The universal Faber–Jackson relation

R. H. Sanders
Kapteyn Astronomical Institute, PO Box 800, 9700 AV Groningen, the Netherlands

ABSTRACT
In the context of modified Newtonian dynamics, the Fundamental Plane, as the observational signature of the Newtonian virial theorem, is defined by high-surface-brightness objects that deviate from being purely isothermal: the line-of-sight velocity dispersion should slowly decline with radius as observed in luminous elliptical galaxies. All high-surface-brightness objects (e.g. globular clusters, ultra-compact dwarfs) will lie, more or less, on the Fundamental Plane defined by elliptical galaxies, but low-surface-brightness objects (dwarf spheroidals) would be expected to deviate from this relation. This is borne out by observations. With Milgrom’s modified Newtonian dynamics (MOND), the Faber–Jackson relation \( L \propto \sigma^4 \), ranging from globular clusters to clusters of galaxies and including both high- and low-surface-brightness objects, is the more fundamental and universal scaling relation in spite of its larger scatter. The Faber–Jackson relation reflects the presence of an additional dimensional constant \( (\text{the MOND acceleration} \quad a_0) \) in the structure equation.

Key words: galaxies: elliptical and lenticular, cD – galaxies: fundamental parameters – galaxies: kinematics and dynamics.

1 INTRODUCTION
The direct observational signature of the Newtonian virial theorem in elliptical galaxies emerged two decades ago with the discovery of the ‘Fundamental Plane’ (FP; Djorgovski & Davis 1987; Dressler et al. 1987). Assuming that ellipticals are reasonably homologous Newtonian pressure-supported systems, they will comprise a two-parameter set: if luminosity is proportional to mass, then the virial theorem implies that \( L \propto R_{\text{eff}} \sigma^2 \) where \( R_{\text{eff}} \) is the effective radius and \( \sigma \) is the line-of-sight (los) velocity dispersion. This is essentially the observed FP relation. Although the connection between the FP and the virial theorem was originally obscured by the fact that the observationally inferred exponents in the above relation were not precisely as expected, it is now established that this deviation is primarily due to a systematic variation in the mass-to-light ratio (M/L) of ellipticals: M/L appears to slowly increase with luminosity (see Faber et al. 1987, for an early realization of the relevance of the virial theorem). The fact that the FP is the observational manifestation of the virial theorem has become indisputable with the direct observational signature of the Newtonian virial theorem in elliptical galaxies (Fabers). The FP relation reflects the presence of an additional dimensional constant \( (\text{the MOND acceleration} \quad a_0) \) in the structure equation.

The striking aspect of the traditional FP is the small scatter about the expected relation. Such a precise relation surely reflects the facts that elliptical galaxies comprise a rather homologous class of objects with a fairly isotropic velocity distribution (at least in the inner regions) and that the random fluctuation in M/L is relatively small. This small fluctuation in dynamical M/L is puzzling given the current view of a galaxy as a luminous baryonic component immersed in a more massive and extensive dark halo composed of non-baryonic particles. Ellipticals are presumably assembled over a Hubble-time-scale via mergers – ‘wet’ or ‘dry’, ‘major’ or ‘minor’ – of smaller mass aggregates; it is not evident that such a fixed proportion of dark and baryon matter within the visible object should survive what must be a rather chaotic process.

The second scaling relation for elliptical galaxies was actually discovered long before the FP – that is the relation between luminosity and los velocity dispersion of the form \( L \propto \sigma^4 \), where \( 3 \leq \sigma \leq 5 \). This correlation was first described in a qualitative way by Morgan & Mayall (1957) who remarked ‘For progressively fainter Virgo cluster ellipticals the spectral lines tend to become narrower, as if there were a line width-absolute magnitude effect for the brighter members’. This effect was given its definite and quantitative form by Faber & Jackson (1976) and has come to be known as the Faber–Jackson relation (FJ). The FJ relation implies that ellipticals comprise a one-parameter family: given the velocity dispersion, the luminosity (or luminous mass) follows. The observed FJ relation does have a much larger dispersion about its mean than does the FP, and this has led to the often-heard assertion that the FJ has superseded the FP – that the FJ represents a non-orthogonal projection of the FP on to a lower dimensional parameter space (Franx & de Zeeuw 1991).

