A new signature of primordial non-Gaussianities from the abundance of galaxy clusters

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ABSTRACT
The evolution with time of the abundance of galaxy clusters is very sensitive to the statistical properties of the primordial density perturbations. It can thus be used to probe small deviations from Gaussianity in the initial conditions. The characterization of such deviations would help distinguish between different inflationary scenarios, and provide us with information on physical processes which took place in the early Universe. We have found that when the information contained in the galaxy cluster counts is used to reconstruct the dark energy equation of state as a function of redshift, assuming erroneously that no primordial non-Gaussianities exist, an apparent evolution with time in the effective dark energy equation of state arises, characterized by the appearance of a clear discontinuity.

Key words: Cosmology: large-scale structure of Universe – Cosmology: dark energy

1 INTRODUCTION
One of the most fundamental predictions of the simplest standard, single field, slow-roll inflationary cosmology, is that the primordial density fluctuations, that seeded the formation of the large-scale structure we see today, were nearly Gaussian distributed (see e.g. Creminelli[2003], Maldacena[2003], Lyth & Rodriguez[2005], Seery & Lidsey[2005], Sefusatti & Komatsu[2007]). Such prediction seems to be in good agreement with current observations of the cosmic microwave background anisotropies (e.g. Slosar et al.2008) and large-scale structure (e.g. Komatsu et al.2011). Nevertheless, a significant, potentially observable level of non-Gaussianity may be produced in some inflationary models where any of the conditions that give rise to the standard single-field, slow-roll inflation fail.

The detection of primordial non-Gaussianities would decrease considerably the number of viable inflationary models, and it would give us an insight on key physical processes that took place in the early Universe. Such detection could be achieved through the statistical characterization of the properties of the large-scale structure, namely the bispectrum and/or trispectrum of the galaxy distribution (e.g. Sefusatti & Komatsu[2007], Matarrese & Verde[2008]), or the determination of the evolution with time of the abundance of massive collapsed objects such as galaxy clusters (see e.g. Matarrese et al.2000, Robinson & Baker2000). These form at high peaks of the density field \( \delta(x) = \delta \rho / \rho \) and their number density as a function of redshift depends on the growth of structure, thus being sensitive to the dynamics and energy content of the Universe and to the statistical properties of the primordial density fluctuations.

In this work, we address the issue of how primordial non-Gaussianities may affect the determination of the effective dark energy equation of state \( w \) using the evolution with time of the galaxy cluster abundance. Throughout, and unless stated otherwise, we consider our fiducial cosmological model to be a flat ΛCDM model with WMAP 7-year cosmological parameters (WMAP+BAO+H0) (Komatsu et al.2011), namely, a Hubble constant, \( H_0 \), equal to 100h km s\(^{-1}\) Mpc\(^{-1}\) with \( h = 0.704 \), fractional densities of matter and baryons today of \( \Omega_m = 0.272 \), \( \Omega_b h^2 = 0.023 \) respectively, a scalar spectral index, \( n_s \), equal to 0.963, and we normalize the power spectrum so that \( \sigma_8 = 0.809 \).

2 PRIMORDIAL NON-GAUSSIANITY
Primordial non-Gaussianity is commonly parametrized by the non-linear parameter \( f_{NL} \) and may be written as follows (Lo Verde et al. 2008),

\[
B_1(k_1, k_2, k_3) = (2\pi)^4 f_{NL} \frac{P^2_2(K)}{(k_1 k_2 k_3)} \mathcal{A}(k_1, k_2, k_3),
\]

where \( K = k_1 + k_2 + k_3 \), \( \zeta \) is the primordial curvature perturbation and \( \mathcal{A} \) is an auxiliary function that contains the shape of the bispectrum, \( B_1 \), of \( \zeta \). Further, \( P^2_2 \propto k^{n_{-1}} \) is the dimensionless power-spectrum, while \( k_i = |k_i| \) with \( k_i \) being the wave vectors. In Fourier space, the functions \( P_1 \) and \( B_1 \) are defined by means of the two and three-point correlation functions,

\[
\langle \delta_{k_1} \delta_{k_2} \rangle = (2\pi)^3 \delta_D(k_{12}) \frac{(2\pi)^4 P_1(k_1)}{4\pi k_1^3},
\]

\[
\langle \delta_{k_1} \delta_{k_2} \delta_{k_3} \rangle = (2\pi)^3 \delta_D(k_{123}) B_1(k_1, k_2, k_3),
\]

where \( k_{1,2,3} \equiv k_1 + k_2 + ... + k_n \). Note that \( f_{NL} \) may or may not be scale-dependent. In this work only the later case is considered.

