Slowly Decaying Ringdown of a Rapidly Spinning Black Hole: Probing the No-Hair Theorem by Small Mass-Ratio Mergers with LISA

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The measurability of multiple quasinormal (QN) modes, including overtones and higher harmonics, with the Laser Interferometer Space Antenna is investigated by computing the gravitational wave (GW) signal induced by an intermediate or extreme mass ratio merger involving a supermassive black hole (SMBH). We confirm that the ringdown of rapidly spinning black holes are long-lived, and higher harmonics of the ringdown are significantly excited for mergers of small mass ratios. We investigate the measurability and separability of the QN modes for such mergers and demonstrate that the observation of GWs from rapidly rotating SMBHs has an advantage for detecting superposed QN modes and testing the no-hair theorem of black holes.

I. INTRODUCTION

We are in a golden age of gravitational-wave (GW) astronomy, where mergers of binary black holes (BHs) are discovered by GW interferometers [1–7]. The end product of a merger is a distorted single BH, which settles down to a Kerr BH by radiating GWs. This ringdown phase is characterized by a set of damped sinusoids called quasinormal (QN) modes, and is an important probe to test general relativity in the strong-gravity regime [8–10]. QN modes from BH merger remnants have been detected for a large number of events, and were used for various tests of general relativity (e.g., [11–14]).

QN modes consist of fundamental modes and overtones, where the latter is short-lived but can be important for characterizing the ringdown signal [15, 16]. Detection of overtones from ringdowns is important for e.g. tests of the no-hair theorem [17]. Evidence of an overtone was claimed in the ringdown of GW 150914 [18], though systematic uncertainties may exist in the fitting, X-ray spectra of local SMBHs in this mass range indicate high spins of > 0.9, consistent with this scenario [30–32].

Using the waveform modeling of a particle plunging into a rapidly spinning BH and extracting the excited QN modes with a fitting analysis, we estimate the measurability of multiple QN modes, including higher overtones and higher angular modes, by LISA.\textsuperscript{3} We find that these modes can be detectable out to cosmological distances, realizing a novel probe of gravity in the near-extreme Kerr spacetime. We also evaluate the error of the mea-

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\textsuperscript{2}The self-force of the plunging object is ignored in our computation, as we consider the orbit of a light object. In other words, dephasing of GWs and the backreaction of the object to the trajectory are assumed to be subdominant.

\textsuperscript{3}For the LISA detectability of the fundamental mode and the first overtone whose amplitude is assumed to be 1/10 that of the fundamental one, see Ref. [33].
survability and separability of the QN modes \cite{34, 35} to assess the feasibility of measuring individual modes.

II. RINGDOWN FOR A SMALL MASS RATIO MERGER

In this work we focus on simulating a merger of small mass ratio, such that its dynamics can be well approximated by a test particle plunging into a SMBH. We numerically compute the GW signal induced by a particle mating by a test particle plunging into a SMBH. We numerically compute the GW signal induced by a particle mating by a test particle plunging into a SMBH. We numerically compute the GW signal induced by a particle mating by a test particle plunging into a SMBH. We numerically compute the GW signal induced by a particle mating by a test particle plunging into a SMBH. We numerically compute the GW signal induced by a particle mating by a test particle plunging into a SMBH.

\[
\left( \frac{d^2}{dr^*^2} - F_{lm} \frac{d}{dr^*} - U_{lm} \right) X_{lm} = \tilde{T}_{lm},
\]

where \(X_{lm}\) is a perturbation variable of the gravitational field, \(r^*\) is the tortoise coordinate, \(F_{lm}\) and \(U_{lm}\) are functions with explicit form given in Ref. \cite{36}, and \(\tilde{T}_{lm}\) is the source term associated with the plunging particle. The form of \(\tilde{T}_{lm}\) is given in Ref. \cite{37} and can be obtained from the geodesic motion of the object. The orbital angular momentum of the plunging orbit is \(\mu \times L_z\), and the infalling geodesic condition is \(-2M(1+\sqrt{1+\tilde{\omega}^2}) < L_z < 2M(1+\sqrt{1-\tilde{\omega}^2})\). We here assume that the value of \(L_z\) for infalling objects is typically \(O(M)\) and take \(L_z = 2M\) throughout the manuscript.\footnote{To simulate a particle plunging from a finite distance from the SMBH (not from infinity as was assumed in Ref. \cite{37}), we modified the source term in Ref. \cite{37}. We suppress the contribution of the source term at \(\omega \ll 1/M\) including at \(\omega = 0\) (originating from the particle motion at infinity), by multiplying \(f_0\) and \(f_1\) in the source term in Appendix B in Ref. \cite{37} by \(2M\omega\).}

