Abstract—Multi-Agent Pathfinding (MAPF) is the problem of finding a set of feasible paths for a set of agents with specific individual start and goal poses. It is considered computationally hard to solve. Conflict-based search (CBS) has shown optimality in developing solutions for multiagent pathfinding problems in discrete spaces. However, neither CBS nor other discrete MAPF techniques can be directly applied to solve Multi-Agent Motion Planning (MAMP) problems, the continuous version on multi-agent pathfinding. In this work, we present the extension of the CBS discrete approach to solve Sampling-based Motion planning problems, and we show its capabilities to produce roadmap-optimal solutions for multirobot motion planning problems.

I. INTRODUCTION

Pathfinding is the problem of finding a path between two vertices over a graph over a discrete space given a start and goal. Multi-agent pathfinding (MAPF) is the problem of finding feasible collision-free paths for a set of agents over the graph. Motion planning is the problem of finding a path in a continuous space. While there are efficient optimal solvers proposed for MAPF, existing techniques for multi-agent motion planning rely on computationally expensive coupled approaches and yield an asymptotical optimality.

CBS [1] is a novel algorithm that produces optimal solutions for MAPF instances for large numbers of agents. Unfortunately, due to the differences of pathfinding and motion planning, CBS cannot be directly applied to MAMP problems.

In this work, we present CBS-MP, an extension of the CBS approach to Sampling-based Motion Planning. We demonstrate its ability for MAMP problems to find roadmap-optimal solutions, or the best set of paths with respect to the current roadmaps. To the best of our knowledge, CBS-MP is the first application of CBS to continuous motion planning problems where agents operate on different state spaces. We validate its performance by solving problems involving high DOF Homogeneous and Heterogenous systems. Our method is compared against Coupled and Decoupled versions of PRM, one of the most relevant algorithms in the field.

II. RELATED WORK

A. Multi-Agent Pathfinding

In recent years, multi-agent pathfinding (MAPF) has received a lot of attention. The problem defined by a set of agents, a graph, and a corresponding start and goal location for each agent. A solution is a set of collision free paths across the graph moving each agent from the start to the goal. The quality of solutions are traditionally measured by either the sum-of-cost, summed cost of all paths, or makespan maximum individual path. An optimal solution is one that minimizes the desired metric.

Generally, all MAPF methods can be classified into three major groups: coupled, decoupled, and hybrid.

1) Coupled Approaches: In coupled approaches, all of the agent paths are computed in unison, step by step. These techniques work in a joint space of all agent states. They tend to provide stronger guarantees on feasible paths and minimum cost because they explore a good part of the joint space, however, this has a high computational cost as the dimensionality of the joint \( c_{space} \) increases with the number of robots and their degrees of freedom.

A simple MAPF solver can be implemented by concatenating all the agents into one single state. A single search algorithm like A* can then be used to traverse the joint space of all agents to get to the state solution [2]. Unfortunately, since the joint-space is the result of the cross-product of all
the single spaces, the number of states grows exponentially with respect to the number of agents (i.e. A* in joint space is \(O(V^{2n}E)\) for \(n\) agents). As a result, this type of approaches only works well for a small number of agents.

In order to deal with this issue and speed up the search, other works attempted to prune the search by using some heuristics and expand fewer nodes than regular A* [3] [4] [5]. However, deterministic algorithms such as A* may take a significant amount of time to solve a problem, or sometimes are not even able to solve it. Alternative approaches have modeled the multi-agent pathfinding problem as Integer Linear Programming (ILP) and Boolean Satisfiability (SAT) problems. In [6] Yu et al. mapped the problem to a network-flow and applied ILP algorithms to optimally solve four objectives: the makespan, the maximum distance, the total arrival time, and the total distance. In [7] the multi-agent pathfinding problem is recast as a SAT problem where logical variables are used to represent time and physical locations for each agent as well as for obstacles. Then logical rules are enforced to solve the problem while avoiding collisions and satisfying other constraints.

