Intermediate phase and pseudo-phase transition in an artificial spin ice model

R.A. Stancioli and L.A.S. Mó
Laboratório de Simulação, Departamento de Física, ICEEx, Universidade Federal de Minas Gerais, 31270-901, Belo Horizonte, MG - Brazil

In this paper we conduct Monte Carlo simulations to investigate the thermodynamic properties of a novel geometry of artificial spin ice recently proposed in the literature that had been termed “rewritable” spin ice, for its experimental realization allows total control over the microstates of the system. Our results show that in the thermodynamic limit a single phase transition between a fully magnetized state and a paramagnetic state exists, whereas for finite systems an intermediate phase also emerges, engendering a low temperature pseudo-phase transition. This intermediate phase is characterized by large magnetic domains separated by domain walls composed of monopole-like excitations, resulting in low net magnetization values. We also show that two types of low energy excitations that behave as magnetic monopoles emerge in the system, both of which are geometrically constrained to move along a predefined path.

I. INTRODUCTION

Artificial spin ices (ASI) are arrays of interacting magnetic nanoislands that can be lithographically patterned in a variety of lattice structures, the most common examples being the square and the kagome lattices. The islands are small enough that they are single-domain, and the elongated shape of each individual island constrains its magnetic moment to point along its longest axis, forcing it to behave as an effective Ising macrospin. In the simplest ASI realization, the square ASI, the lowest energy configurations of each vertex of the lattice, where four spins meet, are characterized by two spins pointing inward and the other two directed outward, obeying an ice rule analogous to the one encountered in some water ice phases.

Initially designed to allow the real-space observation of the exotic states found in geometrically frustrated rare-earth pyrochlore crystals, ASI’s scope have significantly transcended their original purpose, mainly due to their remarkable property of allowing the design of custom lattice structures, which deliberately target the emergence of new behaviors and applications. In addition to the intriguing monopole-like excitations, which are also encountered in their 3D counterparts, ASI have opened windows to the study of novel phenomena such as vertex frustration, monopole-charge screening, topologically protected magnetic charges, transfer and processing of information, among others.

One of the most promising applications of these materials is in the storage and processing of information through the manipulation of their local configurations. In this regard, an important step has been made by Wang et al. who have designed a new lattice geometry derived from the original square artificial spin ice that allows complete experimental control over the microstates of the system. For this reason, the system has been termed rewritable artificial spin ice (RWASI). Interesting is the fact that in this new geometry the same monopole-like excitations picture of the original ASI is expected. However, due to the lattice structure, the monopoles are restricted to move along straight lines, suggesting the possibility of dimensionality reduction.

The experimental realization of this particular geometry was essentially athermal, since the relatively large volume of the nanoislands used in Ref. posed a significant energy barrier for the inversion of spin orientations, blocking the system’s dynamics. However, the fabrication of thinner nanoislands with a much lower blocking temperature is also possible, and has been applied to a number of different spin ice geometries, shedding light on the thermodynamics of this class of materials both in and out of equilibrium.

In order to elucidate the thermal behavior of this new geometry of ASI, we have conducted Monte Carlo simulations of the RWASI using open (OBC) and periodic boundary conditions (PBC). Our results indicate that in the thermodynamic limit the system is expected to behave as the square ASI, exhibiting a phase transition in the Ising universality class between the ordered low temperature phase and the disordered high temperature phase. Nevertheless, for finite systems a pseudo-phase transition appears at low temperature, marking the passing from the fully ordered (magnetized) phase to a low magnetization phase characterized by the presence of large domains separated by domain walls composed of magnetic monopole-like excitations. The appearance of this intermediate phase in finite systems make for interesting possibilities, since the pseudo-transition between the magnetized and low magnetization states is very abrupt, resembling a discontinuous phase transition, in such a way that small temperature variations may completely change the system’s state and properties. The paper is organized as follows: Section II describes our model and simulation methods; in Section III, an analysis of vertices types and monopole excitations is carried out; Section IV is devoted to the system with periodic boundary conditions; Section V contains the results for open boundary conditions, and in Section VI we present our final remarks.
II. MODEL AND SIMULATIONS

