Repeated Imaging of Massive Black Hole Binary Orbits with Millimeter Interferometry: Measuring Black Hole Masses and the Hubble Constant

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Abstract

Very long baseline interferometry (VLBI) at millimeter (mm) wavelengths is being employed to resolve event horizon-scale structure of the environment surrounding the Milky Way black hole at an angular resolution of a few tens of microarcseconds. The same approach could also resolve the orbital separation of a population of massive black hole binaries (MBHBs). Modeling the inspiral of binaries due to gravitational-wave emission and gas and requiring binary orbital periods of less than 10 yr, we estimate that there may exist \( \sim 100 \) resolvable MBHBs that are bright enough to be observed by mm-wavelength VLBI instruments over the entire sky at redshifts \( z \lesssim 0.5 \). We propose to search for these resolvable MBHBs by identifying binaries with the required orbital separations from periodic quasar light curves identified in optical and near-IR surveys. These periodic-light-curve candidates can be followed up with radio observations to determine their promise for observation with VLBI at mm wavelengths. The VLBI observations over the timescale of a binary orbit can allow unprecedented precision in the measurement of the binary mass, to within 30%. In combination with an independent binary mass measurement, VLBI observation would allow a novel \( \mathcal{O}(10\%) \) measurement of the Hubble constant, independent from those currently proposed and employed.

Key words: accretion, accretion disks – distance scale – gravitational waves – quasars: supermassive black holes – submillimeter: galaxies

1. Introduction

The merger of galaxies harboring massive black holes (MBHs; Kormendy & Richstone 1995; Kauffmann & Haehnelt 2000; Ferrarese & Ford 2005; Kormendy & Ho 2013) can lead to the formation of a compact massive black hole binary (MBHB) at the center of the newly formed galaxy (Colpi & Dotti 2011). For moderate MBH mass ratios (\( \gtrsim 1:100 \)), dynamical friction can bring the MBHs together on the galactic dynamical timescale to form a hard binary with an orbital separation of order parsecs (pc; e.g., Callegher et al. 2011; Mayer 2013; Dosopoulou & Antonini 2017). Dynamical friction becomes inefficient at hardening the binary further at smaller orbital separations, and alternative mechanisms for removing binary angular momentum and energy must be employed if the binary is to shrink to \( \lesssim 0.01 \) pc separations, where gravitational radiation will bring the binary to coalescence (e.g., Begelman et al. 1980; Merritt & Milosavljević 2005).

Multiple mechanisms are capable of shrinking the binary orbital separation through this intervening stage, solving the so-called final-parsec problem (Milosavljević & Merritt 2003). Possible solutions include interaction of the binary with a gas disk (Gould & Rix 2000; Armitage & Natarajan 2002; for recent work, see Tang et al. 2017 and references therein), a massive perturber (Goicoechea et al. 2017), or nonaxisymmetric stellar distributions that allow a high interaction rate between stars and the binary (see Gualandris et al. 2017 and references therein). However, to truly understand the mechanisms that drive MBHBs to merge in galactic nuclei, we must find observational tracers of the MBHB population, probing different stages of MBHB evolution. Based on reliable tracers of the MBHB population, the relative fraction of MBHBs at different orbital separations can be translated into the rates at which the binaries are driven together during these stages and hence elucidate the mechanisms driving orbital decay (see, e.g., Haiman et al. 2009).

These observational tracers naturally fall into the realm of multimessenger astronomy. The earliest stages of MBHB formation, where the MBHs have not yet hardened into a binary, have been captured by direct electromagnetic (EM) imaging; two distinct active galactic nuclei (AGNs) with projected separations of tens to thousands of pc have been identified in radio, optical, and X-ray wavelengths (Komossa 2006; Rodriguez et al. 2006; Burke-Spolaor 2011; Fabbiani et al. 2011; Civano et al. 2012; Dotti et al. 2012; Blecha et al. 2013; Comerford et al. 2013; Woo et al. 2014). Low-frequency gravitational radiation will probe the final stages of an MBHB’s life. Particularly, the stochastic background of gravitational waves (GWs) detectable by the Pulsar Timing Arrays (PTAs; Lommen 2012; Hobbs 2013; Kramer & Champion 2013; Manchester & IPTA 2013; McLaughlin 2013; Shannon et al. 2015) will be sensitive to environmental effects determining the binary eccentricity and lifetime at \( \sim \) nHz orbital frequencies (Kelley et al. 2017b), probing the late inspiral of the most massive MBHBs.

The early dynamical friction–driven and late gravitational radiation–driven phases of MBHB evolution are separated by the sub-pc orbital separation regime. At sub-pc orbital separations, it is likely that gas will accompany the MBHB (e.g., Barnes & Hernquist 1996; Barnes 2002), not only aiding in the resolution of the final-pc problem but also providing the potential for bright EM signatures. However, identification of an EM signature with a compact MBHB must surmount the obstacle of disentangling signatures of an accreting MBHB from those of a single accreting MBH. Finding unique EM identifiers of accreting compact MBHBs has been the subject of numerous theoretical studies and corresponding observational searches. Possible signatures...
could arise from emission-line dynamics (e.g., Bogdanović et al. 2009a; Shen & Loeb 2010; Tsalmantza et al. 2011; Eracleous et al. 2012; Decarli et al. 2013; McKernan et al. 2013; Shen et al. 2013; Liu et al. 2014b, 2016), tidal disruptions by a binary (e.g., Liu et al. 2009; Stone & Loeb 2011; Coughlin et al. 2017), peculiar jet morphology (e.g., Gower et al. 1982; Roos et al. 1993; Romero et al. 2000; Merritt & Ekers 2002; Zier & Biermann 2002; Kun et al. 2014, 2015; Kulkarni & Loeb 2016), orbital motion of an unresolved radio core observed with very long baseline interferometry (VLBI; Sudou et al. 2003, similar in goal to the ideas discussed here), or periodic emission and lensing events from quasars (Hayasaki & Mineshige 2008; Haiman et al. 2009; D’Orazio et al. 2013, 2015a; Farris et al. 2014; D’Orazio & Di Stefano 2017). Observational searches motivated by these studies have identified a number of individual MBHB candidates (Valtonen et al. 2008; Bogdanović et al. 2009b; Liu et al. 2014a; Graham et al. 2015a; Li et al. 2017), and recently, time-domain searches for periodically varying quasars have identified ∼140 MBHB candidates (Graham et al. 2015b; Charisi et al. 2016).

In order to use the growing population of MBHB candidates to investigate the drivers of MBHB evolution, further vetting of these candidates must be employed. For this to happen, new ways of identifying compact MBHBs in conjunction with existing methods must be developed. In this work, we suggest the use of VLBI at millimeter (mm) wavelengths to directly image the sky-projected orbital path of sub-pc separation MBHBs.

The outline of this paper is as follows. In Section 2, we present our criteria for tracking an MBHB orbit with VLBI. In Section 3, we present our calculation of the population of resolvable MBHBs and the resulting GW background (GWB) due to this population. We present the results in Section 4. In Section 4.1, we present our main results, while Sections 4.2 and 4.3 provide a detailed analysis of the dependence of our results on the model parameters (the reader primarily interested in the main results and implications may wish to skip Sections 4.2 and 4.3). In Section 5, we discuss the application of MBHB imaging to inferring the MBHB population and measuring the binary mass and Hubble constant. In Section 6, we conclude.

2. Imaging the Orbit of a Compact MBHB

To image an MBHB orbit, we require (i) that the binary orbital separation be larger than the minimum spatial resolution and the size of the emission region at the observing wavelength, and (ii) that both binary components be bright enough to be detectable independently or one component be bright and a calibrator source be nearby. We additionally impose that the binary orbital period be shorter than some maximum baseline timescale, P_{base}. By observing an entire orbit, we ensure that the binary nature of the source can be determined.

The first criterion can be met by mm-wavelength VLBI. The VLBI experiments with maximum baselines the size of the Earth can reach diffraction-limited resolutions on the order of 20 μas when observing in ≤mm wavelengths and subdiffraction-limited (down to 4 μas) resolutions using novel image reconstruction techniques (Akiyama et al. 2017a, 2017b).³ Astrometric tracking of a source can reach 1 μas precision (Broderick et al. 2011). At 1 Gpc, a 10 μas resolution corresponds to a physical binary orbital separation of 0.05 pc and an orbital period of only 10 yr for the most massive, 10^{10} M_{\odot}, binaries. Hence, the first criteria can be satisfied because radio-loud AGNs, which may harbor close MBHBs, are also bright at mm and sub-mm wavelengths (e.g., Elvis et al. 1994).

To determine the validity of the second criterion, we estimate the size of the mm-wavelength emission region. Binaries for which this region is smaller than the separation will be viable targets for sub-mm VLBI imaging, otherwise the photosphere of the mm-emission region could mask the resolvable binary components or emanate from a region in a jet that is larger than the orbital separation. While emission regions that are larger than the binary orbital separation may still provide evidence for a binary via photometric variability or periodically changing geometry, their identification with a binary is possibly more complicated than in the case of two distinguishable sources envisioned here.

Observationally, we can probe the size of the mm-to-sub-mm emission region from variability measurements. Specifically, if the emission region is smaller than the binary separation, then in the most conservative case, causality requires that the light-travel distance over the duration of the shortest mm-variability timescales be smaller than the binary orbital separation,

\[
\frac{c \Delta t_{\text{var}}}{(1 + z)} \leq \theta_{\text{min}} D_A(z) \leq \left( \frac{\sqrt{G M_{\text{base}}}}{2\pi (1 + z)} \right)^{2/3},
\]

where the middle term is the smallest possible binary separation and the rightmost term is the largest binary separation for the maximum allowed binary orbital period P_{base}. Here D_A(z) is the angular diameter distance of the MBHB at redshift z.

Equation (1) requires that the observed mm-variability timescales satisfy \(\Delta t_{\text{var}} \lesssim 1\) day (\(\theta_{\text{min}}/1\) μas) to resolve all possible MBHBs at \(z \geq 0.02\) or \(\Delta t_{\text{var}} \lesssim 54\) days \(M_{\odot}^{1/3} P_{10}^{2/3}\) to resolve only the longest period and most common (see next section) MBHBs at \(z > 0.02\). Here \(M_\odot\) is the total binary mass in units of \(10^9 M_\odot\), and \(P_{10}\) is the maximum baseline period in units of 10 yr.

