Computation of fast linear equations convergent iterations solutions on the Jacobi and Gauss–Seidel methods

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Abstract .This paper present Gauss-Seidel and Jacobi class repetition systems be situated, planned with answering an equations of $AXB = C$ liner produced the cracked organizations of $A$ then $B$. Conjunction plus computational amount with these iterative approaches stay deliberate. In addition, are current preliminaries methods for Gauss-Seidel and Jacobi iterations. Numerical samples are assumed to establish the effectiveness of these approaches and suggested in the example for each. using MATLAB, by explain the schedule. We also show that using iterations of the Jacobi blocks, we can expand the range of tasks for which this approach can be effective. For iterations of Jacobi blocks, it is important, that to knows kind the matrix structure for choosing the best method with accurate fast and short result.

Keywords: linear equations , iterations,  Gauss _Seidel,  Jacobi, convergent .

1. Introduction

In almost college departments of computational mathematics, global physics, natural sciences, etc., it combines to create linear institutions of procedure equations.

$Ax = b, \text{ s.t, } A \in \mathbb{C}(N\times N), x \in \mathbb{C}N\times 1 \text{ and } b \in \mathbb{C}N\times 1,$

Where $\mathbb{C}$ is a set of multi-faceted measures. For simple systems, direct interpretation strategies are preferred, despite their length. However, the Jacobi-Davidson restart technique uses a similar number of tactical convergence; [1,2]. Such as the size of organization grows, it develops necessary to use repetitive methods to look for professional solutions [3]. The modest fixed-point roads to be enhanced for this resolution include Jacobi and Gauss Seidel, respectively, with linear equations and relaxation. [4] Despite this, they suffer from fairly large areas and poor performance with body size. We use different ways to solve the results.

The following repetitive methods were used in limited linear equations: Jacoby's methods [5-9]. And the styles of Gauss Seidel [11,12]. Respectively more relaxation methods [10]. Multigrain style. , Several algorithms to explain the matrix, [16,17]. At the same time we use the methods of Gauss-Seidel and Jacobi repetition [13]. In addition, we offer advance routes such as Gauss-Seidel and Jacobi in [14], this different red-top is prearranged as shadows, one example solved on Gauss Seidel ,we give
the repetition approaches in addition to their meeting possessions, and the consistent procedures are obtainable [15]. With Jacobi’s modification methods with a second example that can be obtained. It can completely reduce computing power and elegance completely and fundamentally related to the types that will be collected very early in Gauss-Seidel. Numerous numerical examples will be presented with solutions demonstrating the effectiveness from repetitive Gauss-Seidel and Jacobi methods. For two main methods from Jacobi and Gauss-Seidel [13], correspondingly.

Direct method discussed upon the previous state pose some problem when the system grow large or when most of the coefficients are zero. They require prohibitively large number of floating point operations and, therefore, not tall simply become time overwhelming but also severely affect inaccuracy of the solution owing near around off errors. In such cases, iterative method provides an alternative in[18,19]. For instance, ill-conditioned systems container to resolved through iterative ways without facing the problem of round off error in details Gauss-Seidel method, the objective of this exertion to choose best iterative technique for solve linear systems with Gauss-Seidel and Jacobi methods study important topic is Jacobi method by using mat lab. [20]. sketch the methods.

2. Mathematical Experiments

In this sector, we will look at double different arithmetical examples of this document. The behavior from an iterative process. Note that we will consider a simple question: problems where the exact solution of the methods is known, so that this solution can be compared with numerical approximation. Now the first part, we check the Gauss algorithm performance when the change. At the second part, we look at Jacobi in examples. In the same sequence, we analyze comparisons with precision and steps. To explain both obstacle problem through repetition with Seidel Gauss and Jacobi.

2. 1 Gassing-Seidel method

In arithmetic on linear algebra, the Gauss-Seidel system, similarly recognized as the Lineman method, the technique for sequential movement and other numerical methods, is an iterative process improved with explain the lined organization of calculations. Titled later of most imperative German’s arithmeticians Karl, Gauss and Philip with Seidel then other mathematicians, it comparable with Jacobi process. Although its useful to some matrix by way of non-zero rudiments in distances, coming together is only guaranteed if the matrix is either leading or quantitatively or positively correlated.