The RWASI lattice is obtained by modifying the square ASI geometry as indicated in Figure 1. In the latter, each vertex is surrounded by two pairs of spins aligned in perpendicular directions, whereas in the former two of those spins are now placed at a 45 degree angle with respect to the other.16 As a result, each vertex of the RWASI lattice is surrounded by three spins, two of which are original, in the sense that they are also present in the square lattice. The third spin, which is the modified one, is placed at a 45 degree angle with respect to the other two. Half of the spins in the lattice are original and half are modified, since each original spin is shared between two vertices whereas the modified one is fully contained within a single plaquette. A more detailed discussion on vertex types will be offered in Section III.

![FIG. 1. (a) A typical square artificial spin ice. (b) The RWASI geometry is obtained by modifying half of the spins in the square lattice.](image)

In our model, nanoislands are treated as point-like Ising spins interacting through a dipolar potential. The magnetic moment of each island is given by Ising spins interacting through a dipolar potential. The dipoles rise to secondary vertices between pairs of slightly misaligned spins. The rows of horizontal spins in the RWASI geometry give rise to secondary vertices between pairs of slightly misaligned dipoles.

In our model, nanoislands are treated as point-like Ising spins interacting through a dipolar potential. The magnetic moment of each island is given by Ising spins interacting through a dipolar potential. The dipoles rise to secondary vertices between pairs of slightly misaligned spins. The rows of horizontal spins in the RWASI geometry give rise to secondary vertices between pairs of slightly misaligned dipoles.

The RWASI lattice is obtained by modifying the square ASI geometry as indicated in Figure 1 in accordance with Ref. 16, so that the separation between two consecutive unit cells is \( l \). The lattice parameter \( l \) is defined in accordance with Ref. 16 so that the separation between two consecutive unit cells is \( l(2 + \sqrt{2}) \) in both \( x \) and \( y \) directions. The total Hamiltonian is written as:

\[
\mathcal{H} = \frac{D}{2} \sum_{i \neq j} \left[ S_i \cdot \hat{e}_i \left( S_j \cdot \hat{e}_j \right) - \frac{3}{r_{ij}^3} (S_i \cdot r_{ij})(S_j \cdot r_{ij}) \right],
\]

where \( r_{ij} \) is the vector connecting spins at sites \( i \) and \( j \), and \( D = \mu_0 \mu^2/(4\pi l^3) \) is the coupling constant of the dipolar interaction, \( \mu_0 \) being the permeability of free space.

We ran simulations with two different boundary conditions: open and periodic. In the lattice with open boundary conditions, the last spin in each row has been removed to avoid an asymmetry between left and right borders. For periodic boundary conditions, several copies of the system have been considered in each direction. As the dipolar potential in two dimensions is generally convergent, we assume that the amount of copies (~10^3) used in the simulations is large enough that the total energy can be computed with reasonable precision. Since the only variable in each term of the summation in Equation 1 is the product \( S_i S_j \), the Hamiltonian can be factored as

\[
\mathcal{H} = \frac{D}{2} \sum_{i \neq j} S_i S_j \left[ \hat{e}_i \cdot \hat{e}_j - \frac{3}{r_{ij}^3} (\hat{e}_i \cdot r_{ij})(\hat{e}_j \cdot r_{ij}) \right].
\]

By calculating the constant term in brackets in Equation 2 for each pair of spins beforehand, the computing time needed to update energy values during simulations is greatly reduced. We used a single spin-flip Metropolis algorithm, combined with some multiple spin-flip steps to speed up the dynamics and avoid trapping in local minima. The results of the simulations were extrapolated to non-simulated temperatures through the multiple histogram reweighting method. For each lattice size and boundary condition, ten samples were simulated to allow the estimation of errorbars. A typical simulation consisted of 10^5 Monte Carlo steps for equilibration and 10^6 steps to compute averages.

