A wall-shear stress predictive model

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Abstract. Following the approach of Marusic et al. (2010b), here we develop a predictive model for the fluctuating wall-shear stress, where the only required input is large-scale information of the streamwise velocity at a location in the outer, logarithmic region of the flow. The model consists of two components, incorporating a superposition and modulation effect of outer region motions that interact with the flow field in the viscous sublayer. The model is seen to capture Reynolds number trends reliably.

1. Introduction

Recent reviews of wall-bounded turbulent flows (Smits et al 2011a, Klewicki 2010, Marusic et al 2010b) have highlighted discussion on the interaction between the inner and outer regions. Classically, the near-wall inner region, which scales with wall shear velocity $U_\tau$ and the viscous length $\nu/U_\tau$, is assumed to be independent of the outer region, which scales with $U_\tau$ and the boundary layer thickness $\delta$. However, a number of studies over the years have questioned this viewpoint, from the discussion of “inactive” motions by Townsend (1961) and Bradshaw (1967), to experimental studies by Rao et al (1971), Wark & Nagib (1991), Ganapathisubramani et al (2005), Hutchins & Marusic (2007a), amongst others; and numerical studies, for example, Abe et al (2004), Hoyas & Jimenez (2006) and Schlatter et al (2009). An important inclusion to this discussion has been the role of the large and very-large scale motions (Kim & Adrian 1999) in the outer region and how they interact with the near-wall region. Many of the above studies were consistent with the viewpoint that some superposition (or “footprint”) of the large-scale motions is experienced right to the wall. Hutchins & Marusic (2007b) went further and proposed that this interaction also involved a modulation of the large scales on the small, and this effect was quantified by Mathis et al (2009). Previous suggestions of modulation effects have also been made by Grinvald & Nikora (1988).

Marusic et al (2010a) extended the observations of a superposition and modulation of the large-scale outer motions in the near-wall region to a predictive model, whereby a statistically representative fluctuating streamwise velocity signal near the wall could be predicted given only a large-scale velocity signature from the logarithmic region of the flow for a zero-pressure-gradient turbulent boundary layer. The model was shown to work very well over a large Reynolds number range for various statistics, including higher-order moments. Their formulation involves a universal signal and universal parameters, which are determined from a once-off calibration experiment at an arbitrarily chosen (but sufficiently high) Reynolds number. Full details of this model are given by Mathis et al (2011).
In this paper we use the same approach as Marusic et al (2010a) and propose a predictive model for the fluctuating wall shear stress given only a measurement of the large-scale streamwise velocity in the logarithmic region. Such a model is desirable in practical problems where one may only have access to a low-frequency velocity measurement away from the wall, yet require information about the wall-shear stress. Near-wall models for large-eddy simulations are one such scenario. Another example of such an application is in environmental flows, where one may require information about the instantaneous bed-shear stress at a sediment-water interface of a stream or river, but only have velocity information at some distance above the interface from an ADV (acoustic Doppler velocimeter) or similar device (see for example, Brand et al 2010). Here, the standard practice to date is to use formulations based on the law of the wall (see Grant & Madsen 1986), but such models are not capable of capturing very energetic events that will be prominent in higher-order statistics. For example, in the DNS of wall-bounded flows, on occasion, the instantaneous maximum wall-shear stress is seen to exceed 5 times the mean value (as evident in the PDF plots of Orlu & Schlatter 2011), and their frequency of occurrence increases with Reynolds number (Klewicki 2010). Such extreme events are the ones that can lead to physical damage or enhanced localised exchange rates, which are important in certain applications.

The model is presented in the remainder of the paper together with details of the experiments used to calibrate the coefficients and test the model.

