Sawtooth Oscillations in EBIT

R. Radtke, C. Biedermann, P. Bachmann, G. Fussmann, and T. Windisch

Institut für Physik der Humboldt-Universität zu Berlin,
Lehrstuhl Plasmaphysik, Newtonstraße 15, 12489 Berlin and
Max-Planck-Institut für Plasmaphysik, EURATOM Association
e-mail address: rainer.radtke@physik.hu-berlin.de

Abstract. The dynamics of mixed ensembles of highly charged argon/xenon and krypton/xenon ions in an electron-beam ion trap (EBIT) was studied by recording the characteristic x-ray emission of the trapped ions. Sawtooth-like signatures manifest in the x-ray spectra for a variety of trap parameters. The effect can be understood as arising from the feedback between low-Z and high-Z ions.

In this paper we report on a new and unexpected effect that is triggered by the non-linear dynamics and interaction of ions confined in electron beam ion traps and was recently demonstrated for the first time at the Berlin EBIT facility [1,2]. The effect manifests by sawtooth-like oscillations of the trap plasma and has the potential to serve as benchmark for testing of theoretical EBIT models. It is caused by the feedback between low-Z and high-Z ions when, in addition to a heavy component, a light element is injected into the trap for cooling purposes. Working with light ions along with heavier ones is termed evaporative cooling [3] and has been proven to be essential for the production and storage of very highly charged ions. A substantial prerequisite for the creation of sawtooth oscillations is that the commonly applied conditions for the cooling technique are changed: The rate at which the light element (coolant gas) is added to the trap is much larger than the influx of the heavy component. Operating in this way, highly charged ions can concentrate in the trap during extended accumulation periods until the cooling from decreased low-Z ions is no longer sufficient and a sudden collapse of the ion population occurs.

While the first measurements described in ref. [1,2] were carried out for a mixture of argon and barium ions, we have used in the present study a continuous flow of xenon (Z=54) atoms mixed with coolant gas argon (Z=18) or krypton (Z=36). The high-Z and low-Z gases were mixed by regulating the absolute pressure in a storage container which then allowed the injection through a differentially pumped gas-jet system. EBIT was operated in a static mode where all experimental parameters (applied voltages, beam current, magnetic field strength, gas flow) were held constant as a function of time for each measurement. The electron-beam energy was fixed at E_b=5 keV, limiting the ionization to the maximum charge state of Ar to 18+, Kr to 34+, and Xe to 44+. Characteristic x-ray spectra of the confined ions were measured in the range 500 eV to 15 keV using a solid-state Ge detector. The pulse-height...
information of the x-ray energy was digitized by an ADC and counted by a multi-
channel scaler as a function of time. This data can be visualized in a scatter plot of x-
ray events versus time as illustrated in Fig. 1 (top) for the system of highly charged
Ar and Xe ions. In the lower part of Fig. 1 time traces representative for Ar and Xe
ions are shown by projecting x-ray events of electron-impact excitation (IE) onto the
time axis. The spectrum was taken for 25 s after closing the trap at t=0. The prominent
feature is the time structure of the x-ray emission making evident that the population
in the trap does not reach a steady state. The signal for Ar (Xe) ions decreases
(increases) steadily over successive time periods. Towards the end of each period, a
sudden drop in the intensity is observed in all radiation channels pointing to rapid ion
expulsion from the trap. It is obvious from the figure that Ar ions are almost
completely driven out, but the Xe inventory is not. This difference in behavior of ion
expulsion is an indication of a strong outflow of energy transferred from the higher-
charge-state Xe ions to the Ar ions leaving the trap.

![Figure 1](image-url)

**FIGURE 1.** X-ray emission of highly charged Ar and Xe ions vs. time (scatter plot). Emission bands
for Ar and Xe from different x-ray production mechanisms are marked (RR = Radiative Recombination,
IE = Impact Excitation). The lower plot is the projection of the Ar IE n = 2-1 and Xe IE n = 3-2 x-ray
events onto the time axis. The feedback between the Ar and Xe ions is expressed by the mirror-like
behaviour of the x-ray intensities.

