Fundamental Limits of Identification System With Secret Binding Under Noisy Enrollment

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Abstract—We study fundamental limits of biometric identification systems with chosen secret from an information theoretic perspective. Ignatenko and Willems (2015) characterized the capacity region of identification, secrecy, and privacy-leakage rates of the system provided that the enrollment channel is noiseless. In the enrollment process, however, it is highly considered that noise occurs when bio-data is scanned. Recently, Yachongka and Yagi (2019) characterized the capacity region of a different system (generated secret system) considering noisy enrollment and template constraint. In this paper, we are interested in characterizing the capacity region of identification, secrecy, template, and privacy-leakage rates of the system with chosen secret under the same settings as Yachongka and Yagi (2019). As special cases, the obtained result shows that the characterization reduces to the one given by Ignatenko and Willems (2015) where the enrollment channel is noiseless and there is no constraint on the template rate, and it also coincides with the result derived by Günlü and Kramer (2018) where there is only one individual.

I. INTRODUCTION

Biometric identification is a process of comparing biological characteristics or data (bio-data) of an individual to that individual’s template already stored in the system database. Some well-known applications are fingerprint-based identification, iris-based identification, voice recognition, etc.

O’Sullivan and Schmid [1] and Willems et al. [2] separately introduced the discrete memoryless biometric identification system (BIS). Willems et al. [2] have clarified the identification capacity of the BIS, which is the maximum achievable rate of the number of individuals when the error probability converges to zero as the length of biometric data sequences goes to infinity. However, the implementation in [2] stores bio-data sequences in the system database in a plain form, leading to a critical privacy-leakage threat. Later, Tuncel [3] has developed their model by incorporating compression of bio-data sequences and clarified the capacity region of identification and coding rates (In this study, a codeword or helper data is called a template, and this coding rate is called the template rate).

Besides, there are several studies dealing with the secrecy rate of the BIS. An example of them is the BIS model with chosen secret. In this model, secret key or key of individual is chosen independently of bio-data and we name the BIS model with chosen secret as the CS-BIS model. Ignatenko and Willems [4] and Lai et al. [5] investigated the fundamental trade-off between secret key and privacy-leakage rates in the CS-BIS model. Ignatenko and Willems [6] extended the model studied in [4] to consider individual’s estimation and characterized the capacity region of identification, secrecy, and privacy-leakage rates in the CS-BIS model. One thing to be noted is that all three studies mentioned above assumed that the enrollment channel is noiseless. However, when bio-data is scanned, it is highly considerable that bio-data sequences are subject to noise so it is important to consider the enrollment channel is noisy like in [2], [3], [8], and [9]. Related to the studies on the CS-BIS model, a generated secret BIS (GS-BIS) model, where secret key is extracted from bio-data sequence, is studied in [6]–[10].

Studies on the CS-BIS model without estimating individual are extensively discussed in, e.g. [4], [5], [7], and [8]. More precisely, Koide and Yamamoto [7] analyzed the model for non-negligible secrecy-leakage. Günlü and Kramer [8] evaluated the model by treating the enrollment channel is noisy (hidden source model). The benefit of having single individual made a successful breakthrough for them to prove the capacity region by one auxiliary random variable (RV) in an elegant way. When the model is extended to the one with considering individual’s estimation, it seems difficult to use the same analyzing techniques, especially the evaluating of privacy-leakage rate, to characterize the capacity region. In the GS-BIS scenario, Yachongka and Yagi [9] have characterized the capacity region of the model with two auxiliary RVs recently. Then, an interesting question is if the same arguments also work for the CS-BIS scenario.

In this paper, we aim to characterize the capacity region of identification, secrecy, template, and privacy-leakage rates for the CS-BIS model. Compared to the model proposed in [6], we analyze the region under conditions that

1) adding noisy enrollment channel,
2) constraining template rate,
3) assuming that the prior distribution of the identified individual is unknown.

We show that it is possible to characterize the capacity region of the CS-BIS model in two different ways. A characterization uses a single auxiliary RV and another requires two auxiliary...
RVs. In this scenario, we will prove the capacity region based on the latter by applying the technique developed in [9]. As special cases, it can be checked that our characterization reduces to the one given by Ignatenko and Willems [6, Theorem 2] where the enrollment channel is noiseless and there is no constraint on the template rate, and it also corresponds to the result derived by Günlü and Kramer [8, Theorem 2] where there is no consideration of individual’s estimation.

