Constraining the Anisotropic Expansion of the Universe

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We study the possibly existing anisotropy in the accelerating expansion universe with the Union2 Type Ia supernovae data and Gamma-ray burst data. We construct a direction-dependent dark energy model and constrain the anisotropy direction and strength of modulation. We find that the maximum anisotropic deviation direction is \((l, b) = (126^\circ, 13^\circ)\) (or equivalently \((l, b) = (306^\circ, -13^\circ)\)), and the current anisotropy level is \(g_0 = 0.030_{-0.003}^{+0.010}\) (1σ confidence level with Union2 data). Our results do not show strong evidence for the anisotropic dark energy model. We also discuss potential methods that may distinguish the peculiar velocity field from the anisotropic dark energy model.

1. INTRODUCTION

The cosmological principle has played an important role in modern cosmology [1]. It tells us that our universe is homogeneous and isotropic on large cosmic scale, which is consistent with currently observational data sets such as the cosmic microwave background (CMB) radiation data from the Wilkinson Microwave Anisotropy Probe (WMAP) [2–4] and Planck satellite [5]. Up to now, current astronomical observations are still in good agreement with ΛCDM model generally [6].

Despite the fact that the concordance cosmological model (ΛCDM model) is confirmed by many observational data, the model is also challenged by some observations [7, 8] (see [9] and references therein for more details). Recently, Appleby and Linder [10] found that an anisotropic dark energy model that preserves isotropic expansion to the level required by CMB still needs to be further considered. Therefore, it is important and necessary to check the cosmological principle with current available observational data. As more Type Ia supernovae (SNIa) data [11, 12] and high-redshift Gamma-ray burst (GRB) data are released [13, 14], it becomes possible to detect the anisotropic direction of cosmic expansion by using the supernovae data and the GRB data.

Indeed, a lot of effects may cause the cosmic anisotropy. For example, peculiar velocities may lead observers to find that the observed acceleration is maximized in one direction but minimized in the opposite [15]. A vector field model of dark energy may lead to a direction-dependent equation of state [16]. Many data analyses have already been made to search for the cosmic anisotropy. Using Union2 of Type Ia supernovae (SNIa) data, Ref. [17] derived the angular covariance function of the standard candle magnitude fluctuations, and the authors did not find any angular scales where the covariance function deviates from 0 in a statistically significant manner. By using 288 SNIa [18], Davis et al. [19] studied the effects of peculiar velocities, taking into consideration of our own peculiar motion, supernova’s motion and coherent bulk motion, and found that it can cause a systematic shift \(\Delta w = 0.02\) in the equation of state of dark energy if one neglected coherent velocities. Gupta et al. [20] introduced a statistics based on the extreme value theory and applied it to the gold data set of SNIa, and they showed that the data is consistent with isotropy and Gaussianity. Cooray et al. [21] used the SNIa to probe the spatial inhomogeneity and anisotropy of dark energy, showing that a shallow, almost all-sky survey can limit the rms dark energy fluctuations at the horizon scale down to a fractional energy density of \(\sim 10^{-4}\). On the other hand, a “residual” statistics was constructed in [22] to search for the preferred direction in different slices of past light-cone, the authors found that at low redshift \((z < 0.5)\) an isotropic model was not consistent with the SNIa data even at 2-3σ. Campanelli et al. [23] found that anisotropy is permitted both in the geometry of the universe and in the dark energy equation of state, if one worked in the framework of an anisotropic Bianchi type I cosmological model and the presence of an anisotropic dark energy equation of state. Furthermore, Refs. [24, 25] used the hemisphere comparison method to fit the ΛCDM model (and \(\omega\)-CDM model) to the supernovae data, and detected a preferred axis at statistically significant level. These results are consistent with many other observations, such as the CMB dipole [26], large scale alignment in the QSO optical polarization data [27] and large scale velocity flows [28]. Ref. [29] obtained the average direction of the preferred axes as \((l, b) = (278^\circ \pm 26^\circ, 45^\circ \pm 27^\circ)\). Further analysis was made by [30], where the authors used different low-redshift \((z < 0.2)\) SNIa samples and employed the the Hubble parameter to quantify the anisotropy level, and the results showed that all the SNIa samples indicated an anisotropic direction at 95% confidence level. Finally, we should mention that an anisotropic universe model or anisotropic dark energy model can potentially solve the
In this paper we study the plausible anisotropy in the accelerated expanding Universe with the Union2 data. We construct an anisotropic dark energy model and aim to detect the maximum anisotropy direction that deviates from the isotropic dark energy model described by ΛCDM model. Furthermore, we consider the impact of redshift on the direction by using the redshift tomography method, with the high-redshift Gamma-ray burst data as a complement data set. Finally, we check two other models as the description of the isotropic background. One is the wCDM model, the other is a dynamical dark energy model represented by the Chevallier-Polarski-Linder (CPL) parametrization. We examine that if our results are dependent on the isotropic background.

