A Novel Hybrid Fuzzy Grey TOPSIS Method: Supplier Evaluation of a Collaborative Manufacturing Enterprise

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Abstract: Recently, there is of significant interest in developing multi-criteria decision making (MCDM) techniques with large applications for real-life problems. Making a reasonable and accurate decision on MCDM problems can help develop enterprises better. The existing MCDM methods, such as the grey comprehensive evaluation (GCE) method and the technique for order preference by similarity to an ideal solution (TOPSIS), have their one-sidedness and shortcomings. They neither consider the difference of shape and the distance of the evaluation sequence of alternatives simultaneously nor deal with the interaction that universally exists among criteria. Furthermore, some enterprises cannot consult the best professional expert, which leads to inappropriate decisions. These reasons motivate us to contribute a novel hybrid MCDM technique called the grey fuzzy TOPSIS (FGT). It applies fuzzy measures and fuzzy integral to express and integrate the interaction among criteria, respectively. Fuzzy numbers are employed to help the experts to make more reasonable and accurate evaluations. The GCE method and the TOPSIS are combined to improve their one-sidedness. A case study of supplier evaluation of a collaborative manufacturing enterprise verifies the effectiveness of the hybrid method. The evaluation result of different methods shows that the proposed approach overcomes the shortcomings of GCE and TOPSIS. The proposed hybrid decision-making model provides a more accurate and reliable method for evaluating the fuzzy system MCDM problems with interaction criteria.

Keywords: grey correlation; topsis; multi-criteria decision making

1. Introduction

Multi-criteria decision making (MCDM) problems are utterly fatal and can be frequently observed academically in many domains, such as in manufacturing [1,2], finance [3], logistics [4], and the supply chain [5]. The rationality and correctness of the decision are quite crucial for a company. The development of an effective and efficient MCDM method to solve the optimization problems correctly is significant.
Conventionally, frequently-used MCDM methods include the goal programming method [6], the multi-attribute value theory [7], the multi-attribute utility method [8], the delaminating sequence method [9], the analytic hierarchy process (AHP) [10], the data envelopment analysis [11], the technique for order preference by similarity to ideal solution (TOPSIS) [12,13], grey comprehensive evaluation (GCE) method [14], fuzzy comprehensive evaluation [15], artificial neural networks [16] and some other fuzzy grey based hybrid MCDM approaches [17–26]. Great progress has been made and compared with the earlier methods. However, there are still some shortcomings when dealing with the problems in real-life systems. It is universally acknowledged that the criteria are interactive [27–29] and for some qualitative criteria, we can only find the fuzzy evaluation rather than numerical values [30]. However, the hypothesis that the criteria are independent is diffusely used in these methods, and the evaluation values are aggregated through linear means, which usually leads to inadequate decisions.

The fuzzy integral is considered as an effective method to improve the limit since it can take into consideration the interactions among criteria expressed by fuzzy measures when aggregating the evaluation data [31–34]. The fuzzy integral is proposed by Sugeno [35] and further developed by many researchers. The fuzzy integral mainly includes the Sugeno fuzzy integral [36], the Choquet fuzzy integral [37], the Y fuzzy integral, the N fuzzy integral, the H fuzzy integral, the uncertain fuzzy integral, the T fuzzy integral, the pseudo-additive and the Pan-integrals fuzzy integral [38]. Among these fuzzy integrals, the Choquet integral is the best for dealing with the nonlinear aggregation of MCDM problems since it can well express the interactions among criteria [39,40]. The fuzzy measures mainly include the nonadditive-measures, the quasi-additive-measures, the k-additive fuzzy-measures, and the λ-fuzzy-measures [38,39]. Although the λ-fuzzy-measures can fully represent the interactions among criteria and are easy to calculate, they can only express one kind of interaction in one group of criteria. The k-additive fuzzy-measure provides us a choice between the good expression ability and the simple computation, but we cannot obtain both simultaneously.

Among the conventional MCDM methods, the TOPSIS evaluates objects according to their difference in the comprehensive closeness degree. This evaluation is determined by the distance between the positive and negative ideal solution schemes [40]. It does not take into consideration the difference of shape of the scheme. The TOPSIS method fails under two conditions. In Figure 1, points A and B are positive and negative ideal solution schemes; Points C, D, E, and F are objects to evaluate. Generally, if the line AB is vertically divided by line CD, the distance between all the points on the straight line CD and point A are equal to that between these points and point B. The comprehensive close degrees of them are all 0.5. If line EF is vertically divided by line AB, the distance of points E and A is equal to that of points F and A. Furthermore, the distance of points E and B is equal to that of points F and B. The comprehensive close degrees of E and F are equal.

![Figure 1. Technique for order preference by similarity to an ideal solution (TOPSIS) failed conditions.](image)

The GCE method depends on the grey correlation degree, which can reflect the closeness degree between the evaluation sequences of objects [41]. It does not take into account the number and distribution of samples and is easy to implement. However, most studies focus on the construction of the grey correlation coefficients. If the shapes of evaluation sequences of some objects are very similar, the GCE method may not be able to evaluate them properly. For example, assume that the evaluation
sequences $U_1$, $U_2$, and $U_3$ are shown in Figure 2. The correlation degree between $U_2$ and $U_1$ is the same as that between $U_3$ and $U_1$.

![Figure 2. Grey comprehensive evaluation (GCE) failed conditions.](image)

Other similar traditional methods, such as TOPSIS and GCE, assume that criteria are independent of each other. In this regard, to a great extent, it leads to limiting them when addressing some practical problems with interactive criteria. By considering these features of the TOPSIS and GCE approaches, this study develops a novel hybrid MCDM method called the fuzzy grey TOPSIS (FGT). By comparing the exiting methods, this research makes the following contributions.

First, the proposed method combines the GCE and TOPSIS, which improves the one-sidedness of the GCE and TOPSIS methods. Decision-makers can make a decision according to their preference through adjusting a parameter that reflects the weights of the shape and the distance factors.

Second, fuzzy measures and fuzzy integral are employed to express and integrate the interaction between the criteria that cannot be properly expressed through the existing methods.

Third, fuzzy numbers are used to help the experts make a more reasonable and accurate evaluation of some qualitative criteria.

The remainder of this paper is organized as follows: Section 2 is a literature review. Section 3 introduces the preliminaries. Section 4 presents the novel hybrid MCDM method that combines the fuzzy integral, the grey correlation, and TOPSIS. Section 5 utilizes an example of supplier evaluation of a collaborative manufacturing enterprise to illustrate the procedure, the feasibility, and effectiveness of the proposed method. Finally, conclusions are reached in Section 6.

2. Literature Review

Academically, many studies have been conducted on MCDM problems, and substantial progress has been made in this field. For example, Bouzouar-Amokrane et al. formulated a consensus bipolar method to solve the collaborative group MCDM problems considering the impact of human behavior, e.g., individualism, fear, caution [42]. The analysis results of the application example about real size wind farm implantation problem showed that this proposed method can be a useful decision-making tool to solve this problem [42]. Tchangani et al. proposed a bipolar aggregation method that combines weighted cardinal fuzzy measures, which can effectively overcome difficulties that dissuade the use of Choquet integral in practices to solve the fuzzy nominal classification in the MCDM problem [43]. Zhang et al. formulated a hybrid optimization approach combining the best worst method, grey relational analysis, and visekriterijumsko kompromisno rangiranje (VIKOR) to solve the MCDM problem in rail transit [44]. A multi-cell thin-walled aluminum energy-absorbing structure was applied to verify that this integrated method is valid and practical [44]. The results proved that this method provides an accurate and effective tool for the structural decision-making problem in rail transit [44]. Mousavi-Nasab and Sotoudeh-Anvari presented a new multi-criteria decision-making approach for the sustainable material selection problem [45]. Ten examples were applied to verify this proposed method [45].

