Cosmic constraint on massive neutrinos in viable $f(R)$ gravity with producing ΛCDM background expansion

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Abstract Tensions between several cosmic observations were found recently, such as the inconsistent values of $H_0$ (or $\sigma_8$) were indicated by the different cosmic observations. Introducing the massive neutrinos in ΛCDM could potentially solve the tensions. Viable $f(R)$ gravity producing ΛCDM background expansion with massive neutrinos is investigated in this paper. We fit the current observational data: Planck-2015 CMB, RSD, BAO, and SNIa to constrain the mass of neutrinos in viable $f(R)$ theory. The constraint results at 95% confidence level are: $\sum m_\nu < 0.202$ eV for the active-neutrino case, $m_{\nu,\text{sterile}}^{\text{eff}} < 0.757$ eV with $N_{\text{eff}} < 3.22$ for the sterile neutrino case. For the effects due to the mass of the neutrinos, the constraint results on model parameter at 95% confidence level become $f_{R0} \times 10^{-6} > -1.89$ and $f_{R0} \times 10^{-6} > -2.02$ for two cases, respectively. It is also shown that the fitting values of several parameters much depend on the neutrino properties, such as the cold dark matter density, the cosmological quantities at matter–radiation equality, the neutrino density and the fraction of baryonic mass in helium. Finally, the constraint result shows that the tension between direct and CMB measurements of $H_0$ gets slightly weaker in the viable $f(R)$ model than that in the base ΛCDM model.

1 Introduction

The base 6-parameter Λ–Cold–Dark–Matter (ΛCDM) model is the most popular one to interpret the accelerating expansion of universe. This model is favored by most “observational probes”, though the fine-tune problem and the coincidence problem in theory exist. However, some tension was found recently between the cosmic observations when one fitted observational data to this model. For example, the tension is found for estimating the values of $H_0$: a lower value of $H_0 = 67.3 \pm 1.0$ is provided by Planck-CMB experiment with an indirect estimate on $H_0$ [1], but the higher value of $H_0 = 74.3 \pm 2.1$ is obtained by SST direct measurements of $H_0$ [2]; this tension also exists between the Planck-CMB experiment and the rich cluster counts, as they provide the inconsistent value of $\sigma_8$ [1,3].

The studies on these tensions are important, since any evidence of a tension may be useful in the search for new physics. One possible interpretation to above tension is that the base 6-parameter ΛCDM model is incorrect or should be extended. Reference [1] shows that introducing $\sum m_\nu$ or introducing $N_{\text{eff}}$ solely in the ΛCDM model cannot resolve the above tensions, but the tensions could be solved in the ΛCDM with including both $\sum m_\nu$ and $N_{\text{eff}}$ or with including the massive sterile neutrinos $m_{\nu,\text{sterile}}^{\text{eff}}$. Here $\sum m_\nu$ denotes the total mass of three species of degenerate massive active neutrinos, and $N_{\text{eff}}$ denotes the effective number of relativistic degrees of freedom, which relates to the neutrinos and the extra massless species. Combined analysis of cosmic data in other references also indicates the existence of the massive neutrinos, for example, joint analysis from CMB and BAO (baryon acoustic oscillation) [4,5], from solar and atmospheric experiments [6–8], or from the reactor neutrino oscillation anomalies [9,10], etc.