Recent studies have employed the SMA calibrator database to characterize AGN variability in the sub-mm regime (Strom et al. 2010; Bower et al. 2015). They quantified the variability timescale by the damped random-walk correlation timescale (MacLeod et al. 2010), finding that the sub-mm variability of these brightest sources has characteristic timescales of \(~1–1000\) days. Notably, Bower et al. (2015) found that the low-luminosity AGNs (LLAGNs) exhibit shorter variability than other blazars and AGNs in the SMA calibrator sample. Furthermore, the characteristic timescale for the variability of these sources appears to track a multiple of the MBH innermost stable circular orbit (ISCO), suggesting that mm emission from LLAGNs tracks the regions very close to the MBH.

That the mm emission from LLAGNs tracks event horizon scales is consistent with standard models for synchrotron emission from jets (Blandford & Königl 1979). In these models, the BH launches a jet, and the mm emission is generated by synchrotron radiation at shocks along the length

³ This fact is a leading driver behind the Event Horizon Telescope, which is currently being employed to resolve Schwarzschild-radius-scale structure of the environment surrounding the Milky Way BH (Doeleman et al. 2008).
of the jet. There is a smallest distance along the length of the jet from which optically thin, bright synchrotron radiation can be emitted. This minimum size scales with the bolometric jet luminosity. Because the jet is launched from a small region that is bound to the BH, the mm emission will necessarily track the BH orbit, regardless of its size compared to the Roche radius. Hence, the size of the mm-emission region need not be truncated close to the BH for VLBI orbit tracking to be viable. Rather, because we wish to consider systems for which the mm-emission regions emanating from the BHs are clearly distinguishable, we compare the size of the emission region with the binary separation. We compute the size of the mm-emission region as a function of AGN luminosity and Eddington ratio (see Appendix B) and so determine which MBHBs have mm-emission regions larger than their binary separation. We exclude these from the population estimates below.

The final criterion, that both binary components be bright, is not only a sensitivity issue (which we address in the next section) but a matter of calibration necessary for VLBI. If only one binary component is bright enough to detect, its orbital path cannot be tracked without a bright source within ~1° for phase reference (e.g., Broderick et al. 2011); i.e., the required ~μas astrometric precision is only possible via relative astrometry. We can make a crude estimate for the probability of finding a bright source within 1° of the target source from the number of ALMA calibrator sources. Taking that there are about 2000 adequate calibrator sources that could be used as phase references, we can estimate a lower limit on the alignment probability by assuming that these calibrators are distributed isotropically on the sky. Then the probability of finding a suitable phase reference within 1° of the source is 2000/(41,252 deg²) ≈ 0.05. This is not zero, but not large enough to be reliable. If, instead, the number of calibrators can be increased by a factor of 10, the probability of finding a nearby phase reference is considerable: 50%.

In the case that both binary components are bright in mm wavelengths, the problem is eliminated, as each component can phase reference its companion. Because we do not know the fraction of binaries for which both components are mm-bright and how this depends on binary parameters, AGN type, or other unknowns, we parameterize this uncertainty with \( f_{\text{bin}} \).

3. A Population of Resolvable MBHBs

3.1. Calculation

We next estimate the number of MBHBs that are emitting bright, mm-wavelength radiation due to accretion and that have an orbital separation large enough to be resolvable by an Earth-sized VLBI array but small enough to have a period observable in a human lifetime.

We assume that a fraction \( f_{\text{bin}} \) of mm-bright AGNs are synonymous with accreting MBHBs. While this fraction is not robustly constrained, a number of theoretical arguments imply that its value may be of order unity (Kauffmann & Haehnelt 2000; Hopkins et al. 2007a). Additionally, the quasar lifetime (Martini 2004) is in agreement with the time for a binary to merge from the edge of a gravitationally stable gas disk down to merger via gas torques and GW losses (Haiman et al. 2009; however, the LLAGN lifetime may be ~10–100 × longer; Hopkins et al. 2007b). Also, recent searches for MBHB candidates as periodically variable quasars estimate values of \( f_{\text{bin}} \) ~ 0.3 (D’Orazio et al. 2015a; Charisi et al. 2016) from the fraction of candidates found at a given binary period. We compute our own constraints on the binary fraction of the population of LLAGNs considered here in Section 3.2 below.

We calculate the time that an MBHB with total mass \( M \) spends at a given orbital period \( P \) during the bright AGN phase. We assume that gas and gravitational radiation drive the binary to merger to compute a residence time at binary separation \( a \),

\[
\tau_{\text{res}}(a) = \frac{a}{\dot{a}} = \left\{ \begin{array}{ll}
20 & \frac{P}{2\pi} \left( \frac{2}{256} \right)^{8/3} \left( \frac{GM}{c^3} \right)^{-5/3} q_s^{-1} P < P_{\text{trans}} \\
\frac{q_s}{4M} & P \geq P_{\text{trans}}
\end{array} \right.
\]

where the first term is the residence time due purely to GW decay (Peters 1964), and the second term is a prescription for orbital decay due to gaseous effects given by Loeb (2010). Here \( q_s = q/(1+q)^2 \) is the symmetric binary mass ratio, where the standard mass ratio is given by \( q = M_2/M_1 \); \( M_2 \leq M_1 \); \( M_2 + M_1 = M \). The Eddington time, \( \tau_{\text{Edd}} \equiv \tau_{\text{Edd}}(M/M_{\text{Edd}} \sim 4.5 \times 10^7 \text{yr} \), is the time it takes to accrete a binary mass of material \( M \) at the Eddington accretion rate, \( M_{\text{Edd}} \equiv L_{\text{Edd}}/(4\pi c^2) \), assuming an accretion efficiency of \( \eta = 0.1 \). The transition orbital period \( P_{\text{trans}} \) delineates gas-driven and GW-driven orbital decay.

In the gas-driven case, the simple assumption is that the binary orbit shrinks via interaction with the environment, either by gas accretion or by application of positive torque to a circumbinary disk (e.g., Rafikov 2016). Because this rate is uncertain (even its sign; e.g., Miranda et al. 2017; Tang et al. 2017), we parameterize the gas-driven orbital decay rate in terms of an Eddington rate \( \dot{M} = M/M_{\text{Edd}} \) ~ 4.5 × 10^7 yr, not the accretion rate that determines the accretion luminosity. Hence, even for the case of LLAGNs, which may not experience gas inflow at the Eddington rate, we still consider mechanisms that drive the MBHB together at a rate comparable to if the binary torques were expelling gas at the Eddington rate. Essentially, due to uncertainties in binary orbital decay rates, we have purposefully not locked together the accretion mechanism and the binary decay mechanism; we have simply parameterized the decay rate in terms of an Eddington rate.

From the binary residence time, we generate a probability distribution function \( F(M, z) \) that provides the probability that a quasar at a given redshift \( z \) and luminosity \( L = f_{\text{Edd}} L_{\text{Edd}}(M) \) harbors a MBHB with orbital period in the specified VLBI range. This probability function is derived by integrating the residence time in Equation (2) over periods and mass ratios that meet the minimum VLBI separation requirement and the maximum period requirement, \( P_{\text{base}} \), and normalizing by the
same integral over all possible binary parameters,
\[
\mathcal{F}(M, z) = f_{\text{bin}} \int_{q_{1,\text{min}}}^{q_{1,\text{max}}} \int_{q_{2,\text{min}}}^{q_{2,\text{max}}} t_{\text{res}}(M, q_1, P) \ dP \ dq_1 \ dP \ dq_2, 
\]
\[
P_{\text{lo}} = \frac{2\pi(\theta_{\text{min}} D_A(z))^{3/2}}{\sqrt{GM}} 
\]
\[
P_{\text{max}} = \frac{2\pi d_{\text{max}}^{3/2}}{\sqrt{GM}} 
\]
\[
P_{\text{hi}} = \text{min}(P_{\text{base}}, (1 + z)P_{\text{max}}).
\] (3)

The normalization introduces three additional parameters. The first two are the minimum (symmetric) mass ratio of the entire MBHB population, \(q_{1,\text{min}}\), and the minimum for the resolvable population, \(q_{V,\text{min}}\). We adopt a flat distribution in mass ratio and fiducially set the two equal to 0.01, a value motivated by the minimum mass ratio for which dynamical friction can form a central binary (Callegari et al. 2011; Mayer 2013). We also vary \(q_{V,\text{min}}\) to larger values to determine the mass ratio dependence of our results. We note that \(q_{V,\text{min}}\) could have a dependence on binary mass, for example, observationally through the Eddington luminosity and flux sensitivity. However, in accordance with our choice of a flat mass ratio distribution, we do not explore this possibility here.

The third new parameter is \(a_{\text{max}}\), the maximum binary separation for which radio-loud quasar activity is triggered. This is required because the residence time due to gas accretion (large \(a_{\text{max}}\)) is independent of the binary separation (the binary spends equal time per \(\ln a\) in the gas-driven phase); hence, we cannot simply set an \(a_{\text{max}}\) in the normalization to correspond to a quasar (or LLAGN) lifetime. As noted above, however, the observationally inferred AGN lifetime is similar to that required for an MBHB to migrate through a gas disk with an outer edge set by the Toomre stability limit (Goodman 2003; Haiman et al. 2009), where the gas disk fragments into stars. We use this separation, corresponding to the outer edge of a gravitationally stable disk, to motivate the fiducial parameter choices below.

The number of MBHBs out to redshift \(z\) over the entire sky with binary separation resolvable by a VLBI array and orbital period limited by \(P_{\text{base}}\) is
\[
N_{\text{VLBI}} \approx 4\pi \int_{z}^{\infty} dV \frac{d^2N}{dL_{\text{mm}} dV} \mathcal{F}(\chi; L_{\text{mm}}, z) dL_{\text{mm}} d\chi,
\]
\[
\chi = (\theta_{\text{min}}, a_{\text{max}}, \mathcal{M}, f_{\text{Edd}}, P_{\text{base}}, q_{V,\text{min}}, f_{\text{bin}}, f_{\text{Edd}}),
\] (4)

where, from left to right, we incorporate the cosmological volume element in a flat universe (e.g., Hogg 1999), a mm-wavelength AGN luminosity function (mmALF; see Appendix A), and the binary probability distribution function discussed above. We have rewritten the binary probability in terms of mm-wavelength luminosity through the relation
\[
M = \frac{L_{\text{bol}}(L_{\text{obs}}/\sigma_T)}{f_{\text{Edd}} 4\pi G M_{\odot}} M_{\odot},
\] (5)

where we have assumed that the accretion on to the binary generates bolometric luminosity equal to a fraction \(f_{\text{Edd}}\) of the Eddington luminosity \((L_{\text{Edd}} = 4\pi GM_{\odot} c/\sigma_T)\), and we estimate the bolometric luminosity from the observed mm-wavelength luminosity (see below).

