Mode areas and field energy distribution in 

honeycomb photonic bandgap fibers

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The field energy distributions and effective mode areas of silica-based pho-
tonic bandgap fibers with a honeycomb airhole structure in the cladding and
an extra airhole defining the core are investigated. We present a generaliza-
tion of the common effective area definition, suitable for the problem at hand,
and compare the results for the photonic bandgap fibers with those of index-
guiding microstructured fibers. While the majority of the field energy in the
honeycomb photonic bandgap fibers is found to reside in the silica, a sub-
stantial fraction (up to ~ 30 %) can be located in the airholes. This property
may show such fibers particularly interesting for sensor applications, especially
those based on nonlinear effects or interaction with other structures (e.g. Bragg
gratings) in the glass.

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1. Introduction

Photonic crystal fibers (PCFs), which guide light in a single-material structure by coherent scattering from an array of $\mu$m-sized airholes (for recent reviews we refer to Refs. [1, 2] and references therein), have in recent years emerged as an attractive alternative to conventional optical fibers within the area of nonlinear fiber devices.[3, 4] The advantages of the PCFs are firstly that very small mode areas can be obtained due to the large refractive index contrast between silica and air, leading to high nonlinearity coefficients. Secondly, the PCFs allow for a more flexible tailoring of the dispersion properties, which are crucial for many applications. PCFs with zero-dispersion wavelengths ranging from 565 to 1550 nm and high nonlinearity coefficients have been demonstrated.[5, 6]

The highly nonlinear PCFs fabricated today are of the index-guiding type,[7] in which a missing hole in a triangular lattice of airholes defines a high-index core, which guides light by total internal reflection. Fibers guiding light in large hollow cores by means of the photonic band gap (PBG) effect have also been demonstrated,[8] with the intention of obtaining very low losses and nonlinearities. However, an alternative PBG fiber design in which a honeycomb airhole lattice is modified by addition of an airhole to form a low-index core region[9] has until now not been investigated thoroughly although its practical feasibility was demonstrated as early as 1998.[10] The design is shown in Fig. [1] and can be characterised by three parameters: The physical distance between nearest-neighbor airholes (commonly denoted the pitch, or $\Lambda$), the diameter of the cladding holes, $d_{cl}$, and the diameter of the hole forming the core defect, $d_c$. 
These fibers guide the majority of the light in silica, just as the conventional index-guiding PCFs, and may, therefore, constitute an alternative way of fabricating highly nonlinear fibers. The purpose of the present work is to investigate the design depicted in Fig. 4 with respect to field energy distribution and nonlinear coefficients. It will be shown that nonlinearities comparable to those obtained in index-guiding PCFs can be achieved in the honeycomb PBG fibers, while at the same time, a substantial fraction of the field energy may be pushed into the airholes. This may make these fibers particularly interesting for applications as sensing devices.

The rest of the paper is organized as follows: In Sect. 2, we describe our theoretical methods, and derive a generalized formula for the fiber nonlinearity coefficient, in terms of an effective area, which is valid for all field distributions, including those where a substantial part of the field energy resides in air. In Sect. 3, we present and discuss our numerical results for some selected honeycomb designs, and compare them to results for index-guiding PCFs. Finally, Sect. 4 summarizes our conclusions.

2. Theoretical approach

We consider a structure, which is uniform along the $z$-axis while structured in the $x$-$y$ plane. The magnetic field vector, $\mathbf{H}$, may then be written:

$$\mathbf{H}(\mathbf{r}) = e^{i(\omega t - \beta z)} H(x, y),$$

for a monochromatic wave. The fundamental equation for $\mathbf{H}$ is:

$$\frac{\omega^2}{c^2} = \frac{\langle \mathbf{H}, \Theta \mathbf{H} \rangle}{\langle \mathbf{H}, \mathbf{H} \rangle},$$

$$\Theta \mathbf{H} = \nabla \times \frac{1}{\varepsilon_r(\mathbf{r})} \nabla \times \mathbf{H}$$

for $\mathbf{r} \neq 0$.\(^{11, 12}\)
where \( \varepsilon(\mathbf{r}) = \varepsilon_0 \varepsilon_r(\mathbf{r}) \) is the dielectric function. For a fixed propagation constant, \( \beta \), this equation may be solved for \( \omega \), which can then be regarded as a function of \( \beta \) and \( \varepsilon(\mathbf{r}) \). From the magnetic field vector, the corresponding electric field is straightforwardly obtained using Amperes law (SI units are used throughout the paper):

\[
\nabla \times \mathbf{H} = i\omega \varepsilon \mathbf{E}
\]

