Heavy Flavour Hadron Spectroscopy: Challenges and Future Prospects

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Abstract. During the last few years, wealth of new experimental results in the heavy flavor hadron sector has become available. The diversity, quantity and accuracy of the data are impressive and include many surprising spectroscopic results. Following the discovery of $\eta_b$ states and the excited $\eta_b(2S)$ states many excited states of open charm and beauty mesons and plethora of exotic states are reported. Discoveries of many new hadrons at B-factories have shed light on a new class of hadrons beyond the ordinary mesons. Many of these states awaits for its right identification. Though we have the theory (QCD) for the strong interaction, we are still far from extracting the major part of the hadron properties from it. These properties at the hadronic scale obviously play relevant role in many searches for new physics and new phenomenon. For obvious reasons, heavy flavour sector offers unique opportunities in this case. For example, the quarkonium systems are crucially important to improve our understanding of QCD as it falls in the low energy region where the non-perturbative effects dominate. Thus the heavy quarks / quark-antiquark bound states are ideal laboratory where our understanding of non-perturbative QCD and its interplay with perturbative QCD may be tested. A comparative review of different model predictions for example in the case of heavy flavor hadronic systems will be highlighted. The quarkonium studies may be used as a benchmark for our understanding of QCD and for the precise determination of standard model strong interaction parameters such as the constituent quark masses, $\alpha_s$, the confinement strength (string tension) etc.

1. Introduction
Three decades after the discovery of charmonium and bottomonium states the hadron spectroscopy within the heavy flavour sector has once again becomes a challenge due to the recent observations of the large number of conventional and nonconventional hadronic states by the different experimental groups at Belle, BaBar, DELPHI, CLEO, CDF, BES-III, SELEX etc. These experimental groups are engaged with more focused experiments at the heavy flavour sector discovering new hadronic states with very high precision and better statistics [1]. At the same time theoretical attempts (Lattice QCD, HQET, NRQCD, NRQM, etc.) are matured enough to provide precise quantitative predictions at the hadronic scales [2, 3, 4, 5, 6, 7, 8]. Apart from the excited states of the known charmonium and bottomonium resonances including their orbital excitations precisely measured, many open charm and open beauty states and large number of $X,Y,Z$ states in the energy range of 3-6 GeV as well as number of $Y_b,Z_b$ states just above 10.0 GeV are being reported recently. Many of these new discoveries shed lights on new types of hadrons like tetra quark states, hadron molecular states or hybrids [9]. The correct identification of them pose new challenges before theoreticians working in the heavy flavour spectroscopy [10].
2. Recent Observations of new hadronic states and Challenges posed by them

Having played a major role in the understanding of QCD at the hadronic scale, heavy hadron spectroscopy has witnessed a renaissance in the last few years driven by recent experimental reports of various hadronic states by the high energy experimental facilities world over. So far the greatest activity has occurred in the charm sector with energy range 3-5 GeV. In the bound state region of energy less than 4 GeV, major contributions in charm sector came from BES and CLEO. While that above 4 GeV, the charmonium like states have reported from CLEO, Belle and BaBar. Until very recently, the spin singlet state of any quarkonium that was known was \( \eta_c(1S) \) and it has provided the hyperfine mass difference between the vector, \( J/\Psi(1S) \) and the pseudoscalar \( \eta_c(1S) \) states to be equal to 172 MeV [11]. However, one did not know how the hyperfine interaction among the excited states behaves. The great excitement, often referred as the renaissance in hadron spectroscopy, has come from the discovery of a host of new states \( X(3872) \), \( X, Y, Z \) (in the range 3940), \( Y(4260) \) from Belle and BaBar and more recently \( X' (4160), X'' (4324), X''' (4660) \) etc.,[11, 12, 13]. The challenges posed by these new states include the right identification with the proper \( J^{PC} \) values and their decay modes. Some of these may be the orbital excited states of the charmonium, for example, \( X(3943) \) as \( \eta_c'(3S) \), \( Z(3929) \) as the \( \chi_{cJ}(2S) \) and the \( Y(3943) \) is speculated to be a hybrid. The \( Y(4260) \) observed by BaBar, CLEO and Belle is a vector meson but not likely to be a charmonium vector state. So it is suggested to be a \( \bar{c}c\bar{b} \) hybrid [9]. If so its partner states \( 0^+ \) and \( 1^- \) ought to be also observed nearby this energy. So far it has not convincingly seen from experiments. In the bottonium sector, the Upsilon (1S-6S), \( \chi_b, \chi_c \) and now the long awaited \( \eta_b \) [14] are all known and identified. However the higher orbital states are yet to be observed. In general, the bottonium is a much better place to get insight to the quarkonium spectroscopy because the running strong coupling constant is much smaller (\( \alpha_s \approx 0.2 \)) and the relativistic effects are less important compared to the charmonium case. For the Upsilon (\( 1^- \)) states, all we know is their masses, total widths, and branching fractions for leptonic, radiative, and \( \Upsilon(nS) \rightarrow \pi^+\pi^-\Upsilon(n'S) \) decays. A scarce \( \Upsilon(3S) \rightarrow \omega\chi_b(2S) \) transition has also been observed, but huge gaps remain.