Investigating other scenarios to solve the above tensions and restricting the mass of neutrinos in different scenarios is significant. Reference [11] shows that the possible discovery of a sterile neutrino with mass $m_{\nu,\text{sterile}}^{\text{eff}} \approx 1.5$ eV, motivated by various anomalies in neutrino oscillation experiments, would favor cosmology based on $f(R)$ gravity rather than the standard ΛCDM. In addition, it is well known that plenty of functions $f(R)$ of the Ricci scalar $R$ [12–28] were presented to modify the Einstein gravity theory, in order to solve puzzles in general relativity. But several forms of $f(R)$ are then found to be nonphysical, since they cannot describe the expansion of universe in matter-dominated time [29,30]. So, studies on observationally viable $f(R)$ theories are necessary. One of...
the viable \( f(R) \) theories has been studied in Refs. [31,32], where the \( f(R) \) theory can realize the most popular \( \Lambda \)CDM universe at background-dynamics level, while the effects of large scale structure with the cosmological perturbation theory in this \( f(R) \) model are different from that in the \( \Lambda \)CDM. In this paper, we investigate the behaviors of massive neutrinos in observationally viable \( f(R) \) theories with producing the \( \Lambda \)CDM background expansion history.

2 Viable \( f(R) \) gravity theory producing \( \Lambda \)CDM background expansion

The action of \( f(R) \) modified gravity theory is written as

\[
I = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} f(R) + \mathcal{L}_u \right].
\]

(1)

\( \mathcal{L}_u \) is the Lagrangian density of universal matter including the radiation and the pressureless matter (baryon matter plus cold dark matter). Using the variation principle, one gets

\[
f_{RR \mu \nu} - \frac{1}{2} f(R) g_{\mu \nu} + (g_{\mu \nu} \Box - \nabla_\mu \nabla_\nu) f_R = 8\pi G T_{\mu \nu}.
\]

(2)

\( f_R = \frac{df(R)}{dR} \), \( R_{\mu \nu} \) and \( T_{\mu \nu} \) denote the Ricci tensor and the energy-momentum tensor of universal matter, respectively. For a universe described by metric \( g_{\mu \nu} = \text{diag}(-1, a(t)^2, a(t)^2) \), the dynamical evolutionary equations of universe in \( f(R) \) theory are

\[
3 f_R H^2 = \frac{f_{RR}}{2} - 3H f_{RR} + 8\pi G (\rho_m + \rho_r),
\]

(3)

\[
2 f_R \dot{H} = H \dot{f}_R - f_R - k^2 \left[ \rho_m + \frac{4\rho_r}{3} \right].
\]

(4)

As shown in Ref. [32], the viable \( f(R) \) theory which realizes the popular \( \Lambda \)CDM universe at background-dynamics level does not have an analytical expression of \( f(R) \) to describe a physical universe from the radiation-dominated epoch to the late-time acceleration, but it really has the analytical solutions of \( f(R) \) in different evolutionary epochs of the universe. Concretely, Ref. [32] gives the forms of \( f(R) \) in two cases: one describes the evolution of the \( \Lambda \)CDM background from the radiation-dominated epoch to the matter-dominated epoch, and the other one represents the evolution of the \( \Lambda \)CDM background from the matter-dominated era to the future expansion. In this paper we focus on studying the \( f(R) \) function producing a \( \Lambda \)CDM background expansion from the matter-dominated epoch to the late-time acceleration,\(^1\) which has a form as follows [31,32]:

\[
f(R) = R - 2\Lambda - \sigma \left( \frac{\Lambda}{R - 4\Lambda} \right)^{p_+ - 1} \times 2F_1 \left[ q_+, p_+ - 1; r_+; - \frac{\Lambda}{R - 4\Lambda} \right],
\]

(5)

where \( \sigma = \frac{3H_0^2 \Omega_\Lambda}{p_+ - 1} (\Omega_\Lambda^{p_+} + 1)^{p_+}, 2F_1[a, b; c; z] \) is the Gaussian hypergeometric function with \( q_+ = \frac{1 + \sqrt{73}}{12}, r_+ = 1 + \frac{\sqrt{73}}{6} \), and \( p_+ = \frac{5 + \sqrt{73}}{12} \). \( D \) is the model parameter in this \( f(R) \) modified gravity, which can relate to the current value \( f_{R0} \) and the current value of the Compton wavelength \( B_0 \) by

\[
f_{R0} = 1 + D \times 2F_1 \left[ q_+, p_+; r_+; - \frac{\Omega_\Lambda}{\Omega_m} \right].
\]

