Instantons, Spin Crisis
and
High $Q^2$ Anomaly at HERA

N.I.Kochelev

Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research,
RU-141980 Dubna, Moscow region, Russia

Abstract

The contribution of the nonperturbative quark-gluon interaction induced by the instantons to $g_1^p(x, Q^2)$ and $F_2^p(x, Q^2)$ structure functions is estimated. It is shown that nontrivial $Q^2$ dependence of the instanton contribution to $g_1^p(x, Q^2)$ allows us to explain the observed decreasing of the part of the proton spin carried by quarks without involving a large positive gluon polarization.

It is demonstrated that the anomalous enhancement of the instanton contribution to $F_2^p(x, Q^2)$ structure function, due to multiple emission of the gluons from instanton vertex, can be one of the reasons for the excess of the DIS events at HERA at high $Q^2$ and $x$.
1 Introduction

At the present time the problem of the proton spin is one of the most exciting problems in QCD. This problem was inspired by the result of the EMC [1], which found out that only the small amount of proton spin is carried by its quarks. This result has been confirmed by more accurate experiments [2].

In spite of many interesting ideas, which have been proposed to explain this surprising result, the mechanism responsible for the decreasing of the value of helicity carried by quarks, is still not clear (see recent review [3]).

A new challenge for Standard Model arose from the recent HERA result for unpolarized DIS events at high $x$ and $Q^2$ [4]. Two collaborations, H1 and ZEUS, observed the large excess of high $Q^2$ and $x$ DIS events in contradiction with the Standard Model expectations. Many interesting explanations of the HERA anomaly beyond Standard Model have been suggested (see for example [5]).

However, from our point of view, in the analysis of these experimental data, the very important feature of QCD has not been taken into account. This particularity is connected with the possibility of new type of the nonperturbative quark-gluon interaction [6], which can be induced by strong vacuum fluctuations of the gluon fields, so-called instantons [6]. These instanton fluctuations describe the quantum tunneling between different gauge rotated classical vacua in QCD and give us the very important information on the complicated structure of the ground state in theory of strong interaction.

In this article we estimate the contribution from the instanton induced quark-gluon chromomagnetic interaction [7] to the $g_1^p(x)$ and $F_2^p(x)$ structure functions in instanton liquid model for QCD vacuum [13], [14].

We show that this contribution to quark depolarization allows to explain the observed decreasing of the spin-dependent structure function $g_1^p(x)$ at $x > 0.001$.

The possibility to explain the excess of high $Q^2$ and large $x$ events at HERA by the anomalous enhancement of the instanton contribution to structure function $F_2^p(x, Q^2)$ is shown.

2 Anomalous Quark-Gluon Interaction Induced by Instantons

One of the models for the description of nonperturbative effects in QCD is the instanton liquid model (see reviews [13], [14]). In the framework of this model many fundamental quantities of the QCD vacuum, such as different types of the quark and gluon condensates were described rather well. Furthermore, this model also gives good description of the important hadron properties, e.g. masses, decay widths, form factors etc.

The existence of instantons leads to a specific quark-quark and quark-gluon interaction through the QCD vacuum, which has the following form [13]

$$L_{eff} = \int \prod_q (m_q \rho - 2\pi^2 \rho^3 \bar{q}_R(1 + \frac{i}{4} \tau^a U_{ad'} \eta_{d'} \gamma \sigma) q_L)$$

$$\cdot \exp \frac{-2\pi^2 \rho^2 U_{ab'} \eta_{d'} \gamma \sigma G_{d'} \rho}{\rho^4} d\rho d_0(\rho) d\hat{o} + R \rightarrow L,$$

where $\rho$ is the instanton size, $\tau^a$ are the matrices of the $SU(2)_c$ subgroup of the $SU(3)_c$ colour group, $d_0(\rho)$ is the density of the instantons, $d\hat{o}$ stands for integration over the instanton orienta-
tion in colour space, \( f d\hat{o} = 1 \), \( U \) is the orientation matrix of the instanton, \( \bar{\eta}_{\mu\dot{\nu}} \) is the numerical t’Hooft symbol and \( \sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2 \).

From this Lagrangian one can find the famous t’Hooft quark-quark interaction \[11\], which is a corner stone for many applications of the instanton physics.

