“Worst-case” Microlensing in the Identification and Modeling of Lensed Quasars

Luke Weisenbach1, Paul L. Schechter1,2, and Sahil Pontula1

1 MIT Department of Physics, Cambridge, MA 02139 USA; weisluke@alum.mit.edu
2 MIT Kavli Institute for Astrophysics and Space Research, Cambridge, MA 02139 USA

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Abstract

Although microlensing of macrolensed quasars and supernovae provides unique opportunities for several kinds of investigations, it can add unwanted and sometimes substantial noise. While microlensing flux anomalies may be safely ignored for some observations, they severely limit others. “Worst-case” estimates can inform the decision whether or not to undertake an extensive examination of microlensing scenarios. Here, we report “worst-case” microlensing uncertainties for point sources lensed by singular isothermal potentials, parameterized by a convergence equal to the shear and by the stellar fraction. The results can be straightforwardly applied to nonisothermal potentials by utilizing the mass sheet degeneracy. We use microlensing maps to compute the fluctuations in image micromagnifications and estimate the stellar fraction at which the fluctuations are greatest for a given convergence. We find that the worst-case fluctuations happen at the stellar fraction \( \kappa_* = \frac{1}{\mu_{\text{macro}}} \). For macrominima, the fluctuations in both magnification and demagnification appear to be bounded (\( 1.5 > \Delta m > -1.3 \), where \( \Delta m \) is the magnitude relative to the average macromagnification). Magnifications for macrosaddles are bounded as well (\( \Delta m > -1.7 \)). In contrast, demagnifications for macrosaddles appear to have unbounded fluctuations as \( 1/\mu_{\text{macro}} \to 0 \) and \( \kappa_* \to 0 \).

Unified Astronomy Thesaurus concepts: Gravitational microlensing (672); Quasar microlensing (1318); Strong gravitational lensing (1643)

1. Introduction

Gravitational lensing has emerged as a powerful tool to probe the potentials of distant galaxies and galaxy clusters, and has further been used to help constrain models of dark matter and dark energy. Lensed quasars have been particularly prominent, and the number of known quadruply lensed quasars has increased by roughly 50% since the first data release from the Gaia satellite (Gaia Collaboration et al. 2016). Approximately 20 quadruply lensed quasars have been discovered using Gaia DR1 and DR2, often in concert with other catalogs. The Gaia GraL group (Krone-Martins et al. 2018) identified 80,000 quartets of candidate point sources for consideration as possible quadruply lensed quasars. The positions and fluxes were analyzed, with great success, using “Extremely Randomized Trees” (Delchambre et al. 2019), with 12 out of the 13 known quadruply lensed quasars with four cataloged images ranked in the top 0.6%. The authors attribute the ranking of the thirteenth, three times further down the list, to microlensing of the quasar images by stars in the lensing galaxy, a phenomenon that had not been taken into account in their training set of roughly 10^8 systems.

Microlensing helps place constraints on the smooth (dark) matter content of the lensing galaxy. Previous work has shown that for highly magnified macroimages, the fluctuations in the flux ratio due to microlensing often increase with the introduction of dark matter to the lens model (Schechter & Wambsganss 2002). Furthermore, it was expected that these microlensing fluctuations would be larger for saddle-points as compared to minima (Witt et al. 1995), which large-scale parameter studies of microlensing magnification probability distributions have confirmed (Wambsganss 1992; Lewis & Irwin 1995; Vernardos & Fluke 2013). Contingent upon the available resources, it would be possible to include the effects of microlensing in the training set of the Extremely Randomized Trees, particularly given the public availability of the GERLUMPH suite of simulations (Vernardos et al. 2014). Here we describe an approximate alternative that could be readily adapted to lens searches, which inevitably involve expectations for lensed flux ratios.

