Graded index multimode fibre as saturable absorber induced by nonlinear multimodal interference for ultrafast photonics

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Abstract
Graded index multimode fibre (GIMF) has emerged as a promising platform for two- and three-dimensional nonlinear optics. Based on the nonlinear multimodal interference technique, GIMF has demonstrated the saturable absorption effect and applied for fibre-based-laser short pulse generation as versatile, wideband ultrafast optical switches. Herein, this review presents the basic principles and the optical properties of the GIMF-based saturable absorber (SA). With this proposed GIMF-based SA device, mode-locking fibre lasers in the wavelength range of 1, 1.55, and 2 µm are realized. Particular focus is on the tunable and multi-wavelength mode-locked fibre lasers, various kinds of soliton generation and large energy soliton generation and a detailed summary of the current advances are given.

1. Introduction
The application fields of ultrafast lasers have vastly extended from optical communication [1] to material processing and manufacturing [2, 3], timing and synchronization [4] and optical frequency combs [5, 6], to name a few. The ultrafast solitons, generally at the femtoseconds (10−15 s) scale, demonstrate excellent characteristics of the finest temporal resolution as well as the highest pulse peak intensities. Fibre lasers with the significant advantages in terms of flexibility, reliability and compact nature are the most reliable sources for ultrafast solitons generation [7–10]. For the achievement of the ultrashort solitons in a fibre laser, the saturable absorber (SA) in the passive approach is the key mode-locking component in the cavity. In a simple form, the saturable absorption mechanism preferentially attenuates the low intensity parts of the propagating solitons in a form of intensity discrimination [11]. The advance of the SA technologies is almost synonymous with the evolution of the mode-locked fibre lasers. With current technology, the SAs can be divided into two categories: real SAs such as semiconductor saturable absorption mirrors (SESAMs) [12–14] and various other low-dimensional materials [15–26], artificial SAs [27–30] induced by the nonlinear optical effects such as nonlinear polarization rotation, nonlinear amplifying/optical loop mirrors. Normally, the desired properties of SAs mainly include appropriate saturation fluence, robustness, durability, ease of use, and small insertion loss. Although these present SA technologies, especially the two-dimensional materials SAs have been extensively explored in the mode locked fibre lasers and have been proved to be promising SA candidates for mode-locking, the materials SA has its intrinsic limitations and disadvantages such as limited service life, poor mechanical strength and low damage threshold. For the SESAMs [12], the small operating bandwidth and the expensive price are the primary disadvantages for the practical applications. For the two-dimensional materials [16–19], apart from the limited service life, the reduced mechanical strength introduced by the side-polished fibre or tapered fibre where the material is deposited on has also limited their applications in ultrafast photonics. The further enhancement of the pulse energy and pulse peak power is still an important restrictive factor. Recently, the multi-mode interferometer have shown great application potential in many fields, such as thermal resistance reduction in high power super luminescent diodes [31], high-power (mW) super luminescent diodes [32, 33], fibre sensor [34, 35], beam shape [36], and so on. More important, it has been demonstrated that the nonlinear multimodal interference (NL-MMI) effect in the multimode fibre has intensity discrimination properties as well [37–41]. Although the step-index
multimode fibres (SIMFs) also allow many intriguing nonlinear optical effects within an ultra-wide spectral band [42, 43], the graded index multimode fibres (GIMFs) with nearly identical group velocities of all the modes at special wavelengths demonstrate richer nonlinear dynamics.

GIMF exhibit unique dispersive, nonlinear, and spatiotemporal properties that are similar to the cases typically in two- and three-dimensional nonlinear optics. The spatiotemporal nonlinear effects in the GIMF can occur not only between the propagating modes, but also some physical quantities such as phase, frequency and energy, and hence the nonlinear strength is significantly enhanced. These distinctive traits and qualities promote the GIMFs an attractive candidate for researching on the novel nonlinear optical phenomena, such as multimode spatiotemporal solitons [44–46], supercontinuum generation [47–49], multimode saturable absorption [40, 41], self-induced beam cleanup [50, 51], geometric parametric instability [52, 53] and photon pair [54]. Here, in this review, we focus on the performance of the GIMF as a function of SA. The SMF-GIMF-SMF device as an embedded SA which possess the intensity discrimination was theoretically predicted by Arash Mafi [40] for the first time. Since then, several theoretical studies have been conducted on nonlinear mode coupling in GMF. In a simple way, high intensity light injected into the GIMF maintains the original state, whereas the low intensity portions are coupled into the high order modes (HOMs) through the NL-MMI. Since only fundamental mode (FM) can be retained into single-mode fibre (SMF), the intensity discrimination is achieved. This dynamic process is similar to the process in the waveguide arrays produced by the nonlinear mode-coupling (NLMC) [55–58]. The first saturable absorption behaviour was experimentally demonstrated at the 1550 nm wavelength range [59]. Since then, there are numerous reports in the literature on the ultrafast solitons generation in the broad wavelength band [60–74] by using this novel GIMF-based SA, which are summarized in table 1. For example, a Tm-doped mode-locking fibre laser using hybrid structure of SIMF-GIMF as an SA achieved lasing at the mode-locked operating wavelength of 1888 nm with the pulse width of around 1.4 ps at a fundamental frequency of 19.82 MHz [72]. The same structure was utilized to generate ultrafast laser pulses with 27 fs pulse duration at 1 μm wavelength [67]. The main advantage of the GIMF-based SA, apart from the simple structure, ease of fabrication and excellent nonlinear characteristics is the high damage threshold. All these advantages promote this all-fibre structured GIMF-based SA as an appealing alternative that help to deal with most of the remaining problems of the common SAs, especially in the field of large energy soliton generation.

