Scientific instrument for creation of effective
Cooper pair mass spectroscopy

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Abstract. We describe electronic instruments for creation of effective Cooper pair spectroscopy. The suggested spectroscopy requires study of electric field effects on the surface of cleaved superconductors. The electronic instrument reacquires low noise amplifier with $10^6$ amplitude amplification which we have formerly used for study of Johnson-Nyquist and Schottky noises. The nonspecific amplifier is followed by high-Q tunable resonance filter based on schematics of general impedance converter topology which is also and innovative device. The work of the device is based on the Manhattan equation of operational amplifier. After a final nonspecific amplification the total amplification can exceed $10^9$ and in such a way sub-nano-volt signals can be reliably detected. In short the observation of new effects in condensed matter physics leads to creation of new generation of electronic equipment.

1. Introduction
Effective masses $m^*$ of charge carriers are important notions of the physics of condensed matter. In the physics of metals and semiconductors they represents basic electronic phenomena related to thermodynamics and transport properties. Alas the physics of supercondutivity is an exception. Except for some episodic hints, effective masses of super-fluid charge carriers have never been systematically determined. The purpose of the present study is to evaluate experimental efforts necessary for the creation of Cooper pair mass spectroscopy: equipment, samples, consumptives, and perspectives for further development. A review of present status of the problem together with detailed theory is given in our recent papers $^{1,2}$ and references therein ($^3$,$^4$,$^{16}$ and so on). In the next Sec. 2 we recall the main theoretical results, then we describe the suggested set-ups in Sec. 3. In Sec. 4 we describe the electronic equipment which has to be build. Later on in Sec. 5 we describe the necessary samples and the experimental data processing for the planned first successful experiments.

2. Theory Review
In this section we recall only the final theoretical formulae which have to be used for the creation of Cooper pair mass spectroscopy.

2.1. Current induced contact potential difference
The initial idea for creation of Cooper pair mass spectroscopy is to use current induced Contact Potential Difference (CPD) or Bernoulli effect in superconductors. For the detailed list of
references, physics and history of the problem see our recent article [1]. We suppose capacitive coupling of the cleaved superconductor by 4 electrodes. Between electrodes (1) and (3) is applied driving voltage, which is a sum of two sinusoidal: one basic \(U_a\) with high frequency \(\Omega\) and another one modulated \(U_b\) with much smaller frequency \(\omega \ll \Omega\)

\[
U_{1,3}(t) = U_a \sin(\Omega t) + U_b \sin(\Omega t) \sin(\omega t) = U_a \sin(\Omega t) + \frac{1}{2} U_b [\cos((\Omega - \omega)t) - \cos((\Omega + \omega)t)] .
\] (1)

Then between those probes (1) and (3) a current flows

\[
I_{1,3}(t) = C_{1,3} d_t U_{1,3} = I_a \cos(\Omega t) - \frac{1}{2} I_b \sin((\Omega - \omega)t) + \frac{1}{2} I_b \sin((\Omega + \omega)t) ,
\] (2)

\[
I_a \approx \Omega C_{1,3} U_a, \quad I_b \approx \Omega C_{1,3} U_b, \quad C_{1,3} \equiv \frac{C_1 C_3}{C_1 + C_3},
\] (3)

where \(C_1\) and \(C_3\) are the capacitances between the electrodes and the superconductor samples. This electrostatically driven current \(I_{1,3}(t)\) in thin superconducting film with thickness \(d_{\text{film}} \ll \lambda(T)\) much smaller than penetration depth \(\lambda(T)\) creates the Bernoulli voltage

\[
U_{2,4}(t) = -B I_{1,3}^2 \equiv -U_B \sin(\omega t) + \ldots, \quad U_B \equiv \frac{1}{2} B I_a I_b,
\]

where the high frequency terms with frequencies \(2\Omega \pm \omega\) are not written. The purpose of the experiment is to use the voltage amplitude \(U_B\) of the demodulated signal with frequency \(f = \omega/2\pi\). The ratio

\[
B(T) \equiv \frac{U_B}{I_a I_b}
\] (4)

of the experimentally measurable amplitudes determines the effective mass of Cooper pairs

\[
m^* = \frac{e^*}{4\pi^2 B R_2} \frac{\ln R_2}{r_2} \left( \frac{\lambda(0) \lambda(T)}{\varepsilon_0 e^2 d_{\text{film}}} \right)^2 ,
\] (5)

\(e^* = 2e\) is the effective Cooper pair charge and \(e\) is the elementary charge, where we suppose that the second electrode is a ring internal and external radii \(r_2\) and \(R_2\). The geometry is described in the next section.

