Discriminating the minimal 3-3-1 models

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We show that due to the $\rho$ parameter bound and the Landau pole limit, the reduced 3-3-1 model is unrealistic, while due to the $\rho$ parameter and FCNCs bounds, the simple 3-3-1 model is experimentally unfavored. All such conditions strictly constrain the gauge symmetry breaking scales of the minimal 3-3-1 model with three scalar triplets.

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Introduction: It is well known that the unsolved questions of the standard model, namely the number of fermion generations, uncharacteristic heaviness of the top quark, strong $CP$ problem, electric charge quantization, neutrino masses, and dark matter, can be addressed by the 3-3-1 models [1–7]. Moreover, the $B - L$ dynamics and resulting $R$-parity, leptogenesis, and inflaton can also be realized by this kind of the theories [8].

Recently, there have emerged three versions of the minimal 3-3-1 model, the reduced 3-3-1 model [9], the simple 3-3-1 model [10], and the minimal 3-3-1 model with three scalar triplets [11], which provide new theoretical and phenomenological aspects beyond the old ones. In this work, we will show experimentally favored degrees for such theories, simply replied on their $\rho$ parameter, FCNCs and Landau pole. The $Z$ and new $Z'$ gauge boson mixing is also analyzed.

The minimal 3-3-1 models: The gauge symmetry is given by $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1), where the first factor is the color group while the last two are the extension of the electroweak symmetry. The electric charge operator takes the form $Q = T_3 - \sqrt{3}T_8 + X$, where $Y = -\sqrt{3}T_8 + X$ is the weak hypercharge. Here, $T_i (i = 1, 2, 3, ..., 8)$ and $X$ are the $SU(3)_L$ and $U(1)_X$ charges, respectively (the color charges will be denoted by $t_i$). The fermions can be arranged as $\psi_{aL} = (\nu_{aL}, e_{aL}, e_{aR}^c) \sim (1, 3, 0)$, $Q_{aL} = (d_{aL}, -u_{aL}, J_{aL}) \sim (3, 3^*, -1/3)$, $Q_{3L} = (u_{3L}, d_{3L}, J_{3L}) \sim (3, 3, 2/3)$, $u_{aR} \sim (3, 1, 2/3), d_{aR} \sim (3, 1, -1/3), J_{aR} \sim (3, 1, -4/3)$, and $J_{3R} \sim (3, 1, 5/3)$, where $a = 1, 2, 3$ and $\alpha = 1, 2$ are generation indices. Note that the values in parentheses present quantum numbers based upon the 3-3-1 symmetries, respectively.

The minimal 3-3-1 model with three scalar triplets works with the following scalar fields $\eta = \ldots$

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Hence, we obtain two physical neutral gauge bosons (besides the photon), \((\eta_1^0, \eta_2^-, \eta_3^+) \sim (1, 3, 0), \rho = (\rho_1^+, \rho_2^0, \rho_3^{++}) \sim (1, 3, 1),\) and \(\chi = (\chi_1^-, \chi_2^-, \chi_3^0) \sim (1, 3, -1)\). The reduced 3-3-1 model works with \((\rho, \chi)\) by excluding \(\eta\), while the simple 3-3-1 model works with \((\eta, \chi)\) by excluding \(\rho\). The VEVs of the scalars are given by \(\langle \eta \rangle = (u/\sqrt{2}, 0, 0), \langle \rho \rangle = (0, v/\sqrt{2}, 0),\) and \(\langle \chi \rangle = (0, 0, w/\sqrt{2})\). The following calculations generally apply for all the models (for the reduced 3-3-1 model, \(u = 0\); for the simple 3-3-1 model, \(v = 0\)).

**Gauge boson masses and mixing:** We now derive the mass spectrum of the gauge bosons, which arises from the Lagrangian \(\sum_{\Phi = \eta, \rho, \chi} (D_\mu(\Phi))^\dagger (D^\mu(\Phi))\), where the covariant derivative takes the form \(D_\mu = \partial_\mu + ig_t t_i G_{i\mu} + ig_T A_{i\mu} + ig_X B_{\mu}\), with the gauge couplings \((g_t, g, g_X)\) and gauge bosons \((G_{i\mu}, A_{i\mu}, B_\mu)\), associated with the respective 3-3-1 groups. We have physical charged gauge bosons with respective masses,

\[
W^\pm = \frac{A_1 \mp iA_2}{\sqrt{2}}, \quad X^\pm = \frac{A_4 \pm iA_5}{\sqrt{2}}, \quad Y^{\pm\pm} = \frac{A_6 \pm iA_7}{\sqrt{2}},
\]

\[
m_W^2 = \frac{g^2}{8}(u^2 + v^2), \quad m_X^2 = \frac{g^2}{8}(u^2 + w^2), \quad m_Y^2 = \frac{g^2}{8}(v^2 + w^2).
\]

