Simple synchronization protocols for heterogeneous networks: beyond passivity

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Abstract: Synchronization among autonomous agents via local interactions is one of the benchmark problems in multi-agent control. Whereas synchronization algorithms for identical agents have been thoroughly studied, synchronization of heterogeneous networks still remains a challenging problem. The existing algorithms primarily use the internal model principle, assigning to each agent a local copy of some dynamical system (internal model). Synchronization of heterogeneous agents thus reduces to global synchronization of identical generators and local synchronization between the agents and their internal models. The internal model approach imposes a number of restrictions and leads to sophisticated dynamical (and, in general, nonlinear) controllers. At the same time, passive heterogeneous agents can be synchronized by a very simple linear protocol, which is used for consensus of first-order integrators. A natural question arises whether analogous algorithms are applicable to synchronization of agents that do not satisfy the passivity condition. In this paper, we study the synchronization problem for heterogeneous agents that are not passive but satisfy a weaker input feedforward passivity (IFP) condition. We show that such agents can also be synchronized by a simple linear protocol, provided that the interaction graph is strongly connected and the couplings are sufficiently weak. We demonstrate how stability of cooperative adaptive cruise control algorithms and some microscopic traffic flow models reduce to synchronization of heterogeneous IFP agents.

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1. INTRODUCTION

As the influential monograph (Strogatz, 2003) states, “the tendency to synchronize is one of the most pervasive drives in the universe, extending from atoms to animals”. Synchrony among subsystems (agents, cells) of a complex system is a basic principle, which explains many natural phenomena (Strogatz, 2003) and has found numerous applications in engineering (Mesbahi and Egerstedt, 2010; Olfati-Saber et al., 2007; Wu, 2007; Ren and Beard, 2008). Establishing synchronization (consensus) is considered now as a benchmark problem in multi-agent control and has been thoroughly examined in the recent decades. Most of the attention has been paid to synchronization among identical agents. The protocols establishing synchronization among single integrators are usually based on the idea of contraction: the convex hull, spanned by the agents’ states, is shrinking until it collapses into a singleton (Münn et al., 2011). An alternative approach is based on convergence criteria for infinite matrix products (Ren and Beard, 2008). The protocols for synchronization of agents, obeying higher order equations, are similar in spirit to first-order algorithms. Synchronization of linear and linearly coupled agents is often analyzed via the spectral decomposition of the Laplacian matrix (Olfati-Saber et al., 2007; Li et al., 2010; Ren and Beard, 2008; Ren and Cao, 2011). Nonlinear protocols are usually employed by Lyapunov methods (Ren and Cao, 2011), employing, among others, the Kalman-Yakubovich-Popov lemma (Zhang et al., 2014; Proskurnikov and Matveev, 2015), contraction theory (DeLellis et al., 2011) and the idea of incremental passivity (Stan and Sepulchre, 2007; Proskurnikov et al., 2015; Liu et al., 2015b).

However, in practice autonomous agents are usually heterogeneous. Algorithms for output synchronization of non-identical agents have been proposed quite recently and most of them employ the internal model principle (Wieland et al., 2011; De Persis and Jayawardhana, 2014; Isidori et al., 2014; Bidram et al., 2014; Liu et al., 2015a), assigning to each agent a virtual copy of some dynamical system, referred to as the internal model or the local reference generator. The control algorithm then consists of two layers: a protocol, synchronizing the (identical) reference generators and local model-matching controllers, synchronizing the agents to their generators.

The general internal model approach has, however, several disadvantages. Being formally decentralized, its implementation assumes that the agents share the same internal model and are able to match it (e.g. in the case of linear agents the Francis regulator equations should be solvable (Wieland et al., 2011; Liu et al., 2015a)). Hence design of an algorithm requires to know the global information about the network. Unlike many synchronization algorithms for identical agents (Olfati-Saber et al., 2007; Li et al., 2010; Ren and Beard, 2008; Ren and Cao, 2011) that use only relative measurements, that is, the deviations between an agent’s output and the outputs of its neigh-
bors, the model-matching controllers need access to the absolute outputs of the agents. Dealing with mobile robots, this implies that agents have to measure their positions and/or velocities in the global frame of reference.

