Wormholes respecting energy conditions and solitonic shells in DGP gravity

Martín G. Richarte

Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina

We build spherically symmetric wormholes within the DGP theory. We calculate the energy localized on the shell, and we find that for certain values of the parameters wormholes could be supported by matter not violating the energy conditions. We also show that it could exist solitonic shells characterized by zero pressure and zero energy; thereafter we make some observations regarding their dynamic on the phase plane.

I. INTRODUCTION

Traversable Lorentzian wormholes [1, 2] are topologically non trivial solutions of the equations of gravity which would imply a connection between two regions of the same universe, or of two universes, by a traversable throat. In the case that such geometries actually exist they could show some interesting peculiarities as, for example, the possibility of using them for time travel [3, 4]. A basic difficulty with wormholes is that the flare-out condition [5] to be satisfied at the throat requires the presence of matter which violates the energy conditions (“exotic matter”) [1, 2, 5, 6]. It was recently shown [7], however, that the amount of exotic matter necessary for supporting a wormhole geometry can be made infinitesimally small. Thus, in subsequent works special attention has been devoted to quantifying the amount of exotic matter respecting the energy conditions [1, 2, 5, 6], and this measure of the exoticity has been pointed as an indicator of the physical viability of a traversable wormhole [10].

A central aspect of any solution of the equations of gravitation is its mechanical stability. The stability of wormholes has been thoroughly studied for the case of small perturbations preserving the original symmetry of the configurations. In particular, Poisson and Visser [11] developed a straightforward approach for analyzing this aspect for thin-shell wormholes, that is, those which are mathematically constructed by cutting and pasting two manifolds to obtain a new manifold [12]. In these wormholes the associated supporting matter is located on a shell placed at the joining surface; so the theoretical tools for treating them is the Darmois–Israel formalism, which leads to the Lanczos equations [13, 14]. The solution of the Lanczos equations gives the dynamical evolution of the wormhole once an equation of state for the matter on the shell is provided. Such a procedure has been subsequently followed to study the stability of more general spherically symmetric configurations (see, for example, Refs. [13]).

Wormholes in theories beyond Einstein framework have gained a lot of interest in the last years because they seem to possess some curious properties regarding the kind of matter which could support them. A few examples of these alternatives theories are the Einstein–Gauss–Bonnet picture [16–18], scalar-tensor theories [19, 20], $F(R)$-theory or massive gravity [21]. In particular, for the Einstein–Gauss–Bonnet theory, it was shown that static thin-shell wormholes could be supported by ordinary matter respecting the energy conditions [16]. Moreover, $C^2$-type wormholes with the latter property can also exist once the nonlinear Gauss-Bonnet term is included in the field equations [17]. Of course, this feature is not only exclusive of the Gauss-Bonnet paradigms; being the Brans-Dicke gravity another set up where the thin-shell wormholes fulfill weak and null energy conditions [19].

In addition, a new type of gravitational model was widely studied in the context of cosmology as well as particle physics, the so called Dvali, Gabadadze and Porrati (DGP) theory. It predicts deviations from the standard 4D gravity over large distances. The transition between four and higher-dimensional gravitational potentials in the DGP model arises because of the presence of both the brane and bulk Hilbert–Einstein (H–E) terms in the action [22]. Cosmological considerations of the DGP model were first discussed in [23, 24] where it was shown that in a Minkowski bulk spacetime we can obtain self-accelerating solutions. In the original DGP model it is known that 4D general relativity is not recovered at linearized level. However, some authors have shown that at short distances we can recover the 4D general relativity in a spherically symmetric configuration (see for example [27]).

It is worth mentioning that an interesting feature of the original DGP model is the existence of ghost-like excitations [26, 28]. Further, the viability of the self-accelerating cosmological solution in the DGP gravity was carefully studied in [27]. For a comprehensive review of the existence of 4D ghosts on the self-accelerating branch of solutions in DGP models see [29].

A common feature among alternative theories is that the junction conditions for the thin-shell wormholes are modified considerably, adding new types of geometrical

---

[1] Electronic address: martin@df.uba.ar

[2] PACS numbers:

Keywords:
objects besides the usual extrinsic curvature. The contributions form the curvature tensors, theoretically, seem to allow the existence of wormholes supported by ordinary matter. For all these reasons, we consider that the construction of wormholes within DGP gravity deserves to be examined in detail to conclude whether or not they could fulfill the energy conditions.

