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Numerical simulation and analysis of five-stage lifting pump by improved turbulence model and catastrophe theory

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ABSTRACT
How to theoretically describe the process of turbulence formation remains an unsolved problem. A new method is presented in this paper for quantitatively researching the turbulence phase transition based on a dimensionless method and catastrophe theory. Then, the formula of energy spectrum density $E_k$, with the wave number and time-scale index strictly derived from the folding catastrophe model, can quantitatively explain the energy in the turbulent system, inherited from large eddies and transferred to small eddies until viscous dissipation occurs, theoretically and physically. As an example, solid–liquid two-phase flow in a five-stage lifting pump for a 6000 m deep-sea mining system is analyzed based on a structured mesh by numerical simulation, in which this turbulence model is validated. This method provides equations that can give a quantitative analysis with a physical explanation for turbulence formation, which not only provides a new insight into the turbulence, but also can be applied to other complex phase transitions.

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Turbulence; catastrophe theory; lifting pump; deep-sea mining; complex phase transitions

1. Introduction
Turbulence is a common phenomenon (Hu, Li, Han, Cai, & Xu, 2016; Omri, Galanis, & Abid, 2008; Tang, Guo, & Ranjan, 2015), occurring in clouds in the atmosphere, rapids in rivers, thick smoke over thermal power plants, blood flowing in arteries, and so on. The energy in a turbulent system is inherited from the large vortices and transferred to the small ones until viscous dissipation occurs. However, it remains unclear how this works, specifically with regard to time and scale.

In previous turbulence research, two notable scientists, Reynolds and Kolmogorov, studied turbulence in different directions, and their results are still of great research value.

Reynolds focused on fluid mechanics, and achieved remarkable results in the study of turbulence from the mechanical and mathematical perspectives (Frisch, 1995; Reynolds, 1883). The Navier–Stokes equation of Reynolds average viscous fluid has been derived, but its general solution has not been solved yet. The conventional Reynolds-averaged Navier–Stokes (RANS) method can solve general flow problems. By performing transient RANS computations, some improvements can be achieved. Unlike the RANS method, the large eddy simulation (LES) method is able to capture the spectrum dynamics of large vortices, which are responsible for the momentum transfer, the associated scalar fields, and turbulence. To overcome the disadvantages of LES, such as its high cost and long time consumption, a hybrid LES/RANS method and a new unsteady RANS method have been developed to solve the problem of flow field resolution. The subsequently developed subgrid scale model has further advantages.

Kolmogorov developed a statistical method based on ideal isotropic turbulence (Frisch, 1995; Kolmogorov, 1941). He derived the scale law and its spectral form by using probabilistic methods. Kolmogorov’s $-5/3$ theory ($E_k = C_k k^{5/3}$) is regarded as the most famous result for turbulent research, but is suitable only for a narrow wave-number slot under the inertial subrange.

With progress in science and technology, the improvement of detection methods (Akbarian et al., 2018; Benferhat, Yahiaoui, Imine, Ladjeedel, & Sikula, 2019; Ramezanizadeh, Nazari, Ahmadi, & Chau, 2019) and the emergence of new measuring instruments, especially
the rapid development of computer science (Ghalandari, Koohshahi, Mohamadian, Shannshirband, & Chau, 2019; Mosavi, Shamshirband, Salwana, Chau, & Tah, 2019), research on fluid flow has also made satisfactory progress. However, our understanding of the nature of turbulence is still based on experiments and observations; that is, on experience. Only a few turbulence predictions are derived from theory.

The turbulence formation process includes laminar flow, turbulence development stage (laminar flow–turbulence transition range for low Reynolds number), and fully developed turbulence stage (dynamic statistical equilibrium). Turbulence can be considered as a typical non-equilibrium phase-transition process. Laminar flow is an equilibrium state (stable phase) in which viscous flow dominates and flow is regular. When the velocity increases and the Reynolds number reaches a certain physical quantity, the flow of fluid begins to be chaotic and vortices occur. Eventually, the system will tend to a state of equilibrium – fully developed turbulence (stable phase), with the smallest scale of vortices. In a phase-change system, if the system is to maintain a stable state, the physical quantity input at a certain position within the system must be equal to the physical quantity dissipated at the same time. That is, the absorption and dissipation in the micro-scale of the system cancel each other at the same time. That is, the absorption and dissipation in the micro-scale of the system cancel each other. That is, the absorption and dissipation in the micro-scale of the system cancel each other. That is, the absorption and dissipation in the micro-scale of the system cancel each other.

According to Equation (1), the equilibrium curve (i.e. non-degenerate critical points set) and the singularity set of the folding catastrophe model are as follows:

\[ V(x) = x^3 + tx \]  
\[ V(x) = x^4 + tx^2 + ux \]  
\[ V(x) = x^5 + tx^3 + ux^2 + vx \]  
\[ V(x) = x^6 + tx^5 + ux^3 + vx^2 + wx \]

where \( x \) is the state variable; \( V(x) \) is potential function; \( t, u, v, \) and \( w \) are the control variables; and \( V''(x) = 0 \) is the critical curve (or critical surface).

