\[(I, J^P) = (1, 1/2^+) \Sigma NN \text{ quasibound state}\]

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Abstract

JLab has recently found indications of the possible existence of a $\Sigma NN$ resonance at $(3.14 \pm 0.84) - i(2.28 \pm 1.2)$ MeV. In the past, using models that exploit symmetries between the two-baryon sector with and without strangeness, hyperon-nucleon interactions have been derived that reproduce the experimental data of the strangeness $-1$ sector. We make use of these interactions to review existing Faddeev studies of the $\Lambda NN - \Sigma NN$ system that show theoretical evidences about a $(I, J^P) = (1, 1/2^+) \Sigma NN$ quasibound state near threshold. The calculated position of the pole is at $2.92 - i2.17$ MeV, in reasonable agreement with the experimental findings.

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Hall A Collaboration at Jefferson Lab has made use of the \((e, e'K^+)\) reaction to study the possible existence of neutral three-body \(\Lambda\) and \(\Sigma\) hypernuclei \([1]\). They reported an excess of events around the \(\Sigma\) thresholds. The most significant enhancement appears \(3.14 \pm 0.84\) MeV below the \(\Sigma^0nn\) threshold and has a width of \(\sigma \approx 2.28 \pm 1.2\) MeV. It possibly hints at a bound \(\Sigma^0nn\) \((I = 1)\) state.

The existing experimental data and the expected forthcoming optimized data call for theoretical studies that could help with their interpretation. In this letter it is our purpose to emphasize the relevant findings of existing Faddeev studies of the \(\Lambda NN - \Sigma NN\) system. The theoretical results obtained are a valuable tool to analyze the Hall A Collaboration data.

We have carried out a detailed study of the \(\Lambda NN - \Sigma NN\) three-body system at threshold looking for bound states or resonances \([2]\). The strangeness \(-1\) two-body interactions have been derived from the chiral quark cluster model (CQCM) \([3]\), by exploiting the symmetries with the two-nucleon sector. In the CQCM hadrons are clusters of massive (constituent) quarks. As color carriers, massive quarks are confined through a confining potential. They interact through a one-gluon exchange potential arising from Quantum Chromodynamics (QCD) perturbative effects. The non-perturbative effects generate one-boson exchange potentials between quarks \([3, 4]\).

The nucleon-nucleon \((NN)\) and hyperon-nucleon \((YN)\) interactions describe reasonably well the \(NN\) and \(YN\) two-body observables \([4]\). In particular, the two-nucleon system low-energy parameters, the \(NN\) \(S\)–wave phase shifts, and the triton binding energy are described correctly \([3]\). Besides, there is a reasonable agreement with the hyperon-nucleon elastic and inelastic scattering cross sections and the hypertriton binding energy. Finally, the isospin one \(\Lambda nn\) system is unbound \([2, 4, 5]\).

At the two-body level, the \(N\Lambda - N\Sigma\) coupling as well as the tensor force, responsible for the coupling between \(S\) and \(D\) waves, have been considered. The \(\Lambda \leftrightarrow \Sigma\) conversion is crucial to have a correct description of the \(\Lambda NN\) system \([2]\). The \(NN\) and \(YN\) interactions contain sizable non-central terms which are responsible, among others, for the deuteron binding energy. The relevance of the \(YN\) tensor force becomes apparent when studying the \(\Sigma^-p \rightarrow \Lambda n\) reaction. Such process is controlled by the \(\Sigma N(\ell = 0) \rightarrow \Lambda N(\ell = 2)\) transition so that if only the central interaction \(\Sigma N(\ell = 0) \rightarrow \Lambda N(\ell = 0)\) is considered, the cross section cannot be described correctly \([6]\). The non-central \(N\Lambda - N\Sigma\) interaction induces
a three-body force through the coupling between $YN$ channels with $(\ell, \lambda) = (0, 0)$ and $(\ell, \lambda) = (2, 2)$, where the $YN$ relative orbital angular momentum is denoted by $\ell$ and $\lambda$ stands for that of the spectator nucleon respect to the $YN$ system.

