Jeans Instability of Palomar 5’s Tidal Tail

Alice C. Quillen & Justin Comparetta
Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA;
aquillen@pas.rochester.edu; jcompare@pas.rochester.edu

25 February 2010

ABSTRACT

Tidal tails composed of stars should be unstable to the Jeans instability and this can cause them to look like beads on a string. The Jeans wavelength and tail diameter determine the wavelength and growth rate of the fastest growing unstable mode. Consequently the distance along the tail to the first clump and spacing between clumps can be used to estimate the mass density in the tail and its longitudinal velocity dispersion. Clumps in the tidal tails of the globular cluster Palomar 5 could be due to Jeans instability. We find that their spacing is consistent with the fastest growing mode if the velocity dispersion in the tail is similar to that in the cluster itself. While all tidal tails should exhibit gravitational instability, we find that clusters or galaxies with low concentration parameters are most likely to exhibit short wavelength rapidly growing Jeans modes in their tidal tails.

1 INTRODUCTION

A thin self-gravitating cylinder of gas can clump into a series of bead like structures (e.g., Chandrasekhar & Fermi 1953; Ostrikov 1964; Elmegreen 1979, 1994). This model has been used to propose explanations for clumps of star formation and star clusters along spiral arms (Elmegreen & Elmegreen 1993) and in tidal tails of galaxies (Elmegreen & Efremov 1996; Duc et al. 2000; Smith et al. 2010). The sausage or varicose instability is a rough analogy to the Plateau-Rayleigh instability where surface tension causes a stream of water to become a series of droplets. A cylinder of stars can also contract into a series of clumps. In this case the stability has been called the Jeans instability (Fridman & Polyachenko 1984) even though this term is more often used to describe the growth and decay of plane waves through a homogeneous stellar medium. The instability for a rotating stellar cylinder has been investigated by Fridman & Polyachenko (1984) (see their Chapter 2). In the limit of infinitely slow rotation their analysis can be used for the non-rotating cylinder.

Near equally spaced clumps observed in the tidal debris of Palomar 5 (Odenkirchen et al. 2001, 2002) has not been interpreted in terms of the Jean instability but instead primarily in terms of oscillations in the cluster (Gnedin et al. 1999) caused by a previous passage through the Galactic disk (Odenkirchen et al. 2002; Dehnen et al. 2004). Alternative explanations for structure in tidal tails include the effect of structure in the dark matter halo, as explored by Mayer et al. (2002) and variations in density along the tail caused by epicyclic motions in the stellar orbits (Küpper et al. 2008; Just et al. 2009; Küpper et al. 2010).

Here we consider the possibility that clumps in a tidal stream could be caused by gravitational instability.

We review the properties of the Palomar 5 cluster and tail system. Its distance is estimated at 23.2 kpc (Harris 1996). The velocity dispersion in the tail is small, measured at 2-4 km/s (Odenkirchen et al. 2009). The dispersion in the cluster itself is small, ~ 0.3 km/s (correcting for binaries Odenkirchen et al. 2002), and the cluster is suspected to have low concentration parameter, \( c_\text{t} \), in the range 0.4-0.8 (Odenkirchen et al. 2002; Dehnen et al. 2004). The width of the tail is 120 pc and the tidal radius of the cluster half of this (Odenkirchen et al. 2002, 2003). The mass of the cluster is about 5000 \( M_\odot \) (Odenkirchen et al. 2002) and that in the tail extending 10° exceeds the mass in the cluster by a factor of a few (Odenkirchen et al. 2002, 2003). The tail has been detected up to 22° from the cluster (Grillmair & Donato 2000). The linear mass density in the two tails as a function of distance along the tail drops slowly as a function of distance from the cluster (Odenkirchen et al. 2003) as would be expected from a tidal evolution model (Johnston et al. 1996; Dehnen et al. 2004).