Briefly, we assume a convergence and shear appropriate to a singular isothermal elliptical potential—a model that works reasonably well for most known quadruply lensed quasar systems—and search for the stellar contribution to the convergence that produces the largest microlensing fluctuations for a point source. We call this “worst-case” microlensing, and give a 95% confidence range for the fluctuations. One can then use these ranges (accounting in some way for the likelihood of the worst-case stellar fraction) to assign a likelihood to a discrepant source flux. The modeling of Gaia quartets is just one of many lens-modeling problems where worst-case estimates could prove useful. One need not completely discount the observed fluxes from quasar images if the expected fluctuations are small, as for low magnification images (particularly macrominima, which as stated previously suffer less from such fluctuations than macrosaddles).

The fluctuations discussed within this work are for that fraction of the source that can be treated as pointlike. Our discussions are not relevant to the flux from the more extended regions of emissions corresponding to longer wavelength observations, which have narrower magnification distributions (Bate et al. 2007).

The organization of this paper is as follows. In Section 2, we give a brief introduction to gravitational lensing, macrolensing models, and the phenomenon of microlensing. In Section 3, we describe the important components of our methodology, including the microlensing maps upon which our analyses depended. Section 4 shows our results, including plots of the worst-case fluctuations. A set of systems that showcase “worst-case” conditions is discussed in Section 5. Conclusions are presented in Section 6.
2. Background

2.1. The Gravitational Lensing Phenomenon

Gravitational lensing is a direct consequence of general relativity. Light passing near a massive object is deflected, much as a light ray obeying Fermat’s principle of least time refracts on passing through an optical lens. Gravitational lensing can be succinctly described using the time delay surface

\[ t = \frac{1 + z_d}{c} \left[ \frac{D_d D_l}{2 D_{dl}} \left( \frac{\xi}{D_d} - \frac{\eta}{D_l} \right)^2 - \Psi(\xi) \right] , \]

where \( z_d \) is the redshift of the lens; \( D_d, D_l, \) and \( D_{dl} \) are the angular diameter distances from observer to lens, observer to source, and lens to source, respectively; \( \eta \) is the position in the source plane; \( \xi \) is the position in the image plane; and

\[ \Psi(\xi) = \frac{2}{c^2} \int_{\text{observer}}^{\text{source}} \phi(\xi, l) \, dl \]

is the Newtonian gravitational potential of the lens \( \phi(\xi, l) \) scaled and integrated along the line of sight (Schneider et al. 1992). The lens equation, which can be simply written as

\[ \nabla t = 0 , \]

or more fully as

\[ \eta = \frac{D_l}{D_d} \xi - \frac{D_d}{D_l} \nabla \Psi(\xi), \]

provides the locations where the images of the source are seen at stationary points of the time delay surface, which can be either minima, maxima, or saddle-points. The magnifications of the images are inversely proportional to the curvature of the time delay surface.

The lens equation can be nondimensionalized into

\[ y = x - \nabla \psi(x), \]

where \( y \) is the position in the source plane, \( x \) is the position in the image plane, and \( \psi \) is the gravitational potential. The magnifications of the images are then

\[ \mu = 1 / \det \left( \frac{\partial y}{\partial x} \right), \]

with the sign of the magnification determining the parity of the image. Saddle-points have \( \mu < 0 \), while minima and maxima have \( \mu > 0 \). Furthermore, minima are always magnified, whereas saddles and maxima can be demagnified.

2.2. Quadruple Lenses and Macrolensing Models

Our analysis is appropriate to lensed sources with any number of images, but as quadruply lensed sources have greater redundancy, we concentrate on these. Quadruply lensed quasars play an important role in extragalactic astrophysics, helping to elucidate the potentials, stellar content, and dark matter content of the lensing galaxy, as well as the structural properties of the background source (Schechter & Wambsganss 2004; Bate et al. 2008). Furthermore, quadruple lenses are of great importance in cosmology because they help constrain the values of parameters such as the Hubble constant (Suyu et al. 2013). Lensing galaxies are often modeled as singular isothermal elliptical potentials (SIEPs) (Kovner 1987), for which observable images are seen at saddle-points and minima of the light travel time surface. An image also forms at the maximum near the center of the lens matter distribution, but the singular nature of the SIEP model at this location means that this image is infinitely demagnified. This model has been reasonably successful at predicting the image positions (Schechter & Wynne 2019). The magnifications of the macromage can be computed as

