A global test of jet structure and delay time distribution of short-duration gamma-ray bursts

Jia-Wei Luo\textsuperscript{1,2}\textsuperscript{*}, Ye Li\textsuperscript{3}\textsuperscript{†}, Shunke Ai\textsuperscript{1,2}, He Gao\textsuperscript{4}, and Bing Zhang\textsuperscript{1,2}\textsuperscript{‡}

\textsuperscript{1}Nevada Center for Astrophysics, University of Nevada, Las Vegas, NV 89154, USA
\textsuperscript{2}Department of Physics and Astronomy, University of Nevada, Las Vegas, NV 89154, USA
\textsuperscript{3}Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 100012, China
\textsuperscript{4}Department of Astronomy, Beijing Normal University, Beijing 100875, China

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ABSTRACT

The multi-messenger joint observations of GW170817 and GRB170817A shed new light on the study of short-duration gamma-ray bursts (SGRBs). Not only did it substantiate the assumption that SGRBs originate from binary neutron star (BNS) mergers, but it also confirms that the jet generated by this type of merger must be structured, hence the observed energy of an SGRB depends on the viewing angle from the observer. However, the precise structure of the jet is still subject to debate. Moreover, whether a single unified jet model can be applied to all SGRBs is not known. Another uncertainty is the delay timescale of BNS mergers with respect to star formation history of the Universe. In this paper, we conduct a global test of both delay and jet models of BNS mergers across a wide parameter space with simulated SGRBs. We compare the simulated peak flux, redshift and luminosity distributions with the observed ones and test the goodness-of-fit for a set of models and parameter combinations. Our simulations suggest that GW170817/GRB 170817A and all SGRBs can be understood within the framework of a universal structured jet viewed at different viewing angles. Furthermore, models invoking a jet plus cocoon structure with a lognormal delay timescale is most favored. Some other combinations (e.g. a Gaussian delay with a power-law jet model) are also acceptable. However, the Gaussian delay with Gaussian jet model and the entire set of power-law delay models are disfavored.

Key words: gamma-ray bursts

1 INTRODUCTION

Gamma-ray bursts (GRBs), as the most energetic transient events in the Universe, are generally categorized into two types based on their durations (Kouveliotou et al. 1993) and multi-wavelength observational criteria (Zhang et al. 2009; Li et al. 2016). Long GRBs (LGRBs) are believed to be originated from core-collapse of massive stars (Woosley 1993), while short GRBs (SGRBs) are deemed to be the result of compact star mergers (Eichler et al. 1989). The core-collapse model for LGRBs is supported by direct observational evidence of the association of some LGRBs with Type Ic supernovae (Galama et al. 1998; Woosley & Bloom 2006). The compact star merger model for SGRBs, on the other hand, has only been supported through indirect evidence such as host galaxy type or position of GRBs within the host galaxies (Gehrels et al. 2005; Nakar 2007; Berger 2014).

In August 2017, the multi-messenger electromagnetic and gravitational wave observations of GW170817/GRB 170817A originated from a binary neutron star merger event (Abbott et al. 2017a,b,c) provided clear evidence for the compact star merger origin of SGRBs (Abbott et al. 2017a; Goldstein et al. 2017; Zhang et al. 2018). This SGRB is special in its relatively weak prompt emission and its peculiar afterglow lightcurve, which suggests that the jet must possess some type of structure and the GRB is viewed off-axis (Troja et al. 2017; Xiao et al. 2017; Zhang et al. 2018; Lazzati et al. 2018; Lyman et al. 2018; Troja et al. 2019; Beniamini et al. 2019; Troja et al. 2020; Cheng et al. 2021).

Some structured jet models have been proposed, including the power-law and Gaussian jet models (Zhang & Meszaros 2002; Rossi et al. 2002) and two-component models invoking a central jet and a surrounding cocoon (Zhang et al. 2004). These models were originally discussed within the context of LGRBs, but were also tested against SGRBs after the discovery of GW170817/GRB 170817A. The current observational data from GW170817 alone is not sufficient to directly differentiate among different models (Nakar & Piran 2018; Troja et al. 2020; Oganesyan et al. 2020; Takahashi & Ioka 2021). Some studies (Beniamini & Nakar 2019; Salafia et al. 2020; Hayes et al. 2020; Guo et al. 2020; Takahashi & Ioka 2020; Beniamini et al. 2020; Tan & Yu 2020; Lloyd-Ronning et al. 2020; Dado et al. 2022; Urrutia et al. 2021; Preau et al.

