Performance of bidimensional location quotients for constructing input-output tables

Abstract
This article seeks to verify the extent at which the formulation of two-dimensional location quotients (2D-LQ) entails a methodological advance in building or generating economic accounts related to sub-territories drawing from basic information. The input-output tables of the Euro Area 19 for 2010 and 2015 are taken as a reference for analysis, and five statistics are used to measure the degree of similarity between the true domestic coefficient matrices for ten countries (namely Austria, Belgium, Estonia, France, Germany, Italy, Latvia, Slovakia, Slovenia and Spain) and the matrices generated from them using non-survey techniques CILQ, FLQ, AFLQ and 2D-LQ. Focus is substantially centred on comparing the results from the four aforementioned techniques to rank methodological efficiency. Standard parameters (associated with 2D-LQ) are also provided as a guide in this scope of work with a view to ascertaining optimal parameters.

Keywords Location quotients · FLQ · 2D-LQ · Non-survey method · Regional input-output tables

JEL Classification C13 · C67 · R19

1 Introduction
Input-output (IO) analysis is commonly used in economics. Indeed, the different associated multisectoral models are useful for quantifying various impacts (economic, environmental, etc.) for territories with IO survey frameworks, usually regions, countries or supranational unions. However, the applicability of this tool becomes complicated when implemented at a sub-territorial level, from small regions to other sorts of areas (e.g. counties), since there are no accounting frameworks for the elevated costs to create, and even lack of basic information to execute, robust projects. Even so, non-survey techniques are still used to generate sub-territorial tables based on the analytical capacity of the characteristic IO sector breakdown.
In general, economic accounts are available for a given territory, which will be used as a reference. There are nevertheless data for sub-territories (resulting from segregation) concerning certain basic magnitudes (industry production, employment or gross added value) and matching the same year and with the same sectoral breakdown. It is thus commonplace to generate an IO table at the local, regional or national level using the data available for a higher territorial level. While different methods are used for this purpose (Morrison and Smith 1974; Schaffer and Chu 1969; Bonfiglio and Chelli 2008), Location Quotients (LQs) are the most commonly used methods, especially Flegg’s location quotient (FLQ) or a modified version thereof, i.e., the augmented FLQ (AFLQ). Different studies have sustained that LQs are an advance in generating IO tables (Flegg and Webber 1997, 2000; Flegg et al. 1995). It is therefore essential to select which LQ will be used, either alone or supplemented by adjustment techniques (Lamonica et al. 2020). While there is no clear majority on which LQ yields the best results, some studies (Bonfiglio and Chelli 2008; Jahn et al. 2020) show the prevalence of FLQ and AFLQ, yet others (Zhao and Choi 2015; Lamonica and Chelli 2018) oppose and favour other ratios.

FLQ and AFLQ techniques have a parameter associated with the size of a certain magnitude of the sub-territory that should be delimited within an interval. Its value varies from one sub-territory to another. Multiple research papers delve into the search for the optimal value of this parameter (Kowalewksi 2015; Flegg and Tohmo 2016; Lamonica and Chelli 2018). This unknown value becomes problematic and its calculation is thus arduous (Lampiris et al. 2019), probably because it is quite sensitive due to the design of the corresponding formulas. Recently and in a context of identical available information, Pereira-López et al. (2020) carried out a two-dimensional reformulation of LQs (for domestic flow tables, though extrapolated to total flows with certain nuances), thus employing two parameters. However, these parameters are not associated with the size of the sub-territories but rather with the degree of specialisation of the various branches of activity and sector size (by rows and columns, respectively). Their sensitivity will thus differ from the FLQ and AFLQ parameters.

In short, the process of generating sub-territorial IO tables has not yet been clearly defined. Researchers are immersed in a search for not only the most suitable LQ but also parameters capable
of yielding robust results. The main purpose of this paper is therefore to obtain a performance of LQs and, in particular, uncover the most effective way to ascertain the standard parameters used in their formulations, with special attention to the 2D-LQ. This paper is structured so that after the present introduction (section 1), section 2 provides a review of the LQs. Section 3 describes the data used. The section 4 contains an analysis of traditional LQs and the 2D-LQ method. Finally, section 5 compares the four examined LQs, identifies guiding parameters for 2D-LQ and indicates the main conclusions drawn.

2 Location quotients

This section contains a brief description of the main LQs. Further details can be found by consulting numerous studies, including Schaffer and Chu (1969), Morrison and Smith (1974), Round (1978), Flegg and Webber (1997, 2000), Miller and Blair (2009) and Pereira-López et al. (2020).

The Simple Location Quotient (SLQ) is the most common approach, which compares the relative weight of a certain sectoral magnitude of a sub-territory with its relative weight in the territory. Analytically,

\[
SLQ_i = \frac{x_i^R}{x_i^N} = \frac{x_i^R}{x_i^N} = \frac{wx_i^R}{wx^R},
\]

where \(x_i^R\) is production (for instance) of sector \(i\) in region \(R\), \(x^R\) is the production in region \(R\), \(x_i^N\) is the production of sector \(i\) in the entire country (\(N\)), and \(x^N\) is the production of the country. Therefore, \(wx_i^R\) represents the weight of the production of sector \(i\) in the total production of the sector; and \(wx^R\) corresponds to the participation of the production of region \(R\) in the total production of the country.

