A commognitive Study on Field-dependent students’ Understanding of Derivative

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Abstract. Cognitive style is a characteristic of students which influences in understanding material learning. The research objectives to explore the student ability to understand the derivatives seen from the commognition perspective. Commognition is defined as a combination of communication and cognition that considers learning as the individualization of patterned collective activities. Moreover, every communication that conducted is called discourse. Keywords, visual mediators, endorsed narrative, and routines are characteristics of mathematical discourse. This descriptive study divided into two groups of students, namely field-independent and field-dependent. This paper only focused on field-dependent students. The data collection was carried out by task-based observation in understanding derivatives using the Focus Group Discussion technique. The steps performed to conduct the data analysis include condensation of data, display of data, and drawing of conclusion and verification. The results showed that the commognitive aspects of students with a dependent-field cognitive style such as keyword, visual mediators, endorsed narrative, and routines appear in each category. However, the scaffolding should be provided by the researcher to obtain the responses from the students.

1. Introduction
Cognitive style is a preferred way for someone to choose, perceive, and process new information, and this has become the focus of research in education [1], [2], [3]. In general, identifying and classifying cognitive structures of student are the purpose in cognitive style research. Moreover, the intended academic achievement of student will be compared to the cognitive structures of student. Furthermore, the instructional methods are matched with the academic achievement of student for better individual learning [3]. Messick [4] states that Herman A. Witkin in the mid-1940s had constructed cognitive styles, namely field-independent (FI) and field-dependent (FD), which were formed based on external influences on individual perceptions and judgments. Furthermore, Messick [4] said FI individuals tend to articulate numbers as discrete from their background and easily distinguish objects from embedded contexts. In contrast, FD people tend to experience events globally in no different way. Furthermore, [5] states that the FI cognitive style can find an upright position relatively more dependent on kinesthetic references. In contrast, those who are more dependent on visual references are FD cognitive styles.

Findings from several previous studies indicate that field-independent style students tend to impose their analytical structure on ambiguous material and ignore external influences. In contrast, students with field-dependent style pay more attention on social and environmental information in learning situations and respond to reinforcement external, both material and social [5], [6]. So, each individual’s
cognitive style is reflected in the individual itself when the individual processes or processes information.

According to the traditional view of individual development, learning can always be seen as a change process. Based on this, in the 20th-century, differences of opinion emerged on what the changed is when learning occurred. According to behaviorists, with learning, there will be a change in student behavior. Simultaneously, the cognitive view says learning is to make the concept of learning as an acquiring process or developing entities of mental said ideas, knowledge or mental vignette. According to [7] and [8], the unique learning process is when students become participants of well-defined forms of activity. Based on that, we cannot focus only on individuals who are learning, but the mechanism of student activity must be considered.

Sfard [9] and [10] stated the combination of the words communication and cognition is commognition which is the main idea of a learning approach based on the presumption that thinking can be conceptualized as a communication of someone with himself. Sfard [10] said the idea of commognition arose from a psychologist and philosopher who claimed his thoughts and expressions were inseparable (Vygotsky 1987; Wittgenstein 1953) and post-modern philosophers about the discursive nature of human knowledge, namely, socio-historical-ontogenetic changes of activity known as science, research, or knowledge development (Lyotard, 1979; Foucault, 1972; Rorty, 1979).

2. Theoretical Frameworks
The following will present the fundamental theories that support this research

2.1. Cognitive style
The concept of cognitive style is about psychological differences, whose roots lie in the fact that the conceptual field around someone affects the perception of individual. This psychological difference is referred to as field-independent and field-dependent [11]. Furthermore, according to [12] and [13], Field independent individuals are more objective and autonomous, which causes they separated from their surroundings. Another advantage is that they are more analytical, while field dependent individuals prefer to interact with persons who are closer. FD individuals are more influenced by external cues. As a result, they find it difficult to sort out the information which they receive and are also not selective in the absorption of that information. Research using the cognitive style of FI and FD has been developed by [14]. According to [14], the perception of FI is only partly affected by changes in the situation because it can choose based on the situation. In consequence, FI is more analytical. For example, FI can analyze a pattern based on its constituent components because what it does first is to separate parts of the pattern. [14] describe the characteristics of the field-independent cognitive style: (1) Analytic, competitive, independent, and individualistic, (2) Has own goals, objectives, strategies, and reinforcement, (3) Intrinsically motivated, (4) Lacking social skills / prefers individual tasks, (5) Structured and well organized in learning. In contrast, individuals who have a field-dependent style tend to have difficulty distinguishing stimuli through their situations so that their perceptions are easily influenced by manipulation from their surroundings. FD tends to view a pattern as a whole, not separating it into parts. According to [14], the perception of FI is only partly affected by changes in the situation because it can choose based on the situation. In consequence, FI is more analytical. For example, FI can analyze a pattern based on its constituent components because what it does first is to separate parts of the pattern.

