Synthesis of the six-part spatial mechanism

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Abstract. The synthesis of flat and spatial mechanisms is of a great importance in the development of various devices, including robotics, automated lines, etc. These questions are set out in the works of Russian scientists (on the structure of the mechanisms, the synthesis) as well as in the foreign science. The purpose of this article is to synthesize a six-part spatial mechanism to reproduce the linear function of the slave link, which takes place at certain angles of rotation of the drive and slave links in devices with linear functions. The authors used the method of uniform best approximation of functions, graphical method, analytical theory of infinitesimal flat displacements of a rigid body, quadratic approximations, and analytical theory of a finite number of positions of a solid body.

1. Introduction

The synthesis of mechanisms is a section of the theory of mechanisms and machines, which examines the methods for designing a mechanism scheme for given kinematic and dynamic properties.

The synthesis of flat and spatial mechanisms is of a great importance in the development of various devices, including robotics, automated lines, etc. These questions are set out in the works of Russian scientists (on the structure of the mechanisms [1,2], the synthesis [3,4,5]) as well as in the foreign science [6,7,8]. Considering the synthesis, we used the analysis of the mechanisms according to P. L. Chebyshev (method of uniform best approximation of functions), graphical method, analytical theory of infinitesimal flat displacements of a rigid body, quadratic approximations, and analytical theory of a finite number of positions of a solid body. Along with this, the general questions of the theory of mechanisms are explained in the following sources [9,10,11,12].

The synthesis of spatial mechanisms with only rotational hinges has its own specifics. First, it is necessary to solve the problem of the mechanism existence, i.e. it is necessary to determine the linear and angular parameters of the links and the relationship between them, the turnability of the links. Then it’s time to solve one or another synthesis problem. Professor P. G. Mudrov made a great contribution to the synthesis of spatial mechanisms with rotational hinges [13].

It should be noted that the kinematics of the output link of the spatial mechanisms with rotational hinges depend primarily on the angular parameters of the links (the angles of crossing of the axes of the hinges of the links). From the linear parameters, the length of one of the links is set according to needs, based on the design of the hinge nodes, while the other links are determined by the structural formulas.
Additional information on spatial mechanisms can be obtained in [14-18].

The purpose of this article is to synthesize a six-part spatial mechanism to reproduce the linear function of the slave link, which takes place at certain angles of rotation of the drive and slave links in devices with linear functions.

2. The main part

The analysis of spatial mechanisms with rotational hinges showed that a mechanism with the following structural parameters [13] (Figure 1) can be used to reproduce a linear function on some part of the rotation of the slave link with its full turnability:

$$\begin{align*}
\alpha_2 &= 180^\circ - 2\alpha_6, \quad \ell_2 = 0, \quad \alpha_s = \alpha_s - \alpha_s, \quad \ell_s = \ell_s - \ell_s, \quad \alpha_4 = 180^\circ - \alpha_s,
\end{align*}$$

$$\begin{align*}
\ell_4 &= \ell_4, \quad \frac{\ell_4}{\sin \alpha_4} = \frac{\ell_4}{\sin \alpha_4} = \frac{\ell_4}{\sin \alpha_4}
\end{align*}$$

(1)

The angle of rotation of the slave link mechanism is

$$\begin{align*}
\psi_s &= \arcsin \left( \cos \alpha_s \cos \alpha_s \sin \phi \right) + \arcsin \left( \cos \alpha_s \cos \alpha_s \sin \phi \right) + \\
&+ \arcsin \left( \cos \alpha_s \cos \alpha_s \sin \phi \right)
\end{align*}$$

(2)

The graph of dependence of the rotation angle of the driven crank on the angle of rotation of the leading one is shown in Figure 2. As can be seen from the graph, the curve is symmetric with respect to point a, therefore, the segment of the line is cb=2ab, and angle is $\phi_p=2\phi_p(\psi_p=2\psi_p)$.

![Figure 1. Scheme of the mechanism.](image-url)
We know the angle $\phi_p$ ($\phi_p = 180^\circ$...300$^\circ$), and the relation of $\phi_p$ and $\psi_p$, i.e.
\[ K = \frac{\psi_p}{\phi_p} = 0,1...0,5. \]

It is necessary to synthesize the mechanism so that the maximum deviation $\Delta_{\text{max}}$ of the curve $\psi_5 = f(\phi_1)$ in the segment $c\theta$ does not exceed the specified value determined by the relative error.
\[ \Delta\psi^0 = \frac{\Delta_{\text{max}}}{\psi_p} 100% \leq \Delta\psi. \]

Considering the accuracy let’s assume $\Delta\psi = (0,2...0,5)\%$.

