Introduction of line contact in frequency-based substructuring process using measured rotational degrees of freedom

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Abstract. The theory of frequency-based substructuring enables us to combine the dynamical behavior of multiple subsystems into the common response of the whole system. Subsystem’s properties can be numerically or experimentally obtained in the form of frequency response functions. Response models are then coupled together taking into account a connectivity conditions. However, when one is dealing with a statically indeterminate multi-point connections such as flanges connected over several bolts, it is difficult to properly define a model in the numerical environment. In order to obtain real contact conditions a few methods like virtual point transformation and multi-point connection were proposed in recent years. Since an indirect approach of obtaining rotational motion struggle to accurately define local dynamic characteristics i.e. antiresonances, it also rely on assumption of rigid interface. In this paper we are introducing an alternative approach, where all six degrees of freedom are experimentally obtained with aim to directly address local joint flexibility in the substructuring process. This procedure represent an alternative to the method that removes the effects of a flexible fixture, however here it is done directly by measuring contact dynamic without the need of additional fixture block. Rotational responses are directly obtained using rotational accelerometer that enable us to gather all six degrees of freedom in each experimental point. Consequently, contact can be fully defined with less measurements points where several of them with only translational responses are required in other methods for reconstruction of rotational motion.

1. Introduction

Structural dynamic analyses can be performed more efficiently whenever complex systems are divided into smaller subsystems, analyzed separately and later coupled together using dynamic substructuring (DS) methods. Depending on the type of numerical or experimental data in
the considered domain we can select the most appropriate DS method. Even though they are already well defined, it is still difficult to provide accurate and reliable dynamic characteristics for the subsystems. Analytical data are consistent, but they make it difficult to accurately define the real material properties. On the other hand, several difficulties appear whenever the entire dynamics of the systems is obtained experimentally [1, 2]. Therefore, due to the lack of reliability, none of them are yet widely used in real-world applications.

Dynamic substructuring methods can be seen as a domain-independent set of tools for combining components’ structural dynamic characteristics. Despite the ability to perform DS in several domains, the majority of the research content relates to a modal and a frequency domain. The modal parameters are implemented in component mode synthesis methods (CMS) [3] and are widely used in numerical approaches. On the other hand, frequency-based substructuring (FBS) [4] methods are normally related to the experimental approach, because it is possible to directly define the real dynamic properties based on the measured frequency-response functions (FRFs). In recent time focus is on the experimental procedures including rotations, where a lot of research has been conducted in recent years. Most of the pioneering methods dealing with rotational motion in combination with a pure moment excitation are summarized in the paper [5]. In [6] the replacement of directly obtained rotational responses is proposed based on their reconstruction from multiple translational accelerometers.

A lot of effort has been invested so far in sensors development to obtain rotational motion as well as procedures to apply a pure moment excitation [7]. Those methods set standards upon which newer procedures were proposed in recent years in order to improve their deficiencies. Procedure of finite differences theory, together with two offset translational accelerometers can be found in [8]. Methods to apply pure moment excitation for mass normalized FRFs has been proposed in [9]. A new type of sensors based on bimorph materials [10], micro electro mechanical systems (MEMS) [11], strain gages [12] and piezoelectric materials [13] has been also developed in recent years. However, extensive research in this field still does not provide reliable procedure that would gain much popularity in the real applications.

Typically, the experimental substructuring process is relying on multiple measurements of translational responses in each joint to ensure the force as well as the torque transmission. Therefore, additional flanges or other joint modifications must be applied to increase the contact surface to enable the positioning of the translational sensors [1, 6]. However, any kind of joint modification can influence the dynamic characteristics of the structure itself, due to the mass and stiffness modification. This is especially the case when thin and light structures are considered.