(6)

\[
B_0 = \frac{2Dp_+}{\Omega_m^2} \left[ 1 + D_2 F_1 \left[ q_+, p_+; r_+; - \frac{\Omega_\Lambda}{\Omega_m} \right] \right]
\]

\[
\times \left\{ \Omega_\Lambda \frac{q_+}{r_+} \times 2F_1 \left[ q_+, p_+ + 1; r_+ + 1; - \frac{\Omega_\Lambda}{\Omega_m} \right] \right. - \Omega_m^2 F_1 \left[ q_+, p_+; r_+; - \frac{\Omega_\Lambda}{\Omega_m} \right] \left. \right\},
\]

(7)

where the Compton wavelength is derived by

\[
\frac{dR}{\Delta a} \frac{H}{\Delta a} = \delta_{fR}/\delta a \frac{H}{\Delta a} \frac{f_R}{\delta f_R}.
\]

Obviously, Eq. (5) can partly realize the background expansion as that of the \( \Lambda \)CDM universe, while the cosmological perturbation behaviors in this \( f(R) \) model are different from that in the \( \Lambda \)CDM model. Given that it is not natural by using two \( f(R) \) functions to mimic one total \( \Lambda \)CDM universe, in this paper we consider our universe including two stages: the early universe \( a < 0.02 \) (including the radiation-dominated epoch and the early stage of the matter-dominated era) is described by the \( \Lambda \)CDM, and the universe \( a \geq 0.02 \) (including the deep matter-dominated epoch and the late-time acceleration) is depicted by the above viable \( f(R) \) model.

3 Cosmological perturbations in viable \( f(R) \) gravity theory producing \( \Lambda \)CDM background expansion

The line element with the perturbation reads

\[
d s^2 = a^2 \left[ -(1 + 2\psi Y^{(s)}) d\tau^2 + 2B Y_i^{(s)} d\tau dx^i + (1 + 2\Phi Y^{(s)}) \gamma_{ij} dx^i dx^j + \varepsilon Y_{ij}^{(s)} dx^i dx^j \right].
\]

(8)

\(^1\) An accelerating cosmological model can be used to interpret the current observations. For the \( \Lambda \)CDM background expansion from the matter-dominated epoch to the late-time acceleration, \( R \) can be written by

\[
R = 3 \Omega_m a^{-3} + 12 \Omega_\Lambda = 3 \Omega_m a^{-3} + 4 \Lambda.
\]
where $\gamma_{ij}$ is the three-dimensional spatial metric in the spherical coordinate

$$[\gamma_{ij}] = \begin{pmatrix}
\frac{1}{1-Kr^2} & 0 & 0 \\
0 & r^2 & 0 \\
0 & 0 & r^2 \sin^2 \theta
\end{pmatrix},
$$

(9)

$$(\Delta + k^2)Y^{(s)} = 0, \quad Y_{ij}^{(s)} \equiv -\frac{1}{k^2} \gamma_{ij} \text{ and } Y_{ij}^{(s)} \equiv \frac{1}{k^2} Y_{ij}^{(s)} + \frac{1}{k^2} \gamma_{ij} Y^{(s)}$$ are the scalar harmonic functions. Considering the synchronous gauge, we have $\psi = 0$, $B = 0$, $h_L = 6 \phi$ and $\eta_T = -(\phi + \epsilon/6)$, where $\eta_T = \frac{\delta R^3}{4k^2+12K} = \frac{6k}{4k^2+12K}$ denotes the conformal 3-space curvature perturbation. The perturbed modified Einstein equations in $f(R)$ theory can be derived as follows [33]:

$$f_R\phi \partial + \frac{1}{2} f_R = \kappa \frac{k^2}{2} a^2 \delta \rho + f_R k^2 \eta_T \beta_2$$

$$- \frac{3}{2} \mathcal{H} \delta f_R - \frac{1}{2} \delta f_R k^2 + \frac{3}{2} \mathcal{H} \delta f_R,$$