\[ \mu_{\text{macro}} = \frac{1}{(1 - \kappa^2) - \gamma^2}, \]

where \( \kappa \) denotes the effective convergence of the lens at the position of a macroimage and \( \gamma \) denotes its shear. \( \kappa \) is related to the gravitational potential through the two-dimensional Poisson equation

\[ \kappa(x) = \frac{1}{2} \nabla^2 \psi(x) = \frac{1}{2} (\psi_{11} + \psi_{22}), \]

while \( \gamma = \frac{\sqrt{\gamma_1^2 + \gamma_2^2}}{2} \), with

\[ \gamma_1 = \frac{1}{2} (\psi_{11} - \psi_{22}), \quad \gamma_2 = \psi_{12} = \psi_{21}. \]

Minima of the time delay surface occur when \( 1 - \kappa - \gamma > 0 \); saddle-points occur when \( 1 - \kappa - \gamma < 0 \); and maxima occur when \( 1 - \kappa + \gamma > 0 \).

For isothermal gravitational potentials, we further have \( \kappa = \gamma \), for which Equation (7) reduces to

\[ \mu_{\text{macro}} = \frac{1}{1 - 2 \kappa}. \]

2.3. Microlensing

“Microlensing” is distinguished from “macrolensing” in that the multiple images produced by the former cannot be resolved with today’s telescopes. Both can occur simultaneously, as when a galaxy produces multiple observable images of a quasar and the stars within the galaxy produce unresolved multiples of those images. It is useful to distinguish between microlensing at low optical depth, where a single star does the lensing, and lensing at high optical depth, where a great many stars contribute to the lensing, producing a great many micromages (Paczynski 1986). The observed flux from the macroimage is then the combined flux from the micromages.

As the source and the microlensing stars move, the micromages change in brightness, introducing fluctuations in the observed fluxes of macromages. As one might expect, this effect depends on the stellar content of the lens. For this reason we define the stellar fraction as

\[ s_* = \frac{\kappa_*}{\kappa}, \]

where \( \kappa_* \) is the convergence due to stars, \( \kappa_s \) is the convergence due to smooth matter, and \( \kappa = \kappa_* + \kappa_s \) is the total convergence.

3. Method

3.1. Microlensing Maps

Our work is based on microlensing magnification maps created using the inverse ray-shooting technique (Kayser et al. 1986; Schneider & Weiss 1986). A multitude of light rays are traced backwards using the lens equation, from image plane to source plane, and collected in a pixelated map. The number of rays per
which itself depends on the macromagnification. The naive implementation uses all the stars in the order that constitutes the shooting region in the image plane. We GPU-parallelized tree-code like that of Alpay where possible. Our GPU implementation is neither a fully direct simulation parameters, such as map size and average ray density, more easily examine areas of interest in the high magnification maps of the GERLUMPH Data Release 1 2014 shooting code that runs on a Graphics Processing Unit (GPU) to more easily examine areas of interest in the high magnification, low stellar density regime, while maintaining a set of consistent simulation parameters, such as map size and average ray density, where possible. Our GPU implementation is neither a fully direct ray tracing method like that of Thompson et al. (2010), nor a fully GPU-parallelized tree-code like that of Alpay (2019). Instead, our naive implementation uses all the stars in the field to directly shoot four rays (one near each of the corners) in each square of a grid that constitutes the shooting region in the image plane. We calculate Taylor coefficients of the deflection angle (up to the third order) within the square and then use those coefficients to shoot ≈700 more rays within the square. The size of the squares depends on the number density of the rays in the image plane, which itself depends on the macromagnification \( \mu_{\text{macro}} = \langle \mu \rangle \), the desired average number of rays per pixel \( \langle n_{\text{rays}} \rangle \), the pixel size, and the number of rays that are shot using the Taylor coefficients. We note that the sizes of the squares in the image plane were never greater than 0.2\( \theta_E \) for our simulations, with the majority being less than 0.05\( \theta_E \) over the parameter space sampled. The top portion of Figure 1 shows sample magnification maps created with our code. We shot rays into a square in the source plane with a side length of 250\( \theta_E \) and an area of 2500 pixels, so that each pixel constituted a 0.001\( \theta_E \times 0.01 \theta_E \) square. We additionally chose \( \langle n_{\text{rays}} \rangle = 1000 \) for all of our simulations.

