Dynamic Volatility Linkages between Developed and Emerging Stock Markets: A MMV-GARCH Approach
Xin-Xia YANG¹,a,*, Xian-Fang SU² and Zu-Xing HE³

¹School of Mathematics and Statistics, Guizhou University of Finance and Economics, Guiyang, Guizhou 550025, China
²School of Finance, Guizhou University of Finance and Economics, Guiyang, Guizhou 550025, China
³Physical Education Department, Guizhou Institute of Technology, Guiyang, Guizhou 550003, China

¹yxx6151130@163.com
*Corresponding author

Keyword: Stock market, Volatility linkage structure, Dynamic conditional correlation, Time-varying unconditional correlation.

Abstract. This paper investigates the short-run and long-run volatility linkage structures among developed and emerging stock markets using a multivariate multiplicative volatility GARCH (MMV-GARCH) model, which fulfills the assumption of a constant unconditional covariance matrix. The empirical results show that in the long-run volatility linkages, both developed and emerging markets are affected by the subprime mortgage crisis, but the developed stock markets are more sensitive to global financial perturbations. Moreover, the concern that the volatility linkages in stock markets are more significant after the subprime mortgage crisis is unfounded. In the short-run volatility linkages, the developed stock markets present stronger volatility correlation than the emerging markets.

Introduction
With the liberalization of markets, rapid technological progress, and financial innovations, the degree of integration of stock markets around the globe has increased significantly, and the global stock market has become an infinitely complex, continuously evolving process that gives a dynamic picture of correlations and volatilities. Therefore, it is imperative for risk management, portfolio allocation and hedging strategies to gain a better understanding of the time-varying volatility correlations between various stock markets.

The developed stock markets and the emerging stock markets represent different types of stock markets that possess different levels of economic development. There is already a substantial literature on studying the link of price level between developed and emerging stock markets. Worthington et al. (2003) used multivariate cointegration and level vector auto-regression procedures to examine the price linkages between three developed Asian stock markets and six emerging Asian stock markets. Their results show that there is a significant causal linkage between price levels. Lim (2009) found that there is evidence of an increase in the level of integration between these five ASEAN markets and that the US market had significant influence on all five ASEAN markets after the Asian financial crisis. Furthermore, Bora et al. (2009) used VAR...
techniques and the Granger causality test to find that the US market had a significant effect on all BRICA (Brazil, Russia, India, China and Argentina) countries. Tripathi et al. (2012) found that the short-run and long-run linkages between the Indian stock market and other emerging markets increased. However, Diamandis (2009) investigated the long-run relationships between four Latin American stock markets and US stock markets. Their results show that although cointegration exists, there are small long-run benefits of these common trends in international portfolio diversification. Horvath et al. (2013) employed multivariate GARCH models to examine the international stock market comovements between Western Europe vis-à-vis central and southeastern Europe. They found that the degree of comovement is much higher for central Europe and that the correlation between the southeastern European stock markets and developed markets is essentially zero.

The goal of this paper is to investigate the short-run and long-run volatility linkage structures between developed stock markets and emerging Asian stock markets using a flexible econometric model. The stock markets in the US, the Chinese mainland and Hong Kong are analyzed over a period from July 21, 2005 to December 30, 2014. Our proposed methodology is based on a two-stage procedure. First, we employ a vector error correction model to specify their conditional mean. Thereby, we filter the index series from the long-run comovement in the level of indexes. Second, we estimate the volatility linkage structures via a MMV-GARCH model. The unconditional covariance matrix are used to estimate the time-varying unconditional covariance matrix, which can describe the long-run dynamic volatility, while the conditional covariance matrix allows us to model the dynamics of the conditional volatilities and their correlations over time, which captures the short-run dynamic volatility structures between different stock markets.

The main contributions of this paper are as follows: Firstly, we use a MMV-GARCH model to model the short-run and long-run dynamic volatility linkage structures among developed and emerging stock markets, which allows relaxing GARCH models assumption of a constant unconditional covariance matrix. Secondly, we investigate the dynamic linkage structures between developed and emerging stock markets and reveal that there exist different volatility linkage structures.

The remainder of this paper is structured as follows. Section 2 outlines the methodology and model used in this paper. The empirical analysis and discussion are provided in Section 3. Section 4 presents the conclusions.

Methodology
Filtering the Data on Long-run Comovement

In a general notation, \( I_t \) is a \((n \times 1)\) vector of index series. If the elements of \( I_t \) are \( I_{i(t)} \) variables, the conventional VECM is represented as

\[
\Delta I_t = c + \Pi I_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta I_{t-i} + \epsilon_t
\]

Where \( \Delta \) is a first-difference operator such that \( \Delta I_t = I_t - I_{t-1} \) denotes the change in the index series \( I \) from time \( t-1 \) to time \( t \), and \( \Pi \) is the matrix containing long-run equilibrium information. If \( \text{rank}(\Pi) = r < p \) and if \( p \) is the number of parameters in the estimation model,
then \( \Pi \) can be written as \( \Pi = \alpha \beta^T \), and the VECM form can be expressed again as

\[
\Delta I_t = c + \alpha \beta^T I_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta I_{t-i} + \varepsilon_t
\]

(2)

where \( \beta^T I_{t-1} \) denotes the cointegration relation, \( \alpha \) gives the speed of adjustment with which index series return to the long-run equilibrium, \( \Gamma \) measures the short-run dynamic relationship between the elements of \( I_t \), and \( \varepsilon_t \) is a residue term that captures potential GARCH effects.

