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Simple two Higgs doublet model

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We study a simple two Higgs doublet model which reflects, in a phenomenological way, the idea of compositeness for the Higgs sector. It is relatively predictive. In one scenario, it allows for a “hidden” usual Higgs particle in the 100 GeV region and a possible dark matter candidate.

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I. INTRODUCTION

Many fascinating models have been suggested for the Higgs sector which is expected to be explored at the new CERN collider, LHC in the next few years. These models are variously motivated by the ideas of supersymmetry \cite{1}, possible Nambu-Goldstone like (or “little”) Higgs particles \cite{2}, possible extra dimensions \cite{3}, “technicolor” binding composite Higgs particles \cite{4} etc.

Here we will be concerned with a model which may be related to the technicolor category but we will motivate it from the assumption that the present Higgs model is not too far from being correct. Namely we want to abstract some properties of the existing Higgs model and apply them to the two doublet case.

It is well known that the ordinary Higgs potential is formally identical to the Gell-Mann Levy SU(2) linear sigma model:\cite{5}

\begin{equation}
V = \alpha_1 I_1 + \alpha_3 (I_1)^2,
\end{equation}

where the SU(2)\textsubscript{L} x SU(2)\textsubscript{R} invariant $I_1$ is simply expressed in terms of the scalar singlet $\sigma$ and the pseudoscalar triplet $\pi$ as $I_1 = \sigma^2 + \pi^2$. Of course the sigma is identified with the Higgs and the $\pi$ with the particles eaten by the W and Z bosons. The analogs of these two particles are the lowest lying ones in ordinary QCD. Clearly a technicolor model which is a straightforward copy of ordinary QCD would be expected to give such a potential as a first approximation. However, it is not easy to rigorously explore the low lying spectrum of an arbitrary strongly interacting gauge theory \cite{6}. Furthermore it is now known that a so-called “walking” technicolor model \cite{7} may be a more reasonable candidate than straightforwardly extended QCD. Thus we will not insist that a technicolor induced Higgs potential be identical to the above and shall not try to estimate the particle masses. Rather we will just ask the effective Higgs potential to satisfy the general properties:

1. SU(2)\textsubscript{L} x SU(2)\textsubscript{R} flavor invariance.
2. Parity invariance and charge conjugation invariance.

These are clearly very reasonable for a strong interaction gauge theory with two massless flavors.

In the present note we introduce a second Higgs doublet based on the fact that the fundamental representation of SU(2) is equivalent to its complex conjugate. This has the consequence that the $(\pi, \sigma)$ multiplet used above is irreducible under the chiral SU(2)\textsubscript{L} x SU(2)\textsubscript{R} group without including the parity reversed partners, denoted as $(a, \eta)$. It seems natural to investigate what happens when these parity reversed partners are included in a second Higgs doublet. Then, the three basic invariants are,

\begin{align}
I_1 &= \sigma^2 + \pi^2, \\
I_2 &= \eta^2 + a^2, \\
I_3 &= \sigma \eta - \pi \cdot a.
\end{align}

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These forms are readily understandable since the two quartet fields may be regarded as 4-vectors in the \( \text{O}(4) \leftrightarrow \text{SU}(2)_L \times \text{SU}(2)_R \) space and these are the three basic invariants which can be made from them. Then the Higgs potential becomes,

\[
V = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_1^2 + \alpha_4 I_2^2 + \alpha_5 I_3^2 + \alpha_6 I_1 I_2
\]  

(3)

The lack of terms linear in \( I_3 \) is due to the assumption of parity invariance. This implies that the fields \( a \) and \( \eta \) each only occur in the potential paired off with either itself or the other. This feature may be expressed as the invariance of the potential under the transformation:

\[
\eta \rightarrow -\eta, \quad a \rightarrow -a
\]  

(4)

while the fields in the multiplet, \((\pi, \sigma)\) are unchanged. Altogether there are six real constants. The present potential is supposed to be an effective one, arising from some underlying renormalizable gauge theory.

