Nuclear matter from effective quark-quark interaction.

M. Baldo and K. Fukukawa

INFN, Sezione di Catania, via S. Sofia 64, I-95123, Catania, Italy

We study neutron matter and symmetric nuclear matter with the quark-meson model for the two-nucleon interaction. The Bethe-Bruckner-Goldstone many-body theory is used to describe the correlations up to the three hole-line approximation with no extra parameters. At variance with other non-relativistic realistic interactions, the three hole-line contribution turns out to be non-negligible and to have a substantial saturation effect. The saturation point of nuclear matter, the compressibility, the symmetry energy and its slope are within the phenomenological constraints. Since the interaction also reproduces fairly well the properties of three nucleons system, these results indicate that the explicit introduction of the quark degrees of freedom for the construction of the nucleon-nucleon interaction strongly reduces the possible role of three-body forces.

PACS numbers: 21.65.-f, 21.65.Cd, 13.75.Cs, 21.30.-x

Introduction. Understanding the properties of the nuclear medium on the basis of the bare interaction among nucleons is one of the fundamental issues of Nuclear Physics. Several methods have been used to model the nucleon interaction and to develop accurate many-body theory to describe the correlations in nuclear systems. It has been established [1-3] that realistic nucleon-nucleon (NN) potentials based on the meson-exchange interaction fail to reproduce the correct saturation point and require the introduction of three-body forces (TBF). The latter can be phenomenological [4, 5] or more fundamental [6, 7]. In any case, within this framework, the effect of TBF is moderate, but it is essential to shift the saturation point inside the phenomenological boundaries. The main problem of this approach is that it appears difficult to device a TBF that describes satisfactorily well few-body systems and at the same time nuclear matter near saturation [8]. The same conclusion has been reached within the variational many-body method [8]. In the framework of this type of nuclear forces, non-local two-body interactions [10] have been constructed that reproduce closely the binding energy of three and four nuclear systems. However, they fail to reproduce the correct saturation point [11].

The Dirac-Brueckner-Hartree-Fock (DBHF) method introduces relativistic effects in the many-body theory. With only two-body forces and two-body correlations, the saturation point is fairly well reproduced [12], but the problem of the few-body systems remains unsolved. It can also be shown [13] that the relativistic effects introduced by the DBHF scheme are equivalent to a particular TBF at the non-relativistic level. More recently the chiral effective forces have been developed [14, 15]. These forces were devised to connect the underlying Quantum Chromodynamics (QCD) theory of strong interaction among quarks to the low energy interaction among nucleons. However no explicit quark degrees of freedom are introduced, but it is based on the expansion of the interaction in the chiral symmetry breaking parameter, i.e. the \( \pi \)-meson mass \( m_\pi \). As such, it is an expansion, in \( k/m_\pi \), where \( k \) is the momentum. It has the fundamental property to classify the forces according to the expected relevance, following a "power counting" rule. Three-body (or higher) forces arise naturally in this expansion. Even if the assignment of the order to the different interaction processes looks tricky [10], this approach has been developed both at the fundamental level [17, 18] and in a wealth of applications to nuclei [20] and nuclear matter [21-24]. The parameters of the forces are fixed by fitting the NN phase shifts and eventually the properties of three-body system. However the procedure looks not unique. In ref. [25] it has been shown that it is possible to construct a realistic chiral two-body force that reproduces the spectroscopic data on light nuclei without invoking TBF. However symmetric nuclear matter was not considered. In ref. [26] it has been shown that a version of chiral force was able to reproduce, by a suitable choice of the momentum cut-off parameter, the few-body binding energies and at the same time a fair saturation point. However, this remarkable result needs confirmation, since two and three-body forces were actually taken at different orders and some correlation diagrams were neglected. In any case, for all chiral forces TBF are the dominant mechanism for saturation. Indeed, with only two-body forces no saturation is apparent in nuclear matter. NN potentials based on the constituent quark model have been developed for some decades, since the resonating-group-method (RGM) equations was firstly solved by Oka and Yazaki [27]. In this model the quark degrees of freedom are explicitly introduced and the NN potential is derived from the quark-quark (qq) interactions. The resulting interaction is highly non-local due to the RGM formalism and contains a natural cut-off in momentum. Realistic quark-model (QM) interactions were proposed...
by Fujiwara and collaborators [28], in which the qq interaction consists of a color-analog of the Fermi-Breit interaction and an effective meson-exchange potential. The most recent model fss2 [29] reproduces the experimental data on the few-body systems (triton, hypertriton [30] and α particle [31] and nucleon-deuteron scattering [32]) fairly well without introducing TBF. In this letter we present results for nuclear matter obtained with the QM force within the Bethe-Brueckner-Goldstone (BBG) many-body expansion up to three hole-line level of approximation. A pedagogical introduction to this many-body method can be found in ref. [33]. The energy dependence inherent to the RGM formalism is eliminated by the off-shell transformation utilizing the norm kernel as in Refs. [34, 35] and the Gaussian representation of fss2 [36] is used. The set of Goldstone diagrams that are used in the calculations is reported in Fig. 1. The BBG expansion classifies the diagrams according to the number of hole-lines that they contain. Diagrams (a) and (b) (direct and exchange) include two hole-lines and they correspond to the well-known Brueckner-Hartree-Fock (BHF) approximation. The wavy line indicates the Brueckner G-matrix [33]. In the BBG expansion