The paper is organized as follows. In the next section we give a general introduction to the anisotropic dark energy model, which is based on the isotropic background described by the ΛCDM model, and the χ^2 statistics of the model with observational data. In Sec. 3, we give the numerical results on the maximum anisotropy directions from the SNIa data and high-redshift GRB data with different slices of redshift. We also give the results for the wCDM and CPL model. Our conclusions are presented in Sec.4.

2. ANISOTROPIC DARK ENERGY MODEL

If dark energy has anisotropic repulsive force, it will directly affect the expansion rate of the universe, leading to the anisotropic luminosity distance. This effect should be observable by the luminosity of SNIa. In this paper, we use the Union2 data set [11], which contains 557 SNIa data covering the redshift range $z = [0.015, 1.4]$. In addition, we incorporate a GRB data set with 67 GRB samples up to $z = 6.6$ (see Table 2).

We try to quantify the deviation from the isotropic background as dipole modulation. By using the luminosity distance we define the deviation from isotropic expansion as

$$d_L(\hat{z}) - d_L^0(\hat{z}) = g(z)(\hat{z} \cdot \hat{n}), \quad (2.1)$$

where, the true luminosity distance of the supernova is $d_L(\hat{z})$, and in an isotropic background, the luminosity distance is $d_L^0(\hat{z})$. $g(z)(\hat{z} \cdot \hat{n})$ is the modulation part of the luminosity distance, which makes the real luminosity function anisotropic. Note that the modulation could be any power-law form of $(\hat{z} \cdot \hat{n})^s$, where $s$ is a constant, but we focus on the dipole modulation here ($s = 1$). In principle, one can use this model to discuss quadrupole modulation ($s = 2$), octupole modulation ($s = 4$) and higher moments ($s > 4$). $\hat{z}$ is the unit direction vector of the supernova, which can be expressed by using the Galactic coordinate system. $\hat{n}$ is the direction of dark energy dipole, which is the maximal expanding direction,

$$\hat{n} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta) \quad (2.2)$$

where $\theta \in [0, \pi)$ and $\phi \in [0, 2\pi)$. For the modulation of strength $g(z)$, one can consider the simplest case, $g(z) = g_0$, which corresponds to the case where the direction-dependent modulation is constant over all redshifts. Of course, we can also parameterize $g(z)$ with linear function of $z$ as

$$g(z) = g_0 + g_1 z. \quad (2.3)$$

where $g_0$ and $g_1$ are two constants, representing the strength of modulation and the time evolution of modulation, respectively. By parameterizing $g(z)$ with linear function, one can detect the redshift dependence of the anisotropy. Here $g_0$ represents the redshift independent part of the deviation and $g_1 z$ stands for the redshift linearly dependent part of the deviation. Of course, in principle, we can include higher order expansion terms of the Taylor expansion of the modulation function, however, including those terms will introduce more free parameters and reduce the constraining ability on the parameters. Therefore we limit to the case with the linear term here. Note that here the linear function is only a representative of various parametrization forms, and it might be invalid at high redshifts. To overcome this, one may take other forms of parametrization, for example, a CPL-like parametrization, $g(z) = g_0 + g_1 z^2$. In this case, the divergence will not appear when $z \to \infty$. Since we mainly focus on the SNIa data with the highest redshift $z = 1.4$, in this paper we therefore consider two cases with modulation function as $g(z) = g_0$ and $g(z) = g_0 + g_1 z$, respectively. We do not expect other parametrization forms will change significantly our main conclusions.

In a spatially flat isotropic cosmological background, the luminosity distance can be expressed as

$$d_L^0(z) = (1 + z) \int_0^z \frac{H_0}{H(z')} dz'. \quad (2.4)$$

where $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$ is the Hubble constant. Accordingly, the theoretical distance modulus $\mu_{th}$ is defined as

$$\mu_{th}(z) = 5 \log_{10} d_L(z) + \mu_0, \quad \mu_0 = 42.384 - 5 \log_{10} h. \quad$$

Since ΛCDM model is consistent with current astronomical observations, it is reasonable to take ΛCDM model as the isotropic base model, in which the Hubble parameter can be expressed as

$$H^2(z) = H_0^2[\Omega_{m0} (1+z)^3 + (1-\Omega_{m0})]. \quad (2.5)$$

where $\Omega_{m0}$ is the current value of the energy density fraction of matter. While in the case of wCDM model, the equation of state of dark energy is parametrized by a constant $w$, $w = p/\rho$, therefore we have

$$H^2(z) = H_0^2[\Omega_{m0} (1+z)^3 + (1-\Omega_{m0})(1+z)^{3+3w}]. \quad (2.6)$$
And if the background is described by the CPL parametrization, the equation of state of dark energy is $w = w_0 + w_1 \frac{z}{1 + z}$. Accordingly one obtains

$$H(z) = H_0^2[\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})(1 + z)^3(1 + w_0 + w_1)] \times \exp(-3w_1 z/(1 + z)).$$ (2.7)

We employ the Union2 data set and the GRB data to constrain the anisotropic dark energy model. The directions to the SNIa we use here are given in Ref. [17], and are described in the equatorial coordinates (right ascension and declination). The 67 GRB data are shown in Table 8. These samples are selected from [13, 14] and we add in the position of each data point. In order to make comparisons with other results, we convert these coordinates to the galactic coordinates ($\theta, \phi$).

