EXTENDING THE MODEL OF KH 15D: ESTIMATING THE EFFECTS OF FORWARD SCATTERING AND CURVATURE OF THE OCCULTING RING EDGE

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ABSTRACT

The periodic eclipses of the pre–main-sequence binary KH 15D have been explained by a circumbinary dust ring inclined to the orbital plane, which causes occultations of the stars as they pass behind the ring edge. We compute the extinction and forward scattering of light by the edge of the dust ring in order to explain (1) the gradual slope directly preceding total eclipse, (2) the gradual decline at the end of ingress, and (3) the slight rise in flux at mideclipse. The size of the forward-scattering halo indicates that the dust grains have a radius of \(a \sim 6(D/3 \text{ AU}) \mu\text{m}\), where \(D\) is the distance of the edge of the ring from the system barycenter. This dust size estimate agrees well with estimates of the dust grain size from polarimetry, adding to the evidence that the ring lies at several AU. Finally, the ratio of the fluxes during an eclipse to those not during an eclipse independently indicates that the ring lies at a few AU.

Subject headings: circumstellar matter — stars: individual (KH 15D) — stars: pre–main-sequence

Online material: color figure, mpeg animation

1. INTRODUCTION

The object KH 15D (V582 Mon) is a binary weak-lined T Tauri star in the cluster NGC 2264. As a class, T Tauri stars are thought to be young stars that are still accreting gas from the remains of their parent molecular clouds. This period of stellar evolution lasts for a few million years and is characterized by optical variability and chromospheric emission lines (see Bertout et al. 2004; Chiang & Murray-Clay 2004), and currently only those variables in young clusters (Joy 1945; Herbst et al. 1994), but the remarkable property of KH 15D is that its variations are also periodic. This depth of eclipse cannot be explained solely with a binary because of the large magnitude and duration of the flux variations, so the earliest theories regarding KH 15D postulated that circumstellar material eclipsed the star.

Since Kearsn & Herbst (1998) first reported the unusual properties of KH 15D, theoretical explanations for its properties have included an edge-on circumstellar disk (Hamilton et al. 2001; Agol et al. 2004; Winn et al. 2003), an asymmetric surrounding envelope (Grinn & Tambaštseva 2002), and an orbiting vortex of solid particles (Barge & Viton 2003). However, it was soon realized that KH 15D was likely an eccentric binary system that is being occulted by the edge of a circumbinary dust ring (Winn et al. 2004; Chiang & Murray-Clay 2004), and currently only one star is visible during part of its orbit, which causes the large change in magnitude. In addition, the advance of the screen as a function of time due to the nodal precession of the ring, which is inclined relative to the binary, explains the lengthening of the duration of the eclipses as a larger portion of the orbit of the visible star is covered. This model provides an excellent fit to the characteristics of the KH 15D light curve and radial velocities, as shown by Winn et al. (2004) and Chiang & Murray-Clay (2004). Chiang & Murray-Clay (2004) also suggested that the ring must be somewhat warped in order to maintain the nodal precession. This picture proved to be correct when KH 15D was confirmed to be an eccentric spectroscopic binary by a two-year, high-resolution, multisite spectroscopic study (Johnson et al. 2004). Archival data showed evidence for the second star, which is completely hidden in recent data (Johnson et al. 2005).

Winn et al. (2006) made a further refinement of the KH 15D model by adding a faint blue halo around each star within the binary, which provides an explanation for the gradual slope of the light curve directly before and just after total eclipse, as well as the slight increase in flux during mideclipse. In their model, they solved for the one-dimensional halo brightness, using four free parameters and an arbitrary functional form that resulted in a strongly asymmetric shape around both stars due to the asymmetric shape of ingress and egress, resulting in a puzzling physical picture: why should identical asymmetric halos surround both stars? Winn et al. (2006) also included a gradual change in the angle of the edge of the ring projected onto the sky, so as to account for longer term variations of the light curve.

In this paper, we focus on fitting the photometric data for KH 15D, which span the years 1995–2004, and the 18 published radial velocity data points. We compute the effects of (1) a gradual change in opacity at the ring edge (prior models treat the ring edge as being sharp) and (2) forward scattering by dust at the ring edge, which together naturally explain the asymmetric shape of the ingress and egress (Winn et al. [2006] acknowledge that forward scattering may be a more sensible explanation for their halo). We do not include the rotation of the edge of the ring (Winn et al. 2006), but instead propose that the edge of the ring is curved as a means of explaining additional variation in the eclipse durations that is not explained by the precession of the ring. We show that these modifications provide a much better fit to the photometric data of KH 15D, and we speculate on what additional insight this gives us about young stellar evolution and protoplanetary processes. In §2 we summarize the data that we use to fit our model. In §3 we discuss the elements included in our model. In §4 we discuss the best-fit model parameters and errors,
and in § 5 we discuss the implications of these results and possible future directions.

