Present status of coupled-channels calculations for heavy-ion sub-barrier fusion reactions

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1. Introduction: H.I. sub-barrier fusion reactions
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Introduction: heavy-ion sub-barrier fusion reactions

Discovery of large sub-barrier enhancement of $\sigma_{\text{fus}}$ (~ the late 70’s)

potential model: $V(r) +$ absorption

cf. seminal work: R.G. Stokstad et al., PRL41('78) 465
Effect of nuclear deformation

$^{154}\text{Sm}$: a deformed nucleus with $\beta_2 \sim 0.3$

\[ \sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta) \]

Fusion: strong interplay between nuclear structure and nuclear reaction
Coupled-Channels method

Coupling between rel. and intrinsic motions

Projectile

Target

Coupled Schrödinger equations for $\psi_k(r)$
C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 (‘99) 143)

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254(‘91))

\[ D_{\text{fus}}(E) = \frac{d^2(E \sigma_{\text{fus}})}{dE^2} \]
Coupled-channels calculations for sub-barrier fusion

Low-lying collective excitations only

- low-lying collective excitations
  - strong coupling to the g.s., strong isotope dep.
- non-collective excitations
- giant resonances → high $E_x$, smooth isotope dep.

Coupling strengths ($B(E\lambda)$) and excitation energies ($E_x$)

identical to those in isolated nuclei
←— colliding nuclei: retain their identity during fusion

Multiple excitations to higher collective states

- multi-phonon excitations
- higher members in the g.s. rotational band
←— simple harmonic oscillator/ rigid rotor

this talk: The validity of each of these assumptions?
Role of non-collective excitations in fusion

- Many non-collective states: weakly coupled, but many levels
  - 35 levels ($< 5$ MeV) for $^{90}$Zr
  - 87 levels ($< 5$ MeV) for $^{92}$Zr

Comparison between $^{90}$Zr and $^{92}$Zr

- $^{90}$Zr ($Z=40$ sub-shell closure, $N=50$ shell closure)
- $^{92}$Zr = $^{90}$Zr + 2n

The coupling strengths for non-collective excitations: poorly known

→ random numbers (cf. random matrix model)
$^{20}\text{Ne} + ^{90}\text{Zr}$

$^{20}\text{Ne} + ^{92}\text{Zr}$

For $^{20}\text{Ne} + ^{90}\text{Zr}$:
- 38 levels (up to 5.7 MeV)
- $\theta_{\text{lab}} = 150$ deg.

For $^{20}\text{Ne} + ^{92}\text{Zr}$:
- 75 levels (upto 5.7 MeV)
- $\theta_{\text{lab}} = 150$ deg.

$D_{\text{qel}}(E) = -\frac{d}{dE} \left( \frac{\sigma_{\text{qel}}(E, \pi)}{\sigma_{\text{Ruth}}(E, \pi)} \right)$

Expt.:
E. Piasecki et al.,
PRC80(‘09)054613

$E_{\text{eff}} = 2E \frac{\sin(\theta_{\text{c.m.}}/2)}{1 + \sin(\theta_{\text{c.m.}}/2)}$

S. Yusa, K.H., and N. Rowley, PRC88(‘13)054621
Damping of collective motions?

$$\lambda_0^+$$

$$\beta_\lambda = \frac{4\pi}{3Z\lambda} \sqrt{\frac{B(E\lambda)}{e^2}}$$

a level scheme in an isolated nucleus

$$V_{\text{coup}}(r) \sim -\frac{\beta_\lambda}{\sqrt{4\pi}} \cdot R \frac{dV_N}{dr}$$
Fusion “hindrance” at deep sub-barrier energies

Theoretical models:

- **Sudden model**
  - S. Misicu and H. Esbensen,
    - PRL96(‘06)112701
  - ✓ frozen density
  - ✓ repulsive inner core
    - ➔ shallow potential

- **Adiabatic model**
  - T. Ichikawa, K.H., and A. Iwamoto,
    - PRL103(‘09)202701
  - ✓ density change after the touching
  - ✓ neck formation
    - ➔ deep and thick potential
Adiabatic model for fusion hindrance (Ichikawa, Hagino, Iwamoto)

One-body and two-body interactions are shown in the diagram. The potential energy of $^{64}\text{Ni}+^{64}\text{Ni}$ is depicted as a function of the distance $R$ (fm). The cross-section for the reaction $^{64}\text{Ni}+^{64}\text{Ni}$ is plotted against the center-of-mass energy $E_{\text{c.m.}}$ (MeV). The graph includes experimental data (Exp.) and theoretical predictions (YPE, YPE(NC), + damping).

