Euler diagrams as an introduction to set-theoretical models

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Abstract
Understanding the notion of a model is not always easy in logic courses. Hence, tools such as Euler diagrams are frequently applied as informal illustrations of set-theoretical models. We formally investigate Euler diagrams as an introduction to set-theoretical models. We show that the model-theoretic notions of validity and invalidity are characterized by Euler diagrams, and, in particular, that model construction can be described as a manipulation of Euler diagrams.

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1 Introduction
Logic is traditionally studied from the different viewpoints of syntax and semantics. From the syntactic viewpoint, formal proofs are investigated using proof systems such as natural deduction and sequent calculus. From the semantic viewpoint, set-theoretical models of sentences are usually investigated. In contrast to a proof, which shows the validity of a given inference, we usually disprove an inference by constructing a counter-model. A counter-model is one in which all premises of a given inference are true, but its conclusion is false. The notions of proofs and models are traditionally defined in the fundamentally different frameworks of syntax and semantics, respectively, and the completeness theorem, one of the most basic theorems of logic, provides a bridge between them. In university courses, logic is usually taught along such lines.

As the notion of a proof appears naturally in mathematics courses, students are, to some extent, familiar with it. However, the notion of a model can be difficult for beginners. A set-theoretical model consists of a domain of entities and sets, and an interpretation function that assigns truth values to sentences. Models are usually defined symbolically in the same way as syntax, and beginners often find it difficult to distinguish between syntax and semantics, as well as to understand the notion of the interpretation of symbols. Thus, some diagrams are used to introduce set-theoretical models. One famous example is Tarski’s World of Barwise and Etchemendy [1], which is designed to introduce model theoretic notions diagrammatically. Another traditional example concerns Euler diagrams, which were originally introduced in the 18th century to illustrate syllogisms. A basic Euler diagram consists of circles and points, and syllogistic sentences are represented using inclusion and exclusion relations between the circles and points. A circle in an Euler diagram can be considered to represent a set, and a point can be considered as an entity of a given domain. Inclusion and exclusion relations between circles and points can then be considered to represent set-theoretical notions such as subset and disjointness relations, respectively.

Besides the traditional informal semantic view, Euler diagrams have recently been investigated syntactically as the counterparts of logical formulas, which constitute formal
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Euler diagrams are rigorously defined as syntactic objects, allowing set-theoretical semantics to be defined. Inference systems are also formalized, and they have been shown to be equivalent to some symbolic logical systems. Consequently, fundamental logical properties such as soundness and completeness have been investigated (e.g., [7, 5, 6]). In this way, Euler diagrams are, on the one hand, regarded as models of given sentences, and on the other hand, they are exploited as syntactic objects that constitute formal proofs.

In this paper, we present a formal investigation of Euler diagrams as set-theoretical models. We characterize the notions of validity and invalidity using our Euler diagrams. (In [2], Barwise and Etchemendy also suggested a possible application of Euler diagrams to construct counter-examples.) The advantages of our view of Euler diagrams as models are as follows: (1) diagrams provide concrete images of set-theoretical models and counter-models; (2) diagrams provide natural images of the interpretation of sentences; (3) model and counter-model construction can be captured as concrete manipulations of diagrams. However, there are the following limitations: (1) the expressive power of our diagrams is restricted to conjunctions of extended syllogistic sentences; (2) not all sets are represented by the circles or simple closed curves of our diagrams; (3) diagrams are sometimes misleading if their meaning and manipulations are not precisely defined. Regardless, Euler diagrams have recently been defined rigorously from logical viewpoints, and we believe that this restricted fragment is sufficient for an introduction to model theory.

In Section 2, we first review the Euler diagrammatic system of [6], in which Euler diagrams are regarded as syntactic objects corresponding to first-order formulas. Then, in Section 3, we investigate a view of Euler diagrams as counterparts of set-theoretical models. Finally, we investigate how Euler diagrams provide counter-models of given inferences in Section 4.

