A PERIODIC REVIEW INVENTORY MODEL WITH STOCK DEPENDENT DEMAND, PERMISSIBLE DELAY IN PAYMENT AND PRICE DISCOUNT ON BACKORDERS

Manisha PAL  
Department of Statistics, University of Calcutta, India  
manishapal2@gmail.com

Sujan CHANDRA  
Department of Statistics, Haldia Govt. College, India

Received: May 2012 / Accepted: May 2013

Abstract: In this paper we study a periodic review inventory model with stock dependent demand. When stock on hand is zero, the inventory manager offers a price discount to customers who are willing to backorder their demand. Permissible delay in payments allowed to the inventory manager is also taken into account. Numerical examples are cited to illustrate the model.

Keywords: Periodic review model; stock dependent demand; shortage; price discount on backorder; delay in payment.

MSC: 90B05.

1. INTRODUCTION

In traditional inventory models, it is generally assumed that the demand rate is independent of factors like stock availability, price of items, etc. However, in actual practice, it is observed that demand for certain items is greatly influenced by the stock level. For example, an increase in shelf space for an item is seen to induce more consumers to buy it owing to its visibility, popularity or variety. Conversely, low stocks of certain goods might raise the perception that they are not fresh. Levin et al. (1972) pointed out that large piles of consumer goods displayed in a supermarket attract the customer to buy more. Silver and Peterson (1985) noted that sales at the retail level tend to be proportional to the stock displayed. Baker and Urban (1988) established an EOQ model for a power-form inventory-level-dependent demand pattern. Padmanabhan and
Vrat (1990) developed a multi-item inventory model for deteriorating items with stock-dependent demand under resource constraints. Datta and Pal (1990) presented an inventory model in which the demand rate is dependent on the instantaneous inventory level until a given inventory level is achieved after which, the demand rate becomes constant. Urban and Baker (1997) deliberated the EOQ model in which the demand is a multivariate function of price, time, and level of inventory. Giri and Chaudhuri (1998) expanded the EOQ model to allow for a nonlinear holding cost. Roy and Maiti (1998) developed multi-item inventory models of deteriorating items with stock-dependent demand in a fuzzy environment. Datta and Paul (2001) analyzed a multi-period EOQ model with stock-dependent, and price-sensitive demand rate. Kar et al. (2001) proposed an inventory model for deteriorating items sold from two shops, under single management dealing with limitations on investment and total floorspace area. Other papers related to this area are Gerchak and Wang (1994), Padmanabhan and Vrat (1995), Ray et al. (1998), Hwang and Hahn (2000), Panda (2010), Chang and Feng (2010), Roy and Chaudhuri (2012), Yadav et al. (2012), among others.

In inventory models with shortages, the general assumption is that the unmet demand is either completely lost or completely backlogged. However, it is quite possible that while some customers leave, others are willing to wait till fulfillment of their demand. In some situations, the inventory manager may offer a discount on backorders and/or reduction in waiting time to tempt customers to wait. Ouyang et al. (1999) considered reduction in lead time and ordering cost in a continuous review model with partial backordering. Chuang et al. (2004) discussed a distribution free procedure for mixed inventory model with backorder discount and variable lead time. Uthayakumar and Parvati (2008) considered a model with only first two moments of the lead time demand known, and obtained the optimum backorder price discount and order quantity in that situation. See also Chung and Huang (1998), Trevino et al. (1993), Kim et al. (1992).

In many real-life situations, the supplier allows the inventory manager a certain fixed period of time to settle his accounts. No interest is charged during this period but beyond it, the manager has to pay an interest to the supplier. During the permitted time period, the manager is free to sell his goods, accumulate revenue and earn interest. Hence, it is profitable to the manager to delay his payment till the last day of the settlement period. Goyal (1985) first developed the EOQ model under conditions of permissible delay in payment. Chand and Ward (1987) analyzed Goyal’s problem under assumptions of the classical economic order quantity model, obtaining different results. Aggarwal and Jaggi (1995) and Hwang and Shin (1997) extended Goyal’s model to the case of deteriorating items. Jamal et al. (1997) and Chang and Dye (2001) extended Aggarwal and Jaggi’s model to allow shortages. Shin et al. (1996) investigated the problem of price and lot size determination under permissible delay in payment and quantity discount on freight cost. Liao et al. (2000) considered an inventory model for initial-stock-dependent consumption rate when a delay in payment is permissible, but no shortages are allowed. Ouyang et al. (2005) developed an inventory model for deteriorating items with partial backlogging under permissible delay in payment. Pal and Ghosh (2007a) considered deterministic inventory models allowing shortage for deteriorating items under stock dependent demand, when delay in payment is allowed. Pal and Ghosh (2006, 2007b) studied quantity dependent settlement period in deterministic inventory models. Ghosh (2007) discussed stochastic inventory model for deteriorating items with permissible delay in payment. Das et al. (2011) developed a
deterministic EOQ inventory model with time dependent demand under permissible delay in payment and the cost parameters are taken as hybrid numbers.

