Observation of $e^+e^- \to \chi_{c0}\omega$ and missing higher charmonium $\psi(4S)$

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Stimulated by the recent BESIII observation of a new resonance in $e^+e^- \to \omega\chi_{c0}$ and by the fact that this state is consistent with our predicted $\psi(4S)$, in this work we estimate the meson loop contribution to $\psi(4S) \to \omega\chi_{c0}$. The evaluation indicates that our theoretical estimate can overlap with the experimental data in a reasonable parameter range. This fact shows that introduction of the missing higher charmonium $\psi(4S)$ provides a possible explanation to the recent BESIII observation. The upper limit of a branching ratio of $\psi(4S) \to \eta J/\psi$ is also predicted as $1.9 \times 10^{-3}$, which can be further tested by BESIII, Belle and forthcoming BelleII.

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I. INTRODUCTION

Although the present $J/\psi$ family has become more and more abundant, there still exist much more puzzling features of charmonia, especially higher than 4 GeV, which are waiting for resolution. In the past decade, experiments have made big progress on searching for the charmonium-like states normally referred to XYZ, where a lot of them come from the $e^+e^-$ annihilation processes, e.g., $Y(4260)$ observed in $e^+e^- \to J/\psi\pi^+\pi^-$ [1], $Y(4360)$ [2] and $Y(4660)$ [3] reported in $e^+e^- \to \psi(2S)\pi^+\pi^-$, and $Y(4630)$ existing in $e^+e^- \to \Lambda_c^+\Lambda_c^-$ [4]. Thus, the $e^+e^-$ annihilation process is a good platform to explore charmonium-like states. By carrying out the study of these experimental observations, which have a close relationship with higher charmonia above 4 GeV, it is helpful to establish the $J/\psi$ family and to definitely identify exotic states. The underlying motivation of this study is to enlarge our knowledge of non-perturbative behavior of quantum chromodynamics (QCD), which is a crucial step to gain a deeper understanding of strong interaction.

Very recently, the BESIII Collaboration performed the search for the $e^+e^- \to \omega\chi_{c0}$ ($J = 0, 1, 2$), which is based on the collected data by BESIII at nine center-of-mass energies from 4.21 to 4.42 GeV [5]. Then, BESIII announced the observation of $e^+e^- \to \omega\chi_{c0}$, where the Born cross sections at nine energy points were measured, among which those at $\sqrt{s} = 4.23$ GeV and 4.26 GeV are (55.4 ± 6.0 ± 5.9) pb and (23.7 ± 5.3 ± 3.5) pb, respectively. However, for the remaining processes $e^+e^- \to \omega\chi_{c1}$ and $e^+e^- \to \omega\chi_{c2}$, there are no significant signals. Additionally, if using the Breit-Wigner function to fit the experimental data of $e^+e^- \to \omega\chi_{c0}$, a resonance structure with mass $M = (4230 \pm 8)$ MeV and width $\Gamma = (38 \pm 12)$ MeV was observed [5]. BESIII indicated that this resonance structure is different from $Y(4260)$ reported in the analysis of $e^+e^- \to J/\psi\pi^+\pi^-$ [1]. It is a challengeable and intriguing task how to understand this novel phenomenon.

To reveal the underlying mechanism behind the above observation, we first answer why only $e^+e^- \to \omega\chi_{c0}$ was observed in BESIII. The enhancement structure around 4230 MeV with a narrow width can provide a valuable hint. Before this BESIII observation, we have once predicted a missing higher charmonium $\psi(4S)$ with the mass 4263 MeV and narrow width utilizing the similarity between the $J/\psi$ and $\Upsilon$ families [6], which is consistent with the mass estimate of $\psi(4S)$ by adopting the screening potential in Refs. [7, 8]. Comparing the resonance parameter of the predicted $\psi(4S)$ with that of the enhancement structure given by BESIII [1], we notice the enhancement structure existing in $e^+e^- \to \omega\chi_{c0}$ is consistent with our predicted missing $\psi(4S)$. Therefore, the enhancement around 4230 MeV in $e^+e^- \to \omega\chi_{c0}$ can be identified as $\psi(4S)$. By further checking the thresholds of $\omega\chi_{c0}$, $\omega\chi_{c1}$, and $\omega\chi_{c2}$, which are 4197 MeV, 4293 MeV, and 4338 MeV, respectively, it is obvious that the central mass of $\psi(4S)$ (if taking the experimental value $M = 4230 \pm 8$ MeV [5]) is just above the $\omega\chi_{c0}$ threshold and below the $\omega\chi_{c1,2}$ thresholds. This fact shows that it is natural to explain why only $e^+e^- \to \omega\chi_{c0}$ was observed for the first time if introducing the contribution of a long-term missing $\psi(4S)$ in experiment, where $\psi(4S) \to \omega\chi_{c1,2}$ are kinematically forbidden.