[14] describe the characteristics of the field-dependent cognitive style as follows: (1) Global and social-sensitive, preferring group assignments, (2) Easily influenced by the field or context, (3) Motivated from the outside, (4) Sensitive to environment, (5) Less structured and less independent.

Based on the above opinion, it can be said that individuals who have a field-independent cognitive style tend to respond to stimuli using their perceptions, are more analytical, and analyze patterns based on their components. Whereas individuals who have a field-dependent cognitive style tend to respond to a stimulus using environmental conditions as a basis in their perceptions and tend to view a pattern as a whole (global), not separating its parts.

2.2. Commognition
[15] says that cognition is based on the assumption that thinking is a form of communication, and learning mathematics is tantamount to modifying and expanding one's discourse. Thinking (cognition) is an inherent individual activity, which can develop into group activities. Furthermore, [9] states that human thought can be considered a form of personal communication activities, namely communicating
with oneself. Co communion is rooted in collective actions that follow specific rules. Different communication types that unite some people even though they do not include several others are called discourses. The characteristics of discourse are considered mathematical if the mathematical word is related to quantities and shapes. Moreover, [16], [15], [9], [17] said that mathematical discourse is characterized by keywords, visual mediators, endorsed narratives, routines. Routine is divided into three types: exploration, rituals, and deeds.

Sfard [10] defines keywords as an important word because it is the same as "a word meaning." Furthermore, according to [17], which includes keywords in mathematics, 'limit,' 'open,' 'continuous,' and 'group.' Visual mediators are objects that operate as part of the communication process. According to [9], "Visual mediators are defined as providers of images with which discussants identify the objects of their talk and coordinate their communication," which means visual mediators are objects such as graphics used by participants in mathematical discourse to identify communication objects. This is in line with the opinion of [18], [19], [17], [20] states that visual mediators are symbolic artifacts such as numbers, algebraic expressions, tables, equations, and graphs. The narrative is any sequence of expressions (utterances) that describe an object, the relationship between objects, or processes with or by objects. In mathematical discourse, narrative solicitation is known as a mathematical theory, which includes discursive concepts as definitions, proofs, and theorems. The mathematical narrative divided into two levels, namely object-level and meta-level. Routine is a pattern that characterizes discourse. In particular, mathematical order can be noted whether someone uses keywords and mathematical mediators to strengthen the narrative. Furthermore, [21] "Routines refer to metarules that determine discursive patterns, including courses of action, and when to start or end specific courses of action," emphasizing patterns as actions. Furthermore, [9] divides routines into three types, namely explorations, rituals, and deeds.

| closing condition/goal | Ekplorasi | Ritual |
|------------------------|-----------|--------|
| By whom the routine is performed | Student- no need for scaffolding | Student- with (scaffolded) others |
| For whom the routine is performed | Student discourse is internally persuasive, depend oneself and mathematical properties. | Student discourse depend outside authoritative, spatial arrangement, or the appearance on entities. |
| Applicability | Wide – the procedure is applicable in various situations | Narrow – because the procedure depends on situation. |
| Flexibility | The student makes the procedure performed variously | The student follows the procedure strictly |
| Acceptability condition | The narrative produced through the performance must be substantiable | The narrative has to be shown strictly to the rules defining the routine procedure. |
| Penggunaan kata dan mediator | Use of keyword mathematics object | Use of keyword driven-phrase |

Source: Adapted from Sfard (2008)

2.3. Understanding

Understanding in the context of mathematics learning is related to the learning process of students in the class. One of the main objectives of learning mathematics is that students have an adequate level of understanding of mathematical knowledge. In this article, understanding facts, concepts, principles, and procedures is an aspect of the learning objectives' cognitive realm. [22] states that someone has
understood facts, ideas, or mathematical procedures if their mental representations become part of the person’s internal representation network.