(10)

$$k^2 f_R (\beta_2 \sigma - Z) = \kappa \frac{k^2}{2} a^2 q + \frac{1}{2} k^2 \delta f_R - \frac{1}{2} k^2 \mathcal{H} \delta f_R,$$

(11)

$$\sigma' + 2 \mathcal{H}_0 \sigma + \frac{f_R}{f_R} = k \eta_T - \kappa^2 a^2 \frac{P_\mathcal{H}}{f_R k} - \frac{k^2 \delta f_R}{f_R},$$

(12)

$$Z' + \left( \frac{1}{2} \frac{f_R}{f_R} + \mathcal{H} \right) Z = \left( - k^2 \beta_2 + \frac{2}{k} + \frac{3 \mathcal{H}^2}{k} \right) \delta f_R$$

$$- \kappa^2 a^2 \frac{2 \delta \rho + 3 \delta p}{2 k f_R} - \frac{2 k f_R}{2 k f_R},$$

(13)

where $q = (\rho + p) v$, $\beta_2 = \frac{k^2 - 3K}{k^2}$, $f_R = 1 + D a^3 v_+ \times 2 F_1[q_+, p_+; r_+; -a^2 (\Omega_0^m + \Omega_0^b) / 2]$, $\mathcal{H} = a / a$ is the conformal Hubble parameter, and the superscript $'$ denotes the derivative with respect to conformal time. In addition, in the CAMB code, the curvature perturbations are characterized by $Z = \frac{\delta f_R}{2k}$ and $\sigma = \frac{(h_L + 6 \epsilon_0 v)}{2k}$ with $\eta_T = \frac{k}{3} (\sigma - Z)$. The evolutionary equation of the perturbed field $\delta f_R$ reads

$$\delta f_R' + 2 \mathcal{H} \delta f_R' + a^2 \left( \frac{k^2}{a^2} + m_0^2 f_R \right) \delta f_R$$

$$= \frac{k^2 a^2}{3} \left( \delta \rho - 3 \delta p \right) - k f_R Z.$$

(14)

The source term of the CMB temperature anisotropy is described by

$$S_T(\tau, k) = e^{-\epsilon} (\alpha'' + \eta_T^s)$$

$$+ g \left( \Delta T(0) + 2 \alpha' + \frac{v_b}{k} + \frac{\zeta}{\sqrt{\beta_2}} + \frac{\zeta''}{4k^2 \sqrt{\beta_2}} \right)$$

$$+ g' \left( \alpha' + \frac{v_b}{k} + \frac{\zeta'}{2k^2 \sqrt{\beta_2}} \right) + \frac{1}{4} \frac{g'' \zeta}{4k^2 \sqrt{\beta_2}},$$

(15)

where $g = -\kappa e^{-\epsilon} = \alpha \sigma T e^{-\epsilon}$ is the visibility function and $\epsilon$ is the optical depth. $\zeta$ is given by $\zeta = \frac{3}{4} I_2 + \frac{9}{8} E_2$, where $I_2, E_2$ indicate the quadrupole of the photon intensity and the E-like polarization, respectively.

### 4 Data fitting and results

#### 4.1 Used data

In this section, we apply the cosmic data to constrain the above viable $f(R)$ model. The used data are as follows.

1. The CMB temperature and polarization information released by Planck 2015 [11]: the high-$l$ $C_l^{TT}$ likelihood (PlikTT), the high-$l$ $C_l^{EE}$ likelihood (PlikEE), the low-$l$ $C_l^{TT}$ likelihood (PlikTE), and the lensing data.