On the other hand, the passively mode-locked fibre lasers, as the dynamically balanced nonlinear dissipative systems, have been verified as such fine platforms for investigating soliton dynamical processes and the nonlinear phenomena. Solitons which refer to an optical field that remains constant in the process of propagation can be generated in the mode-locked fibre lasers [75, 76]. As the dispersion is the intrinsic feature that influence the optical properties of the pulses, solitons can be divided into conventional solitons (CSs) [72–74, 77], stretched solitons [78], self-similar solitons [79] and dissipative solitons (DSs) [67, 68, 80, 81] according to the net dispersion value of the cavity. Besides, another feature in fibre lasers is the multiple soliton generation and interactions. Harmonic mode-locking [82, 83], bound solitons (BSs) [84–86] and synchronous solitons [87, 88] are some typical examples. In the multimode fibre, the optical solitons involve an intricate mix of spatiotemporal phenomenon. As the unique nonlinear mode coupling in the multimode fibres are beneficial to the nonlinear effects, various types of soliton generation and soliton interaction can be induced.

In this review, we discuss the SA properties of the GIMF-based SA and the corresponding applications in mode-locked fibre lasers to generate different types of ultrashort solitons. The review is arranged as follows: Firstly, we go into detail about the principles of the GIMF as SA. Next, we introduce the GIMF-based SA photonic devices and the nonlinear characteristics of the wideband device such as the modulation depth and so on. Finally, we research on the output performance of the ultrafast lasers in multiple wavebands mode-locked by the GIMF-based SA. Specially, we emphasize some special applications, including the tunable and multi-wavelength mode-locked fibre lasers, various kinds of soliton generation and large energy soliton generation. Beneficial from these advances of the SA, we give a brief conclusion and a positive vision on the new potential opportunities of the GIMF-based SA. Overall, the emphasis will be on this all-fibre structured GIMF-based SA.

2. The principles of the GIMF-based SA

2.1. Principles of GIMF as an SA

Figure 1 illustrates the schematic diagram of the GIMF-based SA device which is composed of a section of GIMF with the fibre length of L, and two closely spliced standard SIMFs on both ends of the GIMF [89]. When the laser is launched from SMF into GIMF, multiple fibre optic modes are activated and propagate along the GIMF.
Table 1. Progress in fibre lasers mode-locked by GIMF-based SA.

