Fermion-boson stars with a quartic self-interaction in the boson sector

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Fermion-boson stars are solutions of the gravitationally coupled Einstein-Klein-Gordon-Hydrodynamic equations system. By means of methods developed in previous works, we perform a stability analysis of fermion-boson stars that include a quartic self-interaction in their bosonic part. Additionally, we describe the complete structure of the stability and instability regions of the space of free parameters, which we argue is qualitatively the same for any value of the quartic self-interaction. The relationship between the total mass of mixed stars and their general stability is also discussed in terms of the structure identified within the stability region.

I. INTRODUCTION

Self-gravitating systems are one of the most interesting areas in gravitation, astrophysics and cosmology, mainly because of the characteristic features of this kind of systems that emerge directly from the nature of their main matter constituents. Fermionic self-gravitating objects are the most studied ones, as they help us to understand the properties of stars, given that hydrogen is the most abundant chemical element in the Universe [1, 2].

However, one may also consider the formation of bosonic self-gravitating systems, being boson stars some of the simplest ones. From the first studies in [3], their intrinsic characteristics have also been widely studied [4–6], and their appealing has been recently renewed because of the possibility of explaining the nature of the dark matter in the Universe by means of ultra-light bosons, see for instance the reviews in [7–11].

Given, on one hand, the proved existence of self-gravitating fermions, and on the other hand, the possible presence of self-gravitating bosons, it was just natural to ask for the properties of self-gravitating objects with a mixing of fermions and bosons, which have been known ever since as fermion-boson stars, see the seminal papers in [12–15]. As with any other self-gravitating system, the central question is whether fermion-boson stars are stable. This is not a trivial question at all, as fermion-boson stars are described by two parameters, and then the stability criterions employed for purely fermion or purely boson stars, which are each a one-parameter family of solutions, are no longer valid.

Nonetheless, the authors in [12] were able to establish general guidelines to study the stability of mixed stars, and finding the corresponding region of stability on the two-parameter plane of equilibrium configurations. This was taken as a starting point in [16], where the dynamical evolution of fermion-boson stars was studied. The case considered was a fermion star (neutron star modeled as a perfect fluid), mixed with a boson component modeled with a scalar field, the latter being endowed with only a quadratic potential.

The analysis performed in [16] was based on the behaviour of the bosonic and fermionic particle numbers in mixed configurations with a fixed total mass. Following the guidelines in [13], one looks for the maximum/minimum of the curves associated to the particle numbers, and thereby one can distinguish stable configurations from the unstable ones. The foregoing stability criterion was confirmed by numerically evolving some equilibrium configurations.

In this work, the stability analysis is presented for fermion-boson stars with a quartic self-interaction in the bosonic part, using the methodology developed in [16]. Specifically, we study the influence of the self-interaction term on the total mass, the size, and the number of bosonic and fermionic particles of mixed stars. We will also extend the study of [17] and establish the structure of the stability and instability regions.

This paper is organized as follows. In Sec. II we present the equations of motion and their boundary conditions that allow the construction of equilibrium configuration for mixed stars. Then, Sec. III is devoted to the study of the stability of the mixed stars, and a general discussion on the structure of the two-parameters plane of equilibrium configurations. Finally, Sec. IV presents the conclusions and final remarks.

II. MATHEMATICAL FORMALISM

Boson-Fermion stars can be modeled by a complex scalar field $\phi$ endowed with a scalar potential $V(|\phi|)$, which will represent the bosonic part, and a perfect fluid, which is described by the following primitive physical variables: the rest-mass density $\rho$, the pressure $P$, the internal energy $\epsilon$ and its 4-velocity $u^\alpha$.

