Generation of ultrashort pulses in the THz frequency range

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Abstract. There are results for the spontaneous coherent super-radiative undulator emission in the terahertz frequency range from a short (as compared to the wavelength of the radiated wave) dense electron bunch. If the group velocity of the wave is close to the bunch velocity, this is a process of spontaneous radiation followed by amplification of a single wave cycle. Despite the Coulomb repulsion of electrons inside the bunch, its compactness is provided by the compression of the bunch under the action of its own radiation fields. As a result, formation of an ultra-short (several cycles long) powerful wave packet occurs when the bunch moves through several undulator periods with high (~20% in optimized systems) efficiency of extraction of the electron energy and high intensity (~100 MV/m) of the peak wave field.

1. Introduction

Generation of short ultra-broadband (several wave cycles) terahertz (THz) electromagnetic pulses with high peak fields [1,2] is an important problem from the point of view of various applications of such pulses. As just a few of them we can mention here the use of strong fields for investigations and control of various media [3], including tests of biological media [4], the ultra-fast THz spectroscopy [5] and the so-called pump-probe experiments [6-8], as well as the so-called terahertz (THz) high-gradient acceleration [9,10]. A possible way to make a source of short pulses is to use coherent spontaneous generation of short dense electron bunches produced by photo-injectors and moving in an undulator field. The spontaneous coherent emission regime takes place when the effective axial length of bunches in the radiation section is shorter than the wavelength of the radiated wave, \( L_e < \lambda \). In this situation, the wave packets emitted by each of the electrons add up basically in phase [11]. Advantages of this regime (as compared to the regime of the induced emission used in electron masers conventionally) are the short and simple microwave system (without any wave feedback system and seeding wave signal), possible high efficiency of the radiation process, and the fixed phase of the radiated wave packet.

Modern photo-injectors ensure formation of picosecond electron bunches with charges of the order of 1 nC and particle energy at a level of 3-7 MeV [12]. In the case of 1-2 ps electron bunches (the corresponding bunch lengths are \( L_e = 0.3-0.6 \) mm), the undulator emission at the wavelength \( \lambda \approx 1 \) mm has the spontaneous coherent character. In the case of an electron energy of about 5 MeV (the corresponding relativistic Lorentz factor is \( \gamma \approx 10 \)), the undulator period required to ensure radiation at such a wavelength is \( \lambda_u \approx \gamma^2 \times \lambda \approx 10 \) cm. In order to achieve a high peak field in the radiating wave pulse, one should ensure that the duration of this pulse is as short as possible. Therefore, it is optimal to use the super-radiation regime ensured in the case of group synchronization, when the axial electron velocity is close to the group velocity of the radiated wave packet propagating inside a waveguide. In this regime, the radiated field is accumulated in a short region around the bunch, the short electron bunch is placed inside one cycle of the wave packet (Fig. 1a) and, therefore, interacts (amplifies) just this cycle in the group synchronization regime. However, the total length of the wave pulse is several wavelengths...
(several wave cycles) due to the diffraction effects that are inevitably present, when a wave packet propagates in a waveguide.

Figure 1. Spontaneous coherent super-radiative undulator emission from a short dense electron bunch. The bunch moving in the waveguide is immersed in the periodical undulator field. At different undulator periods, the bunch radiates the waves packet in the same cycle. The used model of the electron bunch (an ensemble of thin charged discs) is also shown. (b): Self-compression of the bunch in its own radiation electric field.

The radiation process continues to remain spontaneous and coherent only if the length of the electron bunch stays shorter than the wavelength (Fig. 1a). In the case of the spontaneous coherent undulator radiation in the group synchronization regime, the problem of a bunch length increase can be solved because of stabilization of the bunch length (or even compression of the bunch) provided by the radiated wave field (super-radiative self-compression) [13]. The peculiarity of the radiation is that during the formation of the wave packet, the center of the electron bunch is shifted from the maximum of the braking phase of the wave by about 1/8 of the wave period (a phase shift of $\pi/4$) (Fig. 1b). In this case, the front of the electron bunch is near the maximum of the decelerating phase of the wave (and, accordingly, is slowed down in the process of radiation), while its “tail” is near zero of the wave field. Due to this fact, spontaneous super-radiation of a wave packet can be accompanied with compression of the bunch by its own radiated field [13].
2. Spontaneous emission of ultrashort pulses