This LQ in some way indicates whether the sector can be self-sufficient or an exporter, or whether the sector imports from other regions. However, it does not take into account the importance of the purchaser section.
The Cross-Industry Location Quotient (CILQ) considers the relative importance of the selling industry with respect to the purchasing industry, as shown below:

\[
CILQ_{ij} = \frac{SLQ_i}{SLQ_j} = \frac{wx_i^R}{wx_j^R},
\]  

(2)

where the subscript \( j \) refers to purchasing sectors.

Given that the formulation above excludes, for the sake of simplification, the size of the region in the process, Flegg and Webber (1997), proposed the FLQ method, which is defined as follows:

\[
FLQ_{ij} = CILQ_{ij} \cdot \left[ \log_2 \left( 1 + \frac{x_j^R}{x_j^N} \right) \right]^\delta, \quad 0 < \delta < 1
\]  

(3)

The effect of region size is usually abbreviated as:

\[
\lambda = \left[ \log_2 \left( 1 + \frac{x_j^R}{x_j^N} \right) \right]^\delta.
\]  

(4)

In this expression, the parameter \( \delta \) is a coefficient associated with interregional imports, after which \( \lambda \) works as a corrective element of the CILQ. Following the standard procedure, the regional technical coefficients \( a_{ij}^R \) are the result of corrections on the national coefficients \( a_{ij}^N \), namely:

\[
a_{ij}^R = a_{ij}^N \cdot FLQ_{ij} \quad \text{if} \quad FLQ_{ij} \leq 1
\]

\[
a_{ij}^R = a_{ij}^N \quad \text{if} \quad FLQ_{ij} > 1
\]  

(5)

McCann and Dewhurst (1998), warned that FLQ does not appropriately address scenarios in which regional industries are more specialised than national industries. Flegg and Webber (2000) then rectified columns (semi-logarithmic smoothing) for specialised purchasing sectors. This resulted in the Augmented FLQ (AFLQ):

\[
AFLQ_{ij} = \begin{cases} 
FLQ_{ij} \cdot \log_2 \left( 1 + SLQ_j \right) & \text{if} \quad SLQ_j > 1 \\
FLQ_{ij} & \text{if} \quad SLQ_j \leq 1 
\end{cases}
\]  

(6)

Greater sectoral specialisation thus leads to an increase in the coefficient and, consequently, a reduction in imports.
As an initial step in designing a generalisation of the Flegg methodology, Pereira-López et al. (2020) proposes a two-dimensional approach (2D-LQ) to estimate domestic coefficients at the sub-territorial level:

\[
\hat{a}_{ij}^R = \begin{cases} 
(SLQ_i)^\alpha a_{ij}^N (wx_j^R)^\beta & \text{if } SLQ_i \leq 1 \\
\frac{1}{2} \tanh(SLQ_i - 1) + 1 \int a_{ij}^N (wx_j^R)^\beta & \text{if } SLQ_i > 1
\end{cases}
\]  

(7)

Scalars $\alpha$ and $\beta$ are corrective parameters associated with rows and columns, respectively. This technique does not use semi-logarithmic smoothing because it seeks to simplify the formula. This method can be extrapolated to other contexts, for instance, for generating flow matrices, total coefficients or multipliers.

3 Data sources

Contrasting estimated coefficients against true coefficients is no easy task for certain regions or small areas, given the insufficiency of data obtained through surveys and even the non-uniformity of the information at different territorial levels, e.g. countries/regions. In this case, we opted to compare and contrast ten (10) Euro Area 19 countries (EA-19). The database downloaded from Eurostat (https://ec.europa.eu/eurostat/web/esa-supply-use-input-tables/data/database) contains symmetric 64x64 matrices at basic prices (product by product) [naio_10_cp1700]. IO tables (2010 and 2015) were then filtered for ten countries, namely, Austria, Belgium, Estonia, France, Germany, Italy, Latvia, Slovakia, Slovenia and Spain. For these purposes, it should be noted that these 10 countries represented 84.39% of EA-19 production (Austria 3.06%, Belgium 4.22%, Estonia 0.17%, France 19.89%, Germany 26.79%, Italy 17.42%, Latvia 0.21%, Slovakia 0.85%, Slovenia 0.39% and Spain 11.40%). Their production volume was 83.79% in 2015: (Austria 3.27%, Belgium 4.33%, Estonia 0.21%, France 19.79%, Germany 28.27%, Italy 15.99%, Latvia 0.24%, Slovakia 0.96%, Slovenia 0.38% and Spain 10.35%). Ireland, Malta, Portugal, Finland, Greece, Lithuania, Netherlands, Cyprus and Luxembourg
did not have the IO tables for one or two years analysed, or they showed confidential or provisional data. For these reasons the remaining nine countries were excluded from the present analysis.