TOPSIS and GCE are two of the several important methods in dealing with MCDM problems. Many researchers have a contribution to hybridization of MCDM techniques with some other method.
to make the best use of their advantages and bypass their disadvantages, as shown in Tables 1 and 2. In addition, the fuzzy integral and fuzzy set are used to address the interaction among criteria which have been applied to many domains, as exposed in Table 3.

**Table 1.** Review of the technique for order preference by similarity to an ideal solution (TOPSIS).

| Typical Reference       | Tool Type                  | Advantage                        | Limitation                      |
|-------------------------|----------------------------|----------------------------------|---------------------------------|
| Hassan et al. [46]      | TOPSIS                     | Flexible framework               | Experts have too many rights    |
| Kazancıoğlu et al. [47] | Fuzzy TOPSIS              | Simple                           | Hard to describe complex systems|
| Bagojić et al. [48]     | TOPSIS and COPRAS, SAW    | Various comparison               | Hard to describe complex systems|
| Yayla et al. [49]       | Fuzzy TOPSIS and Fuzzy-AHP| More realistic and reliable      | Complicated and abstract models |
| John et al. [50]        | Fuzzy TOPSIS and Fuzzy-AHP| More realistic and reliable      | Complicated and abstract models |
| Shanian and Savadogo [51]| Ordinary and Block TOPSIS| Simple and fast                  | Complicated                     |
| Olson [52]              | TOPSIS                     | Simple and fast                  | Not accurate enough             |
| Deng et al. [53]        | Weighted Euclidean distances TOPSIS | Simple and direct               | Not accurate enough             |
| Buyukozkan and Cifci [54,55]| Fuzzy AHP and fuzzy TOPSIS| Solving the uncertainty problem of score evaluation | Hard to describe complex systems|
| Zavadskas et al. [56–58]| TOPSIS, COPRAS, and ARAS  | Aggregate both quantitative and qualitative criteria | Complicated and abstract models |
| Yazdani and Payam [59]  | Ashby, VIKOR, and TOPSIS  | Various comparison               | Complicated and abstract models |

**Table 2.** Literature review of grey evaluation methods.

| Typical Reference       | Tool Type                  | Advantage                        | Limitation                      |
|-------------------------|----------------------------|----------------------------------|---------------------------------|
| Greco et al. [60]       | Rough sets theory and MCDA| The use of the decision rule model and the capacity | Too simple                      |
| Diakoulaki and Karangelis [61]| MCDA and CBA              | Broadening the evaluation perspective | Each method is often disputed    |
| Pohekar and Ramachandran [62]| MCDM, PROMETHEE, and ELECTRE| Compare in a variety of ways     | Thinking not deep enough        |
| Tian et al. [63]        | GRA and AHP                | Simple and fast                  | Dependent on index selection    |
| Hashemi et al. [64]     | Improved grey relational analysis | Novel and convenient             | Hard to describe complex systems|
| Sarucan et al. [65]     | AHP and GRA                | More realistic and reliable      | Complicated and abstract models |
| Ebrahimi and Keshavarz [66]| FA and GIA                | Two-phase comparison results are consistent | Complicated and abstract models |
| Wang et al. [67,68]     | Fuzzy and weighting method| More realistic and reliable      | Theory still flawed             |
| Lee and Lin [69]        | GRA                        | Reasonable and efficient         | The reference factor is not enough|
| Abhang and Hameedullah [70]| Grey relational analysis  | Effectively improve accuracy     | Complicated and abstract models |
Table 3. Literature review of fuzzy integral and fuzzy set.

| Typical Reference       | Tool Type                                | Advantage                  | Limitation                          |
|-------------------------|------------------------------------------|----------------------------|-------------------------------------|
| Aydin et al. [71]       | Fuzzy-AHP, Choquet integral and trapezoidal fuzzy sets | Realistic and reliable     | Complicated and abstract models     |
| Celik et al. [72]       | VIKOR and interval type-2 fuzzy sets     | Effectively improve accuracy | Complicated and abstract models     |
| Feng et al. [73]        | Fuzzy integral                           | Convenient                 | Complicated                         |
| Tian et al. [74]        | Choquet fuzzy integral                   | Effectively improve accuracy| Complicated                         |
| Wu et al. [75]          | Fuzzy Integral with Particle Swarm Optimization | Solving the uncertainty problem | Dependent on index selection        |
| Zhu et al. [76]         | Fuzzy multi-attribute decision making    | Novel and convenient       | Dependent on index selection        |
| Fu and Zhao [77]        | Hesitation institution fuzzy number      | More realistic and reliable| Complicated and abstract model      |
| Lin et al. [78]         | TOPSIS and aggregation approach         | Efficient and robust, realistic and reasonable | Complicated and abstract model      |

3. Preliminaries

3.1. The λ-Fuzzy-Measure and the Choquet Integral

In the FGT method, the weights of all subsets of a criterion set are expressed through the λ-fuzzy-measure.

Fuzzy-measures [39]: Assume that the criterion set \( X \) is nonempty and \( P(X) \) denotes its power set. If the set function \( g : P(X) \rightarrow [0, 1] \) satisfies the following three axioms, then it is called a fuzzy-measure:

(1) Boundness

\[ g(\emptyset) = 0, g(X) = 1. \]

(2) Monotonicity

\[ \forall A, B \in P(X), \text{ if } A \subseteq B, \text{ then } g(A) \leq g(B). \]

(3) Weak continuity:

\[ \text{If } \{A_i\} \subset P(X) \text{ and } \{A_i\} \text{ is monotonic, then } \lim_{i \to \infty} g(A_i) = g\left( \lim_{i \to \infty} A_i \right). \]

λ-fuzzy-measures [39]: If the fuzzy-measure \( g \) satisfy the following axiom, then it is called λ-fuzzy-measures:

\[ \text{If } A, B \in P(X) \text{ and } A \cap B = \emptyset, \text{ then } \exists \lambda \in [-1, \infty), \text{ } g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B). \]

Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a finite set. If the fuzzy density of \( x_i \) is \( g_i \), then \( g_\lambda \) can be calculated as follows:

\[
g_\lambda([x_1, x_2, \ldots, x_k]) = \frac{\sum_{i=1}^{k} g_i + \lambda \sum_{i=1}^{k-1} \sum_{j=1}^{i+1} g_{i_1} g_{i_2} + \ldots + \lambda^{k-1} g_1 g_2 \cdots g_k}{\lambda} \prod_{i=1}^{k} (1 + \lambda g_i) - 1
g_\lambda([x_1, x_2, \ldots, x_k]) = \frac{\sum_{i=1}^{k} g_i + \lambda \sum_{i=1}^{k-1} \sum_{j=1}^{i+1} g_{i_1} g_{i_2} + \ldots + \lambda^{k-1} g_1 g_2 \cdots g_k}{\lambda} \prod_{i=1}^{k} (1 + \lambda g_i) - 1
g_\lambda([x_1, x_2, \ldots, x_k]) = \frac{\sum_{i=1}^{k} g_i + \lambda \sum_{i=1}^{k-1} \sum_{j=1}^{i+1} g_{i_1} g_{i_2} + \ldots + \lambda^{k-1} g_1 g_2 \cdots g_k}{\lambda} \prod_{i=1}^{k} (1 + \lambda g_i) - 1
g_\lambda([x_1, x_2, \ldots, x_k]) = \frac{\sum_{i=1}^{k} g_i + \lambda \sum_{i=1}^{k-1} \sum_{j=1}^{i+1} g_{i_1} g_{i_2} + \ldots + \lambda^{k-1} g_1 g_2 \cdots g_k}{\lambda} \prod_{i=1}^{k} (1 + \lambda g_i) - 1
g_\lambda([x_1, x_2, \ldots, x_k]) = \frac{\sum_{i=1}^{k} g_i + \lambda \sum_{i=1}^{k-1} \sum_{j=1}^{i+1} g_{i_1} g_{i_2} + \ldots + \lambda^{k-1} g_1 g_2 \cdots g_k}{\lambda} \prod_{i=1}^{k} (1 + \lambda g_i) - 1
g_\lambda([x_1, x_2, \ldots, x_k]) = \frac{\sum_{i=1}^{k} g_i + \lambda \sum_{i=1}^{k-1} \sum_{j=1}^{i+1} g_{i_1} g_{i_2} + \ldots + \lambda^{k-1} g_1 g_2 \cdots g_k}{\lambda} \prod_{i=1}^{k} (1 + \lambda g_i) - 1
g_\lambda([x_1, x_2, \ldots, x_k]) = \frac{\sum_{i=1}^{k} g_i + \lambda \sum_{i=1}^{k-1} \sum_{j=1}^{i+1} g_{i_1} g_{i_2} + \ldots + \lambda^{k-1} g_1 g_2 \cdots g_k}{\lambda} \prod_{i=1}^{k} (1 + \lambda g_i) - 1
g_\lambda([x_1, x_2, \ldots, x_k]) = \frac{\sum_{i=1}^{k} g_i + \lambda \sum_{i=1}^{k-1} \sum_{j=1}^{i+1} g_{i_1} g_{i_2} + \ldots + \lambda^{k-1} g_1 g_2 \cdots g_k}{\lambda} \prod_{i=1}^{k} (1 + \lambda g_i) - 1
Choquet integral [79]: Without loss of generality, assume that the evaluation values on \( n \) criteria \( f(x_i)|i = 1, 2, \ldots, n \) satisfy \( f(x_1) \geq \ldots \geq f(x_i) \geq \ldots \geq f(x_n) \). Then, the Choquet fuzzy integral of \( f(x) \) among \( g(x) \) on \( X \) is:

\[
\int f dg = f(x_n)g_1(X_n) + [f(x_{n-1}) - f(x_n)]g_1(X_{n-1}) + \ldots + [f(x_1) - f(x_2)]g_1(X_1)
\]

where \( g_1(X_i) = g_1([x_1, x_2, \ldots, x_i]) \) represents the fuzzy measure of the corresponding criterion set.

3.2. The Correlation Coefficients

Assume that the evaluation matrix is

\[
C_{m \times n} = \begin{bmatrix}
C_1(1) & C_1(2) & \ldots & C_1(n) \\
C_2(1) & C_2(1) & \ldots & C_2(n) \\
\vdots & \vdots & \ddots & \vdots \\
C_m(1) & C_m(2) & \ldots & C_m(n)
\end{bmatrix}
\]

where the \( i \)-th row represents the evaluation sequence of the \( i \)-th object. \( \{C_i()\} = \{C_1(), C_2(), \ldots, C_n()\} \) is selected as the reference sequence where \( C_i(l) \) represents the best of the \( l \)-th criterion. The correlation coefficient vector of the \( i \)-th alternative sequence related to the reference sequence is denoted as \( \xi_i = \{\xi_i(1), \xi_i(2), \ldots, \xi_i(n)\} \). We then have [80]:

\[
\xi_i(k) = \frac{\min\{C_i(k) - C_i(l)\} + \max\{C_i(k) - C_i(l)\}}{|C_i(k) - C_i(l)| + \max\{C_i(k) - C_i(l)\}}
\]

where the resolution ratio \( \rho \in [0, 1] \) is usually assigned as 0.5.

The absolute correlation degree can be found by

\[
r_i = \sum_{j=1}^{n} (w_j \times \xi_i(j)).
\]

The bigger the degree value is, the better the object is.

3.3. The Triangle Fuzzy Number

**Triangle fuzzy number** [81]: The membership function of the fuzzy number \( \tilde{n} = (n_1, n_2, n_3) \) is defined as

\[
f(x) = \begin{cases} 
\frac{x - n_1}{n_2 - n_1}, & n_1 \leq x < n_2 \\
\frac{x - n_2}{n_3 - n_2}, & n_2 \leq x \leq n_3 \\
0, & \text{other}
\end{cases}
\]

where \( 0 \leq n_1 \leq n_2 \leq n_3 \).

Let \( \tilde{m} = (m_1, m_2, m_3) \) and \( \tilde{n} = (n_1, n_2, n_3) \) be fuzzy numbers. Then,

\[
\tilde{m} \oplus \tilde{n} \equiv (m_1 + n_1, m_2 + n_2, m_3 + n_3).
\]

3.4. The Entropy Weight Method

With the premise that there are \( m \) objects and \( n \) criteria for evaluation, which form the evaluation matrix \( Z = [z_{ij}]_{i=1, 2, \ldots, m; j=1, 2, \ldots, n} \), the weights of all criteria can be calculated through the entropy weight method [82], as shown in Equation (7).
\[
\omega_j = \frac{1-H_j}{\sum_{i=1}^{n}(1-H_i)} \\
H_j = \frac{\ln(P_{ij})}{\ln(m)} \\
P_{ij} = \sum_{i=1}^{m} z_{ij} \\
\text{where } z_{ij} \text{ is an entry in the criterion matrix.}
\]

4. The Fuzzy Grey TOPSIS Method

4.1. Background

Assume that there are \( m \) objects to be evaluated. The objects set is \( A = \{a_1, a_2, \ldots, a_m\} \). There are \( p \) experts involved in the evaluation and decision. The experts set is denoted as \( E = \{E_1, E_2, \ldots, E_p\} \).

To evaluate these objects, the experts should construct a reasonable and effective criterion system according to the feature of the considered problem and the purpose and preference of the decision-maker. As it is widely acknowledged, a real-life system usually contains some unknown information, i.e., it is a grey system. Therefore, some qualitative criteria need to be employed to evaluate the system. In addition, we also need a few quantitative criteria for some measurable aspects.

Assume that among \( n \) criteria, some qualitative and some quantitative criteria are considered. The criterion set is \( X = \{x_1, x_2, \ldots, x_n\} \). For the convenience of expression, assume that \( x_1, x_2, \ldots, x_i \) are quantitative criteria and \( x_{i+1}, \ldots, x_n \) are qualitative.

4.2. The Procedure of Fuzzy Grey TOPSIS

Step 1: Find the evaluation data.

Step 1.1: Obtain evaluation data of qualitative criteria.

4.2.1. Obtain the Fuzzy Linguistic Value and Transform it into Fuzzy Numbers

To make the experts’ evaluation more accurate, a set of fuzzy linguistic values is set up and noted as \( \{\text{Superb, Good, Normal, Bad, Terrible}\} \) (\( \{S, G, N, B, T\} \) for short). The evaluation data of qualitative criteria are given by experts in the form of fuzzy linguistic values that correspond to fuzzy numbers. Mapping rules are shown in Table 4.

| Linguistic Variable | Fuzzy Numbers |
|--------------------|---------------|
| Superb (S)         | (0.8, 0.9, 0.9) |
| Good (G)           | (0.6, 0.7, 0.8) |
| Normal (N)         | (0.4, 0.5, 0.6) |
| Bad (B)            | (0.2, 0.3, 0.4) |
| Terrible (T)       | (0.1, 0.1, 0.2) |

The linguistic variable evaluation matrixes are transformed as fuzzy number matrixes, as shown below:

\[
\overline{J}_i = \begin{bmatrix}
\frac{1}{f_{1(n-i+1)}} & \cdots & \frac{1}{f_{1n}} \\
\vdots & \ddots & \vdots \\
\frac{1}{f_{kn(n-i+1)}} & \cdots & \frac{1}{f_{kn}} 
\end{bmatrix}
\]

\[
\overline{J}_m = \begin{bmatrix}
\frac{1}{f_{m(n-i+1)}} & \cdots & \frac{1}{f_{mn}} \\
\vdots & \ddots & \vdots \\
\frac{1}{f_{km(n-i+1)}} & \cdots & \frac{1}{f_{km}} 
\end{bmatrix}
\]

where \( \overline{J}_i \) represents the fuzzy number matrix of the \( i \)-th object \( \overline{f}_{ij} \) represents the fuzzy number corresponding to the \( k \)-th expert’s evaluation of the \( i \)-th object on the \( j \)-th criterion.