In this paper, the Lagrange Multiplier Frequency-Based Substructuring method (LM FBS) [4] is used in the case of coupling two steel plates with collinearly arranged connection points. The presented work is a continuation of the previous experimental research of Drozg et al. [14], where very promising coupling results were obtained by additionally including the experimental rotational DoFs in the substructuring process. The rotational degrees of freedom were measured using direct rotational sensors that are more frequently used for active control of oscillating shafts and car crash testings. However is was shown, that lightweight and robust construction as well as wide dynamic and frequency range seems to be suitable also for structural dynamics applications. As the rotational DoFs are normally subjected to typical data contamination, the proposed approach was limited to simple structures with only one contact point, but proved unreliable in the case of complex structures. In order to extend the applicability of the previous procedure to multiple connection points a set of data-conditioning techniques known as Variability Improvement of Key Inaccurate Node Groups (VIKING) [1] are applied. Smoothing and expansion of the raw experimental data are achieved with the inclusion of a numerical model and the System Equivalent Reduction Expansion Process (SEREP) [15]. By incorporating all six experimentally obtained DoFs in a single connection point it was possible to ensure rigid connection using collinearly arranged connection points. This would not be possible if only
measured translational degrees of freedom would be considered because the connection would behave as a rotational joint. Thus, the proposed method enables efficient contact formulation between plate-like structures with narrow and relatively long contact regions.

2. Overview of the Lagrange multiplier frequency-based substructuring

In this section a brief overview of the coupling method used in this paper is presented. In order to perform the LM FBS process based on the experimentally obtained data, a series of smoothing and expanding processes are performed. Firstly, force-excited translational and rotational FRFs from both test subsystems are experimentally obtained. Then, they are transformed to the modal domain using a poly-reference least-square frequency domain (LSFD) modal parameter estimation method (MPE). Only flexible modes within the acquired frequency range are considered in the smoothing process. Lower- and upper residuals are retained from the equivalent finite-element numerical model and are attached to the smoothed flexible modes before they were synthesized back to the frequency domain and used in the coupling process.

The coupling is performed with the LM FBS method [4]. Derivation of the method starts with the general equation of motion in the time domain, which for an arbitrary subsystem \( s \) is defined as:

\[
M^{(s)} \ddot{u}(t)^{(s)} + C^{(s)} \dot{u}(t)^{(s)} + K^{(s)} u(t)^{(s)} = f(t)^{(s)},
\]

where \( M^{(s)} \), \( C^{(s)} \) and \( K^{(s)} \) stand for the mass, damping and stiffness matrices of the subsystem \( s \), respectively. Vector \( f(t)^{(s)} \) is a load vector applied to the particular subsystem. The Fourier transformation makes it possible to transform Eq. (1) into the frequency domain:

\[
[-\omega^2 M^{(s)} + j \omega C^{(s)} + K^{(s)}] U(\omega)^{(s)} = F(\omega)^{(s)},
\]

where \( U^{(s)} \) represents a response vector. With the introduction of the dynamics stiffness matrix \( Z(\omega) \), Eq. (2) can be written as:

\[
Z(\omega)^{(s)} U(\omega)^{(s)} = F(\omega)^{(s)},
\]

where

\[
Z(\omega)^{(s)} = [-\omega^2 M^{(s)} + j \omega C^{(s)} + K^{(s)}],
\]

represents the dynamic stiffness matrix. Due to the inability to separately obtain the mass, damping and stiffness properties of the subsystems using an experiment, Eq. (3) must be rewritten in the admittance notation:

\[
Y(\omega)^{(s)} F(\omega)^{(s)} = U(\omega)^{(s)}; \quad Y(\omega) = Z(\omega)^{-1}.
\]

The variable \( Y(\omega)^{(s)} \) is the subsystem’s response matrix with FRFs between all the input excitation and the output response experimental points. In addition, we need to define the compatibility conditions based on the Boolean matrix \( B \) that represents the relationship between the connection DoFs. It can be expressed as:

\[
BU = 0,
\]

and its implementation in Eq. (3) results in:

\[
ZU + B^T \lambda = F,
\]
where \( \lambda \) stands for the Lagrange multipliers representing the connection joints’ forces. The matrix notation of the dual assembled system is combined using Eqs. (6) and (7):

\[
\begin{bmatrix}
Z & B^T \\
B & 0
\end{bmatrix}
\begin{bmatrix}
U \\
\lambda
\end{bmatrix} =
\begin{bmatrix}
F \\
0
\end{bmatrix}.
\]

