Heuristic Approach For Robust Counterpart Open Capacitated Vehicle Routing Problem With Time Windows

Evi Yuliza¹, Fitri Maya Puspita¹*, Siti Suzlin Supadi²

¹Mathematics Department, Faculty of Mathematics and Natural Science, University of Sriwijaya, South Sumatera, Indonesia
²Institute of Mathematical Sciences, Faculty of Science, University of Malaya, Malaysia
*Corresponding author: fitrimayapuspita@unsri.ac.id

Abstract
Garbage is one of the environmental problems. The process of transporting garbage sometimes occurs delays such as congestion and engine failure. Robust optimization model called a robust counterpart open capacitated vehicle routing problem (RCOCVRP) with time windows was formulated to get over this delays. This model has formulated with the limitation of vehicle capacity and time windows with an uncertainty of waste volume and travel time. The RCOCVRP model with time windows is solved by a heuristic approach. The heuristic approach used to solve the RCOCVRP model with time windows uses the nearest neighbor and the cheapest insertion heuristic algorithm. The RCOCVRP model with time windows is implemented on the problem of transporting waste in Sako sub-district. The solutions of these two heuristic approaches are compared and analyzed. The RCOCVRP model with time windows to optimize the route problems of waste transport vehicles that is solved using the cheapest insertion heuristics algorithm is more effective than the nearest neighbor method.

Keywords
Optimization Robust, Counterpart Open Capacitated Vehicle Routing Problem, Time Windows, Nearest Neighbour Algorithm, Cheapest Insertion Heuristic Algorithm

1. INTRODUCTION
Garbage is one of the environmental problems. To solve this environmental problem, the department of cleanliness and the environment carries out waste transportation activities. Garbage transport vehicles carry garbage to a temporary dump site (TDS) and end up at the final disposal site (FDS). Classic vehicle routing problem, also known as capacitated vehicle routing problem, has the characteristic that the vehicle departs from the starting point and returns to the starting point for example the route of the vehicle on the distribution of LPG gas (Yuliza and Puspita, 2019). The route of the waste transportation vehicle is an open vehicle routing problem (OVRP) that the garbage transporting vehicle transports waste from the TDS and then takes it to the FDS. The process of transporting garbage sometimes occurs with delay, such as congestion and equipment failure.

Robust optimization is an optimization problem approach for data that has uncertainty. De La Vega et al., 2019 formulated an optimization model for the robust vehicle routing problem (RVRP) with time windows and multiple deliverymen. Sungur et al., 2008 and Sun and Wang, 2015 discuss robust optimization of the vehicle route problem with many uncertain requests. Robust optimization research has experienced developments in the problem of vehicle routes, for example in the transportation and logistics sectors. The robust counterpart open capacitated vehicle routing problem (RCOCVRP) model is a robust optimization that takes into account the uncertainty of demand to minimize the cost of shipping products to costumers (Gounaris et al., 2013). Robust vehicle routing problem with time windows (RVRPTW) is a robust optimization that takes time windows into account. Agra et al., 2013 discussed RVRPTW on transportation problems where delays often occur so that the travel time is uncertain. Braaten et al., 2017 investigated the limit of the maximum amount of delay in the RVRPTW model and this model was solved by a heuristic method based on a large adaptive neighborhood search. Hartono et al., 2018 proposed a robust counterpart vehicle routing problem (RCOCVRP) model for waste transportation by request using the branch and bound algorithm. Puspita et al., 2018a discussed the RCOCVRP model on the problem of waste transportation in Sako and Sukarami Districts. Puspita et al., 2018b discussed the RCOCVRP model with a request on the problem of waste transportation in Sematang Borang District.

Research on the problem of solid waste collection and robust
optimization models has been widely studied (Buhrkal et al., 2012; Campos and Arroyo, 2016; Hannan et al., 2018; Nguyen et al., 2013; Perea et al., 2016). Hartono et al., 2018; Puspita et al., 2018b) discuss the robust counterpart open capacitated vehicle routing problem (RC-OCVRP) model. Puspita et al., 2018a discussed the robust demand counterpart open capacitated vehicle routing problem (DR-CVRP) model on the problem of waste transportation routes.

