Design and construction of 3D Helmholtz coil system to calibrate 3D Hall probes

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Abstract. In this paper we present the design of a system of 3D Helmholtz coils aimed to generate a magnetic field in any direction in a controlled way. The system is intended to be applied to the detailed characterisation of the response of 3D Hall probes as a function of the orientation of the measured field. The system will generate magnetic fields of up to 5 mT with an expected angular precision of 0.2 mrad.

1. Motivation
Accurate measurements of complex magnetic fields in small gap structures, as those generated by elliptical undulators, is becoming crucial to guarantee the right operation of accelerator based light sources. Therefore magnetic measurement systems must be able to determine all three components of the magnetic field with a high degree of precision.

The solution of choice for the mapping of magnetic fields in synchrotron light sources and FELs are measurement benches based on Hall effect sensors. In many laboratories 3D probes are manufactured by combining three orthogonally mounted 1D Hall sensors on a common board. However, the accurate soldering of the sensors is problematic, leading to some unavoidable deviations from orthogonality. These angular misalignments have to be determined in order to enable a precise reconstruction of the magnetic field.

At ALBA magnetic measurements laboratory Hall probes are manufactured in-house using commercial uni-axial Hall sensors [1]. The Hall sensors are soldered on the circuit board with a typical accuracy of ±3°. The resulting misalignment angles between the Hall sensors are estimated by placing the probe inside a calibration dipole at a series of predefined orientations with the help of mechanized pieces. However, this method does not allow determining the mutual misalignments between the sensors with an accuracy better than 0.5°~10 mrad.

The limited space in the air gap of our calibration magnet (with a gap dimension of 15 mm) makes it unfeasible to implement an accurate mechanical system providing a full control of the orientation of the Hall probe inside the magnetic field generated by the dipole. Due to this reason we have decided to design a new system allowing us to generate a magnetic field with an arbitrary and well controlled orientation. Such a system will be based on three orthogonal pairs of Helmholtz coils.

2. System requirements
A Helmholtz pair is constituted by two identical circular coils separated along its common axis by a distance equal to their radius $R$, as shown in Fig. 1. Such a system generates a highly uniform magnetic field in its central region. In addition, given that the setup does not incorporate any ferromagnetic
material, the system is perfectly linear and allows superimposing the fields generated by different sets of coils.

![Figure 1. Cross section of a Helmholtz pair generating a magnetic field along the x axis](image)

In the ideal case of having two identical coils made of vanishing-section conductor, the field at the centre of the system is directed along the axis of the coils and has an intensity given by:

\[
B = \left(\frac{4}{\pi} \right)^{3/2} \frac{\mu_0 NI}{R}
\]

where \(N\) is the number of turns per coil, and \(I\) is the circulating current. Field expressions for a more realistic situation—finite size conductors, small size differences between the two coils, etc—can be found elsewhere [2].

The aim of our system is to generate a magnetic field homogeneous within \(10^{-4}\) in a volume containing all three sensors of our Hall probes. Given that the three sensors are mounted on the circuit board along a common line with a separation of 5.45mm between them, we have decided to impose our homogeneity requirement over a volume of \(15 \times 15 \times 15\) mm\(^3\). This requirement will dictate the overall dimension of the system. A second requirement is to obtain a magnetic field as high as possible whilst keeping the power dissipation and heating of the system at a reasonable level (\(\Delta T < 20^\circ C\)). In connection with this point, we have decided to use air cooled coils so as to avoid increasing the complexity of the system. The third requirement is related to the degree of orthogonality of the magnetic field generated by the different pairs of coils: we call for misalignments smaller than 0.2 mrad. This constraint will have an impact on the mutual alignment between the two coils of each pair and on the assembly tolerances of the overall setup. The set of requirements is summarized in Table 1.

| Parameter                     | Specification                        |
|-------------------------------|--------------------------------------|
| Magnetic field homogeneity    | \(10^{-4}\) within \(15 \times 15 \times 15\) mm\(^3\) |
| Maximum system heating        | 20°C                                 |
| Magnetic field orthogonality  | 0.2 mrad                             |
3. System requirements
The magnetic design of the system has been carried out using RADIA magnetostatic simulation code [3]. The field homogeneity provided by a pair of Helmholtz coils has been characterized by means of the parameter:

$$\sigma_B = \frac{B_{\text{max}} - B_{\text{min}}}{B_{\text{max}}}$$

where $B_{\text{max}}$ and $B_{\text{min}}$ are the maximum/minimum values of the modulus of the magnetic field within the considered volume. Simulations show that the homogeneity parameter depends basically on the radius of the coils $R$, and it is very weakly dependent on the coil thickness. For the homogeneity volume that we are considering, we have found out a numerical expression providing an estimation of $\sigma_B$ as a function of $R$ (in [mm]):

$$\sigma_B \simeq 1.73 \times 10^4 \frac{1}{3435 - 3.375 R^2 + R^4}$$

This expression allows us to set the dimension of the system for a given homogeneity requirement. In our case, with $\sigma_B < 10^{-4}$, we obtain $R > 115$ mm.