Integers \(l\) and \(m\) are, respectively, the angular and azimuthal numbers of the spheroidal harmonics. We here consider a situation where the trajectory of a compact object of mass \(\mu\) is restricted to the equatorial plane\footnote{We plan to investigate the dependence of GW signals on \(L_z\) in a forthcoming paper. Note that when \(|L_{\text{max}}/\text{min} - L_z|/M \ll 1\), where \(L_{\text{max}}\) and \(L_{\text{min}}\) are, respectively, the upper and lower limits of \(L_z\), the object follows a circulating orbit and the self-force would not be negligible.} \((\theta = \pi/2)\) and the total energy of the object (including rest energy) is \(\mu\). The self-force of the object can be neglected, which is valid for a small mass ratio \(q \equiv \mu/M \ll 1\). Using the Green’s function technique, one can solve the SN equation as

\[
\lim_{r^* \to \infty} X_{lm}(\omega, r^*) = X_{lm}^{\text{(out)}}(\omega) e^{i\omega r^*},
\]

where \(G(r', r^*, \omega)\) is the Green’s function that is obtained from the homogeneous solution of the SN equation. We then obtain the GW spectrum

\[
\hat{h} = \sum_{l,m} \hat{h}_{lm}(\omega) = \sum_{l,m} -2 \omega^2 S_{lm}(a\omega, \pi/2) R_{lm}(\omega),
\]

where \(a \equiv J/M\) and \(S_{lm}\) is the spin-weighted spheroidal harmonics, assuming an edge-on observer with argument \(\pi/2\). The time-domain data \(h = h_+ + ih_\times = \frac{\hat{h}}{\sqrt{2}}\).

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perposition of QN modes, each of which is exponentially damped in time, and (ii) overtones may dominate the signal at early times. The exact start time of the ringdown is unknown, but one can obtain a best fit value by fitting multiple QN modes to the GW signal. We then show that the ringdown starts earlier than the strain peak, around the time when the object plunges into the photon sphere.

We perform fitting analysis of QN modes\footnote{The pseudospectrum of QN modes implies [38] that a small modification in the angular momentum potential or the boundary condition at the horizon may destroy the distribution of the Kerr QN frequencies. However, such an instability could be negligible at the early ringdown (e.g., see Refs. [39, 40]), and we are interested in the fit of the standard QN-mode model to the early ringdown in this work.} with a complex frequency $\omega_{lmn}$ including overtones of $n \leq n_{\text{max}}$ and angular modes of $2 \leq l = m \leq 5$. The fitting is done in the frequency domain by using the QN mode model

\[\tilde{h}_{lm} = \sum_{n} \frac{A_{lmn}}{\omega_{lmn}^{2}} e^{iN_{lmn}t^{*} + i\phi_{lmn}},\]

where $\tilde{h}_{lm}$ is obtained by the inverse Laplace transformation of $\tilde{h}$. The function $R_{lm}(\omega)$ is obtained by properly normalizing $X_{lm}$, and its explicit form is provided in Ref. [36]. The GW spectra computed with this scheme is shown in Figure 1. One can see that $(l, m) = (2, 2)$ dominates the GW signal for an intermediate spin ($j = 0.8$ in Figure 1), but higher angular modes are significantly excited for a near-extremal Kerr BH of $j = 0.99$. We are interested in the signal induced by a compact object plunging into a rapidly spinning BH, as more QN modes are long-lived for higher spins (Figure 2).