The equations of motion are the coupled Einstein-
Klein-Gordon-Hydrodynamic equations, given by

\[ G_{\mu\nu} = 8\pi \left( T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(f)} \right) , \quad \square^2 \phi - \phi \frac{dV(\phi)}{d|\phi|^2} = 0 , \quad (1a) \]
\[ \nabla_{\mu} T^{(f)\mu\nu} = 0 , \quad \nabla_\mu (\rho u^\mu) = 0 . \quad (1b) \]

where we use geometric units in which \( c = 1 \). Here, \( T_{\mu\nu}^{(\phi)} \) and \( T_{\mu\nu}^{(f)} \) are the stress-energy tensors of the bosonic and fermionic components, respectively, which are explicitly given by

\[ T_{\mu\nu}^{(\phi)} = \frac{1}{2} \left( \partial_{\mu} \phi^* \partial_{\nu} \phi + \partial_{\mu} \phi \partial_{\nu} \phi^* \right) - \frac{g_{\mu\nu}}{2} \left( \partial^\rho \phi^* \partial_{\rho} \phi + 2V \right) , \]
\[ T_{\mu\nu}^{(f)} = \left[ \rho(1 + \epsilon) + P \right] u_{\mu} u_{\nu} + Pg_{\mu\nu} . \quad (2) \]

The scalar field potential is written as

\[ V(\Phi) = \frac{m^2}{2} |\phi|^2 + \lambda |\phi|^4 . \quad (3) \]

which represents boson particles with mass \( m \) and a self-interaction parameter \( \lambda \).

### A. Equilibrium Configurations

We shall be interested in equilibrium configurations, for which we assume a static and spherically symmetric metric in the form

\[ ds^2 = -\alpha^2 dt^2 + a^2(r) dr^2 + r^2 d\Omega^2 . \quad (4a) \]

For the complex scalar field, we assume the standard harmonic form \( \phi(t, r) = \phi(r) e^{-i\omega t} \), where \( \omega \) is an intrinsic frequency, whereas for the perfect fluid in hydrostatic equilibrium, we take the following four-velocity \( u^\mu = (-1/\alpha, 0, 0, 0) \).

For numerical purposes, we consider the following set of new dimensionless variables:

\[ x = mr , \quad \Omega = \frac{\omega}{m} , \quad \sqrt{4\pi \rho} \rightarrow \phi , \quad (4b) \]
\[ \Lambda = \frac{m^2 \lambda}{4\pi m^2} , \quad \frac{4\pi}{m^2} P \rightarrow \rho , \quad \frac{4\pi}{m^2} P \rightarrow P , \quad (4c) \]

Thus, the equations of motion (1) for equilibrium configurations explicitly read

\[
\begin{align*}
\frac{da}{dx} &= a \left( 1 - \alpha^2 \right) x + a^2 x \left[ \left( \frac{\alpha^2}{\alpha^2} + 1 + \frac{\Lambda \phi^2}{2} \right) \phi^2 + \frac{\Phi^2}{\alpha^2} + 2\rho(1 + \epsilon) \right] , \quad (5a) \\
\frac{\alpha}{dx} &= \alpha \left( \frac{\alpha^2 - 1}{\alpha^2} + a^2 x \left[ \frac{\alpha^2}{\alpha^2} - 1 + \frac{\Lambda \phi^2}{2} \right] \phi^2 + \frac{\Phi^2}{\alpha^2} + 2P \right) , \quad (5b) \\
\frac{d\phi}{dx} &= \phi , \quad (5c) \\
\frac{d\Phi}{dx} &= \left( 1 - \frac{\Omega}{\alpha^2} + \Lambda \phi^2 \right) a^2 \phi - \left( \frac{2}{x} + \frac{\alpha'}{\alpha} - \frac{\alpha'}{a} \right) \Phi , \quad (5d) \\
\frac{dP}{dx} &= -\left[ \rho(1 + \epsilon) + P \right] \frac{\alpha'}{\alpha} . \quad (5e)
\end{align*}
\]

In order to obtain a closed system, we must introduce an equation of state for the perfect fluid. Particularly, we adopt a polytropic equation of state \( P = K\rho^\Gamma \), with polytropic constant \( \Gamma = 2 \) and adiabatic index \( K = 100 \), which corresponds to masses and compactness in the range of neutron stars [18].

