On Designs of Full Diversity Space-Time Block Codes for Two-User MIMO Interference Channels

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Abstract—In this paper, a design criterion for space-time block codes (STBC) is proposed for two-user MIMO interference channels when a group zero-forcing (ZF) algorithm is applied at each receiver to eliminate the inter-user interference. Based on the design criterion, a design of STBC for two-user interference channels is proposed that can achieve full diversity for each user with the group ZF receiver. The code rate approaches one when the time delay in the encoding (or code block size) gets large. Performance results demonstrate that the full diversity can be guaranteed by our proposed STBC with the group ZF receiver.

Index Terms—Two-user MIMO interference channels, space-time block codes, full diversity, group ZF receiver.

I. INTRODUCTION

It is undoubted that how to maximize diversity gain is one of the major concerns in both point-to-point communication systems and wireless networks. As for point-to-point communication systems, remarkable progress has been witnessed in the field of STBC designs to achieve maximum diversity gain over the last decade. In particular, some pioneering works are well known of the full-diversity criteria based on different types of receivers such as maximum likelihood (ML) and linear receivers [1]–[6].

Recently, the contributions aforementioned shed light on the STBC designs for wireless networks such as multiple access channels (MACs) [7]–[12] and X channels [13]. For two-user MACs, various cancellation schemes in [7]–[10] were considered with the help of Alamouti coding to suppress the interference from the neighboring user. In [7], a minimum mean squared error (MMSE) interference cancellation was developed when two receive antennas are equipped at each terminal. Later, a family of interference cancellation schemes based on Bayesian analysis was proposed in [8]. Then, [9] analyzed the diversity order of Alamouti and quasi-orthogonal STBC for multiuser MAC with two decoding algorithms, i.e., joint ML decoding and group ZF decoding. It showed that for joint ML decoding full diversity is obtained and for the group ZF decoding full diversity cannot be achieved by Alamouti and quasi-orthogonal STBC. In order to obtain an optimal power gain, the recent work in [10] employed ZF receiver to eliminate the interference. Unfortunately, all the schemes in [7]–[10] cannot guarantee the maximum possible diversity order after interference cancellation. To achieve full diversity in MACs, the works in [11]–[12] developed a particular precoder for each user with arbitrary number of transmit antenna. Interference alignment was firstly proposed to suppress the interference and achieve the maximum degree of freedom (DoF) in MIMO interference and X channels [14]–[16]. For two-user X channels, a full-diversity transmission scheme in [13] is proposed by borrowing an idea from interference alignment in [14]. Different from the work in [14], the goal of [13] is to achieve the maximum diversity order instead of the maximum DoF in X channels. Note that the interference interference was safely eliminated in [11]–[13], since it can be aligned into an orthogonal subspace to the desired signals with the aid of full channel state information at the transmitters (CSIT). By doing so, interference cancellation is realized without any side impact on the full diversity of the desired user. In practice, however, it is very hard for the transmitters to fully know the channel information. Therefore, it is natural to consider a full-diversity STBC design for wireless networks without CSIT.

In this paper, we propose a design criterion for an STBC to achieve full diversity in two-user MIMO interference channels without CSIT, where the full diversity is defined for a single user pair, i.e., the link between a transmitter and its corresponding receiver. Since the receiver corresponding to the desired user knows nothing about the codebook of the interfering user, a straightforward way to eliminate the interference is using the group ZF receiver, which was widely investigated and applied in space-time coded MIMO systems [5] [6] [7]–[19]. Based on the design criterion, we propose a systematic STBC design for each user with multiple transmit antennas that can obtain full diversity after interference cancellation. For a fixed number of transmit antennas $M$, the code rate approaches one when the time delay in the encoding (or code block size) gets large. In our paper, we focus on two-user interference channels where the codebooks of two users are not shared. Hence, the joint ML decoding cannot be used. Compared to the work in [7]–[10], our contribution lies in the design of full diversity STBC for interference channels with the group ZF decoding. It is worthwhile to mention that the main difference from the works in [11]–[13] is that the channel information is unknown at the transmitter side such that in our paper the inter-user interference is canceled by using the...
group ZF receiver instead of interference alignment. To our best knowledge, this is the first attempt of full diversity STBC design for two-user MIMO interference channels without CSIT.

