I. INTRODUCTION

The developments in cosmology have been influenced to a great extent by the idea of inflation [1–7], which provides a solution of the fundamental puzzles of the old Big Bang paradigm, such as the horizon and the flatness problems. Additionally, inflation was proved crucial in providing a framework for the generation of primordial density perturbations [8, 9]. Although the inflationary scenario is very attractive, it has been recognized that a successful implementation requires special restrictions on the underlying dynamics. The inflation mechanism can be achieved in several different ways considering primordial scalar fields [10–12] or geometric corrections into the effective gravitational action [13–18]. Other scenarios connect the Higgs to inflation by a scale invariant coupling to the Einstein term [19–21]. The last few decades have been very wonderful for the physics of the early universe especially due to the measurements of the Cosmic Microwave Background [22]. [23, 24] study inflationary scenarios from fermions ([24] is based on [25]). However, the potentials that were suggested seem to work up to second order derivative of the potential. This paper establish the foundations of inflationary scenarios from Fermion Tensor Theories (FTT) and suggests couple of possible new solutions that fit with the latest observations. Fermions in cosmology have been widely investigated with non trivial solutions [26–28]. [29] investigates the case where fermions are responsible for accelerated periods during the evolution of a universe. [30] generalizes the Dirac action in curved space time with a coupling to the Einstein term. [31, 32] study the fermi-bounce cosmology from spinors. [33–35] study dark energy emergence from fermionic field.

The plan of the letter is the following: Section II summarizes the formulation of fermions in curved spacetime. Section III studies the theory and the equations of motion. Section IV studies the perturbations and confront the numerical results with the latest observations. Finally, section V summarizes the results.

II. FERMIONS IN CURVED SPACETIME

Fermions in general relativity were studied in detail in [26] also with cosmological applications [36–38]. The tetrad formalism was used to combine the gauge group of general relativity with a spinor matter field. The tetrad $e^a_\mu$ and the metric $g_{\mu\nu}$ tensors are related through

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}, \ a, b = 0, 1, 2, 3, \ (1)$$

with Latin indices refer to the local inertial frame with the Minkowski metric $\eta_{ab}$, while Greek indices denote the local coordinate basis of the manifold. $\gamma^a$ are the Dirac matrices in the standard representation (flat spacetime). The Dirac matrices in curved space $\gamma^\mu = e^a_\mu \gamma^a$ are obtained using the tetrads $e^a_\mu$, labeled with a Latin index. The generalized Dirac matrices obey the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\nu\mu}$. The definition for the covariant derivative for spinors reads:

$$\psi_{;\mu} = \partial_\mu \psi - \Omega_\mu \psi, \quad \bar{\psi}_{;\mu} = \partial_\mu \bar{\psi} + \bar{\psi} \Omega_\mu. \ (2)$$

The metric compatibility condition implies that the spin connection $\Omega_\mu$ is given by:

$$\Omega_\mu = \frac{1}{4} g^{\beta\nu} \left[ \Gamma^\nu_{\alpha\mu} - e^a_\nu e^b_\mu \eta_{ab} \right] \gamma^\beta \gamma^\alpha, \ (3)$$

with $\Gamma^\nu_{\sigma\lambda}$ the Christoffel symbols.

III. FERMION TENSOR THEORIES

The framework of Fermion tensor theories is based on the framework of Scalar Tensor Theories. The FTT read:

$$\mathcal{L} = f(\phi) \frac{R}{2} + \frac{i}{2} \left[ \bar{\psi} \gamma^\mu \psi_{;\mu} - \bar{\psi} \gamma^\mu \gamma^\nu \psi \right] - V(\phi), \ (4)$$

where $R$ is the Ricci scalar, $\psi$ and $\bar{\psi} = \psi^\dagger \gamma^0$ are the spinor field and its adjoint, respectively. The scalar
\[ \phi \equiv |\bar{\psi}\psi| \] multiplies the fermionic field and it's conjugate field. \( f(\phi) \) is the function that couples the Einstein term and \( V(\phi) \) the self-interaction potential density of the fermionic field. The kinetic term of the spinors is the same as the kinetic term from Dirac equation. However, FTT suggest a generic coupling function \( f(\phi) \). For \( f(\phi) = 1 \) and \( V(\phi) = m_\psi \phi \) the FTT reduce to the Dirac equation in curved spacetime.

