Ref. [28]) and skipping intermediate calculations, carried out in the approximation of the leading order in the large nucleon mass $m_N$ expansion, we get

$$\xi T(E_e) = -2 \Re \left( C_V C_A^* + \bar{C}_V \bar{C}_A + |C_A|^2 + |\bar{C}_A|^2 + C_S C_P^* + \bar{C}_S \bar{C}_P - |C_P|^2 - |\bar{C}_P|^2 \right),$$

(26)

where the factor $\xi$ is equal to [23, 28]

$$\xi = |C_V|^2 + |\bar{C}_V|^2 + 3|C_A|^2 + 3|\bar{C}_A|^2 + |C_S|^2 + |\bar{C}_S|^2 + 3|C_P|^2 + 3|\bar{C}_P|^2.$$

(27)

Our result in Eq. (26) agrees well with the result obtained by Ebel and Feldman [63] but without contributions of imaginary parts of the phenomenological scalar and tensor coupling constants, which are proportional to the factor
$am_e/k_e$, caused by the distortion of the Dirac wave function of the electron in the Coulomb field of the proton \[23\].

In this connection, we would like to remind that, according to \[14, 15\], the phenomenological scalar coupling constants should be zero, i.e. $C_S = \bar{C}_S = 0$. Then, we would like to notice that the contribution of the phenomenological pseudoscalar coupling constants $C_P$ and $\bar{C}_P$ vanishes to leading order in the large nucleon mass expansion (see, for example, \[23\]).

At the replacement $C_V = -\bar{C}_V = 1$, $C_A = -\bar{C}_A = g_A$ and $C_S = \bar{C}_S = C_P = \bar{C}_P = C_T = \bar{C}_T = 0$ the factor $\xi$ reduces to $\xi = 2(1 + 3g_A^2)$. In the linear approximation for the phenomenological vector and axial-vector coupling constants $C_V = 1 + \delta C_V, \bar{C}_V = -1 + \delta C_V, C_A = g_A + \delta C_A$ and $\bar{C}_A = -g_A + \delta \bar{C}_A$ the contributions of the vector and axial-vector phenomenological coupling constants can be absorbed by renormalization of the axial coupling constant $g_A$ and the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{ud}$ (it is included into $G_V$) \[18–22\] (see also \[28, 29, 31\]).

Thus, in the linear approximation for the phenomenological vector, axial-vector, scalar and tensor coupling constants \[18–22\] (see also \[28, 29, 31\]) the contribution of interactions beyond the SM, defined by the effective Lagrangian Eq. \[25\], is equal to zero, i.e. $T^{(\text{OSM})}(E_e) = 0$.

V. CONTRIBUTIONS OF THE SECOND CLASS CURRENTS OR THE $G$–ODD CORRELATIONS

The $G$–parity transformation, i.e. $G = C e^{i\pi I_3}$, where $C$ and $I_3$ are the charge conjugation and isospin operators, was introduced by Lee and Yang \[65\] as a symmetry of strong interactions. According to the properties of hadronic currents under $G$–transformation, Weinberg divided hadronic currents into two classes, which are $G$–even first class and $G$–odd second class currents \[66\], respectively. Following Weinberg \[66\], Gardner and Zhang \[73\], and Gardner and Plaster \[74\] (see also \[28, 31\]) the $G$–odd contributions to the matrix element of the hadronic $n \rightarrow p$ transition, caused by the hadronic current, in the $V – A$ theory of weak interactions can be taken in the form

$$\langle p(\vec{k}_\mu, \sigma_p)|J^{(+)\mu}_n(0)|n(\vec{k}_n, \sigma_n)\rangle = \bar{u}_p(\vec{k}_\mu, \sigma_p)\left(\frac{g_A}{m_N} f_3(0) + i\sigma_{\mu\nu}g^{\nu\delta} g_2(0)\right) u_n(\vec{k}_n, \sigma_n),$$