2. DATA

2.1. Photometry

We used the 6694 I-band photometric data points from Hamilton et al. (2005), which were taken between 1995 and 2004 by a dozen different telescopes using CCDs. We ignored older, sparser data, which are neither precise enough nor dense enough to show the effects of forward scattering. We converted the I-band magnitudes to fluxes using the relation $f_\lambda / f_0 = 10^{-0.4 \Delta m}$, where $f_0 \sim 2600 \, \text{Jy}$ is the flux zero point. We enlarged the error bars for all data points in which the star was completely eclipsed or completely un eclipsed, due to observed fluctuations that are larger than the reported errors. Although these fluctuations may be partly due to starspots (Hamilton et al. 2005), for the purposes of model fitting, we treat these fluctuations as an additional source of systematic error and replaced the error bars that accompanied the data with the standard deviation of the scatter of the data either in or out of eclipse, respectively.

In addition to the orbital elements of the binary and the parameters describing the ring (described below), we introduced three primary flux parameters of our model: $\{f_{\text{in}}, f_{\text{1, out}}, f_{\text{2, out}}\}$, where $f_{\text{in}}$ is the nonvariable flux of the system, $f_{\text{1, out}}$ is the flux received when only star 1 is visible, and $f_{\text{2, out}}$ is the flux received when only star 2 is visible.

2.2. Radial Velocities

We fit our model to the published radial velocity data from Hamilton et al. (2003) and Johnson et al. (2004). We did not discard data points taken near eclipse, as these data points did not appear to show the Rossiter-McLaughlin effect (Rossiter 1924; McLaughlin 1924), so we use all 18 values for the data-fitting process. We inflate the error bars for these radial velocity values in order to account for a systematic radial velocity offset due to light from the binary that is scattered off the back and sides of the circumbinary ring, an effect discussed by Herbst et al. (2008). If we assume that the parameter $f_{\text{in}}$ is set by the large-angle scattering by the dust ring, which will have a range of Doppler shifts between $-K$ and $K$, where $K$ is the velocity semiamplitude, then the systematic offset in the measured radial velocity values should be of order $\sqrt{2}(f_{\text{in}}/f_{\text{1, out}})K$. This offset was added in quadrature to the uncertainties published by Winn et al. (2006). Our decision to modify the error bars was intended to reduce the discrepancy between the orbital parameters found from the radial velocity data and those found from the light-curve data, as pointed out by Winn et al. (2006). We implement weighting of the contribution to $\chi^2$ from photometric and radial velocity in the same manner as Winn et al. (2006), which is described in more detail in § 4.

3. THE KH 15D MODEL

3.1. Orbit Model

The binary orbit was computed by solving Kepler’s equation and converting the orbital elements to Cartesian coordinates and velocities, following the procedures described in Murray & Dermott (1999). The parameters describing the binary orbit are $\{P, e, i, \omega, T_p, \gamma\}$, where $P$ is the period, $e$ is the eccentricity, $i$ is the inclination of the system, $\omega$ is the argument of the pericenter, $T_p$ is the time of the pericenter passage, and $\gamma$ is the heliocentric radial velocity of the center of mass. The geometry of the orbit is shown in Figures 1 and 2, where the origin is the barycenter of the binary, the Z-direction is along the line of sight, and we fix the longitude of the ascending node at 180° so that the orbits cross the sky plane on the X-axis, with the visible star (which we call star 1) crossing the sky plane on the positive X-axis as it moves toward positive values of $Z$. We initially fit just the radial velocity data, confirming that our derived model parameters agreed with those of Winn et al. (2004).

3.2. Ring Orientation, Occultation, and Curvature

We define the angle between the X-axis and the ring edge projected onto the X-Y plane as $\beta$. We allow the edge of the ring to move in the Y-direction with velocity $v_{\text{ring}}$ and define the time at which the edge passes the center of mass of the system as $t_{\text{ring}}$. Figure 1 shows the orbits as they would be seen from the observer’s perspective and a snapshot of the position of the edge.
of the ring. To help the reader visualize KH 15D, the approximate three-dimensional geometry of the system is shown in Figure 2.

