Multi-view Subspace Clustering Networks with Local and Global Graph Information

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Abstract

This study investigates the problem of multi-view subspace clustering, the goal of which is to explore the underlying grouping structure of data collected from different fields or measurements. Since data do not always comply with the linear subspace models in many real-world applications, most existing multi-view subspace clustering methods that based on the shallow linear subspace models may fail in practice. Furthermore, underlying graph information of multi-view data is always ignored in most existing multi-view subspace clustering methods. To address aforementioned limitations, we proposed the novel multi-view subspace clustering networks with local and global graph information, termed MSCNLG, in this paper. Specifically, autoencoder networks are employed on multiple views to achieve latent smooth representations that are suitable for the linear assumption. Simultaneously, by integrating fused multi-view graph information into self-expressive layers, the proposed MSCNLG obtains the common shared multi-view subspace representation, which can be used to get clustering results by employing the standard spectral clustering algorithm. As an end-to-end trainable framework, the proposed method fully investigates the valuable information of multiple views. Comprehensive experiments on six benchmark datasets validate the effectiveness and superiority of the proposed MSCNLG.

Keywords: Subspace clustering, Autoencoder, Multi-view clustering

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1. Introduction

Clustering is an important task in unsupervised learning, which can be a prepossessing step to assist other learning tasks or a stand-alone exploratory tool to uncover underlying information of data points \[1\]. The goal of clustering is to group unlabeled data points into corresponding categories according to their intrinsic similarities. Many effective clustering algorithms have been proposed, such as k-means clustering, spectral clustering \[2\], and subspace clustering \[3, 4, 5\]. And some deep-learning based clustering methods are also proposed in recent years \[6, 7\]. However, these methods are designed for single-view rather than multi-view data, which are collected from multiple sources and common in many real-world applications. Unlike single-view data, multi-view data contains both the consensus information and complementary information for multi-view learning \[8\]. Therefore, an important issue of multi-view clustering is how to fuse multiple views properly to mine the underlying grouping information effectively.

Evidently, it is not a good choice to use a single-view clustering algorithm on multi-view data straightforward \[9, 10, 11\]. In this study, we consider the multi-view clustering problem based on the subspace clustering algorithm \[4, 12, 6, 13\], which utilizes the linear subspace model for clustering. To be clear, the linear subspace model assumes that a data point can be represented by a linear combination of other points in the same cluster.

Recently, numerous multi-view subspace clustering methods have been proposed \[14, 11, 15, 16, 17, 18\]. For example, LT-MSC \[14\] boosts the clustering performance by employing a low-rank tensor constraint on multiple views, FCMSC \[17\] obtains the promising multi-view clustering performance by introducing the concept of cluster-specific corruption. CBF-MSC \[18\] introduces the constrained bilinear factorization to explore the consensus and complementary information of multi-view data for clustering. Despite good clustering results can be obtained, there are some deficiencies in these methods. Firstly, aforementioned multi-view subspace clustering methods are all under the linear subspace assumption \[3\], in other words, the data in different views are all conformed
Figure 1: Illustration of the proposed MSCNLG. The autoencoder networks are performed on different views simultaneously to attain latent representations that are conform with linear subspace assumption. $X^{(k)}$ denote the input feature representations, $\tilde{X}^{(k)}$ and $Z^{(k)}$ are output of the decoder network and the latent representation with respect to the $i$-th view, respectively, $C$ indicates the common shared coefficient matrix. By leveraging the 1-st order and 2-nd order graph of all view, the underlying local and global multi-view graph information can be obtained as well. Consequently, A shared smooth representation, which can be leveraged to get clustering results, is achieved by integrating the underlying graph information into the self-expressive layer of autoencoder networks.

to the linear subspace models. However, correlations of data points are preferred to be non-linear rather than linear for data in practice. Some works use the kernel methods to deal with this issue by mapping samples into a high-dimensional feature space and then get subspace clustering results in that space [20, 21]. Whereas, the kernel function is often pre-defined in an ad hoc manner and suffers from the problem of generalization. Secondly, few multi-view subspace clustering methods integrate the graph information of different views into the subspace representation for improving clustering results. Since graph information is vital for clustering, it is highly expected to achieve improvement for multi-view subspace clustering performance by leveraging the underlying graph information of multi-view data.