2. 2 Jacobi method

In mathematical with lines algebraic, Jacobi process (Jacobi repetitive technique) is a procedure to decisive results in italics. For each diagonal component, the entire procedure is not repeated, but converges. This process is an abstract procedure for the way of changing to Jacobi cross-matrix. This method was called Jacob.

3. Results and Discussion

3. 1 numerical results with Gauss-Seidel method and discussion

Trigonometric systems are difficult with solve effectively on computers with high levels synchronization. in these computers, also Jacobi and gauss Seidel complexes have recently been proposed, which are used as very parallel, To solve near quadruple systems of equations processes. The effectiveness of this However, the approach depends on the problem, Jacobi’s frequencies may not always come close enough For all problems. Thus, as a necessary and important step to evaluate this approach will show with solve of systems.

Example1: we consider the liner equations
10x - 2y - z - h = 3
-2x + 10y - z - h = 15
-x - y + 10z - 2h = 2
-x - y - 2z + 10h = -9

Solution 1: by Gauss-Siedel method

\[
\begin{align*}
1 - x &= 0.3 + 0.2y + 0.1z + 0.1h \\
2 - y &= 1.5 + 0.2x + 0.1z + 0.1h \\
3 - z &= 2.7 + 0.1x + 0.1y + 0.2h \\
4 - h &= -0.9 + 0.1x + 0.1y + 0.2z
\end{align*}
\]

Where Initial approximation is

Putting \( x = 0, y = 0, z = 0, h = 0 \).

First Estimate

Putting \( y = 0, z = 0, h = 0 \) \hspace{1cm} \text{in the equation (1) } x = 0.3

Putting \( x = 0.3, z = 0, h = 0 \) \hspace{1cm} \text{in the equation (2) } y = 1.56

Putting \( x = 0.3, y = 1.56, h = 0 \) \hspace{1cm} \text{in the equation (3) } z = 2.886

Putting \( x = 0.3, y = 1.56, z = 2.886 \) \hspace{1cm} \text{in the equation (4) } h = -0.1368

Second Estimate

Putting \( y = 1.56, z = 2.886, h = -0.1360 \) \hspace{1cm} \text{in the equation (1) } x = 0.8369

Putting \( x = 0.8869, z = 2.886, h = -0.1360 \) \hspace{1cm} \text{in the equation (2) } y = 1.9523

Putting \( x = 0.8869, y = 1.9523, h = -0.1360 \) \hspace{1cm} \text{in the equation (3) } z = 2.9566

Putting \( x = 0.8869, y = 1.9523, z = 2.9566 \) \hspace{1cm} \text{in the equation (4) } h = -0.0248

Third Estimate

Putting \( y = 1.9523, z = 2.9566, h = -0.0248 \) \hspace{1cm} \text{in the equation (1) } x = 0.9836

Putting \( x = 0.9836, z = 2.9566, h = -0.0248 \) \hspace{1cm} \text{in the equation (2) } y = 1.9899

Putting \( x = 0.9836, y = 1.9899, h = -0.0248 \) \hspace{1cm} \text{in the equation (3) } z = 2.9924
Putting $x = 0.9836, y = 1.9899, z = 2.9924 \quad \text{in the equation (1)} h = -0.0042$

**Fourth Estimate**

Putting $y = 1.9899, z = 2.9924, h = -0.0042 \quad \text{in the equation (1)} x = 0.9966$

Putting $x = 0.9966, z = 2.9924, h = -0.0042 \quad \text{in the equation (2)} y = 1.9982$

Putting $x = 0.9966, y = 1.9982, h = -0.0042 \quad \text{in the equation (3)} z = 2.9986$

Putting $x = 0.9966, y = 1.9982, z = 2.9986 \quad \text{in the equation (4)} h = -0.0008$

**Fifth Estimate**

Putting $y = 1.9982, z = 2.9986, h = -0.0008 \quad \text{in the equation (1)} x = 0.9994$

Putting $x = 0.9994, z = 2.9986, h = -0.0008 \quad \text{in the equation (2)} y = 1.9997$

