TWO–PHOTON DECAY WIDTHS
OF HIGGS PARTICLES

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ABSTRACT

Two–photon decays of Higgs bosons are important channels for the search of these particles in the intermediate mass range at the \textit{pp} colliders LHC and SSC. Dynamical aspects of the Higgs coupling to two photons can also be studied by means of the $\gamma \gamma$ fusion of Higgs particles at high–energy $e^+ e^-$ linear colliders. Extending earlier analyses which had been restricted to the Standard Model, we present in this note the QCD radiative corrections to the $\gamma \gamma$ decay widths of scalar and pseudoscalar Higgs particles in multi–doublet extensions of the Higgs sector, as realized for instance in supersymmetric theories.
The exploration of the electroweak symmetry breaking mechanism is one of the most important tasks of particle physics. Within the Standard Model, the solution of this problem is associated with the existence of fundamental scalar Higgs particles. If the Higgs particle in the Standard Model \( SM \) is light, with a mass in the intermediate range below \( \sim 200 \) GeV, the theory can be extrapolated perturbatively up to the GUT scale. However, the hierarchy problem suggests the supersymmetric extension of the model in this case, stabilizing the mass of light Higgs bosons in the background of high–energy GUT scales. In contrast to the \( SM \), supersymmetric theories incorporate a spectrum of Higgs particles \( \Phi \), in the minimal version \( [MSSM] \) light and heavy scalar \( [CP–even] \) particles\(^1\) \( h, H \), a pseudoscalar \( [CP–odd] \) particle \( A \), and a pair of charged Higgs particles.

The precise prediction of the \( \gamma\gamma \) decay widths of Higgs particles \( [1] \) is important for two reasons. First, the \( \gamma\gamma \) decay mode plays a crucial rôle for the search of these particles in the lower part of the intermediate mass range at the \( pp \) colliders LHC and SSC \( [2] \). Second, the \( \gamma\gamma \) widths can be measured directly by means of \( \gamma\gamma \) fusion at high–energy \( e^+e^- \) linear colliders \( [3, 5] \). Since the photons couple to Higgs bosons via heavy particle loops, the \( \gamma\gamma \) widths are sensitive to particle masses, standard and also supersymmetric, well above the Higgs masses themselves. However, these effects are small if the particle masses are not generated through the Higgs mechanism. To exploit this method, it is therefore mandatory to control properly the radiative corrections to the \( \Phi\gamma\gamma \) couplings mediated by the standard particles.

In this note, we shall present the QCD radiative corrections of the quark–loop contributions to the \( \gamma\gamma \) widths of the Higgs particles in the [minimal] supersymmetric extension of the Standard Model, Fig. 1. This demands the extension of the calculation for the \( SM \) Higgs decay \( [6, 7] \) in two ways: (i) Since the decay amplitudes in part of the \( SUSY \) parameter space are dominated by \( b \)-quark loops, the QCD corrections are to be analyzed for scalar \( h, H \) masses above the fermion–antifermion threshold\(^2\) and (ii) the QCD corrections are to be determined for pseudoscalar Higgs bosons \( A \).

The \( \gamma\gamma \) couplings to Higgs bosons are mediated by charged heavy particle loops \( [W, \text{fermion}, \text{chargino}, \text{sfermion} \text{and charged Higgs boson loops in the scalar case } h, H, \text{and fermion and chargino loops in the pseudoscalar case } A] \). Denoting the quark amplitudes by \( A_Q \) etc., the \( \gamma\gamma \) decay rates are given by \( [1, 6] \)

\[
\Gamma(\mathcal{H} \rightarrow \gamma\gamma) = \frac{G_F \alpha^2 M_\mathcal{H}^3}{128 \sqrt{2} \pi^3} \left| \sum_Q N_C e_Q^2 g_Q^H A_Q^H + g_W^H A_W^H + \cdots \right|^2 \tag{1}
\]

\[
\Gamma(A \rightarrow \gamma\gamma) = \frac{G_F \alpha^2 M_A^3}{32 \sqrt{2} \pi^3} \left| \sum_Q N_C e_Q^2 g_Q^A A_Q^A + \cdots \right|^2 \tag{2}
\]

\(^1\)The scalar particles will generically be denoted by \( \mathcal{H} \).

