Bag Formation in Quantum Hall Ferromagnets.

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Abstract

Charged skyrmions or spin–textures in the quantum Hall ferromagnet at filling factor \( \nu = 1 \) are reinvestigated using the Hartree–Fock method in the lowest Landau level approximation. It is shown that the single Slater determinant with the minimum energy in the unit charge sector is always of the hedgehog form. It is observed that the magnetization vector's length deviates locally from unity, i.e. a bag is formed which accommodates the excess charge. In terms of a gradient expansion for extended spin–textures a novel \( O(3) \) type of effective action is presented, which takes bag formation into account.

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1 Introduction

Quantum Hall ferromagnets at filling factor \( \nu = 1 \) exhibit charged excitations that are topologically stable objects called skyrmions. Various experiments \[1, 2\] have revealed strong evidence for the existence of these objects in two–dimensional electron systems.

For their theoretical description we employ the microscopic Hartree–Fock (HF) theory with the hedgehog ansatz for charged spin–textures, developed by Fertig et. al. \[3\] and subsequently commonly used in the literature \[4, 5, 6\]. We show that the most general single Slater determinant in the lowest Landau level (LLL) approximation collapses into the hedgehog ansatz, hence supporting this approach. Physical observables like magnetization and charge density are then calculated microscopically in the standard way (section 2). However in contrast to previous approaches, we will analyse the full magnetization vector, not only its third component, the spin–density. It will turn out that the magnetization is not a unit vector, but its length is substantially decreased in regions with non–zero excess charge (bag formation). A signature of this behaviour was already observed in \[6\], namely that the absolute value of the spin–density at the origin as obtained from the HF calculation is reduced compared to that of the non–linear \( O(3) \) model which is always unity. However the conjecture that with quantum corrections included, the discrepancies between effective field theory and microscopic HF results may be resolved, is incorrect. Bag
formation appears already in tree level and accordingly the magnetization vector can no longer be identified directly with the $O(3)$ unit vector field. An additional scalar field is necessary to describe the magnetization vector’s length.

Subsequently, using the gradient expansion, a new $O(3)$ type of effective action for extended spin–textures is derived which takes these effects into account (section 3). It will be demonstrated, that this action reproduces the results of the microscopic HF calculation correctly for zero range forces.

2 Microscopic Hartree-Fock calculations

As usual lengths are measured in units of the magnetic length $\ell = \sqrt{\hbar c/eB}$ and energies in units of the Coulomb energy $e^2/\epsilon\ell$. Then for a given electron-electron interaction in the LLL there exists only one dimensionless parameter in the theory, namely the effective Zeeman coupling $g = g_s\mu_B B/(e^2/\epsilon\ell)$.

In the completely filled LLL (filling factor $\nu = 1$) the ground state is given by the unique single Slater determinant (Vandermonde’s determinant, see e.g. [7])

$$|\Psi\rangle = A \prod_{m=0}^{\infty} |m \downarrow\rangle, \quad \langle r|m\rangle = \frac{1}{2\pi} \frac{1}{2m!} r^m e^{im\varphi} e^{-r^2/4}$$

(1)

with the spins of all single particle states down. Here we are concerned with the description of charged spin-texture excitations, in particular with those in the charge sectors $Q = \pm 1$. Due to the particle-hole symmetry [5] the energies of the positively charged (quasi-hole, antiskyrmion) and negatively charged (quasi-particle and skyrmion) excitations, $E^\pm = E \mp \epsilon_0$, differ only by the exchange energy per electron in the ground state, e.g. $\epsilon_0 = -\sqrt{\pi}/8$ in the pure Coulomb case. Thus it is sufficient to consider the $Q = +1$ sector with the following ansätze for the many-body wave function:

(i) the quasi-hole

$$|\Psi\rangle = A \prod_{m=0}^{\infty} |m + 1 \downarrow\rangle,$$

(quasi-hole) (2)

with just one electron removed from the completely filled LLL. Its energy relative to the ground state is $E^+ = g/2 - 2\epsilon_0$ ($E = g/2 - \epsilon_0$).

