Combining Thermodynamic and Dynamic Perspectives of Tropical Circulation to Constrain the Downdraft Width of the Hadley Cell

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Key Points:

\begin{itemize}
\item Historically, dynamic and thermodynamic frameworks for Hadley cells considered independent
\item In eddy-driven case, combining them leads to novel scaling of Hadley cell width and strength with planetary parameters
\item GCMs of various complexities largely adhere to the new scaling if in the eddy-driven regime, though for different reasons
\end{itemize}
Abstract

In the Hadley circulation downdraft, to leading order vertical potential temperature advection balances the “effective” heating, comprising the sum of diabatic heating and eddy heat flux divergence, placing a thermodynamic constraint on vertical velocity. Insofar as downdraft-averaged effective heating and static stability do not vary with planetary parameters, neither can vertical velocity — an “omega governor.” Separately, in the eddy-driven (i.e. small-Rossby-number) limit, extratropical eddy stresses also constrain the cell strength dynamically. We combine these thermodynamic and dynamic mechanisms to derive new and identical scalings for the downdraft width and overturning strength with rotation rate. With the omega governor maintaining fixed vertical velocity, the downdraft must narrow or widen in order to attain the overturning strength dictated by the eddy stresses. We evaluate the validity of this new scaling using model simulations over a broad range of rotation rates and model forcing schemes.

Plain Language Summary

Theories for the width and strength of the tropical overturning circulation, or Hadley cell, have existed for decades. Hadley cell width and strength generally increase with decreasing rotation rate. If the circulation is angular-momentum-conserving, theory predicts strong but distinct dependencies of Hadley cell width and strength on the rotation rate of the planet. When extratropical eddies break the angular momentum conservation of the Hadley cell, we find a new scaling for the width of the Hadley cell downdraft, which is also identical to that of the Hadley cell strength. In this limit, which is more representative of Earth’s Hadley cell, both the width and the strength inherit the scaling from the dependence of eddy fluxes of momentum on rotation rate.

1 Introduction

The Hadley circulation, which marks the expanse of the Tropics, is likely to widen and weaken under global warming (Lu et al., 2007; Vallis et al., 2015). There are two predominant theories for how the width and strength of the Hadley circulation should scale with planetary parameters. The first treats the atmosphere as axisymmetric, inviscid above the boundary layer, and with ascent out of the boundary layer occurring at a single latitude. The resulting free-tropospheric flow must be angular-momentum-conserving (AMC), with the local Rossby number ($R_o L$, defined below) being unity throughout the Hadley cells (Held & Hou, 1980). The theory compares the vertically integrated thermodynamic equation in radiative-convective equilibrium to its AMC counterpart and then appeals to energy continuity and conservation to predict the Hadley cell width. Notably, the AMC perspective primarily constrains the width of the Hadley circulation; one must appeal to the thermodynamic equation, typically in regions of vertical motion where there is a balance between vertical advection by the mean circulation and diabatic heating, to estimate the circulation strength. In addition, the AMC model requires that eddy stresses and heat flux divergences be negligible throughout the Hadley cells.

Eddy stresses and heat fluxes, however, are not generally negligible in the Tropics, and angular momentum is not generally uniform even along individual streamlines, let alone over the expanse of the entire circulation (Hill et al., 2019). Extratropical eddies propagate equatorward and break, drawing westerly momentum poleward out of the subtropics, and the Hadley circulation transports angular momentum poleward in response. In this $R_o L \ll 1$ limit, the Hadley circulation strength is determined by the momentum budget in the subtropics (e.g., Becker et al., 1997; Kim & Lee, 2001; Walker & Schneider, 2006). Neither limit is perfect — large portions of the tropical free troposphere on Earth and other atmospheres are routinely in an intermediate regime, $R_o L < 1$ but not $R_o L \ll 1$ — but there are times/locations where $R_o L \geq 0.7$, mostly in the deep Tropics, while in the subtropics $R_o L \sim 0.1$ (Schneider, 2006).
Where there are breaking baroclinic eddies, there are likely to be eddy heat fluxes (in addition to stresses) that contribute to the thermodynamic equation. Generally, breaking eddies transport heat poleward, overlapping with the heat transport of the Hadley cell but extending further into the extratropics (Trenberth & Stepaniak, 2003). Thus in the limit $Ro_L \ll 1$, we expect eddy heat flux divergence to be an important contributor to the energetics, especially in the downdraft of the Hadley cell. Because the time-mean flow is primarily vertical in the downdraft, we can expect a balance between the vertical advection of potential temperature and the diabatic heating plus eddy heat flux divergences.