This research discusses the optimization modeling of robust counterpart open capacitated vehicle routing problem (RCOCVRP) with soft time windows, which means that the arrival time of the vehicle in a TDS (or in FDS) can be equal to or more than the final time. Yuliza et al., 2020b has proposed the RCOCVRP model with time windows on the problem of transporting garbage vehicles. This model is solved with an exact approach. The RCOCVRP model with time windows is solved by the cheapest insertion heuristics algorithm then solved using the GAMS software (Yuliza et al., 2021). RCOCVRP model with time windows produces optimal results solution by considering the volume of waste in each TDS carried by the vehicle does not exceed the vehicle capacity, and travel time does not exceed the time of arrival of the vehicle and solved using the nearest neighbor method (Yuliza et al., 2020a).

This model is formulated with the limitation of vehicle capacity ($Q$) and time windows $[a_i,b_i]$ with uncertainty of travel time $t_{ij}$ and volume of waste $q_i$. In this case, the vehicle arriving at a TDS or FDS may exceed the time windows. The RCOCVRP model with soft time windows is applied to the garbage transportation route in Sako District, Palembang. Time windows can be viewed as the time interval from the time the vehicle departs and the time the vehicle arrives at a customer (Agra et al., 2013). This model aims to optimize the route of the waste transport vehicle so as to minimize the distance. This model is solved by a heuristic approach. The heuristic approach used to solve the RCOCVRP model with soft time windows uses the nearest neighbor and the cheapest insertion heuristic algorithm. The nearest neighbor algorithm and the cheapest insertion heuristics algorithm have the same working principle, namely finding the closest point from the initial point.

### 2. EXPERIMENTAL SECTION

#### 2.1 Materials

Primary data in the form of the names of TDS and FDS, vehicle capacity, time windows and service time were obtained through direct interviews with the environmental and sanitation department. Secondary data in the form of distance between each TDS to TDS and FDS and travel time are obtained from the google maps application. Each sub-district has a TDS as a temporary disposal site for waste transported by the department of cleanliness and the environment waste collection vehicles with uncertain waste volume. The type of vehicle for each work area is a dump truck.

The garbage collection vehicle in Sako sub-district has 3 working areas. The garbage collection vehicles serve several TDS in work area 1, work area 2 and work area 3, respectively, as many as 6 TDS, 4 TDS and 11 TDS. Sako district is a densely populated residential area, so the problem of transporting waste is a concern. RCOCVRP model with time windows is simulated on the route of garbage collection vehicles in the working area 1.

Time windows and waste volume in working area 1 is shown in Table 1. The distance between TDS i to TDS j and the distance of each TDS to FDS are expressed in the distance matrix as shown in Table 2. It is assumed that the waste transport vehicle has an average speed of 40 km/hour. Based on the results of interviews with officers in the environmental and sanitation department, the average service time is around 15 minutes.

#### 2.2 Methods

Due to their greedy nature, sometimes the nearest neighbor algorithm can skip a shorter route. However, the nearest neighbor is easy to implement and up quickly (Pop et al., 2011). The cheapest insertion heuristics algorithm is able to solve the problem of vehicle routes. In the case of waste transport routes, each TDS is represented as a point and the travelling from TDS i to TDS j can be represented as an arc.

#### 2.2.1 Nearest Neighbor Algorithm

Following are the steps of the nearest neighbor algorithm for the problem of transporting waste: Step 1: Find the closest distance to each TDS from the initial TDS as the starting node. Step 2: Find the closest node from the starting node provided that the garbage volume from each TDS does not exceed vehicle capacity. Step 3: If the garbage volume exceeds the vehicle then start again from Step 1. Step 4: Repeat step 2 until all TDS (included FDS) are visited.

