Polarimetric Study of Near-Earth Asteroid (1566) Icarus

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Abstract

We conducted a polarimetric observation of the fast-rotating near-Earth asteroid (1566) Icarus at large phase (Sun–asteroid–observer’s) angles $\alpha = 57^\circ - 141^\circ$ around the 2015 summer solstice. We found that the maximum values of the linear polarization degree are $P_{\text{max}} = 7.32 \pm 0.25\%$ at phase angles of $\alpha_{\text{max}} = 124^\circ \pm 8^\circ$ in the V-band and $P_{\text{max}} = 7.04 \pm 0.21\%$ at $\alpha_{\text{max}} = 124^\circ \pm 6^\circ$ in the $R_C$-band. Applying the polarimetric slope–albedo empirical law, we derived a geometric albedo of $p_V = 0.25 \pm 0.02$, which is in agreement with that of Q-type taxonomic asteroids. $\alpha_{\text{max}}$ is unambiguously larger than that of Mercury, the Moon, and another near-Earth S-type asteroid (4179) Toutatis but consistent with laboratory samples with hundreds of microns in size. The combination of the maximum polarization degree and the geometric albedo is in accordance with terrestrial rocks with a diameter of several hundreds of micrometers. The photometric function indicates a large macroscopic roughness. We hypothesize that the unique environment (i.e., the small perihelion distance $q = 0.187$ au and a short rotational period of $T_{\text{rot}} = 2.27$ hr) may be attributed to the paucity of small grains on the surface, as indicated on (3200) Phaethon.

Key words: minor planets; asteroids: individual (Icarus) – polarization – techniques: polarimetric

1. Introduction

The polarimetry of solar system airless bodies (e.g., the Moon, Mercury and asteroids) is a useful diagnostic measure for investigating their surface physical properties, such as the albedo and regolith size. The linear polarization degree $P_\tau$ is defined as

$$P_\tau = \frac{I_- - I_\|}{I_- + I_\|},$$

(1)

where $I_-$ and $I_\|$ denote the intensities of scattered light measured with respect to the scattering plane. In general, $P_\tau$ exhibits a strong dependence on the phase angle (Sun–target–observer’s angle, $\alpha$), consisting of a negative branch at $\alpha \lesssim 20^\circ$ with a minimum value $P_{\text{min}}$ at the phase angle $\alpha_{\text{min}} \sim 10^\circ$ and a positive branch at $\alpha \gtrsim 20^\circ$ with a maximum value $P_{\text{max}}$ at $\alpha_{\text{max}} \sim 100^\circ$ (see, e.g., Geake & Dollfus 1986). It is well known that the albedo (the V-band geometric albedo, $p_V$, is often referenced) of objects has a strong correlation with the polarization degree (so-called Umov’s law) because multiple scattering (which is dependent on the single-scattering albedo) randomizes polarization vectors and eventually weakens the polarization degree of the scattered signal from the bodies. Moreover, it was noted that $P_{\text{max}}$ and $\alpha_{\text{max}}$ have a moderate correlation with grain size (Bowell et al. 1972; Shkuratov & Opanasenko 1992; Dollfus 1998). While intensive polarimetric research on asteroids has been conducted at small phase angles ($\alpha \lesssim 40^\circ$, Bel'skaya et al. 2009; Gil-Hutton & Cañada-Assandri 2011, 2012; Gil-Hutton et al. 2014; Cellino et al. 2016; Bel'skaya et al. 2017), polarimetric studies of asteroids at large $\alpha$ values remain less common, most likely because of fewer opportunities (limited to near-Earth asteroids, NEAs) and observational difficulty (small solar elongation).

Here, we would like to stress the superiority of observatories at middle latitudes ($|l| \sim 45^\circ$) for NEA observations at large $\alpha$ values. During the summer solstice, the Sun does not set at latitudes of $|l| > 66^\circ$ (a phenomenon called the midnight Sun); in addition, astronomical twilight lasts through the night at observatories at $|l| > 48^\circ$, making it difficult to make astronomical observations. When we conduct observations at observatories at longitudes slightly lower than $|l| > 48^\circ$, we are able to observe NEAs around the Sun as if we were using the Earth as a coronagraph. Taking advantage of this location, we conducted a polarimetric observation of an NEA, (1566) Icarus ($q = 1494$ MA), at the Nayoro Observatory ($l = +44^\circ$) in Hokkaido, Japan, around the summer solstice in 2015. Icarus is one of the Apollo asteroids that has a small perihelion distance of $q = 0.187$ au. Such asteroids with a small perihelion distance have gained the attention of solar system scientists interested in understanding the mass erosion mechanisms on these bodies (Jewitt 2013; Granvik et al. 2016). Icarus has a diameter of 0.8–1.3 km (Veeder et al. 1989; Mahapatra et al. 1999; Harris & Lagerros 2002; Nugent et al. 2015) and a short...
rotational period of 2.27 hr (Miner & Young 1969; Gehrels et al. 1970; Harris 1998; Warner 2015). The asteroid is classified as an S-type asteroid (Chapman et al. 1975) or, more specifically, as a Q-type asteroid (DeMeo et al. 2014). Thanks to the favorable location of the observatory, we were able to acquire polarimetric data up to α = 141° and imaging data up to α = 145°. The phase angle is overwhelmingly larger than those of previous polarimetric observations at large phase angles (i.e., α < 106° for (1685) Toro, 115° for (23187) 2000 PN0, and 118° for (4179) Toutatis; Kiselev et al. 1990; Ishiguro et al. 1997; Belskaya et al. 2009). We describe our observations in Section 2, the data reduction in Section 3, and the results in Section 4. Finally, we compare our polarimetric results with those of laboratory samples and solar system airless bodies, and we consider the surface regolith properties of the asteroid in Section 5.

