RADIATION PRESSURE–SUPPORTED STARBURST DISKS AND ACTIVE GALACTIC NUCLEUS FUELING

TODD A. THOMPSON, ELIOT QUATAERT, and NORMAN MURRAY

ABSTRACT

We consider the structure of marginally Toomre-stable starburst disks under the assumption that radiation pressure on dust grains provides the dominant vertical support against gravity. This assumption is particularly appropriate when the disk is optically thick to its own infrared radiation, as in the central regions of ULIRGs. We argue that because the disk radiates at its Eddington limit (for dust), the “Schmidt law” for star formation changes in the optically thick limit, with the star formation rate per unit area scaling as $\Sigma_* \propto \Sigma_d / \kappa$, where $\Sigma_d$ is the gas surface density and $\kappa$ is the mean opacity of the disk. Our calculations further show that optically thick starburst disks have a characteristic flux, star formation rate per unit area, and dust effective temperature of $F \sim 10^{13} L_\odot \text{kpc}^{-2}$, $\Sigma_* \sim 10^3 M_\odot \text{yr}^{-1} \text{kpc}^{-2}$, and $T_{	ext{eff}} \sim 90 \text{K}$, respectively. We compare our model predictions with observations of ULIRGs and find good agreement. We extend our model of starburst disks from many hundred parsec scales to subparsec scales and address the problem of fueling AGNs. We assume that angular momentum transport proceeds via global radiative waves, large-scale magnetic stresses, rather than a local viscosity. We consistently account for the radial depletion of gas due to star formation and find a strong bifurcation between two classes of disk models: (1) solutions with a starburst on large scales that consumes all of the gas with little or no fueling of a central AGN and (2) models with an outer large-scale starburst accompanied by a more compact starburst on 1–10 pc scales and a bright central AGN. The luminosity of the latter models is in many cases dominated by the AGN, although these disk solutions exhibit a broad mid- to far-infrared peak from star formation. We show that the vertical thickness of the starburst disk on parsec scales can approach $h \sim r$, perhaps accounting for the nuclear obscuration in some type 2 AGNs. We also argue that the disk of young stars in the Galactic center may be the remnant of such a compact nuclear starburst.

Subject headings: accretion, accretion disks — galaxies: formation — galaxies: general — galaxies: starburst — Galaxy: center — quasars: general

1. INTRODUCTION

Star formation in galaxies is observed to be globally inefficient; in a sample of local spiral galaxies and luminous starbursts, Kennicutt (1998) showed that only a few percent of the gas is converted into stars each dynamical time. This inefficiency may result from “feedback:” the energy and momentum injected into the interstellar medium (ISM) by star formation can in turn regulate the star formation rate in a galaxy. Models for feedback in the ISM generally invoke energy and momentum injection by supernovae and stellar winds (e.g., McKee & Ostriker 1977; Silk 1997; Efstathiou 2000). However, the momentum supplied to the ISM by the radiation from massive stars is comparable to that supplied by supernovae and stellar winds. The UV radiation from massive stars is absorbed and scattered by dust grains, which reprocess the UV emission into the IR. Because the dust grains are hydrodynamically coupled to the gas, radiation pressure on dust can help stabilize the gas against its own self-gravity and may therefore be an important feedback mechanism.

When the ISM of a galaxy is optically thick to the reradiated IR emission, radiative diffusion ensures that all of the momentum from the photons produced by star formation is efficiently coupled to the gas. We show that this limit is applicable on scales of hundreds of parsecs in luminous gas-rich starbursts, including ultraluminous infrared galaxies (ULIRGs), the most luminous and dust-enshrouded starbursts known (e.g., Genzel & Cesarsky 2000). Indeed, Scoville (2003) has already pointed out that radiation pressure on dust could plausibly be the dominant source of support against gravity in ULIRGs. We quantify this hypothesis by developing models of radiation pressure–supported starburst disks.

Although radiation pressure–supported disks have not been extensively considered in the galactic context, they have been well studied in models of black hole accretion. Most notably for our present purposes, the outer parts of disks around active galactic nuclei (AGNs) are expected to be dominated by radiation pressure on dust (e.g., Sirko & Goodman 2003). In the context of this paper, the distinction between the galactic disk and the AGN disk becomes somewhat unclear: if luminous AGNs are fueled by gas from the cold ISM of their host galaxy, there must be a continuous transition from the star-forming “galactic” disk to the central black hole’s “accretion disk.” The nature of this transition, and indeed whether it occurs at all, remains uncertain. The problem is that the outer parts of AGN disks are strongly self-gravitating with a Toomre stability parameter $Q \ll 1$ (Kolyakhov & Sunyaev 1980; Shlosman & Begelman 1989; Shlosman et al. 1990; Kumar 1999; Goodman 2003; Levin 2005; Tan & Blackman 2005). It is difficult to see how the disk avoids fragmenting almost entirely into stars. One possibility is that in the dense gas-rich nuclear regions of galaxies, angular momentum transport proceeds via global torques (e.g., bars, spiral waves, large-scale magnetic stresses), rather than a local viscosity (Shlosman et al. 1990; Mihos & Hernquist 1996; Goodman 2003). In this case, gas may inflow sufficiently rapidly to avoid

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1 Hubble Fellow.
2 Astronomy Department and Theoretical Astrophysics Center, University of California, 601 Campbell Hall, Berkeley, CA 94720; thompson@astro.berkeley.edu, eliot@astro.berkeley.edu.
3 Canada Research Chair in Astrophysics.
4 Visiting Miller Professor, University of California, Berkeley.
5 Canadian Institute for Theoretical Astrophysics, University of Toronto, 60 St. George Street, Toronto, ON M5S 3H8, Canada; murray@cita.utoronto.ca.
turning entirely into stars. In order to explore the fueling of a
central AGN, we extend our galactic-scale disk models to smaller
radii using a parameterization of the transport of angular mom-
entum by global torques. This provides a framework for un-
derstanding the relationship between AGN activity and nuclear
starbursts.

The plan for this paper is as follows. In § 2 we develop a sim-
ple model of self-regulated star formation in galactic disks. We
distinguish between disks that are optically thick/thin to their
own IR radiation and we present models appropriate to each
of these limits. Throughout § 2, and in particular in § 2.5, we
compare our theoretical models with observations of starburst
galaxies. In § 3 we extend the results of § 2 to include angular
momentum transport and address the problem of AGN fueling.
Finally, § 4 provides a discussion and summary of our con-
clusions and highlights some of the predictions of our models.

2. A ONE-ZONE DISK MODEL

In this section we construct simple dynamical models for the
structure of star-forming disk galaxies. We consider two limits:
(1) the “optically thin” limit, in which the disk is optically thick
to the UV radiation produced by massive stars but optically thin
to the reradiated IR radiation, and (2) the “optically thick”
limit, in which the disk is optically thick to the reradiated IR.

We begin by describing the properties of our models common
to both limits. We make the clear oversimplification that the
disk is a single-phase medium (see, however, the brief discus-
sions in § 2.1 and Appendix B); our model thus describes only
the average properties of the disk. We assume that the disk is in
radial centrifugal balance with a rotation rate \( \Omega = \Omega_K \),
where \( \Omega_K = \sqrt{2} \sigma/r \) is the Keplerian angular frequency in
an isothermal potential with velocity dispersion \( \sigma \). The total dynamical
mass at radius \( r \) is given by \( M_{\text{rot}}(r) = 2 \sigma^2 r^2 / G \), and the associated
surface density is

\[
\Sigma_{\text{tot}} = \frac{\sigma^2}{\pi G r} \sim 0.6 \sigma_{200}^2 r_{\text{kpc}}^{-1} \text{ g cm}^{-2},
\]

where \( \sigma_{200} = \sigma/200 \text{ km s}^{-1} \) and the radial scale is in units of
kpc. For simplicity, we assume that the underlying potential is
spherical, as would be provided by a stellar bulge or the gal-
axy’s dark matter halo.\(^6\) We further assume that the gas mass is a
constant fraction \( f_g = M_g / M_{\text{tot}} = \Sigma_g / \Sigma_{\text{tot}} \) of the total dynamical
mass, where \( \Sigma_g \) is the gas surface density. Although the as-
sumption of a constant \( f_g \) is clearly simplistic, we believe that
this model captures much of the physics of interest on large
scales. In § 3 we relax this assumption and show that a constant
gas fraction is equivalent to a constant accretion rate.

For a thin disk in a spherical potential, the equation for ver-
tical hydrostatic equilibrium, \( \partial p / \partial z = -\rho \Omega^2 z \), can be approx-
imated as

\[
p \approx \rho h^2 \Omega^2,
\]

where \( h \) is the pressure scale height. This implies \( h \approx c_s / \Omega \),
where \( c_s^2 = p/\rho \) is the sound speed. Throughout this paper we
include turbulent pressure in our definition of the sound speed
(in addition to radiation pressure and gas pressure). A term
\( 2 \pi G \Sigma_g \rho h \) should be added to the right-hand side of equation (2)
to account for the self-gravity of the disk. This leads to a
multiplicative correction to equation (2) of \( (1 + 2^{3/2}/Q) \), which
we neglect for simplicity.

We assume that star formation in the disk is governed by local
gravitational instability as described by Toomre’s \( Q \) param-
eter (Toomre 1964). In particular, we argue that the disk self-
regulates such that

\[
Q = \frac{\kappa c_s}{\pi G \Sigma_{\text{g}}} = \frac{\Omega^2}{\sqrt{2 \pi G \rho}}
\]

is maintained close to unity. In equation (3), \( \kappa^2 = 4 \Omega^2 + d \Omega^2 / d \ln r \)
is the epicyclic frequency. The hypothesis of marginal
Toomre stability has been discussed extensively in the literature
(e.g., Paczynski 1978; Gammie 2001; Sirko & Goodman 2003;
Levin 2005) and is based on the idea that if \( Q \gg 1 \), then the disk
will cool rapidly and form stars, while if \( Q \ll 1 \), then the star
formation will be so efficient that the disk will heat up to \( Q \sim 1 \).
There is evidence for \( Q \sim 1 \) in the Milky Way (e.g., Binney &
Tremaine 1987), local spiral galaxies (e.g., Martin & Kennicutt
2001), and starbursts such as ULIRGs (e.g., Downes & Solomon
1998).

From equation (3) it follows that the density distribution of the
gas is determined solely by the local Keplerian frequency:

\[
\rho = \frac{\Omega^2}{\sqrt{2 \pi G Q}} = \frac{\sqrt{2} \sigma^2}{\pi G Q r^2} \Rightarrow n \sim 170 \sigma_{200}^2 r_{\text{kpc}}^{-2} Q^{-1} \text{ cm}^{-3}.
\]

We retain the \( Q \) dependence here and below for completeness,
but our assumption is that \( Q \sim 1 \). From the definition of \( \Sigma_{\text{g}} \) and
equations (1) and (4), it then follows that

\[
(h/r) = f_g Q / 2^{3/2}
\]

and

\[
(c_s / \sigma) = f_g Q / 2.
\]

For constant \( f_g \) and \( Q \), \( h/r \) and \( c_s / \sigma \) are independent of radius.
With \( \sigma = 200 \text{ km s}^{-1} \), a turbulent sound speed of \( 10 \text{ km s}^{-1} \)
corresponds to a gas fraction of \( f_{g_{\text{tot}}} = f_{g}/0.1 \). The scale height
of the disk for this gas fraction is then

\[
h \sim 35 f_{g_{\text{tot}}} r_{\text{kpc}} \text{ pc}.
\]

In our model, the star formation rate per unit area \( \dot{\Sigma}_* \) ad-
justs to maintain \( Q \sim 1 \). We also parameterize star formation as
occurring on a fraction \( \eta \) of the local dynamical timescale (e.g.,
Elmegreen 1997):

\[
\dot{\Sigma}_* = \Sigma_\Omega \eta.
\]

Only values of \( \eta \lesssim 1 \) are physical. For \( \eta \) greater than unity the
disk cannot dynamically adjust to maintain \( Q \sim 1 \). Although \( \Sigma_* \)
is the fundamental derived quantity, \( \eta \) is a useful alternative par-
parameter that characterizes the global star formation efficiency
in the disk. Observationally, \( \eta \) is typically \( \sim 0.02 \) in normal spiral
and starburst galaxies (Kennicutt 1998).

2.1. The Optically Thin Limit

We consider galactic disks to be “optically thin” when the
vertical optical depth to IR photons is \( \lesssim 1 \) but the optical depth to
UV photons is greater than unity. This requires \( \Sigma_{\text{g}} \gtrsim 2 / \kappa_{\text{UV}} \sim
10^{-3} \text{ g cm}^{-2} \), where \( \kappa_{\text{UV}} \sim 10^3 \text{ cm}^2 \text{ g}^{-1} \) is a characteristic UV
opacity.

\(^6\) For a Navarro et al. (1997) dark matter profile, the dark matter mass can be
significant even on the small scales of interest: \( M(r) \sim 10^6 M_{\odot} \). This result
is relatively independent of halo mass so long as \( r \ll r_c \), where \( r_c \) is the scale
radius of the NFW potential.
In our models, feedback from star formation provides the pressure support to maintain $Q \sim 1$. Sources of this pressure support include radiation pressure on dust grains, supernovae, stellar winds, and expanding H II regions. In this section we consider a simple model that relates the pressure in the ISM to the star formation rate $\Sigma_\star$, in the optically thin limit (eq. [15]). To motivate this model and to connect with classic treatments of the ISM (e.g., McKee & Ostriker 1977), we estimate the pressure in both the “cold” ($p_c$) and “hot” ($p_h$) phases of the ISM, focusing on the contributions from radiation pressure and supernovae. The pressure in the cold ISM can thus be written as

$$p_c = \rho c_s^2 \sim p_{\text{mp}} + p_{\text{m}},$$  \hfill (9)

where $c_s$ should be interpreted as a turbulent velocity and $p_c$ as a turbulent pressure.

