Remarks on hyperspaces of soft sets

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Abstract

In this paper we study on some hyperspaces of soft sets such as co-quasi H-closed soft topological spaces and D-soft topological spaces. Then we give the relationship between these hyperspaces and soft Vietoris topological spaces.

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1. Introduction

Molodtsov [1] introduced the concept of soft sets. Soft set theory has rich potential for practical applications in several sciences. Pei and Miao [2] investigated the relationships between soft sets and information systems. Çağman et al. [5] defined a soft topological space. Zorlutuna et al. [14] studied some concepts in soft topological spaces. Çağman and et al. redefined the operations of the soft sets and constructed a uni-int decision making method by using these new operations [7]. Later on, Akdağ and Erol [3, 11] introduced the concept of soft multifunction and studied their properties. Many researcher studied on soft set theory (see [9, 10]).

2. Preliminaries

Definition 2.1. [1] Let $X$ be an initial universe and $E$ be a set of parameters. Let $P(X)$ denote the power set of $X$ and $A$ be a non-empty subset of $E$. A pair $(F, A)$ is called a soft set over $X$, where $F$ is a mapping given by $F : A \rightarrow P(X)$. In other words, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon$-approximate elements of the soft set $(F, A)$.

Definition 2.2. [15] A soft set $(F, A)$ over $X$ is called a null soft set, denoted by $\Phi$, if $F(\varepsilon) = \emptyset$, for all $\varepsilon \in A$.

Definition 2.3. [15] A soft set $(F, A)$ over $X$ is called an absolute soft set, denoted by $\widehat{X}_A$, if $F(\varepsilon) = X$, for all $\varepsilon \in A$.

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If \( A = E \), then the \( A \)-universal soft set is called a universal soft set, denoted by \( \overline{X} \).

**Definition 2.4.** [4] Let \( Y \) be a non-empty subset of \( X \), then \( \overline{Y} \) denotes the soft set \((Y,E)\) over \( X \) for which \( Y(e) = Y \), for all \( e \in E \).

**Definition 2.5.** [15] The union of two soft sets of \((F,A)\) and \((G,B)\) over the common universe \( X \) is the soft set \((H,C)\), where \( C = A \cup B \) and for all \( e \in C \),

\[
H(e) = \begin{cases} 
F(e), & \text{if } e \in A - B, \\
G(e), & \text{if } e \in B - A, \\
F(e) \cup G(e), & \text{if } e \in A \cap B.
\end{cases}
\]

We write \((F,A)\overline{\cup}(G,B) = (H,C)\).

**Definition 2.6.** [2] The intersection \((H,C)\) of two soft sets \((F,A)\) and \((G,B)\) over a common universe \( X \), denoted by \((F,A)\overline{\cap}(G,B)\), is defined as \(H(e) = F(e) \cap G(e)\) for all \( e \in C \) where \( C = A \cap B \).

**Definition 2.7.** [2] Let \((F,A)\) and \((G,B)\) be two soft sets over a common universe \( X \). \((F,A)\overline{\cap}(G,B)\), if \( A \subset B \) and \( F(e) \subset G(e) \) for all \( e \in A \).

**Definition 2.8.** [16] For a soft set \((F,A)\) over \( X \) the relative complement of \((F,A)\) is denoted by \((F,A)^c\) and is defined by \((F,A)^c = (F^c,A)\), where \( F^c : A \to P(X) \) is a mapping given by \( F^c(\alpha) = X - F(\alpha) \) for all \( \alpha \in A \).

**Definition 2.9.** [4] Let \( \tau \) be the collection of soft sets over \( X \), then \( \tau \) is said to be a soft topology on \( X \) if satisfies the following axioms:

1. \( \Phi, \overline{X} \) belong to \( \tau \),
2. the union of any number of soft sets in \( \tau \) belongs to \( \tau \),
3. the intersection of any two soft sets in \( \tau \) belongs to \( \tau \).

The triplet \((X, \tau, E)\) is called a soft topological space over \( X \). Let \((X, \tau, E)\) be a soft topological space over \( X \), then the members of \( \tau \) are said to be soft open sets in \( X \). A soft set \((F,A)\) over \( X \) is said to be a soft closed set in \( X \), if its relative complement \((F,A)^c\) belongs to \( \tau \).

**Definition 2.10.** [12] Let \((G,K)\) be a soft open set in a soft topological space \((Y,\tau,K)\). The soft set families \((G,K)^+\) and \((G,K)^-\) are defined as follows:

\[(G,K)^+ = \{(T,K) \in SS(Y,K) : (T,K)\overline{\cap}(T,K)\}\]

and

\[(G,K)^- = \{(T,K) \in SS(Y,K) : (G,K)\overline{\cap}(T,K) \neq \Phi\} .\]

**Proposition 2.11.** [12] Let \((Y,\tau,K)\) be a soft topological space. For a non null soft sets \((G,K)\) and \((H,K)\) the following statements are true:

1. \((G,K)^+ \cap (H,K)^+ = (\overline{(G,K)\overline{\cap}(H,K)})^+\).
2. \((G,K)^+ \cup (H,K)^+ \subset (\overline{(G,K)\overline{\cup}(H,K)})^+\).
3. \((\overline{(G,K)\overline{\cap}(H,K)})^- \subset (G,K)^- \cap (H,K)^-\).
4. \((G,K)^- \cup (H,K)^- = (\overline{(G,K)\overline{\cap}(H,K)})^-\).
5. \((G,K)\overline{\cap}(H,K)\) if and only if \((G,K)^+ \subset (H,K)^+\).
6. \((G,K)\overline{\cap}(H,K)\) if and only if \((G,K)^- \subset (H,K)^-\).
7. \((G,K)\overline{\cap}(H,K) = \Phi\) if and only if \((G,K)^+ \cap (H,K)^+ = \emptyset\).
8. \((G,K)^- \cap (H,K)^- \neq \emptyset\).
Proposition 2.12. [12] Let \((Y, \tau, K)\) be a soft topological space. Then the soft set families
\[
\beta_{SV}^+ = \{(G, K)^+ : (G, K) \text{ soft open set}\}
\]
and
\[
S_{SV}^- = \{(G, K)^- : (G, K) \text{ soft open set}\}
\]
are soft base and soft sub base for a different soft topological spaces on \(2^{S(Y,K)}\), respectively.