The fuzzy number evaluation values of \( k \) experts need to be integrated into one fuzzy number according to Equation (6). The fuzzy evaluation matrix \( \overline{J} \) can be given as
\[ \bar{J} = \begin{bmatrix} \bar{j}_{1(n-i+1)} & \cdots & \bar{j}_{1n} \\ \vdots & \ddots & \vdots \\ \bar{j}_{m(n-i+1)} & \cdots & \bar{j}_{mn} \end{bmatrix}, \]

where \( \bar{j}_{ij} \) represents the fuzzy number of the \( i \)-th object on the \( j \)-th criterion.

4.2.2. Defuzzify the Fuzzy Numbers

The fuzzy numbers need to be defuzzified into numerical values to take part in the calculation procedure later. There are many defuzzification methods, each of which has its advantages and limits. Three methods are adopted in this paper. In the case that the operation is too simple, and its effectiveness cannot be verified [83], the mean value of the results of the three methods is taken. Fuzzy numbers \( \tilde{j}_{ik} = (a_{ik}, b_{ik}, c_{ik}) \) can be defuzzicated through the three methods stated as follows:

(1) Distance measure method [81]:
\[
M^1_{ik} = \frac{d_{ik}^-}{d_{ik}^- + d_{ik}^*}. \quad (8)
\]

If the optimal fuzzy evaluation value is defined as \( \tilde{j}_{ik}^- = (1, 1, 1) \) and the worst fuzzy evaluation value is defined as \( \tilde{j}_{ik}^- = (0, 0, 0) \), then
\[
d_{ik}^- = \sqrt{\frac{1}{4} (a_{ik}^2 + 2b_{ik}^2 + c_{ik}^2)}, \quad (9)
\]
\[
d_{ik}^* = \sqrt{\frac{1}{4} [(1-a_{ik})^2 + 2(1-b_{ik})^2 + (1-c_{ik})^2]}. \quad (10)
\]

(2) Central value method [84]:
\[
M^2_{ik} = b_{ik} + \frac{(c_{ik}-b_{ik})-(b_{ik}-a_{ik})}{6} = \frac{4b_{ik}+c_{ik}+a_{ik}}{6}. \quad (11)
\]

(3) Gravity method [84]:
\[
M^3_{ik} = \begin{cases} \frac{a_{ik}}{(c_{ik})^2-(a_{ik})^2+3b_{ik}(c_{ik}-a_{ik})}, & a_{ik} = b_{ik} = c_{ik} \\ \frac{(c_{ik})^2-(a_{ik})^2+3b_{ik}(c_{ik}-a_{ik})}{3(c_{ik}-a_{ik})}, & \text{otherwise} \end{cases}. \quad (12)
\]

The numerical value \( j_{ik} \) can be found by taking the mean value of the three results, i.e.,
\[
j_{ik} = \frac{M^1_{ik} + M^2_{ik} + M^3_{ik}}{3}. \quad (13)
\]

The fuzzy evaluation matrix \( \bar{J} \) can be transformed into a numerical evaluation matrix \( J^* \).
\[
J^* = \begin{bmatrix} j_{1(n-i+1)} & \cdots & j_{1n} \\ \vdots & \ddots & \vdots \\ j_{m(n-i+1)} & \cdots & j_{mn} \end{bmatrix}.
\]
Step 1.2: Find evaluation data of quantitative criteria.

The evaluation data of a few numerical criteria \( j_{k1}, j_{k2}, \ldots, j_{kn} | k = 1, 2, \ldots, m \) are obtained through statistics and measurement methods. By seaming the numerical evaluation matrix of qualitative and quantitative criteria, the evaluation matrix \( J \) can be found, as shown below,

\[
J = \begin{bmatrix}
    j_{11} & \cdots & j_{1n} \\
    \vdots & \ddots & \vdots \\
    j_{m1} & \cdots & j_{mn}
\end{bmatrix},
\]

where \( j_{kl} \) represents the evaluation value of the \( k \)-th object on the \( l \)-th criterion, \( k = 1, 2, \ldots, m, l = 1, 2, \ldots, n \).

Step 2: Standardize evaluation matrix.

To eliminate the influence of dimension and order of magnitude, the evaluation matrix \( J \) needs to be standardized according to Equation (14):

\[
c_{ik} = \frac{j_{ik} - \overline{j_k}}{S_k}, \quad (14)
\]

where \( \overline{j_k} \) is the mean and \( S_k \) is the standard deviation of the evaluation values of the \( k \)-th criterion.

In that way, the standardized evaluation matrix \( C \) can be found.

\[
C = \begin{bmatrix}
    c_{11} & \cdots & c_{1n} \\
    \vdots & \ddots & \vdots \\
    c_{m1} & \cdots & c_{mn}
\end{bmatrix},
\]

where \( c_{kl} \) represents the standardized evaluation value of the \( k \)-th object on the \( l \)-th criterion, \( k = 1, 2, \ldots, m, l = 1, 2, \ldots, n \).

**Theorem 1.** The standardization transformation satisfies the following two axioms:

1. **Isotonicity:**

   If \( j_{xk} < j_{yk} \), then \( c_{xk} < c_{yk} \). If \( j_{xk} > j_{yk} \), then \( c_{xk} > c_{yk} \).

   **Proof.** If \( j_{xk} < j_{yk} \), then \( j_{xk} - \overline{j_k} < j_{yk} - \overline{j_k} \). Thus \( \frac{j_{xk} - \overline{j_k}}{S_k} < \frac{j_{yk} - \overline{j_k}}{S_k} \), namely, \( c_{xk} < c_{yk} \). Similarly, if \( j_{xk} > j_{yk} \), then \( c_{xk} > c_{yk} \).

2. **Difference remaining:**

   \[
   \forall x, y, p, q, \quad \frac{j_{xk} - j_{yk}}{j_{pk} - j_{qk}} = \frac{c_{xk} - c_{yk}}{c_{pk} - c_{qk}}.
   \]

   **Proof.** \( \forall x, y, p, q, \frac{c_{xk} - c_{yk}}{c_{pk} - c_{qk}} = \left( \frac{j_{xk} - \overline{j_k}}{S_k} - \frac{j_{yk} - \overline{j_k}}{S_k} \right) / \left( \frac{j_{pk} - \overline{j_k}}{S_k} - \frac{j_{qk} - \overline{j_k}}{S_k} \right) \).

   \( \square \)

Step 3: Obtain weights of criteria.

To aggregate the evaluation data, some series of weights that are reasonable and accurate as much as possible is certainly needed. The weights are considered in two aspects. On the one hand, the objective weights are calculated through the entropy weight method such that the influence of data
can be considered. On the other hand, the subjective weights are given by the experts to express the decision preference. Finally, the weighted average value of objective and subjective weights is taken.

**Step 3.1: Calculate objective weights of criteria.**

The objective initial weights are mainly calculated through the entropy weight method. However, some minor amendments are made to make the formula meaningful. By using the data of standardized evaluation matrix $C = \{c_{ij}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n\}$, the objective weights vector $\omega^1 = [\omega_{1}^1, \omega_{2}^1, \ldots, \omega_{n}^1]$ can be calculated according to Equation (15).

\[
\omega_j^1 = \frac{1 - H_j}{\sum_{j=1}^{n}(1-H_j)} \quad \text{and} \quad H_j = \frac{\sum_{i=1}^{m} (1 + P_{ij}) \ln(1 + P_{ij})}{-\ln(m)}.
\] (15)

**Step 3.2: Obtain subjective weights of criteria.**

The subjective weights vector $\omega^2 = [\omega_{1}^2, \omega_{2}^2, \ldots, \omega_{n}^2]$ is given by experts.

**Step 3.3: Integrate objective and subjective weights.**

The comprehensive weights vector $\omega^0 = [\omega_{1}^0, \omega_{2}^0, \ldots, \omega_{n}^0]$ can be found according to Equation (16).

\[
\omega_j^0 = \theta \omega_j^1 + (1 - \theta) \omega_j^2,
\] (16)

where the parameter $\theta \in (0, 1)$ is determined by the decision maker according to the degree of trust in objective data and subjective judgment of experts. We set it as 0.5 in this paper.

