Research Article

Analytical Solutions to the Caudrey-Dodd-Gibbon-Sawada-Kotera Equation via Symbol Calculation Approach

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Received 24 March 2020; Accepted 22 April 2020; Published 5 May 2020

Guest Editor: Chuanjun Chen

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In this paper, we derive analytical solutions of the Caudrey-Dodd-Gibbon-Sawada-Kotera (CDGSK) equation via symbol calculation approach. Applying the exp(−φ(z))-expansion method, we achieve the trigonometric, exponential, hyperbolic, and rational function solutions for the CDGSK equation. By choosing the appropriate parameters, we give some computer simulation to the analytical solutions of the CDGSK equation.

1. Introduction

Nonlinear fractional and integer order differential equations are widely utilized in fluid dynamics, solid state physics, plasma physics, biology, nonlinear optics, chemistry, etc. It has aroused the strong interest of researchers to these differential equations [1–18]. The study to exact solutions of various NLDEs is extremely important in modern mathematics with ramifications to some areas of physics, mathematics, and other sciences. There are many systematic methods to seek exact solutions of nonlinear differential equations, for example, the Hirota bilinear method [19], Tanh method [20], Lie symmetry [21], modified Kudryashov method [22], Exp-function method [23], sine-Gordon expansion method [24], complex method [25–29], and exp(−φ(z))-expansion method [30–32].

Sawada and Kotera [33] proposed one of basic models in soliton theory as follows:

\[ u_{xxxx} + 15uu_{xx} + 15u_{xxx} + 45u_t + u_t = 0, \quad (1) \]

which is also introduced by Caudrey et al. independently [34]; so in literature, Equation (1) is called the Caudrey-Dodd-Gibbon-Sawada-Kotera equation. Many years have passed by, lots of research results for the CDGSK equation have been developed. As to this equation, the finite dimensional reduction was investigated by Enolski et al. [35], and N-soliton solutions were discovered by Parker [36] via the dressing method. Darboux transformation [37] and Bäcklund transformation in bilinear forms [38] were applied to study the CDGSK equation. Riemann theta function solutions of the CDGSK equation were also established [39]. In this paper, we employ the exp(−φ(z))-expansion method [30–32] to obtain the exact solutions of the CDGSK equation.

2. The exp(−φ(z))-Expansion Method

In this section, we give the main steps of the mentioned method.

Step 1. Inserting traveling wave transform

\[ u(x, t) = u(z), \quad z = \lambda x + \omega t \quad (2) \]

into a nonlinear PDE

\[ P(u, u_t, u_x, \ldots, u_{xx}, u_{yyyy}, \ldots) = 0 \quad (3) \]

yields
\[ F\left(u, u', u'', \ldots\right) = 0, \quad (4) \]

in which \( F \) is the polynomial of \( u \) along with its derivatives.

Step 2. Assume that Equation (4) has analytical solutions as follows:

\[ u(z) = \sum_{r=0}^{m} B_r (\exp (-\varphi (z)))^r, \quad (5) \]

where \( B_r \) (\( 0 \leq r \leq m \)) are constants which will be ascertained subsequently, such that \( B_r \neq 0 \) and \( \varphi = \varphi(z) \) satisfy

\[ \varphi'(z) = \gamma + \exp (-\varphi(z)) + \delta \exp (\varphi(z)). \quad (6) \]

The solutions of Equation (6) are given in the following: When \( \gamma^2 - 4\delta > 0, \delta \neq 0, \)

\[ \varphi(z) = \ln \left( -\sqrt{(\gamma^2 - 4\delta)} \tanh \left( \frac{\sqrt{\gamma^2 - 4\delta}}{2\delta} (z + c) - \gamma \right) \right), \quad (7) \]

\[ \varphi(z) = \ln \left( -\sqrt{(\gamma^2 - 4\delta)} \coth \left( \frac{\sqrt{\gamma^2 - 4\delta}}{2\delta} (z + c) - \gamma \right) \right). \quad (8) \]

