FIRST MASSIVE STATE OF THE SUPERSTRING IN SUPERSPACE

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Abstract

Using the manifestly spacetime supersymmetric description of the four-dimensional open superstring, we construct the vertex operator in superspace for the first massive state. This construction provides an N=1 D=4 superspace representation of the massive spin-2 multiplet.
1 Introduction

The study of massive states using the manifestly spacetime supersymmetric description of the superstring has at least two interesting aspects. Firstly, we want to understand the massive spin-2 multiplet in superspace, which is the first excited state of the superstring. For example, using open superstring field theory\cite{1}, it should be possible to construct a superspace action for this multiplet. Secondly, the vertex operator of this massive state can be used to perform manifestly super-Poincaré invariant calculations of superstring scattering amplitudes\cite{2}.

The manifestly spacetime supersymmetric description of the superstring can be used to describe any four-dimensional compactification of the ten-dimensional superstring which preserves at least $N = 1$ four-dimensional spacetime supersymmetry. Unlike vertex operators in either the Ramond-Neveu-Schwarz or light-cone Green-Schwarz descriptions, vertex operators in this description are manifestly super-Poincaré covariant since they are constructed in $N=1$ $D=4$ superspace.

The plan of the paper is as follows: In section 2, we review the construction of the vertex operator in superspace for the ground state of the open four-dimensional superstring, which is the massless $N=1$ $D=4$ super-Maxwell multiplet. This vertex operator is expressed in terms of the usual scalar superfield $V$ for a super-Maxwell prepotential. In section 3, we construct the vertex operator in superspace for the first excited state of the open four-dimensional superstring, which is the massive $N=1$ $D=4$ spin-2 multiplet. This vertex operator is expressed in terms of a vector superfield $V_m$ for the massive spin-2 prepotential.

2 Review of the Manifestly Spacetime Supersymmetric Description

The left-moving worldsheet fields in the manifestly spacetime supersymmetric description include five bosons $(x^m, \rho)$ and eight fermions $(\theta^\alpha, \bar{\theta}^{\dot{\alpha}}, p^\alpha, \bar{p}^{\dot{\alpha}})$, as well as a $c = 9$ $N=2$ superconformal field theory for the six-dimensional compactification manifold. These worldsheet fields are related to the those in the RNS description by a field-redefinition, and reduce in light-cone gauge to the light-cone Green-Schwarz worldsheet fields.
The worldsheet action for the left-moving fields of the open superstring is given by
\[ S = \frac{1}{2\pi} \int d^2z \left( \frac{1}{2} \partial x^m \partial x_m - p_\alpha \partial \theta^\alpha - \bar{p}_{\dot{\alpha}} \partial \bar{\theta}^{\dot{\alpha}} + \frac{1}{2} \partial \rho \partial \rho \right) + S_C \] (1)
where \( S_C \) is the action for the compactification-dependent fields. Note that we will suppress throughout this paper the right-moving fields of the open superstring, which are related in the usual way by boundary conditions to the left-moving fields.

The above action implies the following free-field OPE's as \( y \to z \):
\[ p_\alpha(y) \theta^\beta(z) = \frac{\delta^\beta_\alpha}{y - z}, \quad \bar{p}_{\dot{\alpha}}(y) \bar{\theta}^{\dot{\beta}}(z) = \frac{\delta^{\dot{\beta}}_{\dot{\alpha}}}{y - z}. \] (2)
\[ \rho(y) \rho(z) = \ln(y - z), \quad x^m(y)x^n(z) = \eta^{mn} \ln|y - z|. \] (3)

We use the conventions that \( \sigma^m_{\dot{\alpha}} \) and \( \bar{\sigma}^{\dot{\alpha}}_m \) are the Pauli matrices and that the Minkowski metric tensor is \( \eta^{mn} = \text{diag}(+1, -1, -1, -1) \). In these conventions, a vector is transformed to a bispinor according to \( v^m = \frac{1}{2}(\bar{\sigma}^m)^{\dot{\alpha}}_{\dot{\alpha}} v_{\alpha\dot{\alpha}} \), and \( v_{\alpha\dot{\alpha}} = (\sigma^m)_{\alpha\dot{\alpha}} v^m \).