In the following, we will study the $e^+e^- \to \omega\chi_{c0}$ process via the intermediate $\psi(4S)$. Since the Born cross sections of $e^+e^- \to \omega\chi_{c0}$ were measured by BESIII [5], we are able to compare our numerical results with the experimental data, which can test whether introduction of $\psi(4S)$ contribution is reasonable to explain this recent BESIII observation. In the next section, we present more details of the calculation of $e^+e^- \to \psi(4S) \to \omega\chi_{c0}$.

The decay $\psi(4S) \to \eta J/\psi$ similar to $\psi(4S) \to \omega\chi_{c0}$ can occur, which is a typical transition accessible by experiment. Hence, in this work we also study the $\psi(4S) \to \eta J/\psi$ decay, where the partial width of this decay and the cross section of $e^+e^- \to \psi(4S) \to \eta J/\psi$ are predicted. These are important informations for experimentalists to further search for the $e^+e^- \to \eta J/\psi$ process, which can be seen as a further test of our understanding of the BESIII observation of $e^+e^- \to \omega\chi_{c0}$.

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This work is organized as follows. After Introduction, we present the detailed calculation of $\psi(4S) \rightarrow \omega\chi_{c0}$ and $\psi(4S) \rightarrow \eta J/\psi$ in Sec. II. In Sec. III, the numerical results are given. The last section ends with the conclusions and discussion.

II. $\psi(4S) \rightarrow \omega\chi_{c0}$ AND $\psi(4S) \rightarrow \eta J/\psi$ TRANSITIONS

For the hidden-charm decays of higher charmonium, the hadronic loop mechanism plays an important role to mediate these decays. In the past decade, there were some discussions of this point, which shows that the novel phenomena existing in the decays of higher charmonia, bottomonia and $B$ mesons can be indeed understood well by the hadronic loop mechanism [9–21].

For the discussed $\psi(4S) \rightarrow \omega\chi_{c0}$ and $\psi(4S) \rightarrow \eta J/\psi$ transitions, $\psi(4S)$ is above the threshold of $D_0^*D_0^*$ and dominantly decays into a pair of charmed mesons or charmed strange mesons [6], where the corresponding partial decay widths were calculated in Ref. [6]. On the other hand, the $\psi(4S) \rightarrow \omega\chi_{c0}$ and $\psi(4S) \rightarrow \eta J/\psi$ transitions can occur via the hadronic loop mechanism. For example, the initial state $\psi(4S)$ can decay into the final state $\omega\chi_{c0}$ via the charmed meson loops, which are shown in Fig. 1.

![Diagram](image)

FIG. 1: Sketchy diagrams of the meson loop contributions to $\psi(4S) \rightarrow \omega\chi_{c0}$: (a) and (b) correspond to $\psi(4S) \rightarrow [D^*0]_D \rightarrow \omega\chi_{c0}$ and $\psi(4S) \rightarrow [D^*0]_{D'} \rightarrow \omega\chi_{c0}$ decays, respectively.

To calculate the hadronic loop contributions to the $\psi(4S) \rightarrow \omega\chi_{c0}$ and $\psi(4S) \rightarrow \eta J/\psi$ decays, we utilize the effective Lagrangian approach, where the the effective interaction Lagrangians are constructed by respecting heavy quark limit and chiral symmetry [22–25]. The involved effective Lagrangians that are related to interactions among charmonium and charmed mesons and among vector/pseudoscalar meson and charmed mesons read as

$$L_{\psi D^*D^*V} = -ig_{\psi D^*D^*V} \partial_{\mu} D^*_\mu D^*_\nu D^*_\rho D^*_\sigma \epsilon_{\mu \nu \rho \sigma},$$

