Basis independent parametrisations
of R parity violation in the soft SUSY breaking sector

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Abstract

The magnitude of R-parity violating coupling constants depends on which direction in the space of weak doublets with hypercharge $= -2$ corresponds to the Higgs. To address this "basis dependence", one can construct combinations of coupling constants that are invariant under these basis transformations, and which parametrise how much R parity violation is present in the Lagrangian (analogous to Jarlskog invariants for CP violation). This has previously been done for the Higgs vev and the R parity violating couplings constants in the superpotential. In this letter, I build invariants that include soft SUSY breaking interactions, and briefly discuss their relation to invariants involving the Higgs vev. This completes the construction of invariants based on the MSSM with baryon parity.

In the Standard Model, it is not possible to write down any renormalisable interactions that violate either baryon number ($B$) or lepton number ($L$)\(^1\). This is a consequence of the gauge symmetries and the particle content. In the supersymmetric extension of the Standard Model\(^2\) there are many new particles, and it becomes possible to have renormalisable $B$ or $L$ non-conserving interactions. However, since neither $B$ nor $L$ violation has been observed in the laboratory, these interactions are often removed by imposing a symmetry.

There are various symmetries that prevent the $B$ or $L$ violating renormalisable interactions in the supersymmetric Standard Model\(^3\). The most common is a discrete symmetry called $R$-parity, referred to as $R$ in this letter. It alots each particle a multiplicative quantum number $:(−1)^{2S+3B+L}$, where $S$ is the particle spin.

One can also allow the renormalisable $B$ or $L$ violating interactions to be present, but require the coupling constants to be sufficiently small to satisfy experimental bounds\(^4\). In this case, it is not desirable for the $B$ and $L$ violating interactions to be simultaneously present, because they mediate rapid proton decay. The bound on the product (lepton number violating Yukawa-type coupling)$×$(baryon number violating Yukawa-type coupling) varies from $10^{-10}$ to $10^{-25}$\(^5\), depending on the generation indices of the coupling constants. One therefore usually requires that either $B$ or $L$ be conserved. In this paper, I will assume that baryon number is exactly conserved in the renormalisable interactions.

The superpotential for the Minimal Supersymmetric Standard Model (MSSM) with $R$-parity imposed is

\[ W = \mu H_1 H_2 + h^u_{pq} H_2 Q_p U_q^c + h^d_{ij} H_1 Q_p D_c^i + h^e_{ij} H_1 L_c E_j^c . \]  

(1)
The Lagrangian also contains kinetic terms, gauge interactions, D-terms and soft SUSY breaking terms of the form

\[ \text{soft masses} + B_i H_1 H_2 + A^p_p H_2 Q_p U_q^c + A^{pq}_q H_1 Q_p D_q^c + A^{ij}_i H_1 L_i E_j^c . \]  

I am abusively using capital letters for both superfields (as in eqn [1]) and scalar component fields (as in eqn [2]). Quark generation indices are \( p, q, r, s... \) and lepton indices are \( i, j, k,... \). Whether indices are up or down makes no difference.

If instead of imposing \( R \)-parity, one merely requires that baryon number be conserved, there can be lepton number violating interactions in the superpotential:

\[ W_L = \epsilon H_2 L_i + \lambda^i j k L_i L_j E_k^c + \lambda^{ipq} L_i Q_p D_q^c \]  

and in the soft terms:

\[ \text{soft masses mixing } L^\dagger \text{ and } H_1 + B^i H_2 L_i + A^i j k L_i L_j E_k^c + A^{ipq} L_i Q_p D_q^c . \]  

There are experimental upper bounds on these new couplings from various processes [4], such as Flavour-Changing-Neutral-Currents (FCNC), lepton flavour violation and lepton number non-conservation. However, some of these coupling constants can be made zero by a basis choice, so it is important to remember in which basis the bounds apply. In this letter I would like to approach this problem in a different way; I construct combinations of coupling constants that are “basis independent” and that parametrise the amount of \( R \)-parity violation present in the Lagrangian. They are zero if \( R \), or equivalently lepton number, is conserved.

