PREDICTION OF SOLAR FLARES USING UNIQUE SIGNATURES OF MAGNETIC FIELD IMAGES

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ABSTRACT

Prediction of solar flares is an important task in solar physics. The occurrence of solar flares is highly dependent on the structure and topology of solar magnetic fields. A new method for predicting large (M- and X-class) flares is presented, which uses machine learning methods applied to the Zemike moments (ZM) of magnetograms observed by the Helioseismic and Magnetic Imager on board the Solar Dynamics Observatory for a period of six years from 2010 June 2 to 2016 August 1. Magnetic field images consisting of the radial component of the magnetic field are converted to finite sets of ZMs and fed to the support vector machine classifier. ZMs have the capability to elicit unique features from any 2D image, which may allow more accurate classification. The results indicate whether an arbitrary active region has the potential to produce at least one large flare. We show that the majority of large flares can be predicted within 48 hr before their occurrence, with only 10 false negatives out of 385 flaring active region magnetograms and 21 false positives out of 179 non-flaring active region magnetograms. Our method may provide a useful tool for the prediction of solar flares, which can be employed alongside other forecasting methods.

Key words: sunspots – Sun: activity – Sun: flares – Sun: magnetic fields

Supporting material: tar.gz file

1. INTRODUCTION

It is accepted that the energy release mechanism of solar and stellar flares is based on magnetic field reconfiguration; however, the exact underlying chain of processes remains ambiguous (Priest & Forbes 2002). Accurate forecasting of solar flares is an extremely important task due to their effect on space weather (Rust 1993; Wheatland 2005; Schwenn 2006; Pulkkinen 2007; Barnes & Leka 2008). Many forecasting methods—e.g., those based on sunspot classification, time series analysis, avalanche models, machine learning algorithms, and others—have been proposed. In recent years the quality and frequency of observations have increased, e.g., due to the availability of data from the Solar Dynamics Observatory (SDO) and other satellites. The new data should enable more accurate prediction. However, that requires prediction methods which identify, and take advantage of, additional information in the data.

McIntosh (1986, 1990) presented a flare forecasting method named THEO (Theophrastus), which is an expert system based on sunspot classification. In the extended approach, the McIntosh classification is primary, and some additional information, including the magnetic field properties and time series of former large flares, is used (McIntosh 1990). Wheatland (2004, 2005) investigated a flare prediction method that exploits solar flare statistics using Bayesian analysis. In this method, predictions are made based on the observed time series of flares and the phenomenological distributions of events in energy and time. The method was shown to produce forecasts comparable in accuracy to those issued by the National Oceanic and Atmospheric Administration (NOAA), which have been based on THEO (Wheatland 2005). Bélanger et al. (2007) applied a four-dimensional variational data assimilation method and an avalanche model for the prediction of large solar flares. Avalanche models have also been proposed as a basis for solar flare forecasting by Strugarek & Charbonneau (2014), who suggested that such models could lead to significant improvement in the prediction of large solar flares, since solar flares are stochastic in nature. Guerra et al. (2015) presented a method called ensemble flare prediction, in which three flaring probabilities derived from three different methods used by the Community Coordinated Modeling Center (NASA-GSFC) and the flare forecasting results provided by the NOAA are linearly combined to find a final flaring probability.

Because of the magnetic origin of large solar energetic events (i.e., flares and coronal mass ejections), most large-event prediction methods use measured properties of the photospheric vector magnetic field, including the magnetic flux of active regions (ARs) (Künzel 1960; Sammis et al. 2000; Leka & Barnes 2003; Georgoulis & Rust 2007; Schrijver et al. 2007; Falconer et al. 2008; Mason & Hoeksema 2010; Falconer et al. 2011; Georgoulis et al. 2012; Abramenko 2015). Leka & Barnes (2007) used discriminant analysis applied to a set of photospheric magnetic quantities computed from the vector field data observed by the University of Hawaii Imaging Vector Magnetograph, and showed that only a few variables and/or their combinations are related to the flare productivity of ARs. Barnes & Leka (2008) debated how the performance of different solar flare forecasting methods which incorporate different data sources should be compared. They used skill scores to compare the ability of those methods that are based on a number of parameters computed for photospheric vector magnetic field data to forecast the flaring time of large flares. At a flare forecasting workshop held in 2009, a variety of prediction methods were tested on a common data set. The participating methods were not found to perform substantially better than “climatological” forecasts, i.e., predictions based on long-term averages (Barnes et al. 2016).