At the same time, synchronization among heterogeneous passive agents (e.g., mechanical systems in the Euler-Lagrange form) can be established by the same simplest protocols (Pogromsky and Nijmeijer, 2001; Arcak, 2007; Hatana et al., 2015) as used to synchronize single integrator agents (Olfati-Saber et al., 2007). Such a protocol does not require any knowledge of the agents’ dynamics (except for their passivity) and uses only deviations between the agents’ outputs, but not the outputs themselves.

Thus a visible gap exists between the problems of synchronization in networks of passive heterogeneous agents, provided that the couplings among them are sufficiently weak. The class of IFP systems is much broader than the class of passive systems (and contains, in particular, all asymptotically stable linear systems). We demonstrate applications of our results to the design of cooperative adaptive cruise control (CACC) for platoons of automated vehicles and stability of a microscopic traffic flow model with delayed drivers’ responses, both of which can be reduced to synchronization of IFP agents. Proskurnikov and Mazo Jr (2017).

2. PRELIMINARIES

In this section, we introduce basic concepts from graph theory and define input-feedforward passivity (IFP).

2.1 Graphs and their connectivity properties

A (weighted directed) graph is a triple $G = (V, E, A)$, where $V = \{v_1, \ldots, v_N\}$ stands for the set of nodes, $E \subseteq V \times V$ is a set of arcs and $A = (a_{jk})_{N}^{N}$ is a non-negative adjacency matrix, such that $a_{jk} > 0$ if $(v_k, v_j) \in E$ and otherwise $a_{jk} = 0$. We always assume that the number of nodes $N$ and their indices are fixed, so $V = \{1, \ldots, N\}$, there is a one-to-one correspondence between such graphs and their adjacency matrices $A \rightarrow G[A] \triangleq (V, E[A], A)$, where $E[A] \triangleq \{(j, k) : a_{jk} \neq 0\}$. Henceforth all graphs have no self-loops $a_{jj} = 0 \forall j$. A graph is called undirected if $A = A^{T}$. For any node $j$ we introduce the weighted in- and out-degrees $d_{j}^{+}[A] \triangleq \sum_{k=1}^{N} a_{kj}$ and $d_{j}^{-}[A] \triangleq \sum_{k=1}^{N} a_{jk}$.

A walk connecting nodes $v$ and $v'$ is a sequence of nodes $v_0 \triangleq v, v_1, \ldots, v_{n-1}, v_n \triangleq v'$ ($n \geq 1$) such that $(v_{k-1}, v_k) \in E$ for $k = 1, \ldots, n$. A graph is strongly connected if there exists a walk between any two distinct nodes exists. A graph is quasi-strongly connected, or has a directed spanning tree, if one of its nodes is connected by walks to all other nodes. For an undirected graph these conditions are equivalent (such a graph is simply called connected).

2.2 Passivity and input-feedforward passivity

Consider the dynamical system

$$\dot{x}(t) = f(x(t), u(t)), \quad y(t) = h(x(t), u(t)), \quad t \geq 0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^m$ stand, respectively, for the state, control and output.

The system (1) is passive (Khalil, 1996; Willems, 1972) if there exists a storage function $V(x) \geq 0$ such that

$$V(x(T)) - V(x(0)) \leq \int_{0}^{T} \dot{y}(t) \top u(t) \, dt \quad \forall T \geq 0 \tag{2}$$

(here $T$ varies in the interval where the solution exists). Assuming $V$ to be $C^{1}$-smooth, (2) can be rewritten as

$$\dot{V}(x, u) = \frac{\partial V}{\partial x} f(x, u) \leq b(x, u) \top u \quad \forall x \in \mathbb{R}^n, u \in \mathbb{R}^m. \tag{3}$$

In this paper, we primarily deal with systems, satisfying a “relaxed” passivity condition, defined as follows.

