Experimental analysis of drop-size density distribution during dropwise condensation of steam

M Mirafiori1, R Parin1, S Bortolin1 and D Del Col1*

1University of Padova, Department of Industrial Engineering
Via Venezia 1, 35131 Padova, Italy
*Corresponding author e-mail: davide.delcol@unipd.it

Abstract. Dropwise condensation (DWC) has attracted research interest since its first description in 1930 due to the high heat transfer coefficients that can be obtained by promoting this mechanism as compared to the case of filmwise condensation. The modelling of dropwise condensation heat transfer is complex since it must account for droplets nucleation, growth and removal: researchers proposed different methods and the first semi-empirical model was presented in 1966 by Le Fevre and Rose. Their model predicts the total heat transfer across a surface during DWC by estimating the drop-size density distribution and the heat transfer through a single droplet. The goal of the present work is to investigate the large drop-size density distribution during dropwise condensation considering different operating conditions (heat flux and surface coating). In this work, a new illumination system (torus-shaped LED) has been designed and built to allow droplets detection with diameters from millimetres down to few microns. The torus-shaped LED is used as light source for a high-speed camera coupled with microscope lens. The recorded images have been analysed by a homemade software in MATLAB® code, and the measured droplet population has been compared with the theoretical formula proposed by Le Fevre and Rose [1]. More than six million droplets have been detected and an excellent agreement with the Le Fevre and Rose theory has been found.

1. Introduction

Condensation of steam is a phase-change process involved in several industrial applications including energy production, sea water desalinization and environmental control. A vapor can condense on a surface in two different ways: dropwise condensation (DWC) mode or filmwise condensation (FWC) mode. During FWC, the condensate forms a continuous liquid film over the cooled surface which results in a heat transfer resistance. On the contrary, during DWC, the condensate forms discrete droplets that are swept away leading to a reduction of the conduction thermal resistance through the liquid phase. The heat transfer coefficient (HTC) measured during DWC of steam is typically 5-7 times higher than the one of FWC [2]. In order to promote DWC on metallic surfaces (having in general high surface energy), the deposition of coatings with low wettability (hydrophobic coatings) is a viable solution. The wettability can be characterized by measuring the advancing \( \theta_a \) and the receding \( \theta_r \) contact angle. Since the discovery of DWC mechanism by Schmidt et al. [3] in 1930, researchers tried to describe this heat transfer mode. The first semi-empirical model was proposed by Le Fevre and Rose [1] in 1966 and after that many researchers try to describe the phenomena that take place during dropwise condensation: the nucleation of a droplet until its departure, the heat exchanged by a single drop during its lifetime and the droplet population on the surface. In particular, the droplet population is subdivided in small droplet population, where the dominant growth mechanism is direct condensation and large droplet population, where the dominant growth mechanism is coalescence. In 1966, Le Fevre and Rose [1] introduced the
2. Drop-size density distribution

The power-law equation proposed by Le Fevre and Rose is here reported:

\[ N(r) = \frac{1}{3\pi r_{\text{max}}^{1/3}} r^{-8/3} \]  

(1)

The model assumes that the distribution does depend only on the maximum drop radius \( r_{\text{max}} \). The maximum drop radius is the result of a force balance between the adhesion force and the gravity force, plus some other forces which promote the droplet movement. The formulation for the maximum radius proposed by Le Fevre and Rose is the following:

\[ r_{\text{max}} = K_3 \left( \frac{\sigma}{\rho g} \right)^{1/2} \]  

(2)

where \( K_3 \) is a constant [2], \( \sigma \) is the liquid-vapor surface tension, \( g \) is the gravity acceleration, and \( \rho \) is the density of the condensate. It must be pointed out that Eq. 2 does not consider the surface wettability. Different formulas for the maximum droplet radius have been proposed in the literature; hereinafter the equation proposed by Kim and Kim [5] is considered:

\[ r_{\text{max}} = \left( \frac{6c(\cos \theta_r - \cos \theta_a) \sin \theta \sigma}{\pi(2 - 3 \cos \theta + \cos^2 \theta) \rho g} \right)^{1/2} \]  

(3)

where \( \theta_r \) and \( \theta_a \) are respectively the receding contact angle and the advancing contact angle, whereas \( \theta \) is the angle at which the drops growth, and \( c \) is a numerical constant that depends on the shape of the drop and on the steepness of the substrate surface. Assuming \( r_{\text{max}} = 1.2 \) mm, the drop-size density function \( N(r) \) is described in Fig. 1 by the red line. If the departing radius is changed from 1 mm to 1.4 mm, the big droplet distribution variation remains well below 20% with respect to the \( N(r) \) distribution calculated considering \( r_{\text{max}} = 1.2 \) mm. Therefore, to obtain an appreciable effect on the droplet population \( N(r) \), the maximum drop radius \( r_{\text{max}} \) must change significantly.

