A Comparative Study of GPA Calculation Methods in the Academic Credit System

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Abstract. Academic credit system (ACS) is an instructional management system in higher education. Based on this system, grade point average (GPA) is the main measurement to summarize undergraduates’ overall performance. The paper aims to compare commonly used GPA calculation methods, then analyzes the different GPA calculation results based on the unique data of some university. In this study, experimental results show that GPA calculation method based on the standard scores is the most scientific and reasonable. At last, the paper provides information and advice to use GPA for acknowledgement, awards, and as partial evidence for admission to other colleges and universities.

Introduction

The academic credit system (ACS) was originated from the elective courses of Germany in the 18th century and freedom in learning is the basic concept[1]. This system affects many universities around the world, especially in the late 19th century, many American universities begin to accept this elective courses system. In 1872, the president of Harvard University established the academic credit system and began to implement it in Harvard University, which uses a certain amount of credits as an indicator to measure students' learning [2]. After that, many universities in the world try to adopt ACS. Although ACS has many advantages, it also has an obvious shortcoming, that is, it can't effectively reflect students’ learning quality. The grade point average (GPA) becomes one of most important elements in ACS, and it uses grade points and credits to measure a student's study completion and achievement. Because it is simple and easier to compare, GPA is widely used in many universities [3]. The main objective of the present study is to describe different GPA calculation methods in detail and compare the effect of different GPA calculation methods. At last, this study provides information and advice to use GPA for acknowledgement, awards, and as partial evidence for admission to other colleges and universities.

Calculation Methods of GPA

Calculation of a student’s GPA in colleges or universities usually involves a process as follows: (i) Set GP of each course (ii) Calculate weighted average of all GPs and credits.

Set GPA of Each Course

No matter what scale, a transition is made from a marks (letter or percentage) to grade point (GP). There are three frequently-used transition approaches: (i) Section-Point Mapping (SPM) (ii) Linear Transformation (LT) (iii) Standard Score Notation (SSN) [4].

Section-Point Mapping (SPM). This method adopts a piecewise function to map the marks to grade point. Different colleges or universities have different criterions, so the piecewise function is also different. Most Chinese universities use Peking University standard [5], as shown in Table 1. Most American universities WES criterion, as shown in Table 2. Besides, the scale of United Kingdom higher education is a little different, which is illustrated in Table 3. On the other side, India
has different criterion, which uses 10 grade points, and an example of Central University of Bihar[6], is described in Table 4.

Table 1. Peking University Criterion (PUC).

| Percentage | 90-100 | 85-89 | 82-84 | 78-81 | 75-77 | 72-74 | 68-71 | 64-67 | 60-63 | 0-59 |
|------------|--------|-------|-------|-------|-------|-------|-------|-------|-------|------|
| GPA        | 4.0    | 3.7   | 3.3   | 3.0   | 2.7   | 2.3   | 2.0   | 1.5   | 1.0   | 0    |

Table 2. WES Criterion (WESC).

| Letter | A       | A-     | B     | B+    | B-    | C+    | C    | C-    | D     | F    |
|--------|---------|--------|-------|-------|-------|-------|------|-------|-------|------|
| GPA    | 4.0     | 3.67   | 3.33  | 3.0   | 2.67  | 2.33  | 2.0  | 1.67  | 1.0   | 0    |

Table 3. UK Criterion.

| Grade | Mark | Grade Point |
|-------|------|-------------|
| A+    | >= 75 | 4.25 |
| A     | 71-74 | 4.0 |
| A-    | 67-70 | 3.75 |
| B+    | 64-66 | 3.50 |
| B     | 61-63 | 3.25 |
| B-    | 57-60 | 3.0 |
| C+    | 54-56 | 2.75 |
| C     | 50-53 | 2.50 |
| C-    | 48-49 | 2.25 |
| D+    | 43-47 | 2.0 |
| D     | 40-42 | 1.50 |

Table 4. India Criterion.

| Grade | Mark | Grade Point |
|-------|------|-------------|
| A+    | >= 85 | 10 |
| A     | 75-84 | 9 |
| A-    | 65-74 | 8 |
| B+    | 60-64 | 7 |
| B     | 55-59 | 6 |
| B-    | 50-54 | 5 |
| C     | 40-49 | 4 |
| F     | <40  | 0 |

Actually, the smaller the range of section function is, and the higher the mapping precision is, so the final result is more approximate to real assessment. But, after conversion, this kind of methods always exist deviation.