We assume that the stars are limb-darkened following a quadratic limb-darkening law given by

\[ I(r) = 1 - \gamma_1(1 - \epsilon) - \gamma_2(1 - \epsilon)^2, \]

where \( \epsilon \) is the cosine of the angle between the normal to the stellar surface and the line of sight to the observer. We set \( \gamma_1 = 0.4478 \) and \( \gamma_2 = 0.2091 \), as is appropriate for a star with a value of \( T_{\text{eff}} \approx 4300 \) K (Agol et al. 2004) in the I band, according to the tables of Claret (2000).

Initially we treat the ring as a “knife edge” that changes abruptly from transparent to opaque, and thus any part of the binary system seen above the edge is unobscured, while that below the edge is completely occulted. For this sharp-edged model, the flux during ingress and egress is given by

\[
F(z) = 1 + \langle I \rangle^{-1} \left[ z \sqrt{1 - z^2} \left( 1 - \gamma_1 - \frac{11}{6} \gamma_2 + \frac{1}{3} \gamma_2 z^2 \right) - \left( 1 - \gamma_1 - \frac{3}{2} \gamma_2 \right) \cos^{-1} z \right],
\]

where \( z \) is the distance between the center of either star and the edge of the occulting ring, in units of the stellar radius, and \( \langle I \rangle = \pi \left( 1 - (1/3) \gamma_1 - (1/6) \gamma_2 \right) \). Equation (2) is only valid for \( |z| < 1 \) and takes on values of \( F(z) = 0 \) for \( z \leq -1 \) and \( F(z) = 1 \) for \( z \geq 1 \). Although the knife-edge model provides a fairly accurate set of initial parameters, the quantitative agreement with the data is poor, giving a value of \( \chi^2 \) of \( \sim 12,991 \) for 6697 degrees of freedom, where 6697 = 6694 photometric points + 18 velocity points – 15 free parameters. Most of the discrepancy in this fit occurs at the beginning of ingress and the end of egress, where the model has a much steeper slope than the data, as is shown in Figure 3.

The next step in our calculation is to add “fuzziness” to the edge of the ring by defining the optical depth to vary as a power law as a function of the sky-plane distance from the ring edge (it has infinite optical depth below the edge). This power-law behavior seems physically plausible if we assume that there is a drop-off in density at the edge of the ring or due to the vertical structure of the ring. We compute the transmitted flux during ingress and egress by convolving the knife edge with the power-law optical depth,

\[
F^* = \frac{\alpha}{\gamma_0} \int_{\max(0, z-1)}^{\max(0, z+1)} dy \ e^{-\gamma_0^{-1} y^\alpha} \left( \frac{\gamma_0}{y} \right)^{1+\alpha} F(z - y),
\]

where \( \gamma_0 \) is the scale length of the fuzzy edge (projected on the sky), in units of the stellar radius, \( R_{\text{star}} \) (we take both stars to have the same radius); \( \alpha \) is the power-law exponent of the optical depth variation; and \( y \) is perpendicular to the ring edge. In this model, we initially used a fixed value of \( \alpha = 2 \), but after establishing a reasonable fit to the data, we allowed \( \alpha \) to vary freely. We report the value of \( \alpha \), which is \( \gamma_0 \) transformed to physical units: \( w \equiv \gamma_0 R_{\text{star}} \) (in units of meters). This addition reduces the value of \( \chi^2 \) to 12,890 for 6695 degrees of freedom, improving the fit by \( \Delta \chi^2 = 101 \).

We next add curvature of the ring edge to the model, which improves the fit to the mideclipse data. We parameterize this as \( y_{\text{ring}} = \mu x_{\text{ring}} \), where \( y_{\text{ring}} \) and \( x_{\text{ring}} \) are the \( y \)- and \( x \)-coordinates perpendicular and parallel to the edge of the ring at the origin when it crosses the barycenter of the system. We find that by adding curvature, the rate at which the central rebrightenings fade during the years 1995–1998 is slower than that without it, since the hidden star’s orbit remains visible for a slightly longer period of time, agreeing better with the light-curve data. Again we see an incremental improvement in the model as the value of \( \chi^2 \) drops to 9582 with 6694 degrees of freedom, a reduction of \( \Delta \chi^2 = 3308 \). We neglected precession of the ring, as the model of Winn et al. (2006) indicates that the edge projected on the sky plane would have rotated only \( \sim 4.5^\circ \) over the 9 yr data set.

