Applications of a curvature correction turbulent model for computations of unsteady cavitating flows

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Abstract. A Curvature Correction model (CCM) based on the original k-ε model is proposed to simulate unsteady cavitating flows. The objective of this study is to validate the CCM model and further investigate the unsteady vortex behaviors of cavitating flows around a Clark-Y hydrofoil. Compared with the original k-ε model, predicted results are improved in terms of the cavity detachment and hydrofoil fluctuations. Results show that streamline curvature correction of CCM model overcomes the over-predictions of turbulence kinetic energy and eddy viscosity in cavitating vertical region with the original k-ε model, which leads to better simulation abilities for the unsteady cavitating flow computations. Based on computations, it is proved that the vortex structure is significantly modified by the transient cavitation, especially with respect to the cavity shedding behaviors. Complex vortex interactions and corresponding cavity shedding process near hydrofoil trailing edge lead to various load frequencies.

1. Introduction
Cavitation is a dynamic phase-change phenomenon that occurs in liquids when the static pressure drops below the vapor pressure of liquid[1][2]. It is well known that the unsteady cavitation in turbo-machinery and marine control surfaces will lead to problems such as material damage, vibration, noise and reduced efficiency[3][4]. Actually, cavitation physical mechanisms are not well understood due to the complex, unsteady flow structures associated with turbulence and phase change.

Cavitating flows are generally relatively high Reynolds number flows and hence the turbulence modeling plays an important role in the capture of unsteady behaviors. Conventional eddy-viscosity RANS turbulence models, such as the k-ε model, have been widely used in many turbulent flows of practical interest. However, many researchers[5][6] have indicated that high eddy viscosity of the original Launder–Spalding version of the k-ε model may dampen the vortex shedding behavior and excessively restrain the cavitation instabilities. To improve the predicted results for unsteady cavitating flows with conventional eddy viscosity models, many approaches have already been developed. A widely used approach is the density correction model (DCM) [7], which takes into account the compressibility characteristics of cavitating flows. Better predictions of unsteadiness of cavitating flows with DCM model have been proved [8][9][10]. Similarly, the present paper is devoted to the refinement of conventional eddy-viscosity RANS model considering the streamline curvature effect in strong vertical cavitating flows.

It’s well known that instabilities of cavities often result in the formation of large-scale vortex
structures and various researches have proved the strong correlation between cavitation and vortex structures. Gopalan and Katz observed that the collapse of the vapor structure is a primary mechanism of vorticity production leading to the generation of hairpin vortices in the downstream region. Lyer and Ceccio found the streamwise velocity fluctuations increased with cavitation, which is confirmed that the collapse of the vapor cavities is a source of vorticity generation. An inherent shortcoming of the conventional eddy-viscosity RANS turbulence models is their property of material frame indifference. In other words, these models do not contain streamline curvature production terms, and hence are not capable of capturing effects of streamline curvature in strong vertical cavitating flows. An effective approach for resolving this issue is proposed by Spalart and Shur, which has been introduced to many conventional eddy-viscosity RANS turbulence models, including the Spalart–Allmaras (S-A) model and the Shear Stress Transport (SST) model. With revised models, better predictions of streamline curvature effect in turbulent flows with conventional eddy-viscosity RANS models have been documented including 2D flow in a channel with U-duct, 3D foil tip vortex flow and so on.

In the present paper, the streamline curvature correction is introduced to the \( k-e \) model to achieve the curvature correction model (CCM) for simulations of sheet/cloud cavitating flows around a Clark-Y hydrofoil. Validations of CCM model are obtained comparing with experimental data as well as the baseline model, and hence advantages of CCM model are highlighted. Based on the numerical results, unsteadiness of vortex structures and induced lift fluctuations in cloud cavitating flows are discussed.