#### 2.2.2 Cheapest Sequential Insertion Algorithm

The principle of the cheapest sequential insertion algorithm is to make the initial route first. Starting from arc $(i,j)$ from the starting vertex to the destination vertex and try to insert vertex $k$ at a certain position gives the best criteria (Alatartsev et al., 2013; Scutella, 2015). The arc $(i, j)$ with which node k is inserted is calculated using the formula in the following formula:

$$c_{i,k} + c_{k,j} - c_{ij}$$






Table 1. Time windows and waste volume in working area 1

| Time Windows   | Volume of waste (kg) |
|----------------|----------------------|
| 07:00, 07:30   | $q_1 = 800$          |
| 07:30, 08:00   | $q_2 = 1500$         |
| 08:00, 08:30   | $q_3 = 600$          |
| 08:30, 09:00   | $q_4 = 4000$         |
| 09:00, 09:30   | $q_5 = 700$          |
| 09:30, 10:00   | $q_6 = 2000$         |
| 10:00, 11:00   | $q_7 = 6000$         |
Table 2. Matrix of distance (km)

|       | TDS1 | TDS2 | TDS3 | TDS4 | TDS5 | TDS6 | FDS |
|-------|------|------|------|------|------|------|-----|
| TDS1  | 0    | 0.4  | 0.35 | 0.45 | 1.3  | 4.5  | 2.3 |
| TDS2  | 0    | 0.55 | 0.85 | 1.3  | 4.8  | 2.3  |     |
| TDS3  | 0    | 1.1  | 1.4  | 5.2  | 2.2  |      |     |
| TDS4  |      | 0    | 1.7  | 4.1  | 2.3  |      |     |
| TDS5  |      | 0    | 5.8  | 1.2  |      |      |     |
| TDS6  |      |      | 0    | 5.6  |      |      |     |
| FDS   |      |      |      |      |      |      | 0   |

Table 3. Variables

| Variables | Description |
|-----------|-------------|
| xij      | the vehicle traveling from TDS i to TDS j |
| yij      | vehicle load when leaving TDS i |
| tij      | arrival time to TDS i |

3. RESULTS AND DISCUSSION

Given a directed graph $G = (V, A)$ where $V = \{1, 2, 3, ..., n + 1\}$ as the set of nodes and $A = \{(i, j) \mid i = 1, 2, ..., n + 1, i \neq j\}$ as a set of arcs, where nodes $1, 2, ..., n$ represent each TDS and node $n + 1$ represents FDS. Denoted $N$ with $N \subset V$ as a set of nodes that do not start from an origin (o) and destination (d) The set $T$ represents the set of all time windows for each node $T = [a_i, b_i] \in V$. The set ext($T$) represents all the extreme points on T. The variables and parameters proposed is the following:

The RCOCVRP with time windows in working area 1 is formulated as follows:

Minimize

$$
0.4x_{12} + 0.35x_{13} + 0.45x_{14} + 1.3x_{15} + 4.5x_{16} + 2.3x_{17} + 0.4x_{21} + 0.55x_{23} + 0.85x_{24} + 1.3x_{25} + 4.8x_{26} + 2.3x_{27} + 0.35x_{31} + 0.55x_{32} + 1.1x_{34} + 1.4x_{35} + 5.2x_{36} + 2.2x_{37} + 0.45x_{41} + 0.85x_{42} + 1.1x_{43} + 1.7x_{45} + 4.1x_{46} + 2.3x_{47} + 1.3x_{51} + 1.3x_{52} + 1.4x_{53} + 1.7x_{54} + 5.8x_{56} + 1.2x_{57} + 4.5x_{61} + 4.8x_{62} + 4.1x_{64} + 5.8x_{65} + 5.6x_{63} + 5.6x_{67} + 2.3x_{71} + 2.3x_{72} + 2.3x_{73} + 2.3x_{74} + 1.2x_{75} + 5.6x_{76}
$$

(2)

