The dynamic research and position estimation of the towed array during the U-turn process

J X Yang1,2,3, C G Shuai1,2,4, L He1,2, S K Zhang1,2 and S T Zhou1,2
1 Institute of Noise & vibration, Naval University of Engineering, Wuhan, China
2 National Key Laboratory on Ship Vibration & Noise, Wuhan, China

E-mail: 3yangjiaxuan89@126.com; 4chgshuai@163.com

Abstract. A dynamic model for estimating position of ship towed array during U-turn manoeuvre is introduced and developed. Based on this model, the influences of the parameters such as time step and segment length on the numerical simulation are analysed. The results indicate that decreasing the time step has little effect on the simulation accuracy but will increase the computational time. The selection of segment length has a great influence on the estimation of ship towed array position during U-turn manoeuvre. Reducing the segment length somewhat increases the computational complexity and significantly improves simulation precision.

1. Introduction
Measuring ship radiated noise by using its own towed array is more flexible, more convenient, and at lower cost, if compared with the traditional methods based on a fixed array or an auxiliary measurement ship. Ship’s towed array normally dragged after the stern is usually applied to measure the radiated noise of other vessels. It is very difficult to measure its own radiated noise due to the existing measurement blind zones near the bow. Theoretically the deficiency can be overcome when ship makes a special manoeuvre such as U-turn during measurement. In this case, it is significant to accurately predict the position of ship towed array composed of unequally distributed hydrophones during U-turn manoeuvre, because the spatial location and spacing of hydrophones will greatly affect the beam formation and matched field inversion during measurement and analysis of ship own radiated noise.

Many theoretical and experimental researches on the localization of ship towed array system have been done by some researchers such as Srivastava [1] and Duncan [2,3]. In 1998, Srivastava conducted an experiment to verify the developed model of the underwater towed cable-array system during loop manoeuvre, in which the X, Y, Z coordinates of the cable-array system and tow-point tension for different radii and speeds during the circular manoeuvre could be recorded. The results showed that the experimental results of tails, lateral shift, depth of array, and tow-point tension were in good agreement with the theoretical values. Based on this, in 2002, Duncan carried out a hydrodynamic simulation and experiment to estimate the location of hydrophones in the towed array during U-turn manoeuvre of the tow-vessel. The results showed that the path of towed array was far more sensitive to the tangential and normal drag coefficients than it was to any of other parameters. Meanwhile, the vessel speed had little effect on the array path.
In this paper, a simplified dynamic model for calculating the localization of towed array system during U-turn manoeuvre is established. And the influences of different time steps and nodes’ number on simulation results are discussed.

2. Dynamic model
Generally, as shown in Figure 1, the towed array system is composed of a tow cable, forward vibration isolating mass (VIM), instrument module, acoustic module and aft module [4]. The forward VIM is used as a buffer and vibration isolator. The instrument module including temperature sensor, depth sensor, and so on can acquire the environmental parameters for acoustic calculation. The acoustic module composed of tens or hundreds of hydrophones distributed at different intervals along the towed cable is the main part of towed array system. The aft module, in addition to isolating vibration, can increase the resistance and keep the state of towed array. Normally, a small towed array only consists of two parts: towed cable and acoustic module.

The towed array system is approximately assumed as a uniform towed cable neglecting the difference of modules’ material and diameter. And starting with the stern, the towed cable can be equally divided into \( n \) sections, each of which is modelled as a mass-spring. The force analysis of the \( i^{th} \) section is shown in Figure 2, where \( M_i \) (kg) is the mass of the \( i^{th} \) segment, \( T_i \) and \( T_{i+1} \) are the tensions acting on the \( i^{th} \) segment, and \( F_{dn} \) and \( F_{dt} \) are the normal and tangential hydrodynamic drag forces respectively. The cable end at the stern can be considered as hinge joint and the other end is free [5].

![Figure 1. The typical layout of a towed array.](#)

![Figure 2. The towed cable model and force analysis of the \( i^{th} \) section.](#)
The equilibrium equations of forces for the $i^{\text{th}}$ segment can be expressed as

$$
\begin{align*}
T_i - T_{i+1} - Fd_i &= M_i a_i \quad (1 < i \leq n) \\
T_i - Fd_i &= M_i a_i \quad (i = n + 1)
\end{align*}
$$

(1)

The tension acting on the $i^{\text{th}}$ segment of cable related to its coordinates and velocities on either end of the segment can be represented as

$$
T_i = A_i E_i \left( \frac{l_i}{l_{0i}} - 1 \right) + \alpha_i A_i \frac{\dot{l}_i}{l_{0i}}
$$

(2)

