Neural-Rendezvous: Learning-based Robust Guidance and Control to Encounter Interstellar Objects

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Interstellar objects (ISOs), astronomical objects not gravitationally bound to the Sun, are likely representatives of primitive materials invaluable in understanding exoplanetary star systems. Due to their poorly constrained orbits with generally high inclinations and relative velocities, however, exploring ISOs with conventional human-in-the-loop approaches is significantly challenging. This paper presents Neural-Rendezvous – a deep learning-based guidance and control framework for encountering any fast-moving objects, including ISOs, robustly, accurately, and autonomously in real-time. It uses pointwise minimum norm tracking control on top of a guidance policy modeled by a spectrally-normalized deep neural network, where its hyperparameters are tuned with a newly introduced loss function directly penalizing the state trajectory tracking error. We rigorously show that, even in the challenging case of ISO exploration, Neural-Rendezvous provides 1) a high probability exponential bound on the expected spacecraft delivery error; and 2) a finite optimality gap with respect to the solution of model predictive control, both of which are indispensable especially for such a critical space mission. In numerical simulations, Neural-Rendezvous is demonstrated to achieve a terminal-time delivery error of less than 0.2 km for 99% of the ISO candidates with realistic state uncertainty, whilst retaining computational efficiency sufficient for real-time implementation.

Nomenclature

\( \mathcal{A} \quad \text{weak infinitesimal operator of stochastic processes} \)

\( C_{x2v} \quad \text{matrix that maps } x \text{ to } \dot{p}, \text{ i.e., } C_{x2v} = [O_{3x3} \ I_{3x3}] \in \mathbb{R}^{3x6} \)

\( \mathbb{E} \quad \text{expected value operator} \)

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\[ \mathbb{E}_Z = \text{conditional expected value operator given } Z \]

\[ I_{m \times n} = m \times n \text{ identity matrix} \]

\[ L_\ell = 2\text{-norm Lipschitz constant of } u_\ell \]

\[ m = \text{dimension of spacecraft control input (}= 3) \]

\[ m_{sc}(t) = \text{mass of spacecraft at time } t \]

\[ \mathbb{N} = \text{set of natural numbers} \]

\[ N_d = \text{number of neural network training samples} \]

\[ n = \text{dimension of spacecraft and ISO state (}= 6) \]

\[ O_{m \times n} = m \times n \text{ zero matrix} \]

\[ \omega(t), \hat{\omega}(t), \omega_d(t) = \text{true, estimated, and desired orbital elements of ISO at time } t \]

\[ \mathbb{P} = \text{probability measure} \]

\[ p(t), \hat{p}(t), p_d(t) = \text{true, estimated, and desired position of spacecraft relative to ISO in LVLH frame at time } t \]

\[ \mathbb{R} = \text{set of real numbers} \]

\[ \mathbb{R}_{>0} = \text{set of positive real numbers} \]

\[ \mathbb{R}_{\geq 0} = \text{set of non-negative real numbers} \]

\[ x(t), \hat{x}(t), x_d(t) = \text{true, estimated, and desired state of spacecraft relative to ISO in LVLH frame at time } t \]

\[ t = \text{time} \]

\[ t_d = \text{time to compute desired state trajectory} \]

\[ t_f = \text{terminal time at ISO encounter} \]

\[ t_s = \text{time to activate } u_{\ell}^* \]

\[ \mathcal{U}(t) = \text{set containing admissible control input at time } t \]

\[ u = \text{control policy} \]

\[ u_\ell = \text{SN-DNN guidance policy} \]

\[ u_{\ell}^* = \text{SN-DNN min-norm control policy} \]

\[ u_{\text{max}} = \text{maximum admissible control input} \]

\[ u_{\text{mpc}} = \text{MPC policy} \]

\[ y(t) = \text{state measurement at time } t \]

\[ \theta_{\text{nn}} = \text{hyperparameter of } u_\ell \]

\[ \rho = \text{desired terminal position of spacecraft relative to ISO} \]

**Acronyms**

DNN = deep neural network
I. Introduction

Interstellar objects (ISOs) represent one of the last unexplored classes of solar system objects. They are active or inert objects passing through our solar system on an unbound hyperbolic trajectory about the Sun, which could sample planetesimals and primitive materials that provide vectors to compare our solar system with neighboring exoplanetary star systems [1]. To date, two such objects have been identified and observed: 1I/’Oumuamua [2] discovered in 2017 (Fig. 1); and 2I/Borisov [3] discovered in 2019 (Fig. 2). In 2022, the United States Department of Defense confirmed that a third ISO impacted Earth in 2014, three years before the identification of ʿOumuamua. ISOs are physical laboratories that can enable the study of exosolar systems in-situ rather than remotely using telescopes such as the Hubble or James Webb Space Telescopes.

While remote observation can aid in constraining the level of activity, shape, and spectral signature of ISOs, limiting scientists to Earth-based telescopic observation prevents some of the most impactful science that can be performed. Using a dedicated spacecraft to flyby an ISO opens the doors to high-resolution imaging, mass or dust spectroscopy, and a larger number of vantage points than Earth observation. It could also resolve the target’s nucleus shape and spin,
characterize the volatiles being shed from an active body, reveal fresh surface material using an impactor, and more [4].

The discovery and exploration of ISOs are “once in a lifetime” or maybe “once in a civilization” opportunities, and their exploration is challenging for three main reasons: 1) they are often not discovered until they are close to Earth, meaning that launches to encounter them often require high launch energy; 2) their orbital properties are poorly constrained at launch, generally leading to significant on-board resources to encounter; and 3) the encounter speeds are typically high (> 10 km/s) requiring fast response autonomous operations. This paper presents an offline deep learning-based robust nonlinear guidance and control (G&C) approach, called Neural-Rendezvous, to autonomously encounter them even in the presence of such large state uncertainty and high-velocity challenges. As outlined in Fig. 3, the guidance, navigation, and control (GNC) of spacecraft for the ISO encounter are split into two segments: 1) the cruise phase, where the spacecraft utilizes state estimation obtained by ground-based telescopes and navigates via ground-in-the-loop operations, and; 2) the terminal phase, where it switches to fully autonomous operation with existing onboard navigation frameworks. The current state of practice and performance of autonomous navigation systems for small bodies like ISOs are discussed in [5, 6]. Neural-Rendezvous is for performing the second phase of the autonomous terminal G&C with the on-board state estimates and is built upon the spectrally-normalized deep neural network (SN-DNN), a neural network that has been successfully used for providing stability guarantees of learning-based feedback controllers [7–9]. It is constructed by the combination of the following new deep learning-based approaches.

We first propose a dynamical system-based SN-DNN for designing a real-time guidance policy that approximates model predictive control (MPC), which is known to be near-optimal in terms of dynamic regret, i.e., the MPC performance minus the optimal performance in hindsight [10–12]. This is to avoid solving MPC optimization at each time instant and compute a spacecraft’s control input autonomously even with its limited online computational capacity. Consistent with our objective of encountering ISOs, it is an SN-DNN [13] with a novel loss function for directly imitating the MPC state trajectory performing dynamics integration, as well as indirectly imitating the MPC control input as in existing methods [14]. We then introduce learning-based min-norm feedback control to be used on top of this guidance policy, which provides an optimal and robust control input, expressed in an analytical form, that minimizes its instantaneous deviation from that of the SN-DNN guidance policy under the incremental stability condition as in the

Fig. 2  2I/Borisov near and at perihelion (credit: NASA, ESA, and D. Jewitt (UCLA)).
one of contraction theory [15, 16].

In particular, we rigorously show that: 1) the SN-DNN guidance policy possesses a verifiable optimality gap with respect to the computationally expensive MPC policy; and that 2) when it is equipped with the learning-based min-norm control, the state tracking error bound with respect to the desired state trajectory decreases exponentially in expectation with a finite probability, robustly against the state uncertainty. The latter indicates that the terminal spacecraft deliver error at the ISO encounter, i.e., the shaded blue region in Fig. 3, is probabilistically bounded in expectation, where its size can be modified accordingly to the mission requirement by tuning its feedback control parameters. In numerical simulations with 100 ISO candidates for possible exploration obtained using [17], it is demonstrated that Neural-Rendezvous outperforms other G&C algorithms including (i) the SN-DNN guidance policy; (ii) proportional-derivative (PD) control with a pre-computed desired trajectory; (iii) robust nonlinear tracking control of [18, pp. 397-402] with a precomputed desired trajectory; and (iv) MPC with linearized dynamics [19]. It achieves a delivery error less than 0.2 km under the realistic ISO state uncertainty [6] for 99% of the ISO candidates, with $8.0 \times 10^{-4}$ s computational time for its one-step evaluation and control effort less than 0.6 km/s. It is also validated that, when compared with the conventional SN-DNN that imitates the MPC control input, our dynamical system-based SN-DNN indeed reduces the delivery error thanks to the presence of the state trajectory imitation loss. A YouTube video which visualizes these simulation results can be found at https://youtu.be/8h60B_p1fyQ.

Such verifiable and real-time optimality, stability, and robustness guarantees offer indispensable analytical insight on determining whether or not we should utilize Neural-Rendezvous based on the size of the state uncertainty, thereby enhancing conventional black-box machine learning and AI-based G&C approaches with few theoretical guarantees. It is worth noting that our proposed approach and its guarantees are general enough to be used not only for encountering ISOs, but also for solving various nonlinear autonomous rendezvous problems with fast-moving objects, accurately in real-time under external disturbances and various sources of uncertainty resulting from, e.g., state measurement noise, process noise, control execution error, unknown parts of dynamics, or parametric/non-parametric variations of dynamics and environments.

**Related Work**

The state of practice in realizing asteroid and comet rendezvous missions is to pre-construct an accurate spacecraft state trajectory to the target before launch, and then perform a few trajectory correction maneuvers (TCMs) along the way based on the state measurements obtained by ground-based and onboard navigation schemes [6]. Such a G&C approach is only feasible for targets with sufficient information on their orbital properties in advance, which is not realistic for ISOs visiting our solar system traveling through interstellar space.

The ISO rendezvous problem can be cast as a robust motion planning and control problem that has been investigated in numerous studies in the field of robotics. The most well-developed and commercialized of these G&C methods is
On-board Navigation (OpNav)
LAUNCH CRUISE G&C TERMINAL G&C – NEURAL-RENDZVOUS

Fig. 3 Illustration of cruise and terminal GNC, where $t$ is time, $\alpha(t)$ is ISO state, $x(t)$ is spacecraft state relative to $\alpha(t)$, $y(t)$ is state measurement of ISO and spacecraft, $u$ is control input, and $\hat{\alpha}(t)$ and $\hat{x}(t)$ are estimated ISO and spacecraft relative state, respectively. Neural-Rendezvous enables obtaining verifiable delivery error bound even under large ISO state uncertainty and high-velocity challenges.

robust MPC [20, 21], which extensively utilizes knowledge about the underlying nonlinear dynamical system to design an optimal control input at each time instant, thereby allowing a spacecraft to use the most updated ISO state information that should get more accurate as it gets closer to the target. When the MPC is augmented with feedback control, robustness against the state uncertainty and various external disturbances can be shown using standard techniques in systems and control theory, such as Lyapunov and contraction theory [16, 22–24]. However, as mentioned earlier, the spacecraft’s onboard computational power is not necessarily sufficient to solve the MPC optimization at each time instant of its TCM, which could lead to failure in accounting for the ISO state that changes dramatically in a few seconds due to the body’s high velocity.

Learning-based control designs have been considered a promising solution to this problem, as they allow replacing these computationally-expensive G&C algorithms with computationally-cheap mathematical models, e.g., neural networks [14, 25–31]. Neural-Rendezvous we present in this paper is a novel variant of a learning-based control design. Its benefits lie in the formal derivation of its optimality gap with respect to the MPC policy as a function of the dynamical system-based SN-DNN learning error, and the robustness and stability guarantee even under the presence of state uncertainty and learning errors, which can be shown by extending the results of [16] for general nonlinear systems to the case of the ISO rendezvous problem.

Notation

We let $\|x\|$ denote the Euclidean norm for a vector $x \in \mathbb{R}^n$, and let $A > 0$, $A \geq 0$, $A < 0$, and $A \leq 0$ denote the symmetric positive definite, positive semi-definite, negative definite, negative semi-definite matrices, respectively, for a square matrix $A \in \mathbb{R}^{n \times n}$. Also, the target celestial bodies will be considered to be ISOs in the subsequent sections for the sake of consistency, but our proposed method should work for long-period comets (LPCs) [32–34], drones, and other fast-moving objects in robotics and aerospace applications, with the same arguments to be presented. In this paper, the new and important results will be stated in Theorems 1–3, and key concepts for understanding our contributions will
be illustrated in Fig. 4–8.

II. Technical Challenges in ISO Exploration

In this paper, we consider the following translational dynamical system of a spacecraft relative to an Interstellar Object (ISO), equipped with feedback control $u \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^m$:

$$
\dot{x}(t) = f(x(t), a(t), t) + B(x(t), a(t), t)u(\hat{x}(t), \hat{a}(t), t)
$$

where $t \in \mathbb{R}_{\geq 0}$ is time, $a \in \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^n$ are the ISO orbital elements evolving by a separate equation of motion (see [35, 36] for details), $x \in \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^n$ is the state of the spacecraft relative to $a$ in a local-vertical local-horizontal (LVLH) frame centered on the ISO [37, pp. 710-712], $f : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^n$ and $B : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^{n \times m}$ are known smooth functions [15, 38] (see (2)), and $\hat{a} \in \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^n$ and $\hat{x} \in \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^n$ are the estimated ISO and spacecraft relative state in the LVLH frame given by an on-board navigation scheme, respectively. Particularly when we select $x$ as $x = [p^T, \dot{p}^T]^T$ as its state, where $p \in \mathbb{R}^3$ is the position of the spacecraft relative to the ISO, we have that

$$
f(x, a, t) = \begin{bmatrix} p \\
-m_{sc}(t)^{-1}(C(a)p + G(p, a)) \end{bmatrix}, \quad B(x, a, t) = \begin{bmatrix} O_{1 \times 3} \\
m_{sc}(t)^{-1}I_{3 \times 3} \end{bmatrix}
$$

where $m_{sc}(t)$ is the mass of the spacecraft described by the Tsiolkovsky rocket equation, and the functions $G$ and $C$ are as given in [15, 36].

