Triton photodisintegration in three-dimensional approach

M. A. Shalchi and S. Bayegan

Department of Physics, University of Tehran, P.O.Box 14395-547, Tehran, Iran
e-mail: shalchi@khayam.ut.ac.ir

Received: date / Revised version: date

Abstract. Two- and three-particle photodisintegration of the triton is investigated in a three-dimensional (3D) Faddeev approach. For this purpose the Jacobi momentum vectors for three particles system and spin-isospin quantum numbers of the individual nucleons are considered. Based on this picture the three-nucleon Faddeev integral equations with the two-nucleon interaction are formulated without employing the partial wave decomposition. The single nucleon current as well as \( \pi \) - and \( \rho \)-like exchange currents are used in an appropriate form to be employed in 3D approach. The exchange currents are derived from AV18 NN force. The two-body t-matrix, Deuteron and Triton wave functions are calculated in the 3D approach by using AV18 potential. Benchmarks are presented to compare the total cross section for the two- and three-particle photodisintegration in the range of \( E_\gamma < 30 \text{ MeV} \). The 3D Faddeev approach shows promising results.

PACS. 21.45.+v, 21.30.-x, 21.10.Dr, 27.10.+h, 21.10.Hw, 25.20.-x

1 Introduction

The study of 3N system with the electromagnetic interaction shed light on the ambiguity of different kinds of nuclear forces participating in the few body systems. Following the introduction of Faddeev formulation for the three-body system [11-2], a new effort using this scheme was started. The electrodisintegration [3] and photodisintegration [4] of \( ^3\text{He} \) and \( ^3\text{H} \) in the Faddeev scheme were performed as early attempts in this respect. The photodisintegration calculation based on Faddeev formalism for 3N bound and continuum with the same 3N Hamiltonian were performed with quite simple separable NN interactions [5]. The other approaches in this field can be considered to be Green Function Monte-Carlo [6] and Lorentz Integral Transform [7] methods.

The photodisintegration of \( ^3\text{He} \) was investigated by pair-correlated hyperspherical harmonics method using AV18 NN and Urbana 3N forces [8]. Also, a systematic application of Faddeev formalism was introduced, based on the partial wave decomposition, for the calculation of the photodisintegration of \( ^3\text{He} \) and \( ^3\text{H} \) with the two- and three-body forces [9].

We use the novel three-dimensional (3D) approach [10] [11,12,13,14,15,16,17,18,19,20]. In this approach we employ the momentum-vector variables as basis states. We derive the Faddeev integral equation in a realistic 3D scheme as a function of Jacobi momentum vectors and the spin-isospin quantum numbers. We implement this 3D scheme in the two- (Nd) and three-body (3N) photodisintegration of \( ^3\text{H} \). This 3D approach avoids truncation problems and the necessity of complicated recoupling algebra that accompanies partial wave based calculations.

It is our aim to calculate the nuclear matrix element which is given by

\[
N^{3N} = \frac{1}{2} \langle \phi_0 | P | U \rangle \quad \text{or} \quad N^{Nd} = \frac{1}{2} \langle \phi_0 | P | U \rangle \quad \text{for} \quad \gamma^3H \rightarrow 3N \quad \text{or} \quad \gamma^3H \rightarrow Nd, \quad \text{respectively.}
\]

We introduce \( \langle -| \phi_0 \rangle \) and \( \langle \phi_4 | \) as outgoing 3N and Nd system, \( P \) as permutation operator and \( |U\rangle = (1+P)\langle O|\psi \rangle + (tG_\rho P + \ldots)|U\rangle \) as an auxiliary state with \( O \) as the current operator. The wave function \( \psi \) is the three-body bound state, \( t\) is the two body transition operator and \( G_\rho \) is free propagator. The second part of \( |U\rangle \) is related to the solution of \( |U\rangle \) of this equation. We introduce a formalism to solve this equation directly in the momentum-vector variables avoiding the partial wave (PW) decomposition scheme.

We use the AV18 potential for the calculation of wave functions of the triton and deuteron as well as the two-body t-matrix. For nuclear electromagnetic current operator we choose, in addition to the single-nucleon current, the two-body contribution in the form of the \( \pi \) - and \( \rho \)-meson exchanges with connection to NN force AV18.

This manuscript is organized as follow: In section 2 formulation of nuclear matrix equation in the 3D approach is introduced and \( |U\rangle \), as an auxiliary state, is derived for the two-body interaction with handling of singularity problem. In this section the appropriate formulation is constructed for the one- and two-body currents to be implemented in the 3D formalism. In section 3 numerical method and results are presented and section 4 is concluded by a summary and outlook.
2 Formulation of N matrix equation in a 3D approach

Our aim is to calculate the total cross section of the Triton photodisintegration. Final states can be either Nd or free 3N state. For each we can write the cross section as a function of the tensor components of the nuclear matrix element, which is the nuclear current sandwiched between initial bound and final scattering states:

\[
\sigma_t^{Nd} = \int dk_n(2\pi)^4 \frac{k_n^2}{2Q |k_n|} \sum_{m,m_1,m_2} \frac{1}{(N_{+}^{Nd}|^2 + |N_{-}^{Nd}|^2)},
\]

(1)

\[
\sigma_t^{3N} = \int dk_1dk_2dk_3 \frac{1}{2Q} \sum_{m,m_1,m_2,m_3} \frac{2\pi^2}{Q^\alpha} \frac{k_1^2k_2^2m_N}{|Q \cos \theta_2 - k_1 \cos \theta_1 - 2k_2|} (|N_+^{3N}|^2 + |N_-^{3N}|^2). \]

(2)

In the above equations, \( m \) is the nucleon mass, \( \alpha \approx \frac{1}{177} \) is the fine-structure constant, \( k_n \) and \( k_d \) are the neutron and deuteron momenta, respectively. \( k_s \) is free particle momenta and \( \theta_1 \) are their angles with \( Q \) as the momentum of the photon. \( \theta_2 \) is the angle between momenta \( k_1 \) and \( k_2 \). There is also a summation over all the spins of ingoing and outgoing particles which are indicated by \( m, m_d, m_1, m_2 \) and \( m_3 \).

To obtain the nuclear matrix element we follow the formalism in [9].

\[
N^{3N} = \frac{1}{2} \langle |\phi_0|P|U\rangle,
\]

(3)

\[
N^{Nd} = \frac{1}{2} \langle |\phi_d|P|U\rangle,
\]

(4)

where \( |\phi_0\rangle \) is the three particles scattering state as;

\[
|\phi_0\rangle^{(-)} = (1 + G_0 t)|\psi_0\rangle,
\]

(5)

so we can write:

\[
N^{3N} = \frac{1}{2} [\langle |\phi_0|P|U\rangle + \langle |\phi_0|tG_0P|U\rangle]
\]

(6)

and \( |\phi_d\rangle \) is Nd state

\[
|\phi_d\rangle = |qm_1\nu_1\rangle|\psi_d\rangle,
\]

(7)

where \( \psi_d \) is the deuteron wave function, and \( |U\rangle \) is the auxiliary state that can be calculated by solving the following equation.

\[
|U\rangle = (1 + P)J|\psi\rangle + tG_0P|U\rangle.
\]

(8)

|\psi\rangle is the three-body bound state, \( J \) is the current, \( G_0 \) is the 3N free propagator, \( t \) is the two-body transition operator and \( P \) is the permutation operator.