The rest of the paper is organized as follows. The system model is outlined in Section II. Then, a design criterion of STBC under the group ZF receiver is given in Section III. In Section IV, a systematic STBC design is proposed for each user and the full diversity is proved when the group ZF receiver is utilized. Simulation results are presented in Section V. Finally, we conclude the paper.

**Notations:** Superscripts $^T$ and $^\dagger$ stand for transpose and conjugate transpose, respectively. $\mathbb{C}$ denotes the complex number field. $I_n$ denotes an $n \times n$ identity matrix and $0_{m \times n}$ denotes an $m \times n$ matrix whose elements are all 0. $\text{diag}\{\mathbf{v}\}$ produces a diagonal matrix with entries of a vector $\mathbf{v}$ in main diagonal. $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_M\}$ denotes a subspace spanned by all the vectors of $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_M\}$. $E\{\cdot\}$ denotes the statistical expectation. $\otimes$ denotes Kronecker product. $|| \cdot ||$ represents the Frobenius norm. $\text{tr}\{\mathbf{A}\}$ denotes the trace of a square matrix $\mathbf{A}$. $\min\{\mathbf{v}\}$ denotes the minimum value in a vector $\mathbf{v}$.

## II. SYSTEM MODEL

Fig. 1 shows a two-user interference channel composed of 2 transmitters (i.e., user 1 and user 2) and 2 receivers (i.e., R1 and R2), where each user is equipped with $M$ transmit antennas and its corresponding receiver has $N$ receive antennas.

At the transmitter side, two symbol vectors $s = [s_1, s_2, \ldots, s_L]^T$ and $\mathbf{c} = [c_1, c_2, \ldots, c_L]^T$ are firstly encoded to STBCs $\mathbf{S} \in \mathbb{C}^{T \times M}$ and $\mathbf{C} \in \mathbb{C}^{T \times M}$, respectively. The independent symbols $s_l$ and $c_l$ for $l = 1, 2, \ldots, L$ are selected from two independent codebooks $A_1$ and $A_2$ such as quadrature amplitude modulation (QAM). Then, the codewords $\mathbf{S}$ and $\mathbf{C}$ are transmitted from user 1 for R1 and user 2 for R2 over $T$ time slots, respectively.

The received signals $\mathbf{Y}_1 \in \mathbb{C}^{T \times N}$ and $\mathbf{Y}_2 \in \mathbb{C}^{T \times N}$ at R1 and R2 are given by, respectively

\[
\mathbf{Y}_1 = \sqrt{\frac{\rho}{\mu}} \mathbf{H}_1 \mathbf{s} + \sqrt{\frac{\rho}{\mu}} \mathbf{G}_1 \mathbf{c} + \mathbf{n}_1, \quad \text{(1)}
\]

\[
\mathbf{Y}_2 = \sqrt{\frac{\rho}{\mu}} \mathbf{H}_2 \mathbf{s} + \sqrt{\frac{\rho}{\mu}} \mathbf{G}_2 \mathbf{c} + \mathbf{n}_2, \quad \text{(2)}
\]

where the matrices $\mathbf{H}_r$ and $\mathbf{G}_r$ stand for the channel matrices from user 1 to R$r$ and user 2 to R$r$ for $r = 1, 2$, respectively. Moreover, the entries of the channel matrices $\mathbf{H}_r \in \mathbb{C}^{M \times N}$ and $\mathbf{G}_r \in \mathbb{C}^{M \times N}$ for $r = 1, 2$ are assumed to be independently and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance, i.e., $\mathcal{CN}(0, 1)$, and the channels are assumed to experience the quasi-static fading. $\mathbf{N}_r \in \mathbb{C}^{T \times N}$ is the noise matrix at R$r$ for $r = 1, 2$, whose elements are also i.i.d. $\mathcal{CN}(0, 1)$. Additionally, $\rho$ is the average signal-to-noise ratio (SNR) per receive antenna at each receiver and $\mu$ is the normalization factor to ensure that the average energy of the coded symbols transmitting from all antennas of each user during one symbol period is 1.

