Resolving the octant of $\theta_{23}$ via radiative $\mu$-$\tau$ symmetry breaking

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Abstract

We point out that the observed neutrino mixing pattern at low energies is very likely to originate from the $3 \times 3$ Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton flavor mixing matrix $U$ which possesses the exact $\mu$-$\tau$ permutation symmetry $|U_{\mu i}| = |U_{\tau i}|$ (for $i = 1, 2, 3$) at a superhigh energy scale $\Lambda_{\mu\tau} \sim 10^{14}$ GeV. The deviation of $\theta_{23}$ from $45^\circ$ and that of $\delta$ from $270^\circ$ in the standard parametrization of $U$ are therefore a natural consequence of small PMNS $\mu$-$\tau$ symmetry breaking via the renormalization-group equations (RGEs) running from $\Lambda_{\mu\tau}$ down to the electroweak scale $\Lambda_{EW} \sim 10^2$ GeV. In fitting current experimental data we find that the RGE-corrected value of $\theta_{23}$ is uniquely correlated with the neutrino mass ordering: $\theta_{23} \simeq 42.4^\circ$ reported by Capozzi et al (or $\theta_{23} \simeq 48.9^\circ$ reported by Forero et al) at $\Lambda_{EW}$ can arise from $\theta_{23} = 45^\circ$ at $\Lambda_{\mu\tau}$ in the minimal supersymmetric standard model if the neutrino mass ordering is inverted (or normal). Accordingly, the preliminary best-fit results of $\delta$ at $\Lambda_{EW}$ can also evolve from $\delta = 270^\circ$ at $\Lambda_{\mu\tau}$ no matter whether the massive neutrinos are Dirac or Majorana particles.

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1 Introduction

From the discovery of atmospheric neutrino oscillations in 1998 [1] until the observation of the smallest neutrino mixing angle $\theta_{13}$ in 2012 [2], experimental neutrino physics was in full flourish. Today the era of precision measurements has come. A number of undergoing and upcoming neutrino oscillation experiments aim to determine the neutrino mass ordering, to probe the octant of the largest neutrino mixing angle $\theta_{23}$, and to measure the Dirac CP-violating phase $\delta$. Such knowledge will be fundamentally important, as it can help identify the underlying flavor symmetry or dynamics behind the observed pattern of lepton flavor mixing.

As far as the octant of $\theta_{23}$ is concerned, it is desirable to know whether this “atmospheric neutrino mixing” angle deviates from $45^\circ$ or not, and if it does, how large or small the deviation is and in what direction the deviation evolves. A global analysis of current neutrino oscillation data done by Capozzi et al [3] yields the best-fit result $\theta_{23} \approx 41.4^\circ$ (normal neutrino mass ordering) or $\theta_{23} \approx 42.4^\circ$ (inverted neutrino mass ordering), which has a preference for the first octant (i.e., $\theta_{23} < 45^\circ$). In contrast, another best-fit result reported by Forero et al [4] is $\theta_{23} \approx 48.8^\circ$ (normal ordering) or $\theta_{23} \approx 49.2^\circ$ (inverted ordering), by which the second octant (i.e., $\theta_{23} > 45^\circ$) is favored. In both cases $\theta_{23} = 45^\circ$ will be allowed when the 1$\sigma$ or 2$\sigma$ error bars are taken into account. Hence the octant of $\theta_{23}$ remains an open issue, and a resolution to this puzzle awaits more accurate experimental data.

On the other hand, the best-fit results of $\delta$ in both Ref. [3] and Ref. [4] are close to an especially interesting value, $270^\circ$, although the confidence level remains quite low. In fact, the T2K measurement of a relatively strong $\nu_\mu \rightarrow \nu_e$ appearance signal [5] plays a crucial role in the global fit to make $\theta_{13}$ consistent with the Daya Bay result [2] and drive a slight but intriguing preference for $\delta \approx 270^\circ$ [3, 4]. If this preliminary expectation turns out to be true, there will be no problem to observe significant effects of leptonic CP violation in the forthcoming long-baseline neutrino oscillation experiments.