2.1 Introducing free basis states

We introduce \( |\phi_0\rangle \) as our free basis states where the particles 2 and 3 are in subsystem and particle 1 is the spectator. The state is antisymmetric under permutation of subsystem particles.

\[
|\phi_0\rangle = |pm_2m_3\nu_2\nu_3\rangle|qm_1\nu_1\rangle = |pqm_1m_2\nu_2\nu_3\nu_1\rangle^a,
\]

(9)

where \( m_1 \) and \( \nu_1 \) are spin and isospin of the individual nucleons and \( p \) and \( q \) are Jacobi momentum for a three-particle system.

\[
p = \frac{1}{2}(k_2 - k_3),
\]

(10)

\[
q = \frac{2}{3}[k_1 - \frac{1}{2}(k_2 + k_3)],
\]

(11)

so we have:

\[
|pqm_1m_2\nu_2\nu_3\nu_1\rangle = \frac{1}{2}(|pqm_1m_2\nu_1\nu_2\nu_3\rangle + |pqm_1m_2\nu_2\nu_3\nu_1\rangle).
\]

(12)

2.2 Nuclear matrix in free basis states

We now apply these free basis states to the left hand side of each term of the Eq. [5].

\[
\langle \phi_0|U\rangle = a^*(pqm_1m_2m_3\nu_1\nu_2\nu_3|U|)
\]

(13)

In the next step we need to know the effect of the permutation operator on the free basis states.

\[
P|pqm_1m_2m_3\nu_1\nu_2\nu_3\rangle^a
\]

\[
= |(\frac{1}{2} - \frac{3}{4}q)(p - \frac{1}{2}q)m_2m_3\nu_2\nu_3\nu_1\rangle^a
\]

\[
+ |(\frac{1}{2} - \frac{3}{4}q)(-p - \frac{1}{2}q)m_3m_2\nu_2\nu_1\nu_3\rangle^a,
\]

(14)

so the first part of Eq. [13] is

\[
\langle \phi_0|U\rangle = a^*(pqm_1m_2m_3\nu_1\nu_2\nu_3|U|)
\]

(15)

\[
= a^*(-(\frac{1}{2} - \frac{3}{4}q)(p - \frac{1}{2}q)m_2m_3m_1\nu_2\nu_3|U|)
\]

In the evaluation of the first term of Eq. [13] we can write

\[
a^*(pqm_1m_2m_3\nu_1\nu_2\nu_3|1 + P|J|\Psi\rangle
\]

(16)
Evaluation of each term of Eq. (16) is similar and represents generally, by using completeness relation and inserting the free basis states in a suitable position:

\[ a(pq m_1 m_2 m_3 \nu_1 \nu_2 \nu_3) J[\bar{\psi}] = \sum_{m_\nu} \int dp' dq' \times \langle \bar{p} q' m_1' m_2' m_3' \nu_1' \nu_2' \nu_3' | a \rangle, \]

\[ \times \psi_{m_1' m_2' m_3'}(p', q'), \]

(17)

triton wave function in the free basis states is introduced symbolically as follows:

\[ \psi_{m_1 m_2 m_3}(p, q) = \langle \bar{p} q m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 | \bar{\psi} \rangle. \]

(18)

For the second part of Eq. (13) we have:

\[ \langle \phi_0 | tG_0 P | U \rangle = \langle \bar{p} q m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 | tG_0 P | U \rangle \]

\[ = \sum_{m'_\nu, \nu} \int dp dq dq'' dq''' \times \langle \bar{p} q m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 | tG_0 P | U \rangle \]

\[ \times \sum_{m'_\nu, \nu} \int dp'' dq'' dq''' dq'''' \times \langle \bar{p} q m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 | tG_0 P | U \rangle \]

\[ \times \psi_{m_1' m_2' m_3'}(p'', q''), \]

(19)

two important terms in Eq. (19) to be evaluated. The first term including \( tG_0 \) operators can be written in terms of two-body t-matrices in the free basis-states multiplied by free propagator, \( G_0 = \frac{1}{E - m} \), in the energy of subsystem. For the second term of Eq. (19), one can proceed to the final form by considering the properties of the permutation operator, \( P \) (Eq. (14)), and the symmetry properties of the free basis states. The final form of the second part of Eq. (13) is evaluated as follows:

\[ \langle \phi_0 | tG_0 P | U \rangle = \langle \bar{p} q m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 | tG_0 P | U \rangle \]

\[ = 2 \sum_{m'_\nu, \nu} \int dq'' dq''' \times \langle \bar{p} q m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 | tG_0 P | U \rangle \]

\[ \times \langle \bar{\nu} m_3 \nu_1 \nu_2 \nu_3 | \bar{p} q' m_1' m_2' m_3' \nu_1' \nu_2' \nu_3' | U \rangle \]

\[ \times \psi_{m_1' m_2' m_3'}(p', q'), \]

(20)

where the shifted argument \( \Pi_1 \) and \( \Pi_2 \) are defined as follows:

\[ \Pi_1 = q' + \frac{1}{2} q'', \quad \Pi_2 = q'' + \frac{1}{2} q'. \]

(21)

We write for Eq. (13) with the help of Eq. (21):

\[ \langle \bar{p} q m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 | tG_0 P | U \rangle \]

\[ = 2 \sum_{m'_\nu, \nu} \int dq'' dq''' \times \langle \bar{p} q m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 | (1 + P) J[\bar{\psi}] \rangle \]

\[ \times \psi_{m_1' m_2' m_3'}(p'', q'''), \]

(22)

where the first term has been described before in Eq. (16). Finally the calculation of \( N^{3N} \) after inserting Eqs. (15) and (20) in Eq. (19) leads to:

\[ N^{3N} = \frac{1}{2} \langle a | (-\frac{1}{2} p - \frac{3}{4} q) (p - \frac{1}{2} q) m_2 m_1 m_3 \nu_1 \nu_2 \nu_3 | U \rangle \]

\[ + \langle a | (-\frac{1}{2} p + \frac{3}{4} q) (p - \frac{1}{2} q) m_3 m_1 m_2 \nu_1 \nu_2 | U \rangle \]

\[ + \sum_{m'_\nu, \nu} \int dq'' dq''' \times \langle \bar{p} q m_2 m_3 \nu_2 \nu_3 | t - q'' - 3q, m_2 m_3 \nu_1 \nu_2 | U \rangle \]

\[ \times \psi_{m_1 m_2 m_3'}(p', q'), \]

(23)

we proceed to \( N^{Nd} \) calculation by inserting the completeness relation of the free basis states and Eq. (15) into:

\[ N^{Nd} = \frac{1}{2} \sum_{m'_\nu, \nu} \int dp dq dq'' dq''' \times \langle \bar{p} q m_1 m_3 \nu_1 \nu_2 \nu_3 | P | U \rangle \]

\[ \times \langle \bar{\nu} m_3 \nu_1 \nu_2 \nu_3 | \bar{p} q' m_1' m_3' \nu_1' \nu_2' \nu_3' | U \rangle \]

\[ \times \psi_{m_1' m_3'}(p', q'), \]

(24)

2.3 Singularity problem and rewriting Eq. (22)

We know that the two-body t-matrix has a simple singularity at \( E = E_d \), which is the deuteron binding energy. There is also a moving singularity in Eq. (20), which is very difficult to handle. To solve this moving singularity problem we use the method introduced in Ref. [21] and we separate the radial part from the angle part in the Dirac delta functions:

\[ \delta(p' + \Pi_2) \delta(p'' - \Pi_1) \]

\[ = \frac{\delta(p' - \Pi_2) \delta(p'' - \Pi_1) \delta(p' + \Pi_2) \delta(p'' + \Pi_1)}{p'^2 - p''^2}, \]

(25)

then we write:

\[ \langle \bar{p} q m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 | tG_0 P | U \rangle \]

\[ = 2 \sum_{m'_\nu, \nu} \int dq'' dq''' t_{m_1 m_2 m_3 \nu_1 \nu_2 \nu_3} m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 \]

\[ \times U_{m_1' m_2' m_3'}(p'', q'') \times \delta(p' - \Pi_2) \delta(p'' - \Pi_1) \]

\[ \times \psi_{m_1' m_2' m_3'}(p'', q''), \]

in the above equation we used:

\[ t_{m_1 m_2 m_3 \nu_1 \nu_2 \nu_3} m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 \]

\[ = \langle \bar{p} q m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 | t' \bar{\Pi}_2 \rangle, \]

(27)
and
\[ U_{m_1m_2m_3}^{\nu_1\nu_2\nu_3}(p(\vec{n}I_1),q) = a(p(\vec{n}I_1))q m_1 m_2 \nu_1 \nu_2 \nu_3 |U\rangle, \] (28)
we have for the radial delta function:
\[ \delta(p' - \Pi_2)\delta(p'' - \Pi_1) = \delta(p' - \frac{1}{4}q^2 + q'^2 + qq''x'') \]
\[ \times \delta(p'' - \sqrt{p'^2 + \frac{3}{4}q^2 - \frac{3}{4}q'^2}), \] (29)
we can write by using the property of the delta function:
\[ \delta(p' - \sqrt{\frac{1}{4}q^2 + q'^2 + qq''x''}) = \frac{2p'}{qq''} \delta(x'' - x_0), \] (30)
where
\[ x_0 = \frac{p'^2 - \frac{1}{4}q^2 - q'^2}{qq''}. \] (31)
\[ x_0 \text{ is } \cos \theta_{q''} \text{ and it is expressed in this interval: } -1 \leq x_0 \leq +1. \]