In choosing \(f_{\text{Edd}}\), recall that LLAGNs are expected to have mm-wavelength emission emanating from a small region around each BH, making them well-suited for the imaging study proposed here. To incorporate LLAGNs, we choose a distribution \(P(x)\) of the log Eddington fraction, \(x \equiv \log_{10} f_{\text{Edd}}\), of \(L_{\text{Edd}}\) to \(0.0 \leq x < 3\) that consists of a power law with a slope \(\alpha = -0.3\) and minimum value of \(x_{\text{min}} = -5.5\) plus a Gaussian in \(x\) with mean at \(x_0 = -0.6\) and standard deviation \(\sigma = 0.3\),
\[
P(x) = \left(10^{-3}\right)^x + \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-(x - x_0)^2/(2\sigma^2)\right] + \frac{1 - 10^{x_{\text{min}}}}{\ln 10}.
\] (6)

We plot \(P(x)\) in Figure 1. This choice is based on observations of a normally distributed population of AGNs accreting near Eddington and a power-law tail of LLAGNs (Kauffmann & Heckman 2009; Shankar et al. 2013; but see also Weigel et al. 2017). The value of \(x_{\text{min}}\) is based on Figure 5 of Eracleous et al. (2010). While this \(f_{\text{Edd}}\) distribution comprises our fiducial model, we also employ a simpler delta function \(f_{\text{Edd}}\) distribution for comparison.

The lower limit on the observed specific luminosity is written in terms of the specific flux sensitivity \(E_{\nu,\text{min}}\) of the mm-VLBI instrument,
\[
L_{\nu,\text{min}}^\nu = \frac{L_{\nu} D_{\nu}^2 E_{\nu,\text{min}}^\nu}{1 + z},
\] (7)

where the first term is the \(K\)-correction, which accounts for a luminosity difference of the quasar in the emitted \(((1+z)\nu)\) and observed \(\nu\) bands. We use a fiducial value of \(E_{\nu,\text{min}}^\nu = 1\) mJy for \(\nu \sim 300\) GHz (mm-wavelength), motivated by the near-future capabilities of the Event Horizon Telescope (Broderick et al. 2011).

To compute the \(K\)-correction and construct the mm-wavelength luminosity function from a radio luminosity function, we assume that the spectral energy distribution of...
radio-loud AGNs is a power law, $\nu L_\nu \approx \nu^{0.9}$, over the frequency range $9 \lesssim \log_{10}(\nu/\text{Hz}) \lesssim 12$ (Elvis et al. 1994). This approximation is valid for LLAGNs, as well as regular radio-loud AGNs, because the radio-to-mm portion of radio-loud AGN spectra is similar for LLAGNs and normal AGNs (Fernández-Onteriz et al. 2012).

To scale from $L_{\text{mm}}$ to a bolometric luminosity for LLAGNs, we use a median bolometric correction from 2 to 10 keV of 50 (Erlaclos et al. 2010). Because $\nu L_\nu$ in the mm is within a factor of a few of $\nu L_\nu$ in the 2–10 keV range (Fernández-Onteriz et al. 2012), and because there is a large scatter in the value of the bolometric correction in the X-ray range, we adopt $L_{\text{bol}} = 50L_{\text{mm}}L_{\text{mm,v}}$.

By calculating $N_{\text{VLBI}}$ in this way, the mass distribution of binaries is provided through the AGN luminosity function. We stress again that the parameter $f_{\text{Edd}}$ relates the total mass of the binary to the observed bolometric luminosity. This is assumed to be independent of $M_\ast$, which parameterizes the binary decay rate.

For each computation of the integrand in Equation (4) corresponding to a value of the bolometric luminosity, we draw a value of $\log_{10} f_{\text{Edd}}$ from the chosen Eddington fraction distribution. The value of $\log_{10} f_{\text{Edd}}$ is used to convert the luminosity for which the mmALF is evaluated to a binary mass for which the probability $\mathcal{F}$ is evaluated. The probability also depends on the size of the mm-wavelength emission region. This is calculated from the luminosity and compared to the size of the binary separation, which depends on the binary mass through the value of the Eddington ratio (Appendix B).

One caveat of our implementation is that the Eddington fraction distribution is constructed so as to match observations of low-luminosity nearby AGNs, not necessarily radio-loud AGNs (i.e., we have assumed that the $f_{\text{Edd}}$ distribution is the same for both radio-loud and radio-quiet populations). Meanwhile, the mmALF we use is aimed at large redshifts for only radio-loud AGNs. The use of one with the other can cause extrapolation to very large BH masses. This is because, at large $z$, only the most luminous AGNs can be observed. For a given Eddington fraction distribution, the most luminous AGNs must be powered by the most massive MBHs. Because the LLAGN Eddington distribution samples very low values, the mapping from luminosity to binary mass can result in very large values of the MBH mass, above $10^{10} M_\odot$. It is likely that such MBHs do not exist, but rather that there would be a preference for the Eddington distribution to sample larger values of $f_{\text{Edd}}$ at higher redshifts. To get around this artificial MBH mass inflation, we simply make a cut in the mass distribution so that, effectively, low values of $f_{\text{Edd}}$ are not sampled at high luminosities and high redshifts. Essentially, we are requiring that the LLAGN Eddington distribution we have employed is correct for low redshifts where it is derived, but then to extrapolate to high redshifts, we enforce that the resulting MBH masses are consistent with observed maximum masses. We implement this by multiplying the binary probability $\mathcal{F}$ by a factor $\exp[-(M/M_{\text{max}})^2]$. As a fiducial value, we choose $M_{\text{max}} \sim 10^{10} M_\odot$.

Choosing a maximum binary orbital period of $P_{\text{base}} = 10$ yr and a minimum binary mass ratio of the entire MBHB population of $q_{\text{min}} = 0.01$, we are left with the free parameters $\theta_{\text{min}}, f_{\text{min}}, q_{\text{min}}^*, a_{\text{max}},$ and $M$. For mm VLBI, the value of $\theta_{\text{min}}$ is dominated by uncompensated propagation delays caused by the troposphere, rather than contributions due to thermal noise that scale with the inverse of the signal-to-noise ratio (Broderick et al. 2011; Reid & Honma 2014). Hence, we do not adopt a signal-to-noise-ratio-dependent resolution. Instead, we treat $\theta_{\text{min}}$ as a parameter that elucidates the change in the number of resolvable MBHBs with the change in astrometric precision. As motivated by the GWB upper limits in the next section, we set $f_{\text{min}} = 0.05$. We set $f_{\text{max}} = 1$ throughout, as the result scales linearly with this factor. Then we determine the dependence of $N_{\text{VLBI}}$ on the turn-on separation $a_{\text{max}}$, minimum (symmetric) mass ratio of the resolvable population $q_{\text{min}}^*$, and gas-driven migration rate $\dot{M}$ out to a given redshift. The parameters of our model and their fiducial values are given in Table 1 for reference.

3.2. GW Background

Before examining the population of resolvable MBHBs, we use the results of the previous section to compute the contribution to the stochastic GWB of all MBHBs in this mock population and enforce consistency with current limits from the PTAs.

The frequency-dependent characteristic strain due to the GWB is Phinney (2001),

$$h_c^2(f) = \frac{G}{c^2 \pi f_r^2} \int_0^\infty \int_0^\infty \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{d^3 n}{dz dM dq} \times \frac{1}{1 + z d \ln f_r} dE_{\text{GW}} dq dM dz.$$

Here $f_r$ is the rest-frame frequency of the GWs. Assuming circular binaries, $f_r$ is equal to twice the Keplerian orbital period of the binary. The first term under the integral is the comoving number density of inspiraling MBHBs per redshift $z$, binary mass $M$, and mass ratio $q$. Continuing the assumption of circular binary orbits, each MBHB emits GW energy per log frequency (Sesana et al. 2008)

$$\frac{dE_{\text{GW}}}{d \ln f_r} = \frac{d t_r}{d \ln f_r} \frac{32 G^7/3}{c^5} (\pi M f_r)^{1/3}$$

$$= \frac{64 G^7/3}{15} (\pi M f_r)^{10/3} f_{\text{res}},$$

where we have rewritten the rest-frame time per log frequency in terms of the residence time of the binary at a given separation (see, e.g., Kocsis & Sesana 2011). This residence time is given by Equation (2) for $a \leq a_{\text{max}}$ and by the GW residence time for $a > a_{\text{max}}$. Here $M_\ast(M, q) \equiv M q^{1/3}/(1 + q)^{1/3}$ is the chirp mass of the binary and $q$ is related to the symmetric mass ratio via the expression in the discussion below Equation (2).

For the comoving number density, we use the luminosity function from the previous section (see Appendix A), mapping luminosity to binary mass via Equation (5). We further assume a flat distribution of mass ratios from $q_{\text{min}}^*$ to 1. Then the GWB strain becomes

$$h_c^2(f) = \frac{G^{10/3}}{c^7} \frac{256}{15 \pi f_r^2} \int_0^\infty \int_0^\infty \frac{d^3 n}{dz dM}$$

$$\times \left( \frac{(\pi M f_r)^{10/3} f_{\text{res}}}{1 + z} \right) q_{\text{min}}^* dM,$$
where $\langle \cdot \rangle_{a}$ denotes an average over the symmetric mass ratio, $q_a$.

The GWB characteristic strain associated with the radio-loud MBHBs is plotted versus GW frequency in Figure 2 for four different sets of parameters dictating the gas-driven decay and assuming the power law–plus–Gaussian Eddington ratio distribution described in the previous section. The dashed gray line plots the approximate current PTA upper limits assuming GW decay alone and scaled to the currently most stringent PTA upper limits at a single frequency (Shannon et al. 2015; Sesana et al. 2018). The hatched orange region represents the approximate range of GW frequencies for which mm-VLBI-resolvable MBHBs will reside, while the gray shaded region represents the approximate PTA frequency range.