(8)

By eliminating \( \lambda \) from Eq. (8) gives us the final notation of the LM FBS method:

\[
Y^{(\text{tot})} = Y - YB(BYB)^{-1}BY; \quad Y \triangleq Z^{-1}.
\]

(9)

Admittance matrices \( Y \) can be obtained solely with an experiment or in our case as a hybrid combination between the numerical model and the experimental data combined within the VIKING process.

3. Introduction of line contact with all six experimental DoFs in FBS process

In this paper the idea of including all six experimentally obtained DoFs in a substructuring process is presented in order to properly define the characteristics of the line contact. Since there are several difficulties with the measurements of the rotational DoFs, in most cases only translational responses are used. Even though it is possible to indirectly reconstruct the rotational motion based on the multiple translational responses, they often poorly define a local joint’s flexibility and the positions of anti-resonances. Those are greatly influenced by the location of the measurement transducers, which must be placed at the exact coupling positions. Whenever a rigid joint connection between substructures is established, an indirect approach might be sufficient. However, this is not the case with flexible joints, where the connection points are arranged in a straight line. In order to constrain all the DoFs in the joint a translational as well as rotational DoFs must be obtained for each experimental point. In this manner the proposed procedure poses no limitations in term of joint geometry as it is more general compared to existing formulations. Furthermore, we still need to smooth the experimental data’s contaminations, relying on the implementation of an equivalent numerical model. Generally, two sources of errors can be introduced during the measurements. First, there are the random errors that introduce uncertainty into the measured data that are normally not under control. The second type are the bias or systematic errors. Here, the errors introduce systematically shifted values of the resonances and anti-resonances. During the signal processing an additional error can be introduced. Modal truncation is certainly one of the major issues when dealing with EMA. It is impossible to obtain all modal shapes and include them into the substructuring process. Beside flexible modes, also rigid-body modes and upper residuals need to be considered.

The proposed process in this paper consists of four major steps that are schematically presented in Fig. 1. In the first step, only the force-excited FRFs are obtained from all the subsystems. This applies to the inner and connection nodes, the driven and transfer FRFs as well as the translational and rotational DoFs. As the frequency-response model is later transformed into the modal domain, it is possible to reconstruct this missing information. All the existing combinations between the nodes and the directions must be obtained consistently. However, some of the combinations are impossible to obtain due to the inability to perform the excitation or attach the sensors. Those points are omitted from the measurement procedure and later expanded with the SEREP [15].

In the second step a numerical modal analysis is performed on the equivalent FE model. The analysis applies a much denser mesh than the mesh of the experimental nodes. Therefore, the results from only coincident nodes are exported and later used in the smoothing process. It is important to take into account that the number of modes in the SEREP expansion is conditioned by the number of nodes on the subsystem.
Both experimental and numerical modal parameters enter the VIKING process in the third step. First, they are compared using the set of criteria to determine the correlation and to eliminate all the poorly correlated DoFs from the modal shapes. The retained or considered as well correlated DoFs are marked as active "a" and not measured or eliminated based on the criteria as deleted "d". Using this notation a SEREP transformation matrix is established and an expansion performed. Because only flexible modes are included in the process, rigid-body modes and upper residuals must be additionally included before the FRF synthesis is applied.

In the last step the experimentally obtained and numerically smoothed response models are used as part of the LM FBS coupling process. Both subsystems A and B are rigidly connected over five nodes in a line. In order to validate our procedure, a comparison with the experimentally obtained FRFs of the coupled system was performed.