Interactions violating the invariances in 1. and 2. above are introduced as perturbations in the model when the chiral fields are coupled to the \( \text{SU}(2) \times \text{U}(1) \) gauge fields in the usual way. The two quartets of the chiral group are conveniently written for this purpose as two spinors,

\[
\Phi = \begin{bmatrix} i\pi^+ \\ \sigma - i\pi_0 \sqrt{2} \end{bmatrix}, \quad \Psi = \begin{bmatrix} -ia^+ \\ \eta + i\sigma_0 \sqrt{2} \end{bmatrix},
\]  

(5)

and their conjugates. Furthermore,

\[
\pi^+ = \frac{\pi_1 - i\pi_2}{\sqrt{2}}, \quad a^+ = \frac{a_1 - ia_2}{\sqrt{2}}.
\]  

(6)

The gauged kinetic terms for these fields give the usual Lagrangian contribution:

\[
\mathcal{L} = -D_\mu \Phi^\dagger D_\mu \Phi - D_\mu \Psi^\dagger D_\mu \Psi
\]  

(7)

where

\[
D_\mu \Phi = \partial_\mu \Phi - ig W_\mu \Phi + \frac{ig'}{2} B_\mu \Phi,
\]

\[
D_\mu \Phi^\dagger = \partial_\mu \Phi + ig \Phi^\dagger W_\mu - \frac{ig'}{2} B_\mu \Phi^\dagger,
\]  

(8)

with similar forms containing \( \Psi \). Here \( B_\mu \) is the \( \text{U}(1) \) gauge boson and the \( \text{SU}(2) \) gauge bosons are expanded as:

\[
W_\mu = \frac{1}{2} \pi^a \cdot W^a_\mu = \frac{1}{2} \begin{bmatrix} W^0_\mu & \sqrt{2} W^+_\mu \\ \sqrt{2} W^-_\mu & -W^0_\mu \end{bmatrix}
\]

The presence of the pure \( \text{SU}(2) \times \text{U}(1) \) gauge field kinetic terms in \( \mathcal{L} \) is to be understood. Finally consider the Yukawa terms containing the coupling of the quarks and leptons to the Higgs field. For this purpose, it seems natural to demand the symmetry in Eq.(4), which can be rewritten as,

\[
\Phi \rightarrow \Phi, \quad \Psi \rightarrow -\Psi.
\]  

(9)

We also assume here that the quarks and leptons do not change under this symmetry transformation. Then only the original Higgs multiplet \( \Phi \) can couple to the fermions and the Yukawa couplings are just the usual ones.

II. DISCUSSION

Of course, there has been a very extensive discussion of various two Higgs doublet models in the literature. Recent related work includes that of Randall [9], who considers a model with a heavy extra doublet in which the mixing between singlet states is very small (ie, large \( \tan \beta \)), Ma [10] who stresses the connection with the dark matter problem, Gerard and Herquet [11] who consider connections with the custodial symmetry and Lopez Honorez, Nezri, Oliver and Tytgat [12] who discuss the dark matter application extensively.
In the present work we emphasize that the idea of compositeness for the Higgs bosons motivates both the SU(2)$_L \times$ SU(2)$_R$ as well as the P and C invariance of the Higgs potential. This contains the usual custodial SU(2)$_V$ symmetry together with a discrete Z$_2$ symmetry. If it is true that the model arises from some underlying technicolor theory, it is reasonable to think that the Higgs potential is an approximation to the underlying theory describing the interactions of its lowest lying scalar states. From this point of view the electroweak interactions represent a perturbation to this “strong” interaction. Then it seems natural to classify the symmetries of the Higgs potential according to the larger “strong” interaction symmetry. This stands in contrast to discussing the symmetry from the point of view of the spinors $\Phi$ and $\Psi$ in Eq. (4). In that language, our invariant $I_1$ is identified as $2\Phi^\dagger \Phi$ while $I_2$ is identified as $2\Psi^\dagger \Psi$. Also our $I_3$ corresponds to the combination $[\Phi^\dagger \Psi + \Psi^\dagger \Phi]$. On the other hand, the combination $i[\Phi^\dagger \Psi - \Psi^\dagger \Phi]$ is easily seen to violate the proposed SU(2)$_L \times$ SU(2)$_R$ invariance and will not be included. This gives an additional simplification of the potential.

It is interesting to remark that the “minimal walking technicolor theory” [13] automatically respects the symmetries 1. and 2. which we are advocating. That theory contains the Higgs bosons we are studying but also contains other effective fields associated with the technicolor interactions.

In section III we will discuss the potential part of this model and list all its terms. We will explicitly give the expressions for the scalar boson masses and two relevant coupling constants. In section IV we will explicitly give all the Lagrangian terms for the interactions of the scalar bosons with the gauge bosons. The formulas in these sections can be read to get an idea of the interactions of the extra bosons $\eta$, $\sigma$, $a^{\pm}$ beyond the usual Higgs ($\sigma$).