Recently, machine learning algorithms have been applied to the forecasting of both solar flares (Colak & Qahwaji 2009; Yuan et al. 2010; Huang et al. 2013; Yang et al. 2013; Boucheron et al. 2015; Shin et al. 2016) and coronal mass ejections (Bobra & Ilonidis 2016). Ahmed et al. (2013) developed a solar flare prediction method using a feature selection of 21 magnetic element properties produced by the Solar Monitor Active Region
Tracker and a machine learning base classifier. They identified that a diminished set of six magnetic features produced forecasting results similar to those of the whole set of 21 magnetic features. Bobra & Couvidat (2015) computed 25 quantities from four years of vector magnetic field data from 2017 active regions recorded by the SDO and examined the relationship with flaring. They used the f-score feature selection algorithm to select the parameters with the highest score. They concluded that using four parameters—namely, the total unsigned current helicity, the total photospheric magnetic free energy density, and the total unsigned vertical current—resulted in nearly the same forecasting efficiency as the whole set of 25 parameters. Using the four parameters listed above and a machine learning algorithm, the support vector machine (SVM), they grouped ARs into two separate classes. They defined a positive class, which encompasses all those ARs that will produce at least one large flare within a given time interval, and a negative class, which contains all those ARs that will not produce any flare in the same time interval.

Zernike moments (ZMs) provide a decomposition of image data which is invariant under scaling, translation, and rotation and hence in this sense is unique (Zernike 1934). These moments have previously been applied, together with machine learning algorithms, to the task of identifying and tracking solar photospheric and coronal bright points and mini-dimmings (Alipour et al. 2012; Javaherian et al. 2014; Alipour & Safari 2015). In this paper, these methods are adopted as a predictor algorithm for solar flares. Following the approach of Bobra & Couvidat (2015), magnetograms for ARs are categorized into two distinct classes, namely, positive and negative, corresponding to whether the ARs have or have not produced large flares, respectively. ZMs are calculated for the AR magnetograms in the two categories. Then, by using a well-trained machine learning algorithm, we attempt to identify the corresponding class (positive or negative) for any given AR magnetogram. The motivation for implementing ZMs in solar flare forecasting is to provide a set of unique features for each magnetogram treated as an image. It is anticipated that this will improve the performance of the classification process in comparison with that of classifiers trained with just a few global parameters (e.g., total flux, current helicity, etc.) extracted from vector magnetic fields.

The paper is organized as follows: First, the data processing and the method are discussed in Section 2, and then the results are given in Section 3. A discussion is presented in Section 4, followed by an Appendix with additional details of the machine learning methods.

2. DATA PROCESSING AND METHOD

2.1. Data

The Helioseismic and Magnetic Imager (HMI) instrument on board the SDO has been returning full-disk solar photospheric vector magnetic field data since 2010 (Schou et al. 2012). In the present study, we use the cylindrical equal area (CEA) version of the Spaceweather HMI Active Region Patch (SHARP) data hmi.sharp_cea_720s (http://solar-data.jsoc.stanford.edu/ajax/lookdata.html?ds=hmi.sharp_cea_720s), including magnetic field data for 422 NOAA ARs. The ARs used were observed in the time period 2010 June 2 to 2016 August 1. The CEA SHARP vector magnetic data are projections of magnetograms in CCD coordinates onto heliographic CEA coordinates after rotation to the disk center. Here, we use only the radial component of the vector magnetic field, namely, $B_r$. For more information about SHARP vector magnetic field data, see Hoeksema et al. (2014). Using the Geostationary Operational Environmental Satellite (GOES) flare catalog (ftp://ftp.ngdc.noaa.gov/STP/space-weather/solar-data/solar-features/solar-flares/x-rays/goes/xrs/), we identify 113 NOAA ARs (out of the 422 collected ARs) which generate large (M- and X-class) flares during the abovementioned period. Magnetograms dated 2010 June 2 up to 2014 June 1 and 2014 June 1 to 2016 August 1 are chosen for the training and test sets, respectively.

