Simulation of the cable part of the load-lifting device drive, represented as a rod system

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Abstract. In this paper, the cable part of a lifting device operating on a marine vessel was studied. When the object is being towed, the “the vessel – the lifting device – the cargo system” makes non-stationary motion. In this case, the parameters of the cable part of the device: the distance between the points of the wire cable hanging, the length of the wire cable, its shape, and hydrodynamic resistance are variable. In the simulation of wire rope, it is proposed to replace it in the form of a rod system. During researches the mathematical model of small fluctuations of rod system on the basis of the Lagrange equation of II sort has been developed. Using the example of two-, three- and four-rod systems from age-old equations, dependences for their own frequencies are obtained and natural forms of small vibrations are constructed. However, as the number of rods in the system increases, the number of elements and degrees of freedom increases, leading to a significant complication. Therefore, at modeling similar types of load-lifting elements, it is recommended to limit the number of rods.

1. Introduction
Towage of objects is one of purposes of the load-lifting equipment. In the maritime industry, towed objects are used for military purposes, for geological exploration, oceanographic research, survey and prospecting works, as well as for fishing [1-3].

A characteristic feature of most deck hoisting equipment is work in special conditions. As special conditions, it is necessary to consider the effect of wind, rolling, changes in the parameters of the “cable-cargo” system (its shape, mass, hydrodynamic and inertial coefficients, cable length), the transition from one medium to another [4-9].

When the speed of the ship changes, the length of the etched wire cable, the system “ship - load-lifting device - cargo system” makes a non-stationary movement. In this case, the variables will be: the distance between the points of the wire cable hanging, the length of the wire cable, its shape, and hydrodynamic resistance. The resulting non-stationary loads have a negative affect on the operability and reliability of the drive of load-lifting equipment. Then the description of the cable part of the system by a flexible inextensible thread becomes practically impractical and it becomes necessary to create another model. Therefore, the research aimed at improving the drive system of deck lifting devices is relevant and caused by requests [10-12].
2. Materials and Methods

Objective of work is research of small free fluctuations of the cable part of a drive of the load-lifting device, presented in the form of rod system.

To model such a system, replace the flexible thread with a rod system with \( n \) links. We will assume that the point of the wire cable hanging at the aft end of the vessel. Imagine a flexible thread \( AD \) of length \( L \) as a system of rods \( AD_1,..., D_{i-1}D_i \) \( (i = 1, n) \), in which the links are pivotally connected to each other (Figure 1). Let the rods be uniform, of the same length \( 2l \) and the same weight \( P=2ql \), where \( q \) is the mass of the unit length of the rod. Consider the movement of a system of rods in a vertical plane about the equilibrium position \( Ay \), where the first rod \( AD_1 \) performs small vibrations around the axis \( A \), and all subsequent rods around the hinges \( D_i \). We assume that all connections are holonomic, ideal, and stationary, and the forces acting on the points of the system are potential.

![Figure 1. A flexible thread as a system of rods.](image)

The system under consideration has \( n \) degrees of freedom. For the generalized coordinates we will accept angles of deviation of the rods from a vertical \( \varphi_i \) which in position of stable equilibrium settle down on \( Ay \) and have zero value.

According to the d’Alembert principle, during the motion, the external generalized inertial forces, together with elastic restoring forces, satisfy the conditions of equilibrium. [13-16]. Therefore, the Lagrange equations for small oscillations, which are also equilibrium conditions, can be written as follows

\[
\left[ -\frac{d}{dt} \left( \frac{\partial K}{\partial \dot{\varphi}_i} \right) \right] + \left[ \frac{\partial U}{\partial \varphi_i} \right] = 0, \ (i = 1, n).
\]

Here, the kinetic energy of the system is equal to the sum of the kinetic energies of each rod. Thus, the kinetic energy of the rod \( AD_1 \), rotating around the \( A \) axis will be

\[
K_1 = \frac{1}{2} I_A \dot{\varphi}_1^2 - \frac{2P}{3g} \Gamma_1^2 \dot{\varphi}_1^2,
\]

where \( I_A \) is the moment of inertia of the rod \( AD_1 \) relative to the \( A \) axis.

