evaluation of seepage under hydraulic structures using gene expression programming

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Abstract. The problem of seepage under concrete dams was studied numerically using finite difference code in MATLAB, after verification with GeoStudio (2018 R2) to determine the lift force under the dams, the head at the third floor of each of the dams, and the exit gradients, which all affect the functionality of these dams. The main goal of this study was to investigate the effects of cutoff locations and depths on the exit gradient and uplift pressure. An empirical equation was thus developed to predict the exit gradient by employing gene expression programming (GEP) and non-linear regression using IBM SPSS based on parameters obtained from the finite difference code. Multiple (327) runs were executed in the finite difference code for a set difference between head upstream and downstream (H=10 m) with various cutoffs depth (d/b), with \(k_x/k_y = 1\) representing isotropic soil. For each d/b value, various cutoffs locations (x/b) were also used, with results indicating that the minimum exit gradient is observed when the cutoff location ratio at the downstream is \(x_1/b=1\) with a maximum relative depth of \(d_1/b=0.6\), while the minimum uplift pressure is observed when the cutoff location ratio at the upstream is \(x_1/B=0\), with a minimum relative depth of \(d_1/B=0.1\). The results further indicate that the maximum exit gradient is observed when the ratio of the length of upstream cutoff to the length of downstream cutoff is \(d_1/d_2 = 1\). This exit gradient decrease as the ratio of \(d_1/d_2\) increases. Additionally, the exit gradient decreases with increases in cutoff location and depth, while increasing with uplift pressure. Based on the simulation results, the equation obtained using the genetic expression programming model performed better in terms of predicting the exit gradient than the SPSS model, with a coefficient of determination \(R^2\) of 0.892 for training and 0.90 for testing. An empirical equation was thus derived to predict the exit gradient based on cutoff\(1\) depth, cutoff\(1\) location, cutoff\(2\) depth and cutoff\(2\) location.

Keywords: seepage, finite difference code, verification, piping, uplift pressure

1. Introduction

Dams are designed to hold water in reservoirs over long periods of time; any factors that may affect this functionality in dams and reduce it thus need to be carefully studied and minimised in order to obtain...
proper dam performance. Compared to other parts of the structure, the foundation should be therefore of most interest in analysis and design, as any failure in the foundation undermines the entire structure. Whenever such a structure is located on a pervious foundation, the movement of water under the structure will produce uplifting pressure on its floor; where the thickness of the floor is inadequate, it will not be able to withstand this pressure, and the floor will be broken, resulting in structural failure. In addition, if the exit gradient at the base of a dam reaches a critical value, the water may begin to drain, carrying soil particles from the base and creating a hole underneath the dam; this is known as the piping phenomenon, and it generally causes the structure to collapse [1].

To obtain the necessary safety factors against piping from both exit gradient and uplift pressure, engineers typically supply hydraulic stability by introducing cutoffs at the upstream (U.S.) and downstream (D.S.) sides of the base. In general, upstream cutoffs lower the lifting pressure and exit gradient; however, they may reduce the uplift pressure at a rate higher than that required for exit gradient influence, and a downstream cutoff should thus always be provided, as this has a more significant impact on the exit gradient. In order to achieve the necessary safety factors, engineers must thus determine the appropriate depths for both U.S. and D.S. cutoffs [2].

Many researchers [3-9] have studied the effects of upstream or downstream cutoffs on the differences in exit gradient D.S. of the structure and on uplift pressure. Such results are usually provided in the form of dimensionless curves that assist the design process. Several researchers [10-13] have also used finite element models to determine the head distribution under a structure with angled cutoffs under various soil properties and flow conditions.

The researchers in [14] examined the efficiency of cutoff walls based on various design parameters in an assumed diversion dam cross-section. For this purpose, different placements of cutoff wall with various angles of inclination were used in the dam foundation. The researchers found that the minimum uplift pressure occurred when the cut off wall was in the heel (upstream) of the dam. With a fixed longitudinal cut off wall placement, inclination of the cutoff wall with respect to the vertical position was also found to result in a reduction of uplift pressure. The researchers in [15] presented a numerical analysis of the impact of an angled cutoff on the distribution of the hydraulic system and the exit gradient due to uplift pressure. They also graphically determined the best cutoff inclination angle and position through using ANSYS11.0 for three selection locations (U.S., D.S., and midstream). Similarly, [16] focused on developing a numerical method to investigate the performance of cutoff walls’ systems with regard to uplift pressure and the piping phenomenon.