In the next section, we show that a number of highly damped modes dominate the early ringdown of a near-extremal BH by fitting multiple overtones and fundamental modes to the GW signal. Then we show that rapidly spinning BHs are better targets to perform a high-precision detection of multiple QN modes, including higher angular modes and higher overtones.

III. EXCITATION OF OVERTONES

The measurability of the QN modes is highly sensitive to the start time of the ringdown $t^{*}$ because (i) it is a su-
an analytic function \( \tilde{f} \sim \sim \) no-hair theorem. We here consider detections of them

is the sky/polarization average of the antenna pattern

\( F \) the single-link optical metrology noise and the single test

modes. [43], we include many overtones to model the waveforms. For

\( j = 0.99 \), we include QN modes up to \( n = 21 \) for each mode

\( l = m = 2, 3, 4, \) and 5. For \( j = 0.8 \), on the other hand, QN

modes up to \( n = 16 \) are included for each mode \( l = m = 2, 3 \) and 4.

significantly excited for a near-extremal BH. It has an

advantage for measuring multiple QN modes, including

overtones and higher harmonics, and for accurately testing

general relativity. Indeed, it is reported that a frac-

tion of SMBHs can have near-extremal spin parameters of

\( j \approx 0.99 \) [30–32]. In the following, we show such SMBHs

are suitable observation targets to detect multiple QN

modes.

IV. NO-HAIR TEST OF SMBHS WITH LISA

A. Measurability of superposed QN modes

Let us evaluate the measurability of multiple QN

modes of rotating BHs, and its feasibility for tests of the

no-hair theorem. We here consider detections of them

with LISA, which is sensitive to signals of \( \sim 0.01 \) Hz and

is suitable for detecting ringdown of SMBHs with mass

\( \sim 10^8 \sim 10^9 \) M\(_\odot\). The sensitivity curve can be modeled by

an analytic function \( \tilde{S}_n(f) \) [44]:

\[
\tilde{S}_n = \frac{1}{\mathcal{F}} \left( \frac{P_{\text{OMS}}}{L^2} + (1 + \cos^2(f/f_\star)) \frac{2P_{\text{acc}}}{(2\pi f)^4 L^2} \right),
\]

\[
\mathcal{F} \equiv \frac{3}{10} \left( 1 + 0.6 \times (f/f_\star)^2 \right)^{-1},
\]

where \( 2L \) is the round-trip light travel distance, \( f_\star \equiv c/(2\pi L) \) is the transfer frequency, \( P_{\text{OMS}} \) and \( P_{\text{acc}} \) are the single-link optical metrology noise and the single test

mass acceleration noise, respectively. The function \( \mathcal{F} \) is the sky/polarization average of the antenna pattern

functions. We include the galactic confusion noise from compact binaries,

\[
S_c(f) = Af^{-7/3}e^{-f_{\alpha}^\beta} + \beta f \sin(\kappa f) [1 + \tanh(\gamma (f_k - f))] \text{Hz}^{-1},
\]

where \( A = 9 \times 10^{-45} \) and the parameter set \( \{ \alpha, \beta, \gamma, \kappa, f_k \} \) is fixed with the values for a four-year mission (we use

Table 1 in Ref. [44]). Then we obtain the full sensitivity

curve as \( S_n(f) = S_c(f) + S_e(f) \). Using \( S_n \), we evaluate the SNR

of the GW signal and the likelihood ratio, to investigate the support for the model of the no-hair QN

modes \( H_0 \) over a modified model \( H_\delta \) consisting of a set of complex frequencies deviated from the no-hair values.