In this paper, we consider a periodic review inventory model with stock dependent demand. The supplier allows the inventory manager a fixed time interval to settle his dues and the manager offers his customer a discount in case he is willing to backorder his demand when there is a stock-out. The paper is organized as follows. Assumptions and notations are presented in Section 2. In Section 3, the model is formulated and the optimal order quantity and backorder price discount determined. In Section 4, numerical examples are cited to illustrate the policy and to analyze the sensitivity of the model with respect to the cost parameters. Concluding remarks are given in Section 5.

2. NOTATIONS AND ASSUMPTIONS

To develop the model, we use the following notations and assumptions.

Notations

(a) Given variables

- \( K \) = ordering cost per order
- \( P \) = purchase cost per unit
- \( h \) = holding cost per unit per unit time
- \( s_1 \) = backorder cost per unit backordered per unit time
- \( s_2 \) = cost of a lost sale
- \( \pi_0 \) = marginal profit per unit
- \( I_e \) = interest that can be earned per unit time
- \( I_r \) = interest payable per unit time beyond the permissible delay period (\( I_r > I_e \))
- \( M \) = permissible delay in settling the accounts
- \( b_0 \) = upper bound on backorder ratio, \( 0 \leq b_0 \leq 1 \).

(b) Decision variables

- \( b \) = fraction of the demand during stock-out period which is allowed or accepted to be backlogged
- \( \pi \) = price discount on unit backorder offered
- \( T \) = length of a replenishment cycle
- \( T_1 \) = time taken for stock on hand to be exhausted, \( 0 < T_1 < T \)
$S$ = maximum stock height in a replenishment cycle.

Further, let

$I(t) = \text{inventory level at time point } t, \ 0 \leq t \leq T.$

**Assumptions**

1. The model considers only one item in inventory.
2. Replenishment of inventory occurs instantaneously on ordering, that is, lead time is zero.
3. Shortages are allowed, and a fraction $b$ of unmet demands during stock-out is backlogged.
4. Demand rate $R(t)$ at time $t$ is

$$R(t) = \alpha + \beta I(t) \quad \text{for } 0 < t < T_1,$$

$$= \alpha \quad \text{for } T_1 < t < T,$$

where $\alpha = \text{fixed demand per unit time, } \alpha > 0$ and $\beta = \text{fraction of total inventory demanded per unit time under the influence of stock on hand, } 0 < \beta < 1.$

5. During the stock-out period, the backorder fraction $b$ is directly proportional to the price discount $\pi$ offered by the inventory manager. Thus,

$$b = \frac{b_0 \pi}{\pi_0}, \text{ where } 0 \leq \pi \leq \pi_0.$$

3. **MODEL FORMULATION**

The planning period is divided into reorder intervals, each of length $T$ units. Orders are placed at time points $0, T, 2T, 3T, \ldots,$ the order quantity being just sufficient to bring the stock height to a certain maximum level $S$. Assuming that at the beginning of the first reorder interval the stock level is zero just before ordering, the order quantity in this interval is equal to $S$.

Depletion of inventory occurs due to demand during the period $(0, T_1), T_1 < T,$ and in the interval $(T_1, T)$ shortage occurs, of which a fraction $b$ is backlogged. Hence, the variation in inventory level with respect to time is given by

$$\frac{d}{dt} I(t) = -\alpha - \beta I(t), \text{ if } 0 \leq t \leq T_1$$

$$= -b\alpha, \quad \text{if } T_1 < t \leq T.$$
Since \( I(T_1) = 0 \), we get

\[
I(t) = \frac{\alpha}{\beta} (e^{\beta(T-t)} - 1), \quad \text{if } 0 \leq t \leq T_1
\]

\[
= b\alpha(T_1 - t), \quad \text{if } T_1 < t \leq T.
\]

Hence, \( S = \frac{\alpha}{\beta} (e^{\beta T} - 1) \).