Clumps are most easily seen on the southern side of the tidal tail with distance to the first clump about 1.7 degrees from the cluster center and that between the first and second clump about 2 degrees (using Figure 14 by Odenkirchen et al. 2003 that also shows that extinction cannot account for the density variations). At a distance of 23.2 kpc, 2 degrees corresponds to 800 pc and we can treat this as the wavelength of a possible growing unstable mode. It is useful to take the ratio of the wavenumber to the tidal tail radius, \( r_0 \), (diameter/2). We estimate \( kr_0 \sim 0.5 \) using \( r_0 \sim 60 \) pc.
2 JEANS INSTABILITY OF A STELLAR CYLINDER

We briefly review the gravitational instability of a stellar cylinder to varicose or sausage like compressive modes of oscillation. The dispersion relation is derived more rigorously by Fridman & Polyachenko (1984) (in their Chapter 2) and covering the case of a homogenous rotating stellar cylinder. We can adopt the simplest assumption of a constant linear mass density, \( \mu_0 \), but a deformed boundary of radius \( R(z) \) and a corresponding interior density, \( \rho \), that is independent of radius interior to \( R \). We are working in cylindrical coordinates with \( z \) oriented along the cylinder. To first order in a perturbation amplitude \( a \)

\[
\rho(z) = \rho_0 (1 + a \cos kz) \\
R(z) = r_0 \left(1 - \frac{a}{2} \cos kz\right) 
\]

(1)

Here we have assumed that our perturbation amplitude, \( a \), is small. The relation between the perturbation amplitude in radius \( R \) and density \( \rho \) is determined to first order by the assumption that the linear mass density does not depend on \( z \) and \( \mu_0 = \rho_0 \pi r_0^2 \).

To estimate the perturbation to the gravitational potential we can consider the potential perturbation caused by a massive wire with mass density \( \rho_0 + \mu_1 \cos(kz) \). The part of the gravitational potential fluctuating with \( z \) at a distance \( r \) from the wire and at \( z \) is

\[
\Phi(r,z) = -\int_0^\infty G\mu_1 \frac{\cos(kz')}{\sqrt{r^2 + (z-z')^2}} dz' = -2G\mu_1 \cos(kz) K_0(\kappa r) 
\]

(2)

where \( K_0 \) is a modified Bessel function of the second kind.

To estimate the gravitational potential perturbation we must integrate the Bessel function over radius interior to the cylinder boundary. For \( k r < 1 \) the asymptotic limit \( K_0(\kappa r) \to -\ln(\kappa r/2) - 0.5772 \ldots \) where the constant is the Euler-Mascheroni constant. Integrating equation (2) out to our boundary \( R \) (given in equation (1)) the potential perturbation

\[
\Phi_1(z, r = 0) \approx \int_0^R 4\pi G\rho_0a \cos kz \left[ \ln \left( \frac{kr}{2} \right) + 0.5572 \right] r \, dr 
\]

(3)

Deviations on the boundary only contribute to second order so

\[
\Phi_1(z) \approx -2G\mu_0a|1 - \ln kr_0| \cos kz 
\]

(4)

where we have used a constant of 1 inside the absolute value as it is a more accurate match to the integral of the Bessel function than the constant 0.62 given by the asymptotic limit. To first order in \( a \) the same relation would have been estimated if we had assumed no boundary deformation but a longitudinal variation in linear density \( \mu \). This situation corresponds to longitudinal modes rather than sausage or varicose modes (e.g., Ostriker 1964).

We now consider density and potential perturbations that are proportional to \( e^{i(kz - \omega t)} \). We assume a phase space stellar distribution function in the cylinder

\[
f(z,v,t) = \rho_0 \left[1 + ae^{i(z-vt-kz)}\right] f_0(v) 
\]

(5)

consistent with our density perturbation (equation (1)). Here \( f_0(v) \) is a Gaussian velocity distribution and we need only consider the distribution as a function of the \( z \) velocity component,

\[
f_0(v) = \frac{1}{\sqrt{2\pi \sigma_z^2}} \exp\left(-\frac{v^2}{2\sigma_z^2}\right) 
\]

(6)

where \( \sigma_z \) is the stellar velocity dispersion in the \( z \) direction.