The effective area is a concept originating in the theory of third-order nonlinearity effects in optical waveguides. In a homogeneous material, such as amorphous silica, the third-order part of the nonlinear susceptibility gives rise to an amplitude dependent shift in the material refractive index:

\[
n = n_0 + \Delta n = n_1 + n_2 \left| \mathbf{E} \right|^2
\]

where \( n_1 \) is the refractive index of silica in the limit of zero field, and \( n_2 \) is a nonlinear coefficient related to the nonlinear-susceptibility tensor, \( \chi^{(3)} \), through:

\[
n_2 = \frac{3}{8n_1} \text{Re} \chi^{(3)}_{xxxx}.
\]

In an optical fiber, the change in material index leads to a corresponding change in the effective index \( (n_{\text{eff}} = \frac{\beta}{\omega}) \). In first-order perturbation theory, the mode-field distribution can be considered unchanged by the index perturbation, and the change in \( \omega \) for a (Kerr-induced) \( \Delta \varepsilon \) for fixed \( \beta \) is from Eq. 2 found to be:

\[
\Delta \omega = -\frac{\omega \varepsilon_0 c^2 \langle \mathbf{E}, \Delta \varepsilon_r \mathbf{E} \rangle}{2 \langle \mathbf{H}, \mathbf{H} \rangle}.
\]
Usually, the experimental situation is that light is launched at a fixed frequency $\omega_0$ and the Kerr-induced change in refractive index effects a change in the propagation constant $\beta$ from $\beta_0$, say, to $\beta_0 + \Delta \beta$. $\Delta \beta$ is then determined by:

$$\omega(\beta_0 + \Delta \beta) + \Delta \omega = \omega_0, \ \omega(\beta_0) = \omega_0.$$  \hspace{1cm} (8)

Here, $\omega(\beta)$ is the relation between $\omega$ and $\beta$ in the absence of the Kerr effect, and we have neglected the change in $\Delta \omega$ arising from the shift in $\beta$, assuming that both $\Delta \omega$ and $\Delta \beta$ are small. We can then obtain $\Delta \beta$ by linear expansion:

$$\frac{\partial \omega}{\partial \beta} \Delta \beta = -\Delta \omega \Rightarrow \Delta \beta = -\frac{\Delta \omega}{v_g^0}$$ \hspace{1cm} (9)

Here, $v_g^0$ is the waveguide group velocity in the absence of material dispersion effects (since we consider propagation at a fixed frequency, the frequency dependence of $\varepsilon$ should not be taken into account when evaluating $\frac{\partial \omega}{\partial \beta}$). The change in $n_{\text{eff}}$ arising from the Kerr effect is then:

$$\Delta n_{\text{eff}} = -\frac{c \Delta \omega}{v_g^0 \omega_0}.$$ \hspace{1cm} (10)

For $\Delta \varepsilon_r$, we have:

$$\Delta \varepsilon_r = 2\sqrt{\varepsilon_r} n_2 \left| E \right|^2 + O \left( n_2^2 \left| E \right|^4 \right),$$ \hspace{1cm} (11)

and thereby, neglecting the small $n_2^2 \left| E \right|^4$ term and using $\sqrt{\varepsilon_r} = n_1$:

$$\Delta n_{\text{eff}} = \frac{\varepsilon_0 c^3 \int n_1 n_2 \left| E \right|^4 dA}{v_g^0 \langle H, H \rangle} = \frac{\varepsilon_0 c \int n_1 n_2 \left| E \right|^4 dA}{v_g^0 \langle E, D \rangle}.$$ \hspace{1cm} (12)
where, in the last equality, the inner product of $\mathbf{H}$ with itself has been rewritten using Eq. (2) and (4). The integral is taken over the $xy$-plane. Note that, for a microstructured fiber, $n_1, n_2$ are position-dependent quantities.