While it is certainly true that the FP, because of its much smaller scatter, is superior to the FJ as a distance indicator for elliptical galaxies, it is not evident that the FJ is a simple projection of the FP;
nothing like FJ is required by the virial relation. FJ implies that the mass of a pressure-supported system is correlated with the velocity dispersion more or less independently of the size of the object. I have argued (Sanders 1994) that this same correlation extends to the great clusters of galaxies in the form of the gas-mass–temperature relation \(M \propto T^2\). Indeed, the relation \(M \propto \sigma^4\) appears to broadly apply to all near-isothermal pressure-supported astronomical systems from globular star clusters to clusters of galaxies.

In the context of the standard cold dark matter–based cosmology, this correlation must arise from aspects of structure formation. In the standard scenario, galaxies, or pre-galactic bound objects, form at a definite epoch which implies that there is, more or less, a characteristic density for protogalactic objects. The existence of a characteristic density combined with the virial theorem yields a mass–velocity-dispersion relation for haloes of the form \(M \propto \sigma^4\) which seems to be borne out by cosmological N-body simulations (e.g., Frenk et al. 1988). However, such a relatively shallow power law cannot be extrapolated from globular cluster scale objects to clusters of galaxies. In addition, globular clusters and clusters of galaxies form at different epochs via different processes; the emergence of this apparently universal correlation is not obviously implied.

Milgrom’s modified Newtonian dynamics (MOND) does give rise to a mass–velocity-dispersion relation of the form \(M \propto \sigma^4\) as an aspect of existent dynamics rather than the contingencies of structure formation (Milgrom 1983a). This was an early prediction of the hypothesis (Milgrom 1983b). MOND does not, however, inevitably predict existence of the FP; in fact, one might ask if the FP as a manifestation of the Newtonian virial theorem is consistent with MOND. Here I consider this question and discuss how the FP arises in the context of MOND. I review these two global scaling relations and their extension to the wide class of pressure-supported, near-isothermal objects and I compare the universality of the FP to that of FJ. I point out that objects which define the FP should have high surface brightness (and thus be essentially Newtonian) within an effective radius, and I highlight a MOND prediction that such objects must deviate from an isothermal state with an observed velocity dispersion that declines with radius. Further, very low-surface-brightness objects, such as the dwarf spheroidal galaxies, might not be expected to lie on the FP defined by ellipticals and globular clusters. I compare these expectations with the observations.

2 GLOBAL SCALING RELATIONS IN THE CONTEXT OF MOND

One of the original motivations for MOND as an acceleration-based modification of Newtonian dynamics was to subsume the Tully–Fisher relation for spiral galaxies (Sanders & McGaugh 2002). This is the observed correlation between the luminosity and asymptotic constant rotation velocity in spiral galaxies, but has since been more accurately described as an exact relation between the baryonic mass and rotation velocity (McGaugh et al. 2000), \(M \propto V_{\text{rot}}^4\). Milgrom (1983b) pointed out that a similar correlation should exist for pressure-supported systems such as elliptical galaxies: the total (baryonic) mass is roughly proportional to the fourth power of the velocity dispersion, but non-homology as well as variations in the degree of anisotropy of the velocity field introduces much more scatter than is evident in the Tully–Fisher relation.

Milgrom (1984) explored this further in his paper on MONDian isothermal spheres. Solving the spherically symmetric Jeans equation with the MOND expression for the effective gravitational acceleration, he was able to draw several powerful conclusions as follows.