The rest of this paper is organized as follows. In Sect. III, we define notation used in this paper and describe the details of the system model. In Sect. IV, we present the problem formulation and main result. Next, we highlight the proof of the main result in Sect. V. Finally, in Sect. VI, we give some concluding remarks and future works.

II. NOTATION AND MODEL DESCRIPTIONS

A. Notation

Calligraphic $\mathcal{A}$ stands for a finite alphabet. Upper-case $A$ denotes a RV taking values in $\mathcal{A}$ and lower-case $a \in \mathcal{A}$ denotes its realization. $P_A(a) := \Pr[A = a]$, $a \in \mathcal{A}$, represents the probability distribution on $\mathcal{A}$, and $P_{A^n}$ represents the probability distribution of RV $A^n = (A_1, \cdots, A_n)$ in $\mathcal{A}^n$, the $n$-th Cartesian product of $\mathcal{A}$. $P_{A^n|B^n}$ represents the joint probability distribution of a pair of RVs $(A^n, B^n)$ and its conditional probability distribution $P_{A^n|B^n}$ is defined as

$$P_{A^n|B^n}(a^n|b^n) = \frac{P_{A^n,B^n}(a^n,b^n)}{P_{B^n}(b^n)} \quad (\forall a^n \in \mathcal{A}^n, \forall b^n \in \mathcal{B}^n \text{ such that } P_{B^n}(b^n) > 0).$$

The entropy of RV $A$ is denoted by $H(A)$, the joint entropy of RVs $A$ and $B$ is denoted by $H(A,B)$, and the mutual information between $A$ and $B$ is denoted by $I(A;B)$ [11]. Throughout this paper, logarithms are of base two. For integers $a$ and $b$ such that $a < b$, $[a, b]$ denotes the set $\{a, a+1, \cdots, b\}$. A partial sequence of a sequence $c^n$ from the first symbol to the $k$th symbol $(c_1, \cdots, c_k)$ is represented by $c^k$.

A sequence $x^n \in \mathcal{X}^n$ is said to be $\delta$-strongly typical with respect to a distribution $P_{X}$ on $\mathcal{X}$ if $\frac{1}{n} N(a|x^n) - P_X(a) \leq \delta$ and $P_X(a) = 0$ implies $\frac{1}{n} N(a|x^n) = 0$ for all $a \in \mathcal{X}$, where $N(a|x^n)$ is the number of occurrences of $a$ in the sequence $x^n$, and $\delta$ is an arbitrary positive number. The set of sequences $x^n \in \mathcal{X}^n$ such that $x^n$ is $\epsilon$-strongly typical is called the strongly typical set and is denoted by $A_{\epsilon^n}(\mathcal{X})$ (cf. [11], [12]). This concept is easily extended to joint distributions.

B. Model Descriptions

The CS-BIS model considered in this paper is shown in Fig. 1. Basically, it consists of two phases: (I) Enrollment Phase and (II) Identification Phase. Next we explain the details of each phase.

(I) Enrollment Phase:

Let $I = [1, M_f]$ and $\mathcal{X}$ be the set of indexes of individuals and a finite source alphabet, respectively. For any $i \in I$, we assume that $x_i^n = (x_{i1}, \cdots, x_{in}) \in \mathcal{X}^n$, an $n$-length bio-data sequence of individual $i$, is generated i.i.d. from a stationary memoryless source $P_{X_i}$. The generating probability for each sequence $x_i^n \in \mathcal{X}^n$ is

$$P_{X_i^n}(x_i^n) := \Pr[X_i^n = x_i^n] = \prod_{k=1}^{n} P_{X_i}(x_{ik}).$$

(2)

Now let $J = [1, M_j]$ and $S = [1, M_S]$ be the set of indexes of templates stored in the database and individuals’ secret key, respectively. All bio-data sequences are observed via a discrete memoryless channel (DMC) $(Y, P_{Y|X}, \mathcal{X})$, where $\mathcal{Y}$ is a finite output-alphabet of $P_{Y|X}$. The corresponding probability that $x_i^n \in \mathcal{X}^n$ is observed as $y_i^n = (y_{i1}, y_{i2}, \cdots, y_{in}) \in \mathcal{Y}^n$ via the DMC $P_{Y|X}$ is

$$P_{Y_i^n|X_i^n}(y_i^n|x_i^n) = \prod_{k=1}^{n} P_{Y_i|X_i}(y_{ik}|x_{ik}).$$

(3)

for all $i \in I$.