### 3.3. Forward Scattering

Forward scattering by dust is the most important new addition to our model: the same dust at the edge of the ring that causes extinction will also diffract light, leading to an apparent halo around the star. The same effect can be seen on nights when the Moon passes behind a thin layer of clouds. The water droplets in the clouds diffract light from the Moon, producing a halo of forward-scattered light about the Moon. For both the Moon and KH 15D, the halo is present when the optical depth is near unity; at very high optical depths, multiple scatterings will cause the radiation to eventually be absorbed, while at very low optical depths there is almost no scattering. As the star passes behind the edge of the ring, this forward-scattering halo softens the shape of the ingress and egress, as is observed in KH 15D.

The angular distribution of scattered light for spherical grains can be modeled approximately as an Airy disk. Since the wings of the Airy disk are much weaker than the central peak, we approximate the Airy disk by a Gaussian angular distribution in order to allow us to make a faster computation of the scattered flux,

\[
\frac{d\sigma_F}{d\Omega} = \sigma_0 e^{-\omega^2/(2\sigma^2)},
\]

where \( \omega \) is the angular spread of the scattered light and \( \sigma \) is the angular radius of the Airy disk.
where $d\sigma_\Omega/d\Omega$ is the differential scattering cross section into a solid angle $d\Omega$. A Gaussian with a standard deviation of $\sigma_\theta = 0.43 a/(2\alpha a)$, where $a$ is the radius of the scattering particle, gives a good approximation to the peak of an Airy disk. There may be additional large-angle scattering by the ring; however, if the size of the ring is much larger than the orbital size of the binary, then this radiation should have a much weaker time dependence than the forward-scattered light, so we treat this as a constant flux contribution during the eclipse; namely, the quantity $f_\text{in}$ mentioned in § 2.1.

As the optical depth becomes large, multiple forward scatterings can occur. We make the assumption that the scattering region (due to the ring surrounding the two stars) can be treated in the thin-screen approximation; that is, (1) the scattering angle is small enough that a photon can be approximated as going straight as it passes through the dust ring, $\cos(\theta_{\text{scat}}) \sim 1$, and thus the total optical depth for each forward-scattered photon is simply the line-of-sight optical depth to the star; and (2) the dust is at approximately a constant distance from the source. Since both absorption and large-angle scattering can occur in addition to forward scattering, we add an additional parameter, $\kappa_{\text{abs}}$, which is the ratio of the sum of the absorption and large-angle scattering opacity to the forward-scattering opacity.

With these approximations, we can compute the specific intensity of the scattered radiation, $I(x, y) = \sum_{N=1}^{N_{\text{max}}} I_N(x, y)$, at a position $(x, y)$ relative to the source position $(x_0, y_0)$, where both positions are projected on the sky plane and relative to the ring edge (which is rotated relative to the $X$-$Y$ plane). For radiation that has undergone $N$ scatterings, the specific intensity of the scattered radiation, $I_N(x, y)$, is given by a Gaussian, with standard deviation $\sqrt{N} \sigma_\theta$, times the probability of scattering $N$ times:

$$I_N(x, y) = \frac{F_0}{2\pi N \sigma_\theta^2} \frac{\sigma_\theta^N(x, y)}{N!} e^{-\tau(x, y)} e^{-[(x-x_0)^2+(y-y_0)^2]/(2D^2 N \sigma_\theta^2)}, \quad (5)$$

where $F_0$ is the flux of the star at the distance of the scattering screen of dust, $D$ is the star-dust separation, and $\tau$ is the forward-scattering optical depth through the slab at position $(x, y)$. We treat the stars as point sources for the purposes of computing the forward-scattered light. When $\kappa_{\text{abs}} \sim 1$, truncating the sum at $N_{\text{max}} = 5$ scatterings gives a converged distribution of scattered light. We report the value of $\sigma_\theta \equiv D \sigma_\theta$ in units of meters rather than units of $\sigma_\theta$.