\textsuperscript{*} luoj7@unlv.nevada.edu
\textsuperscript{†} yeli@pmo.ac.cn
\textsuperscript{‡} bing.zhang@unlv.edu

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2 MODELS FOR SGRB SIMULATION

2.1 Delay time distribution models

SGRBs are believed to be originated from neutron star mergers, either NS-NS or NS-BH mergers. These mergers take an extra time with respect to star formation to allow the two compact objects to form and more importantly, to allow the binary to lose orbital energy and angular momentum via gravitational wave radiation before merging (Faber & Rasio 2012; Burns 2020).

If we assume that the fraction of mass to form compact binary systems over all the mass to form new-born stars remains constant in cosmological time, then the SGRB rate density $R_{\text{SGRB}}$ (in units of $\text{Mpc}^{-3} \text{Gyr}^{-1}$) can be estimated as a convolution of star formation rate density (SFRD) $\phi$ (in units of $\text{M}_{\odot} \text{Mpc}^{-3} \text{Gyr}^{-1}$) and the probability density function of delay time distribution $f(\tau)$ (Sun et al. 2015):

$$R_{\text{SGRB}}(z) = \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} \phi(z') f(\tau) d\tau,$$

where $z'(\tau)$ is the redshift at the formation of the binary star system while $z$ is the redshift of the SGRB event. The delay time between star formation and merger is $\tau = t(z) - t(z')$, where $t(z)$ and $t(z')$ are the age of universe at redshifts $z$ and $z'$, respectively. Here $\phi(z')$ is star formation rate density (SFRD) at the formation of the binary star system.

Since the progenitors of neutron stars usually have $10 \text{M}_{\odot} - 29 \text{M}_{\odot}$ initial mass, and the lifetimes of them are 10 Myr–30 Myr. A minimum delay time $\tau_{\text{min}} = 10 \text{Myr}$ is required. The age of universe at redshift $z$, $t(z)$ is used as $\tau_{\text{max}}$. We use Equation 5 of Yüksel et al. (2008) as SFRD $\phi(z)$:

$$\phi(z) = \rho_0 \left[ (1 + z)^{a\eta} + \left( \frac{1 + z}{B} \right)^{b\eta} + \left( \frac{1 + z}{C} \right)^{c\eta} \right]^{1/\eta},$$

where $\rho_0 = 0.02$, $a = 3.4$, $b = -0.3$, $c = -3.5$, $\eta = -10$, $B = 5000$, $C = 9$.

As for the delay time distribution, we consider the following three function forms that have been discussed in the literature, with examples shown in Fig. 1:

(i) Gaussian:

$$f(\tau) \propto \exp \left( -\frac{(\tau - t_G)^2}{2\sigma_G^2} \right)/\sqrt{2\pi}\sigma_G. \quad (3)$$

A previous analysis suggested $t_G = 2 \text{Gyr}$ and $\sigma_G = 0.3 \text{Gyr}$ (Virgili et al. 2011).

(ii) Lognormal:

$$f(\tau) \propto \exp \left( -\frac{\ln(\tau - \ln t_{LN})^2}{2\sigma_{LN}^2} \right)/\sqrt{2\pi}\sigma_{LN}. \quad (4)$$

A previous analysis suggested $t_{LN} = 2.9 \text{Gyr}$ and $\sigma_{LN} = 0.2 \ln(\text{Gyr})$ (Wanderman & Piran 2015).

(iii) Power-law

$$f(\tau) \propto \tau^{-\alpha}. \quad (5)$$
Such power-law model is supported by the study of host galaxy mass distribution and stellar age distributions (Berger 2014). It is also consistent with the merger time distribution of Galactic NS-NS binaries (Piran 1992). By comparing the ratio between early-type host galaxies and late-type host galaxies, Zheng & Ramirez-Ruiz (2007) suggested that $\alpha = 1$. On the other hand, Wanderman & Piran (2015) suggested $\alpha = 0.81$, even though this model is not as good as the lognormal model to interpret the SGRB data.