Based on the data in Table 1 and Table 2, the RCOCVRP model with time windows aims to minimize the distance traveled by waste transport vehicles so that the objective function is formulated as Equation 2, subject to

$$
x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} = 1,
0.21 + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} = 1,
0.31 + x_{32} + x_{34} + x_{35} + x_{36} + x_{37} = 1,
0.41 + x_{42} + x_{43} + x_{45} + x_{46} + x_{47} = 1,
0.51 + x_{52} + x_{53} + x_{54} + x_{56} + x_{57} = 1,
0.61 + x_{62} + x_{63} + x_{64} + x_{65} + x_{67} = 1,
0.71 + x_{72} + x_{73} + x_{74} + x_{75} + x_{76} = 1
$$

(3)

Garbage transport vehicles visit each node only as Constraints (3) is defined as at least one trip by a waste transport vehicle from node i to node j and at least one trip by a waste transport vehicle from node j to node i.

$$
x_{21} + x_{31} + x_{41} + x_{51} + x_{61} + x_{71} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} = 1; \ x_{71} + x_{27} + x_{37} + x_{47} + x_{57} + x_{67} - x_{71} - x_{72} - x_{73} - x_{74} - x_{75} - x_{76} = 1; \ x_{12} + x_{32} + x_{42} + x_{52} + x_{62} + x_{72} - x_{21} - x_{23} - x_{24} - x_{25} - x_{26} = 0; \ x_{13} + x_{23} + x_{43} + x_{53} + x_{63} + x_{73} - x_{31} - x_{32} - x_{34} - x_{35} - x_{36} - x_{37} = 0; \ x_{14} + x_{24} + x_{34} + x_{54} + x_{64} + x_{74} - x_{41} - x_{42} - x_{43} - x_{45} - x_{46} - x_{47} = 0; \ x_{15} + x_{25} + x_{35} + x_{55} + x_{65} + x_{75} - x_{51} - x_{52} - x_{53} - x_{54} - x_{56} - x_{57} = 0; \ x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} - x_{61} - x_{62} - x_{64} - x_{65} - x_{67} = 0
$$

(4)

Constraints (4) ensure that no sub-tours are cut off for each node visited by the garbage transport vehicle and the vehicles traveling from TDS i to TDS j are the same as vehicles from TDS j to TDS i.

$$
800 \leq y_1 \leq 8000, 1500 \leq y_2 \leq 8000, 600 \leq y_3 \leq 8000, 400 \leq y_4 \leq 8000
$$
$$
700 \leq y_5 \leq 8000, 2000 \leq y_6 \leq 8000, 0 \leq y_7 \leq 8000
$$

(5)

Constraint (5) states that the load of waste transport vehicles is limited by the volume of waste and the capacity of the vehicle.

$$
y_1 - y_2 + 8000x_{12} \leq 4500, y_2 - y_3 + 8000x_{23} \leq 5400, y_3 - y_4 + 8000x_{34} \leq 2000, y_4 - y_5 + 8000x_{45} \leq 5300, y_5 - y_6 + 8000x_{56} \leq 4000, y_6 - y_7 + 8000x_{57} \leq 0
$$

(6)
The volume of waste is uncertain so that the load of garbage vehicles at each node is formulated as Constraints (6).

\[ 7 \leq t_1 \leq 7.5, 7.5 \leq t_2 \leq 8, 8 \leq t_3 \leq 8.5, 8.5 \leq t_4 \leq 9, 9 \leq t_5 \leq 9.5, \\
9.5 \leq t_6 \leq 10.5, 10.5 \leq t_7 \leq 11, \]

\[ (7) \]

The vehicle travel time when visiting node i is limited by the time windows formulated by Constraints (7).

\[ t_1 - t_2 + 0.01x_{12} \leq 0, t_2 - t_3 + 0.01375x_{23} \leq 0, t_3 - t_4 + 0.0275x_{34} \leq 0, t_4 - t_5 + 0.0425x_{45} \leq 0, t_5 - t_6 + 0.145x_{56} \leq 0, t_6 - t_7 + 0.14x_{67} \leq 0 \]

\[ (8) \]

Vehicle arrival time at node i is influenced by departure time, travel time and vehicle arrival time at node j formulated by Constraints (8).

