Black holes and dark energy from gravitational collapse on the brane

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Abstract. The gravitational collapse of a pressureless fluid in general relativity (Oppenheimer–Snyder collapse) results in a black hole. The study of the same phenomenon in the brane-world scenario has shown that the exterior of the collapsing dust sphere cannot be static. We show that by allowing for pressure, the exterior of a fluid sphere can be static. The gravitational collapse on the brane proceeds according to the modified gravitational dynamics, turning the initial nearly dust-like configuration into a fluid with tension. The tension arises from the nonlinearity of the dynamical equations in the energy–momentum tensor, and it vanishes in the general relativistic limit. Below the horizon the tension turns the star into dark energy. This transition occurs right below the horizon for astrophysical black holes and far beyond the horizon for intermediate mass and galactic black holes. Further, both the energy density and the tension increase towards infinite values during the collapse. The infinite tension, however, could not stop the formation of the singularity.

Keywords: dark energy theory, black holes, extra dimensions

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1. Introduction

According to brane-world models our observable universe is a four-dimensional (4D) hypersurface (the brane) with tension \( \lambda \), embedded into a five-dimensional (5D) curved space–time (the bulk). Standard model fields act on the brane; however, gravitation spreads out into all five dimensions and it evolves according to the 5D Einstein equation. While in the original Randall–Sundrum (RS) second model [1] a flat brane is embedded into an anti-de Sitter (AdS) bulk in a \( \mathbb{Z}_2 \)-symmetric way, later generalizations have evolved into considering curved branes, embedded both symmetrically and asymmetrically into a bulk characterized by both Weyl and Ricci curvatures. A negative bulk cosmological constant has a warping effect. The bulk can host non-standard model fields, like moduli or dilatonic scalar fields or a radiation of quantum origin. Such models are motivated by string/M-theory.

In brane-world models the projection of the 5D Einstein gravity onto the brane leads to a 4D gravitational dynamics [2], which is different from the Einstein gravity. An essential modification appears at high energies in the form of a new source term in the effective Einstein equation, which is quadratic in the brane energy–momentum tensor. This source term becomes negligible at low energies. Therefore early cosmology is modified: however, late-time cosmology is not. Another modification arises whenever the bulk has a Weyl curvature with non-vanishing projection onto the brane. This is known as the electric part of the bulk Weyl tensor. The possible asymmetric embedding contributes with a third unconventional source term in the effective Einstein equation [3]. So does the non-standard model bulk matter, via the pull-back to the brane of its energy–momentum tensor. Solar System tests, all in the weak gravity regime, could not confirm or disprove the quadratic source term, but can put strong limits on the other types of modifications. Quantum corrections arising from the coupling between bulk gravity and brane matter, known as induced gravity, were introduced originally in [4] and presented in the most generic covariant form (including asymmetry) in [5]. Corrections from higher-order curvature invariants, more specifically Gauss–Bonnet type modifications, which are motivated by the AdS/CFT correspondence, were discussed covariantly in [6].

Even in the simplest of these models, with the Gauss–Bonnet and induced gravity contributions switched off, the possible brane-worlds can be of great variety, according
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to the symmetries of the brane. Friedmann branes, for example, are known to be embeddable into any of the Schwarzschild/Reissner–Nordström/Vaidya–anti-de Sitter bulks (depending on the energy–momentum in the bulk; for a systematic treatment see [3]). Cosmological evolution has been extensively studied in this scenario (for symmetric embeddings see [7,8] and references therein; for asymmetric embeddings [3,9] and references therein). The matter in the bulk affects the cosmological evolution on the brane through a 'comoving mass' and a bulk pressure [10,11].

Other branes of cosmological type, like the Einstein brane [12] or the Kantowski–Sachs type homogeneous brane [13], are embedded into a vacuum bulk which is not Schwarzschild–anti-de Sitter (SAdS).

Black hole type branes again are not embedded into a SAdS bulk. The spherically symmetric, static black hole on the brane [14] is given by the Reissner–Nordström metric of general relativity, with the (square of the) electric charge replaced by a tidal charge. The tidal charge \( q \) can take both positive and negative values, in contrast with the general relativistic case, when \( q = Q^2 \) represents the square of the electric charge, \( q \) being always positive. Due to the \( q \) term, the tidal charged black hole presents \( r^{-2} \) type corrections to the Schwarzschild potential. This has to be contrasted with the \( r^{-3} \) type correction to the Schwarzschild potential [1,15,16], arising in the weak field analysis of the spherically symmetric gravitational collapse on the brane in the original RS set-up. As such corrections are related to the electric part of the bulk Weyl curvature (the Kaluza–Klein (KK) modes of gravity), the bulk in which the tidal charged black hole can be embedded cannot be SAdS. Neither is the bulk containing a Schwarzschild black hole (\( q = 0 \)) on the brane. In fact, the bulk containing the tidal charged brane black hole remains unknown.