$$L_{\chi_{c0} D^*D^*V} = -g_{\chi_{c0} D^*D^*V} \partial_{\mu} D_{\rho} \partial_{\nu} D_{\sigma} D_{\rho} D_{\sigma} \epsilon_{\mu \nu \rho \sigma},$$

$$L_{\psi D^*D^*V} = -ig_{\psi D^*D^*V} \partial_{\mu} D^*_\mu D^*_\nu D^*_\rho D^*_\sigma \epsilon_{\mu \nu \rho \sigma},$$

$$L_{\chi_{c0} D^*D^*V} = -g_{\chi_{c0} D^*D^*V} \partial_{\mu} D_{\rho} \partial_{\nu} D_{\sigma} D_{\rho} D_{\sigma} \epsilon_{\mu \nu \rho \sigma}. (1)$$

$$L_{\psi D^*D^*V} = -ig_{\psi D^*D^*V} \partial_{\mu} D^*_\mu D^*_\nu D^*_\rho D^*_\sigma \epsilon_{\mu \nu \rho \sigma}, L_{\chi_{c0} D^*D^*V} = -g_{\chi_{c0} D^*D^*V} \partial_{\mu} D_{\rho} \partial_{\nu} D_{\sigma} D_{\rho} D_{\sigma} \epsilon_{\mu \nu \rho \sigma}. (2)$$

$$L_{\psi D^*D^*P} = -ig_{\psi D^*D^*P} \partial_{\mu} D^*_\mu D^*_\nu D^*_\rho D^*_\sigma \epsilon_{\mu \nu \rho \sigma},$$

$$L_{\chi_{c0} D^*D^*P} = -g_{\chi_{c0} D^*D^*P} \partial_{\mu} D_{\rho} \partial_{\nu} D_{\sigma} D_{\rho} D_{\sigma} \epsilon_{\mu \nu \rho \sigma}. (3)$$

$$L_{\psi D^*D^*V} = -ig_{\psi D^*D^*V} \partial_{\mu} D^*_\mu D^*_\nu D^*_\rho D^*_\sigma \epsilon_{\mu \nu \rho \sigma},$$

$$L_{\chi_{c0} D^*D^*V} = -g_{\chi_{c0} D^*D^*V} \partial_{\mu} D_{\rho} \partial_{\nu} D_{\sigma} D_{\rho} D_{\sigma} \epsilon_{\mu \nu \rho \sigma}. (4)$$

where $V$ and $P$ denote the matrix of the vector octet and pseudoscalar octet, respectively. The explicit expressions for $V$ and $P$ are

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}}(\eta^0 + \omega) & 0 & K^+ & K^0 \\ \frac{1}{\sqrt{2}}(\eta^0 - \omega) & 0 & K^0 & K^+ \\ 0 & 0 & K^+ & K^0 \\ 0 & 0 & K^0 & K^+ \end{pmatrix},$$

$$P = \begin{pmatrix} \sqrt{2} \eta^0 + \alpha \eta + \beta \eta' & \pi^+ & K^0 \\ -\sqrt{2} \eta^0 + \alpha \eta + \beta \eta' & \pi^- & K^0 \\ 0 & 0 & K^+ \\ 0 & 0 & K^- \end{pmatrix}. (5)$$

where the corresponding mixing angles are defined as

$$\alpha = \frac{\cos \theta - \sqrt{2} \sin \theta}{\sqrt{6}}, \beta = \frac{\sin \theta + \sqrt{2} \cos \theta}{\sqrt{6}},$$

$$\gamma = \frac{-2 \cos \theta - \sqrt{2} \sin \theta}{\sqrt{6}}, \delta = \frac{-2 \sin \theta + \sqrt{2} \cos \theta}{\sqrt{6}}. (6)$$

In the present calculations, we adopt $\theta = -19.1^\circ$ determined in Refs. [27, 28]. Since $\psi(4S)$ and $J/\psi$ have the same $J^{PC}$ quantum numbers, the interaction Lagrangians for $\psi(4S)D^*_0D^*_0$ have the same forms as those describing the interactions for $J/\psi D^*_0D^*_0$, where the coupling constants will be given later.

With these effective Lagrangians listed in Eqs. (1)-(4), we can obtain the Feynman rules, which are collected in Appendix.

