DISK GALAXY MODELS DRIVEN BY STOCHASTIC SELF-PROPAGATING STAR FORMATION

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Abstract. We present a model of chemical and spectrophotometric evolution of disk galaxies based on a stochastic self-propagating star formation scenario. The model incorporates galaxy formation through the process of accretion, chemical and photometric evolution treatment, based on simple stellar populations (SSP), and parameterized gas dynamics inside the model. The model reproduces observational data of a late-type spiral galaxy M 33 reasonably well. Promising test results prove the applicability of the model and the adequate accuracy for the interpretation of disk galaxy properties.

Key words: galaxies: evolution – galaxies: disk, individual (M 33)

1. INTRODUCTION

Over the recent decades galaxy evolution models became sophisticated and based on large state of art codes (Hensler 2009). Despite this, the main processes controlling the evolution of galaxies, e.g., star formation rate and star formation feedback must be taken into account more carefully. Therefore, there are still advantages of using relatively simple models of galactic evolution based on parameterized physical processes. Due to the fast computation of these models, one can explore a wide range of galactic parameters and galaxy properties.

A part of such simple models apply a stochastic self-propagating star formation (SSPSF) scenario, which has been used with success to explain the flocculent spiral patterns in late-type disk galaxies (Gerola & Seiden 1978). Initially these models were applied to explain galactic disk spiral patterns by applying the percolation phenomena to star formation propagation. More advanced models include gaseous disks and perform galaxy modeling self-consistently. Successful attempts have been made to explain properties of dwarf galaxies: Gerola et al. (1980) have explained a highly diverse star formation rate in these systems as a characteristic behavior of SSPSF models on a spatially small disk. SSPSF models were developed further by introducing anisotropic propagating star formation probabilities, to account for different galaxy morphologies (Jungwiert & Palouš 1994), and physical groundings for the self-propagating star formation process (Palouš et al. 1994). By incorporating SSPSF ideas into the models of disk galaxy evolution, Sleath & Alexander (1995) were able to reproduce the Kennicutt-Schmidt law (Kennicutt 1998). Later on, following a rapid evolution of computational power, galaxy
evolution models have been developed mainly by applying N-body methods. We have developed a model based on the ideas of Seiden & Gerola (1982) with updated input physics and modern-day knowledge of the star formation process in disk galaxies. This model was applied to interpret observational data of the M 33 galaxy.

2. THE MODEL

We use the disk galaxy model scheme proposed by Seiden & Gerola (1982) and supplement it with a prescription of physical processes characterizing star formation and galaxy evolution. The disk of a galaxy is subdivided into regions (cells) of characteristic size, which, to some degree, are independent entities evolving according to chemical evolution scenarios. The most practical representation of such a disk structure is a two-dimensional (2-D) polar grid (Figure 1), which makes the implementation of the galaxy rotation curve simple and produces realistic galaxy disk models.

![Fig. 1. The galaxy disk model subdivided into cells. The neighboring cells (open circles) to a cell under consideration (filled circle) and the fraction of its perimeter (λ_{i,j}) contacting with a particular neighbor are marked.](image)

2.1. Model geometry

A galaxy is approximated by a 2-D disk subdivided into concentric rings of the same width (see Figure 1). A ring with a running number \( i \) contains \( 6 \times i \) cells. This division produces cells of the same area and perimeter, except for the central cell with an area smaller by a factor of \( 3/4 \). Cells are the basic structure elements in the model. The main parameter describing galactic disks is the number of rings. Each ring rotates according to a given rotation curve, which remains constant during the simulation procedure.

2.2. Disk formation

The simulated galaxy consists of two main parts, i.e., the galaxy itself and the reservoir, where all the gas is initially located. The galaxy formation proceeds gradually by accreting gas from the reservoir. The rate of accretion onto the galaxy disk, \( A(t, R) \), gradually decreases in time and is assumed to be proportional to gas
density in the reservoir at a particular galactocentric distance, \( R \):

\[
A(t, R) = A(0, R) \cdot \exp \left( -\frac{t}{\tau_{\text{acc}}} \right),
\]

(1)

where \( \tau_{\text{acc}} \) is an accretion timescale and \( A(0, R) \) is an initial accretion rate at the galactocentric distance \( R \).

