SUPERSYMMETRIC HYBRID INFLATION

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Abstract. The non-supersymmetric and supersymmetric versions of hybrid inflation are summarized. In the latter, the necessary inclination along the inflationary trajectory is provided by radiative corrections. Supersymmetric hybrid inflation (with its extensions) is an extremely 'natural' inflationary scenario. The reasons are that it does not require 'tiny' parameters, its superpotential has the most general form allowed by the symmetries, and it can be protected against radiative or supergravity corrections. Concrete supersymmetric grand unified theories which lead to hybrid inflation, solve the $\mu$ problem via a Peccei-Quinn symmetry and generate seesaw masses for the light neutrinos can be constructed. As an example, we present a theory with unified gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The 'reheating' which follows hybrid inflation is studied. It is shown that the gravitino constraint on the 'reheat' temperature can be 'naturally' satisfied. Also, the observed baryon asymmetry of the universe can be generated via a primordial leptogenesis consistently with the requirements from solar and atmospheric neutrino oscillations. Extensions of the standard supersymmetric hybrid inflationary scenario which are still consistent with all these requirements but can also avoid the cosmological disaster from the possible copious monopole production at the abrupt termination of standard hybrid inflation are constructed. They rely on utilizing the leading non-renormalizable correction to the standard hybrid inflationary superpotential and are necessary for higher unified gauge groups such as $SU(4)_c \times SU(2)_L \times SU(2)_R$ which predict the existence of monopoles. In one extension, which we call shifted hybrid inflation, the relevant part of inflation takes place along a 'shifted' classically flat direction on which the unified gauge symmetry is already broken. In the other extension, called smooth hybrid inflation, the trilinear term of the standard hybrid inflationary superpotential is removed by a discrete symmetry. The inflationary path then possesses a classical inclination and the termination of inflation is smooth.
1. Hybrid Inflation

1.1. THE NON-SUPERSYMMETRIC VERSION

The most important disadvantage of inflationary scenarios such as the ‘new’ [1] or ‘chaotic’ [2] ones is that they require ‘tiny’ coupling constants in order to reproduce the measurements of the cosmic background explorer (COBE) [3] on the cosmic microwave background radiation (CMBR). This difficulty was overcome by Linde [4] who proposed, in the context of non-supersymmetric grand unified theories (GUTs), the hybrid inflationary scenario. The basic idea was to use two real scalar fields $\chi$ and $\sigma$ instead of one that was normally used. $\chi$ provides the ‘vacuum’ energy density which drives inflation, while $\sigma$ is the slowly varying field during inflation. This splitting of roles between two fields allows us to reproduce the observed temperature fluctuations of the CMBR with ‘natural’ (not too small) values of the relevant parameters in contrast to previous realizations of inflation.

The scalar potential utilized by Linde is

$$V(\chi, \sigma) = \kappa^2 \left( M^2 - \frac{\chi^2}{4} \right)^2 + \frac{\lambda^2 \chi^2 \sigma^2}{4} + \frac{m^2 \sigma^2}{2},$$

where $\kappa$, $\lambda$ are dimensionless positive coupling constants and $M$, $m$ are mass parameters. The vacua lie at $\langle \chi \rangle = \pm 2M$, $\langle \sigma \rangle = 0$. Putting $m=0$, for the moment, we observe that $V$ possesses an exactly flat direction at $\chi = 0$ with $V(\chi = 0, \sigma) = \kappa^2 M^4$. The mass$^2$ of the field $\chi$ along this flat direction is $m^2_\chi = -\kappa^2 M^2 + \lambda^2 \sigma^2/2$. So, for $\chi = 0$ and $|\sigma| > \sigma_c = \sqrt{2}\kappa M/\lambda$, we obtain a flat valley of minima. Reintroducing $m \neq 0$, this valley acquires a non-zero slope and the system can inflate as it rolls down this valley. This scenario is called hybrid since the ‘vacuum’ energy density ($= \kappa^2 M^4$) is provided by $\chi$, while the slowly rolling field (inflaton) is $\sigma$.

The $\epsilon$ and $\eta$ criteria (see e.g., Ref.[5]) imply that, for the relevant values of parameters (see below), inflation continues until $\sigma$ reaches $\sigma_c$, where it terminates abruptly. It is followed by a ‘waterfall’, i.e., a sudden entrance into an oscillatory phase about a global minimum. Since the system can fall into either of the two minima with equal probability, topological defects (monopoles, walls or cosmic strings) are copiously produced [6] if they are predicted by the particular particle physics model employed. So, if the underlying GUT gauge symmetry breaking (by $\langle \chi \rangle$) leads to the existence of monopoles or walls, we encounter a cosmological catastrophe.

The onset of hybrid inflation requires [7] that, at a cosmic time of order $H^{-1}$, $H$ being the Hubble parameter during inflation, a region exists in the universe with size greater than about $H^{-1}$, where $\chi$ and $\sigma$ happen to be almost uniform with negligible kinetic energies and values close to
the bottom of the valley of minima. Such a region, at the Planck time $t_P = M_P^{-1}$ ($M_P \approx 1.22 \times 10^{19}$ GeV is the Planck mass), would have been much larger than the Planck length $\ell_P = M_P^{-1}$ and it is, thus, very difficult to imagine how it could emerge so homogeneous. Moreover, as it has been argued [8], the initial values (at $t_P$) of the fields in this region must be strongly restricted in order to obtain adequate inflation. Several possible solutions to this problem of initial conditions for hybrid inflation have been already proposed (see e.g., Refs.[9, 10, 11]).

The quadrupole anisotropy of CMBR produced during hybrid inflation can be estimated, using the standard formulae (see e.g., Ref.[5]), to be

$$\left(\frac{\delta T}{T}\right)_Q \approx \left(\frac{16\pi}{45}\right)^\frac{1}{2} \frac{\lambda \kappa^2 M^5}{M_P^2 m^2}. \quad (2)$$

The COBE [3] result, $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$, can then be reproduced with $M \approx 2.86 \times 10^{16}$ GeV, the supersymmetric (SUSY) GUT vacuum expectation value (vev), and $m \approx 1.3 \kappa \sqrt{\lambda} \times 10^{15}$ GeV. Note that $m \sim 10^{12}$ GeV for $\kappa, \lambda \sim 10^{-2}$.

1.2. THE SUPERSYMMETRIC VERSION

Hybrid inflation turns out [12] to be ‘tailor made’ for application to globally SUSY GUTs except that an intermediate scale mass for $\sigma$ cannot be obtained in this context. Actually, all scalar fields acquire masses of order $m_{3/2} \sim 1$ TeV (the gravitino mass) from soft SUSY breaking.

Let us consider the renormalizable superpotential

$$W = \kappa S(-M^2 + \tilde{\phi}\phi), \quad (3)$$

where $\tilde{\phi}, \phi$ is a conjugate pair of $G_S$ (the standard model gauge group) singlet left handed superfields belonging to non-trivial representations of the GUT gauge group $G$ and reducing its rank by their vevs, and $S$ is a gauge singlet left handed superfield. The parameters $\kappa$ and $M$ ($\sim 10^{16}$ GeV) can be made positive by field redefinitions. The vanishing of the F-term $F_S$ implies that $\langle \phi \rangle = \langle \tilde{\phi} \rangle = M^2$, whereas the D-terms vanish for $|\langle \tilde{\phi} \rangle| = |\langle \phi \rangle|$. So, the SUSY vacua lie at $\langle \phi \rangle^* = \langle \tilde{\phi} \rangle = \pm M$ and $\langle S \rangle = 0$ (from $F_{\tilde{\phi}} = F_\phi = 0$). We see that $W$ leads to the spontaneous breaking of $G$.

