Λ polarization from unpolarized quark fragmentation

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Abstract:
The longstanding problem of explaining the observed polarization of Λ hyperons inclusively produced in the high energy collisions of unpolarized hadrons is tackled by considering spin and $k_\perp$ dependent quark fragmentation functions. The data on Λ’s and Ā’s produced in $p-N$ processes are used to determine simple phenomenological expressions for these new “polarizing fragmentation functions”, which describe the experiments remarkably well.
1. Introduction

It is well known since a long time that Λ hyperons produced with $x_F \gtrsim 0.2$ and $p_T \gtrsim 1$ GeV/$c$ in the collision of two unpolarized hadrons, $AB \to \Lambda^\uparrow X$, are polarized perpendicularly to the production plane, as allowed by parity invariance; a huge amount of experimental information, for a wide energy range of the unpolarized beams, is available on such single spin asymmetries [1]:

$$P_\Lambda = \frac{d\sigma^{AB \to \Lambda^\uparrow X} - d\sigma^{AB \to \Lambda^\downarrow X}}{d\sigma^{AB \to \Lambda^\uparrow X} + d\sigma^{AB \to \Lambda^\downarrow X}}.$$  \hfill (1)

Similar effects have been observed for several other hyperons, but we shall consider here only Λ’s and ¯Λ’s.

Despite the wealth of data and the many years they have been known, no convincing theoretical explanation or understanding of the phenomenon exist [2, 3]. The perturbative QCD dynamics forbids any sizeable single spin asymmetry at the partonic level [4]; the polarization of hyperons resulting from the strong interaction of unpolarized hadrons must then originate from nonperturbative features, presumably in the hadronization process. A number of models attempting some understanding of the data in this perspective [2]-[9] only achieve partial explanations.

In the last years other single spin asymmetries observed in $p^\uparrow p \to \pi X$ reactions [10] have attracted a lot of theoretical activity [11]-[20]; a phenomenological description of such asymmetries appears now possible with the introduction of new distribution [21, 11, 14, 22] and/or fragmentation [12, 19, 23] functions which are spin and $k_{\perp}$ dependent; $k_{\perp}$ denotes either the transverse momentum of a quark inside a nucleon or of a hadron with respect to the fragmenting quark.

In particular the effect first discussed by Collins [12] – that is, the azimuthal angle dependence of the number of hadrons produced in the fragmentation of a transversely polarized quark – has been recently observed [24, 25]; were such results confirmed the role of these new fragmentation functions would be of great phenomenological importance.

We consider here an effect similar to that suggested by Collins, namely a spin and $k_{\perp}$ dependence in the fragmentation of an unpolarized quark into a polarized hadron: a function describing this mechanism was first introduced in Ref. [23] and denoted by $D_{1T}^{+}$. This function is introduced in a frame defined by two light-like four-vectors $n^+$ and $n^-$, satisfying $n^+ \cdot n^- = 1$, and by the plane transverse to them. The four-momentum $P$ of the outgoing hadron – a Λ hyperon in the present investigation – is in the $n^-$ direction (up to a mass term correction). The function $D_{1T}^{+}$ is then defined as (displayed in the $n^+ \cdot A = 0$ gauge)

$$\frac{e^{ij} k_{T_i} S_{T_j}}{M_h} D_{1T}^{+}(z, k_{\perp}) \equiv \sum_X \int \frac{d^2 y_T}{4 \pi} e^{ik_{\perp} y_T}$$

$$\times \Tr \langle 0|\psi(y)|P, S_T; X\rangle \langle P, S_T; X|\bar{\psi}(0)\gamma^-|0\rangle \bigg|_{y^- = 0},$$  \hfill (2)
where the final state depends on the transverse part \((S_T)\) of the spin vector \(S\) of the produced \(\Lambda\) only, \(i.e.\) one should interpret it as \(|P, S; T; X\rangle\equiv(|P, S = S_T; T; X\rangle-|P, S = -S_T; T; X\rangle)/2\), such that the contribution from unpolarized fragmentation cancels out. Furthermore, \(k_\perp = |k_\perp|\) is the modulus of the transverse momentum of the hadron in a frame where the fragmenting quark has no transverse momentum. More details on this type of definition of fragmentation (or decay) functions can be found in Refs. [26, 12, 23].

In the notations of Ref. [19] a similar function is defined as:

\[
\Delta^N D_{h^{\uparrow}/a}(z, k_\perp) \equiv \hat{D}_{h^{\uparrow}/a}(z, k_\perp) - \hat{D}_{h^{\downarrow}/a}(z, k_\perp) = \hat{D}_{h^{\uparrow}/a}(z, k_\perp) - \hat{D}_{h^{\uparrow}/a}(z, -k_\perp),
\]

and denotes the difference between the density numbers \(\hat{D}_{h^{\uparrow}/a}(z, k_\perp)\) and \(\hat{D}_{h^{\downarrow}/a}(z, k_\perp)\) of spin \(1/2\) hadrons \(h\), with longitudinal momentum fraction \(z\), transverse momentum \(k_\perp\) and transverse polarization \(\uparrow\) or \(\downarrow\), inside a jet originated by the fragmentation of an unpolarized parton \(a\). From the above definition it is clear that the \(k_\perp\) integral of the function vanishes.

The exact relation between \(D^{\uparrow}_{1T}\) and \(\Delta^N D_{h^{\uparrow}/a}\) is given by (notice that also \(D^{\uparrow}_{1T}\) should have labels \(h\) and \(a\) which are often omitted):

\[
\Delta^N D_{h^{\uparrow}/a}(z, k_\perp) = \frac{k_\perp}{z M_h} \sin \phi \ D^{\uparrow}_{1T}(z, k_\perp),
\]

where \(\phi\) is the angle between \(k_\perp\) and the transverse polarization vector of the hadron, which shows that the function \(\Delta^N D_{h^{\uparrow}/a}(z, k_\perp)\) vanishes in case the transverse momentum and transverse spin are parallel.

In the sequel we shall refer to \(\Delta^N D_{h^{\uparrow}/a}\) and \(D^{\uparrow}_{1T}\) as “polarizing fragmentation functions”.

In analogy to Collins’ suggestion for the fragmentation of a transversely polarized quark we write [12, 27]:

\[
\hat{D}_{h^{\uparrow}/q}(z, k_\perp) = \frac{1}{2} \hat{D}_{h/q}(z, k_\perp) + \frac{1}{2} \Delta^N D_{h^{\uparrow}/q}(z, k_\perp) \frac{\hat{P}_h \cdot (p_q \times k_\perp)}{|p_q \times k_\perp|}
\]

for an unpolarized quark with momentum \(p_q\) which fragments into a spin 1/2 hadron \(h\) with momentum \(p_h = z p_q + k_\perp\) and polarization vector along the \(\uparrow = \hat{P}_h\) direction; \(\hat{D}_{h/q}(z, k_\perp) = D_{h^{\uparrow}/q}(z, k_\perp) + D_{h^{\downarrow}/q}(z, k_\perp)\) is the \(k_\perp\) dependent unpolarized fragmentation function. Notice that \(\hat{P}_h \cdot (p_q \times k_\perp) = p_q \cdot (k_\perp \times \hat{P}_h) \sim \sin \phi\).