Definition 2.13. [12] The soft topological spaces which mentioned in above proposition are called soft upper Vietoris, soft lower Vietoris and denoted by \(\tau_{SV}^+\), \(\tau_{SV}^-\), respectively.

Definition 2.14. [12] Let \((U_1, K), (U_2, K), \ldots, (U_n, K)\) be open soft sets in a soft topological space \((Y, \tau, K)\). The family of all soft sets which
\[
B((U_1, K), (U_2, K), \ldots, (U_n, K)) = \{(T, K) \in S(Y, K) : (T, K) \subseteq \bigcup (U_i, K) \text{ and } (T, K) \cap (U_i, K) \neq \emptyset, \ i = 1, 2, \ldots, n\}
\]
is the soft base for a soft topological space. This space is called Vietoris soft topological space and denoted by \(\tau_{SV}\). For the soft Vietoris topology \(\tau_{SV} = \tau_{SV}^+ \cup \tau_{SV}^-\).

Definition 2.15. [11] Let \(S(X,E)\) and \(S(Y,K)\) be two soft classes. Let \(u : X \to Y\) be multifunction and \(p : E \to K\) be mapping. Then a soft multifunction \(F : S(X,E) \to S(Y,K)\) is defined as follows: For a soft set \((G, E)\) in \(S(X,E)\), \((F(G, E), K)\) is a soft set in \(S(Y,K)\) given by
\[
F(G, E)(k) = \begin{cases} 
\bigcup_{e \in p^{-1}(k) \cap E} u(G(e)), & \text{if } p^{-1}(k) \cap E \neq \emptyset, \\
\emptyset, & \text{otherwise}
\end{cases}
\]
for \(k \in K\). \((F(G, E), K)\) is a soft image of a soft set \((G, E)\). Moreover,
\[
F(G, E) = \bigcup \{F(E_x^i) : E_x^i \in \widetilde{X}, \ \text{for a soft subset } (G, E) \text{ of } X\}.
\]

Definition 2.16. [11] Let \(F : S(X,E) \to S(Y,K)\) be a soft multifunction. The soft upper inverse image of \((H, K)\) denoted by \(F^+(H, K)\) and defined as \(F^+(H, K) = \{E_x^i \in \widetilde{X} : F(E_x^i) \subseteq (H, K)\}\). The soft lower inverse image of \((H, K)\) denoted by \(F^-(H, K)\) and defined as \(F^-(H, K) = \{E_x^i \in \widetilde{X} : F(E_x^i) \cap (H, K) \neq \emptyset\}\).

Definition 2.17. [12] Let \(F,G : S(X,E) \to S(Y,K)\) be two soft multifunctions. For \(E_x^i \in \widetilde{X}\), the union and intersection of \(F\) and \(G\) is denoted by
\[
(F \cup G)(E_x^i) = F(E_x^i) \cup G(E_x^i) \quad \text{and} \quad (F \cap G)(E_x^i) = F(E_x^i) \cap G(E_x^i).
\]

Definition 2.18. [11] Let \(F : S(X,E) \to S(Y,K)\) and \(G : S(X,E) \to S(Y,K)\) be two soft multifunctions. Then \(F\) equal to \(G\) if \(F(E_x^i) = G(E_x^i)\), for each \(E_x^i \in \widetilde{X}\).

Definition 2.19. [11] The soft multifunction \(F : S(X,E) \to S(Y,K)\) is called surjective if \(p\) and \(u\) are surjective.

Theorem 2.20. [11] Let \(F : S(X,E) \to S(Y,K)\) be a soft multifunction. Then, for soft sets \((G, E), (H, E)\) and for a family of soft sets \((G_i, E)\) in the soft class \(S(X,E)\) the following are hold:

1. \(F(\emptyset) = \emptyset\).
2. \(F(X) \subseteq Y\).
3. \(F((G, E) \cup (H, E)) = F(G, E) \cup F(H, E)\) in general \(F\left(\bigcup_{i \in I}(G_i, E)\right) = \bigcup_{i \in I}F(G_i, E)\).
4. \(F((G, E) \cap (H, E)) \subseteq F(G, E) \cap F(H, E)\) in general \(F\left(\bigcap_{i \in I}(G_i, E)\right) \subseteq \bigcap_{i \in I}F(G_i, E)\).
(5) If \((G, E) \subseteq (H, E)\) then \(F(G, E) \subseteq F(H, E)\).

**Theorem 2.21.** [11] Let \(F : S(X, E) \rightarrow S(Y, K)\) be a soft multifunction Then for soft sets \((G, K), (H, K), (G_i, K)\) in the soft class \(S(Y, K)\) for each \(i \in I\) the following statements are hold:

1. \(F^-(\{\emptyset\}) = \emptyset\) and \(F^+(\{\emptyset\}) = \emptyset\).
2. \(F^-(\emptyset) = \emptyset\) and \(F^+(\emptyset) = \emptyset\).
3. \(F^-(\bigcup_{i \in I} (G, K)) = F^-(\bigcup_{i \in I} (G_i, K))\) and in general \(F^-(\bigcup_{i \in I} (G_i, K)) = \bigcup_{i \in I} F^-(G_i, K)\).
4. \(F^+(\bigcup_{i \in I} (G, K)) = \bigcup_{i \in I} F^+(G_i, K)\) and in general \(F^+(\bigcup_{i \in I} (G_i, K)) = \bigcup_{i \in I} F^+(G_i, K)\).
5. \(F^-(\bigcap_{i \in I} (G, K)) = \bigcap_{i \in I} F^-(G_i, K)\) and in general \(F^-(\bigcap_{i \in I} (G_i, K)) = \bigcap_{i \in I} F^-(G_i, K)\).
6. \(F^+(\bigcap_{i \in I} (G, K)) = \bigcap_{i \in I} F^+(G_i, K)\) and in general \(F^+(\bigcap_{i \in I} (G_i, K)) = \bigcap_{i \in I} F^+(G_i, K)\).
7. If \((G, K) \subseteq (H, K)\), then \(F^-(G, K) \subseteq F^-(H, K)\) and \(F^+(G, K) \supseteq F^+(H, K)\).

