Counterdiabatic route for preparation of state with long-range topological order

Sanjeev Kumar, Shekhar Sharma, and Vikram Tripathi
Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Navy Nagar, Mumbai 400005, India

(Dated: July 20, 2021)

We propose here a counterdiabatic (CD) strategy for fast preparation of a state with long-range topological order by magnetic field tuning of an initial separable state. For concreteness, we consider the ground state of the honeycomb Kitaev model whose long-range topological order together with the anyonic excitations make it an interesting candidate for fault-tolerant universal quantum computation and storage. The required CD perturbation is found to be local, having the form of the off-diagonal exchange interactions reminiscent of trigonal deformations in Kitaev Hamiltonians. We show that the counterdiabatically produced state can have high fidelity and retain numerous desired entanglement properties.

I. INTRODUCTION

The usefulness of a quantum computer depends on the ability to exploit the quantum entanglement and linear superposition absent in their classical counterparts. How well the entanglement property of a quantum many body state can be utilised in applications depends on purity of the state, details of entanglement and how feasibly it can be prepared in a short time since long duration preparation protocols may lead to environment induced decoherence. Ideally one should be able to prepare such states from an easily accessible initial quantum state through adiabatic tuning of a suitable Hamiltonian. The quantum adiabatic theorem is a no-go theorem for fast protocols as they would result in non-adiabatic excitations. In contrast to the adiabatic evolution of Hamiltonian, shortcut to adiabaticity (STA) or counterdiabatic (CD) are transitionless driving protocols which provide the means to ramp up the evolution without compensating the purity of the desired final state. CD driving suppresses these non-adiabatic excitations by adding an auxiliary field, \( \hat{H}_1(t) \) to the system Hamiltonian, \( \hat{H}_0(t) \). With this auxiliary field, even for a very rapid protocol (as compared to its adiabatic counterpart), the system always traverses the adiabatic manifold of \( \hat{H}_0(t) \) and certainly not of \( \hat{H}_0(t) + \hat{H}_1(t) \). The explicit expression for the counterdiabatic perturbation is given by

\[ \hat{H}_1(t) = i\hbar \sum_{m \neq n} \frac{|m\rangle \langle n| \partial_t \hat{H}_0(t) |n\rangle \langle m|}{E_n - E_m}, \]

where \( |m\rangle \) denotes the instantaneous eigenstate of \( \hat{H}_0(t) \) with eigenvalue \( E_m \). Physically, the CD assistance does not work through increasing the spectral gap (thereby reducing Landau-Zener transitions) but through suppression of matrix elements that would connect the states in the adiabatic manifold to those outside. The expression Eq. (1) is reminiscent of the Berry curvature. Indeed, the transitionless CD driving compensates for the Berry curvature resulting in higher fidelity at the end of the protocol. The entire spectrum of \( \hat{H}_0(t) \) is required to contract \( \hat{H}_1(t) \). Moreover, the above expansion suffer from the issue of vanishing denominators in quantum many-body chaotic systems. Non-locality of the CD term and exponential sensitivity to any perturbation in the many-body Hamiltonian is a consequence of constraining the large number of degrees of freedom of the system to the transitionless manifold. This limits the applicability of fast protocols to small few-level systems and the thermodynamic limit is out of question. The perturbation \( \hat{H}_1(t) \) suppress excitations for all \( |m\rangle \)'s and not just for some special state, for e.g., well separated ground state of the system.

As was pointed out in Ref. [15], restriction of exponentially large degrees of freedom in many-body dynamics is not always the goal. Practically, the exact and formal rigidity of Eq. (1) is relaxed by focusing on a specific state only and considering some local operators as an approximation for transitionless driving. Here we focus on the fast preparation of the ground state of the Kitaev Hamiltonian using the CD protocol. The Kitaev model is an integrable 2D system of spin-1/2 particles on the honeycomb lattice interacting with peculiar Ising-like direction dependent local interactions. The model exhibits the spin fractionalization phenomenon with no magnetic order, and elementary excitations consisting of free Majorana fermions and gapped, quantized half-vortices (visons). Depending on the interaction parameters, the half-vortices are Abelian or non-Abelian anyons. For isotropic Kitaev interactions, the Majorana fermions are linearly dispersing and massless, but they become massive above a small characteristic magnetic field. The ground state has long range topological order, signified by a finite topological entanglement entropy \( \gamma = \ln 2 \), which is not destroyed by small magnetic fields. Owing to the non-local entanglement and long-range topological order, both types of anyons are useful resources for universal quantum computation, providing fault-tolerant quantum memory and quantum gates realisation: the Abelian ones are likely to be easily accessible in experiments while the non-Abelian kind is more useful (although less readily realizable) for the quantum computation purposes.