With the redshift-dependent SGRB rate density specified, the number of SGRBs observed during an observation time duration $T$ within redshift range $z$–$dz$ can be derived as

$$dN_{\text{SGRB}}(z) = \frac{R_{\text{SGRB}}(z) T}{1+z} \frac{dV}{dz} dz,$$

where

$$\frac{dV(z)}{dz} = \frac{4\pi D_L^2}{(1+z)^2} \left[ \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda \right]^{-1/2},$$

is the comoving volume of the universe at redshift $z$, $D_L$ is the luminosity distance, $c$ is speed of light. We take the following Planck cosmological parameters in our calculations: $H_0 = 67.66$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.3111$, $\Omega_k = 0$ and $\Omega_\Lambda = 0.6889$ (Aghanim et al. 2020). In our simulations, we do not need to consider the normalization of the delay time distribution function but vary the number of SGRBs we simulate to ensure the simulated samples are comparable to the observed samples.

2.2 Structured jet models

After simulating the distances of SGRB sources, we then simulate the luminosity distribution of the SGRBs within the framework of a quasi-universal structured jet model. For each structured jet model, we assume that all SGRBs have the same jet structure, so that the angular peak luminosity per solid angle, which is the proxy of the isotropic equivalent peak luminosity, depends on the viewing angle $\theta$.

Assuming that the jet structure has a rotational symmetry around the jet axis, the angular peak luminosity per solid angle as a function of viewing angle $\theta$ is expressed as $L(\theta)$. For SGRB jets with random orientations, the viewing angle should be isotropically distributed. So in our simulations, we can draw $\sin \theta$ randomly from a uniform distribution and deduce $\theta$ from it. Then for a viewing angle $\theta$, the observed isotropic peak luminosity is

$$L_{p, \text{iso}} = 4\pi l(\theta).$$

For convenience, hereafter we will use $L$ to denote the isotropic peak luminosity $L_{p, \text{iso}}$.

There are three main structured jet models discussed in the literature, with examples shown in Fig. 2:

(i) Gaussian jet model (Zhang & Meszaros 2002)

$$l(\theta) = l_0 e^{\frac{-\theta^2}{2\theta_0^2}},$$

where $l_0$ is the maximum luminosity density when the jet is pointed directly at the observer. When the viewing angle $\theta$ is larger than $\theta_0$, the isotropic peak luminosity $L$ falls rapidly.

(ii) Power-law jet model (Meszaros et al. 1998; Rossi et al. 2002; Zhang & Meszaros 2002)

$$l(\theta) = \begin{cases} l_0 & \theta < \theta_0, \\ l_0 (\theta/\theta_0)^{-k} & \theta \geq \theta_0, \end{cases}$$

where $l_0$ is the energy density at or smaller than a small characteristic angle $\theta_0$. When $\theta > \theta_0$, the angular peak luminosity decreases as a power-law. Note that $l_0$ is needed to avoid divergence to infinity at small angles.

(iii) Two-component Gaussian jet+cocoon model (Bromberg et al. 2011)

$$l(\theta) = l_0 e^{\frac{-\theta^2}{2\theta_0^2}} + l_1 e^{\frac{-\theta^2}{2\theta_1^2}}.$$ (11)

This model consists of two Gaussian components: a narrower, brighter jet defined by $l_0$ and $\theta_0$, and a wider, fainter cocoon defined by $l_1$ and $\theta_1$.

Since we are testing a quasi-universal model for all SGRBs including GRB 170817A, all our parameters are required to reproduce the observed luminosity and viewing angle of GRB 170817A. The constraints from GRB 170817A are shown as an orange rectangle in Fig. 2, which represents the uncertainty ranges of $1.2 \times 10^{57}$ erg s$^{-1} < L < 4.1 \times 10^{57}$ erg s$^{-1}$ and $13.18^\circ < \theta < 36.10^\circ$ (Zhang et al. 2018; Troja et al. 2020), we require the luminosity distributions calculated from the parameter combinations intersect with this rectangle.

3 TEST MODELS AGAINST DATA

For an easy comparison with the observed values, we will report the jet central luminosity parameter with isotropic peak luminosity $L$ instead of luminosity per solid angle $l$ in the rest of the paper (e.g. $L_0 = 4\pi l_0$ and $L_1 = 4\pi l_1$ in Tables 1 and 2). The peak bolometric flux $P_\gamma$, in units of erg s$^{-1}$ cm$^{-2}$, is

$$P_\gamma = L/(4\pi D_L^2).$$

Because gamma-ray detectors have limited energy ranges,
we can directly compare with the observational data. To correct for this, we define a $k$ correction factor from the lab frame to the bolometric rest frame as

$$
 k = \frac{\int_{E_{\text{min}}}^{E_{\text{max}}} N(E) dE}{\int_{E_{\text{min}}}^{E_{\text{max}}} EN(E) dE}.
$$