Another challenging area of spectroscopic interest lie in the open flavour sector. Only the lowest \( 0^- \) and \( 1^- \) states are listed in the Particle Data Group (PDG) [11]. However, the L3 collaboration [18] reported first measurement of masses of the \( 1^3P_1 \) and \( 1^3P_2 \) of \( B_s \) mesons at \( 5670 \pm 10 \pm 13 \) MeV and \( 5768 \pm 5 \pm 6 \) MeV respectively. Two years ago, DØ and CDF collaborations have reported results on the spectroscopy of orbitably excited beauty mesons [19]. CDF found two states, \( 1^1P_1 \) and \( 1^3P_2 \), with masses \( M(1^1P_1)=5734 \pm 3 \pm 2 \) MeV and \( M(1^3P_2)=5738 \pm 6 \pm 1 \) MeV. DØ also found the same states but with slightly different masses, \( M(1^1P_1)=5720\pm2.5\pm5.3 \) MeV and \( M(1^3P_2) - M(1^1P_1)=25.2 \pm 3.0 \pm 1.1 \) MeV. In the strange sector, CDF reported two narrow \( B_s(1^3P_1) \) and \( B_s(1^3P_2) \) states with masses \( M(B_s(1^3P_1))=5829.4 \) MeV and \( M(B_s(1^3P_2))=5839 \) MeV while DØ measured only the \( B_s(1^3P_2) \), with mass of \( 5839.1 \pm 1.4 \pm 1.5 \) MeV.

Similar progress has been observed in the open charm sector. The BaBar Collaboration reported the observation of a charm-strange state, the \( D_{sJ}^*(2317) \) [20]. It was confirmed by CLEO Collaboration at the Cornell Electron Storage Ring [21] and also by Belle Collaboration at KEK [22]. Besides, BaBar had also pointed out to the existence of another charm-strange meson, the \( D_{sJ}(2460) \) [20]. This resonance was measured by CLEO [21] and confirmed by Belle [22]. Belle results are consistent with the spin-parity assignments of \( J^P = 0^+ \) for the \( D_{sJ}^*(2317) \) and \( J^P = 1^+ \) for the \( D_{sJ}(2460) \).

Many heavy flavour baryons are also being observed in recent times at CLEO, BaBar and Belle [1]. Even the positive parity excited states are being observed. However more refined high
luminosity measurements are required to identify their $J^{PC}$ values. The progress in this sector is more encouraging as more number of charmed and beauty baryons are predicted long ago by the extension of the Gell-Mann’s SU(3) quark model. The B- factories have already reported large number of these baryon states. For example BaBar reported $\Lambda_c(2940)$, $\Omega_{cc}(2770)$, Belle has observed $\Sigma_c(2800)$, $\Sigma_{c'}(2980)$ etc. Before 2006 also, only one bottom baryon ($\Lambda_b$) was known, now we have the $\Sigma_b$ and $\Xi_b$. These are extremely challenging measurements resolving states at about 6 GeV separated by just 20 MeV also. In short, these high precision measurements really pose challenges before the theorists who are trying to extract the basic QCD properties at these hadronic scales.

