Applying DCOP to User Association Problem in Heterogeneous Networks with Markov Chain Based Algorithm

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Abstract—Multi-agent systems (MAS) is able to characterize the behavior of individual agent and the interaction between agents. Thus, it motivates us to leverage the distributed constraint optimization problem (DCOP), a framework of modeling MAS, to solve the user association problem in heterogeneous networks (HetNets). Two issues we have to consider when we take DCOP into the application of HetNet including: (i) How to set up an effective model by DCOP taking account of the negative impact of the increment of users on the modeling process (ii) Which kind of algorithms is more suitable to balance the time consumption and the quality of solution. Aiming to overcome these issues, we firstly come up with an ECA\(\eta\) model in which a parameter \(\eta\) with an adequate assignment (\(\eta = 3\) in this paper) is able to control the scale of the model. After that, a Markov chain (MC) based algorithm is proposed on the basis of log-sum-exp function. Experimental results show that the solution obtained by DCOP framework is better than the one obtained by the Max-SINR algorithm. Comparing with the Lagrange dual decomposition based method (LDD), the solution performance has been improved since there is no need to transform original problem into a satisfied one. In addition, it is also apparent that DCOP based method has better robustness than LDD when the number of users increases but the available resource at base stations are limited.

I. INTRODUCTION

of agent and the cooperation between agents in Multi-agent System (MAS), a framework, named distributed constraint optimization problem (DCOP) in terms of constraints that are known and enforced by distinct agents comes into being with it. In last decade, the research effort of DCOP has been dedicated on the following three directions: 1) the development of DCOP algorithms which are able to better balance the computational complexity and the accuracy of solution, such as large neighborhood search method [11]; Markov Chain Monte Carlo sampling method [2] [3] and distributed junction tree based method [4] 2) the extension of classical DCOP model in order to make it more flexible and effective for practical application, such as expected regret DCOP model [5], multi-variable agent decomposition model [6] and dynamic DCOP model [7] 3) the application of DCOP in modeling environmental systems, such as sensor networks [8], [9], [10], disaster evacuation [12], traffic control [13], [14] and resource allocation [15], [16], [3]. In this paper, we take more attention to the application of DCOP. More precisely, we leverage DCOP to solve user association problem in the downlink of multi-tier heterogeneous networks with the aim to assign mobile users to different base stations in different tiers while satisfying the QoS constraint on the rate required by each user.

is generally regarded as a resource allocation problem [17], [18], [19], [20] in which the resource is defined by the resource blocks (RBs). In this case, the more RBs allocated to a user, the larger rate achieved by the user. The methods to solve the user association problem are divided into centralized controlled and distributed controlled. With regard to the centralized way, a central entity is set up to collect information, and then used to decide which particular BS is to serve which user according to the collected information. A classical representation of centralized method is Max-SINR [21]. Distributed controlled methods attract considerable attention in last decade since they do not require a central entity and allow BSs and users to make autonomous user association decisions by themselves through the interaction between BSs and users. Among all available methods, the methods based on Lagrange dual decomposition (LDD) [20] and game theory [22] have better performance. Hamidreza and Vijay [20] put forward a unified distributed algorithm for cell association followed by RBs distribution in a \(k\)-tier heterogeneous network. With aid of LDD algorithm, the users and BSs make their respective decisions based on local information and a global QoS, expressed in terms of minimum achievable long-term rate, is achieved. However, the constraint relaxation and the backtrack in almost each iteration are needed to avoid overload at the BSs. In addition, as we will show later, the number of out-of-service users will increase since a user always selects a best-rate thereby taking up a
large number of RBs and leaving less for others. Nguyen and Bao [22] proposed a game theory based method in which the users are modeled as players who participate in the game of acquiring resources. The best solution is the one which can satisfy Nash equilibrium (NE). Loosely speaking, such solution is only a local optima. In addition, it is difficult to guarantee the quality of the solution.

There is no research of modeling user association problem as MAS. However, some similar works have been done focusing on solving the resource management problem in the field of wireless networks or cognitive radio network. [23], [24], [25] by DCOP framework. These methods can not be directly applied to user association problem mainly due to the scale of the models for these practical applications is relatively small.