3.2. Worst-case Analysis

The magnification maps show the total source magnification as a function of position. The magnification at a specific source position is the sum of the magnifications of the many microimages, and in general will fluctuate around the average macromagnification \( \langle \mu \rangle \) as the source moves. We create histograms of the magnifications, expressed as magnitude differences from the theoretical average \( \langle \mu \rangle \), using

\[
\Delta m = -2.5 \log \frac{\mu}{\langle \mu \rangle} = -2.5 \log \frac{n_{\text{rays}}}{\langle n_{\text{rays}} \rangle}.
\]

We choose to examine worst-case microlensing by calculating the 97.5 and 2.5 percentiles\(^3\) of the magnitude distributions, i.e., we find the ray counts for which 2.5\% of all the pixels contain more rays and for which 2.5\% of all the pixels contain fewer rays, and transform them into magnitudes. We designate these limits by \( \Delta_{++} \) and \( \Delta_{--} \), respectively.\(^4\) The bottom portion of Figure 1 shows the histograms for the magnification maps of the top portion, with the values of \( \Delta_{++} \) and \( \Delta_{--} \) marked. For every point sampled in \( (\kappa, s_\star) \) space, we run 10 microlensing simulations and calculate the average \( \Delta_{++} \) and \( \Delta_{--} \) from these 10 simulations.

3.3. The Mass Sheet Degeneracy

The mass sheet degeneracy (Falco et al. 1985) allows for an arbitrary scaling of lens mass distributions, without affecting any of the observable quantities like position and magnification. In the context of microlensing, this degeneracy allows for any triplet of \( (\kappa, \gamma, s_\star) \) to be converted into an effective doublet\(^5\) through a judicious choice of effective \( \kappa' \), reducing the parameter space by one dimension while scaling the magnifications and source plane coordinates (Paczynski 1986; Kochanek 2004; Vernardos et al. 2014; Schechter et al. 2014). Our simulations, which sample the parameter space \( (\kappa, \gamma, s_\star) \) within the square where \( 0 < \kappa < 1 \) and \( 0 < s_\star < 1 \), are immediately applicable to the SIEP model. However, they are still relevant for a wider range of parameter values when appropriately scaled. More specifically, given values for \( (\kappa, \gamma, s_\star) \), the transformations

\[
1 - \kappa' = \frac{1 - \kappa}{1 - \kappa + \gamma},
\]

\[
\gamma' = \frac{\gamma}{1 - \kappa + \gamma} = \kappa',
\]

and

\[
s_\star' = s_\star \frac{\kappa}{\gamma}
\]

allow comparison with our results.

4. Results

We created contour plots of our averaged \( \Delta_{++} \) and \( \Delta_{--} \) as functions of \( (\kappa, s_\star) \).\(^6\) The left half of Figure 2 shows \( \Delta_{--} \), while the right half shows \( \Delta_{++} \). The “+” symbols on the plots denote the points in the parameter space for which we ran microlensing simulations. Data points are symmetrical around \( \kappa = 0.5 \) for the majority of the plots, with the symmetry broken closer to \( \kappa = 0.5 \) to explore regions of interest. The top portions of the plots show the entirety of the parameter space sampled, while the bottom portions of the plots zoom in around \( \kappa = 0.5 \) for low values of \( s_\star \).