3. A Theoretical Attempt Using coulomb Plus Power Potential (CPP$_\nu$)

The investigation of the properties of mesons composed of a heavy quark and antiquark ($c\bar{c}$, $b\bar{c}$, $b\bar{b}$) gives very important insight into heavy quark dynamics and to the understanding of the constituent quark masses. At the hadronic scale, the nonperturbative effects connected with complicated structure of QCD vacuum necessarily play an important role. All this leads to a theoretical uncertainty in the $Q\bar{Q}$ potential at large and intermediate distances. So the success of theoretical model predictions of the hadronic properties with respect to the new experimental results can provide important information about the quark-antiquark interactions. Such information is of great interest, as it is not possible to obtain the $Q\bar{Q}$ potential starting from the basic principle of the quantum chromodynamics (QCD) at the hadronic scale.

Among the various scheme to describe the inter quark interaction inside the hadrons of different flavour compositions, the Cornell potential has become quite successful and it has been deduced from the lattice description of QCD at the hadronic scale. However, for the description of the excited states of the well established quarkonium states and for the description of open charm or open beauty hadrons, it was felt that the string tension in the confinement part of the potential become energy depended. This energy dependent on the string tension can also be viewed in terms of an exponent on the inter-quark separation corresponding to the linear potential of the Cornell type. This has led to the choice of Coulomb plus power potential with exponent $\nu$ (CPP$_\nu$). Such a choice of the phenomenological potential for the confinement part would allow us to study the variations in the nature of the inter quark interaction at the formation of the bound states as well as at the different decay processes.

The theoretical predictions of the masses of heavy-light system for ground state as well as excited state are very few [24, 25, 26, 27, 28, 29]. Spectroscopy of heavy flavour mesons ($Q\bar{Q}$, $Qq$ systems) have been studied using coulomb plus power potential (CPP$_\nu$) in both relativistic and nonrelativistic formalism with different choices of the potential index $\nu$ ($0.1 \leq \nu \leq 2.0$). A comprehensive study based on the CPP$_\nu$ model of the heavy flavour hadrons containing one or more heavy flavour quarks with minimum number of free parameters are being studied by us in recent years [30, 31, 32, 33, 34]. The Hamiltonian of the system is described as

$$H = M_Q + \sqrt{p^2 + m^2_q} + \frac{p^2}{2M_Q} + V(r) + V_{SD}(r)$$

in the case of light-heavy mesonic systems and

$$H = 2M_Q + \frac{p^2}{M_Q} + V(r) + V_{SD}(r)$$

in the case of light-heavy mesonic systems.
in the case of quarkonia states. Here,

\[ V(r) = -\frac{4\alpha_s}{3r} + Ar^n \]  

(3)

and the spin depended part is taken as

\[ V_{SD}(r) = V_{SS}(r) \left[ S(S+1) - \frac{3}{2} \right] + V_{LS}(r) \left( \vec{L} \cdot \vec{S} \right) + V_T(r) \left[ S(S+1) - \frac{3(S \cdot \vec{r})(\vec{S} \cdot \vec{r})}{r^2} \right] \]  

(4)

with

\[ V_{LS}(r) = \frac{1}{2 m_Q m_{\bar{Q}}} \left( 3 \frac{dV_v}{dr} - \frac{dV_S}{dr} \right) \]  

(5)

\[ V_T(r) = \frac{1}{6 m_Q m_{\bar{Q}}} \left( 3 \frac{d^2V_v}{dr^2} - \frac{1}{r} \frac{dV_v}{dr} \right) \]  

(6)

\[ V_{SS}(r) = \frac{1}{3 m_Q m_{\bar{Q}}} \nabla^2 V_v = \frac{16\pi\alpha_s}{9 m_Q m_{\bar{Q}}} \delta^{(3)}(\vec{r}) \]  

(7)

\[ \alpha_s(\mu^2) = \frac{4\pi}{(11 - \frac{2}{3}n_f)\ln(\mu^2/\Lambda^2)} \]  

(8)