Then,

\[
H(T_1, T, b) = \text{inventory carried during a cycle}
\]

\[
= \int_0^{T_1} I(t) dt
\]

\[
= \frac{\alpha}{\beta} \left[ \frac{1}{\beta} (e^{\beta T} - 1) - T_1 \right]
\]

\( S(T_1, T, b) = \text{number of backorders during a cycle} \)

\[
= \int_{T_1}^T I(t) dt
\]

\[
= b\alpha(T - T_1)^2 / 2
\]

\( E(T_1, T, b) = \text{number of lost sales during a cycle} \)

\[
= (1 - b)\alpha(T - T_1)
\]

As regarding the permissible delay in payment, there can be two possibilities: \( M \leq T_1 \) and \( M \geq T_1 \).

We consider the two cases separately.

Case 1: \( M \leq T_1 \)

For \( M \leq T_1 \), the inventory manager has stock on hand beyond \( M \), and so he can use the sale revenue to earn interest at a rate \( I_e \) during \((0, T_1)\). The interest earned by the buyer is, therefore,

\[
IE_1(T_1, T, b) = PI_e \int_0^{T_1} I(t) dt = \frac{PI_e \alpha}{\beta} \left[ \frac{1}{\beta} (e^{\beta T_1} - 1) - T_1 \right]
\]

Beyond the fixed settlement period, the unsold stock is financed with an interest rate \( I_r \), so that the interest payable by the inventory manager is

\[
IP_1(T_1, T, b) = PI_r \int_{T_1}^T I(t) dt = \frac{PI_r \alpha}{\beta} \left[ \frac{1}{\beta} (e^{\beta(T_1, M)} - 1) - (T_1 - M) \right]
\]

Hence, the cost per unit length of a replenishment cycle is given by
$C(T_i, T, b) = \frac{1}{T} \{K + hH(T_i, T, b) + s_1 S(T_i, T, b) + s_2 E(T_i, T, b) + IP_i(T_i, T, b) - IE_i(T_i, T, b)\}$

$$= \frac{1}{T} \{K + \frac{\alpha}{\beta} \left( \frac{1}{\beta} (e^{\beta T_i} - 1) - T_i \right) + \frac{h \alpha}{2} (T - T_i)^2 + s_1 (1 - b) \alpha (T - T_i)$$

$$+ \frac{Pl_i \alpha}{\beta} \left( \frac{1}{\beta} (e^{\beta(T_i - M)} - 1) - (T_i - M) \right) - \frac{Pl_i \alpha}{\beta} \left( \frac{1}{\beta} (e^{\beta T_i} - 1) - T_i \right) \}$$

$$= \frac{N_2(T_i, T, b)}{T}, \text{ say.}$$

**Case 2**: $M \geq T_i$

Since $M \geq T_i$, the inventory manager pays no interest, but earns interest in the interval $(0, M)$ at a rate $I_e$.

The interest earned is given by

$$IE_2(T_i, T, b) = Pl_i \alpha \left( \frac{1}{\beta} (e^{\beta T_i} - 1) - T_i \right) + Pl_i \frac{h \alpha}{2} (M - T_i)^2$$

Hence, the cost per unit length of a replenishment cycle is given by

$$C_2(T_i, T, b) = \frac{1}{T} \{K + hH(T_i, T, b) + s_1 S(T_i, T, b) + s_2 E(T_i, T, b) - IE_2(T_i, T, b)\}$$

$$= \frac{1}{T} \{K + \frac{\alpha}{\beta} \left( \frac{1}{\beta} (e^{\beta T_i} - 1) - T_i \right) + \frac{h \alpha}{2} (T - T_i)^2 + s_1 (1 - b) \alpha (T - T_i)$$

$$- \frac{Pl_i \alpha}{\beta} \left( \frac{1}{\beta} (e^{\beta T_i} - 1) - T_i \right) - Pl_i \frac{h \alpha}{2} (M - T_i)^2 \}$$