In the remainder of the paper, we discuss the possible application of this model to “shielding” a relatively light ordinary Higgs boson from detection. This is motivated from the unusually low value of its mass expected from precision analyses of electroweak corrections. Section V deals with a first model in which the decay of the usual Higgs into $\eta\eta$ is a competing mode which might prevent seeing the Higgs in an experiment which searches for $b\bar{b}$ pairs in combination with a $\mu^+\mu^-Z$ boson indicator. The eta has no decay interactions in this model. That means that it could not “hide” the Higgs in an experiment which just identifies events in which a $Z$ is made and no other identifiable particles emerge. In section VI, we propose an alternate way in the present framework to shield the Higgs from such an experiment too.

### III. HIGGS POTENTIAL TERMS

Recent discussions of general two Higgs doublet potentials are given in [14]. The present case, in which the field variables comprise two O(4) vectors, is simpler than the general case. First, we note the constraints which follow from the requirement that the Higgs potential be positive for large field configurations. This implies that the quartic terms of the potential,

$$V = \cdots + \alpha_3(I_1)^2 + \alpha_4(I_2)^2 + [\alpha_5 \cos^2 \theta + \alpha_6]I_1I_2,$$

where we used the O(4) property that $I_3^2 = I_1I_2 \cos^2 \theta$ for some angle $\theta$, be positive for large field configurations. Then taking either $I_1$ or $I_2$ to be dominant for large fields we get the requirements:

$$\alpha_3 > 0, \quad \alpha_4 > 0.$$

There is a possibility that $\alpha_5$ and/or $\alpha_6$ may be negative. In such cases there is an additional discriminant condition which is obtained by forbidding real roots of the quadratic form obtained by dividing through by $(I_1)^2$. It has the form:

$$(\alpha_5 \cos^2 \theta + \alpha_6)^2 < 4\alpha_3 \alpha_4,$$

for any $\theta$. As examples,

$$(\alpha_5 + \alpha_6)^2 < 4\alpha_3 \alpha_4, \quad \alpha_6^2 < 4\alpha_3 \alpha_4.$$

Stronger information on the $\alpha$ coefficients arises, as to be discussed next, from calculating the particle masses and interactions by expanding the potential around the physical minimum $<\sigma> \neq 0$, $<\eta> = 0$. The latter corresponds to our assumed underlying parity invariance. A simple calculation verifies that $<\partial V/\partial \sigma> = <\partial V/\partial \eta> = 0$ for this minimum.

The $\alpha_1$ and $\alpha_3$ terms in Eq. (13) correspond to the usual single Higgs model. In the present case, parity invariance prevents the $\sigma$ from mixing with the $\eta$ so $\alpha_1$ and $\alpha_3$ are determined just as in the standard model. Then $\alpha_1$ is negative and related to $\alpha_3$ by the minimization equation:

$$\alpha_1 + 2\alpha_3 v^2 = 0,$$
where the vacuum value, \( v \) is given as
\[
v = \langle \sigma \rangle \approx 246 \text{GeV}.
\] (15)

The Higgs squared mass is obtained as
\[
m^2_{\sigma} = 8\alpha_3 v^2.
\] (16)

The potential also yields \( m^2_{\pi} = 0 \) for all three “pions”, which, in the unitary gauge get absorbed into massive gauge bosons. For the particles in the \( \Psi \) multiplet, the squared masses are obtained as,
\[
m^2_{\eta} = 2[\alpha_2 + (\alpha_5 + \alpha_6)v^2],
\]
\[
m^2(a^0) = m^2(a^\pm) \equiv m^2 = 2[\alpha_2 + \alpha_6 v^2].
\] (17)

Notice that the three “a” particles are degenerate in mass. Furthermore there is no mixing between the two Higgs multiplets.

Defining a shifted Higgs field \( \sigma = v + \tilde{\sigma}, \) the interaction terms in the Lagrangian resulting from the Higgs potential are:
\[
-\alpha_3(\tilde{\sigma}^4 + 4v\tilde{\sigma}^3) - \alpha_4(a^2 + \eta^2)^2
\]
\[
-\alpha_5\eta^2(2v\tilde{\sigma} + \tilde{\sigma}^2) - \alpha_6(a^2 + \eta^2)(2v\tilde{\sigma} + \tilde{\sigma}^2).
\] (18)

The interaction vertices for Feynman rules can be read off from this equation. For later convenience we identify the coupling constants for the \( \sigma\eta\eta \) and \( \sigma a^0 a^0 \) vertices,
\[
g_{\sigma\eta\eta} = 4v(\alpha_5 + \alpha_6), \quad g_{\sigma a^0 a^0} = 4v\alpha_6.
\] (19)

It may be noted from Eqs. (14) and (10) that specifying the Higgs mass, \( m_\sigma \) will fix the coefficients \( \alpha_1 \) and \( \alpha_3. \) Furthermore specifying \( m_\eta, m_a \) and \( g_{\sigma\eta\eta} \) will fix \( \alpha_2, \alpha_5 \) and \( \alpha_6. \) Information about \( \alpha_4 \) is related to the a-\( \eta \) scattering amplitude. We will not need \( \alpha_4 \) in the present paper.

