We summarize the main characteristics and recent results on $B \to X_u \ell \nu \ell$ decays of a model based on soft–gluon resummation and an analytic time–like QCD coupling.

1. Introduction

By comparing various spectra in the semileptonic inclusive $B$ decays $B \to X_u \ell \nu \ell$, where $\ell$ is a fixed lepton species ($\ell = e, \mu$) and $X_u$ the fragmented $u$ quark, with the predictions of a model including non–perturbative corrections to soft–gluon dynamics through an effective QCD coupling [1], we obtain a value for the $|V_{ub}|$ Cabibbo–Kobayashi–Maskawa (CKM) matrix element [2,3]:

$$|V_{ub}| = (3.76 \pm 0.13 \pm 0.22) \times 10^{-3}$$

being the errors experimental and theoretical, respectively.

The model involves the insertion, inside standard threshold resummation formulae, of an effective QCD coupling $\tilde{\alpha}_S(k^2)$, based on an analyticity requirement and resumming absorptive effects in gluon cascades [4]. By construction, $\tilde{\alpha}_S(k^2)$ has no Landau pole and saturates at small scales:

$$\lim_{k^2 \to 0} \tilde{\alpha}_S(k^2) = \frac{1}{\beta_0} \approx O(1).$$

This model, which has no free parameters, describes $B$–meson fragmentation data at the $Z^0$ peak rather well [5], where — unlike $B$ decays — accurate data are available and there is no uncertainty coming from the CKM matrix elements.

In the following, we describe the phenomenological model used, then the extraction of $|V_{ub}|$, and finally the results, followed by the conclusions.

2. Threshold resummation with an effective coupling

Factorization and resummation of threshold logarithms in semileptonic decays leads to an expression for the triple–differential distribution, the most general distribution, of the following form [6]:

$$\frac{1}{\Gamma} \frac{d^3 \Gamma}{dxdwdu} = C[x, w; \alpha_S(Q)] \sigma[u; Q] + D[x, u, w; \alpha_S(Q)],$$

where:

$$x = \frac{2E_\ell}{m_b}, \quad w = \frac{Q}{m_b}, \quad u = \frac{1 - \sqrt{1 - (2m_X/Q)^2}}{1 + \sqrt{1 - (2m_X/Q)^2}}$$

and the hard scale $Q = 2E_X$, with $E_\ell$, $E_X$ and $m_X$ being the charged lepton energy, the total hadron energy and the hadron mass, respectively. $\Gamma = \Gamma(\alpha_S)$ is the inclusive width of decay $B \to X_u \ell \nu \ell$. Furthermore, $C[x, w; \alpha_S]$ is a short–distance, process dependent hard factor; $\sigma[u; Q]$ is the universal QCD form factor for heavy–to–light transitions, resumming to any order in $\alpha_S$ the series of logarithmically enhanced terms to some logarithmic accuracy; $D[x, u, w; \alpha_S]$ is a short–distance, process dependent, remainder function, vanishing in the threshold region $u \to 0$ and in lowest–order in $\alpha_S$. The heavy flavor decay form factor has an exponential form in Mellin moments $N$–space [7]. We apply a change of renormalization scheme for the coupling constant $\alpha_S \to \tilde{\alpha}_S$. 

The QCD form factor $\sigma[w;Q]$ has been numerically computed for different values of $\alpha_S(m_2^2)$ in [1].

Threshold suppression — the main theoretical ingredient for the measurement of $|V_{ub}|$ — is represented by the factor

$$W(a,b) \equiv \frac{\Gamma[B \to X_u \ell \nu_\ell, p \in (a,b)]}{\Gamma[B \to X_u \ell \nu_\ell]} = \int_{a<p(x,w,u)<b} \frac{1}{\Gamma} \frac{d^3\Gamma}{dxdwdu} \leq 1. \quad (1)$$

where $W(a,b) = 1$ when integrated over all the phase-space of $x, w$ and $u$. For threshold resummed spectra of $B \to X_u \ell \nu_\ell$ decays at next-to-leading order see Ref. [8].