Remark 1. In general, the estimation errors $\|\hat{x}(t) - x(t)\|$ and $\|\hat{a}(t) - a(t)\|$ are expected to decrease with the help of the state-of-the-art onboard navigation schemes as the spacecraft gets closer to the ISO, utilizing more accurate ISO state measurements obtained from an onboard sensor as detailed in [6]. For example, if the extended Kalman filter [39] or contraction theory-based estimator [40–42] is used for navigation, their expected values can be shown to be bounded and exponentially decreasing in $t$. Although G&C are the focus of our study and thus developing such a navigation technique is beyond our scope, those interested in this field can also refer to, e.g., [5, 6], to tighten the estimation error bound using practical knowledge specific to the ISO dynamics.

The major design challenges in realizing the ISO encounter, i.e., achieving a sufficiently small spacecraft delivery error with respect to a given desired relative position to flyby or impact the ISO, can be decomposed into the following G&C problems.

(P1) Autonomous terminal guidance responding rapidly to radically changing ISO state

Since the ISO state and its onboard estimate in (1) change dramatically in time due to their poorly constrained orbits with high inclinations and high relative velocities, using a fixed desired trajectory computed at some point earlier in
time could fail to utilize the radically changing ISO state information as much as possible. Therefore, we construct a guidance policy to optimally achieve the smallest spacecraft delivery error for given estimated states \( \hat{x}(t) \) and \( \hat{a}(t) \) in (1) at time \( t \), and then design a learning-based guidance algorithm that approximates it with a verifiable optimality gap, so the spacecraft can update its desired trajectory autonomously in real-time using the most recent state estimates \( \hat{x}(t) \) and \( \hat{a}(t) \), which become more accurate as the spacecraft gets closer to the ISO as discussed in Remark 1.

(P2) Feedback tracking control robust against state uncertainty

Although the learning-based terminal guidance in (P1) is useful in exploiting the radically changing state information in real-time, evaluating its robustness property against the ISO and spacecraft relative state uncertainty is difficult in general. We thus design an autonomous nonlinear feedback control algorithm that robustly tracks the desired trajectory with zero spacecraft delivery error computed using (P1), and that guarantees a finite tracking error bound even under the presence of the state uncertainty and the guidance learning error in (P1), while still having a verifiable optimality gap with respect to the optimal guidance policy of (P1).

III. Autonomous Guidance via Dynamical System-Based Deep Learning

This section proposes dynamical system-based deep learning to solve the autonomous terminal guidance problem (P1) in Sec. II. As to be seen in the following, it utilizes the known dynamics of (1) and (2) in approximating a computationally-expensive optimal guidance policy, thereby directly minimizing the deviation of the learned state trajectory from the optimal state trajectory with the smallest spacecraft delivery error at the ISO encounter, possessing a verifiable optimality gap with respect to the optimal guidance policy.

A. Model Predictive Control Problem

Let us first introduce the following definition of an ISO state flow, which maps the ISO state at any given time to the one at time \( t \), so we can account for the rapidly changing ISO state estimate of (1) in our proposed framework.

Definition 1. A flow \( \varphi(t) \) of \( a_0 \), where \( a_0 \) is some given ISO state, defines the solution trajectory of the autonomous ISO dynamical system [35, 36] at time \( t \), which satisfies \( \varphi(0) = a_0 \) at \( t = 0 \).

Utilizing the ISO flow given in Definition 1, we consider the following optimal guidance problem for the ISO encounter, given estimated states \( \hat{x}(\tau) \) and \( \hat{a}(\tau) \) in (1) at \( t = \tau \):

\[
\begin{align*}
\hat{u}^*(\hat{x}(\tau), \hat{a}(\tau), t, \rho) &= \arg \min_{u(t) \in \mathcal{U}(t)} \left( c_0 \| p_{\xi}(t_f) - \rho \|^2 + c_1 \int_{\tau}^{t_f} \mathcal{P}(u(t), \xi(t)) \, dt \right) \\
\text{s.t.} \quad &\dot{\xi}(t) = f(\xi(t), \varphi^{\tau}(\hat{a}(\tau)), t) + B(\xi(t), \varphi^{\tau}(\hat{a}(\tau)), t)u(t), \quad t \in (\tau, t_f], \quad \xi(\tau) = \hat{x}(\tau)
\end{align*}
\]

(3) (4)

where \( \tau \in [0, t_f] \) is the current time at which the spacecraft solves (3), \( \hat{\xi} \) is the fictitious spacecraft relative state of the
dynamics (4), \( \int_\tau^{t_f} P(u(t), \xi(t))dt \) is some performance-based cost function, such as \( L^2 \) control effort, \( L^2 \) trajectory tracking error, and information-based cost [43]. \( p_\xi(t_f) \) is the terminal relative position of the spacecraft satisfying \( \xi(t_f) = [p_\xi(t_f)^\top, \dot{p}_\xi(t_f)^\top]^\top \). \( \rho \) is a mission-specific predefined terminal position relative to the ISO at given terminal time \( t_f \) (\( \rho = 0 \) for impacting the ISO), \( U(t) \) is a set containing admissible control inputs, and \( c_0 \in \mathbb{R}_{>0} \) and \( c_1 \in \mathbb{R}_{\geq 0} \) are the weights on each objective function. This paper assumes that the terminal time \( t_f \) is not a decision variable but a fixed constant, as varying it is demonstrated to have a small impact on the objective value of (3) in our simulation setup in Sec. VII. Note that \( \rho \) is explicitly considered as one of the inputs to \( u^* \) to account for the fact that it could change depending on the target ISO.

**Remark 2.** Since it is not realistic to match the spacecraft velocity with that of the ISOs due to their high inclination nature, the terminal velocity tracking error is intentionally not included in the cost function, although it could be with an appropriate choice of \( P \) in (3). Also, we can set \( c_0 = 0 \) and augment the problem with a constraint \( \|p_\xi(t_f) - \rho\| = 0 \) if the problem is feasible with this constraint.

Since the spacecraft relative state changes dramatically due to the ISO’s high relative velocity, as discussed in the problem (P1), and because the actual dynamics are perturbed by the ISO and spacecraft relative state estimation uncertainty, which decreases as \( t \) gets closer to \( t_f \) (see Remark 1), it is expected that the delivery error at the ISO encounter (i.e., \( \|p_\xi(t_f) - \rho\| \)) becomes smaller as the spacecraft solves (3) more frequently onboard using the updated state estimates in the initial condition (4) as in (1). More specifically, it is desirable to apply the optimal guidance policy solving (3) at each time instant \( t \) as follows as in model predictive control (MPC) [19, 44]:

\[
 u_{mpc}(\hat{x}(t), \hat{a}(t), t, \rho) = u^*(\hat{x}(t), \hat{a}(t), t, \rho) \tag{5}
\]

where \( u \) is the control input of (1). Note that \( \tau \) of \( u^* \) in (5) is now changed to \( t \) unlike (3), implying we only utilize the solution of (3) at the initial time \( t = \tau \). Due to the predictive nature of the MPC, which leverages future predictions of the states \( x(t) \) and \( a(t) \) for \( t \in [\tau, t_f] \) obtained by integrating their dynamics given \( \hat{x}(\tau) \) and \( \hat{a}(\tau) \) as in (4), the solution of its linearized and discretized version can be shown to be near-optimal [10] in terms of dynamic regret, i.e., the MPC performance minus the optimal performance in hindsight [11]. This would imply that the nonlinear MPC also enjoys similar optimality guarantees when it is solved by the sequential convex programming approach [45–47], which is proven to converge to a point satisfying the KKT conditions [48, pp. 243-244] (see [12]). However, solving the nonlinear optimization problem (3) at each time instant to obtain (5) is not realistic for a spacecraft with limited computational power.
B. Imitation Learning of MPC State and Control Trajectories

In order to compute the MPC of (5) in real-time with a verifiable optimality guarantee, our proposed learning-based terminal guidance policy models it using an SN-DNN [13] defined as follows.

**Definition 2.** A neural network is the following nonlinear mathematical model to approximately represent training data points \( \{(a_i, b_i)\}_{i=1}^{N_d} \) of \( b = \psi(a) \), generated by a function \( \psi \), by optimally tuning its hyperparameters \( W_\ell \), \( \ell \in \mathbb{N} \cup [1, N_\ell + 1] \):

\[
b_{\ell} = \psi_{nn}(a_{\ell}; W_\ell) = T_{N_{\ell}+1} \circ \phi \circ T_{N_\ell} \circ \cdots \circ \phi \circ T_1(a_{\ell})
\]

(6)

where \( T_{\ell}(x) = W_\ell x, \phi \) denotes composition of functions, and \( \phi \) is an activation function. A neural network with more than two layers (i.e., \( N_\ell \geq 2 \)) is called a deep neural network (DNN). A spectrally-normalized DNN (SN-DNN) is a DNN with its weights \( W_\ell \) normalized as \( W_\ell = (C_{nn} \Omega_\ell)/\|\Omega_\ell\| \), where \( C_{nn} \in \mathbb{R}_{\geq 0} \) is a given constant.

The SN-DNN is known to have the following useful properties [7, 13].

**Lemma 1.** An SN-DNN is Lipschitz continuous by design with its 2-norm Lipschitz constant \( C_{nn}^{N_\ell+1} L_\phi^{N_\ell} \), where \( L_\phi \in \mathbb{R}_{> 0} \) is the Lipschitz constant of the activation function \( \phi \) in (6). Also, it is robust to perturbation in its input.

*Proof.* See Lemma 6.2 of [16].

Let us denote the proposed learning-based terminal guidance policy as \( u_\ell(\hat{x}(t), \hat{\alpha}(t), t, \rho; \theta_{nn}) \), which models the MPC policy \( u_{\text{mpc}}(\hat{x}(t), \hat{\alpha}(t), t, \rho) \) of (5) using the SN-DNN of Definition 2, where \( \theta_{nn} = \{W_\ell\}_{\ell=1}^{N_\ell+1} \) is its hyperparameter given in (6). The following definition of the process induced by the spacecraft dynamics with \( u_{\text{mpc}} \) and \( u_\ell \), which map the ISO and spacecraft relative state at any given time to their respective spacecraft relative state at time \( t \), is useful for simplifying notation in our framework.

**Definition 3.** Mappings denoted as \( \varphi^\ell(t, \alpha_\tau, \tau, \rho; \theta_{nn}) \) and \( \varphi^\text{mpc}(t, \alpha_\tau, \tau, \rho) \) (called processes [49, p. 24]) define the solution trajectories of the following non-autonomous dynamical systems at time \( t \), controlled by the SN-DNN and MPC policy, respectively:

\[
\dot{\xi}(t) = f(\xi(t), \varphi^{\ell-\tau}(\alpha_\tau), t) + B(\xi(t), \varphi^{\ell-\tau}(\alpha_\tau), t)u_\ell(\xi(t), \varphi^{\ell-\tau}(\alpha_\tau), t, \rho; \theta_{nn}), \quad \xi(\tau) = x_\tau
\]

(7)

\[
\dot{\xi}(t) = f(\xi(t), \varphi^{\ell-\tau}(\alpha_\tau), t) + B(\xi(t), \varphi^{\ell-\tau}(\alpha_\tau), t)u_{\text{mpc}}(\xi(t), \varphi^{\ell-\tau}(\alpha_\tau), t, \rho), \quad \xi(\tau) = x_\tau
\]

(8)

where \( \tau \in [0, t_f] \), \( t_f \) and \( \rho \) are the given terminal time and relative position at the ISO encounter as in (4), \( \alpha_\tau \) and \( x_\tau \) are some given ISO and spacecraft relative state at time \( t = \tau \), respectively. \( f \) and \( B \) are given in (1) and (2), and \( \varphi^{\ell-\tau}(\alpha_\tau) \) is the ISO state trajectory with \( \varphi^0(\alpha_\tau) = \alpha_\tau, t = \tau \) as given in Definition 1.
Let \((\bar{x}, \bar{a}, \bar{t}, \bar{\rho}, \Delta \bar{f})\) denote a sampled data point for the spacecraft state, ISO state, current time, desired terminal relative position, and time of integration to be used in (9), respectively. Also, let \(\text{Unif}(S)\) be the uniform distribution over a compact set \(S\), which produces \((\bar{x}, \bar{a}, \bar{t}, \bar{\rho}, \Delta \bar{f}) \sim \text{Unif}(S)\). Using Definition 3, we introduce the following new loss function to be minimized by optimizing the hyperparameter \(\theta_{\text{nn}}\) of the SN-DNN guidance policy \(u_{\ell}(\hat{x}(t), \hat{a}(t), t, \rho; \theta_{\text{nn}})\):

\[
\mathcal{L}_{\text{nn}}(\theta_{\text{nn}}) = \mathbb{E} \left[ \|u_{\ell}(\bar{x}, \bar{a}, \bar{t}, \bar{\rho}; \theta_{\text{nn}}) - u_{\text{mpc}}(\bar{x}, \bar{a}, \bar{t}, \bar{\rho})\|^2_{C_{u}} + \|\varphi^+_\ell(\bar{x}, \bar{a}, \bar{t}, \bar{\rho}; \theta_{\text{nn}}) - \varphi^+_\text{mpc}(\bar{x}, \bar{a}, \bar{t}, \bar{\rho})\|^2_{C_{x}} \right] 
\]

(9)

where \(\|\cdot\|_{C_{x}}\) and \(\|\cdot\|_{C_{u}}\) are the weighted Euclidean 2-norm given as \(\|\cdot\|^2_{C_{u}} = (\cdot)^T C_{u} (\cdot)\) and \(\|\cdot\|^2_{C_{x}} = (\cdot)^T C_{x} (\cdot)\) for symmetric positive definite weight matrices \(C_{u}, C_{x} > 0\), and \(\varphi^+_\ell(\bar{x}, \bar{a}, \bar{t}, \bar{\rho}; \theta_{\text{nn}})\) and \(\varphi^+_\text{mpc}(\bar{x}, \bar{a}, \bar{t}, \bar{\rho})\) are the solution trajectories with the SN-DNN and MPC guidance policy given in Definition 3, respectively.