### 2.3. Star formation prescription

A cell is able to experience star formation events (SF events) of two types: spontaneous and stimulated. The spontaneous star formation process is assumed to be related to collisions of giant molecular clouds (GMCs). However, we do not consider the formation of GMCs explicitly, and assume that the number of GMC collisions in the cell is proportional to \( \sigma_{\text{gas},i}^2 \). Therefore, the likelihood that a cell experiences a spontaneous SF event is proportional to \( \sigma_{\text{gas},i}^2 / \sigma_{\text{SP}}^2 \), where \( \sigma_{\text{SP}} \) is a parameter controlling spontaneous star formation, assumed to be equal to \( 320 M_\odot \text{pc}^{-2} \), i.e., double the density of GMCs in M33 (Bolatto et al. 2008).

Stimulated star formation occurs when the cell \( i \), which forms stars in the present time step, induces SF events in the neighboring cells during the next time step. This process can be parameterized by the likelihood of stimulated star formation, \( L_{\text{ST}} \), defined as the average number of new cells in which SF events can be induced by the SF event in the \( i \) cell. Therefore, the likelihood of stimulated star formation in the neighboring cell \( j \) is proportional to \( L_{\text{ST}} \times \lambda_{i,j} \), where \( \lambda_{i,j} \) is the fraction of the \( i \) cell’s perimeter contacting the cell \( j \) (see Figure 1). In our model we assume \( L_{\text{ST}} = 2 \) (Seiden & Gerola 1982).

Galaxy disks gradually grow due to gas accretion, therefore, initially within the entire galaxy and later on at the disk edge star formation is weak due to the critical gas surface density, \( \sigma_c \), which is assumed for star formation. This density reduces the likelihood of a SF event by a factor of \( \sigma_{\text{gas},i} / \sigma_c \), where \( \sigma_{\text{gas},i} \) is the average gas surface density of a particular cell.

Schaye (2004) models support the critical surface gas density, \( \sigma_c = (3 - 10) M_\odot \text{pc}^{-2} \), independent of the galactocentric distance. We applied \( \sigma_c = 7 M_\odot \text{pc}^{-2} \), a value adopted for the investigation of the disk evolution in our Galaxy (Chiappini et al. 2001).

During a SF event in a cell lasting one time step (e.g., 10 Myr), the fraction of gas converted to stars is defined by star formation efficiency, \( \epsilon \):

\[
\epsilon = \epsilon_0 \cdot \left( \frac{\sigma_{\text{gas},i}}{\sigma_{\text{gas},0}} \right),
\]

(2)

where \( \epsilon_0 \) is the efficiency of star formation at a gas surface density used for calibration, \( \sigma_{\text{gas},0} \). A linear dependency on \( \sigma_{\text{gas},i} \) for self-regulated star formation was suggested by Köppen et al. (1995). In our model we assume \( \sigma_{\text{gas},0} = 10 M_\odot \text{pc}^{-2} \) derived for the Milky Way galaxy (Wolfire et al. 2003), which produces the observed star formation efficiency in GMCs (Myers et al. 1986), \( \epsilon_0 \sim 0.02 \).

In the next time step after a SF event in the cell, star formation ceases due to the energy injected by high-mass stars. Another SF event in the same cell becomes highly improbable since some time (the refractory time) is needed to settle and cool the disturbed hot gas. Throughout the simulation we use a constant refractory
time of 100 Myr, a value derived for irregular galaxies (Quillen & Bland-Hawthorn 2008).

2.4. Chemical evolution

The cells in the galaxy model experience discrete star formation bursts. During the burst in a particular cell, gas (considered to be well-mixed within the cell) is converted into stars, and the formed stellar population can be represented satisfactorily by a simple stellar population (SSP) approach. Using SSP properties, calculated with the PÉGASE software (v. 2.0, Fioc & Rocca-Volmerange 1997; see Table 1 for the parameters), and following the track of a cell’s star formation history we compute the chemical evolution of the cell $i$. The equation of chemical evolution includes the three main contributions:

$$\frac{\Delta (Z_i \cdot \sigma_{\text{gas},i})}{\Delta t} = A(t, R) \cdot Z_0 + \sum_j F_{i,j} \cdot Z_{i,j} + \sum_k G_{i,k} \cdot Z_{i,k}. \quad (3)$$

