INTERMITTENCY FOR
COHERENT
AND INCOHERENT CURRENT
ENSEMBLE MODEL *

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Abstract

We investigate the origin of intermittency for multiparticle distribution in momentum space, following the idea that there is a kind of power law distribution of the space-time region of hadron emission. Using the formalism of current ensemble model to describe boson sources we discuss intermittency exponents for the coherent and incoherent (chaotic) particle production scheme.

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1 Introduction

Recently, several experiments [1],[2],[3] found that the phenomenon of intermittency [4],[5] is dominated by very short range correlations between momenta of identical hadrons. As it is well known, the HBT correlations [6] reflect the size and shape of the space-time region from which the observed identical particles are emitted [7]. Remembering that intermittency is equivalent to a power law dependence of hadronic correlation functions (multiplicity distributions), one may conclude that the power law dependence must also be present in the distribution of space-time shapes and sizes of the region of hadron emission [8]. The first analysis of this problem was done in Ref. [9] using the HBT formalism for purely incoherent hadron source. The considered description of hadron emission was simplified: totally incoherent production from the space region was assumed, and therefore the single particle distribution could not be adequately described. Nevertheless, intermittency exponents obtained in this model linearly increased with the rank of multiparticle distributions.

In order to correct description of the single particle distribution the pion source should not only have a power-law density profile but also some space-time correlations between particle emitting points. In this paper we would like to consider the model for HBT correlations worked out by Gyulassy, Kauffmann and Wilson in Ref. [10], known as the covariant current formalism or the current ensemble model. It has been often used to describe many different aspects of multiparticle production (see eg. Ref. [12], [13], for review Ref. [7] and references therein). Covariant current formalism is a generalization of the HBT formalism formulated in the language of field theory. Pion emitting regions are described as ”currents” with different phases, and incorporated into the pion field equation in the form of current ensemble. Each current is localized around some point called a collision site, and the collision sites are distributed in space-time with some probability density. Pion emission depends on current phases: if phases are identical, particles are emitted
coherently, if they change randomly, the pion production will be incoherent.

Current ensemble model suits our purposes very well. On the one hand, it is a natural continuation of the treatment presented in [9]. The quantum-mechanical source density is replaced by the power-law distribution of collision sites. The case of coherent and incoherent emission can be described easily. On the other hand, current ensemble formalism operates already on pions (hadrons). This fact removes one serious difficulty which plagues the interpretation of intermittency in terms of parton distribution. The problem is that, even if one explains the intermittent behaviour of the parton system, the experiment observes hadrons, not partons. Consequently, one should take into account the hadronization process. Usually, one invokes at this point the principle of parton-hadron duality, the phenomenon which is in fact not understood. However, if intermittency is caused by the HBT phenomenon, there is no problem of hadronization and the decay of resonances: the HBT effect works already on hadron level, reflecting the structure of the source density. And so does it in the current ensemble model.

Alternative approaches to the calculation of Bose-Einstein correlations (for review see [9] and references therein) incorporate also the reaction dynamics. Hence the current ensemble model can be considered as a plausible ”first approximation” concentrated only on the effects of source geometry on the shape of correlation functions.

In our previous studies of the current ensemble formalism [11] we have already proved that assuming purely coherent production of particles, multiparticle distributions are influenced by the scaling space-time structure of bosonic source, and give observed intermittent results if the source number is large enough.

In the present paper we analyze the case of incoherent (chaotic) particle production. We apply the method of random phase averaging described in Ref. [7] and [10] to generate incoherence. This formalism is presented in section 2. In section 3 we explain briefly how to measure intermittency parameters. In section 4 we incorporate the scaling dependence of the pro-
duction region into the model. At first we discuss the simple power law
distribution of collision sites following \[9\] and prove that it does give ob-
servable intermittent result. Later we show that the obtained results can be
generalized for other source distributions showing the scaling behaviour. At
the end of this section we describe the special case of the fixed source number
for correlation functions.

The aim of our discussion is to prove that in a well working physical
model of boson production like the current ensemble model, multiparticle
distributions in momentum space can be strongly influenced by the scal-
ing behaviour of the space-time structure of bosonic source, and can show
intermittent behaviour.

