Fast and robust particle shuttling for quantum science and technology

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PACS numbers: 37.10.Ty, 03.67.Lx, 37.10.Gh

I. INTRODUCTION

Taking advantage of quantum phenomena to develop sensors, information processing, metrology, or secure communications needs in general an exquisite control of the internal and/or motional state of some quantum system. This control is challenging, because of environmental decoherence of superpositions, and because the scale gap between macroscopic control and microscopic objects often leads to imperfect preparation or driving. Thus, “robustness” of the manipulation is a much needed quality, even more so to scale up the control to many qubits, as for quantum information processing. Robustness is a multifaceted concept, relative to initial conditions, noise, and loosely known or imperfectly controlled Hamiltonian parameters. It is usually measured in terms of final quantum-state fidelities or acquired energy.

Ideas and techniques have been put forward to avoid or mitigate the effects of noise, perturbations and imperfections, such as decoherence-free subspaces, error correction strategies, or dissipation engineering. The “process time” is also an important control element which affects robustness. Times longer than the diabatic/adiabatic transition time scale allow for diabatic dynamics, which are intrinsically robust with respect to smooth, on-transit deviations of the control parameters. The trouble is that slow processes also increase decoherence of superpositions, and because the scale gap between macroscopic control and microscopic objects often leads to imperfect preparation or driving. Thus, “fast” is defined with respect to adiabatic transport times. Special attention is paid to shortcut-to-adiabaticity protocols that achieve, in faster-than-adiabatic times, the same results of slow adiabatic driving.

We review methods to shuttle quantum particles fast and robustly. Ideal robustness amounts to the invariance of the desired transport results with respect to deviations, noisy or otherwise, from the nominal driving protocol for the control parameters; this can include environmental perturbations.

A seminal demonstration of STA-mediated diabatic transport used optical tweezers to transport cold atom clouds and deliver them in different experimental platforms. Individual atoms are also transported in optical-lattices for on-demand positioning, quantum simulators, integration in photonic platforms, or to study quantum random walks, see references therein. Another example of a context in which robust transport is needed is “STA-enhanced interferometry”, or more generally “guided interferometry” based on moving two branch potentials along some predetermined separated trajectories to measure e.g. rotations, unknown forces or fields, or to implement quantum gates. The interferometric phase is “geometrical” which provides robustness with respect to area preserving deviations of the trajectories.

1 For a particle of mass $m$ driven by harmonic trap of angular frequency $\omega$, the standard adiabaticity criterion gives $t_f \gg m \omega^2/(2\omega_1)$, namely, a characteristic kinetic energy smaller than the vibrational quantum $\hbar \omega$.

2 While our systems and motivation are “quantum”, the shuttling itself may well be akin, or even identical to “classical” transport, in particular for harmonic potentials.
Further motivation is provided by the “quantum charge-coupled device” (QCCD) architecture [16–19], which aims at making quantum information processing with trapped ions scalable by keeping only small groups of ions in processing and storing sites and shuttling ions among them diabatically [20, 21].

Even by narrowing down the scope of this review to robust shuttling, there is quite a broad landscape of systems, settings (e.g. with optical, magnetic, radio-frequency traps, hybrid traps, or spin-chains), and potentials (lattices, quadratic, Gaussians and others). Common elements and concepts will be emphasized first, and specificities will be also pointed out later on. We assume an effective one-dimensional description unless stated otherwise, and leave aside variants of the linear shuttling operations such as turns at junctions, or separations and mergings.

