Methods of the analytical construction of diagrams of the vibratory drum vertical movements into the soil for the compacted soil strength assessment

V N Tarasov¹, I V Boyarkina¹, G N Boyarkin² and V S Serebrennikov¹
¹Siberian State Automobile and Highway University, 5, Mira Ave., Omsk, 644080, Russia
²Omsk State Technical University, 11, Mira Ave., Omsk 644050, Russia

Abstract. The soil compaction process by a vibratory drum is performed during a slow horizontal progressive movement and high-frequency vertical movements of the vibratory drum by a vibration exciter. Let’s consider the process of a single vertical movement of the vibratory drum into the soil on a horizontal compacted surface. As a result of the vibratory drum progressive movement vertical movements take place on new horizontal areas of compacted soil. The problem of the vibratory drum cylindrical surface vertical indentation into the soil is first presented in a differential form. The stress distribution law as per the soil depth is set taking into consideration the compacted soil layer thickness \( h_0 \) and general soil deformation modulus \( E_0 \). The relative vertical deformation of elementary soil columns according to Hooke’s law is determined as the ratio of the vertical movement of the vibratory drum elements to the thickness of the layer to be compacted. The analytical integration of equations connecting the vertical load on the vibratory drum with elementary forces of the soil reactions on the vibratory drum bearing surface has been performed. The state of soil strength under the vibratory drum is determined as the initial state \( \sigma_0 \) and additional state \( \Delta \sigma \), caused by an increase of the soil strength due to the compaction \( \sigma = \sigma_0 + \varepsilon E_0 \), caused by relative deformation. The problem of the determination of the vibratory drum vertical movement by vertical forces has been solved. Obtained analytical dependences and method allow to construct the dependence diagrams of the vibratory drum movement into the soil on different types of soils. The problem of the vibratory drum vertical movement into the soil has been analytically solved on the basis of the deformation of vertical soil columns in the form of Hooke’s law using the general soil deformation modulus \( E_0 \). The vertical interaction of the vibratory drum with the soil has been formulated as the equilibrium problem of the vertical reaction forces along the contact arc. Finite integrals have been obtained connecting the vertical load on the vibratory drum with the parameters of the soil and vibratory drum. The calculation method of the dependence diagrams of loads on the vibratory drum and vertical movements (settlement) of the vibratory drum into the soil has been developed.

Key-words: soil strength, cylindrical vibratory drum, soil class, soil deformation modulus.

1. Introduction
The strength of construction soils in earthwork structures is determined by the resistance to the deformations of compression and shift. For the soil strength measurement the indentation into the soil of round or rectangular punches is made. The soil compression deformations are mainly irreversible processes, as elastic phenomena during the soil compression are very insignificant, and the deformation process is accompanied by the soil compaction phenomenon.

Another line of research of the soil strength is based on the indentation into the soil of bodies of different geometric shapes: rods, balls, cones.
Numerous works of foreign and Russian scientists are devoted to the improvement of the vibration compaction process of materials and soils [1-17].

This study solves the problem of the analytical description of the cylindrical rigid vibratory drum indentation process into the soil, and the problem of the indicator system development characterizing the soil strength during the vibration compaction. The peculiarity of the soil compaction process by the vibratory drum is characterized by a low speed of the vibratory drum progressive movement (about 6 km/h) during the vibratory drum high-frequency vertical impact on the soil with the angular frequency of about \( n = 3000 \) rpm.

2. Problem statement
To create the method for the strength determination of surfaces to be compacted by the cylindrical vibratory drum penetration into the soil.

3. Theory
Rated conditions of the vibratory drum interaction with the soil are determined as follows. During the vibratory drum uniform rolling, practically immediate vertical impact of the vibratory drum on the soil horizontal surface is performed. During the time of a vertical movement the vibratory road roller moves to a new position, that’s why the second impact of the vibratory drum on the soil and all subsequent vertical impacts are also fulfilled on new horizontal areas arranged in succession without overlapping.

The average specific pressure in the contact of the vibratory drum with the soil is determined by the formula

\[
\sigma = \frac{Q}{A} = \frac{Q}{bL},
\]

(1)

where \( Q \) is the vertical force on the vibratory drum; \( A \) is the contact area of the vibratory drum surface with the soil; \( L \) is the vibratory drum length; \( b \) is the contact width.