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The bispectrum of $\zeta$ is the lowest order statistics sensitive to non-Gaussian features. Depending on the underlying physical mechanism responsible for generating non-Gaussianities, different triangular configurations (shapes) will arise. There are broadly four classes of triangular shapes or, equivalently, four different bispectrum parametrizations that can be found in the literature: Local, Equilateral, Folded and Orthogonal. Here we shall only consider the first two. They are defined as follows:

- **Local shape**: It arises from multi-field, inhomogeneous reheating, curvaton and ekpyrotic models. Mathematically, the Local shape can be characterized by a simple Taylor expansion around the Gaussian curvature perturbation, $\zeta_G$. (Salopek & Bond 1990; Komatsu & Spergel 2001),

$$\zeta(x) = \zeta_G(x) + \frac{3}{2} f_{NL}^{\text{local}} (\zeta_G^2(x) - \langle \zeta_G^2(x) \rangle).$$

(4)

The WMAP 7-year estimate for $f_{NL}^{\text{local}}$ is (Komatsu et al. 2011)

$$f_{NL}^{\text{local}} = 32 \pm 21 \text{(68\% C.L.)}.$$ 

(5)

The bispectrum for this shape can be derived using Eq. (4) and it is given by (Lo Verde et al. 2008; Komatsu 2010)

$$\mathcal{A}_{\text{local}} = \frac{3}{10} K^{-2(\nu-1)} \left[ k_1^3 (k_2 k_3)^{\nu-1} + k_2^3 (k_1 k_3)^{\nu-1} + k_3^3 (k_1 k_2)^{\nu-1} \right].$$

(6)

This quantity is maximized for the so-called squeezed triangle configuration, i.e., $k_3 < k_2 \approx k_1$.

- **Equilateral shape**: It is characteristic of inflationary models where scalar fields have a non-canonical kinetic term (for example Dirac-Born-Infeld inflation, see Alishahiha et al. 2004). The mathematical expression for the Equilateral shape is (Lo Verde et al. 2008; Komatsu 2010)

$$\mathcal{A}_{\text{equi}} = \frac{9}{10} K^{-2(\nu-1)} \left[ -k_1^3 k_2 k_3 \right]^{\nu-1} + \text{perm}. -2(2k_1 k_3)^{1-2\nu-1/3} + k_1^{2(\nu-1)/3} k_2^{1+2\nu-1/3} k_3^{\nu-1} + \text{perm}. ].$$ 

(7)

This quantity reaches a maximum at $k_1 \approx k_2 \approx k_3$. Constraints from WMAP 7-year set the level of non-Gaussianity for this shape at (Komatsu et al. 2011)

$$f_{NL}^{\text{equi}} = 26 \pm 14 \text{(68\% C.L.)}.$$ 

(8)

As mentioned before, the abundance of rare objects such as galaxy clusters holds relevant information that can be used to probe the initial conditions. For such information to be of use, the statistics of the density perturbation, $\delta_R$, smoothed on a scale $R$, have to be characterized. However, we have previously defined the non-Gaussianity in the primordial curvature perturbation, $\zeta$, rather than in the smoothed linear density field. The relation between $\zeta$ and the linear perturbation to the matter density today smoothed on a scale $R$ is given by,

$$\delta_R(k, z) = D(z) W(k, R) M(k) T(k) \zeta(k, z),$$

(9)

where

$$M(k) = \frac{2}{5} \frac{c^2}{\Omega_m H_0^2} k^2,$$

(10)

with $D(z)$ being the linear growth factor (Komatsu et al. 2009), $T(k)$ is the transfer function adopted from Bardeen et al. (1986), and $W(k, R)$ is the smoothing top-hat window. We use the shape parameter given by Sugiyama (1995),

$$\Gamma = \Omega_m h \exp \left[ -\Omega_m \left( 1 + \sqrt{2} \Omega_m \right) \right].$$