1. MOND isothermal spheres, unlike their Newtonian counterparts, have finite mass.
2. Asymptotically the density falls as power law, \(\rho(r) \propto r^{-\alpha}\) where \(\alpha_c \approx 4\).
3. The ratio of the total mass to the fourth power of the radial velocity dispersion, \(\sigma_t\), depends primarily upon the anisotropy parameter

\[
\beta = 1 - \frac{\sigma_t^2}{\sigma_i^2}
\]

where \(\sigma_t\) (the tangential velocity dispersion); specifically,

\[
M = \left(\frac{G a_0}{c^2}\right)^{-1/2} \left(\alpha_i - 2\beta\right) \sigma_i^4,
\]

where \(a_0\) (approximately \(10^{-8} \text{ cm s}^{-2}\)) is the fundamental new parameter of the theory. This forms the basis for the FP relation.

4. For all ‘deep MOND’ objects (having internal acceleration everywhere less than the \(a_0\)) it is the case that \(M/\sigma^4 = 9/(4Ga_0)^{-1}\) independently of \(\beta\), where \(\sigma\) is the three-dimensional velocity dispersion [as an aside I mention that the deep MOND analytic solution for \(\beta = 1/2\) corresponds exactly to the Hernquist (1990) model for spherical systems, a model which projects to a surface density distribution similar to that observed for elliptical galaxies].

None of these conclusions anticipated the FP because it was not then fully appreciated that luminous elliptical galaxies are essentially Newtonian systems within an effective radius, whereas MONDian isothermal spheres are in transition between Newton and MOND at \(R_{\text{eff}}\). That is to say, isothermal spheres are not good representations of actual elliptical galaxies; they are too MONDian. We may characterize the degree to which an object is Newtonian within the bright inner regions (within \(R_{\text{eff}}\)) by the parameter \(\eta = g_{\text{eff}}/a_0\), where \(g_{\text{eff}}\) is the gravitational acceleration at \(R_{\text{eff}}\). By this criterion, \(\eta \approx 0.7\) for MONDian isotropic isothermal spheres whereas \(\eta \approx 6\) typically for real ellipticals. This means that actual luminous elliptical galaxies should show little evidence for dark matter within an effective radius, as was borne out when detailed kinematic data became available for several of these objects (Romanowsky et al. 2003). This result is completely consistent with the expectations of MOND for such high-surface-brightness systems (Milgrom & Sanders 2003).

Viewed in terms of MOND, the Newtonian nature of elliptical galaxies requires that these objects are not isothermal; the velocity dispersion should be observed to decrease with radius. An implication is that such systems should be consistent with the Newtonian virial theorem within the effective radius; that is to say, they will define an FP relation. On the other hand, deep MOND (low-surface-brightness) objects might be expected to deviate from this FP while being generally consistent with the universal FJ relation.

3 THE MOND BASIS FOR THE FUNDAMENTAL PLANE

MONDian isotropic, isothermal spheres possess a fixed relation between effective radius and central los velocity dispersion of the form \(\sigma^2 = 870R_{\text{eff}}, \) where \(\sigma\) is in km s\(^{-1}\) and \(R_{\text{eff}}\) is in kpc. However, actual elliptical galaxies exhibit a fairly broad distribution by effective radius and velocity dispersion (Jørgensen, Franx & Kjærgaard 1996; Sanders 2000), and, for a given velocity dispersion, all are much more compact (smaller \(R_{\text{eff}}\)) than the isothermal sphere.

In order to reproduce the observed distribution of elliptical galaxies by \(R_{\text{eff}}\) and \(\sigma\) in the context of MOND, I previously considered...
non-isothermal non-isotropic models: specifically, high order polytropic spheres with an increasing radial orbit anisotropy in the outer regions (Sanders 2000). To be consistent with the observed scatter in these two quantities, it was necessary to consider a range of polytropic indices \((n = 12–16\) with \(\sigma^2 \propto \rho^{1/n}\)) and anisotropy radii (scaled in terms of the effective radius); i.e., the models must deviate from strict homology. Matching the observed joint distribution of \(R_{\text{eff}}\) and \(\sigma\) with this range of models, I found that the observed FP is reproduced. In particular, the implied high surface brightness means that the models are essentially Newtonian within an effective radius, and this yields a small scatter about the mean (virial) relation. A weak systematic increase of \(M/L\) with luminosity is also required, but the ensemble provides a good representation of the lensing (mass-based) FP discovered by Bolton et al. (see Sanders & Land 2008, for a discussion of the lensing-based FP in the context of MOND). The FJ relation is also present in this ensemble of polytropic models but with much larger scatter because the relation is more sensitive to the necessary deviations from homology.