The angle \( \alpha \) of the contact arc (Figure 1) can be determined by the formula

\[
\alpha = \arcsin\left(\frac{b}{D}\right).
\]

(2)

\[\text{Figure 1. The process of the cylinder indentation into the soil.}\]

Cylindrical vibratory drums for the soil compaction have a large diameter and length \( D \times L \), that’s why the chord \( b \) of the cylinder contact surface length with the soil practically coincides with the arc length. With sufficient certainty the contact surface can be regarded as a flat punch, the area of which is

\[
A = DL\sin\alpha.
\]

The depth of the cylinder settlement into the deformable soil can be determined by the formula
\[ z_s = 0.5D(1 - \cos \alpha). \quad (3) \]

In Figure 1, for the cylindrical vibratory drum to be penetrated into the soil, the determination of the settlement value \( z_s \) in the compaction conditions also represents certain technical difficulties, however, it is expedient to take into consideration the parameter \( z_s \) for the clarification of the physical nature of compaction processes.

The width of the indent \( b \) can be determined by the mathematical formula

\[ b = \sqrt{D^2 - (2z_s - D)^2}. \quad (4) \]

The soil compaction by cylindrical drums of static and vibrational action has found universal application in world practice, that’s why the vertical penetration is important for vibratory drums, as it determines the soil settlement during compaction, penetration of the vibratory drum into the soil is fulfilled many times. The period of one lifting and lowering of the vibratory drum is determined by the formula

\[ T = \frac{2\pi}{\omega} = \frac{1}{f}, \quad (5) \]

where \( \omega \) is the rotation frequency of the vibration exciter eccentric weight, rad/s; \( f \) is the frequency, Hz.

For example, for a heavy vibratory drum of the type \( f = 40 \) Hz the period \( T = 0.157 \) s.

Figure 2 shows the design diagram of a single vertical interaction of the vibratory drum with the horizontal surface of the soil to be compacted.

Figure 2. The design diagram the cylindrical vibratory drum indentation into the deformable soil.

The initial thickness of the soil to be compacted is characterized by the height \( h_0 \). The value of the vibratory drum settlement into the soil is marked as \( z_s \). The length of the contact \( b \) of the vibratory drum with the soil is shown as the projection of the contact arc with the angle \( 2\alpha \) on the horizontal direction of the \( x \) axis.

On the design diagram (Figure 2) during the vibratory drum indentation into the soil the vertical soil crushing takes place. The soil segment volume, limited by section points 1–2–3–4 and the vibratory drum length \( L \), is crushed and passes downwards to the compacted soil half-space. The soil compaction process happens. The process of the vibratory drum vertical penetration into the soil starts at point 1 at the moment of the vibratory roller contact with the soil, when the stress at contact point 1
is equal to the initial soil strength $\sigma_0$. During the vibratory drum lowering in Figure 2 to the position coinciding with point 3, the vertical compression of the soil columns by all points of the arc $2\alpha$ occurs. The Law of variation of normal vertical stresses $\sigma$ during compaction is to be suggested to write down in the study as the function of the angle $\phi$ of the vibratory drum contact arc

$$\sigma = \sigma_0 + \frac{R - R \cos \phi}{h_0} E_0,$$

where $R$ is the vibratory drum radius; $E_0$ is the general soil deformation modulus; $h_0$ is the depth of the layer to be compacted; $\sigma_0$, $\sigma$ are accordingly normal stresses at the end points of the vibratory drum contact arc with the soil and under the vibratory drum at point 3.

Thus, during the vibratory drum vertical movement the compaction of the soil volume takes place having the section of segment $1-2-3-4$ as a result of a relative deformation of the vertical elementary soil columns. Normal stresses as per formula (6) can be written down in the form of Hooke’s law

$$\sigma = \sigma_0 + \varepsilon E_0,$$

The relative deformation $\varepsilon$ in equation (7) is the function of the angle $\phi$ for the contact points of the vibratory drum with the soil according to (6)

$$\varepsilon = \frac{z_{cr}}{h_0},$$

where $z_{cr}$ is the average value of the vertical soil crushing on the arc $2\alpha$.