Incorporating black hole singularities in brane-world models is a difficult task. In the original, simplest RS model [1], containing a flat brane, there are no black holes at all. In its curved, cosmological generalizations no black holes can exist on the brane—except as test particles. Among the black hole space–times of general relativity, remarkably, the Schwarzschild solution still solves the modified gravitational equations on the brane, under the assumptions of vacuum and no electric bulk Weyl source term. (This is the tidal charged brane black hole with \( q = 0 \).) It was conjectured in [17] that a Schwarzschild brane can be embedded in the bulk only by extending the singularity into the bulk. In this way one obtains a black string with singular AdS horizon. Due to the Gregory–Laflamme instability [18] the black string can decay into a black cigar [19], although other arguments show that, under very mild assumptions, classical event horizons cannot pinch off [20]. Stable black string solutions with no Weyl source arise in the two-brane model of [21]. Recently, the gravity wave perturbations of such a black-string brane-world were computed [22]. Only if the bulk contains exotic matter is the Schwarzschild brane black hole allowed to have a regular AdS horizon [23]. In any case the brane black hole is not localized on the brane. More generically, in brane-worlds any event horizon on the brane can hide a singularity which may not even be on the brane.

Rotating stationary axisymmetric black holes with tidal charge, localized on the brane in the RS brane-world model, were presented in [24] and brane black hole solutions in a simple model with induced gravity were given in [25,26]. The gravitational collapse on the brane in the presence of curvature corrections was also studied in [27]. The formation of black holes is quite different in brane-worlds, as compared to general relativity. This is because well-known processes from general relativity are modified due
to the unconventional brane-specific sources. However, the ‘energy–momentum squared’ source term becomes dominant at high energies. Such high energies are certainly occurring in the final stages of the gravitational collapse, therefore serious modifications are to be expected in comparison with the general relativistic gravitational collapse.

Based on the tidal charged brane black hole solution, which is set as the exterior of the collapsing object, the gravitational collapse of a dust sphere was analysed [28] and, indeed, sharp differences as compared to the general relativistic Oppenheimer–Snyder collapse [29] were found. First it was shown that the idealized collapse of homogeneous KK energy density with static exterior leads to either a bounce, a black hole or a naked singularity. This result has no counterpart in general relativity. Second, in the absence of the tidal charge (no KK energy density), the vacuum surrounding the collapsing sphere of dust could not be static. This is in sharp contrast with general relativity, where the Birkhoff theorem yields the Schwarzschild solution outside any spherically symmetric configuration as the unique vacuum. The non-static exterior of the collapsing brane star could be the Vaidya radiating solution on the brane [30]. Moreover, this can be regarded as an intermediate radiation layer, and matched from the exterior to the tidal charged brane black hole solution [31]. According to this model, the spherically symmetric collapse on the brane is accompanied by radiation, in contrast with general relativity. Alternatively, in a special toy model a Hawking flux was shown to appear from a collapsing spherically symmetric dust object on the brane [32]. A related result states that the vacuum exterior of a spherical star has radiative-type stresses, and again the exterior is not a Schwarzschild space–time [33]. An effective Schwarzschild solution on the brane was, however, shown to exist if there is energy exchange between the bulk and the brane collapsing star [34].

The above mentioned results refer to collapsing spherically symmetric matter configurations with vanishing surface pressure. But as pointed out first in [35], the junction conditions on the boundary of a star in brane-world theories do not necessarily require a vanishing pressure on the junction surface. This is because the multitude of source terms in the effective Einstein equation can conspire in such a way that the effective pressure still vanishes on the junction surface with the vacuum exterior, in spite of a non-vanishing ordinary pressure.

The same conclusion on the junction surface emerged in [36], where the possibility of a Swiss-cheese brane-world was raised. The generic junction conditions between the Schwarzschild vacua embedded in the FLRW brane along spheres of constant comoving radii were exploited in [37] and [38] by discussing models of black strings penetrating a cosmological brane.