With the above preparation, we can easily write out the decay amplitudes for $\psi(4S) \rightarrow \omega\chi_{c0}$ and $\psi(4S) \rightarrow \eta J/\psi$, where a general form of the decay amplitude is

$$M = \int dq \frac{V_1 V_2 V_3}{(2\pi)^4 P_1 P_2 P_E} F^2(q, m_E), (8)$$

where $V_i$ are triple couplings corresponding to Eqs. (13)-(25) and $1/P_i$ correspond to the propagators defined in Eq. (26)-(27) with $1/P_E$ expressing the exchanged-meson propagator. The concrete expressions for the amplitude $\psi(4S) \rightarrow \omega\chi_{c0}$ with the meson loop contributions are similar to those in our former work relevant to $\Upsilon(5S) \rightarrow \omega\chi_{c0}$ [16]. In Eq. (8), the form factor $F(q, m_E)$ is taken as a monopole expression $F(q, m_E) = (m^2_E - \Lambda^2)/(q^2 - \Lambda^2)$, which is introduced to describe the structure effect of interacting vertices and off-shell effect that results from the exchanged meson. Here the cutoff $\Lambda$ can be further parameterized as $\Lambda = \alpha_A \Lambda_{QCD} + m_E$.
with \( \Lambda_{QCD} = 220 \text{ MeV} \) [29] and \( m_E \) denotes the exchanged-meson mass. In addition, the introduced form factor plays a role of regularization to get rid of the UV divergence of the loop integrals, which is similar to the Pauli-Villas regularization scheme.

In the following, we elucidate how to determine the values of the involved coupling constants in our calculation. With the Feynman rules listed in Eqs. (13)-(16), we get the partial decay widths of the open charm decays of \( \psi(4S) \), i.e.,

\[
\Gamma_{\psi(4S) \to D\bar{D}^*} = \frac{g^2_{\psi(4S)D\bar{D}^*}(m^2_{\psi(4S)}, m^2_{D^*}, m^2_{\bar{D}^*})^{3/2}}{24\pi m^5_{\psi(4S)}},
\]

where \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \) denotes the Källén function. We can obtain the partial decay width related to charm mesons by replacing the charm meson mass with the corresponding masses of charm strange mesons and multiplying a factor 1/2 caused by isospin. With the relations obtained above and the partial decay widths estimated in Ref. [6], we can evaluate the coupling constants related to the interactions \( \psi(4S) D^{(*)} \overline{D}^{(*)} \). Since the partial decay widths in Ref. [6] are dependent on the parameter \( R \), which is introduced in the harmonic oscillator wave function for \( \psi(4S) \), the extracted coupling constants also vary with the parameter \( R \), which is presented in Fig. 2.

**TABLE I**: The concrete values of coupling constants of charmonium (\( J/\psi \) and \( \chi_{c0} \)) interacting with charm mesons, and those of charm mesons interacting with light pseudoscalar/vector mesons [22–25].

| Coupling | Value | Coupling | Value |
|----------|-------|----------|-------|
| \( g_{J/\psi DD} \) | 7.44 | \( g_{J/\psi D\bar{D}^*} \) | 2.49 GeV\(^{-1} \) |
| \( g_{DD D} \) | 3.47 | \( g_{D\bar{D}^* D\bar{D}^*} \) | 2.32 GeV\(^{-1} \) |
| \( f_{D\bar{D}^* D\bar{D}^*} \) | 4.67 | \( g_{\chi_{c0} DD} \) | -25.00 GeV |
| \( g_{D\bar{D}^* D\bar{D}^*} \) | 8.94 | \( g_{\chi_{c0} D\bar{D}^*} \) | -8.96 GeV |

Other coupling constants \( \chi_{c0} D^{(*)} \overline{D}^{(*)}, J/\psi D^{(*)} D^{(*)}, D^{(*)} D^{(*)} P \), and \( D^{(*)} D^{(*)} V \) can be estimated by considering heavy quark limit and chiral symmetry, which are given in Refs. [22–25]. In Table 1, we list the concrete values of these coupling constants adopted in our calculation. The coupling constant involved in charm strange mesons can be obtained by the relations \( g_{Y D^{(*)} D^{(*)}} = \sqrt{m^2_D m^2_{D^*} m^2_{D^*}} g_{Y D^{(*)} D^{(*)}} \) with \( Y = J/\psi, \chi_{c0}, V, P \) [22–25].