The reliance upon such specific polytropic models somewhat obscures the dynamics underlying the appearance of an FP in the context of MOND; that is to say, the polytropic assumption is in no sense necessary for matching the FP. The necessary ingredient is a deviation from an isothermal state. This becomes more evident when considering a specific model for the mass distribution in elliptical galaxies, i.e., the spherically symmetric Jaffe model (Jaffe 1983) with a radial dependence of density given by

\[
\rho(r) = \frac{\rho_0 \eta^4}{r^2(r + r_j)^2}, \tag{3}
\]

where \(r_j\) is the characteristic length-scale of the model and \(\rho_0 = M/(4\pi r_j^3)\) is the characteristic density \((M\) being the total mass). The Jaffe model projects very nearly into the empirically fitted surface density of elliptical galaxies (de Vaucouleurs 1958) where \(R_{\text{eff}} = 0.763 r_j\).

Given this density distribution, I numerically solve the Jeans equation for the run of radial velocity dispersion:

\[
\frac{d \sigma_L^2}{dr} + \frac{d \log(\rho)}{d \log(r)} \sigma^2 = -g. \tag{4}
\]

Here I have assumed that the velocity field is isotropic \((\beta = 0)\).

The interpolating function is taken to be \(\mu(x) = x/(x^2 + 1)^{0.5}\) with \(a_0 = 10^{-8}\) cm s\(^{-2}\) as implied by the rotation curves of spiral galaxies (Bottema et al. 2002).

In Fig. 1, I show the results of such an integration for three different values of the parameter \(\eta = g_{\text{eff}}/\sigma_0\). This is the mean intensity-weighted los velocity dispersion within a circular aperture, \(\sigma_{\text{e}}\), scaled to its value within one-half of an effective radius plotted as a function of radius in units of the effective radius. Three cases are shown: \(\eta = 7.8\) which is highly Newtonian within an effective radius (labelled \(N\)), \(\eta = 0.9\) which is in transition between Newton and MOND at an effective radius \((T)\) and \(\eta = 0.2\) which is in the deep MOND limit \((M)\). In all cases, for an isotropic velocity distribution, the velocity dispersion approaches a constant value at large radii.

Fig. 1 illustrates the point that Newtonian objects must deviate from isothermal with a velocity dispersion that declines within an effective radius. Transitional Jaffe models are almost isothermal; the MOND isothermal sphere is such a transitional object. Deep MOND objects exhibit a velocity dispersion rising to an almost constant value if the velocity field is isotropic; indeed, for small values of \(\eta\) this form of the velocity dispersion profile becomes independent of \(\eta\). This is all reminiscent of the form of rotation curves in high- to low-surface-brightness disc galaxies.

We would expect objects which define the FP (i.e., consistent with the Newtonian virial theorem) to be well into the Newtonian regime, i.e., \(\eta \gg 1\). This is apparently the case as is shown by the dashed curve in Fig. 1, which is a power-law fit by Cappellari et al. (2006) to the mean observed run of \(\sigma_1 (\propto r^{-0.096})\) for 25 early-type galaxies. That is to say, these observations are consistent with the MOND expectation of a declining velocity dispersion profile for high-surface-brightness elliptical galaxies, defining the FP.

The galaxies in the sample of Cappellari et al. are plotted as an FP relation in Fig. 2. The crosses show the luminosity versus the virial quantity \(\sigma^2_{\text{e}} R_{\text{eff}}/G\), where \(\sigma_{\text{e}}\) is the intensity-weighted mean los velocity dispersion within an effective radius. Units are \(10^{11}\) L\(_\odot\) and M\(_\odot\).