Let us suppose the experiment error between each measurement is completely independent, so the covariance we add in the position of each data point $\sigma_{ij} = \delta_{ij}$.

$$
\chi^2 = \sum_{i=1}^{N} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma^2(z_i)},
$$

where $N$ is the number of data ($N = 557$ for SNIa, $N = 67$ for GRB, thus $N = 624$ for combined data set). $\mu_{\text{obs}}(z_i)$ is the measured distance modulus from the data, and $\mu_{\text{th}}(z_i)$ is the direction-dependent theoretical distance modulus.

We can eliminate the nuisance parameter $\mu_0$ by expanding $\chi^2$ with respect to $\mu_0$ [37]:

$$
\chi^2 = A + 2B\mu_0 + C\mu_0^2, (2.8)
$$

where

$$
A = \sum_{i} \frac{[\mu_{\text{th}}(z_i; \mu_0 = 0) - \mu_{\text{obs}}(z_i)]^2}{\sigma^2(z_i)},
$$

$$
B = \sum_{i} \frac{\mu_{\text{th}}(z_i; \mu_0 = 0) - \mu_{\text{obs}}(z_i)}{\sigma^2(z_i)},
$$

$$
C = \sum_{i} \frac{1}{\sigma^2(z_i)}.
$$

The $\chi^2$ has a minimum as

$$
\tilde{\chi}^2 = A - B^2/C,
$$

which is independent of $\mu_0$. This technique is equivalent to performing a uniform marginalization over $\mu_0$ [37]. We will adopt $\tilde{\chi}^2$ as the goodness of fitting instead of $\chi^2$.

Combining with Eq. (2.4) and substituting each anisotropic model in the $\chi^2$, one can easily calculate the likelihood function of each parameter by performing the Markov Chain Monte Carlo analysis. The parameters to be constrained are $(g_0, \theta, \phi)$ and $(g_0, g_1, \theta, \phi)$, respectively, where $(\theta, \phi)$ is the direction of modulation. We then convert $(\theta, \phi)$ into galactic coordinate $(l, b)$.

### 3. Redshift Tomography

Following the procedure introduced in Sec.2, one can obtain the best-fitting values and errors of parameters by performing the Markov Chain Monte Carlo analysis in the multidimensional parameter space. During the procedure, we use the best-fitting values obtained by WMAP group to calculate $d_n^2(z)$ in the isotropic background. We choose $\Omega_{m0} = 0.274$ for ΛCDM model, $\Omega_{m0} = 0.274$, $w = -1.037$ for wCDM model, and $\Omega_{m0} = 0.274$, $w_0 = -1.17$, $w_1 = 0.35$ for CPL parametrization, which are obtained by using WMAP+cMB+BAO+H$_0$+SNe data sets [4].

First, we consider the constant modulation function $g(z) = g_0$. By using the full Union2 data, we obtain the results listed in Table 1 which shows that the maximum deviation from the ΛCDM model is 0.024, and the maximum anisotropy direction is $(\sim l, b) = (128^\circ, 16^\circ)$, but we can not exclude the case $g_0 = 0$ at $1\sigma$ confidence level. We also use the GRB data to explore the high-redshift behavior, which can extend our detection to $z = 6.6$. In order to check the consistency between the SNIa data and GRB data, we show the fitting results by using GRB data, SNIa data and SNIa+GRB data in Table 1. The result shows that GRB data prefer larger longitude and negative latitude, compared with the results of SNIa data. But the maximum deviation axes are both consistent within $1\sigma$ error, and the anisotropy level are both close to 0, which means that the GRB data and the Union2 data are consistent on this issue.

Next, in order to explore the possible redshift dependence of the anisotropy, we consider the linear modulation function and implement a redshift tomography analysis, taking the same procedure as before for the following redshift slices: 0-0.2, 0-0.4, 0-0.6, 0-0.8, 0-1.0, 0-1.2, 0-1.4. Our results for ΛCDM model are summarized in Table 2.