In this paper we introduce catastrophe theory to study and analyze turbulence theoretically and quantitatively. By adopting the folding catastrophe model, we strictly derive the quantitative relationship between the energy spectrum and wave number concerning a new time-scaling index factor \( g \) to explain the energy in the turbulent system, inherited from the large eddies and transferred to the small eddies until viscous dissipation occurs, theoretically and physically. The turbulence model was validated by simulating the solid–liquid flow in the five-stage lifting pump of a 6000 m deep-sea mining system.

2. Research method of phase transition based on catastrophe theory

According to Equation (1), the equilibrium curve (i.e. non-degenerate critical points set) and the singularity set of the folding catastrophe model are as follows:

\[ V' (x) = 3x^2 + t = 0 \]  
\[ V'' (x) = 6x = 0 \]

The bifurcation set \( B (t = 0) \) can be obtained by Equations (5) and (6) as shown in Figure 1(a) and (c). Moreover, the bifurcation set bifurcates the potential function into a system state with gradual change (\( t > 0 \)) and a system state with phase transition (\( t < 0 \)) in Figure 1(b) and (c). When \( t > 0 \), Equation (5) has no practical solution, and therefore no critical point set; and the system behavior has no phase change. When \( t < 0 \), the potential function \( V(x) \) has two equilibrium states, which are unstable \((D_1)\) and stable \((D_2)\) in Figure 1(b), corresponding to negative and positive branches, respectively, in the equilibrium curve \( V'(x) = 3x^2 + t = 0 \) in Figure 1(a), bifurcated by the bifurcation set. The unstable equilibrium state \( D_1 \) is the starting point of phase transition, which is easily broken. \( D_2 \) is the end-point of phase transition. Some external factors will cause the system to rise from \( D_2 \) to \( D_2' \), undergoing mechanical instability, but it will eventually return to \( D_2 \).

For example, laminar flow is regular \((D_1)\) dominated by viscous. When the velocity increases, which is called
control variable change, the flow of fluid begins to be chaotic and vortices occur from $D_1$ state. At this time, the fluid changes from laminar flow to turbulent flow. Eventually, it will tend to a state of equilibrium – fully developed turbulence ($D_2$), with the smallest scale of vortices and isotropic turbulence.

The research outline of this method is as follows. First, a suitable catastrophe model is identified on the basis of the type of phase transition and the number of state variables. Second, the main parameters, such as energy and dissipation, which affect the process of phase transition, are selected and written as control variables in a certain function form. Third, the control variables are brought into the critical curve or surface expression to find the quantitative relationship between the parameters. Finally, the critical curve or surface expression is solved to obtain the extremum of different phases.

3. Quantitative catastrophe method analysis of turbulence phase-transition process

Many physicists use wave number to explain the energy transfer in turbulent systems and research the turbulence phase-transition process (Gao, Guo, & Vinen, 2016; Gioia, Ott-Monsivais, & Hill, 2006). Thus, scale and wave number are almost interchangeable concepts, and they are inversely proportional to each other (Adams, Chesler, & Liu, 2014; Cardesa, Vela-Martin, & Jimenez, 2017; Lin & Shen, 2014). Kolmogorov believed that turbulence is composed of vortices of different scales. When the Reynolds number is large enough, the fluid can develop turbulence. Kolmogorov made two famous assumptions: turbulence phase transition is affected by the mean energy dissipation rate $\epsilon$ and the kinematic viscosity $\nu$, considered to be the basis of turbulence research (Frisch, 1995). However, the pulsating energy has never been quantified.

Many researchers have studied turbulence and discovered the law of turbulence, which is the variation of energy fluctuation with time-scale or wave number (Chen, Chen, Eyink, & Sreenivasan, 2003; Falkovich & Ryzhenkova, 1992; Frisch, 1995; Liang, Wu, & Zhong, 2017; Young & Read, 2017). So, the other two important variables for the turbulent phase transition are the energy spectrum ($E_k$) and time index factor ($g$) (Frisch, 1995; Layek & Sunita, 2018; Montoya, Nieto, Alvarez, Hernandez, & Jurado, 2018; Romero-Gomez, Harding, & Richmond, 2017).

Therefore, $k$ is regarded as the state variable; $\epsilon (L^2 T^{-3})$, $\nu (L^2 T^{-1})$, and the energy spectrum density $E_k (L^3 T^{-2})$ are identified as the control variables; and $t$ is in the power exponent form in accordance with $E_{kk} = C_k \epsilon^{2/3} k^{-5/3}$ and the Reynolds number $Re = LV \nu^{-1}$ (Frisch, 1995; Thampi, Doostmohammadi, Shendruk, Golestanian, & Yeomans, 2016):}

$$t = 3 A \epsilon^{g_1} \nu^{g_2} E_k^g$$

(7)
where $A$ is a constant less than 0; and $g_1, g_2,$ and $g$ denote time-scaling indices.