II. DIFFERENT STA PARADIGMS FOR FAST PARTICLE SHUTTLING

There are several approaches to STA-mediated shuttling of a single particle or condensate in a trap, moving the trap center from \( x_0(0) = 0 \) at initial time \( t = 0 \) to \( x_0(t_f) = d \) in a transport time \( t_f \). Assuming a rigid transport, without deformation, of a harmonic trap of (angular) frequency \( \omega \), \( x_0(t) \) may be inverse engineered using quadratic invariants of motion [22] (alternatively “scaling” for condensates [22, 23]) or, equivalently, using methods based on the Fourier transform of the trap velocity or acceleration [20], so that the excess energy of the final state compared to an adiabatic evolution vanishes regardless of the initial state of the particle. This independence of the initial state is clearly a desirable robustness feature. Virtually all treatments presuppose a semiclassical driving Hamiltonian with \( x_0(t) \) as a time-dependent “external” control not affected by the transported particle because of the scale gap between the macroscopic control and the shuttled system, a requisite for STA to be independent of the driven state [21, 25]. The trajectory \( x_0(t) \) is hardly unique. In the invariant approach, it is found from a designed auxiliary classical trajectory \( x_c(t) \) by solving inversely a forced-harmonic-oscillator equation, \( \ddot{x}_c(t) + \omega^2 x_c(t) = \omega^2 x_0(t) \). The only conditions to guarantee an excitation-free final state are \( x_c(0) = 0 \), \( x_c(t_f) = d \) and that the time derivative \( \dot{x}_c \) vanishes at boundary times. Other than that there is full freedom to interpolate \( x_c(t) \). (Higher derivatives may also be nullified to impose continuity of the trap position and velocity at boundary times, which is usually desirable in practice). This freedom, characteristic of STA methods, is a powerful asset to optimize robustness with respect to different perturbations or errors. Invariant theory also provides explicit expressions for the dynamics when the frequency depends on time [28], see also [30], which are useful to calculate excitations and robustness with respect to such variations [4]. If both the frequency and the trap position are controllable, protocols may be designed inversely, e.g. to launch a highly monochromatic wavepacket [29].

An alternative for excitation-free shuttling is to add control terms to some reference Hamiltonian \( H_0 = p^2/(2m) + U(q - x_0(t)) \), which includes a rigidly transported potential \( U \) not necessarily harmonic (\( q \) and \( p \) are position and momentum operators). This addition can be done in different ways: The “counterdiabatic (CD) approach” applied to transport [22] assumes that the displacement of the initial eigenvectors \( |n(0)\rangle \) of \( H_0(0) \) according to \( e^{-i\omega x_0(t)/\hbar} e^{-iE_n t/\hbar} |n(0)\rangle \), corresponds to an actual dynamical evolution. From the implied unitary evolution operator, the driving Hamiltonian is deduced to be \( H_0 + \tilde{p} \tilde{x}_0 \). Simple indeed, but it has not been realized so far, apart from a transport simulation in an interaction picture [31]. It is in principle physically feasible in systems with actual or simulated spin-orbit coupling interactions \( \sigma \gamma \tilde{x} \), but note that the two spin components suffer opposite forces, see [4] and references therein.

Assuming a dynamical evolution which shifts both the positions and the momenta, i.e., a unitary evolution \( e^{i\lambda x_0(t)/\hbar} e^{-i\omega x_0(t)/\hbar} e^{-iE_{\text{int}} t/\hbar} |n(0)\rangle \), implies instead the easier-to-realize driving Hamiltonian \( H = p^2/(2m) + U(q - x_0(t)) - m\tilde{q}\tilde{x}_0 \). The added term compensates the inertial force. In a “moving frame”, the interaction picture wavefunction is that for a potential at rest. The compensating force may as well be found as a result of the “fast forward approach” [32], “invariant-based inverse engineering” within a (Lewis-Leach [33]) family of Hamiltonians [22, 27], or by unitary transformation from the CD approach [29].

The main contribution of the compensating force to “robustness” is that the trap can be arbitrary. As long as it is complemented by an adequate linear term, the details of the potential \( U \), and specifically its anharmonicities, do not affect the excitation-free final result. In interferometers using the compensating force, the interferometric phase does not depend on the initial state, or on perturbations that could affect the stability of the pivot point for the compensating potentials [14]. Moreover, the interferometric loop can be traversed quickly without adversely affecting sensitivity, and excitations that grow with process time can be mitigated. (Other excitation types are possible as discussed below.)

Applying the compensating force is an elegant, simple solution for fast shuttling. However, the practical implementation of the potential gradients depends on the system, experimental setting, transport distance and time.