For the fiducial parameters (solid orange line), the GWB is dominated by gravitational decay, as can be seen by the $f_{GW}^{-2/3}$ decline in the background at low frequencies. The increase in slope above $\sim 10^{-7}$ Hz is due to the removal of MBHBs that would be at separations smaller than that of the last stable orbit corresponding to the binary mass $\left( a = 6GM/c^2 \right)$.

Increasing both $\mathcal{M}$ and $a_{\text{max}}$ increases the impact of the gas-driven decay on the GWB at low frequencies, in agreement with the studies of Kocsis & Sesana (2011) and Kelley et al. (2017b). This can be seen by the turnover of the green dashed and pink dot-dashed lines in Figure 2 at low frequencies. The gas moves the binary through this larger separation regime more quickly than GW-driven decay alone.

We find that the population of radio-loud AGNs must have an MBHB fraction of $\lesssim 0.05$ for consistency of our model with the upper limits on the PTA GWB. Hence, we use $f_{\text{bin}} = 0.05$ as our fiducial MBHB fraction throughout.

We note that there are many studies of the GWB from model MBHB populations that find agreement with the PTA upper limits but typically with a less stringent constraint on the fraction of MBHB-harboring AGNs (see Kocsis & Sesana 2011; Kelley et al. 2017a, 2017b; Mingarelli et al. 2017 and references therein). The stringency of the limit derived here is the inclusion of LLAGNs via the Eddington distribution plotted in Figure 2; the many AGNs at low Eddington ratios implies more MBHBs at higher masses. This is evidenced in the nontrivial dependence of the GWB on the choice of Eddington fraction distribution. If, for example, we choose $x_{\text{min}} = -4$ instead of the fiducial $x_{\text{min}} = -5.5$ in Equation (6), then the inferred GWB strain is an order of magnitude smaller, and hence $f_{\text{bin}}$ is not constrained at all. We further note that this GWB is calculated using the radio-loud luminosity function, which accounts for 10% of the AGNs. However, as discussed above, our value of $f = 0.05$ is already quite conservative and dependent on the Eddington distribution fraction. Hence, we simply use the fiducial value of $f = 0.05$, which is the minimal constraint taking into account all of the MBHBs in our model. We move forward within our toy model using the already motivated choice of the Eddington fraction distribution and
leave exploration of this distribution and the consequences for
the GWB to future work.

4. Results

4.1. Number of Resolvable MBHBs: Redshift Distribution and
Dependence on Angular Resolution

We begin by presenting the total number of resolvable
MBHBs for which an entire orbit can be tracked with VLBI.

Figure 3 explores the dependence of the total number of
resolvable binaries on maximum redshift and minimum
instrument angular resolution. The binary fraction in AGNs is
assumed to be $f_{\text{bin}} = 0.05$, as constrained in Section 3.2. The
left column fixes the flux sensitivity to the fiducial value but increases the minimum binary mass ratio. The top right panel enhances the instrument flux sensitivity, the middle right panel worsens the instrument flux sensitivity, and the bottom right panel shows the gain due to increasing the maximum baseline period at the best-case flux sensitivity.

Figure 3. Number of resolvable MBHBs over the entire sky as a function of minimum instrument angular resolution $\theta_{\text{min}}$ for different maximum redshifts (labeled). The left column fixes the flux sensitivity to the fiducial value but increases the minimum binary mass ratio. The top right panel enhances the instrument flux sensitivity, the middle right panel worsens the instrument flux sensitivity, and the bottom right panel shows the gain due to increasing the maximum baseline period at the best-case flux sensitivity.
maximum binary period at the optimal minimum mass ratio and flux sensitivity.

In the fiducial case (top left panel), which assumes no mass ratio preference for the resolvable binaries ($q_{\text{min}} = 0.01$) and a flux sensitivity of $F_{\text{min}} = 1 \text{ mJy}$, we could resolve binaries with angular separations as large as $\sim 25 \mu\text{as}$. This is within the diffraction limit of present-day mm VLBI. At a best-case angular resolution of $\theta_{\text{min}} = 1 \mu\text{as}$, a few tens of MBHBs would be resolvable with mm VLBI. These resolvable MBHBs lie between redshift 0.05 and 0.5.

The left column of Figure 3 shows that in order to resolve at least one MBHB, one must have $q_{\text{min}} \lesssim 0.9$ at the best-case angular resolution and fiducial flux sensitivity. This implies that a resolvable population for the fiducial binary decay model cannot consist of preferentially equal-mass binaries (but it can contain equal-mass binaries).

The top right panel of Figure 3 shows that an order-of-magnitude improvement in flux sensitivity provides an order-of-magnitude increase in $N_{\text{VHBI}}$. This improvement in sensitivity also increases the redshift range out to $z \sim 1.0$. The improved sensitivity does not increase the range of $\theta_{\text{min}}$ at which resolvable MBHBs can be found. The middle right panel of Figure 3 shows that an order-of-magnitude worse flux sensitivity provides an order-of-magnitude decrease in $N_{\text{VHBI}}$. The middle right panel also shows that for $N_{\text{VHBI}} \gtrsim 1$, one must have $F_{\text{min}} \lesssim 10 \text{ mJy}$ for the best-case $q_{\text{min}}$.

Finally, the bottom right panel of Figure 3 considers a best-case scenario: including MBHBs with orbital periods up to 20 yr, an optimal flux sensitivity of $F_{\text{min}} = 0.1 \text{ mJy}$ and otherwise fiducial model parameters. We see that the number of resolvable MBHBs can be increased to a few thousand at the best-case angular resolution. Including longer-period binaries also yields a population of large angular separation, $\gtrsim 30 \mu\text{as}$ separation binaries.

### 4.2. Visualization of Result

We now investigate the dependence of our results presented in Figure 3 upon the parameters of our model. This allows us to determine the demographic of MBHBs that can be tracked with VLBI, and it also allows us to determine the range of results that our model could produce.

We begin by visualizing the MBHB demographics that contribute to the integral in Equation (4). Each panel of Figure 4 plots contours of the integrand of Equation (4) as a function of maximum binary orbital period and observed mm-wavelength flux. We scale the integrand by $4\pi L_{\text{mm}}$, taking a representative value of $L_{\text{mm}}^{*} = 10^{44}/\nu_{\text{mm}} \sim 3 \times 10^{32} \text{ erg s}^{-1} \text{ Hz}^{-1}$ to correspond roughly to the cumulative value of the integral at that point. The contours of this scaling of the integrand of Equation (4) are colored chartreuse to purple, denoting many MBHBs and zero MBHBs, respectively. On top of the chartreuse–purple contours, we shade regions with different colors corresponding to the regions where our criteria for a resolvable orbit break down. That is, the overplotted shaded regions delineate the space of resolvable (unshaded) and nonresolvable (shaded) binaries. The left middle panel labels these regions. The resulting trapezoid-shaped window at the center of each panel, which contains only the chartreuse–purple contours, frames the parameter space of MBHBs that are resolvable and have orbits that can be tracked in a human lifetime.

On the top horizontal axis of each panel in Figure 4, we relate the observed flux (bottom axis) to the binary total mass via the Eddington ratio. The left column of Figure 4 assumes a delta function value of the Eddington ratio, $f_{\text{Edd}} = 10^{-4.1}$, that corresponds to the expectation value of the distribution of Equation (6). The right column samples from the full Eddington ratio distribution. Because the latter case requires a random draw of the Eddington ratio for each point in period–flux space, we generate 200 realizations and plot the average. In the right panels, we also plot the average binary mass that corresponds to the flux on the lower horizontal axis (using the same $200f_{\text{Edd}}$ draws). In each panel, we record the fixed parameters in the bottom left corner.

The darkest regions to the left of the thick black lines represent binaries that are dimmer than the limiting flux $F_{\text{lim}}^\text{mm}$. For the fixed $f_{\text{Edd}} = 10^{-4.1}$ Eddington ratio case (left column of Figure 4), at $z = 0.1$, MBHBs with total mass $\gtrsim 10^{8.5} M_{\odot}$ are detectable at a limiting flux of $F_{\text{lim}}^\text{mm} = 1 \text{ mJy}$. By $z = 0.5$, there are no more MBHBs below $\sim 10^{10} M_{\odot}$ that are detectable above 1 mJy.

In the right column, the darker shaded region corresponds to the same flux limit as in the left column, but this corresponds to a different binary mass via the Eddington ratio distribution in Equation (6). In this case, using the power law–plus–Gaussian distribution, the same fluxes correspond to higher-mass binaries. At $z = 0.1$, MBHBs with a total mass of $\gtrsim 10^{9.5} M_{\odot}$, on average, are detectable at a limiting flux of $F_{\text{lim}}^\text{mm} = 1 \text{ mJy}$. By $z = 0.5$, only rare MBHBs with mass $\gtrsim 10^{10.5} M_{\odot}$ can be bright enough to exceed the flux limit. Hence, binaries can be resolved out to $z \sim 0.5$, limited by the minimum detectable mm-wavelength flux. These binaries will preferentially be more massive at higher redshifts.

Note that in the right column, the contours for the scaled integrand of Equation (4) do not plunge abruptly to the maximum cutoff mass as they do in the left panels. This is due to the averaging of $M$ for many draws of $f_{\text{Edd}}$.

The red regions lying outside of the thick red trapezoid represent the space of restricted binary orbital parameters. Below the bottom left red line, for smaller masses and periods, the binary is too compact to be resolvable at the quoted redshift and minimum instrument angular resolution (this being the $P_{\text{min}}$ limit of integration in Equation (4)). The bottom right red line is the orbital period at $a = 6GM/c^2$, which we label ISCO. The top red line is the imposed maximum observed orbital period ($P_{\text{base}}$). To the far top right, the small restricted region at the longest periods and largest binary masses is where the period at turn-on separation $a_{\text{max}}$ is shorter than the maximum baseline period (see the definition of $P_{\text{hi}}$ in Equation (4)).

