Ionization of Binary Bound States in a Strongly Coupled Quark-Gluon Plasma

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(October 30, 2018)

Although at temperatures $T \gg \Lambda_{\text{QCD}}$ the quark-gluon plasma (QGP) is a gas of weakly interacting quasiparticles (modulo long-range magnetism), it is strongly interacting (sQGP) in the temperature range $(1 - 3) T_c$. One aspect of these interactions is the existence of many binary bound states of quasiparticles. Only $q\bar{q}$ ones have been so far directly seen on the lattice, for charmed and light quarks, but other attractive channels in $qg, gg$ are likely to have them as well. It was argued in our previous paper that such bound states account for a significant part of the bulk properties such as density and pressure. Using the same model, we evaluate the energy loss $dE/dx$ due to the ionization of these states. We found that it is substantial, but only in the narrow interval of temperatures $T = (1.4 - 1.7) T_c$. In contrast to that, we show that radiative and elastic losses are not likely to be modified much by binding, as the total density of color charges is close to what it is for weakly coupled quasiparticles. These distinctions would be important for understanding the energy dependence of jet quenching.

Jet quenching is a sort of “tomography” of the prompt excited phase triggered in high energy heavy ion collisions. Even very hard jets are expected to lose some energy during their passage through the system, thereby providing information about the early stages of the collision. The quenching factor $Q(p_t)$ is defined as the observed number of jets normalized to the expected number of jets calculated in the parton model\footnote{Effects due to initial state interaction (nuclear shadowing) are included. In other words, the parton distribution functions are nuclear rather than hadronic, and follow from lepton-nuclei experiments. Parton rescattering in nuclei at the origin of the so called Cronin effect is included in the expected yield. Final state interactions are excluded.}. Experimentally, jet reconstruction in a heavy ion environment is very difficult to achieve. Therefore, all currently reported results for jet quenching refer to the observed/expected ratio of the yields of single hadrons. Two-particle correlations have also been studied experimentally, confirming existence of forward and backward (in azimuth) correlations expected from jets.

In the early theoretical assessments of jet quenching \cite{1} the mechanism considered was a jet re-scattering on quasiparticles in the Quark-Gluon Plasma (QGP). Taking into account radiation effects leads to larger quenching \cite{2} for hard jets. However, in dense enough matter one has to correct the radiative energy loss due to destructive interferences (the so-called Landau-Pomeranchuk-Migdal or LPM effect), which modifies and reduces the effect, see e.g. \cite{3} and references therein. For a recent brief summary see also \cite{4}.

One way to enhance the quenching in the prompt phase is through a synchrotron-like QCD radiation \cite{5}. However, this only takes place during a very short initial time in the heavy ion collision, when (and if) the gluons can be treated as a coherent classical field (the color glass condensate \cite{6} or a set of QCD sphalerons \cite{7}).

Experimentally, a relatively modest quenching of rather high energy jets going through cold nuclear matter has been first observed in deep inelastic scattering, see e.g. \cite{8}. In contrast to that, the very first RHIC data at large $p_t$ have shown quite spectacular jet quenching, by about one order of magnitude. It implies that only jets originating from the surface of the nuclear overlap region reach the detector. Strong azimuthal anisotropy characterized by $v_2 = \langle \cos(2\phi) \rangle$ is observed, close or even somewhat exceeding the “geometrical limit” \cite{9}.

Addressing the origin of this strong quenching, one may naturally start with a question: Is this quenching just proportional to the parton density involved, as so many mechanisms predict? The apparently near-absence of jet quenching at CERN SPS energies\footnote{One more argument, not emphasized enough, is the ratio of direct photons to $\pi^0$ decays: it is about 2-3 at RHIC while only 0.2 or less at the SPS.} with the fact that the relevant multiplicity $dN/dy$ at $y = 0$ at these two energies is different by less than a factor of 2, it seems likely that a new mechanism of quenching opens up at RHIC energies. Future experiments will investigate this further, and at the time of this write-up, experimentalists are busy analyzing the latest $\sqrt{s} = 62$ GeV AuAu run at RHIC.

In this letter we evaluate a contribution to jet quenching, induced by the “ionization” of the recently found binary bound states at $T > T_c$. One motivation for that is that in ordinary matter, the QED $dE/dx$ for charged projectiles with gamma factors in the range $\gamma = 1$-$10^3$, is known to be dominated by the ionization losses.
FIG. 1. (a) The lines show twice the effective masses for quarks and gluons versus temperature $T/T_c$. Note that for $T < 3T_c$ we have $M_q > M_g$. Circles and squares are the estimated masses of the pion-like and rho-like $\bar{q}q$ bound states, while the crosses stand for all colored states. (b) Density of various components of sQGP, normalized to $T/T_c$, versus the temperature $T/T_c$ in units of the critical temperature. Circles and squares correspond to colorless mesons and colored bound states, respectively. The diamonds and crosses correspond to quark and gluon quasiparticles. The upper and lower lines are the total density of “states”, binaries and quasiparticles, and the total number of “charges”.

