Manipulating the frequency entangled states by
acoustic-optical-modulator

Bao-Sen Shi*, Yun-Kun Jiang and Guang-Can Guo†

Lab. of Quantum Communication and Quantum Computation
Department of Physics
University of Science and Technology of China
Hefei, 230026 P. R. China

Abstract

In this paper, we describe how to realize conditional frequency entanglement swapping and to produce probabilistically a three-photon frequency entangled state from two pairs of frequency entangled states by using an Acoustic-Optical-Modulator. Both schemes are very simple and may be implementable in practice.

03.65.Bz; 42.79.Sz

*E-mail: drshi@ustc.edu.cn
†E-mail: gcguo@ustc.edu.cn
I. INTRODUCTION

Ever since the seminal work of Einstein, Podolsky, and Rosen [1] there has been a quest for generating entanglement between quantum particles. The resource of entanglement [2] has many useful applications in quantum information processing, such as secret key distribution [3], quantum teleportation [4], dense coding [5], and so on. Two polarization entangled photons, which are produced by spontaneous parametric down-conversion (SPDC) with type II phase matching in nonlinear crystal [6] have been used to realize both dense coding [7] and quantum teleportation [8]. Entanglement swapping [9], which enables one to entangle two quantum systems that have never interacted directly with each other, has been demonstrated experimentally [10]. Maximally entangled state of three or more particles, so called Greenberger-Horne-Zeilinger (GHZ) state [11], has been fascinating quantum systems to reveal the nonlocality of the quantum world. There are some methods to produce the maximally entangled multiply particles states [12, 13]. The proposals for three particles entangled states have been made for experiments with photons [13] and atoms [14]. Three nuclear spins within a single molecule have been prepared such that they locally exhibit three-particle correlations [15]. Recently, Bouwmeester et al [16] have observed successfully the entanglement of three-photon GHZ state. The main idea of this scheme is to transform two pairs of polarization entangled photons into three polarization entangled photons and a fourth independent photon. In all above proposals, only polarization or momentum entanglement of multiply particles is considered. The frequencies or energies of these particles in an entangled state are assumed to be equal. Molotkov and Nazin [17] considered the case of the frequency entangled state and presented a simple nonlinear crystal based optical scheme for experimental realization of the frequency entanglement swapping between the photons belonging to independent biphotons. But this scheme is unpractical because of the lower nonlinear susceptibility of the nonlinear crystal. In this paper, the frequency entangled state is considered. We find that the frequency entanglement swapping can be realized only by Acoustic-Optical-Modulator (AOM). In contrast to the scheme of Ref. [10], no Bell
state measurement is needed. Furthermore, we can produce a three-photon frequency entangled state from two pairs of frequency entangled states by AOM. These schemes are simply and may be implementable in practice. One disadvantage of our scheme is both schemes are probabilistically realizable.

The paper is outlined as follows. In Sec. II, we give the frequency transformation done by AOM; In Sec. III, we present a proposal for frequency entanglement swapping; In Sec. IV, we show how to produce a three-photon frequency entangled state from two pairs of frequency entangled states; Finally, we give a brief conclusion in Sec. V.

II. FREQUENCY TRANSFORMATION DONE BY AOM

In this paper, all proposals are realized by AOM, so before proceeding, we will give the frequency transformation done by AOM. Suppose there is an AOM (for example, Acousto-Optic Bragg cell), which is driven at radio frequency (rf) $\delta$. If a monochromatic beam of frequency $\omega$ is introduced into the AOM, then it will be separated into two beams, one is a transmitted beam and the other is a diffracted beam. The frequency of the transmitted beam holds unchanged and the frequency of the diffracted wave shifts. The shift may be $\delta$ or $-\delta$, depending on the directions of the incident beam and the sound wave in AOM. For example, in Fig. 1, if the incident beam transmits along the path 1 ($1'$), and the direction of the sound wave in AOM is shown by arrow $\rightarrow$, the diffracted beam $d(t)$ will experience a frequency shift of $\delta$ ($-\delta$).[18]