In the case of ordinary silica fibers, the parameters $n_1, n_2$ have little variation over the fiber cross section, and it is common practice to express the Kerr-induced change in the guided-mode effective index as:

$$\Delta n_{\text{eff}} = P \frac{n_2^P}{A_{\text{eff}}}$$  \hspace{1cm} (13)

where $P$ is the power launched into the fiber, $n_2^P = \frac{n_2}{n_1 \varepsilon_0 c}$ and $A_{\text{eff}}$ is the effective mode area. In a microstructured fiber, both $n_1$ and $n_2$ can have a strong position dependence, and a generalization of Eq. (13) is not straightforward. However, in the case where $n_1, n_2$ are piecewise constant functions, taking on $N$ different values over the fiber cross section, we can modify the above definition to:

$$\Delta n_{\text{eff}} = P \sum_{i=1}^{N} \frac{n_2^P}{A_{\text{eff}}^i}$$  \hspace{1cm} (14)

where $n_2^P_i$ denote the value of $n_2^P$ in the different sections of the fiber. It can be shown that $P$ and $v_g^0$ are connected by:

$$P = \int (\mathbf{E} \times \mathbf{H})_z \, dA = v_g^0 \langle \mathbf{E}, \mathbf{D} \rangle$$  \hspace{1cm} (15)

From Eqs. (12), (15) we now obtain:

$$\Delta n_{\text{eff}} = P \sum_{i=1}^{N} n_2^P i \frac{(n_1^i \varepsilon_0 c)^2 f_i | \mathbf{E} |^4 \, dA}{(v_g^0 \langle \mathbf{E}, \mathbf{D} \rangle)^2} = P \sum_{i=1}^{N} n_2^P i \frac{(n_1^i n_0^0)^{2} f_i | \mathbf{E} |^4 \, dA}{(\mathbf{E}, \mathbf{D}_r)^2}$$  \hspace{1cm} (16)
In the last step, we have introduced $D_r = \varepsilon_r E$ and the effective group index of the guided mode, $n_g^0 = \frac{c}{v_g}$. Note that the integral over $|E|^4$ in each term is restricted to the regions with $n_1 = n_1^i$. Comparing Eqs. (14) and (16), it can be seen that:

$$A_{\text{eff}}^i = \frac{\langle E, D_r \rangle^2}{(n_1^i n_g^0)^2 \int \ |E|^4 \ dA} \left( \frac{n_1^i}{n_g^0} \right)^2 \int \ |E \cdot D_r|^2 \ dA$$

(17)

In the present paper, we shall only be concerned with the case of pure silica/air fibers, in which $n_1, n_2$ are equal to 1 and 0, respectively, in the air regions while having the values appropriate for silica in the rest of the transverse plane. In this case, the nonlinear coefficient will be entirely determined by the effective area relating to the silica regions, that is:

$$A_{\text{eff}} = \left( \frac{n_1}{n_g^0} \right)^2 \frac{\langle E, D_r \rangle^2}{\int_{\text{SiO}_2} |E \cdot D_r|^2 \ dA}$$

(18)

The values for $n_1, n_2$ are now understood to be those of pure silica.

At this point, two comments are in order: Firstly, note that of the two $n_1$ factors in the denominator of Eq. (18), one comes from the definition of $n_2^P$ in terms of $n_2$, whereas the other one comes from the fundamental wave equation. Therefore, if one uses a table value of $n_2^P$ derived with a $n_1$ different from that at which experiments are done, it may be necessary to use two different $n_1$ values in the product. Secondly, Eq. (18) differs somewhat from the formula commonly used for $A_{\text{eff}}$, namely:

$$\tilde{A}_{\text{eff}} = \left( \frac{\int |E|^2 \ dA}{\int |E|^4 \ dA} \right)^2$$

(19)
However, if we assume that all the field energy resides in the silica regions of the fiber, and that $n_g^0 \approx n_1$, we obtain:

$$\left( \frac{n_1}{n_g^0} \right)^2 \frac{\langle E, D_r \rangle^2}{\int_{\text{SiO}_2} |E \cdot D_r|^2 dA} = \frac{(f n_1^2 |E|^2 dA)^2}{(n_1 n_g^0)^2 \int |E|^4 dA} \approx \frac{(f |E|^2 dA)^2}{f |E|^4 dA}$$

