New $G$-parity violating amplitude in the $J/\psi$ decay?

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The $J/\psi$ meson has negative $G$ parity so that, in the limit of isospin conservation, its decay into $\pi^+\pi^-$ should be purely electromagnetic. However, the measured branching fraction $B(J/\psi \to \pi^+\pi^-)$ exceeds by more than 4.5 standard deviations the expectation computed according to BABAR data on the $e^+e^- \to \pi^+\pi^-$ cross section. The possibility that the two-gluon plus one-photon decay mechanism is not suppressed by $G$-parity conservation is discussed, even by considering other multipion decay channels. As also obtained by phenomenological computation, such a decay mechanism could be responsible for the observed discrepancy. Finally, we notice that the BESIII experiment, having the potential to perform an accurate measurement of the $e^+e^- \to \pi^+\pi^-$ cross section in the $J/\psi$ mass energy region, can definitely prove or disprove this strong $G$-parity-violating mechanism by confirming or confuting the BABAR data.

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I. INTRODUCTION

The $J/\psi$ meson, like all the isoscalar vector mesons, having total angular momentum $J = 1$, negative $C$-parity, $C = -1$, and isospin zero, $I = 0$, possesses a well-defined $G$ parity, i.e., $G = -1$. Indeed, particles that are eigenstates of the charge conjugation with eigenvalue $C$, are also eigenstates of $G$ parity with eigenvalue $G = C(-1)^I$, where $I$ is the isospin.

$G$ parity is particularly useful because it is well defined also for those particles, which are not $C$-parity eigenstates, as those belonging to isospin multiplets, that have all the same value of $G$. Moreover, being a multiplicative quantum number, states containing particles eigenstates of $G$ parity are themselves eigenstates of $G$ parity with eigenvalue equal to the product of those of each particle. A state with $n$ pions and no other particles has total $G$ parity, $G_n = (G_p)^n = (-1)^n$, since each pion, belonging to the same isospin multiplet, has the same $G$ parity, i.e., $G_p = -1$.

The strong interaction conserves $G$ parity, so that $G$ is a good quantum number in QCD; on the contrary, the electromagnetic interaction can violate the isospin conservation and hence $G$ parity.

II. $J/\psi$ DECAY AMPHITUDES

The amplitude for the decay $J/\psi \to \mathcal{H}_q$, where $\mathcal{H}_q$ represents a final state containing only light hadrons, is usually parametrized as the sum of the three main contributions, $A_{3\gamma}$, $A_{2\gamma\gamma}$, and $A_\gamma$, whose Feynman diagrams are shown in Fig. 1.

In general the amplitude $A_\mathcal{I}$ describes the decay chain $J/\psi \to \mathcal{I} \to \mathcal{H}_q$, i.e., the $J/\psi$ decay mediated by the virtual state $\mathcal{I}$ that could be three gluons, $\mathcal{I} = 3g$, two gluons plus one photon $\mathcal{I} = 2g + \gamma$, and a single photon, $\mathcal{I} = \gamma$. The branching fractions for these $J/\psi$ decays, except for $\mathcal{I} = \gamma$, for which the one in the on-shell $\mu^+\mu^-$ final state is reported, are

$$B(J/\psi \to 3g) = \frac{|A_{3\gamma}|^2 \cdot PS_{3g}}{\Gamma_{J/\psi}}$$

$$= \frac{40(\pi^2 - 9)}{81\Gamma_{J/\psi}} \alpha^3(M_{J/\psi}) \left| \frac{\Psi(0)}{m_c^2} \right|^2$$

$$\times \left( 1 + 4.9 \frac{\alpha(M_{J/\psi})}{\pi} \right),$$

(1)
usually considered negligible \[1\] with respect to the mass of the charm quark, \(\Psi(r)\) is the \(c\bar{c}\) wave function, and the quantities in parentheses are the first-order QCD corrections at the \(J/\psi\) mass. Equations (1) and (2) represent the branching fractions for the decays of the \(J/\psi\) into the intermediate states \(3g\) and \(2g + \gamma\) considered as on shell. The decay mode of Eq. (2) is usually considered negligible \[1\] with respect to the purely electromagnetic one of Eq. (3) and it has been ignored so far. This assumption is reconsidered later on. In Eq. (3) the amplitude \(A_{J/\psi}\) is contracted with the pointlike electromagnetic current \(J_{\mu+\mu^-}\). The branching fraction \(B(J/\psi \rightarrow \mu^+\mu^-)\) can be related to that of the one-photon exchange decay of \(J/\psi\) into a hadronic final state, \(\mathcal{B}_\gamma(J/\psi \rightarrow \mu^+\mu^-)\), by considering the nonresonant dressed \[2\] cross section\[^1\] at the \(J/\psi\) mass, as

\[
B(J/\psi \rightarrow \gamma) = \frac{|A_{2g2f}|^2 \cdot PS_{2g2f}}{\Gamma_{J/\psi}} = \frac{128(\alpha^2 - 9)}{81\Gamma_{J/\psi}} \alpha^2 (M_{J/\psi}) \frac{\Psi(0)}{M_{c}^2} \times \left(1 - 0.9 \frac{\alpha_s(M_{J/\psi})}{\pi}\right), \tag{2}
\]

\[
B(J/\psi \rightarrow \mu^+\mu^-) = \frac{|A_{J/\psi \mu^+\mu^-}|^2 \cdot PS_{\mu^+\mu^-}}{\Gamma_{J/\psi}} = \frac{64\pi}{9\Gamma_{J/\psi}} \frac{\alpha^2}{M_{J/\psi}^2} \left(1 - \frac{16\alpha_s(M_{J/\psi})}{3\pi}\right), \tag{3}
\]

