Constraints of students in drawing logarithmic graphs based on Polya steps and scaffolding implementation

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Abstract. This study aimed to describe the constraints of high school students in drawing logarithmic graphs based on Polya steps and scaffolding implementation. This study used descriptive explorative approach. This study was involved 6 students of grade X. The instruments were sheets of problems and interview guides. The six students were asked to draw logarithmic graphs. Two respondents were selected because they answered all problems then interviewed to clarify the answers based on Polya steps and to identify the constraints they faced. Constraints experienced by the respondents based on Polya steps were in understanding the problem and devising a plan. The results of this study were (1) constraints experienced by the respondents based on Polya steps were in understanding the problem and devising a plan; (2) In understanding the problem, the respondents experienced difficulties in understanding logarithmic symbols; (3) In devising a plan, they were constrained in choosing the technique to be used so that they used table. The results of this study can help teachers know the constraints of students in drawing logarithmic graphs and determine the appropriate scaffolding for students who experienced the same constraints with the respondents.

1. Introduction

Mathematics learning in schools often uses graphs as a solving way. The graphs in mathematics can be interpreted as an image showing a state relating to mathematical concept. Some graphs studied in schools include graph of straight-line equations [1], graphs of quadratic functions [2], exponential graphs, and logarithmic graphs [3]. Graphs can be used to determine the solve system of equations. In addition graph is also one form of representation in mathematics. The ability to draw a graph can show a deep knowledge of its function.

Graphs have been studied by several studies. There was a research that studied the construction and interpretation of pre-service teachers on graphs and found that the pre-service teacher did not succeed in drawing graphs for the given problem while in this study, it involved the students [4]. There was research that studied the student understanding of functions, one of which is seen by determining the types of function based on the given graph [5]. There was research that studied how to determine and overcome student misconceptions about graphs in calculus using GeoGebra software as a form of scaffolding and found that students learn better with software than students who studied without software while in this study we applied scaffolding according to constraints experienced by students [6]. There was research that studied students’ thinking and reasoning when analyzing quadratic functions and drawing graphs [7]. This is different from this study which focuses only on drawing graphics. There was research that studied the college students’ error in drawing function graphs [8].
There was research that studied students’ response to information presented in symbols, tables, graphs, and diagrams in logarithms [9].

One of the graphs studied in school is the logarithmic graph. This graph is studied after students learn about the concept of logarithms [10]. This graph is the result of the reflection of the exponent graph towards the line y = x. Logarithms are an inverse function of exponentials [11]. Student can use graph method to one of ways to solve logarithm problem.

Logarithmic became a topic studied by several researchers. One of researchers found that some students made mistakes in solving logarithmic problems [12]. These errors include (1) error reading problems, caused by lack of understanding of students in reading questions; (2) the mistake of choosing a method of settlement, caused by a lack of understanding of the properties of logarithms. Another researcher studied the understanding of students of grade X about logarithms [13]. There was research that studied about students' understanding and misconception on logarithmic materials and found that students were skilled in performing routine calculations but were less skilled in answering questions that required a higher level of thinking [14]. From that research, logarithms become quite difficult material for some students by being seen from the error in doing logarithmic problem. This study looks at how students draw logarithmic graphs.

Errors that students do in working on a problem is an indication that the student is having constraints in doing it. Constraints experienced by students can be interpreted as a factor that restricts students in solving math problems. Some studies established constraints as topics. One of researchers studied constrains of Indonesian student in algebra learning and found that student experience constraints in mathematization, understand problem that contain algebraic expressions, and use arithmetic operations to solve the problem [15]. They also studied constraints of Indonesian students and found that student experienced constraints in understanding the algebraic expression and concept of variable [16]. They also studied constraints of Indonesian students and found that students experienced constraints in making a mathematics model to solve algebraic problem [17]. There was research that studied the students' constraints to the enumeration rules, which showed that students’ constraints included difficulty in defining and understanding procedures in solving problems [18]. Another researchers studied students’ constraints to learn ratios and proportions and found that students’ constraints included difficulty in understanding material and difficulty in solving problems using different contexts [18]. There was a research that studied the students’ constraints to inverse learning functions and found students’ constraints between internal factors (motivation and ability) and external factors (curriculum) [20].