**Linear Transformation (LT).** Linear transformation is commonly-used method in percentage scale. Assume that there are n courses, $g_i$ is the grade point of the course $i$, and $s_i$ is the percentage score of the course $i$, so $g_i$ can be calculated through Eq.1. When the percentage is 60, the corresponding grade point is 1.0. When the score reaches to 100, the maximal grade point is 5.

$$g_i = \begin{cases} \frac{s_i}{10} - 5 & 60 \leq s_i \leq 100 \\ 0 & s_i < 60 \end{cases}$$

(1)

Since the top grade point of most universities is 4, the Eq.1 can be changed to Eq.2, as follows:

$$g_i = \begin{cases} 4 & s_i > 90 \\ \frac{s_i}{10} - 5 & 60 \leq s_i < 90 \\ 0 & s_i < 60 \end{cases}$$

(2)

Compared to the SPM method, LT is more scientific and reasonable. Because different teachers have different scales, some teachers usually evaluate higher marks, but other teachers maybe evaluate lower marks.

**Standard Score Notation (SSN).** Standard score is calculated in the scope of certain students groups, which is relative transition scale. Through calculating the mean and variance of a course, the
percentage score is transformed to standard score which obeys the standard normal distribution[7]. Assume that \( s_i \) is the percentage score of the course \( i \), \( \mu_i \) is the mean of the course \( i \), and \( \sigma_i \) is the variance of the course \( i \), the standard score \( y_i \) of the course \( i \) is calculated as Eq.3.

\[
y_i = \frac{s_i - \mu_i}{\sigma_i}
\]  
(3)

Assume that \( n \) is the number of students who are learning course \( i \), so \( \mu_i \) and \( \sigma_i \) can be calculated as Eq.4 and Eq.5 respectively.

\[
\mu_i = \frac{1}{n} \sum_{j=1}^{n} s_i
\]  
(4)

\[
\sigma_i = \sqrt{\frac{1}{n} \sum_{j=1}^{n} (s_i - \mu_i)^2}
\]  
(5)

Because the standard score contains many decimal digits and sometimes it is negative, which is not conform to commonly-used mark notation. In order to avoid these defects, the standard score can be transformed to percentage scale by using T model[7], as shown in the Eq.6.

\[
s_i' = \sigma_i y_i + \mu_i
\]  
(6)

Although the calculation of standard score is complicated and the computation cost is large, it can be completed by computer. After calculating standard score, grade point can be obtained by using SPM or LT method. Applying standard score may adjust a student’s grade point reasonably, which doesn’t change the rank of the student in the same group.

**Calculation Methods of GPA**

When calculating GPA, the learning quality (grade points) and learning quantity (credits) can be combined to assess a student’s overall learning outcome. GPA is a weighted average of their grade points and credits. Assume that \( g_i \) is the grade point of the course \( i \), \( c_i \) is the credit of the course \( i \), and the number of courses are \( m \), GPA is calculated as Eq.7.

\[
GPA = \frac{\sum_{i=1}^{m} g_i \cdot c_i}{\sum_{i=1}^{m} c_i}
\]  
(7)

Some universities calculate the weighted average score (WAS) by using their percentage scores and credits, then transform the weighted average to grade point according to some scale. Assume that \( s_i \) is the percentage score of the course \( i \), \( c_i \) is the credit of the course \( i \), and the number of courses are \( m \), WAS is calculated as Eq.8.

\[
WAS = \frac{\sum_{i=1}^{m} s_i \cdot c_i}{\sum_{i=1}^{m} c_i}
\]  
(8)
Experimental Results and Analysis

Dataset

This work chooses real data from some Chinese university. The data is based on percentage scores, which includes 1,055 junior undergraduates (Grade 2014) and 2004 sophomore undergraduates (Grade 2015). This study adds up the last three year courses for grade 2014, and the last two years for grade 2015.

Comparison of Different GPA Calculation Results

This study combines different GP methods and CGPA methods based on original scores and standard scores respectively. Different methods can be illustrated in Fig.1 and Fig. 2.

Compare the Percentage of Distribution of GPA. From Fig.3 and Fig.4, the results demonstrate that the ratio of grade 2015's GPA which is greater than 3.0 is higher than that of grade 2014's. Especially, when GPA is between to 3.0 to 3.3, all results of grade 2015 are more than 18%. Moreover some are near to 30%, such as SPM_GPA (28.49%) and WAS_SPM (26.60%). But in the same section, results of grade 2014 are just about 15%, and only WAS_SPM(22.75%) and SSN_WAS_SPM(25.12%) are more than 20%. Through observation, these four methods all adopt SPM method, which shows that SPM process is beneficial to higher marks. Moreover, the proportion which is less than GPA 2.0, grade 2015’s is lower than 2014’s, which illustrates grade 2015's learning performance is overall superior to grade 2014's, but grade 2015's polarization is more obvious. However, shown in Fig. 4, all results using SSN are mainly between GPA 2.3-3.0, nearly more than 60%, which shows that standard score can effectively reveal actual situation of learning outcomes.

Figure 1. Methods Based on Original Scores.

Figure 2. Methods Based on Standard Scores.

Figure 3. Percentage of distribution of grade 2014’s GPA.
Figure 4. Percentage of distribution of grade 2015’s GPA.