**Proposition 2.22.** [11] Let \(F : (X, \tau, E) \rightarrow (Y, \sigma, K)\) be a soft multifunction. Then the following statements are true:

1. \((G, E) \subseteq F^+(F(G, E)) \subseteq F^-(F(G, E))\) for a soft subset \((G, E)\) in \(X\). If \(F\) is surjective then \((G, E) = F^+(F(G, E)) = F^-(F(G, E))\).
2. \((F^+(H, K)) \subseteq F^+(F^-(H, K))\) for a soft subset \((H, K)\) in \(Y\).
3. For two soft subsets \((G, K)\) and \((H, K)\) in \(Y\) such that \((G, K) \cap (H, K) = \emptyset\) then \(F^+(G, K) \cap F^-(H, K) = \emptyset\).

**Definition 2.23.** Let \(F : S(X, E) \rightarrow S(Y, K)\) and \(G : S(Y, K) \rightarrow S(Z, L)\) be two soft multifunction. Then the combination of \(G\) and \(F\) is the soft multifunction denoted by \(G \circ F : S(X, E) \rightarrow S(Z, L)\) and defined as \((G \circ F)(H^\sharp_L) = G(F(H^\sharp_E))\).

**Proposition 2.24.** [11] Let \(F : (X, \tau, E) \rightarrow (Y, \sigma, K)\) and \(G : (Y, \sigma, K) \rightarrow (Z, \eta, L)\) be two soft multifunction. Then the follows are true:

1. \((F^-)^- = F\).
2. For a soft subset \((T, L)\) in \(Z\), \((G \circ F)^- (T, L) = F^- (G^- (T, L))\) and \((G \circ F)^+ (T, L) = F^+ (G^+ (T, L))\).

**Proposition 2.25.** [11] Let \((G, K)\) be a soft set over \(Y\). Then the followings are true for a soft multifunction \(F : (X, \tau, E) \rightarrow (Y, \sigma, K)\):

1. \(F^+(\overline{Y} - (G, K)) = \overline{X} - F^-(G, K)\).
2. \(F^-(\overline{Y} - (G, K)) = \overline{X} - F^+(G, K)\).

**3. Hyerspaces of soft sets**

**3.1. Co-quasi H-closed soft topological spaces defined on \(2^{S(Y,K)}\)**

**Definition 3.1.** Let \((Y, \tau, K)\) be a soft topological spaces and \((H, K)\) be a soft set.

1. \((H, K)\) is said to be soft quasi H-closed if any open cover \(\{G_i, K\} : i \in \Lambda\) of \((H, K)\) has a finite subfamily \(\{G_{\lambda_1}, K), (G_{\lambda_2}, K), ..., (G_{\lambda_n}, K)\}\) such that \((H, K) \subseteq \bigcup_{i=1}^{n} \text{cl} \big(G_{\lambda_i}, K\big)\).
2. \((Y, \tau, K)\) is said to be soft quasi H-closed topological spaces if \(\overline{Y}\) is soft quasi H-closed (or any open cover \(\{G_i, K\} : i \in \Lambda\) of \(\overline{Y}\) has a finite subfamily \(\{G_{\lambda_1}, K), (G_{\lambda_2}, K), ..., (G_{\lambda_n}, K)\}\) such that \(\bigcup_{i=1}^{n} \text{cl} \big(G_{\lambda_i}, K\big) = \overline{Y}\).
3. If \((Y, \tau, K)\) is soft Hausdorff and soft quasi H-closed topological space then \((Y, \tau, K)\) is called soft H-closed topological space.
4. If every soft quasi H-closed set of \(Y\) is closed then \((Y, \tau, K)\) is called HC soft topological space.
(5) If every soft closed set of $Y$ is soft quasi H-closed then $(Y, \tau, K)$ is called C-compact topological space.

**Proposition 3.2.** [14] Let $(Y, \tau, K)$ be a soft topological space. Then the following statements are true for a soft sets $(C, K)$ in $(Y, \tau, K)$.

1. $2^{S(Y,K)} - (C, K)^+ = \left(\overline{Y} - (C, K)^-\right)$.
2. $2^{S(Y,K)} - (C, K)^- = \left(\overline{Y} - (C, K)^+\right)$.

**Proposition 3.3.** Let $(Y, \tau, K)$ be a soft topological space. Then the following statements are true for a soft sets $(C_1, K)$ and $(C_2, K)$ in $(Y, \tau, K)$.

1. $\left[2^{S(Y,K)} - (C_1, K)^-\right] \cap \left[2^{S(Y,K)} - (C_2, K)^-\right] = 2^{S(Y,K)} - [(C_1, K)^- \cup (C_2, K)^-]$.
2. $\left[2^{S(Y,K)} - (C_1, K)^+\right] \cap \left[2^{S(Y,K)} - (C_2, K)^+\right] = 2^{S(Y,K)} - [(C_1, K)^+ \cup (C_2, K)^+]$.

**Proof.**

(1) \[(H, K) \in \left[2^{S(Y,K)} - (C_1, K)^-\right] \cap \left[2^{S(Y,K)} - (C_2, K)^-\right] \iff (H, K) \in 2^{S(Y,K)} - (C_1, K)^- \text{ and } (H, K) \in 2^{S(Y,K)} - (C_2, K)^- \]

\[(\iff (H, K) \notin (C_1, K)^- \text{ and } (H, K) \notin (C_2, K)^-) \]

\[(\iff (H, K) \cap (C_1, K) = \emptyset \text{ and } (H, K) \cap (C_2, K) = \emptyset) \]

\[(\iff (H, K) \notin \left[(C_1, K) \cup (C_2, K)\right]^{-1}) \]

\[(\iff (H, K) \in 2^{S(Y,K)} - \left[(C_1, K) \cup (C_2, K)\right]^{-1}) \]

\[(\iff (H, K) \in 2^{S(Y,K)} - [(C_1, K)^- \cup (C_2, K)^-]). \]

(2) \[(H, K) \in 2^{S(Y,K)} - [(C_1, K)^+ \cup (C_2, K)^+] \iff (H, K) \notin [(C_1, K)^+ \cup (C_2, K)^+] \]

\[(\iff (H, K) \notin (C_1, K)^+ \text{ and } (H, K) \notin (C_2, K)^+) \]

\[(\iff (H, K) \notin (C_1, K)^+ \iff (H, K) \in 2^{S(Y,K)} - (C_1, K)^+) \]

and

\[(H, K) \in 2^{S(Y,K)} - (C_2, K)^+ \iff (H, K) \in 2^{S(Y,K)} - (C_1, K)^+ \cap 2^{S(Y,K)} - (C_2, K)^+]. \]

**Proposition 3.4.** Let $(Y, \tau, K)$ be a soft topological space. Then the soft set families

\[\beta_{SH}^+ = \left\{2^{S(Y,K)} - (H, K)^- : (H, K) \text{ is soft quasi H-closed set}\right\}\]

and

\[S_{SH}^- = \left\{2^{S(Y,K)} - (H, K)^+ : (H, K) \text{ is soft quasi H-closed set}\right\}\]

are soft base and soft sub base for a different soft topological spaces on $2^{S(Y,K)}$, respectively.