The system of equations (5) represents an eigenvalue problem for the frequency of the bosonic part \( \Omega \), as a function of the central values of the fluid density \( \rho_0 \) and the scalar field \( \phi_0 \). We solve this system by using the shooting method [19] and under boundary conditions corresponding to regularity at the origin and asymptotic flatness at infinity. The latter are:

\[
\begin{align*}
a(0) &= 1 , \quad \lim_{x \to \infty} a(x) = 1 , \quad (6a) \\
\alpha(0) &= 1 , \quad \lim_{r \to \infty} \alpha(r) = \lim_{x \to \infty} a(x) , \quad (6b) \\
\phi(0) &= \phi_0 , \quad \lim_{x \to \infty} \phi(x) = 0 , \quad \Phi(0) = 0 , \quad (6c) \\
\rho(0) &= \rho_0 , \quad P(0) = K\rho_0^\Gamma , \quad \lim_{x \to \infty} P(x) = 0 . \quad (6d)
\end{align*}
\]

Additionally, we use the Schwarzschild mass to obtain the mass of equilibrium configurations,

\[ M_T = \lim_{x \to \infty} \frac{x}{2} \left( 1 - \frac{1}{\alpha^2} \right) . \quad (7) \]

Also, due to the symmetry \( U(1) \) in the Lagrangian of the scalar field and the Noether theorem, the scalar field charge is conserved, which can be associated with the
number of bosons \( N_B \), whereas the number of fermions \( N_F \) is defined by the conservation of the baryonic number. \( N_B \) and \( N_F \) are then calculated by means of the following expressions,

\[
\frac{\partial N_B}{\partial r} = \frac{4\pi\alpha \omega^2 r^2}{\alpha}, \quad \frac{\partial N_F}{\partial r} = 4\pi \rho r^2. \tag{8}
\]

Finally, we define the radius of a star \( R_T \) as the value of \( r \) containing 95% of the total mass. Correspondingly, the radius of the bosonic (fermionic) component \( R_B \) (\( R_F \)), will be the value of \( r \) containing 95% of the corresponding particles.

### B. Numerical solutions

In this section, we construct the equilibrium configurations of the mixed boson-fermion stars by solving numerically the system of equations \( 5 \). In Fig. 1, we can see a typical example, which satisfies our conditions of regularity at the origin and asymptotically flatness at infinity. This particular configuration was constructed with a central scalar field \( \phi(0) = \phi_0 = 0.01 \), a central fluid density \( \rho_0 = 0.005 \) and \( \Lambda = 10 \).

Fig. 2 shows the total mass of fermion-boson stars configurations as a function of \( \rho_0 \) and \( \phi_0 \), that is \( M_T = M_T(\rho_0, \phi_0; \Lambda) \), with a self-interaction parameter \( \Lambda = 10 \). In the plane \( \rho_0 = 0 \), one may see the typical curve \( M_B = M_B(\phi_0) \) for a purely boson star, whereas in the plane \( \phi_0 = 0 \), one may see the typical curve \( M_F = M_F(\rho_0) \) of a purely fermion star.

It is possible to verify that, if we set \( \phi_0 = 0 \), then we get back to the mass curve of neutron stars with a critical mass \( M_{Fc} = 1.64 \), that corresponds to \( K = 2 \) and \( \Gamma = 100 \). This mass separates the stable configurations \( \{ M_T < M_{Fc} \} \) from the unstable ones \( \{ M_T > M_{Fc} \} \) in the case of purely-fermion stars. On the other hand, if we set \( \rho_0 = 0 \), we recover the mass curve of boson stars, which have a critical mass \( M_{Bc} = 0.92 \) corresponding to the value \( \Lambda = 10 \) \( 20 \).

In Table I we show the resultant numerical values of different equilibrium configurations obtained from Eqs. \( 5 \), taking into account the self-interaction in the bosonic part: \( \Lambda = 0, 10, 30 \). We also indicate the stability of each numerical case, the details of which we explain in Sec. III below.