4.1. \( \Delta_{++} \)

The plot of \( \Delta_{++} \) displays several interesting properties. First, for both the macrominima (the left half, \( 0 < \kappa < 0.5 \)) and the macrosaddles (the right half, \( 0.5 < \kappa < 1 \)), there appears to be a ridge along which the value of \( \Delta_{++} \) is extremized. For each respective macromage type, the brightest value of \( \Delta_{++} \) appears to be constant along this ridge. For the macrominima we found this value to be \( \Delta m \approx -1.3 \) mag, while for the macrosaddles we found it to be \( \Delta m \approx -1.7 \) mag. The dashed black line on the plot denotes the locus where \( \kappa_s = 1/|\mu_{\text{macro}}| \), which traces the ridge of the brightest \( \Delta_{++} \). Figure 3 displays the values of \( \Delta_{++} \) along this ridge.

\(^3\) Chosen to reflect a quasi-two standard deviation spread.

\(^4\) We use \( \Delta_{++} \) and \( \Delta_{--} \) to refer to the cutoffs as well as the number of rays, the magnification, and the magnitude at those cutoffs, for ease of reference. Which quantity is meant at a particular time should be evident from the context.

\(^5\) The choice of doublet is arbitrary. A common alternative is to choose \( (\kappa = \kappa_s, \gamma) \).

\(^6\) Contour plots were created using Mathematica’s ListContourPlot function. They are simple linear interpolations of the data points provided.
Figure 1. Top: example magnification maps for (left) $\kappa = \gamma = 0.45$, $\sigma = 0.5$; and (right) $\kappa = \gamma = 0.55$, $\sigma = 0.5$. The physical size of the maps is 256 pixels per side. Middle: the magnification maps converted to very low contrast. Black pixels denote $\mu < \Delta_{-}$, white pixels denote $\mu > \Delta_{+}$, and gray pixels cover everything between. Bottom: histograms for the magnification maps. The left and right dashed lines in each mark the values of $\Delta_{-}$ and $\Delta_{+}$, respectively, while the central dotted-dashed line marks the magnitude of the simulated average magnification. The magnitude bins have a width of $\Delta m = 0.03$ mag.
Figure 2. Contour plots of (left) $\Delta_- \gamma$ and (right) $\Delta_+ \gamma$, as functions of $\kappa = \gamma$ and $s_\gamma$. The “+” symbols denote the points in the parameter space for which we ran microlensing simulations, while the dashed black line denotes the locus where $\kappa = 1/|\mu_{\text{macro}}|$. The contours are separated by 0.1 mag, with the thicker black contours occurring in intervals of 0.5 mag. The top portion of each plot shows the entire parameter space covered, with the dashed red rectangle denoting the zoomed-in region that is shown in the bottom portion of each plot.
4.2. $\Delta_{-}$

The left half of Figure 2 shows the contour plot for $\Delta_{-}$. Note the difference in the range of magnitudes shown compared to that for $\Delta_{+}$. The contours of this plot are markedly different than those for $\Delta_{+}$, and display different features for the macrominima as opposed to the macrosaddles. The macrominima, there again appears to be a ridge along which the value of $\Delta_{-}$ is extremized. We find this extremal value to be $\Delta m \approx 1.5$ mag. The dashed black line no longer describes where this ridge is located though—the ridge appears to have some other locus describing it.