**Definition 1.** The system (1) is IFP($\alpha$) (input-feedforward passive with the passivity index $\alpha$) if it is passive with respect to the output $\hat{y} = y + \alpha u$, i.e.

$$V(x(T)) - V(x(0)) \leq \int_{0}^{T} \dot{y}(t) \top u(t) + \alpha |u(t)|^2 \, dt \tag{4}$$

In the case $\alpha = 0$ an IFP($\alpha$) system is passive; if $\alpha < 0$ the condition (4) is referred to as the strict input passivity.

In this paper, we are primarily interested in systems that are not passive but IFP($\alpha$) with $\alpha > 0$. Examples of such systems are discussed in Section 3.3.

Although this is not required by the formal definition, the conditions of passivity and IFP usually hold for systems with zero equilibrium: $f(0,0) = 0$ and $h(0,0) = 0$. For systems without equilibria points a modification of passiv- ity criteria for nonlinear systems (Hill and Moylan, 1976) are not passive but IFP($\alpha$) with $\alpha > 0$. Examples of such systems are discussed in Section 3.3.

Consider a group of $N$ agents obeying the equations:

$$\dot{x}_j(t) = f_j(x_j(t), u_j(t)), \quad y_j(t) = h_j(x_j(t)), \quad t \geq 0, \tag{6}$$

3. MAIN RESULTS: SYNCHRONIZATION PROTOCOLS FOR IFP AGENTS
for \( j \in \{1, \ldots, N\} \). Here \( x_j(t) \in \mathbb{R}^n \), \( u_j(t) \in \mathbb{R}^m \), \( y_j(t) \in \mathbb{R}^m \) stand respectively for the \( j \)th agent’s state, control and output.

In this paper, we study distributed protocols, synchronizing the outputs \( y_j \) asymptotically or in \( L_2 \)-norm.

**Definition 4.** Solutions \( \{x_j(t), u_j(t), y_j(t)\}_{j=1}^N \) of the systems (6), defined on \( t \in [0; \infty) \), are output synchronized if
\[
|y_i(t) - y_j(t)| \rightarrow 0 \quad \forall i, j = 1, \ldots, N.
\]

More specifically, the solutions are output synchronized with a predefined reference signal \( \bar{y}_i : [0; \infty) \rightarrow \mathbb{R}^m \) if
\[
|y_i(t) - \bar{y}_i(t)| \rightarrow 0 \quad \forall i = 1, \ldots, N.
\]

**Definition 5.** Solutions \( \{x_j(t), u_j(t), y_j(t)\}_{j=1}^N \) of the systems (6), defined for \( t \geq 0 \), are output \( L_2 \)-synchronized if
\[
\int_0^\infty |y_i(t) - y_j(t)|^2 dt < \infty \quad \forall i, j = 1, \ldots, N.
\]

The solutions are output \( L_2 \)-synchronized with a predefined reference signal \( \bar{y}_i : [0; \infty) \rightarrow \mathbb{R}^m \) if
\[
\int_0^\infty |y_i(t) - \bar{y}_i(t)|^2 dt < \infty \quad \forall i = 1, \ldots, N.
\]

In practice, the difference between the asymptotical and \( L_2 \)-synchronization is minor. Mathematically, none of these conditions implies the other one. However, in some special situations it is possible to prove that \( L_2 \)-synchronization implies asymptotic synchronization.

**Proposition 6.** Let \( y_j(t) \) be absolutely continuous and \((\bar{y}_i - y_j) \in L_p[0; \infty)\) for some \( p > 1 \) and for any \( i, j \). Then (9) implies (7). If, additionally, \( \bar{y}_i(t) \) is absolutely continuous and \((\bar{y}_i - y_j) \in L_p[0; \infty]\) then (10) entails (8).