Throughout this paper, there are two assumptions as follows

1) The channel matrices $\mathbf{H}_1$ and $\mathbf{G}_1$ are perfectly known at R1, and $\mathbf{H}_2$ and $\mathbf{G}_2$ are known at R2. However, no channel information is known at any transmitter.

2) The codebook of each user is only known at its respective receiver.

To decode the desired information at each receiver, (1) and (2) can be rewritten as the equivalent forms by using some linear transformations, respectively

\[
\mathbf{y}_1 = \sqrt{\frac{\rho}{\mu}} \mathbf{H}_1 \mathbf{s} + \sqrt{\frac{\rho}{\mu}} \mathbf{G}_1 \mathbf{c} + \mathbf{n}_1 \quad \text{(3)}
\]

\[
\mathbf{y}_2 = \sqrt{\frac{\rho}{\mu}} \mathbf{H}_2 \mathbf{s} + \sqrt{\frac{\rho}{\mu}} \mathbf{G}_2 \mathbf{c} + \mathbf{n}_2, \quad \text{(4)}
\]

where $\mathbf{y}_r \in \mathbb{C}^{T \times N}$ is the equivalent form of $\mathbf{Y}_r$ for $r = 1, 2$. Likewise, $\mathbf{N}_r$ is equivalently transformed into $\mathbf{n}_r$ for $r = 1, 2$. Moreover, the equivalent channel matrices are given by $\mathbf{H}_r \in \mathbb{C}^{T \times L}$ and $\mathbf{G}_r \in \mathbb{C}^{T \times L}$ for $r = 1, 2$, both of which are determined by the code matrices $\mathbf{S}$ and $\mathbf{C}$ and the original channel matrices $\mathbf{H}_r$ and $\mathbf{G}_r$, respectively.

From now on, we focus on the interference cancellation and decoding at R1. The process at R2 follows similarly and is omitted.

As we mentioned, the desired information at R1 is interfered by the information of user 2. The group ZF receiver is used to cancel the interference and decode the desired information of the respective user. Firstly, the cancellation matrix of the group ZF receiver at R1 is written as

\[
\mathbf{Q}_1 = \mathbf{I}_{TN} - \mathbf{G}_1 (\mathbf{G}_1^\dagger \mathbf{G}_1)^{-1} \mathbf{G}_1^\dagger, \quad \text{(5)}
\]

where $\mathbf{Q}_1$ is the projection matrix that projects a vector in $\mathbb{C}^N$ on to the orthogonal complementary subspace of $\mathbf{V}_2$ spanned by the column vectors of $\mathbf{G}_1$, i.e.,

\[
\mathbf{V}_2 = \text{span}\{\mathbf{G}_1^1, \mathbf{G}_1^2, \ldots, \mathbf{G}_1^L\}, \quad \text{(6)}
\]

where $\mathbf{G}_1^l$ is the $l$th column of $\mathbf{G}_1$ for $l = 1, 2, \ldots, L$.

Then, multiplying $\mathbf{y}_1$ by $\mathbf{Q}_1$ to the left, we obtain that

\[
\mathbf{Q}_1 \mathbf{y}_1 = \frac{1}{\sqrt{\mu}} \mathbf{Q}_1 \mathbf{H}_1 \mathbf{s} + \mathbf{Q}_1 \mathbf{n}_1, \quad \text{(7)}
\]

such that the interference from user 2 is completely removed, i.e., $\mathbf{Q}_1 \mathbf{G}_2 = \mathbf{0}$.

After interference cancellation, ML decoding is utilized to decode $\mathbf{s}$ of user 1 at R1, and the decision metric is given by

\[
\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in A_1} \left\| \mathbf{Q}_1 \mathbf{y}_1 - \sqrt{\frac{1}{\mu}} \mathbf{Q}_1 \mathbf{H}_1 \mathbf{s} \right\|^2. \quad \text{(8)}
\]