2. Numerical model in cavitation computation

2.1. Favre-averaged continuity and momentum equations

The set of governing equations comprises the conservative form of the incompressible Favre-averaged Navier-Stokes equations, coupling with classical cavitation model and turbulence closure. The mass continuity, momentum equations are given below.

\[
\frac{\partial \rho_m}{\partial t} + \frac{\partial (\rho_m u_j)}{\partial x_j} = 0
\]  

\[
\frac{\partial (\rho_m u_i)}{\partial t} + \frac{\partial (\rho_m u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ (\mu_l + \mu_v) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \right) \right] - \frac{\partial}{\partial x_j} \left[ \left( \mu_l + \mu_v \right) \left( \frac{\partial u_k}{\partial x_k} \right) \right] + \frac{\partial}{\partial x_j} \left( \frac{\partial k}{\partial x_j} \right)
\]

The mixture property \( \varphi_m \), can be expressed as

\[
\varphi_m = \varphi \alpha_i + \varphi (1 - \alpha_i)
\]

where \( \rho_m \) is the mixture density, \( u \) is the velocity, \( p \) is the pressure, \( \mu_l \) and \( \mu_v \) are the laminar and turbulent viscosity, subscripts \( i, j, k \) are the directions of the axes, subscripts \( l \) and \( v \) present liquid and vapor, and \( \varphi \) can be density, viscosity and so on.

2.2. Curvature correction turbulent model (CCM)

2.2.1. Baseline model. The original \( k-e \) turbulence model has become the workhorse of practical engineering flow calculations since it was proposed by Launder and Spalding. The \( k-e \) model shows two partial differential equations for the transport of the turbulence kinetic energy \( k \) and dissipation rate \( \varepsilon \):

\[
\frac{\partial (\rho_m k)}{\partial t} + \frac{\partial (\rho_m k u_j)}{\partial x_j} = P_k - \rho_m \varepsilon + \frac{\partial}{\partial x_j} \left[ \left( \mu_l + \frac{\mu_v}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right]
\]
\[
\frac{\partial (\rho_m e)}{\partial t} + \frac{\partial (\rho_m e u_j)}{\partial x_i} = C_{ei} \frac{e}{k} P_k - C_{ej} \rho_m \frac{e^2}{k} + \frac{\partial}{\partial x_i} \left[ \left( \mu + \frac{\mu_f}{\sigma_f} \right) \frac{\partial e}{\partial x_i} \right]
\]

where the production term of turbulence kinetic energy \( (P_k) \) and the Reynolds stress tensor \( (\tau_{ij}) \) are defined as:
\[
P_k = \tau_{ii} = \frac{\partial u_i}{\partial x_i} \quad \text{and} \quad \tau_{ij} = -\rho \mu' \mu_j' = \frac{2}{3} \rho k \delta_{ij} - \mu_i \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),
\]
with \( C_{ei} = 1.44, \ C_{ej} = 1.92, \ C_{\varepsilon} = 1.3, \) and \( \sigma_k = 1.0. \) The turbulent eddy viscosity is defined as:
\[
\mu_i = \frac{C_\mu \rho_k k^2}{\varepsilon}, \quad C_\mu = 0.09
\]

2.2.2. Curvature correction method. Shur\cite{16} suggested a modification to describe the streamline curvature effect in strong vertical flows. Correction function \( f_{Rotation} \) is defined as:
\[
f_{Rotation} = \left( 1 + C_{r1} \right) \frac{2r^*}{1 + r} \left[ 1 - C_{r3} \tan^{-1} (C_{r2} R) \right] - C_{r1}
\]

\( R \) and \( r^* \) here are defined as
\[
r = 2\omega \alpha S_{yy} \left( \frac{dS_{yy}}{d\omega} \right), \quad r^* = \frac{S}{\omega}
\]
The strain rate and vorticity tensor are defined, respectively, using Einstein summation convention as
\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad \omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)
\]
where \( S = \sqrt{2S_{yy}S_{ii}}, \ \omega = \sqrt{2\omega_{yy} \omega_{yy}}, \ D = \max \left( S^2, 0.09 \omega^2 \right). \) The empirical constants \( C_{r1}, \ C_{r2}, \) and \( C_{r3} \) are set to 1.0, 2.0 and 1.0, respectively. The curvature correction function \( f_i \) is used as a multiplier of the production term \( P_k \) in Eq.(4).
\[
P_k \rightarrow P_k \cdot f_i
\]