One of the causes of students having constraints is the difficulty level of the matter is higher than the student's ability. Vygotsky introduces Zone of Proximal Development (ZPD) which is a zone where students cannot do their own work, but the students can do it when assisted by others [21]. Then appears the term scaffolding which has meaning of the help given so that students can solve the problem given. With the existence of ZPD and scaffolding, students who experience constraints are expected to be able in solving the problem given. There are studies which studied scaffolding. There was research that studied about scaffolding that was used to solve students’ error in solving PISA 2012 and found the type of scaffolding that was used in this study which were understanding the problem by explaining the problem and checking the answers [22]. Another researcher studied about media scaffolding i.e. chart, visual aids & figure to increase understanding of geometry concept [23].

One of frameworks that is most commonly used for solving problems is Polya framework. There were four steps in finding a solution namely (1) understanding the problem; (2) devising a plan; (3) carrying out the plan; (4) looking back [24]. Several studies have studied the Polya framework. There was research that studied the errors of students in solving permutations and combinations based on Polya's steps [25]. Another researcher studied the processes involved in problem solving during collaborative learning in the classroom based on Polya's steps [26]. There was research that introduced the Polya problem-solving method in the statistic classroom to improve students' skills [27]. Another researcher studied the construction process of symbolic and verbal representation made by students in solving problems based on Polya's steps [28]. There was research that studied the difficulties of
students in solving problems and ways to handle them using the Polya step [29]. Another researcher studied the types of student errors and their causes in solving problems [30].

Based on the explanation, this study aimed to describe the constraints of high school students in drawing logarithmic graphs based on Polya steps and scaffolding implementation to handle these constraints.

2. Method Another section of your paper

This study used a descriptive explorative approach because this study described the constraints of high school students in drawing logarithmic graphs based on Polya steps and scaffolding forms according to the constraints. This study involved 6 tenth grade students of Surya Buana High School Malang City.

2.1. Data Collection and Study Steps

The main instrument is researchers because researchers are the ones who plan, carry out research, and report the results of research. The supporting instruments used were 3 questions with logarithmic materials and interview guidelines. Questions used in this study were presented in figure 1 as follows.

Given a graph of function \( f(x) = \log_2 x \).

- a. Sketch the graph of function \( g(x) = \log_2 x^3 \) with the help of a known graph. Then explain how you sketched the graph.
- b. Sketch the graph of function \( h(x) = \log_2 2x \) with the help of a known graph. Then explain how you sketched the graph.
- Sketch the graph of function \( p(x) = \log_2 \frac{x}{a} \) with the help of a known graph. Then explain how you sketched the graph.

![Figure 1. Example of problems](image)

Six students were asked to draw a logarithmic graph and then the respondents were selected based on the most complete answers (answered all the problem based on Polya criterion). The respondents were interviewed to clarify the answers based on Polya's steps and to identify the constraints faced by the respondents. The Polya steps used in this study were 4 steps (understanding problems, devising plans, carrying out plans, looking back).

Based on the results of the given tasks to six students, the researchers set two respondents. S1 and S2 were the respondents who answered all the questions. They were interviewed one by one. They were interviewed about understanding the problem such as symbol and prior knowledge that can be used to solve the problem, plan to solve the problem and activity of looking back.

2.2 Data Analysis

Data analysis was done by data reduction, data presentation, and then withdrawal of a conclusion. Data reduction was performed to select data that can be used to answer research questions. The selected data was then represented before it was deduced from the data.

The indicators used to analyze the results of student work were presented in Table 1.
Table 1. Indicator of Polya Steps

| Polya’s Steps       | Indicators                                                                 |
|---------------------|-----------------------------------------------------------------------------|
| Understanding the problem | - Telling what is understood from the given problem                            |
|                     | - Recognizing the symbols contained in the problem                            |
|                     | - Knowing the given information to answer the problem                         |
| Devising a plan     | - Explain the chosen ways to answer the problem                                |
| Carrying out a plan | - Writing the processes that correspond to the chosen ways to answer the problem |
| Looking back        | - Checking answers                                                            |
|                     | - Explaining the suitability of answers with problems                         |

3. Result and Discussion

Researchers use 4 steps Polya to know the constraints experienced by S1 and S2 in drawing the graph.

