Crossover Between Universality Classes in the Statistics of Rare Events in Disordered Conductors

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The crossover from orthogonal to the unitary universality classes in the distribution of the anomalously localized states (ALS) in two-dimensional disordered conductors is traced as a function of magnetic field. We demonstrate that the microscopic origin of the crossover is the change in the symmetry of the underlying disorder configurations, that are responsible for ALS. These disorder configurations are of weak magnitude (compared to the Fermi energy) and of small size (compared to the mean free path). We find their shape explicitly by means of the direct optimal fluctuation method.

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Statistical properties of the wave functions, $\psi(r)$, in disordered conductors have been the subject of an intensive theoretical study during the last decade \[1-3\]. Considerable attention was devoted to the so-called anomalously localized states (ALS). These states constitute the large-$|\psi|^2$ “tail” of the wave function distribution, where this distribution deviates strongly from the prediction of the random-matrix theory. Analytical results obtained to date are summarized in the comprehensive review \[4\]. The bulk of these results are obtained using different versions \[7-10\] of the nonlinear $\sigma$-model. The latest works on the subject report on numerical tests of the theoretical predictions \[1-3, 11\]. Calculations of the ALS density, based on non-linear $\sigma$ model \[1-3\], yield exponentially small tails with different numerical factors in the exponent, depending on the universality class. The drawback of the non-linear $\sigma$-model approach is that it fails to pinpoint the actual disorder realizations responsible for ALS. This is because within the machinery of the non-linear $\sigma$-model the averaging over the disorder is performed at an early stage of the calculation. In an attempt to overcome this deficiency, an alternative approach, the direct optimal fluctuation method, was proposed in Ref. \[11\]. In two dimensions the optimal disorder configuration obtained in Ref. \[11\] had a shape of a compact potential well with a radius of the order of the Fermi wavelength, supplemented by a periodic modulation at large distances. An important accomplishment of Ref. \[11\] is the observation that, within a numerical factor in the exponent, the non-linear-$\sigma$-models-based results can be reproduced by considering a simplest configuration yielding the ALS, namely a circular (in two dimensions) region of space surrounded by a high and thick potential barrier. Then the anomalously large values of the wave function, $\psi$, can be characterized by a single, intuitively transparent, parameter

$$T = |\psi_{\text{out}}|^2/|\psi_{\text{in}}|^2, \tag{1}$$

where "in" and "out" refer to the inner and outer boundaries of the barrier. As a direct consequence of the compactness of the fluctuations, the probability of their formation is insensitive to weak magnetic fields, in apparent contradiction to the $\sigma$-model predictions. On the basis of this contradiction the authors of Ref. \[11\] questioned the ability of the non-linear $\sigma$-models to handle the ALS.

The goal of the present paper is to demonstrate that the direct optimal fluctuation approach captures the dependence of the density of ALS on the universality class. Moreover, it allows to trace the crossover between the orthogonal and unitary classes with increasing magnetic field. We restrict our consideration to the two-dimensional case.

The sensitivity to a weak magnetic field within the direct optimal fluctuation approach is restored in two steps. First, instead of circle-shape fluctuations of Ref. \[11\] we consider fluctuations of ring-shape, schematically depicted in Fig. 1. This step is in accord with the paper by Karpov \[12\], who has demonstrated that in three dimensions fluctuations of a toroidal shape, providing a given value of $T$, are more probable than spherical fluctuations, considered in Ref. \[11\]. Ring-shape fluctuations, having much bigger area, are sensitive to the magnetic field. However, this class of fluctuations alone does not exhibit a crossover between the orthogonal and unitary classes. To reveal this crossover, as a second step, we note that, on a ring, the states with angular momenta $m$ and $-m$ are degenerate in the absence of magnetic flux. This suggests that the fluctuations lifting this degeneracy cause the increase of $P(E_F, T)$, which is the probability to find a state with energy $E_F$ and a given value of $T$. The underlying reason for this increase is the following. A weak coupling, $\kappa$, between the states $|m\rangle$ and $|-m\rangle$ leads to decrease in the formation probability of the fluctuation. This decrease is quadratic in $\kappa$. On the other hand, the $|m\rangle$, $|-m\rangle$ splitting due to this coupling is proportional to $|\kappa|$. This splitting results in a “gain” in $T$, which is also proportional to $|\kappa|$. Thus, for small
coupling, the “gain” overweighs the “penalty” for creating the coupling. The above argument is completely analogous to the Jahn-Teller effect [13]. Kusmartsev and Rashba [14] have employed this argument to demonstrate that the fluctuations responsible for the tail states of a four-fold degenerate semiconductor valence band are of ellipsoidal rather than spherical shape. Summarizing, in a zero magnetic field $P(E_F, T)$ is determined by the warped ring-shape fluctuations of the random potential. In a finite magnetic field the degeneracy $|m|, |−m|\rangle$ is lifted; the warp does not lead to increase of $P$ anymore. Then the fluctuations contributing to $P$ are close to rings. Thus, in the statistics of rare events, the crossover between the orthogonal and unitary classes can be viewed as switching from warped to almost perfect rings. This is illustrated in Fig. 1(a).