where \(\text{PS}_f\) is the phase space for the final state \(f\), \(m_c\) is the mass of the charm quark, \(\Psi(r)\) is the \(c\bar{c}\) wave function, and the quantities in parentheses are the first-order QCD corrections at the \(J/\psi\) mass. Equations (1) and (2) represent the branching fractions for the decays of the \(J/\psi\) into the intermediate states \(3g\) and \(2g + \gamma\) considered as on shell. The decay mode of Eq. (2) is usually considered negligible \[1\] with respect to the purely electromagnetic one of Eq. (3) and it has been ignored so far. This assumption is reconsidered later on. In Eq. (3) the amplitude \(A_{J/\psi}\) is contracted with the pointlike electromagnetic current \(J_{\mu+\mu^-}\). The branching fraction \(B(J/\psi \rightarrow \mu^+\mu^-)\) can be related to that of the one-photon exchange decay of \(J/\psi\) into a hadronic final state, \(\mathcal{B}_\gamma(J/\psi \rightarrow \mu^+\mu^-)\), by considering the nonresonant dressed \[2\] cross section\[^1\] at the \(J/\psi\) mass, as

\[
B_{\gamma}(J/\psi \rightarrow \mathcal{H}_q) = \mathcal{B}(J/\psi \rightarrow \mu^+\mu^-)
\]

\[
\times \frac{\sigma(e^+e^- \rightarrow \mathcal{H}_q)}{\sigma^0(e^+e^- \rightarrow \mu^+\mu^-)} \sqrt{q^2 - m_{J/\psi}^2}, \tag{4}
\]

where \(\sigma^0(e^+e^- \rightarrow \mu^+\mu^-)\) stands for the bare (vacuum-polarization corrected) cross section.\[^2\] The detailed derivation of Eq. (4) is reported in the Appendix. An upper limit can be obtained by considering all possible hadronic final states

\[
B_{\gamma}(J/\psi \rightarrow \mathcal{H}_q) < \mathcal{B}(J/\psi \rightarrow \mu^+\mu^-) \times \left| \frac{\sigma(e^+e^- \rightarrow \mathcal{H}_q)}{\sigma^0(e^+e^- \rightarrow \mu^+\mu^-)} \right| \sqrt{q^2 - m_{J/\psi}^2}
\]

\[
= \mathcal{B}(J/\psi \rightarrow \mu^+\mu^-) R_{\text{had}}(M_{J/\psi}), \tag{5}
\]

where \(R_{\text{had}}\) is the ratio of the bare or dressed nonresonant hadronic and muon cross sections in \(e^+e^-\) collisions and \(\alpha^2(\sqrt{q^2})\) is the QED running coupling constant \[3\]. Such inequality is saturated once the sum over all possible hadronic final states for the \(J/\psi\) decay is considered, so that

\[
\mathcal{B}_\gamma(J/\psi \rightarrow \text{hadrons}) = \sum_{\mathcal{H}_q} B_{\gamma}(J/\psi \rightarrow \mathcal{H}_q)
\]

\[
= \mathcal{B}(J/\psi \rightarrow \mu^+\mu^-) R_{\text{had}}(M_{J/\psi}) \times \left| \frac{\sigma(e^+e^- \rightarrow \mathcal{H}_q)}{\sigma^0(e^+e^- \rightarrow \mu^+\mu^-)} \right| \sqrt{q^2 - m_{J/\psi}^2}
\]

\[
= 2.63 \cdot \mathcal{B}(J/\psi \rightarrow \mu^+\mu^-). \tag{6}
\]

The value 2.63 has been obtained by using the nonresonant values \(R_{\text{had}}(M_{J/\psi}) = 2.5\) \[4\] and \(\left| \frac{\sigma(M_{J/\psi})}{\sigma^0(e^+e^- \rightarrow \mu^+\mu^-)} \right|^2 = 1.05\) \[3,5\]. In the case of a hadronic final state with negative \(G\) parity, as those containing only an odd number of pions, the strong amplitude \(A_{3g}\) is the dominant one. Moreover, by using the value \(\alpha_s(M_{J/\psi}) = 0.135 \pm 0.015\), as extracted from the data on the ratio \(\mathcal{B}(J/\psi \rightarrow 3g)/\mathcal{B}(J/\psi \rightarrow 2g + \gamma)\) \[4\] and Eqs. (1)–(3), we obtain the following ratios of branching ratios:

\[^1\]By dressed cross section we mean a cross section that includes the vacuum-polarization effects. The dressed Born cross section for the annihilation \(e^+e^- \rightarrow \mathcal{H}_q\) is obtained by multiplying the corresponding bare cross section by \(\left| \alpha(\sqrt{q^2}) \right|^2/\alpha^2\), i.e.,

\[
\sigma(e^+e^- \rightarrow \mathcal{H}_q)(\sqrt{q^2}) = \sigma^0(e^+e^- \rightarrow \mathcal{H}_q)(\sqrt{q^2}) \frac{\left| \alpha(\sqrt{q^2}) \right|^2}{\alpha^2},
\]

where the superscript 0 stands for bare cross section, \(\alpha = e^2/(4\pi) = 1/137\) is the QED fine-structure constant, and \(\alpha(\sqrt{q^2})\) is the QED running coupling constant, which includes vacuum-polarization effects.

\[^2\]See footnote 1.
proceed mainly through the intermediate states.