Existing learning methods [16, 28] indirectly imitate a given desired state trajectory by imitating the desired control policy using only the first term of (9). As can be seen from the loss function (9) and as illustrated in Fig. 4, a key distinction of our proposed learning-based guidance from existing work is that we directly train the SN-DNN to also minimize the deviation of the state trajectory with the SN-DNN guidance policy from the desired MPC state trajectory itself using the second term of (9). This is to properly account for the fact that the learning objective is not only to minimize \(\|u_{\ell} - u_{\text{mpc}}\|\), but to imitate the optimal trajectory with the smallest spacecraft delivery error at the ISO encounter as discussed in the problem (P1). We will see how the selection of the weight matrices \(C_{u}\) and \(C_{x}\) affects the control performance of \(u_{\ell}\) in Sec. VII.

**Remark 3.** As in standard learning algorithms for neural networks including stochastic gradient descent (SGD) [50, 51], the expectation of the loss function (9) can be approximated using sampled data points as follows:

\[
\mathcal{L}_{\text{emp}}(\theta_{\text{nn}}) = \frac{1}{N_{\text{d}}} \sum_{i=1}^{N_{\text{d}}} \left[ \|u_{\ell}(\bar{x}_i, \bar{a}_i, \bar{t}_i, \bar{\rho}_i; \theta_{\text{nn}}) - u_{\text{mpc}}(\bar{x}_i, \bar{a}_i, \bar{t}_i, \bar{\rho}_i)\|^2_{C_{u}} + \|\varphi^+_\ell(\bar{x}_i, \bar{a}_i, \bar{t}_i, \bar{\rho}_i; \theta_{\text{nn}}) - \varphi^+_\text{mpc}(\bar{x}_i, \bar{a}_i, \bar{t}_i, \bar{\rho}_i)\|^2_{C_{x}} \right] 
\]

(10)
Table 1 Notations in Theorem 1.

| Symbol   | Description                                                                 |
|----------|-----------------------------------------------------------------------------|
| $L_\ell$ | 2-norm Lipschitz constant of $u_\ell$ in $\mathbb{R}_{>0}$, guaranteed to exist due to Lemma 1 |
| $N_d$    | Number of training data points                                              |
| $S_{\text{test}}$ | Any compact test set containing $(x, \alpha, t, \rho)$, not necessarily training set $S_{\text{train}}$ itself |
| $u_\ell$ | Learning-based terminal guidance policy that models $u_{\text{mpc}}$ by SN-DNN using loss function given in (9) |
| $u_{\text{mpc}}$ | Optimal MPC terminal guidance policy given in (5) |
| $(\bar{x}_i, \bar{\alpha}_i, \bar{t}_i, \bar{\rho}_i)$ | Test data point for S/C relative state, ISO state, current time, and desired terminal relative position of (4), respectively, where $i \in [1, N_d]$ |
| $\Pi_{\text{train}}$ | Training dataset containing finite number of training data points, i.e., $\Pi_{\text{train}} = \{(\bar{x}_i, \bar{\alpha}_i, \bar{t}_i, \bar{\rho}_i)\}_{i=1}^{N_d}$ |

where the training data points $\{(\bar{x}_i, \bar{\alpha}_i, \bar{t}_i, \bar{\rho}_i, \Delta \bar{t}_i)\}_{i=1}^{N_d}$ are drawn independently from $\text{Unif}(S)$.

C. Optimality Gap of Deep Learning-Based Guidance

Since an SN-DNN Definition 2 is Lipschitz bounded by design and robust to perturbation as in Lemma 1, the optimality gap of the guidance framework $u_\ell$ introduced in Sec. III.B can be bounded as in the following theorem, where the notations are summarized in Table 1 and the proof concept is illustrated in Fig. 5.

**Theorem 1.** Suppose that $u_{\text{mpc}}$ is Lipschitz with its 2-norm Lipschitz constant $L_{\text{mpc}} \in \mathbb{R}_{>0}$. If $u_\ell$ is trained using the empirical loss function (10) of Remark 3 to have $\exists \epsilon_{\text{train}} \in \mathbb{R}_{\geq 0}$ s.t.

$$\sup_{i \in [1, N_d]} \|u_\ell(\bar{x}_i, \bar{\alpha}_i, \bar{t}_i, \bar{\rho}_i; \theta_{\text{mn}}) - u_{\text{mpc}}(\bar{x}_i, \bar{\alpha}_i, \bar{t}_i, \bar{\rho}_i)\| \leq \epsilon_{\text{train}}$$

then we have the following bound:

$$\|u_\ell(x, \alpha, t, \rho; \theta_{\text{mn}}) - u_{\text{mpc}}(x, \alpha, t, \rho)\| \leq \epsilon_{\text{train}} + r(x, \alpha, t, \rho)(L_\ell + L_{\text{mpc}}) = \epsilon_{\ell u}, \forall (x, \alpha, t, \rho) \in S_{\text{test}}$$

where $r(x, \alpha, t, \rho) = \inf_{\eta \in S_{\text{test}}} \sqrt{||\bar{x}_i - x||^2 + ||\bar{\alpha}_i - \alpha||^2 + ||\bar{t}_i - t||^2 + ||\bar{\rho}_i - \rho||^2}$.

**Proof.** Let $\eta = (x, \alpha, t, \rho)$ be a test element in $S_{\text{test}}$ (i.e., $\eta \in S_{\text{test}}$) and let $\bar{\zeta}_j(\eta)$ be the training data point in $\Pi_{\text{train}}$ (i.e., $\bar{\zeta}_j(\eta) \in \Pi_{\text{train}}$) that achieves the infimum of $r(\eta)$ with $i = j$ for a given $\eta \in S_{\text{test}}$. Since $u_\ell$ and $u_{\text{mpc}}$ are Lipschitz by design and by assumption, respectively, we have for any $\eta \in S_{\text{test}}$ that

$$\|u_\ell(\eta; \theta_{\text{mn}}) - u_{\text{mpc}}(\eta)\| \leq \|u_\ell(\bar{\zeta}_j(\eta); \theta_{\text{mn}}) - u_{\text{mpc}}(\bar{\zeta}_j(\eta))\| + \|u_\ell(\eta; \theta_{\text{mn}}) - u_\ell(\bar{\zeta}_j(\eta); \theta_{\text{mn}})\| + \|u_{\text{mpc}}(\eta) - u_{\text{mpc}}(\bar{\zeta}_j(\eta))\|$$

$$\leq \|u_\ell(\bar{\zeta}_j(\eta); \theta_{\text{mn}}) - u_{\text{mpc}}(\bar{\zeta}_j(\eta))\| + r(\eta)(L_\ell + L_{\text{mpc}}) \leq \epsilon_{\text{train}} + r(\eta)(L_\ell + L_{\text{mpc}})$$

where the second inequality follows from the definition of $\bar{\zeta}_j(\eta)$ and the third inequality follows from (11) and the fact...
that \( \bar{\zeta}(\eta) \in \Pi_{\text{train}} \). This relation leads to the desired result (12) as it holds for any \( \eta \in S_{\text{test}} \).

The major strength of the SN-DNN is that it provides a verifiable optimality gap even for data points not in its training set, using spectral normalization for Lipschitz regularization, which indicates that it still benefits from the near-optimal guarantee of the MPC in terms of dynamic regret [10–12] as discussed below (5). As illustrated in Fig. 5 for clarifying the proof of Theorem 1, each term in the optimality gap (12) can be interpreted as follows:

1) \( \epsilon_{\text{train}} \) of (11) is the training error of the SN-DNN, expected to decrease as the learning proceeds using SGD.
2) \( r(\eta) \) is the closest distance from the test element \( \eta \in S_{\text{test}} \) to a training data point in \( \Pi_{\text{train}} \), expected to decrease as the number of data points \( N_d \) in \( \Pi_{\text{train}} \) increases and as the training set \( S_{\text{train}} \) gets larger.
3) The Lipschitz constant \( L_{\ell} \) is a design parameter we could arbitrarily choose when constructing the SN-DNN.
4) We could also treat \( L_{\text{mpc}} \) as a design parameter by adding a Lipschitz constraint, \( \Vert \partial u_{\text{mpc}} / \partial \eta \Vert \leq L_{\text{mpc}}, \) in solving (3).

Note that \( \epsilon_{\text{train}} \) and \( r(\eta) \) can always be computed numerically for a given \( \eta \) as the dataset \( \Pi_{\text{train}} \) only has a finite number of data points. Furthermore, since the proposed guidance framework trains the SN-DNN to also imitate the desired state trajectory using the second term of (9), we could achieve a bound similar to (12) of Theorem 1 even for the optimality gap of the learned state trajectory with the SN-DNN guidance policy, assuming the whole network as in (7), which outputs integrated states given initial states, is Lipschitz. The pseudo-code for constructing the proposed dynamical system-based SN-DNN for autonomous guidance is given in Algorithm 1, where we utilize SGD and back-propagation techniques in standard neural network training on top of the dynamics integration given in (7) and (8).

Remark 4. Obtaining a tighter and more general optimality gap of the learned control and state trajectories has been an active field of research [52–55]. Particularly for a neural network equipped with a dynamical structure as in our proposed approach, we could refer to generalization bounds for the neural ordinary differential equations [56–61], or use systems and control theoretical methods to augment it with stability and robustness properties [62, 63]. We could also consider combining the SN-DNN with neural networks that have enhanced structural guarantees of robustness,
including robust implicit networks [64, 65] and robust equilibrium networks [66, 67].

**Algorithm 1: Dynamical System-Based SN-DNN Guidance**

**Inputs**: Training set $S_{\text{train}}$ and test set $S_{\text{test}}$

**Outputs**: Trained dynamical system-based SN-DNN

**A. Training Data Sampling**

for $i \leftarrow 1$ to $N_d$ do

Sample $\tilde{\zeta}_i = (\bar{x}_i, \bar{a}_i, \bar{t}_i, \bar{\rho}_i, \Delta \bar{t}_i) \sim \text{Unif}(S_{\text{train}})$

Sample $\tilde{\eta}_i = (\bar{x}_i, \bar{a}_i, \bar{t}_i, \bar{\rho}_i, \Delta \bar{t}_i) \sim \text{Unif}(S_{\text{test}})$

Evaluate $u_{\text{mpc}}(\bar{x}_i, \bar{a}_i, \bar{t}_i, \bar{\rho}_i)$ of (5) for $\tilde{\zeta}_i$ and $\tilde{\eta}_i$

Integrate (8) with $\tilde{\zeta}_i$ and $\tilde{\eta}_i$ to get $\phi_{\text{mpc}}^{\bar{t} + \Delta \bar{t}}(\bar{x}_i, \bar{a}_i, \bar{t}_i, \bar{\rho}_i)$

Save training and test data as $D_{\text{train}}$ and $D_{\text{test}}$

**B. Dynamical System-Based SN-DNN Training**

for epoch $\leftarrow 1$ to number of epochs do

for training batch $\in D_{\text{train}}$ do

Evaluate $u_{\ell}(\bar{x}_i, \bar{a}_i, \bar{t}_i, \bar{\rho}_i; \theta_{\text{nn}})$ for training batch

Integrate (7) for training batch to get $\varphi_{\ell}^{\bar{t} + \Delta \bar{t}}(\bar{x}_i, \bar{a}_i, \bar{t}_i, \bar{\rho}_i)$

Train SN-DNN with loss (10) using SGD

Compute test loss with (10) for $D_{\text{test}}$

Compute error bounds (12) of Theorem 1 and Remark 4 for $D_{\text{test}}$

if test loss & error bounds are small enough then

break


Although the optimality gap discussed in Theorem 1 and in Remark 4 are useful in that they provide mathematical guarantees for learning-based frameworks only with the Lipschitz assumption (e.g., it could prove safety in the learning-based MPC framework [14]). However, when the system is perturbed by the state uncertainty as in (1), we could only guarantee that the distance between the state trajectories controlled by $u_{\text{mpc}}$ of (5) and $u_{\ell}$ of Theorem 1 is bounded by a function that increases exponentially with time [16, 28]. In Sec. IV and V, we will see how such a conservative bound can be replaced by a decreasing counterpart.

**IV. Deep Learning-Based Optimal Tracking Control**

This section proposes a feedback control policy to be used on top of the SN-DNN terminal guidance policy $u_{\ell}$ of Theorem 1 in Sec. III. In particular, we utilize $u_{\ell}$ to design the desired trajectory with zero spacecraft delivery error and then construct pointwise optimization-based tracking control with a Lyapunov stability condition, which can be shown to have an analytical solution for real-time implementation. In Sec. V, we will see that this framework plays an essential part in solving the control problem (P2) of Sec. II.

**Remark 5.** For notational convenience, we drop the dependence on the desired terminal relative position $\rho$ of (4) in $u_{\ell}$, $u_{\text{mpc}}$, $\varphi_{\ell}$, and $\varphi_{\text{mpc}}$ in the subsequent sections, as it is time-independent and thus does not affect the arguments to be made in the following. Note that the SN-DNN is still to be trained regarding $\rho$ as one of its inputs as outlined in Algorithm 2, so we do not have to retrain SN-DNNs every time we change $\rho$. 