To address above-mentioned limitations, we proposed the multi-view subspace clustering networks with local and global graph information (MSCNLG) in this paper. As shown in Fig. 1 to conform with the linear subspace assumption, autoencoder networks are simultaneously performed on multiple views re-
spectively, and the self-expressive layer [6] is employed between the encoder and decoder. The coefficient matrix of self-expressive layer, i.e. the subspace representation, is shared by all views. It is noteworthy that the self-expressive layer leveraged in our method is much different from existing works [6, 22, 7, 23], such as DSC-Net [6], DPSC [7], and DMSCN [23]. Specifically, the self-expressive layer employed in DSC-Net, DPSC, and DMSCN leverages the $\ell_1$ norm or Frobenius norm to constrain the learning process of subspace representation. Actually, the utilization of $\ell_1$ norm or Frobenius norm assumes that the subspace representation is sparse [3] or dense [24], and ignores the important graph information of data. Unlike most existing methods, the proposed MSCNLG utilizes the underlying local and global graph information of multi-view data to guide the subspace representation learning process in the self-expressive layer of autoencoders in multiple views. As can be observed in Fig. 1, local and global graph information can be mine by the first-order and second-order graph, then are leveraged in the shared self-express layer to constrain the learning process of subspace representation. Once the desired subspace representation is obtained, clustering results can be calculated by employing the standard spectral clustering algorithm. Comprehensive experiments conducted on six real-world datasets illustrate the effectiveness of MSCNLG.

The main contributions of this study are as follows:

• By performing autoencoder networks on all view simultaneously, latent representations that conform to the linear subspace model can be achieved and leveraged in our method for multi-view subspace clustering.

• To fully utilizing the graph information, in the proposed method, the local and global graph information of multi-view data can be obtained and employed to guide the learning process of the desired subspace representation.

• Extensive experiments are performed on six benchmark dataset, and experimental results shows the superiority of our method over several state-of-the-arts.
2. Related Work

In recent years, many multi-view clustering approaches have been proposed. Although various multi-view subspace clustering methods based on different theories are conducted, the key issue of them all is one, i.e., achieving promising clustering results by combining multiple views properly and exploring the underlying clustering structures of multi-view data fully.

Based on the way of views combination, most existing multi-view clustering methods can be classified roughly into three groups [8, 25]: co-training or co-regularized [26, 9], graph based [27, 28, 29, 30, 31, 32], and subspace clustering based methods [33, 34, 14, 11, 15, 17, 18, 19]. For example, in the first category, the method proposed in [26] searches to for the clustering results agreed among multiple views by the idea of co-training, the approach proposed in [9] uses the the co-regularized strategy to combine graphs information of different views implicitly to achieve multi-view clustering results. For the graph based methods, RMSC [27] pursues a latent transition probability matrix of all views via low rank and sparse decomposition, and then obtains clustering results based on the standard Markov chain. AMGL [29] achieve multi-view clustering results by assigning auto-weighted factors to multiple views. MCGC [35] achieves clustering results by learning a common shared graph of all views with a constrained Laplacian rank constraint. Regarding subspace clustering based methods, LMSC [36] seeks an underlying latent representation, which is the origin of all views, and runs the low-rank representation algorithm on the learning latent representation simultaneously. MLRSSC [15] aims to learn a joint subspace representation and constructs a shared subspace representation with both the low-rank and sparsity constraints.