We use a linearized version of the collisionless Boltzmann equation,

\[
\left[ \frac{\partial}{\partial t} + v \frac{\partial}{\partial z} \right] f_1 = -\frac{\partial \Phi_1}{\partial z} + \frac{\partial f_0}{\partial v} = 0 
\]

(7)

where we have denoted the unperturbed and first order perturbation to the distribution function as \( f_0 \) and \( f_1 \) (see equation (4)). Using equation (4) for the potential perturbation we find

\[
1 + 2G\mu_0|1 - \ln(kr_0)| \int dv \frac{kv}{\sqrt{2\pi \sigma_z^2}} \exp\left(-\frac{v^2}{2\sigma_z^2}\right) - k = 0 
\]

(8)

valid for \( kr_0 \lesssim 1 \). We can rewrite this as

\[
1 - q^{-1}|1 - \ln kr_0| \int_0^\infty \frac{1}{\sqrt{\pi}} u e^{-u^2} du = 0 
\]

(9)

where \( Z = \omega/\sqrt{2k\sigma_z} \) and

\[
q \equiv \frac{\sigma_z^2}{2G\mu_0} = \frac{2}{(kjr_0)^2} 
\]

(10)

where \( k_j \equiv \frac{2\pi c}{\sqrt{2G\mu_0}} \) is the Jeans wavenumber and \( k_j = 2\pi/\lambda_J \) with \( \lambda_J \) the Jeans wavelength.

The value \( q \) is analogous to the Toomre Q instability parameter. The dispersion relation is similar to that for the Jeans instability for a plane wave propagating through a homogenous stellar medium. The above dispersion relation that we have estimated roughly from the potential perturbation at the center of the cylinder is consistent with that in equation (14) more rigorously derived by Fridman & Polyachenko (1984) for the homogenous rotating cylinder.

Assume that \( \omega = i\gamma \) leading to unstable solutions. Then we can integrate equation (4) finding

\[
1 - q^{-1}|1 - \ln kr_0| \left[\sqrt{\pi} e^{-\gamma^2 q} \text{erf}(\gamma' - 1) + 1\right] = 0 
\]

(11)

where \( \gamma' = \gamma/\sqrt{2k\sigma_z} \). This dispersion relation implies that a “long cylinder is unstable to any large but finite thermal scatter of particles in longitudinal velocities” (Fridman & Polyachenko 1984 just before their equation 21; also see their Figure 10 showing a numerically measured stability boundary for the non-rotating cylinder with numerical calculations done by S. M. Churilov).

The above equation gives a relation between growth rate and wavelength. As \( \gamma' \) depends on \( k \) the growth rate is most easily discussed in terms of \( \gamma' k r_0 = \gamma r_0 / \sigma_z \) or the growth rate in units of the tidal tail’s radius and velocity dispersion. Interestingly the growth rate seems to be of order \( \sigma_z / r_0 \) rather than of order \( \sqrt{G\mu_0}k \). However if \( q \sim 1 \) and \( kr_0 \sim 1 \) then the two growth rates are similar.

In Figure we show growth rate versus wavenumber for different values of \( q \). We see that at each \( q \) there is a mode that is maximally unstable or grows fastest. In this figure we have plotted \( \gamma r_0 / \sigma_z \) versus \( kr_0 \) so that the growth rate
3 APPLICATION TO PALOMAR 5

In the introduction we estimated from the distance between the clumps on the southern tail of Palomar 5 that the fastest growing mode has $k r_0 \sim 0.5$. Using Figure 1 we estimate that $q \sim 0.9$ in Palomar 5’s tail as that value gives the correct wavelength for the fastest growing mode. As $q$ depends on both linear mass density and longitudinal velocity dispersion we have a relation between these two quantities. We first review estimates of the linear mass density based on observations and then we place constraints on both these quantities.