For instance, Monteiro [24] formalized the channel allocation in a wireless network as a DCOP with no more than 10 agents considered in the simulation parts. However, the amount of users and resource included in a HetNet is always hundreds and thousands. In this case, a good modeling process along with a suitable DCOP algorithm is necessary. According to the in-depth analysis above, it motivates us to explore a good way to solve user association problem by DCOP. The main contributions of this paper are as follows:

- An ECAV (Each Connection As Variable) model is proposed for modeling user association problem using DCOP framework. In addition, we introduce a parameter $\eta$ with which we can control the scale (the number of variables and constraints) of the ECAV model.
- A DCOP algorithm based on Markov chain (MC) is proposed which is able to balance the time consumption and the quality of the solution.
- The experiments are conducted which show that the results obtained by the proposed algorithm have superior accuracy compared with the Max-SINR algorithm. Moreover, it has better robustness than the LDD based algorithm when the number of users increases but the available resource at base stations are limited.

The rest of this paper is organized as follows. In Section II the definition of DCOP and the system model of user association problem along with its mixed integer programming formulation are briefly introduced. In Section III we illustrate the ECAV-$\eta$ model. After that, a MC based algorithm is designed in Section IV. We explore the performance of the DCOP framework by comparing with the Max-SINR and LDD methods in Section V. Finally, Section VI draws the conclusion.

II. Preliminary

This section expounds the DCOP framework and system model of user association problem along with its mixed integer programming formulation.

A. DCOP

The definitions of DCOP have a little difference in different literatures [26], [4], [5]. In this paper, we formalize the DCOP as a four tuples model $< A, \mathcal{V}, D, C >$ where $A = \{a_1, a_2, \ldots, a_{|A|}\}$ consists of a set of agents, $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ is the set of variables in which each variable $v_i \in \mathcal{V}$ only belongs to an agent $a \in A$. Each variable has a finite and discrete domain where each value represents a possible state of the variable. All the domains of different variables consist of a domain set $D = \{d_1, d_2, \ldots, d_m\}$, where $d_i$ is the domain of $v_i$. A constraint $c \in C = \{c_1, c_2, \ldots, c_{|C|}\}$ is defined as a mapping from the assignments of $m$ variables to a positive real value:

$$R(c) : d_1 \times d_2 \times \cdots \times d_m \rightarrow \mathbb{R}^+$$ (1)

The purpose of a DCOP is to find a set of assignments of all the variables, denoted as $\mathcal{X}^*$, which maximize the utility, namely the sum of all constraint rewards:

$$\arg\max_{\mathcal{X}^*} \sum_c R(c)$$ (2)

B. System Model of User Association Problem

Consider a $k$-tier HetNet where all the BSs in the same tier have the same configurations. For example, a two-tier network including a macro BS ($B_1$) and a femto BS, ($B_2$), is shown in Fig. 1. The set of all BSs is denoted as $\mathcal{B} = \{B_1, B_2, \ldots, B_{NB}\}$ where $\mathcal{N}B$ is the total number of BSs. All the BSs in the $k$th tier transmit with the same power $P_k$. The total number of users is denoted by $\mathcal{N}U$ and the set of all users is $\mathcal{U} = \{U_1, U_2, \ldots, U_{\mathcal{N}U}\}$.

With OFDMA technology in LTE-Advanced networks, the resource, time-frequency, is divided into blocks where each block is defined as a resource block (RB) including a certain time duration and certain bandwidth [17]. In this paper, the resource configured at each BS is in the format of RB so that its available RBs are decided by the bandwidth and the scheduling interval duration allocated to that BS. We assume the BSs in the HetNet share the total bandwidth such that both intra- and inter-tier interference exist when the BSs allocate RBs to the users instantaneously.

