Morphed Learning: Towards Privacy-Preserving for Deep Learning Based Applications

Juncheng Shen
College of Electrical Engineering
Zhejiang University

Juzheng Liu
Department of Physics
Tsinghua University

Hai Li and Yiran Chen
Department of Electrical and Computer Engineering
Duke University

Abstract
The concern of potential privacy violation has prevented efficient use of big data for improving deep learning based applications. In this paper, we propose Morphed Learning, a privacy-preserving technique for deep learning based on data morphing that allows data owners to share their data without leaking sensitive privacy information. Morphed Learning allows the data owners to send securely morphed data and provides the server with an Augmented Convolutional layer to train the network on morphed data without performance loss. Morphed Learning has these three features: (1) Strong protection against reverse-engineering on the morphed data; (2) Acceptable computational and data transmission overhead with no correlation to the depth of the neural network; (3) No degradation of the neural network performance. Theoretical analyses on CIFAR-10 dataset and VGG-16 network show that our method is capable of providing $10^{-30}$ morphing possibilities with only 5% computational overhead and 10% transmission overhead under limited knowledge attack scenario. Further analyses also proved that our method can offer same resilience against full knowledge attack if more resources are provided.

Introduction
Recent years witnessed rapid development in deep learning algorithms. Nowadays, deep neural networks are able to present close to or even better than human performance in contained environments, such as image classification (He et al. 2016), speech recognition (Amodei et al. 2016), and decision making of the game of Go (Silver et al. 2016). However, real-world applications based on the same algorithms rarely perform as good. As (Halevy, Norvig, and Pereira 2009) has pointed out that data is as important as the algorithm, one of the main reasons for the performance gap between lab datasets and real-world applications is the abundance of data. Different from standard and publicly available datasets, developing real-world deep learning based applications usually require a great amount of time and resource for data collection. On the other hand, due to the massive adoption of the internet, nowadays users generate data at a faster rate than ever. We consider the concern of potential privacy violation is an important reason that causes the contradiction of high-cost data collection and abundant big data. Since these data may contain sensitive personal information, they are prevented from contributing to training better intelligent algorithms.

To address the concern of privacy violation, the research community urges deep learning techniques with privacy-preserving features. Several prior privacy-preserving deep learning methods have been proposed. Generally, these methods take one of the four following approaches: collaborative learning (Shokri and Shmatikov 2015), homomorphic encrypted learning (Rouhani, Riazi, and Koushanfar 2018), learning with differential privacy, (Abadi et al. 2016) and feature transmission (Osia et al. 2017). Collaborative learning tries to train a model by sharing the gradients between collaborators while each of them does not have access to the original data. However, this method can cause huge computation and transmission overhead, and the training process is vulnerable to adversarial collaborators. Homomorphic encrypted learning tries to adopt the method of homomorphic encryption for the field of deep learning, but the nature of homomorphic encryption also introduces computational overhead proportionally to the neural network’s computational complexity, which is considered to be impractically for every deep neural network. Deep learning with differential privacy tries to keep privacy by adding noise to the original data or the training gradient, but the accuracy can be influenced by the noise. On the other hand, feature transmission is able to overcome the compromise on the accuracy, but it introduces the potential vulnerability of exposing the original data by reverse-engineering on the features.

For the purpose of providing strong privacy with fewer drawbacks, we proposed a new privacy-preserving method for deep learning based applications, namely Morphed Learning. In Morphed Learning, the original data are morphed before being sent to the server, and a customized layer called Augmented Convolutional (aug-conv) layer is designed to eliminate accuracy loss introduced by data morphing. Morphed Learning provides the three following features: (1) Strong privacy protection; (2) Acceptable computational and data transmission overhead with no correlation to the depth of the neural network; (3) No compromise of the network performance. Analyses on CIFAR dataset (Krizhevsky 2009) with VGG-16 neural network model show (Simonyan and Zisserman 2013) that Morphed...
Learning provides $10^{40}$ morphing possibilities with only 5% computational overhead and 10% transmission overhead under limited knowledge attack scenario. Further theoretical proofs are provided to elaborate Morphed Learning’s security in both full knowledge attack scenario and limited knowledge attack scenario.

**Related Work**

To dispel the concern about sensitive information leakage while collecting the personal data for deep learning applications, several prior privacy-preserving deep learning methods were proposed. The existing approaches to preserve data privacy mainly fall into four approaches as will be introduced detailedly in this section.