(ii) the hedgehog

$$|\Psi\rangle = A \prod_{m=0}^{\infty} (u_m|m \uparrow\rangle + v_m|m + 1 \downarrow\rangle), \quad u_m^2 + v_m^2 = 1,$$

(hedgehog) (3)

introduced by Fertig et. al. [3]. For $u_m = 0$, $v_m = 1$, $\forall m$, this ansatz reduces to the quasi–hole (2).
(iii) the general single Slater determinant

\[ |\Psi\rangle = A \prod_\alpha |\alpha\rangle, \quad |\alpha\rangle = \sum_{m\sigma} C_{m\sigma}^\alpha |m\sigma\rangle, \quad \text{(general single SD)} \]  

whose single particle states \( |\alpha\rangle \) with \( \langle \alpha | \alpha \rangle = 1 \) are given by the most general linear combination of LLL-states labeled by magnetic quantum number \( m = 0, \ldots, N - 1 \) and spin \( \sigma \) (up and down).

A HF calculation for the general Slater determinant (ansatz (4)) was performed with \( N - 1 \) particles in order to guarantee \( Q = +1 \) in a basis up to \( N = 300 \) states involving the diagonalization of \( 600 \times 600 \) matrices. Independently from the initial conditions chosen, the general Slater determinant always collapsed

\[ C_{m\downarrow}^\alpha = u_\alpha \delta_{m,\alpha}, \quad C_{m\uparrow}^\alpha = v_\alpha \delta_{m,\alpha+1} \]  

into the specific hedgehog ansatz (3) during the iteration process. This happened for various Zeeman coupling strengths \( g \) and also for different (repulsive) interactions. Thus, we may consider this finding as a strong indication that the lowest energy single Slater determinant for \( Q = 1 \) in the LLL is indeed always of the hedgehog form. I am not aware of a rigorous proof of that statement, although it is certainly related to the fact that in the ansatz (3) only single particle states with the same grand-spin \( m + \sigma \) couple.

For the reasons just explained we may use the hedgehog ansatz (3) in the \( Q = +1 \) sector which simplifies the calculation considerably. Instead of diagonalizing matrices there are only coupled algebraic equations for \( u_m \) and \( v_m \) to solve for each \( m \) separately. We give the expression for the HF energy derived in [3] relative to the completely filled LLL

\[ E_{HF}^+ = \frac{g}{2} \sum_m (u_m^2 - v_{m-1}^2 + 1) \]

\[ + \frac{1}{2} \sum_{mm'} (u_m^2 + v_{m-1}^2 - 1)(u_{m'}^2 + v_{m'-1}^2 - 1)\langle mm'|V|m m'\rangle \]

\[ - \frac{1}{2} \sum_{mm'} (u_m^2 u_{m'} + v_{m-1}^2 v_{m'-1} - 1)\langle mm'|V|m' m\rangle \]

\[ - \sum_{mm'} u_m v_m u_{m'} v_{m'} \langle mm' + 1|V|m' m + 1\rangle. \]  

The individual terms are the Zeeman energy and the direct and exchange terms of the two-body interaction \( V \). The corresponding HF equations are immediately derived. The (excess) charge density

\[ \rho^C(r) = \frac{1}{2\pi} - \langle |\Psi\rangle \sum_i \delta(r - r_i)|\Psi\rangle \]

\[ = \frac{1}{2\pi} \sum_m (1 - u_m^2 - v_{m-1}^2) \frac{x^m}{m!} e^{-x}, \quad x = \frac{r^2}{2} \]  

\[ \]
is radially symmetric and normalized to $Q^C = +1$ as may be checked using the condition $u_m^2 + v_m^2 = 1$. The magnetization vector

$$m(r) \equiv -2\pi \langle \Psi | \sum_i \sigma(i) \delta(r - r_i) | \Psi \rangle = (f(r) \cos \varphi, f(r) \sin \varphi, h(r))$$

$$f(r) = -\sqrt{2} r \sum_m \frac{u_m v_m}{\sqrt{m+1}} \frac{x^m}{m!} e^{-x}, \quad h(r) = \sum_m (v_{m-1}^2 - u_m^2) \frac{x^m}{m!} e^{-x}, \quad (8)$$

$(\sigma(i)$ are the Pauli matrices for particle $i)$ normalized to $\lim_{r \to \infty} m = e_3$ is of the hedgehog form as expected. However, it is certainly not a unit vector: there are two radial functions involved for which in general $f^2 + h^2 \neq 1$. In order to make closer contact to effective field theories we may rewrite the magnetization vector in the form

$$m(r) = \sigma(r) \hat{m} = \sigma(r) \left( \sin F(r) \cos \varphi, \sin F(r) \sin \varphi, \cos F(r) \right) \quad (9)$$

with $\hat{m}$ a unit vector. The scalar function $\sigma$ describes the deviation of the magnetization vector’s length from 1 (bag formation) and the angle function $F$ determines the orientation of the unit vector $\hat{m}$. It is clear that these functions are readily expressed in terms of the original functions $f$ and $h$.