However, all those techniques suffer from being susceptible to increase their computational cost as the number of robot increases or even when dealing with a few robots with high DOF since they navigate in the joint state space.

2) Decoupled Approaches: To mitigate this potential computational cost, attention has turned to decoupled algorithms. Instead of planning all paths in unison, each path is planned individually. Once all paths are computed, they are adjusted according to defined priorities in order to avoid inter-agent collisions. Decoupled approaches work in lower-dimensional spaces allowing these strategies to rapidly compute feasible paths for problems with a large number of agents. However, these techniques do not explore the entire joint state space. As a result, there is the possibility of failing to find feasible paths on solvable problems.

To achieve the group coordination of multiple robots, [8] computes a set of pre-specified paths over a roadmap, considering motion safety and minimizing the traveling time. Then, possible collision points of the local paths of the robots are estimated. Finally, for each robot path, a fixed velocity is computed in order to avoid all conflict points. In [9] [10] [11] [12] [13], priorities are assigned to each agent based on different heuristics, and then their individual paths are computed in decreasing priority order while treating higher priority agents as dynamic obstacles. For example, in [14] a Genetic Algorithm implementation is used, where the optimal individual paths serve to both obtaining the priority order of the robots and a fitness function for a path genetic mutation. Then, when a conflict is detected, the less priority path is mutated several times until the conflict is resolved, once solved the path is added as a dynamic obstacle. Unfortunately for decoupled approaches, given the drawback of just exploring a reduced portion of the joint-space, decoupled planners are incomplete. They may fail to find a solution when one exists.

Not all decoupled approaches suffer from that issue. CBS [1] alleviates the incompleteness issue by considering all the possible ways in which conflicts between individual paths can be resolved. This is done by using a two-level search. The low-level generates individual robot paths in a decoupled manner. The high-level detects conflicts in these paths and generates constraints for the the low-level search to modify the individual paths. This is the method we extend to multi-agent motion planning in this paper.

Recent improvements and variations on the CBS algorithm have been proposed. In [15], ByPass-CBS allows internal modifications to conflicts found in the high-level search that compute alternative paths without expanding the high-level search. Additionally, the authors in [16] Continuous-Time-CBS extend the state-space search to plan in continuous-times.

3) Hybrid Approaches: Due to the tradeoff between faster computation times and finding optimal cost solutions, researchers explored new ways of leveraging the strengths of both Coupled and Decoupled techniques. These techniques are also known as hybrid approaches. In [17], M* solves the MAPF problem by initially planning in a fully decoupled manner. Then when inter-robot conflict arises, the individual search is backtracked until the last collision-free state, at which point, the conflicting agents are joined into a coupled meta-agent, and new collision-free paths are computed by using a coupled planner for those agents. In the worst of the cases, if all agents are in collision at the same place and time, MAPF is solved in a fully coupled manner.

Recent extension to the CBS algorithm also use this meta-agent concept. In [18], MetaAgent-CBS, when a conflict is found, rather expanding the high-level search, the conflicting agents are merged into a meta-agent and coupled planner is used in the low-level search to compute its path. In [19], an improved version of MetaAgent-CBS is proposed, where the decision of merging agents has only local effects and conflicting agents are treated again as single agents in further high-level exploration.

B. Multi-Agent Motion Planning

Motion planning is the problem of finding a feasible path between a start and goal in a continuous state space that is usually intractable to represent explicitly. This is a superset of the pathfinding problem that searches over a discretized state space. In motion planning, the state space is comprised of the set of all possible agent configurations known as the configuration space \(C_{space}\) [20]. A solution to the motion planning problem is a continuous path in a subset of \(C_{space}\)-called free space \(C_{free}\) consisting of valid configurations.

In response to the complexity of motion planning [21], sampling-based motion planners were developed as an efficient means of discovering valid paths in \(C_{free}\). These methods, such as the Probabilistic Roadmap Method (PRM) [22] attempt to create a roadmap, or graph, approximating \(C_{free}\). Paths are found by querying this roadmap.