2. The 10 datapoints of the redshift space distortion (RSD): the RSD measurements from 6dFGS ($f\sigma_8(z = 0.067) = 0.42 \pm 0.06$) [34], 2dFGRS ($f\sigma_8(z = 0.17) = 0.51 \pm 0.06$) [35], WiggleZ ($f\sigma_8(z = 0.22) = 0.42 \pm 0.07$, $f\sigma_8(z = 0.41) = 0.45 \pm 0.04$, $f\sigma_8(z = 0.60) = 0.43 \pm 0.04$, $f\sigma_8(z = 0.78) = 0.38 \pm 0.04$) [36], SDSS LRG DR7 ($f\sigma_8(z = 0.25) = 0.39 \pm 0.05$, $f\sigma_8(z = 0.37) = 0.43 \pm 0.04$) [37], BOSS CMASS DR11 ($f\sigma_8(z = 0.57) = 0.43 \pm 0.03$) [38], and VIPERS ($f\sigma_8(z = 0.80) = 0.47 \pm 0.08$) [39]. Here $f = \frac{d \ln D}{d \ln a}$, $D$ is the linear growth rate of matter fluctuations, $\sigma_8$ is the RMS matter fluctuations in linear theory, RSD reflects the coherent motions of galaxies, so it provides information as regards the formation of large-scale structure [40–42].

3. The BAO data: the 6dFGS [43], the SDSS-MGS [44], the BOSSLOWZ BAO measurements of $D_V = r_{\text{drag}}$ [44] and the CMASS-DR11 anisotropic BAO measurements [44]. Since the WiggleZ volume partially overlaps that of the BOSSCAMASS sample, we do not use the WiggleZ results in this paper. 6dFGS denotes the six-degree-FIELD Galaxy survey (6dFGS) at $z_{\text{eff}} = 0.106$ [43], SDSS-MGS denotes the SDSS Main Galaxy Sample (MGS) at $z_{\text{eff}} = 0.15$ [44], BOSSLOWZ denotes the Baryon Oscillation Spectroscopic Survey (BOSS) “LOWZ” at $z_{\text{eff}} = 0.32$ [44], and CMASS-DR11 denotes the BOSS CMASS at $z_{\text{eff}} = 0.57$ [44]. The recent analysis of the latter two
BAO data use peculiar velocity field reconstructions to sharpen the BAO feature and reduce the errors on $D_V = r_{\text{drag}}$. The point labeled BOSS CMASS at $z_{\text{eff}} = 0.57$ shows $D_V = r_{\text{drag}}$ from the analysis of [45], updating the BOSS-DR9 analysis.

(4) The supernova Ia (SN Ia) data from SDSS-II/SNLS3 jointly, extra parameters are discussed in some references [1, 59–64]. Given that the constraints on $\sum m_\nu$ (or $m_{\nu,\text{sterile}}$) are model-dependent, we fit the cosmic data to limit the mass of neutrinos in the above viable $f(R)$ model by using the MCMC method [65–70]. Obviously, extra parameters $f_{R0}$ and $\sum m_\nu$ (or $m_{\nu,\text{sterile}}$ with the required $N_{\text{eff}}$) are added, relative to the base $\Lambda$CDM model. Table 1 and Fig. 1 show the 95% limits of basic parameters in $f(R)$ model with the massive neutrino by fitting the Planck TT, TE, EE+lowP and the low-redshift data: Planck lensing+BAO+JLA.