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Table 2  Summary of ISO state flow and spacecraft state processes in Definitions 1 and 3, where $u_f$ and $u_{mpc}$ are SN-DNN and MPC guidance policies, respectively. Note that the dependence on the terminal position $\rho$ is omitted as explained in Remark 5.

| $\psi_t (x, \alpha_t, \tau, \theta_{in})$ | Solution trajectory of ISO dynamics at time $t$ which satisfies $\psi^0(\alpha_t) = \alpha_t$ |
| $\psi_t^f (x, \alpha_t, \tau ; \theta_{in})$ | Solution trajectory of S/C relative dynamics at time $t$, controlled by $u_f$ with no state estimation error, which satisfies $\psi_t^f(x, \alpha_t, \tau ; \theta_{in}) = x_f$ at $t = \tau$ and $\alpha(t) = \psi_t (x, \alpha_t)$ |
| $\psi_t^{mpc} (x, \alpha_t, \tau)$ | Solution trajectory of S/C relative dynamics at time $t$, controlled by $u_{mpc}$ with no state estimation error, which satisfies $\psi_t^{mpc}(x, \alpha_t, \tau) = x_f$ at $t = \tau$ and $\alpha(t) = \psi_t^{-1}(\alpha_t)$ |

A. Desired State Trajectory with Zero Delivery Error

Let us recall the definitions of the ISO state flow and spacecraft state processes of Definitions 1 and 3 summarized in Table 2. Ideally at some given time $t = t_d \in [0, t_f)$ when the spacecraft possesses enough information on the ISO, we would like to construct a desired trajectory using the estimated states $\hat{x}(t_d)$ and $\hat{\xi}(t_d)$ such that it ensures zero spacecraft delivery error at the ISO encounter assuming zero state estimation errors for $t \in [t_d, t_f]$ in (1). This can be achieved by obtaining a desired trajectory for $\phi_t^f(x_f, \alpha_f, t_f)$ of Table 2 with $\alpha_f = \phi_t^{-t_d}(\hat{\alpha}(t_d))$, i.e., by solving the following dynamics with the SN-DNN terminal guidance policy $u_f$ (7) backward in time:

$$\dot{\xi}(t) = f(\xi(t), \phi_t^{-t_d}(\hat{\alpha}(t_d)), t) + B(\xi(t), \phi_t^{-t_d}(\hat{\alpha}(t_d)), t) u_f(\xi(t), \phi_t^{-t_d}(\hat{\alpha}(t_d)), t ; \theta_{in}), \quad \xi(t_f) = x_f$$

(13)

where the property of the ISO solution flow in Table 2 introduced in Definition 1, $\phi_t^{-t_f}(\alpha_f) = \phi_t^{-t_f}(\phi_t^{-t_d}(\alpha(t_d))) = \phi_t^{-t_d}(\alpha(t_d))$, is used to get (13). $t_f$ is the given terminal time at the ISO encounter as in (4), and the ideal spacecraft terminal relative state $x_f$ is defined as

$$x_f = \left[ C_{2v} \psi_t^f (\hat{\xi}(t_d), \hat{\alpha}(t_d), t_d) \right]$$

(14)

where $\rho$ is the desired terminal relative position given in (4), $\phi_t^f (\hat{\xi}(t_d), \hat{\alpha}(t_d), t_d)$ is the spacecraft relative state at $t = t_f$ obtained by integrating the dynamics forward as in Table 2, and $C_{2v} = [O_{3 \times 3} I_{3 \times 3}] \in \mathbb{R}^{3 \times 26}$ is a matrix that maps the spacecraft relative state to its velocity vector. Figure 6 illustrates the construction of such a desired trajectory.

Remark 6. Although the desired state trajectory design depicted in Fig. 6 involves backward and forward integration (13) and (14) as in Definition 3, the SN-DNN approximation of the MPC policy allows performing it within a short period. In fact, when measured using the Mid 2015 MacBook Pro laptop, it takes only about $3.0 \times 10^{-4}$ s for numerically integrating the dynamics for one step using the fourth-order Runge–Kutta method (e.g., it takes about 3 s to get the desired trajectory for $t_d = 0$ s and $t_f = 10000$ s with the discretization time step 1 s). The computational time should decrease as $t_d$ becomes larger. Section V.C delineates how we utilize such a desired trajectory in real-time with the feedback control to be introduced in Sec. IV.B.
B. Deep Learning-Based Pointwise Min-Norm Control

Using the results given in Sec. IV.A, we present one approach to designing pointwise optimal tracking control resulting in verifiable spacecraft delivery error, even under the presence of state uncertainty. For notational simplicity, let us denote the desired spacecraft relative state and ISO state trajectories of (13), constructed using $\hat{x}(t_d)$ and $\hat{\alpha}(t_d)$ at $t = t_d$ with the terminal state (14) as illustrated in Fig. 6, as follows:

$$x_d(t) = \varphi_t^f(x_f, \alpha_f, t_f), \quad \alpha_d(t) = \varphi_t^{\alpha_d}(\hat{\alpha}(t_d)).$$  \hspace{1cm} (15)

Also, assuming that we select $x$ as $x = [p^T, \dot{p}^T]^T$ as the state of (1) as in (2), where $p \in \mathbb{R}^3$ is the position of the spacecraft relative to the ISO, let $f(x, \alpha, t)$ and $B(x, \alpha, t)$ be defined as follows:

$$f(x, \alpha, t) = C_{s2v} f(x, \alpha, t), \quad B(x, \alpha, t) = C_{s2v} B(x, \alpha, t),$$  \hspace{1cm} (16)

where $f$ and $B$ are given in (1) and $C_{s2v} = [O_{3 \times 3} I_{3 \times 3}] \in \mathbb{R}^{3 \times 6}$ as in (14). Note that the relations (16) imply

$$\dot{p} = f(x, \alpha, t) + B(x, \alpha, t)u(\hat{x}, \hat{\alpha}, t)$$  \hspace{1cm} (17)

Given the desired trajectory (15), $x_d = [p_d^T, \dot{p}_d^T]^T$, which achieves zero spacecraft delivery error at the ISO encounter when the state estimation errors are zero for $t \in [t_d, t_f]$ in (1), we propose to design a controller $u$ of (1) and (17) as follows:

$$u^*_f(\hat{x}, \hat{\alpha}, t; \theta_{mn}) = u_f(x_d(t), \alpha_d(t), t; \theta_{mn}) + k(\hat{x}, \hat{\alpha}, t)$$  \hspace{1cm} (18)

$$k(\hat{x}, \hat{\alpha}, t) = \begin{cases} 0 & \text{if } Y(\hat{x}, \hat{\alpha}, t) \leq 0 \\ \frac{-Y(\hat{x}, \hat{\alpha}, t)(\dot{p}_d - \dot{p}_d(\hat{p}, t))}{\|\dot{p}_d - \dot{p}_d(\hat{p}, t)\|^2} & \text{otherwise} \end{cases}$$  \hspace{1cm} (19)
with Υ and 𝑔, defined as

\[ \Upsilon(\dot{\hat{x}}, \dot{\hat{v}}, t) = (\dot{\hat{p}} - \dot{\hat{q}}(\dot{\hat{p}}, t))^T M(t)(f(\dot{\hat{x}}, \dot{\hat{v}}, t) - f(x_d(t), \alpha_d(t), t) + \Lambda(\dot{\hat{p}} - \dot{\hat{q}}(\dot{\hat{p}}, t))) + \alpha(\dot{\hat{p}} - \dot{\hat{q}}(\dot{\hat{p}}, t)) \]

\[ \dot{\hat{q}}(\dot{\hat{p}}, t) = -\Lambda(\dot{\hat{p}} - p_d(t)) + \dot{\hat{p}}_d(t) \]  

(20)

where \( \hat{p} \in \mathbb{R}^3 \) is the estimated position of the spacecraft relative to the ISO s.t. \( \dot{\hat{x}} = [\dot{\hat{p}}^T, \dot{\hat{v}}^T]^T \), \( u_\ell \) is the SN-DNN terminal guidance policy of Theorem 1, \( \alpha_d, f, \) and \( \mathcal{B} \) are given in (15) and (16), \( \Lambda > 0 \) is a given symmetric positive definite matrix, \( \alpha \in \mathbb{R}_{>0} \) is a given positive constant, and \( M(t) = m(t)I_{3 \times 3} \) for the spacecraft mass \( m(t) \) given in (2).

This control policy can be shown to possess the following pointwise optimality property, in addition to the robustness and stability guarantees to be seen in Sec. V.

Consider the following optimization problem, which computes an optimal control input that minimizes its instantaneous deviation from that of the SN-DNN guidance policy at each time instant, under an incremental Lyapunov stability condition:

\[ u^*_\ell(\dot{\hat{x}}, \dot{\hat{v}}, t; \theta_m) = \arg \min_{u \in \mathbb{R}^m} \|u - u_\ell(x_d(t), \alpha_d(t), t; \theta_m)\|^2 \]

\[ \text{s.t.} \quad \frac{\partial V}{\partial \hat{p}}(\dot{\hat{p}}, \dot{\hat{q}}(\dot{\hat{p}}, t), t)(f(\dot{\hat{x}}, \dot{\hat{v}}, t) + \mathcal{B}(\dot{\hat{x}}, \dot{\hat{v}}, t)u)
\]

\[ + \frac{\partial V}{\partial \dot{\hat{q}}}(\dot{\hat{p}}, \dot{\hat{q}}(\dot{\hat{p}}, t), t)(\dot{\hat{q}}_d(t) - \Lambda(\dot{\hat{p}} - \dot{\hat{q}}_d(t))) \leq -2\alpha V(\dot{\hat{p}}, \dot{\hat{q}}(\dot{\hat{p}}, t), t) \]  

(22)

where \( V \) is a non-negative function defined as

\[ V(\dot{\hat{p}}, \dot{\hat{q}}(\dot{\hat{p}}, t)) = (\dot{\hat{p}} - \dot{\hat{q}}(\dot{\hat{p}}))^T M(t)(\dot{\hat{p}} - \dot{\hat{q}}(\dot{\hat{p}})) \]

and the other notation is as given in (18).

**Lemma 2.** The optimization problem (21) is always feasible, and the controller (18) defines its analytical optimal solution for the spacecraft relative dynamical system (2). Furthermore, substituting \( u = u_\ell(x_d(t), \alpha_d(t), t; \theta_m) \) into (22) yields \( Y(\dot{\hat{x}}, \dot{\hat{v}}, t) \leq 0 \), which implies the controller (18) modulates the desired input, \( u_\ell(x_d(t), \alpha_d(t), t; \theta_m) \), only when necessary to ensure the stability condition (22).

**Proof.** See Appendix.

Let us emphasize again that, as proven in Lemma 2, the deviation term \( k(\dot{\hat{x}}, \dot{\hat{v}}, t) \) of the controller (18) is non-zero only when the stability condition (22) cannot be satisfied with the SN-DNN terminal guidance policy \( u_\ell \) of Theorem 1. This result can be viewed as an extension of the control methodology of [68, 69], where the Lagrangian system-type structure of the spacecraft relative dynamics is used extensively to obtain the analytical solution (18) of the quadratic
optimization problem (21), for the sake of its real-time implementation.

C. Assumptions for Robustness and Stability

Before proceeding to the next section on proving the robustness and stability properties of the SN-DNN min-norm control of (18) of Lemma 2, let us make a few assumptions with the notations given in Table 3, the first of which is that the spacecraft has access to an on-board navigation scheme that satisfies the following conditions.

**Assumption 1.** Let the probability of the error vectors remaining in $E_{iso}$ and $E_{nc}$ be bounded as follows for $\exists \varepsilon_{est} \in \mathbb{R}_{\geq 0}$:

$$\mathbb{P}\left[ \bigcap_{t \in [0, t_f]} (\hat{x}(t) - a(t)) \in E_{iso} \cap (\hat{x}(t) - x(t)) \in E_{nc} \right] \geq 1 - \varepsilon_{est}. \quad (24)$$

We assume that if the event of (24) has occurred, then we have the following bound for any $t_1, t_2 \in [0, t_f]$ s.t. $t_1 \leq t_2$:

$$\mathbb{E}_{Z_1} \left[ \|\hat{x}(t) - a(t)\|^2 + \|\hat{x}(t) - x(t)\|^2 \right] \leq \varsigma^t(Z_1, t_1), \; \forall t \in [t_1, t_2]. \quad (25)$$

We also assume that given the 2-norm estimation error satisfies $\sqrt{\|\hat{x}(t) - a(t)\|^2 + \|\hat{x}(t) - x(t)\|^2} < c_{\varepsilon}$ for $c_{\varepsilon} \in \mathbb{R}_{\geq 0}$ at time $t = t_s$, then it satisfies this bound for $\forall t \in [t_s, t_f]$ with probability at least $1 - \varepsilon_{err}$, where $\exists \varepsilon_{err} \in \mathbb{R}_{\geq 0}$.

If the extended Kalman filter [39] or contraction theory-based estimator [40–42] is used for navigation with disturbances expressed as the Gaussian white noise processes, then we have $\varsigma^t(Z_1, t_1) = e^{-\beta(t-t_1)} \sqrt{\|\hat{x}(t) - a(t)\|^2 + \|\hat{x}(t) - x(t)\|^2} + c$, where $Z_1 = (a(t), \hat{a}(t), x(t))$ and $\beta$ and $c$ are some given positive constants (see Example 1). The last statement of the boundedness of the estimation error is expected for navigation schemes resulting in a decreasing estimation error, and can be shown formally using Ville’s maximal inequality for supermartingales [71, pp. 79-83] with an appropriate Lyapunov-like function for navigation synthesis [71, pp. 79-83] (see Lemma 3 in Appendix for similar computation in control synthesis). Note that if $E_{iso} = E_{nc} = \mathbb{R}^n$, i.e., the bound (25) holds globally, then we have $\varepsilon_{est} = 0$. Let us further

---

| $C_{iso}(r)$, $C_{nc}(r)$ | Tubes of radius $r \in \mathbb{R}_{\geq 0}$ centered around desired trajectories $x_d$ and $a_d$ given in (15), i.e., $C_{iso}(r) = \bigcup_{t \in [0, t_f]} \{ x \in \mathbb{R}^n \mid \| x - a_d(t) \| < r \} \subset \mathbb{R}^n$ and $C_{nc}(r) = \bigcup_{t \in [0, t_f]} \{ x \in \mathbb{R}^n \mid \| x - x_d(t) \| < r \} \subset \mathbb{R}^n$ |
|---|---|
| $E_{iso}$, $E_{nc}$ | Subsets of $\mathbb{R}^n$ that have ISO and S/C state estimation error vectors $\| \hat{x}(t) - a(t) \|$ and $\| \hat{x}(t) - x(t) \|$, respectively, where given on-board navigation scheme is valid (e.g., region of attraction [70, pp. 312-322]) |
| $Z_s[t]$ | Conditional expected value operator s.t. $\mathbb{E}[\cdot | x(t_1) = x_1, \hat{x}(t_1) = \hat{x}_1, a(t_1) = a_1, \hat{a}(t_1) = \hat{a}_1]$ |
| $t_f$ | Given terminal time at ISO encounter as in (4) |
| $t_s$ | Time when S/C activates SN-DNN min-norm control policy (18) of Lemma 2 |
| $Z_1$ | Tuple of true and estimated ISO and S/C state at time $t = t_1$, i.e., $Z_1 = (a_1, \hat{a}_1, x_1, \hat{x}_1)$ |
| $\varsigma^t(Z_1, t_1)$ | Expected estimation error upper bound at time $t = t_1$ given $Z_1$ at time $t = t_1$, which can be determined based on choice of navigation techniques (see Remark 1) |

---

Table 3  Notations in Sec. IV.C.
make the following assumption on the SN-DNN min-norm control policy (18).

