The $\mu$-Problem and Seesaw-type Mechanism in the Higgs Sector

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Abstract

We explore a new solution to the $\mu$-problem. In the scenario of SUSY-breaking mediation through anti-generation fields, we find that the $B\mu$ term has its origin in a seesaw-type mechanism as well as in a loop diagram through gauge interactions. It is shown that the dominant contributions to the $B\mu$ term are controlled by the flavor symmetry in the model.

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1 Introduction

Supersymmetry (SUSY) is expected to become an important ingredient of new physics beyond the Standard Model. In particular, SUSY provides the most attractive mechanism to stabilize the scale hierarchy between the Planck scale, $M_{Pl}$, and the electroweak scale $M_w$. Because we have not yet observed superparticles up to the order of $M_w$, SUSY should be broken at scales above $M_w$, but not far from $M_w$. The phenomenology of SUSY depends on the mechanism of SUSY-breaking and on the manner in which SUSY-breaking in the hidden sector is transmitted to the observable sector.

Recently, in Ref. [1], a new scenario of SUSY-breaking mediation was proposed in the framework of a weakly-coupled heterotic string on the Calabi-Yau compactification. In this scenario, the breakdown of supersymmetry in the hidden sector is transmitted to anti-generation fields by gravitational interactions, and subsequent transmission of the breaking to the observable sector occurs through gauge interactions. It has been shown that the spectra of the superparticles given by the model are phenomenologically viable.

In the MSSM, we are confronted with the $\mu$-problem. The supersymmetric $\mu$ term is the only dimensional parameter in the superpotential

$$W_{MSSM} \supset \mu H_u H_d.$$ (1)

The Higgs scalar potential is given by

$$V_{Higgs} = m_1^2 |H_u|^2 + m_2^2 |H_d|^2 - (B_H u H_d + h.c) + \frac{g_2^2}{8} \left( H_u^\dagger \tilde{\sigma} H_d + H_d^\dagger \tilde{\sigma} H_u \right)^2 + \frac{g_1^2}{8} \left( |H_u|^2 - |H_d|^2 \right)^2$$ (2)

with $m_1^2 = m_{H_u}^2 + \mu^2$ and $m_2^2 = m_{H_d}^2 + \mu^2$, where $m_{H_u}$ and $m_{H_d}$ are soft SUSY-breaking Higgs masses, and $g_1$ and $g_2$ are $U(1)$ and $SU(2)$ gauge coupling constants, respectively. The $B_H$ term is the soft SUSY-breaking term for Higgs bilinear couplings. The two Higgs fields acquire the vacuum expectation values (VEVs)

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix},$$ (3)

with $v_u^2 + v_d^2 \simeq (174 \text{ GeV})^2$. The minimization condition of the scalar potential leads to the relations

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1},$$ (4)

$$2B_H = \left( 2\mu^2 + m_{H_d}^2 + m_{H_u}^2 \right) \sin 2\beta,$$ (5)
with $\tan \beta = v_u/v_d$. The electroweak symmetry breaking occurs naturally in the MSSM, because $m_{H_u}^2$ becomes negative due to the dominant contribution of top Yukawa coupling $h_t$ in the RG evolution as

$$m_{H_u}(M_Z) = m_{H_d}(M_Z) - \frac{6h_t^2}{16\pi^2}m_t^2 \log \left( \frac{\Lambda_{RG}}{M_Z} \right)^2.$$  (6)

Here $m_t$ is the stop mass and $\Lambda_{RG}$ is the renormalization scale. For $\tan \beta \gtrsim 2$, Eq. (6) becomes

$$\frac{M_Z^2}{2} \sim -\mu^2 - m_{H_u}^2.$$  (7)

Therefore, phenomenologically, the values of $\mu$ and $B\mu$ should satisfy the condition

$$\mu^2 \sim B\mu = \mathcal{O}((100 \text{ GeV})^2).$$  (8)

An important question in the mediation of SUSY-breaking is whether or not we can obtain $\mu$ and $B\mu$ terms to be consistent with Eq. (5). This is the so-called $\mu$-problem. We would like to determine the dynamical mechanism by which $\mu$ and $B\mu$ terms are generated. In the model studied in Ref. [1], the $B\mu$ term is generated by a two-loop diagram through gauge interactions, as shown in Fig. 1, where anti-generation fields propagate the inner loop, and the remaining internal line has the fermion mass insertion, the $\mu$ term. It has been shown that this model exhibits a phenomenologically viable solution to the $\mu$-problem.