The ML metric can be used in (8) since the noise term $\mathbf{Q}_1 \mathbf{n}_1$ after interference cancellation is proved to be a degenerated Gaussian white noise in [5, Lemma 2].
Definition 1: (Full diversity in two-user MIMO interference channel) Consider a transmission scheme for two-user MIMO interference channel in (3), where each user has $M$ transmit antennas and its respective receiver has $N$ receive antennas. User 1 is said to obtain full diversity of order $MN$ if its average pairwise error probability (PEP) $P(\Delta s)$ decays like $\rho^{-MN}$, i.e.,

$$P(\Delta s) \leq c \cdot \rho^{-MN}, \quad (9)$$

where $c$ is a constant and an error vector is defined as $\Delta s = s - \bar{s}$ and $s$ is erroneously decoded as $\bar{s}$. From (9), the full diversity of user 1 is defined when user 1 only considers the link between the corresponding receiver and itself.

Remark 1 (Difference from Guo-Xia’s work [5] [6]): In [5] [6], the point-to-point MIMO systems are considered. From the equivalent channel model $y = Hs + n$, the received signal is divided into $K$ groups which results in

$$y = H_1s_1 + H_2s_2 + \ldots + H_Ks_K + n,$$

where $s_k$ denotes the symbol vector in the $k$th group corresponding to the channel matrix $H_k$ for $k = 1, 2, \ldots, K$. Suppose we want to decode $s_1$. The group ZF receiver is used to cancel the interference from all the other groups $\{s_2, s_3, \ldots, s_K\}$. Note that the channel matrix $H_1$ in the desired group and the channel matrices $\{H_2, H_3, \ldots, H_K\}$ in the interfering groups may be dependent on each other.

In our work, the interference from a different and independent transmitter (i.e., user 2) is eliminated by the group ZF receiver, thereby the channel matrices $G_1$ and $H_1$ are independent of each other.

Remark 2 (Independence between $Q_1$ and $H_1$): In Guo-Xia’s work [5] [6], the cancellation matrix $Q_1$ is constructed by the channel matrices $\{H_2, H_3, \ldots, H_K\}$ in the interfering groups, which may depend on the desired channel $H_1$ since all the elements in $Q_1$ and $H_1$ are the linear combinations of $h_{1,j}$ and $h_{i,j}$ with $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, N$.

In our work, $Q_1$ and $H_1$ in (7) are independent, since $Q_1$ in (5) is derived from $G_1$ and $G_1$ is independent of $H_1$.

III. CODE DESIGN CRITERION

In this section, we propose a design criterion for user 1 to achieve full diversity with the group ZF receiver. Firstly, let us introduce the following lemma.

Lemma 1: Consider a transmission scheme for two-user MIMO interference channel in (3). By using the group ZF receiver in (5), user 1 achieves full diversity of order $MN$ if the following inequality can be satisfied,

$$\|Q_1H_1\Delta s\|^2 \geq \alpha \sum_{i=1}^{M} \sum_{j=1}^{N} |\{h_{1,i,j}\}|^2, \quad \forall \Delta s \in \Delta A_i^L, \quad (10)$$

where an error vector is defined as $\Delta s = s - \bar{s}$ and $s$ is erroneously decoded as $\bar{s}$. Moreover, $\{h_r\}_{i,j}$ and $\{g_r\}_{i,j}$ with $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, N$ are the $\{i,j\}$th entry of $H_r$ and $G_r$ for $r = 1, 2$, respectively. $\alpha$ is a positive constant independent of $\{h_{1,i,j}\}$.

Proof:

For convenience, $\{h_1\}_{i,j}$ and $\{g_1\}_{i,j}$ are simplified to be $h_{1,j}$ and $g_{1,j}$, respectively. In addition, we let $h_1 = [h_{1,1}, h_{2,1}, \ldots, h_{M,1}]^T$ and $g_1 = [g_{1,1}, g_{2,1}, \ldots, g_{M,1}]^T$. Moreover, we define $h = [h_1^T, h_2^T, \ldots, h_N^T]^T$ and $g = [g_1^T, g_2^T, \ldots, g_N^T]^T$.