$$= \frac{N_2(T_i, T, b)}{T}, \text{ say.}$$

The total expected cost per unit length of a replenishment cycle is, therefore, given by

$$C(T_i, T, b) = C_1(T_i, T, b), \text{ if } T_i \geq M$$

$$= C_2(T_i, T, b), \text{ if } T_i \leq M.$$

The optimal values of the decision variables $(T_i, T, b)$ minimizing $C(T_i, T, b)$ will be the set of values minimizing $C_1(T_i, T, b)$ if $\min C_1(T_i, T, b) \leq \min C_2(T_i, T, b)$, or the set of values minimizing $C_2(T_i, T, b)$ if $\min C_2(T_i, T, b) \leq \min C_1(T_i, T, b)$.

To find the optimal values of $T_i, T$ and $b$, we note that for given $b, (T_i, T)$ minimizing $C_i(T_i, T, b)$ satisfy

$$\frac{\alpha}{\beta} e^{\beta T_i} [h + P (I_e e^{\beta M} - I_e)] + s_1 b \alpha T_i = \frac{h \alpha}{\beta} + s_1 b \alpha T + s_2 (1 - b) \alpha + \frac{P \alpha}{\beta} (I_e - I_e)$$

(3.1)
\( s_b \alpha (T - T_i) + s_1 (1 - b) \alpha = C_i(T, T, b) \)

(3.2)

and 

\( (T_i, T) \) minimizing \( C_i(T, T, b) \) satisfy

\[
\frac{\alpha}{\beta} e^{\beta T} (h - P I_i) + (s_i - P I_i) b \alpha T_i = h \frac{\alpha}{\beta} + s_b \alpha T + s_1 (1 - b) \alpha - P I_i \left( \frac{\alpha}{\beta} + b \alpha M \right)
\]

(3.4)

\[
s_b \alpha (T - T_i) + s_1 (1 - b) \alpha = C_i(T_i, T, b)
\]

(3.5)

Clearly, equations (3.1)-(3.2) and (3.3)-(3.4) give solutions to \( (T_i, T) \) that are non-linear in \( b \). If these solutions are obtainable in closed form, one can substitute these in \( C_1(T, T, b) \) and \( C_2(T_i, T, b) \) respectively to get the cost functions as functions of \( b \) alone. Then, minimizing the cost functions with respect to \( b \), one can find the optimal value of \( b \), and hence of \( T_i, T \). However, as closed form solutions are difficult to obtain, the following theorems may be helpful in finding the optimal solution to the problem.

**Theorem 3.1:** For given \( T \) and \( b \), \( C_1(T, T, b) \) is a convex function of \( T_i \) if 

\( \frac{M}{\beta} \sigma_P e I \sigma_I \beta - + - \geq 0 \), and is concave in \( T_i \) if 

\( \frac{M}{\beta} \sigma_P e I \sigma_I \beta - + - \leq 0 \), while 

\( C_2(T, T, b) \) is 

a convex function of \( T_i \) if 

\( \min(s_1, h) - P I_i \geq 0 \), and is concave in \( T_i \) if 

\( \min(s_1, h) - P I_i \leq 0 \).

**Proof:** We have

\[
\frac{\partial^2}{\partial T_i^2} C_i(T_i, T, b) = \frac{\alpha}{T} \left[ e^{\beta T_i} \{ h + P(I_i e^{-\beta T_i} - I_i) \} + s_b \right]
\]

(3.6)

\[
\frac{\partial^2}{\partial T_i^2} C_2(T_i, T, b) = \frac{\alpha}{T} \left( e^{\beta T_i} (h - P I_i) + (s_i - P I_i) b \right)
\]

(3.7)

(3.6) is \( \geq \) or \( \leq 0 \) according as 

\( h + P(e^{-\beta T_i} I_i - I_i) \geq 0 \) or \( \leq 0 \), while (3.7) is \( \geq \) or \( \leq 0 \) according as 

\( \min(s_1, h) - P I_i \geq 0 \) or \( \leq 0 \). Hence, the theorem.