In the first row of the first column of this table (for \( \Lambda = 0 \)) we have one value of \( \rho_0 = 0.04 \) employed to construct a purely stable fermion star (whose mass \( M_F \), radius \( R_F \) and number of particles \( N_F \) are displayed in the second column), and three values of \( \phi_0 = 0.01, 0.27, 0.6 \) employed to construct three purely boson stars, two stable and one unstable (whose masses \( M_B \), radius \( R_B \) and number of particles \( N_B \), are displayed in the third column).

In the fourth column, features of three mixed stars formed from \( (\rho_0, \phi_0) = (0.04, 0.01), (0.04, 0.271) \) and \( (0.04, 0.6) \) are displayed. As long as both individual stars (fermionic and bosonic type) are stable, the corresponding mixed star is stable, but it is unstable if either individual star is not stable (as it is shown in the second row where \( \rho_0 = 0.06 \) and \( \phi_0 = 0.01, 0.27, 0.6 \) were used). We observe that \( M_T \leq Max\{M_F, M_B\} \) that is to say, the total mass of a mixed star is typically smaller than the maximum value between \( M_F \) and \( M_B \) of individual stars.

Results for \( \Lambda = 10, 30 \) are also presented in Table I. For a fixed value of \( \Lambda \), when a mixed star is formed from individual stable stars (purely fermionic/bosonic) its characteristics are closer to those of the more massive individual star (either fermionic or bosonic). However, if a mixed star is formed from a bosonic unstable star regardless what the value of \( M_F \) is, its characteristics are closer to those of the individual boson star. For \( \Lambda \neq 0 \), \( M_T \leq Max\{M_F, M_B\} \) still stands.
TABLE I. Properties of fermion, boson and mixed stars. The columns report, from left to right: the central density of the mixed stars; the mass $M_B$, radius $R_B$ and particles number $N_B$ of the purely-fermion stars; the total mass $M_T$, total radius $R_T$, radii $R_F$, $R_B$ and particle numbers $N_F$, $N_B$ of the fermionic and bosonic components, respectively, of the mixed stars. See the text for details.