In both the left and right columns, it is evident that the trapezoid-shaped window defined by the red regions gets smaller and moves to lower fluxes at higher redshifts. Both behaviors are a combination of three effects: (i) the minimum angular resolution corresponds to a larger physical binary separation at greater distances, (ii) higher flux systems do not exist at larger distances because of the limiting intrinsic luminosity set by the maximum binary mass and the Eddington ratio distribution, and (iii) at larger distances, higher flux systems correspond to more massive binaries; for a fixed binary

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This is the case at the redshifts of interest here. For $z \gtrsim 1.6$, this relation reverses due to cosmology.
Figure 4. Visualization of the integral of Equation (4). The left middle panel is labeled as a guide to the different regions plotted in each panel. The left column assumes a delta function value of the Eddington ratio, $f_{\text{Edd}} = 10^{-4.1}$. The right column assumes a power law–plus–Gaussian distribution of $f_{\text{Edd}}$ (Equation (6)) and averages over 200 $f_{\text{Edd}}$ draws. From top to bottom, each row is for redshift $z = 0.1, 0.2,$ and $0.5$. The labeled contours, shaded from chartreuse to purple, are a scaling of the integrand in Equation (4), with chartreuse indicating many MBHBs and purple indicating none. The differently colored shaded regions overplotted on these contours denote where our criteria for an MBHB to be a VLBI tracking target are not met. Hence, the trapezoid-shaped window in the center of each panel delineates the region of parameter space where MBHBs can be resolved and tracked along their orbits with VLBI. The darker shaded regions, bounded on the right by the thick black line, are where emission is too dim to detect, given the labeled minimum flux sensitivity. The red shaded regions external to the thick red lines are where the binary is not resolvable because it is either below the minimum resolvable separation, above the maximum baseline orbital period, or at merger. The yellow shaded regions are where the mm-emission region is larger than the binary separation.
period, these systems have smaller binary separations and are less likely to lie above the limiting angular resolution.

The yellow region in the bottom right of each panel represents where the mm-wavelength emission region is larger than the binary separation. This occurs for the closest separation (shortest orbital period) and brightest systems because brighter emission corresponds to a larger photosphere (see Appendix B). For the low Eddington ratios chosen here, only the more rare, short-period, high-mass binaries are obscured by a large photosphere. This is, of course, the reason that we consider LLAGNs for this study. Figure 7 shows that for Eddington ratios larger than ~0.1 (and even lower for worse angular resolution limits), obscuration by a large mm photosphere engulfs what would otherwise be a resolvable binary.

Summarizing Figure 4, we find the following.

1. The number of resolvable MBHBs becomes flux limited at $z \gtrsim 0.5$ (for $F_{\text{min}} = 1\text{ mJy}$).
2. The MBHBs with a total mass of $10^7$–$10^8\,M_\odot$ and the longest orbital periods contribute the most to $N_{\text{VLBI}}$. However, the relevant flux sensitivity results in a preference for the brightest, highest-mass systems. Increasing $P_{\text{base}}$ always yields more resolvable MBHBs.
3. The most massive and shortest-period binaries are obscured by a large mm-wavelength emission region.
4. Lower-mass and shorter-period binaries are excluded by angular resolution requirements.
5. For the small Eddington ratios considered here, the mm-wavelength emission region size constraints do not exclude much of the parameter space. Size constraints do become important, however, for Eddington ratios closer to unity (see Figure 7 below).

4.3. Parameter Dependencies

Now that we have explored the makeup of MBHBs that contribute to the integral in Equation (4), we explore the dependence of the result, $N_{\text{VLBI}}$, on the major model parameters. The most important parameters controlling the binary decay model are the turn-on separation $a_{\text{max}}$, the minimum binary mass ratio $q_{r_{\text{min}}}$, and the gas-driven decay rate $\mathcal{M}$. In Figure 5, we explore the dependence of $N_{\text{VLBI}}$ on these parameters for three values of the minimum angular resolution. On the left vertical axis of each panel, we plot the angular scale corresponding to $a_{\text{max}}$ at $z = 0.2$. In each panel, we record the fixed parameters in the top left corner. We graphically mark the fiducial values of the parameters being varied by green lines intersecting at a green point. The white lines are contours of constant residence time in units of years. These are drawn for a representative binary with $M = 10^9\,M_\odot$ and $q_r = 0.1$ (left column) or $M = 10^8\,M_\odot$ and $\mathcal{M} = 1$ (right column) and labeled in units of years.

To demonstrate the full parameter dependencies of $N_{\text{VLBI}}$, we draw contours of $N_{\text{VLBI}}$ in Figure 5, assuming a value of $f_{\text{bin}} = 1$. Note, however, that the GW analysis of the previous section requires $f_{\text{bin}} \lesssim 0.05$. Hence, Figure 5 shows that, for minimum resolutions ranging from 1 to $20\,\mu\text{as}$, the binary decay model predicts a total of $N_{\text{VLBI}} \sim f_{\text{bin}} a_{\text{max}} 10^3$ resolvable binaries at the fiducial parameter values of $F_{\text{min}} = 1\text{ mJy}$, $a_{\text{max}} \sim 0.1\,\text{pc}$, $q_{r_{\text{min}}} = 0.01$, and $\mathcal{M} \sim 1.0$. For the range of plausible parameter space, and relaxing the GWB limit, this number can range up to $\sim 10^5$. Next, we discuss the dependence of $N_{\text{VLBI}}$ on these parameter values.

1. $a_{\text{max}}$ dependence.—Above and below $a_{\text{max}} \sim 10^{-1.75}\,\text{pc}$, $N_{\text{VLBI}}$ decreases. For $a_{\text{max}} \gtrsim 10^{-1.75}\,\text{pc}$, this results from the assumption that $a_{\text{max}}$ corresponds to where the binary becomes bright and, simultaneously, where gas-driven orbital decay begins. For larger values of $a_{\text{max}}$, the range of binary separations over which the binary is bright becomes larger, while the range of binary separations over which it is resolvable stays the same. Put another way, for larger $a_{\text{max}}$, the space of possible binary parameters for which the binary is bright increases in size while the target range does not, so the probability that any bright MBHB system is in the resolvable range decreases.

For $a_{\text{max}} \lesssim 10^{-1.75}\,\text{pc}$, the space of possible binary parameters over which the binary is resolvable decreases because the binaries with the longest allowed periods ($P \rightarrow P_{\text{base}}(1 + z)$) have separations larger than $a_{\text{max}}$ and hence are not bright for all of their resolvable lifetime. The $N_{\text{VLBI}}$ drops to zero at the value of $a_{\text{max}}$ that falls below the minimum resolvable angular resolution at a given redshift. This can be seen by comparing the location of the small-$a_{\text{max}}$ cutoff of $N_{\text{VLBI}}$ between the $\theta_{\text{min}} = 1, 10,$ and $20\,\mu\text{as}$ panels in Figure 5.

Physically motivated values of $a_{\text{max}}$ are suggested by the red shaded regions in Figure 5. This region is the range of the (binary mass–dependent) outer radii of a gravitationally stable gas disk (Goodman 2003; Haiman et al. 2009). To illustrate this range, we choose a binary mass of $10^9\,M_\odot$ (motivated by Figure 6 below) and compute the range of gravitationally stable outer disk radii assuming electron scattering and free–free-absorption-dominated opacity.

However, because the bright lifetime of an AGN is not necessarily determined by the size of a Toomre-stable disk, we leave $a_{\text{max}}$ as a free parameter. We choose a fiducial value of $a_{\text{max}} = 0.1\,\text{pc}$ to be consistent with these gravitationally stable disks but also to correspond to a value near the peak of the $N_{\text{VLBI}}$ distribution. It is encouraging that the most optimistic predictions for $N_{\text{VLBI}}$ lie within this region. We do not choose a smaller $a_{\text{max}}$ because of possible tension with the sub-pc separations of known MBHB candidates (e.g., Graham et al. 2015; Charisi et al. 2016). Smaller $a_{\text{max}}$, and hence larger inferred values of $N_{\text{VLBI}}$, are of course not ruled out.

2. $q_{r_{\text{min}}}$ dependence.—In the left column of Figure 5, we plot the contours of $N_{\text{VLBI}}$ in $a_{\text{max}}$ versus $q_{r_{\text{min}}}$ space. That is, we assume a minimum mass ratio $q_{r_{\text{min}}} = 0.01$ of the population of all MBHBs and vary the minimum mass ratio of resolvable MBHBs, $q_{r_{\text{min}}}$. The utility of the $q_{r_{\text{min}}}$ parameter is to elucidate which mass ratios are contributing to the resolvable population. It is also useful if we wish to restrict the population of resolvable MBHBs to only those with near-unity mass ratios, in order to increase the probability that both will be bright. The left panels of Figure 5 clearly show that the value of $N_{\text{VLBI}}$ only begins to strongly depend on $q_{r_{\text{min}}}$ when the latter
approaches unity, at which point $N_{\text{VLBI}}$ drops to zero (at $q_s \sim 0.7$ or $q \sim 0.3$). This is simply a result of positng a flat distribution of binary mass ratios.

3. $\dot{M}$ dependence.—In the right column of Figure 5, we plot the contours of $N_{\text{VLBI}}$ in $a_{\text{max}}$ versus $\dot{M}$ space. To understand the dependence of $N_{\text{VLBI}}$ on $\dot{M}$, we first
consider the purely gas-driven scenario. In this case, \( N_{\text{VLBI}} \) does not depend on \( \dot{\mathcal{M}} \); rather, it sets the average active lifetime of the MBHB indicated by the white contours in Figure 5. The steep turnaround in these white contours is where the binary changes from gas-driven to GW-driven orbital decay. In the gas-driven regime, there is a decrease in MBHB lifetime with increasing \( \dot{\mathcal{M}} \), as expected, but this is not reflected in the contours of \( N_{\text{VLBI}} \) because the binary decreases its time spent in the resolvable regime proportionally to its total lifetime. This is essentially a result of the duty-cycle argument that we use in this computation, i.e., our assertion that a fraction \( f_{\text{bin}} \) of AGNs are MBHBs. Reassuringly, our total MBHB lifetimes are consistent or slightly lower than the observationally constrained AGN lifetime (Martini 2004). We note, however, that the bright lifetime of an LLAGN may be longer (\( \sim 10-100 \times \) than the bright Eddington-limited quasar lifetime (Hopkins et al. 2007b). Hence, choices of \( \dot{\mathcal{M}} \) (and \( a_{\text{max}} \)) that lead to \( \sim \)Gyr bright binary residence times may be relevant for the LLAGN. However, as can be seen from the right column of Figure 5, this does not greatly affect our results at the fiducial value of \( a_{\text{max}} \).