The allowed ranges of the alpha parameters are constrained by the requirement that the squared masses \( m^2_\sigma, m^2_\eta \) and \( m^2_a \) be positive definite. This agrees with the requirement that \( V(\sigma, \eta) \) have a minimum, rather than a maximum or saddle point at the point \( (\sigma, \eta) = (v, 0). \) Specifically, we have:
\[
A = \frac{\partial^2 V}{\partial \sigma^2}(v, 0) = 2\alpha_1 + 12v^2\alpha_3 = m^2_\sigma,
\]
\[
B = \frac{\partial^2 V}{\partial \sigma \partial \eta}(v, 0) = 0,
\]
\[
C = \frac{\partial^2 V}{\partial \eta^2}(v, 0) = 2\alpha_2 + 2v^2(\alpha_5 + \alpha_6) = m^2_\eta.
\] (20)

The condition for no saddle point, \( B^2 - AC < 0 \) as well the condition for a minimum rather than a maximum, \( A + C > 0 \) are both clearly satisfied for positive definite squared masses.

### IV. GAUGE-HIGGS INTERACTIONS

No undetermined parameters are introduced here. It is necessary to first give conventions for the \( W^0_\mu - B_\mu \) mixing matrix:
\[
\begin{bmatrix}
Z_\mu \\
A_\mu
\end{bmatrix} =
\begin{bmatrix}
c & s \\
-s & c
\end{bmatrix}
\begin{bmatrix}
W^0_\mu \\
B_\mu
\end{bmatrix},
\] (21)

where \( s \) and \( c \) are respectively the sine and cosine of the mixing angle. They are connected to the proton charge, \( e \) and the coupling constants in Eq. (5) by \( g = -e/s \) and \( g' = -e/c. \)

Since there are many terms we present them in four parts. First the terms in the Lagrangian containing a single gauge boson are:
\[
-ieA_\mu(a^- \bar{\mu} a^+) - eZ_\mu \left[ -i(s^2 - c^2) \bar{a} \bar{\mu} a^+ + \frac{1}{2sc}(a^0 \bar{\mu} \eta) \right]
\]
\[
-\frac{e}{2s} \left[ W^+_\mu(a^- \bar{\mu} (\eta + ia^0)) + W^-_\mu(a^+ \bar{\mu} (\eta - ia^0)) \right].
\] (22)
The terms involving both $W^+$ and $W^-$ are:

$$- W^+ W^- \left[ \frac{m_W^2}{v^2} (2v\bar{\sigma} + \bar{\sigma}) + \frac{e^2}{s^2} (a^+ a^- + \frac{\eta^2}{2} + a^0 a^0) \right].$$  \hfill (23)

The terms with two gauge bosons but no $W$'s are:

$$- e^2 A_\mu A_\mu a^+ a^-$$

$$- Z_\mu Z_\mu \left[ \frac{m_Z^2}{2v^2} (2v\bar{\sigma} + \bar{\sigma}) + \frac{e^2}{8c^2s^2} (\eta^2 + a^0 a^0) + \frac{e^2(c^2 - s^2)^2}{4s^2c^2} a^+ a^- \right]$$

$$- A_\mu Z_\mu e^2 (s^2 - c^2) a^+ a^-.$$  \hfill (24)

Finally, the terms with two gauge bosons containing a single $W$ take the form:

$$- e^2 (\frac{Z_\mu}{c} + A_\mu) \left[ i\eta (a^+ W^-_\mu - a^- W^+_\mu) + a^0 (a^+ W^- + a^- W^+_\mu) \right].$$  \hfill (25)

Feynman rules for gauge particle-Higgs particle vertices in the unitary gauge can be read off from the above expressions. There is one observation about the additional Higgs particles which is immediate. Since there is a $Za^+a^-$ vertex and the width of the $Z$ is already well accounted for, the $Z$ should not decay into $a^+ + a^-$; this suggests considering the mass range:

$$m_a > m_Z. \hfill (26)$$

On the other hand there is no $Z\eta\eta$ vertex so the $\eta$ mass has no lower bound from an analogous decay. There is a $Za^0\eta$ vertex so the bound,

$$m_a + m_\eta > m_Z, \hfill (27)$$

is suggested. However, this does not prevent $\eta$ from being very light if $a$ is of the order or somewhat heavier than the $Z$.