2.2. Electrostatically modulated thin film kinetic inductance

Historically, the first experimental determination of effective mass of Cooper pairs has been performed [17] using the formula [19]

\[
m^* = -2|e| \text{sgn}(e^*) L(0) L(T) \frac{\delta Q}{\delta L(T)} , \quad C_s(T) = \frac{L(0)}{L(T)} ,
\] (6)

where \(L(T)\) is the temperature dependent kinetic inductance per square sample, \(L(0)\) analogously to \(\lambda(0)\) is extrapolated to zero temperature value, \(\delta L(T)\) is the change of the kinetic inductance under the electrostatic modulation by extra electric charge per unit area \(\delta Q\), i.e. the electrostatic induction \(D_2\).

2.3. Electric field induced surface magnetization of the vortex phase

Last possibility for creation of Cooper pair mass spectroscopy is to study sinusoidal modulation of the excess surface magnetization per unit area \(M\) under the modulation of surface charge density \(Q\)

\[
m^* = \frac{e^* B \lambda^2(0)}{4(dM/dQ)} = \frac{e B \lambda^2(0) Q_0}{2 \text{sgn}(e^*) M_0} , \quad Q(t) = Q_0 \sin(\omega t), \quad M(t) = M_0 \sin(\omega t),
\] (7)

where \(B\) is the external magnetic field which is perpendicular to the superconductor surface of extreme type-II superconductor which are, for example, all high-\(T_c\) cuprates.
3. Experimental Set-ups

Here we describe the set-ups for the experiments, which theory we review in the former section. The set-ups corresponding to the described 3 methods are depicted in figures 1, 2 and 3.

**Figure 1.** Experimental set-up for electrostatic excitation of current induced contact potential difference [1, Fig. 1]. Drive electrodes (1) and (3) and detector electrodes (2) and (4) are coaxial normal metal rings. All four electrodes are plates of plane capacitors. The other plane is the cleaved surface of a superconducting film. The effective mass of cooper pairs is determined according to (5) by the voltages $U_{1,3}$ and $U_{2,4}$.

**Figure 2.** Schematics (not to scale) representation of the experimental set-up for the first measurement [17–20] of $m^*$ determined according to (6). The eddy currents in the superconducting film are induced by the current through the drive coil $I_d$ and detected by the voltage $U_r$ induced in the detector coil. The modulating voltage $U_{ms}$ changes the mutual inductance between the coils parameterized by the kinetic inductance per square sample $L(T)$.

After this brief description of the theory and set-ups in the next section we will focus on the electronic necessary to be build for the suggested Cooper pair mass spectroscopy.

4. Electronics of the instrument for Cooper pair mass spectroscopy

All considered in the former section experiments require measurement of nano-volt range signals which is below the standard electronics equipment and requires development on
Figure 3. Set-up for study of surface magnetization of the vortex phase of superconductors induced by AC electric field applied to the surface [2, Fig. 1]. Magnetic field $B \ll B_c^2(T)$ is created by a Nd permanent magnet. Bi$_2$Sr$_2$CaCu$_2$O$_8$ crystal is grown on a substrate. The cleaved upper surface of the crystal is one plate of a plane capacitor with insulator layer. The other plate is a normal metal. An AC electric voltage is applied between the crystal and the normal electrode. The theoretically predicted AC magnetization is measured as a voltage in the detector coil. The new effect is parameterized by the effective mass of Cooper pairs $m^*$ according to (7).

unique electronics. For example the block-scheme for observation of current induced CPD is schematically represented in figure 4. The symbolically denoted there voltmeter starts with

![Block diagram](image)

Figure 4. Effective schematics for measurement of current induced contact potential difference [1, Fig. 3]. Capacitively induced current $I(t)$ creates CPD according Bernoulli theorem. The AC CPD is measured after two capacitors with sequential capacitance $C_2C_4/(C_2 + C_4)$. The pre-amplifier and the lock-in are represented symbolically as a voltmeter.

a pre-amplifier shown in figure 5. This pre-amplifier is reliable and we used it for education experiments for determination of the Boltzmann constant [21] and the electron charge [22]. The device is so robust that we reproduced in more than 300 samples for the set-ups of 5-th and 6-th high-school Experimental physics olympiads [21,22]. The pre-amplifier consists of a buffer with a double operational amplifiers. The buffer has a frequency dependence of the amplification as for a non-inverting amplifier $Y_{NIA}$. After the buffer we have a difference amplifier with frequency dependence of the amplification coinciding with the last inverting amplifier $Y_{IA} = Y_{IA}$. In such a way the total amplification is given by the product

$$ Y(\omega) = Y_{NIA}(\omega)Y_{\Delta}(\omega)Y_{IA}(\omega). \quad (8) $$

The offset voltages of the operational amplifiers are stopped by the large capacitors $C_G$ followed by large gain resistors $R_G$. In such a way for relatively low frequencies for which nevertheless $\omega R_G C_G \gg 1$ the amplification is approximately constant

$$ Y(\omega \to 0) \equiv Y = Y_{NIA}Y_{\Delta}Y_{IA} = \left(1 + 2 \frac{R_F}{r_G}\right) \left(\frac{R_F}{R_G}\right)^2 $$
Figure 5. A reliable pre-amplifier for measurements of CPD after Ref. [21]. One can see a buffer 1) and a difference amplifier 2) forming an instrumental amplifier which is sequenced by a non-inverting amplifier 3). The pre-amplifier calculated frequency response is given in figure 6.