(20)

Thus, the commonly used formula (19) appears as a limiting case of the more general result (18). The approximations leading from Eq. (18) to (19) are reasonably well fulfilled in standard fibers and in most index-guiding PCFs. This is, however, not the case for the fiber designs examined in the present work. In Fig. 2 we show the relative difference between the $A_{\text{eff}}$ definitions in Eqs. (18), (19) defined as:

$$\Delta A_{\text{eff}} = A_{\text{eff}} - \tilde{A}_{\text{eff}}$$

(21)

with $A_{\text{eff}}$, $\tilde{A}_{\text{eff}}$ given by Eqs. (18), (19) respectively. The differences have been calculated for some fibers designed to have a substantial part of the field energy in air, as discussed in the next section. It is evident that differences of 10-20% between the two definitions can easily occur. Also, the difference varies in both sign and magnitude over the transmission window of the fibers, so that no simple scaling rule between the two definitions can be extracted.

In this work, we solve Eq. (2) by expanding the magnetic field and the dielectric function in plane waves, using a freely available software package. Eq. (4) can be used to derive the electric field vector from the magnetic, and the effective area can then be found from Eq. (18). Since it is important for many applications of nonlinear effects to be close to a wavelength at which the group velocity dispersion is zero,
we have also investigated the dispersion properties of the fibers. In order to scan a
range of physical pitches efficiently we have used a recently developed perturbative
approach to the inclusion of material dispersion effects. We have found this scheme
to be both efficient and accurate in the case of silica/air microstructured fibers.

3. Numerical results and discussion

In the present investigation, we will primarily focus on structures with $\frac{d}{\Lambda}$ lying in
the interval between 0.3 and 0.8 for both core and cladding holes. Such structures
are by now routinely fabricated by fusing and drawing hand-stacked capillary tubes
and rods of silica. However, in order to investigate the limitations of the hole-defect
honeycomb PBG fibers, we have also studied a design with $\frac{d_{cl}}{\Lambda}=0.95$ and $\frac{d_{c}}{\Lambda}$ in the
range between 0.1 and 0.3. Results for $\frac{d_{cl}}{\Lambda}=0.1$, 0.2 and 0.3 are shown in Fig. 3. The
lowest value of $A_{\text{eff}}$ is 0.76$\lambda^2$ and is obtained for $\frac{d_{c}}{\Lambda}=0.2$. The occurrence of a minimum
$A_{\text{eff}}$ as a function of core hole diameter can be understood as follows: Since nonlinear
effects only occur in the silica part of the fiber, reduction of core hole size increase
the region of integration in the denominator of Eq. (18), thereby acting to decrease
the effective area. On the other hand, as is evident from Fig. 3, reduction of the core
hole size also decreases the values of $\frac{\lambda}{\Lambda}$, where the guided mode becomes localized.
In other words, the fiber dimensions becomes larger relative to the wavelength of the
guided mode, and this acts to increase the effective area, when measured relative to
$\lambda^2$.

In the case of index-guiding PCFs, effective areas as low as 1.7 $\mu m^2$ at a wave-
length of 1.55 $\mu m$ have been reported experimentally, corresponding to $A_{\text{eff}} \sim 0.75\lambda^2$. 
The minimal mode area that can be obtained in a silica rod in air has been proposed as a theoretical lower limit on the effective area in silica-based index-guiding fibers, and has been found to be $1.48 \, \mu m^2$ at $1.55 \, \mu m$ (or $0.62\lambda^2$). Thus, in spite of the fact that the guided mode is localized around an airhole in the honeycomb structures considered here they are able to obtain mode areas, which are only slightly larger than what is possible in index-guiding PCFs.

In Fig. 4 we show the effective area as a function of wavelength for $d_{cl}\Lambda = 0.56$, 0.68 and 0.80, and $d_c\Lambda$ around 0.3-0.5. It can be seen that $A_{eff} \sim 1.5 - 3\lambda^2$ is readily obtained, and that the minimal area decreases with increasing size of the cladding holes and increases for increasing size of the core hole.

