On $U_V(1) \times U_A(1)$ gauge invariance in a Lorentz-violating QED

M. A. Anacleto, F. A. Brito and E. Passos
Departamento de Física, Universidade Federal de Campina Grande,
Caixa Postal 10071, 58109-970 Campina Grande, Paraíba, Brazil

We introduce a Lorentz-symmetry violating master quantum electrodynamics which preserves the $U_V(1) \times U_A(1)$ gauge symmetry. The master fermionic sector can radiatively induce a master effective action which simultaneously displays the same electromagnetic terms present in the Carroll-Field-Jackiw, Myers-Pospelov and Aether actions.

I. INTRODUCTION

Recent investigations consider the possibility of Lorentz-symmetry violation from a modification of the usual Standard Model of particle physics [1]. This procedure is called the Standard Model Extension (SME) which provides a quantitative description of the Lorentz-symmetry violation, controlled by a set of coefficients whose values can be determined or constrained by experiments [2]. Although, this approach introduces the Lorentz-symmetry violation it preserves all the conventional properties of quantum field theory such as the local vector gauge transformation $U_V(1)$.

As we know, the properties of the gauge symmetry plays a decisive role at interesting investigations such as radiative induction of Lorentz-symmetry violating electromagnetic effective action [3, 4]. In this work, we introduce a Lorentz-violating master quantum electrodynamics compatible with the vector and axial gauge transformation $U_V(1) \times U_A(1)$ [4].

The main purpose of this paper is to investigate how such special gauge symmetry acts on the one-loop quantum corrections. The fermionic sector (and interactions) of the master QED in four-dimension is given by

$$\mathcal{L}_\psi = \bar{\psi} (i\partial - m) \psi - q \bar{\psi} A^\mu R \psi - q \bar{\psi} A^\mu L \psi,$$

where $R^\mu$ and $L^\mu$ are right- and left-handed projection operators: $R^\mu = (1 + \gamma_5)/2$ and $L^\mu = (1 - \gamma_5)/2$ and the quantities $A^\mu$ and $\tilde{A}^\mu$ are effective fields through which we introduce the Lorentz-symmetry violation. As an example, such fields can be written as

$$A^\mu = A^\mu + M^{-1} F^\mu + \frac{M}{q} n^\mu \text{ and } \tilde{A}^\mu = A^\mu + M^{-1} F^\mu - \frac{M}{q} n^\mu. \quad (2)$$

Note that the structure of the expression (2) is in accord with the SME. In this approach, $q$ is a coupling coefficient, $n^\mu$ is a massless constant vector which control the Lorentz-symmetry violation and $F^\mu = \varepsilon^{\mu\nu\alpha\beta} F^\nu_{\alpha\beta}$ represents an interaction between the electromagnetic field strength and the constant vector background field. The quantity $M$ is the mass where new physics such as Lorentz symmetry violation emerges. The Lagrangian (1) can be considered as the fermionic sector of the usual extended QED written in more general terms. By substituting the expression (2) in (1) leads to the usual interaction term $q \bar{\psi} A^\mu \psi$ between the fermions, a axial coupling $\bar{\psi} \gamma_5 \tilde{A} \psi$ which can be recognized as a usual term of the SME [1] and a non-minimal coupling $q (M^{-1}) \bar{\psi} F^\mu \psi$ [5].

Notice that the Lagrangian (1) admits the vector gauge transformation $U_V(1)$:

$$\psi \to \exp [i q \alpha(x)] \psi, \quad \bar{\psi} \to \bar{\psi} \exp \left[ -i q \alpha(x) \right],$$

$$A^\mu \to A^\mu + \partial^\mu \alpha(x), \quad \tilde{A}^\mu \to \tilde{A}^\mu + \partial^\mu \alpha(x). \quad (3)$$

and also the axial gauge transformation $U_A(1)$:

$$\psi \to \exp [i q \gamma_5 \alpha(x)] \psi, \quad \bar{\psi} \to \bar{\psi} \exp [i q \gamma_5 \alpha(x)],$$

$$A^\mu \to A^\mu + \partial^\mu \alpha(x), \quad \tilde{A}^\mu \to \tilde{A}^\mu - \partial^\mu \alpha(x). \quad (4)$$

for massless fermions $(m = 0)$. Therefore, we see that the SME can have a $U_V(1) \times U_A(1)$ gauge symmetry where the vector and axial currents are separately conserved, i.e., $\partial^\mu j^\mu = 0$ and $\partial^\mu j^5_5 = 0$ (for $m = 0$). We take advantage of the fermionic sector of the master QED (1) to generate a master effective action in terms of the quantities $A^\mu$ and $\tilde{A}^\mu$ via one-loop radiative corrections.

