The spectral theory of graphs at researches of fuzzy models of the determined regular structures of network information systems

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Abstract. Operation is devoted the decision of the task increase of structural stability and information efficiency of network information systems on the basis of mathematical apparatus of the theory of fuzzy sets and the spectral theory of graphs. For this purpose introduction of two variants fuzzy structural (channel and nodal) is offered to redundancy of the determined regular topology of network information systems. Functions of an accessory of separate structural elements to which in correspondence the modes of an information exchange which are representing itself as requirements to structural characteristics of system are defined. Analytical and imitative modeling for research of influence fuzzy redundancy on the main structural parameters of the determined topology and estimation of information efficiency of the received structures of network information systems is conducted. Results of modeling showed perspectivity of the offered approach at implementation of structural adaptation of systems to changing conditions of an information exchange.

1. Introduction

The decision of the task increase of efficiency of an information exchange (information efficiency) network information systems (NIS) demands maintenance of their structural stability taking into account high dynamics of change of operating conditions and influence of destabilizing factors. The analysis of the known publications devoted to given subjects, shows that in a mode of high intensity of input information highways, characteristic for modern NIS, usage of the regular determined structures, as application of casual topology in these conditions was extremely ineffectively is necessary. The results of researches presented in operations [1, 2], confirm that for maintenance of high information efficiency NIS with the determined topology, there is enough support of value of $k$-connectivity $a(G) = r = 3$ at the regular character of structure while in casual topology presence 6 - 8 neighbors for each switching node (SN) is required.

Thus for increase of structural stability in many operations the approach based on application of structural models NIS with usage of mathematical apparatus of the theory of fuzzy sets [3, 4] is offered. In this case functions of an accessory of separate communication channels (CC) at the given amount SN or the fuzzy status separate SN (so-called background subscribers) on all connected to them CC for NIS with the regular determined structures, allow to define a level of topological redundancy of systems necessary for increase of structural stability and survivability. Therefore determined NIS (for example, to level of $k$-connectivity $a(G) = r = 6...8$ of casual topology),
described with usage of the theory of fuzzy sets, it is possible to consider increase of redundancy of structures as a method of increase of structural stability of systems for support of demanded efficiency of an information exchange in the conditions of high intensity of an input information highway and influence of destabilizing factors. At the same time procedure of serial lowering of structural redundancy to an admissible minimum (3 adjacent SN) can be used at formation of the boundary criteria, allowing to achieve essential saving of a network resource at support of demanded efficiency of functioning NIS. The presented mechanisms of change of redundancy (k-connectivity) of structures SIS on the basis of the theory of fuzzy sets are offered to be described and researched with usage of mathematical apparatus of the spectral theory of graphs [5 - 7]. Spectrum analysis application to the description of structural parameters of fuzzy graphs in the field of researches NIS is the new and nonconventional approach, however he allows to receive set of interesting results which cannot achieve with application of the standard methods of structure analysis [8].

Thus, the purpose of the given operation is research of application of fuzzy representation of channel and nodal redundancy of the determined regular structures for increase of structural stability NIS, and also an estimation of their influence on efficiency of an information exchange (information efficiency) on the basis of the generalized general-purpose index - efficiency of information transfer and function of information efficiency NIS [9, 10].

2. Materials and methods

As basic structures SIS for researches it is offered to consider following variants of the elementary topology:

- at nodal redundancy NIS - the complete connection structure (figure 1a see, it is selected by fat solid lines), consisting of four SN (1, 3, 5, 7);
- at channel redundancy - the regular topology from eight SN with values of k-connectivity (figure 1b see, it is selected by fat solid lines) [10].

![Figure 1. Usage of fuzzy representation of redundancy for increase structural stability of basic topology NIS.](image)

For increase of structural stability introduction of fuzzy redundancy and the extension of structure SIS to eight SN for the first variant of nodal redundancy (additional four SN - 2, 4, 6, 8 and in a limiting case obtaining complete connection - eight-nodal structure, dotted lines on figure 1a see) or introduction additional CC for the purpose of support of the maximum k-connectivity \(a(G) = r = 7\) in the second variant of channel redundancy (dotted lines on figure 1b see) is offered.
Functions of an accessory $\mu_{ij}$ separate CC (or SN as a whole) vary depending on a priority of information directions or impairment of conditions of an information exchange and are arranged in following limits:

- $\mu_{ij} = 1$ corresponds to basic topology NIS at the minimum k-connectivity $a(G) = r = 3$ providing steady and effective in sense of an information exchange system operation with determined structure - «a standard mode» (corresponding to a state of feeble information loading in not strained a mode of an information exchange without destructive influences);
- $\mu_{ij} = 0.9$ in this case introduction additional (reserve) CC or SN is caused by increase of intensity of an input information highway that demands involvement of additional network resources for support of a steady information exchange - «a mode of the raised information loading» ($a(G) = r = 4$ - not strained a mode in loaded state NIS);
- $\mu_{ij} = 0.8$ application reserve CC or SN has for an object additional reliability augmentation and stability NIS at a high input information highway (appearance of losses of packets) and displays of destructive influences - «a mode of high information loading» ($a(G) = r = 5$ corresponds to a recommended interval of high information efficiency in a quasioptimal mode of an information exchange);
- $\mu_{ij} = 0.7$ introduction reserve CC or SN owing to growth of destabilizing factors (possible failure of separate elements) at stably high intensity of an information exchange - «a mode of an information exchange with destabilizing actions» ($a(G) = r = 6$ in not recommended interval of high information efficiency quasioptimal and an initial part restrained modes NIS);
- $\mu_{ij} = 0.6$ corresponds to introduction in system reserve CC and SN in worst case conditions of an information exchange and the strong influence of destabilizing factors - «an extremal mode» (at k-connectivity $a(G) = r = 7$ - a state of information overload NIS in a rebusy schedule of an information exchange) [9].

The gradation of functioning NIS marked above in terms of functions of an accessory $\mu_{ij}$ separate CC or SN at fuzzy character of the description of structures of systems can be spread analytically out on the levels leading to representation of topology NIS normal determined graph [4], for example, for structure in a picture 1b is had:

$$
\begin{array}{cccccccc}
0 & 0.7 & 0.9 & 1 & 1 & 1 & 0.8 & 0.6 \\
0.7 & 0 & 0.6 & 0.9 & 1 & 1 & 1 & 0.8 \\
0.9 & 0.6 & 0.7 & 0.8 & 1 & 1 & 1 & 1 \\
0.9 & 0.7 & 0 & 0.6 & 0.8 & 1 & 1 & 1 \\
1 & 0.8 & 0.6 & 0 & 0.7 & 0.9 & 1 & 1 \\
1 & 1 & 0.8 & 0.7 & 0 & 0.6 & 0.9 & 1 \\
0.8 & 1 & 1 & 0.9 & 0.6 & 0 & 0.7 & 0 \\
0.6 & 0.8 & 1 & 1 & 0.9 & 0.7 & 0 & 1 \\
\end{array}
= \max_{0.6}
$$

$$
\begin{array}{cccccccc}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
\end{array}
$$

A
Thus there are following tasks of structure analysis of systems with usage of the spectral theory of graphs within the limits of the decision of the general research problem - increases of structural stability and information efficiency NIS [10] (analytical modeling in the environment of Maple 15):

- the description and the analysis of structural parameters NIS at usage of channel redundancy and fuzzy representation CC (as an example, table 1);
- the description and the analysis of structural parameters NIS at usage of channel redundancy and determined representation CC;
- the description and the analysis of structural parameters NIS at usage of nodal redundancy and fuzzy representation SN;
- the description and the analysis of structural parameters NIS at usage of nodal redundancy and determined representation SN.

For researches of offered variants of topological creation NIS following structural parameters have been used: amount SN ($N_{SN}$), number CC ($M$), number of routes of length $k$ ($N_k$ unit length routes), number of spanning trees $t_G$ (for models with nodal redundancy, trees for the coherent components of graphs), diameter NIS ($d$), the generalized structural index ($GSI$), k-connectivity were defined ($\alpha(G)$), a constant of Chiger (in the form of an interval of minimum and maximum values $h_{\text{min}}(G) - h_{\text{max}}(G)$ and also average value $h(G)$) and number of internal stability $\alpha(G)$.

On the basis of the presented parameters criteria of structural synthesis NIS which mathematical represent the decision of minimax tasks have been formulated and used for an estimation of efficiency of correction (optimization) of basic structure [10]

\[
d \to \min ; \quad GSI \to \min ; \quad \alpha(G) \to \min ; \quad t(G) \to \max ;
\]

\[
a(G) \to \max ; \quad r, \tilde{r} \to \max ; \quad h(G) \to \max ; \quad N_k \to \max \quad (\text{при} \quad k \to \min ).
\]
Table 1. Research of the regular structures NIS at usage of channel redundancy with fuzzy representation CC.