**Assumption 2.** We assume that $k$ of (19) is locally Lipschitz in its first two arguments, i.e., $\exists r_{sc}, r_{iso}, L_k \in \mathbb{R}_{>0}$ s.t.

$$
\|k(x_1, a_1, t) - k(x_2, a_2, t)\| \leq L_k \sqrt{\|a_1 - a_2\|^2 + \|x_1 - x_2\|^2}, \forall a_1, a_2 \in C_{iso}(r_{iso}), x_1, x_2 \in C_{sc}(r_{sc}), t \in [t_s, t_f]. (26)
$$

We also assume that $r_{iso}$ and $r_{sc}$ are large enough to have $r_{iso} - 2c_e \geq 0$ and $r_{sc} - 2c_e \geq 0$ for $c_e$ of Assumption 1, and that the set $C_{iso}(r_{iso} - c_e)$ is forward invariant, i.e.,

$$
\alpha(t_s) \in C_{iso}(r_{iso} - c_e) \Rightarrow \varphi^{t-t_s}(\alpha(t_s)) \in C_{iso}(r_{iso} - c_e), \forall t \in [t_s, t_f]
$$

(27)

where $\varphi^{t-t_s}(\alpha(t_s))$ is the ISO state trajectory with $\varphi^0(\alpha(t_s)) = \alpha(t_s)$ at $t = t_s$ as given in Table 2 and defined in Definition 1.

If $f$ of (16) is locally Lipschitz in $x$ and $\alpha$, $k$ of (19) can be expressed as a composition of locally Lipschitz functions in $x$ and $\alpha$, which implies that the Lipschitz assumption (26) always holds for finite $r_{iso}$ and $r_{sc}$. Since the estimation error is expected to decrease in general, $c_e$ of Assumption 1 can be made smaller as $t_s$ gets larger, which renders the second condition of Assumption 2 less strict (see Appendix for details in how we select $c_e$).

**V. Neural-Rendezvous: Learning-based Robust Guidance and Control to Encounter ISOs**

This section finally presents Neural-Rendezvous, a deep learning-based terminal G&C approach to autonomously encounter ISOs, thereby solving the problems (P1) and (P2) of Sec. II that arise from the large state uncertainty and high-velocity challenges. It will be shown that the SN-DNN min-norm control (18) of Lemma 2 verifies a formal exponential bound on expected spacecraft delivery error, with a finite probability and finite optimality gap, which provides valuable information in determining whether we should use the SN-DNN terminal guidance policy or enhance it with the SN-DNN min-norm control, depending on the size of the state uncertainty.

**A. Robustness and Stability Guarantee**

The assumptions introduced in Sec. IV.C allows bounding the mean squared distance between the spacecraft relative position of (1) controlled by (18) and the desired position $p_d(t)$ given in (15), even under the presence of the state uncertainty (see Fig. 5). We remark that the additional notations in the following theorem are summarized in Table 4, where the others are consistent with the ones in Table 3.

**Theorem 2.** Suppose that Assumptions 1 and 2 hold, and that the spacecraft relative dynamics with respect to the ISO, given in (1), is controlled by $u = u^*_t$. If the estimated states at time $t = t_s$ satisfy $\hat{\alpha}_s \in C_{iso}(\hat{R}_{iso})$ and $\hat{\dot{x}}_s \in C_{sc}(\hat{R}_{sc})$, then
the spacecraft delivery error is explicitly bounded as follows with probability at least \( 1 - \varepsilon_{\text{cut}} \):

\[
\mathbb{E} \left[ \|p(t_f) - \rho\| \right] \leq \sup_{(x_s, x_e) \in \mathcal{D}_{\text{cut}}} e^{-\frac{2\gamma}{L_k} (t_f - t_s)} \|p_s - p_d(t_s)\| + e^{-\frac{2\gamma}{m_{\text{dc}}(t_f)} \int_{t}^{t_f} \sigma^\tau(Z_s, t_s) dt} \sigma^\tau(Z_s, t_s)
\]

\[
+ \int_{t}^{t_f} e^{-\frac{2\gamma}{m_{\text{dc}}(t_f)} \int_{t}^{t_f} \sigma^\tau(Z_s, t_s) dt} \sigma^\tau(Z_s, t_s) dt \frac{(t_f - t_s)e^{-\alpha(t_f - t_s)}v(x_s, t_s)}{\sqrt{m_{\text{dc}}(t_f)}} \text{ if } \alpha \neq \frac{\Lambda}{2},
\]

\[
\text{if } \alpha = \frac{\Lambda}{2}.
\]

where \( \varepsilon_{\text{cut}} \in \mathbb{R}_{\geq 0} \) is the probability given in (50) of Appendix, \( \sigma^\tau(Z_s, t_s) \) is a time-varying function defined as

\[
\sigma^\tau(Z_s, t_s) = e^{\frac{2\gamma - \gamma}{L_k} \int_{t}^{t_f} v(x_s, t_s) dt} \sigma^\tau(Z_s, t_s).
\]

Note that (28) yields a bound on \( \mathbb{P} \left[ \|p(t_f) - \rho\| \leq d \right] = \mathbb{E} \left[ \sup_{(x_s, x_e) \in \mathcal{D}_{\text{cut}}} e^{-\frac{2\gamma}{L_k} (t_f - t_s)} \|p_s - p_d(t_s)\| + e^{-\frac{2\gamma}{m_{\text{dc}}(t_f)} \int_{t}^{t_f} \sigma^\tau(Z_s, t_s) dt}\right] \)

as in Theorem 2.5 of [16], where \( d \in \mathbb{R}_{\geq 0} \) is any given distance of interest, using Markov’s inequality [72, pp. 311-312].

**Proof.** A complete proof of this theorem can be found in Appendix and we thus give a sketch of the proof here. The dynamics (17) controlled by (18) can be rewritten as

\[
\dot{\hat{p}} = f(x, \alpha, t) + \hat{\mathcal{B}}(x, \alpha, t)u^*_{\ell}(x, \alpha, t; \theta_{\text{nn}}) + \hat{\mathcal{B}}(x, \hat{\alpha}, t)\hat{u}
\]

where \( \hat{u} = u^*_{\ell}(\hat{x}, \hat{\alpha}, t; \theta_{\text{nn}}) - u^*_{\ell}(x, \alpha, t; \theta_{\text{nn}}) \). Qualitatively speaking, since the learning-based control is constructed to make the closed-loop part \( \tilde{p} = f(x, \alpha, t) + \hat{\mathcal{B}}(x, \alpha, t)u^*_{\ell}(x, \alpha, t; \theta_{\text{nn}}) \) robust and exponentially stable with respect to the desired trajectory \( x_d \) of (8), we can view \( \hat{u} \), which arises from the ISO state uncertainty, as external disturbance to get an exponential bound on \( \mathbb{E}_{Z_s} [||p(t) - p_d(t)||] \) as in Theorem 2.5 of [16], where \( \mathbb{E}_{Z_s} [\cdot] = \mathbb{E} [\cdot | x(t_s) = x_s, \hat{x}(t_s) = \hat{x}_s, \alpha(t_s) = \alpha_s, \hat{\alpha}(t_s) = \hat{\alpha}_s] \). The rest follows from deriving the probability of the
Fig. 7  Illustration of expected position tracking error bound (28) of Theorem 2, where \( \zeta^t(Z_s,t_s) \) is assumed to be non-increasing function in time.

states \( \alpha(t), \hat{\alpha}(t), x(t), \) and \( \hat{x}(t) \) remaining in the sets where the estimation bound (25) and the Lipschitz condition (26) hold for \( \forall t \in [t_s, t_f] \) under Assumptions 1 and 2.

As derived in Theorem 2, the SN-DNN min-norm control policy (18) of Lemma 2 enhances the SN-DNN terminal guidance policy of Theorem 1 by providing the explicit spacecraft delivery error bound (28), which holds even under the presence of the state uncertainty. This bound is valuable in modulating the learning and control parameters to achieve a verifiable performance guarantee consistent with a mission-specific performance requirement as to be seen in Sec. V.C.

B. Examples and Optimality Guarantee

As illustrated in Fig. 7, the expected state tracking error bound (28) of Theorem 2 decreases exponentially in time if \( \zeta^t(Z_s,t_s) \) is a non-increasing function in \( t \). The following examples demonstrate how we compute the bound (28) in practice.

Example 1. Suppose that the estimation error is upper-bounded by a function that exponentially decreases in time, i.e., we have \( \zeta^t(Z_s,t_s) = e^{-\beta(t-t_s)} \sqrt{||\hat{\alpha}_s - \alpha_s||^2 + ||\hat{x}_s - x_s||^2} + c \) for (25) in Assumption 1 as in [39–42], where \( c \in \mathbb{R}_{\geq 0} \) and \( \beta \in \mathbb{R}_{>0} \). Assuming that \( \alpha \neq \beta, \beta \neq \lambda, \) and \( \lambda \neq \alpha \) for simplicity, we get

\[
RHS \text{ of (28)} \leq e^{-\bar{\beta}t} \left( \|\hat{\alpha}_s - p_d(t_s)\| + c_e \right) + \frac{e^{-\bar{\beta}t} - e^{-\alpha t}}{\alpha - \lambda} \frac{(\lambda + 1)(\|\hat{x}_s - x_d(t_s)\| + c_e)}{m_{sc}(t_f)} + \frac{L_k}{m_{sc}(t_f)} \left( \frac{c_e}{\alpha - \beta} \left( \frac{e^{-\bar{\beta}t} - e^{-\beta t}}{\beta - \lambda} - \frac{e^{-\bar{\alpha}t} - e^{-\alpha t}}{\alpha - \lambda} \right) + \frac{c}{\alpha} \left( \frac{1 - e^{-\bar{\beta}t}}{\lambda} - \frac{e^{-\bar{\alpha}t} - e^{-\alpha t}}{\alpha - \lambda} \right) \right)
\]

(30)

where \( \bar{t}_f = t_f - t_s \). Note that the bound (30) can be computed explicitly for given \( \hat{x}_s \) and \( \hat{\alpha}_s \) at time \( t = t_s \) as long as \( \hat{\alpha}_s \in C_{iso}(\tilde{R}_{iso}) \) and \( \hat{x}_s \in C_{sc}(\tilde{R}_{sc}) \). Its dominant term in (30) for large \( \bar{t}_f \) is \( L_k c / (m_{sc}(t_f) \alpha \lambda) \) due to the following relation:

\[
\lim_{\bar{t}_f \to \infty} RHS \text{ of (30)} = \frac{L_k c}{m_{sc}(t_f) \alpha \lambda}.
\]
Example 2. For the case where we can only guarantee that \( E_z \left[ \sqrt{\|\hat{x}(t) - x(t)\|^2 + \|\hat{\xi}(t) - x(t)\|^2} \right] \leq \hat{\sigma} \) for \( \hat{\sigma} \in \mathbb{R}_{\geq 0} \) in (25) of Assumption 1, we get

\[
\text{RHS of (28)} \leq e^{-2\hat{\sigma}_f} (\|\hat{p} - p_d(t)\| + \hat{\sigma}) + \frac{e^{-2\hat{\sigma}_f} - e^{-\alpha \hat{\sigma}_f}}{\alpha - \hat{\sigma}} (\hat{\lambda} + 1)(\|\hat{s}_s - x_d(t)\| + \hat{\sigma})
\]

\[
+ \frac{L_{kc}}{m_{sc}(t_f)} \alpha \left( 1 - e^{-2\hat{\sigma}_f} \frac{\alpha - \hat{\sigma}}{\alpha - \hat{\sigma}} \right)
\]

which can be obtained using (30).

In addition to the stability and robustness property shown in Theorem 2, the SN-DNN min-norm control policy (18) of Lemma 2 possesses a finite optimality gap with respect to the optimal MPC policy of (5) as in (12) of Theorem 1, thanks to the objective function of (21).

Theorem 3. Suppose that \( f \) of (16) is Lipschitz in the first two arguments with its 2-norm Lipschitz constant \( L_f \in \mathbb{R}_{> 0} \). The SN-DNN min-norm control policy (18) of Lemma 2 has the following bound, which reduces to the optimality gap \( \epsilon_{\ell u} \) of Theorem 1 as the state tracking error and estimation error tend to zero:

\[
\|u^*_\ell (\hat{x}(t), \hat{\theta}(t), t; \theta_{\text{m}}) - u_{\text{mpc}}(\hat{x}(t), \hat{\theta}(t), t)\| \leq \epsilon_{\ell u} + (L_{\ell} + m_{sc}(0)L_f)\|\hat{x}(t) - x_d(t)\| + (L_{\ell} + m_{sc}(0)\|L_f + \hat{\lambda}(\alpha + 1 + \alpha)\|\hat{\xi}(t) - x_d(t)\|
\]

where \( u_{\text{mpc}} \) is the optimal MPC policy given by (5), \( L_{\ell} \) is the Lipschitz constant of the SN-DNN guidance policy \( u_{\ell} \) of Theorem 1, and the other notations are given in Table 4.

Proof. Using triangle inequality, we get \( \|u^*_\ell (\hat{x}(t), \hat{\theta}(t), t; \theta_{\text{m}}) - u_{\text{mpc}}(\hat{x}(t), \hat{\theta}(t), t)\| \leq \|u_{\ell}(x_d(t), \alpha_d(t), t; \theta_{\text{m}}) - u_{\text{mpc}}(\hat{x}(t), \hat{\theta}(t), t)\| + \|k(\hat{x}(t), \hat{\theta}(t), t)\| \). For the first term, the Lipschitz property of \( u_{\ell} \) along with the result of Theorem 1 implies

\[
\|u_{\ell}(x_d(t), \alpha_d(t), t; \theta_{\text{m}}) - u_{\text{mpc}}(\hat{x}(t), \hat{\theta}(t), t)\| \leq \epsilon_{\ell u} + L_{\ell} (\|\hat{x}(t) - x_d(t)\| + \|\hat{\xi}(t) - x_d(t)\|)
\]

The second term can be bounded as follows by the definition of \( k \) given in (19) of Lemma 2

\[
\|k(\hat{x}(t), \hat{\theta}(t), t)\| \leq m_{sc}(0)L_f (\|\hat{x}(t) - x_d(t)\| + \|\hat{\xi}(t) - x_d(t)\|) + m_{sc}(0)\|\alpha \Lambda \hat{\theta}(t) + (\Lambda + \alpha I_{3 \times 3}) \hat{\xi}(t)\|
\]

where \( \hat{\theta}(t) = \hat{p}(t) - p_d(t) \) and the Lipschitz assumption for \( f \) is also used to obtain the inequality. Bounding the last term in (32) by \( m_{sc}(0)(\bar{\lambda}(\alpha + 1 + \alpha)\|\hat{\xi}(t) - x_d(t)\| \) gives the desired result (31).