**Compare the Number of Distribution of GPA.** In order to compare these methods, some points are considered: i) GPA is 2.0, because some universities or colleges award a bachelor degree on the condition that GPA must be great than 2.0. ii) GPA is 3.0, because some universities provide offer for undergraduate program admission or graduate admission only when GPA is greater than 3.0. As shown in Table 5 and Table 6, the number got from SSN_WAS_SPM method is least, just 102 (grade 2014) and 128 (grade 2015) respectively. On the other side, WAS_LT method produces the most of students, 211 (grade 2014) and 247 (grade 2015) respectively. When calculating GPA greater than 3.0, SSN_WAS_SPM method has the maximum, 438 (grade 2014) and 1014 (grade 2015) respectively. But WAS_LT method gets the minimum, just 288 (grade 2104). SPM_GPA method gets the minimum, just 677 (grade 2015), and WAS_LT method is also small, only 688 (grade 2015). Experimental results show that WAS_LT method will result in lower GPA. Besides, SSN_WAS_SPM method can get more higher GPA.

### Table 5. The number of distribution of grade 2014’s GPA.

| ----- | 4   | 3.7-4.0 | 3.3-3.7 | 3.0-3.3 | 2.7-3.0 | 2.3-2.7 | 2.2-2.3 | 1.5-2.0 | <1.5 |
|-------|-----|---------|---------|---------|---------|---------|---------|---------|------|
| WAS_SPM | 1  | 59      | 107     | 240     | 180     | 169     | 153     | 93      | 54   |
| SPM_GPA | 0  | 4       | 111     | 180     | 182     | 254     | 139     | 133     | 52   |
| WAS_LT  | 1  | 14      | 112     | 161     | 178     | 234     | 144     | 139     | 72   |
| LT_GPA  | 1  | 16      | 119     | 168     | 177     | 253     | 137     | 133     | 51   |
| SSN_WAS_SPM | 1  | 61      | 111     | 265     | 188     | 184     | 143     | 70      | 32   |
| SSN_SPM_GPA | 0  | 9       | 134     | 196     | 213     | 252     | 130     | 99      | 22   |
| SSN_WAS_LT | 1  | 14      | 114     | 171     | 202     | 249     | 155     | 106     | 43   |
| SSN_LT_GPA | 1  | 16      | 145     | 186     | 203     | 249     | 133     | 105     | 19   |

### Table 6. The number of distribution of grade 2015’s GPA.

| ----- | 4   | 3.7-4.0 | 3.3-3.7 | 3.0-3.3 | 2.7-3.0 | 2.3-2.7 | 2.2-2.3 | 1.5-2.0 | <1.5 |
|-------|-----|---------|---------|---------|---------|---------|---------|---------|------|
| WAS_SPM | 8  | 148     | 274     | 533     | 373     | 290     | 209     | 82      | 87   |
| SPM_GPA | 0  | 20      | 266     | 391     | 418     | 468     | 428     | 155     | 61   |
| WAS_LT  | 8  | 52      | 261     | 367     | 415     | 445     | 209     | 140     | 107  |
| LT_GPA  | 8  | 52      | 261     | 367     | 415     | 464     | 225     | 154     | 58   |
| SSN_WAS_SPM | 8  | 148     | 287     | 571     | 418     | 275     | 169     | 66      | 62   |
| SSN_SPM_GPA | 0  | 38      | 311     | 431     | 437     | 481     | 170     | 106     | 30   |
| SSN_WAS_LT | 8  | 52      | 266     | 391     | 439     | 472     | 188     | 114     | 74   |
| SSN_LT_GPA | 11 | 62      | 304     | 430     | 422     | 471     | 168     | 108     | 28   |

**Summary**

It is very meaningful to study the determination of universities’ GPA because it plays a significant role in assessing an undergraduate’s complete academic achievements and future potential such as awarding of scholarship, awarding of bachelor degree, applying of graduate programs and future
performance in the occupations. Since different universities have different requirements, this paper aims to describe all these different methods in detail and compare their calculation results. Based on two real datasets, experimental results find out the advantages and disadvantages of different methods. Firstly, compared to LT, SPM is beneficial to higher marks. For example, as long as the score equals to and greater than 75, the GP can be set as 2.7. Secondly, when calculating final result, WAS is superior to GPA method. Adopting weighted average of percentage scores and credits, conversion errors can be avoided obviously. Lastly, when percentage scores are converted to standard scores, all methods can obtain better results, which demonstrates that SSN can effectively reflect difference of learning outcome, such GPA is more scientific and reasonable. Moreover, SSN_WAS_SPM surpass other methods, which can obtain least number of lower GPA and maximum number of higher GPA. Moreover, GPA results can provide practical suggestions for education management. For example, when finding some student’s GPA is less than 2.0, the education management department should give according warning and strategies. Besides, some weights should be considered in calculating GPA, for example, the coefficient of difficulty or importance of some course, which should be researched from theory and practice in the future.

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