\(^2\)This problem has first been solved by one of the authors \( [8] \). The analysis would agree with the results of a recent preprint \( [9] \) if the large logarithms in the QCD corrections were not mapped into the running quark mass [see Appendix A].
where the quark and $W$ amplitudes read at lowest order
\[
A_Q^H \Rightarrow 2\tau^{-1} \left[ 1 + (1 - \tau^{-1})f(\tau) \right]
\]
\[
A_W^H \Rightarrow -\tau^{-1} \left[ 3 + 2\tau + 3(2 - \tau^{-1})f(\tau) \right]
\]
\[
A_Q^A \Rightarrow \tau^{-1}f(\tau)
\]
(3)

The scaling variable is defined as $\tau = M_\Phi^2/4m_i^2$ with $m_i$ denoting the loop mass, and
\[
f(\tau) = \begin{cases} 
\arcsin^2 \sqrt{\tau} & \text{for } \tau \leq 1 \\
-\frac{1}{4} \left[ \log \frac{\sqrt{\tau} + \sqrt{1 - \tau}}{\sqrt{\tau} - \sqrt{1 - \tau}} - i\pi \right]^2 & \text{for } \tau > 1
\end{cases}
\]
(4)

The coefficients $g_i^\Phi$ denote the couplings of the Higgs bosons [normalized to the SM Higgs couplings] to top, bottom quarks and $W$ bosons, recollected for the sake of convenience in Table 1.

**Table 1:** Coefficients $g_i^\Phi$ of the Higgs couplings to quarks and $W$ bosons in the MSSM. $\alpha, \beta$ are mixing angles, $\tan\beta = v_2/v_1$ being the ratio of the Higgs vacuum expectation values. Numerical values of the coefficients are presented in Ref. [10], for instance.

| $\Phi$ | $g_t$ | $g_b$ | $g_W$ |
|---|---|---|---|
| $h$ | $\cos \alpha / \sin \beta$ | $-\sin \alpha / \cos \beta$ | $\sin(\beta - \alpha)$ |
| $H$ | $\sin \alpha / \sin \beta$ | $\cos \alpha / \cos \beta$ | $\cos(\beta - \alpha)$ |
| $A$ | $1/\tan\beta$ | $\tan\beta$ | 0 |

The cross section for the $\gamma\gamma$ fusion of Higgs bosons is found by folding the parton cross section with the $\gamma\gamma$ luminosity, see e.g. [3]. The parton cross section is determined by the $\gamma\gamma$ width so that $\langle \sigma(\gamma\gamma \to \Phi) \rangle \propto \Gamma(\Phi \to \gamma\gamma)/M_\Phi^2$. The photon beams are generated either automatically as initial-state Weizsäcker–Williams and bremsstrahl photons in $e^\pm$ collisions, or they may be generated, in a dedicated effort, by Compton back-scattering of laser light [5].

The QCD corrections to the quark amplitudes can be parametrized as
\[
A_Q = A_Q^{LO} \left[ 1 + C \frac{\alpha_s}{\pi} \right]
\]
(5)

The coefficient $C$ depends on $\tau = M_\Phi^2/4m_i^2(\mu^2)$ where the running quark mass $m_Q(\mu^2)$ is defined at the renormalization point $\mu$ which is taken to be $\mu = M_\Phi/2$ in our analysis; this value is related [11] to the pole mass $m_Q(m_Q^2) = m_Q$ in the on-shell renormalization scheme by
\[
m_Q([M_\Phi/2]^2) = m_Q \left[ \frac{\alpha_s([M_\Phi/2]^2)}{\alpha_s(m_Q^2)} \right]^{12/(33-2N_F)} \left\{ 1 + \mathcal{O}(\alpha_s^2) \right\}
\]
(6)
The lowest order amplitude $A_{LO}^Q$ is to be evaluated for the same mass value $m_Q([M_\Phi/2]^2)$. The choice $\mu = M_\Phi/2$ of the renormalization point avoids large logarithms $\log M_\Phi^2/m_Q^2$ in the final results for Higgs masses much larger than the quark mass [for details see Appendix A]. $\alpha_s$ is taken at $\mu$ for $\Lambda = 200$ MeV.

We have evaluated the diagrams of the type shown in Fig. 1 plus the corresponding counter terms for the running quark mass at $\mu = M_\Phi/2$. The 't Hooft–Veltman $\gamma_5$ prescription [12] has been adopted for the dimensional regularization of the amplitudes. The 5–dimensional Feynman parameter integrals have been reduced analytically to 1–dimensional integrals which have been calculated numerically.