Foundation design is the most essential part of these structures, and this requires the following dimensions to be determined: floor length (B); depth of cutoff 1 (d1) and depth of cutoff 2 (d2), both for a given depth of foundation (D); and head difference (H). Figure (1) shows how these dimensions are measured.
The values are influenced by the overall predicted head difference between the U.S. and D.S. sides of the dam (H), and the soil layers properties (Kx and Ky). In this study, kx=ky is proposed in three cases (without cutoff, with one cutoff, with two cutoffs). For each case, the exit gradient, the head at the third floor, and the pressure force under the dam were determined using finite difference code [17-21].

The primary aim of this study was to investigate the effects of cutoff location and depth on the exit gradient and the uplift pressure, as these in turn affect the functionality of the dam. Inductive modelling tools, based on GEP-based models, were then applied to predict the exit gradient using the data generated by the finite difference code, and the model performance was compared with the performance of a model in SPSS.

2. Methodology

2.1 Controlling equations

The principle of flow across a porous layer depends on Laplace’s equation of continuity, which defines the state of constant flow in a soil segment for a specified place. The following equation is applicable across any incompressible anisotropic layer with two dimensional constant flows in the x, y plane:

\[ k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \]  

(1)

where (h) is the total head, Kx is the hydraulic conductivity in the X-direction, and Ky is the hydraulic conductivity in the Y-direction. The seepage pattern can thus be fully calculated by solving Equation (1) using finite difference code.

For steady and confined flow, limit conditions are defined as shown by the following states.

- **Boundaries of the reservoirs (Constant Head):** The piezometric head spread along these boundaries is constant.
  
  \[ p = \gamma_w h_0 \]  
  
  (2)
  
  \[ h = h_0 = \frac{p}{\gamma_w} + z \]  
  
  (3)
  
  Thus, all boundaries of the reservoir have equipotential lines.

- **Impervious boundary (no flow):** with respect to these limits, water cannot pass across the surface, making qn equal to zero, as shown in
(4)

where $I_x$ and $I_y$ show the direction of cosine for the normal surface vector in the x and y directions, respectively.

2.2 Finite difference seepage modelling

The problem under analysis, as discussed in the preceding section, was outlined in MATLAB programming using finite difference code modelling (FDM). This code was used to resolve the problem of seepage under a concrete dam and to determine the uplift force under dam head at the third floor and the exit gradient directly based on specific input data: total head at upstream, total head at downstream, depth of cutoff 1 ($d_1$), depth of cutoff 2 ($d_2$), cutoff 1’s location on the floor ($x_1$), and cutoff 2’s location on the floor ($x_2$). For different cases, the input data vary in terms of different depths of cutoffs, cutoffs' locations in the floor (measured from U.S. for cutoff 1 and D.S. for cutoff 2), head, depth of foundation, and length of floor. The principle of FDM is to substitute the partial derivatives of the dependent variable with partial differential formulae (PDE) with $O(h^n)$ errors that utilise finite difference estimations. This process thus converts the zone into a mesh network of nodes in which the predictor variables are estimated, with PDE determining the independent variables. Replacing partial derivatives with formulas of difference estimation relies on Taylor's theorem, and can be used to solve elliptic PDEs. A linear equation system was thus developed and solved for calculating heads at nodes using several common iterative techniques such as a successive over relaxation (SOR) Jacobi, Gauss Seidel, and combined gradient methods [22].

To obtain the formula of approximation of finite difference for Laplace’s equation, three points, i, i+1, and i-1, separated by distance $h$ on the X axis, were considered, as indicated in Figure 2:

**Figure 2:** Three point on the X axis used to obtain the formula of approximation of finite difference.

Where the values of the $u (x, y)$ equation at points (i-1, j), (i, j), and (i+1, j) are u (i-1, j), u (i, j), and u (i+1, j), the form of Taylor series to represent $u (i+1, j)$ and $u (i-1, j)$ around point i is

$$ u_{i+1,j} = u_{i,j} + \frac{h}{1!} \frac{\partial u}{\partial x} |_l + \frac{h^2}{2!} \frac{\partial^2 u}{\partial x^2} |_l + \frac{h^3}{3!} \frac{\partial^3 u}{\partial x^3} |_l + o(h^5) \tag{6} $$

$$ u_{i-1,j} = u_{i,j} - \frac{h}{1!} \frac{\partial u}{\partial x} |_l + \frac{h^2}{2!} \frac{\partial^2 u}{\partial x^2} |_l - \frac{h^3}{3!} \frac{\partial^3 u}{\partial x^3} |_l + \frac{h^4}{4!} \frac{\partial^4 u}{\partial x^4} |_l + o(h^5) \tag{7} $$