| N | Structure, spectrum | \( N_{SN} \) | \( M \) | \( N_k \) | \( t_G \) | \( d \) | GSI | \( a(G) \) | \( h_{min}(G) \) | \( h_{min}(G) \) | \( \alpha(G) \) |
|---|---------------------|-------------|-------------|-------------|-------------|-------------|-------|-----------|---------------|---------------|-----------|-----------|
| 1 | ![Structure 1](image1) | 8           | 21.2        | 16.97       | 228800      | 2           | 0.05  | 6; 5.3    | 3.37 – 8.46; 5.92 | 1             |
|   | \( Sp(G) = [6; 0.43; 0.55; 0.6; 0.6; 0.8; 1.45; 1.57] \) |             |             |             |             |             |       |           |               |               |           |           |
| 2 | ![Structure 2](image2) | 8           | 19.8        | 15.27       | 76404       | 3           | 0.09  | 5.4; 4.94 | 3.48 – 8.29; 5.89 | 2             |
|   | \( Sp(G) = [5.4; 0.08; 0.02; 0; 0; 1.4; 2.02; 2.08] \) |             |             |             |             |             |       |           |               |               |           |           |
| 3 | ![Structure 3](image3) | 8           | 17.8        | 13.29       | 20175       | 3           | 0.11  | 4.7; 4.45 | 3.4 – 7.78; 5.59 | 2             |
|   | \( Sp(G) = [4.7; 0.7; 0.49; 0.35; 0.7; 0.7; 2.35; 2.49] \) |             |             |             |             |             |       |           |               |               |           |           |
| 4 | ![Structure 4](image4) | 8           | 15.2        | 11.03       | 3813        | 4           | 0.19  | 3.9; 3.81 | 2.91 – 6.66; 4.78 | 3             |
|   | \( Sp(G) = [3.9; 1.15; 0.10; 1.005; 0.1; 2.005; 3.15] \) |             |             |             |             |             |       |           |               |               |           |           |
| 5 | ![Structure 5](image5) | 8           | 12          | 24          | 392         | 4           | 0.25  | 3; 3     | 2 – 4.9; 3.45 | 3             |
|   | \( Sp(G) = [3.1; 0.41; 0.41; 0; 1; 2.41; 2.41] \) |             |             |             |             |             |       |           |               |               |           |           |

Research of the regular structures NIS at usage of channel redundancy with fuzzy representation CC.
3. Results and discussion. The analysis of the received results shows that at introduction of channel redundancy with fuzzy (table 1 see) and determined representation CC according to the formulated criteria of structural synthesis NIS (2) greatest effect is watched on parameters: the number of spanning trees which already on the first iteration (at \( a(G)=4 \)) increases practically by the order and a constant of Chiger \( h(G) \) (especially for a case fuzzy CC in 1.4 times) that speaks magnification of an amount of possible routes of information transfer (the extension of "a narrow throat" systems) and, as consequence, increase of structural stability NIS. Structures NIS with channel redundancy possess a great number of routes of unit length \( N_k \) (at \( k=1 \)), however at fuzzy representation CC this value considerably below the determined status of channels because of the registration at calculations of spectra of functions of accessories \( m_{ij} \) of reserve channels (value \( N_k = 16.97 \) at \( a(G) = 7 \) that insignificantly exceeds lower bound corresponding to the determined channels \( M = 12 \) of basic structure). The same fact fuzzy value of number CC (\( M \)), k-connectivity (\( a(G) \)) and average value of an index models of graphs NIS \( r \), which in process of reduction of redundancy of topology as to their values in the determined structures speaks. Thus diameter NIS \( d \) (at fuzzy representation \( d_1 = d_{p_{ij}} = 4 > d_2 = 2 \), at comparing of parameters, an index «1» we will designate the structural parameters received in models with channel redundancy, and an index «2» - in structures with nodal redundancy), \( GSI_1 = 0.05 > GSI_2 = 0.027 \) and the number of internal stability at channel redundancy NIS is worse than the GSI of similar parameters in structures with nodal redundancy that mathematical speaks character of change of spectra of graphs, and physically - smaller intersection information transfer ways to structures at introduction of nodal redundancy and, as consequence, capacity growth and survivability of system.