The droplet population density is necessary to calculate the heat flux \( q \). Knowing the heat flow rate \( Q_d(r) \) exchanged through a single drop and the drop-size density distribution \( N(r) \), the heat flux \( q \) transferred during DWC can be calculated as follows:

\[ q = \int_{r_{\text{min}}}^{r_{\text{max}}} Q_d(r)N(r)dr \]  

(4)

Eq. 4 does not consider the small droplet population, which was introduced by Tanaka [6] through the population balance theory and it leads to the new formulation of total heat flux as:
\[ q = \int_{r_{\min}}^{r_e} Q_d(r)n(r)dr + \int_{r_e}^{r_{\max}} Q_d(r)N(r)dr \]  

(5)

where \( n(r) \) is the drop-size distribution for small droplets. The most recent models by Kim and Kim [5] (2011) and by Miljkovic [7] (2013) use the formulation proposed by Tanaka [6].

![Figure 1](image1.png)

**Figure 1.** (a) Drop-size density distribution \( N(r) \) calculated with the equation proposed by Le Fevre and Rose [1]. The effect of the maximum droplet radius (\( r_{\max} \) ranges between 1 mm and 1.4 mm) on the droplet population is reported. (b) Enlarged portion of the drop-size density distribution showing the ±20% deviation band.

### 3. Video analysis

Videos have been recorded by a high-speed camera (Photron FASTCAM UX100) directly connected to the test rig to reduce the asynchronous vibrations between the experimental apparatus and the camera. In this work, two different illumination techniques have been employed: a square-shaped high intensity single LED (Fig. 2) and a torus-shaped home-made LEDs array. Considering the single LED, there is a displacement between the center of the light source and the center of the camera lens leading to an illumination of the sample that is not symmetric. Instead, in the second case, lens can be accommodated in correspondence of the center of the torus and this guarantee an axisymmetric illumination of the field of view. When using the single-LED, the high speed camera is coupled with macro-lens and this configuration allows to detect only droplets with radii higher than 200 μm. To overcome this limitation, a new toroidal illumination system has been developed according to [8].

![Figure 2](image2.png)

**Figure 2.** Picture of the single high intensity LED used for DWC visualizations.

![Figure 3](image3.png)

**Figure 3.** The torus-shaped LED. The plastic base is 3D printed to obtain different inclinations for the two rows of LEDs.
The torus-shaped LED guarantees a more uniform illumination of the droplet contour (which is essential for precise droplet dimensions detection) and it can be also coupled with microscope camera lens allowing a considerable extension of the droplet radius detection that, in this configuration, ranges from 600 μm down to 15 μm.

The torus-shaped light has been designed considering the dimensions of the DWC test section [9] and it is made of two rows of LEDs for a total of 32 LEDs. External and internal rows have two different inclinations to focus the light of both LEDs rows on the droplets. The torus-shape light projects its torus pattern onto each drop which is proportional to the drop’s radius itself. An example of the resulting illumination is showed in Fig. 4a.

Depending on the type of the illumination system, a specific MATLAB® program has been developed for images processing. In the case of the torus-shaped LED, the main steps of the algorithm are below reported.

- As a first step, the original image is loaded in a graphical user interface (GUI) for evaluating a characteristic area in which the torus pattern is clearly projected on the drops (Fig. 4a).
- The captured image is binarized with a particular threshold value of brightness to make the torus-shaped pattern detectable.
- An algorithm finds closed circles in correspondence of each droplet in the image.
- The detected circles do not represent exactly the droplets dimensions since the reflection inside the droplets is not attached to the droplets’ contour. A non-linear interpolation was performed to evaluate the real droplet radius as follows:

\[
r_{\text{real}} = k \cdot (r_{\text{found}})^2
\]  

where \(r_{\text{real}}\) is the real droplet radius obtained during calibration by manually detecting the actual droplet contour, \(k\) is a constant evaluated in order to obtain the best fit with real droplet radius, and the \(r_{\text{found}}\) is the radius detected by the program. The relationship between the two radii is not linear because the torus-shaped pattern reflected on drops depends on droplets size. An example of the reconstructed image is reported in Fig. 4b.

![Figure 4](image)

**Figure 4.** a) Original image obtained by the high-speed camera coupled with torus-shaped LED (6.2 x 6.2 mm selected area). b) Reconstructed image in the selected area. c) Original image obtained with the single LED illumination system (investigated area 14.2 x 14.2 mm). d) Reconstructed image obtained with the MATLAB® program.
When using the single LED illumination system, the main steps of the algorithm are the following.

- The original image is loaded into a GUI and cropped to the desired size (Fig. 4c).
- The droplets are detected by subtracting the background from the original image. Once the droplets are extracted, the image is binarized and processed to highlight the real drops contour.
- The real droplets dimensions are evaluated by an algorithm which provides positions and dimensions of connected white pixels clusters. An example of the reconstructed image is shown in Fig. 4d.