In this paper we drop the assumption of vanishing surface pressure on the boundary of the collapsing matter configuration. Obviously then the dust model for the collapsing matter is given up. The gravitational collapse of spherically symmetric perfect fluid matter configuration can be modelled by immersing a FLRW sphere into a static vacuum. Mathematically the problem of a collapsing perfect fluid sphere immersed into a static vacuum becomes very similar to the reverse problem of embedding of the Schwarzschild vacua into FLRW branes.

By lifting the requirement of vanishing pressure we can achieve a static exterior of the collapsing star even in the absence of the KK energy density and even without inserting an intermediate radiation layer. This will be shown in the following section. In the original RS set-up the assumption of a static exterior without tidal charge can be justified.
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whenever the radius of the collapsing sphere is much higher then the characteristic scale $L$ of the extra dimension [39]. With $L = 0.1 \text{ mm}$ from table-top experiments [40] and for astrophysical or galactic black holes, this is certainly the case.

In section 3 we discuss the results and comment on the formation of the black hole and singularity. Finally, section 4 contains the concluding remarks.

Throughout the paper we use units $G = 1 = c$.

2. Spherically symmetric collapse of a fluid in a static exterior

We choose a simple scenario, with vanishing cosmological constant on the brane, $\Lambda = 0$ (Randall–Sundrum fine-tuning). The collapsing star is described by the FLRW metric with flat spatial sections, $k = 0$. The static vacuum exterior is the Schwarzschild metric. This simplification arises by switching off the KK modes. Table-top experiments [40] on possible deviations from Newton’s law currently probe gravity at submillimetre scales and as a result they constrain the characteristic curvature scale of the bulk to $L = 0.1 \text{ mm}$. Therefore our results will apply to collapse situations which may end in black holes with radii $r_H \gg L$. With this choice we focus on the effect of the nonlinear source terms on the gravitational collapse.

The junction condition of these two space–times along spheres of constant comoving radius $\chi_0$ was derived in [36]:

$$a a^2 = \frac{2m}{\chi_0^3}, \quad (1)$$

where $a(\tau)$ is the scale factor in the FLRW metric. We apply the above result for a stellar model with boundary surface in free fall, given by $\chi = \chi_0 = \text{constant}$. Then $m$ is the Schwarzschild mass of the collapsing star. The integration of equation (1) gives the evolution of the scale factor of the collapsing star in comoving time $\tau$:

$$a^{3/2} = a_0^{3/2} - \left(\frac{9m}{2\chi_0^3}\right)^{1/2} \tau. \quad (2)$$

In order to describe a collapse situation we have chosen the ‘$-$’ root of equation (1) and we have kept the integration constant $a_0$, which represents the scale factor at the beginning of the collapse (at $\tau = 0$). It is immediate to see that the collapse ends when $a = 0$ is reached, after finite time $\tau_1 = (2\chi_0^3 a_0^3/9m)^{1/2}$.

How is the Schwarzschild mass $m$ related to the integral of the energy density over the volume of the star? The latter, denoted $M$, is

$$M = \frac{4\pi\chi_0^3 a_0^3}{3} \rho. \quad (3)$$

From equation (2) we easily derive

$$\dot{a}^2 = \frac{2m}{\chi_0^3} \left[a_0^{3/2} - \left(\frac{9m}{2\chi_0^3}\right)^{1/2} \tau\right]^2 = \frac{8\pi m \rho}{3M}. \quad (4)$$

The stellar perfect fluid obeys the brane Friedmann equation as well:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi \rho}{3} \left(1 + \frac{\rho}{2\lambda}\right). \quad (5)$$

1 In the cosmological case the corresponding choices were ‘$+$’ and $a_0 = 0$, see [36].
By comparing the two expressions for \( \dot{a}^2 \) we find the relation between the ‘physical’ mass \( M \) and the mass \( m \) seen from the exterior, Schwarzschild region of the brane:

\[
m = M \left( 1 + \frac{\rho}{2\lambda} \right).
\]

(6)

Obviously, both \( m \) and \( M \) cannot be constants, except for the trivial case \( m = M = 0 \), or in the general relativistic limit \( \rho/\lambda \to 0 \), when the masses become equal. That both masses cannot be constant in the brane-world collapsing star model was already pointed out in [28].