By adding together Eq. (6) and Eq. (7), Eq. (8) is obtained:

$$ u_{i+1,j} + u_{i-1,j} = 2u_{i,j} + h^2 \frac{\partial^2 u}{\partial x^2} |_l + \frac{h^4}{12} \frac{\partial^4 u}{\partial x^4} |_l + o(h^5) \tag{8} $$

Eq. (8) can then be modified to get Eq. (9):

$$ \frac{\partial^2 u}{\partial x^2} |_l = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + o(h^2) \tag{9} $$

where Eq. (9) is the finite difference approximating form for the second order error $o(h^2)$ for $\frac{\partial^2 u}{\partial x^2} |_l$.
Figure 3: Three point on the Y axis used to obtain the formula of approximation of finite difference.

Where the value of the \( u(x,y) \) equation at the points \((i,j+1)\), \((i,j)\), and \((i,j-1)\) are \( u_{i,j} \), \( u_{i,j+1} \) and \( u_{i,j-1} \), the form of the Taylor series to represent \( u_{i,j-1} \) and \( u_{i,j+1} \) at point \( j \), the finite difference approximating form with error term \( o(h^2) \) of second order for \( \frac{\partial^2 u}{\partial x^2} \) at point \( j \), is

\[
\frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{h^2} + o(h^2) \tag{10}
\]

The star shape (5-point stencil) area around point \((i,j)\) is thus obtained by combining Figure 2 and Figure 3, as indicated in Figure 4.

Figure 4: The star shape for point \((i,j)\) in the finite difference method.

Inserting Eq. (9) and Eq. (10) into Eq. (5) yields:

\[
\left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)_{(i,j)} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0 \tag{11}
\]

where \( h = k \). By rearranging Eq. (11), the following equation is obtained

\[
(u_{i+1,j} + u_{i,j+1}) - 4u_{i,j} + (u_{i-1,j} + u_{i,j-1}) = 0
\]

so that

\[
u_{i,j} = \frac{1}{4} [u_{i+1,j} + u_{i,j+1} + u_{i-1,j} + u_{i,j-1}] \tag{12}
\]

In general, for the 5-point sketch, the value of \( u \) at the middle point is equal to the mean of the \( u \) values at the four neighbouring points, as shown in Figure 4. This equation can be applied for each node in the domain and represents the dependent equation in the code. Figure 5 shows a flow chart for the relevant programming in MATLAB (finite difference code) to solve the Laplace equation.
3. Statistical Analysis using GEP and SPSS

Statistical analysis was used to determine the relationships between dependent variables in the form of mathematical formulas and independent variables. In this paper, two modelling techniques were used to achieve the best estimate of dam exit gradient: gene expression programming (GEP) and IBM SPSS (Version 26). The best strategies for predicting the exit gradient was then identified by comparing three common error measures: coefficient of determination ($R^2$), root mean square error (RMSE), and mean absolute error (MAE) [23-25]. In this study, four parameters (depth of cutoff1, depth of cutoff2, location of cutoff1, and location of cutoff2) were selected as input (independent) variables, while the exit gradient, obtained from numerical simulation in MATLAB, was selected as the output (dependent) variable.
The primary purpose of this work was to improve the available inductive modelling tools used to predict exit gradients by using GEP-based models for such prediction based on data optioned from numerical simulation, comparing such performance with standard SPSS modelling.

### 3.1 Gene expression programming (GEP)

Genetic algorithms are used in genetic programming (GP) as an effective tool to forecast results and to identify working relationships. GP allows determination of the most “fit” computer program by instituting artificial evolution [26]. Multiple mathematical functions, including √x, tan x, sin x, and x² are combined with mathematical operations (+, -, /, *) and logical functions. The alphabet used to create these structures is the 0 and 1 values applied to GA individual factors. The main purpose of GEP is to construct a mathematical function that can be customised to input data. The GEP method was thus used to develop a symbolic regression for the mathematical equation using multiple genetic operators.