Using Eq. (9) and the definitions given in Eqs. (2) and (3), one may compute the variance

$$\sigma^2(R) = \delta_R^2 = \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} T_1 T_2 \zeta_{k_1} \zeta_{k_2} =$$

$$= \int_0^\infty \frac{dk}{k} F^2(k) P_L(k),$$

(11)

and the three-point function for the smoothed density field (Lo Verde et al. 2008)

$$\langle \delta_R^3 \rangle = f_{NL} \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} \int \frac{d^3k_3}{(2\pi)^3} T_1 T_2 T_3 \langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle =$$

$$= \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} \int \frac{d^3k_3}{(2\pi)^3} T_1 T_2 T_3 \langle P_L(K) \rangle^2 \mathcal{A}(k_1, k_2, k_3)^{3/2}.$$ 

(12)

with $\zeta_i = \zeta(k_i), T_i = W(k_i, R) M(k_i) T(k_i), K = k_1 + k_2 + k_3$ and $k_{12} = \sqrt{k_1^2 + k_2^2 + 2k_1 k_2 \cdot k_3}$ (here a dot represents the scalar product).

Eqs. (11) and (12) will be of special relevance in the next section, since in order to incorporate non-Gaussian initial conditions in the prediction of rare objects, one has to derive a non-Gaussian probability density function (PDF) for the smoothed density field, $\delta_R$. This can be done by using a mathematical procedure that enables us to construct the PDF from its cumulants (see Lo Verde et al. 2008; Matarrese & Verde 2008 for details).

### 3 THE DARK ENERGY EQUATION OF STATE FROM THE GALAXY CLUSTER ABUNDANCE

#### 3.1 Halo Mass Function

The comoving number density of virialized halos per unit of volume at a given redshift $z$, $dn/dM(z, M)$, with a mass, $M$, in the range $[M, M + dM]$ is called the mass function. In the presence of non-Gaussian initial conditions, the mass function has been estimated using extensions of the Press-Schechter (PS) formalism (Press & Schechter 1974). This formalism asserts that the fraction of matter ending up in objects of mass $M$ is proportional to the probability that the density fluctuations smoothed on the scale $R \equiv (3M/4\pi\sigma^2)^{1/3}$, and above a certain threshold value, $\delta_c$, can be written as,

$$\frac{dn}{dM}(z, M) = -\frac{2}{\rho M} \frac{d}{dM} \int_0^\infty \frac{dvP(v, M)}{(v^3/\Omega_m)^{1/3}}.$$ 

(13)

where $\rho$ is the mean density of the universe, $\sigma_M$ is the matter density in the spherical collapse model and the PDF of the smoothed density field. The redshift dependence of the threshold for spherical collapse as been incorporated in $\delta_c(z) = 1.686D(0) D^{-1}(z)$. For Gaussian initial conditions the mass function acquires the following form,

$$\frac{dn}{dM}(z, M) = \frac{2}{\pi \rho M^2} \frac{d}{dM} \int \frac{dvP(v, M)}{v^3/\Omega_m^{1/3}}.$$ 

(14)

The cosmological parameters enter Eq. (13) essentially through the variance and the linear growth factor, as well as, through the critical density contrast $\delta_c(z)$.

There are several prescriptions for changing Eq. (13) in the presence of non-Gaussian initial conditions (Chu et al. 1998; Robinson et al. 2000; Avelino & Viana 2000; Fosalba et al. 2000; Matarrese et al. 2000). Here we will adopt that which has been proposed by Lo Verde et al. (2008), and which has been shown to

\[ \Omega_m H_0^2 \approx 0.02. \]

\[ \Omega_b H_0^2 \approx 0.002. \]
provide a good fit to results from N-body simulations (see Wagner et al. [2008] and references therein),

\[
\frac{dn}{dM}(M, z) = -\sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M} e^{-d\chi(z)^2/2\sigma_M^2} \left( \frac{d \ln \sigma_M}{d \ln M} \frac{\delta(z)}{\sigma_M} + \frac{S_{3M} \sigma_M}{6} \right) \times \left( \frac{\delta(z)}{\sigma_M} - 2 \frac{\delta(z)}{\sigma_M} - 1 \right) + \frac{1}{6} S_{3M} \frac{d \ln \sigma_M}{d \ln M} \frac{\delta(z)}{\sigma_M} - 1 \right),
\]

where \(S_{3M} = (\delta_M^3)/(\sigma_M^2)^2\) and \(f_{NL}\) is the skewness of the smoothed density field. If \(f_{NL} = 0\), then \(S_{3M} = 0\) and Eq. (15) reduces to the Gaussian mass function.