To enlarge our understanding of the sawtooth effect and survey the conditions
where sawtooth-like features can be observed in EBIT, the time evolution was
measured as a function of various trap parameters. In addition, the Z-dependence of
the effect was studied by working with argon and krypton as coolant gas. Figure 2
presents data showing the influence of the axial trap potential $V_{ax}$ on the time profiles.$V_{ax}=V_{bias}+V_{ip}$ is the sum of the upper drift tubes’ bias plus the image potential formed
by the electron beam and the geometry of the trap. The trap consists of three
cylindrical tubes, where the center drift tube has an inner diameter of 10 mm and the
outer ones are 3 mm at minimum. In addition to $V_{ip}$, the trapped ions produce a space charge potential ($V_{ion}$) that causes the true axial trap depth to deviate from $V_{ax}$. The value of $V_{ion}$ can only roughly be estimated in the experiment. Most prominent in Fig. 2 is the change in the shape of the profiles as $V_{ax}$ is raised. At lower trapping voltage, Ar ions escape the trap essentially immediately after being created while the intensity of Xe increases. The trapping process is connected with a complex interplay between the ions expressed by fast oscillations in the intensities. Beyond a certain value of $V_{ax}$ a greater abundance of ions is confined in the trap (which proceeds with an enhanced ion-ion collision rate) and a threshold for the excitation of strong sawteeth is reached. The growth of the sawteeth as $V_{ax}$ is further raised is seen in the 27.8-V and 35.8-V plots. Note also that with increasing $V_{ax}$ the system takes longer to reach sawtooth activity. For potentials above approximately 40 V a transition to a new mode is observed where the periodic mechanism is turned off definitely after the first collapse. This single ion expulsion occurs even at higher trapping voltage ($V_{ax} \geq 300$ V), while the periodic sawtooth mechanism requires shallow trap conditions.

![FIGURE 2. Time profile of n = 2-1 and n = 3-2 emission spectra of Ar and Xe ions for different axial trap depths $V_{ax}$. The value of $V_{ax}$ is labeled in each panel. The beam current ($I_b=86$ mA) and energy ($E_b=5$ keV) and the ratio of components ($p_{Ar}/p_{Xe} = 7.3$) are fixed.](image)

In Fig. 3, time profiles as a function of the electron-beam current ($I_b$) are shown. The most dramatic aspect of the $I_b$ scan are the thresholds at 66 and 100 mA. The
transition from single to periodic ion expulsion occurs within a very small current interval of ~1 mA. It is this critical dependence that could establish a base for a detailed comparison with theoretical predictions and help to detect deficiencies in existing EBIT models. The growing of the tooth length in Fig. 3 is caused by the increasing trap potential with greater $I_b$. This becomes also obvious in Fig. 2 where the effect is more pronounced because of the larger variation of $V_{ax}$.

**FIGURE 3.** Time profile of $n = 2-1$ and $n = 3-2$ emission spectra of Ar and Xe ions for eight electron-beam currents. Since the bias voltage $V_{bias}$ was held constant and because of $V_{ip} \propto I_b$, the value of $V_{bias} + V_{ip}$ changes with $I_b$ as indicated in each panel. Parameters: $E_b = 5$ keV, $V_{bias} = -10$ V, $p_{Ar}/p_{Xe} = 10.0$.

Substituting the coolant krypton for argon resulted in the profiles of Fig. 4. In contrast to operating with Ar gas (Fig. 3), we see less variation in the time structure which points to weaker exchange between the confined ions. In particular, thresholds for switching the periodic mechanism on and off could not be detected in this case. This observation is important because it supports our explanation of the sawtooth effect in terms of a strong feedback between lower-charged coolant ions and highercarged trapped ions of different gas species.

We have also studied the influence of the magnetic field ($B$) which compresses the electron beam in the drift tube region and contributes to the radial trapping of ions. A series of $n=2-1$ and $n=3-2$ emission spectra for Ar and Xe ions, obtained at 90 and
100-mA-electron beam current, is plotted in Fig. 5 for different magnetic fields. The measurements show a dramatic variation of the time structure with B, which is much stronger than expected from the $V_{ax}$ and $I_b$ scans. Using the characteristic electron-beam radius calculated with the expression given by Herrmann [4], the electron-beam current in the B scan was adjusted to keep the electron density for two different fields approximately constant. This corresponds to similar rates of electron-beam heating and production of Ar and Xe ions in the trap. Nevertheless, we notice differences between the profiles with approximately equal electron density indicating a direct influence of the B-field.