We use the Eqs. (29) and (30) in Eq. (20), therefore, the final form of Eq. (22) can be rewritten as follows:
\[ a(pq_{m_1m_2m_3\nu_1\nu_2\nu_3}) = a(pq_{m_1m_2m_3\nu_1\nu_2\nu_3})(1 + P)J|\Psi\rangle \]
\[ = \frac{4}{q - E - E_d - \frac{3q}{4m} m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6} \]
\[ \times \int \frac{dp'}{E - \frac{3q}{4m} p' - p''} \int_1^{1 + p'} dq'' q'' \int_1^{2\pi} dx'' \int_0^{2\pi} d\phi_{q''} \delta(x'' - x_0) \]
\[ \times \sum_{m_1m_2m_3m_4m_5m_6} \left(p', p(\vec{n}I_2)\right) \psi_{m_1m_2m_3}^{\nu_1\nu_2\nu_3\nu_4\nu_5\nu_6}(p'') (\vec{n}I_1), q''), \] (32)
where
\[ p'' = \sqrt{p'^2 + \frac{3q'^2 + 3q^2}{4}}, \] (33)
we have used the following relation to extract the singularity from two-body t-matrix:
\[ \bar{t}(E) = \frac{t(E)}{E - E_d}. \] (34)

### 2.4 Current

The current is consisted of single-nucleon and two-nucleon currents
\[ J = J^I + J^{II}, \]
\[ J^I = J^I(1) + J^I(2) + J^I(3), \]
\[ J^{II} = J^{II}(1, 2) + J^{II}(2, 3) + J^{II}(3, 1). \] (35)

According to the following symmetry relation one can only consider \( J^I(1) \) and \( J^{II}(2, 3) \) in Eq. (22), i.e.
\[ a(pq_{m_1m_2m_3\nu_1\nu_2\nu_3})(1 + P)J|\Psi\rangle \]
\[ = 3q(pq_{m_1m_2m_3\nu_1\nu_2\nu_3})(1 + P)J^I(1) + J^{II}(2, 3)|\Psi\rangle, \] (36)
the matrix elements of the single-nucleon current i.e. \( J^I(1) \) in the free basis states is independent of the initial and final two-body subsystem, and the initial and final spectators Jacoby momentum vectors are related by the delta function \( \delta(q - q' - \frac{3}{4}Q) \). So the matrix element of the single-nucleon current takes the following expression with the help of Eq. (13) and Eq. (36) and is evaluated as follows:
\[ a(pq_{m_1m_2m_3\nu_1\nu_2\nu_3})(1 + P)J^I|\Psi\rangle \]
\[ = \delta(q' - q + \frac{1}{3}Q)\delta_{\nu_1\nu_2} J_{m_1m_2m_3m_4m_5m_6}^{I|\psi}\left(p, p'\right). \] (38)

For two-body current we have:
\[ a(pq_{m_1m_2m_3\nu_1\nu_2\nu_3})(1 + P)J^{II}|\Psi\rangle \]
\[ = \frac{3}{4m} \int dp' \]
\[ \times \left\{ J_{m_1m_2m_3m_4m_5m_6}^{II, \nu_1\nu_2\nu_3\nu_4\nu_5\nu_6}(p, p') \psi_{m_1m_2m_3m_4m_5m_6}(p', q - \frac{1}{3}Q) \right\}. \] (39)

For the meson exchange current we have taken into account the momentum transfer of the two nucleons:
\[ q_2 = k_2 - k'_2 = \frac{1}{2}Q + p - p', \]
\[ q_3 = k_3 - k'_3 = \frac{1}{2}Q - p + p', \] (40)
where \( k_2, k'_2, k_3, k'_3 \) are the initial and final momenta of the nucleons 2 and 3.

Single nucleon current in nonrelativistic limit as well as the two-body meson exchange currents are introduced and evaluated in the free basis states by suitable expressions in the Appendix A.
\textbf{3 Numerical methods and results}

In order to calculate the total cross sections by Eqs. (1) and (2), we need to obtain the nuclear matrix elements using Eqs. (23) and (24). In fact, we have to calculate the auxiliary state $|U\rangle$ in the free basis states using Eq. (32). To solve this integral equation and also Eqs. (1), (2), (23) and (24), we first need to choose the appropriate coordinate system. Because of the current properties, We have to choose the photon momentum vector, $Q$, to be in z direction, which is also the spin-quantization axis. This selection does not impose any restrictions because in all of the above equations $Q$ is a constant vector. The $p$ and $q$ vectors like any free vectors will be determined with three components. Finally, except for the energy of photon which is constant, we need six independent variables to specify $U$ function in Eq. (32) in terms of $p$ and $q$ vectors.

\begin{equation}
U(p, q) = U(p, q, \theta_p, \theta_q, \phi_p, \phi_q).
\end{equation}

In appendix A The relation between variables of each term of Eq. (32) and six variables of $U$ function is demonstrated.

For the polar angles which vary from 0 to $\pi$ the sign of the sine is always positive and we can introduce $x_0 = \cos \theta$ as an independent variable. However, for the azimuthal angle, which is in the interval $[0, 2\pi]$, the sign of the sine can not be specified as a function of $x_0 = \cos \phi$ because in each intervals of $0 < \phi < \pi$ or $\pi < \phi < 2\pi$ we have different signs for sine. So it is difficult to specify the value of any function in terms of $x_0 = \cos \phi$.

In order to save computing time and computational memory we need to define two functions for $U$ in each interval.

\begin{equation}
U(\phi) = \begin{cases} 
U_1(\phi) & \text{if } 0 < \phi < \pi \\
U_2(\phi) & \text{if } \pi < \phi < 2\pi 
\end{cases}.
\end{equation}

$U_1$ and $U_2$ are defined as:

\begin{equation}
U_1(\phi) = F(\cos \phi, \sqrt{1 - \cos^2 \phi}),
\end{equation}

\begin{equation}
U_2(\phi) = F(\cos \phi, -\sqrt{1 - \cos^2 \phi}),
\end{equation}

We write Eq. (32) in terms of the independent variables of the $U$ function in an appropriate expression for general numerical iteration. We consider the $U$ operator in a general form:

\begin{equation}
|U\rangle = |U_0\rangle + K|U\rangle = |U_0\rangle + K|U_0\rangle + K^2|U_0\rangle + ...,
\end{equation}

where $|U_0\rangle$ is the current term and $K$ is an integral kernel, then we apply the kernel $K$ to generate the finite Neumann series up to $(i - 1)$th order in $K$. This Neumann series can be summed up using Padé approximation to get $U_{\text{Pade}}^{(i)}$. We get $U_{\text{Pade}}^{(i)}$ from one more iteration. The definition of the distance between $U_{\text{Pade}}^{(i-1)}$ and $U_{\text{Pade}}^{(i)}$ is:

\begin{equation}
\Delta_i = \frac{\sum |U_{\text{Pade}}^{(i)} - U_{\text{Pade}}^{(i-1)}|^2}{\sum |U_{\text{Pade}}^{(i)}|^2},
\end{equation}

where, the summation runs over all six-dimensional grid points. We continue the iteration to reach $\Delta_i^2 < \epsilon$. $\epsilon$ is a small number determined by the desired accuracy. In our work $n = 10$ and $\epsilon = 10^{-2}$.

According to Eqs. (32) and (13) we need the two-body t-matrix and triton wave function in the free basis states. The extraction of singularity from two-body t-matrix in deuteron binding energy (appendix D) shows that we also need the deuteron wave function in the free basis states. For calculation of the two-body t-matrix, deuteron and triton wave functions we follow the 3D approach introduced in appendix C and we recalculate them in the free basis states using AV18 potential [23]. In the calculation of the Eq. (32) we interpolate the calculated data of the t-matrix, deuteron and triton wave functions using Cubic-Hermite spline method [24]. In the numerical treatment the momenta and angles variables should be transformed to certain discrete values. For the AV18 potential we use the Gaussian quadrature grid points with the hyperbolic mapping for the lower momentum and linear mapping for the higher momentum. The numerical Fourier-Bessel transformation of the potential encounters difficulties in handling at the very high momentum, so it is necessary to use a cut off in the integration interval at $150 \text{fm}^{-1}$.

