Active and sterile neutrino phenomenology with $A_4$ based minimal extended seesaw

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Abstract

We study a model of neutrino within the framework of minimal extended seesaw (MES), which plays an important role in active and sterile neutrino phenomenology in (3+1) scheme. The $A_4$ flavor symmetry is augmented by additional $Z_4$ symmetry to constraint the Yukawa Lagrangian of the model. We use non-trivial Dirac mass matrix, with broken $\mu - \tau$ symmetry, as the origin of leptonic mixing. Interestingly, such structure of mixing naturally leads to the non-zero reactor mixing angle $\theta_{13}$. Non-degenerate mass structure for right-handed neutrino $M_R$ is considered so that we can further extend our study to Leptogenesis. We have also considered three different cases for sterile neutrino mass, $M_S$ to check the viability of this model, within the allowed 3σ bound in this MES framework.

Keywords: Beyond Standard Model, Minimal extended seesaw, Sterile neutrino, Flavor symmetry

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I. INTRODUCTION

Followed by the discovery of the Higgs Boson, the Standard Model (SM) of particle physics is essentially complete, although there are some insufficiencies in the theory. One needs to extend the SM in order to address problems like origin of neutrino mass, dark matter, strong CP problem and matter-antimatter asymmetry, etc. Several neutrino oscillation experiments like MINOS[1], T2K[2], RENO[3], DOUBLE CHOOZ[4], DAYABAY[5], SK[6], SNO[7] etc. have established the fact that neutrinos produced in a well-defined flavor eigenstate can be detected as a different flavor eigenstate while they propagate. This can be interpreted as, like all charged fermions, neutrinos have mass and mixing because their flavor eigenstates are different from mass eigenstates. The existence of neutrino mass was the first evidence for the new physics beyond the Standard Model (BSM). Some recent reviews on neutrino physics are put into references [8, 9, 17].

In standard neutrino scenario three active neutrinos are involved with two mass square differences\(^1\), three mixing angles (\(\theta_{ij}; i, j = 1, 2, 3\)) and one Dirac CP phase (\(\delta_{13}\)). Earlier it was assumed that the reactor mixing angle \(\theta_{13}\) is zero but later in 2012 it was measured with incredible accuracy: \(\theta_{13} \sim 8.5^0 \pm 0.2^0\[5\]. If neutrinos are Majorana particles then there are two more CP violating phases (\(\alpha\) and \(\beta\)) come into the 3-flavor scenario. Majorana phases are not measured experimentally as they do not involve in the neutrino oscillation probability. The current status of global analysis of neutrino oscillation data [18–20] give us the allowed values for these parameters in 3\(\sigma\) confidence level, which is shown in Table I. Along with the Majorana phases, the absolute mass scale for the individual neutrino is still unknown as the oscillation experiments are only sensitive to the mass square differences, even though Planck data constrained the sum of the three neutrinos, \(\Sigma m_{\nu} < 0.17eV\) at 95% confidence level [21]. Due to the fact that absolute scale of the neutrino mass is not known yet, as the oscillation probability depends on the mass square splittings but not the absolute neutrino mass. Moreover, neutrino oscillation experiments tell that the solar mass square splitting is always positive, which implies \(m_2\) is always greater than \(m_1\). However, the same confirmation we have not yet received regarding the atmospheric mass square splittings from the experiments. This fact allows us to have two possible mass hierarchy

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1 order of \(10^{-5}eV^2\) and \(10^{-3}eV^2\) for solar (\(\Delta m^2_{21}\)) and atmospheric (\(\Delta m^2_{23}/\Delta m^2_{13}\)) neutrino respectively.
patterns for neutrinos; Normal Hierarchy (NH: $m_1 \ll m_2 < m_3$) as well as Inverted Hierarchy (IH: $m_3 \ll m_1 < m_2$).

In past few decades, there has been successful achievements in solar, reactor and accelerator experiments whose results are in perfect agreement with only three active neutrino scenario meanwhile there are some anomalies which need explanation. The very first and most distinguished results towards new physics in the neutrino sector were from LSND results [22–24], where electron anti-neutrino ($\bar{\nu}_e$) were observed in the form of muon anti-neutrino ($\bar{\nu}_\mu$) beam seemingly $\bar{\nu}_e$ was originally $\bar{\nu}_\mu$. Moreover, data from MiniBooNE [25] results overlap with LSND results and give an indication towards extra neutrino hypothesis. To make sure that these data are compatible with current picture one needs new mass eigenstates for neutrinos. These additional states must relate to right-handed neutrinos (RHN) for which bare mass term are allowed by all symmetries i.e. they should not be present in $SU(2)_L \times U(1)_Y$ interactions, hence are Sterile. Recently observed Gallium Anomaly observation[26–28] is also well explained by sterile neutrino hypothesis. Although there are few talks about the non-existence of extra neutrino, but finally reactor anti-neutrino anomaly results [29, 30] give a clear experimental proof that the presence of this fourth non-standard neutrino is mandatory. Moreover, cosmological observation [31] (mainly CMB\textsuperscript{2} or SDSS\textsuperscript{3}) also favor the existence of sterile neutrino. From cosmological consequences, it is said that the sterile neutrino has a potential effect on the entire Big-Bang Nucleosynthesis [32]. Thus, hints from different backgrounds point a finger towards the presence of a new generation of neutrinos.

Sterile neutrino is a neutral lepton which does not involve itself in weak interactions, but they are induced by mixing with the active neutrinos that can lead to observable effect in the oscillation experiments. Furthermore, they could interact with gauge bosons which lead to some significant correction in non-oscillation processes e.g., in the neutrinoless double beta decay (NDBD) amplitude[33, 34], beta decay spectra. Since RH neutrinos are SM gauge singlets[35], so it is possible that sterile neutrinos could fit in the canonical type-I seesaw as the RH neutrino if their masses lie in the eV regime. Some global fit studies have been carried out for sterile neutrinos at eV scale being mixed with the active neutrinos [36, 37, 64]. While