### 3.1. Test system

The selected test system consists of two steel plates. They are rigidly connected perpendicular to each other with four M12 bolts in the line contact. The moment of the attachment was 75 Nm. The subsystem A is in a vertical position and is slightly larger than the horizontally lying subsystem B. All the dimensions are shown in Fig. 2. Both subsystems and later also the system were free-free supported with ropes.

### 3.2. Experimental setup

The real structural dynamic properties can only be obtained based on the experimental data. The excitation was performed with a modal hammer (PCB 086C03) and the response obtained with quartz-based piezoelectric translational and rotational accelerometers (PCB 208C01 and Kistler 8840). The measurement nodes (25 points) were equally distributed along both subsystems (Fig. 2). Five of them represent line connection nodes. It is not necessary to obtain the complete response model in order to perform the experimental modal analysis (EMA) for all 6 directions. Ideally, it would be sufficient to perform only driving-point measurements for all the directions. However, moment excitation is very difficult to apply, and thus only force excitation...
Figure 2. Subsystems; a) Geometry and dimensions, b) Test rig with rotational accelerometer.

is performed. The excitation in y and z direction is preformed by impacting the structure along its edge. Table 1 demonstrates the measurement procedure to obtain the experimental FRFs.

Table 1. Excitation and response directions for both subsystems.

| subsystem A | subsystem B |
|-------------|-------------|
| excit. | resp. | roving | nodes | excit. | resp. | roving | nodes |
| x | hammer | 1-25 | x | sensor | 1-25 |
| r_y | sensor | 1-25 | r_y | sensor | 1-5 |
| r_z | sensor | 1-25 | y | hammer | 1-25 |
| y | sensor | 1-25 | r_x | sensor | 1-25 |
| r_x | sensor | 1-5 | r_z | sensor | 1-25 |
| z | sensor | 1-25 | z | sensor | 1-25 |

Due to the inability to attach the sensor or perform the excitation in all the directions for each node, a few of them were omitted (Table 1). Moreover, it would be possible to directly omit the weakly correlated DoFs as they are subjected to high noise level. However, all the DoFs associated with the connection nodes were measured, as they are of great importance for a reliable substructuring process. The omitted DoFs are later expanded and smoothed with the SEREP process. Each measurement was averaged 5 times before applying the LSFD MPE method. The frequency range for the translational responses was 6.4 kHz and the rotational was 4 kHz. The rotational sensor is calibrated up to 2 kHz, and so the modal parameters associated with the rotations will be used in this frequency range during the smoothing process.

Subsystem A is more flexible in the x and subsystem B in the y direction. Consequently, a high modal density on the first subsystem is always combined with a low modal density on the second subsystem. Examples of force-excited FRFs are shown in Fig. 3, where the indexes refer to the node number and the direction (Fig. 2) for different input/output combinations on subsystem A and B.

3.3. Numerical model

Subsystems A and B are defined by four node-shell finite elements (Fig. 4). Each node has 6 DoFs. The slightly larger subsystem A has approximately 11k and subsystem B 10k DoFs. They are rigidly coupled in five nodes. The beam elements were used for the connection, where the length was conditioned by an offset that takes into account the plate thickness. By adjusting the stiffness of beam element, it directly affects the joint stiffness. The stiffness of the beam elements was deduced experimentally. Then eigen-frequencies of the system were measured and
the stiffness of beam elements was deduced in the way that the quadratic difference between the numerically and experimentally obtained natural frequencies was minimized.

The modal analysis is obtained on a free-free supported system. The modal damping for all modal shapes was set to 0.3 % of critical. In order to compare the numerical model with an experiment, only responses from the coincident nodes were considered in the smoothing process.