The structure of the work is organized as follows: In the section II we propose the master effective action to be radiatively induced and thus we construct the fermion determinants associate to the master QED. In the section III we isolate the self-energy tensor via derivative expansion method. The results show that the effective action can be generated and offer high derivative Lorentz-violating operators which may be added in the usual electrodynamics for future investigations. Finally, in section IV we present our conclusions.
II. THE EFFECTIVE ACTION INDUCED

In this section, we revisit the problem on induced effective theory via radiative corrections at one loop contribution via derivative expansion method. We will address the specific problem of radiatively inducing the following Lorentz-violating term:

$$S_{\text{eff}} \sim \int d^4x \varepsilon^{\alpha\beta\mu\nu} (\partial_\alpha \tilde{A}_\mu) \tilde{A}_\nu \cdots$$  \hspace{1cm} (5)

where (⋯) means the possible permutations between the fields $A_\mu$ and $\tilde{A}_\mu$. As we shall see it is possible to generate the previous effective action. It may offer important contributions to the usual electrodynamics. We have the appearance of quantities that depend only on the field strength and not on the potential, i.e.,

$$S_{\text{eff}}(F_\mu) \sim (M^{-1}) \int d^4x n_\alpha \varepsilon^{\alpha\beta\mu\nu} (\partial_\beta F_\mu) F_\nu$$  \hspace{1cm} (6)

which is completely gauge invariant. By fixing the gauge as $\partial \cdot A = 0$ and $n \cdot A = 0$, the expression (6) can be rewritten as a higher derivative Chern-Simons-like extension

$$S_{\text{eff}}(F_\mu) \rightarrow S_{\text{MP}} = (M^{-1}) \int d^4x \varepsilon^{\mu\nu\lambda\sigma} n_\mu ((n \cdot \partial)^2 - n^2 \partial^2) A_\nu \partial_\lambda A_\sigma$$  \hspace{1cm} (7)

given in terms of the potential. This is the extended version of the Myers-Pospelov electrodynamics [6] which has been recently considered to address the issue of superluminal particles [7]. As we shall see later we can also simultaneously radiatively induce other contributions such as the Carroll, Field and Jackiw term [8].

Let us now start the process of radiative induction of the effective action (5) by using the derivative expansion of fermion determinants. The one loop effective action of the fields $A_\mu$ and $\tilde{A}_\mu$ associated with the theory (1) can be expressed in the form of the following functional trace:

$$S_{\text{eff}}[A_\mu, \tilde{A}_\mu] = -i \text{Tr} \ln [\hat{p} - m - q (\mathcal{A} P_R + \tilde{\mathcal{A}} P_L)],$$  \hspace{1cm} (8)

where the symbol Tr stands for the trace over Dirac matrices, trace over the internal space as well as for the integrations in momentum and coordinate spaces. The first nondynamical determinant factor has been absorbed into normalization of the path integral such that

$$S_{\text{eff}}[A_\mu, \tilde{A}_\mu] = S_{\text{eff}} + S_{\text{eff}}^{(l)}[A_\mu, \tilde{A}_\mu].$$  \hspace{1cm} (9)

The term dependent on the effective fields is explicitly written in the form

$$S_{\text{eff}}^{(l)}[A_\mu, \tilde{A}_\mu] = i \text{Tr} \sum_{l=1}^{\infty} \frac{1}{l} \left[-i q S(p) (\mathcal{A} P_R + \tilde{\mathcal{A}} P_L)\right]^l,$$  \hspace{1cm} (10)

where

$$S(p) = \frac{i}{\hat{p} - m}$$  \hspace{1cm} (11)

is the usual fermion propagator. All the terms of the series (10) are one-loop contributions. In the following we shall focus only on the term associate to $l = 3$. Thus, we rewrite this contribution as

$$S_{\text{eff}}^{(3)}[A_\mu, \tilde{A}_\mu] = -q^3 \text{Tr} \left[ S(p) A P_R S(p) A P_R S(p) A P_L S(p) A P_L S(p) A P_L S(p) A P_R \right].$$  \hspace{1cm} (12)

Notice these are one-loop contributions.

III. THE SELF-ENERGY TENSOR

Now applying the main property of derivative expansion method [9] – see also [10]. This method consider that any function of momentum can be converted into a coordinate dependent quantity as

$$(A_\mu, \tilde{A}_\mu) S(p) = (S(p - i\partial)(A_\mu, \tilde{A}_\mu)),$$  \hspace{1cm} (13)
Thus, this confirms our previously assumption that the Lagrangian (13) which is a finite and determined contribution. In the following, we substitute the fields defined in equation (12) to give

$$S^{(3)}_{\text{eff}}[A_\mu, \tilde{A}_\mu] = -q^3 \int d^4x (\partial_\lambda A_\mu) A_\nu \tilde{A}_\alpha I^\mu_{\lambda\nu\alpha} + (\partial_\lambda \tilde{A}_\mu) \tilde{A}_\nu A_\alpha I^\mu_{\nu\lambda\alpha}$$