![FIG. 1: Different Goldstone diagrams contributing to the Nuclear Matter EOS. The wavy line indicates Brueckner G-matrix. The box labelled $T^{(3)}$ is the intermediate three-body scattering matrix. Diagram (c) is the first term of the sum of diagrams obtained once the expansion of $T^{(3)}$ is inserted in (f) and it has been singled out for numerical convenience.

TABLE I: Three hole-line contributions to the neutron matter EOS for different Fermi momenta $k_F$ in fm$^{-1}$. $E_3$ is the total three hole-line contribution, $B$ is the “bubble diagram” of Fig. 1(c), BU is the U-insertion diagram of Fig. 1(d), R is the “ring diagram” of Fig. 1(e) and H indicates the “higher order” diagrams, as defined in the text. Energies are in MeV.

| $k_F$ | $T + E_2$ | $B$ | BU | R | H | $E_3$ | EOS |
|-------|-----------|-----|----|---|---|-------|-----|
| 1.1   | -17.090   | -7.020 | 10.655 | -0.750 | 0.177 | 3.072 | -14.028 |
| 1.2   | -19.680   | -6.351 | 11.407 | -1.270 | 0.157 | 3.943 | -15.737 |
| 1.3   | -22.154   | -4.669 | 11.647 | -1.761 | 0.144 | 5.361 | -16.793 |
| 1.4   | -24.393   | -2.689 | 12.340 | -2.030 | -0.079 | 7.542 | -16.851 |
| 1.5   | -26.183   | 0.223 | 12.781 | -2.122 | 0.050 | 11.022 | -15.161 |
| 1.6   | -27.498   | 4.162 | 13.759 | -2.280 | 0.029 | 15.670 | -11.828 |

TABLE II: The same as in Table I, but in the gap choice for the single particle potential.

| $k_F$ | $T + E_2$ | $B$ | BU | R | H | $E_3$ | EOS |
|-------|-----------|-----|----|---|---|-------|-----|
| 1.1   | -11.605   | -0.556 | -1.003 | 0.063 | -1.496 | -13.101 |
| 1.2   | -13.525   | -0.029 | -1.119 | 0.040 | -1.108 | -14.633 |
| 1.3   | -15.439   | 0.846 | -1.251 | 0.033 | -0.372 | -15.721 |
| 1.4   | -16.959   | 2.213 | -1.301 | 0.021 | 0.933 | -16.026 |
| 1.5   | -18.212   | 4.234 | -1.296 | 0.012 | 2.575 | -15.272 |
| 1.6   | -18.974   | 7.233 | -1.328 | 0.006 | 5.911 | -13.063 |