A QCD factorization theorem gives for the high energy and large \(p_T\) process \(AB \rightarrow \Lambda^+ X\), at leading twist with collinear parton configurations:

\[
d\sigma^{AB \rightarrow \Lambda^+ X} = \sum_{a,b,c,d} f_{a/A}(x_a) \otimes f_{b/B}(x_b) \otimes d\hat{\sigma}^{ab \rightarrow cd} \otimes D_{\Lambda^+/c}(z)
\]
and
\[ d\sigma^{AB\rightarrow\Lambda X} = \sum_{a,b,c,d} f_{a/A}(x_a) \otimes f_{b/B}(x_b) \otimes d\hat{\sigma}_{ab\rightarrow cd} \otimes D_{\Lambda/c}(z). \]  

(7)

Here and in the sequel we shall fix the scattering plane as the x-z plane, with hadron A moving along +\( \hat{z} \) and the detected \( \Lambda \) produced in the first x-z quadrant; the \( \uparrow, \downarrow \) directions are then respectively +\( \hat{y} \) and −\( \hat{y} \).

In the absence of intrinsic \( k_{\perp} \) (or rather when integrated over) the fragmentation functions \( D_{\Lambda/c}(z) = \int d^2 k_{\perp} \hat{D}_{\Lambda/q}(z, k_{\perp}) \) (or \( D_{\Lambda/c}(z) \)) cannot depend on the hadron polarization, so that one has \( d\hat{\sigma}^\uparrow = d\hat{\sigma}^\downarrow \), which implies \( P_\Lambda = 0 \).

However, by taking into account intrinsic \( k_{\perp} \) in the hadronization process, and assuming that the factorization theorem holds also when \( k_{\perp} \)'s are included \[12\], one has, using Eq. (1) instead of \( D_{\Lambda/c}(z) \) in Eqs. (4), (5) and (7):

\[ \frac{E_\Lambda d^3\sigma^{AB\rightarrow\Lambda X}}{d^3p_\Lambda} P_\Lambda = \sum_{a,b,c,d} \int \frac{dx_a dx_b dz}{\pi z^2} d^2 k_{\perp} f_{a/A}(x_a) f_{b/B}(x_b) \]
\[ \times \hat{s} \delta(\hat{z} + \hat{t} + \hat{u}) \frac{d\hat{\sigma}_{ab\rightarrow cd}}{dt}(x_a, x_b, k_{\perp}) \Delta^N D_{\Lambda/c}(z, k_{\perp}) \]  

(8)

where \( \hat{s}, \hat{t} \) and \( \hat{u} \) are the Mandelstam variables for the elementary process: for collinear configurations \( \hat{s} = x_a x_b s, \hat{t} = x_a t/z \) and \( \hat{u} = x_b u/z \) and the modifications due to intrinsic \( k_{\perp} \) will be taken into account in the numerical evaluations.

\[ \frac{E_\Lambda d^3\sigma^{AB\rightarrow\Lambda X}}{d^3p_\Lambda} = \sum_{a,b,c,d} \int \frac{dx_a dx_b dz}{\pi z^2} d^2 k_{\perp} f_{a/A}(x_a) f_{b/B}(x_b) \]
\[ \times \hat{s} \delta(\hat{z} + \hat{t} + \hat{u}) \frac{d\hat{\sigma}_{ab\rightarrow cd}}{dt}(x_a, x_b, k_{\perp}) \hat{D}_{\Lambda/c}(z, k_{\perp}) \]  

(9)

In Eq. (8) \( k_{\perp} \) is considered only where its absence would lead to zero polarization: that is, leading collinear configurations are assumed for partons \( a \) and \( b \) inside unpolarized hadrons \( A \) and \( B \), while transverse motion is considered in the fragmentation process. The final hadron, detected with momentum \( p_\Lambda \), is generated by the hadronization of a parton \( c \) whose momentum, \( p_c = (p_\Lambda - k_{\perp})/z \), varies with \( k_{\perp} \); also the corresponding elementary process, \( ab \rightarrow cd \), depends on \( k_{\perp} \).

\( P_\Lambda \) is a function of the hyperon momentum \( p_\Lambda = p_L + p_T \) and is normally measured in the \( AB \) c.m. frame as a function of \( x_F \equiv 2p_L/\sqrt{s} \) and \( p_T \).

Notice that, in principle, there might be another contribution to the polarization of a final hadron produced at large \( p_T \) in the high energy collision of two unpolarized hadrons; in analogy to Sivers’ effect \[11, 14\] one might introduce a new spin and \( k_{\perp} \) dependent distribution function:

\[ \Delta^N f_{a/\Lambda}(x_a, k_{\perp a}) \equiv \hat{f}_{a/\Lambda}(x_a, k_{\perp a}) - \hat{f}_{a/\Lambda}(x_a, -k_{\perp a}) \]
\[ = \hat{f}_{a/\Lambda}(x_a, k_{\perp a}) - \hat{f}_{a/\Lambda}(x_a, -k_{\perp a}) \]  

(10)
\[ \text{i.e. the difference between the density numbers } f_{a\uparrow}^{\uparrow}/A(x_a, k_{\perp a}) \text{ and } f_{a\downarrow}^{\downarrow}/A(x_a, k_{\perp a}) \text{ of partons } a, \text{ with longitudinal momentum fraction } x_a, \text{ transverse momentum } k_{\perp a} \text{ and transverse polarization } \uparrow \text{ or } \downarrow, \text{ inside an unpolarized hadron } A. \]

This idea was first applied to unpolarized lepto-production [22] and to single spin asymmetries in \( pp^{\uparrow} \) scattering [28]; the corresponding function, related to \( \Delta N f_{a\uparrow}/A(x_a, k_{\perp a}) \), was denoted by \( h_{\perp 1}^{\uparrow} \). In the present case of transversely polarized \( \Lambda \) production this function would enter the cross-section accompanied by the transversity fragmentation function \( H_{1}(z) \) (or \( \Delta D h_{a\uparrow}^{\uparrow}/a^{\uparrow} \)). We shall not consider this contribution here; not only because of the problems concerning \( \Delta N f_{a\uparrow}/A(x_a, k_{\perp a}) \) discussed below, but also because the experimental evidence that the hyperon polarization is somewhat independent of the nature of the hadronic target suggests that the mechanism responsible for the polarization is in the hadronization process\(^1\). A clean test of this should come from a measurement of \( P_{\Lambda} \) in unpolarized DIS processes, \( \ell p \to \Lambda^{\uparrow} X \) [30].