Recently, it was shown that from Eq. \((\ref{eq:1})\) a new type of the nonperturbative quark-gluon interaction can be obtained. This interaction has the form of anomalous chromomagnetic quark-gluon interaction \[7\]

\[
\Delta L_A = -i\mu_a \sum_q \frac{g}{2m_q^*} \bar{q} \sigma_{\mu\nu} t^a q G_{\mu\nu}^a.
\]

\[\text{(2)}\]

The value of the quark anomalous chromomagnetic moment in the liquid instanton model is

\[
\mu_a = -\frac{f\pi}{2\alpha_s}.
\]

\[\text{(3)}\]

where \( f = n_c\pi^2\rho_c^4 \) is the so-called packing fraction of instantons in vacuum, \( m_q^* = m_q - 2\pi^2\rho_c^2 < 0 \mid \bar{q}q \mid 0 /3 \) is the effective quark mass. The value of \( n_c \) is connected with the value of the gluon condensate by the formula:

\[ n_c = \langle 0 \mid \alpha_s G_{\mu\nu}^a G_{\mu\nu}^a \mid 0 /16\pi \approx 7.5 \times 10^{-4} \text{ GeV}^4. \]

\[\text{(4)}\]

The following estimate for the value of the anomalous quark chromomagnetic moment has been obtained for \( \rho_c = 1.6 \text{ GeV}^{-1} \) in \[6\]

\[
\mu_a = -0.2.
\]

\[\text{(5)}\]

The principal difference between the instanton induced interactions \((\ref{eq:1}), (\ref{eq:2})\) and the perturbative quark-gluon vertex

\[
\mathcal{L}_{\text{pert}} = g\bar{q}\gamma_\mu t^a q A_\mu^a,
\]

\[\text{(6)}\]

is the quark helicity flip at instanton vertex. Therefore this interaction can give a contribution to different spin-dependent cross sections, in particular to the spin-dependent structure function \( g_1(x, Q^2) \).

### 3 Instantons and ”Spin Crisis”

The diagram, which gives rise to the quark structure functions from interaction \((\ref{eq:2})\) is presented in Fig.1a. The contribution to quark spin-dependent structure function \( g_1^q(x, Q^2) \) can be obtained by the projection of the imaginary part of the forward Compton scattering amplitude \( T_{\mu\nu} \)

\[
g_1^q(x, Q^2) = -\frac{i\epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma}{2p.q} \text{Im} T_{\mu\nu},
\]

\[\text{(7)}\]

where \( p \) is the momentum of the initial valence quark in nucleon. The straightforward calculation of the contribution of diagram Fig.1a leads to the result

\[
g_1^q(x, Q^2) = -\frac{e_q^2}{8} |\mu_a| \rho_c^2 \int_0^{\frac{Q^2(1-x)}{4x}} dk_1^2 \int \frac{F^2(k\rho_c/2)}{\sqrt{1 - \frac{4xk_1^2}{(1-x)Q^2}}}.
\]

\[\text{(8)}\]

where

\[
F(z) = z \frac{d}{dz} [I_0(z)K_0(z) - I_1(z)K_1(z)],
\]

\[\text{(9)}\]
Figure 1: The instanton contributions to proton structure function from the anomalous chromomagnetic interaction (a), and from the multiple gluons emission from instanton vertex (b). The label I(\bar{I}) denotes instanton(antiinstanton).

is the Fourier transformation of the quark zero mode in instanton field, $k^2 = k_\perp^2/(1 - x)$, $x = Q^2/2p.q$ and the relation $\alpha_s\mu^2_\pi/m_q^2\rho_c^2 = 3\pi|\mu_a|/8$ [12] has been used.

The very interesting feature of the instanton contribution \cite{8} is its specific $Q^2$ dependence. Namely, at small $Q^2 \ll 1/\rho_c^2$ it is proportional to $Q^2$ and for $Q^2 \gg 1/\rho_c^2$ it is the constant. Therefore, the $Q^2$ dependence of the instanton contribution to polarized structure functions should be different from the ordinary perturbative $\log(Q^2/\Lambda^2)$ evolution. Due to definite value of the quark helicity induced by the instantons, the same conclusion applies to the instanton contribution to the unpolarized structure functions as well. The fundamental reason for such a behavior is the quark spin-flip induced by instanton, which adds the extra power of $k_\perp$ to the matrix element for forward Compton scattering amplitude. As result, at $Q^2 = 0$ the instanton contribution to $g_1(x, Q^2)$ is zero. Therefore the value of the helicity of quarks at $Q^2 \gg 1/\rho_c^2$ should differ from the value of helicity carried by proton quarks, that was extracted from baryon spectroscopy at very small $Q^2$. This anomalous $Q^2$ dependence of the instanton contribution to quark polarization can be one of the reasons for the success of the simple three quark constituent model in the description of nucleon magnetic moments at $Q^2 = 0$.