To keep consistency with the standard model, we impose \(u, v \ll w\). The field \(W\) is identical to the standard model charged gauge boson, which implies \(v_W^2 = u^2 + v^2 = (246 \text{ GeV})^2\), while \(X\) and \(Y\) are new gauge bosons with large masses in \(w\) scale.

For the neutral gauge bosons, the photon, \(Z\), and new \(Z'\) can be identified as

\[
A = s_W A_3 + c_W \left(-\sqrt{3} t_W A_8 + \sqrt{1 - 3 t_W^2} B\right),
\]

\[
Z = c_W A_3 - s_W \left(-\sqrt{3} t_W A_8 + \sqrt{1 - 3 t_W^2} B\right),
\]

\[
Z' = \sqrt{1 - 3 t_W^2} A_8 + \sqrt{3} t_W B,
\]

where \(s_W = e/g = t/\sqrt{1 + 4t^2}\), with \(t = g_X/g\), is the sine of the Weinberg angle \(\text{[12]}\). The photon field \(A\) is physical \((m_A = 0)\) and decoupled, whereas \(Z\) and \(Z'\) mix as given by the mass matrix,

\[
\begin{pmatrix}
m_Z^2 & m_{ZZ'}^2 \\
m_{ZZ'} & m_{Z'}^2
\end{pmatrix},
\]

where

\[
m_Z^2 = \frac{g^2}{4c_W^2}(u^2 + v^2), \quad m_{ZZ'}^2 = \frac{g^2 [2(1 - 2s_W^2)u^2 - 2s_W^2v^2]}{4\sqrt{3} c_W^2 \sqrt{1 - 4s_W^2}},
\]

\[
m_{Z'}^2 = \frac{g^2 [2(1 - 2s_W^2)^2u^2 + 2s_W^2v^2 + 4c_W^4w^2]}{12c_W^2 (1 - 4s_W^2)}.
\]

Hence, we obtain two physical neutral gauge bosons (besides the photon),

\[
Z_1 = c_\varphi Z - s_\varphi Z', \quad Z_2 = s_\varphi Z + c_\varphi Z',
\]
with the $Z$-$Z'$ mixing angle,

$$t_{2\varphi} = \frac{2m_{ZZ'}^2}{m_{Z'}^2 - m_Z^2} \simeq \frac{\sqrt{3(1 - 4s_W^2)}}{2c_W^4} \frac{[(1 - 4s_W^2)u^2 - (1 + 2s_W^2)v^2]}{w^2},$$  

and their masses,

$$m_{Z_1}^2 = \frac{1}{2}[m_Z^2 + m_{Z'}^2 - \sqrt{(m_Z^2 - m_{Z'}^2)^2 + 4m_{ZZ'}^4}] \simeq \frac{g_W^2}{4c_W^4}(u^2 + v^2),$$

$$m_{Z_2}^2 = \frac{1}{2}[m_Z^2 + m_{Z'}^2 + \sqrt{(m_Z^2 - m_{Z'}^2)^2 + 4m_{ZZ'}^4}] \simeq \frac{g_W^2c_W^2}{3(1 - 4s_W^2)}w^2.$$

The approximations for the masses are given at the leading order. Because the mixing angle $\varphi$ is small, we have $Z_1 \simeq Z$ and $Z_2 \simeq Z'$, which imply that the $Z_1$ is like the standard model $Z$ boson, while $Z_2$ is a new neutral gauge boson with a large mass in $w$ scale.

**$\rho$-parameter**: The experimental $\rho$ parameter (or $\Delta \rho \equiv \rho - 1$ used below) that is contributed (or induced) only by the new physics comes from the following sources. The first one is given at the tree-level due to the $Z$-$Z'$ mixing, which can be evaluated as

$$\langle \Delta \rho \rangle_{\text{tree}} \equiv \frac{m_W^2}{c_W^2m_{Z_1}^2} - 1 \simeq \frac{m_{ZZ'}^4}{m_Z^2m_{Z'}^2} \simeq \frac{[(1 - 4s_W^2)u^2 - (1 + 2s_W^2)v^2]^2}{4c_W^4v_w^2w^2}. \quad (10)$$

The second one arises from the one-loop contributions of a heavy gauge boson doublet ($X^-, Y^-)$.