The \( k_{\perp} \) dependent functions considered in Refs. [11, 14, 19, 22, 12, 23] (\( \Delta N f_{a\uparrow}^{\uparrow}/A, \Delta N f_{a\uparrow}/A, \Delta N D_{h/a\uparrow}^{\uparrow} \) and \( \Delta N D_{h^{\uparrow}}/a \), or, respectively, \( f_{T}^{\uparrow}, h_{T}^{\uparrow}, H_{1}^{\uparrow} \) and \( D_{T}^{\uparrow} \)) have the common feature that the transverse momentum direction is correlated with the direction of the transverse spin of either a quark or a hadron, via a \( \sin \phi \) dependence, as can be seen from Eq. (2) for example. The reason for considering these functions is that this “handedness” of the transverse momentum compared to the transverse spin is displayed by the single spin asymmetry data in, for instance, \( pp^{\uparrow} \to \pi X \) and \( pp \to \Lambda^{\uparrow} X \). However, the problem of using such functions is that naively they appear to be absent due to time reversal invariance. This conclusion would be valid if the hadronic state appearing in the definition of such functions is treated as a plane wave state. One can then show that the functions are odd under the application of time reversal invariance, whereas hermiticity requires them to be even. If, however, initial or final state interactions are present, then time reversal symmetry will not prevent the appearance of nonzero (naively) T-odd functions. In the case of a state like \( |P_{h}, S_{h}; X\rangle \) final state interactions are certainly present and nonzero fragmentation functions \( \Delta N D_{h/a^{\uparrow}}^{\uparrow} \) and \( \Delta N D_{h^{\uparrow}}/a \) are expected. However, for distribution functions this issue poses severe problems and since we will only consider fragmentation functions here, we refer to Refs. [12, 14, 22] for more detailed discussions on this topic.

The main difference between the function \( \Delta N D_{h/a^{\uparrow}}^{\uparrow} \) as originally proposed by Collins, and the function under present investigation \( \Delta N D_{h^{\uparrow}}/a \), is that the former is a so-called chiral-odd function, which means that it couples quarks with left- and right-handed chiralities, whereas the latter function is chiral-even. Since the pQCD interactions conserve chirality, chiral-odd functions must always be accompanied by a mass term or appear in pairs. Both options restrict the accessibility of such functions and make them harder to be determined separately. On the other hand, the

\(^1\)This is not in contradiction with the observed spin transfer \( (D_{NN}) \) in \( pp^{\uparrow} \to \Lambda^{\uparrow} X \) [29], which in the factorized approach can be described in terms of the ordinary transversity distribution and fragmentation functions, rather than in terms of \( \Delta N D_{h^{\uparrow}}/a \).
chiral-even fragmentation function can simply occur accompanied by the unpolarized (chiral-even) distribution functions, which are the best determined quantities, allowing for a much cleaner extraction of the fragmentation function itself. Moreover, since chiral-even functions can appear in charged current mediated processes (as opposed to chiral-odd functions), more methods of extraction are available [31].

As it was studied in Ref. [32] the Collins function $H_1^\perp$ (or $\Delta^N D_{h/a^\perp}$) satisfies a sum rule arising from momentum conservation in the transverse directions. The same holds for the other $k_\perp$-odd, T-odd fragmentation function $D_1^\perp$ [33],

$$\sum_h \int dz \int d^2k_\perp \frac{k_\perp^2}{zM_h} D_{1T}^\perp(z,k_\perp) = 0,$$

(11)
or, in terms of the function $\Delta^N D_{h/a^\perp}$,

$$\sum_h C_h M_h \equiv \sum_h \int dz \int d^2k_\perp k_\perp \sin \phi \Delta^N D_{h^\perp/a}(z,k_\perp) = 0,$$

(12)

which is equivalent to Eq. (11) via Eq. (4).

Notice that the above sum rule can be written as

$$\sum_h \int dz \int d^2k_\perp \hat{D}_{h^\perp/a}(z,k_\perp) = 0,$$

(13)

and the same holds independently for $\hat{D}_{h^\perp/a}(z,k_\perp)$. Eq. (13) has a clear nontrivial physical meaning: for each polarization direction ($\uparrow$ or $\downarrow$) the total transverse momentum carried by spin 1/2 hadrons is zero.

The sum over hadrons prevents a straightforward application of the sum rule to the case of $\Lambda$ production alone. It can not be used as a constraint on the parameterization of the function to be fitted to the data. However, we note that the integral $C_h M_h$ for each hadron type $h$ is the same function of the energy scale (implicit in all expressions), apart from as yet unknown normalization. In this sense it closely resembles the tensor charge. In other words, the running of the functions are the same for any type of hadron and there is no mixing with other functions. The ratios for different types of hadrons are constants, which allow for checks of consistency between sets of data obtained at different energies, without the need for evolution. These constants are universal, if indeed the T-odd fragmentation functions are universal. This universality is the main point of interest here: one wants to see if $\Lambda$ polarization from unpolarized quark fragmentation is independent of the initial state, as it is implicit when writing down the factorized cross-sections Eqs. (8) and (9). At the present time, this can not be verified due to lack of data, but some predictions can be given [30] that will allow tests of this universality.

We only consider the quark fragmenting into a $\Lambda$ and not into other hyperons, like the $\Sigma^0$. The latter actually produces a significant amount (20-30%) of the $\Lambda$'s

\[2\text{Strictly speaking, the sum over } h \text{ is over all hadrons that carry transverse polarization, which might be true also for higher spin hadrons.}\]
via the decay $\Sigma^0 \to \Lambda \gamma$. The reason we do not introduce a separate $\Sigma^0$ fragmentation function at this stage is that the factorized approach by itself does not address such a separation (it is about a generic spin-1/2 hadron of type $h$), unless one introduces some additional input, like a model based on $SU(3)$, or unless one applies it to separate sets of data for each hyperon (which are not available yet). Apart from that, for each quark flavour such a $\Sigma^0$ fragmentation function would evolve in the same way as the $\Lambda$ fragmentation function, which implies that their relative fraction stays constant. In this way we can view the $\Lambda$ fragmentation function as an effective fragmentation function that includes the contamination due to $\Sigma^0$'s. Indeed, the fragmentation functions we shall use in next Section have been obtained from fits to inclusive $\Lambda$ productions in $e^+e^-$ processes, independently of their origin.

