Effects of the $\Lambda(1405)$ on the Structure of Multi-Antikaonic Nuclei

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Abstract

The effects of the $\Lambda(1405)$ ($\Lambda^*$) on the structure of the multi-antikaonic nucleus (MKN), in which several $K^-$ mesons are embedded to form deeply bound states, are considered based on chiral symmetry combined with a relativistic mean-field theory. It is shown that additional attraction resulting from the $\Lambda^*$ pole has a sizable contribution to not only the density profiles for the nucleons and $K^-$ mesons but also the ground state energy of the $K^-$ mesons and binding energy of the MKN as the number of the embedded $K^-$ mesons increases.

Key words: multi-antikaonic nuclei, chiral symmetry, kaon condensation, subthreshold resonance $\Lambda(1405)$

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1. Introduction

Exploring multi-strangeness systems is an important aspect of understanding hadron dynamics in dense matter. Kaon condensation in neutron stars may exist as a strangeness-nonconserving system, from normal matter through weak processes, $N + n \to N + p + K^-$, $N + e^- \to N + K^- + \nu_e$ ($N = p, n$)\(^\text{[1]}\). Recently multi-antikaonic nuclei (abbreviated as MKN), where several antikaons ($K^-$ mesons) are bound in the ground state of the nucleus, have been investigated\(^\text{[2, 3]}\), stimulated by the proposal to explore deeply bound kaonic nuclear states and subsequent theoretical and experimental studies\(^\text{[4]}\). The MKN is a strangeness-conserving system and should be formed by embedding a $K^-$ meson in the nucleus through strong processes. Both the kaon-condensed state in neutron-star matter and the MKN formed in experiments are cold, dense objects originating from the common $\bar{K} - N$ and $\bar{K} - \bar{K}$ interactions in dense matter, so that they may be closely related with each other.

We have considered properties of the MKN within the framework of a relativistic mean-field theory (RMF) coupled with the nonlinear effective chiral Lagrangian\(^\text{[5]}\). It has been shown that the lowest $K^-$ energy, $\omega_{K^-}$, increases as the number of embedded $K^-$ mesons, $|S|$, becomes large and that it enters into the subthreshold resonance region of the $\Lambda(1405)$ ($\Lambda^*$), where $\omega_{K^-} = m_{\Lambda^*} - m_N = 467$ MeV. This is because the contribution to the energy from the repulsive $\bar{K} - \bar{K}$ interaction becomes sizable with the increase in $|S|$ as compared with the attractive $\bar{K} - N$ interaction. In this paper, we take into account the $\Lambda^*$-pole contribution as well as range terms and study these effects on the structure of the MKN.
2. Formulation

A spherical symmetry is assumed for the MKN, and the mass number $A$, the number of protons $Z$, and the number $|S|$ of embedded $K^-$ mesons with the lowest energy $ω_{K^-}$ are kept fixed. We start with the effective chiral Lagrangian, which incorporates $s$-wave interactions between the (nonlinear) $K$ mesons and nucleons of the scalar type simulated by the $KN$ sigma term, $Σ_{KN}$, and of the vector type (Tomozawa-Weinberg term). The nonlinear $K^-$ field $Σ$ is given as $Σ = \exp[2i(K^+T_{4+5} + K^-T_{4-5})/f]$, where $T_{4+5,4-5}$ is the SU(3) generator and $f$ (= 93 MeV) the meson decay constant. The $K^-$ field is represented as $K^{-}(r) = fθ(r)/\sqrt{2}$ with $θ(r)$ being the chiral angle in the condensate approximation[2]. These $K^{-}N$ interactions are replaced by those generated by the $σ$ and $ω$, $ρ$ mesons-exchanges, respectively, within the RMP[2].