In order to provide a more complete characterization of the sawtooth effect, we have begun measurements of the ions’ escape from the trap during the collapse. For the conditions chosen here, axial escape should dominate the radial escape because of $V_{ax} < V_{rad}$ ($V_{rad} \approx 10.9\times I_b/(4\pi\varepsilon_0 v_e)$ is the radial trapping potential at the drift tube wall relative to the beam axis). We could prove this supposition experimentally by locating a Faraday cup at the upper end of EBIT monitoring the ion escape in axial direction. Figure 6 displays for a Kr/Xe-ion inventory the evolution of the x-ray emission along with the trace of the measured ion current. The ion current exhibits features that in fact

**FIGURE 4.** Time profile of $n = 2-1$ and $n = 3-2$ emission spectra of Kr and Xe ions for eight electron-beam currents. Fixed parameters: $E_b = 5$ keV, $V_{bias} = -10$ V, $p_{Ar}/p_{Xe} = 85.0$. 
FIGURE 5. Time profile of $n = 2-1$ and $n = 3-2$ emission spectra of Ar and Xe ions for different magnetic field strengths (B) and beam currents. The value of B is marked in each panel. The values of $I_b$ and the calculated electron density $n_e$ are 90 mA & $3.6 \cdot 10^{12}$ cm$^{-3}$, 90 mA & $3.3 \cdot 10^{12}$ cm$^{-3}$, 100 mA & $3.6 \cdot 10^{12}$ cm$^{-3}$, and 100 mA & $3.3 \cdot 10^{12}$ cm$^{-3}$ (from top to bottom panel). Fixed parameters: $E_b = 5$ keV, $V_{\text{bias}} = -22$ V, $p_{\text{Ar}}/p_{\text{Xe}} = 10.0$.

FIGURE 6. Upper plot: Measured current of ions expelled from the trap in axial direction. Lower plot: Corresponding time profile of $n = 2-1$ and $n = 3-2$ emission spectra of Kr and Xe ions. Fixed parameters: $E_b = 5$ keV, $V_{\text{bias}} = -10$ V, $p_{\text{Kr}}/p_{\text{Xe}} = 85.0$. 

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accurately coincide with the quickly changing intensity during the collapse period of the sawteeth.

Theoretical analysis of the sawtooth effect was made by performing modeling calculations for conditions similar to those used in the experiment. The modeling physics commonly rests on coupled rate equations for the ion density and temperature and includes (1) electron-impact ionization, (2) radiative recombination, (3) charge exchange, (4) electron-beam heating, (5) ion-ion energy exchange, (6) axial ion escape, (7) radial ion escape (commonly treated incorrectly similar to axial escape, but it is due to cross-field diffusion), (8) overlap factors (electron beam-ion and ion-ion), and (9) trap neutralization. In order to correlate the sawtooth effect with certain processes in EBIT time profiles were calculated making two assumptions: (i) supposing the ions to be uniformly distributed throughout a top-hat electron-beam profile and taking the processes (1,2,4,5,6) into account, (ii) supposing a top-hat electron-beam profile and taking (1,2,4,5,6) as well as (8,9) into account. The evolution of the ion density is described by

\[ \frac{dn_q}{dt} = n_{q-1}v_{q-1}^{\text{ion}} - n_{q+1}v_{q+1}^{\text{rec}} - n_q \left( v_{q-1}^{\text{ion}} - v_{q-1}^{\text{rec}} - v_{q-1}^{\text{esc}} \right) \]