We can use a quasi-maximum likelihood method to estimate the parameters of the VECM under the assumption of homoscedastic errors. Bauwens et al. (2013) proposed that the estimation results are still consistent under the presence of heteroscedasticity errors. Therefore, we can estimate VECM parameters and GARCH parameters separately.

**Modeling the Time-varying Unconditional Correlation Structure**

The MMV-GARCH model can decompose the covariance matrix into a conditional component and an unconditional component. The covariance matrix of the VECM residual term \( \varepsilon_t \) is decomposed as

\[
H_t = \sum(t/T)^{1/2} G_t \left\{ \sum(t/T)^{1/2} \right\}^T
\]

(3)

where \( \sum(t/T) \) is the unconditional covariance matrix that captures the long-run dynamics and \( G_t \) is a DCC process to capture the short-run GARCH dynamic. Let \( E(G_t) = I_n \). We can obtain the following:

\[
\text{Var}(\varepsilon_t) = \sum(t/T)^{1/2} E(G_t) \left\{ \sum(t/T)^{1/2} \right\}^T = \sum(t/T)
\]

(4)

Hence, \( \sum(t/T) \) denotes the time-varying unconditional covariance matrix of the residual term \( \varepsilon_t \). The unconditional covariance matrix \( \sum(t/T) \) can be estimated by the nonparametric Nadaraya-Watson estimator (Hafner~and~Linton, 2010).

\[
\hat{\Sigma}(\tau) = \frac{\sum_{t=1}^{T} K_h \left( \tau - \frac{t}{T} \right) \varepsilon_t \varepsilon_t^T}{\sum_{t=1}^{T} K_h \left( \tau - \frac{t}{T} \right)}
\]

(5)

Where \( \tau \in [0,1], K_h(\cdot) \) is a kernel function and \( h \) is a positive bandwidth parameter. It is critical to select the bandwidth parameter. The bandwidth \( h \) can be obtained using a likelihood cross-validation criterion (Yilmaz et al., 2010). The optimal bandwidth is determined by minimizing the following:

\[
CV(h) = \frac{1}{n} \sum_{i=1}^{T} \left[ \varepsilon_i^T \sum_{-i} \left( \frac{t}{T} \right) \varepsilon_i + \log \left( \left| \sum_{-i} \left( \frac{t}{T} \right) \right| \right) \right]
\]

(6)

where \( \sum_{-i} \) is the leave-one-out estimator of the unconditional covariance matrix.
After estimated the unconditional covariance matrix $\Sigma(t/T)$, let $u_t = \Sigma(t/T)^{1/2} \varepsilon_t$ be the vector of residuals standardized by its time-varying unconditional covariance. It follows that $\text{Var}(u_t) = I_n$ and $\text{Var}(u_t | F_{t-1}) = G_t$. Then $G_t$ indicates the conditional covariance matrix of the standardized term $u_t$ with a constant unconditional covariance $I_n$.

**Empirical Analysis**

**Data Description and Filtering**

As proxies for the Chinese mainland, US and Hong Kong stock markets, we use the Shanghai Composite index $I_{SP}$, the S&P 500 index $I_{HS}$ and the Hang Seng index $I_{SHC}$. We exclude the days when any of the three markets are closed, which yields 2,194 observations for each market. The daily closing index series of the three markets are shown in Figure 1. We take logarithmic transformations for the three stock index series. The ADF test results, as well as the KPSS test results, are shown in Table 1. Both the ADF test and KPSS test provide evidence of the presence of a unit root in all three index series. The optimal lag order of VECM is selected as 1 based on the Hannan Quinn information criterion. Table 2 shows the estimation results of VECM.

Table 1. Results of ADF test and KPSS test.

|       | ADF Test | KPSS Test |
|-------|----------|-----------|
|       | The statistic | 5% Critical Value | The statistic | 5% Critical Value |
| SP500 | 1.4717 | -1.95 | 3.0510 | 0.463 |
| HIS   | 0.1864 | -1.95 | 1.9289 | 0.463 |
| SHCI  | 0.1801 | -1.95 | 0.8387 | 0.463 |

Table 2. Estimates of the vector error correction model.