A simple example of this approach is to take the superpotential of equations (1) and (3) with one quark and lepton generation. It appears to have two \( R \)-violating interactions: \( \epsilon H_2 L \) and \( \lambda^i j k L_i L_j E_k^c \). (There is no \( \lambda^i j k L_i L_j E_k^c \) interaction because \( \lambda^i j k \) is antisymmetric on the \( i j \) indices.) It is well known that one of these can be rotated into the other by mixing \( H_1 \) and \( L \). If

\[ H'_1 = \frac{\mu}{\sqrt{\epsilon^2 + \epsilon^2}} H_1 + \frac{\epsilon}{\sqrt{\epsilon^2 + \epsilon^2}} L \]
\[ L' = \frac{\epsilon}{\sqrt{\epsilon^2 + \epsilon^2}} H_1 - \frac{\mu}{\sqrt{\epsilon^2 + \epsilon^2}} L , \]

then the Lagrangian expressed in terms of \( H'_1 \) and \( L' \) contains no \( H_2 L' \) term. One could instead dispose of the \( \lambda^i j k E_k^c \) term. The coupling constant combination that is invariant under basis redefinitions in \( H_1, L \) space, zero if \( R \) parity is conserved, and non-zero if it is not is \( \mu \lambda' - h_d \epsilon = (\epsilon, \mu) \wedge (\lambda', h_d) \).

“Basis-independent” parametrisations of \( R \) parity violation have previously been constructed from subsets of the parameters of the \( R \) parity non-conserving Minimal Supersymmetric Standard Model (MSSM). An invariant parametrisation of \( R \) violation due to the misalignment between the neutral vev and the superpotential \( \mu \) term was constructed in [3] and subsequently much discussed [7]. Invariants measuring the \( R \) parity violation between superpotential couplings, including the one discussed in the previous paragraph, were discussed in [6, 8]. The aim of this letter is to construct the “missing” invariants involving soft terms.

The invariants in [6, 8] measure lepton number violation, whatever the lepton flavour; the singlet lepton family index is summed. I will here follow the approach of [6] and build invariants that parametrise lepton number violation in each family. Note that these invariants do not measure lepton flavour violation when lepton number is conserved. Invariants with lepton flavour indices are more numerous, but have the advantage that their relation to \( R \) violating coupling constants (which have lepton flavour indices) is more direct.

In this letter I will first introduce some notation, review the geometric interpretation of the invariants, and then construct invariants parametrising the \( R \) parity violation among the soft terms, and between the soft terms and the superpotential terms. Finally I will briefly discuss the relation
of the invariants introduced here to the one of \( \mathbb{R} \), and calculate cosmological bounds on \( R \) violating soft terms.

The lepton number violating interactions in equations (2) and (3) arise because the Higgs \( H_1 \) has the same gauge quantum numbers as the doublet sleptons \( L_i \). If lepton number is not a symmetry of the Lagrangian, then there is no longer any distinction based on quantum numbers between the Higgs \( H_1 \) and a slepton \( L_i \), or between the higgsino and a doublet lepton. One can therefore assemble the superfields \( H_1 \) and the \( L_i \) in a four component vector

\[
\phi^I = (L_1, L_2, L_3, H_1) \quad , \quad I : 1..4
\]

and rewrite the superpotential as

\[
W = \mu^I \phi_I H_2 + h^{|pq}_u H_2 Q_p U_q^c + \lambda^{Ipq} \phi_I Q_p D_q^c + \lambda_e^{IJk} \phi_I \phi_J E_k^c .
\]

where \( \phi^I \) is now composed of scalar fields. The reason for introducing this notation is that it makes the geometrical significance of the couplings constants clearer. There is a four dimensional vector space spanned by the hypercharge = \( \mu \), and rewrite the superpotential as

\[
\mu^I = (\epsilon_i, \mu), \quad \text{and } \lambda^{Ipq} = (\lambda^{ijpq}, h^{|pq}_d) \text{ are vectors in } \phi^I \text{ space. } \lambda_e^{IJk} \text{ is an anti-symmetric matrix, with }
\]

\[
\lambda_e^{ijk} = h^{|ijk}_{c} , \quad \lambda_e^{ijk} = \lambda^{ij}.k.
\]