**Proof.** Since $(H, K) = \emptyset$ is a soft quasi H-closed set and $(H, K)^- = \emptyset$, then $2^{S(Y,K)} - (H, K)^- = 2^{S(Y,K)} \in \beta_{SH}^+$. Thus

\[2^{S(Y,K)} = \bigcup_{(B, K) \in \beta_{SH}^+} (B, K). \]

Let $(H, K) \in 2^{S(Y,K)} - (H_1, K)^+ \in \beta_{SH}^+$ and $(H, K) \in 2^{S(Y,K)} - (H_2, K)^+ \in \beta_{SH}^+$. Then $(H, K) \in \left[2^{S(Y,K)} - (H_1, K)^+\right] \cap \left[2^{S(Y,K)} - (H_2, K)^+\right] = 2^{S(Y,K)} - [(H_1, K)^+ \cup (H_2, K)^+] \in \beta_{SH}^+$. Thus $\beta_{SH}^+$ is soft base for a soft topological space. Similarly it can be show that $S_{SH}^-$ is soft sub base for a soft topological space. □
Definition 3.5. The soft topological spaces which mentioned in above proposition are called upper co-quasi H-closed soft topological space, lower co-quasi H-closed soft topological space and denoted by $\tau_{SH}^+$, $\tau_{SH}^-$ respectively.

Example 3.6. Let $Y = \{y\}$ be a universal set, $K = \{k_1, k_2\}$ be a parameter set, $\tau = \{\Phi, \overline{Y}, (G, K)\}$ be a soft topological space and $\tau' = \{\Phi, \overline{Y}, (H, K)\}$ be family of closed soft sets. Where $(G, K) = (\{k_1, \{y\}\})$ and $(H, K) = (\{k_2, \{y\}\}) = (G, K)^c$. Then $\Phi' = \emptyset$, $(G, K)' = \{(G, K), \overline{Y}\}$, $(H, K)' = \{(H, K), \overline{Y}\}$, $\overline{\overline{Y}} = \{(G, K), (H, K), \overline{Y}\} = 2^{\mathcal{S}(Y)\setminus\mathcal{K}}$. Therefore we have $2^{\mathcal{S}(Y)\setminus\mathcal{K}} - \Phi' = 2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (G, K)' = 2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (H, K)' = 2^{\mathcal{S}(Y)\setminus\mathcal{K}}$. Then $\Phi' = \emptyset$. Thus the family

$$\beta_{SH}^+ = \left\{2^{\mathcal{S}(Y)\setminus\mathcal{K}} - \Phi', 2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (G, K)' - (H, K)'ight\}$$

is a soft base for a soft topology on $2^{\mathcal{S}(Y)\setminus\mathcal{K}}$. This topology (called upper co-quasi H-closed soft topology) is

$$\tau_{SH}^+ = \left\{2^{\mathcal{S}(Y)\setminus\mathcal{K}} - \Phi', 2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (G, K)' - (H, K)'ight\}$$

Example 3.7. Let $\tau = \{\Phi, \overline{Y}, (G, K)\}$ be a soft topological space in previous example. Then

$$\Phi^+ = \{\Phi\}, \quad (G, K)^+ = \{\Phi, (G, K)\}, \quad (H, K)^+ = \{\Phi, (H, K)\} \cup \{\Phi, (G, K), \overline{Y}\} = S(Y, K).$$

Therefore we have $2^{\mathcal{S}(Y)\setminus\mathcal{K}} - \Phi^+ = 2^{\mathcal{S}(Y)\setminus\mathcal{K}} = \{(G, K), (H, K), \overline{Y}\}$, $2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (G, K)^+ = \{(G, K), \overline{Y}\}$, $2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (H, K)^+ = \{(G, K), (H, K), \overline{Y}\}$. Then

$$\tau_{SH}^- = \left\{2^{\mathcal{S}(Y)\setminus\mathcal{K}} - \Phi', 2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (G, K)' - (H, K)'ight\}$$

is a soft base for a soft topology on $2^{\mathcal{S}(Y)\setminus\mathcal{K}}$. Hence $\beta_{SH}^- = \left\{2^{\mathcal{S}(Y)\setminus\mathcal{K}} - \Phi', 2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (G, K)' - (H, K)'ight\}$ is soft base for a soft topology on $2^{\mathcal{S}(Y)\setminus\mathcal{K}}$. This topology (called lower co-quasi H-closed soft topology) is

$$\tau_{SH}^- = \left\{2^{\mathcal{S}(Y)\setminus\mathcal{K}} - \Phi', 2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (G, K)' - (H, K)'ight\}$$

Proposition 3.8. Let $(Y, \tau, K)$ be a soft topological space. Then the following statements are true for the soft topological spaces $\tau_{SH}^+$, $\tau_{SH}^-$, $\tau_{SV}^+$, $\tau_{SV}^-$ on $2^{\mathcal{S}(Y)\setminus\mathcal{K}}$.

1. If $(Y, \tau, K)$ HC soft topological space then $\tau_{SH}^+ \leq \tau_{SV}^+$ and $\tau_{SH}^- \leq \tau_{SV}^-$. 
2. If $(Y, \tau, K)$ HC and C-compact soft topological space then $\tau_{SH}^+ = \tau_{SV}^+$ and $\tau_{SH}^- = \tau_{SV}^-$. 

Proof. (1) Let $2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (H, K)^- \in \beta_{SH}^+$. Since $(H, K)$ quasi H-closed soft set and $(Y, \tau, K)$ HC soft topological space then $(H, K)$ is a soft closed set. Thus $\overline{Y} - (H, K)$ is soft open set. Then $2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (H, K)^- = (\overline{Y} - (H, K))^+ \in \tau_{SV}^+$. Since $2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (H, K)^- = (\overline{Y} - (H, K))^+$ then $2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (H, K)^- \in \tau_{SV}^+$ and thus $\tau_{SH}^+ \leq \tau_{SV}^+$. Similarly by use $2^{\mathcal{S}(Y)\setminus\mathcal{K}} - (H, K)^+ = (\overline{Y} - (H, K))^-$ it can be show that $\tau_{SH}^- \leq \tau_{SV}^-$. 