The macrosaddles have no such ridge where $\Delta_{-}$ is extremized; instead, the values form a “basin” with slowly sinking contours that grow fainter and fainter. However, for a given vertical cut at a specific $\kappa^c$ value, the value of $\Delta_{-}$ grows fainter as $s_\kappa$ increases before reaching an extremum and growing brighter again. This faintest value for a specific $\kappa^c$ is described once more by the locus where $\kappa^c = 1/|\mu_{\text{macro}}|$. As $\kappa \rightarrow 0.5$ and $s_\kappa \rightarrow 0$, the contours of $\Delta_{-}$ gradually sink further and further, with the dimmest possible fluctuations becoming fainter and fainter as the macromage becomes brighter and the stars move farther apart. We were limited by computer power and time, but we found values of demagnification up to $\Delta m \approx 5.4$ mag in this regime.

4.3. Simulation Errors and Uncertainties

The Appendix contains a discussion of some of the errors associated with microlensing simulations, our mitigations of such errors, and the uncertainties in our measured values for $\Delta_{+}$ and $\Delta_{-}$.

5. Observed “Worst-case” Scenarios

“Worst-case” scenarios require the confluence of three factors: the random configuration of microlensing stars, the ratio of stellar to total mass surface density, and the ratio of the half-light radius of the source to the Einstein ring radius of the microlenses. The likelihoods of the last two of these will vary from one circumstance to another. There are many quadruply lensed sources for which the observed flux ratio anomalies at one or another wavelength are much smaller than those of the worst-case scenario, either by virtue of the source size or the microlens surface density. In this section we examine three examples of lensed systems that are consistent with worst-case scenarios.

5.1. Quad Lens RX J1131-1231

The quad RX J1131-1231 was the thirteenth and last-ranked GraL system described in Section 1. We have used Keeton’s lensmodel program (Keeton 2001) to model the HST positions for the four images as a singular isothermal sphere with external shear.

Minimizing the rms residuals of Gaia magnitudes from the model’s predictions, we find that image C, the faintest of the cusp images, is 0.34 magnitudes fainter than predicted. This is only one quarter of the $\Delta_{-}$ excursion shown in Figure 2. Image D, isolated on the far side of the lens, is 1.20 magnitudes brighter than predicted. This is roughly three-quarters of the $\Delta_{-}$ excursion shown in 2.

These fluctuations are not extreme. They raise the question of why the 12 other known lenses exhibited fluctuations that were so much smaller. The widely appreciated answer is that the optical continuum-emitting regions of quasars are at least as extended as the Einstein rings of microlensing stars. Pooley et al. (2007) show that the rms deviations of optical fluxes from those predicted by lens models only are half as large as those observed in X-ray fluxes, which have been found to arise from smaller regions than the optical (Morgan et al. 2008).

5.2. Quad Lens SDSS J0924+0219

Keeton et al. (2006) call SDSS J0924+0219 “the most anomalous lensed quasar.” Using the Keeton et al. model, with the HST V-band source flux determined from the B and C
images, we find the D image is 2.79 magnitudes fainter than predicted, compared with a $\Delta_\gamma$ excursion of 3.1 mag found from Figure 3. Using the X-ray fluxes from Pooley et al. (2007), again calibrated by B and C, we find image D is 2.58 magnitudes fainter than predicted.

One might wonder whether the faintness of image D is due to substructure millilensing, but Badole et al. (2020) have found radio flux ratios that are consistent with the model predictions, ruling out the millilensing hypothesis. SDSS J0924+0219 would appear to be a quad in which our worst-case estimate is borne out.

5.3. SN 1a iPTF16geu

Mortsell et al. (2020) present a model for the observations of the quadruply lensed supernova iPTF16geu. Their model has a potential that is slightly shallower than isothermal, so the convergence will typically be larger than the shear. We take the shortcut of using their magnifications and our Figure 2 to estimate the expected range of microlensing fluctuations, rather than finding the magnifications corresponding to $\kappa = \gamma$. Their image 1 is 1.00 mag brighter than the model’s prediction, and their image 4 is 0.70 mag fainter than predicted. Their estimated stellar fraction is $s_\ast \sim 0.2$. Figure 2 gives $\Delta_{\gamma_\ast} = -1.5$ mag and $\Delta_\gamma = 1.3$ mag. Taken together the fluctuations are roughly what one would expect if the supernova were pointlike at the time of observation.