Figure 4 shows the distribution of scattered light as a function of distance of the center of the star from the edge $(\tau = \infty)$ of the ring. The best-fit parameters (see below) for the KH 15D system were used in the creation of this figure. When the star is far above the ring edge (see the $y = 6.0$ frame), the forward...
scattering is weak, as the optical depth is small within the angular size of the scattering halo. As the star approaches the edge, forward scattering increases as the optical depth increases (see the \( y = 3.5 \) and \( 1.0 \) frames), but the stellar light starts to decrease because of extinction by the dust. As the star becomes eclipsed, the halo can still be seen above the edge, but it is much weaker because of extinction of the scattered halo (see the \( y = -1.5 \) frame).

For computing the sky-plane separation of the star and the ring edge, we take into account the curvature of the edge of the ring, since the distance traveled by the star and the edge of the ring is comparable to the radius of curvature. However, to simplify the scattering calculation, we treat the edge as being straight (since the radius of curvature is much larger than the edge scale length), and the optical depth varies only perpendicular to the ring edge, which allows us to perform an analytic integration of the scattered specific intensity parallel to the edge (along \( x \)), while integration perpendicular to the edge is carried out numerically.

After we add the forward scattering, the model light curve matches the more gradual slope on either end of the eclipses, as well as the small increases in flux during mideclipse. The value of \( \chi^2 \) is reduced to 7162 for 6692 degrees of freedom, which is an overall improvement of \( \Delta \chi^2 = 5829 \) as compared to the knife-edge model with no forward scattering. In particular, we note that the combination of forward scattering with a curved ring edge produces mideclipse “bumps” in the model that are more pronounced and widened, which agrees well with the data. Furthermore, we see the noticeable effects of the hidden star decrease as the heights of the mideclipse bumps diminish over time (Fig. 3). This supports the idea that as time passes, an increasing portion of the orbit of the binary system is obscured by the ring of material.

As a comparison between our model and the “halo” model of Winn et al. (2006), we take the flux data as a function of the separation between star 1 and the edge of the ring (as in Fig. 3) and run a \( \chi^2 \) minimization using the halo model on the data points for which star 2 contributes an insignificant flux. Using

| Parameter                     | Estimated Value |
|-------------------------------|-----------------|
| \( M_1 (M_\odot) \)          | 0.6 ± 0.1       |
| \( M_2 (M_\odot) \)          | 0.72 ± 0.1      |
| \( P \) (days)               | 48.359 ± 0.0012 |
| \( e \)                      | 0.51 ± 0.008    |
| \( \omega \) (deg)           | 5.1 ± 1.3       |
| \( \gamma \) (deg)           | 18.6 ± 1.5      |
| \( r_{\text{ring}} \) (m)    | 58.46 ± 5.95    |
| \( \rho_{\text{ring}} \) (m) | -1289.64 ± 340.23 |
| \( f_{\text{out}} (f_\text{in}) \) | 1.2 ± 0.23 |
| \( f_{\text{in}} (f_\text{out}) \) | (5.075 ± 0.123) \times 10^{-8} |
| \( \omega_{\text{out}} (\omega_\text{in}) \) | (1.59 ± 0.03) \times 10^{-6} |
| \( \rho_{\text{out}} (\rho_\text{in}) \) | (3.28 ± 0.28) \times 10^{-6} |
| \( w (m) \)                  | (1.30 ± 0.67) \times 10^{9} |
| \( \sigma (m) \)             | (4.35 ± 0.34) \times 10^{9} |
| \( \nu \) (rad)              | 1.91 ± 0.18     |
| \( \beta \) (rad)            | 1.18 ± 0.09     |
| \( \mu (AU^{-1}) \)          | 2.00 ± 0.40     |
| \( \alpha \)                 | 1.53 ± 0.59     |

![Fig. 5.—Light curve of KH 15D as a function of phase. Each plot shows 1 yr worth of data, with the best-fit model overplotted. The most notable features are the drop-off in mideclipse rebrightenings from 1995 to 1998, the gradual deepening of the eclipses from start to finish, and the mideclipse bumps from 1998 to 2004, which are produced by forward scattering in our model as the ring edge moves across the orbit of the binary.](image-url)
the seven parameters necessary for modeling the occultation curve, which are \{R_{\text{star, i}}, f_{\text{in}}, f_{\text{out}}, \sigma_w, \kappa, \alpha \} for our model versus \{R_{\text{star, i}}, f_{\text{in}}, f_{\text{out}}, \epsilon_1, \epsilon_2, \xi_1, \xi_2 \} for the model of Winn et al. (2006), we find that forward scattering produces a value of \( \chi^2 = 5849.4 \) and the halo model produces a value of \( \chi^2 = 5903.7 \) for 5484 degrees of freedom. The difference in \( \chi^2 \) between the two models is \( \Delta \chi^2 = 54 \), which indicates that the forward-scattering model is favored over the halo model.