[23] states seven cognitive processes in the category of understanding include (1) interpreting (changing one form of representation to another), e.g., changing the formula of a function into graphical form, (2) exemplifying (finding a specific example or illustration of a concept or principle), e.g., providing an example of the derivative of a function; (3) classifying (determining that something belongs to a category), e.g., grouping functions based on the existence of their derivatives at a point, (4) summarizing (abstracting a general theme or major point(s)), e.g., stating that all non-continuous functions at a point do not have a derivative at that point, (5) inferring (drawing a logical conclusion from presented information), e.g., concluding that all linear function have constant derivative, (6) comparing (detecting correspondences between two ideas, objects and the like), e.g., identifying that all increasing functions have positive derivative; (7) explaining (constructing a cause-and-effect model of a system), explaining some non-continuous function having no derivative at a point as every differentiable function must be continuous. This article will describe the seven cognitive processes in the category of understanding derivative of a function.

3. Method

This type of research is qualitative. According to [24], qualitative research is an approach to explore and understand meaning by individuals or groups relating to social or humanitarian issues. The subjects in this study were students in the 3rd semester of the academic year 2018/2019. This paper discusses subjects who have a field-dependent cognitive style. Furthermore, sampling is also based on gender and mathematics ability tests are relatively similar. Mathematical discourse is the main object of research because changes will occur as a form of communication activities. Learning is understood as a phenomenon that is inherently collective, or social, and not individual. Researchers can act as participants and as insider/outsider. The data analysis stage used is the three stages proposed by [25], namely condensation of data, display of data, and drawing of conclusion and verification.

4. Results of Students’ Work and Discussion

Students are given two different Understanding of Derivative Tasks in each cognitive category, consisting of Understanding of Derivative Task1 (TMT 1) and Understanding of Derivative Task 2 (TMT 2). The following will discuss the commognitive discourse (keywords, visual mediators, endorsed narratives, routines) that appear in each category.

4.1. Keywords

Keywords appear in categories: Interpreting, Classifying, and Summarizing. In the Interpreting category, the subject is asked to define derivatives in their own words. TMT 1 asked the subject to change the form of representation from images to words. In this case, they are asked to mention the elements contained in the image. The subject can say "curves", "tangents" and the "secant" (line $PQ_1$). TMT 2 asked the subject to change the form of representation from derivative formulas to words in expressing derivative definitions. They read the symbol $f'(c)$ as ‘f prime c,’ which states "the first derivative of f at c", the derivative of the function $f(c)$.

In the Classifying category, the subject is asked to determine which function has a derivative from some of the given functions. The results show that they can use formal keywords, namely "absolute function" and "step function," and they can distinguish from other forms of function.

In the Summarizing category, the question is, What are the characteristics of a function that has a derivative? Subjects can only say the keywords "continuous function" and "discontinuous function" based on the stimulus given to the question. The keywords mentioned by the subject above include nouns.
4.2. **Visual mediator.**

Visual mediators appear in categories: Interpreting, Classifying, Comparing, and Explaining. In the Interpreting category, points $PQ_1, PQ_2, PQ_3, \ldots, PQ_n$ which are the intersection points of the secant with curves, can be symbolized by the subject as $(c, f(c))$ and $(c + h, f(c + h))$ and the $PQ_1$ line gradient is symbolized by them with $m_{PQ_1}$. In the Classifying category, the subject is more familiar with using the symbol $f(x) = |x|$ by mentioning it as a step function. The subject expresses symbol $f(x) = |x|$ as an absolute function. In the Comparing category, from the TMT 1 given, the subject can only interpret the fragment of the question, namely “derivative of the sum of two functions and the sum of derivatives of each function” which is written with the symbol $(f + g)'(x)$ and $f'(x) + g'(x)$. Furthermore, from the TMT 2 given in the “limit number” part of the question, the subject can only write the limit notation. The Explaining category, in the question piece "the derivative of this odd function is an even function” the subject symbolizes $f(-x) = f(x)$ for even functions and $f(-x) = -f(x)$ for odd functions. Furthermore, in TMT 2 in the part of the problem, “function $f$ has a first derivative,” the subject can only write the first derivative formula symbol, namely $f'(x)$. In this part of the visual mediator, to facilitate communication, the subject uses more symbols.