2. Model formulation

Following Marusic et al (2010a), the predicted fluctuating wall shear stress normalized by wall variables, $\tau_p^+ = \tau_p/(\rho U_\tau^2)$, is modelled by

$$\tau_p^+ = \tau^* \{ 1 + \beta u_{OL}^+(z_O^+, \theta_L) \} + \alpha u_{OL}^+(z_O^+, \theta_L).$$ (1)

Here, $u_{OL}$ is the fluctuating large-scale signal from the log-region, $\tau^*$ is referred to as the statistically “universal” wall shear signal (normalized in wall units), and $\alpha$ and $\beta$ are, respectively, the superimposition and modulation coefficients. It is noted that the model consists of two parts: the first part models the amplitude modulation at the wall by the large-scale motion, and the second part ($\alpha u_{OL}^+$) models the superimposition of the large-scale motion felt at the wall.

The large-scale signal, $u_{OL}$, is the only user input required for equation (1), and is obtained from the $u$-signal in the log region. Here, following Mathis et al. (2009), we choose the point in the logarithmic region that corresponds to the outer-spectral-peak location, $z_O^{+} = 3.9Re^{1/2}$. Two steps are involved to obtain $u_{OL}$. First, the $u$-signal is low-pass filtered to retain only large scales (here streamwise wavelengths of $\lambda_U^+ > 7000$ are retained), and second, since we are equating a log-region signal (from $z_O^+$) to the wall, the measured $u$-signal is shifted to account for the structure inclination angle, $\theta_{LS}$, between these two wall-normal positions, which previous studies have shown to be effectively invariant with Reynolds number.

3. Experimental details

The experiments were carried-out in the high Reynolds number boundary layer wind-tunnel (HRNBLWT) at the University of Melbourne. This open return blower facility consists of a working test section of $27 \text{ m} \times 2 \text{ m} \times 1 \text{ m}$, in which the pressure-gradient is maintained to zero by bleeding air through adjustable slots from the ceiling. The free-stream turbulence intensity is nominally 0.05%. Further details about the facility are available in Nickels et al. (2005). Measurements were performed using single normal hot-wire anemometry made from platinum Wollaston wire of diameter $d = 2.5 \mu m$, and a ratio $l/d = 200$ (where $l$ is the etched length.
of the wire). The frequency and sampling duration have been carefully chosen to adequately resolve the small-scale energy content as well as the large-scale motions. Two-point simultaneous measurements were conducted: a wall probe measuring the fluctuating wall shear-stress $\tau$ is mounted in the viscous sublayer at 45 $\mu$m above the wall, and an outer probe measuring the fluctuating streamwise velocity component $u$ is positioned in the logarithmic region. A schematic of the experimental setup is shown in figure 1. It should be noted that while the outer-probe is calibrated in the free-stream flow using a classical technique, the wall-probe is calibrated in-situ directly in terms of $\tau$. This has been performed using asymptotic similarity formulations for $Re_\delta^*$ versus $Re_x$ as given by Monkewitz et al. (2007), and $U_\tau$ versus $Re_\delta^*$ as given by Nagib et al. (2009):

$$U_\tau = U_\infty \left( \frac{1}{\kappa} \ln (Re_\delta^*) + C_* \right)^{-1},$$

(2)

where $\kappa = 0.384$ and $C_* = 3.3$.

The experiments were carried out for four Reynolds numbers and a summary of the experimental conditions is given in table 1.

![Figure 1. Experimental setup.](image)

| $Re_\tau$ | $x$ | $z^+$ | $U_\infty$ | $\delta$ | $U_\tau$ | $\nu/U_\tau$ | $l^+$ | $l/d$ | $\Delta T^+$ | $TU_\infty/\delta$ |
|---------|-----|-------|-----------|--------|---------|-------------|------|-----|----------------|-----------------|
| 4480    | 12.8| 1.03  | 10.17     | 0.195  | 0.350   | 43.5        | 12   | 200 | 0.16           | 46 900          |
| 5990    | 12.8| 1.48  | 14.90     | 0.182  | 0.501   | 30.5        | 16   | 200 | 0.33           | 48 900          |
| 7560    | 12.8| 1.93  | 19.93     | 0.176  | 0.657   | 23.3        | 21   | 200 | 0.56           | 67 800          |
| 11100   | 12.8| 2.77  | 29.67     | 0.180  | 0.947   | 16.2        | 31   | 200 | 0.58           | 98 600          |

Table 1. Experimental condition for simultaneous single-normal hot-wire experiments in the Melbourne wind tunnel.