For a homogeneous and isotropic spacetime, it is natural to consider the Friedman-Lemaitre Robertson Walker (FLRW) metric

\[
ds^2 = -dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right),
\]

with the scale factor \( a(t) \). For the metric (5) the action (4) reduces to the form:

\[
\mathcal{L} = 3a\dot{a} \left( \dot{\phi} f(\phi) + a\dot{f}(\phi) \right) + \frac{i}{2} a^3 \left( \bar{\psi}_\mu \gamma^\mu \psi - \bar{\psi}_\mu \gamma^\mu \psi \right) + a^3 V(\phi).
\]

The solution is obtained via the complete set of variations: the scale factor \( a(t) \) and the spinor field \( \psi \). However, using Noether's symmetry we can avoid the variation with respect to the fields. The Noether symmetry w.r.t. \( U(1) \) phase transformations yields:

\[
\psi \rightarrow e^{i\theta} \psi, \quad \bar{\psi} \rightarrow e^{-i\theta} \bar{\psi}, \quad \phi \rightarrow \phi,
\]

through the Noether's Symmetry yields a conserved current:

\[
j^\mu = \bar{\psi}_\mu \gamma^\mu \psi, \quad j_{\mu}^\mu = 0
\]

which proves convenient in simplifying our analysis. In FLRW metric only the zeroth component in (8) survives. Therefore the covariant conservation gives:

\[
\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} j^\mu \right) = \frac{1}{a^3} \frac{\partial}{\partial t} (a^3 j^0) = 0,
\]

which then leads to a dust-like behavior for the absolute value of the spinor field, i.e.

\[
\phi = |\bar{\psi}\psi| = n_\psi / a^3,
\]

with the particle number density \( n_\psi > 0 \). Impose that the energy function associated with the Lagrangian (4) is null yields the Friedmann equation:

\[
3H^2 = \frac{V(\phi)}{f(\phi) - 3\dot{\phi} f'(\phi)}
\]

The weak energy condition \( (\rho > 0) \) forces the condition \( f(\phi) > 0 \), \( \alpha \phi^{1/3} \), where \( \alpha \) is some constant. [39] uses the Noether symmetry in order to find the coupling function \( f(\phi) \). The paper suggests \( f(\phi) \sim \phi \) or \( f(\phi) \sim \phi^{1/3} \). However, for a cosmological solution it is possible to choose \( f(\phi) = 1 \) and redefine the potential \( V(\phi) \). It is similar to use the conformal transformation in \( f(R) \) gravity to obtain the Einstein frame with a scalar field [40].

![Graph](image)

**FIG. 1:** Numerical solution of the Hubble parameter with the potential (15) and the parameters \( m = \xi = 1 \). The Hubble parameter begins with a constant value which refers to De Sitter solution. At the final stage of inflation the Hubble parameter is reduced.

**IV. PERTUBATIONS**

The calculation of the above observables requires a detailed perturbation analysis. To this end, one can obtain approximate expressions by imposing the slow-roll assumptions under which all inflationary information is encoded in the slow-roll parameters. In particular, following [41, 42] let us introduce:

\[
\epsilon_n = \frac{d^n \log |H(N)|}{dN^n} = \frac{3}{2} \frac{d^n \log |V(\phi)|}{d\log \phi^n},
\]

with the solution of Eq. (11) and \( f(\phi) = 1 \). \( N \equiv \ln(a/a_i) \) is the e-folding number, \( a_i \) is the scale factor at the beginning of inflation and \( n \) a positive integer. The inflation ends at a scale factor \( a_f \) where \( \epsilon_1(a_f) = 1 \) and the slow-roll behavior breaks down. The inflationary observables are expressed as:

\[
r \approx 16 \epsilon_1, \quad n_s \approx 1 - 2\epsilon_1 - 2\epsilon_2, \quad \alpha_s \approx -2\epsilon_1 \epsilon_2 - 2\epsilon_3, \quad n_T \approx -2\epsilon_1,
\]

where all quantities are calculated at \( a_i \). \( r \) is the scalar to tensor ratio and \( n_s \) is the primordial tilt. The slow roll condition means \( \epsilon_n \ll 1 \). In order to find an appropriate potential that satisfies the slow roll condition it is natural to use Taylor expansion in Eq. (12):

\[
\frac{3}{2} \log V(\phi) = \log V_0 + \epsilon_1 \log \phi + \frac{\epsilon_2}{2} \log \phi^2 + ...
\]

Because of the relation (10) the initial state of inflation should be governed by the highest power of \( \log \phi \). As a model we examine the form:

\[
\log V(\phi) = \log V_0 + \xi \log^{-m} \phi
\]

where \( m \) and \( \xi \) are real values. Fig (1) shows the numerical solution of the Friedmann equation with this potential. The Hubble parameter begins with a constant value
which refers to De-Sitter solution. At the final stage of inflation the Hubble parameter is reduced.