2.2. ZM Representation

ZMs are derived from Zernike polynomials (Zernike 1934), which are defined in a unit circle $(x^2 + y^2 \leq 1)$ and are given in polar coordinates $(r, \theta)$ by

$$ U_{n,m}(r, \theta) = R^n_m(r) \exp(i m \theta), \quad (1) $$

where $n$ and $m$ are positive integers and where $R^n_m(r)$ is given by

$$ R^n_m(r) = \sum_{s=0}^{\frac{1}{2}(n-m)} (-1)^s \frac{(n-s)!}{s! \left( \frac{1}{2}(n+m) - s \right) ! \left( \frac{1}{2}(n-m) - s \right) !} r^{n-2s}. \quad (2) $$

Zernike polynomials have three fundamental properties: they satisfy orthogonality conditions and form a complete set or vector space basis; their absolute values are invariant under rotation; and they force constraints on the $n$ and $m$ indices, namely, $n \geq 0$, where $n \geq |m|$ and where $n \pm m$ is an even number.

With Zernike polynomials, a 2D magnetogram image $B_s(x, y)$ can be mapped onto a complex feature space, but first the image must be transformed from Cartesian coordinates to polar coordinates. To do this, a square magnetogram image is mapped onto a unit disk, with the center of the image mapping onto the origin of the polar coordinates. A thorough explanation about transforming images from Cartesian to polar coordinates is given by Hosny (2010). The ZMs for the feature space are defined by (Hosny 2010)

$$ Z_{n,m} = \frac{n + 1}{\pi} \int_{0}^{2\pi} \int_{0}^{r_{max}} U_{n,m}(r, \theta) B_s(r, \theta) r dr d\theta, \quad (3) $$

where the asterisk denotes the complex conjugate.

The magnitudes of ZMs are invariant under rotation because of the exponential angular factor $\exp(i m \theta)$ in Equation (1), but they can also be made invariant under translation and scaling. This can be done by transforming an arbitrary image $I(x, y)$ into a new image $I(x/a + x', y/a + y')$, with $x$ and $y$ being the location of the image centroid and $a$ the scale factor computed from the first-order normal moments (Khoitanzad 1990; Hosny 2010). With a proper normalization, this produces ZMs which are invariant to scale. These properties of ZMs mean that they uniquely characterize any two-variable function. Here we calculate ZMs for magnetogram images of ARs, as a basis for classifying whether the ARs produce large flares (positive class) or do not (negative class).
can be made using $ij$ where, ideally, $N$ is weighted by $n$, the only possible number for $N = 31$ it means that $n$ takes values from 1 through 31. Applying the third confinement rule of Zernike polynomials yields 528 pairs of $(n, m)$ in the following way: if $n = 0$, the only possible number for $m$ are 0; if $n = 1$, the acceptable numbers for $m$ are $+1$, 0, and $1$; and so on.

Figure 1 depicts different terms of ZMs for magnetic field data for ARs belonging to the positive class (flare producing) and to the negative class (non-flare producing). The figure illustrates how the ZMs describe an image. The radial part of the Zernike polynomials is bounded to unity ($R_n^m(r) \leq 1$) inside the unit disc. In Equation (3), the magnetogram image $B_r$ is weighted by $r R_n^m(r)$, which is bounded to $r$ inside the unit disc. This means that pixels closer to perimeter of the disc have more weight than those closer to the center of the disc. Increasing the polynomial order $n$ leads to an increase in the frequency of oscillations of the polynomial along the radial direction. This provides enhanced capability to describe the details of a magnetogram image with a set of ZMs due to the polynomial oscillation. As we see in Figure 1, the magnitude values of ZMs have different oscillations and shapes for the two magnetograms from the flaring and non-flaring ARs.

Based on the orthogonality of the Zernike polynomials and using the ZM coefficients ($Z_{n,m}$), a digital image reconstruction $\hat{B}_r(r, \theta)$ can be made using

$$\hat{B}_r(r, \theta) = \sum_{n=0}^{N} \sum_{\substack{m=0 \\ |n-m| \text{even}}}^{n} Z_{n,m} U_{n,m}(r, \theta),$$  

where, ideally, $N$ is infinity. Using Equation (4) and a finite number of terms defined by $n \leq N$, we can reconstruct the magnetic field image from the ZMs. The optimal value of $N$ is determined empirically and found to be 31. This is decided based on the image reconstruction error (Javaherian et al. 2014):

$$E^2(N) = \frac{\sum_i \sum_j |B_r(i, j) - \hat{B}_r N(i, j)|^2}{\sum_i \sum_j |B_r(i, j)|^2},$$

where $B_r(i, j)$ represents an element of the original magnetogram array, $\hat{B}_r N(i, j)$ is an element of the reconstructed magnetogram array, and the sum is over all possible $i$ and $j$. The minimum reconstruction error defines the best value for $N$. In practice this is determined by trial and error. More information about image reconstruction and associated relative errors is given by, e.g., Khotanzad (1990), Hosny (2010), and Javaherian et al. (2014).