(28)

where $J^{(+)\mu}_n(0) = V^{(+)\mu}_n(0) - A^{(+)\mu}_n(0)$, $\bar{u}_p(\vec{k}_\mu, \sigma_p)$ and $u_n(\vec{k}_n, \sigma_n)$ are the Dirac wave functions of the proton and neutron \[30\]. Then, $f_3(0)$ and $g_2(0)$ are the phenomenological coupling constants defining the strength of the second class currents in the weak decays. Following \[30, 31\] and skipping intermediate calculations we get the contribution of the second class currents or the $G$–odd correlations to the correlation coefficient $T(E_e)$

$$T^{(G\text{–odd})}(E_e) = \text{Reg}_2(0) \frac{1}{1 + 3g_A^2} \frac{E_0}{m_N} \left(4(g_A + 1) + 2(g_A - 1) \frac{E_e}{E_0}\right).$$

(29)

Apart from the dependence of $T^{(G\text{–odd})}(E_e)$ on the axial coupling constant $g_A$, the contribution of the second class currents or the $G$–odd correlations to the correlation coefficient $T(E_e)$ is represented by the phenomenological coupling constant $\text{Reg}_2(0)$ only.

VI. DISCUSSION

We have analyzed the correlation coefficient $T(E_e)$, caused by the correlation structure $(\vec{e}_n \cdot \vec{k}_e)(\vec{e}_e \cdot \vec{k}_e)/E_e E_0$ invariant under discrete P, C and T symmetries. Such a correlation structure was introduced by Ebel and Feldman \[65\] in addition to the set correlation structures proposed by Jackson et al. \[22, 25\]. The correlation coefficient $T(E_e)$, calculated to leading order in the large nucleon mass $m_N$ expansion within the standard effective $V – A$ theory of weak interactions \[54\], is equal to $T(E_e) = -B_0$, where $B_0 \sim 1 \times 10^{-3}$. Having calculated the correlation coefficient $T(E_e)$ to leading order in the large nucleon mass $m_N$ expansion, we have given within the SM a complete description of the correlation coefficient $T(E_e)$ at the level of $10^{-3}$ by taking into account the outer model-independent radiative corrections of order $O(\alpha/\pi)$, calculated to leading order in the large nucleon mass expansion, and the corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil. In addition we have calculated the contributions of interactions beyond the SM, expressed in terms of i) the phenomenological coupling constants of the effective phenomenological interactions proposed by Jackson et al. \[23\], and ii) the phenomenological coupling constants of the second class currents, measuring the strength of $G$–odd correlations \[66, 73, 74\] (see also \[30, 31\]). We have found that in the linear approximation for the phenomenological vector, axial-vector, scalar and tensor coupling constants \[18–22\] (see also \[28, 29, 31\]) the contribution of interactions beyond the SM, defined by the effective Lagrangian Eq. \[25\], is equal to zero, i.e. $T^{(\text{OSM})}(E_e) = 0$. As a result, in such an approximation the interactions beyond the SM are represented in the correlation coefficient $T(E_e)$ by the second class currents or $G$–odd correlations only. Our result in Eq. \[29\] for the contribution of interactions beyond the SM, defined by the phenomenological coupling constants of
the phenomenological interactions proposed by Jackson et al. [25], agrees well with the result obtained by Ebel and Feldman [26], but without contributions of the imaginary parts of the phenomenological scalar and tensor coupling constants proportional to the factor $a_{mc}/k_e$, caused by the distortion of the Dirac wave function of the electron in the Coulomb field of the proton. Then, we would like to remind that, according to [14][13], the contributions of the phenomenological scalar coupling constants should vanish.

Summing up all corrections we obtain the following expression for the correlation coefficient $T(E_e)$:

$$
T(E_e) = -B_0 \left( 1 + \frac{\alpha}{\pi} f_\alpha(E_e) \right) + \frac{1}{1 + 3g_A} \frac{E_0}{m_N} \left( 2g_A (g_A + (\kappa + 1)) - (g_A^2 + g_A (3\kappa + 2) + (\kappa + 1)) \frac{E_0}{E_e} \right) + \frac{B_0}{1 + 3g_A} \frac{E_0}{m_N} \left( -2g_A (g_A + (\kappa + 1)) + (10g_A^2 + 4g_A (\kappa + 1) + 2) \frac{E_0}{E_e} - 2g_A (g_A + (\kappa + 1)) \frac{m_N^2 E_0}{E_e} \right) + \text{Reg}_2(0) \left( \frac{1}{1 + 3g_A} \frac{E_0}{m_N} \left( 4(g_A + 1) + 2(2 - g_A - 1) \frac{E_0}{E_e} \right) \right).
$$