The expectation from the virial theorem is

\[
M = c_1 \sigma^2_{\text{e}} R_{\text{eff}}/G, \tag{5}
\]

where \(c_1\) is the structure constant which depends upon the MOND parameter \(\eta\) or, alternatively, upon the contribution of dark matter to the total mass within an effective radius. The two parallel lines in Fig. 2 show this relation for two different values of \(c_1\). For the upper curve, labelled \(N\), the structure constant, \(c_1 = 4.54\), is that appropriate for the Newtonian Jaffe model \((\eta = 10)\) or an equivalent dark matter fraction within an effective radius of about 0.1. If all systems were represented perfectly by isotropic Newtonian Jaffe models with \(M/L = 1\), then they would lie on this line. For the lower line labelled \(M\), the structure constant, \(c_1 = 0.74\), corresponds to a deep MOND Jaffe model \((\eta = 0.1)\) or an equivalent dark matter fraction within \(R_{\text{eff}}\) of 0.9. Note that deep MOND Jaffe models with a fixed value of \(\eta\) will also define a virial FP relation because they correspond to homologous objects with a fixed fraction of dark matter within the effective radius. Because lower accelerations (lower \(\eta\)) correspond to more dark matter (higher effective M/L), the MOND line lies to the right of the Newtonian line in this figure.
The solid points in Fig. 2 are the same systems but with stellar mass plotted against the virial quantity. The masses are estimated from the luminosity multiplied by M/L derived from population synthesis models (Cappellari et al. 2006). Now we see that the points lie very near the virial relation for pure Newtonian Jaffe models (small discrepancy or little dark matter within an effective radius). With MOND, this is the expected result for high-surface-brightness galaxies.

There are other classes of high-surface-brightness objects with a radially declining velocity dispersion which, apart from small shifts due to variations in homology, should lie on the same FP: specifically, dwarf elliptical galaxies, the ultra-compact dwarfs (UCDs) and the globular star clusters.

In Fig. 3, a wide range of objects are plotted on a virial representation of the FP. Here again the logarithm of the luminosity ($10^{11} \, L_\odot$), a proxy for the mass, is plotted against the logarithm of the virial quantity $\sigma^2 R_{eff}/G(10^{11} \, M_\odot)$. If the objects are Newtonian (satisfying the Newtonian virial theorem) and homologous with constant M/L, they should lie on the FP defined by the elliptical galaxies. On this plot, the ellipticals drawn from the larger sample of Jørgensen, Franx & Kjaergaard (1996) are shown as crosses; the globular clusters are solid points, the triangles are dwarf ellipticals, the open points are UCDs and the stars are the dwarf spheroidal companions of the Milky Way. The open squares are X-ray emitting clusters of galaxies (Croston et al. 2008). The data are quite heterogeneous between the various classes of objects: luminosity is measured in different photometric bands, and the velocity dispersion is, in some cases, the central velocity dispersion within a fixed diaphragm (globular clusters), or the intensity-weighted velocity dispersion within an effective radius (dwarf ellipticals and UCDs) or the mean velocity dispersion within a fixed linear radius (ellipticals). For the clusters, it is the gas mass that is plotted on the vertical axis. As in Fig. 2 the parallel lines correspond to the virial quantity $M = c_1 \sigma^2 R_{eff}/G$ where $\sigma$ is the intensity-weighted los velocity dispersion within an effective radius. The upper line is appropriate to a near-Newtonian system ($\eta = 10$) and the lower line to deep MOND objects ($\eta = 0.1$), or, alternatively, to homologous objects with a fixed fraction of dark matter within $R_{eff}$.

The essential feature of this plot is that the high-surface-brightness objects ranging from ellipticals to globular clusters define a fairly narrow FP which is consistent with the virial theorem. There are, of course, deviations due to non-homology and, more seriously, to the heterogeneity of the data samples. But the point is that the same FP is delineated by all high-surface-brightness (i.e. high internal acceleration) objects.

The low-surface-brightness dwarf spheroidals, as expected, lie significantly below this FP defined by the high-surface-brightness elliptical galaxies. The conventional explanation is that these objects are dominated by dark matter in the inner regions. With MOND, this is precisely the expectation for low-surface-brightness objects where a large discrepancy is predicted and the size has disappeared as a parameter.