The meson exchange currents (MECs) were restricted to $\pi$-like and $\rho$-like exchanges. These MECs are derived from AV18 Based on the Riska’s recipe [25].

We compare the 3D Faddeev calculation, including the explicit MECs and the PW representation of Faddeev calculation, with Siegert theorem [26]. We have shown that the 3D approach with the continuous angle variables instead of the discrete angular momentum quantum numbers in evaluation of the nuclear matrix elements for $\gamma^3H \rightarrow Nd$ and $\gamma^3H \rightarrow 3N$ leads to less complicated expressions but with higher dimensionality of integral equations in comparison with the PW representation.

Calculation of the one-body current and the two-body current for $\sigma^{Nd}_t$ and $\sigma^{3N}_t$ are compared together, with experimental data in Figs. 1 and 3. The comparison re-confirms the enhancement of the three-nucleon photodisintegration cross section in the peak region and at the higher energies. The addition of the three-nucleon force can significantly lower the peak in $\sigma^{Nd}_t$ and $\sigma^{3N}_t$ and as a result gives better agreement with the available data. We have displayed in Figs. 3 and 4 that the Siegert and MEC predictions are too close in the lower energies of the photodisintegration of $3^1H \rightarrow Nd$ and $3^1H \rightarrow 3N$. However, the 3D calculation uses the single nucleon current with explicit use of $\pi$- and $\rho$-like mesons. We found discrepancies between the two predictions in the higher energies of the photodisintegration. We expect that in the higher energies of the photon, the use of meson exchange in 3D approach produces more sensible results.

In comparison with the experimental data (Ref [27]), Fig. 3, the 3D total cross section of $\gamma^3H \rightarrow Nd$ shows less agreement at low energies ($E_\gamma < 20 \text{MeV}$). The over-estimation shows the need for a three-nucleon force (3NF) effect. The results of Golak et. al [28] indicate the improvement by adding 3NF. The contribution of 3NF in the 3D...
calculations can be implemented by modifying the $|U|$ as an auxiliary state with the term appropriate for the 3NF adding to the two-body forces.

The data of (Ref. 27) at $E_\gamma = 20 - 30$ MeV due to the insufficient precision can not be compared with the theoretical calculations and no concrete conclusions can be reached.

The Skopic et. al data and Kosiak et al data (29,30) although nearly agree with the 3D calculations in $(E_\gamma = 15 - 28$ MeV), however more experimental data is needed to reach better conclusion.

In the case of $\gamma^3 H \rightarrow 3N$ the overestimation of the calculated total cross section in the low and medium energies in Fig. (4) is also predicted to be due to the absence of 3NF. At higher energies more experimental data is needed to overcome the discrepancies with the present day theories.

4 Summary and outlook

In this paper we have formulated the Faddeev integral equations for calculating the two- and three-body photodisintegration cross sections of the triton in a 3D approach. To this aim we have used the free basis states which contain Jacobi momentum vectors as well as individual spin and isospin of the nucleons. So we avoid to decompose the angular dependent in terms of the angular momentum quantum numbers, traditionally used to solve these kind of equations, i.e. partial wave approach. The final integral equations are less complicated than the similar partial wave integral equations and are unique in number of the equations in all energies. We have also explained how to overcome the moving singularity in Eq. (20) by the separation of the radial and angle parts of the Dirac delta function.

Using Eqs. (1) and (2) we have calculated the total cross section for 3N and Nd photodisintegration of the triton. Benchmarks for the three-nucleon total photodisintegration cross sections are presented in Figs. (3) and (4).

Although the classical approximation of photodisintegration cross section (predicted by Golak et. al. 28) and the 3D calculation with MECs are nearly in agreement for both $\gamma^3 H \rightarrow Nd$ and $\gamma^3 H \rightarrow 3N$ at low energies, the significance of the 3D based calculation with MECs can be tested further with the inclusion of the 3NF calculation.

Adding the three-body current as well as the three-body forces in our calculations are the other major future works to be done. We have calculated the two-body t-matrices using chiral potential in the 3D approach [17]. Therefore the calculation of three-body photodisintegration by this potential using the chiral currents is another area for consideration. The similar calculation for the radiative capture is also under preparation.

Acknowledgments

We would like to thank J. Golak and R. Skibinski for providing us the results of their calculations. This work was supported by the research council of the university of Tehran.

A matrix elements of current

Single nucleon current in nonrelativistic limit consists of convection and spin current terms:

$$J(1) = G_E(Q)\frac{k_1+k'_1}{2m_N} + \frac{i}{2m_N}G_M(Q)\sigma \times (k'_1-k_1),\quad (47)$$

Where $G_E$ and $G_M$ are electric and magnetic form factors of the nucleon, respectively. $k_1$ and $k'_1$ are the initial and final momentum of nucleon 1.

We used the matrix element of the one-body currents in the tensor component representation. Considering this equality:

$$k_1 + k'_1 = 2q - Q + \frac{2}{3}(k_1 + k_2 + k_3)$$

$$= 2q - Q + \frac{2}{3}K,$$  \quad (48)

we can rewrite the single nucleon current in the representation of the tensor components as follows:

$$J^{conv}_{\pm} = \frac{q_{1\pm}}{m_N}(G_E^R(Q)\Pi^P + G_E^I(Q)\Pi^I),\quad (49)$$

$$J^{spin}_{\pm} = -\frac{\sqrt{2Q}}{2m_N}S^P_{\pm}(G_M^R(Q)\Pi^P + G_M^I(Q)\Pi^I),\quad (50)$$

where

$$\Pi^P = |p/p|, \quad \Pi^I = |n/n|, \quad (51)$$

$$S^P_+ = |+\rangle\langle +|, \quad S^I_- = |-\rangle\langle -|, \quad (52)$$

and finally:

$$J^{+,\nu\nu'}_{m_1m'_1}(q, Q) = \left\{ \begin{array}{ll}
0 & \text{for } \nu \neq \nu' \\
\frac{q_{m1\nu}}{\sqrt{2m_N}} & \text{for } m' = m + 1 \\
-\frac{2m_N}{q_{m1\nu}}G_E^I(Q) & \text{for } m' = m - 1 \\
\end{array} \right., \quad (53)$$

$$J^{-,\nu\nu'}_{m_1m'_1}(q, Q) = \left\{ \begin{array}{ll}
0 & \text{for } \nu \neq \nu' \\
-\frac{2m_N}{q_{m1\nu}}G_M^R(Q) & \text{for } m' = m + 1 \\
0 & \text{for } m' = m - 1 \\
\end{array} \right.. \quad (54)$$

The following $\pi$ and $\rho$– meson exchange currents are introduced in (5) as follows:

$$j_\pi(q_2, q_3) = i\left[ G_E^R(Q) - G_E^I(Q) \right] (\tau_2 \times \tau_3) \frac{d_3}{d_3}$$

$$\times \left[ \sigma_0 \sigma_2 \cdot q_2v_x(q_2) - \sigma_2 \sigma_3 \cdot q_3v_x(q_3) \right]$$

$$+ \frac{q_2 - q_3}{q_2^2 - q_3^2} \left( v_x(q_3) - v_x(q_2) \right) \sigma_2 \cdot q_3 \sigma_3 \cdot q_3, \quad (55)$$

\text{where} q_2 \text{ and } q_3 \text{ are the momenta of the nucleons.}$$
and
\[ j_\rho(q_2, q_3) = i \left( G_E^p(Q) - G_E^n(Q) \right) (\tau_2 \times \tau_3)_3 \]
\[ \frac{q_2 - q_3}{q_2^2 - q_3^2} \left( v_\rho^S(q_3) - v_\rho^S(q_2) \right) \]
\[ - v_\rho(q_3) \sigma_2 \times (\sigma_3 \times q_3) - v_\rho(q_2) \sigma_3 \times (\sigma_2 \times q_2) \]
\[ - \frac{v_\rho(q_3) - v_\rho(q_2)}{q_2^2 - q_3^2} \left( (\sigma_2 \times q_2) \cdot (\sigma_3 \times q_3) (q_2 - q_3) \right) \]
\[ - \sigma_2 \cdot (q_2 \times q_3) (\sigma_3 \times q_3) \]. \hspace{8cm} (56)

The functions \( v_\pi(q), v_\rho^S(q) \) and \( v_\rho(q) \) can be extracted from the phenomenological AV18 two-nucleon interaction \[9\].