3. $|V_{ub}|$ extraction

Experimentally, given a kinematical variable $p$, such as for example the energy $E_\ell$ of the charged lepton, one measures the number of $B$'s decaying semileptonically to $X_u$ with $p$ in some interval $(a,b)$, divided by the total number of produced $B$'s (decaying into any possible final state):

$$B[p \in (a,b)] \equiv \frac{N[B \to X_u \ell \nu_\ell, p \in (a,b)]}{N[B \to \text{(anything)}]} \quad (2)$$

This branching ratio can be written as:

$$B[p \in (a,b)] = \frac{B_{\text{SL}}}{1 + R_{c/u}} W(a,b), \quad (3)$$

where we have defined the semileptonic branching ratio:

$$B_{\text{SL}} = \frac{\Gamma[B \to X_c \ell \nu_\ell] + \Gamma[B \to X_u \ell \nu_\ell]}{\Gamma[B \to \text{(anything)}]}$$

and the ratio of $(b \to c)/(b \to u)$ semileptonic widths:

$$R_{c/u} = \frac{\Gamma[B \to X_c \ell \nu_\ell] / \Gamma[B \to X_u \ell \nu_\ell]}{1 + R_{c/u}} \cdot (4)$$

Since $B_{\text{SL}}$ is rather well measured, we use the experimental determination [9]:

$$B_{\text{SL}} = 0.1066 \pm 0.0020.$$  

With this method there is no $m_c^2$ dependence (with the related uncertainty) which would appear using the theoretical expression of $\Gamma(B \to X_u \ell \nu_\ell)$, because one has to compute only the ratio of widths $R_{c/u}$ and not the absolute widths. The semileptonic $b \to c$ width is written as:

$$\Gamma(B \to X_c \ell \nu_\ell) = \frac{G_F^2 m_b^3 |V_{cb}|^2}{192\pi^3} \times I(\rho) F(\alpha_S) G(\alpha_S, \rho), \quad (5)$$

where $\rho \equiv \frac{m_c^2}{m_b^2} \approx 0.1$. The function $I(\rho)$ accounts for the suppression of phase-space because of $m_c \neq 0$ [10]:

$$I(\rho) = 1 - 8\rho + 12\rho^2 \log \frac{1}{\rho} + 8\rho^3 - \rho^4.$$

Note that there is an (accidental) strong dependence on the charm mass $m_c$, because of the appearance of a large factor in the leading term in $\rho$, namely $-8$. As far as inclusive quantities are concerned, the largest source of theoretical error comes indeed from the the uncertainty in $\rho$. Most of the dependence is actually on the difference $m_b - m_c$, which can be estimated quite reasonably with the Heavy Quark Effective Theory (HQET). Finally, the factor $G(\alpha_S, \rho)$ contains corrections suppressed by powers of $\alpha_S$ and $\rho$:

$$G(\alpha_S, \rho) = 1 + \sum_{n=1}^\infty G_n(\rho) \alpha_S^n,$$

with $G_n(0) = 0$. Note that $G(0, \rho) = G(\alpha_S, 0) = 1$. By inserting the above expressions for the semileptonic rates, one obtains for the perturbative expansion of $R_{c/u}$:

$$R_{c/u} = R_{c/u}(\rho, \alpha_S, |V_{cb}|/|V_{ub}|) = \frac{|V_{cb}|^2}{|V_{ub}|^2} I(\rho) G(\alpha_S, \rho).$$

This method actually provides a measurement of the ratio $|V_{ub}|/|V_{cb}|$, but since the error on $|V_{cb}|$ is rather small and theoretically well understood, one is basically measuring $|V_{ub}|$.\footnote{The average of determinations of $|V_{ub}|$ coming from a global fit to the $B \to X_c \ell \nu_\ell$ and $b \to s \ell \nu$ moments in the kinetic and 1S schemes, in good agreement with each other, is $|V_{ub}| = (41.6 \pm 0.6) \times 10^{-3}$ [11].}

3.1. Quark masses

The approach we are following to compute $|V_{ub}|$ can be subdivided into two parts: in the first
part we compute the triple–differential distribution and in the second part the $|V_{ub}|$ value. As far as the triple differential distribution is concerned, as the whole process is described in a perturbative framework, we do not distinguish between the mass of the $B$ meson and the pole mass of the $b$ quark, i.e. we consistently assume $m_b = m_B$.

Once we compute the ratio $|V_{ub}|/|V_{ub}|$, we use the standard HQET formulas, based on the $b$–quark mass. Thus, the $b$–quark mass is introduced in our formulation of the ratio. Since quarks are confined inside observable hadrons, their masses cannot be directly measured and their values are biased by the selected theoretical framework. We have performed the calculation in the $\overline{\text{MS}}$ mass scheme. The $\overline{\text{MS}}$ masses for the $b$ and the $c$ quark are taken $m_b(\overline{m}_b) = 4.243 \pm 0.042 \text{ GeV}$ and $m_c(\overline{m}_c) = 1.25 \pm 0.09 \text{ GeV}$ [119], respectively. However, we have considered the pole–mass scheme as well in order to take into account the uncertainties coming from a different scheme definition.