6. Conclusions

We have conducted an analysis of the effects of microlensing on the brightness of gravitationally lensed images. We used microlensing maps to obtain “worst-case” uncertainties (which we defined as the boundaries of the 95% confidence range) in the micromagnification of different combinations of the convergence $\kappa$ and stellar fraction $s_\ast$ for lenses characterized by isothermal potentials, where $\kappa = \gamma$. On the faint end ($\mu < \Delta_\gamma$), demagnification can be attributed to the absence of extra image pairs—a macrominimum has only one microminimum and a saddle-point for every star, while a macrosaddle has one microsaddle corresponding to the global saddle-point and fainter microsaddles for every star. The fact that minima must be of at least unit magnification while saddle-points can be arbitrarily demagnified leads to some of the differences between the left and right halves of the $\Delta_\gamma$ plot in Figure 2. At the bright end ($\mu > \Delta_{\gamma_\ast}$), high magnifications are largely due to caustic crossing events or passage near cusps. The bottom half of Figure 1 shows the magnification maps of the top half converted into a three-color scheme: black for $\mu < \Delta_{\gamma_\ast}$, white for $\mu > \Delta_{\gamma_\ast}$, and gray for our 95% interval in between. The caustics largely occur within the region of $\mu > \Delta_{\gamma_\ast}$, and they would be completely recovered in white in the figure if the pixel resolution were to be increased.

While our purpose was to determine the “worst-case” microlensing fluctuations, we can comment on the cause of the deviations within the 95% confidence range as well. The deviations within can, for the most part, be attributed to variations in the number of microminima. Granot et al. (2003) assert that these variations are greatest when the average number of extra image pairs $\langle n \rangle \sim 1$. If the average area of the caustic due to a star scales with $|\mu_{\text{macro}}|$, then the covering factor of the caustics (which gives the expected number of extra image pairs) that maximizes the variations can be found as roughly $\kappa_\ast \cdot |\mu_{\text{macro}}| \sim 1$, which recovers the locus $\kappa_\ast = 1/|\mu_{\text{macro}}|$. Alternatively, one can consider the contributions of smooth and grainy matter to the magnification tensor. The stars introduce fluctuations into the magnification tensor on top of the smooth matter component. One might expect these fluctuations to be most impactful when $1 - \kappa_\ast - \gamma = 0$, which (for macrosaddles at least) recovers $\kappa_\ast = 1/|\mu_{\text{macro}}|$ as well.

The appendix of Liao et al. (2015) examines the role of $s_\ast$ in the rms fluctuations of magnification maps for a macrominimum and a macrosaddle. For their macrominimum, which has $\kappa = 0.475$ and $\gamma = 0.425$, we would expect the fluctuations to peak at $\kappa_\ast = 1/|\mu_{\text{macro}}| \approx 0.1$. Accounting for the somewhat sparse sampling in their Figure 14, we find this value of $\kappa_\ast$ to be consistent with their results. Similarly for their macrosaddle, which has $\kappa = 0.475$ and $\gamma = 0.425$ resulting in the same macromagnification, a peak in the rms near $\kappa_\ast \approx 0.1$ is clearly seen. Indeed, Figure 14 of Liao et al. (2015), after appropriate conversion using the mass sheet degeneracy, bears a striking resemblance to a vertical cut of $\Delta_{\gamma_\ast}$ or $\Delta_\gamma$ in our Figure 2 at $\kappa = \gamma = 0.45$ or 0.55 (for the minimum or saddle, respectively).

Under the worst set of conditions, corresponding to the microlensing of a highly magnified saddle with low stellar surface mass density, our analysis showed that microlensing introduces uncertainties of at least three magnitudes, with the uncertainty appearing to increase unbounded as $\kappa \rightarrow 0.5$ and $s_\ast \rightarrow 0$. Elsewhere, (de)magnifications appear to be bounded. The fluctuations in the magnifications for macrominima and macrosaddles peak when the stellar fraction $\kappa_\ast = 1/|\mu_{\text{macro}}|$.