The tidal tail linear mass density for a cluster that is losing mass at a rate $\dot{M}$ is

$$\mu = \frac{dM}{dz} = \dot{M} \frac{dt}{dz}$$

(12)

where $\frac{dt}{dz}$ is the rate at which stars are drifting away from the cluster and $z$ gives distances along the tail.

At a radius $R$ from the Galaxy center,

$$\frac{dz}{dt} \approx \frac{d\Omega}{dR} R t$$

(13)

where $\Omega(R)$ is the angular rotation rate of a particle in a circular orbit in the galaxy radius $R$. This is approximately consistent with the more precise semi-analytical model by Johnston et al. (1999). For a galaxy with a flat rotation curve $\frac{dt}{dz} \sim \frac{\sigma}{R}$ so

$$\frac{dz}{dt} \approx \Omega R.$$

(14)

For a tidal radius of $60$ pc, a galactic rotational velocity of $v_c = 220$ km/s and distance of $23.2$ kpc, the drift rate in Palomar 5’s tidal tail is $dz/dt \approx 0.6$ pc Myr$^{-1}$. The above estimate is only approximate as a better estimate would take into account the cluster orbit and adopt a more realistic Galactic potential (e.g., as explored in the appendix by Odenkirchen et al. 2003; also see Johnston et al. 1999; Just et al. 2009; Kükper et al. 2010).

Using the estimated drift rate of $0.6$ pc Myr$^{-1}$ the mass loss rate of $\dot{M} \sim 5 M_\odot$ Myr$^{-1}$ estimated by Odenkirchen et al. (2009) corresponds to a linear density of $\mu \sim 8 M_\odot$ pc$^{-1}$. This is three times lower than a linear mass density estimated from stellar number counts, with the number counts in the tails over 10 degrees exceeding that in the cluster by a factor of about 2 and the estimated total cluster mass of $\sim 5000 M_\odot$ (Odenkirchen et al. 2002). We note that mass segregation may affect estimates of the tail mass density based on star counts (Koch et al. 2004).

We can write for $q$

$$q \equiv \frac{\sigma^2}{2G\mu} \approx 1 \left( \frac{\sigma_*}{0.3 \text{ km s}^{-1}} \right)^2 \left( \frac{\mu_0}{10 M_\odot \text{ pc}^{-1}} \right)^{-1}$$

(15)

where we have inserted the estimated velocity dispersion of the cluster (Odenkirchen et al. 2002) and a scaling value for the linear density. We can see that $q$ may be of order 1 for Palomar 5’s tail just by looking at the above scaling values.

The position of the first clump gives us an estimate of the growth timescale for the instability. The first clump is visible only about 2 degrees away from the cluster center. This corresponds to $800$ pc or a time $1.3$ Gyr using the above drift velocity. This implies that the growth rate is approximately $\gamma \sim 0.7$ Gyr$^{-1}$ (assuming a single exponential growth timescale to the first clump).

The growth rate is related to the longitudinal velocity dispersion in the tail so the estimated growth rate gives an estimate for the stellar velocity dispersion in the tail. For $q \approx 0.9$ we expect that the growth rate has $\gamma \frac{\sigma}{\sigma_*} = 0.2$ (using Figure 1) consequently we can estimate

$$\sigma_* \sim \frac{\gamma \sigma}{0.2} \sim 0.2$ \text{ km s}^{-1}$$

(16)

This dispersion and our previous estimate for $q$ implies that
the mass density in the tail is about $\mu \sim 4M_\odot \text{pc}^{-1}$. This is in between the density estimated from the mass loss rate, $\mu \sim 8M_\odot \text{pc}^{-1}$, and that estimated from star counts $\mu \sim 2-3M_\odot \text{pc}^{-1}$. Had we used a lower value of the constant inside the absolute value in equation 4 we would have estimate a slightly higher growth rate and linear density.