V. FIRST MODEL FOR A HIDDEN HIGGS SCENARIO

Stimulated by precision calculations in the standard model giving the Higgs mass prediction\cite{15},

$$m_\sigma = 89^{+38}_{-28} \text{GeV}, \hfill (28)$$

a number of groups have revived\cite{16} an older idea\cite{17} that the Higgs might be light and not yet detected because of a competitive decay mode to some hard to observe new particles. It would seem that a decay mode in the present model, $\sigma \rightarrow \eta\eta$ is a reasonable candidate for such a competing channel. As we observe above, the $\eta$ occurs only in quadratic form in the Higgs potential and only together with a conceivably much heavier $a$ particle in the gauge-Higgs part of the Lagrangian. Thus it could have escaped detection.

For the present purpose we need the formula for the predicted Higgs width for its decay into $\eta\eta$:

$$\Gamma(\sigma \rightarrow \eta\eta) = \frac{g_{\sigma\eta\eta}^2}{32\pi m_\sigma} \sqrt{1 - \frac{4m_\eta^2}{m_\sigma^2}}, \hfill (29)$$

wherein $g_{\sigma\eta\eta}$ and $m_\eta$ are given in Eqs.\cite{19} and \cite{20} respectively. It can be seen that these two quantities are determined by the parameters $\alpha_2$ and $\alpha_5 + \alpha_6$. The typical Higgs search involves the reaction:

$$Z \rightarrow Z^* + \sigma, \hfill (30)$$

wherein the virtual $Z^*$ decays into $\mu^+\mu^-$ and the Higgs decays primarily into $b\bar{b}$ jets. The formula for $\Gamma(\sigma \rightarrow b\bar{b})$ is:

$$\Gamma(\sigma \rightarrow b\bar{b}) = \frac{3m_\sigma m_b^2}{8\pi v^2} \left( 1 - \frac{4m_b^2}{m_Z^2} \right)^{3/2}, \hfill (31)$$
where $m_b \approx 4.2$ GeV is a conventional estimate for the b quark mass. We need the ratio,

$$R = \frac{\Gamma(\sigma \rightarrow \eta \eta)}{\Gamma(\sigma \rightarrow bb)}.$$  

(32)

Now if $P_{\text{standard}}$ gives the strength of the Higgs signal in the standard model scenario, the reduced strength due to the existence of the competitive $\eta \eta$ decay mode in the present scenario would be,

$$P_{\text{new}} = \frac{\Gamma(\sigma \rightarrow bb)}{\Gamma(\sigma \rightarrow bb) + \Gamma(\sigma \rightarrow \eta \eta)} P_{\text{standard}} = \frac{1}{1 + R} P_{\text{standard}}.$$  

(33)

It was noted [16] that a value, $R = 0.8$ would decrease the presently expected Higgs signal below the detection threshold. Using the numbers just given we have,

$$R = 2184 y \sqrt{1-x},$$  

(34)

where $x = (2m_\eta/m_\sigma)^2$ and $y = (g_{\sigma \eta \eta}/v)^2$. A plot of $y$ vs $x$ for the value $R = 0.8$ is shown in Fig. 1. Any point on that curve is a solution for suppression of the $bb$ Higgs signal.

![FIG. 1: $y$ vs. $x$.](image)

Three typical points, together with the corresponding values of the Higgs potential parameters $\alpha_2$ and $\alpha_5 + \alpha_6$ are given in Table I.

| $m_\eta$ (GeV) | $g_{\sigma \eta \eta}$ (GeV) | $\alpha_5 + \alpha_6$ | $\alpha_2$ (GeV$^2$) |
|---------------|-----------------|-----------------|-----------------|
| 14.1          | 4.8             | 4.91 x 10^{-3}  | -198            |
| 31.5          | 5.6             | 5.69 x 10^{-3}  | +151            |
| 42.2          | 8.4             | 8.51 x 10^{-3}  | +376            |

TABLE I: Values of $m_\eta$, $g_{\sigma \eta \eta}$ and Higgs potential parameters which give suitable suppression of the Higgs signal.