| Laser type | Ref. | GIMF-based SA type                  | SA characteristics (modulation depth; saturation fluence, nonsaturable loss) | Cavity parameters | Output characteristics |
|------------|------|------------------------------------|------------------------------------------------------------------------------|------------------|-----------------------|
|            |      |                                    |                                                                             | Dispersion | Gain fibre length | Cavity length | Soliton type | 3 dB bandwidth | Average power | Pulse duration |
| Erbium     | [44] | Hybrid Structure of SIMF-GIMF      | 3.16%, 2.43 μJ cm\(^{-2}\), 87%                                          | −0.176 ps\(^2\) | 2.5 m        | ~17.6 m   | CS           | 4.48 nm       | 0.15 mW (12.8 pJ) | ~446 fs       |
|            | [45] | Hybrid Structure of no core fiber (NCF)-GIMF | 4.77%, 0.59 μJ cm\(^{-2}\), 52.7%                                     | −0.139 ps\(^2\) | 2.5 m        | ~10.5 m   | CS           | 3.8–5.6 nm    | 0.53 (27.8 pJ)  | 1.70 ps        |
|            | [46] | Hybrid Structure of NCF-GIMF       | 4.7%, 0.14 μJ cm\(^{-2}\), 65.5%                                        | 0.467 ps\(^2\) | 2 m          | ~12.1 m   | DS and BS   | 5.8–14.6 nm   | 2.8 mW (0.17 nJ) | 7.7–23 ps      |
|            | [47] | Hybrid Structure of NCF-GIMF       | 4.57%, 1.92 μJ cm\(^{-2}\), 69.71%                                      | Net negative dispersion | 3 m          | ~13 m     | BS           | ~4 nm         | –                   | ~883 fs        |
|            | [48] | Cascaded dual GIMF                | –                                                                          | Negative dispersion | 0.2 m          | 10.6 m   | CS           | 2.9 nm        | 7.62 mW         | 930 fs         |
|            | [49] | Stretched GIMF                   | 10.37%–22.27%, 588.2 μJ cm\(^{-2}\), ~20%                              | Negative dispersion | ~0.024 ps\(^2\) | ~18.5 m | CS and stretched pulse (SP) | 6.35 nm/10.54 nm | 0.38 nJ/4 nJ | 2.3 ps/387 fs |
|            | [50] | GMIF                              | 1.8%, 0.75 GW cm\(^{-2}\)                                              | –                                                                  | –               | –         | –           | –             | –                   | –                |
|            | [51] | PC-coiled GIMF                   | 9%–21.7%, 14.1–16.1 μJ cm\(^{-2}\), 20.4%–8.6%                         | Negative dispersion | 0.2 m          | 7.6 m     | CS           | 6.84 nm       | 4.69 mW         | 559 fs         |
| Ytterbium  | [52] | Hybrid Structure of SIMF-GIMF     | –                                                                          | Normal dispersion | 0.3 m          | ~4.6 m   | DS           | 11.8 nm       | 5.8 mW (0.13 nJ) | 236 fs         |
|            | [53] | Offset-spliced GIMF              | 15.28%, 8.18 MW cm\(^{-2}\), ~64%                                       | Normal dispersion | 0.4 m          | ~7 m      | DS           | 12.3 nm       | 3.11 mW         | 11 ps          |
|            | [54] | Hybrid Structure of SIMF-GIMF     | –                                                                          | Normal dispersion | –              | –         | noise-like pulse (NLP) | 7.5 nm       | 37 mW           | 23.9 fs        |
|            | [55] | Hybrid Structure of SIMF-GIMF     | 5.9%, 1.56 GW cm\(^{-2}\), 90.4%                                        | 0.125 ps\(^2\)   | –              | 5.7 m     | BS           | 11 nm         | 29 mW            | 275 fs         |
|            | [56] | Offset GIMF                      | 10%, 11 MW cm\(^{-2}\), ~80%                                            | Normal dispersion | 3 m            | 109 m    | DSR          | 0.62 nm       | 0.14 μJ         | 2.5 ns–25 ns   |
| Thulium    | [57] | Hybrid Structure of SIMF-GIMF     | –                                                                          | Negative dispersion | 2 m            | ~10 m    | CS           | 3.68 nm       | –                | 1.4 ps         |
|            | [58] | Hybrid Structure of NCF-GIMF      | –                                                                          | Negative dispersion | 2 m            | 10.6 m   | CS           | 3.3 nm        | ~1 mW            | 1.25 ps        |
|            | [59] | Stretched GIMF                   | 9.5%, 142.8 μJ cm\(^{-2}\), 61.6%                                     | Negative dispersion | 1.8 m         | ~11 m    | CS           | 2.3 nm        | 0.25 mW (12.5 pJ) | 1.2 ps        |
In the linear regime where the pulse intensity is weak, the fundamental mode exchanges optical power with the HOMs due to the MMI effect with a periodicity of beat length $2L_c$, which is the so-called self-imaging effect. The half beat length $L_c$ where a $\pi/2$ phase difference is generated means the distance where the energy of the fundamental mode are transferred to the HOMs. The value of half beat length is determined by the core radius of the fibre cores and the refractive index distribution. However, when the peak intensity of the input light is large enough, the refractive index and the equivalent propagation constant of every mode propagating in the GIMF can be altered by the self-phase effect and cross-phase modulation effect, and thus, the group-velocity mismatch between modes is balanced by the nonlinear mode-coupling. In other words, the essentially same accumulated phase of every mode finally decreases the power conversion efficiency between the fundamental mode and the HOMs and thus, the light in the fundamental mode is retained. Therefore, such a SMF-GIMF-SMF element with a half beat length could demonstrate the saturable absorption effect.

For a further comprehension of the saturable absorption effect in the GIMF, we demonstrate a theoretical description of the nonlinear multimode propagating in the GIMF in detail. When the fundamental mode is coupled into the GIMF, multiple $\text{LG}_{n0}$ ($n = 0, 1, 2\ldots$) HOMs are activated. A simplified equation is given as following to describe the electrical field distribution of the transmitting $n$th HOM [37]

$$\frac{\partial E_p}{\partial z} = i\delta\beta_{(0)}^{(p)} E_p - \delta\beta_{(1)}^{(p)} \frac{\partial E_p}{\partial T} - \frac{i}{2}\beta_{(2)}^{(p)} \frac{\partial^2 E_p}{\partial T^2} + i\frac{\gamma}{\pi\omega_0} \sum_{l,m,n} \eta_{p,l,m,n} A_l A_m A_n^* = D^{(p)} (z, t) + N^{(p)} (z, t)$$

where $D^{(p)} (z, t)$ and $N^{(p)} (z, t)$ are the dispersion term and the nonlinear term of mode $p$, respectively, $E_p$ is the electric field distribution of the $\text{LG}_{n0}$. $\delta\beta_{(0)}^{(p)}$ $(\delta\beta_{(1)}^{(p)})$ means the difference value between the $p$th mode and the FM of the propagation constant $\beta_{(0)} (\beta_{(1)})$. $\beta_{(2)}$ and $\gamma$ mean the group velocity dispersion and the nonlinear coefficient, respectively. The nonlinearity couples the energy of the mode to every combination of modes. The propagation of general ultrafast multimode pulses can be described as a series of coupled nonlinear partial differential equations. When the peak intensity of the input light is large enough, nonlinear portion $N^{(p)} (z, t)$ reaches to be the same order of magnitude with the dispersion portion $D^{(p)} (z, t)$, the nonlinearity balances the material dispersion.

The transmission of the light refers to the optical power back into the following SMF from the GIMF divided by the total optical power coupled into the GIMF, written as [40]

$$\tau = \frac{1}{\tilde{P}} \left| \sum_n A_n^* (0) A_n (z) \right|^2$$

where $\tilde{P}$ means the total light energy which can be written as $\tilde{P} = \sum_{n} |A_n (z)|^2$. The nonlinear switching behaviour of the GIMF is demonstrated in figure 2 where GIMF length is equal to the half beat length for $p_0$.  

