Extraction of Polarized Gluon Distributions from Large-$p_T$ Light Hadron Pair Production

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Abstract

We propose a new formula for extracting the polarized gluon distribution from the large-$p_T$ light hadron pair production for semi-inclusive processes in polarized deep inelastic scattering. In general, a large-$p_T$ hadron pair is produced via photon-gluon fusion (PGF) and QCD Compton at the lowest order of QCD. The PGF gives us a direct information on $\Delta g$ in the nucleon, while QCD Compton becomes background to the signal process for extracting $\Delta g$. We show that the contribution from this background, i.e. the QCD Compton process, can be removed by using symmetry relation among fragmentation functions and taking an appropriate combination of various light hadron pair production processes, and thus the double spin asymmetry can be described in terms of $\Delta g/g$ alone.

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1 Introduction

The nucleon is not an elementary but compound particle. Accordingly, its spin is carried by its constituents and is described by a sum rule,

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta g + L_z , \]  

(1)

where \( \frac{1}{2} \) on the left hand side denotes the spin of a longitudinally polarized nucleon, while \( \Delta \Sigma \), \( \Delta g \) and \( L_z \) on the right hand side represent the amount of the spin carried by the quarks, gluons and their orbital angular momenta, respectively. In the static quark model with SU(6) symmetry, the nucleon’s spin is totally carried by quarks alone and thus, each term on the right hand side of eq.(1) becomes \( \Delta \Sigma = 1 \), \( \Delta g = 0 \) and \( L_z = 0 \). Moreover, in the naive quark–parton model, the value of \( \Delta \Sigma \) is predicted to be \( \sim 0.7 \), which means that the quarks almost carry the nucleon’s spin\[1\]. However, triggered by the first measurement by the EMC in 1988 of the longitudinally polarized structure function of the proton \( g_1^p \) for wide kinematic range of Bjorken \( x \) and \( Q^2 \), the situation has drastically changed, that is, the quarks carry the proton spin little and the strange quark is negatively polarized to the proton spin\[2\]. These results are in significant contradiction to the traditional understandings based on the static quark model and/or the quark–parton model. Since then, a great deal of efforts have been made experimentally and theoretically for disclosing the origin of the nucleon spin\[3\]. So far, based on the next–to–leading order QCD analyses on the \( x \) and \( Q^2 \) dependence of the longitudinally polarized structure function \( g_1 \), we have got a number of excellent parametrization models of the polarized parton distribution functions (pol–PDFs) from fitting to experimental data, with high precision, of polarized deep inelastic scattering (pol–DIS) for various targets\[4\]–\[7\]. All of these parametrization models tell us that \( \Delta \Sigma \) is around 0.3 or smaller, that is, the contribution of the quark spin to the nucleon spin is rather small. The remainder should be compensated by \( \Delta g \) and \( L_z \). To understand the physical ground of these results is the main subject of the so-called proton spin puzzle, which is still challenging problem to be solved. To solve this problem, it is very important to precisely know how the gluon polarizes in the nucleon. It is well-known that the next–to–leading order QCD analyses on \( g_1 \) bring about information on the first moment of the polarized gluon \( \Delta g \)[8]. However, there are large uncertainties in \( \Delta g \) extracted from \( g_1 \) alone. Knowledge of \( \Delta g \) is still limited because it is very difficult to directly obtain such information from existing data.

