Mathematical Model Design for a 7-DOF robot and 6-DOF comparison simulation

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Abstract. Currently, serial manipulators are the boom in the industry. Most companies are looking for solutions that improve production processes and that are human collaborative. They seek to simulate the skills that humans perform. Nowadays there is a constant question of comparison between a redundant robot of 7-DOF and one that meets the necessary joints to perform complex jobs useful in the industry as is the 6-DOF. In this paper we design a mathematical model that describes the forwards kinematics of the 7-DOF manipulator using the famous D-H convention and determine if the kinematic chain of the manipulators interferes in the performance of their work and thus reach a comparison with its predecessor of 6-DOF in the industry by means of a simulation in CoppeliaSim. As the main results the 6-DOF is more useful for repetitive production assignments and the 7-DOF for human collaboration.

1. Introduction
This article shows first the development of a mathematical model developed in Matlab and then the simulation in CoppeliaSim of the manipulator of seven degrees of freedom (7-DOF) in comparison with one of 6 degrees of freedom (6-DOF). For this, references were consulted about the last 5 years from three libraries (IEEEExplore, IOPScience and ACM Digital Library). Figure 1 shows the line time of the investigations that have as title "Seven Degrees of Freedom" in the last two years this topic has increase its interest [1-10].

Figure 1. Five years 7-DOF research timeline.
Within this review of references, 10 articles were found, which were divided into three main topics. This highlights the development in Matlab of the angular displacements of the actuators through genetic algorithms, through a graphical interface and the construction of the manipulator [3]. Therefore, the articles describing applications of 7-DOF [1-3] were placed in the first topic. The study of the kinematics and dynamics of these manipulators is the most developed topic, proposing templates and methods based on 40% of the Denavit Hartenberg (D-H) algorithm [4-8]. An example is the proposed method for redundant manipulator configuration, obtaining a complete analytical inverse kinematics equation in the position space, which eliminates the Jacobian matrix [6]. Finally, 20% of the articles make comparisons between robots with 7-DOF and those with 6-DOF [9-10]. Matlab for both applications, kinematics and dynamics is the most used software with 70%, followed by ROS with 20% to simulate the models developed in Matlab as shown in figure 2.

![Research topics and softwares](image)

**Figure 2.** Five years 7-DOF research topics and software.

These papers are not enough to show the advantages and disadvantages of these manipulators. So, the objective of this research is to compare 7-DOF and 6-DOF manipulators in industrial settings to understand their characteristics. This comparison also includes a mathematical model of forward kinematics.

2. Methodology
In order to develop a research project in robotics in an effective way, a methodical study is required to evaluate the risks present in the different stages of development and take into account the implementation of solutions for the delivery of an efficient work that meets the objectives set. To achieve this objective, it is determined to use a spiral methodology in this article, a methodology that helps to consider the risks presented in the different iterations, a methodology that solves the risks that arise, it is a methodology that manages to implement the proposed solutions achieving a successful conclusion of the project if it is applied in the best way.
2.1. Kinematic diagram design
This stage focuses on using the D-H convention to create the kinematic diagram of the Kuka manipulator based on the joint structure and structure of its links, a sequence of cylindrical representations is determined to denote the degrees of freedom in revolute, in case the degree of freedom was prismatic, it would be exposed as a cube. The kinematic diagram of the degrees of freedom is shown in figure 3a, representing the rotation of each joint with the theta parameter.

2.2. Implementation of the D-H algorithm
Having made a kinematic diagram of the joints and solved the risks of the previous stage, we proceeded to perform the series of steps of the D-H algorithm where it is instructed how to obtain the position of the end effector with respect to its base corresponding to the relative positions of the rigid body of its links that $0T7$ resulting in a more concrete diagram that includes the coordinate systems. The D-H algorithm has high degree of accuracy, although it has two constraints that refer where $xi$ axis intersects $zi-1$ and $xi$ axis is perpendicular to $zi-1$, the consequence of this constraint refers in the occasions that the coordinate frames of the links are not quite in the physical links of the manipulator. Another consequence is the robot to be placed in a specific configuration known as the zero-angle configuration, which all angles must be placed on 0 degrees [11]. It is necessary to take into account different aspects of the dimensional structure of the manipulator links, where the result of the mathematical model interferes due to its working space. Table 1 shows the D-H parameters of the working space of the Kuka manipulator.