Here, $E_{\text{max}}$ and $E_{\text{min}}$ are the maximum and minimum observational energy of the detector. $N(E)$ denotes the photon spectrum of GRBs. We use a typical Band function spectrum (Band et al. 1993) with $\alpha = -0.5$, $\beta = -2.3$ (Preece et al. 2000) to simulate the bursts. The peak energy $E_p$ is estimated with the $E_p - E_{\text{iso}}$ correlation (Amati et al. 2002, 2009; Kumar & Zhang 2015) but specifically for SGRBs (Zhang et al. 2012) as:

$$
 E_p = 2455 \text{keV} \left( \frac{E_{\text{iso}}}{10^{52} \text{erg}} \right) \left( \frac{0.59}{1 + z} \right).
$$

The isotropic energy $E_{\text{iso}}$ is calculated with $E_{\text{iso}} = LT_0$, where the intrinsic duration $T_0$ of SGRBs is drawn from a lognormal distribution with $\mu = -0.3$ and $\sigma = 0.57$ (Li et al. 2016).

To convert the observed peak flux $P_v$ to peak photon flux $P_p$, reported by gamma-ray detectors, we further introduce a $p$ correction factor, denoting the ratio of energy flux and photon flux as

$$
 p = \frac{\int_{E_{\text{min}}}^{E_{\text{max}}} N(E) dE}{\int_{E_{\text{min}}}^{E_{\text{max}}} EN(E) dE},
$$

then

$$
 P_p = \frac{P_v}{k \cdot p}.
$$

The simulated peak photon flux $P_p$ is one of the quantities we can directly compare with the observational data. Because the sensitivity of gamma-ray detectors are limited, only a fraction of all the simulated SGRBs that are bright enough can be “observed”. Therefore, we must also select SGRBs in our simulation according to their detectability. For this study, we apply a photon flux threshold in line with the detection threshold of the gamma-ray detectors. Considering the non-uniform detection thresholds related to the relative position of the SGRB with respect to the detector, there is a “gray zone” inside which the detection sensitivity does not reach a full capacity. We introduce an empirical soft detection probability function to mimic this effect. The detection probability function is a logistic function $1/1 + \exp[-s(P_p - b)]$ with bias factor $b = 0.75$ and scale factor $s = 16$. We also set a hard detection limit of 0.5 photon cm$^{-2}$ s$^{-1}$ according to the Fermi-GBM sensitivity (von Kienlin et al. 2020).

We confront our simulations with two samples. The first sample is the large SGRB sample detected by Fermi-GBM, whose redshifts are largely not measured. For this sample, we mainly compare the simulation results against the observed peak flux distribution, i.e. $\log N - \log P$, which is a convolution of the luminosity function (related to jet structure) and the redshift distribution (related to delay time distribution).

The second sample is a smaller sample of SGRBs whose redshifts have been measured. These SGRBs are mostly Swift GRBs, but also include some Fermi GRBs. For these SGRBs, one can use equations 12–16 and the $k$ and $p$ correction factors described therein in reverse to estimate isotropic luminosities. One can then compare the simulated bursts and the observed bursts in the $z-L$ two-dimensional plane, which carries additional information not available from the $z$-unknown sample.

Fig. 3 shows an example of our testing results. Given the same set of model (a delay time model plus a jet structure model), one can utilize two plots, a log $N - \log P$ plot and a $z-L$ plot to compare the model against the data. For the 2D plot, one can also compare the histograms for each dimension, as shown in the upper and right sides of the plot.

To test how well our simulations reproduce the observed parameter distribution, we utilize the two-sample Kolmogorov–Smirnov test (Massey 1951). The two-sample KS test compares the cumulative distribution histograms of the observed and simulation data. The maximum difference of the two histograms are taken as the test statistic:

$$
 D = \max_i \left| p(i) - q(i) \right|,
$$

where $p(i)$ and $q(i)$ are normalized cumulative distributions of the observed and simulated parameters. A significance level can be also calculated following Hodges (1958). In this study, we use the two-sample KS test implemented in Scipy (Virtanen et al. 2020).