For the modification of the no-hair model, we con-

sider two types of modifications:

i) a model \( H_\delta^{(F+O)} \) for which all QN frequencies, including fundamental modes and overtones, are modified as

\[
\omega_{lmn}^{(GR)} = (1 + \delta f_R) + i\delta q (1 + \delta f_l), \quad (8)
\]

where \( \omega_{lmn}^{(GR)} \) are the QN frequencies in general relativity, and

ii) a model \( H_\delta^{(O)} \) for which only overtones are modified with the above expressions. Depending on the

parameters of the remnant mass and spin, the whole QN mode frequencies coherently change like the model

\( H_\delta^{(F+O)} \). As such, we can estimate the feasibility of the test of the no-hair theorem, which states that the fre-

cquency and decay rate of QN modes are uniquely set by the remnant mass and spin. On the other hand, compar-

ing the likelihood ratio with \( H_\delta^{(F+O)} \) and the one with \( H_\delta^{(O)} \), we can see the efficiency of the inclusion of over-

tones in the test of the no-hair theorem. The models \( H_0, H_\delta \) are respectively given by the superposition of

GR or modified QN modes, and the model parameters, i.e., an amplitude and phase, are assigned to each QN

mode. The best fit values of the amplitudes and phases are determined by the \textit{Mathematica} function “Fit”. Our artificial modification to QN modes affects the model pa-

rameters and the likelihood ratio. The SNR \( \rho \) and the likelihood ratio \( L \) are [45]

\[
\rho = \sqrt{\langle \hat{h}_+ \hat{h}_+ \rangle + \langle \hat{h}_\times \hat{h}_\times \rangle},
\]

\[
L = \frac{p(\hat{h}_+ | H_0)}{p(\hat{h}_+ | H_\delta)} \frac{p(\hat{h}_\times | H_0)}{p(\hat{h}_\times | H_\delta)}, \quad (10)
\]

8 Note that the information of the antenna pattern is already in-

cluded in the noise curve, and we do not need to include this in the

signal (see [44]).

9 The source parameters of the SMBH (\( M \) and \( j \)) are assumed to

be fully known. We leave a more realistic inference with LISA,

considering measurement errors of these parameters, to a future

study.
erved, the uncertainty in the damping rate $\delta f$ may cause the large uncertainty in $\delta f_I$ as shown in Figure 5. A similar conclusion and a large uncertainty in the imaginary part was reported in [18], where the no-hair test was performed for GW150914 [46].

Figure 6 summarizes the expected distance out to which we can measure multiple QN modes with high precision. In this section we discuss the prospects for LISA, for moderate and extreme mass ratios.

For moderate mass ratios of $q \sim 10^{-3}$, it corresponds to a merger between a SMBH and an intermediate-mass BH (IMBH). A scenario usually considered for such mergers is clusters hosting IMBHs falling into the galactic nuclei [47–51]. The event rate is uncertain, but recent N-body simulations find a range 0.003–0.03 Gpc$^{-3}$ yr$^{-1}$ [50, 51], or 2–20 yr$^{-1}$ within $z < 1$ ($D_L \lesssim 7$ Gpc) [25]. For the $H_\delta^{(F+O)}$ model with $j = 0.99$ multiple QN frequencies can be measured within $\sim 5\%$ ($\sim 1\%$) for sources at $D_L \lesssim 10$ Gpc ($\lesssim 3$ Gpc), and thus no-hair tests of SMBHs are promising. For the $H_\delta^{(O)}$ model, one may constrain the real frequencies within $\lesssim 10\%$ for sources out to a few Gpc, corresponding to an event rate of 0.1–1 yr$^{-1}$.

The likelihood ratio for the model of $H_\delta^{(O)}$ with $\delta f_R > 0$ is more significant than that with $\delta f_R < 0$ (see Figure 6). Being sensitive to the modification of $\delta f_R > 0$ is reasonable since the higher-frequency modes of $\omega \gtrsim \omega_{\text{min}}$ in the GW signal are exponentially suppressed (see Ref. [22] for more details).

For extreme mass ratios of $q < 10^{-5}$, it corresponds to a stellar-mass BH plunging into a SMBH. Such plunges are expected not to be strong GW emitters, as we also deduce from Figure 6. The EMRI rate for a Milky-Way like Galaxy is estimated to be $10^{-6}$–$10^{-5}$ yr$^{-1}$, i.e. an event rate of $10^{-8}$–$10^{-7}$ Mpc$^{-3}$ yr$^{-1}$ (52) and references therein). Plunge orbits can be up to 100 times more likely than EMRIs [53, 54], so we expect plunges of ${\sim 10} M_\odot$ BHs within the detectable distance ($\lesssim 10$ Mpc) at a rate of $< 0.1$ yr$^{-1}$. Recently a new formation channel of IMBHs in galactic nuclei was proposed, where stellar-
mass BHs grow in situ up to \( \sim 10^4 M_\odot \) by collisions with surrounding stars [55]. If such growth is efficient, this would likely enhance the above rates.