We now give some comments on the $\mu$-problem. The $\mu$ and $B\mu$ terms are of different characters. Since the $\mu$ term is supersymmetric, it cannot be generated by quantum effects, as shown by the non-renormalization theorem. Namely, $\mu$ arises only at tree level. On the other hand, since the $B\mu$ is a soft SUSY-breaking term, it can be generated by quantum effects associated with SUSY-breaking.

For example, a simple model for generating these terms has been proposed in the gauge-mediation scenario. In this model, the superpotential contains the term $XH_uH_d$, where $X$ is the singlet superfield. When $X$ develops a nonzero VEV, i.e., $\langle X \rangle = X + \theta^2 F^X$, then we obtain $\mu \sim X$ and $B\mu \sim F^X$. In this scenario, the gaugino mass is given by $m_{\lambda} \sim (g^2/16\pi^2)F^X/X$. When we take $m_{\lambda} \sim \mu = \mathcal{O}(100 \text{ GeV})$, this...
implies $B\mu \gg \mu^2$, which is phenomenologically unacceptable. This is because the $\mu$ and $B\mu$ terms have their origin in the same coupling. Thus, it is difficult to avoid this problem, unless we introduce a new symmetry and additional fields by hand.\footnote{2}

However, we choose not to introduce new symmetry and additional fields only for the purpose of solving the $\mu$-problem. Rather, we rely on the flavor symmetry, which is favorable for explaining the quark/lepton mass hierarchy. In this situation, $\mu$ and $B\mu$ are represented by independent parameters. As discussed below, we show that the magnitude of each term is controlled by flavor symmetry.

For supergravity to be a low energy effective theory of superstrings, all soft terms must depend strongly on the form of the Kähler potential. Although it is difficult to study the definite form of the Kähler potential in generic Calabi-Yau compactification, typical forms are known in special cases, such as the large radius limit and orbifold compactification ($Z_N$ or $Z_N \times Z_M$), and the relation $B\mu \sim \mu^2 \sim (m_{3/2})^2$ can be derived by approximating the scalar potential. For this reason, approaches to the $\mu$-problem have been tried only by carrying out suitable modification of forms of the Kähler potential. In contrast to such approaches, assuming that Kähler potential is of the minimal form, we explore an alternative approach to the $\mu$-problem.

\section{Mechanism for generating $B\mu$ term}

In this paper we consider the scenario in which SUSY-breaking is mediated by anti-generation fields, proposed in Ref.\footnote{1}, and discuss the possibility of generating a $B\mu$ term with a seesaw-type mechanism as well as with a loop diagram through gauge interactions, as shown in Fig.\footnote{1}. Further, we study the $\mu$-problem in this scenario.

The model discussed here is based on a string-inspired model.\footnote{3} The effective theory in the observable sector from the weakly-coupled Calabi-Yau string is characterized by $N = 1$ SUSY, the $E_6$ gauge group, and massless matter fields which belong to $27$ and $27^*$ representations in $E_6$. The massless chiral superfields consist of