Concerning the $J = \pi^+\pi^-$ cross section, the only set of data that reaches $\sqrt{q^2} = 3$ GeV is the one collected by the BABAR experiment in 2006 [2], by means of the initial state radiation techniques (ISR). However, because of the large errors and the presence of structures nearby, the local fitting procedure, used in the previous cases, gives unreliable results. To avoid this limitation, the fitting procedure of Ref. [2] has been exploited. The fit function of the cross section, based on the Gounaris-Sakurai model [9] for the pion form factor, is shown in Fig. 3 superimposed to the data. Even though the BABAR data are on the bare cross section, the dressed value is recovered by using directly the pion form factor parametrization, which indeed is related to the dressed cross section. Moreover, since the Gounaris-Sakurai model does not embody the $J/\psi$ contribution, the cross section value, extrapolated at $\sqrt{q^2} = M_{J/\psi}$, reported in Table I, is consequently not resonant. The one-photon contribution is considered negligible with respect to the single-photon one and is therefore ignored so far.



III. EVEN MULTIPION FINAL STATES

As already discussed in Sec. I, multipion final states, having well-defined $G$ parity, represent useful and clean channels to test different models to parametrize the decay amplitudes and hence hypotheses about the dynamical mechanisms that rule the decay.

In particular, amplitudes of $J/\psi$ decay into even numbers of pions; i.e., final states with $G = +1$ are assumed to be dominated by $A_3$, because $G$-parity conservation does not allow pure gluonic intermediate states.

Some $G$-parity-violation decay, not related to an electromagnetic contribution, has been observed, being interpreted as due to $G$-parity violation in the produced mesons, like in the case of $\rho - \omega$ or $f_0 - a_0$ mixing.

Figure 2 shows the nonresonant dressed cross section data and fits\(^3\) in the $\sqrt{q^2} \geq 2.4$ GeV region, in case of $3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$ and $2(\pi^+\pi^-)$ [6] final states. To extract the nonresonant cross section values, reported in the first three rows of Table I, the data points lying in the $J/\psi$ resonance region have been removed and hence not considered in the fitting procedures.

Of particular interest are the decays of $J/\psi$ into final states with positive $G$ parity, $G = +1$, as for instance those consisting in an even number of pions. Indeed, since the strong interaction conserves $G$ parity, the three-gluon contribution, $A_{3g}$, is suppressed and such decays proceed mainly through the intermediate states $\gamma$ and $2g + \gamma$ that, due to the presence of the photon, can violate the isospin conservation and hence $G$ parity. Let us stress again that the $2g + \gamma$ contribution is considered negligible with respect to the single-photon one and is therefore ignored so far.

\[
\frac{B(J/\psi \to 3g)}{B(J/\psi \to 2g + \gamma)} = \frac{5}{16}\frac{\alpha_s(M_{J/\psi})}{\alpha} \frac{\pi + 4.9\alpha_s(M_{J/\psi})}{\pi - 0.9\alpha_s(M_{J/\psi})} \approx 7.3 \pm 0.9,
\]

\[
\frac{B(J/\psi \to 3g)}{B(J/\psi \to \mu^+\mu^-)} = \frac{5}{72\pi}\frac{\alpha_s^2(M_{J/\psi})}{\alpha^2} \frac{\pi + 4.9\alpha_s(M_{J/\psi})}{\pi - 16\alpha_s(M_{J/\psi})/3} \approx 8 \pm 3. \tag{6}
\]

\(^3\)Such a value has been obtained by fitting the cross section data in the energy range $\sqrt{q^2} \geq 2.4$ GeV, with the power law $\sigma_{\text{fit}}(q^2; P_1, P_2, P_3) = P_1[(P_2^2 + M_{J/\psi}^2)/(P_2^2 + q^2)^P_3]^{P_1}$, where $P_1, P_2, P_3$ are free parameters. In particular, $P_1$ represents the cross section value at $\sqrt{q^2} = M_{J/\psi}$.\n
FIG. 2. Data and fit on the dressed cross sections: $3(\pi^+\pi^-)$ [6], left panel; $2(\pi^+\pi^-\pi^0)$ [6], central panel; $2(\pi^+\pi^-)$ from Ref. [7], empty circles; and Ref. [8], solid circles, right panel. The fits, shown as colored curves, have been performed in the region from $\sqrt{q^2} = 2.4$ GeV up to the highest $\sqrt{q^2}$ available data point, which, in all the cases, is at $\sqrt{q^2} = 4.5$ GeV. In the four-pion case, right panel, two sets of data and three fits have been considered: 2012 data, blue, upper curve, 2005 data, red, lower curve, 2005 and 2012 data together, magenta, middle curve. The vertical dashed lines indicate the sets of data and three fits have been considered: 2012 data, blue, upper curve, 2005 data, red, lower curve, 2005 and 2012 data together, magenta, middle curve.

