Formation and disruption of current filaments in a flow-driven turbulent magnetosphere

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Abstract. Recent observations have established that the magnetosphere is a system of natural complexity. The co-existence of multi-scale structures such as auroral arcs, turbulent convective flows, and scale-free distributions of energy perturbations has lacked a unified explanation, although there is strong reason to believe that they all stem from a common base of physics. In this paper we show that a slow but turbulent convection leads to the formation of multi-scale current filaments reminiscent of auroral arcs. The process involves an interplay between random shuffling of field lines and dissipation of magnetic energy on sub-MHD scales. As the filament system reaches a critical level of complexity, local current disruption can trigger avalanches of energy release of varying sizes, leading to scale-free distributions over energy perturbation, power, and event duration. A long-term memory effect is observed whereby the filament system replicates itself after each avalanche. The results support the view that that the classical and inverse cascades operate simultaneously in the magnetosphere. In the

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former, the high Reynolds-number plasma flow disintegrate into turbulence through successive breakdowns; in the latter, the interactions of small-scale flow eddies with the magnetic field can self-organize into elongated current filaments and large-scale energy avalanches mimicking the substorm.

1. INTRODUCTION

Energy release in the magnetosphere manifests itself as geomagnetic and auroral perturbations. Detailed analyses have shown that these perturbations follow the so-called scale-free distributions (Consolini, 1997; Lui et al., 2000; Uritsky et al., 2002; 2009; Kozelov et al., 2004). For instance, Uritsky et al. (2002) found that the probability density function over auroral brightness integrated over space and time (called $E$) has a power-law form $E^{-\alpha}$, where $\alpha$ is a constant. What scale-free distributions mean in the context of magnetospheric physics has drawn considerable interest of late. One interpretation is that the active magnetosphere is in a state of self-organized criticality (SOC); energy releases in a SOC state can have different sizes, but the governing physics is the same. A number of theoretical and simulation studies have been carried out, in which scale-free distributions of magnetospheric perturbations were reproduced (Chapman et al., 1998; Klimas et al., 2000, 2004; Uritsky et al., 2001; Valvidia et al., 2003; Liu et al., 2006; Valliere-Nollet et al., 2010).

While scale-free dynamics may be mathematically elegant and conceptually appealing, a deeper inspection brings us to an apparent contradiction: The structures that are associated with or responsible for energy release do not follow scale-free statistics. It is
well-known that active aurora is dominated by discrete arcs, and the disruption of
equatorward arcs lies at the heart of auroral substorm onsets (Akasofu, 1964). The
relationship of the disruption to propagation of substorm perturbations in the
magnetosphere was recently elaborated by Donovan et al. (2008). Knudsen et al. (2001)
performed a quantitative study of the thickness of the 557.1 nm green line excited by 1-
10 keV electrons and found a centered distribution with a mean thickness of \( \sim 18 \) km.
Embedded in the Knudsen distribution are finer-scale arc populations with thicknesses \( \sim 1 \)
km (Partamies et al., 2010), \( \sim 100 \) m (Trondsen et al., 1998) and \( \sim 10 \) m (Maggs and
Davis, 1968). Although the structuring of auroral arcs has not been completely resolved
as an observational problem, it is generally agreed that the scale distribution of aurora is
not a smooth continuum but has multiple peaks. How do we reconcile the discrete
structuring of arcs with scale-free dynamics of energy release? The incongruity of this
question led Knudsen et al. (2001) to assert that “the arc width spectrum argues against
the notion of a turbulent cascade of energy from larger to small scales.”

The formation of auroral arcs is by no means a settled question. As will be elaborated
in a separate study, arcs in the Knudsen population typically have longitudinal lengths of
several thousand km, which maps to a scale comparable to the size of the magnetosphere.
Moreover, the lifetime of these arcs is typically well over 1 min, which is approximately
the Alfvén transit time. These properties hint strongly that these arcs are regulated by the
magnetosphere. While processes in the auroral acceleration region 1-2 Re above Earth
can explain the observed thickness of Knudsen arcs (e.g., Borovsky (1993)), it is unlikely
that long arcs are formed without any organization on the part of the magnetosphere, for
otherwise one would be forced to concoct theories why an aurora arc align itself so perfectly over the magnetospheric scale without the magnetosphere playing a role. From the temporal point of view, auroral features lasting longer than the Alfvén transit time must maintain some equilibrium with equivalent features in the magnetosphere. Last but not least is the 18-km average thickness. At the approximate 67° magnetic latitude where the Knudsen population was sampled by the CANOPUS all-sky camera in Gillam, the latitudinal mapping factor has the order ~50; a 18-km thick arc should map to the central plasma sheet (CPS) as a filament ~900 km in width. In comparison, a 10 keV proton in a 20-nT magnetic field has a gyroradius ~500 km. Therefore, while the cross-tail length of an arc mapped to the magnetosphere is definitely of the MHD scale, its width is likely controlled, in part, by dissipation effects on the ion scale.