In many experiments involving nonlinear effects it is important to work at a wavelength at which the chromatic dispersion of the fiber is close to zero. In these situations, it is the minimal effective area obtainable at a given wavelength, under the condition that the dispersion coefficient be zero, which is of interest. Both index- and PBG-guiding PCFs can have complicated dispersion curves with several zero-dispersion points. Some examples of what can be achieved with the honeycomb design for $d_{cl}/\Lambda = 0.68$ are shown in Fig. 5. Following the three curves with $d_c/d_{cl}=0.45$ it can be seen that the zero-dispersion point can have a discontinuous behaviour as a function of $\Lambda$. The curve for $d_c/d_{cl}=0.55$ is an example of a fiber with several dispersion zeros. To investigate the effective area at the zero-dispersion point we have chosen to focus on the longest zero-dispersion wavelength for a given design, since this is where one will usually have the smallest effective area relative to the wavelength of the light. For the PBG fiber we have, therefore, investigated the location of the longest zero-
dispersion wavelength, $\lambda_0$ over a range of physical fiber dimensions from $\sqrt{3}\Lambda=1.5$ to 5 $\mu$m. In Fig. 6 we report $\lambda_0$ versus the physical pitch, $\Lambda$, and in Fig. 7 the effective area at $\lambda_0$ is plotted versus $\lambda_0$. Broken curves indicate a discontinuity in $\lambda_0$ as a function of $\Lambda$. In some cases, the zero-dispersion wavelength can sweep over the same frequency several times as $\Lambda$ is varied, which is why some of the curves in Fig. 7 are multi-valued. Generally, the honeycomb fibers tend to have dispersion zeros falling in the range between 0.8 and 2 $\mu$m.

For comparison we have also considered index-guiding PCFs with a triangular array of airholes constituting the cladding and a core-defect formed by a missing air hole. For air hole diameters $d \leq d^*$ (with $d^* \sim 0.45\Lambda$) this class of PCFs is endlessly single mode and by scaling the pitch both large-mode area PCFs as well as small-core non-linear PCFs can be formed. In addition to the mode-size the dispersion properties may also be engineered. In the top panel of Fig. 8 we show the zero-dispersion wavelength $\lambda_0$ as a function of the pitch $\Lambda$ for 5 hole diameters. Depending on the pitch the PCF may have none, one, or two (or even three if also considering the near-infrared regime) dispersion zeros. For the situation with two dispersion zeros it is seen that the second dispersion zero (counting from the short wavelength limit) depends strongly on both the pitch and the hole diameter whereas for the first dispersion zero the dependence on hole diameter dominates over the much weaker dependence on pitch. One of the exciting properties of these PCFs is that by increasing the hole diameter the lowest dispersion zero can be moved toward the visible regime. In the lower panel of Fig. 8 we show the effective area versus zero-dispersion wavelength. In general we find that when the hole size is increased the mode becomes more tightly confined.
with a smaller effective area. The plot also illustrates a highly desired property; when shifting the first dispersion-zero toward the visible the effective area also decreases so that the intensity thresholds for various non-linear phenomena also decreases. We note that the first dispersion zero may be moved to the visible and the effective area may be decreased by further increasing the air hole diameter (so that \( d > d^* \)), but then care must be taken that the PCF remain single-mode near the dispersion zero.

Comparing Figs. 7 and 8 it can be seen that the honeycomb PBG fibers investigated here do not offer substantially improved flexibility in the tailoring of mode areas and dispersion properties compared to the index-guiding PCFs. The PBG designs with large cladding holes do seem to offer smaller mode areas at the longer values of \( \lambda_0 \) but this could also be obtained in index-guiding PCFs by increasing the size of the airholes, thereby going further out of the endlessly single-mode regime. Determining the single-mode regime for PBG fibers with large cladding airholes is difficult due to the appearance of multiple bandgaps even at relatively long values of \( \lambda \). For the fibers with \( \frac{d_{cl}}{\Lambda} = 0.8 \) we have found that a guided second-order mode is present at \( \lambda_0 \) over most of the \( \Lambda \) range investigated. For the fibers with smaller values of \( \frac{d_{cl}}{\Lambda} \) the second-order modes mostly appears at wavelengths shorter than \( \lambda_0 \) for \( \Lambda \)-values smaller than \( \sim 2 \) \( \mu \)m meaning that a useful range of \( \lambda_0 \) values without second-order mode guidance is available. We have not, however, checked for the presence of guided modes in higher-order gaps.