an auxiliary single-particle potential $U(k)$ is introduced, and calculated self-consistently according to the Brueckner prescription. However the auxiliary potential is not unique. Two somehow opposite choices are possible. In the so called "standard" or gap choice (GC), the potential is assumed to be zero above the Fermi momentum, while in the "continuous" choice (CC) the potential is calculated self-consistently also above the Fermi momentum. In principle the final result should be independent of $U$, which is introduced in order to rearrange the perturbation series for a faster convergence. The comparison of the result obtained with the gap and the continuous choice can be used to estimate the degree of convergence of the expansion [2,3]. The rearrangement of the expansion is embodied in a series of "U-insertion" diagrams. The first one is diagram (d), while diagram (c) is a self-energy insertion. Notice that the various self-energy or $U$-insertion diagrams must follow from the BBG expansion, otherwise an arbitrary number of insertions would spoil the hole-line ordering of the diagrams. Diagram (e) is generally indicated as "ring diagram". It describes long range correlations in the matter. Diagram (f) describes the full scattering process of three particles that are virtually excited above the Fermi sphere and its evaluation requires the solution of the Bethe-Fadeev equations [1, 33, 37]. The sum of the diagrams (e-f) gives the three hole-line contribution, which, according to the BBG hole-line expansion, is expected to be substantially smaller than the two hole-line (Brueckner) contribution. **Results and Discussion.** The results for symmetric matter (SM) in the CC are reported in Table I for a set of values of Fermi momenta around saturation. The breakdown of the contributions of each of the diagrams discussed above is also specified.

The last column reports the final EOS obtained summing up the two hole and three hole-line contributions. In the column before the last the total contribution of the three hole-line diagrams is reported. Table II is similar but for the gap choice. Notice that in this case the U-insertion diagram (d) vanishes, because the auxiliary potential is zero above the Fermi momentum. The comparison between the two sets of results for the EOS of SM is summarized in Fig. 2 for a wider range of density.
One can see that at the Brueckner (two hole-lines) level of approximation the continuous and gap choices differ by a few MeV, being the continuous one more attractive. However, as the three hole-line contribution is added, the two EOS are quite close. The discrepancy around saturation is not exceeding 1 MeV and it is vanishing small just at saturation. We consider this result as a strong indication of the convergence of the BBG expansion. However, at high density the discrepancy tends to increase, which indicates a lower degree of convergence, but no divergence of the expansion is really apparent. It has to be noticed that for other NN interactions, in particular the Argonne $v_{18}$ potential, and the corresponding simplified versions $v_8$, $v_6$ and $v_4$, the three hole-line contribution is much smaller in the CC than in the GC. For the present QM potential it is the opposite, in the gap choice the convergence of the energy looks faster. However, it has to be kept in mind that the continuous choice appears more physical, since the potential has no discontinuity, and it can be more appropriate for the calculations of phenomenological quantities like the optical potential. To be more quantitative one can notice that around saturation the ratio between the correlation energy due to the three hole-line diagrams and the two hole-line ones (BHF) is 0.15 for the CC and 0.02 for the GC. This is in line with the expectation of the hole-line expansion and supports the validity of the BBG expansion. Notice that the second columns of Tables I and II include the free kinetic energy $T$. If the EOS around saturation density are fitted with a form of the type $E/A = a\rho + b\rho^2$ the saturation point turns out to be $e_0 = -16.9$ MeV and $\rho_0 = 0.166$ fm$^{-3}$ for the CC and $e_0 = -16.06$ MeV and $\rho_0 = 0.177$ fm$^{-3}$ for the GC. This establishes the range of the uncertainty on the predicted saturation point and of the whole EOS in the considered density range. From the same fits one can extract the compressibility at saturation, which turns out to be $K = 228$ MeV for the CC and $K = 192$ MeV for the GC. These values can be considered compatible with the range encompassed by the phenomenological constraints, the GC value being at the lower edge. A similar analysis can be performed for pure Neutron Matter (PNM). The corresponding EOS for the CC and GC are reported in Fig. 3.

For comparison two EOS which include also three-body forces are also reported, one obtained within the BHF approach and one from the variational method. A blow up of the low density region is reported in Fig. 4, where in addition the EOS obtained from the chiral force approach of ref. [26] is reported. The latter two includes three-body forces for comparison the EOS from the chiral expansion of ref. [26] (dashed-dotted line) has been reported. The area delimited by the thick full line indicates the region where the EOS predicted from the chiral approach of ref. [26] should be enclosed.
the derivative of asymmetry at saturation, usually embodied in the parameter $L = 3\rho(\partial S/\partial \rho)$. At saturation one finds $S_0 = 34$ MeV and $L = 54$ MeV for the CC, and $S_0 = 33.7$ MeV and $L = 53$ MeV for the GC, again compatible with phenomenology \[39\].

