Study on Saturation Estimation of Initial Perturbations in Ensemble Marine Forecasting based on BGM Method

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Abstract. Because the ocean is a highly complex nonlinear chaotic system, its numerical prediction results are strongly uncertain due to the influence of the initial values and the model itself. Ensemble forecasting is an effective way to reduce the uncertainty of forecasting. The initial perturbations quality, especially the saturation estimation, is of practical significance for improving ensemble forecasting techniques. In this paper, the factors affecting initial perturbation saturation of ensemble forecasting are analyzed by BGM based on ROMS model, and the perturbation saturation velocities of the two methods are compared. The results show that the saturation velocity of the initial perturbation is faster than that of the initial perturbation space structure. The larger the initial perturbation is, the higher the saturation degree and velocity are, and the saturation time of the same variable varies at different layers. The saturation of different variables is also different; the perturbation saturation speed of the regional adjust perturbation method is faster than that of the classical static perturbation method.

1. Introduction

As a chaotic dynamical system, the temporal and spatial evolution of the ocean is highly random\textsuperscript{11}. Considering the randomness of initial conditions and model errors in the process of ocean numerical forecasting, ensemble forecasting and probability forecasting are commonly used to quantitatively estimate the uncertainty of forecasting and to reduce the influence of forecasting errors caused by nonlinear process\textsuperscript{2-3}.

As the core problem of ensemble forecasting, the quality of initial perturbation sets determines the skill of ensemble forecasting, and the saturation estimation process of perturbation is the key technology. At present, the main dynamic methods used to generate initial perturbations in ensemble forecasting are Breeding Growing Method\textsuperscript{4}, Singular Vector Method\textsuperscript{5} and Conditional Non-linear Optimal Perturbation Method\textsuperscript{6}. Among them, the Breeding Growing Method has such advantages as clear dynamic meaning and low computational cost, and is widely used in atmospheric and oceanic ensemble forecasting.

Aimed at the problem of initial perturbation saturation estimation in ensemble prediction, this paper analyses the saturation of BGM perturbation based on ROMS model, and compares the saturation velocity of the two kinds of perturbation methods.
2. Models and Methods

2.1 ROMS Numerical Prediction Model
Using the Rutgers version of ROMS, this paper calculates the area of East China Sea (23.5°N-41.5°N,117°E-133°E) with the spatial resolution of 5 km, and the grid points are 372 (longitudinal)×506×(latitudinal)×32 (vertical). Appropriate densification has been done in the surface and bottom layers. The GLS vertical mixing model is used for the turbulence closure model. The radiation boundary condition for the open boundary and the HYCOM data are used for the model, and the forced field data are derived from the monthly average data of the European Centre for Medium-Term Weather Forecasting (ECMWF). The results of surface temperature prediction of the model under this configuration are similar to those of AVHRR data in the same period, which verify the reliability of the model results.

2.2 Classical Static BGM Method
The classical static BGM method introduces random perturbation at the initial time and generates fast-growing high-energy errors by mode cyclic integration. The initial random perturbation \( P(z) \) introduced at first is determined by the following formula:

\[
P(z) = \omega R E(z)
\]

\( z \) is the state variable of the numerical model (\( z = T, U, V \) representing the thermal and dynamic fields respectively). \( \omega \) is an adjustment coefficient to control the size of the initial random perturbation. \( R \) is a random number that obeys the uniform perturbation on the \([-1,1]\) interval. \( E(z) \) is the root mean square error of each layer of variables in numerical prediction. The cycle integral period is 12 hours. At the end of each cycle, the control prediction results without initial random perturbation are subtracted from the perturbation prediction results and adjusted by the perturbation limiting coefficient. The adjusted perturbation is then introduced to the analysis field, and the cyclic integral is complete when the perturbation is saturated.

In the classical static BGM method, the initial random perturbation adjustment coefficient does not change with time and space, so it belongs to a static constant perturbation method.

2.3 Regional adjusting BGM Method
The vertical mixing process of the ocean is an important way of transferring energy and momentum between the ocean and the atmosphere, which has an important influence on the perturbation of ocean temperature, salt and flow field structure\(^7\). Observations show that the mixing is stronger in steep seabed areas and weaker in flat seabed areas\(^8\). In the process of perturbation propagation of the regional adjust BGM method, the perturbation adjustment coefficient of discontinuous time-space perturbation is used to disturb different regions with different intensity, and the spatial-temporal variation of perturbation is appropriately increased. That is to say, the perturbation is strengthened in the region with large depth gradient while the propagation of perturbation is promoted by means of vertical mixing.
3. Perturbation Saturation Measure

According to the introduction of BGM method, when the perturbation reaches the saturation state, the perturbation propagation ends, and the perturbation obtained at this time is the initial perturbation qualified for cultivation. The saturation of perturbation refers to the saturation of perturbation size and spatial form.

3.1 Saturation Measure of Perturbation Size

For the saturation measure of perturbation size, the perturbation growth size and the perturbation growth rate are generally chosen to be considered. The perturbation growth size represents the size of perturbation, and the perturbation growth rate represents the speed of the perturbation growth. The perturbation growth rate is specifically defined as:

$$r_t = \frac{M(p_t)}{M(p_{t-1})}$$

$M(p_t)$ and $M(p_{t+1})$ represent the initial perturbation growth size of the two adjacent steps in the mode cyclic integration process, respectively. When the perturbation growth size reaches a certain value or when the perturbation growth rate approaches 1, the magnitude of the perturbation can be considered to be saturated. The larger the growth mode of saturated perturbation is, the more rapid the perturbation is.
3.2 Saturation Measure of Perturbation Spatial Form