Not much work has been proposed for sampling-based multi-agent motion planning (MAMP. Much of the work that
has been done modifies or extends two widely used single agent sampling based motion planning algorithms, PRM [22] and RRT [23]. In [24] [25] optimizations and improvements to regular RRT are proposed to enable it to solve multi-agent problems in Joint-Cspace. In [26], the multi-agent version of RRT* (MA-RRT*) searches for the shortest path in a group graph that represents the joint-state space of all agents. The returned solution is then a collision-free joint plan containing a path for each agent. In [27] [28], PRM is used to compare the performance of centralized and decoupled planning for multi-robot systems. In [29], a coupled planner called MRP-JC solves the MAMP problem in an incremental way. First, an individual path for the first agent is found and this serves as the basis for recursively guiding a coupled search in an incremented state-space to find the path of the next k+1 agent by reusing the paths of the first k agents which were already computed.

In [30], several ideas of how discrete MAPF techniques can be adapted to continuous problems are provided. "Relocating Roadmap Nodes", "Merging nodes" and "Addressing Node/Node and Node/Edge interactions" are some of these ideas, however no experimentation is provided to validate their applicability.

III. PRELIMINARIES
A. Conflict-Based Search

Conflict-Based Search (CBS) is an optimal decoupled MAPF method [1]. The approach utilized a low-level search to plan individual agent paths and a high-level search to resolve conflicts between the agent paths.

The authors use A* over a state space consisting of location and time [1]. The locations correspond to vertices on a grid roadmap shared by all agents. All vertices are one unit distance from their neighbors, and moving between neighboring vertices constitutes one timestep.

The high-level search utilizes a Conflict Tree (CT) [1]. Nodes in this tree contain a set of paths for all agents, a set of path constraints for the various agents, and a cost of the solution. The set of initial paths planned for all agents individually is used to create the root node. The root node contains no constraints, and the cost is relative either the SOC or makespan for the solution.

The paths in this node are checked for collision amongst themselves. A collision constitutes to agents occupying the same vertex or traversing the same edge simultaneously [1]. Upon discovering a conflict, a pair of constraints is generated, one for each agent. Simply put, the constraint indicates that the agent can not utilize the vertex/edge at the timestep of the conflict.

A child node is spawned for either constraint. All constraints in the parent node are also passed down. The path of the corresponding agent is recomputed with the low-level search given the new constraint, and the cost of the child node is computed. The remaining unexplored node with the minimum cost is selected next to be checked for conflicts, and the process is repeated.

When a node is found to contain a solution free of conflicts it is deemed a solution node. The search keeps track of the minimum solution node and its cost. As all child nodes have a cost at least as great as their parent, once all remaining unexplored nodes have a cost greater than the minimum solution node the search stops and returns the minimum solution. By continuing to search until all remaining solutions have a greater cost, CBS guarantees an optimal solution.

B. Adapting CBS to Sampling-based Motion Planning

The path computation, conflict detection, and conflict resolution is made simpler in the MAPF problem by the uniform distance and time represented by the edges in the grid roadmap. In sampling-based motion planning, the length of an edge in the roadmap is no longer uniform. Additionally, heterogeneous agents may move at differing speeds, such that the same distance along a path does need indicate a collision by occupying the same space at the same time. Agents also often do not share a roadmap, so conflicts do not occur neatly along shared vertices and edges.

To alleviate this, we discretize the roadmap edges into uniform time resolution. This allows the conflict detection to be tractable as vertex-to-vertex checking can be adapted to check the agent configurations each discretized segment at a time. These checks can be done with standard motion planning collision detection methods.

IV. METHOD

The original CBS method is a MAPF method which is given a graph to search over. In a sampling-based motion planning approach to MAMP, this constitutes the query stage. As such, our adaption of CBS is related only to the query stage, and we use standard decoupled PRM techniques for constructing the individual agent roadmaps. The initial plan is found by querying each of these individual roadmaps for a valid path. If all agents have a viable path, then the method proceeds to detect and resolve conflicts until a roadmap-optimal solution is found.