At a later stage one might make the distinction that the $\Sigma^0$ fragmentation is a different fraction of the effective $\Lambda$ fragmentation function for different quark flavours. One can insert all this information with hindsight and correct for it, but the present approach cannot be used to acquire this information unless the data would clearly distinguish the $\Lambda$'s coming from $\Sigma^0$'s. Our approach of using an effective $\Lambda$ fragmentation function would be more problematic if the $\Sigma^0$ would decay into other final states than $\Lambda \gamma$ (which branching ratio happens to be 100%): then only a part of the total $\Sigma^0$ fragmentation function would be included into the effective $\Lambda$ fragmentation function and this would be energy dependent.

In the case of longitudinally polarized $\Lambda$ production the $\Sigma^0 \to \Lambda \gamma$ background gives rise to a depolarizing effect of about 10% \cite{34}, but in the present situation of transversely polarized $\Lambda$ production this is not the case, since the photon does not carry away any of the transverse polarization and it hardly affects the definition of the plane compared to which the transverse polarization is measured. Therefore, the $\Sigma^0 \to \Lambda \gamma$ background does not produce a significant depolarizing effect for the transverse $\Lambda$ polarization and an effective $\Lambda$ polarizing fragmentation function can be used also.

We shall now consider both $\Lambda$ and $\bar{\Lambda}$ production and attempt a determination of the polarizing fragmentation functions $\Delta^N D_{\Lambda^\uparrow q}$ by comparing results for $P_\Lambda$ and $P_{\bar{\Lambda}}$ from Eqs. (8) and (9) with data \cite{35}-\cite{39}.

2. Numerical fits of data on $P_\Lambda$ from $pN \to \Lambda X$ processes

Eq. (8), for proton-nucleon processes, can be rewritten as:

$$
\frac{E_\Lambda d^3 \sigma^{pN \to \Lambda X}}{d^3 p_\Lambda} P_\Lambda = \sum_{a,b,c,d} \int d^2 k_\perp \left[ \int_{z_{\min}}^{1} dz \int_{x_{a_{\min}}}^{1} dx_a \frac{1}{\pi z} \frac{-\bar{x}_b^2 s}{t\Phi_t} \right] \left( \Delta^N D_{\Lambda^\uparrow c}(z, k_{\perp}) \right)
$$

which deserves several comments and some explanation of notations.

- In deriving Eq. (14) from Eq. (8) we have used the fact that $\Delta^N D_{\Lambda^\uparrow c}(z, k_{\perp})$,
Eq. (3), is an odd function of $k_{\perp}$; the $(+k_{\perp})$ integration region of $k_{\perp}$ runs over one half-plane of its components.

- The $x_b$ integration has been performed by exploiting the $\delta(\hat{s} + \hat{t} + \hat{u})$ function; the resulting value of $x_b$ is given by

$$\bar{x}_b = -\frac{x_a t \Phi_t}{x_a z s + u \Phi_u},$$

where $\Phi_t$ and $\Phi_u$ are defined below.

- $k_{\perp}$ could have any direction in the plane perpendicular to $p_c$; however, due to parity conservation in the hadronization process – Eq. (5) – the only $k_{\perp}$ component contributing to the polarizing fragmentation function is that perpendicular to $P_\Lambda$, i.e. the component lying in the production plane, the $x - z$ plane in our conventions. To simplify the kinematics we shall then consider only the leading contributions of $k_{\perp}$ vectors in the $x - z$ plane.

- $s$, $t$ and $u$ are the Mandelstam variables for the $pN \rightarrow \Lambda X$ process; in the simple planar configuration discussed above ($p_c$ and $k_{\perp}$ both lying in the $x - z$ production plane) they are related to the corresponding variables for the elementary processes by:

$$\hat{s} = 2 p_a \cdot p_b = x_a x_b s$$

$$\hat{t} = -2 p_a \cdot p_c = (x_a/z) t \Phi_t(\pm k_{\perp})$$

$$\hat{u} = -2 p_b \cdot p_c = (x_b/z) u \Phi_u(\pm k_{\perp})$$

with

$$t \Phi_t(\pm k_{\perp}) = g(k_{\perp}) \left\{ \frac{t + 2 k_{\perp} \sqrt{stu}}{t + u} - \left[ 1 - g(k_{\perp}) \right] \frac{t - u}{2} \right\}$$

$$u \Phi_u(\pm k_{\perp}) = g(k_{\perp}) \left\{ \frac{u + 2 k_{\perp} \sqrt{stu}}{t + u} + \left[ 1 - g(k_{\perp}) \right] \frac{t - u}{2} \right\}$$

where $g(k_{\perp}) = \sqrt{1 - (k_{\perp}/p_\Lambda)^2}$ and where $\pm k_{\perp}$ refers respectively to the configuration in which $k_{\perp}$ points to the left or to the right of $p_c$. At leading order in $k_{\perp}/p_\Lambda$ one has:

$$\Phi_t(k_{\perp}) = 1 - \frac{k_{\perp}}{p_\Lambda} \sqrt{\frac{u}{t}} \quad \Phi_u(k_{\perp}) = 1 + \frac{k_{\perp}}{p_\Lambda} \sqrt{\frac{t}{u}}.$$  

- The lower integration limits in Eq. (14) are given by:

$$x_{a\min} = -\frac{u \Phi_u(\pm k_{\perp})}{zs + t \Phi_t(\pm k_{\perp})},$$

$$z \geq -\frac{t \Phi_t(\pm k_{\perp}) + u \Phi_u(\pm k_{\perp})}{s}.$$
Notice that the integration limits are slightly different for $+k_\perp$ and $-k_\perp$; when replacing $k_\perp$ with $-k_\perp$ inside the square bracket of Eq. (14), one should not forget to change accordingly also the $z$ and $x_a$ integration limits, and the value of $\bar{x}_b$, Eq. (15), although the results are only marginally affected by this.

• Eq. (14) can be schematically written as

$$d\sigma^{pN\to\Lambda X} P_\Lambda = d\sigma^{pN\to\Lambda^+ X} - d\sigma^{pN\to\Lambda^- X} = \sum_{a,b,c,d} f_a/p(x_a) \otimes f_b/N(x_b) \otimes [d\hat{\sigma}^{ab\to cd}(x_a, x_b, k_\perp) - d\hat{\sigma}^{ab\to cd}(x_a, x_b, -k_\perp)] \otimes \Delta^N D_{\Lambda/c}(z, k_\perp)$$

(21)

which shows clearly that $P_\Lambda$ is a higher twist effect, despite the fact that the polarizing fragmentation function $\Delta^N D_{\Lambda^+/a}$ is a leading twist function: this is due to the difference in the square brackets, $[d\hat{\sigma}(+k_\perp) - d\hat{\sigma}(-k_\perp)] \sim k_\perp/p_T$, similarly to what happens for the single spin asymmetries in $p^+p\to\pi X$ [14, 19].