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A remarkable classical-quantum convergence occurs for the Lewis-Leach Hamiltonians \( H = p^2 /(2m) - F(t)q + \omega_0^2 t^2 (t)q^2 /2 + \rho(t)^{-2} U(q - x_0(t))/p(t) \), where the scaling factor \( \rho \) and the force \( F \) satisfy Ermakov and Newton equations, which are common for quantum or classical systems. The quadratic invariant takes the same form, up to the correspondence \( 2\rho_0 \) (classical) \( \sim q \rho + p \rho_0 \) (quantum), so classical and quantum systems can be inverse engineered in the same way [33–37]. STA approaches are as well applicable to stochastic processes described e.g. by Fokker-Planck equations [38].
To estimate feasibility, a lower bound from the maximum of $\bar{x}_0$ from the mean value theorem is $2d/t_f^2$. For shuttling trapped ions in multisegmented Paul traps control voltages are upper-bounded to avoid the breakdown of the insulating regions between the electrodes [21, 22]. For shuttling neutral atoms in weaker optical or magnetic traps, achieving a proper compensation may be challenging [23, 42]. Approximations to the ideal compensation are also possible, for example, retaining only the harmonic part of $U(q-x_0(t))$, the compensation amounts to implementing a shifted trajectory $x_0(t) = x_0(t) + \bar{x}_0/\omega^2$ [39]. The approximation will fail for very fast processes as the particle escapes from the finite-depth trap. For specific potential forms, such as a Gaussian for optical tweezers, this limitation may be quantified in the form of “speed limit” relations [42].

The compensating method can be extended to condensates in the mean field regime [25] and formally to transport $N$ particles [43], but particles of different mass would need different forces [44]. For neutral atoms, different optical potentials could be set for the different species, whereas within the current trapped-ion technology, forces are proportional to the charge, so the needed mass dependence of the forces is not feasible [44]. And yet transporting different species together is of much interest. With a pair of different ions it is possible to sympathetically cool one species without perturbing the qubit in the other one, and furthermore, the use of ions of a different species as entanglement or measurement ancillas is widely considered to be necessary for large-scale quantum computing [2]. An approach to shuttle different-species ion pairs is to express the Hamiltonian in “dynamical normal modes” [45, 46, 47], i.e., independent motions for time-dependent harmonic oscillators in the small-oscillations regime. For a transport operation they can always be found by simple point transformations [48]. Invariant-based inverse engineering the trap trajectory can then be worked out to satisfy the boundary conditions imposed on both independent oscillators for excitation-free transport. This needs in general numerical optimization, but analytical approximations perform satisfactorily [44] up to very short times for which the small-oscillation approximation breaks down. Lu et al. [49] used perturbation theory to design for pairs of different ions, trap trajectories robust with respect to slow frequency drifts in the scale of the shuttling time. For ions of equal mass in a harmonic trap the center of mass motion is separable, but not if the trap is anharmonic. Palmero et al. [48] found robust perturbation theory-based trap trajectories noticing that the dominant effect of a quartic anharmonicity is to shift the effective trap frequency for the center of mass.

III. THE “FOURIER METHOD”

If shuttling is performed in a harmonic oscillator—with constant frequency—and zero boundary conditions for the trap velocity, the final excitation energy depends on the Fourier transform (FT) of the trap velocity or acceleration at the trap frequency $\omega_0$ [8, 21, 22, 46, 47]. This was known for classical systems long ago [50], and it can be applied to the quantum case. Fast transport which is excitation-free at the destination may thus be achieved by trajectories that nullify the transform at $\omega_0$.

An important consequence is that the final excitation effect of deviations from some ideal, excitation-free trajectory, only depends on the FT of the acceleration deviation. The excitation is formally independent of the ideal trajectory, and indeed in practice it will be independent when the deviation is itself independent, for example if it is due to background noise. An indirect dependence may occur if the deviation is due to driving imperfections affected by the ideal trajectory. Thus, smooth, band-limited ideal trajectories should be preferable, although some settings allow for nearly-discontinuous, bang-bang protocols [51].