Here, we combine these dynamic and thermodynamic considerations to develop a new scaling for the width of the Hadley cell downdraft. The key insight is that, provided the static stability and combined diabatic-plus-eddy heating stay fixed, vertical velocities must be fixed also. In the $Ro_L \ll 1$ limit, this “omega governor” closely links eddy stresses to the strength and width of the Hadley cell downdraft. In Section 2 we develop the theory for the strength and downdraft width of the Hadley circulation. In Section 3, we describe a suite of model simulations over a broad range of rotation rates and forcing schemes to test the validity of our scaling.

2 Theory combining dynamic and thermodynamic perspectives

We define the Hadley cell strength as the maximum within the cell of the zonal-mean mass streamfunction,

$$
\Psi(\varphi, p) = 2\pi a \cos \varphi \int_{p_s}^{p} \bar{v} \frac{dp}{g},
$$

with planetary radius $a$, latitude $\varphi$, surface pressure $p_s$, meridional velocity $v$, gravity $g$, and an overbar denoting a temporal and zonal average. Equivalently, the streamfunction is

$$
\Psi(\varphi, p) = -\frac{2\pi a^2}{g} \int_{\varphi_0}^{\varphi} \bar{\omega} \cos \varphi d\varphi,
$$

with vertical (pressure) velocity $\omega$ and $\varphi_0$ being a latitude where the streamfunction vanishes, i.e. at the cell edges.

2.1 The dynamic perspective

In the upper, poleward-flowing branch of the Hadley circulation near the latitude of the maximum mass flux (i.e. the cell center), we expect a dominant balance between meridional advection of absolute vorticity and eddy momentum flux divergence,

$$(f + \bar{\zeta})\bar{v} = (1 - Ro_L)f\bar{v} \simeq s,$$

where

$$s = \frac{1}{a \cos^2 \varphi} \frac{\partial}{\partial \varphi}(\cos^2 \varphi \bar{u}'\varphi') + \frac{\partial}{\partial p}(\bar{u}'\varphi')$$

is the eddy momentum flux divergence, $\bar{\zeta}$ is relative vorticity, $f$ is the planetary vorticity, $u'$ is zonal velocity, primes denote deviations from the time- and zonal-mean, and

$$Ro_L = -\bar{\zeta}/f$$

is a local Rossby number (Walker & Schneider, 2006). If $Ro_L = 1$, then necessarily $s = 0$, and (3) is degenerate. Otherwise, integrating (3) vertically from the level of max streamfunction, $p_m$, to the cell top level, $p_t$,

$$\int_{p_m}^{p_t} (1 - Ro_L)f\bar{v} \frac{dp}{g} \simeq \int_{p_m}^{p_t} s \frac{dp}{g}.$$

Multiplying through by $2\pi a \cos \varphi$ and following Singh and Kuang (2016) defining a mass-weighted bulk Rossby number\(^1\), $Ro$, yields

$$\Psi_{\text{vmax}}(1 - Ro) \simeq S/f$$

where $S = 2\pi a \cos \varphi \int_{P_t}^{P_m} s \, dp / g$ and $\Psi_{\text{vmax}}(\varphi)$ is the value of the streamfunction at the level of the overall Hadley cell maximum mass flux.\(^2\) While (6) is valid at any latitude meeting the given assumptions, henceforth we use it only at the latitude of the maximum streamfunction, $\varphi = \varphi_{\text{max}}$. (6) makes clear the dependence of the max streamfunction on the (bulk) Rossby number: if $Ro \ll 1$, the demand for momentum by breaking extratropical eddies drives the Hadley circulation; if $Ro \sim 1$, the momentum budget does not constrain the Hadley circulation strength.

\subsection*{2.2 The thermodynamic perspective (the omega governor)}

Consider the steady-state, zonal-mean thermodynamic equation in regions of purely vertical zonal-mean flow (rather than purely horizontal flow as for the dynamical arguments above):

$$\tilde{\omega} \frac{\partial \theta}{\partial p} + \frac{1}{a \cos^2 \varphi} \frac{\partial}{\partial \varphi} (\cos^2 \theta') + \frac{\partial}{\partial p} (\omega' \theta') = \bar{Q} \left( \frac{p}{p_s} \right)^{-\kappa}$$

where

$$Q_{\text{eff}} = Q \left( \frac{p}{p_s} \right)^{-\kappa} - \frac{1}{a \cos^2 \varphi} \frac{\partial}{\partial \varphi} (\cos^2 \theta') - \frac{\partial}{\partial p} (\omega' \theta') \quad \text{if} \quad Ro \ll 1$$

$Q_{\text{eff}}$ is the “effective” heating experienced by the mean circulation from the combination of diabatic heating and eddy heat flux divergence. In the strongly eddying, $Ro \ll 1$ limit, the eddy terms in (9) may be important, but need not be; in principle (especially barotropic) eddies could effect large stresses but small eddy heat flux divergences. In the axisymmetric, $Ro \sim 1$ limit, (9) reduces to the diabatic heating provided transient symmetric instabilities do not generate appreciable heat flux divergences.