### Table 1 The 95% confidence level of basic parameters in viable $f(R)$ model with the massive neutrino by fitting the Planck TT, TE, EE+lowP and the low-redshift data: Planck lensing+BAO+JLA

| Parameters | Active | Sterile |
|------------|--------|---------|
| $\sum m_\nu$ | $<0.202$ | $-$ |
| $m_{\nu,\text{sterile}}$ | $-$ | $<0.757$ |
| $N_{\text{eff}}$ | $-$ | $<3.22$ |
| $f_{R0} \times 10^{-6}$ | $>-1.89$ | $>-2.02$ |
| $\Omega_0 h^2$ | $0.02233_{-0.00028}^{+0.00028}$ | $0.02228_{-0.00029}^{+0.00031}$ |
| $\Omega_0 h^2$ | $0.1178_{-0.0022}^{+0.0022}$ | $0.1147_{-0.0068}^{+0.0063}$ |
| $100\theta_{MC}$ | $1.04099_{-0.00059}^{+0.00060}$ | $1.04096_{-0.00065}^{+0.00063}$ |
| $\tau$ | $0.053_{-0.022}^{+0.026}$ | $0.060_{-0.028}^{+0.028}$ |
| $\ln(10^{10} A_s)$ | $3.035_{-0.048}^{+0.048}$ | $3.049_{-0.053}^{+0.052}$ |
| $n_s$ | $0.9683_{-0.0079}^{+0.0080}$ | $0.9713_{-0.0087}^{+0.0097}$ |

4.2 Constraint on neutrino mass and the base parameters in viable $f(R)$ model producing $\Lambda$CDM expansion

Constraints on the neutrino mass in $\Lambda$CDM model or in dynamical dark energy models or in $f(R)$ theory have been discussed in some references [1, 59–64]. Given that the constraints on $\sum m_\nu$ (or $m_{\nu,\text{sterile}}$) are model-dependent, we fit the cosmic data to limit the mass of neutrinos in the above viable $f(R)$ model by using the MCMC method [65–70]. Obviously, extra parameters $f_{R0}$ and $\sum m_\nu$ (or $m_{\nu,\text{sterile}}$ with the

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2 Measurements provide some different results on $H_0$, which are almost in the region $(60–80)$ km s$^{-1}$ Mpc$^{-1}$, such as the higher values: $H_0 = 74.3 \pm 2.6$ km s$^{-1}$ Mpc$^{-1}$ [2], the lower values: $H_0 = 63.7 \pm 2.3$ km s$^{-1}$ Mpc$^{-1}$ [48] and the concordance value: $H_0 = 69.6 \pm 0.7$ km s$^{-1}$ Mpc$^{-1}$ [49], etc. For other measurement values of $H_0$, one can see Refs. [50–56].
models [61], respectively. Table 1 also exhibits the constraint result \( m_{\nu, \text{sterile}} < 0.757 \) with \( N_{\text{eff}} < 3.22 \) for the sterile-neutrino case in viable \( f(R) \) model. One can compare these results with other ones. For example, fitting the different cosmic data gives \( m_{\nu, \text{sterile}} < 0.52 \) eV with \( N_{\text{eff}} < 3.7 \) [1], \( m_{\nu, \text{sterile}} < 0.479 \) eV with \( \Delta N_{\text{eff}} = < 0.98 \) [60], or \( m_{\nu, \text{sterile}} < 0.43 \) with \( N_{\text{eff}} < 3.96 \) in \( \Lambda \)CDM model [62], and \( m_{\nu, \text{sterile}} < 0.61 \) with \( N_{\text{eff}} < 3.95 \) in the \( f(R) \) model [62]. Obviously, a higher upper limit on \( m_{\nu, \text{sterile}} \) and a lower limit on \( N_{\text{eff}} \) are obtained in our study. Some inconsistent results on the sterile-neutrino mass can also be found, for example, the sterile neutrino mass \( 0.47 \) eV < \( m_{\nu, \text{sterile}} < 1 \) eV (2\( \sigma \)) is given in a \( f(R) \) model and \( 0.45 \) eV < \( m_{\nu, \text{sterile}} < 0.92 \) eV is given in the \( \Lambda \)CDM model [63], or the active-neutrino mass \( \sum m_\nu = 0.35 \pm 0.10 \) is presented in the \( \Lambda \)CDM model [4]. The constraint results on the model parameter in a viable \( f(R) \) theory are \( f_{R0} \times 10^{-6} > -1.89 \) for the active-neutrino case and \( f_{R0} \times 10^{-6} > -2.02 \) for the sterile-neutrino case at the 95% limit. Though the fitting results on \( f_{R0} \) are affected by the additional parameters \( \Sigma m_\nu \) (or \( m_{\nu, \text{sterile}} \) with \( N_{\text{eff}} \)), for using the Planck 2015 data in this paper it has a more stringent constraint than the result given by Ref. [71]: \( f_{R0} \times 10^{-6} = -2.58^{+2.14}_{-0.58} \) in the 1\( \sigma \) region.