with the quantities $I^\mu_{\lambda\nu\alpha}$ and $I^\mu_{\nu\lambda\alpha}$ given as

$$I^\mu_{\lambda\nu\alpha} = \int \frac{d^4p}{(2\pi)^4} \text{tr} [S(p)\gamma^\mu P_R S(p)\gamma^\lambda S(p)\gamma^\nu P_L + S(p)\gamma^\nu P_R S(p)\gamma^\lambda S(p)\gamma^\mu P_L + S(p)\gamma^\mu P_L S(p)\gamma^\nu P_R S(p)\gamma^\lambda S(p)\gamma^\alpha P_L + S(p)\gamma^\nu P_L S(p)\gamma^\mu P_R S(p)\gamma^\lambda S(p)\gamma^\alpha P_R],$$

and

$$I^\mu_{\nu\lambda\alpha} = \int \frac{d^4p}{(2\pi)^4} \text{tr} [S(p)\gamma^\mu P_L S(p)\gamma^\lambda S(p)\gamma^\nu P_R S(p)\gamma^\alpha P_R + S(p)\gamma^\nu P_L S(p)\gamma^\lambda S(p)\gamma^\mu P_R S(p)\gamma^\alpha P_R].$$

where the symbol tr denotes the trace of the product of the gamma matrices. Therefore, we can calculate the trace of gamma matrices in the expressions (16) and (17) and then we substitute the results in the equation (15) to find

$$S^{(3)}_{\text{eff}}[A_\mu, \tilde{A}_\mu] = 2i q^3 \int d^4x (\partial_\lambda A_\mu) A_\nu \tilde{A}_\alpha (\partial_\lambda \tilde{A}_\mu) \tilde{A}_\nu A_\alpha \left[ \frac{2\varepsilon^{\mu\nu\lambda\alpha} p^\lambda - \varepsilon^{\lambda\mu\alpha\nu} (p^2 - m^2)}{(p^2 - m^2)^4} \right].$$

Note that all integrals in the above expression are manifestly finite and does not require any regularization. The exact result, regularization independent, is given by

$$S^{(3)}_{\text{eff}}[A_\mu, \tilde{A}_\mu] = \frac{q^3}{24\pi^2} \int d^4x \varepsilon^{\mu\lambda\alpha\nu} (\partial_\lambda A_\mu) A_\nu \tilde{A}_\alpha + (\partial_\lambda \tilde{A}_\mu) \tilde{A}_\nu A_\alpha),$$

which is a finite and determined contribution. In the following, we substitute the fields defined in equation (2) into the induced effective action (19) to find

$$S^{(3)}_{\text{eff}}[A_\mu, \tilde{A}_\mu] \rightarrow S^{(3)}_{\text{eff}}[A_\mu, F_\mu, \eta_\nu]$$

$$= \frac{q^2}{6\pi^2} \int d^4x \left[ M n_\alpha \varepsilon^{\alpha\lambda\nu\mu} (\partial_\lambda A_\mu) A_\nu + M^{-1} \eta_\alpha \varepsilon^{\alpha\lambda\mu\nu} (\partial_\lambda F_\mu) F_\nu + 2 \eta_\alpha \varepsilon^{\alpha\lambda\mu\nu} (\partial_\lambda F_\mu) F_\nu \right].$$

Thus, this confirms our previously assumption that the Lagrangian (1) in fact may be able to radiatively induce a master effective action which displays uniqueness of the result due to the absence of the divergences and then without the use of any regularization scheme.

IV. CONCLUSION

In summary, we introduce the fermion sector of a Lorentz-symmetry violating master QED invariant under the $U_V(1) \times U_A(1)$ gauge symmetry. We apply this theory to radiatively induce a master effective action which shows aspects of Lorentz violation in the electromagnetic sector. In our main result (20), it is shown that the first term is the well-known CPT-odd Chern-Simons-like term and is an important result of the literature [11]. In this case, the factor $q^2/6\pi^2$ is a result that has also been obtained in the literature in a regularization independent model [4]. The second term is a new contribution, which needs to be further explored. In addition, as we previously mentioned, this quantity may generates another higher derivative term such as the Myers-Pospelov effective action [12]. The third contribution represents a CPT-even effective action that when is algebraically manipulated reads

$$\mathcal{L}_{F_2} = n_\alpha F_{\mu\nu} F^{\mu\nu} - 2 \eta_\alpha n_\lambda F^{\rho\lambda} F_{\rho\mu}$$

being the first term similar to the usual Maxwell term and the second term being the aether-like Lorentz-violating term [13]. One can still identify in the Lagrangian (21), in case of linear, non-dispersive dielectric medium, the following correspondence $\varepsilon \sim n_\alpha^2$ and $1/\mu \sim n_\lambda^2$, where $\varepsilon$ and $\mu$ are the dielectric and magnetic constants of the medium.
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