ROTATIONAL CORRECTIONS TO AXIAL CURRENTS IN SEMIBOSONIZED SU(3) NAMBU–JONA-LASINIO MODEL

A. BLOTZ, M. PRASZALOWICZ and K. GOEKE

Institute for Theor. Physics II, Ruhr-University
44780 Bochum, Germany

ABSTRACT

We examine $O(1/N_c)$ rotational corrections to axial couplings $g_A^{(3)}$ and $g_A^{(8)}$ in the framework of the semibosonized SU(3) Nambu–Jona-Lasinio model. A novelty is due to the observation that the rotational (cranking) velocity and the rotation matrix itself do not commute within semiclassical quantization scheme. If time ordering in the quark loop is maintained new contributions, which have no analogue in the Skyrme model, appear. They substantially improve numerical results for the axial couplings which without the present corrections are badly underestimated.

1 Alexander von Humboldt Fellow, on leave of absence from the Institute of Physics, Jagellonian University, ul. Rey- monta 4, 30-059 Kraków, Poland
1 Introduction

There are now three different measurements of the spin asymmetry in polarized lepton–nucleon deep inelastic scattering [1]–[3]. Although the original experimental papers seemed to show some discrepancies between SLAC and CERN experiments, the recent work of Ellis and Karliner [4] reconciles the three experiments within one standard deviation. The message of this work is that whereas the Bjorken sum rule [5] is in agreement with the data, the Ellis–Jaffe sum rule [6] is violated. Their results read:

\[ g_A^{(0)} = 0.24 \pm 0.09, \quad g_A^{(8)} = 0.35 \pm 0.04 \quad \text{and} \quad g_A^{(3)} = 1.25. \] (1)

What was at first called a spin crisis has been subsequently understood as a crisis of the nonrelativistic quark model [7]. Although the model was quite successful in reproducing \( g_A^{(3)} \) (the well known value of \((N_c + 2)/3\)) it failed badly for \( g_A^{(0)} \), predicting – in our normalization – \( g_A^{(0)} = \sqrt{3} g_A^{(8)} \) (Ellis–Jaffe sum rule result). Since the quarks in the nucleon are by no means free, it soon became clear that the total spin, even if one neglects gluons and anomaly, has also an orbital component which screens the nucleon matrix element of the singlet axial current.

In an immediate response to the first EMC result Brodsky, Ellis and Karliner published a paper [8] in which it was shown that the Skyrme model gives zero for \( g_A^{(0)} \) unless some extra terms responsible for the mixing with \( \eta' \) were added. Although it was in a sense a welcome result its actual significance was very much weakened by the fact that the model underestimated \( g_A^{(3)} \) by a factor of 2 [9], (see however [10]).

In the present paper we report on the calculation of the three axial couplings: \( g_A^{(0)} \), \( g_A^{(3)} \) and \( g_A^{(8)} \) in the semibosonized Nambu–Jona-Lasinio model (NJL or chiral quark model). In this model \( N_c \) valence quarks are bound in a self-consistent solitonic field (see Ref.[11] and references therein). The model interpolates between the quark model (small soliton size), where the valence quarks are almost unbound and the Skyrme model, where for large soliton sizes the valence level disappears in the Dirac sea. Similarly to the Skyrme model, also in the NJL model \( g_A^{(3)} \) was badly underestimated [12, 13]. The recent result of Refs.[14, 15] where the \( O(1/N_c) \) correction to \( g_A^{(3)} \) was calculated in the SU(2) version of the model seems to solve this long lasting problem.

The new result of the present work consists of a calculation of \( O(1/N_c) \) contributions to \( g_A^{(3)} \) and \( g_A^{(8)} \) in the SU(3) NJL model. In fact one expects large \( O(1) \) corrections to the leading \( O(N_c) \) term of for \( g_A^{(3)} \) and \( g_A^{(8)} \); indeed the SU(2) constituent quark model predicts \( g_A^{(3)} = (N_c + 2)/3 \). In the present model these corrections are of two types: antisymmetric ones which have no counterpart in the Skyrme model since they crucially depend on the time ordering within the quark loop, and the symmetric ones which can be attributed to the Wess-Zumino term. Altogether with \( g_A^{(0)} \), which by itself gets a contribution only at the \( O(1/N_c) \) level [16], the full results for the three axial couplings are in relatively good agreement with experimental data.
2 Axial Current in the NJL model