The soil compaction is characterized by increasing normal stresses from the value $\sigma_0$ at point 2 to the value $\sigma$ at point 3 at the account of the vertical deformation of elementary vertical soil columns under the vibratory drum, which are subject to the relative compression $\varepsilon$ in the form of Hooke’s law.

The nature of the normal vertical stress diagram change in Figure 2 allows us to express the vertical load $Q$ on the vibratory drum axis in the form of two components

$$Q = Q_o + \Delta Q,$$

where $Q_o$ is the component of the vertical load caused by the initial soil strength $\sigma_0$; $\Delta Q$ is the load component caused by increasing the soil strength as a result of its compaction.

The load $Q_o$ can be determined by the formula

$$Q_o = \sigma_0 bL,$$

where $bL$ is the projection of the vibratory drum bearing surface area with the soil on the horizontal plane.

The equilibrium of the vertical reaction forces and forces acting on the vibratory drum can be written down by the equation

$$Q_o + \Delta Q = 2 \int_0^\alpha \sigma LR \cos \varphi d\varphi.$$

Taking into consideration expressions (6) and (9), the vertical load $Q$ on the vibratory drum axis can be written down as the doubled integral of the vertical reaction forces along the contact arc of the vibratory drum with the soil

$$Q_o + \Delta Q = 2 \int_0^\alpha \left( \sigma_0 + \frac{R - R \cos \phi}{h_0} E_0 \right) LR \cos \varphi d\varphi.$$

The right-hand side of expression (12) can be represented in the form of two integrals

$$Q_o + \Delta Q = 2 \left[ LR \sigma_0 \int_0^\alpha \cos \varphi d\varphi + \frac{LR^2 E_0}{h_0} \int_0^\alpha \left( 1 - \cos \varphi \right) \cos \varphi d\varphi \right].$$
The component $Q_0$ on the vibratory drum axis is numerically equal to the first integral of expression (13)

$$Q_0 = \sigma_0 b L = 2LR\sigma_0 \int_0^\alpha \cos \varphi d\varphi = 2LR\sigma_0 \sin \alpha .$$  \hspace{1cm} (14)

This means that the vertical component of the load $Q_0$ on the vibratory drum according to equation (14) is balanced by the first integral of the right-hand side of equation (13). And the second integral of equation (13) is nothing more than the additional vertical load $\Delta Q$ caused by increasing the soil carrying capacity due to the additional compaction by the vibratory drum

$$\Delta Q = \frac{2LR^2E_0}{h_0} \int_0^\alpha (\cos \varphi - \cos^2 \varphi) d\varphi .$$  \hspace{1cm} (15)

After integrating expression (15), we obtain

$$\Delta Q = \frac{2LR^2E_0}{h_0} \left( \sin \alpha - \frac{\alpha}{2} - \frac{1}{4} \sin 2\alpha \right) .$$  \hspace{1cm} (16)

Equation (16) is obtained for calculating the additional vertical carrying capacity of the soil caused by the additional compaction of the soil by the vibratory drum, where the vibratory drum geometric parameters, technological parameters of the compaction process and soil strength parameters are taken into consideration, in particular the general soil deformation modulus $E_0$.

Thus, taking into consideration formula (1), the actual contact length of the vibratory drum in the form of projection of the contact arc on the horizontal plane can be determined by the formula

$$b = \frac{Q - \Delta Q}{L\sigma_0} .$$  \hspace{1cm} (17)

The angle $\alpha$ of the contact arc is determined by formula (2)

$$\alpha = \arcsin \left( \frac{b}{D} \right) .$$

In equation (17) the initial strength of the compacted soil $\sigma_0$ is determined as per the table.

**Table.** Mechanical characteristics of soil strength classes

| Soil strength classes | I    | II   | III  | IV   | V    |
|-----------------------|------|------|------|------|------|
| Initial $\sigma_0$ and final $\sigma$ stresses, MPa | 0.524÷ 2.096 | 2.62÷ 4.192 | 4.718÷ 8.384 | 8.908÷ 18.34 | 18.34÷ 36.68 |
| Number of the instrument density gage strokes for the measurement of $\sigma$ | 1÷4 | 5÷8 | 9÷16 | 17÷35 | 36÷70 |

The soil classification table according to A.N. Zelenin [13] is supplemented by the value of the initial stresses $\sigma_0$ obtained by formula of paper [14]

$$\sigma_0 = 0.524n_{imp}, \text{ MPa},$$  \hspace{1cm} (18)

where $n_{imp}$ is the number of strokes of the soil strength determination instrument.