Hence, if we accept the premise of magnetospheric origin for auroral arcs, as observations compel us to, we must deal with conceptual problems on several fronts. One has to do with the metastability of arcs. By metastable we mean that the arcs maintain a steady form for a period longer than the Alfvén transit time (~1 min for the CPS). Under this condition, one would be tempted to view arcs as a characteristic solution of the quasistatic convection problem. However, even in the latest edition of the Rice Convection Model (e.g., Lemon et al., 2004), arc-like solutions do not exist; neither do these structures arise naturally in global MHD simulations. In fact, the actual condition of the magnetosphere poses an even more confounding problem. In-situ observations of plasma flows in the plasma sheet paint a system that is rather turbulent, with the rms speed much larger than the average speed (Angelopoulos et al., 1992; 1999; Borovsky et
al., 1997; Borovsky and Funsten, 2003). How can metastable, arc-like structures survive in, let alone be produced by, a turbulent magnetosphere? Little consideration has been given to this question in the literature. The stationary Alfvén wave theory of Knudsen (1996) predicts arcs with thickness a few times the electron inertial length in the topside ionosphere (~1 km), but requires some ionospheric irregularity (i.e., proto-arc) to anchor the resulting structure. Field-line resonances (FLRs) (Southwood, 1974; Chen and Hasegawa, 1974) give arc-like structures, and observations showed that some arcs indeed oscillate at ULF frequencies predicted by FLR theories (e.g., Xu et al., 1993; Liu et al., 1995). However, for those arcs which oscillate, the fluctuation is typically a small fraction of the overall brightness (e.g., Uritsky et al., 2009). We are still left with the task of explaining the dominant non-oscillating part of the arcs.

The brief review above points to significant gaps in our knowledge of the relationship between magnetospheric structures and dynamics of energy release usually associated with the collapse of these structures. Of particular interest are the following questions: How do metastable arc-like structures form in a turbulent magnetosphere? What makes these structures collapse? What is the distribution of energy release from the collapse? At present we lack a clear program to formulate answers to these questions, a task we embark upon from the point of view of nonlinear multi-scale coupling.

As a first step, we develop a new framework whose salient properties are investigated with a simplified model. As a point of departure, we begin with a magnetosphere in a state of weak turbulence (in the sense that the flow speed is much smaller than the speeds of MHD modes). We track the change of the magnetic field frozen in the flow and
observe the current structures resulting from the random shuffling of field lines. In a surprising twist, we will show that the resulting current distribution does not have the uncorrelated random appearance of its turbulent driver but exhibits elongated filamentary structures reminiscent of arcs. In section 2, we give the basic outline of the theory, as well as key assumptions of the model. In section 3, we present simulation results from select runs of the model, including time series of energy avalanche, probability density functions of energy release, and morphology of representative current distributions. In section 4, we discuss the implications of the results in the context of multiscale magnetospheric dynamics and propose an interpretation of magnetospheric dynamics based on the idea of natural complexity.

2. THEORY

Bright auroral arcs are generated by energetic electron precipitation and associated principally with upward field-aligned currents (FACs) denoted as $j_\parallel$. By virtue of current continuity, a FAC is related to the magnetospheric current $j_\perp$ perpendicular to magnetic field as

$$j_\parallel = -B_i \int \nabla \cdot j_\perp \frac{ds}{B}$$

where $ds$ denotes integration along a field line, and the subscript $i$ denotes value at the ionospheric footprint. For metastable arcs with lifetime longer than the Alfvén transit time, (1) implies that, after adjustment for mapping, auroral structures associated with $j_\parallel$
should correspond to similar structures in $j_\perp$. Elphinstone et al. (1991) showed that there is indeed a close correlation between aurora arcs observed by the Viking UV imager and cross-tail current in the magnetosphere. In this paper we direct our attention to how arc-like structures can be formed as the magnetospheric $B$ field evolves in a turbulent convection. It bears further notice that the smaller the scale length of $j_\perp$, the larger the magnitude of $j_\parallel$, explaining why thin arcs tend to be brighter.