• In the computation of the unpolarized cross-section $E_\Lambda d^3\sigma^{pN\to\Lambda X}/d^3p_\Lambda$ intrinsic transverse motion is significant only in limited phase space regions: we have checked that the values obtained in most of the kinematical regions of available data do not vary whether or not taking into account $k_\perp$. Notice that when taking into account $k_\perp$ the expression for $E_\Lambda d^3\sigma^{pN\to\Lambda X}/d^3p_\Lambda$ is the same as Eq. (14) with the $-$ inside the square brackets replaced by a $+$ and $\Delta^N D_{\Lambda^+/c}(z, k_\perp)$ replaced by $D_{\Lambda^+/c}(z, k_\perp)$.

• When computing the cross-sections for scattering off nuclei, $pA\to\Lambda^+ X$, for which plenty of data are available, we have adopted the most simple incoherent sum, neglecting nuclear effects. That is, for the scattering off a nucleus with $A$ nucleons and $Z$ protons we use:

$$d\sigma^{pA\to\Lambda X} = Z \cdot d\sigma^{pp\to\Lambda X} + (A - Z) \cdot d\sigma^{pn\to\Lambda X}.$$  (22)

We have checked that different ways of taking into account nuclear effects leave results for $P_\Lambda$ – a ratio of cross-sections – almost unchanged. The partonic distribution functions in a neutron are obtained from the usual proton distribution functions by applying isospin invariance.

• Eq. (14) holds for any spin 1/2 baryon; we shall use it also for $\bar{\Lambda}$’s, using charge conjugation invariance to obtain $\bar{q} \to \bar{\Lambda}$ fragmentation properties from $q \to \Lambda$ ones, which implies $D_{\Lambda/q} = D_{\Lambda'/q}$ and $\Delta^N D_{\Lambda^+/q} = \Delta^N D_{\Lambda'^+/q}$.

We now use Eq. (14) in order to see whether or not it can reproduce the data and in order to obtain information on the new polarizing fragmentation functions. To do so we introduce a simple parameterization for these functions and fix the parameters by fitting the existing data on $P_\Lambda$ and $P_\bar{\Lambda}$ [37, 39].
We assume that $\Delta^N D_{\Lambda^+/c}(z, k_\perp)$ is strongly peaked around an average value $k_\perp^0$ lying in the production plane, so that we can expect:

$$
\int d^2k_\perp \Delta^N D_{\Lambda^+/c}(z, k_\perp) F(k_\perp) \simeq \Delta^N_0 D_{\Lambda^+/c}(z, k_\perp^0) F(k_\perp^0).$

(23)

Consistently, since in this case $F(k_\perp)$ depends weakly on $k_\perp$ when $k_\perp \ll p_T$, in the computation of the unpolarized cross-section we use:

$$
\int d^2k_\perp \hat{D}_{\Lambda/c}(z, k_\perp) F(k_\perp) \simeq \frac{1}{2} D_{\Lambda/c}(z) F(k_\perp^0).

(24)

The average $k_\perp^0$ value depends on $z$ and we parameterize this dependence in a most natural way:

$$
\frac{k_\perp^0(z)}{M} = K z^a (1 - z)^b,

(25)

where $M$ is a momentum scale ($M = 1$ GeV/c); in performing the fit we demand that $K$, $a$ and $b$ are constrained so that they satisfy the kinematical bound $p_\perp^2 = (p_\Lambda^2 - k_\perp^2)/z^2 \geq p_\Lambda^2$, from which $k_\perp^2 \leq (1 - z^2) p_\Lambda^2$ and

$$
k_\perp^0(z) \leq (p_\Lambda)_{\min} \sqrt{1 - z} \simeq (1 \text{ GeV}/c) \sqrt{1 - z},

(26)

which implies $a \geq 0$ and $b \leq 0.5$. The values of $K$, $a$ and $b$ resulting from our best fit will have a clear physical meaning.

We parameterize $\Delta^N_0 D_{\Lambda^+/c}(z, k_\perp^0)$ in a similar simple form: we know that it has to be zero when $k_\perp = 0$ and $z = 1$; in addition, the positivity of the fragmentation functions – Eq. (3) – requires, at any $k_\perp$ value, $|\Delta^N D_{h/q}(z, k_\perp)| \leq D_{h/q}(z, k_\perp)$. However, according to Eqs. (23), (24) and to take into account the mentioned [see comment after Eq. (24)] difference of the integration regions $(+k_\perp)$ and $(-k_\perp)$, which is significant at the boundaries of the kinematical ranges (when $p_T \simeq k_\perp$ and when $p_T \simeq p_{T_{\max}}$) we prefer to impose the more stringent bound $|\Delta^N_0 D_{\Lambda^+/c}(z, k_\perp^0)| \leq D_{\Lambda/c}(z)/2$. Following Ref. [40], this is automatically satisfied by taking:

$$
\Delta^N_0 D_{\Lambda^+/q}(z, k_\perp^0) = N'_q \frac{k_\perp^0(z)}{M} \left[ z^\alpha_q (1 - z)^\beta_q \frac{c_q d_q (c_q + d_q) c_q + d_q}{c_q d_q (c_q + d_q) c_q + d_q} D_{\Lambda/q}(z) \right] \frac{2}{2}

= N'_q K \frac{z^\alpha_q (1 - z)^d_q}{c_q d_q (c_q + d_q) c_q + d_q} \frac{D_{\Lambda/q}(z)}{2}

\equiv N_q z^\alpha_q (1 - z)^d_q \frac{D_{\Lambda/q}(z)}{2},

(27)

where we have used Eq. (25), and we require $c_q = a + \alpha_q > 0$, $d_q = b + \beta_q > 0$, and $|N'_q| K \leq 1$.

We are now almost fully equipped to compute $P_\Lambda$ and $P_\Lambda$; let us briefly discuss the remaining quantities appearing in Eq. (14).
- We sum over all possible elementary interactions, computed at lowest order in pQCD. The polarizing fragmentation functions – parameterized as in Eq. (27) – are supposed to be non vanishing only for Λ valence quarks, u, d and s; similarly for Λ valence antiquarks ¯u, ¯d and ¯s. All contributions to the unpolarized fragmentation functions – from quarks, antiquarks and gluons – are taken into account.

- We adopt the unpolarized distribution functions of MRST [41]. We have explicitly checked that a different choice makes no difference in our conclusions. We fix the QCD hard scale of distribution (and fragmentation) functions at 2 (GeV/c)^2, corresponding to an average value p_T ≃ 1.5 GeV/c. Since the range of p_T values of the data is rather limited, no evolution effect would be visible anyway.