**4. Results discussion**

In this study all values included in the considered formulas are discussed and scientifically justified. This allows to create methods for constructing diagrams of dependences of vertical loads $Q$ on vertical movements of the vibratory drum $z_s$ for soils of different initial strength $\sigma_0$.

For soil with the set initial strength $\sigma_0$ with the load $Q$ it’s possible to determine the preliminarily contact length $b'$ of the vibratory drum with the soil
Further we preliminarily determine the contact arc angle

\[ \alpha' = \arcsin \frac{b'}{D}. \]

The first approximation of the settlement value is determined by the formula

\[ z_s' = 0.5D(1-\cos \alpha') \quad \text{or} \quad z_s' = \frac{D - \sqrt{D^2 - (b')^2}}{2}. \]  

If \( \alpha' > 4^\circ \), then we proceed to the clarification of the values \( b' \), \( \alpha' \), \( z_s' \).

We calculate \( \Delta Q \) applying formula (16)

\[ \Delta Q = \frac{2LR^2E_0}{h_0} \left( \sin \alpha' - \frac{\alpha'}{2} - \frac{1}{4} \sin 2\alpha' \right). \]  

The soil deformation modulus is calculated by the formula from papers [14, 15]

\[ E_0 = \frac{QK_\omega(1-\mu^2)}{z_s'L} \quad \text{or} \quad E_0 = C \frac{K_\omega(1-\mu^2)}{L}, \]

where \( C \) is the soil stiffness factor; \( \mu \) is the Poisson’s ratio, \( \mu = 0.3 \); \( z_s' \) is the vertical settlement of the vibratory drum in deformable soil; \( K_\omega \) is the shape factor of the rectangular punch area of the size \( L \times b \), it’s determined by the formula of paper [14]

\[ K_\omega = 0.9409(L/b)^{0.3568}. \]  

We determine the actual contact length of the vibratory drum

\[ b = \frac{Q - \Delta Q}{\sigma_0 L}. \]

We specify the contact arc angle \( \alpha \) by the formula \( \alpha = \arcsin (b/D) \), the vibratory drum settlement

\[ z_s = 0.5D(1-\cos \alpha) \quad \text{or} \quad z_s = \frac{D - \sqrt{D^2 - b^2}}{2}. \]  

Thus, the problem of determining the deformation value \( z_s \), corresponding to the set load \( Q \) on the vibratory drum axis and soil strength \( \sigma_0 \) has been solved. If at high values of \( \sigma_0 \) and low loads \( Q \) the condition \( \alpha > 4^\circ \) is not satisfied, then it is necessary to solve the problem regarding the sufficient accuracy of the parameter values \( z_s' \) and \( b' \). If the values \( z_s' \), \( \alpha' \) do not require any clarification, then we accept \( z_s = z_s' \); \( \alpha = \alpha' \). In another case, it is possible to clarify the deformation values \( b' \), \( \alpha' \), \( z_s' \) by calculating \( \Delta Q \) and further calculations according to the developed methods.

In Figure 3 the dependences \( Q=f(z_s) \) for a heavy-type vibratory drum with the parameters \( L \times D = 2.13 \times 1.73 \) m for weak soils \( \sigma_0 = 0.152 \pm 2.096 \) MPa with large soil deformations \( z_s > 0.12 \) m, which are typical for the static soil compaction modes by vibratory drums.
Figure 3. Dependence diagram of the vertical load $Q$ on vertical movements of the vibratory drum $z_s$ on loose soils during a single impact of the vibratory drum on the soil.