We focus on dry-statically stable atmospheres; see Caballero et al. (2008) for a detailed analysis of the dry-neutral case. The stability is typically quite uniform in the deep tropics, since any diabatic heating is communicated effectively by wave dynamics (e.g., Sobel et al., 2001). We have confirmed this holds in all of our models as the rotation rate is varied (see Supporting Information, Figure S8-S10).

The diabatic heating, meanwhile, will depend on the circulation itself as well as (in models) the physical parameterizations of radiation and convection. However, as we will show in subsequent sections $Q_{\text{eff}}$ is remarkably constant across a broad range of rotation rates in at least one of our model configurations. In this case, (8) implies a value for $[\tilde{\omega}]$ that is independent of rotation rate. This “omega governor” applies no matter the value of $Ro$ as long as the static stability and total heating (diabatic plus eddy) are constant.

\(^1\) $Ro$ is to be distinguished from the thermal Rossby number, $Ro_{th}$, which is a control parameter in the theory of (Held & Hou, 1980) and others. The thermal Rossby number has a specified dependence on rotation rate, $Ro_{th} \sim \Omega^{-2}$, while the bulk Rossby number, $Ro$, is diagnosed from a given simulation.

\(^2\) (6) differs from that of Singh and Kuang (2016) in that the mass streamfunction is in kg s\(^{-1}\) rather than kg m\(^{-1}\) s\(^{-1}\).
2.3 The combined perspective

Provided the omega governor conditions are met, the Hadley circulation can only increase (decrease) its mass flux by expanding (contracting) in latitude, so that the downdraft area increases (decreases). Thus, in the eddy-driven limit, the eddy stresses control both the Hadley cell strength and downdraft width.

More formally, equating (2) and (6) at $p_m$ yields

$$S/f \simeq -(1 - Ro) \frac{2\pi a^2 [\bar{\omega}]_{vmax}}{g} (\sin \varphi - \sin \varphi_0),$$

with zonal- and meridional-mean vertical velocity at $p_m$

$$[\bar{\omega}]_{vmax} = \frac{\int_{\phi_0}^{\phi} \bar{\omega}(\varphi, p_m) \cos \varphi d\varphi}{\sin \varphi - \sin \varphi_0}.$$  \hspace{1cm} (11)

Setting $\varphi = \varphi_{max}$ and $\varphi_0 = \varphi_h$, where $\varphi_h$ is the cell’s poleward edge, (10) becomes

$$S/f \simeq -\frac{2\pi a^2 [\bar{\omega}]_{vmax}}{g} (\sin \varphi_{max} - \sin \varphi_h) \text{ if } Ro \ll 1.$$  \hspace{1cm} (12)

If $\bar{\omega}_{vmax}$ is constant and in the small-angle limit this further reduces to

$$S/f \simeq \frac{2\pi a^2 \bar{\omega}_{vmax}}{g} (\varphi_h - \varphi_{max}) \text{ if } (Ro, \varphi_h, \varphi_{max}) \ll 1.$$  \hspace{1cm} (13)

By (6) and (13), respectively, the cell strength and downdraft width are both proportional to $S/f$: eddy stresses determine the overturning strength, which for a fixed descent rate can only be altered by making the cell downdraft narrower or wider.

This is not a closed theory, since both $S/f$ and $Ro$ must be diagnosed from simulations. But given $S/f$ and sufficiently small $Ro$, the width and the strength have the same scaling with rotation rate, inherited from that of the eddy momentum flux divergence. Another caveat is that the downdraft width does not directly constrain the total Hadley cell width; only if condensation, eddy heat fluxes, and any other non-radiative forcings are absent must the updraft and downdraft areas be equal. That does not hold generally (most notably in moist atmospheres), and what controls the updraft width remains actively researched (Byrne et al., 2018). But a theory for downdraft width combined with a theory for the updraft width (or their ratio) would constitute a complete theory for the total cell width.