In this way we are going to analyse the full magnetization vector $m$, not only its third component, which is the main difference to previous presentations. It will immediately lead us to an important relation of the formed bag with the charge. In order to better understand the situation we will study the analytical hard-core model before we present numerical results for the more realistic Coulomb interaction.

### 2.1 Hard-core interaction

For a zero range interaction $V(r_1 - r_2) = t\delta(r_1 - r_2)$ and vanishing Zeeman splitting, $g = 0$, the HF equations may be solved analytically

$$u_m = \sqrt{a \over m + 1 + a}, \quad v_m = \sqrt{m + 1 \over m + 1 + a}, \quad a = \lambda^2 \over 2. \quad (10)$$

Here, $\lambda$ is a free parameter which fixes the soliton size. The radial functions determining the magnetization $m$ in (8) are

$$f(r) = -\lambda r p(x, a), \quad h(r) = (x - a) p(x, a), \quad x = r^2 \over 2 \quad (11)$$

with the single valued analytical function

$$p(x, a) = \sum_m \frac{1}{m + 1 + a} \frac{x^m}{m!} e^{-x},$$

$$x^{a+1} p(x, a) e^x = \int_0^x dt \, t^a \, e^t, \quad (12)$$

$$\frac{d}{dx} \left( xp(x, a) \right) + (a + x) p(x, a) = 1,$$
related to the incomplete gamma function \([8]\). From (9) and (11) we obtain

\[ F(r) = 2 \arctan \frac{\lambda}{r}, \quad \sigma(r) = (x + a)p(x, a) \quad (13) \]

the angle function of the familiar Belavin–Polyakov soliton \([8]\), but the modulus function \(\sigma\) is not equal to 1. The topological and physical charge densities are

\[ \rho^T(r) = \frac{1}{4\pi} \frac{d}{dr} (\cos F) = \frac{1}{\pi} \frac{\lambda^2}{(r^2 + \lambda^2)^2}, \quad (14) \]
\[ \rho^C(r) = \frac{1}{2\pi} \left[ 1 - (a + x)p(x, a) \right] = \frac{1 - \sigma}{2\pi}. \quad (15) \]

It is readily checked with (12) that both charges are \(Q^T = Q^C = +1\), however the densities do look quite different in general. For example for the quasi–hole, \(\lambda \to 0\), we obtain from (12) \(\sigma = 1 - \exp(-r^2/2)\), i.e.

\[ \rho^T(r) = \delta(r), \quad \lambda \to 0, \quad (16) \]
\[ \rho^C(r) = \frac{1}{2\pi} \exp(-r^2/2), \quad \lambda \to 0. \quad (17) \]

It is only in the opposite limit of very large skyrmions \(\lambda \to \infty\) that the two densities become similar. But note, that although \(\sigma \xrightarrow{\lambda \to \infty} 1\) in that limit, the integral over the physical charge density, i.e. over \((1 - \sigma)/2\pi\), always remains 1.

Thus, the conclusion of this simple exercise is twofold: (i) \(f^2 + g^2 = \sigma^2 \neq 1\), a bag is formed which does go beyond the standard \(O(3)\) description and (ii) the physical charge density is not equal to the \(O(3)\) model’s topological density, rather it is connected with the bag via (15).

It should be added, that for hard-core forces at finite Zeeman splitting the size parameter \(\lambda \to 0\) and the (anti-)skyrmion collapses into the quasi–hole \([16,17]\). There are no skyrmions for hard-core forces. In the following we are going to discuss the situation for Coulomb forces.

### 2.2 Coulomb interaction

Although a realistic calculation would have to take into account Landau level mixing, the finite thickness of the layer \([4,7,9,11]\) and also impurities, we consider the electrons in the LLL interacting in the plane via pure Coulomb forces \(V(r_1 - r_2) = 1/|r_1 - r_2|\). We are interested in bag formation and its relation to the physical charge density. The corresponding HF equations are solved numerically using a basis up to several thousand states for large skyrmions. The soliton energy in dependence of the Zeeman coupling strength \(g\) is depicted in Fig. 1. We observe a second order phase-transition at \(g \simeq 0.053\) below which (anti-)skyrmions are energetically
Figure 1: Skyrmion and quasi-hole energies as a function of the Zeeman splitting calculated in Hartree Fock for a pure Coulomb interaction. For $g < 0.053$ skyrmions are energetically favored. In the limit $g \to 0$ the Bogomol’nyi bound is approached.