**Theorem 3.2:** For given \( T \) and \( b \), optimal \( T_i \) minimizing \( C_i(T_i, T, b) \) is an increasing function of \( T \) if 

\( h + P(e^{-\beta T} I_i - I_i) \geq 0 \), and optimal \( T_i \) minimizing \( C_2(T_i, T, b) \) is an increasing function of \( T \) if 

\( \min(s_1, h) - P I_i \geq 0 \).

**Proof:** Differentiating (3.1) w.r.t. \( T \) we get that if 

\( h + P(e^{-\beta T} I_i - I_i) \geq 0 \),

\[
\frac{\partial T_i}{\partial T} = \frac{s_b}{s_b + e^{\beta T_i} (h + P(e^{-\beta T_i} I_i - I_i))} > 0.
\]

Again, differentiating (3.3) w.r.t. \( T \) we have that if 

\( \min(s_1, h) - P I_i \geq 0 \),
Hence, the theorem.

4. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS

Since it is difficult to find closed form solutions to the sets of equations (3.1)-(3.2) and (3.3)-(3.4), we numerically find optimal solutions to the problem for given sets of model parameters, using the statistical software MATLAB. The following tables show the change in optimal inventory policy with change in a model parameter, when the other parameters remain fixed. We assume that $\alpha = 70$, $\beta = 0.7$, $b_0 = 1$.

| Table 1: Showing the optimal inventory policy for different values of $s_1$, when $K=50$, $P = 100$, $I_r = 0.05$, $I_e = 0.03$, $M = 0.1$, $s_2 = 70$ and $h = 40$. |
|---|---|---|---|---|
| $s_1$ | $T_1$ | $T$ | $b$ | $C(T_1, T, b)$ |
| 40 | 1.0069 | 4.5069 | 0.9980 | 4230.03 |
| 45 | 0.9961 | 4.1072 | 0.9992 | 4166.68 |
| 50 | 0.9858 | 3.7858 | 0.5000 | 4106.49 |
| 60 | 0.9664 | 3.2997 | 0.9995 | 3994.60 |
| 70 | 0.9485 | 2.9485 | 0.9993 | 3892.55 |
| 80 | 0.9319 | 2.6819 | 0.9856 | 3798.93 |
| 100 | 0.9018 | 2.3018 | 0.9749 | 3632.60 |
| 120 | 0.8754 | 2.0420 | 0.9653 | 3488.72 |
| 125 | 0.8692 | 1.9892 | 0.9620 | 3455.66 |

| Table 2: Showing the optimal inventory policy for different values of $s_2$, when $K=50$, $P = 100$, $I_r = 0.05$, $I_e = 0.03$, $M = 0.1$, $s_1 = 80$ and $h = 40$. |
|---|---|---|---|---|
| $s_2$ | $T_1$ | $T$ | $b$ | $C(T_1, T, b)$ |
| 60 | 0.8288 | 2.3288 | 0.9977 | 3242.32 |
| 70 | 0.9319 | 2.6819 | 0.9856 | 3798.93 |
| 80 | 1.0289 | 3.0289 | 0.9993 | 4360.93 |
| 90 | 1.1205 | 3.3705 | 0.9923 | 4927.75 |
| 100 | 1.2072 | 3.7072 | 0.9904 | 5498.96 |
| 110 | 1.2895 | 4.0395 | 0.9885 | 6074.21 |
| 120 | 1.3678 | 4.3678 | 0.5000 | 6653.20 |
| 125 | 1.4056 | 4.5306 | 0.5000 | 6944.02 |
### Table 3: Showing the optimal inventory policy for different values of $h$, when $K=50$, $P = 100$, $L = 0.05$, $I_r = 0.03$, $M=0.1$, $s_1=80$ and $s_2 = 70$.

| $h$ | $T_1$ | $T$ | $b$ | $C(T_1,T,b)$ |
|-----|-------|-----|-----|--------------|
| 25  | 1.2114| 2.9614| 0.9995| 3525.52      |
| 30  | 1.0988| 2.8488| 0.9995| 3632.45      |
| 40  | 0.9319| 2.6819| 0.9856| 3798.93      |
| 50  | 0.8125| 2.5625| 0.9995| 3924.07      |
| 60  | 0.7221| 2.4721| 0.9985| 4022.43      |
| 70  | 0.6509| 2.4009| 0.9985| 4102.18      |
| 80  | 0.5931| 2.3431| 0.9984| 4168.38      |
| 100 | 0.5045| 2.2545| 0.9982| 4272.35      |