The same superpotential $W$ gives rise to hybrid inflation. The potential derived from $W$ in Eq.(3) is

$$V(\tilde{\phi}, \phi, S) = \kappa^2 |M^2 - \tilde{\phi}\phi|^2 + \kappa^2 |S|^2 (|\tilde{\phi}|^2 + |\phi|^2) + D - \text{terms.} \quad (4)$$

D-flatness implies $\tilde{\phi}^* = e^{i\theta} \phi$. We take $\theta = 0$, so that the SUSY vacua are contained. Note that $W$ possesses a $U(1)_R$ R-symmetry: $\tilde{\phi}\phi \to \tilde{\phi}\phi$,.
$S \rightarrow e^{i\alpha}S$, $W \rightarrow e^{i\alpha}W$. Actually, $W$ is the most general renormalizable superpotential allowed by $U(1)_R$ and $G$. Performing appropriate $G$ and $R$-transformations, we bring $\phi, \bar{\phi}, S$ on the real axis, i.e., $\phi = \phi \equiv \chi/2$, $S \equiv \sigma/\sqrt{2}$ where $\chi, \sigma$ are normalized real scalar fields. $V$ then takes the form in Eq.(1) with $\kappa = \lambda$ and $m = 0$. So, Linde’s potential for hybrid inflation is almost obtainable from SUSY GUTs but without the mass term of $\sigma$ which is, however, crucial for driving the inflaton towards the vacua.

One way to generate the necessary slope along the inflationary trajectory is [13] to include the one-loop radiative corrections on this trajectory ($\bar{\phi} = \phi = 0$, $|S| > S_c \equiv M$). In fact, SUSY breaking by the ‘vacuum’ energy density $\kappa^2 M^4$ along this valley causes a mass splitting in the supermultiplets $\bar{\phi}, \phi$. We obtain a Dirac fermion with mass $\kappa^2 |S|^2$ and two complex scalars with mass $\kappa^2 |S|^2 \pm \kappa^2 M^2$. This leads to the existence of important one-loop radiative corrections to $V$ on the inflationary valley which can be found from the Coleman-Weinberg formula [14]:

$$\Delta V = \frac{1}{64\pi^2} \sum_i (-)^{F_i} M_i^4 \ln \frac{M_i^2}{\Lambda^2},$$

(5)

where the sum extends over all helicity states $i$, $F_i$ and $M_i^2$ are the fermion number and mass$^2$ of the $i$th state, and $\Lambda$ is a renormalization mass scale. We find that $\Delta V(|S|)$ is given by

$$\kappa^2 M^4 \frac{\kappa^2 N}{32\pi^2} \left( 2 \ln \frac{\kappa^2 |S|^2}{\Lambda^2} + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1}) \right),$$

(6)

where $z = x^2 = |S|^2/M^2$ and $N$ is the dimensionality of the representations to which $\bar{\phi}, \phi$ belong. For $z \gg 1$ ($|S| \gg S_c$), the effective potential on this trajectory can be expanded as [13, 15]

$$V_{\text{eff}}(|S|) = \kappa^2 M^4 \left[ 1 + \frac{\kappa^2 N}{16\pi^2} \left( \ln \frac{\kappa^2 |S|^2}{\Lambda^2} + \frac{3}{2} - \frac{1}{12z^2} + \cdots \right) \right].$$

(7)

Note that the slope along the inflationary valley which is provided by these radiative corrections is $\Lambda$-independent.

From Eq.(6) and using the standard formalism (see e.g., Ref.[5]), we find that the quadrupole anisotropy of CMBR is

$$\left( \frac{\delta T}{T} \right)_Q \approx \frac{8\pi}{\sqrt{N}} \left( \frac{N_Q}{45} \right)^{\frac{1}{2}} \left( \frac{M}{M_P} \right)^2 x_Q^{-1} y_Q^{-1} \Lambda(x_Q^2)^{-1},$$

(8)

with

$$\Lambda(z) = (z + 1) \ln(1 + z^{-1}) + (z - 1) \ln(1 - z^{-1}),$$

(9)
\[ y_Q^2 = \int_1^{x_Q} \frac{dz}{z} \Lambda(z)^{-1}, \quad y_Q \geq 0. \] (10)

Here, \( N_Q \) is the number of e-foldings suffered by our present horizon scale during inflation, and \( x_Q = |S_Q|/M \), with \( S_Q \) being the value of \( S \) when our present horizon scale crossed outside the inflationary horizon. For \( |S_Q| \gg S_c \), \( y_Q = x_Q(1 - 7/12 x_Q^2 + \cdots) \). Finally, from Eq.(6), one finds

\[ \kappa \approx \frac{8\pi^{\frac{3}{2}}}{\sqrt{NN_Q}} y_Q \frac{M}{M_P}. \] (11)

The slow roll conditions (see e.g., Ref.[5]) for SUSY hybrid inflation are \( \epsilon, |\eta| \ll 1 \), where

\[ \epsilon = \left( \frac{\kappa^2 M_P}{16\pi^2 M} \right)^2 \frac{N^2 x^2}{8\pi} \Lambda(x^2)^2, \] (12)

\[ \eta = \left( \frac{\kappa M_P}{4\pi M} \right)^2 \frac{N}{8\pi} \left( (3z + 1) \ln(1 + z^{-1}) + (3z - 1) \ln(1 - z^{-1}) \right). \] (13)

Note that \( \eta \to -\infty \) as \( x \to 1^+ \). However, for most relevant values of the parameters (\( \kappa \ll 1 \)), the slow roll conditions are violated only 'infinitesimally' close to the critical point at \( x = 1 (|S| = S_c) \). So, inflation continues practically until this point is reached, where the ‘waterfall’ occurs.

From the COBE [3] result, \( (\delta T/T)_Q \approx 6.6 \times 10^{-6} \), and eliminating \( x_Q \) between Eqs.(8) and (11), we obtain \( M \) as a function of \( \kappa \). For \( x_Q \to \infty \), \( y_Q \to x_Q \) and \( x_Q y_Q \Lambda(x_Q^2) \to 1^- \). Thus, the maximal \( M \) is achieved in this limit and equals about \( 10^{16} \) GeV (for \( N = 8, N_Q \approx 55 \)). This value of \( M \), although somewhat smaller than the SUSY GUT scale, is quite close to it. As a numerical example, take \( \kappa = 4 \times 10^{-3} \) which gives \( M \approx 9.57 \times 10^{15} \) GeV, \( x_Q \approx 2.633, y_Q \approx 2.42 \). The slow roll conditions are violated at \( x - 1 \approx 7.23 \times 10^{-5} \), where \( \eta = -1 (\epsilon \approx 8.17 \times 10^{-8} \text{ at } x = 1) \). The spectral index of density perturbations \( n = 1 - 6\epsilon + 2\eta \) [16] is \( \approx 0.985 \).