Figure 2: Profiles for the (anti-)skyrmion with $g = 0.03$ versus the radius in magnetic lengths. A bag is formed.
Figure 3: Topological and physical charge densities for the soliton with \( g = 0.03 \) (solid lines). Although both total charges are normalized to 1, the physical charge density deviates significantly from the topological one. Instead it is extremely well approximated by the bag’s shape \( 1 - \sigma \) (dashed line).

favored compared to the quasi–hole (dashed line). For extremely small Zeeman couplings (large skyrmions) the soliton energy approaches the Bogomol’nyi bound \( \sqrt{\pi/32} \simeq 0.3133 \), indicating that in this limit the Belavin-Polyakov soliton of the non–linear sigma model is obtained.

The profiles are calculated microscopically via the magnetization (11) and plotted in Fig. 2 for Zeeman coupling \( g = 0.03 \). The formation of a bag is clearly indicated. With decreasing Zeeman coupling the bag begins to flatten out. But notice, that in the Belavin-Polyakov limit \( g \to 0 \) the integral over \( (1 - \sigma)/2\pi \) is 1. The physical charge density, calculated microscopically according to (7), and the topological density defined via the angle function \( F \) (Fig. 2) are depicted in Fig. 3 again for Zeeman coupling \( g = 0.03 \). It is noticed that the physical charge density is much flatter compared to the topological density. The bag shape function \( 1 - \sigma \) (dashed line) is hardly distinguishable from the physical charge density. This supports our findings for hard-core forces: it is the bag and not the topological charge density which is closely related to the physical charge density.

3 Effective field theory

There has been a lot of effort to derive an effective field theory for the quantum Hall ferromagnet [12, 13, 14, 15, 16]. The result in terms of a gradient expansion of the unit magnetization vector \( \mathbf{m} \) is essentially an \( O(3) \) model, i.e. the non–linear sigma
model augmented by higher order terms stemming from the interaction. However from the above considerations it is expected that the full magnetization vector $\mathbf{m}$, namely its modulus $\sigma$, should show up in the effective action. In particular the relation between the physical charge density and the bag (12) should be respected. The derivation of such an effective action in full generality is a challenging task. Therefore, in the following subsection we resort to the gradient expansion for extended spin-textures quite as in previous approaches, however we take bag formation into account.

3.1 $O(3) - \text{model}$

Starting point is the full magnetization vector and the physical charge density which in lowest order gradient expansion read

$$\mathbf{m} = \sigma \hat{\mathbf{m}} = \left( 1 - \frac{1}{4} \partial_i \hat{\mathbf{m}} \partial_i \hat{\mathbf{m}} \right) \hat{\mathbf{m}} + \mathcal{O}(4), \quad (18)$$

$$\rho^C = -\frac{1}{8\pi} \epsilon_{ij} \hat{\mathbf{m}} (\partial_i \hat{\mathbf{m}} \times \partial_j \hat{\mathbf{m}}) + \mathcal{O}(4). \quad (19)$$

Because of the charge conjugation properties of the magnetization vector, i.e. the scalar field is the same for skyrmions and anti–skyrmions, it is clear that in (18) the energy density of the non–linear sigma model rather than the topological density enters. It is remarkable that its coefficient, which follows e.g. from (15), is independent from the specific two-body interaction. From the discussion in section 2 (cf. Fig. 3) it is evident that the physical charge density in general is not well approximated by the topological density, nevertheless in terms of the gradient expansion for extended spin-textures eq. (19) holds. In accordance with eq.(18) the energy functional could be chosen

$$E[\mathbf{m}] = \frac{\rho_s}{2} \int d^2r [\partial_i \hat{\mathbf{m}} \partial_i \hat{\mathbf{m}} + (1 - \hat{\mathbf{m}} \hat{\mathbf{m}})^2] + \text{interaction terms}$$

$$= \frac{\rho_s}{2} \int d^2r [\sigma^2 \partial_i \hat{\mathbf{m}} \partial_i \hat{\mathbf{m}} + (1 - \sigma^2)^2] + \ldots, \quad (20)$$

with $\rho_s$ representing the spin–stiffness, e.g. $4\pi \rho_s = \sqrt{\pi}/32$ for the Coulomb interaction. Of course other potentials for the scalar field are possible similar to the one derived from the QCD scaling behaviour to describe bag formation in hadron physics [17]. However up to fourth order in the gradient expansion the exact form of the potential will not matter provided that it reproduces (18) correctly. Note in this context that gradients on the scalar field are of higher order and may be neglected.