On the theoretical side, $\theta_{23} = 45^\circ$ and $\delta = 270^\circ$ are a straightforward consequence of the $\mu$-$\tau$ permutation symmetry manifesting itself in the $3 \times 3$ Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton flavor mixing matrix $U$ [6]: $|U_{\mu i}| = |U_{\tau i}|$ (for $i = 1, 2, 3$), which can easily be embedded in an explicit flavor symmetry model. Hence the deviation of $\theta_{23}$ from $45^\circ$ and that of $\delta$ from $270^\circ$ must be related to small PMNS $\mu$-$\tau$ symmetry breaking effects. This observation is important and suggestive, implying that the observed pattern of the PMNS matrix $U$ should have an approximate $\mu$-$\tau$ symmetry of the form $|U_{\mu i}| \simeq |U_{\tau i}|$ at low energies [7]. In comparison, the Cabibbo-Kobayashi-Maskawa (CKM) quark flavor mixing matrix $V$ [8] does not possess such a peculiar structure.

In this work we pay particular attention to a very real possibility: the PMNS $\mu$-$\tau$ symmetry is exact at a superhigh energy scale $\Lambda_{\mu\tau}$ where both tiny neutrino masses and large neutrino mixing angles could naturally be explained in a well-founded theoretical framework (e.g., with the canonical seesaw mechanism [9] and proper flavor symmetry groups [10]). In this case we find that it is possible to resolve the octant of $\theta_{23}$ and the quadrant of $\delta$ via radiative $\mu$-$\tau$ symmetry breaking effects. Namely, the equalities $|U_{\mu i}| = |U_{\tau i}|$ are more or less violated when they evolve from $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to the electroweak scale $\Lambda_{\text{EW}} \sim 10^2$ GeV via the relevant renormalization-group equations (RGEs), such that the correct octant of $\theta_{23}$ and the correct quadrant of $\delta$ can consequently be obtained. We carry out a numerical analysis of the issue for both Dirac and Majorana neutrinos based on the one-loop RGEs in the minimal supersymmetric standard model.
A striking finding of ours in fitting current neutrino oscillation data is that the RGE-corrected value of $\theta_{23}$ is uniquely correlated with the neutrino mass ordering: $\theta_{23} \simeq 42.4^\circ$ reported in Ref. 3 (or $\theta_{23} \simeq 48.9^\circ$ reported in Ref. 4) at $\Lambda_{\text{EW}}$ can evolve from $\theta_{23} = 45^\circ$ at $\Lambda_{\mu\tau}$ only when the neutrino masses have an inverted (or normal) ordering. Accordingly, the preliminary best-fit results of $\delta$ at $\Lambda_{\text{EW}}$ can also originate from $\delta = 270^\circ$ at $\Lambda_{\mu\tau}$ thanks to radiative $\mu$-$\tau$ symmetry breaking. Such remarkable results are independent of any specific models of neutrino mass generation and lepton flavor mixing, and they will soon be tested in the upcoming precision experiments of neutrino oscillations.

2 The RGEs of $\mu$-$\tau$ symmetry breaking

The PMNS lepton flavor mixing matrix can be parametrized in the following “standard” way [12]:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\delta} & s_{13}c_{23}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & s_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_{\nu},$$ (1)

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ (for $ij = 12, 13, 23$), $\delta$ is referred to as the Dirac CP-violating phase, and $P_{\nu} = \text{Diag} \{e^{i\beta}, e^{i\gamma}, 1\}$ contains two extra phase parameters if massive neutrinos are the Majorana particles. Up to now $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$ have all been measured to a good degree of accuracy, and some preliminary hints for a nontrivial value of $\delta$ have also been obtained from a global analysis of current neutrino oscillation data [3, 4]. Here we are concerned about the three PMNS $\mu$-$\tau$ “asymmetries”:

$$\Delta_1 \equiv |U_{\mu1}|^2 - |U_{e1}|^2 = (\cos^2 \theta_{12} \sin^2 \theta_{13} - \sin^2 \theta_{12}) \cos 2\theta_{23} - \sin 2\theta_{12} \sin \theta_{13} \sin 2\theta_{23} \cos \delta,$$
$$\Delta_2 \equiv |U_{\mu2}|^2 - |U_{\mu3}|^2 = (\sin^2 \theta_{12} \sin^2 \theta_{13} - \cos^2 \theta_{12}) \cos 2\theta_{23} + \sin 2\theta_{12} \sin \theta_{13} \sin 2\theta_{23} \cos \delta,$$
$$\Delta_3 \equiv |U_{\tau3}|^2 - |U_{\mu3}|^2 = \cos^2 \theta_{13} \cos 2\theta_{23},$$ (2)

which satisfy the sum rule $\Delta_1 + \Delta_2 + \Delta_3 = 0$. All the three $\Delta_i$ vanish when the exact $\mu$-$\tau$ permutation symmetry holds.