The aforementioned extraction is based on the classification system the European System of Accounts (ESA) 2010, specifically on the Classification of products by Activity (CPA) 2008. We then opted to use sector outputs instead of the employment vector or gross added value, because, according to Flegg and Tohmo (2019) “It should be noted that the SLQ and CILQ are defined in terms of output rather than the more usual employment. Using output is preferable to using a proxy such as employment because output figures are not distorted by differences in productivity across regions”.

4 Analysis

The following statistics were used to measure the similarity between estimated domestic coefficient matrices (CILQ, FLQ, AFLQ and 2D-LQ) and true matrices with a view to ascertaining the most appropriate LQ approach to execute projections of the sub-territorial IO tables. These statistics are Standardized Total Percentage Error (STPE), Mean Absolute Difference (MAD), Mean Absolute Percentage Error (MAPE), Standard Deviation of the Mean Absolute Difference (SD-MAD) and Theil’s Index (U). The following equations are used to calculate these statistics.

\[
STPE = 100 \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} |\tilde{a}_{ij}^R - a_{ij}^R|}{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^R} \quad (8)
\]
\[
MAD = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} |\tilde{a}_{ij}^R - a_{ij}^R| \quad (9)
\]
\[
MAPE = \frac{100}{n^2} \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} |\tilde{a}_{ij}^R - a_{ij}^R|}{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^R} \quad (10)
\]
\[
SD - MAD = \sqrt{\frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} (|\tilde{a}_{ij}^R - a_{ij}^R| - MAD)^2} \quad (11)
\]
\[
U = 100 \sqrt{\frac{\sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij}^R - \tilde{a}_{ij}^R)^2}{\sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij}^R)^2}} \quad (12)
\]
Where $a_{ij}^R$ is the true sub-territorial coefficient—usually regional—and $\tilde{a}_{ij}^R$ is the estimated sub-territorial coefficient; $n$ is the number of products/sectors.

STPE is used to calculate the relative distance in absolute terms between the estimated coefficient and the true coefficient. Multiplying it by one hundred yields error as a percentage (Jalili 2000; Jackson and Murray 2004; Bonfiglio 2005; Flegg et al. 2016; Lampiris et al. 2019). MAD calculates the difference (in absolute value) between estimated and true coefficients, yielding the absolute mean of the differences when divided by the total number of elements in the matrix (Morrison and Smith 1974; Jackson and Murray 2004; Bonfiglio 2005; Bonfiglio and Chelli 2008; Miller and Blair 2009; Kowalewksi 2015; Wiebe and Lenzen 2016; Lamonica and Chelli 2018; Lampiris et al. 2019; Lamonica et al. 2020). MAPE is practically the average of STPE (Oosterhaven et al. 2003; Minguez et al. 2009; Miller and Blair 2009; Lampiris et al. 2019; Flegg and Tohmo 2019; Jahn et al. 2020). SD-MAD is the standard deviation to the median absolute deviation between the estimated and true coefficients (Lamonica and Chelli 2018). Theil index is known as the inequality index, as it estimates the overall distance ratio, and thus indicates perfect equality when equal to zero (Jalili 2000; Lahr and Stevens 2002; Jackson and Murray 2004; Bonfiglio 2005; Flegg and Tohmo 2013; Kowalewksi 2015; Flegg et al. 2016; Flegg and Tohmo 2019; Lampiris et al. 2019; Jahn et al. 2020). This study compares matrices element by element, unlike other works, which focus solely on sums by rows or columns through a matrix of coefficients or the Leontief inverse matrix. There is inaccuracy when working with sum vectors (rows or columns), since errors can be offset easily.

4.1 Sensitivity Analysis of traditional location quotients

The starting point begins with sub-territorial coefficients generated by CILQ, FLQ and AFLQ. As we have seen from (3) to (6), the last two equations incorporate the parameter $\delta$ (as an exponent), which is somehow associated with interregional imports. There have been numerous discussions regarding the optimal value for this parameter, though it is logical to have it vary according to the size of the sub-territory, since, in reality, the goal is to ascertain a suitable $\lambda$ that depends on $\delta$. For example,
Flegg and Webber (2000) consider that in the absence of information they suggest assigning 0.3 as the value for $\delta$. However, in a study for the Italian region of le Marche through the Monte Carlo simulation, Bonfiglio (2009) maintains that this parameter is centred on 0.3 (for FLQ) with an associated probability of 33% (if the width of the interval it is set at 0.1), and between 0.3 and 0.4 for AFLQ, with a probability of 38%. In a study for 20 regions in Finland, Flegg and Thomo (2013) set this figure between 0.15 and 0.35. The results concurred with the Bonfiglio study that an optimal value of 0.3 can only be expected in a third of the regions and that an optimal value has yet to be found. Kowalewksi (2015) applied an extension of the Flegg methodology and revealed values between 0.11 and 0.17, which are relatively low compared to previous studies. Lampiris et al. (2019) compared technical coefficient matrices and estimated Leontief inverse matrices using traditional LQs for several EU countries. Their results allow us to affirm that AFLQ and FLQ provide better results for the values of $\delta$ between 0.1 and 0.3, yet prove unsatisfactory for values higher than 0.3.