B. Separability and measurability of individual QN modes

In the previous section, we studied the measurability of superposed QN modes, where we required that the SNR for the secondary QN mode is above a given detectability threshold. However, to assess LISA’s potential for no-hair tests with ringdown signals, it also is important to evaluate the separability and measurability of individual QN modes (BH spectroscopy).

Let us evaluate the measurability and separability of the fundamental QN mode and the first overtone to see the feasibility of the BH spectroscopy [34, 35]. The statistical errors on a model parameter \( \theta_a \) are given by

\[
\sigma_{\theta_a} = \sqrt{(\Gamma^{-1})_{aa}}
\]

where \( \Gamma^{-1} \) is the inverse of the Fisher matrix,

\[
\Gamma_{ab} = \left\langle \frac{\partial \tilde{h}}{\partial \theta_a} \frac{\partial \tilde{h}}{\partial \theta_b} \right\rangle,
\]

with \( \langle x, y \rangle \) defined as in equation (11). We here compute the Fisher matrix with the following parameter set

\[
\theta_a = \bigcup_{l,m,n} \{ f_{R|lmn}, f_{I|lmn}, A_{I|lmn}, \phi_{I|lmn} \},
\]

where the parameter set has the fundamental mode \( (n = 0) \) and the first overtone \( (n = 1) \). The angular modes in the parameter set are \( (l, m) = (2, 2), (3, 3), (4, 4), (5, 5) \) for \( j = 0.99 \). For \( j = 0.8 \), it has \( (l, m) = (2, 2), (3, 3), (4, 4) \). We use the waveform we numerically obtained in Sec. II and use a Mathematica function “Fit” to obtain the best fit model of (4). We then compute the Fisher matrix (14) by analytically computing the derivative of (4) and estimate the statistical errors from the inverse matrix \((\Gamma^{-1})_{ab}\). From the statistical errors, we can evaluate the separability based on the Rayleigh criterion [34, 35]:

\[
s[\theta_a, \theta_b] = \max[\sigma_a, \sigma_b] / |\hat{\theta}_a - \hat{\theta}_b| < 1,
\]

where \( \hat{\theta}_a \) is the true value of \( \theta_a \). Also, we can estimate the measurability (i.e. measurement error) with [35]

\[
\Delta \theta_a = \sigma_{\theta_a} / \hat{\theta}_a.
\]

The signal has \( x \% \) measurability if the set of \( \{ \Delta \theta_a \} \) satisfies

\[
\max_i [\Delta \theta_i] < \frac{x}{100}.
\]

From this quantity, we can also examine the hierarchy of measurability among the modes we are interested in.

Figures 7 and 8 show the value of \( \Delta \theta_a \) and \( s[\theta_a, \theta_b] \), respectively. We can read that the errors in the measurability and that in the separability for \( j = 0.99 \) are
generally smaller than those for $j = 0.8$ at the same luminosity distance $D_L$, remnant mass $M$, and the mass ratio of $q \leq 0.005$. The real parts of the QN mode frequencies for $j = 0.8$ all have larger measurement errors. In the case of $j = 0.99$, the error of the real part of the QN frequencies with $n = 0$ and $(l, m) = (4, 4)$ and $(5, 5)$ take the smallest values (i.e., highest precision) among them. The real part of QN frequencies of the first overtones can be still measurable in the level of $\Delta f_{\text{fit m1}} \lesssim 0.01$. On the other hand, the imaginary parts of QN frequencies for higher harmonics are measurable with $\Delta f_{\text{fit m1}} \sim 0.1$. The error of the imaginary part of QN frequencies in the separability is smaller and can be resolvable for the modes of higher harmonics (see Figure 8). On the other hand, the real parts of QN frequencies for $n = 0$ and $n = 1$ are too close to resolve especially for $j \sim 0.99$ (see Figures 2 and 8). The QN modes for $j = 0.8$ are difficult to distinguish each other in our setup as the damping rates in QN modes are larger.

