Thermodynamic control of soliton dynamics in liquid-core fibers: supplementary material

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Published 29 May 2018

This document provides supplementary information to "Thermodynamic control of soliton dynamics in liquid-core fibers," https://doi.org/10.1364/OPTICA.5.000695.

Goodness of fit of temperature-dependent Sellmeier coefficients

Fig S1. Temperature dependence of (a) amplitude coefficient $B_1$ and (b) resonance coefficient $C_1$ of the first Sellmeier term (i.e. UV term) used to describe the refractive index dispersion of CS$_2$.

Laser specifications and model pulse

Fig S2. (a) Output spectrum and (b) autocorrelation (AC) trace of the output pulse of the thulium fiber laser system in comparison with the calculated AC trace from the assumed model pulse used in the nonlinear simulations.
Experimental setup for temperature tuning

![Photograph of the experimental setup used for the temperature tuning of the dispersive wave. OFM: opto-fluidic mount, LiCOF: liquid-core optical fiber, OSA: optical spectral analyzer.]

**Generalized nonlinear Schrödinger equation**

The GNLSE used in our simulations is the same as in [1] and defined as follows

\[
\partial_z \tilde{A}(z; \omega) + \frac{i}{2} \alpha_m(\omega) \tilde{A} - i \tilde{\beta}(\omega) \tilde{A} = i \tilde{\gamma}(\omega) \mathcal{F} \left\{ A(z, t) \int_{-\infty}^{\infty} h(t') |A(t - t')|^2 dt' \right\}
\]

with the spectral field envelope \( \tilde{A}(z; \omega) \) and its time-domain counterpart \( A(z; t) = \mathcal{F} \{ \tilde{A}(z; \omega) A_{\text{eff}}^{-1/4}(\omega) \} \) (normalized to the effective mode area \( A_{\text{eff}} \) to account for the dispersion of the mode field area as introduced by [2]), the general response function \( h(t) \) (normalised using \( \int_{-\infty}^{\infty} h(t) dt = 1 \)), the propagation constant in the moving time frame \( \tilde{\beta}(\omega) = \beta(\omega) - \beta(\omega_0) - \omega \partial_\omega \tilde{\beta}|_{\omega_0} \) (with pump frequency \( \omega_0 \)), and the modal attenuation coefficient \( \alpha_m \). The operator \( \mathcal{F} \{ \cdot \} \) denotes the Fourier transformation. No noise was added if not stated otherwise. The modified nonlinear coefficient \( \tilde{\gamma} \) is defined as follows

\[
\tilde{\gamma}(\omega) \approx \frac{\omega n_{2,CS_2}}{c_0 A_{\text{eff}}} = \frac{\gamma(\omega) \cdot \frac{1}{A_{\text{eff}}^{3/4}}}{A_{\text{eff}}^{-1/4}}
\]

where the effective mode field area \( A_{\text{eff}} \) was calculated following the semi-analytic solution for cylindrical step-index waveguides described in [3]. We isolate the factor \( A_{\text{eff}}^{-3/4} \) from the conventional definition of \( \tilde{\gamma} \) since it serves as normalization factor of the field amplitudes in the nonlinear term of the GNLSE. The total nonlinear refractive index of CS2 \( n_{2,CS_2} \) is calculated to be

\[
n_{2,CS_2} = n_{2,el} + \frac{\int I(t) \int R(t - t') I(t') dt' dt}{\int I(t)^2 dt} = n_{2,el} + n_{2,m}
\]

with \( n_{2,el} \) as the nonlinear refractive index originating from the instantaneous electronic contribution, which is calculated using \( n_{2,el} = 3 \chi^{(3)}(4\varepsilon(\omega)\varepsilon_0 c_0)^{-1} \) with the electric and the relative permittivity \( \varepsilon_0 \) and \( \varepsilon \), and the speed of light \( c_0 \). The convolution between the noninstantaneous nonlinear response function \( R(t) \) and the temporal intensity distribution \( I(t) \) of the incoming optical pulse makes the nonlinear refractive index dependent on the pulse duration. A detailed description of the \( R(t) \) model of CS2 used for our calculations can be found in [4]. The GNLSE was solved with a split-step Fourier transform method where the nonlinear step was solved with a Runge-Kutta integrator of the 4th order.
Note that, the general response function $h(t)$ includes both the electronic nonlinear contribution (assumed to be instantaneous using Dirac delta function) and the weighted nonlinear molecular response function $R(t)$, leading to $h(t) = (1 - f_m)\delta(t) + f_mR(t)/\int R(t)dt$. The weight between the two contributions is the molecular fraction factor $f_m = n_{2,m}/(n_{2,el} + n_{2,m})$.

It is important to note that the normalization in $h(t)$ takes out the weights from the response function $R(t)$, which would be necessary to account for the change of the liquid nonlinearity with evolution of the pulse shape as described with the equation for $n_{2,CS_2}$ above. As a consequence of the pulse-shape independent normalization of $h(t)$, the convolution integral of the GNLSE imposes noninstantaneous phase changes only relative to the initial nonlinear refractive index calculated at the beginning of the simulation (i.e., on basis of the initial pulse intensity $I(t; z = 0)$ using the equation for $n_{2,CS_2}$ (and $\gamma$)). Hence, the nonlinear contributions of the standard GNLSE might deviate strongly from the real contributions (i.e., calculations based on $n_{2,CS_2}$ incorporating the intermediate pulse intensities $I(t; z > 0)$). These deviations might become significant in particular cases where the pulse width changes over an order of magnitude as the pulse propagates, e.g., in case of anomalously dispersive pulse compression before soliton fission. To account for the pulse width dependent change of the nonlinear refractive index an adaptive modification of the strength of the convolution (i.e., the modified nonlinear parameter $\tilde{\gamma}$) along propagation was included into the split-step solver of the GNLSE.

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