Because we assume that the optical depth to UV photons is greater than unity, the radiation pressure on cold gas can be related to the star formation rate by

$$p_{\text{mp}} \sim n \Sigma_\star c,$$  \hfill (10)

where $\epsilon$ is the efficiency with which star formation converts mass into radiation ($\epsilon \sim 10^{-3}$ for a Salpeter initial mass function [IMF] of $1$–$100 M_\odot$). Equation (10) assumes that there is no significant cancellation of oppositely directed momentum. This becomes an increasingly better assumption as the disk becomes optically thick to the IR (as in starburst galaxies). In addition to radiation pressure, supernova explosions deposit momentum into the ISM via swept-up shells of cold gas. From Thornton et al. (1998) we estimate that each supernova has an asymptotic momentum of $3 \times 10^{51} \frac{E_{51}}{\text{ergs}}$ cm s$^{-1}$, where $E_{51} = E/10^{51}$ ergs is the initial energy of the supernova and $n_1 = n/1$ cm$^{-3}$ is the density of the ambient medium. The contribution to the ISM pressure from supernovae may be estimated by taking $p_{\text{sn}} = P_{\text{sn}}/(2\pi r^2)$, where $P_{\text{sn}}$ is the total momentum injection rate. Alternatively, one can estimate $p_{\text{sn}}$ by balancing the net volumetric energy injection rate by supernovae, $q_{\text{sn}} \sim p_{\text{sn}c_s/(2\pi r^2)}$, with energy losses due to turbulent decay. Assuming that turbulence decays on a crossing time so that $q_{\text{sn}} \sim \rho c_s^3/h_c$ (e.g., Stone et al. 1998), we see that $q_{\text{sn}} = q_{\text{sn}}$ implies that $P_{\text{sn}}/(2\pi r^2) = \rho c_s^2 = p_{\text{sn}}$. Thus, the contribution to the pressure of the cold ISM from supernovae is

$$p_{\text{sn}} \sim 1.5 \times 10^8 n_1^{-1/4} E_{51}^{13/4} \Sigma_\star \sim 5 n_1^{-1/4} E_{51}^{13/4} P_{\text{mp}}.$$  \hfill (11)

Equations (10) and (11) show that radiation pressure and supernovae contribute comparably to the pressure of the cold ISM.

Supernovae also generate a shocked hot ISM, whose pressure $p_{\text{h}}$ can be estimated using the model of McKee & Ostriker (1977). Individual supernovae initially expand adiabatically to a cooling radius $R_\text{c} \sim 20 E_{51}^{3/4} n_1^{-1/4}$ pc, where a cool dense shell forms (Chevalier 1974; Cioffi et al. 1988; Thornton et al. 1998). The subsequent remnant evolution is approximately momentum conserving. In the absence of interaction with neighboring supernova remnants, the remnant expands to a maximum radius $R_p \sim 60 E_{51}^{3/4} \Sigma_\star^{-1/2} P^{-1/2}$ pc, where the pressure of the supernova remnant is equal to the ambient pressure ($P$) and $P_{-12} = P/10^{-12}$ ergs cm$^{-3}$. In fact, the remnant typically does not reach $R_p$ before encountering a neighboring supernova remnant. That is, the overlap radius for supernova remnants ($R_\odot$) is $R_\odot < R_p$. In this model, the average pressure of the hot ISM is not known a priori but can be estimated using the pressure of a supernova remnant at $R_\odot$ (this assumes that there is no significant energy loss after the supernova remnants overlap and mix). Using results from Chevalier (1974), McKee & Ostriker (1977) showed that the pressure at overlap can be expressed in terms of the pressure at $R_\odot$ as

$$p_{\text{h}} \sim P_{\text{sn}} \Sigma_\star c.$$  \hfill (12)

where $\Sigma_\star = S/c$, is the number of supernovae that occur in the volume $V_\star \sim (4/3)\pi R_\star^3$, $t_\star$ is the time for the remnant to reach $R_\star$, and $S$ is the supernova rate per unit volume. The time $t_\star$ has been estimated by Cox (1972) and Cioffi et al. (1988): $t_\star \sim 4 \times 10^7 E_{51}^{3/4} n_1^{-4/7}$ yr for solar metallicity. Using $p_c = E/V_c$, we find that

$$p_{\text{h}} \sim 10^{-12} E_{51}^{17/14} n_1^{-4/7} S_{-13} \text{ ergs cm}^{-3},$$  \hfill (13)

where we have scaled the number density to a value comparable to that of the hot ISM ($n_1 = n/10^{-2}$ cm$^{-3}$) and the supernova rate to the Galactic value: $S_{-13} = S/10^{-13}$ pc$^{-3}$ yr$^{-1}$. Equation (13) is in reasonable agreement with observations and more detailed models of the local pressure of the ISM (McKee & Ostriker 1977; Boulanger & Cox 1990).

In order to compare $p_h$ and $p_{\text{mp}}$ from equation (10) directly, we note that $S \sim 10^{-2} \Sigma_\star/2(h_\star)$ with $\Sigma_\star$ in $M_\odot$ yr$^{-1}$ per unit area. The ratio of these two components of the pressure can then be written as

$$\frac{p_h}{p_{\text{mp}}} \sim 3 h_{100}^{-1/2} E_{51}^{17/14},$$  \hfill (14)

where $h_{100} = h/100$ pc. Because the total volume occupied by remnants with $R \leq R_\star$ decreases as the density of the ISM increases, the contribution from supernovae to the total pressure decreases with increasing density. Equations (9)–(14) show that, to order of magnitude, $p_c = p_{\text{mp}} + p_{\text{sn}} \sim p_{\text{h}}$ for conditions appropriate to the Galaxy (as is observed).

In the luminous starbursts we focus on in this paper, the density of the ISM is much larger than in the local ISM. For example, in the inner few hundred parsecs of ULIRGs the average gas density reaches $10^3$–$10^4$ cm$^{-3}$, comparable to the density of a local molecular cloud (e.g., Downes & Solomon 1998; see also eq. [4]). Several lines of evidence suggest that the cold molecular gas may fill a significant fraction of the volume in ULIRGs, unlike in the local ISM (Downes et al. 1993; Solomon et al. 1997). The high fraction and high luminosity of radio supernovae in Arp 220 are also consistent with an environment much denser than the ISM of normal spiral galaxies (Smith et al. 1998). Taking $n \sim 10^3$ cm$^{-3}$ as a characteristic value, we find that $R_\odot$ is just $\sim 1$ pc, $p_h/p_{\text{mp}} \sim 10^{-2} h_{100}^{-1}$, and that the total asymptotic thermal energy of a supernova remnant is $\sim 4 \times 10^{46}$ ergs $\ll E$ (Thornton et al. 1998). This argues against a dynamically dominant, volume-filling hot ISM. Even in the limit of strong radiative losses, however, supernovae are still important for generating the random motion of cold gas ($p_{\text{sn}} \sim p_{\text{mp}} \gg p_{\text{h}}$; eq. [11]).

In the simple estimates above, all of the contributions to the pressure of the ISM scale roughly linearly with the star formation rate. Moreover, the ratio $p_{\text{mp}}/p_{\text{sn}}$ is of order unity, and while $p_{\text{h}}/p_{\text{mp}}$ is of order unity in normal spiral galaxies, it may be significantly smaller in the dense nuclei of the most luminous

\footnote{More carefully, $\Psi_c$ should be raised to the 10th/11th power in eq. (12), which we approximate as unity for simplicity.}
starbursts. For this reason we choose to express the effective pressure of the ISM in the “optically thin” limit as
\[ p \sim p_c = p_{mp} \left[ 1 + \left( \frac{p_{in}}{p_{mp}} \right) \right] = \epsilon \Sigma_\star c, \]  
where the last equality defines the parameter \( \xi \). In what follows we retain the dependence of our results on \( \xi \) but scale to \( \xi \sim 1 \) for the reasons given above.

Using equations (4), (6), and (15), it is straightforward to solve for the physical parameters of our disk model. The star formation rate per unit area required to support the disk with \( Q \sim 1 \) is given by
\[ \Sigma_\star \sim \frac{35 f_\delta^2 \sigma_{200}^4 r_{kpc}^{-2} \epsilon^{-1} Q M_\odot \text{yr}^{-1} \text{kpc}^{-2}}{f_\delta \sigma \epsilon^{-1}}. \]  

Scaling equation (16) for typical \( f_\delta \) and \( \sigma \), we find that
\[ \Sigma_\star \sim 35 f_\delta^2 \sigma_{200}^4 r_{kpc}^{-2} \epsilon^{-1} Q M_\odot \text{yr}^{-1} \text{kpc}^{-2}, \]  
where \( \epsilon_3 = \epsilon/10^{-3} \) and \( f_\delta = f_\delta/0.5 \) is appropriate for gas-rich starbursts. The star formation rate can also be expressed in terms of the efficiency \( \eta \):
\[ \eta = \frac{1}{2} \frac{\epsilon_3 c}{\epsilon \epsilon_3 c} = \frac{Q}{4} \left( \frac{f_\delta \sigma}{\epsilon \epsilon_3 c} \right) \sim 0.1 f_\delta \sigma_200 \epsilon_3^{-1} \xi^{-1}. \]  
The second equality in equation (18) is meant to show explicitly that one may write \( \eta \) in terms of \( c_0 \) or \( f_\delta \sigma \) (eq. [6]). Using equation (16), the flux and luminosity of the disk viewed face-on are given by
\[ F = \epsilon \Sigma_\star c^2 \sim 5 \times 10^{11} f_\delta^2 \sigma_{200}^4 r_{kpc}^{-2} \epsilon^{-1} L_\odot \text{kpc}^{-2}, \]  

and
\[ L = \pi r^2 F = \frac{f_\delta^2 c Q}{2 \sqrt{2} \xi} \sigma_4^4 \sim 2 \times 10^{12} f_\delta \sigma_{200}^4 Q \xi^{-1} L_\odot. \]  
Up to logarithmic corrections, equation (20) implies that for constant \( f_\delta \), all radii contribute equally to the total luminosity.

The effective temperature is defined by the relation \( \sigma T^4 \equiv \epsilon \Sigma_\star c^2 \). In the optically thin limit, the observed dust temperature \( T_{dust} \) is related to \( T_{eff} \) by \( T_{dust} \sim T_{eff}^4 \), where \( T_{dust} \) is the vertical optical depth to IR radiation. This relation between \( T_{dust} \) and \( T_{eff} \) assumes that the sources of UV radiation are uniformly distributed vertically throughout the disk. With equation (16),
\[ T_{dust} = \left( \frac{f_\delta \sigma^2}{r} \frac{Q c}{2 \sqrt{2} \xi \epsilon \epsilon_3 c_{SB}} \right)^{1/4} \sim 60 f_\delta^{1/4} \sigma_{200}^{1/2} \left( \kappa_i r_{kpc} \right)^{-1/4} Q^{1/4} \xi K, \]
where \( \kappa_i = \kappa/1 \text{g cm}^{-2} \) is a representative value for the IR opacity of the disk (Fig. 1) and we have assumed that an individual dust grain radiates as a blackbody.

There are several interesting properties of the disk model presented in this section. First, the efficiency of star formation required to maintain \( Q \sim 1 \) is \( \eta \sim 0.02 \) for a canonical turbulent velocity of \( c_t \sim 10 \text{ km s}^{-1} \) (equivalent in our model to having \( f_\delta \sim 0.1 \) for \( \sigma \sim 200 \text{ km s}^{-1} \), as in the Galaxy). This value for \( \eta \) is in good agreement with observations compiled by Kennicutt (1998).

Second, the first two equalities in equation (16) yield \( \Sigma_\star \propto \Sigma_g \). This Schmidt-like star formation law is somewhat steeper than the \( \Sigma_\star \propto \Sigma_g^{1.4} \) favored by Kennicutt (1998) but comparable to the scaling \( \Sigma_\star \propto \Sigma_g^{0.75} \) obtained by Gao & Solomon (2004) using a sample that includes more luminous starburst galaxies. Given the simplicity of the model presented here, this agreement is satisfactory. Third, for \( f_\delta = 0.1 \) and \( \sigma = 200 \text{ km s}^{-1} \), the dust temperature (eq. [21]), turbulent velocity, pressure, flux, luminosity, and scale height (eq. [7]); compare with Fig. 9.25 from Binney & Merrifield (1998) are all in fair agreement with observations of the Milky Way. Thus, despite its simplicity, the model presented in this section provides a useful characterization of galactic-scale star formation supported by the turbulent pressure produced by supernovae and the radiation from massive stars.

2.2. The Optically Thick Limit

The nuclei of gas-rich starbursts are optically thick to their own infrared radiation. Radiative diffusion then ensures that radiation pressure provides the dominant vertical support against gravity. In this section we describe disk models appropriate to this limit. The vertical optical depth of the disk is given by \( \tau_{\nu} = \Sigma_\star K/2 \), where \( K \) is the Rosseland mean opacity to dust. Evaluating this expression yields
\[ \tau_{\nu} = \frac{\kappa \sigma^2 f_\delta}{2 \pi G r} \sim 0.15 \sigma_{200}^2 f_\delta \xi_{kpc}^{1/4}. \]  
The radius at which \( \tau_{\nu} = 1 \) is then
\[ R_{\nu=1} = \frac{\kappa \sigma^2 f_\delta}{2 \pi G} \approx 150 \kappa_i \sigma_{200} f_\delta \xi_{kpc}, \]  
and thus for the largest most gas-rich starbursts (\( \sigma \sim 300 \text{ km s}^{-1} \) and \( f_\delta \sim 1 \)) the inner \( \sim 700 \text{ pc} \) are optically thick.
In the optically thick limit, the effective temperature is given by
\[ \sigma_{SB} T_{\text{eff}}^4 = \frac{1}{2} \xi \Sigma_{\star} \epsilon^2, \]  
(24)
where the factor of \( \frac{1}{2} \) arises because both the top and bottom surfaces of the disk radiate. The midplane temperature is related to the effective temperature by \( T = \frac{1}{2} \tau_T T_{\text{eff}} \), where the opacity \( \kappa(T, \rho) \) in \( \tau_T \) should be evaluated using the central temperature and mass density of the disk. The temperature dependence of the opacity is important for this problem and is discussed in the next section. For now we simply normalize \( \kappa \) to 1 cm\(^2\) g\(^{-1}\).