4.3. **Endorsed narratives**

The endorsed narrative appears in the Interpreting and Comparing categories. In the Interpreting category, the subject has mentioned the keyword "bowstring" and has written the point symbols $P$ and $Q$. The researcher then directs the subject by stating that if $n$ goes to $\infty$ and $Q_n$ goes to $P$, then they respond by saying $h_n$ goes to zero, and they can write $n \to \infty$ and $Q_n \to P$, then $h_n \to 0$. In this case, we can say that the subject can use an endorsed narrative. Relating to the question fragment, "derivative of a sum of two functions and the sum of derivatives of each function,” the subject replaces the word “and” with the word "equal to.” Writing by the subject itself is expressed in the form $(f + g)'(x) = f'(x) + g'(x)$. This form is the definition of the quantity rule in derived material. The description carried out by the subject in this form appears in the Comparing category.

4.4. **Routines**

Routines appear in the categories Exemplifying, Classifying, Summarizing, Inferring, and Explaining. This paper discusses the routine that appears based on understanding categories. In the Exemplifying category, the routine that emerged included the “flexibility” ritual. In TMT1, the subject is asked to name two examples of functions that have derivatives. They can provide examples, but the examples given by them do not vary. It is very rigid because it only follows the examples mentioned and written by other subjects. TMT 2, two examples of functions that do not have derivative, the subject tries to give an example, but the example given is still wrong. The subject says a function that has no derivative is a constant function, for example $f(x) = 2$ and the other function is $f(x) = x^0$. In this category, it can be said that the subject has not fully understood the concept of derived functions because they cannot provide examples of functions that do not have derivatives, and the subject needs to be scaffolded from other people.

In the Classifying category, in TMT 1 and TMT 2, the subject can only classify functions that have derivatives and functions that have no derivatives. Functions which according to them have no derivative are $f(x) = |x|$ at $x = 0$; $f(x) = |x|$, where $x = 0$ and $f(x) = |x - 1|$, where $x = 1$ while the functions have derivatives according to the subject is $f(x) = x^2$, where $x = 0$; $f(x) = x|x|$, at $x = 0$; $f(x) = x^2 - 1$, where $x = 0$. The subject can already answer correctly but the subject can be said not to fully understand the concept of the derivative of a function. This is because they cannot explain why she classifies these functions. The subject needs to be scaffolded by other people.

In the Summarizing category, the subject cannot distinguish between continuous functions and functions that have derivatives. In this category, they also say that a continuous function is a function that has a limit value. On the other hand, the subject also said the characteristic of a function with a derivative if the limit value exists, even though the requirement for a continuous function is

$$\lim_{x \to c} f(x) = f(c).$$
Based on this, the terms used by the subject cannot be used in general because, in this case, one must pay close attention to what the subject means by “the left limit equals the right limit.” In this case, the subject needs to be scaffolded from others because the subject can be said to have not fully understood the concept of derived functions.

In the Inferring category, there is also confusion where the subject says that a function having a derivative means that the limit and the function are also continuous. It shows that the subject mixes up the term “the limit exists” to say a function has a derivative and a continuous function. So, it is necessary to emphasize the difference between the characteristic of a function having a derivative and a continuous function. Although in the end, the subject can make correct conclusions from the information provided. This category is also included in the applicability ritual.

In the Explaining category builds a causal model or a specific system, for example, by knowing a particular function, then using the relationship of derived functions with other functions. In TMT 1 and TMT 2, the subject could hardly do it because, at every step, she had to be guided and directed. Based on this, students need to be scaffolded, which is given by the researcher.

5. Conclusion

The results showed that not all of the students' discourse characteristics appeared in every category of the seven cognitive processes. Keywords of formal nouns and words occur during interpreting, classifying, comparing, and explaining. Visual mediators appear in interpreting, classifying, comparing, and explaining where students use symbols as a medium of communication. The definition as a narrative characteristic of solicitation emerges during interpreting and classifying. Routines appear in the categories Exemplifying, Classifying, Summarizing, Inferring, and Explaining. Based on three types of routine, namely exploration, ritual, and deed, only the type of ritual appeared. The part of the ritual that appears is applicability, flexibility, and scaffolded. In this research, for the category of exemplifying, classifying, summarizing, and explaining, the subject needs to be scaffolded.

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