Numerical simulations have shown that the PS form of the mass function under-predicts the abundance of high-mass objects and over-predicts low-mass ones. Therefore, to be in better agreement with the results of numerical simulations, we follow Lo Verde et al. (2008) and model the departures from Gaussianity using the mass function suggested by Sheth et al. (2001). The non-Gaussian mass function then becomes,

\[
\frac{dn_{NG}}{dM}(z, M, f_{NL}) = \frac{dn_{ST}}{dM} \frac{dn_{PS}/d\ln(M(z, M, f_{NL}))}{dn_{PS}/d\ln(M(z, M, f_{NL} = 0))}. \tag{16}
\]

The Seth-Tormen mass function has been calibrated using numerical simulations with Gaussian initial conditions and it is given by (Sheth et al. 2001).

\[
\frac{dn_{ST}}{dM}(z, M) = -\sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M} \left( 1 + \left( \frac{a_d(z)}{\sigma_M} \right)^p \right) \frac{d \ln \sigma_M}{d \ln M} \frac{\delta(z)}{\sigma_M} e^{-a_d(z)^2/(2\sigma_M^2)} \tag{17}
\]

with \(a = 0.707, \sigma = 0.322184\) and \(p = 0.3\).

Given Eqs. (16) and (17), one may now compute the number of clusters of galaxies per unit of redshift above a certain mass threshold, \(M_{lim}\),

\[
\frac{dN}{dz}(z, M > M_{lim}) = f_{sky} \frac{dV}{dz}(z) \int_{M_{lim}}^{\infty} dM \frac{dn}{dM}(z, M, f_{NL}), \tag{18}
\]

where \(f_{sky}\) is the fraction of sky being observed in the survey and \(dV/dz(z)\) is the comoving volume element which, for a flat cosmology, is given by

\[
\frac{dV}{dz}(z) = 4\pi \chi(z)^2 \frac{dz}{dz}, \tag{19}
\]

with \(\chi(z)\) being the comoving radial distance.

Figures 1a and 1b illustrate the impact of the primordial non-Gaussianities on the number density of galaxy clusters with mass threshold \(M > M_{lim} = 5 \times 10^{14} h^{-1} M_{\odot}\) and \(w = -1\), as a function of redshift, for the Local and Equilateral parametrizations, respectively. Figure 1c shows how the cluster number density is affected when a constant dark energy equation of state parameter \(w\) is changed for the same mass threshold, always assuming Gaussian initial conditions. The effect of \(f_{NL}\) and \(w\) on the abundance of galaxy clusters is quite different. On one hand, increasing \(w\) above \(w = -1\) flattens the slope of the cluster abundance above \(z = 0.5\), which translates in an increase in the number of high-z clusters. On the other hand, changes in \(f_{NL}\) modify the cluster abundance more uniformly in redshift. The difference in behaviour occurs because \(w\) affects both the volume factor and the mass function, while \(f_{NL}\) changes only the tail of the distribution of the density perturbations, thus modifying just the mass function.

3.2 Estimation of \(w_{eff}\)

Figures 1a, 1b, and 1c suggest that the redshift evolution of the number density of galaxy clusters in non-Gaussian models could be wrongly taken to be the result of an effective dark energy equation of state different from the real one, under the assumption of Gaussian initial conditions. In order to test this hypothesis, we have generated mock catalogues with the expected redshift evolution of the cluster number density for different non-Gaussian initial conditions in bins of redshift with width \(\Delta z = 0.1\) up to redshift 2, assuming a mass threshold of \(M_{lim} = 5 \times 10^{14} h^{-1} M_{\odot}\) and a nearly full sky survey area of 40,000 square degrees. We further consider \(f_{NL, loc} = (-20, 20)\) and \(f_{NL, eff} = (-60, 60)\), respectively for the Local and Equilateral parametrizations mentioned in section 2. The reason for choosing these specific values for \(f_{NL}\) will become clear in the next subsection.