Since the spacecraft delivery error (28) and the optimality gap (31) both depend on the state estimation error, which
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\[
\begin{align*}
\{\bar{x}_t\}_{i=1}^{N_d} & \quad \text{Computationally-Expensive Motion Planner (MPC)} \\
\{\bar{u}_t\}_{i=1}^{N_d} & \quad \text{Min-norm control policies online as in Algorithm 2, thereby enabling fast response autonomous operation under large ISO state uncertainty and high-velocity challenges. Note that SN-DNN is trained offline using Algorithm 1.}
\end{align*}
\]

is expected to become smaller as the spacecraft gets closer to the ISO as mentioned in the problem (P1) of Sec. II, they can be considered as a tool to determine whether we should use the SN-DNN terminal guidance policy of Theorem 1 or the SN-DNN min-norm control policy (18) of Lemma 2, based on the trade-off between them as to be seen in the following.

C. Neural-Rendezvous

We summarize the pros and cons of the aforementioned terminal G&C techniques.

- The SN-DNN terminal guidance policy of Theorem 1, which solves the problem (P1) of See. II, can be implemented in real-time and possesses a small optimality gap as in (12), resulting in the near-optimal guarantee in terms of dynamic regret [10–12] as discussed below (5) and Theorem 1. However, obtaining any quantitative bound on the spacecraft delivery error with respect to the desired relative position, to either flyby or impact the ISO, is difficult in general [16, 28].
- In contrast, the SN-DNN min-norm control policy (18) of Lemma 2, which solves the problem (P2) of Sec. II, can also be implemented in real-time and provides an explicit upper-bound on the spacecraft delivery error that decreases in time as proven in Theorem (2). However, the desired trajectory it tracks is subject to the large state uncertainty initially for small \(t_s\), resulting in a large optimality gap as can be seen from (31) of Theorem 3.

Based on these observations, it is ideal to utilize the SN-DNN terminal guidance policy without any feedback control initially for large state uncertainty, to avoid having a large optimality gap as in (31) of Theorem (3), then activate the SN-DNN min-norm control once the verifiable spacecraft delivery error of (28) of Theorem (2) becomes smaller than a mission-specific threshold value for the desired trajectory (15), which is to be updated at \(t_d \in [0, t_f] \) along the way as discussed in Remark 6. The pseudo-code for the proposed learning-based approach for encountering the ISO is given in Algorithm 2, and its mission timeline is shown in Fig. 8.

Fig. 8  Mission timeline for encountering ISO, where \(y(t)\) is state measurement as in Fig. 3 and the other notation follows that of Theorems 1 and 2. Terminal G&C (Neural-Rendezvous) is performed using SN-DNN guidance and min-norm control policies online as in Algorithm 2, thereby enabling fast response autonomous operation under large ISO state uncertainty and high-velocity challenges. Note that SN-DNN is trained offline using Algorithm 1.
Algorithm 2: Neural-Rendezvous

Inputs: \( u_\ell \) of Theorem 1 and \( u_\ell^* \) of Lemma 2

Outputs: Control input \( u \) of (1) for \( t \in [0, t_f] \)

\( t_0 \leftarrow \text{current time} \)
\( \Delta t \leftarrow \text{control time interval} \)
\( t_k, t_d, t_{\text{int}} \leftarrow \text{current time} - t_0 \)
\( \text{flagA, flagB} \leftarrow 0 \)

while \( t_k < t_f \)
  \( \text{do} \)
  Obtain \( \hat{\omega}(t_k) \) and \( \tilde{x}(t_k) \) using navigation technique
  if \( \text{flagA} = 0 \)
    \( u \leftarrow u_\ell(\tilde{x}(t_k), \hat{\omega}(t_k), t_k) \)
  else
    if \( \text{flagB} = 1 \)
      \( u \leftarrow u_\ell^*(\tilde{x}(t_k), \hat{\omega}(t_k), t_k) \)
    else
      Compute RHS of (28) in Theorem 2 with \( t_s = t_k \)
      if RHS of (28) > threshold then
        \( u \leftarrow u_\ell(\tilde{x}(t_k), \hat{\omega}(t_k), t_k) \)
      else
        \( u \leftarrow u_\ell^*(\tilde{x}(t_k), \hat{\omega}(t_k), t_k) \)
    flagB \leftarrow 1
  Apply \( u \) to (1) for \( t \in [t_k, t_k + \Delta t] \)
while current time - \( t_0 < t_k + \Delta t \)
  \( \text{do} \)
  if \( \text{flagB} = 0 \)
    \( \text{Integrate dynamics to get } x_d \text{ and } \alpha_d \text{ of (15) form } t_{\text{int}} \)
  if integration is complete then
    Update \( x_d \) and \( \alpha_d \)
    \( t_d, t_{\text{int}} \leftarrow t_k + \Delta t \)
    flagA \leftarrow 1
  else
    \( t_{\text{int}} \leftarrow \text{time when S/C stopped integration} \)
    \( t_k \leftarrow t_k + \Delta t \)
Remark 7. We use $t_{\text{int}}$ in Algorithm 2 to account for the fact that computing $x_d$ could take more than $\Delta t$ for small $\Delta t$ and $t_k$ as pointed out in Remark 6, and thus the spacecraft is sometimes required to compute it over multiple time steps.

It can be seen that the SN-DNN terminal guidance policy $u_\ell$ is also useful when the spacecraft does not possess $x_d$ yet, i.e., when flagA = 0. Also, the impact of discretization introduced in Algorithm 2 will be demonstrated in Sec. VII.C, where the detailed discussion of its connection to continuous-time stochastic systems can be found in [73].

VI. Extensions

In this section, we present several extensions of the proposed G&C techniques for encountering ISOs, which can be used to further improve certain aspects of their performance.

A. Optimal Lyapunov Functions and Other Types of Disturbances

The Neural-Rendezvous approach in Algorithm 2 is based on the feedback control of Lemma 2, constructed using the non-negative function $V$ of (23). In general, we can always construct such a feedback control policy as long as there exists a Lyapunov function $V: \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0}$ [70, p. 154] and control policy $u = \mu(x, \alpha, t)$ that satisfies

\begin{align}
  k_1 \|(x - x_d)\|^2 &\leq V(x, \alpha, t) \leq k_2 \|(x - x_d)\|^2 \\
  \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \alpha} \alpha + \frac{\partial V}{\partial x} (f(x, \alpha, t) + B(x, \alpha, t)\mu(x, \alpha, t)) &\leq -2\alpha V(x, \alpha, t)
\end{align}

(33) (34)

$\forall x, \alpha \in \mathbb{R}^n$ and $t \in \mathbb{R}_{\geq 0}$, where $k_1, k_2, \alpha \in \mathbb{R}_{>0}$, and $u, f, \text{ and } B$ are given in (1). The combination of such $V$ and $\mu$ is not necessarily unique, and many studies have discussed optimal and numerically efficient ways to find them, as partially summarized in [16]. For example, contraction theory [16, 23, 74, 75] uses a squared differential length $V = \delta x^T M(x, \alpha, t) \delta x$ as a Lyapunov-like function, allowing the systematic construction of $V$ and $\mu$ of (33) and (34) via convex optimization to minimize an upper bound of the steady-state distance between the controlled and desired system trajectories [73, 75]. The computational burden of these approaches can be significantly reduced by using machine learning techniques [16, 28, 41, 42].

It is worth emphasizing that since our proposed feedback control of Lemma 2 is also categorized as a Lyapunov-based approach (which can be verified for $V = V + \epsilon (p - p_d)^T \Lambda (p - p_d)$, where $V$ and $\Lambda$ are given in (21) and $\epsilon \in \mathbb{R}_{>0}$ is as defined in [76, pp. 54-55]), we can show that it is robust not only against the state uncertainty of (1), but also deterministic and stochastic disturbances resulting from e.g., process noise, control execution error, parametric uncertainty, and unknown parts of dynamics as shown in [16, 77, 78].
B. Stochastic MPC with Terminal Chance Constraints

We could utilize the expectation bound (28) in the guidance problem to compute a risk-constrained policy using terminal chance-constrained stochastic optimal control problem formulation in [79], with probabilistic guarantees on reaching the terminal set. This approach improves the quality of the solution that is approximated by an SN-DNN as in Theorem 1. Using the same notation as in (3) of Sec. III.A, the chance-constrained stochastic optimal control problem is described as follows:

\[
\begin{align*}
\hat{u} &= \arg \min_{u(t) \in U(t)} \mathbb{E} \left[ \int_{\tau}^{t_f} J_S(t, u(t)) dt + J_{S_f}(\xi(t_f)) \right] \\
\text{s.t.} & \quad d\xi(t) = f(\xi(t), \varphi^{t-t}(\hat{\omega}(t)), t) dt + B(\xi(t), \varphi^{t-t}(\hat{\omega}(t)), t) dt + G(\xi(t), \varphi^{t-t}(\hat{\omega}(t)), u(t)) d\mathcal{W}(t), \ \forall t \in [\tau, t_f] \\
& \quad \mathbb{E}(\xi(t)) = \hat{\xi}(t), \ \xi(t_f) \in X_{S_f}
\end{align*}
\]

where \( J_{S_f} \) and \( J_S \) are given transient and terminal cost functions, \( G : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n \times w} \) is a matrix-valued diffusion coefficient for stochastic disturbance, \( \mathcal{W} : \mathbb{R}_{\geq 0} \to \mathbb{R}^w \) is a \( w \)-dimensional Wiener process [80, p. 100] (see also [80, p. xii] for the notations used), and the terminal constraint set \( X_{S_f} \) is defined using a quadratic chance constraint as follows:

\[
X_{S_f} = \{ x \in \mathbb{R}^n \mid \mathbb{P} \left[ (x - \rho)^T Q_{S_f} (x - \rho) \leq c_f \right] \geq 1 - \epsilon_f \},
\]

where \( Q_{S_f} > 0, c_f \in \mathbb{R}_{\geq 0}, \) and \( \epsilon_f \) is the risk measure. The terminal set defines the encounter specifications (position, velocity, and their variance, respectively) with the ISO. Unlike the formulation given in (3), the problem (35) explicitly account for the stochastic disturbance resulting from the ISO state uncertainty in its dynamics, leading to a more sophisticated offline solution that can be obtained using the generalized polynomial chaos-based sequential convex programming method [79].

C. Additional Remarks

1. Multi-Agent Systems in Cluttered Environments

The optimization formulation with chance constraints in Sec. VI.B is also useful in extending our proposed approach to a multi-agent setting with obstacles, where each spacecraft is required to achieve its mission objectives in a collision-free manner. It allows expressing stochastic guidance problems as deterministic counterparts [79] so we could exploit existing methods for designing distributed, robust, and safe control policies for deterministic multi-agent systems, which can be computed in real-time [28, 29].
2. Discrete-Time Systems

Although this paper considers a continuous-time system and we discretize it when implementing the proposed algorithm as discussed in Remark 7, where the impact of discretization is to be demonstrated in Sec. VII.C, we could also start from a discrete-time system and analyze robustness and stability in a discrete sense, which can be performed by replacing the stability constraint (34) with its discrete versions introduced in [73, 74, 81–85].

3. Robustness of Neural Networks

As discussed in Remark 4, there are several ongoing studies in the field of machine learning that view a neural network as one form of a dynamical system, so their robustness performance could be analyzed using the techniques of Lyapunov and contraction theory described in Sec. VI.A. These approaches typically rewrite a neural network as a discrete-time dynamical system, and utilize the stability constraint of Sec. VI.C.2 as either a regularization loss or a structural constraint of neural networks [62–67]. This permits us to augment them with stronger robustness and stability guarantees that could further tighten the optimality gap of Theorem 1.

4. Online Learning

The proposed learning-based algorithm is based solely on offline learning and online guarantees of robustness and stability, but there could be situations where the parametric or non-parametric uncertainty of underlying dynamical systems is too large to be treated robustly. As shown in [16], robust control techniques, including our proposed approach in this paper, can always be augmented with adaptive control techniques with formal stability [7, 9, 86–88] and with static or dynamic regret bounds [89, 90] for online nonlinear control problems [91].

VII. Simulation

Our proposed framework, Neural-Rendezvous, is demonstrated using the data set that contains ISO candidates for possible exploration [6, 17] to validate if it indeed solves Problems (P1) and (P2) introduced earlier in Sec. II. PyTorch [92] is used for designing and training neural networks and NASA’s Navigation and Ancillary Information Facility (NAIF) [93, 94] is used to obtain relevant planetary data. A YouTube video which visualizes these simulation results can be found at https://youtu.be/8h60B_p1fyQ.

A. Simulation Setup

All the G&C frameworks in this section are implemented with the control time interval $1$ s unless specified, and their computational time of which is measured using the MacBook Pro laptop (2.2 GHz Intel Core i7, 16 GB 1600 MHz DDR3 RAM). The terminal time $t_f$ of (4) for terminal guidance is selected to be $t_f = 86400$ (s), and the wet mass of the spacecraft at the beginning of terminal guidance is assumed to be 150 kg. Also, we consider the SN-DNN min-norm
control (18) of Theorem 2 designed with \( \Lambda = 1.3 \times 10^{-3} \) and \( \alpha = 8.9 \times 10^{-7} \). The maximum control input is assumed to be \( u_{\text{max}} = 3 \) (N) in each direction with the total admissible delta-V (2-norm S/C velocity increase) being 0.6 km/s.

**B. Dynamical System-Based SN-DNN Training**

This section delineates how we train the dynamical system-based SN-DNN for solving the autonomous terminal guidance problem (P1) in Sec. II using Algorithm 1.