2 Euler diagrams as syntactic objects

Our sentences have the following extended syllogistic form:

\[
\begin{align*}
a \text{ is } B, & \quad a \text{ is not } B, \\
\text{All } A \text{ are } B, & \quad \text{No } A \text{ are } B, \\
\text{Some } A \text{ are } B, & \quad \text{Some } A \text{ are not } B,
\end{align*}
\]

where \(a\) is a constant and \(A, B\) are predicates. These basic sentences are denoted by \(S, S_1, S_2, \ldots\), and we also consider their conjunctions.

From a semantic viewpoint, every constant \(a\) is interpreted as an element \(I(a)\), and a predicate \(B\) is interpreted as a nonempty set \(I(B)\) in a set-theoretical domain. Then, our syllogistic sentences are interpreted as usual. For example, \(a \text{ is } B\) is true iff \(I(a) \in I(B)\); \(\text{All } A \text{ are } B\) is true iff \(I(A) \subseteq I(B)\), and so on (cf. [6]). Note that we impose the so-called existential import, i.e., the interpretation of \(B\) is nonempty. See Remark after Example 12.

We first review the Euler diagrammatic system of [6], in which Euler diagrams are regarded as syntactic objects corresponding to formulas.

Definition 1. Our Euler diagram, called EUL-diagram, is defined as a plane with named circles (simple closed curves, denoted by \(A, B, C, \ldots\)) and named points \((p, p_1, p_2, \ldots)\). Named points are divided into constant points \((a, b, c, \ldots)\), which correspond to constants of the first-order language, and existential points \((x, y, z, \ldots)\), which correspond to bound variables associated with the existential quantifier. Diagrams are denoted by \(D, E, D_1, D_2, \ldots\).

Each diagram is specified by the following inclusion, exclusion, and crossing relations maintained between circles and points.
Definition 2. EUL-relations consist of the following reflexive asymmetric binary relation \( \sqsubset \), and the irreflexive symmetric binary relations \( \sqsupset \) and \( \sqsubseteq \):

- \( A \sqsubset B \) “the interior of \( A \) is inside of the interior of \( B \),”
- \( A \sqsupset B \) “the interior of \( A \) is outside of the interior of \( B \),”
- \( A \sqsupseteq B \) “there is at least one crossing point between \( A \) and \( B \),”
- \( p \sqsubset A \) “\( p \) is inside of the interior of \( A \),”
- \( p \sqsupset A \) “\( p \) is outside of the interior of \( A \),”
- \( p \sqsupseteq q \) “\( p \) is outside of \( q \) (i.e., \( p \) is not located at point \( q \)).”

EUL-relations are denoted by \( R, R_1, R_2, \ldots \).

The set of EUL-relations that hold on a diagram \( D \) is uniquely determined, and we denote this set by \( \text{rel}(D) \). For example, \( \text{rel}(S_1 + S_2) \) in Example 3 is \( \{ x \sqsubset A, x \sqsubset B, x \sqsubset C, A \sqsupset B, A \sqsupset C, B \sqsubset C \} \). In the description of \( \text{rel}(D) \), we omit the reflexive relation \( s \sqsubset s \) for each object \( s \). Furthermore, we often omit relations of the form \( p \sqsupset q \) for points \( p \) and \( q \), which always hold by definition. In the following, we consider the equivalence class of diagrams in terms of the EUL-relations.

To clarify the intended meaning of our diagrams, we describe the translation of diagrams into the usual first-order formulas. See [10, 11] for a detailed description of our translation. Each named circle is translated into a unary predicate, and each constant (resp. existential) point is translated into a constant symbol (resp. variable). Then, each EUL-relation \( R \) is translated into a formula \( R^\circ \) as follows:

\[
\begin{align*}
(p \sqsubset A)^\circ & := A(p); & (p \sqsupset A)^\circ & := \neg A(p); \\
(A \sqsubset B)^\circ & := \forall x(A(x) \rightarrow B(x)); & (A \sqsupset B)^\circ & := \forall x(A(x) \rightarrow \neg B(x)); \\
(A \sqsupseteq B)^\circ & := \forall x((A(x) \rightarrow A(x)) \land (B(x) \rightarrow B(x))).
\end{align*}
\]