The thermodynamic potential $Ω$ for the MKN is derived under a local density approximation for the nucleons[2]. The correction to the energy density, $Δε(r)$, from the $Λ^+$ is introduced through the second-order perturbation with respect to the axial current of hadrons, $A(r)$, the Coulomb potential $Ω_{Coul}$. The $Ω_{Coul}$ is given by

$$Ω_{Coul} = \frac{4}{3}πe^2\rho(r) - \frac{2}{3}πe^2\rho(r),$$

where the smooth parts $= d{ρ}_p, d{ρ}_n$ are the range terms with $ρ_p(r)$ ($ρ_n(r)$) being the scalar density of the proton (neutron) and the pole contribution comes from the $Λ^+$ with $γ_{Λ^+}$ being the width. These terms are absorbed into the effective nucleon masses. We call these contributions to the energy second-order effects (SOE)[1]. In Eq. (1), $\vec{ω}_{K} - r$ [$ω_{K^-} - V_{Coul}(r)$] is the lowest energy of the $K^-$ shifted in the presence of the Coulomb potential. The parameters, $d_p$, $d_n$, $g_{Λ^+}$, and $γ_{Λ^+}$ are determined so as to reproduce the on-shell $s$-wave $K^-N$ scattering lengths[3].

The classical $K^-$ field equation is given from $δΩ/δθ = 0$ as

$$\nabla^2 θ(r) = \sin θ(r) \left( m_{K}^2(r) - 2ω_{K} - rX_0(r) - \vec{ω}_{K} - r \cos θ(r) \right) - \vec{ω}_{K} - r \cos θ(r) \left( ρ_p(r) + \frac{g_{Λ^+}}{2f} \left( \frac{m_{Λ^+} - m_N - ω_{K^-}}{m_{Λ^+} - m_N - ω_{K^-}} + γ_{Λ^+}^2 \right) \right) d{ρ}_p(r).$$

where $m_{K}^2(r) = m_{K}^2 - 2g_{σKx}mk(r)σ(r)$ is the square of the effective mass of the $K^-$, and $X_0(r) = g_{σx}\left( ω_{x}(r) + g_{ρx}R_{x}(r) \right)$ represents the $K^-N$ vector interaction. In these quantities, $g_{σx} (i = σ, ω, ρ)$ are the coupling constants, while $σ(r), ω(r)$, and $R_{x}(r)$ are the mean fields of the $σ$ meson and the time components of the $ω$ and $ρ$ mesons, respectively. Together with Eq. (4) one obtains the coupled equations of motion (EOM) for the other mesons $σ, ω, ρ$, and the Poisson equation for the Coulomb potential $V_{Coul}(r)$:

$$\begin{align*}
-\nabla^2 σ(r) + m_σ^2σ(r) &= -\frac{dU}{dσ}(r) + g_{σN}(ρ_p^*(r) + ρ_n(r)) + 2g_{σKx}mf^2(1 - \cos θ(r)), \quad (3a) \\
-\nabla^2 ω_0(r) + m_ω^2ω_0(r) &= g_{ωN}(ρ_p^*(r) + ρ_n(r)) - 2g_{ωK}(ω_{K} - r) f^2(1 - \cos θ(r)), \quad (3b) \\
-\nabla^2 R_0(r) + m_ρ^2R_0(r) &= g_{ρN}(ρ_p^*(r) - ρ_n(r)) - 2g_{ρK}(ω_{K} - r) f^2(1 - \cos θ(r)), \quad (3c) \\
\n\n\n\n\n\n\n\n}\begin{align*}
-\nabla^2 V_{Coul}(r) &= 4πe^2(ρ_p(r) - ρ_N(r)). \quad (3d)
\end{align*}$$
where $\rho_i(r) (i = p, n, K^-)$ are the number densities and $g_{iN} (i = \sigma, \omega, \rho)$ the coupling constants. The coupled equations (2) and (3a) − (3d) are solved self-consistently, and the density distributions $\rho_i(r)$ and other quantities are obtained as functions of the radial distance $r$.