**Bound states in sQGP.** Recently a radically new view of the QCD matter in the temperature range $T = (1-3)T_c$ has emerged [10], in which the interaction between quasiparticles is strong enough to generate multiple binary bound states of quasiparticles. This picture provides a consistent description [11,16] of several previously disconnected lattice observations, such as (i) bound states for charmonium and some light $\bar{q}q$ states [12]; (ii) static potentials [13]; (iii) quasiparticle masses [14]; and (iv) bulk thermodynamics [15]. For studies of binary states in $\mathcal{N} = \triangle$ SUSY YM theory, in which a parametrically strong QGP-like phase is possible, see [17]. Last but not least, a liquid-like picture of matter also provides a natural explanation [10] for a successful description of collective phenomena at RHIC by ideal hydrodynamics.

In this letter we will not go over the detailed arguments and calculations, which can be found in the above-mentioned papers. However and for definiteness we need to explain the parameters of a particular model used for the estimates to be carried below. Table 1 lists all binary attractive channels of quarks and gluons, indicating the number of states and their corresponding squared effective dipole charge.

For the effective ($T$-dependent) number of flavors, we will use

$$N_f = 2 + e^{-m_s/T} \quad m_s = 120 \text{MeV} \quad (1)$$

Although listed in the table, two channels which have smaller attraction than $\bar{q}q$, namely $qg, gg$, are ignored in estimates to follow.

In Fig.1a we show the (doubled) quasiparticle masses and those for bound states, as determined in a model used in [16] for global thermodynamics. For the purposes of this paper we need the particle densities, shown in Fig.1b. The highest $T$ on that plot roughly corresponds to that at which $qg, gg$ states gets unbound. One can see that quark and gluon quasiparticles do not dominate the density: the binary bound states do. It is however amusing to note, that the total number of “charges”, defined as twice the density of composites plus those of quasiparticles, oscillate $\dagger$ around $n_{\text{charges}}/T^3 \approx 4$ in the whole interval. This is close to the total number for 16 massless gluons and 24 $\bar{q} + q$ ($N_f = 2$), the same within the uncertainties involved.

| channel | rep. | dipole factor $C_d$ | no. of states |
|---------|------|---------------------|--------------|
| $gg$    | 1    | 3                   | $9_s$        |
| $gg$    | 8    | 9/4                 | $9_s \times 16$ |
| $qg + \bar{g}g$ | 3 | 11/6                | $3_s \times 6_s \times 2 \times N_f$ |
| $gg + \bar{q}g$ | 6 | 4/3                 | $6_s \times 6_s \times 2 \times N_f$ |
| $\bar{q}q$ | 1 | 4/3                 | $8_s \times N_f^2$ |
| $qg + \bar{g}q$ | 3 | 1                   | $4_s \times 3_s \times 2 \times N_f$ |

**TABLE I.** Binary attractive channels, the subscripts $s, c, f$ mean spin, color and flavor, and $N_f$ is the number of relevant flavors.

$\dagger$Note that a minimum around $T = 1.2T_c$ is where the neglected contributions of $qq, qgs$ will peak.

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Thus, the parameters of our model for sQGP (fixed from the solution of the Klein-Gordon equation in [16] with lattice-based potentials and masses) possess an interesting but mysterious fine tuning, leading to a rather accurate duality between the total density of “charges” with a naive weakly coupled QGP made of massless quasiparticles.

Photodissociation. We start by recalling some textbook results in QED. In the dipole approximation, the ionization cross section relates to the matrix element of the momentum operator between the initial and final states,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2|\vec{p}|^2}{2\pi m_0 \omega} < i|\vec{p}|f>|^2.$$ \hspace{1cm} (2)

There are two cases in which the corresponding cross section has been analytically calculated: i. Coulomb systems, with a long-range interaction in the outgoing channel, and ii. systems bound by a short-range interaction, where the outgoing state can be approximated by a free plane wave.

For Coulomb systems the cross section is known to be

$$\sigma(\omega) = 2\alpha^2 \frac{\pi^2}{3} |\alpha|^2 \left(\frac{\Delta}{\omega}\right)^4 \frac{e^{-4\nu \arccot \nu}}{1 - e^{-2\pi \nu}}$$ \hspace{1cm} (3)

where the binding energy is Δ, the Bohr radius is a = 1/mZα, the Gamow parameter is ν = Zα/ν, with ν being the outgoing velocity. The ionization energy is \(\Delta = Z^2 \alpha^2 m/2\). The remarkable feature of this cross section (due to long-range Coulomb final state interactions) is its finite value at the threshold \(\omega = \Delta\), where the outgoing velocity \(\nu \rightarrow \infty\) with the last factor in (3) becoming finite \(1/e^4\). In the opposite limit \(\omega \gg \Delta\) one finds \(\nu \rightarrow 0\) and the cross section rapidly decreases \(\sigma(\omega) \approx 1/\omega^{7/2}\).