The soft terms can be similarly rewritten as

\[
\frac{1}{2} \phi^I m^2_{J,I} \phi^J + B^I H_2 \phi_I + A^{|pq}_u H_2 Q_p U_q^c + A^{Ipq} \phi_I Q_p D_q^c + A_e^{IJk} \phi_I \phi_J E_k^c + h.c .
\]

where \( \phi^I \) is now composed of scalar fields. The reason for introducing this notation is that it makes the geometrical significance of the couplings constants clearer. There is a four dimensional vector space spanned by the hypercharge = \(-2\) doublets (\( H_1 \) and \( L_i \)). \( \mu^I \) and \( (\lambda_d^{pq})^I \) are vectors in this space, and they correspond to directions that would like to be the Higgs—i.e., if one chooses a basis where the Higgs is parallel to \( \mu^I \), then the \( R \) violating masses \( \mu^1, \mu^2, \mu^3 \) are absent.

\[ [\lambda_e^k]^{IJ} \] is a little harder to visualise. It is an antisymmetric \( 4 \times 4 \) matrix, and geometrically corresponds to one or two planes which would like to be spanned by a Higgs and a lepton (see \[ \mathbb{R} \] for a discussion of this). More practically, \[ [\lambda_e^k]^{IJ} \] is a two index object (in \( \phi \) space) that, when contracted with a Higgs, becomes a vector corresponding to a lepton. For instance, if the Higgs direction is

\[ \hat{H}^I \propto \mu^I \]

then the lepton directions can be taken to be

\[ (\hat{L}_k)^J \propto \mu_I [\lambda_e^k]^{IJ} \]

(\( \hat{L}_k \) with a hat is a basis vector, not necessarily of unit length. \( L_k \) without a hat is a quantum number, or sometimes a superfield or scalar field.) \[ [\lambda_e^k]^{IJ} \] is anti-symmetric, so \( \mu^I \) is automatically perpendicular in \( \phi \) space to \( (\hat{L}_k)^J \). If the singlet lepton basis is chosen such that \( \hat{L}_k \cdot \hat{L}_m \propto \mu_I [\lambda_e^k]^{IJ}[\lambda_e^m]^{JK} \mu^K \propto \delta^{km} \) (in the absence of \( R \) violation, this would be \( h_e^{cij} h_e^{mj} \propto \delta^{km} \)), then the lepton directions in \( \phi \) space are also orthogonal. Note that transformations amongst the singlets \( E^c_k \) rotate the \[ [\lambda_e^k]^{IJ} \] into each other on their singlet index \( k \).

I have now constructed a geometrically motivated orthogonal basis in \( \phi \) space using a subset of coupling constants from the Lagrangian. There is \( R \) violation if this basis conflicts with the one chosen by a different collection of coupling constants. For instance, there are nine \( (\lambda_d^{pq})^I \) vectors which could be chosen as the Higgs direction. If they have components in the directions labelled as leptons by equation (10), then \( R \) is not conserved, and the scalar quantity that parametrises \( R \) violation in the \( k \)th lepton flavour is

\[
\sqrt{\frac{\delta^{kpq}}{\mu^{kq}}} \hat{L}_k \cdot \frac{\lambda_d^{pq}}{|\lambda_d^p|} = \frac{\mu_I [\lambda_e^k]^{IJ} \lambda_d^{pq}}{|\mu||\lambda_e^k||\lambda_d^p|}. 
\]

