On the Equivalence between 2D Induced Gravity 
and a WZNW System *

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Abstract

A general method of constructing canonical gauge invariant actions is used to establish the equivalence between 2D induced gravity and a WZNW system, defined by a difference of two simple WZNW actions for the $SL(2, R)$ group. The diffeomorfism invariance of the induced gravity is generated by the $SL(2, R)$ Kac–Moody structure of the WZNW system.

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1. Introduction

A dynamical structure of quantum gravity in 2D, which is induced by string theory, is of great importance for understanding string dynamics [1]. The induced gravity action in the light–cone gauge possesses a hidden chiral $SL(2, R)$ symmetry [2,3], while in the conformal gauge it becomes the Liouville theory [4,5]. Having in mind the dynamical importance of the $SL(2, R)$ symmetry, it is natural to try to understand the way in which this symmetry is associated to the induced gravity, and, thereby, to the Liouville theory.

In this paper we shall show that the induced gravity action,

$$S(\phi, g_{\mu\nu}) = \int d^2\xi \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + \frac{1}{2} \alpha \phi R + M^2 \left( e^{2\phi/\alpha} - 1 \right) \right],$$

(1)

is gauge equivalent to the $SL(2, R)$ Wess–Zumino–Novikov–Witten (WZNW) system,

$$S(g_1, g_2) = S_L(g_1) - S_R(g_2) \quad g_1, g_2 \in SL(2, R),$$

(2)

defined by a difference of two, left and right, WZNW actions (L–R). In the process of establishing this equivalence, the connection between the $SL(2, R)$ Kac–Moody (KM) structure of the WZNW system and the diffeomorfism invariance of the induced gravity will become much more clear.

Our result generalizes Polyakov’s work, who found the connection between the WZNW action for $SL(2, R)$, and the induced gravity in the light–cone gauge [6]. Similar results in the light–cone gauge have been obtained in Refs.[7,8], using the methods of conformal field theory and coadjoint orbits of the Virasoro group, respectively. The same problem was also discussed in the conformal gauge, in Ref. [9], where the related connection is used to obtain the general solution of the Liouville theory from that of the WZNW model. Due to the presence of two simple WZNW actions in (2), we are able to demonstrate the equivalence of (1) and (2) in a covariant way, fully respecting the diffeomorfism invariance of the induced gravity.

We shall use the general method of constructing gauge invariant actions based on the Hamiltonian formalism [10]. The method uses the fact that the Lagrangian equations of motions are equivalent to the Hamiltonian equations derived from the action

$$S(q, \pi, u) = \int d\xi (\pi_i \dot{q}^i - H_0 - u^m G_m),$$

(3a)

where $G_m$ are primary constraints, and $H_0$ is the canonical Hamiltonian. If $G_m$ are first class constraints, satisfying the Poisson bracket algebra

$$\{G_m, G_n\} = U_{mn}^r G_r,$$

$$\{G_m, H_0\} = V_m^r G_r,$$

(3b)
than the action $S(q, p, u)$ is invariant under the following gauge transformations:

$$
\delta F = \varepsilon^m \{ F, G_m \}, \quad F = F(q, \pi)
$$

$$
\delta u^m = \varepsilon^m + u^r \varepsilon^s U_{sr}^m + \varepsilon^r V_r^m.
$$

(4)

Using this idea we shall start with the action (2), make a convenient gauge extension of this theory by using (3a), and show that the resulting formulation of (2) reduces to (1) after a convenient gauge fixing.

2. The WZNW model for $SL(2, R)$

We shall begin by recalling some facts about the WZNW model, described by the action

$$
S_L(g) = S_0(g) + n \Gamma(g) = \frac{n}{8\pi} \int_{\Sigma} (*v, v) + \frac{n}{24\pi} \int_B (v, [v, v]).
$$

(5)

The first term is the action of the non–linear $\sigma$–model for a field $g$, defined over a two dimensional spacetime $\Sigma$ and taking values in a semisimple group $G$, $v = g^{-1} dg$ is the Maurer–Cartan one–form, $*v$ is the dual of $v$, and $(X, Y) = \frac{1}{2} \text{Tr}(XY)$ is the Cartan–Killing bilinear form (the trace operation is taken in the adjoint representation of $G$). The second term is the topological Wess–Zumino term, where $B$ is a three dimensional manifold with boundary $\Sigma$.