The SUSY hybrid inflationary scenario can be considered ‘natural’ for the following reasons:

i. There is no need of ‘tiny’ coupling constants (\( \kappa \sim 10^{-3} - 10^{-2} \)).

ii. The superpotential \( W \) in Eq.(3) has the most general renormalizable form allowed by the gauge and R-symmetries. Moreover, the coexistence of the \( S \) and \( S\phi\phi \) terms in \( W \) implies that the combination \( \phi\phi \) is ‘neutral’ under all possible symmetries of \( W \) and, thus, all the non-renormalizable terms of the form \( S(\phi\phi)^n, n \geq 2 \), are also necessarily present in \( W \) [17]. The leading term of this type \( S(\phi\phi)^2 \), if its dimensionless coefficient is of order unity, can be comparable to \( S\phi\phi \) (recall
that $\kappa \sim 10^{-3}$) and, thus, play an important role in inflation (see Sec.3). All higher order terms of this type with $n \geq 3$ give negligible contributions to the inflationary potential. The presence of $U(1)_R$ is crucial since it guarantees the linearity of $W$ in $S$ to all orders excluding terms such as $S^2$ which could generate an inflaton mass $\geq H$, thereby ruining inflation by violating the slow roll conditions.

iii. SUSY guarantees that the radiative corrections do not invalidate [18] inflation, but rather provide [13] a slope along the inflationary trajectory, needed for driving the inflaton towards the SUSY vacua.

iv. Supergravity (SUGRA) corrections can be brought under control leaving inflation intact. The scalar potential in SUGRA is given [19] by

$$V = \exp \left( \frac{K}{m_P^2} \right) \left[ (K^{-1})^i_j F^i F_j - 3 \frac{|W|^2}{m_P^2} \right], \quad (14)$$

where $K$ is the Kähler potential, $m_P = M_P/\sqrt{8\pi} \approx 2.44 \times 10^{18}$ GeV in the ‘reduced’ Planck scale, $F^i = W^i + K^i W/m_P^2$, and upper (lower) indices denote differentiation with respect to the scalar field $\phi_i (\phi^{*i})$. $K$ can be expanded as $K = |S|^2 + |\phi|^2 + |\bar{\phi}|^2 + \alpha |S|^4/m_P^2 + \cdots$, where the leading (quadratic) terms constitute the ‘minimal’ Kähler potential. The term $|S|^2$, whose coefficient is necessarily normalized to unity, could generate a mass $\sim \kappa^2 M^4/m_P^2 \sim H^2$ for $S$ along the inflationary trajectory from the expansion of the exponential prefactor in Eq.(14). This would ruin inflation. Fortunately, with this particular form of $W$ (including all the higher order terms) this mass $\sim H^2$ is exactly cancelled in $V$ [12, 20]. The linearity of $W$ in $S$, which is guaranteed to all orders by the R-symmetry, is crucial for this cancellation to take place. This is an important property of this scheme. The $|S|^4$ term in $K$ also generates a mass $\sim \kappa^2 M^4/m_P^2 \sim H^2$ for $S$ via the factor $(\partial^2 K/\partial S \partial S^*)^{-1} = 1 - 4\alpha |S|^2/m_P^2 + \cdots$ in Eq.(14), which is however not cancelled (see e.g., Ref.[21]). In order to avoid ruining the present inflationary scheme, one has then to assume [10, 15] that $|\alpha| < \sim 10^{-3}$. All other higher order terms in $K$ are harmless since they give suppressed contributions ($|S| \ll m_P$ on the inflationary path). So, we see that a mild tuning of just one parameter is adequate for controlling SUGRA corrections. This is a great advantage of this model since in other cases tuning of infinitely many parameters is required. Moreover, note that with special forms of the Kähler potential one can solve this problem even without a mild tuning. An example is given in Ref.[11], where the dangerous mass $\sim \kappa^2 M^4/m_P^2$ could be cancelled in the presence of fields without superpotential but with large vevs generated via D-terms. This property practically persists even in the extensions of the model we will consider in Sec.3.
In summary, for all these reasons, we consider SUSY hybrid inflation (with its extensions) as an extremely ‘natural’ inflationary scenario.

2. Hybrid Inflation in Concrete SUSY GUTs

We will now discuss how the SUSY hybrid inflationary scenario can be ‘embedded’ in concrete SUSY GUTs. As an example, let us consider a moderate extension of the minimal supersymmetric standard model (MSSM) based on the left-right symmetric gauge group

\[ G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \]  

(see Refs.[15, 22, 23]). The breaking of \( G_{LR} \) to \( G_S \) is achieved via a conjugate pair of \( SU(2)_R \) doublet superfields \( \tilde{l}_c, l_c \) with \( B-L \) (baryon minus lepton number) equal to -1, 1, which acquire vevs along their right handed neutrino directions \( \bar{\nu}_c H, \nu_c H \) corresponding to the superfields \( \bar{\phi}, \phi \) in Sec.1.2.

The (renormalizable) superpotential for the breaking of \( G_{LR} \) is

\[ W = \kappa S( -M^2 + \bar{l}^c l^c ) \]  

where \( \kappa, M \) can be made positive by field redefinitions. This superpotential leads to hybrid inflation exactly as \( W \) in Eq.(3). The quadrupole anisotropy of CMBR, \( (\delta T/T)_Q \), and the coupling constant \( \kappa \) are given by Eqs.(8) and (11) with \( N = 2 \) since \( \bar{l}^c, l^c \) have two components each.

An important shortcoming of MSSM is that there is no understanding of how the SUSY \( \mu \) term, with the right magnitude of \( |\mu| \sim 10^2 - 10^3 \text{ GeV} \), arises. One way [24] to solve this \( \mu \) problem is via a Peccei-Quinn (PQ) symmetry \( U(1)_{PQ} \) [25], which also solves the strong CP problem. This solution is based on the observation [26] that the axion decay constant \( f_a \), which is the symmetry breaking scale of \( U(1)_{PQ} \), is (normally) ‘intermediate’ \( (\sim 10^{11} - 10^{12} \text{ GeV}) \) and, thus, \( |\mu| \sim f_a^2/m_P \). The scale \( f_a \) is, in turn, \( \sim (m_{3/2}m_P)^{1/2} \), where \( m_{3/2} \sim 1 \text{ TeV} \) is the gravity-mediated soft SUSY breaking scale (gravitino mass). In order to implement this solution of the \( \mu \) problem, we introduce a pair of gauge singlet superfields \( \bar{N}, N \) with PQ charges 1, -1 and the non-renormalizable couplings \( \lambda_1 \bar{N}^2 h^2/m_P, \lambda_2 \bar{N}^2 N^2/m_P \) in the superpotential. Here, \( h = (h^{(1)}, h^{(2)}) \) is the electroweak Higgs superfield, which is a bidoublet under \( SU(2)_L \times SU(2)_R \), and \( \lambda_{1,2} \) are taken positive by redefining the phases of \( \bar{N}, N \). After SUSY breaking, the \( N^2 \bar{N}^2 \) term leads to the scalar potential:

\[ V_{PQ} = \left( m_{3/2}^2 + 4\lambda_1^2 \left( \frac{\bar{N}N}{m_P} \right)^2 \right) \left[ (|\bar{N}| - |N|)^2 + 2|ar{N}||N| \right] 
+ 2|A|m_{3/2}^2 \left( \frac{\bar{N}N}{m_P} \right)^2 \cos(2\theta + 2\theta), \]  

(17)
where $A$ is the dimensionless coefficient of the soft SUSY breaking term corresponding to the superpotential term $\bar{N}^2 N^2$ and $\epsilon, \theta, \bar{\theta}$ are the phases of $A, \bar{N}, N$ respectively. Minimization of $V_{PQ}$ then requires $|\bar{N}| = |N|$, $\epsilon + 2\bar{\theta} + 2\theta = \pi$ and $V_{PQ}$ takes the form

$$V_{PQ} = 2|N|^2 m_{3/2}^2 \left( 4\lambda_2^2 \frac{|N|^4}{m_{3/2}^2 m_P^2} - |A|\lambda_2 \frac{|N|^2}{m_{3/2}^2 m_P} + 1 \right). \quad (18)$$

For $|A| > 4$, the absolute minimum of the potential is at

$$|\langle \bar{N} \rangle| = |\langle N \rangle| \equiv \frac{f_a}{2} = (m_{3/2} m_P)^{1/2} \left( \frac{|A| + (|A|^2 - 12)^{1/2}}{12\lambda_2} \right)^{1/2} \sim (m_{3/2} m_P)^{1/2}. \quad (19)$$

The $\mu$ term is generated via the $N^2 h^2$ superpotential term with $|\mu| = 2\lambda_1 |\langle N \rangle|^2 / m_P$, which is of the right magnitude.