When \( \gamma^2 - 4\delta < 0, \delta \neq 0, \)

\[ \varphi(z) = \ln \left( \frac{\sqrt{(4\delta - \gamma^2)}}{2\delta} \tan \left( \frac{\sqrt{4\delta - \gamma^2}}{2\delta} (z + c) - \gamma \right) \right), \quad (9) \]

\[ \varphi(z) = \ln \left( \frac{\sqrt{(4\delta - \gamma^2)}}{2\delta} \cot \left( \frac{\sqrt{4\delta - \gamma^2}}{2\delta} (z + c) - \gamma \right) \right). \quad (10) \]

When \( \gamma^2 - 4\delta > 0, \gamma \neq 0, \delta = 0, \)

\[ \varphi(z) = -\ln \left( \frac{\gamma}{\exp (\gamma(z + c)) - 1} \right). \quad (10) \]

When \( \gamma^2 - 4\delta = 0, \gamma \neq 0, \delta \neq 0, \)

\[ \varphi(z) = \ln \left( -\frac{2(\gamma(z + c) + 2)}{\gamma^2 (z + c)} \right). \quad (11) \]

When \( \gamma^2 - 4\delta = 0, \gamma = 0, \delta = 0, \)

\[ \varphi(z) = \ln (z + c). \quad (12) \]

In Equations (7)-(12), \( B_r \neq 0, \gamma, \delta, c \) are constants. Taking the homogeneous balance between the nonlinear terms and highest order derivatives of Equation (4) yields the positive integer \( m \).

Step 3. Insert Equation (5) into Equation (4), and collect the function \( \exp (-\varphi(z)) \) to yield the polynomial to \( \exp (-\varphi(z)) \). Let all coefficients with the same power of \( \exp (-\varphi(z)) \) be zero to obtain a system of algebraic equations. In solving these equations, we achieve the values of \( B_r \neq 0, \gamma, \delta \) and substitute them into Equation (5) as well as Equations (7)-(12) to accomplish the determination for analytical solutions of the original PDE.

3. Exact Solutions of the CDGSK Equation

Inserting

\[ u(x, t) = u(z), \quad z = \lambda x + \omega t \quad (13) \]

into Equation (1) and then integrating it, we obtain

\[ \lambda^5 u''' + 15\lambda^3 u'' + 15\lambda u' + \omega u + \delta = 0, \quad (14) \]

where \( \delta \) is the integration constant.

Taking the homogeneous balance between \( u^3 \) and \( u''' \) of Equation (14) yields

\[ u(z) = B_0 + B_1 \exp (-\varphi(z)) + B_2 (\exp (-\varphi(z)))^2, \quad (15) \]

where \( B_2 \neq 0 \) and \( B_1 \) and \( B_0 \) are constants.

Substituting \( u''', u'', u^3, u \) into Equation (14) and letting the coefficients about \( \exp (-\varphi(z)) \) be zero, we obtain

\[ e^{\varphi(z)} : \lambda^5 B_1 \gamma^3 \theta + 14 \lambda^3 B_1 \gamma^2 \theta^2 + 8 \lambda^2 B_1 \gamma \theta^3 \]

\[ + 16 \lambda B_1 \gamma \theta^4 + 15 \lambda \theta \gamma^3 B_0 + 7 \gamma + 30 \lambda^3 B_0 B_2 \theta^2 \]

\[ + 15 \lambda^2 \theta^2 + \omega \delta = 0, \]

\[ e^{\varphi(z)} : B_1 \omega + B_1 \gamma \lambda^5 + 30 B_2 \gamma \lambda^5 \theta + 22 B_1 \gamma^2 \lambda^5 \theta \]

\[ + 120 B_2 \gamma^2 \lambda^5 \theta^2 + 30 B_1 \theta^2 B_1 \lambda^3 \theta \]