Thus, the scale index satisfies

$$2g_1 + 2g_2 + 3g = -2$$
$$-3g_1 - g_2 - 2g = 0$$

on the basis of Table 1 and Equations (5) and (7), and Equation (8) is derived from Equations (5) and (7):

$$k^2 + A\epsilon^{(2-g)/4}\nu^{(-6-5g)/4}E_k^g = 0 \quad (8)$$

The expression of energy spectrum density $E_k$ strictly derived from the folding catastrophe model can quantitatively explain the energy in the turbulent system inherited from the large eddies and transferred to the small eddies until viscous dissipation occurs, theoretically and physically. In particular, the energy spectrum density $E_k$ is obtained by Equation (8) as follows:

$$E_k = -A^{-(1/5)}\epsilon^{(g-2)/(4g)}\nu^{(5g+6)/(4g)}k^{2g} \quad (9)$$

Figure 2 shows various points in the turbulence phase transition: $g = -2$ (starting stage), $g = -9/5$ (maximum value), $g = -6/5$ (derived from $(\partial^2 E)/(\partial u^2) = 0$, regarded as the singularity set), and $g = -2/3$ (derived from $(\partial^2 E)/(\partial \epsilon^2) = 0$, regarded as the singularity set). $g = -2$ is the starting point of the change in energy in the turbulence phase transition. $g = -9/5$ is maximum value for $E_k$, defined as the energy-containing scale $l_0$, or the energy-containing wave number $k_0 = 1/l_0$. According to Kolmogorov’s theory, the range near $g = -9/5$ is the energy-containing range. In this range, the energy spectrum density increases rapidly, mainly affected by the continuous input of external energy, until the maximum energy spectrum is generated at $g = -9/5$, reaching its peak. The range near $g = -6/5$ is the inertial subrange, where $E_{k(g=-6/5)} = -A^{5/6}k^{2/3}k^{-5/3}$. The range near $g = -2/3$ is the dissipation range, where turbulence develops into fully developed turbulence. As explained by folding catastrophe theory, $g = -2$ is the laminar flow phase, regarded as the unstable equilibrium state, $g = -2/3$ is the fully developed phase, regarded as the stable equilibrium state, and the range $-2 < g < -2/3$ is considered as a turbulent development process (transition stage).

The range near $g = -6/5$ is the inertial subrange, where $E_{k(g=-6/5)} = -A^{5/6}k^{2/3}k^{-5/3}$ is in accord with $E_{kk} = C_4k^{2/3}k^{-5/3}$ in Kolmogorov’s theory. The energy-containing range and the inertial subrange are separated, and the inertial subrange and the dissipation range are also separated, as shown in Figure 2, consistent with Kolmogorov’s theory.

This novel turbulence model is a universal form used to study energy in the laminar and turbulence transition process. As an example, a five-stage lifting pump for 6000 m deep-sea mining is simulated and studied, then the turbulent kinetic energy at the outlet with time is extracted to verify this improved model.

## 4. Model design and mesh generation of five-stage lifting pump

In the 6000 m deep-sea mining system, the multi-stage lifting pump not only provides the power to transport ore, but also ensures that ore can pass through the lifting pump, as the key equipment (Han, Wang, Wu, Liu, & Zou, 2002; Yan et al., 2017). A simplified model of the hydraulic pipe lifting system of the lifting pump is shown in Figure 3(a). The pressure energy required to lift the mixed fluid from the intermediate bin ($-5100$ m) to the sea surface is provided solely by the lifting pump.

### 4.1. Flow design method of five-stage lifting pump

According to a research report on sea mining test systems (Zou, 2007), the main technical indicators of a mining system are: the output of dry nodules is $W_g = 30$ t/h; according to the water content of the nodules, the output of wet nodules is $W_s = 45$ t/h; the concentration of ore transported $C_v = 5 \sim 10$%; the density of polymetallic nodules $\rho_s = 2040$ kg/m$^3$; the average diameter of nodules $d_s = 20 \sim 50$ mm; the density of seawater $\rho_l = 1025$ kg/m$^3$; and the depth of intermediate storage in a 6000 m deep-sea mining system is $L = 5400$ m. The production capacity of the mining system can be determined by these technical indicators, and the workflow of the conveying system can be obtained.

The volume flow rate of ore $Q_s$ is:

$$Q_s = \frac{W_s}{\rho_s} \quad (10)$$

where $\rho_s$ is density (kg/m$^3$), and the output of wet nodules within time $t_0$ is $W_s$.

The ratio of the volume flow rate $Q_s$ of ore to the volume flow rate $Q_m$ of solid–liquid fluid is the volume concentration $C_v$:

$$C_v = \frac{Q_s}{Q_m} \quad (11)$$

The solid–liquid flow rate $Q_m$ of the conveying system can be obtained by combining Equations (10) and (11):

$$Q_m = \frac{W_s}{t\rho_s C_v} \quad (12)$$

### Table 1. Relationship between the scale indices.

| $g_1$ | $g_2$ | $g(E_k)$ | $t$ |
|-------|-------|----------|-----|
| $L$   | 2     | 2        | 3   |
| $T$   | $-3$  | $-1$     | $-2$ |
|       | 0     | 0        | 0   |
It should be noted that the design flow $Q$ of the lifting pump must be greater than the conveying flow $Q_m$, so that the nodule mining capacity can meet the technical indicators.