The velocity dispersion we estimate above is significantly lower than the 2-4 km/s measured by Odenkirchen et al. (2009). We note that if the dispersion is isotropic a velocity dispersion of 1 km/s would cause the tidal stream to spread at a rate of about 1 kpc Gyr$^{-1}$. This would exceed the width of the observed tail a few degrees from the cluster (taking into account the estimated drift rate). Consequently a velocity dispersion in the tail is expected to be similar to that estimated for the cluster or $\sim 0.3$ km/s (Odenkirchen et al. 2002) and so consistent with our estimate. Numerical simulations by Dehnen et al. (2004) have also concluded that the tail is “cold.” Küpper et al. (2010) suggest that stars escape the cluster from Lagrange points and so the tail could have velocity dispersion lower than the cluster dispersion. In short we find that the tail density and velocity dispersion could be consistent with an interpretation of clumps in Palomar 5 as the fastest growing Jeans unstable mode of a stellar cylinder. Here we used a distance of 2 degrees between clumps to estimate the wavelength of the fastest growing mode but there could also be structure on 1 degree scales (see Figure 14 by Odenkirchen et al. 2003). We used a 2 degree scale because our dispersion relation is appropriate for $kr_0 > 1$. Future work could improve calculation of the dispersion relation for the $kr_0 \sim 1$ regime.

4 ESTIMATING THE TAIL DENSITY DURING TIDAL STRIPPING

The mass loss rate due to tidal disruption depends on the radial density distribution of the cluster and on the Galactic tidal field. Thus tidal evolution models should be able to predict the linear mass density and velocity dispersion in tidal tails. To estimate these quantities we explore the simplest model, that where both background galaxy and cluster are described as spherically symmetric isolated spheres with dispersions $\sigma_g$ and $\sigma_d$, respectively. It may also be useful to write things in terms of velocities of particles in circular orbits $v_0 = \sqrt{2}\sigma_g$. A balance of tidal force versus self gravity gives a relation for the tidal radius $r_t$ of the cluster

$$\frac{r_t}{R} = \frac{\sigma_d}{\sigma_g}$$

(17)

where the cluster is at a radial distance $R$ from the galaxy center. This gives a mass loss rate for the cluster

$$\dot{M} = \frac{2\sigma_g^3}{G} \frac{\sigma_d}{\sigma_g}$$

(18)

Material stripped from the cluster at the tidal radius will be on orbits that move away from the cluster because they have a different angular momentum. We let $R = bv_0$ with unit-less parameter $b$ that depends on the cluster orbit with $b = 0$ if on a circular orbit and $b$ of order unity for a radial orbit. Using equation 12 and 13 and the above relation for $\dot{M}$, we can estimate the mass density in the tidal tails as

$$\mu = \frac{dM}{dz} = \dot{M} \frac{dz}{d\phi} = \frac{2\sigma_g^3}{G} b$$

(19)

The dispersion in the tail should be similar to the velocity dispersion of the cluster, $\sigma_d$, and so using our estimate for $\mu$, $q \equiv \frac{\sigma_d^2}{2G\mu} \approx \frac{1}{4b}$

(20)

The above value for $q$ is of order unity for a moderately eccentric orbit. We expect that streams from tidally disrupting isolated spheres with a moderately eccentric orbit have $q$ of order 1 and so are likely to have fastest growing mode with wavelength of order a few times the radius of the stream. In this regime the growth rate can either be estimated with a velocity $\sqrt{G\mu}$ or a velocity $\sigma_d$.

The estimates in this section are for an isothermal sphere which has radial density profile $\rho \propto r^{-2}$. This can be compared to the core of a King model that has constant density. The concentration of the cluster can be described by a concentration parameter $c_t \equiv \log_{10}(r_t/R_0)$ where $R_0$ is the core radius and $r_t$ the tidal radius. This parameter is used to describe King models. Exterior to $R_0$, clusters with high concentration parameter have density dropping more steeply with radius than those with low concentration parameter. If the density drops less steeply with radius than an isothermal sphere, as would be true for a cluster with low concentration parameter, then we would expect that the linear mass density in the tidal tail is higher than predicted from an isothermal sphere. This in turn we expect would lower the $q$ of the tail, allowing shorter wavelength modes to grow. If the cluster or galaxy is centrally concentrated then $q$ would be high, only very long wavelength modes would grow, and they would grow very slowly.