### Table 4: Showing the optimal inventory policy for different values of $M$, when $K=50$, $P = 100$, $L = 0.05$, $I_r = 0.03$, $h=40$, $s_1=80$ and $s_2 = 70$.

| $M$ | $T_1$ | $T$ | $b$ | $C(T_1,T,b)$ |
|-----|-------|-----|-----|--------------|
| 0.01| 0.9240| 2.6740| 0.9980| 3813.09      |
| 0.05| 0.9275| 2.6775| 0.9983| 3806.57      |
| 0.1 | 0.9319| 2.6819| 0.9856| 3798.93      |
| 0.3 | 0.9484| 2.6984| 0.5000| 3773.65      |
| 0.5 | 0.9638| 2.7138| 0.9982| 3756.02      |
| 0.7 | 0.9781| 2.7281| 0.9983| 3745.08      |
| 1   | 0.9977| 2.7477| 0.8556| 3739.42      |
| 1.5 | 0.9745| 2.7245| 1.0000| 3729.29      |
| 2   | 0.9466| 2.6966| 1.0000| 3698.71      |
| 2.5 | 0.9137| 2.6637| 1.0000| 3647.00      |

### Table 5: Showing the optimal inventory policy for different values of $I_r$, when $K=50$, $P = 100$, $L = 0.05$, $h=40$, $s_1=80$ and $s_2 = 70$.

| $I_r$ | $T_1$ | $T$ | $b$ | $C(T_1,T,b)$ |
|-------|-------|-----|-----|--------------|
| 0.010 | 0.9050| 2.6550| 0.9994| 3826.67      |
| 0.015 | 0.9115| 2.6615| 0.9818| 3819.88      |
| 0.020 | 0.9182| 2.6682| 0.9827| 3812.99      |
| 0.025 | 0.9250| 2.6750| 0.9852| 3806.01      |
| 0.030 | 0.9319| 2.6819| 0.9856| 3798.93      |
| 0.035 | 0.9389| 2.6889| 0.9994| 3791.75      |
| 0.040 | 0.9460| 2.6960| 0.9989| 3784.47      |
| 0.045 | 0.9532| 2.7032| 0.9988| 3777.08      |
Table 6: Showing the optimal inventory policy for different values of $I_e$, when $K=50$, $P = 100$, $I_e = 0.03$, $h=40$, $s_1=80$ and $s_2 = 70$.

| $I_e$ | $T_1$  | $T$  | $b$  | $C(T_1, T, b)$ |
|-------|--------|------|------|----------------|
| 0.05  | 0.9319 | 2.6819 | 0.9856 | 3798.93 |
| 0.06  | 0.9199 | 2.6699 | 0.9828 | 3809.85 |
| 0.07  | 0.9083 | 2.6583 | 0.9992 | 3820.48 |
| 0.08  | 0.8971 | 2.6471 | 0.9993 | 3830.82 |
| 0.10  | 0.8756 | 2.6256 | 0.5000 | 3850.72 |
| 0.15  | 0.8270 | 2.5770 | 0.5000 | 3896.27 |
| 0.20  | 0.7845 | 2.5345 | 0.9990 | 3936.72 |
| 0.25  | 0.7469 | 2.4969 | 0.9986 | 3972.92 |

The above tables show that, for other parameters remaining constant,
(a) both $T_1$ and $T$ are decreasing in $s_1$, $h$ and $I_e$, but increase as $s_2$ and $I_e$ increase;
(b) $b$, and hence $\pi$, decreases with increase in $s_1$, $s_2$ and $h$, but increases with $M$;
(c) the minimum cost per unit length of a reorder interval increases as $h$, $s_2$ and $I_e$ increase, but decreases with increase in $M$, $s_1$ and $I_e$.

The above observations indicate that, with the aim to minimizing total cost, the policy should be to maintain high inventory level for low backorder and holding costs but high lost sales cost and interest earned. Also, higher the backorder cost, lower should be the price discount offered on a backorder.