The amplitudes of the transmitted and diffracted waves can be adjusted by the amplitude of the sound wave. If only one photon of frequency $\omega$ enters this AOM along path 1, and we adjust the amplitude of the sound $\delta$ so that the amplitudes of the transmitted and diffracted waves are equal, then the transformation done by AOM can be written as the following:

$$|\omega\rangle_1 \xrightarrow{AOM} \frac{1}{\sqrt{2}} [ |\omega\rangle_t + |\omega + \delta\rangle_d].$$

(1)

If a photon of frequency $\omega + \delta$ enters AOM along path $1'$, the transformation done by AOM is
\[ |\omega\rangle_1^{AOM} \rightarrow \frac{1}{\sqrt{2}}[|\omega\rangle_1 + |\omega + \delta\rangle_d]. \]  \hspace{1cm} (2)

With the demonstration above in mind, we proceed to the following.

**III. PHOTON FREQUENCY ENTANGLEMENT SWAPPING**

Suppose we are given two biphotons frequency entangled states \(|\Phi\rangle\) and \(|\Psi\rangle\):

\[ |\Phi\rangle = \frac{1}{\sqrt{2}}[|\omega\rangle_1|\omega + \delta\rangle_2 + |\omega + \delta\rangle_1'||\omega\rangle_2'], \]  \hspace{1cm} (3)

and

\[ |\Psi\rangle = \frac{1}{\sqrt{2}}[|\omega\rangle_3|\omega + \delta\rangle_4 + |\omega + \delta\rangle_3'||\omega\rangle_4'], \]  \hspace{1cm} (4)

Where, the subscripts represent beams taken by photons. In an experiment, these states can be easily obtained for example by SPDC in type I with noncolinear and nondegenerate phase matching. Obviously, photons in state \(|\Phi\rangle\) are independent of photons in state \(|\Psi\rangle\). In order to make one of photon in \(|\Phi\rangle\) state entangle with one of photon in state \(|\Psi\rangle\), we consider the arrangement of Fig. 2, in which, two AOMs are needed. Beams 2 and 3 enter AOM1 driven at a radio frequency \(\delta\), and beams 2’ and 3’ enter AOM2, which is also driven at the radio frequency \(\delta\). We arrange the diffracted beam of the frequency \(\omega\) along the transmitted beam of the frequency \(\omega + \delta\) in each AOM. According to Sec. II, if the frequencies of the beams 2, 2’, 3 and 3’ are \(|\omega + \delta\rangle_2, |\omega\rangle_2', |\omega\rangle_3\) and \(|\omega + \delta\rangle_3'\) respectively, then the following transformations are obtained:

\[ |\omega + \delta\rangle_2^{AOM1} \rightarrow \frac{1}{\sqrt{2}}[|\omega + \delta\rangle_{T_1} + |\omega\rangle_{T_1'}], \]  \hspace{1cm} (5)

\[ |\omega\rangle_2'^{AOM2} \rightarrow \frac{1}{\sqrt{2}}[|\omega\rangle_{T_2} + |\omega + \delta\rangle_{T_2'}], \]  \hspace{1cm} (6)

\[ |\omega\rangle_3^{AOM1} \rightarrow \frac{1}{\sqrt{2}}[|\omega + \delta\rangle_{T_1} + |\omega\rangle_{T_1'}], \]  \hspace{1cm} (7)
\[ |\omega + \delta\rangle_{3'} \xrightarrow{\text{AOM} 2} \frac{1}{\sqrt{2}}[|\omega\rangle_{T_2} + |\omega + \delta\rangle_{T'_2}] . \] (8)

where, \( T_1, T'_1 \) are the directions of transmitted (diffracted) and diffracted (transmitted) beams of the incident wave \(|\omega + \delta\rangle_2\) through AOM 1 , \( T_2, T'_2 \) are the directions of transmitted (diffracted) and diffracted (transmitted) beams of the incident wave \(|\omega\rangle_{2'}\) through AOM 2.