The potential $V_{PQ}$ also has a local minimum at $\bar{N} = N = 0$, which is separated from the global PQ minimum by a sizable potential barrier preventing a successful transition from the trivial to the PQ vacuum. This situation persists at all cosmic temperatures after the ‘reheating’ which follows hybrid inflation, as has been shown [17] by considering the one-loop temperature corrections [27] to the potential. We are, thus, obliged to assume that, after the end of inflation, the system emerges in the PQ vacuum since, otherwise, it will be stuck for ever in the trivial vacuum.

The gauge group $G_{LR}$ implies the presence of right handed neutrino superfields $\nu^c_i$ ($i = 1, 2, 3$ is the family index), which form $SU(2)_R$ doublets $L^c_i = (\nu^c_i, e^c_i)$ with the $SU(2)_L$ singlet charged antileptons $e^c_i$. In order to generate ‘intermediate’ scale masses for the $\nu^c$s, we introduce the non-renormalizable superpotential couplings $\gamma_i \bar{\ell}^c L_i L^c_i / m_P$ (in a basis with diagonal and positive $\gamma$’s). The $\nu^c$ masses are then $M_i = 2\gamma_i m^2 / m_P$ (with $\langle \ell^c \rangle$, $\langle \ell \rangle$ taken positive by a $B-L$ transformation). Light neutrinos acquire hierarchical masses via the seesaw mechanism and, thus, cannot play the role of hot dark matter (HDM) in the universe. They are more appropriate for a universe with non-zero cosmological constant ($\Lambda \neq 0$) favored by recent observations [28]. The presence of HDM in such a universe is not necessary [29, 30]. Note that the couplings which generate the $\nu^c$ masses are also responsible for the inflaton decay (see Sec.2.1).

From our discussion above, it is clear that the superpotential of the model must contain the following extra couplings:

$$N^2 h^2, \quad \bar{N}^2 N^2, \quad hQQ^c, \quad hLL^c, \quad \bar{\ell}^c L^c L^c. \quad (20)$$

Here $Q_i$ and $L_i$ are the $SU(2)_L$ doublet left handed quark and lepton superfields, whereas $Q^c_i = (u^c_i, d^c_i)$ are the $SU(2)_R$ doublet antiquarks. The
quartic terms in Eq. (20) carry a factor $m_P^{-1}$ which has been left out together with the dimensionless coupling constants and family indices.

The continuous global symmetries of this superpotential are $U(1)_B$ (and, consequently, $U(1)_L$) with the extra superfields $l^c, l^c, S, N, N$ carrying zero baryon number, an anomalous PQ symmetry $U(1)_{PQ}$, and a non-anomalous R-symmetry $U(1)_R$. The $PQ$ and $R$ charges of the superfields are as follows ($W$ carries one unit of $R$ charge):

$$
\begin{align*}
PQ: & \bar{l}^c, l^c, S, Q^c, L^c (0), h, \tilde{N} (1), Q, L, N (-1); \\
R: & \bar{l}^c, l^c, h, \tilde{N} (0), S (1), Q, Q^c, L, L^c, N (1/2).
\end{align*}
$$

(21)

Note that $U(1)_B$ (and, thus, $U(1)_L$) invariance is automatically implied by $U(1)_R$ even if all possible non-renormalizable terms are included in the superpotential. This is due to the fact that the $R$ charges of the products of any three color (anti)triplets exceed unity and cannot be compensated since there are no negative $R$ charges available.

In order to avoid undesirable mixing of the components of the $L$’s with the ones of $l^c$ or $h^{(2)}$ via the allowed superpotential couplings $NNLL^c$, $NNLh^c$, we impose a $Z_2$ symmetry (‘lepton parity’) under which $L, L^c$ change sign. This symmetry is equivalent to $Z_2$ ‘matter parity’ under which $L, L^c, Q, Q^c$ change sign since $U(1)_B$ is also present and contains ‘baryon parity’ under which $Q, Q^c$ change sign. The only superpotential terms which are permitted by $U(1)_{PQ}, U(1)_R$ and ‘matter parity’ are the ones already included and $LL^cL^cN^2L^c$ and $LL^cL^chh$ modulo arbitrary multiplications by non-negative powers of the combination $l^c l^c$. The vevs of $\bar{l}^c, l^c, \tilde{N}, N$ leave unbroken only the symmetries $G_S, U(1)_B$ and ‘matter parity’.

### 2.1. ‘REHEATING’ AND THE GRAVITINO CONSTRAINT

A complete inflationary scenario should be followed by a successful ‘reheating’ satisfying the gravitino constraint [31] on the ‘reheat’ temperature, $T_r \lesssim 10^9$ GeV, and generating the observed baryon asymmetry of the universe (BAU). After the end of inflation, the system falls towards the SUSY vacuum and performs damped oscillations about it. The inflaton (oscillating system) consists of the two complex scalar fields

$$
\theta = (\delta \bar{\nu}_H^c + \delta \nu_H^c)/\sqrt{2},
$$

$$
(\delta \bar{\nu}_H^c = \bar{\nu}_H^c - M, \delta \nu_H^c = \nu_H^c - M) \text{ and } S,
$$

with equal mass $m_{\text{infl}} = \sqrt{2\kappa M}$.

The oscillating fields $\theta$ and $S$ decay into a pair of right handed neutrinos ($\psi_{\nu_i}$) and sneutrinos ($\nu_{\nu_i}^c$) respectively via the superpotential couplings $\bar{l}^c l^c L^c L^c$ and $S \bar{l}^c l^c$. The relevant Lagrangian terms are:

$$
L_{\text{decay}}^\theta = -\sqrt{2} \gamma_i \bar{M} \theta \psi_{\nu_i}^c \psi_{\nu_i}^c + h.c.,
$$

(22)
\[ L^{S}_{\text{decay}} = -\sqrt{2}\gamma_i \frac{M}{m_P} S^* \nu^c_i \bar{\nu}^c_i m_{\text{infl}} + h.c. , \]  

(23)

and the common, as it turns out, decay width is given by

\[ \Gamma = \Gamma_{\theta \rightarrow \bar{\psi}_i \nu^c_i} = \Gamma_{S \rightarrow \nu^c_i \bar{\nu}^c_i} = \frac{1}{8\pi} \left( \frac{M_i}{M} \right)^2 m_{\text{infl}} , \]  

(24)

provided that the mass \( M_i = 2\gamma_i M^2 / m_P \) of the relevant \( \nu^c_i \) satisfies the inequality \( M_i < m_{\text{infl}} / 2 \).

To minimize the number of small coupling constants, we assume that

\[ M_2 < \frac{1}{2} m_{\text{infl}} \leq M_3 = \frac{2M^2}{m_P} \]  

(with \( \gamma_3 = 1 \)),  

(25)

so that the inflaton decays into the second heaviest right handed neutrino superfield \( \nu^c_2 \) with mass \( M_2 \). The second inequality in Eq.(25) implies that \( y_Q \leq \sqrt{2N_Q / \pi} \approx 3.34 \) for \( N_Q \approx 55 \). This gives \( x_Q \approx 3.5 \). As an example, choose \( x_Q \approx 1.05 \) (bigger values cannot lead to an adequate BAU) which yields \( y_Q \approx 0.28 \). From the COBE [3] result, we then obtain \( M \approx 4.06 \times 10^{15} \) GeV, \( \kappa \approx 4 \times 10^{-4} \), \( m_{\text{infl}} \approx 2.3 \times 10^{12} \) GeV and \( M_3 \approx 1.35 \times 10^{13} \) GeV.