The calculation of the RCOCVRP model with time windows is solved by the Nearest Neighbor algorithm simulated in the working area 1. Furthermore, the calculation of the RCOCVRP model with time windows is solved by the cheapest insertion heuristic algorithm. The route of waste transport vehicles in working area 1 uses the nearest neighbor algorithm and the cheapest insertion heuristics algorithm is shown in Table 5.

The total distance of waste transport vehicles in working area 1, respectively, using the nearest neighbor method and the cheapest insertion heuristic algorithm are 11.65 km and 11.7 km. The results of calculations using these two heuristic approaches are almost the same. This is influenced by the working principle of these two heuristic approaches is to find the closest point. The results for working area 2 and working area 3 use the nearest neighbor algorithm and the cheapest insertion heuristics algorithm as shown in Table 6.

**4. CONCLUSIONS**

RCOCVRP with time windows can be used to solve waste transportation route problems. The results of the calculation of this model using the nearest neighbor method and the cheapest insertion heuristics algorithm produce almost the same solution because the principle of these two heuristic approaches is to find the closest distance. The cheapest insertion heuristics algorithm is more efficient than the nearest neighbor algorithm for determining the route of the waste transport vehicle. From the calculation results, it is found that the cheapest insertion heuristic algorithm is more efficient than the nearest neighbor algorithm for determining the route of the waste transport vehicle.

**REFERENCES**

Agra, A., M. Christiansen, R. Figueiredo, L. M. Hvattum, M. Poss, and C. Requejo (2013). The robust vehicle routing problem with time windows. *Computers & operations research*, 40(3); 856–866

Alatartsev, S., M. Augustine, and F. Ortmeier (2013). Constric-
Table 6. The solution of the RCOCVRP model with time windows in working area 2 and working area 3

| Route                      | Distance (km) |
|----------------------------|---------------|
| Working Area 2             |               |
| TDS 1 – TDS 2 – TDS 3 – FDS| 5.8           |
| TDS 4 – FDS                | 6.2           |
| Working Area 3             |               |
| TDS 1 – TDS 2 – TDS 3 – FDS| 5.86          |
| TDS 4 – TDS 5 – TDS 6 – FDS| 5.75          |
| TDS 7 – TDS 8 – FDS        | 6.18          |
| TDS 9 – TDS 11 – FDS       | 6.4           |
| TDS 10 – FDS               | 6.4           |

Heuristic algorithms for solving the generalized vehicle routing problem. *International Journal of Computers Communications & Control, 6*(1); 158–165

Puspita, F. M., E. S. Cahyono, S. Rahayu, and B. L. Sintia (2018a). Model of Demand Robust Counterpart Open Capacitated Vehicle Routing Problem (DRC-OCVRP) Simplification by Applying Preprocessing Techniques in Rubbish Controlling in Sematang Borang District, Palembang. *E3S Web of Conferences, 68;* 1–6

Scutellà, M. G. (2015). Logistics. *In Lecture Note;* 91–103

Sun, L. and B. Wang (2015). Robust optimisation approach for vehicle routing problems with uncertainty. *Mathematical Problems in Engineering, 2015;* 1–8

Sungur, I., F. Ordóñez, and M. Dessouky (2008). A robust optimization approach for the capacitated vehicle routing problem with demand uncertainty. *IIE Transactions, 40*(5); 509–523

Yuliza, E. and F. Puspita (2019). The Branch and Cut Method for Solving Capacitated Vehicle Routing Problem (CVRP) Model of LPG Gas Distribution Routes. *Science and Technology Indonesia, 4*(4); 105–108

Yuliza, E., F. Puspita, S. Supadi, and S. Octaria (2020a). The robust counterpart open capacitated vehicle routing problem with time windows. *Journal of Physics: Conference Series, 1663*(1); 1–7

Yuliza, E., F. M. Puspita, and S. S. Supadi (2020b). The robust counterpart open capacitated vehicle routing problem with time windows on waste transport problems. *Bulletin of Electrical Engineering and Informatics, 9*(5); 2074–2081

