The Polyakov line, the Nishimori line and polymer networks standing a smectic liquid crystal

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(Dated: March 30, 2010)

We once more specify an universality of a phase transition from smectic-A to nematic phase in the crosslinked polymer network with a smectic liquid crystal, basing on the X-ray and 1H-NMR spectroscopy experiments. Comparing the superconducting 3D XY model and typical spin-glass models for such systems, it is possible to clarify a type of a phase transition, which might be the reason of percolation.

PACS numbers:

There are well known encountering problems in soft materials, polymers and liquid crystals, when at a quantum phase transitions, a system reveals percolation properties [1, 8]. The presence of observations the phase transition from a smectic-A (SmA) to a nematic (N) phase in crosslinked elastomers by X-ray [1] and 1H-NMR spectroscopy [4], paves the way for speculations on the percolation threshold predictions and measurements.

This report is devoted to the analysis of percolation at the observations cited above, in which a first riddle is formulated as follows. During which of phase transitions (thermal or quantum, i.e. caused by an increase of a crosslink density) percolation is happened?

Elastomers are polymer networks composed by crosslinked polymer chains. They consist nodes and links obeying ascertained rules of self organization [1, 7, 8]. In such a systems, an increase of a crosslink density may carry out to percolation [3, 4, 10].

For mere polymers, the percolation problem is considered in the light of analogy of dimers with connective conductors [9, 11, 12, 13], revealing the fractal dimensionality of a system.

For a polymer networks, there are applicable spin glass models, resolving with replica methods (see the review [16]), the cradles of which are sufficiently ancient [14, 15].

Here we face with a more complicated combination of such fluids, namely, the smectic bulk phase and elastomer network with their crosslinks (saying in a sketch of the aerogels description [19] and nomenclature accepted there and references therein).

In a smectic, the linear nontrivial disclinations (3D vortexes or dislocation loops [20]) may appear [13, 21], as well as a point monopole-type defects, suspecting the system are consisting monomers and dimers particles, corresponding to the liquid crystal molecules and polymeric fragments respectively. Percolation associates with condensation of vortexes in the high temperature phase, in the deconfinement phase (β < βc, where temperature denotes as β = 1/kT, k_B is the Boltzmann’s constant, and T is an absolute temperature, β_c labels a critical temperature) [22]. Fractal-dimension domains of this phase are existing on a non-integer lattice [22], where is the Polyakov line plays role of the order parameter [24].

For smectic-A liquid crystals surviving a tilt second order transition to a nematic phase, Dasgupta proposed the loop inverted analog of the superconductive XY model [21]. The action of this model is equivalent of the Villain’s one [21], if in the XY model, the de Gennes’s coefficient K_1 = 0 (what means an anisotropic case) [21, 27, 28].

Vortexes in the XY 3D model form loops (in the XY 4D model of compact electrodynamics [22] they corresponds to strings). The XY model is a quantized variant of a scalar field theory Φ with broken global U(1) symmetry. The potential of this theory is λ(|Φ|^2 − Φ_0^2)^2, at λ → ∞, the radial term of |Φ| is freezing, and the residual dynamical variable Φ is compact (Φ = Φ_0 |exp(iϕ)|) [23]. A vortex, as well as a dislocation with its similar properties, is non-trivial classical minimum of the action [20].

If a crosslinked elastomer system with smectic liquid crystal is considered in the 3D superconductivity class [19, 29] with a scalar gauge field k = 0, a dimensionality of defects (vortexes) then is D − k − 2 = 1. [23]. As it was shown numerically [30] for a bulk nematic, due to the coupling between smectic and nematic order parameters and director fluctuations, the SmA − N transition deviates from 3D XY universality to a nonuniversal crossover value between 3D XY and tricritical behavior with an effective heat capacity critical exponent α ranging between 0.26 and 0.31 confinements. In 3D, similar percolation holds below a critical temperature, β < β_c, β_c ≈ 0.4542 [23]. The calculation procedure for D_f is contained in [23]: the gauge field A [22], however, makes difficult a quantitative characterization of the order parameter in view of an ambiguity in A [21].

The order parameter (TrL) in terms of the Polyakov
where $\mathcal{P}$ denotes path ordering, $x_4$ and $A_4$ are a time coordinate component and a time component of the Yang-Mills field for 3D, in our case these are a temperature components) is zero in the confined phase below the critical temperature, and, otherwise, its a nonzero value denotes the broken symmetry. But in contrast to usual Polyakov loops, our system is uncentered (such as $SmA-N$ transition carries out from $T(2) \times D_{\infty h}$ to $D_{\infty h} \times T(3)$) \[24\]. However, amid this transition, an exact sequence with $Z(N)$ should exist to provide $U(1)$ in the limit case $N \to \infty$. For instance, $Z(N)$ has to be a group of crosslinks of a dual lattice.

The Polyakov line is not invariant under spatial gauge transformations. A periodical gauge transformation reads here as follows:

$$A_4(x) \rightarrow U(x)A_4(x)U(x)^\dagger,$$

$$A_i(x) \rightarrow U(x)A_iU(x)^\dagger + iU(x)\partial_iU(x)^\dagger.$$ 

And some a gauge field $A_\mu$ means a background field \[24, 31\], it has gauged in the same way:

$$A_\mu \rightarrow SA_\mu S^\dagger + iS\partial_\mu S^\dagger.$$ 

As it was shown, the general $SU(N)$ gauge group has the explicitly symmetric $Z(N)$ resulting action. So, for $Z(3)$, the maxima of the potential energy of the system corespond to the Polyakov line $L_{\text{max}} = \text{diag}(1, e^{2\pi i/3}, e^{-2\pi i/3})$, $\text{Tr}L_{\text{max}}=0$, etc. \[24\].

Alternative models of the random pinning potential \[1\], with the appointed role of topological defects in polymers are treatable as spin glassy approximations \[32\], which may be resolvable by the replica tricks. Also, a random graph method \[32\] is applied, which allows us to take into account both a long-wavelength and short-wavelength fluctuations. For $n > 1$ component spin system, a modification of the Polyakov method is used. These well developed ideas are not confirmed in the concrete case of the order parameter of polymers, complicated by smectic states \[34\].

Progressing in the problem of $SmA-N$ phase transitions, several 2D cases of quantum smectics, breaking their symmetry into a nematic phase, were discussed in \[35\]. Properties of 2+1D dislocation loops there were corresponded to the quantum double Hopf symmetry.

On the other hand, the spin-glass theory of polymers via its accessible results of a replica approximation gives a criterion of a temperature phase transition in the region of a spin-glass phase diagram, where replicas are not present, this is so called the Nishimori line $e^\beta = \frac{1-p}{p}$ \[36, 37\]. ($p$ is concentration of ferromagnetic bonds).
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