The density of solid–liquid is the mass of solid–liquid fluid per unit volume:

$$\rho_m = \rho_l Q_l + \rho_s Q_s \over Q_m$$  \hspace{1cm} (13)

where $Q_l$ is volume flow of seawater (m$^3$/h); $Q_s$ is the volume flow of solid and liquid fluids (m$^3$/h); and $Q_m$ is the volume flow of solid and liquid fluids (m$^3$/h).

$$Q_m = Q_l + Q_s$$  \hspace{1cm} (14)

Therefore,

$$\rho_m = C_V \rho_s + (1 - C_V) \rho_l$$  \hspace{1cm} (15)

According to the theory of particle sedimentation, the sedimentation velocity of solid particles $u_t$ is:

$$u_t = \frac{\pi d_s g (\rho_s - \rho_l)}{6 \varphi \rho_l}$$  \hspace{1cm} (16)

where $\varphi$ is the spherical drag coefficient, and its drag coefficient $\varphi = (\pi/8) C$, $C = 0.4 \sim 0.5$; and $d_s$ is the average diameter of ore particles (mm). So,

$$u_t = \sqrt{\frac{8}{3} d_s g \left( \frac{\rho_s - \rho_l}{\rho_l} \right)}$$  \hspace{1cm} (17)

Considering that the shape of solid ore is polygonal, a shape coefficient $\psi$ is introduced, where $\psi = 0.67 \sim 0.82$. Equation (17) can be converted to:

$$u_t = (0.67 \sim 0.82) \sqrt{\frac{8}{3} d_s g \left( \frac{\rho_s - \rho_l}{\rho_l} \right)}$$  \hspace{1cm} (18)

According to Govier’s theory (Zou, 2007), the axial velocity of solid–liquid flow must be at least twice that
of ore settling to ensure the system’s ability to transport nodules.

4.2. Head design method of five-stage lifting pump

The first phase is seawater, and the second phase is the formation of ore particles. The Bernoulli equation in the lifting system is:

$$\frac{P_1}{\rho_w g} + \frac{\alpha_{m1} u_{m1}^2}{2g} + Z_1 = \frac{P_2}{\rho_w g} + \frac{\alpha_{m2} u_{m2}^2}{2g} + Z_2 + \Delta h_m$$  \hspace{1cm} (19)

where $P_1$ and $P_2$ are the pressures corresponding to sections 1 and 2, respectively (Pa); $\alpha_{m1}$ and $\alpha_{m2}$ are kinetic energy correction factors for sections 1 and 2; $u_{m1}$ and $u_{m2}$ are the average flow rates in sections 1 and 2; and $Z_1$ and $Z_2$ are the vertical distances between sections 1 and 2; and $\rho_m$ is solid–liquid fluid density ($\rho_m = \rho_v \times \rho_s$ or $(1 - \rho_v) \times \rho_s$).

The multi-stage lifting pump provides the required pressure energy to lift the mixed fluid from the intermediate bin to the surface of mining ship (Figure 3), so Equation (19) can be calculated as:

$$H_{\text{min}} = \frac{P_{\text{out}}}{\rho_w g} + \frac{\rho_m L}{\rho_w} - \frac{\rho_L L}{\rho_w} + \frac{\rho_m u_{\text{out}}^2}{2\rho_w g} + \frac{\Delta P_m}{\rho_w g}$$  \hspace{1cm} (20)

where $P_{\text{out}}/(\rho_w g)$ is the pressure energy at the outlet; $(\rho_m L)/\rho_w$ is the potential energy of solid–liquid fluid; $(\rho_L L)/\rho_w$ is the potential energy of seawater; $(\rho_m u_{\text{out}}^2)/(2\rho_w g)$ is the kinetic energy of solid–liquid fluid at the outlet; and $(\Delta P_m)/(\rho_w g)$ is hydraulic loss of solid–liquid fluid in the pipeline.

The higher the speed, the higher the head of the multi-stage pump and the lower the efficiency. The choice of speed has a large effect on the wear of the components and the vibration of the lifting pump system. Through simulation, we found that $n = 1450$ rpm is the most appropriate rotation speed.

In summary, the working performance of the multi-stage pump is small flow, high lift. The multi-stage pump is designed by the amplifying flow design method to enlarge the flow channel width of the lifting pump, and the working flow is lower. Therefore, the design flow of the multi-stage pump is $Q = 800$ m$^3$/h and the design speed is $n = 1450$ rpm. The design head of the single-stage pump is $H = 35$ m; therefore, the design head of the five-stage pump is $H_5 = 175$ m. Four sets of five-stage pulp pumps are connected in series in the 6000 m mining system. The lifting pump is a segmental multi-stage centrifugal pump and its structure is shown in Figure 3(b).
4.3. Computer-aided design method of blades of impeller and guide vane

The design method of the impeller blade is as follows.

The inlet diameter of impeller \( D_j \) is calculated as:

\[
D_j = \sqrt{\left( K_0 \sqrt{\frac{Q}{n}} \right)^2 + D_h^2}
\]  

(22)

where \( K_0 \) is the empirical coefficient, \( K_0 = 3.8 \sim 4.5 \); and \( D_h \) is the hub diameter.

The outer diameter \( D_2 \) of the impeller is as follows:

\[
D_2 = (9.35 \sim 9.6) \left( \frac{n_s}{100} \right)^{-0.5} \sqrt{\frac{Q}{n}}
\]  

(23)

where \( n_s = (3.65n\sqrt{Q})/H^{3/4} \).

The blade width \( b_2 \) at the impeller exit is:

\[
b_2 = (1.0 \sim 1.4) \left( \frac{n_s}{100} \right)^{5/6} \sqrt{\frac{Q}{n}}
\]  

(24)

The blade profile is designed by the logarithmic helix equation, as follows:

\[
r = r_1 e^{\frac{(\tan \beta_2 - \tan \beta_1)}{K} \left( \frac{\theta}{r_1} \right) + \tan \beta_1}
\]  

(25)

where \( \theta \) is the polar angle; \( K \) is the empirical coefficient, where \( K = \frac{(\tan \beta_2 - \tan \beta_1)\psi}{\ln(r_2/r_1)} - \varphi \tan \beta_1 - 1; r_1 \) is the impeller inlet radius; and \( r_2 \) denotes the impeller exit radius.