which can be clearly seen at the frequency dependence of the frequency calculated according to our formulae [21]

\[ G^{-1}(\omega) = G_0^{-1} + j\omega \tau, \quad \tau \equiv 1/2\pi f_0, \]

\[ \Upsilon_{\text{NIA}}(\omega) = \frac{1}{G^{-1}(\omega) + y^{-1}(\omega)}, \quad y(\omega) = \frac{Z_F(\omega)}{r_G} + 1, \]

\[ \Upsilon_{\Delta}(\omega) = \frac{-1}{\Lambda(\omega) + G^{-1}(\omega)[1 + \Lambda(\omega)]}, \quad \Lambda(\omega) \equiv \frac{Z_G(\omega)}{R_F^1}, \quad Z_G(\omega) = R_G + \frac{1}{j\omega C_G}, \]

\[ \Upsilon_{\text{IA}}(\omega) = -\frac{1}{\Gamma(\omega) + G^{-1}(\omega)[1 + \Gamma(\omega)]}, \quad \Gamma(\omega) \equiv \frac{Z_G(\omega)}{Z_F^1}, \quad \frac{1}{Z_F(\omega)} = \frac{1}{R_F^1} + j\omega C_F. \]

and shown in figure 6. For this study we re-derived the Manhattan equation of OpAmps [23], applied it in the theory of amplifiers [24], and studied statistical properties of the crossover frequency of the crossover frequency \( f_0 \) of the used low-noise OpAmp ADA4898-2 [25].

Concerning the nonspecific amplifier, we have reached saturation using one of the best low-noise OpAmp after \( 10^6 \) amplification the signal is slightly below the overload regime. In order to go further for observation of new physical effects we need to construct a new type resonance filter having Q-factor comparable with quartz resonator but having advantage to be tunable. We have solved this long standing endeavor using the topology of general impedance converter (GIC) [26] drawn in figure 7. The general theoretical formula we have derived (\( \alpha, \beta \equiv G^{-1}(\omega) \))

\[ Z_{\text{GIC}}(\omega) = \frac{U}{T} = \frac{Z_S}{1 - \frac{(Z_1 + Z_2) - (Z_3 + Z_4)K}{(1 + \beta)(Z_1 + Z_2) - Z_3K}}, \quad K \equiv \frac{Z_2 + (Z_1 + Z_2)\alpha}{Z_3 + (Z_3 + Z_4)\alpha}, \]

perfectly matches the experimental data depicted in figure 8. The detailed description of the resonator will be published elsewhere [27]. Now we can describe in short the planned experiment.
Figure 6. Calculated frequency dependence of the amplification $|\Upsilon(\omega)|$ of the schematics represented in figure 5. The theoretical formulae for the transmission of every block are given in equations 8 and 9.

Figure 7. Left: GIC [26, 27]. Right: Effective schematics for calculation of low frequency behavior when operation amplifier output currents equalize the input voltages. General theoretical formula for the effective impedance of GIC is given by 10 and both experiment and theory are graphically represented in figure 8.

5. Experiment
All details of the planned experiment are already tested from the fundamental theory of a new effect in superconductors. The last missing line of the chain was the resonator by GIC and now we can start creation of the scientific instrument for the new Cooper pair mass spectroscopy. In order to alleviate the success in the initial stage we recommend the use of 90 K $\text{Bi}_2\text{Sr}_2\text{Ca}_1\text{Cu}_2\text{O}_8$ mono-crystal which can be easily cleaved. In conclusion we believe that within one year we will
Figure 8. Experimental data points and theoretical curves (10) for the frequency dependence of a resonator implemented by GIC. The upper figure is the frequency dependence of the resistance $R(f) = \Re(Z)$, while the lower one is the frequency dependence of the inductance $L(f) = \Im(Z)/\omega$. At small frequencies we have an almost ideal inductance, then we have a high-Q resonance and after it the inductance turns into capacitance. The real part of the impedance, which is de facto the resistance is almost symmetric with respect to the resonance.

have first intentionally done experiment for determination of $m^*$ in high-$T_c$ cuprates.

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