QUASI-EQUILIBRIUM SEQUENCES OF BINARY STRANGE QUARK STARS IN GENERAL RELATIVITY

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Abstract.

Inspiraling compact binaries are expected to be the strongest sources of gravitational waves for VIRGO, LIGO and other laser interferometers. We present the first computations of quasi-equilibrium sequences of compact binaries containing two strange quark stars (which are currently considered as a possible alternative to neutron stars). We study a precoalescing stage in the conformal flatness approximation of general relativity using a multidomain spectral method. A hydrodynamical treatment is performed under the assumption that the flow is irrotational.

1 Introduction

One of the most important prediction of Einstein theory of relativity is gravitational radiation. Since the very precise measurement of the orbital decay in the binary pulsar B1913+16 system by Hulse and Taylor, the existence of gravitational waves (GW) has been indirectly proved and general relativity has passed another quite constraining test. Due to the emission of GW, binary neutron stars (NS) decrease their orbital separation and finally merge. The evolution of a binary system can be separated into three phases: point-like inspiral where orbital separation is much larger than the NS radius, hydrodynamical inspiral where orbital separation is just a few times larger than the radius of the NS so that hydrodynamics play an important role, and merger in which the two stars coalesce dynamically.

The GW signal of the terminal phases (the hydrodynamical phase or the merger phase) of inspiraling binary can bring the information about the stellar structure. In particular it may be possible to impose constraints on the equation of state of NS. It is still an open question whether the core of NS consists mainly of superfluid neutrons or an exotic matter like kaon condensations, pion condensations or

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Strange quark matter. As suggested by Bodmer (1971) the absolute true ground state of nuclear matter may be a state of deconfined up, down and strange quarks (since energy per baryon for strange matter is lower than the energy per baryon for Fe$^{56}$). If it is true then objects made of such matter so called strange stars (SS) could exists (Witten 1984). SS are currently considered as a possible alternative to NS as compact objects (see e.g. Madsen 1999 for a review and Gondek-Rosińska et al. 2003).

Up to now, majority of the relativistic calculations of the terminal phase of inspiral have been done for binary systems containing NS described by a simplified equation of state of dense matter so called polytropic EOS (Taniuchi & Gourgoulhon 2003). In the paper we present the results of our studies on the hydrodynamical phase of inspiraling binary systems containing equal mass strange stars (according to the recent population synthesis calculations (Bulik, Gondek-Rosińska and Belczyński, 2004) a significant fraction of the observed binary NS in GW will contain stars with masses $\sim 1.4 \, M_{\odot}$). We compare the evolution of SS-SS systems with NS-NS systems in order to find any characteristic features in the GW waveform that will help to distinguish between SS and NS.

2 Strange quark stars and stellar models

As already mentioned, SS are composed of deconfined up, down and strange quarks. Typically, they are modeled with an equation of state based on the MIT-bag model in which quark confinement is described by an energy term proportional to the volume (Fahri et al., 1984). SS are self-bound objects, having high density ($> 10^{14}$ g cm$^{-3}$) at the surface. There are three physical quantities entering the MIT-bag model: the mass of the strange quarks, $m_s$, the bag constant, $B$, and the strength of the QCD coupling constant $\alpha$. In the numerical calculations reported in the present paper we consider three different MIT-bag models (corresponding to three different sets of the model parameters):

- **SQS0** - the standard MIT bag model: $m_s c^2 = 200$ MeV, $\alpha = 0.2$, $B = 56$ MeV/fm$^3$;
- **SQS1** - the simplified MIT bag model with $m_s = 0$, $\alpha = 0$; $B = 60$ MeV/fm$^3$;
- **SQS2** - the "extreme" MIT bag model (relatively low strange quark mass and $B$ but high $\alpha$) : $m_s c^2 = 100$ MeV, $\alpha = 0.6$, $B = 40$ MeV/fm$^3$.

3 Basic assumptions

In the hydrodynamical phase, since the timescale of orbital shrinking due to the emission of GW is longer than the orbital period, one may consider a binary NS system to be in quasiequilibrium state. For given EOS, we construct so called evolutionary sequence by calculating a sequence of quasiequilibrium configurations with constant baryon mass for decreasing orbital separation. The second assumption is to consider a conformally flat metric, which corresponds to the Isenberg-Wilson-Mathews approximation of general relativity. In this approximation, the
spacetime metric takes the form:

\[ ds^2 = -(N^2 - B_i B^i) dt^2 - 2 B_i dt dx^i + A^2 f_{ij} dx^i dx^j, \]

where \( N \) is the lapse, \( A \) the conformal factor, \( B^i \) the shift vector and \( f_{ij} \) the flat spatial metric. Another assumption concerns the fluid motion inside each star, here we considered irrotational binaries.

### 4 Results and conclusions

![Fig. 1. Orbital binding energy of an equal mass binary system (\( M_1 = M_2 = 1.35 \, M_\odot \)) versus frequency of GW along two irrotational equilibrium sequences. Solid and long-dashed lines correspond to strange quark stars described by MIT bag model and NS described by polytropic EOS respectively. A diamond indicates the marginally stable orbit. A dotted line corresponds to 3rd post-Newtonian calculations for point masses.](image)

In order to calculate the last orbits of inspiral phase of binary NS and SS we use highly accurate numerical code which solves the five elliptic equations for the gravitational field (for \( A, N \) and \( B^i \)), supplemented by an elliptic equation for the velocity potential in the case of irrotational flows (see Limousin, Gondek-Rosinska & Gourgoulhon 2004 for boundary conditions in the case of SS). In Fig. 1 we show the evolution of equal mass binary NS (a long-dashed line) and SS described by SQS1 model (a solid line) having total gravitational mass \( 2.7 M_\odot \) at infinity. The binding energy is defined as the difference between \( M_{ADM} \) (see Taniguchi & Gourgoulhon 2003) and the total mass of the system at infinity. We see a good agreement with 3PN calculations for big distances i.e. small frequencies, where the internal structure of the stars is not the predominant effect.

In order to compare results for SS and NS we take a polytrope giving the same gravitational mass and the same compaction parameter \( GM/Rc^2 \) at infinity as obtained for static SS. The minimum of energy (shown as a diamond) for the evolutionary sequence of SS corresponds to the appearance of a dynamical instability for binaries (the innermost stable circular orbit (ISCO)). The frequency of
the ISCO is a potentially observable parameter by the GW detectors. We don’t see the ISCO for NS. The sequence of NS terminates by the mass-shedding limit (corresponding to exchange of matter between two stars). Different evolutions of NS and SS stem from the fact that SS are principally bound by an additional force, strong interaction between quarks (for the same distances there are less deformed than NS).

![Diagram](image)

**Fig. 2.** Orbital binding energy versus frequency of GW along evolutionary sequences of irrotational strange quark star binaries described by three different MIT bag models.

In Fig 2. we present the evolution of binary SS, with total mass 2.7 $M_\odot$, described by three different sets of EOS parameters of the MIT bag model. We see that the frequency of GW at the ISCO strongly depends on the compaction parameter - the higher it is, the higher the frequency at the ISCO. Detection of GW may help to impose constraints on the EOS of NS and SS and to find the ground state of matter at high densities.

Partially supported by the KBN grants 5P03D.017.21 and PBZ-KBN-054/P03/2001; by the “Bourses de recherche 2004 de la Ville de Paris” and by the Associated European Laboratory Astro-PF (Astrophysics Poland-France).

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