Magnetic Fields, Accretion, and the Central Engine of Gamma–Ray Bursts

A.R. King

Theoretical Astrophysics Group, University of Leicester, Leicester LE1 7RH

Abstract.
I briefly review magnetic effects in accretion physics, and then go on to discuss a possible central engine for gamma–ray bursts. A rotating black hole immersed in a non–axisymmetric magnetic field experiences a torque trying to align spin and field. I suggest that gamma–ray burst hosts may provide conditions where this effect allows rapid extraction of a significant fraction of the hole’s rotational energy. I argue that much of the electromagnetic emission is in two narrow beams parallel and antiparallel to the asymptotic field direction. This picture suggests that only a mass $\sim 10^{-5} M_\odot$ is expelled in a relativistic outflow, as required by the fireball picture.

Keywords: magnetic fields – accretion – gamma rays: bursts – gravitational waves – black hole physics – supernovae, general

ACCRETION

Accretion – the conversion of gravitational potential energy to other forms – is the most efficient energy source in nature. Accordingly it is a prime candidate for powering the most luminous systems: at the galactic scale these are X–ray binaries and ultraluminous X–ray sources (ULXs), while the most powerful distant sources are quasars and active galactic nuclei (AGN). Dropping a mass $\Delta M$ on to a compact object of mass $M$ and radius $R$ releases energy

$$\Delta E = \frac{G M \Delta m}{R}$$

where $G$ is the gravitational constant. Thus the accretion yield increases with compactness $M/R$. But matter does not in general want to fall into a very small radius $R$, just as the Earth does not fall into the Sun. Just like the Earth, matter which might otherwise accrete tends to orbit at a radius given by its angular momentum. The major problem in accretion theory is getting matter to lose enough angular momentum to accrete. In an active galaxy, the matter supply probably has specific angular momentum of order $j \simeq (G M a)^{1/2}$, where $M$ is the supermassive black hole mass and $a$ is a length order parsecs or more. To get this matter to accrete to the black hole one must reduce $j$ to a value $j_{\text{acc}} \simeq (G M r)^{1/2}$ where $r$ is of order the hole’s Schwarzschild radius $\sim 10^{14}$ cm. Thus $j$ has to be reduced by a factor at least $(a/r)^{1/2} \simeq 0.003$. Similar considerations hold in close binaries.

The agency which transports angular momentum outwards to allow matter to move inwards is usually (and perhaps misleadingly) called viscosity. Until recently there was little progress in identifying a suitable mechanism. The main problem is that purely hydrodynamical processes are generally sensitive to the angular momentum itself, and thus
transport angular momentum from large values to small, i.e. inwards. Instead we need a process which is sensitive to the matter velocity, which does increase inwards, and so would lead to angular momentum transport outwards. Magnetic fields naturally suggest themselves, as they offer a way of directly connecting fast–moving (low a.m.) matter at small radii to slow–moving (high a.m.) matter at larger radii. As shown by Balbus and Hawley (1991), the magnetorotational instability (MRI) discovered originally by Velikhov (1959) and Chandrasekhar (1961) offers a mechanism for this process. The physics of the process is – with hindsight – straightforward. A fieldline connecting fast–rotating matter close in to the accretor with slower matter further out is dragged in such a way that it rotates more slowly (rapidly) than its surroundings at small (large) radii, and so tends to move further in (out). Eventually differential rotation creates an azimuthal field which is now unstable to the Parker instability – the gas pressure and density inside its flux tubes are lower than outside, so they are buoyant and tend to rise, recreating a vertical field so that the cycle starts again.