So far, a number of interesting proposals such as direct prompt photon production in polarized proton–polarized proton collisions\[9\], open charm\[10\] and \( J/\psi \)[11] productions in polarized lepton
scattering off polarized nucleon targets, were presented for studying longitudinally polarized gluon distributions. Recently, HERMES group at DESY reported the first measurement of the polarized gluon distribution from di–jet analysis of semi–inclusive processes in pol–DIS, though only one data point was given as a function of Bjorken $x$ [12]. In general, a large–$p_T$ hadron pair is produced via photon–gluon fusion (PGF) and QCD Compton at the lowest order of QCD (Fig. 1). The PGF gives us a direct information on the polarized gluon distribution in the nucleon, while QCD Compton becomes background to the signal process for extracting the polarized gluon distribution [13, 14]. When we consider only the case of heavy hadron pair productions, we could safely neglect the contribution of QCD Compton processes [15] because the contents of heavy quarks in the proton are extremely small and furthermore the probability of fragmenting of light quarks to heavy hadrons is also small. However, in this case the cross section itself is rather small at the energy in the running(HERMES) or forthcoming(COMPASS) experiments and thus, we could not have enough data for doing detailed analysis. Furthermore, if we consider only small $x$ region where the PGF dominates over the QCD Compton, we could also extract the polarized gluon distribution without difficulty [3]. However, here we are interested in the polarized gluon distribution for wide region of $x$ to calculate the first moment of the polarized gluon. For light hadron pair productions in wide $x$ region, QCD Compton contribution is not necessarily small and hence, it is rather difficult to unambiguously extract the behavior of the polarized gluon from those processes.

In this work, getting over these obstacles, we propose a new formula for clearly extracting the polarized gluon distribution from the light hadron pair production of pol–DIS by removing the QCD Compton component from the cross section. As is well-known, the cross section of the hadron pair production being semi-inclusive process, can be calculated based on the parton model with various fragmentation functions. Then, by using symmetry relations among fragmentation functions and taking an appropriate combination of various hadron pair production processes, we can possibly remove the contribution of QCD Compton components from the cross section and thus, get clear information of the polarized gluon distribution from the remaining PGF components. Here, to show how to do this practically, we consider the light hadron pair production with large transverse momentum as shown in Fig. 1.
2 Cross sections and double spin asymmetry for large-$p_T$ pion pair production

Let us consider the process of $\ell + N \rightarrow \ell' + h_1 + h_2 + X$ in polarized lepton scattering off polarized nucleon targets (Fig. 1), where $h_1$ and $h_2$ denote light hadrons in a pair. As mentioned above, the spin–dependent differential cross section at the leading order of QCD can be given by the sum of the PGF process and QCD Compton as follows,

$$d\Delta\sigma = d\Delta\sigma_{PGF} + d\Delta\sigma_{QCD}.$$  \hspace{1cm} (2)

with

$$d\Delta\sigma = d\sigma_{-+} - d\sigma_{++} + d\sigma_{+-} - d\sigma_{++}.$$  \hspace{1cm}

Here, for example, $d\sigma_{-+}$ denote that the helicity of an initial lepton and the one of a target proton is negative and positive, respectively. Each term on the right hand side of eq.(2) is given by

$$d\Delta\sigma_{PGF} \sim \Delta g(\eta, Q^2)d\Delta\sigma_{PGF} \sum_{i=u,d,s,\bar{u},\bar{d},\bar{s}} e_i^2 \{ D_i^{h_1}(z_1, Q^2)D_i^{h_2}(z_2, Q^2) + (1\leftrightarrow 2) \},$$  \hspace{1cm} (3)

$$d\Delta\sigma_{QCD} \sim \sum_{q=u,d,s,\bar{u},\bar{d},\bar{s}} e_q^2 \Delta f_q(\eta, Q^2)d\Delta\sigma_{QCD} \{ D_q^{h_1}(z_1, Q^2)D_g^{h_2}(z_2, Q^2) + (1\leftrightarrow 2) \},$$  \hspace{1cm} (4)

where $\Delta g(\eta, Q^2)$, $\Delta f_q(\eta, Q^2)$ and $D_i^h(z, Q^2)$ denote the polarized gluon and $q$–th quark distribution functions with momentum fraction $\eta$ and the fragmentation function of a hadron $h$ with momentum fraction $z$ emitted from a parton $i$, respectively. $d\Delta\sigma_{PGF}$ and $d\Delta\sigma_{QCD}$ are the polarized differential cross sections of hard scattering subprocesses for $\ell g \rightarrow \ell' q\bar{q}$ and $\ell (\gamma) \rightarrow \ell' g(\gamma)$ at the leading order QCD, respectively.