![FIG. 2: (color online). The \( R \) dependence of the extracted coupling constants of \( \psi(4S) \) interacting with charm or charm-strange mesons.](image)

### III. NUMERICAL RESULTS

Before presenting the numerical results, we first focus on the BESIII measurement of \( e^+e^- \to \omega\chi_{c0} \) [5]. If explaining the observed enhancement structure existing in \( e^+e^- \to \omega\chi_{c0} \) to be \( \psi(4S) \), the BESIII data indicates [5]

\[
\Gamma(\psi(4S) \to e^+e^-) \mathcal{B}(\psi(4S) \to \omega\chi_{c0}) = (2.7 \pm 0.5 \pm 0.4) \text{ eV},
\]

which can be applied to extract the branching ratio of \( \psi(4S) \to \omega\chi_{c0} \). In order to get the branching ratio \( \mathcal{B}(\psi(4S) \to \omega\chi_{c0}) \) from Eq. (12), we have to rely on the theoretical evaluation of the width \( \Gamma(\psi(4S) \to e^+e^-) \). In Refs. [7, 8], the screen potential was considered when studying the mass spectrum of the charmonium family. The mass of \( \psi(4S) \) is calculated in Refs. [7, 8] and is given by 4274 MeV and 4273 MeV, respectively, both of which are close to the mass of the enhancement structure reported by BESIII [5]. In Refs. [7, 8], the partial width of \( \psi(4S) \to e^+e^- \) was also estimated, i.e., \( \Gamma(\psi(4S) \to e^+e^-) = 0.63 \pm 0.04 \text{ keV} \) [7] and \( \Gamma(\psi(4S) \to e^+e^-) = 0.66 \pm 0.04 \text{ keV} \) [8]. If taking theoretical range \( \Gamma(\psi(4S) \to e^+e^-) = 0.63 \approx 0.66 \text{ keV} \), we can obtain the branching ratio \( \mathcal{B}(\psi(4S) \to \omega\chi_{c0}) = (3.1 \sim 5.3) \times 10^{-3} \), which is the same order of the upper bound for \( \mathcal{B}(\Upsilon(5S) \to \omega\chi_{c0}) \) [26]. This extracted \( \mathcal{B}(\psi(4S) \to \omega\chi_{c0}) \) ratio will be compared with our calculation.

Using the formula listed in Sec. II, we calculate the \( \psi(4S) \to \omega\chi_{c0} \). In Fig. 3, we present the branching ratio of \( \psi(4S) \to \omega\chi_{c0} \) dependent on \( \alpha_c \) and \( R \) values, where we use the theoretical total decay width of \( \psi(4S) \) to transfer the obtained partial decay width to the branching ratio [6]. For comparison, the upper and lower limits of branching ratio obtained from the experimental measurement are also given in Fig. 3. From our present calculation, we find that our theoretical result overlaps with the experimental data in a reasonable parameter range of \( 2.6 < \alpha_c < 4.0 \) and \( 1.83 < R < 2.17 \). This study shows that the \( e^+e^- \to \omega\chi_{c0} \) observation can be understood through introduction of the predicted \( \psi(4S) \) contribution.
In the same way as $\psi(4S) \rightarrow \omega \chi_{c0}$, we can also study the hidden charm decay process $\psi(4S) \rightarrow \eta J/\psi$ via the hadronic loop mechanism\(^{1}\). The phase space of $\psi(4S) \rightarrow J/\psi \eta$ is larger than that of $\psi(4S) \rightarrow \omega \chi_{c0}$. $\psi(4S) \rightarrow \eta J/\psi$ occurs via $P$-wave while $\psi(4S)$ decays into $\omega \chi_{c0}$ through $S$-wave, which is the difference between $\psi(4S) \rightarrow \eta J/\psi$ and $\psi(4S) \rightarrow \omega \chi_{c0}$.