Proposition (6), as well as all other statements of this paper, is proved in Appendix. In the following subsections we examine synchronization algorithms.

### 3.1 Synchronization without reference signal

We start examining the linear controller:
\[
\dot{u}_j(t) = \sum_{k=1}^N a_{jk}(y_k(t) - y_j(t)),
\]
where \( a_{jk} \geq 0 \) are the coupling gains. The matrix \( A = (a_{jk}) \) determines the interaction graph (or the network’s topology) \( G[A] \), where node \( k \) is connected to node \( j \) by an arc if and only if \( a_{jk} \neq 0 \), that is, the control input of agent \( j \) is directly influenced by the output of agent \( k \).

It is widely known (Olfati-Saber et al., 2007; Ren and Beard, 2008; Münz et al., 2011) that single integrators \( \dot{y}_j = u_j \), coupled via the protocol (11) reach consensus (that is, a common limit \( y_\ast = \lim_{t \to \infty} y_j(t) \) exists) whenever \( G[A] \) has a directed spanning tree. Output synchronization (7) is retained replacing single integrators by general passive systems (6) and assuming strong connectivity of \( G[A] \) (Hatana et al., 2015, Theorem 8.3). Our first result extends this to IFP agents.

**Theorem 7.** Assume that agent \( j \) (for \( j = 1, \ldots, N \)) is IFP(\( \alpha_j \)) with a storage function \( V_j(x_j) \geq 0 \). Let \( G[A] \) be strongly connected and the couplings be “weak”, i.e.
\[
\alpha_j d_j^+ [A] = \alpha_j \sum_{k=1}^N a_{jk} < 1/2 \quad \forall j = 1, \ldots, N.
\]

Then the following statements hold.

(1) Any solution of the system (6),(11), which is prolongable to \( \infty \), is output \( L_2 \)-synchronized (9);

(2) Suppose that for any \( j \) the function \( V_j \) is radially unbounded \( \lim_{\|x_j\| \to \infty} V_j(x_j) = \infty \), the map \( f_j \) is continuous and \( h_j \) is \( C^1 \)-smooth. Then, any solution of the closed-loop system (6),(11) is prolongable to \( \infty \), bounded, and output synchronized (7).

The proofs of Theorem 7 and other results of this section are omitted due to the page limit and can be found in (Proskurnikov and Mazo Jr., 2017). In the case of \( \alpha_j = 0 \) the inequalities (12) hold for any matrix \( A \), and Theorem 7 coincides with Theorem 8.3 in (Hatana et al., 2015). We proceed with two remarks, regarding the assumptions.

**Remark 8.** Unlike passive agents, for general IFP agents the requirement of weak coupling (12) cannot be disregarded, as demonstrated by the following example. For any \( p, q > 0 \) the system:
\[
\bar{y}_j(t) + p\dot{y}_j(t) + q\dot{y}_j(t) = u_j(t) \in \mathbb{R}, \quad t \geq 0
\]

is IFP(\( \alpha \)) with some \( \alpha = \alpha(p, q) > 0 \) (cf. Subsect. 3.3). Applying the protocol (11) with all-to-all coupling \( a_{ij} = \alpha > 0 \) \( \forall i, j \) to a group of identical agents (13), output synchronization is guaranteed (Olfati-Saber et al., 2007; Li et al., 2010) only when the polynomial \( s^2 + ps^2 + qg + \alpha(N - 1) = 0 \) is Hurwitz. Accordingly to the Routh-Hurwitz criterion, this is possible only if \( \alpha(N - 1) < pq \), i.e. the gain \( \alpha \) is small.