**Step 4: Identification of $\lambda$-fuzzy-measures.**

It is generally believed that many interactions exist among criteria. If we integrate the evaluation matrix by weights only, the interactions are ignored, which will lead to an inadequate decision. Thus, the $\lambda$-fuzzy-measures are employed here to express the interactions.

Let us regard the comprehensive weights vector $\omega^0 = [\omega_{1}^0, \omega_{2}^0, \ldots, \omega_{n}^0]$ as the fuzzy densities. The parameter $\lambda$ can be obtained by Equation (17):

\[
\frac{1}{\lambda} \prod_{i=1}^{n} \left( 1 + \lambda \omega_i^0 \right) - 1 = 1.
\] (17)

Note that the comprehensive weights vector $\omega^0 = [\omega_{1}^0, \omega_{2}^0, \ldots, \omega_{n}^0]$ and the parameter $\lambda$ are obtained. All the fuzzy-measures $\{\omega_Q; Q \in P(X)\}$ can be calculated according to Equation (18), where $P(X)$ is the power set of $X$.

\[
\omega_Q = \sum_{i=1}^{p} \omega_i^0 + \lambda \sum_{i=1}^{p-1} \sum_{i=1}^{p} \omega_i^0 \omega_{i+1}^0 + \ldots + \lambda^{p-1} \omega_1^0 \omega_2^0 \ldots \omega_n^0
\] (18)

**Step 5: Calculate the grey fuzzy correlation degree.**

**Step 5.1: Calculate correlation coefficients.**

To describe the difference in shapes of all objects, their positive and the negative correlation coefficients need to be calculated. First of all, the positive and the negative ideal solution schemes $C^+ = [c_{1}^+, c_{2}^+, \ldots, c_{n}^+]$ and $C^- = [c_{1}^-, c_{2}^-, \ldots, c_{n}^-]$ should be selected. With the premise that $J^+$ represents the set of some criteria which are the bigger the better, and $J^-$ represents the set of some criteria which are the smaller the better, the vector $C^+$ and $C^-$ can be determined according to Equations (19) and (20).
**Definition 1.** (Positive grey fuzzy Choquet integral): Let correlation coefficients \( \xi \) be reordered in an ascending order such that

\[
\xi_j^+ = \begin{cases} 
\max\{c_{ij}\}, & \text{if } j \in J^+ \\
\min\{c_{ij}\}, & \text{if } j \in J^- 
\end{cases}
\]

\[
\xi_j^- = \begin{cases} 
\min\{c_{ij}\}, & \text{if } j \in J^+ \\
\max\{c_{ij}\}, & \text{if } j \in J^- 
\end{cases}
\]

The positive correlation coefficients \( \xi_{ij}^+, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \) and the negative correlation coefficients \( \xi_{ij}^-, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \) of the \( j \)-th criterion can be calculated according to Equations (21) and (22):

\[
\xi_{ij}^+ = \min_{i} \left| c_{ij}^+ - c_{ij} \right| + \rho \max_{i} \left| c_{ij}^+ - c_{ij} \right| \\
\xi_{ij}^- = \min_{i} \left| c_{ij}^- - c_{ij} \right| + \rho \max_{i} \left| c_{ij}^- - c_{ij} \right|
\]

Then the positive correlation matrix \( \xi^+ \) and the negative correlation matrix \( \xi^- \) can be obtained as

\[
\xi^+ = \begin{bmatrix} \xi_{11}^+ & \cdots & \xi_{1n}^+ \\
& \ddots & \vdots \\
\xi_{m1}^+ & \cdots & \xi_{mn}^+ \end{bmatrix}, \\
\xi^- = \begin{bmatrix} \xi_{11}^- & \cdots & \xi_{1n}^- \\
& \ddots & \vdots \\
\xi_{m1}^- & \cdots & \xi_{mn}^- \end{bmatrix}
\]

**Step 5.2: Calculate the grey fuzzy Choquet integral.**

To take the interactions among criteria into account, the Choquet fuzzy integral is used to integrate the correlation coefficients of every object. The positive grey fuzzy integral vector \( R^+ = \left[ R_{1}^+, R_{2}^+, \ldots, R_{m}^+ \right] \) and the negative grey fuzzy integral vector \( R^- = \left[ R_{1}^-, R_{2}^-, \ldots, R_{m}^- \right] \) can be calculated as follows.

**Definition 2.** (Negative grey fuzzy Choquet integral): Let correlation coefficients \( \xi_{ij}^-; i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \) be reordered in an ascending order such that \( \xi_{i1}^- \geq \cdots \geq \xi_{ij}^- \geq \cdots \geq \xi_{in}^- \). The negative grey fuzzy Choquet integral of the \( i \)-th object is defined as

\[
R_i^- = \xi_{i0}^- \omega_{12 \ldots n} + \left[ \xi_{i(n-1)}^- - \xi_{in}^- \right] \omega_{12 \ldots (n-1)} + \cdots + \left[ \xi_{i1}^- - \xi_{i2}^- \right] \omega_1.
\]

where \( \omega_{12 \ldots k} \) represents the weight of the set \( \{x_1, x_2, \ldots, x_k\} \) \( (k \leq n) \) after rearrangement.
Step 6: Calculate fuzzy distance to ideal solution.

To embody the difference in distance of all objects, the distance between all objects to the positive and negative ideal solution schemes should be obtained. The Choquet fuzzy integral should also be employed to amend the Euclidean distance such that the interactions can be expressed in the distance.

**Definition 3.** (Positive fuzzy ideal solution distance): Let positive distances of all criteria \(d_{ij}^+ = (c_{ij} - c_j^+)^2; i = 1, 2, \ldots, m, j = 1, 2, \ldots, n\) be reordered in an ascending order such that \(d_{i1}^+ \geq \ldots \geq d_{im}^+ \geq \ldots \geq d_{in}^+\). The positive fuzzy ideal solution distance of the \(i\)-th object is defined as

\[
D_i^+ = \left[d_{i1}^+\omega_{12n} + \left(d_{i1}^+ - d_{in}^+ight)\omega_{12(n-1)}\right] + \ldots + \left(d_{i1}^+ - d_{i2}^+\right)^2\omega_1^2.
\]

**Definition 4.** (Negative fuzzy ideal solution distance): Let negative distances of all criteria \(d_{ij}^- = (c_{ij} - c_j^-)^2; i = 1, 2, \ldots, m, j = 1, 2, \ldots, n\) be reordered in an ascending order such that \(d_{i1}^- \geq \ldots \geq d_{i1}^- \geq \ldots \geq d_{im}^- \geq \ldots \geq d_{in}^-\). The positive fuzzy ideal solution distance of the \(i\)-th object is defined as

\[
D_i^- = \left[d_{i1}^-\omega_{12n} + \left(d_{i1}^- - d_{in}^-ight)\omega_{12(n-1)}\right] + \ldots + \left(d_{i1}^- - d_{i2}^-\right)^2\omega_1^2.
\]

Step 7: Get grey fuzzy TOPSIS.

The positive grey fuzzy Choquet integral and the negative ideal solution distance depend on the close degree to the positive ideal solution of each scheme on the shape and distance, respectively. Their combination can represent the comprehensive close degree to the positive ideal solution of each scheme. Analogously, the combination of negative grey fuzzy Choquet integral and the positive ideal solution distance can characterize the close degree to the negative ideal solution of each scheme. To express the effects of \(R^+, R^-, D^+,\) and \(D^-\), they need to be normalized before combination.

\[
N_{ij}^{\text{new}} = \frac{N_i}{\max N_j}; i = 1, 2, \ldots, m,
\]

where \(N_i\) represent \(R^+(R^-; D^+; D^-)\), respectively.