For short range interactions, an instructive example is the deuteron photodissociation. Using the deuteron wave function in a simple form \(\psi(r) \approx e^{-\kappa r}\), where \(\kappa^2 = M\Delta\) with \(M\), \(\Delta\) are the nucleon mass and binding energy, the photodissociation cross section is

\[\sigma(\omega) = \frac{8\pi\alpha}{3} \frac{\sqrt{\Delta}}{M}\frac{(\omega - \Delta)^{3/2}}{\omega^{3}}.\] \hspace{1cm} (4)

In this case the cross section at threshold vanishes with the cube of the momentum following from (2), and peaks at \(\omega = 2\Delta\).

In the QCD context the same effect has been discussed for the disintegration of heavy quarkonia by incoming gluons. This process was originally discussed by one of us [18], and subsequently by Bhanot and Peskin [19] using (4) to discuss charmonium disintegration by hadrons. A relativistic version of (4) with detailed balance built in was discussed by one of us [20] in the context of charmonium disintegration by gluo-effect in the quark-gluon plasma.

Our case is intermediate between (3-4): the bound states in the QGP have the effective Coulomb interaction at small \(r\), which is however Debye screened at large \(r\). One thus expect the threshold behavior of (4) and large energy behavior of (3). Let us also note that in the cases considered here, like for deuteron, no internal excitation possible as there is only one bound state.

Energy Loss. For a high energy partons is estimated using the well known Weizsacker-Williams (WW) approximation, which in QED views the Coulomb field of rapidly moving charges as a set of “equivalent photons”. Their number per frequency is

$$n(\omega) = \frac{2\alpha}{\pi} \ln \left(\frac{E}{\omega}\right) \frac{d\omega}{\omega}$$ \hspace{1cm} (5)

where the logarithm originates from the integral over the transverse momenta. In the QCD context this idea is known as “splitting functions” where one-kind of parton can split into two. At small relative energy the gluon-in-quark and gluon-in-gluon number per frequency is just (5) where \(\alpha \rightarrow \alpha_s\xi\), with \(\xi = C(3)\) and \(\xi = C(8)\) respectively.

The ensuing energy loss per length \(dE/dz\) may be written as

$$\frac{dE}{dz} = \int_\Delta^E d\omega \frac{n(\omega)}{\lambda(\omega)} \frac{\omega}{\lambda(\omega)}.$$ \hspace{1cm} (6)

The energy dependent mean-free path

$$\lambda(\omega) = \frac{1}{\rho \sigma_{\text{ph}}(\omega)}$$ \hspace{1cm} (7)

involves the parton-bound-state cross section \(\sigma_{\text{ph}}\) and the density of bound states of mass \(M\). At temperature \(T > T_c\) the latter is just \((\hbar = 1)\)

$$\rho = \frac{e^{-M/T}}{\lambda_f^2} = e^{-M/T} (MT/2\pi)^{3/2}.$$ \hspace{1cm} (8)

The integral in (6) leads \(dE/dx \approx \ln(E/\Delta)\) for relativistic particles, which is well known and well tested in QED in atomic ionization losses.

The ionization cross section follows from the forward gluon-bound-state amplitude by the optical theorem, \(\sigma_{\text{ph}} = \text{Im} M_{gb}/\omega\). For energies \(\omega \approx \Delta\), the forward amplitude is dipole dominated. Standard second order perturbation theory yields

$$\text{Im} M_{gb} = \langle gb| (d^{A^I} E^{A^I}) \pi \delta (\hbar + \Delta - \omega) (d^{B^J} E^{B^J}) |gb\rangle$$ \hspace{1cm} (9)

with the colored dipole moment for the bound state.
\[ d^{Ai} = \frac{g}{2} (T^{A_1}_i - T^{A_2}_i) x^i \] (10)

for two constituents each of mass \( m \), respectively in the representation 1, 2. The effective factor in cross section can be rewritten solely in terms of Casimirs

\[
c_d = \frac{1}{2} (C_1 + C_2) - \frac{1}{4} C_R . \] (11)

and it is listed in the Table 1.