**Remark 9.** Dealing with general heterogeneous agents, the condition of strong connectivity cannot be replaced by the existence of a directed spanning tree in \( G[A] \). Consider, for instance, a pair \( (N = 2) \) of harmonic oscillators
\[
\xi_1 + \omega_1^2 \xi_1 = u_1, \quad \xi_2 + \omega_2^2 \xi_2 = u_2, \quad \omega_1 \neq \omega_2.
\]

that are passive with respect to the outputs \( y_1 = \xi_1 \) and \( y_2 = \xi_2 \). Consider the protocol \( u_1 = k(\xi_2 - \xi_1), u_2 = 0 \), which corresponds to the graph with \( N = 2 \) nodes and the only arc \( 2 \to 1 \). It can be shown that the system has a family of solutions \( \xi_1(t) = Re[W(\omega_2)e^{i\omega_2 t}], \xi_2 = Re[ce^{i\omega_2 t}] \), where \( c \in \mathbb{C} \) is constant and \( W(s) = ks/(s^2 + ks + \omega_1^2) \). The corresponding outputs are \( y_1(t) = Re[\omega_2 W(\omega_2) e^{i\omega_2 t}] \) and \( y_2(t) = Re[\omega_2 W(\omega_2) e^{i\omega_2 t}] \). Since
\[
|W(\omega_2)| = \frac{k\omega_2}{|\omega_1^2 - \omega_2^2| + k\omega_2} < 1,
\]

the outputs are harmonic signals with the same frequency \( \omega_2 \) but different amplitudes and cannot be synchronous.

### 3.2 Reference-tracking synchronization

We now consider the more complex problem of output synchronization with a reference signal (8). In this paper we confine ourselves to a special situation: when the desired trajectory is generated as the output of an agent for some appropriate control input and initial condition.

**Assumption 10.** For any \( j \) system (6) has a solution \( (\bar{x}_j(t), \bar{y}_j(t), \bar{u}_j(t)) \) such that \( \bar{y}_j(t) = \bar{y}(t) + \omega \) \( \forall t \geq 0 \) (in particular, the solution is prolongable to \( \infty \)). At any time agent \( j \) is aware of the value \( \bar{u}_j(t) \), however the reference \( \bar{y}(t) \) may be available only to a few “dedicated” agents.

\[\text{If } \bar{u}_j(t) \text{ is not unique, agent } j \text{ knows one of such solutions.} \]
Assumption 10 is often adopted implicitly or explicitly in reference-tracking synchronization problems. For linear agents (Li et al., 2010; Liu et al., 2015a) the reference signal \( y(t) \) is usually supposed to be an output of a reference system, whose model is known and included by the models of other agents. Dealing with first-order integrator agents \( \dot{y}_j = u_j \). Assumption 10 implies that the agents know the derivative \( \dot{y}(t) \); this holds e.g. if \( \dot{y}(t) = \dot{v} + \ddot{y}(0) \), where \( \dot{v} \) is known, but the initial condition \( \ddot{y}(0) \) is uncertain. A practical example of this type is discussed in Section 4. Note that the solution \( (\ddot{y}_j(t), \ddot{u}_j(t), \dot{y}_j(t)) \) is not assumed to be asymptotically stable, so the control \( u_j(t) = \ddot{u}_j(t) \) does not guarantee the reference signal tracking (8). In general, only some of the agents are able to measure the tracking error \( \ddot{y}(t) - \ddot{y}_j(t) \), whereas the remaining agents measure only deviation between theirs and their neighbors' outputs.

Consider the following modification of the algorithm (11)

\[
u_i(t) = \bar{u}_i(t) + b_i(\ddot{y}(t) - \ddot{y}_i(t)) + \sum_{j=1}^{N} a_{ij}(y_j(t) - y_i(t)) \tag{14}
\]

Here \( b_i > 0 \) if agent \( i \) has access to the reference signal, and otherwise \( b_i = 0 \). The following result is a counterpart of Theorem 7 for reference-tracking synchronization.

