Observation of Strong and Weak Thermalization in a Superconducting Quantum Processor

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We experimentally study the ergodic dynamics of a 1D array of 12 superconducting qubits with a transverse field, and identify the regimes of strong and weak thermalization with different initial states. We observe convergence of the local observable to its thermal expectation value in the strong-thermalization regime. For weak thermalization, the dynamics of local observable exhibits an oscillation around the thermal value, which can only be attained by the time average. We also demonstrate that the entanglement entropy and concurrence can characterize the regimes of strong and weak thermalization. Our work provides an essential step toward a generic understanding of thermalization in quantum systems.

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Thermalization of an isolated quantum many-body system under nonequilibrium dynamics is a central topic in statistical mechanics [1–7]. For classical systems, statistical mechanics claims that with a longtime average, the Maxwell velocity distribution, independent of the initial condition, is satisfied [1]. In quantum cases, the choice of initial states plays an important role in the thermalization of closed nonintegrable many-body systems driven out of equilibrium. Numerical works [8,9] have shown that in a nonintegrable Ising model, when the effective inverse temperature of initial states is close to 0, the quenched states rapidly converge to their thermal expectations at a time after a short relaxation, and different from the classical counterpart, the time average is not necessary for achieving thermal equilibration. This phenomenon is regarded as strong (rapid) thermalization. In contrast, if the effective inverse temperature of initial states is sufficiently far away from 0, the temporal evolution of the local observable exhibits an obvious oscillation, with the longtime average attaining the thermal expectation value. Moreover, the decay of local observables and the growth of entanglement entropy (EE) are slow [10]. This phenomenon is known as weak (slow) thermalization. Recently, it has been numerically shown that regimes of strong and weak thermalization exist in the long-range Ising model describing trapped ions [10]. Nevertheless, a direct experimental comparison between strong thermalization and weak thermalization remains absent.

On the basis of the high-precision control, long coherence time, and the accurate readout, a superconducting quantum processor is an excellent platform for generating multipartite entangled states [11–13], characterizing quantum supremacy [14–16], and demonstrating variational quantum computation [17,18]. Moreover, by performing analog quantum simulations, the platform is also employed to study the phenomena in quantum many-body systems out of equilibrium, including quantum walks [19], many-body localization [20–22], dynamical phase transitions [23], and ergodic-localized junctions [24].

Here, a superconducting circuit, consisting of 12 transmon qubits, arranged in a 1D array with nearest-neighbor capacitive couplings, is employed to implement our experiments [see Fig. 1(a)]. The array of transmon qubits can be described by the 1D Bose-Hubbard model, where the anharmonicity of transmons is sufficiently large, preventing the doubly occupied state of transmons [19,25–27]. Thus, in the hard-core limit, the Bose-Hubbard model reduces to the integrable XX model [28–30]. To experimentally study...
strong and weak thermalization, we realize a nonintegrable system, using the superconducting qubit chain with a controllable transverse field. We observe the signatures of strong and weak thermalization via measuring the local observable with different initial states. Since the description of the local observable, using statistical mechanics, relies on the local entropy created by entanglement, the dynamics of the EE plays a key role in thermalization [31–35]. Thus, we study the EE of the single-qubit subsystems and show that the EE can distinguish the strong-thermalization regime from the weak one. Furthermore, we measure the concurrence [36] of the reduced density matrices of two nearest qubits, employing the tomographic readout, and observe thermal entanglement [37] in the presence of weak thermalization.

When we impose resonant microwave drives with a magnitude \( g \approx \lambda \) on all qubits, generating a local transverse field [23], the effective Hamiltonian of the superconducting circuit reads

\[
\hat{H} = \lambda \sum_{j=1}^{11} (\hat{\sigma}^x_j \hat{\sigma}^z_{j+1} + \hat{\sigma}^z_j \hat{\sigma}^x_{j+1}) + g \sum_{j=1}^{12} \hat{\sigma}^y_j,
\]

where \( \lambda = J/2 \) with \( J \) being the nearest-neighbor coupling strength shown in Fig. 1(a). For details regarding the Hamiltonian (1), including its experimental realization and breakdown of integrability, as well as the performance of our device, see Supplemental Material [38].

To observe strong and weak thermalization, we initialize the system by preparing each qubit in the direction \((\theta_0, \phi_0)\), which can be described as the spin coherent state

\[
|\theta_0, \phi_0\rangle = \prod_{j=1}^{12} \left( \cos \frac{\theta_0}{2} |+\rangle_j + e^{-i\phi_0} \sin \frac{\theta_0}{2} | -\rangle_j \right),
\]

where \(| +\rangle_j, (-\rangle_j\rangle\) denotes the eigenstate of \( \hat{\sigma}^z_j \) with the eigenvalue \(+1\) \((-1)\). Next, all qubits are biased to the working point to start the quench dynamics, \( |\Psi_i\rangle = e^{-i\hat{H}_0}\). We then tune the qubits to their idle points, and perform the quantum state tomography to reconstruct the one- and two-qubit density matrices. The experimental pulse sequence and control waveforms are shown in Figs. 1(b) and 1(c), respectively. There are three essential experimental requirements to be satisfied: (i) To realize the time evolution, all qubits should be tuned to the same frequency; (ii) The initial states of all qubits should be uniform at the start point of the time evolution; (iii) The local transverse fields of all qubits should be uniform during the evolution.
These requirements are fulfilled with specific calibrations (see Supplemental Material for details [38]).

