Concomitant Larkin-Ovchinikov states in polarized atomic gases

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Abstract. Fermionic superfluids continue to be a topic of current interest and debate in condensed matter and sub-nuclear physics. Recent experiments with laser-cooled atomic gas clouds of fermions seek to explore this physics and point out that discrepancies might arise due to different confining geometries. We reported on realistic numerical simulations with up to 100\,000 atoms (a number comparable to that in the actual experiments) that shed light into what might be going on in the laboratory realizations.

1. Introduction

Recent developments in laser cooling allow experimentalists to trap clouds of atomic gases and, in tandem with evaporative cooling techniques, reach the limit of quantum degeneracy. This capability in combination with the use of Feshbach resonances [1], allows to produce systems in which the interactions and the confining potentials are controllable. Thus, new vistas open up to experimentally produce systems with interesting quantum behavior and, in particular, exotic superfluid phases. In particular, there exist strong motivations to seek for the realization of states similar to the long-sought FFLO (Ferrel-Fulde-Larkin-Ovchinnikov) states, in which fermions pair up with non-zero center-of-mass momentum [2, 3]. This unusual superconducting states are not seen in standard metals, but are thought to be realized in the context of organic conductors [4] and heavy fermions [5]. Not only that, they are also probably important for the understanding of the expected superfluid states of nuclear and quark matter [6].

Recent state-of-the-art experiments studying the effects of spin imbalance on pairing in unitary Fermi gases encountered mutual qualitative and quantitative discrepancies which we interpret to be a function of the confining geometry. To substantiate this idea we presented, and report in this proceeding, numerical results based on solutions of the Bogoliubov-de Gennes (BdG) equations for the actual experimental setups. In particular, we do not resort to the use of a local density approximation or LDA (extensively used in studying trapped gases, though its violation has been documented in some of the afore mentioned experiments in elongated 3d atomic clouds [7] due to the presence of surface/interface effects [8, 9]).

2. Modeling

We model the system of trapped fermionic atoms starting from the following one-body Hamiltonian:

\[ H_\sigma = -\frac{\hbar^2}{2m} \left( \nabla_r^2 + \frac{\partial^2}{\partial z^2} \right) + V(r, z) \]
[where $V(r, z) = \frac{m}{2} (\omega_\perp^2 r^2 + \omega_z^2 z^2)$ is the trapping potential; $m$ is the atomic mass and $\omega_\perp, z$ are the respective trapping frequencies] and taking into account fermion-fermion interactions using a fully spatial-dependent self-consistent paring approximation:

$$
\begin{pmatrix}
H_\uparrow - \mu_\uparrow & \Delta(r, z) \\
\Delta(r, z) & -H_\downarrow + \mu_\downarrow
\end{pmatrix}
\begin{pmatrix}
u_j \\
v_j
\end{pmatrix} = E_j
\begin{pmatrix}
u_j \\
v_j
\end{pmatrix}.
$$

One has to find all the solutions of this eigensystem and the superconducting order parameter needs to be defined self-consistently via $\Delta(r, z) \propto \sum_j u_j(r, z)v_j^* (r, z)f(E_j)$, with $f$ the Fermi function. The self-consistency is reached via an iterative solution of the full problem.

3. Discussion and Results

We are concerned here with the fate of the superfluid state in the case of a polarized gas. Following the experiments, we fix $N = N_\uparrow + N_\downarrow$ (total particle number) and $P = \frac{N_\uparrow - N_\downarrow}{N}$ (global polarization). We take $P$ in the 30%-40% range and try to have $N$ as large as possible to approach the experimental number of about a quarter of a million atoms. We find that the nature of the solution evolves as the number of atoms is increased (see Fig. 1). While for a small number the solution is unique and displays a trap-induced phase separation with a superfluid core and polarized wings, as the number increases (and approaches the experimental values) the most stable solution turns out to be close to the lda prediction with a uniform superfluid phase. New solutions are also found which are close in energy to the lowest one (within our error estimates) and that display an intermediate axial layer with an oscillating order parameter as in the Larkin-Ovchinikov ansatz solution. This solution becomes more stable and the LO region better defined as the number of particles augments. Also, as the aspect ratio of the trapping potential increases, the energy barrier between the different solutions seem to get larger. For large number of particles, which solution one converges to depends strongly on the initial ansatz, which suggests that which state is realized in the experiments would depend heavily on the initial/boundary conditions and cooling procedure.

We carried out numerical solutions of the BdG equations with up to 100'000 atoms and we obtained column-density profiles in good agreement with the experimental ones. A detailed report of the comparison between theory and experiments will be presented elsewhere [10]. Our results indicate that the confining geometry plays an important role in deciding what type of solution is stabilized. This is consistent with the scenario from one dimension, since in that limit it is known that one can stabilize a polarized liquid with paired states and many characteristics akin to an fflo superfluid state (see Ref. [11] for an example and more references; see also Ref. [12] for preliminary experimental results).

4. Conclusions

To summarize, we have studied attractive spin-imbalanced ultracold atomic clouds of fermions in 3d elongated geometries. Our results are consistent with the possible experimental observation of polarized superfluid states. One auspicious aspect of the results worth commenting on is the radial alignment of the nodes of the FFLO phase. A fact which is promising for the prospects of experimental detection and characterization, because it implies that the Larkin-Ovchinikov state could yield a measurable signal in the density profile. It also suggests that for an array of 1d tubes, such as are being used in current experiments [12], if the inter-tube coupling is substantial then the nodes in the order parameter at each tube (and thus the peaks in the density difference between the two species) are likely to be radially aligned among tubes and yield a measurable signal ‘easy’ to access in the experiments.

Our approach is well suited to explore interesting crossover effects, which are challenging to capture with other theoretical methods. Moreover, our inhomogeneous numerical scheme
Figure 1. Axial profiles of the order parameter at the center of an elongated trap for different numbers of trapped atoms (from top to bottom: 200, 1'100 and 10'000, respectively). The dotted line represents the local-density solution and is given for comparison. The different solid lines correspond to different self-consistent solutions of the BdG equations corresponding to different free-energy local minima. The colors correlate with free energy so that the blue line indicates the most stable solution and so forth (see the discussion in the text).
could be applied to answer questions about dynamics that are currently still open. Efforts in this direction are underway. Such calculations would further address the possible experimental signatures of the inhomogeneous, partially-polarized, superfluid phases of attractive atomic Fermi gases near the unitary limit. The synergy between theory and experiments is likely to produce new insights into the physical properties of polarized superfluid matter with potentially far reaching implications.

Acknowledgments
We would like to thank the conference organizers for giving us the opportunity to present this work during the 2010 edition of the International Conference on Strongly Correlated Electron Systems (SCES2010). This work was supported in part by DARPA-ARO (grant no. W911NF-07-1-0464) and by the W. M. Keck and Welch foundations (C-1669 and C-1681). Part of this work was performed at NERSC, Navy DSRC and the ARSC.

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