$$N_f \Phi(27) + \delta \left( \Phi_0(27) + \overline{(27^*)} \right),$$

where $N_f$ represents the family number at low energies. Here, $\delta$ sets of vector-like multiplets are included in the massless sector. The numbers $N_f + \delta$ and $\delta$ are the generation number and anti-generation number, respectively. We set $N_f = 3$ and $\delta = 1$ for the sake of simplicity. In the effective theory, in general, there appear certain discrete symmetries $G_{st}$ as a stringy selection rule associated with symmetric structure of the compactified space.\footnote{3, 4} Let us consider the case $G_{st} = G_{fl} \times Z_2$, where $G_{fl}$ is the flavor symmetry and $Z_2$ is the $R$-parity. The $R$-parity distinguishes $\Phi_i$ ($i = 1, 2, 3$) from a vector-like set. It is assumed that ordinary quarks and leptons are included in the chiral multiplets $\Phi_i$ ($i = 1, 2, 3$) and that the $R$-parity of all $\Phi_i$ are odd. Because light Higgs scalars are even under $R$-parity, light Higgs doublets must
reside in the set \( \{ \Phi_0, \overline{\Phi}\} \), whose elements are vector-like multiplets. For this reason we assign even \( R \)-parity to \( \Phi_0 \) and \( \overline{\Phi} \). Hence, through the spontaneous breaking of gauge symmetry, gauge superfields possibly mix with the vector-like multiplets \( \Phi_0 \) and \( \overline{\Phi} \) but not mix with the chiral multiplets \( \Phi_i \) \( (i = 1, 2, 3) \). No mixing occurs between the vector-like multiplets and the chiral multiplets. Hence, in the low energy region, ordinary quarks and leptons arise from \( \Phi_i \) \( (i = 1, 2, 3) \) and ordinary Higgs fields from \( \Phi_0 \). Moreover, there appear a dilaton field \( D \), Kähler class moduli fields \( T_i \), and complex structure moduli fields \( U_i \). The VEV of the dilaton field \( D \) determines the gauge coupling constant and the VEVs of the moduli fields \( U_i \) and \( T_i \) parametrize the size and shapes of the compactified manifold.

We choose \( SU(6) \times SU(2)_R \) as a typical example of the unification gauge group. With this gauge group, chiral superfields \( (\Phi) \) in the 27 representation of \( E_6 \) are decomposed as

\[
E_6 \supset SU(6) \times SU(2)_R \\
= (15, 1) Q, L, g, g^c, S, \\
(6^*, 2) (U^c, D^c), (N^c, E^c), (H_u, H_d).
\]

(10)

The \( SU(6) \times SU(2)_R \) model has the following attractive features. The first feature is that without introducing additional adjoint fields, the gauge symmetry is spontaneously broken to the Standard Model gauge group in two steps at scales \( \langle S_0 \rangle = \langle S \rangle \sim 10^{17} \text{ GeV} \) and \( \langle N_0 \rangle = \langle N \rangle \sim 10^{16} \text{ GeV} \). The second feature is that this model is free from triplet-doublet splitting problem, because the light Higgs doublets \( H_u \) and \( H_d \) and the color-triplet Higgs \( g \) and \( g^c \) belong to different irreducible representations.

It is assumed that the SUSY-breaking resulting from gaugino condensation in the hidden sector is only transmitted to the \( F \)-component of the moduli field \( T \). Specifically, the moduli field \( T \) develops a SUSY-breaking VEV, \( \langle T \rangle = T + \theta^2 F^T \), while both \( D \) and \( U \) have vanishing \( F \)-components \( (FD = FU = 0) \). This situation is the same as that in the moduli-dominated SUSY-breaking scenario. At present, it is unknown how to specify the seeds of SUSY-breaking through non-perturbative dynamics in superstring theory. Therefore, we make an assumption regarding the source of SUSY-breaking.

The superpotential \( W \) in the observable sector is described in terms of \( \Phi(27) \) and \( \overline{\Phi}(27^*) \). Since we are interested in the \( \mu \)-problem, we concentrate our attention on the terms incorporating \( SU(2)_L \)-doublet Higgss,

\[
W \sim h(T, U) \left( \frac{S_0 S}{M_5^2} \right)^s S_0 H_u H_d + f(T, U) \left( \frac{S_0 S}{M_5^2} \right)^T S H_u H_d \\
+ g(T, U) M_s \left( \frac{S_0 S}{M_5^2} \right)^b (H_u H_d + H_d H_u),
\]

(11)
where the exponents \( s, \bar{s} \) and \( b \) are non-negative integers, which are governed by the flavor symmetry. Hereafter we assume \( h(T, U), f(T, U), g(T, U) \sim \mathcal{O}(1) \). For a Calabi-Yau string, generation fields and anti-generation fields in the trilinear terms couple separately to the \( U \) moduli and the \( T \) moduli, respectively. In view of this feature of the trilinear terms, we assume that \( h(T, U) \) and \( f(T, U) \) depend only on the complex structure moduli \( U \) and the Kähler moduli \( T \), respectively.