A secret key $s(i) \in S$ is chosen uniformly at random and independent of all other RVs. Encoder mapping $f$ encodes $y_i^n$ and $s(i)$ into template $j(i) \in J$ as $j(i) = f(y_i^n, s(i))$. The template $j(i)$ is stored at position $i$ in the database, which can be accessed by the decoder.

(II) Identification Phase:

Bio-data sequence $x_w^n \in \mathcal{X}^n$ (w ∈ I) of an unknown $w$ (index of individual has already enrolled in the database) is observed via a DMC $(Z, P_{Z|X}, \mathcal{X})$, where $\mathcal{Z}$ is a finite output-alphabet of $P_{Z|X}$. The probability that $x_w^n \in \mathcal{X}^n$ is output as $z^n = (z_1, z_2, \cdots, z_n) \in \mathcal{Z}^n$ via $P_{Z|X}$ is given by

$$P_{Z^n|X^n}(z^n|x_w^n) = \prod_{k=1}^{n} P_{Z|X}(z_k|x_{wk}).$$

(4)

The sequence $z^n$ is passed to the decoder $g : \mathcal{Z}^n \times J^{M_j} \rightarrow I \times S$, comparing $z^n$ with templates in the database and outputs the pair of estimated value $(\hat{w}, \hat{s}(w))$.

Remark 1. Note that the distribution of $P_X$, $P_{Y|X}$, and $P_{Z|X}$ are assumed to be known or fixed and RV $W$ is independent of $(x_i^n, y_i^n, j(i), S(i), Z^n)$ for all $i \in I$ like previous studies. But, in this paper we assume neither that
The capacity region achievable if for any $\delta > 0$ and large enough $n$ there exist pairs of encoders and decoders that satisfy for all $i \in I$

$$\max_{i \in I} \Pr((\widehat{W}, \widehat{S}(W)) \neq (W, S(W)) | W = i) \leq \delta,$$  \hspace{1cm} (5)

$$\frac{1}{n} \log M_I \geq R_I - \delta,$$  \hspace{1cm} (6)

$$\frac{1}{n} \log M_J \leq R_J + \delta,$$  \hspace{1cm} (7)

$$\frac{1}{n} \log M_S \geq R_S - \delta,$$  \hspace{1cm} (8)

$$\max_{i \in I} \frac{1}{n} I(X_i^n; J(i)) \leq R_L + \delta,$$  \hspace{1cm} (9)

$$\max_{i \in I} \frac{1}{n} I(S(i); J(i)) \leq \delta.$$  \hspace{1cm} (10)

Moreover, the capacity region $\mathcal{R}$ is defined as the closure of the set of all achievable rate tuples.

In Definition 1, (5) is the condition of error probability of individual $i$ which should be arbitrarily small. Equations (6), (7) and (5) are the constraints related to identification, template and secrecy rates, respectively. In terms of the privacy protection perspective, we measure the information leakage of individual $i$ by (9) and (10). Condition (9) measures the amount of privacy-leakage of original bio-data $X_i^n$ from template $J(i)$ in the database and it must be smaller than or equal to $R_L + \delta$. Condition (10) measures the secrecy-leakage between the template and the secret key of individual $i$ and it requires that the leaked amount is arbitrarily small.

**Theorem 1.** The capacity region for the CS-BIS model is given by

$$\mathcal{R} = \mathcal{A}_1,$$  \hspace{1cm} (11)

where $\mathcal{A}_1$ is defined as

$$\mathcal{A}_1 = \{(R_I, R_S, R_J, R_L):$$

$$R_I + R_S \leq I(Z; V),$$

$$R_J \geq I(Y; U),$$

$$R_L \geq I(X; U) - I(Z; V) + R_I,$$

$$R_I \geq 0, R_S \geq 0,$$

for some $U$ s.t. $Z - X - Y - U$},  \hspace{1cm} (12)

where auxiliary RV $U$ takes values in a finite alphabet $\mathcal{U}$ with $|\mathcal{U}| \leq |\mathcal{V}| + 2$. \hspace{1cm} $\square$