III. VERTICES AND MONOPOLES

As can be noticed in Figure 1, each vertex of the square ASI has its counterpart in RWASI. However, the ice rule does not hold in the latter, at least in its original formulation. In the modified lattice, the net magnetic charge of a vertex is kept neutral whenever one of the original spins points in and the other one points out. The violation of this ice rule analog yields an excess of magnetic charge in the vertex, in a similar way to what happens in the square lattice when the ice rule is violated. The orientation of the modified spin is important for energy considerations but does not contribute to the magnetic charge in the vertex, because the two opposite poles of this spin are equidistant to the vertex.

Since a vertex is now defined by only three spins, it can assume eight possible configurations, which can be divided in three different groups according to their energy levels. It is important to notice, though, that these configurations may appear reflected about the horizontal axis, as shown in Figure 2, because the modified spin appears above the other two in half of the vertices and below them in the other half. Both type I and type II vertices obey the modified ice rule and are charge neutral, but type I vertices are less energetic than type II.
FIG. 2. The eight possible configurations of the primary vertex are classified in three types, in ascending order of interaction energy. The vertices shown in the second row are in the same configuration as the vertices immediately above them, but they are reflected about the $x$ axis. In a random configuration of the system, the expected values of vertex populations are 25% for types I and II and 50% for type III vertices. The secondary vertices are not present in square ASI and are classified in types A and B. The head-to-tail alignment of spins in type A vertices makes them less energetic than type B.

because all of its local interactions are satisfied. Type III vertices are the most energetic and exhibit an excess of magnetic charge. The doubly charged vertices present in square ASI do not occur in the modified lattice.

Contrary to what happens in square ASI, in the RWASI lattice not all spins point towards their respective vertices. Indeed, the modified spins in RWASI form structures very similar to one-dimensional Ising arrays, with a small offset in the $y$ direction between nearest neighbors. The midpoint between two of these spins can be treated a secondary vertex of coordination number $z = 2$. Energy is minimized when both spins have the same orientation in the $x$ direction (type A vertices), whereas excited vertices (type B) appear when spins have opposite orientations (see Figure 2).

In square ASI, it has been shown that elementary excitations above the ground state appear as a pair of oppositely charged vertices, whose interaction energy exhibits an effective Coulombic term. The separation of the excited vertices across the lattice result in a string of charge-neutral yet energetically disfavored vertices. Thus, the energy of these excitations can be written as:

$$V(r) = -\frac{q}{r} + ax(r) + b,$$

(3)

where $q$ is the magnetic charge magnitude, $r$ is the distance between the excited vertices, $a$ is the string tension, $x(r)$ is the string length, and $b$ is the energy required for monopole pair formation. In the RWASI lattice, this behavior is observed not only for type III vertices, which are analogous to the single-charged monopoles of the square lattice, but also for type B vertices, that are specific to this geometry. In both cases, the separation of a pair of monopoles across the lattice results in a string of type II vertices, that are charge-neutral but considerably more energetic than type I (see Figure 3(a)). By adjusting Equation 3 to the energy of excited configurations, we were able to calculate the effective magnetic charge of type III and type B vertices, which are, respectively, $q_{III} = 1.65(2)Dl$ and $q_{B} = 1.21(1)Dl$ (see Figure 4).

One important consequence of this new geometry is the fact that monopoles are now constrained to move along a predefined path. From Figure 3(a), it is clear that a pair of type III monopoles created in the $i^{th}$ row of the lattice can only follow a zigzag path that either leads them further apart or brings them closer together in the same row until they meet and annihilate. The same happens with type B monopoles, which follow a straight line when hopping between secondary vertices (Figure 3(b)). This allows for the possibility of controlled magnetic currents which is absent in conventional ASI arrays.