- We use the set of unpolarized fragmentation functions of Ref. [42], which allows a separate determination of D_Λ/q and D_¯Λ/q, and which includes Λ’s both directly and indirectly produced; it also differentiates between the s quark contribution and the u and d ones: the non strange fragmentation functions D_Λ/u = D_Λ/d are suppressed by an SU(3) symmetry breaking factor λ = 0.07 as compared to D_Λ/s. In our parameterization of Δ_N D_Λ/q(z, k_T^2), Eq. (27), we use the same D_Λ/q as given in Ref. [42], keeping the same parameters α_q and β_q (c_q and d_q) for all quark flavours, but allowing for different values of N_u = N_d and N_s.

We can now use pQCD dynamics, together with the chosen distribution and fragmentation functions, and the parameterized expressions of the polarizing fragmentation functions, in Eq. (14) to compute P_Λ and P_¯Λ; by comparing with data we obtain the best fit values of the parameters K, a, b, N_u = N_d, N_s, c_q and d_q introduced in Eqs. (25) and (27). Notice that we remain with 7 free parameters, c_q and d_q being the same for all flavours.

Our best fit results (χ^2/d.o.f. = 1.57), taking into account all data [35]-[39], are shown in Figs. 1-5. They correspond to the best fit parameter values:

\begin{equation}
K = 0.69 \quad a = 0.36 \quad b = 0.53 \quad (28)
\end{equation}

\begin{equation}
N_u = -4.30 \quad N_s = 1.13 \quad c_q = 6.58 \quad d_q = 0.67. \quad (29)
\end{equation}

Let us comment in some details on our results.

In Fig. 1 and 2 we present our best fits to P_Λ as a function of p_T for different x_F values, as indicated in the figures: the famous approximately flat p_T dependence, for p_T greater than 1 GeV/c, is well reproduced. Such a behaviour, as expected, does not continue indefinitely with p_T and we have explicitly checked that at larger values of p_T the values of P_Λ drop to zero: the shape of such a decrease, contrary to what happens in the p_T region of the data shown here, strongly depends on the assumptions about the nuclear corrections. It may be interesting to note that this
fall-off has not yet been observed experimentally, but is expected to be first seen in the near-future BNL-RHIC data.

Also the increase of $|P_\Lambda|$ with $x_F$ at fixed $p_T$ values can be well described, as shown in Fig. 3; the two curves correspond to $p_T = 1.5$ GeV/$c$ (solid) and $p_T = 3$ GeV/$c$ (dashed-dotted).

The best fits of Figs. 1 and 2 are compared to $p$–$Be$ data [30]-[33]; these are collected at two different energies, in the $p$–$Be$ c.m. frame, $\sqrt{s} \approx 82$ GeV [30]-[32] and $\sqrt{s} \approx 116$ GeV [33]. Our calculations are performed at $\sqrt{s} = 80$ GeV; we have explicitly checked that by varying the energy between 80 and 120 GeV, our results for $P_\Lambda$ vary, in the kinematical range considered here, at most by 10%, in agreement with the observed energy independence of the data.

Some data from $p$–$p$ collisions are also available; in Ref. [35] a linear fit to $P_\Lambda(x_F)$ is performed, collecting all data with $p_T \geq 0.96$ GeV/$c$, for an average value $\langle p_T \rangle = 1.1$ GeV/$c$. In Fig. 4 we show these data and the linear fit (central line); the upper and lower lines show the fit error band. The solid line is our computation, at $p_T = 1.1$ GeV/$c$, with all parameters fixed as in Eqs. (28) and (29): our fit reproduces the data with good accuracy.

In Fig. 5 we show our best fit results for $P_\Lambda$ as a function of $p_T$ for different $x_F$ values, as indicated in the figure: in this case all data [34, 38] are compatible with zero, with large errors, and the measured $x_F$ range is not as wide as for $P_\Lambda$, as expected from the lack of overlapping between $\bar{\Lambda}$ and nucleon valence quarks.

The resulting values of the parameters, Eqs. (28) and (29), are very realistic; notice in particular that $b$ essentially reaches its kinematical limit 0.5 and the whole function (25) giving the average $k_\perp$ value of a $\Lambda$ inside a jet turns out to be very reasonable.

Mostly $u$ and $d$ quarks contribute to $P_\Lambda$, resulting in a negative value of $N_u$; instead, $u$, $d$ and $s$ quarks all contribute significantly to $P_\Lambda$ and opposite signs for $N_u$ and $N_s$ are found, inducing cancellations.

In Fig. 6 we plot $|\Delta^N D_{\Lambda^+/u,d}|$ and $\Delta^N D_{\Lambda^+/s}$ as given by Eq. (27) with the best fit parameters (28) and (29). We show, for comparison, also $D_{\Lambda/u,d}$ and $D_{\Lambda/s}$: notice how a tiny value of the polarizing fragmentation functions, in a limited large $z$ region, is enough to allow a good description of the data. This also shows that taking into account only valence quark contributions to $\Delta^N D_{\Lambda^+/q}$ as we have done – is a justified assumption.

A different set of unpolarized fragmentation functions can be found in the literature [43]: it holds for the quark fragmentation into $\Lambda + \bar{\Lambda}$ and gives $D_{(\Lambda + \bar{\Lambda})/q}$ rather than a separate expression of $D_{\Lambda/q}$ and $D_{\bar{\Lambda}/q}$; it would be appropriate to compute the $\Lambda + \bar{\Lambda}$ single spin asymmetry:

$$P_{\Lambda + \bar{\Lambda}} = \frac{d\sigma^{\Lambda^+} + d\sigma^{\bar{\Lambda}^+} - d\sigma^{\Lambda^-} - d\sigma^{\bar{\Lambda}^-}}{d\sigma^{\Lambda^+} + d\sigma^{\bar{\Lambda}^+} + d\sigma^{\Lambda^-} + d\sigma^{\bar{\Lambda}^-}} = \left( P_{\Lambda} + \frac{d\sigma^{\bar{\Lambda}}}{d\sigma^{\Lambda}} P_{\bar{\Lambda}} \right) \left( 1 + \frac{d\sigma^{\bar{\Lambda}}}{d\sigma^{\Lambda}} \right)^{-1}. \quad (30)$$
However, since one knows from experiments on $p - N$ reactions that in the kinematical range of interest $d\sigma^\Lambda \ll d\sigma^A$ (and this is confirmed in our scheme, simply due to the dominance of $q$ over $\bar{q}$ in a nucleon), one can safely assume

$$P_{\Lambda + \Lambda} \simeq P_{\Lambda} \cdot$$

(31)

This set – differently from the one of Ref. [42] – assumes $SU(3)$ symmetry and takes $D_{\Lambda/u} = D_{\Lambda/d} = D_{\Lambda/s}$.