(2) In a HC and C-compact soft topological space, the quasi H-closed soft sets are soft closed and the soft closed sets are soft quasi H-closed. Therefore we have $\tau_{SH}^+ = \tau_{SV}^+$ and $\tau_{SH}^- = \tau_{SV}^-$. 

□
Remark 3.9. In a soft topological space, the compact soft sets are soft quasi H-closed. Therefore we have
\( \tau^+_SC \leq \tau^+_SH \) and \( \tau^-SC \leq \tau^-SH \).

Corollary 3.10. Let \((Y, \tau, K)\) be HC soft topological space then \( \tau^+_SC \leq \tau^+_SH \leq \tau^+_SV \) and \( \tau^-SC \leq \tau^-SH \leq \tau^-SV \).

Definition 3.11. Let \((X, \sigma, E)\), \((Y, \tau, K)\) be two soft topological spaces and \( E^E \) be soft point in \( X \). Then the soft multifunction \( F : (X, \sigma, E) \rightarrow (Y, \tau, K) \) is called:

1. soft upper H continuous at \( E^E \) if for each set \((V, K)\) with \( \overline{\overline{V}} - (V, K) \) is soft quasi H-closed set such that \( F(E^E) \cap (V, K) = \Phi \), there exists \((P, E)\) a soft open neighbourhood of \( E^E \) such that \( F(E^E) \cap (V, K) = \Phi \) for every \( E^E \in \mathcal{E}(P, E) \).
2. soft lower H continuous at \( E^E \) if for each set \((V, K)\) with \( \overline{\overline{V}} - (V, K) \) is soft quasi H-closed set such that \( F(E^E) \cap (V, K) = \Phi \), there exists \((P, E)\) a soft open neighbourhood of \( E^E \) such that \( F(E^E) \cap (V, K) = \Phi \) for every \( E^E \in \mathcal{E}(P, E) \).

Proposition 3.12. Let \((X, \sigma, E)\), \((Y, \tau, K)\) be two soft topological spaces and \( F : (X, \sigma, E) \rightarrow (Y, \tau, K) \) be a soft multifunction. Then:

1. \( F \) is soft upper H continuous at \( E^E \) if and only if the mapping \( f : (X, \sigma, E) \rightarrow 2^{S(Y,K)} \) is continuous at \( E^E \). Where \( F(E^E) = f(E^E) \) for all \( E^E \in X \).
2. \( F \) is soft lower H continuous at \( E^E \) if and only if the mapping \( f : (X, \sigma, E) \rightarrow 2^{S(Y,K)} \) is continuous at \( E^E \). Where \( F(E^E) = f(E^E) \) for all \( E^E \in X \).

Proof. (1) \( \Rightarrow \) Let \( f(E^E) \in 2^{S(Y,K)} - (H, K)^- \in \beta^+_SH \). Then \( f(E^E) \in 2^{S(Y,K)} - (H, K)^- = (\overline{\overline{Y}} - (H, K))^+ \) and \( f(E^E) \subset \overline{\overline{Y}} - (H, K) \). Then \( f(E^E) \cap (H, K) = \Phi \). Since \( F \) is soft upper H continuous at \( E^E \), then there exists \((P, E)\) a soft open neighborhood of \( E^E \) such that \( F(E^E) \cap (V, K) = \Phi \) for every \( E^E \in \mathcal{E}(P, E) \). Thus \( F(E^E) = f(E^E) \notin (H, K)^- \) and \( F(E^E) \in 2^{S(Y,K)} - (H, K)^- \). Thus we have \( f : (X, \sigma, E) \rightarrow 2^{S(Y,K)} \) is continuous at \( E^E \).

(\( \Leftarrow \)) Let \((H, K)\) be a soft quasi H-closed set such that \( F(E^E) \cap (H, K) = \Phi \). Then \( f(E^E) \notin (H, K)^- \) and \( f(E^E) \in 2^{S(Y,K)} - (H, K)^- \in \beta^+_SH \). Since \( f : (X, \sigma, E) \rightarrow 2^{S(Y,K)}, \tau^+_SH \) is continuous at \( E^E \), then there exists \((P, E)\) a soft open neighborhood of \( E^E \) such that \( f(P, E) \subset 2^{S(Y,K)} - (H, K)^- \). Thus \( f(E^E) \in 2^{S(Y,K)} - (H, K)^- \) for every \( E^E \in \mathcal{E}(P, E) \). Therefore \( F(E^E) \cap (H, K) = \Phi \). Hence \( F \) is soft upper H continuous at \( E^E \).

(2) Similarly it can be proof.

\( \square \)

Proposition 3.13. For a soft multifunction \( F : (X, \sigma, E) \rightarrow (Y, \tau, K) \) the following statements are equal:

1. \( F \) is soft upper H continuous,
2. \( F^+ (V, K) \) is soft open set for every set \((V, K)\) such that \( \overline{Y} - (V, K) \) is quasi H-closed.
3. \( F^- (H, K) \) is soft closed set for every soft quasi H-closed set \((H, K)\).

Proof. (1) \( \Rightarrow \) (2): Let \((V, K)\) be a soft set such that \( \overline{Y} - (V, K) \) is quasi H-closed and \( E^E \in F^+(V, K) \). Then \( F(E^E) \subset \overline{Y} - (V, K) \) and thus \( F(E^E) \cap (\overline{Y} - (V, K)) \neq \Phi \). Since \( F \) is soft upper H continuous at \( E^E \) then there exists \((P, E)\) a soft open neighborhood of \( E^E \) such that \( F(E^E) \subset (V, K) \) for every \( E^E \in \mathcal{E}(P, E) \). Then \( E^E \in \mathcal{E}(P, E) \) is soft open set. Thus \( F^+ (V, K) \) is soft open set.

(2) \( \Rightarrow \) (3): Let \((H, K)\) be a soft quasi H-closed set. Then by hypothesis, \( F^+ (\overline{Y} - (H, K)) \) is soft open. Since \( F^+ (\overline{Y} - (H, K)) = \overline{X} - F^- (H, K) \), then \( \overline{X} - F^- (H, K) \) is soft open and thus \( F^- (H, K) \) is soft closed set.