### 3.2 GEP modelling for exit gradient at the toe of the dam

A powerful software package, GeneXproTools 5.0, was used to develop GEP-based models for exit gradient prediction. This program permitted the creation of compact and explicit mathematical expressions for the exit gradient model. The problem, which can be resolved by the programming of gene expressions, is a symbolic regression (function finding), whereby an expression can be identified as appropriately describing the dependent variable. Initially, the available datasets (total of 327) of possible exit gradients at the toe of dam were obtained from numerical simulation of the exit gradient in different cases, and the parameters \( \frac{X_2}{B}, \frac{X_1}{B}, \frac{d_1}{B}, \) and \( \frac{d_2}{B} \) were assigned to columns as independent input variables, while the exit gradient \( \frac{i_{gH}}{B} \) was used as the dependent output variable. A model of the output variable \( \frac{i_{gH}}{B} \) was developed by using GEP. The datasets were split for use as training and validation data. The training data utilised 278 observations (about 85%) that were randomly chosen and used to build the GEP model. The validation data thus consisted of 49 observations (about 15%), used to test or verify the resulting GEP model. Various parameters for model construction were determined after data division as illustrated in the following six stages:

- **First stage:** GEP begins with an initial majority of individual data points, known as chromosomes, that can be single or multi genic. Populations range from 30 to 100 chromosomes generally give positive outcomes [27,28]. The optimum population size for this model was 50 chromosomes; this value was obtained after several trials to decide optimum population size.
- **Second stage:** after initializing the population, each individual is tested, and its fitness function (here, RMSE) measured.
- **Third stage:** in this stage, the set of functions and the set of terminals for each gene in the chromosome are selected. The arithmetic operators used in this study to prepare this model were \{+, -, *, /, power\}, and the terminal set, including the independent variable and random numerical constant, was given as \( T= \{ \frac{X_2}{B}, \frac{X_1}{B}, \frac{d_1}{B}, \frac{d_2}{B} \} \).
- **Fourth stage:** in this stage, the number of genes and their head length are determined. This begins with a single gene and then a gradual increase in the number. After many trials, three
genes were used in each chromosome in this work, creating multi genic chromosomes with heads equal to eight (h=8).

- Fifth stage: the linking function is selected in this stage. As three genes were used in each chromosome in this work, the results can be generated from this fact, with Expression trees linked by the addition operator (+).

- Sixth step: finally, selecting a set of genetic operators causes variation in the rates. The principles of the genetic operators are given in Table 1.

The model was simulated using GeneXproTools after all these model parameters were determined.

| Parameters                      | Values   |
|---------------------------------|----------|
| Population size                 | 50       |
| Mathematical operators          | *, /, -, +, power |
| Independent var.                | (x1/B, x2/B, d1/B, d2/B) |
| Random numerical constant (RNC) | 5        |
| RNC sort                        | Float point |
| RNC scope                       | (-10, 10) |
| Head length                     | 8        |
| NO. of genes                    | 3        |
| Linking                         | +        |
| Fitness function                | RMSE     |
| Genetic operators’ strategy     | Optimal Evolution |

4. Analysis and Discussion of Results

4.1 Verification the finite difference code

Verification of the finite difference code in MATLAB 7.11.0 (R2010b) was carried out by comparing the total water head at a specific position (such as at 1/3 of the dam’s floor) and the exit gradient value obtained by the computer program Geo Studio 2018R2 SEEP/W with that obtained by the finite difference code. An example of flow under a concrete dam with one cutoff (H=10m, D=30m, B=15m, x1/b = 0 to 1, d1/B=0.2) is illustrated in figures 6 and 7:
The results of the comparison between the finite difference code and GeoStudio show good agreement between these methods.

4.2 Effect of main parameters on exit gradient and uplift pressure

Altogether, 327 runs were carried out in the numerical model using differential head (H)=10m, floor length (B)=15m, depth of impervious layer (D=30), and isotropic soil (kx/ky = 1). Six depths (d/B=0.1, 0.2, 0.3, 0.4, 0.5, and 0.6) were assessed, and for each depth ratio, various cutoff locations (b/B=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1) were used.

4.2.1 Effect of cutoff location ratio on exit gradient and uplift pressure

The cutoff location on the floor has a direct influence on both exit gradient and the uplift pressure. Amending the cutoff upstream is particularly effective in reducing the uplift pressure, while the downstream cutoff is more effective in reducing the hydraulic gradient at the exit, as shown in figures 8 and 9 below:

4.2.1.1 Effect of cutoff depth ratio on exit gradient and uplift pressure

Figure 6. Scatter plot for exit gradient in GeoStudio versus code.

Figure 7. Scatter plot for total water head in GeoStudio versus code.