The amplitudes $C_H$ for scalar loops and $C_A$ for pseudoscalar loops are shown in Fig. 2a/b as functions of $\tau$. The coefficients are real below the quark threshold $\tau < 1$, and complex above. Very close to the threshold, within a margin of a few GeV, the present perturbative analysis cannot be applied anymore. [It may account to some extent for resonance effects in a global way.] Since $QQ$ pairs cannot form $0^{++}$ states at the threshold, $\text{Im}C_H$ vanishes there; $\text{Re}C_H$ develops a maximum very close to the threshold. By contrast, since $QQ$ pairs do form $0^{-+}$ states, the imaginary part $\text{Im}C_A$ develops a step which is built–up by the Coulombic gluon exchange [familiar from the Sommerfeld singularity of the QCD correction to $QQ$ production in $e^+e^-$ annihilation]; $\text{Re}C_A$ is singular at the threshold. For large $\tau$, both coefficients approach a common numerical value, as expected from chiral invariance in this limit. In the opposite limit, the QCD corrections can be evaluated analytically,

$$m_Q \gg M_\Phi : \quad C_H \to -1 \quad \text{and} \quad C_A \to 0$$

These results can easily be traced back to the form of the $\gamma\gamma$ anomaly in the trace of the energy–momentum tensor [13] and to the non-renormalization of the axial–vector anomaly [14], as demonstrated in Appendix B.

In Fig. 3a–d the QCD corrected $\gamma\gamma$ widths for $h, H, A$ Higgs bosons are displayed in the minimal supersymmetric extension of the Standard Model [taking into account only quark and $W$ boson loops] for two values $tg\beta = 2.5$ and $tg\beta = 20$. While in the first case top loops give a significant contribution, bottom loops are the dominant component for large $tg\beta$. The overall QCD corrections are shown in the lower part of the figures. The corrections to the widths are small, $\sim \mathcal{O}(\alpha_s/\pi)$ everywhere. [Artificially large $\delta$ values occur only for specific large Higgs masses when the lowest order amplitudes vanish accidentally as a consequence of the destructive interference between $W$ and quark–loop amplitudes, see also [8, 9].]

In conclusion. We have calculated the QCD corrections to the decays $h, H, A \to \gamma\gamma$ of the neutral scalar and pseudoscalar Higgs bosons in the minimal supersymmetric extension of the Standard Model, that is taken for illustration. These corrections are well under

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3By choosing the renormalization point $\mu = M_\Phi/2$ the perturbative threshold $E_{th} = 2m_Q(m_Q^2)$ coincides with the on-mass shell value proper. A shift between $M_\Phi/2$ and $M_\Phi$, for instance, affects the widths very little away from the threshold.
control across the physically interesting mass ranges, if the running of the quark masses is properly taken into account.

APPENDIX A

The QCD corrected quark amplitude for $\Phi \to \gamma\gamma$ may be written in the general form

$$A_Q = A_Q^{LO}(m_Q) \left\{ 1 + \left[ c_1(m_Q) + c_2(m_Q) \log \frac{\mu^2}{m_Q^2} \right] \frac{\alpha_s}{\pi} + O(\alpha_s^2) \right\}$$

where $m_Q \equiv m_Q(\mu^2)$ is the quark mass defined at the renormalization point $\mu$. The scale in $\alpha_s$ may in principle be chosen different from $\mu$. For large $m_Q$, $c_1$ approaches $-1$ for $h, H$, and $0$ for $A$, while $c_2$ vanishes $\sim 1/m_Q^2$, and no large logarithms appear in this limit. For $m_Q \to 0$, however, $c_2$ approaches a finite non-zero value while $c_1$ develops a large logarithm the coefficient of which is given by $c_2$,

$$c_2 \to 2$$
$$c_1 \to -2c_2 \log(M^2_\Phi/4m_Q^2) + \text{const}$$

By choosing $\mu = M_\Phi/2$, all large logarithms are eliminated from the coefficient of $\alpha_s$ and mapped into the effective quark mass of the lowest–order amplitude. This is reminiscent of the corresponding procedure for Higgs decays to fermion pairs [16]. Had we chosen $\mu = m_Q$ instead [9], we would be left with unnaturally large corrections not taking advantage of renormalization group improvements.