For the rest of the rods \( D_{i-1}D_i \) hingedly connected to each other, the kinetic energy is equal to the sum of the kinetic energy of the center of inertia of the rod \( C_i \), in which all its mass is concentrated, and the kinetic energy in its relative movement around the center of inertia.

\[
K_i = \frac{p}{8g} (\dot{x}_i^2 + \dot{y}_i^2) + \frac{1}{2} I_{C_i} \dot{\varphi}_i^2,
\]

where \( I_{C_i} \) is the moment of inertia of the rod \( D_{i-1}D_i \) relative to the center \( C_i \), \( I_{C_i} = \frac{p^2}{g} \frac{1}{3} \),
\[ x_{ci} \text{ and } y_{ci} - \text{coordinates of the middle of } i\text{-th rod } C_i \]
\[ x_{ci} = l(2\sin\phi_{i-1} + \sin\phi_i) \]
\[ y_{ci} = l(2\cos\phi_{i-1} + \cos\phi_i), (i=1, n). \]

So, for the second rod, the kinetic energy will be

\[ K_2 = \frac{p_1^2}{2g} \left[ (2\cos\phi_1 \cdot \dot{\phi}_1 + \cos\phi_2 \cdot \dot{\phi}_2)^2 + (2\sin\phi_1 \cdot \dot{\phi}_1 + \sin\phi_2 \cdot \dot{\phi}_2)^2 + \frac{1}{3} \dot{\phi}_1^2 \right] = \]
\[ = \frac{p_1^2}{2g} \left[ 4\dot{\phi}_1^2 + \dot{\phi}_2^2 + 4\dot{\phi}_1 \dot{\phi}_2 \cos(\phi_1 - \phi_2) + \frac{1}{3} \dot{\phi}_2^2 \right] \]

Taking into account, that the system makes small fluctuations in decomposition \( \cos(\phi_1 - \phi_2) \) it is possible to be limited only to one member \( \cos(\phi_1 - \phi_2) \approx 1 \), and then after transformations we get

\[ K_2 = \frac{p_1^2}{2g} \left[ 4\dot{\phi}_1^2 + \dot{\phi}_2^2 + 4\dot{\phi}_1 \dot{\phi}_2 + \frac{1}{3} \dot{\phi}_2^2 \right] \]

Similarly, the kinetic energy for the \( i\)-th rod will be

\[ K_i = \frac{p_i^2}{2g} \left[ (2\cos\phi_i \cdot \dot{\phi}_i)^2 + \frac{1}{3} \dot{\phi}_i^2 \right] \]

Then the total kinetic energy of the entire rod system is equal to

\[ K = \frac{2p_1^2}{3g} \dot{\phi}_1^2 + \frac{p_i^2}{2g} \sum_{i=1}^{n} \left[ (2\cos\phi_i \cdot \dot{\phi}_i)^2 + \frac{1}{3} \dot{\phi}_i^2 \right] \]

Potential energy is determined by the work of the scales when the system moves from its current position to the upright position of the equilibrium. Thus, the potential energy of the first and second rods is equal to

\[ \Pi_1 = \Pi(1 - \cos\phi_1) \]
\[ \Pi_2 = \Pi[2(1 - \cos\phi_1) + (1 - \cos\phi_2)] \]

In view of it, potential energy for the \( i\)-th rod and the entire rod system, respectively, will be equal to

\[ \Pi_i = \Pi \left[ 2 \left( 1 - \cos\phi_{i-1} \right) + (1 - \cos\phi_i) \right] \]
\[ \Pi = \Pi \sum_{i=1}^{n} \left[ \frac{1}{2} \left( 2i - (2i - 1) \cos\phi_i \right) \right] \]

After decomposing into Taylor's series by degree \( \phi_i \), starting with the second member relative \( \phi_i \), we will receive

\[ \Pi = \Pi \sum_{i=1}^{n} \frac{9 - 2i}{2} \phi_i^2 \]

Write the oscillation equations in reverse form

\[ \frac{9 - 2i}{2} \phi_i = \frac{1}{g} \left[ \frac{d}{dt} \left( \frac{\dot{\phi}_1}{3} \phi_1^2 + \frac{1}{2} \sum_{i=1}^{n} \left[ (2\cos\phi_{i-1} + \phi_i)^2 + \frac{1}{3} \phi_i^2 \right] \right) \right] \]

3. Results

Further, for example, the research will be conducted for a rod system consisting of 2, 3 and 4 rods. Accordingly for such systems the oscillation equations will be
The solution of such systems should be found by assuming that the rod system performs one of the main oscillations, which can be written as

\[ \varphi_i = \lambda_i \sin(pt + \epsilon), \]

where \( \lambda_i \) – angular amplitudes of vibrations; \( p \) – natural frequencies of vibrations.