3.1 Description of Constraints Experienced S1 in Drawing of Logarithmic Graphs

The first step in solving the problem was to understand the problem. In order to know how S1 understands the given problem, the researcher conducts the interview. Here is the interview quotes.

\[ P : \text{Your answer likes this. How could you draw a graph like that? Have you ever worked on a similar problem?} \]

\[ S1 : \text{I never did it before. In the school, teacher only gave the table like this but never gave the way to solve it. The way to prove this graph is by using table. So, that is how I draw a graph, make a table first, then Table of x and next substitute into formula to get y} \]

\[ P : \text{This symbol (pointing to the symbol } \log_2 x \text{) ... do you understand that it is same as } 2 \log x? \]

\[ S1 : \text{At the first I do not understood. But the teacher said that it is same. It is same as } 2 \log x. \]

\[ P : \text{What do you understand from this (pointing to the given graph) from this graph. What do you understand from this graph? Tell me what you know from it?} \]

\[ S1 : \text{What I understand is that there is a value of x, there is a function, then there is a graph x y then value of x let I substitute into formula getting } 2^{\log 1} 0, \text{ right? Value y is 0. Then each of these numbers is paired with its y so draw a graph draw line to connect it.} \]

\[ P : \text{What do you think about the function of graph? Do you think that the function of graph is to answer the problem?} \]

\[ S1 : \text{Oh. I didn’t think so. Because I was taught the way to answer it by using table in the school so if I was asked to draw graph, I use table.} \]

Based on the interview quote, S1 could understand the given problem which is shown from the explanation of what is known from the given graph. Explanation that is bold shows that S1 did not recognize the symbol in the problem written as \( \log_a b \). Logarithm form that understood is \( 2^{\log x} \). This finding is in line with Piaget’s theory of learning [21] that students of tenth grade should have undergone a formal operational stage in which students are able to think abstractly using symbols. In the step of devising a plan, S1 stated that the step used is to create a table because the problem in this study is included in open ended problem. The open-ended approach can also be interpreted to get many answers to a problem [31]. However in this study, the problem can be worked out with different
strategies to get the same answer. The tables used by S1 were presented in Figure 2, Figure 3, and Figure 4.

Based on the table used by S1, it is shown that S1 understood the relation between logarithm and exponent because S1 selected the numbers corresponding to $2^x$. In performing the step of carrying out the plan, S1 plotted the dots corresponding to the created table then connected it to the logarithmic graph. This result is in line with Piaget's theory of learning [21] that S1 had scheme of steps to draw graph then S1 did assimilation because the problem can be solved by that scheme. The graphs made by S1 were presented in Figure 5 and Figure 6.
In the looking back step, S1 did not check the answers they are already given. This result is in line with the research findings of two researches that student did not have habit to check the answers after doing a problem [32], [33]. S1 realized that he did not draw graphs according to the command in the question. S1 did not think of any other way to draw graphics besides using table. At this step, researchers gave scaffolding so that S1 could draw graphs in other ways. Here is the interview quotes to provide scaffolding for problem 1.

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P : Do you remember logarithmic properties?
S1 : I don’t think so
P : Write one you remember!
S1 : (writing $a \log b^n = n \cdot a \log b$)
P : This is $2 \log x^3$.
S1 : So this is (writing $3 \cdot 2 \log x$)
P : Right. That’s true. Do you realize that if it is related to this one (question 1),
this given a graph $2 \log x$. Then a simple way? You must calculate again.
S1 : Wait a minute.
P : If it is related to number 3?
S1 : What do you mean?
P : If 1 to 3, 2 to 6. If it is related to 3?
S1 : Just multiple by 3

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Based on the interview, S1 could conclude that the requested graph on problem 1 can be obtained by enlarging the graph $2 \log x$ by a scale factor 3. Then researcher gave a similar problem with problem 1. The answer is presented in figure 7.
Based on that graph, S1 could connect logarithmic properties and logarithmic graphs. Here is the interview quotes to provide scaffolding for problem 2.

\[ P : \text{If this (problem 2). What properties that used?} \]
\[ S1 : \text{It is same as } 2 \log 2x \]
\[ P : \text{Right. Remember other properties?} \]
\[ S1 : \text{This. Is it right? (writing } 2 \log 2x = 2 \log 2 + 2 \log x) \]
\[ P : \text{Yes, that’s right. If } 2 \log 2? \]
\[ S1 : \text{What is the power of 2 that is equal to 2? It means that this is added to 1} \]
\[ P : \text{If added to 1, what do you do to the graph?} \]
\[ S1 : \text{Up} \]