In this description, the superstring possesses a critical N=2 superconformal invariance which is related to the topological N=2 superconformal invariance of the RNS superstring. In terms of the above fields, the \( c = 6 \) N=2 superconformal generators \((T, G, \bar{G}, J)\) are:
\[ T = \frac{1}{2} \partial x^m \partial x_m - p_\alpha \partial \theta^\alpha - \bar{p}_{\dot{\alpha}} \partial \bar{\theta}^{\dot{\alpha}} + \frac{1}{2} \partial \rho \partial \rho + T_C \] (4)
\[ G = e^{i\rho} d^m d_\alpha + G_C \] (5)
\[ \bar{G} = e^{-i\rho} \bar{d}^m \bar{d}_{\dot{\alpha}} + \bar{G}_C \] (6)
\[ J = -\partial \rho + J_C, \] (7)
where
\[ d_\alpha = p_\alpha + \frac{i}{2} \bar{\theta}^{\dot{\alpha}} \partial x_{\alpha\dot{\alpha}} - \frac{1}{4} (\bar{\theta})^2 \partial \theta_\alpha + \frac{1}{8} \theta_\alpha \partial (\bar{\theta})^2 \] (8)
\[ d_\dot{\alpha} = \bar{p}_\dot{\alpha} + \frac{i}{2} \theta^\alpha \partial x_{\alpha \dot{\alpha}} - \frac{1}{4} (\theta)^2 \partial \theta_{\dot{\alpha}} + \frac{1}{8} \bar{\theta}_{\dot{\alpha}} \partial (\theta)^2 \]  

(9)

\[ \Pi_{\alpha \dot{\alpha}} = \partial x_{\alpha \dot{\alpha}} + i \theta_\alpha \partial \bar{\theta}_{\dot{\alpha}} + i \bar{\theta}_{\dot{\alpha}} \partial \theta_\alpha, \]  

(10)

are spacetime supersymmetric combinations of the worldsheet fields and \([T_C, G_C, \bar{G}_C, J_C]\) are the \(c = 9\) N=2 superconformal generators of the compactification dependent fields. In this paper, the N=2 superconformal generators are untwisted so \(G\) and \(\bar{G}\) have conformal weight \(\frac{3}{2}\) while \(e^{\pm i \rho}\) has conformal weight \(-\frac{1}{2}\).

Note that \([T_C, G_C, \bar{G}_C, J_C]\) contain no singular OPE's with the compactification independent fields. Furthermore, the OPE \(d_\alpha(y)\bar{d}_\alpha(z)\) is regular and

\[ d_\alpha(y)\bar{d}_\alpha(z) \rightarrow i \frac{\Pi_{\alpha \dot{\alpha}}}{y - z}, \quad d_\alpha(y)\Pi_{\beta \dot{\beta}}(z) \rightarrow \frac{-2i \epsilon_{\alpha \beta} \partial \bar{\theta}_\dot{\beta}}{y - z}. \]  

(11)

Vertex operators \(\Phi\) are physical if they satisfy the N=2 super-Virasoro conditions:

\[ L_n |\Phi\rangle = 0, \quad n \geq 0 \]  

(12)

\[ G_r |\Phi\rangle = 0, \quad r > 0 \]  

(13)

\[ \bar{G}_r |\Phi\rangle = 0, \quad r > 0 \]  

(14)

\[ J_n |\Phi\rangle = 0, \quad n \geq 0 \]  

(15)

Note that \(L_0 |\Phi\rangle = 0\) since the ground state is massless in critical N=2 superconformal field theories. Because of the N=2 algebra, these conditions are satisfied if

\[ L_0 |\Phi\rangle = G_{\frac{1}{2}} |\Phi\rangle = \bar{G}_{\frac{1}{2}} |\Phi\rangle = J_0 |\Phi\rangle = J_1 |\Phi\rangle = 0, \]  

(16)

which imply that \(\Phi\) has no double pole with \(G, \bar{G}, T\) and \(J\), and has no single pole with \(J\).

Gauge transformations of \(\Phi\) are described by

\[ \delta |\Phi\rangle = G_{-\frac{1}{2}} |\Lambda\rangle + \bar{G}_{-\frac{1}{2}} |\bar{\Lambda}\rangle, \]  

(17)

or equivalently, \(\delta \Phi = G(\Lambda) + \bar{G}(\bar{\Lambda})\) where \(G(\Lambda)\) means to take the contour integral of \(G\) around \(\Lambda\). These gauge transformations leave invariant the integrated form of the vertex operator, which is given by

\[ \int d^2 z G(\bar{G}(\Phi)) \]  

(18)
where $G(\bar{G}(\Phi))$ means to take the contour integral of $G$ and $\bar{G}$ around $\Phi$.