Guéry-Odelin and Muga [26], see also [36], worked out a systematic method to design trap trajectories that guarantee a vanishing transform at a chosen set of trap frequencies. This method may be useful for multispecies transport, and also to increase the frequency window for which the transform vanishes around some $\omega_0$, thus allowing for robustness with respect to errors in the actual trap frequency $\omega$ with respect to the nominal one $\omega_0$. As $\omega$ is supposed constant, these trajectories are of interest when drift times of the trap frequency are larger than $t_f$. In ion traps, such variations can be caused by voltage drifts, thermal expansion, and charging of the trap surfaces [11, 52].

Quite often robustness with respect to one parameter comes with a price. In particular, increasing the frequency window width implies trap trajectories with stronger oscillations and also increasing spatial domains, and therefore higher transient energies. These effects have been observed but not fully characterized [20]. Therefore, in a realistic scenario the window size cannot be increased arbitrarily since the trap is effectively harmonic only up to a certain energy. An open question is to what extent these broad-window trajectories are robust with respect to time-dependent deviations of the frequency within the window, or anharmonicities. Since the trajectory design leaves room for different solutions, optimization strategies may be implemented.

IV. NOISE IN DIFFERENT PARAMETERS

Explicit analysis of the effect of noise in different parameters on final energy excitation, using perturbative techniques proposed in [5], were worked out in Refs. [8, 47, 52, 53]. Here we focus on the work by Lu et al. [9]...
as it covers the most complete set of scenarios. Even if it falls short of addressing the broad span of complex noises that may be found in different settings, it provides basic guidance and useful insights. The trap is assumed to be an optical lattice $A \sin^2[K x + \Phi]$, in the deep lattice limit where anharmonicities may be neglected, and for weak noise, which allows the perturbative treatment. Random time-dependent fluctuations $\xi(t)$ may affect the three parameters independently: $K(t) = k[1 + \lambda \xi(t)]$ (wavenumber “accordion” noise); $A(t) = a[1 + \lambda \xi(t)]$ (amplitude or trap depth noise which amounts to a spring-constant noise); and $\Phi = \phi(t) - \lambda \xi(t)$ (trap position noise), where $\lambda$ is the perturbative parameter. The fluctuating parameters are modeled by stochastic zero-mean and “stationary” deviations, i.e., the average over noise realizations is $\langle \xi(t) \rangle = 0$, and the correlation function depends only on time difference, $\langle \xi(t) \xi(s) \rangle = \alpha(t-s)$.

The “noise sensitivities” are the second order terms of the final energy expansion in powers of $\lambda$ and may be computed analytically for specific noises, such as white noise, or Ornstein-Uhlenbeck (OU) colored noise, a simple model for examining non-zero correlation time effects. It is the natural noise in a resistor-capacitor low-pass filter output voltage if noise in the input voltage is white; it may also be used to produce flicker noise by combining different OU-noises in different relative amplitudes $\{A_n\}$. The sensitivity can generally be split into “static”—independent on the nominal (ideal) trap trajectory—and “dynamical” contributions that depend on the choice of ideal trajectory. For fixed transport time only the later allow for some optimization. An important finding is that position noise only leads to static effects, so that the choice of nominal STA trap trajectory does not formally have any effect, consistent with the FT analysis in the previous section. In the white-noise limit the only way to decrease the sensitivity for position noise is to shorten $t_f$. Colored OU noise implies sensitivity oscillations with respect to $t_f$ so that some time optimization is possible. The sensitivities have different behavior with respect to $t_f$ for the different noisy parameters. In particular static and dynamical sensitivities for spring-constant noise behave in opposite ways which leads to optimal transport times. These optimal times demonstrate that smaller process times are not necessarily the best strategy to fight noise effects. It all depends on which parameter is noisy.

For transport times larger than the noise correlation time the excitation rates are proportional to the power spectral density at the trap frequency $\omega_0$ (noise in $\Phi$) or at $2\omega_0$ (“parametric heating” due to noise in $K$ or $A$). Changing the trap frequency may thus be worthwhile if the shift diminishes the power spectral density.