At the analysis of spectral characteristics and results of modeling NIS with nodal redundancy following singularities are revealed. In comparison with channel redundancy stronger uncertainty between value of \( k \)-connectivity (\( a(G) \) which equally or aspires (for fuzzy nodes) to the rated value) and average value of an index models of graphs NIS \( r \), that is saved even in models with determined (guaranteed) reservation SN (for example, at \( a(G)=4 \), \( r = 2.5 \)) is visible. The number of spanning trees \( t_c \) grows much more slowly, than in structures with channel redundancy. The greatest saluts is reached only on the last iteration (\( a(G) = 7 \)) at passage to complete connection NIS (increases in 15.5 times). Narrower interval \( h_{min}(G) - h_{max}(G) \) and considerably smaller values of average values of constants of Chiger \( h(G) \) testify to presence of «a narrow throat» (values come nearer to structures with channel redundancy \( h(G) = 3.59 < h(G) = 4.16 \), since \( a(G) = 5 \) for a case of guaranteed reservation SN). Thus, spectrums NIS \( S_{P}(G) \) and \( S_{PC}(G) \) at nodal redundancy have more monotonous character with a considerable quantity of repeating components (own values) that affects determination of such structural parameters as: number of internal stability \( a_2(G) = 2;2;2;2.1 \), diameter NIS \( d_2 = 2;2;2;2;1 \) and the GSI \( GSI_1 = 0.25:...;0.027 \) which appear better by the gated in criteria of synthesis (2).

The number of routes of unit length \( N_k \) (at \( k=1 \)) with nodal redundancy grows in structures SIS more slowly, than at reservation CC thus their considerable lowering in case of fuzzy reserve SN (\( N_k = 15.6 \) for \( a(G) = 7 \) also is watched, at lower bound in basic model \( M = 6 \)).

The general tendency which can be marked in independence of a used method of nodal or channel redundancy that noticeable improving of structural characteristics is provided at \( k \)-connectivity \( a(G) = 5 \), for «a mode of high information loading» or according to classification [9] recommended intervals of high information efficiency in a quasioptimal mode of an information exchange. The further magnification structural (channel or nodal) redundances NIS (up to complete connection a variant, in this case \( a(G) = 7 \)) leads to minor improvement of structural characteristics, but thus demands the additional expenditure of a channel-central resource, increases structural complexity,
intersection information routes and reduces reliability of functioning of separate elements and system as a whole.

With a research objective of influence of structural characteristics NIS on efficiency of an information exchange for the variants of structural models NIS received during topological conversions analytical tensor modeling on advanced orthogonal model taking into account destabilizing factors [11] and simulation modeling on the basis of the standard approach with application of models CMO [12] has been led. Thus for estimation of information efficiency NIS the new system of the generalized indexes and parameters was used: cybernetic capacity $P_{NIS}$, performance coefficient (efficiency) of information transfer $\eta_{NIS}$, efficiency taking into account influence of noises (destabilizing factors) $\eta_{noi}$ and bandpass range NIS on input $\Pi_{\text{thres}}(\eta_{\text{thres}})$, intranetwork $\Pi_{\text{thres}}(\eta_{\text{thres}})$ information highways and a temporal time delay $\Pi_{\text{thres}}(\eta_{\text{thres}})$, a tangent of angle of band efficiency [9]. Programs of analytical models are fulfilled in Maple 15 environments and Delphi 7, and simulation modeling programs in GPSS/PC environments and LiteIDE X.

As criteria of structural synthesis (conversions of basic structure) at modeling following values of structural parameters were used: $d_{\text{max}} = 2$, $\text{GSI} = \bar{d}/M = 0.25$, $t(G)_{\text{min}} = 16$, $r = f = 3$, $h(G) \geq 1.4$, $\alpha(G) \leq 3$, $a(G)_{\text{min}} = 3$ and $N_i \geq 12$. Criterion of information efficiency threshold value of efficiency of information transfer taking into account influence of the noises $\eta_{\text{thres}} = 20\%$, received on a technique presented in operation [9] is accepted at parameters of input information highways and the destabilizing factors selected taking into account support of a mode of high loading of researched structures SIS ($\gamma_\mu = (3..17) \text{Mbit/s}$).

At the first stage of modeling by means of the adaptive algorithm of conversion of basic structure SIS synthesis of structurally steady topology with the minimum structural redundancy, criteria of structural synthesis providing performance set as the initial data is carried out. Then for the synthesized topological models with usage of a method of determination cospectral structures (switchings of Zajdel) there is alternative topology with identical structural characteristics [5]. During the second stage of modeling the estimation of efficiency of an information exchange in received structural models NIS on the basis of presented above the generalized general-purpose parameters and indexes is produced. Results of modeling in the form of functions of information efficiency $E_{NIS} = \eta_{noi}(\gamma_{in})$ are presented on figure 2.