For the conciseness, the normalized values of them are still denoted as \(R^+, R^-, D^+,\) and \(D^-\), respectively. The positive comprehensive proximity degree vector \(S^+ = \left[S_1^+, S_2^+, \ldots, S_m^+\right]\) and the negative comprehensive proximity degree vector \(S^- = \left[S_1^-, S_2^-, \ldots, S_m^-\right]\) can be obtained by combining \(R^+, R^-, D^+,\) and \(D^-\) with a proper weight parameter \(p\) according to Equation (28).

\[
\begin{align*}
S_i^+ &= pR_i^+ + (1-p)D_i^-; i = 1, 2, \ldots, m \\
S_i^- &= pR_i^- + (1-p)D_i^+; i = 1, 2, \ldots, m
\end{align*}
\]

where parameter \(p \in (0, 1)\) is determined by the decision-maker’s preference.

The comprehensive evaluation vector \(CS = [CS_1, CS_2, \ldots, CS_m]\) can be calculated by Equation (29).

\[
CS_i = \frac{S_i^+}{S_i^+ + S_i^-}; i = 1, 2, \ldots, m.
\]
The objects can be evaluated reasonably according to the comprehensive evaluation vector \( CS = [CS_1, CS_2, \ldots, CS_m] \). The object whose comprehensive evaluation value is big is better than the one whose comprehensive evaluation value is small.

The flow chart of the proposed method is given in Figure 3.

![Flow Chart](image)

**Figure 3.** The flow chart of Grey Fuzzy TOPSIS.

5. **An Illustrative Example**

To verify the feasibility and effectiveness of the proposed method, an illustrative example of supplier evaluation of a collaborative manufacturing enterprise is given in this paper. The result was compared with GCE and TOPSIS. This example has six alternatives \( A = \{A_1, A_2, A_3, A_4, A_5, A_6\} \) and 12 experts \( E = \{E_1, E_2, \ldots, E_{12}\} \).

According to the feature of the supplier evaluation of the collaborative manufacturing enterprise and the purpose and preference of the decision-maker, a reasonable and effective criterion system was constructed by experts, as shown in Figure 4.
5.1. Conducting the Proposed Method

The detailed process of the proposed method to solve this example is illustrated below. Each level is addressed one by one. We illustrate the quality level in detail only for economy of space.

**Step 1: Get the evaluation matrix.**

The fuzzy linguistic value evaluation matrices \( \tilde{J}_1, \tilde{J}_2, \tilde{J}_3, \tilde{J}_4, \tilde{J}_5, \tilde{J}_6 \) are shown as follows, in which the mapping rules of linguistic variables and fuzzy number are given in Table 4.

\[
\tilde{J}_1 = \begin{bmatrix}
S & S & S & G & S & G & S & G & S & S & G & S & S & S & S \\
G & N & G & G & N & N & G & G & G & G & S & G & G & G & G \\
S & G & S & S & S & S & N & G & G & G & S & G & S & S & S \\
G & G & S & G & G & S & G & S & S & G & S & S & G & S & S \\
\end{bmatrix}
\]

\[
\tilde{J}_2 = \begin{bmatrix}
B & B & B & T & T & B & B & B & T & B & B & T \\
N & N & N & G & N & N & N & G & N & N & G & G \\
N & G & G & N & G & S & G & G & G & G & N & G \\
G & G & G & G & G & G & G & G & S & G & N & G \\
\end{bmatrix}
\]

\[
\tilde{J}_3 = \begin{bmatrix}
S & S & G & S & S & S & G & S & G & S & S & S & S & S & S \\
N & N & N & G & N & N & N & G & N & B & N & N & N & N & N \\
N & N & N & B & N & B & B & B & N & N & N & B & N & N & N \\
N & N & N & B & N & N & N & G & G & N & N & N & N & N & N \\
\end{bmatrix}
\]
where each matrix represents the evaluation data of an alternative, and each row in the matrix represents the evaluation data of one qualitative criterion given by 12 experts expect for the second row of each matrix that is the evaluation data of the quantitative criterion and has just one value. In fact, we can give the evaluation of qualitative criteria and quantitative criteria, respectively, but for the convenience of writing, we give them in one matrix.

We transformed the fuzzy linguistic values into fuzzy numbers according to (6). Then these fuzzy numbers were aggregated according to Equation (6) and defuzzified according to Equations (8)–(13). Then the evaluation matrix $\tilde{J}$ was found as follows:

$$J = \begin{bmatrix}
0.8295 & 0.9800 & 0.6635 & 0.7839 & 0.7858 \\
0.2579 & 0.8300 & 0.5662 & 0.6635 & 0.7111 \\
0.8150 & 0.9200 & 0.5166 & 0.4172 & 0.5166 \\
0.0679 & 0.0580 & 1.3439 & 0.5502 & 0.2263 \\
0.5763 & 0.9854 & 0.6225 & 0.4755 & 0.8334 \\
0.5129 & 0.2898 & 0.5002 & 0.3452 & 0.6819 
\end{bmatrix},$$

where each row represents the evaluation data of an alternative, and each column represents the evaluation data of a criterion.

The 3-dimensional histogram of $J$ is portrayed in Figure 5.

Step 2: Standardize evaluation matrix.

The evaluation matrix $J$ can be standardized as $C$ according to Equation (14), as shown as below.

$$C = \begin{bmatrix}
- & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - 
\end{bmatrix}.$$

The 3-dimensional histograms of $C$ is depicted in Figure 6.

Step 3: Obtain weights of criteria.

Step 3.1: Calculated the objective weights of criteria.

The objective weight vector which was calculated by Equation (15) on the standardized evaluation matrix was $\omega_1 = [0.2216, 0.2098, 0.2159, 0.1818, 0.1709]$.

Step 3.2: Obtain the subjective weights of criteria.

The subjective weight vector which was given by experts was $\omega_2 = [0.1990, 0.6040, 0.2080, 0.2990, 0.3100]$.

Step 3.3: Integrate objective and subjective weights.

The 3-dimensional histogram of $J$ is portrayed in Figure 5.
Step 2: Standardize evaluation matrix.
The evaluation matrix \( J \) can be standardized as \( C \) according to Equation (14), as shown as below.

\[
C = \begin{bmatrix}
0.5129 & 0.8115 & 0.6977 & 1.3681 & 1.4790 \\
-1.9837 & -1.7970 & -0.6225 & 0.4202 & 0.7949 \\
0.4494 & -0.2319 & -1.2968 & -1.5178 & -0.9849 \\
-0.0679 & -0.0580 & 1.3439 & 0.5502 & 0.2263 \\
0.5763 & 0.9854 & -0.6225 & -0.4755 & -0.8334 \\
0.5129 & 0.2898 & 0.5002 & -0.3452 & -0.6819
\end{bmatrix}.
\]

The 3-dimensional histograms of \( C \) is depicted in Figure 6.

Step 3: Obtain weights of criteria.

Step 3.1: Calculated the objective weights of criteria.
The objective weight vector which was calculated by Equation (15) on the standardized evaluation matrix was \( \omega^1 = [0.2216, 0.2098, 0.2159, 0.1818, 0.1709] \).

Step 3.2: Obtain the subjective weights of criteria.
The subjective weight vector which was given by experts was \( \omega^2 = [0.1990, 0.6040, 0.2080, 0.2990, 0.3100] \).

Step 3.3: Integrate objective and subjective weights.
The comprehensive weights vector can be obtained by combining the objective weights vector and the subjective weights vector according to Equation (16). It was \( \omega^0 = [0.2103, 0.4069, 0.2119, 0.2404, 0.2405] \).