Adiabatic preparation of highly entangled quantum states are typically hard to implement owing to the prob-
lem of exponentially small (in system size $N$) spectral gaps near the ground state (see e.g. Ref. [20]). However, the excitation gap near the ground state of the isotropic Kitaev model vanishes only as $1/\text{poly}[N]$, and, upon the introduction of a magnetic field above a certain small threshold, a finite (Majorana) gap separates its ground state from the rest of the spectrum [17, 21], while the vison gap is essentially unchanged. This will be our regime of interest. Since the vison excitations tend to destroy the long range topological order, the finite vison gap is a desirable feature.

We present a local CD protocol for high fidelity preparation of the Kitaev ground state, on timescales significantly shorter than that permitted by the quantum adiabatic theorem. The topological entanglement entropy, $\gamma$, and the (half-vortex) plaquette fluxes $W$ at the end of the protocol are found to be much closer to the equilibrium values compared to that obtained in the same time without the CD protocol. The CD interactions in our model have the form of certain off-diagonal exchange interactions [22–24] in Kitaev systems commonly associated with trigonal deformations. Besides their physical realisation in Kitaev materials, such interactions in addition to the Kitaev coupling can be implemented using superconducting quantum circuits [25, 26].

The rest of the paper is organised as follows. In Sec. II we introduce our model Hamiltonian and CD protocol. Section III presents our calculations of the state fidelity and other properties such as the topological entanglement entropy and the plaquette flux expectation value. This highlights the difference between our protocol and the naive unassisted protocol in the results section. Section IV summarizes our findings and contains a discussion.

II. MODEL AND COUNTERDIABATIC PERTURBATION

In our study, we consider the following Hamiltonian,

$$\hat{H}_0(\lambda) = \hat{H}_k + (\lambda + \delta)\hat{H}_m, \quad (2)$$

where $\hat{H}_k$ the Kitaev Hamiltonian on the honeycomb lattice given by

$$\hat{H}_k = -J \sum_{\langle ij \rangle_{\gamma \in \text{link}}} \sigma_i^{(\gamma)} \sigma_j^{(\gamma)}, \quad (3)$$

Here $i$ labels the sites and $\langle ij \rangle_{\gamma \in \text{link}}$ denotes the nearest neighbours $i, j$ on a link $\gamma = x, y, z$ as shown in Fig. 1. $\hat{H}_m$ corresponds to an external Zeeman field,

$$\hat{H}_m = B \sum_i (\sigma_i^x + \sigma_i^y + \sigma_i^z), \quad (4)$$

and $\lambda(t) \in [0, 1]$ parametrizes the protocol for the Hamiltonian evolution. We set the Kitaev interaction scale $J = 1$. The system is initiated in a high external magnetic field, $B \gg 1$, such that the ground state is a product state, and gapped. The time dependent part, $\lambda(t)$ is a smooth yet fast ramp evolving from 1 to 0 in a time $\tau$, and $\delta$ is a small positive constant such that the magnetic field at the end of our protocol is finite, but small, $B\delta \ll 1$. The spectrum remains gapped throughout the parametric evolution (see below), decreasing monotonously as $\lambda$ decreases.