2. Observation
The journal of our observations is given in Table 1.

We conducted observations for eight nights from UT 2015 June 11 to UT 2015 June 20 using the 1.6-m Pirka telescope at the Nayoro Observatory (142°28′58″0, +44°22′25″1, 192.1 m, observatory code number Q33). The observatory has been operated since 2011 by the Faculty of Science, Hokkaido University, Japan. We utilized a Multi-Spectral Imager (MSI) mounted at the f/12 Cassegrain focus of the Pirka telescope. The combination of the telescope and the instrument enables the acquisition of images that cover a 3′3 × 3′3 field of view (FOV) with a 0″39 pixel resolution (Watanabe et al. 2012). We conducted an imaging observation on the first night, June 11, to test the non-sidereal tracking of the fast moving object at a low elevation (~10°). These data were used for the study of photometric functions presented in Section 4.3. After the second night, we made an imaging polarimetric observation from June 12–20 using a polarimetric module comprising a Wollaston prism and a half-wave plate. To avoid the blending of ordinary and extraordinary rays, a two-slit mask was placed at the focal plane for the polarimetric observation. With the mask, the FOV was subdivided into two adjacent sky areas of 3′3 × 0′7 each and separated by 1′7 (see Figure 1). We chose standard Johnson–Cousins V and RC-band filters for this study to examine the wavelength dependence of the polarization degree.

We took polarimetric data with exposure times of either 30 s or 60 s (depending on the apparent magnitude of the asteroid) for a single frame. Between exposures, the half-wave plate was routinely rotated from 0° to 45°, from 45° to 22°5, and from 22°5 to 67°5 in sequence to acquire one subset of polarimetric data. Once we acquired the subset of data, the pointing direction of the telescope was shifted by +10° and −10° in turn along the east–west axis (the longer axis of the polarization mask) to acquire the other two subsets. This technique (called dithering) can reduce the effects of pixel-to-pixel inhomogeneity that were not substantially corrected by flat-field correction. Accordingly, each set of data consists of 12 exposures (4 exposures with different half-wave plate angles × 3 locations on the CCD chip with the ±10°-dithering mode). We took bias frames before and after the asteroid observations in approximately 3 hr intervals. At the end of nightly observation, we obtained dome flat-field data at the same focal position of the telescope as that with which we observed the asteroid.

3. Data Analysis
The observed data were analyzed in the same manner as in Kuroda et al. (2015) and Itoh et al. (2017). The raw observational data were preprocessed using flat images and bias frames by the MSI data reduction package (MSIRED). Cosmic rays on the images were erased using the L.A.Cosmic tool (van Dokkum 2001). After these processes, we extracted source fluxes on ordinary and extraordinary parts of the images (see Figure 1) using the aperture photometry package in IRAF.

Table 1
Observation Journal

| Date       | UT     | Filter | texp   | N  | r0c | Δd | αc | θc | δc | Mode |
|------------|--------|--------|--------|----|-----|----|----|----|----|------|
| 2015 Jun 11| 11:23-15:27 | RC     | 60     | 212| 0.928 | 0.104 | 145.1| 86.3 | 356.3 | Phot, Pol |
| 2015 Jun 12| 13:22-16:18 | RC     | 60     | 92 | 0.945 | 0.089 | 141.3| 98.9 | 8.9  | Phot, Pol |
| 2015 Jun 14| 13:48-15:50 | V      | 60     | 36 | 0.975 | 0.064 | 127.4| 144.7| 54.7 | Phot, Pol |
| 2015 Jun 15| 13:10-15:09 | RC     | 60     | 48 | 0.975 | 0.065 | 127.7| 143.7| 53.7 | Pol   |
| 2015 Jun 12| 11:34-15:27 | V      | 30     | 152| 0.989 | 0.057 | 116.1| 176.0| 86.0 | Pol   |
| 2015 Jun 14| 11:42-15:18 | RC     | 30     | 156| 0.989 | 0.057 | 116.1| 176.0| 86.0 | Pol   |
| 2015 Jun 16| 12:30-17:20 | V      | 30     | 220| 1.005 | 0.054 | 100.2| 20.6 | 110.6| Pol   |
| 2015 Jun 17| 11:19-12:43 | V      | 30     | 68 | 1.018 | 0.056 | 86.5 | 29.8 | 119.8| Pol   |
| 2015 Jun 19| 11:08-12:35 | RC     | 30     | 76 | 1.018 | 0.056 | 86.6 | 29.8 | 119.8| Pol   |
| 2015 Jun 19| 11:13-11:20 | RC     | 30     | 12 | 1.046 | 0.073 | 64.0 | 32.2 | 122.2| Pol   |
| 2015 Jun 11| 11:02-11:12 | RC     | 30     | 16 | 1.046 | 0.073 | 64.0 | 32.2 | 122.2| Pol   |
| 2015 Jun 20| 11:22-11:29 | V      | 30     | 12 | 1.060 | 0.086 | 57.2 | 30.1 | 120.1| Pol   |
| 2015 Jun 22| 11:12-11:22 | RC     | 30     | 16 | 1.060 | 0.086 | 57.2 | 30.1 | 120.1| Pol   |