Here, we describe mainly our results on the properties of quarkonia only. Our predictions on the S-wave quarkonia are shown in Fig.1 as against the different choices of the potential index \( \nu \). The quark mass parameters employed in our calculations are \( m_u = m_d = 330 \text{MeV} \), \( m_c = 1.28 \text{GeV} \) and \( m_b = 4.4 \text{GeV} \). The experimental (PDG average) values are drawn as the horizontal lines. We also find the standard deviation of the predicted S-wave masses with reference to the experimental values and are shown in Fig.2. They show a minimum around \( \nu \sim 1.0 \) in the case of charmonia and around \( \nu \sim 0.8 \) in the case of bottomonium states. The low lying \( P^- \)-wave masses are also shown in Fig.3 against the potential index. The numerical solution of the radial wave function of these states can further be employed to compute the mean square radii, the decay constants \( f_{\rho \nu/V} \), the di-lepton, di-gamma and di-gluon widths as well as the E1 and M1 transitions rates. The di leptonic decay width of the \( c\bar{c}(1S-4S) \) and \( b\bar{b}(1S-6S) \) states based in the NRQCD formalism are shown in Fig.4 and Fig.5 respectively against the potential index \( \nu \). Corresponding two photon and di gluon decay width are shown in Fig.6 and Fig.7 respectively. The shaded region in all these plots represents the range of \( \nu \) at which the predicted masses of the states agree with the experimental results. In the heavy-light flavour sector, the model extended to predict the masses of few low-lying states of \( Qq \) systems \( (D,D_s,B,B_s) \), the decay constants \( f_{\rho \nu/V} \), the inclusive semi-leptonic and leptonic branching ratios and the neutral flavour oscillations of \( B^0 - \bar{B}^0 \) and \( B_s^0 - \bar{B}_s^0 \) mesons [34].

Unlike in the case of mesons, for baryons, the magnetic moments become an additional property to be studied. Though many of the theoretical attempts successfully predict the masses, there is no consensus among the theoretical predictions of the properties like spin-parity, the form factors, magnetic moments etc. Heavy baryons further provide excellent laboratory to understand the dynamics of light quarks in the vicinity of heavy flavour quarks. The present model potential \( C PvP_v \) has also been extended to study the heavy flavour baryonic properties within a hypercentral scheme [35, 30]. In the hypercentral model, the hyper spherical coordinates are given by the angles

\[ \Omega_p = (\theta_p, \phi_p) ; \Omega_\lambda = (\theta_\lambda, \phi_\lambda) \]  

(9)
the hyper radius, \( x \) and hyper angle \( \xi \) as,
\[
x = \sqrt{\rho^2 + \lambda^2} ; \quad \xi = \arctan\left(\frac{\rho}{\lambda}\right)
\]
(10)

Where \( \rho \) and \( \lambda \) are the Jacobi Co-ordinates to describe a three body system of three quarks of masses \( m_1, m_2 \) and \( m_3 \), given by [36]
\[
\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) ; \quad \vec{\lambda} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 - (m_1 + m_2)\vec{r}_3)}{m_1^2 + m_2^2 + (m_1 + m_2)^2}
\]
(11)

where
\[
m_\rho = \frac{2m_1m_2}{m_1 + m_2} ; \quad m_\lambda = \frac{2m_3(m_1^2 + m_2^2 + m_1m_2)}{(m_1 + m_2)(m_1 + m_2 + m_3)}
\]
(12)

The model Hamiltonian corresponds to the three body system (baryons) in the hyper central co-ordinates now be expressed as
\[
H = \frac{P_x^2}{2m} + V(x)
\]
(13)

where
\[
m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}
\]
(14)

and
\[
V(x) = -\frac{x}{x} + \beta x' + \kappa + V_{spin}
\]
(15)

Here the potential \( V(x) \) is not purely a two body interaction but it contains three-body effects also. Within this scheme we have studied the ground state masses, mean square radii, hyperfine mass splitting and the magnetic moments of light flavour (\( qqq \)), single heavy flavour (\( \chi \)), double heavy flavour (\( \chi_{QQ} \)) and triple heavy flavour (\( \chi_{QQQ} \)) baryons. Details and results can be found in [30, 37, 38, 39].