While the two-sample KS test can be directly applied to the log $N - \log P$ plot, an extension of this test to 2 dimensional data is needed to test the 2D $z-L$ plot. (Peacock 1983; Fasano & Franceschini 1987; Press & Teukolsky 1988; Lopes et al. 2007; Xiao 2017). In this study, we use a modified version of the 2D KS test used by Zhang et al. (2021).

$$
 D = \max_i \max\left( |p_1(i) - q_1(i)|, |p_2(i) - q_2(i)| \right),
$$

$$
 \left| p_3(i) - q_3(i) \right|, \left| p_4(i) - q_4(i) \right|,
$$

where $p_1(i)$ to $p_4(i)$ and $q_1(i)$ to $q_4(i)$ are the normalized cumulative distribution functions of the observed and simulated data points at the $i$th point defined from the four quadrants, respectively.

Figure 2. Angular luminosity distributions of different jet models. Gaussian jet: $L_0 = 2.4 \times 10^{51}$ erg s$^{-1}$, $\theta_0 = 0.1$. Power-law jet: $L_0 = 1.0 \times 10^{52}$ erg s$^{-1}$, $\theta_0 = 0.01$, $k = 3.0$. Jet+cocoon jet: $L_0 = 1.0 \times 10^{52}$ erg s$^{-1}$, $\theta_0 = 0.06$, $L_1 = 1 \times 10^{49}$ erg s$^{-1}$, $\theta_1 = 0.2$. The constraint on jet parameters from GW170817 are shown as an orange rectangle.
4 RESULTS

We gather SGRB data from the Fermi GRB catalog (von Kienlin et al. (2020)) and filter out the short GRBs with $T_{90} < 2s$. There are 522 SGRBs in our sample.

We then match the Fermi SGRBs with redshift known SGRBs in the Greiner’s GRB catalog (Greiner (2022)) to obtain a list of redshift known Fermi SGRB. For the SGRBs with redshift reported in Greiner’s GRB catalog but are not reported in the Fermi catalog, we complement with the Swift/GRB catalog (Lien et al. (2016)). We also add GRB170817A to this redshift-known sample. This leaves us a sample of 37 $z$-known SGRBs. The isotropic luminosity of the $z$-known SGRBs are estimated with the method described in Section 3 in reverse. We use $10\text{keV}–1000\text{keV}$ as the detector energy range for Fermi (von Kienlin et al. 2020), and $15\text{keV}–150\text{keV}$ for Swift (Barthelmy et al. 2005).

For each possible model and parameter combination listed in Table 1, we first test the jet model to ensure that it is enclosed by the constraint of jet parameters from GW 170817 as shown in Fig. 2.

To generate more usable data in the same simulation size, we set a minimum isotropic luminosity of $1 \times 10^{50} \text{erg s}^{-1}$. A maximum viewing angle $\theta$ limit is calculated with the jet model parameters. We only simulate viewing angles smaller than this limit.

Subsequently, we simulate SGRB events in batches of 10000 and apply a sensitivity model with an empirical grey zone. After the detection filtering, we count the number of simulated GRBs that can be detected. We keep simulating batches until we have more than the observed number of GRBs. Finally, we randomly select the same number (522) of GRBs from the simulated sample. We also further randomly select the same number (37) of $z$-known sample. This sub-sampling process makes sure that we have the same number of simulated data points across the parameter space, so it is more straightforward in comparing the goodness-of-fit for each model and parameter combination.

Finally, we compare the simulated log $N - \log P$ and 2D $z - L$ distributions with the observed distributions as described in Section 2 and obtain test statistics $D_{NP}$ and $D_{zL}$ and the corresponding p-values $p_{NP}$ and $p_{zL}$. We multiply the two significance levels $p_{NP}$ and $p_{zL}$ to obtain a global p-value $p_{\text{global}}$. The parameter set with highest global p-value is deemed as the best-fit parameter for each delay and jet model combination.

The best-fit results are shown in Table 2, Figure 4, and Figure 5. Out of the 9 models we have tested, 5 models are consistent with the data. In particular, the lognormal delay with cocoon jet model performs the best and the Gaussian delay with power-law jet model comes second. On the other hand, 4 models, including all power-law delay time models and the Gaussian delay + Gaussian jet models have $p_{\text{global}} < 0.05$, and hence, are disfavored. We also test a positive power-law index in the power-law delay model (negative $\alpha$ in the nomenclature of this study). We find that while such a model can reproduce the log $N - \log P$ distribution, it fails to reproduce the $z - L$ distribution, and hence, is also disfavored.