Further research on the effects of noise and its mitigation may go along different lines, such as making STA trajectories “noise resistant” by imposing additional constraints on the invariant, e.g., that its eigenbasis coincides as much as possible with that of the noise operator $\{A_n\}$; also, considering other noises (correlated for the different parameters, rather than independent, non-Markovian, position-dependent, ...); optimizing trap trajectories to minimize dynamical effects; or tracking the origin and form of the noises down to its setting-dependent source. In a multisegmented Paul trap, for example, the modeling could go more deeply by making the potential a function of electrode voltages and the circuits to produce them. In the Paul trap, a quasi noise, e.g., in the trap frequency, may appear not due to actual electrode noise but to the numerical optimization to set the electrode voltages. Further research to minimize and characterize this type of deviation and its effects is needed.

V. SHORTCUTS TO ADIABATICITY AND OPTIMAL CONTROL THEORY

Shortcuts to adiabaticity based on invariants or inverse engineering approaches, and Optimal Control Theory (OCT) blend quite well \cite{7, 23, 20, 59, 50}, usually via Pontryagin’s maximal principle. While STA techniques provide families of ideal trap trajectories, OCT helps to select the best among them to minimize some cost function, restricting the selection space if necessary to domains imposed by physically motivated constraints. Imposing constraints is a way to make the transport “robust”, as problematic domains are avoided. The constrained optimizations also set useful “speed limits”, even if the optimal trajectories found are hard to realize because of discontinuities. For example, \cite{7} explores, indirectly, different ways to avoid the effect of anharmonicities by finding, for harmonic traps, minimal-time trajectories with constrained relative displacement $|x_c - x_0|$, $x_c$ being the state center, as well as minimal (time-integrated) displacement trajectories for given time, and minimal (integrated) potential energy trajectories for a given $t_f$. Also, Torrontegui et al \cite{24} find minimal-time trajectories that stay inside the domain $[0, d]$, and Lau and Daniel minimize the Stark shift \cite{60} to set ultimate time limits of shuttling ion qubits because of dephasing and decoherence effects due to the driving field. In current ion-shuttling experiments these field effects are not significant \cite{61}. To avoid the practical difficulties implied by the sudden jumps in $x_0$ or its velocity, Ding et al. \cite{58} work out smooth protocols adding further constraints on first and second derivatives of $x_c - x_0$, which imply larger shuttling times.

The anharmonicities may as well be included explicitly in the modeling and optimization: Zhang et al. \cite{62}, assuming that the compensating force approach is not viable, minimize for fixed time the time-averaged anharmonic potential energy for a potential with cubic and quartic terms. The strategy was to work out the family of STA for a purely harmonic trap, and then combine perturbation theory and OCT to choose the one that minimizes the perturbation. In \cite{63}, a different approach is proposed based on a gradient method, called “enhanced shortcuts to adiabaticity”, which provides trajectories...
which are not necessarily shortcuts for the harmonic problem. Instead of these quantum treatments, in Refs. [64, 65], optimizations using classical trajectories are performed. Interestingly, “magic times” with very small excitation and remarkable robustness versus the parameter multiplying the anharmonic terms are found [64], which are approximately valid in corresponding quantum calculations. Li et al. [64], minimized the excitation for some assumed trigonometric ansätze and compared their performance.

Mortensen et al. [50] suggest a generic strategy to deal with robustness optimization that could be quite useful for shuttling. They define a cost function that takes into account the sensitivity to perturbations [5] as well as resource requirements. The optimization is performed among the STA control parameter trajectories which guarantee no excitation so it is less computationally expensive than other optimal control methods. Simple cost functions can be quite effective to improve robustness [46], and different numerical optimization subroutines or approaches may be used [66].

OCT numerical approaches maximizing the final fidelity or minimizing the final energy without using invariant theory or inverse engineering as an intermediate step are also possible, see e.g. [40, 41, 57]. They share with analytical STA methods the goal to achieve final excitation-free states via diabatic processes.

VI. EXPERIMENTS AND SOME SYSTEM-SPECIFIC ASPECTS

For a list of STA-mediated transport experiments see [1]. More recent experiments are described in Refs. [19, 41, 67–69]. We comment here on works that have addressed robustness aspects in specific experimental settings. Each setting involves particular three-dimensional potentials and a preliminary issue is to see if and how an effective 1D theory makes sense. This is often the case, but 3D effects may be relevant, in particular if the trap is expanded, compressed [70], or rotated, and also due to nonlinearities.

