Geometric thinking level of the Indonesian seventh grade students of junior high school

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Abstract. Enhancement of students’ ability to think geometry at level 0 until level 4 will configure the comprehensive geometric knowledge. The aim of this study was to analyze the geometric thinking level of seventh grade students of junior high school. This research is part of a series of research conducted for 2 years with the Design Didactical Research (DDR) method. The geometric thinking ability test was given to 32 students in the VIIA class. The questions used are 25 questions which contain 5 levels of thinking geometry proposed by Van Hiele. The visualization ability data of the visualization level ability test held by students was obtained by 32% of students in pre level 0, 47% of students in level 0 and 21% of students at level 1. While at level 2, 3, 4 there were no students who arrived to that level.

1. Introduction

Learning outcomes cannot be separated from the learning process by student’s experience. Teachers need to consider the difficulties experienced by students during the learning process. There are three types of difficulties that are usually faced by students. First, the difficulties caused by discrepancies in the child's ability level with the demands of thought contained in teaching materials. Second, the difficulties caused by the limited context in understanding a concept. Third, difficulties caused by mistakes or weaknesses related to the design of teaching materials [1].

Learning difficulties as much as possible are reduced to maximize student learning outcomes. Tiered learning in accordance with the order of students' thinking ability is a process that must be passed by students. The ability test of the National Examination in mathematics that contain 1) Elements and properties of 2-dimensional shape, 2) Elements and properties of 3-dimensional shape, 3) Number operations, social arithmetic, sequence / series, 4) The concept of Probability theory, 5) Statistics: data presentation and measures of center and variability. A total of two ability from 5 ability that must be mastered by junior high school students related to geometry. The average national exam questions have around 17 questions out of 40 questions relating to geometry material. Students do not experience changes in the level of geometry thinking because one of them is a test that is done directly after the final semester exam, so that some students feel tired [2]. Students could not perform activities at the level of formal deduction. This means that students cannot be at level 4, namely rigor. The higher class got higher Van Hiele level values. There were no students from grades 1 to 4 at level 3 and there were...
no classes 1 through 2 at level 2. Most of the class 1 to class 2 at level 1, and grades 3 through 6 are at level 2. Only grades 5 and 6 can meet level 3 [3].

According to [4] students need thinking at a relatively high level to study geometry formally. Students must have deeper thinking experience at the lower level before learning formal geometry concepts. The process of increasing students' level of thinking needs intensive guidance from the teacher. [5] investigated the geometrical abilities of 15-year-old learners to write formal evidence. According to Dimakos et al, the success of students to be able to write formal evidence cannot be separated from the teacher's strategy to use the level of thinking geometry comprehensively. [6] examined the errors of thinking of 13-year-old students, one of the students' mistakes in writing formal evidence about geometrical problems was that students made a leap of thinking. Students do not sequence the geometry thinking phases especially in the visualization phase. Students tend to focus on how to calculate numbers to solve problems. Analysis of geometry thinking ability is very important to know the right learning pattern but the results of observations in one of the junior high schools in Indonesia, learning activities in geometry material do not pay attention to the steps that are in accordance with the steps of learning geometry.

2. Literature review

2.1. Learning taxonomy

The learning taxonomy in the cognitive domain according to [7] consists of knowledge, comprehension, application, analysis (decomposition), synthesis, and evaluation. [8] conducted research to develop and improve Bloom’s taxonomy. [8] argue that procedural knowledge if coupled with the ability to analyze it will produce the students’ integration capabilities. While the ability to analyze is the ability of the 4th of 6th level abilities by Anderson. So, Van Hiele's learning level, which is until integration only, must be developed so the aims to mastery the Van Hiele thinking level can be mastered completely.

![Anderson's taxonomic learning model](image)

**Figure 1.** Anderson's taxonomic learning model

2.2 Geometry Thinking Level

The research was carried out in line with the standpoint of thinking geometry according to Van Hielle. According to [9] in the learning geometry, students' thinking development occurs through 5 (five) levels as follows.
1) Level 0 (Visualization). The student reasons about basic geometric concepts, such as simple shapes, primarily by means of visual considerations of the concept as the whole without explicit regard to properties of its components.

2) Level 1 (Analysis). The students reasons about geometric concepts by means of an informal analysis of component parts and attributes. Necessary properties of the concept are established.

3) Level 2 (Abstraction). The student logically orders the properties of the concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of a set of properties in determining a concept.

4) Level 3 (Deduction). The student reasons formally within the context of a mathematical system, complete with undefined terms, axiom, an underlying logical system, definition, and theorems.

5) Level 4 (Rigor). The student can compare systems based on different axioms and can study various geometries in the absence of concrete models.