We conjecture that the exact PMNS $\mu$-$\tau$ symmetry (i.e., $\Delta_i = 0$) can be realized at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV in a given neutrino mass model with a proper flavor symmetry group [10]. In view of the facts that a nonzero and relatively large $\theta_{13}$ has been observed and the preliminary best-fit value of $\delta$ is not far from $270^\circ$ at the electroweak scale [3, 4], we infer that the condition for all the three $\Delta_i$ to vanish should naturally be $\theta_{23} = 45^\circ$ and $\delta = 270^\circ$ at the $\mu$-$\tau$ symmetry scale. In this case $\Delta_i \neq 0$ can therefore be achieved at $\Lambda_{\text{EW}} \sim 10^2$ GeV through the RGE running effects. The one-loop RGEs of $|U_{\alpha i}|^2$ (for $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$) have been derived by one of us in Ref. 14. So it is straightforward to write out the RGEs of $\Delta_i$ in the MSSM as follows.

1In this connection the standard-model RGEs are less interesting for two reasons: (a) it will be difficult to make the deviation of $\theta_{23}$ from $45^\circ$ appreciable even if the neutrino masses are nearly degenerate; (b) the standard model itself will largely suffer from the vacuum-stability problem for the measured value of the Higgs mass ($\simeq 125$ GeV) as the energy scale is above $10^{10}$ GeV [11].

2Although $\Delta_i = 0$ might also result from $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$ or $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$ [13], neither of them is close to the best-fit results of the lepton flavor mixing parameters reported in Refs. 3 and 4. These two possibilities are much less likely because they have to invoke violent RGE running effects between $\Lambda_{\mu\tau}$ and $\Lambda_{\text{EW}}$ in order to fit the present experimental data, which actually favor slight $\mu$-$\tau$ symmetry breaking [7]. That is why we concentrate our interest only on the possibility of $\theta_{23} = 45^\circ$ and $\delta = 270^\circ$ in this paper.
2.1 Dirac neutrinos

If massive neutrinos are the Dirac particles, we find

\[
16\pi^2 \frac{d\Delta_1}{dt} = -y^2 \left\{ \xi_{21} \left( |U_{r1}|^2 \Delta_2 + |U_{r2}|^2 \Delta_1 + |U_{e3}|^2 \right) + \xi_{31} \left( |U_{r1}|^2 \Delta_3 + |U_{r3}|^2 \Delta_1 + |U_{e2}|^2 \right) \right\},
\]
\[
16\pi^2 \frac{d\Delta_2}{dt} = +y^2 \left\{ \xi_{21} \left( |U_{r1}|^2 \Delta_2 + |U_{r2}|^2 \Delta_1 + |U_{e3}|^2 \right) - \xi_{32} \left( |U_{r2}|^2 \Delta_3 + |U_{r3}|^2 \Delta_2 + |U_{e1}|^2 \right) \right\},
\]
\[
16\pi^2 \frac{d\Delta_3}{dt} = +y^2 \left\{ \xi_{31} \left( |U_{r1}|^2 \Delta_3 + |U_{r3}|^2 \Delta_1 + |U_{e2}|^2 \right) + \xi_{32} \left( |U_{r2}|^2 \Delta_3 + |U_{r3}|^2 \Delta_2 + |U_{e1}|^2 \right) \right\},
\]