\textsuperscript{2} cosmic microwave background
\textsuperscript{3} Sloan Digital Sky Survey
doing this the Yukawa Coupling relating lepton doublets and right-handed neutrinos should be of the order $10^{-12}$ which implies a Dirac neutrino mass of sub-eV scale to observe the desired active-sterile mixing. These small Dirac Yukawa couplings are considered unnatural unless there is some underlying mechanism to follow. Thus, it would be captivating to choose a framework which gives low-scale sterile neutrino masses without the need of Yukawa coupling and simultaneously explain active-sterile mixing. In order to accommodate sterile neutrino in current SM mass pattern, various schemes were studied. In $(2+2)$ scheme, two different classes of neutrino mass states differ by $eV^2$, which is disfavored by current solar and atmospheric data [39]. Current status for mass square differences, corresponding to sterile neutrinos, dictates sterile neutrinos to be either heavier or lighter than the active ones. Thus, we are left with either $(1+3)$ or $(3+1)$ scheme. In the first case, three active neutrinos are in eV scale and sterile neutrino is lighter than the active neutrinos. However, this scenario is ruled out by cosmology [31, 40]. In the latter case, three active neutrinos are in sub-eV scale and sterile neutrino is in eV scale [41, 42]. Numerous studies have been exercised taking this $(3+1)$ framework with various prospects [29, 43, 44, 64].

The seesaw mechanism is among one of the most prominent theoretical mechanism to generate light neutrino masses naturally. Various types of see-saw mechanisms have been put in literature till date (for detail one may look at [9, 10, 12–16, 45–48, 50, 51]). In our study, we will focus our model to fit with $(3+1)$ framework where the sterile neutrino is in the eV range and the active neutrinos in sub-eV range. Study of eV sterile neutrino in Flavor symmetry model have been discussed by various authors in [52–56]. There has been plenty of exercises performed in order to study eV scale sterile neutrino phenomenology through the realization of Froggatt-Nielsen (FN) mechanism[57] adopting non-Abelian $A_4$ flavor symmetry in seesaw framework [53, 59–61]. Similar approaches using type-I seesaw framework have been evinced by some authors[53, 59, 62], where type-I seesaw is extended by adding one extra singlet fermion, which scenario is popularly known as the minimal extended seesaw (MES) model. This extension gives rise to tiny active neutrino mass along with the sterile mass without the need of small Yukawa couplings.

In this paper, we have studied the active and sterile neutrino mixing scheme within the MES framework based on $A_4$ flavor symmetry along with the discrete $Z_4$ symmetry. In spite of having $M_D$, $M_R$, $M_S$ matrices, we have introduced a new matrix $M_P$ which is produced from a similar kind of coupling term that produces the Dirac mass matrix ($M_D$). $M_P$ is
| Parameters          | NH (Best fit)       | IH (Best fit)       |
|---------------------|---------------------|---------------------|
| $\Delta m_{21}^2[10^{-5}eV^2]$ | 6.93-7.97(7.73)     | 6.93-7.97(7.73)     |
| $\Delta m_{31}^2[10^{-3}eV^2]$ | 2.37-2.63(2.50)     | 2.33-2.60(2.46)     |
| $\sin^2\theta_{12}/10^{-1}$    | 2.50-3.54(2.97)     | 2.50-3.54(2.97)     |
| $\sin^2\theta_{13}/10^{-2}$    | 1.85-2.46(2.14)     | 1.86-2.48(2.18)     |
| $\sin^2\theta_{23}/10^{-1}$    | 3.79-6.16(4.37)     | 3.83-6.37(5.69)     |
| $\delta_{13}/\pi$           | 0-2(1.35)           | 0-2(1.32)           |
| $\Delta m_{LSND}^2(\Delta m_{41}^2 or \Delta m_{43}^2)eV^2$ | 0.87-2.04(1.63)     | 0.87-2.04(1.63)     |
| $|V_{e4}|^2$               | 0.012-0.047(0.027)  | 0.012-0.047(0.027)  |
| $|V_{\mu 4}|^2$             | 0.005-0.03(0.013)   | 0.005-0.03(0.013)   |
| $|V_{\tau 4}|^2$            | <0.16(-)            | <0.16(-)            |

**TABLE I:** The latest global fit 3\(\sigma\) range and best fit results from recent active neutrino parameters[20]. The current sterile neutrino bounds are from [62, 64].

added to \(M_D\), such that there is a broken \(\mu - \tau\) symmetry which leads to the generation of the non-zero reactor mixing angle. The \(M_D\) matrix constructed for NH does not work for IH, the explanation of which we have given in the model section. Thus, we have reconstructed \(M_D\) by introducing a new flavon (\(\varphi'\)) to the Lagrangian to study the case of IH pattern. A most general case also has been introduced separately where the non-zero \(\theta_{13}\) is automatically generated by a different \(M_D\) constructed with the help of a most general kind of VEV alignment. A non-degenerate mass structure is considered for the diagonal \(M_R\) matrix so that we can extend our future study towards Leptogenesis. We have extensively studied the consequences brought out by taking the sterile mass pattern via altering the position for the non-zero entry in \(M_S\). All these \(M_S\) structures have been studied independently for both the mass ordering.

This paper is organized as follows. In section II brief review of the minimal extended seesaw is given. In section III we have discussed the \(A_4\) model and generation of the mass matrices in the leptonic sector. We keep the section IV and its subsections for numerical analysis in NH and IH case respectively. Finally, the summary of our work is concluded in the section V.
II. THE MINIMAL EXTENDED SEESAW

In the present work we have used Minimal extended seesaw (MES) which enable us to
connect active neutrino with sterile neutrino of a wider range \cite{53}. In this section, we describe
the basic structure of MES, where canonical type-I seesaw is extended to achieve eV-scale
sterile neutrino without the need of putting tiny Yukawa coupling or any small mass term.
In MES scenario along with the SM particle, three extra right-handed neutrinos and one
additional gauge singlet chiral field $S$ is introduced. The Lagrangian of the neutrino mass
terms for MES is given by:

$$-\mathcal{L}_M = \bar{\nu}_L M_D \nu_R + \frac{1}{2} \bar{\nu}_R M_R \nu_R + \bar{S} M_S \nu_R + h.c.,$$

where $M_D$ and $M_R$ are $3 \times 3$ Dirac and Majorana mass matrices respectively whereas $M_S$ is
the $1 \times 3$ matrix. The neutrino mass matrix will be a $7 \times 7$ matrix, in the basis $(\nu_L, \nu^c_R, S^c)$,
reads as

$$M^7_{\nu} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^T \\ 0 & M_S & 0 \end{pmatrix}$$