Here we consider the following 4 pairs of a combination for the produced hadrons $h_1$ with $z_1$ and $h_2$ with $z_2$,

(i) $(\pi^+, \pi^-)$,  \hspace{1cm} (ii) $(\pi^-, \pi^+)$,  \hspace{1cm} (iii) $(\pi^+, \pi^+)$,  \hspace{1cm} (iv) $(\pi^-, \pi^-)$,

where (particle 1, particle 2) corresponds to ($h_1$ with $z_1$, $h_2$ with $z_2$). Then, the differential cross section of eq.(4) for each pair can be written as

(i)

$$d\Delta\sigma_{\pi^+\pi^-} \sim \Delta g(\eta, Q^2)d\Delta\sigma_{PGF} \{ \frac{4}{9} D_u^{\pi^+}(z_1, Q^2)D_u^{\pi^-}(z_2, Q^2) + \frac{1}{9} D_d^{\pi^+}(z_1, Q^2)D_d^{\pi^-}(z_2, Q^2)$$

$$+ \frac{1}{9} D_s^{\pi^+}(z_1, Q^2)D_s^{\pi^-}(z_2, Q^2) + (\pi^+ (z_1) \leftrightarrow \pi^-(z_2)) \}$$
various fragmentation functions in eqs. (5)–(8) can be classified into the following 4 functions:\[16\],

Based on the isospin symmetry and charge conjugation invariance of the fragmentation functions,

\[ d\Delta\sigma^{\pi^+\pi^+} \sim \Delta g(\eta, Q^2)d\Delta\hat{\sigma}_{QCD} \{ \frac{4}{9}D_{u}^\pi(z_1, Q^2)D_{\bar{u}}^\pi(z_2, Q^2) + \frac{1}{9}D_{d}^\pi(z_1, Q^2)D_{\bar{d}}^\pi(z_2, Q^2) \}
\]

\[ + \frac{4}{9}\Delta u(\eta, Q^2)d\Delta \hat{\sigma}_{QCD} \{ D_{u}^\pi(z_1, Q^2)D_{\bar{u}}^\pi(z_2, Q^2) + D_{d}^\pi(z_1, Q^2)D_{\bar{d}}^\pi(z_2, Q^2) \} \]

\[ + (\text{contributions from } \Delta d, \Delta s, \Delta \bar{u}, \Delta \bar{d} \text{ and } \Delta \bar{s}) , \] (5)

\[ d\Delta\sigma^{\pi^+\pi^+} \sim \Delta g(\eta, Q^2)d\Delta\hat{\sigma}_{QCD} \{ \frac{4}{9}D_{u}^\pi(z_1, Q^2)D_{\bar{u}}^\pi(z_2, Q^2) + \frac{1}{9}D_{d}^\pi(z_1, Q^2)D_{\bar{d}}^\pi(z_2, Q^2) \}
\]

\[ + \frac{4}{9}\Delta u(\eta, Q^2)d\Delta \hat{\sigma}_{QCD} \{ D_{u}^\pi(z_1, Q^2)D_{\bar{u}}^\pi(z_2, Q^2) + D_{d}^\pi(z_1, Q^2)D_{\bar{d}}^\pi(z_2, Q^2) \} \]

\[ + (\text{contributions from } \Delta d, \Delta s, \Delta \bar{u}, \Delta \bar{d} \text{ and } \Delta \bar{s}) , \] (6)

\[ d\Delta\sigma^{\pi^-\pi^-} \sim \Delta g(\eta, Q^2)d\Delta\hat{\sigma}_{QCD} \{ \frac{4}{9}D_{u}^\pi(z_1, Q^2)D_{\bar{u}}^\pi(z_2, Q^2) + \frac{1}{9}D_{d}^\pi(z_1, Q^2)D_{\bar{d}}^\pi(z_2, Q^2) \}
\]