If \( M \) would be constant, the energy density of the star would scale as \( \rho(\tau) \sim a^{-3} \), cf equation (3). The star therefore would consist of dust, as the pressure would vanish by virtue of the continuity equation

\[
\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = 0.
\]

(7)

For such a dust sphere the exterior cannot be static, and we arrive to the no-go result presented in [28].

However, as remarked earlier, it is not compulsory to impose a vanishing pressure in the star. The energy density of an ideal fluid with pressure does not evolve as \( a^{-3} \). This means that \( M \) varies. Then the exterior can be held static, provided \( M \) varies in the proper way.

In order to interpret the Schwarzschild mass \( m \) in terms of the interior metric, we transform the FLRW metric

\[
ds_{\text{FLRW}}^2 = -d\tau^2 + a^2(\tau) \left[ d\chi^2 + \chi^2 \left( d\theta^2 + \sin^2 \theta \, d\varphi^2 \right) \right]
\]

(8)

into the standard form of spherically symmetric metrics:

\[
ds_{\text{FLRW}}^2 = -e^{2\psi(R)} F(R) \, dT^2 + F(R)^{-1} \, dR^2 + R^2 \left( d\theta^2 + \sin^2 \theta \, d\varphi^2 \right).
\]

(9)

The relation between the two sets of coordinates is \( T = T(\tau, \chi) \) and \( R = R(\tau, \chi) = a(\tau) \chi \). Therefore \( dT = T \, d\tau + T' \, d\chi \), and \( dR = a \chi \, d\tau + a \, d\chi \). From the \( \chi-\tau \) block of the metric we obtain:

\[
\dot{a}^2 \chi^2 + F = e^{2\psi} F^2 T'^2,
\]

(10)

\[
a \dot{a} \chi = e^{2\psi} F^2 T' T,
\]

(11)

\[
a^2 (1 - F) = e^{2\psi} F^2 T'^2.
\]

(12)

By multiplying the first equation with the third one and eliminating the derivatives of \( T \) with the second one we obtain the metric function \( F = 1 - \dot{a}^2 \chi^2 \). Now we define the mass \( m \) contained inside radius \( R_0 = a \chi_0 \) with the metric coefficient \( F \) as

\[
F(R_0) = 1 - \frac{2m}{R_0},
\]

(13)

thus the mass at \( \chi_0 \) is given by

\[
\frac{2m}{a^3 \chi_0^3} = \frac{\dot{a}^2}{a^2}.
\]

(14)
Finally, by applying equation (5) the mass $m$ emerges as

$$m = \frac{4\pi a^3 \chi_0^3 \rho}{3} \left(1 + \frac{\rho}{2\lambda}\right).$$

(15)

This is different from the usual relation among the mass, density and volume, represented by equation (3), but reduces to it in the general relativistic limit. By applying equation (3), we recover the relation (6) between the masses $m$ and $M$, which justifies the notation $m$ in equation (13).

It is easy to show that $m$ is the quasilocal mass appearing in the Bondi-type coordinates used by Bardeen [41]. For this we need to further transform the metric (9) into either the advanced or the retarded Bardeen coordinates $(v, R, \theta, \phi)$. The null coordinate $v$ is given as $dv = dT + ce^{-\psi} F^{-1} dR$, with $c = \pm 1$ (the sign $+$ holds for advanced, $-$ for retarded). We obtain

$$ds_{FLRW}^2 = - e^{2\psi} F dv^2 + 2 ce^{\psi} dR dv + R^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

(16)

and conclude that $m$ defined by equation (13) is the Bardeen quasilocal mass.

The mass of the star is its Bardeen quasilocal mass $m$, rather than the ‘physical’ mass $M$. It is not surprising that these differ. The Bardeen mass contains contributions not only from matter, but from gravitational energy as well. Therefore it should be different compared to the general relativistic Bardeen mass (which agrees with the ‘physical’ mass $M$) because in brane-worlds the gravitational dynamics is modified (in the present case by $\rho^2$ source terms).

In the chosen simple scenario, with no bulk matter and no Weyl contribution to the sources from the bulk, the effective Einstein equation for the exterior region is the vacuum Einstein equation of general relativity. For spherical symmetry therefore the Birkhoff theorem applies. Then the exterior Schwarzschild solution with mass parameter $m$ is the unique exterior for the collapsing spherically symmetric matter configuration. As its Schwarzschild mass $m$ agrees with the Bardeen quasilocal mass in the star, the Bardeen mass $m$ rather than the ‘physical’ mass $M$ should be taken as constant.