In the analogy of type-I seesaw the mass spectrum of these mass matrices are considered as
$M_R \gg M_S > M_D$, so that the heavy neutrinos decoupled at low scale. After diagonalizing,
$4 \times 4$ neutrino mass matrix in the basis $(\nu_L, S^c)$, is given by,

$$M^4_{\nu} = - \begin{pmatrix} M_D M^{-1}_{R} M_D^T & M_D M^{-1}_{R} M_S^T \\ M_S (M^{-1}_{R})^T M_D^T & M_S M^{-1}_{R} M_S^T \end{pmatrix}$$

Here in $M^4_{\nu}$ matrix (3), there are three eigenstates exists for three active neutrinos and
one for the light sterile neutrino. Taking the determinant of Eq.(3), we get,

$$\det(M^4_{\nu}) = \det(M_D M^{-1}_{R} M_D^T) \det[-M_S M^{-1}_{R} M_S^T + M_S (M^{-1}_{R})^T M_D^T (M_D M^{-1}_{R} M_D^T)^{-1} (M_D M^{-1}_{R} M_S^T)]$$

$$= \det(M_D M^{-1}_{R} M_D^T) \det[M_S (M^{-1}_{R} - M^{-1}_{R}) M_S^T]$$

$$= 0$$

Here the zero determinant indicates that one of the eigenvalue is zero. Thus, the MES
formalism demands one of the light neutrino mass be exactly vanished.

Proceeding for diagonalization, we face three choices of ordering of $M_S$:
• $M_D \sim M_S$: This indicates a maximal mixing between active and sterile neutrinos which is not compatible with the neutrino data.

• $M_D > M_S$: The light neutrino mass is obtained same as type-I seesaw i.e., $m_\nu \simeq -M_DM_R^{-1}M_D^T$ and the sterile neutrino mass is vanishing. Moreover, from the experimental active-sterile mass squared difference result, the active neutrino masses would be in the eV scale which would contradict the standard Planck limit for the sum of the active neutrinos.

Finally, we have the third choice,

• $M_S > M_D$: which would give the possible phenomenon for active-sterile mixing.

Now applying the seesaw mechanism to Eq. (3), we get the active neutrino mass matrix as

$$m_\nu \simeq M_DM_R^{-1}M_SM_S^{-1}M_D^T - M_DM_R^{-1}M_D^T M_S^T M_D,$$

and the sterile neutrino mass as

$$m_s \simeq -M_SM_R^T M_S^T.$$

The first term of the active neutrino mass does not vanish since $M_S$ is a vector rather than a square matrix. It would lead to an exact cancellation between the two terms of the active neutrino mass term if $M_S$ were a square matrix.

III. THE MODEL

A. Normal Hierarchy

Non-Abelian discrete flavor symmetry like $A_4, S_4$ etc. along with $Z_n$ have played an important role in particle physics. In particular, $A_4$ is more popular in literature in explaining neutrino mass [52–56, 58–61, 63]. $A_4$ being the discrete symmetry group of rotation leaving a tetrahedron invariant. It has 12 elements and 4 irreducible representation denoted by $1, 1', 1''$ and $3$. The product rules for these representations are given in appendix A. Our present work is an extension of $A_4 \times Z_4$ flavor symmetry. Here, we have assigned left-handed (LH) lepton doublet $l$ to transform as $A_4$ triplet whereas right-handed (RH) charged leptons ($e^c, \mu^c, \tau^c$) transform as $1, 1''$ and $1'$ respectively. The flavor symmetry is broken by
the triplets $\zeta, \varphi$ and two singlets $\xi$ and $\xi'$. Besides the SM Higgs $H$, we have introduced two more Higgs ($H', H''$)\cite{58, 62} which remain invariant under $A_4$. We also have restricted non-desirable interactions while constructing the mass matrices. The particle content and the $A_4 \times Z_4$ charge assignment are shown in the table II.

The leading order invariant Yukawa Lagrangian for the lepton sector is given by,

$$\mathcal{L} = \mathcal{L}_{M_l} + \mathcal{L}_{M_D} + \mathcal{L}_{M_R} + \mathcal{L}_{M_S} + h.c.$$  \hspace{1cm} (7)$$

Where,

$$\mathcal{L}_{M_l} = \frac{y_{e}}{\Lambda} (\bar{l}H\zeta)_{1} e_{R} + \frac{y_{\mu}}{\Lambda} (\bar{l}H\zeta)_{1} \mu_{R} + \frac{y_{\tau}}{\Lambda} (\bar{l}H\zeta)_{1} \tau_{R}$$

$$\mathcal{L}_{M_D} = \frac{y_{1}}{\Lambda} (\bar{l}H\varphi)_{1} \nu_{R1} + \frac{y_{2}}{\Lambda} (\bar{l}H'\varphi)_{1} \nu_{R2} + \frac{y_{3}}{\Lambda} (\bar{l}H''\varphi)_{1} \nu_{R3}$$

$$\mathcal{L}_{M_R} = \frac{1}{2} \lambda_{1} \xi \nu_{R1} \nu_{R1} + \frac{1}{2} \lambda_{2} \xi' \nu_{R2} \nu_{R2} + \frac{1}{2} \lambda_{3} \xi \nu_{R3} \nu_{R3}$$  \hspace{1cm} (8)$$

We have extended our study with three variety of $M_S$ structures, which is generated by the interaction of a singlet field $S_i$ and the right-handed neutrino $\nu_{Ri}$. The $A_4 \times Z_4$ charge alignment for the scalar fields are given in table III. The effective mass term for each of the above three cases are as follows,

$$\mathcal{L}_{M_{S1}} = \frac{1}{2} \rho \chi S_{R1} \nu_{R1}$$

$$\mathcal{L}_{M_{S2}} = \frac{1}{2} \rho \chi S_{R2} \nu_{R2}$$

$$\mathcal{L}_{M_{S3}} = \frac{1}{2} \rho \chi S_{R3} \nu_{R3}$$  \hspace{1cm} (9)$$