The correlation coefficient $T(E_e)$, calculated for $g_A = 1.2764$ [30], $E_0 = 1.2926$ MeV, $m_N = 938.9188$ MeV, $m_e = 0.5110$ MeV and $\kappa = 3.7059$ [14], is equal to

$$
T(E_e) = - \left( 0.987 - 4.63 \times 10^{-5} - 2.13 \times 10^{-3} \text{Reg}_2(0) \right) - 2.29 \times 10^{-3} f_\alpha(E_e) + \left( 4.37 \times 10^{-3} + 1.29 \times 10^{-4} \text{Reg}_2(0) \right) \frac{E_0}{E_e} - 5.51 \times 10^{-4} \frac{E_0}{E_e}.
$$

According to [71][74] and [30][31], in the correlation coefficient $T(E_e)$ the contribution of the second class currents may be estimated at the level of a few parts of $10^{-5}$ and even smaller. Thus, the numerical analysis of the correlation coefficient $T(E_e)$ shows that such a correlation coefficient can be a nice tool for experimental probes of contributions of the second class currents or $G$-odd correlations in terms of the phenomenological coupling constant $\text{Reg}_2(0)$ in Eq. (28). However, it is obvious that successful experimental searches of such contributions it is required the description of the correlation coefficient $T(E_e)$ within the SM at the level of $10^{-5}$ and as well as experimental uncertainties at the level of a few parts of $10^{-5}$. We are planning to carry out such a theoretical description of the correlation coefficient $T(E_e)$ in our forthcoming publication by taking into account the results, obtained in [33][35][38] and also in [11] in terms of Wilkinson’s corrections (see also [28][29][31]).

A rather complicated correlation structure $(\vec{\xi}_n \cdot \vec{p}) (\vec{\xi}_e \cdot \vec{p}_e)/E_e E_p$ responsible for the correlation coefficient $T(E_e)$, which entangles the spin–polarization vectors of the neutron and electron and 3-momenta of the electron and antineutrino, makes difficult its experimental analysis. Indeed, the experimental investigation of the correlation coefficient $T(E_e)$ should be performed for polarized neutrons and longitudinally polarized decay electrons. This is unlike the experiments i) on the neutron beta decay for polarized neutrons and transversally polarized electrons [15][24][77][78] and ii) on the nuclear beta decays for unpolarized nuclei and longitudinally polarized decay positrons [12][73][74]. Moreover, the dependence of the correlation structure on the antineutrino 3–momentum demands a simultaneous detection of decay electrons and protons, i.e. electron-proton pairs, similar to the measurements of the antineutrino asymmetry [81][82]. The theoretical analysis of the antineutrino asymmetry in the neutron beta decay, related to the correlation coefficient $B(E_e)$, was carried out by Glück et al. [84] (see also [28]). We are planning to perform an analogous theoretical analysis of the asymmetry, related to the correlation coefficient $T(E_e)$, in our forthcoming publication.

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23 (FH-Call 16) under the project “Photonik - Stiftungsprofessur für Lehre”.

25 [28, 29, 31]
VIII. THE SUPPLEMENTAL MATERIAL

Appendix A: The electron-photon-energy and angular distribution of the neutron radiative beta decay for a polarized neutron, a polarized electron and unpolarized proton and photon

In order to remove the dependence of the radiative corrections to the neutron lifetime and correlation coefficients of the neutron beta decay on the infrared cut–off \( \mu \) we have to add the contribution of the neutron radiative beta decay \( n \to p + e^- + \bar{\nu}_e + \gamma \) \(^{68,69}\) (see also \(^{53,55,56,57,58,28,29,31}\)). Following \(^{28,29,31}\) we define the electron-photon-energy and angular distribution of the neutron radiative beta decay for a polarized neutron, a polarized electron, a polarized photon and an unpolarized proton as follows

\[
\frac{d^3\lambda_{\beta^-}}{d\omega dE_{\text{e}} d\Omega_{\text{e}} d\Omega_\gamma} = \left(1 + 3 \lambda^2 \right) \frac{\alpha}{\pi} \frac{|G|^2}{(2\pi)^6} \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) \left(E_0 - E_e - \omega \right)^2 \frac{1}{\omega} \\
\times \sum_{\text{pol.}} \frac{|M(n \to p e^- \bar{\nu}_e \gamma)|^2}{(1 + 3 g_\lambda^2)\epsilon^2 |G|^2 64m_e^2 E_e E_0},
\]