Having generated the mock catalogues with the expected redshift distribution of the number density of galaxy clusters with non-Gaussian initial conditions, we then computed an effective dark energy equation of state, \(w_{eff}\), using Eq. (18) with \(f_{NL} = 0\), that mimics the distribution of the number of galaxy clusters in the presence of non-Gaussian initial conditions at the \(i\)-th redshift bin, by solving the following equation,

\[
N_i^{\text{eff}} = \int_{z_i - \Delta z/2}^{z_i + \Delta z/2} \frac{dN}{dz}(z, w = w_{eff}, f_{NL} = 0) dz =
\]

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Figure 2. Figure 2a shows the value of $w_{\text{eff}}$ that maximizes the abundance of clusters as a function of redshift, with Gaussian initial conditions and $M_{\text{lim}} = 5 \times 10^{14} h^{-1} M_\odot$ (the vertical dotted line corresponds to the redshift $z_* \sim 0.575$ where $w_* = -1$). Figure 2b shows the dependence of the of the value of $z_*$ on the mass threshold $M_{\text{lim}}$. Figures 2c and 2d show the reconstructed effective dark energy equation of state $w_{\text{eff}}$ for $M > M_{\text{lim}} = 5 \times 10^{14} h^{-1} M_\odot$ and different values of $f_{NL}$ (Local and Equilateral parametrizations, respectively).

![Figure 2a](image1)

![Figure 2b](image2)

![Figure 2c](image3)

![Figure 2d](image4)

**Table 1.** The computed $\sigma_8$ for the Local parametrization and Equilateral parametrization, obtained by demanding the present-day number density of galaxy clusters is recovered.

| Model          | Local | Equilateral |
|----------------|-------|-------------|
| $f_{NL}$       | 20    | 60          |
| $\sigma_{8,NG}$| 0.812 | 0.806       |

Table 1 shows the computed $\sigma_8$ for different values of $f_{NL}$ and Local and Equilateral parametrizations.

The normalization of the power-spectrum, for models with non-Gaussian initial conditions, was done by demanding the present-day number density of galaxy clusters in the concordance model ($\Lambda$CDM cosmology with Gaussian initial conditions and $\sigma_8 = 0.809$) is recovered. Table 1 shows the computed $\sigma_8$ for different values of $f_{NL}$ and Local and Equilateral parametrizations.

Figures 2c and 2d show the reconstructed dark energy equation of state, $w_{\text{eff}}$, as a function of the redshift for $M_{\text{lim}} = 5 \times 10^{14} h^{-1} M_\odot$ and different values of $f_{NL}$ (Local and Equilateral parametrizations, respectively). At low redshift, our reconstructed $w_{\text{eff}}$ is very close to our fiducial $w = -1$. But, as we move towards higher redshifts, our computed $w_{\text{eff}}$ deviates from $-1$, with this effect being more evident for higher values of $f_{NL}$ in both parametrizations. Further, for $f_{NL} > 0$ there is a redshift interval where $w_{\text{eff}}$, is undefined, which widens with increasing $f_{NL}$.
\[ M_{\text{bin}} = 5 \times 10^{14} h^{-1} M_{\odot} \] this happens in a redshift range centered at the redshift \( z_\ast \approx 0.575 \), at which the value of \( w_{\text{eff}} \), which we will call \( w_{\ast} \), that maximizes the cluster abundance is equal to \( -1 \) (see Figure 2a). In this interval, there is no \( w_{\text{eff}} \) that solves Eq. (20), since the product of the comoving volume with the integral of the mass function, when assuming Gaussian initial conditions, is always smaller than the same quantity for non-Gaussian initial conditions, i.e. \( f_{\text{NL}} \neq 0 \). If our fiducial cosmological model had a different value for \( w \), then the discontinuity would appear at the redshift at which \( w = w_{\ast} \). On the other hand, for models with \( f_{\text{NL}} < 0 \), we have also a discontinuous \( w_{\text{eff}} \) but there are two values of \( w_{\text{eff}} \) capable of reproducing the number of clusters in non-Gaussian models in a small redshift interval centered at \( z_\ast \). The dependence of the value of \( z_\ast \) on the mass threshold \( M_{\text{bin}} \) is shown in figure 2b.