2 Classical current formalism

2.1 Pion fields

In this section we would like to recall the main ideas of current ensemble
model \[10\]. To obtain the final pion state produced by a classical current
source one should solve the following field equation for the scalar pion field
\( \Phi(x) \) with the source current operator \( J(x) \) :

\[
(\partial_\mu \partial^\mu + m_\pi^2)\Phi(x) = J(x)
\]

(1)

In principle, the source current \( J(x) \) is an operator coupled to the pion fields,
and treating it as a complex space-time function is only an approxima-
tion. Because we will not specify the conditions of production process it is difficult
to define when this approximation could be used. We assume simply that
we are allowed in our model to replace the pion current operator \( J(x) \) by its
expectation value \[10\].

Solution of (1) gives the coherent final multipion state :

\[
| \Phi > = e^{-\Phi} exp(i \int d^3k J(k)a^+(k)) | 0 > , < \Phi | \Phi > = 1
\]

(2)
where $a(k)$, $a^+(k)$ are creation and annihilation operators, $n$ is the average pion multiplicity and $J(k)$ is the on-mass-shell Fourier transform of $J(x)$ \cite{7}, \cite{10}. Pion density matrix $\rho_\pi$ constructed from (2) is:

$$\rho = | \Phi \rangle \langle \Phi |$$  \hspace{1cm} (3)

Multipion-inclusive distributions $P_m(k_1, \ldots, k_m)$ are then defined by the formula:

$$P_m(k_1, \ldots, k_m) = \frac{1}{\sigma_\pi d^3k_1 \ldots d^3k_m} Tr[\rho_\pi a^+(k_1) \ldots a^+(k_m) a(k_m) \ldots a(k_1)]$$  \hspace{1cm} (4)

### 2.2 Pion source as current ensemble

Now we are ready to consider the following description of pion emission: pions are produced in space-time centers $x_1, \ldots, x_N$, distributed with some probability density $\rho(x)$. These centers can be for example an effect of $N$ separate collisions "producing" pions \cite{10}. In this picture, the total pion source $J(x)$ would thus be a sum of $N$ different currents $J_i(x)$:

$$J(x) = \sum_{i=1}^{N} J_i(x) e^{i\varphi_i}$$  \hspace{1cm} (5)

where we allowed the possibility that the individual currents have different phases. It reduces easily to the purely coherent case described in \cite{11} assuming that:

$$\varphi_1 = \ldots = \varphi_N \equiv \varphi$$  \hspace{1cm} (6)

We shall consider the situation when the $J_i(x)$ depend only on distance from the individual collision site $x_i$. It means that, if a collision centered in $x = 0$ is parametrized by $j_\pi(x)$ we get:

$$J(x) = \sum_{i=1}^{N} j_\pi(x-x_i) e^{i\varphi_i}$$  \hspace{1cm} (7)
and the on-mass-shell Fourier transform is given by:

\[ J(k) = j_\pi(k) \sum_{i=1}^{N} e^{i\phi_i} \exp(i\omega_k t_i - ikx_i), \quad \omega_k = \sqrt{k^2 + m^2} \]  \hspace{1cm} (8)

### 2.3 Multipion distributions for current ensemble. The coherent and incoherent case.

The density matrix \( \rho_\pi \) averaged over the number of sources \( N \) and positions \( x_i \) of the pion sources is:

\[ \rho_\pi = \sum_{N} P(N) \int d^4x_1 \ldots d^4x_N \rho(x_1) \ldots \rho(x_N) \mid \Phi \rangle < \Phi \mid \]  \hspace{1cm} (9)

where \( P(N) \) denotes the probability to find exactly \( N \) pion sources, and \( \rho(x) \) is the probability density to find the source placed at the point \( x \). The density \( \rho(x) \) is normalized to 1. Using the formulae (2), (4) and (9) one can obtain the inclusive multipion distribution \( P_m(k_1, \ldots, k_m) \) in the form:

\[ P_m(k_1, \ldots, k_m) = \sum_{N} P(N) \int d^4x_1 \ldots d^4x_N \rho(x_1) \ldots \rho(x_N) \mid J(k_1) \rangle ^2 \ldots \mid J(k_m) \rangle ^2 \]  \hspace{1cm} (10)