3 Simulation results

3.1 Model and simulation descriptions

We now describe a suite of model simulations using the same dynamical core (FMS; Gordon & Stern, 1982) but three distinct sets of forcings and physical parameterizations. Briefly, the first model uses a Held-Suarez-like Newtonian cooling and linear boundary layer friction (Held & Suarez, 1994). The second model also uses Newtonian cooling, but to a dry-unstable state, along with a convectively adjustment parameterization (Schneider, 2004). The third model is an idealized aquaplanet with gray radiation and moist physics (Frierson et al., 2006, 2007). The simulations are run over a variety of planetary rotation rates (although not the same ones for each model) to test our theory of the Hadley cell, specifically the scalings predicted by equations (6) and (12). Further details of the simulations and data processing and latitude-height plots of all simulated zonal-mean streamfunctions, zonal winds, and eddy momentum and heat flux convergence fields are provided in the Supporting Information.
In short, \( \omega v_{\text{max}} \) is indeed fixed, and the cell width and strength share the same scaling in the two dry models, even when the \( Ro \ll 1 \) condition isn’t met. In the aqua-planet simulations, \( \omega v_{\text{max}} \) does vary with rotation rate, but nevertheless behaviors are consistent with the more general expression of our scalings, (10). The scalings in all three models are inconsistent with axisymmetric, nearly inviscid theory, in which Hadley cell strength and total width scale as \( \Omega^{-3} \) and \( \Omega^{-1} \), respectively (Held & Hou, 1980).

### 3.2 Held-Suarez-like model

In Held-Suarez-like simulations, the effective heating, static stability, their ratio, and therefore \( \omega \) averaged over the downdraft depends very weakly on \( \Omega \) (Figure 1c,S1,S8). The eddy heat flux divergence and Newtonian cooling both depend on rotation rate, but nearly exactly cancel each other’s dependence. One possible explanation for this compensation is the following. For the eddy heat flux divergence, we expect a general increase in the strength of baroclinic heat fluxes with increasing rotation rate. For the diabatic heating, while at Earth’s rotation rate the edge of the Hadley cell sits in a region of strong meridional gradients in the Newtonian cooling profile, at faster rotation rates Hadley cell contracts into the “flatter” temperatures of the low latitudes. Thus at fast rotation rates, the Hadley cell does less “flattening” (in latitude) of the temperature structure, and the Newtonian cooling is comparatively weak.

Figure 1(a)-(d) displays the scaling of the width of the Hadley cell downdraft, its strength measured by the maximum mass flux, vertical pressure velocities averaged over the downdraft, and bulk eddy stresses divided by the Coriolis parameter, \( S/f \), at the location of the maximum Hadley cell mass flux. As in other studies (e.g., Williams, 1988a, 1988b; Walker & Schneider, 2006; Pinto & Mitchell, 2014), the Hadley cell becomes wider and stronger as the rotation rate decreases (Figure 1(a),(b)). The downdraft velocity is nearly constant, albeit with some scatter (Figure 1(c)), except \( \Omega^* = 4.3 \). This is consistent with the thermodynamic constraint implied by (8) and fixed downdraft heating rates. Figure 1(d) shows the scaling of the bulk eddy stresses divided by the coriolis parameter, \( S/f \), at the location of the maximum Hadley circulation mass flux, which roughly follow a power-law \( \sim \Omega^{-0.25} \), except at the largest value of rotation rate. In our theory, \( S/f \) is the fundamental driver of the Hadley cell mass flux. If vertical velocities are fixed as Figure 1(c) demonstrates, then according to equations (6) and (12), both the strength of the Hadley cell, \( \Psi_{\text{max}} \), and the width of the downdraft, \( \sin \varphi_h - \sin \varphi_m \), should have the same power-law scaling, inherited from the eddy stresses, \( S/f \). Figures 1(a), (b) and (d) show that the downdraft width \( \sim \Omega^{*-0.27} \), Hadley cell mass flux \( \sim \Omega^{*-0.30} \) and eddy stresses \( \sim \Omega^{*-0.25} \) have similar power-law scalings, except for the largest rotation rate case.