The redshift tomography analysis here shows that the preferred axes at different redshifts are all located in a relatively small region of the Galactic Hemisphere $(\sim l, b) = (126^\circ, 13^\circ)$ for Union2 data). Note that the meaning of this best-fitting direction is the same as $(l, b) = (306^\circ, -13^\circ)$, because both directions are located on the same axis, and both are directions of maximum deviation from the ΛCDM model. The best-fitting direction is consistent with the result in Ref. [38] at $1\sigma$ confidence level. We also re-examined the dark energy dipole

\footnote{For more details, please visit the website http://www.mpe.mpg.de/~jcg/grbgen.html}
by using the Union2 data with the same method proposed in Ref. [38], showing that the dark energy dipole is indeed aligned with the corresponding fine structure constant cosmic dipole, where the dark energy dipole direction is ($l = 320.5^\circ \pm 11.8^\circ$, $b = -11.7^\circ \pm 7.5^\circ$).

The effect of the GRB data and the consistency between the SNIa data and GRB data are shown in Table 3 for the linear modulation case. One can see from the table that the GRB data and the Union2 data are consistent at 1$\sigma$ confidence level.

Using the full Union2 data, we plot the likelihood of the parameters ($g_0, g_1, l, b$) in Fig. 1. It is obvious from Eq (2.1) that there are two maximum anisotropic directions, which are settled on the same axis, accordingly $g(z)$ can be positive or negative. We also plot the joint posterior probability distribution of parameters ($g_0, g_1$) in

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**Fig. 1:** Likelihoods for parameters ($g_0, g_1, l, b$) using the full Union2 data, with the best-fitting parameters ($g_0 = 0.030, g_1 = -0.031, l = 126^\circ, b = 13^\circ$).

**Fig. 2:** 68% and 95% joint posterior probability distribution of parameters ($g_0, g_1$). The red, green and blue contours represent the results using the GRB, SNIa and SNIa+GRB data, respectively.

**Fig. 3:** Best-fitting $g(z)$ with 1$\sigma$ error at different redshifts.
Fig. 2, showing that $g_0$ and $g_1$ are negatively correlated and that the GRB data give a much weaker constraint compared with the SNIa data. Note that since we consider dipole modulation ($s = 1$ in Eq. (2.1)), the positive $g(z)$ at one direction is equivalent to the negative $g(z)$ at opposite direction ($\hat{z} \rightarrow -\hat{z}$). Therefore we restrict $g_0 > 0$ in our likelihood analysis.

Furthermore, we constrain the redshift dependence of the anisotropy by using SNIa data located only in different redshift bins: $0 - 0.2, 0.2 - 0.4, 0.4 - 0.6, 0.6 - 0.8, 0.8 - 1.0$, thus to avoid influences from other bins. Accordingly, we obtain $N = 220, 124, 102, 50$ and $41$ data points in different redshift bins, respectively. Our results for $\Lambda$CDM model are summarized in Table 4. Fig. 3 shows the best-fitting $g(z)$ with $1\sigma$ error at different redshifts. It is clear that the anisotropy level $g(z)$ is close to 0, which means that the $\Lambda$CDM model is still consistent with the Union2 data very well. The error increases as redshift increases, and we can not exclude the case $g(z) = 0$ even at $68.3\%$ confidence level. Note that the anisotropic direction changes as redshift bin changes, but theses directions are consistent with each other at $68.3\%$ confidence level.

We also consider the $w$CDM and the CPL parameterized dark energy models as the isotropic background. We use $w$ and $(w_0, w_1)$ as the isotropic background dark energy parameters and fit our anisotropic parameters using Union2 and GRB data, and the results are summarized in Table 5 and Table 6, respectively. The results of constraints on $(l, b, g_0, g_1)$ are not much different from the case of the $\Lambda$CDM model. This means that the best-fitting value of the maximum deviation direction from the isotropic background is not sensitive to the details of isotropic dark energy models. In addition, we can also see that using the GRB data only one would obtain a larger longitude and negative latitude, compared with the result using the SNIa data, but both data are still consistent at $1\sigma$ confidence level.

Finally, we make Bayesian statistical comparison between the isotropic dark energy models and the anisotropic dark energy models, in order to see which model is more favored by observational data. The Bayesian Evidence $E$ provides us with a good metric to quantify the level of consistency between each model and observational data [40],

$$E = \int \Pr(D|\theta)\Pr(\theta)d\theta,$$

where $\Pr(D|\theta) = \mathcal{L}(\theta)$ is the likelihood function, $\Pr(\theta)$ is the prior distribution of the model parameter vector $\theta$, which is usually assumed to be uniform or Gaussian. The logarithmic ratio of Evidences between the two models $\log(B_{AB}) = \log(E(A)/E(B))$, also known as the Bayes factor, measures the goodness of fit of the models. In the following, we calculate the Bayesian Evidence ratio of the anisotropic model $(A)$ and the isotropic model $(B)$ by assuming the multivariate Gaussian distribution of likelihood function and the uniform distribution of the prior. According to the Jeffreys grades [40], the result is summarized as follows (see Table 7 as well).