The ‘reheat’ temperature \( T_r \), for the MSSM spectrum, is given by [15]

\[ T_r \approx \frac{1}{T}(\Gamma M_P)^{\frac{1}{2}} \]  

(26)

and must satisfy the gravitino constraint [31], \( T_r < \approx 10^9 \) GeV, for gravity-mediated SUSY breaking with universal boundary conditions. To maximize the ‘naturalness’ of the model, we take the maximal value of \( M_2 \) (and, thus, \( \gamma_2 \)) allowed by this constraint. This is \( M_2 \approx 2.7 \times 10^{10} \) GeV (\( \gamma_2 \approx 2 \times 10^{-3} \)). Note that, with this \( M_2 \), the first inequality in Eq.(25) is well satisfied.

2.2. BARYOGENESIS VIA LEPTOGENESIS

In hybrid inflationary models, it is [32] generally not so convenient to generate the observed BAU in the usual way, i.e., through the decay of super-heavy color (anti)triplets. Some of the reasons are:

i. Baryon number is practically conserved in most models of this type.

In some cases [33], this is a consequence of a discrete ‘baryon parity’ symmetry. In the left-right model under consideration, \( B \) is exactly conserved due to the presence of the R-symmetry.

ii. The gravitino constraint would require that the mass of the relevant color (anti)triplets does not exceed \( 10^{10} \) GeV. This suggests strong deviations from the MSSM gauge coupling unification and possibly leads into problems with proton stability.
It is generally preferable to produce first a primordial lepton asymmetry [34] which is then partly converted into baryon asymmetry by the non-perturbative electroweak sphaleron effects [35]. Actually, in the left-right model under consideration as well as in many other models, this is the only way to generate the observed BAU since the inflaton decays into right handed neutrino superfields. The subsequent decay of these superfields into lepton (antilepton) $L$ ($\bar{L}$) and electroweak Higgs superfields can only produce a lepton asymmetry. It is important to ensure that this lepton asymmetry is not erased [36] by lepton number violating $2 \to 2$ scattering processes such as $LL \to h^{(1)\ast}h^{(1)\ast}$ or $Lh^{(1)} \to Lh^{(1)\ast}$ at all temperatures between $T_r$ and 100 GeV. This is automatically satisfied since the lepton asymmetry is protected [37] by SUSY at temperatures between $T_r$ and $T \sim 10^7$ GeV and, for $T \lesssim 10^7$ GeV, these scattering processes are well out of equilibrium provided [37] $m_{\nu_L} \lesssim 10$ eV, which readily holds in our case (see below). For MSSM spectrum, the observed BAU $n_B/s$ is related [37] to the primordial lepton asymmetry $n_L/s$ by $n_B/s = (-28/79)n_L/s$.

As already mentioned, the lepton asymmetry is produced through the decay of the superfield $\nu_2^c$, which emerges as decay product of the inflaton. This superfield decays into electroweak Higgs and (anti)lepton superfields. The relevant one-loop diagrams are both of the vertex and self-energy type [38] with an exchange of $\nu_3^c$. The resulting lepton asymmetry is [39]

$$n_L/s \approx 1.33 \frac{9T_r}{16\pi m_{\text{inf}}} \frac{M_2}{M_3} \frac{c^2 s^2 \sin 2\delta (m^D_3^2 - m^D_2^2)^2}{|\langle h^{(1)} \rangle|^2 (m^D_3^2 s^2 + m^D_2^2 c^2)},$$

(27)

where $|\langle h^{(1)} \rangle| \approx 174$ GeV, $m^D_{2,3}$ ($m^D_2 \leq m^D_3$) are the ‘Dirac’ neutrino masses (in a basis where they are diagonal and positive), and $c = \cos \theta$, $s = \sin \theta$, with $\theta$ and $\delta$ being the rotation angle and phase which diagonalize the Majorana mass matrix of the right handed neutrinos. Note that Eq. (27) holds [40] provided that $M_2 \ll M_3$ and the decay width of $\nu_3^c$ is $\ll (M_3^2 - M_2^2)/M_2$, and both conditions are well satisfied in our model. Here, we concentrated on the two heaviest families ($i = 2, 3$) and ignored the first one. We were able to do this since the analysis [41] of the CHOOZ experiment [42] shows that the solar and atmospheric neutrino oscillations decouple.

The light neutrino mass matrix is given by the seesaw formula:

$$m_\nu \approx -\tilde{m}^D \frac{1}{M} m^D,$$

(28)

where $m^D$ is the ‘Dirac’ neutrino mass matrix and $M$ the Majorana mass matrix of right handed neutrinos. The determinant and the trace invariance of the light neutrino mass matrix imply [39] two constraints on the
(asymptotic) parameters which take the form:

\[ m_2 m_3 = \frac{(m_2^D m_3^D)^2}{M_2 M_3}, \]  

(29)

\[ m_2^2 + m_3^2 = \frac{(m_2^D c^2 + m_3^D s^2)^2}{M_2^2} + \]

\[ \frac{(m_3^D c^2 + m_2^D s^2)^2}{M_3^2} + \frac{2(m_3^D c^2 + m_2^D s^2)^2 c^2 s^2 \cos 2\delta}{M_2 M_3}, \]  

(30)

where \( m_2 = m_{\nu_\mu} \) and \( m_3 = m_{\nu_\tau} \) are the (positive) eigenvalues of \( m_{\nu} \).

The \( \mu - \tau \) mixing angle \( \theta_{23} = \theta_{\mu\tau} \) lies \([39]\) in the range

\[ |\varphi - \theta^D| \leq \theta_{\mu\tau} \leq \varphi + \theta^D, \text{ for } \varphi + \theta^D \leq \pi/2, \]  

(31)

where \( \varphi \) is the rotation angle which diagonalizes the light neutrino mass matrix in the basis where the ‘Dirac’ mass matrix is diagonal and \( \theta^D \) is the ‘Dirac’ mixing angle (i.e., the ‘unphysical’ mixing angle with zero Majorana masses for the right handed neutrinos).

We take \( m_{\nu_\mu} \approx 2.6 \times 10^{-3} \text{ eV} \) which is the central value from the small angle MSW resolution of the solar neutrino problem \([43]\). The \( \tau \)-neutrino mass is taken to be \( m_{\nu_\tau} \approx 7 \times 10^{-2} \text{ eV} \) which is the central value implied by SuperKamiokande \([44]\). We choose \( \delta \approx -\pi/4 \) to maximize \(-n_L/s\). Finally, we assume that \( \theta^D \) is negligible, so that maximal \( \nu_\mu - \nu_\tau \) mixing, which is favored by SuperKamiokande \([44]\), corresponds to \( \varphi \approx \pi/4 \).

From the determinant and trace constraints and the diagonalization of \( m_{\nu_\mu} \), we determine the value of \( m_3^D \) corresponding to maximal \( \nu_\mu - \nu_\tau \) mixing (\( \varphi \approx \pi/4 \)) for any given \( \kappa \). We find that a solution for \( m_3^D \) exists provided that \( M_2 \leq 0.037 M_3 \). For the numerical example in Sec.2.1, we find \( m_3^D \approx 8.3 \text{ GeV} \) and \( m_2^D \approx 0.98 \text{ GeV} \). The lepton asymmetry turns out to be \( n_L/s \approx -2.23 \times 10^{-10} \) and, thus, the baryogenesis constraint is satisfied. We see that, with not too ‘unnatural’ values of \( \kappa \) (\( \approx 4 \times 10^{-4} \)) and the other relevant parameters (\( \gamma_2 \approx 2 \times 10^{-3}, \gamma_3 \approx 1 \)), we can not only reproduce the results of COBE \([3]\) but also have a successful ‘reheating’ satisfying the gravitino and baryogenesis requirements together with the constraints from solar and atmospheric neutrino oscillations.