The above discussion suggests the following analytical steps. Firstly, the fluctuation $V(r)$ of the white-noise random potential is presented as a sum of the angular harmonics $V(r) = \sum_m V_m(\rho)e^{im\phi}$, where $\rho = |r|$. Secondly, in the probability $P\{V(r)\}$ of the fluctuation $V(r)$ determined by

$$\ln P = \frac{\pi \nu T}{2\hbar}\int dr V^2(r) = \frac{\pi^2 \nu T}{\hbar}\sum_{m=-\infty}^{\infty} \int d\rho |V_m(\rho)|^2,$$  \hspace{1cm} (2)

only the angular harmonics $V_0(\rho)$ and $V_{\pm 2m}(\rho)$ are retained. This is because the $|m|, |−m|$ coupling is provided by the harmonics $\pm 2m$ of the random potential. In Eq. (2) $\tau$ and $\nu$ stand for the scattering time and the density of states, respectively. Then the system of equations relating the components $\chi_m$ and $\chi_{−m}$ of the wave function $\rho^{1/2}\psi(\rho, \phi)$ takes the form

$$\hat{H}_0\chi = -\frac{\hbar^2}{2M} \frac{d^2\chi}{d\rho^2} + \left(V^{(m)}_{\text{eff}}(\rho) + V_2\sigma_z\right)\chi = E\chi,$$  \hspace{1cm} (3)

where $M$ is the electron mass, $\chi = (\chi_m, \chi_{−m})$ is a two-component wave function, the matrix $V_2 = \text{diag}(V_{2m}, V_{−2m})$ is diagonal, and $\sigma_i$ are the Pauli matrices. The effective potential in (3) is a sum of a centrifugal potential and $V_0(\rho)$

$$V^{(m)}_{\text{eff}}(\rho) = \frac{\hbar^2(m^2 - 1/4)}{2M\rho^2} + V_0(\rho).$$  \hspace{1cm} (4)

The turning point, $\rho_c$, is determined by $V^{(m)}_{\text{eff}}(\rho_c) = E\psi$ [see Fig. 1(b)]. As we will see below, the characteristic width, $\nu$, of the fluctuation $V_0(\rho)$ and the barrier thickness, $d$, [see Fig. 1(b)] are related as $\nu \ll d \ll \rho_c$. Similarly to Ref. [12], this separation of spatial scales allows a number of crucial simplifications: (a) since $\nu \ll d$, it follows that, within the barrier, $V^{(m)}_{\text{eff}}(\rho)$ is dominated by the centrifugal term; (b) since $d \ll \rho_c$, this term can be linearized within the barrier region as

$$V^{(m)}_{\text{eff}}(\rho) = E\psi + \frac{\rho_c - \rho}{d},$$  \hspace{1cm} (5)

where $\nu = h^2m^2d/M\rho_c^3$ is the barrier height. Using the above simplifications the ratio $\tau = |\psi_E(\rho_c - d)|^2/|\psi_E(\rho_c)|^2$ can be readily calculated

$$|\ln \tau| = 2\frac{\sqrt{2M}}{\hbar} \int_{\rho_c - d}^{\rho_c} d\rho \left[|V_{\text{eff}}(\rho) - E\psi|^2\right]^{1/2} = \frac{2m}{3} \left(2d\rho_c \right)^{3/2} = \frac{2m}{3} \left(\frac{\nu}{E\psi} \right)^{3/2}.$$  \hspace{1cm} (6)