See figure \[ \mathbb{R} \]. The norms are defined in the obvious way; see \[ \mathbb{R} \]. The first reason for dividing by the magnitude of \( \lambda_d^{pq} \) is to avoid privileging \( \lambda_d^{pq} \) over \( \mu \). Both are equally good choices for the Higgs.
direction, and this invariant just parametrises the difference between the two. The second reason is to give a normalised measure of how much $R$ violation is present; (11) ranges from 0, when $\mu$ and $\lambda_{pq}^d$ are parallel, to 1 when they are orthogonal in the plane of $\lambda_e$.

One can square the invariant (11), in which case the coupling constant combination corresponds to a closed supergraph; see [8]. One can also use two $\lambda_k^e$ matrices (geometrically planes) to define a Higgs direction (the intersection of the planes) and two leptons (the orthogonal directions within the planes); in this case only the squared invariant can be constructed, so it is arguably better to choose the square of (11) as the invariant. A more complete discussion of this geometry and the invariants that can be constructed from the superpotential can be found in [8, 9].

There are more invariants than $R$ violating coupling constants, because the invariants are not all independent. To see this, consider a model where there are (at least) three different definitions of the Higgs: $H$, $H'$ and $H''$. There are invariants corresponding to the components of the vectors $H - H'$, $H' - H''$ and $H - H''$, but since one of these vectors can be written as the sum of the two others, some of the invariants can be expressed in terms of the others. The counting of independent invariants constructed out of real coupling constants is discussed in [8]; there and here, I neglect possible CP violating phases. See [11] for a counting of the number of free parameters, including phases, in the renormalisable Lagrangian of the MSSM with baryon parity imposed.

I would like to extend this geometric construction to include the soft breaking terms. For the $B$ and $A$ terms, this is straightforward, because one can build the same invariants as was done with the $\mu$ and $\lambda$ terms in the superpotential [9]. There are many possibilities. A minimal set, in terms of which the rest can be expressed, and which is complete in the sense that if all the invariants are zero, then there is no $R$ violation among the $A$ and $B$ terms, could be the 27 invariants that measure $L_\ell$
violation between \( B \) and the \( A_d \):

\[
\tilde{\delta}_{B_A}^{pq} = \frac{|B^{I*} A_e^{JKL} A_d^{Jpq*}|^2}{|B^2| |A_e^{KL} A_d^{pq*}|^2} \sim \left| \hat{L}^I \cdot \frac{A_d^{pq}}{|A_d^{pq}|} \right|^2
\]  

(12)

and the nine invariants that parametrise \( L_j \) violation in \( A_e^c \):

\[
\tilde{\delta}_{B_A}^{I} = \frac{B^{I*} A_e^{JK} A_e^{LM} A_d^{I*} B_M - 5B^{I*} A_e^{JK} B^{KL} T_R [A_e^{I*} A_d^{Jpq}]}{|B^2 T_R [A_e^{I*} A_d^{Jpq}]} \sim \left| \hat{L}^I \cdot \frac{A_d^{pq}}{|A_d^{pq}|} \right|^2
\]  

(13)

where I am using a definition of lepton based on the soft terms: \( \hat{L}^k \propto A_e^c \cdot B \). There are 36 independent invariants constructed out of superpotential couplings, and the 36 invariants of similar form constructed out of \( B \) and \( A \) terms listed above. To form a complete set, one needs invariants associated with possible lepton number violating soft masses mixing \( H_1 \) with the sleptons, and one needs invariants to measure \( R \) violation between the superpotential and the soft breaking terms.