Apparently, combination of functions of efficiency of separate structural models NIS (since the determined topology of minimum connectivity $a(G) = 3$ at a variation channel (CC5), see figure 2a and nodal redundancy (SN5), figure 2b see and finishing complete connection structures $a(G) = 7$, accordingly CC1 and SN1) provides overlapping of broad range of change of an input flow in which NIS effectively functions in sense of efficiency of information transfer $\eta_{noi}$ by the given criterion $\eta_{\text{thres}}$. The greatest information efficiency and stability to changes of input information highways and interfering conditions structures with the greatest value $t(G)$ and minimum $d$ possess. It speaks reduction of conflicts and losses of packets in a type of distribution of flows on various independent ways to the first case and reduction of probability of their origin in the second.

The maximum values of efficiency which managed to be received at modeling are in limits from 40 to 65 % (for structure with minimum connectivity $a(G) = 3$ and complete connection NIS, accordingly), with possible lowerings of indexes depending on set parameters of an interfering flow (modeling influence of destabilizing actions on NIS) on values from 10 to 15 %.
Thus, the variation of channel redundancy NIS (figure 2a see) is accompanied by more monotonous (smooth) passage between functions of efficiency of separate structures while change of nodal redundancy (figure 2b see) leads to essential non uniformities (bursts) $\eta_{\text{noi}}$. Apparently, usage even three variants of structural creation NIS basic ($a(G) = 3$) and two reserve ($a(G) = 5$, $a(G) = 7$)
without intermediate \((a(G) = 4, \ a(G) = 6)\) allows to solve the task of broad banding of passage NIS on an input flow taking into account support of demanded information efficiency and simultaneously to optimize an amount of used reserve topology.

4. Conclusion

Thus, imitative and analytical modeling confirmed serviceability of an offered method of structural conversions fuzzy models of graphs NIS with usage of the spectral theory of graphs, adequacy and reliability of estimations of information system effectiveness received on its basis. Thus finding of the best (quasioptimal) operating conditions NIS is possible on the basis of conciliatory proposals on support of demanded efficiency of an information exchange and a choice of structurally steady topology of system taking into account destructive influences. By results of the led researches, specific practical recommendations for application of variants of structural creation NIS depending on values of parameters of an information exchange and influence of exterior destabilizing factors with which registration formation of a database of reserve topology NIS is carried out can be formulated. It allows to say about necessity of its application at the organization of complex multiple loop adaptation of modern NIS in real time to changing conditions of an information exchange and destabilizing actions that allows to provide increase (maintenance) of demanded structural stability and information system effectiveness [10].

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References

[1] Narraway J J 1998 Shortest paths in regular grids (IEE Proc. – Circuits Devices System) 10 pp 289 – 296
[2] Kleinrock L, Silvester 1978 J Optimum transmission radii for packet radio networks or why six is a magic number (Proc. IEEE Natl. Telecomm. Conf. ) 12 pp 431 – 435
[3] Kofman A 1982 Introduction in the theory of fuzzy sets (Moscow: Radio and communication) p 432
[4] Pasechnikov I I 2004 Metodology of the analysis and synthesis of extreme loaded information networks: monography (Moscow: Mechanical engineering-1) p 216
[5] Tsvetkovich D, Dub M, Zaho H 1984 Spectrums of graphs. The theory and application (Kiev: Sciences Thought) p 384
[6] Toshikazu Sunada 1985 Riemannian coverings and isospectral manifolds (Ann. of Math) 21 pp 169 – 186
[7] Butler S, Chung F 2017 Spectral graph theory. In Handbook of Linear Algebra: 2nd edition (Ed. L. Hogben: CRC Press) p 1904
[8] Svami M, Thulasiraman K 1984 Graphs, networks and algorithms (Moscow: the World) p 455
[9] Mezhuev A M, Pasechnikov I I, Korennoj A V 2017 Analysis of function of efficiency of an information network and algorithm of an estimation of modes of an information exchange on the basis of derivatives of the generalized index. Electromagnetic waves and electronic systems 5 pp 12 – 22
[10] Mezhuev A M, Korennoj A V, Pasechnikov I I 2019 Method of formation structurally steady also it is information effective network information systems. Radio engineering 4 pp 84 – 94
[11] Mezhuev A M, Pasechnikov I I, Korennoj A V 2018 Tensor orthogonal model taking into account influence of interfering conditions at an estimation of information efficiency information-communication networks. Radio engineering 10 pp 96 – 108
[12] Bertsekas D, Gallager R 1992 Data Networks: 2nd ed. (rentice-Hall, Englewood Cliffs. NJ) p 556