A technical remark ought to be added on a subtle problem related to the 't Hooft–Veltman implementation of $\gamma_5$ in the dimensional regularization scheme which reproduces the axial–vector anomaly to lowest order [12]. The multiplicative renormalization factor of the scalar $Q\bar{Q}$ current is well–known to be given by $Z_{HQQ} = 1 - Z_2 + \delta m_Q/m_Q$ where $Z_2$ is the wave–function renormalization factor and $\delta m_Q$ the additive mass shift. To insure the chiral symmetry relation $\Gamma_5(p', p) \to \gamma_5 \Gamma(p', p)$ in the limit $m_Q \to 0$ for the fermionic matrix element of the pseudoscalar and scalar currents, the renormalization factor of the pseudoscalar current has to be chosen [17]

$$Z_{AQQ} = Z_{HQQ} + 8\alpha_s/(3\pi)$$

The additional term supplementing the naive expectation is caused by spurious anomalous contributions that must be subtracted by hand [18].
APPENDIX B

For large quark masses compared to the Higgs mass $m_Q \gg M_{\Phi}$, low energy theorems can be exploited to calculate the corrections $C_{\mathcal{H},A}$ in this limit.

The scalar coupling $\mathcal{H}\gamma\gamma$ can be derived $^{[13]}$ from the requirement that the matrix element $\langle 0|\theta_{\mu\mu}|\gamma\gamma\rangle$ of the trace of the QCD corrected energy–momentum tensor $^{[13]}$,

$$\theta_{\mu\mu} = [1 + \delta]m_Q \bar{Q}Q + \frac{1}{4} \frac{\beta_0}{\alpha} e_Q^2 F_{\mu\nu} F^{\mu\nu}$$

vanish in the low energy limit; $\delta = 2\alpha_s/\pi$ and $\beta_0 = 2\alpha^2/\pi(1+\alpha_s/\pi+\cdots)$ is the QED/QCD $\beta$–function. Since the Higgs bosons are coupled to the mass operator $(m_Q/v) \bar{Q}Q$, the QCD corrected effective Lagrangian can be determined immediately,

$$\mathcal{L}_{\text{eff}}(\mathcal{H}\gamma\gamma) = \frac{\alpha}{2\pi} \left(\sqrt{2} G_F\right)^{1/2} e_Q \left(1 - \frac{\alpha_s}{\pi}\right) \mathcal{H} F_{\mu\nu} F^{\mu\nu}.$$ 

From the non–renormalization of the anomaly of the axial–vector current $^{[14, 15]}$ follows the non–renormalization of the $A\gamma\gamma$ coupling,

$$\partial_{\mu} A_{\mu} = 2m_Q \bar{Q} i\gamma_5 Q + \frac{\alpha}{4\pi} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

so that the effective Lagrangian

$$\mathcal{L}_{\text{eff}}(A\gamma\gamma) = \frac{\alpha}{8\pi} \left(\sqrt{2} G_F\right)^{1/2} \alpha F_{\mu\nu} \tilde{F}^{\mu\nu}$$

is valid to all orders of $\alpha_s$ for the two–photon irreducible part of the diagrams.

Note added. As a corollary to this result we observe that the irreducible part of the $Agg$ coupling will not be renormalized either, and only the reducible diagrams need be calculated for the QCD corrections to the production process $gg \to A$ in the range $m_A \leq 2m_Q$. Only the virtual corrections are different from the scalar case, $C \to \pi^2 + 6 + \frac{1}{8} (33 - 2N_F) \log \mu^2/m_{H}^2$ in the notation of eq.(9) in Ref. $^{[19]}$. [A. D., Madison SSC Workshop '93; see also R. Kauffman and W. Schaffer, BNL preprint).]

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FIGURE CAPTIONS

Fig.1. Generic diagram of the QCD radiative corrections to the Higgs coupling to two photons.

Fig.2. (a) Real and imaginary part of the radiative corrections to the quark amplitude of the scalar $H\gamma\gamma$ coupling, normalized to the lowest–order amplitude; (b) the same for the pseudoscalar $A\gamma\gamma$ coupling.

Fig.3. (a) Two–photon widths of the MSSM Higgs bosons $h, H, A$ for $tg\beta = 2.5$, and (b) size of the QCD radiative corrections; (c), (d) the same for $tg\beta = 20$. 