4. Results and Discussions

The predicted results on the \( P \)-wave masses of \( c\bar{c} \) mesons, \( 1^P_1(3514 - 3542 \text{ MeV}) \), \( 1^P_3(3514 - 3542 \text{ MeV}) \), \( 1^P_2(3524 - 3552 \text{ MeV}) \) for the potential index between \( \nu = 1.0 \) to 1.1 are in good agreement with the experimental values of \( h_c(3526) \), \( \chi_{c1}(3511) \), \( \chi_{c2}(3556) \) [11] while the \( 1^P_0(3414) \) at \( \nu = 0.8 \) exactly matches with the experimental value of \( \chi_{c0}(3415) \) [11]. Similar agreement for \( b\bar{b} \) states \( 1^P_0(9817 - 9909 \text{ MeV}) \), \( 1^P_1(9831 - 9929 \text{ MeV}) \) and \( 1^P_2(9838 - 9938 \text{ MeV}) \) for the potential index \( \nu = 0.5 \) to 0.7 are in agreement with the experimental average values of \( \chi_{b0}(9859) \), \( \chi_{b1}(9893) \), \( \chi_{b2}(9912) \) [11]. In the same range of \( \nu \), the model predicts the \( h_b \) state around (9834 - 9932 MeV), which is in close agreement with the recently reported 9898.25±1.06 MeV by Belle group [40].

The predictions for \( 2^P_2(3887 - 3970 \text{ MeV}) \), \( 2^P_1(3875 - 3958 \text{ MeV}) \), \( 2^P_0(3835 - 3912 \text{ MeV}) \) and \( 1^P_1(3877 - 3960 \text{ MeV}) \) within the potential index between 1.0 and 1.1 for the \( 2P \)-states of \( c\bar{c} \) systems lie close to the experimental states reported by Belle group around 3940 [41]. In the same range of the potential index, \( 1.0 \leq \nu \leq 1.1 \) the results for \( 3^1S_0(3895 - 3991 \text{ MeV}) \) is closer to the experimental charmonium state of \( X(3938) \) reported recently by Belle [41] and for \( 4^1S_0(4180 - 4325 \text{ MeV}) \) is close to the \( Y(4260) \) state reported by BaBar [42]. The predicted \( 2^P_3 \) states (4130 - 4245 MeV) of \( c\bar{c} \) system in the same range of \( 1.0 \leq \nu \leq 1.1 \) is closer to the experimental \( \psi(4160, J^P = 1^-) \) state [43]. The lone known \( 1^P_3(3770) \) is found to be closer
Figure 1. S-wave masses of quarkonia against the potential exponent $\nu$. The horizontal line represents the respective experimental values (PDG). The encircled region shows the intersected region of predictions with the experiment.

Figure 2. The standard deviation with respect to the experimental values of the predicted masses for each choice of potential exponent $\nu$ against the exponent $\nu$. 
Figure 3. Predicted P-wave masses and their experimental values (horizontal line) of the quarkonia against the potential exponent $\nu$.

Figure 4. The di-lepton decay widths of $c\bar{c}$ system with potential index $\nu$ in NRQCD formalism. The horizontal lines are the respective experimental values.
Figure 5. The di-lepton decay widths of $b\bar{b}$ system with potential index $\nu$ in NRQCD formalism. The horizontal lines are the respective experimental values.

Figure 6. Two photon and two gluon decay width of Charmonia states against potential index $\nu$. 
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Figure 7. Two photon and two gluon decay width of bottomia states against potential index \( \nu \).

to the value of 3796 MeV at \( \nu = 0.9 \). The \( D \)-wave masses obtained here for the potential index \( \nu = 0.9 \) are close to the lattice predictions [44] for \( c\bar{c} \).

The predictions for the \( 2P \)-spin triplet states of \( b\bar{b} \) meson, \( 2^3P_0 \) (10214 - 10329 MeV), \( 2^3P_1 \) (10208 - 10322 MeV) and \( 2^3P_0 \) (10193 - 102304 MeV), for the potential index \( \nu \) in the range 0.7 to 0.8 are nearer to the corresponding experimental \( \chi_b \) states. Its spin singlet state \( 2^1P_1 \) (10210 - 10234) in the same range of \( \nu \) is close to the lattice predictions as well as the recently reported values of 10.259±0.64GeV of the \( h_b(2P) \) by Belle [40]. This shift towards the lower index for the higher angular momentum states probably suggests the orbital energy \( (n, \ell) \) dependence on the string tension \( A \).