1. If the background is described by the $\Lambda$CDM model, the parameters for isotropic and anisotropic models are $(\Omega_{m0}, \omega)$ and $(l, b, g_0, g_1)$, respectively. The Bayes factor is $-0.35$, which means that there is “weak” evidence that the $\Lambda$CDM model is better than the anisotropic model.

2. If the background is described by the $w$CDM model, the parameters for isotropic and anisotropic models are $(\Omega_{m0}, w)$ and $(l, b, g_0, g_1)$, respectively. The Bayes factor is $1.08$, which means that there is a “significant” evidence that the anisotropic model is better than the $w$CDM model.

3. If the background is described by the CPL parametrization, the parameters for isotropic and anisotropic models are $(\Omega_{m0}, w_0, w_1)$ and $(l, b, g_0, g_1)$, respectively. The Bayes factor is $3.17$, which means that there is “strong to very strong” evidence that the anisotropic dark energy model is better than the CPL parametrization.

A. Distinguish Anisotropic Dark Energy With Peculiar Velocity Field Model

In this paper, we consider the model of anisotropic dark energy, which may potentially be degenerated with the peculiar velocity field model in an isotropic background. For example, the best-fitting bulk flow direction found in Ref. [41] is quite close to the direction $(l, b) = (306^\circ, -13^\circ)$. This is because, the peculiar motion of galaxies or supernovae can also produce a direction-dependent luminosity distance, which may indicate some preferred direction. With the current data at low redshift, we are not able to distinguish them, but we propose the following method which is useful for doing this in advance of new data.

First, if the anisotropy is caused by the peculiar velocity, the anisotropic direction should be randomly distributed on different cosmic scales, because peculiar velocity is driven by emergent of large scale structure, but if the anisotropy is caused by the dark energy dipole, the anisotropic direction should be a constant on all cosmic scale [39]. But the dark energy dipole should not change with the redshift. So by average the galaxy luminosity over a large volume, one can distinguish where the direction-dependence of luminosity is due to peculiar velocity or dark energy dipole.

In addition, the Integrated Sachs-Wolfe effect (ISW) can be used to detect the dark energy dipole, because if the accelerating expansion is anisotropic, the photons
that travel from different distances in different directions are potentially observable. On the other hand, peculiar motion of galaxies can only re-scatter the CMB photons and produces secondary anisotropic effect, which acquires its maximum at galaxy and cluster scales.

B. Comparison With Other Methods

As discussed in Sec.1, many studies have been made on the issue of cosmic anisotropy, and those results are compatible at certain confidence level [30], the anisotropic directions are in the vicinity to the WMAP cold spot [3]. Different from other methods, here we used the luminosity distance as the diagnostics to search for the maximum anisotropic direction, because anisotropy can directly affect the expansion rate and lead to anisotropic luminosity distance, no matter what it is caused physically. In addition, since the deviation from isotropy is tiny, we therefore parameterize the deviation as a linear function of redshift. This method is simple but can be applied to analyze many different anisotropic models.

For example, if the anisotropy caused by an anisotropic equation of state of dark energy, the Hubble parameter shall also be anisotropic, leading the luminosity distance to deviate from the isotropic universe, which can be detected with our method. And if the anisotropy is due to the anisotropic background geometry, such as the Bianchi I model, the anisotropic scale factor will lead the luminosity distance to be direction dependent. By taking the method we proposed here and using SNIa data in different sky patches, one can also detect the scale factors in different directions and quantify the anisotropic level.

4. CONCLUSIONS

There is a tentative evidence that the anisotropic direction on the cosmic expansion exists. If such a cosmological preferred axis indeed exists, one has to consider an anisotropic expanding universe, instead of the isotropic cosmological model.

In this paper, we investigated the plausible anisotropy in the accelerating expansion universe with the Union2 data and high-z GRB data. We constructed an anisotropic dark energy model, where we quantified the strength of modulation as a constant over all redshift and the direction of maximum expansion as (l, b). By using the full Union2 data, we found that the maximum deviation from the ΛCDM universe is 0.024+0.024−0.024, and the maximum anisotropy direction is (l, b) = (128°, 16°), but we can not exclude the case g₀ = 0 at 1σ confidence level. We also compared the results by using the GRB data and SNIa data, showing that both data are consistent at 1σ confidence level, and that the GRB data give a much weaker constraint compared with the SNIa data.

If the modulation is a linear function g(z) = g₀ + g₁z, we found that the maximum anisotropic deviation direction is (l, b) = (126°, 13°) (or equivalently (l, b) = (306°, −13°)), and the maximum anisotropy level is described by the parameters g₀ = 0.030+0.040−0.010, g₁ = −0.031+0.042−0.012 (obtained using Union2 data, at 1σ confidence level). Furthermore, we used the redshift tomography method by adding in the GRB data, and we found that the anisotropy strength g(z) is close to 0 within 1σ confidence level, which indicates that there is no strong evidence against isotropic ΛCDM model.