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**Figure 1.** The diagram of the GIMF-based saturable absorber [39] [reprinted].
(the normalized proportion of the fundamental mode) = 0.5, 0.25, 0.75. The saturable absorption results from the mutual competition between the dispersion term $D^{(p)}(z,t)$ and the nonlinear term $N^{(p)}(z,t)$ and the transition from small transmission to large transmission is achieved while the nonlinear coupling between fundamental modes and HOMs researches the same level as the linear coupling, and hence, a lower power threshold for this nonlinear switching behaviour requires a small linear coupling coefficient and a big nonlinear coefficient.

2.2. Photonic devices based on SA

The key macro-parameters of the GIMF-based SA that can determine the NL-MMI are mainly attributed to the mode-field diameter of the LG$_{00}$ mode in GIMF to the mode-field diameter of the SMF which corresponds to the input mode distribution for a certain kind of GIMF, the GIMF length, the relative nonlinear coefficient and the amount of the existing modes along the GIMF [40]. Typically, the dominant factors that determine the nonlinear conversion properties behinds the GIMF-based SA element include the input of the suitable mode pattern combination at the GIMF entrance from the SMF and the corresponding aggregation of the modes from the GIMF to the following SMF.

Hence, the saturable absorption can be achieved mainly in two methods: (a) change the GIMF length to a half of beat length; (b) change the mode field of the input light or in the GIMF. To be mentioned, directly obtaining the right GIMF is difficult since the self-focusing length of the GIMF is in the order of micrometres. In order to precisely choose the length of GIMF, it can be realized by the means of stretching the fibre or precisely cutting the fibre to a certain length. Figure 3(a) shows the schematic diagram of the stretched GIMF [64]. The length of SMF-GIMF-SMF element can be precisely controlled with an accuracy of a few microns in a range of $\sim$600 $\mu$m by using two translation stages. In order to change the input mode distribution, various designs have been implemented. For example, a section of SIMF [59, 67] (or NCF [60–62]) or an inner microcavity [90] at the entrance of the GIMF is added to inspire more HOMs, as shown in figure 3(b); an offset-GIMF SA is achieved through the partial misplaced fusing method to choose an appropriate offset. The GIMF-based SA demonstrates a good characteristic of flexible construction.

2.3. Nonlinear optical properties

For a typical SA device, the transmission increase as the light intensity until to reach saturation state. The main optical properties of a SA mainly include saturation fluence, modulation depth, nonlinear saturable loss and operational spectral range. One of the main methods to gauge the optical absorption characteristics of a SA element is the balanced twin-detector technique (the so-called 1-scan technique). In this technique, an ultrashort laser source at the wavelength to be measured is divided into two equal sections by using a 3 dB coupler, where one beam is used as a reference. The nonlinear optical characteristics of the SA device as a function of the input pulse intensity which is tuned by a variable modulator can be calibrated. Figure 4(a) gives a typical 1-scan schematic diagram.

The parameters of the GIMF-based SA can be simplistically described by a two level model. In this fit, it is assumed that the SA device possesses an instantaneous response. The function of the transmittance as the incident light can be written as

![Figure 2. The power transmittance $\tau$ versus the $\tilde{\gamma}$ when $p_0$ is set as 0.5 (solid), 0.25 (dashed), and 0.75 (dotted), respectively [40] [reprinted].](image-url)
Figure 3. The implementation of the GIMF-based SA device: (a) the stretched GIMF fixed on adjustment frames; (b) the introduction of a section of SIMF at the entrance of the GIMF.

Figure 4. (a) The twin-detector system for the detection of the nonlinear optical absorption characteristics. (b) [89] and (c) [74] [reprinted] are the measured nonlinear transmission curve of the GIMF-based SA at 1.5 µm and 2 µm, respectively.

\[ T(I) = 1 - \Delta T \times \exp \left( -\frac{I}{I_{\text{sat}}} \right) - \alpha_{\text{ns}}. \]

where \( T \) means the normalized transmissivity, \( \Delta T \) means the modulation depth, \( I \) means the input pulse energy, \( I_{\text{sat}} \) means the saturation intensity, and \( \alpha_{\text{ns}} \) means the non-saturable insertion loss. From this equation, we can learn that both the nonlinear effects and the linear effects contribute to the final absorption. Figure 8(a) present an example of the saturable absorption property of the stretched GIMF-based SA. Here, the ultrashort fibre source is centred at the wavelength of 1561 nm with the pulse width of 590 fs at a fundamental rate of 23.7 MHz [89]. The absorption modulation depth and saturation fluence of the GIMF-based SA obtained is 29.6% and \(~7.19 \times 10^{-3} \, \mu J \, cm^{-2} \) (\(~1.21 \times 10^{-2} \, MW \, cm^{-2} \)), respectively, as illustrated in figure 4(a). The GIMF-based SA properties have also been demonstrated in other wavelength regions. In the 2 µm band, the modulation depth of is measured to be \(~9.5\% \), as shown in figure 4(c).