It is seen from Eq. (6) that the condition $\rho_c \gg d$ can be rewritten as $m \gg |\ln \tau|$. Equation (6) relates $\tau$ to the energy level position $\nu\tau$ in the potential well centered around $\rho_c, d - \nu/2$ [see Fig. 1(b)]. Thus the problem of finding the distribution of $\tau$ reduces to the conventional problem of finding the most probable configuration of random potential which confines the energy level of a depth $-\nu\tau$. In our case $-\nu\tau$ is the eigenvalue of the matrix Hamiltonian $\hat{H}$, defined as

$$\hat{H} = -\frac{\hbar^2}{2M} \frac{d^2}{dx^2} + V_0(x) + V_2\sigma_x,$$  \hspace{1cm} (7)

where $x = \rho - \rho_c + d + \nu/2$. The form of the Hamiltonian Eq. (7) follows from the system Eq. (3) and the condition $\nu \ll d$ which allows to neglect the change of the centrifugal potential within the potential well [see Fig. 1(b)].

The straightforward generalization of the approach of Ref. [13,14] to the matrix Hamiltonian (7) implies minimizing the functional $F\{V_0, V_{\pm 2m}\} = |\ln P| - \lambda(\chi^\dagger \hat{H}\chi)$. With an appropriate choice of the Lagrange multiplier, $\lambda$, we obtain the following expressions for $V_0$ and $V_{\pm 2m}$ in terms of two-component wave function, $\chi$,.
\[ V_0 = \chi^+ \chi, \quad V_{\pm 2m} = \chi^+ \sigma_{\pm} \chi. \]  

At this point it is convenient to introduce dimensionless variables as \( z = x(2M \varepsilon_0)^{1/2}/\hbar \) and \( \tilde{\chi} = \varepsilon_0^{-1/2} \chi \). With new notations, the matrix Schrödinger equation, \( \hat{H}_\chi = -\varepsilon_0 \chi \), reduces to

\[ \ddot{\chi}' + \frac{3}{2} \left( \ddot{\chi} + \chi \right) \dot{\chi} - \frac{1}{2} \left( \ddot{\chi} + \sigma_z \dot{\chi} \right) \dot{\sigma}_z \dot{\chi} - \dot{\chi} = 0, \]  

and probability (Eq. (3)) of the fluctuation, providing a given value of \( \mathcal{T} \), takes the form

\[ |\ln P_m| = \pi^2 \frac{\nu_T \rho_c}{\hbar} \int dx \left( |V_0(x)|^2 + 2 |V_{2m}(x)|^2 \right) = (3/16)\pi k_F l \ln \mathcal{T} C_m, \]  

where \( k_F = (2M E_F)^{1/2}/\hbar \) is the Fermi momentum, \( l = \hbar k_F \tau/\lambda \) is the mean free path; dimensionless factor \( C_m \) in Eq. (10) is defined as

\[ C_m = \frac{1}{2} \int dz \left[ 3 \left( \ddot{\chi} + \chi \right)^2 - \left( \ddot{\chi} - \sigma_z \ddot{\chi} \right)^2 \right]. \]  

Equations (1)-(11) express analytically the Jahn-Teller physics discussed in the introduction. Indeed, this system has a conventional instanton solution \( \tilde{\chi}_{-m} = 0 \) and \( \tilde{\chi}_m = 2^{1/2} \cosh^{-1} z \), corresponding to the absence of warp \( (V_{\pm 2m} \equiv 0) \). This solution yields \( C_m = 16/3 \). On the other hand, the Jahn-Teller-type solution \( \tilde{\chi}_m = \tilde{\chi}_{-m} = (2/3)^{1/2} \cosh^{-1} z \) leads to a smaller value \( C_m = 32/9 \), which corresponds to exponentially higher probability \( P_m \). Thus, the final result for orthogonal case reads

\[ \ln P_m = -(2/3)\pi g \ln \mathcal{T}, \]  

where \( g = k_F l \) is the dimensionless conductance. Remarkably, \( \ln P_m \) does not depend on the value of the angular momentum \( m \), provided that \( m \gg |\ln \mathcal{T}| \). In fact, the condition \( m \gg |\ln \mathcal{T}| \) justifies all the assumptions made in deriving the result Eq. (12). To see this, we write explicitly the potentials \( V_0 \) and \( V_{\pm 2m} \):

\[ V_0 = 2V_{\pm 2m} = \frac{4\varepsilon_0}{3 \cosh^2 \left[ x(2M \varepsilon_0)^{1/2}/\hbar \right]} \]  