Notes.

a Individual effective exposure time in seconds.
b Number of exposures.
c Median heliocentric distance in au.
d Median geocentric distance in au.
e Median Solar phase angle (Sun–Asteroid–Observer angle) in degrees.
f Mode position angle of normal vector with respect to the scattering plane in degrees.
g Median position angle of the scattering plane in degrees.
h Observation mode: Photometry (Phot) or Polarimetry (Pol).
The obtained fluxes were used to derive the Stokes parameters after completing the necessary procedures for the Pirka/MSI data (see Appendix). These procedures contain corrections for polarization efficiencies, the subtraction of instrumental polarization, and the conversion into the standard celestial coordinate system.

The linear polarization degree \( P \) and the position angle of polarization \( \theta_p \) were derived with the following equations:

\[
P = \sqrt{(q_{\text{pol}}')^2 + (u_{\text{pol}}')^2},
\]

and

\[
\theta_p = \frac{1}{2} \tan^{-1} \left( \frac{u_{\text{pol}}'}{q_{\text{pol}}'} \right),
\]

where \( q_{\text{pol}}' \) and \( u_{\text{pol}}' \) are the Stokes parameters \( Q \) and \( U \), respectively, normalized by \( I \) after correcting for instrumental effects. We derived the linear polarization degree with respect to the scattering plane:

\[
P_t = P \cos (2\theta_t),
\]

where \( \theta_t \) is given by

\[
\theta_t = \theta_p - (\phi \pm 90^\circ),
\]

where \( \phi \) is the position angle of the scattering plane on the sky.

The polarization degree of each set of four exposures has an error of 0.2\%–5\% (depending largely on the apparent magnitudes of the nights). We combined these sets of \( (q''', u''') \) values to obtain nightly averaged \( q''' \) and \( u''' \) values, addressing systematic noise (\( \delta q''' \) and \( \delta u''' \)) and random noise (\( \sigma_{q'''} \) and \( \sigma_{u'''} \)), separately. Regarding systematic noise, we took the arithmetic averages \( \delta q''' \) and \( \delta u''' \). Regarding the synthesized random errors, we calculated the variances of the weighted means, given by

\[
\sigma^2_{q'''} = \frac{1}{\sum_{i=1}^{n}(\sigma_{q_i'''}^2)}, \quad \sigma^2_{u'''} = \frac{1}{\sum_{i=1}^{n}(\sigma_{u_i'''}^2)},
\]

where \( \sigma^2_{q'''i} \) and \( \sigma^2_{u'''i} \) are the synthesized random errors for \( q''' \) and \( u''' \), respectively. The resultant values for \( q''' \) and \( u''' \) are given by

\[
\bar{q}''' = \sigma^2_{q'''} \sum_{i=1}^{n} \frac{q_{i'''}'}{\sigma_{q_i'''}^2}, \quad \bar{u}''' = \sigma^2_{u'''} \sum_{i=1}^{n} \frac{u_{i'''}'}{\sigma_{u_i'''}^2},
\]

with total errors of

\[
\epsilon_{q'''} = \sqrt{\sigma^2_{q'''} + \delta^2_{q'''}}, \quad \epsilon_{u'''} = \sqrt{\sigma^2_{u'''} + \delta^2_{u'''}}.
\]

Similarly, using Equations (1)–(5), we obtained synthesized \( P_t \) and \( \theta_p \) values, as shown in the following sections.

4. Results

We summarize our polarimetric results in Table 2. We describe our findings below.

4.1. Phase Angle Dependence and Polarimetric Color

Figure 2 shows the nightly averaged polarization degrees with respect to the phase angles. At first glance, we noted that the polarization degree increases almost linearly with increasing phase angle at \( \alpha = 60^\circ\text{--}100^\circ \), has a peak \( \alpha \sim 120^\circ \), and drops at \( \alpha = 125^\circ\text{--}140^\circ \). We also found that the \( V \) polarization degrees are higher than the \( R_C \) polarization degrees regardless of the observed phase angles. The average difference is \( \Delta P_v = -0.37 \pm 0.04\% \). This trend is similar to that observed for other S-type asteroids (Lupishko et al. 1995; Mukai et al. 1997;
constraining the inversion angle

and Muinonen function

parameters by weighting with the square of the errors. We

asteroids; Belskaya et al. 2017

Figure 2.

Figure 3. Time-dependence of polarization degree in the V-band (top) and the $R_c$-band (bottom) using data taken on UT 2015 June 16 ($\alpha = 100^\circ$). For reference, we show averaged values (dashed lines). The length of arrows corresponds to one rotational period.