Figure 1(e) shows \( \Psi_{\text{max}} \) plotted against \( S/f \) color-coded by the values of \( \Omega^* \) (c.f. Singh & Kuang, 2016). From (6), if \( Ro = 0 \), the bulk eddy stresses and Hadley cell strength are equal (solid line, Figure 1(e)); if \( Ro = 1 \), the eddy stresses and Hadley cell strength are independent (dashed line, Figure 1(e)). All rotation rates stay closer to the \( Ro = 0 \) line than the \( Ro = 1 \) line. The value of the bulk Rossby number as diagnosed from (6), \( Ro = 1 - S/(f \Psi_{\text{max}}) \), is shown as a function of \( \Omega^* \) in Figure 1(f). There is considerable scatter in \( Ro \) values, however all have \( Ro \leq 0.5 \). The reason for the scatter in \( Ro \) is not obvious, however the sequence of mass streamfunctions and eddy momentum flux divergences in the Supporting Information suggest circulation changes – for instance a stacked, double-maximum structure in the streamfunction – corresponding to the jumps in \( Ro \) are the culprit. Indeed, the stacked cells appear to be split at or near the top of the boundary layer, 700hPa, suggesting an important role for boundary layer

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3 This outlier case exhibits multiple overturning cells and jets in each hemisphere (Figure S2), a regime beyond our scope. We therefore exclude it from our analyses, including the power-law fits.
processes, although we do not pursue it further here. Regardless of the origin, it is remarkable that this range of simulated $Ro$ values have the same scaling with $\Omega^*$, which should only hold in the $Ro \ll 1$ limit.

### 3.3 Dry convective adjustment model

Figure 2 repeats Figure 1 but for the model featuring dry convective adjustment. The vertical velocities averaged over the downdraft are roughly independent of rotation rate (Figure 2(c)). The Hadley cell width and strength increase with decreasing rotation rate, with similar power laws ($\sim \Omega^*^{-0.33}$ and $\sim \Omega^*^{-0.42}$), except for deviations at the largest rotation rate (Figure 2(a),(b)). Note importantly that the power-law scaling for the width is very similar to that for the Held-Suarez-like forcing simulations, indicating a possible universal scaling for Hadley cell width (we will see it also approximately holds for the aquaplanet simulations). However, contrary to our small-$Ro$ theory, $S/f$ does not have the same power-law scaling; it is non-monotonic with a maximum at Earth’s rotation rate (Figure 2(d)). As such, unlike the Held-Suarez-like case, the eddy stresses do not scale with the width and strength of the Hadley cell.

The reason is the circulation is becoming more angular-momentum-conserving, with increasing values of $Ro$ at small rotation rates. Though the four largest rotation rates are near $Ro = 0$ (Figure 2(e),(f)), the slower rotation rates all have $Ro \sim 1$, with the switch between the two $Ro$ regimes, at $\Omega^* = 1$, coinciding with the change in the sign of the eddy stress dependence on $\Omega^*$. Remarkably, the increase in $Ro$ exactly offsets the decrease in $S/f$ so that the power laws of the widths and strengths remain the same across low and high values of $\Omega^*$ (Figure 2(a),(b)), as we would expect for our small-$Ro$ theory. We leave an explanation for this compensation between $Ro$ and eddy flux convergence to future work.

These results illustrate an important feature of the omega governor, namely that the fixed-velocity argument doesn’t depend on the value of $Ro$. By (2), if the omega governor applies, the strength and downdraft width will have the same scaling with rotation rate. It then remains to determine the scaling of the downdraft width or strength by some other constraint.

### 3.4 Aquaplanet model

Figure 3 repeats Figures 1 and 2 but for the aquaplanet model. Unlike in the two dry models, the downdraft velocity is not independent of rotation rate (Figure 3(c)), and the Rossby number decreases rapidly with rotation rate (Figure 3(f)). The eddy stresses scale as $S/f \sim \Omega^*^{-0.25}$, somewhat similar to the downdraft width [$\sim \Omega^*^{-0.36}$; Figure 2(a)] but not the cell strength ($\sim \Omega^*^{-0.76}$; Figure 2(b)).

Because the downdraft velocity is not fixed across rotation rates in this model, we can no longer expect the width, strength, and eddy stresses to have the same scaling. However, it can be seen that the downdraft velocity times the width has approximately the same scaling as $\Psi_{\text{max}}$, as is always required by (2). Accounting for the scaling of eddy stresses, $S/f$, relative to $\Psi_{\text{max}}$ requires knowledge of the scaling of $(1-Ro)$, which can be inferred from Figure 3(f) to be $\Omega^*^{0.45}$. By (6), we expect the product of $\Psi_{\text{max}}$ and $(1-Ro)$ to scale with $S/f$. The product of the first two is $\Omega^*^{-0.31}$ which is a similar power law to $S/f \propto \Omega^*^{-0.25}$ (Figure 3(b) and (d)). Note importantly that the power-law scaling for the width is very similar to that for the other two model forcing simulations, indicating a possible universal scaling for Hadley cell width.
Figure 1. Results from Held-Suarez-like forcing experiments with varying rotation rates (scaled to the Earth’s value). (a) Hadley cell downdraft width (black dots), best-fit power-law (black line; excludes $\Omega^* = 4$) and total width (red dots). (b) Hadley cell strength [kg s$^{-1}$] (black dots) and best-fit power-law (excluding $\Omega^* = 4$). (c) Hadley cell downdraft-averaged vertical pressure velocity [Pa s$^{-1}$] (black dots) and best-fit power-law (black line). (d) Bulk eddy stresses divided by the Coriolis parameter at the Hadley cell center (black dots) and best-fit power-law (black line, excluding $\Omega^* = 4$). (e) $S/f$ averaged at the Hadley cell center plotted against the Hadley cell strength. The solid line corresponds to $Ro = 0$, and the dashed line to $Ro = 1$. (f) The value of the bulk Rossby number, $Ro = 1 - S/(f \Psi_{\text{max}})$, vs. rotation rate scaled to Earth’s value, $\Omega^*$. 