None of the previously proposed effective actions allows for bag formation. The collective approaches [13, 14, 15, 7] operate with unitary rotations which restrict the magnetization to the unit vector $\hat{\mathbf{m}}$ right from the beginning. Derivations from
Chern-Simons theories have a scalar field included which however is not taken seriously as a dynamical field [16].

In order to arrive at a non–linear $O(3)$ type of model, the scalar field may now be eliminated from (20) in favour of a symmetric fourth order term with definite strength related to the spin–stiffness $E[\hat{m}] = \rho_s^2 \int d^2r \left[ \partial_i \hat{m} \partial_i \hat{m} - \frac{1}{4} (\partial_i \hat{m} \partial_i \hat{m})^2 + O(6) \right] + \text{interaction terms}$. (21)

It should be emphasized that although the above $O(3)$ model deals with a unit vector $\hat{m}$ the bag is still formed via the relation to the magnetization (18). Eqs. (18) and (21) are at variance with the corresponding equations in [15]. The full energy functional, including interaction and Zeeman terms is given by

$$E[\hat{m}] = \frac{\rho_s}{2} \int d^2r \left[ \partial_i \hat{m} \partial_i \hat{m} - \frac{1}{4} (\partial_i \hat{m} \partial_i \hat{m})^2 \right] + \frac{1}{2} \int d^2r_1 d^2r_2 \rho^C(r_1) V(r_1 - r_2) \rho^C(r_2)$$

$$+ \frac{g}{4\pi} \int d^2r \left[ 1 - \left( 1 - \frac{1}{4} \partial_i \hat{m} \partial_i \hat{m} \right) \hat{m}_3 \right],$$

with $\rho^C$ replaced by $\rho^T$ [19] in lowest order gradient expansion. This energy functional is clearly charge conjugation invariant as it should be. It is unique up to $O(4)$ in the gradients and linear terms in the Zeeman coupling. The essential difference to previously proposed effective field theories is the additional symmetric fourth order term and of course the nontrivial connection of the $O(3)$ field with the magnetization [18].

The energy functional (22) captures the essential features of the microscopic approach discussed in section 2. For zero range forces, $4\pi \rho_s = t/4\pi$, and vanishing Zeeman coupling it simplifies with the hedgehog-ansatz (1) inserted

$$E[F] = \frac{\rho_s}{2} \int d^2r \left[ \left( F'^2 + \frac{\sin^2 F}{r^2} \right) - \frac{1}{4} \left( F'^2 - \frac{\sin^2 F}{r^2} \right)^2 \right].$$

(23)

It is immediately verified that the Belavin-Polyakov soliton [13] solves the corresponding stability condition. With this solution inserted the second term in (23) vanishes leaving only the pure non–linear sigma model. Hence, the soliton energy is $E = 4\pi \rho_s = t/4\pi$ as in the microscopic calculation. Also, the charge is related to the integral over the bag

$$\frac{1}{2\pi} \int d^2r (1 - \sigma) = \frac{1}{8\pi} \int d^2r \left( F'^2 + \frac{\sin^2 F}{r^2} \right) = 1$$

(24)

as in subsection 2.1. Again, for finite Zeeman coupling the soliton collapses. None of the previously proposed effective actions has this simple property that it gives
the correct result for extended spin-textures with hard-core forces. It is clear that in order to obtain these results, the symmetric fourth order term in (22) is essential.

In the Coulomb case, we obtain also solitons for non-zero but weak Zeeman couplings. Because the repulsive interaction term scales with the inverse soliton size $1/\lambda$ it competes with the attractive symmetric fourth order term proportional to $-1/\lambda^2$ and there exists a critical soliton size, determined by the Zeeman coupling strength, below which the soliton ceases to exist. In this way the effective theory models qualitatively the phase-transition discussed in subsection 2.2. However, because of the gradient expansion employed, it is not expected that the energy functional (22) remains valid for too small solitons. Down to which soliton sizes (22) may give a fair description of the microscopic HF calculation has still to be investigated.