(A-1)

where we sum over polarizations of massive fermions. The photon state is determined by the 4–momentum \( q^\mu = (\omega, \vec{q}) \) and the 4-vector of polarization \( \epsilon^\mu(q) \) with \( \lambda = 1, 2 \), obeying the constraints \( \epsilon^\mu(q) \cdot \epsilon_{\lambda}(q) = -\delta_{\lambda\mu} \) and \( q \cdot \epsilon_{\lambda}(q) = 0 \). The sum over polarizations of the massive fermions is defined by \(^{28,29,31}\)

\[
\sum_{\text{pol.}} |M(n \to p e^- \bar{\nu}_e \gamma)|^2 = \frac{1}{(1 + 3 g_\lambda^2) |G|^2 64m_e^2 E_e E_0} \left( \epsilon^\mu \epsilon_{\lambda} \right) \left( \epsilon^\nu \epsilon_{\sigma} \right) \left( \epsilon^\rho \epsilon_{\delta} \right) \left( \epsilon^\theta \epsilon_{\gamma} \right) \left( \epsilon^\nu \epsilon_{\sigma} \right) \left( \epsilon^\rho \epsilon_{\delta} \right) \left( \epsilon^\theta \epsilon_{\gamma} \right) \left( \epsilon^\mu \epsilon_{\lambda} \right).
\]

(A-2)

where we have denoted a unit 3-vector \( \vec{n} = \vec{q}/\omega, Q_\lambda = 2 \epsilon^\lambda_\nu(q) \cdot k_e + \epsilon^\nu_\lambda(q) \bar{q} \) and \( Q_\gamma = \gamma^0 Q_\lambda \gamma^0 = 2 \epsilon_{\lambda\nu}(q) \cdot k_e + \epsilon_{\nu\lambda}(q) \bar{q} \) \(^{28,29,31}\). Calculating the traces over the nucleon degrees of freedom and using the properties of the Dirac matrices

\[
\gamma^\alpha \gamma^\beta \gamma^\mu = \gamma^0 \eta^{\mu\alpha} - \gamma^\nu \eta^{\alpha\nu} + \gamma^\mu \eta^{\mu\nu} + i \epsilon^{\alpha\nu\beta} \gamma^\nu \gamma^\gamma \gamma^\lambda,
\]

(A-3)

where \( \eta^{\mu\nu} \) is the metric tensor of the Minkowski space–time, \( \epsilon^{\alpha\nu\beta} \) is the Levi–Civita tensor defined by \( \epsilon^{0123} = 1 \) and \( \epsilon^{\alpha\nu\beta} = -\epsilon^{\alpha\beta\nu} \) \(^{39}\), we transcribe the right-hand–side (r.h.s.) of Eq.(A-2) into the form \(^{28,29,31}\)

\[
\sum_{\text{pol.}} |M(n \to p e^- \bar{\nu}_e \gamma)|^2 = \frac{1}{(1 + 3 g_\lambda^2) |G|^2 64m_e^2 E_e E_0} \left( \epsilon^\mu \epsilon_{\lambda} \right) \left( \epsilon^\nu \epsilon_{\sigma} \right) \left( \epsilon^\rho \epsilon_{\delta} \right) \left( \epsilon^\theta \epsilon_{\gamma} \right) \left( \epsilon^\nu \epsilon_{\sigma} \right) \left( \epsilon^\rho \epsilon_{\delta} \right) \left( \epsilon^\theta \epsilon_{\gamma} \right) \left( \epsilon^\mu \epsilon_{\lambda} \right).
\]