A. Conflict Detection

The original CBS checks if two agents share the same discrete state at the same timestep. Operating in a continuous space, we have to perform a model simulation to checking whether their models are in collision at any point during their path. To achieve this we must model the exact configuration of each agent for the same point in time. By applying a time-based discretization to the edges of each individual roadmap with respect to an individual parameter of each agent (i.e. an aerial robot may move faster than a robotic arm) we can obtain segments of uniform time resolution. This allows comparing two paths by checking the endpoints of each segment, point-to-point. If at some point a collision is found, we can return the points and the time when the collision occurred, and if no collision is found we can declare the paths are collision-free.
B. Conflict Object

In CBS, the authors define a conflict by the graph vertex, the timestep, and the agents involved \( c = \langle v, t, a_i, a_j \rangle \). However, under MAMP features, we need a different structure to represent a conflict object. As each agent has its own roadmap, our conflict representation has the structure \( c = \langle p_i, t_i, p_j, t_j \rangle \) where the points correspond to the time-based discretization segment end points. This allows us to report the exact points and times where the collision occurred. It is important to note, that the reported points are the exact endpoints of the discretized segments where the collision occurred and not the actual roadmap vertices. In other words, what the reported endpoints represent is the exact agents’ configurations that caused the collision.

C. Conflict Resolution

When examining a new CT node, the low-level planner computes new individual paths which are consistent with the current set of conflicts. In the MAPF problem, this is a trivial task since all agents lie on the same graph, and a conflict object can be easily mapped to an “invalid” state to be avoided during the low-level search. In CBS-Query, the conflicts cannot be as easily mapped to invalid states for the low-level search.

In our approach, avoiding a conflict means to compute a collision-free path while avoiding the other agent configuration that is contained in the conflict object. This means in the Conflict Checking procedure, the edges we examine during the graph search must be discretized into uniform time resolution segments to correctly verify if the edge we are checking during graph search collides with the other agent configuration contained in the conflict object. Doing this involves performing a large number of collision detection calls, so to alleviate this issue, we first verify if the edge we are checking during graph search contains the timestep of the conflict object, and if so, we perform the collision detection against the other agent configuration. Otherwise we proceed with the search.

D. Algorithm

Query-CBS receives as input a set of roadmaps \( R \), a set of agents \( A \), and a set of start \( S \) and goal \( G \) configurations. Its task is to produce an optimal set of collision-free paths with respect to the given roadmaps. It is important to emphasize that the roadmaps were built in previous stages of a MAMP planner, and we assume that each roadmap contains at least one feasible path to satisfy its corresponding query.

Query-CBS follows the same structure as the original CBS method. A initial solution is generated by planning for each agent individually. This solution is used to create the root node of the conflict tree (CT). The paths in the root node are checked for collision, and upon detecting a collision, the constraints are generated for the two agents involved. A child node is generated for both of the agents and the corresponding constraint is added to the appropriate child node. Both child nodes are then added to the CT.

The unexplored node in the CT with the lowest solution cost is explored next as the currentNode. The collision detection processes is repeated with new nodes being added to the CT until a node is explored where the solution contains no conflict. This is a valid solution to the MAMP problem, and the solution is stored as the optimalPaths. The exploration of the CT must continue to evaluate, updating the optimalPaths solution whenever a better cost solution is discovered until all nodes have been explored or have a cost greater than the optimal solution cost. As any child node will have a cost at least as expensive as its parent, the exploration can be stopped once all remaining nodes have a more expensive cost than the discovered optimal solution. By exploring until all remaining solutions are more expensive than the currently discovered best solution, we produce the roadmap-optimal solution.