For the compactification-independent massless states, $\Phi$ must be U(1)-invariant and conformal weight zero, which means it is a scalar super field $V(x, \theta, \bar{\theta})$. No double poles with the N=2 generators implies

$$(D)^2V = (\bar{D})^2V = \partial_m \partial^m V = 0,$$

which are the equations of motion in Lorentz gauge for the N=1 D=4 super-Maxwell prepotential. The gauge transformations implied by (17) take the usual form $\delta V = (D)^2 \lambda + (\bar{D})^2 \bar{\lambda}$ where $\Lambda = e^{-i \rho} \lambda$ and $\bar{\Lambda} = e^{i \rho} \bar{\lambda}$.

### 3 Vertex Operator for First Massive State

Now we will construct the vertex operator for the first massive states of the four-dimensional open superstring which are independent of the compactification. This N=1 D=4 multiplet of states contains eight on-shell bosons and eight on-shell fermions which include a massive spin-2 tensor, spin-1 vector, and spin-3/2 Rarita-Schwinger field.

Since the vertex operator represents massive states, it should contain conformal weight one at zero momentum. In other words, the $e^{-ik^\mu x_\mu}$ dependence contributes conformal weight $-1$, so the zero-momentum part of the vertex operator should contribute conformal weight $+1$ in order that the vertex operator has no second-order pole with the stress-energy tensor.

The most general such vertex operator which is U(1)-invariant and independent of the compactification fields is

$$\Phi = d^\alpha W_\alpha(x, \theta, \bar{\theta}) + \bar{d}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}}(x, \theta, \bar{\theta}) + \Pi^m V_m(x, \theta, \bar{\theta})$$

$$+ \partial \theta^\alpha V_\alpha(x, \theta, \bar{\theta}) + \partial \bar{\theta}^{\dot{\alpha}} \bar{V}_{\dot{\alpha}}(x, \theta, \bar{\theta}) + \partial \rho V(x, \theta, \bar{\theta}).$$

The condition that $\Phi$ is a physical state of the superstring implies that $\Phi$ has no double poles with $J$, $G$, $\bar{G}$, and $T$. The vanishing of the double poles of the OPE of $J$ with $\Phi$ gives the condition:

$$V = 0.$$

The vanishing of the double pole with $G$ gives the following constraint equations on the superfields:

$$D^2V_\alpha = 0$$

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\[ D^2 \bar{W}_\dot{\alpha} = 0 \]  
\[ D^2 V_\alpha - 2i(\sigma_n)_{\alpha\dot{\alpha}} D^\alpha \bar{W}^{\dot{\alpha}} = 0 \]  
\[ D^2 W_\alpha - 2i(\sigma^n)_{\alpha\dot{\alpha}} \partial_n \bar{W}^{\dot{\alpha}} + 2V_\alpha = 0 \]  
\[ D^2 \bar{V}_\dot{\alpha} - 2i(\sigma^n)_{\dot{\alpha}\alpha} D^\alpha V_\alpha + 4\bar{W}_\dot{\alpha} = 0, \]  
where the covariant derivatives are defined by

\[ D_\alpha = \frac{\partial}{\partial \theta^\alpha} + \frac{i}{2} \bar{\theta}^{\dot{\alpha}} (\sigma^n)_{\alpha\dot{\alpha}} \partial_n \]  
\[ \bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + \frac{i}{2} \theta^\alpha (\sigma^n)_{\alpha\dot{\alpha}} \partial_n. \]

The vanishing of the double pole with \( \bar{G} \) gives the complex conjugate of the above equations. Finally, the vanishing of the double pole with \( T \) gives the expected mass-shell condition:

\[ (\partial^\alpha \partial_m + 2) \Phi = 0. \]  

The gauge invariance for the massive state is defined analogously to the massless case. We define the gauge parameter as

\[ \Lambda = e^{-i\rho} \lambda, \quad \bar{\Lambda} = e^{i\rho} \bar{\lambda}, \]  
\[ \lambda = d^\alpha C_\alpha + \bar{d}^{\dot{\alpha}} \bar{E}_{\dot{\alpha}} + \Pi^m B_m + \partial \rho F + \partial \theta^\alpha B_\alpha + \partial \bar{\theta}^{\dot{\alpha}} \bar{H}_{\dot{\alpha}}. \]

and the gauge transformation is \( \delta \Phi = G(\Lambda) + \bar{G}(\bar{\Lambda}). \)