Figs. 1 and 2 show the STPEs related to traditional LQs for the ten countries studied (2010 and 2015). Both figures show that FLQ and AFLQ curves are convex around the optimum, yet exceed the (constant) value of CILQ considerably from certain thresholds marked by values of $\delta$, although when $\delta$ tends to 1, the curves behave nearly asymptotic (horizontal) and virtually converge. Once breaching thresholds, these two techniques must be ruled out to the detriment of the CILQ equation, even though the latter is much more elemental in nature. As a general guideline, one can conclude that $\delta$ is quite sensitive when it tends to 1 on the left (values between 0 and 1) and the values of the statistics would shoot off if selecting the wrong value, i.e. the results would be questionable.

However, the substantial is clearly given by the degree of approximation of the different matrices. The larger countries therefore seem to behave better than smaller ones, which should not be surprising given that the higher their proportion, the more productive structures will resemble the reference area. The STPEs for France, Germany, Italy and Spain (2010 and 2015), are lower than those

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1 The charts are all associated with the STPE statistic. The results of the remaining statistics (MAD, MAPE, SD-MAD and Theil) are specified in Table 1 in Appendix.
for the other six countries analysed. These results concur with the figures indicated for 2005 in Pereira-López et al. (2020). There is a similar diagnosis in relation to the other four statistics. Refer to the Appendix.

While the mentioned figures are considered to be quite explicit, certain $\delta$ parameters of the two curves under study intersect the CILQ line (not depending on $\delta$). Thus, out of the ten countries analysed in 2010, only Belgium allows us to assign the maximum value to the parameter for FLQ, which must be less than or equal to 0.47 and for AFLQ, which must be less than or equal to 0.5. France, Germany, Italy and Spain yielded smaller relative distances between CILQ and the optimum associated with AFLQ, improving results by 4.46%, 2.86%, 8.23% and 3.20% respectively. The other countries meanwhile show greater distances, as clearly seen in Fig. 1. Nearly the same curves and corresponding intersections with the CILQ line reappear for 2015. For instance, the following extreme values: 0.52 in Belgium (for FLQ) and 0.57 in Germany (for AFLQ). France, Germany, Italy and Spain yielded smaller relative distances between CILQ and the optimum for AFLQ (figures relatively similar to 2010 figures, namely 5.15%, 3.91%, 3.93% and 3.51% respectively). Once again, the other countries mark wider distances, though there is also more room for improvement since the STPEs are higher.
Compared to FLQ, AFLQ slightly reduces errors in matrix estimates. This circumstance repeats for virtually all the countries in 2010 and 2015. The sole exceptions are Slovenia (2010 and 2015) and Estonia (2015), where positions are exchanged between both techniques. This leads us to work with AFLQ as the most efficient traditional technique, albeit aware of the need to ascertain an optimal $\delta$, which is conditioned by the size of the sub-territories. In light of the aforementioned figures, when the value of $\delta$ exceeds 0.3, FLQ and AFLQ are no longer effective techniques and CILQ thus becomes preferred to forestall estimation errors. One may somehow surmise that the Flegg equation incorporates basic information (overall size of sub-territory) concretely in the estimation process and
alternatives could be sought to address this information efficiently and thus avoid using the highly sensitive $\delta$, particularly from a given value (as indicated above). This is the key to 2D-LQ design, construed as one of the possible generalisations of the Flegg’s formula.

4.2 Estimating of parameters of the 2D-LQ method

The 2D-LQ method is characterised by its use of the sectoral degrees of specialisation at the sub-territorial level (by rows), yet with an alternative formulation which excludes the effect of the sub-territorial size at the global level. In other words, it seeks to circumvent the sensitivity of parameter
\( \delta \). This section graphically demonstrates the method’s robustness and also indicates pairs of suitable parameters to apply in future LQ applications. The Appendix contains the values of the global minimum statistics and associated pairs.

Figs. 3 and 4 show three-dimensional, country-by-country representations of the STPE statistic against parameters \( \alpha \) and \( \beta \) for 2010 and 2015. The corresponding contour lines are also highlighted with a fixed gradation by country and year. The optimal pair of parameters and behaviour of the scalar field in its environment are thus clearly visible. The Appendix contains information on the global minimums on each scalar field for STPE and the other four statistics. The scalar fields have a convex behaviour.
Fig. 3 Estimating 2D-LQ for ten EA-19 countries in 2010
Fig. 4 Estimating 2D-LQ for ten EA-19 countries in 2015
The graphical representations for MAD and MAPE are identical, and virtually similar for SD-MAD and U, though the values of the statistics change when relativising distances in another way. In the scalar fields, common patterns are not clearly detected according to the size of the countries. Of course there is a nearly perfect country-by-country match in the fields for the two years studied.