Substituting \( J(k) \) in (10) in the form (8) one obtains:

\[ P_m(k_1, \ldots, k_m) = \mid j_\pi(k_1) \rangle ^2 \ldots \mid j_\pi(k_m) \rangle ^2 \sum_{N} P(N) \int d^4x_1 \ldots d^4x_N \rho(x_1) \ldots \rho(x_N) \sum_{i_1=1}^{N} \ldots \sum_{i_2m=1}^{N} \sum_{i_1=1}^{N} \ldots \sum_{i_2m=1}^{N} e^{ik_1(x_{i_1} - x_{i_2})} e^{ik_2(x_{i_3} - x_{i_4})} \ldots e^{ik_m(x_{i_{2m-1}} - x_{i_{2m}})} e^{i(\phi_{i_1} - \phi_{i_2})} e^{i(\phi_{i_3} - \phi_{i_4})} \ldots e^{i(\phi_{i_{2m-1}} - \phi_{i_{2m}})} \]  \hspace{1cm} (11)

One should remember that in the scalar product of 4-dimensional vectors \( k_i \) and \( x_j \) the first component of \( k_j \) is equal to \( \omega_{k_j} \) as a result of the on-mass-shell Fourier transform taken in (8). For current phases satisfying the relation (6) we recover multiplicity distributions for the coherent particle production from [11]. If the sources add incoherently, we should also average (11) over...
the set of phases. This method of incoherence generation has led to the name "chaotic" production because we assume that the current phases $\varphi_i$ fluctuate randomly from 0 to $2\pi$. After phase averaging we obtain the formulae for the single- and double pion distribution in the form:

$$P_1(k_1) = |j_\pi(k_1)|^2 < N >$$

$$P_2(k_1, k_2) = |j_\pi(k_1)|^2 |j_\pi(k_2)|^2 ( < N^2 > + < N(N-1) > |\rho(k_1 - k_2)|^2)$$

One should notice that for $N = 1$ one gets the coherent field results Ref. [11]. The produced particles are then uncorrelated. Hence we exclude the case $N = 1$ from our further investigations.

### 3 Intermittency exponents

Intermittency is equivalent to the power law dependence of hadronic moments in momentum space. The hadronic moments we would like to consider in this paper are factorial moments obtained by integration of multiparticle distributions and cumulants obtained by integration of correlation functions in the finite region of size $\delta$. Generally, one studies the behaviour of the factorial moments which tests the whole n-particle distribution function. However, the cumulants have the advantage of testing the genuine n-particle correlations, and so it is always interesting to investigate their contribution to the higher order correlations (see e.g. [14]).

Scaling dependence of the moments is described by the intermittency exponents. After integration of pion multiplicities $P_m(k_1, \ldots, k_m)$ in the finite region of size $\delta$, one obtains a kind of series in $\delta$:

$$F_m(\delta) = \int_0^\delta P_m(k_1, \ldots, k_m)d^3k_1 \ldots d^3k_m = \sum_j a_j(L\delta)^{-\alpha_j}$$

where $\alpha_j$ are intermittency indices for the factorial moment $F_m$, $a_j$ are the weights and the length $L$ is introduced for dimensional reasons. In experiment we observe the term dominating in (14).
So, for our purposes we define the intermittency exponent \( f_m \) to be equal to \( \alpha_j \) taken from the term dominating in (14). We should notice it need not to be the term with max/min(\( \alpha_j \)) because of the weights \( a_j \). In the similar way one can get the cumulant intermittency exponent \( \nu_m \).

There is not much experimental data concerning the higher order moments. The existing ones show intermittency exponents increasing with rank of multiparticle distributions \[14\]. The NA22 group measured also the third order cumulant with the preliminary result \( \nu_3 = 2\nu_2 \) \[14\].

4 Scaling behaviour of the distribution \( \rho(x) \)

4.1 Scaling distribution of production centers

In this section we show that the scaling distribution of collision sites \( \rho(x) \) (defined in (9)) can produce the scaling of momentum distributions \( P_m \). We consider the simplest example of the distribution \( \rho(x) \) with the assumption that all particles were produced at the same moment \( t_0 \):

\[
\rho(x) = \delta(t - t_0) \rho_S(|x|)
\]

and the space part follows a power law:

\[
\rho_S(x) = L^{-\alpha} |x|^{|\alpha-3|}, 0 < \alpha < 2, \alpha \neq 1
\]

where the length L was introduced for dimensional reasons.