\[ + \frac{4}{9}\Delta u(\eta, Q^2)d\Delta \hat{\sigma}_{QCD} \{ D_{u}^\pi(z_1, Q^2)D_{\bar{u}}^\pi(z_2, Q^2) + D_{d}^\pi(z_1, Q^2)D_{\bar{d}}^\pi(z_2, Q^2) \} \]

\[ + (\text{contributions from } \Delta d, \Delta s, \Delta \bar{u}, \Delta \bar{d} \text{ and } \Delta \bar{s}) . \] (7)

\[ d\Delta\sigma^{\pi^+\pi^-} \sim \Delta g(\eta, Q^2)d\Delta\hat{\sigma}_{QCD} \{ \frac{4}{9}D_{u}^\pi(z_1, Q^2)D_{\bar{u}}^\pi(z_2, Q^2) + \frac{1}{9}D_{d}^\pi(z_1, Q^2)D_{\bar{d}}^\pi(z_2, Q^2) \}
\]

\[ + \frac{4}{9}\Delta u(\eta, Q^2)d\Delta \hat{\sigma}_{QCD} \{ D_{u}^\pi(z_1, Q^2)D_{\bar{u}}^\pi(z_2, Q^2) + D_{d}^\pi(z_1, Q^2)D_{\bar{d}}^\pi(z_2, Q^2) \} \]

\[ + (\text{contributions from } \Delta d, \Delta s, \Delta \bar{u}, \Delta \bar{d} \text{ and } \Delta \bar{s}) . \] (8)

Based on the isospin symmetry and charge conjugation invariance of the fragmentation functions, various fragmentation functions in eqs. (5)–(8) can be classified into the following 4 functions:\[16\],

\[ D \equiv D_{u}^\pi = D_{d}^\pi = D_{u}^\pi = D_{d}^\pi ; \]

\[ \tilde{D} \equiv D_{d}^\pi = D_{u}^\pi = D_{d}^\pi = D_{u}^\pi ; \]

\[ D_{s} \equiv D_{s}^\pi = D_{s}^\pi = D_{s}^\pi = D_{s}^\pi ; \]

\[ D_{g} \equiv D_{g}^\pi = D_{g}^\pi . \]

where \( D \) and \( \tilde{D} \) are called favored and unfavored fragmentation functions, respectively. Considering the suppression of the s quark contribution to the pion production compared with the u and d quark contribution, we do not identify \( D_{s} \) with \( \tilde{D} \). This seems to be confirmed by 'leading particle' measurements:\[17, 18\]. By using these 4 kinds of pion fragmentation functions, we can
make an interesting combination of cross sections which contains only the PGF contribution as follows:

\[
   d\Delta \sigma^{\pi+\pi^-} + d\Delta \sigma^{\pi^-\pi^+} - d\Delta \sigma^{\pi^+\pi^+} - d\Delta \sigma^{\pi^-\pi^-} \sim \frac{10}{9} \Delta g(\eta, Q^2) d\Delta \hat{\sigma}_{\text{PGF}} \\
   \times \left\{ D(z_1, Q^2)D(z_2, Q^2) + \widetilde{D}(z_1, Q^2)\widetilde{D}(z_2, Q^2) - D(z_1, Q^2)\widetilde{D}(z_2, Q^2) - \widetilde{D}(z_1, Q^2)D(z_2, Q^2) \right\} .
\]

From this combination, we can calculate the double spin asymmetry \(A_{LL}\) defined by

\[
   A_{LL} = \frac{d\Delta \sigma^{\pi^+\pi^-} + d\Delta \sigma^{\pi^-\pi^+} - d\Delta \sigma^{\pi^+\pi^+} - d\Delta \sigma^{\pi^-\pi^-}}{d\sigma^{\pi^+\pi^-} + d\sigma^{\pi^-\pi^+} - d\sigma^{\pi^+\pi^+} - d\sigma^{\pi^-\pi^-}} = \frac{\Delta g(\eta, Q^2)}{g(\eta, Q^2)} \cdot \frac{d\Delta \hat{\sigma}_{\text{PGF}}}{d\hat{\sigma}_{\text{PGF}}} ,
\]

where the factor of the fragmentation function in eq.(9) is dropped out from the numerator and the denominator of \(A_{LL}\). Therefore, from the measured \(A_{LL}\), one can get clear information of \(\Delta g/g\) with reliable calculation of \(d\Delta \hat{\sigma}_{\text{PGF}}/d\hat{\sigma}_{\text{PGF}}\).