For transitionless evolution, we now introduce the CD perturbation to $\hat{H}_0$. The CD term of Eq. (1) is expressed through a gauge potential, $\hat{A}_\lambda$, such that $\hat{H}_1(\lambda) = \hat{\lambda}\hat{A}_\lambda$:

$$\hat{H}_{\text{CD}} = \hat{H}_0 + \hat{\lambda}\hat{A}_\lambda. \quad (5)$$

We require the prefactor $\hat{\lambda}$ to vanish at the end points of the protocol; this condition ensures we begin and end in the ground state manifold of $\hat{H}_0$. The gauge potential can be expressed as a sum of nested commutators [27],

$$\hat{A}_\lambda = i \sum_{k=1}^{\infty} \alpha_k \langle \hat{H}_0, \hat{H}_0, ...[\hat{H}_0, \partial_\lambda \hat{H}_0]|\rangle, \quad (6)$$

where we have suppressed $\hbar$. The above series expansion gives the exact gauge potential for a gapped system, and higher order commutators generate increasingly nonlocal contributions. As an approximation, Eq. (6) is truncated after a certain order to ensure local interactions, while still suppressing excitations to give a reasonable fidelity of a quantum state, which in this paper is set at $\geq 0.5$. The variational parameters, $\alpha_k$’s are then found by minimizing $S = \langle \hat{G}^2 \rangle - \langle \hat{G} \rangle^2$, where

$$\hat{G} = \partial_\lambda \hat{H}_0 - i[\hat{H}_0, \hat{A}_\lambda], \quad (7)$$
value, $\delta B$ using the CD assisted protocol. In both cases, we choose an initial large Zeeman field, $B = 50$, which decreases to a small value, $\delta B = 0.05$, at the end of the protocol. Durations varying from $\tau = 0.01$ to $\tau = 20$ are shown, spanning both sides of the validity condition for the adiabatic theorem. For smaller values of $\tau$, CD greatly aids in suppressing the transitions to excited states, resulting in much larger fidelities compared to the unassisted case. For $\tau = 20$, it can be seen that there is no significant difference in the fidelities obtained. The inset in (a) shows the dimensionless smooth ramp $\lambda(t) = \cos^2 \left( \frac{\pi}{2} \sin^2 \left( \frac{\pi t}{\tau} \right) \right)$ used in our calculations. Inset in (b) shows the energy gap between the ground state and the first excited state in the two protocols as a function of time $t$. Clearly, CD does not assist in reducing the minimum energy gap.

and $\langle \rangle$ denotes averaging with respect to the Boltzmann weight, $\exp(-\beta H_0)$. The minimization condition ensures transitions due to non-zero off-diagonal elements in the instantaneous Hamiltonian are suppressed [8]. For ease of calculation we focus on the infinite temperature limit ($\beta \to 0$), where the problem reduces to minimizing $\Delta = \text{Tr} [\hat{G}^2]$. Note the infinite temperature is not ideal for ground state preparation as it treats the excited states on the same footing. We show the CD assistance produces desirable results even in this worst scenario limit. It has been shown that for the one dimensional Kitaev model, the CD Hamiltonian is of $M$-body interaction type thus limiting its practicality [28] for cluster state generation. Limiting ourselves to two-body interactions only, we retain in Eq. (6) only the leading term and obtain

$$
\hat{A}_\lambda^{(1)} = \frac{B/J}{18(\lambda + \delta B^2)(B/J)^2} + 10 \left\{ \sum_{\langle ij \rangle_{\text{link}}} (\sigma_i^x \sigma_j^y - \sigma_i^y \sigma_j^x) + \sum_{\langle ij \rangle_{\text{link}}} (\sigma_i^y \sigma_j^x - \sigma_i^x \sigma_j^y) + \sum_{\langle ij \rangle_{\text{link}}} (\sigma_i^z \sigma_j^z - \sigma_i^\delta \sigma_j^\delta) + i \leftrightarrow j \right\}.
$$

(8)

The above gauge potential, $\hat{A}_\lambda^{(1)}$, resembles the $\Gamma'$-interaction in Kitaev systems with the difference that Eq. (8) has asymmetric terms while $\Gamma'$-interaction has symmetric ones [22-24], and associated with trigonal distortions in the lattice. Using Eq. (8) in Eq. (5) gives the CD Hamiltonian which we use in the rest of the paper. Numerical calculations are performed by exact diagonalisation of a 24-site cluster (see Fig. 11) using Quspin 29, 30. For benchmarking, we compare the energy gap in our system with that of a larger 144-site cluster (obtained via DMRG, see Appendix B), and find that the two are in agreement. We thus conclude that the system is gapped throughout the range of magnetic fields we study - this is in contrast with some calculations [31] in the recent literature (based on apparently power-law decay of ground state spin correlators obtained using DMRG) that claim the existence of a gapless phase in the range $0.2 \lesssim B \lesssim 0.3$. The finite spectral gap even in the thermodynamic limit is of relevance to our problem since a vanishing gap at intermediate magnetic fields would invalidate the counterdiabatic protocol.