Using the relation \( \varepsilon_0 = E_F \left( 3 |\ln \mathcal{T}|/2m \right)^{2/3} \), it is seen that under the condition \( m \gg |\ln \mathcal{T}| \) we have \( V_0(x) \ll E_F \), i.e. the potential well is shallow.

Other assumptions used in the calculation are less restrictive. Indeed, the typical width of the potential well \( w \sim \hbar/(M \varepsilon_0)^{1/2} \sim k_F^{-1} (m/|\ln \mathcal{T}|)^{1/3} \). On the other hand, for barrier width \( d \) we have from Eq. (11)

\[ d = \rho_c (\varepsilon_0/2)^{1/3} \left( |\ln \mathcal{T}|/2m \right)^{2/3}. \]

Since \( \rho_c = m/k_F \), we have \( d = m^{1/3} (3 |\ln \mathcal{T}|/2)^{2/3} \) \( 2k_F \), so that \( w/d \sim |\ln^{-1} \mathcal{T}| \ll 1 \). We have also assumed that the change of the centrifugal potential within the potential well is much smaller than \( \varepsilon_0 \). This change can be estimated as \( \varepsilon_0 w/d \ll \varepsilon_0/|\ln \mathcal{T}| \ll \varepsilon_0 \).

We now turn to the case of a finite magnetic field, \( B \). Due to the azimuthal symmetry of the problem the system of equations Eq. (4) relating the radial functions \( \chi_m, \tilde{\chi}_{-m} \) can be easily modified to

\[ \tilde{H}_0 \chi + \frac{\hbar^2}{2M} \left( \frac{m}{l_B^2} \sigma_z + \frac{\rho^2}{4l_B^2} \right) \chi = E_F \chi, \]  

where \( l_B = (\hbar/c B)^{1/2} \) is the magnetic length. As can be seen from Eq. (14) there exists a following hierarchy of magnetic field strengths. When \( l_B \gg m^{1/2}/k_F \), then the “confining” term \( \rho^2/l_B^2 \) can be neglected. The latter condition is equivalent to \( R_L \gg \rho_c \), where \( R_L = k_F l_B^2 \) is the Larmour radius for an electron with energy \( E_F \). On the other hand, it is obvious that the term \( \pm \hbar^2 m/(2M l_B^2) \), lifting the \( |m \rangle, | - m \rangle \) degeneracy, becomes important when it is of the order of \( \varepsilon_0 \). This yields \( R_L \sim \rho_c (|\ln \mathcal{T}/m|^{-2/3} \). Since \(|\ln \mathcal{T}| \ll m \), we conclude that there exists an interval of magnetic fields which affect the potential well but do not affect the barrier. Condition \( \varepsilon_0 \sim \hbar^2 m/(2M l_B^2) \) determines the characteristic magnetic field for orthogonal-unitary crossover: \( B_0^{(m)} = (\Phi_0/2\pi) k_F^2 (3 |\ln \mathcal{T}|/2)^{2/3} m^{-5/3}, \) where \( \Phi_0 \) is the flux quantum. The remaining task is to find the dependence \( C_m(\delta) \), where \( \delta = B/B_0^{(m)} \), which describes the change of \( C_m \) between “orthogonal”, \( C_m(0) = 32/9 \), and “unitary”, \( C_m = 16/3 \), values with increasing magnetic field, \( \delta \). At finite \( \delta \) the system Eq. (14) takes the form