When \( S_0 \) and \( \bar{S} \) develop nonzero VEVs, the above superpotential induces the bilinear terms

\[
W_h = h(U) x^s \langle S_0 \rangle H_u H_d + f(T) x^\bar{s} \langle \bar{S} \rangle H_u H_d + m \left( H_u H_u + H_D H_d \right),
\]

where

\[
x = \frac{\langle S_0 \rangle \langle \bar{S} \rangle}{M_s} = \mathcal{O}(10^{-2})
\]

and

\[
m = g(T, U) M_s x^b.
\]

We now consider the \( \mu \) term, which is the first term on the r.h.s. of Eq. (12). From this we obtain

\[
\mu = h(U) x^s \langle S_0 \rangle.
\]

For an appropriate value of \( s \), the \( \mu \) term can be taken to be on the electroweak scale. The choice of the exponent \( s \) is linked to the flavor charge assignment for \( \Phi_0(15, 1) \), \( \Phi_0(6*, 2) \), \( \Phi(15, 1) \) and \( \Phi(6*, 2) \). For the second term in Eq. (11), we assume \( s = 0 \). The function \( f(T) \) is determined by the string worldsheet instanton effects as

\[
f(T) \propto e^{-2\pi T/M_s}.
\]

Next, we turn to the seesaw-type mechanism of generating the \( B\mu \) term. From Eq. (12), we have the \( 4 \times 4 \) mass-squared matrix

\[
M^2 = \begin{pmatrix}
|p|^2 + |m|^2 & q^* & 0 & p^* m + \mu m^* \\
q & |p|^2 + |m|^2 & pm^* + \mu^* m & 0 \\
0 & pm^* + \mu^* m & |m|^2 + |\mu|^2 & 0 \\
0 & 0 & 0 & |m|^2 + |\mu|^2
\end{pmatrix}
\]

for Higgs scalar fields, where \( p \) and \( q \) are defined as

\[
p = f(T) \langle \bar{S} \rangle, \quad q = f'(T) F^T \langle \bar{S} \rangle.
\]

In Eq. (17), \( p, q, \mu \) and \( m \) can be complex numbers. Here \( f'(T) \) represents the derivative of \( f(T) \) with respect to the moduli \( T \). The magnitude of \( p \) becomes

\[
|p| = |f(T) \langle S_0 \rangle| \sim \mathcal{O}(10^{17} \text{ GeV})
\]
Figure 2: Diagram for generation of the $B\mu$ term via the seesaw-type mechanism.

The scale $m$ depends on the exponent $b$ and becomes smaller than $\langle S_0 \rangle$. Here we assume

$$|p| \gg |m| \gg |\mu| \sim \mathcal{O}(100 \text{ GeV}).$$

(21)

Since the anti-generation fields become superheavy, they decouple at low energies. Further, we assume $|F^T| \ll M^2_s$. Then we have $|p| \gg |q|$.

Although we have no Higgs bilinear coupling, i.e., the $H_{u0} - H_{d0}$ and $H_{u0}^{\dagger} - H_{d0}^{\dagger}$ components of $M^2$ vanish, after integrating out the heavy modes $\overline{H}_u$ and $\overline{H}_d$, Higgs bilinear couplings are induced at low energies due to the seesaw-type mechanism. Concretely, a $B\mu$ term is induced through propagation of the heavy anti-generation fields $\overline{H}_u$ and $\overline{H}_d$, as shown in Fig. 2, in which the dotted line between the circled crosses represents a linear combination of the heavy anti-generation scalar fields $\overline{H}_u$ and $\overline{H}_d$. The matrix Eq. (17) is equivalently represented by the block matrix

$$M^2 = \begin{pmatrix} X & Y \\ Y & Z \end{pmatrix},$$

(22)

with

$$X = \begin{pmatrix} |p|^2 + |m|^2 & q^* \\ q & |p|^2 + |m|^2 \end{pmatrix},$$

(23)

$$Y = \begin{pmatrix} 0 & p^* m + \mu m^* \\ p m^* + \mu^* m & 0 \end{pmatrix},$$