(A-4)

where the ellipsis denotes the contributions of the terms, which possess the correlation structures different to the correlation structure responsible for the correlation coefficient \( T(E_e) \). In Eq.(A-4) in the covariant form the traces over Dirac matrices were calculated in \(^{28,31}\). The result is

\[
\frac{1}{16} \left( \epsilon^\lambda_\nu \epsilon^\mu \epsilon_{\gamma} + \epsilon^\nu_\lambda \epsilon_{\gamma} \epsilon^\mu \epsilon_{\nu} \epsilon_{\lambda} \right) q^\mu - \frac{1}{2} \left( \epsilon^\lambda_\nu \epsilon^\mu \epsilon_{\gamma} \epsilon_{\lambda} \epsilon_{\nu} + \epsilon^\nu_\lambda \epsilon_{\gamma} \epsilon^\mu \epsilon_{\lambda} \epsilon_{\nu} \right) \epsilon_{\nu} - \frac{1}{2} \epsilon^\nu \epsilon_{\gamma} \epsilon_{\lambda} \epsilon_{\nu} \epsilon_{\lambda} \epsilon^\mu \epsilon_{\gamma} \epsilon_{\nu} \epsilon_{\lambda} \epsilon_{\nu}.
\]

(A-5)

where the right hand side \( q^\mu \) and \( \epsilon_{\lambda} \) obeys the constraint

\[
q^\nu \epsilon_{\lambda}^\nu = 0, \epsilon^\lambda_\nu \epsilon_{\lambda} \epsilon_{\nu} = 0, \sum_{\lambda=1,2} \epsilon^\lambda_i \epsilon_{\lambda}^i = \delta^{ij} - \frac{k^i k^j}{\omega^2} = \delta^{ij} - \vec{n} \cdot \vec{n}, \sum_{j=1,2,3} \sum_{\lambda=1,2} \epsilon_{\lambda}^j \epsilon_{\lambda}^i = 2.
\]

(A-6)
In the physical gauge $\varepsilon_\lambda = (0, \varepsilon_\lambda)$ we obtain for the r.h.s. of Eq.\,[A5] the following expression

$$\sum_{\text{pol}} \frac{|M(n \rightarrow p e^- \bar{v}_e \gamma)|^2}{(1 + 3g^2_n)^2 |G_{VF}|^2 16m^2_{\bar{\nu}} E_e E_0} = \frac{-B_0}{(E_e - \bar{n} \cdot \bar{k}_e)^2} \frac{m_e}{E_e} \left\{ \left( \varepsilon_\lambda \cdot \bar{k}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) 0^0_{\lambda, \lambda} + \frac{1}{2} \left( \varepsilon_\lambda \cdot \bar{K}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \right\} + \frac{1}{2} \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \omega + \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \omega + \ldots \right\}, \quad (A-7)$$

where the ellipsis denotes the contribution of the terms, which vanish after the summation over polarizations of the photon. Plugging Eq.\,[A7] into Eq.\,[A3] we obtain the contribution to the electron-photon-energy and angular distribution of the neutron radiative beta decay for a polarized neutron, a polarized electron, a polarized photon and an unpolarized proton, which should be responsible for a cancellation of the dependence of the radiative corrections to the correlation coefficient $T(E_e)$ on the infrared cut-off. We get

$$\frac{d^3 \lambda_{\beta\gamma} (E_e, \omega, \bar{k}_e, \bar{\nu}_e, \bar{\eta}_e, \bar{\xi}_e, \bar{\zeta}_e, \bar{\epsilon}_e)}{d\omega dE_e d\Omega_e} = \frac{(1 + 3g^2_n) \alpha |G_{VF}|^2}{16\pi^5} \sqrt{E^2_e - m^2_e} E_e F(E_e, Z = 1) \left( E_0 - E_e - \omega \right)^2 \frac{1}{\omega} \times \left\{ -B_0 \frac{\bar{k}_e \cdot \bar{\nu}_e}{E_e} \frac{m_e}{E_e} \left( \varepsilon_\lambda \cdot \bar{k}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) 0^0_{\lambda, \lambda} + \frac{1}{2} \left( \varepsilon_\lambda \cdot \bar{K}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \omega + \frac{1}{2} \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \omega \ldots \right\}, \quad (A-8)$$

where ellipses denote the contributions of the terms, which vanish after the summation over polarizations of the photon and which define the contributions of other correlation structures [28, 29, 31], respectively. Summing up over polarizations of the photon we determine the electron-photon-energy and angular distribution of the neutron radiative beta decay in terms of the integrals over directions of the 3-momentum of the photon