In Fig. 4, the $\alpha_{\Lambda}$ and $R$ dependence of the branching ratio $\psi(4S) \rightarrow \eta J/\psi$ is presented. If taking the same parameter range as that of $\psi(4S) \rightarrow \omega \chi_{c0}$, we find that there is large variation between the upper and lower limits of the branching ratio of $\psi(4S) \rightarrow \eta J/\psi$. Thus, in this work we incline to predict the upper limit of the branching ratio, i.e.,

$$B(\psi(4S) \rightarrow \eta J/\psi) < 1.9 \times 10^{-3},$$

which can be tested in future experiment, especially BESIII and forthcoming BelleII.

## IV. Conclusions and Discussion

In this work, we have proposed that the newly observed $e^+e^- \rightarrow \omega \chi_{c0}$ by BESIII [1] can be due to the contribution from the missing charmonium $\psi(4S)$, which was predicted in Ref. [6] by the similarity between charmonium and bottomonium families. This proposal is supported by the comparison between the resonance parameter of the reported structure in $e^+e^- \rightarrow \omega \chi_{c0}$ [1], the theoretical results in Ref. [6], and the estimated mass of $\psi(4S)$ via the screen potential [7, 8].

If the above assumption is correct, we must understand $e^+e^- \rightarrow \omega \chi_{c0}$ process by studying the $\psi(4S) \rightarrow \omega \chi_{c0}$ decay and comparing this calculation with the corresponding extracted experimental data. Accordingly, in the present work we have studied the $\psi(4S) \rightarrow \omega \chi_{c0}$ decay mediated by the hadronic loop mechanism. Our theoretical calculation has shown that the extracted branching ratio of the $\psi(4S) \rightarrow \omega \chi_{c0}$ transition can be well described, which provides a direct support to our proposal. As a prediction, the upper limit of the branching ratio of $\psi(4S) \rightarrow \eta J/\psi$ has been given, which is similar to the discussed $\psi(4S) \rightarrow \omega \chi_{c0}$. The predicted upper limit of $\psi(4S) \rightarrow \eta J/\psi$ indicates that $\psi(4S) \rightarrow \eta J/\psi$ can be accessible at future experiment, especially at BESIII, Belle and forthcoming BelleII, where we have also suggested a measurement of $e^+e^- \rightarrow \eta J/\psi$ to be carried out.

Before closing this section, we should mention the measurement of the cross section of $e^+e^- \rightarrow \pi^+\pi^- h_c$ at $\sqrt{s} = 3.90 \sim 4.42$ GeV done by BESIII [30]. Here, the measured cross section of $e^+e^- \rightarrow \pi^+\pi^- h_c$ is of the same order of magnitude as that of the $e^+e^- \rightarrow \pi^+\pi^- J/\psi$. However, the lineshape of $e^+e^- \rightarrow \pi^+\pi^- h_c$ is different from that of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$. By fitting the available experimental data of $e^+e^- \rightarrow \pi^+\pi^- h_c$ from 3.90 to 4.42 GeV, a narrow structure around 4.2 GeV was discovered, where the mass and width are reported to be $M = 4216 \pm 7$ MeV and $\Gamma = 39 \pm 17$ MeV or $M = 4230 \pm 10$ MeV and $\Gamma = 12 \pm 36$ MeV [31], which depend on the different assumptions of lineshape trend above 4.42 GeV. In Ref. [6], the authors have once suggested that this narrow structure existing in $e^+e^- \rightarrow \pi^+\pi^- h_c$ can be due to the predicted missing $\psi(4S)$.

### TABLE II: Summary of resonance parameters of the structures reported in the $e^+e^- \rightarrow \omega \chi_{c0}$ and $e^+e^- \rightarrow \pi^+\pi^- h_c$ processes.

| Process          | Mass (MeV) | Width (MeV) |
|------------------|------------|-------------|
| $e^+e^- \rightarrow \omega \chi_{c0}$ [1] | 4230 ± 8 | 38 ± 12 | 2 |
| $e^+e^- \rightarrow \pi^+\pi^- h_c$ [31] | 4216 ± 7 | 39 ± 17 | 2 |
|                  | 4230 ± 10 | 12 ± 36 | 2 |

\(^{1}\) In Ref. [15], we have estimated the meson loop contributions to the $\eta$ transition between $\psi(4040)/\psi(4160)$ and $J/\psi$, which are consistent with the experimental measurements.
At present, we can find two experimental evidences for existence of a narrow charmonium $\psi(4S)$, where we collect the experimental information of the narrow structures near 4.2 GeV in Table II. Comparing the experimental data listed in Table II, we notice that these data are comparable with each other when considering the experimental errors. Combining them with our prediction of the branching ratio of $\psi(4S) \to \eta J/\psi$, we expect that there exists a similar narrow structure near 4.2 GeV in the $\eta J/\psi$ distribution of $e^+e^- \to \eta J/\psi$, which can be tested in future.