E. Size-Limited Conflict Trees

Placing a maximum size on the conflict tree reflects the maximum acceptable complexity of the changes needed to the individual agents’ plans to achieve coordination. Large numbers of conflict nodes represent the agents needing a lot of information about each other to coordinate, whereas fewer nodes imply simpler coordination. The conflict tree size limitation thus is a parameter in CBS which allows one to describe this, with one representing a single adjustment and infinity representing unlimited. Finite conflict trees are not necessarily complete, as problems requiring more complex coordination would not be discovered. However in the motion planning setting, the roadmaps can be improved if the query fails, thus opening new paths to attempt. This causes the planner to search for paths which meet the maximum complexity requirement by continually refining the roadmaps to discover one.

F. Theoretical Properties

In this subsection, theoretical details about Query-CBS are discussed. We start by formally defining the Multi-agent Motion Planning problem and what constitutes a solution for it.

Definition 1: A multi-agent motion planning (MAMP) problem consists of an environment \( E \), set of agents \( A \), and initial and goal conditions for each agent \( a_i \in A \). Let the free configuration space for agent \( a_i \) be denoted as \( \mathcal{C}_i \). Then for each agent \( a_i \in A \) the initial state is a configuration \( s_i \in \mathcal{C}_i \) and the goal condition is reaching a set \( G_i \subset \mathcal{C}_i \).

Definition 2: Let \( P \) be a set of paths \( \rho \) for each agent \( a_i \in A \). We say that \( P \) is a solution to an MAMP problem if for each path \( \rho(t) \) no collisions occur at any time \( t \) when all robots are configured at their respective positions \( \rho(t) \).

Definition 3: Let \( P' \) be a solution to an MAMP problem extracted from a set of roadmaps \( r_i \in R \) for each agent \( a_i \in A \). \( P' \) is said to be roadmap-optimal with respect to \( R \) and a cost metric \( C(R, P) = C^* \) if there is no set of valid paths \( P'' \) with lower cost \( C(M, P'') < C^* \).

Lemma 1: If for the given current set of roadmaps \( R \) no solution exists, Query-CBS will terminate in finite time without returning any paths.
Given a MAMP instance, where the current set of roadmaps \( R \) does not contain a solution, Query-CBS will end exploring all possible paths. Loops are not allowed in graph search, including self-loops which is used to make the agents wait, and therefore the number of possible paths is finite. Hence, Query-CSB will terminate in finite time.

**Lemma 2:** If a roadmap-optimal path without cycles or waiting exists in the current \( R \), Query-CBS will find it in a finite time.

Query-CBS builds and manages CT in the same manner as the original CBS. The primary difference is that Query-CBS enforces additional rules on path validity, which is equivalent to restricting the agents’ action space (for which the CBS proofs still hold).

**Lemma 3:** If a solution exists, Query-CBS will find it.

Given a MAMP instance in which a real solution exists, but it cannot be found with the current \( R \), then by Lemma 1 Query-CBS will terminate without finding a solution. Nevertheless, given individual paths are computed with a singular sampling-based motion planner, the method is probabilistically complete. This is because as the number of samples approaches infinity, the probability of finding the valid path that satisfies the group plan approaches 1. As the number of vertices of all roadmaps in \( R \) is increased, \( R \) will eventually contain the solution. Finally, by Lemma 2 Query-CBS will find the solution.

**Lemma 4:** Query-CBS can coordinate MAMP plans for agents using different world graphs and with different transition lengths.

Query-CBS maps the heterogeneous representations with world-graph transitions to a common representation with time-step transitions where it is possible to detect the conflicts. The conflicts are mapped back and can be individually resolved.

**Lemma 5:** CBS can use any admissible cost function.

Given all individual roadmaps in \( R \) have strictly non-negative edge weights, which in our context represents motion time. Hence, adding and resolving a conflict will cause the team path lengths to increase monotonically. Thus, any cost metric which is an increasing function of path length will be admissible for the CT method of CBS which is used here.