\begin{table}[h]
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Field & $l$ & $e_R$ & $\mu_R$ & $\tau_R$ & $H$ & $H'$ & $H''$ & $\zeta$ & $H$ & $H'$ & $H''$ & $\nu_{R1}$ & $\nu_{R2}$ & $\nu_{R3}$ \\
\hline
SU(2) & 2 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\hline
$A_4$ & 3 & 1 & 1 & 1 & 1 & 1 & 1 & 3 & 3 & 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\hline
$Z_4$ & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 \\
\hline
\end{tabular}
\caption{Particle content and their charge assignments under SU(2), $A_4$ and $Z_4$ groups.}
\end{table}

\begin{table}[h]
\begin{tabular}{|c|c|c|c|c|}
\hline
Charges & $S_1$ & $S_2$ & $S_3$ & $\chi$ \\
\hline
$A_4$ & 1 & i & $1'$ & 1 \\
\hline
$Z_4$ & -1 & 1 & i & 1 \\
\hline
\end{tabular}
\caption{Scalar singlet fields and their transformation properties under $A_4$ and $Z_4$ groups.}
\end{table}
In the Lagrangian, $\Lambda$ represents the cut-off scale of the theory and $\tilde{H} = i\tau_2 H$. Following VEV alignments of the extra flavons are required to generate the desired light neutrino mass matrix.

$$\langle \zeta \rangle = (v, 0, 0),$$
$$\langle \varphi \rangle = (v, v, v),$$
$$\langle \xi \rangle = \langle \xi' \rangle = v,$$
$$\langle \chi \rangle = u.$$

Following the $A_4$ product rules and using the above mentioned VEV alignment, one can obtain the charged lepton mass matrix as follows,

$$M_l = \frac{\langle H \rangle v}{\Lambda} \text{diag}(y_e, y_\mu, y_\tau) \tag{10}$$

The Dirac and Majorana neutrino mass matrices are given by,

$$M'_D = \begin{pmatrix} a & b & c \\ a & b & c \\ a & b & c \end{pmatrix}, M_R = \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix} \tag{11}$$

where, $a = \frac{\langle H \rangle v}{\Lambda} y_1, b = \frac{\langle H \rangle v}{\Lambda} y_2$ and $c = \frac{\langle H \rangle v}{\Lambda} y_3$. The elements of the $M_R$ are defined as $d = \lambda_1 v, e = \lambda_2 v$ and $f = \lambda_3 v$.

Three different structures for $M_S$ reads as,

$$M^1_S = \begin{pmatrix} g & 0 & 0 \end{pmatrix}, M^2_S = \begin{pmatrix} 0 & g & 0 \end{pmatrix} \text{ and } M^3_S = \begin{pmatrix} 0 & 0 & g \end{pmatrix} \tag{12}$$

Considering only $M^1_S$ structure, the light neutrino mass matrix takes a symmetric form as,

$$m_\nu = \begin{pmatrix} -\frac{b^2}{e} - \frac{c^2}{f} & -\frac{b^2}{e} - \frac{c^2}{f} & -\frac{b^2}{e} - \frac{c^2}{f} \\ -\frac{b^2}{e} - \frac{c^2}{f} & -\frac{b^2}{e} - \frac{c^2}{f} & -\frac{b^2}{e} - \frac{c^2}{f} \\ -\frac{b^2}{e} - \frac{c^2}{f} & -\frac{b^2}{e} - \frac{c^2}{f} & -\frac{b^2}{e} - \frac{c^2}{f} \end{pmatrix} \tag{13}$$

As we can see, this $m_\nu$ is a symmetric matrix. It can produce only one mixing angle and one mass square difference. This symmetry must be broken in order to generate two mass square differences and three mixing angles. For breaking the symmetry we introduce two new $SU(2)$ singlet flavon fields $(\eta, \eta')$ the coupling of which give rise to a matrix (15) which later on makes the matrix (13) asymmetric after adding to it, hence by braking the earlier
symmetry. These additional flavons and thereby the new matrix (15) have a crucial role to play in reproducing nonzero reactor mixing angle. The Lagrangian responsible for generating the matrix (15) can be written as,

$$L_{MP} = \frac{y_1}{\Lambda} (\tilde{H}_1 \eta)_{1 \nu} R_1 + \frac{y_2}{\Lambda} (\tilde{H}_2' \eta')_{1 \nu} R_2 + \frac{y_3}{\Lambda} (\tilde{H}_3'' \eta')_{1 \nu} R_3$$  \hspace{1cm} (14)

The singlet flavon fields \((\eta, \eta')\) are supposed to take \(A_4 \times Z_4\) charges as same as \(\phi\) (as shown in the table II). Now, considering VEV for the new flavon fields as \(\langle \eta \rangle = (0, v, 0)\) and \(\langle \eta' \rangle = (0, 0, v)\), we get the matrix as,

$$M_P = \begin{pmatrix} 0 & 0 & p \\ 0 & p & 0 \\ p & 0 & 0 \end{pmatrix}$$  \hspace{1cm} (15)

Hence \(M_D\) from eq. (11) will take new structure as,

$$M_D = M_D' + M_P = \begin{pmatrix} a & b & c + p \\ a & b + p & c \\ a + p & b & c \end{pmatrix}$$  \hspace{1cm} (16)

B. Inverted Hierarchy

Earlier in the work[59], author have explained the necessity of a new flavon in order to realize the IH within the MES framework. In our present work, we also have modified the Lagrangian for the \(M_D\) matrix by introducing a new triplet flavon \(\phi'\) with VEV alignment as \(\langle \phi' \rangle \sim (2v, -v, -v)\), which affects only the Dirac neutrino mass matrix and give desirable active-sterile mixing in IH. The invariant Yukawa Lagrangian for the \(M_D\) matrix will be,

$$L_{MD} = \frac{y_1}{\Lambda} (\tilde{H}_1 \phi)_{1 \nu} R_1 + \frac{y_2}{\Lambda} (\tilde{H}_2 \phi')_{1 \nu} R_2 + \frac{y_3}{\Lambda} (\tilde{H}_3 \phi')_{1 \nu} R_3$$  \hspace{1cm} (17)

Hence the Dirac mass matrix will have the form,

$$M_D' = \begin{pmatrix} a & -b & c \\ a & -b & c \\ a & 2b & c \end{pmatrix}$$  \hspace{1cm} (18)

with, \(a = \frac{(H)_v}{\Lambda} y_1, b = \frac{(H)_v}{\Lambda} y_2\) and \(c = \frac{(H)_v}{\Lambda} y_3.\)
This Dirac mass matrix will also give rise to a symmetric $m_\nu$ like the NH case. Thus, the modified $M_D$ to break the symmetry will be given by,

$$M_D = M_D' + M_P = \begin{pmatrix} a & -b & c + p \\ -b & a & c \\ c + p & 2b & c \end{pmatrix}$$

(19)

Other matrices like $M_R, M_P, M_1^S, M_2^S, M_3^S$ will retain their same structure throughout the inverted mass ordering.