In the large $Q^2$ limit the contribution to $g_1^q(x)$ is constant

$$g_1^q(x) = -\frac{e^2_q}{4}|\mu_a|. \quad (10)$$

It should be mentioned that the sign of the correction to $g_1^q$ structure function is negative and comes from negative valence quark polarization induced by instantons inside proton (Fig.1). Furthermore, this contribution does not depend on $x$ and therefore can give rise to proton structure functions at rather large values of Bjorken variable $x$.

To estimate the contribution to the proton structure functions, we will use the simple convolution model

$$g_1^p(x) = \sum_q \int_x^1 \frac{dy}{y} g_1^q \left( \frac{x}{y} \right) \Delta q_V(y), \quad (11)$$
where $\Delta q_V(y)$ are the initial valence quark polarizations, which are taken in the form

$$\Delta u_V(x) = 3.7(1-x)^3, \quad \Delta d_V(x) = -1.3(1-x)^3,$$

normalized to the experimental data on the weak decay coupling constants of hyperons

$$g_A^3 = \Delta u_V - \Delta d_V = 1.25; \quad g_A^8 = \Delta u_V + \Delta d_V = 0.6.$$  

In Fig. 2 the result of the calculation of instanton contribution to $g_1^p(x)$ in the region of Bjorken variable $0.001 < x < 1$ is presented. As we see, the quark depolarization induced by the quark-gluon interaction through instantons leads to a rather strong decrease of the value of the $g_1^p(x)$, in particularly at large values of $x$.

Therefore, the considerable part of the observed decreasing of the part of the proton spin carried by quarks can be explained by the contribution from anomalous quark-gluon chromo-magnetic interaction induced by instantons. The additional decreasing of the value of the proton helicity carried by quarks, especially at low $x$ region, can be connected with the contribution to quark depolarization which comes from the quark-quark t’Hooft interaction induced by instantons (see also [8], [9]).

The instanton mechanism of the “spin crisis” solution is opposite to the approach which is based on the assumption about the large positive gluon polarization inside proton [3]. It was shown recently [12] that instantons lead to a negative gluon polarization and therefore instanton model for QCD vacuum rules out the mechanism based on positive gluon polarization.

4 Instantons and High $Q^2$ anomaly at HERA

The contribution of instantons to the unpolarized structure function $F_2^p(x, Q^2)$ can be estimated in the similar way. Due to definite helicity of the quarks in instanton field (see Fig.1a), the
instanton contributions to quark structure functions $f_1^q(x)$ and $g_1^q(x)$ are related to each other by the following expression

$$f_1^q(x, Q^2) = -g_1^q(x, Q^2), \quad (14)$$

The proton structure function is given by the formula

$$F_{1,\text{Inst}}(x, Q^2) = \int_x^1 \frac{dy}{y} f_1^q\left(\frac{x}{y}, Q^2\right) q_V(y, Q^2), \quad (15)$$

where valence unpolarized distribution function $q_V$ is taken in the simple form

$$q_V(x) = Ax^{-0.5}(1-x)^3 \quad \text{(16)}$$

with constant $A$ normalized to the number of $u$- and $d$-quarks in proton.

The result of the calculation of the instanton contribution to $F_{2}^p(x)$ in the region $0.1 < x < 1$ is shown in Fig.3. This contribution is rather large $\approx 5\%$ in this region and therefore should be taken into account in the analysis of $Q^2$ dependence of $F_2(x, Q^2)$. The special interest is the different $Q^2$ dependence of the instanton and perturbative gluon contributions to $F_2(x, Q^2)$, which was mentioned above.

The first results of search for the instanton induced events in DIS have been published by H1 Collaboration [20]. This collaboration obtained very small upper limit for the instanton contribution to structure function $F_{2}^p(x, Q^2)$. However, this limit was obtained without taking into account the possible features of the hadronization of the soft gluons from instanton vertex.

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Footnote: In [18],[19] some contribution from the instantons to the coefficient functions of the structure function of unpolarized DIS, which was connected with exponentially suppressed on $Q^2$ part of the quark propagator in instanton field, was calculated in the dilute instanton gas approximation. The very small value, which was found in [18],[19], comes from the cut-off of the contribution from the instantons with the large size $\rho \geq 1/Q$. 

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Figure 3: The ratio of contributions from the instantons and valence quarks to proton structure function $F_{2}^p(x)$. 

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One of such features is the strong Mueller’s interaction of the gluons \( [10] \) and therefore the assumption on the independent fragmentation of the gluons emitted from instanton, which was used in \[20\], is probably incorrect.