### III. RESULTS

Below we show the numerical results for the overlap of the time evolved state, $|\psi(\lambda(t)) = U(t)|\psi(\lambda(0))\rangle$, with the ground state $|\phi_{\text{GS}}(\lambda(t))\rangle$ of the instantaneous Hamiltonian. The fidelity, $F$, is defined as $F = |\langle \psi | \phi_{\text{GS}} \rangle|^2$. A smooth ramp with vanishing time-derivative at the end points ensures the initial and final Hamiltonians are the same in the unassisted and CD protocols. For concreteness we choose $\lambda(t) = \cos^2 \left( \frac{\pi}{2} \sin^2 \left( \frac{\pi t}{\tau} \right) \right)$ for $t \in [0, \tau]$ as the

![Figure 2](image-url)
Figure 3. Plots showing the fidelity ($F$), entanglement entropy ($\gamma$) and expectation value of the $\hat{W}$ (plaquette flux) operator, ($W$) calculated for the quantum state obtained via CD assisted and naive, unassisted protocol. In (a) these quantities are shown as a function of a wide range of protocol time duration, $\tau$ spanning both sides of the validity condition of the adiabatic theorem. Dashed line shows the pure ground state value corresponding to $\delta \hat{H}_m + \hat{H}_k$. Unassisted protocol yield close-to zero values for $\gamma$ and $W$ owing to lesser fidelity as compared with the CD assisted protocol. $\gamma$ values smaller than $10^{-6}$ are suppressed to zero by switching from log scale to linear scale. (b) Show the same quantities as a variation of final magnetic field in the system $B\delta$. The quantities measure start converging for $\tau = 10$. 

shown in the inset of Fig. 2h. Figure 2 shows the fidelity of the evolving quantum state in the (a) unassisted and (b) CD assisted protocols for various time-durations, $\tau$. We note that for $\tau \ll 1$, the fidelity in the CD assisted protocol falls sharply around $t \gtrsim \tau/2$ (even falling to zero for the shorter durations) before jumping to values significantly larger than the unassisted case towards the end of the protocol. The vanishing fidelity during intermediate times is not on account of any closure of the spectral gap (see inset of Fig. 2h) but rather shows that the time evolved state in the CD protocol overlaps poorly with the ground state of the instantaneous Hamiltonian for these intermediate times. However, for $\tau \gg 1$, the two protocols do not show a significant difference. This is in accordance with the fact that for slow variation of the system parameters, the CD Hamiltonian approaches the adiabatically varying Hamiltonian. From Fig. 2 we see that the fidelity remains approximately unity even for times near the middle of the protocol owing to the still large Zeeman gap. This implies one can start from an initial product state and yet attain large fidelities for the Kitaev ground state at the end of protocol.

We next compare the usefulness of the quantum state obtained via the two protocols by studying their en-
tanglement entropy and plaquette fluxes. The Kitaev ground state is associated with a finite topological entanglement entropy, which is the part of the von Neumann bipartite entropy, $S_A = \text{Tr} \rho_A \log \rho_A = \alpha L - \gamma$, remaining after subtracting the area law contribution. Here $L$ is the perimeter of a 2D subsystem $A$ whose bipartite entanglement entropy is $S_A$. For the Kitaev ground state, $\gamma = \ln 2$. Appropriately choosing four partitions of the lattice (see Fig. 3) and taking a linear combination of entropies of three of the partitions yields $\gamma$, which is free from the boundary term [18]:

$$- \gamma = S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC}. \quad (9)$$