Next, we proceed to derive $\rho(\tau)$ under the assumption $m = \text{constant}$. This is immediate by inserting $a(\tau)$ given by equation (2) into (3). The energy density $\rho(\tau)$ is determined by a quadratic equation

$$\frac{\rho^2}{\lambda^2} + 2\frac{\rho}{\lambda} - \frac{3m}{2\pi \lambda \chi_0^3 \left[a_0^{3/2} - (9m/2\chi_0^{3})^{1/2}\right]} \tau^2 = 0,$$

(17)

with the solutions

$$\rho_\pm = -1 \pm \sqrt{\frac{3m}{2\pi \lambda \chi_0^3 \left[a_0^{3/2} - (9m/2\chi_0^{3})^{1/2}\right]} \tau^2}.$$

(18)

The physical solution is $\rho_+$, this being positive for any $\tau < \tau_1$. The energy density $\rho_+$ is increasing in time, reaching an infinite value at $\tau_1$, at the end of collapse. During the collapse, the ‘physical’ mass $M$, scaling with $(1 + \rho/\lambda)^{-1}$, decreases continuously towards zero.
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Figure 1. The two branches of energy density $\rho_{\pm}$ in the collapsing star, plotted for a mass $m = 2\pi\lambda\chi_0^3/3$. The collapse starts at $a_0 = 1$ and the time $\tau$ is given in units of $(9m/2\chi_0^3)^{1/2}$. An infinite density singularity is reached on the $\rho_+$ branch at $\tau = \tau_1 = 1$.

From the junction condition (1) we find

$$\frac{\ddot{a}}{a} = -\frac{m}{\chi_0^3 a^3} a^3,$$

while the Raychaudhuri equation for the FLRW fluid sphere gives another expression for $\ddot{a}$:

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} \left[ \rho \left( 1 + \frac{2\rho}{\lambda} \right) + 3p \left( 1 + \frac{\rho}{\lambda} \right) \right].$$

Combining these, we obtain the equation describing the evolution of the fluid pressure and density with radius:

$$\rho \left( 1 + \frac{2\rho}{\lambda} \right) + 3p \left( 1 + \frac{\rho}{\lambda} \right) = \frac{3m}{4\pi\chi_0^3 a^3}.$$

The pressure then emerges as the algebraic root of the quadratic equation (21):

$$\frac{p_+}{\lambda} = 1 \pm \frac{1}{2} \left[ 1 + \frac{3m}{2\pi\lambda\chi_0^3 \left[ a_0^{3/2} - (9m/2\chi_0^3)^{1/2} \tau \right]^2} \right]^{1/2} \mp \frac{1}{2} \sqrt{\frac{3m}{2\pi\lambda\chi_0^3 \left[ a_0^{3/2} - (9m/2\chi_0^3)^{1/2} \tau \right]}}.$$

Thus our original assumption of a static exterior leads to the conclusion that the fluid is not dust. Time evolution of its energy density and pressure are represented in figures 1 and 2, respectively.
3. Discussion

By lifting the condition of vanishing pressure in the spherically symmetric collapsing object we have arrived at the conclusion, that a static vacuum exterior is possible at the price of an unexpected behaviour of the fluid. What does the fluid composing this brane-world star consist of? From equations (18) and (22) we find a simpler form of the equation describing the evolution of the pressure with density as the radius of the star shrinks:

$$\frac{p_{\pm}}{\lambda} = \frac{1}{2} \left(1 - \frac{\rho_{\pm}}{\lambda}\right) - \frac{1}{2} \left(1 + \frac{\rho_{\pm}}{\lambda}\right)^{-1}. \quad (23)$$

This equation need not necessarily be interpreted as an equation of state for the collapsing fluid. Rather it expresses the run of pressure with density during the collapse. It is similar to the assumption of polytropes as Newtonian pseudo-stellar models, where a simple power law dependence $p = K \rho^{1 - 1/n}$ is chosen as an expression of the evolution of $p$ with radius, in terms of the evolution of $\rho$ with radius. Such an equation satisfies the mass equation and the equation for hydrostatic equilibrium, but no reference to heat transfer or thermal balance is made [42]. Even so, polytropes were found useful in the study of many aspects of real stellar structure.