For \( \tau_T \gtrsim 1 \), the radiation pressure is given by
\[ p_{\text{rad}} = \frac{4 \sigma_{SB}}{3c} T^4 = \frac{\sigma_{SB}}{c} T^4 \tau_T = \frac{1}{2} \tau_T \epsilon \Sigma_{\star} \epsilon. \]  
(25)
Comparing equations (15) and (25) shows that radiation pressure exceeds the turbulent pressure due to supernovae by a factor of \( \sim \tau_T \) (assuming \( \xi \sim 1 \) for the reasons given in § 2.1). Radiation pressure support will thus dominate the vertical support of compact optically thick starbursts. The exact surface density (or \( \Sigma_{\star} \)) at which the transition to radiation pressure support occurs is somewhat uncertain and requires a better understanding of the pressure support provided by supernovae, stellar winds, etc. We will explore this in more detail in future work.

With equation (25) it is again straightforward to solve for the disk’s physical parameters, \( T, T_{\text{eff}}, \Sigma_{\star}, \) etc., in terms of our model variables: \( \sigma, f_{\text{g}}, \epsilon, \) and the radius in the disk \( R \). The midplane temperature of the disk is
\[ T = \left( \frac{f_{0} \sigma_{200}^2}{r} \right)^{1/2} \left( \frac{3 \epsilon Q}{2^{7/2} \pi G \sigma_{SB}} \right)^{1/4} \sim 41 \sigma_{200} f_{0,3}^2 \tau_{\text{eff}}^{-1/4} Q^{1/4} \text{ K}, \]  
and the effective temperature is
\[ T_{\text{eff}} = \left( \frac{f_{0} \sigma_{200}^2}{r} \right)^{1/4} \frac{c Q}{\sqrt{2} \kappa \sigma_{SB}} \sim 70 \sigma_{200} f_{0,3}^2 \tau_{\text{eff}}^{-1/4} \kappa_{\text{opt}}^{-1/4} Q^{1/4} \text{ K}. \]  
(27)
For our fiducial numbers, \( T_{\text{eff}} \) is somewhat larger than \( T \), implying that the vertical optical depth is less than unity. Our assumption that the disk is optically thick in the far-IR (FIR) is therefore only marginally applicable on kiloparsec scales. It is an increasingly better assumption on smaller scales (see eqs. [22] and [23]).

The total star formation rate per unit area required to support the disk with radiation pressure is
\[ \dot{\Sigma}_{\star} = \frac{\sqrt{2} f_{0} \sigma_{200}^2}{\epsilon \kappa_{\text{opt}}} \frac{Q}{r} \sim 400 f_{0,3}^2 \sigma_{200}^2 Q r^{-1} \kappa_{1}^{-1} \epsilon_{1}^{-1} M_{\odot} \text{ yr}^{-1} \text{ kpc}^{-2}. \]  
(28)
Integrating, we derive the total star formation rate:
\[ \dot{M}_{\star} = \frac{2^{3/2} \pi f_{0} \sigma_{200}^2 r^2}{\epsilon \kappa_{\text{opt}}} \sim 3000 f_{0,3}^2 \sigma_{200}^2 Q r^{-1} \kappa_{1}^{-1} M_{\odot} \text{ yr}^{-1}. \]  
(29)
Note that if the disk is optically thick \( (2 \tau_T \sim 1) \), the observed flux is only from half of the disk and therefore the observationally inferred star formation rate would be one-half of the true total star formation rate (given in eqs. [28] and [29]).

The star formation efficiency \( \eta \) is
\[ \eta = \frac{\pi G Q r}{\kappa_{\text{opt}} \epsilon} \sim 1 r_{\text{kpc}} \sigma_{200}^2 \epsilon_{1}^{-1} \kappa_{1}^{-1} Q. \]  
(30)
For \( r \leq R_{\sigma} = 1 \) we see that \( \eta \) is \( < 1 \). It follows that the disk can adjust to maintain \( Q \sim 1 \) on subkiloparsec scales. In addition, we find that the ratio of the cooling timescale to the orbital timescale is much less than unity, so the disk should self-regulate to maintain \( Q \sim 1 \) (see Appendix B).

Finally, the surface brightness of the disk viewed face-on is
\[ F = \frac{f_{0} Q c}{\sqrt{2} \kappa} \frac{\sigma^2}{r} \sim 3 \times 10^{12} f_{0,3}^2 \sigma_{200}^2 \tau_{\text{eff}}^{-1} r_{\text{kpc}} Q r^{-1} L_{\odot} \text{ kpc}^{-2}, \]  
and the total luminosity for a single side of the disk is
\[ \dot{L} = \frac{\sqrt{2} f_{0} Q c r \sigma^2}{\kappa} \sim 2 \times 10^{13} f_{0,3}^2 \sigma_{200}^2 r_{\text{kpc}} Q r^{-1} L_{\odot}. \]  
(31)
Equations (27)–(32) show that the observable properties of the disk depend sensitively on the magnitude of the opacity. In particular, the star formation rate per unit area is proportional to \( \Sigma_{\star}^2 \tau_{\text{eff}} / \tau_T \sim \Sigma_{\star} / \kappa \) (eq. [28]). Therefore, in regions of the disk where the opacity is low, there must be more star formation to maintain \( Q \sim 1 \). Conversely, where the optical depth is high, less star formation is required. The functional dependence of \( \Sigma_{\star} \) on the optically thick limit should be compared with equation (16), which shows that in the optically thin limit \( \Sigma_{\star} \propto \Sigma_{\odot} \). Therefore, the “Schmidt law” for star formation changes in the dense optically thick inner regions of starburst galaxies.

Equation (31) can be rewritten as
\[ \frac{L}{\dot{M}} = \frac{\dot{F}}{\Sigma_{\text{int}}} = \frac{\pi Q G \sigma c}{\sqrt{2} \kappa} = \frac{2 \pi G c h}{r} \sim 10^4 f_{0,3} Q \kappa_{1} L_{\odot} M_{\odot}^{-1}. \]  
(33)
The third equality above is the classical Eddington limit, modified by the factor \( (h/r) \sim f_{0} \) and with the Rosseland mean opacity taking the place of the usual electron scattering opacity.8 This way of expressing the flux highlights the fact that each disk annulus radiates at its local Eddington limit. The value derived here for the luminosity per unit mass from the disk is similar to that estimated by Scoville (2003), who argues that this limit is observed in both young star clusters such as M51 and ULIRGs such as Arp 220 (see also Scoville et al. 2001). Note that for a given gas fraction, the mass-to-light ratio is proportional to the dust opacity and is therefore metallicity dependent.

2.3. Opacity Dependence

Figure 1 shows the Rosseland mean opacity as a function of temperature for several densities using the publicly available opacities of Semenov et al. (2003) and Bell & Lin (1994). There are two important features. First, for temperatures \( T \lesssim 100–200 \text{ K} \), the opacity is essentially independent of density and can be approximated by \( \kappa = \kappa_0 T^2 \), with \( \kappa_0 \approx 2.4 \times 10^{-4} \text{ cm}^2 \text{ g}^{-1} \text{ K}^{-2} \). The scaling of \( \kappa \) with \( T^2 \) follows from the fact that the dust absorption cross section scales as \( \lambda^{-2} \) with.

8 The factor \( 2 \pi \) rather than the more standard \( 4 \pi \) in eq. (33) is because the luminosity in eq. (32) is only from one-half of the disk.
\( \delta \rightarrow 2 \) in the Rayleigh limit (Pollack et al. 1985). The normalization \( \kappa_0 \) is somewhat uncertain and depends on grain physics and the dust-to-gas ratio; the latter may vary systematically as a function of radius and metallicity in starburst disks. In fact, in our own Galaxy there is evidence that the dust-to-gas ratio increases within the central few kiloparsecs (Sodroski et al. 1997). In what follows we set \( \kappa_{-3.6} = \kappa_0 / 2.4 \times 10^{-4} \text{ cm}^2 \text{ g}^{-1} \text{ K}^{-2} \) and retain the scaling with \( \kappa_0 \). The second important feature of Figure 1 is the dramatic decrease in the opacity for \( 10^3 \text{ K} \leq T \leq 10^4 \text{ K} \), the “opacity gap.” Here the temperature is larger than the sublimation temperature of dust but smaller than the temperature at which hydrogen is significantly ionized.

Equation (26) shows that, even for the largest galaxies with the highest gas fractions, the temperature at \( \sim 10^5 \text{ K} \) is \( \leq 100 \text{ K} \), and so the opacity on large scales can be approximated by \( \kappa = \kappa_0 T^2 \). We may then eliminate the opacity dependence from the disk properties derived above. Remarkably, because \( T \propto \Sigma_g^{1/2} \) and \( \Sigma_* \propto \Sigma_g/\kappa \), with \( \kappa \propto T^2 \) we find that the star formation rate per unit area, effective temperature, and flux are all independent of virtually all model parameters:

\[
\Sigma_* = \left( \frac{2^{9/2} \pi G \kappa_0 Q}{3 \kappa_0^2 g} \right)^{1/2} \propto \left( \frac{10^3}{e^3} \right) \kappa_{-3.6}^{-1/2} Q^{1/2} \text{ M}_\odot \text{ yr}^{-1} \text{ kpc}^{-2},
\]

(34)

\[
F = \left( \frac{2^{5/2} \pi G \kappa_0 Q G}{3 \kappa_0^2 g} \right)^{1/2} \propto \left( \frac{10^{13}}{e^3} \right) \kappa_{-3.6}^{-1/2} L_\odot \text{ kpc}^{-2},
\]

(35)

and

\[
T_{\text{eff}} = \left( \frac{25^{2/3} \pi G Q G}{3 \kappa_0 \sigma_{\text{SB}}} \right)^{1/8} \propto 88 \kappa_{-3.6}^{-1/8} Q^{1/8} \text{ K}.
\]

(36)

In particular, note that neither \( \Sigma_* \), \( F \), nor \( T_{\text{eff}} \) depends on \( r \), \( \sigma \), or \( f_g \). Our model thus predicts that starburst disks have roughly constant flux and effective temperature over a range of radii.

The constancy of these disk observables follows from three ingredients: (1) the disk is supported by radiation pressure and \( \tau \geq 1 \); (2) the disk self-regulates with \( \Omega \sim 1 \); and (3) \( \kappa \propto T^2 \) at low \( T \). Above \( T \sim 100–200 \text{ K} \), \( \kappa \) ceases to increase monotonically with \( T \) (see Fig. 1) and equations (34)–(36) no longer hold. Because \( T \propto r^{-1/2} \) (eq. [26]), the temperature exceeds \( \sim 200 \text{ K} \) at a radius \( R_{200} \sim 40 \sigma_{200}^2 G_9 T_{1/2} \Omega_{-1/2} \text{ pc} \). This radius should be compared with the radius at which \( \tau \geq 1 \). Using the \( \kappa \propto T^2 \) scaling, we find that

\[
R_{\tau = 1} = \sigma f_g \left( \frac{3 \kappa_0 Q}{211 \sigma_{\text{SB}} (\pi G)} \right)^{1/4} \sim 250 \sigma_{200}^2 G_9 T_{1/2} \Omega_{-1/2}^{1/4} \text{ pc}
\]

(37)

(compare with eq. [23]). Therefore, we expect \( \Sigma_* \), \( T_{\text{eff}} \) and \( F \) to be roughly constant in the radial range \( R_{200} \lesssim r \lesssim R_{\tau = 1} \), roughly hundreds of parsecs for fiducial parameters.

For completeness we note that the scaling for the dust temperature in the optically thin limit can also be rewritten using \( \kappa \propto T^2 \) (eq. [21]):

\[
T_{\text{dust}} \sim 60 \sigma_{200}^{1/6} \sigma_{200}^{-1/6} T_{1/6}^2 \Omega_{-1/6}^{-1/6} \text{ K}.
\]

(38)

The weak scaling with model parameters in equation (38) implies that the dust temperature should not vary significantly from system to system, as appears to be observed (e.g., Yun & Carilli 2002).

2.4. Combining the Optically Thin and Optically Thick Limits

The optically thin and optically thick limits can be combined by expressing the pressure as

\[
p = c \Sigma_* c \left( \frac{1}{2} \tau_T + \xi \right)
\]

(39)

using the full temperature-dependent opacity curve (Fig. 1). Recall that in the limit \( \tau_T \gg 1 \) equation (39) describes true radiation pressure support while in the limit \( \tau_T \ll 1 \) it describes turbulent support with contributions from supernovae and radiation pressure. Because the opacity depends on temperature, we must connect the central temperature with the effective temperature in order to solve equation (39). By interpolating between the optically thin and optically thick regimes, we obtain (see also Sirko & Goodman 2003)

\[
T^4 = \frac{3}{4} T_{\text{eff}}^4 \left( \frac{1}{2} \tau_T + \frac{2}{3} \right).
\]

(40)

In the optically thick limit, equations (39) and (40) combine to yield \( (4 \sigma_{\text{SB}}/3c) T^4 \sim \varphi \), whereas in the optically thin limit \( (4 \sigma_{\text{SB}}/3c) \tau_T T^4 \sim \varphi \). In solving equations (39) and (40), we find multiple solutions because of the complicated temperature dependence of the opacity. In Appendix B we address the thermal and viscous stability of these solutions. We argue that there is a single stable physical low-temperature solution and focus on this solution throughout the paper.