One interesting feature of the honeycomb PBG design compared to the index-guiding PCFs is that a relatively large fraction of the field energy resides in the airholes of the fiber. This is illustrated in Fig. 9 where the fraction of electric field energy
present in air has been plotted for the fiber designs discussed above. It is evident that energy fractions of 10-15% in air are readily obtained, even for holes of moderate size, and that the fraction increases with increasing size of the core hole defect. To estimate the range of energy distributions accessible, we have investigated some designs, in which the core hole defect has been further enlarged. In Fig. 10 we show results for effective areas and the energy fraction in air for fibers with $\frac{d_c}{\Lambda}$ around 0.6 and varying size of the cladding holes. It can be seen that the energy fraction in air can be as large as 30%, while still having a range of possible zero-dispersion wavelengths and fairly small mode areas. Further increase of $d_c$ does not push appreciably more field energy into the airholes, however, the transmission windows and accessible zero-dispersion ranges are quickly diminished. The somewhat counterintuitive fact that the minimum of the effective area curves (i.e., the maximum of the nonlinear coefficient at a particular wavelength) occurs approximately at the same $\frac{\Lambda}{\lambda}$ values as the maximum of the energy fraction in the airholes is due to the fact that the energy fraction in air is high at long wavelengths, where the fiber size relative to wavelength is small.

In Fig. 11 we show some radial mode profiles obtained by integrating the electric field energy density over the angular coordinate in a polar coordinate system around the core center. The curves are calculated for fibers with $\frac{d_c}{\Lambda}$ = 0.8 and varying size of the core hole, and have been normalized so that their radial integral is unity. The wavelength has been chosen so as to maximize the fraction of field energy in air. Only 5-10% of the field energy is present in the central hole defining the core. This shows that the increase in the energy fraction present in air for increasing $\frac{d_c}{\Lambda}$ is not so much due to the field energy being pushed into the central core hole, but rather
to an increase of the field energy present in the cladding region as is also evident from Fig. 11. The presence of a substantial part of the field energy in air not only influences the integrals in Eq. (18), but also makes the group velocity of the guided mode deviate substantially from the material refractive index, thus influencing the prefactor \( \left( \frac{n_1}{n_0} \right)^2 \). In Fig. 12, this fraction is depicted for the designs with \( \frac{d\Lambda}{\Lambda} = 0.8 \). Its importance for a correct evaluation of the effective area in these structures is evident.

The power fraction in air (which is not completely equivalent to the quantity calculated here) for index-guiding PCFs was investigated by Monro and co-workers\(^{18}\) who found that large airhole diameters and \( \frac{\lambda}{\Lambda} \)-values were needed to obtain appreciable power fractions in air. In Fig. 13, we report the fraction of field energy in air for some index-guiding PCFs with the same cladding structure as those discussed above but somewhat larger airholes. Comparing Figs. 9 and 10 with Fig. 13 it is evident that the \( \lambda/\Lambda \) values needed to push a given fraction of field energy into the air region are considerably larger in the index-guiding fibers, calling for smaller values of the physical pitch for operation at a given wavelength. This may make the honeycomb fibers particularly relevant for evanescent field devices\(^{11,12}\) such as gas sensors, based on interactions with the glass, through, e.g., nonlinear effects or inscribed Bragg gratings.

4. Conclusion

In conclusion, we have investigated the field energy distribution and nonlinear coefficients of honeycomb photonic bandgap fibers and compared them to index-guiding photonic crystal fibers with a cladding structure consisting of a triangular array of
airholes. A generalized concept of effective mode area, which is adequate for the treat-
ment of fibers with a substantial part of the field energy present in the airholes, has
been derived for this purpose. While the honeycomb fibers do not seem to offer in-
creased flexibility in the design of dispersion properties and mode areas they do offer
the same possibilities as the index-guiding fibers, at wavelengths above \( \sim 1 \mu m \). In
addition, the honeycomb fibers have a larger fraction of the field energy present in the
airholes which may make these fibers particularly interesting for sensor applications
based on interactions with the glass, through e.g. nonlinear effects and/or inscribed
Bragg gratings.