**Remark 2.** We define a region $\mathcal{A}_2$ as

$$\mathcal{A}_2 = \{(R_I, R_S, R_J, R_L):$$

$$0 \leq R_I \leq I(Z; V),$$

$$0 \leq R_S \leq I(Z; U) - I(Z; V),$$

$$R_J \geq I(Y; U),$$

$$R_L \geq I(X; U) - I(Z; V) + I(Z; V),$$

for some $U$ and $V$ s.t. $Z - X - Y - U - V$},  \hspace{1cm} (13)

where auxiliary RVs $U$ and $V$ take values in some finite alphabets $\mathcal{U}$ and $\mathcal{V}$ with $|\mathcal{U}| \leq (|\mathcal{Y}| + 2)(|\mathcal{X}| + 3)$ and $|\mathcal{V}| \leq |\mathcal{Y}| + 3$. Then, it can be verified that

$$\mathcal{A}_1 = \mathcal{A}_2.$$  \hspace{1cm} (14)

The proof can be done by similar arguments shown in [9]. Appendix A] therefore omitted. In this paper, we will prove Theorem 1 based on the rate constraints of the region $\mathcal{A}_2$ instead of $\mathcal{A}_1$.

**Remark 3.** Likewise the observation in [8], the capacity region of GS-BIS model (cf. [9, Theorem 1]) is clearly wider than $\mathcal{R}$, which is due to the bound on $R_J$. A remark given in [10] indicated that in case where the enrollment channel is noiseless ($X = Y$), the fundamental limit of $R_L$ and $R_J$ is identical to the result in [6, Theorem 2] where the enrollment region is noiseless ($X = Y$) and the template rate can be arbitrarily large. Also, this characterization corresponds to the region given by Günlü and Kramer [12, Theorem 2] with only one individual. It is easy to check this claim by just setting $R_I = 0$.

**IV. PROOF OF THEOREM 1**

In this section, we only give a guideline of how to prove Theorem 1 For detailed proofs, we recommend the readers refer to check [9, Proof of Theorem 1].

A. Achievability (Direct) Part

In order to avoid the confusion in the following arguments, we introduce some new notations which are used only in this part. The pairs $(J_C(i), S_C(i))$ and $(J_G(i), S_G(i))$ denote the template and the secret key of individual $i$ for CS-BIS and GS-BIS encoders, respectively. Moreover, $M_{J_C}$ and $M_{J_G}$ denote the number of templates of the CS-BIS and GS-BIS models.

1 Normally, $J_C(i), S_C(i)$, and $M_{J_C}$ are denoted by $J(i), S(i)$, and $M_J$ in other sections of this paper.
Overviews:

The proof idea of this part is based on the achievability proof of the GS-BIS model provided in [9]. The difference is that the encoder and decoder of the GS-BIS model are used as components inside the encoder and decoder of the CS-BIS model as shown in Fig. 2. For encoding in the CS-BIS model, a so-called masking layer (one-time pad operation) is used to mask $s_C(w) \in S$ for secure transmission by using $s_G(w) \in S$ as $s_C(w) \oplus s_G(w)$. The template $j_C(w)$ is the combined information of $j_C(i)$ and the masked data $s_C(w) \oplus s_G(w)$, i.e.,

$$j_C(w) = (j_C(i), s_C(w) \oplus s_G(w)).$$

(15)

For decoding, it first uses the decoder of the GS-BIS model to estimate the pair $(\hat{w}, s_C(\hat{w}))$ and afterward the chosen secret key is retrieved by

$$s_C(\hat{w}) = s_C(\hat{w}) \oplus s_G(\hat{w}) \oplus s_G(i),$$

(16)

where $\oplus$ and $\ominus$ denote addition and subtraction modulo $M_S$. This technique is also used in [4], [6], [9], and so on.

Parameter Settings:

First, we define $R_{j_G}$ and $R_{j_C}$ as the template rates in the GS-BIS and the CS-BIS models encoders, respectively. Let $\delta$ be a small enough positive and fix a block length $n$. We choose test channels $P_{U \mid Y}$ and $P_{V \mid U}$. Next, We set $R_I = I(Z; V) - \delta$, $R_S = I(Z; U) - I(Z; V) - \delta$, $R_{j_G} = I(Y; U) + \delta$, and $R_L = I(X; U) - I(Z; U) + I(Z; V) + 2\delta$. We also set the number of individuals $M_I = 2^n R_I$, the number of secret key $M_S = 2^n R_S$, and the number of templates $M_{j_G} = 2^n R_{j_G}$ for the CS-BIS encoder and $M_{j_D} = \frac{M_{j_G}}{M_S} = 2^n (I(Y; U) - I(Z; U) + I(Z; V) + 2\delta)$ for the GS-BIS encoder, respectively.