Firstly, we evaluate the conditional PEP $P_{\{h,g\}}(\Delta s)$ of user 1 for two given vectors $h$ and $g$. From (10), we obtain that

$$P_{\{h,g\}}(\Delta s) = \mathcal{Q}\left(\sqrt{\frac{\rho}{\mu}} \|Q_1H_1\Delta s\|^2\right) \leq \mathcal{Q}\left(\frac{\alpha \rho}{2\mu} \sum_{i=1}^{M} \sum_{j=1}^{N} |h_{1,j}|^2\right) \leq \frac{1}{2} \exp\left(-\frac{\alpha \rho}{2\mu} \sum_{i=1}^{M} \sum_{j=1}^{N} |h_{1,j}|^2\right), \quad (11)$$

where $\mathcal{Q}(\cdot)$ denotes $Q$-function and the last inequality is obtained since $\mathcal{Q}(x) \leq \frac{1}{2} \exp\left(-\frac{x^2}{2}\right)$.

By taking average over $h$ in (11), we further have that

$$P_{\{h\}}(\Delta s) = \mathcal{E}_h\left(P_{\{h,g\}}(\Delta s)\right) \leq \mathcal{E}_h\left\{\frac{1}{2} \exp\left(-\frac{\alpha \rho}{2\mu} \sum_{i=1}^{M} \sum_{j=1}^{N} |h_{1,j}|^2\right)\right\} \leq \frac{1}{2} \left(\frac{2\mu}{2\mu + \alpha \rho}\right)^{MN}, \quad (12)$$

where the last equality holds since

$$\mathcal{E}_h\{\exp(-a|h|^2)\} = \frac{1}{1 + e^a}, \quad h \sim \mathcal{CN}(0,1) \text{ and } a > 0.$$

Furthermore, when $\rho$ approaches infinity, the conditional PEP of user 1 for the given $g$ can be upper bounded by

$$P_{\{g\}}(\Delta s) \leq \frac{2^{MN-1}\mu^M}{\sigma^M \rho^{-MN}}. \quad (13)$$

Next, we take expectation over $g$ in (13), which has no impact on the power of $\rho$ (see Remark 7). Therefore, the average PEP $P(\Delta s)$ is upper bounded by

$$P(\Delta s) \leq \sigma \rho^{-MN},$$

where $\sigma$ is a positive constant independent of $h$. According to Definition 1 we can conclude that user 1 achieves full diversity of order $MN$ after interference cancellation.

Consequently, we complete the proof of Lemma 1.

In what follows, we propose a design criterion for user 1 to satisfy the inequality (10) with an STBC in two-user MIMO interference channels with the group ZF receiver.

Theorem 1: Consider a transmission scheme for two-user MIMO interference channel in (3) where each user has $M$ transmit antennas and the receiver has $N$ receive antennas. At the receiver $R_1$, the group ZF receiver is used to cancel the interference from user 2 with the STBC $C$. After interference cancellation of $Q_1 \in \mathbb{C}^{TN \times TN}$ in (5), user 1 with the STBC $S$ can satisfy the inequality (10) in the following system

$$Q_1y_1 = \sqrt{\frac{\rho}{\mu}} Q_1H_1s + Q_1n_1, \quad (14)$$
if and only if the matrix $Q_1(I_N \otimes \Delta S)$ has the full column rank of $MN$, where the matrix $\Delta S \in \mathbb{C}^{T \times M}$ shares the same structure as the code matrix $S$ and every entry is selected from the erroneous vector $\Delta s$.

Proof:

1) Sufficiency: Firstly, we prove that the full-rank condition is sufficient to ensure the inequality (10) and thereby user 1 can guarantee full diversity after interference cancellation. From (1) and (3), it is not hard to obtain that

$$||Q_1 \mathcal{H}_1 \Delta s||^2 = ||Q_1(I_N \otimes \Delta S) \mathbf{h}||^2,$$

where $\Delta s$ corresponds to a code matrix $\Delta S$ and a column vector $\mathbf{h} = [h_1^T \ h_2^T \ldots h_T^T]^T$. Both $\Delta s$ and $\mathbf{h}$ have been defined in Lemma 7.