Yuliza, E., F. M. Puspita, and S. S. Supadi (2021). Robust Optimization for the Counterpart Open Capacitated Vehicle Routing Problem With Time Windows. *Advances in Social Science, Education and Humanities Research, 513;* 556–560

Heuristic algorithms for solving the vehicle routing problem with multiple deliverymen. *Central European Journal of Operations Research, 27*(4); 905–936

Gounaris, C. E., W. Wiesemann, and C. A. Floudas (2013). The robust capacitated vehicle routing problem under demand uncertainty. *Operations Research, 61*(3); 677–693

Hannan, M., M. Akhtar, R. Begum, H. Basri, A. Hussain, and E. Scavino (2018). Capacitated vehicle-routing problem model for scheduled solid waste collection and route optimization using PSO algorithm. *Waste management, 71;* 31–41

Hartono, Y., F. M. Puspita, D. I. Permatasari, and B. Arisha (2018). LINGO-based on robust counterpart open capacitated vehicle routing problem (RC-OCVRP) model of waste transportation in Palembang. *International Conference on Information and Communications Technology (ICOIACT), 18;* 429–435

Nguyen, P. K., T. G. Crainic, and M. Toulouse (2013). A tabu search for time-dependent multi-zone multi-trip vehicle routing problem with time windows. *European Journal of Operational Research, 231*(1); 43–56

Perea, F., R. Ruiz, K. Katragjini, and G. d. S. de Optimización Aplicada (2016). Integer programming, clustering, and local search approaches for grouping urban waste collection sites. *Boletín de Estadística E Investigacion Operativa, 32*(3); 203–224

Pop, P. C., I. Zelina, V. Lupșe, C. P. Sitar, and C. Chira (2011). Integer programming, clustering, and local search algorithms for grouping urban waste collection sites. *Procedia-Social and Behavioral Sciences, 23*(1); 2–9

Braaten, S., O. Gjønnes, L. M. Hvattum, and G. Tirado (2017). Heuristics for the robust vehicle routing problem with time windows. *Expert Systems with Applications, 77;* 136–147

Buhkal, K., A. Larsen, and S. Ropke (2012). The waste collection vehicle routing problem with time windows in a city logistics context. *Procedia-Social and Behavioral Sciences, 39;* 241–254

Campos, A. A. and J. E. C. Arroyo (2016). An ILS Heuristic for the Waste Collection Vehicle Routing Problem with Time Windows. *Advances in Intelligent Systems and Computing, 557;* 889–899

De La Vega, J., P. Munari, and R. Morabito (2019). Robust optimization for the vehicle routing problem with multiple deliverymen. *Central European Journal of Operations Research, 27*(4); 905–936

Gounaris, C. E., W. Wiesemann, and C. A. Floudas (2013). The robust capacitated vehicle routing problem under demand uncertainty. *Operations Research, 61*(3); 677–693

Hannan, M., M. Akhtar, R. Begum, H. Basri, A. Hussain, and E. Scavino (2018). Capacitated vehicle-routing problem model for scheduled solid waste collection and route optimization using PSO algorithm. *Waste management, 71;* 31–41

Hartono, Y., F. M. Puspita, D. I. Permatasari, and B. Arisha (2018). LINGO-based on robust counterpart open capacitated vehicle routing problem (RC-OCVRP) model of waste transportation in Palembang. *International Conference on Information and Communications Technology (ICOIACT), 18;* 429–435

Nguyen, P. K., T. G. Crainic, and M. Toulouse (2013). A tabu search for time-dependent multi-zone multi-trip vehicle routing problem with time windows. *European Journal of Operational Research, 231*(1); 43–56

Perea, F., R. Ruiz, K. Katragjini, and G. d. S. de Optimización Aplicada (2016). Integer programming, clustering, and local search approaches for grouping urban waste collection sites. *Boletín de Estadística E Investigacion Operativa, 32*(3); 203–224

Pop, P. C., I. Zelina, V. Lupșe, C. P. Sitar, and C. Chira (2011).