In this work we explore thin-shell wormholes within the DGP gravity theory. Our research is focus on configurations supporting by non-exotic matter which satisfy the energy conditions. Then, we show the existence of solitonic vacuum shells and make some comment about their dynamic.

II. FIVE-DIMENSIONAL BULK SOLUTION

We start from the action for the DGP theory in five-dimensional manifold $\mathcal{M}_5$ with four-dimensional boundary $\partial \mathcal{M}_5 = \Sigma$ (cf. [28]),

$$S = 2M_5^2 \int_{\mathcal{M}_5} d^5 x \sqrt{-g} R(g_{\mu \nu}) + \int_{\Sigma} d^4 x \sqrt{-\gamma} 2M_4^2 R(\gamma_{ab})$$

$$+ \int_{\Sigma} d^4 x \sqrt{-\gamma} \left( -4M_3^2 K(\gamma_{ab}) + L_m \right),$$

where $g_{\mu \nu}$ is the five-dimensional metric, $\gamma_{ab}$ is the four-dimensional induced metric on the boundary $\Sigma$, and $K$ is the trace of extrinsic curvature. The extra term in the boundary introduces a mass scale $m_c = 2M_5^2/M_4^2 = r_c^{-1}$, that is, the model has one adjustable parameter, namely $m_c$ which determines a scale that separates two different regimes of the theory. For distances much smaller than $m_c^{-1}$ one would expect the solutions to be well approximated by General Relativity and the modifications to appear at larger distances. This is indeed the case for distributions of matter and radiation which are homogeneous and isotropic at scales $\gtrsim r_c$. Typically, $m_c \sim 10.42\text{GeV}$, so it sets the distance/time scale $r_c = m_c^{-1}$ at which the Newtonian potential significantly deviates from the conventional one [35].

It is a well known fact that the DGP scheme is a five-dimensional model where gravity propagates throughout an infinite bulk, and matter fields in $\mathcal{L}_m$ are confined to a 4-dimensional boundary. The action for gravity at lowest order in the derivate expansion is a bulk Einstein-Hilbert term and a boundary one, generically with two different planck masses $M_5, M_4$, plus a suitable Gibbons-Hawking term. In the bulk the DGP equations are the Einstein ones in vacuum: $G^{(5)}_{\mu \nu} = 0$. Then in this case, the Birkhoff’s theorem forces the bulk metric to be static, and of the Schwarzschild form:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_3^2, \quad (1)$$

$$f(r) = 1 - \frac{\mu}{r^2} \quad (2)$$

where the parameter $\mu$ is related to the five dimensional Arnowitt–Deser–Misner (ADM) mass, $M_{ADM} = 3\pi^2 \mu M_4^2$. The above spacetime has only one horizon placed at $r_+ = \sqrt{\mu}$ with $\mu > 0$. Besides, when $\mu < 0$ the manifold only presents a naked singularity at the origin $r = 0$.

III. WORMHOLES IN DGP

A. Thin-shell construction

Employing the metric Eqs. (1) we build a spherically thin-shell wormhole in DGP theory. We take two copies of the spacetime and remove from each manifold the five-dimensional regions described by

$$\mathcal{M}_\pm = \{ x/r_\pm \leq a, a > r_b \}. \quad (3)$$

The resulting manifolds have boundaries given by the timelike hypersurfaces

$$\Sigma_\pm = \{ x/r_\pm = a, a > r_b \}. \quad (4)$$

Then we identify these two timelike hypersurfaces to obtain a geodesically complete new manifold $\mathcal{M} = \mathcal{M}_+ \cup \mathcal{M}_-$. We take values of $a$ large enough to avoid the presence of singularities and horizons in the case that the geometry [2] has any of them. The manifold $\mathcal{M}$ represents a wormhole with a throat placed at the surface $r = a$, where the matter supporting the configuration is located. This manifold is constituted by two regions which are asymptotically flat (see Fig. 1). The wormholes throat $\Sigma$ is a synchronous timelike hypersurface, where we define locally a chart with coordinates $\xi^a = (\tau, \chi, \theta, \phi)$, with $\tau$ the proper time on the shell. Though we shall first focus in static configurations, in the subsequent we could allow the radius of the throat be a function of the proper time for studying the dynamics evolution of the wormholes, then in general we have that the boundary hypersurface reads:

$$\Sigma : \mathcal{H}(r, \tau) = r - a(\tau) = 0. \quad (5)$$

It is important to remark that the geometry remains static outside the throat, regardless the radius $a(\tau)$ can vary with time, so no gravitational waves are present. This is naturally guaranteed because the Birkhoff theorem holds for the original manifold.

Our starting point is to list the main geometric objects which shall appear in the junction condition associated with the field equation for $\Sigma$. The extrinsic curvature, namely $K_{ab}$, associated with the two sides of the shell are defined as follows:

$$K_{ab}^\pm = -n^\pm_{\kappa} \left( \frac{\partial^2 X^\kappa}{\partial \xi^a \partial \xi^b} + \Gamma^\kappa_{\mu \nu} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b} \right)_{\tau = a}, \quad (6)$$
\[ n^\pm_\kappa = \pm \left| g^{\mu\nu} \frac{\partial \mathcal{H}}{\partial x^\mu} \frac{\partial \mathcal{H}}{\partial x^\nu} \right| \frac{\partial \mathcal{H}}{\partial \Sigma^\kappa} \]  

The field equations projected on the shell \( \Sigma \) are the generalized junction (or Darmois–Israel) conditions \[ r_c \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) - 2 \left( \mathcal{K}_{ab} - \mathcal{K} \gamma_{ab} \right) = \frac{\mathcal{S}_{ab}}{8M_5^3}, \]  

where the bracket (,) stands for the jump of a given quantity across the hypersurface \( \Sigma \) and \( \gamma_{ab} \) is the induced metric on \( \Sigma \). Notice that the first term in [5] is not enclosed with the brackets because this contribution comes from the four dimensional E-H term in the DGP action [11] which already lives in the boundary so it does not need to be projected on \( \Sigma \). By taking the limit \( r_c \to 0 \) we recover the standard Darmois–Israel junction condition found in [13].

Now, let us calculate some quantities that we shall need later. The mixed components of the four-dimensional Einstein tensor are given by

\[ G^0_0 = -3 \left( \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} \right), \]
\[ G^i_j = -\left( \frac{1}{a^2} + \frac{\ddot{a}}{a^2} + \frac{\dot{a}}{a} \right) \delta^i_j. \]  

where dot means derivate with respect to the proper time on \( \Sigma \). The extrinsic curvature components read

\[ \langle \kappa^0_0 \rangle = \frac{2\dot{a} + f'(a)}{\sqrt{f(a) + \dot{a}^2}}, \]
\[ \langle \kappa^i_j \rangle = \frac{2}{a} \sqrt{f(a) + \dot{a}^2} \delta^i_j \]  

where the prime indicates the derivates with respect to \( a \). The most general form of the stress energy tensor on shell compatible with the simetries is

\[ S^a_b = \text{diag} (-\sigma, p \delta^2_j) \]  

where \( \sigma \) is the energy density and \( p \) is the pressure. Replacing Eqs. [11],[13] into the DGP junction condition [5] we obtain that the energy density and the pressure can be recast as

\[ \frac{\sigma}{8M_5^3} = 3r_c \left( \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} \right) - \frac{12}{a} \sqrt{f(a) + \dot{a}^2}, \]
\[ \frac{p}{8M_5^3} = -r_c \left( \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} + \frac{2\dot{a}}{a} \right) + \frac{8}{a} \sqrt{f(a) + \dot{a}^2} \]  

where the DGP contributions are encoded in the \( r_c \) factor of the above equations. If we take \( r_c \to 0 \) in both equations [11] and [13] we recover the expression for the energy density \( \sigma \) and the pressure \( p \) found in [16]; ignoring the Gauss-Bonnet contribution.