Simple comparisons of the observed magnitude differences in lensed systems to the “worst-case” fluctuations in Figure 2 can serve as a guide for determining whether more detailed microlensing analyses are worthwhile.

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The data from this work can be made available via a reasonable request to the corresponding author.

Appendix

Simulation Errors and Uncertainties

Any microlensing ray tracing simulations are subject to the effects of using a finite number of stars. Rays passing close to a star are deflected out of the receiving square, but there are no stars far away that deflect rays in. Katz et al. (1986) and Schneider & Weiss (1987) provide expressions for the number of stars seen in a solid angle containing a desired percentage of the total flux; our code uses enough stars and appropriately sized shooting regions to account for 99% of the flux.

Any given realizations of a star field will still exhibit slight differences from one another as well. Our decision to average over 10 simulations for each point in $(\kappa_\ast, s_\ast)$ attempts to mitigate such differences. Figure 2 shows the average $\Delta_{\gamma_\ast}$ and $\Delta_\gamma$ from each set of 10 simulations. Here, we discuss the spread of each set.

What we actually measure is the number of rays corresponding to our $\Delta_{\gamma_\ast}$ and $\Delta_\gamma$ limits in each simulation, so the standard deviation $\sigma$ of each set of 10 simulations is found in
Figure 4. The $\sigma_{--}$ uncertainty in the number of rays corresponding to $\Delta_{--}$ measured relative to $\Delta_{--}$ (left), and the $\sigma_{++}$ uncertainty in the number of rays corresponding to $\Delta_{++}$ measured relative to $\Delta_{++}$ (right).
units of the number of rays as well. We must then account for the fact that $\Delta_-$ is demagnified and will have fewer rays than the magnified $\Delta_+$—a standard deviation $\sigma_-$ of 10 rays for $\Delta_-$ is not necessarily better than a standard deviation $\sigma_+$ of 100 rays for $\Delta_+$. The obvious comparators for $\sigma_+$ and $\sigma_-$ are the number of rays giving the $\Delta_+$ or $\Delta_-$ limits themselves. Figure 4 shows density plots of the uncertainties in the number of rays for $\Delta_+$ and $\Delta_-$ measured relative to themselves. We find that $\sigma_- \leq 0.1 \Delta_-$ for the majority of the parameter space covered. This increases slightly to $\sigma_- \leq 0.3 \Delta_-$ closer to $\kappa \approx 0.5$, particularly for higher values of $s_*$. There are some additional instances where $\sigma_- \approx 0.5 \Delta_-$, for much lower values of $s_* \approx 0.001$, but these can be attributed to the low surface mass density and the fact that our source plane region of $25 \theta_E$ is likely to be somewhat smaller than necessary to adequately sample the caustics. We find that $\sigma_+ \leq 0.23 \Delta_+$ for the entirety of the parameter space, with the majority having $\sigma_+ \leq 0.1 \Delta_+$. We note here that our choice of $\langle n_{\text{rays}} \rangle = 1000$ plays a role in the errors. For low macromagnifications, this choice of $\langle n_{\text{rays}} \rangle$ is more than adequate, but for high macromagnifications, a larger number of rays may have been more appropriate (at the expense of more computing time). Some of our magnification maps contained pixels with zero rays, although unsurprisingly this only occurred in the high macromagnification, low stellar density regime of the macrosaddles, where there is a possibility of strong demagnification. Increasing the average number of rays would have removed any zero-count pixels and decreased our $\sigma_-$ uncertainties relative to $\Delta_-$. 

ORCID iDs

Luke Weisenbach @ https://orcid.org/0000-0003-1175-8004
Paul L. Schechter @ https://orcid.org/0000-0002-5665-4172

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