| $\Lambda = 0$ | Fermion Star ($\rho_0 \neq 0, \phi_0 = 0$) | Boson Star ($\rho_0 = 0, \phi_0 \neq 0$) | Mixed Star ($\rho_0 \neq 0, \phi_0 \neq 0$) |
|--------------|---------------------------------|---------------------------------|---------------------------------|
| $\rho_0$ | $\phi_0$ | $M_F$ | $R_F$ | $N_F$ | $M_B$ | $R_B$ | $N_B$ | $M_T$ | $R_T$ | $R_F$ | $R_B$ | $N_F$ | $N_B$ |
| 0.01 | | 0.022 | 0.022 | stable | 1.636 | 7.20 | 6.757 | 4.587 | 1.800 | 7.08 (10^{-4}) | stable |
| 0.04 | 0.27 | 1.637 | 6.750 | 1.798 | 0.633 | 5.929 | 0.65 | stable | 1.035 | 6.33 | 5.931 | 4.223 | 0.784 | 0.329 | stable |
| 0.60 | | 0.525 | 3.335 | 0.516 | unstable | 0.513 | 4.68 | 1.455 | 3.301 | 0.016 | 0.489 | unstable |
| 0.01 | | 0.022 | 0.022 | stable | 1.594 | 6.36 | 5.955 | 3.818 | 1.740 | 4.33 (10^{-4}) | unstable |
| 0.06 | 0.27 | 1.594 | 5.947 | 1.739 | 0.633 | 5.929 | 0.653 | stable | 1.255 | 6.22 | 5.814 | 3.611 | 1.130 | 0.234 | unstable |
| 0.60 | | 0.525 | 3.335 | 0.516 | unstable | 0.508 | 4.46 | 2.085 | 3.170 | 0.053 | 0.448 | unstable |
| $\Lambda = 10$ | | | | | | | | | | | | | | |
| $\rho_0$ | $\phi_0$ | $M_F$ | $R_F$ | $N_F$ | $M_B$ | $R_B$ | $N_B$ | $M_T$ | $R_T$ | $R_F$ | $R_B$ | $N_F$ | $N_B$ |
| 0.01 | | 0.210 | 0.211 | stable | 1.636 | 7.20 | 6.757 | 4.587 | 1.800 | 7.08 (10^{-4}) | stable |
| 0.04 | 0.22 | 1.637 | 6.750 | 1.798 | 0.920 | 6.796 | 0.964 | stable | 1.084 | 6.13 | 5.716 | 4.665 | 0.799 | 0.370 | stable |
| 0.35 | | 0.830 | 4.835 | 0.852 | unstable | 0.758 | 5.94 | 2.825 | 4.537 | 0.116 | 0.665 | unstable |
| 0.01 | | 0.210 | 0.211 | stable | 1.594 | 6.36 | 5.955 | 3.818 | 1.740 | 4.33 (10^{-4}) | unstable |
| 0.06 | 0.22 | 1.594 | 5.951 | 1.739 | 0.920 | 6.796 | 0.964 | stable | 1.288 | 6.11 | 5.708 | 3.881 | 1.170 | 0.232 | unstable |
| 0.35 | | 0.830 | 4.835 | 0.852 | unstable | 0.796 | 4.86 | 3.902 | 3.925 | 0.329 | 0.500 | unstable |
| $\Lambda = 30$ | | | | | | | | | | | | | | |
| $\rho_0$ | $\phi_0$ | $M_F$ | $R_F$ | $N_F$ | $M_B$ | $R_B$ | $N_B$ | $M_T$ | $R_T$ | $R_F$ | $R_B$ | $N_F$ | $N_B$ |
| 0.01 | | 0.228 | 0.229 | stable | 1.636 | 7.20 | 6.757 | 4.591 | 1.800 | 7.10 (10^{-4}) | stable |
| 0.04 | 0.16 | 1.637 | 6.750 | 1.798 | 1.336 | 8.652 | 1.413 | stable | 1.255 | 6.42 | 6.000 | 5.144 | 1.050 | 0.315 | stable |
| 0.25 | | 1.177 | 6.244 | 1.215 | unstable | 0.989 | 7.11 | 3.463 | 5.671 | 0.228 | 0.801 | unstable |
| 0.01 | | 0.228 | 0.229 | stable | 1.594 | 6.36 | 5.954 | 3.819 | 1.740 | 4.34 (10^{-4}) | unstable |
| 0.06 | 0.16 | 1.594 | 5.947 | 1.739 | 1.336 | 8.652 | 1.413 | stable | 1.397 | 6.17 | 5.765 | 4.135 | 1.350 | 0.170 | unstable |
| 0.25 | | 1.177 | 6.244 | 1.215 | unstable | 0.990 | 5.32 | 4.355 | 4.560 | 0.533 | 0.510 | unstable |

FIG. 2. The total mass of mixed stars as a function of the central values $\Lambda = 10$. On the plane $\rho_0 = 0$, one can see the typical curve (in black) $M_B = M_B(\phi_0)$ for a purely-boson star, whereas on the plane $\phi_0 = 0$ one can see the typical curve (in red) $M_F = M_F(\rho_0)$ for a purely-fermion star.

III. STABILITY ANALYSIS

Because mixed stars are parameterized by the two quantities $\phi_0$ and $\rho_0$, we cannot use the stability theorems for single parameter solutions to carry out the stability analysis of these stars. We then use the method developed in [10] to study the stability of mixed configurations.

A. General stability regions

The foregoing method is based on the behavior of $N_B$ and $N_F$, for configurations with the same value of $M_T$. Beginning with a purely fermionic star (by providing $\rho_0$ and setting $\phi_0 = 0$), mixed configurations are then built up by increasing the value of $\phi_0$ from zero in such a way that $M_T$ is kept fixed. From the curve of the number of particles in terms of $\phi_0$ and $\rho_0$, it can be observed that $N_F$ decreases to a minimum while $N_B$ increases until reaching a maximum, at exactly the same values of $\phi_0$ and $\rho_0$.