In the present scenario, with $m_\alpha > m_\eta$, the $\eta$ boson has “annihilation” modes but not decay modes. On the other hand the $\alpha$ particles have leading decay modes of the forms,

$$a^0 \rightarrow Z + \eta, \quad a^+ \rightarrow W^+ + \eta,$$  

(35)

wherein it has been assumed that the $\alpha$’s are sufficiently heavier than the massive gauge bosons. If the $\alpha$’s are lighter than the massive gauge bosons but still heavier than the $\eta$, one would expect important decays like,

$$a^0 \rightarrow \eta + \mu^+ + \mu^-, \quad a^+ \rightarrow \eta + \pi^+.$$  

(36)
These two decays are mediated by virtual $Z$ and $W$ bosons respectively. The formula for the decay width of a heavy $a^+$ by the reaction in Eq. (35) is readily found to be:

$$\Gamma(a^+ \to W^+ + \eta) = \frac{k}{8\pi m_a} \mathcal{F}(a^+ \to W^+ + \eta),$$  \hspace{1cm} (37)

where the momentum, $k$ of each of the two daughter particles in the $a^+$ rest frame is:

$$k = \frac{1}{2m_a} \sqrt{|m_a^2 - (m_\eta + m_W)^2||m_a^2 - (m_\eta - m_W)^2|},$$  \hspace{1cm} (38)

and the squared amplitude summed over the final $W^+$ polarization states is:

$$\mathcal{F}(a^+ \to W^+ + \eta) = \left(\frac{e}{2s}\right)^2 \frac{m_a^2 - m_\eta^2}{m_W^2} - m_a^2 - m_\eta^2 - 2m_a \sqrt{k^2 + m_\eta^2}.$$  \hspace{1cm} (39)

We may use the same formula for $\Gamma(a^0 \to Z + \eta)$ if we replace $m_W$ by $m_Z$ and the overall factor $(e/(2s))^2$ by $(e/(2sc))^2$. These $a$ widths are listed in Table II for a characteristic range of $a$ masses in cases where they are heavy enough to decay into the gauge boson modes. The $\eta$ mass is taken to be 31.5 GeV, the central value in Table I. It is seen that the widths are in the range 0.2 to 2 MeV for the $a$ masses shown. This may be compared to the width, 2.5 MeV, for the Higgs (sigma) to decay into two $\eta$'s according to Eq. (29) taking $m_\eta = 31.5$ GeV.

Also listed in Table II are the associated dimensionless coupling constants $\alpha_5$ and $\alpha_6$ in the Higgs potential. These are all less than unity, indicating that for the mass range under discussion, the new part of the Higgs sector is not very “strongly coupled”.

| $m_a$ (GeV) | $\Gamma(a^+ \to W^+ \eta)$ (GeV) | $\Gamma(a^0 \to Z \eta)$ (GeV) | $\alpha_5$ | $\alpha_6$ |
|------------|-------------------------------|------------------------------|-----------|-----------|
| 150        | $2.14 \times 10^{-4}$         | $1.52 \times 10^{-4}$       | -0.178    | 0.235     |
| 200        | $8.70 \times 10^{-4}$         | $7.69 \times 10^{-4}$       | -0.322    | 0.379     |
| 250        | $2.67 \times 10^{-3}$         | $1.94 \times 10^{-3}$       | -0.508    | 0.565     |

TABLE II: Widths of the $a$ bosons for various mass values and associated Higgs potential parameters.

It is amusing to remark that the quartic coupling constant $\alpha_5$ is negative. The discussion at the end of section III implies that this is of no concern, since the squared masses of all the Higgs particles are positive. Note that the positive $\alpha_6$ is larger than the magnitude of $\alpha_5$.

Since the $\eta$ under study in the present scenario does not have any decay modes, it would appear to be another candidate for the “dark matter” required to understand galactic structures. Work in this direction will be presented elsewhere.

VI. SECOND HIDDEN HIGGS MODEL

It was stressed in [16] that Higgs search experiments [18] which look for an appropriate $Z$ (say by tagging $\mu^+ \mu^-$ pairs) together with the absence of any other particle signals could eliminate the possibility of a light Higgs. They point out that the Higgs can therefore be shielded only if there is a “cascade” decay of the decay products ($\eta$’s in the first model) to final states containing a recognizable particle. The $\eta$’s have no decays in our model, however.

