Observation of a Dislocation-Related Interfacial Friction Mechanism in Mobile Solid $^4$He

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Abstract We report a study of the temperature and stress dependence of the friction associated with a relative motion of crystallites of solid $^4$He in contact with each other. A situation where such motion exists emerges spontaneously during a disordering of a single crystal contained inside an annular sample space of a torsional oscillator (TO). Under the torque applied by the oscillating walls of the TO these crystallites move relative to each other, generating measurable dissipation at their interface. We studied this friction between 0.5 and 1.8 K in solid samples grown from commercially pure $^4$He and from a 100 ppm $^3$He–$^4$He mixture. The data were analyzed by modeling the TO as a driven harmonic oscillator. In this model, an analysis of the resonant frequency and amplitude of the TO yields the temperature dependence of the friction coefficient. By fitting the data to specific forms, we found that over our temperature range, the dominant friction mechanism associated with the interfacial motion of the crystallites results from climb of individual dislocations. The characteristic energy scale associated with this friction can be 3 or 6 K, depending on the sample. The motion of the solid in the presence of such friction can perhaps be described as the low limit of “slip–stick” motion.

Keywords Solid helium · Friction · Slip–stick · Dislocation climb

1 Introduction

Investigation of the unusual properties of solid $^4$He as a quantum solid has been a subject of intense study in recent years. The ongoing search for supersolidity at low
temperatures [1] which is currently under debate [2–4], stimulated an effort to explore and understand the unique elastic [5] and plastic [6–8] properties of crystalline He at very low temperatures. Our group is using a torsional oscillator (TO) to investigate solid He, as in the search for supersolidity. We however conduct experiments at much higher temperatures, between 0.5 and 2 K, so that our work appears unrelated to this issue, which has been searched for at temperatures of \(~0.2\) K and lower. Another important difference is that we are growing single crystals, while the majority of the supersolidity-related research is done with polycrystalline samples. We are particularly interested in understanding the mechanism of friction between crystallites of He moving past each other. The reason why this problem may be interesting is that a priori, it is not obvious that friction between quantum solids would be the same as between classical solids. For example, there are additional predictions regarding what are the friction mechanisms in the quantum regime. One such proposed quantum friction mechanism comes from the Van der Waals force, due to zero point charge fluctuations [9]. Another dissipation mechanism involves exchange of phonons between the two solids, where the phonons populating one of the masses are Doppler shifted due to the relative motion [10]. Internal friction in static polycrystalline solid He was investigated in the past, and was found to result from dynamics of pinned dislocations as described by the Lucke–Granato model [11,12]. However, the classical friction problem of one solid mass moving against another, was not yet addressed in solid He. Since solid He exists only under pressure, the classical layout of a friction experiment with two masses on a tabletop cannot be realized. By chance, we have found a way to perform such experiments.

2 Experimental Background

Our understanding of the physical situation in which crystallites of solid He can move past each other was developed over several years of systematic experiments done in our lab [13–16]. Here, we highlight the main points which are relevant for understanding of the experimental system.

The present work is a part of an ongoing program which began with our discovery [17] that disordered He crystals are mobile, in the sense that some of the crystallites making up the solid can spontaneously change their orientation without any external intervention. These observations were done on bcc solid \(^4\)He using neutron diffraction [17] and later on hcp solid \(^4\)He with X-ray diffraction [18]. The next logical step in our program was to see how this spontaneous mobility manifests itself if the solid He is subject to external stress. To this end, we grew single crystals inside an annular sample cell of a TO, shown schematically in Fig. 1. To enable us to grow a single crystal filling the whole cell, we add He through a heated filling line at the top, rather than through a hole drilled in the torsion rod (see Fig. 1). Once the sample space is filled with solid, we block the filling line and cool the TO by 50–100 mK. Cooling generates stress on the crystal, since the solid contracts around the central post of the sample chamber (see Fig. 1). Additional time-dependent shear stress is applied to the solid inside the TO by the oscillating wall of the sample chamber. We observe that simultaneously with the cooling, the resonant frequency of the TO increases spontaneously, as if part of
the solid He became decoupled from the motion of the TO. This effect is reproducible with different crystals and TO’s. The frequency change of the TO is equivalent to a reduction of the moment of inertia of the solid He by 10–30 %, depending on the sample. Such reduction would occur if part of the solid He becomes decoupled from the oscillatory motion of the TO. We call it the “decoupled mass effect” [13].