Now, we turn our attention to the case of $G = SL(2, R)$. The generators of $SL(2, R)$ are taken as $t_a = (t_-, t_0, t_+)$ = $(\sigma_-, \sigma_3/2, \sigma_+)$, where $\sigma_i$ are the Pauli matrices. They define the structure constants $f_{abc}$ ($f_{-0} = 1, f_{+0} = -1, f_{+0} = 2$), and the Cartan metric $\gamma_{ab} = (t_a, t_b)$ (with nonvanishing components $\gamma_{-+} = \gamma_{++} = 2, \gamma_{00} = 1$). In $SL(2, R)$, any group element $g$ admits the Gauss decomposition, $g = e^{xt} e^{\varphi t_0} e^{yt}$, where $q^a = (x, \varphi, y)$ are group coordinates in a neighbourhood of the identity,

$$
g = \begin{pmatrix}
  e^{\varphi/2} + xy e^{-\varphi/2} & xe^{-\varphi/2} \\
  ye^{-\varphi/2} & e^{-\varphi/2}
\end{pmatrix},
$$

and, also, $q$’s are the usual fields on $\Sigma$, $q = q(\xi)$. With the above expression for $g$, the WZNW action for $SL(2, R)$ takes the simple local form

$$
S_L(q) = \kappa \int \Sigma d^2 \xi \left[ \frac{1}{2} \eta^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + 2(\eta^{\mu\nu} - \varepsilon^{\mu\nu}) \partial_\mu x \partial_\nu ye^{-\varphi} \right],
$$

(6)

where $\kappa \equiv n/8\pi$.

The dynamical structure of the theory (6) is characterized by the existence of two sets of currents. In the phase space with coordinates $(q^\alpha, \pi_\alpha)$, where $\pi_\alpha = (\pi_x, \pi_\varphi, \pi_y)$ are
momenta canonically conjugate to $q^\alpha$, the left–handed currents $J_{(-)a}$ are given as [11]

$$
\begin{align*}
J_{(-)+} &= \pi_x, \\
J_{(-)0} &= x\pi_x + (\pi\varphi - \kappa\varphi'), \\
J_{(-)-} &= -x^2\pi_x - 2x(\pi\varphi - \kappa\varphi') - 4\kappa x' + \pi_y e^\varphi,
\end{align*}
$$

(7a)

while the right–handed ones, $J_{(+a)}$, are

$$
\begin{align*}
J_{(+)+} &= y^2\pi_y + 2y(\pi\varphi + \kappa\varphi') - 4\kappa y' - \pi_x e^\varphi, \\
J_{(+)0} &= -y\pi_y - (\pi\varphi + \kappa\varphi'), \\
J_{(+)-} &= -\pi_y.
\end{align*}
$$

(7b)

Using the basic Poisson brackets $\{q^\alpha(\sigma_1), \pi_\beta(\sigma_2)\} = \delta^\alpha_\beta \delta(\sigma_1 - \sigma_2)$, one finds that the currents $J_{(\mp)a}$ satisfy two independent KM algebras:

$$
\{J_{(\mp)a}, J_{(\mp)b}\} = f_{abc} J_{(\mp)c} \delta + 2\kappa \gamma_{ab} \delta',
$$

(8)

where $\delta = \delta(\sigma_1 - \sigma_2)$, $\delta' = \partial_{\sigma_1} \delta$, and $\{J_{(-)a}, J_{(+b)}\} = 0$.

In a similar way one can define $S_R(g) = S_0(g) - n\Gamma(g)$, whose local coordinate expression $S_R(q)$ is obtained from (6) by changing the sign of the $\varepsilon^{\mu\nu}$–term.

3. Covariant extension of the WZNW model for $SL(2, R)$

Now, as a preparation for the main problem of proving the gauge equivalence between the induced gravity and the WZNW system (2), we shall first study the problem of the covariant extension (with respect to diffeomorfisms) of the WZNW models (6) and (2) [12].