IV. PERIODIC BOUNDARY CONDITIONS

Contrary to the square ASI, whose ground state is characterized by an alternate tiling of type I vertices that results in no net magnetization, the ground state of the RWASI was determined to be fully magnetized. This is due to the fact that the RWASI geometry removes the frustration of the square lattice at the local level, since it allows a head-to-tail arrangement of all nearest-neighbor spins, as can be inferred from the configuration of its least energetic vertices. In the ground state, all vertices are in type I and type A configurations, as shown in Figure 1(b). As the temperature rises, the magnetization drops rapidly and a peak in the specific heat curve is
observed (see Figure 5) as the system transitions into a paramagnetic phase.

We simulated lattices of sizes ranging from \( L = 8 \) to \( L = 40 \) to investigate the nature of this phase transition. The finite-size scaling of the specific heat peak revealed a logarithmic divergence, as shown in the inset of Figure 6 indicating that the critical exponent that governs the specific heat behavior, denoted by \( \alpha \), equals zero for this transition. A power-law fit was also tried, but resulted in a worse fit (not shown). This is in accordance with previous results for the square ASI\(^{30}\), where a logarithmic divergence was also found.

In order to estimate the critical temperature we applied a method based on the energy probability distribution (EPD) zeros recently reported in the literature\(^{30}\). This method consists of a modification of the Fisher zeros\(^{31}\) approach and allows the calculation of the complex zeros of the partition function from a single histogram constructed at a certain inverse temperature \( \beta_0 \). As in the Fisher zeros method, phase transitions are characterized by the zeros that touch the real positive axis in the thermodynamic limit. For finite systems, however, there can be no real positive zeros, so that one expects that the zeros nearest to the positive real axis, called leading or dominant zeros, are indicative of phase transitions. This method presents some advantages over conventional ones, such as not requiring the definition of an order parameter and allowing the obtention of the critical temperature directly from the partition function—and not from the behavior of derived thermodynamic quantities such as specific heat and susceptibility. In addition, an estimate of the transition exponent \( \nu \) can also be easily obtained. More details about this method can be found in the Appendix and in Refs. \(^{30,32,33}\).

The inset of Figure 6 shows a log-log plot of the imaginary part of the leading zero as a function of lattice size. The linear fit of the specific heat peak as a function of the natural logarithm of the lattice size \( L \) suggests a logarithmic divergence, resulting in a critical exponent \( \alpha = 0 \). The points in the graphs are the direct results of simulations, whereas the lines were obtained by the multiple histogram reweighting technique. When not shown error bars are smaller than the symbol sizes.

The critical temperature of the square ASI has been determined by Silva et al\(^{29}\) to be approximately \( T_c \approx 7.2D'/k_B \), where \( D' = \mu_0 \mu^2/(4\pi l^3) \) is the coupling constant of the dipolar interaction used in that work and \( l' \)
is the lattice parameter. A comparison between the critical temperatures of the square ASI and the rewritable ASI is only possible if we notice that the definition of the lattice parameter \( l \) used in our study is different from the lattice parameter \( l' \) defined in Ref. 29. While \( l' \) represents the distance between two adjacent vertices in the square lattice, the corresponding vertices in the rewritable lattice are separated by a distance of \( l(1+\sqrt{2}) \). Therefore, in order for the position of the corresponding vertices in both lattices to coincide, we should have \( l = l'/\sqrt{2} \), which yields a ratio between coupling constants of \( D = (1+\sqrt{2})/D' \). The critical temperature \( T_c = 0.671(1)/D/k_B \) of the RWASI model would thus result in \( T_c = 9.44(1)/D'/k_B \) in the units used in Ref. 29 which is higher than that of the square ASI. A possible explanation is the higher level of frustration present in the square ASI, which makes ordering more difficult.