Figure 1a is a representation of the magnetosphere. The plasma sheet situated on the night side is generally considered as the source of discrete aurora arcs in the oval. Particularly, the equatorward arcs sampled by Knudsen et al. (2001) map mostly to the central plasma sheet (CPS) located earthward of 15 Re. In Figure 1b, the CPS is abstracted as a collection of discrete flux tubes identified by their foot points through equatorial plane. In a weakly turbulent magnetosphere, the foot prints undergo slow quasi-random motions (by quasi-random we mean that the motions appear random and uncorrelated beyond the correlation length of the turbulent field). To simplify the problem and make the salient points more transparent, we take the field lines as straight. This approximation removes field line curvature, which accounts for a large part of the perpendicular current that feeds the FAC in (1), hence limiting the literal use of the model in its present form. This caveat notwithstanding, we expect that the salient features emphasized by the present study, namely, the relationship between current filaments and turbulence, as well as the scale-free nature of energy release, should survive this approximation. At this point, the objective of our treatment is to substantiate the
We use the magnetic field $B_z$ as the primary variable. At the start of simulation, $B_z$ is initialized as a linearly decreasing function of $x$. The electric field in the plane is given by

$$E = -v \times B + \eta \nabla \times B$$

where $\eta$ is the plasma resistivity. *Lui et al.* (2007) analyzed the Vlasov-averaged version of generalized Ohm’s law in a neutral sheet crossing event observed by the Cluster satellites and found that the resistivity term accounted for most of the deviation from the ideal MHD condition, with a magnitude comparable to the $E \times B$ terms individually. For the typical parameters given in the event of *Lui et al.* (2007) and assuming a current sheet thickness 1000 km, we find that $\eta$ has an order of magnitude $\sim 10^{11} \text{ m}^2/\text{s}$, which is a significant value. Formally the resistivity term written by *Lui et al.* (2007) represents the effects of electromagnetic turbulence and was found to be predominantly dissipative (i.e., $j \cdot E > 0$). This finding is consistent with the following interpretation: As the shuffling of field lines create more and more complex structures in $B_z$, electromagnetic turbulence on the ion scale and below is excited. These turbulent excitations are a conduit which transfers energy from the magnetic field to thermal energy of particles. In this manner, the dissipation prevents the formation of excessively sharp structures. Faraday’s law, coupled with the incompressibility condition, gives the rate of change of the magnetic field as

$$\frac{\partial B_z}{\partial t} = -v \cdot \nabla B_z + \eta \nabla^2 B_z$$

Equation (3) is solved on a two-dimensional coupled lattice. Simulations are performed
on a 256×256 grid. If the size of the physical system, is $20 \, R_E \times 20 \, R_E$, one grid spacing $\Delta$ at the 256×256 resolution has the approximate length 500 km, comparable to the ion gyroradius cited earlier. Physics below this scale is represented by kinetic dissipation through $\eta$.

We take $v$ as given. At each time step, the velocity is prescribed randomly at each node. In a realistic turbulence, flow velocities become independent only beyond a finite correlation length. The above implementation, adopted mainly for its convenience, implies that the correlation length is less than the grid spacing. In truth, this condition does not typically apply to Earth’s magnetosphere. *Borovsky and Funsten* (2003), for example, estimated that the correlation length of magnetospheric turbulence is of the order 1-2 $R_E$. As these authors pointed out, the size of the CPS (whose thickness is also a few $R_E$) is comparable to the inferred correlation distance, giving a sort of “turbulence-in-a-box” which deviates from the classical turbulence with well-separated injection, inertial and dissipation scales. To bring clarity to the problem at hand, we defer this detail for future consideration and assume that the turbulence following a power-law distribution of energy density, $\varepsilon(k) \propto k^{-a}$, where $\varepsilon(k)$ is energy per wave number $k$. (The classical Kolmogorov turbulence has $a = 5/3$.) The velocity at scale $k$ is $v_k \propto k^{1-a}$. It can be shown that the first term on the right-hand side of (3), which drives the formation of structure in $B_z$, varies as $k^{3-a}$, whereas the dissipation term varies as $k^2$. If the driving turbulence has $a < 3$, equation (2) predicts that small-scale structures grow
faster than large-scale ones. Since the current density at scale $k$ is $j_k \propto k B_k \propto k^{\frac{5\pi}{2}}$, the process will quickly lead to the formation of small-scale current structures. Eventually, the dissipation $\eta$ kicks in and the formation of structures stops at a scale $k_c \propto \eta^{-\frac{1}{2}}$. Because of the faster growth of small-scale structures, it is a reasonable first approximation to retain only the uncorrelated flow components at the scale $\Delta$ and below; this flow component is a fraction of the observed flow speed at any given point. Effectively, our present implementation implies that flow components at scales larger than $\Delta$ do not contribute significantly to the formation of current structures. By the same token, the velocity fields between successive time steps are also uncorrelated and prescribed randomly.