Putting $x = 0.9994, y = 1.9997, h = -0.0008 \quad \text{in the equation (3)} z = 2.9998$

Putting $x = 0.9994, y = 1.9997, z = 2.9998 \quad \text{in the equation (4)} h = -0.0001$

**Sixth Estimate**

Putting $y = 1.9997, z = 2.9998, h = -0.0001 \quad \text{in the equation (1)} x = 0.9999$

Putting $x = 0.9999, z = 2.9998, h = -0.0001 \quad \text{in the equation (2)} y = 1.9999$

Putting $x = 0.9999, y = 1.9999, h = -0.0001 \quad \text{in the equation (3)} z = 3.0$

Putting $x = 0.9999, y = 1.9999, z = 3.0 \quad \text{in the equation (4)} h = 0.0000$

**Seventh Estimate**

Putting $y = 1.9999, z = 3.0, h = 0 \quad \text{in the equation (1)} x = 1.0$

Putting $x = 1.0, z = 3.0, h = 0 \quad \text{in the equation (2)} y = 2.0$

Putting $x = 1.0, y = 2.0, h = 0 \quad \text{in the equation (3)} z = 3.0$

Putting $x = 1.0, y = 2.0, z = 3.0 \quad \text{in the equation (4)} h = 0.0$

**Table 1:** The result of numerical approximate with gauss Seidel method in example 1.

| N | X     | Y     | Z     | H     |
|---|-------|-------|-------|-------|
| 1 | 0.3   | 1.56  | 2.886 | -0.1368 |
| 2 | 0.8869| 1.9523| 2.9566| -0.0248 |
| 3 | 0.9836| 1.9899| 2.9924| -0.0042 |
| 4 | 0.9968| 1.9982| 2.9987| -0.0008 |
Dissuasion of gauss Seidel results. Note the values of the Gauss solution in the first step are illustrated with different results for all the variables, the second step of the values of x, y converge of the solutions added to the values of z, h. From the second step to the fifth step, all values began to converge more, and in the sixth step, the results of a very close approximation for the final solution in the last stage of the solution appeared, although h, z, had obtained convergence in the fifth stage of the solutions.

3.2. Numerical results with Jacobi method and dissension:

Using high resolution triangulation solutions for its Jacobi method requires execution in seconds compared to the use of the Jacobi scan, and the acceleration factor relative to the Gaussian method used. Note that. This runtime does not include this time, which can be great for accurate triangle solutions as a level of breed, scheduling data structures in general a sequential process. As a solve of both methods, the behavior here differs from the Gauss observed in the past such as the results will be here using Jacobi method of solution.

Example2: solve the liner system using Jacobi numerical method

\[ \begin{align*}
1 - x &= 0.3 + 0.2y + 0.1z + 0.1h \\
2 - y &= 1.5 + 0.2x + 0.1z + 0.1h \\
3 - z &= 2.7 + 0.1x + 0.1y + 0.2h \\
4 - h &= -0.9 + 0.1x + 0.1y + 0.2z
\end{align*} \]

With Initial approximation

Putting \( x = 0, y = 0, z = 0, h = 0 \)

First Estimate

\[\begin{align*}
\text{Putting } y &= 0, z = 0, h = 0 \quad \text{in the equation (1) } x = 0.3 \\
\text{Putting } x &= 0, z = 0, h = 0 \quad \text{in the equation (2) } y = 1.5 \\
\text{Putting } x &= 0, y = 0, h = 0 \quad \text{in the equation (3) } z = 2.7 \\
\text{Putting } x &= 0, y = 0, z = 0 \quad \text{in the equation (4) } h = -0.9
\end{align*}\]

Second Estimate

\[\begin{align*}
\text{Putting } y &= 0, z = 0, h = 0 \quad \text{in the equation (1) } x = 0.78 \\
\text{Putting } x &= 0.3, z = 2.7, h = -0.9 \quad \text{in the equation (2) } y = 1.74 \\
\text{Putting } x &= 0.3, y = 1.5, h = -0.9 \quad \text{in the equation (3) } z = 2.7
\end{align*}\]
Putting $x = 0.3, y = 1.5, z = 2.7$ in the equation (4) $h = -0.18$