Substituting the potential (15) into (12) yields:

$$\epsilon_1 = \frac{3}{2} \frac{m \xi}{\log \phi^{m+1}}, \quad \epsilon_2 = -\frac{3}{2} \frac{m(m+1) \xi}{\log \phi^{m+2}}.$$  \hspace{1cm} (16)

For a very small initial scale parameter $a_i \ll 1$ the dimensionless parameters $\epsilon_n$ approach zero. Therefore the slow roll condition is satisfies.

[23, 24] suggest different potentials that have a slow roll behavior in the first order $\epsilon_1$. However, higher derivatives of $\epsilon_n$ break the slow roll condition $\epsilon_n \ll 1$.

A simplification of the difference between the initial and the final value of $\phi$ uses the identity: $\log \phi_i - \log \phi_f = 3N$. The condition for the final state of inflation $\epsilon(\phi_f) = 1$ gives the final value of $\log \phi_f^{m+1} = 3\xi/2$. Therefore the observables for this form of potential read:

$$r \approx \frac{24\xi}{(3N + (3\xi/2)^{m+1})^2},$$  \hspace{1cm} (17)

and

$$n_s \approx 1 - \frac{r}{4} - (m + 1) \left( \frac{3mr}{8m\xi} \right)^{1/(m+1)}.$$  \hspace{1cm} (18)

The latest Planck observations [22] give the constraint on the observables:

$$r < 9 \cdot 10^{-3}, \quad n_s = 0.968 \pm 0.006.$$  \hspace{1cm} (19)

Fig (2) presents the predicted values of $r$ and $n_s$ for $m = 1$ with different values of $\xi$ within the range $0 < \xi < 2$ and $50 < N < 60$ for the number of $e$-folds. The constraint from Planck observations is presented with gray background. The parameters $m = 1$ and $\xi \sim 1$ leads to a good fit with the current observations. In particular, for $N = 60$ the predicted observables are shown in table 1. From the generalized Friedmann equation (11) it is clear that there is an essential difference between the behavior of scalar fields and fermion fields in inflationary scenarios. During inflation, the scalar field slightly changes due to the slow-roll condition (see [43, 44] for further discussion). However, for fermions, the composite field $\psi \bar{\psi}$ changes 78 orders of magnitude while the scale parameter evolves 60 $e$-folds.

Moreover, the same form of potential produce different physical behavior. For a constant scalar field potential the contribution of the kinetic term yields a Stiff equation of state that behaves as $a^{-6}$. However, for a fermionic field with any potential the kinetic term does not contribute to Friedmann Eq. (11).

\section{Conclusions}

This paper constructs a model of inflation scenario, driven by fermions. Incorporation of Dirac fermions in a framework of FTT in the FLRW metric yields a simple Friedmann equation. The fermions produce a conserved Noether current that impose the solution for the fermionic bilinear condensate field. At the perturbation level the observables we study the scalar spectral index and the tensor-to-scalar ratio. Analytical expressions for the observables with the correct parameters yield the values in agreement with Planck observations. The efficiency of inflation from FTT at both the background and perturbation level reveals the capabilities of the scenario and makes it a good candidate for the description of nature.

As a next task we may try to investigate extensions of the above basic scenario. In particular, in this work the spinor fields assumed to be classical lying on a minimal state or being cold. Nevertheless, incorporation of high-temperature effects, as for example in the case of warm inflation [45, 46] could lead to the appearance of an additional friction term with different values of the observables.

Another important direction is to allow dependence of $f(\phi)$ on $\phi$. This paper assumes $f(\phi) = 1$ and impose a potential that gives the slow roll behavior. However, the potential may have a different form with different $f(\phi)$. 

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$\xi$ & $r (10^{-3})$ & $n_s$ \\
\hline
0.5 & 0.367 & 0.966735 \\
1 & 0.731 & 0.966709 \\
1.5 & 1.093 & 0.966669 \\
\hline
\end{tabular}
\caption{The observable vs. different values of $\xi$ for $m = 1$ and $N = 60$ $e$-folds.}
\end{table}
In this case the effective Newtonian Constant depends on the scale parameter $G_{\text{eff}} = G_N/f(\phi)$. The constraints on the effective Newtonian Constant has to be taken into account [47].

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