\[ \ddot{\chi}' + \frac{3}{2} (\ddot{\chi} + \chi) \dot{\chi} - \frac{1}{2} (\ddot{\chi} + \sigma_z \dot{\chi}) \dot{\sigma}_z \dot{\chi} - \dot{\chi} = \delta(1 - \sigma_z)\dot{\chi}, \]  

\[ 3 \]
whereas Eqs. (10, 11) retain their form. At small $\delta \ll 1$ the dependence $C_m(\delta)$ can be found by solving Eq. (15) perturbatively and substituting $\tilde{\chi}(\delta)$ into Eq. (11). This program can be carried out analytically, yielding

$$C_m(\delta) = C_m(0) \left(1 + \frac{3\delta}{2}\right). \quad (16)$$

Our important observation is that perturbative expression Eq. (16) remains valid with high accuracy up to $\delta = 1/3$, when the “unitary” solution $\tilde{\chi}_{-m} = 0$, $\tilde{\chi}_m = 2^{1/2}\cosh^{-1} z$ takes over. The crossover behaviour $C_m(\delta)$ is illustrated in Fig. 2. In the same figure we show the “orthogonal” solution of Eq. (13) at crossover point.

Overall, the direct optimal fluctuation approach yields in two dimensions $|\ln P| \sim g |\ln T|$ rather than $|\ln P| \sim g \ln^2 T$ in Ref. [11]. This is because the truly optimal fluctuations are close to rings rather than to circles. The ring area, $S_m \sim \rho \omega \tau$, being much bigger than the area of a circle $\sim k_F^{-2}$ in Ref. [11] is the origin of a high sensitivity of rings to the magnetic field. On the other hand, due to the same relation, $S_m \gg k_F^{-2}$, the rings are much more “vulnerable” to the perturbations caused by the harmonics with small angular momenta, for which the centrifugal barrier is low. To demonstrate this we write down the expression for the decay rate of quasilocal state in a ring

$$\text{Im} \ E = 2\pi \sum_{\mu} \left(\langle m|V_0|\mu \rangle^2 + \sum_{m \neq 0} \langle m|V_n|\mu \rangle^2\right) \delta(E - E_\mu) = \frac{\hbar}{\tau_1} + \frac{\hbar}{\tau_2}, \quad (17)$$

where $\langle m|V_n|\mu \rangle$ are the matrix elements of the harmonics $V_n(\rho)$ [see Eq. (2)]; $|\mu \rangle$ is the state of the continuous spectrum in the absence of random potential (the corresponding energy is $E_\mu$). For simplicity in Eq. (17) we have neglected the warp. Above we were looking for the fluctuations with anomalously large $T$. Note, that within a numerical factor $T = \hbar/\tau_0 \tilde{\tau}_1$. It is seen from Eq. (17) that our consideration is justified only if $\tau_2 \gtrsim \tau_1$. Within the optimal fluctuation approach harmonics with small $n$, contributing to $\hbar/\tau_2$, do not affect the principal exponent. They enter at the stage of the prefactor calculation [17]. A typical value of $\tau_2$ is the scattering time $\tau$, which is much shorter than $\tau_1$. The relation $\tau_2 \gtrsim \tau_1$ is satisfied only for sparse configurations, in which the harmonics $V_n$ are suppressed within the ring.

Therefore, our result for $P_m$ should be multiplied by the probability, $P$, to find such a configuration. To calculate this probability we consider the distribution function of time $\tau_2$:

$$P(\tau_2) = \left\langle \delta \left(\frac{1}{\tau_2} \int dr dr' S(r, r')V(r)V(r') \right) \right\rangle_{V(r)} \quad (18)$$

where the expression for $S(r, r')$ follows from Eq. (17)

$$S(r, r') = \frac{2\pi}{\hbar} \rho c \chi_m(\rho)\chi_m^*(\rho') \sum_{\mu} \psi_\mu(r)\psi_\mu^*(r') \delta(E - E_\mu). \quad (19)$$