Recently, the very interesting results on the structure function \( F_2^g(x, Q^2) \) at high values of \( x \) and \( Q^2 \) from HERA have been published \[1\]. The analysis of the data, which was based on the assumption of its perturbative \( \text{Log}(Q^2/\Lambda^2) \) evolution of quark distribution functions in this region, discovered large excess of events, as compared with perturbative QCD predictions.

From our point of view, the natural explanation of this excess can be the anomalous enhancement of the instanton contribution to \( F_2^g(x, Q^2) \) structure function at high \( Q^2 \) and large \( x \) region, which comes from multiple creation of soft gluons from instanton vertex Fig.1b.

It should be mentioned that the similar mechanism of enhancement of the electro-weak instantons contribution to the cross sections with violation of the baryon number conservation due to multiple creation of gauge bosons by instanton, is widely discussed \[17\].

We can estimate the factor of enhancement in the case of the large number of the created gluons. When the distance between \( I \) and \( \bar{I} \) is large, \( R_{II} \gg \rho_c \), and \( n_g \to \infty \), the correction can be written by formula \[18\]

\[
F_{enh} \approx \exp(-\delta S_{II}) \approx \exp\left(\frac{24\pi}{\alpha_s(\rho_c)} \frac{1}{\xi^2}\right),
\]

where \( \xi \) is the so-called conformal parameter

\[
\xi = \frac{R_{II}^2 + 2\rho_c^2}{\rho_c^2},
\]

and \( \delta S_{II} \) is the variation of the value of the action for the instanton-antiinstanton configuration due to the dipole-dipole \( I\bar{I} \) interaction. For \( \alpha_s(\rho_c) \approx 0.36 \) \[3\], and \( R_{II} \approx 3\rho_c \) \[13\], the enhancement is \( F_{enh} \approx 10^{11} \). Therefore we should have approximately 50% larger value of \( F_2^g(x, Q^2) \) at \( x > 0.1 \) as compared with the perturbative QCD predictions if the large number of the gluons can be created.

This enhancement comes from the large phase space allowed for gluons, which are created by the instanton in \( \gamma^*(Z, W)q \) collision at high energy. So, one can write for Bjorken variable \( x \) the following formula \( x = Q^2/(M_X^2 + Q^2) \), where \( M_X \) is the mass of the produced hadron system. The value of energy of the gluon that is emitted by instanton equals approximately to \( E_g \approx 1/\rho_c \approx 1\text{GeV} \). At some fixed value of \( x \), for example at \( x \approx 0.5 \), and high \( Q^2 \approx 10^4\text{GeV}^2 \), where excess of events at HERA was found, we have \( M_X \approx 100\text{GeV} \) and therefore a large amount of gluons can be emitted. In this case the large enhancement of instanton contribution takes place. On the other hand, at \( Q^2 \approx 10\text{GeV}^2 \), and high \( x > 0.5 \) region, where the good experimental data on \( F_2(x, Q^2) \) are available, \( M_X < 10\text{GeV} \) and as the result the number of the possible additional gluons is rather small. Therefore at low \( Q^2 \) and large \( x \) the additional enhancement of the instanton contribution to structure functions is absent.

The instanton mechanism for the high \( Q^2 \) anomaly at HERA can be checked by using the future possible polarized option at HERA, because strong helicity dependence of the instanton contribution should lead to large double spin asymmetry of these events. For example, due to negative quark polarization induced by instantons, at high \( x \) and \( Q^2 \) region we expect large negative double spin asymmetry \( A_L \) for the cross sections in the scattering of longitudinal polarized lepton and hadron beams at HERA. This prediction is opposite to the positive value of the asymmetry, which could be expected if the perturbative QCD works in this kinematical region.
5 Summary

In summary, the instanton induced quark-gluon interaction leads to a large negative contribution to the proton spin-dependent structure function $g_1^p(x, Q^2)$. This allows us to explain the observed decreasing of the $g_1^p(x, Q^2)$ structure function in comparison with the prediction of the naive quark model.

It is shown that the same interaction can be responsible for the high $Q^2$ excess of DIS events at HERA.

Thus, from our point of view, these two phenomena, ”spin crisis” and high $Q^2$ and $x$ anomaly at HERA, can be the first direct manifestation of the complicated structure of QCD vacuum in deep inelastic scattering: a vacuum filled with intensive nonperturbative fluctuations of gluon fields, instantons.

The future polarized beams at HERA can be a unique place for investigation of the instanton induced events in deep-inelastic scattering because it will allow us to investigate the strong spin and $Q^2$ dependence of the instanton contribution to structure functions in the very wide kinematic region.

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