Figure 2 shows an example of a reconstructed image of a positive-class magnetogram belonging to NOAA active region number 11504 on 2012 June 14 at 12:00 UT. It should be noted that there are artifacts and errors in the reconstructed image, so that the two panels in the figure do not fully correspond. One error is due to mapping the original image onto polar coordinates, and another is due to intrinsic defects in numerical methods (Liao & Pawlak 1998). The reconstructed image is not used for the process of classification and is included only to illustrate the image reconstruction process.

2.3. Prediction Method

Here, we propose a flare prediction method using the invariant and unique properties of ZMs and the SVM classifier. The SVM classifier is a supervised statistical machine learning
method which is based on Lagrange multiplier optimization (Vapnik 1995) and is defined specifically for two-class problems (e.g., Gunn 1997). In supervised learning, the labeled training data set consists of training examples (pairs of typical vectors in an l-dimensional space as the input objects). The SVM classifier attempts to find a separating hyperplane with a maximum margin between the two classes inside the training set. The maximum margin ensures the least possible error in classification. The process to find this hyperplane can be simplified to solving an optimization problem (Equation (14) in the Appendix). The SVM code used in this work is the SVM-KM MATLAB toolbox (http://asi.insa-rouen.fr/enseignants/~arakoto/toolbox/SVM-KM.zip). The regularization parameter $c$ (Equation (14) in the Appendix) is set to 1, and the kernel function $K(x_i, x_j)$ (Equation (16) in the Appendix) used here is Gaussian. After these required procedures, the learning algorithm can infer (predict) the probable relative class for unseen cases. Further details of the SVM are discussed in the Appendix and also in Tan et al. (2006).

As noted above, we divide the magnetograms for the ARs into two classes, namely, a positive and a negative class, corresponding to all the ARs that produce at least one large flare (M- and X-class) within a certain time interval and those ARs which do not produce any large flares within the same time interval, respectively. The ZMs of each magnetogram are distinctive enough to be separated using the SVM classifier, as illustrated in Figure 1. Figure 3 depicts the flowchart of our flare prediction algorithm for reference.

3. RESULTS

In this paper, using the unique and invariant properties of the ZMs of the photospheric magnetogram images and the SVM classifier, we attempt to predict which of the ARs at hand will produce at least one large flare within 48 hr. We divide the whole data set into a training set and a test set. A supplement to this paper provides electronic tables which contain the ZMs calculated for each magnetogram in the training and test data sets as MATLAB structures, with the exact time and the NOAA AR numbers given for each. The ARs used in this paper consist of a total of 422 different NOAA active regions observed during the time period 2010 June 2 to 2016 August 1. The training set consists of a total of 85 different NOAA ARs belonging to the positive class observed in the time period 2010 June 2 to 2014 June 1, meaning they produced at least one large flare within 48 hr, and 208 different NOAA ARs belonging to the negative class observed in the same period, meaning they did not produce a large flare within the same time interval. Empirically, in the process of training the positive class to the SVM, we use 6, 2, 2, 2, 2, 2, and 2 magnetogram images (20 in total) from 1, 5, 18, 20, 22, 24, 25, and 48 hr, respectively, before the flaring time of each of the 85 ARs (Table 1). Also, the data used to train the negative class to the SVM consist of almost seven magnetogram images for each of the 208 ARs that did not produce any large flare within the past 48 hr. The SVM was trained on the ZMs extracted from this data set.

The rest of the data are taken as the test set, which consists of 129 ARs. We pretend that we do not know whether these ARs are positive or negative. There are at most four magnetogram images at four different times for each of the ARs inside the test set. The goal is to identify the corresponding class for every magnetogram image in the test set.

An analysis of the output of the classifier is presented in Table 2, in which TP (true positive) denotes the number of flaring ARs that are correctly classified as a member of the positive class (375), FP (false positive) denotes the total number of non-flaring ARs that are incorrectly classified as a member of the positive class (21), TN (true negative) denotes the total number of non-flaring ARs that are correctly classified as a member of the negative class (158), and FN (false negative) denotes the total number of flaring ARs that are incorrectly classified as a member of the negative class (10).

