Short-time quantum correlations in the atom optics kicked rotor

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Abstract. - We experimentally verify the analytical expressions that exist for the diffusion rate in the quantum delta kicked rotor system for small numbers of kicks. We show development of diffusion resonances from two to five kicks, and of multiple resonances for high kick strengths. Furthermore, we show that, in contrast to classical predictions, the results are purely periodic in the kick period, and reproduce the predicted quantum- and diffusion resonances.

The atom optics implementation of the delta kicked rotor system has been the vehicle of a number of ‘Quantum Chaos’ experiments: experiments that study a quantum system whose classical equivalent exhibits chaotic behavior. One such system is the classical delta kicked rotor system. The atom optics implementation of this system consists of a two-level atom placed in a pulsed standing wave of laser light, which is detuned from resonance [1]. The AC Stark shift induces a potential which depends sinusoidally on the position in the standing wave. The atoms generally have a momentum distribution that is much wider than the recoil of a single photon, and are distributed over a large range of positions in the standing wave. The chaotic nature of the classical equivalent of this system means that the energy growth would be constant for each kick. The quantum mechanical nature of the atomic motion gives rise to coherences, which curb the energy growth.

The Hamiltonian for the atomic motion in the pulsed standing wave can be written as [2]

\[ \hat{H} = \frac{\hat{p}^2}{2m} - \frac{\hbar \Omega_R}{2} \cos(2k_l \hat{x}) \sum_{n=1}^{N} f(t - nT) \]  

(1)

where \( \hat{p} \) and \( \hat{x} \) are the atomic momentum and position operators, \( \Omega_R = \Omega^2/(4\Delta) \) is the effective Rabi frequency, \( k_l \) is the laser wave number, \( m \) is the atomic mass and \( T \) is the kick period. The function \( f(t) \) describes the laser pulse, and is generally given by a square pulse with a length \( \tau_p \ll T \).

The Hamiltonian can be scaled to dimensionless parameters to read

\[ \hat{H}' = \frac{\hat{\rho}^2}{2} + \kappa \cos(\hat{\phi}) \sum_{n=1}^{N} f(\tau - n) \]  

(2)

where the scaled momentum operator \( \hat{\rho} = 2k_l T \hat{p}/m \), the scaled position operator \( \hat{\phi} = 2k_l \hat{x} \), \( \tau = t/T \) and \( \hat{H}' = (4k_l^2 T^2/2m)\hat{H} \). The commutator \([\hat{\phi}, \hat{\rho}] = 8i\omega_r T \equiv ik\) where \( \omega_r = \hbar k_l^2/(2m) \) is the recoil frequency. Consequently \( k \), which can be experimentally altered by
changing the kick period, plays the role of Planck’s constant in the uncertainty principle. Here, we have also introduced the classical kick strength \( \kappa = 4\Omega R T \).

This allows us to map out the transition from ‘classical’ (\( \kappa \to 0 \) at constant \( \kappa \)) to ‘quantum’ (\( \kappa \gg 1 \)) using a convenient experimental parameter, the kick period. For convenience, we will also introduce the experimental kick strength \( \phi_d = \kappa / k = \Omega R T / 2 \).

Analytical results [3] exist for the values of the energy after small numbers of kicks. For the energies after the first five kicks we obtain:

\[
\begin{align*}
E_1 &= E_0 + \phi_d^2 \\
E_2 &= E_0 + 2\phi_d^2 \\
E_3 &= E_0 + \phi_d^2 (3 - 2J_2(\kappa_q)) \\
E_4 &= E_0 + \phi_d^2 (4 - 4J_2(\kappa_q) + 2J_3(\kappa_q) - 2J_1^2(\kappa_q)) \\
E_5 &\approx E_0 + \phi_d^2 (5 - 6J_2(\kappa_q) + 4J_3^2(\kappa_q) - 4J_1^2(\kappa_q) + 2J_2^2(\kappa_q))
\end{align*}
\]

where \( E_0 \) is the initial cloud energy, \( \kappa_q = 2\phi_d \sin(k/2) \) and \( J_n(x) \) are the Bessel J functions. The energies are given in units of the recoil energy \( E_R = \hbar^2 k_0^2 / (2m) \), and are periodic in \( k \).

This periodicity of the final energy is a feature that is not reproduced in classical physics, and is a quantum phenomenon. For values of \( k \) which are multiples of \( 2\pi \), a ‘resonance’ appears, which becomes sharper for larger numbers of kicks.