Random Code Generation:

Sequences $v_m^n$ are generated i.i.d. from $P_V$ for $m \in [1, N_V]$, where $N_V = 2^{n(I(Y; V) + \delta)}$. For each $m$, sequences $u_{k|m}^n$ are generated from the memoryless channel $P_{U^n \mid V^n = v_m^n}$ for $k \in [1, N_U]$, where $N_U = 2^{n(I(Y; U) + \delta)}$. The indexes of these $N_U$ codewords are permuted by an uniformly distributed permutation $\pi_m$ on $[1, N_U]$ and divided equally into $N_B = 2^{n(I(Y; U) - I(Z; U) + 2\delta)}$ bins. That is, the first bin contains $\{\hat{u}_{k|m}^n\}$, the second bin contains $\{\hat{u}_{k|2m}^n\}$, and so on, where $\hat{u}_{k|m}^n = u_{\pi_m^{-1}(k)|m}$ denotes the $k$th codeword after the permutation. Consequently, each bin contains $M_S$ codewords in a random order. Bins are indexed by $b \in [1, N_B]$ and codewords inside a certain bin are indexed by $s \in S$. Without loss of generality, there exists a one-to-one mapping between $k$ and the pair $(b, s)$.

Encoding (Enrollment):

When the GS-BIS encoder, used as a component inside the CS-BIS encoder, observes the bio-data sequence $y_m^n \in Y^n$, the component looks for $(m, k)$ such that $(y_m^n, v_m^n, u_{k|m}^n) \in A^{(n)}(Y^n U^n)$. In case there are more than one such pairs, the component picks one of them uniformly at random. Assume that the component found a corresponding pair $(m, k)$, denoted as $(m(i), k(i)) = (m(i), b(i), s(i))$, satisfying the jointly typical condition above. Then, the component sets $j_C(i) = (m(i), b(i))$ and $s_C(i) = s(i)$ and shares them to the CS-BIS encoder. After that, the CS-BIS encoder uses $s_C(i)$ to mask the chosen secret $s_C(i)$ by $s_C(i) \oplus s_G(i)$. This masked information is combined with $j_G(i)$ to form the template $j_C(i) = j_C(i) \ominus s_C(i) \oplus s_G(i) = (m(i), b(i), s_C(i) \oplus s_G(i))$. The template is stored at position $i$ in the database. If there do not exist such $m$ and $k$, the component shares $j_G(i) = (1, 1)$ and $s_C(i) = 1$ to the CS-BIS encoder. In this case, the CS-BIS encoder declares error.

Decoding (Identification):

The GS-BIS decoder, embedded as a component inside the CS-BIS decoder, has access to all records in the database $\{(m_1, b(1), s_C(1) \oplus s_G(1)), \cdots, (m(M_I), b(M_I), s_C(M_I) \oplus s_G(M_I))\}$.
component shares the index of the template

\([\text{CS-BIS}]\) decoder declares error. When the component receives \(z^n\) (the noisy version of identified individual sequence \(x^n_i\)), it checks if the codeword pair \((v_{m(i)}^n, u_{m(i),s,m(i)}^n)\) is jointly typical with \(z^n\) for all \(i \in \mathcal{I}\) with some \(s \in S\), i.e. \((z^n, v_{m(i)}^n, u_{m(i),s,m(i)}^n) \in A_{\mu}^n(\mathcal{ZVU})\). If there exists a unique pair \((i,s)\), for which this condition holds, then the component sets \((\hat{w}, sG(w)) = (i,s)\) and forwards the pair \((\hat{w}, sG(w))\) to the CS-BIS decoder. After getting it, the CS-BIS decoder outputs \(\hat{\Pi}\) as the result of indexes of the sequences \(x^i, z\). Otherwise, the component shares the index of the template \((1,1)\) and \(sG(w) = 1\) to the CS-BIS decoder. Upon detecting these information, the CS-BIS decoder declares error.

Next we check that the conditions of \((5) - (10)\) in Definition \([1]\) are satisfied for the CS-BIS model. The detailed proof is provided in \([9, \text{Appendix B-C}]\). Then, the error probability of individual \(w\) for the CS-BIS model can also be bounded by

\[
\Pr\{x^n_i = \hat{x}^n_i\} \leq 4\delta
\]

for large enough \(n\).