We have also computed $P_\Lambda$ with this second set of fragmentation functions; as in the previous case we have parameterized $\Delta^N D_{\Lambda^+/q}$ according to Eq. (27), with the same values of $c_q$ and $d_q$ for each flavour, but different values of $N_{u,d}$ and $N_s$. We can equally well ($\chi^2$/d.o.f. = 1.85) fit the data on $P_\Lambda$, obtaining fits almost indistinguishable from those of Figs. 1-4, with the best fit values:

$$K = 0.66 \quad a = 0.37 \quad b = 0.50 \quad (32)$$

$$N_u = -28.13 \quad N_s = 57.53 \quad c_q = 11.64 \quad d_q = 1.23 \quad (33)$$

Notice that, also in this case of $SU(3)$ symmetric fragmentation functions $D_{\Lambda/q}$, and using only data on $P_\Lambda$, one reaches similar conclusions about the polarizing fragmentation functions $\Delta^N D_{\Lambda^+/q}$: $N_{u,d} \neq N_s$ and not only is there a difference in magnitude, but once more one finds negative values for $\Delta^N_0 D_{\Lambda^+/u,d}$ and positive ones for $\Delta^N_0 D_{\Lambda^+/s}$. This seems to be a well established general trend. Plots analogous to those of Fig. 6, would show also in this case, Eqs. (27) and (33), $\Delta^N_0 D_{\Lambda^+/u,d} > |\Delta^N_0 D_{\Lambda^+/u,d}|$.

Very recently, new sets of quark and antiquark fragmentation functions into a $\Lambda$, based on a bag model calculation and a fit to $e^+e^-$ data, have been published [44]. Both a $SU(3)$ flavour symmetric and a $SU(3)$ symmetry breaking set are available, although at a rather too low energy scale ($\mu = 0.25$ GeV). Nevertheless, we have also tried using these sets in our scheme to fit the data on $P_\Lambda$ and $P_{\bar{\Lambda}}$: once more we can fit the data, better with the symmetric than the asymmetric set, and, again, negative $\Delta^N_0 D_{\Lambda^+/u,d}$ and positive $\Delta^N_0 D_{\Lambda^+/s}$ are found, with $\Delta^N_0 D_{\Lambda^+/s} > |\Delta^N_0 D_{\Lambda^+/u,d}|$.

3. Conclusions

A phenomenological approach has been developed towards a consistent explanation and predictions of transverse single spin effects in processes with inclusively produced hadrons; we work in a kinematical region where pQCD and the factorization scheme can be used, but $p_T$ is not much larger than intrinsic $k_\perp$, so that higher twist contributions are still important. This applies to several processes for which data are available, like $p^1p \rightarrow \pi X$ [16, 19] and $pN \rightarrow \Lambda^+X$. The single spin effect originates in the fragmentation process, either of a transversely polarized quark into an unpolarized hadron – Collins’ effect [12] – or of an unpolarized quark into a transversely polarized hadron – the polarizing fragmentation functions [23]. Single spin effects in quark distribution functions [14] have also been discussed [14, 17].
We have considered here the well known and longstanding problem of the polarization of $\Lambda$ hyperons, produced at large $p_T$ in the collision of two unpolarized hadrons. We have assumed a generalized factorization scheme – with the inclusion of intrinsic transverse motion – with pQCD dynamics; the new, spin and $k_\perp$ dependent, polarizing fragmentation functions $\Delta^N D_{\Lambda^+/q}$ have been parameterized in a simple way and data on $pBe \rightarrow \Lambda^+X$, $pBe \rightarrow \bar{\Lambda}^+X$ and $pp \rightarrow \Lambda^+X$ have been used to determine the values of the parameters which give a best fit to the experimental measurements.

The data can be described with remarkable accuracy in all their features: the large negative values of the $\Lambda$ polarization, the increase of its magnitude with $x_F$, the puzzling flat $p_T \gtrsim 1$ GeV/$c$ dependence and the $\sqrt{s}$ independence; data from $p-p$ processes are in agreement with data from $p-Be$ interactions and also the tiny or zero values of $\bar{\Lambda}$ polarization are well reproduced. The resulting functions $\Delta^N D_{\Lambda^+/q}$ are very reasonable and realistic.

Different sets of unpolarized fragmentation functions – either $SU(3)$ symmetric or not – lead to very similar conclusions about these new polarizing fragmentation functions describing the hadronization process of an unpolarized quark into a polarized $\Lambda$: they have opposite signs for $u$ and $d$ quarks as compared with $s$ quarks and their magnitudes are larger for $s$ quarks. They are sizeable – with respect to the unpolarized fragmentation functions – only in limited $z$ regions, yet they can describe remarkably well the experiments.

These polarizing fragmentation functions have a partonic interpretation and a formal definition, Eq. (4); they are free from the ambiguities related to initial state interactions which might affect analogous distribution functions and we expect them to be universal, process independent functions. Our parameterization of $\Delta^N D_{\Lambda^+/q}$ should allow us to give prediction for $\Lambda$ polarization in other processes; a study of $\ell p \rightarrow \Lambda^+X$ and $e^+e^- \rightarrow \Lambda^+X$ is in progress [30].

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References

[1] For a review of data see, e.g., K. Heller, in Proceedings of Spin 96, C.W. de Jager, T.J. Ketel and P. Mulders, Eds., World Scientific (1997); or A.D. Panagiotou, Int. J. Mod. Phys. A5 (1990) 1197.

[2] For a recent and complete review of all theoretical models see J. Félix, Mod. Phys. Lett. A14 (1999) 827

[3] J. Soffer, e-Print Archive: hep-ph/9911373

[4] W.G.D. Dharmaratna and G.R. Goldstein, Phys. Rev. D53 (1996) 1073; Phys. Rev. D41 (1990) 1731

[5] J. Soffer and N.E. Törnqvist, Phys. Rev. Lett. 68 (1992) 907

[6] Y. Hama and T. Kodama, Phys. Rev. D48 (1993) 3116

[7] R. Barni, G. Preparata and P. Ratcliffe, Phys. Lett. B296 (1992) 251

[8] S.M. Troshin and N.E. Tyurin, Phys. Rev. D55 (1997) 1265

[9] L. Zuo-Tang and C. Boros, Phys. Rev. Lett. 79 (1997) 3608

[10] D.L. Adams et al., Phys. Lett. B264 (1991) 462; Phys. Rev. Lett. 77 (1996) 2626