As the magnetic field evolves in accordance with (3), more and more complex structures form, and the current density increases. When the local current density exceeds the starting current by a factor $M$, we assume that some form of current-driven instability takes place, and the current distribution is relaxed with a certain amount of energy released. Observationally, the cross-tail current has been observed at values as high as $100 \, \mu\text{A/m}^2$ (Asano et al., 2003; Nakamura et al., 2010), while the quiet-time current density in equatorial plane has the order of $1 \, \mu\text{A/m}^2$. In our simulation, we have used $M = 2 \sim 20$ as the instability threshold. Once an instability occurs, we assume that it reduces the local current density to zero. This means that, after the instability, the unstable node and its four nearest neighbors (labeled 0-4) have the same magnetic field equal to the 5-point average before onset, viz, $\langle B \rangle = (B_0 + B_1 + B_2 + B_3 + B_4)/5$. This
procedure conserves magnetic flux and releases an amount of energy equal to

\[ \Delta E = \frac{1}{2\mu_0} \sum (B_i - \langle B \rangle)^2 \]

where the sum is over all nodes on the grid.

As in Liu et al. (2006), a fraction \( \delta \) of the energy release goes into Alfvén waves to excite aurora. The rest, \((1 - \delta)\Delta E\), stays in the magnetosphere. We make the simple assumption that the retained energy release feeds a plasma flow that blasts out radially from the unstable node. The velocity on the four nearest neighbors has the magnitude

\[ v_b = \sqrt{(1 - \delta)\Delta E / 2\rho} \]

where \( \rho \) is the plasma mass density. The effect of the blasts on the magnetic field is solved through (3). Once the system is settled, we implement the next iteration of the turbulent \( v \). A free boundary condition is imposed in the simulation runs; that is, when an avalanche hits the boundary, the energy freely exits the system without any impediment.

Takalo et al. (1999) studied a coupled-lattice model which at first glance looks similar to ours. A close examination indicates that the two models invoke different physical assumptions. We note the following distinctions in our model: 1) The full induction equation is solved, rather than assuming a source function generating magnetic flux. This allows a direct link to magnetospheric turbulence. 2) The magnetic resistivity is a constant, rather than a function of local current and plays a different role in our model. It can be shown that, if there is only resistivity and no flow, the solution of (2) is simply the decay of the initial \( B_z \), without any emergent complexity. It is the turbulent \( v \) (which, through its product with \( B \), constitutes the nonlinearity in our model) that leads to the
formation of structures and release of energy; the role of \( \eta \) is merely to dissipate energy on the sub-MHD scale. In Takalo et al. (1999), the hysteresis of \( \eta \) was the nonlinearity responsible for the resultant complexity. 3) Energy partition in our model is more realistic, with particle heating associated with \( \eta \), bulk flows associated with \( v \), and energy flux to the auroral ionosphere associated with the partition of (3). In Takalo et al. (1999), only particle heating was present.

3. RESULTS

We have run the model under different combinations of parameters. These runs showed a consistent general pattern in terms of structure formation, avalanche, and statistical distributions. In this section, we present samples of the simulation runs to highlight some of the more interesting aspects of this pattern. The dimensionless parameters for these runs were chosen to be \( M = 2.5 \), \( \eta = 10^{-3} \), \( v_{\text{rms}} = 10^{-6} \), and \( \delta = 0.1 \). The choice of parameters was verified \textit{a posteriori} to give filamentary structures with thickness between 1 and 10 \( \Delta \), the estimated width of mapped arcs suggested by our previous calculation. More extended analyses and discussion of our model for a broader range of parameters will be reported elsewhere.

3.1. Energy avalanches and self-organized criticality

Figure 2 gives the time series of total lattice energy and total liberated energy (namely the sum of (4) over all active nodes) from the coupled lattice over \( 4 \times 10^6 \) iterations of a
particular run. For the first $2.5 \times 10^6$ iterations, the system slowly approaches a critical state, as there is an increasing trend of the total magnetic energy stored on the lattice. Afterwards, the system settles on a statistically stationary state, where the average energy, as well as other statistical properties, does not change with time. Whether this state represents a self-organized criticality is a technical matter for future consideration, what is clear is that, once driven into this state, the system spontaneously slips into energy avalanches of varying sizes.

Figure 3 shows a typical avalanche in detail. From a lull of no active node, the avalanche starts abruptly, reaching its peak power in a dozen or so iterations. The initial onset of avalanche removes a large amount of free energy from the system, but the system is not completely relaxed, with unstable current structures forming in neighboring nodes that led to further avalanches and secondary peaks of energy release. It takes $\sim 10$ times longer than the initial peak release for the system to settle, and free energy to be completely removed. This pattern is similar to the profile of an aurora substorm; that is, the initial expansion phase that is typically the brightest and lasts a few minutes, followed by up to 1 hour of recovery phase where auroral brightness undergoes ebbs and flows before finally dying down.