The matrix elements of the two-body current are also written in the representation of the tensor component. For the pion-exchange we can write:
\[ (p'q' | m_1 m_2 | m_3 m_4 | J_{\pi}^\alpha | pq m_1 m_2 m_3 m_4) = \delta_{m_1 m_3} \delta_{m_2 m_4} \delta(q' - q - \frac{1}{3} Q) \left( G_E^p(Q) - G_E^n(Q) \right) \]
\[ \left( \delta_{\nu_2 \nu_1} - \delta_{\nu_2 \nu_3 + 1} + \delta_{\nu_2 \nu_3 + 1} \right) \times \left\{ \sum_{m''} m'' D_{m'' m^\prime} (q_3 \hat{q}) D_{m^\prime m}^{+} (q_2) \right\} v_\pi(q_3) \]
\[ \pm 2 \sqrt{2} \delta_{m_3 - m_2, \pm 1} \left( \sum_{m''} m'' D_{m'' m^\prime} (q_3 \hat{q}) D_{m^\prime m}^{+} (q_2) \right) v_\pi(q_3) \]
\[ + (q_2 - q_3) \left( v_\rho(q_3) - v_\rho(q_2) \right) \]
\[ \times \left\{ \sum_{m''} m'' D_{m'' m^\prime} (q_3 \hat{q}) D_{m^\prime m}^{+} (q_2) \right\}. \hspace{8cm} (57) \]

For the matrix elements of \( \rho \)-exchange current in the tensor components form we can write:
\[ (p'q' | m_1 m_2 | m_3 m_4 | J_{\rho}^{\nu_1 \nu_2} | pq m_1 m_2 m_3 m_4) = \delta_{m_1 m_3} \delta_{m_2 m_4} \delta(q' - q - \frac{1}{3} Q) \left( G_E^p(Q) - G_E^n(Q) \right) \]
\[ \left( \delta_{\nu_2 \nu_1} - \delta_{\nu_2 \nu_3 + 1} + \delta_{\nu_2 \nu_3 + 1} \right) \times \left\{ \sum_{m''} m'' D_{m'' m^\prime} (q_3 \hat{q}) D_{m^\prime m}^{+} (q_2) \right\} v_\rho(q_3) \]
\[ - \left( \sum_{m''} m'' D_{m'' m} (q_3 \hat{q}) D_{m'' m}^{+} (q_2) \right) v_\rho(q_3) \]
\[ - (\delta_{m_3 - m_2, +1} + \delta_{m_3 - m_2, +1} + \delta_{m_3 - m_2, +1}) \left( 2 q_2 \pm 1 v_\rho(q_2) - 2 q_2 \pm 1 v_\rho(q_2) \right) \]
\[ \pm 2 \sqrt{2} \delta_{m_3 - m_2, \pm 1} \left( \sum_{m''} m'' D_{m'' m} (q_3 \hat{q}) D_{m'' m}^{+} (q_2) \right) v_\rho(q_3) \]. \hspace{8cm} (58)

In the above equations \( q_2 \) and \( q_3 \) are introduced in Eq. \[10\] and \( D_{m' m}(\hat{q}) \) is rotation matrix element for \( j = \frac{1}{2}, \alpha = \phi_\pi, \beta = \theta_\pi \) and \( \gamma = 0 \):
\[ D_{m' m}(\alpha \beta \gamma) = \langle jm'| R(\alpha \beta \gamma) | jm \rangle \]
\[ = e^{-im' \alpha} \delta_{m' m}(\beta) e^{-im \gamma}. \hspace{8cm} (59) \]

**B U function equation in details**

The U function Eq. \[32\] consists of single nucleon current (SNC), two-nucleon current (TBC) and a complicated part denoting by(I):
\[ U_{m_1 m_2 m_3}^{p q x p x q} (p, q, x_p, x_q, x_{\phi_p}, x_{\phi_q}) = SNC + TBC + I, \hspace{8cm} (60) \]
For the SNC part we can incorporate the one-nucleon current as follows:

\[
SNC = 3 \sum_{m'_{1}m'_{2}} \left\{ J^{I_{1}v_{1}v_{1}'}_{\pm m_{1}m_{2}}(q, x_{p}, x_{\phi}, y_{p}, x_{\phi}) \psi_{m_{1}m_{2}m_{3}}(p, q', x_{p}, x_{\phi}, x_{\phi}) + J^{I_{1}v_{1}v_{1}'}_{\pm m_{2}m_{1}}(q, x_{p}, x_{\phi}, y_{p}, x_{\phi}) \psi_{m_{3}m_{2}m_{1}}(p, q', x_{p}, x_{\phi}, x_{\phi}) \right\} \times \psi_{m_{1}'m_{1}2}(p, q', x_{p}, x_{\phi}, x_{\phi}),
\]

(61)

In term of the \( p, q, x_{p}, x_{\phi}, x_{\phi} \) we can write all the variable as follows:

\[
q' = |q - \frac{2}{3}Q| = (q^2 + \frac{4}{9}Q^2 - \frac{4}{3}pqq_{x})^{\frac{1}{2}},
\]

(62)

\[
p_{2} = \left| -\frac{1}{2}p - \frac{3q}{4} \right| = \left( \frac{1}{4}p^2 + \frac{9}{16}q^2 + \frac{3}{4}pq \cos \gamma \right)^{\frac{1}{2}},
\]

(63)

\[
q_{2} = |p - \frac{1}{2}q| = (p^2 + \frac{1}{4}q^2 + pq \cos \gamma)^{\frac{1}{2}},
\]

(64)

\[
\cos \gamma = x_{p}x_{q} + \sqrt{1 - x_{p}^2 \sqrt{1 - x_{q}^2}} \left( x_{\phi}x_{\phi} + \sqrt{1 - x_{\phi}^2 \sqrt{1 - x_{\phi}^2}} \right),
\]

(65)

\[
x_{2q} = \frac{p_{x_{p}} - \frac{1}{2}q_{x_{q}}}{q_2},
\]

(66)

\[
x_{2p} = -\frac{1}{2}p_{x_{p}} - \frac{2}{3}q_{x_{q}}.
\]

(67)

\[
q'_{2} = |p - \frac{1}{2}q - \frac{2}{3}Q| = (p^2 + \frac{1}{4}q^2 + \frac{4}{9}Q^2 - pq \cos \gamma)
\]

\[-\frac{4}{3}pQx_{p} + \frac{2}{3}q(Qx_{q})^{\frac{1}{2}},
\]

(68)

\[
x_{2q'} = \frac{p_{x_{p}} + \frac{1}{2}q_{x_{q}} - \frac{2}{3}Q}{q_2},
\]

(69)

\[
x_{2\phi} = \cos \phi_{2p}
\]

\[= \frac{-\frac{1}{2}p\sqrt{1 - x_{p}^2x_{\phi}^2} - \frac{2}{3}q\sqrt{1 - x_{q}^2x_{\phi}^2}}{p_{2}\sqrt{1 - x_{2p}^2}},
\]

(70)

\[
\sin \phi_{2p} = \left( -\frac{1}{2}p\sqrt{1 - x_{p}^2}\sqrt{1 - x_{\phi}^2} - \frac{3}{4}q\sqrt{1 - x_{q}^2}\sqrt{1 - x_{\phi}^2} \right) / \left( p_{2}\sqrt{1 - x_{2p}^2} \right),
\]

(71)

\[
x_{2\phi_{q}} = \cos \phi_{2q} = \frac{p\sqrt{1 - x_{p}^2x_{\phi}^2} - \frac{1}{2}q\sqrt{1 - x_{q}^2x_{\phi}^2}}{q_{2}\sqrt{1 - x_{2q}^2}},
\]

(72)

\[
\sin \phi_{2q} = \left( p\sqrt{1 - x_{p}^2}\sqrt{1 - x_{\phi}^2} - \frac{1}{2}q\sqrt{1 - x_{q}^2}\sqrt{1 - x_{\phi}^2} \right) / q_{2}\sqrt{1 - x_{2q}^2},
\]