Assuming the channel state information is available at the BSs, the SINR experienced by user $U_j$, served by $B_i$ in the $k$th tier is given by

$$\text{SINR}_{ij} = \frac{P_k g_{ij}}{\sum_{B_l \in \mathcal{B} / \{B_i\}} P_k g_{lj} + B N_0}$$ (3)

In (3), $g_{ij}$ is the channel power gain between $U_j$ and $B_i$, $\mathcal{B} / \{B_i\}$ represents all the BSs in $\mathcal{B}$ except $B_i$, $B$ is the bandwidth and $N_0$ is noise power spectral density. The channel power gain includes the effect of both path loss and fading. Path loss is assumed to be static and its effect is captured in

[26] formalized the DCOP as a three tuples model. [4] adopted a four-tuples model while [5] used a five tuples model.
the average value of the channel power gain, while the fading is assumed to follow the exponential distribution.

From the above, the efficiency of user $U_j$ powered by BS $B_i$, denoted as $e_{ij}$, is calculated as

$$e_{ij} = \log_2(1 + \text{SINR}_{ij})$$

(4)

Given the bandwidth $B$, time duration $T$ and the scheduling interval $\Gamma$ configured at each RB, we attain the unit rate at $U_j$ upon one RB as follows

$$u_{ij} = \frac{BT e_{ij}}{\Gamma}$$

(5)

On the basis of formula (5), the rate received at $U_j$ with $n_{ij}$ RBs provided by $B_i$ in the $k$th tier is

$$r_{ij} = n_{ij} u_{ij}$$

(6)

Associated with each user is a quality-of-service (QoS) constraint. This is expressed as the minimum total rate the user should receive. Denoting the rate requirement of the $j$th user by $\gamma_j$, the minimum number of RBs required to satisfy $\gamma_j$ is calculated by:

$$n_{ij}^{\min} = \lceil \frac{\gamma_j}{u_{ij}} \rceil$$

(7)

in which $\lceil \cdot \rceil$ is a ceiling function.

C. Mixed Integer Programming Formulation

The formulations of user association problem by mixed linear programming are similar in a series of papers (see the survey literature [19]). In this paper, we present a more commonly used formulation as follows

$$\text{maximize } \mathcal{F} = \sum_{i \in B} \sum_{j \in U} x_{ij} r_{ij}$$

(8a)

s.t. $\sum_{i \in B} x_{ij} r_{ij} \geq \gamma_j, \forall U_j \in U$ (8b)

$\sum_{j \in U} x_{ij} n_{ij} \leq N_i, \forall B_i \in B$ (8c)

$\sum_{i \in B} x_{ij} \leq 1, \forall U_j \in U$ (8d)

$n_{ij} \in \{0, 1, ..., N_i\}, \forall B_i \in B, \forall U_j \in U$ (8e)

$x_{ij} \in \{0, 1\}, \forall B_i \in B, \forall U_j \in U$ (8f)

The first constraint ensures the rate QoS requirement from users. Constraint (8c) indicates that the amount of RBs consumed at the same BS is no more than the total RBs $N_i$ configured at the BS. Constraint (8e) guarantees one user associated with a unique BS. Constraint (8e) guarantees the number of RBs a BS allocates to a user falls within the range from zero and $N_i$. The last constraint (8f) guarantees the connection between a user and a BS has two states denoted by a binary variable. The objective function (8a) refers to the sum of rate rather than a function acted on the rate such as $U(x) = \log(1 + x)$ in some references. Generally, two phases are needed to gain the solution including: 1) transforming original problem into a satisfied one through relaxing Constraint (8e) by $n_{ij} \in \{0, n_{ij}^{\min}\}$; 2) the left RBs in each BS will be allocated to users in order to maximize the objective function.

III. FORMULATION WITH DCOP

In this section, we expound and illustrate the ECA V model along with its modified version ECAV-$\eta$.

A. ECAV Model

Before giving the formulation based on DCOP, we firstly introduce the definition of candidate BS:
Definition 1. we declare $B_i, i \in B$ is a candidate BS of $U_j, j \in U$ if the rate at $U_j$ is above the threshold $\gamma$ with $n_{ij}^{\text{min}}$ RBs provided by $B_i$. Simultaneously, $n_{\text{min}}^{ij}$ should be less than the total number of RBs ($N_i$) configured at $B_i$.