2.3 Computational setup

Computational set-up is shown in figure 1. The computational domain and boundary conditions are given according to the experimental set-up\cite{9}. The Clark-Y hydrofoil with the chord length of \( c = 0.07 \text{m} \) is located at the center of the test section with the angle of attack of \( 8^\circ. \) The two important dimensionless parameters are the Reynolds number \( Re \) and the cavitation number \( \sigma, \) which is defined based on the outlet pressure \( p, \) the saturated vapor pressure \( p_v, \) and the inlet velocity \( U_\infty. \)
\[
Re = \frac{U_\infty c}{v}, \quad \sigma = \frac{p - p_v}{\rho U_\infty^2 / 2}
\]
Computations are performed for cloud cavitation condition (\( \sigma = 0.8 \)). Constant velocity is imposed
at the inlet, $U_c = 10\text{m/s}$, with the corresponding Reynolds number of $Re = 7 \times 10^5$. The vapor pressure of water at $25^\circ\text{C}$ is $p_v = 3169\ \text{Pa}$. Details of the grid solutions are shown in Huang’s previous work[9]. Basically, the numerical grids used here are enough to the wall function requirement.

![Figure 1](image)

**Figure 1.** Boundary conditions for Clark-Y hydrofoil

### 3. Results and discussions

#### 3.1. Time-averaged cavity visualization and flow structures

The time-averaged flow structure and cavity shape with different models are shown in figure 2. The cavitation structures consist of two parts, which are attached and detached cavity respectively. The attached cavity is located in the leading edge of the hydrofoil, while the detached cavity is near the trailing edge. In the figure, attached cavity can be predicted with both the original $k$-$\varepsilon$ and CCM models. The detached cavity cannot be captured with the original $k$-$\varepsilon$ models, while it can be well predicted by the CCM models with appropriate scaling coefficient ($C_{\text{Scale}}=10, 20$).

![Figure 2](image)

**Figure 2.** Time-averaged vapor volume fraction contour and representative streamlines

Previous researches[9][18] indicate that the detached cavity is formed due to the re-entrant jet and overlaps with the vertical zone near the trailing edge. The formation and significance of the re-entrant jet with CCM model ($C_{\text{Scale}}=10$) are first highlighted. In Figure 3(a), a vertical zone consists of the re-entrant jet in the upper part and incoming flow from upstream in the lower part and the front of the re-entrant jet will determine the cavity end. The vertical zone will grow in size while the re-entrant jet pushing the attached cavity toward upstream will also become stronger, and then the cavity is detached in Figure 3(b) with a low density region near the center of the vertical zone.

It is known that the original $k$-$\varepsilon$ turbulence model over-predicts the turbulent viscosity, causing the re-entrant jet in cavitating flows to lose momentum. Thus, the re-entrant jet is not able to cut across the cavity sheet, which significantly modifies the predictions of detached cavity. To assess the CCM models, time averaged turbulence eddy viscosity contours are shown in Figure 4. In the figures, significant decrease of time averaged turbulence eddy viscosity in vertical zone with CCM models can be observed. It is due to the decrease of predicted time averaged turbulence kinetic energy discussed
before. As over-predictions of eddy viscosity with the original $k$-$\varepsilon$ model can be well solved, better predictions of detached cavity can be achieved with CCM model.

![Formation of the re-entrant jet](image1.png) ![Highlight of the detached cavity](image2.png)

**Figure 3.** The generation of re-entrant jet and detached cavity (by CCM model ($C_{\text{Scale}}=10$) with streamline)

![Original $k$-$\varepsilon$ model](image3.png) ![CCM ($C_{\text{Scale}}=1.0$)](image4.png)

(a) Original $k$-$\varepsilon$ model (c) CCM ($C_{\text{Scale}}=1.0$)

(b) CCM ($C_{\text{Scale}}=10$) (d) CCM ($C_{\text{Scale}}=20$)