The coil’s characteristics (conductor section, number of turns, etc.) and operating current, which will determine the maximum field attainable by the system, will be dictated by thermal considerations. The power dissipated on each coil will be equal to the increase rate of its internal energy plus the rate of energy transferred from the coil to its surroundings through its surface ($S$), i.e.

$$P = I^2 R_{el} = \frac{dU}{dt} + \oint_S \mathbf{j} \cdot d\mathbf{s} = C_{\text{coil}} \frac{dT}{dt} + h_c S \Delta T$$

where $R_{el}$ is the electrical resistance of one coil, $C_{\text{coil}}$ is its heat capacity, and the heat exchange term has been estimated using Newton’s law of cooling, with $h_c$ standing for the convective heat transfer coefficient and $\Delta T$ for the temperature increase of the coil with respect to the surrounding air. When the stationary state is reached only the second term in Eq. (4) remains, and the maximum temperature increase of the coil is given by:

$$\Delta T_{\text{max}} = \frac{I^2 R_{el}}{h_c S}$$

If we put Eq. (5) in terms of the parameters of the coil, it can be rewritten as:

$$\Delta T_{\text{max}} = \frac{(IN)^2 \rho}{4 h_c f_c A^{3/2}}$$

where $\rho$ is the electrical resistivity of the coil’s wire (1.68×10^{-8} Ω·m for copper), $A$ is the total section of the coil, and $f_c$ is the fill factor indicating the fraction of the coil’s section which is occupied by conductor material, $f_c = N \pi (\varnothing^2/4)/A$ for a wire with a conductor diameter $\varnothing$. Values for $f_c$ typically are in the range 0.6-0.8, depending on the packing of wires within the coil and on the insulation thickness.

For the coil manufacturing we have decided to use grade 2 enamelled copper wire with a conductor/total diameter of 1.5mm/1.571mm. This wire has been selected because it is thin enough so as to enable the usage of standard coil winding equipment, and at the same time it provides a higher conductor/insulator ratio in comparison to thinner wires, thus delivering a higher filling ratio $f_c$. In addition, we have estimated that the maximum operating current ($\leq 5$ A) and power ($<100$W) required for a coil made of this wire will sit comfortably inside the range provided by the power supplies available at our laboratory. Using an orthocyclic winding layout, we expect attaining a filling ratio close to 0.77.

The combination of Eqs. (1) and (6) allows us to estimate the maximum magnetic field generated and the corresponding temperature increase for a given set of parameters of the coil. Assuming a heat transfer coefficient to air of 10 W/m²K [4], we have looked for a compromise between the obtained field and the induced heating, coming up finally to the parameters listed in “Coil 1” column of Table 2. It can be seen that the magnetic field generated at a nominal current of 2.3 A is of 5 mT, for a maximum temperature increase of the coils of 15ºC.
In order to generate a magnetic field in an arbitrary direction we need to add two pairs of orthogonal coils to our system. However, these two additional pairs of coils have to be enlarged with respect to the first one in order to avoid any mechanical interference. In our case the chosen scale factor from pair to pair has been 1.37, leaving enough space between the different pairs of coils to accommodate the elements of the support system. The number of turns of the 2nd and 3rd pair of coils has also been scaled accordingly in order to provide the same magnetic field intensity at nominal current. The complete set of parameters for all three pairs of coils is listed in Table 2.

Table 2. Design parameters for the 3 sets of coils of the 3D Helmholtz system.

| Parameter              | Coil 1 | Coil 2 | Coil 3 |
|------------------------|--------|--------|--------|
| Number of turns $N$    | 304    | 418    | 572    |
| Radius $R$ [mm]        | 125.74 | 172.89 | 236.59 |
| Coil width [mm]        | 26.40  | 31.20  | 36.00  |
| Coil height [mm]       | 26.66  | 30.83  | 36.40  |
| Nominal Current [A]    | 2.3    | 2.3    | 2.3    |
| Magnetic Field [mT]    | 5      | 5      | 5      |
| Resistance/coil [Ω]    | 2.31   | 4.37   | 8.18   |
| Power/coil [W]         | 12.22  | 23.11  | 43.27  |
| Estimated $\Delta T$ [ºC] | 15   | 17     | 20     |

3.1. Mechanical tolerances

If the two coils of each pair are not perfectly aligned, the generated magnetic field will display an angular deviation with respect to the nominal axis of the pair and its homogeneity will be degraded. Since we want to keep these two parameters within the requirements for the whole system defined in Table 1, this will set a tolerance on the mechanical alignment of each pair of coils. We have analysed the effect of two possible sources of error, illustrated in Fig. 2: a rotation $\alpha$ of one coil with respect to the other, and a displacement $\tau$ between their centres. By means of simulations we have analysed the effect of these sources of error, and the tolerances listed in Table 3 have been obtained.

![Figure 2](image-url). Sources of misalignment of each pair of coils.
Table 3. Mechanical tolerance for the alignment of each pair of coils.

| Parameter                  | Coil 1 | Coil 2 | Coil 3 |
|----------------------------|--------|--------|--------|
| Angular deviation $\alpha$ [mrad] | 0.2    | 0.2    | 0.2    |
| Centres offset $\tau$ [mm]    | 0.08   | 0.12   | 0.16   |

4. Mechanical design

For the mechanical implementation of the system we have designed a set of interlocking aluminium pieces. Each one of the pieces has a rectangular groove with the appropriate dimensions to allocate the foreseen number of wire windings for one of the coils. The precise machining of the pieces will guarantee that upon assembly the relative positioning of the coils will be close to the design values. Besides this, the pieces provide a flat surface parallel to the plane of the coil and a cylindrical surface concentric with its axis. Therefore, once assembled it will be possible to measure any misalignment between the coils by means of a laser tracker or a portable coordinate measuring machine and, if necessary, to correct it by means of the introduction of shims. Figure 3 shows a view of the assembled system.

![Figure 3. Drawing of the assembled system of 3D Helmholtz coils.](image)

5. Conclusion

We have presented the design of a system of 3D Helmholtz coils. The mechanical parts will be manufactured in-house, and the coil winding will be outsourced to a specialized local company. The system is expected to be ready by Summer 2019, and tests will be carried out during the second half of the year.

It is worth mentioning that the system has been designed to generate 5 mT in a sustainable way without overheating. However, given the long time constant of the heating process (estimated to be >20 min) it will be possible to generate larger fields for short periods of time.

References

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