(24)

$$Z = \begin{pmatrix} |m|^2 + |\mu|^2 & 0 \\ 0 & 1 \end{pmatrix}.$$

(25)

In comparison with the usual gauge mediation scenario, the seesaw-type mechanism depicted in Fig. 2 is worthy of note. The anti-generation fields $\overline{H}_u$ and $\overline{H}_d$ behave as the messenger fields in the usual gauge mediation, in which scenario messenger fields do not directly couple to the ordinary Higgs fields. Therefore, in the usual gauge mediation scenario we have vanishing $Y$ for the mass matrix Eq. (22) and the $B\mu$ term arises only from the two-loop diagram illustrated in Fig. 1. By contrast, in the present model, the sub-matrix $Y$ is non-vanishing and causes the seesaw-type mechanism to occur. Because the anti-generation fields couple to the observed Higgs fields through the non-renormalizable interactions, we need to take
account of the seesaw-type mechanism as well as the two-loop diagram through gauge interactions.

The mass-squared values of the heavy anti-generation Higgs fields are given approximately by the sub-matrix $X$, and the eigenvalues become $|p|^2 + |m|^2 \pm |q|$. The mass splitting in the heavy modes can be attributed to the SUSY-breaking source $F^T$.

For light modes, the mass-squared values are obtained after integrating out the heavy modes. Explicitly, we have

\[ Z - Y X^{-1} Y = \begin{pmatrix} x & y \\ y^* & x \end{pmatrix}, \tag{26} \]

where $x$ and $y$ are written as

\[
x = |m|^2 + |\mu|^2 - \frac{|p|^2 + |m|^2}{(|p|^2 + |m|^2) - |q|^2} |p^* m + \mu m^*|^2, \tag{27} \]

\[
y = \frac{q(p^* m + \mu m^*)^2}{(|p|^2 + |\mu|^2) - |q|^2}. \tag{28} \]

The off-diagonal elements $y$ and $y^*$ result from the SUSY-breaking $F^T$ and represent $B\mu$ and $B\mu^*$. From Eq. (21), the quantity $B\mu^{(\text{seesaw})}$ generated by the seesaw-type mechanism can be approximated as

\[
B\mu^{(\text{seesaw})} = y \sim q \frac{m^2}{p^2}. \tag{29} \]

The ratio of $B\mu^{(\text{seesaw})}$ to $\mu^2$ becomes

\[
\frac{B\mu^{(\text{seesaw})}}{\mu^2} = \frac{1}{\mathcal{O}(10^3 \text{ GeV}^2)} \times \frac{\Lambda m^2}{p}, \tag{30} \]

where we take $\mu \sim \mathcal{O}(100 \text{ GeV})$. Here we have used $q = 2p\Lambda$, and $\Lambda$ is defined as

\[
\Lambda = \frac{F^T}{M_s}. \tag{31} \]

As mentioned above, in the present model, the $B\mu$ term is also generated by the two-loop diagram through gauge interactions, as shown in Fig. 1. The contribution of loop diagram is given by

\[
\frac{B\mu^{(\text{loop})}}{\mu^2} = \mathcal{O}(10) \times \frac{g_5^2}{16\pi^2} \frac{m_{\lambda_2}}{\mu} \ln \left( \frac{m_{\lambda_2}}{\mu} \right) = \frac{1}{\mathcal{O}(10^4 \text{ GeV})} \times \Lambda, \tag{32} \]

where $g_5$ is the gauge coupling and $m_{\lambda_2}$ is the mass of the two-loop diagram.
where the $SU(2)$ gaugino mass $m_{\lambda_2}$ comes from the one-loop diagram of the anti-generation fields and is expressed as

$$m_{\lambda_2} \sim \frac{g_2^2 F^T}{8\pi M_s} = \frac{1}{\mathcal{O}(10)} \times \Lambda.$$  (33)