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**Note added:** When submitting our paper to arXiv, we noticed that a very recent work of $e^+e^- \to \omega \chi_{c0}$ appeared in arXiv:1411.2952 [32]. In this work, the authors suggested that $e^+e^- \to \omega \chi_{c0}$ can be due to the intermediate $\psi(4160)$ contribution.

**Appendix**

The Feynman rules corresponding to effective Lagrangians listed in Eq. (1) are

\[
\psi(4S)^\mu \cdot p_0 \rightarrow \begin{cases} 
ig_D D\bar{D}(ip_2^\mu - ip_1^\mu), & (13) \\
g_D D\bar{D}\epsilon_{\mu\nu\rho\sigma}(ip_0^\rho - ip_1^\rho)(ip_2^\sigma - ip_1^\sigma), & (14) \\
g_D D\bar{D}\epsilon_{\mu\nu\rho\sigma}(ip_0^\rho - ip_1^\rho)(ip_2^\sigma - ip_1^\sigma), & (15) \\
ig_D D\bar{D}^\prime(ip_2^\mu g_{\mu\nu} - ip_1^\mu g_{\mu\nu} - ip_2^\nu g_{\mu\nu} - ip_1^\nu g_{\mu\nu}) \nonumber \label{eq:psi4s}
\end{cases}
\]

With Eq. (2), we can obtain the Feynman rules related to the $\chi_{c0}D^{(*)}D^{(*)}$ couplings, i.e.,

\[
\begin{align*}
\psi(4S)^\mu & \rightarrow \begin{cases} 
-g_{\chi_{c0}DD}, & (17) \\
ig_{\chi_{c0}D^*D}(ip_2^\mu g_{\mu\nu} - ip_1^\mu g_{\mu\nu} - ip_2^\nu g_{\mu\nu} - ip_1^\nu g_{\mu\nu}) & (18)
\end{cases}
\end{align*}
\]

Next, we get the Feynman rules from Eq. (3), which depicts the coupling of charmed mesons with $\omega$ meson,

\[
\begin{align*}
\begin{array}{ll}
\psi(4S)^\mu & \rightarrow \begin{cases} 
-ig_{DDV}(\mu(p_1 + q_\nu))e_\nu^V, & (19) \\
-2f_{DDV}\epsilon_{\mu\nu\rho\sigma}(ip_3^\rho)e_\nu^V(ip_1^\sigma + q_\nu), & (20) \\
-2f_{DDV}\epsilon_{\mu\nu\rho\sigma}(ip_3^\rho)e_\nu^V(-ip_1^\sigma - q_\nu), & (21) \\
ig_{DDV^\prime}e_\nu^V(-ip_1^\nu - iq_\nu)g_{\mu\nu} & (22)
\end{cases}
\end{array}
\end{align*}
\]

Similarly, the Feynman rules corresponding to the interaction of charmed mesons with $\eta$ meson can be easily extracted from Eq. (4), i.e.,

\[
\begin{align*}
\begin{array}{ll}
\psi(4S)^\mu & \rightarrow \begin{cases} 
-ig_{DD}\epsilon_{\mu\nu}(ip_3^\nu), & (23) \\
ig_{DD}\epsilon_{\mu\nu}(ip_3^\nu), & (24) \\
ig_{\frac{1}{2}DD^\prime}\epsilon_{\mu\nu\rho\sigma}(ip_3^\nu)(ip_1^\rho - ip_1^\rho), & (25)
\end{cases}
\end{array}
\end{align*}
\]

In addition, the involved propagators are given by

\[
\begin{align*}
\begin{array}{ll}
\frac{p}{D} & \rightarrow \begin{cases} 
\frac{i}{p^2 - m_D^2}, & (26) \\
\frac{i(-g_{\mu\nu} + p^\mu p^\nu/m_D^2)}{p^2 - m_D^2} & (27)
\end{cases}
\end{array}
\end{align*}
\]
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