Step 4: Identify \( \lambda \)-fuzzy-measures.
The parameter \( \lambda \) is identified according to Equation (17), which was \(-0.5262\). The weights of all subsets of the power set of the criterion set can be calculated according to Equation (18), as shown in Table 5.
Table 5. Fuzzy measures.

| Sets  | Measures | Sets  | Measures | Sets  | Measures |
|-------|----------|-------|----------|-------|----------|
| 1     | 0.2103   | 2     | 0.4069   | 3     | 0.2119   |
| 4     | 0.2404   | 5     | 0.2405   | 12    | 0.5722   |
| 13    | 0.3988   | 14    | 0.4241   | 15    | 0.4242   |
| 23    | 0.5734   | 24    | 0.5958   | 25    | 0.5959   |
| 34    | 0.4255   | 35    | 0.4256   | 45    | 0.4504   |
| 123   | 0.7203   | 124   | 0.7402   | 125   | 0.7402   |
| 134   | 0.5888   | 135   | 0.5888   | 145   | 0.6109   |
| 234   | 0.7413   | 235   | 0.7413   | 245   | 0.7609   |
| 345   | 0.6121   | 1234  | 0.8696   | 1235  | 0.8696   |
| 1245  | 0.8870   | 1345  | 0.7547   | 2345  | 0.8880   |
| 12345 | 1.0000   |       |          |       |          |

where 1, 2, 3, 4 represent the sets of $x_{11}, x_{12}, x_{13}$ and $x_{14}$, respectively (Similarly hereinafter).

Step 5: Calculate the grey fuzzy correlation degree.

Step 5.1: Calculate correlation coefficients.

Select $\{C^+\} = \{0.5763, 0.9854, 1.3439, 1.3681, 1.4790\}$ as the positive ideal solution scheme and $\{C^-\} = \{-1.9837, -1.7970, -1.2968, -1.5178, -0.9849\}$ as the negative ideal solution scheme. According to Equations (21) and (22), the positive correlation matrix $\xi^+$ and the negative correlation matrix $\xi^-$ were found as follows.

$$\xi^+ = \begin{bmatrix}
0.9579 & 0.8924 & 0.6907 & 1.0000 & 1.0000 \\
0.3605 & 0.3415 & 0.4232 & 0.6035 & 0.6784 \\
0.9192 & 0.5424 & 0.3533 & 0.3333 & 0.3693 \\
0.6914 & 0.5803 & 1.0000 & 0.6383 & 0.5353 \\
1.0000 & 1.0000 & 0.4232 & 0.4391 & 0.3842 \\
0.9579 & 0.6747 & 0.6310 & 0.4572 & 0.4004
\end{bmatrix}$$

$$\xi^- = \begin{bmatrix}
0.3663 & 0.3562 & 0.4198 & 0.3333 & 0.3693 \\
1.0000 & 1.0000 & 0.6815 & 0.4268 & 0.4477 \\
0.3723 & 0.4797 & 1.0000 & 1.0000 & 1.0000 \\
0.4296 & 0.4535 & 0.3533 & 0.4110 & 0.5437 \\
0.3605 & 0.3415 & 0.6815 & 0.5806 & 0.9050 \\
0.3663 & 0.4088 & 0.4454 & 0.5517 & 0.8265
\end{bmatrix}$$

Step 5.2: Calculate grey fuzzy Choquet integral.

The positive and negative grey fuzzy integral vectors can be obtained by applying Equations (23) and (24) on the correlation matrix and the comprehensive weights vector, which, respectively, were $R^+ = [1.0000, 0.5314, 0.5824, 0.7486, 0.8085, 0.7103]$ and $R^- = [0.4665, 1.0000, 0.9881, 0.5708, 0.7381, 0.6792]$.

Step 6: Calculate fuzzy distance to ideal solution.

The positive and negative ideal solution distance vectors can be calculated by applying Equations (25), (26) on the standardized evaluation matrix and the comprehensive weights vector, which, respectively, were $D^+ = [0.1119, 0.7536, 0.7771, 0.3408, 1.0000, 0.9583]$ and $D^- = [0.4690, 0.3669, 0.4180, 0.4585, 1.0000, 0.9597]$.

Step 7: Calculate the comprehensive proximity degree.

Normalize $R^+, R^-, D^+, D^-$ according to Equation (27). Set the parameter $p$ as 0.5, which means the importance of shape and distance is the same. The positive and negative comprehensive proximity degree vectors can be calculated by applying Equation (28) on the grey fuzzy integral vector and the ideal solution distance vector, which, respectively, were $S^+ = [0.7345, 0.4491, 0.5002, 0.6036, 0.9043, 0.8350]$ and $S^- = [0.2892, 0.8768, 0.8826, 0.4585, 0.8691, 0.8188]$.

The final comprehensive evaluation vector can be obtained by aggregating the positive and negative comprehensive proximity degree vector according to Equation (29) which was $CS = [0.7175, 0.3387, 0.3617, 0.5697, 0.5099, 0.5049]$. The sequence of objects was $1 \succ 4 \succ 5 \succ 6 \succ 3 \succ 2$. 
Intuitively, Figure 6 shows that enterprise 1 performed very well on all criteria; while enterprise 2 performed very badly on all criteria. Enterprise 4 performed averagely on criteria 1, 2, 4, and 5 but performed excellently on criterion 3. Enterprises 5, 6, and 3 all performed averagely and enterprise 5 performed slightly better than enterprise 6, and enterprise 6 performed slightly better than enterprise 3 on most criteria. The sequence coincided with our intuitionistic judgment.

The bar chart of $R^+$, $R^-$, $D^+$, and $D^-$ is depicted in Figure 7. It shows that the values of $R^+$, $R^-$, $D^+$, and $D^-$ were not coincident. If we concentrate on different aspects, we can obtain different evaluation results.

5.2. Comparisons and Sensitivity Analysis

To compare the effects of the proposed method with the existing methods, the result of GCE and TOPSIS is given below.

By standardizing $w^0$, the weights of GCE can be obtained. The absolute correlation degree (ACD) can be calculated according to the weights and the positive correlation matrix. The ideal solution distance $D^+_\text{TOPSIS}$ and $D^-\text{TOPSIS}$ can be found by the standardized evaluation matrix. The relative closeness degree (RCD) can be obtained through $D^+_\text{TOPSIS}$ and $D^-\text{TOPSIS}$.

To carry out a sensitivity analysis of parameter $p$, a series of experiments was conducted and the value of $p$ was between 0 and 1, and the interval was 0.2. The result is shown in Table 6 and Figure 8.

5.3. Result Analysis

Table 6 and Figure 8 show that the sequence sorted according to GCE was always $1 \succ 5 \succ 4 \succ 6 \succ 3 \succ 2$ while the sequence sorted due to TOPSIS was always $1 \succ 4 \succ 6 \succ 5 \succ 3 \succ 2$. The result means that enterprise 4 is bad in terms of shape but good in terms of distance since it performed well on criterion 3 but badly on criteria 1 and 2. Enterprise 5 is bad in terms of distance, but good in terms of shape since it performed slightly better than average on most criteria, which is similar to enterprise 1.

The sequence sorted according to FGT is slightly related to parameter $p$. When $p = 0$, the sequence was $1 \succ 4 \succ 5 \succ 6 \succ 2 \succ 3$. When $p > 0$, the sequence was $1 \succ 4 \succ 5 \succ 6 \succ 3 \succ 2$. Enterprise 2 has slight advantages on distance, since it performed barely satisfactorily on criteria 4 and 5. However, its performance was instable. When the factor of shape is taken into consideration, enterprise 3 will be better than enterprise 2. Figure 8 clearly shows that the comprehensive evaluation value of enterprise 1 ($C_1$) obviously decreased, $C_4$ slightly decreased, $C_5$, $C_6$, and $C_7$ obviously increased, and $C_6$ slightly increased with the increase of parameter $p$, i.e., the increase consideration of shape.