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Fig. 1 The generic PCF structure investigated in the present work. The core and innermost cladding holes are shown along with the defining parameters $d_c$, $d_{cl}$, and $\Lambda$.

Fig. 2 Relative difference between the effective area definition proposed here, Eq. (18), and the commonly used definition, Eq. (19).

Fig. 3 Effective area, relative to wavelength, calculated from Eq. (18) for a fiber with $\frac{d_{cl}}{\Lambda}=0.95$ and various values of the core hole diameter, $d_c$.

Fig. 4 Effective area relative to the wavelength of the guided mode for honeycomb PBG fibers with $\frac{d_{cl}}{\Lambda}=0.56$ (a), 0.68 (b) and 0.8 (c) and various values of $\frac{d_c}{d_{cl}}$.

Fig. 5 Plots of the chromatic dispersion coefficient, D, in units of ps/nm/km for various values of the pitch in a honeycomb PBG fiber with $\frac{d_{cl}}{\Lambda}=0.68$ and $\frac{d_c}{d_{cl}}=0.45$.

Fig. 6 Longest zero-dispersion wavelength, $\lambda_0$, as a function of the physical pitch, $\Lambda$. Structures and labeling as in Fig. 4.

Fig. 7 Effective area at the zero-dispersion wavelength, $\lambda_0$ as a function of $\lambda_0$. Structures and labeling as in Fig. 4.

Fig. 8 Effective area and dispersion zeros for index-guiding PCFs. Top panel: Zero-dispersion wavelength versus pitch. Lower panel: Effective area at the zero-dispersion wavelength as a function of the zero-dispersion wavelength.

Fig. 9 The fraction of the electric field energy of the guided mode present in the airholes. Structures and labeling as in Fig. 4.

Fig. 10 Effective area relative to wavelength (a), effective area at the zero-
dispersion wavelength (b) and energy fraction in air (c) for some fiber designs in which a substantial part of the field energy resides in the airholes.

Fig. 11 Radial profile of the electric field energy density, obtained by integration over the angular coordinate in a coordinate system with origin at the core center. The curves are normalized to have unit radial integrals. The thin vertical lines indicate the position of the first ring of cladding airholes.

Fig. 12 Variation of the prefactor $\frac{n_1}{n_0}$ in Eq. (18) with wavelength for a fiber with $\frac{d_{cl}}{\Lambda}=0.8$ and various values of the core hole diameter, $d_c$.

Fig. 13 Energy fraction in air of index guiding fibers with a cladding structure as shown in Fig. 8 and various airhole diameters $d$. 
Fig. 1. LÆGSGAARD
\[ \Delta A_{\text{eff}} / A_{\text{eff}} \]

Fig. 2. LÆGSGAARD
Fig. 3. LÆGSGAARD
Fig. 4. LÆGSGAARD
Fig. 5. LÆGSGAARD
Fig. 6. LÆGSGAARD
Fig. 7. LÆGSGAARD
Fig. 8. LÆGSGAARD
Fig. 9. LÆGSGAARD
Fig. 10. LÆGSGAARD
$d_{cl}/\Lambda=0.8$

![Graph showing the relationship between $E\cdot D$ (arb. units) and $r/\Lambda$ for different values of $r_c/\Lambda$.](image)

- $r_c/\Lambda=0.16$
- $r_c/\Lambda=0.24$
- $r_c/\Lambda=0.32$

Fig. 11. LÆGSGAARD
Fig. 12. LÆGSGAARD

\[ \frac{d_{cl}}{\Lambda} = 0.8 \]

\[ \left( \frac{n_1}{n_g} \right)^2 \]

\[ \frac{d_c}{d_{cl}} = 0.4 \]
\[ \frac{d_c}{d_{cl}} = 0.5 \]
\[ \frac{d_c}{d_{cl}} = 0.6 \]
\[ \frac{d_c}{d_{cl}} = 0.8 \]
Fig. 13. LÆGSGAARD