When including GW emission, \( N_{\text{VLBI}} \) does depend on \( \dot{\mathcal{M}} \) because it sets the binary separation where GW-driven decay takes over. The consequence of this is that larger values of \( \dot{\mathcal{M}} \) correspond to binaries that spend relatively more time in the gas-driven regime than in the more short-lived GW-driven regime. This affects the value of \( N_{\text{VLBI}} \) if the transition separation \( a_{\text{trans}} \) falls within \( a_{\text{max}} \). The wider the range of separations that the binary spends in the gas-driven stage and above the minimum separation \( a_{\text{min}} = \theta_{\text{min}} D(z) \), the more resolvable binaries we expect to find.

This explains why \( N_{\text{VLBI}} \) is constant below the values of \( a_{\text{max}} \) that fall on a line parallel to the line marking the transition from gas- to GW-driven decay (the line connecting the elbows of the white contours). Below this line, \( a_{\text{max}} < a_{\text{trans}} \), and the binary is already in the GW-driven regime when it enters the disk and becomes bright, so gas does not affect the decay. That \( N_{\text{VLBI}} \) obtains an \( \dot{\mathcal{M}} \) dependence approximately above the line connecting the elbows of the white \( M = 10^9 \, M_\odot \) contours means that these most massive MBHBs dominate the resolvable population. We show that this is indeed the case in Figure 6.

The increasing dependence of \( N_{\text{VLBI}} \) on \( \dot{\mathcal{M}} \) with larger \( \dot{\mathcal{M}} \) stems from the fact that larger \( \dot{\mathcal{M}} \) implies smaller \( a_{\text{trans}} \) and hence causes the binary to spend more of its resolvable lifetime in the gas-dominated stage. A similar increasing importance of gas effects for larger \( a_{\text{max}} \) and smaller \( a_{\text{trans}} \) (larger \( \dot{\mathcal{M}} \)) can also been seen in the different GWB realizations of Figure 2.

4. Maximum period and mass dependence.—Figure 6 illustrates the demographics of resolvable MBHBs by plotting the contours of \( N_{\text{VLBI}} \) as a function of an imposed maximum observed binary orbital period and a maximum binary mass. From top to bottom, the panels show that the minimum angular resolution is varied from \( \theta_{\text{min}} = 1 \) to 10 to 20 \( \mu \)as. The white lines show the contours of constant binary separation, and the cyan lines are the contours of constant GW strain.

![Figure 6](image-url)

*Figure 6. Log contours of the number of resolvable MBHBs over the entire sky as a function of the maximum temporal baseline (maximum observed binary period) and binary mass. From top to bottom, we vary the minimum angular resolution \( \theta_{\text{min}} \) from 1 to 10 to 20 \( \mu \)as. The white lines are the contours of constant binary separation. The cyan lines are the contours of constant (dimensionless) GW strain.*
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redshift of z = 0.2. The most massive and lowest-period resolvable binaries are approaching those that could be resolvable as individual GW sources. We comment on this possibility in Section 5.5. As expected and pointed out in the discussion surrounding Figure 4, Figure 6 illustrates that a smaller minimum spatial resolution implies a population of resolvable binaries with periods and masses that extend to lower values.

The smallest binary masses at a maximum orbital period of ~10 yr are 3 \times 10^6 M_\odot for \( \theta_{\text{min}} = 1 \mu \text{as} \), \~3 \times 10^7 M_\odot for \( \theta_{\text{min}} = 10 \mu \text{as} \), and \~10^8 M_\odot for \( \theta_{\text{min}} = 20 \mu \text{as} \). The minimum observed binary periods for the largest, \( 10^{10} M_\odot \), MBHBs are \~1.5 yr for \( \theta_{\text{min}} = 1 \mu \text{as} \), \~3 yr for \( \theta_{\text{min}} = 10 \mu \text{as} \), and \~5 yr for \( \theta_{\text{min}} = 20 \mu \text{as} \).

5. Discussion

5.1. MBHB Population

Imaging of an MBHB with VLBI over the course of an orbit could provide the first definitive proof of an MBHB at sub-pc separations. It will also probe a regime of MBHB evolution inaccessible to GW observations. Sub-pc separations represent the poorly understood stage in MBHB evolution where GW radiation plus environmental effects are competing for dominance in binary orbital decay but GW emission is not yet loud enough to identify the system as an individually resolved GW source with observatories such as the PTAs or LISA (Amaro-Seoane et al. 2017). Hence, not only is this regime one of the most important to understand for deciphering, e.g., the final-parsec problem, it is also directly accessible only in the EM sector.

VLBI imaging could attain the definitive detection of multiple MBHBs, but importantly, work as a verification tool for more indirect methods of MBHB identification. Linking indirect MBHB signatures, such as quasar periodicity, variable broad lines, or jet morphology, with a secure detection of an MBHB via imaging would aid in the confident use of these indirect but more easily employed identification methods. Identification of at least a few MBHB systems with VLBI imaging could even aid in identifying yet-undiscovered signatures of MBHBs in galactic nuclei.

If a population of MBHBs can be identified, the work presented in Section 4.3 can be inverted to infer the binary residence times given the observed number of MBHBs at given separations and redshifts. In essence, a plot similar to Figure 3 could be made from observations. From this, models for the residence times can be ruled out, and the parameters within these models can be constrained. This would provide a powerful approach to understanding the mechanisms that drive MBHBs through the final pc to centi-pc separations of their existence.

The MBHB population that can be accessed through the methods proposed here will be preferentially low Eddington ratio systems. Figure 7 shows the dependence of \( N_{\text{VLBI}} \) on the Eddington ratio in the case where \( f_{\text{edd}} \) takes on a single value. The most striking feature of Figure 7 is the steep falloff of \( N_{\text{VLBI}} \) for \( f_{\text{edd}} \gtrsim 0.1 \). As alluded to in the discussion surrounding Figure 4, this falloff is due solely to our criteria that the mm-wavelength emission region be smaller than the binary separation.

The inset of Figure 7 shows how the size of the mm-wavelength emission region grows to envelope the binary for large Eddington fractions. This inset is identical to the left middle panel of Figure 4 but for \( f_{\text{edd}} = 0.1 \) and without enforcing emission region size constraints when drawing the chartreuse-to-purple shaded contours. The yellow region labeled “joint emission region” is shaded with lower opacity than in Figure 4 for clarity.

The inset of Figure 7 also shows that any near-Eddington sources that can be resolved as two individual sources will preferentially be low-mass long-period binaries. This means that the resolvable MBHB population would have a mass and period correlation with Eddington ratio.

Figure 8 shows the distribution of such circumbinary mm-emission systems assuming a single Eddington ratio of \( f_{\text{edd}} = 0.1 \) and not excluding systems with mm-emission regions larger than their separation. We see that at minimum angular resolutions approaching 1 \( \mu \text{as} \), \( f_{\text{edd}} \gtrsim 0.1 \times 10^3 \) of such systems could exist out to redshifts of z \~ 0.5. Future work should clarify what is expected from the mm emission of these systems. It is intriguing to note that the MBHB candidates PG 1302 (Graham et al. 2015a) and OJ 287 (Valtonen et al. 2008) fall into this latter circumbinary mm-emission region category, with angular separations of 4 and 12 \( \mu \text{as} \), using estimated total binary masses of \( 10^{8.4} \) and \( 10^{10.3} M_\odot \) for PG 1302 and OJ 287, respectively.
distribution of \( f_{\text{edd}} = 0.1 \) and not removing MBHBs with emission regions surrounding the binary.

5.2. Binary Mass Determination

Beyond probing the MBHB population, VLBI imaging of an MBHB orbit would allow a precise measurement of the binary mass. Consider resolving the orbital separation of an MBHB with each component active. If the binary is on a circular orbit, its separation \( a \) can be measured from the maximum resolved angular separation \( \theta_a \) of the two orbiting sources and the redshift of the host galaxy \( a = \theta_a D_a(z) \), where \( D_a(z) \) is the angular diameter distance of the source given the redshift \( z \). By tracking the binary for a large enough fraction of an orbit needed to fit an orbital solution (this need not be an entire orbit), the binary orbital period \( P \) can be used to measure the total mass of the binary,

\[
M = \frac{1}{G} \left( \frac{2\pi}{P(1+z)} \right)^2 (\theta_a D_a(z))^3, \tag{11}
\]

where the observed period is \( P(1+z) \).

Assuming Gaussian errors on the measurements of \( P, \theta_a, \) and comparatively negligible errors on \( z \) and the Hubble constant \( H_0 \) (which factors into \( D_a(z) \)), the uncertainty in this mass measurement is

\[
\frac{\delta M}{M} \approx \left[ 2 \left( \frac{\delta P}{P} \right)^2 + 3 \left( \frac{\delta \theta_a}{\theta_a} \right)^2 \right]^{1/2}. \tag{12}
\]

We estimate the error in the measurement of \( \theta_a \) by the minimum instrument angular resolution. When we are limited by a minimum angular resolution of \( \sim 10 \) \( \mu \)as, the calculations of the previous section show that most resolvable binaries are at this limiting angular separation, so \( \delta \theta_a/\theta_a \sim 1 \). If, however, the minimum resolution can reach \( \sim 1 \) \( \mu \)as, we find that \( \delta \theta_a/\theta_a \sim 0.1 \).

From VLBI astrometry alone, the error in \( P \) is set by the cadence of observations \( \Delta T_{\text{obs}} \), and the precision at which the centroid of emission from each binary component can be determined,

\[
\delta P \approx \left[ \frac{\Delta T_{\text{obs}}^2}{P\theta_{\text{min}}^2 \theta_a} \right]^{1/2}, \tag{13}
\]

which in the worst-case scenario can be of order \( P \). However, as we will discuss below, identification of VLBI resolvable candidates can be carried out through searches for quasar periodicity caused either by the relativistic Doppler boost or by variable circumbinary accretion. In these cases, the binary period can be identified to within a few percent (e.g., Graham et al. 2015b). Taking into account the above best- and worst-case scenarios for \( \delta \theta_a/\theta_a \) and a best-case \( \delta P/P \sim 0.05 \), we estimate the precision in the MBHB mass measurement to fall in the range

\[
0.3 \lesssim \frac{\delta M}{M} \text{ (VLBI)} \lesssim 4. \tag{14}
\]

The present-day state-of-the-art technique for measuring MBH masses, reverberation mapping, can typically measure central MBH masses to within 0.5 dex, or \( \delta M/M_{\text{RM}} \sim 3 \) (e.g., Shen 2013 and references therein); hence, the mass measurement put forth here would rival the precision of those found through reverberation-mapping techniques and, in the best-case scenario, provide the most precise MBH masses to date. Furthermore, the mass measurement proposed here is much cleaner in that it only requires Newtonian orbit fitting and does not rely upon the unknown geometric factors related to AGN broad-line regions, which contribute much of the uncertainty in the reverberation-mapping analysis. The mass measurements found via VLBI orbit tracking could probe MBHBs out to redshifts of \( z \sim 0.5 \) in a mass range \( 10^6 \rightarrow 10^{10} M_\odot \).