Let \( D \) be an EUL-diagram with the following set of relations: \( \text{rel}(D) = \{ R_1, \ldots, R_i, x_1 \sqsubset A_1, \ldots, x_l \sqsubset A_k \} \), where \( \sqsubset \) is \( \sqsubset \) or \( \sqsupset \), and no existential point appears in \( R_1, \ldots, R_i \). Then, the diagram \( D \) is translated into the following conjunctive formula:

\[
D^\circ := R_1^\circ \land \cdots \land R_i^\circ \land \exists x_1(\overline{A_1}(x_1) \land \cdots \land \overline{A_k}(x_1))) \land \cdots \land \exists x_l(\overline{A_1}(x_l) \land \cdots \land \overline{A_k}(x_l)),
\]

where \( \overline{A}(x) \) is \( A(x) \) or \( \neg A(x) \) depending on \( \sqsubset \).

Note that our diagram is abstractly a “conjunction of relations.” Note also that we interpret the \( \sqsupseteq \)-relation so that it does not convey any specific (inclusion or exclusion) information about the relationship between circles. (This interpretation is based on the convention of Venn diagrams. See [6] for further details.) Thus, \( A \sqsupseteq B \) is translated into a tautology as above.

From a semantic viewpoint, by interpreting circles (resp. points) as nonempty subsets (resp. elements) of a set-theoretical domain, each “syntactic object” EUL-diagram is interpreted in terms of the relations that hold on it. See [12] for details.

Remark. Our system is essentially the same as the region connection calculus RCC [8]. RCC is a topological approach to qualitative spatial representation and reasoning, and is applied, for example, to Geographic Information System (GIS). RCC investigates eight basic relations, including our inclusion, exclusion, and crossing (partially overlapping), between spatial regions (circles in our framework). Although RCC investigates general \( n \)-dimensional spaces of spatial regions, we concentrate on 2-dimensional diagrams. Thus, without any named points, our system can be considered as a subsystem of RCC. See, for example, [8] for surveys of RCC.
We next review the Euler diagrammatic inference system of [6], called the Generalized Diagrammatic Syllogistic inference system GDS. GDS consists of two kinds of inference rules: Deletion and Unification. Deletion allows us to delete a circle or point from a given diagram. Unification allows us to unify two diagrams into one diagram in which the semantic information is equivalent to the conjunction of the original two diagrams.

Example 3. Some A are B, All B are C \models Some A are C

The validity of the given inference is shown by the Unification of the two diagrams \( S_1 \) and \( S_2 \), which represent the given premises. In this unification, circle B in \( S_1 \) and \( S_2 \) is identified, and C is added to \( S_1 \) so that B is inside C and C overlaps with A without any implication of a specific relationship between A and C. \( S_3 \), which represents the given conclusion, is then obtained by deleting B.

Two types of constraint are imposed on Unification. One is the constraint for determinacy, which blocks disjunctive ambiguity with respect to the location of points, and the other is the constraint for consistency, which blocks representing inconsistent information in a single diagram. Unification can only be applied when these constraints are satisfied.

The notion of diagrammatic proof, or d-proof for short, is defined inductively as a tree structure consisting of Unification and Deletion steps. The provability relation \( \vdash \) is defined as usual in terms of the existence of a d-proof. See [6, 9] for details.

GDS is shown to be sound and complete with respect to our set-theoretical semantics. To avoid introducing the diagrammatic counterpart of the so-called absurdity rule in our system, we impose a consistency condition on the set of premise diagrams \( D_1, \ldots, D_n \) in our formulation of completeness, i.e., the set of premise diagrams has a model. See [6] for further discussion and a detailed proof.