\[ + 9 B_0 B_2 \gamma \lambda^3 \theta + 15 B_1 \gamma^2 \lambda^3 \theta + 30 B_1 B_2 \lambda^3 \theta^2 \]

\[ + 16 B_1 ^2 \lambda^3 \theta + 45 B_0 ^2 B_1 \lambda = 0, \]

\[ e^{\varphi(z)} : 16 B_1 ^2 \gamma \lambda^5 + 15 \lambda \theta^2 B_1 \gamma^3 + 232 B_2 \gamma^2 \lambda^5 \theta \]

\[ + 60 B_1 \gamma \lambda^3 \theta + 136 B_2 \lambda^3 \theta^2 + 15 B_1 ^2 \gamma \lambda^3 \theta \]

\[ + 60 B_0 B_2 \gamma^2 \lambda^3 + 105 B_1 \gamma \lambda^3 \theta + 30 B_2 ^2 \lambda^2 \theta^2 \]

\[ + 45 B_0 B_1 \gamma \lambda^3 + 120 B_2 \gamma \lambda^3 \theta + 30 \gamma B_2 ^2 \lambda \theta \]

\[ + 45 B_0 ^2 B_1 \lambda ^2 + 45 B_1 ^2 \lambda + 2 B_1 \lambda + \delta = 0, \]

\[ e^{\varphi(z)} : 130 B_1 \gamma \lambda^5 + 50 B_1 \gamma^2 \lambda^5 + 440 B_2 \gamma \lambda^5 \theta \]

\[ + 75 B_1 \gamma^2 \lambda^3 + 40 B_1 \gamma \lambda^3 \theta + 15 B_1 ^2 \lambda \]

\[ + 150 B_0 B_1 \gamma \lambda^3 + 45 B_2 ^2 \gamma \lambda^3 + 150 B_1 B_2 \lambda \theta \]

\[ + 30 B_0 \lambda + 90 B_0 B_1 \lambda + 90 B_2 \lambda \gamma \lambda^3 \theta = 0, \]
\( e^{\lambda(z)} \) : \( 330 B_2 y^2 \lambda^5 + 60 B_1 y \lambda^5 + 60 B_2^2 y^2 \lambda^3 + 240 B_2 \lambda^9 + 195 B_1 B_2 y \lambda^3 + 30 B_1^2 \lambda^3 + 120 B_2^2 \lambda^3 \theta + 90 B_0 B_2 \lambda^3 + 45 B_0 B_2^2 \lambda + 45 B_1^2 \lambda, \lambda = 0, \)

\( e^{\delta(z)} \) : \( 336 B_2 y \lambda^5 + 24 B_1 y \lambda^5 + 150 B_2^2 y \lambda^3 + 120 B_1 B_2 \lambda^3 + 45 B_1 B_2^2 \lambda = 0, \)

\( e^{\phi(z)} \) : \( 120 y \lambda^5 + 90 B_2^2 \lambda^3 + 15 B_2^3 \lambda = 0. \)

(16)

We solve the above algebraic equations and then obtain two different cases:

**Case 1.**

\[ B_3 = -2 \lambda^2, \]
\[ B_1 = -2 \gamma \lambda^2, \]
\[ B_0 = \frac{-5 \lambda^3 (y^2 + 8 \theta) + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda}, \]
\[ \delta = \frac{\lambda^7 (y^2 - 4 \theta)^3}{9} - \frac{\sqrt{5 \lambda (y^2 - 4 \theta)^4 - 4 \omega} \left(2 \lambda^5 (y^2 - 4 \theta)^2 + \omega \right)}{45 \lambda}, \]

(17)

where \( \theta \) and \( \gamma \) are arbitrary constants.