As seen in the figures, the GIMF-based SA possesses unique properties, such as the controllable modulation depth, low saturable absorption, small insertion loss and so on. Here, we give a brief introduction on the structural features which contribute to these unique nonlinear properties. Firstly, we focus on the origination of the large modulation depth. To act as a SA, the fibre length of GIMF should be chosen precisely (to be the half beat length) to ensure that the energy with low peak power are mostly distributed in the HOMs when the input power intensity is small. The coupling efficiency to the following SMF is small at this time. However, the situation is on the contrary when the input power intensity is large. And thus, the GIMF-based SA can guarantee a large modulation depth. Secondly, let us transfer into the
small saturable intensity of the GIMF-based which is a few orders of magnitude lower than that of the traditional material SA [15–20]. The small saturable intensity benefits from two aspects, including the large GIMF length and highly Ge-doping [91, 92]. The GIMF length is usually set to be tens of centimetres which is several hundred times as the beating length of the GIMF. A relative interaction length provides a strong modulation of the light. The high Ge concentration in the core can enlarge the refractive-index which finally advances the nonlinear coefficient and hence the nonlinear strength.

3. Mode-locked fibre lasers using GIMF-based SA

The ultimate and central goal of exploiting these GIMF-based SA properties is to employ the SA element into a fibre cavity to initiate the ultrashort soliton generation. In this part, we initially give a systematic review on the passively mode-locked fibre lasers with the GIMF-based SA. And then we summarize the application of this device in some special aspects (include the tunable and multi-wavelength mode-locked fibre lasers, various kinds of soliton generation and large energy soliton generation) due to its unique linear and nonlinear properties.

Since 1.5 μm is the universal telecom optical wavelength, the GIMF-based SA was firstly employed in the Er-doped fibre laser [59–66, 93–96]. Zhaokun Wang et al [59] first experimentally report the optical properties of SIMF-GIMF SA device and its application in soliton generation from an all fibre Er-doped laser cavity. The all fibre laser generated 446 fs CSs centered at the wavelength of 1561 nm. And then, the mode-locking operation in an Er-doped fibre laser has also been proposed and demonstrated based on the stretched SMF-GIMF-SMF device as a SA [64]. Since the lower power modes propagate in a periodic interference pattern in the GIMF, the aggregation of the modes from the GIMF to the following SMF can be adjusted by changing the GIMF length. And thus, the modulation depth of the GIMF-based SA is tuned. In this paper, the GIMF length can be precisely tuned by being stretched and thus, the modulation depth of the device is tuned from 10.37% to 22.27% by changing the stretch length. Compared with Wang et al [89], the GIMF-based SA [64] demonstrates different modulation depth, since the accelerated nonlinearity strengths are quite different when the GIMF are with different fibre length. An offset-spliced GIMF is proposed to improve the SA performances in the method of coating the GIMF into the polarization controller [66]. These ultrashort lasers operate in the net anomalous dispersion regime, generating CSs. Further researches through cavity dispersion engineering result in DSs generation which can enable higher pulse energies. By adding the dispersion compensated fibre into an Er-doped fibre laser cavity mode-locked by a hybrid NCF-GIMF element to adjust the cavity net dispersion, the DSs with controllable pulse-width from 7.7 ps to 23 ps are obtained [61].

One main advantage of the GIMF-based SA is its operation bandwidth. The operating wavelength of the GIMF-based SA has been extended since then, covering from 1 μm band in the Yb-doped lasers [67–71] to 2 μm band in the Tm-doped fibre lasers [72–74, 97]. These achievements manifest the extensive applicability of the GIMF-based SA as a broadband device. By using the SIMF-GIMF SA device, the all fibre Tm-doped laser generates 1.4 ps solitons at 1888 nm and Yb-doped laser generates 276 fs DSs at 1030 nm. Figure 5 demonstrates the performances of Er-doped, Yb-doped and Tm-doped fibre laser by using this GIMF-based SA, respectively. The former table 1 illustrates the recent researches on the mode-locked fibre lasers mode-locked by GIMF-based SA of different construction.

To be mentioned, in addition to the cavity gain, loss and the balance between the dispersion and nonlinearity, the mode-locking operation can be also affected by the gain filtering effect, especially in the normal dispersion region. The GIMF-based SA can act simultaneously as a bandpass filter which can stabilize the mode-locked operation.

3.1. Tunable or multiwavelength mode-locked fibre laser

As mentioned above, the GIMF-based device provides an ideal, ultra-low-loss platform for tunable or multiwavelength mode-locking operation simultaneously as a SA and a spectral bandpass filter [98–102]. The functional filter effect originates from the optical interaction between the various modes. The transmission intensity I of a sandwiched-structure SMF-MMF-SMF device with the MMF length of L can be written as followed [98–103]

$$I = \sum_{i=1}^{n} I_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 2 \sqrt{I_i I_j} \cos \left( \frac{2\pi (n_i - n_j) L}{\lambda} \right)$$

where $I_i$ and $n_i$ mean the intensity distribution and the corresponding refractive index of the ith-order modes, $I_j$ and $n_j$ are the intensity distribution and the corresponding refractive index of the jth-order modes,
λ means the operating wavelength. The constructive interference condition of phase matching for two different modes can be given as followed:

\[(n_i - n_j)\lambda = m\lambda_i\]

The periodic interference pattern of this all-fibre filter originates from the superposition of all interference terms of the various modes. However, the different modes propagating along the MMF is susceptible to the birefringence intensity induced by bending or stretching the MMF, which can introduce an additional phase shift, as expressed below:

\[T = 1 + 2\sqrt{k(1-k)}\cos\left(\frac{2\pi(n_1 - n_2)\lambda}{\lambda} + \Phi\right)\]

where \(k\) means the coupling coefficient of the FM and \(\Phi\) means the additional phase shift which affects the coupling coefficients of the principle propagating modes. By bending or stretching the MMF, this unique feature can promote an ordinary multimode filter to a periodical bandpass filter with tunable ability. Figure 6 shows the transmission spectrum of a GIMF-based SA (SIMF-GIMF structured) and a periodically modulated effective gain spectrum with a period of \(\sim 40\) nm can be observed [97].