In summary we find that tidally stripped isothermal spheres have $q$ of order 1, with lower and higher concentration disrupting objects having tails with higher values and lower values of $q$, respectively. We expect the separation between clumps to be set by the fastest growing wavelength which we expect should be of order a few times the tidal radius of the tidally disrupting cluster. If the cluster is small then the timescale for stars to move away from the cluster or galaxy is long. But in this case the instability takes longer to grow because the density is lower. The two effects cancel out to leave an e-folding length scale for the growth of the instability in the tidal-tail that is only dependent on the tidal radius.

5 SUMMARY AND CONCLUSION

We have investigated Jeans instability of a stellar cylinder in the context of tidal tails. As had previous works found (e.g., Fridman & Polyachenko 1984), we find there are always modes that can grow. The wavelength and growth rate of the fastest growing mode depends on a $q$ parameter $q = \frac{\sigma_d^2}{2G\mu}$ that is analogous to the Toomre Q parameter. Here $\mu$ is the linear mass density, $\sigma_d$ is the longitudinal velocity dispersion, $r_t$ is the tail radius and $k_J$ the Jeans wave-number. For $q \sim 1$, the fastest growing mode has wavelength a few times the diameter of the tail and growth rate approximately equal to the tail velocity.

\[ \text{\mu} \equiv \frac{dM}{dz} = \frac{\dot{M}}{G} \frac{dz}{d\phi} = \frac{2\sigma_g^3}{G} b \]
tidal tails with lower tails with.

For a cluster or galaxy that is approximately an isothermal sphere we find that the disrupting system will have tidal tails with lower parameters and so their modes of maximum growth rate should have shorter wavelengths and faster growth rates. More concentrated objects would have tails for which the mode of maximum growth rate is more slowly growing and longer wavelength than for low density or low concentration parameter objects. All tidal tails could exhibit Jean instability, however if the cluster or galaxy is sufficiently concentrated then the wavelength of the fastest growing mode in the tail would be large and growth rate sufficiently long that the instability may be difficult to detect. The opposite is true for objects that are close to being completely disrupted; Jeans instability should be particularly prominent.

The tidal tails of Palomar 5 exhibit clumps with spacing approximately a few times the tail width. From this we have estimated a tail density and find it approximately consistent with observational estimates. This suggests that the clumps in Palomar 5’s tails are consistent with the fastest growing Jeans unstable mode. Our estimated velocity dispersion is lower than measured by Odenkirchen et al. (2009) but consistent with the estimated cluster velocity dispersion. Palomar 5 has a low concentration parameter and so its tails would be expected to exhibit a quickly growing, short wavelength, fastest growing Jeans unstable mode.

We find that tidal tails from disrupting clusters or dwarf galaxies should display gravitational instabilities and from measurements of the wavelengths of the fastest growing motion, properties of the tail can be directly measured in a fashion only weakly dependent on the orbit or host galaxy dark matter profile. More detailed understanding of the instability may yield constraints on the orbit and tidal mass loss rate that are complimentary to those from studies of shape of the tail on the sky or its velocity gradient.

Here we have ignored the nature of the stellar orbits in the tails as they move through the galaxy. However density variations will be induced in the tail by its orbit. For Palomar 5 the growth timescale for the instability exceeds the orbital period so it may be sufficient to consider the tail density averaged over the orbit. In other systems the orbit period may be shorter than the growth timescale for the fastest growing mode. In this case at orbit apocenters a linear density increase could cause shorter wavelength modes to grow. In both cases as the tail expands and density drops, shorter wavelength unstable modes may become stable.