As shown in Fig. 4, the adoption of the torus-shaped LED allows a significative improvement of the droplets detection capabilities of the optical system. In fact, comparing the reconstructed images in Figs. 4b and 4d, it can be observed that, even if the selected area is reduced in the case of the torus-shaped LED (6.2 mm x 6.2 mm), smaller droplets are detected. Furthermore, comparing Fig. 4a and Fig. 4b, it can be noticed that the single LED provides an illumination that is not uniform for each droplet. As a result, with the simple LED only droplets with a diameter larger than 0.6 mm can be correctly reconstructed.

4. Experimental results and discussion

The experimental apparatus is a thermostyphon loop where steam is used as operative fluid. The system consists of four main components (boiling chamber, test section, cooling water loop and post-condenser) and it allows simultaneous visualization and thermal measurements of the dropwise condensation phenomenon. A detailed description of the experimental apparatus, the data reduction technique and the uncertainty analysis can be found in [9] and [10]. All the tests have been performed at around 105 °C saturation temperature, vapor velocity equal to 2.7 m s$^{-1}$ and varying the heat flux.

4.1. Experimental data set

Experimental data have been obtained on four aluminium samples treated with different hydrophobic layers. Table 1 reports advancing and receding contact angles, heat flux range, maximum departing radius measured during tests, and maximum departing radius calculated by Eqs. 2-3.

| Coating  | $\theta_a$ [deg] | $\theta_r$ [deg] | $q$ [kW m$^{-2}$] | $r_{\text{max}}$ [mm] | $r_{\text{max,Rose}}$ [mm] | $r_{\text{max,Kim}}$ [mm] |
|---------|------------------|------------------|-------------------|------------------------|---------------------------|-------------------------|
| Coating 1 | 89 ± 1 | 64 ± 3 | 167 - 509 | 1.04 ± 0.08 | 1.00 | 1.05 ± 0.07 |
| Coating 2 | 83 ± 3 | 57 ± 6 | 144 - 544 | 1.30 ± 0.08 | 1.00 | 1.23 ± 0.16 |
| Coating 3 | 77 ± 4 | 45 ± 5 | 89 - 346 | 1.44 ± 0.12 | 1.00 | 1.59 ± 0.19 |
| Coating 4 | 76 ± 5 | 53 ± 11 | 111 - 464 | 1.23 ± 0.10 | 1.00 | 1.35 ± 0.36 |

The droplet departing radius is the result of a force balance between gravity, vapor drag force and adhesion force whereas it is not affected by the heat flux. The measurement of the droplet departing radius is not an easy task. In fact, during DWC, droplets can reach their maximum radius in different ways: a smoothly growing drop increases its size coalescing with very tiny droplets and then it slowly starts to sweep; alternatively, a droplet can be abruptly swept way when it coalesces with neighbour droplets of the same size (Fig. 5). The procedure to identify the droplet departure radius is illustrated in Fig. 5. Video frames refer to a time interval of 5 ms. At $t = 0$, a droplet is detected (yellow circle). The droplet is growing by coalescence but its center is not yet moving. At $t = 15$ ms, after coalescing with a neighbour big droplet, it suddenly starts to move: at this time the diameter is measured.
4.2. Drop-size density distribution obtained by image analysis

During the experimental campaign, droplet population has been investigated considering all the coatings listed in Table 1 and different values of heat flux. In particular, coatings #1 and #2 have been investigated with the single powerful LED and the high-speed camera was coupled with a macro lens. Due to the asymmetrical illumination of the sample and to the use of macro lens, detectable droplets are limited to larger diameters as compared to the results obtained with the torus-shaped light. Once the video apparatus has been upgraded as reported in Section 3 (torus-shaped LED and microscope lens) the measuring range has been significantly expanded (Fig. 6). In particular, coatings #3 and #4 have been investigating with the new optical system.

In Table 2, experimental data have been interpolated with a power-law equation following the Le Fevre and Rose formula [1]:

\[ N(r) = A \cdot r^B \] (7)

where the coefficient \( A \) corresponds to the constant term in Eq. 1 \( (3\pi r_{max}^{1/3})^{-1} \) and the coefficient \( B \) corresponds to the exponent of the diameter in Eq. 1. The coefficients \( A \) and \( B \) resulting from the interpolation are reported, together with their uncertainty, in Table 2. The uncertainty on the coefficients \( A \) and \( B \) has been calculated following the method reported in [11]. The coefficients of the interpolation curve are very close to the values calculated from the theory proposed by Le Fevre and Rose [1] (Eq.1).