\[
B(J/\psi \to 3g) = \frac{5}{16}\frac{\alpha_s(M_{J/\psi})}{\alpha} \frac{\pi + 4.9\alpha_s(M_{J/\psi})}{\pi - 0.9\alpha_s(M_{J/\psi})} \approx 7.3 \pm 0.9,
\]

\[
B(J/\psi \to 3g) = \frac{5}{72\pi}\frac{\alpha_s^2(M_{J/\psi})}{\alpha^2} \frac{\pi + 4.9\alpha_s(M_{J/\psi})}{\pi - 16\alpha_s(M_{J/\psi})/3} \approx 8 \pm 3. \tag{6}
\]
TABLE I. The dressed nonresonant cross section values (third column) have been obtained, as described in the text, by fitting or extrapolating the data, which are from Ref. [6] for the six pions, Ref. [7,8] for the four pions, and Ref. [2] for the two pions. The values of the last column are from Ref. [4]. The last row has been inserted to highlight a similar $G$-parity-violation phenomenon also for the $\psi(2S)$.

| Decaying particle | $n\pi$ channel | $\sigma(e^+e^- \rightarrow 2n\pi)$ (nb) at $\sqrt{q^2} = M_{J/\psi}$ | $B_{\gamma}(J/\psi \rightarrow 2n\pi)$ | $B_{\text{PDG}}(J/\psi \rightarrow 2n\pi)$ |
|------------------|----------------|-----------------------------------|--------------------------------|--------------------------------|
| $J/\psi$ | $3(\pi^+\pi^-)$ | 0.55 ± 0.02 | $(3.62 \pm 0.12) \times 10^{-3}$ | $(4.3 \pm 0.4) \times 10^{-3}$ |
| | $2(\pi^+\pi^-\pi^0)$ | 2.03 ± 0.06 | $(1.34 \pm 0.04) \times 10^{-2}$ | $(1.62 \pm 0.21) \times 10^{-2}$ |
| | $2(\pi^+\pi^-\pi^0)$ | 0.612 ± 0.005 | $(4.04 \pm 0.04) \times 10^{-3}$ | $(3.57 \pm 0.30) \times 10^{-3}$ |
| | $\pi^+\pi^-$ | $(7.2 \pm 2.6) \times 10^{-3}$ | $(4.7 \pm 1.7) \times 10^{-5}$ | $(1.47 \pm 0.14) \times 10^{-4}$ |
| $\psi(2S)$ | $\pi^+\pi^-$ | $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ (nb) extrapolated at $\sqrt{q^2} = M_{\psi(2S)}$ | $(3.0 \pm 1.0) \times 10^{-6}$ | $(7.8 \pm 2.6) \times 10^{-6}$ |

![Graph showing data and fit on $\pi^+\pi^-$ dressed nonresonant cross section](image)

At least in the $\pi^+\pi^-$ case it might be that the one-photon amplitude does not dominate over the other $G$-parity-violating $2g + \gamma$ contribution; that indeed should be of the same order as $A_{\gamma}$, not foreseen by previous estimates [10]. Unfortunately, it is quite difficult to compute such an amplitude in the framework of QCD, even exploiting the formulas of Eqs. (1) and (2). Information about the relative strength of the $2g + \gamma$ amplitude with respect to the others might be inferred by considering odd-multipion decay channels, where $G$ parity is conserved.

TABLE II. One-photon contributions to the decay rate of $J/\psi$ into $2(\pi^+\pi^-)$ from 2005 and 2015 BABAR data.

| Year and reference | $\sigma(e^+e^- \rightarrow 2(\pi^+\pi^-))$ (nb) at $\sqrt{q^2} = M_{J/\psi}$ | $B_{\gamma}(J/\psi \rightarrow 2(\pi^+\pi^-))$ |
|-------------------|-------------------------------------------------|--------------------------------|
| 2005 [7] | $0.463 \pm 0.020$ | $(3.05 \pm 0.13) \times 10^{-3}$ |
| 2012 [8] | $0.645 \pm 0.006$ | $(4.24 \pm 0.05) \times 10^{-3}$ |
Finally, in the last row of Table I we also considered the $\psi(2S)$ decay into $\pi^+\pi^-$. To estimate the electromagnetic contribution, since there are no data, we extrapolate the $e^+e^- \rightarrow \pi^+\pi^-$ BABAR cross section at the $\psi(2S)$ mass, always by using the pion form factor parametrization of Ref. [2]. Even in the case as in that of the $J/\psi$, the electromagnetic contribution is responsible for less than the 40% of the measured branching fraction.

A. The $G$-parity-conserving channels

As a reference, the $G$-parity-conserving decays $J/\psi \rightarrow \pi^+\pi^-\pi^0$ and $J/\psi \rightarrow 2(\pi^+\pi^-)\pi^0$ are considered. The corresponding dressed production cross sections in $e^+e^-$ annihilation have been measured by the BABAR experiment [11,12], again by means of the ISR, up to center of mass energies of $\sqrt{s} = 3$ and $\sqrt{s} = 4.5$ GeV, respectively. The values of such dressed nonresonant cross sections at $\sqrt{s} = M_{J/\psi}$, i.e.,

$$
\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0)(M_{J/\psi}) = 0.044 \pm 0.025 \text{ nb},
\sigma(e^+e^- \rightarrow 2(\pi^+\pi^-)\pi^0)(M_{J/\psi}) = 0.277 \pm 0.013 \text{ nb},
$$

are obtained by means of the fitting procedure\textsuperscript{4} used in Sec. III and shown in Fig. 4 together with the cross section data.