Note that the other new particles such as the exotic quarks, $Z'$, and new Higgs bosons do not contribute [13]. Generalizing the results in [13] and using the $X$, $Y$ masses in [1], we obtain

$$\langle \Delta \rho \rangle_{\text{rad}} = \frac{3\sqrt{2}G_F}{16\pi^2} \left( \frac{m_Y^2 + m_X^2 - 2m_Y^2m_X^2}{m_Y^2 - m_X^2} \ln \frac{m_Y^2}{m_X^2} \right)$$

$$+ \frac{\alpha}{4\pi s_W^4} \left( \frac{m_Y^2 + m_X^2}{m_Y^2 - m_X^2} \ln \frac{m_Y^2}{m_X^2} - 2 + 3t_W^2 \ln \frac{m_Y^2}{m_X^2} \right)$$

$$= \frac{3g^2}{64\pi^2v_w^2} \left( \frac{v_w^2 + 2w^2 - 2(u^2 + w^2)(u^2 + w^2)}{u^2 - u^2} \ln \frac{v^2 + w^2}{u^2 + w^2} \right)$$

$$+ \frac{g^2}{16\pi^2} \left( \frac{v_w^2 + 2w^2 - 2(u^2 + w^2)(u^2 + w^2)}{u^2 + w^2} \ln \frac{v^2 + w^2}{u^2 + w^2} - 2 + 3t_W^2 \ln \frac{v^2 + w^2}{u^2 + w^2} \right), \quad (11)$$

where $\sqrt{2}G_F = 1/v_w^2$ and $\alpha = g^2s_W^2/(4\pi)$. Summarizing the above results, we get the $\Delta \rho$ deviation due to the new physics contributions up to one-loop level,

$$\Delta \rho = \langle \Delta \rho \rangle_{\text{tree}} + \langle \Delta \rho \rangle_{\text{rad}}. \quad (12)$$

**New physics constraints**: Because $Z'$ nonuniversally couples to the ordinary quarks, it gives rise to tree-level FCNCs. These processes can be evaluated that are completely identical to those in [10] and give a bound: $w > 3.6$ TeV (see also [14] for other discussions and constraints on the
3-3-1 breaking scale). On the other hand, since $s_W^2 = g_X^2/(g^2 + 4g_X^2) < 1/4$, the model encounters a low Landau pole ($\Lambda$), at which $s_W^2(\Lambda) = 1/4$ or $g_X(\Lambda) = \infty$, that is roundly $\Lambda = 4 - 5$ TeV, depending on the unfixed 3-3-1 breaking scale ($\mu_{331} < \Lambda$) [15]. Hereafter, $\Lambda = 5$ TeV will be taken into account. From the global fit, the $\rho$ parameter is $\rho = 1.00040 \pm 0.00024$, which is 1.7 $\sigma$ above the standard model expectation $\rho = 1$ [16].

Three remarks are in order

1. The reduced 3-3-1 model ($u = 0$, $v = v_w$): The deviation $\Delta \rho$ can be approximated as

$$\Delta \rho \simeq \left( \frac{1 + 2s_W^2}{2c_W^2} \right)^2 \frac{v_w^2}{w^2},$$

which yields $9.243 \text{ TeV} < w < 18.487 \text{ TeV}$, provided that $0.00016 < \Delta \rho < 0.00064$ and $s_W^2 = 0.231$ [16]. The model is invalid due to the limit of the Landau pole $w < 5$ TeV. In other words, due to the Landau pole limit $w < 5$ TeV (assumed the model works), we have $\Delta \rho > 0.0022$, which is too large to be consistent with the experimental data [16].

2. The simple 3-3-1 model ($v = 0$, $u = v_w$): The leading order for the $\Delta \rho$ deviation is

$$\Delta \rho \sim \left[ \left( \frac{1 - 4s_W^2}{2c_W^2} \right)^2 + \frac{3\alpha}{4\pi s_W^2} \left( \frac{1}{4} - c_W^2 \right) \right] \frac{v_w^2}{w^2},$$

which yields $w \sim 555 \text{ GeV}$ (by using the central value $\Delta \rho = 0.0004$, $s_W^2 = 0.231$ and $\alpha = 1/128$ [16]). The new physics is well defined below the Landau pole. However, as mentioned the FCNCs constrain $w > 3.6$ TeV, which opposes the above regime. Thus, the model encounters an experimental discrepancy.