The occurrence of strong or weak thermalization relates closely to the effective inverse temperature \( \beta \) of \( \langle \theta_0, \phi_0 \rangle \), which can be obtained by solving \( \text{Tr}[\hat{H}(\theta_0, \phi_0) \langle \theta_0, \phi_0 \rangle - \hat{\rho}_\beta] = 0 \), with \( \hat{\rho}_\beta = e^{-\beta \hat{H}} / \text{Tr}(e^{-\beta \hat{H}}) \) being the thermal state [8]. Moreover, the quasiparticle explanation of weak thermalization indicates that initial states in the weak-thermalization regime are near the edge of the energy spectrum [9]. Here, we first consider two initial states \( \langle \theta_0, \phi_0 \rangle = \langle \pi/2, \pi/4 \rangle \) and \( \langle \pi, 0 \rangle \) whose effective inverse temperature is numerically estimated as \( J \beta \approx -1.034 \) and 0, lying in the weak- and strong-thermalization regime, respectively. In addition to the effective inverse temperature, the regimes of strong and weak thermalization can be identified by defining the normalized energy of the initial state \( \langle \theta_0, \phi_0 \rangle \)

\[
\epsilon = \frac{\langle \theta_0, \phi_0 \rangle \hat{H} \langle \theta_0, \phi_0 \rangle - E_{\text{min}}}{E_{\text{max}} - E_{\text{min}}},
\]

with \( E_{\text{max}} \) and \( E_{\text{min}} \) being the maximum and minimum eigenvalue of \( \hat{H} \), respectively. In Fig. 1(d), the \( \epsilon \) of the initial state \( \langle \pi, 0 \rangle \) corresponds to the maximum density of states (DOS) \( \rho(\epsilon) \), while the \( \epsilon \) of the initial state \( \langle \pi/2, \pi/4 \rangle \) is close to the edge with \( \rho(\epsilon) \approx 0 \). Moreover, in Fig. 1(e), we plot the normalized energy of different initial states \( \langle \theta_0, \phi_0 \rangle \), i.e., \( \epsilon(\theta_0, \phi_0) \).

We start by characterizing strong and weak thermalization employing the local observable \( \overline{\rho(t)} = 1/12 \sum_{j=1}^{12} \langle \Psi_j | \hat{\sigma}_j^z | \Psi_j \rangle \). Figures 2(a) and 2(c) present the experimental results of the time evolution of \( \overline{\rho(t)} \) with initial states \( \langle \pi, 0 \rangle \) and \( \langle \pi/2, \pi/4 \rangle \), respectively. It is shown that for the initial state \( \langle \pi, 0 \rangle \) in the strong-thermalization regime, \( \overline{\rho(t)} \) stably achieve the thermal value \( \text{Tr}(\hat{\rho}_\beta \hat{\sigma}_j^z) = 0 \) after \( t \approx 150 \text{ ns} \). In contrast, for the initial state \( \langle \pi/2, \pi/4 \rangle \) in the weak-thermalization regime, \( \overline{\rho(t)} \) strongly oscillates around the thermal value 0. It is noted that the anomalously persistent oscillation has also been observed in the Rydberg-atom quantum simulator [42]. This oscillation arises from the quantum many-body scars, that is a mechanism different from weak thermalization [43]. In addition, we measure the dynamics of \( \overline{\rho(t)} \) with the initial state \( \langle \pi/2, 8\pi/5 \rangle \), which also lies in the strong-thermalization regime, since its effective inverse temperature is \( J \beta \approx 0 \). The results, depicted in Fig. 2(b), show that even the behavior of short relaxation is different from that with the initial state \( \langle \pi, 0 \rangle \), the local observable also has a stationary value near the thermal value 0 after \( t \approx 150 \text{ ns} \), which is a hallmark of strong thermalization.

Next, we consider the von Neumann EE, \( S = -\text{Tr}(\hat{\rho}_j \ln(\hat{\rho}_j)) \), where \( \hat{\rho}_j \) is the reduced density matrix of the \( j \)th qubit. We average the EE over all qubit sites.
The dynamics of the EE, with the initial states $|\pi,0\rangle$ and $|\pi/2,8\pi/5\rangle$ in the regime of strong thermalization, are displayed in Figs. 2(d) and 2(e), respectively. We observe that for strong thermalization, the EE rapidly reaches the Page value $S_{\text{Page}} \approx 0.692$ as the maximum EE of a single-qubit subsystem of a total system in the random pure state [44]. However, for weak thermalization, the non-equilibrium dynamics gains the EE smaller than the Page value [Fig. 2(f)].