Proposition 1. Let \( D_1, \ldots, D_n \) be consistent. \( \mathcal{E} \) is a semantic consequence of \( D_1, \ldots, D_n \) \( (D_1, \ldots, D_n \models \mathcal{E}) \) if and only if \( \mathcal{E} \) is provable from \( D_1, \ldots, D_n \) \( (D_1, \ldots, D_n \vdash \mathcal{E}) \) in GDS.

Note that the above completeness is obtained by regarding diagrams as “syntactic objects”, corresponding to first-order formulas, and by defining appropriate set-theoretical semantics.

We next investigate a view of Euler diagrams as counterparts of set-theoretical models.

3 Euler diagrams as models

By regarding our diagrams as models, we define the truth condition for our sentences.

Definition 4. Constants are interpreted as named points, predicates are circles. Then, for a diagram \( \mathcal{D} \):

- \( a \) is \( B \) holds on \( \mathcal{D} \) iff named point \( a \) is located inside circle \( B \);
- \( a \) is not \( B \) holds on \( \mathcal{D} \) iff named point \( a \) is located outside circle \( B \);
- All \( A \) are \( B \) holds on \( \mathcal{D} \) iff \( A \subseteq B \) holds on \( \mathcal{D} \);
- No \( A \) are \( B \) holds on \( \mathcal{D} \) iff \( A \nsubseteq B \) holds on \( \mathcal{D} \);
- Some \( A \) are \( B \) holds on \( \mathcal{D} \) iff there exists an existential point \( x \) in the intersection of \( A \) and \( B \);
- Some \( A \) are not \( B \) holds on \( \mathcal{D} \) iff there exists an existential point \( x \) inside \( A \) and outside \( B \).
We write \( D \vDash S \) when sentence \( S \) holds on diagram \( D \).

**Example 5.** Some \( A \) are \( B \) holds on each of the following diagrams, for example.

\[
\begin{align*}
D_1 & : A \quad B \\
D_2 & : B \quad A \\
D_3 & : A \quad B \\
D_4 & : B \quad A \\
D_5 & : A \quad B
\end{align*}
\]

We define the notion of **Euler diagrammatic validity** in the same way as model-theoretic validity.

**Definition 6.** For \( n \neq 0 \), \( S_1, \ldots, S_n \vDash S \) if \( S \) holds on any diagram \( D \) on which \( S_1, \ldots, S_n \) hold (i.e., \( \forall D, D \vDash \bigwedge S_i \Rightarrow D \vDash S \)).

In our **Unification** of GDS, as illustrated in Example 3, when the relationship between circles cannot be determined as \( \sqsubset \) or \( \sqsupset \) from the given premises, we assign this to be a \( \vDash \)-relation without any implication. However, if we extend the notion of “unification” to allow additional implications (and hence, such that this “unification” is invalid) as long as all relations on premises are preserved, then we can describe model construction in terms of the extended “unification” as follows.

**Example 7.** Some \( A \) are \( B \), All \( B \) are \( C \) \( \vdash \) Some \( A \) are \( C \)

Various diagrams can represent the given premises, and there are various ways to “unify” these diagrams (using **Deletion** if necessary). The following describes three possible diagrams, and the conclusion Some \( A \) are \( C \) holds in each of them. Cf. Example 3, where the representations of sentences are fixed canonically in a one-to-one correspondence depending on each sentence, and the way of unification is also fixed uniquely.

It is shown that the canonical form of the diagram for every sentence (e.g., \( D_1 \) in Example 3 for Some \( A \) are \( B \)) is sufficient to consider the usual model-theoretic validity. Then, through the completeness of GDS, it is shown that Euler diagrammatic validity of Definition 6 is equivalent to the usual model-theoretic validity.

### 4 Euler diagrams for counter-models

Models can be practically applied to disprove given inferences. Let us consider how to disprove a given inference using Euler diagrams.