It is common to assign scores to assess the accuracy of prediction (Wheatland 2005; Barnes & Leka 2008). Several skill scores have been proposed and applied for solar flare predictions. Table 3 presents different skill scores and their related formulae. These metrics are gathered from different papers on the subject of flare forecasting (Woodcock 1976; Barnes & Leka 2008; Mason & Hoeksema 2010; Bloomfield et al. 2012, 2016; Bobra & Couvidat 2015).

Table 4 lists the prediction metrics achieved by the present algorithm compared to the scores of other works. Bobra &

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**Figure 2.** Reconstructed image using the first 528 ($N = 31$) ZM terms for a magnetogram image of NOAA active region number 11504 on 2012 June 14 at 12:00 UT, which is in the positive class. Left: radial component ($B_r$) magnetogram. Right: reconstruction of the left-hand side image using the first 528 ZM terms.
Couvidat (2015) provided two tables (Tables 2 and 3 therein) to compare the values of skill scores obtained by different forecasting methods. Other than the second column of Table 4, all columns are a copy of Tables 2 and 3 of Bobra & Couvidat (2015).

Barnes & Leka (2008) concluded that even if different databases are used for prediction, comparison of the skill scores for different methods is meaningful. However, unless the data sets are identical, there is no completely meaningful comparison between two or more different methods that one could make. Hence, one should not consider the results of Table 4 as an absolute reference for comparison between the methods.

The second column of Table 4 lists the skill scores of the present algorithm for the classification process (see the two supplementary electronic tables). The third and fourth columns of Table 4 list the performance metrics achieved by Bobra & Couvidat (2015) for specifically tuned SVMs which result in the highest HSS2 and TSS, respectively. Their method was demonstrated to predict large solar flares within 48 hr before occurrence with a TSS of 0.817. The second-highest TSS in the table, 0.671, belongs to Ahmed et al. (2013). As Table 4 shows, the TSS achieved in the present work is 0.856. Also, the highest HSS2 among all other previous methods, given by Ahmed et al. (2013), is 0.751, and the second-highest HSS2, 0.737, belongs to Bobra & Couvidat (2015). The HSS2 obtained with the present method is 0.871. Another metric of interest here is Recall+. As shown in Table 3, the Recall+ score is associated with the number of FNs and TPs, which characterize the ability of the classifier to achieve the least number of FNs. The reason for this interest is that if a positive event is falsely reported as a negative one, the resulting costs for this lack of accuracy in prediction could be devastating. Misprediction of negative events (i.e., FPs) may only require, for example, powering off a power plant or rotating a satellite’s shields toward the Sun, but when it is reported to an astronaut in deep space that they are unlikely to be hit by a large flare within some time, the consequence of error is more serious. The highest Recall+ score among former methods is 0.869, for Bobra & Couvidat (2015), and the second-highest Recall+ is 0.817, for Yu et al. (2009). The Recall+ score gained by the present method is 0.974.

4. DISCUSSION

Here, we propose a method based on the properties of the ZMs of magnetogram images for ARs and on the SVM for the prediction of large (M- and X-type) solar flares.
We use the TSS 0.856 0.703 instead of the Heidke Skill Score 0.871 0.737. The precision and recall for the abovementioned parameters? Suppose we have two arbitrary three-dimensional vector magnetic fields, denoted by $B_1$ and $B_2$. The total current helicity for these two vector magnetic fields can be expressed as

$$\sum B_2 \cdot J_2 = \sum \frac{1}{\mu_0} (\nabla \times B_2) \cdot B_2 = \sum \frac{1}{\mu_0} (\nabla \times B_1) \cdot B_1 = \sum B_1 \cdot J_1, \quad (7)$$

and the total flux

$$\sum B_2 (x, y) dxdy = \sum B_2 (x, y) dxdy. \quad (8)$$

Since the total free energy density and the total Lorentz force are both proportional to $B^2$, applying an additional constraint

$$2 \frac{\partial \phi}{\partial x} B_{1x} + \left( \frac{\partial \phi}{\partial y} \right)^2 = -2 \frac{\partial \phi}{\partial y} B_{1x} - \left( \frac{\partial \phi}{\partial y} \right)^2, \quad (9)$$