**Theorem 11.** Let Assumption 10 hold and further assume that: for all \( j \in \{1, \ldots, N\} \) agent \( j \) is IFP(\( \alpha_j \), \( G[A] \)) strongly connected, at least one agent has access to the reference signal, i.e. \( \sum_i b_i > 0 \), and the couplings are sufficiently weak, i.e.

\[
\alpha_j(d_j^+[A] + 2b_j) < 1/2 \quad \forall j = 1, \ldots, N. \tag{15}
\]

Then, the following two statements hold:

1. Any solution of the system (6),(14), prolongable to \( \infty \), is output \( L_2 \)-synchronized with the reference signal (10); in particular, \( \int_0^\infty |u_i(t) - u_j(t)|^2 dt < \infty \).
2. If for all \( j \) the functions \( v_j \) are radially unbounded, the maps \( f_j \) are \( C^1 \)-smooth, the Jacobians \( \frac{\partial f_j}{\partial y_j}, \frac{\partial f_j}{\partial \bar{y}_j} \) are uniformly bounded, and the maps \( h_j \) are linear: \( h_j(\xi_1 - \xi_2) = h_j(\xi_1) - h_j(\xi_2) \); then, any solution of the closed-loop system (6),(14) is prolongable to \( \infty \) and output synchronized (8) with the reference signal.

### 3.3 Examples of IFP agents

In this subsection, examples of IFP agents are provided.

**SISO agents with a pole at zero**

Consider a SISO system

\[
s\rho(s)\zeta(t) = u(t) \in \mathbb{R}, \quad s = \frac{d}{dt}, \quad \rho(\lambda) = \sum_{k=0}^{r} \rho_k \lambda^k; \tag{16}
\]

\[
y(t) = \eta(s)\zeta(t), \quad \eta(\lambda) = \sum_{k=0}^{r} \eta_k \lambda^k
\]

**Lemma 12.** Assume that \( \rho(s) \) is a Hurwitz polynomial and \( \eta_0 \rho_0 \geq 0 \). Then the system (16) is IFP(\( \alpha \)) for sufficiently large \( \alpha \geq 0 \). Denoting the transfer function from \( u \) to \( y \) by \( W(\lambda) = \eta(\lambda)/(\lambda \rho(\lambda)) \), the passivity index can be found as

\[
\alpha = -\inf_{\omega \in \mathbb{R}} \text{Re } W(i\omega). \tag{17}
\]

For instance, Lemma 12 implies that the system (13) is IFP (in this case, \( \rho(\lambda) = \lambda^2 + p\lambda + q \) is Hurwitz since \( p, q > 0 \) and \( y(t) = \xi(t) \)).

**First-order delayed integrators**

Consider now a delayed system:

\[
\ddot{y}(t) = u(t - \alpha) \in \mathbb{R}^m. \tag{18}
\]

Here \( \alpha \geq 0 \) is a constant delay and we assume, by definition, that \( u(t) \equiv u_0(t) \) for \( t \in [-\alpha; 0] \), where \( u_0 \in L_2([-\alpha, 0] \rightarrow \mathbb{R}^m) \) is a given function. The vector \( \ddot{y}(0) \) and the function \( u_0 \) are the initial conditions for the system (18). Formally, our definition of IFP deals with ordinary differential equations (1) only and is not applicable to delay systems. However, the following weaker condition holds for (18), see (Proskurnikov, 2016, p.141, proof of Lemma 7.3).

**Lemma 13.** For any solution of (18) one has

\[
\int_0^T (y(t)^\top u(t) + \alpha |u(t)|^2) dt \geq -\mathcal{V} \quad \forall T \geq 0, \tag{19}
\]

where \( \mathcal{V} = \mathcal{V}(y(0), u_0(\cdot)) \geq 0 \) is independent of \( T \).