$\sin \phi_h - \sin \phi_{\text{max}} \Omega - 0.32$

$\bar{\omega}$ [Pa s$^{-1}$] $\Omega^*$. $0.2$

$\Psi_{\text{max}}$ [kg s$^{-1}$] $\Omega^*$. 0.30

$\Omega = 1 - S/(f \Psi_{\text{max}})$, vs. rotation rate scaled to Earth’s value, $\Omega^*$. 

$\Omega^*$. $1/16$

$\Omega^*$. $1/8$

$\Omega^*$. $1/4$

$\Omega^*$. $1/2$

$\Omega^*$. 1

$\Omega^*$. $1/16$
Figure 2. Same as in Figure 1, but for the “dry convective” model forcing. Negative values in panels (a)-(d), if present, are not displayed because of the logarithmic axes.
Figure 3. Same as in Figure 1, but for the (Frierson et al., 2006) model forcing.
3.5 Results for the updraft

The same arguments leading to an omega governor in the downdraft are in principle applicable in the updraft, since the same leading order balance should apply. However, taking the dry convective model as an example, though the updraft-averaged static stability remains fixed across rotation rates in our simulations, $\omega$ varies strongly with rotation (not shown), an explanation for which we leave to future work.

The overlaid red dots in Figures 1(a), 2(a), and 3(a) show the total width of the Hadley cell. In all three models, the total cell width and the downdraft width share, to first order, a similar scaling though with the total width scaling being somewhat shallower. This amounts to the downdraft widening at a faster rate than the updraft does as rotation rate decreases and the overall cell expanse grows (Supporting Information). Multiple lines of argument link the relative updraft and downdraft widths (Watt-Meyer & Frierson, 2019), and controls on the width of the updraft alone remain actively investigated (Byrne et al., 2018).

4 Discussion and conclusions

We have presented a new perspective for the $Ro \ll 1$ limit that relies on vertical velocities in the Hadley cell downdraft being roughly independent of rotation rate. In this limit, the Hadley cell is driven by the momentum demand of breaking extratropical eddies, and both the width and the strength scale with the bulk eddy stresses divided by the planetary vorticity, $S/f$. The latter quantity scales relatively weakly with rotation rate, and as a result so do the Hadley cell width and strength.

In simulations with Held-Suarez-like forcing, our scaling works well over roughly two orders of magnitude in rotation rate. Vertical velocities in the downdraft are constant for all but the fastest rotation, which is consistent with the near-constant total heating rate and lapse rate in these simulations. Remarkably, though Newtonian cooling decreases with increasing rotation rate, this is nearly exactly offset by an increase in eddy heat flux convergence. The exact cause of this compensation is not clear, however decreasing Newtonian cooling is consistent with increasing rotation rate and decreasing Hadley cell width; similarly, we expect eddy heat flux convergence to increase with increasing rotation rate. The Hadley cell downdraft width and strength both have the same rotation rate scaling as $S/f$ as the theory predicts. Notably, $Ro$ is not small for all rotation rates, but despite this our scaling approximately holds.

In similar simulations but including a dry convective adjustment parameterization, $S/f$ varies non-monotonically with rotation rate, with a maximum value at Earth’s rotation rate. Despite this, the width and the strength follow a single power-law in rotation rate, because the positive power law in $S/f$ at weak rotation rates is compensated by an increase in $Ro$ at weak rotation rates. This $Ro$-number compensation is currently not well understood. Perhaps as the Hadley circulation strengthens and widens with decreasing $\Omega^*$, it becomes shielded from the influence of extratropical eddies in a manner similar to the monsoonal circulations (Bordoni & Schneider, 2008; Schneider & Bordoni, 2008), but this is left to future work.