**Analysis of Error Probability:**

For individual \(W = w\), the operation at the decoder \([16]\) means that \(S_C(w) = S_C(w)\) only if \(sG(w) = 1\). It is shown that the error probability of individual \(w\) for the GS-BIS model can be made that \(\Pr\{(\hat{w}, sG(W)) \neq (W, S_C(W)) | W = w\} \leq 4\delta\). The detailed proof is provided in \([9, \text{Proof of Theorem 1}]\). Therefore, it follows that the error probability of individual \(w\) for the CS-BIS model can also be bounded by

\[
\Pr\{(\hat{w}, sG(W)) \neq (W, S_C(W)) | W = w\} \leq 4\delta
\]

for large enough \(n\).

**Analysis of Identification, Secrecy, and Template Rates:**

It is easy to confirm that \((6), (7), \text{and} (8)\) hold from the parameter settings.

**Analysis of Privacy-Leakage Rate:**

It is shown in \([6, \text{Appendix B-A}]\) that

\[
\frac{1}{n} I(X^n_i; J_C(i) | C_n) = I(X^n_i; J_C(i), S_C(i) \oplus S_C(i)) \leq 1\]

By using a result shown in \([9]\), the privacy-leakage of the GS-BIS model can be bounded by \(1\) and \(\frac{1}{n} I(X^n_i; J_C(i) | C_n) \leq I(X; U) - I(Z; U) + I(Z; V) + 3\delta\) for large enough \(n\). The detail proof is provided in \([9, \text{Appendix B-C}]\). Then, the privacy-leakage of the CS-BIS model can also be made that

\[
\frac{1}{n} I(X^n_i; J_C(i) | C_n) \leq I(X; U) - I(Z; U) + I(Z; V) + 3\delta
\]

for large enough \(n\).

We invoke the following relation on secrecy-leakage between the CS-BIS and the GS-BIS models \([6, \text{Appendix B-A}]\):

\[
\frac{1}{n} I(J_C(i); S_C(i) | C_n)
\]

\[
= \frac{1}{n} I(J_G(i), S_C(i) \oplus S_C(i); S_C(i) | C_n)
\]

\[
\leq \frac{1}{n} \log M_S - \frac{1}{n} H(S_C(i)) + \frac{1}{n} I(J_G(i); S_C(i) | C_n).
\]

In \([9, \text{Appendix B-B}]\) and \([9, \text{Appendix B-C}]\), it is shown that

\[
\frac{1}{n} H(S_C(i)) \geq \log M_S - 2\delta
\]

\[
\frac{1}{n} I(J_G(i); S_C(i) | C_n) \leq 2\delta
\]

for large enough \(n\). Substituting \((21)\) and \((22)\) into \((20)\), the secrecy-leakage of the CS-BIS model is bounded by

\[
\frac{1}{n} I(J_C(i); S_C(i) | C_n) \leq 4\delta
\]

for large enough \(n\).

Finally, by applying the selection lemma \([13, \text{Lemma 2.2}]\) to above results, there exists at least a good codebook satisfying all conditions in Definition \([5]\) for large enough \(n\).

**B. Converse Part**

We consider a more relaxed case where identified individual index \(W\) is uniformly distributed over \(\mathcal{I}\) and \((5)\) is replaced with the average error criterion

\[
\Pr\{(\hat{w}, S(\hat{w})) \neq (W, S(W))\} \leq \delta
\]

We shall show that the capacity region, which is not smaller than the original one \(R_n\), is contained in the right-hand side of \((13)\).

We assume that a rate tuple \((R_U, R_S, R_J, R_L)\) is achievable so that there exists a pair of encoder and decoder \((f,g)\) such that all conditions in Definition \([1]\) with replacing \((5)\) by \((24)\) are satisfied for any \(\delta > 0\) and large enough \(n\).

Here, we provide other key lemmas used in this part. For \(t \in [1, n]\), we define auxiliary RVs \(U_t\) and \(V_t\) as \(U_t = (Z^{t-1}, J(W), S(W))\) and \(V_t = (Z^{t-1}, J(W))\), respectively. We denote a sequence of RVs \(Y^n = (Y_1(W), \cdots, Y_n(W))\).

**Lemma 1.** The following Markov chain holds

\[
Z^{t-1} - (Y^{t-1}(W), J(W), S(W)) - Y_t(W).
\]

(Proof) The proof is given in \([9, \text{Appendix C-A}]\).