In the subsequent analysis we assume that the ionized outgoing constituents are free and non-relativistic with \( h \approx p^2/m \). Averaging over the gluon spin and color yields

\[
\langle g|E^A_i E^B_j|g \rangle = \frac{\omega^2}{24} \delta^{ij} \delta_{AB} . \] (12)

For Coulomb bound states with a deuteron-type wave function, straightforward algebra using (9-11) yield the ionization cross section

\[
\sigma_{gb}(\omega) = \frac{g^2 c_d}{3} \frac{\sqrt{\Delta}}{m} \frac{(\omega - \Delta)^{3/2}}{\omega^3} . \] (13)

This result is in agreement with (4) modulo color factors. The temperature dependence of the inverse absorption length \((\lambda \ast T_c)^{-1}\) for a gluon of 4 different energies is shown in Fig.2(a).

In terms of (13) the energy loss (6) is simply

\[
\frac{dE}{dz} = \frac{c_d}{3} \frac{g^2 \rho}{m} \int_1^{E/\Delta} dx \left( \xi \frac{\alpha}{\pi} \ln \left( \frac{E}{x\Delta} \right) \right) \frac{(x-1)^{3/2}}{x^3} . \] (14)

Using the density of binaries discussed above, and performing the integral over the energy of “equivalent gluons” we get our final results for the energy loss \( dE/dx \) for gluon jets as shown in Fig.2b by the thick solid lines. (As usual, for quarks the results are 4/9 times smaller.)

For comparison, we plotted (thin dashed lines) a contribution from elastic losses, using Bjorken’s expression

\[
\frac{dE}{dx} = \frac{1}{2} C_2 3\pi \alpha_s^2 T^2 \ln(3ET/(2\mu^2)) \] (15)

where we used the Debye mass for \( \mu \approx 2T \). Although this formula was derived for massless quasiparticles, due to the duality mentioned above it holds approximately for our model with binaries as well. We note that although the magnitude is comparable, the \( T \)-dependence of \( dE/dx \) due to these two mechanisms is quite different.

**Radiative losses.** A comparison to the radiative losses can be better done not in terms of \( dE/dx \) but in total losses after a path of length \( L \), since the LPM effect makes \( dE/dx \sim L \) [3]. Using the same simplifications as discussed in [4], we can write radiative energy loss on gluons as

\[
\Delta E_{rad} \approx \frac{27\pi \alpha_s^3}{8} n_{gluons} L^2 \ln(E/\mu) \] (16)

A comparison to our result written in a similarly approximate form yields the ratio

\[
\frac{\Delta E_{\text{photo}}}{\Delta E_{\text{rad}}} = \frac{2}{9 \alpha_s} \sum_i C_i n_i \frac{1}{LM} = (l_0/L) \] (17)

![Fig. 2.](image-url)
radiative losses are thus larger than ionization ones if the length the jet passes in matter is larger than \( l_0 \).

However, one can easily see that this difference is hardly important in practice. Indeed, already from the magnitude of the experimental jet quenching \( Q(p_t) \approx 0.1-0.2 \) and the rather steep slope of the \( p_t \) spectra in the relevant \( p_t \) region, it follows that a loss of about \( \Delta E/E \approx 1/5 \) is sufficient at RHIC conditions to get a suppression of the order of that observed, making such jets invisible anyway. As one can see from Fig. 2, such losses will take place for a gluon (or quark) passing through only \( L \approx 0.5 \) fm (or 1 fm) of matter with \( T \approx 1.6 T_c \approx 300 \) MeV. For such length both quenching mechanisms are about equally important.

**Discussion.** It is well known that partonic jets of quarks and gluons, promptly produced in RHIC collisions, can be quenched via gluonic radiation. We have shown that additional “photodissociation” mechanism of quenching can appear, provided there are binary bound states at \( T > T_c \). The energy loss is few times \( \Delta \) the binding energy of the \( gg \), \( qq \) or \( gq \) per collision. We evaluated the total energy loss due to it, and found that it is comparable to the radiation loss in magnitude for lengths \( L \approx 1 \) fm.

Due to the “duality” between the total density of charges in our model and the naive gas of massless quarks and gluon noticed above, the radiative energy loss is the same for both (strong) sQGP and naive (weak) wQGP models. The “photodissociation” we have discussed, on the other hand, is present only in the sQGP scenario, and is only there in a restricted interval of temperatures \( T_c < T < 1.7 T_c \approx 300 \) MeV. Such \( T \) are reached at RHIC but not at the SPS. This distinction, as well as very restricted interval of \( T \) where new mechanism exists, will help us to decide which scenario is the case.

Another important distinction between the two mechanisms of quenching is that radiation remains in the forward cone, and so it can be found there experimentally. The energy lost by ionization of binaries obviously is simply dissipated in the heat bath.

**Acknowledgments.** We thank G.Brown for useful discussions initiating this work. It was partially supported by the US-DOE grants DE-FG02-88ER40388 and DE-FG03-97ER4014.

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