**V. Experiments**

Given the roadmap-based nature of our method, we decide to compare it against the coupled and decoupled versions of PRM, CompositePRM and DecoupledPRM, respectively. For our experiments, DecoupledPRM was implemented as a dynamic-obstacle-based approach. As the coupled approach provides an optimal solution, and the decoupled approach an improvement in performance, we compare against the characteristics of both methods.

We verify the performance of our method in three main scenarios, large sets of agents, agent teams with high DOF, and heterogeneous systems. Agents are not allowed to perform wait or looping actions. Also, they are always physically present during the whole group plan (i.e. an agent never disappears even if it finishes its plan before to the rest of the group) and they start their plans simultaneously. CT trees are size-limited to 64 nodes. All methods are given 1000s seconds to plan at which the attempt is considered a failure.

**A. Scenario I**

Fig. 2: a) The agents on the bottom must find paths to the top, and the agents on the left must find paths to the right. The paths for each agent making up the solution are shown in the various colors. (b) A crane occupies the central region of the space while a ground-based and aerial robot must navigate through the same space.

This scenario examines how each method scales with the number of robots when a large number of planning conflicts will occur. This scenario consists of an open environment with simple shape box-like agents, where half of the robots move from the left to the right side and the other half go from the bottom to the top side (Fig. 2a). This forces half of the paths to be orthogonal to the other half creating a problem full of potential inter-robot collisions.

Besides simply analyzing the scalability of the three methods, this scenario studies the performance of our method when its conflict tree grows significantly. Using simple shape robots inside a free environment without obstacles minimizes extraneous factors that may affect the performance of the planners. In addition, it is important to mention that when increasing the number of robots, we also increased the size of the environment. This allows us to have the same density per robot in all the tests thus ensuring this has no effect on the performance of the planners.

With small numbers of agents, the CompositePRM is able keep relative low planning times, taking roughly three times as long to plan as CBS-MP and planning faster than DecoupledPRM. After the number of agents reaches eight, the coupled approach becomes drastically more expensive as the state space grows too large. DecoupledPRM scales better, but still takes significantly longer to plan as the number of agents increases. Neither method is able to solve in the allotted time for 16 or 32 agents.

The DecoupledPRM method often produces higher quality plans than CBS-MP. This is expected as CBS-MP is able to find solutions with sparser roadmaps as indicated by the average number of nodes in the agent roadmaps. Decoupled-PRM continues to sample and build denser roadmaps until it finds a solution as opposed to resolving conflicts within the
### Table