Figure 2 shows the numerically calculated structure of our disk models for \( \sigma = 200 \text{ km s}^{-1} \) and with \( f_g = 0.03 \) and 1. There are three regimes to notice in Figure 2. First, at large radii the disk is optically thin. In this region \( \Sigma_* \propto r^{-2} \) and \( \eta \propto \text{const} \) (see § 2.1). At smaller radii the disk becomes optically thick. There is then a range of radii \( (50 \text{ pc} \leq r \leq 300 \text{ pc}) \) for \( f_g = 1 \) where \( \Sigma_* \) and \( T_{\text{eff}} \) are roughly constant, in good agreement with the estimates in § 2.3. Note that where the disk is optically thin \( T_{\text{dust}} \sim T_{\text{eff}}^2 / (2 \tau_T) \). At very small radii \( r \sim 10 \) pc the opacity decreases dramatically when dust grains sublimate (the “opacity gap,” Fig. 1). In this region the disk becomes optically thin, and the star formation rate required to maintain \( Q \sim 1 \) increases significantly (see also Sirko & Goodman 2003). We present a detailed discussion of this part of the disk in § 3 and Appendix A but note here that it is unphysical to assume that the gas fraction is constant throughout the region where the star formation rate increases so markedly.

2.5. Application to ULIRGs

To focus the discussion, we emphasize the application of our optically thick disk models to Arp 220, a prototypical ULIRG. Arp 220 consists of two merging nuclei separated by about 350 pc (Graham et al. 1990). The total FIR luminosity of the system is \( \sim 10^{12} \text{ L}_\odot \). The 2–10 keV X-ray luminosity is only \( \sim 3 \times 10^9 \text{ L}_\odot \), however, and the column density of X-ray absorbing material must exceed \( 10^{23} \text{ cm}^{-2} \) if an obscured AGN is to contribute significantly to the bolometric luminosity (Iwasawa et al. 2001). Thus, there is little evidence for an energetically important AGN. The detection of numerous radio supernovae also supports a starburst origin for most of the radiation from Arp 220 (Smith et al. 1998).

The stellar velocity dispersion of Arp 220 is \( \sim 165 \text{ km s}^{-1} \) (Genzel et al. 2001), and it has an extended CO disk with a scale
The data are also consistent with the increase in flux we noted that for a gas fraction of \( f \) in agreement with that estimated by Soifer et al. (1999). Lastly, the dust sublimates and the opacity drops precipitously. Such large star formation rate per unit area is roughly constant at intermediate radii, suggesting that the nuclear region is probably optically thick even in the FIR (as implied by eq. [35]).

Our model predicts an effective temperature \( T \approx 0.03 \) when the disk is optically thick (see also eq. [18]). The model with \( f_0 = 1 \) demonstrates that the star formation rate per unit area is roughly constant at intermediate radii \( \sim 100 \) pc (see \( \S \) 2.3). At small radii \( \sim 0.01-10 \) pc, \( T \) is extremely large in the region where the dust sublimates and the opacity drops precipitously. Such large star formation rates are unphysical since the mass accreting through the starburst disk would quickly be exhausted. In \( \S \) 3 we construct more realistic models, taking into account the depletion of the gas locally as a result of star formation. Including this effect significantly reduces \( \Sigma_* \) on few parsec scales (compare with Fig. 5).

An alternative way to present these data is shown in Figure 4, where we plot the inferred flux as a function of the size of the resolved radio source. Superimposed on the data we plot the predictions of our disk models for \( \sigma = 200 \) km s\(^{-1}\) and several \( f_0 \) (solid lines) and for \( \sigma = 300 \) km s\(^{-1}\) and \( f_0 = 1 \) (dashed line). The latter is a plausible upper limit to the emission expected in our starburst models. This figure demonstrates the excellent agreement between our models and the observations for \( f_0 \approx 0.3-1 \). The data are also consistent with the increase in flux we predict for more compact starbursts. Taken together, the results of Figures 3 and 4 provide strong support for our interpretation of ULIRGs as Eddington-limited starbursts.

Note that our conclusions are not significantly changed if only, say, \( \sim 0.2 \) of the bolometric power is produced by the compact nuclei resolved in the radio (as some models of Arp 220 suggest; e.g., Soifer et al. 1999).
It is also worth comparing these results to an empirical surface brightness limit for starburst disks found by Meurer et al. (1997), who argued that starburst fluxes satisfy $F \lesssim 2 \times 10^{11} L_\odot$ kpc$^{-2}$, which corresponds to $\Sigma_{\star} \lesssim 13 \Sigma_{\odot} \text{yr}^{-1} \text{kpc}^{-2}$. Our calculations, as well as the observations summarized above, suggest that the Meurer et al. (1997) limit does not represent a fundamental limit to the surface brightness of starburst galaxies.

The characteristic flux $\sim 10^{13} L_\odot$ kpc$^{-2}$ found in Figure 3 is equivalent to a blackbody temperature of $\sim 90$ K (eq. [36]). This is noticeably larger than the typical color temperature of $\sim 60$ K inferred from the FIR spectra of ULIRGs. Another way to state this result is that using the observed FIR spectra and luminosities, the blackbody size of the FIR-emitting region is typically larger (by a factor of few) than the radio sizes observed by Condon et al. (1991). This is likely because the compact nuclei of many ULIRGs are optically thick even at $\sim 30 \mu$m, so that radiative diffusion degrades the $\sim 90$ K emission and ensures that the FIR size can be larger than the true size of the nuclear starburst. This interpretation requires sufficient obscuring gas at large radii, but also that star formation in this gas does not dominate the bolometric power of the source (or else the radio source would be more extended than is observed). In our models we find that if the gas fraction in the nuclear region increases at small radii, as Downes & Solomon (1998) infer for several systems, then most of the luminosity is produced near the radius where $r_T \sim 1$, rather than in the extended optically thin portion of the disk at larger radii.

To conclude this section, we compare our model predictions with the observations of high-redshift ULIRGs ("submillimeter sources") presented in L. Tacconi et al. (2005, in preparation). Tacconi et al. describe CO observations of four high-$z$ systems with the Plateau de Bure Interferometer. They find an average velocity dispersion of $\sigma \simeq 290 \pm 35$ km s$^{-1}$, gas fraction of $f_g \simeq 0.4 \pm 0.2$, and half-power radius for CO of $R_{CO} \simeq 1.6 \pm 0.3$ kpc. The global properties of the disks in the high-$z$ ULIRGs appear to be scaled up versions of their local counterparts, with luminosities $\sim 10$ times higher and CO disks $\sim 3$ times larger, implying comparable fluxes and $\Sigma_{\star}$. For the $f_g$ and $\sigma$ inferred from the observations, we predict that the radius at which the disk becomes optically thick to its own IR radiation is $R_{T1} \simeq 400-500$ pc. Our disk calculations yield a total luminosity of $(1-2) \times 10^{13} L_\odot$ and a flux that varies by $6 \times 10^{11} L_\odot$ kpc$^{-2}$ at 1.6 kpc to $10^{13} L_\odot$ kpc$^{-2}$ at 200 pc. We also predict a dust temperature of $\sim 57$ K at $\sim 1.6$ kpc, a sound speed of $c_s \simeq 60$ km s$^{-1}$, and a gas surface density of $\Sigma_g \simeq 0.8$ g cm$^{-2}$. These predictions are in excellent agreement with the observations of Tacconi et al. (2005): $L \sim 10^{13} L_\odot$, $F \sim 10^{12} L_\odot$ kpc$^{-2}$, $c_s \simeq 95 \pm 30$ km s$^{-1}$, and $\Sigma_g \simeq 0.6 \pm 0.1$ g cm$^{-2}$. As discussed extensively above, in local ULIRGs the nuclear starbursts appear significantly concentrated with respect to the large-scale CO disks. It remains to be seen if the same is true in their high-redshift counterparts (Chapman et al. [2004] find radio sizes of $\sim 3$ kpc in several systems, suggesting that star formation may be more extended in some high-redshift ULIRGs).

3. DISK MODELS WITH ACCRETION

In the previous section we considered the properties of large-scale starburst disks with constant gas fraction. In this section we address the problem of AGN fueling by connecting our kiloparsec-scale starburst disks with AGN disks on subparsec scales. This requires a consistent treatment of the gas fraction, which must evolve with radius as a result of star formation.

On large scales, models with constant gas fraction are equivalent to models with a constant mass accretion rate. To see this, consider a Shakura-Sunyaev accretion disk with viscosity $\nu = \alpha c_s h$. The mass accretion rate in such a disk is given by

$$M = 2 \pi \nu \Sigma_g \frac{d \ln \Omega}{d \ln r} = \frac{2^{3/2} \alpha \nu^{1/2} \Omega^{3/2}}{GQ} \frac{d \ln \Omega}{d \ln r}.$$  

Equating the scale height determined by equation (41) with that defined in equation (5) yields a one-to-one correspondence between the gas fraction and the mass accretion rate:

$$f_g = \left( \frac{8}{\alpha \nu Q} \frac{d \ln \Omega}{d \ln r} \right)^{-1} \left( \frac{MG}{\Omega r^3} \right)^{1/3}.$$  

Since $\Omega \propto r^{-1}$ in an isothermal potential, equation (42) shows that a constant gas fraction implies a constant accretion rate. On small scales, where the black hole dominates the gravitational potential, $f_g \propto r^{1/2}$ for constant $M$.

In reality, the accretion rate (or $f_g$) is not constant. As gas accretes from larger to smaller radii and some of the gas is converted into stars to maintain $\Omega \sim 1$, the accretion rate decreases monotonically. At the outer radius of the disk $R_{out}$, we assume that gas is supplied at a rate $M_{out}$. At any radius $r < R_{out}$ the accretion rate $M(r)$ is

$$M(r) = M_{out} - \int_{R_{out}}^r 2 \pi r'^2 \Sigma_{\star} \, dr'.$$  

If the star formation rate required to maintain $\Omega \sim 1$ is large enough, all of the gas goes into stars and the accretion rate becomes vanishingly small. To quantize this, it is useful to consider two timescales characterizing the disk: (1) the advection
timescale $\tau_{\text{adv}} = r/V_r$, and (2) the star formation timescale $\tau_s = 1/(\rho \Omega)$. At any radius in the disk, there is a critical accretion rate $\dot{M}_c$ for which $\tau_{\text{adv}} = \tau_s$. Taking the optically thick limit and assuming $\kappa = \kappa_0 T^2$ (§ 2.3) yields

$$\dot{M}_c = r^2 \left( \frac{2^{13/2} \pi^3 G \rho \sigma_{\text{SB}}}{3 \kappa_0^2 c^3} \right)^{1/2} \sim 280 r_{200}^{-2/3} \kappa_{-1} Q_{1.6}^{1/2} M_\odot \text{ yr}^{-1},$$

(44)

where we have scaled the radius to 200 pc in anticipation of numerical calculations described below. If the gas accretion rate in the disk satisfies $\dot{M} > \dot{M}_c$, then $\tau_{\text{adv}} < \tau_s$ and the gas is able to accrete inward to smaller radii. By contrast, if $\dot{M} < \dot{M}_c$, then $\tau_{\text{adv}} > \tau_s$ and most of the gas is converted into stars at radius $r$ without flowing significantly inward. We note that gas supply rates exceeding $\dot{M}_c$ seem plausible in view of the fact that many ULIRGs have star formation rates greater than $\dot{M}_c$.

Equation (44) has several important properties. First, $\dot{M}_c$ is seemingly independent of the viscosity. This results from the fact that we have assumed $\kappa \propto T^2$, which is valid only at low temperatures and thus large radii. For general $\kappa$ or in the optically thin limit, $\dot{M}_c$ depends explicitly on the viscosity (see Appendix D). Second, $\dot{M}_c$ is an increasing function of radius. Thus, if gas is supplied at the outer radius $R_{\text{out}}$ at a rate $\dot{M}_{\text{out}} > \dot{M}_c$, significant gas accretion can continue to smaller radii. However, on scales smaller than $\sim 10$ pc, the central disk temperature is high enough that dust sublimates, $\kappa$ decreases dramatically, and the star formation rate required to maintain $Q \sim 1$ increases (Fig. 2; see also Sirk & Goodman 2003). The large star formation rate at small radii makes it difficult to fuel a central accretion disk at a rate sufficient to explain bright AGNs. This competition between star formation ($\tau_s$) and inflow ($\tau_{\text{adv}}$), particularly throughout the opacity gap on $0.1$–$10$ pc scales, determines the rate at which a central AGN is fueled.

3.1. A Model of AGN Fueling

In Appendix C we collect the equations and parameters used in this section to derive the properties of starburst and AGN disks. As in § 2, we assume that $\Omega = \Omega_K$, but here we account for the gravitational potential of the black hole: $\Omega_K = GM_B/r^2 + 2\pi^2 \rho^2 / r^2$. The black hole mass is assumed to be given by the $M_B - \sigma$ relation: $M_B \simeq 2 \times 10^8 \sigma_{200}^2 M_\odot$ (Tremaine et al. 2002; Ferrarese & Merritt 2000; Gebhardt et al. 2000). In calculating vertical hydrostatic equilibrium, we include gas pressure, which is important for $r \lesssim 1$ pc. The most important change relative to the model of § 2 is the use of a consistent accretion rate that accounts for the loss of gas locally to star formation (eq. [43]).