Some deep mult-view clustering methods [37, 24, 38] are also proposed recently. DCCAE [37] uses two autoencoders and tries to maximize the canonical correlation between two different views. DMSCN [24] proposes an affinity fusion-based model by employing the self-expressive layer and leverages Frobenius norm to constrain the subspace representation. AE$^2$-Nets propose a nested
Table 1: Main symbols used in this paper.

| Symbol | Meaning |
|--------|---------|
| $n$    | The number of samples. |
| $v$    | The number of views. |
| $c$    | The number of clusters. |
| $d_k$  | The dimension of the original $k$-th view. |
| $\hat{d}_k$ | The dimension of the latent $k$-th view. |
| $A_{i}$ | The $i$-th column of matrix $A$. |
| $X^{(k)} \in \mathbb{R}^{d_k \times n}$ | The data matrix of the $k$-th view. |
| $X^{(k)}_{i} \in \mathbb{R}^{d_k}$ | The $i$-th data point from the $k$-th view. |
| $W^{(k)}$ | The first-order proximity of the $k$-th view. |
| $\hat{W}^{(k)}$ | The second-order proximity of the $k$-th view. |
| $Z^{(k)} \in \mathbb{R}^{\hat{d}_k \times n}$ | The latent representation the $k$-th view. |
| $C \in \mathbb{R}^{n \times n}$ | The shared smooth representation matrix. |

An autoencoder framework to learn an intact representation of multi-view data. It is worth noting that most existing multi-view subspace clustering methods, including deep-based multi-view subspace clustering, ignore the graph information of multiple views during the subspace representation learning process.

3. The Proposed Approach

In this section, we introduce the proposed method in detail. Fig. presents the whole framework of our MSCNLG, Table lists the main symbols employed in this paper. Given a multi-view dataset $X = \{X^{(k)}\}_{k=1}^{v}$, samples of which is collected from $v$ multiple views, we start with the whole objective function of the proposed MSCNLG in this section. To be clear, the objective function is formulated as follows:

$$
L = \sum_{k=1}^{v} (L_1^{(k)} + \alpha L_2^{(k)}) + \beta L_3,
$$

where $L_1^{(k)}$ and $L_2^{(k)}$ denote the reconstruction loss and the self-representation loss of the $k$-th view respectively, and the Frobenius-norm is employed here. $L_3$
stands for a regularizer term, which employs the multi-view local and global graph information to guide the learning process of multi-view subspace representation. $\alpha$ and $\beta$ are the trade-off parameters.

For $L_1^{(k)}$ and $L_2^{(k)}$, they are utilized to learn the new feature representations, i.e., $\{Z^{(k)}\}_{k=1}^v$, for multiple views. Specifically, the autoencoder framework with a self-express layer is employed in $L_1^{(k)}$ and $L_2^{(k)}$, and they have the following formulas:

$$L_1^{(k)} = \frac{1}{2} \| \tilde{X}^{(k)} - X^{(k)} \|_F^2,$$
$$L_2^{(k)} = \frac{1}{2} \| Z^{(k)} - \tilde{Z}^{(k)}C \|_F^2,$$

where $\tilde{X}^{(k)}$ and $Z^{(k)}$ are output of the decoder network and the latent representation with respect to the $k$-th view, respectively. $C$ indicates the common shared coefficient matrix. Obviously, $L_1^{(k)}$ promises that the information of the $k$-th view can be reserved in maximum to learn the latent new representation $Z^{(k)}$. For $L_2^{(k)}$, it leverages $\{Z^{(k)}\}_{k=1}^v$ and aims to learn the desired subspace representation $C$ for clustering. It is clear that a specific constraint should be added to guide the learning process of $C$. In the proposed MSCNLG, we leverage the underlying local and global information of multi-view data by introducing $L_3^{(k)}$ to achieve this goal. It is emphasized that $L_3^{(k)}$ employed here is much different from existing deep based subspace clustering, which employs the $\ell_1$ norm or Frobenius norm to regularize the subspace representation learning process and graph information is neglected.