It is noted that, in order to reach a SOC-like state, the system has to be driven slowly (in comparison to the rate of avalanche), and the driver itself is statistically stationary. Neither condition is necessarily fulfilled in the actual magnetosphere. Therefore, Figures 3 and 4 represent a theoretical limit that may not be perfectly realized but is instructive in terms of providing insight on how intermittent energy release can result from persistent
actions of a turbulent flow.

3.2. Probability density distributions

In Figure 4, probability distribution functions of total energy release ($E$), event duration ($T$), and peak power ($P$) are presented. The sample consists of 8676 avalanches. All PDFs are fit to a power law $X^{-\alpha}$, represented by the red line through the corresponding histograms in Figure 5. A visual inspection confirms that distributions of the three parameters have excellent fits to the power laws. Table 1 lists the power law exponents obtained for two different lattice sizes: $128 \times 128$ and $256 \times 256$. We conclude from the table that the results shown in Figure 5 are statistically robust based on the convergence of $\alpha$.

Due to the approximations made in the current implementation of the model, we do not make direct comparisons of the power-law exponents obtained through simulation to those estimated from real data. It is, however, interesting to note that the power exponent $\alpha_E = 1.14$, for example, is identical to that obtained by Liu et al. (2006) obtained through a different approximation of the CPS dynamics.

| N   | $\alpha_E$     | $\alpha_T$     | $\alpha_P$     |
|-----|----------------|----------------|----------------|
| 128 | 1.15±0.03       | 0.97±0.06      | 1.41±0.05      |
| 256 | 1.15±0.02       | 1.09±0.06      | 1.37±0.05      |

Table 1. Simulations parameters and results for the PDF's of avalanches.
3.3. Current filaments

Figure 5 shows four plots of the current density distribution taken at random points of a simulation run. The current density is calculated as $\mathbf{j} = \hat{z} \times \nabla B_z$. In order to highlight the filamentary current structures, we use a form of contour plot to identify nodes where there is an enhancement of current magnitude, without regard to direction. By connecting the dots, we get a sense of the overall structure of the current distribution. Also, to see the relationship between current distribution and energy release in an avalanche, we plot on the right-hand side of the current distribution the avalanche event in which it found itself, with the arrow indicating the moment when the current distribution was collected.

As indicated earlier, the driver to the system is a turbulent flow field that is completely uncorrelated and random on the coupled lattice. It would not be unreasonable to suppose that the current distribution that results should be similarly uncorrelated and random. The actual results defy this expectation. The common feature of the four plots is that the current distribution is highly filamentary, with the length of the filament much greater than the width. In detail the four plots differ, determined largely by their phasing in relation to the energy release at the moment.

In general, we expect that a highly structured current distribution should presage a major energy release event, as there is more energy contained in such a configuration. This expectation is largely borne out in Figure 5. Figure 5d has the most complex structuring, with well-defined system-wide filaments. The current distribution is indeed
found to be just before the onset of a large secondary peak in an avalanche. Next in level of complexity is Figure 5c. The current distribution in this case is collected between two secondary peaks, as the system was rebuilding free energy for a significant release. The current filaments are weaker than Figure 5c, and there is a new morphological feature which we call patches, marked as hatches in the middle. Further down the scale of complexity comes Figure 5a, where the current distribution is collected from the downward slope of an energy peak. There is a further weakening of the filaments to be barely visible. Figure 4b shows the current distribution collected right at an energy peak. As expected, it is the least structured of the four plots, as the current filaments have practically disappeared. Replacing them are the prominent patches in the middle. We do not have an answer as to why current patches seem more stable than filaments and leave it as a topic for future investigation.

It is interesting to note that the four avalanches in Figure 5 were collected at random. One might expect that the current distributions should have no semblance to each other, as each was rebuilt after the system was cleared of free energy, and there should be no long-term memory effect. However, when we inspect the underlying current distributions for the four events, it is clear that they have a significant degree of similarity. Despite waxes and wanes of the current density, and the presence or absence of patches, the overall pattern is slanted at a $\sim 45^\circ$ angle to the cross-tail line; even the number of filaments does not seem to vary greatly. Hence the system does retain memory. After a more careful observation of the current distribution, we offer the explanation as follows: Once the general pattern of current distribution is formed, randomly at first, in the build-
up phase of a simulation run, it cannot be completely erased by an avalanche. Just as in Figure 5b, at the peak of energy release, there are still remnants of the filaments that precede the event. Then, as the system enters into the next period of energy buildup, the surviving current enhancements serve as the seed to rebuild a current distribution similar to the previous one. The reason is that the current increment per iteration is proportional to the local current density, according to (2). Thus, the surviving current enhancements have the advantage, and the probability of recurrence of the initial distribution is high, even though the driver is random. In a manner of speaking, this behavior is not fundamentally different from the fact that facture tends to happen where the bone has already been broken before or an earthquake is more likely to hit where there is already a fault.