(73)

\[
p_{3} = | -\frac{1}{2}b + \frac{3q}{4} | = \left( \frac{1}{4}p^{2} + \frac{9}{16}q^{2} - \frac{3}{4}pq \cos \gamma \right)^{\frac{1}{2}},
\]

(74)

\[
q_{3} = | -p - \frac{1}{2}q | = (p^{2} + \frac{1}{4}q^{2} + pq \cos \gamma)^{\frac{1}{2}},
\]

(75)

\[
x_{3q} = -\frac{px_{p} - \frac{1}{2}q_{x_{q}}}{q_{3}},
\]

(76)

\[
x_{3p} = -\frac{1}{2}px_{p} + \frac{3}{4}q_{x_{q}},
\]

(77)

\[
q'_{3} = | -p - \frac{1}{2}q - \frac{2}{3}Q | = (p^{2} + \frac{1}{4}q^{2} + \frac{4}{9}Q^{2} + pq \cos \gamma)
\]

\[+\frac{4}{3}pqQx_{p} + \frac{2}{3}q(Qx_{q})^{\frac{1}{2}},
\]

(78)

\[
x_{3\phi_{p}} = \cos \phi_{3p}
\]

\[= \left( -\frac{1}{2}p\sqrt{1 - x_{p}^2x_{\phi}^2} + \frac{3}{4}q\sqrt{1 - x_{q}^2x_{\phi}^2} \right) / \left( p_{2}\sqrt{1 - x_{3p}^2} \right),
\]

(79)

\[
\sin \phi_{3p} = \left( -\frac{1}{2}p\sqrt{1 - x_{p}^2}\sqrt{1 - x_{\phi}^2} + \frac{3}{4}q\sqrt{1 - x_{q}^2}\sqrt{1 - x_{\phi}^2} \right) / p_{2}\sqrt{1 - x_{3p}^2},
\]

(80)
\[ x_{3q} = \cos \phi_{3q} \]
\[ = \frac{\left(- p \sqrt{1 - x_{p}^2} x_{p} - \frac{1}{2} q \sqrt{1 - x_{q}^2} x_{q}\right)}{\sqrt{1 - x_{3q}^2}}, \]  
\[ \sin \phi_{3q} = \frac{\left(- p \sqrt{1 - x_{p}^2} \sqrt{1 - x_{p}^2} x_{q} - \frac{1}{2} q \sqrt{1 - x_{q}^2} \sqrt{1 - x_{q}^2}\right)}{\sqrt{1 - x_{3q}^2}}. \]  
\[ (81) \]
\[ (82) \]

For two-body current, i.e. the second term in Eq. (60) one can write as follows:

\[ TBC = 3 \sum_{m_{1}^{m_{2}}} \int_{0}^{\infty} \int_{-1}^{1} \int_{-1}^{1} \frac{dx_{p}}{\sqrt{1 - x_{p}^2}} \times \left\{ F_{TBC}(x_{p}, \sqrt{1 - x_{p}^2}) + F_{TBC}(-x_{p}, -\sqrt{1 - x_{p}^2}) \right\}, \]

Where

\[ F_{TBC}(x_{p}, \sqrt{1 - x_{p}^2}) = j_{2p_{m_{1}m_{2}m_{3}}}^{2p_{m_{1}m_{2}m_{3}}}(q_{2b}, x_{q_{2b}}, x_{3q_{2b}}) \times \psi_{m_{1}m_{2}m_{3}}^{2p_{m_{1}m_{2}m_{3}}}(p'_{m}, q'_{m}, x_{p'}, x_{q'}, x_{3q_{2b}}, x_{3q_{2b}}) + j_{2p_{m_{1}m_{2}m_{3}}}^{2p_{m_{1}m_{2}m_{3}}}(q_{2b}, x_{q_{2b}}, x_{3q_{2b}}) \times \psi_{m_{1}m_{2}m_{3}}^{2p_{m_{1}m_{2}m_{3}}}(p', q', x_{p'}, x_{q'}, x_{3q_{2b}}, x_{3q_{2b}}) + j_{2p_{m_{1}m_{2}m_{3}}}^{2p_{m_{1}m_{2}m_{3}}}(q_{2b}, x_{q_{2b}}, x_{3q_{2b}}) \times \psi_{m_{1}m_{2}m_{3}}^{2p_{m_{1}m_{2}m_{3}}}(p', q', x_{p'}, x_{q'}, x_{3q_{2b}}, x_{3q_{2b}}), \]

In term of \( p, q, x_{p}, x_{q}, x_{p'}, x_{q'} \) we write the \( j \)’s argument as follows:

\[ q_{2b} = \left( - \frac{1}{2} p \sqrt{1 - x_{p}^2} x_{p} - \frac{1}{2} q \sqrt{1 - x_{q}^2} x_{q}\right) \]
\[ = \left( - \frac{1}{2} p \sqrt{1 - x_{p}^2} x_{p} - \frac{1}{2} q \sqrt{1 - x_{q}^2} x_{q}\right) \]
\[ - 3 \left( \frac{1}{2} p \sqrt{1 - x_{p}^2} x_{p} - \frac{1}{2} q \sqrt{1 - x_{q}^2} x_{q}\right) \]
\[ \frac{1}{2} p \sqrt{1 - x_{p}^2} x_{p} - \frac{1}{2} q \sqrt{1 - x_{q}^2} x_{q}\right) \]
\[ (83) \]
\[ (84) \]

\[ \cos \beta = x_{p} x_{p'} + \sqrt{1 - x_{p}^2} \sqrt{1 - x_{p'}^2} \]
\[ \left( x_{p}, x_{p'} + \sqrt{1 - x_{p}^2} \sqrt{1 - x_{p'}^2}\right), \]
\[ (86) \]
\[ x_{q_{2b}} = \frac{1}{2} Q + px_{p} - p'x_{p'}, \]
\[ (87) \]
\[ x_{\phi_{2b}} = \cos \phi_{2b} = \frac{p \sqrt{1 - x_{p}^2} x_{p} - p' \sqrt{1 - x_{p}^2} x_{p'}}{q_{2} \sqrt{1 - x_{q_{2b}}^2}}, \]
\[ (88) \]

\[ \sin \phi_{2b} = \left( p \sqrt{1 - x_{p}^2} x_{p} - p' \sqrt{1 - x_{p}^2} x_{p'}\right) \]
\[ / q_{2} \sqrt{1 - x_{q_{2b}}^2}. \]
\[ (89) \]

\[ q_{2b} = \left| \frac{1}{2} Q - \frac{1}{2} p - \frac{3}{4} q - p'\right| \]
\[ = \left( - \frac{1}{2} p \sqrt{1 - x_{p}^2} x_{p} - \frac{1}{2} q \sqrt{1 - x_{q}^2} x_{q}\right) \]
\[ + \left( \frac{3}{4} Q x_{q} + pp' \cos \beta - \frac{3}{4} pp' \cos \beta \right) \]
\[ \frac{1}{2} p \sqrt{1 - x_{p}^2} x_{p} - \frac{1}{2} q \sqrt{1 - x_{q}^2} x_{q}\right) \]
\[ / q_{2} \sqrt{1 - x_{q_{2b}}^2}. \]
\[ (90) \]
\[ \cos \beta' = x_{p} x_{q} + \sqrt{1 - x_{p}^2} \sqrt{1 - x_{q}^2} \]
\[ \left( x_{p} x_{q} + \sqrt{1 - x_{p}^2} \sqrt{1 - x_{q}^2}\right), \]
\[ (91) \]
\[ x_{q_{2b}} = \frac{1}{2} Q - \frac{1}{2} p x_{p} - \frac{1}{2} q x_{q} - p' x_{p'}, \]
\[ (92) \]
\[ \sin \phi_{2b} = \left( - \frac{1}{2} p \sqrt{1 - x_{p}^2} x_{p} - \frac{1}{2} q \sqrt{1 - x_{q}^2} x_{q}\right) \]
\[ + \left( \frac{3}{4} Q x_{q} + pp' \cos \beta - \frac{3}{4} pp' \cos \beta \right) \]
\[ / q_{2} \sqrt{1 - x_{q_{2b}}^2}. \]
\[ (94) \]
Where the momentum-helicity basis states is defined by:

\[ \ket{\hat{p}S\Lambda T} = (1 - \eta \langle -1 \rangle^{\Lambda + T}) \ket{pS\Lambda T} \]