Table 2

| Date       | Filter | $P^d$ | $\theta^e$ | $\theta^f$ | $\alpha^g$ | $\sigma^h$ |
|------------|--------|-------|------------|------------|------------|------------|
| 2015 Jun 12 | $R_c$  | 6.29  | -82.8      | 2.7        | 6.28       | -1.7       |
| 2015 Jun 14 | V      | 7.14  | -37.8      | 1.1        | 7.11       | -2.5       |
| 2015 Jun 15 | $R_c$  | 6.92  | -36.7      | 0.7        | 6.92       | -0.4       |
| 2015 Jun 16 | V      | 7.26  | -5.8       | 0.5        | 7.24       | -1.8       |
| 2015 Jun 17 | $R_c$  | 7.02  | -5.3       | 0.4        | 7.01       | -1.3       |
| 2015 Jun 19 | V      | 6.77  | 18.4       | 0.3        | 6.75       | -2.2       |
| 2015 Jun 20 | $R_c$  | 6.33  | 18.8       | 0.3        | 6.32       | -1.7       |
| 2015 Jun 17 | V      | 5.78  | 28.2       | 0.4        | 5.77       | -1.6       |
| 2015 Jun 19 | $R_c$  | 5.44  | 31.5       | 1.1        | 5.43       | -1.6       |
| 2015 Jun 20 | V      | 4.02  | 31.5       | 1.1        | 4.02       | -0.7       |
| 2015 Jun 20 | $R_c$  | 3.47  | 34.6       | 1.7        | 3.46       | 2.4        |
| 2015 Jun 20 | V      | 3.15  | 27.3       | 1.5        | 3.13       | -2.8       |
| 2015 Jun 20 | $R_c$  | 2.71  | 27.1       | 1.4        | 2.69       | -3.0       |

Notes.

$^a$ Polarization degree in percent.

$^b$ Error of $P$ in percent.

$^c$ Position angle of the strongest electric vector in degrees.

$^d$ Error of $\theta$ in degrees.

$^e$ Polarization degree with respect to the scattering plane in percent.

$^f$ Polarization degree in percent.

$^g$ Error of $\alpha$ in degrees.

Belskaya et al. 2009 and a Q-type asteroid (Fornasier et al. 2015).

To obtain $\alpha_{\text{max}}$ and $P_{\text{max}}$, we fit our data using the Lumme and Muinonen function (Goidet-Devel et al. 1995; Penttilä et al. 2005):

$$P_r = b \sin^c(\alpha) \cos^2(\alpha) \sin^2(\alpha - \alpha_0),$$

where $b$, $c_1$, $c_2$, and $\alpha_0$ are parameters for fitting. As our data are not covered at lower phase angles, we fixed the inversion angle $\alpha_0 = 20^\circ$ (a typical value for S-type and Q-type asteroids; Belskaya et al. 2017) and derived the other three parameters by weighting with the square of the errors. We obtained $\alpha_{\text{max}} = 124 \pm 8^\circ$ and $P_{\text{max}} = 7.32 \pm 0.25\%$ in the V-band and $\alpha_{\text{max}} = 124 \pm 6^\circ$ and $P_{\text{max}} = 7.04 \pm 0.21\%$ in the $R_c$-band. However, we noted that $c_2$ has a negative value, which does not make sense per the original definition of the trigonometric function (Penttilä et al. 2005). We discuss this insufficiency and describe the error analysis in Section 5.

4.2. Rotational Variation in $P_r$

Figure 3 shows the polarization degrees with respect to time from UT 2015 June 16 ($\alpha = 100^\circ$). We choose the data from this night not only because the sky was clear and stable but also because the time coverage was long enough to see a rotational variability in the polarization degree. We combined each set of data taken at three different positions on the detector (see Section 2), excluding several images where field stars overlapped the asteroid. The data cover approximately two rotational periods of the asteroid. From this, we found that the polarization degree was notably constant over the quadrant ($\sim 1/4$ because $\alpha \sim 90^\circ$) of the surface. We determined the upper limit of the rotational variation in $P_r$ as 0.3% in the V-band and 0.2% in the $R_c$-band with a 1σ confidence level.

It has been reported that some large asteroids show rotational variations of $P_r$. A notable example is (4) Vesta (Dollfus et al. 1989), which showed a 0.1% polarimetric variation, and the maximum of the polarization coincides with the light curve minimum, suggesting that albedo variation exists on the surface per the controlled polarization degree and visible magnitude. Similarly, (3) Juno, (9) Metis, and (216) Kleopatra showed rotational variations of 0.15%–0.27%, ~0.1%, and ~0.2%, respectively (Nakayama et al. 2000; Takahashi et al. 2004, 2009). Although the measurement accuracy is too limited to detect such small variations in the polarization degree, we suggest that Icarus has a quite homogeneous albedo in contrast with these asteroids because our measurement was made at a large phase angle, while these previous detections were made at a small phase angle where the polarization degree itself has
small values (1/6 ~ 1/10 of Icarus’s \(|P|\), i.e., 0.5 \(\lesssim |P| \lesssim 1.0\%\)). We will discuss this homogeneity in Section 5.

4.3. Photometric Function and Macroscopic Roughness

As a byproduct of our polarimetric observation, we took images without using the polarimetric module. These images were obtained mostly in the \(R_C\)-band filter when we tested the non-sidereal tracking of the telescope or set the position of the asteroid in the narrow FOV of the polarization mask. Through comparison with the fluxes of field stars with magnitudes listed in the third U.S. Naval Observatory CCD Astograph Catalog (UCAC3; Zacharias et al. 2010), we derived the \(R_C\)-band magnitude of Icarus. Applying the \(V-R_C\) color index of 0.57 \(\pm 0.08\) (Gehrels et al. 1970), the magnitude was converted into the \(V\)-magnitude. The observed magnitude, \(m_V\), was converted into the reduced \(V\)-magnitude, \(m_V(1, 1, \alpha)\), a magnitude at unit heliocentric and observer’s distances that is given by

\[
m_V(1, 1, \alpha) = m_V - 5 \log(r_h \Delta),
\]

where \(r_h\) and \(\Delta\) are the heliocentric and observer’s distances in au. Figure 4 is the reduced \(V\)-magnitude with respect to the phase angle. In the figure, the magnitude data at \(\alpha > 120^\circ\) were obtained by us, while the data at \(\alpha < 110^\circ\) were obtained from Gehrels et al. (1970) and Warner (2015).