5. CONCLUSIONS

The paper studies a periodic review inventory model with stock dependent demand, allowing shortages. When there is a stock out, the inventory manager offers a discount to each customer who is ready to wait till fulfillment of his demand. On the other hand, the replenishment source allows the inventory manager a certain fixed period of time to settle his accounts. No interest is charged during this period but beyond it, the manager has to pay an interest. The optimum ordering policy and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval. Through numerical study, it is observed that for low backorder cost, it is beneficial to the inventory manager to offer the customers high discount on backorders.

ACKNOWLEDGEMENT

The authors thank the anonymous referee for the fruitful suggestions, which helped to improve the presentation of the paper.
REFERENCES

[1] Aggarwal, S.P., Jaggi, C.K., “Ordering policies of deteriorating items under conditions of permissible delay in payments”, *Journal of Operational Research Society*, 46 (1995) 658-662.

[2] Baker, R.C., Urban, T.L., “A deterministic inventory system with an inventory-level dependent demand rate”, *Journal of Operational Research Society*, 39 (1988) 823–825.

[3] Chang, C.-T., “Inventory model with stock-dependent demand and nonlinear holding costs for deteriorating items”, *Asia-Pacific Journal of Operational Research*, 21 (2004) 435-446.

[4] Chang, H.J., Dye, C.Y., “An inventory model for deteriorating items with partial back logging and permissible delay in payments”, *International Journal of System Sciences*, 32 (2001) 345-352.

[5] Chang, H.J., and Feng Lin, W., “A partial backlogging inventory model for non-instantaneous deteriorating items with stock-dependent consumption rate under inflation”, *Yugoslav Journal of Operations Research*, 20(1) (2010) 35-54.

[6] Chand, S., Ward, J., “A note on economic order quantity under condition of permissible delay in payments”, *Journal of Operational Research Society*, 38 (1987) 83-84.

[7] Chuang, B.R., Ouyang, L.Y., Chuang, K.W., “A note on periodic review inventory model with controllable setup cost and lead time”, *Computers and Operations Research*, 31(4) (2004) 549–561.

[8] Chung, K.J., Huang, C.K., “Economic manufacturing quantity model involving lead time and setup cost reduction investment as decision variables”, *International Journal of Operations & Quantitative Management*, 4 (1998) 209–216.

[9] Das, D., Roy, A., Kar, S., “Optimum payment time for retailer under permitted delay of payment by the wholesaler with dynamic demand and hybrid number cost parameters”, *Opsearch*, 48(3) (2011) 171-196.

[10] Datta, T.K., and Pal, A.K., “A note on an inventory model with inventory level dependent demand rate”, *Journal of Operational Research Society*, 41 (1990) 971-975.

[11] Datta, T.K., and Paul, K., “An inventory system with stock-dependent, price-sensitive demand rate”, *Production Planning and Control*, 12 (2001) 13-20.

[12] Ghosh, S.K., “A probabilistic inventory model for deteriorating items under condition of permissible delay in payment”, *IAPQR Transactions*, 32(2) (2007) 103 – 115.

[13] Giri, B.C., and Chaudhuri, K.S., “Deterministic models of perishable inventory with stock dependent demand rate and nonlinear holding cost”, *European Journal of Operational Research Society*, 105 (1998) 467-474.

[14] Gerchak, Y. and Wang, Y., “Periodic-review inventory models with inventory-level dependent demand”, *Naval Research Logistics*, 41 (1994) 99-116.

[15] Goyal, S.K., “Economic Order Quantity under conditions of permissible delay in payments”, *Journal of Operational Research Society*, 36 (1985) 335-338.

[16] Hwang, H., and Hahn, K. H., “An optimal procurement policy for items with an inventory level-dependent demand rate and fixed lifetime”, *European Journal of Operational Research Society*, 127 (2000) 537-545.

[17] Hwang, H., Shinn, S.W., “Retailer’s pricing and lot sizing policy for exponentially deteriorating product under condition of permissible delay in payments”, *Computers and Operations Research*, 24 (1997) 539-547.

[18] Jamal, A.M., Sarker, B.R., Wang, S., “An ordering policy for deteriorating items with allowable shortage and permissible delay in payment”, *Journal of Operational Research Society*, 48 (1997) 826-833.

[19] Kar, S., Bhunia, A.K., and Maiti, M., “Inventory of multi-deteriorating items sold from two shops under single management with constraints on space and investment”, *Computers and Operations Research*, 28 (2001) 1203-1221.

