Large deformation failure analysis of slopes using the smoothed particle finite element method

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Abstract. Slope instability and landslides can be catastrophic events leading to loss of lives and properties. To prevent and assess the risks of slope failures, it is often desired that the dynamic process of the slope failure can be simulated, which is difficult with the classic Finite Element Method (FEM). In this study, the smoothed particle finite element method is developed based on the popular and efficient FEM. A numerical example with a slope model is employed to demonstrate the capacity of the proposed approach. An elastoplastic material model based on the Mohr–Coulomb yield criterion is used. The run out distance and failure mass is recorded which paves a way of being able to better quantify slope failure consequence and risk.

1. Introduction
For slope stability analysis, the limit equilibrium method (LEM) is often used in practice due to its simplicity and efficiency. In order to address the statically indeterminate problem, many assumptions on slip surfaces and internal force distribution have been made [1]. As an alternative, the finite element method (FEM) does not require these assumptions and the factor of safety is given naturally from the results [2]. Its reliability and robustness in dealing with slope stability analyses are proved in both academe and practice [3-9]. However, these two conventional methods are restricted to the pre-failure stage.

It is clear that the risk of slope failure is strongly related to the failure consequence. To model the failure processes, numerical methods dedicated to modelling large deformation of geomaterials have been developed and applied. The discrete element method (DEM) [10] is formulated with discrete particles and free from the restriction of the deformation compatibility. It has been applied to model the progressive failure of slopes or investigate their failure mechanisms [11-15]. One shortcoming of the DEM is that its micro-parameters need to be calibrated, which is not trivial. The material point method (MPM) [16, 17] represents the material as a collection of material points and uses moving material points and computational nodes on a background mesh. The complex failure mechanism such as the development of successive shear bands can be captured [18-22]. The smooth particle hydrodynamics (SPH) [23] has been applied in geomechanics for large deformation and post-failure flows of geomaterials [24-26]. A good agreement with experimental observations and previously simulated results in terms of the profile and internal deformation has been found in [25]. The particle finite element method (PFEM) [27-29] is another meshfree method but solves the governing equations via a standard FEM procedure. It inherits both the solid mathematical foundation of the classic FEM and the capability of meshfree particle methods for handling extremely large deformation and free-surface evolution [30]. In PFEM, all the information of the continuum medium is carried by the particles, and the state variables such as stresses are calculated at Gauss points. Therefore, information
transferring between particles and Gauss points is frequently performed during the calculation process, inevitably leading to numerical error \[31, 32\].

In this study, a strain smoothing technique \[33, 34\] developed in the smoothed finite element method (SFEM) \[33, 35, 36\] is implemented in the framework of the PFEM. The associated governing equations are converted into a standard second-order cone programming problem that can be solved efficiently using standard off-the-shelf solvers. Typical advantages of such a solution strategy include the straightforward treatment of singularities in some yield criteria, such as the Mohr–Coulomb model for geomaterials \[37-39\], the extension from single-surface plasticity to multi-surface plasticity without additional computational effort \[37\] and the capacity of dealing with interfaces and contacts \[11, 40\].

2. Principle of smoothed particle finite element method

After FEM meshes are conducted, smoothing domains associated with FEM nodes are created, as shown in Figure 1. As depicted, the smoothing domain \(\Omega^s_k\) assigned to node \(k\) is the coloured polygon covering one-third of all the node’s adjacent elements. The smoothing domain is bounded by multiple straight boundary segments which connect the midpoint of an element edge to a centroid of a triangular element.

Following the classic FEM, for each finite element the strain-displacement relation is given as:

\[
\varepsilon = B_u \hat{u}
\]

where \(\varepsilon\) is the strain field, \(B_u\) is the strain-displacement matrix and \(\hat{u}\) is a vector consisting of nodal displacements.

In the smoothing domain \(\Omega^s_k\) (Figure 1), the smoothed strain \(\bar{\varepsilon}_k\) at node \(k\) is calculated by:

\[
\bar{\varepsilon}_k = \int_{\Omega^s_k} \Phi_k(x)\varepsilon(x)d\Omega = \int_{\Omega^s_k} \Phi_k(x)B_u \hat{u}d\Omega \tag{2}
\]

where \(\Phi_k(x)\) is the smoothing function and, in this study, the local constant smoothing function \[34, 41\]

\[
\Phi_k(x) = \begin{cases} 
1/A^i_k, & x \in \Omega^i_k \\
0, & x \notin \Omega^i_k
\end{cases}
\tag{3}
\]

is used where \(A^i_k\) is the area of the smoothing domain \(\Omega^i_k\).

