Breakup dynamics in $^6$Li elastic scattering with four-body and three-body CDCC

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Abstract. We have investigated projectile breakup effects on $^6$Li + $^{209}$Bi elastic scattering near the Coulomb barrier with the four-body and three-body continuum-discretized coupled-channels methods. In the analysis, the elastic scattering is well described by the $p + n + ^4$He + $^{209}$Bi four-body model. Four-body dynamics of the elastic scattering is precisely investigated and we then found that $d$ ($p + n$) in $^6$Li may hardly break up during the scattering.

1. Introduction

In reactions of weakly-bound nuclei, projectile breakup processes are essential and we need to treat the effect properly in the calculation. The continuum-discretized coupled-channels method (CDCC) has been proposed as an accurate method of treating breakup effects [1, 2]. Nowadays, CDCC has become a standard model known as three-body CDCC in which the total system is assumed to be a three-body system (two-body projectile + target). Furthermore, three-body CDCC has been extended to four-body CDCC in which the total system is assumed to be four-body system (three-body projectile + target) [3]. Thus, CDCC is a powerful method to describe not only three-body scattering [4] but also four-body scattering [5]. The recent developments of CDCC are shown in Ref. [6, 7, 8].

$^6$He + $^{209}$Bi scattering near the Coulomb barrier was analyzed with three-body CDCC based on a $^2n + ^4$He + $^{209}$Bi three-body model [9]; that is to say a pair of extra neutrons in $^6$He was treated as a single particle, dineutron ($^2n$). The three-body CDCC calculation, however, did not reproduce the angular distribution of the measured elastic cross section and overestimated the measured total reaction cross section by a factor of 2.5. This problem has been solved by four-body CDCC in which the total system is described as a $n + n + ^4$He + $^{209}$Bi four-body system [3]. Also for $^6$Li + $^{209}$Bi scattering near the Coulomb barrier, three-body CDCC was applied [9]. In the three-body CDCC, a $d + ^4$He + $^{209}$Bi three-body model was assumed. However, the calculation could not reproduce the data without normalization factors for the potential between $^6$Li and $^{209}$Bi. These studies [3, 9] strongly suggest that $^6$Li + $^{209}$Bi scattering should also be treated with four-body CDCC.

In this work, we analyze $^6$Li + $^{209}$Bi elastic scattering at 29.9 and 32.8 MeV with four-body CDCC by assuming the $p + n + ^4$He + $^{209}$Bi four-body model. This is the first application of four-body CDCC to $^6$Li scattering. We deal with four-body dynamics explicitly and compare with the results of three-body CDCC. This poster presentation is mainly based on our recent work of Ref. [10].
2. Theoretical framework and Model Hamiltonian

Breakup dynamics of the $^6\text{Li} + ^{209}\text{Bi}$ scattering is governed by the Schrödinger equation

$$(H - E)\Psi = 0$$

(1)

for the total wave function $\Psi$, where $E$ is a total energy of the system. In order to clarify the difference between four-body ($p + n + ^4\text{He} + ^{209}\text{Bi}$) and three-body ($d + ^4\text{He} + ^{209}\text{Bi}$) dynamics, we set two types of the model Hamiltonian. One is for four-body CDCC defined by

$$H^{(4b)} = K_R + U_n(R_n) + U_p(R_p) + U_\alpha(R_\alpha) + \frac{e^2Z_{\text{Li}}Z_{\text{Bi}}}{R} + h,$$

(2)

where $h$ denotes the internal Hamiltonian of $^6\text{Li}$ described by three-cluster model, $R$ is the center-of-mass coordinate of $^6\text{Li}$ relative to $^{209}\text{Bi}$, $K_R$ stands for the kinetic energy and $U_x$ describes the nuclear part of the optical potential between $x$ and $^{209}\text{Bi}$ as a function of the relative coordinate $R_x$. Since the Coulomb breakup effect is negligible for the $^6\text{Li}$ elastic scattering [9], we approximate the Coulomb part of $^{p-209}\text{Bi}$ and $^4\text{He}-^{209}\text{Bi}$ interactions by $e^2Z_{\text{Li}}Z_{\text{Bi}}/R$, where $Z_A$ is the atomic number of the nucleus $A$. The other is for three-body CDCC defined by

$$H^{(3b)} = K_R + U_d(R_d) + U_\alpha(R_\alpha) + \frac{e^2Z_{\text{Li}}Z_{\text{Bi}}}{R} + h',$$

(3)

where $h'$ denotes the internal Hamiltonian of $^6\text{Li}$ described by two-cluster model.

In general, the total wave function $\Psi$ is expanded in terms of the orthonormal set of eigenstates $\phi$ of $h$ ($h'$)

$$\Psi(R, \xi) = \chi_0(R)\phi_0(\xi) + \int_0^\infty d\varepsilon \chi_\varepsilon(R)\phi_\varepsilon(\xi),$$

(4)

where $\xi$ is the Jacobi coordinate in $^6\text{Li}$, subscripts 0 and $\varepsilon$ denote the ground state and the continuum state with the internal energy $\varepsilon$ of $^6\text{Li}$ respectively. The expansion coefficient $\chi_0$ ($\chi_\varepsilon$) describes the relative motion between $^{209}\text{Bi}$ and $^6\text{Li}$ in its ground state (continuum state with $\varepsilon$). In CDCC, the continuum state is truncated at an upper limit $\varepsilon_{\text{max}}$, and the continuum up to $\varepsilon_{\text{max}}$ is discretized into a finite number of discrete states

$$\Psi(R, \xi) \approx \chi_0(R)\phi_0(\xi) + \int_0^{\varepsilon_{\text{max}}} d\varepsilon \chi_\varepsilon(R)\phi_\varepsilon(\xi) \approx \sum_{i=0}^{i_{\text{max}}} \tilde{\chi}_i(R)\tilde{\phi}_i(\xi).$$

(5)

Readers are directed to Ref. [10] for more details.