(3) \( \Rightarrow \) (1): Let \((C, K)\) be a soft quasi H-closed set such that \( F(E^E) \cap (C, K) \neq \Phi \). Then \( F(E^E) \subset \overline{\overline{Y}} - (C, K) \). By hypothesis \( F^- (C, K) \) is soft closed set. Also, since \( \overline{X} - F^- (C, K) = F^+ (\overline{Y} - (C, K)) \) then \( F^+ (\overline{Y} - (C, K)) \) is
soft open set and \( E^c_e \in F^+ (\bar{Y} - (C, K)) \). If we take \((P, E) = F^+ (\bar{Y} - (C, K))\) soft open neighbourhood of \( E^c_e \), then \( F( E^c_e ) \cap (C, K) \) for every \( E^c_e \in F^+ (\bar{Y} - (C, K)) \) and \( F( E^c_e ) \cap (C, K) = \Phi \). Thus \( F( E^c_e ) \cap (C, K) = \Phi \). Therefore \( F \) is soft upper \( H \) continuous at \( E^c_e \).

**Proposition 3.14.** For a soft multifunction \( F : (X, \sigma, E) \to (Y, \tau, K) \) the following statements are equal:

1. \( F \) is soft lower \( H \) continuous.
2. \( F^- (V, K) \) is soft open set for every soft set \((V, K)\) such that \( \bar{Y} - (V, K) \) is soft quasi \( H \)-closed set.
3. \( F^+ (H, K) \) is soft closed set for every soft quasi \( H \)-closed set \((H, K)\).

**Proof.** (1) \( \Rightarrow \) (2): Let \((V, K)\) be a soft set such that \( \bar{Y} - (V, K) \) is quasi \( H \)-closed and \( E^c_e \in F^-(V, K) \). Then \( F( E^c_e ) \cap (V, K) = \Phi \). Since \( F \) is soft lower \( H \) continuous at \( E^c_e \) then there exists \((P, E)\) a soft open neighbourhood of \( E^c_e \) such that \( F( E^c_e ) \cap (V, K) = \Phi \) for every \( E^c_e \in F^+(V, K) \). Hence \( F^- (V, K) \subseteq \text{int} (F^- (V, K)) \). Thus \( F^- (V, K) \) is soft open set.

(2) \( \Rightarrow \) (3): Let \((H, K)\) be a soft quasi \( H \)-closed set. Then by hypothesis, \( F^- (\bar{Y} - (H, K)) = \bar{X} - F^+ (H, K) \) is soft open and thus \( F^- (H, K) \) is soft closed.

(3) \( \Rightarrow \) (1): Let \((V, K)\) be a soft set such that \( \bar{Y} - (V, K) \) is soft quasi \( H \)-closed set and \( F( E^c_e ) \cap (V, K) = \Phi \). By hypothesis \( F^+ (\bar{Y} - (V, K)) = \bar{X} - F^- (V, K) \) is soft closed then \( F^- (V, K) \) is soft open set and \( E^c_e \in F^-(V, K) \). If we take \((P, E) = F^+(\bar{Y} - (V, K))\) is soft open neighbourhood of \( E^c_e \), then \( F( E^c_e ) \cap (V, K) = \Phi \) for every \( E^c_e \in F^-(V, K) \). Therefore \( F \) is soft lower \( H \) continuous at \( E^c_e \).

3.2. *D soft topological spaces defined on* \( 2^{\langle \gamma \rangle} \)

**Definition 3.15.** Let \((G, K)\) be a soft set in a soft topological space \((Y, \tau, K)\).

1. If \((G, K)\) is equal to union of countable closed soft sets then it called \( F_\sigma \) soft set namely \((G, K)\) is called \( F_\sigma \) soft set if \((G, K) = \bigcup_{i=1}^{\infty} (F_i, K) \) and \((F_i, K)\) is soft closed set.

2. If \((G, K)\) is equal to intersection of countable open soft sets then it called \( G_\delta \) soft set namely \((G, K)\) is called \( G_\delta \) soft set if \((G, K) = \bigcap_{i=1}^{\infty} (F_i, K) \) and \((F_i, K)\) is soft open set.

**Proposition 3.16.** For \( F_\sigma \) and \( G_\delta \) soft sets the following statements are hold.

1. Every soft closed set is \( F_\sigma \) soft set.
2. Every soft open set is \( G_\delta \) soft set.
3. The soft complement of any \( F_\sigma \) soft set is \( G_\delta \) soft set.
4. The soft complement of any \( G_\delta \) soft set is \( F_\sigma \) soft set.

**Proof.** (1) and (2) are obvious from the definition of \( F_\sigma \) soft set and \( G_\delta \) soft set.

(3) Let \((G, K)\) be a \( F_\sigma \) soft set. Then \((G, K) = \bigcup_{i=1}^{\infty} (F_i, K) \), where \((F_i, K)\) are soft closed set. \( \bar{Y} - (G, K) = \bar{Y} - \bigcup_{i=1}^{\infty} (F_i, K) = \bigcap_{i=1}^{\infty} (\bar{Y} - (F_i, K)) \) and \( \bar{Y} - (F_i, K) \) are soft open set. Thus \( \bar{Y} - (G, K) \) is \( G_\delta \) soft set.

(4) Let \((G, K)\) be a \( G_\delta \) soft set. Then \((G, K) = \bigcap_{i=1}^{\infty} (F_i, K) \), where \((F_i, K)\) are soft open set. \( \bar{Y} - (G, K) = \bar{Y} - \bigcap_{i=1}^{\infty} (F_i, K) = \bigcup_{i=1}^{\infty} (\bar{Y} - (F_i, K)) \) and \( \bar{Y} - (F_i, K) \) are soft closed set. Thus \( \bar{Y} - (G, K) \) is \( F_\sigma \) soft set.
Proposition 3.17. Let \((Y, \tau, K)\) be a soft topological space. Then the soft set families

\[ \beta_{SD}^+ = \left\{ 2^{S(Y,K)} - (V, K)^- : (V, K) \text{ is } G_\delta \text{ soft closed set} \right\} \]

and

\[ S_{SD}^- = \left\{ 2^{S(Y,K)} - (V, K)^+ : (V, K) \text{ is } G_\delta \text{ soft closed set} \right\} \]

are soft base and soft subbase for a different soft topological spaces on \(2^{S(Y,K)}\), respectively.