Finite topological entanglement entropy ensures the quantum state is fault-tolerant to local disturbances. In addition to the entanglement entropy, the plaquette flux operator $\hat{W}$ is another characteristic quantity in pure Kitaev system. With reference to Fig. 1, we choose the plaquette, $p = \{1, 4, 9, 13, 10, 5\}$ for computing expectation value of the flux operator defined as $\hat{W}_p = \sigma_1^p \sigma_2^p \sigma_3^p \sigma_4^p \sigma_5^p$. The flux operator commutes with the Kitaev Hamiltonian, $\hat{H}_k$ and has eigenvalues $W = \pm 1$. Ground state is characterised by $W = 1$. In Fig. 3, we show the dependence of these quantities on (a) protocol duration, $\tau$ and (b) final Zeeman field, $B\delta$. The green coloured line in Fig. 4 shows the value of these quantities corresponding to the ground state of the final Hamiltonian for comparison. We find that for $\tau \lesssim 1$, the topological entanglement entropy of the quantum state evolved using transitionless driving is an order of magnitude higher than that of the state evolved without it. A finite nonzero $\gamma$ implies the state is useful for quantum computation. Similarly, CD assisted protocol yields values of $W$ closer to unity compared to unassisted driving. Because of better fidelity through counterdiabatic approach, the final time-evolved quantum state stays close to the true ground state. Quantitatively, this can be expressed in terms of the support size $\xi$ of the quantum state (obtained at the end of the protocol) in the many-body Hilbert space of the exact eigenstates of the final Hamiltonian. The support size $\xi$ is defined as $\xi^{-1} = \sum_i |\langle i | \xi \rangle|^2$, where $\langle i |$ is the overlap of the quantum state with the many-body eigenstates $|i\rangle$. We observe in Fig. 4(a) support sizes of 2.5 states or less when CD assisted for a wide range of protocol durations while unassisted driving gives support sizes of the order of 10 states or more. In CD protocol, $\xi$ shows a relatively slower variation on changing the final Zeeman field as opposed to unassisted protocol where $\xi$ rapidly rises for decreasing $B\delta$ as shown in Fig. 4(b). Along with $\xi$ in Fig. 4(a) we have plotted the energy difference of the obtained quantum state with the pure ground state. We find the energy difference is small for the CD case. We thus claim that the CD aided approach yields the quantum state with high localisation (the dimensionality of the many-body Hilbert space for our 24-spin cluster is $2^{24}$) near the ground state, with the ground state having the maximum share equal to its fidelity.

**IV. DISCUSSION**

The Kitaev ground state is useful for universal quantum computing and storage due to its long-range topological order and anyonic excitations. We demonstrated using a counterdiabatic strategy, the feasibility of preparing the Kitaev ground state (in the presence of a small Zeeman field sufficient for introducing a bulk spectral gap) with high fidelity on time scales significantly smaller than that permitted by the adiabatic theorem. Our proposed method relies on local two-body interactions which makes it practicable for implementation. Features such as topological entanglement entropy and nonzero flux expectation were shown to be preserved much better than unassisted protocols in the same time duration. Remarkably, these properties are found to be rather insensitive to the (short) duration of the protocol and instead depend
which are given by $B$ with interaction, we expand eq. (6) to leading order:

\[ \text{Let the Zeeman field coupling constant } B \text{ neglected in our approximate CD protocol.} \]

We sketch the steps involved to obtain the counterdiabatic perturbation to the Hamiltonian $\hat{H}_0$. For 2-body local interaction, we expand eq. (3) to leading order:

\[ \hat{A}^{(1)}_\lambda = i\alpha_1 [\hat{H}_0, \partial_\lambda \hat{H}_0] = i\alpha_1 [\hat{H}_k, \hat{H}_m]. \]  

(A1)

Let the Zeeman field coupling constant $B = (B_x, B_y, B_z)$. In our analysis, we considered the field along $[111]$ direction with $B_x = B_y = B_z = B$. The commutator in eq. (3) can be written as a sum of 3 terms $[\hat{H}_k, \hat{H}_m] = \hat{\mathcal{X}} + \hat{\mathcal{Y}} + \hat{\mathcal{Z}}$ which are given by

\[ \hat{\mathcal{X}} = 2iB_x J \left\{ \sum_{(jk)_{x-link}} (\sigma_j^y \sigma_k^z + \sigma_j^z \sigma_k^y) - \sum_{(jk)_{y-link}} (\sigma_j^y \sigma_k^z + \sigma_j^z \sigma_k^y) \right\}, \] 