### C. General Case

Moreover, we have come up with a most general structure for $M_D$, where the trivial $\mu - \tau$ symmetry is broken automatically and hence generating non-zero reactor angle. Unlike the previous cases, we have constructed the Lagrangian for the desired mass term by introducing two extra triplet flavon $\varphi'$ and $\varphi''$. The minimal field content required to arrive at the desired structure for the light neutrino mass is shown in Table IV.

The invariant Lagrangian will be

$$\mathcal{L} = \frac{y_e}{\Lambda} (\bar{l}H \zeta)_1 e_R + \frac{y_\mu}{\Lambda} (\bar{l}H \zeta)_1 \mu_R + \frac{y_\tau}{\Lambda} (\bar{l}H \zeta)_1 \tau_R + \frac{y_1}{\Lambda} (\bar{l}H \varphi')_1 \nu_{R1} + \frac{y_2}{\Lambda} (\bar{l}H \varphi')_1 \nu_{R2} + \frac{y_3}{\Lambda} (\bar{l}H \varphi'')_1 \nu_{R3} + \frac{1}{2} \lambda_1 \xi \bar{\nu}_{R1} \nu_{R1} + \frac{1}{2} \lambda_2 \xi \bar{\nu}_{R2} \nu_{R2} + \frac{1}{2} \lambda_3 \xi \bar{\nu}_{R3} \nu_{R3} + \frac{1}{2} \rho S \bar{\nu}_{R1} + h.c.,$$

(20)
We choose a specific VEV alignment for the flavons as
\[ \langle \zeta \rangle = (v, 0, 0), \]
\[ \langle \varphi \rangle = \langle \varphi' \rangle = \langle \varphi'' \rangle = (v_1, v_2, v_3), \]
\[ \langle \xi \rangle = \langle \xi' \rangle = v, \]
\[ \langle \chi \rangle = u. \]

The mass matrices like \( M_l \) and \( M_R \) will retain their same structure like earlier cases. Only the Dirac mass matrix will take a new form as follows,
\[
M_D = \begin{pmatrix}
    a_1 & b_1 & c_1 \\
    a_2 & b_2 & c_2 \\
    a_3 & b_3 & c_3
\end{pmatrix}
\]  
(21)

where, \( a_i = \frac{<H>v_i}{Y_1}, b_i = \frac{<H>v_i}{Y_2}, c_i = \frac{<H>v_i}{Y_3} \) with \( i = 1, 2, 3 \).

In this most general case we have studies only one \( M_S \) structure with \( M_S = \begin{pmatrix} g & 0 & 0 \end{pmatrix} \) (where \( g = \rho u \)) in NH.

The light neutrino mass matrix is obtained as,
\[
m_\nu = - \begin{pmatrix}
    \frac{b_1^2}{e} + \frac{c_1^2}{f} & \frac{b_1b_2}{e} + \frac{c_1c_2}{f} & \frac{b_1b_3}{e} + \frac{c_1c_3}{f} \\
    \frac{b_2b_1}{e} + \frac{c_2c_1}{f} & \frac{b_2^2}{e} + \frac{c_2^2}{f} & \frac{b_2b_3}{e} + \frac{c_2c_3}{f} \\
    \frac{b_3b_1}{e} + \frac{c_3c_1}{f} & \frac{b_3b_2}{e} + \frac{c_3c_2}{f} & \frac{b_3^2}{e} + \frac{c_3^2}{f}
\end{pmatrix}
\]  
(22)

This is a self-sufficient light neutrino mass matrix. The trivial symmetry is broken by itself hence within this paradigm, the active neutrino mixing approves current global fit 3\( \sigma \) bound.

Here, the active-sterile mixing matrix is given by,
\[
R^T \simeq \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} = \begin{pmatrix} V_{e4} & V_{\mu 4} & V_{\tau 4} \end{pmatrix}^T
\]  
(23)

If we are to check the active-sterile mixing, we must vary the elements of \( R \) matrix which contain \( a_1, a_2, a_3 \) and \( g \). The light neutrino matrix \( (m_\nu) \) does not have any contribution on \( R \), and \( a_i = \frac{<H>v_i}{Y_1} \), which solely depends on the VEV \( v_i \). Even if we solve for the elements of \( R \), it needs to put the VEV \( v_i \) by hand. So, the numerical analysis for this case has not been carried out in this paper, and we are looking forward for some alternate solving mechanism in our future work.
IV. NUMERICAL ANALYSIS

The leptonic mixing matrix for active neutrinos depends on three mixing matrices $\theta_{13}, \theta_{23}$ and $\theta_{12}$ and one CP-violating phase ($\delta$) for Dirac neutrinos and two Majorana phases $\alpha$ and $\beta$ for Majorana neutrino. Conventionally this Leptonic mass matrix for active neutrino is parameterized as,

$$U_{PMNS} = \begin{pmatrix} 
  c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
  -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\
  s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} 
\end{pmatrix}. \quad (24)$$

The abbreviations used are $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and $P$ would be a unit matrix $1$ in the Dirac case but in Majorana case $P = \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$.

The light neutrino mass matrix $M_\nu$ is diagonalized by the unitary PMNS matrix as,

$$M_\nu = U_{PMNS} \text{diag}(m_1, m_2, m_3) U_{PMNS}^T, \quad (25)$$

where $m_i$ (for $i = 1, 2, 3$) stands for three active neutrino masses.

Since we have included one extra generation of neutrino along with the active neutrinos in our model thus, the final neutrino mixing matrix for the active-sterile mixing takes $4 \times 4$ form as,

$$V \simeq \begin{pmatrix} 
  (1 - \frac{1}{2}RR^\dagger)U_{PMNS} & R \\
  -R^\dagger U_{PMNS} & 1 - \frac{1}{2}R^\dagger R 
\end{pmatrix}. \quad (26)$$

where $R = M_D M_R^{-1} M_S^T (M_S M_R^{-1} M_S^T)^{-1}$ is a $3 \times 1$ matrix governed by the strength of the active-sterile mixing i.e., the ratio $\frac{\langle M_D \rangle}{\langle M_S \rangle}$.

