Calculation of the end form the rotating electrode in the liquid environment

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Abstract. Electroslag Remelting Process (further – ESR) is an advanced technology of ingots’ production, which are used in applications such as aviation, power generation, medicine etc. [1, 2]. However, the purity of the resulting metal and its structural homogeneity depend on the selected remelting regime. This article provides mathematical calculation, which allows estimating the optimal shape of the electrode’s end and this will contribute to the creation of optimal conditions for obtaining the maximum performance values of electroslag remelting. Such conditions can be provided at radial flow of molten metal on the even melted-off surface, that is reached by rotation of the melted electrode with an optimum speed.

1. Introduction

The passage of an alternating current from the electrode to the base plate creates Joule heating in the highly resistive calcium fluoride based slag which is sufficient to melt the electrode. Electrode metal drops while passing a layer of less dense slag, from a thub of liquid metal, followed by the formation of an ingot in a water-cooled crystallizer. Due to the heat loss solidifies the metal gets crystallized, forming an ingot with dendrites structure and maintaining a shallow liquid metal pool (depth of 1/3 to 1 diameter) throughout the process [3, 4] ‘figure 1’.

![Figure 1. Diagram of the ESR.](image_url)
The main disadvantages of the electroslag process are a relatively low output, high specific energy consumption, scarce materials used for flux production, and the high cost of a remelted metal. It restricts the field of its application. Therefore, the problems of a further improvement in ESR are relevant and important [5]. To ensure continuous energy saving, the heat transfer must be optimized to a heated metal by improving the thermal operation of a furnace unit [6]. Novel commercial processes and technological regimes imply an enhancement of the set of thermal, mechanical, and chemical actions on metallic and slag melts. New solutions can be obtained during ESR with the rotation of a consumable electrode about its axis, which is a rather simple technical procedure [7], Electrode rotation mechanism ‘figure 2’.

![Electrode rotation mechanism](image)

**Figure 2.** Electrode rotation mechanism: 1 – engine; 2 – transmission; 3 – shaft; 4 – body; 5 – collet; 6 – contact brushes.

In this process, instead of just moving downward, additionally the electrode is revolving around its axis. As a result, an ascending slag flow is induced under the electrode zone. The slag heats up while rising and that leads to an enhanced heat transfer with the electrode, which can improve the efficiency with nearly the same power.

Optimum conditions for obtaining the maximum values of efficiency and the refining ability of process will be reached when can be ensured of a film on all melted-off surfaces, the uniform thickness what is possible in the presence of the flat melted-off electrode end.

2. **Mathematical formulation**

Formation of the melted-off part of the consumable electrode in case of its rotation is of interest not only from hydrodynamics positions, as it was shown above, but also process from thermal physics. The distribution of flows in the slag bath, shown in ‘figure 3’. For this purpose the mathematical model of formation of a fluid film at a end of the rotating cylinder has developed and investigated, and also a liquid thermal physics in this film.

Let's accept the following conditions: thick liquid, incompressible, uniphase; stable process liquid, from the cylinder enters all throw the section (1), driving in a film lamina; the cylinder rotates in liquid
(II) at a speed $\omega$. The strimming of thick incompressible liquid under the rotating disk is considered in work [8]. Numerical results are received, without gravity for the corresponding values of speed liquid’s on a cylindrical coordinate $(r, \theta, t)$. They are provided in type of schedules.

Assuming that film thickness $\delta(r)$ under end of the rotating cylinder is small in comparison with its radius $R$, we apply process of alignment, which is provided in work [9] to curves for speed components $(V_r, V_z)$ with method of least squares with the subsequent correcting of results, using an equation of continuity [10].

$$\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z} = 0.$$  \hspace{1cm} (1)

After that we will receive the following expressions for speed components near the formed cylinder:

$$V_r = 0.21 K_r \sqrt{Z};$$  \hspace{1cm} (2)

$$V_z = -0.28 K_z \sqrt{Z};$$
where $K = \sqrt{\frac{\omega}{\nu}}$; $\omega$ – angular velocity of cylinder rotation; $\nu$ – entering liquid kinematic viscosity (I) from the cylinder.

Minus sign at the making speed $V_z$ shows that liquid in a film and liquid, adjacent to a film (II) move to a cylinder end, to fill a rate of flow in the radial direction.

Taking into account symmetry of rotation and a smallness $\delta (\delta \ll R)$ the heat conduction equation will look like [10]:

$$0.21Kr \sqrt{Z} \frac{\partial T}{\partial r} - 0.28Kz \sqrt{Z} \frac{\partial T}{\partial z} = a \frac{\partial^2 T}{\partial z^2},$$

(3)

where $a$ – liquid thermal diffusivity (I); $T$ – temperature in a fluid film.