Averaging over random potential $V(r)$ in Eq. (18) can be carried out explicitly. The result is expressed in terms of eigenvalues, $\lambda_n$, of the integral operator with the kernel $S(r, r')$: $\int dr S(r, r')\phi_n(r) = \lambda_n\phi_n(r)$. With the accuracy of a numerical factor the result can be obtained qualitatively. This is because $P(\tau_2)$ is determined by the first $N_0$ eigenvalues, which are almost equal to each other: $\lambda_n \approx \lambda_0$ for $n < N_0$. The value $N_0$ can be estimated as the number of squares with a side $k_F^{-1}$ within the area of a ring $S_m$, i.e. $N_0 \sim \rho_c c k_F^2$. On the other hand, there is an exact sum rule $\sum_n \lambda_n = 2\pi\hbar$ for the eigenvalues $\lambda_n$. Since $\sum_n \lambda_n \approx \lambda_0 N_0$, we can find $\lambda_0$. This leads to the following result for the distribution Eq. (18): $P(\tau_2) \propto (\nu \tau / \lambda_0 \tau_2)^{N_0} \propto (\tau / \tau_2)^{N_0}$. Then the sought product $\tilde{P}_m = P_m P(\tau_2 \sim \tau_1)$, describing the probability to find ALS with “preexponential” accuracy, can be presented as

$$\ln \tilde{P}_m = \ln P_m - c_2 N_0 \ln(T/g) = -\frac{2}{3} \pi g |\ln T| - c_2 \frac{m^{4/3}}{\ln^{1/3} T} \ln(T/g), \quad (19)$$

where $c_2 \sim 1$ is a numerical factor. For concreteness we choose in Eq. (14) the orthogonal expression for $\ln P_m$. The above expression for $\tilde{P}_m$ allows to find the optimal value of the angular momentum, $m$. Indeed, the prefactor falls off rapidly with $m$, while the main term to the first order is independent on $m$. We note that the $m$-dependent correction to the principal term in $\ln P$, which originates from the corrections to the barrier potential Eq. (2), is of the order of $|\ln P|/|\rho c|$, i.e., it has the form $-c_1 g |\ln T/m^{2/3}| |\ln T|$, where $c_1 \sim 1$ is another numerical coefficient. The sign of the correction is determined by the fact that, due to deviation of $V_{eff}^{(m)}(\rho)$ from the linear form Eq. (5), preserving the value of $\ln T$ requires an increase in the binding energy, $\varepsilon_0$. Thus, the correction is negative, and increases with $m$. Probability $\tilde{P}_m$ is maximal for $m = m_{opt} = c_0(g |\ln T|)^{1/2}$, where $c_0 = (c_1/2c_2)^{1/2}$. The resulting expression for $\tilde{P}_m$ with the improved accuracy reads
\[
|\ln \mathcal{P}_m| = \frac{2}{3} \pi g |\ln T| \left( 1 + \frac{9}{8} c_0^{4/3} \left[ \frac{|\ln T|}{g} \right]^{1/3} \right).
\]  

(20)

It is seen from Eq. (20) that the above consideration is valid within the domain \(1 \ll \ln T \ll g\). For the optimal \(m\) the ring radius is \(\rho_c = c_0 k_F^{-1} (g |\ln T|)^{1/2} \ll l\), whereas the ring width is \(w = c_0^{1/3} k_F^{-1} (g |\ln T|)^{1/6}\).

Note in conclusion, that the function \(\mathcal{P}(T)\) studied in the present paper describes the distribution of the ALS widths. Therefore, among asymptotical distributions of various characteristics of ALS (see Ref. [4]), \(\mathcal{P}(T)\) corresponds to the distribution of the relaxation times. The random potential fluctuations that determine this distribution are shallow compared to \(E_F\). Whether these fluctuations emerge within \(\sigma\)-model-based calculations remains to be seen.

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FIG. 1. (a) Illustration of the crossover between orthogonal and unitary universality classes. Equipotential lines of the warped and ring-shaped fluctuations are shown schematically. (b) Effective potential for the angular harmonics $m$. Optimal $V_0(\rho)$ at $\delta = 0$ (zero magnetic field) and at critical $\delta \approx \frac{1}{3}$ are plotted with solid and dashed lines, respectively.
FIG. 2. Normalized logarithm of the ALS density is shown versus the dimensionless magnetic field, $\delta$. The inset shows the “orthogonal” optimal wave function $(\tilde{\chi}_m, \tilde{\chi}_{-m})$ for crossover value $\delta \approx 1/3$. 