Furthermore, when \(z_1 = z_2\), we have another formula

\[
   d\Delta \sigma^{\pi^+\pi^-} - d\Delta \sigma^{\pi^-\pi^+} - d\Delta \sigma^{\pi^-\pi^-} \sim \frac{10}{9} \Delta g(\eta, Q^2) d\Delta \hat{\sigma}_{\text{PGF}} \\
   \times \left\{ D(z, Q^2)D(z, Q^2) + \widetilde{D}(z, Q^2)\widetilde{D}(z, Q^2) - 2 D(z, Q^2)\widetilde{D}(z, Q^2) \right\} .
\]

In this case, we can also define the double spin asymmetry \(A_{LL}\) as follows;

\[
   A_{LL} = \frac{d\Delta \sigma^{\pi^+\pi^-} - d\Delta \sigma^{\pi^-\pi^+} - d\Delta \sigma^{\pi^-\pi^-}}{d\sigma^{\pi^+\pi^-} - d\sigma^{\pi^-\pi^+} - d\sigma^{\pi^-\pi^-}} = \frac{\Delta g(\eta, Q^2)}{g(\eta, Q^2)} \cdot \frac{d\Delta \hat{\sigma}_{\text{PGF}}}{d\hat{\sigma}_{\text{PGF}}} .
\]

The \(A_{LL}\) defined in eq.(11) and eq.(12) results in the same physical quantity. From eq.(12), we can also extract \(\Delta g/g\) as well as in eq.(11).

### 3 Numerical calculations of the cross sections and the double spin asymmetry

In this section, to see how the above formula works, we numerically calculate the double spin asymmetry for the large–\(p_T\) pion pair production of pol–DIS. The spin–independent (spin–dependent) differential cross sections for producing hadrons \(h_1\) and \(h_2\) are given by

\[
   \frac{d(\Delta)\sigma_{h_1h_2}}{dz_1d\cos \theta_1dz_2d\cos \theta_2dxdy} = \frac{d(\Delta)\sigma_{\text{PGF}}^{h_1h_2}}{dz_1d\cos \theta_1dz_2d\cos \theta_2dxdy} + \frac{d(\Delta)\sigma_{QCD}^{h_1h_2}}{dz_1d\cos \theta_1dz_2d\cos \theta_2dxdy} .
\]

\(^1\)Here we take account of the light quarks alone in eqs.(3)–(8). For large \(Q^2\) regions, heavy quarks might be generated for PGF process and QCD compton process might also have contributions of heavy quarks. Even then, the \(A_{LL}\) is reduced to eq.(10) if \(D_{Q}^{\pi} = D_{\bar{Q}}^{\pi} = D_{Q}^{\pi} = D_{\bar{Q}}^{\pi}\).
Each term in the right hand side of eq. (13) is written as

\[
\frac{d(\Delta)\sigma_{h_1h_2}^{h_1h_2}}{dz_1d\cos\theta_1dz_2d\cos\theta_2dz_3dy} = (\Delta)g(\eta, Q^2) C(\theta_1, \theta_2) \frac{d(\Delta)\sigma_{h_1h_2}^{h_1h_2}}{dz_1d\cos\theta_1dz_3d\cos\theta_2dz_4dy} \\
\times \sum_{i=u,d,s,\bar{u},\bar{d},\bar{s}} e_i^2 \{ D_i^{h_1}(z_1', Q^2) D_i^{h_2}(z_2', Q^2) + (1 \leftrightarrow 2) \},
\]