The predicted masses of \( P \)-wave of \( D \)-mesons provide \( 1^3P_2 \) (2342 - 2472 MeV) within the range of 1.0 \( \leq \nu \leq 1.7 \), \( 1^3P_1 \) (2361 - 2464 MeV) for 1.0 \( \leq \nu \leq 1.5 \), \( 1^3P_0 \) (2312 - 2398 MeV) for 0.9 \( \leq \nu \leq 1.3 \) and \( 1^1P_1 \) (2269 - 2337 MeV) for 1.0 \( \leq \nu \leq 1.5 \). While the experimental candidate for \( J^P = 1^+ \) state of \( D \)-meson (2420 - 2460 MeV) observed by CLEO [45] and Belle [46] and \( J^P = 0^+ \) state observed in the range 2300 - 2400 MeV by Belle and Focus [47] lie within the predicted range.

In the case of open strange-charm mesons (\( D_s \)), the predictions for \( P \)-states provide \( 1^3P_2 \) (2416 - 2552 MeV) for 1.0 \( \leq \nu \leq 2.0 \) as against the latest experimental average value (PDG2010) of 2573 MeV, \( 1^3P_1 \) (2397 - 2484 MeV) for 1.0 \( \leq \nu \leq 1.5 \) as against the PDG(2010) value of 2460 MeV, \( 1^3P_0 \) (2317 - 2350 MeV) for 0.8 \( \leq \nu \leq 1.0 \) as against the recently reported value of 2317 by BaBar, CLEO and Belle group [1]. The radial excitation of \( D_s^*(2112) \) observed by Belle collaboration [48] at 2715 MeV is found to be close to the \( 3^3S_1 \) state predicted in this model and at \( \nu = 1.0 \) as the \( 2^3S_1 \) values predicted here lie in the range of 2474 - 2730 MeV for the choice of 1.0 \( \leq \nu \leq 1.7 \), which are below the experimental value in the expected range of the potential index around \( \nu = 1.0 \). Even higher excited states of \( c\bar{s} \) system has been observed by the BaBar collaboration [49] with spin parity 0+ , 1+ and 2+ with mass at 2856±1.5±5.0 which in our case corresponds to the \( 2P \) state with the predicted mass range of \( 2^3P_2 \) (2668 - 2842 MeV)
MeV) for $1.0 \leq \nu \leq 1.3$, $2^3P_1(2651 - 2820 \text{ MeV})$ for $1.0 \leq \nu \leq 1.3$, $2^3P_0(2612 - 2858 \text{ MeV})$ for $1.0 \leq \nu \leq 1.5$ and $2^1P_1(2656 - 2935 \text{ MeV})$ for $1.0 \leq \nu \leq 1.5$. Thus, present study on the open charm and open strange-charm mesons are being identified with the recently discovered $D$- and $D_s$ meson states. Other predicted high angular momentum states $\ell \geq 2$ of these mesons are expected to be seen in the future experiments at BES-III, BaBar, Belle and CLEO collaborations.

While in the case of open beauty systems ($B, B_s$), the spectral predictions are in better agreement with the known experimental states and with other theoretical model predictions at potential index lying between 0.7 $\leq \nu \leq 1.1$. The predicted mass for the $1^1P_1(5724 \text{ MeV})$ and $1^3P_2 (5431 \text{ MeV})$ at $\nu = 0.7$ of $B-$mesons are very close to the recently observed $1^1P_1(5721 \pm 2.5 \pm 5.3 \text{ MeV})$ and $1^3P_2 (5738 \pm 6 \pm 1 \text{ MeV})$ states by CDF and D0[19]. While in the case of $B_s$ meson the recent CDF observation of $1^3P_1 (5829 \text{ MeV})$ and $1^3P_2 (5839 \text{ MeV})$ lie well within the range of values predicted by $1^3P_2 (5816 - 5850 \text{ MeV})$ and $1^3P_2 (5809 - 5842 \text{ MeV})$ in the potential index between 0.8 to 0.9. Unfortunately there exist only very few experimental data for $B - B_s$ systems [11]. Future, high luminosity $B-$factories are expected to provide more clean and high precision data in the open heavy flavour mesons.