As seen in table 1, the size of the sensing length in viscous wall units increases with increasing Reynolds number and this leads to attenuation of the small-scale signals. Even at the lowest Reynolds number, with $l^+ = 12$, the spatial resolution may not be enough to capture all of the signal (Smits et al 2011b suggest $l^+ < 5$). However, the measured $\tau$ signal has all the features that have previously been reported in the literature, e.g. $\tau_{rms}^+ \approx 0.4$, skewness of 1.0 and Kurtosis of 4.78 as shown in figure 2(b) (see Alfredsson et al 1988). Figure 2(a) shows
the cross-correlation profile between $\tau$ at the wall and $u$ measured in the logarithmic region (red solid line). This correlation compares well with the trends in previously reported results (Marusic & Heuer 2007).

![Graph](image)

**Figure 2.** (a) Cross correlation coefficient between $\tau^+$ and $u^+$. Dot-dashed blue lines are from ASL measurements at $Re_\tau = O(10^6)$ (Marusic & Heuer, 2007) for $z/\delta = 0.0024, 0.005, 0.0091, 0.0165,$ and $0.0293$. The solid red line is for present data at $Re_\tau = 4480$ for $z/\delta = 0.058$. Dashed black lines are laboratory wind tunnel results (Marusic *et al.*, 2001; Kunkel & Marusic, 2003) at $Re_\tau = 1350$ for $z/\delta = 0.073, 0.091, 0.115, 0.145,$ and $0.183$. (b) Probability density function of the total wall shear stress $\tau^+$. The vertical dashed line marks the mean value $\tau^+

4. Model calibration

The parameters in equation (1) were obtained from the calibration experiment at $Re_\tau = 4480$, from which $\tau^+ (= \tau^+_p$ in the equation) and $u^+_OL$ are known. Figure 3 shows a sample of the measured $u^+_OL$ signal from the logarithmic region for a turbulent boundary layer at $Re_\tau = 4480$, together with the corresponding simultaneously measured $\tau$-signal. The high degree of correlation of $u^+_OL$ with the low frequency content of $\tau^+$ (denoted by $\tau^+_L$) is shown in figure 3(e).

The superposition coefficient $\alpha$ is found from the maximum of the cross-correlation between the large-scale components at the inner and outer locations, multiplied by the ratio of the r.m.s. values to account for the combination of two signals with fluctuations of different orders of magnitude,

$$\alpha = \max \left( R \left\{ \tau^+_L, u^+_OL \right\} \right) \frac{\langle \tau^+_L \rangle_{r.m.s.}}{\langle u^+_OL \rangle_{r.m.s.}} \tag{3}$$

and is found to be $\alpha = 0.0898$. The mean inclination angle of the large-scale structures $\theta_L$, used in equation (1), corresponds to the streamwise shift at the maximum of the correlation (assuming Taylor’s hypothesis), and is found to be $\theta_L = 14.1^\circ$.

With $\alpha$ and $\theta_L$ now known, the ‘footprint’ effect of the large-scale log-region events can be removed from the calibration wall-shear stress signal, leading to a ‘de-trended’ signal $\tau^+_d$ of the form:

$$\tau^+_d = \tau^+ - \alpha u^+_OL(z^+_O, \theta_L) = \tau^+ \left\{ 1 + \beta u^+_OL(z^+_O, \theta_L) \right\} \tag{4}$$

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Figure 3. Representative fluctuating signals for \( Re = 4480 \); (a) raw fluctuating streamwise velocity component at \( z_O^+ = 3.9 Re^{1/2} \); (b) raw fluctuating wall shear stress and; (c) – (e) large-scale fluctuations.

where \( u_{OL}^+ (z_O^+, \theta_L) \) is the filtered outer signal shifted forward in the streamwise direction for the corresponding value of \( \theta_L \). The ‘de-trended’ signal \( \tau_d^+ \) represents the inner-scaled signal without the superposition or mean shift imposed by the large-scale log-region events. Samples of instantaneous fluctuating signals including the raw signal \( \tau^+ \), the large-scale components, and the de-trended signal \( \tau_d^+ \), are shown in figure 4(a – c). The de-trended signal, as shown in figure 4(c), is seen to have the long wavelength trends effectively removed by the process described in equation 4.