Two-body calculations in the 3D approach are performed with the three following properties to change the argument from \( x'' \) to \( x''' \):

\[ f(x''') = ax'' + \sqrt{1 - x''^2} \sqrt{1 - x'''} \]

\[ x'' = x_q x'' + \sqrt{1 - x_q^2} \sqrt{1 - x'''} \] \( x_q \neq 0 \)

\[ x''' = x_{q'''} x_q + \sqrt{1 - x_{q'''}^2} \sqrt{1 - x_q^2} \]

For delta function we have:

\[ \delta(x'' - x_0) = \delta[f(x''')] \]

where:

\[ f(x''') = ax'' + \sqrt{1 - a^2} \sqrt{1 - x''^2} - x_0 \] \( a = x_q, c = x_{q'''} \text{ and } x_0 = \frac{p''}{x'''} - \frac{q''}{x''} \)

For handling the Eq. \( \ref{111} \) we have to use delta function properties to change the argument from \( x'' \) to \( x''' \). So we need zero points of \( f(x) \), i.e. \( x_i \):

\[ x_i = \frac{a^2 x_0^2 \pm \sqrt{\Delta}}{c^2(a^2 - 1) - a^2} \]

\[ \Delta = c^2(a^2 - 1)[c^2(a^2 - 1) + x_0^2 - a^2] \]

**C Two-body t-matrix, Deuteron and Triton wave functions in free basis states**

Two-body calculations in the 3D approach are performed in Refs. \( \ref{111} \)-\( \ref{112} \). We briefly introduce the two-body t-matrix in the momentum-helicity basis states as:

\[ \langle p\hat{p}S\Lambda T | t | p\hat{p}S\Lambda T \rangle_a = \alpha_a \langle p\hat{p}S\Lambda T (p', p) \]

Where the momentum-helicity basis states is defined by:

\[ | p\hat{p}S\Lambda T \rangle_a = (1 - \eta_a (-1)^{\Lambda + T}) | p\hat{p}SA)_{x} | T \]
where $S$ is spin, $A$ is its projection to $\hat{p}$ and $T$ is the isospin of the two-body system. If we choose the direction of the vector $p$ to be in the $z$-direction we can write:

$$t^{\pi S T}_{\Lambda A}(p', p, x_{p'}) = e^{i\Lambda \phi_p} t^{\pi S T}_{\Lambda A}(p', p, x_{p'}) ,$$

and the Lippmann-Schwinger equation in the momentum-helicity basis states for any energy is written as:

$$t^{\pi S T}_{\Lambda A}(p', p, x_{p'}) = V^{\pi S T}_{\Lambda A}(p', p, x_{p'}) + \frac{1}{4} \sum_{A'} \int_{-\infty}^{\infty} q'' q'^2 \int_{-1}^{1} dx'' v^{\pi S T A'}_{\Lambda'}(p', p'', x_{p''}, x_{p''}) \times G_{\Lambda}^{\nu}(E_p) t^{\pi S T}_{\Lambda A'}(p'', p, x'_{p'}) .$$

Where

$$v_{\Lambda A'}^{\pi S T A'}(p', p'', x_{p'}, x'_{p''}) = \int_{0}^{\pi} d\hat{p}' e^{-i\Lambda (\hat{p}' \hat{\phi}' - \hat{p} \hat{\phi})} V^{\pi S T}_{\Lambda A'}(p', p'') .$$

The relation between t-matrix in the momentum-helicity basis states and those in the free basis states is:

$$a^{\Lambda}(p' m_2 m_3 \nu_3 \nu_3 | p m_2 m_3 \nu_3 \nu_3)^a = \frac{1}{4} \delta_{(p_2 + \nu_3), (p' + \nu_3)} C^{(\hat{A}_\Lambda \phi_p - A \phi_0)}_{\Lambda ST} \sum_{A' ST} (1 - \eta_{\nu}(1)^{S+T}) \times C \left( \frac{1}{2} T, \nu_2 \nu_3 \right) C \left( \frac{1}{2} T, \nu_2 \nu_3 \right) C \left( \frac{1}{2} S, m_2 m_3 \Lambda_0 \right) C \left( \frac{1}{2} S, m_2 m_3 \Lambda_0 \right) \sum_{A' \Lambda} d^{\Lambda}_{S}(\Lambda_0(x_p)) d^{\Lambda}_{A'}(x_{p'}),$$

$$t^{\pi S T}_{\Lambda A}(p', p) ,$$

rotation matrix element, $d^{S}_{A}(x_p)$ is introduced in [59]. The deuteron wave function in the free basis states is also related to the momentum-helicity basis states:

$$\langle \hat{\phi}_{d}^{M_2}\rangle | p m_2 m_3 \nu_2 \nu_3) = \frac{1}{2} \frac{1}{2} \phi_{d}^{M_2}(p) d^{1}_{A=0}(x_p) - \frac{1}{2} \phi_{d}^{M_2}(-p) d^{1}_{A=0}(x_p) + \phi_{d}^{M_2}(p) d^{1}_{A=0}(x_p) - \phi_{d}^{M_2}(-p) d^{1}_{A=0}(x_p) \times C \left( \frac{1}{2} T, \nu_2 \nu_3 \right) C \left( \frac{1}{2} T, \nu_2 \nu_3 \right) C \left( \frac{1}{2} S, m_2 m_3 \Lambda_0 \right) C \left( \frac{1}{2} S, m_2 m_3 \Lambda_0 \right) e^{i\Lambda \phi_p} d^{1}_{A=0}(x_p) .$$

Where $M_d$ is spin projection of the deuteron, we have summed over $\Lambda = -1, 0, 1$, and we have used this fact that the terms with $\Lambda = -1$ and $\Lambda = 1$ are the same. Also we define:

$$\phi_{A}^{M_2}(q) = \langle \hat{q} 1 A; 0| \hat{\phi}_{d}^{M_2}\rangle = \phi_{A}^{M_2}(q, x_q) e^{iM_2 \phi_q},$$

(122)

deuteron wave function in the momentum-helicity basis states is calculated using eigenvalue equation:

$$\left( \frac{q^2}{m} - E_d \right) \phi^{M_2}(q, x_q) + \frac{1}{4} \int_{-\infty}^{\infty} dq q^2 \int_{-\infty}^{\infty} dq' q'^2 \int_{-1}^{1} dx' e^{iM_2 \phi_q} \phi^{M_2}(q, x_q) = \frac{1}{4} \int_{-\infty}^{\infty} dq q^2 \int_{-1}^{1} dx' e^{iM_2 \phi_q} \phi^{M_2}(q, x_q) = \frac{1}{4} \int_{-\infty}^{\infty} dq q^2 \int_{-1}^{1} dx' e^{iM_2 \phi_q} \phi^{M_2}(q, x_q) = 0.$$ (123)

The triton wave function is calculated by applying the formalism of Ref.[16]. In this formalism the triton wave function has been evaluated in the basis states:

$$\langle \hat{p} q q | \hat{\phi}_{t} \rangle = \langle \hat{p} q q | \hat{\phi}_{t} \rangle | S_{23}(\frac{1}{2}) S M S_{T}(\frac{1}{2} T M T) | \hat{\phi}_{t} \rangle ,$$

(124)

In the above equation $s_{23}$ is total spin of the subsystem $S$ and $M_S$ are the total spin of the three particles and its projection along the z axis respectively. The same explanation is used for the isospin.