Regarding to $L_3^{(k)}$, it investigates the underlying local and global graph information of multi-view data and guide the learning process of desired subspace representation. Graph information of data is vital for clustering, however, it is a challenging to explore graph information of multiple views effectively. Here, the first-order proximity and second-order proximity [39] of different views are employed to attain a fused graph. Taking the $k$-th view for example, we have the following formula:

$$S_{ij}^{(k)} = \exp\left(-\frac{\| X_i^{(k)} - X_j^{(k)} \|_2^2}{\sigma^2}\right),$$
where $S_{ij}^{(k)}$ denotes the similarity between the $i$-th and $j$-th data points in the $k$-th view, and $\sigma$ is the median Euclidean distance. Mutual $k$ nearest neighbor (mkNN) strategy is employed, which means that the elements of the first-order proximity are:

\[
W^{(k)} = \begin{cases} 
S_{ij}^{(k)}, & \text{if } X^{(v)}_j \text{ and } X^{(v)}_i \text{ are mkNN}, \\
0, & \text{otherwise}
\end{cases}
\] (4)

where $W^{(k)}$ is the first-order proximity matrix of the $k$-th view. Clearly, $W^{(k)}$ captures the local graph structures, and it is oversimplified to fully capture the graph information for clustering. To this end, the second-order proximity is employed and can be formulated as follows:

\[
\hat{S}_{ij}^{(k)} = \exp\left(-\frac{\|W_i^{(k)} - W_j^{(k)}\|^2}{\sigma^2}\right),
\] (5)

where $\hat{S}_{ij}^{(k)}$ denotes the similarity of the $i$-th row and $j$-th row in the first-order
proximity of the $k$-th view. Similarly, the corresponding second-order proximity $\hat{W}^{(k)}$ can be obtained by conducting mkNN on $\hat{S}_{ij}^{(k)}$. Under the intuition that data points with more shared neighbors are more likely to be similar, it is evident that the second-order proximity matrices of multiple views capture the global graph information.

To integrate local and global graph information, we construct the intrinsic multi-view graph as follows:

$$ W = \sum_{k=1}^{v} W^{(k)} + \sum_{k=1}^{v} \hat{W}^{(k)}, $$

(6)

where $\odot$ denotes the Hadamard product, $W$ contains the local and global graph information of multi-view data. In $L_3^{(k)}$, the learning process of desired subspace representation is regularized as follows:

$$ L_3 = \frac{1}{2} \sum_{i,j} W_{ij} \| C_i - C_j \|_2^2 $$

$$ = Tr(C^T L C), $$

(7)

where $L$ denotes the Laplacian matrix of $W$.

Therefore, the objective function of the proposed MSCNLG can be rewritten as follows:

$$ L = \frac{1}{2} \sum_{k=1}^{v} \left( \| X^{(k)} - \bar{X}^{(k)} \|_F^2 + \alpha \| Z^{(k)} - Z^{(k)} C \|_F^2 \right) $$

$$ + \beta Tr(C^T L C). $$

(8)

As an end-to-end framework, the proposed MSCNLG can be trained effectively based on Adam [40] with learning rate fixed to $1 \times 10^{-3}$. Once the shared subspace representation $C$ is optimized, desired multi-view clustering results can be achieved by conducting the standard spectral clustering algorithm on the following matrix:

$$ C^* = \frac{1}{2}(|C| + |C^T|). $$

(9)
Table 2: Statistic information of six multi-view datasets.

| Dataset       | # Samples | # Clusters | # Views |
|---------------|-----------|------------|---------|
| Yale Face     | 165       | 15         | 3       |
| ORL           | 400       | 40         | 3       |
| MSRCV1        | 210       | 7          | 6       |
| BBC           | 685       | 5          | 3       |
| Caltech101-20 | 2386      | 20         | 6       |
| LandUse-21    | 2100      | 21         | 3       |