After confirming the set of candidate BSs of $U_j, j \in U$, denoted by $CB_j$, sends messages to its candidate BSs so that each $B_i, i \in NB$ gets knowledge of its possible connected users. We define each possible connection between $U_j$ and its candidate BS $B_i$ as a variable, denoted by $V_i^{j}$. In this case, all the variables are divided into $NB$ groups according to the potential connection between users and different BSs. The domain of each variable $V_i^{j}$, denoted by $D_i^j = \{0, n_{ij}^{\text{min}}, ..., N_i\}$, where $V_i^{j} = 0$ if no RB is allocated to $U_j$, otherwise, $V_i^{j} \geq n_{ij}^{\text{min}}$. We define each group as an agent. Thus, an $n$-ary constraint exists among $n$ variables (intra-constraint) to guarantee that there is no overload at $B_i$. Note that a user may have more than one candidate BS, there are constraints (inter-constraints) connecting the variables affiliated to different agents on account of the assumption that a unique connection exists between a user and a BS. Generally speaking, the utility (objective) function in the DCOP model is the sum of constraint rewards which reflects the degree of constraint violations. We define the reward $R(c)$ of inter- and intra-constraints in the ECAV model as follows. For $\forall c \in C_{\text{inter}}$

$$R(c) = \begin{cases} \infty, & \exists \exists V_i^{j_1}, V_i^{j_2} \in \psi(c), Val(V_i^{j_1}) > (9a) \\ 0, & \text{Otherwise} \\ \end{cases}$$

For $\forall c \in C_{\text{intra}}$

$$R(c) = \begin{cases} \infty, & \sum_{V_i^{j} \in \psi(c)} Val(V_i^{j}) > (10b) \\ 0, & \text{Otherwise} \\ \end{cases}$$

In constraint $(9a)$, $\psi(c)$ is the subset of variables connected by constraint $c$. $Val(V_i^{j})$ represents the assignment of $V_i^{j}$. A reward (we use $\infty$ in this paper) is assigned to the constraints if there at least two variables are non-zero at the same time (unique connection between a user and a BS). Otherwise, the reward is equal to zero. In constraint $(10a)$, the reward is $\infty$ once there is a overload at the BS. Otherwise, the reward is the sum of the rates achieved at users.

It is easy to find that a variable in the ECAV model with non-zero assignment covers constraint $(8b)$ and $(8c)$ in the mixed integer programming formulation. Moreover, intra and inter-constraints respectively cover constraint $(8c)$ and constraint $(8d)$. Therefore, The global optimal solution $X^*$ obtained from the ECAV model is consistent with the one obtained from the mixed integer programming formulation, denoted as $X^*_{\text{ECAV}}$.

To better understand the modeling process, we recall the instance in Fig.1 where the candidate BSs of $U_1$ and $U_2$ are the same, denoted as $\{B_1, B_2\}$, while the candidate BSs of $U_3$ and $U_4$ are respectively $\{B_1\}$ and $\{B_2\}$. We assume the total RBs configured at $B_1$ and $B_2$ is 8 and 10. For simplicity, we assume the rate of each user served by one RB provided by $B_1$ is 0.8 bit/s. And 1 bit/s of each user is served by $B_2$. Then, the ECAV model is shown in Fig.1(b). There are two agents named $A_1$ and $A_2$. The variables in $A_1$ are $V_1^{1}, V_1^{2}$ and $V_1^{3}$ where $V_i^{j}$ refers to a connection between user $U_j$ and $B_i$. Similarly, the variables in $A_2$ are $V_2^{1}, V_2^{2}$ and $V_2^{3}$. Assuming the threshold rate is 3 bit/s, we can calculate that at least $\frac{3}{10} = 4$ RBs needed for the users served by $B_1$, thus the domain of each variable in $A_1$ is $\{0, 4, ..., 8\}$. Also, the domain of each variable in $A_2$ is $\{0, 3, ..., 10\}$. The black lines in each agent are two 3-nry intra-constraints, thus $C_{\text{intra}} = \{C_{\text{intra}}^1, C_{\text{intra}}^2\}$. The red lines connecting two agents are two intra-constraints, thus $C_{\text{inter}} = \{C_{\text{inter}}^1, C_{\text{inter}}^2\}$. We use $C_{\text{intra}}^1$ and $C_{\text{intra}}^2$ to illustrate how the reward of constraint works in different conditions. Considering $C_{\text{intra}}^1$, the reward is $\infty$ when all the variables associated with $C_{\text{intra}}^1$ have the same assignment 4. Thus the total number of RBs consumed by three users is 12 which is more than 8 RBs configured at $B_1$. Otherwise, the reward is $0.8 \times 4 \times 3 = 9.6$ (bit/s) calculated according to $(6)$. Considering $C_{\text{intra}}^2$, the reward is $\infty$ when the assignment of $V_2$ is 3 and the assignment of $V_1$ is 4 because it means $U_2$ will connect with more than one BSs ($B_1$ and $B_2$), which violates the assumption of unique connection. Otherwise, the reward is $0$ ($\sigma_5$). If there is no constraint violated, the final utility calculated by the objective function is the total rate in the whole HetNet (constraint $(10b)$).