**Figure 4.** Time averaged turbulence eddy viscosity contours

### 3.2. Instantaneous Cavity Shapes and flow structures

Comparisons of the temporal evolutions of the computational and experimentally observed cavity structures are shown respectively in Figure5. The results demonstrate that the CCM model ($C_{\text{Scale}}=10$) is capable of capturing the time-dependent cavity performances, in accordance with the experimental observations. The vertical zone growing ($t_0$~$t_0+28.6\%$ Cycle) and hence detached cavity formatting process shown in Figure2 can be captured. Moreover, the detached cavity will be dissipated when it travels toward downstream ($t_0+55.5\%$~$t_0+74.5\%$ Cycle). Then, the re-entrant jet and the vertical zone will become weaker, and meanwhile the attached cavity will grow up again to form the next cycle. On the contrary, the original $k$-$\varepsilon$ model fails to capture the time-dependent cavity performances.

The curvature correction function $f_c$, and hence revised turbulence kinetic energy predicted with the CCM model ($C_{\text{Scale}}=10$) are shown in Figure6. Predicted turbulence kinetic energy contours at representative times with the original $k$-$\varepsilon$ model are also shown as a comparison. From the figures, it is shown that as the attached cavity expands towards hydrofoil trailing edge ($t_0$~$t_0+28.6\%$ Cycle), large value of curvature correction function can be observed inside of the vertical zone. Curvature correction during this process results in the decrease of predicted turbulence kinetic energy. And hence turbulence eddy viscosity decreases, which keeps predicted attached cavity stretching until covering the whole hydrofoil suction side. As the attached cavity breaks up and partially sheds downstream ($t_0+55.5\%$~$t_0+74.5\%$ Cycle), curvature correction function is remarkable in the vertical zone. Decrease of predicted turbulence kinetic energy hence turbulence eddy viscosity can be seen during this process, which leads to better capture of the unsteadiness of cavitation.
Figure 5. Instantaneous vapor fraction contours and streamlines

Figure 6. Time evolutions of turbulence kinetic energy contours
In order to compare the space-time evolution of the cavity shapes predicted with different schemes, the vapor volume fractions of the cavity are plotted as a function of the time and space. Comparisons between the shedding processes predicted with different models are presented in Figure 7. The x-axis is dimensionless time, $T_{ref}$, defined as:

$$T_{ref} = \frac{c}{U_{\infty}}$$

Where $c$ is the chord length and $U_{\infty}$ is the free stream velocity. The y-axis is dimensionless position at the foil suction side. $L/L_{ref}$ = 0 corresponds to the leading edge, and $L/L_{ref}$ = 1.0 corresponds to the trailing edge.

In Figure 7, it is found that time evolutions of cavity volumes with two models are both periodic, and the predicted periods are almost the same. However, the cavity volume distributions along the foil suction side are significantly different. The maximum length of cavities predicted with the original $k-\varepsilon$ model is just nearly half of the hydrofoil chord, and no shedding cavity is observed. Predicted attached cavity with the CCM model ($C_{Scale}$ = 10) can develop until covering the whole hydrofoil suction side. Cloud cavity shedding behavior can be captured. At the same time, partial cavity shedding process near the trailing edge of hydrofoil can also be observed.

4. Conclusions
In this paper, a curvature correction model (CCM) is proposed, which takes into account the strong streamline curvature effect in cavitating vortex structures and modifies the production term of turbulence kinetic energy. Details of the modifications are discussed. The CCM models with different scaling coefficients are assessed by experimental data as well as those with the original $k-\varepsilon$ model, interactions between cavitation and vortex structures are also discussed to get an insight of improved predictions with the CCM model. Compared with the original $k-\varepsilon$ model, predicted results are improved, including the cavity detachment and hence more hydrofoil load fluctuations. Streamline curvature correction of CCM model overcomes the over-predictions of turbulence kinetic energy and eddy viscosity in cavitating vertical region with the original $k-\varepsilon$ model, which leads to better simulation abilities for the unsteady cavitating flow computations.

Acknowledgment
This work was supported by the National Natural Science Foundation of China (Grant No. 11172040) and the state Key Program of National Natural Science Foundation of China (Grant No. 51239005).
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