3. Extensions of SUSY Hybrid Inflation

In trying to apply (SUSY) hybrid inflation to higher GUT gauge groups which predict the existence of monopoles, we encounter the following problem. Inflation is terminated abruptly as the system reaches the critical point
on the inflationary trajectory and is followed by the ‘waterfall’ regime during which the scalar fields $\phi, \bar{\phi}$ develop their vevs starting from zero and the spontaneous breaking of the GUT gauge symmetry occurs. The fields $\phi, \bar{\phi}$ can end up at any point of the vacuum manifold with equal probability and, thus, monopoles are copiously produced \cite{6} via the Kibble mechanism \cite{45} leading to a cosmological catastrophe (see e.g., Ref.\cite{46}).

One of the simplest GUT models predicting monopoles is the Pati-Salam (PS) model \cite{47} based on the gauge group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$. These monopoles carry two units of ‘Dirac’ magnetic charge \cite{48}. We will discuss possible solutions \cite{6, 17} of the magnetic monopole problem of hybrid inflation within the SUSY PS model, although our mechanisms can be readily extended to other semi-simple gauge groups such as the ‘trinification’ group $SU(3)_c \times SU(3)_L \times SU(3)_R$, which emerges from string theory and predicts \cite{49} monopoles with triple ‘Dirac’ charge, and possibly to simple gauge groups such as $SO(10)$.

3.1. SHIFTED HYBRID INFLATION

One idea \cite{17} for solving the monopole problem is to include into the standard superpotential for hybrid inflation the leading non-renormalizable term, which, as explained in Sec.1.2, cannot be excluded by any symmetries. If its dimensionless coefficient is of order unity, this term can be comparable with the trilinear coupling of the standard superpotential (whose coefficient is $\sim 10^{-3}$). The coexistence of these terms reveals a completely new picture. In particular, there appears a non-trivial (classically) flat direction along which $G_{PS}$ is spontaneously broken with the appropriate Higgs fields $(\phi, \bar{\phi})$ acquiring constant values. This ‘shifted’ flat direction can be used as inflationary trajectory with the necessary inclination obtained again from one-loop radiative corrections \cite{13}. The termination of inflation is again abrupt followed by a ‘waterfall’ but no monopoles are formed in this transition since $G_{PS}$ is already spontaneously broken during inflation.

The spontaneous breaking of the gauge group $G_{PS}$ to $G_S$ is achieved via the vevs of a conjugate pair of Higgs superfields

$$
\bar{H}^c = (4, 1, 2) \equiv \left( \bar{u}^c_H, \bar{d}^c_H, \bar{e}^c_H, \bar{\nu}^c_H \right),
$$

$$
H^c = (\bar{4}, 1, 2) \equiv \left( u^c_H, d^c_H, e^c_H, \nu^c_H \right),
$$

in the $\bar{\nu}_H, \nu_H^c$ directions. The relevant part of the superpotential, which includes the leading non-renormalizable term, is

$$
\delta W = \kappa S (-M^2 + \bar{H}^c H^c) - \beta S (\bar{H}^c H^c)^2 M^2_S,
$$

\cite{32}.
where $M_S \approx 5 \times 10^{17}$ GeV is the string scale and $\beta$ is taken positive for simplicity. D-flatness implies that $\bar{H}^c* = e^{i\theta} H^c$. We restrict ourselves to the direction with $\theta = 0$ ($\bar{H}^c* = H^c$) containing the non-trivial inflationary path (see below). The scalar potential derived from $\delta W$ then takes the form

$$V = \left[ \kappa (|H^c|^2 - M^2) - \beta \frac{|H^c|^4}{M_S^2} \right]^2 + 2\kappa^2 |S|^2 |H^c|^2 \left[ 1 - \frac{2\beta}{\kappa M_S^2} |H^c|^2 \right]^2. \quad (34)$$

Defining the dimensionless variables $w = |S|/M$, $y = |H^c|/M$, we obtain

$$\tilde{V} = \frac{V}{\kappa^2 M^4} = (y^2 - 1 - \xi y^4)^2 + 2w^2 y^2 (1 - 2\xi y^2)^2, \quad (35)$$

where $\xi = \beta M^2 / \kappa M_S^2$. This potential is a simple extension of the standard potential for SUSY hybrid inflation (which corresponds to $\xi = 0$) and appears in a wide class of models incorporating the leading non-renormalizable correction to the standard hybrid inflationary superpotential.

For constant $w$ (or $|S|$), $\tilde{V}$ in Eq.(35) has extrema at

$$y_1 = 0, \quad y_2 = \frac{1}{\sqrt{2}\xi}, \quad y_3 \pm = \frac{1}{\sqrt{2}\xi} \sqrt{(1 - 6\xi w^2) \pm \sqrt{(1 - 6\xi w^2)^2 - 4\xi(1 - w^2)}}. \quad (36)$$

Note that the first two extrema (at $y_1$, $y_2$) are $|S|$-independent and, thus, correspond to classically flat directions, the trivial one at $y_1 = 0$ with $\tilde{V}_1 = 1$, and the non-trivial one at $y_2 = 1/\sqrt{2}\xi = constant$ with $\tilde{V}_2 = (1/4\xi - 1)^2$, which we will use as our inflationary path. The trivial trajectory is a valley of minima for $w > 1$, while the non-trivial one for $w > w_0 = (1/8\xi - 1/2)^{1/2}$, which is its instability (critical) point. We take $\xi < 1/4$, so that $w_0 > 0$ and the non-trivial trajectory is destabilized before $w$ reaches zero (the destabilization is in the chosen direction $\bar{H}^c* = H^c$). The extrema at $y_3 \pm$, which are $|S|$-dependent and non-flat, do not exist for all values of $w$ and $\xi$, since the expressions under the square roots in Eq.(36) are not always non-negative. These two extrema, at $w = 0$, become the SUSY vacua. The relevant SUSY vacuum (see below) corresponds to $y_{3-}(w = 0)$ and, thus, the common vev $v_0$ of $\bar{H}^c$, $H^c$ is given by

$$\left( \frac{v_0}{M} \right)^2 = \frac{1}{2\xi} (1 - \sqrt{1 - 4\xi}). \quad (37)$$

We will now discuss the structure of $\tilde{V}$ and the inflationary history in the most interesting range of $\xi$, which is $1/4 > \xi > 1/6$. For fixed $w > 1$, there exist two local minima at $y_1 = 0$ and $y_2 = 1/\sqrt{2}\xi$, which corresponds to lower potential energy density, and a local maximum at $y_{3+}$ lying between the minima. As $w$ becomes smaller than unity, the extremum at $y_1$ turns
into a local maximum, while the extremum at $y_{3+}$ disappears. The system can freely fall into the non-trivial (desirable) trajectory at $y_2$ even if it started at $y_1 = 0$. As we further decrease $w$ below $(2 - \sqrt{36\xi - 5})^{1/2}/3\sqrt{2} \xi$, a pair of new extrema, a local minimum at $y_{3-}$ and a local maximum at $y_{3+}$, are created between $y_1$ and $y_2$. As $w$ crosses $(1/8\xi - 1/2)^{1/2}$, the local maximum at $y_{3+}$ crosses $y_2$ becoming a local minimum. At the same time, the local minimum at $y_2$ turns into a local maximum and inflation along the ‘shifted’ trajectory is terminated with the system falling into the local minimum at $y_{3-}$ which, at $w = 0$, develops into a SUSY vacuum.