For this study different models have been designed by choosing sets of spin-singlet and spin-triplet $\Lambda N$ scattering lengths describing correctly the available experimental data. In particular, besides a reasonable description of the $YN$ cross sections, the hypertriton binding energy corresponds to its experimental value within the error bars $B_{0,1/2} = 0.130 \pm 0.050$ MeV [7]. The upper limit of the $\Lambda N$ spin-triplet scattering length, $a_{1/2,1}^{\Lambda N}$, has been established by requiring that the $(I, J^P) = (0, 3/2^+)$ $\Lambda NN$ state does not become bound [2]. The lower limit was set by requiring a correct description of the $YN$ cross sections, which deteriorates markedly as the $\Lambda N$ spin-triplet scattering length decreases. Thus, it was found that $1.41 \leq a_{1/2,1}^{\Lambda N} \leq 1.58$ fm. Once the $\Lambda N$ spin-triplet scattering length has been defined, the $\Lambda N$ spin-singlet scattering length, $a_{1/2,0}^{\Lambda N}$, was constrained by demanding that the hypertriton binding energy is in the experimental interval $B = 0.130 \pm 0.050$ MeV, leading to $2.33 \leq a_{1/2,0}^{\Lambda N} \leq 2.48$ fm. Without loss of generality we take the model with $a_{1/2,1}^{\Lambda N} = 1.41$ and $a_{1/2,0}^{\Lambda N} = 2.48$ as the reference model. All calculations have been performed for several models within the scattering lengths intervals and the conclusions remain unchanged. For the reference model the hypertriton binding energy obtained is 129 keV. Just to illustrate the relevant role played by the $D$ waves of the three-body system, it is worth to note that considering only $S$ wave three-body channels the hypertriton binding energy is 89 keV, out of the experimental range.

The solution of the three-body problem have been described elsewhere [2] and are out of the scope of this letter. We focus on the results concerning the possible existence of a $\Sigma NN$ $(I, J^P) = (1, 1/2^+)$ resonance [4].

Let us first discuss the attractive or repulsive character of the different $J^P = 1/2^+$ $\Sigma NN$ channels. Fig. 1 shows the Fredholm determinant of the $J^P = 1/2^+$ $\Sigma NN$ channels below the $\Sigma d$ threshold, where the continuum starts. The Fredholm determinant of the $I = 0$ and 1 channels is complex because the $\Lambda NN$ channels are open. The imaginary part is small and uninteresting. It can be seen that the channel showing the most attractive character is the $(I, J^P) = (1, 1/2^+)$. For an attractive channel the Fredholm determinant, $D_F$, is smaller than 1 and it becomes negative if a bound state exists [5]. Thus, as the Fredholm determinant is very close to zero at the $\Sigma d$ threshold is a clear indication of a quasibound state. The $(I, J^P) = (0, 1/2^+)$ channel is also attractive, but far less than the $I = 1$ one.
This can be easily understood as follows. We show in Table I the two-body channels that contribute to a given $J^P = 1/2^+ \Sigma NN - \Sigma NN$ state with isospin $I$. The most attractive two-body channels, in particular the $\Sigma N$ $^3S_1(I = 1/2)$ and $^1S_0(I = 3/2)$ and the $NN$ $^3S_1(I = 0)$, contribute to the $(I, J^P) = (1, 1/2^+) \Sigma NN$ state, however, the last two are forbidden for the $(I, J^P) = (0, 1/2^+) \Sigma NN$ state, one of them being the deuteron channel.