where \( t \equiv \ln(\mu/\Lambda_{\text{EW}}) \) with \( \mu \) being an arbitrary scale between \( \Lambda_{\text{EW}} \) and \( \Lambda_{\text{MS}} \), \( y^2 = (1 + \tan^2 \beta) m^2_\tau/v^2 \) is the Yukawa coupling eigenvalue of the tau lepton in the MSSM with \( \tan \beta \) and \( v \) being self-explaining, and \( \xi_{ij} \equiv (m_i^2 + m_j^2)/\Delta m^2_{ij} \) with \( \Delta m^2_{ij} \equiv m_i^2 - m_j^2 \) being the neutrino mass-squared differences. Given the fact \( |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \sim 30 \Delta m^2_{21} \) with \( \Delta m^2_{21} \simeq 7.5 \times 10^{-5} \text{eV}^2 \), \( \xi_{21} \gg |\xi_{31}| \simeq |\xi_{32}| \) is expected to hold in most cases. But this does not necessarily mean that the \( \mu-\tau \) asymmetry \( \Delta_3 \) should be more stable against radiative corrections than the other two asymmetries. The reason is simply that the running behaviors of \( \Delta_1 \) depend also on the initial inputs of all the nine \( |U_{ai}|^2 \). In general, however, an appreciable deviation of \( \theta_{23} \) from 45° (i.e., an appreciable deviation of \( \Delta_3 \) from zero) requires a sufficiently large value of \( \tan \beta \), and its evolving direction is governed by the neutrino mass ordering or equivalently the sign of \( \Delta m^2_{31} \) or \( \Delta m^2_{32} \).

2.2 Majorana neutrinos

If massive neutrinos are the Majorana particles, we arrive at

\[
16\pi^2 \frac{d\Delta_1}{dt} = -y^2 \left\{ \xi_{21} \left( |U_{r1}|^2 \Delta_2 + |U_{r2}|^2 \Delta_1 + |U_{e3}|^2 \right) + \xi_{31} \left( |U_{r1}|^2 \Delta_3 + |U_{r3}|^2 \Delta_1 + |U_{e2}|^2 \right) \right\},
\]
\[
16\pi^2 \frac{d\Delta_2}{dt} = +y^2 \left\{ \xi_{21} \left( |U_{r1}|^2 \Delta_2 + |U_{r2}|^2 \Delta_1 + |U_{e3}|^2 \right) - \xi_{32} \left( |U_{r2}|^2 \Delta_3 + |U_{r3}|^2 \Delta_2 + |U_{e1}|^2 \right) \right\},
\]
\[
16\pi^2 \frac{d\Delta_3}{dt} = +y^2 \left\{ \xi_{31} \left( |U_{r1}|^2 \Delta_3 + |U_{r3}|^2 \Delta_1 + |U_{e2}|^2 \right) + \xi_{32} \left( |U_{r2}|^2 \Delta_3 + |U_{r3}|^2 \Delta_2 + |U_{e1}|^2 \right) \right\},
\]

where \( \zeta_{ij} \equiv 2m_i m_j/\Delta m^2_{ij} \), \( \cos \Phi_{ij} \equiv \text{Re}(U_{\tau_i}U^*_{\tau_j})/|U_{\tau_i}U^*_{\tau_j}|^2 \), \( \sin \Phi_{ij} \equiv \text{Im}(U_{\tau_i}U^*_{\tau_j})/|U_{\tau_i}U^*_{\tau_j}|^2 \), and the leptonic Jarlskog invariant \( J \) [15] is defined through

\[
\text{Im} \left( U_{ai}U_{\beta j}U^*_{\alpha j}U^*_{\beta i} \right) = J \sum_{\gamma} \epsilon_{\alpha \beta \gamma} \sum_k e_{ijk}
\]

with the Greek and Latin subscripts running over \((e, \mu, \tau)\) and \((1, 2, 3)\), respectively. Since the sign of \( \zeta_{ij} \) is always the same as that of \( \xi_{ij} \), it is possible to adjust the evolving direction of \( \Delta_3 \) without much fine-tuning of the other relevant parameters. Hence, similar to the Dirac neutrino case, the RGE-triggered deviation of \( \theta_{23} \) from 45° might be closely correlated with the neutrino mass ordering in the Majorana case.
In Eq. (4) it should be noted that the two Majorana CP-violating phases \( \rho \) and \( \sigma \) in the standard parametrization of \( U \) affect the running behaviors of \( \Delta_i \) via \( \cos \Phi_{ij} \) and \( \sin \Phi_{ij} \). One should also note that \( \mathcal{J} = \left( \sin 2\theta_{12} \sin 2\theta_{13} \cos \theta_{13} \sin 2\theta_{23} \sin \delta \right) / 8 \), which only depends on the Dirac CP-violating phase \( \delta \), measures the strength of CP violation in neutrino oscillations. Therefore, \( \delta \sim 270^\circ \) is especially favorable for significant CP-violating effects in the lepton sector, no matter whether the massive neutrinos are Dirac or Majorana particles.