Consider first these second invariants. In principle it could be possible for the superpotential couplings to make a unique choice of the Higgs, and for the \( B \) and \( A \) terms to do the same, but for these two directions to be different. It is sufficient to have three invariants, for instance

\[
\tilde{\delta}_{B\mu}^\ell = \frac{|B^{I*} \mu^{J*} \mu^{J*}|^2}{|B^2| |\lambda_e^{JKL} |^2 |\mu|^2} \sim \left| \hat{L}^I \cdot B \right|^2
\]  

(14)

where \( \ell : 1..3 \). In the basis of equations (1) and (11), these measure the projection of \( B \) along the three lepton directions.

Finally we need invariants that parametrise lepton number violation in the soft mass matrix for the sleptons and Higgs \( H_1 \): \( \phi^I \mu^2_m \phi^J \cdot m^2_m \phi^J \) is a hermitian matrix; the \( R \) violating couplings are the off-diagonal elements that mix \( H_1 \) with the sleptons. If I identify the Higgs direction as \( \mu^I \), and the sleptons as \( (\hat{L}_k)^J \propto \mu^I \lambda_e^{JK} \), then the fractional amount of \( R \) violation in lepton family \( k \) is

\[
\tilde{\delta}_{\mu m^2}^k = \frac{|\mu^I \lambda_e^{JK} m^2_m \mu^2_K|}{|\mu|^2 |\lambda_e^{JK} |^2 |m^2|^2} \sim \left| \hat{L}_k \cdot m^2 \cdot \hat{H} \right|^2
\]  

(15)

where \( k : 1..3 \).

Note that these invariants measure lepton number violation in each family (using the definition of family of (11)), but not lepton flavour violation where lepton number is conserved. The latter would be proportional to \( \hat{L}_k \cdot m^2 \cdot \hat{L}_i \).

Equations (15), (14), (12), and (13), plus equations (12) and (13) with \( B \) replaced by \( \mu \) and \( A \) by \( \lambda \) should form a complete set of real invariants parametrising \( R \) violation in the Lagrangian. There are 78 of them. This is as expected; there are naively 81 new coupling constants in equations (1) and (11), and three of them (commonly the \( \epsilon_i H_2 L_i \)) can be removed when choosing the Higgs direction in \( \phi \) space. Of course, all these new coupling constants can have phases, so the number of new parameters is closer to \( 2 \times 78 \) (see (1) for an exact count).

The list of invariants that I have constructed does not include the wedge product of \( \mu \) and the vev of \( \phi \), which was introduced in (3). If the direction in \( \phi \) space that gets a vev (\( \equiv v^I \)) is not parallel to \( \mu^I \), i.e. \( \mu \wedge v \neq 0 \), then the neutrino gets a mass (proportional to \( \mu \wedge v \)). This invariant is phenomenologically relevant, so I would like to check that it can be expressed in terms of the invariants listed here.

The wedge product between \( \mu \) and the vev measures the sum of lepton number violation in all generations. Since the other invariants I have discussed measure \( R \) violation in a specific lepton generation, it is useful to have a flavour dependent version of the invariant of (1):

\[
\frac{\mu \cdot \lambda_e^I \cdot v}{|\mu||\lambda_e^I||v|}
\]  

(16)
Assuming that the neutral vev does not break $R$ spontaneously, it can only be misaligned with $\mu^I$ if some coupling constants in the scalar potential are misaligned with $\mu^I$. $R$ violation between $\mu^I$, $B^I$ and $m^2_{I,J}$ is parametrised by equations (14) and (15) so there should be some relation to $\mu \wedge v$. This is straightforward to solve if only one sneutrino gets a vev.