V. OPEN BOUNDARY CONDITIONS

Remarkably, the system’s behavior is different when open boundary conditions are used, suggesting that finite-size effects give rise to some features that become suppressed in the thermodynamic limit. From Figure 7(a), it is clear that the total magnetization exhibits three different regimes, and not only two as in the PBC case. The ground state is still maximally magnetized, as one would expect, but as the temperature rises the magnetization curve reaches a plateau before finally decaying into the paramagnetic phase. The specific heat curve is also qualitatively different, as another local maximum arises at a much lower temperature than the formerly observed peak (Figure 7(b)). This local maximum is significantly less pronounced for smaller lattices, and appears as a mere elbow in the curve for \( L = 8 \) and \( L = 12 \). By looking at typical configurations for these three regimes, we notice that between the fully magnetized and the paramagnetic phases there is an intermediate phase that is characterized by the emergence of a domain wall of type B and type II vertices separating clusters of opposite spin orientations. These clusters, however, contain almost no magnetically charged vertices (Figure 7(c)).

Although phase transitions can only be rigorously de-
fined in the thermodynamic limit, this behavior of the finite system suggests that it undergoes two pseudo-phase transitions. This is further confirmed by the EPD zeros method. Figure 9 shows a comparison between the zeros maps for PBC and OBC lattices of the same size and at the same temperature. In the OBC lattice’s map, two zeros clearly stand out, both of which converge to a real part of one upon iteration, indicating two pseudo-phase transitions. The PBC lattice, on the other hand, presents only one dominant zero as expected. We simulated OBC lattices ranging from \( L = 8 \) to \( L = 28 \). The temperatures at which these transitions occur, shown in Table I, satisfactorily agree with the temperatures of the specific heat peaks for each system size.

### Table I. Temperature of pseudo-phase transitions for different system sizes.

| \( L \) | \( T_{c1}(D/k_B) \) | \( T_{c2}(D/k_B) \) |
|--------|-----------------|-----------------|
| 8      | 0.2439(1)       | 0.569(1)        |
| 12     | 0.2645(1)       | 0.588(1)        |
| 16     | 0.2765(7)       | 0.600(2)        |
| 20     | 0.2843(5)       | 0.611(2)        |
| 24     | 0.2884(4)       | 0.620(3)        |
| 28     | 0.293(2)        | 0.622(2)        |

This low-temperature pseudo-phase transition appears to be first order in nature. Indeed, the Binder cumulant of the magnetization,

\[
U_4 = 1 - \frac{\langle m^4 \rangle}{3 \langle m^2 \rangle^2}
\]

exhibits a sharp drop that begins near the transition temperature and reaches negative values (Figure 10(a)), suggesting a first order transition. Although the energy histogram near the transition is not clearly double-peaked, we can infer that it is formed by the sum of two unimodal distributions as can be seen in the histograms above and below the transition temperature (see Figure 10(b)).

To gain more insight on the mechanism that drives these different behaviors, we looked at the population of vertices that belong to each type as a function of temperature (see Figure 11). The PBC system presents a trivial behavior in which low energy vertices (types I and A) are preferred at low temperature and higher energy vertices are increasingly common as the temperature rises. With OBC, however, we see that vertices of types II and B increase more rapidly in the region of the intermediate phase, as shown in Figure 11(c). This is actually a requirement for the formation of the domain.
FIG. 11. (a) Normalized vertex populations for the PBC lattice, $L = 8$, as a function of temperature. Type B vertices appear at a lower temperature with OBC than with PBC. As the temperature rises, all vertex populations converge to their correspondent multiplicities. (b) Comparison of types II and III populations for OBC and PBC. Vertical lines correspond to transition temperatures for $L = 8$ with OBC (light gray) and PBC (dark gray). In the finite system, type II and type B vertices are slightly favored in the intermediate phase. (c) The susceptibility of vertex populations (given in arbitrary units) with OBC show two peaks for types II and B, and only one peak for type III. This extra peak appears as a consequence of the formation of domain barriers in the low-temperature transition.