(14)

\[
\frac{d(\Delta)\sigma_{h_1h_2}^{h_1h_2}}{dz_1d\cos\theta_1dz_2d\cos\theta_2dz_3dy} = \sum_{q=u,d,s,\bar{u},\bar{d},\bar{s}} e_q^2(\Delta) f_q(\eta, Q^2) C(\theta_1, \theta_2) \frac{d(\Delta)\sigma_{h_1h_2}^{h_1h_2}}{dz_1d\cos\theta_1dz_3d\cos\theta_2dz_4dy} \\
\times \{ D_q^{h_1}(z_1', Q^2) D_q^{h_2}(z_2', Q^2) + (1 \leftrightarrow 2) \},
\]

(15)

where

\[
\begin{align*}
\eta &= x + (1 - x)\tau_1\tau_2 , \quad Q^2 = y s , \\
\begin{bmatrix} z_1' \\ z_2' \end{bmatrix} &= \begin{bmatrix} \frac{\tau_2 + \tau_1}{\tau_2} \\ \tau_1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} , \\
C(\theta_1, \theta_2) &= \frac{\tau_1 + \tau_2}{\eta \tau_1 \tau_2} \frac{(1 - x)}{8\cos^2\frac{1}{2}\theta_1 \cos^2\frac{1}{2}\theta_2 \sin^2\frac{1}{2}(\theta_1 + \theta_2)} ,
\end{align*}
\]

(16)

with

\[
\tau_{1,2} = \tan \frac{1}{2}\theta_{1,2} .
\]

Here we simply assume the scattering angle of outgoing hadrons \( \theta_{1, 2} \) to be the same with the one of scattered partons in the virtual photon–nucleon c.m. frame. This assumption might not be unreasonable if observed particles are light hadrons with high energy. \( s \) is the total squared energy of the lepton scattering off the nucleon. \( x, y \) and \( z_{1, 2} \) in eqs. (13)–(16) are familiar kinematic variables for semi–inclusive processes in DIS and are defined as

\[
\begin{align*}
x &= \frac{Q^2}{2P \cdot q} , \quad y = \frac{P \cdot q}{P \cdot \ell} , \quad z_{1, 2} = \frac{P \cdot P_{1,2}}{P \cdot q} ,
\end{align*}
\]

where \( \ell, q, P \) and \( P_{1,2} \) are the momentum of the incident lepton, virtual photon, target nucleon and outgoing hadrons, respectively. The differential cross sections of hard scattering subprocesses with outgoing two partons having opposite in an azimuth angle for \( \ell g \to \ell' q\bar{q} \) and \( \ell' q \to \ell' q' \bar{q}' \) at the leading order QCD are given as

\[
\frac{d(\Delta)\sigma_{\text{PGF}(QCD)}}{dz_1d\cos\theta_1dz_2d\cos\theta_2dz_3d\cos\theta_3dz_4d\phi_4} = \frac{\alpha^2\alpha_s}{128\pi^2(p_0 \cdot \ell)Q^2} \frac{y(\eta - x)(1 - \eta)^2}{x} \\
\times B(\theta_1, \theta_2) e_i^2 e_j^2 |(\Delta)M_{\text{PGF}(QCD)}|^2 ,
\]

(17)

with

\[
1 = \frac{B(\theta_1, \theta_2)}{B(\theta_1 + \theta_2)} \left[ \frac{\{ z_i(1 - \eta) + (\eta - x) \} \sin \theta_1 + \{ z_i(1 - \eta) + (\eta - x) \} \sin \theta_2}{\sin \theta_1 \sin \theta_2} \right] ,
\]
where $z_i$, $z_\ell$ and $z_g$ are the momentum fraction of the outgoing parton $i$, $\tilde{i}$ and $g$, respectively, to the incoming parton, and are given as \[ 19 \]

$$z_i = \frac{\tau_2}{\tau_1 + \tau_2}, \quad z_{i(g)} = \frac{\tau_1}{\tau_1 + \tau_2}. $$