We also discussed the cases where the dark energy equation of state is described by a constant w and w(z) = w₀ + w₁z/(1 + z), respectively, but the results show a similar anisotropic direction. This indicates that the best-fitting value of the maximum deviation direction from the isotropic background is not sensitive to the isotropic dark energy models.

Finally, by using the Bayesian Evidence, we found that the anisotropic dark energy model does not show great statistical evidence better than isotropic wCDM model, except that there is a slightly greater evidence of anisotropic dark energy than CPL isotropic dark energy model.

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| data sets | \(l[\text{degree}]\) | \(b[\text{degree}]\) | \(g_0\) | \(g_1\) |
|-----------|-----------------|-----------------|--------|--------|
| SNIa      | 128^{+226}_{-224} & 16^{+19}_{-21} & 0.024 & -0.009^{+0.008}_{-0.002} |
| GRB       | 149^{+116}_{-107} & -10^{+35}_{-34} & 0.115 & -0.036^{+0.033}_{-0.013} |
| SNIa + GRB| 131^{+256}_{-133} & 12^{+18}_{-17} & 0.027 & +0.009^{+0.016}_{-0.016} |

Table 1: Constraints of the directions and amplitude of maximum anisotropy for the constant modulation, using SNIa data, GRB data and SNIa+GRB data, respectively. The error-bars quoted are 1σ and 2σ errors.

| redshift range | \(l[\text{degree}]\) | \(b[\text{degree}]\) | \(g_0\) | \(g_1\) |
|----------------|-----------------|-----------------|--------|--------|
| 0 – 0.2        | 134^{+212}_{-213} & 4^{+10}_{-10} & 0.045^{+0.041}_{-0.041} & -0.289^{+0.410}_{-1.013} |
| 0 – 0.4        | 125^{+212}_{-213} & 8^{+8}_{-8} & 0.030^{+0.016}_{-0.016} & -0.093^{+0.163}_{-0.163} |
| 0 – 0.6        | 121^{+81}_{-82} & 10^{+24}_{-24} & 0.033^{+0.018}_{-0.018} & -0.077^{+0.099}_{-0.099} |
| 0 – 0.8        | 128^{+88}_{-88} & 15^{+19}_{-19} & 0.030^{+0.013}_{-0.013} & -0.0022^{+0.014}_{-0.014} |
| 0 – 1.0        | 125^{+79}_{-80} & 14^{+20}_{-20} & 0.031^{+0.012}_{-0.012} & -0.040^{+0.056}_{-0.056} |
| 0 – 1.2        | 129^{+90}_{-90} & 12^{+29}_{-29} & 0.032^{+0.012}_{-0.012} & -0.042^{+0.050}_{-0.050} |
| 0 – 1.4        | 126^{+92}_{-92} & 13^{+30}_{-30} & 0.030^{+0.010}_{-0.010} & -0.031^{+0.039}_{-0.039} |

Table 2: Constraints of the directions and amplitude of maximum anisotropy for different redshift bins of the Union2 data. The error-bars quoted are 1σ and 2σ errors.

| data sets | \(l[\text{degree}]\) | \(b[\text{degree}]\) | \(g_0\) | \(g_1\) |
|-----------|-----------------|-----------------|--------|--------|
| SNIa      | 126^{+206}_{-205} & 13^{+20}_{-20} & 0.030^{+0.010}_{-0.010} & -0.031^{+0.049}_{-0.049} |
| GRB       | 336^{+289}_{-290} & 5^{+20}_{-20} & 0.011^{+0.011}_{-0.011} & -0.074^{+0.082}_{-0.082} |
| SNIa + GRB| 129^{+256}_{-256} & 16^{+17}_{-17} & 0.025^{+0.013}_{-0.013} & -0.006^{+0.025}_{-0.025} |

Table 3: Constraints of the directions and amplitude of maximum anisotropy for the linear modulation, using SNIa, GRB data and SNIa+GRB data, respectively. The error-bars quoted are 1σ and 2σ errors.

| redshift range | \(l[\text{degree}]\) | \(b[\text{degree}]\) | \(g_0\) | \(g_1\) |
|----------------|-----------------|-----------------|--------|--------|
| 0 – 0.2        | 134^{+212}_{-213} & 4^{+10}_{-10} & 0.045^{+0.041}_{-0.041} & -0.289^{+0.410}_{-1.013} |
| 0.2 – 0.4      | 253^{+88}_{-88} & 4^{+10}_{-10} & 0.036^{+0.016}_{-0.016} & -0.093^{+0.163}_{-0.163} |
| 0.4 – 0.6      | 334^{+80}_{-80} & 18^{+20}_{-20} & 0.048^{+0.037}_{-0.037} & -0.077^{+0.099}_{-0.099} |
| 0.6 – 0.8      | 149^{+224}_{-224} & 63^{+10}_{-10} & 0.0163^{+0.010}_{-0.010} & -0.191^{+0.191}_{-0.191} |
| 0.8 – 1.0      | 146^{+234}_{-234} & 6^{+46}_{-46} & 0.097^{+0.003}_{-0.003} & -1.111^{+1.111}_{-1.111} |