Third Estimate

Putting $y = 1.74, z = 2.7, h = -0.18$ in the equation (1) $x = 0.9$

Putting $x = 0.78, z = 2.7, h = -0.9$ in the equation (2) $y = 1.908$

Putting $x = 0.78, y = 1.74, h = -0.18$ in the equation (3) $z = 2.916$

Putting $x = 0.78, y = 1.74, z = 2.7$ in the equation (4) $h = -0.108$

Fourth Estimate

Putting $y = 1.908, z = 2.917, h = -0.108$ in the equation (1) $x = 0.9624$

Putting $x = 0.9, z = 2.917, h = -0.108$ in the equation (2) $y = 1.9608$

Putting $x = 0.9, y = 1.908, h = -0.108$ in the equation (3) $z = 2.9592$

Putting $x = 0.9, y = 1.908, z = 2.917$ in the equation (4) $h = -0.036$

Fifth Estimate

Putting $y = 1.9608, z = 2.9592, h = -0.036$ in the equation (1) $x = 0.9845$

Putting $x = 0.9624, z = 2.9592, h = -0.036$ in the equation (2) $y = 1.9848$

Putting $x = 0.9624, y = 1.9608, h = -0.036$ in the equation (3) $z = 2.9851$

Putting $x = 0.9624, y = 1.9608, z = 2.9592$ in the equation (4) $h = -0.0158$

Sixth Estimate

Putting $y = 1.9848, z = 2.9851, h = -0.0158$ in the equation (1) $x = 0.9939$

Putting $x = 0.9845, z = 2.9851, h = -0.0158$ in the equation (2) $y = 1.9938$

Putting $x = 0.9845, y = 1.9848, h = -0.0158$ in the equation (3) $z = 2.9938$

Putting $x = 0.9845, y = 1.9848, z = 2.9851$ in the equation (4) $h = -0.006$

Seventh Estimation

Putting $y = 1.9938, z = 2.9938, h = -0.006$ in the equation (1) $x = 0.9975$

Putting $x = 0.9939, z = 2.9938, h = -0.006$ in the equation (2) $y = 1.9975$

Putting $x = 0.9939, y = 1.9938, h = -0.006$ in the equation (3) $z = 2.9976$
Putting $x = 0.9939, y = 1.9938, z = 2.9938$ in the equation (4) $h = -0.0025$