5 CONCLUSIONS AND DISCUSSION

In this paper, we have systematically tested the quasi-universal structured jet idea for SGRBs by confronting 9 different delay time and structured jet combinations with the observations including the observed flux, luminosity and redshift distributions. We come up with three conclusions:

- The SGRB data is consistent with a quasi-universal jet structure model that interpret all SGRBs and GRB 170817A. The latter is simply a large-viewing-angle event with respect to a standard jet.
- The jet structure of this universal model can be loosely constrained. Our results favor the cocoon jet model, whilst the Gaussian and power-law jet models are plausible in some
hypothesis that all SGRBs share a similar quasi-universal jet then there is a direct conflict and one may need to drop the power-law delay models cannot pass the host galaxy constraints, conflict between the two claims. If the lognormal and Gaussian delay models, it remains unclear whether there is a direct such as the host galaxy properties of SGRBs. For example, under real-world circumstances.

The data used in this paper are public and are available in future can pose important constraints on the hypothesis and the tested jet and delay models.

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DATA AVAILABILITY

The data used in this paper are public and are available in corresponding references. The code can be shared upon request to the authors.

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Table 1. List of model parameters used in simulation. The luminosities shown in this table are the equivalent isotropic luminosities. For Jet+cocoon jet model, \( L_1 \) and \( \theta_0 \) are set to be smaller than \( L_0 \) and \( \theta_1 \) respectively.

| Delay model | Jet model | Delay parameter | Jet parameter | \( D_{NP} \) | \( P_{NP} \) | \( D_{zL} \) | \( \rho_{zL} \) | \( P_{global} \) | Judgment |
|-------------|-----------|-----------------|--------------|-------------|-------------|-------------|-------------|-------------|----------|
| Gaussian    | Gaussian  | \( t_G = 3.0, \sigma_G = 0.1 \) | \( L_0 = 3.36 \times 10^{42}, \theta_0 = 0.12 \) | 0.044 | 0.692 | 0.405 | 0.009 | 0.006 | ✗ |
| Gaussian    | Power-law | \( t_G = 4.0, \sigma_G = 0.3 \) | \( L_0 = 2.15 \times 10^{42}, \theta_0 = 0.02, k = 3.7 \) | 0.059 | 0.316 | 0.23 | 0.354 | 0.112 | ✓ |
| Gaussian    | Cocoon    | \( t_G = 4.0, \sigma_G = 0.5 \) | \( L_0 = 4.64 \times 10^{42}, \theta_0 = 0.02, \theta_1 = 0.15 \) | 0.067 | 0.191 | 0.243 | 0.288 | 0.055 | ✓ |
| Power-law   | Gaussian  | \( \alpha = 0.8 \) | \( L_0 = 3.36 \times 10^{42}, \theta_0 = 0.09 \) | 0.067 | 0.191 | 0.392 | 0.013 | 0.003 | ✗ |
| Power-law   | Power-law | \( \alpha = 0.85 \) | \( L_0 = 4.64 \times 10^{42}, \theta_0 = 0.02, k = 4.0 \) | 0.084 | 0.049 | 0.189 | 0.601 | 0.029 | ✗ |
| Power-law   | Cocoon    | \( \alpha = 0.8 \) | \( L_0 = 4.64 \times 10^{42}, \theta_1 = 0.15 \) | 0.056 | 0.396 | 0.338 | 0.047 | 0.019 | ✓ |
| Lognormal   | Gaussian  | \( t_{LN} = 3.5, \sigma_{LN} = 0.1 \) | \( L_0 = 7.85 \times 10^{42}, \theta_0 = 0.10 \) | 0.059 | 0.316 | 0.27 | 0.187 | 0.059 | ✓ |
| Lognormal   | Power-law | \( t_{LN} = 3.0, \sigma_{LN} = 0.2 \) | \( L_0 = 1 \times 10^{42}, \theta_0 = 0.02, k = 4.0 \) | 0.054 | 0.441 | 0.27 | 0.178 | 0.078 | ✓ |
| Lognormal   | Cocoon    | \( t_{LN} = 3.5, \sigma_{LN} = 0.2 \) | \( L_0 = 1 \times 10^{42}, \theta_0 = 0.04, \theta_1 = 0.15 \) | 0.044 | 0.692 | 0.216 | 0.428 | 0.296 | ✓ |
Figure 4. Best fit $\log N - \log P$ distribution for every jet+delay combination model.

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Figure 5. Best fit 2D $z - L$ distribution for every jet+delay combination model.
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