(A) By using explicit expressions for the KM currents (7) associated to the simple WZNW model (6), we can construct the related $SL(2, R)$ invariant expressions,

$$
\begin{align*}
T_- &= \frac{1}{4\kappa} \gamma^{ab} J_{(-)a} J_{(-)b} = \frac{1}{4\kappa} \left[ \pi_x \pi_y e^\varphi - 4\kappa x' \pi_x + (\pi\varphi - \kappa\varphi')^2 \right], \\
T_+ &= -\frac{1}{4\kappa} \gamma^{ab} J_{(+a)} J_{(+b)} = -\frac{1}{4\kappa} \left[ \pi_x \pi_y e^\varphi + 4\kappa y' \pi_y + (\pi\varphi + \kappa\varphi')^2 \right],
\end{align*}
$$

(9)

representing components of the energy–momentum tensor. From the KM algebra of currents we obtain two independent Virasoro algebras for $T_-$ and $T_+$:

$$
\{T_+(\sigma_1), T_+(\sigma_2)\} = -[T_+(\sigma_1) + T_+(\sigma_2)] \delta'.
$$

(10)

Now, we wish to construct the canonical action (3a) for a theory in which $H_0 = 0$, and $G_m = (T_-, T_+)$ are first class constraints:

$$
\mathcal{L}_L(q, \pi, h) = \pi_\alpha q^\alpha - h^- T_- - h^+ T_+.
$$

(11a)
After eliminating the momentum variables with the help of the equations of motion, and introducing a change of variables \((h^-, h^+) \rightarrow \tilde{g}^{\mu\nu}\),

\[
\begin{align*}
\tilde{g}^{00} &= \frac{2}{h^- - h^+}, & \tilde{g}^{01} &= \frac{h^- + h^+}{h^- - h^+}, & \tilde{g}^{11} &= \frac{2h^- + h^+}{h^- - h^+},
\end{align*}
\tag{12a}
\]

with \(\det \tilde{g}^{\mu\nu} = -1\), the Lagrangian becomes

\[
\mathcal{L}_L(q, h) = \kappa \left[ \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + 2(\tilde{g}^{\mu\nu} - \varepsilon^{\mu\nu}) \partial_\mu x \partial_\nu ye^{-\varphi} \right].
\tag{11b}
\]

If we make the identification \(\tilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}\), the above expression is seen to represent the covariant generalization of the WZNW theory; for \(h^\mp = \pm 1\) it reduces to (6).

The transformation properties of \(\tilde{g}^{\mu\nu}\) are consistent with this interpretation. Indeed, the transformation rules for the multipliers \(h^\mp\) are

\[
\delta h^\mp = \partial_0 \varepsilon^\mp + h^\mp \partial_1 \varepsilon^\mp - \varepsilon^\pm \partial_1 h^\mp.
\tag{12b}
\]

Then, after introducing new parameters, \(\varepsilon^\mp = \varepsilon^1 - \varepsilon^0 h^\mp\), one finds

\[
\delta \tilde{g}^{\mu\nu} = \tilde{g}^{\mu\rho} \partial_\rho \varepsilon^\nu + \tilde{g}^{\nu\rho} \partial_\rho \varepsilon^\mu - \partial_\rho (\varepsilon^\rho \tilde{g}^{\mu\nu}),
\tag{12c}
\]

which is the diffeomorphism transformation of a metric density.

One should observe that the Virasoro generators \(T_\mp\) are constructed in terms of the KM currents (7). This construction yields a simple explanation of how the diffeomorphisms can be obtained out of the KM structure of the WZNW model.

**B**  The next step in our discussion is to combine two simple WZNW actions, as in Eq.(2), and construct the related covariant extension. The KM currents related to the L sector, \(J^{(1)}\), are the same as in (7a, b), while those related to the R sector, \(J^{(2)}\), are of opposite chiralities:

\[
\begin{align*}
J^{(1)}(q_1, \pi_1) &= J^{(1)}(q_1, \pi_1), & J^{(2)}(q_2, \pi_2) &= J^{(2)}(q_2, \pi_2),
\end{align*}
\tag{13a}
\]

We also define the energy–momentum tensors of the L and R sectors as

\[
\begin{align*}
T^{(1)}_\mp(q_1, \pi_1) &= T^{(1)}_\mp(q_1, \pi_1), & T^{(2)}_\mp(q_2, \pi_2) &= -T^{(2)}_\mp(q_2, \pi_2).
\end{align*}
\tag{13b}
\]

It is easy to check that the components of the complete energy–momentum,

\[
T_\mp = T^{(1)}_\mp + T^{(2)}_\mp,
\tag{14}
\]
satisfy two independent Virasoro algebras (10).