Proof. Since \((V, K) = \emptyset\) is \(G_\delta\) soft closed set and \((V, K)^- = \emptyset\), then \(2^{S(Y,K)} - (V, K)^- = 2^{S(Y,K)} \in \beta_{SD}^+\). Thus

\[ 2^{S(Y,K)} = \bigcup_{(B, K) \in \beta_{SD}^+} (B, K). \]

Let \((H, K) \in 2^{S(Y,K)} - (V_1, K)^-\) and \((H, K) \in 2^{S(Y,K)} - (V_2, K)^-\). Then

\[ (H, K) \in \left[ 2^{S(Y,K)} - (V_1, K)^- \right] \cap \left[ 2^{S(Y,K)} - (V_2, K)^- \right] = 2^{S(Y,K)} - \left( (V_1, K)^- \cup (V_2, K)^- \right). \]

Thus \(\beta_{SD}^+\) is soft base for a soft topological space. Also,

\[ S_{SD}^- \subseteq \beta_{SD}^+ = \left\{ \bigcap_{i=1}^n (2^{S(Y,K)} - (V_i, K)^+) : 2^{S(Y,K)} - (V_i, K)^+ \in S_{SD}^- \right\} \]

\[ = \left\{ \bigcap_{i=1}^n (2^{S(Y,K)} - (V_i, K)^+) : (V_i, K) \text{ } G_\delta \text{ soft closed set} \right\}. \]

For \((V, K) = \emptyset\) since \((V, K)^+ = \emptyset\) then \(2^{S(Y,K)} - (V, K)^+ = 2^{S(Y,K)} \in \beta_{SD}^-\). Let

\[ (H, K) \in \left[ \bigcap_{i=1}^n (2^{S(Y,K)} - (V_i, K)^+) \right] \cap \left[ \bigcap_{i=1}^m (2^{S(Y,K)} - (V_i, K)^+) \right]. \]

Then \((H, K) \in \left[ \bigcap_{i=1}^n (2^{S(Y,K)} - (V_i, K)^+) \right]\) and \((H, K) \in \left[ \bigcap_{i=1}^m (2^{S(Y,K)} - (V_i, K)^+) \right]\). Thus \((H, K) \in \left( 2^{S(Y,K)} - (V_i, K)^+ \right)\) for every \(i \in I\) and \((H, K) \in \left( 2^{S(Y,K)} - (V_j, K)^+ \right)\) for every \(j \in J\). Then \((H, K) \notin (V_i, K)^+\) and \((H, K) \notin (V_j, K)^+\) for every \(i \in I\) and \(j \in J\). There exists \(i_0 \in I\) and \(j_0 \in J\) such that \((H, K) \notin (V_{i_0}, K)^+\) and \((H, K) \notin (V_{j_0}, K)^+\). Then \((H, K) \notin (V_{i_0}, K)^+ \cup (V_{j_0}, K)^+\). Therefore

\[ (H, K) \in 2^{S(Y,K)} - \left( (V_{i_0}, K)^+ \cup (V_{j_0}, K)^+ \right) = 2^{S(Y,K)} - (V_{i_0}, K)^+ \cap 2^{S(Y,K)} - (V_{j_0}, K)^+. \]

Then \(S_{SD}^-\) is soft subbase for a soft topological space. \(\square\)

Definition 3.18. The topological spaces which mentioned in above proposition are called upper D soft topological space, lower D soft topological spaces and denoted by \(\tau_{SD}^+, \tau_{SD}^-\) respectively.

Example 3.19. Let \(Y = \{y_1, y_2\}\) be a universal set, \(K = \{k\}\) be a parameter set, \(\tau = \{\emptyset, \bar{Y}, (G, K)\}\) be a soft topological space and \(\tau' = \{\emptyset, \bar{Y}, (H, K)\}\) be family of closed soft sets. Where \((G, K) = \{(k, \{y_1\})\}\) and \((H, K) = \{(k, \{y_2\})\}\) be \((G, K)\). Then \(\Phi^- = \emptyset, (G, K)^- = \{(G, K), \bar{Y}\}, (H, K)^- = \{(H, K), \bar{Y}\}, (G, K), (H, K), \bar{Y}\). Therefore we have \(2^{S(Y,K)} - \Phi^- = \{(G, K), (H, K), \bar{Y}\}, 2^{S(Y,K)} - (G, K)^- = \{(H, K), \bar{Y}\}, 2^{S(Y,K)} - (H, K)^- = \{(G, K), (H, K), \bar{Y}\}.\) Thus the family \(\beta_{SD}^+ = \{(G, K), (H, K), \bar{Y}\}, \{(H, K), (G, K), \emptyset\}\) is a soft base for a soft topology on \(2^{S(Y,K)}\). This topology (called upper D soft topology) is

\[ \tau_{SD}^+ = \{(G, K), (H, K), \bar{Y}\}, \{(H, K), (G, K), \emptyset\}. \]
Example 3.20. Let $\tau = \{\emptyset, \overline{Y}, (G, K)\}$ be a soft topological space in previous example. Then $\Phi^+ = \{\emptyset\}$, $(G, K)^+ = \{\emptyset, (G, K), (H, K), \overline{Y}\}$, $(H, K)^+ = \{\emptyset, (H, K), \overline{Y}\}$. Therefore we have $2^{(Y, K)} - \Phi^+ = \{(G, K), (H, K), \overline{Y}\}$, $2^{(Y, K)} - (G, K)^+ = \{(H, K), \overline{Y}\}$, $2^{(Y, K)} - (H, K)^+ = \{(G, K), \overline{Y}\}$, $2^{(Y, K)} - \overline{Y}^+ = \emptyset$. Thus the family
\[
S^\tau_{SD} = \left\{\{(G, K), (H, K), \overline{Y}\}, \{(H, K), \overline{Y}\}, \{(G, K), \overline{Y}\}, \emptyset\right\}
\]
is a soft subbase for a soft topology on $2^{(Y, K)}$. Hence
\[
\beta^\tau_{SD} = \left\{\{(G, K), (H, K), \overline{Y}\}, \{(H, K), \overline{Y}\}, \{(G, K), \overline{Y}\}, \{\overline{Y}\}, \emptyset\right\}
\]
is soft subbase for a soft topology on $2^{(Y, K)}$. This topology (called lower D soft topology) is
\[
\tau^\tau_{SD} = \left\{\{(G, K), (H, K), \overline{Y}\}, \{(H, K), \overline{Y}\}, \{(G, K), \overline{Y}\}, \{\overline{Y}\}, \emptyset\right\}.
\]

Proposition 3.21. Let $(Y, \tau, K)$ be a soft topological space. Then $\tau^+_{SD} \leq \tau^+_{SV}$ and $\tau^-_{SD} \leq \tau^-_{SV}$.