Thus, the contributions to the $B\mu$ term arises from the loop diagram shown in Fig. 1 and also from the seesaw-type mechanism depicted in Fig. 2. It is important for us to study whether or not the loop contribution is dominated by the seesaw-type contribution. From Eqs. (30) and (32), the ratio of the magnitudes of these contributions is given by

$$\frac{B\mu^{(\text{loop})}}{B\mu^{(\text{seesaw})}} = \mathcal{O}\left(\frac{1}{10} \text{ GeV}\right) \times \frac{p}{m^2} = \frac{p}{p_0},$$  (34)

where $p_0$ is defined as

$$p_0 = \mathcal{O}(10 \text{ GeV}^{-1}) \times m^2.$$  (35)

To investigate the cases of different dominant contributions to the $B\mu$ term, let us consider the following three cases of $p/p_0$ separately.

**Case (i) $p \sim p_0$**

Here, the two contributions to the $B\mu$ term are comparable. Putting $\Lambda \sim \mathcal{O}(10^4 \text{ GeV})$, we have $B\mu^{(\text{loop})} \sim B\mu^{(\text{seesaw})} \sim \mu^2$ and $m_{\lambda_2} \sim \mathcal{O}(1 \text{ TeV})$. Thus the $\mu$-problem can be solved. This case is phenomenologically viable.

**Case (ii) $p \gg p_0$**

Here, the contribution of loop diagram is dominant. Putting $\Lambda \sim \mathcal{O}(10^4 \text{ GeV})$, we have $B\mu^{(\text{loop})} \sim \mu^2 \gg B\mu^{(\text{seesaw})}$ and $m_{\lambda_2} \sim \mathcal{O}(1 \text{ TeV})$. In this case the $\mu$-problem can be solved.

**Case (iii) $p \ll p_0$**

Here, the contribution of seesaw-type mechanism is dominant. However, the condition $B\mu^{(\text{seesaw})} \sim \mu^2 \gg B\mu^{(\text{loop})}$ implies $\Lambda \ll \mathcal{O}(10^4 \text{ GeV})$. Then, from Eq. (33), we obtain $m_{\lambda_2} \ll \mathcal{O}(1 \text{ TeV})$. This case is phenomenologically unacceptable.

From the above consideration, we find that the contribution to the $B\mu$ term depends on $p/p_0$ (or $p/m^2$) and that the contribution of the seesaw-type mechanism cannot become dominant. The magnitude of $m$ is controlled by the exponent $b$, which is closely related to the flavor symmetry. We now discuss the issue of phenomenologically acceptable values of $b$. From Eqs. (14), (20) and (35) we have

$$m = \mathcal{O}(10^{18-2b} \text{ GeV})$$  (36)
\[
\frac{P}{P_0} = \mathcal{O}(10^{4b-20}),
\]  
(37)

where we take \(\langle S \rangle = \langle S_0 \rangle \sim \mathcal{O}(10^{17} \text{ GeV})\) and \(M_s \sim \mathcal{O}(10^{18} \text{GeV})\). Therefore, we can obtain phenomenologically viable values of \(m\) and \(p_0\) for non-negative integers \(b\). For \(b \geq 8\), the assumption \(|m| \gg |\mu|\) is not satisfied. For \(b = 1 - 4\), we have \(p_0 \gg p\), which corresponds to case (iii). In the case that \(b = 5, 6\) or \(7\), we obtain phenomenologically acceptable solutions to the \(\mu\)-problem, which correspond to case (i) or case (ii). If \(b = 5\), then both the seesaw-type mechanism and loop diagram are dominant contributions to the \(B\mu\) term. If \(b = 6\) or \(7\), the dominant contribution is only the loop diagram.