4. Results

We calculate $|V_{ub}|$ for all the experimental analyses. They are categorized according to the kinematical distribution looked at, where selection criteria are applied to define the limited phase–space on which the branching ratio is computed. They are: the lepton energy ($E_\ell$), the invariant mass of the hadron final state ($M_X$), the light–cone distribution ($P^+ \equiv E_X - |\vec{p}_X|$, $E_X$ and $\vec{p}_X$ being the energy and the magnitude of the 3–momentum of the hadronic system), a two dimensional distribution in the electron energy and $s^{\text{max}}$, the maximal $M_X^2$ at fixed $q^2$ and $E_\ell$. Moreover, as described in Ref. [1], we will look only at the range where data are not affected by potential $b \to c \ell \nu_\ell$ background: $2.3 \text{ GeV} < E_\ell < 2.6 \text{ GeV}$, although the method works for lower lepton energy as well.

We compute $|V_{ub}|$ for each of the analyses starting from the corresponding partial branching fractions. Then, we determine the average $|V_{ub}|$ value using the HFAG methodology [11].

Table 1 reports the extracted values of $|V_{ub}|$ for all the uncorrelated analyses and their corresponding average. The errors are experimental (i.e. statistical and systematic) and theoretical, respectively. The average is:

$$V_{ub} = (3.76 \pm 0.13 \pm 0.22) \times 10^{-3},$$

consistent with the measured value of $|V_{ub}|$ from exclusive decays [11] and the indirect measurement [12].

The table shows also the criteria used for the determination of the partial branching ratio ($\Delta B$). The theoretical errors are considered completely correlated among all the experimental analyses, when performing the average. The $|V_{ub}|$ values and the corresponding average are plotted in Figure 1.

![Figure 1](image-url)
Table 1
The first column in the table shows the uncorrelated analyses, the second column shows the corresponding values of $|V_{ub}|$, and finally the last column shows the criteria for which $\Delta B$ is available. The final row shows the average value of $|V_{ub}|$. The errors on the $|V_{ub}|$ values are experimental and theoretical, respectively. The experimental error includes both the statistical and systematic errors.

| Analysis            | $|V_{ub}|(10^{-3})$ | $\Delta B$ criteria                                      |
|---------------------|---------------------|---------------------------------------------------------|
| BaBar ($E_\ell$) [13]   | 3.46±0.14 ±0.23     | $E_\ell > 2.3$ GeV                                     |
| Belle ($E_\ell$) [14]   | 3.20±0.17 ±0.23     | $E_\ell > 2.3$ GeV                                     |
| CLEO ($E_\ell$) [15]    | 3.49±0.20 ±0.23     | $E_\ell > 2.3$ GeV                                     |
| BaBar ($M_X$) [16]      | 4.04±0.19 ±0.24     | $M_X < 1.55$ GeV                                       |
| Belle ($M_X$) [17]      | 3.93±0.26 ±0.23     | $M_X < 1.7$ GeV                                        |
| BaBar ($M_X, q^2$) [16] | 4.14±0.26 ±0.23     | $M_X < 1.7$ GeV, $q^2 > 8$ GeV²                       |
| Belle ($M_X, q^2$) [18] | 3.95±0.42 ±0.22     | $M_X < 1.7$ GeV, $q^2 > 8$ GeV²                       |
| BaBar ($E_\ell, s^{max}$) [19] | 3.87±0.26 ±0.24 | $E_\ell > 2.0$ GeV, $s^{max} < 3.5$ GeV² |
| BaBar ($P^+$) [16]      | 3.45±0.22 ±0.24     | $P^+ < 0.66$ GeV                                       |
| Average              | 3.76±0.13 ±0.22     |                                                         |

Finally, using the value of $\Delta B$ corresponding to the lowest lepton energy cut for the endpoint analyses, the value of $|V_{ub}|$ is 3% higher than what we quote adopting a cut at 2.3 GeV, qualitatively consistent with the expectations due to the observation of a larger number of events in that region than predicted [11].

5. Conclusions

We have analyzed semileptonic $B$ decay data in the framework of a model for QCD non-perturbative effects based on an effective time-like QCD coupling, free from Landau singularities.