For the Er-doped fibre laser mode-locked by GIMF-based SA, a tunable ultrashort fibre laser in a wide range has been obtained. By employing an intracavity loss modulator to Er-doped fibre laser cavity mode-locked by NCF-GIMF SA device, larger tuning range of the operating wavelength and dual wavelength mode-locked operation can be achieved. Figure 7 shows the corresponding characteristics. The mode-locked wavelength is continuously tuned from 1574 to 1601.1 nm with the pulse width of \(\sim 1.7\) ps, as shown in figure 7(e) [60]. Moreover, the dual-wavelength mode-locking operation at 1575.4 and 1603 nm, with the pulse widths of 1.2 and 1.7 ps is also generated from the cavity, as shown in figures 7(a)–(d).

Furthermore, by adjusting the total intracavity insertion loss, the tunable and dual-wavelength mode-locking operation demonstrates more flexible output characteristics. For the tunable mode-locking operation, by increasing the total insertion loss, a more widely range of 47 nm from 1555.9 to 1602.6 nm is obtained when the pulse width of \(\sim 850\) fs is achieved [104]. The centre wavelength of the output pulses can be tuned at the basically whole gain bandwidth of the Er-doped fibre. In the dual-wavelength operation, the spectral spacing of the dual wavelength can be tuned from 11 to 33.01 nm. Based on a stretched-GIMF SA,
Figure 6. The transmission spectroscopy at 2 $\mu$m band of the SIMF-GIMF device [97] [reprinted].

Figure 7. The tunable or multiwavelength mode-locking output characteristics: (a) the dual-wavelength optical spectrum at 1575.4 and 1603.0 nm; (b) the RF spectrum with peaks at 19.06380 MHz for 1575.4 nm and 19.06545 MHz for 1603.0 nm; (c) the autocorrelation trace of 1.2 ps at 1574.4 nm and 1.7 ps at 1603.0 nm, respectively; (d) the scope chart of single soliton and the corresponding pulse train; (e) the tunable wavelength in a range of 27 nm from 1574 to 1601.1 nm.

Zhaokun Wang et al report the generation of a ring Er-doped mode-locked fibre laser with switchable wavelength where ultrashort solitons give the pulse durations of 506 fs and 416 fs at the two different wavelengths of 1572 nm and 1591 nm, respectively [64]. Similarly, the CD-SMS enable the mode-locking operation tuned in a range of 40 nm by mechanically adjusting the two PCs [63].

In addition to the pulse generation at the telecom band, the tunable or multiwavelength ultrafast lasers have also been demonstrated in other wavelength region, assisted by the filtering effect of the GIMF-based SA. By employing a cascaded SIMF-GIMF device as the SA and the bandpass filter simultaneously, a stable tri-wavelength mode-locking operation with the spectral wavelengths centered at 1855 nm, 1895 nm and 1933 nm was obtained, as shown in figure 8 [97]. In the literature, it has been confirmed that the multi-wavelength operation can be easily obtained regardless of the GIMF length. Such wideband tunability of the GIMF-based SA suggests that the excellent promises of this device for ultrafast photonics technology.

3.2. Different kinds of solitons generation

By using this GIMF-based device in the cavity and governing the cavity dispersion and nonlinearity properly, different kinds of ultrafast soliton can be experimentally achieved under certain conditions. In this section, we will present a comprehensive review of experimental investigations on the solitons formed based on this novel SA, including the dissipative soliton resonance (DSR), BSs and the synchronous pulse at different centre wavelengths.

3.2.1. Flat-top DSR

In 2008, a novel type of soliton which is named as DSR was theoretically predicted by Chang et al [105], which indicates that the pulse width increase indefinitely while the pulse amplitude remained constant with the increasing pump intensity according to the complex cubic-quintic Ginzburg–Landau equation.
Figure 8. (a) The RF spectrum of triple-wavelength mode-locking operation centered at 19.708 MHz (1933 nm), 19.713 MHz (1895 nm) and 19.718 MHz (1855 nm); (b) the RF spectrum of dual-wavelength mode-locking operation centered at 19.709 MHz (1934 nm) and 19.716 MHz (1857 nm); (c) the pulse train with three independent solitons of the triple-wavelength operation; (d) the pulse train with two independent solitons of the dual wavelength operation; (e) the spectra of the controllable multi-wavelength mode locking operation [97] [reprinted].

Figure 9. (a) The schematic diagram of the offset-GIMF SA: the offset of the two GIMFs with different length of 3 cm and 8 cm is about 10 µm; (b) the nonlinear transmission curve of the SA device where the reverse saturable absorption is observed in the dotted region; (c) the characteristics of the square-shaped solitons. The Guass-shaped spectral wavelengths centered at 1063 nm at different pump power; the rectangular single pulse with tunable pulse duration; the RF spectrum in a range of 300 MHz with a modulation of amplitude envelope; the pulse energies and the pulse widths at the different pump intensities [71] [reprinted].

The generation of the DSR is attributed to the reverse saturable absorption effect which indicates the transmission of the SA device decreases as the input light intensity increases.