For a proper reconstruction of the FRFs it is also necessary to obtain the rigid-body modes and the upper residuals. Optionally, they can be obtained from the experimental data; however, due to the measurement contamination and the omitted DoFs this is not possible. Therefore, smoothed flexible modes will be combined with numerical residuals and in this form used in the FRF synthesis equation:

$$h_{ij}^{\text{syn}}(\omega) = -\frac{R_{ij}^{LR}}{\omega^2} + \sum_{r=m_1}^{m_2} \frac{\Phi_{ir} \Phi_{jr}}{(\omega_r^2 - \omega^2) - j2\xi_r\omega_r\omega} + R_{ij}^{UR},$$

where the lower and upper residuals (LR and UR) are defined by:

$$-\frac{R_{ij}^{LR}}{\omega^2} = \sum_{r=1}^{m_1-1} \frac{\Phi_{ir} \Phi_{jr}}{(\omega_r^2 - \omega^2) - j2\xi_r\omega_r\omega},$$

and

$$h_{ij}^{\text{syn}}(\omega) = -\frac{R_{ij}^{LR}}{\omega^2} + \sum_{r=m_1}^{m_2} \frac{\Phi_{ir} \Phi_{jr}}{(\omega_r^2 - \omega^2) - j2\xi_r\omega_r\omega} + R_{ij}^{UR},$$

Figure 3. Experimental FRFs; a) FRF$^A_{1x1x}$, b) FRF$^B_{2y8rz}$.

Figure 4. Numerical model with line contact.
\[
R_{ij}^{UR} = \sum_{r=m_2+1}^{N} \phi_{ir} \phi_{jr} (\omega_r^2 - \omega_j^2) - j2\xi_r\omega_r\omega_j. \tag{12}
\]

3.4. Smoothing and expanding process based on the VIKING method

The modal parameters from EMA are typically contaminated by random and systematic errors. Additionally, all the directions in each node were not measured due to the inability to attach the sensor or perform an excitation (Table 1). Consequently, incomplete and contaminated data leads to the erroneous coupling results. In order to improve the result, the modal shapes must be smoothed and expanded. The test modes must first be reduced to the active DoFs that are considered as accurate measurements. Reduced model is then expanded back to a full set of DoFs with the SEREP method. In this way the complete response model is based on the smoothed experimental data and can be used in the FRF synthesis and later in the coupling process.

The raw experimental data entering the smoothing process must initially be compared to the data from the reference numerical model. Even though it has been shown in [1] that a slightly perturbed numerical model can also still smooth the experimental data, a good correlation is desired. This can be verified by comparing the natural frequencies and the Modal Assurance Criterion (MAC) [16] for the combined modal shapes from the translational and the modal shape slopes from the rotational responses. Here, the modal shape and the modal shape slopes will both be addressed as modal shapes.

Table 2 compares the results for the numerical reference and the experimental data for subsystems A and B. In both cases the experimentally obtained natural frequencies are in good correlation with the numerical model. The experimental modal shapes within the 4 kHz frequency range were obtained in the same order as the numerical ones. However, despite the good correlation of the natural frequencies, several experimental DoFs are contaminated with too much error, which leads to the relatively low MAC values (see Fig. 3b). Therefore, it is necessary to apply the smoothing process to minimize this error.

The main idea of the VIKING method is to address the contamination from outliers in the data. A set of criteria must be established in order to retain only the accurately measured DoFs and delete the others. This is one of the crucial steps to ensure the quality of the coupling process. The most difficult part is to select the trigger value that determines whether a particular DoF is accurate enough or should be omitted. Nićorski [1] proposed a procedure that seems to work well, whenever solely translational DoFs are analyzed. However, in this paper the rotational responses are also included in order to better define the local joint’s flexibility as
well as the positions of anti-resonances. The ability to deduce the translational and rotational DoFs in each measurement node enables the establishment of a flexible connection using the line contact formulation. Because they are obtained directly, fewer measurements points are required in order to rigidly connect two substructures. For each rotational DoF at least three spatially offset translational responses are needed [6], which requires the appropriately sized contact area. Unfortunately, such a condition cannot always be satisfied in practice, especially for thin plates. In these cases additional flanges are mounted onto the structure to enable the measurement of translations. This kind of structural modification also alters the basic dynamic properties due to the mass and a joint stiffness modification. With all six experimentally obtained DoFs at each connection point, additional flanges are not required and almost any size of contact area can be considered in the coupling process.