Based on the interview, S1 could conclude that the requested graph on question 2 can be obtained by shifting the graph \(2 \log x\) upwards. Then researchers gave a similar problem with problem 2. The answer is presented in figure 8.
Based on that graph, S1 could connect logarithmic properties and logarithmic graphs. Here is the interview quotes to provide scaffolding for problem 3.

\[ P \quad : \quad \text{So this?} \]
\[ S1 \quad : \quad 2 \log_8 x. \text{It means that like this} \quad (\text{writing} \quad 2 \log_8 x = 2 \log x - 2 \log 8) \quad \text{because it is multiplication, this division. It means that the result} \quad 2 \log x \quad \text{then the table} \]

\[ P \quad : \quad \text{If subtracted by 3 then what do you do with the graph?} \]
\[ S1 \quad : \quad \text{Lowered. Each point is subtracted by 3.} \]

Based on the interview, S1 could conclude that the requested graph on problem 3 can be obtained by shifting the graph \(2 \log x\) down by 3. Then researchers gave a similar problem with problem 3. The answer is presented in figure 9.

![Figure 9](image)

**Figure 9.** Student’s answer after applying scaffolding

Based on that graph, S1 could connect logarithmic properties and logarithmic graphs.

### 3.2 Description of Constraints Experienced S2 in Drawing of Logarithmic Graphs

The first step in solving the problem was to understand the problem. In order to know how S2 understands the given problem, the researchers conducted the interview. Here is the interview quotes.

\[ P \quad : \quad \text{You can draw the logarithmic graph, where do you know that graph? Have you ever drawn on a logarithmic graph?} \]
\[ S2 \quad : \quad \text{In the previous material, I have drawn a logarithmic graph. Indeed in the first semester there is learning about the material and I have learned how to draw logarithm in outline.} \]

\[ P \quad : \quad \text{So the first step you used to draw graph is make a table} \]
\[ S2 \quad : \quad \text{Yes} \]
\[ P \quad : \quad \text{So why did you use 1, 2, 4, 8, 16?} \]
\[ S2 \quad : \quad \text{Find the easy one} \]
\[ P \quad : \quad \text{Why do you say that it is easy? Why the multiple by 2?} \]

In the step of creating a plan, the step used in drawing a graph was to create a table. The tables used by S1 are presented in Figure 10 and figure 11.
Based on the results of S2’s work, it appears that S2 changed the logarithmic form on problem 1 and problem 3 using logarithmic properties (shown in the box) before creating the table. In problem 3, S2 made a mistake in changing the form $\log_2 \frac{x}{8}$. The error in this step caused the S2 to misrepresent the logarithmic graph shown in Figure 11. This error is in the line with the research findings of one research that students still had difficulties in altering the logarithmic form and S2 underwent a theorem distortion [34]. S2 tried to use the rule $\log_a a = 1$ to change the logarithmic form.

In carrying out the plan, S2 plotted the points corresponding to the created table and then related them to the logarithmic graph. The graphs made by S2 were presented in figure 12 and figure 13.

Figure 10. Table made by S2 for question 1 (10a) and question 2 (10b)

Figure 11. Table made by S2 for question 3

Figure 12. Graphs made by S2 for question 1 (12a) and question 2 (12b)
In the looking back step, S2 has no other way to draw graphics by using the logarithmic properties. Here is the interview quotes to provide scaffolding for problem 1.

- **P**: You already know, this one means which properties?
- **S2**: This, this one (writing $a \log b^n = n \cdot a \log b$)
- **P**: If it is related to number 3. That means, what do you do from the initial graph?
- **S2**: Hmm, are this (pointing to the given question) and this (question 1) related?
- **P**: Look at the order to draw the graph of $2 \log x^3$ with the help of the known graph.
- **S2**: Yes
- **P**: The graph $2 \log x$ is this, is not it? (pointing the graph of $2 \log x$). Then when you want to draw $2 \log x^3$ if using this (pointing the graph of $2 \log x$) this means what will you do?
- **S2**: Enlarge
- **P**: How many?
- **S2**: With ratio 2, eh 3, the ratio is 3.
- **P**: How many is the n?
- **S2**: n is 3.
- **P**: So it means enlarge?
- **S2**: With multiply to 3.