4. Experiments

Comprehensive experiments are performed on six benchmark datasets in this section. To be specific, Yale Face\(^1\) which contains 165 images of 15 individuals, and ORL\(^2\) which contains 400 images of 40 individuals, are both face images datasets, and each image is described by three features, namely intensity, LBP, and Gabor. MSRCV1\(^3\) consists of 210 image samples collected from 7 clusters with 6 views, including CENT, CMT, GIST, HOG, LBP, and SIFT. BBC\(^4\) consists of 685 new documents from BBC and each of which is divided into 4 sub-parts. Caltech101-20\(^5\) consists of 2386 samples collected from 20 clusters with 6 different features, i.e. Gabor, WM, CENT, HOG, GIST, and LBP. LandUse-21\(^6\) has 2100 samples collected from 21 categories, and it has three types of feature, including GIST, PHOG, and LBP. We summary the statistic information of these multi-view datasets in Table 2. Additionally, four evaluation metrics \([36]\) are utilized, including Normalized Mutual Information (NMI), ACCuracy (ACC), F-measure, and Rand Index (RI). Higher values of all metrics indicate the better clustering performance.

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\(^1\)http://cvc.yale.edu/projects/yalefaces/yalefaces.html
\(^2\)https://www.cl.cam.ac.uk/research/dtg/
\(^3\)http://research.microsoft.com/en-us/project/
\(^4\)http://mlg.ucd.ie/datasets/3sources.html
\(^5\)http://www.vision.caltech.edu/Image Datasets/Caltech101/
\(^6\)http://weegee.vision.ucmerced.edu/datasets/landuse.html
Table 3: Clustering results of the comparison experiments on Yale Face, ORL, and MSRCV1. Four metrics and average rank (Avg. Rank) are reported here. Values in the bold type denote the best clustering results.

| Dataset   | Method     | NMI      | ACC      | F-measure | RI      | Avg. Rank |
|-----------|------------|----------|----------|-----------|---------|-----------|
| Yale Face | AMGL [29]  | 0.6437(7)| 0.6046(7)| 0.3986(7)| 0.9087(7)| 7.00      |
|           | LMSC [36]  | 0.7011(3)| 0.6691(5)| 0.5031(5)| 0.9337(5)| 4.50      |
|           | MLRSSC [15] | 0.7065(4)| 0.6733(4)| 0.5399(4)| 0.9420(3)| 3.75      |
|           | DMSCN [23] | 0.6889(6)| 0.7091(3)| 0.5859(3)| 0.9392(4)| 4.00      |
|           | GMC [31]   | 0.6892(5)| 0.6545(6)| 0.4801(6)| 0.9257(6)| 5.75      |
|           | MSCNLG     | 0.8712(2)| 0.8910(2)| 0.8199(2)| 0.9738(2)| 2.00      |
|           | MSCNLG     | 0.9012(1)| 0.9152(1)| 0.8551(1)| 0.9795(1)| 1.00      |
| ORL       | AMGL [29]  | 0.8830(6)| 0.6046(7)| 0.3986(6)| 0.9087(7)| 6.50      |
|           | LMSC [36]  | 0.9215(4)| 0.8193(4)| 0.7623(4)| 0.9884(4)| 4.00      |
|           | MLRSSC [15] | 0.9102(5)| 0.8042(5)| 0.7459(5)| 0.9879(5)| 5.00      |
|           | DMSCN [23] | 0.9532(3)| 0.8658(3)| 0.8885(2)| 0.9926(3)| 2.75      |
|           | GMC [31]   | 0.8571(7)| 0.6325(6)| 0.3599(7)| 0.9357(6)| 6.50      |
|           | MSCNLG     | 0.9538(2)| 0.8700(2)| 0.8873(3)| 0.9929(2)| 2.25      |
|           | MSCNLG     | 0.9561(1)| 0.8975(1)| 0.9000(1)| 0.9939(1)| 1.00      |
| MSRCV1    | AMGL [29]  | 0.7357(4)| 0.7171(6)| 0.6445(6)| 0.8807(7)| 5.75      |
|           | LMSC [36]  | 0.6149(7)| 0.6948(7)| 0.5909(7)| 0.8826(6)| 6.75      |
|           | MLRSSC [15] | 0.6709(6)| 0.7775(5)| 0.6524(5)| 0.9025(5)| 5.25      |
|           | DMSCN [23] | 0.6999(5)| 0.8000(4)| 0.6951(4)| 0.9028(4)| 4.25      |
|           | GMC [31]   | 0.8200(2)| 0.8952(3)| 0.7997(3)| 0.9434(3)| 2.75      |
|           | MSCNLG     | 0.8192(3)| 0.9000(2)| 0.8254(2)| 0.9463(2)| 2.25      |
|           | MSCNLG     | 0.8724(1)| 0.9381(1)| 0.8867(1)| 0.9666(1)| 1.00      |