(A2)

\[ \hat{\mathcal{Y}} = 2iB_y J \left\{ \sum_{(jk)_{x-link}} (\sigma_j^z \sigma_k^x + \sigma_j^x \sigma_k^z) - \sum_{(jk)_{y-link}} (\sigma_j^z \sigma_k^x + \sigma_j^x \sigma_k^z) \right\}, \] 

(A3)

\[ \hat{\mathcal{Z}} = 2iB_z J \left\{ \sum_{(jk)_{y-link}} (\sigma_j^y \sigma_k^x + \sigma_j^x \sigma_k^y) - \sum_{(jk)_{x-link}} (\sigma_j^y \sigma_k^x + \sigma_j^x \sigma_k^y) \right\}. \] 

(A4)

The variational parameter $\alpha_1$ is found by constructing the operator $\hat{G}$ as defined in eq. (4) and minimizing $S = \text{Tr} \hat{G}^2$. The leading order of $\hat{G}$ can be expressed as

\[ \hat{G}^{(1)} = \partial_\lambda \hat{H}_0 - i \left[ \hat{H}_0, \hat{A}^{(1)}_\lambda \right] \]

\[ = \hat{H}_m - \alpha_1 (\lambda + \delta) \left[ \hat{H}_m, \left[ \hat{H}_m, \hat{H}_k \right] \right] - \alpha_1 \left[ \hat{H}_k, \left[ \hat{H}_m, \hat{H}_k \right] \right]. \]  

(A5)

The minimization condition $\delta S/\delta \alpha_1 = 0$ yields

\[ \alpha_1 = \frac{(\lambda + \delta) \text{Tr} \left( \hat{H}_m \left[ \hat{H}_m, \left[ \hat{H}_m, \hat{H}_k \right] \right] \right) + \text{Tr} \left( \hat{H}_m \left[ \hat{H}_k, \left[ \hat{H}_m, \hat{H}_k \right] \right] \right)}{(\lambda + \delta)^2 \text{Tr} \left( \left[ \hat{H}_m, \left[ \hat{H}_m, \hat{H}_k \right] \right] \right)^2 + \text{Tr} \left( \left[ \hat{H}_k, \left[ \hat{H}_m, \hat{H}_k \right] \right] \right)^2 + 2 (\lambda + \delta) \text{Tr} \left( \left[ \hat{H}_m, \left[ \hat{H}_m, \hat{H}_k \right] \right] \left[ \hat{H}_k, \left[ \hat{H}_m, \hat{H}_k \right] \right] \right)} \]

(A6)

\( \text{The traces are evaluated numerically resulting in} \)

\[ \alpha_1 = \frac{-1/4}{9 (\lambda + \delta)^2 B^2 + 5 J^2} \] 

(A7)

Substituting $\alpha_1$ in eq. (A1) gives the gauge potential $\hat{A}^{(1)}_\lambda$. 

ACKNOWLEDGMENTS

We thank Aman Kumar for sharing the DMRG result of energy gaps.

Appendix A: Gauge Potential, $\hat{A}_\lambda$

We sketch the steps involved to obtain the counterdiabatic perturbation to the Hamiltonian $\hat{H}_0$. For 2-body local interaction, we expand eq. (3) to leading order:
Figure 5. Ground state energy gap for the Hamiltonian, $\hat{H} = \hat{H}_m + \hat{H}_k$ as calculated for 24–sites using exact diagonalisation and 144–sites via DMRG as a function of Zeeman field coupling $B$ is shown. The 2 cases coincide for a wide range of magnetic field values encountered in the naive as well as CD assisted protocol (see inset of Fig. (2b)).