In general, the modal shapes here include all six directions. Firstly, they were separately compared with the numerical modal shapes to evaluate which direction is the most responsive. The translational responses were compared to the rotational responses more accurately for both the in- and out-of-plane directions. Referring to the MAC values in Table 2 we can conclude that contamination has a strong influence and must be minimized as much as possible. In order to delete the inaccurate DoFs for each modal shape individually, a criterion must be defined for all six directions. The DoFs are selected based on their contribution to the unity value on the diagonal of an orthogonality matrix (Pseudo-Orthogonality Criterion (POC)). The POC criterion is defined as:

$$\text{POC}_{ij} = \sum_k \sum_l e_{ki} m_{kl} u_{lj}, \quad (13)$$

where $e_i$ is an $i$-th experimental and $u_j$ a $j$-th numerical modal vector and $m$ numerical mass matrix. Values closer to zero mean a weak, and closer to one, a strong correlation. However, POC values greater than one can exist because of the effects of the mass scaling. Furthermore, the contribution of an individual DoF is calculated using the Coordinate Orthogonality Check (CORTHOG), which can be calculated as:

$$\text{CORTHOG}_{ijkl} = \frac{e_{ki} m_{kl} u_{lj} - u_{ki} m_{kl} u_{lj}}{\sum_k \sum_l (e_{ki} m_{kl} u_{lj} - u_{ki} m_{kl} u_{lj})}. \quad (14)$$

The trigger value that separates the active from the deleted DoFs is set intuitively. The proposed procedure is applied to both subsystems. Reduced or retained experimental DoFs are considered as accurate measurements that will be used within the SEREP expansion process. In Table 3 the results of the MAC and POC values for both subsystems after the elimination of poorly correlated DoFs are presented.

| Mode | Freq. exp. A [Hz] | MAC A [%] | POC A | Freq. exp. B [Hz] | MAC B [%] | POC B |
|------|------------------|-----------|-------|------------------|-----------|-------|
| 1    | 964.83           | 99.9      | 1.02  | 1054.30          | 99.7      | 1.00  |
| 2    | 1383.49          | 99.6      | 1.01  | 1567.69          | 99.7      | 1.02  |
| 3    | 1803.71          | 99.7      | 0.99  | 1894.36          | 99.9      | 0.99  |

Table 3. Correlation between the numerical and reduced experimental modal shapes.

Table 3 shows the correlation between the reduced numerical and experimental modal shapes. The DoFs with the smallest contribution to the orthogonality were eliminated. Approximately 88 % of the translational and 32 % of the rotational DoFs were retained on subsystem A and 84
10% of the translational and 36% of the rotational on subsystem B. Even though the translational DoFs are much more correlated with the numerical modes, compared to the rotational, they could still not be used directly from an experiment. The measurement points with a small response amplitude for a particular modal shape contain too much error and therefore could not be used for further processing. The latter issue is even more evident in the case of the rotational responses, where the application of the proposed set of reduction criteria is crucial due to the noisy measurements. The trigger value that eliminates poorly correlated values was iteratively accommodated until a good correlation was observed. The final MAC values that compare the shapes of the modal shapes and the POC values that compare the overall scale of the modal shapes are both close to one. At this stage the data are now prepared for the smoothing and the expansion process.