| i | $\theta_i$ | $d_i$ | $a_i$ | $\alpha_i$ |
|---|---|---|---|---|
| 1 | $\theta_1^*$ | $d_1$ | 0 | -90 |
| 2 | $\theta_2^*$ | 0 | 0 | 90 |
| 3 | $\theta_3^*$ | $d_2$ | 0 | -90 |
| 4 | $\theta_4^*$ | 0 | 0 | 90 |
| 5 | $\theta_5^*$ | $d_3$ | 0 | -90 |
| 6 | $\theta_6^*$ | 0 | 0 | 90 |
| 7 | $\theta_7^*$ | $d_4$ | 0 | 0 |

2.3. Matrix resolution
In this stage we have the help of Matlab software for its great capacity to solve the linear algebra, in this stage a code is generated where all the individual matrices are obtained based on the generalized matrix (1) obtaining the matrices $0T1$, $1T2$ .... Having all the individual matrices, we proceed to obtain the matrix $0T7$ by solving (2) and thus obtain the mathematical model of direct kinematics with the matrix named in $Tt$, which is the same as $0T7$ in the literature. (3) shows the last matrix without evaluation and without being simplified. Figure 3b shows the Matlab simulation for the manipulator.

$$0T_7 = 0T_1^1T_2^2T_3^3T_4^4T_5^5T_6^6T_7^7.$$  

(1)

$$X = L_4^4(C(th6)^*S(th4)^*S(th3)^*S(th1 - pi/4) - C(th2)^*C(th3)^*C(th1 - pi/4)) + C(th4)^*C(th1 - pi/4)^*S(th2))$$

(2)
\[ L_3 \ast (S(\theta_4) \ast (S(\theta_3) \ast S(\theta_1 - \pi/4) - C(\theta_2) \ast C(\theta_3) \ast C(\theta_1 - \pi/4)) + C(\theta_4) \ast C(\theta_1 - \pi/4) \ast S(\theta_2)) + L_2 \ast C(\theta_1 - \pi/4) \ast S(\theta_2) \]

\[ Y = L_4 \ast (S(\theta_6) \ast (C(\theta_5) \ast (C(\theta_4) \ast (C(\theta_1 - \pi/4) \ast S(\theta_3) + C(\theta_2) \ast C(\theta_3) \ast S(\theta_1 - \pi/4)) + S(\theta_2) \ast S(\theta_4) \ast (\theta_1 - \pi/4)) + S(\theta_5) \ast (C(\theta_3) \ast (\theta_1 - \pi/4) - C(\theta_2) \ast S(\theta_3) \ast S(\theta_1 - \pi/4))) - C(\theta_6) \ast (S(\theta_4) \ast (C(\theta_1 - \pi/4) \ast S(\theta_3) + C(\theta_2) \ast C(\theta_3) \ast S(\theta_1 - \pi/4)) - C(\theta_4) \ast S(\theta_2) \ast S(\theta_1 - \pi/4))) - L_3 \ast (S(\theta_4) \ast (C(\theta_1 - \pi/4) \ast S(\theta_3) + C(\theta_2) \ast C(\theta_3) \ast S(\theta_1 - \pi/4)) - C(\theta_4) \ast S(\theta_2) \ast S(\theta_1 - \pi/4)) + L_2 \ast S(\theta_2) \ast S(\theta_1 - \pi/4) \]

\[ Z = L_1 + L_3 \ast (C(\theta_2) \ast C(\theta_4) + C(\theta_3) \ast S(\theta_2) \ast S(\theta_4)) + L_4 \ast (S(\theta_6) \ast (C(\theta_5) \ast (C(\theta_2) \ast S(\theta_4) - C(\theta_3) \ast C(\theta_4) \ast S(\theta_2)) + S(\theta_2) \ast S(\theta_3) \ast S(\theta_5)) + C(\theta_6) \ast (C(\theta_2) \ast \cos(\theta_4) + C(\theta_3) \ast S(\theta_2) \ast S(\theta_4))) + L_2 \ast C(\theta_2) \]

(3)

2.4. Implementation of the D-H algorithm

At this stage it is proposed to simulate the mathematical model in Matlab, the programming codes establish to replace the dimensions of the links in table 1, and thus obtain a numerical result when validating the prototype of the mathematical model in (3). When simulating the mathematical model in Matlab we will obtain the coordinates of the final effector based on the joints configurations where its workspace is defined. The manipulators were also simulated in an industrial environment using the CoppeliaSim software. This simulation will be done to achieve the results of four tests (avoiding obstacles, different orientations, velocity, acceleration, and productivity).