In Eqs. (2) and (3), the optical potentials ($U_n, U_p, U_\alpha, \text{and } U_d$) are taken from Refs. [11, 12, 13]. The proton optical potential $U_p$ is assumed to be the same as $U_n$, and parameters of $U_\alpha$ are refitted to reproduce experimental data on $n + ^{209}\text{Bi}$ elastic scattering [14], where only the central interaction is taken for simplicity. We confirmed that these optical potentials well reproduce the experimental data for each subsystem of $^6\text{Li} + ^{209}\text{Bi}$ as shown in Fig. 1 [(a) $n + ^{209}\text{Bi}$ scattering at 5 MeV, (b) $^4\text{He} + ^{209}\text{Bi}$ scattering at 19–22 MeV, (c) $d + ^{209}\text{Bi}$ scattering at 12.8 MeV].

3. Results

Figure 2 shows the angular distribution of elastic cross section for $^6\text{Li} + ^{209}\text{Bi}$ scattering at 29.9 MeV and at 32.8 MeV. The dotted line shows the result of three-body CDCC calculation with the $d$-optical potential $U_{d}\text{OP}$. This calculation is similar to the one in the previous study [9] and underestimates the measured cross section. The solid (dashed) line, meanwhile, stands for...
the result of four-body CDCC calculation with (without) projectile breakup effects. In CDCC calculations without $^6$Li-breakup, the model space is composed only of the $^6$Li ground state ($\phi_0$). The solid line reproduces the measured cross section but the dashed line does not, indicating the projectile breakup effects are thus significant. As just described, the present $^6$Li scattering is well described by the $p + n + ^4$He + $^{209}$Bi four-body model.

In order to understand breakup dynamics in the $^6$Li scattering, we investigate how $d$ breakup affects the cross section in the framework of three-body CDCC. As a calculation in the limit of no $d$-breakup effect, the optical potential between $d$ and $^{209}$Bi ($U_{d}^{\text{OP}}$) should be replaced by the single-folding potential ($U_{d}^{\text{SF}}$) which is obtained by folding $U_n$ and $U_p$ in the ground-state deuteron density. Note that we use the same $U_n$ and $U_p$ for as for four-body CDCC (see Eq. 2). In Fig. 2, the dot-dashed line shows the result of the three-body CDCC calculation with $U_{d}^{\text{SF}}$. The result well simulates that of four-body CDCC calculation (the solid line). This result suggests $d$ breakup is suppressed in the $^6$Li scattering.

Finally, we would like to mention about $d + ^{209}$Bi scattering at 12.8 MeV which is the same reaction as in Fig. 1(c). For this scattering, we apply three-body CDCC in which the $p + n + ^4$He + $^{209}$Bi model is assumed and both Coulomb and nuclear breakup effects are taken into account. The results are as follows. The CDCC calculation with breakup effects reproduces the experimental data but the CDCC calculation without breakup (one-channel calculation with $U_{d}^{\text{SF}}$) does not. $d$-breakup effect is thus quite important for “normal” $d$ scattering. As already mentioned, the one-channel calculation with $U_{d}^{\text{OP}}$ well reproduces the data [see Fig. 1(c)] and this is because the optical potential ($U_{d}^{\text{OP}}$) implicitly includes the $d$-breakup effect. For $^6$Li + $^{209}$Bi scattering, the reason why three-body CDCC with $U_{d}^{\text{OP}}$ does not work may be because we have overcounted the $d$-breakup effect which is almost absent in $d$ in $^6$Li scattering.

4. Summary
The $^6$Li + $^{209}$Bi scattering at 29.9 MeV and 32.8 MeV near the Coulomb barrier is well described by four-body CDCC based on the $p + n + ^4$He + $^{209}$Bi model. This is the first application of four-body CDCC to $^6$Li scattering. Through the three-body CDCC analysis based on $d + ^4$He + $^{209}$Bi model, we found that three-body CDCC can reproduce the measured cross section if the single-folding potential $U_{d}^{\text{SF}}$ is taken, although three-body CDCC with the optical potential $U_{d}^{\text{OP}}$ cannot. This result suggests that $d$ in $^6$Li hardly breaks up during the scattering. In other words, the failure of three-body CDCC with $U_{d}^{\text{OP}}$ might come from overcounting the $d$-breakup effect in the four-body CDCC model.
Figure 2. Angular distribution of the elastic cross section for $^6$Li + $^{209}$Bi scattering at 29.9 MeV (a) and at 32.8 MeV (b) [10]. The cross section is normalized by the Rutherford cross section. The dotted (dot-dashed) line stands for the result of three-body CDCC calculation in which $U_d^{OP}$ ($U_d^{SF}$) is taken as $U_d$. The solid (dashed) line represents the result of four-body CDCC calculations with (without) breakup effects. The experimental data are taken from Refs. [15, 16].

The effect which is almost absent in $d$ in $^6$Li scattering. However, we need to discuss carefully whether the $d$ breakup is always suppressed in the $^6$Li scattering. Further analysis such as energy and target dependences of $d$ breakup should be done.

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