3. Numerical Results

We take the $^{15}$O ($A=15$, $Z=8$) as a reference nucleus. The $K^-$ optical potential depth $U_K$ is chosen to be $U_K = -80$ MeV.

3.1. Density profiles

The density distributions of the protons, neutrons, and the distribution of the strangeness density $[\rho_K(r) - \rho(r)]$ are shown for $|S|=4$ and $8$ in Fig. 1. The solid lines are for the previous result without the SOE, and the dashed-dotted lines for the present result with the SOE. Due to the

\begin{align*}
\text{SOE, the } K^- \text{ mesons and the protons are attracted more to each other than the case without the SOE, since in the former the } K^- \text{ lies below the resonance region of the } \Lambda^* \text{ and feels an additional attraction through coupling with the } \Lambda^* \text{ pole. As a result, the central densities of the protons and } K^- \text{ mesons become larger. On the other hand, neutrons are pushed outward from the center of the MKN due to the weakly repulsive effect from the range term (} \propto d_n \rho_n s_n, d_n < 0 \text{ in Eq. (2)} \text{). These features become remarkable for a large value of } |S| (\text{Compare the cases of } |S| = 4 \text{ and } 8). \text{ The central baryon density } \rho_B(0) (=\rho_p(r = 0) + \rho_n(r = 0)) \text{ becomes } \rho_B^{(0)} \sim 3.5 \rho_0 \text{ with } \rho_0 = 0.153 \text{ fm}^{-3} \text{ for } |S| \sim 8. \text{ One can see a “neutron skin” structure with a thickness (1–2) fm for } |S| \sim 8. \text{ In addition, for a larger } |S|, \text{ the proton and } K^- \text{ density distributions tend to be more uniform near the center.}
\end{align*}

3.2. $|S|$-dependence of the lowest $K^-$ energy and binding energy

In Fig. 2 the lowest energy of the $K^-$, $\omega_K^-$, is shown as a function of $|S|$. The energy difference per unit of strangeness, $[E(A,Z,|S|) - E(A,Z,0)]/|S| (=m_K - B(A,Z,|S|)/|S|)$ with $B(A,Z,|S|)$ being the binding energy of the MKN, is shown as a function of $|S|$ in Fig. 3. In these figures the solid lines are for the result without the SOE, and the dashed-dotted lines for the result with the SOE. From Fig. 2 the $\omega_K^-$ is shown to be lowered by $\sim 40$ MeV from that without the SOE due to the additional attraction brought about from the $\Lambda^*$ pole. Nevertheless, $\omega_K^-$ increases with
an increase in $|S|$ since the repulsive $\bar{K} - \bar{K}$ interaction overwhelms the attractive $\bar{K} - N$ interactions at large $|S|$. For $|S| \geq 12$ (in the case of $U_K = -80$ MeV), $K^-$ mesons become unbound, where $\omega_K \gtrsim m_{\Lambda^*} - m_N$ above the $\Lambda^*$-resonance region.

From Fig. 3, the $B/|S|$ steadily increases with $|S|$ in the case in which the SOE is included, while it shows little dependence upon $|S|$ without the SOE. One finds that $m_K - B/|S| > m_{\Lambda(1116)} - m_N$, where $m_{\Lambda(1116)}$ is the free mass of the lightest hyperon $\Lambda(1116)$. Hence the MKN decays through strong processes such as $K^- NN \rightarrow \Lambda(1116) N$, so that it is not stable as a self-bound object. This result qualitatively agrees with that in Gazda et al.\cite{3}.

4. Concluding remarks

With regard to creating self-bound objects for the MKN, hyperon-mixing effects may be responsible for formation of more strongly bound states. It has been shown in a liquid-drop picture that coexistence of antikaons and hyperons leads to highly dense self-bound objects, which may decay only through weak processes\cite{7}. There is a controversy about the possible existence of such objects depending on the adopted models and approximations\cite{6}. A realistic framework including antikaons and hyperons as well as nucleons beyond the local density approximation for baryons is necessary for further investigation.

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