The sterile neutrino of mass of order eV, can be added to the standard 3-neutrino mass states in NH: $m_1 \ll m_2 < m_3 \ll m_4$ as well as IH: $m_3 \ll m_1 < m_2 \ll m_4$. One can write the diagonal light neutrino mass matrix for NH as $m_\nu^{NH} = \text{diag}(0, \sqrt{\Delta m^2_{21}}, \sqrt{\Delta m^2_{21} + \Delta m^2_{31}}, \sqrt{\Delta m^2_{41}})$ and for IH as, $m_\nu^{IH} = \text{diag}(\sqrt{\Delta m^2_{31}}, \sqrt{\Delta m^2_{21} + \Delta m^2_{31}}, 0, \sqrt{\Delta m^2_{43}})$. The lightest neutrino mass is zero in both the mass ordering as demanded by the MES framework. Here $\Delta m^2_{41}$ (or $\Delta m^2_{43}$) is the active-sterile mass square difference for NH and IH respectively. As explained in previous section, the non-identical VEV alignment for the Dirac mass matrix in NH and IH produces distinct pattern for the active neutrino mass matrix. The active neutrino mass matrix is obtained
using equation (5) and the sterile mass is given by equation (6). The complete matrix picture for NH and IH are presented in table V and table VII respectively.

For numerical analysis we have first fixed non-degenerate values for the right-handed neutrino mass parameters as $d = e = 10^{13}\, GeV$ and $f = 5 \times 10^{13}\, GeV$ so that they can exhibit successful Leptogenesis without effecting the neutrino parameters, which is left for our future study. The mass matrix arises from eq. (25) give rise to complex quantities due to the presence of Dirac and the Majorana phases. Since the leptonic CP phases are still unknown, we vary them within their allowed $3\sigma$ ranges $(0, 2\pi)$. The Global fit $3\sigma$ values for other parameters like mixing angles, mass square differences are taken from [20]. The active neutrino mass matrix emerges from our model matrices is left with three parameters for each case. Comparing the model mass matrix with the one produced by light neutrino parameters given by eq. (25), we numerically evaluate the model parameters satisfying the current bound for the neutrino parameters and establish correlation among various model and oscillation parameters within $3\sigma$ bound. Three assessment for each distinct structures of $M_D$ for both normal and inverted hierarchy cases are carried out in the following subsections.

A. NORMAL HIERARCHY

For the diagonal charged lepton mass we have chosen a non-trivial VEV alignment resulting a specific pattern in Dirac mass hence a broken $\mu - \tau$ symmetry along with non-zero reactor mixing angle is achieved. The complete picture for active neutrino mass matrices and the sterile sector for different cases are shown in table V and VI respectively. For each $M_S$ structure, three variables are there in the light neutrino mass matrix. After solving them by comparing with the light neutrino mass, we obtain some correlation plots which redefines our model parameters with more specific bounds. Correlation among various model parameters in NH are shown in fig. 1.

As $m_s$ depends only on $M_R$ and $M_S$, so due to the non-degenerate value of $M_R$, the $m_s$ structure let us study the active-sterile mixing strength $R$. The active-sterile mixing matrix also have a specific form due to the particular $M_S$ structure.
| Mass ordering | Structures | $m_\nu$ |
|---------------|------------|--------|
| NH(Case-I)    | $M_R = \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix}$ | $m_\nu = - \begin{pmatrix} \frac{b^2}{c} + \frac{(c+p)^2}{f} & \frac{b(b+p)}{e} + \frac{c(c+p)}{f} & \frac{b^2}{e} + \frac{c(c+p)}{f} \\ \frac{b(b+p)}{e} + \frac{c(c+p)}{f} & \frac{b^2}{e} + \frac{c^2}{f} + \frac{b(b+p)}{e} + \frac{c^2}{f} \\ \frac{b^2}{e} + \frac{c(c+p)}{f} & \frac{b^2}{e} + \frac{c^2}{f} + \frac{b(b+p)}{e} + \frac{c^2}{f} \end{pmatrix}$ |
| $M_D = \begin{pmatrix} a & b & c + p \\ a & b + p & c \\ a + p & b & c \end{pmatrix}$ | $M_3^1 = \begin{pmatrix} 0 \\ g \\ 0 \end{pmatrix}$ |
| $M_3^1 = \begin{pmatrix} g & 0 & 0 \end{pmatrix}$ | |

| NH(Case-II)   | $M_R = \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix}$ | $m_\nu = - \begin{pmatrix} \frac{a^2}{d} + \frac{(c+p)^2}{f} & \frac{a^2}{d} + \frac{c(c+p)}{f} & \frac{a(a+p)}{d} + \frac{c(c+p)}{f} \\ \frac{a^2}{d} + \frac{c(c+p)}{f} & \frac{a^2}{d} + \frac{c^2}{f} + \frac{a(a+p)}{d} + \frac{c^2}{f} \\ \frac{a(a+p)}{d} + \frac{c(c+p)}{f} & \frac{a(a+p)}{d} + \frac{c^2}{f} + \frac{(a+p)^2}{d} + \frac{c^2}{f} \end{pmatrix}$ |
| $M_D = \begin{pmatrix} a & b & c + p \\ a & b + p & c \\ a + p & b & c \end{pmatrix}$ | $M_3^2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ |
| $M_3^2 = \begin{pmatrix} 0 & g & 0 \end{pmatrix}$ | |

| NH(Case-III)  | $M_R = \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix}$ | $m_\nu = - \begin{pmatrix} \frac{a^2}{d} + \frac{(b^2)}{e} & \frac{b^2}{e} + \frac{b(b+p)}{e} & \frac{a(a+p)}{d} + \frac{b^2}{e} \\ \frac{b^2}{e} + \frac{b(b+p)}{e} & \frac{b^2}{e} + \frac{b^2}{e} + \frac{b(b+p)}{e} & \frac{a(a+p)}{d} + \frac{b(b+p)}{e} \\ \frac{a(a+p)}{d} + \frac{b^2}{e} & \frac{a(a+p)}{d} + \frac{b(b+p)}{e} & \frac{(a+p)^2}{d} + \frac{b^2}{e} \end{pmatrix}$ |
| $M_D = \begin{pmatrix} a & b & c + p \\ a & b + p & c \\ a + p & b & c \end{pmatrix}$ | $M_3^3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ |
| $M_3^3 = \begin{pmatrix} 0 & 0 & g \end{pmatrix}$ | |

TABLE V: The light neutrino mass matrices and the corresponding $M_D$ and $M_R$ matrices for three different structures of $M_S$ under NH pattern.