Now, the canonical action in which $H_0 = 0$ and $G_m = (T_-, T_+)$ has the form

$$\mathcal{L}(q_i, \pi_i, h) = \pi_{1\alpha} \dot{q}_{1\alpha} + \pi_{2\alpha} \dot{q}_{2\alpha} - h^- T_- - h^+ T_+, \quad (15a)$$

As in the case (A), the elimination of momenta $\pi_{1\alpha}$ and $\pi_{2\alpha}$ leads to the result

$$\mathcal{L}(q_1, q_2, h) = \mathcal{L}_L(q_1, h) - \mathcal{L}_R(q_2, h), \quad (15b)$$

in analogy to (11b). It describes the covariant extension of the theory (2). Transition to the flat space is achieved by $h^\mp \rightarrow \pm 1$.

**4. Gauge equivalence between the WZNW system and induced gravity**

As a final step, we shall now construct a more general gauge invariant extension of the WZNW system (2), and show that it reduces to the induced gravity action (1) after a suitable gauge fixing.

We first note that the currents $J_{(\mp)a}$ and $*J_{(\mp)a}$, defined by

$$*J_{(\mp)\pm} \equiv J_{(\mp)\mp} \quad *J_{(\mp)0} \equiv -J_{(\mp)0}, \quad (16a)$$

satisfy the same KM algebras. Now, we can use two sets of the KM currents, corresponding to the L and R sectors of the WZNW theory (2), to define new quantities

$$I_{(\mp)a} = J_{(\mp)a}^{(1)} + *J_{(\mp)a}^{(2)} .$$

The Poisson bracket algebra between $I_{(\mp)a}$ and $T_{\mp}$ has the form

$$\{T_{\pm}(\sigma_1), T_{\mp}(\sigma_2)\} = -[T_{\pm}(\sigma_1) + T_{\mp}(\sigma_2)]\delta', \quad (16b)$$

$$\{T_{\mp}(\sigma_1), I_{(\mp)a}(\sigma_2)\} = -I_{(\mp)a}(\sigma_1)\delta',$$

$$\{I_{(\mp)a}(\sigma_1), I_{(\mp)b}(\sigma_2)\} = f_{ab}^c I_{(\mp)c}(\sigma_2)\delta,$$

representing two independent (left and right) semi–direct products of the Virasoro and $SL(2, R)$ algebras. We see that the currents $I_{(\mp)a}$ satisfy two $SL(2, R)$ algebras, since the central charges of $J^{(1)}$ and $*J^{(2)}$ mutually cancel. Therefore, the set $(T_{\mp}, I_{(\mp)a})$ can be taken as a set of first class constraints, needed in the construction of the canonical action.

In the theory (2) defined by a difference of two WZNW actions, one is able to gauge the full $SL(2, R)_{(-)} \times SL(2, R)_{(+)}$ symmetry, whereas for the simple WZNW model for $SL(2, R)$ this is not possible.
We display here the complete set of constraints, multipliers and gauge parameters:

\[
\begin{align*}
G_m &= T_\mp I_{(\mp)a} \\
u^m &= h_\mp A_{(\mp)a} \\
\varepsilon^m &= \varepsilon_\mp \eta_{(\mp)a}
\end{align*}
\]

In the canonical theory defined by the full set of constraints, the gauge transformations of the multipliers \(h_\mp\) are the same as in (12b), while those of \(A_{(\mp)a}\) are given as

\[
\delta A_{(\mp)a} = \partial_0 \eta_{(\mp)a} + h_\mp \partial_1 \eta_{(\mp)a} + f_{bc}^\ a A_{(\mp)c} \eta_{(\mp)b} - \varepsilon_\mp A_{(\mp)a}.
\]

For our purposes, it will be enough to consider the restriction of the above theory, defined by the subset of first class constraints \(T_\mp, I_{(-)}, I_{(-)0}, I_{(+)}, I_{(+0)}\), representing a subalgebra of (16b). The canonical action of the restricted theory takes the form

\[
\mathcal{L}(q_i, \pi_i, h) = \pi_{1a} \dot{q}_1^a + \pi_{2a} \dot{q}_2^a - h^\mp T_\mp - h^+ T_+
- A^{(-)} + I_{(-)} - A^{(-)0} I_{(-)0} - A^{(+)} - I_{(+)} - A^{(+0)} I_{(+0)}.
\]

The transformation rules for the multipliers are easily obtained from the general expressions, by imposing the restriction \(\eta_{(\mp)\mp} = 0, A_{(\mp)\mp} = 0\).