Substituting Equation (17) into Equation (15) yields

\[ u(z) = \frac{-5 \lambda^3 (y^2 + 8 \theta) + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} - 2 \gamma \lambda^2 \exp(-\phi(z)) - 2 \lambda^2 (\exp(-\phi(z)))^2. \]

(18)

Applying Equations (7)–(12) into Equation (18), respectively, yields analytical solutions of the CDGSK equation as follows:

**Case 1.1.** When \( y^2 - 4 \theta > 0, \theta \neq 0, \)

\[ u_{11}(z) = \frac{-5 \lambda^3 (y^2 + 8 \theta) + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} + \frac{4 \lambda^3 \gamma \theta}{\sqrt{(y^2 - 4 \theta)^2} \tanh \left(\left(\sqrt{(y^2 - 4 \theta)^2} \right)(z + c) \right) + \gamma} - \frac{2 \lambda^2 \gamma^2}{\left(\exp\left(y(z + c)\right) - 1\right)^2}. \]

(21)

**Case 1.2.** When \( y^2 - 4 \theta < 0, \theta \neq 0, \)

\[ u_{12}(z) = \frac{-5 \lambda^3 (y^2 + 8 \theta) + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} + \frac{4 \lambda^3 \gamma \theta}{\sqrt{(y^2 - 4 \theta)^2} \coth \left(\left(\sqrt{(y^2 - 4 \theta)^2} \right)(z + c) \right) + \gamma} - \frac{2 \lambda^2 \gamma^2}{\left(\sqrt{(y^2 - 4 \theta)^2} \coth \left(\left(\sqrt{(y^2 - 4 \theta)^2} \right)(z + c) \right) + \gamma\right)^2}. \]

(19)

**Case 1.3.** When \( y^2 - 4 \theta > 0, \gamma \neq 0, \theta = 0, \)

\[ u_{13}(z) = \frac{-5 \lambda^3 (y^2 + 8 \theta) + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} - \frac{2 \lambda^2 \gamma^2}{\left(\exp\left(y(z + c)\right) - 1\right)^2}. \]

(20)

**Case 1.4.** When \( y^2 - 4 \theta = 0, \gamma \neq 0, \theta \neq 0, \)

\[ u_{14}(z) = \frac{-5 \lambda^3 \gamma^2 + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} + \frac{\lambda^2 \gamma^2 (z + c)}{\gamma(z + c)^2} + 2 \lambda^2 \gamma^2 \left(\frac{z + c}{\gamma(z + c)^2} \right)^2. \]

(22)

**Case 1.5.** When \( y^2 - 4 \theta = 0, \gamma = 0, \theta = 0, \)

\[ u_{15}(z) = \frac{-5 \lambda^3 \gamma^2 + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} + \frac{\lambda^2 \gamma^2 (z + c)}{\gamma(z + c)^2} + 2 \lambda^2 \gamma^2 \left(\frac{z + c}{\gamma(z + c)^2} \right)^2. \]

(23)
Where $\theta$ and $\gamma$ are arbitrary constants.

Substituting Equation (24) into (15) yields

$$u(z) = -\frac{5 \lambda^3 (y^2 + 8 \theta) + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} - 2 \lambda^2 \gamma \exp (-\varphi(z)) - 2 \lambda^2 ( \exp (-\varphi(z))^2).$$

Applying Equations (7)–(12) into Equation (25), respectively, yields analytical solutions of the CDGSK equation as follows:

Case 2.1. When $\gamma^2 - 4 \theta > 0$, $\theta \neq 0$,

$$u_{21}(z) = -\frac{5 \lambda^3 (y^2 + 8 \theta) + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} + \frac{4 \lambda^2 \gamma \theta}{\sqrt{(y^2 - 4 \theta) \tan \left(\left(\sqrt{y^2 - 4 \theta}/2\right)(z + c)\right) + \gamma}} - \frac{(y^2 - 4 \theta)^2 \tan \left(\left(\sqrt{y^2 - 4 \theta}/2\right)(z + c)\right) + \gamma}{8 \lambda^2 \theta^2},$$

$$u_{22}(z) = -\frac{5 \lambda^3 (y^2 + 8 \theta) + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} + \frac{4 \lambda^2 \gamma \theta}{\sqrt{(y^2 - 4 \theta) \coth \left(\left(\sqrt{y^2 - 4 \theta}/2\right)(z + c)\right) + \gamma}} - \frac{(y^2 - 4 \theta)^2 \coth \left(\left(\sqrt{y^2 - 4 \theta}/2\right)(z + c)\right) + \gamma}{8 \lambda^2 \theta^2}.$$