To confirm this explanation, we show in Figure 6 the results from a different run of the model. The current distributions just before and after an avalanche are plotted. As our argument above implies, this run initialized a different current pattern from Figure 5. Furthermore, the avalanche did remove energy from the coupled lattice but did not completely erase the underlying pattern, as the current distribution after the avalanche (Figure 6b) is essentially a weakened facsimile of that before the avalanche (Figure 6a).

While a first glance at Figure 5 may suggest that the highly structured current distribution is incongruent to the smooth and scale-free energy releases in Figure 4, further reflection indicates that the two can be reconciled. For argument’s sake, suppose the system before disruption has $n$ current filaments. Suppose further that the system is near criticality everywhere, and the ensuing avalanche causes all filaments to disrupt, the
so-called system-wide discharge. The total energy release under this scenario would have a normalized value $n$. However, it is also possible that only half of the filaments are near criticality, yielding a release of $n/2$. We can follow this logic to the case where only one filament is near criticality, with energy release equal to 1. In fact, it is possible that avalanches occur only in part of a filament, leading to releases that are any fractions of unity. It is also reasonable to suppose that, in a system without built-in preference and selection effect, the smaller the event the higher the probability. For this reason, we expect that the probability density function increases monotonically toward the small releases, although we cannot quite predict that the specific form should be power-law without further analysis or actual simulation.

4. DISCUSSION

Filamentary structures are very common in nature. From the cosmic microwave background, to mass distribution in galaxies, to active regions involved in solar flares, to seismic faults, we find matter or energy concentrated in elongated, asymmetric forms. While physics responsible for these phenomena certainly vary, that different physics give rise to similar structures has been cited by many as a sign of universal laws which we do not quite yet grasp but could well exist to govern how complex systems appear and work. Studying aurora and the underlying magnetospheric system from this perspective is an example of this search for potential universality.

As an interesting side note, one cannot escape noticing a similarity of auroral phenomena to the seismic system. The distribution of earthquake energy (the Richter
Scale) has the scale-free power-law form, whereas the scale distribution of earthquake faults is certainly centered, just like aurora arcs. In the literature, terms such as magnetoseismology and substorm epicenter are seeing regular use. Admittedly, there are areas where aurora and earthquakes differ; for example, seismic faults form mostly along the boundaries of different tectonic plates, whereas aurora arcs can form in a medium that is homogeneous. Nonetheless, the co-existence of centered scale distribution and scale-free energy distribution in both phenomena point to the possibility of a multiscale coupling that features both turbulence and self-organized criticality.

The foremost concern of this study was the relationship between magnetospheric turbulence and filamentary current structures which, as we have argued, must underlie metastable auroral arcs. The model we used to establish this potential relationship was simple and should not be used literally to describe the actual magnetospheric physics. However, the salient point concerning the formation of filaments in a totally random flow field is something that transcends the various approximations. What we did in this study was to bring unity to several seemingly unrelated, even contradictory features. We started with a constant (i.e., structureless) current distribution. We drove the system with a completely random flow field. We yielded highly filamentary current distributions from the primordial uniformity. And, finally, we found that the energy release from the filaments is scale-free, returning to a lack of structure many take as a sign of universality. The simplicity of the model with which we unified the disparate strands should be considered a strength, rather than weakness in this regard.

Looking forward, there are several aspects of the model that need improvements. We
cite a few that are receiving current attention. Magnetic field lines are strongly curved in equatorial plane, so much so that field line curvature $\mathbf{c}$ can dominate the current density

$$j = \mu_0^{-1} \nabla \times \mathbf{B} = \hat{\mathbf{b}} \times \nabla B + \mathbf{B} \hat{\mathbf{b}} \times \mathbf{c}.$$ In this study, only the first term was considered.

Incorporation of the curvature term requires a two-dimensional or field-line integrated model. We anticipate that many of the salient features of the interplay between turbulence and magnetic field should persist in the more realistic implementations, as a turbulent flow would distort the shape of a field line much in the same way as it transports it.

We are also looking at a more realistic prescription of $\mathbf{v}$. Turbulent flows are to be specified with arbitrary correlation time and length. In this paper we considered only the extreme case of zero correlation time and correlation length. It will be interesting to see how the results might change when the driver maintains a finite correlation in space and time.