Table 4: Constraints on direction and strength of modulation for several redshift bins of the SNIa data fitting with the \(\Lambda\)CDM model, together with the 1σ and 2σ errors of parameters \((l, b, g_0, g_1)\).

| datasets | \(l[\text{degree}]\) | \(b[\text{degree}]\) | \(g_0\) | \(g_1\) |
|-----------|-----------------|-----------------|--------|--------|
| SNIa      | 128^{+256}_{-256} & 14^{+17}_{-17} & 0.034^{+0.026}_{-0.026} & -0.045^{+0.049}_{-0.049} |
| GRB       | 340^{+206}_{-206} & 1^{+34}_{-34} & 0.009^{+0.012}_{-0.012} & -0.024^{+0.072}_{-0.072} |
| SNIa + GRB| 129^{+179}_{-179} & 15^{+18}_{-18} & 0.028^{+0.012}_{-0.012} & -0.006^{+0.026}_{-0.026} |

Table 5: Directions of maximum anisotropy fitting with the wCDM model, together with the 1σ and 2σ errors of parameters \((l, b, g_0, g_1)\), using the SNIa data and GRB data.

| datasets | \(l[\text{degree}]\) | \(b[\text{degree}]\) | \(g_0\) | \(g_1\) |
|-----------|-----------------|-----------------|--------|--------|
| SNIa      | 127^{+212}_{-212} & 15^{+17}_{-17} & 0.035^{+0.015}_{-0.015} & -0.049^{+0.049}_{-0.049} |
| GRB       | 339^{+406}_{-406} & 16^{+34}_{-34} & 0.008^{+0.013}_{-0.013} & -0.073^{+0.073}_{-0.073} |
| SNIa + GRB| 131^{+426}_{-426} & 16^{+15}_{-15} & 0.025^{+0.010}_{-0.010} & -0.006^{+0.025}_{-0.025} |

Table 6: Directions of maximum anisotropy fitting with the CPL model, together with the 1σ and 2σ errors of parameters \((l, b, g_0, g_1)\), using the SNIa data and GRB data.
| Models | Parameters | Bayes factor | Comparison |
|--------|------------|--------------|------------|
| ΛCDM   | \(\Omega_{m0}\) | \(-0.35\) | Weak |
| wCDM   | \(\Omega_{m0}, w\) | 1.08 | Significant |
| CPL    | \(\Omega_{m0}, w_0, w_1\) | 3.17 | Strong to very strong |

Table 7: Results of comparison by using the Bayes factor for anisotropic and isotropic dark energy models.