For sufficiently large kick strength \( \phi_d \) and for values of \( k \) between zero and the quantum resonance, or between the quantum resonances, equations [4] describe a series of increases and decreases in the final energy as a function of \( k \), forming diffusion resonances [4], which are due to quantum correlations in the atomic motion. Note that these resonances are transient in nature, as after a finite Heisenberg time, dynamic localisation will prohibit further energy growth and that at larger kick numbers, these ‘resonances’ will hence be less visible. As the argument of the Bessel functions \( \kappa_q \) depends on the intensity of the laser light, any variation of the laser intensity over the atomic cloud will smear out these ‘resonances’, also effectively lowering them.

A large body of experimental [5, 6] and theoretical [7–10] work exists, which explores the dynamics of this system in the limit of a large number of kicks. Generally, the energy growth is measured for different kick periods and kick strengths, and compared to numerical simulations. The experiments start with an ensemble of atoms, typically caesium or rubidium, in magneto-optical trap at temperatures in the microKelvin range. This means that the average momentum of the atoms before kicking is much larger than a single photon recoil. Starting from a single momentum state, the kicks distribute the atoms over many momentum states, spaced by two photon recoils. Interference between these momentum states eventually curb the growth of the momentum spread. It is therefore convenient to consider a quasimomentum \( q = p \mod 2\hbar k \). At microKelvin temperatures, the atoms have a flat, even distribution of quasimomenta. The absolute momentum \( p \), is of minor importance for the evolution of the energy spread. This means that the initial temperature of the MOT has very little effect on the results of the measurements. A detailed study of the effect of the initial momentum distribution can be found in reference [11].

In further experiments, the effects of amplitude and phase noise have been studied [12, 13], as well as the effect of spontaneous emission [14, 15]. It was found that amplitude noise smoothed out the diffusion resonances, while preserving the quantum resonance [16]. Phase noise in the kick sequence was found to destroy the effects of dynamic localisation [17]. Spontaneous emission was found to broaden the quantum resonance peak, making it easier to observe experimentally for large numbers of kicks, which was discussed in a recent paper by Wimberger et.al. [18]. They also mention a remaining questions in the atom optics kicked rotor system, which concerns the observation of higher order quantum resonances.

Here, we present the first experimental results showing the development of the diffusion
resonances as a function of kick number and for different kick strengths, and compare these
directly with the analytical equations. Although oscillations in the mean energy have been
observed before, they have only been studied as a function of the kick strength [19]. When
the mean energy was studied as function of the kick period, attention was focused on the
quantum resonance [20]. Furthermore, we extend the range of experimentally investigated
kick periods to $k = 20\pi$ and demonstrate that the final energy is periodic in $k$ to that value.

In our experiment we trap rubidium 85 atoms in a standard magneto-optical trap [21],
loaded from a background vapour. After an accumulation phase, the anti-Helmholtz mag-
netic field is turned off and the trap lasers are switched to a larger detuning for a 3 ms
cooling phase, after which the trap lasers are extinguished. The velocity distribution of the
atoms at that moment can be described by a Gaussian curve with a FWHM of $\sim 4$ cm$^{-1}$,
which corresponds to a temperature of $\sim 10\, \mu$K. The atoms are then subjected to a
periodic sequence of kicks by the, linearly polarised, standing wave laser field. The atomic
ensemble is subsequently allowed to expand for 15 ms, after which the ensemble fluores-
cence is imaged on a CCD camera (Apogee AP47p) by flashing on the molasses lasers for 5 ms. We
obtain the positional variances of the atomic cloud, in both the direction of the kick laser
(‘kicked’) and orthogonal to that (‘non-kicked’), numerically from the image and convert
these to average velocity squared, and from that to kinetic energy. We take the energy in
the non-kicked direction as the initial energy. The experiment is then repeated for different
kick periods or kick numbers. The average initial energy is determined from an experimental
run over a range of parameters, and is used to rescale the energy ratio to an energy. The
kick laser beam was obtained from a Toptica DLX110 laser system, which was locked to the
$D_2$ transition in $^{87}$Rb, obtaining a laser detuning of 1.3 GHz from the $F = 3 \rightarrow F = 4$
transition in $^{85}$Rb. The beam was passed through an AOM, which was controlled by a home-built
programmable pulse generator, allowing us to pulse the laser with adjustable pulse number,
period and amplitude. The pulse length for most experiments was set to 300 ns. To spa-
tially filter the laser beam, it was then passed through a polarisation preserving single mode
fibre. After collimation, the radius $(1/e^2)$ of the Gaussian laser beam was 3.0 mm. The
initial radius of the atomic cloud was 0.3 mm $(1\sigma)$, small compared to the laser beam size.
The laser beam was passed through a polarising beam splitter to ensure linear polarisation.
The large detuning and linear laser polarisation yield equal kick strengths for all magnetic
sublevels of the $F = 3$ ground state. The maximum kick laser power was 100 mW, which for
a 300 ns pulse and a 1.3 GHz laser detuning yields a scaled kick strength $\phi_d = 5.2$. Larger
kick strengths could be obtained by lengthening the kick pulse.