**Collaborative Deep Learning**

In order to protect the privacy of user data, one straightforward method is to share the training gradients or the model parameters instead of the original data. This method has been implemented by (Shokri and Shmatikov 2015), in which a deep learning system was designed with several collaborators training their local models using their local data, and share their gradients with a centralized parameter exchange server. The advantage of this method is that the data are completely separated between collaborators. Therefore, each collaborator only has the access to the data generated by himself, providing privacy-preserving feature for the data generated by other collaborators. However, this method introduces a great amount of data transmission overhead between several systems, which poses a potential bottleneck for the training process. In addition, it also requires each collaborator to have enough computational capacity for deep learning training which may not always be practical. Furthermore, the decentralized nature of this method brings in the threat of poisoning attack by malicious collaborators.

**Deep Learning with Differential Privacy**

After differential privacy was first proposed by (Dwork 2006), it was then introduced in to the field of deep learning privacy-preserving by (Abadi et al. 2016). Differential privacy in deep learning combines certain noise into the gradient during the training process to prevent adversaries from gaining the original data via reverse-engineering. However, this method contributes to privacy preserving in an indirect way, and also adding noise to the gradient can cause significant deterioration on training and testing accuracy. (Phan et al. 2018) tried to solve the problems above by adding adaptive noise directly to the original data. With certain privacy budget, the details of the original data can be kept secure on a certain probability level when the statistical information of the dataset is queried by others. This method can partly reduce the accuracy loss by adding more noise to a selected unimportant part of the data. However, the data stays recognizable to human after applied with noise. Meanwhile, adding noise adaptively means it requires extra computational overhead.

**Homomorphic Encrypted Learning**

To keep the accuracy while protecting the data privacy, fully homomorphic encryption (Gentry 2009) has been introduced into the field of deep learning. In homomorphic encryption, the plaintext of the original data is encrypted into a ciphertext. Then the encrypted results can be calculated independently using the ciphertexts, and the decrypted results can be gained from the encrypted results nondestructively. This algorithm can achieve a high-level security without changing the computing result. Unfortunately, up to now, fully homomorphic encryption is still impractical for computation-intensive applications including deep learning, due to its heavy computational and data transmission overhead, as shown in (Aslett, Esperança, and Holmes 2013) and (Rouhani, Riazi, and Koushanfar 2018). Furthermore, practical homomorphic encryption algorithms usually are only capable of addition and multiplication operations. Therefore activation functions that introduce nonlinearity to deep neural networks models cannot be reproduced by homomorphic encryption accurately. For example, ReLu function can only be approximated by polynomials (Chabanne et al. 2017) which in comparison is more computation-intensive. In addition, homomorphic encryption also introduces bloating of data size. Therefore, currently this method is not practical for training deep neural networks due to high computational and transmission overhead.

**Feature Transmitting**

Feature Transmitting is another approach for deep learning privacy-preserving. It eliminates the compromise on accuracy, as proposed in (Osia et al. 2017). Instead of transmitting the data, the data owner runs several feature extraction layers of the neural network and then send the extracted features to the network trainer. However, the feature extraction layers run by data owner have to be assigned by the server provider. This enables the possibility for the server provider to retrieve the original data by inverse operations such as deconvolution. To defend the potential threat, several feature preprocessing methods have been introduced by (Osia et al. 2017) to avoid the server provider finding out the accurate inverse function. However, the preprocessing method can cause a severe influence on the performance of the neural network, especially the performance on complex tasks such as emotion detection. Lastly, the transmission cost of the feature is tens of times larger than transmitting the original data, which also lowered the practicality of this technique.

**Method**

The workflow of Morphed Learning is shown in Figure 1. In Morphed Learning, the data owner applies data morphing on the training set and then send the morphed training set to the server provider. The server provider needs to replace the first convolutional layer of the original model with the aug-conv layer and threat the aug-conv layer as a fixed feature extraction layer. The rest of the training process remains unchanged. The trained model from Morphed Learning can achieve the same accuracy on morphed test data.
as its original model on original test data. Morphed Learning secures the data by separating the server provider with the original data. The design of data morphing ensures the server provider cannot retrieve the original data via statistical methods, while the design of aug-conv layer ensures the server provider cannot retrieve the original data via reverse-engineering the inverse function of the aug-conv layer.