The electromagnetic decay rates can be computed by exploiting Eq. (4), as

$$
\begin{align*}
B_{\gamma}(J/\psi \rightarrow \pi^+\pi^-\pi^0) &= B(J/\psi \rightarrow \mu^+\mu^-) \frac{\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0)}{\sigma^0(e^+e^- \rightarrow \mu^+\mu^-)} \sqrt{q^2=M_{J/\psi}} \\
&= (2.9 \pm 1.6) \times 10^{-4},
\end{align*}
$$

these values have to be compared with the PDG data

$$
\begin{align*}
B_{\text{PDG}}(J/\psi \rightarrow \pi^+\pi^-\pi^0) &= (2.11 \pm 0.07) \times 10^{-2}, \\
B_{\text{PDG}}(J/\psi \rightarrow 2(\pi^+\pi^-)\pi^0) &= (4.1 \pm 0.5) \times 10^{-2}.
\end{align*}
$$

Assuming that such decays are dominated by the three-gluon exchange mechanism, whose Feynman diagram is shown in the left panel of Fig. 1, the branching fractions can be parametrized following Eq. (1) as

$$
\begin{align*}
B_{3\gamma}(J/\psi \rightarrow 3\pi,5\pi) &= B(J/\psi \rightarrow 3g) \left[ \frac{4}{3} \alpha_s(M_{J/\psi}) \right]^3 \cdot \text{PS}_{3\pi,5\pi} \\
&= 40(\pi^2 - 9) \frac{\alpha_s(M_{J/\psi})}{8\pi \Gamma_{J/\psi}} \frac{|\Psi(0)|^2}{m_{\psi}^2} \left( 1 + 4.9 \frac{\alpha_s(M_{J/\psi})}{\pi} \right) \\
&\times \left[ \frac{4}{3} \alpha_s(M_{J/\psi}) \right]^3 \cdot \text{PS}_{3\pi,5\pi},
\end{align*}
$$

where the factor $[4\alpha_s(M_{J/\psi})/3]^3$ accounts for the three-gluon vertices in the final state, while $\text{PS}_{3\pi(5\pi)}$ represents the three-pion (five-pion) phase space. In the same line of reasoning, the $2g + \gamma$ contributions, central panel of Fig. 1, are obtained from Eq. (2) as
of the dressed nonresonant cross sections in Eq. (7), the five-pion-conserving channels are of the same order in the case of $\gamma$ and $\pi^0$ are of the same order of $\pi$ in the case of $\pi^0$. However, the dominance of the three-gluon amplitude in the $G$-parity-violating channels hides this effect. On the contrary, in the $G$-parity-violating decays of the $J/\psi$, where the $A_{3g}$ amplitudes are suppressed, the effect of the drop of $B_{2g}/B_{2gf}$ as the pion multiplicity decreases becomes important being that $A_{2gf}$ and $A_3$ are the dominant amplitudes.

In light of that, it is plausible that for the $\pi^+\pi^-$ final state, i.e., the multipion channel with the lowest multiplicity, the amplitudes $A_{3g}$ and $A_{2g}$ are similar and hence by considering $A_3$ only, as it has been done in Sec. III and shown in Table I, the decay rate is underestimated.

A computation of the $A_{3g}$ contribution, made by means of a procedure based on a phenomenological description of the $2g + \gamma$ coupling, the Cutkosky rule [13], and the dispersion relations, has been made in Ref. [14]. By considering the only imaginary part of the amplitude $A_{3g}$ and maximum interference with the one-photon amplitude it is obtained that

$$B_{2gf}(J/\psi \to 3\pi, 5\pi)$$

$$= B(J/\psi \to 2g + \gamma) \left[ \frac{4}{3} \alpha_s(M_{J/\psi}) \right] \alpha \cdot \text{PS}_{3\pi, 5\pi}$$

$$= \frac{128(\alpha^2 - 9)}{81 \Gamma_{J/\psi}} \frac{\alpha_s^2(M_{J/\psi}) \alpha \left[ \Psi(0) \right]^2}{m_G^2} \left( 1 - 0.9 \frac{\alpha_s(M_{J/\psi})}{\pi} \right)$$

$$\times \left[ \frac{4}{3} \alpha_s(M_{J/\psi}) \right] \alpha \cdot \text{PS}_{3\pi, 5\pi},$$

where, with respect to the previous case, there is only the exchange of a gluon propagator with a photon propagator; hence there are two powers of $\alpha_s(M_{J/\psi})$ and one of the electromagnetic coupling constant $\alpha$, while the phase space is the same. Using the value $\alpha_s(M_{J/\psi}) = 0.135 \pm 0.015$ obtained in Sec. II, the first ratio of Eq. (6), and assuming that the PDG value is dominated by the three-gluon exchange contribution one gets

$$B_{2gf}(J/\psi \to \pi^+\pi^0)$$

$$= B_{PDG}(J/\psi \to \pi^+\pi^0)$$

$$\times B(J/\psi \to 2g + \gamma) \frac{\alpha}{B(J/\psi \to 3g)} \frac{\alpha_s^2(M_{J/\psi})}{4 \alpha_s(M_{J/\psi})/3}$$

$$= (1.2 \pm 0.2) \times 10^{-4},$$

$$B_{2gf}(J/\psi \to 2(\pi^+\pi^-)\pi^0)$$

$$= B_{PDG}(J/\psi \to 2(\pi^+\pi^-)\pi^0)$$

$$\times B(J/\psi \to 2g + \gamma) \frac{\alpha}{B(J/\psi \to 3g)} \frac{\alpha_s^2(M_{J/\psi})}{4 \alpha_s(M_{J/\psi})/3}$$

$$= (3.2 \pm 0.3) \times 10^{-4}.$$
This result is unexpected because, following perturbative QCD [1] (pQCD), at high $q^2$, the pion form factor should vanish with the power law $(q^2)^{-1}$; as a consequence, the cross section scales as

$$\sigma(e^+e^- \rightarrow \pi^+\pi^-) \left(\sqrt{q^2}\right) \propto \frac{1}{q^2} .$$