Appendix B: Energy gap

Higher fidelities in CD assisted protocols are aided by the mass gap present in the Kitaev system in presence of a Zeeman field. Here, we illustrate this gap is not a finite size effect. Figure (5) shows the comparison of energy gap between ground state and first excited state for the Hamiltonian, $\hat{H} = \hat{H}_m + \hat{H}_k$ in 24–sites and 144–sites lattice. All 24–sites calculations are performed via exact diagonalistion in Quspin while the larger 144–sites energy gap calculation is done using finite size DMRG. We note the gap coincides for the 2 cases for a wide range of magnetic field encountered in the protocol (see inset of Fig. (2b)).

[1] A. Y. Kitaev, Annals of Physics 303, 2 (2003).
[2] D. Gross, S. T. Flammia, and J. Eisert, Phys. Rev. Lett. 102, 190501 (2009)
[3] S. He, S.-L. Su, D.-Y. Wang, W.-M. Sun, C.-H. Bai, A.-D. Zhu, H.-F. Wang, and S. Zhang, Scientific reports 6, 1 (2016).
[4] S. Bandyopadhyay and A. Dutta, Phys. Rev. B 102, 094301 (2020)
[5] A. Hamma and D. A. Lidar, Phys. Rev. Lett. 100, 030502 (2008)
[6] A. C. Santos and M. S. Sarandy, Journal of Physics A: Mathematical and Theoretical 51, 025301 (2018)
[7] D. Guéry-Odelin, A. Ruschhaupt, A. Kiely, E. Torrentegui, S. Martínez-Garaot, and J. G. Muga, Rev. Mod. Phys. 91, 045001 (2019)
[8] M. Kolodrubetz, D. Sels, P. Mehta, and A. Polkovnikov, Physics Reports 697, 1 (2017)
[9] X. Chen, A. Ruschhaupt, S. Schmidt, A. del Campo, D. Guéry-Odelin, and J. G. Muga, Phys. Rev. Lett. 104, 063002 (2010)
[10] S. Deffner, C. Jarzynski, and A. del Campo, Phys. Rev. X 4, 021013 (2014)
[11] C. Jarzynski, Phys. Rev. A 88, 040101 (2013)
[12] M. V. Berry, Journal of Physics A: Mathematical and Theoretical 42, 365303 (2009).
[13] C. W. Duncan and A. Del Campo, New Journal of Physics 20, 085003 (2018).
[14] A. Hartmann and W. Lechner, New Journal of Physics 21, 043025 (2019)
[15] D. Sels and A. Polkovnikov, Proceedings of the National Academy of Sciences 114, E3909 (2017).
[16] E. J. Meier, K. Njan, D. Sels, and B. Gadway, Phys. Rev. Research 2, 043201 (2020)
[17] A. Kitaev, Annals of Physics 321, 2 (2006).
[18] A. Kitaev and J. Preskill, Phys. Rev. Lett. 96, 110404 (2006)
[19] S. Lloyd, Quantum Information Processing 1, 13 (2002).
[20] D. Sels, H. Krovi, and J. Roland, Proceedings of the National Academy of Sciences 107, 12446 (2010).
[21] M. Gohike, R. Moessner, and F. Pollmann, Phys. Rev. B 98, 014418 (2018)
[22] J. G. Rau and H.-Y. Kee, arXiv:1408.4811 (2014).
[23] D. Takikawa and S. Fujimoto, Phys. Rev. B 102, 174414 (2020)
[24] D. Takikawa and S. Fujimoto, Phys. Rev. B 99, 224409 (2019)
[25] M. Sameti and M. J. Hartmann, Phys. Rev. A 99, 012333 (2019)
[26] J. Q. You, X.-F. Shi, X. Hu, and F. Nori, Phys. Rev. B 81, 014505 (2010)
[27] P. W. Claeys, M. Pandey, D. Sels, and A. Polkovnikov, Phys. Rev. Lett. 123, 090602 (2019)
[28] T. H. Kyaw and L.-C. Kwek, New Journal of Physics 20, 045007 (2018).
[29] P. Weinberg and M. Bukov, SciPost Phys. 2, 003 (2017).
[30] P. Weinberg and M. Bukov, SciPost Phys. 7, 20 (2019).
[31] N. D. Patel and N. Trivedi, Proceedings of the National Academy of Sciences 116, 12199 (2019).