### 3 Numerical results for \( \Delta_i \), \( \theta_{23} \) and \( \delta \)

We proceed to numerically illustrate the effects of \( \mu-\tau \) symmetry breaking regarding the PMNS matrix \( U \) — namely, the quantities \( \Delta_i \) run from \( \Delta_i = 0 \) (i.e., \( \delta_{23} = 45^\circ \) and \( \delta = 270^\circ \)) at \( \Lambda_{\mu r} \sim 10^{14} \) GeV down to \( \Lambda_{EW} \sim 10^2 \) GeV via the one-loop RGEs obtained in Eq. (3) or (4). Given a proper value of \( \tan \beta \), the values of \( m_1 \), \( \Delta m^2_{21} \), \( \Delta m^2_{31} \), \( \theta_{12} \) and \( \theta_{13} \) at \( \Lambda_{\mu r} \) should be carefully chosen such that the best-fit results of \( \Delta m^2_{21} \), \( \Delta m^2_{31} \), \( \theta_{12} \), \( \theta_{13} \), \( \theta_{23} \) and \( \delta \) at \( \Lambda_{EW} \) as listed in Table 1 can all be reproduced to a good degree of accuracy. If this strategy is workable, then the deviation of \( \theta_{23} \) from \( 45^\circ \) and that of \( \delta \) from \( 270^\circ \) will be purely attributed to the RGE-triggered PMNS \( \mu-\tau \) symmetry breaking effects.

| Reference       | Mass ordering | \( \Delta m^2_{21} \) (eV\(^2\)) | \( \Delta m^2_{31} \) (eV\(^2\)) | \( \theta_{12} \) | \( \theta_{13} \) | \( \theta_{23} \) | \( \delta \) |
|-----------------|---------------|-------------------------------|-------------------------------|-----------------|-----------------|-----------------|------------|
| Capozzi et al [3]| Normal        | 7.54 \times 10^{-5}           | +2.47 \times 10^{-3}          | 33.7            | 8.8             | 41.4            | 250°       |
|                 | Inverted      |                               | −2.34 \times 10^{-3}         |                 | 8.9             | 42.4            | 236°       |
| Forero et al [4]| Normal        | 7.60 \times 10^{-5}           | +2.48 \times 10^{-3}          | 34.6            | 8.8             | 48.9            | 241°       |
|                 | Inverted      |                               | −2.38 \times 10^{-3}         |                 | 8.9             | 49.2            | 266°       |

#### 3.1 Dirac neutrinos

For simplicity, we fix \( \tan \beta = 31 \) and input \( m_1 = 0.1 \) eV at \( \Lambda_{\mu r} \), where \( \Delta_1, \Delta_2 \) and \( \Delta_3 \) are vanishing (or equivalently, \( \theta_{23} = 45^\circ \) and \( \delta = 90^\circ \)), in our numerical calculations. Table 2 shows the input and output values of all the relevant parameters for two examples, which are based on the best-fit results reported by Capozzi et al [3] and Forero et al [4], respectively. Figure 1 illustrates how \( \Delta_i \) evolve in either example. Some comments and discussions are in order.

1. Given the inverted neutrino mass ordering, the best-fit results of the six neutrino oscillation parameters \( \Delta m^2_{21}, \Delta m^2_{31}, \theta_{12}, \theta_{13}, \theta_{23} \) and \( \delta \) at \( \Lambda_{EW} \) in Example I [3] can successfully be reproduced from the proper inputs at \( \Lambda_{\mu r} \). In this case \( \theta_{23}(\Lambda_{EW}) \) lies in the first octant, and \( \theta_{23}(\Lambda_{\mu r}) - \theta_{23}(\Lambda_{EW}) \approx 2.6^\circ \) holds thanks to the RGE running effect. At the same time, we obtain \( \delta(\Lambda_{\mu r}) - \delta(\Lambda_{EW}) \approx 34^\circ \). Hence the RGE evolution can also provide a resolution to the quadrant of \( \delta \).

Note that the notations \( \delta m^2 \equiv m^2_3 - m^2_1 \) and \( \Delta m^2 \equiv m^2_3 - (m^2_1 + m^2_2) / 2 \) have been used in Ref. [3]. They are related with \( \Delta m^2_{21} \) and \( \Delta m^2_{31} \) as follows: \( \Delta m^2_{21} = \delta m^2 \) and \( \Delta m^2_{31} = \Delta m^2 + \delta m^2 / 2 \).