In order to carry on let us comment that we still have the usual energy conservation, \( \nabla_\mu S^{ab} = 0 \) by virtue of \( \nabla^a (\mathcal{K}_{ab} - \gamma_{ab} \mathcal{K}) = 0 \), coming from the momentum constraint implicit in the five-dimensional Einstein equations. Further it is easy to see from \( \sigma \) and \( p \) that the energy conservation equation is fulfilled:

\[ \frac{d(a^3 \sigma)}{d\tau} + p \frac{d(a^3)}{d\tau} = 0, \]

the first term in Eq. [17] represents the internal energy change of the shell and the second the work by internal forces of the shell. The dynamical evolution of the wormhole throat is governed by the generalized Lanczos equations and to close the system we must supply an equation of state \( p = p(\sigma) \) that relates \( p \) and \( \sigma \). Notice that the reason why one obtains exact conservation, i.e., no energy flow to the bulk, is that the normal-tangential components of the stress tensor in the bulk is the same on both side of the junction hypersurface.

\[ \text{IV. MATTER SUPPORTING THE WORMHOLES} \]

Recently, classical solutions within the DGP model were found when the stress-energy tensor on the brane satisfies the dominant energy condition, yet the brane has
negative energy from the bulk point of view (see 28). Within this frame, the study of superluminal propagation indicates that superluminozosity occurs whenever the stress tensor on the shell is a pure cosmological constant, irrespective of the value of the shell density (cf. 28). All these elements are good reasons to consider a careful discussion about the nature of matter supporting wormholes in the DGP model. Moreover, motivated by the results within Einstein-GaussBonnet gravity (i.e. with R2-like terms) in 19, here we evaluate the amount of exotic matter and the energy conditions, following the approach presented above where the four-dimensional H--E term generalizes the standard junction, adding a few geometrical terms, which indeed represents the Einstein tensor projected on the shell. Consequently, coming the DGP contribution from the curvature tensor, the next approach is clearly the most suitable to give a precise meaning to the characterization of matter supporting the wormhole.

The weak energy condition (WEC) states that for any timelike vector \( U^\mu \) it must be \( T_{\mu\nu} U^\mu U^\nu \geq 0 \); the WEC also implies, by continuity, the null energy condition (NEC), which means that for any null vector \( k^\mu \) it must be \( T_{\mu\nu} k^\mu k^\nu \geq 0 \) [3]. In an orthonormal basis the WEC reads \( \rho \geq 0 \), \( \rho+p_\ell \geq 0 \) \( \forall \ell \) while the NEC takes the form \( \rho+p_\ell \geq 0 \) \( \forall \ell \). Besides, the strong energy condition states that \( \rho \geq 0 \) \( \forall \ell \), and \( \rho + 3p_\ell \geq 0 \) \( \forall \ell \).

In the case of thin-shell wormholes the radial pressure \( p_r \) is zero, within Einstein gravity, the surface energy density must fulfill \( \sigma < 0 \), so that both energy conditions would be violated. The sign of \( \sigma + p_\ell \), where \( p_\ell \) is the transverse pressure is not fixed, but it depends on the values of the parameters of the system. In what follows we restrict to static configurations. The surface energy density \( \sigma_0 \) and the transverse pressure \( p_0 \) for a static configuration (\( a = a_0, \dot{a} = \ddot{a} = 0 \)) are given by

\[
\frac{\sigma_0}{8M_5^2} = \frac{3r_c}{a_0} \frac{12}{a_0} \sqrt{f(a_0)},
\]

\[
\frac{p_0}{8M_5^2} = -\frac{r_c}{a_0} + \frac{8}{a_0} \sqrt{f(a_0)} + 2 \frac{f'(a_0)}{\sqrt{f(a_0)}}.
\]

Now the sign of the surface energy density as well as the pressure is, in principle, not fixed. The most usual choice for quantifying the amount of exotic matter in a Lorentzian wormhole is the integral 3:

\[
\Omega = \int (\rho + p_r) \sqrt{-g_5} \, d^4x.
\]