An offset-spliced GIMF-based SA device was proposed and the reverse saturable absorption effect of the device was observed, which is demonstrated in figure 9(a). The reverse saturable absorption region can be obviously obtained at an optical density of 11 MW cm$^{-2}$, as shown in figure 9(b). Square-shaped pulses centered at 1063 nm with tunable pulse duration from 350 ps to 52.6 ns are generated from the Yb-doped fibre laser. The maximum single pulse energy is advanced to 139.1 nJ. The characteristics of the rectangular solitons are shown in figure 9(c).

3.2.2. Bound soliton

Bound solitons which refers to the multiple solitons transmitting in the fibre as a steady and integrative whole with a relatively close temporal interval are theoretically proposed in the 1990s [84, 108, 109]. Multiple types of BSs, such as soliton pairs with two separated solitons, soliton triplets with three separated solitons and so on have been obtained from the all-fibre mode locked oscillators by accurately adjustment [86, 110–114]. Since a large amount of complex nonlinear phenomena can be produced in the GIMF in the process of the ultrafast soliton generation, the GIMF-based SA offers an efficient candidate for the research on the BSs.
Figure 10. (a) The typical characteristics of tight BSs. Optical spectrum centered at 1573.72 nm with a spatial interval of about 3.78 nm; (b) the corresponding AC trace which demonstrates three peaks with the width of 662 fs and the interval of ∼2.07 ps; (c) pulse train with a fundamental repetition of 16.63 MHz; (d) RF spectrum with the SNR of over 74 dB; (e) the typical characteristics of the loose BSs: the left column (red) shows optical spectra with tunable intervals of regular spectral modulation and the right column (blue) are the corresponding AC traces with pulse width in the insert [62] [reprinted].

By employing a NCF-GIMF SA with the modulation depth, saturation fluence and nonlinear loss of 4.57%, 1.92 µJ cm⁻² and ∼69.71% into an all-fibre-structured Er-doped oscillator with the total cavity length of 13 m in the anomalous dispersion regime, both the single-soliton mode-locked operating state at the fundamental frequency and the BSs at tight and loose states can be obtained. The tightly BS pairs are composed of a pair of solitons with the similar pulse width of 700 fs and an interval of 2.07 ps, which matches well with the spectral modulation period. By stretching the SA element, the mode-locking wavelength is continuously tuned from 1567.48 nm to 1576.20 nm. For the loose soliton pairs, the optical spectrum centered at has a high-contrast stable modulation interference fringes with a tunable period (corresponding to the pulse interval) from 0.236 nm (37.57 ps) to 0.145 nm (56.46 ps). The corresponding results are shown in figure 10 [62].

The BSs are also observed in the normal dispersion regime. In an Er-doped fibre laser in the normal dispersion regime with the dispersion of 0.467 ps², mode-locked by NCF-GIMF device [61], BSs centered at 1560 nm with regular spectral modulation on the top are generated. The corresponding BSs with two separated molecules are observed from the autocorrelation traces, as shown in figure 11. BS states are also reported in a SIMF-GIMF-based Yb-doped mode-locked all-fibre laser operating in the normal dispersion regime. The SIMF-GIMF device is used for the pulse shaping in the scopes of both the frequency and time domains and acts as the SA and the bandpass filter simultaneously [70].

3.2.3. Synchronous pulse at different centre wavelengths

Synchronous pulse means the two pulse trains at different carrier wavelengths are perfectly overlap in time with the same repetition rate. The synchronous pulses have been extensively used in many fields, such as terahertz (THz) beating generation [115], and coincidence single-photon frequency up-conversion [116]. Normally, the synchronous pulse is generated from two different resonant cavities [87, 88]. In an SM fibre system, since the multi-wavelengths share a common physical length, an optical path difference will be introduced by the refractive index difference of the different wavelengths. Thus, the two pulse trains cannot be synchronized. However, in the fibre lasers with multimode section, the physical path length of every mode is different. When the pulses at different wavelengths are distributed on different mode states which could be realized by adjusting the cavity gain profile, the initial optical path difference in cavity length between
different wavelengths can be compensated. Consequently, in the fibre lasers, the multimode oscillator can operate at the dual-wavelength synchronous mode-locked state once the proper orientation is reached.

By employing a hybrid NCF-GIMF device in a ring cavity Er-doped fibre laser with overall net anomalous dispersion of around $-0.149 \text{ ps}^2$ and cavity length of 13 m, dual wavelength synchronous solitons with tunable property are obtained, benefiting from the change of the intracavity loss/gain distribution introduced by the adjustment of the variable optical attenuator [104]. Figure 11 illustrates the spectra and the AC traces of dual-wavelength mode locking operation. Interestingly, clear temporal modulation on the autocorrelation trace which results from the interaction of the solitons at two different wavelengths appears and this is the typical characteristic of the dual wavelength synchronous solitons. Furthermore, this dual-wavelength fibre laser has a controllable wavelength interval from 19.9 nm to 33.01 nm while maintaining the synchronization of two solitons. We believe that this multimode device can be used in other wavelength generation and is an excellent candidate for many researches and applications.