If the initial state of four photons to be \(|\Phi\rangle \otimes |\Psi\rangle\), it will transform into:

\[
|\Phi\rangle \otimes |\Psi\rangle = \frac{1}{2} \{ |\omega\rangle_1 |\omega + \delta\rangle_4 \otimes \frac{1}{\sqrt{2}}[|\omega + \delta\rangle_{T_1} + |\omega\rangle_{T'_1}] \otimes \frac{1}{\sqrt{2}}[|\omega + \delta\rangle_{T_2'} + |\omega\rangle_{T'_2}] + |\omega + \delta\rangle_{1'} |\omega + \delta\rangle_4 \otimes \frac{1}{\sqrt{2}}[|\omega + \delta\rangle_{T_1} + |\omega\rangle_{T'_1}] \otimes \frac{1}{\sqrt{2}}[|\omega + \delta\rangle_{T_2'} + |\omega\rangle_{T'_2}] + |\omega + \delta\rangle_{1'} |\omega + \delta\rangle_4 \otimes \frac{1}{\sqrt{2}}[|\omega + \delta\rangle_{T_1} + |\omega\rangle_{T'_1}] \otimes \frac{1}{\sqrt{2}}[|\omega + \delta\rangle_{T_2'} + |\omega\rangle_{T'_2}] \} .
\] (9)

The first and the last terms of the right of Eq. (9) mean that there are two photons which transmit through the AOM1 or AOM2, we discard these two cases. The other two terms mean that there is one photon which enters the AOM1 and another photon enters the AOM2 respectively, we only discuss these two terms. Obviously, these two terms can be rewritten as the following:

\[
\frac{1}{2} \{ |\omega\rangle_1 |\omega + \delta\rangle_{4'} + |\omega + \delta\rangle_{1'} |\omega + \delta\rangle_4 \} \otimes \frac{1}{\sqrt{2}}[|\omega + \delta\rangle_{T_1} + |\omega\rangle_{T'_1}] \otimes \frac{1}{\sqrt{2}}[|\omega + \delta\rangle_{T_2} + |\omega\rangle_{T'_2}] .
\] (10)

For example, if one photon enters AOM1 and at the same time another photon enters AOM2, photon 1 and photon 4 will be projected to a maximally frequency entangled state. The effect of AOMs in our scheme is that erasing all information about frequency and paths taken by photons. The probability of success is only 50%. This can be regarded as the probabilistic entanglement swapping. One advantage of this scheme is no Bell state measurement is needed, which may make this scheme implementable easily in practice. One disadvantage of our scheme is photons 2 and 3 are completely disentangled pair if successful entanglement swapping occurs, which is part of entanglement loses. Furthermore, this scheme can be used to realize photon frequency entanglement purifying in Ref. [19].
IV. PRODUCING A THREE-PHOTON FREQUENCY ENTANGLED STATE
FROM TWO PAIRS OF FREQUENCY ENTANGLED STATES

In order to produce a three-photon frequency entangled state, we consider the arrangement of Fig. 3, in which, only one AOM driven at rf $\delta$ is needed. Suppose we are also given two biphotons frequency entangled states $|\Phi\rangle$ and $|\Psi\rangle$ shown in Eqs. (1) and (2). The beams 2 and 3 enter the AOM. If the frequencies of beams 2 and 3 are $\omega + \delta$ and $\omega$ respectively, according to Sec. II, the following transformations can be obtained

\[ |\omega + \delta\rangle \rightarrow \frac{1}{\sqrt{2}}[|\omega\rangle_T + |\omega + \delta\rangle_{T'}], \]  
\[ |\omega\rangle \rightarrow \frac{1}{\sqrt{2}}[|\omega\rangle_T + |\omega + \delta\rangle_{T'}], \]  

where, $T$ and $T'$ are the directions of transmitted (diffracted) and diffracted (transmitted) beams of the incident beam 3 (beam 2). This kind of change can be obtained by the following: arranging the diffracted beam of the frequency $\omega$ along the transmitted beam of the frequency $\omega + \delta$.