Our starting point is a semibosonized Euclidean action of the NJL model with scalar degrees of freedom kept constant [11]:

\[ S_{\text{eff}}[U] = -\text{Sp} \log \{-i \theta + m + M U^\gamma\}, \]

where \( \gamma \) matrices are antihermitian, the SU(3) matrix \( U^\gamma \) describes chiral fields \( \pi^-K^-\eta \), \( M \) is the constituent quark mass and \( m \) denotes the current quark matrix, which we will put equal to 0. The influence of the finite quark masses will be investigated elsewhere [17]. Eq.(2) has to be regularized. Following Ref.[11], we will use the two step cut-off function, with 2 parameters \( \Lambda_1, \Lambda_2 \) to regularize the real part of the Euclidean action by means of the Schwinger proper-time procedure. The imaginary part, which is finite to start with, remains unregularized. This has been at length discussed in Refs.[11, 18]. From the gradient expansion of Eq.(2) in the meson sector one fixes \( \Lambda_1, \Lambda_2 = \Lambda_1, \Lambda_2(M) \). Therefore the model is very economical: there is in fact only one explicit parameter, namely \( M \).

The model has solitonic solutions (see Ref.[11] and references therein) which have been successfully interpreted as baryons. The existence of the soliton is guaranteed due to the interplay between the valence level and the energy of the continuum states. All quantities have therefore typically two contributions which we subsequently call valence and sea respectively. The sea contributions have usually their counterparts in the Skyrme model.

The splittings within the SU(3) multiplets of baryons as well as the isospin splittings have been shown to be in very good agreement with experiment [11, 19]. To this end one uses the hedgehog Ansatz and performs collective quantization in terms of the rotation matrix \( A(t) \):

\[ U(x) = A(t)U_0(\vec{x})A^\dagger(t), \]

with

\[ U_0 = \text{diag}(U_0, 1) \]

and \( U_0 = \cos P(r) + i\vec{n}\vec{r}\sin P(r) \) with the profile function satisfying the boundary conditions: \( P(0) = \pi \) and \( P(\infty) = 0 \). The collective SU(3) octet wave functions are given in terms of Wigner matrices:

\[ D_{ab} = \frac{1}{2} \text{Tr}(A^\dagger \lambda_a A \lambda_b), \]

where left index goes over all 8 flavor states and the right index \( b = (Y^R, T^R, T^R_3) \) is confined to the states with \( Y^R = 1 \) [11, 20]. The corresponding right isospin has interpretation of spin (\( T^R_3 = -J_3 \)).

The space-like collective axial current operator can be expressed by the following functional trace in Euclidean space:

\[ A^a_j = i \text{Sp} \left\{ \Gamma^b_j D_{ab} \frac{1}{\partial_t + H + 2\frac{i}{2} \Omega^R_\lambda \lambda_c} \right\} \]

with \( H = -i\gamma_4(-i\gamma_k \partial_k + MU^\gamma) \) and \( \Gamma^b_j = \gamma_4\gamma_j \gamma_5 \lambda_b \). For a singlet axial current \( \lambda_b \) should be replaced by a unit matrix and one should also replace \( D_{ab} \to 1 \) in Eq.(4).

All quantities in the NJL model are given as power series in the cranking velocities \( \Omega \), where each power of \( \Omega \) counts as \( 1/N_c \). In the zeroth order one gets [12]:

\[ (A^a_j)^{(0)} = i \text{Sp} \left\{ \Gamma^b_j D_{ab} \frac{1}{\partial_t + H} \right\} \equiv -\frac{1}{2} D_{aj} A. \]

Note that at this level the singlet axial current is equal to 0 in agreement with the Skyrme model result [8].
The next term in $\Omega$ (or in $1/N_c$) is of great importance. Expanding in $\Omega$ we get:

$$(A^a_j)^{(1)} = \frac{1}{2} \text{Sp} \left\{ \Gamma_j^b D_{ab} \frac{1}{\partial_t + H} \Omega^E c_{\lambda} \frac{1}{\partial_t + H} \right\}.$$ (6)

There are two subtleties connected with Eq.(6): first it should be remembered that because of the collective quantization $\Omega$ is no longer a c-number but rather an operator which does not commute with $D_{ab}$. Second, the trace in Eq.(6) should be understood as time-ordered \[14, 15\].