| Scenario | Method            | Running Time | Path Cost | Roadmap Size |
|----------|-------------------|--------------|-----------|--------------|
|          |                   | Avg | Std. Dev. | Avg | Std. Dev. | Avg | Std. Dev. | Success Rate |
| Scal2    | CBS-MP            | 0.1 | 0.04 | 14.92 | 2.81 | 17 | 0.07 | 100 |
|          | DecoupledPRM      | 0.1 | 0.04 | 14.92 | 2.81 | 17 | 0.07 | 100 |
|          | CompositePRM      | 0.1 | 0.04 | 14.92 | 2.81 | 17 | 0.07 | 100 |
| Scal4    | CBS-MP            | 0.48 | 1.18 | 20.01 | 2.12 | 17.8 | 0.9 | 100 |
|          | DecoupledPRM      | 2.34 | 0.21 | 25.73 | 2.12 | 17.8 | 0.9 | 100 |
|          | CompositePRM      | 0.5 | 0.57 | 135.46 | 24.97 | 25.35 | 0.08 | 100 |
| Scal8    | CBS-MP            | 15.82 | 20.75 | 92.37 | 14.36 | 14 | 6.38 | 100 |
|          | DecoupledPRM      | 253.25 | 297.18 | 84.71 | 3.44 | 53.33 | 32.81 | 90 |
|          | CompositePRM      | 796.3 | 2.06 | 1.01 | 1122 | 3257.6 | 1098.3 | 1374.69 | 50 |
| Scal16   | CBS-MP            | 73.12 | 40.46 | 489.6 | 58.73 | 26.3 | 10.84 | 100 |
|          | DecoupledPRM      | 0 | 0 | 0 | 0 | 0 | 0 |
|          | CompositePRM      | 0 | 0 | 0 | 0 | 0 | 0 |
| Scal32   | CBS-MP            | 7.28 | 3.04 | 18.00 | 26.73 | 29.4 | 7.78 | 10 |
|          | DecoupledPRM      | 0 | 0 | 0 | 0 | 0 | 0 |
|          | CompositePRM      | 0 | 0 | 0 | 0 | 0 | 0 |
| Man2     | CBS-MP            | 0.26 | 0.08 | 5.24 | 1.47 | 14 | 4.88 | 100 |
|          | DecoupledPRM      | 2.52 | 0.35 | 5.24 | 1.76 | 14 | 4.81 | 100 |
|          | CompositePRM      | 0.47 | 0.51 | 3.77 | 1.62 | 0 | 0 |
| Man4     | CBS-MP            | 0.19 | 0.08 | 5.24 | 2.68 | 12.9 | 4.16 | 0 |
|          | DecoupledPRM      | 50.0 | 21.64 | 12.28 | 1.96 | 15.2 | 4.73 | 100 |
|          | CompositePRM      | 10.9 | 1.28 | 13.96 | 3.12 | 22 | 0 | 100 |
| Man8     | CBS-MP            | 29.9 | 14.76 | 25.82 | 3.29 | 57 | 15.81 | 100 |
|          | DecoupledPRM      | 0 | 0 | 0 | 0 | 0 | 0 |
|          | CompositePRM      | 0 | 0 | 0 | 0 | 0 | 0 |
| Heter    | CBS-MP            | 747.5 | 201.08 | 15.1 | 18.28 | 30.86 | 40.42 | 20 |
|          | DecoupledPRM      | 0 | 0 | 0 | 0 | 0 | 0 |
|          | CompositePRM      | 0 | 0 | 0 | 0 | 0 | 0 |

This experiment shows how our algorithm performs problems with multiple high-DOF robots moving through a shared space. In this second scenario, several robotic arms start in a confined initial position where each arm is nearly in contact with all the rest of the robots. This requires a coordinated approach to transition from the group start configuration to the goal configuration. The conflict tree was limited to sixteen nodes to favor simpler solutions because there are quite a few ways for the manipulators to become entangled. CBS-MP performs well in this problem even with eight robots.

### C. Scenario III

This experiment demonstrates our algorithm's ability to plan for heterogeneous robot teams. A heterogeneous multirobot system, composed of an aerial, a ground-based, and a crane-like robot, must coordinate their motions in a constrained environment (Fig. 2b).

Only CBS-MP and DecoupledPRM were able to solve the heterogeneous problem within the 1000 seconds. Decoupled-PRM was only successful on 20% of the trials while CBS-MP was able to solve in all the trials. This demonstrates the effectiveness of the CBS-MP approach and the uniform-time discretization of roadmaps for finding and resolving conflicts in heterogeneous systems.

### VI. Conclusions and Future Work

In this paper, we presented Query-CBS, the extension of CBS to Sampling-based Motion planning field, and we apply it to solve the multi-agent motion planning problem. Due to the efficient transition to the continuous time-space, and leveraging the potential of CBS, Query-CBS is the first decoupled approach capable of providing a degree of optimality. We validate our work in different scenarios to show the strengths of our method to deal with sets of numerous agents, sets with high-DOF agents and sets of heterogeneous agents.

Query-CBS showed it can solve MAMP instances without having to grow large roadmaps. There are still several challenges that can be met to improve the performance of Query-CBS. One of these is designing a better conflict-based individual planner to compute individual paths while avoiding conflicts. Another challenge is to understand better CT of Query-CBS as well as coming up with heuristics to explore and build it more efficiently.
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