By using singular value decomposition (SVD) of the matrix $Q_1(I_N \otimes \Delta S)$, we further obtain that

$$||Q_1(I_N \otimes \Delta S) \mathbf{h}||^2 = \text{tr}\{\mathbf{h}^H \mathbf{U}^H \Lambda \mathbf{U} \mathbf{h}\},$$

where $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{MN}\}$ and $\mathbf{U}$ is a unitary matrix. Since the matrix $Q_1(I_N \otimes \Delta S)$ has the full column rank of $MN$, all the singular values $\lambda_i$ are positive. Let $\lambda_{\text{min}} = \min\{\lambda_1, \lambda_2, \ldots, \lambda_{MN}\}$ and $\alpha = \min_{\Delta S \in \Delta S} \lambda_{\text{min}}$. Since the signal constellation $S$ is finite, the erroneous set $\Delta S$ is finite as well. Since every $\lambda_{\text{min}} > 0$, we conclude that $\alpha > 0$. Thus, $||Q_1(I_N \otimes \Delta S) \mathbf{h}||^2$ can be lower bounded by

$$||Q_1(I_N \otimes \Delta S) \mathbf{h}||^2 \geq \alpha \sum_{i,j=1}^{MN} |h_{i,j}|^2,$$

where the second inequality holds since $\mathbf{U}$ is a unitary matrix and $\alpha$ is the minimum among all $\lambda_{\text{min}}$. Since the matrix $Q_1(I_N \otimes \Delta S)$ is independent of the channel $\mathcal{H}_1$ (see Remark 2) and therefore independent of the channel $\mathbf{h}$, we obtain that $\alpha$ is independent of the channel $\mathbf{h}$. This proves that user 1 can fulfill the inequality (10) under the group ZF receiver if the full-rank condition is satisfied.

2) Necessity: A proof by contradiction is considered to prove that the full-rank condition is necessary for user 1 to guarantee (10) after the group ZF receiver. We assume that the matrix $Q_1(I_N \otimes \Delta S)$ is rank deficient, which means that it must exist a nonzero vector $a = [a_1, a_2, \ldots, a_{MN}]$ with $a_v \in \mathbb{C}$ for $v = 1, 2, \ldots, MN$ such that

$$a_1Q_1\Delta S_1' + a_2Q_1\Delta S_2' + \ldots + a_{MN}Q_1\Delta S_M'N = 0,$$

where $\Delta S_v'$ stands for the $v$th column vector of the matrix $I_N \otimes \Delta S$. Consequently, it is possible to find a nonzero $\mathbf{h} = [a_1, a_2, \ldots, a_{MN}]^T$ such that we obtain $||Q_1(I_N \otimes \Delta S) \mathbf{h}||^2 = 0$, which does not satisfy the inequality (10) for any positive constant $\alpha$. Thus, the assumption contradicts with the condition that user 1 with the STBC $S$ can fulfill the inequality (10), thereby the full-rank condition must hold.

The proof of Theorem 7 is completed.

By following Lemma 7, we conclude that user 1 can achieve full diversity of order $MN$ under the group ZF receiver if the full-rank condition in Theorem 7 is satisfied.

IV. PROPOSED STBC DESIGN

In this section, a systematic STBC design is proposed for each user and two code examples are presented. Based on the proposed design criterion, we prove that the full diversity can be guaranteed with our proposed STBC designs.

A. STBC Designs

The STBC designs $S$ and $C$ for user 1 and user 2, respectively, are as follows

$$S = \begin{bmatrix}
\tilde{s}_1 & 0 & \ldots & 0 \\
\tilde{s}_2 & \tilde{s}_1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots \\
0 & \tilde{s}_L & \ldots & \tilde{s}_1 \\
0 & 0 & \ldots & 0 \\
\end{bmatrix}_{(L+2M-1) \times M},$$

and $C = \begin{bmatrix}
\tilde{c}_1 & 0 & \ldots & 0 \\
\tilde{c}_2 & \tilde{c}_1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \tilde{c}_L & \ldots & \tilde{c}_1 \\
0 & 0 & \ldots & 0 \\
\end{bmatrix}_{(L+2M-1) \times M},$

where $L$ stands for the number of layers in each codeword. Moreover, $\tilde{s}_i$ and $\tilde{c}_i$ are the $i$th elements of the rotated symbol vectors $\tilde{s}$ and $\tilde{c}$, respectively, which are given by

$$\tilde{s} = \Theta s = [\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_L]^T,$$

and $\tilde{c} = \Theta c = [\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_L]^T,$

where the rotation matrix $\Theta$ is given in 20 and 21 such that both $\tilde{s}$ and $\tilde{c}$ have non-zero product distance, i.e., full rank if they are used to form the diagonal space-time codes 22. Similar to 3, the proposed codes are featured as Toeplitz coding structure. However, in order to achieve full diversity after interference cancellation, the independent symbols in each layer of the proposed codes are transformed by a rotation matrix $\Theta$, which will be specified in the following.