To this end let us consider the Euclidean propagator:

$$< x | \frac{1}{\partial_t + H} | y > = \theta(t_x - t_y) \sum_{E_n > 0} \Phi_n(x) \Phi_n^\dagger(y) \exp(-E_n(t_x - t_y))$$
$$- \theta(t_y - t_x) \sum_{E_n < 0} \Phi_n(x) \Phi_n^\dagger(y) \exp(-E_n(t_x - t_y)).$$ (7)

If two such propagators are multiplied and time ordering in Eq.(6) is assumed, then the correct expression reads:

$$(A^a_j)^{(1)} = \frac{1}{2} \sum_{E_n < 0, E_m > 0} \frac{1}{|E_m| + |E_n|} \left\{ < m | \Gamma_j^b | n > < n | \gamma_5 | m > D_{ab} \Omega^E c \right\}$$

$$+ \sum_{E_n < 0} \left\{ D_{ab} \Omega^E c \right\}.$$ (8)

Taking into account symmetry properties of the matrix elements and the order of $\Omega$ and $D_{ab}$ in Eq.(8) we get back in Minkowski space:

$$(A^a_j)^{(1)} = [\Omega_{c}, D_{ab}] i \left\{ \frac{1}{2} B \epsilon_{jbc} + C (f_{jbc} - \epsilon_{jbc}) \right\}$$

$$+ \frac{1}{2} \left\{ D_{ab}, \Omega_{c} \right\} + \left\{ D f_{jbc} (\delta_{\gamma_5} r_{b} - \delta_{\gamma_5} \delta_{b}) (\delta_{\gamma_5} c - \delta_{\gamma_5} \delta_{c}) + E \delta_{j} \delta_{bc} \right\}. $$ (9)

The above structure comes from the general symmetry arguments and from the specific form of the hedgehog Ansatz \[9\]. For the singlet current we get simply:

$$(A^0_j)^{(1)} = \sqrt{3} \Omega_j E.$$ (10)

As we have already mentioned the explicit expressions for constants $A \ldots F$ have to be regularized if they come from the square of the Dirac operator in the proper-time scheme or left unregularized otherwise. With this in mind we get:

$$A_{\text{val}} = - N_c < \text{val} | \gamma_5 \lambda_3 \sigma_3 | \text{val} >,$$

$$A_{\text{sea}} = \frac{N_c}{2} \sum_{n} < n | \gamma_5 \lambda_3 \sigma_3 | n > \text{sign}(E_n) R_{\Sigma}(E_n).$$ (11)
and

\[ \frac{N_c}{2} \sum_{m \neq n} \mathcal{R}_Q(E_m, E_n) \frac{<m | \sigma_3 \lambda_a | n > < n | \lambda_b | m >}{|E_m - E_n|} = \begin{cases} B \text{ if } a = 1, b = 2, \\ C \text{ if } a = 4, b = 5, \end{cases} \]

\[ -N_c \sum_{m \neq n} \mathcal{R}_M(E_m, E_n) \frac{<m | \sigma_3 \lambda_a | n > < n | \lambda_b | m >}{|E_m - E_n|} = \begin{cases} D \text{ if } a = b = 4, \\ 2E \text{ if } a = 8, b = 3 \end{cases} \]

\[(12)\]

with

\[ \mathcal{R}_Q(E_n, E_m) = \frac{1}{2} (\text{sign}(E_n - \mu_F) - \text{sign}(E_m - \mu_F)), \]

\[ \mathcal{R}_M(E_n, E_m) = \frac{1}{2} (1 - \text{sign}(E_n - \mu_F) \text{sign}(E_m - \mu_F)). \]

\[(13)\]

The chemical potential \(\mu_F\) is chosen in such a way, that it always lies between the valence level and the positive continuum of states. In this way \(B \ldots E\) contain both the valence and the sea parts simultaneously. The regularization function:

\[ \mathcal{R}_\Sigma(E_n) = \frac{1}{\sqrt{\pi}} \int_0^\infty d\tau \sqrt{\tau} e^{-\tau \phi(\frac{\tau}{E_n^2})} \]

\[(14)\]

is given in terms of the two-step function \(\phi(\frac{\tau}{E_n^2})\) of Ref.\[1\] and is identical as in the SU(2) case \[12\].