Another phenomenologically viable solution to the \(\mu\)-problem has been proposed by Giudice and Masiero (G-M). In the G-M mechanism, both \(\mu\) and \(B\mu\) terms are generated by the Kähler potential with non-renormalizable interactions. Contrastingly, in the present model the Kähler potential is assumed to be of the minimal form. Consequently, the G-M mechanism does not act, and, instead, \(\mu\) and \(B\mu\) terms come from the superpotential. In conventional models with the minimal Kähler potential containing the moduli, both the \(\mu_{(\text{gra})}\) term and the \(B\mu_{(\text{gra})}\) term are generated by gravitational effects, and we have \(B\mu_{(\text{gra})} \sim \mu_{(\text{gra})}^2\). From the first term in Eq. (11), we then find \(\mu_{(\text{gra})} \sim \partial_U h F^U x \langle S_0 \rangle\), where \(U\) is a complex structure moduli and \(x \equiv (\langle S_0 \rangle / \langle S \rangle) / M_s^2\). By contrast, in the present model, since we make the assumption \(F^U = 0\), neither the \(\mu\) term nor the \(B\mu\) term can be generated by gravitational interactions. Thus, the \(B\mu\) term comes only from the loop diagram and the seesaw-type mechanism.

In the present model, the seesaw-type mechanism also generates the soft SUSY-breaking Higgs masses \(m^2_{H_u}\) and \(m^2_{H_d}\), which contribute to the diagonal element \(x\) of Eq. (26). Under the condition Eq. (21), the magnitude of \(x\) can be approximated as

\[
x \sim \mu^2 - 2 \frac{m^2}{p} \mu = \mu^2 \left(1 - \frac{1}{\mathcal{O}(10^3)} \times \frac{p_0}{p}\right),
\]  
(38)

where we have used Eq. (35). The first term here comes from the supersymmetric \(\mu\) term, and the second term from the seesaw-type mechanism. Cases (i) and (ii) are of interest here. Note that the second term is significantly smaller than the first term. Thus, the contribution of seesaw-type mechanism is negligible. On the other hand, since anti-generation fields behave as messengers, as in the gauge mediation scenario, two-loop diagrams through gauge interactions generate soft scalar masses \(m_0\) of the same order as the gaugino masses:

\[
m_0 \sim m_\lambda \sim \frac{g^2 F^T}{8\pi M_s} = \frac{1}{\mathcal{O}(10)} \times \Lambda.
\]  
(39)
Therefore, in the cases (i) and (ii) with $\Lambda \sim \mathcal{O}(10^4 \text{ GeV})$, the contributions to the soft scalar masses of the two-loop diagrams through gauge interactions become dominant.

3 Conclusion

In conclusion, we have proposed a phenomenologically viable solution to the $\mu$-problem in the framework of SUSY-breaking mediation through anti-generation fields. In supergravity, a solution to the $\mu$-problem can be derived by approximating the scalar potential by using a special form of the Kähler potential. We explored an alternative approach to the $\mu$-problem that does not involve gravitational interactions. We found that the $B\mu$ term is generated by a seesaw-type mechanism as well as by a loop diagram through gauge interactions and that the solution to the $\mu$-problem is closely related to the flavor symmetry of the model. In the present model, although the contribution of the seesaw-type mechanism cannot become dominant, the relative magnitudes of the two contributions to the $B\mu$ term depend on the charge assignment of the flavor symmetry to the fields. To find a solution to the $\mu$-problem in the framework of string-inspired model, we need to consider contributions of the superpotential, in addition to the Kähler potential. Moreover, the discrete symmetry (flavor symmetry) as a stringy symmetry places strong constraints on the low energy physics, such as the fermion mass hierarchies, mixing angles, and so on. We would like to emphasize that the discrete symmetry plays an important role also in solving the $\mu$-problem. Furthermore, the seesaw-type mechanism also generates the soft Higgs masses. However, we find that this contribution is negligible.

The model considered here may seem to regard only a particular case. However, general perturbative string theories contain both matter fields of the fundamental representation (generation fields) and those of the anti-fundamental representation (anti-generation) under the gauge group. The numbers of generation and anti-generation fields correspond to the Hodge number in a compact manifold. Along the path to low energy physics, the generation fields and anti-generation fields of sets of vector-like multiplets decouple, and the remaining generation fields lead to the MSSM at low energy. From the viewpoint of string phenomenology, the present model is not specific to a particular case, and it sheds light on an important role of anti-generation fields.

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