B. ECAV-\eta Model

The scale of an ECAV model, referring to the number of agents and constraints, is related to the number of users, BSs and the candidate BSs hold at each user. However, some candidate BSs of the user can be ignored because these BSs are able to satisfy the requirement of the user but with massive RBs consumed. It can be illustrated by the number of RBs a BS allocate to a user is inversely proportional to the geographical distance between them. In this way, we introduce a parameter $\eta$ with which we limit the number of candidate BSs of each user is no more than $\eta$. The following algorithms present the selection of top $\eta$ candidate BSs (denoted by $CB$) and the modeling process of ECAV-\eta.

Algorithm 1 is the pseudo code for determining $CB$. It is executed by each user distributely. More precisely, a user estimates its total candidate BSs $CB$ by the procedure from line 5 to 9. Based on 4 to 7, the candidate BSs of a user is ordered according to the unit number of RBs consumed at such user served by different BSs (from line 22 to 28). The time consumption of Algorithm 1 mainly consists of two parts. One is the derivation of $CB$ with time complexity $O(NB^2)$. The other is the ordering operation with time complexity $O(NB^2)$. As a result, the total time expended of Algorithm 1 is $O(NB + NB^2)$. With $CB$, we present the pseudo code in relation to the building of ECAV-\eta model.

As for Algorithm 2, it firstly sets up the agents distributely
Algorithm 1 CB$_j$ of user $U_j$, $j \in \mathcal{U}$ based on $\eta$

Input: The information of HetNet ($\mathcal{B}$, $\mathcal{U}$, $\gamma$, $\eta$)

Output: The set of candidate BS CB based on $\eta$

Initialize: $CB_j \leftarrow \emptyset$, $CB_j \leftarrow \emptyset$

procedure GETALLCANDIDATEBS
  for $i \in B$, $j \in U$
    if $r_{ij} \geq \gamma$ then
      $CB_j \cup \{B_i\}$
  end if
end procedure

procedure GETPARTIALCANDIDATEBS
  BubbleSort($CB_j$) >> sorting by SINR
  if $|CB_j| > \eta$ then
    for $n$ from 1 to $\eta$
      $CB_j^n \leftarrow CB_j^n$ >> get $\eta$ candidate BSs
    end for
  else
    $CB_j \leftarrow CB_j$
  end if
end procedure

procedure BUBBLESORT($CB_j$)
  for $m$ from 1 to $|CB_j|$
    for $n$ from $|CB_j|$ to $m + 1$
      if $SINR(CB_j^n) > SINR(CB_j^{n-1})$
        exchange $CB_j^n$ and $CB_j^{n-1}$
      end if
    end for
  end for
end procedure

Algorithm 2 ECAV-$\eta$

Initialize: 
  $\{A, V, D, R\} \leftarrow \emptyset$ >> elements in DCOP model
  $R \leftarrow \emptyset$ >> utility upon each constraint

procedure SETECAV
  for $i \in B$
    $A \cup \{A_i\}$
  end for
  for $\forall j \in U$, $m \in CB_j$
    $V \cup \{V_j^m\}$
    $D_j^i \leftarrow \{0, 1, \ldots, n_{min}^j\}$
    $D \cup \{D_j^i\}$
    $C \cup \{C_j^i\}$
    $R \cup \{R(C_j^i)\}$
    >> based on (9b), (9a)
  end for
  for $i \in A$
    $C \cup \{C_i^j\}$
    $R \cup \{R(C_i^j)\}$
    >> based on (10a), (10b)
  end for
end procedure
with zero duality gap. By solving the Karush-Kuhn-Tucker to construct a MC with the state space being important thing is to design the nonnegative transition rate in a better or best solution with most of the time. Another be time-shared according to $p$ MC converges. In this way, the assignments of variables will

$$
\max_{p,s} \sum_{s \in S} p_s \sum_{i \in |B|, j \in |U|} Val_s(V^i_j) - \frac{1}{\beta} \sum_{s \in S} p_s \log p_s
$$