3 Results for Axial Couplings

Under the collective quantization scheme angular velocities are promoted to spin operators \[11, 20\]: \(\Omega_a \rightarrow J_a/I_1\) for \(a = 1, 2, 3\). For \(a = 4 \ldots 7\) \(\Omega_a \rightarrow J_a/I_2\) where operators \(J_a\) fulfill SU(3) algebra, but do not correspond to any symmetry, since the physical states are constrained to \(Y^R = -2/\sqrt{3}J_8 = 1\). Here \(I_{1,2} = O(N_c)\) are moments of inertia, and can be found in Ref.\[11\]. We get: \([J_a, D_{jk}] = i f_{abc} D_{jc}\). Left generators: \(T_a = -D_{ab} J_b\) correspond to flavor. The proton axial coupling constants: \(g_A^{(a)} \equiv <A_3^{(a)}>\) defined for \(J_3 = \frac{1}{2}\) and \(T_3 = \frac{1}{2}\) read:

\[ g_A^{(a)} = - \left( A + \frac{B}{I_1} + \frac{C}{I_2} \right) D_{a3} + \frac{7}{I_2} \sum_{m=4}^{7} d_{3mm} D_{am} J_m + \frac{E}{I_1} D_{a8} J_3 \quad \text{for } a = 3, 8; \]

\[ g_A^{(0)} = \sqrt{3} \frac{E}{I_1} J_3. \]

\[(15)\]

Constants \(A \ldots F\) are calculated numerically for the profile function \(P(r)\) which fulfills self-consistent time-independent equation of motion for the soliton mass. In our notation they are positive. Matrix elements of the collective operators can be evaluated with the help of the SU(3) Clebsch–Gordan coefficients and for the nucleon they read:

\[ <D_{88}> = \frac{3}{10}, \quad <D_{38}> = \frac{\sqrt{3}}{10} T_3, \]
\[
\begin{align*}
\langle D_{83} \rangle &= -\frac{\sqrt{3}}{15} J_3, & \langle D_{33} \rangle &= -\frac{14}{15} T_3 J_3, \\
\langle d_{3mn} D_{8n} J_n \rangle &= \frac{\sqrt{3}}{30} J_3, & \langle d_{3mn} D_{3n} J_n \rangle &= \frac{14}{30} T_3 J_3.
\end{align*}
\] (16)

For the purpose of numerical illustration we present in Table 1 results for \( M = 420 \) MeV (this value follows from the overall fit to the hyperon spectra \[11\]) and, for comparison, also for \( M = 380 \) MeV.

| Constants of Eq.(15) | \( M = 380 \) MeV | \( M = 420 \) MeV |
|----------------------|------------------|------------------|
|                      | \( g_A^{(3)} \) | \( g_A^{(8)} \) | \( g_A^{(0)} \) | \( g_A^{(3)} \) | \( g_A^{(8)} \) | \( g_A^{(0)} \) |
| \( A \)             | 0.63             | 0.15             | -               | 0.59             | 0.15             | -               |
| \( D \) and/or \( E \) | 0.28             | 0.14             | 0.42            | 0.26             | 0.13             | 0.37            |
| \( B \) and \( C \) | 0.69 (0.43)      | 0.17 (0.11)      | -               | 0.73 (0.45)      | 0.18 (0.11)      | -               |
| Total               | 1.60 (1.34)      | 0.46 (0.40)      | 0.42            | 1.58 (1.30)      | 0.45 (0.39)      | 0.37            |

Table 1: Various contributions to \( g_A \), figures in parenthesis correspond to the valence part of constants \( B \) and \( C \).

The above calculation shows that \( 1/N_c \) cranking corrections to \( g_A \) are not negligible and, in accordance with the naive expectations drawn from the quark model, relatively large. Constants \( B \) and \( C \) emerge due to the time ordering within the quark loop. Although their sea contributions are not small, they have no corresponding counterparts in the Skyrme model. These new contributions split almost equally between the purely SU(2) correction \( (B/I_1) \) and the purely SU(3) part \( (C/I_2) \). Let us stress that we have calculated constants \( B \) and \( C \) from the unregularized expressions of Eq.(12). We postpone the discussion of possible regularization schemes of \( B \) and \( C \) which would be compatible with the time ordering of the path integral to the forthcoming paper \[17\]. Here we content ourselves with simply removing the sea part from \( B \) and \( C \) and quote in parenthesis in Table 1 the contributions corresponding only to the valence parts of \( B \) and \( C \). Constants \( D \) and \( E \) need not to be regularized; in the Skyrme model they follow from the Wess-Zumino term \[10\], (however \( E = 0 \) for the hedgehog Ansatz).