Note that the fit group’s best-fit results [16] are not taken into account in our numerical examples, because they happen to correspond to the disfavored cases listed in Table 1 (i.e., \( \theta_{23} < 45^\circ \) for the normal neutrino mass ordering or \( \theta_{23} > 45^\circ \) for the inverted ordering, in conflict with our expectations shown in Tables 2 and 3, respectively).
Table 2: The RGE-triggered PMNS $\mu$-$\tau$ symmetry breaking effects for Dirac neutrinos running from $\Delta_i = 0$ at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{EW} \sim 10^2$ GeV in the MSSM with $\tan \beta = 31$.

| Parameter            | Example I (Capozzi et al [3]) | Example II (Forero et al [4]) |
|----------------------|--------------------------------|--------------------------------|
|                      | Input ($\Lambda_{\mu\tau}$)   | Output ($\Lambda_{EW}$)        | Input ($\Lambda_{\mu\tau}$)   | Output ($\Lambda_{EW}$)        |
| $m_1$ (eV)           | 0.100                          | 0.093                          | 0.100                          | 0.093                          |
| $\Delta m_{21}^2$ (eV$^2$) | $1.82 \times 10^{-4}$         | $7.54 \times 10^{-5}$         | $1.96 \times 10^{-4}$         | $7.60 \times 10^{-5}$         |
| $\Delta m_{31}^2$ (eV$^2$) | $-2.60 \times 10^{-3}$       | $-2.34 \times 10^{-3}$       | $3.00 \times 10^{-3}$         | $2.48 \times 10^{-3}$         |
| $\theta_{12}$        | $10.8^\circ$                   | $33.6^\circ$                  | $10.3^\circ$                   | $34.6^\circ$                  |
| $\theta_{13}$        | $9.4^\circ$                    | $8.9^\circ$                   | $8.4^\circ$                    | $8.8^\circ$                   |
| $\theta_{23}$        | $45.0^\circ$                   | $42.4^\circ$                  | $45.0^\circ$                   | $48.4^\circ$                  |
| $\delta$             | $270^\circ$                    | $236^\circ$                   | $270^\circ$                    | $237^\circ$                   |
| $\mathcal{J}$        | $-0.015$                       | $-0.029$                      | $-0.013$                       | $-0.029$                      |
| $\Delta_1$           | 0                              | 0.053                         | 0                              | 0.114                         |
| $\Delta_2$           | 0                              | $-0.141$                      | 0                              | 0.001                         |
| $\Delta_3$           | 0                              | 0.088                         | 0                              | $-0.115$                      |

(2) In contrast, only the normal neutrino mass ordering allows us to obtain $\theta_{23}(\Lambda_{EW}) \approx 48.4^\circ$ from $\theta_{23}(\Lambda_{\mu\tau}) = 45^\circ$ via the RGE evolution as shown in Example II [4]. Moreover, we obtain $\delta(\Lambda_{EW}) \approx 237^\circ$ from $\delta(\Lambda_{\mu\tau}) = 270^\circ$, and this result is also consistent very well with the corresponding best-fit value $\delta \approx 241^\circ$ as listed in Table 1. The future experimental data will only verify one of the above two possibilities for the octant of $\theta_{23}$, but it will be interesting to test the expected correlation between the neutrino mass ordering and the deviation of $\theta_{23}$ (or $\delta$) from $45^\circ$ (or $270^\circ$).

(3) Figure 1 shows the behaviors of three PMNS $\mu$-$\tau$ asymmetries $\Delta_{i}$ evolving from $\Lambda_{\mu\tau}$ down to $\Lambda_{EW}$ for the two examples under discussion. In view of $\Delta_3 = \cos^2 \theta_{13} \cos 2 \theta_{23}$ in Eq. (2), one must have $\Delta_3(\Lambda_{EW}) > 0$ for $\theta_{23}(\Lambda_{EW}) < 45^\circ$ in Example I, and $\Delta_3(\Lambda_{EW}) < 0$ for $\theta_{23}(\Lambda_{EW}) > 45^\circ$ in Example II. In comparison, the running behaviors of $\Delta_1$ and $\Delta_2$ are not so straightforward, because they depend on all the three flavor mixing angles and the CP-violating phase $\delta$. But $\Delta_1 + \Delta_2 + \Delta_3 = 0$ holds at any energy scale between $\Lambda_{EW}$ and $\Lambda_{\mu\tau}$, as one can see in Figure 1.