4 THE UNIVERSAL FABER–JACKSON RELATION

Fig. 4 is the FJ relation for the early-type galaxies shown in Fig. 2 (from Cappellari et al. 2006). Here the luminosity is plotted against $\log(\sigma^2/Ga_0)$; the parallel lines are the relation $M = c_2 \sigma_c^4/Ga_0$. 

$$\sigma = \frac{\text{half-light radius}}{\text{effective radius}} \text{ and } M = \frac{\text{intensity-weighted los velocity dispersion}}{\text{within an effective radius}}.$$
The symbols and sources of data are the same for the 11 c = L⊙ for the FP relation. The plotted quantity is proportional to M/L as only, is a factor of 4 larger than for the FP relation.

A log–log plot of the luminosity versus the FJ quantity σ/Lσ⊙ or a Jaffe model with a very low dark matter η/Σ1, applies to highly Newtonian σ10 (or to a visible matter-dominated system). Most objects, ranging from from 10 to 10 L⊙ or M⊙, do not exhibit such a large and systematic offset from the mean relation as they do in the case of the FP relation.

The upper line (labelled M), with c2 = 20.25, is relevant to the deep MOND limit or equivalently the complete dominance of dark matter. The lower curve, with c2 = 1, applies to highly Newtonian objects (η = 10) or a Jaffe model with a very low dark matter content within Reff. Objects with a fixed value of η correspond to homologous structures having a fixed mean surface density, Σ, within an effective radius. Constant surface density and the virial theorem yield σ2 ≈ G M/Σ or an M ∝ σ4 relation even for the Newtonian systems. Large internal acceleration (high η) implies a large Σ which implies that the Newtonian line, for the FJ relation, lies to the right of the MOND line, opposite to the FP relation. Again, the solid points are the same objects with luminosity multiplied by M/L from the population synthesis models. Although the scatter is larger than for the FP relation, most of these points lie near the relation for Newtonian Jaffe models (low discrepacy).

The FJ relation for the wider class of objects, globular clusters to clusters of galaxies, is shown in Fig. 5. Most objects, ranging from Newtonian to deep MOND, in so far as they can be approximated by the isotropic Jaffe model, should lie in the range between the two parallel lines if M/L is near unity. The predicted scatter, due to a variation of η only, is a factor of 4 larger than for the FP relation. As in Fig. 3, for the clusters of galaxies (open squares), it is the gas mass that is plotted against the MOND parameter (σ/L⊙). As has been pointed out elsewhere (e.g. Sanders 2003), the cluster masses calculated with MOND still require about three times more mass than is directly observed in stars and hot gas. Including this ‘missing mass’ would move the cluster points up by about 0.5 in the logarithm in this plot.

Broadly speaking, this wide range of objects is consistent with the FJ relation implied by MOND; that is to say, there exists a Universal FJ relation applying to all near-isothermal pressure-supported objects. In particular, the dwarf spheroidal systems, deep MOND objects, do not exhibit such a large and systematic offset from the mean relation as they do in the case of the FP relation.
and UCDs taken together. The dwarf spheroidals deviate by about 2σ from those systems lying on the FP (this probably understates the significance as the offset becomes more pronounced at lower luminosities). Fig. 7 is a plot of \( \log(\sigma^2/G\sigma_0 L) \) versus \( \log(L) \) for these same systems. This is proportional to \( M/L \) as determined from the FJ relation (MOND). Again, the two points with error bars are the mean and dispersion of this quantity for the dwarf spheroidals (triangle) and separately the globulars and UCDs (square). Here we see that there is no significant difference between these classes of objects; the dwarf spheroidals lie on the universal FJ relation defined by the more compact pressure-supported objects.