The final step to complete the model is to solve for \( \beta \) and \( \tau^* \) in equation (4). This is similar to that described by Mathis et al 2011, and involves an iterative procedure using the de-trended signal \( \tau_d^+ \) to find \( \beta \) and \( \tau^* \) that gives zero degree of amplitude modulation of the universal signal \( \tau^* \) (using the amplitude modulation tool developed in Mathis et al. (2009)). From this we find \( \beta = 0.0867 \). A sample of \( \tau^* \) is seen in figure 4(d). Overall, little differences are seen between \( \tau_d^+ \) and \( \tau^* \). However, plotting the difference between the square of both signals and comparing this with the outer large-scale component \( u_{OL}^+ \) highlights the ‘de-amplitude modulation’ effect (figure 4e). It can be seen that the difference of the squares follows the behaviour of large-scale fluctuations \( u_{OL}^+ \) fairly well. A positive value of the difference of the square means that energy has been removed from \( \tau_d^+ \) to create \( \tau^* \), and \textit{vice versa} for the negative values. Therefore, the process of ‘de-amplitude modulating’ the de-trended signal means that near the wall the small-scale energy is removed where positive large-scale fluctuations occur, and added where negative large-scale fluctuations occur. The corresponding pre-multiplied energy spectra of the universal signal \( \tau^* \), along with the original spectra of \( \tau^+ \) for \( Re = 4480 \), is shown in figure 5. It is observed that the universal signal \( \tau^* \) is one with minimal large-scale influence, with most of the long wavelength energy content removed, as would occur for a low Reynolds number case. This was expected since \( \tau^+ \) corresponds to a universal wall shear-stress signal that would exist in the absence of any large-scale influence, and therefore following the behaviour of a low Reynolds
number flow. This is consistent with numerous studies reporting that high Reynolds number effects are associated with increasing large-scale activity (Townsend, 1976; Adrian et al., 2000; Metzger & Klewicki, 2001; del Álamo et al., 2004; Marusic et al., 2010a, and others).

Figure 4. Representative fluctuating signals for $Re_\tau = 4480$: (a) raw fluctuating wall shear stress; (b) large-scale signatures $\tau_L^+$ and $u_{OL}^+$; (c) de-trended signal $\tau_d^+$; (d) universal signal $\tau^*$; and; (e) difference between square of de-trended and universal signal along with $u_{OL}^+$.

Figure 5. Pre-multiplied energy spectra of the original wall shear stress $\tau^+$ for $Re_\tau = 4480$, and of the obtained universal wall shear stress signal $\tau^*$. 
5. Results and discussion

The model is now calibrated, with all parameters $\tau^*, \alpha, \beta$ and $\theta_L$ resolved, and so it is now possible to reconstruct a statistically representative wall shear-stress signal $\tau_p^+$ at any Reynolds number using equation (1), where the only required input is the large-scale signal $u_{OL}^+(z_O^+, \theta_L)$ from the outer-spectral peak location $z_O^+ = 3.9Re_T^{1/2}$. To make the prediction the measured outer-peak signal $u_O^+$ is filtered to retain only the large-scale energy content, and shifted forward in the streamwise direction to account for the structural angle $\theta_L$, forming $u_{OL}^+(z_O^+, \theta_L)$.

![Figure 6](image.png)

Figure 6. Predicted pre-multiplied spectra of the wall shear stress for all Reynolds numbers.