The result shows that the proposed method can reflect both the difference of shape and distance of objects, which overcomes the one-sidedness of the GCE method and TOPSIS method.
which cannot take the interaction of criteria into consideration. In the view of this, a novel MCDM method that combines the GCE, TOPSIS, and fuzzy integral, called the fuzzy grey TOPSIS, was proposed to improve the deficiency mentioned above. The effectiveness of the proposed method was verified by the illustrative example of supplier evaluation of a collaborative manufacturing enterprise as well as some sensitivities.

| p   | A1  | A2  | A3  | A4  | A5  | A6  | Sequence   |
|-----|-----|-----|-----|-----|-----|-----|------------|
| CS  | 0   | 0.8073 | 0.3274 | 0.3498 | 0.5737 | 0.5000 | 0.5003 | 1,4,6,5,3,2 |
| ACD | 0   | 0.9097 | 0.4677 | 0.5022 | 0.6684 | 0.6907 | 0.6228 | 1,5,4,6,3,2 |
| RCD | 0   | 0.8929 | 0.3807 | 0.3769 | 0.6949 | 0.5286 | 0.5685 | 1,4,6,5,3,2 |
| CS  | 0.2 | 0.7588 | 0.3324 | 0.3550 | 0.5718 | 0.5037 | 0.5020 | 1,4,5,6,2,3 |
| ACD | 0.2 | 0.9097 | 0.4677 | 0.5022 | 0.6684 | 0.6907 | 0.6228 | 1,5,4,6,3,2 |
| RCD | 0.2 | 0.8929 | 0.3807 | 0.3769 | 0.6949 | 0.5286 | 0.5685 | 1,4,6,5,3,2 |
| CS  | 0.4 | 0.7286 | 0.3368 | 0.3596 | 0.5704 | 0.5077 | 0.5039 | 1,4,5,6,3,2 |
| ACD | 0.4 | 0.9098 | 0.4677 | 0.5022 | 0.6684 | 0.6907 | 0.6228 | 1,5,4,6,3,2 |
| RCD | 0.4 | 0.8929 | 0.3807 | 0.3769 | 0.6949 | 0.5286 | 0.5685 | 1,4,6,5,3,2 |
| CS  | 0.6 | 0.7081 | 0.3406 | 0.364  | 0.5692 | 0.5122 | 0.5060 | 1,4,5,6,3,2 |
| ACD | 0.6 | 0.9098 | 0.4677 | 0.5022 | 0.6684 | 0.6907 | 0.6228 | 1,5,4,6,3,2 |
| RCD | 0.6 | 0.8929 | 0.3807 | 0.3769 | 0.6949 | 0.5286 | 0.5685 | 1,4,6,5,3,2 |
| CS  | 0.8 | 0.6932 | 0.3440 | 0.3675 | 0.5682 | 0.5172 | 0.5084 | 1,4,5,6,3,2 |
| ACD | 0.8 | 0.9098 | 0.4677 | 0.5022 | 0.6684 | 0.6907 | 0.6228 | 1,5,4,6,3,2 |
| RCD | 0.8 | 0.8929 | 0.3807 | 0.3769 | 0.6949 | 0.5286 | 0.5685 | 1,4,6,5,3,2 |
| CS  | 1   | 0.6819 | 0.3470 | 0.3708 | 0.5674 | 0.5228 | 0.5112 | 1,4,5,6,3,2 |
| ACD | 1   | 0.9098 | 0.4677 | 0.5022 | 0.6684 | 0.6907 | 0.6228 | 1,5,4,6,3,2 |
| RCD | 1   | 0.8929 | 0.3807 | 0.3769 | 0.6949 | 0.5286 | 0.5685 | 1,4,6,5,3,2 |

Figure 8. The result of sensitivity analysis.

6. Conclusions

Generally speaking, the MCDM problems are essential in modern society for businesses and private individuals. The existing MCDM methods, such as GCE and TOPSIS, have some disadvantages which cannot take the interaction of criteria into consideration. In the view of this, a novel MCDM method that combines the GCE, TOPSIS, and fuzzy integral, called the fuzzy grey TOPSIS, was proposed to improve the deficiency mentioned above. The effectiveness of the proposed method was verified by the illustrative example of supplier evaluation of a collaborative manufacturing enterprise as well as some sensitivities.

There are some recommendations for future studies. One idea is to focus on other types of fuzzy numbers and correlation coefficients and even redefine them. We will compare them by conducting more experiments and refine the proposed method [85–87]. As such, hybridizing the proposed MCDM method with recent advances in heuristics and metaheuristics is another good continuation of this work [88–93].

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Notations and Nomenclature:

\[ A = \{a_1, a_2, \ldots, a_m\} \]
\[ X = \{x_1, x_2, \ldots, x_n\} \]
\[ \tilde{J}_k \]
\[ M^1, M^2, M^3 \]
\[ J^* = \begin{bmatrix} j_1^* & \cdots & j_n^* \\ \vdots & \ddots & \vdots \\ j_m^* & \cdots & j_{mn}^* \end{bmatrix} \]
\[ \tilde{J}_k \]
\[ J = \begin{bmatrix} j_{11} & \cdots & j_{1n} \\ \vdots & \ddots & \vdots \\ j_{m1} & \cdots & j_{mn} \end{bmatrix} \]
\[ c^+_{ik} \]
\[ C^+ = \begin{bmatrix} c^+_{11} & \cdots & c^+_{1n} \\ \vdots & \ddots & \vdots \\ c^+_{m1} & \cdots & c^+_{mn} \end{bmatrix} \]
\[ C^- = \begin{bmatrix} c^-_{11} & \cdots & c^-_{1n} \\ \vdots & \ddots & \vdots \\ c^-_{m1} & \cdots & c^-_{mn} \end{bmatrix} \]
\[ \xi^+ = \begin{bmatrix} \xi^+_{11} & \cdots & \xi^+_{1n} \\ \vdots & \ddots & \vdots \\ \xi^+_{m1} & \cdots & \xi^+_{mn} \end{bmatrix} \]
\[ \xi^- = \begin{bmatrix} \xi^-_{11} & \cdots & \xi^-_{1n} \\ \vdots & \ddots & \vdots \\ \xi^-_{m1} & \cdots & \xi^-_{mn} \end{bmatrix} \]
\[ \omega^1 \]
\[ \omega^2 \]
\[ \lambda \]
\[ R^- = \begin{bmatrix} R^-_1, R^-_2, \ldots, R^-_n \end{bmatrix} \]
\[ S^+ = \begin{bmatrix} S^+_1, S^+_2, \ldots, S^+_m \end{bmatrix} \]
\[ S^- = \begin{bmatrix} S^-_1, S^-_2, \ldots, S^-_m \end{bmatrix} \]
\[ CS = \begin{bmatrix} CS_1, CS_2, \ldots, CS_1 \end{bmatrix} \]

The objects set.
The criterion set.
The evaluation value of the \( i \)-th object on the \( k \)-th qualitative criterion in the form of fuzzy number, \( i \in A, k \in \{x_1, x_2, \ldots\} \) is a qualitative criterion. For the convenience, \( j_{ik} \) is presented in a simplified form as \( \tilde{J}_k \).

The numerical results of defuzzification of \( \tilde{J}_k \) through methods 1, 2, and 3.

Numerical evaluation matrix of qualitative criteria.
The evaluation value of the \( i \)-th object on the \( k \)-th criterion, \( i = 1, 2, \ldots, m, k = 1, 2, \ldots, n \).

Evaluation matrix.
The standardized evaluation value of the \( i \)-th object on the \( k \)-th criterion, \( i = 1, 2, \ldots, m, k = 1, 2, \ldots, n \).

Standardized evaluation matrix.
Positive ideal solution scheme.
Negative ideal solution scheme.
Positive correlation matrix.
Negative correlation matrix.
Objective weights vector.
Subjective weights vector.
Comprehensive weights vector.
Parameter of \( \lambda \)-fuzzy-measures.
The fuzzy measure of the subset \( Q \) of power set of \( X \). For example, represents a set consisting of \( x_1, x_3, \) and \( x_5 \). For convenience, \( \omega_{1335} \) is presented in a simplified form as \( \omega_{1335} \).

Positive grey fuzzy integral vector.
Negative grey fuzzy integral vector.
Positive fuzzy ideal solution distance.
Negative fuzzy ideal solution distance.

Positive comprehensive proximity degree vector.
Negative comprehensive proximity degree vector.
Comprehensive evaluation vector.
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