1. **State Uncertainty Assumption**

For the sake of simplicity, we assume that the spacecraft has access to the estimated ISO and its relative state generated by the respective normal distribution \( \mathcal{N}(\mu, \Sigma) \), with \( \mu \) being the true state and \( \Sigma \) being the navigation error covariance, where \( \text{Tr}(\Sigma) \) exponentially decaying in \( t \) as in Example 1 similar to [39–42]. In particular, we assume that the standard deviations of the ISO and spacecraft absolute along-track position, cross-track position, along-track velocity, and cross-track velocity, expressed in the ECLIPI2000 frame as in JPL’s SPICE toolkit [94], are \( 10^4 \) km, \( 10^2 \) km, \( 10^{-2} \) km/s, and \( 10^{-2} \) km/s initially at time \( t = 0 \) (s), and decays to \( O(10^1) \) km, \( O(10^0) \) km, \( O(10^{-4}) \) km/s, and \( O(10^{-4}) \) km/s finally at time \( t = t_f = 86400 \) (s) in the along-track and cross-track direction, respectively, which can be achieved by using, e.g., the extended Kalman filter [39] with full state measurements and estimation gains given by \( R = I_{n\times n} \) and \( Q = I_{n\times n} \times 10^{-10} \).

These uncertainties are derived from an autonomous optical navigation orbit determination filter, as is used in the AutoNav system [5]. A Monte-Carlo analysis simulating AutoNav performance was run to provide state estimation error versus time results. The distribution of errors at each time step then provides the input uncertainty profile.

**Remark 8.** As discussed in Remark 1, G&C are the focus of our study and navigation is beyond our scope, we have simply assumed the ISO state measurement uncertainty given above following the discussion of [6]. The assumption can be easily modified accordingly to the state estimation schemes to be used in each aerospace and robotic problem of interest.

2. **Training Data Generation**

We generate 499 candidate ISO and spacecraft ideal trajectories based on the ISO population given in [17] and analyzed in [6], and utilized the first 399 ISOs for training the SN-DNN and the other 100 for testing its performance later in this section. We then obtain 10000 time and ISO index pairs \((t_i, I_i)\) uniformly and randomly from \([0, t_f) \times [1, 399] \cup \mathbb{N}\) and perturbed the ISO and spacecraft ideal state with the uncertainty given in Sec. VII.B.1 to produce the training samples \((\bar{x}_i, \bar{a}_i, \bar{t}_i)\) of Algorithm 1 for the control input loss (i.e., the first term of (9)). The training data samples for the state trajectory loss (i.e., the second term of (9)) are obtained in the same way using the pairs generated uniformly and randomly from \([0, 3600] \times [1, 399] \cup \mathbb{N}\), with \(\Delta \bar{t}_i\) of Algorithm 1 fixed to \(\Delta \bar{t}_i = 10\) (s). The desired relative positions \(\bar{\rho}_i\)
in (10) are also sampled uniformly and randomly from the surface of a ball with radius 100 km.

The desired control inputs \( u_{\text{mpc}}(\tilde{x}_i, \tilde{\alpha}_i, \tilde{t}_i, \tilde{\rho}_i) \) and desired state trajectories \( \Phi_{\text{mpc}}^{\tilde{t}+\tilde{\rho}_i}(\tilde{x}_i, \tilde{\alpha}_i, \tilde{t}_i, \tilde{\rho}_i) \) of Algorithm 1 are then sampled by solving (5) using the sequential convex programming approach [45–47] and by numerically integrating (8) using the fourth-order Runge–Kutta method, respectively, where the terminal position error is treated as a constraint \( \| \rho \xi(t_f) - \rho \| = 0 \) in (4), the cost function of (3) is defined with \( c_0 = 0, c_1 = 1 \), and \( P(u(t), \xi(t)) = \| u(t) \|^2 \) as in Remark 2, and the control input constraint \( u(t) \in \mathcal{U}(t) = \{ u \in \mathbb{R}^m | |u| \leq u_{\text{max}} \} \) is used with \( u_i \) being the \( i \)-th element of \( u \) and \( u_{\text{max}} = 3 \) (N). Note that the problem is discretized with the time step 1 s, consistently with the control time interval.

3. Training Data Normalization

Instead of naively training the SN-DNN with the raw data generated in Sec. VII.B.2, we transform the SN-DNN input data \((\tilde{x}_i, \tilde{\alpha}_i, \tilde{t}_i, \tilde{\rho}_i)\) as follows, thereby accelerating the speed of learning process and improving neural network generalization performance:

\[
\text{SN-DNN input} = \left( \hat{p}_i - \bar{p}_i, \frac{\hat{\rho}_i - \bar{\rho}_i}{t_f - \tilde{t}_i}, \hat{p}_i, t_f - \tilde{t}_i, \tilde{\omega}_{z,i}, G(p, \tilde{\alpha}) \right) \tag{36}
\]

where \( \tilde{x}_i = [\hat{p}_i^T, \hat{\rho}_i^T]^T \), \( G(p, \tilde{\alpha}) \) is given in (2), and \( \tilde{\omega}_{z,i} \) is the \( i \)-th training sample of the orbital element \( \omega_z \) given in [15, 38], which is the dominant element of the matrix function \( C(\tilde{\alpha}) \) of (2). We further normalize the input (36) and output \( u(t, x, \alpha, \rho; \theta_{\text{nn}}) \) of the SN-DNN by dividing them by their maximum absolute values in their respective training data.

4. SN-DNN Configuration and Training

We select the number of hidden layers and neurons of the SN-DNN as 6 and 64 as a result of the performance analysis similar to [41, 42], with the spectral normalization constant \( C_{nn} \) of Definition 2 being \( C_{nn} = 25 \). The activation function is selected to be tanh, which is also used in the last layer not to violate the input constraint \( u(t) \in \mathcal{U}(t) = \{ u \in \mathbb{R}^m | |u| \leq u_{\text{max}} \} \) by design. Figure 9 shows the terminal spacecraft position error (delivery error) and control effort (total delta-V) of the SN-DNN terminal guidance policy trained for 10000 epochs using several different weights \( c_u, c_x \in \mathbb{R}_{\geq 0} \) of the loss function (9) in Sec. III, where its weight matrices are given as \( C_x = c_x \text{diag}(I_{3\times3}, 10^7 \times I_{3\times3}) \) and \( C_u = c_u I_{3\times3} \). The results are averaged over 50 simulations for the ISOs in the test set without any state uncertainty. Consistently with the definition of (9), this figure indicates that

- as \( c_x/c_u \) gets smaller, the loss function (9) penalizes the imitation loss of the control input more heavily than that of the state trajectory, and thus the spacecraft yields smaller control effort but with larger delivery error, and
- as \( c_x/c_u \) gets larger, the loss function (9) penalizes the imitation loss of the state trajectory more heavily than that
of the control input, and thus the spacecraft yields smaller delivery error but with larger control effort.

The weight ratio $c_x/c_u$ of the SN-DNN to be implemented in the next section is selected as $c_x/c_u = 10^2$, which achieves the smallest delivery error with the smallest standard deviation of all the weight ratios in Fig. 9, while having the control effort smaller than the admissible delta-V of 0.6 km/s. Note that we could further optimize the ratio with the delta-V constraint based on this trade-off discussed above, but this is left as future work. The SN-DNN is then trained using SGD [50, 51] for 10000 epochs with 10000 training data points obtained as in Sec. VII.B.2, following the pseudo-code outlined in Algorithm 1.

C. Neural-Rendezvous Performance

Figure 10 shows the spacecraft delivery error and control effort of Neural-Rendezvous of Algorithm 2, SN-DNN terminal guidance of Algorithm 1, PD and robust nonlinear tracking control of [18, pp. 397-402] with respect to a pre-computed and fixed desired trajectory, and MPC with linearized dynamics [19], where the SN-DNN min-norm control (18) of Theorem 2 is activated at time $t = t_s = 26400$ (s). It can be seen that Neural-Rendezvous achieves $\lesssim 0.2$ km delivery error for 99% of the ISOs in the test set, even under the presence of the large ISO state uncertainty given in Sec. VII.B.1. Also, its error is indeed less than the dominant term of the expectation bound on the tracking error (28) of Theorem 2, which is computed assuming the estimation error is upper-bounded by a function that exponentially decreases in time as in Example 1. Furthermore, the SN-DNN terminal guidance can also achieve $\leq 1$ km delivery error for 86% of the ISOs, and as expected from the optimality gaps given in (12) of Theorem 1 and (31) of Theorem 3, the control effort of Neural-Rendezvous is larger than that of the SN-DNN guidance and MPC with linearized dynamics, but it is still less than the admissible delta-V of 0.6 km/s for all the ISOs in the test set.

Figure 11 then shows the spacecraft delivery error and control effort of Neural-Rendezvous of Algorithm 2 and SN-DNN terminal guidance of Algorithm 1, averaged over 100 ISOs in the test set, versus the control time interval. Although both of these methods involve discretization when implementing them in practice as pointed out in Remark 7, it can be seen that Neural-Rendezvous enables having the delivery error smaller than 5 km with its standard deviation always smaller than that of the SN-DNN guidance, even for the control interval 600 s (10 min). Since the MPC problem

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Fig. 9 Control performances versus weight ratio of SN-DNN loss function. Shaded area denotes standard deviations ($\pm 2.5 \times 10^{-1} \sigma$ for spacecraft delivery error and $\pm 5 \times 10^{-2} \sigma$ for control effort).
is solved by discretizing it with the time step $1\text{s}$ as explained in Sec. VII.B.2, and the SN-DNN min-norm control (18) of Theorem 2 is designed for continuous dynamics, Neural-Rendezvous yields less optimal control inputs for larger control time intervals, resulting in larger delivery error and control effort as expected from [73].

Finally, as shown in Table 5, we can see that all the methods presented in this section, including our proposed approaches, can be computed in $\leq 1$ (s) and thus can be implemented in real-time. The observations so far imply that the proposed approach indeed provides one of the promising solutions to Problems (P1) and (P2) of Sec. II.

**Table 5  Computational time of each method for ISO encounter averaged over 100 evaluations.**
VIII. Conclusion

This paper proposes Neural-Rendezvous – a deep learning-based terminal G&C framework for achieving ISO encounter under large state uncertainty and high-velocity challenges, where we develop a minimum-norm tracking controller with an optimal MPC-based guidance policy imitated by a spectrally-normalized deep neural network. As derived in Theorems 1–3 and illustrated in Fig. 4–8, its major advantages include 1) stability and robustness guarantees, resulting in a spacecraft delivery error bound that decreases exponentially in expectation with a finite probability; and 2) real-time performance guarantees with a verifiable optimality gap. The simulation results validate these guarantees for some possible ISO candidates visiting our solar system, which also implies that our proposed method should work for long-period comets, drones, and other fast-moving objects in robotics and aerospace applications.

Appendix

Proof of Lemma 2. Let \( u_n(\hat{x}, \hat{v}, t) \) be defined as follows:

\[
u_n(\hat{x}, \hat{v}, t) = M(t)\hat{g}_d(\hat{\rho}, t) + C(\hat{\rho})\hat{g}_d(\hat{\rho}, t) + G(\hat{\rho}, \hat{\nu}) - \alpha M(t)(\hat{\rho} - \hat{g}_d(\hat{\rho}, t))\]

where \( \hat{g}_d \) is as given in (20). Substituting this into the left-hand side of (22) as \( u = u_n(\hat{x}, \hat{v}, t) \) gives

\[2(\hat{\rho} - \hat{g}_d(\hat{\rho}, t))^T(-C(\hat{\rho}) - \alpha M(t))(\hat{\rho} - \hat{g}_d(\hat{\rho}, t)) = -2\alpha V(\hat{\rho}, \hat{g}_d(\hat{\rho}, t), t)\]

where \( f \) of (16) is computed with (2), and the equality follows from the skew-symmetric property of \( C \), i.e.,

\[C(\hat{\rho}) + C(\hat{\nu})^T = O_{3 \times 3} \] [15, 38]. The relation indeed indicates that \( u = u_n(\hat{x}, \hat{v}, t) \) is a feasible solution to the optimization problem (21). Furthermore, applying the KKT condition [48, pp. 243-244] to (21), we have that

\[
\begin{align*}
\mu_{\text{KKT}}((\hat{\rho} - \hat{g}_d(\hat{\rho}, t))^T(u^*(\hat{x}, \hat{v}, t; \theta_{\text{mn}}) - u_f(x_d(t), \alpha_d(t), t; \theta_{\text{mn}})) + Y(\hat{x}, \hat{v}, t)) &= 0 \\
u^*(\hat{x}, \hat{v}, t; \theta_{\text{mn}}) - u_f(x_d(t), \alpha_d(t), t; \theta_{\text{mn}}) + \mu_{\text{KKT}}(\hat{\rho} - \hat{g}_d(\hat{\rho}, t)) &= 0_{m \times 1}
\end{align*}
\]

for \( \mu_{\text{KKT}} \in \mathbb{R}_{\geq 0} \). Solving them for \( u^*(\hat{x}, \hat{v}, t; \theta_{\text{mn}}) \) and \( \mu_{\text{KKT}} \) yields (18). Also, we get the condition \( Y(\hat{x}, \hat{v}, t) \leq 0 \) by substituting \( V \) of (23) and \( \hat{\rho} \) into the stability condition (22) with \( u = u_f(x_d(t), \alpha_d(t), t; \theta_{\text{mn}}) \)