The tree level potential for the vevs of $H_2$ and $\phi^I$, respectively $\eta$ and $v^I$, is

\[
V(v^I, \eta) = \frac{1}{2} m_2^2 \eta^2 + B^I v_I \eta + \frac{1}{2} (m_{I,J}^2 + \mu_I \mu_J) v^I v^J + \frac{g^2 + g^{'2}}{32} (\eta^2 - v^2)^2. \tag{17}
\]

This is minimised when

\[
\left[ \frac{g^2 + g^{'}^2}{8} (\eta^2 - v^2) - m_2^2 \right] \eta = -B^I v^I \tag{18}
\]

and

\[
\left[ \frac{g^2 + g^{'}^2}{8} (\eta^2 - v^2) - (m_{I,J}^2 + \mu_I \mu_J) \right] v^J = \eta B^J. \tag{19}
\]

Taking the inner product of (19) with $\mu \cdot \lambda^k_e$, dropping indices, and assuming that the vev only overlaps with the $k$th lepton generation, gives

\[
\frac{\mu \cdot \lambda^k_e \cdot v}{|\mu||\lambda^k_e||v|} = \frac{\alpha \beta \pm \gamma \sqrt{(\alpha^2 + \gamma^2)\rho - \beta^2}}{\alpha^2 + \gamma^2}, \tag{20}
\]

where

\[
\alpha = \frac{g^2 + g^{'2}}{8} (\eta^2 - v^2) - \frac{\mu \cdot \lambda^k_e m^2 \lambda^k_e \cdot \mu}{|\lambda^k_e \cdot \mu|^2}, \tag{21}
\]

\[
\rho = \frac{|\mu \cdot \lambda^k_e|^2}{|\mu|^2 |\lambda^k_e|^2}, \tag{22}
\]

and $\beta$ and $\gamma$ parametrise $R$ violation:

\[
\beta^2 = \frac{\eta^2}{|v|^2} \delta_{B\mu}^k B^2, \tag{23}
\]

\[
\gamma^2 = \frac{|\mu|^2 |\lambda^k_e|^2}{|\mu \cdot \lambda^k_e|^2} \delta_{\mu n}^k |m^2|^2. \tag{24}
\]

The obvious question to ask, once these invariants are constructed is “what are they good for?”. In principle, they clarify how $R$ parity violation can be moved around the Lagrangian. In practise, they are messy and their relevance to phenomenology is unclear because $SU(2)$ is spontaneously broken; the invariants respect the $SU(2) \times U(1)$ gauge symmetry of the Lagrangian, but the propagating mass eigenstates do not.

Invariants can be useful in calculating cosmological bounds on $R$ violation [10], because the thermal mass eigenstate basis above the electroweak phase transition is not the same as the zero temperature one. One must therefore take some care in identifying which interactions violate $R$, or work in a basis independent formalism. This is discussed in [3] for the $L$ violating superpotential couplings. Including soft breaking interactions changes this analysis slightly, because there are two additional mass terms ($B$ and $m^2$) that would like to choose a direction in $\phi$ space for the Higgs. I will briefly discuss the modifications here. Bounds on $A$ terms and trilinears were discussed in [10].

The cosmological bounds on $L$ violation arise in models where the observed baryon asymmetry was generated before the electroweak phase transition. For the asymmetry to survive in the presence of $B + L$ violating non-perturbative electroweak effects, it must be an asymmetry at least one of the $B/3 - L_i$. Therefore interactions violating at least one of the $B/3 - L_i$ must be out of equilibrium just above the electroweak phase transition. This gives a bound on the $B$ violating trilinear $\lambda''$ couplings, and on the lepton number violating couplings in one generation.
The reason naive rate estimates can give the wrong bound on $L$ violating couplings can be understood in the one generation exactly supersymmetric toy model where lepton number violation can be rotated between the $\epsilon LH_2$ term and the $\lambda' \lambda QD^c$ term. If one puts the $L$ violation in the mass term, the rate for $L$ violation is $\Gamma \sim \epsilon^2/T$. However if one rotates $\epsilon$ away, one generates a a trilinear of order $\lambda' \sim h_d \times \epsilon/\mu$, for which the rate should be of order $\Gamma \sim \lambda'^2 T$. Requiring the first of these rates to be out of equilibrium gives $\epsilon < 10 \, \text{keV}$, requiring the second to be out of equilibrium gives $\epsilon/\mu < 10^{-5}$. For $\mu \sim 100 \, \text{GeV}$, these are not the same. The point is that the mass interaction is stronger, so it determines what is the Higgs, and lepton number violation is in the trilinear, with the basis independent magnitude $y = \sqrt{|\delta_{\mu\lambda d}| h_d^2 + \lambda'^2}$. $\delta_{\mu\lambda d}$ is from equation (11) for quark and lepton generation. The $R$ violating rate at temperatures above the electroweak phase transition is therefore of order $\Gamma_y \sim 10^{-2} y^2 T$ where the $10^{-2}$ counts for various numerical factors [1]. The constraint $\Gamma_y < H$, where $H \sim 10 T^2/m_\mu$ is the expansion rate of the Universe, gives