The amplitude $|M|^2_{PGF(QCD)}$ in eq. (17) is given by \[ 21 \]

$$|M|^2_{PGF} = 16(\ell \cdot \ell') \left[ \frac{(\ell \cdot p_1)^2 + (\ell' \cdot p_1)^2 + (\ell \cdot p_2)^2 + (\ell' \cdot p_2)^2}{(p_0 \cdot p_1)(p_0 \cdot p_2)} \right], \quad |M|^2_{QCD} = \frac{128}{3}(\ell \cdot \ell') \left[ \frac{(\ell \cdot p_0)^2 + (\ell' \cdot p_0)^2 + (\ell \cdot p_1)^2 + (\ell' \cdot p_1)^2}{(p_0 \cdot p_2)(p_1 \cdot p_2)} \right]$$

for the spin–independent case and

$$|\Delta M|^2_{PGF} = 16(\ell \cdot \ell') \left[ \frac{(\ell' \cdot p_1)^2 - (\ell \cdot p_1)^2 + (\ell' \cdot p_2)^2 - (\ell \cdot p_2)^2}{(p_0 \cdot p_1)(p_0 \cdot p_2)} \right], \quad |\Delta M|^2_{QCD} = \frac{128}{3}(\ell \cdot \ell') \left[ \frac{(\ell \cdot p_0)^2 - (\ell' \cdot p_0)^2 - (\ell \cdot p_1)^2 + (\ell' \cdot p_1)^2}{(p_0 \cdot p_2)(p_1 \cdot p_2)} \right]$$

for the spin–dependent case. The $p_0, p_1, p_2$ and $\ell'$ are the momentum of the incoming parton, outgoing parton $i$, $\tilde{i}$, $g$ and the outgoing lepton, respectively, and are written by

$$p_0^\mu = |P|(1, 0, 0, -\eta), \quad p_1^\mu = |p_1|(1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta),$$

$$p_2^\mu = |p_2|(1, \sin \theta_2 \cos(\phi_1 - \pi), \sin \theta_2 \sin(\phi_1 - \pi), \cos \theta_2),$$

$$\ell^\mu = |\ell|(1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta),$$

$$q^\mu = |P|(1, 2x, 0, 0, 1), \quad \ell'_\mu = \ell_\mu - q_\mu,$$

with

$$|P| = \sqrt{\frac{Q^2}{4x(1-x)}}, \quad |p_{1,2}| = |P| \{z_i, z_{i(g)}(1-\eta) + (\eta-x)\},$$

$$\cos \theta_1 = \frac{z_{i, i(g)}(1+\eta-2x)-(\eta-x)}{z_{i, i(g)}(1-\eta)+(-\eta-x)}, \quad |\ell| = |P| \frac{1-xy}{y}, \quad \cos \theta_\ell = \frac{1-2x+xy}{1-xy},$$

in the c.m. frame of the virtual photon–nucleon system \[ 19 \]. By using these formulas and newly analyzed pion fragmentation functions \[ 17 \], we have calculated the spin–dependent and spin–independent cross sections of the large–$p_T$ pion pair production and estimated the double spin asymmetry of eq. (10) at the energy of HERMES experiments. Here, we have taken the AAC \[ 7 \] and GS96 \[ 5 \] parametrizations at LO QCD as polarized parton distribution functions and GRV98 \[ 21 \] and MRST98 \[ 22 \] as unpolarized ones. At $\sqrt{s} = 7.25$GeV, $y = 0.75$, $Q^2 \geq 1$GeV$^2$ and $W^2 \geq 10$GeV$^2$ with two different sets of kinematical values of $\theta_{1,2}$ and $z_{1,2}$ for the produced pion.
pair, the calculated results of the spin–independent (spin–dependent) differential cross sections and $A_{LL}$ are shown as a function of $\eta$ in Figs. 2 and 3, respectively. From Fig. 3, one can see a big difference of the behavior of $A_{LL}$ depending on the models of $\Delta g/g$ and hence, we can extract the behavior of $\Delta g$ rather clearly from this analysis.