The relation (23) was obtained by solving the brane-world generalization for a collapse situation of the mass equation and hydrostatic equilibrium equations, and as we will see in what follows, its interpretation is also related to the polytropic pseudo-stellar models.

In the initial, low-energy regime of the collapse ($|\rho_{\pm}| \ll \lambda$) the collapsing brane-world fluid approaches a polytrope with the run of pressure versus energy density given by

$$p_{\pm} \approx -\frac{\rho_{\pm}^2}{2\lambda}. \quad (24)$$
This polytrope is characterized by the constant $K = -1/2\lambda$, and polytropic index $n = 1$. The strongest bound on the minimal value of $\lambda$ was derived by combining the results of table-top experiments on possible deviations from Newton’s law, which probe gravity at submillimetre scales [40] with the known value of the four-dimensional Planck constant. In the 2-brane model [21] this gives [43] $\lambda > 138.59 \text{ TeV}^4$. A much milder limit $\lambda \gtrsim 1 \text{ MeV}^4$ arises from the constraint that the dominance of the quadratic effects ends before the big bang nucleosynthesis [44]. From astrophysical considerations on brane neutron stars an intermediate value $\lambda > 5 \times 10^8 \text{ MeV}^4$ was derived [33]. (Note that all these limiting values are given for $c = 1 = \hbar$, while the expressions of this paper are given in units $c = 1 = G$.

In this latter system of units the corresponding minimal values of the brane tension are $\lambda_{\text{tabletop}} = 4.2 \times 10^{-119} \text{ eV}^{-2}$, $\lambda_{\text{BBN}} = 3 \times 10^{-145} \text{ eV}^{-2}$ and $\lambda_{\text{astro}} = 1.5 \times 10^{-136} \text{ eV}^{-2}$, respectively.) Thus the constant becomes either a tiny or a huge number, depending on the chosen system of units. However, with customary values of the stellar density (for the Sun $\rho_\odot = 1408 \text{ kg m}^{-3}$, which in units $c = 1 = G$ gives $\rho_\odot = 1.8 \times 10^{-150} \text{ eV}^{-2}$), the condition $\rho/\lambda \ll 1$ is obeyed no matter which one of the available lower bounds and which system of units is chosen. The collapsing fluid in the star is then indistinguishable from ordinary dust.

At the final stage of the collapse, when $|\rho_\pm| \gg \lambda$, the pressure tends to

$$p_\pm \approx -\frac{\rho_\pm}{2}. \quad (25)$$

This is again a polytrope with constant $K = -1/2$ and polytropic index $n \to \infty$. The condition for dark energy $\rho_+ + 3p_+ \approx -\rho_+/2 < 0$ is then satisfied on the physical branch. This provides a mechanism of how an initial configuration of (nearly) pressureless fluid turns into dark energy due to gravitational collapse in the brane-world scenario.

In the general relativistic limit ($\rho/\lambda \to 0$) the fluid sphere degenerates into a spherically symmetric dust cloud and it remains dust (no pressure) until the end of the collapse, which of course is the highly idealized picture of the Oppenheimer–Snyder collapse.

The collapse is different in a brane-world. On the physical branch, as the end of the collapse is approached ($\tau \to \tau_1$), the pressure $p \to -\infty$. This means that an enormous isotropic tension appears in the brane-world star as the collapse proceeds towards its final stage. The role of any tension (like in solids) is to restore the original configuration. The enormous tension appearing in the latter stages of collapse therefore by analogy could be regarded as the backreaction of the brane towards the stretching effect of the collapsing matter. We will comment on the interpretation of the emerging tension later in this section.

The tension exceeding considerably the brane tension $\lambda$ is still incapable to stop the collapse and the formation of the singularity. Why is the tension, or equivalently, the emerging dark energy, incapable to stop the collapse, especially as $M \to 0$? The answer lies in the source terms quadratic in the energy–momentum. Towards the end of the collapse the linear source terms in the Raychaudhuri equation (20) sum up to $2\pi p_\pm^2/3$ (dark energy type source on the physical branch). However, the quadratic source terms give $-2\pi p_\pm^2/3\lambda$. The latter is dominant and the collapse proceeds until the singularity is formed.

The black hole is formed much earlier. This happens at $\tau_H$, when the radius $R(\tau) = a(\tau)\chi_0$ of the collapsing fluid sphere reaches the horizon, which is at $r_H = 2m$. 