where \( n_q \) is the density of trapped ions in charge state \( q \) and \( v_q \) the collision frequency for ionization (\( \text{ion} \)), recombination (\( \text{rec} \)), and axial escape (\( \text{esc} \)). Ionization and recombination are calculated using the ionization and recombination cross-sections devised by Lotz [5] and Kim and Pratt [6], respectively. The collision frequency for axial ion escape was calculated using the expression from ref. [7]. In addition, a modified expression for \( v^{\text{esc}} \) was derived (see Appendix) taking into account that the ions’ velocity distribution function must vanish beyond the maximum velocity \( u_a = (2qAV_{\text{ax}}/m_a)^{1/2} \). The new expression predicts increased loss rates, but does not alter the structure of the calculated time evolution significantly. The ion temperature is obtained from the balance between electron-beam heating of ions (4), ion-ion energy exchange (5), and energy loss due to escaping ions (6)

\[ \frac{d}{dt} \left( \frac{1}{2}kT_q n_q \right) = -dW_q^{(4)}/dt - dW_q^{(5)}/dt - \Sigma_p dW_p^{(5)}/dt \]

For the energies \( W^{(4,5,6)} \) we have employed the expressions in ref. [7]. The coupled non-linear rate equations were solved by a FORTRAN code using stiff stable NAG routines [8]. The calculation started with neutral xenon and argon (or krypton) and covered all rungs of the charge ladder to determine the time evolution. In our earlier analysis [1] only a representative sequence of ionization stages was considered in the simulation. We stress that solving the complete set of rate equations is important to warrant the dynamics of the many-particle system. Reducing the problem to only few representatives of each component resulted in qualitatively similar time evolution, but slower oscillations of the ion density and temperature in Fig. 7.
FIGURE 7. Calculated time evolution of Ar and Xe ions at $E_b = 5$ keV, $I_B = 50$ mA, and $V_{bias} = -10$ V. The value of $V_{ax}$ determined from these parameters is 15.8 V. The calculations are based on Eqs. (1) and (2) and assumptions (i). Number density of neutrals: $n_{0,Ar} = 4 \times 10^6$ cm$^{-3}$ and $n_{0,Xe} = 5 \times 10^3$ cm$^{-3}$. Only the results for Xe$^{10+}, 20+, 30+, 37+, 44+$ and Ar$^{18+}$ are shown in the figure.

Figure 7 is an example showing the predicted evolution for a typical set of trap parameters and injection of Ar/Xe gas. Although our calculations contain only a fraction of the complexities of the EBIT device [assumptions (i)], they reproduce the feedback between low-Z Ar and high-Z Xe ions and exhibit features that are quite similar to part of the experimental findings. From the data presented in Fig. 7, the sawteeth arise according to the following scheme: Xe ions accumulate in the trap at low temperature and decreasing density of Ar ions. The process continues until the effect of cooling is reduced due to the reduction of the Ar density. This leads to the rise of the ion temperatures and substantially more particles are lost from the trap, such that the temperature increase and particle loss is further enhanced. The lower-charge-state Ar ions are bound weaker to the trapping potential and are almost completely lost by axial escape. Xe ions are driven out from the trap to lesser extent. Once their density decreases as a function of time, the rate at which energy is transferred to the Ar ions slows down. This allows fresh Ar ions to populate the trap and stay in the electron beam longer. They are heated less by the electron beam and cool the Xe ions at declining temperature. The rise in the density curves for Xe in Fig. 7 is a consequence of the improved cooling. The ions are confined at the low initial temperature and concentrate in the trap until their density becomes sufficiently high that the competition between Ar and Xe starts again.

Our calculations did not reproduce the strength of the collapse in the trap (expulsion of Xe ions) and accordingly the measured tooth lengths or the threshold behavior observed in the time profiles in Figs. 2 and 3. This underlines the complexity of the ion dynamics in EBIT which is difficult to predict with such simple assumptions.
Although overlap factors may not be a good concept to describe the spatial distribution of a mixed ensemble of heavy and light ions (which are displaced perpendicularly to the field \( B \) through ion-ion collisions), an improvement was achieved in making calculations based on the assumptions (ii). These calculations are still being developed and only preliminary results were obtained so far. We could generate oscillations of the ion density and temperature and identify parameter ranges where the ion plasma bifurcates (Hopf bifurcation). This bifurcation corresponds to extremes in the density and temperature evolution. It is likely that the experimental thresholds for switching the sawtooth activity on and off correspond to bifurcation points of the many-particle system. However, the discrepancy in the amplitude of the ion collapse still remains. This indicates that more sophisticated simulations will be required to gain physical insight in the dynamics of ions confined in electron-beam ion traps and sources.