We can introduce a new radial coordinate \( R = \pm(r - a_0) \) with \( \pm \) corresponding to each side of the shell. Then, because in our construction the energy density is located on the surface, we can also write \( \rho = \delta(R) \sigma_0 \), and because the shell does not exert radial pressure the amount of exotic matter reads

\[
\Omega = \int \int \int \int \delta(R) \sigma_0 \sqrt{-g_5} \, d\xi \, d\theta \, d\phi = 2\pi^2 a_0^3 \sigma_0.
\]

Replacing the explicit form of \( \sigma_0 \) and \( g_5 \), we obtain the exotic matter amount as a function of the parameters that characterize the configurations:

\[
\Omega = 16M_5^2 \pi^2 \left( 3r_c a_0 - 12a_0^2 \sqrt{f(a_0)} \right).
\]

where \( f \) is given by the bulk solution. For \( r_c \to 0 \) we obtain the exotic amount for Schwarzschild geometries as if it was calculated with the standard junction conditions. Far away from the General Relativity limit we now find that there exist positive contributions to \( \sigma_0 \); these come from the different signs in the expression for the surface energy density, because is proportional to \( \sigma_0 \). We stress that this would not be possible if the standard Darmois-Israel formalism was applied, treating the DGP contribution as an effective energy-momentum tensor, because this leads to \( \sigma_0 \propto -\sqrt{f(a_0)/a_0} \). Now, once the explicit form of the function \( f(a_0) \) is introduced in Eq. (22), we focus on what are the conditions that lead to wormholes with \( \sigma_0 > 0 \) or \( \Omega > 0 \). Then, it can be proved that wormholes with a non-negative surface density located at the shell are allowable when the following inequalities are simultaneously satisfy:

\[
\frac{r_c}{a_0} - \frac{4}{a_0} \left( \frac{\mu}{a_0} \right) \frac{1}{\sqrt{f(a_0)}} > 0,
\]

\[
\frac{a_0^2}{\mu} > 0.
\]

so it is always possible to choose \( a_0 \) such that the existence of thin-shell wormholes is compatible with positive surface energy density (see Fig.2.), more precisely its radius must belong to the interval given below:

\[
\sqrt{\mu} < a_0 \leq \left( \mu + \frac{r_c^2}{16} \right)^{1/2}
\]

Notice that \( r_c \)-term is essential to have positive energy density; as one would expect, in the limit \( r_c \to 0 \), this possibility completely vanishes. Besides, from Eq. (18) and Eq. (19) we have that the sum of the pressure and energy density takes the form

\[
\sigma_0 + p_0 = 8M_5^3 \left( \frac{2r_c}{a_0} + \frac{2a_0 f'(a_0) - 4f(a_0)}{a_0 \sqrt{f(a_0)}} \right)
\]

because the first term in (26) is positive the sign of \( \sigma_0 + p_0 \) depends on the second term, implying that the sum is positive for \( \sqrt{\mu} < a_0 \leq \frac{2\sqrt{\mu}}{\sqrt{\mu}} \). Therefore, the remarkable result is that we have a region with \( \sigma_0 \geq 0 \) and besides \( \sigma_0 + p_0 \geq 0 \), so the WEC and the NEC are satisfied (see Fig.3 and Fig.4). Additionally, it is easy to corroborate that \( \sigma_0 + 3p_0 = 12 \times 8M_5^3/(a_0 \sqrt{f(a_0)}) \), then SEC holds
in the interval \( a_0 \in (\sqrt{\mu}, \sqrt{2\mu}) \) (see Fig.3 and Fig.4). Thus, by treating the DGP contribution as a geometric object, the generalized junction conditions \(^{18}\) provide a clear meaning to the matter in the shell leading to a central finding that in the DGP gravity the violation of the energy conditions could be avoided and wormholes could be supported by ordinary matter.

![Fig. 2](image2.png)

**FIG. 2:** We plot the zones in the plane \( r_c - a_0 \) where the condition \( \sigma_0 > 0 \) for several values of \( \mu \).

However, note that one could choose another route because Eq.15 can be formally recast as follows

\[
-16M_5^3 (\mathcal{K}_{ab} - \gamma \gamma_{ab}) = S^{ef}_{ab},
\]  
(27)

although this identification is also possible; physically we would be treating curvature objects as an effective source for the junction condition. Moreover, based on effective energy-momentum tensor approach we inevitably would obtain that the energy density is negative definite because the flare-out condition is fulfilled. For a review of junction conditions within the DGP theory see \(^{20}\) and references therein.