Here the left-hand side represents the change in the metal content of the cell $i$. The first right-hand side term is a contribution by the accreted primordial gas with metallicity $Z_0$ from the reservoir at the galactocentric distance $R$. The second term is a sum of gas flows ($F_{i,j}$) from all neighboring cells to the cell $i$ (see below). The last term is the sum of metal contributions through all SSPs formed in the cell $i$ ($G_{i,k}$ is the expelled gas from stars in the stellar population $k$ with metallicity $Z_{i,k}$). The evolution of gas content is described by the equation:

$$\frac{\Delta \sigma_{\text{gas},i}}{\Delta t} = A(t, R) + \sum_j F_{i,j} + \sum_k G_{i,k}. \quad (4)$$

### Table 1. The parameter sources for the PÉGASE software.

| Parameter                       | Value          | Reference                        |
|---------------------------------|----------------|----------------------------------|
| IMF                             |                | Kroupa (2002)                    |
| SN II yields                    | Model B        | Woosley & Weaver (1995)          |
| Fraction of close binary systems| 0.05           | PÉGASE default value             |

2.5. Gas dynamics

The galactic disk model, subdivided into isolated cells of $\sim 100$ pc in size, would be an unrealistic approach, as typical multi-supernovae powered super-bubbles reach a similar size (e.g., Chu 2008). The isolated cells could also lead in some cases to the highly inhomogeneous metallicity of interstellar matter in galaxy disks, which clearly contradicts observations (Scalo & Elmegreen 2004).

The gas dynamics, implemented in our model, is based on assumptions that the equilibrium gas distribution is defined by an exponential disk scale length and that every SF event in a cell is powerful enough to inflate a super-bubble beyond the cell’s boundaries, leaving in it only the rarefied gas of high temperature. Assuming the initial mass function (IMF) (see Table 1) and the energy released by supernovae, $E_{\text{SN}} = 10^{44}$ J, the super-bubble radius after 10 Myr is equal to $\sim 140$ pc (McCray & Kafatos 1987) at the edge of the star forming disk ($\sigma_{\text{gas},i} = 7 M_\odot \text{pc}^{-2}$). This proves that SF events in the cells are powerful enough to drive
inter-cell gas dynamics. As a consequence, the gas, flowing from the cell \( i \) to the cell \( j \) is proportional to the mass of gas in the cell \( i \) and the fraction of its perimeter, contacting the cell \( j \): 

\[
F_{i,j} \sim m_{\text{gas},i} \cdot \lambda_{i,j}.
\]

The cavity in the cell \( i \), produced by a SF event and supported by a SN II, the main driving mechanism of the super-bubble (Mac Low & McCray 1988), starts to vanish after about 40 Myr due to refilling with gas through diffusion processes or SF events in the neighboring cells. For our simulation we assume a diffusion timescale of 350 Myr for a cell size region, following the numerical simulations by Recchi & Hensler (2005), who found the refill time to be ranging from 125 to 600 Myr.

3. APPLICATION OF THE MODEL

The described model is most appropriate for late-type spiral galaxies for the following reasons. Firstly, because the models based on the SSPSF scenario produce patchy star forming region patterns, which are similar to late-type flocculent spiral galaxies (Seiden & Gerola 1982). Secondly, we do not model the formation and evolution of the bulge, therefore, the results are most adequate to galaxies with a negligible bulge contribution.

According to those model properties, one of the best candidates for model tests is the Local Group galaxy M 33. It is the nearest late-type spiral having a moderate inclination (see Table 2 for the adopted parameters). As shown by Ferguson et al. (2007) the lack of substructures in the outer regions of this galaxy indicates its evolution in relative isolation. The small distance to the galaxy ensures plenty of observational data and makes it the ideal target for chemical and spectrophotometric evolution studies in terms of the proposed model.

| Table 2. The parameters of M 33 |
|---------------------------------|
| Parameter                        | Value | Reference     |
|---------------------------------|-------|---------------|
| Morphological type              | Sc    | Paturel et al.(2003)\(^1\) |
| Disk inclination                | 54°   | Paturel et al. (2003)\(^1\) |
| Position angle of major axis    | 22.5° | Paturel et al. (2003)\(^1\) |
| Distance                        | 840 kpc | Freedman et al. (1991) |
| Optical disk radius \((B = 25 \text{mag/arcsec}^2)\) | 7.3 kpc | Paturel et al. (2003)\(^1\) |
| Baryonic mass \(\leq 10^{10} M_{\odot}\) | Corbelli (2003) |