The most interesting result in connection with the results reported in [1] is the prediction of a $\Sigma NN$ $(I, J^P) = (1, 1/2^+)$ quasibound state in the region near threshold. We show in Fig. 2 the real, $\text{Re}(A_{1,1/2})$, and imaginary, $\text{Im}(A_{1,1/2})$, parts of the $\Sigma d$ scattering length as a function of the attraction in the three-body channel. The real part becomes negative while

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
$I$ & $(i_\Sigma, s_\Sigma)$ & $(i_\Lambda, s_\Lambda)$ & $(i_{N(\Sigma)}, s_{N(\Sigma)})$ & $(i_{N(\Lambda)}, s_{N(\Lambda)})$ \\
\hline
0 & (1/2,0),(1/2,1) & (1/2,0),(1/2,1) & (1,0) & (0,1) \\
1 & (1/2,0),(3/2,0),(1/2,1),(3/2,1) & (1/2,0),(1/2,1) & (0,1),(1,0) & (1,0) \\
2 & (3/2,0),(3/2,1) & & (1,0) & \\
\hline
\end{tabular}
\caption{Two-body $\Sigma N$ channels $(i_\Sigma, s_\Sigma)$, $\Lambda N$ channels $(i_\Lambda, s_\Lambda)$, $NN$ channels with $\Sigma$ spectator $(i_{N(\Sigma)}, s_{N(\Sigma)})$, and $NN$ channels with $\Lambda$ spectator $(i_{N(\Lambda)}, s_{N(\Lambda)})$ that contribute to a given $J^P = 1/2^+ \Sigma NN - \Sigma NN$ state with total isospin $I$.}
\end{table}
the imaginary part has a maximum, which are the typical signals of a quasibound state [8]. The position of the pole almost does not change for the different models, being at $2.92 - i 2.17$ MeV for the reference model. The width of this state comes mainly from the coupling to a $D$ wave $\Lambda NN$ channel. It is worth to emphasize that the enhancement suggested as a possible $\Sigma NN$ resonance by the Hall A Collaboration at Jefferson Lab appears at about $(3.14 \pm 0.84) - i (2.28 \pm 1.2)$ MeV [1], in very good agreement with the theoretical results reported in this study.

The existence of a $(I, J^P) = (1, 1/2^+)$ $\Sigma NN$ quasibound state was suggested in a variational calculation investigating the structure of the $A = 3$ $\Sigma$-hypernuclei [9]. Similar results were obtained by Harada and Hirabayashi [10] using a distorted-wave impulse approximation within a coupled $(2N - \Lambda) + (2N - \Sigma)$ model with a spreading potential. Afnan and Gibson found a near threshold $I = 0$ resonance while exploring $\Lambda d$ elastic scattering by a continuum Faddeev calculation [11]. Recent preliminary calculations [12] have suggested that the pole for the $I = 1$ resonance is also located near the $\Sigma NN$ threshold, but the two resonances are unlikely to be distinguished experimentally.

To conclude, Hall A Collaboration at Jefferson Lab [1] has found indications of the possible existence of a $\Sigma NN$ resonance at $(3.14 \pm 0.84) - i (2.28 \pm 1.2)$ MeV. The state is likely a $\Sigma^0 nn$ state, although this has to be confirmed by future experiments. We have presented a detailed study of $\Lambda NN - \Sigma NN$ system using the hyperon-nucleon and nucleon-
nucleon interactions derived from a chiral constituent quark model with full inclusion of
the $\Lambda \leftrightarrow \Sigma$ conversion and taking into account all three-body configurations with $S$ and
$D$ wave components. In the case of the $\Sigma NN$ system there exists a narrow quasibound
state near threshold in the $(I, J^P) = (1, 1/2^+)$ channel. The position of the pole is at
$2.92 - i 2.17$ MeV. There is a reasonable agreement with the enhancement suggested as a
possible $\Sigma NN$ resonance by the Hall A Collaboration at Jefferson Lab, appearing at about
$(3.14 \pm 0.84) - i (2.28 \pm 1.2)$ MeV, being our result inside the error bar of the experimental
data.

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