The natural question to ask is whether this effect represents real decoupling of a part of the solid from the motion of the TO, or there are other reasons responsible for the increase of the resonant frequency. Possible such reasons include: (a) the presence of some liquid in the cell, or (b) changes of the elastic properties of the solid with temperature. Consequently, the next step in our study program was to determine whether the decoupled mass effect can result from a presence of liquid in the cell.

We remark that our cells are carefully designed to have no sharp corners where residual liquid can be trapped. Since the filling line enters the cell through the top of the cell rather than through the torsion rod (see Fig. 1), we can heat the filling line along its entire length. The heating prevents solid from blocking the filling line, which can give an erroneous impression that the cell is full of solid before it actually is. The measured decrease of the frequency of the TO upon filling with solid He agrees very well with the calculated value.

Regarding the Presence of Some Liquid in the Cell The obvious test is to introduce liquid into the cell while it is in the “mass decoupled” state and see what happens. The result of this test is that as soon as liquid enters the cell, the frequency of the TO decreases back to what it was before decoupling took place, and the “decoupled mass” effect vanishes. We have also cooled the TO partially filled with solid, with some liquid left in the sample space. There was no decoupled mass effect in this case. These experiments show that there is absolutely no decoupled mass effect if there is any liquid in the cell. We have also used Finite Element Analysis to simulate the resonant frequency of the cell with some residual liquid present. This frequency was compared to that of the TO filled with solid, which would happen once cell was cooled and the liquid solidified. Details of the analysis are presented in the Appendix of Ref. [16]. The result is that the calculated frequency difference has an opposite sign and is at least one order of magnitude smaller than what we observe in the experiment.
Hence, frequency changes which we call a decoupled mass effect are not caused by solidification of residual liquid. Consequently, all our experiments in which there is a finite decoupled mass, are done without any liquid.

We now discuss the other possibility, that the increase of the resonant frequency of the TO (the decoupled mass effect) is caused by changes of the elastic properties of solid He. An increase of the shear modulus of solid He will increase the effective torsion constant and with it the resonant frequency. The influence of the elastic properties of solid He on the resonant period of a TO was discussed by several authors [19,20], in conjunction with the search for supersolidity [1–4] at low temperatures. To see whether this possibility applies in our case, we simulated the change of the effective shear modulus of our TO filled with solid over the whole temperature range using finite element analysis [16]. The key result is that effect of the temperature dependence of the shear modulus of solid He on the resonant frequency is 1–2 orders of magnitude less than the observed decoupled mass effect. Consequently, effects related to the shear modulus cannot explain our results either.

From the tests described above, we conclude that in the disordered state, part of the solid He does not move as a rigid body with the TO and becomes effectively decoupled from the motion of the walls. In the frame of reference of the TO, this decoupled solid moves relatively to the body of the TO.

The next logical step of our study was to search directly for the existence of such motion. The motion is supposed to take place inside the TO, and so we needed a method to detect what is going on inside the cell. Specifically for this purpose, we developed an in-situ acoustic AFM-like sensor (“microphone”) [16]. The “microphone,” described in detail in [16], is essentially a plate capacitor with one of its electrodes capable of moving under very small stress. The capacitor is embedded in solid He near the inner wall of the annular channel of the sample cell. It can detect vibrations down to the $10^{-11}$ m range, namely a few % of a lattice spacing. Vibrations of this amplitude would be excited by motion of solid He inside the cell. If there was no motion or if the motion was random, such vibrations would either not exist or average to zero. Indeed, we did not detect any such vibrations when the cell was filled with liquid He (random motion) or with a single crystal (single crystal is rigid, no relative motion). Excess vibrations proportional to the speed of the wall were detected only when the cell contained a disordered crystal in the “mass decoupled” state. The detection of these vibrations confirmed the existence of relative motion inside the cell.