We can shield a light Higgs in such an experiment if we assume that the three $a$ particles are lighter than half the Higgs mass and that the $\eta$ is lighter still. For example, with a Higgs mass of 115 GeV, $a$ masses of 50 GeV could do the job. The $a$’s would be heavy enough that they would not alter the well known $Z$ width. (This mechanism is clearly suitable for shielding Higgs bosons which are roughly more massive than the $Z$). Then the decay modes

$$\sigma \to a^+ + a^-, \quad \sigma \to a^0 + a^0,$$  \hspace{1cm} (40)

are possible. Furthermore, the Eqs. (36) show that the $a$’s decay into the inert $\eta$ as well as the recognizable particles $\pi^\pm$ or $\mu^+ \mu^-$. It is still possible of course for there to be some $\sigma \to \eta + \eta$ in addition to these modes. To illustrate the present scenario we will assume for simplicity that the coupling constant, $g_{\sigma \eta \eta}$ has been tuned to be negligible. Then the relevant decay width is:

$$\Gamma(\sigma \to a^+ a^-) + \Gamma(\sigma \to a^0 a^0) =$$

$$3\Gamma(\sigma \to a^0 a^0) = \frac{3g_{\sigma \eta \eta}^4}{32\pi m_\sigma^3} \sqrt{1 - \frac{4m_\sigma^2}{m_\eta^2}},$$  \hspace{1cm} (41)
Proceeding as before we define,

\[ R' = \frac{3\Gamma(\sigma \rightarrow a^0\bar{a}^0)}{\Gamma(\sigma \rightarrow bb)} = 1319y'\sqrt{1-x'^2}, \]  

(42)

where \( x' = (2m_a/m_\sigma)^2 \) and \( y' = 3(g_{\sigma a_0 a_0}/v)^2 \). A plot of \( y' \) vs \( x' \) for the value \( R' = 0.8 \) is shown in Fig. 2. Any point on that curve is a solution for suppression of the \( bb \) Higgs signal. In contrast to Fig. 1, the \( x' \) variable is not displayed down to zero, indicating that the shielding is only operative for roughly \( m_a > m_Z/2 \).

\[ \text{FIG. 2: } y' \text{ vs. } x'. \]

Three typical points, together with the corresponding values of the Higgs potential parameters \( \alpha_2 \) and \( \alpha_6 \) are given in Table III.

| \( m_a \) (GeV) | \( g_{\sigma a^0 a^0} \) (GeV) | \( \alpha_6 \) | \( \alpha_2 \) (GeV\(^2\)) |
|----------------|------------------|-------------|------------------|
| 48.1           | 4.7              | 4.82 x 10^{-3} | 576              |
| 51.4           | 5.2              | 5.32 x 10^{-3} | 674              |
| 54.5           | 6.2              | 6.32 x 10^{-3} | 723              |

TABLE III: Values of \( m_a \), \( g_{\sigma a^0 a^0} \) and Higgs potential parameters which give suitable suppression of the Higgs signal. Here we take \( m_\sigma = 115 \) GeV.

One notices, as in the previous shielding model, that the dimensionless coupling constant \( \alpha_6 \) is much less than one, so the Higgs bosons are not strongly coupled. If we want to tune the \( \sigma \rightarrow \eta\eta \) contribution to be small, Eq. (19) indicates that \( \alpha_5 \) should be taken negative and slightly less in magnitude than \( \alpha_6 \).

Effectively, the present “cascade” type shielding mechanism would have characteristic signals of a \( \pi^+\pi^- \) pair together with two unobservable \( \eta \)'s or two \( \mu^+\mu^- \) pairs together with two unobservable \( \eta \)'s.

VII. SUMMARY

We noted that a technicolor theory underlying the standard electroweak model is likely to result in a Higgs potential which possesses standard “strong” interaction symmetries like chiral SU(2), parity and charge conjugation. This is obvious for the single Higgs doublet model. Imposing the same requirement for a two doublet model results in an interesting picture, which is rather constrained compared to a general two doublet model.

In particular the second doublet doesn’t mix with the first one although it interacts with it. This leads to at least one possible dark matter candidate.

A number of very interesting Higgs scenarios can be constructed. The most conservative one would make the second doublet heavier than the first. In this paper we considered an opposite picture with lighter second doublet members.
This provides extra decay modes for the usual Higgs boson and enables us to construct models which might hide the usual Higgs from being observed in certain experiments. These models involve all, but one, of the parameters in our Higgs potential. Information about the remaining one, $\alpha_4$ might be found by considering the connection with dark matter observations.