To ensure excitation in the group synchronism regime, we consider an ensemble of electrons moving in a cylindrical waveguide immersed in the periodical transverse magnetic field of an undulator (Fig. 1a). We consider grazing of the lowest transverse mode TE_{11} of a cylindrical waveguide and particles in a planar undulator. We study formation of an ultra-broadband wave packet during motion of a short (on the scale of the wavelength) electron bunch through several undulator periods. In order to describe longitudinal Coulomb field we use the model of thin charged discs [14].

Figure 2. Efficiency of radiation as a function of the axial coordinate normalized by the undulator period in the regular waveguide (R_w=3.9 mm, blue dashed curves) and in the profiled waveguide (solid purple curves) in the cases of a total bunch charge of (a): 0.5 nC and (b): 2 nC. The waveguide radius R_w profile is the same for both (red curves): 3.9 mm → 3.2 mm → 3.9 mm.

We describe the process of spontaneous coherent undulator emission from an electron bunch with the parameters typical for modern photo-injectors, specifically, an initial duration of 1 ps, the transverse size R_e = 2 mm, total bunch charges 0.5 and 2.0 nC, and an energy of 5 MeV [12]. The radiation occurs in the undulator with the period \( \lambda_u = 10 \) cm, and the undulator parameter \( K = 0.7 \). In the case of the group synchronism regime, the resonant wavelength is determined as \( \lambda = \lambda_u(1 + K^2)/\gamma^2 \approx 1.2 \) mm. Figure 2 illustrates the results of the simulations for the cases, where the spontaneous radiation is organized in a regular waveguide (the optimal radius is \( R_w = 3.9 \) mm) and in a waveguide with an irregular input section. In the case of the regular waveguide, a radiated electron efficiency of 5-7% is achieved at the length of the electron-wave interaction region corresponding to \( N_u = 4 \sim 7 \) undulator periods. The use of irregular input sections provides a significant (up to 18-23%) increase in the efficiency, which is accompanied with the corresponding lengthening of the interaction region.
Such high efficiencies are achieved since the axial size of the bunch is maintained at a sufficiently small level (to ensure the coherent spontaneous emission regime) over a relatively large length of the interaction region due to the compression of the electron bunch by the radiation field (Fig. 3).

**Figure 3.** Dynamics of the charge density inside the electron bunch. The linear charge versus the axial coordinate after the electronic bunch passes $N_u$ undulator periods. Black curves: only the Coulomb field is considered, whereas the radiated wave field is absent. Green fills: both the radiated and Coulomb fields are taken into account.

Figure 4 illustrates process of formation of the radiated wave pulse in the optimized waveguide with an irregular input section. The spontaneous radiation process starts at the first undulator period, where a quasi-unipolar wave pulse (one cycle of the wave) is radiated (Fig. 1a). Since the group wave velocity coincides with the bunch velocity, in the next undulator periods we see amplification of this central wave cycle. However, in a waveguide with the dispersion characteristic $k = \omega/c = \sqrt{\omega^2 + k_z^2}$, there is frequency dispersion of the wave group velocity, $V_{gr} = c \times dk/d\omega = c h / k$. In this situation, a quasi-unipolar pulse with the frequency band $\delta k/k \sim 1$ is stretched to the length $L_w = \delta V_{gr} \delta z / V_z$ in the process of moving along the segment $\delta z = N_u \lambda_u$ due to the difference in the group velocities of the components of the wave packet $\delta V_{gr} = \delta k \times (dV_{gr}/dk)$. If the centre frequency of this wave packet corresponds to the undulator resonance with electrons, $\omega = (h + h_u)V_z$, and the translational velocity of electrons coincides with the group velocity, $V_{gr} = V_z$, then in the ultra-relativistic case $V_z \sim c$ the number of wave cycles in the stretched wave pulse is of the order of $L_w / \lambda \sim N_u$. At the same time, the central cycle, in which the bunch is located, stands out against the general background, since it is the bunch field that is amplified by the electrons. The peak field in this wave cycle continues to exceed noticeably the amplitudes of the other cycles, if the growth rate of the electron efficiency with the coordinate remains sufficiently high (compare Figs. 4 and 2), that is, if the central wave cycle extracts from the electron
bunch more energy than it is leave this cycle due to the diffraction spreading of the wave pulse. For instance, in the case of the 0.5 nC electron bunch, the highest amplitude in the central cycle is achieved in the point \( z = 11 \lambda_u \) (Fig. 4a), which corresponds to the region preceding the transition of the dependence of the electronic efficiency on the coordinate to the saturation stage (Fig. 2a). The efficiency continues to grow, the energy stored in the wave pulse continues to grow due to the increase in the field in the neighboring cycles, rather than to the strengthening of the central cycle.