The number of blades \( Z_i \) is generally 3–6, and the wrap angle of blade is generally 85–150°. When the number of blades is large, the blade wrap angle takes a small value.

The larger the inlet angle \( \beta_1 \) of the blade, the larger the flow area of the impeller inlet, and the wear of the blade at the entrance is reduced. So, \( \beta_1 \) generally takes a value of 20–40°. The blade exit angle \( \beta_2 \) is generally taken to be 10–30° in consideration of wear at the blade at the exit. The wooden patterns of the impeller blade are shown in Figure 4.

The guide vane is an important energy-conversion component, and its blade design method is as follows.

The maximum diameter \( D_3 \) of the inner streamline is \( D_3 = D_2 + (2 \sim 5) \), where \( D_2 \) is the outer diameter of the impeller. The maximum diameter \( D_4 \) of the outer streamline is \( D_4 = (1.2 \sim 1.45)D_2 \). The axial length \( L \) is \( L = (0.5 \sim 0.7)D_2 \). The number of guide vanes is \( Z = Z_i \pm 1 \).

The inlet angle of the blade of the guide vane is \( \alpha_3 = \alpha_3' + \Delta\alpha \), where \( \Delta\alpha = 3° \sim 10° \) and \( \alpha_3' = \arctan(v_m/v_u) \). The outlet angle of the blade of the guide vane is \( \alpha_4 = 80 \sim 95° \). The blade profile is designed by \( d_\varphi = (1/r \tan \alpha)d \theta \). Its wooden patterns are shown in Figure 5.

The main geometric parameters of the blades of the impeller and space guide vane are listed in Table 2.

For the convenience of installation, the motor and the multi-stage lifting pump are fixed, respectively, in the motor cylinder and the pump barrel. The import and export flanges, the motor barrel, and the pump barrel are fixed on the same axis by the bolt. The import and export flanges are connected with the lifting hard pipe in series, as shown in Figure 3(b).

4.4. The spatial discretization process

The calculation area of the lifting pump includes the inlet and outlet extension section (static zone), the impeller passage (rotating zone), and the space guide vane passage (static zone).
This model uses structured mesh to improve the accuracy of the simulation. According to the number of blades, the structured grids of the 1/3 impeller runner and 1/4 guide vane runner are made as shown in Figure 6(a) and (b). Closer to the walls and boundary layers, denser grids are used to improve the simulation accuracy. The first layer grids for the walls and boundaries are at $\Delta y = 0.0005$, guaranteeing that the first layer grids for the wall are situated nearly everywhere at $y^+ = u_c \Delta y / \nu = 1.0$. More than two grid faces are made in the laminar subregion ($y^+ < 5.0$).

4.5. Mathematical models and the numerical algorithm

We verified the accuracy of the new turbulence model based on catastrophe theory and Kolmogorov’s theory, by comparing the energy changes between the traditional numerical simulation method and the new turbulence model.

4.5.1. Mathematical models

Mou, He, Zhao, and Chau (2017) present the effects of wind on various square high-rise buildings in detail.
using computational fluid dynamics (CFD) technology. In Mosavi et al. (2019), the multi-input bubble column reactor is simulated and predicted based on the hybrid model of CFD. Consequently, this paper adopts the $k$–$\epsilon$ turbulence model and the Navier–Stokes equations, including the continuity equation and the momentum equation (Issakhov, Bulgakov, & Zhandaulet, 2019).

$$\frac{\partial u_j}{\partial x_j} = 0$$  (26)

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + f_i \right]$$  (27)

The kinematic relationship between the position and the velocity of particles is as follows:

$$dx_p/d_t = u_p$$  (28)

$$m_p \frac{du_p}{dt} = F_D + F_G$$  (29)

Four turbulence models are determined for testing:

1. The $k$–$\epsilon$ turbulence model consists of an eddy viscosity and two equations (Jones & Launder, 1972):

$$\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right]$$  (30)

$$+ P_k - \rho \varepsilon + P_{kb}$$

$$\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho \varepsilon u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right]$$  (31)

$$+ \frac{\varepsilon}{k} (C_{1\varepsilon} P_k - C_{2\varepsilon} \rho \varepsilon + C_{1\varepsilon} P_{eb})$$

where constants are defined as follows (Launder & Spalding, 1974): $C_{1\varepsilon} = 1.44$, $C_{2\varepsilon} = 1.9$, $\sigma_\varepsilon = 1.2$, $\sigma_k = 1$, $C_3 = 1$.