![Fig. 3](image3.png)

**FIG. 3:** We show \( \sigma_0, \sigma_0 + p_0 \) and \( \sigma_0 + 3p_0 \) versus the wormhole radius \( a_0 \) for several values of \( (\mu, r_c) \).

![Fig. 4](image4.png)

**FIG. 4:** We show \( \sigma_0, \sigma_0 + p_0 \) and \( \sigma_0 + 3p_0 \) versus the wormhole radius \( a_0 \) for different values of \( (\mu, r_c) \).

\[
S_{ab} - 8M_5^3 r_c \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) = S^{eff}_{ab}
\]  
(28)

V. SOLITONIC WORMHOLES/SHELL

In general to obtain the dynamic picture of the wormholes within the DGP gravity is a very complicated task. As it can see from the Eqs. \(^{14}\) nonlinear character of these expressions make the standard procedure exposed in \(^{32}\) very hard to implement. So, we are going to focus in a particular type of wormholes/shell. To be precise we desire to examine if it is possible to have dynamical solitonic wormholes/shells characterized by a zero pressure \( (p = 0) \) and zero energy density \( (\sigma = 0) \). Unlike the standard Darmois–Israel junction condition, nontrivial solutions may be possible even when \( S^0_a = 0 \). That is, the extrinsic curvature can be discontinuous across the throat with no matter on the shell to serve a source; turning the discontinuity a self-supported gravitational system. Of course, these configurations are impossible in the Einstein gravity but not in the Einstein-Gauss-Bonnet gravity (cf. \(^{18}\)).

For \( \dot{a} \neq 0 \) the Eq.17 shows that if \( \sigma = 0 \) then \( p = 0 \); so we are going to work with the most useful expression which in this case is given by \( \sigma \). Following the procedure mentioned in \(^{20}\) we shall plot trajectories in the phase space spanned by \( (\dot{a}, a) \). Because of the energy constraint
\[ (\sigma = 0) \text{ is invariant under the symmetry } \dot{a} \leftrightarrow -\dot{a} \text{ we can work on a two-dimensional plane which is defined as a non-compact domain, namely } B = (0, +\infty) \times (r_+, +\infty). \]

The curves which represent the dynamics of solitonic wormholes are obtained by imposing the following conditions:

\[ r_c (a^2 + 1) - 4 \left( a^2 - \mu + a^2 \dot{a}^2 \right) = 0, \quad (29) \]

\[ a^2 - \mu > 0 \quad (30) \]

such that the first inequality guarantees zero energy whereas the second one ensures that the wormholes radius is larger than event horizon. According to Fig. 5, the

For all \( \mu \) and \( r_c \) considered in this section we obtain that the kinematic of the shell has four possible types of dynamical evolution. More precisely, the solitonic solution could suffer an accelerated (\( \ddot{a} > 0 \)) or decelerated (\( \ddot{a} < 0 \)) expansion (\( \dot{a} > 0 \)) as well as an accelerated or decelerated contraction (\( \dot{a} < 0 \)) regimes.

Unlike the Einstein–Gauss-Bonnet case studied in \[18\] it turns that the existence of solitonic shells in DGP gravity does not require the presence of a cosmological constant term in the bulk spacetime.

**VI. SUMMARY**

The generalization of Einstein gravity in the way proposed by Dvali, Gabadadze and Porrati (DGP) introduces a new parameter, which allows for more freedom in the framework of determining the most viable wormhole configurations. If wormholes could actually exist, one would be interested in those which are require as little amount of exotic matter as possible. Of course, the case could be that a given change of the theory leads to a worse situation, i.e. that configurations require more matter violating the energy conditions as the departure from the standard theory becomes relevant. However, for suitable wormhole radius, this seems not to be the case with DGP gravity: Here we have examined the “exotic” matter content of thin-shell wormholes using the generalized junction condition, and we have found that for large values of the DGP parameter, corresponding to a situation far away from the General Relativity limit, the amount of exotic matter is reduced in relation with the standard case because it can be positive definite. Moreover, the remarkable result is that we have a region with \( \sigma_0 \geq 0 \) and besides \( \rho_0 + \rho_0 \geq 0 \), so the WEC and the NEC are satisfied. Further the SEC condition holds also. Thus if the requirement of exotic matter is considered as the hardest objection against wormholes, our results suggest that in a physical scenario with small crossover scale \((r_c \sim O(1))\) or far away from the General Relativity limit where the DGP becomes dominant \((r_c \gtrsim 10^2)\) these types of wormholes could be possible. Finally, we showed the existence of gravitational solitonic wormholes/shell characterized by \( \sigma = p = 0 \) within the DGP model. Unlike the case of Einstein–Gauss-Bonnet theory we found that the existence of solitonic shells in DGP gravity does not require the presence of a cosmological constant term in the bulk solution.