Although the critical accretion rate $\dot{M}_c$ at which $\tau_{\text{adv}} = \tau_s$ does not depend that strongly on the viscosity in the disk at large radii (eq. [44]), the fate of gas at small radii in the opacity gap is a very strong function of the rate of angular momentum transport. The efficiency of angular momentum transport is important because, for fixed $\dot{M}$, higher viscosity implies lower surface density and thus self-gravity is comparatively less problematic. Indeed, it is well known that local angular momentum transport, such as is produced by the magnetorotational instability (Balbus 2003), is incapable of supplying sufficient gas to a central black hole to account for luminous AGNs (e.g., Shlosman & Begelman 1989; Shlosman et al. 1990; Goodman 2003).

One possible solution to this problem is that angular momentum transport proceeds by global torques such as would be provided by stellar bars, spiral waves, or large-scale magnetic stresses (Shlosman et al. 1990; Goodman 2003). In this section we use a phenomenological prescription to describe this process: we assume that the radial transport of gas by a global torque allows the radial velocity to approach a constant fraction $m$ of the local sound speed (Goodman 2003). In this case,

$$\dot{M} = 2 \pi r \Sigma_g V_r = \frac{2^{3/2} \Omega^2 r h^2 m}{GQ}$$

and the relationship between accretion rate and gas fraction is given by

$$f_g = \left[ \frac{2^{1/2} M G}{Qm (\Omega r)^3} \right]^{1/2}$$

instead of equation (42). Our hope is that, much as the Shakura-Sunyaev prescription provides a useful zeroth-order model for local angular momentum transport in disks, the above model captures some of the essential physics of disks in which angular momentum transport is dominated by global torques. With equation (45) to relate the gas surface density to the gas accretion rate, we solve the equations of Appendix C to determine the structure of the disk.

3.2. Results

The dashed lines in Figure 5 show the mass accretion rate $\dot{M}$ as a function of radius for $M_{\text{out}} = 80, 160, 220, 320, \text{and} 640 M_\odot \text{ yr}^{-1}$ and $M_B = 320 M_\odot$. The latter produce outer ($r \sim R_{\text{out}}$) and inner ($r \sim 1$–$10$ pc) starbursts but also fuel a bright central AGN with $M_B \sim 4 M_\odot \text{ yr}^{-1}$ (Appendix A). Fig. 7 shows the computed spectra for each model. Fig. 6 shows the full disk structure for the model with $M_{\text{out}} = 320 M_\odot \text{ yr}^{-1}$.

\[^{10}\text{An alternative possibility not considered here is that AGNs are fueled by low angular momentum gas (Goodman 2003), including perhaps the hot ISM in clusters of galaxies (e.g., Nulsen & Fabian 2000).}\]
640 $M_\odot$ yr$^{-1}$ in a model with $\sigma = 300$ km s$^{-1}$ ($M_{BH} \simeq 10^8 M_\odot$), $R_{out} = 200$ pc, and with dynamical angular momentum transport specified by $m = 0.2$. The black hole dominates the gravitational potential for $r \lesssim 50$ pc. The solid lines show the local star formation rate $\dot{M}$, defined by $\dot{M} = \pi r^2 \Sigma_*$.

There are two types of solution represented in Figure 5. The two models with the smallest $M_{out}$ lose essentially all of the supplied gas at large radii to star formation. At $r \sim 20-40$ pc in these two models the star formation rate becomes so low that $M$ approaches a constant near $-0.1 M_\odot$ yr$^{-1}$. This occurs when the central temperature decreases sufficiently that gas pressure dominates. These models have $\dot{M}_{adv} > \tau_s$ at $R_{out}$ and $M_{out} < M_{c}(R_{out})$. They are starburst dominated and the star formation occurs predominantly at $R_{out}$.

For the three models shown with $M_{out} \geq 220 M_\odot$ yr$^{-1}$ the results are qualitatively different. In these models, $M_{out}$ is large enough that star formation persists for more than a decade in radius from $R_{out} = 200$ pc down to $r \sim 1-10$ pc. At these small scales, the temperature is sufficiently hot that dust sublimes and the opacity decreases sharply (see Fig. 1). Because $\Sigma_* \propto \Sigma_g/\kappa$ in the optically thick limit, the star formation rate must increase dramatically to maintain $Q \sim 1$. The increase in $M_{c}$ as the opacity gap is encountered is typically an order of magnitude. Although these star formation rates are large, they are much smaller than those computed for the constant $M$ (or constant $f_\dot{M}$) solutions in Figure 2 ($\S$ 2). In the models with variable $f_\dot{M}$ an equilibrium between advection and star formation, expressed by the equality $\dot{M}_{adv} = \tau_s$, limits the star formation rate in the opacity gap (Appendix A). Each of these high-$f_\dot{M}$ disk models has a gas accretion rate of $\sim 4 M_\odot$ yr$^{-1}$ at $r \sim 0.01$ pc, sufficient to power a luminous AGN. That these models have nearly identical accretion rates at small radii follows from the fact that $\dot{M}_{adv} = \tau_s$ in the opacity gap. In Appendix A we present an analytical model for the structure of the disk in this region and provide simple formulae for the black hole accretion rate.

As Figure 5 shows, there is a strong bifurcation between models that fuel a central AGN and those that do not. The critical mass supply rate distinguishing these two classes of solutions is close to $M_{c}$ as estimated in equation (44) by equating $\dot{M}_{adv}$ and $\tau_s$.

As an aside we note that all models presented in Fig. 5 have small $M_{c}$ at small radii and one may worry about discreteness; that is, for very small $M_{out}$ only a few massive stars may be formed in a given annulus, and because an individual massive star only supports a volume $\sim 4\pi r^3$ around it, the disk may not be capable of self-regulation. The “continuum” approximation employed throughout this work is only valid when $r/h \ll N_{\nu}(r) \sim M_{c}(r) T_{m\nu} 10^{-11.35} M_\odot^{-1}$, where $N_{\nu}(r)$ is the number of massive stars at radius $r$, $T_{m\nu}$ is the lifetime of a massive star, and we have assumed a Salpeter IMF and defined “massive stars” as those with mass greater than $\sim 10 M_\odot$. For $T_{m\nu}$ in the range $10^7-10^8$ yr the continuum approximation is valid for the models displayed in Fig. 5. For a discussion of when the continuum limit breaks down, see footnote 14.
For comparison with these calculations assuming dynamical angular momentum transport, we have also calculated disk solutions employing a local $\alpha$ viscosity (eq. [41]). With $\alpha = 0.3$, the disk structure exterior to the opacity gap is very similar to the models with dynamical angular momentum transport because $h/r$ is large on scales of $R_{\text{AGN}}$ in the models with high $M_{\text{out}}$. Therefore, a local viscosity can transport gas from several hundred parsecs down to $\sim 10$ pc scales in gas-rich starbursts. However, we find that a local viscosity cannot transport gas through the opacity gap at a rate sufficient to fuel bright AGNs; typical black hole accretion rates are several orders of magnitude smaller than those with dynamical angular momentum transport (see Appendix A).

In the three models with $M_{\text{out}} \geq 220 M_\odot \, \text{yr}^{-1} > M_*$, we can distinguish between the outer “starburst disk,” where the heating of the disk is dominated by star formation ($\sigma_{\text{SB}} T_\text{eff} = \frac{1}{4} \Sigma_c c^2$), and the inner “AGN disk,” where the heating is dominated by accretion ($\sigma_{\text{SB}} T_\text{eff} = (3/8\pi) M [1 - (R/r)^{1/2}]^{1/2}$). The transition between these two regimes occurs at $R_{\text{AGN}} \sim 0.5$ pc in the models presented in Figure 5. At this radius, accretion heating is sufficient to produce $Q > 1$ and star formation ceases.

By combining the equations describing the “starburst” disk at $r > R_{\text{AGN}}$ with those describing the “AGN disk” at $r < R_{\text{AGN}}$, we have computed the full radial structure of models from hundreds of parsecs down to the central black hole. We note again that there are multiple solutions at some radii and that we have selected what we believe is the physical disk solution as described in Appendix B. As an example of the full radial disk structure, Figure 6 shows the results for the model with $M_{\text{out}} = 320 M_\odot \, \text{yr}^{-1}$ (see also Fig. 5). In the middle left panel, note that gas pressure dominates radiation pressure over $\sim 1$ decade in radius inside the opacity gap at $r \leq 1$ pc (Appendix A) but that radiation pressure dominates at all other radii.

Assuming that every annulus in the disk radiates as a blackbody and that the disk is viewed face-on, the spectral energy distribution of our disk models can be computed using

$$\lambda L_\lambda = \frac{2\pi c^2}{\lambda^4} \int_{R_{\text{in}}}^{R_{\text{out}}} \frac{2\pi r \, dr}{\exp[hc/\lambda k_B T_{\text{eff}}(r)] - 1}.$$  \hspace{1cm} (47)

The resulting multicolor blackbody spectra for the models presented in Figure 5 are shown in Figure 7, assuming an inner disk radius of $R_{\text{in}} = 3(2GM_{\text{BH}}/c^2)$. The models with small mass supply rate ($M_{\text{out}} < M_*$) have the lowest bolometric luminosity and virtually all of their flux is generated by the starburst at $R_{\text{out}}$. The spectra of these models peak at $\sim 50 \mu$m, corresponding to an outer disk dust temperature of $\sim 70$ K. For higher $M_{\text{out}}$, the dust temperature at $R_{\text{out}}$ increases somewhat and the spectral peak moves to shorter wavelengths.

Higher $M_{\text{out}}$ models have similar FIR spectra. However, at $\sim 8 \mu$m they exhibit an additional spectral peak. This peak is produced by the inner ring of star formation on $1-10$ pc scales shown in Figure 5, which is caused by the decrease in the opacity of the disk at the dust sublimation temperature, $T_{\text{sub}} \sim 1000$ K. Although $T_{\text{sub}}$ sets the central temperature of the disk where this sudden increase in $\Sigma_c$ occurs, the disk is typically optically thick at these radii and the effective temperature can be a factor of $\sim 3-4$ smaller, accounting for the fact that the spectral peak associated with star formation in the opacity gap is at $\sim 8 \mu$m. Together, the FIR peak from the outer starburst ring and the peak from the inner starburst ring create a broad IR peak spanning $\sim 1.5$ decades in wavelength.

The distinguishing feature of the models with large $M_{\text{out}}$ is the significant contribution of the AGN to the bolometric luminosity: the UV emission in Figure 7 is comparable to or dominates the peaks at $\sim 8$ and $\sim 50 \mu$m. Figure 7 suggests that quasar accretion disks can plausibly be fed by radiation pressure-supported starburst disks at larger radii. Indeed, the broad but subdominant IR peak and the magnitude of the AGN luminosity in our models are reasonably consistent with composite quasar spectra (Elvis et al. 1994; Haas et al. 2003).

It should be noted that the spectra shown in Figure 7 are photospheric and do not include AGN reprocessing. Because the disk may intercept some of the AGN emission, we expect that reprocessing could lead to an additional emission component at $\sim 1-2 \mu$m if dust in the surface layers of the disk is heated to the sublimation temperature.

3.3. Vertical Structure and Nuclear Obscuration in AGNs

At radii $\sim 1$ pc the models shown in Figure 6 have the interesting property that the central temperature of the disk is above the sublimation temperature of dust while the effective (surface) temperature of the disk is below the sublimation temperature. As a result, there is a large vertical gradient in the opacity of the disk. Because the disk is radiation pressure dominated, this large gradient in the opacity has important implications for the disk’s vertical structure. Here we argue that
the photosphere of the disk can lie a distance \( h_{\text{ph}} \sim r \) off of the midplane even though the midplane scale height is small, with \( h \ll r \) (where \( h = c_s/\Omega \) is evaluated using the midplane properties of the disk). We then discuss the implications of this result.

The equations governing the vertical structure of the disk include those of vertical hydrostatic equilibrium and vertical energy transport:

\[
\frac{dp}{dz} = -\rho\Omega^2 z, \tag{48}
\]

where \( p = p_{\text{gas}} + p_{\text{rad}} \) and we assume that \( \Omega \) is independent of the height \( z \) in the disk. Assuming that energy is transported diffusively by photons implies

\[
\frac{dT}{dz} = -\frac{3\kappa\rho F}{16T^3\sigma_{\text{SB}}}, \tag{49}
\]

where \( F \) is the vertical flux of energy. If the disk is radiation pressure dominated, as our models are at most radii, then equations (48) and (49) imply that the flux in the disk must be everywhere equal to the Eddington flux:

\[
F = F_{\text{Edd}} = \frac{\Omega^2 cz}{K}, \tag{50}
\]

The presence of a large vertical opacity gradient suggests that convection might develop in the outer atmosphere of the disk. However, two arguments show that convection is unlikely to be important in the present context: (1) Our disk models have \( \tau_{\text{th}}\Omega \ll 1 \), where \( \tau_{\text{th}} \) is the cooling time of the disk (Appendix B). This implies that the diffusion time through the disk is much shorter than the characteristic timescale of buoyant convective motions. In the presence of such strong radiative diffusion, convective modes driven by radiation entropy gradients grow very slowly, if at all (e.g., Blaes & Socrates 2003). (2) Even if convection develops, it is unlikely to carry a significant fraction of the vertical energy flux. Using \( F \approx \rho V_c^3 \) to estimate the convective velocity \( V_c \) required to carry the energy flux, we find very high Mach numbers:

\[
V_c \approx 40M_y^{1/3}(\frac{r}{z})^{1/3}(\frac{F}{F_{\text{Edd}}})^{1/3}(\frac{1}{3})^{-1/3}T^{-2/3}c_{s}^{-2/3}z^{-2/3}1000\kappa_{1}^{-1/3}\rho_{-15}^{-1/6}, \tag{51}
\]

where \( \rho_{-15} = \rho/10^{-15} \) g cm\(^{-3} \) and we have scaled our estimate to parameters appropriate to parsec-scale disks (see Fig. 6). These considerations strongly suggest that radiative diffusion dominates the vertical transport of energy throughout the disk.