### Figure 7

A log–log plot of \( \sigma^2/G\sigma_0 L \) versus \( L \) for dwarf spheroidals, globular clusters, and UCDs over the same range of luminosities. The triangular point with error bar is the mean and dispersion of this quantity for the dwarf spheroidals and the square point is the same for the globulars and UCDs taken together. The result shows that there is no significant deviation of the dwarf spheroidals from the FJ as defined by other classes of pressure-supported objects.

\[
\log(\sigma^2/G\sigma_0 L) \quad \log L
\]

#### 5 CONCLUSIONS

Luminous elliptical galaxies, dwarf elliptical galaxies, UCDs and globular star clusters are high-surface-brightness objects. High surface brightness implies high internal acceleration; i.e. the gravitational acceleration at \( R_{eff} \) is roughly 10 times larger than \( a_0 \), the fundamental MOND acceleration parameter. This means that, in the context of MOND, such objects should be described essentially by Newtonian dynamics within an effective radius. In that respect, it is not surprising that this wide range of gravitationally bound pressure-supported structures fall on an FP that reflects the Newtonian virial relation. Of course, there are variations: the globular clusters do not lie precisely upon the same FP as the ellipticals, but then the data shown here are heterogeneous, and the various classes of objects are probably not homologous. But the fact that the dwarf spheroidal galaxies deviate significantly from this FP is consistent with MOND. These are deep MOND low-surface-brightness objects with a large discrepancy; in this deep MOND limit the length-scale has dropped out as a parameter, and the mass is related only to the velocity dispersion.

The high-surface-brightness objects defining the Newtonian FP must deviate from an isothermal state, as they do in the case of the bright ellipticals and the globular clusters; both classes of objects exhibit a los velocity dispersion that declines with radius, at least within \( 1.5R_{eff} \). At larger radii, the radial profile of the \( \sigma \) is highly dependent upon the form of the anisotropy parameter \( \beta \), but a continuing decline is consistent with a trend towards more radial orbits in the outer regions.

In order to produce a high surface density Newtonian object, the required deviation from an isothermal state is relatively small. Typically, the Jaffe models which are Newtonian within an effective radius are equivalent to polytropes of index from 12 to 16; thus the density changes by many orders of magnitude while the velocity dispersion changes by a factor of 2. In other words, the requirements for producing objects which obey the Newtonian virial relation are that they should have high internal accelerations (\( \geq a_0 \)) and be near-isothermal with a radially declining velocity dispersion.

One might argue that this again begs the question: why are ellipticals and globular clusters near-isothermal objects with high internal accelerations? Does this not push the existence of the FP back to contingencies of structure formation? To some extent, this is the case, but the requirement of a near-isothermal state is rather mild. Violent relaxation, even in spherical collapse, typically produces a near-isothermal virialized structure. For example, with modified dynamics just such objects with high internal accelerations do result from the spherically symmetric dissipationless collapse out of a medium initially expanding with the Hubble flow (Sanders 2008). In non-spherical MONDian dissipationless collapse calculations, the final virialized objects with low internal accelerations (\( \leq a_0 \)) are essentially isothermal, but also those with higher internal accelerations are near-isothermal with a radial velocity dispersion which, compared with density, declines slowly with radius (Nipoti, Londrillo & Ciotti 2007). Viewed in this context, the dwarf spheroidal galaxies become the anomalous objects requiring an alternative formation scenario.

It is the FJ relation, in spite of its large scatter, that emerges as the more fundamental and more universal scaling relation for hot systems, embodying both high- and low-surface-brightness systems. The amplitude of the relation is even more significant than its slope because this is related directly to the magnitude of \( a_0 \). It is easily demonstrated that pure Newtonian systems (obeys the Newtonian virial theorem) with a constant mean surface density will also fall on an \( M \propto \sigma^4 \) relationship. The essential point is that \( a_0 \) sets this characteristic value of the surface density \( (a_0/G) \) for all near-isothermal systems.

The appearance of a universal mass–velocity-dispersion relation as an aspect of dynamics directly reflects the presence of this new dimensional constant in the structure equation. The fact that the magnitude of this constant (approximately \( 10^{-8} \text{ cm s}^{-2} \)) is the same as that required by the scale of spiral galaxy rotation curves (the Tully–Fisher relation) is one more powerful indication that such a fundamental acceleration scale exists in the Universe and is operative in gravitational physics.

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