It is easy to check that \( \delta W^\alpha = 2B^\alpha + \ldots \) and \( \delta \bar{W}^{\dot{\alpha}} = 2\bar{B}^{\dot{\alpha}} + \ldots \) under this gauge transformation. Therefore, one can choose the gauge where

\[ W^\alpha = \bar{W}^{\dot{\alpha}} = 0. \]  

In this gauge, the equations (23)-(27) and (30) simplify to

\[ V^\alpha = \bar{V}^{\dot{\alpha}} = W^\alpha = \bar{W}^{\dot{\alpha}} = V = 0, \]  

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\[ (\partial^n \partial_n + 2)V_m = 0 \quad (35) \]
\[ (\sigma^n)_{\alpha\dot{\alpha}} D^\alpha V_n = 0 \quad (36) \]
\[ (\sigma^n)_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} V_n = 0 \quad (37) \]

To show that equations (35), (36) and (37) for \( V_m \) describe an \( N=1 \) \( D=4 \) massive spin-2 multiplet, we expand the superfield \( V_m \) in components as

\[ V_m = C_m + i\theta \chi_m - i\bar{\theta} \bar{\chi}_m \quad (38) \]

In components, equations (36) and (37) are given by

\[ (\sigma_m)_{\alpha\dot{\alpha}} \dot{\chi} \alpha \dot{\alpha} C_m = 0 \quad (39) \]

\[ \frac{i}{2} (\bar{\sigma}^n \sigma^m)_{\dot{\alpha} \dot{\alpha}} \partial_n C_m + (\bar{\sigma}^n \sigma^m)_{\dot{\alpha}} v_{mn} = 0 \quad (40) \]

\[ (\sigma^m)_{\alpha\dot{\alpha}} (M_m + iN_m) = 0 \quad (41) \]

\[ \frac{1}{4} (\bar{\sigma}^n \sigma^m)_{\dot{\alpha}} \partial_n \bar{\chi}_m + i(\sigma^m)_{\alpha\dot{\alpha}} \lambda^\alpha_m = 0 \quad (42) \]

\[ (\bar{\sigma}^n \sigma^m)_{\dot{\alpha}} \partial_n (M_m + iN_m) = 0 \quad (43) \]

\[ -\frac{i}{4} (\bar{\sigma}^n \sigma^m)_{\alpha\dot{\alpha}} \partial_n v_{mn} + 2(\sigma^m)_{\alpha\dot{\alpha}} (D_m - \frac{1}{16} \partial^n \partial_n C_m) = 0 \quad (44) \]

\[ (\bar{\sigma}^n \sigma^m)_{\dot{\alpha}} \partial_n \bar{\lambda}_m = 0. \quad (45) \]

We now analyse the propagating degrees of freedom of the superfield \( V_m \). Firstly, we analyse the bosonic sector. Equation (41) implies \( M_m = N_m = 0 \). Taking the trace of the equation (40), we get the conditions \( \partial^m C_m = v_m^m = 0 \). Contracting equation (40) with \( (\bar{\sigma}^p \sigma^q)_{\dot{\alpha}} \), we obtain a relation between the antisymmetric part of \( v_{mn} \) and \( C_m \), namely

\[ v_{[mn]} = \frac{1}{4} \epsilon_{mnpq} \partial^p C^q. \quad (46) \]
Equations (44) implies $\partial^m v_{(mn)} = 0$ and, together with (46), implies $D_m = 0$. So the bosonic fields in $V_m$ describe a massive vector $C_m$ and a massive symmetric traceless tensor $v_{(mn)}$ satisfying the Lorentz gauge conditions $\partial^m C_m = \partial^m v_{(mn)} = 0$.

For the fermionic sector, equations (39) and (42) imply

$$\partial^m \bar{\chi}_m \dot{\alpha} = \partial^m \chi_m \alpha = 0,$$

(47)

and

$$(\sigma^m)_{\alpha \dot{\alpha}} \chi_m^\alpha = (\sigma^m)_{\dot{\alpha} \alpha} \bar{\chi}_m^{\dot{\alpha}} = 0,$$

(48)

Equations (47)-(49) and (35) describe a massive spin-3/2 Dirac fermion,

$$\Psi^{A}_m = (\chi_m^\alpha, 2\sqrt{2} \bar{\chi}_m^{\dot{\alpha}}),$$

(50)

satisfying the Lorentz gauge condition $\partial^m \Psi^{A}_m = 0$.

The superfield $V_m$ therefore provide a superspace representation of the 8+8 on-shell degrees of freedom of the N=1 D=4 massive spin-2 multiplet.

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