The code rate of an STBC is defined as $R = \frac{L}{T}$ symbols per channel use, where $L$ independent symbols selected from a finite constellation in the STBC are transmitted over $T$ time slots. Accordingly, the code rate of the proposed STBC for each user is $\frac{L}{L+2M-1}$ symbols per channel use with $T = L + 2M - 1$. For a fixed $M$, the rate approaches 1 symbol per channel use when $L$ in the time delay in the encoding (or code block size) goes to infinity.

Remark 3 (Difference from the multilayer STBC in 17): The multilayer STBC $X$ in 17, Eq. (13)] was proposed
to achieve full diversity for MIMO systems based on the
criterion in [5], [6]. In [17], the STBC design is given by
\[
X = \begin{bmatrix}
X_{1,1} & 0 & \cdots & 0 \\
X_{2,1} & X_{1,2} & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
X_{L,1} & \cdots & \cdots & X_{1,M} \\
0 & X_{L,2} & \cdots & X_{2,M} \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & 0 \\
\end{bmatrix}, \tag{23}
\]
where the symbol vector \([X_{1,1}, X_{1,2}, \ldots, X_{L,M}]^T\) in the \(l\)th layer is given by
\[
\begin{bmatrix}
X_{1,1} \\
X_{1,2} \\
\vdots \\
X_{L,1} \\
0 \\
\vdots \\
0 \\
\end{bmatrix} = \Theta s_l,
\tag{24}
\]
and the transmitted symbol vector \(s_l\) is given by
\[
\begin{bmatrix}
s_{1,(l-1)M+1} \\
s_{1,(l-1)M+2} \\
\vdots \\
s_{1,M} \\
\end{bmatrix}^T,
\tag{25}
\]
for \(l = 1, 2, \ldots, L\).

Similar to our STBC designs in [19], the design in [23] has \(L\) layers and the transmitted symbols in each layer is performed by a rotation matrix \(\Theta\). Different from [19], the code structure of [23] is not featured as a Toeplitz matrix since the entries in each layer are different. With the code design in [23], user 1 may not obtain the full diversity after interference cancellation since full-rank condition in Theorem 7 cannot be satisfied.

B. Full Diversity Property

Now, we show that each user can achieve full diversity after interference cancellation with the proposed STBCs. Without loss of generality, we focus on user 1 only.

**Theorem 2:** Consider two-user MIMO interference channels where each user has \(M\) transmit antennas and each receiver is equipped with \(N\) receive antennas. The STBC designs in [19] can obtain full diversity of order \(MN\) under the group ZF receiver.

**Proof:**

To prove that user 1 with the proposed STBC \(S\) in [19] obtains full diversity, we show that \(Q_1(I_N \otimes \Delta S)\) has full rank of \(MN\) by following Lemma 7 and Theorem 7.

