Excited-State Hadron Masses from Lattice QCD

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Abstract. Progress in computing the spectrum of excited baryons and mesons in lattice QCD is described. Large sets of spatially-extended hadron operators are used. The need for multi-hadron operators in addition to single-hadron operators is emphasized, necessitating the use of a new stochastic method of treating the low-lying modes of quark propagation which exploits Laplacian Heaviside quark-field smearing. A new glueball operator is tested and computing the mixing of this glueball operator with a quark-antiquark operator and multiple two-pion operators is shown to be feasible.

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In a series of papers[1, 2, 3, 4, 5, 6], we have been striving to compute the finite-volume stationary-state energies of QCD using Markov-chain Monte Carlo integration of the QCD path integrals formulated on a space-time lattice. Such calculations are very challenging. Computational limitations cause simulations to be done with quark masses that are unphysically large, leading to pion masses that are heavier than observed and introducing systematic errors in all other hadron energies. The use of carefully designed quantum field operators is crucial for accurate determinations of low-lying energies. To study a particular state of interest, the energies of all states lying below that state must first be extracted, and as the pion gets lighter in lattice QCD simulations, more and more multi-hadron states lie below the masses of the excited resonances. The evaluation of correlations involving multi-hadron operators contains new challenges since not only must initial to final time quark propagation be included, but also final to final time quark propagation for a large number of times must be incorporated. The masses and widths of resonances must be deduced from the discrete spectrum of finite-volume stationary states for a range of box sizes.

To compute the QCD stationary-state energies, the matrices \( C_{ij}(t) \) of temporal correlations of sets of single-hadron and multi-hadron operators are estimated using the Monte Carlo method. For an \( N \times N \) matrix, the \( N \) eigenvalues of \( C(t_0)^{-1/2}C(t)C(t_0)^{-1/2} \) tend to \( \exp(-E_k(t-t_0)) \) for large \( t \) and fixed \( t_0 \), where the decay rates \( E_k \) are the \( N \) lowest-lying stationary-state energies that can be produced from the vacuum by the operators used. To compute the correlations involving isoscalar mesons and good multi-hadron operators, quark propagation from all spatial sites on one time slice to all spatial sites on another time slice are needed, and propagation from the sink time to the sink time are needed for a large number of sink times. A new method known as the stochastic LapH method[6] has been introduced to make such computations accurate and practical in large volumes.

Our first results for the isovector mass spectrum on a large \( 24^3 \times 128 \) anisotropic lattice are shown in Fig. 1. The pion mass is about \( m_\pi \sim 390 \) MeV here. These results are not finalized since only single-hadron operators were used. The shaded region indicates the threshold locations for multi-hadron energy levels. We emphasize that extractions of energies in the shaded regions could be complicated by “false plateaux” unless multi-hadron operators are used. The inclusion of the multi-hadron operators is in progress.

Some initial results that incorporate multi-hadron operators are shown in Fig. 2. A \( 2 \times 2 \) correlation matrix was evaluated involving a single-site \( \rho \)-meson operator and a total isospin \( I = 1 \) \( \pi \pi \) operator in a \( P \)-wave with minimal relative momentum. The stochastic LapH method enables very accurate estimates of all elements (both diagonal and off-diagonal) of this correlation matrix such that diagonalization can be done. The effective masses associated with the diagonalized correlator are shown in Fig. 2. The \( \rho \) operator dominates the lowest-lying level, while the \( \pi \pi \) operator dominates the first-excited state. Although the mixing of the operators is small, it is not negligible. This is certainly a warning about the dangers of extracting high-lying resonance energies using only single-hadron operators.