**Data Morphing**

To ensure the security of the data privacy, the data morphing applied by the data owner needs to satisfy two properties: (1) The morphed data must be unrecognizable for humans. (2) The original data cannot be retrieved using statistical methods. In our work, we took linear transformation by a randomly generated matrix as our morph method.

For the first property, the original data from CIFAR-10 and the morphed data are shown in Figure 2. The transformation matrix size is chosen to match the size of the image as can be seen in the figure that larger transformation matrix makes morphed data more heterogeneous and difficult for humans to recognize.

For the second property, the morphing can be represented as:

\[ DM = T \]  

(1)

where \( D \) stands for a 1D or 2D tensor from the original data (for instance, a channel from an original CIFAR-10 image), \( M \) stands for the transformation matrix, and \( T \) stands for the transformed pixel tensor. The inverse matrix \( M^{-1} \) can be gained through:

\[ M^{-1} = T^{-1}D \]  

(2)

Since only the morphed data is transmitted to the server, the server provider can only get \( T \), and has no access to \( D \). If the server provider tries to gain \( M^{-1} \) through (where \( \bar{T} \) and \( \bar{D} \) represent the statistical average of the original channel and the transformed channel):

\[ M^{-1} = \bar{T}^{-1}\bar{D} \]  

(3)

and an estimation of every element in \( \bar{D} \) and the inverse matrix of \( \bar{T} \) are needed. The server provider cannot get a precise and invertible estimation of \( \bar{D} \), for there is no access to the original data. In case that the server provider can get a very close estimation of \( \bar{D} \) from another similar dataset, the data provider only need to make sure \( \bar{T} \) is not a square matrix to make \( \bar{T} \) irreversible. As a result, the server provider cannot retrieve the original data in a statistical way.

**Augmented Convolutional Layer**

Though the data morphing can keep the original images from being recognized by humans, it makes the training process more difficult to converge, and lead to a decrement of the test accuracy. The reason for this phenomenon is that the clustered original data in the primary space becomes un-clustered in the image space after the transformation, as is illustrated in Figure 3. In order to prevent the degradation of the test accuracy, a custom layer namely Augmented Convolutional (aug-conv) layer is proposed to restore the accuracy while keeping the privacy at the same time.

The aug-conv layer needs to meet the following three requirements: (1). To completely restore the test accuracy, the features extracted by the aug-conv layer from the morphed data should be equivalent to the features extracted by the original network from the original data. (2). The new feature
tures should not be useful for the server provider to retrieve the original data by inverse operation of the feature extract operation such as deconvolution. (3). The aug-conv layer should not provide a way for the server provider to retrieve the original data.

To satisfy the first and the second requirement, the aug-conv layer is designed to extract the features that have the same content as the original features that extracted by the first convolutional layer of the original model from the original data, but the channel sequence is randomly reset. This means the aug-conv layer is equal to an inverse morphing combined with a convolutional layer that its channel sequence is different from the original one. In the training and testing process, the new features have the same performance as the original ones, for all of the information from the original features is preserved. Meanwhile, the new features are not useful for the server provider to retrieve the original data, because the new channel sequence needs to be guessed out before the deconvolution, and the extremely low probability make it unattainable. For instance, if a feature has 64 channels, the probability of guessing out the correct sequence is 

\[ P_{64} = \frac{1}{64!} \approx 10^{-89}. \]

As for the third requirement, the function of the inverse morphing and the channel-randomized convolutional layer must be compiled into a single operation. Normal linear transformation cannot realize the function of convolutional layer, so if the aug-conv layer is realized by an inverse linear layer and a convolutional layer discretely, the linear layer needs to be exactly the inverse matrix of the transform matrix, and the convolutional layer that is used to generate the new feature is also exposed to the server provider. The server provider can gain the original data either using inverse linear transformation or deconvolution. To combine the linear layer and the convolutional layer, the two layers need to be reconstructed using bigger matrices that can be multiplied together to construct a single layer.