3.3. Large energy soliton generation

Large energy soliton generation is always one of the important research directions for ultrafast fibre lasers. Generally, the maximum power of the material-SA fibre laser is limited by both the accumulation of nonlinear effects inside the fibre oscillator and damage threshold of fibre devices (commonly the most vulnerable device is the SA). Benefiting from the large damage threshold and ideal nonlinear characteristics (e.g. large modulation depth, small saturable intensity), the SMF-GIMF-SMF device can be designed to promote the pulse energy in the mode-locked fibre lasers. The modulation depth with large value enables the SA providing more absorption to continuum wave (CW) which can decrease the soliton stability than to the solitons with high peak intensity in the laser oscillator. The GIMF-based SA with large modulation depth can restrain the CW effectively and hence contribute to the improvement of the pulse energy. The saturable intensity with small value ensures the mode-locking operation can still be initiated in the condition where a large coupling output is adopted. Overall, this GIMF-based SA demonstrates excellent specific properties for the generation of solitons with large energy.

For obtaining the solitons with large energy experimentally, a GIMF-based SA is constructed. The modulation depth and saturation fluence is measured to be 29.6% and $\sim 7.19 \times 10^{-3} \text{ mJ cm}^{-2}$, respectively. As discussed in section 2 of this paper, large modulation, and small saturation fluence would be beneficial to the realization of large energy solitons. In an Er-doped ring cavity as shown in figure 13(a), the pulse energy...
Figure 12. The spectra of the dual-wavelength operation and the corresponding autocorrelation traces of the synchronous pulses [104] [reprinted].

Figure 13. (a) The schematic of the all fibre oscillator with a ring cavity; (b) [1] output power (black) and pulse energy (red) as a function of the pump intensity in the anomalous regime which is in a linear increase with different slope efficiencies at different pump wavelengths; [2] the corresponding spectrum and AC trace (Inset) centered at 1564.4 nm; [3] output power (black) and pulse energy (red) as a function of the pump intensity in the normal regime; [4] the corresponding H-shaped spectrum at 1563 nm with the spectral width of about 9.1 nm and AC trace with pulse width of 11 ps (Inset) [89] [reprinted]; (c) the schematic of the all fibre oscillator with a linear cavity; (d) the corresponding output power, the spectrum and AC trace (Inset) [117].

is scaled up to 13.65 nJ (corresponding to the average power of 212.4 mW) in the anomalous regime with the dispersion of $-0.233 \text{ ps}^2$ and cavity length of 14 m and 6.25 nJ (the average power of 72.5 mW) in the normal regime with the dispersion of 0.336 ps$^2$ and cavity length of $\sim 17$ m [89]. Figure 13(b) demonstrates the output characteristics of the laser. In the anomalous dispersion regime, the average output power increases to 212.4 mW with an efficiency slope of 16.3% @ 980 nm and 51.3% @ 1480 nm. The smooth spectral file with 3 dB bandwidth of 1.7 nm centred at 1564.4 nm and the clear AC trace confirms that CS operation. In the normal dispersion regime, the maximum average output power obtained is 72.6 mW with the efficiency slope of 10.6%@ 980 nm and 28.5% @ 1480 nm. The rectangular spectral profile with the bandwidth of 9.1 nm is a typical characteristic of the DSs.

Similarly, in a linear Er-doped cavity with the total dispersion of $-0.038 \text{ ps}^2$, the 675 fs, CSs with the centre wavelength of 1571 nm and spectral width of 4 nm are generated at the repetition rate of 11.69 MHz. The maximum pulse energy (peak power) obtained is 4.72 nJ (7.79 KW), corresponding to the average
power of 61.4 mW. The results obtained from these experiments prove that this GIMF-based SA is a good candidate for wave-breaking free soliton with large energy generation [117].

4. Summary and outlook

In summary, we have presented divergent aspects of the all-fibre structured GIMF-based SA for ultrafast pulsed applications. This GIMF-based SA has drawn considerable attention as the advantages of all fibre structure and the controllable nonlinear properties. By choosing the appropriate length or changing the input light distribution of the GIMF, the SA effect can be achieved in the GIMF-based photonic element which enables the widely application in mode-locked fibre sources to achieve the ultra-short solitons with pulse width of ns-fs in a wideband wavelength range. In terms of the output characteristics, the GIMF-based SA can be used in the tunable and multi-wavelength mode-locked fibre lasers, various kinds of soliton generation and large energy soliton generation. The types of solitons have ever been reported include CS, DS, DSR, BS, NLP and SP. The maximum output power obtained to date is promoted to be 13.65 nJ/212.4 mW. This value is expected to be further improved by exploiting emerging fibre laser structures (such as the Mamyshhev oscillators) or using the cascaded master oscillator power amplifier (MOPA) schemes.

There is also growing interest in exploring broadband photonic SA device. While the GIMF-based SAs have been reported to act as ultrafast wideband photonic elements for initiating the mode-locking operation throughout the near-infrared region, we hope the application band of this SA can be expanded. In the fluoride or chalcogenide optical fibres with the graded core, ultrafast mid-infrared solitons should be achieved at an unprecedented longer wavelength.

Furthermore, many literatures on the ultrafast lasers utilizing the GIMF-based SA have been issued in the past several years, which implies this is a hot topic for photonics applications. However, these works rush to focus on the outstanding performances of this photonic device, very little literatures have focused on the long-term stability of the fibre oscillators, which is one of the important factors for the commercial applications. The commercial application of a reliable photonic device relies on the robustness, durability and the mass manufacturable possibility. The greatest challenge we are facing now is how to transfer these controllable experimental works to the practical applications.

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