In aquaplanet simulations, our theory breaks down because vertical velocities in the downdraft vary significantly with rotation rate. Eddy stresses, the Hadley cell strength, and Hadley cell downdraft width each possess distinct power law scalings. However, the value of $Ro$ systematically varies with rotation rate, and accounting for the dependence of $Ro$ on rotation rate gives a consistent scaling according to (6) provided $Ro < 1$. Thus, despite the omega governor being invalid for the moist convective model forcing case, the theory provides a good empirical fit to the width and strength given the vertical velocities, eddy stresses, and $Ro$. 
Interestingly, downdraft widths possess similar power-laws in all three models (compare Figures 1(a), 2(a) and 3(a)), potentially hinting at a universal scaling. The Hadley cell strength has nearly the same power law in the two dry models but not the aquaplanet, which could be attributable to the complicating effects of moist thermodynamics that may be responsible for the steep dependence of the Rossby number and vertical velocities on rotation rate.

However, the validity of the omega governor is not limited to $Ro \ll 1$, and this had important consequences for the dry convective model simulations (Figure 2). Because downdraft velocities were nearly independent of rotation rate, and the omega governor applied across the entire range of simulations, which included both $Ro \ll 1$ and $Ro \sim 1$, the downdraft width and strength shared the same scaling across the entire range. It remains to determine what sets the width and/or strength in the $Ro \sim 1$ regime, which do not follow predictions from axisymmetric, inviscid theory.

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Supporting Information for "Combining Thermodynamic and Dynamic Perspectives of Tropical Circulation to Constrain the Downdraft Width of the Hadley Cell"

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1. Figures S1 to S10

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1. Description of models and forcings used for simulations

We now describe a suite of model simulations using the same dynamical core but three distinct sets of forcings and physical parameterizations, which we use to test the theory developed in the previous section. The simulations are run over a variety of planetary rotation rates (although not the same ones for each model).

1.1. Held-Suarez-like forcing

The first model we employ is a dry GCM based on the FMS spectral dynamical core (Gordon & Stern, 1982) and forced with simple, linearized diabatic and frictional terms as in the Held-Suarez benchmark (Held & Suarez, 1994). The equilibrium temperature profile to which temperatures are relaxed via Newtonian cooling, taken from (Mitchell & Vallis, 2010), differs from the original Held-Suarez benchmark in its vertical structure; in our model, we specify a uniform lapse rate of 6 K km$^{-1}$ to approximate a moist adiabat in the troposphere and cap it with an isothermal stratosphere. The horizontal structure of the forcing profile is the same as Held-Suarez, following a specified surface temperature meridional distribution $T_o = \bar{T}[1 + \Delta_H/3(1 - 3\sin^2\varphi)]$, where $\bar{T} = 285$K is the global-mean surface temperature and $\Delta_H = 0.2$ is a non-dimensional equator-to-pole temperature gradient. The stratospheric cap is specified by not allowing temperatures to drop below 70% of $\bar{T}$. The Newtonian cooling timescale distribution is identical to Held-Suarez.

We systematically vary the rotation rate spanning from 1/16 to 4 times Earth’s rotation, spaced by factors of two. Model simulations are run at T42 resolution for rotation rates below that of Earth and at T85 for those with larger rotation rates, because the horizontal
scale of circulations contract with increasing rotation rate, as is evident in the multiple-jet structure of the $\Omega^* = 4$ case (Figure S2). All runs have 20 vertical levels spaced evenly in pressure. We use the final 360 days of 1080 simulated days to perform our analysis.

1.2. Dry convective forcing

The second model used is the dry idealized GCM presented by Schneider (2004) and that has been used in a number of prior studies investigating eddy influences on the Hadley cells (Walker & Schneider, 2005, 2006; Schneider & Bordoni, 2008; Bordoni & Schneider, 2010). The primary differences between it and the Held-Suarez-like model are, first, the Newtonian cooling relaxation profile is statically unstable in this case, and, second, it includes a simple convective adjustment scheme that relaxes temperatures toward a lapse rate of $\gamma \Gamma_d$, where $\Gamma_d$ is the dry adiabatic lapse rate and $\gamma = 0.7$ mimics the stabilizing effect of condensation (although, again, the model is dry). The convective adjustment time varies from 7 days near the equator to 50 days near the poles. The reader is referred to Schneider (2004) for further details of the model. (Note that Hill, Bordoni, and Mitchell (2019) made additional modifications to the model that were not made here.) Simulations were performed for a range of rotation rates from 1/64 to 4 times that of Earth, spaced by factors of two. All simulations were performed at T42 resolution.