We thus have a picture of disc accretion in which tangled chaotic magnetic fields transport angular momentum outwards and drive matter in. In general the field directions of neighboring disc annuli are uncorrelated, so now large–scale organised field arises. However if we wait long enough it must happen by chance that all the field directions in a patch of disc happen to line up. The effect of this was recently studied by King et al. (2004). Such an organised field is more effective in transporting angular momentum, so more matter moves in. But this motion drags in the field lines and tends to strengthen the field. Eventually this process amplifies the field to the point where instead of diffusing inwards, matter moves inwards as a wave (formally the diffusion equation for mass inflow changes into a wave equation). This wave of matter is accompanied by a strong magnetic field, and moves in a relatively non–dissipative manner. It is tempting to see these conditions as suitable for launching a jet. This may be how accretion discs are able to produce jets, and indeed sometimes alternate the processes of accretion and jet production, as in the microquasar GRS 1915+105 (Belloni et al., 1997).

**ACCRETION IN MAGNETIC BINARY SYSTEMS**

In the last Section I described how magnetic fields may drive accretion. If the compact accretor itself possesses an intrinsic magnetic field this itself can act as an obstacle to the flow. This occurs when the typical radius $R_{\text{mag}}$ at which the field is still dynamically important is comparable with various lengthscales within the binary, all related to the separation $a$. As a measure of $R_{\text{mag}}$ we can take the Alfvén radius given by equating magnetic stresses $\mu^2/8\pi \gamma \delta$ with material ones $\rho v^2$, where $\mu = BR^3$ is the magnetic moment of the accretor.

The no–hair theorems forbid intrinsic fields for black holes, so these ideas are only applicable in binaries containing neutron stars and white dwarfs. It is important to realise that the latter have stronger magnetic moments $\mu = BR^3$ than the former: for white dwarfs we can have $B$ as large as $10^7$ – $10^9$ G and hence $\mu \sim 10^{34} – 10^{36}$ Gcm$^3$ (although the latter are rare), whereas a neutron star with $B = 10^{12}$ G will only have $\mu \sim 10^{30}$ Gcm$^3$.

The relative importance of the magnetic moment in a binary is measured by the
quantity \( \mu/a^3 \), where \( a \) is the binary separation. Ranking systems by this quantity defines a hierarchy of accretion flows. Observationally they are distinguished by the ratio of spin to orbital periods.

**Small \( \mu/a^3 \)**

For systems with small \( \mu/a^3 \) (essentially all neutron star binaries, binary white dwarfs with weak fields, or wide WD systems with strong \( \mu \)) disc formation is unaffected by the field, because \( R_{\text{mag}} < < R_{\text{circ}} \), where \( R_{\text{circ}} \) is the circularization radius – the Kepler orbit with the same specific a.m. as the matter transferred from the donor star. In this situation the accretor gains only low specific a.m. \( \sim (GMR_{\text{circ}})^{1/2} \) and spins up to short spin periods \( P_{\text{spin}} \sim 10 – 100 \) s where \( R_{\text{mag}} = R_{\text{co}} \), the corotation radius, where the field lines rotate at the local Kepler value.

This is the typical situation in high–mass X–ray binaries, and also occurs in just two cataclysmic variables: the wide CV GK Per (orbital period 2 days) and the very weak–field system DQ Her. There is one exceptional CV: AE Aquarii has \( P_{\text{spin}} = 33 \) s, but no accretion disc. The accretion rate has dropped sharply in the recent past (as the system’s secondary/primary mass ratio decreased below \( \sim 1 \)) and the rapid WD spin can now expel the transferred matter via propeller action.

**Modest \( \mu/a^3 \)**

When \( \mu/a^3 \) is large enough that \( R_{\text{mag}} > R_{\text{min}} = 0.5R_{\text{circ}} \) the accretion stream from the donor hits the accretor’s magnetic field before it can orbit and make an accretion disc. In this case the WD spins up until \( R_{\text{co}} = R_{\text{circ}} \). This implies a relation between spin and orbital periods of the form \( P_{\text{spin}} \sim 0.08P_{\text{orb}} \), the precise coefficient depending on the binary mass ratio. This situation is possible only for white dwarfs, and these systems are called intermediate polars.