**B. INVERTED HIERARCHY**

In this section we will discuss the inverted mass ordering (i.e., $m_2 > m_1 > m_3$) of the neutrinos. Referring to [59], we have introduced a new flavon as, $\langle \phi \rangle = (2v, -v, -v)$ in
TABLE VI: Sterile neutrino mass and active-sterile mixing matrix for three different $M_S$ structures under NH pattern.

| Case | $M_S$ | $m_s$ | $R$ |
|------|-------|-------|-----|
| I    | $M_1^S = (g \ 0 \ 0)$ | $m_s \simeq \frac{g^2}{10^4}$ | $R^T \simeq \left( \frac{a}{g} \ \frac{a+e}{g} \ \frac{a+e}{g} \right)$ |
| II   | $M_2^S = (0 \ g \ 0)$ | $m_s \simeq \frac{g^2}{10^4}$ | $R^T \simeq \left( \frac{b}{g} \ \frac{b+e}{g} \ \frac{b}{g} \right)$ |
| III  | $M_3^S = (0 \ 0 \ g)$ | $m_s \simeq \frac{g^2}{5 \times 10^4}$ | $R^T \simeq \left( \frac{c+e}{g} \ \frac{c+e}{g} \ \frac{c}{g} \right)$ |

FIG. 1: Variation of model parameters among themselves for the NH pattern.
FIG. 2: Correlation plots among various model parameters and light neutrino parameters (within $3\sigma$ bound) in NH. The Dirac CP phase shows a good correlation with the model parameters than the other light neutrino parameters.
FIG. 3: Variation of Sine of reactor mixing angle with $p$, which is responsible for the generation of reactor mixing angle ($\theta_{13}$) for NH.

the Yukawa Lagrangian for the Dirac mass term, so that this model can exhibit inverted hierarchy. A detailed discussion has already been carried out in previous section III.

Numerical procedure for IH is analogous to the NH. Here also we have considered three distinguished cases for $M_S$, which is responsible for three separate $m_\nu$ matrices. A brief picture for the matrices has shown in table VII.

In table VIII, three different $M_S$ structures are shown, which lead to various $m_s$ and $R$ values. Unlike the normal ordering, a deviation from the common track is observed in $R$ matrix for the second case ($M_S^2 = (0, g, 0)$). This occurs due to the change in $M_D$ matrix structure for non-identical VEV alignment.
FIG. 4: Allowed bound for the active-sterile mixing matrix elements in NH. The green line is the lower bound for $|V_{e4}|^2$ and the blue line in the first plot gives the upper bound for $|V_{\tau 4}|^2$ while in the third plot it gives the lower bound for $|V_{\mu 4}|^2$. 
| Mass ordering | Structures                                                                 | $m_\nu$                                                                 |
|---------------|---------------------------------------------------------------------------|-------------------------------------------------------------------------|
| IH (Case-I)   | $M_R = \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix}$, $M_D = \begin{pmatrix} a & -b & c+p \\ a & -b+p & c \\ a+p & 2b & c \end{pmatrix}$ | $m_\nu = - \begin{pmatrix} \frac{b^2}{e} + \frac{(c+p)^2}{f} & \frac{b(b-p)}{e} & \frac{c(c+p)}{f} \\ \frac{b(b-p)}{e} & \frac{(b-p)^2}{e} + \frac{c^2}{f} & \frac{-2b^2}{e} + \frac{c(c+p)}{f} \end{pmatrix}$ |
|               | $M_S^I = \begin{pmatrix} g & 0 & 0 \end{pmatrix}$                                           |                                                                                   |
| IH (Case-II)  | $M_R = \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix}$, $M_D = \begin{pmatrix} a & -b & c+p \\ a & -b+p & c \\ a+p & 2b & c \end{pmatrix}$ | $m_\nu = - \begin{pmatrix} \frac{a^2}{d} + \frac{(c+p)^2}{f} & \frac{a^2}{d} + \frac{c(c+p)}{f} & \frac{a(a+p)}{d} + \frac{c(c+p)}{f} \\ \frac{a(a+p)}{d} + \frac{c(c+p)}{f} & \frac{a^2}{d} + \frac{c^2}{f} & \frac{(a+p)^2}{d} + \frac{c^2}{f} \end{pmatrix}$ |
|               | $M_S^I = \begin{pmatrix} 0 & g & 0 \end{pmatrix}$                                           |                                                                                   |
| IH (Case-III) | $M_R = \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix}$, $M_D = \begin{pmatrix} a & -b & c+p \\ a & -b+p & c \\ a+p & 2b & c \end{pmatrix}$ | $m_\nu = - \begin{pmatrix} \frac{a^2}{d} + \frac{b^2}{e} & \frac{a^2}{d} & \frac{b(b-p)}{e} \\ \frac{b(b-p)}{e} & \frac{(b-p)^2}{e} + \frac{(a+p)^2}{d} & \frac{2b(b-p)}{e} \\ \frac{a(a+p)}{d} & \frac{a(a+p)}{d} - \frac{2b(b-p)}{e} & \frac{2b(b-p)}{e} + \frac{(a+p)^2}{d} \end{pmatrix}$ |
|               | $M_S^I = \begin{pmatrix} 0 & 0 & g \end{pmatrix}$                                           |                                                                                   |

**TABLE VII:** The light neutrino mass matrices and the corresponding $M_D$ and $M_R$ matrices for three different structures of $M_S$ under IH pattern.
| Case | $M_S$ | $m_s$ | $R$ |
|------|-------|-------|-----|
| I    | $M_S^1 = (g, 0, 0)$ | $m_s \simeq \frac{g^2}{16\pi}$ | $R^T \simeq \left( \frac{a}{g}, \frac{a}{g}, \frac{a+p}{g} \right)^T$ |
| II   | $M_S^2 = (0, g, 0)$ | $m_s \simeq \frac{g^2}{16\pi}$ | $R^T \simeq \left( \frac{-b}{g}, \frac{-b+p}{g}, \frac{2b}{g} \right)^T$ |
| III  | $M_S^3 = (0, 0, g)$ | $m_s \simeq \frac{g^2}{5\times10^4}$ | $R^T \simeq \left( \frac{c+p}{g}, \frac{c}{g}, \frac{c}{g} \right)^T$ |

TABLE VIII: Sterile neutrino mass and active-sterile mixing matrix for three different $M_S$ structures under IH pattern.