The exact power law behaviour in (16) implies that the \( \rho(x) \) in (15) cannot be normalized. Following \[13\] we introduce a simple cut-off to get rid of the problem. It could be also done in a more elegant way but for our purposes of rather qualitative analysis it is enough to use the cut-off:

\[
\rho_S(x) = L^{-\alpha} |x|^{|\alpha-3|} \Theta(L - |x|) \alpha(4\pi)^{-1}
\]

In this case L can be interpreted as a size of the pion production region. The assumption of simultaneous particle production in (15) allows to consider in
and respectively in (12), (13) only 3-dimensional Fourier transforms of $\rho_S$. Using the relations for 3-dimensional Fourier transform:

$$\rho_S(k) = \int d^3xe^{-ik\cdot x}\rho_S(x)$$

$$\rho_S(k = 0) = \int d^3x\rho_S(x)$$

one obtains the distribution in momentum space:

$$\rho_S(k) = \alpha(L | k |)^{-\alpha}\int_0^{L|k|} du u^{\alpha-2}\sin u$$

One can observe the power law behaviour in (20) only for $L | k | \geq 1$. For $L | k | \leq 1$ the singularity will be cut off, and $\rho(k)$ in (20) tends smoothly to 1 for $L | k | \to 0$. We notice also that $\rho_S(k) \leq 1$ for any $k$.

4.2 Leading terms for multiplicities in the chaotic source current ensemble

Now we will discuss leading terms in (14) for pion multiplicities defined in (11). At first we consider the lowest order distributions (12) and (13). After integrating the one-, two-, and three-pion distribution from (15) in the region $(0, \delta)$ we get respectively:

$$F_1(\delta) = <N > c_{11}(\delta)$$

$$F_2(\delta) = <N^2 > c_{21}(\delta) + <N(N-1) > c_{22}(\delta)$$

$$F_3(\delta) = <N^3 > c_{31}(\delta) + <N^2(N-1) > c_{32}(\delta) + <N(N-1)(N-2) > c_{33}(\delta)$$

where $c_{ij}$ are results of integrating $P_1(k_1)$, $P_2(k_1, k_2)$, $P_3(k_1, k_2, k_3)$ obtained from (11): they depend on $\delta$ but they are independent of $N$. It is also important to notice that both terms in (22) which survive after phase averaging have the same rank of $N$. From (21) we obtain obviously $\alpha_1 = 0$ for $P_1(k_1)$. From the distribution $P_2(k_1, k_2)$ we will get the power singularity $\delta^{-2\alpha}$ only
in $c_{22}(\delta)$. There are two singularities: $c_{32} \sim \delta^{-2\alpha}$ and $c_{33} \sim \delta^{-3\alpha}$ in $F_3$ of the same rank in $N$, so the singularity $\delta^{-2\alpha}$ will dominate here. Hence the intermittency exponent is:

$$f_2 = f_3 = 2\alpha$$

(24)

Now we will analyze the general form of multipion distribution $P_m(k_1, \ldots, k_m)$ to get the dominating term there. At first let us consider terms which can survive in (11) after phase averaging. The answer will be obtained from the analysis of the expression:

$$< e^{i(\phi_{i1} - \phi_{i2})} e^{i(\phi_{i3} - \phi_{i4})} \ldots e^{i(\phi_{i2m-1} - \phi_{i2m})} >_{[\phi_i]}$$

(25)

where the average is taken over the set of phases $\phi_i$. The exponent product in brackets should be equal to 1. *All the surviving terms have the same rank $N^m$ of $N$.*

So we are ready to consider leading terms in $P_m(k_1, \ldots, k_m)$ for any $m > 1$. Because all the terms have the same rank of $N$, and $\rho_S(k)$ shows the power law behaviour only for $L\delta \geq 1$ as mentioned in the previous section we conclude that the term with the smallest intermittency exponent will dominate and therefore:

$$f_m = 2\alpha$$

(26)

One can check this result holds on for both large and small $N$. It means that *intermittency exponents for factorial moments will not change with the rank of multiplicity for the chaotic production.* The result for scaled factorial moments will be the same.