[11] D. Sivers, Phys. Rev. D41 (1990) 83; D43 (1991) 261

[12] J.C. Collins, Nucl. Phys. B396 (1993) 161

[13] C. Boros, Liang Zuo-tang, and Meng Ta-chung, Phys. Rev. Lett. 67, 1751 (1993)

[14] M. Anselmino, M. Boglione and F. Murgia, Phys. Lett. B362 (1995) 164

[15] A.V. Efremov, V.M. Korotkiyan, and O. Teryaev, Phys. Lett. B348 (1995) 577

[16] X. Artru, J. Czyzewski and H. Yabuki, Z. Phys. C73 (1997) 527

[17] M. Anselmino and F. Murgia, Phys. Lett. B442 (1998) 470

[18] J.W. Qiu and G. Sterman, Phys. Rev. D59 (1999) 014004

[19] M. Anselmino, M. Boglione and F. Murgia, Phys. Rev. D60 (1999) 054027

[20] K. Suzuki, N. Nakajima, H. Toki and K.-I. Kubo, Mod. Phys. Lett. A14 (1999) 1403

[21] J.P. Ralston and D.E. Soper, Nucl. Phys. B152 (1979) 109

[22] D. Boer and P.J. Mulders, Phys. Rev. D57 (1998) 5780

[23] P.J. Mulders and R.D. Tangerman, Nucl. Phys. B461 (1996) 197; Nucl. Phys. B484 (1997) 538 (E)

[24] HERMES Collaboration (A. Airapetian et al.), Phys. Rev. Lett. 84 (2000) 4047

[25] A. Bravar (on behalf of the SMC collaboration), Nucl. Phys. B79 (Proc. Suppl.) (1999) 520

[26] J.C. Collins and D.E. Soper, Nucl. Phys. B194 (1982) 445

[27] M. Anselmino and F. Murgia, Phys. Lett. B483 (2000) 74

[28] D. Boer, Phys. Rev. D60 (1999) 014012

[29] E704 Collaboration (A. Bravar et al.), Phys. Rev. Lett. 78 (1997) 4003.

[30] M. Anselmino, D. Boer, U. D’Alesio and F. Murgia, in preparation

[31] D. Boer, R. Jakob and P.J. Mulders, Nucl. Phys. B564 (2000) 471

[32] A. Schäfer and O.V. Teryaev, Phys. Rev. D61 (2000) 077903

[33] D. Boer, e-Print Archive: hep-ph/9912311
[34] M. Burkardt and R.L. Jaffe, *Phys. Rev. Lett.* **70** (1993) 2537
[35] A.M. Smith et al., R608 Collaboration, *Phys. Lett.* **B185** (1987) 209
[36] K. Heller *et al.*, *Phys. Rev. Lett.* **41** (1978) 607
[37] K. Heller *et al.*, *Phys. Rev. Lett.* **51** (1983) 2025
[38] B. Lundberg *et al.*, *Phys. Rev.* **D40** (1989) 3557
[39] E.J. Ramberg *et al.*, *Phys. Lett.* **B338** (1994) 403
[40] M. Boglione and E. Leader, *Phys. Rev.* **D61** (2000) 114001
[41] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *Eur. Phys. J.* **C14** (2000) 133
[42] D. Indumathi, H.S. Mani and A. Rastogi, *Phys. Rev.* **D58** (1998) 094014
[43] D. de Florian, M. Stratmann and W. Vogelsang, *Phys. Rev.* **D57** (1998) 5811
[44] C. Boros, J.T. Londergan and A.W. Thomas, *Phys. Rev.* **D62** (2000) 014021
**Fig. 1:** Our best fit to $P_\Lambda$ data from $p$–$Be$ reactions, as a function of $p_T$ and for different $x_F$ bins, as indicated in the figure. Only some of the bins are shown; see Fig. 2 for complementary bins. The experimental results, [37]–[39], are collected at two different c.m. energies, $\sqrt{s} \simeq 82$ GeV and $\sqrt{s} \simeq 116$ GeV. For each $x_F$–bin, the corresponding theoretical curve is evaluated at the mean $x_F$ value in the bin, and at $\sqrt{s} = 80$ GeV; the results change very little with the energy. See the text for further details.
Fig. 2: Our best fit to $P_{\Lambda}$ data from $p$–$Be$ reactions, as a function of $p_T$ and for different $x_F$ bins, as indicated in the figure. Only some of the bins are shown; see Fig. 1 for complementary bins. The experimental results,[37]-[39], are collected at two different c.m. energies, $\sqrt{s} \simeq 82$ GeV and $\sqrt{s} \simeq 116$ GeV. For each $x_F$-bin, the corresponding theoretical curve is evaluated at the mean $x_F$ value in the bin, and at $\sqrt{s} = 80$ GeV; the results change very little with the energy. See the text for further details.
Fig. 3: $P_\Lambda$ data for $p$–Be reactions, as a function of $x_F$ and for different $p_T$ bins, as indicated in the figure. The data are collected at two different c.m. energies, $\sqrt{s} \simeq 82$ GeV and $\sqrt{s} \simeq 116$ GeV. The two theoretical curves, evaluated at $\sqrt{s} = 80$ GeV, correspond to $p_T = 1.5$ GeV/c (solid) and $p_T = 3$ GeV/c (dot-dashed).
Fig. 4: Experimental results for $P_\Lambda$ in $p-p$ reactions, as a function of $x_F$, from Ref. [36]. All data with $p_T \geq 0.96$ GeV/$c$ are collected, and $\langle p_T \rangle = 1.1$ GeV/$c$. Also shown is a linear fit to the data, taken from Ref. [36] (central line); the upper and lower dot-dashed lines show the corresponding fit error band. The solid curve shows the theoretical computation, at $p_T = 1.1$ GeV/$c$, with all parameters fixed as in Eqs. (28) and (29).
Fig. 5: Our best fit to $P_{\Lambda}$ data from $p$–$Be$ reactions, as a function of $p_T$ and for different $x_F$ bins, as indicated in the figure. The experimental results [36, 38] are collected at the c.m. energies $\sqrt{s} \simeq 82$ GeV. For each $x_F$-bin, the corresponding theoretical curve is evaluated at the mean $x_F$ value in the bin, and at $\sqrt{s} = 80$ GeV; the results change very little with the energy.
Fig. 6: Plot of $|\Delta_0^N D_{\Lambda/u}|$ (= $|\Delta_0^N D_{\Lambda/d}|$) and $\Delta_0^N D_{\Lambda/s}$, as given by Eq. (27) with the best fit parameters (28) and (29). For comparison we also show the unpolarized fragmentation functions $D_{\Lambda/u}$ (= $D_{\Lambda/d}$) and $D_{\Lambda/s}$ [42].