**Slightly higher \( \mu/a^3 \)**

For slightly larger \( \mu/a^3 \), the field influences the matter directly issuing through the inner Lagrange point \( L_1 \). The white dwarf spins up until the corotation radius is equal to the distance to \( L_1 \). This leads to a range of equilibria with \( P_{\text{spin}} \sim 0.6P_{\text{orb}} \) or higher. These are the EX Hydrae systems.

**High \( \mu/a^3 \)**

At the highest values of \( \mu/a^3 \), the field interacts directly with the donor, and forces the whole dwarf to corotate (or nearly so) with the binary orbit, i.e. \( P_{\text{spin}} \sim P_{\text{orb}} \). These are the AM Herculis systems.
THE CENTRAL ENGINE OF GAMMA–RAY BURSTS

Gamma–ray bursts liberate a significant fraction of the rest–mass energy of a star (i.e. $E_{\text{burst}} > 10^{51}$ erg s$^{-1}$) over intervals ranging from a few seconds to minutes. The fireball picture (Rees & Meszaros 1992) explains the otherwise puzzling ability of such sources to vary on short timescales by arguing that the energy of the burst drives a relativistic outflow with a bulk Lorentz factor $\gamma \sim 100$. Rest–frame variations of the central engine on timescale $t_{\text{var}}$ are then seen in the lab frame to have timescales

$$t_{\text{lab}} \simeq \frac{1}{2\gamma^2} t_{\text{var}}$$

To produce such behaviour the baryonic mass $M_{\text{out}}$ of the outflow must obey $E_{\text{burst}} \sim \gamma M_{\text{out}} c^2$, so that

$$M_{\text{out}} \sim 10^{-5} M_\odot$$

This ‘baryon–loading’ constraint is quite stringent, as many models of the central engine suggest instead that the prompt energy release may energise a larger mass. The most promising way of satisfying appears to involve prompt energy release as pure Poynting flux, such as may result from the Blandford–Znajek (BZ) process (Blandford & Znajek, 1977).

Other mechanisms are possible. The torque between a spinning black hole and a nonaxisymmetric magnetic field (King & Lasota, 1977) releases a large fraction of the rotational and thus rest–mass energy of the hole, and was recently reconsidered by Kim et al. (2003). Here I study this process further. I argue that electromagnetic energy $E_{\text{burst}} \sim 10^{51}$ erg s$^{-1}$ is released largely in two narrow oppositely–directed jets, with characteristic diameter of order the ergosphere radius. This arrangement satisfies the constraint (3).

THE ALIGNMENT TORQUE

Press (1972) deduced the existence of a torque on a spinning black hole immersed in a non–axisymmetric magnetic field as a corollary of Hawking’s (1972) theorem that a stationary black hole is either static or axisymmetric. Press was able to calculate the torque for the simpler case of a scalar field uniform at infinity, assuming that the hole and field passed through a sequence of stationary states, and inferred an answer for the magnetic case. King and Lasota (1977) calculated the magnetic torque explicitly, with the result

$$N = \frac{2G^2}{3c^5} M (J \wedge B) \wedge B$$

(where $M$ is the hole mass, $B$ the magnetic field at infinity and $J$ the hole’s angular momentum), a factor 2 larger than Press’s estimate.

The expression shows that a black hole aligns its spin with a stationary magnetic field by suppressing the angular momentum component $J_\perp$ exponentially on a timescale $\tau \simeq J/N$, i.e.

$$J_\perp = J_\perp, 0 e^{-t/\tau}$$
with

$$\tau = \frac{3c^5}{2G^2MB^2}$$  \hspace{1cm} (6)

The form of (4) means that there is no precession as this occurs. The parallel component \( J_\parallel \) remains fixed, so that the total angular momentum \( J = |J| \) decreases on the timescale (6). This process extracts rotational energy \( E \) from the hole, since

$$\dot{E} \propto J \cdot \mathbf{N} \propto (J \wedge \mathbf{B}) \wedge \mathbf{B} = -[J^2B^2 - (J \cdot B)^2] = -J^2 B^2. \hspace{1cm} (7)$$