**APPENDIX**

One considers the loss of test particles of type “a” due to Coulomb collisions with field particles of type “b”. The particles “a” (ions of charge \( q_a \), mass \( m_a \), and temperature \( T_a \)) are confined in EBITs’ axial potential \( V_{ax} \) up to a maximum velocity \( u_a = (2q_aV_{ax}/m_a)^{1/2} \). Their distribution function \( f_a \) must therefore vanish for \( v_\parallel > u_a \). Because of the high rate of pitch angle scattering \( f_a \) has still nearly spherical symmetry. For \( v_\perp \leq u_a \) it can be approximated by the following model function

\[
f_a = \frac{n_{a0}}{\pi^{3/2} v_a} \left( e^{-v_\perp^2/v_a^2} - e^{-(v-2u_a)^2/v_a^2} \right)
\]

where \( v_a = (2T_a/m_a)^{1/2} \) is the thermal velocity. For \( v_a \ll u_a \) a Maxwellian with temperature \( T_a \) and density \( n_{a0} \) is obtained. However, with rising temperature the deviations in the wings of the distribution function are becoming substantial and the true density \( n_a \) will then differ from \( n_{a0} \). \( n_a \) and \( n_{a0} \) are linked by the normalization relation

\[
\frac{n_a}{n_{a0}} = \text{Erf}\left[\frac{u_a}{v_a}\right] + \left(1 + \frac{8u_a^2}{v_a^2}\right) \text{Erf}\left[\frac{u_a}{v_a}\right] - \text{Erf}\left[\frac{2u_a}{v_a}\right] + \frac{4u_a}{v_a^{1/2}} \left( e^{-u_\perp/v_a^2} - e^{-4u_\perp/v_a^2} \right)
\]

Using the model functions for test and field particles, the Fokker-Planck Collision operator (i.e. the Rosenbluth potentials \( \varphi \) and \( \psi \), see Ref. [9]) can be analytically determined. The loss rate of the test particles \( \frac{dn_a}{dt} = \Sigma_b L_{ab} \) is then given by the diffusion in velocity space at the threshold velocity

\[
L_{ab} = \alpha \ln \Lambda \left\{ \frac{q_a q_b}{m_a e_0} \right\}^2 4\pi u_a^2 (\partial \varphi / \partial v_a) (\partial \psi / \partial v_b) \left|_{v=v_a} \right.
\]
where $\ln \Lambda \approx 10$ is the Coulomb logarithm and $\alpha \leq 1$ a factor taking into account that particles are lost in a preferential direction. With $z = u_a/v_b$ and $y = u_b/v_b$ we get (using Mathematica)

$$L_{ab} = \alpha \ln \Lambda \frac{8n_a n_b}{3\pi^{3/2} e_0^2} \left( \frac{q_a q_b}{m_a v_a} \right)^2 \frac{F(z,y)}{v_b} \frac{u_a^3}{v_a} e^{-u_a/v_a},$$

where $F(z,y)$ equals

$$\frac{y(5+8y^2)}{z^4} \frac{e^{-(z-2y)^2}}{\pi^{1/2}} - \frac{3}{2z^2} \frac{e^{-(z-2y)^2}}{\pi^{1/2}} - \frac{2y(2y+z)}{z^2} \frac{e^{-(z-2y)^2}}{\pi^{1/2}} +$$

$$2y[Erf(y) - Erf(2y-z)] + \left[ 3 + 48y^2 + 64y^4 \right] \left[ Erf(2y-z) - Erf(2y) \right] + 3\text{Erf}(z)$$

for $z < y$ and

$$\frac{1}{z} \left[ 6y^3 \frac{e^{-y^2}}{\pi^{1/2}} + y(5+8y^2) \left( \frac{e^{-y^2}}{\pi^{1/2}} \right) + \frac{3}{4} \text{Erf}(2y) + (1.5 + 12y^2 + 16y^4) \left[ \text{Erf}(y) - \text{Erf}(2y) \right] \right]$$

for $z \geq y$.

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