Assuming the power-law behavior and relying on the only CLEO point, the nonresonant dressed cross section extrapolated at the $J/\psi$ mass, blue curve in Fig. 5, is

$$\sigma(e^+e^- \rightarrow \pi^+\pi^-)(M_{J/\psi})_{\text{CLEO}} = (0.0251 \pm 0.0063) \text{ nb}.$$ 

This value is more than three times higher than that reported in Table I, obtained by the extrapolation of the BABAR data, red curve in Fig. 5; and, through the formula of Eq. (4), it gives the electromagnetic branching fraction

$$\mathcal{B}^\gamma_{\text{CLEO}}(J/\psi \rightarrow \pi^+\pi^-) = (1.65 \pm 0.42) \times 10^{-4},$$

which, being in agreement with the PDG value $\mathcal{B}_{\text{PDG}}(J/\psi \rightarrow \pi^+\pi^-) = (1.47 \pm 0.14) \times 10^{-4}$, confirms $G$-parity conservation, i.e., the one-photon-exchange dominance in the decay $J/\psi \rightarrow \pi^+\pi^-$. 

However, as can be seen in Fig. 5, the two extrapolations, from BABAR data to higher $\sqrt{q^2}$ and from the CLEO point back to lower $\sqrt{q^2}$, are not compatible, that is, BABAR and CLEO data do not follow the pQCD behavior.

There are then three possibilities.

(i) The BABAR measurement underestimates the cross section in the region 2.3–3.0 GeV by a factor of 3; (ii) the CLEO datum overestimates the cross section at $\sqrt{q^2} = 3.671$ GeV by a factor of 3; (iii) the high-$\sqrt{q^2}$ regime at which pQCD is expected to hold is still not reached, i.e., other prominent structures (strongly coupled high-mass resonances) are present and then BABAR and CLEO data are actually compatible.

The last possibility has been considered in Ref. [17], where the authors fit all the pion form factor data, including not only the CLEO point, but also a theoretical value [16] at the $J/\psi$ mass, star symbol in Fig. 5, obtained from the branching ratio $\mathcal{B}_{\text{PDG}}(J/\psi \rightarrow \pi^+\pi^-)$, assuming $G$-parity conservation. The cross section obtained in Ref. [17] is shown as a black dashed curve in Fig. 5. The structure found at $\sqrt{q^2} = 2.8$ GeV is due to the model used to fit the pion form factor data, which accounts not only for the “visible” resonances, but also for the infinite possible $\rho$ radial excitations [18]. Nevertheless the last three BABAR points, with $\sqrt{q^2} \geq 2.7$ GeV, are hardly described.

V. THE WEIRD CASE OF $J/\psi \rightarrow \omega\pi^0$

The decay $J/\psi \rightarrow \omega\pi^0$, with a branching fraction $\mathcal{B}_{\text{PDG}}(J/\psi \rightarrow \omega\pi^0) = (4.5 \pm 0.5) \times 10^{-4}$ [4], could be another channel where $G$ parity is violated. Unfortunately there are no data on the cross section $\sigma(e^+e^- \rightarrow \omega\pi^0)$ at $\sqrt{q^2} = M_{J/\psi}$ that can be used to estimate, through Eq. (4), the electromagnetic contribution, $\mathcal{B}^\gamma_{\gamma}(J/\psi \rightarrow \omega\pi^0)$. Nevertheless, data on such a cross section are available in other energy regions. In particular, as shown in Fig. 6, at low $\sqrt{q^2}$, the DM2 experiment [19] collected data in the range $(1.05 \leq \sqrt{q^2} \leq 2.00)$ GeV, while the SND experiment [20] covered the interval $\sqrt{q^2} \geq 2.67$ GeV.
(1.35 ≤ √q² ≤ 2.40) GeV. Moreover, the cross section \(\sigma(e^+e^− → ωπ^0)\) has been measured around the \(ψ(2S)\) and \(ψ(3770)\) masses by the BES [21] and CLEO [22] experiments and in proximity of the \(Υ(4S)\) mass by the Belle experiment [23]. All the data, together with fits, are shown in Fig. 6. Following pQCD the expected asymptotic behavior for the cross section \(\sigma(e^+e^− → ωπ^0)\) as a function of \(q^2\) is [1,24]

\[
\sigma(e^+e^− → ωπ^0)(\sqrt{q^2}) \propto |F_{ωπ^0}(q^2)|^2 \propto (q^2)^{-4}.
\]

In light of this, to obtain the value at \(\sqrt{q^2} = M_{J/ψ}\), the high energy data are fitted with\(^5\)

\[
σ_{asy}(q^2; P_1, P_2, P_3) = P_1 \left( \frac{P_2^2 + M_{J/ψ}^2}{P_2^2 + q^2} \right)^{P_3},
\]

where \(P_1, P_2,\) and \(P_3\) are free parameters and \(P_1\) represents the desired value of the cross section. Moreover, since the high energy tails of DM2 and SND data disagree, two fits have been performed by considering at low energy either the only DM2 data with \(\sqrt{q^2} ≥ 1.9\) GeV or the only SND data with \(\sqrt{q^2} ≥ 1.825\) GeV. These two lower limits have been chosen to have the same number of points from both DM2 and SND data sets. At higher energies, in both cases, all the available data from the BES, CLEO, and Belle experiments have been included. The two results, called DM2 and SND cases, are shown in Fig. 6 as curves (blue and red) superimposed to the data. The parameters and normalized \(χ^2\)'s are reported in Table IV.