4.1. Experimental Settings

To demonstrate the effectiveness and superiority of the proposed MSCNLG, five state-of-the-art multi-view clustering methods are used for comparison, including AMGL [29], LMSC [36], MLRSSC [15], DMSCN [23], and GMC [31]. Specifically, For LMSC, the dimension of latent representation is fixed to 100, and the optimal $\lambda$ is selected from $\{0.01, 0.1, 1, 10, 100\}$. For MLRSSC, parameters are referred to the recommended setting. For the DMSCN, autoencoder networks with six layers are employed, $\lambda_1$ and $\lambda_2$ are fixed to 1 and $1 \times 10^{-3}$.
Table 4: Clustering results of the comparison experiments on BBC, Caltech101-20, and LandUse-21. Four metrics and average rank (Avg. Rank) are reported here. Values in the bold type denote the best clustering results.

| Dataset        | Method    | NMI     | ACC     | F-measure | RI      | Avg. Rank |
|----------------|-----------|---------|---------|-----------|---------|-----------|
| BBC            | AMGL [29] | 0.5185(7) | 0.6261(7) | 0.6050(7) | 0.7370(7) | 7.00      |
|                | LMSC [36] | 0.5594(6) | 0.7394(5) | 0.6291(6) | 0.8336(5) | 5.50      |
|                | MLRSSC [15] | 0.6935(2) | 0.8556(2) | 0.7897(2) | 0.9022(2) | 2.00      |
|                | DMSCN [24] | 0.5773(4) | 0.7796(4) | 0.6602(4) | 0.8425(4) | 4.00      |
|                | GMC [31]  | 0.5628(5) | 0.6934(6) | 0.6333(5) | 0.7664(6) | 5.50      |
|                | MSCNLG    | 0.6251(3) | 0.8423(3) | 0.7275(3) | 0.8731(3) | 3.00      |
| Caltech101-20  | AMGL [29] | 0.5224(6) | 0.4988(1) | 0.3837(5) | 0.7245(6) | 4.50      |
|                | LMSC [36] | 0.6144(4) | 0.4729(3) | 0.3991(4) | 0.8641(2) | 3.25      |
|                | MLRSSC [15] | 0.5746(5) | 0.4174(7) | 0.3564(6) | 0.8601(5) | 5.75      |
|                | DMSCN [24] | 0.6249(3) | 0.4447(5) | 0.5067(3) | 0.8637(3) | 3.50      |
|                | GMC [31]  | 0.4809(7) | 0.4564(4) | 0.3403(7) | 0.5756(7) | 6.25      |
|                | MSCNLG    | 0.6333(2) | 0.4559(6) | 0.5153(2) | 0.8629(4) | 3.50      |
|                | MSCNLG    | 0.6669(1) | 0.4983(2) | 0.5393(1) | 0.8763(1) | 1.25      |
| LandUse-21     | AMGL [29] | 0.2772(7) | 0.1733(6) | 0.1126(6) | 0.6050(6) | 6.25      |
|                | LMSC [36] | 0.3434(4) | 0.2967(3) | 0.2021(4) | 0.9192(5) | 4.00      |
|                | MLRSSC [15] | 0.3188(5) | 0.2898(5) | 0.1828(5) | 0.9199(4) | 4.75      |
|                | DMSCN [24] | 0.3691(3) | 0.2924(4) | 0.2340(3) | 0.9223(3) | 3.25      |
|                | GMC [31]  | 0.2845(6) | 0.1371(7) | 0.1074(7) | 0.3954(7) | 6.75      |
|                | MSCNLG    | 0.3898(2) | 0.3443(1) | 0.2545(2) | 0.9274(1) | 1.50      |
|                | MSCNLG    | 0.4028(1) | 0.3362(2) | 0.2631(1) | 0.9265(2) | 1.50      |