The detuning of the kick laser yields a spontaneous emission probability of $1\%$ per atom
per kick for this kick strength, which we deem negligible. This was verified by observing the
momentum variance in the kicked direction for a range of kick numbers, up to 80 kicks. These
experiments showed negligible energy growth after a certain number of kicks. This represents
strong evidence that the quantum correlations, which curb the energy growth, survive that
number of kicks, justifying the neglect of spontaneous emission in our experiments.

The first feature in eqns that is of interest is the development of the structure in the
final energy, from non-existent for two kicks, to quite pronounced for five kicks. In figure the
energy difference between the ‘kicked’ and ‘non-kicked’ dimensions is shown as a function
of the kick period for two to five kicks. For two kicks, the spectrum is flat as expected. This
provides us with a reference value of $\phi_d$. For three kicks structure starts to appear, which
is well reproduced by the analytic expressions in equation. The quantum resonance at
$k = 2\pi$ is starting to appear, and an onset of the ‘diffusion resonances’ can be observed
for smaller $k$. For four and five kicks, these structures become more and more pronounced,
but stay at the same position. The value of the kick strength $\phi_d = 4.8$ for the analytical
line is determined from the energy after two kicks. The analytical line takes a variation of
10% in kick strength into account, which could be due to the Gaussian laser beam profile,
or imperfect wavefronts in the kick laser beam. We also note the deviation of the first
few points from the theoretical curve, as this is where the system exhibits more classical
behaviour. This phenomenon is detailed in ref [22].

As the argument of the Bessel functions $\kappa_q$ in eqns 3 is proportional to the scaled kick strength $\phi_d$, distinctly, we expect distinct qualitative differences in the final energy curves for different values of $\phi_d$. In figure 2 the energy difference is shown as a function of $k$ for five kicks for a range of values for $\phi_d$. Again, the analytical lines take a variation of 10% in kick strength into account. For $\phi_d = 3.4$, virtually no diffusion resonances are observed. For larger values of $\phi_d$, maxima are starting to appear, which move out at increasing $\phi_d$. An additional maximum starts to form in the centre for $\phi_d = 5.7$, in agreement with the analytical formulae. At the same time, the quantum resonance gets more narrow, as can be seen from the movement of the minima in the energy towards the quantum resonance. As the energy at the minima remains close to the energy after two kicks, which is proportional to $\phi_d^2$, this energy increases with increasing $\phi_d$.

At even larger values of $\phi_d$, multiple oscillations of the final energy as a function of $k$
are expected. For the results in figure 2(b) the pulses were lengthened to enable larger
kick strengths. We clearly see multiple oscillations in the final energy as a function of \( k \)
developing for these large kick strengths. Again, the lines represent eqns \( 3 \) averaged over a
10\% spread in \( \phi_d \). To our knowledge, this represents the first direct observation of multiple
oscillations in the final energy, or diffusion resonances.

As indicated by Wimberger et. al. \cite{18}, an interesting question concerns the periodicity
of the final energies that is apparent from equations \( 3 \). This periodicity has been observed
for relatively large kick numbers and in the presence of spontaneous emission up to a value
for \( k = 6\pi \) in reference \cite{20}. From a classical point of view, long periods between the kicks
should put the system in a chaotic regime, where kicks are completely unrelated. Quantum-
mechanically, the induced coherence in the atomic motion should remain indefinitely, and
there should be no difference in the physics between \( k \) and \( k + 2M\pi \), where \( M \) is a (positive)
integer. In figure 3 we display the energy difference as a function of \( k \) for ten periods
in \( k \), along with the analytical line, for \( \phi_d = 5 \). The structure appears to be completely
periodic, in sharp contrast to classical predictions, but in agreement with Shepelyansky’s
equations. Note that both the quantum resonance, at integer multiples of \( 2\pi \), which is a
quantum phenomenon, and the diffusion resonances, which are thought to be classical in
origin, appear to be completely periodic in \( k \). There is no apparent degradation of either
the quantum- or diffusion resonances up to \( k = 20\pi \). This demonstrates the quantum-
mechanical nature of the system, with no apparent decoherence.

In summary, we have experimentally verified the validity of Shepelyansky’s expressions
for the diffusion rate for the first five kicks in a quantum kicked rotor system. We find good
agreement between experiment and the analytical expressions, we verify the periodicity of
the results, and observe the predicted diffusion- and quantum resonances up to \( k = 20\pi \).

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The authors would like to acknowledge stimulating discussions with S. Parkins, M. Sad-
grove, T. Mullins, A. Hilliard and R. Leonhardt, as well as financial support from the
University of Auckland.

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