The phase curve was fitted with the disk-integrated Hapke model (Hapke 1993). \(m_V(1, 1, \alpha)\) data are converted into the logarithm of \(I/F\) (where \(F\) is the incidence solar irradiance divided by \(\pi\), and \(I\) is the intensity of reflected light from the asteroid surface) as

\[
-2.5 \log \left(\frac{I}{F}\right) = m_V(1, 1, \alpha) - m_{V_\odot} - \frac{5}{2} \log \left(\frac{\pi}{S}\right) + m_c,
\]

where \(m_{V_\odot} = -26.74\) (Allen 1973) is the solar magnitude at 1 au, \(S\) is the geometrical cross section of the asteroid in \(m^2\), and \(m_c = -5 \log(1.4960 \times 10^{11}) = -55.87\) is a constant to adjust the length unit. The disk-integrated Hapke function is given by

\[
\frac{I}{F} = \left[\frac{w}{8} (1 + B(\alpha)) P(\alpha) - 1 + \frac{r_0}{2} (1 - r_0)\right] \\
\times \left[1 - \sin \left(\frac{\alpha}{2}\right) \tan \left(\frac{\alpha}{2}\right) \ln \left(\cot \left(\frac{\alpha}{4}\right)\right)\right] \\
+ \frac{2}{3} \sigma c \frac{\sin(\pi - \alpha) \cos(\alpha)}{\pi} K(\alpha, \bar{\theta}),
\]

where \(w\) is the single-particle scattering albedo. \(K(\alpha, \bar{\theta})\) is a function that characterizes the surface roughness parameterized by \(\bar{\theta}\) (Hapke 1984). The term \(r_0\) is given by

\[
r_0 = \frac{1 - \sqrt{1 - w}}{1 + \sqrt{1 - w}}.
\]

The opposition effect term \(B(\alpha)\) is given by

\[
B(\alpha) = \frac{B_0}{1 + \frac{\sin(\alpha/2)}{h}},
\]

where \(B_0\) denotes the amplitude of the opposition effect, and \(h\) characterizes the width of the opposition effect.

Two parameters of a double Henyey–Greenstein function, \(P(\alpha)\) (see, e.g., Lederer et al. 2008), was employed:

\[
P(\alpha) = \frac{1 - c_{HG}(1 - b_{HG}^2)}{1 - 2 b_{HG} \cos(\alpha) + b_{HG}^2)^{3/2}} + \frac{c_{HG}(1 - b_{HG}^2)}{1 + 2 b_{HG} \cos(\alpha) + b_{HG}^2)^{3/2}}.
\]

For the fitting, we fixed opposition parameters as \(B_0 = 0.02\) and \(h = 0.141\) (from 25143 Ikawa’s values, Lederer et al. 2008) as an analog of S/Q-type asteroid. By changing the initial values of \(b_{HG}\), \(c_{HG}\), \(\alpha\), and \(\bar{\theta}\) to the range of \(0.01 \leq b_{HG} \leq 0.8, 0.01 \leq c \leq 0.8, 0.1 \leq w \leq 0.8,\) and \(5^\circ \leq \bar{\theta} \leq 55^\circ\), we searched for the best-fit parameters. From the fitting, we obtained \(b_{HG} = 0.42 \pm 0.08, \) \(c_{HG} = 0.41 \pm 0.20,\) \(w = 0.48 \pm 0.10,\) and \(\bar{\theta} = 48^\circ \pm 6^\circ.\). Although there are large uncertainties in \(b_{HG}\) and \(c_{HG}\), we found that \(\bar{\theta}\) is significantly larger for the 10–30 km sized S-type asteroids (243) Ida and (951) Gaspra (Helfenstein et al. 1994, 1996). Note that we assumed the diameter of the asteroid to be 1440 m. If we change the assumed size, \(w\) would be different, while \(\bar{\theta}\) is nearly constant. Considering the large uncertainty (~18%) in the size (Greenberg et al. 2017), the fitting provides a reliable result only for \(\bar{\theta}\). The large value of \(\bar{\theta}\) may suggest that there are few small particles equivalent to the wavelength (i.e., micrometer or smaller), resulting in the large macroscopic roughness.

5. Discussion

5.1. Description for Deriving \(P_{max}\) and \(\alpha_{max}\) and Their Errors

Lumme & Muinonen’s equation, Equation (9), has been widely used for the fitting of polarimetric phase curves because it produces several key features of the phase curve, including \(P_p = 0\%\) at \(\alpha = \alpha_0\), \(\alpha_0\) and 180°, a negative branch at \(0^\circ < \alpha < \alpha_0\), and a positive branch at \(\alpha > \alpha_0\). By definition, the power components \(c_1\) and \(c_2\) should be positive. The
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NEA, Toutatis, at $\alpha = 74^\circ-111^\circ$ and derived $P_{\text{max}} = 7.0 \pm 0.2\%$ at $\alpha_{\text{max}} = 107 \pm 10^\circ$. Belskaya et al. (2009) observed 2000 PN$_9$ at $\alpha = 90^\circ.7$ and 115° and posited that $P_{\text{max}} = 7.7_{-0.7}^{+0.5}\%$ at $\alpha_{\text{max}} = 103 \pm 12^\circ$. Among these asteroids, the Toutatis data have good coverage around the maximum phase (Figure 5(c)), showing a clear drop beyond $\alpha_{\text{max}}$. Once again, our Icarus data clearly show $\alpha_{\text{max}}$ values larger than those for Toutatis.