Figure 8. Exit gradient versus cutoff1 location ratio (d1/B=0.2).

Figure 9. Uplift force versus cutoff1 location ratio (d1/B=0.2).
When the cutoff depth increases, the exit gradient decreases while the uplift pressure increases, as seen in Figure 10, which further indicates that the minimum exit gradient occurs when $d_1/B=0.6$, while Figure (11) indicates that minimum uplift force occurs at $d_1/B=0.1$. Accordingly, the exit gradient decreases when the cutoff location ratio and the cutoff depth ratio increase, while the uplift pressure increases with increases in the cutoff location ratio and the cutoff depth ratio.

![Figure 10](image1.png)

**Figure 10.** Exit gradient versus cutoff1 location ratio at different values of cutoff depths for $d_2/B=0$, $x_2/B=0$.

![Figure 11](image2.png)

**Figure 11.** Uplift force versus cutoff1 location ratio at different values of cut off depths for $d_2/B=0$, $x_2/B=0$.

### 4.2.3 Effect of two cutoffs on exit gradient

The exit gradient with a fixed value of depth of cutoff1 ($d_1/B=0.1$, $x_1/B=0$) and different values of depth of cutoff2 is shown in Figure 12. This figure indicates that the maximum exit gradient occurs when $d_2/B=0.1$, and that a considerable reduction results from any increase of the value of $d_2/B$. 

![Figure 12](image3.png)
Figure 12. Exit gradient versus cutoff2 location ratio at different values of cutoff depths for $d_1/B=0.1$, $x_1/B=0$

4.2.4 Effect of $d_1/d_2$ ratio on exit gradient

The results indicate that the maximum exit gradient occurs when the ratio of the length of upstream cutoff to the length of downstream cutoff equals one ($d_1/d_2 = 1$). This value decreases as the ratio of $d_1/d_2$ increases, as shown in Figure 13, and significantly reduces as the ratio of $d_1/d_2$ decreases, as shown in Figure 14.

Figure 13. Exit gradient versus $(cutoff1 \text{ length}/cutoff2 \text{ length})$ ratio

Figure 14. Exit gradient versus $(cutoff1 \text{ length}/cutoff2 \text{ length})$ ratio
4.3 GEP prediction model

The available datasets (327) of exit gradients at the toe of the dam were obtained from the numerical simulation of exit gradients under different cases. These datasets were divided into two groups, the first one being training data and the second validation data [29-32]. The training data consisted of 278 observations (about 85%), which were randomly chosen and used to build the model. The validation data consisted of 49 observations (about 15%), which were used for testing or validating the GEP model.

The equation of the exit gradient \( \frac{i.g^*H}{B} \) is a function of the expression tree (ET) as indicated in Figure 6 and stated in Eq. (13).

\[
\frac{i.g^*H}{B} = ET1 + ET2 + ET3
\]

where

Gene1:

Sub ET1=[2.154 − \( d_1 \)]

(14)

Gene2:

Sub ET2={\((-6.537) − \left\{ (d_1 − d_2) − ((-1.151) − d_0) \right\} \times \left( \frac{(-1.151)+d_0}{2.758} \right)\}}

(15)

Gene3:

Sub ET3=\left\{ (d_1 − d_3) \times \frac{d_3}{4.818} \right\} \times \left( (4.818 − d_3) − d_3 + 4.818 \right)

(16)

Thus, the exit gradient \( \frac{i.g^*H}{B} \) formula is

\[
\frac{i.g^*H}{B} = [2.154 − \( d_1 \)] + \{\(-6.537) − \left\{ (d_1 − d_2) + 1.151 + d_0 \right\} \times \left( \frac{(-1.151)+d_0}{2.758} \right)\} + \left\{ (d_1 − d_3) \times \frac{d_3}{4.818} \right\} \times \left( (4.818 − 2d_3) + 4.818 \right)
\]

(17)
Figure 15. Expression Trees (ET) of the GEP formulation for exit gradient.

The definitions of the parameters used in Eq. (14 to 17) are represented in Table 2.
Table 2: Definition of parameters in ET.

| Parameters | definition          |
|------------|---------------------|
| $d_0$      | $\frac{x^2}{b}$     |
| $d_1$      | $\frac{b}{d^2}$    |
| $d_2$      | $\frac{x^1}{d}$    |
| $d_3$      | $\frac{b}{d}$      |
| $c_0$ (gene1) | 2.154             |
| $c_3$ (gene 2) | -6.537            |
| $c_4$ (gene 2) | -1.151            |
| $c_0$ (gene 2) | 2.758              |
| $c_0$ (gene 3) | 4.818              |

Figure 16. Curve fitting between predicted (yellow) and measured (green) exit gradients (Training data)
Figure 17. Scatter plot of measured $\frac{lg+H}{B}$ versus predicted $\frac{lg+H}{B}$ (Training data).