VI. CONCLUSION

In summary, we have investigated the thermodynamic properties of a novel artificial spin ice geometry that had been proposed in Ref. [16] by means of Monte Carlo Simulations. The prospects of technological applications related to the possibility of total control over the system’s microstates allowed by this new type of ASI, as well as its suitability for experimental studies addressing the equilibrium and non-equilibrium properties of magnetic arrays of nanoislands, justify the exploration of its thermal properties.

By analyzing low energy excitations in the system, we see that the magnetic monopole picture of the square ASI [9] is present in this geometry, even though the local frustration is absent. In addition, we observe that the secondary vertices of the RWASI lattice also sup-
port the presence of monopole-like excitations, which are connected by an energetic string and whose interactions are characterized by a Coulombic term. These secondary monopole-like excitations (type B monopoles) were found to have a slightly smaller magnetic charge, $q_B = 1.21(1)L$, when compared to the monopoles in the main vertices (type III monopoles), $q_{III} = 1.65(2)L$. This difference might be associated with the misalignment of the spins that form type B vertices.

By using a method based on the zeros of the energy probability distribution (EPD) \[ Z(\beta) = \sum_E g(E)e^{-\beta E} = \sum_n h_0(n)e^{-\beta E}, \] where the summation runs over all possible energy values $E$, $g(E)$ is the density of states, $\Delta_{\beta} = \beta - \beta_0$, and $h_0(n) = g(E)e^{-\beta_0 E}$ is the non-normalized energy probability distribution—or, in other words, the histogram at an inverse temperature $\beta_0$. By assuming a discrete set of energies $E_n = \epsilon_0 + n\epsilon$ and defining $x \equiv e^{-\Delta_{\beta_0}c}$, the partition function can be rewritten in polynomial form:

\[ Z = e^{-\Delta_{\beta_0}c} \sum_n h_0(n)x^n, \]

The zeros of this polynomial are complex numbers. Since all polynomial coefficients, $e^{-\Delta_{\beta_0}c}h_0(n)$, are real positive numbers, there are no real positive zeros and all roots appear as conjugated pairs. However, in the thermodynamic limit, one expects to find a zero at the point $(1,0)$ in the complex plane for $\beta_0 = \beta_c$, signaling the non-analytic behavior of the free energy $f = -k_B T \ln(Z)$ at the phase transition temperature, $\beta_c$. For finite systems, the zero that is closest to this point is considered the dominant zero, and its real part can be used to estimate the critical temperature through the expression:

\[ \beta_c = \beta_0 - \frac{\ln(\Re(x_c))}{\epsilon}, \]

where $x_c$ is the dominant zero at inverse temperature $\beta_0$. By making a new histogram at $\beta_0 = \beta_c$, we can further refine our estimate for the critical temperature, making this an iterative process. It is also worth noticing that the imaginary part of the dominant zero is expected to scale with system size $L$ as

\[ \Im(x_c) \sim L^{-1/\nu}, \]

where $\nu$ is the critical exponent defined by the divergence of the correlation length.

\section*{Acknowledgments}

We would like to thank B.V. Costa and J.C.S. Rocha for valuable discussions. The authors thank CAPES, FAPEMIG and CNPq for financial support.