From equation (2) we get

$$\tau_H = \frac{4m}{3} \left[ \left( \frac{a_0 \chi_0}{2m} \right)^{3/2} - 1 \right].$$  (26)

The energy density and pressure of the fluid at horizon crossing is, cf equations (18) and (22):

$$\frac{(\rho \pm)_H}{\lambda} = -1 \pm \frac{1}{2} \sqrt{1 + \frac{3}{16\pi\lambda m^2}}.$$  (27)

$$\frac{(p \pm)_H}{\lambda} = 1 \pm \frac{1}{2} \sqrt{1 + \frac{3}{16\pi\lambda m^2}} \mp \frac{1}{2} \sqrt{1 + 3/(16\pi\lambda m^2)}.$$  (28)

Therefore

$$\frac{(\rho \pm)_H + 3(p \pm)_H}{\lambda} = 2 \mp \frac{1}{2} \sqrt{1 + \frac{3}{16\pi\lambda m^2}} \mp \frac{3}{2} \sqrt{1 + 3/(16\pi\lambda m^2)}.$$  (29)

This expression, on the physical branch is positive between the roots $(\lambda m^2)_1 \to \infty$ and $(\lambda m^2)_2 = 3/128\pi$ and negative for any $\lambda m^2 < 3/128\pi$.

For astrophysical or galactic black holes the dark energy condition could be obeyed only below the horizon. From equation (21) we can find the density–radius relation, at which $\rho + 3p = 0$ occurs:

$$\frac{\rho_H^2}{\lambda} = \frac{3m}{4\pi r^3_{de}}.$$  (30)

Combined with equations (2) and (18), this gives the radius of dark energy crossing

$$r_{de} = \frac{A^{1/3}}{\mu^{2/3}r_H},$$  (31)

with $\mu$ the mass of the black hole expressed in units of solar masses $M_\odot = 1.1154 \times 10^{66}$ eV (for $c = 1 = G$) and

$$A = \frac{3}{128\pi \lambda M_\odot^2}.$$  (32)

a dimensionless constant.

With the astrophysical limit set on the brane tension, which in units $c = 1 = G$ is $\lambda_{\text{astro}} = 1.5 \times 10^{-136}$ eV$^{-2}$ we obtain $A = 40$ (from $\lambda M_\odot^2 = 1.8662 \times 10^{-4}$) and

$$r_{de} = 3.42 \mu^{-2/3}r_H = \mu^{1/3} \times 7.6293 \times 10^{66}$$ eV.  (33)

The first expression shows that the bigger the black hole, the more the fluid has to collapse below the horizon before the dark energy condition is obeyed. For example, for astrophysical black holes with $\mu = 10$, 100 and galactic black holes with $10^4$, $10^6$, $10^8$ the ratio $r_{de}/r_H$ is 0.737, 0.159 and 7.37 $\times 10^{-3}$, 3.42 $\times 10^{-4}$, 1.59 $\times 10^{-5}$ respectively.

The second expression in equation (33) shows that the radius where the dark energy condition is obeyed increases with the cubic root of $\mu$. For the above examples $r_{de}$ expressed in units $10^{66}$ eV takes the values 1.64, 3.54 and 16.44, 76.29 and 354.12.
We note from the equations (30)–(32) that the transition occurs at the extreme high density $\rho = 2\lambda$, where the perfect fluid approximation may break down. For comparison with the known lower limits for the brane tension $\lambda_{\text{tabletop}} = 4.2 \times 10^{-119} \text{ eV}^{-2}$, $\lambda_{\text{BBN}} = 3 \times 10^{-145} \text{ eV}^{-2}$ and $\lambda_{\text{astro}} = 1.5 \times 10^{-136} \text{ eV}^{-2}$, we give here the density range for the densest known stellar objects, the neutron stars, extending from $\rho_{\text{ns}} = 8 \times 10^{16}$ up to $2 \times 10^{18} \text{ kg m}^{-3}$, which in units $c = 1 = G$ give $\rho_{\text{ns}}^{\text{min}} = 1 \times 10^{-136} \text{ eV}^{-2}$ and $\rho_{\text{ns}}^{\text{max}} = 2.6 \times 10^{-135} \text{ eV}^{-2}$. We see that the astrophysical limit is of the same order of magnitude as the density of the neutron stars, which in turn is comparable to the density of the atomic nucleus.