In the DM2 case, despite the large \(χ^2/\text{d.o.f.}\), the value of the \(P_3\) parameter, which defines the power-law behavior, is in perfect agreement with the pQCD expectation that, on the contrary, is disregarded in the SND case. Finally, the electromagnetic contributions to the \(J/ψ\) branching fraction in the two cases are obtained by using the values of the \(P_1\) parameter, which represents \(\sigma(e^+e^− → ωπ^0)(M_{J/ψ})\), in Eq. (4),

\[
B_J(J/ψ → ωπ^0)
\]

\[
= \frac{B(J/ψ → μ^+μ^-) \sigma(e^+e^− → ωπ^0)}{\sigma(e^+e^− → μ^+μ^-)} \bigg|_{√q^2=M_{J/ψ}}
\]

\[
= \begin{cases} 
(3.53 ± 0.18) \times 10^{-4} & \text{DM2 case} \\
(2.29 ± 0.40) \times 10^{-4} & \text{SND case} 
\end{cases}
\]

to be compared with \(B_{PDG}(J/ψ → ωπ^0) = (4.5 ± 0.5) \times 10^{-4}\).

\(^5\)See footnote 3.
accuracy level, a precise measurement by BESIII, close to and at the \( J/\psi \) mass, can certainly be achieved.

**VII. CONCLUSIONS**

The \( G \)-parity-violating decay \( J/\psi \to \pi^+\pi^- \) behaves differently with respect to the other \( J/\psi \) decays into even-multipion final states. There is a non-negligible disagreement, more than 4.5 standard deviations, between what is expected from the measurement of the dressed nonresonant cross section at the \( J/\psi \) mass and the measured branching ratio.

The \( J/\psi \) decay mechanism mediated by \( 2g + \gamma \), usually neglected, or better considered negligible being that it is \( G \)-parity violating, might be responsible for this discrepancy. Indeed, it happens that for this channel the one-photon contribution is so low that it might be of the same order of the \( 2g + \gamma \) one. The fact that the one-photon contribution becomes lower and lower as the pion multiplicity decreases has been shown in Table I, in the case of an even number of pions and in Table III in the case of an odd number of pions.

In Sec. III A, it has been noticed that, for the \( G \)-parity-conserving channels, the contribution to the branching fraction due to the \( 2g + \gamma \) intermediate state, which in this case can be estimated by exploiting its relation with the 3\( g \) contribution [Eqs. (1) and (2)], turns out to be comparable with \( B_\gamma \), especially in the case of the decay \( J/\psi \to \pi^+\pi^- \).

The phenomenological computation of \( B_{2g}(J/\psi \to \pi^+\pi^-) \) [14] corroborates the hypothesis about the softening and even the cancellation of the hierarchy between the two main contributions \( B_\gamma \) and \( B_{2g} \) in the case of lower multiplicity multipion final states. However, as a matter of fact, all the estimates, done until now, found the \( 2g + \gamma \) amplitude totally negligible with respect to the one-photon decay.

Finally, it has been shown that the BESIII experiment has the tools to repeat this measurement with high precision, to prove or disprove the discrepancy between \( B_\gamma(J/\psi \to \pi^+\pi^-) \) and \( B_{PDG}(J/\psi \to \pi^+\pi^-) \) pointed out by the BABAR data.

If such a discrepancy is confirmed, the existence of this \( G \)-parity-violating amplitude can have heavy consequences for the attempts to get the relative phase between the strong and the electromagnetic \( J/\psi \) decay amplitudes, already in the case of branching ratios at the \( 10^{-3} \) level.

**APPENDIX: THE MASTER FORMULA**

The partial rate of the decay \( J/\psi \to \gamma^* \to \mathcal{H}_q \), where the hadronic final state consists in a set of \( n \) hadrons \( h_j \) with 4-momenta \( p_j, j = 1, \ldots, n \), reads

\[
d\Gamma_j(J/\psi \to \mathcal{H}_q) = \frac{(2\pi)^4}{2M_{J/\psi}} |M_j(J/\psi \to \mathcal{H}_q)|^2 d\Phi_n \times \left( M_{J/\psi}; p_1, p_2, \ldots, p_n \right),
\]

where \( M_j(J/\psi \to \mathcal{H}_q) \) is the Lorentz-invariant matrix element and \( d\Phi_n \) is an element of \( n \)-body phase space.

The Feynman diagram is shown in Fig. 8 and the matrix element is

\[
M_j(J/\psi \to \mathcal{H}_q) = e^\mu(J/\psi) \frac{G_{j/\psi}}{q^2} H_\mu(p_j). \quad (A1)
\]

where \( e^\mu(J/\psi) \) is the \( J/\psi \) polarization vector, \( G_{j/\psi} \) is the \( J/\psi-\gamma \) coupling, the hadronic structure function, described in the text, and the hadronic final state, respectively.