From the code structure in [19], we have that
\[
\Delta S = [\Delta S_1 \ \Delta S_2 \ \ldots \ \Delta S_M],
\]
where \(\Delta S_i\) is the \(i\)th column vector of \(\Delta S\) for \(i = 1, 2, \ldots, M\) and
\[
\Delta \tilde{s} = \Theta \Delta s = [\Delta \tilde{s}_1, \ \Delta \tilde{s}_2, \ \ldots \ \Delta \tilde{s}_L]^T,
\tag{27}
\]
where \(\Delta \tilde{s}_l \neq 0, 1 \leq l \leq L\), from the design of the rotation matrix \(\Theta\) in [20] and [21]. Generally, the equivalent channel matrix of a linear dispersion STBC is expressed as a stack version of the equivalent channel matrices of all individual receive antennas [23]. Hence, the equivalent channel matrix \(G_1\) is given by
\[
G_1 = \begin{bmatrix}
G_{1,1,1}^T & G_{1,1,2}^T & \cdots & G_{1,1,N}^T
\end{bmatrix}^T,
\tag{28}
\]
where \(G_{1,j}\) denotes the equivalent channel matrix of user 2 corresponding to the \(j\)th receive antenna. From [3], [19] and [23], we have
\[
G_{1,j} = G_{1,j}^T \Theta,
\tag{29}
\]
with
\[
G_{1,j} = \begin{bmatrix}
G_{1,1,j} & G_{1,2,j} & \cdots & G_{1,L,j}
\end{bmatrix}
\]
\[
= \begin{bmatrix}
g_{1,j} & 0 & \cdots & 0 \\
g_{2,j} & g_{1,j} & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
g_{M,j} & \cdots & \cdots & g_{1,j} \\
0 & g_{M,j} & \cdots & g_{2,j} \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & 0 \\
\end{bmatrix} \in (L+2M-1) \times (L+2M-1)
\tag{30}
\]
where \(G_{1,j}^l\) denotes the \(l\)th column vector of \(G_{1,j}\) for \(l = 1, 2, \ldots, L\). By substituting (29) into (28), the equivalent channel \(G_1\) is rewritten as
\[
G_1 = \begin{bmatrix}
G_{1,1,1}^T & G_{1,1,2}^T & \cdots & G_{1,1,N}^T
\end{bmatrix}^T \Theta,
\tag{31}
\]
where we let \(G_{1,j} = \begin{bmatrix}
G_{1,1,j}^T & G_{1,2,j}^T & \cdots & G_{1,L,j}^T
\end{bmatrix}^T\).

From [5], the cancellation matrix \(Q_1\) is given by
\[
Q_1 = I_{TN} - G_1 (G_1^\dagger G_1)^{-1} G_1^\dagger,
\]
where the second equality holds since \(\Theta\) is a full rank matrix [20] and [21]. In this scenario, \(Q_1\) can be regarded as a projection matrix that projects a vector in \(\mathbb{C}\) onto the orthogonal complementary subspace of \(V_2\). The subspace \(V_2\) is spanned by all the column vectors \(G_{1,j}^l\) of \(G_{1,j}\) for \(l = 1, 2, \ldots, L\).

Then, we are ready to check whether the matrix \(Q_1(I_N \otimes \Delta S)\) has full rank. For a clear explanation, the following matrix is used to see the relationship between
Therefore, all column vectors of the matrix \( Q_1(I_N \otimes \Delta S) \) are linear independent over \( C \), which justifies that the matrix has the full column rank of \( M.N \). By following Theorem 7 user 1 with the proposed STBC \( S \) in (19) retains the full diversity after interference cancellation.

The proof of Theorem 2 is completed.

\[ \]
achieve full diversity under the group ZF receiver. Based on the criterion, we proposed a STBC for each user to obtain full diversity after the group ZF receiver. For a fixed $M$, the coding gain with the AO receiver is about 1dB better than the AO receiver always outperforms the group ZF receiver, the group ZF and AO/MMSE receivers. Since the performance of the AO receiver always outperforms the group ZF receiver, the proposed STBC when it is decoded by the group ZF receiver. It can be observed that user 1 with our proposed STBC is capable of achieving full diversity of order $M$ under both group ZF and AO/MMSE receivers. Since the performance of the AO receiver always outperforms the group ZF receiver, the proposed codes can also achieve the full diversity as evidenced from this simulation. Moreover, when both $M$ and $L$ are fixed, the coding gain with the AO receiver is about 1dB better than the one with the group ZF receiver.

VI. CONCLUSION

In this paper, a design criterion of STBC design was proposed for two-user MIMO interference channels under the group ZF receiver. Based on the criterion, we proposed a systematic STBC for each user to obtain full diversity after interference cancellation. The proposed STBC approaches a code rate of one symbol per channel use when the time delay in the encoding (or code block size) gets large. Simulation results were presented to show that our proposed STBCs can achieve full diversity under the group ZF receiver.

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