Proof of Theorem 2. Since the third term of (29) is subject to stochastic disturbance due to the state uncertainty of Assumption 1, we consider the weak infinitesimal operator \( \mathcal{A} \) given in [71, p. 9], instead of taking the time derivative, for analyzing the time evolution of the non-negative function \( \nu \) defined as \( \nu(x, t) = \sqrt{V(\hat{\rho}, \hat{g}_d(p, t), \hat{\rho} - \hat{g}_d(\hat{\rho}, t))} = \sqrt{m_{\text{sc}}(t)}\|A(p - p_d(t)) + \hat{\rho} - \hat{\rho}_d(t)\| \) for \( V \) of (23). To this end, let us compute the following time increment of \( \nu \).
evaluated at the true state and time \((x, \alpha, t)\):

\[
\Delta v = v(x(t + \Delta t), t + \Delta t) - v(x, t) = \frac{1}{2v(x, t)} \left( \frac{\partial V}{\partial \hat{p}} \dot{\rho}(t) + \frac{\partial V}{\partial \hat{g}_d} \hat{g}_d(p(t), t) + \frac{\partial V}{\partial t} \right) \Delta t + O(\Delta t^2) \tag{37}
\]

where \(\Delta t \in \mathbb{R}_{\geq 0}\) and the arguments \((\hat{p}(t), \hat{g}_d(p(t), t))\) of the partial derivatives of \(V\) are omitted for notational convenience. Dropping the argument \(t\) for the state variables for simplicity, the dynamics decomposition (29) gives

\[
\frac{\partial V}{\partial \hat{p}} \dot{\rho} + \frac{\partial V}{\partial \hat{g}_d} \hat{g}_d(p, t) + \frac{\partial V}{\partial t} (f(x, \alpha, t) + \mathcal{B}(x, \alpha, t)u^*(\hat{\xi}, \hat{\alpha}, t; \theta_{\text{est}})) + \frac{\partial V}{\partial \hat{g}_d}(\hat{p}_d - \Lambda(\hat{p} - \hat{p}_d)) \\
\leq -2\alpha V(\hat{p}, \hat{g}_d(p, t), t) + 2\frac{\partial V}{\partial \hat{p}} \mathcal{B}(x, \alpha, t)\dot{\alpha} = -2\alpha v(x, t)^2 + 2v(x, t) \frac{\|\dot{\alpha}\|}{\sqrt{m_{sc}(t)}} \tag{38}
\]

where the first inequality follows from the fact that the spacecraft mass \(m_{sc}(t)\), described by the Tsiolkovsky rocket equation, is a decreasing function and thus \(\partial V/\partial t \leq 0\), and the second inequality follows from the stability condition (22) evaluated at \((x, \alpha, t)\), which is guaranteed to be feasible due to Lemma 2. Since \(u^*\) is assumed to be Lipschitz as in (26) of Assumption 2, we get the following relation for any \(x_s, \hat{x}_s \in C_{sc}(r_{sc}), \alpha_s, \hat{\alpha}_s \in C_{iso}(r_{iso})\), and \(t_s \in [0, t_f]\), by substituting (38) into (37):

\[
\mathcal{A}v(x_s, t_s) = \lim_{\Delta t \to 0} \mathbb{E}_{Z_s} \left[ \frac{\Delta v}{\Delta t} \right] \leq -\alpha v(x_s, t_s) + \frac{L_k}{\sqrt{m_{sc}(t_f)}} \sqrt{\|\dot{x}_s - \alpha_s\|^2 + \|\hat{\xi}_s - x_s\|^2} \tag{39}
\]

where \(Z_s = (x_s, \hat{x}_s, \alpha_s, \hat{\alpha}_s), \mathbb{E}_{Z_s} [\cdot] = \mathbb{E} [\cdot | x(t_s) = x_s, \hat{x}(t_s) = \hat{x}_s, \alpha(t_s) = \alpha_s, \hat{\alpha}(t_s) = \hat{\alpha}_s]\), and the relation \(m_{sc}(t) \geq m_{sc}(t_f)\) by the Tsiolkovsky rocket equation is also used to obtain the inequality. Applying Dynkin’s formula [71, p. 10] to (39) gives the following with probability at least \(1 - \varepsilon_{\text{est}}\) due to (25) of Assumption 1:

\[
\mathbb{E}_{Z_s} [\|\Lambda(p(t) - p_d(t)) + \hat{p}(t) - \hat{p}_d(t)\|] \leq \frac{e^{-\alpha(t-t_s)}v(x_s, t_s)}{\sqrt{m_{sc}(t_f)}} + \frac{e^{-\alpha t}L_k}{m_{sc}(t_f)} \int_{t_s}^{t_f} e^{\alpha \tau} s^*(Z_s, t_s) \text{d} \tau, \forall t \in [t_s, \bar{t}_e] \tag{40}
\]

where \(s^*(Z_s, t_s)\) is as given in (25) and \(\bar{t}_e\) is defined as \(\bar{t}_e = \min\{t_e, t_f\}\), with \(t_e\) being the first exit time that any of the states leaves their respective set, i.e.,

\[
t_e = \inf \{t \geq t_s : \alpha(t) \notin C_{iso}(r_{iso}) \cup \hat{\alpha}(t) \notin C_{iso}(r_{iso}) \cup x(t) \notin C_{sc}(r_{sc}) \cup \hat{x}(t) \notin C_{sc}(r_{sc}) \} \tag{41}
\]

given that the event of (24) in Assumption 1 has occurred. Since we further have that \(\mathcal{A}\|p - p_d\| = \frac{d}{dt}\|p - p_d\| \leq \|\dot{\alpha}\| - \Lambda(p - p_d)\) for \(\dot{\alpha} = \Lambda(p - p_d) + \hat{p} - \hat{p}_d\) due to the hierarchical structure of \(\dot{\alpha}\), utilizing Dynkin’s formula one
more time and then substituting (40) result in
\[
\mathbb{E}_x [\| p(t) - p_d(t) \|] \leq e^{-\Delta(t-t_*)} \| p_x - p_d(0) \| + e^{-\Delta t} \int_{t_*}^t \frac{e^{(\alpha-a)\tau + at_x} v(x, t_x)}{\sqrt{m_{sc}(t_f)}} + \frac{L_k}{m_{sc}(t_f)} e^{\tau} (Z_s, t_s) d\tau, \forall t \in [0, t_e]
\]  
(42)

where \( x_s = [p_s^T, p_s^T]^T \).

What is left to show is that we can achieve \( t_e > t_f \) with a finite probability for the first exit time \( t_e \) of (41). For this purpose, we need the following lemma.

**Lemma 3.** Suppose that the events of Assumption 1 has occurred, i.e., the event of (24) has occurred and the 2-norm estimation error satisfies \( \sqrt{\| \hat{\epsilon}(t) - \alpha(t) \|^2 + \| \hat{\chi}(t) - \chi(t) \|^2} < c_e \) for \( \forall t \in [t_s, t_f] \). If Assumption 2 holds and if we have
\[
\hat{\mathscr{A}} v(x_s, t_s) \leq -\alpha v(x_s, t_s) + \frac{L_k}{\sqrt{m_{sc}(t_f)}} \sqrt{\| \hat{\alpha}_s - \alpha_s \|^2 + \| \hat{\chi}_s - x_s \|^2}
\]  
(43)

for any \( x_s, \hat{x}_s \in C_{sc}(r_{sc}), \alpha_s, \hat{\alpha}_s \in C_{iso}(r_{iso}), \) and \( t_s \in [0, t_f] \) as in (39), then we get the following probabilistic bound for \( \exists x_{\text{exit}} \in \mathbb{R}_{\geq 0} \):
\[
\mathbb{P}_x \left[ \bigcap_{t \in [t_s, t_f]} x(t) \in C_{sc}(\tilde{r}_{sc}) \right] \geq 1 - e_{\text{exit}} = \begin{cases} 
1 - \frac{E_s}{\alpha} e^{H(t_f)} & \text{if } \tilde{v} \geq \frac{\sup_{t \in [t_s, t_f]} h(t)}{\alpha} \\
1 - \frac{\hat{\alpha} e^{H(t_f)}}{\hat{\alpha} v_0 e^{H(t_f)}} & \text{otherwise}
\end{cases}
\]  
(44)

where \( Z_s = (\alpha_s, \hat{\alpha}_s, x_s, \hat{x}_s) \) denotes the states at time \( t = t_s \), which satisfy \( \alpha_s \in C_{iso}(r_{iso}), \hat{\alpha}_s \in C_{iso}(\tilde{r}_{iso}), x_s \in C_{sc}(\tilde{r}_{sc}), \) \( \hat{x}_s \in C_{sc}(\tilde{r}_{sc}) \) for \( r_{iso} = r_{iso} - c_e \geq 0, \tilde{r}_{iso} = r_{iso} - 2c_e \geq 0, r_{sc} = r_{sc} - c_e \geq 0, \) and \( \tilde{r}_{iso} = r_{iso} - 2c_e \geq 0, E_s = v(x_s, t_s) + \delta_p \| p_x - p_d(t_s) \|, H(t) = \int_{t_s}^t h(\tau) d\tau, \tilde{H}(t) = 2H(t)\tilde{\alpha}/\sup_{t \in [t_s, t_f]} h(t), \) and \( h(t) = L_k s^\theta (Z_s, t_s) / \sqrt{m_{sc}(t_f)} \).

The suitable choices of \( \tilde{v}, \tilde{\alpha}, \delta_p \in \mathbb{R}_{\geq 0} \) are to be defined below.

**Proof of Lemma 3.** Let us define a non-negative and continuous function \( E(x, t) \) as
\[
E(x, t) = v(x, t) + \delta_p \| p - p_d(t) \|
\]  
(45)

where \( x = [p^T, \hat{p}^T]^T \) and \( \delta_p \in \mathbb{R}_{\geq 0} \). Note that \( E(x, t) \) of (45) is 0 only when \( x = x_d(t) \). Since we have \( \hat{\mathscr{A}} \| p - p_d(t) \| \leq v(x, t) / \sqrt{m_{sc}(t_f)} - \frac{\alpha}{\sqrt{m_{sc}(t_f)}} (p - p_d(t)) \), designing \( \delta_p \) to have \( \alpha - \delta_p / \sqrt{m_{sc}(t_f)} > 0 \), the relation (43) gives
\[
\hat{\mathscr{A}} E(x_s, t_s) \leq -\tilde{\alpha} E(x_s, t_s) + \frac{L_k}{\sqrt{m_{sc}(t_f)}} \sqrt{\| \hat{\alpha}_s - \alpha_s \|^2 + \| \hat{\chi}_s - x_s \|^2}
\]  
(46)

for any \( x_s, \hat{x}_s \in C_{sc}(r_{sc}), \alpha_s, \hat{\alpha}_s \in C_{iso}(r_{iso}), \) and \( t_s \in [0, t_f] \), where \( \tilde{\alpha} = \min \{ \alpha - \delta_p / \sqrt{m_{sc}(t_f)}, 0 \} > 0 \). Let us define
another non-negative and continuous function \( W(x,t) \) as
\[
W(x,t) = E(x,t)e^{\gamma H(t)} + \frac{e^{\gamma H(t_f)} - e^{\gamma H(t)}}{\gamma}
\]
where \( \gamma \) is a positive constant that satisfies \( 2\tilde{a} \geq \gamma \sup_{t \in [0,t_f]} h(t) \). If \( \tilde{v}, \tilde{w} \in \mathbb{R}_{>0} \) is selected to have \( W(x,t) < \tilde{w} \Rightarrow E(x,t) < \tilde{v} \Rightarrow x \in C_{sc}(\tilde{r}_{sc}) = \bigcup_{t \in [0,t_f]} \{ s \in \mathbb{R}^{\mathbb{R}} \mid \| x - x_d(t) \| < \tilde{r}_{sc} \} \subset \mathbb{R}^{\mathbb{R}} \), which is always possible since \( E(x,t) \) of (45) is 0 only when \( x = x_d(t) \), then we can utilize (46) to compute \( \partial W \) as follows for any \( x_s = [p_s^T, \tilde{p}_s^T]^T \in C_{sc}(\tilde{r}_{sc}) \) and \( t_s \in [0,t_f] \) that satisfies \( W(x_s,t_s) < \tilde{w} \):
\[
\partial W(x_s,t_s) = (\gamma h(t_s)E(x_s,t_s) + \partial E(x_s,t_s))e^{\gamma H(t_s)} - h(t_s)e^{\gamma H(t_s)} \leq (\partial \gamma(\hat{\gamma}, \hat{\alpha}, \hat{\beta}, \hat{\epsilon})) e^{\gamma H(t_s)} \quad (47)
\]
where \( \partial \gamma(\hat{\gamma}, \hat{\alpha}, \hat{\beta}, \hat{\epsilon}) = L_k \sqrt{\sup_{t \in [t_s,t_f]} \frac{\| \hat{\gamma} \| + \| \hat{\alpha} \|}{\| \hat{\beta} \|}} \) and the inequality follows from the relation \( \tilde{a} \geq \gamma \sup_{t \in [0,t_f]} h(t) \). Applying Dynkin’s formula [71, p. 10] to (47), it can be verified that \( W(x(t),t) \) is a non-negative supermartingale due to the fact that \( \mathbb{E}_{\mathcal{Z}_t} \sup_{t \in [t_s,t_f]} \gamma(\hat{\gamma}(t), \hat{\alpha}(t), \hat{\beta}(t), \hat{\epsilon}(t)) \leq L_k \gamma(\hat{\gamma}(t), \hat{\alpha}(t), \hat{\beta}(t), \hat{\epsilon}(t)) \) for \( \mathcal{Z}_t \) in (25) of Assumption 1, which gives the following due to Ville’s maximal inequality [71, p. 26]:
Assumption 1 along with the result of Lemma 3, we have \( \exists \varepsilon_{\text{ctrl}} \in \mathbb{R}_{\geq 0} \) s.t.

\[
P(\mathcal{A} | \hat{x}(t_s) = \hat{x}_x \cap \check{x}(t_s) = \check{x}_x) \geq 1 - \varepsilon_{\text{ctrl}} = (1 - \varepsilon_{\text{ext}})(1 - \varepsilon_{\text{est}})(1 - \varepsilon_{\text{err}})
\]

where \( \mathcal{A} = \bigcap_{t \in [t_s, t_f]} \alpha(t) \in C_{\text{iso}}(r_{\text{iso}}) \cap \hat{x}(t) \in C_{\text{iso}}(r_{\text{iso}}) \cap x(t) \in C_{\text{sc}}(r_{\text{sc}}) \cap \check{x}(t) \in C_{\text{sc}}(r_{\text{sc}}) \cap \mathcal{E} \) with \( \mathcal{E} \) being the event of (24) in Assumption 1. The desired relation (28) follows by evaluating the first term of the integral in (42), and by observing that if \( \mathcal{A} \) of (50) occurs, we have \( t_e > t_f \) and \( \sqrt{||\hat{x}_x - \alpha_{\hat{x}}||^2 + ||\check{x}_x - \hat{x}_x||^2} < k_e \mathbb{E} [s^\alpha (Z_0, 0)] \) due to (49).

Note that if \( E_{\text{iso}} = E_{\text{sc}} = \mathbb{R}^n \) in Assumption 1 and \( k \) is globally Lipschitz in Assumption 2, the estimation bound (25) and the Lipschitz bound (26) always hold without the second condition of Assumption 1. This indicates that the bound (28) holds as long as \( (a_x, x_x) \in \mathcal{D}_{\text{est}} \) for given \( \hat{x}_x, \check{x}_x \in \mathbb{R}^n \), which occurs with probability at least \( 1 - k_e^{-1} \) due to (49).

\[\Box\]

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