It is clear that the action (17) represents a generalization of the (L–R) theory (2). Indeed, if we fix the gauge so that \(A^{(\mp)} \pm = A^{(\mp)0} = 0\), the above theory reduces to (15). Now, we shall demonstrate that a different gauge choice reduces the action (17) to the form (1), proving thereby the gauge equivalence between the WZNW theory (2) and the induced gravity (1).

Let us fix the gauge symmetry corresponding to the first class constraints \(I_{(\mp)\pm}, I_{(\mp)0}\), by choosing the following gauge conditions:

\[
\Omega_{(\mp)\pm} = J_{(\mp)0} - \mu_{(\mp)} = 0, \quad \Omega_{(\mp)0} = J_{(\mp)0} - \lambda_{(\mp)} = 0.
\]

To impose these gauge conditions in the functional integral, we introduce a set of ghost fields \(c^{\alpha} = (e^{\mp}, c^{(\mp)a})\), antighosts \(\bar{c}^{(\mp)a}\), and multipliers \(b^{(\mp)a}\). After introducing the gauge fermion \(\Psi\) in the usual way,

\[
\Psi = \bar{c}^{(-)0} \Omega_{(-)0} + \bar{c}^{(-)} \Omega_{(-)} + \bar{c}^{(+)} \Omega_{(+)0} + \bar{c}^{(+)} \Omega_{(+)}.
\]

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the gauge fixing and the Fadeev–Popov parts in the quantum action are determined by

\[ s\Psi = \mathcal{L}_{GF} + \mathcal{L}_{FP}, \]

\[ \mathcal{L}_{GF} = b(-) + \Omega_{(-)} + b(-) \Omega_{(-)} + b(+) \Omega_{(+)} + b(+) \Omega_{(+)} , \]

\[ \mathcal{L}_{FP} = -c(-) [s\Omega_{(-)}] - c(+) [s\Omega_{(+)0}] - c(+) [s\Omega_{(-)}] - c(+) [s\Omega_{(+)}0] , \]

where \( sX \) denotes the BRST transformation of \( X \), obtained by replacing gauge parameters in \( \delta X \) with ghosts. Explicit form of the Faddeev–Popov term is obtained with the help of

\[ s\Omega_{(\mp)} = - (e^\mp J^{(1)}_{(\mp)\pm})' + c(\mp) J^{(1)}_{(\mp)\pm} , \]

\[ s\Omega_{(0)} = - (e^\mp J^{(2)}_{(\mp)0})' + c(\mp) J^{(2)}_{(\mp)0} \pm 2\kappa (c(\mp)0)' . \]

The integration over the multipliers \( A(\mp) \) and \( b(\mp) \) yields

\[ \mathcal{L}(\varphi, \pi_i, h) = \pi_1 \delta_1^\alpha + \pi_2 \delta_2^\alpha - h^\mp T_\mp - h^+ T_+ + \mathcal{L}_{FP} \bigg|_{I=\Omega=0} . \]  \( (19) \)

To evaluate this Lagrangian it is convenient to rewrite the first class constraints \( I_{(\mp)\pm} = 0 \), \( I_{(\mp)0} = 0 \) and the related gauge conditions in the form

\[ J^{(1)}_{(\mp)\pm} = \mu_{(\mp)} = - J^{(2)}_{(\mp)0} , \quad J^{(1)}_{(\mp)0} = \lambda_{(\mp)} = J^{(2)}_{(\mp)0} . \]

From here we see that \( \pi_{1x}, \pi_{1y} \) and \( \pi_{2x}, \pi_{2y} \) are constants, therefore the related \( \pi_1 \delta \) terms in the action can be ignored as total time derivatives. The above conditions also ensure that the contribution of the Faddeev–Popov term is decoupled from the rest, so that the integration over antighosts and ghosts can be absorbed into the normalization of the functional integral. The calculation of \( T_{\mp} \) leads to

\[ \kappa T_{\mp} = [\pm (K_{1\mp})^2 + 2\kappa (K_{1\mp})'] + [\mp (K_{2\mp})^2 + 2\kappa (K_{2\mp})'] \mp \frac{1}{4} \mu^2 (e^{\varphi_1} - e^{\varphi_2}) , \]

where \( K_{1\mp} = (\pi_{1\mp} + \mp \kappa \varphi_1')/2 \), and \( \mu^2 = \mu_{(-)} = \mu_{(+)} \).