The same behavior is observed if we consider first a purely bosonic configuration (by providing $\phi_0$ and setting $\rho_0 = 0$) and subsequently adding fermions by increasing the value of $\rho_0$ from zero. In this case, $N_B$ decreases to a minimum while $N_F$ increases until reaching a maximum. This is exemplified in the two cases shown in Fig. 3, which were obtained with $\Lambda = 10$ and fixed total mass $M_T = 0.83$.

The stability analysis of mixed stars is based on this fact, and we can summarize the criterion developed in [10] as follows. The stars whose particle number is lo-
cated to the left of that point where the maximum and minimum of $N_B, N_F$ coalesce are considered stable configurations, whereas stars with particle numbers on the right of that point correspond to unstable configurations. Using this criterion, we can construct stability boundary curves on the plane $(\rho_0, \phi_0)$ by considering different values of the total mass $M_T$ (see also Fig. 3), which then split the space of possible configurations in two well defined regions.

In Fig. 3 we show the boundary curves for different values of the scalar field self-interaction $\Lambda$. It must be noticed that the total mass of the configurations was varied in between two values, namely $M_\ast \leq M_T \leq M_{Fc}$, where $M_\ast$ is a particular value that we explain in detail in Sec. III B below. For instance, in the case $\Lambda = 0$ the range of variation was $0.613 \leq M_T \leq 1.637$, and correspondingly for $\Lambda = 30$ the range was $1.05 \leq M_T \leq 1.637$.

In summary, mixed stars configurations constructed from $(\rho_0, \phi_0)$ that lie within the stable region will be stable, and those outside will be unstable. It can also been noticed, in the Fig. 4 that the stability region shrinks as $\Lambda$ increases. The boundary curves intersect the $\phi_0$-axis at different points, because the critical field value decreases for $\Lambda \neq 0$ (see Table I), while they intersect the $\rho_0$-axis at the same point corresponding to $M_T = 1.637$. In consequence, the maximum value of $M_T$ of stable mixed stars do not go beyond the critical mass $M_{Fc}$ of purely neutron stars.

Taking for reference the case $\Lambda = 0$, there are stable and unstable configurations with total mass in the range $0.633 \leq M_T \leq 1.637$, as they correspond precisely to the curves used to determine the boundary curve shown in Fig. 4. Following the above reasoning, it was reported in [16] that, for $\Lambda = 0$, all mixed stars with total mass smaller than the critical mass of purely bosonic stars, $M_T < M_c = 0.633$, were stable, which was not quite precise. This is because curves corresponding to the aforementioned configurations may also have points inside and outside the stability regions.

B. Inner structure of the stability region

Similarly to the case of the boundary curves described above, one can also draw curves on the plane $(\rho_0, \phi_0)$ representing boson-fermion stars with the same total mass $M_T$, but now considering an extended range of values $0 < M_T \leq 1.637$. Examples of such curves are shown in the top panel of Fig. 5 for the case $\Lambda = 10$. For instance, all black (yellow) squares in the plot correspond to mixed stars with the same total mass $M_T = 0.91$ ($M_T = 0.87$), which is less than the critical boson mass $M_{Bc} = 0.92$, but they are represented by two different curves: one that
starts and ends at the vertical axis $\phi_0$, and another one that does the same but with respect to the horizontal axis $\rho_0$.

Because of these behaviors, the curves must necessarily cross the boundary (red) curve, and then some of the points are within the stable region and others are outside of it. This means that there are stable and unstable mixed configurations with total mass $M_T = 0.91$ ($M_T = 0.87$). Other curves are shown that depict the same behavior, and the common feature in all is that they have a total mass such that $M_T > M_* = 0.827$. For the particular value $M_T = M_*$, the two resultant curves (cyan and purple) meet at the same point on the boundary (red) curve, that we denote by $(\rho_0^*, \phi_0^*)$, and then they represent the extreme cases of a curve that starts and ends at the same axis. Furthermore, these lines seem to determine the boundary of an inner region inside the stability one, within which all curves start on the $\phi_0$ axis and end up on the $\rho_0$ axis (or viceversa). Moreover, all configurations with total mass $M_T < M_*$ lie inside the stability region, and then for such values of the total mass there are only stable configurations. Thus, the stability region of boson-fermion configurations has a non-trivial inner structure, as depicted in the bottom panel of Fig. 5 with three well distinctive sub-regions that have the following properties.