In the 1-soliton sector it appears that the effective field theory has not much advantage over the microscopic approach. The numerical difficulty in solving the non-linear (integro-) differential equation for the profile function deduced from (22) is comparable to that of the HF calculation. However for multi-soliton systems such as skyrmion gases and crystals [18, 19, 20] the virtues of the effective field theory become apparent. In terms of collective coordinates the dynamics of such multi-soliton systems may efficiently be described. Also beyond tree approximation when loop corrections are considered the effective field theory approach might have advantages. On the microscopic level this corresponds to go beyond HF which is complicated.

In the following we give a brief argument why properties like bag formation discussed in the context of the HF calculation are not explained by loop corrections.

3.2 Loop corrections

Intuitively it is clear that the HF calculation on the microscopic level should correspond to the tree approximation of the effective theory. Accordingly, bag formation (18) and the symmetric fourth order term in (21) appear already classically. Could quantum corrections be responsible for these effects as suggested in [6]?

In the absence of an appropriate expansion parameter like the number of colors in hadron physics the role of loop corrections is difficult to judge. In fact, loop corrections may lead to bag formation by softening the constraint $\hat{m}^2 = 1$. Also the energy is lowered and the charge distribution broadened. Using heat kernel expansion methods one can even show that loop corrections effectively lead to a symmetric fourth order term with the correct (attractive) sign. Nevertheless, we argue that these effects observed in the HF calculation cannot be attributed to loop corrections for the subsequent argument.

Let us for simplicity consider only the non-linear sigma model part in the effective action. The equations for the fluctuations are independent of the strength of that term, i.e. the spin-stiffness $\rho_s$. The only dimensionful parameter which appears is
the size $\lambda$ of the BP soliton. For that reason the effective symmetric fourth order term produced by the heat kernel expansion comes with a coefficient proportional to $\lambda$ such that the total term scales like $1/\lambda$. In fact, the whole Casimir energy scales like $1/\lambda$ as was explicitly demonstrated in [21]. In any case, the spin–stiffness does not appear at all, which implies that the symmetric fourth order term found in [21] necessary to model the microscopic HF calculation cannot be explained by quantum fluctuations. From that argument it becomes clear that the bag is already formed in tree level according to (18) and (21).

The above reasoning does not imply that fluctuations are unimportant. In fact, we expect significant loop corrections [21], which among other things also will intensify the bag formation. But in the microscopic picture this would correspond to go beyond HF [10] performing e.g. a RPA calculation.

4 Conclusions

Charged skyrmions in quantum Hall ferromagnets at filling factor $\nu = 1$ were re-analysed in the framework of the HF theory. It was demonstrated, that within the LLL approximation the hedgehog ansatz leads to the lowest energy configuration in the unit charge sector. Calculating all three components of the magnetization vector it was noticed that its length deviates locally from unity, i.e. a bag is formed which accommodates the excess charge. The physical charge density is extremely well approximated by the bag, but in general deviates considerably from the topological density. Hence, the commonly accepted and celebrated equivalence of physical and topological charge does not hold locally.

In terms of a gradient expansion an improved $O(3)$ type of effective action is proposed for the quantum Hall ferromagnet, which takes bag formation into account. Accordingly, the magnetization vector is nontrivially related to the $O(3)$ unit vector field. In addition to previous approaches, the effective action contains an attractive symmetric fourth order term. For hard-core forces it was explicitly demonstrated that with these amendments made the correct microscopic result for extended spin–textures is obtained. For more realistic forces the effective action models the phase–transition anti–skyrmion to quasi–hole (skyrmion to quasi–particle) observed in the microscopic calculation when the Zeeman splitting is increased. The additional fourth order term also modifies the soliton–soliton interaction, e.g. it influences the dipole–strength in the dipole–dipole force for skyrmions. By this means the properties of multi–skyrmion systems such as skyrmion gas and lattice are affected.

The present investigation can be completed in several respects. The range of validity of the proposed energy functional for realistic two–body forces may carefully be tested by comparison with the microscopic HF calculation. Also the time–derivative part of the effective action [15] is of interest and will be subject to changes due to the bag. Beyond that, the derivation of a corresponding effective field theory without
resorting to the gradient expansion is desirable. Such a theory should, in addition to the $O(3)$ unit vector field, contain a scalar field for the magnetization vector’s length as a dynamical field. Finally, loop corrections may be calculated along the lines of ref. [21]. On the microscopic level this can be complemented by RPA type calculations which take fluctuations into account.

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