Obviously, if one of these two detectors $D_T$ and $D_{T'}$ detects a photon, we can not get any information from which source this photon comes by frequency, i.e., information about the source of this photon by frequency is erased. This make the state of other three photons collapse into a superposition state. If all information about the source of this photon is erased, a three-photon frequency entangled state will be obtained.

Now, we discuss this scheme in detail. The product of state $|\Phi\rangle \otimes |\Psi\rangle$ is

\[
\frac{1}{2}\{ |\omega\rangle_1 |\omega + \delta\rangle_2 |\omega\rangle_3 |\omega + \delta\rangle_4 + |\omega\rangle_1 |\omega + \delta\rangle_2 |\omega + \delta\rangle_{3'} |\omega\rangle_{4'} + \\
|\omega + \delta\rangle_{1'} |\omega\rangle_{2'} |\omega\rangle_3 |\omega + \delta\rangle_4 + |\omega + \delta\rangle_{1'} |\omega\rangle_{2'} |\omega + \delta\rangle_{3'} |\omega\rangle_{4'} \}. \]

The first term of Eq. (13) means that there is a photon in beam 2 and beam 3 respectively, the fourth term means that there is no photon in both beams. We discard these two cases (which can be distinguished from the other cases by the following: observe whether both detectors fire, or both detectors are dark, or not) and only discuss the other two terms.
\[ \frac{1}{2} \{|\omega\rangle_1 |\omega + \delta\rangle_2 |\omega + \delta\rangle_3 |\omega\rangle_4' + |\omega + \delta\rangle_2' |\omega\rangle_3' |\omega + \delta\rangle_4 \}. \] (14)

By AOM driven at \( \delta \), Eq. (14) changes into:

\[ \frac{1}{2\sqrt{2}} \{|\omega\rangle_1 |\omega + \delta\rangle_3 |\omega\rangle_4' + |\omega + \delta\rangle_2' |\omega\rangle_3' |\omega + \delta\rangle_4 \}
\]

\[ + \{|\omega\rangle_1 |\omega + \delta\rangle_2' |\omega + \delta\rangle_4' + |\omega + \delta\rangle_2' |\omega + \delta\rangle_4' |\omega + \delta\rangle_4 \}. \] (15)

Equation (15) can be rewritten as:

\[ \frac{1}{2\sqrt{2}} \{|\omega\rangle_1 |\omega + \delta\rangle_2 |\omega\rangle_3' |\omega\rangle_4' + |\omega + \delta\rangle_2' |\omega\rangle_3' |\omega + \delta\rangle_4 \}
\]

\[ + \{|\omega\rangle_1 |\omega + \delta\rangle_2' |\omega + \delta\rangle_4' + |\omega + \delta\rangle_2' |\omega + \delta\rangle_4' |\omega + \delta\rangle_4 \}. \] (16)

Obviously, if only one photon is detected by any one of detectors, the state of the Eq. (16) collapses into a frequency superposition state.

\[ |\omega\rangle_1 |\omega + \delta\rangle_2 |\omega\rangle_3' |\omega\rangle_4' + |\omega + \delta\rangle_2' |\omega + \delta\rangle_4. \] (17)

To form a frequency-entangled GHZ state from the superposition state of Eq. (17), one must erase all ways by which one might in principle identify true pairs. The pair produced from one source will in general carry correlation in polarization, energy and time. Any of these may be exploited to identify the true sibling and hence prevent a GHZ state from forming. However, polarization correlation can never be exploited if all photons from the two sources carry the same polarization. This is very easy to realize. For example, we let both sources be SPDC with type I phase matching. By AOM, the energy correlation of true pairs (emitted by the same source) are indistinguishable from mixed pairs (one photon from each source). The temporal correlation can never be exploited if all four photons are produced or detected at the same time or, more generally, if the temporal correlation of true pairs and one of mixed pairs are indistinguishable. In order for that, it is necessary that the coherence time of the photons is substantially longer than the duration of the pulse. We can achieve this by placing two narrow filters \( F(\omega) \) and \( F(\omega + \delta) \) in front of detector \( D_T \) and \( D_{T'} \) respectively, and the bandwidths \( \sigma_1 \) of \( F(\omega) \) and \( \sigma_2 \) of \( F(\omega + \delta) \) satisfy the relation.
\[ \sigma_p \geq \sigma_1, \sigma_2 \]  

where, \( \sigma_p \) is the bandwidth of the pump pulse. By these above, a frequency entangled GHZ state is formed, which is also a beam entangled GHZ state. The probability of obtaining the GHZ state is about 50%.