3. The minimal 3-3-1 model with three scalar triplets: Because of $u^2 + v^2 = v_w^2$, we can make a contour for $\Delta \rho$ (where $0.00016 < \Delta \rho < 0.00064$) as a function of only two variables ($u, w$). The Landau pole limit $w < 5$ TeV and the FCNCs bound $w > 3.6$ TeV are also imposed.

The result is shown in Fig. [1] For completeness, the mixing angle $\varphi$ is shown in Fig. [2]

**Conclusion:** The reduced 3-3-1 model should be ruled out because it encounters either a large $\Delta \rho$ deviation or being mathematically inconsistent. The simple 3-3-1 model is experimentally unfavored due to the discrepancy between the FCNCs and $\rho$ parameter bounds. The minimal 3-3-1 model with three scalar triplets is consistent when $3.6 \text{ TeV} < w < 4 - 5$ TeV and $162.5 \text{ GeV} < u < 215.6 \text{ GeV}$ (or $0.55 < v/u < 1.14$). In all cases, we can always obtain the corresponding ($u, w$) values so that the $Z-Z'$ mixing angle is small, consistent with the precision data.
FIG. 1: The $(u, w)$ region that is bounded by $0.00016 < \rho < 0.00064$ and $3.6 \text{ TeV} < w < 5 \text{ TeV}$. Note that $u$ runs from 0 to 246 GeV.

FIG. 2: The $(u, w)$ region that is bounded by $-0.001 < \varphi < 0.001$ (the typical limits imposed by the electroweak measurements [16]) and $3.6 \text{ TeV} < w < 5 \text{ TeV}$. Note also that $u$ runs from 0 to 246 GeV.

The class of 3-3-1 models with $\beta = \pm \sqrt{3}$ and basic scalar triplets (that particularly consists of the mentioned ones and the 3-3-1 model with exotic charged leptons [17]) could be the subject of these constraints.
Acknowledgments

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[1] F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992); P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992); R. Foot, O. F. Hernandez, F. Pisano, and V. Pleitez, Phys. Rev. D 47, 4158 (1993).

[2] M. Singer, J. W. F. Valle, and J. Schechter, Phys. Rev. D 22, 738 (1980); J. C. Montero, F. Pisano, and V. Pleitez, Phys. Rev. D 47, 2918 (1993); R. Foot, H. N. Long, and Tuan A. Tran, Phys. Rev. D 50, R34 (1994).

[3] D. Ng, Phys. Rev. D 49, 4805 (1994); D. G. Dumm, F. Pisano, and V. Pleitez, Mod. Phys. Lett. A 09, 1600 (1994); H. N. Long and V. T. Van, J. Phys. G 25, 2319 (1999).

[4] P. B. Pal, Phys. Rev. D 52, 1659 (1995); P. V. Dong, H. T. Hung, and H. N. Long, Phys. Rev. D 86, 033002 (2012).

[5] F. Pisano, Mod. Phys. Lett A 11, 2639 (1996); A. Doff and F. Pisano, Mod. Phys. Lett. A 14, 1133 (1999); C. A. de S. Pires and O. P. Ravinez, Phys. Rev. D 58, 035008 (1998); C. A. de S. Pires, Phys. Rev. D 60, 075013 (1999); P. V. Dong and H. N. Long, Int. J. Mod. Phys. A 21, 6677 (2006).

[6] M. B. Tully and G. C. Joshi, Phys. Rev. D 64, 011301 (2001); Alex G. Dias, C. A. de S. Pires, and P. S. Rodrigues da Silva, Phys. Lett. B 628, 85 (2005); D. Chang and H. N. Long, Phys. Rev. D 73, 053006 (2006); P. V. Dong, H. N. Long, and D. V. Soa, Phys. Rev. D 75, 073006 (2007); P. V. Dong and H. N. Long, Phys. Rev. D 77, 057302 (2008); P. V. Dong, L. T. Hue, H. N. Long, and D. V. Soa, Phys. Rev. D 81, 053004 (2010); P. V. Dong, H. N. Long, D. V. Soa, and V. V. Vien, Eur. Phys. J. C 71, 1544 (2011); P. V. Dong, H. N. Long, C. H. Nam, and V. V. Vien, Phys. Rev. D 85, 053001 (2012); S. M. Boucenna, S. Morisi, and J. W. F. Valle, Phys. Rev. D 90, 013005 (2014).