Table 1 also lists the values of six basic cosmological parameters. \( \Omega_\rho h^2 \) is the current baryon density, \( \Omega_\chi h^2 \) is the cold dark matter density at present, \( \theta_{MC} \) denotes the approximation to \( r_s/D_A \), \( \tau \) represents the Thomson scattering optical depth due to reionization, \( \log(10^{10} A_s) \) is the Log power of the primordial curvature perturbations, and \( n_s \) is the scalar spectrum power-law index. From Table 1 and Fig. 2, it can be seen that the neutrino properties to a much higher extent affect the fitting value of the cold dark matter density than the fitting values of the other parameters. These results could be interpreted as follows. Since the massive neutrinos are considered as one kind of dark matter in the universe, the mass of the neutrino (active or sterile) would directly affect the dimensionless energy density of dark matter. According to the constraint results on \( \Omega_\rho h^2 \) and \( \Omega_\chi h^2 \), one can see that the larger uncertainty of \( \Omega_\chi h^2 \) value is caused by the looser constraint on the dimensionless energy density of sterile neutrino \( \Omega_\chi h^2 \), which maybe reflects that there is less information on sterile neutrino from cosmic observations. However, the constraint on \( \Omega_\rho h^2 \) is stricter for the active-neutrino case, since the constraint

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{The 1-D distributions of basic cosmological parameters in viable \( f(R) \) model with massive neutrino}
\end{figure}
Table 2 The 95% confidence level of derived parameters in viable $f(R)$ model with the massive neutrino by fitting the Planck TT, TE, EE, lowP and the low-redshift data: Planck lensing+RSD+BAO+JLA

| Parameters | Active | Sterile |
|------------|--------|---------|
| $H_0$      | $67.9^{+1.1}_{-1.1}$ | $68.4^{+1.1}_{-0.99}$ |
| $\Omega_m$ | $0.306^{+0.014}_{-0.013}$ | $0.301^{+0.012}_{-0.013}$ |
| $\Omega_{m0}h^2$ | $0.09579^{+0.00085}_{-0.00096}$ | $0.0964^{+0.0026}_{-0.0013}$ |
| $\sigma_8\Omega_m^{0.25}$ | $0.665^{+0.015}_{-0.015}$ | $0.601^{+0.017}_{-0.017}$ |
| $10^9A_s$  | $2.08^{+0.10}_{-0.10}$ | $2.11^{+0.11}_{-0.11}$ |
| $n_s$(Gyr) | $13.803^{+0.068}_{-0.061}$ | $13.762^{+0.074}_{-0.12}$ |
| $\tau$     | $145.02^{+0.51}_{-0.49}$ | $144.87^{+0.88}_{-1.4}$ |
| $z_{\text{drag}}$ | $1059.68^{+0.58}_{-0.58}$ | $1059.61^{+0.78}_{-0.72}$ |
| $k_D$      | $0.14018^{+0.00058}_{-0.00060}$ | $0.1401^{+0.0012}_{-0.00092}$ |
| $k_{\text{eq}}$ | $0.01022^{+0.00015}_{-0.00015}$ | $0.01001^{+0.00035}_{-0.00038}$ |
| log$(10(B_0))$ | $-5.74^{+0.92}_{-1.00}$ | $-5.69^{+0.90}_{-1.00}$ |
| $Y_P$      | $0.24537^{+0.00012}_{-0.00013}$ | $0.2460^{+0.0018}_{-0.00071}$ |