### 3.2 Majorana neutrinos

In this case we simply fix $\tan \beta = 30$ and input $m_1 = 0.1$ eV at $\Lambda_{\mu\tau}$, where $\Delta_1 = \Delta_2 = \Delta_3 = 0$ holds, in our numerical calculations. Table 3 is a brief summary of the input and output values of all the relevant parameters for Example I [3] and Example II [4], respectively. In addition, Figure 2 illustrates how the three PMNS $\mu$-$\tau$ asymmetries evolve from $\Lambda_{\mu\tau}$ down to $\Lambda_{EW}$ in either example.

Although the present case involves two extra CP-violating phases $\rho$ and $\sigma$, the running behaviors of $\Delta_{i}$ in Figure 2 are quite similar to those in Figure 1. Of course, one has to adjust the initial values of $\rho$ and $\sigma$ at $\Lambda_{\mu\tau}$ in a careful way, such that the best-fit results of the six neutrino oscillation parameters can correctly be reproduced at $\Lambda_{EW}$. We find that it is really possible to resolve the octant of $\theta_{23}$ and the quadrant of $\delta$ at the same time via radiative PMNS $\mu$-$\tau$ symmetry breaking. Very similar to the Dirac neutrino case, the RGE-triggered deviation of $\theta_{23}$ from $45^\circ$ in the Majorana case is also closely correlated with the neutrino mass ordering. Namely, $\theta_{23} \approx 42.4^\circ$ reported in
Ref. [3] (or $\theta_{23} \simeq 48.9^\circ$ reported in Ref. [4]) at $\Lambda_{\text{EW}}$ can evolve from $\theta_{23} = 45^\circ$ at $\Lambda_{\mu\tau}$ only when the neutrino masses have an inverted (or normal) ordering.

Table 3: The RGE-triggered PMNS $\mu$-$\tau$ symmetry breaking effects for Majorana neutrinos running from $\Delta_i = 0$ at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{\text{EW}} \sim 10^2$ GeV in the MSSM with $\tan\beta = 30$.

| Parameter      | Example I (Capozzi et al [3]) | Example II (Forero et al [4]) |
|----------------|-------------------------------|--------------------------------|
| $m_1$ (eV)     | 0.100                         | 0.100                          |
| $\Delta m_{21}^2$ (eV$^2$) | $1.70 \times 10^{-4}$                         | $2.12 \times 10^{-4}$                          |
| $\Delta m_{31}^2$ (eV$^2$) | $-2.98 \times 10^{-3}$                        | $3.50 \times 10^{-3}$                        |
| $\theta_{12}$  | $35.2^\circ$                  | $32.1^\circ$                   |
| $\theta_{13}$  | $10.1^\circ$                  | $6.9^\circ$                    |
| $\theta_{23}$  | $45.0^\circ$                  | $45.0^\circ$                   |
| $\delta$       | $270^\circ$                   | $270^\circ$                    |
| $\rho$         | $-82^\circ$                   | $-76^\circ$                    |
| $\sigma$       | $19^\circ$                    | $17^\circ$                     |
| $J$            | $-0.040$                      | $-0.027$                       |
| $\Delta_1$     | 0                             | 0                             |
| $\Delta_2$     | 0                             | 0                             |
| $\Delta_3$     | 0                             | 0                             |

In view of the fact that the present best-fit results of $\theta_{23}$ and $\delta$ are still quite preliminary, we foresee that they must undergo some changes before they are well determined by the more precise experimental data in the near future. Hence our numerical analysis is not targeted for a complete parameter-space exploration but mainly for the purpose of illustration [17]. Its outcome supports our original conjecture: the slight $\mu$-$\tau$ symmetry breaking behind the observed pattern of lepton flavor mixing can originate from the RGE evolution from a superhigh flavor symmetry scale down to the electroweak scale. Note that there are two adjustable unknown parameters in our calculations: the absolute neutrino mass $m_1$ and the MSSM parameter $\tan\beta$. Once $m_1$ is experimentally determined and $\tan\beta$ is theoretically fixed, for example, it will be interesting to see whether one can still resolve the octant of $\theta_{23}$ and the quadrant of $\delta$ with the help of radiative PMNS $\mu$-$\tau$ symmetry breaking effects.