Figure 3. 7-DOF structure diagram Matlab simulation.

3. Results

In order to give a result to the design of the mathematical model we used Matlab with Peter Corke’s Toolbox. Figure 3b shows a change in its second and third joint of 45 degrees, giving as valid the joint structure of the mathematical model. The proposed diagram was simulated in Matlab with the D-H algorithm where the mathematical model was validated. The accuracy of the mathematical model to obtain the Cartesian coordinates was validated using the CoppeliaSim.
This validation was carried out three times to best validate the mathematical model. Once the mathematical model was validated, the Kuka arm was compared to the ABB manipulator. The test consists of testing them in industrial pick and place processes, where they will have to pass a series of comparison challenges, the mathematical model will be used to find a point that the end effector of both manipulators be reachable at the deposit point in the process, the mathematical model validated the coordinates.

The process consists of choosing objects of different colors. The objects will pass through a conveyor belt, a sensor will detect if it is the selected color, and the robotic arm will come into action and transfer it to the second belt. A proximity sensor is located on this second belt. When the sensor receives information, the second belt will begin to move the object.

3.1. Avoiding Obstacles Test

By placing an obstacle in the same position for both manipulators as shown in figure 4. It is noted that the 6-DOF serial manipulator suffers collisions at its joint number 5. The 7-DOF manipulator can avoid this obstacle, due to its extra degree of freedom.

![Figure 4. KUKA (7-DOF) avoiding obstacles test.](image)

3.2. Different Orientations Test

In this test the orientation of the robots is changed to take the objects in the same position. The KUKA robot succeeded in replacing its previous inverse kinematics with a new joint configuration that avoided the obstacle and gave the human even more workspace (figure 5a). This 7-DOF manipulator provides an extra solution in scenarios where the final orientation of the object needs to be changed. In contrast, the ABB robot in the simulation failed to dodge the obstacle, nor did it achieve a change in inverse kinematics to achieve new joint configurations and reach the goal (figure 5b).
3.3. Velocity and Acceleration Test

Three iterations were performed for each 0.8935m manipulator. The difference between the two is 0.00012s in favor of ABB. In terms of acceleration, ABB also has a small difference in its favor. Figure 6 shows the behavior of ABB in the iteration with the highest speed 0.04593m / s in 19.45s. Table 2 shows the average of the velocities and accelerations after the three iterations.

3.4. Productivity Test

The productivity of both manipulators was evaluated by conducting a real-time test of the industrial process with different speeds. Three tests were carried out, at 25%, 45% and 65% of their joint velocities, during a period of 20 minutes. The ABB robot was the most productive at each of the speeds. The average of the iterations in each speed is shown in table 2.
### Table 2. Test results.

| Tests                  | KUKA 7-DOF | ABB 6-DOF |
|------------------------|------------|-----------|
| Avoiding Obstacles     | Yes        | No        |
| Different Orientations | 2          | 1         |
| Velocity (m/s)         | 0.04517    | 0.04529   |
| Acceleration (m/s²)    | 0.00218    | 0.00229   |
| Productivity (25%)     | 34         | 58        |
| Productivity (45%)     | 207        | 297       |
| Productivity (65%)     | 270        | 346       |

### 4. Conclusions

In this article, results were achieved with respect to a mathematical model that solved the forward kinematics of the Kuka manipulator using the D-H algorithm that was validated in certain ways with the help of simulators that met the needs of the research, such as CoppeliaSim conformed the industrial process and helped in the validation of the accuracy of the forward kinematics data provided by the mathematical model. Matlab was of great help in solving the homogeneous transformation matrices generated by the algorithm, the whole validation process was a success with the simulators and the algorithm.

Verifying the analysis made, it was possible to understand the behavior of the redundant manipulators with respect to their kinematics by understanding the motion of rigid bodies. The analysis based on the study made by [12] was of great help due to the analogy made of the human arm with the robotic arm. It was also possible to understand the kinematics thanks to the studies made by [11] where its being examples of different manipulators with different degrees of freedom using the Toolbox elaborated by the author of the book.

A concrete comparison was made by average of several tests with respect to joint movements and production speeds of a redundant manipulator of 7-DOF in contrast with 6-DOF. The results shows that the 6-DOF is more useful for repetitively process where humans have no interaction with the manipulator and no obstacle avoiding is needed. This 6-DOF has more speed and more acceleration that increase its productivity. The 7-DOF is more useful to collaborate with humans in production process and in process where is very difficult to pick an object due to orientation issues.

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