|       | $c$ | $\beta^T$ | $\Delta I_{SP(t-1)}$ | $\Delta I_{HS(t-1)}$ | $\Delta I_{SHC(t-1)}$ |
|-------|-----|----------|-----------------|-----------------|-----------------|
| $\Delta I_{SP(c)}$ | 0.0135 | 0.0014 | -0.0884*** | 0.0017 | -0.0154 |
|       | (0.0161) | (0.0017) | (0.0221) | (0.0205) | (0.0193) |
| $\Delta I_{HS(c)}$ | 0.0778*** | 0.0082*** | 0.5376*** | -0.1338*** | 0.0851*** |
|       | (0.0180) | (0.0019) | (0.0247) | (0.0229) | (0.0215) |
| $\Delta I_{SHC(c)}$ | 0.0708** | 0.0074** | 0.2081*** | -0.0301 | -0.0272 |
|       | (0.0203) | (0.0021) | (0.0279) | (0.0259) | (0.0243) |

** and *** Statistically significant at the 5% and 1% significance levels, respectively. P-values in parentheses.

The estimates indicate that at a 5% significance level, the Hang Seng index and the Shanghai composite index react to deviations from the cointegration relation, while the S&P 500 index is exogenous with respect to the long-run relationship. This can be explained by the fact that the level of internationalization in the US stock market is higher. To consider the short-term index changes, the S&P 500 index, when considering a 1% significance level, is affected by the lagged index but
not by the Hang Seng index or the Shanghai composite index. In contrast, the Hang Seng index reacts not only to changes in the S&P 500 index and the Shanghai composite index but also to the lagged index in the short run.

Figure 1. Daily indexes of stock markets for the period from 20050721 to 20141230.

Time-varying Unconditional Correlation Structure Estimations

The estimate results of unconditional variances and correlations, together with 95% confidence intervals, are shown in Figure 2.

The unconditional correlation between the volatility of the US stock market and the volatility of the Hong Kong stock market was significant at approximately 0.3 at the beginning of the sample period but showed a tendency to decrease until 2008. In 2008, the correlation sharply rose and reached a peak of approximately 0.5. The sudden inversion of the correlations in 2008 corresponds to the unstable phase of the subprime mortgage crisis. After 2008, it increased again and reached approximately 0.2 in 2010. Since the end of 2013, it has increased again. The correlation between the US stock market and the Chinese mainland stock market is volatile, but its correlation amplitude decreased from -0.1 to 0.2. This indicates that the volatility linkage between emerging and developed stock markets is smaller than the volatility linkages between the developed stock markets. The unconditional correlation estimate for the Chinese mainland stock market and the Hong Kong stock market is relatively high and significant during the whole sample period.

Figure 2. Unconditional variance and correlation estimates with 95% point wise confidence intervals.

Conclusion

This paper investigates the short-run and long-run volatility linkage structures between developed and emerging stock markets. The evolution of volatilities and their correlations over time can be
captured by specific volatility model. In the MMV-GARCH framework, the time-varying unconditional covariance component can be used to capture the long-run unconditional volatility and correlation. This strategy provides a flexible and accurate fitting procedure to capture the linkage structures of dynamic volatility.

We find that there exist different linkage structures in both long-run volatility and short-run volatility among developed and emerging Asian stock markets. In the long run, the unconditional correlations of the volatility in both developed stock markets and emerging Asian markets were affected by the subprime mortgage crisis from 2007 to 2010. In the short-run volatility linkages, there exist a high degree of persistency in the volatility in emerging stock markets. This means that high volatility currently implies high volatility in the future for different stock markets.

Acknowledgements

This research is supported by the talent introduction project of Guizhou University of finance and economics in 2017.

References

[1] Worthington AC, Katsuura M, Higgs H, 2003. Price Linkages in Asian Equity Markets: Evidence Bordering the Asian Economic, Currency and Financial Crises. Asia -Pacific Finan. Mark. 10(1): 29 -44.

[2] Lim L K., 2009. Convergence and interdependence between ASEAN-5 stock markets. Mathematics and Computers in Simulation, 79(9), 2957-2966.

[3] Bora A, Pinar EM, Baris SK, Bülent E. 2009. Behaviour of Emerging Stock Markets in the Global Financial Meltdown: Evidence from Brica. Afr. J. Bus. Manage., 3(7), 396-404.

[4] Tripathi, V., Sethi, S. 2012. Inter linkages of Indian stock market with advanced emerging markets. The Asian Economic Review, 54, 507-528.

[5] Diamandis, P. F., 2009. International stock market linkages: Evidence from Latin America. Global Finance Journal, 20,13-30.

[6] Horvath R, Petrovski D. 2013. International stock market integration: Central and South Eastern Europe compared. Economic Systems, 37(1), 81-91.

[7] Bauwens, L., Hafner, C. M., Pierret, D., 2013. Multivariate volatility modeling of electricity futures. Journal of Applied Econometrics, 28(5), 743-761.

[8] Hafner C, Linton O., 2010. Efficient estimation of a multivariate multiplicative volatility model. Journal of Econometrics, 159, 55-73.

[9] Yilmaz, K., 2010. Return and volatility spillovers among the East Asian equity markets. Journal of Asian Economics 21 (3), 304-313.