FIG. 5: Constrained region of model parameters in case of IH pattern.
FIG. 6: Correlation plots among various model parameters with light neutrino parameters in IH pattern.
FIG. 7: Variation of $p$ with the $\sin$ of reactor mixing angle for IH. The third structure of $M_S$ shows a constrained region for the model parameter.

FIG. 8: Allowed bound for active-sterile mixing matrix elements in IH. The blue solid line gives the upper and lower bound for $|V_{\mu 4}|^2$ along the y-axis while solid green line gives the lower bound for $|V_{e4}|^2$ along the x-axis.
V. SUMMARY AND CONCLUSION

In this paper we have investigated the extension of low scale SM type-I seesaw \textit{i.e.}, the minimal extended seesaw, which restricts active neutrino masses to be within sub-eV scale and generates an eV scale light sterile neutrino. $A_4$ based flavor model is extensively studied along with a discrete Abelian symmetry $Z_4$ to construct the desired Yukawa coupling matrices. Under this MES framework the Dirac mass $M_D$ is a $3 \times 3$ complex matrix. The Majorana mass matrix $M_R$, which arises due to the coupling of right-handed neutrinos with the anti-neutrinos is also a $3 \times 3$ complex symmetric diagonal matrix with non-degenerate eigenvalues. A singlet $S_i$ (where $i = 1, 2, 3$) is considered which couples with the right-handed neutrinos ($\nu_{Ri}; i = 1, 2, 3$) and produces a singled row $1 \times 3$ $M_S$ matrix with one non-zero entry. Three separate cases are carried out for both NH and IH for three $M_S$ structures. Within the active neutrino mass matrix, the common $\mu - \tau$ symmetry is broken along with $\theta_{13} \neq 0$ by adding a new matrix ($M_P$) to the Dirac mass matrix.

Both normal and inverted cases are analyzed independently for three $M_S$ structures in this work. We have used similar numerical techniques for solving model parameters in both the cases (NH & IH) and plotted them among themselves as well as with the light neutrino parameters. The plots in fig. 1, 2, 5, 6 show constrained parameter space in the active neutrino sector in case of NH and IH for various $M_S$ structure. In most of the cases the parameter space is narrow, which can be verified or falsified in future experiments. In the $m_{\nu}$ matrix, the $\mu - \tau$ symmetry is broken due to the extra term added to the Dirac mass matrix. The variation of $\sin^2 \theta_{13}$ with $p$ plotted in fig. 3 and 7 for NH & IH respectively. Within NH, the first structure of $M_S$ shows a better constrained region for the model parameter ($p$) than the other two structures. Whereas in IH case, the third $M_S$ structure gives a relatively narrower region than that obtained for the other two structures.

The active-sterile mixing phenomenology is also carried out under the same MES framework. The fourth column of the active-sterile mixing matrix is generated and solved the elements with an acceptable choice of Yukawa coupling. Apart from generating non-zero $\theta_{13}$, the matrix element of $M_P$ has an important role to play in the active-sterile mixing. As we can see in table VI and VIII, $p$ has an active participation in differentiating the elements of $R$ matrix. We have plotted the mixing matrix elements ($V_{e4}, V_{\mu 4}, V_{\tau 4}$) within themselves as shown in fig. 4 and 8. In NH case, the first and the third $M_S$ structure show an allowed
range for the mixing parameters but no such mutual allowed range is obtained for the second structure of $M_S$. The plots in fig. 8 shows the IH case for the mixing elements. The first structure of $M_S$ covers a wider range of allowed data points within the 3σ bound than the other two case.

In conclusion, the low scale MES mechanism is analyzed in this work. This model can also be used to study the connection between effective mass in neutrinoless double beta decay in a wider range of sterile neutrino mass from $eV$ to few $keV$. Study of $keV$ scale sterile neutrino can be a portal to explain origin of dark matter and related cosmological issues in this MES framework.

VI. ACKNOWLEDGEMENTS

We would like to thank Department of Computer Science and Engineering, Tezpur University for giving us the full access to The High performance computing facility (a Joint project by Tezpur university and C-DAC Pune) for our simulation work.

Appendix A: $A_4$ product rules

$A_4$, the symmetry group of a tetrahedron, is a discrete non-Abelian group of even permutations of four objects. It has four irreducible representations: three one-dimensional and one three-dimensional which are denoted by $1, 1', 1''$ and $3$ respectively, being consistent with the sum of square of the dimensions $\sum_i n_i^2 = 12$. Their product rules are given as,

$$1 \otimes 1 = 1$$
$$1' \otimes 1' = 1''$$
$$1' \otimes 1'' = 1$$
$$1'' \otimes 1'' = 1'$$
$$3 \otimes 3 = 1 \otimes 1' \otimes 1'' \otimes 3_a \otimes 3_s$$

where $a$ and $s$ in the subscript corresponds to anti-symmetric and symmetric parts respectively. Denoting two triplets as $(a_1, b_1, c_1)$ and $(a_2, b_2, c_2)$ respectively, their direct product can be decomposed into the direct sum mentioned above as,

$$1 \sim a_1a_2 + b_1c_2 + c_1b_2$$
\[ 1' \sim c_1 c_2 + a_1 b_2 + b_1 a_2 \]

\[ 1'' \sim b_1 b_2 + c_1 a_2 + a_1 c_2 \]

\[ 3_s \sim (2a_1 a_2 - b_1 c_2 - c_1 b_2, 2c_1 c_2 - a_1 b_2 - b_1 a_2, 2b_1 b_2 - a_1 c_2 - c_1 a_2) \]

\[ 3_a \sim (b_1 c_2 - c_1 b_2, a_1 b_2 - b_1 a_2, c_1 a_2 - a_1 c_2) \]

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