| ID   | \(z\)    | \(\mu \pm \sigma_{\mu}\) | Equatorial Coordinate | ID   | \(z\)    | \(\mu \pm \sigma_{\mu}\) | Equatorial Coordinate |
|------|----------|---------------------------|-----------------------|------|----------|---------------------------|-----------------------|
| 070128 | 0.70     | 42.72 \pm 0.68            | 5\(^{h}\)57\(^{m}\)57\(^{s}\), 11\(^{h}\)46\(^{m}\)41\(^{s}\) | 030329 | 0.17     | 39.75 \pm 0.29            | 10\(^{h}\)44\(^{m}\)50\(^{s}\), 21\(^{h}\)31\(^{s}\) |
| 070508 | 0.84     | 43.76 \pm 0.35            | 6\(^{h}\)59\(^{m}\)28\(^{s}\), 79\(^{h}\)17\(^{m}\)4.1 |
| 071211 | 3.42     | 47.54 \pm 0.59            | 11\(^{h}\)56\(^{m}\)30\(^{s}\), 65\(^{h}\)12\(^{m}\) |
| 080613 | 1.10     | 44.75 \pm 1.22            | 10\(^{h}\)17\(^{m}\)46\(^{s}\), 71\(^{h}\)29\(^{m}\)9.1 |
| 080703 | 0.97     | 43.84 \pm 0.32            | 23\(^{h}\)50\(^{m}\)7\(^{s}\), 8\(^{h}\)35\(^{m}\)6.2 |
| 090123 | 1.61     | 44.36 \pm 0.37            | 15\(^{h}\)48\(^{m}\)14\(^{s}\), 51\(^{h}\)31\(^{m}\) |
| 090506 | 1.31     | 43.76 \pm 0.33            | 11\(^{h}\)54\(^{m}\)41\(^{s}\), -26\(^{h}\)45\(^{m}\) |
| 090510 | 1.62     | 45.36 \pm 0.31            | 13\(^{h}\)38\(^{m}\)51\(^{s}\), -80\(^{h}\)32\(^{m}\) |
| 090705 | 0.84     | 43.40 \pm 0.38            | 5\(^{h}\)39\(^{m}\)32\(^{s}\), -72\(^{h}\)9\(^{m}\) |
| 090712 | 0.43     | 41.76 \pm 0.44            | 22\(^{h}\)31\(^{m}\)50\(^{s}\), -73\(^{h}\)24\(^{m}\) |
| 091208 | 0.71     | 41.65 \pm 0.65            | 16\(^{h}\)33\(^{m}\)55\(^{s}\), 46\(^{h}\)26\(^{m}\) |
| 091216 | 1.02     | 43.12 \pm 0.35            | 1\(^{h}\)59\(^{m}\)31\(^{s}\), 11\(^{h}\)11\(^{m}\) |
| 000131 | 4.50     | 47.14 \pm 0.68            | 6\(^{h}\)13\(^{m}\)33\(^{s}\), -51\(^{h}\)55\(^{m}\) |
| 000210 | 0.85     | 42.27 \pm 0.65            | 1\(^{h}\)59\(^{m}\)15\(^{s}\), -40\(^{h}\)40\(^{m}\) |
| 000911 | 1.06     | 44.27 \pm 0.67            | 2\(^{h}\)18\(^{m}\)42\(^{s}\), 7\(^{h}\)48\(^{m}\) |
| 000926 | 2.07     | 45.09 \pm 0.68            | 17\(^{h}\)4\(^{m}\)15\(^{s}\), 51\(^{h}\)46\(^{m}\) |
| 010222 | 1.48     | 44.62 \pm 0.29            | 1\(^{h}\)52\(^{m}\)17\(^{s}\), 43\(^{m}\) |
| 010921 | 0.42     | 42.53 \pm 0.54            | 22\(^{h}\)55\(^{m}\)35\(^{s}\), 40\(^{h}\)56\(^{m}\) |
| 011211 | 2.14     | 45.53 \pm 0.43            | 11\(^{h}\)15\(^{m}\), -21\(^{h}\)56\(^{m}\) |
| 020124 | 3.20     | 46.73 \pm 0.37            | 9\(^{h}\)33\(^{m}\)8\(^{s}\), -11\(^{h}\)35\(^{m}\)6.2 |
| 020405 | 0.70     | 43.47 \pm 0.46            | 5\(^{h}\)19\(^{m}\)31\(^{s}\), 11\(^{h}\)11\(^{m}\) |
| 020813 | 1.25     | 43.95 \pm 0.33            | 19\(^{h}\)46\(^{m}\)38\(^{s}\), -19\(^{h}\)35\(^{m}\) |
| 020903 | 0.25     | 42.23 \pm 1.16            | 22\(^{h}\)49\(^{m}\), -20\(^{h}\)56\(^{m}\) |
| 021004 | 2.32     | 46.60 \pm 0.48            | 0\(^{h}\)27\(^{m}\)5\(^{s}\), -19\(^{h}\)61\(^{m}\) |
| 021211 | 1.01     | 43.49 \pm 0.55            | 8\(^{h}\)55\(^{m}\)6\(^{s}\)44\(^{m}\) |
| 030155 | 2.50     | 46.25 \pm 0.57            | 11\(^{h}\)18\(^{m}\)30\(^{s}\), 15\(^{h}\)2\(^{m}\) |
| 030226 | 1.98     | 46.50 \pm 0.40            | 11\(^{h}\)33\(^{m}\)36\(^{s}\), 25\(^{h}\)49\(^{m}\)9.2 |
| 030233 | 3.37     | 47.65 \pm 0.96            | 11\(^{h}\)33\(^{m}\)36\(^{s}\), 25\(^{h}\)49\(^{m}\)9.9 |
| 030328 | 1.52     | 44.68 \pm 0.37            | 12\(^{h}\)10\(^{m}\)46\(^{s}\), -9\(^{h}\)22\(^{m}\)5.4 |
| 060210 | 3.94     | 48.59 \pm 0.47            | 3\(^{h}\)50\(^{m}\)55\(^{s}\), 27\(^{h}\)2\(^{m}\) |
| 060223 | 4.41     | 47.64 \pm 0.54            | 3\(^{h}\)40\(^{m}\)52\(^{s}\), -17\(^{h}\)8\(^{m}\) |
| 060418 | 1.49     | 45.00 \pm 0.51            | 15\(^{h}\)45\(^{m}\)41\(^{s}\), -3\(^{h}\)38\(^{m}\)6.2 |
| 060502 | 1.31     | 44.90 \pm 0.62            | 16\(^{h}\)3\(^{m}\)26\(^{s}\), 66\(^{h}\)36\(^{m}\) |
| 060510 | 4.90     | 48.60 \pm 0.93            | 6\(^{h}\)23\(^{m}\)29\(^{s}\), -1\(^{h}\)10\(^{m}\) |

Table 8: Selected 67 GRB samples with positions described in the equatorial coordinates (right ascension and declination).