**Eighth Estimation**

Putting $y = 1.9975, z = 2.9976, h = -0.0025$ in the equation (1) $x = 0.9990$

Putting $x = 0.9975, z = 2.9976, h = -0.0025$ in the equation (2) $y = 1.9990$

Putting $x = 0.9975, y = 1.9975, h = -0.0025$ in the equation (3) $z = 2.9990$

Putting $x = 0.9975, y = 1.9975, z = 2.9975$ in the equation (4) $h = -0.0010$

**Ninth Estimation**

Putting $y = 1.9990, z = 2.9990, h = -0.0010$ in the equation (1) $x = 0.9996$

Putting $x = 0.9990, z = 2.9990, h = -0.0010$ in the equation (2) $y = 1.9996$

Putting $x = 0.9990, y = 1.9990, h = -0.0010$ in the equation (3) $z = 2.9996$

Putting $x = 0.9990, y = 1.9990, z = 2.9990$ in the equation (4) $h = -0.0004$

**Tenth Estimation**

Putting $y = 1.9996, z = 2.9996, h = -0.0004$ in the equation (1) $x = 0.9998$

Putting $x = 0.9996, z = 2.9996, h = -0.0004$ in the equation (2) $y = 1.9998$

Putting $x = 0.9996, y = 1.9996, h = -0.0004$ in the equation (3) $z = 2.9998$

Putting $x = 0.9996, y = 1.9996, z = -2.9996$ in the equation (4) $h = -0.002$

**Eleventh Estimate**

Putting $y = 1.9998, z = 2.9998, h = -0.0002$ in the equation (1) $x = 0.9999$

Putting $x = 0.9998, z = 2.9998, h = -0.0002$ in the equation (2) $y = 1.9999$

Putting $x = 0.9998, y = 1.9998, h = -0.0002$ in the equation (3) $z = 2.9999$

Putting $x = 0.9998, y = 1.9998, z = 2.9998$ in the equation (4) $h = -0.0001$

**Twelfth Estimate**

Putting $y = 1.9999, z = 2.9999, h = -0.0001$ in the equation (1) $x = 1.0$

Putting $x = 0.9999, z = 2.9999, h = -0.0001$ in the equation (2) $y = 2.0$

Putting $x = 0.9999, y = 1.9999, h = -0.0001$ in the equation (3) $z = 3.0$

Putting $x = 0.9999, y = 1.9999, z = 2.9999$ in the equation (4) $h = 0.0$
Table 2: numerical approximate results for Jacobi method in example two

| N | X  | Y   | Z   | H   |
|---|----|-----|-----|-----|
| 1 | 0.3| 1.5 | 2.7 | -0.9|
| 2 | 0.78| 1.74| 2.7 | -0.18|
| 3 | 0.9 | 1.908| 2.916| -0.108|
| 4 | 0.9624| 1.9608| 2.9592| -0.036|
| 5 | 0.9845| 1.9845| 2.9851| -0.0158|
| 6 | 0.9939| 1.9938| 2.9938| -0.006|
| 7 | 0.9975| 1.9975| 2.9975| -0.0025|
| 8 | 0.9990| 1.9990| 2.9990| -0.0010|
| 9 | 0.9996| 1.9996| 2.9996| -0.0004|
| 10| 0.9998| 1.9998| 2.9998| 0.0002|
| 11| 0.9999| 1.9999| 2.9999| -0.0001|
| 12| 1   | 2   | 3   | 0   |

The results dispute of the Jacobi method compared to the results of Gauss accurate but it is longer in the steps for this takes longer time in the solution to the twelve steps, in Gauss we obtained the same results in seven steps and the same accuracy of the solutions on all steps, for this reason we prefer the way Gauss Seidel because it is easy to use and limited to add to its accuracy.

In this solution we tested the use of repetitive methods to approximate the numerical solutions that arise when solving the other methods, we used with the methods Seidel Gauss and Jacobi, we analyzed the best way in a short and quick way also with accurate results.

We started from one with red line to about 0.3 of Jacobi by red the color form Gauss-Siedel represented by blue colure in sort line, we see the blue line common with the red line from the seventh step to the twelfth step means that the number of error is small, but from step five to second, We see the font differently and shows a different value.

Figure 1 through the illustration by Mat lab,
This discussion starts from the values we get in the column that appears from 1.5 to value 2, note that the value of the fifth step of the solution represents very closed lines and appears in red and blue in Figure 2.

**Figure 2**

This figure shows the results we get from column z for Jacobi and Gauss. It started from 2.7 to 3. We note that from the seventh step till the twelfth step, the values for both methods are very close, as shown by the line.

**Figure 3**

This figure shows the results we get from column z for Jacobi and Gauss. It started from 2.7 to 3. We note that from the seventh step till the twelfth step, the values for both methods are very close, as shown by the line.
We will discuss the result of the last column of the Jacobi method provided by the value of two lines of Gauss - Cedil and Jacobi, which are clear in red and blue, note that the step four to six values are separated from each other, while the results close and excellent at the other steps note that the errors are very small, The adjacent lines also give almost the best approximation but they are lengthy making it a complicated way.

4. Conclusion:

We also mentioned the Gauss-Seidel method with the proposed method in order to obtain variable values for linear equation systems rather than the previous Jacobi method, so we found values for the linear equation variables in the manner suggested in each solution step.

In this work, we have some equations of Gauss-Seidel. Using linear system of equations and , we demonstrated (Example 1) that we can obtain an analytical answer for a given differential equation with an individual primitive value. We have demonstrated Gauss-Seidel applicability in finding solutions to a number of different problems. In the numerical part, we noticed that we can reduce the level of accounting difficulties using the Jacobi conversion method. Although all the examples were of the Gauss and Jacobi equations, there are other problems that could benefit well from Gauss. The results show that values can converge more rapidly than Jacobi it was twelve steps and obtain semi-analytical solutions in fewer steps with Gauss.

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