Furthermore, we study the time-averaged EE between 100 and 200 ns with different initial states $|\theta_0, \phi_0\rangle$. In Fig. 3(a), we show the experimental data of time-averaged EE with different initial states $|\theta_0, \phi_0\rangle$, which bears a close resemblance to the normalized energy in Fig. 1(e). Specifically, with $\theta_0 = \pi/2$, around $\phi_0 \approx 3\pi/2, \epsilon \approx 0.4$, and the DOS $\rho(e)$ becomes the maximum [see Figs. 1(d) and 1(e)]. Thus, it can be predicted that strong thermalization occurs in this regime. Additionally, according to the results in Figs. 1(d) and 1(e), the normalized energy is near 1 at $\phi_0 = \pi/2$, where the DOS is close to 0, and the degree of thermalization is the weakest. The experimental data of the time-averaged EE, with $\theta_0 = \pi/2$ and different $\phi_0$, are presented in Fig. 3(b).

There is a minimum of the EE around $\phi_0 = \pi/2$, corresponding to the weakest thermalization. Moreover, the maximum EE reveals a regime of strong thermalization with $\theta_0 = \pi/2$ and $\phi_0 \in [1.3\pi, 1.9\pi]$.

The trace distance between the nonequilibrium state $\hat{\rho}_i = |\Psi_i\rangle \langle \Psi_i|$ and the thermal state $\hat{\rho}_\beta$, with $\beta$ being the effective inverse temperature of the initial state, i.e., $1/\beta \approx \text{Tr}(\hat{\rho}_i - \hat{\rho}_\beta)$, can diagnose quantum thermalization [31]. It has been numerically shown that the distance monotonically decays to 0 for strong thermalization. With initial states in the weak-thermalization regime, the decay of the distance can also be observed but with a strong fluctuation [8].

We measure the reduced density matrix $\hat{\rho}_{i,j+1}$ of the subsystem consisting of the $i$th and $(j+1)$th qubit, using the quantum state tomography. For the initial state $|\pi,0\rangle$, the effective inverse temperature is $J\beta = 0$, and the corresponding two-qubit thermal state is $\hat{I}/4$ with $\hat{I}$ being an identity matrix. Then, the trace distance between $\hat{\rho}_{i,j+1}$ and $\hat{I}/4$, averaged over the qubit site $j$, can be directly obtained. Similarly, by considering the thermal states $\hat{\rho}_{\beta,j+1}$ with $J\beta \approx -1.034$, the dynamics of the trace distance, with the initial state $|\pi/2,\pi/4\rangle$, can also be measured. As shown in Figs. 4(a) and 4(b), the trace distance decays during the time evolution for both strong and weak thermalization, indicating the tendency $\hat{\rho}_{i,j} \approx \hat{\rho}_{\beta,j+1}$. However, for weak thermalization, the trace distance strongly oscillates [Fig. 4(b)].

Finally, we experimentally investigate the concurrence of the two-qubit reduced density matrix $\hat{\rho}_{i,j+1}$, which is
defined as $E(\rho) = \max\{0, \sqrt{\gamma_1} - \sqrt{\gamma_2} - \sqrt{\gamma_3} - \sqrt{\gamma_4}\}$, where $\gamma_1, ..., \gamma_4$ are eigenvalues listed in decreasing order of the matrix $\Gamma = \hat{\rho}(\hat{\sigma}^x \otimes \hat{\sigma}^x)\hat{\rho}^*(\hat{\sigma}^x \otimes \hat{\sigma}^x)$ [36]. The time evolution of the concurrence, with the initial state $|x, 0\rangle$ and $|\pi/2, \pi/4\rangle$, is presented in Figs. 4(c) and 4(d), respectively. We observe that the concurrence vanishes after $t \approx 50$ ns for strong thermalization. Whereas, the concurrence preserves a finite value with the initial state in the weak-thermalization regime, which can be interpreted as the thermal entanglement, i.e., the concurrence in thermal states $\hat{\rho}_\beta$ [37], according to $\hat{\rho}_\beta^{j+1} \approx \hat{\rho}_\beta^{j+1}$ as a result of the ergodic dynamics in the weak-thermalization regime. The numerics of the concurrence in thermal states $\hat{\rho}_\beta$ with different $\beta$ are presented in Supplemental Material [38].

In summary, we have provided clear experimental evidence for characterizing the regimes of strong and weak thermalization. Weak thermalization, with a slow growth of the EE, has the potential for generating states with long-lived coherence and stabilizing the phases of matter far from equilibrium, such as Floquet symmetry-protected topological phases [45], discrete time crystals [46], many-body localized phase [6,47,48], and dynamical paramagnetic and ferromagnetic phases [23,49]. Our work also indicates that the 1D array of superconducting qubits can be a promising platform for exploring the issues at the foundation of quantum thermodynamics.

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