respectively as suggested. Regarding to AMGL and GMC, multi-view clustering results can be achieved without parameter selection. Additionally, to further illustrate the effectiveness of the proposed method, the following baseline is also defined for comparison: MSCNLG\textsubscript{1-st}, the fused multi-view graph of which is obtained only by the first-order proximity only. For the fair comparison, autoencoder networks of MSCNLG\textsubscript{1-st} and MSCNLG have six layers, the optimal values of $\alpha$ and $\beta$ are fixed to 0.1 and 100 respectively, 20 nearest neighbors are employed for both the first-order and second-order graph construction.

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| Dataset | View | NMI    | ACC    | F-measure | RI    |
|---------|------|--------|--------|-----------|-------|
| Yale Face | View 1: $X^{(1)} | Z^{(1)}$ | 0.5948 | 0.5584 | 0.5733 | 0.4103 | 0.3569 | 0.9263 | 0.9191 |
| Yale Face | View 2: $X^{(2)} | Z^{(2)}$ | 0.2483 | 0.6376 | 0.6010 | 0.0695 | 0.4479 | 0.8736 | 0.9310 |
| Yale Face | View 3: $X^{(3)} | Z^{(3)}$ | 0.2543 | 0.6508 | 0.6034 | 0.0674 | 0.4712 | 0.8802 | 0.9343 |
| ORL | View 1: $X^{(1)} | Z^{(1)}$ | 0.7903 | 0.6909 | 0.6275 | 0.4753 | 0.5064 | 0.3282 | 0.9765 | 0.9682 |
| ORL | View 2: $X^{(2)} | Z^{(2)}$ | 0.4810 | 0.7484 | 0.5758 | 0.0969 | 0.4388 | 0.9545 | 0.9735 |
| ORL | View 3: $X^{(3)} | Z^{(3)}$ | 0.4366 | 0.7579 | 0.5741 | 0.0536 | 0.4439 | 0.9522 | 0.9737 |
| MSRCV1 | View 1: $X^{(1)} | Z^{(1)}$ | 0.3768 | 0.4857 | 0.4052 | 0.5460 | 0.3694 | 0.4467 | 0.8221 | 0.8408 |
| MSRCV1 | View 2: $X^{(2)} | Z^{(2)}$ | 0.0839 | 0.3667 | 0.2811 | 0.4810 | 0.1632 | 0.3364 | 0.7537 | 0.8136 |
| MSRCV1 | View 3: $X^{(3)} | Z^{(3)}$ | 0.2001 | 0.6182 | 0.3524 | 0.6714 | 0.2714 | 0.5935 | 0.7766 | 0.8854 |
| MSRCV1 | View 4: $X^{(4)} | Z^{(4)}$ | 0.2655 | 0.5889 | 0.4238 | 0.6873 | 0.2782 | 0.5640 | 0.7897 | 0.8772 |
| MSRCV1 | View 5: $X^{(5)} | Z^{(5)}$ | 0.0623 | 0.4822 | 0.1667 | 0.5492 | 0.2410 | 0.4338 | 0.1994 | 0.8334 |
| MSRCV1 | View 6: $X^{(6)} | Z^{(6)}$ | 0.1549 | 0.1598 | 0.2905 | 0.3111 | 0.2066 | 0.2041 | 0.7759 | 0.7757 |

Table 5: Subspace clustering results performed on original data representations $X^{(v)}$ and latent representations $Z^{(v)}$ via the low-rank representation. For the fair comparison, LRR with a fixed tradeoff parameter ($\lambda = 0.01$) is employed for all datasets and all views. Values in the bold type denote the best clustering.