We see that, no matter where the system starts from, it always passes from the ‘shifted’ trajectory, where the relevant part of inflation takes place, before falling into the SUSY vacuum. So, $G_{PS}$ is already broken during inflation and no monopoles are produced at the ‘waterfall’.

It should be noted that, after inflation, the system could fall into the minimum at $y_{3+}$ instead of the one at $y_{3-}$. This, however, does not happen since in the last e-folding or so the barrier between the minima at $y_{3-}$ and $y_2$ is considerably reduced and the decay of the ‘false vacuum’ at $y_2$ to the minimum at $y_{3-}$ is completed within a fraction of an e-folding before the $y_{3+}$ minimum even comes into existence. This transition is further accelerated by the inflationary density perturbations.

The mass spectrum on the ‘shifted’ trajectory can be evaluated [17]. We find that the only mass splitting in supermultiplets occurs in the $\tilde{\nu}_H^c, \nu_H^c$ sector. Namely, we obtain one Majorana fermion with mass $2$ equal to $4\kappa^2 |S|^2$, which corresponds to the direction $(\tilde{\nu}_H^c + \nu_H^c)/\sqrt{2}$, and two normalized real scalars $\text{Re}(\delta\nu_H^c + \delta\nu_H^c)$ and $\text{Im}(\delta\nu_H^c + \delta\nu_H^c)$ with $m^2 = 4\kappa^2 |S|^2 + 2\kappa^2 m^2$. Here, $m = M(1/4\xi - 1)^{1/2}$ and $\delta\nu_H^c = \nu_H^c - v$, $\delta\nu_H^c = \nu_H^c - v$ with $v = (\kappa M^2 / 2\beta)^{1/2}$ being the common value of $\bar{H}^c, H^c$ on the trajectory.

The radiative corrections on the non-trivial trajectory can then be found from the Coleman-Weinberg formula [14] in Eq.(5) and $(\delta T/T)Q$ and $\kappa$ can be evaluated. We find the same expressions as in Eqs.(8) and (11) with $N = 2$ ($N = 4$) in the formula for $(\delta T/T)Q$ (\kappa) and $M$ generally replaced by $m$. The COBE [3] result can be reproduced, for instance, with $\kappa \approx 4 \times 10^{-3}$, which corresponds to $\xi = 1/5$, $v_0 \approx 1.7 \times 10^{16}$ GeV (for $\beta = 1$). The scales $M \approx 1.45 \times 10^{16}$ GeV, $m \approx 7.23 \times 10^{15}$ GeV and the ‘inflationary scale’, which characterizes the inflationary ‘vacuum’ energy density, $v_{\text{inf}} = \kappa^{1/2} m \approx 4.57 \times 10^{14}$ GeV. The spectral index $n = 0.954$.

The model can be completed by adding the superpotential terms

$$N^2 h^2, \bar{N}^2 N^2, \ hF F^c, \ \bar{H}^c H^c F^c F_c, \ \ G\bar{H}^c \bar{H}^c, \ \ G\bar{H}^c H^c.$$  \hfill (38)

In analogy with the model of Sec.2, the first two terms are needed for solving the $\mu$ problem via a PQ symmetry, the third term represents the Yokuwa couplings, with $F = (4,2,1)$, $F_c = (4,1,2)$ being the ‘matter’
superfields, and the fourth term is required for generating masses for the $\nu^c$'s (and, thus, for the $\nu$'s) and allowing the inflaton to decay. The last two terms, with $G = (6, 1, 1)$, are novel and were introduced \cite{50} in order to give masses to the superfields $\tilde{d}_H^c$, $\tilde{H}_H^c$ in $H^c$, $H^c$.

The model possesses a PQ and a $U(1)$ R-symmetry with charges

$$
PQ : \ H^c, \ H^c, \ S, \ F^c, \ G \ (0), \ h, \ \tilde{N} \ (1), \ F, \ N \ (-1);
$$

$$
R : \ H^c, \ H^c, \ h, \ \tilde{N} \ (0), \ S, \ G \ (1), \ F, \ F^c, \ N \ (1/2).
$$

(39)

We further impose a $Z_2^{mp}$ symmetry (‘matter parity’), under which $F, F^c$ change sign. Additional superpotential terms allowed by the symmetries of the model are $FFH^cH^cN^2$, $FFH^cH^chh$, $FFH^cH^cN^2$, $FFH^cH^chh$, $F^cF^cH^cH^c$. All superpotential terms can be multiplied by arbitrary non-negative powers of the combinations $H^cH^c$, $(H^c)^4$, $(H^c)^4$. The symmetry which is left unbroken by the vevs of $H^c$, $H^c$, $\tilde{N}$ and $N$ is $G_S \times Z_2^{mp}$.

In contrast to the model in Sec.2, $B$ (and $L$) violation is present here. It comes from the last three additional superpotential terms mentioned above (and the combinations $(H^c)^4$, $(H^c)^4$) which give couplings like $u^c d^c \nu_H^c$, $u^c d^c e^c \nu_H^c$, $u^c d^c H^c \nu_H^c$, $u^c d^c H^c \nu_H^c$ with appropriate coefficients. The couplings $G H^c H^c$ and $G H^c H^c$ also give rise to $B$ (and $L$) violation. Proton can decay via effective dimension five operators generated by one-loop graphs with two of the $u^c_H^c$, $d^c_H^c$ or one of the $u^c_H^c$, $d^c_H^c$ and one of the $\nu_H^c$, $\nu_H^c$ in the loop. Its lifetime, however, turns out to be long enough to make it practically stable.

The ‘reheating’ proceeds as in Sec.2.1 with $m_{\text{infl}}^2 = 2\kappa^2 v_0^2 (1 - 2\xi v_0^2 / M^2)^2$.

The infaton again decays into the superfield $\nu_2^c$ with mass $M_2 = 2\gamma_2 v_0^2 / M_S$, which is evaluated by saturating the gravitino constraint. The observed BAU is again generated via a primordial leptogenesis from the subsequent decay of $\nu_2^c$. The gravitino constraint together with the restrictions on the primordial lepton asymmetry ($1.8 \times 10^{-10} \lesssim -n_L / s \lesssim 2.3 \times 10^{-10}$) from the low deuterium abundance constraint \cite{51} on the BAU ($0.017 \lesssim \Omega_B h^2 \lesssim 0.021$) can be satisfied with ‘natural’ values of the relevant parameters and in accord with the neutrino oscillation data.

A typical solution, for $\kappa = 4 \times 10^{-3}$ ($m_{\text{infl}} \approx 4.1 \times 10^{13}$ GeV), is $M_2 \approx 5.9 \times 10^{10}$ GeV, $M_3 \approx 1.1 \times 10^{15}$ GeV ($\gamma_3 = 0.5$), $m_{\nu_\mu} \approx 7.6 \times 10^{-3}$ eV, $m_{\nu_\tau} \approx 8 \times 10^{-2}$ eV, $m_D^2 \approx 1.2$ GeV, $m_D^2 = 120$ GeV, $\sin^2 2\theta_{\mu\tau} \approx 0.87$, and $n_L / s \approx -1.8 \times 10^{-10}$ ($\theta \approx 0.016$ for $\delta \approx -\pi / 3$).