Additionally, it is necessary to create an equation relating the input and output parameters of the synthesis mechanism. The equation is derived by solving a vector equation (Figure 3),
\[ \overline{AB} = \overline{AF} + \overline{FE} + \overline{ED} + \overline{DC}. \]

Solving the vector equation in the projections on the $Y$ axis, substituting the value of unit vectors and taking into account that
\[
\begin{align*}
\phi_1 &= 180^\circ + \phi_r = 180^\circ + \phi_p/2, \\
\psi_1 &= 180^\circ + \psi_r = 180^\circ + K\phi_p/2, \\
\phi_4 &= \phi_r, \\
\psi_4 &= \psi_r, \\
\phi_3 &= \phi_r, \\
\psi_3 &= \psi_r, \\
\phi_2 &= \phi_r, \\
\psi_2 &= \psi_r, \\
\sin \gamma &= \frac{-(\cos \alpha_e - \cos \alpha_i) \sin \phi_1}{1 + \cos \alpha_e \cos \alpha_i - \sin \alpha_e \sin \alpha_i \cos \phi_1}, \\
\psi_0 &= \frac{(1 + \cos \alpha_e \cos \alpha_i \cos \phi_1 - \sin \alpha_e \sin \alpha_i)}{1 + \cos \alpha_e \cos \alpha_i - \sin \alpha_e \sin \alpha_i \cos \phi_1}.
\end{align*}
\]
and
\[ \ell_1 \cos \varphi_1 = \ell_2 + \ell_3 \cos \psi_2 - \ell_4 (\cos \varphi_1 \cos \psi_2 - \cos \alpha_2 \sin \varphi_1 \sin \psi_2) + \]
\[ + (\ell_3 - \ell_1) \left\{ \left[ \cos \alpha_2 \sin \varphi_1 \cos \gamma_3 - (\sin \alpha_2 \sin \varphi_1 - \cos \alpha_2 \cos \varphi_1) \sin \gamma_3 \right] \sin \psi_2 - \left[ -(\cos \varphi_1 \cos \gamma_3 - \cos \alpha_2 \sin \varphi_1 \sin \gamma_3) \cos \psi_2 \right] \right\}. \]  
(4)

The angle \( \alpha_i \) is determined
\[ \sin \alpha_i = \arcsin \left\{ \frac{1 - \cos \varphi_i}{2} \left( \cos \alpha_i \cos \alpha_i \sin \alpha_i + \sin \alpha_i \sin \alpha_i \cos \varphi_i \right) \right\} + \arcsin \left\{ \frac{1 + \cos \alpha_i \cos \alpha_i + \sin \alpha_i \sin \alpha_i \cos \varphi_i}{2} \right\} - K \varphi_i \]
\[ + \arcsin \left\{ \frac{1 + \cos \alpha_i \cos \alpha_i + \sin \alpha_i \sin \alpha_i \cos \varphi_i - K \varphi_i}{2} \right\}. \]
(5)

Maximum deviation \( \Delta_{\text{max}} \) of the curve will be written as
\[ \Delta_{\text{max}} = \arcsin \left\{ \frac{1 - \cos \alpha_i - \cos \alpha_i \sin \varphi_i}{2} \right\} + \arcsin \left\{ \frac{1 + \cos \alpha_i \cos \alpha_i + \sin \alpha_i \sin \alpha_i \cos \varphi_i}{2} \right\} - K \varphi_i \]
\[ + \arcsin \left\{ \frac{1 + \cos \alpha_i \cos \alpha_i + \sin \alpha_i \sin \alpha_i \cos \varphi_i - K \varphi_i}{2} \right\}. \]
(6)

The angle \( \varphi_h \) is found from the equality of the partial derivative \( \frac{d\psi}{d\varphi_1} \) to the slope \( K \), i.e.
\[ \frac{d\psi_2}{d\varphi_1} = \frac{\cos \alpha_5 - \cos \alpha_1}{1 - \cos \alpha_i \cos \alpha_i - \sin \alpha_i \sin \alpha_i \cos \varphi_i} + \frac{\cos \alpha_5 + \cos \alpha_1}{1 + \cos \alpha_i \cos \alpha_i - \sin \alpha_i \sin \alpha_i \cos \varphi_i} \]
\[ - \frac{\cos \alpha_6 - \cos \alpha_5}{1 + \cos \alpha_i \cos \alpha_i - \sin \alpha_i \sin \alpha_i \cos \varphi_i} = K, \]

where \( \varphi_h = 180^0 + \varphi_0 \).

The sequence of the synthesis is as follows:

We set the working section of the angle \( \varphi_0 \) of the driving crank, as well as the extreme upper values of the coefficient \( K \) and the angles \( \alpha_5 \) and \( \alpha_6 \). The angle \( \alpha_i \) is determined by the formula (5), with the check of the restriction \( \alpha_i \geq 3^0 \). Then we calculate the objective function \( \Delta \psi^0 \), which is compared with the lower value of \( \Delta \psi \), and the condition
\[ \Delta \psi^0 \leq \Delta \psi \text{ or } \Delta \psi^0 \geq 0. \]  
(7)

If this condition is not met, then the values of the angles \( \alpha_5 \) and \( \alpha_6 \) are sequentially reduced by a certain amount and the process is repeated until one of the conditions (7) is obtained.