The Jeans instability model for clumps in tidal tails differs from that proposed by Kipper et al. (2008) who interpreted clumps in the tidal tails of Palomar 5 in terms of epicyclic over-densities. By studying systems of different densities and with different orbits, future studies should be able to differentiate between the possible explanations for clumping in stellar tidal tails. Future work will determine if the clumping seen in N-body simulations of higher density tidal tails from dwarf galaxies (Comprretta & Quillen 2010) is due to Jeans instability.

We thank Joss Bland-Hawthorn for helpful comments. Support for this work was provided by NSF through award AST-0907841. Support for this work was provided by the National Aeronautics and Space Administration through Chandra Award Number Cycle 10-800424, and Chandra Cycle 11-GO0-11012C issued by the Chandra X-ray Observatory Center, which is operated by the Smithsonian Astrophysical Observatory for and on behalf of the National Aeronautics Space Administration under contract NAS8-03060. Support for this work, part of the Spitzer Space Telescope Theoretical Research Program, was provided by NASA through a contract issued by the Jet Propulsion Laboratory, California Institute of Technology under a contract with NASA.

REFERENCES

Chandrasekhar, S., & Fermi, E. 1953, ApJ, 118, 116

Comparetta, J. & Quillen, A. C. 2010, in preparation

Dehnen, W., Odenkirchen, M., Grebel, E. K., & Rix, H.-W. 2004, AJ, 127, 2753

Duc, P.-A., Brinks, E., Springel, V., Pichardo, B., Weilbacher, P., Mirabel, I. F. 2000, AJ, 120, 1238

Elmegreen, B. G., & Efremov, Y. N. 1996, ApJ, 466, 802

Elmegreen, B. G., & Elmegreen, D. M. 1983, MNRAS, 203, 31

Elmegreen, B. G. 1994, ApJ, 433, 39

Elmegreen, B. G. 1979, ApJ, 231, 372

Fridman, A. M. & Polyachenko, V. L., 1984, “Physics of Gravitating Systems,” Volume 1, (New York: Springer-Verlag)

Gnedin, O. Y., Lee, H. M., &Ostriker, J. P. 1999, ApJ, 522, 935

Grillmair, C. J., Dionatos, O. 2006, ApJ, 641, L37

Harris, W. E. 1996, AJ, 112, 1487

Johnston, K. V., Sigurdsson, S., & Hernquist, L. 1999, MNRAS, 302, 771

Just, A., Berczik, P., Petrov, M. I., & Ernst, A. 2009, MNRAS, 392, 969

Koch, A., Grebel, E. K., Odenkirchen, M., Martinez-Delgado, D., & Caldwell, J. A. R. 2004, AJ, 128, 2274

Kipper, A. H. W., MacLeod, A., & Heggie, D. C. 2008, MNRAS, 387, 1248

Kipper, A. H. W.; Kroupa, P., Baumgardt, H., Heggie, D. C. 2010, MNRAS, 401, 105

Mayer, L., Moore, B., Quinn, T., Governato, F., & Stadel, J. 2002, MNRAS, 336, 119

Odenkirchen, M. et al. 2001, ApJ, 548, L165

Odenkirchen, M., Grebel, E. K., Dehnen, W., Rix, H.-W., Cudworth, K. M. 2002, AJ, 124, 1497

Odenkirchen, M., Grebel, E. K., Dehnen, W., Rix, H.-W., Yanny, B., Newberg, H. J., Rockosi, C. M., Martinez-Delgado, D., Brinkmann, J., & Pier, J. R. 2003, AJ, 126, 2385

Odenkirchen, M., Grebel, E. K., Kayser, A., Rix, H.-W. & Dehnen, W. 2009, AJ, 137, 3378

Ostriker, J. 1964, ApJ 140, 1529

Smith, B. J., Giroux, M. L., Struck, C., Hancock, M., & Hurlock, S. 2010, arXiv1001.0989, ApJS in press.