Movements through $\beta$ ($y$ axis) entail greater errors than movements through $\alpha$ ($x$ axis). In general, the minimums tend to stay between 0.12 and 0.84 for $\alpha$ and 0.05 and 0.23 for $\beta$ (in 2010). The ranges in 2015 are quite similar, respectively between 0.28 and 0.80 and 0.10 and 0.23. In light of the obtained STPEs ($z$ axis), the behaviours of France, Germany, Italy and Spain are better than the rest, most likely because of their size in the EA-19. It is moreover understood that generating IO tables for sub-territories with a reduced proportion within the total could be misleading, particularly if no post-adjustment techniques are implemented.

5 Discussion and conclusions

We compare the four studied LQs in this section. This entails extracting combined information from Figs. 1 and 3 on the one hand, and from Figs. 2 and 4 on the other. Scalar fields intercessions are basically carried out based on the traditional LQs for the different countries and the two years studied. This yields areas delimited by contour lines conditioned by the CILQ, FLQ and AFLQ values. The order of validity of the techniques (from lowest to highest) has so far appeared as follows: CILQ, FLQ, AFLQ and 2D-LQ. However, positions are exchanged between FLQ and AFLQ in some cases for a minimal difference from the statistics, namely Slovenia (2010 and 2015) and Estonia (2015), as indicated above.

We have opted for mapping the different countries (2010 and 2015) with a view to condensing results. Figs. 5 and 6 focus on rendering an effectively staggered 2D-LQ compared to the other techniques: CILQ, FLQ and AFLQ. The figures are clearly interpretable. The central core expresses the superiority of 2D-LQ over the next most efficient technique, which is almost always AFLQ. An intermediate ring appears to mark the distance between AFLQ and FLQ (though this ring clearly does not exist in the three noted exceptions). Finally, an outer ring reflects the superiority of FLQ over
CILQ. The shapes of the areas have some homogeneity and the global minima given by optimal pairs (2D-LQ) are more or less centred. There are numerous combinations of parameters that ensure better statistics compared to the other techniques. This merely requires looking at the epicentres and recalling the convexity of the scalar fields in Figs. 3 and 4. In relation to the degree of rigidity of parameters $\alpha$ and $\beta$, $\beta$ is clearly more sensitive; i.e., small changes lead to bigger errors. In effect, the ratio used between $\alpha$ and $\beta$ to design the charts is 4/1.

These figures exclude the STPE values, though it is evident that the lower they are, the more difficult it is to reduce them. For comparison, the largest country, Germany (year 2010), reduces the STPE from 56.54 (CILQ) to 54.49 (2D-LQ), so the improvement in stages from CILQ is 1.88% (FLQ), 2.86% (AFLQ) and 3.63% (2D-LQ). This gradual reduction is shown in the corresponding chart. In relation to another much smaller country, Slovakia (year 2010), its STPE went from 80.19 (CILQ) to 72.33 (2D-LQ). The improvements are 6.16% (FLQ), 8.48% (AFLQ) and 9.81% (2D-LQ) as shown in the illustration.

Only in relation to the AFLQ, Figs. 7 and 8 reveal the range of $\alpha$ values (associated with the optimal $\beta$ value) for 2010 and 2015, respectively. Optimal $\beta$ values and less errors in 2D-LQ vs. AFLQ. The intervals are characterised by having a considerable amplitude, i.e. the parameter linked to row rectifications does not excessively incur estimated penalties. This is important, since an average value can be set regardless of the size of the sub-territory, yet still ensure errors lower than the AFLQ. The width of $\beta$ intervals is much smaller than $\alpha$ intervals. In principle, it is possible to work with an average value of $\beta$ around 0.10, with the exception of Germany (larger country). By way of synthesis, it should be noted that the comparison between AFLQ and 2D-LQ techniques affords us a guide to parameters that can be used in this field of work.
Fig. 5 Mapping for ten EA-19 countries in 2010
Fig. 6 Mapping for ten EA-19 countries in 2015
There is no clear relationship between the width of 2D-LQ method parameter ranges and its relative distance with the AFLQ method. Of course, for all the sub-territories studied, the 2D-LQ method has a wide range of parameters that guarantee fewer errors than the AFLQ in the optimal $\delta$ (generally unknown).

In conclusion, in this study matrices were contrasted element by element but not by vector sums for rows or columns. It is considered appropriate to work in this way to avoid possible compensation.
for errors. The results of the statistics are consistent with those of other similar studies. The 2D-LQ method demonstrably improves the estimates of prior LQs (CILQ, FLQ and AFLQ), and this technique is therefore useful yet requires a longer journey, at least for the sake of parameter contrasting. It is nevertheless recommended to supplement IO tables (via 2D-LQ or another LQ) with optimisation processes, so long as there is additional information, e.g. other macroeconomic magnitudes not used in the LQ equations. In this sense, it is pointed out that resorting to basic RAS or cross-entropy (Lamonica et al. 2020) could be somewhat misleading, since LQs are applied in contexts lacking in information. Adjustments are thus suggested for projections secured through the Euromethod or Path-RAS (Mahajan et al. 2018). Both techniques are, in a way, generalisations of the basic RAS and characterised by implementing other types of adjustments in light of the lack of available information. This was in any case not the purpose of this article yet should nevertheless be the object of a future and necessary research.