3 Results and Discussion

We use an electrostatically driven TO shown in Fig. 1. The body of the TO is made of Be–Cu. The inner radius of the annular sample space is 6.5 mm, the height is 10 mm, and its width is 2 mm (for additional details see [13]). The applied torque $\tau_0$ is related to the AC driving voltage $V_0$ through $\tau_0 = V_{DC} r_0 C V_0 / d$, where $V_{DC} = 200$ V is a constant bias on one of the capacitor plates; $r_0 = 8.5$ mm is the radius of the outer wall of the cell; $C = 2$ pF; and $d = 400 \mu$m is the distance between the capacitor plates. With a $V_0 = 1$ mV rms, the applied torque is $10^{-11}$ Nm.
Over the last few years, we studied many tens of solid samples prepared as single crystals under a variety of growth conditions [13–15]. Depending on the temperature and pressure at which a crystal is grown, we could grow either bcc or hcp solid. Most crystals were grown using commercially pure $^4$He, and some were grown using a 100 ppm $^3$He–$^4$He mixture. While studying the response of the TO containing the solid, we noticed that the variation of the resonant frequency and dissipation with temperature and applied drive seemed similar for all of these samples. This observation indicates that the mechanism responsible for the dissipation may be common for all these solid samples. To try and understand this mechanism, in this study we systematically collected and analyzed data of 5 hcp crystals grown under a variety of conditions: (a) grown in the hcp phase from a normal fluid at some temperature above the upper triple point of 1.772 K. (b) grown from the superfluid in the hcp phase below the lower triple point of 1.464 K. (c) grown in the bcc phase at 1.65 K, and then transformed into the hcp structure after having been cooled below the triple point at 1.46 K. (d) same as (a) or (c), but grown from a $^3$He–$^4$He mixture containing 100 ppm of $^3$He. The selection of a variety of initial conditions for the crystal growth was intended to look for quantitatively similar features in their response to temperature and drive changes, independent of the growth conditions. For the crystals and TO used in this experiment (Fig. 1), the decoupling process caused the moment of inertia of the solid to decrease by typically 30%. We remark in this context that when we grow a polycrystalline solid rather than a single crystal [15], the decoupled mass decreases to ~1 %, similar to what is measured by other groups who study polycrystals grown with the blocked capillary technique [22–25].

An important clue which led us to consider damping by dislocations came from studying crystals grown from a $^3$He–$^4$He mixture. Figure 2 shows the temperature dependence of the resonant amplitude of such a crystal, for two values of the applied drive. At small drive, the amplitude as a function of temperature shows a peak around $T = 0.8$ K. At high drive, there is no peak and the resonant amplitude decreases

![Figure 2](https://example.com/figure2.png)

**Fig. 2** Temperature dependence of the oscillation amplitude under a constant drive for a crystal grown from a 100 ppm $^3$He–$^4$He mixture. Note that the voltage driving the TO differs by almost 2 orders of magnitude for the two datasets shown (Color figure online)
monotonically with temperature. The particular temperature at which the amplitude at low drive shows a maximum has been associated with the onset of $^3$He atoms to dislocations [6,27]. Impurity atoms bound to dislocations contribute to their pinning, by reducing the mobility of kinks or jogs. It is likely that the reduced mobility of dislocations below 1 K results from the presence of $^3$He atoms acting as pinning centers. At high enough drive, dislocations break away from these pinning sites and the presence of $^3$He does not influence the oscillation amplitude strongly. Unbinding of dislocations from $^3$He atoms under high excitation levels was observed in measurements of shear modulus by other groups [11,27]. At high drive, the presence of $^3$He is felt in other ways which we discuss later on. At any rate, it seems that mobility of dislocations strongly affects the dissipation of the TO. On the basis of these observations, we decided to try and analyze the data using a dislocation-based dissipation mechanism.