4 Conclusion and discussion

We proposed a new formula for extracting the polarized gluon distribution from the large–$p_T$ light hadron pair production in pol–DIS by making an appropriate combination of hadron pair productions. Since the double spin asymmetry $A_{LL}$ for this combination is directly proportional to $\Delta g/g$, the measurement of this quantity is quite promising for getting rather clear information on the polarized gluon distribution in the nucleon.

In this work, we calculated only the case of the large–$p_T$ pion pair production. The same analysis can be applied also for the kaon or the proton pair productions by considering the reflection symmetry along the V–spin axis, the isospin symmetry and charge conjugation invariance of the fragmentation functions as follows\[10\]

\[
\begin{align*}
D^K_s &\equiv D^K_s^+ = D^K_s^- , \\
\tilde{D}^K_s &\equiv D^K_{\bar{s}}^+ = D^K_{\bar{s}}^- , \\
D^K_u &\equiv D^K_u^+ = D^K_u^- , \\
\tilde{D}^K_{u,d} &\equiv D^K_{\bar{u}}^+ = D^K_{\bar{d}}^+ = D^K_{\bar{d}}^- = D^K_{\bar{u}}^- = D^K_u^- = D^K_{\bar{d}}^- = D^K_{\bar{u}}^- = D^K_{\bar{d}}^- , \\
D^K_g &\equiv D^K_g^+ = D^K_g^- ,
\end{align*}
\]
for the kaon pair production case and

\[
\begin{align*}
D^p &\equiv D^p_u = D^p_d = D^p_{\bar{u}} = D^p_{\bar{d}} , \\
\tilde{D}^p &\equiv D^p_{\bar{u}} = D^p_{\bar{d}} = D^p_u = D^p_d , \\
D^p_s &\equiv D^p_s = D^p_{\bar{s}} = D^p_{\bar{s}} = D^p_s , \\
D^p_g &\equiv D^p_g = D^p_g ,
\end{align*}
\]
for the proton pair production case.

\[\text{Here the polarized parton distributions were evolved from } Q_0^2 = 1\text{GeV}^2 \text{ to any } Q^2 \text{ value. Though the initial } Q_0^2 \text{ value of the GS96 model is taken to be } 4\text{GeV}^2 \text{ in original literature, we simply assumed their parton distributions to be in scaling for } 1 \leq Q^2 < 4\text{GeV}^2.\]
In general, it might not be so easy to precisely extract a physical quantity from a combination of data on the different kind of physical quantities. However, if we have enough data with high precision for wide kinematical region, it could be possible to determine the physical quantity concerned with rather precisely from those data, just as in the case of Bjorken sum rule in which two different quantities, $g_1^p$ and $g_1^n$, were combined. We believe that the formulas proposed here are suitable for those analyses, since we can expect to have rather many and precise data because of large cross sections for the light hadron pair production. The HERMES RICH detector at DESY which will provide good particle identification in the wide momentum range could allow to measure $A_{LL}$ for the processes discussed here.

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Figure 1: Lowest order Feynman diagrams for the large-$p_T$ hadron pair production.
Figure 2: Combined spin–independent and spin–dependent differential cross sections defined at the denominator and numerator, respectively, of eq.(10) as a function of $\eta$ at $\sqrt{s} = 7.25$ GeV, $y = 0.75$ for the deep inelastic regions ($Q^2 \geq 1$GeV$^2$ and $W^2 \geq 10$GeV$^2$) with two different sets of kinematical values($\theta_{1,2}, z_{1,2}$) of the produced pion pair. $\eta$ is the momentum fraction of the gluon and obtained from $x$ by eq.(16).
$s^{1/2} = 7.25$ GeV, $y = 0.75$

Figure 3: The $\eta$ dependence of $A_{LL}$ at $\sqrt{s} = 7.25$ GeV, $y = 0.75$ for two different sets of kinematical values($\theta_{1,2}, z_{1,2}$) of the produced pion pair. Solid line and dotted line are for AAC LO and GS96LO–C parametrization models, respectively. $\Delta g/g$ itself is also presented for both parametrization models.