3.3 Estimation of \( w_{\text{eff}} \) with statistical uncertainties

The observational estimation of the number density of galaxy clusters is affected by two sources of statistical uncertainty: the shot-noise and the cosmic variance (e.g. [Valageas et al. 2011]). The statistical uncertainty associated with the former increases, for example, with the cluster mass threshold, as clusters then become more rare, while the statistical uncertainty associated with the later increases, for example, as the cosmic volume surveyed gets smaller. However, assuming primordial Gaussian density perturbations, and for a cluster mass threshold above \( 5 \times 10^{14} h^{-1} M_{\odot} \), it has been shown that the magnitude of the contribution of the cosmic variance to the statistical uncertainty, in the observed number density of galaxy clusters, is always at least an order of magnitude smaller than the contribution due to shot-noise, almost independently of the surveyed sky area (e.g. see figures 5 and 9 of [Valageas et al. 2011]).

The existence of statistical uncertainties in the observed number density of galaxy clusters will to some extent mask the apparent discontinuity on the evolution of \( w_{\text{eff}} \) described in the previous section, which could be used to identify the presence of non-Gaussian density perturbations. In order to determine the impact of such uncertainties, we will re-estimate \( w_{\text{eff}} \) in similar fashion to what was done in the previous section, but with the inclusion of shot-noise, \( \sigma_N = \sqrt{N_i} \), at the 1\( \sigma \) level in each redshift bin. We are able to neglect the contribution from cosmic variance by setting the cluster mass threshold to \( 5 \times 10^{14} h^{-1} M_{\odot} \), and noting that the assumed level of non-Gaussianity is relatively small, not affecting much the average number density of galaxy clusters with respect to the Gaussian case (see Fig. 1).

Figures 3 and 4 show the 1\( \sigma \) statistical uncertainty in the reconstructed \( w_{\text{eff}} \), as a function of redshift, when shot-noise is taken into account, and as before for a nearly full sky survey area of 40,000 square degrees. As can be seen, even when including the effect of such uncertainty, the apparent discontinuity on the evolution of \( w_{\text{eff}} \) found in the previous subsection is still present (at least at the 2\( \sigma \) confidence level) for values of \( f_{\text{NL}} \) as small as \( \pm 20 \) for the Local parametrization, and \( \pm 60 \) for the Equilateral parametrization. Clearly, decreasing the survey area would mean that only values of \( f_{\text{NL}} \) with larger modulus could be detected.

4 CONCLUSION

In this work, we investigated whether the presence of primordial non-Gaussianities has an impact on the estimation of the effective dark energy equation of state, when one uses the abundance of galaxy clusters as a tool to probe different cosmological scenarios. We computed the effective dark energy equation of state, \( w_{\text{eff}} \) per redshift bin, assuming Gaussian initial conditions, that is capable of reproducing the galaxy cluster counts expected in several non-Gaussian models, thus constructing a correspondence \( f_{\text{NL}} \leftrightarrow w_{\text{eff}} \) for each redshift bin. The most important result of this work is the discovery of a redshift interval where no value for the effective dark energy equation of state is capable of reproducing the non-Gaussian cluster abundance for models with \( f_{\text{NL}} > 0 \), which is the result of there being a \( w_{\text{eff}} \) that maximizes the cluster number density at each redshift, while for models with \( f_{\text{NL}} < 0 \) a discontinuous \( w_{\text{eff}} \) is obtained. The appearance of such type of features may thus constitute a new diagnostic of the presence of primordial non-Gaussianities. Although, even under the ideal situation where only statistical uncertainties are present, their detection is only possible for values of \( f_{\text{NL}} \) not too far away from those permitted by analysis of the WMAP 7-year data ([Komatsu et al. 2011]), it should be remembered that \( f_{\text{NL}} \) may not be invariant with scale. In particular, there isn’t any particular reason why \( f_{\text{NL}} \) could not increase as the scale diminishes (see e.g. [Riotto & Slosar 2011]).

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Figure 3. The reconstructed effective dark energy equation of state for the Local parametrization with statistical observational uncertainties taken into account.

Figure 4. The same as in Fig. 3 but for the Equilateral parametrization.