For the saturation measure of perturbation spatial form, the correlation coefficients of perturbation at two time points are generally considered, and the correlation degree can be quantitatively expressed by solving the coincidence index and Pearson correlation coefficients. Symbolic index is specifically defined as:

\[ F = \frac{n^+ - n^-}{n^+ + n^-} \]

\( n^+ \) denotes the number of lattices with the same sign of positive and negative perturbation values at the same position in the two-time perturbation field, \( n^- \) denotes the number of lattices with the opposite sign of positive and negative perturbation values. Pearson correlation coefficient is defined as:

\[ \rho_{X,Y} = \frac{N \sum(XY) - \sum(X) \sum(Y)}{\sqrt{N \sum(X^2) - (\sum(X))^2} \sqrt{N \sum(Y^2) - (\sum(Y))^2}} \]

\( X \) and \( Y \) denote the perturbation fields \( p_t \) and \( p_{t-1} \) of two adjacent output steps, respectively. \( N \) denotes the number of grid points. When the perturbation fields before and after propagation are very similar, the sign index reaches a certain value or the Pearson correlation coefficient approaches 1. It can be considered that the spatial structure of the initial random perturbation has reached saturation.

4. Perturbation Saturation Estimation

When introducing random initial perturbation, the size of initial perturbation, the layer of initial perturbation and the variables of perturbation should be considered comprehensively. In this paper, we only give the saturation estimation results under different initial settings when the time of 0h on January 1 is taken as the initial time. The estimation results at other times are similar.

4.1 Influence of Perturbation Initial Size on Saturation

The mean square deviation of random initial perturbation is about 0.5, because it obeys the uniform perturbation in [-1,1] interval. By taking 0.2, 1 and 2 respectively, the random initial perturbation is set as 10%, 50% and 100% of the root mean square error of each layer of the variables in numerical prediction. Figure 2 shows the development of initial temperature perturbations of different initial size at 10m: the larger the initial size is, the greater the saturation degree of the perturbation space form and the faster the corresponding saturation speed. Different initial modes have little influence on the development of perturbations size, which shows that the perturbations growth size and the perturbations growth rates tend to be the same. The results of saturation estimation for other variables (like \( U \) and \( V \)) are similar to those of temperature. Therefore, from the point of view of perturbations saturation estimation, the larger the initial perturbations size, the better. However, considering the reliability of the prediction results after introducing the initial perturbation, the initial perturbation model cannot be blindly large, and it needs to be within a certain range of constraints, so the adjustment coefficient in the follow-up test is taken as 1. The results at the rest of the layers are similar.
4.2 The Influence of Perturbation Initial Layers on Saturation

Figure 3 shows that the development of initial temperature perturbations in different perturbations layers is different. Specifically, with the development of prediction time, the initial perturbation at surface (1m) and subsurface (10m) increases greatly, while the initial perturbation in the middle (250m) and at the bottom (1000m) increases slightly, and the perturbation growth rate does not show obvious differences between layers. Regarding the saturation of perturbation spatial form, the results show that the saturation velocity of perturbation spatial Form at surface (1m), subsurface (10m) and bottom (1000m) is faster than that in the middle (250m). This may be due to the fact that the Kuroshio system in the simulated sea area is in the mid-level and the circulation structure is complex, which results in the slow development saturation of the spatial form of perturbation and makes it difficult to find out the rapid growth mode of perturbation. The results that correspond to the initial size of the remaining perturbations are similar.
4.3 The Saturation Velocity of Different Variables

Figure 4 shows the evolution of initial perturbation of different variables at 10m. Compared with the dynamic variables $U$ and $V$, the magnitude of temperature perturbation and the degree of spatial saturation are higher. When the perturbation is saturated, the temperature growth size is about 6 times that of the velocity growth size of $U$ and $V$. In the saturation estimation of the spatial form of perturbation, the initial temperature perturbation reaches spatial saturation in about 4 days, and the corresponding saturation time of $U$ and $V$ velocities is about 8 days. The saturation value in temperature space tends to be 1, and the saturation degree is high, while the saturation value in $U$ and $V$ velocity space is about 0.6 and the saturation degree is relatively low. That is to say, the saturation degrees of different physical variables are different, so the saturation criteria are also different. The results under other configurations are similar.
Figure 4. Development of initial perturbations of different variables under the same initial size and the same layer

4.4 Perturbation Saturation Velocity Corresponding to Two Perturbation Methods
Taking the initial temperature perturbation at 10m and PN section (Figure.5) as examples, the saturation velocities of the perturbation obtained by classical static BGM method and regional enhanced BGM method are compared from plane and section aspects (Figure.6-7). By increasing the initial perturbation size in the strong vertical mixing region, the region-enhanced BGM method makes the initial perturbation develop rapidly, and the corresponding perturbation size and spatial morphological saturation velocity are higher than those of the classical static BGM method. Especially for the classical BGM method at PN section, the perturbation saturation velocity is slower, and the saturation velocity and degree are improved significantly after vertical mixing is enhanced.
Figure 5. Depth gradient at PN section

Figure 6. Development of perturbations for two perturbation methods at 10m
5. Conclusion

In this paper, the initial perturbation saturation estimation of ensemble ocean forecasting is studied based on BGM method, and the following conclusions are obtained. Perturbation saturation includes both the size of perturbation and the shape of perturbation space, in which the saturation speed of the initial perturbation is faster than that of the initial perturbation space structure. The saturation estimation of perturbation is related to the size of initial perturbation, the location of perturbation layer and perturbation variables. The larger the initial perturbation mode, the higher the saturation degree and speed. The saturation time at different layers is different. Perturbation surface layers and subsurface layers are more easily saturated; the saturation of different variables is also different. The saturation degree and speed of temperature are higher than that of velocity. The perturbation saturation speed of the regional adjusting perturbation method is faster than that of the classical static perturbation method.

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