Based on the interview, S1 could conclude that the requested graph on problem 1 can be obtained by enlarging the graph $2 \log x$ by a scale factor 3. Here is the interview quotes to provide scaffolding for problem 2.

- **P**: ... Logarithmic properties again. Remember?
- **S2**: Not really for this one. (writing $a \log b x = a \log b + a \log x$)
- **P**: If it’s become like this $2 \log 2x$
- **S2**: $2 \log 2x$ oohh I see 1 + $2 \log x$
- **P**: So it means what do you do with this graph?
- **S2**: Add 1.

Based on the interview, S1 could conclude that the requested graph on problem 2 can be obtained by shifting the graph $2 \log x$ upwards. Here is the interview quotes to provide scaffolding for question 3.

- **P**: If this is multiplication, if this is division, could it be related?
S2 : If I write like this, could it be $2\log_2 2 \times 3$? (writing $2\log 2$) ooh so it couldn’t be. (writing $a\log \frac{b}{x} = a \log b - a \log x$)

P : So how then?

S2 : Reduced by 3.

P : So what will you do?

S2 : That graph is reduced by 3.

Based on the interview, S2 realized that the graphs made were wrong. With scaffolding, S2 can conclude that the graph requested in question 3 can be obtained by shifting the graph $2\log x$ downward by 3. This result is in line with the research founding of one research that students used accommodation in solving the problem [25]. In this study, students used accommodation when they received scaffolding then they will make a new scheme. This scheme can be used to solve similar problems.

Based on the information from the class teacher, two respondents are students who had high mathematical competence so that the experienced constraints were slight. In accordance, the applied scaffolding strategies were also minor. This is consistent with finding of one of research who found that students were skilled in performing routine calculations but were less skilled in answering questions that required higher levels of thinking [14]. It is because respondents having high mathematical ability can answer the problem.

4. Conclusion and Recommendations

Based on students’ answers and interview results, the constraints experienced by students in drawing logarithm graphs viewed based on Polya steps and the application of scaffolding are as follows.

4.1 Understanding the Problem

In the step of understanding the problem, students experienced constraints in understanding logarithmic symbols. Students did not recognize the symbols in the question. The logarithm form that is understood by the students is $a \log b$ while in the questions it was written as $\log a \cdot b$. Students felt unfamiliar with the symbol which caused the students did not understand the problem. Another side, students had less practice in drawing graphs. In the learning of logarithms at school, students are taught logarithmic graph by telling students about what the logarithmic graph looks like. The applied scaffolding technique inform the students that the symbol is the same as $a \log b$. In addition, the students could explain the dots passed by the logarithmic graphs.

4.2 Devising a Plan

In the step of devising a plan, students decided to create tables to draw logarithmic graphs. They selected the number corresponding to $2^x$. It shows that they knew the correlation between logarithmic and exponential. It was not match with the instructions but the given answers were true. This problem was included in open ended questions. Logarithmic graphs could be drawn using tables and could also be obtained by shifting graphs using logarithmic properties. At this step, there was no need for implementing scaffolding strategies.

4.3 Carrying Out a Plan

In the step of carrying out a plan, the students plotted the points correctly based on the table that were previously made in the step of plan making. Thus, they connected the point to make graph. In this step, scaffolding was not implemented.

4.4 Looking Back

In the step of looking back, the students did not check the answers they were already given. The students realized that the answers they were given did not match the instructions on the questions.
They also did not check the results of calculations. At this step, the applied scaffolding was asking the students to remember the logarithmic properties and then connect with the logarithmic form included in the questions.

4.5 Recommendations

Teachers can use the results of this study as a reference in determining appropriate scaffolding for students who experience the similar constraints with the respondents in this study. In understanding the problem step, the teachers are suggested to familiarize the students to reread the provided information and relate it to the prerequisite knowledge. Besides teacher can improve students’ skill in understanding a problem through drill. In devising a plan step, the teachers are suggested to give students opportunity to choose the strategies that suit their knowledge. In carrying out plan step, the teachers are suggested to familiarize student to make an analogy into simpler form and use translation or other transformations. In looking back step, the teachers are suggested to familiarize the students to do looking back step in order to train students’ reasoning. Students who are used to look back will have better reasoning than students who are not used to check back. For the further study, it is suggested to conduct the study by involving students with medium and low mathematical competence.

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