Boundary conditions for a liquid film (I) at the end face of the cylinder we will set as following:

$$Z = 0, \; T = T_c; \quad (4)$$

$$Z = \delta(r), \; T = T_i; \quad (5)$$

$$r = R, \; \frac{T - T_c}{T_1 - T_c} = \frac{Z}{\delta}; \quad (6)$$

where $T_c$ – cylinder temperature; $T_i$ –environmental liquid temperature (II).

We will find dependence $\delta(r)$ using kinematic condition

$$V_r \frac{d\delta}{dr} = V_z,$$

(7)

which expresses a equality condition of normal components of speed of liquid (I) and liquid (II) on limit of their section, that is at $z = \delta(r)$.

Substituting (2) in (7), we will receive:

$$\delta = \delta_{\text{max}} \left( \frac{r}{R} \right)^{4/3}.$$  

(8)

Here $\delta_{\text{max}}$ – the maximum value of thickness of a film on cylinder’s end face edge, received in account of inertial forces.

Influence of the edge on processes in a film is not considered. In (3) and (6) let’s pass to the dimensionless variables:

$$\eta = \frac{r}{R}, \quad \xi = \frac{z}{\delta}, \quad u = \frac{T - T_c}{T_1 - T_c} - \xi.$$  

(9)

Then we will have the following heat conduction equation in the dimensionless parameters:

$$3\eta^{13} \xi^3 \frac{\partial \hat{u}}{\partial \eta} - 8\xi^3 \eta^2 \frac{\partial \hat{u}}{\partial \xi} - A \frac{\partial^2 \hat{u}}{\partial \xi^2} = 8\xi^5 \eta^3,$$

(10)

at the homogeneous boundary conditions of a look:

$$\xi = 0, \; u = 0; \quad (11)$$

$$\xi = 1, \; u = 0; \quad (12)$$
\[ \eta = 1, \ u = 0, \]  \tag{13}

where \[ A = \frac{100}{7\delta_{\text{max}} K} \] - the dimensionless parameter.

For the solution of the (10) under (11)...(13) we will use Galerkin's method \[ [11, \ 12] \], meaning that we will look for the decision in a look:

\[ u = \xi (1-\eta)(1-\xi) \sum_{i=0}^{\infty} a_i (1-\eta)^i. \]  \tag{14}

For \( i = 2 \) we will receive the following approximate expression of distribution of temperature in the liquid film, entering from the cylinder:

\[ u^* = \frac{\xi (1-\eta)(1-\xi)}{A} \left[ 0.88 + 1.28(1-\eta) - 2.34(1-\eta^2) \right] + \xi. \]  \tag{15}

Temperature gradient at the cylinder end at \( \xi = 0 \) takes in this case a form:

\[ g(n) = 1 + \left( \frac{1-\delta}{\omega} \right) \left[ 0.88 + 1.28(1-\eta) - 2.34(1-\eta^2) \right], \]  \tag{16}

where \[ g(\eta) = \frac{\partial u}{\partial \xi(\xi=0)}, \ A = \frac{1000\omega}{7\delta_{\text{max}} \omega} \sqrt{\omega}. \]

3. Results and discussion

Given below is the qualitative picture of influence of cylinder’s rotation speed \( \omega \) on the form of its end face at constant \( \nu \) and \( a \). For example, for values \( \omega = 6.28 \text{ rad/s (‘figure 4’, curve 1), and } \omega = 10.42 \text{ rad/s (‘figure 4’, curve 2).} \)

![Figure 4](image)

**Figure 4.** Design form end of the cylinder at different speed.

If we assume, that the gradient of temperature is proportional to liquid thickness under the cylinder, we can define that the cylinder end form depending on an angular velocity of rotation, liquid’s viscosity and coefficient of thermal diffusivity.
In that way, knowing the speed of rotation of the cylinder, it is possible to predict a form of its end, and also to calculate a temperature profile in the entering liquid.

4. Conclusion
The intention to receive a minimum thickness, and evenly distributed on the melted-off electrode surface a fluid metal film is dictated by the fact, that the main refinement of the melted metal, as it was noted above, happens exactly in this layer, and the less thickness of a layer is, the cleaning process is more effective.

The beneficial factor is that technology of manufacturing electrodes for secondary remelting is already quite advanced.

Intake of electrode metal from top of its conic part in a fluid metal tub leads to increase in depth of its central part; in return that promotes violation of configurationally regularity of an ingot. In this regard creation of such conditions at electroslag remelting, at which thickness of a layer of molten metal on an electrode would be minimum, and metal tub, the front of a crystallization and end face of an electrode in the course of remelting would remain flat, would allow to increase effectiveness of refinement of metal. Such conditions can be provided at radial flow of molten metal on the even melted-off surface, that is reached by rotation of the melted electrode with an optimum speed. Based on the above calculation we found out that it is possible to provide optimal conditions for carrying out the electroslag remelting.

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