Unfortunately, equations (48) and (49) are insufficient to fully specify the vertical structure of the disk. An equation for the vertical generation of energy, \( dF/dz \), is also needed (e.g., Davis et al. 2005). The vertical heating profile is particularly uncertain in the present context where star formation, rather than accretion, dominates the heating of the disk. Given this significant uncertainty, we restrict ourselves to the following simple considerations about the disk’s vertical structure.

We assume that the calculations described in the previous sections, which apply to the midplane of the disk, provide an adequate estimate of the star formation rate \( \Sigma_s \) and, thereby, the flux \( F \) required to maintain \( \dot{Q} \sim 1 \). This assumes that the atmosphere of the disk does not contribute much flux, a reasonable assumption. Because the disk is radiation pressure dominated near its photosphere, we can estimate the location of the photosphere using \( F \approx F_{\text{Edd}} \). This yields

\[
h_{\text{ph}} \approx \frac{F_{\text{ph}}}{\Omega c} \approx h\left(\frac{\kappa_{\text{ph}}}{\kappa_{\text{mid}}}\right), \tag{52}
\]

where \( \kappa_{\text{ph}} \) is the opacity at the photosphere, evaluated using the effective temperature, and \( \kappa_{\text{mid}} \) is the opacity at the midplane, evaluated using the central temperature. Equation (52) shows that if the opacity at the surface of the disk is much larger than the opacity near the midplane, as will inevitably occur in the vicinity of the sublimation temperature of dust, then \( h_{\text{ph}} > r \) and the midplane scale height does not provide a good estimate of the location of the photosphere of the disk.

Figure 8 shows our estimates of the midplane scale height \( h/r \) and the photospheric scale height \( h_{\text{ph}}/r \) for models with \( M_{\text{out}} = 320 \) and 640 \( M_y \) yr\(^{-1} \) (whose other properties have been described in Figures 5–7). These estimates of the photospheric scale height show that \( h_{\text{ph}} \) can readily approach \( \sim 0.1 \)–10 pc, even though the midplane scale height of the disk satisfies \( h \ll r \). Because the effective temperature of the disk is \( T_{\text{sub}} \), the opacity near the photosphere is \( \kappa \sim 1 \) g cm\(^{-2} \) and the hydrogen column through the disk’s atmosphere can approach \( \sim 10^{24} \) cm\(^{-2} \). We suggest that this extended dusty atmosphere may account for the presence of obscuring material inferred from observations of AGNs (e.g., Antonucci 1993). In particular, although AGN disks are typically classified as being “thin” because \( h \ll r \), Figure 8 shows that the photosphere may nonetheless puff up substantially and reach \( h_{\text{ph}} \sim r \). Some of this material is likely to be unbound and the origin of a dusty outflow.

The disk’s extended atmosphere is supported by radiation pressure from a parsec-scale nuclear starburst (in contrast to Pier & Krolik 1992), who argued that dusty gas could be
supported at $h \sim r$ by radiation pressure from the AGN). Estimating the luminosity of the disk at these radii, $L \equiv \pi r^2 F$, we find

$$L \approx \frac{\pi GMc}{\kappa_{ph}} \left( \frac{h_{ph}}{r} \right) \approx \frac{L_\odot}{4} \left( \frac{\kappa_{es}}{\kappa_{ph}} \right) \left( \frac{h_{ph}}{r} \right),$$

(53)

where $L_\odot$ is the canonical Eddington luminosity defined using the electron scattering opacity $\kappa_{es}$. Taking $\kappa_{ph} \sim 10\kappa_{es}$, equation (53) implies that in order to have $h_{ph} \sim r$, as suggested by the relative number of type 1 and 2 Seyfert galaxies, the luminosity of the parsec-scale disk must be $\approx 0.01L_\odot - 0.1L_\odot$. Therefore, in order for the disk to have $h_{ph} \sim r$, we predict the presence of a very compact nuclear starburst with a luminosity similar to that of the AGN.

Because the luminosity from star formation required to “puff up” the disk is substantial, it is unlikely that this model is relevant to low-luminosity Seyfert 2 galaxies such as NGC 4258 (where there is also strong evidence that the nuclear obscuration is due to a warped disk; e.g., Fruscione et al. 2005). A compact starburst may, however, dynamically support obscuring material in more luminous AGNs. In particular, there is evidence for a compact starburst in the prototypical Seyfert 2 galaxy NGC 1068. IR interferometry of NGC 1068 has also directly resolved warm dusty gas on the parsec scales predicted by our model (Jaffe et al. 2004; Rouan et al. 2004).

4. DISCUSSION AND CONCLUSIONS

In standard models of galactic-scale star formation, energy and momentum injected by supernovae and stellar winds are assumed to drive turbulent motions in the ISM of a galaxy (e.g., Silk 1997). This turbulent pressure helps stave off the self-gravity of the disk and maintain marginal stability to gravitational perturbations (Toomre’s $Q \sim 1$). In this paper we have focused instead on the role of radiation pressure on dust grains in regulating the structure and dynamics of star formation in galaxies. To order of magnitude, the turbulent pressure from supernovae, stellar winds, and the radiation from massive stars are all comparably important when the galactic disk is optically thin to its own IR radiation, as in normal star-forming galaxies. By contrast, when the disk is sufficiently optically thick to the IR, radiation pressure provides the dominant vertical support against gravity. This condition is met in the inner few hundred parsecs of luminous gas-rich starbursts, most notably in ULIRGs. In addition, the outer parts of accretion disks around AGNs are expected to be dominated by radiation pressure on dust. Understanding the dynamics of disks in the radiation pressure–dominated limit is therefore crucial for understanding nuclear starbursts and AGN fueling.

We have constructed simple quantitative models of self-regulated disks appropriate to both the optically thin and optically thick limits. From these models, we derive the star formation rate per unit area required to maintain $Q \sim 1$. In the optically thin limit, we find that $\Sigma_* \propto \Sigma_g^2$ (eq. [16]), a slightly steep version of the Schmidt law for star formation, whereas in the optically thick limit, $\Sigma_* \propto \Sigma_g^2/\gamma \propto \Sigma_g/\kappa$ (eq. [28]). Because radiation pressure dominates the vertical pressure support in the optically thick limit, the star formation rate is sensitive to the mean opacity of the disk and thus also to the metallicity. In fact, each annulus of the disk radiates at its local Eddington limit, defined using the Rosseland mean opacity $\kappa$. This criterion can be written as (see also Scoville 2003) $L/M \sim 10^3 f_g \kappa L_\odot M_\odot^{-1}$, where the gas fraction $f_g$ is proportional to the disk thickness $h/r$.

Our prediction that the Schmidt law changes in the optically thick limit (with $\Sigma_* \propto \Sigma_g/\kappa$) is currently difficult to test and requires further high-resolution observations. It is important to stress that in our models the star formation efficiency in the optically thick nuclei of starbursts is actually lower than what would be inferred from an extrapolation of the star formation efficiency in normal star-forming galaxies because of the extra support from radiation pressure in the optically thick limit.

Our calculations show that in the optically thick portion of starburst disks near $\sim 100$ pc, the flux, star formation rate per unit area, and effective temperature of the disk are roughly constant with characteristic values of $F \sim 10^{13}$ $L_\odot$, $\kappa_{ph}^{-2}$, $\Sigma_* \sim 10^3 M_\odot$ yr$^{-1}$ kpc$^{-2}$, and $T_{eff} \sim 90$ K ($\gamma \sim 1$). To test these predictions, we have estimated the fluxes (temperatures) in the emission regions of ULIRGs using the resolved radio images of Condon et al. (1991) as indicative of the sizes of nuclear starbursts in these systems (the fluxes inferred using FIR blackbody temperatures may not accurately characterize the fluxes in the emission region if there is still significant obscuration at $\sim 20–30$ $\mu$m). We find excellent agreement with the predictions of our models (Figs. 3 and 4). This supports our interpretation that a significant fraction of the radiation from ULIRGs is produced by an Eddington-limited starburst. In the future, imaging at optically thin FIR wavelengths will test our assumption that the radio emission traces star formation in ULIRGs.

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In our model of starburst disks, the central temperature exceeds the dust sublimation temperature ($T_{sub} \sim 1000$ K) at radii $\sim 1–10$ pc. The opacity of the disk then drops dramatically and thus self-gravity is comparatively less problematic. For this reason, we have focused on the possibility that angular momentum transport proceeds via a global torque, as would be provided by a bar, spiral waves, or a large-scale wind. With this prescription for angular momentum transport we have extended our starburst disk models to smaller radii where they connect consistently with AGN disks on parsec scales. A key ingredient to these calculations is an equation that accounts for the radial loss of gas due to star formation (eq. [43]). This limits the star formation rate required to support the disk in the opacity gap region and, therefore, allows for significant gas accretion and AGN fueling (Appendix A). In this respect our model differs from that of Sirko & Goodman (2003), who also considered star formation–supported AGN disks but assumed constant $M$. Our model provides a way of quantifying the connection between starbursts and AGN activity. In particular, we find two classes of disk models (Figs. 5–7): (1) For mass supply rates at the outer edge of the disk less than a critical rate $M_\ell$ (eq. [44]), a starburst occurs primarily in a narrow ring at $R_{loop}$. Nearly all of the gas is converted into stars at large radii and the spectrum is dominated by an FIR starburst peak. This class of solutions corresponds to the limit in which the star formation time in the disk is shorter than the viscous time ($\tau_\ell < \tau_{adv}$). (2) On the other hand, for $M_{out} > M_\ell$, a fraction of the gas accretes inward to smaller radii because the advection timescale is shorter than the star formation timescale. These disk solutions have an outer
starburst at $\sim R_{\text{out}}$ and an inner nuclear starburst ring on $\sim 1$–10 pc scales at the opacity gap (Fig. 5). In addition, for fiducial parameters we find that the accretion rate onto the central black hole is $\sim 1$–10 $M_\odot$ yr$^{-1}$ (Appendix A). Models with $M_{\text{out}} > M_\bullet$ can be dominated by AGN emission, although there is a prominent contribution in the mid- and far-IR from the inner and outer starbursts (Fig. 7). Both the broad but subdominant IR peak and the magnitude of the AGN luminosity in our models are reasonably consistent with composite AGN spectra.

One prediction of our models is that a significant fraction of the IR emission in luminous AGNs must be from star formation rather than reprocessing. The star formation is required to support the accretion disk at large radii against its own self-gravity (for an observational discussion of this point see Rowan-Robinson 2000). We have identified an additional observational consequence of the parsec-scale starburst that is predicted to occur when the temperature of the disk reaches the sublimation temperature of dust. Our estimates suggest that for luminous AGNs, the nuclear starburst is able to inflate the photosphere of the inner disk to a height $h_{\text{ph}} \sim r$ ($\S$ 3.3 and Fig. 8). This extended atmosphere of the disk contains little mass but produces significant obscuration, and it may account for some of the nuclear obscuration observed in type 2 AGNs.

As a final application of our calculations, we note that there is direct observational evidence for compact starbursts near massive black holes. In particular, observations of the Galactic center reveal a population of young O and B stars within the central parsec of our Galaxy. Most of these stars appear to lie in a thin disk located within 0.1–0.3 pc of the black hole (Levin & Beloborodov 2003; Genzel et al. 2003). The origin of these stars remains uncertain, but their disklike kinematics suggests that they might have formed in a dense self-gravitating accretion disk, if the black hole in the Galactic center were accreting at a much higher rate several million years ago (e.g., Levin & Beloborodov 2003). To explore this hypothesis in the context of our models, Figure 9 shows the star formation rate and accretion rate as a function of radius in models with $\sigma = 75$ km s$^{-1}$, $M_{\text{BH}} = 4 \times 10^6$ $M_\odot$, $R_{\text{out}} = 3$ pc, and an $\alpha$ viscosity with $\alpha = 0.3$ (we also increased the opacity by a factor of 3 relative to the curves presented in Figure 1 to account for the supersolar metallicity of the Galactic center). The chosen outer radius is appropriate if the circumnuclear disk in the Galactic center, a central reservoir of $\sim 10^5$–$10^6$ $M_\odot$ of gas (e.g., Jackson et al. 1993; Shukla et al. 2004), were accreted onto the central black hole. The mass supply rates we have used, $M_{\text{out}} = 0.015$–0.15 $M_\odot$ yr$^{-1}$, are in the range expected if the circumnuclear disk were accreted on a viscous timescale $\sim 10^8$ yr. Figure 9 shows that the star formation rate in this hypothesized burst of accretion typically has a strong peak at $r \sim 0.1$ pc. This is the “inner nuclear starburst” caused by the decrease in opacity when the central disk temperature exceeds the sublimation temperature of dust. The location of this starburst is strikingly similar to the current location of the stellar disk in the Galactic center.$^{14}$

Several aspects of our calculations require further study. The first is the stability of our disk solutions. It is well known that radiation pressure–supported disks are prone to numerous instabilities: viscous, thermal, convective, and photon bubble (e.g., Lightman & Eardley 1974; Piran 1978; Blaes & Socrates 2003). Such instabilities have been extensively studied in the context of the central $\sim 10$–100 Schwarzschild radii of black hole accretion disks, where the radiation pressure is on free electrons. An important difference between our solutions and these is that at large radii the disk is heated primarily by star formation, not accretion. We find that as a result, the disk is globally thermally and viscously stable (Appendix B). As in the local ISM, however, the disk may fragment into a multiphase medium. A second aspect of our model that requires further study is the physics of star formation under the unusual conditions appropriate to gravitationally unstable accretion disks, particularly at radii $\sim 1$ pc where the gas is much denser and hotter than in normal star-forming regions. Because $Q \ll 1$ in the absence of star formation, and because the cooling time of the disk is much shorter than the orbital time (Appendix B), we believe that fragmentation of the disk is inevitable. But beyond this, the properties of star formation under these conditions are poorly understood (Goodman & Tan 2004 argue that the disk will form supermassive stars). Interestingly, the power-law solutions for the star formation rate in the opacity gap region (Appendix A) are independent of the IMF-dependent parameter $e$ (eq. [10]). On larger scales ($r \gtrsim 1$–10 pc), however, a top-heavy IMF (larger $e$) can suppress the star formation rate (eqs. [16] and [28]), decrease the star formation efficiency $\eta$ (eqs. [18] and [30]), and decrease the critical mass accretion rate required for AGN fueling (eq. [44]). Another component of our model requiring further study is the interaction of the AGN emission with the flared starburst disk and the tenuous dusty obscuring atmosphere predicted in $\S$ 3.3.
Finally, we note that large-scale outflows of cold dusty gas are commonly observed from starbursting galaxies, including ULIRGS (Alton et al. 1999; Heckman et al. 2000; Martin 2005). We have recently proposed that radiation pressure on dust is an important mechanism in driving such winds (Murray et al. 2005, hereafter MQT05). The disk models presented here provide support for this idea because it is natural to suspect that a radiation pressure–supported starburst disk will drive a wind from its photosphere, much as near-Eddington stars drive strong outflows. In MQT05 we further argued that both starbursts and AGNs have a maximum Eddington-like luminosity given by \( L_{\text{M}} = 4f \sigma^4 c^4 G/M \). One can show that the disk models presented in this paper have luminosities \( \lesssim L_{\text{M}} \), with the equality obtained (approximately) when \( h \sim r \) and the spherical limit employed in MQT05 is realized. We also note that the results in this paper, together with those of MQT05, imply that fueling a luminous AGN requires an accretion rate in the range \( M_c \leq M_{\text{out}} \leq M_{\text{max}} = L_{\text{M}}/c^2 \). With the black hole accretion rate given by equation (A4), this yields a ratio of star formation rate to black hole accretion rate of \( \sim 100–1000 \), similar to the observed ratio of stellar mass to black hole mass in nearby galaxies (Magorrian et al. 1998; Häring & Rix 2004).