Our inclusive measurement of the $|V_{ub}|$ CKM matrix element is:

$$|V_{ub}| = (3.76 \pm 0.13 \pm 0.22) \times 10^{-3}.$$  

The errors on the $|V_{ub}|$ value are experimental and theoretical, respectively. The experimental error includes both the statistical and systematic errors. This value is fully consistent with the determination from exclusive decays ($|V_{ub}| = (3.51 \pm 0.21^{+0.66}_{-0.42}) \times 10^{-3}$) [2] and from an indirect estimate ($|V_{ub}| = (3.44 \pm 0.16)$ [12] $\times 10^{-3}$), whilst other methods show a discrepancy up to

\footnote{The value is obtained from an average of the Lattice QCD determinations [11].}
Table 2
The first column of the table shows the different contributions to the theoretical errors, the second column shows the corresponding variation, and finally the third column shows the percentage contribution with respect to the $|V_{ub}|$ value.

| Contribution | Variation | Error (%) |
|--------------|-----------|-----------|
| $\alpha_S$   | 0.1176 $\pm$ 0.024 | $\pm$0.6 $\rightarrow$ 3.5 |
| $|V_{cb}|$    | (41.6 $\pm$ 6.6) $\times$ 10$^{-3}$ | $\pm$1.4 |
| $m_b$ (GeV)  | 4.20 $\pm$ 0.07  | $\pm$0.6 |
| $m_c$ (GeV)  | 1.25 $\pm$ 0.09  | $\pm$4.4 |
| $B(B \rightarrow X_u\ell\nu)$ | 0.1066 $\pm$ 0.0020 | $\pm$1.0 |
| $|V_{ub}|$ method | 4.7 $< m_b < 5.0$, $1.47 < m_c < 1.83$ | $-1.3 \rightarrow -5.2$ |
| pole mass (GeV) | $3.34 < m_b - m_c < 3.41$ | $+0.8$ |
| approx. NNLO rate | | $+2.0$ |

Table 3
The table contains the $|V_{ub}|$ values for several analyses and the corresponding averages. The errors on the $|V_{ub}|$ values are experimental and theoretical, respectively. The experimental error includes both the statistical and systematic errors.

| $|V_{ub}|$ for endpoint analyses ($10^{-3}$) |
|------------------------------------------|
| BaBar ($E_\ell$) [13] 3.46$\pm$0.14 $^{+0.24}_{-0.23}$ | $E_\ell > 2.3$ GeV |
| Belle ($E_\ell$) [14] 3.20$\pm$0.17 $^{+0.21}_{-0.23}$ | $E_\ell > 2.3$ GeV |
| CLEO ($E_\ell$) [15] 3.49$\pm$0.20 $^{+0.23}_{-0.23}$ | $E_\ell > 2.3$ GeV |
| Average 3.42$\pm$0.15 $^{+0.23}_{-0.22}$ | |

| $|V_{ub}|$ for $M_X$ analyses ($10^{-3}$) |
|------------------------------------------|
| BaBar ($M_X$) [16] 4.04$\pm$0.19 $^{+0.24}_{-0.23}$ | $M_X < 1.55$ GeV |
| Belle ($M_X$) [17] 3.93$\pm$0.26 $^{+0.23}_{-0.22}$ | $M_X < 1.7$ GeV |
| Average 4.00$\pm$0.16 $^{+0.23}_{-0.23}$ | |

| $|V_{ub}|$ for $(M_X, q^2)$ analyses ($10^{-3}$) |
|------------------------------------------|
| BaBar ($M_X, q^2$) [16] 4.14$\pm$0.26 $^{+0.23}_{-0.23}$ | $M_X < 1.7$ GeV, $q^2 > 8$ GeV$^2$ |
| Belle ($M_X, q^2$) [17] 4.21$\pm$0.37 $^{+0.23}_{-0.23}$ | $M_X < 1.7$ GeV, $q^2 > 8$ GeV$^2$ |
| Belle ($M_X, q^2$) [18] 3.95$\pm$0.42 $^{+0.23}_{-0.22}$ | $M_X < 1.7$ GeV, $q^2 > 8$ GeV$^2$ |
| Average 4.13$\pm$0.21 $^{+0.23}_{-0.23}$ | |

| $|V_{ub}|$ for $P^+$ analyses ($10^{-3}$) |
|------------------------------------------|
| BaBar ($P^+$) [16] 3.45$\pm$0.22 $^{+0.24}_{-0.23}$ | $P^+ < 0.66$ |
| Belle ($P^+$) [17] 3.73$\pm$0.32 $^{+0.24}_{-0.29}$ | $P^+ < 0.66$ |
| Average 3.55$\pm$0.19 $^{+0.24}_{-0.23}$ | |

$\approx +2\sigma$ [11] from those. We argue that the main difference of our model with respect to previous ones is a smaller suppression of the threshold region.

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