- **Region I.** The configurations have a total mass in the range $M_* < M_T \leq M_{Bc}$, the latter value corresponding to the critical mass of purely-boson configurations. Additionally, their main characteristic is that the number of bosons is always larger than the number of fermions, $N_B > N_F$.

- **Region II.** The configurations have a total mass in the range $M_* < M_T \leq M_{Fc}$, the latter value corresponding to the critical mass of pure-fermion configurations. Additionally, their main characteristic is that the number of fermions is always larger than the number of bosons, $N_F > N_B$.

- **Region III.** The configurations have a total mass in the range $0 < M_T < M_*$. In contrast to the configurations in Regions I and II, this time the mixed stars can be either boson or fermion dominated, as indicated by their end points: one is on the vertical axis $\phi_0$ and another one is on the horizontal axis $\rho_0$.

- The boundary lines of the three inner sub-regions are: the boundary (red) curve, and the curves of the configurations with exactly the total mass $M_T = M_*$. For these latter configurations, they start as a purely-boson (fermion) star (or viceversa), and then change their nature by becoming a purely-fermion (boson) star. Then, they have the same number of bosons and fermions, $N_B = N_F$, when they meet at the boundary curve at the point $(\rho_0^*, \phi_0^*)$.

In our numerical experiments, we have found that the same inner structure of the stability region of mixed stars exist for any given value of the self-interaction parameter $\Lambda$, see Fig. 6, but just with a different value of $M_*$. For instance, for $\Lambda = 0$, we find $M_* = 0.613$, whereas for $\Lambda = 30$ the corresponding value is $M_* = 1.05$. This also seems to suggest that the value of $M_*$ increases for larger values of $\Lambda$, although one must recall that likewise the stability region becomes smaller, see Fig. 6.

### C. Structure of the instability region

Just as in the case of the stability zone of the plane $(\rho_0, \phi_0)$, we can see in Fig. 6 that there is also a structure on the instability region, ie for the points beyond the
In this paper we have applied the criterion developed in [16] to determine the stability of fermion-boson stars with quartic self-interaction for the bosonic part. Using this criterion, stability boundary curves in the \((\rho_0, \phi_0)\) plane were constructed. We have again found a main curve that splits the plane in two well defined regions, one that contains stable configurations and another with unstable ones. It turns out that as the value of the self-interaction parameter \(\Lambda\) increases, the stability region shrinks. It was also shown that the maximum value of the total mass of mixed stars do not go beyond the critical mass of purely neutron stars, a results that stands for non-zero values of \(\Lambda\).

Additionally, we have been able to unravel the structure of both the stable and unstable regions, by means of the special curves corresponding to equilibrium configurations with total mass \(M_\star\). These curves are special, they meet at the boundary curve at the same point, which means that at such point the equilibrium configuration has the same number of bosons and fermions. We have then argued that the value of \(M_\star\) is a true discriminant for the existence of stable equilibrium configurations. Our study suggests that equilibrium configurations with total mass \(M_T < M_\star\) are intrinsically stable (whether boson or fermion dominated), whereas one can construct either stable or unstable configurations in the opposite case \(M_T > M_\star\) (again, whether boson or fermion dominated). The value of \(M_\star\) depends on the value of the self-interaction parameter \(\Lambda\), but its role as discriminant for intrinsically stable configurations is always the same. Our study considers the effects coming form the addition of a bosonic self-interaction, but it would be interesting to study the effects of varying the polytropic and adiabatic index on the stability of mixed stars. This is a topic of an ongoing research that we expect to report elsewhere.

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**IV. FINAL REMARKS AND DISCUSSION**

Although we have not been able to do a thorough exploration of all possible solutions, we can see that the structure of the unstable region we have just described is consistent with that of the stable region explained in Sec. III B above. In that consistency the special value \(M_\star\) plays a central role and seems to be the common element of the overall stability and instability regions.
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