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APPENDIX A

THE BLACK HOLE ACCRETION RATE

An interesting feature of the results shown in Figure 5 is that all of the models with \( M_{\text{out}} > M_\star \) deliver precisely the same amount of mass to the AGN disk at small radii. More specifically, all of the dashed lines for \( M_{\text{out}} \geq 220 M_\odot \text{yr}^{-1} \) in Figure 5 come together and attach to a single power-law solution in the opacity gap between \( r \sim 1 \) and \( 10 \text{ pc} \) and then settle onto a constant \( \dot{M} \approx 4 M_\odot \text{yr}^{-1} \) solution at smaller radii. In this appendix we explain why this is the case.

The steep drop in the opacity in Figure 1 occurs when the central disk temperature reaches the sublimation temperature \( T_{\text{sub}} \) at a radius \( R_{\text{sub}} \). In the models presented in Figure 5 with \( M > M_\star \), \( R_{\text{sub}} \) corresponds to the sudden increase in \( M_\star \) between \( \sim 1 \) and \( 10 \text{ pc} \). The rapid decrease in \( \kappa \) inside the opacity gap can be approximated by a power law, \( \kappa \propto T \). For example, in the opacity models of Bell & Lin (1994), \( \beta = -24 \) and \( T_{\text{sub}} \approx 900 \text{ K} \) (see Fig. 1). Because the star formation rate required to maintain \( Q \sim 1 \) in the optically thick limit is given by \( \Sigma_\star \propto \Sigma_\delta/\kappa \) (eq. [28]), one might expect \( \Sigma_\star \) to increase by \( \sim 5 \) orders of magnitude inside the opacity gap because \( \kappa \) decreases by this amount at \( T_{\text{sub}} \) (see Fig. 1). In the models with constant \( f_\delta \) shown in Figure 2, this is precisely what happens. However, when we self-consistently solve for \( M_\star \) (eq. [43]), as in the models presented in Figure 5, the results are qualitatively different. The star formation rate increases by only \( \sim 1 \) order of magnitude at \( r \sim R_{\text{sub}} \) because of an important physical effect. As \( \kappa \) decreases and \( M_\star \) increases, the accretion rate must decrease because the gas is depleted as a result of star formation. This leads to a lower surface density and, therefore, a lower star formation rate required to maintain \( Q \sim 1 \). Thus, an equilibrium is established between star formation and accretion in the opacity gap (\( \tau_{\text{adv}} = \tau_\star \)). An analytic solution representing this balance between star formation and accretion can be derived by solving equation (43) for \( M_\star \) using \( \kappa \propto T^3/4 \), \( T \propto \Sigma_\delta/\kappa \) (eq. [26]), \( \Sigma_\star \propto \Sigma_\delta/\kappa \) (eq. [28]), and the fact that there is a one-to-one relationship between \( f_\delta \) and \( M_\star \). If angular momentum transport proceeds via global torques, the \( f_\delta-M_\star \) relation is given by equation (46), and the analytic solution for \( M_\star \) takes the form

\[
\dot{M}(r)^{1/2+\beta/4} - \dot{M}(R_{\text{sub}})^{1/2+\beta/4} = A \left( r^{5/4+3/8} - R_{\text{sub}}^{5/4+3/8} \right),
\]

where \( A \) is a \( \beta \)-dependent constant that is specified below. Because \( \beta = -24 \) in the opacity gap, the solution quickly loses memory of the initial conditions at \( R_{\text{sub}} \). As a result, a single universal equilibrium power-law solution for \( M \approx M_\star \) exists in the opacity gap. Because of this equilibrium, the increase in \( M_\star \) at \( R_{\text{sub}} \) is much less pronounced in these solutions than in the constant \( f_\delta \) models (compare Figs. 2 and 5).

The general expression for the constant \( A \) for arbitrary \( \beta \) is a bit awkward, but the limit \( \beta \rightarrow -\infty \) (as appropriate for a step function in opacity) provides a useful approximation. In this limit, the accretion rate inside the opacity gap is given by

\[
\dot{M}(r) \approx \frac{4\Sigma_{\text{sub}}}{3c} T_{\text{sub}}^{\frac{4}{3}} \frac{4m_{\text{in}}}{\Omega} \approx 720T^4 \frac{m_0}{m_{\text{in}}} M_9^{5/2} M_\odot^{-1/2} \text{yr}^{-1},
\]

where \( T_{1000} = T_{\text{sub}}/1000 \text{ K}, m_0 = m/0.1, r_{10} = r/10 \text{ pc}, \) and \( M_9 = M_{\odot}/10^9 M_\odot \). Note that the normalization of \( \dot{M} \) in equation (A2) depends only on the black hole mass \( M_{\text{BH}} \), the Mach number \( m = V/c_s \), and the dust opacity law through \( T_{\text{sub}} \). This explains why all of the solutions with \( M_{\text{out}} > M_\star \) in Figure 5 are the same inside the opacity gap. Note also that because \( M(r) \) is a power law inside the opacity gap, the star formation rate \( M_\star(r) \) is as well (see eq. [43]). Indeed, for these solutions, \( M_\star(r) \propto r^{5/2} \) in the opacity gap, both \( T_{\text{sub}}^{\frac{4}{3}} \) and \( \Sigma_\star \) are proportional to \( r^{-1/2} \). The central temperature \( T \) and the surface density \( \Sigma_\delta \), on the other hand, are independent of radius.
As is clear from Figure 5, the equilibrium power-law solution for $\dot{M}$ given in equation (A2) does not persist to arbitrarily small radii. Because $\rho$ increases and $T$ is constant in the opacity gap, gas pressure eventually dominates radiation pressure. The two pressures are comparable at a radius $R_{\mathrm{EOS}}$ given by

$$R_{\mathrm{EOS}} \sim 1 M_9^{5/9} \dot{M}_1^{-2/3} m_{0.1,1}^{-2/3} Q^{-8/9} \text{~pc}, \quad (A3)$$

where $\dot{M}_1 = \dot{M}/M_9 \text{~yr}^{-1}$. For $r < R_{\mathrm{EOS}}$, one can show that star formation is no longer a significant impediment to accretion: $\dot{M}$ drops precipitously and $\dot{M}$ approaches a constant. As a result, we can estimate the accretion rate onto the black hole by evaluating equation (A2) at $r \sim R_{\mathrm{EOS}}$. This yields

$$\dot{M}_{\text{BH}} = 2^{9/4} T_{\text{sub}}^{3/2} m m_{1/3} \dot{M}_{1/3} G^{-1/2} \left(\frac{\sigma_{\text{SB}} \pi}{3c}\right)^{1/6} \left(\frac{k_B}{Q m_p}\right)^{5/6} \sim 2 T_{1000}^{1/2} m_{0.1,1} M_9^{1/3} Q^{-5/6} M_\odot \text{~yr}^{-1}, \quad (A4)$$

where we emphasize that $m$ is the Mach number in the opacity gap region; even if angular momentum transport becomes inefficient for $r < R_{\mathrm{EOS}}$ (e.g., $m \ll 0.1$ or a transition to local viscosity with $\alpha \lesssim 0.1$ occurs), $\dot{M}_{\text{BH}}$ remains unchanged. Using equation (A4) and taking $L_{\text{BH}} = \zeta \dot{M}_{\text{BH}} c^2$, we find that

$$L_{\text{BH}} \sim 10^{46} \dot{\zeta}_{0.1} T_{1000}^{3/2} m_{0.1,3} M_9^{1/3} Q^{-5/6} \text{~ergs s}^{-1}, \quad (A5)$$

where $\dot{\zeta}_{0.1} = \zeta/0.1$. This result is in excellent agreement with the models presented in Figures 5 and 7. With $L_{\text{Edd}} = \zeta L_{\text{Edd}} c^2 = 4 \pi G M_{\text{BH}} c / c_{\text{esc}}$, where $c_{\text{esc}}$ is the electron scattering opacity, we find that $\dot{M}_{\text{BH}} / L_{\text{Edd}} \sim 0.1 m_{0.1,1} M_9^{-2/3}$. Therefore, for typical parameters, the black hole is fed at a reasonable fraction of its Eddington rate. Because of the scaling with $M_{\text{BH}}$, we expect black holes of smaller mass to be preferentially super-Eddington.

Note again that from equation (A4) the accretion rate through the opacity gap into the AGN disk depends primarily on the dust sublimation temperature and on the rate of angular momentum transport in the opacity gap (via the Mach number $m$). Because of the latter dependence, it is of interest to compare the above expressions with the analogous results assuming that angular momentum transport in the opacity gap is via a local viscosity, for which the $f_g \dot{M}$ relation is given by equation (42). In this case the accretion rate in the opacity gap is given by

$$\dot{M}(r) = \frac{3 \sqrt{2} \alpha}{G Q} \left(\frac{4 \sigma_{\text{SB}}}{3c} T_{\text{sub}}^4 \pi G Q\right) \frac{3/2}{L} \approx 210 \alpha_{0.1} T_{1000}^6 Q^{-1/2} M_9^{3/2} T_{1000}^{9/2} M_\odot \text{~yr}^{-1}, \quad (A6)$$

and the accretion rate onto the black hole is given by

$$\dot{M}_{\text{BH}} = \frac{3 \sqrt{2} \alpha}{G Q} \left(\frac{k_B T_{\text{sub}}}{m_p}\right)^{3/2} \approx 2 \times 10^{-3} \alpha_{0.1} Q^{-1} T_{1000}^{3/2} M_\odot \text{~yr}^{-1}. \quad (A7)$$

Equations (A6) and (A7) highlight the fact that in the opacity gap at a rate capable of fueling a bright AGN.

**APPENDIX B**

**DISK STABILITY**

Appendix C lists the full set equations we solve for calculating the disk models presented in this paper. Because of the complicated temperature dependence of the opacity (Fig. 1), together with equations (C3) and (C5), we find multiple solutions at some radii. These solutions have the same pressure (by eq. [C3]) but different temperatures and opacities. In order to determine whether all of these solutions are physical, we assess the thermal and viscous stability of our disk solutions in this appendix. A second motivation for doing so is that radiation pressure–dominated disks close to black holes are known to be thermally and viscously unstable in certain circumstances (e.g., Piran 1978), and so it is important to check whether the same is true for our solutions.

**B1. TIMESCALES**

There are four relevant timescales characterizing the disk: (1) the dynamical timescale, $\tau_{\text{dyn}} \sim \Omega^{-1}$; (2) the advection timescale, $\tau_{\text{adv}} = r/V_c$; (3) the star formation timescale, $\tau_s = (\rho \Omega)^{-1}$; and (4) the thermal or cooling timescale, $\tau_{\text{th}} = \Sigma c_s^2/F$, where $F$ is the flux. The latter is also the photon diffusion time across the disk.

In the optically thick limit, and assuming that radiation pressure dominates gas pressure ($p_{\text{rad}} / p_{\text{gas}} \gg 1$), the ratio of the cooling timescale to the dynamical timescale is very small:

$$\frac{\tau_{\text{th}}}{\tau_{\text{dyn}}} \sim \left(\frac{c_s}{c}\right) \tau_F. \quad (B1)$$
For the disk solutions presented in Figure 5, the optical depth is always greater than unity but not larger than ~1000, even for $r \sim 1$ pc. Typically, the optical depth is closer to ~10–100. This, together with the fact that $c_s \sim f_g\sigma$ (eq. [6]), implies that $\tau_{\text{th}}/\tau_{\text{dyn}} \ll 1$. This is an important difference relative to canonical accretion disk models. For a local $\alpha$ viscosity an accretion-heated disk has $\tau_{\text{th}} \sim \alpha^{-1}\tau_{\text{dyn}} \gg \tau_{\text{dyn}}$.