Proof. (1) Let $2^{(Y, K)} - (V, K)^- \in \beta^+_{SD}$, where $(V, K)$ is soft closed $G_\delta$ set. Then $(V, K)$ is soft closed set and $\overline{\tau} - (V, K)$ is soft open set. Since $2^{(Y, K)} - (V, K)^- = (\overline{\tau} - (V, K))^+$ and $(\overline{\tau} - (V, K))^+ \in \tau^+_{SV}$. Then $2^{(Y, K)} - (V, K)^- \in \tau^+_{SV}$. Thus $\tau^+_{SD} \leq \tau^+_{SV}$.

Similarly it can be show that $\tau^-_{SD} \leq \tau^-_{SV}$ by use $2^{(Y, K)} - (V, K)^+ = (\overline{\tau} - (V, K))^-$.

Definition 3.22. Let $(X, \sigma, E), (Y, \tau, K)$ be two soft topological spaces and $E^\tau_\emptyset$ be soft point in $X$. Then the soft multifunction $F : (X, \sigma, E) \rightarrow (Y, \tau, K)$ is called:

1. soft upper D continuous at $E^\tau_\emptyset$ if for each $\overline{\tau} - (V, K)$ soft open $F_\emptyset$ set with $F(E^\tau_\emptyset) \cap (V, K) = \emptyset$, there exists $(P, E)$ a soft open neighbourhood of $E^\tau_\emptyset$ such that $F(E^\tau_\emptyset) \cap (V, K) = \emptyset$ for every $E^\tau_\emptyset \in (P, E)$.
2. soft lower D continuous at $E^\tau_\emptyset$ if for each $\overline{\tau} - (V, K)$ soft open $F_\emptyset$ set with $F(E^\tau_\emptyset) \cap (V, K) = \emptyset$, there exists $(P, E)$ a soft open neighbourhood of $E^\tau_\emptyset$ such that $F(E^\tau_\emptyset) \cap (V, K) = \emptyset$ for every $E^\tau_\emptyset \in (P, E)$.

Proposition 3.23. Let $F : (X, \sigma, E) \rightarrow (Y, \tau, K)$ be a soft multifunction.

1. $F$ is soft upper D continuous at $E^\tau_\emptyset$ if and only if the soft mapping $f : (X, \sigma, E) \rightarrow (2^{(Y, K)}, \tau^+_{SD}, K)$ is continuous at $E^\tau_\emptyset$.
2. $F$ is soft lower D continuous at $E^\tau_\emptyset$ if and only if the soft mapping $f : (X, \sigma, E) \rightarrow (2^{(Y, K)}, \tau^-_{SD}, K)$ is continuous at $E^\tau_\emptyset$.

Proof. (1) (\implies) Let $(V, K)$ be soft closed $G_\delta$ set and $f(E^\tau_\emptyset) \in 2^{(Y, K)} - (V, K)^-$. Then $f(E^\tau_\emptyset) \in (\overline{\tau} - (V, K))^+$ and $f(E^\tau_\emptyset) \cap (V, K) = \emptyset$. Since $\overline{\tau} - (V, K)$ soft open $F_\emptyset$ set and $F$ is soft upper D continuous then there exists $(P, E)$ a soft open neighborhood of $E^\tau_\emptyset$ such that $F(E^\tau_\emptyset) \cap (V, K) = \emptyset$ for every $E^\tau_\emptyset \in (P, E)$. Thus $F(E^\tau_\emptyset) \notin (V, K)^-$ for every $E^\tau_\emptyset \in (P, E)$. Thus $F(E^\tau_\emptyset) \in 2^{(Y, K)} - (V, K)^-$ for every $E^\tau_\emptyset \in (P, E)$. Thus $f : (X, \sigma, E) \rightarrow (2^{(Y, K)}, \tau^+_{SD}, K)$ is continuous at $E^\tau_\emptyset$.

\((\iff)\) Let $\overline{\tau} - (V, K)$ be soft open $F_\emptyset$ set with $F(E^\tau_\emptyset) \cap (V, K) = \emptyset$. Then $F(E^\tau_\emptyset) \subseteq \overline{\tau} - (V, K)$ and thus $F(E^\tau_\emptyset) \in (\overline{\tau} - (V, K))^+$. Then $F(E^\tau_\emptyset) \in 2^{(Y, K)} - (V, K)^-$ and $f(E^\tau_\emptyset) \in 2^{(Y, K)} - (V, K)^-$. Since $f$ is continuous at $E^\tau_\emptyset$ there exists $(P, E)$ a soft open neighborhood of $E^\tau_\emptyset$ such that $f(E^\tau_\emptyset) \subseteq 2^{(Y, K)} - (V, K)^-$ for every $E^\tau_\emptyset \in (P, E)$. Thus $f(E^\tau_\emptyset) \subseteq (\overline{\tau} - (V, K))^+$ and $F(E^\tau_\emptyset) \subseteq (\overline{\tau} - (V, K))^+$. Thus $F(E^\tau_\emptyset) \subseteq \overline{\tau} - (V, K)$ and $F(E^\tau_\emptyset) \cap (V, K) = \emptyset$. Thus $F$ is soft upper D continuous at $E^\tau_\emptyset$.

(2) It can be show that similarly to (1).

\(\square\)
4. Conclusion

Recently, many researches had been done various works in the soft set theory and in practices. In this paper, first, we introduce upper and lower co-quasi H-closed soft topological space which one of the hyperspaces of soft sets. Then we study some characterization of these spaces. Second, we introduce upper and lower D soft topological space which one of the hyperspaces of soft sets. Then we study some characterization of these spaces. Then we give the relationship between of hyperspaces of soft sets. We expect that results in this paper will be basis for further applications of soft multifunctions in soft sets theory.

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