$$\sqrt{|\delta_{\mu\lambda d}| \lambda_d} < 10^{-7}. \quad (25)$$

For $|\lambda_d| \sim m_b/(\sqrt{2} v)$, this gives $\delta_{\mu\lambda d} < 10^{-11}$.

Now consider the case when there are three possible mass terms that can determine what is the Higgs: $\mu$, $B$, and $m_{1J}^2$. Above the electroweak phase transition, assuming $\mu^2 \sim B \sim \lambda^2 \sim T^2$, one can estimate an interaction rate for the $\mu \phi^I H_2$ from mass corrections to a gauge boson-fermion-fermion vertex. This gives

$$\Gamma_\mu \sim 10^{-2} |\mu|^2 \frac{T}{T} \quad (26)$$

Similarly, one can estimate rates for $B$ and $m_{1J}^2$ from the decay of a scalar boson to two fermions to be of order

$$\Gamma_B \sim 10^{-2} |B|^2 \frac{T}{T}, \quad \Gamma_{m^2} \sim 10^{-2} m^2 \frac{T}{T} \quad (27)$$

where $m^2$ is some eigenvalue of $m_{1J}^2$, and the $B$ rate is in the basis where $B^\ell$ is the direction of the Higgs. Each of these rates is calculated as if the others were absent, and none of them alone violates lepton number. However, if they disagree on what direction is the Higgs, then the lepton number violating rates can be estimated as in the toy model discussed above.

Suppose, for instance, that $\delta_{B\mu}^\ell$ [14] is non-zero, and $|B|$ is slightly larger than $|\mu|^2$. Then $B$ determines what is the Higgs, and the cosmological bound implies $\delta_{B\mu}^\ell \Gamma_\mu < H$. Since $\Gamma_\mu \sim \Gamma_B$, the bound from requiring $\delta_{B\mu}^\ell \Gamma_B < H$ is approximately the same. A similar argument can be applied to $\Gamma_B, \Gamma_{m^2}$ and $\delta_{Bm^2}^\ell$ giving the bounds

$$\delta_{B\mu}^\ell, \delta_{Bm^2}^\ell < 10^{-14} \quad (28)$$

for one given lepton generation index $\ell$. Notes that these bounds are more stringent than (25); the cosmological bounds require that the mass terms be more aligned with each other than with the trilinears.

In the $R$ violating MSSM, lepton number may not be conserved, in which case there is no distinction between the down-type Higgs doublet $H_1$ and doublet leptons $L_i$. These fields can be rotated into each other, changing the definition of lepton number and the magnitude of $L$ violating coupling constants. In [8, 9], we constructed basis independent combinations of coupling constants from the MSSM superpotential that parametrised $L$ violation in the renormalisable interactions. In this letter, I have completed this process, constructing “invariants” that parametrise $R$ violation in the soft terms, and between the soft terms and the superpotential. A particular invariant is zero if there is no lepton number violation among the coupling constants used to construct it. If all the invariants are zero, there is no $L$ violation in the Lagrangian. The invariants are built by using a minimal subset of the Lagrangian to define lepton number, and then seeing if the remaining interactions conserve this definition of $L_k$. 


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