Figure 3 shows the calculation results for points 1 and 2 of the diagram. For point 1 we have the following data: $Q=300$ kN; $\sigma_0=0.25$ MPa; $L\times D=2.13\times 1.73$ m; $h_0=0.5$ m; $b'=0.5634$ m; $\alpha'=19.0053^\circ$; $z'_s=0.04715$ m; $L/b=3.7808$; $K_0=1.5114$; $\mu=0.3$; $E_0=4.108442$ MPa; $\Delta Q=77$ kN; $b=0.4195$ m; $\alpha=14.033^\circ$; $z_s=0.02581429$ m. For point 2 we have the following data: $Q=400$ kN; $\sigma_0=0.25$ MPa; $L\times D=2.13\times 1.73$ m; $h_0=0.5$ m; $b'=0.7512$ m; $\alpha'=25.734683^\circ$; $z'_s=0.085796$ m; $L/b'=2.8356$; $K_0=1.364$; $\mu=0.3$; $E_0=2.716884$ MPa; $\Delta Q=122$ kN; $b=0.5225$ m; $\alpha=17.581^\circ$; $z_s=0.04040281$ m. Connecting points 1 and 2 in Figure 3, we obtain a part of the general curve $\sigma_0=0.25$ MPa. All points of the graph in Figure 3 are obtained in the same way.

For small general deformations $z_s<0.004$ m, Figure 4 shows the dependences $Q=f(z_s)$ typical for vibratory drums with a limited amplitude of vertical vibrations $A(z_s)$.

Figure 4. Dependence diagram of the vertical load $Q$ on vertical movements of the vibratory drum $z_s$ during a single impact of the vibratory drum on the soil.
For Figure 4 we’ll show the calculation results according to the developed methods.

For point 1 we have: \( Q = 300 \text{ kN} \); \( \sigma_0 = 1.572 \text{ MPa} \); \( L \times D = 2.13 \times 1.73 \text{ m} \); \( h_0 = 0.5 \text{ m} \); \( b' = 0.0896 \text{ m} \); \( \alpha' = 2.968656^\circ \); \( \zeta_0' = 0.001161 \text{ m} \); \( L/b' = 23.773 \); \( K_\omega = 2.91274 \); \( \mu = 0.3 \); \( E_0 = 321.603752 \text{ MPa} \); \( \Delta Q = 24 \text{ kN} \); \( b = 0.0825 \text{ m} \); \( \alpha = 2.7335^\circ \); \( z_0 = 0.00098425 \text{ m} \).

For point 1 in Figure 4 the contact arc angle \( \alpha' = 3.357135^\circ \), i.e. the condition is not satisfied. However, taking into consideration that the value \( \alpha' = 3.357135^\circ \) is close to the assigned limit, it is necessary to introduce the tolerance on this condition \( \alpha' \geq 4^\circ \) minus \( 1^\circ \div 2 \), i.e. we adopt a decision regarding the necessity of clarifying preliminary calculations of parameters \( b' \), \( \alpha' \), \( z_0' \). After calculating \( \Delta Q \), we obtain the exact value of the results for point 1 in Figure 4 \( z_0 = 0.000984 \text{ m} \).

The diagrams presented in Figures 3, 4 contain useful information for in-depth knowledge of the soil compaction process by the vibratory drum.

For a real vibratory drum the load \( Q \) on the drum according to known methods consists of two components

\[
Q = m_r g + P_d K_v, \tag{27}
\]

where \( m_r \) is the vibratory drum weight, kg; \( P_d \) is the radial force module of a vibration exciter, N; \( K_v \) is the module of the harmonic force reduction \( P_d \) to the vertical static force, \( K_v = 0.6366 \) [14].

5. Consideration of the results

For soils of low strength (Figure 3) with increasing the vertical load \( Q \) the vibratory drum settlement \( z_s \) increases and acquires large values for all values of the initial soil strength.

In Figure 3 for soil with the initial strength \( \sigma_0 = 2.096 \text{ MPa} \) the vibratory drum settlement \( z_s \) is a small value. This means that for a given soil the load \( Q = 700 \text{ kN} \) gives a slight compaction. For the compaction of such soil it’s necessary to make an intensive multiple vibrational impact of the vibratory drum on the soil.

In Figure 4 for soils with a large initial strength, it can be noted that for all values of the initial strength \( \sigma_0 \), with an increase of the load \( Q \), the vibratory drum settlement is increased with different intensity during a single vertical impact of the vibratory drum on the soil. For a road roller road with given geometric dimensions with the load \( Q \), the ultimate soil strength is \( \sigma_0 = 8.38 \text{ MPa} \), for which the vertical movement is about 0.001 m during a single vertical impact.