**Lemma 2.** There exist some RVs \(U\) and \(V\) which satisfy Z – X – Y – U – V and

\[
\sum_{t=1}^{n} I(Y_t(W); U_t) = nI(Y; U),
\]

\[
\sum_{t=1}^{n} I(Y_t(W); V_t) = nI(Y; V).
\]
Proof: The proofs are provided in [9, Appendix C-B].

In the following arguments, we fix auxiliary RVs $U$ and $V$ specified in Lemma 2.

**Analysis of Identification and Secrecy Rates:**

It can be shown that

$$R_I \leq I(Z;V) + \delta + \delta_n, \quad (28)$$

$$R_S \leq I(Z;U) - I(Z;V) + 2\delta + \delta_n, \quad (29)$$

where $\delta_n = \frac{1}{n}(1 + \frac{1}{n}\log M_J M_S)$ and $\delta_n \downarrow 0$ as $n \to \infty$.

The proofs can be done by similar arguments of the analysis of identification and secrecy rates in the converse part of [9, Proof of Theorem 1].

**Analysis of Template Rate:**

From (7), it holds that

$$n(R_J + \delta) \geq \log M_J \geq H(J(W))$$

$$\geq I(J(W);Y^n_W,S(W))$$

$$= H(Y^n_W|S(W)) - H(Y^n_W|J(W),S(W))$$

$$= a \sum_{t=1}^{n} \left\{ H(Y_t(W)) - H(Y_t(W)|J(W),S(W),Y^{t-1}(W)) \right\}$$

$$b \sum_{t=1}^{n} \left\{ H(Y_t(W)) - H(Y_t(W)|J(W),S(W),Y^{t-1}(W),Z^{t-1}) \right\}$$

$$\geq \sum_{t=1}^{n} I(Y_t(W);Z^{t-1},J(W),S(W))$$

$$= \sum_{t=1}^{n} I(Y_t(W);U_t) \overset{(d)}{=} nI(Y;U), \quad (30)$$

where

(a) holds because $S(W)$ is independent of $Y^n_W$ and each symbol of $Y^n_W$ is i.i.d.,

(b) is due to (23) in Lemma 1,

(c) follows because conditioning reduces entropy,

(d) holds due to (26) in Lemma 2.

Thus, we obtain

$$R_J \geq I(Y;U) - \delta. \quad (31)$$

**Analysis of Privacy-Leakage Rate:**

It can be proved that

$$R_L + \delta \geq I(X;U) - I(Z;U) + I(Z;V) - \delta_n. \quad (32)$$

For detailed proof, the readers should refer to the analysis of privacy-leakage rate in the converse part of [9, Proof of Theorem 1] since similar approach is taken.

By letting $n \to \infty$ and $\delta \downarrow 0$, we obtain that the capacity region is contained in the right-hand side of (12) from (28), (29), (31), and (32).

To derive the bound on the cardinality of alphabet $U$ in the region $A_1$ (cf. [12, Appendix C]), we use the support lemma in [12, Appendix C] to show that RV $U$ should have $|\mathcal{Y}| - 1$ elements to preserve $P_Y$ and add three more elements to preserve $H(Z|U)$, $H(Y|U)$, and $H(X|U)$. This implies that it suffices to take $|U| \leq |\mathcal{Y}| + 2$ for preserving $A_1$. Similarly, for bounding the cardinalities of alphabets $U$ and $V$ in the region $A_2$ (cf. [13]), we also utilize the same lemma to show that $|V| \leq |\mathcal{Y}| + 3$ and $|U| \leq |\mathcal{Y}| + 2(|\mathcal{Y}| + 3)$ suffice to preserve the regions $A_1$ and $A_2$.

**V. Conclusion and Future Works**

In this study, we characterized the capacity region of identification, secrecy, template, and privacy-leakage rates in the CS-BIS model. Compared to the model proposed in [6], we imposed a noisy channel in the enrollment phase as seen in [2], [3], [8], [9] and assumed that the prior distribution of the identified individual is unknown. As special cases, the characterization reduces to the result given by Ignatenko and Willems [6, Theorem 2] when the enrollment channel is noiseless and there is no constraint on the template rate, and also matches with the one given by Günülü and Kramer [8, Theorem 2] where there is only one individual. For future work, we plan to analyze the capacity regions of the GS-BIS and CS-BIS models under strong secrecy criterion in terms of secrecy-leakage.

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