3. Method
The method used is a qualitative research method in the form of a Didactical Design Research [10]. First stage prospective analysis, we started this stage by doing reflection for action made before the implementation of teaching and learning focuses. We look for data about concepts in the material contained in mathematics subjects especially quadrilateral geometry. We Analyze predetermined mathematical concepts, namely the subject of seventh grade geometry concepts. We Make the initial instrument in the form of a test of ability to think geometric level and observations sheet to find out the obstacle learning that exists in the concept. We observe a learning by using observations sheet and tests of thinking geometry. Then we continue re-contextualization and re-personalization teaching materials underlying the design of didactic hypotheses include ADP. We make learning trajectory of geometry matter become to design didactic. Second stage metapedagogic, this stage exploration action during the process of teaching and learning. The analysis includes three integrated components, namely unity, flexibility, and coherence that dynamically grow as the basis for making the best decisions. Third stage retrospective analysis, activity after the teaching and learning where teachers conducted a retrospective analysis related to the results of analysis of the hypothetical didactic situation designed through the results of the analysis of meta pedagogic didactic. In this study, the focus of this research is to determine the level of geometry thinking in class VII. The population of this study were seventh grade students of SMP out of the junior high schools in Indonesia, while the sample used was class VIIA students. Data collection techniques used in this study is triangulation, which is a combination of interviews, observation, and documentation. First, from the observation, we select one among 6 participants having criteria in pra level 0, level 0, and level 1. We interviewed the selected participant and then confirmed the data with the documents of worksheet.

4. Result and Finding
The geometry thinking level test was conducted on 32 students of seventh Grade of junior high school. This problem consists of 25 questions that contain 5 levels of thinking from level 0 to level 4. Problem number 1 to number 5 contains questions about visualization, questions number 6 to 10 contain analysis questions, questions number 11 to number 15 contain abstraction, number problems 16 to 20 contains deduction questions, and number problems 21 to 25 contains rigor questions. Here are the results.

| No | Name | Ability criteria | LEVEL 0 | LEVEL 1 | LEVEL 2 | LEVEL 3 | Conclusion |
|----|------|------------------|--------|--------|--------|--------|------------|
| 1  | S1   | Pass             | Not Pass | Not Pass | Not Pass | Not Pass | Level 0    |
| 2  | S2   | Pass             | Not Pass | Not Pass | Not Pass | Not Pass | Level 0    |
| 3  | S3   | Pass             | Not Pass | Not Pass | Not Pass | Not Pass | Level 0    |
Table 1. Geometry thinking level test

| No | Name | Ability criteria | Conclusion |
|----|------|------------------|------------|
|    |      | LEVEL 0 | LEVEL 1 | LEVEL 2 | LEVEL 3 |
| 4  | S4   | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 5  | S5   | Not Pass | Not Pass | Not Pass | Not Pass | pra Level 0 |
| 6  | S6   | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 7  | S7   | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 8  | S8   | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 9  | S9   | Not Pass | Not Pass | Not Pass | Not Pass | pra Level 0 |
| 10 | S10  | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 11 | S11  | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 12 | S12  | Not Pass | Not Pass | Not Pass | Not Pass | pra Level 0 |
| 13 | S13  | Not Pass | Not Pass | Not Pass | Not Pass | pra Level 0 |
| 14 | S14  | Pass    | Not Pass | Not Pass | Not Pass | Level 1 |
| 15 | S15  | Pass    | Not Pass | Not Pass | Not Pass | Level 1 |
| 16 | S16  | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 17 | S17  | Not Pass | Not Pass | Not Pass | Not Pass | pra Level 0 |
| 18 | S18  | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 19 | S19  | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 20 | S20  | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 21 | S21  | Pass    | Not Pass | Not Pass | Not Pass | Level 1 |
| 22 | S22  | Pass    | Not Pass | Not Pass | Not Pass | Level 1 |
| 23 | S23  | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 24 | S24  | Not Pass | Not Pass | Not Pass | Not Pass | pra Level 0 |
| 25 | S25  | Not Pass | Not Pass | Not Pass | Not Pass | pra Level 0 |
| 26 | S26  | Not Pass | Not Pass | Not Pass | Not Pass | pra Level 0 |
| 27 | S27  | Tidak   | Not Pass | Not Pass | Not Pass | pra Level 0 |
| 28 | S28  | Pass    | Not Pass | Not Pass | Not Pass | Level 1 |
| 29 | S29  | Pass    | Not Pass | Not Pass | Not Pass | Level 1 |
| 30 | S30  | Pass    | Not Pass | Not Pass | Not Pass | Level 0 |
| 31 | S31  | Pass    | Not Pass | Not Pass | Not Pass | Level 1 |
| 32 | S32  | Not Pass | Not Pass | Not Pass | Not Pass | pra Level 0 |