In a previous work [35], such measurement errors by LISA were computed for mergers of nonspinning BHs with mass ratio of $0.1 \lesssim q \lesssim 1$ and total mass of $10^6 M_\odot$. While varying $q$ just changes the overall scale of the measurement error, varying the remnant mass and spin may change even the hierarchy among different modes. Indeed, our result shows that the higher angular modes, i.e., $(4, 4, 0)$ and $(5, 5, 0)$, have the first three smallest measurement errors for $j = 0.99$ whereas $(2, 2, 0)$ mode takes the smallest error for the case considered by [35] (their Figure 4). As the higher angular modes may dominate the ringdown signal for a rapidly spinning BH as shown in Figure 1, the rapid spin of the remnant BH may affect the hierarchy.

V. CONCLUSION

In this paper, we studied the measurability and separability of multiple QN modes emitted by near extremal SMBHs, which may exist at the center of galaxies according to the X-ray observation of the accretion disks [26–28]. The measurability of superposed QN modes is estimated by the SNR of a ringdown signal that is obtained by the fit of QN modes to the whole GW data (Figure 6). The goodness of the fit with the GR QN modes was assessed by the likelihood ratio (Figures 5 and 6). To assess the ability of the BH spectroscopy, we computed the statistical error to obtain the errors in the separability (16) and in the measurability (17). We then found that the separability and measurability for mergers involving near-extremal SMBHs of $j = 0.99$ are generally better than those with SMBHs of moderate spins of $j = 0.8$ (Figures 7 and 8). The measurement error of the real part of QN frequencies can be $\lesssim 1\%$ and the separability condition is satisfied for the imaginary part when $D_L \lesssim 1 \text{ Gpc}$ and $q \sim 0.005$. We thus conclude that intermediate (and possibly extreme) mass ratio mergers can be unique targets for LISA to probe multiple QN modes of rapidly spinning BHs, and an important target for tests of gravity in a near-extreme Kerr spacetime.

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[1] B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. X 6, 041015 (2016), [Erratum: Phys.Rev.X 8, 039903 (2018)], 1606.04856.
[2] B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. X 9, 031040 (2019), 1811.12907.
[3] R. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. X 11, 021053 (2021), 2010.14527.
[4] R. Abbott et al. (LIGO Scientific, VIRGO) (2021), 2108.01045.
[5] R. Abbott et al. (LIGO Scientific, VIRGO, KAGRA) (2021), 2111.03606.
[6] A. H. Nitz, C. D. Capano, S. Kumar, Y.-F. Wang, S. Kastha, M. Schäfer, R. Dhurkunde, and M. Cabero, Astrophys. J. 922, 76 (2021), 2105.09151.
[7] S. Olsen, T. Vennumadhav, J. Mushkin, J. Roulet, B. Zachay, and M. Zakkariaga (LIGO Scientific Collaboration, the Virgo), Phys. Rev. D 106, 043009 (2022), 2201.02252.
[8] K. D. Kokkotas and B. G. Schmidt, Living Rev. Rel. 2, 2 (1999), gr-qc/9909058.
[9] H.-P. Nollert, Class. Quant. Grav. 16, R159 (1999).
[10] E. Berti, V. Cardoso, and A. O. Starinets, Class. Quant. Grav. 26, 163001 (2009), 0905.2975.
[11] B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 116, 221101 (2016), [Erratum: Phys.Rev.Lett. 121, 129902 (2018)], 1602.03841.
[12] B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. D 100, 104036 (2019), 1903.04467.
[13] R. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. D 103, 122002 (2021), 2010.14529.
[14] R. Abbott et al. (LIGO Scientific, VIRGO, KAGRA) (2021), 2112.06861.
[15] M. Giesler, M. Isi, M. A. Scheel, and S. Teukolsky, Phys. Rev. X 9, 041060 (2019), 1903.08284.
[16] N. Oshita, Phys. Rev. D 104, 124032 (2021), 2109.09757.