4.2. Comparison Experiments

As displayed in Table 3 and 4, clustering results in metrics of NMI, ACC, F-measure, and RI with corresponding ranks are reported. In a big picture, the proposed deep multi-view smooth representation clustering network obtains promising clustering results, and outperforms other comparison approaches with a large margin. And Comparison experimental results illustrate the following observations:

1) Compared with traditionally multi-view subspace clustering methods, the proposed MSCNNLG achieves significant improvements. For example, on MSRCV1 dataset, 20.15% and 6.41% increments are attained in metrics of NMI and RI respectively with respect to MLRSSC. Since autoencoder networks used in the proposed method can reconstruct data of all views, and reconstructed data can be more suitable for the linear subspace model, it is expected that the proposed method can achieve better clustering results than these traditionally approaches.
2) The proposed method also gains remarkable progress over other deep-based algorithm, i.e., DMSCN, owing to the underlying local and global multi-view graph information are used in our method.

3) Compared with MSCNLG_{1-st}, the proposed MSCNLG investigates multi-view graph information by the first-order and second-order proximity, and it achieves better clustering results than MSCNLG_{1-st} in most cases, which means that the local and global multi-view graph information play an important role in our method.

Are Autoencoder Networks Necessary? The autoencoder networks employed in multiple views can reconstruct the input data, that is to say that data attained from the output of encoder networks are more suitable for the linear subspace model. Specifically, we conduct a low-rank subspace clustering algorithm [4] on original data of multiple views and latent representations obtained from the output of encoder networks to illustrate the rationality and necessity of leveraging autoencoder networks.

Experiments on Yale Face, ORL and MSRCV1 are taken for examples. As can be observed in Table 5, clustering results demonstrate that the reconstructed data $\mathbf{Z}^{(k)}$ are much suitable for subspace clustering than original input data $\mathbf{X}^{(k)}$ in general. It is worth noting that although clustering results of latent representations in the first view of Yale Face and ORL is slightly worse, clustering results of the rest views gain remarkable improvements in all metrics. Consequently, it is necessary to utilize the autoencoder networks in the proposed MSCNLG to reconstruct the multi-view for clustering.

4.3. Model Analysis

Taking the experiments on Yale Face for example, the parameter sensitivity and convergence property are also discussed in this section to further verify the effectiveness of our method.
4.3.1. Parameter Sensitivity

Moreover, parameter sensitivity is also explored. In metrics of NMI and ACC, we obtain clustering results by tuning different $\alpha$ and $\beta$ selected from \{0.001, 0.01, 0.1, 1, 10, 100, 1000, 10000\}. As shown in Fig. 3(a) and Fig. 3(b), it can be observed that the proposed method is relatively insensitive to $\beta$, and can achieve promising results when $\alpha$ is less than 0.1 and $\beta$ belongs to \{1, 10, 100, 1000\}. Regarding to the number of nearest neighbors in multi-view graph fusion, Fig. 3(c) illustrates that the proposed approach is pretty robust to the number of nearest neighbors. Therefore, the proposed MSCNLG is robust for different parameters.

4.3.2. Convergence Property

Taking experiments conducted on Yale Face for example. As depicted in Fig. 3(d), the normalized loss and clustering results (in metrics of NMI and ACC) with respect to epochs are reported here. Actually, experimental results on other datasets have the similarity performance. It is clear that the proposed method has a stable convergence property.

5. Conclusion

In this paper, the novel multi-view subspace clustering networks with local and global graph information is proposed to achieve promising multi-view clustering results. By leveraging the autoencoder and exploring the underlying local and global graph information, a desired subspace representation can be obtained in our method. Extensive experiments conducted on six benchmark datasets illustrate the effectiveness and competitiveness of the proposed method in comparison to several state-of-the-art multi-view clustering methods.

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Figure 3: Model analysis of the proposed method. (a) and (b) present clustering results in metrics of NMI and ACC with respect to different $\alpha$ and $\beta$. (c) illustrates the influence of the nearest neighbors number in the multi-view graph fusion. (d) shows the convergence properties of the proposed MSCNLG.

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