[20] Kim, K.L., Hayya, J.C. and Hong, J.D., “Setup cost reduction in economic production quantity model”, *Decision Sciences*, 23 (1992) 500-508.

[21] Lee, W.C., Wu, J.W., Lei, C.L., “Optimal inventory policy involving back-order discounts and variable lead time demand”, *International Journal of Advanced Manufacturing Technology*, 34(9-10) (2007) 958–967.
[22] Levin, R.I., McLaughin, C.P., Lemone, R.P., Kottas, J.F., Production/Operations Management: Contemporary Policy for Managing Operating Systems, McGraw-Hill, New York, 1972.

[23] Liao, H.C., Tsai, C.H., Su, C.T., “An inventory model with deteriorating items under inflation when a delay in payments is permissible”, International Journal of Production Economics, 63 (2000) 207-214.

[24] Ouyang, L., Cheng, C., Chang, H., “Lead time and ordering cost reductions in continuous review inventory systems with partial backorders”, Journal of Operational Research Society, 50 (1999) 1272–1279.

[25] Padmanabhan, G., and Vrat, P., “Analysis of multi-item inventory systems under resource constraints: A non-linear goal programming approach”, Engineering Costs and Production Economics, 20 (1990) 121-127.

[26] Padmanabhan, G., and Vrat, P., “EOQ models for perishable items under stock dependent selling rate”, European Journal of Operational Research, 86 (1995) 281-292.

[27] Pal, M., and Ghosh, S.K., “An inventory model with shortage and quantity dependent permissible delay in payment”, Australian Society of Operations Research Bulletin, 25(3) (2006) 2-8.

[28] Pal, M., and Ghosh, S.K., “An inventory model with stock dependent demand and general rate of deterioration under conditions of permissible delay in payments”, Opsearch, 44(3) (2007a) 227-239.

[29] Pal, M., and Ghosh, S.K., “An inventory model for deteriorating items with quantity dependent permissible delay in payment and partial backlogging of shortage”, Calcutta Statistical Association Bulletin, 59(235-236) (2007b) 239-252.

[30] Panda S., “An EOQ model with stock dependent demand and imperfect quality items”, Yugoslav Journal of Operations Research, 20(2) (2010) 237-247.

[31] Roy, T., and Chaudhuri, K. S., “An EPLS model for a variable production rate with stock-price sensitive demand and deterioration”, Yugoslav Journal of Operations Research, 22 (1) (2012) 19-30.

[32] Roy, T.K., and Maiti, M., “Multi-objective inventory models of deteriorating items with some constraints in a fuzzy environment”, Computers and Operations Research, 25 (1998) 1085-1095.

[33] Ray, J., Goswami, A., and Chaudhuri, K.S., “On an inventory model with two levels of storage and stock-dependent demand rate”, International Journal of System Science, 29 (1998) 249-254.

[34] Trevino, J., Hurley, B.J, Friedrich, W., “A mathematical model for the economic justification of setup time reduction”, International Journal of Production Research, 31 (1993)191–202.

[35] Uthayakumar, R., Parvathi, P., “Inventory models with mixture of backorders involving reducible lead-time and setup cost”, Opsearch, 45(1) (2008) 12-33.

[36] Ouyang, L.Y., Teng, J.T., Chen, L.H., “Optimum ordering policy for deteriorating items with partial backlogging under permissible delay in payments”, Journal of Global Optimization, 34 (2005) 245-271.

[37] Shinn, S.W., Hwang, H.P., Sung, S., “Joint price and lot size determination under conditions of permissible delay in payments and quantity discounts for freight cost”, European Journal of Operational Research, 91 (1996) 528-542.

[38] Silver, E.A., Peterson, R., Decision Systems for Inventory Management and Production Planning, 2nd Edition. Wiley: New York, 1985.

[39] Urban, T.L., and Baker, R.C., “Optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns”, European Journal of Operational Research Society, 103 (1997) 573-583.

[40] Yadav, D., Singh, S. R., and Kumari, R., “Inventory model of deteriorating items with two warehouse and stock dependent demand using genetic algorithm in fuzzy environment”, Yugoslav Journal of Operations Research, 22 (1) (2012) 51-78.