The transport in a Gaussian-beam optical trap or lattice includes a radial-longitudinal coupling term [70] which can be made small but implies some effects. In an approximate theory fitting well with experimental results for transport in a Gaussian-beam trap, an averaging over radial direction shifts the trap frequency of the effective axial trap [71]. Ness et al. [71] noted that trap trajectories designed via invariant-based inverse engineering could have velocity discontinuities at the boundaries, but these discontinuities are difficult to implement. A solution is to use a higher order polynomial ansatz for \( x_z \).

Hickman et al. [67] work out a 1D theory for a moving lattice that takes into account that the transverse direction induces a slow variation in trap frequency and trap depth. These effects are modeled by applying dephasing to the density matrix with an empirical parameter.

The transport of Bose-Einstein condensates has also been studied with STA techniques for mean-field descriptions [24, 25, 13, 57, 72]. Three dimensional aspects are important because of the couplings implied by the non-linear term, and non-trivial potential evolution in some traps. The experiments performed in magnetic traps [24, 73] involve both transport and a simultaneous frequency change, and the treatments and protocols are strongly trap dependent. In [24], vertical transport is a side effect of the combination of gravity and decompression of a Ioffe-Pritchard trap. A generalization of invariant theory based on scaling laws in the Thomas-Fermi regime allowed to design a complete STA protocol for radial and axial frequencies suppressing in principle final excitations. Measured residual excitations were attributed to several experimental imperfections. Ref. [73] deals with a dedicated transport experiment in an atom-chip trap with a Z-shaped configuration in the absence of gravity. Transport is carried out in the \( z \) direction, with trap rotation in the \( xy \) plane. As the center-of-mass transport in the \( z \) direction is uncoupled to the \( xy \) directions, an approximate shortcut may be designed just for the center of mass [72], that takes into account cubic anharmonicities semiclassically, and the \( z \)-dependence of the frequency in that direction. The results are shown to be robust with respect to timing errors and offsets in the magnetic bias field used to drive the trap within current experimental resolutions. However, since the shape of the condensate and the 3D couplings are disregarded some residual collective excitations occur, even in ideal conditions. The condensate shape in the Thomas-Fermi regime could be taken into account using OCT [57], and the extension of a full inverse engineered protocol as in [24] to this setting is an open question.

Henson et al. [74] implement a machine learning (ML) algorithm to control the coupled decompression and transport of a metastable helium condensate. After each experimental iteration the excitation is measured and the algorithm adjusts its empirical model of the system to improve the control. This optimization circumvents the need for an accurate theoretical modelling of a complex system. However, black-box data-science models have their limitations too, e.g. a dependence on spurious temporal relations, or the lack of a mechanistic understanding [75]. A largely unexplored but promising paradigm is a theory-guided data-science approach hybridizing the unique capabilities of machine learning with theoretical knowledge of the underlying physics.

In linear Paul traps, the radial directions are decoupled from the axial potential [70], but transverse-axial couplings can occur due to non-ideal trap geometries, in particular for motionally excited states [19]. To shuttle ions in the axial direction, time-dependent voltages are applied to a set of DC trap electrodes. The transport potential results from superposing the voltages and unit potentials of each trap electrode. Once a desired transport potential is identified, solving for the electrode voltages to achieve such a potential is generally an ill-posed
Several methods exist, including the pseudoinverse of a constraint matrix and least-squares minimization. While the pseudoinverse method applied on a perfectly-matched problem (number of constraints equals the degrees of freedom) may result in voltage sets that exactly achieve the target potential, additional constraints such as voltage limits are unable to be included. This process for determining voltages can result in imperfect transport potentials.