Number of pass: 22 7 0 0

The table 1 illustrates that 10 students have a level 0 of geometry ability, 15 students have level 0 of geometry ability, and 7 students have level 1 of geometry ability. The ability of 7th grade students with an average age of 13 years should be more than pravisualization, visualization and analysis. Because according to [11] that children aged 7-11 years are able to think the concrete operations and at the age of 12 years and above students can think up to formal spatial operations. Other facts are also expressed in the findings [12], seventh grade students should have passed the 0 thinking level, even though students experience epitimological obstacles at the thinking level 1. The result of student thinking level categories are presented in figure 1.
The next analysis was carried out in the learning process in the class for 8 meetings about plane. Students are given learning according to the geometric thinking phases to form comprehensive geometric thinking skills. The learning process emphasizes student activities on lower level geometry skills [3]. The teaching aids that used are visualization props, rulers, bows, scissors and student activity sheets as one of the scaffolding. Students do discussion activities in groups, grouping students does not pay attention to gender because according to [13] that sex/gender does not affect the results of learning geometry.

Students can’t mastery the geometric thinking smoothy as easily. Students at the age of 9-11 years are mostly at level 0 [14]. [15] found that students often slipped to level 0 again when they were in transition to level 1. This was because students were faced with a foreign form they had not met. Therefore [16] suggested in the results of his research that the teacher really pay attention to the scaffolding of all the activities of students and guide them diligently. The diagonal concept is not a new thing for 7th grade material, but most of students did the error while analyzing the data (level 1).
After completed the learning, the next data collection is interviewing the 6 students. This interview aims to find out how students respond after being given learning that matches the geometric thinking level. In addition, data was also obtained about the use of visual aids and student activity sheets. The following is a cut of the interview with one of the students.

Q : here is a worksheet containing questions, what do you think about the question, is it to over? Or not?
S1 : no
Q : can you finish this worksheet in one meeting?
S1 : yes, i can
Q : in the activity sheet there are underlined words or terms. Do you know about this? or never know about the term I underlined?
S1 : yes, i know
Q : this is a word / term angles. Can you spot where is the angle? (pointing at the props?)
S1 : this (while pointing to some angles)
Q : If there is a diagonal term, which one is diagonal?
S1 : this is the same (pointing the first diagonal and the second diagonal)
Q : that means it's easy ... so I give you the problem, it's not difficult, does it mean that question number 1-13 has no difficult words?
S1 : none
Q : there is a display sheet, do you think the size of the display sheet is not big enough or big?
S1 : medium
Q : means what is the right size?
S1 : yes right
Q : for what color is not striking, is this not the favorite color or what does it mean later if I make a prop again what’s the good color?
S1 : the color has no effect
Q : it means that the the color is not important, the important thing is the planes?
S1 : yes
Q : it means if there is a visual form ... you are not only think in the mind, but there is a picture to visualize?
S1 : yes
Q : cthe, if I compare using media of not using media, for example if I ask you to fill this (worksheet) without props, then on the other time I ask you to fill this (worksheet) with props. Do you think this display helps you or not?
S1 : help
Q : better if there are props?
S1 : yes (while nodding)
Q : if for example you learn more comfortable using a tool that is made (props) or around the school, for example when you learn about square. Do you want to see ceramics only? Or do you want to use these props or it is enough to just see the ceramic below?
S1 : use props
Q : using props, yes? (confirms) ... okay ... keep on
......
Q : it means that when I deliver it is not so difficult, does it mean that the results of the discussion can be delivered properly. Then ... this is when the teacher teaches you usually give information first, like "what is a difficult word or not?" When I was taught to be given information, what was it or not?
S1 : yes given
Q : When students fill out (worksheet) the teacher goes around helping the hard ones, right?
S1 : yes
Visual teaching aids and student activity sheets that designed to help level 0 thinking processes are designed with the good sentences, words and phrases that are most easily understood by students. The process of interviewing the selection of shapes, objects, colors, question sentences and quantity of questions is dug up to find activities and visual aids that students prefer. The improvements made to these learning tools and resources will help students reduce epistemological learning obstacles to students, so that the students competencies are expected to well achieve [17]. Students have a tendency to relate the same traits to quadrilateral to construct knowledge in a new quadrilateral. The process of constructing knowledge of the new quadrilateral will follow an iconic procedure, where visualization of various forms of quadrilateral will be connected [18]

5. Conclusion
The geometric thinking level of 7th grade junior high school students is at the lower level, namely level 0 and 1. This study contributes to mathematics teachers who teach geometry at 7th grade. The teacher must increase his teaching attention to visualization and analysis. Improved visualization can be done by active learning processes that students like. Visual aids that are well designed and used appropriately can reduce students' errors in visualizing geometric objects. While the level analysis process carried out after the visualization level requires intensive scaffolding from the teacher. At the level of analysis, it is necessary to pay attention to the obstacles to epistemological learning in students. However, this study has a number of limitations that need to be taken into account in interpreting the findings. The use of research samples is students who have intermediate abilities, so these findings need to be studied further in students who have low abilities and high abilities.

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