**Acknowledgments**

MGR thanks the University of Buenos Aires for partial support under project X044. MGR is also supported by the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET).
[1] M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).
[2] M. Visser, *Lorentzian Wormholes* (AIP Press, New York, 1996).
[3] M. S. Morris, K. S. Thorne and U. Yurtsever, Phys. Rev. Lett. 61, 1446 (1989).
[4] V. P. Frolov and I. D. Novikov, Phys. Rev. D 42, 1057 (1990).
[5] D. Hochberg and M. Visser, Phys. Rev. D 56, 4745 (1997).
[6] D. Hochberg and M. Visser, Phys. Rev. Lett. 81, 746 (1998); D. Hochberg and M. Visser, Phys. Rev. D 58, 044021 (1998).
[7] M. Visser, S. Kar and N. Dadhich, Phys. Rev. Lett. 90, 201102 (2003).
[8] C. Barceló and M. Visser, Int. J. Mod. Phys. D 11, 1553 (2002); T. A. Roman, *Some Thoughts on Energy Conditions and Wormholes*, [arXiv:0409090 [gr-qc]](http://arxiv.org/abs/gr-qc/0409090).
[9] K. K. Nandi, Y.-Z. Zhang, and K. B. Vijay Kumar, Phys. Rev. D 70, 127503 (2004); K.K. Nandi, Y.Z. Zhang, R.G. Cai, A. Panchenko, Phys.Rev.D 79 024011 2009.
[10] K. K. Nandi, Y.-Z. Zhang and N. G. Migranov, J. Nonlinear Phenomena in Complex Systems 9, 61 (2006).
[11] E. F. Eiroa, Phys. Rev. D 52, 7318 (1995).
[12] M. Visser, Phys. Rev. D 39, 3182 (1989); M. Visser, Nucl. Phys. B328, 203 (1989).
[13] N. Sen, Ann. Phys. (Leipzig) 73, 365 (1924); K. Lanczos, *ibid.* 74, 518 (1924); G. Darmois, Mémorial des Sciences Mathématiques, Fascicule XXV ch V (Gauthier-Villars, Paris, 1927); W. Israel, Nuovo Cimento 44B, 1 (1966); 48B, 463(E) (1967).
[14] P. Musgrave and K. Lake, Class. Quantum Grav. 13, 1885 (1996).
[15] E. F. Eiroa and G. E. Romero, Gen. Relativ. Gravit. 36, 651 (2004); F. S. N. Lobo and P. Crawford, Class. Quantum Grav. 21, 391 (2004); M. Thibeault, C. Simeone and E. F. Eiroa, Gen. Relativ. Gravit. 38, 1593 (2006); E. F. Eiroa and C. Simeone, Phys.Rev.D 71 127501 (2005); F. Rahaman, M. Kalam and S. Chakraborty, Gen. Relativ. Gravit. 38, 1687 (2006); M. G. Richarte and C. Simeone, Int. J. Mod. Phys. D17, 1108 (2008); G. Dotti, J. Oliva, R. Troncoso, Phys.Rev.D 75 024002 (2007); A.A. Usmani, F. Rahaman, Sanab Ray, Sk.A. Rakib, Z. Hasan, Peter K.F. Kuhfittig, (e-print: arXiv:1001.1415); F. Rahaman, K A Rahman, Sk.A Rakib, Peter K.F. Kuhfittig, (e-print: arXiv:0909.1071).
[16] M. G. Richarte and C. Simeone Phys. Rev. D 76, 087502 (2007); Erratum-ibid 77, 089903 (2008) (e-Print: arXiv:0710.2041 [gr-qc]).
[17] Hideki Maeda, Masato Nozawa,Phys.Rev.D 78 024005 (2008).
[18] C. Garraffo, G. Giribet, E. Gravanis and S. Willson, J. Math. Phys. 49 (2008) 042503.