As a consequence of the current conservation, \( P^\mu F_{\alpha\beta} = P^\mu F_{\alpha\beta} = 0 \), the tensor structure of \( F_{\mu\nu}(p_j) \) can be defined as

\[
F_{\mu\nu}(p_j) = \sum_{h} H_\mu(p_j) H_\nu(p_j),
\]

where \( P^\mu \) is the \( J/\psi \) 4-momentum and the tensor \( F_{\mu\nu}(p_j) \) is defined as

\[
F_{\mu\nu}(p_j) = (g^\mu p^\nu - P^\mu P^\nu) F_\mu(p_j). \quad (A3)
\]

where \( F_\mu(p_j) \) is a Lorentz scalar structure function, depending on all the scalar quantities that can be obtained by using the 4-momenta and the Lorentz tensors related to the spin structure of the final hadrons (e.g., in case of a baryon-antibaryon final state, \( \mathcal{H}_q = \{B, \bar{B}\} \), these Lorentz tensors are the Dirac gamma matrices). Using such a
where the multiparticle structure is described by the Feynman diagram shown in Fig. 9, whose squared of the matrix element is

\[ |\mathcal{M}(\psi \to H_q)|^2 = \gamma \frac{\pi^2}{J^2_{\gamma}} |G_{J/\psi}|^2 \frac{3\pi^2 F_{H_q}(q^2, p_j)}{3M^2_{J/\psi}} \]

and the partial decay rate becomes

\[ d\Gamma_f(J/\psi \to H_q) = \frac{(2\pi)^4 |G_{J/\psi}|^2}{2M^2_{J/\psi}} F_{H_q}(M^2_{J/\psi}, p_j) d\phi_n \]

\[ \times (M_{J/\psi}; p_1, p_2, \ldots, p_n) \]

The total rate is obtained by integrating in \( d\phi_n \), i.e.,

\[ \Gamma_f(J/\psi \to H_q) = \frac{(2\pi)^4 |G_{J/\psi}|^2}{2M^2_{J/\psi}} \int F_{H_q}(M^2_{J/\psi}, p_j) \]

\[ \times d\phi_n(M_{J/\psi}; p_1, p_2, \ldots, p_n) \] (A4)

where the multiparticle structure is described by the function \( F_{H_q}(M^2_{J/\psi}, p_j) \).

The differential cross section for the annihilation process \( e^+ e^- \to \gamma^* \to H_q \) is

\[ d\sigma(e^+ e^- \to H_q) = \frac{(2\pi)^4 |\mathcal{M}(e^+ e^- \to H_q)|^2}{4\sqrt{(k_1k_2)^2 - m^2_e}} \]

\[ \times d\phi_n(k_1 + k_2; p_1, p_2, \ldots, p_n), \]

where \( k_1(k_2) \) is the electron (positron) 4-momentum. In the \( e^+ e^- \) center of mass frame and neglecting the electron mass, the previous expression reads

\[ d\sigma(e^+ e^- \to H_q) = \frac{(2\pi)^4 |\mathcal{M}(e^+ e^- \to H_q)|^2}{2q^2} \]

\[ \times d\phi_n(q; p_1, p_2, \ldots, p_n), \]

with \( q = k_1 + k_2 \).

In Born approximation of the annihilation process is described by the Feynman diagram shown in Fig. 9, whose matrix element reads

\[ \mathcal{M}(e^+ e^- \to H_q) = i\bar{\psi}(k_2)\gamma^\mu u(k_1) \frac{\gamma^\nu}{q^2} H_{\mu}(p_j), \]

where \( u(k_1) \) and \( v(k_2) \) are the electron and positron spinors. Using, for the contraction of the hadronic vectors, the expression of Eq. (A3), the spin-averaged modulus squared of the matrix element is

FIG. 9. Feynman diagram of the annihilation \( e^+ e^- \to H_q \) in Born approximation. The gray hexagon and the lined area represent the hadronic structure function and the hadronic final state, respectively.
Using in Eq. (A6) the value $|G_{J/\psi}|^2$ extracted form Eq. (A7),

$$\frac{\Gamma_{\psi}(J/\psi \to H_q)}{\sigma(e^+e^- \to H_q)(M_{J/\psi})} = \frac{3M^2_{J/\psi}}{4\pi\alpha^2} \frac{\Gamma(J/\psi \to \mu^+\mu^-)}{\Gamma(J/\psi \to \mu^+\mu^-)} = \frac{\Gamma(J/\psi \to \mu^+\mu^-)}{4\pi\alpha^2/(3M^2_{J/\psi})} = \frac{\Gamma(J/\psi \to \mu^+\mu^-)}{\sigma^0(e^+e^- \to \mu^+\mu^-)(M_{J/\psi})},$$

(A8)

where the $e^+e^- \to \mu^+\mu^-$ bare Born cross section has been used,

$$\sigma^0(e^+e^- \to \mu^+\mu^-)(\sqrt{q^2}) = \frac{4\pi\alpha^2}{3q^2}.$$

Neither the decay rate $\Gamma_{\psi}(J/\psi \to H_q)$ nor the total cross section $\sigma(e^+e^- \to H_q)$ has been corrected by the vacuum-polarization effects, so that they are dressed observables; indeed these effects are embodied in the hadronic structure function $F_H(q^2, p_j)$, evaluated at the $J/\psi$ mass.

Finally, dividing Eq. (A8) by the $J/\psi$ total width $\Gamma_{J/\psi}$, in order to have a relation between branching fractions, we obtain Eq. (4), i.e.,

$$B_{\psi}(J/\psi \to H_q) = B(J/\psi \to \mu^+\mu^-) \frac{\sigma(e^+e^- \to H_q)}{\sigma^0(e^+e^- \to \mu^+\mu^-)} \mid_{q^2=M^2_{J/\psi}}.$$

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