T. Ichikawa, K.H., and A. Iwamoto, PRL103(‘09)20270
RPA calculation at each separation

T. Ichikawa and K. Matsuyanagi
PRC88(‘13) 011602(R)

damping after the touching
Semi-microscopic modeling of sub-barrier fusion

K.H. and J.M. Yao, PRC91(‘15)064606

multi-phonon excitations

![Graph showing fusion cross section vs. center-of-mass energy for $^{58}$Ni + $^{58}$Ni. The graph includes data points and curves for different phonon excitations: no coupling, 1 phonon, and 2 phonon.](image)

simple harmonic oscillator

$0^+, 2^+, 4^+$
Anharmonic vibrations

- Boson expansion
- Quasi-particle phonon model
- Shell model
- Interacting boson model
- Beyond-mean-field method

\[ \langle JM \rangle = \int d\beta \ f_J(\beta) \hat{P}_M^J \Phi(\beta) \]

- MF + ang. mom. projection
- + particle number projection
- + generator coordinate method (GCM)

M. Bender, P.H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 (‘03) 121
J.M. Yao et al., PRC89 (‘14) 054306
Recent beyond-MF (MR-DFT) calculations for $^{58}\text{Ni}$

K.H. and J.M. Yao, PRC91 (‘15) 064606
J.M. Yao, M. Bender, and P.-H. Heenen, PRC91 (‘15) 024301

- A large fragmentation of $(2^+ \times 2^+)_J=0$
- A strong transition from $2_2^+$ to $0_2^+$

Effects on sub-barrier fusion?

**Diagram:**

- $^{58}\text{Ni}$
- Energy levels: $0^+, 2^+, 4^+, 6^+$
- B(E2): $I_{2\text{ph}}^+ \rightarrow 2_1^+$
- $= 2 \times B(E2: 2_1^+ \rightarrow 0_1^+)$

**Figure:**

- Experimental (Exp.)
- PC-PK1 model
Semi-microscopic coupled-channels model for sub-barrier fusion

K.H. and J.M. Yao, PRC91 (‘15) 064606

- $V_{\text{coup}}(r, \alpha \lambda_0) \rightarrow V_{\text{coup}}(r, \tilde{Q} \lambda_0)$
- $M(\text{E2})$ from MR-DFT calculation
- scale to the empirical $B(\text{E2}; 2_1^+ \rightarrow 0_1^+)$
- still use a phenomenological potential
- use the experimental values for $E_x$
- $\beta_N$ and $\beta_C$ from $M_n/M_p$ for each transition
- axial and reflection symmetries (no $3^+$ and $3^-$ states)

$^{58}\text{Ni}$

$E_x$ (MeV)

$6^+$

$4^+$

$2^+$

$0^+$

$126(8)$

$0_1^+$

$2^+_1$

$4^+_1$

$6^+_1$

$150$

$206$

$270$

$333$

$0_2^+$

$2_2^+$

$0_3^+$

$273$

$82$

$229$

$5$

$\text{Exp.}$

$\text{PC-PK1}$

Microscopic multi-pole operator among higher members of phonon states
\( a = 0.9 \text{ fm} \)

\[ D_{\text{fus}}(E) = \frac{d^2(E \sigma_{\text{fus}})}{dE^2} \]
Experimental data:
D. Bourgin, S. Courtin et al.,
PRC90(‘14)044601.
c.f. S. Courtin’s talk,
Tue. afternoon
Coupled-channels calculations for sub-barrier fusion

- Role of non-collective excitations
  - difference between $^{20}\text{Ne} + ^{90}\text{Zr}$ and $^{20}\text{Ne} + ^{92}\text{Zr}$
  - heavy systems relevant to SHE

- Damping of collective excitations
  - overlapping region
  - deep sub-barrier fusion hindrance

- C.C. calculations with MR-DFT method
  - anharmonicity
  - truncation of phonon states
  - octupole vibrations and tri-axiality: in progress

more flexibility:
- application to transitional nuclei
- a good guidance to a Q-moment of excited states