Taking the convolution of images as an example, the replacement of the convolutional layer is shown in Figure 4(a). The original convolutional layer is replaced by a matrix (denoted as \( \mathbf{M}_{\text{conv}} \)). The RGB image needs to be resized into a one-dimension vector, and the vector is multiplied by \( \mathbf{M}_{\text{conv}} \) to gain the feature tensor. Every single element in the feature vector is corresponding to one column in \( \mathbf{M}_{\text{conv}} \). The elements in the corresponding original convolution kernels are inserted into certain positions of the column. As is shown in Figure 4(b), each value in the feature can be regarded as the weighted sum of all the pixels from all three channels in the image. But only the weights of those pixels that need be involved in the convolution operation is set as the value of the original convolution kernels, other weights are set as zero. For the \((m, n)\) value in the \(i^{th}\) channel of the feature, the nonzero elements of the corresponding column can be present as:

\[
\mathbf{M}_{\text{conv},x,y} = k_{a+1,b+1},
\]

\[
x = H_f W_i x + W_f m + n,
\]

\[
y = H_i W_j x + W_i (m + a) + n + b
\]

where \(a, b \in \{-1,0,1\}, j \in \{0,1,2\}\) (image channel), \(H_i, W_i, H_f, W_f\) mean the height and width of the input image
and the feature, \( k_{a+1,b+1} \) stands for the \((a+1, b+1)\) element in the convolution kernel, \( x \) stands for the column index, and \( y \) stands for the pixel index.

To combine the inverse linear transformation with the convolutional layer, the original inverse matrix needs to be reconstructed in to \( M_{\text{line}} \), of which the number of columns is equal to the number of rows of \( M_{\text{conv}} \), to be able to multiply with \( M_{\text{conv}} \). The reconstruction and the replacement of the inverse linear matrix is shown in Figure 5. In this way, the convolutional layer and the inverse linear layer can be combined into the augmented convolutional layer \( M_{\text{aug-conv}} = M_{\text{line}} M_{\text{conv}} \) that can be applied to extract useful features from the morphed image.

Though the data morphing and augmented convolutional layer are illustrated using images as an example, the application of our privacy-preserving method is not limited to image classification scenarios. This method can be applied to any neural network model with a convolutional layer as a feature extractor.

### Analysis

#### Threat Model

The main purpose of Morphed Learning is to prevent the server provider from accessing the original data. In this case, we refer an ill-intended server provider as an attacker, who tries to break Morphed Learning’s data morphing and retrieve the original data. The threat model of Morphed Learning falls into two categories depending on the information accessible to the attacker: Limited Knowledge Attack (LKA) and Full Knowledge Attack (FKA). In an LKA, the attacker only has the access to the morphed data and their labels. This scenario is most common in real-world applications, where the aug-conv layer is delivered to the server provider in binary executable files, and in this way, the details of aug-conv layer remains a black-box to the server provider. On the other hand, in an FKA the attacker has the access to the details of aug-conv layer in additional the labels. This scenario happens when the attacker manages to reverse-engineer the binary file or the application has open-source requirements.

For the FKA, if the size of the transformation matrix is not big enough, the server provider can easily retrieve the inverse matrix and the convolutional layer using training methods. To prevent this threat, the matrix of the linear transformation in the FKA scenario needs to be the max-size matrix to prevent the data provider from getting a correct inverse matrix. The max-size matrix is a \( \eta \times \eta \) matrix with random nonzero elements, where \( \eta \) is the number of elements in an original data. The original data will be resized into a \( 1 \times \eta \) tensor and multiplied by the transform matrix. In the LKA scenario, without the leakage of the type and size of the original transformation, the server provider cannot gain the inverse matrix, and transformation matrix in a smaller size (in comparison with the max-size matrix) can be applied to reduce computational and transmission cost while the original is still securely protected. The mathematical proof and the practical demonstration of the security of both FKA and LKA will be shown in the next subsection.

#### Security Analysis

In the FKA, there are two possible ways for the server provider to try to retrieve the original data. Since the aug-conv layer will be sent to the server provider, the inverse matrix may be gained directly through the aug-conv layer. That means the server provider may try to solve the equations that represent the construction of the aug-conv layer to regain the inverse matrix and the convolutional layer. The construction of the aug-conv layer can be present as \( A_x \) stands for the \( x^{th} \) column in \( M_{\text{aug-conv}} \), and the column is corresponding to the \((m, n)\) value in the \( i^{th} \) channel of the feature:

\[
A_x = \sum_{j=0}^{C_i} \sum_{a=-1}^{W_k-1} \sum_{b=-1}^{H_k-1} k_{i,j,a+1,b+1} T_{F,PI}^{-1}
\]  

(5)

where \( T_{F,PI}^{-1} \) means the \( PI^{th} \) column of the max-size inverse matrix in the FKA scenario, \( W_k \) and \( H_k \) stand for the width and height of the convolution kernel, and \( x \) and \( y \) are presented in equation (4).