results in

$$B_1^2 = B_2^2, \quad (10)$$

and hence, the total free energy density and the total Lorentz force for the two magnetic fields are the same. Assume that these two vector magnetic fields represent the photospheric magnetic field for two arbitrary ARs. It may happen that one of the magnetic fields corresponds to a flare-productive AR and the other corresponds to a non-flaring AR. In this case a classification process based on helicity, total flux, total free energy, and total Lorentz force will not discriminate between the two ARs, since these two different magnetic fields have the same values for the abovementioned parameters. In other words, there could be two different vector magnetic fields for two ARs having an identical vector in the feature space. This can obviously affect the results of the classification. It can be seen that the ZMs for these two magnetic fields given by Equation (3) represent two different sets of values.\(^3\)

\(^3\) This example is not intended to be realistic: two real vector magnetograms will not have identical values of $J$ and $B$. However, the example demonstrates the principle that two different magnetic fields may have the same values of these parameters.
Moreover, as discussed in Barnes et al. (2016), performance comparisons between different flare forecasting methods based on extracting a few parameters out of AR magnetograms indicate that there is no clearly superior method, and it was pointed out that this might be due to correlations between the parameters. Also, the methods were found to have a rather weak performance in achieving high positive skill scores. An advantage of the present method is that the ZMs provide unique information as a basis for the classification of an AR by comparison with a few global parameters (e.g., total flux, current helicity etc.). Further, the present method is demonstrated to be able to predict solar flares with a small number of FNs rather than just reducing the number of FPs. This has important practical consequences for reducing the costs of errors in prediction (e.g., Bobra & Couvidat 2015).

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APPENDIX

A.1. The Support Vector Machine

The purpose of the SVM classifier is to find a decision boundary with a margin as large as possible to reduce the classification error. Suppose that $D$ is a binary-class training set with $N$ data points in the $l$-dimensional feature space — that is,

$$D = \{(x_i, y_i) | x_i \in \mathbb{R}^l, y_i \in \{-1, +1\} \}, \quad i = 1, \ldots, N.$$  (11)

Constructing a decision boundary, which is a separating hyperplane in a high-dimensional space, the SVM can segregate classes. This hyperplane is given by

$$w \cdot \Phi(x) + b = 0,$$  (12)

where

$$\Phi: \mathbb{R}^l \rightarrow \mathbb{R}^{L}, \quad L \geq l,$$  (13)

and where $w$ and $b$ are the weight vector and bias, respectively, and $\Phi(x)$ is a linear or nonlinear function that maps each data point $x_i$ onto the feature space in high-dimensional space. These parameters, namely, $w$ and $b$, can be computed by solving the following optimization problem:

$$\min \left\{ \frac{1}{2} ||w||^2 + c \sum_{i=1}^{N} \xi_i \right\}$$  (14)

subject to (the constraint)

$$y_i(w \cdot x_i + b) \geq 1 - \xi_i, \quad \xi_i \geq 0,$$  (15)

where $\xi_i$ and $c$ are the error value for the decision boundary and the regularization parameter, respectively. The regularization parameter controls the trade-off between the margin width and the model complexity and is determined by the user. The equations given above can be converted into the following dual form:

$$\max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j K(x_i, x_j),$$  (16)

subject to

$$\sum_{i=1}^{N} y_i \alpha_i = 0, \quad \alpha_i \geq 0, \quad \forall i; 0 \leq i \leq c,$$  (17)

where $\alpha_i$ is the Lagrange multiplier corresponding to the $i^{th}$ training sample and $K(x_i, x_j)$ is a kernel function which maps the input vectors onto a suitable feature space to achieve a better representation. So, we have $K(x_i, x_j) = \Phi(x_i) \cdot \Phi(x_j)$. This is a constrained optimization problem, and it can be solved by a Lagrangian multiplier method. The output of the SVM for each input data point is equal to

$$y(x) = \text{sign}(f(x)),$$  (18)

where

$$f(x) = \sum_{i=1}^{N} y_i \alpha_i K(x_i, x_j) + b.$$  (19)

Usually, after the SVM is trained, the value of the Lagrange multiplier is zero for many training points. Support vectors are input vectors that just touch the boundary of the margin (see, e.g., Qu et al. 2003; Theodoridis & Koutroubouas 2009; Hsu et al. 2011).

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