These are, however, by no means the only \( 1/N_c \) corrections one can think of. First of all pion loop corrections have to be in principle taken into account. In the Skyrme model they have been shown to be large and negative \[21\]. There is also some dispute in the literature concerning the value of the axial coupling of the constituent quark \[22\], which in the present formulation of the model is equal to 1 (see however Refs\[23, 24\] and references therein). Many different calculations indicate that it might be actually smaller than 1. Our results have to be therefore understood as the first step towards \( 1/N_c \) corrections. It is however encouraging that we get a positive contribution which slightly overestimates the values of axial couplings, so that there is place for negative corrections from other sources. Another problem is the departure from the chiral limit. Here the situation as far as cranking corrections are concerned is rather straightforward: one has to expand Eq.(4) not only in terms of cranking velocity but also in terms of the current mass matrix \( m \). Our preliminary calculations indicate that these corrections are small although not always negligible; this will be a subject of a separate paper \[17\].

Let us also note that taking for our Ansatz \( U \) the SU(3) matrix rather than U(3), we have neglected the contribution of \( \eta' \). It was shown in Ref.\[25\] that the influence of the additional U(1) factor and the
't Hooft interaction on the solionic quantities is for all practical purposes rather small.

Acknowledgements
We would like to thank V.Yu. Petrov for discussion which stimulated this work and P.V. Pobylitsa for discussion and numerous technical remarks. We also thank M.V. Polyakov, M. Wakamatsu, T. Watabe and Ch. Christov. The work was partially supported by Polish Research Grant KBN-2.0091.91.01 (M.P.), Graduiertenstipendium des Landes NRW and Ruth und Gerd Massenberg-Stiftung (A.B.).

References

[1] EMC Collaboration, J. Ashman et al., Phys. Lett. B206 (1988) 364; Nucl. Phys. B328 (1989) 1.
[2] SMC Collaboration, B. Adeva et al., Phys. Lett., B302 (1993) 533.
[3] E142 Collaboration, E. Hughes Determination of the neutron structure function, SLAC-PUB-6217, 1993, to be published in: Proceedings, Rencontres des Morionds: QCD and High Energy Interactions.
[4] J. Ellis and M. Karliner, Analysis of data on polarized lepton-nucleon scattering, CERN-TH-6898/93 or hep-ph-9305306, 1993.
[5] J. Bjorken, Phys. Rev. 148 (1966) 1467; D1 (1970) 1376.
[6] J. Ellis and R.L. Jaffe, Phys. Rev. D9 (1974) 1444; D10 (1974) 1669.
[7] R.L. Jaffe and A.V. Manohar, Nucl. Phys., B337 (1990) 509.
[8] S. Brodsky, J. Ellis, and M. Karliner, Phys. Lett., 206B (1988) 309.
[9] G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys., B228 (1983) 552.
[10] N.W. Park, J. Schechter, and H. Weigel, Phys. Rev., D43 (1991) 869.
[11] A. Blotz, D. I. Dyakonov, K. Goeke, N. W. Park, V. Petrov, and P. V. Pobylitsa, Phys. Lett. B287 (1992) 29; Nucl. Phys A555 (1993) 765.
[12] Th. Meissner and K. Goeke, Z. Phys., A339 (1991) 513.
[13] D. I. Diakonov, V. Yu. Petrov, and M. Praszałowicz, Nucl. Phys., B323 (1989) 53.
[14] M. Wakamatsu and T. Watabe, Phys. Lett., B312 (1993) 184.
[15] Ch. Christov, K. Goeke, V.Yu. Petrov, P.V. Pobylitsa, M. Wakamatsu, and T. Watabe, RUB-TH-53/93, in preparation.
[16] A. Blotz, M.V. Polyakov, and K. Goeke, Phys. Lett., B302 (1993) 151.
[17] A. Blotz, M. Praszałowicz, and K. Goeke, in preparation.
[18] D. I. Dyakonov, V. Yu. Petrov, and P. V. Pobylitsa, Nucl. Phys., B306 (1988) 809.
[19] M. Praszałowicz, A. Blotz, and K. Goeke, Phys. Rev., D47 (1992) 1127.
[20] E. Guadagnini, Nucl. Phys., B236 (1984) 35.
[21] I. Zahed, A. Wirzba, and U.-G. Meißner, Phys. Rev., D33 (1986) 830.
[22] S. Weinberg, Phys. Rev. Lett., 67 (1991) 3473.
[23] M. Praszałowicz, Phys. Rev., D42 (1990) 216.
[24] P. Sieber, M. Praszałowicz, and K. Goeke, Dependence of baryonic observables on the quark axial-vector coupling in chiral quark model, Bochum Univ. preprint RUB-TP2-38/93, 1993.
[25] M. Kato, W. Bentz, K. Yazaki, and K. Tanaka, Nucl. Phys., A551 (1993) 541.