**Conclusion.** We have presented the calculation of the EOS both for symmetric matter and pure neutron matter based on the NN interaction by Fujimura and collaborators. Since the interaction is derived from an effective quark-quark interaction within the RGM, the EOS are directly related to the QCD inner structure of the nucleons. The BBG many-body theory has been used up to the three hole-line level of approximation. The expansion shows a good degree of convergence, and the saturation point, the compressibility, the symmetry energy and its derivative compare well with the phenomenological data, with an error that is within the phenomenological uncertainty. The same NN interaction is able to reproduce fairly well the $^3H$ binding \[30\] and the scattering data on proton-deuteron system \[32\]. These results suggest that, if the NN interaction is modelled from the quark-quark interaction, the need of three-body forces is reduced to a minimum. The consistency of this conclusion could be checked by deriving explicitly the three-body forces from the three quark model, which is left to a future long term project.

Partial support from NewCompStar, COST Action MP1304 is gratefully acknowledged.

[1] B.D. Day, Phys. Rev. C24, 1203 (1981) and Phys. Rev. Lett. 47, 226 (1981).
[2] H. Q. Song, M. Baldo, G. Giansiracusa, and U. Lombardo, Phys. Rev. Lett. 81, 1584 (1998).
[3] M. Baldo, G. Giansiracusa, U. Lombardo and H.Q. Song, Phys. Lett. B473, 1 (2000); M. Baldo, A. Fiasconaro, G. Giansiracusa, U. Lombardo and H.Q. Song, Phys. Rev. C65, 017303 (2001).
[4] J. Carlson, V.J. Pandharipande and R.B. Wiringa, Nucl.Phys. A401, 59 (1983), R. Schiavilla, V.J. Pandharipande and R.B. Wiringa, Nucl. Phys. A449, 219 (1986).
[5] G. Taranto, M. Baldo and G.F. Burgio, Phys. Rev. C87, 045803 (2013).
[6] P. Grange, A. Leujeune, M. Martzolf and J.-F. Mathiot, Phys. Rev. C40, 1040 (1989); W. Zuo, A. Leujeune, U. Lombardo and J.-F. Mathiot, Nucl. Phys. A706, 418 (2002); W. Zuo, A. Leujeune, U. Lombardo and J.-F. Mathiot, Eur. Phys. J. A14, 469 (2002).
[7] Z.H. Li, U. Lombardo, H.-J. Schulze, W. Zuo, L.W. Chen and H.R. Ma, Phys. Rev. C74, 047304 (2006).
[8] S.C. Pieper and R.B. Wiringa, Ann. Rev. Nucl. Part. Sci. 51, 53 (2001).
[9] A. Akmal, V.R. Pandharipande and D.G. Ravenhall, Phys. Rev. C58, 1804 (1998).
[10] P. Dolleschall and I. Borbely, Phys. Rev. C62, 054004 (2000); P. Dolleschall, I. Borbely, Z. Papp and W. Plessas, Phys. Rev. C67, 064005 (2003); P. Dolleschall, Phys. Rev. C69, 054001 (2004).
[11] M. Baldo and C. Maieron, Phys. Rev. C72, 034005 (2005).
[12] T. Gross-Boelting, C. Fuchs and A. Faessler, Nucl. Phys. A648, 105 (1999).
[13] G.E. Brown, W. Weise, G. Baym and J. Speth, Comm. Nucl. Part. Phys. 17, 39 (1987).
[14] S. Weinberg, Phys. Lett. B251, 288 (1990); Nucl. Phys. B363, 3 (1991); Phys. Lett. B295, 114 (1992); Phys. Rev. Lett. 166, 1568 (1968).
[15] D.R. Entem and R. Machleidt, Phys. Rev. C68, 041004 (2003).
[16] M.P. Valderrama and D.R. Phillips, [arXiv:1407.0437 [nucl-th]] (2014).