**s.t.** $\sum_{s \in S} p_s = 1$

Assuming $s^*$ and $\lambda^*$ are the primal and dual optimal points with zero duality gap. By solving the Karush-Kuhn-Tucker (KKT) conditions [27], we can obtain the following equations:

$$
\sum_{i \in |B|, j \in |U|} Val_s(V^i_j) - \frac{1}{\beta} \log p_{s^*} - \frac{1}{\beta} + \lambda = 0, \forall s \in S
$$

$$
\sum_{s \in S} p_{s^*} = 1
$$

$$
\lambda \geq 0
$$

Then we can get the solution of $p_{s^*}$ as follows:

$$
p_{s^*} = \frac{\exp(\beta \sum_{i \in |B|, j \in |U|} Val_s(V^i_j))}{\sum_{s \in S} \exp(\beta \sum_{i \in |B|, j \in |U|} Val_s(V^i_j))}
$$

On the basis of above transformation, the objective is to construct a MC with the state space being $S$ and the stationary distribution being the optimal solution $p_{s^*}$ when MC converges. In this way, the assignments of variables will be time-shared according to $p_{s^*}$ and the system will stay in a better or best solution with most of the time. Another important thing is to design the nonnegative transition rate $q_{s,s'}$ between two states $s$ and $s'$. According to [28], a series of methods are provided which not only guarantee the resulting MC is irreducible, but also satisfy the balance equation: $p_s q_{s,s'} = q_{s'} q_{s,s'}$. In this paper, we use the following method:

$$
q_{s,s'} = \alpha \left[ \exp(\beta \sum_{i \in |B|, j \in |U|} Val_s(V^i_j)) \right]^{-1}
$$

The advantage of (15) is that the transition rate is independent of the performance of $s'$. A distributed algorithm, named Wait-and-Hp [3] in [28], is used to get the solution after we transform DCOP into a MC. However, as the existence of inter- and intra- constraints in, a checking through the way of message passing is made in order to avoid constraint violation.

**V. EXPERIMENTAL EVALUATION**

**A. Experimental Setting**

In this section, we test the performance of the MC based algorithm with different assignments of $\eta$ in the ECAV model. A simulated environment including a three-tiers HetNet created within a $1000m \times 1000m$ square is considered. In the system, there is one macro BS, 5 pico BSs and 10 femto BSs with their transmission powers respectively 46, 35, and 20 dBm. The macro BS is fixed at the center of the square, and the other BSs are randomly distributed. The path loss between the macro (pico) BSs and the users is defined as $L(d) = 34 + 40 \log_{10}(d)$, while the pass loss between femto BSs and users is $L(d) = 37 + 30 \log_{10}(d)$. The parameter $d$ represents the Euclidean distance between the BSs and the users in meters. The noise power refers to the thermal noise at room temperature with a bandwidth of 180kHz and equals to -111.45 dBm. One second scheduling interval is considered. Without special illustration, 200 RBs are configured at macro BS, as well as 100 and 50 RBs are configured at each pico and femto BS. In addition, all the results are the mean of 10 instances.

**B. Experimental Results**

We firstly discuss the impact of different assignments of $\eta$ on the performance of ECAV model from the point of view of the runtime and the quality of solution. More precisely, we generate different number of users ranging from 20 to 100 with the step interval of 10. The time consumed by the MC based algorithm is displayed in Fig.2. It is clear to see that more time is needed when the number of users increases. Also, the growth of runtime is depended on the value of $\eta$. Specially, there is an explosive growth of runtime when we set $\eta$ from four to five. As previously stated, this is caused by more candidate BSs considered by each user. However, the quality of the solutions with different values of $\eta$ is not obviously improved according the results in Table I. For instance, the average rate achieved at each user is only improved no more than 0.1 bit/s when the number of users are 100 with the values of $\eta$ are 3 and 5. It is difficult to make a theoretical analysis of the realationship between $\eta$ and the quality of the solution. We leave this research in future works.