If all values of \( K \), \( \alpha_5 \) and \( \alpha_6 \) are enumerated, and condition (7) is not met, then the value of \( \Delta \psi \) increases by a certain amount and the process is repeated again until the required condition (7) is fulfilled.

After determining \( K \), \( \alpha_1 \), \( \alpha_5 \) and \( \alpha_6 \), we find the other parameters of the mechanism (\( \alpha_2 \), \( \alpha_3 \), \( \alpha_4 \), linear parameters) using the above formulas (1).

The size of the working, straight-line section of the driven crank turn is determined by \( \psi_p = K \varphi_p \).

The beginning of new system’s \( \psi \varphi_0 \) coordinates (and point \( C \) – beginning of the straight-line section) \( \psi_50 = 1800 \), \( \varphi_{10} = 1800 \), \( \psi_{10} = 0.5 \psi \).

Example. We need to synthesize a mechanism, which reproduces linear function on the segment determined by the angle \( \varphi_p = 1800 \), with the accuracy \( \Delta \psi = 0.5 \% \) and \( K = 0.5 \ldots 0.1 \).

Let’s accept the values \( \alpha_6 = 750 \), \( \alpha_5 = 1100 \) and \( K = 0.25 \). Then we determine the angle \( \alpha_1 \) using (5)
3. Conclusion

The six-part spatial mechanism with rotational hinges with the specified parameters ensures the reproduction of a linear function with sufficiently high accuracy and can be recommended for practical use in engineering, for example, in robots or manipulators.

Since the angle φ should lie within 0°...90°, then the angle φ = 50.75° will be valid.

Determining angle φ after intermediate value conversions we can see φ equals:

\[ \phi_1 = \arccos \left( \frac{0.632664}{1.06} \right) = 50.75°, \]

\[ \phi_2 = \arccos \left( \frac{0.695432}{1.341016} \right) = 134.06°. \]

The obtained value of the relative error is greater than the required, \( \Delta \psi = 0.5\% \), i.e. \( \Delta \psi = 0.005 \).

Now, by expression (6), \( \Delta \psi = \frac{\Delta \alpha}{\alpha} \), it is determined as:

\[ \Delta \psi = \frac{0.225}{180} = 0.125 \%

\[ \Delta \psi = \frac{0.25}{180} = 0.139 \%

\[ \Delta \psi = \frac{0.410507}{75} = 0.547 \%

\[ \Delta \psi = \frac{0.75}{75} = 1.00\%

\[ \Delta \psi = \frac{0.91}{75} = 1.15\% \]

The relative error will be

\[ \Delta \psi = \frac{0.202909}{100} = 0.202909 \%

The relative error will be

\[ \Delta \psi = \frac{0.70148}{100} = 0.70148 \%

The relative error will be

\[ \Delta \psi = \frac{0.039388}{100} = 0.039388 \%

The relative error will be

\[ \Delta \psi = \frac{0.06863}{100} = 0.06863 \%

The relative error will be

\[ \Delta \psi = \frac{0.08063}{100} = 0.08063 \%

The relative error will be

\[ \Delta \psi = \frac{0.100063}{100} = 0.100063 \%

The relative error will be

\[ \Delta \psi = \frac{0.125}{100} = 0.125 \%

The relative error will be

\[ \Delta \psi = \frac{0.139}{100} = 0.139 \%

The relative error will be

\[ \Delta \psi = \frac{0.547}{100} = 0.547 \%

The relative error will be

\[ \Delta \psi = \frac{1.00}{100} = 1.00\%

The relative error will be

\[ \Delta \psi = \frac{1.15}{100} = 1.15\% \]

The obtained value of the relative error is greater than the required, \( \Delta \psi = 0.5\% \), i.e. \( \Delta \psi = 0.005 \).

Having accepted \( \psi = 108° \) and \( K = 0.225 \), we leave the angle \( \alpha = 75° \) the same.

The error corresponds to the specified.
The synthesis of spatial mechanisms with rotational joints is carried out in a certain sequence. First of all, the question of the conditions for turning the links of the mechanisms is solved by analyzing their average gear ratio. If it is zero, then the slave link will be the balance bar, if one - it will be a crank, if two, it will be the crank that makes two turns per master drive, etc.

The multi-functionality of the mechanisms also generates various problems of their synthesis that can be solved in one way or another.

In addition to reproducing a linear function, we can solve some other problems with the synthesis of mechanisms, for example: to ensure the movement of a slave link with a given variable law, characterized by a given degree of non-uniform rotation; synthesis of mechanisms for the given positions of the master and slave units; synthesis according to the elliptic law of motion of the slave link; ensuring the movement of an excavator shovel from the condition of undermining of soil; synthesis by axial displacement of the working shaft of the screw mixer from the middle position, etc.

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