Lemma 13 allows for the group of linear delayed agents

\[
\ddot{y}_i(t) = u_i(t - \alpha_i), \quad i = 1, \ldots, N, \tag{20}
\]

the protocol (11) provides output synchronization (7) and \( L_2 \)-synchronization (9), whenever the graph \( G[A] \) is strongly connected and (12) holds.

**Remark 15.** In the monograph Tian (2012) a more general result is formulated without a complete proof (Theorem 7.10), stating that under assumptions of Theorem 14 synchronization is retained if the graph is not strongly connected but has a directed spanning tree.

**Remark 16.** Theorem 11 also holds for agents (20). However, Assumption 10 becomes impractical since each agent has to be aware of \( \ddot{u}_i(t) = \ddot{y}(t + \alpha_i) \) at time \( t \), which makes the controller (14) non-causal. The protocol (14) may still be used in the case where the reference signal is linear \( \ddot{y}(t) = v_0 t + \ddot{y}(0) \) and \( v_0 \) is known, but \( \ddot{y}(0) \) is uncertain.

## 4. SYNCHRONIZATION IN VEHICLE PLATOONING AND TRAFFIC FLOW MODELING

In this section, we consider two practical applications of the synchronization criteria from Section 3.

### 4.1 Stability of a microscopic traffic flow model

A basic problem in vehicular traffic is the prevention of congestions and accidents. Microscopic traffic flow models are often employed to represent the traffic flow as a result of cooperation between individual drivers. Since the pioneering work of Chandler et al. (1958), the delay in drivers reaction has been recognized as a crucial factor participating into the overall flow dynamics. The simplest model of this kind (Chandler et al., 1958; Sipahi et al., 2007) deals with \( N \) vehicles, indexed 1 through \( N \), traveling along a common straight or circular single lane road (their order remains unchanged since overtaking is not possible). Each

\[
\frac{d}{dt} v(t) = -p(v(t) - \bar{v})(v(t) - \bar{v}) + q(v(t) - \bar{v}), \quad v(t) \geq \bar{v}
\]
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Connected and (12) holds. Then the vehicles’ velocities are bidirectional. In this subsection, we examine a CACC algorithm with each vehicle has information only about the predecessor. The model (22) allows, in particular some drivers to make each vehicle keep the safe distance to the predecessor vehicle. In the case of straight road, \( v_0(t) \equiv v_0 \) is the desired velocity with respect to the leading vehicle 1; for a circular road, \( v_0(t) \equiv v_N(t) \), i.e. vehicle 1 follows vehicle \( N \). A key issue addressed via this model Chandler et al. (1958) is that of the stability of the “synchronous” manifold: \( v_1 = \ldots = v_N \).

For the straight road case a necessary and sufficient condition for such a synchronization: \( 2aK < 1 \) was found in (21). We extend this classical result to the traffic flow model with a general directed interaction topology and heterogeneous delays and sensitivities of the drivers.

\[ \dot{v}_i(t) = u_i(t) - \alpha_i, \quad u_i(t) = \sum_{j=1}^{N} a_{ij}(v_j(t) - v_i(t)), \quad \forall i. \]  

The model (22) allows, in particular some drivers to respond to the change not only in the predecessor’s, but also in the follower’s velocity, or use the information about several predecessors and followers. The following theorem gives a criterion of velocity synchronization in (22) under the assumption of a strongly connected topology, which holds e.g. for uni- and bidirectional ring coupling (circular road). The gain \( a_{ij} \geq 0 \) in (22) stands for the sensitivity of driver \( i \) to changes in the speed of vehicle \( j \). Theorem 14, applied to \( y_i = v_i \), yields the following corollary:

**Corollary 17.** Suppose that the graph \( G[A] \) is strongly connected and (12) holds. Then the vehicles’ velocities are asymptotically synchronized \( v_i(t) - v_j(t) \to 0 \) \( t \to \infty \).