**Example 8.** Some \( A \) are \( B \), All \( B \) are \( C \) \( \not\vdash \) All \( A \) are \( C \)

These premises are the same as in Example 3. To construct a diagram to falsify the given conclusion All \( A \) are \( C \), we add a fresh existential point \( y \) to the unified diagram \( S_1 + S_2 \) of Example 3 as follows.
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The existential point \( y \) denotes that “there exists something that is \( A \) but not \( C \),” i.e., the negation of All \( A \) are \( C \). Note that even after the addition of \( y \), all of the \( \sqsubset, \sqsupset \) relations that hold on the premises \( \{ x \sqsubset A \land x \sqsubset B \land S_1, A \sqsupset C \land S_1 \} \) are maintained. Hence, the above \( S_1 + S_2 + y \) neatly illustrates a counter-model of the given inference in which all premises are true but the conclusion is false.

Let us define our counter-diagrams.

**Definition 9.** A diagram \( E \) is a **counter-diagram** of \( D \), and vice versa, when one of the following holds between \( D \) and \( E \):
- \( A \sqsubset B \) holds on \( D \), and \( A \sqsupset B \) holds on \( E \);
- \( A \sqsubset B \) holds on \( D \), and, for some \( x \), \( x \sqsubset A \) and \( x \sqsupset B \) hold on \( E \);
- \( A \sqsupset B \) holds on \( D \), and, for some \( x \), \( x \sqsubset A \) and \( x \sqsubset B \) hold on \( E \).

Note that our diagram is abstractly a conjunction of relations, and the above definition lists the so-called “counter-relation” for every relation. As illustrated in Example 11, there may be a number of counter-diagrams for a given diagram \( D \), because a counter-diagram of \( D \) may contain circles and points that are irrelevant to \( D \). Thus, in general, the notions of a counter-diagram of \( D \) and the negation of \( D \), which should be uniquely determined, are different.

**Example 10.** Let \( D \) be the following diagram on the left. Then, all of \( D_6, D_7, D_8, \) and \( D_9 \) are counter-diagrams of \( D \). Thus, a diagram and its counter-diagram do not necessarily share the same circles and points, and it is sufficient that a relation that holds on a counter-diagram falsifies a relation of the given diagram.

We can list all of the operations to construct counter-diagrams, such as the addition of a new point illustrated in Example 8. However, let us consider another way to construct counter-diagrams. Theoretically speaking, \( S_1 + \ldots + S_n \) means that there exists a \( D \) such that \( D \models \neg S_1 \land \ldots \land S_n \land \neg T \). Based on this fact, we construct a diagram that falsifies the given conclusion, and, by unifying the diagram with all premises, we construct a counter-diagram for the given inference. We formalize our counter-diagram construction according to this idea. Let us reconsider the inference in Example 8.

**Example 11.** Some \( A \) are \( B \), All \( B \) are \( C \) \( \not\models \) All \( A \) are \( C \)

Let \( E' \) be a diagram such that \( \text{rel}(E') = \{ y \sqsubset A, y \sqsupset C, A \dashv C \} \), which represents Some \( A \) are not \( C \), and which falsifies the given conclusion All \( A \) are \( C \). We unify this \( E' \) with \( S_1 \) and \( S_2 \), as illustrated in Fig. 4. Thus, the resulting diagram \( S_1 + S_2 + E' \) is the same as \( S_1 + S_2 + y \) in Example 8 and disproves the given inference.

Let us examine another example.

**Example 12.** No \( A \) are \( B \), There is something \( C \) \( \not\models \) No \( A \) are \( C \)

Let \( E' \) be a diagram such that \( \text{rel}(E') = \{ y \sqsubset A, y \sqsubset C, A \not\models C \} \), which represents Some \( A \) are \( C \), and which falsifies the given conclusion No \( A \) are \( C \). We then unify this \( E' \) with the given premise \( S_3 \), representing No \( A \) are \( B \), as illustrated in Fig. 4. When we further attempt to unify diagram \( S_4 \), in which \{ \( x \sqsubset C \) \} holds, representing There is something \( C \), we find...
three possibilities for the position of \( x \), that is, \( x \) is indeterminate with respect to circles \( A \) and \( B \), and we cannot unify this from our valid Unification rules. However, whichever of the three possibilities gives the position of \( x \), we obtain a counter-diagram to disprove the given inference. For example, \( E' \) in Fig. 2 is a required counter-diagram.