The triton wave function in Eq. (124) is reproduced using AV18 potential and is related to the one in the free basis states as follows:

$$\langle \hat{p} q q | M_1 M_2 M_3 \nu_1 \nu_2 \nu_3 \rangle = \sum_{\alpha} \langle \hat{p} q q | \hat{\phi}_{t} \rangle \langle \hat{\phi}_{t} | \hat{\phi}_{t} \rangle \langle \hat{\phi}_{t} | \hat{\phi}_{t} \rangle \langle \hat{\phi}_{t} | \hat{\phi}_{t} \rangle ,$$

(125)

and are defined by Eqs. (126) and (127) and the Clebsch-Gordan coefficients, $g_{\gamma \alpha}$, are introduced in Ref. [16].

$$| \alpha \rangle = | (s_{23}, \frac{1}{2}) S M S_{T}(\frac{1}{2} T M T),$$

(126)

$$| \gamma \rangle = | m_1 m_2 m_3 \nu_1 \nu_2 \nu_3 \rangle .$$

(127)

D Extraction of Deuteron singularity

To extract singularity of the two-body t-matrix in deuteron binding energy we need to evaluate the following term in the momentum-helicity basis states.

$$\lim_{E \to E_d} | \hat{p} | \hat{\phi}_{d}^{M_2}\rangle \langle \hat{\phi}_{d}^{M_2}| \hat{p} | \hat{\phi}_{d}^{M_2}\rangle = \frac{1}{2} \frac{1}{2} \phi_{d}^{M_2}(p) d^{1}_{A=0}(x_p) - \frac{1}{2} \phi_{d}^{M_2}(-p) d^{1}_{A=0}(x_p) \times C \left( \frac{1}{2} T, \nu_2 \nu_3 \right) C \left( \frac{1}{2} T, \nu_2 \nu_3 \right) C \left( \frac{1}{2} S, m_2 m_3 \Lambda_0 \right) C \left( \frac{1}{2} S, m_2 m_3 \Lambda_0 \right) e^{i\Lambda \phi_p} d^{1}_{A=0}(x_p),$$

(128)

two terms on the right hand side of the above equation evaluate separately.

$$\int dp' 1 A'; 0 | \hat{\phi}_{d}^{M_2}\rangle = \frac{1}{4} \int_{\Lambda \nu} dp' 1 A'; 0 | \hat{\phi}_{d}^{M_2}\rangle \langle \hat{\phi}_{d}^{M_2}| \hat{p} | \hat{\phi}_{d}^{M_2}\rangle ,$$

(129)
and
\[
\langle \psi_{M_A} | \mathbf{p} | \Lambda A \rangle_{1a} = \frac{1}{4} \sum_{A'} \int d\mathbf{p}' d\mathbf{p}'' d\mathbf{x}' d\mathbf{x}'' (V_{10}^{110, M_A} (\mathbf{p}', \mathbf{x}', \mathbf{p}'', \mathbf{x}'') \Phi_0^{M_d} (\mathbf{p}'', \mathbf{x}'')) \
\times 2 \Phi_1^{110, M_A} (\mathbf{p}', \mathbf{x}', \mathbf{p}'', \mathbf{x}'') \Phi_1^{M_d} (\mathbf{p}'', \mathbf{x}'') \int_{2\pi} d\mathbf{p}'' e^{i(\Lambda - M_d) (\Phi'' - \Phi')} \
\times 2 V_{110}^{110, \mathbf{p}''} (\mathbf{p}, \mathbf{x}'') \Phi_1^{M_d} (\mathbf{p}'', \mathbf{x}'') \int_{2\pi} d\mathbf{p}'' e^{i(\Lambda - M_d) (\Phi'' - \Phi')} = \frac{1}{4} \sum_{A'} \int d\mathbf{p}' d\mathbf{p}'' d\mathbf{x}' d\mathbf{x}'' (V_{10}^{110, A} (\mathbf{p}', \mathbf{x}', \mathbf{p}'', \mathbf{x}'') \Phi_0^A (\mathbf{p}'', \mathbf{x}'') \
\times 2 \Phi_1^{110, A} (\mathbf{p}', \mathbf{x}', \mathbf{p}'', \mathbf{x}'') \Phi_1^A (\mathbf{p}'', \mathbf{x}'') \int_{2\pi} d\mathbf{p}'' e^{i(\Lambda - M_d) (\Phi'' - \Phi')}.
\]

In the above equation we have used this equality:
\[
\int_{0}^{2\pi} d\Phi'' e^{i(\Lambda - M_d) (\Phi'' - \Phi')} = 2\pi \delta_{\Lambda M_d}.
\]

References

1. L. D. Faddeev, Zh. Eksp. Teor. Fiz. 39, 1459 (1960).
2. E. O. Alt, P. Grassberger, W. Sandhas, Nucl. Phys. B2, 167 (1967).
3. D. R. Lehman, Phys. Rev. Lett. 23, 1339 (1969).
4. I. R. Barbour, A.C. Phillips, Phys. Rev. Lett. 19, 1388 (1967).
5. B. F. Gibson and D. R. Lehman, Phys. Rev. C 11, 29 (1975).
6. J. Carlson, Phys. Rev. C 36, 2026 (1987).
7. V. D. Efros, W. Leidemann, G. Orlandini, Phys. Lett. B 338, 130 (1994).
8. M. Viviani, A. Kievsky, L. E. Marcucci, S. Rosati, R. Schiaffini, Phys. Rev. C 61, 064001 (2000).
9. J. Golak et al., Phys. Rept. 415, 89 (2005).
10. R. A. Rice, Y. E. Kim, Few-Body Syst. 14, 127 (1993).
11. I. Fachruddin, Ch. Elster, W. Glöckle, Phys. Rev. C 62, 044002 (2000).
12. I. Fachruddin, Ch. Elster, W. Glöckle, Phys. Rev. C 63, 054003 (2001).
13. M. R. Hadizadeh and S. Bayegan, Eur. Phys. J. A 36, 201 (2008).
14. S. Bayegan, M. R. Hadizadeh, and W. Glöckle, Prog. Theor. Phys. 120, 887 (2008).
15. S. Bayegan, M. Harzchi and M. R. Hadizadeh, Nucl. Phys. A 814, 21 (2008).
16. S. Bayegan, M. R. Hadizadeh, and M. Harzchi, Phys. Rev. C 77, 064005 (2008).
17. S. Bayegan, M. A. Shalchi, M. R. Hadizadeh, Phys. Rev. C 79, 057001 (2009).
18. S. Bayegan, M. Harzchi and M. A. Shalchi, Nucl. Phys. A 832, 1 (2010).
19. M. Harzchi, S. Bayegan, Eur. Phys. J. A 46, 271 (2010).
20. M. R. Hadizadeh, Lauro Tomio, S. Bayegan, Phys. Rev. C 83, 054004 (2011).
21. Ch. Elster, W. Glöckle, H. Witala, Few-Body Syst. 45, 1 (2009).
22. H. Liu, Ph.D thesis. Ohio University, USA, (2005).
23. R. B. Wiringa, V. G. J. Stoks, R. Schiavilla, Phys. Rev. C 51, 38 (1995).
24. D. Huber, H. Witala, A. Nogga, W. Glöckle and H. Kamada, Few-Body Systems 22, 107 (1997).
25. D.O. Riska, Phys. Scr. 31, 107 (1985).
26. A.J.F. Siegert, Phys. Rev. 52, 787 (1937).
27. D. D. Faul, B. L. Berman, P. Meyer, D. L. Olson, Phys. Rev. C 24, 849 (1981).
28. J. Golak et al. Nucl. Phys. A 707, 365 (2002).
29. D. M. Skopik, D. H. Beck, J. Asai, J. J. Murphy II, Phys. Rev. C 24, 1791 (1981).
30. R. Kosiek, D. Müller, R. Pfeiffer, O. Merwitz, Phys. Lett. 21, 199 (1966).
Fig. 1. Comparison of the 3D calculations for the total cross section of the two-body (Nd) photodisintegration of Triton using the single nucleon current (dashed line) and the two-body current (solid line). The experimental values are taken from Ref. [27] as EXP. DATA1, [29] as EXP. DATA2 and [30] as EXP. DATA3.

Fig. 2. Comparison of the 3D calculations for the total cross section of the three-body (3N) photodisintegration of Triton using the single nucleon current (dashed line) the two-body current (solid line). The experimental values are taken from Ref. [27].
Fig. 3. Total cross section for Nd photodisintegration of Triton. The solid line is the 3D calculation with AV18 potential. The dashed line represents the partial wave calculation using AV18 potential and dot-dashed line represents the partial wave result using AV18 potential and 3NF (Urbana IX). The experimental values are the same as Fig. 1.

Fig. 4. Total cross section for 3N photodisintegration of Triton. The solid line is the 3D calculation with AV18 potential. The dashed line represents the partial wave calculation using the AV18 potential and dot-dashed line represents the partial wave result using AV18 potential and 3NF (Urbana IX). The experimental values are the same as Fig. 2.