\section*{Appendix: EPD zeros}

Here we present the main ingredients necessary to understand the EPD zeros scheme. For further details we refer the reader to Refs. [30] [32] and [33]. Let us start by writing the partition function as

\[ Z(\beta) = \sum_E g(E)e^{-\beta E} = \sum_n h_0(n)e^{-\Delta_{\beta}E}, \] where the sumation runs over all possible energy values $E$, $g(E)$ is the density of states, $\Delta_{\beta} = \beta - \beta_0$, and $h_0(n) = g(E)e^{-\beta_0 E}$ is the non-normalized energy probability distribution—or, in other words, the histogram at an inverse temperature $\beta_0$. By assuming a discrete set of energies $E_n = \epsilon_0 + n\epsilon$ and defining $x \equiv e^{-\Delta_{\beta_0}c}$, the partition function can be rewritten in polynomial form:

\[ Z = e^{-\Delta_{\beta_0}c} \sum_n h_0(n)x^n, \]

The zeros of this polynomial are complex numbers. Since all polynomial coefficients, $e^{-\Delta_{\beta_0}c}h_0(n)$, are real positive numbers, there are no real positive zeros and all roots appear as conjugated pairs. However, in the thermodynamic limit, one expects to find a zero at the point $(1,0)$ in the complex plane for $\beta_0 = \beta_c$, signaling the non-analytic behavior of the free energy $f = -k_B T \ln(Z)$ at the phase transition temperature, $\beta_c$. For finite systems, the zero that is closest to this point is considered the dominant zero, and its real part can be used to estimate the critical temperature through the expression:

\[ \beta_c = \beta_0 - \frac{\ln(\Re(x_c))}{\epsilon}, \]

where $x_c$ is the dominant zero at inverse temperature $\beta_0$. By making a new histogram at $\beta_0 = \beta_c$, we can further refine our estimate for the critical temperature, making this an iterative process. It is also worth noticing that the imaginary part of the dominant zero is expected to scale with system size $L$ as

\[ \Im(x_c) \sim L^{-1/\nu}, \] where $\nu$ is the critical exponent defined by the divergence of the correlation length.
M. Tanaka, E. Saitoh, H. Miyajima, T. Yamaoka, and Y. Iye, Phys. Rev. B 73, 052411 (2006)

3 J. D. Bernal and R. H. Fowler, J. Chem. Phys. 1, 515 (1933)

4 L. Pauling, J. Am. Chem. Soc. 57, 2680 (1935)

5 M. J. Harris, S. T. Bramwell, D. F. McMorrow, T. Zeiske, and K. W. Godfrey, Phys. Rev. Lett. 79, 2554 (1997)

6 S. T. Bramwell and M. J. P. Gingras, Science 294, 1495 (2001)

7 C. Nisoli, R. Moessner, and P. Schiffer, Rev. Mod. Phys. 85, 1473 (2013)

8 C. Nisoli, V. Kapaklis, and P. Schiffer, Nat. Phys. 13, 200 (2017)

9 L. A. Mól, R. L. Silva, R. C. Silva, A. R. Pereira, W. A. Moura-Melo, and B. V. Costa, J. Appl. Phys. 106, 063913 (2009)

10 C. Castelnovo, R. Moessner, and P. Schiffer, Phys. Rev. Lett. 98, 057204 (2013)

11 I. Gilbert, G.-W. Chern, S. Zhang, L. O’Brien, B. Fore, C. Nisoli, and P. Schiffer, Nat. Phys. 10, 670 (2014)

12 A. Farhan, A. Scholl, C. F. Petersen, L. A. S. Mól, W. A. Moura-Melo, and B. Alava, Nat. Commun. 7, 12635 (2016)

13 Y. Lao, F. Caravelli, M. Shekht, J. Sklenar, D. Gardeaza-bal, J. D. Watts, A. M. Albrecht, A. Scholl, K. Dahmen, C. Nisoli, and P. Schiffer, Nat. Phys. 14, 723 (2018)

14 H. Arava, P. M. Derlet, J. Vijayakumar, J. Cui, N. S. Bingham, A. Kleibert, and L. J. Heyderman, Nanotechnology 29, 265205 (2018)

15 Y.-L. Wang, Z.-L. Xiao, A. Snezhko, J. Xu, L. E. Ocola, R. Divan, J. E. Pearson, G. W. Crabtree, and W.-K. Kwok, Science 352, 962 (2016)