Case 2.2. When $\gamma^2 - 4 \theta < 0$, $\theta \neq 0$,

$$u_{23}(z) = -\frac{5 \lambda^3 (y^2 + 8 \theta) + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} - \frac{4 \lambda^2 \gamma \theta}{\sqrt{(4 \theta - y^2)^2 \tan \left(\left(\sqrt{(4 \theta - y^2)/2}\right)(z + c)\right) - \gamma} - \frac{(4 \theta - y^2)^2 \tan \left(\left(\sqrt{(4 \theta - y^2)/2}\right)(z + c)\right) - \gamma}{8 \lambda^2 \theta^2},$$

$$u_{24}(z) = -\frac{5 \lambda^3 (y^2 + 8 \theta) + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} - \frac{4 \lambda^2 \gamma \theta}{\sqrt{(4 \theta - y^2)^2 \cot \left(\left(\sqrt{(4 \theta - y^2)/2}\right)(z + c)\right) - \gamma} - \frac{(4 \theta - y^2)^2 \cot \left(\left(\sqrt{(4 \theta - y^2)/2}\right)(z + c)\right) - \gamma}{8 \lambda^2 \theta^2}. $$

Case 2.3. When $\gamma^2 - 4 \theta > 0$, $\theta \neq 0$, $\gamma \neq 0$,

$$u_{25}(z) = -\frac{5 \lambda^3 y^2 + \sqrt{5 \lambda^6 (y^2 - 4 \theta)^2 - 20 \lambda \omega}}{30 \lambda} - \frac{2 \lambda^2 y^2}{\exp (\gamma^2 (z + c)) - 1}. $$

Case 2.4. When $\gamma^2 - 4 \theta = 0$, $\gamma \neq 0$, $\theta \neq 0$,

$$u_{26}(z) = -\frac{30 \lambda^3 \theta + \sqrt{-5 \lambda \omega}}{15 \lambda} - \frac{\lambda^2 y^2 (z + c)}{y(z + c) + 2} - \frac{\lambda^2 y^2 (z + c)^2}{2(y(z + c) + 2)^2}. $$

Case 2.5. When $\gamma^2 - 4 \theta = 0$, $\theta = 0$, $\gamma \neq 0$,

$$u_{27}(z) = -\frac{\sqrt{-5 \lambda \omega}}{15 \lambda} - \frac{2 \lambda^2 y^2}{(z + c)^2}. $$

Figures 1–5 show the properties of the solutions.

4. Conclusions

In this paper, abundant analytical solutions of the CDGSK equation are obtained via symbol calculation approach. The properties of the solutions are shown by some graphs. It shows that the exp(-\varphi(z))-expansion method is an effective method to seek analytical solutions for nonlinear differential equations.
Figure 1: The 3D and 2D graphics of $u_{11}(z)$ by choosing the values $\gamma = 5$, $\beta = 6$, $\lambda = 1$, $\omega = 1/4$, $c = 1$, and $t = 0$ for the 2D graphic.

Figure 2: The 3D and 2D graphics of $u_{12}(z)$ by choosing the values $\gamma = 5$, $\beta = 6$, $\lambda = 1$, $\omega = 1/4$, $c = 1$, and $t = 0$ for the 2D graphic.

Figure 3: The 3D and 2D graphics of $u_{13}(z)$ by choosing the values $\gamma = 4$, $\beta = 5$, $\lambda = 1$, $\omega = 1$, $c = 1$, and $t = 0$ for the 2D graphic.
Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgments
This work is supported by the NSF of China (11901111) and the Visiting Scholar Program of Chern Institute of Mathematics.

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