Fig. 3 The 1-D distributions of derived cosmological parameters in viable $f(R)$ model with massive neutrino

on the dimensionless energy density of the active neutrino $\Omega_\nu h^2$ is tighter than the case of the sterile neutrino, which maybe reflects that there is more information on the active neutrino from cosmic observations. Except for $\Omega_\nu h^2$, the other basic parameters in Table 1 are not directly related to the neutrino density, so the effects on the fitting values of the other basic parameters from the neutrino characters are smaller.
The values of the derived parameters of interest are listed in Table 4 of Ref. [1] and three other parameters addition, it is found from Fig. 3 that the neutrino properly, for the active-neutrino case, and otherwise, from Table 2 we can see that the constraint results denotes the fraction of baryonic mass in helium. Obviously, we Table 2 we can see that the constraint results on $H_0$ and $\sigma_8$ are: $H_0 = 67.9 \pm 1.1$ and $\sigma_8 = 0.813 \pm 0.023$ for the active-neutrino case, and $H_0 = 68.4 \pm 1.9$ and $\sigma_8 = 0.811 \pm 0.023$ for the sterile-neutrino case, which are compatible with results given by [59]. For these constraint results on $H_0$, it is also shown that the tension between direct and CMB measurements of $H_0$ gets slightly weaker in our considered model than that in the base $\Lambda$CDM model, where $H_0 = 67.6 \pm 0.6$ is given by Ref. [1]. In addition, it is found from Fig. 3 that the neutrino properties much affect the fitting value of parameters: $z_{eq}$, $k_{eq}$, $100\theta_s$, $\Omega_v h^2$, and $Y_p$, which could be partly explained by the dependence of the parameters on the cold dark matter density and might be useful for testing the neutrino properties in experiments. The values of $\sigma_8$ in a viable $f(R)$ model are almost the same for the cases of different-species neutrinos, and the same result is also suitable for the parameters $f \sigma_8$, $\Lambda_x e^{-2\tau}$, and $\theta_s$, as exhibited in Figs. 3 and 4.

5 Conclusion

Tension between several observations was found recently. The studies of tensions are important, since they are useful to search for new physics. The massive neutrinos are introduced in cosmological models to solve the tensions concerning the inconsistent values of $H_0$ (or $\sigma_8$). Investigating other scenarios to solve these tensions and restricting the mass of neutrinos in different scenarios are significant. Given that several forms of $f(R)$ are found to be nonphysical, we study the viable $f(R)$ gravity with the massive neutrinos in this paper. We fit the current observational data: Planck-2015 CMB, RSD, BAO, and SNIa to constrain the mass of neutrinos in viable $f(R)$ theory. The constraint results at 95% confidence level are $\Sigma m_{\nu} < 0.202$ eV for the active-neutrino case and $m_{\nu, \text{eff}} < 0.757$ eV with $N_{\text{eff}} < 3.22$ for the sterile-neutrino case, which are comparable with some other results. For the effects by the mass of the neutrinos, the constraint results on model parameter become $f_{R0} \times 10^{-6} > -1.89$ and $f_{R0} \times 10^{-6} > -2.02$ for the two cases, respectively. It is also shown that the fitting values of several parameters strongly depend on the neutrino properties, such as the cold dark matter density $\Omega_c h^2$, the cosmological quantities at matter–radiation equality: $z_{eq}$, $k_{eq}$, and $100\theta_{eq}$, the neutrino density $\Omega_{\nu} h^2$, and the fraction of baryonic mass in helium $Y_p$. Finally, the constraint result shows that the tension between direct and CMB measurements of $H_0$ gets slightly weaker in the viable $f(R)$ model than that in the base $\Lambda$CDM model.

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