There are several possibilities resulting in the large $\alpha_{\text{max}}$ values. Lunar data show a moderate dependence on albedo (Korokhin & Velikodsky 2005). Thus, smaller albedo values tend to show larger $\alpha_{\text{max}}$ values. Icarus has an albedo typical of stony materials in the solar system (see Section 5.3). In addition, lunar data cover an $\alpha_{\text{max}}$ in the range of $92^\circ-106^\circ$, which is much smaller than the Icarus values. Accordingly, the high $\alpha_{\text{max}}$ values cannot be explained by the albedo. Shkuratov & Opanasenko (1992) examined the size dependence of the polarization properties of laboratory samples and suggested that $\alpha_{\text{max}}$ would increase with increasing size, even up to $\alpha_{\text{max}} \sim 150^\circ$. Although there may be other factors increasing the $\alpha_{\text{max}}$ values, we hypothesize that one possible explanation for the large $\alpha_{\text{max}}$ of Icarus is that the asteroid could be covered with large grains.

5.3. Albedo

The geometric albedo ($p_V$) of Icarus was determined by different measurements, but these results do not match well: 0.42 (Veeder et al. 1989), 0.33–0.70 (Harris & Lagerros 2002), 0.14±0.16 (Thomas et al. 2011), and 0.29 ± 0.05 (Nugent et al. 2015). We now derive the geometric albedo based on our polarimetric measurement using the so-called slope–albedo law, which is given by

$$\log_{10} p_V = C_1 \log_{10} h_{\text{SLP}} + C_2,$$  

where $h_{\text{SLP}}$ is the phase slope near the inversion angle (i.e., $dP/d\alpha$ at $\alpha = \alpha_0$). $C_1$ and $C_2$ are constants that have been determined by several authors. Lupishko et al. (1995) derived $C_1 = -0.98$ and $C_2 = -1.73$ in an early study, and these values were updated to $C_1 = -1.21 \pm 0.07$ and $C_2 = -1.89 \pm 0.14$ (Masiero et al. 2012) and to $C_1 = -0.80 \pm 0.04$ and $C_2 = -1.47 \pm 0.04$ (when $p_V \leq 0.08$; Cellino et al. 2015). We fit our $V$–polarimetric data at $\alpha = 57.2\rlap/-86.5$ constraining the inversion phase angle $\alpha_0 = 20^\circ$, which is a typical value for Q-type asteroids (Belskaya et al. 2017), and obtained $h_{\text{SLP}} = 0.0874 \pm 0.0017$. With $C_1$ and $C_2$ as in Masiero et al. (2012) and Cellino et al. (2015), we acquired a geometric albedo of $p_V = 0.25 \pm 0.02$. This albedo value is consistent with those of Q-type and S-type asteroids (Thomas et al. 2011; DeMeo & Carry 2013; Usui et al. 2013). Note that the fitted phase angle range is larger than those of previous studies. However, we believe that this range is reasonable for fitting the data not only because our phase curve shows a linear profile at $\alpha \lesssim 86.5^\circ$ but also because a study of Iokawa at similar phase angles of $\alpha = 41.5^\circ-79.2^\circ$ demonstrated a good match for the albedo ($p_V = 0.24 \pm 0.01$ via polarimetry Cellino et al. (2005) versus $p_V = 0.24 \pm 0.02$, Ishiguro et al. (2010), via remote–sensing observation by the Hayabusa onboard camera).

In Section 4.2, we examined the rotational change in $P$ and found no variability to the accuracy of 0.2%–0.3% at $\alpha = 100^\circ.2$. Extrapolating the linear slope to the phase angle (although the phase curve slightly deviated from the line), the
Figure 5. Comparison with (a) the Moon (Shkuratov et al. 2011), (b) Mercury (Dollfus & Auriere 1974) and asteroids, (c) (4179) Toutatis (Ishiguro et al. 1997), (d) (1685) Toro, (e) (23187) 2000 PN_{5} (Belskaya et al. 2009), and (f) (214869) 2007 PA_{8} (Fornasier et al. 2015). The curves fit with Equation (9) are shown in (a)–(c) but not shown in (d)–(f) because of insufficient phase coverage.
upper limit of the polarization variability (0.2%–0.3%) is converted into the upper limit of the \( k_{SLIP} \) variability of \( \sim 0.0025 \). With Equation (16), we put the upper limit of the albedo variation on the quadrant surface at \( \sigma P_V = 0.02 \). The upper limit would suggest that the surface of the asteroid is quite homogeneous in albedo from a large-scale viewpoint (one-fourth of surface resolution).