4. RESULTS

The complete model has 18 free parameters, consisting of the six orbital elements, \{P, e, i, \omega, T_p, \gamma \}, four parameters that define the orientation, shape, and motion of the occulting screen, \{f_{\text{ring}}, f_{\text{out}}, \beta, \mu \}, and eight parameters that modify the structure of the light curve, \{R_{\text{star, i}}, f_{\text{in}}, f_{\text{out}}, \sigma_w, \kappa, \alpha \}. In addition, we assume that the masses of the stars are \( M_1 = 0.6 \pm 0.1 M_\odot \) and \( M_2 = 0.72 \pm 0.1 M_\odot \), respectively. This is based on the theoretical pre–main-sequence evolutionary tracks and the mass-luminosity relation indicating that the mass of the visible star should be \( 0.6 \pm 0.1 M_\odot \) and the mass ratio, \( M_2/M_1 \), should be \( 1.2 \pm 0.1 \) (Winn et al. 2006).

For the optimization of these parameters, we choose to minimize with respect to a \( \chi^2 \) that is defined in a similar way to that of Winn et al. (2006), such that

\[
\chi^2 = \sum_{i=1}^{N_f} \left( \frac{f_{\text{mod, i}} - f_{\text{data, i}}}{\sigma_{f, i}} \right)^2 + \lambda \sum_{i=1}^{N_e} \left( \frac{\epsilon_{\text{mod, i}} - \epsilon_{\text{data, i}}}{\sigma_{\epsilon, i}} \right)^2, \tag{6}
\]

where \( N_f = 6694 \) and \( N_e = 18 \). The presence of \( \lambda \) allows us to apply a certain amount of weight to the relatively few radial velocity measurements so that equally good fits are found to both fluxes and radial velocities. To determine what value of \( \lambda \) to use for our model, we attempted to find a value that would produce \( \chi^2 / N_f \approx \chi^2 / N_e \), where \( \chi^2_f \) and \( \chi^2_e \) are the separate nonreduced \( \chi^2 \) values for the fluxes and radial velocities, respectively. This procedure gives a value of \( \lambda = 9 \) for our final model. Using that value of \( \lambda \), we find that our best-fit parameter set produces values of \( \chi^2_f = 6992.6 \) and \( \chi^2_e = 18.8 \). A list of the values of the best-fit parameters and their errors (which we describe next) is presented in Table 1. In addition, Figures 5 and 6 show the model and corresponding data for the light curve and the radial velocity, respectively.

To estimate the uncertainties in each of our 18 parameters, we followed the procedure of Winn et al. (2006). We repeatedly fitted our model to artificial data sets composed of random noise added to our best-fit model. To model the noise, we took the residuals of the best-fit model, normalized them by the model flux, randomized them, and then added them back to the best-fit model (again scaling to the flux) to create an artificial data set. Since the number of radial velocity points is so small, we instead added Gaussian noise with a standard deviation equal to the error bars of the radial velocity data (Winn et al. 2006). For each artificial data set, we also allowed \( M_1 \) and \( M_2 \) to vary with Gaussian distributions that had means of 0.6 and 0.72, respectively, and standard deviations of 0.1, so that the resulting error bars would have the \( \pm 0.1 M_\odot \) constraint placed on them as mentioned above. To determine the error bars on the remaining parameters, which are quoted in Table 1, we took the standard deviations of 2000 parameter sets that were produced from fits to 2000 random realizations of the noisy light curve. We found that subsequent iterations did not appear to produce significant changes in the errors.

5. SUMMARY AND DISCUSSION

We find that our best-fit model is in good agreement with the data, with a reduced \( \chi^2 \) of order unity. The model accurately reproduces the falloff in the emergence of star 2 during the 1995–1998 time frame, as well as the gradual deepening of the total eclipses from 1998 to 2004 (Fig. 5). Furthermore, we find one of the most satisfying results to be the success of forward scattering as the source for the mideclipse bumps in the light curve once star 2 no longer passes beyond the edge of the ring, as well as explaining the gradual falloff and rises in the curve as star 1 enters and exits the eclipses (Fig. 3). Our model also predicts that around the beginning of 2008, the photosphere of star 1 will no longer move beyond the edge of the occulting screen: this is the same time frame as that predicted by Winn et al. (2006). After that point, the maximum light we will receive from KH 15D will be due to forward scattering as star 1 nears the edge of the ring, which will last roughly 5–7 yr.