**Table 2.** Interpolation coefficients of the power-law equation (7) calculated from the experimental data considering the four different coatings.

| Coating   | \( A \) \( m^{-1/3} \) | \( B \) | [\( - \) ] |
|-----------|------------------------|------|---------|
| Coating #1| 1.08 ± 0.39            | -2.703 ± 0.056 |
| Coating #2| 1.53 ± 1.12            | -2.758 ± 0.105 |
| Coating #3| 0.88 ± 0.23            | -2.599 ± 0.061 |
| Coating #4| 0.60 ± 0.09            | -2.597 ± 0.034 |
| Overall data | 1.00 ± 0.14            | -2.681 ± 0.027 |

The results of the droplet population analysis are shown in Fig 6. The data, in total around six million droplets, are grouped according to the type of coating and, for each coating, different heat fluxes have been considered. Although both the optical systems lead to a good agreement with the theory proposed by Le Fevre and Rose [1], the torus LED allows to extend the droplet analysis from around 250 μm to 15 μm droplet radius. The experimental uncertainty of the measured drop-size density distribution \( N_r \),
has been estimated by Monte Carlo simulations [12]: the mean uncertainty is lower than 5%. In Fig. 6, the experimental data are also compared with the theoretical distribution by Le Fevre and Rose [1].

The relative deviations between the experimental distribution and the theoretical distribution proposed by Le Fevre and Rose [1] have been calculated with Eq. 8:

\[
e = \left| \frac{N(r)_{\text{exp}} - N(r)_{\text{theory}}}{N(r)_{\text{theory}}} \right| \cdot 100
\]  

(8)

For each coating, the theoretical drop-size distribution has been calculated with three different departing radii: the measured maximum drop radius, the calculated radius with Eq. 2 proposed by Rose [2] and the radius calculated considering Eq. 3 proposed by Kim and Kim [5]. The results are reported in Table 3 considering the arithmetical mean of the relative deviations \(e\) calculated for each coating.

**Table 3.** Relative deviations between the experimental drop-size density distribution and the theoretical distribution proposed by Le Fevre and Rose [1] considering three different departure radii (measured maximum drop radius, calculated radius with Eq. 2, and calculated radius with Eq. 3).

| Coating #1 | Coating #2 | Coating #3 | Coating #4 | Overall data |
|------------|------------|------------|------------|--------------|
| \(e_{r,\text{max}}\) [%] | \(e_{r,\text{max,Rose}}\) [%] | \(e_{r,\text{max,\text{Kim}}}\) [%] | \(e_{r,\text{max,\text{Kim}}}\) [%] | \(e_{r,\text{max,\text{Kim}}}\) [%] |
| 17.86 | 18.89 | 17.56 | 12.59 | 19.48 | 13.76 | 32.88 | 24.81 | 35.94 | 19.42 | 23.71 | 17.84 | 18.61 | 21.72 | 18.15 |

As reported in Table 1, the equation proposed by Kim and Kim [5] for the maximum drop radius is able to well describe the surface behaviour. For this reason, in Table 3, the lower deviation is obtained when the formula proposed by Kim and Kim [5] is used to determine the droplet departing radius in the drop-size distribution function.

In must be mentioned that the wetting properties of the tested coatings do not allow to extend the maximum departing radius analysis to a large range and thus to show a more evident effect of the maximum radius on the drop-size distribution (Fig. 1). Furthermore, as reported in Section 2, some authors introduced the small droplet population where the droplets growing mechanism is by direct
vapor condensation and not by coalescence. In the present work, the minimum measurable radius is around 15 μm (Fig. 6) and droplets are still growing by coalescence (as can be inferred from videos). Therefore only the big droplet population can be observed. To estimate the threshold radius $r_e$ between the small and big droplet population, the nucleation site density should be known [7]. In the present case, since $N_s$ cannot be directly measured, supposing a random nucleation sites distribution [7], it can be stated that there are more than $10^9$ nucleation sites per square meter at the present operating conditions and thus the radius $r_e$ is around 5 μm, lower than the minimum measurable radius.

5. Conclusions
In the present work, drop-size density distribution during dropwise condensation of steam has been investigated over an aluminium vertical sample. The wettability of the surface has been modified using four different coatings. Tests have been performed inside a thermosyphon loop at a fixed value of vapor velocity (2.7 m s$^{-1}$) and varying the heat flux. A new optical system based on a torus-shaped LED has been designed and built. The optical system is able to detect droplets with radius ranging between 1 mm and 15 μm. The results show a good agreement between experimental data and the equation for the large size droplet population (Eq. 1) proposed by Le Fèvre and Rose [1], in which the maximum droplet radius has been calculated with the model proposed by Kim and Kim [5]. It has been found that, for all the four investigated coatings, the drop-size density distribution is not affected by the heat flux and the influence of the surface wettability on the droplet maximum radius can be taken into account using the equation proposed by Kim and Kim [5].

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