Since the TO is a weakly damped harmonic oscillator, it is natural to use such an equation to try to analyze the data. We denote the angular velocity of the TO and of the solid rigidly coupled to the walls by $\dot{\theta}$. We assume that the mobile part of solid He is coupled only by friction with the static part and moves at an angular velocity $\dot{\theta}_m$ proportional to the velocity of the cell. The relative motion between the mobile and static solid parts in our model is represented by a phase lag $\alpha$, namely $\dot{\theta}_m = \beta \theta e^{-i\alpha}$ ($0 < \beta < 1$ is a constant). The friction force between the two parts is assumed to depend linearly on the relative velocity $\dot{\theta} - \dot{\theta}_m$, with an effective friction coefficient $\gamma_2$. Denoting the moment of inertia of the cell including the solid rigidly attached to it by $I$, we can write the equation of motion of the cell

$$I \ddot{\theta} + \gamma_1 \dot{\theta} + \gamma_2 (\dot{\theta} - \beta \dot{\theta}_m e^{-i\alpha}) + \kappa \theta = \tau_0 e^{i\omega t}.$$ 

(1)

Here $\tau_0$ is the external driving torque, $\kappa$ is the torsion constant of the rod, and $\gamma_1 \dot{\theta}$ is the internal friction of the TO itself and of the static solid attached to it.

We consider the case where the friction associated with the intergrain motion is much lower than the internal friction within the solid moving with the cell, namely $\gamma_1 \gg \gamma_2 (1 - \beta \cos(\alpha))$. We also take $\omega_0 \gg \gamma_2 \beta \sin(\alpha)/2I$. Both these assumptions hold for our system. In this case, the solution of Eq. (1) yields the resonant frequency $\omega_\alpha$ and the amplitude $\theta_0$:

$$\omega_\alpha \approx \omega_0 - \beta \gamma_2 \sin(\alpha)/2I$$

(2)

$$\theta_0 \approx \frac{\tau_0}{\omega_0 \gamma_1} - \frac{\tau_0}{\omega_0 \gamma_1 (\gamma_2/\gamma_1)} (1 - \beta \cos(\alpha)).$$

(3)

Here $\omega_0 = \sqrt{\kappa/I}$. In our model, the phase lag $\alpha$ should increase with the relative velocity which in turn increases with the applied torque. To find this dependence, in Fig. 3, we plot the resonant frequency of the cell as function of the applied AC voltage. The small offsets between different curves are due to temperature dependence of $\omega_0$, which are taken as given. The data were fit to Eq. (2). We found that datasets at different temperatures all have the same dependence on the external driving torque. The solid lines shown in Fig. 4 were obtained by taking the phase lag $\alpha$ to depend linearly on $\sqrt{\tau_0}$, with no temperature dependence. Obviously, the fit is very good. We remark in this context that in a classical model of a dislocation gliding with a speed $v_d$ through a
Fig. 3  Resonant frequency of the TO containing an hcp crystal grown of a 100 ppm $^3$He–$^4$He mixture, as function of the driving AC voltage. This particular crystal was initially grown in the bcc phase at 1.65 K and transformed into the hcp structure at 1.46 K. Datasets are labeled by the different temperatures. **Solid lines** are fits to Eq. (2) with $\alpha \propto \sqrt{\tau_0}$ (Color figure online)

periodic (Peierls) potential [28], the dislocation radiates energy in the form of sound. This mechanism causes the speed $v_d$ to increase as the square root of the applied stress.

Keeping this in mind, we next fit the data of the resonant amplitude $\theta_0$. The first term in Eq. (3), $\tau_0/\omega_0\gamma_1$, is the amplitude for the case where the solid moves rigidly with the cell. From fitting the data we found that this term is linear in the external torque and has the same magnitude as the measured resonant amplitude of the TO with the cell full with a single crystal. Hence, the first term in Eq. (3) represents what would be the resonant amplitude if the TO included only of the cell and the static solid helium attached to its walls. The second term in Eq. (3) decreases the resonant amplitude due to additional friction between the mobile and static solid. In our model, the temperature dependence of this term is contained in the friction coefficient $\gamma_2$. We found that we can fit the temperature dependence of the oscillation amplitude at a fixed external torque using $\gamma_2$ given by