The expression we have got above differs from intermittency exponents obtained in [11] for the coherent source ensemble. In [11] the intermittency exponents fulfilled the relation:

$$f'_m = 2m\alpha$$

(27)

The calculations we have done to get $f'_m$ requested the assumption $N$ to be large enough. For small $N$ the formulae were very complicated, and it was
difficult to see how they could provide the simple power law behaviour as observed in experiment. There were also no scaling in the scaled factorial moments.

The result (26) can be actually derived under much less restrictive conditions. To see this, let us first formulate general conditions that the function $\rho(x)$ must fulfill to be considered as a probability distribution and to show the scaling behaviour:

$$\rho_S(x) \geq 0 \quad (28)$$
$$\int \rho_S(x) d^3x = 1 \quad (29)$$
$$\rho_S(k) \propto \text{const}(L \mid k \mid)^{-\alpha} \text{in some interval} L \mid k \mid \in (a, b) \quad (30)$$

One can notice that conditions (28), (29) give the following inequality for the 3-dimensional Fourier transform:

$$\rho_S(k) \leq \rho_S(k = 0) = 1 \text{ for any } k \quad (31)$$

So we can generalize the results obtained for the collision sites distribution $\rho(x)$ defined in (15), (16). For any function $\rho(x)$ which fulfills conditions (28), (29), (30) intermittency exponents behave like (26).

### 4.3 Correlation functions

Analyzing the results from Ref. [11] for the coherent source ensemble one can observe that intermittency exponents (27) calculated from multiplicities are equal to intermittency exponents obtained from correlation functions. For the chaotic production we can make an interesting remark. If the source number $N$ is fixed – in our formalism it means that $N$ is given by a ”spike” distribution $P(N)$ and $<N> \approx N$, the scaled correlation functions look like:

$$C_2(k_1, k_2) = \frac{N - 1}{N} | \rho(k_1 - k_2) |^2 \quad (32)$$
$$C_3(k_1, k_2, k_3) = \frac{(N - 1)(N - 2)}{N^2} [\rho(k_1 - k_2)\rho(k_3 - k_1)\rho(k_3 - k_2) + c.c.] \quad (33)$$
It seems to be possible to generalize easily the above results for correlation functions of any rank $m > 1$. Then intermittency exponents will grow with the rank $m$ following the rule:

$$\nu_m = m\alpha$$  \hspace{1cm} (34)

identically with the behaviour of intermittency exponents calculated in Ref. \[9\]. And for multiplicities we get as usually intermittency exponents following (26). There is no contradiction here. The term with the exponent (34) appears also in the multiplicity but it will be small compared with the term with the smallest exponent (26). So correlation functions seem to be a better tool to investigate the scaling properties of the pion production. In the above case the correlation function simply extracts the term with the largest exponent.

5 Conclusions

We have investigated the relation between intermittency and the scaling behaviour of the space-time distribution of collision sites in current ensemble model. Our conclusions can be summarized as follows:

- there is a possibility of intermittency in current ensemble model provided that
  I. source number $N$ is not fixed and :
  (a) one assumes a power law singularity in the distribution of collision sites
  (b) the particle production is totally coherent
  (c) the number of coherent sources $N$ is large enough. Then intermittency exponents grow linearly with the increasing rank of multiplicities following the Eq. (27). For small number of sources $N$ intermittency is generally not observed. There are no well defined leading terms which can give intermittency exponents growing with the rank of moments/
multiplicities. In the scaled factorial moments/cumulants scaling does not appear.

II. source number $N$ is fixed and:

(a) one assumes a power law singularity in the distribution of collision sites
(b) the particle production is incoherent

Then the intermittency exponents for scaled and not scaled factorial moments do not change with the rank of multiplicity. However, intermittency exponents for scaled cumulants follow the result obtained in Ref. [9] for one incoherent source, i.e. the formula (34).

- the above results only partially agree with the experiment. The intermittency exponents can grow with the rank of the multiplicity/correlation function, except the intermittency exponents for the factorial moments in the case of incoherent production. In this case correlation functions better detect scaling properties of the emission region than multiparticle densities. However, the result for cumulants $\nu_3 = 2\nu_2$ obtained by the NA22 group cannot be confirmed.

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