2. The shear stress transport (SST) $k$–$\omega$ turbulence model (Goldberg & Batten, 2015; Hu et al., 2016) consists of an eddy viscosity and two equations. Because the turbulence model inside the boundaries allows it to be directly related to the wall by the viscous layer, the SST $k$–$\omega$ turbulence model is available to the low Reynolds turbulence model without adding damping functions.

$$\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{k}{\partial x_j} \right]$$  (32)

$$+ P_k - \beta^* \rho \omega + P_{kb}$$

$$\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho \omega u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\omega} \right) \frac{\omega}{\partial x_j} \right]$$

$$+ \alpha \frac{\omega}{k} P_k - \beta \rho \omega^2 + P_{\omega b}$$  (33)

where the constants have the following values (Wilcox, 1988):

$$\sigma_k = 2, \sigma_\omega = 2, \beta = 0.075, \beta^* = 0.09, \alpha = 5/9.$$

3. The detached eddy simulation (DES) model (Haque, Katsuchi, Yamada, & Nishio, 2016; Tang et al., 2015) is combined with the RANS (calculating the near-wall parts) and LES (calculating the remaining areas) for numerical simulation:

$$\varepsilon = \beta \star k \omega \rightarrow k^{3/2}/L_t \rightarrow k^{3/2}/(C_{DES} \Delta)$$

$$\Delta = \max(\Delta t); L_t = (\sqrt{k})/\beta \star \omega$$

The DES revision is expressed as a factor in the destructive term in the $k$ equation (Issakhov, Zhandaulet, & Nogaeva, 2018):

$$\varepsilon = \beta \star k \omega \rightarrow \beta \star k \omega \star F_{DES}$$

where

$$F_{DES} = \max \left( \frac{L_t}{C_{DES}} \right) \quad \text{and} \quad C_{DES} = 0.61$$

4. Turbulence involves a wide variety of scales. The LES can solve large-scale fluctuating motions and use the subnet-scale turbulent model for small-scale eddies (Issakhov, 2017).

### 4.5.2. The numerical algorithm

As a numerical algorithm, the semi-implicit method for pressure-linked equations (SIMPLE) has proved to be successful in solving some numerical solution problems of Equations (26) and (27) for the calculation of incompressible fluid flow. The entrance boundary condition is velocity inlet $v_0 = (Q/A) = (4Q/\pi D_i^2)$, and the outflow is used for the outlet boundary condition. The standard wall functions are used to calculate the influence of the walls on flow (Launder & Spalding, 1974). Equations (28) and (29) were computed by explicit time discretization, and the additional Equations (30) and (31) or (32) and (33) were computed by explicit time discretization. All calculations were completed in ANSYS FLUENT. Designs were produced on a personal computer (Intel® Core™ CPU i5-7500, 3.40 GHz, 8 GB RAM).
and simulations were performed in the cluster system (Intel® Xeon® CPU E5-2670, 2.50 GHz, 128 GB RAM) to save computing time.

5. Numerical simulation results and analysis at the fully developed turbulence stage based on the universal turbulence model

Under the conditions of rotation speed \( n = 1450 \text{ rpm} \), particle size \( d = 30 \text{ mm} \), and concentration \( C_v = 8\% \), the solid–liquid two-phase flow field with different flow rates, 420, 560, and 700 m\(^3\)/h, is numerically simulated. To verify the universal turbulence model, the particle clogging problem and the hydraulic performance of the new designed lifting pump are studied through the analysis of velocity distribution and pressure distribution at the fully developed turbulence stage.

5.1. Velocity distribution of full flow runner with different flow rates

From the velocity distribution, we can directly see the efficiency, trajectories, and outflow time of solid particles. The velocity distributions of the flow field in five-stage pumps with different flow rates (\( Q = 420, 560, \text{ and } 700 \text{ m}^3/\text{h} \)) are shown in Figure 7(a)–(c).

The impeller accelerates the solid–liquid flow to provide kinetic energy. The maximum velocity in the whole flow path appears at the tail of the impeller passage. The fluid with maximum flow velocity flows out of the impeller runner and flows into the space guide vane flow passage through the steering action of the outer cover plate. The velocity of the solid liquid flow decreases rapidly in the space guide vane region.

The speed at the impeller exit exceeds 30 m/s, so the rotating acceleration effect of the impeller blade on solid–liquid flow is obvious, as shown in Figure 7(a). The movement of the particles is smooth and the blade design is reasonable. The velocity difference of the space guide vane is over 20 m/s, so the effect of the diffuser is improved.

Comparison of the velocity nephograms in Figure 7(a)–(c) reveals that the particle trajectory becomes more stable as the velocity increases. Stable particles are not susceptible to turbulence of water flow, so the length of the actual trajectory is shorter. On the other hand, as the velocity increases, the time required for particles to flow through the same distance is shorter, which reduces the outflow time for particles. A stable trajectory and shorter outflow time will improve the blockage situation. The smaller the work flow, the less serious the blockage.

The design of the amplification flow method and working at a smaller flow rate can enlarge the width of the impeller flow path and improve the flow capacity of particles, so the performance of the lifting pump can be improved.

5.2. Pressure distribution in full flow passage with different flow rates

If there is a blockage of solid particles, the pressure and the energy transfer of the impeller will be affected, and then the performance of the lifting pump will be affected. The nephograms of the absolute pressure distribution in the five-stage pump with different flow rates (\( Q = 420, 560, \text{ and } 700 \text{ m}^3/\text{h} \)) are shown in Figure 8(a)–(c).
Figure 8. Pressure distribution of flow field in the five-stage lifting pump: (a) $Q = 420 \text{ m}^3/\text{h}$; (b) $Q = 560 \text{ m}^3/\text{h}$; (c) $Q = 700 \text{ m}^3/\text{h}$.