In order to smooth the reduced modal shapes the SEREP expansion matrix $T_u$ must be generated based on a set of numerical modal vectors. Derivation of $T_u$ begins with relationship of physical and modal spaces as follows:

$$X_n = \left[ \begin{array}{c} X_a \\ X_d \end{array} \right] = \left[ \begin{array}{c} U_a \\ U_d \end{array} \right] p,$$

where $X_n$ is a full set displacement vector, $U_a$ and $U_d$ are the reduced set modal shape matrix and the $p$ modal displacement vector. With a generalized inverse this is transformed into:

$$p = (U_a^T U_a)^{-1} U_a^T X_a = U_a^g X_a,$$

where $U_a^g$ stands for the generalized inverse. The relation between the full $n$ and only the active $a$ DoFs can be written as:

$$X_n = U_n U_a^g X_a = T_u X_a,$$

The transformation matrix is accommodated to include all the relevant DoFs. Expansion process smooths the reduced experimental modal shapes and constructs the omitted DoFs based on a numerical model. As was stated above, approximately 33% of all the measured rotational DoFs were used in the expansion process, which is relatively small compared to the translations. However, as it was possible to include 25 numerical modal shapes into the smoothing and the expansion process, the number of retained rotational DoFs was not as critical as their accuracy. Therefore, we intentionally select only the rotations with the largest amplitudes and, consequently, the smaller influence of noise.

Even though the numerically smoothed, experimental, flexible modes from a limited frequency range are included in the FRF synthesis, it does not represent a major issue. The ability to additionally attach rigid-body modes and upper residuals from an equivalent numerical model makes it possible to construct the so-called hybrid set of the modal parameters. It is desirable that the experimentally obtained modes cover the frequency range of interest as they contain real dynamic properties and the smoothing process can be efficiently performed only within this frequency band. In our case the technical specifications of the measurement equipment enable us to perform FRF synthesis using Eq. (10) up to 2 kHz, which is even above the classic EMA frequency range. In Fig. 5 the synthesized responses of both subsystems are compared to the numerical and experimental FRFs. It is clear that the influence of both the numerical and experimental data are synergized into the hybrid solution. The experimentally obtained driving-point translational FRFs (Fig. 5) are already very accurate, and thus only a small correction is needed. The rotational DoFs contain a higher level of noise (Fig. 5), which presents a problem in the coupling process. The noise is present in the signal due to the sensor low sensitivity. However, after applying the smoothing and the expansion procedure, the reconstructed FRFs
are free of noise and natural frequencies, and their amplitudes are well correlated with the experimental results.

The reconstructed response models of both subsystems can be used along with the compatibility conditions in the LM FBS method. However, it is very difficult to properly define the connection characteristics due to the local joint flexibility. Whenever only translational DoFs are considered, additional flanges in order to increase the contact surface must be used to ensure the coupling of the rotational DoFs. Nevertheless, in this paper a relatively rigid line contact was defined based on all six experimentally obtained and numerically smoothed DoFs in five nodes. The rigid connection between particular DoFs can be expressed by the sign Boolean matrix, as it is defined in Eq. (6). The final results are shown in Fig. 6 where the numerical, experimental and coupled responses of the system AB are compared.

The final coupling results in Fig. 6 are in good agreement with the experimental results. There are some misalignments and small shifts of the natural frequencies, but in general the difference is acceptable. The FRFs are stripped of noise and the resonance peaks are closer to the experimental ones. The smoothing and the expansion were performed based on the equivalent numerical model, which was especially important for more noisy rotational DoFs. Additionally, the attached numerical rigid body modes and the upper residuals sufficiently define the static response and minimize the modal truncation. Nevertheless, the proposed procedure of smoothing all six DoFs clearly shows the potential in coupling more complex system using the line contact.

The local joint’s flexibility is of primary concern, whenever a line or surface contact is analyzed.
connection area makes it possible to account for all the joint’s dynamics properties.

4. Conclusions
In this paper the procedure for inclusion of directly measured rotational DoFs is presented in order to couple two structures with collinearly arranged contact points. Our approach to measure rotations using a direct quartz based rotational piezoelectric accelerometer enables the extraction of accurate rotational FRFs. As in each connection point all six DOFs were obtained using direct measurement it was possible to ensure rigid connection even if the connection points are arranged collinearly. The method has a great potential especially whenever plate-like structures are considered where it is impossible to perform multi-point planar translational measurements without introduction of additional flanges or by using other more advanced methods like transmission simulator or virtual point transformation.

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