What would be the mass of a black hole for which the dark energy crossing occurs exactly on the horizon? From equation (29) this is the root $m = (3/128\pi\lambda)^{1/2} = A^{1/2}M_\odot = 6.32 M_\odot$. The same result stems from equation (33) by imposing $r_{\text{de}}/r_H = 1$. Remarkably, this is about the minimal mass required for black hole formation, right above the maximally allowed mass for neutron stars, given by the Tolman–Oppenheimer–Volkoff limit $m_{\text{max}}^{\text{ns}} = 1.5–3 M_\odot$ [49]–[51].

Whenever $\lambda m^2 \gg 1$ can be assumed (thus $\mu \gg 73$), to second order in the small parameter $(\lambda m^2)^{-1}$ on the physical branch

$$\rho_H = \frac{3}{32\pi m^2} \left( 1 - \frac{3}{64\pi \lambda m^2} \right),$$

(34)

$$p_H = -\frac{9}{2048\pi^2 \lambda m^2}$$

(35)

and

$$\rho_H + 3p_H = \frac{3}{32\pi m^2} \left( 1 - \frac{3}{16\pi \lambda m^2} \right).$$

(36)

This barely differs from the general relativistic value, as the second, brane-world-induced term represents a tiny negative correction. The composition of the collapsing fluid remains basically dust at horizon crossing. For galactic black holes thus $\rho + 3p > 0$ everywhere above the horizon and also in the greatest part of the collapse below the horizon.

It is to be expected that, due to the existence of a fundamental length scale, the Planck length (which in units $c = 1 = G$ is $L_P = 1.221 \times 10^{28} \text{ eV}$), the brane model as presented here, with the brane as an infinitesimal hypersurface, will break down. Rather, a finite thickness should be assumed for the brane. Such thick brane models were already studied, either with a scalar field on the thick brane [45, 46] or another matter form with transverse pressure component [47, 48], meant to replace the brane tension. We expect that the emergence and increase without bounds of the tension in the collapsing fluid (23) could be derived in a suitable limit of some thick brane model, from the interaction of the fluid with the matter configuration of the thick brane.

4. Concluding remarks

We have considered a simple brane-world model containing a collapsing, spherically symmetric stellar configuration on the brane. By considering only objects with gravitational radius much higher than the characteristic curvature scale of the bulk we have suppressed the KK modes. Then the exterior of the collapsing star is described by
the general relativistic vacuum Einstein equation and, due to the Birkhoff theorem, the exterior should be the Schwarzschild solution.

Under these circumstances we have shown that the gravitational collapse of a near-dust-like perfect fluid star can occur with a static exterior. Due to the modified brane-world dynamics a tension rises during the collapse, and the ‘physical’ mass of the star gradually diminishes, while its Schwarzschild mass, which coincides with the Bardeen quasilocal mass, which stays constant.

For large collapsing objects, leading to galactic or intermediate mass black holes, the collapsing fluid even at horizon crossing can be considered to a remarkably good accuracy as dust. Tensions rise slowly, so that below the horizon the fluid remains ordinary matter for a long time, but in the very latest stages of the collapse it turns into dark energy.

By contrast, for the lowest mass astrophysical black holes (with the astrophysical limit set for the brane tension), an important tension appears already at the horizon crossing so that the dark energy condition could be obeyed shortly after the horizon crossing.

This peculiar behaviour of the collapsing star stays hidden below the horizon, such that a distant outside observer will sense nothing but an usual black hole with mass $m$. In spite of the dark energy condition being satisfied somewhere below the horizon for all black holes, the collapse of the fluid will not be stopped or even slowed down. Due to the energy-squared source term, appearing in the modified brane dynamics, the fluid further evolves into a central singularity. Thus as in general relativity, the singularity is formed in brane-world theory too.

The appearance of the tension in the collapsing fluid is a pure brane-world effect. As we have disregarded the Weyl-source term of the bulk, the rising tension is due solely to the source term quadratic in the fluid energy–momentum tensor. This source term modifies the early cosmology (before BBN) in an essential way. The same happens in the case of the gravitational collapse: a tension rises, induced by the nonlinearity of the dynamical equations in the energy–momentum tensor. In this sense, the tension in the fluid is an expression of the interaction of the fluid with the brane.

However as long as the brane is considered as a hypersurface in the mathematical sense (with only one characteristic, the brane tension) such an interaction could not be described phenomenologically. This feature of the model may be an indication for the need of a description based on thick-branes.

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