1.3. Aquaplanet forcing

A final suite of four simulations were performed with an aquaplanet model with idealized, gray radiation and simplified moist physics, which is also based on the same FMS spectral dynamical core (Frierson et al., 2006, 2007). Briefly, the model uses a fixed, constant longwave optical depth to solve the radiative transfer equation, a simplified con-
vective relaxation scheme for handling moist physics, and a simplified Monin-Obukhov
turbulence parameterization. The surface is assumed to be a slab of uniform thickness
and unlimited water supply.

For the aquaplanet configuration, simulations were performed at 1/8, 1/4, 1/2, and 1
times Earth’s rotation. The simulations are run at T42 resolution with 25 vertical levels.
We use the final 360 of 1080 days to perform our analysis.

1.4. Data processing

We perform the following data processing on all simulations: (1) all quantities are sym-
metrized about the equator; (2) the Hadley cell strength, $\Psi_{\text{max}}$, is taken as the maximum
over the cell of the overturning mass flux; (3) the quantity $S/f$ is also taken at the latitude
and level of $\Psi_{\text{max}}$ (4) vertical velocities are averaged over the downdraft at the level of
maximum Hadley cell mass flux. Latitude-pressure contour plots of zonal-mean stream-
functions, zonal winds, eddy momentum flux divergences, eddy heat flux divergences, and
static stability averaged over Hadley cell downdrafts are shown for all simulations in the
Supplement. The Hadley cell edge is defined to be the latitude where, moving poleward,
the mass flux drops to 10% of the maximum value; the top of the Hadley cell is similarly
defined, except moving upward rather than poleward.

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tions with an idealized gcm. *Journal of the atmospheric sciences*, 63(12), 3333–3350.
Figure S1. Heating rates for the Held-Suarez-like simulations at the level of the maximum Hadley cell mass flux and averaged over the downdraft. Total heating rate ($Q_{\text{eff}}$, black squares); eddy heat flux convergence (EHFC, blue circles); and Newtonian cooling (NC, red stars). Small, negative values are not shown.
**Figure S2.** Hemispherically symmetrized, Held-Suarez-like-forced model diagnostics, zonally and temporally averaged over the last 360 days of 1080-day simulations. Values of $\Omega^*$ are given along the right border. Eddy momentum flux divergences (both columns, colored contours [m s$^{-1}$ day$^{-1}$]); zonal winds (right column, black contours spaced 0 to 50 by 8 m s$^{-1}$, bolded contour is the zero-wind line), and mass streamfunction (left column, black contours, positive values are solid and rotating clockwise, spaced $-2\times10^{11}$ to $2\times10^{11}$ by $2\times10^{10}$, bolded contour is the zero-mass-flux line). The left “x” marks the location of the maximum mass flux of the Hadley cell and the right “x” marks our definition of the Hadley cell edge.
Figure S3. Same as in Figure S2, but for the “dry convective” forcing. Eddy momentum flux divergences (both columns, colored contours with spacing \( [\text{m s}^{-1} \text{ day}^{-1}] \)); zonal winds (right column, black contours spaced 0 to 50 by \( 8 \text{ m s}^{-1} \)), and mass streamfunction (left column, black contours, positive values are solid and rotating clockwise, spaced \(-3 \times 10^{11}\) to \(3 \times 10^{11}\) by \(3 \times 10^{10}\)).
Figure S4. Same as in Figure S2, but for the “aquaplanet” configuration. Eddy momentum flux divergences (both columns, colored contours with spacing [m s\(^{-1}\) day\(^{-1}\)]); zonal winds (right column, black contours spaced 0 to 30 by 4 m s\(^{-1}\)), and mass streamfunction (left column, black contours, positive values are solid and rotating clockwise, spaced \(-20\times10^{11}\) to \(20\times10^{11}\) by \(10^{11}\)).
Figure S5. (left) Eddy heat flux divergence (colors, [K day$^{-1}$]) and mass streamfunction (as in Figure S2) for the Held-Suarez-like model forcing. (right) Newtonian cooling (colors, [K day$^{-1}$]) and mass streamfunction (as in Figure S2)
Figure S6. As in Figure S5 but for the “dry convective” model forcing.
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Figure S7. As in Figure S5 but for the “aquaplanet” model configuration.
Figure S8. Zonal-mean static stability, \( \partial_p \bar{\theta} \), at the level of the maximum streamfunction and averaged over the width of the downdraft for each of the Held-Suarez-like simulations.
Figure S9. Same as in Figure S8 but for the “dry convective” forcing.
Figure S10. Same as in Figure S8 but for the “aquaplanet” forcing.