The pressure of the flow field is obviously increased from the inlet to the outlet, as shown in Figure 8(a). There is no large low-pressure area at the impeller inlet area. Comparing the nephograms in Figure 8, with the increase in the lifting pump series, the pressure increases sharply, as shown in Figure 8(a). Therefore, the energy transfer effect of the space guide vane is improved. As the flow decreases, the absolute pressure at the outlet is significantly greater, as shown in Figure 8(a)–(c). Under the condition in Figure 8(a), the absolute pressure at the outlet exceeds 2000 kPa, while the absolute pressure in Figure 8(b) is close to 2000 kPa. Under the condition in Figure 8(c), the absolute pressure at the outlet is significantly less than 2000 kPa. The pressure difference is up to 2000 kPa, which can satisfy the requirements for lifting. Therefore, these conditions are favorable for the transport of solid particles at a lower flow rate.

5.4. Pressure distribution of impeller blade pressure surface with different flow rates

During the operation of the five-stage pump, the solid–liquid fluid obtains kinetic energy under the rotation of the blade. The blade pressure distribution can reflect the effect of the impeller on solid–liquid fluid. Absolute pressure distributions of the impeller blade pressure surface under different flow conditions ($Q = 420, 560, \text{ and } 700 \text{ m}^3/\text{h}$) in the five-stage pump flow field are shown in Figure 10(a)–(c). As the radial radius of the blade increases, the transmission force to the solid–liquid flow increases, and the pressure of the blade increases. The pressure gradient is approximately perpendicular to the central axis of the five-stage pump, and the pressure field on the blade changes uniformly, so the solid particles move smoothly, and the function of impeller is effective. The blade linear design is reasonable, and the energy transfer is more efficient.

The pressure increases sharply as the flow rate decreases. Under the condition in Figure 10(a), the pressure value of the blade pressure surface at the pump outlet is more than 2000 kPa, whereas pressure in Figure 10(b) is close to 2000 kPa, and the absolute pressure under the condition in Figure 10(c) is less than 2000 kPa. Under the condition of large flow rate, the movement of solid particles is prone to disorder, which directly affects the pressure effect of the lifting pump impeller on solid–liquid flow and the hydraulic performance of the lifting pump. As can be seen from Figure 10, the hydraulic performance of the five-stage lifting pump is best under the condition of $Q = 420 \text{ m}^3/\text{h}$.
5.5. Velocity vector pictures of Z cross-section with different flow rates

The velocity vector pictures of the five-stage lifting pump under different flow conditions ($Q = 420$, 560, and 700 m$^3$/h) are shown in Figure 11(a)–(c). From the surface vector diagram of the impeller blade, we can see the specific trend of the particles in the three-dimensional space of the pump chamber. The fifth stage pressure distribution is the largest in Figures 7 and 8, so we only take the fifth stage pump to study the velocity vector of the Z cross-section.
Along the inlet of the impeller to the exit, the relative velocity of solid–liquid flow increases gradually, and the whole flow line is smooth. There is no large area of velocity gradient, no flow phenomenon occurs, and there is no large eddy. This shows that the design of the impeller blade is reasonable, in general. Particles in the impeller move along with the liquid, so the movement of solid particles is improved, and their movement is basically the same. There are no obvious eddies or reflux in the area, and there is no jet wake structure. Energy loss is very small. The numerical simulation preferably reflects the actual change in the relative velocity. The performance of the impeller is good for the solid–liquid flow, and the efficiency and hydraulic efficiency of the five-stage pump are improved. The blade linear design of the pump with the enlarged flow design method is reasonable, and it can work well under different flow conditions.

6. Simulation verification of turbulent flow energy derived by the improved method

Figure 12 shows the variation in the turbulent kinetic energy spectrum with time in the process of turbulence formation. The abscissa is time factor \( t_i \). The turbulent kinetic energy spectrum \( E_k \) under different flow conditions is shown in Figure 9. We can see that the \( E_k \) varies with time. The highest point of the turbulent kinetic energy spectrum is \( t_i = 2 \). With \( t_i \) from 0 to 1, the region is called the large eddy range; with \( t_i \) from 1 to 5.6, the region is the energy-containing range; with \( t_i \) from 5.6 to 8.8, the region is the inertial subrange; with \( t_i \) from 4 to 8.8, the region is called the dissipation range; and after 8.8, the system reaches fully developed turbulence. These values are in accordance with the changing trends derived from Equation (9). One of the important characteristics of catastrophe theory is that it can quantitatively predict the phase transition of a system even without knowing the differential equation of the system. The numerical results of turbulence transformation obtained by catastrophe theory are in agreement with the numerical simulation of the differential equations.

Figure 12 shows the variation in the turbulent kinetic energy spectrum with the time-scale factor at different points. Time is the reciprocal of frequency, and the relationship between frequency and wave number is \( k = 2\pi f_i / u_i \). Figure 2 shows that the wave number \( k \) and index \( g \) are in one-to-one correspondence, so the 2 point on the abscissa of Figure 9 is consistent with the \( g = -1.8 \) point of theoretical deduction in Figure 2. The energy of the fluid system increases gradually, and positive energy accumulation occurs in the region of \(-2 < g < -1.8\), that is, 0–2 in Figure 12. In the region of \(-1.8 < g \), i.e. where the time \( t_i \) is greater than 2 in Figure 12, the amount of energy dissipated is greater than the amount obtained from the outside, so there is a negative accumulation of energy. At this point, the large-scale vortex gradually ruptures and forms a small-scale vortex, and transfers energy to the small-scale vortex. In this way, we prove the accuracy of this turbulence model in the five-stage lifting pump for the 6000 m deep-sea mining system.