5.4. Grain Size Estimate

It is known that \( P_{\text{max}} \) is inversely correlated with the geometric albedo \( P_V \) (Umov law). \( P_{\text{max}} \) also depends on the grain size. Shkuratov & Opanasenko (1992) examined these relationships using lunar soil samples and gave the following equations:

\[
d = 0.03 \exp(2.9 \, b),
\]

and

\[
b = \log(10^2 \, A_{\alpha=5}) + a \log(10 \, P_{\text{max}}),
\]

where \( d \) denotes the grain size in \( \mu m \), \( a \) is 0.795 at 0.43 \( \mu m \) and 0.845 at 0.65 \( \mu m \) (Shkuratov & Opanasenko 1992). \( A_{\alpha=5} \) is the albedo at \( \alpha = 5^\circ \). Using the phase function we determined in Section 4.3, we derived \( A_{\alpha=5} = 0.215 \pm 0.018 \) for Icarus. Applying Equation (18) to our polarimetric result, we obtained \( d = 100–130 \mu m \). In addition, we plotted our data onto the \( P_{\text{max}} \)-albedo relation for different sizes of laboratory samples (Figure 6). Similarly, the plot (Figure 6) shows a trend indicating that Icarus may be covered with particles hundreds of microns in size.

This result is consistent with the fact that Icarus has a large macroscopic roughness. The large values of \( \alpha_{\text{max}} \) also imply a large particle size. Furthermore, the asteroid exhibits a Q-type spectrum, which is bluer than an S-type spectrum. The blueness can be explained not only by the freshness in terms of the space weathering but also by large grains. It is known that an increase in grain size yields a bluer spectral slope regardless of the types of asteroid (e.g., Miyamoto et al. 1981; Reddy et al. 2016; Vernazza et al. 2016). Therefore, these optical properties consistently suggest a large grain size on the asteroid. Why is the particle size so large? How did the asteroid lose the small particles from the surface?

5.5. Consideration of Mass Ejection around Perihelion

Icarus has a critical rotational period (2.273 hr) in which the centrifugal force exceeds the self-gravitational force on the equator. Assuming an Itolawa-like bulk density of \( \sim 2000 \, \text{kg} \, \text{m}^{-3} \) (Fujitake et al. 2006; Scheeres et al. 2010) and a spherical body with a 1440-m diameter (Greenberg et al. 2017), the ambient gravitational acceleration is approximately 80 micro-G’s at the pole and minus 5 micro-G’s at the equator, suggesting that granular materials may be ejected from the equatorial region (around a latitude within 30° from the equator) via centrifugal acceleration. In contrast, the rotational axis of Icarus nearly aligns to the ecliptic pole (Greenberg et al. 2017), meaning that it is roughly perpendicular to the orbital plane with a moderate inclination to the orbital plane \( (i = 22°.3) \). Under this geometry, the Sun shines almost parallel to the polar region. Although regolith grains can remain in the high-latitude region, the oblique sunshine can strip small grains off from the polar region when the asteroid passes through perihelion. Such an idea was suggested for the surface of (3200) Phaethon to explain the dust emission near perihelion (Jewitt & Li 2010; Jewitt et al. 2013). The solar radiation pressure is given by \( F_s = \beta_s F_s \), where \( F_s = 0.169 \, \text{m} \, \text{s}^{-2} \) is the solar gravity at the perihelion of Icarus \( (q = 0.187 \, \text{au}) \). \( \beta_s \) is the ratio of the solar radiation pressure to the solar gravity, given approximately by \( \beta_s = 1.14/\rho_d d \), where \( \rho_d \) and \( d \) are the particle mass density and diameter, respectively. Thus, the solar radiation pressure exceeds the ambient gravity in the polar region when \( d \lesssim 240 \, \mu m \) (a mass density, \( \rho_d = 1.0 \, \text{g} \, \text{cm}^{-2} \), was assumed). Although some cohesive forces, such as van der Waals forces, would work to prevent mass ejection from the surface, we conjecture that the environment of the “fast-rotating” body at a “small solar distance” would be responsible for the paucity of small grains and its unique polarimetric properties.

Intriguingly, Ohtsuka et al. (2007) noted that 2007 MK6 has a strong dynamical connection to Icarus, suggesting that these two asteroids share a common origin. Such groupings of asteroids are also recognized for Phaethon and (155140) 2005 UD (Ohtsuka et al. 2006). It is still unclear if these two bodies were split due to the tidal force of a planet during a close encounter, thermal stress, rotational breakup via YORP acceleration, or other mechanisms. It is important to note that these two bodies have similarities in two aspects: their rapid rotational periods and small perihelion distances. Supposing that these groups of asteroids experienced large-scale splittings that produced their current bodies, they may have had the chance to lose small dust grains during splitting due to strong solar radiation pressure quickly sweeping small dust grains from their orbits before they had the chance to accumulate, producing bodies that lack small dust grains.

Figure 6. Albedo vs. \( P_{\text{max}} \) plot for lunar and terrestrial samples in (Geake & Dollfus 1986). We also plotted Icarus data from our measurements. The albedo of these samples \((A_{\alpha=5})\) is defined at a phase angle \( \alpha = 5^\circ \), slightly lower than the geometric albedo \((P_V)\).
6. Summary

We made photopolarimetric observations of Icarus at large phase angles $\alpha = 57^\circ$–$141^\circ$ during its apparition in 2015 and found the following.