$$\frac{d^3 \lambda_{\beta\gamma} (E_e, \omega, \bar{k}_e, \bar{\nu}_e, \bar{\eta}_e, \bar{\xi}_e, \bar{\zeta}_e, \bar{\epsilon}_e)}{d\omega dE_e d\Omega_e} = \frac{(1 + 3g^2_n) \alpha |G_{VF}|^2}{16\pi^5} \sqrt{E^2_e - m^2_e} E_e F(E_e, Z = 1) \left( E_0 - E_e - \omega \right)^2 \frac{1}{\omega} \times \left\{ -B_0 \frac{\bar{k}_e \cdot \bar{\nu}_e}{E_e} \frac{m_e}{E_e} \int d\Omega_e \frac{\langle 0^2_{\lambda, \lambda} (E_e - \bar{n} \cdot \bar{k}_e)^2 \rangle}{(E_e - \bar{n} \cdot \bar{k}_e)^2} \left\{ \varepsilon_\lambda^2 - (\bar{n} \cdot \bar{k}_e)^2 \right\} \left( \varepsilon_\lambda \cdot \bar{k}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \omega + \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \left( \varepsilon_\lambda \cdot \bar{\nu}_e \right) \omega \ldots \right\}. \quad (A-9)$$

For the calculation of the integrals over the directions of the 3-momentum of the photon we use the results, obtained in [29]. Having integrated over the directions of the 3-momentum and energy of the photon we get

$$\frac{d^3 \lambda_{\beta\gamma} (E_e, \omega, \bar{k}_e, \bar{\nu}_e, \bar{\eta}_e, \bar{\xi}_e, \bar{\zeta}_e, \bar{\epsilon}_e)}{d\omega dE_e d\Omega_e} \left( \bar{e}_{\beta\gamma} \right) (E_e, \omega_{\min}) + \ldots \right\}, \quad (A-10)$$

where the function $g_{\beta\gamma}^{(2)} (E_e, \omega_{\min})$ is defined by the integral

$$g_{\beta\gamma}^{(2)} (E_e, \omega_{\min}) = \int_{\omega_{\min}}^{E_0 - E_e} d\omega \frac{\left( E_0 - E_e - \omega \right)^2}{\omega \left( E_0 - E_e \right)^2} \left[ 1 + \frac{1}{\beta^2} \omega \left( 1 + \frac{\omega}{2 E_e} \right) \right] \left[ \frac{1}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 2 \right]. \quad (A-11)$$

The function $g_{\beta\gamma}^{(2)} (E_e, \mu)$, regularized by the covariant infrared cut-off $\mu$, was calculated in [28] (see Eq.(B-28) in Appendix B in Ref.[28]). We adde the functions $g_{\beta\gamma}^{(1)} (E_e, \mu)$ and $g_{\beta\gamma}^{(2)} (E_e, \mu)$ for completeness

$$g_{\beta\gamma}^{(1)} (E_e, \mu) = \left[ \frac{1}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 2 \right]$$

$$+ \frac{1}{4 \beta} \ln \left( 1 + \frac{\beta}{1 - \beta} \right) - \frac{1}{4 \beta^2} \ln \left( 1 + \frac{\beta}{1 - \beta} \right) - \frac{1}{2} \ln \left( 1 + \frac{\beta}{1 - \beta} \right) + \frac{1}{12} \frac{\left( E_0 - E_e \right)^2}{E_e^2},$$

$$g_{\beta\gamma}^{(2)} (E_e, \mu) = \left[ \frac{1}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 2 \right]$$

$$+ \frac{1}{4 \beta} \ln \left( 1 + \frac{\beta}{1 - \beta} \right) - \frac{1}{4 \beta^2} \ln \left( 1 + \frac{\beta}{1 - \beta} \right) - \frac{1}{2} \ln \left( 1 + \frac{\beta}{1 - \beta} \right) + \frac{1}{12} \frac{\left( E_0 - E_e \right)^2}{E_e^2},$$

(A-12)
where $\text{Li}_2(z)$ is the PolyLogarithmic function \[84\]. Taking into account the contribution of the neutron radiative beta decay, given by Eqs. (A-12) and (A-11), we remove the dependence of the radiative corrections to the correlation coefficient $\zeta(E_e)T(E_e)$ on the infrared cut-off (see Eqs. (19) - (23)).
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