Note that the node-based integration is used, i.e. the equilibrium of the continuum medium is achieved at the strain smoothing cells.
From the Hellinger-Reissner variational principle, the min-max program corresponding to the elastoplastic governing equation can be derived [27]. It is then discretised using above mentioned smoothed particle finite element method, leading to the final optimisation problem to be solved, in which the standard $\theta$-method is employed to discretise the governing equations [42]. The range of $\theta$ of $0.5 \leq \theta \leq 1$ can be used. For $\theta = 0.5$, numerical damping effect is eliminated, while damping effect is considered in the model with $\theta > 0.5$. The standard efficient second-order cone program (SOCP) solver may be used for this purpose. Interested readers are referred to [27, 39, 43]. A typical computational cycle of the smoothed particle finite element method is simply written as:

1. Initialise the program with the particles’ position, filed variables and boundary conditions;
2. Loop over the load increment steps;
3. Generate the Delaunay triangles and obtain element topology;
4. Construct the strain smoothing cells and compute particles’ smoothed strains;
5. Solve the discrete governing equations to obtain the displacement of the nodes to obtain particles’ incremental solutions;
6. Update the positions and field variables of particles;
7. End looping over the load increment steps;
8. Output results.

3. Numerical examples

3.1 A cantilever beam

To verify the proposed approach, the dynamic response of an end-loaded elastic cantilever beam is used, as shown in Figure 2. It is discretised with 2184 linear triangular elements. The Poisson’s ratio is 0.3, and Young’s modulus is 300 MPa. The weight of the beam is neglected. The time step is 0.1s. Both the cases with and without damping are considered. The point force load is applied at the tip of the beam and its value is changed over time, as shown in Figure 3.

![Figure 2. The model setup of the cantilever model.](image)
Figure 3. The history data of the point force.

The results of this study in comparison with the commercial FEM software Abaqus are shown in Fig. 4. A good agreement between the results of this study and Abaqus has been achieved. Furthermore, because of the numerical damping, the beam’s kinetic energy was dissipated. After sufficient computation time, the beam can reach the static balance position.

Figure 4. Numerical results of the proposed approach and Abaqus

3.2 slope failure process

After the approach’s dynamic formulations are validated in the first example, the slope failure process is modelled in this section. The model is shown in Figure 5. The unit weight, friction angle and cohesion are set as $\gamma = 20$ kN/m$^3$, $\phi = 20^\circ$ and $c = 63.7$ kPa, respectively. Other model parameters are Young’s modulus of 1 GPa, the Poisson’s ratio of 0.25 and a time step of 0.02 s.

Figure 5. The slope model

Figure 6 shows the numerical results of the slope failure at different computation time. The slope surface can be identified by the equivalent plastic strain increment. After 6.1 seconds, the collapsed slope reaches its final stable configuration, as shown in Figure 7. The run-out distance is 18.0 m. It is possible to determine the hazardous area of slope failures based on the numerical results of the post-failure.
Figure 6. The failure process for the slope at (a) 2 s, (b) 4 s and (c) 6 s. Colours are proportional to the equivalent plastic strain increment.

Figure 7. Final configuration of the collapsed slope. Colours are proportional to the slope’ horizontal displacements.

4. Conclusion
A smoothed particle finite element method for the slope failure process is presented. All variable states (e.g. displacements, strains and stresses) are stored on mesh nodes, meaning that variable mapping from old meshes to new meshes is not required anymore in the particle finite element analysis of
history-dependent problems despite remeshing operations. The numerical example of a cantilever beam is employed to validate the dynamic formulations. The second example, a slope failure simulation shows that the method can be used for post-failure analysis. This would help to better quantify the consequences of slope failures by calculating the sliding mass, run-out distance and impacting forces. Therefore, engineers and policymakers can better assess the risk associated with each slope.

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