Determining meson masses in the interesting scalar isoscalar sector will ultimately involve including a scalar glueball operator, so we began looking into the feasibility of such calculations. LapH quark-field smearing involves the
FIGURE 1. Masses of the isovector mesons in terms of the nucleon mass using the stochastic LapH method with 170 gauge configurations on a $24^3 \times 128$ anisotropic lattice. The pion mass is about $m_\pi \sim 390$ MeV. In the irrep labels, the letters with numerical subscripts refer to the point group $O_h$ irreps, the subscripts $g$ and $u$ refer to even and odd parity, respectively, and the superscripts $\pm$ refer to $G$-parity. Only single-hadron operators were used with dilution scheme $(T_F, S_F, L_I)$. The shaded region indicates the threshold locations for multi-hadron energy levels. We emphasize that extractions of energies in the shaded regions could be complicated by "false plateaux" unless multi-hadron operators are used.

covariant spatial Laplacian $\tilde{\Delta}$. The eigenvalues of the Laplacian are invariant under rotations and gauge transformations so are appropriate for a scalar glueball operator. The lowest-lying eigenvalue was studied, as well as other functions of the eigenvalues. We found that any combination of the low-lying eigenvalues worked equally well for studying the scalar glueball. In particular, the operator defined by $G_\Delta(t) = \frac{-1}{16} Tr(\Theta(\sigma_2^2 + \tilde{\Delta} \tilde{\Delta}))(t)$ was used. The effective mass associated with this operator is compared to that of the smeared plaquette glueball operator in Fig. 3. Similarity of the results shows that $G_\Delta$ is just as useful as the familiar smeared plaquette for studying the scalar glueball.

Effective masses corresponding to the diagonalized 4 × 4 correlator matrix involving a scalar isoscalar single-site

FIGURE 2. Effective masses corresponding to the diagonal elements of the rotated 2 × 2 correlator matrix involving a single-site $\rho$-meson operator and an $I = 1 \pi\pi$ operator in a $P$-wave with minimal relative momentum. The $\rho$ operator dominates the lowest-lying level (left), while the $\pi\pi$ operator dominates the first-excited state (right). Although the mixing of the operators is small, it is not negligible. These results were obtained on 584 configurations of the $24^3 \times 128$ lattice with pion mass $m_\pi \sim 240$ MeV.
FIGURE 3. Comparison of the effective mass associated with the correlator of the standard smeared plaquette glueball operator (left) with that of our new glueball operator $G_{\Delta}$ defined in the text (right). Similarity of the two results shows that the new glueball operator is just as useful as the familiar smeared plaquette for studying the scalar glueball. These effective masses do not reach a plateau at the glueball mass since various $\pi\pi$ states and other multi-hadron states have smaller energies than the glueball mass. These results were obtained using 584 configs of the $24^2$ ensemble for pion mass $m_\pi \sim 240$ MeV.

quark-antiquark meson operator, the new glueball operator $G_{\Delta}$, and two $I = 0 \pi\pi$ operators in an $S$ wave (one with zero relative momentum and the other with minimal nonzero relative momentum) are shown in Fig. 4. Mixing of these operators is sizeable. More $\pi\pi$ operators must be included to reliably extract the glueball mass.

These results demonstrate that the stochastic LapH method is useful for accurately estimating all of the temporal correlations needed for a full study of the QCD stationary-state energy spectrum, which we are currently pursuing. This work was supported by the U.S. NSF under awards PHY-0510020, PHY-0653315, PHY-0704171, PHY-0969863, and PHY-0970137, and through TeraGrid/XSEDE resources provided by TACC and NICS under grant numbers TG-PHY100027 and TG-MCA075017.

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FIGURE 4. Effective masses corresponding to the diagonal elements of the rotated $4 \times 4$ correlator matrix involving a scalar isoscalar single-site quark-antiquark meson operator, the new glueball operator $G_{\Delta}$, and two $I = 0 \pi\pi$ operators in an $S$ wave (one with zero relative momentum and the other with minimal nonzero relative momentum). Mixing of these operators is sizeable. More $\pi\pi$ operators must be included to reliably extract the glueball mass. These results were obtained on 100 configurations of a $16^3 \times 128$ lattice with pion mass $m_\pi \sim 390$ MeV.