3.1. Model calibration

We model M 33 by simulating a disk composed of 99 rings. A cell width of 100 pc corresponds to a 10 kpc disk radius and fits well with the optical disk radius of the galaxy. The disk formation timescale was chosen to correspond to a slow accretion scenario. As recent studies have shown, such a scenario is in better agreement with observations (Magrini et al. 2007) and the up-to-date infall rate (Grossi et al. 2008). An accretion timescale, \(\tau_{\text{acc}}\), of 6 Gyr, typical to Sc-type galaxies (Arimoto et al. 1992), and a disk scale-length of 2 kpc (Freeman 1970), were applied. The distribution of mass in the reservoir was chosen to fit the baryonic mass radial distribution calculated from the gas and star surface densities derived by Corbelli (2003). The galaxy rotation data are taken from Corbelli &

\(^1\)http://leda.univ-lyon1.fr
Based on the best estimate from our modeling of M 33, we assume its age to be 12 Gyr.

3.2. Comparison with observations

Our model predicts gas and star disk surface densities, which are in good agreement with observations (Figures 2 and 3). A good agreement with the gas surface density indicates that the adopted parametrization of gas dynamics is adequate for this study. However, the predicted relatively sharp cut-off in the stellar density profile could be an artifact of our model, because there is no dispersion of stars between the cells introduced, and stars remain in situ positions. Roskar et al. (2008) have shown that such an assumption is incorrect, however, this does not change the galactic parameters significantly.

![Fig. 2. The profiles of gas surface density. The solid line denotes the average of 30 models; the circles represent observational data from Corbelli (2003).](image1)

![Fig. 3. The surface density profiles of stars in the disk. The solid line denotes the average of 30 models; the circles represent observational data from Corbelli (2003). The arrow indicates the location of a sharp break in the stellar density profile (Ferguson et al. 2007).](image2)
The radial profile of star formation rate predicted by the model (Figure 4) is also in good agreement with observations.

For the comparison of M33 with the model predictions we use oxygen abundances in HII zones and metallicities of blue supergiants. The model oxygen abundances are derived assuming the scaled solar metallicity by Asplund et al. (2005). Recently, Rosolowsky & Simon (2008) provided the largest homogeneous sample of HII zone abundances in M33 (Figure 5). The metallicities of blue supergiants (Figure 6) are taken from Urbania et al. (2005) and U et al. (2009). It is evident that both samples display different behavior. The supergiants show a steeper metallicity gradient and systematically higher metallicity values. The determination of the radial metallicity gradient in HII zones is uncertain due to a high intrinsic scatter of the data.

**Fig. 4.** Star formation rate (SFR) density profiles. The solid line denotes the average of 30 models; the triangles and circles represent data derived from observations (Verley et al. 2009) using different tracers (FUV and Hα respectively).

**Fig. 5.** The oxygen abundance model profile of gas in M33. The solid line denotes the average of 30 models; the circles represent abundances of the HII zones (Rosolowsky & Simon 2008).
A systematic H II zone “under-abundance” was discussed by Stasińska (2005) – at the solar metallicity the measured abundance can be underestimated by 0.2 dex. Therefore, bearing in mind systematic differences in abundances between the H II zones and blue supergiants, our model is in reasonable agreement with both datasets. Judging from a few observation data points (Urbaneja et al. 2005, U et al. 2009) at large radii, our model produces too sharp cutoff in the outer disk, which could be explained by the neglected star exchange between the model’s cells. Spitoni et al. (2008) have shown that during a SF event there is a possibility to contaminate regions at distances of up to 1 kpc by a super-bubble blow-out which can considerably smooth the abundance gradients in real disks.

4. CONCLUSIONS

We propose a new model of chemical and spectrophotometric evolution of disk galaxies, based on the stochastic self-propagating star formation scenario. We have extended the disk galaxy model by Seiden & Gerola (1982) and supplemented it with the disk formation through the accretion process, the parameterized gas dynamics inside the disk, and the chemical and photometric evolution treatment based on a simple stellar population approach.

The model of the late-type galaxy M 33 is in good agreement with the observed radial profiles of gas and star surface density, oxygen abundance, metallicity and star formation rate surface density.

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Fig. 6. The metallicity profile of gas in M 33. The solid line denotes the average of 30 models; the circles with error bars indicate the abundances in blue supergiants from Urbaneja et al. (2005) and U et al. (2009).
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