Where, $A_i$ is the cross-sectional area of towed cable ($\text{m}^2$), $E_i$ is the Young’s modulus of cable material ($\text{Pa}$), $\alpha_i$ is the damping coefficient, $l_{0i}$ is the initial length of the $i^{\text{th}}$ segment ($\text{m}$). $l_i$, the real length of the $i^{\text{th}}$ segment ($\text{m}$) can be written as

$$
l_i = (\delta x_i^2 + \delta y_i^2)^{1/2}
$$

(3)

$$
\dot{l}_i = \frac{\delta x_i}{l_i} \dot{x}_i + \frac{\delta y_i}{l_i} \dot{y}_i = (v_{x,i} - v_{x,i+1}) \cos \theta_i + (v_{y,i} - v_{y,i+1}) \sin \theta_i
$$

(4)

Bedendender [6,7] presented a model to calculate the normal and tangential hydrodynamic drag forces in which the fluid is assumed to be stationary. The equations are given as follows

$$
Fd_n = 0.5 C_{dn} \rho_0 D v^2 \left( a \sin \varphi \left| \sin \varphi \right| + b \sin \varphi \right)
$$

(5)

$$
Fd_t = 0.5 C_{dt} \rho_0 D v^2 \left( d \cos \varphi \left| \cos \varphi \right| + e \sin \varphi + g \left| \cos \varphi \right| / \cos \varphi + h \sin \varphi \left| \sin \varphi \right| \right)
$$

(6)

Where $C_{dn}$ is the normal drag coefficient, $C_{dt}$ is the tangential drag coefficient, $\rho_0$ is the density of the fluid ($\text{kg} \cdot \text{m}^{-3}$), $D$ is the diameter of cable ($\text{m}$), $v$ is the velocity of flow, $\varphi$ is the angle between cable and flow. When $a=1$, $d = \pi C_{dt} / C_{dn}$, and $b, e, g, h=0$, the equation (5) and (6) can be simplified as Wilson model [8], as shown below

$$
\begin{align*}
Fd_{ni} &= -0.5 C_{dn} \rho_0 D l_{i-1} v_{n,i} v_{n,i} \\
Fd_{ti} &= -0.5 C_{dt} \rho_0 \pi D l_i v_{t,i} v_{t,i}
\end{align*}
$$

(7)

In which $D_i$ is the diameter of the $i^{\text{th}}$ segment ($\text{m}$), $v_{n,i}$ is the mean normal velocity of the $i^{\text{th}}$ segment ($\text{m/s}$) and $v_{t,i}$ is the mean tangential velocity of the $i^{\text{th}}$ segment ($\text{m/s}$).

The mass of the $i^{\text{th}}$ segment is given by

$$
M_i = M_{i0} + M_a
$$

(8)

Where, $M_a$ is additional mass causing by the drag force of the fluid, and can be expressed as [9,10]

$$
M_a = \rho_0 \pi r^2
$$

(9)

In which, $r$ is radius of the towed cable.

3. Simulation and analysis

The parameters of ship U-turn manoeuvre and its own towed array are shown in Table 1.
### Table 1. Parameters of simulation

| Parameters                  | Value                        |
|-----------------------------|------------------------------|
| Overall section length      | 800 m                        |
| Segment length              | 20 m                         |
| Number of segments          | 40                           |
| Section diameter            | 0.05m                        |
| Young’s modulus             | $7.8 \times 10^{10}$ Pa      |
| Tension damping coefficient | $1 \times 10^9$ Pa.s         |
| Normal drag coefficient     | 1.2                          |
| Tangential drag coefficient | $2.5 \times 10^{-3}$         |
| Tow vessel length           | 120 m                        |
| Turning radius              | 200 m                        |
| Vessel speed                | 5 m/s                        |

According to the developed model, the simulated motion paths and relative positions of vessel and towed array are shown in Figure 3(a) and Figure 3(b) respectively in which the black bold line denotes the position of vessel, the blue dotted line refers to the towed array, and red dashed line is the motion trail of vessel U-turn. Next, it is very important and practical for high-accurate and real-time simulation to analyse and optimize the parameters such as time step and segment length on the numerical simulation.

![Figure 3](image)

**Figure 3.** Simulation results of U-turn manoeuvre.  
(a) Path of vessel and towed array, (b) Relative Position of vessel and towed array.