5.3. Determination of the Hubble Constant

If, instead, there is a measurement of the central binary mass, VLBI orbit tracking of an MBHB allows a novel measurement of the Hubble constant. This can be achieved by solving Equation (11) for the Hubble constant, which determines \( D_a(z) \). As for the mass measurement of the previous section, one must again know the redshift of the MBHB host galaxy. Using the same best-case estimates for \( \delta \theta_a/\theta_a \sim 0.1 \) and \( \delta P/P \sim 0.05 \) as in the previous subsection and choosing an optimistic \( \delta M/M \sim 0.5 \), the uncertainty in the Hubble constant measured from VLBI orbit tracking is approximately

\[
\frac{\delta H_0}{H_0} \gtrsim 0.2 \left[ \left( \frac{\delta \theta_a/\theta_a}{0.1} \right)^2 + \frac{4}{9} \left( \frac{\delta P/P}{0.05} \right)^2 + \frac{1}{9} \left( \frac{\delta M/M}{0.5} \right)^2 \right]^{1/2}, \tag{15}
\]

down to a 20% relative error. If the relative error in \( \theta_a \) and \( M \) could be decreased to 5%, the measurement uncertainty in \( H_0 \) would drop to 6%.

If the binary generates periodically variable continuum emission due to the relativistic Doppler boost (D’Orazio et al. 2015b), then simultaneous VLBI monitoring of MBHB astrometry and Doppler-boosted fluxes can determine the Hubble constant even without a binary mass measurement.

For the range of binary periods and masses for which we predict more than one resolvable MBHB (e.g., the left panel of Figure 6), the orbital velocity can range from \( \sim 0.01c \) for \( M \sim 10^7 M_\odot \) and \( P \sim 10 \) yr to \( \sim 0.19c \) for \( M \sim 10^{10} M_\odot \) and

\text{The strongest observed periodicity could be higher or lower than the orbital period (Charisi et al. 2015; D’Orazio et al. 2015a), but in each scenario, the true orbital period could still be discernible with a long enough observation.}
P ∼ 1.5 yr, meaning that significant Doppler modulation at the tens of percent level is possible.

Assume again that we have observations of a resolved MBHB with both MBHs active. Then, mm VLBI can observe the orbital motion projected onto the sky. The observed angular velocity of the ith binary component is

$$\dot{\theta}_i = (1 + z) \frac{v_i(q)}{D_h(z)} \sqrt{\cos^2[\Omega t] \sin^2 I + \sin^2[\Omega t]},$$  

where $I$ is the inclination of the binary orbital plane to the line of sight, $\Omega = 2\pi/P$ is the observed angular orbital frequency of the binary, and $v_i$ is the rest-frame angular velocity of the ith MBH. We assume a binary with mass ratio $q$ on a circular orbit, 

$$v_p = \frac{q}{1 + q} \left( \frac{GM\Omega}{c} \right)^{1/3} = q v_r,$$  

where $s$ denotes secondary and $p$ denotes primary, and

$$D_h(z) = \frac{c}{H_0(1 + z)} \int_0^z \frac{dt'}{\sqrt{\Omega_M(1 + z')^3 + \Omega_\Lambda}}$$

is the angular diameter distance of the source at redshift $z$ for Hubble constant $H_0$ and matter and dark energy density parameters $\Omega_M$ and $\Omega_\Lambda$.

If the binary is on a circular orbit, the inclination of the binary can be discerned from the projected shape of the orbit alone. The inclination can also be recovered for an eccentric orbit. However, the change in proper motion along the path of the eccentric orbit must also be included to break the degeneracy between, for example, a face-on elliptical orbit and a tilted circular orbit. Assuming a circular orbit for simplicity of demonstration,

$$I = \tan^{-1} \left( \frac{\theta^b}{\theta^a} \right),$$

where $\theta^a$ and $\theta^b$ are the semimajor and semiminor axes of the ellipse that the binary orbital motion traces. In the more general case, tracking the proper motion of the binary components will allow measurement of the orbital eccentricity.

Similarly, the mass ratio could be measured if the binary orbit is resolved on the sky. For example, a binary on a circular orbit with semimajor axis $a$ has a secondary that orbits at a distance $r_2 = a/(1 + q)$ from the binary center of mass, while the primary orbits at a distance $r_p = aq/(1 + q)$. The measurement of the ratio of these angular distances yields the binary mass ratio.

The orbital velocity can be measured independently from monitoring the periodically changing flux caused by the relativistic Doppler boost. Generally, the emission from both MBHs must be taken into account,

$$\frac{F_{\nu}^\text{obs}(t)}{F_{\nu}^0} = f_s \left[ \gamma_0 \left( 1 - \frac{v_{||}}{c} \right) \right]^{3/2} + (1 - f_s) \left[ \gamma_p \left( 1 + q \frac{v_{||}}{c} \right) \right]^{3/2},$$

where $v_{||}$ denotes the velocity component along the line of sight, and we have used $v_{||} = -q v_{||}$. This introduces a new quantity, $f_s = \langle F_s/(F_s + F_p) \rangle$, the fraction of time-averaged flux detected from the secondary compared to the total. From measurements of $\alpha, I, q$, and the time-dependent flux, we solve the above equation for the orbital velocity of the secondary and primary MBH.

Hence, the two measurements of the Hubble constant are given by

$$(H_0)_i = \frac{c^3 \int_{v_i}^{\infty} \frac{dc'}{\sqrt{\Omega_M(1 + z')^3 + \Omega_\Lambda}}}{v_i \sqrt{\cos^2[\Omega t] \sin^2 I + \sin^2[\Omega t]}}$$

which must be averaged over some fraction of the binary orbit in order to measure $\dot{\theta}_i$. This measurement of $H_0$ could then be used in conjunction with the method using Equation (11) to further improve the uncertainty in the VLBI-measured Hubble constant.

Note that this method for measuring the Hubble constant with proper motions and orbital velocities can also be carried out without detection of the Doppler boost but rather with a measurement of the binary mass and mass ratio. In this case, the orbital velocity of each MBH yields via Equation (17).

As an example for the precision at which the Hubble constant can be measured in the Doppler-boost method, we envision the simplest case. We consider the case where only the Doppler boost from the secondary is important and $\gamma_s \sim 1$. Then the secondary orbital velocity can be written as

$$\frac{v_s}{c} \cos I = [\Delta_+ + 1]^{1/3} - 1$$

$$\Delta_+ = \frac{F_{\nu}^\text{obs}(0)}{F_{\nu}^0} - \frac{F_{\nu}^\text{obs}(P/4)}{F_{\nu}^0},$$

where $\Delta_+$ is the modulation amplitude between peak and average flux (between $t = 0$ and $t = P/4$). Assuming that the uncertainty in the binary inclination dominates, we can write $\delta v/v \approx I \tan I (\delta I/I)$. Assuming a small inclination angle to the line of sight (needed for the Doppler boost to be significant), we approximate $I \approx \theta_0/\theta_a$ (from Equation (19)), and $\delta I/I \approx \sqrt{2} (\delta \theta_0/\theta_a)$. Taking optimistic values of $I \lesssim 0.5 \text{ rad}$, $\delta \theta/\theta = 0.1$, $\delta \theta_a / \theta_a = 0.1$, and $\delta \Omega / \Omega = 0.05$,

$$\frac{\delta H_0}{H_0} \gtrsim 0.1 \quad (\text{Doppler case}),$$

which is valid for $I \lesssim 0.5$, where this estimate does not vary greatly over the course of the orbit. If these measurements can be made out to redshifts $z > 0.1$, as Figure 3 suggests is possible, then they could provide an independent measure of the Hubble constant that could aid in resolving discrepancies in other independent measurements, such as the current mismatch between $H_0$ measured by Planck via the cosmic microwave background (Planck Collaboration et al. 2016) and the value measured by the Hubble Space Telescope via the Cepheid variables (Riess et al. 2016).
5.4. Observational Strategy

As mm VLBI is not suited for all-sky surveys, we require pilot observations that can identify MBHB mm-bright candidates for VLBI follow-up. We have shown that the majority of resolvable MBHBs would be at the heart of mm-bright LLAGNs. This means that there will be only a dim optical/UV component to these sources (LLAGNs do not exhibit a big blue bump in their spectra). LLAGN spectra, however, are brighter in the near-IR (e.g., Fernández-Ontiveros et al. 2012) and hence accessible via upcoming time-domain surveys such as the Large Synoptic Survey Telescope (LSST; Ivezić et al. 2008). Hence, we propose the following general strategy to find VLBI MBHB candidates.

1. Search for periodicity with all-sky time-domain surveys in the near-IR, e.g., LSST. Such a survey will identify a list of MBHB candidates from which we can select those with the required binary separations given a mass estimator and the observed variability period (see, e.g., Graham et al. 2015b; Charisi et al. 2016). We note that the lifetime of these surveys will set the size of the maximum binary period $P_{\text{base}}$. Currently, surveys such as the Catalina Real Time Transient Survey have a 10 yr temporal baseline (Djorgovski et al. 2011).

2. Determine whether each periodic-light-curve candidate is radio-loud (and hence bright in mm wavelengths), either via archival searches or follow-up with single radio observations.

3. Observe candidates that pass stages 1 and 2 with mm VLBI and determine if they consist of two compact sources or one compact source and a nearby phase calibrator. If they do, monitor these sources for orbital motion. Even for the longest-period binaries considered here (10 yr), two or three observations spaced by a year would be sufficient to test for orbital motion.