**Availability of data and materials**

The datasets used and analysed during the current study are available from the corresponding author on request.

**Author’s contributions**

All authors have made substantial contributions. All authors read and approved the final manuscript.

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The authors declare that they have no competing interests.

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## Appendix

### Table 1 Assessment of IO table projections via LQs for EA-19 countries (2010 and 2015)

| Countries | CILQ  | FLQ ($\delta$) | AFLQ ($\delta$) | 2D-LQ ($\alpha; \beta$) | CILQ  | FLQ ($\delta$) | AFLQ ($\delta$) | 2D-LQ ($\alpha; \beta$) |
|-----------|-------|---------------|----------------|----------------------|-------|---------------|----------------|----------------------|
| Austria   | 73.4995 | 69.6879 (0.18) | 66.3139 (0.36; 0.13) | 75.4435 (0.19) | 70.9785 (0.21) | 69.0646 (0.40; 0.15) | 67.0386       |
| Belgium   | 75.4562 | 68.6528 (0.21) | 67.2126 (0.52; 0.16) | 77.0734 (0.25) | 69.3534 (0.26) | 68.7097 (0.68; 0.17) | 67.4587       |
| Estonia   | 92.2069 | 81.6705 (0.13) | 79.6816 (0.40; 0.13) | 94.3536 (0.12) | 82.6840 (0.18) | 82.7006 (0.28; 0.13) | 79.5765       |
| France    | 51.6088 | 50.1284 (0.17) | 48.0810 (0.68; 0.11) | 55.0476 (0.19) | 53.1773 (0.20) | 52.2111 (0.80; 0.13) | 50.2849       |
| Germany   | 56.5391 | 55.4739 (0.18) | 54.4885 (0.12; 0.23) | 58.0644 (0.20) | 56.8606 (0.32) | 55.7934 (0.36; 0.23) | 54.5182       |
| Italy     | 55.5537 | 52.0088 (0.08) | 49.6537 (0.68; 0.09) | 55.2133 (0.12) | 54.2537 (0.15) | 53.0440 (0.68; 0.11) | 51.5449       |
| Latvia    | 91.0184 | 83.7678 (0.15) | 79.2991 (0.44; 0.10) | 90.7480 (0.17) | 82.8537 (0.18) | 79.7378 (0.40; 0.12) | 77.3945       |
| Slovakia  | 80.1889 | 75.2500 (0.15) | 72.3254 (0.56; 0.12) | 83.9898 (0.19) | 77.8713 (0.20) | 76.0804 (0.56; 0.13) | 75.6412       |
| Slovenia  | 82.1660 | 78.6179 (0.09) | 75.4679 (0.20; 0.11) | 84.6056 (0.12) | 78.7981 (0.14) | 78.8749 (0.28; 0.12) | 76.7242       |
| Countries | 2010 | | | 2015 | | | |
|-----------|------------|------------|---|----------------|------------|---|---|
|           | CILQ | FLQ (𝛿) | AFLQ (𝛿) | 2D-LQ (α; β) | CILQ | FLQ (𝛿) | AFLQ (𝛿) | 2D-LQ (α; β) |
| Austria   | 0.0044 | 0.0041 | 0.0040 | **0.0039** | 0.0043 | 0.0040 | 0.0039 | **0.0038** |
|           | (0.18) | (0.20) | (0.36; 0.13) | | (0.19) | (0.21) | (0.4; 0.15) | |
| Belgium   | 0.0046 | 0.0042 | 0.0041 | **0.0041** | 0.0045 | 0.0041 | 0.0040 | **0.0039** |
|           | (0.21) | (0.24) | (0.52; 0.16) | | (0.25) | (0.26) | (0.68; 0.17) | |
| Estonia   | 0.0048 | 0.0042 | 0.0042 | **0.0041** | 0.0048 | 0.0042 | 0.0042 | **0.0040** |
|           | (0.13) | (0.17) | (0.40; 0.13) | | (0.12) | (0.18) | (0.28; 0.13) | |
| France    | 0.0033 | 0.0033 | 0.0032 | **0.0031** | 0.0034 | 0.0033 | 0.0032 | **0.0031** |
|           | (0.17) | (0.20) | (0.68; 0.11) | | (0.19) | (0.20) | (0.80; 0.13) | |
| Germany   | 0.0036 | 0.0036 | 0.0035 | **0.0034** | 0.0036 | 0.0035 | 0.0035 | **0.0034** |
|           | (0.18) | (0.31) | (0.12; 0.23) | | (0.20) | (0.32) | (0.36; 0.23) | |
| Italy     | 0.0038 | 0.0037 | 0.0037 | **0.0036** | 0.0040 | 0.0039 | 0.0038 | **0.0037** |