#### 3.1 Different time step

Figure 4 shows the influences of different time steps ($dt=5.0 \times 10^{-4}$, $2.0 \times 10^{-4}$, $1.0 \times 10^{-4}$, $5.0 \times 10^{-5}$, $2.0 \times 10^{-5}$, $1.0 \times 10^{-5}$, $5.0 \times 10^{-6}$, $2.0 \times 10^{-6}$, and $1.0 \times 10^{-6}$s) on the simulation results, in which the bottom axis is time step (s), the left axis is computing time (s), and the right axis is maximum node position.
change (m) in reference to the simulation result of minimum time step ($dt=1.0 \times 10^{-6}$s). It can be found that the selection of time step has a certain influence on the simulation result. If the time step is too large, the simulation will overflow. As the time step is reduced, the computing time will increase but its simulation accuracy rises slowly. Therefore, it is very important to select an optimum time step in simulation.

Figure 4. Influences of different time steps on simulation results.

3.2 Various meshing
Figure 5 show six meshing cases of the towed array which is divided into 4, 8, 16, 32, 64 and 128 sections respectively. In these cases, there are five invariant nodes in which node 1 is at the junction between ship and its towed array and not for analysis, and node 5 is farthest from the ship. Supposing $dt=2.0 \times 10^{-5}$s, the simulation paths of the corresponding node 2 to node 5 for six meshing cases during ship U-turn manoeuvre are demonstrated in Figure 6. It can be known that the meshing way of towed array have a great effect on the position estimation accuracy of its nodes.

Figure 5. Different segmentation methods.
Table 2 lists much more information about the position differences of each node. It illustrates that there is an upward trend in computing time for the reason that the decline of segment length leads to the number of sections increases. Besides, the max node position changes of 4 common nodes are given by using the results of min segment length as a reference value. It can be known that the farther away from the towed ship, the larger the max position change. On the other hand, with the segment length decreases, the max position change diminishes. The max value in the table reaches 88m. In addition, compared with the calculation of different time step, simulation results are more sensitive to the change of different segment length.

| Segment length (m) | Computing time (s) | Max node position change (m) |
|--------------------|-------------------|----------------------------|
|                    | Note 2 | Note 3 | Note 4 | Note 5 |
| 200                | 10.375 | 33.435 | 47.744 | 64.284 | 88.118 |
| 100                | 17.469 | 17.459 | 26.647 | 34.211 | 41.857 |
| 50                 | 30.516 | 8.043  | 12.170 | 14.967 | 17.510 |
| 25                 | 56.047 | 3.040  | 4.514  | 5.464  | 6.197  |
| 12.5               | 108.109| 0.835  | 1.216  | 1.464  | 1.636  |
| 6.25               | 213.086| 0.000  | 0.000  | 0.000  | 0.000  |

4. Conclusions
In this paper, a dynamic model for estimating position of towed array is put forward basing on former research. The model is simple and intuitive, and has a strong flexibility and universality. According to the theoretical model, a series of numerical simulation are carried out by altering the parameters of time step and segment length. The results indicate that the selection of time step has a certain influence on the simulation result. When the time step is too large, the calculation overflows. When the time step decreases, the computing time increases, but the upward trend of calculation accuracy is not obvious. On the other hand, the selection of segment length has a great influence on the simulation result. Both calculation quantity and calculation precision rises with the segment length reduces.

Acknowledgments
This work was supported by the National Natural Science Foundation of China and the Operation
Expenses for Chinese Universities' Basic Scientific Research of Central Authorities.

References

[1] Srivastaava S K and Ganapathy C 2011 Experimental investigations on loop manoeuvre of underwater towed array cable-array system J. Ocean Engng. 25 (1) pp 85-102

[2] Duncan A J 2003 The measurement of the underwater acoustic noise radiated by a vessel using the vessel's own towed array (Curtin University of Technology)

[3] Duncan A J and McMahon D R 2002 Using a towed array to characterize the underwater acoustic noise radiated by the tow-vessel Acoustics 2002 - Innovation in Acoustics and Vibration Australia AAS pp 165-175

[4] Yang J X, Shuai C G and He L 2015 Application review on underwater radiated noise measurement by using a vessel’s own towed array Vibroengineering Procedia 5 p 585-590

[5] Duncan A J and McMahon D R 2004 Using a towed array to localise and quantify underwater sound radiated by the tow-vessel J. Proceedings of Acoustic S 1

[6] Zhu K Q 1993 The nonlinear numerical simulation of ship and underwater towed body system coupling motions J. Journal of East China Shipbuilding Institute 7(4) pp 51-58

[7] Zhu K Q 1999 Comparison of Hydrodynamic Properties Effects of Bare and Faired Cable on Towing System J. Journal of East China Shipbuilding Institute 13(6) pp 13-18

[8] Wilson B W 1960 Characteristics of Anchor Cables in Uniform Ocean Currents (Texas A & M University, Dept. of Oceanography)

[9] Zhang Z H and Gu J N 2015 Fluid dynamics (China Science Press)

[10] Huo C F 2011 Cables and its application to cable-remotely operated vehicle system (Shanghai Jiao Tong University)