Several of the predicted parameters are in relatively good agreement with the values that we expect. First, \( \gamma \) is in agreement with both the value published by Winn et al. (2006) and the median heliocentric radial velocity of the cluster NGC 2264, which is \( 20 \pm 3 \text{ km s}^{-1} \) (Soderblom et al. 1999). Second, we allowed the radius of star 1, \( R_{\text{star, i}} \), to be a free parameter determined by the shape of the light curve, and we found a best-fit value of \( 1.2 \pm 0.23 R_\odot \), which is consistent with the value used in Winn et al. (2006), who held the parameter fixed at 1.3 \( R_\odot \) on the basis of the mass-radius relation as predicted by pre–main-sequence evolutionary tracks. Finally, we find that our value for the eccentricity, 0.51, is consistent with the pseudosynchronization upper limit of 0.66 as discussed by Winn et al. (2006).

Our derived value for the angle between the edge of the ring and the horizontal axis of the system, \( \beta = 68^\circ \pm 5^\circ \), was not what we expected. Although our error bars seem to indicate that the value is relatively well constrained, since the system is nearly edge-on, changes in \( \beta \) will not produce significant changes in the model light curve, so we are not confident in the best-fit value or uncertainty of this parameter.

We now compare our best-fit model with the model in Winn et al. (2006). Our best-fit period, eccentricity, and inclination are relatively close to those presented by Winn et al. (2006), agreeing
to within 0.1%, 10%, and 5%, respectively. However, formally there is quantitative disagreement (>1 σ) between the values of Winn et al. (2006) and our own, which most likely results from physical differences between their model and ours, as well as fitting different subsets of the published data with different assumed errors. Our best-fit flux for star 2 is nearly twice that of star 1, which is somewhat larger than the ratio of 1.36 found by Winn et al. (2006).

From the forward-scattering width in our model, σθ, we can estimate the average size of the dust grains that dominate the forward-scattering opacity. Using the parameter σθ and our Gaussian approximation, σθ = 0.43λ/(2πa) with λ = 8140 Å, we find that the radius a of the dust grains is ~6(D/3 AU) μm, where D is the distance between the edge of the ring and the stars. This agrees extremely well with the ~6–8 μm size of the dust grains estimated by Agol et al. (2004) on the basis of the weak variation of the observed polarization with wavelength (smaller/larger grains cause a stronger/weaker variation of polarization with wavelength than is observed). A distance of 3 AU was estimated by Chiang & Murray-Clay (2004) and Winn et al. (2004) to explain a rate of precession that matches the observed velocity of the ring edge across the binary, indicating internal consistency between the forward-scattering, polarization, and precession constraints on a and D.

Beyond our confirmation that forward scattering is the correct physical process by which the light-curve features are produced, we can also use the flux parameters to estimate the apparent area of the ring. One possible interpretation of the residual flux in the infrared observations. Since we observe a value of f in the forward-scattered to large-angle scattered light, which changes with wavelength, and this is an additional reason to make infrared observations.

To finish our discussion of model parameters, we comment on the value of κsat ~ 2. As the wavelength of light is λ = 0.8 μm and we have estimated that the radius of the dust particles is a = 6 μm, then the particle-scattering parameter is x = 2π(a/λ) ~ 50. In this limit, the forward-scattering cross section and reflectance (large-angle scattering plus absorption) cross section equal the area of the particle, so one would expect a value of κsat = 1. Therefore, our value of κsat ~ 2 indicates that there may be grains smaller than 6 μm mixed in with a = 1. These smaller grains would not have as narrow a forward-scattering peak and would thus contribute to the large-angle scattering/absorption component, resulting in an increase in the value of κsat. Since large-angle scattering causes polarization and forward scattering does not, a possible test of the model proposed in this paper would be to measure the degree of polarization as a function of the ratio of forward-scattered to large-angle scattered light, which changes during the ingress/egress. Infrared, radial velocity, and polarimetric measurements, combined with more detailed modeling of the dust ring, should help to put tighter constraints on the properties of the system and unlock further mysteries of KH 15D. We conclude by noting that a similar effect of forward scattering may have been seen in the system β Pictoris (Lamers et al. 1997).

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