The combination of the maximum polarization degree and the geometric albedo is in accordance with terrestrial rocks with a diameter of several hundreds of micrometers. The photometric function indicates a large macroscopic roughness. We posit that the unique environment metric function suggests a large macroscopic roughness. We found the following.

1. The maximum values of the linear polarization degree are $P_{\text{max}} = 7.32 \pm 0.25\%$ at a phase angle of $\alpha_{\text{max}} = 124^\circ \pm 8^\circ$ in the $V$-band and $P_{\text{max}} = 7.04 \pm 0.21\%$ at $\alpha_{\text{max}} = 124^\circ \pm 6^\circ$ in the $R_C$-band.
2. Applying the polarimetric slope–albedo law, we derived a geometric albedo $\rho_V = 0.25 \pm 0.02$, which is consistent with that of Q-type asteroids. The albedo would be globally constant, showing no significant rotational variation in the polarization degree.
3. $\alpha_{\text{max}}$ is significantly larger than those of Mercury, the Moon, and the S-type asteroid Toutatis but consistent with laboratory samples hundreds of microns in size.
4. The $P_{\text{max}}$–albedo relation suggests that Icarus is covered with particles hundreds of microns in size.
5. The photometric function suggests a large macroscopic roughness, supporting the dominance of large grains.

To explain the dominance of large grains on the asteroid, we conjecture that a strong radiation pressure around the perihelion passage would strip small grains off of the fast-rotating asteroid.

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Appendix

Pirka/MSI Polarimetric Data Analysis Procedures

The observed ordinary and extraordinary fluxes at the half-wave plate angle $\Psi$ in degrees, $I_d(\Psi)$ and $I_o(\Psi)$, were used to derive

$$q'_{\text{pol}} = \left( \frac{R_q - 1}{R_q + 1} \right)/p_{\text{eff}}.$$

and

$$u'_{\text{pol}} = \left( \frac{R_u - 1}{R_u + 1} \right)/p_{\text{eff}},$$

where $R_q$ and $R_u$ are obtained from the observation using the following equations:

$$R_q = \sqrt{\frac{I_d(0)/I_o(0)}{I_o(45)/I_o(45)}},$$

and

$$R_u = \sqrt{\frac{I_o(22.5)/I_o(22.5)}{I_o(67.5)/I_o(67.5)}},$$

where $p_{\text{eff}}$ is a polarization efficiency, which was examined by taking a dome flat image through a pinhole and a Polaroid-like linear polarizer, which produces artificial stars with $P = 99.97 \pm 0.02\% (V)$ and $99.98 \pm 0.01\% (R_C)$. $p_{\text{eff}}$ was measured approximately two months prior to our observation and was determined to be $p_{\text{eff}} = 0.9967 \pm 0.0003$ in the $V$-band and $0.9971 \pm 0.0001$ in the $R_C$-band.

The instrumental polarization of Pirka/MSI is known to depend on the instrument angle of rotation and can be corrected with the following equation:

$$\begin{align*}
q''_{\text{pol}} &= \left( q'_{\text{pol}} - \cos 2\theta_{\text{rot1}} - \sin 2\theta_{\text{rot1}} \right) - \left( \sin 2\theta_{\text{rot2}} \cos 2\theta_{\text{rot2}} \right) \\
u''_{\text{pol}} &= \left( u'_{\text{pol}} - \cos 2\theta_{\text{rot1}} - \sin 2\theta_{\text{rot1}} \right) - \left( \sin 2\theta_{\text{rot2}} \cos 2\theta_{\text{rot2}} \right)
\end{align*}$$

where $\theta_{\text{rot1}}$ denotes the average instrument rotator angle during the exposures with $\Psi = 0^\circ$ and $45^\circ$, while $\theta_{\text{rot2}}$ denotes the average angle with $\Psi = 22.5^\circ$ and $67.5^\circ$. $q_{\text{inst}}$ and $u_{\text{inst}}$ are two components of the Stokes parameters for the instrumental polarization and were determined to be $q_{\text{inst}} = 0.963 \pm 0.029\%$ in the $V$-band and $0.703 \pm 0.033\%$ in the $R_C$-band and $u_{\text{inst}} = 0.453 \pm 0.043\%$ in the $V$-band and $0.337 \pm 0.020\%$ in the $R_C$-band, respectively, by observing the unpolarized stars HD 212311 and BD+32 3739 (see Table 3, on page 1566, Schmidt et al. 1992).

The instrument position angle in celestial coordinates was determined by measuring the polarization position angles of strongly polarized stars for which position angles are reported in Schmidt et al. (1992). The instrument position angle can be corrected using the following equations:

$$\begin{align*}
q''_{\text{pol}} &= \left( \cos 2\theta_{\text{off}} - \sin 2\theta_{\text{off}} \right) q'_{\text{pol}} \\
u''_{\text{pol}} &= \left( -\sin 2\theta_{\text{off}} \cos 2\theta_{\text{off}} \right) u'_{\text{pol}}
\end{align*}$$

and

$$\theta'_{\text{off}} = \theta_{\text{off}} - \theta_{\text{ref}},$$

where $\theta_{\text{ref}}$ is a given parameter for specifying the position angle of the instrument. Through an observation of strongly polarized stars (HD 204827, HD 154445, and HD 155197) in 2015 May, we derived $\theta_{\text{off}} = 3.82 \pm 0.38$ in the $V$-band and $3.38 \pm 0.37$ in the $R_C$-band.

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