|           | (0.08) | (0.13) | (0.68; 0.09) | | (0.12) | (0.15) | (0.68; 0.11) | |
| Latvia    | 0.0058 | 0.0053 | 0.0052 | **0.0050** | 0.0050 | 0.0046 | 0.0044 | **0.0043** |
|           | (0.15) | (0.17) | (0.44; 0.10) | | (0.17) | (0.18) | (0.40; 0.12) | |
| Slovakia  | 0.0047 | 0.0045 | 0.0043 | **0.0043** | 0.0050 | 0.0046 | 0.0045 | **0.0045** |
|           | (0.15) | (0.18) | (0.56; 0.12) | | (0.19) | (0.20) | (0.56; 0.13) | |
| Slovenia  | 0.0047 | 0.0045 | 0.0045 | **0.0043** | 0.0047 | 0.0044 | 0.0044 | **0.0043** |
|           | (0.09) | (0.14) | (0.20; 0.11) | | (0.12) | (0.14) | (0.28; 0.12) | |
| Countries  | 2010 |     | 2015 |     |
|-----------|------|-----|------|-----|
|           | CILQ | FLQ | AFLQ (δ) | 2D-LQ (α; β) | CILQ | FLQ (δ) | AFLQ (δ) | 2D-LQ (α; β) |
| Austria   | 0.0191 | 0.0181 | 0.0177 | 0.0173 | 0.0196 | 0.0185 | 0.0180 | 0.0174 |
|          | (0.18) | (0.20) | (0.36; 0.13) | (0.4; 0.15) | (0.19) | (0.21) | (0.25) | (0.26) |
| Belgium   | 0.0196 | 0.0179 | 0.0178 | 0.0175 | 0.0201 | 0.0180 | 0.0179 | 0.0175 |
|          | (0.21) | (0.24) | (0.52; 0.16) | (0.68; 0.17) | (0.25) | (0.26) | (0.26) | (0.26) |
| Estonia   | 0.0240 | 0.0212 | 0.0211 | 0.0207 | 0.0245 | 0.0215 | 0.0215 | 0.0207 |
|          | (0.13) | (0.17) | (0.40; 0.13) | (0.28; 0.13) | (0.12) | (0.18) | (0.18) | (0.18) |
| France    | 0.0134 | 0.0130 | 0.0128 | 0.0125 | 0.0143 | 0.0138 | 0.0136 | 0.0131 |
|          | (0.17) | (0.20) | (0.68; 0.11) | (0.80; 0.13) | (0.19) | (0.20) | (0.20) | (0.20) |
| Germany   | 0.0147 | 0.0144 | 0.0143 | 0.0139 | 0.0151 | 0.0148 | 0.0145 | 0.0142 |
|          | (0.18) | (0.31) | (0.12; 0.23) | (0.36; 0.23) | (0.20) | (0.32) | (0.32) | (0.32) |
| Italy     | 0.0137 | 0.0135 | 0.0133 | 0.0129 | 0.0144 | 0.0141 | 0.0138 | 0.0134 |
|          | (0.08) | (0.13) | (0.68; 0.09) | (0.68; 0.11) | (0.12) | (0.15) | (0.15) | (0.15) |
| Latvia    | 0.0237 | 0.0218 | 0.0213 | 0.0206 | 0.0236 | 0.0216 | 0.0207 | 0.0201 |
|          | (0.15) | (0.17) | (0.44; 0.10) | (0.40; 0.12) | (0.17) | (0.18) | (0.18) | (0.18) |
| Slovakia  | 0.0209 | 0.0196 | 0.0191 | 0.0188 | 0.0218 | 0.0203 | 0.0198 | 0.0197 |
|          | (0.15) | (0.18) | (0.56; 0.12) | (0.56; 0.13) | (0.19) | (0.20) | (0.20) | (0.20) |
| Slovenia  | 0.0214 | 0.0205 | 0.0207 | 0.0196 | 0.0220 | 0.0205 | 0.0205 | 0.0200 |
|          | (0.09) | (0.14) | (0.20; 0.11) | (0.28; 0.12) | (0.12) | (0.14) | (0.14) | (0.14) |
| Spain     | 0.0147 | 0.0147 | 0.0143 | 0.0140 | 0.0159 | 0.0156 | 0.0153 | 0.0152 |
|          | (0.06) | (0.09) | (0.84; 0.05) | (0.76; 0.10) | (0.10) | (0.13) | (0.13) | (0.13) |
| Countries    | 2010          |                | 2015          |                |
|--------------|---------------|----------------|---------------|----------------|
|              | CILQ | FLQ (δ) | AFLQ (δ) | 2D-LQ (α; β) | CILQ | FLQ (δ) | AFLQ (δ) | 2D-LQ (α; β) |
| Austria      | 0.0137 | 0.0137 | **0.0134** | 0.0134 | 0.0135 | 0.0135 | **0.0130** | **0.0129** |
|              | (0.00) | (0.05) | (0.40; 0.02) |          | (0.01) | (0.06) | (0.48; 0.03) |          |
| Belgium      | 0.0151 | 0.0144 | **0.0144** | 0.0144 | 0.0153 | 0.0141 | **0.0140** | 0.0141 |