As before, we eliminate the remaining momenta by using their equations of motion,

\[ \pi_1 \varphi = \kappa \frac{\varphi_1'}{h^- + h^+} [2\varphi_1 + \varphi_1'(h^- + h^+) + 2(h^- - h^+)'] , \]

and \( \pi_2 \varphi = -\pi_{1\varphi} |_{\varphi_1 \rightarrow \varphi_2} \), and obtain the effective Lagrangian

\[ \mathcal{L}(\varphi_1, \varphi_2, h) = \Lambda(\varphi_1, h) - \Lambda(\varphi_2, h) , \]

\[ \Lambda(\varphi, h) \equiv \kappa \frac{\varphi_1'}{h^- + h^+} \left\{ (\varphi + h^- \varphi')(\varphi + h^+ \varphi') + 2[(h^-)'(\varphi + h^+ \varphi') + (h^+)'(\varphi + h^- \varphi')] \right\} + \frac{\mu^2}{\kappa} (h^- - h^+) e^\varphi . \]  \( (21a) \)
Now, if we introduce new variables $\phi$ and $F$,

$$\phi = \frac{\alpha}{2}(\varphi_1 - \varphi_2), \quad F = \frac{1}{2}\varphi_2, \quad \alpha \equiv 2\sqrt{\kappa},$$

the Lagrangian $L(\varphi_1, \varphi_2, h)$ can be written as a sum of three terms:

$$L_1 = \frac{1}{h^- - h^+}(\dot{\phi} + h^- \phi')(\dot{\phi} + h^+ \phi'),$$

$$L_2 = \alpha(\omega_0 \phi' - \omega_1 \dot{\phi}),$$

$$L_3 = \mu^2 \frac{1}{\alpha^2}(h^- - h^+)e^{2F}(e^{2\phi/\alpha} - 1),$$

where

$$\omega_0 = \frac{1}{h^- - h^+}[(h^- h^{})' + 2h^- h^+ F'(h^- + h^+)]F,$$

$$\omega_1 = -\frac{1}{h^- - h^+}[(h^- h^{})' + (h^- + h^+) F' + 2\dot{F}].$$

To recognize the geometrical meaning of the action (21b) we now introduce, in addition to the metric density $\tilde{g}^{\alpha\beta}$, Eq.(12a), the determinant of the metric:

$$\sqrt{-g} = \frac{1}{2}(h^- - h^+)e^{2F}.$$

The transformation rule for $\sqrt{-g}$ is of the correct form. By noting the identity $\sqrt{-g}R = 2(\partial_0 \omega_1 - \partial_1 \omega_0)$, one finds, after a partial integration in $L_2$, that the final form of the action coincides with the induced gravity action (1), with $M^2 = 2\mu^2/\alpha^2$.

5. Concluding remarks

The general method of constructing canonical gauge invariant actions is used to prove that the $SL(2, R)$ invariant WZNW system of the (L–R) type, Eq.(2), is gauge equivalent to the induced 2D gravity (1).

We first obtained the covariant extension (15) of the (L–R) WZNW theory, working with the local form of the action, and using the energy–momentum components $T_{\mp}$ as the generators of the diffeomorphisms. Then, we constructed a more general gauge invariant action (17), based on the set of first class constraints $T_{\mp}$ and $(I_{(-)} +, I_{(-)} 0, I_{(+)} -, I_{(+)} 0)$, where $I$’s represent a subalgebra of $SL(2, R)(-) \times SL(2, R)(+)$. If the gauge symmetry is fixed in the simplest way, $A^{(\mp)} = 0$, $h^{\mp} = \pm 1$, the theory (17) goes over into the original (L–R) WZNW action (2), while a different gauge fixing of the same symmetry, given by Eq.(18), leads to the induced gravity action (1). The induced gravity and the (L–R) WZNW system for $SL(2, R)$ are thus shown to be gauge equivalent.
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