Ultimately, the turbulent flow $\mathbf{v}$ should be given self-consistently, rather than specified externally. Just like the kinematic theory of solar dynamo establishes that it is possible to generate magnetic field in the convection zone, and it takes a dynamic theory to know exactly how a dynamo works, a central task facing us is to integrate $\mathbf{v}$ into the model as a co-variable. There are two possible sources of $\mathbf{v}$. One is through magnetic reconnection in the tail; the turbulence could be a result of reconnection itself or of the interaction of the flow with local plasma (e.g., Liu (2001)). Another possibility is that the flow is the product of local instability. In the latter connection, it is useful to envisage an integration between the present model and the model developed by Liu et al. (2006) and Vallières-Nollet et al. (2010) (called LVN). These authors took the pressure (internal energy) as the
primary variable, and increased it deterministically to simulate the energization of the plasma sheet in the growth phase. Noting that the current density is related to the pressure gradient by \( j = B \times \nabla p / B^2 \), they made a node topple when \( |\nabla p| \) exceeded a prescribed limit. The only random factor in LVN is the energy partition ratio \( \delta \); yet scale-free avalanches were a defining characteristic of this system. As mentioned before, the slope of the energy distribution from our model was identical to that predicted by the model of Liu et al. (2006). This could mean that scale-free distributions are not sensitive to the choice of primary variable or driver. In its current implementation, the LVN model redistributes all the released energy to neighboring nodes as internal energy (pressure). A modification can be attempted so that the free energy is redistributed into flow \( v \) (as we did with the present model), which can serve as the flow driver to the magnetic field. For an incompressible fluid, the flow would change the pressure distribution through the equation \( \partial p / \partial t = -v \cdot \nabla p \), which can be solved in much the same way as (3). This approach would maintain the self-consistency between \( p \) and \( B_z \), as both evolve in time.

Despite the various limitations of our model, it is not entirely premature, given the results here and in some of the references, to sketch out a complexity perspective of magnetospheric dynamics, including the nature of substorms. The enunciation of this perspective is not meant to be the final words on the question, as evidence so far has been sketchy, nor a repudiation of other points of view, which all have their basis in facts and logic. Rather, we intend it to be an injection of new ideas that should help broaden our perspective. Key to our outlook are four aspects which merit greater attention: 1)
hysteresis, 2) energy storage in multiscale structures, 3) scale-free avalanches associated
with the collapse of multi-scale structures, and 4) insensitivity to “triggers.” We discuss
each in turn, highlighting, where applicable, differences from the traditional view of
substorm.

Hysteresis (also known as irreversibility) means that in a properly constructed phase
space, a system's path of evolution is different from point A to B, as compared to B to A.
The area enclosed by the A→B→A loop is usually proportional to a physical quantity
(e.g., energy) that is irreversibly released. For store-and-release processes such as the
substorm, hysteresis must exist so that the system can accumulate energy without
spontaneously relaxing into a lower-energy state. For multiscale problems, the loop can
have a wide range of sizes, resulting in scale-free distributions alluded to earlier. In the
literature, the hysteretic nature of substorm is implicitly acknowledged (e.g., growth
phase vs expansion phase) but seldom emphasized. In our model, the energy storage and
release processes are governed by two clearly different processes (the storage represented
by the induction equation (2), and release process by current-driven instability and energy
redistribution, respectively). For studies of complex systems, explicit reference to
hysteresis is a needed step to conceptual clarity and quantitative treatment.

In terms of energy storage, the existing theories are biased toward producing large-
scale distributions rather than multi-scale ones. Consideration of a simple example
demonstrates the point. Suppose that the solar wind-magnetosphere interaction imposes a
boundary condition at the magnetopause. The distributions of pressure $p$ and magnetic
field $B$ can be solved in principle. A general property of boundary-value problems of the
above sort is that small-scale features on the boundary decay quickly. Hence, one would expect predominance of large-scale features in the CPS which is far away from the outer magnetopause boundary. This expectation is inconsistent with the actual observation of the CPS and the scale-free energy distribution which suggests a multiscale process at play. In our model, energy is stored in multi-scale filamentary structures. As our simulation showed, scale-free distributions resulted as a matter of course, without appealing to extraneous factors or special circumstances.

The energy avalanche also warrants special attention. The traditional theory usually invokes a substorm trigger at a special location, and the trigger excites a fast-mode MHD wave that further disturbs the neighboring points (e.g., Friedriech et al., 2000). While similar to avalanche in appearance, the wave process implies that the expansion is at a fixed speed, the pattern of propagation is regular (e.g., circular wave fronts), and the reach of the expansion is global. In contrast, the avalanche model differs in these important details. An avalanche occurs, in principle, in an irregular, often fractal area; the network of nodes that are excited cannot be predicted beforehand, nor can the speed at which the avalanche spreads on this network. Moreover, the avalanche can terminate at any size; most in fact do not evolve into global events. This is the fundamental reason why the avalanche model can naturally reproduce power-law distributions over energy, size, and event time, while there is no such obvious path to scale-free distributions with the traditional theory.