|              | (0.12) | (0.15) | (0.32; 0.08) |          | (0.15) | (0.18) | (0.48; 0.09) |          |
| Estonia      | 0.0150 | 0.0132 | 0.0134 | **0.0133** | 0.0151 | 0.0129 | 0.0132 | **0.0129** |
|              | (0.09) | (0.15) | (0.20; 0.07) |          | (0.10) | (0.16) | (0.16; 0.09) |          |
| France       | 0.0086 | 0.0084 | 0.0081 | **0.0075** | 0.0092 | 0.0087 | 0.0082 | **0.0075** |
|              | (0.09) | (0.13) | (0.68; 0.05) |          | (0.15) | (0.19) | (0.64; 0.08) |          |
| Germany      | 0.0102 | 0.0102 | 0.0101 | **0.0098** | 0.0101 | 0.0101 | 0.097 | **0.0093** |
|              | (0.00) | (0.09) | (0.40; 0.02) |          | (0.00) | (0.11) | (0.52; 0.04) |          |
| Italy        | 0.0108 | 0.0100 | 0.0097 | **0.0096** | 0.0112 | 0.0102 | 0.0100 | **0.0096** |
|              | (0.19) | (0.24) | (0.44; 0.11) |          | (0.19) | (0.23) | (0.48; 0.14) |          |
| Latvia       | 0.0168 | 0.0161 | 0.0156 | **0.0152** | 0.0167 | 0.0166 | 0.0159 | **0.0152** |
|              | (0.06) | (0.13) | (0.24; 0.04) |          | (0.02) | (0.10) | (0.32; 0.03) |          |
| Slovakia     | 0.0140 | 0.0139 | 0.0137 | **0.0135** | 0.0155 | 0.0154 | 0.0150 | **0.0150** |
|              | (0.02) | (0.10) | (0.28; 0.04) |          | (0.03) | (0.10) | (0.28; 0.03) |          |
| Slovenia     | 0.0145 | 0.0140 | 0.0145 | 0.0141 | 0.0162 | 0.0156 | 0.0157 | **0.0154** |
|              | (0.05) | (0.10) | (0.04; 0.06) |          | (0.06) | (0.11) | (0.12; 0.06) |          |
| Spain        | 0.0110 | 0.0109 | 0.0105 | **0.0103** | 0.0125 | 0.0123 | **0.0120** | 0.0121 |
|              | (0.06) | (0.11) | (0.72; 0.03) |          | (0.08) | (0.13) | (0.48; 0.04) |          |
| Countries  | 2010       | 2015       |
|-----------|------------|------------|
|            | CILQ (\(\delta\)) | AFLQ (\(\delta\)) | 2D-LQ (\(\alpha; \beta\)) | CILQ (\(\delta\)) | AFLQ (\(\delta\)) | 2D-LQ (\(\alpha; \beta\)) |
| Austria    | 61.1455 (0.00) | 59.6314 (0.06) | 59.8916 (0.40; 0.03) | 61.4834 (0.01) | 59.0615 (0.07) | 59.0347 (0.48; 0.04) |
| Belgium    | 72.3667 (0.13) | 68.6360 (0.15) | 68.7739 (0.32; 0.09) | 74.7101 (0.16) | 68.7492 (0.19) | 68.4625 (0.48; 0.09) |
| Estonia    | 82.3278 (0.09) | 73.7562 (0.15) | 73.0498 (0.20; 0.07) | 87.1358 (0.10) | 76.3152 (0.16) | 74.4023 (0.16; 0.09) |
| France     | 47.0037 (0.09) | 44.0643 (0.13) | 41.3462 (0.68; 0.06) | 51.6500 (0.16) | 46.6069 (0.19) | 42.7677 (0.64; 0.08) |
| Germany    | 49.6259 (0.00) | 49.3755 (0.10) | 47.8575 (0.36; 0.04) | 50.3366 (0.00) | 48.8274 (0.12) | 46.8283 (0.52; 0.06) |
| Italy      | 58.2963 (0.18) | 53.0215 (0.23) | 52.0694 (0.48; 0.11) | 61.7249 (0.18) | 55.6855 (0.18) | 53.5532 (0.48; 0.13) |
| Latvia     | 78.4507 (0.07) | 72.7200 (0.13) | 70.7718 (0.24; 0.05) | 72.9161 (0.03) | 69.0737 (0.10) | 66.1865 (0.32; 0.04) |
| Slovakia   | 65.9687 (0.03) | 64.3763 (0.10) | 63.6510 (0.28; 0.04) | 69.3574 (0.04) | 66.8900 (0.11) | 66.9878 (0.32; 0.04) |
| Slovenia   | 74.6075 (0.05) | 74.3736 (0.10) | 72.2452 (0.08; 0.06) | 79.5425 (0.07) | 77.0159 (0.12) | 75.8576 (0.16; 0.07) |
| Spain      | 52.9878 (0.06) | 50.4862 (0.11) | 49.5333 (0.72; 0.03) | 58.4636 (0.08) | 55.9103 (0.13) | 56.5230 (0.52; 0.05) |
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