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[1] See the 2007 Particle Data Group reviews by H.E. Haber and J.-F.Grivaz.
[2] N.Arkani-Hamed, A.G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001)[arXiv:hep-ph/ 0105239. For recent reviews, see N.Arkani-Hamed, A.G. Cohen, E. Katz and A.E. Nelson, JHEP, 0208, 019 (2002)[arXiv:hep-ph/0206023]; T. Gregoire and J.G. Wacker, JHEP 0208, 019 (2002)[arXiv:hep-ph/0206023].
[3] For a recent review, see C. Csaki, J. Hubisz and P. Meade, [arXiv:hep-ph/0510275].
[4] For a recent review, see C.T. Hill and E.H. Simmons, Phys. Rept. 381, 235 (2003); E 390, 553 (2004),[arXiv:hep-ph/ 2003079].
[5] M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).
[6] Discussions are given by G.F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, JHEP 0706:045 (2007)[arXiv: hep-ph/0703164] and B. Grinstein and M. Trott, Phys. Rev. D 76, 073002 (2007)[arXiv: 0704.1503].
[7] For a recent review see F. Sannino [arXiv:0804.0182].
[8] See for example A. Salomone, J. Schechter and T. Tudron, Phys. Rev. D 24, 492 (1981).
[9] L. Lopez Honorez, E. Nezri, J.F. Oliver and M.H.G. Tytgat, JCAP 0702, 028 (2007),[arXiv:hep-ph/0612275]. See also R. Barbieri, L.J. Hall and V.S. Rychkov, Phys. Rev. D 74, 015007 (2006)[arXiv:hep-ph/0603188] and M. Cirelli, N. Fornengo and A. Strumia, [arXiv:hep-ph/0512090].
[10] E. Ma, [arXiv:0802.2017][hep-ph]. See also N.G. Deshponde and E. Ma, Phys. Rev. D 18, 2574 (1978). In this paper, a model similar to the present one is mentioned as a special case of a more general two Higgs scheme.
[11] J.-M. Gerard and M. Herquet, Phys. Rev. Letts. 98, 251802 (2007), arXiv:hep/0703051.
[12] L. Lopez Honorez, E. Nezri, J.F. Oliver and M.H.G. Tytgat, JCAP 0702, 028 (2007),[arXiv:hep-ph/0612275]. See also R. Barbieri, L.J. Hall and V.S. Rychkov, Phys. Rev. D 74, 015007 (2006)[arXiv:hep-ph/0603188] and M. Cirelli, N. Fornengo and A. Strumia, [arXiv:hep-ph/0512090].
[13] R. Foadi, M.T. Frandsen, T.A. Ryttov and F. Sannino, Phys. Rev. D 76:055005, 2007; (arXiv:0706.1696[hep-ph]). This paper contains a detailed discussion of the minimal walking technicolor model. An historical review of “walking technicolor” is given by K. Yamawaki, Proceedings of the international symposium pnΛ50, Eds. M. Harada, Y. Ohnuki, S. Sawada and K. Yamawaki, PTP supplement no. 167 (2007) p. 127.
[14] See, for examples, S. Nie and M. Sher, Phys. Lett. B449, 89 (1999), arXiv:hep-ph/ 9811234 and I.F. Ginzburg and K.A. Kanishev, Phys. Rev. D 76, 095013 (2007), arXiv:0704.3664[hep-ph]. It seems worthwhile to remark, as briefly noted in section II above, that our potential is significantly simpler than the general ones discussed in these references. This follows because of the imposition of the technicolor inspired requirements of “strong” SU(2)$_L \times$ SU(2)$_R$ symmetry as well as P and C conservation on the Higgs potential. To compare with these references we note that our $\alpha_3, \alpha_4, \alpha_5$ correspond to their $\lambda_1, \lambda_2, \lambda_3$ respectively. On the other hand they have two more quartic invariants while we have just one more. Our final SU(2)$_L \times$ SU(2)$_R$ quartic invariant has the coefficient $\alpha_5$. Note that the square of $i[\Phi^\dagger \Psi - \Psi^\dagger \Phi]$ has the electroweak SU(2)$_L$ invariance but does not satisfy the larger “technicolor” invariance we are requiring. If we take linear combinations of our $\alpha_5$ term with this disallowed piece we could recover both the usual $\lambda_1$ and $\lambda_3$ terms. This also demonstrates that the conventional $\lambda_1$ and $\lambda_3$ terms do not obey the O(4) invariance being imposed in the present model. Our simplified potential results in the very much simplified mass formulas given in Eqs. (17) for the masses of the additional Higgs particles.
[15] See page 128 of Particle data group, J. Phys.G: Nucl. Part. Phys. 33, 1 (2006).
[16] For a recent review, see S. Chang, R. Dermisek, J.F. Gunion and N. Weiner, [arXiv:0801.4554][hep-ph].
[17] J.F. Gunion and H.E. Haber, Nucl. Phys. B 272, 1 (1986); L.-F. Li, Y. Liu and L. Wolfenstein, Phys. Lett. B 159, 45 (1985).
[18] A detailed discussion of the relevant experiments is given in [10] above.