To reduce high-frequency technical noise at the ions’ locations, the trap electrodes are low-pass filtered, which results in output voltages that differ from inputs. This imposes limitations if the electrode voltage ramps need frequencies above the cutoff [61]. Accounting for technical constraints such as electrode filter response, voltage limits, and slew rate requires optimization subroutines. A machine learning approach to account for electrode filtering and systematic noise effects was used for optimizing voltages for rotating an ion crystal [74]. A similar method can be extended for shuttling. Other optimization techniques include feedback control to find optimal control parameters on segments of a transport operation and slow splitting and merging [80, 81]. Local search techniques have also been used to optimize a robustness cost functional on the free parameters in a particular STA solution [40, 56]. Optimization on cost functions that include experimental data naturally lead to more robust shuttling operations, however full real-time feedback has not been implemented for trapped ions. While static experimental errors can be calibrated, time-dependent errors are not so simply accounted for. In this regard, further analysis of the relative effect of different time-dependent perturbations in transport controls would be useful.

Electric-field noise that heats ions from the motional ground state is a consideration for robust ion shuttling, as noise near the trap frequency will exist during protocols that maintain the curvature of the trap-potential well. Noise originating on the trap-electrode surfaces [82] is of current interest, as it can limit multi-qubit gate fidelity in small traps when technical noise sources are eliminated [84]. Some methods for electric-field-noise mitigation have been determined [40, 84], but a complete picture is lacking as the likely multiple underlying mechanisms are currently unknown [83]. Protocols robust to mis-calibration of trap frequency can address noise on a timescale much longer than the transport time, but the ion is heated by noise close to the trap frequency, typically on the megahertz scale, faster than typically achievable transport times. This will in general be separable from transport, with the trade off being that shuttling times as small as possible minimize heating from the surface while very short times lead to direct excitation of the ion motion. For transport protocols in which the trap frequency is not maintained (“non-rigid”) [86], parametric excitation must be taken into account, and additional surface-noise heating may accrue if the potential is weakened, due to the ion heating rate’s roughly inverse-square dependence on the trap frequency [84]; 3D effects, as described above in the non-rigid case, are also a potential issue here.

Besides temporal variation of the trap frequency over slow and fast timescales as described above, spatial variation of the trap frequency along the transport path is also of concern in more complex ion-trap-array geometries. The trap frequency as a function of distance along the trajectory can vary by more than 5% due to local trap imperfections, patches of charged insulator or adsorbate, or stray fields from other parts of the trap environment [20]; while these fields may be compensated at a sampling of points along the trajectory through calibration, this procedure adds overhead that scales with transport distance. Robustness to potential-curvature variation is thus technologically beneficial for long trajectories in multi-electrode arrays.

In interferometers [12, 13], or two-qubit gates [12, 87], ions may be subjected to hybrid traps, making use of the Paul trap to implement a stationary harmonic potential, and of a time- and internal-state-dependent displacing forces caused by a gradient of detuned laser light. Protocols with smooth field changes will favor robust operations. The intensity of the laser field might induce undesired internal state transitions, and the scaling of this effect with process time was examined in [12].

Fast transport of thousands of ions has been also carried out and implies strong trap frequency variations and anharmonicities [88, 89]. Transfer efficiencies are found to be quite sensitive to $t_f$ but in general the best transfer efficiencies are at short transport durations. There is room for improvement on the theoretical description to find efficient and stable protocols for large ion clouds.

### VII. FINAL COMMENTS

We have pointed out existing and promising avenues towards fast and safe shuttling of quantum particles. They are in general “open loop”, without feedback control, easier to implement than closed loop approaches. However, open loop approaches suffer problems such as sensitivity to imperfect control functions, and rely on an accurate modeling of the systems. This suggests a concerted effort by theorists to achieve robust designs and by experimentalists to characterize systems and dominant control errors. A further step would be to integrate these protocols into closed loop approaches based e.g. on hybrid theory-guided machine learning.

### Acknowledgments

We thank I. Chuang, X. Chen, E. Torrontegui, and D. Guéry-Odelin for useful comments. This work was supported by the Basque Country Government (Grant No. IT986-16) and PGC2018-101355-B-I00 (MCIU/AEI/FEDER,UE). This material is based upon
work supported by the Department of Defense and Under Secretary of Defense for Research and Engineering under Air Force Contract No. FA8702-15-D-0001. Any opinions, findings, conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the Department of Defense and Under Secretary of Defense for Research and Engineering.

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