A frequency entangled GHZ state can also be produced from two pairs of non-maximally frequency entangled states by the same way. For example, from

\[ |\Phi\rangle = \cos \alpha |\omega\rangle_1 |\omega + \delta\rangle_2 + \sin \alpha |\omega + \delta\rangle_1' |\omega\rangle_2' \]  

and

\[ |\Psi\rangle = \cos \alpha |\omega\rangle_3 |\omega + \delta\rangle_4 + \sin \alpha |\omega + \delta\rangle_3' |\omega\rangle_4' \].

The probability of getting a GHZ state is about \( \sin^2 \alpha \cos^2 \alpha \). The extension to producing the frequency entangled state of a higher number of photons from frequency entangled states of a lower number of photons is very easy, see Ref. [12].

V. CONCLUSION

In this paper, we show some manipulations of frequency entangled state by AOM. As two examples, we discuss how to realize frequency entanglement swapping and how to produce a three-photon frequency entanglement GHZ state. These schemes can be extended to realizing frequency entanglement purifying and to producing a frequency entangled states of multiply photons using the schemes in Refs. [12] and [19]. All schemes are very simple, and may be implementable in practice. The weakness of our schemes is that all proposals are probabilistically realizable.

This subject is supported by the National Natural Science Foundation for Youth of China.
REFERENCES

[1] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev., 47, 777 (1935)
[2] A. Zeilinger, Phys. World, 11, 35 (1998)
[3] A. K. Ekert, Phys. Rev. Lett., 67, 661 (1991)
[4] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres and W. K. Wootters, Phys. Rev. Lett., 70, 1895 (1993)
[5] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett., 69, 2881 (1992)
[6] Y. H. Shih and A. V. Sergienko, Phys. Rev. A., 50, 2564 (1994)
[7] K. Mattle, H. Weinfurter, P. G. Kwiat and A. Zeilinger, Phys. Rev. Lett., 76, 4656 (1996)
[8] D. Bouwmeester, J-W. Pan, K. Mattle, M. Eibl, H. Weinfurter and A. Zeilinger Nature, 390, 575 (1997)
[9] M. Zukowski, et. al, Phys Rev. Lett., 71, 4287 (1993)
[10] J-W. Pan, et. al., Phys Rev. Lett., 80, 3891 (1998)
[11] A. Zeilinger, M. Horne, H. Weinfurter, M. Zukowski, Phys. Rev. Lett., 78, 3031 (1997)
[12] S. Bose, et.al, Phys. Rev. A., 57, 822 (1998)
[13] M. Zukowski, et.al, Ann. N. Y. Acad. Sci., 755, 91 (1995)
[14] S. Haroche, Ann. N. Y. Acad. Sci., 755, 73 (1995)
[15] S. H. Lloyd, Phys. Rev. A., 57, R1473 (1998)
[16] D. Bouwmeester, J-W. Pan, M. Daniell, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett., 82, 1345 (1999)
[17] S. N. Molotkov and S. S Nazin, Phys. Lett. A, 252, 1 (1999)
[18] A. Yariv, *Quantum Electronics* 2nd ed. (John Wiley & Sons, New York, 1975), Chap. 14 (in chinese).

[19] S. Bose, et. al., Phys Rev. A., 60, 194 (1999)
Figure captions

Fig.1 Transformation done by AOM.

Fig.2 Setup of frequency entanglement swapping.

Fig.3 Schematic drawing of producing a three-photon GHZ state.
Fig. 1
Fig. 3