Because we cannot obtain the above \( E'' \) from \( E' + S_3 \) and \( S_4 \) using our valid Unification rules of GDS, we construct counter-diagrams by introducing another “invalid” rule: invalid Point Insertion (iPI). This arbitrarily fixes the position of a point from several possibilities. Then, we obtain \( E'' \) from \( E' + S_3 \) and \( S_4 \).

In this way, a given inference is shown to be invalid by: (1) constructing a diagram that falsifies the given conclusion; and (2) unifying the diagram of (1) with all premises. In the resulting diagram, the relation falsifies the given conclusion and all of the \( \sqsubseteq \) and \( \sqsupseteq \) relations of the given premises hold. Hence, it is a counter-diagram for the given inference.

**Remark.** In our system, we postulate the so-called existential import in the literature of syllogisms. With this postulate, for example, we have All \( A \) are \( B \), No \( B \) are \( C \) \( \models \) Some \( A \) are not \( C \), even though this is not valid from the usual logical viewpoint without the existential import. Without this postulate, two diagrams, say \( D \) in which \( A \sqsubseteq B \) holds and \( E \) in which \( A \sqsupseteq B \) holds, are consistent when \( A \) denotes the empty set. However, it is difficult to express \( D \) and \( E \) in a single diagram, as our system lacks a device to express the emptiness of circles.

Even if we introduce another device such as “shading”, which expresses the emptiness of the corresponding region (cf. [7]), there remains a question as to whether we draw the shaded circle \( A \) inside or outside \( B \) (whichever is legal in an abstract sense). Thus, it seems that the assumption of the existential import is natural for our Euler diagrammatic system.

Let us define our counter-diagrammatic proofs.
Definition 13. A counter-diagrammatic proof, or counter-d-proof for short, of $E$ under $D_1, \ldots, D_n$ is a tree structure:

- consisting of valid Unification rules and invalid iPl;
- whose premises are $D_1, \ldots, D_n, E'$, where $E'$ is a counter-diagram of $E$ such that no existential point of $E'$ appears on $D_1, \ldots, D_n$;
- in whose conclusion, all $\sqsubseteq, \vdash$-relations of $D_1, \ldots, D_n, E'$ hold.

We write $D_1, \ldots, D_n \rightarrow E$ when there exists a counter-d-proof of $E$ under $D_1, \ldots, D_n$.

Note that we may not use the invalid iPl as illustrated in Example 11. Additionally, we can freely choose a counter-diagram $E'$ to add to the premises of a counter-d-proof. Hence, it may be the case that $E'$ is itself a required conclusion of a counter-d-proof. In our proof of Proposition 2 below, we present a canonical method to choose $E'$ and construct counter-d-proofs. See [11] for further details.

Although space limitations prevent us from going into detail, there exists an invalid inference for which we cannot construct a counter-d-proof in our framework of inclusion, exclusion, and crossing relations. Thus, we need to restrict the position of existential points appearing in a given conclusion. We call a diagram “relational” if the position of every existential point in the diagram is determined by relations with at most two circles. See [11] for a detailed discussion. Thus, we have shown that our counter-d-proofs sufficiently characterize the notion of invalidity.

Proposition 2. Let $D_1, \ldots, D_n$ be consistent, and $E$ be relational. $E$ is not a valid conclusion of $D_1, \ldots, D_n$ (i.e., $D_1, \ldots, D_n \nvdash E$) if and only if there exists a counter-d-proof of $E$ under $D_1, \ldots, D_n$ (i.e., $D_1, \ldots, D_n \rightarrow E$).

There is no need to worry about relational diagrams when we consider the usual syllogisms or transitive inference, where a conclusion diagram consists of only two circles, and it is already relational. We require the restriction when we consider a more general diagram as a conclusion, in which only a particular existential point makes the given inference invalid. See [11].

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