5. Conclusions and Summary

At the end, we summarize that the nonrelativistic Coulomb plus power potential with varying power index using numerical approach to solve the Schrödinger equation is an attempt to understand the nature of the interquark potential and their parameters that provide us the spectroscopic properties as well as the decay properties of the $Q\bar{Q}$ system with the potential index between 0.7 and 1.1. It also provides us the importance of the quark mass parameters and the energy dependence on the potential strength for the study of the spectral properties. The radial wave functions obtained as the solution from the study are not only important for the determination of hyperfine and fine splitting of their mass spectra but also essential inputs for evaluating decay constants, decay rates, NRQCD parameters and production cross sections for quarkonium states. The spectroscopic mass difference due to the hyperfine/fine splitting are found to be sensitive to the choice of quark mass parameters. A closer look at the different properties of the heavy flavour mesons studied using phenomenological models reveals strong correlation between the model quark mass($m_{Q,q}$) parameter and the confinement strength ($A$).

The study on the spectroscopy of heavy flavour mesons clearly indicate the dependence on the energy scales for the nature of inter-quark potential. For instance below 3.0 GeV energy scale the confinement part of the interquark potential seems to be above $\nu = 1.0$ while for the energy scales beyond 3.0 GeV, the confinement part of the potential seemed to be flattened with $\nu < 1.0$. The deviation from the linear behavior ($\nu = 1$) indicates the relative importance of nonperturbative behavior of QCD below and above 3.0 GeV scale. It indicates 3 GeV scale as a transition energy between the perturbative and nonperturbative domains.

The spectroscopic parameters of the CPP$_\nu$ model, are also being employed to compute the neutral the ($B^0, B^0_s$) oscillation parameters quite satisfactorily [30]. The predictions for $\Delta m_q$ and other parameters are very close to values observed from the recent experiments.

The most challenging problem at present is the description of the recent observation of states such as $X(1835)$, observed at BES, the observations of $X(3940)$, $Y(3940)$, $Z_{b1}(10.61,10.65)$, $Y_b(10.88)$ at Belle, the observations of Y states in the (4.1 to 4.6). Many extremely interesting questions in hadron spectroscopy remain unanswered at present. However, there is every hope that the upcoming facilities, PANDA at GSI, JPARC at KEK, and the 12 GeV upgrade at
Jefferson Laboratory (JLab), will rise to meet the challenges and pose new challenges to the theorists and phenomenologist to have serious attention in heavy flavour spectroscopy.

The Large Hadron Collider (LHC), with 14 TeV center of mass energy proton-proton collisions, will offer an opportunity for the study of QCD at unprecedented energy scales. The CMS facility at the LHC will be able to detect hadron production at an unprecedented center-of-mass energy of 14 TeV. The CMS physics program is mostly devoted to searches related to Higgs and new physics beyond standard model. Heavy flavor physics is also a field where many interesting observations are expected, both in terms of hadron production and decays, especially in the low-luminosity scenario. Heavy flavor processes are also interesting because they can constrain indirectly transitions that involve scales much higher than $m_b$, through loop propagation of new particles. The CMS Experiment will also study the $J/\psi$ reconstruction and decay of $B_c$ meson.

The ATLAS facilities at LHC will cover central proton-proton collisions and plans to study heavy flavor hadrons including quarkonium states. The expected statistics will permit high precision measurements of production polarization of the $\Lambda_b$ and $J/\psi$. ATLAS is well instrumented for $B$ Physics, having been designed with precision vertexing and tracking, good muon identification, high resolution calorimetry, and a flexible dedicated $B$ trigger. A rich $B$ Physics program is planned, including CP violation studies (especially for the $\Lambda_b$ and $B_s$ systems, which are not accessible to the $B$ factories), rare decays sensitive to new physics (including $b \rightarrow s\ell^+\ell^-$ and $b \rightarrow d\ell^+\ell^-$). The ATLAS Collaborations planned high statistics studies of heavy flavor hadrons, including quarkonia, will throw light on the properties of bound states, the spin dependence of quark confinement, the nature of the strong interaction potential, factorization in heavy quark effective theory, the source of CP Violation, and perhaps the resolution of some puzzles in heavy flavour physics [60]. The LHC will be the first opportunity for significant statistics on excited states in the $B_c$ family. These statistics can constrain models of the strong potential and cast light on the inter-dependence between the electroweak and strong forces.

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