Note that the mass scale $v_0$ is of order $10^{16}$ GeV which is consistent with the unification of the gauge coupling constants of MSSM. Also, the ‘Dirac’ mass parameter $m_D^2$, after including renormalization effects with large $\tan \beta$ and MSSM spectrum, becomes consistent with ‘asymptotic’ Yukawa coupling unification which is implied by $SU(4)_c$ (see Ref.\cite{39}). Finally, the $\mu$-neutrino mass turns out to be consistent with the large rather than the small angle MSW resolution of the solar neutrino problem.
3.2. SMOOTH HYBRID INFLATION

An alternative solution to the monopole problem of hybrid inflation has been proposed [6] some years ago. We will present it here within the SUSY PS model of Sec.3.1, although it can be applied to other semi-simple (and possibly some simple) gauge groups too. The idea is to impose an extra $Z_2$ symmetry under which $\bar{H}^c H^c \rightarrow -\bar{H}^c H^c$ (say $H^c \rightarrow -H^c$). The whole structure of the model remains unaltered except that now only even powers of the combination $\bar{H}^c H^c$ are allowed in the superpotential terms.

The inflationary superpotential in Eq.(33) becomes

$$\delta W = S \left( -\mu^2 + \left( \frac{\bar{H}^c H^c}{M_S^2} \right)^2 \right),$$

where we absorbed the dimensionless parameters $\kappa, \beta$ in $\mu, M_S$. The resulting scalar potential $V$ is then given by

$$\tilde{V} = \frac{V}{\mu^4} = (1 - \tilde{\chi}^4)^2 + 16\tilde{\sigma}^2\tilde{\chi}^6,$$

where we used the dimensionless fields $\tilde{\chi} = \chi/(\mu M_S)^{1/2}, \tilde{\sigma} = \sigma/(\mu M_S)^{1/2}$ with $\chi, \sigma$ being normalized real scalar fields defined by $\bar{\nu}^c H, \nu^c_H, S$ to the real axis.

The emerging picture is completely different. The flat direction at $\tilde{\chi} = 0$ is now a local maximum with respect to $\tilde{\chi}$ for all values of $\tilde{\sigma}$, and two ‘new’ symmetric valleys of minima appear [6, 52] at

$$\tilde{\chi} = \pm \sqrt{6}\tilde{\sigma} \left[ \left( 1 + \frac{1}{36\tilde{\sigma}^4} \right)^{1/2} - 1 \right]^{1/2}.$$

They contain the SUSY vacua which lie at $\tilde{\chi} = \pm 1, \tilde{\sigma} = 0$. It is important to note that these valleys are not classically flat. In fact, they possess an inclination already at the classical level, which can drive the inflaton towards the vacua. Thus, there is no need of radiative corrections in this case. The potential along these paths is given by [6, 52]

$$\tilde{V} = 48\tilde{\sigma}^4 \left[ 72\tilde{\sigma}^4 \left( 1 + \frac{1}{36\tilde{\sigma}^4} \right) \left( \left( 1 + \frac{1}{36\tilde{\sigma}^4} \right)^{1/2} - 1 \right) - 1 \right]$$

$$= 1 - \frac{1}{216\tilde{\sigma}^4} + \cdots, \text{ for } \tilde{\sigma} \gg 1.$$
Inflation does not come to an abrupt end in this case since the inflationary path is stable with respect to \( \tilde{\chi} \) for all \( \tilde{\sigma} \)'s. The value \( \tilde{\sigma}_0 \) at which inflation is terminated smoothly is found from the \( \epsilon \) and \( \eta \) criteria (see e.g., Ref.[5]), and the derivatives [52] of the potential along the inflationary path:

\[
\frac{d\tilde{V}}{d\tilde{\sigma}} = 192\tilde{\sigma}^3 \left( 1 + 144\tilde{\sigma}^4 \right) \left( \left( 1 + \frac{1}{36\tilde{\sigma}^4} \right)^{\frac{3}{2}} - 1 \right) - 2,
\]

\[
\frac{d^2\tilde{V}}{d\tilde{\sigma}^2} = \frac{16}{3\tilde{\sigma}^2} \left[ (1 + 504\tilde{\sigma}^4) \left( 72\tilde{\sigma}^4 \left( \left( 1 + \frac{1}{36\tilde{\sigma}^4} \right)^{\frac{3}{2}} - 1 \right) - 1 \right) \right. \\
\left. \quad - (1 + 252\tilde{\sigma}^4) \left( \left( 1 + \frac{1}{36\tilde{\sigma}^4} \right)^{-\frac{3}{2}} - 1 \right) \right].
\]

The quadrupole anisotropy of CMBR, \( (\delta T/T)_Q \), and the number of e-foldings, \( N_Q \), of our present horizon during inflation can be found from the standard formulae (see e.g., Ref.[5]) and using Eq.(44). One advantage of this scenario is that the common vev of \( \bar{H}_c, H_c \), which is equal to \( (\mu M_S)^{1/2} \), is not so rigidly constrained and, thus, can be chosen equal to the SUSY GUT scale \( M_G \approx 2.86 \times 10^{16} \) GeV. \( (\delta T/T)_Q \) can be approximated [6] as

\[
\left( \frac{\delta T}{T} \right)_Q \approx \frac{1}{\sqrt{5}} \left( \frac{6}{\pi} \right)^{\frac{1}{3}} N_Q^{\frac{2}{3}} M_G^{\frac{10}{3}} M_P^{-\frac{4}{3}} M_S^{-2}.
\]

From the results of COBE [3] and taking \( N_Q \approx 57 \), we obtain \( M_S \approx 7.89 \times 10^{17} \) GeV, \( \mu \approx 1.04 \times 10^{15} \) GeV, which are quite ’natural’. The relevant part of inflation takes place between \( \sigma_Q \approx (9N_Q/2)^{1/6} \sigma_0 \approx 2.72 \times 10^{17} \) GeV and \( \sigma_0 \approx (2M_P/9\sqrt{\pi}M_G)^{1/3} M_G \approx 1.08 \times 10^{17} \) GeV.

The inflaton with mass \( m_{\text{infl}} = 2\sqrt{2}(\mu/M_S)^{1/2} \mu \approx 1.07 \times 10^{14} \) GeV decays again into \( \nu^c \)'s. One can show [52] that the gravitino and leptogenesis constraints can be satisfied with ’natural’ values of the parameters together with the restrictions from solar and atmospheric neutrino oscillations and \( SU(4)_c \) invariance. However, this model requires slightly higher \( T_i \)'s (up to \( 10^{10} \) GeV), which are perfectly acceptable [53] provided that the branching ratio of the gravitino to photons is somewhat smaller than unity and \( m_{3/2} \) is relatively large (of the order of a few hundred GeV).

4. Conclusions

We have shown that, in a wide class of SUSY GUTs, hybrid inflation arises ‘naturally’, in the sense that no ‘tiny’ coupling constants are needed, the superpotential is restricted only by symmetries, and inflation can be protected against radiative and SUGRA corrections.
This inflationary scenario can be readily incorporated in concrete SUSY GUT models which simultaneously meet a number of other requirements such as the solution of the strong CP and $\mu$ problems (via a PQ symmetry), and the generation of (seesaw) masses and mixing for light neutrinos. Moreover, in these concrete models, hybrid inflation is followed by a successful ‘reheating’ leading to adequate baryogenesis (via a primordial leptogenesis) and satisfying the gravitino constraint on the ‘reheat’ temperature together with the available solar and atmospheric neutrino oscillation data. We give an example of such a SUSY GUT model based on the left-right symmetric gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

Natural extensions of the standard SUSY hybrid inflationary scenario, which still meet all the above requirements, can also solve the problem of the possible monopole overproduction at the end of inflation. This is vital for the application of hybrid inflation to higher GUT gauge groups such as the PS group $SU(4)_c \times SU(2)_L \times SU(2)_R$ or the ‘trinification’ group $SU(3)_c \times SU(3)_L \times SU(3)_R$ predicting the existence of monopoles.

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