7. Test verification

The School of Mechanical and Electrical Engineering of Central South University and the Research Institute of Mining and Metallurgy jointly established the State Key Laboratory for deep-sea mineral resource exploitation and utilization. The laboratory has vertical deep-sea mining lift test equipment equipped with flow, pressure, and ore volume fraction testing equipment for lifting transportation. The experimental equipment simulates the deep-sea pressure environment by adding back-pressure at the output. Part of the experimental equipment is shown in Figures 13 and 14.

7.1. Simulated calculation method of head and efficiency

The total pressure difference is the pump head, and the calculation formula for the predicted head of the pump is as follows:

\[
H_m = \frac{1}{A_b} \int_{A_b} \left( \frac{p_b}{\rho_m g} + \frac{v_b^2}{2g} \right) dA - \frac{1}{A_a} \int_{A_a} \left( \frac{p_a}{\rho_m g} + \frac{v_a^2}{2g} \right) dA
\]  

(34)
where $A_a$ and $A_b$ are the inlet and outlet area of the lifting pump; $p_a, p_b$ are the inlet pressure and outlet pressure; and $v_a$ and $v_b$ are the inlet and outlet velocities, respectively. So,

$$H_m = \frac{p_{mb} - p_{ma}}{\rho mg} + \Delta z$$

(35)

where $p_{ma}$ is the total pressure of the outlet section; and $\Delta z$ is the vertical distance of the inlet and outlet sections.

The hydraulic efficiency formula of the lifting pump is as follows:

$$\eta_m = \frac{\rho mgQH_m}{M \omega} \times 100\%$$

(36)

where $M$ is the impeller torque; and $\omega$ is the impeller angular velocity.

### 7.2. Test calculation method of head and efficiency

The total pressure difference is the pump head, ignoring the total head loss of the inlet and outlet. The formula for calculating the experimental head of the lifting pump is as follows:

$$H = \frac{p_b - p_a}{\rho mg} + \frac{v_b^2 - v_a^2}{2g} + \Delta z$$

(37)

where $p_b$ and $p_a$ are values of the outlet and inlet pressure gauge; $v_b$ is average velocity at the outlet section, $v_b = (Q/A_b)$; $v_a$ is average velocity at the inlet section; and $v_a = (Q/A_a)\Delta z$ is the distance from the center position of the pressure gauge to the inlet and outlet in the vertical direction, where $\Delta z = 4.8m$.

The experimental efficiency calculation formula of the lifting pump is as follows:

$$\eta = \frac{P_B}{P_M} \times 100\%$$

(38)

where $P_B$ is the effective power of the pump, $P_B = (\rho mgQH/3600) \times 10^{-3}$; and $P_M$ is motor power, $P_M = 2\pi n_w \cdot M_w$.

The single-stage lifting pump was installed in the pump test platform to check the correctness of the simulation through experimentation. The pressure transmitter, torque sensor, and torque measuring instrument were used to measure the pressure of the pump, the rotation shaft torque, and the rotation speed, respectively. The measured head ($H$) and efficiency ($\eta$) curves and the numerical simulation results are compared in Figure 15. The hollow triangles and squares represent the simulated values, and the solid triangles and squares represent test values. The error mainly concerns energy loss at clearance joints, dynamic–static joints, and bearings. The numerical calculations are basically in accordance with the test results, which proves the feasibility and accuracy of the numerical simulation method.

**Figure 13.** Water supply pipe.

**Figure 14.** Lifting pump.

**Figure 15.** Head and efficiency curves between numerical simulation results and experimental results: 1 = head simulation value; 2 = head test value; 3 = efficiency simulation value; 4 = efficiency test value.
8. Conclusions

(1) The universal process of turbulence formation can be exactly derived based on catastrophe theory and non-dimensional analysis. This method can quantitatively analyze the relationship between the energy pulsation spectrum and wave number with respect to a time index factor $g$ to explain the energy in the turbulent system inherited from the large eddies and transferred to the small eddies until viscous dissipation occurs, theoretically and physically.

(2) For the universal turbulence model, solid–liquid two-phase flow in a five-stage lifting pump in the fully developed turbulence stage is studied. The outlet velocity of the impeller is up to 30 m/s, so the linear design of the blade is reasonable for the new design pump. The smaller the work flow, the less serious the blockage. The design of the amplification flow method and working at a smaller flow rate can ameliorate the flow capacity of particles and the function of the lifting pump. The pressure difference is up to 2000 kPa, which can satisfy the requirement of lift. It is conducive to the improvement of the particle blocking problem to work with a small flow rate.

(3) Comparing the turbulent kinetic energy and theoretical deduction under different flow conditions, it is found that the theoretical deduction is consistent with the simulation results.

(4) Our method provides equations that enable a quantitative analysis with a physical explanation for turbulence formation, which not only provides new insight into turbulence, but also can be applied to other complex phase transitions.

(5) The particle shape of polymetallic nodules is irregular and the range of particle sizes varies greatly. In the simulation and experiment, we assume that nodules are single spherical particles. In future research, assuming improved computing capability, we could introduce multiple phases and solve multiple Euler equations in the simulations and calculations.

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