In agreement with these ideas we note that since

$$J = MGa/c = Ma_*/c = Ma_* cR_g$$  \hspace{1cm} (8)

where \( a_* = a/M \) is the dimensionless Kerr parameter and \( R_g = GM/c^2 \) the gravitational radius, we can write

$$N \sim R_g^3B^2 \sim R_g \frac{L}{c}, \quad \tau \sim \frac{J}{N} \sim \frac{a_*Mc^2}{L} \hspace{1cm} (9)$$

where

$$L \sim R_g^2B^2c \hspace{1cm} (10)$$

is a luminosity which carries off the available rotational energy \( a_*Mc^2 \) on the timescale \( \tau \).

For a non-negligible Kerr spin parameter \( a_* < 1 \) the black hole’s rotational energy is comparable with its total rest-mass energy \( Mc^2 \). Hence a significant misalignment of \( \mathbf{J} \) and \( \mathbf{B} \) offers an energy reservoir sufficient to power a gamma-ray burst. However this energy is released only if the sources of the magnetic field remain essentially fixed on the timescale (6). This depends on the relative importance of the hole and source angular momenta \( J, J_{\text{sources}} \). If \( J_{\text{sources}} < J \), the timescale (6) and the corresponding energy release both reduce by a factor \( \sim J/J_{\text{sources}} \) (King & Lasota, 1977). Hence significant extraction of black hole spin energy occurs only if

$$J_{\text{sources}} > J$$  \hspace{1cm} (11)

**GAMMA-RAY BURSTS**

Condition (11) limits the applicability of the alignment torque. Although Kim et al. (2003) note that the alignment timescale (6) is dimensionally the same as the BZ timescale, very little of the black hole spin energy would emerge in a misaligned version of the BZ picture before alignment was complete, because the sources of the field (an accretion disc) have so little angular momentum compared with the hole.

One obvious case where the sources of a strong external field have high angular momentum and inertia occurs in a gamma-ray burst. In the hypernova picture (Woosley 1993; McFadyen & Woosley, 1999; Paczyński 1998) the degenerate core of an evolved and rapidly-spinning star collapses. The central regions support equipartition magnetic fields \( \sim 10^{15} \) G within a few gravitational radii of the spinning black hole forming
in the centre, and have high mass and angular momentum. Observations of magnetars with fields of this order support this idea. The well–attested occurrence of neutron–star kicks shows that core collapse can be significantly anisotropic. Moreover the kick must produce a spin (Spruit & Phinney, 1998) uncorrelated with the rotation of the stellar core. If the collapse forms a black hole rather than a neutron star, one can expect cases where the hole’s spin is misaligned with the core rotation and thus presumably the magnetic field. Unless this field varies over a lengthscale $\sim R_g$ we can regard it as approximately uniform near the hole. Under such conditions the alignment timescale (6) is

$$\tau_{\text{GRB}} = 4 \times 10^3 m^{-1} B_{15}^{-2} \text{s},$$

where $m = M/M_\odot$ and $B_{15} = B/10^{15}$ G. Even with weaker fields $B_{15} \sim 0.1$ this is of order the rest–frame duration of a gamma–ray burst with $\gamma \sim 100$ (cf Kim et al. (2003).

**THE PROMPT RADIATION PATTERN**

Press (1972) and King & Lasota (1977) calculate the alignment torque of a Kerr black hole, for stationary scalar and magnetic fields respectively. In reality both the gravitational field and the perturbing scalar or magnetic field must themselves vary on the timescale (5), implying the emission of both gravitational and electromagnetic radiation. (This is obvious from the fact that both fields change significantly between the start and end of the alignment process.) The stationary approximation is still correct for the purpose of calculating the torque; both fields adjust on the light–travel time, and thus pass through a sequence of stationary states. However this approach cannot give us the detailed form of the radiation fields. Direct calculation of these fields is a formidable undertaking, as testified by the extremely laborious nature of the torque calculation in the stationary magnetic case (the author’s handwritten algebra for the 1977 paper with Lasota covers some 65 pages of large–format computer paper). However we can deduce the nature of the electromagnetic emission by a simple argument.