### 4.2 An application to cooperative adaptive cruise control

In this subsection we demonstrate an application of Theorem 11 to the stability of a platoon of vehicles (Fig. 2), constituted by the leading vehicle 0 and \( N \) follower vehicles, indexed 1 through \( N \) (Fig. 1). Cooperative adaptive cruise control (CACC) system implements a control algorithm, making each vehicle keep the safe distance to the predecessor and follow the leader’s velocity. The interaction topology between the vehicles may be different (Zheng et al., 2016); the most studied is a unidirectional topology, where each vehicle has information only about the predecessor.

In this subsection, we examine a CACC algorithm with bidirectional interactions. The advantages of bidirectional platooning algorithms over unidirectional ones are discussed e.g. in (Zhang et al., 1999; Barooah et al., 2009; Zheng et al., 2016) (see also references therein); in many senses such algorithms are more robust against disturbances propagating through the platoon (“string-stable”). We examine the CACC algorithm, proposed in (Barooah et al., 2009). The leader’s speed \( v_0(t) \equiv v_0 \) is broadcasted to every follower (Fig. 1). Besides this, the vehicles 1 through \( N-1 \) measure the distances to both their predecessors and followers, and the rear vehicle \( N \) measures the distance to its predecessor. Denoting the position of vehicle \( i \)’s rear bumper by \( q_i \in \mathbb{R} \) (see Fig. 2), the goal of the CACC algorithm is to keep the desired distance to the predecessor and the desired velocity, i.e. \( q_{i-1}(t) - q_i(t) \to s_i, \quad v_i(t) = \dot{q}_i(t) \to v_0 \).

As usual in CACC problems (Zhang et al., 1999; Zheng et al., 2016), the follower vehicles obey linear models

\[ \tau_i \ddot{q}_i + \dot{q}_i = a_{i,\text{des}}(t), \]

where \( a_{i,\text{des}} \) is the desired acceleration and \( \tau_i \) is a time constant, depending on the vehicle’s powertrain. The vehicles 1 through \( N-1 \) apply the following controller:

\[ a_{i,\text{des}}(t) = \mu_i(v_0 - v_i(t)) + \eta_i(q_{i-1}(t) - q_i(t) - s_i) + v_i(q_{i+1}(t) - q_i(t) + s_{i+1}), \quad 1 \leq i \leq N-1, \]

Vehicle \( N \) is controlled similarly, but has no follower

\[ a_{N,\text{des}}(t) = \mu_N(v_0 - v_N(t)) + \eta_N(q_{N-1}(t) - q_N(t) - s_N). \]

**Theorem 18.** Let \( \mu_i, \tau_i < \frac{1}{2} \) and \( \eta_i, \nu_i > 0 \) satisfy

\[ \frac{\mu_i^2}{\tau_i} > \begin{cases} \frac{\eta_i + \nu_i}{2}, & 1 < i < N; \\ \eta_i, & i = 1; \\ \eta_N, & i = N. \end{cases} \]

Then the algorithm (25), (26) provides (23).

The result of Theorem 18 can be extended to some cases of nonlinear vehicles’ dynamics, where the inner-loop engine and torque controllers (Zhang et al., 1999) fail to attenuate the nonlinearities. Notice that Theorem 18 does not address the string stability problem, i.e. the robustness of CACC against small disturbances in measurements as \( N \) becomes large; the analysis of string stability is based on other techniques and is beyond the scope of this paper.

### 5. CONCLUSIONS AND FUTURE WORK

In this paper, we offer simple distributed protocols for synchronization of heterogeneous non-passive agents that satisfy an IPF property. We apply the obtained results to analysis of microscopic traffic flow models and CACC algorithms for heterogeneous platoons. The results can be extended to some nonlinearly coupled networks, time-varying graphs and “coopetitive” networks, where agents can both cooperate and compete (Proskurnikov and Cao, 2016). The robustness of synchronization algorithms against delays and noises is a subject of ongoing research.
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