We admit that the present best-fit result $\delta \sim 270^\circ$ remains too preliminary. In fact, there is not any nontrivial region associated with the allowed values of $\delta$ at the 2$\sigma$ level [3, 4]. Hence it also makes sense to look at the RGE-triggered corrections to $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$ for the energy scale to evolve from $\Lambda_{\mu\tau}$ down to $\Lambda_{\text{EW}}$. This possibility has already been discussed in some literature (see, e.g., Refs. [17, 18]). Once the CP-violating phase $\delta$ is measured or constrained to a better degree of accuracy in the near future, it will be possible to examine whether the quantum corrections can really accommodate the observed effect of PMNS $\mu$-$\tau$ symmetry breaking or not.
4 Summary and further discussions

To summarize, we have conjectured that the PMNS $\mu$-$\tau$ permutation symmetry is exact at a superhigh energy scale $\Lambda_{\mu\tau} \sim 10^{14}$ GeV, where the origin of neutrino masses and flavor mixing has a good dynamic reason, and its slight breaking happens via the RGE running down to the electroweak scale $\Lambda_{EW} \sim 10^2$ GeV. This idea is particularly interesting in the sense that it can help resolve the octant of $\theta_{23}$ and the quadrant of $\delta$ at the same time thanks to radiative PMNS $\mu$-$\tau$ symmetry breaking in the MSSM. In fitting current neutrino oscillation data we have found that the RGE-triggered deviation of $\theta_{23}$ from 45° is uniquely correlated with the neutrino mass ordering: $\theta_{23} \simeq 42.4°$ [3] (or $\theta_{23} \simeq 48.9°$ [4]) at $\Lambda_{EW}$ can naturally originate from $\theta_{23} = 45°$ at $\Lambda_{\mu\tau}$ if the neutrino mass ordering is inverted (or normal). Accordingly, the preliminary best-fit results of $\delta$ at $\Lambda_{EW}$ can also evolve from $\delta = 270°$ at $\Lambda_{\mu\tau}$. Such remarkable findings are independent of any specific models of neutrino mass generation and lepton flavor mixing, and they will soon be tested in the upcoming neutrino oscillation experiments.

Note that some previous studies of the RGE evolution of lepton flavor mixing parameters have more or less involved the $\mu$-$\tau$ symmetry breaking effects [19]. In this connection a few constant neutrino mixing patterns which possess $|U_{\mu i}| = |U_{\tau i}|$, such as the bimaximal [20] and tri-bimaximal [21] ones with $\theta_{13} = 0°$ and $\theta_{23} = 45°$, have been assumed at a superhigh energy scale; and their RGE running behaviors have been investigated mainly to see whether a finite $\theta_{13}$ can be radiatively generated at low energies [18]. The closest example of this kind should be the work [22] on radiative corrections to the tetra-maximal neutrino mixing pattern [23], in which $\theta_{13} \simeq 8.4°$, $\theta_{23} = 45°$ and $\delta = 90°$ or $270°$ have been predicted. Our present work is different from the previous ones in several aspects: (a) it is not subject to any explicit neutrino mixing pattern; (b) it focuses on the PMNS $\mu$-$\tau$ asymmetries $\Delta_i$ and its RGE evolution; (c) it provides a reasonable resolution to the octant of $\theta_{23}$ by attributing it to the PMNS $\mu$-$\tau$ symmetry breaking effect; (d) it may also resolve the quadrant of $\delta$ in a similar way. All in all, we have established the RGE connection between a given neutrino mass model with the exact $\mu$-$\tau$ symmetry at superhigh energies and the neutrino oscillation parameters at low energies. Such a connection is expected to be very useful for neutrino phenomenology in the era of precision measurements.

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Figure 1: The RGE-triggered $\mu$-$\tau$ symmetry breaking effects for Dirac neutrinos running from $\Delta_i = 0$ at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{\text{EW}} \sim 10^2$ GeV in the MSSM with $\tan \beta = 31$. 
Figure 2: The RGE-triggered $\mu$-$\tau$ symmetry breaking effects for Majorana neutrinos running from $\Delta_i = 0$ at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{EW} \sim 10^2$ GeV in the MSSM with $\tan \beta = 30$. 