6. Conclusion

1. Methods and algorithm have been developed for the construction of diagrams of the vibratory drum vertical movement from a vertical load during a single impact.

2. Obtained diagrams have theoretical and practical significance for in-depth knowledge of the soil compaction process by vibratory drums.

7. References

[1] Xu Tong, Chen Siwei, Wang Dong, Wu Ti, Xu Yang and Zhang Weigong 2019 A Novel Path Planning Method for Articulated Road Roller Using Support Vector Machine and Longest Accessible Path With Course Correction IEEE Access Volume 7 Electronic ISSN 2169-3536 pp 182784 – 182795 doi 10 1109 ACCESS 2019 2959346

[2] Tarasov V N, Boyarkina I V and Serebrennikov V S 2019 Analytical study of oscillating horizontal vibrations of a road roller IOP Conf Series Journal of Physics Conference Series 1260 2019 112027 doi 10.1088/1742-6596/1260/11/112027

[3] Mikheyev V V, Saveliev S V and Permyakov V B 2019 Comprehensive approach to the selection of the optimal energy-effective mode of operation of vibration rollers Problems of mechanical engineering. Materials of the 3rd International Scientific Conference pp 158-165

[4] Zhi Jinning, Zhang Hong and Li Jie 2011 Dynamic Modeling and Simulation Analysis of the Cushioning System of the Impact Roller Based on ADAMS Third International
Conference on Measuring Technology and Mechatronics Automation INSPEC Accession Number 11850294
[5] Chun-feng Guo and Chang-tan Xu 2010 Research and utilizing of multidisciplinary co-simulation for vibrating system of vibrating YZ18JA-type road roller 5th IEEE Conference on Industrial Electronics and Applications INSPEC Accession Number 11434001
[6] Syed Asif Imran, Sesh Commuri and Musharraf Zaman 2015 A 2 dimensional dynamical model of asphalt-roller interaction during vibratory compaction 12th International Conference on Informatics in Control Automation and Robotics (ICINCO) INSPEC Accession Number 15677275
[7] Wang G F, Hu Y B, Zhu W Q, Bao Z Y, Chen Z J, Huang Z H and Wang W 2017 The design of a compaction parameters management system for intelligent vibratory roller IEEE 3rd Information Technology and Mechatronics Engineering Conference (ITOEC) pp 634 – 638
[8] Jing Baode, Lin Dongbing, Liu Meiyu and Zhu Xilin 2010 Design of non-impact and independent exciting chamber of vibratory roller International Conference on Intelligent Computation Technology and Automation vol 2 pp 44 – 46
[9] Yan Tao-ping 2011 Vibration frequency vibratory roller stepless design and analysis of the hydraulic system International Conference on Consumer Electronics Communications and Networks (CECNet) pp 4621 –4624
[10] Zhang Yi, Zhang Jun, Shu XingZu, Guo Lei, Shi Yong and Liu XinBo 2009 Optimization of intelligent compactness control rule of vibratory roller based on genetic algorithm method Fifth International Joint Conference on INC IMS and IDC pp 1943 – 1947
[11] Heqing Li and Qing Tan 2008 Recognition of reliability model of vibratory roller based on artificial neural network International Conference on Intelligent Computation Technology and Automation (ICICTA) Vol 1 pp 231 – 234
[12] Syed Imran, Fares Beainy, Sesh Commuri and Musharraf Zaman 2012 Transient response of a vibratory roller during compaction IEEE 51st IEEE Conference on Decision and Control (CDC) Maui H USA pp 4378 – 4383
[13] Zelenin A N, Balovnev VI and Kerov I P 1975 Earth moving machines (Moskow Mechanical engineering) p 424
[14] Tarasov V N, Boyarkina I V and Serebrennikov V S 2019 Soil deformation module during compaction with a light road roller Construction and road construction machinery no 3 pp 25 – 30
[15] Tsitovich N A 1979 Soil Mechanics (Moscow Higher School) p 272
[16] Ivovich V A and Onishchenko V Ya 1990 Protection against vibration in mechanical engineering (Moscow Mechanical Engineering Publishers) p 272
[17] Bykhovsky I I 1980 Fundamentals of vibration engineering (New York Robert Krieger Publishing Co) p 382