We may also compare the star formation timescale with the dynamical timescale. Again assuming that $p_{\text{rad}}/p_{\text{gas}} \gg 1$ and that $\tau_\gamma > 1$, we find that

$$\frac{\tau_\gamma}{\tau_{\text{dyn}}} \sim \left(\frac{c_s}{c_i}\right)^2 \epsilon \tau_\gamma.$$  \hfill (B2)

As a consequence, throughout our disk models $\tau_\gamma/\tau_{\text{dyn}} > 1$. This result is, of course, in keeping with our assumption that star formation occurs on a timescale longer than the local free-fall timescale (that is, $\gamma \leq 1$).

Lastly, we compare the advection timescale with the dynamical timescale. If angular momentum transport is produced by a global torque (as in Fig. 5), $\tau_{\text{adv}} \sim \tau_{\text{dyn}}(r/h)^{-1}$, which is always greater than unity. If angular momentum transport is instead driven by local viscosity, $(\tau_{\text{adv}}/\tau_{\text{dyn}}) \sim (r/h)^{-1}$. As discussed in § 3, the advection timescale can be larger or smaller than the star formation timescale, depending on the ratio of $\dot{M}$ to $M_c$.

These considerations lead to the following well-defined hierarchy of timescales in $Q \sim 1$ disks:

$$\tau_{\text{th}} \ll \tau_{\text{dyn}} \ll \tau_\gamma.$$ \hfill (B3)

B2. THERMAL STABILITY

The fact that $\tau_{\text{th}}$ is much less than $\tau_{\text{dyn}}$ has important implications for the thermal stability of the disk. In particular, because of this disparity of timescales, the star formation rate per unit area $\Sigma_\gamma$ is constant on a timescale $\tau_{\text{th}}$. This implies that the volumetric heating rate $q^+ \sim F/h \propto \Sigma_\gamma/h$ is a constant on $\tau_{\text{th}}$ and, hence, $\ln q^+/d\ln T = 0$. However, the volumetric cooling rate in the optically thick limit is $q^- \sim \sigma_{SB}T^4/(\tau_\gamma h)$. Taking $\kappa \propto T^2$, $d \ln q^+/d\ln T \propto (4 - \beta)$. Therefore, if $\beta > 4$, the solution is thermally unstable. This condition is only obtained when $\kappa \propto T^{10}$ at $T \sim 2000$ K (see Fig. 1; Bell & Lin 1994). All other solutions are thermally stable. In particular, our $Q \sim 1$ radiation pressure–supported starburst disk with $T \sim 100$ K is globally thermally stable, in contrast to radiation pressure–supported disks heated by accretion energy.

In the optically thin limit, the cooling rate is $q^- \sim j \sim \kappa B \propto T^{4+\beta}$, where $j$ is the emissivity. Therefore, if $\beta < -4$, the solution is unstable. There are only two regions where such a condition might obtain and they correspond to sudden decreases in opacity [nearly step functions in $\kappa(T)$] at $T \sim 100–200$ K and in the opacity gap at $T \sim 1000$ K in Figure 1. Otherwise, all optically thin solutions are globally thermally stable.

B3. VISCOUS STABILITY

To evaluate the viscous stability of our disks, we must compute the partial derivative $\partial M/\partial \Sigma_\gamma$ (e.g., Lightman & Eardley 1974). For $\partial M/\partial \Sigma_\gamma < 0$, the disk is viscously unstable and we expect perturbations to grow and disrupt the disk on a timescale $\tau_{\text{adv}}$. If angular momentum transport is driven by global torques, $M \propto \Sigma_\gamma c_s$. But as a consequence of our assumption of marginal stability against self-gravity ($Q \sim 1$), at any radius, $c_s \propto f_g M^{1/2}$. Moreover, because $\tau_{\text{adv}} \gg \tau_{\text{dyn}}$, it is reasonable to maintain $Q \sim 1$ during perturbations to the surface density. These results imply that $\partial M/\partial \Sigma_\gamma$ is always greater than zero and the disk is viscously stable. A similar argument applies if angular momentum transport is instead driven by local viscosity ($M \propto \Sigma_\gamma c_s^2$). The $Q \sim 1$ disks thus appear to be viscously stable.

B4. PHYSICAL DISK SOLUTIONS

In the region where $Q \sim 1$, we often find three solutions to our disk equations, one low-temperature optically thick solution and two high-temperature optically thin solutions (see also Sirko & Goodman 2003). Throughout this paper we have focused on the low-temperature optically thick solution as the physical solution. Here we justify this choice.

All three of the disk solutions appear viscously stable. The two high-temperature solutions bracket the opacity gap: one has $T = T_{\text{sub}} \sim 1000$ K, and the other occurs at $T \sim 2000–5000$ K. By the optically thin thermal stability criteria of § 3, the solution with $T \sim 1000$ K is thermally unstable but its line is very large and negative. This solution is therefore probably unphysical. The solution with $T \sim 2000–5000$ K is formally thermally stable because $\beta$ is positive (Fig. 1). However, the opacity curve employed here does not take into account important line cooling processes in this temperature range. Indeed, studies of the ISM have shown that gas with $T \sim 2000–5000$ K is thermally unstable (Wolfire et al. 1995). Although the ISM of dense starburst nuclei may be interestingly different than that of normal star-forming galaxies, it is still likely that optically thin gas at several thousand kelvin is thermally unstable. For this reason we do not believe that the $T \sim 2000–5000$ K solution represents a physical global equilibrium of the disk. It is, however, likely that the existence of multiple solutions at the same pressure implies that the ISM is prone to breaking up into a multiphase medium. A detailed investigation of the multiphase structure of the ISM in starburst galaxies is clearly of interest but beyond the scope of this paper.
APPENDIX C

DISK EQUATIONS

The equations employed in constructing the models shown in Figures 5–7 and described in § 3 are

\[ \Omega(r) = \Omega_k(r) = \left( \frac{GM_{\text{BH}}}{r^3} + \frac{2\sigma^2}{r^2} \right)^{1/2}, \]  
(Eq. C1)

\[ \dot{\Sigma}_* = \Sigma_g \Omega, \]  
(C2)

\[ p_{\text{gas}} + \epsilon \dot{\Sigma}_* c^2 (\frac{1}{2} \tau_V + \xi) = \rho h^2 \Omega^2, \]  
(C3)

\[ p_{\text{gas}} = \rho k_B T / m_p, \]  
(C4)

\[ T^4 = \frac{3}{4} \sigma_{\text{eff}} \left( \frac{\tau_V + 2}{3\tau_V} + \frac{4}{3} \right), \]  
(C5)

\[ \tau_V = \kappa \Sigma_g / 2, \]  
(C6)

\[ \Sigma_g = 2\rho h, \]  
(C7)

\[ \dot{M} = 4\pi R h \rho V_r = 4\pi R h \rho m_c, = 4\pi R h^2 \rho \Omega, \]  
(C8)

\[ \dot{M} = \dot{M}_{\text{out}} - \int_{r_{\text{in}}}^{r_{\text{out}}} 2\pi r \dot{\Sigma}_* \, dr. \]  
(C9)

In the outer part of the disk where accretion heating is insufficient to maintain \( Q \sim 1 \), these equations are solved subject to the conditions

\[ \rho = \frac{\Omega^2}{\sqrt{2\pi G Q}} \]  
(C10)

(with \( Q = 1 \)) and

\[ \sigma_{\text{SB}} \sigma^4 = \frac{1}{2} \epsilon \dot{\Sigma}_* c^2 + \frac{3}{8\pi} \dot{M} \left( 1 - \sqrt{\frac{R_{\text{in}}}{r}} \right) \Omega^2. \]  
(C11)

In the inner part of the disk where accretion heating maintains \( Q > 1 \), equations (C10) and (C11) are replaced by

\[ \sigma_{\text{SB}} \sigma^4 = \frac{3}{8\pi} \dot{M} \left( 1 - \sqrt{\frac{R_{\text{in}}}{r}} \right) \Omega^2. \]  
(C12)

The radius at which \( Q > 1 \) is denoted \( R_{\text{AGN}} \). Throughout this work we set \( \xi = 1 \) in equation (C3) and \( \epsilon = 10^{-3} \) in equation (C11). \( M_{\text{BH}} \) is specified by \( M_{\text{BH}} = 2 \times 10^8 \sigma_{200}^2 M_\odot \). The inner edge of the AGN disk is taken to be at \( R_{\text{in}} = 3(2GM_{\text{BH}}/c^2) \), and \( \dot{M}_{\text{out}} \) and \( R_{\text{out}} \) are the mass supply rate at the outer boundary and the location of the outer boundary, respectively. In equation (C8), we generally take \( m = 0.1 \text{--} 0.2 \).

When calculating disks whose accretion is driven by a local viscosity, as in Figure 9, we replace equation (C8) with

\[ \dot{M} = 2\pi \nu \dot{\Sigma}_g \left| \frac{d \ln \Omega}{d \ln r} \right| = \frac{2^{3/2} \alpha h^2 \Omega^3}{G Q} \left| \frac{d \ln \Omega}{d \ln r} \right|. \]  
(C13)

APPENDIX D

THE CRITICAL MASS SUPPLY RATE

As shown in Figure 5 and Appendix A, when \( \dot{M}_{\text{out}} >\) exceeds a critical rate \( \dot{M}_c \), gas can accrete to small radii fueling a bright AGN. By contrast, for \( \dot{M}_{\text{out}} <\) \( \dot{M}_c \), star formation consumes the gas in a narrow range of radii at \( \sim R_{\text{out}} \). In the optically thick limit, and to the extent that \( \kappa \) may be approximated by \( \kappa_0 T^2 \), the critical mass supply rate is given in equation (44).

The balance between star formation and accretion is different in the optically thin limit than it is in the optically thick limit. To show this, we note that equation (43) admits a simple power-law solution when angular momentum transport is driven by a global torque (Eq. [45]):

\[ \dot{M}(r) \propto \rho^{\beta/(\Sigma_0 \xi_{\text{cm}})}. \]  
(D1)
Because

\[
\frac{\sigma}{\sqrt{2 e \xi cm}} \sim 7 \sigma_{300} (\xi \Omega_{0.1})^{-1} \tag{D2}
\]

we see that unless there is a very efficient global torque ($m \geq 1$), gas will be converted into stars rather than accreting to smaller radii.

REFERENCES

[References list is not provided, but the text likely refers to various scientific works in the field of starburst disks and AGN fueling.]

Levin, Y. 2005, MNRAS, submitted (astro-ph/0307084)
Levin, Y., & Beloborodov, A. M. 2003, ApJ, 590, L33
Lightman, A. P., & Eardley, D. M. 1974, ApJ, 187, L1
Magorrian, J., et al. 1998, AJ, 115, 2285
Martin, C. L. 2005, ApJ, 621, 227
Martin, C. L., & Kennicutt, R. C., Jr. 2001, ApJ, 555, 301
McKee, C. F., & Ostriker, J. P. 1977, ApJ, 218, 148
Meurer, G. R., Heckman, T. M., Lehnert, M. D., Leitherer, C., & Lowenthal, J. 1997, AJ, 114, 54
Mihos, J. C., & Hernquist, L. 1996, ApJ, 464, 641
Murray, N., Quataert, E., & Thompson, T. A. 2005, ApJ, 618, 569 (MQT05)
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Nulsen, P. E. J., & Fabian, A. C. 2000, MNRAS, 311, 346
Paczyński, B. 1978, Acta Astron., 28, 91
Pier, E. A., & Krolik, J. H. 1992, ApJ, 399, L23
Piran, T. 1978, ApJ, 211, 652
Pollack, J. B., McKay, C. P., & Christofferson, B. M. 1985, Icarus, 64, 471
Rowan-Robinson, M., et al. 2004, A&A, 417, L1
Rowan-Robinson, M. 2000, MNRAS, 316, 885
Sakamoto, K., Scoville, N. Z., Yun, M. S., Crossa, M., Genzel, R., & Tacconi, L. J. 1999, ApJ, 514, 68
Scoville, N. Z. 2003, J. Korean Astron. Soc., 36, 167
Scoville, N. Z., Polletta, M., Ewald, S., Stolovy, S. R., Thompson, R., & Rieke, M. 2001, AJ, 122, 3017
Semenov, D., Henning, T., Helling, C., Illgner, M., & Sedlmayr, E. 2003, A&A, 410, 611
Shlosman, I., & Begelman, M. C. 1989, ApJ, 341, 685
Shlosman, I., & Begelman, M. C., & Frank, J. 1990, Nature, 345, 679
Shlosman, I., Frank, J., & Begelman, M. C. 1989, Nature, 338, 45
Shukla, H., Yun, M. S., & Scoville, N. Z. 2004, ApJ, 616, 231
Silk, J. 1997, ApJ, 481, 703
Sirkö, E., & Goodman, J. 2003, MNRAS, 341, 501
Smith, H. E., Lonsdale, C. J., Lonsdale, C. J., & Diamond, P. J. 1998, ApJ, 493, L17
Soifer, B. T., Neugebauer, G., Matthews, K., Becklin, E. E., Ressler, M., Werner, M. W., Weinberger, A. J., & Egami, E. 1999, ApJ, 513, 207
Solomon, P. M., Downes, D., Radford, S. J. E., & Barrett, J. W. 1997, ApJ, 478, 144
Stone, J. M., Ostriker, E. C., & Gammie, C. F. 1998, ApJ, 508, L99
Tan, J. C., & Blackman, E. G. 2005, MNRAS, in press (astro-ph/0409413)
Thornton, K., Gaudlitz, M., Janka, H.-T., & Steinmetz, M. 1998, ApJ, 500, 95
Toomre, A. 1964, ApJ, 139, 129
Tremaine, S., et al. 2002, ApJ, 574, 740
Wolfe, M. G., Ho, L., Heckman, T. M., & Elvis, M. 1999, ApJ, 519, 599
Yun, M. S., & Carilli, C. L. 2002, ApJ, 568, 88