We point out that an advantage to searching for sub-pc separation, dual-source binaries over searching for wide, pc−kpc dual AGNs is that these smaller-separation binaries could be identified for follow-up via the imprint of their orbital period.

5.5. GW Single-source Detection

As shown in the cyan contours of Figure 6, the nearest MBHBs at the high-mass and low-period end of the resolvable population could be detectable as single GW sources.5 In this case, the EM discovery of an MBHB could aid the PTAs in digging out a weak signal of the binary. Such identification could also allow a precision binary mass measurement that could be used to increase the precision of the Hubble constant measurement of Section 5.3 or to corroborate the mass measurement of Section 5.2. A simultaneous GW detection could also contribute a second independent measurement of the Hubble constant through the standard sirens approach (see Schutz 1986; Abbott et al. 2017 and references therein).

Finally, VLBI orbit tracking plus GW detection of an MBHB could be used to measure the relative speed of light and GWs. This can be achieved by tracking the binary orbital phase through both the EM and GW messengers. The detection of the relativistic Doppler boost would further enhance the capability to make such a measurement (see Haiman 2017 for a similar scenario).

6. Conclusions and Future Work

By constructing simple models for gas- and GW-driven binary orbital decay and an mmALF, we estimate the number of MBHBs over the entire sky that reside at separations directly resolvable by mm-wavelength VLBI and with orbital periods of less than 10 yr. We show that 1−10 μas resolution sub-mm VLBI with flux sensitivity better than 1 mJy can resolve the orbital separation of tens of MBHBs at fiducial model parameters and up to $\sim 10^4$ at parameter values tuned to maximize this number. These MBHBs are found out to redshift $z \approx 0.5$ and have total masses above $\sim 10^7 M_{\odot}$.

For a minimum flux sensitivity of 10 mJy, closer to current capabilities, there are of order a few resolvable MBHBs out to redshift $z \lesssim 0.2$. For an enhanced sub-mm flux sensitivity of 0.1 mJy, our fiducial MBHB-decay model predicts that a few $\times 10^2$ MBHBs will be resolvable at 1 μas resolution out to $z \lesssim 1.0$, while of order one MBHB will be resolvable with 25 μas resolution. Further extending the maximum temporal baseline to include 20 yr binary orbital periods increases the all-sky number of resolvable MBHBs to a few thousand at the best-case 1 μas resolution and a few tens even at a minimum angular resolution of 30 μas.

We determine that resolvable MBHBs, for which each component is tracked by a mm-emission region smaller than the binary orbit, preferentially reside in LLAGNs. These objects could be identified through periodicity signatures in near-IR/optical time-domain surveys and followed up with radio observations as a precursor for VLBI orbit tracking.

Resolvable MBHBs will be emitting gravitational radiation in the PTA band (~Hz). If they are within a redshift of 0.1, the closest, most massive MBHBs could be detectable as individual binary sources. We show that the total binary fraction of these LLAGNs is constrained to be $\lesssim 0.05$ by PTA limits on the GWB.

Beyond providing a definitive existence of sub-pc MBHBs, VLBI tracking of a complete binary orbit would allow a measurement of the binary mass (to within ~30%) or a novel, O(10%), measurement of the Hubble constant if the binary mass is known.

Future work should consider improved models for the orbital decay of MBHBs and their mm-bright lifetimes. This is important for understanding the range in predictions for the number of resolvable MBHBs and determining how well such models can be ruled out by future population estimates. Additionally, future work should understand the population of periodic-light-curve candidates that could be identified with current and future time-domain surveys, specifically, the subset of these that are VLBI imaging candidates. Further analysis of the existing MBHB candidates in the literature may already provide interesting targets for VLBI orbit tracking. For example, PG 1302-102, OJ 287, and 3C 273 are each sub-mm-bright AGNs within redshift 0.3 that have been reported as MBHB candidates with observed periods of 5, 12, and 16 yr (Abraham & Romero 1999; Romero et al. 2000; Valtonen et al. 2008; Graham et al. 2015a), respectively. From total binary mass estimates of $10^{9.4}$, 10$^{10.3}$, and 10$^{9.9}$, this corresponds to putative angular separations of the binary orbits of 4, 12, and 10 μas.

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5 See Schutz & Ma (2016) for constraints on the mass ratios of a nearby most massive population of putative MBHBs.
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Appendix A

Millimeter-wavelength AGN Luminosity Function

We use a redshift-dependent luminosity function of radio-loud AGNs that takes into account both density evolution and luminosity evolution (Yuan et al. 2017),

\[
d\mathcal{N}/dLdV = e_1(z) \frac{\phi}{L_{408}/e_2(z)} \ln 10 \left( \frac{L_{408}}{L_*} \right)^{-\beta} 
\times \exp \left[ - \frac{L_{408}}{e_2(z)} \right] 
\]

\[
e_1(z) = \left( \frac{1 + z}{1 + z_c} \right)^{p_1} + \left( \frac{1 + z}{1 + z_c} \right)^{p_2}, 
\]

\[
e_2(z) = 10^{k_1 z + k_2 z^2}, 
\]

\[
L_* = 10^{24.79} \text{ W Hz}^{-1}, \quad \phi = 10^{-4.72} \text{ Mpc}^{-3}, 
\]

\[
p_1 = -1.29, \quad p_2 = 6.80, 
\]

\[
\beta = 0.45, \quad \gamma = 0.31, \quad k_1 = 1.44, 
\]

\[
k_2 = -0.16, \quad z_c = 0.78, 
\]

\[ (24) \]

where \( L_{408} \) is the specific luminosity at 408 MHz. This is scaled to a mm-wavelength luminosity function as described in the main text. We note for ease of reproducibility that in Table 1 of Yuan et al. (2017), the values of \( k_1 \) and \( k_2 \) are erroneously swapped, while the values of \( p_1 \) and \( p_2 \) are erroneously stated as their negatives (needed to reproduce Figures 3 and 4 in Yuan et al. 2017).

Appendix B

The Size of the Mm-wavelength Emission Region in LLAGNs

We require that the size of the mm-wavelength emission region be smaller than the binary separation. We estimate the size of the mm-emission region with the jet model of Blandford & Königl (1979).

The radio and mm emission from MBHs is likely from synchrotron emission generated by shocks in a jet. At low frequencies, synchrotron radiation is optically thick to self-absorption, and its spectrum rises as \( F_\nu \propto \nu^{5/2} \) until the radiation becomes optically thin at frequency \( \nu_{\text{star}} \). For higher frequencies, the optically thin spectrum falls off as \( F_\nu \propto \nu^{-(\beta-1)/2} \), where the electrons are assumed to be distributed as \( N_e = K_e \gamma^{-p} \). At even higher frequencies, however, the synchrotron losses are great enough to cool the radiation within a jet expansion time. Then, above some \( \nu_{\text{loss}} \), the synchrotron flux drops more steeply as \( F_\nu \propto \nu^{-(\beta-1)/2-0.5} \).

Both limiting frequencies, and hence the shape of the synchrotron spectrum, depend on radial position within the jet. The self-absorption frequency \( \nu_{\text{star}} \), for which only higher-frequency synchrotron photons can escape, becomes larger at larger radii in the jet. The maximum frequency for which synchrotron losses are negligible, \( \nu_{\text{loss}} \), decreases with radius in the jet. Hence, there is a minimum jet radius \( r_{\text{min}} \) below which no bright, optically thin synchrotron spectrum exists. This minimum radius is given by equating \( \nu_{\text{star}}(r) = \nu_{\text{loss}}(r) \).

Then, for the mm-emission regions to be smaller than the binary separation, we must have \( \nu_{\text{star}}(a) < \nu_{\text{mm}} < \nu_{\text{loss}}(a) \). In practice, we evaluate this inequality for a given binary separation \( a \) and set the resolvable MBBH probability \( \mathcal{F} \) equal to zero if it is not satisfied.

The self-absorption frequency is given by (Equation 28 of Blandford & Königl 1979)

\[
\nu_{\text{star}} = \frac{300}{1 + z} k_e^{1/3} \left( \frac{\Delta}{1 + 2 k_e} \right)^{2/3} \gamma^{-4/3} \beta^{-2/3} D_j^{2/3} 
\times (\sin \theta)^{-1/3} \phi_{\text{obs}}^{-1/3} \frac{r_{\text{obs}}}{0.01 \text{ pc}} \text{ GHz}, 
\]

\[ (25) \]

where \( k_e \ll 1 \), \( \Delta = \ln(105) \), \( \gamma_j = (1 - \beta_j^2)^{-1/2} \) is the Lorentz factor of the jet, \( D_j \) is the jet Doppler factor, \( \theta \) is the viewing angle of the jet, \( \phi_{\text{obs}} = \phi/\sin \theta \) is the observed opening angle of the jet, \( L_{44} \) is the bolometric luminosity of the jet in units of \( 10^{44} \text{ erg s}^{-1} \), and \( r_{\text{obs}} = r \sin \theta \) is the observed distance from the jet launching point to the emission region. This suggests that sources with \( L \sim 10^{44} \text{ erg s}^{-1} \) are optically thin at mm wavelengths down to size scales of \( r \sim 0.01 \text{ pc} \). Lower-luminosity sources are visible down to even smaller scales.

The maximum frequency for which synchrotron losses are negligible,

\[
\nu_{\text{loss}} = \frac{0.07}{1 + z} \gamma_j^{\frac{2}{3}} \beta^{-1/2} D_i \sin \theta B_i^2 r_{\text{obs}}, 
\]

\[ (26) \]

where \( B_i \) is the magnetic field strength at \( r = 1 \text{ pc} \),

\[
B_i = 2 \left[ \Delta \left( 1 + \frac{2}{3} k_e \right) \right]^{1/2} \phi (\beta_j c)^{-1/2} \gamma_j 
\times \left( \frac{r}{1 \text{ pc}} \right)^{-1/2} \left( \frac{L}{10^{44} \text{ erg s}^{-1}} \right)^{1/2}, 
\]

\[ (27) \]

is found by equating the total power in the jet to that carried away by relativistic electrons and the magnetic field (Equation 23 of Blandford & Königl 1979).

Throughout, we use \( k_e = 1 \), \( \Delta = \ln(105) \), \( \gamma_j = 10 \), \( \phi = 1/\gamma_j \), and \( \theta = 0.1 \). The choice of \( \theta \) is conservative, as it yields the maximum value of \( \nu_{\text{star}} \).

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