Figure 6 shows the predicted pre-multiplied energy spectra $k_x \Phi_{\tau_p^+ \tau_p^+}$ for all sets of measurements. Overall, it can be observed that the main effect of the Reynolds number, the increase of the large-scale energetic content, is captured well by the model over the range of Reynolds numbers considered. The model captures the Reynolds trend for the spectra, suggesting that the predicted $\tau_{\text{rms}}^+$ will also be good. This is confirmed in figure 7 in comparisons with the original measurements and available data for zero-pressure gradient turbulent boundary layer from Österlund (1999), Komminaho & Skote (2002) and Schlatter & Örlü (2010). Also included in this figure are the predictions using the high Reynolds number dataset of Mathis et al. (2009). The overall trend of the predicted wall shear stress r.m.s., $\tau_{\text{rms}}^+$, using the present dataset and the dataset set from Mathis et al. (2009) agree reasonably well with available data and recent findings of Schlatter & Örlü (2010), which reported a slight increase with Reynolds number, e.g. $0.298 + 0.018 \ln(Re_T)$ (the solid line in figure 7). However, although the Reynolds number trend is correctly captured by the model, there is a slight overall underestimation of the energy (the dash-dotted line in figure 7) due the relatively large sensor element used, $l^+ = 12$, which means that the smallest scale wall-shear stress events are not correctly resolved.

For the original measurements, as noted in section 3, there is a significant attenuation of the energy as the Reynolds number increases due to the size of the sensing length, which leads to a misleading Reynolds number trend. However, using the model we are able to reconstruct the correct trend, showing the capability of the model in correction of spatially under-resolved measurements (the prediction being only dependent on the spatial resolution of the calibration experiment, here $l^+ = 12$). This is explained further in figure 8, in which predictions have been
performed using models calibrated at different Reynolds number, where \( l^+ \) varies from 12 to 31. It can be seen that, whatever the sensing element \( l^+ \) considered, the Reynolds trend of the wall shear stress is correctly captured. Using these results it can also be shown that for each Reynolds number considered here the energy attenuation can be estimated, which allows us to determine a correction scheme depending on the Reynolds number \( Re_\tau \) and \( l^+ \).

![Figure 7. Fluctuation magnitude of the wall shear stress \( \tau_{\text{rms}}^+ \) versus Reynolds number \( Re_\tau \), for the prediction, present dataset and available data for zero-pressure gradient turbulent boundary layer. The solid line indicates the Reynolds number trend reported by Schlatter & Örlü (2010) \( 0.298 + 0.018 \ln(Re_\tau) \). The dash-dotted line indicates the trend \( 0.245 + 0.018 \ln(Re_\tau) \). The horizontal dashed line marks the classical value of 0.4 suggested by Alfredsson et al. (1988).]

6. Conclusion

A predictive model for the wall shear stress fluctuations \( \tau \) is proposed for the smooth-wall zero-pressure-gradient turbulent boundary layer. To make a prediction at any Reynolds number, the model requires only a single measurement point taken in the logarithmic region, here chosen to be the location of the outer-spectral-peak, \( z^+ = 3.9 Re_\tau^{1/2} \) (Mathis et al., 2009). The model is based on the recent predictive model for the streamwise velocity component developed by Marusic et al. (2010b). It is found that the reconstructed fluctuating wall shear-stress capture particularly well the Reynolds number trend, with only a slight overall underestimation, which in the future could be corrected by simply re-calibrating the model using a smaller sensing element \( l^+ \). As discussed in the introduction to the paper, this model has potentially broad applications, from environmental to laboratory flows, where wall-shear stress information is inaccessible or inaccurate.

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Figure 8. Fluctuation magnitude of the wall shear stress $\tau_{\text{rms}}^+$ versus Reynolds number $Re_\tau$ for the present dataset and predictions obtained using a different model constructed from each of the Reynolds numbers. The horizontal dashed line marked the classical value of 0.4 suggested by Alfredsson et al. (1988).

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