Finally, in the complexity paradigm, the exact nature or location of the trigger has lesser import than in traditional models. Of course, the exact plasma physics that
contributes to the local instability which releases energy is important. What the above statement alludes to, rather, is that the system’s susceptibility to, global evolution, and statistical properties of substorm may not be sensitive to the trigger. If a substorm is large, it is likely due to the fact that the magnetic field structure out of which the substorm erupts is more complex, rather than because it was triggered by a certain process. On a more qualitative level, the present work argues for an important, if somewhat subtle change of perspective. If a substorm is a global phenomenon, its underlying cause must be global. The last snowflake that “triggers” a mountain avalanche is no different from previous drops; it is thus incorrect to give it any special physical significance. The reason why avalanches occur is that the overall snow cover has reached a critical state in a global sense. This analogy encapsulates the point why trigger is not necessarily the central problem in substorm. That the flu can trigger fatality is not a medically interesting discovery; why the patient is susceptible to this trigger is. Similarly, the magnetotail has a complex pattern of reaction to different disturbances (triggers). Most of these triggers do not lead to a substorm. Those which do may not be fundamentally different from those which do not. Therefore the study of substorm should be a study of how the magnetotail behaves as a system, not merely about unstable modes which have a much higher probability of occurrence, if not happening all the time.

Another new tapestry woven into the fabric of substorm theory is the role of the so-called cross-scale coupling. The focus and forte of the traditional theory is transport processes in the configurational (x) space. In this paper, our model was deliberately set up so that it had no built-in structure in the initial current distribution, and a driver that
was also statistically constant and uncorrelated in space and time. Without any preconditioning, the coupling of the two gave rise to a level of complexity that was not anticipated. The physics behind these results is best elucidated in the Fourier-transformed k-space.

Our results pointed to an interplay between flow $\mathbf{v}$ and current $\mathbf{j}$, which may render the debate about the primacy of one over the other a secondary issue, if not altogether irrelevant. We demonstrated that a turbulent and spatially uncorrelated $\mathbf{v}$ can lead to highly filamendened current structures. In turn, a disruption in current $\mathbf{j}$ can set off secondary flows, which helped unleash the avalanches.

**CONCLUSION**

Structuring of aurora is an unsolved problem important not only to magnetospheric physics, but also to other problems of broad scientific interest. What we did in this paper was not the provision of a solution, but a sketch that could help fashion a solution that takes into account the fact that magnetospheric processes exhibits such complexity that ideas and techniques developed in the study of nonlinear, non-equilibrium systems should be used. Through simple but physically motivated argument and simulation, we have explored an alternate view of energy storage and release in the CPS. This view distinguishes itself from existing theoretical ideas in its emphasis of complexity and reproduces several observed features which are mostly absent in traditional theories. The highlights of our findings are:

1. Turbulent magnetospheric convection creates elongated current filaments in the
central plasma sheet. The energy stored in these structures is multi-scale.

2. The filaments have an arc-like appearance and may explain the formation of meso-scale arcs reported by Knudsen et al. (2001);

3. If the turbulence is strong enough or lasts long enough, the filamentary current distribution reaches a criticality where energy avalanches are excited in the CPS;

4. The distributions of avalanches over total released energy, peak power, and event duration are scale-free. It is possible that phenomena we variously call substorms, pseudo-breakups, saw-tooth events, etc, are subpopulations on this continuum subjugate to common physics.

5. There is a memory effect that governs the re-formation of filaments. An energy avalanche does not completely erase the memory of current distribution preceding the event. As a consequence, the remnant current distribution has a tendency to replicate itself after the system starts the buildup phase again. This may explain why auroral arcs tend to recur in the same general region of space.

These results hint strongly that energy storage and release processes in the magnetotail, including the substorm, are multiscale involving both the classical cascade (which gives rise to the turbulent flow) and inverse cascade featuring self-organization of small-scale perturbations into larger-scale avalanches.

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Figure 1. Approximation of the magnetosphere (1a) as a collection of flux tubes moving on a coupled lattice (1b). The motion is prescribed as a random, uncorrelated, and slow shuffle to simulate the turbulent condition encountered in the central plasma sheet.

Figure 2. Time series of total magnetic energy stored on the lattice (top line) and energy that is released through avalanche. Shown in the inset is a typical avalanche event and the definition of total energy release ($E$), peak power ($P$), and event duration ($T$).

Figure 3. A typical avalanche event.

Figure 4. Probability density functions of energy release, peak power and event duration. All three exhibit a power-law distribution suggesting scale-free dynamics.

Figure 5. Four examples of current distributions taken from the run in Figure 2. Plotted alongside each distribution is the avalanche event it was in. The arrow in the plots on the right-hand side indicates the exact moment when the current distribution was taken.

Figure 6. Current distributions from a different run of the model. The current distribution is structurally different from Figure 5. Plot a is taken just before the onset of an avalanche,
and plot b right after. It can be seen that the avalanche does not completely remove the
memory the system has of the current distribution.