Figure 18. Curve fitting between predicted (yellow) and measured (green) exit gradients (Testing data).
Figure 19. Scatter plot of measured $\frac{\dot{i}_g}{B}$ versus predicted $\frac{\dot{i}_g}{B}$ (Testing data)

These results show that GEP achieved a high value of $R^2$ for the testing data, suggesting excellent estimation of the exit gradient, with very little discrepancy between measured and predicted exit gradients and a low value of RMSE and MAE implying good performance of the applied method.

Table 3: Statistical results for the GEP model.

| NO. | Error Measure | Gep Model |
|-----|---------------|-----------|
|     |               | Training data | Testing data |
| 1   | $R^2$         | 0.892      | 0.901       |
| 2   | RMSE          | 0.065      | 0.057       |
| 3   | MAE           | 0.050      | 0.045       |
| 4   | Correlation factor | 0.944  | 0.949       |

4.4 SPSS prediction model
For the SPSS prediction model, the same division of the dataset seen in GEP was used. The standard SPSS protocols were run to achieve the required analysis and model building[33-36]. After several trials, the following final formula for the exit gradient model was obtained, with $R^2$ values equal to 0.864 for the training data and 0.860 for testing data:

$$\frac{i_{g^{*}H}}{B} = [0.948 + 0.369 \cdot \left(\frac{x_2}{B}\right) - 1.065 \cdot \left(\frac{d_2}{B}\right) - 0.0947 \cdot \left(\frac{x_1}{B}\right) - 0.589 \cdot \left(\frac{d_1}{B}\right) - 0.747 \cdot \left(\frac{x_2}{B}\right)^2 + 0.995 \cdot \left(\frac{d_2}{B}\right)^2 - 0.309 \cdot \left(\frac{x_1}{B}\right)^2]$$

(18)

The comparison between the measured exit gradient and the predicted exit gradient ($i_{g^{*}H}/B$) for the training and validation data is represented in the scatterplots in Figures 11 and 12.

![Scatter plot of measured exit gradient versus predicted exit gradient for training data.](image)

Figure 20. Scatter plot of measured exit gradient versus predicted exit gradient for training data.
4.5 Comparison of GEP and SPSS

The GEP model specification was evaluated by determining its efficiency and comparing this with that of the SPSS model. The predicted exit gradient was also computed using the GEP model and compared that produced by the SPSS model and with the measured exit gradient. The coefficient of determination, $R^2$, was calculated for both models, being equal to 0.892 and 0.864 for GEP and SPSS, respectively, for the training data, and to 0.90 and 0.86 for the testing data. As a result, the performance of the GEP model can be said to be better than that of SPSS, as the $R^2$ values for GEP are greater than those of SPSS. The SPSS equation, despite its lower performance than the GEP equation, in general offers relatively good results. Overall, the statistical tests and scatter diagrams suggest that the performance of the GEP model is better than that of the SPSS model, however, and that GEP provides an explicit objective description and compactness that could be useful to engineers.

5. Conclusion

1. The results of verification between the finite difference code and Geostudio seep/w showed good agreement.

2. The exit gradient decreases with increases in the cutoff location ratio and depth ratio, while the uplift pressure increases with such increases.

3. Using the cutoff upstream is very effective at reducing uplift pressure, while the cutoff downstream is more effective in reducing the exit gradient.

4. The minimum exit gradient was observed when the cutoff location ratio at the downstream was $x_1/b=1$, with a maximum relative depth of $d_1/b=0.6$, while the minimum uplift pressure was observed when the cutoff location ratio at the upstream was $x_1/B=0$, with a minimum relative depth of $d/B=0.1$.

5. The results indicate that the maximum exit gradient is observed when the ratio of the length of upstream cutoff to the length of downstream cutoff is $d_1/d_2 = 1$. This gradient decreases as the ratio of $d_1/d_2$ increases and is significantly reduced as the ratio of $d_1/d_2$ decreases.
6. The performance of the GEP model in computing the exit gradient sizes better than that of the SPSS model, as the value of $R^2$ obtained from the GEP model was greater than that obtained from the SPSS model.

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