For the first feature channel, \( i = 0 \) and \( CI \in [0, H_f W_f] \). There are \( H_f W_f \) independent equations and \( C_1 H_f W_f + C_1 \kappa^2 \) unknowns variables, where \( \kappa \) represents the size of a convolutional kernel, and \( C_1 (\geq 1) \) stands for the number of channels of the original data. As a result, there are not enough independent equations for the server provider to solve out the unknowns variables. If another channel of the feature is taken into account, for the \((m, n)\) value in the \((i + 1)^{th} \) channel of the feature the equation then change in to:

\[
A_{x+H_f W_f} = \sum_{j=0}^{C_i} \sum_{a=-1}^{W_k-1} \sum_{b=-1}^{H_k-1} k_{i,j,a+1,b+1} T_{F,PI}^{-1}
\]  

(6)

It seems that there are \( H_f W_f \) new equations, and only \( C_1 \kappa^2 \) new unknown variables. However, only \( C_1 \kappa^2 \) of the new equations are independent, for there are only \( C_1 \kappa^2 \) new unknowns variables. Therefore including new channels into the system of equations does not help the data provider to solve out the variables. That means the whole system of equations has infinite solutions and cannot be solved by the data provider, and the precise inverse matrix stays unknown. This explains why a max-size transformation matrix ensures the security of Morphed Learning.

Other than solving the equation directly, the server provider may try to use the fitting method to train an inverse max-size matrix and a convolutional layer. However, as proven above, the infinite solutions of the system of equations mean there are infinite global optimal points of the training process, so the trained inverse matrix cannot be the original one.

In the LKA, with a smaller transformation matrix, the original data are still well protected under the two retrieve method discussed in the FKA scenario. Without the values of the element in the aug-conv layer, the server provider cannot gain the inverse matrix by solving the equation group. On the other hand, if the server provider tries to retrieve the inverse matrix by the training method, missing the information of the size of the inverse matrix, it needs to use a max-size matrix in the training process, for a max-size matrix can
In the LKA scenario, a small inverse matrix means only a small portion of elements in $M_{\text{aug-conv}}$ are nonzero. As is shown in Figure 2 for one channel in the input image, only the weights of the shadowed pixels are nonzero. For a $3 \times 3$ convolution kernel and a $32 \times 32$ inverse matrix, the nonzero weight pixels contain three rows, and the weights of each row can be present as:

$$W_{m+a} = \sum_{b=-1}^{1} k_{a+1,b+1} T^{-1}_{L,n+b}, \ a \in \{-1,0,1\} \quad (7)$$

where $T^{-1}_{L,n+b}$ means the $n+b$ column in the inverse matrix in the LKA scenario. So, for every value in the feature, the positions of the nonzero elements are fixed in the corresponding column. This means only the nonzero elements need to be compiled into binary executable files and sent to the server provider. While training and testing, only the weight sum of the nonzero weights need to be calculated. Take the example that the size of an input image is $C_i \times W_i \times H_i$, the feature size is $C_f \times W_f \times H_f$ and convolution kernel size is $W_k \times W_k$, then the transmission cost for the aug-conv layer is:

$$P_{\text{trans}} = C_f W_f H_f W_k C_i \theta \quad (8)$$

where $P_{\text{trans}}$ means the transformation cost, and $\theta$ stands for the size of the data type used by the aug-conv layer. For images from CIFAR-10 and the aug-conv layer combined with the inverse matrix and the first convolutional layer of the VGG16 model, the transmission cost is approximately equal to 300 features or 6000 images. For the whole dataset, it only increases 10% of transmission cost. For a bigger dataset such as ImageNet, this method will become more cost-efficient.