The natural frequencies and amplitudes, the number of which corresponds to the number of degrees of freedom of the rod system under study, are obtained from the age-old equations. Graphically, small fluctuations in the system can be represented as corresponding own forms (Figure 2-4).

Thus, for a two-rod system, the secular equation has the form

\[
\begin{vmatrix}
\frac{16l}{3g} & \frac{1}{p^2} & \frac{2l}{g\sqrt{3}} \\
\frac{2l}{g\sqrt{3}} & \frac{4l}{3g} & 1 \\
\frac{2l}{g\sqrt{3}} & \frac{4l}{3g} & \frac{1}{p^2}
\end{vmatrix} = 0.
\]

Where the natural frequencies are equal \( p_1 = 0.61 \sqrt{\frac{g}{l}}, p_2 = 1.62 \sqrt{\frac{g}{l}} \). The ratio of angular amplitudes will be

- for the first form of oscillation \( \varphi_{11}: \varphi_{12} = 2:2.73; \)
- for the second form of oscillation \( \varphi_{21}: \varphi_{22} = 2:-4.2. \)
Figure 2. The first (a) and second (b) proper form for a two-rod system.

Accordingly for a three-rod system we have

\[
\begin{vmatrix}
\frac{28l}{15g} & \frac{l}{p^2} & 6\frac{l}{5g} & 2\frac{l}{5g} \\
2\frac{l}{g} & 16\frac{l}{9g} & 2\frac{l}{3g} \\
2\frac{l}{g} & 2\frac{l}{g} & 4\frac{l}{3g} & \frac{1}{p^2}
\end{vmatrix} = 0; \quad p_1 = 2.33 \sqrt{\frac{g}{l}}, \quad p_2 = 0.46 \sqrt{\frac{g}{l}}, \quad p_3 = 1.24 \sqrt{\frac{g}{l}}
\]

- for the first form of oscillation $\varphi_{11}$, $\varphi_{12}$, $\varphi_{13}$, $\varphi_{14}$ = 0.45: 0.6: 0.67;
- for the second form of oscillation $\varphi_{21}$, $\varphi_{22}$, $\varphi_{23}$ = 0.33: -0.02: -0.95;
- for the third form of oscillation $\varphi_{31}$, $\varphi_{32}$, $\varphi_{33}$ = 0.35: -0.7: 0.63.

Figure 3. The first (a), the second (b), and the third (c) proper forms for a three-rod system.

For a four-rod system, we similarly obtain $p_1 = 0.9 \sqrt{\frac{g}{l}}$, $p_2 = 1.5 \sqrt{\frac{g}{l}}$, $p_3 = 0.42 \sqrt{\frac{g}{l}}$, $p_4 = 2.9 \sqrt{\frac{g}{l}}$

- for the first form of oscillation $\varphi_{11}$, $\varphi_{12}$, $\varphi_{13}$, $\varphi_{14}$ = 0.32: 0.4: 0.5: 0.7;
- for the second form of oscillation $\varphi_{21}$, $\varphi_{22}$, $\varphi_{23}$, $\varphi_{24}$ = 0.2: 0.15: -0.07: -1;
- for the third form of oscillation $\varphi_{31}$, $\varphi_{32}$, $\varphi_{33}$, $\varphi_{34}$ = 0.4: -0.02: -0.8: 0.5;
• for the fourth form of oscillation $\varphi_{31} \cdot \varphi_{32} \cdot \varphi_{33} \cdot \varphi_{44} = 0.4 \cdot 0.7 \cdot 0.5 \cdot 0.2$

![Figure 4. The first (a), the second (b), the third (c) and the fourth (d) own forms for a four-rod system.](image)

4. Conclusions

As you can see, the obtained proper values describe the dynamics of the flexible thread movement quite well. Therefore, this principle of its modeling by replacing the rod system can be used in the study of such elements of load-lifting devices.

When modeling any mechanical device, it is desirable to detail the schematization for a more accurate reflection of the object under study. However, this often leads to the study of a mechanical system with a large number of elements and degrees of freedom. And, although the expediency of representing a flexible cable as a rod system is undoubtedly significant, it is still necessary to limit the number of links in the system and at the same time model the object with acceptable accuracy.

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