The increase in the bunch charge from 0.5 nC up to 2 nC leads to shortening of the effective electron-wave interaction region, as well as to an increase in the amplitude of the central wave cycle \( (E^2 \sim Q) \) in the spontaneous coherent radiation process). In this case, the highest amplitude of the central wave cycle is achieved at the length \( z = 7 \lambda_u \), which corresponds again to the end of the fast growth of the electron efficiency with the coordinate (compare Figs. 4b and 2b). Note that in both cases, the radiated wave fields provide stabilization of the electron bunch in spite of the Coulomb repulsion (Fig. 4). Moreover, in those parts of the interaction regions where the wave fields reach the maximum, the bunch is compressed. It is important that the use of an irregular input waveguide section ensures both an increase in the length of the effective electron-wave interaction and a significant increase in the efficiency of extraction of the electron energy by the radiated wave (Fig. 2). Note that the optimal length of the irregular section corresponds to saturation of the radiation process in the case of using the regular waveguide. In the case of using an input section with a smaller radius, a wave packet with a slightly longer wavelength is formed. As a result, the difference \( \gamma_0 - \gamma_{\text{res}} \) between the initial electron energy and the energy corresponding to the exact electron-wave resonance, \( \gamma_{\text{res}} = \sqrt{\lambda_u (1 + K^2) / \lambda} \), becomes greater. Thus, in fact, the use of the irregular input section is analogous to the use of the electron-wave resonance mismatch step, which is a well-known method of increasing the efficiency of electronic masers (see, e.g., [14]).

Alternative method of increasing efficiency is using of particle movement in the negative mass regime. This regime is realized in the undulator (circular polarized) with the guiding magnetic field when the frequency of cyclotron \( \Omega_c = \frac{eH_0}{m c} \) slightly exceeds the frequency of the forced bounce-oscillation \( \Omega_u = \hbar u V_z \), here \( \hbar u = 2 \pi / \lambda_u \) in the transverse periodical undulator field \( H_u \). In this case the undulator period \( K \rightarrow K_{\text{eff}} = K / |\Delta| \), here \( \Delta = 1 - \Omega_c / \Omega_u \) [10]. Consider the system with the same parameters, \( K = 0.3 \) and \( \Delta = 0.3 \). In this case the efficiency exceeds 30% for both considered bunch charges (see fig. 5, a). The amplitude of the central cycle \( \sim 70 \text{ MV/m} \) and \( \sim 140 \text{ MV/m} \) for bunches with charges 0.5 nC and 2 nC, accordingly (see fig. 5, b). However in the case of the total bunch charge 2 nC, particles leaves the stationary trajectory according to the negative-mass regime after just two undulator periods (green fills in the fig. 5, b).
Figure 5. (a): Efficiency of radiation as a function of the axial coordinate normalized by the undulator period in the negative mass regime in the cases of the total bunch charge 0.5 nC (purple dashed curve) and 2 nC (blue solid curve). (b): Radiated electric field and charge distribution inside the electron bunch at various points of the electron-wave interaction region in the cases of a total bunch charge of 0.5 nC (purple curves) and 2 nC (blue curves).

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