As for the computational cost, we can estimate the cost as the quantity of add and multiplication involved in the computation of the aug-conv layer, then we get:

$$P_{\text{ac.add}} = P_{\text{ac.mul}} = C_f W_f H_f W_k W_i C_i \quad (9)$$

For a VGG-16 model, the extra computational cost of the aug-conv layer is equal to 10 times of the first convolutional layer and $\frac{1}{4}$ of the second convolutional layer from the original model. For the whole network, the computational cost only increases by 5%. So this data transform and accuracy restore method in the LKA scenario does not cost significant extra transmission and computational cost.

In the FKA scenario, however, computational cost and transmission are larger than that in the LKA scenario. There is no zero element in the aug-conv layer in the FKA scenario, and all the parameters need to be transmitted and calculated. Therefore, the transmission cost increases by 128%, and computational cost increases by 57% in the FKA scenario. Even though, this method is still much more efficient than previous privacy preserving approaches. Specific efficiency comparison of this method and other approached will be listed in the next section.

**Experiment**

To demonstrate that our method can keep the accuracy while the data privacy is preserved, several experiments using different datasets have been performed.
The aug-conv layer is combined using the inverse matrix of the transform matrix and the first convolutional layer of a pre-trained VGG-16 model trained on the CIFAR-10 dataset. To prove that the feature extraction method is correct, the experiment that extracted features from transformed CIFAR-10 data using aug-conv layer and trained the rest layers of the VGG-16 model was performed. When the VGG-16 model is trained by the morphed data, the test accuracy can only reach 60.5%. With the aug-conv layer, the test accuracy can be restored back to 87.6%, which is approximately the same with the test accuracy of the original dataset and the original model. However, in real use, the aug-conv layer need to extract features from data that is different from the pre-train data, so the experiment using CIFAR-100 dataset was carried out as well, and the test accuracy can be restored back to 59.9% from 28.7% by the aug-conv layer. The results of both experiments are shown in Figure 8. Obviously, after replacing the first layer of VGG-16 with the aug-conv layer, test accuracy can be perfect restored using different training and testing dataset.

Besides accuracy, computational and transmission costs of this method were assessed and compared with other existing approaches, and the result is shown in Table 1. It has been clearly shown in Table 1. that our work has outperformed all other previous work in a comprehensive manner, and is practical in real privacy-preserving deep learning scenarios.

**Conclusion and Future Work**

In this work, a privacy-preserving deep learning method without a significant increment in transmission and computational cost has been proposed. The original data has been transformed using randomly generated invertible matrix before being transmitted to the server. Training the model directly using the transformed data will lead to a severe influence on the accuracy. The augmented convolution layer is designed to restore the training and testing accuracy without leaking the original data. It is combined with the inverse matrix and the first pre-trained convolution layer of the model. By using the combined aug-conv layer, equivalent features can be extracted from the morphed data and the test accuracy on morphed data can be restored in this way.

| Existing methods | Accuracy decrement | Computational cost increment(rate) | Transmission cost increment(rate) | Potential threat |
|------------------|--------------------|------------------------------------|----------------------------------|-----------------|
| Our work (LKA)   | 0(cifar10, cifar100) | 0.05                              | 0.10                             | None            |
| Our work (PKA)   | 0(cifar10, cifar100) | 0.56                              | 1.28                             | None            |
| Collaborative deep learning (best performance situation) | 0 (Mnist) | n (number of collaborators) | 471n | Poisoning attack by malicious collaborators |
| Deep learning with differential privacy | 0 to 8% (Mnist) | Not clear | 0 | Original data remains unencrypted |
| Deep learning with adaptive differential privacy | 0 to 10% (Mnist) | Not clear | 0 | Data remains recognizable after noise adding |
| Homomorphic encrypted learning | 0 | 16400 | 1.00E+06 | None |
| Feature transmitting | 0(without feature processing) | 0 | > 5(depends on network structure) | Server has inverse operation to retrieve original data |

Table 1: Performance comparison with relevant prior works.

Figure 8: Test accuracy comparison.

It has been mathematically proven that the server provider cannot retrieve the original data by reverse-engineering the aug-conv layer. In addition, there are no significant computational and transmission overhead introduced. The advantages above have made Morphed Learning a practically useful method to address privacy violation concerns for deep learning. Morphed Learning is based on the convolutional operation, thus it should work for deep convolutional networks regardless of their application. However, only computer vision tasks are presented for evaluation in this works, as they provide more intuitive elaboration. In the future, we look forward to extending our experiment to other applications.

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