16 I. Gilbert, Y. Lao, I. Carrasquillo, L. O’Brien, J. D. Watts, M. Manno, C. Leighton, A. Scholl, C. Nisoli, and P. Schiffer, Nat. Phys. 12, 162 (2015)

17 V. Kapaklis, U. B. Arnalds, A. Harman-Clarke, E. T. Papaoannou, M. Karimipour, P. Korelis, A. Taroni, P. C. W. Holdsworth, S. T. Bramwell, and B. Hjörvarsson, New J. Phys. 14, 035009 (2012)

18 J. M. Porro, A. Bedoya-Pinto, A. Berger, and P. Vavassori, New J. Phys. 15, 055012 (2013)

19 A. Farhan, P. M. Derlet, A. Kleibert, A. Balan, R. V. Chopdekar, M. Wyss, L. A. S. Mól, W. A. Moura-Melo, G. M. Wyss, and L. J. Heyderman, Nat. Phys. 9, 375 (2013)

20 D. Shi, Z. Budrikis, A. Stein, S. A. Morley, P. D. Olmsted, G. Burnell, and C. H. Marrows, Nat. Phys. 14, 309 (2018)

21 S. Gliga, G. Hrkac, C. Donnelly, J. B. Böttcher, A. Kleibert, J. Cui, A. Farhan, E. Kim, R. V. Chopdekar, Y. Masaki, N. S. Bingham, A. Scholl, R. L. Stamps, and L. J. Heyderman, Nature Materials 16, 1106 (2017)

22 U. B. Arnalds, J. H. Stopfel, V. Kapaklis, O. Bärenbold, M. A. Verschuuren, U. Wolff, V. Neu, A. Bergman, and B. Hjörvarsson, New J. Phys. 18, 023008 (2016)

23 C. Nisoli, R. Moessner, and P. Schiffer, Rev. Mod. Phys. 85, 1473 (2013)

24 M. J. Harris, S. T. Bramwell, D. F. McMorrow, T. Zeiske, and K. W. Godfrey, Phys. Rev. Lett. 79, 2554 (1997)

25 M. Tanaka, E. Saitoh, H. Miyajima, T. Yamaoka, and Y. Iye, Phys. Rev. B 73, 052411 (2006)

26 J. D. Bernal and R. H. Fowler, J. Chem. Phys. 1, 515 (1933)

27 L. Pauling, J. Am. Chem. Soc. 57, 2680 (1935)

28 A. Farhan, A. Scholl, C. F. Petersen, L. A. S. Mól, W. A. Moura-Melo, and B. V. Costa, J. Appl. Phys. 106, 063913 (2009)

29 C. Nisoli, V. Kapaklis, and P. Schiffer, Nat. Phys. 13, 200 (2017)

30 C. Castelnovo, R. Moessner, and S. L. Sondhi, Nature 451, 42 (2008)

31 M. J. Morrison, T. R. Nelson, and C. Nisoli, New J. Phys. 15, 045009 (2013)

32 I. Gilbert, G.-W. Chern, S. Zhang, L. O’Brien, B. Fore, C. Nisoli, and P. Schiffer, Nat. Phys. 10, 670 (2014)

33 A. Farhan, A. Scholl, C. F. Petersen, L. A. S. Mól, W. A. Moura-Melo, and B. V. Costa, J. Appl. Phys. 106, 063913 (2009)

34 H. Arava, P. M. Derlet, J. Vijayakumar, J. Cui, N. S. Bingham, A. Kleibert, and L. J. Heyderman, Nanotechnology 29, 265205 (2018)

35 Y.-L. Xie, Z.-Z. Du, Z.-B. Yan, and J.-M. Liu, Sci. Rep. 5, 15875 (2015)

36 R. C. Silva, R. J. C. Lopes, L. A. S. Mól, W. A. Moura-Melo, G. M. Wyss, and L. J. Heyderman, Nat. Phys. 9, 375 (2013)