[7] D. Fregolente and M. D. Tonasse, Phys. Lett. B 555, 7 (2003); H. N. Long and N. Q. Lan, Europhys. Lett. 64, 571 (2003); S. Filippi, W. A. Ponce, and L. A. Sanches, Europhys. Lett. 73, 142 (2006); C. A. de S. Pires and P. S. Rodrigues da Silva, JCAP 0712, 012 (2007); J. K. Mizukoshi, C. A. de S. Pires, F. S. Queiroz, and P. S. Rodrigues da Silva, Phys. Rev. D 83, 065024 (2011); J. D. Ruiz-Alvarez, C. A. de S. Pires, F. S. Queiroz, D. Restrepo, and P. S. Rodrigues da Silva, Phys. Rev. D 86, 075011 (2012); P. V. Dong, T. Phong Nguyen, and D. V. Soa, Phys. Rev. D 88, 095014 (2013); S. Profumo and F. S. Queiroz, Eur. Phys. J. C 74, 2960 (2014); C. Kelso, C. A. de S. Pires, S. Profumo, F. S. Queiroz, and P. S. Rodrigues da Silva, Eur. Phys. J. C 74, 2797 (2014).

[8] P. V. Dong, T. D. Tham, and H. T. Hung, Phys. Rev. D 87, 115003 (2013); P. V. Dong, D. T. Huong, F. S. Queiroz, and N. T. Thuy, Phys. Rev. D 90, 075021 (2014); D. T. Huong, P. V. Dong, C. S. Kim, and N. T. Thuy, “Inflation and leptogenesis in the 3-3-1-1 model”, in preparation.
[9] J. G. Ferreira Jr, P. R. D. Pinheiro, C. A. de S. Pires, and P. S. Rodrigues da Silva, Phys. Rev. D 84, 095019 (2011); D.T. Huong, L.T. Hue, M.C. Rodriguez, and H. N. Long, Nucl. Phys. B 870, 293 (2013); W. Caetano, C. A. de S. Pires, P. S. Rodrigues da Silva, D. Cogollo, and F. S. Queiroz, Eur. Phys. J. C 73, 2607 (2013); C.-X. Yue, Q.-Y. Shi, and T. Hua, Nucl. Phys. B 876, 747 (2013); J.G. Ferreira, C.A. de S.Pires, P.S. Rodrigues da Silva, and A. Sampieri, Phys. Rev. D 88, 105013 (2013); D. Cogollo, F. S. Queiroz, and P. Vasconcelos, Mod. Phys. Lett. A 29, 1450173 (2014); C. Kelso, P. R. D. Pinheiro, F. S. Queiroz, and W. Shepherd, Eur. Phys. J. C 74, 2808 (2014).

[10] P. V. Dong, N. T. K. Ngan, and D. V. Soa, Phys. Rev. D 90, 075019 (2014).

[11] C. A. de S. Pires, F. Queiroz, and P. S. Rodrigues da Silva, Phys. Rev. D 82, 065018 (2010).

[12] P. V. Dong and H. N. Long, Eur. Phys. J. C 42, 325 (2005).

[13] K. Sasaki, Phys. Lett. B 308, 297 (1993); P. H. Frampton and M. Harada, Phys. Rev. D 58, 095013 (1998); H. N. Long and T. Inami, Phys. Rev. D 61, 075002 (2000).

[14] See, for other constraints, D. Cogollo, A. V. de Andrade, F. S. Queiroz, and P. R. Teles, Eur. Phys. J. C 72, 2029 (2012); Y. A. Coutinho, V. S. Guimaraes, and A. A. Nepomuceno, Phys. Rev. D 87, 115014 (2013).

[15] Alex G. Dias, R. Martinez, and V. Pleitez, Eur. Phys. J. C 39, 101 (2005).

[16] K. A. Olive et al. (Particle Data Group), Chin. Phys. C 38, 090001 (2014).

[17] V. Pleitez and M. D. Tonasse, Phys. Rev. D 48, 2353 (1993).