[19] E. F. Eiroa, M. G. Richarte, and C. Simeone, Phys.Lett.A 373 1-4 (2008). Erratum-ibid.373:2399-2400,2009, (e-Print: arXiv:0809.1623 [gr-qc]).
[20] K. A. Bronnikov, A. A. Starobinsky, Mod.Phys.Lett.A 24 1559-1564 (2009), (e-print: arXiv:0903.5173 [gr-qc]); E. Ebrahimi, and N. Rüzi, (e-print: arXiv:0905.4116 [hep-th]); F.S.N. Lobo, M. A. Oliveira, (e-print: arXiv:1001.0995 [gr-qc]).
[21] V. P. Frolov and I. D. Novikov, Phys. Rev. Lett. 80 14012 (2003).
[22] E. F. Eiroa, G. Gabadadze, and M. Porrati, Phys.Lett.B 485 208-214 (2000); G. R. Dvali and G. Gabadadze, Phys. Rev. D 63, 065007 (2001) (e-Print: gr-qc/0008054).
[23] C. Deffayet, Phys. Lett. B 502, 199 (2001).
[24] C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D 65, 044023 (2002) (e-Print: astro-ph/0105068).
[25] T. Tanaka, Phys. Rev. D 69 (2004) 024001.
[26] A. Nicolis and R. Rattazzi, JHEP 06 (2004) 059; K. Koyama, Phys. Rev. D 72 123511; K. Izumi, K. Koyama and T. Tanaka, JHEP 0704 (2007) 053.
[27] D. Gorbunov, K. Koyama, and S. Sibiryakov,Phys Rev. D 73 044016 (2006).
[28] M. A. Luty, M. Porrati, and R. Rattazzi, JHEP 0309 029 (2003), (e-Print: hep-th/0303116); K. Hinterbichler, A. Nicolis, and M. Porrati, JHEP 0909 089 (2009), (e-Print: arXiv:0905.2359 [hep-th]).
[29] R. Gregory, N. Kaloper, R. Myers, and A. Padilla, JHEP 0701 060 (2007), (e-Print: arXiv:0701.2666 [hep-th]); C. Charmousis, R. Gregory, N. Kaloper, and A. Padilla, JHEP 0610 066 (2006), (e-Print: hep-th/0604086).
[30] K. Maeda, S. Mizuno and T. Torii, Phys.Rev. D68 (2003) 024033; A. Davidson and I. Gurwich, Phys.Rev. D74 (2006) 044023; A.O. Barvinsky, C. Deffayet and A.Yu. Kamenshchik, (e-print: arXiv:0912.4604).
[31] G. Gabadadze and A. Iglesias Class.Quant.Grav.25 154008 (2008), ( e-Print: arXiv:0712.4086 [hep-th]); G. Gabadadze and A. Iglesias,Phys.Rev.D72084024 (2005), (e-Print: hep-th/0407049); A. Lue, Phys.Rept.423 48 (2006),(e-Print: astro-ph/0510068); A. Lue, R. Scoccimarro and G. D. Starkman, Phys.Rev.D 69 124015 (2004), (e-Print: astro-ph/0401515).
[32] E. F. Eiroa, Phys. Rev. D 78, 024018, (2008).
[33] However, a more careful analysis indicates that when compact static source of the mass M and radius r0 are taken into account, such that r_{M} < r_{0} < r_{c} (r_{c} = 2G\mu_{N} / M_{N} is the Schwarzschild radius) a new scale, combination of r_{c} and r_{M}, emerges (the so-called Vainshtein scale) : r_{*} = (r_{c}^{2}r_{M})^{1/3}. Below this scale the predictions of the theory are in good agreement with the GR results and above it they deviate considerably(cf.[31]).