The torque $N$ acts entirely on the misaligned component $J_\perp$. The net torque exerted by the Poynting field therefore consists of a couple acting about this axis. We can express this as two oppositely–directed beams on each side of the axis, each carrying radiation pressure force $L_p/c$ in a direction orthogonal to $J_\perp$, separated by a lever arm. Since the torque $N$ vanishes if the hole does not rotate, the lever–arm associated with the $L/c$ beams must be of order the ergosphere radius, which is itself of order the gravitational radius $R_g = GM/c^2$. The total radiation field may also have a component $L_\theta$ with no net lever arm (e.g. isotropic, or reflection–symmetric).

Since the $L_p$–beams exert the torque with lever–arm $R_g$ we have from (9) that

$$R_g \frac{L_p}{c} \sim N \sim R_g \frac{L}{c}$$

i.e. the luminosity $L_p$ in the two opposing beams is of order the total luminosity $L \sim a_\ast M c^2/\tau$ released by the alignment process. By symmetry the beams must be either parallel or orthogonal to $B$ as well as orthogonal to $J_\perp$. Alignment must leave the far field unchanged and simply affect the pinching of the field by the hole (cf King, Lasota
& Kundt, 1975), i.e. the component orthogonal to the asymptotic field direction. Since the magnetic field of an electromagnetic wave is transverse, this requires the beams to be emitted parallel to this asymptotic field direction (and with a definite polarization pattern).

This argument shows that a large fraction of the electromagnetic luminosity released during magnetic alignment is emitted in an opposed pair of narrow radiation beams (width $\sim$ ergosphere radius $\sim R_g$) along the asymptotic magnetic field direction, which in practice must have a slight intrinsic spread because of the deviation from a completely uniform field. (The radiation does not of course travel along field lines, but is simply emitted in directions parallel to the far field.) The photon picture of this emission is extremely simple: the misaligned angular momentum component $J_\perp$ ‘spins off’ photons from each side of the ergosphere, parallel to the asymptotic field lines.

**OBSERVATIONAL APPEARANCE**

For a black hole mass $M \sim 10M_\odot$ forming in a degenerate core of radius $\sim 10^9$ cm the prompt luminosity beams are directed over solid angle $\sim 10^{-5}$. They presumably drive out core matter from a pair of tubes with roughly these diameters, i.e. about $10^{-5}$ of the core mass. An important point here is that the beams are automatically directed along fieldlines in the region of strong field, making it much easier to drive matter out. The alignment process thus naturally satisfies the baryon–loading constraint (3). Even if the isotropic luminosity $L_0$ is of the same order as the beam luminosity $L$, geometrical dilution means that it must produce much lower Lorentz factors and thus be observationally insignificant.

**DISCUSSION**

I have suggested that in a hypernova the black hole may form with its spin misaligned with the strong magnetic field anchored in matter further out in the core. Under these conditions magnetic alignment can extract a significant fraction of the hole’s rest–mass energy. I have argued that much of this energy appears in two narrow, oppositely–directed beams. These beams have properties making them very suitable as the prompt emission for this type of gamma–ray burst.

Similar conditions might well hold in a gamma–ray burst caused by the coalescence of a black hole and a neutron star with a magnetar–strength field. In both cases the misaligned angular momentum component $J_\perp$ fixes the total energy reservoir. The energy therefore comes from the kick process producing this misalignment, and thus ultimately from the stellar spin and the asymmetry of the core collapse.

One never observes the prompt emission (as opposed to the fireball) from a gamma–ray burst, which makes observational tests of this idea necessarily indirect. The considerable range of observed neutron–star kick velocities does however suggest that if alignment is the engine for gamma–ray bursts, these form at least an one–parameter family.
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