$$\gamma_2 = \eta \frac{e^{-W_0/k_B T}}{k_B T}. \quad (4)$$

Here, $\eta$ and $W_0$ are fitting parameters. The dominant dependence of $\gamma_2$ on drive is through $W_0$ which we discuss below. This particular form describes the rate at which dislocations move during climb via vacancy emission/absorption by jogs [29]. The energy $W_0$ is the characteristic energy scale associated with this dissipative process. In Fig. 4, we show the fit of the temperature dependence of $\theta_0$ for a typical set of data, here for an hcp crystal grown at 1.83 K and cooled by 0.15 K steps down to 0.5 K.

Different sets of data in Fig. 4 are for different values of the external torque. Equation (3), with $\gamma_2$ given by Eq. (4), fits our data very well over the whole temperature range (0.5–1.6 K), over 2 orders of magnitude in the driving torque, and for all the crystals which were used in this work.
Fig. 4 Resonant amplitude of the TO as function of temperature, for a hcp $^4$He crystal grown at 1.83 K. Each dataset is associated with a constant value of the drive shown in the legend. Solid lines are fits to Eq. (3), with the friction coefficient $\gamma_2$ having the form given by Eq. (4) (Color figure online)

Regarding the energy scale $W_0$ which emerges from the fit, Fig. 5 shows the values of $W_0$ for all the crystals in this study. It appears that these values fall into two groups. For hcp crystals grown using commercially pure $^4$He we find $W_0 \simeq 3.0$ K at small stress. For crystals grown initially as bcc or from a $^3$He–$^4$He mixture, $W_0 \simeq 5.5$ K. The fact that $W_0$ takes only two distinct values indicates that crystals grown under similar conditions (crystal symmetry, composition) grow with a similar orientation. By the same token, crystals grown under different conditions can have somewhat different orientations and a different value of $W_0$. In the elastic approximation, the calculated energy of a jog in solid He is 5.8 K [12]. This is similar to the values we found. A climb of dislocations offers a natural way to describe the plastic deformation required of the mobile solid during relative motion. The fact that we can fit the amplitude vs. temperature using a single value of $W_0$ supports the idea that the friction between the crystallites is rather similar inside the cell. This result also supports the use of a discrete dissipative term in Eq. (1).

For completeness, we tried to fit the temperature dependence of the amplitude using other functional forms of $\gamma_2$, including among others an Arrhenius-type exponential dependence and a power law in temperature. The extended temperature range over which we can fit the data enabled us to clearly discriminate between these various forms, and state that none of these fit the data nearly as well as that given in Eq. (4).

The data in Fig. 5 show that $W_0$ decreases with stress. The solid lines in Fig. 5 are fits to a square root dependence on the stress (or external drive); $W_0 = A - B \sqrt{\text{drive}}$. A square root dependence is predicted by the classical theory of dislocations [29], resulting from the dependence of the vacancy formation energy on the local stress. Recently, Suhel and Beamish [30] found an activation energy of 5.1 K characterizing the structural relaxation of an hcp polycrystal. This value, together with the values of $W_0$ which we found, set the characteristic energy scale associated with structural changes in the solid. Since the mechanism of friction described here involves only
elementary crystalline defects, one can perhaps call it the low limit of a slip–stick motion.

In conclusion, we measured the temperature dependence of the dissipation inside the TO containing disordered single crystals of $^4$He or a 100 ppm $^3$He–$^4$He mixture.

We modeled the dissipation using an equation of motion of a damped harmonic oscillator. An excellent fit of our data was obtained by taking the friction to result from climb of dislocations. A well-defined energy scale $W_0$ emerged from fitting the data. In that sense, the dissipation due to climb of individual dislocations can be perhaps regarded as a low limit of slip–stick motion. The magnitude of the friction forces which are measurable in our system is very small compared with that in usual solids [31], which opens the possibility to look for even weaker mechanisms of friction, such as quantum friction [10].

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