From above analysis, we set $\eta = 3$ in the following experiments in order to balance the runtime and performance of the solution. In addition, we test the performance of the MC based algorithm comparing with its counterparts Max-SINR and LDD based algorithms.

| TABLE I | THE AVERAGE RATE (BIT/S) ACHIEVED AT EACH USER |
|---------|---------------------------------------------|
| Users   | $\eta = 1$ | $\eta = 2$ | $\eta = 3$ | $\eta = 4$ | $\eta = 5$ |
| 50      | 12.47      | 12.82      | 13.10      | 13.22      | 13.59      |
| 80      | 8.12       | 8.37       | 8.55       | 8.68       | 8.77       |
| 100     | 6.11       | 6.37       | 6.73       | 6.81       | 6.83       |

In Fig.3, we check the connection state between 200 users and BSs in different tiers. A phenomenon we can observe from the figure is that there are more or less some users out of service even we use different kinds of algorithms. It is not only caused by the limited resource configured at each BS, but also related to the positions of such kinds of users. They are located at the edge of the square and hardly served by any BS in the system. Further, more users are served by macro BS in Max-SINR algorithm because a larger SINR always

\[ \frac{1}{\beta} \log(\sum_{s \in S} \exp(\beta \sum_{i \in |B|, j \in |U|} Val_s(V^i_j)) \right) \]
exists between the users and macro BS. As a result, the total non-served users in Mmax-SINR algorithms are more than the other two if there is no scheme for allocating the left resource. On the other hand, the number of non-served users in MC are less than LDD when \( \eta = 3 \) since the user \( U_j \) will select a BS \( B_i \) with the maximal \( QI_{ij} \) in each iteration of the LDD algorithm. In other words, the users prefer to connect with a BS which can offer better QoS even when more resources are consumed. Therefore, some BSs have to spend more RBs which leads to the resource at these BSs being more easily used up.

In Fig 4, we produce a statistic of the number of non-served users when we change the total number of users configured in the HetNet. The average number of non-served users for each algorithm along with the standard deviation is presented in the figure. Compared with Fig 3, a more clear results imply that more than 60 (at worst, around 70) non-served users in the Max-SINR algorithm. The LDD based algorithm comes the second with approximate 20 users. The best results are obtained by the MC algorithm with no more than 20 users even the total users in the HetNet is 240.

In Fig 5, the CDFs of the rate acheived at users

Fig. 2. The runtime of ECAV-\( \eta \) (\( \eta = 1 - 5 \)) with different number of users in the HetNet

Fig. 3. The connection between users and BSs according to the allocation scheme obtained through different algorithms

Fig. 4. Non-served users in the HetNet with a different number of users

Fig. 5. The CDFs of the rate acheived at users

Fig. 6. The total rate against the number of RBs held at the macro base station

In Fig 4, we produce a statistic of the number of non-served users when we change the total number of users configured in the HetNet. The average number of non-served users for each algorithm along with the standard deviation is presented in the figure. Compared with Fig 3, a more clear results imply that more than 60 (at worst, around 70) non-served users in the Max-SINR algorithm. The LDD based algorithm comes the second with approximate 20 users. The best results are obtained by the MC algorithm with no more than 20 users even the total users in the HetNet is 240.
In Fig 5, we compare the cumulative distribution function (CDF) of the rate. The rate of the users seldomly drops below the threshold (3 bit/s) when we use the distributed algorithms (LDD and MC based algorithms), while Max-SINR algorithm is unable to satisfy the rate QoS constraints. Moreover, the rate CDFs of the MC based algorithm never lie above the corresponding CDFs obtained by implementing the Max-SINR algorithm (the gap is between 6% − 20%). Likewise, At worst 5% gap exists between the MC based algorithm and LDD when we set η = 3.

At last, another interesting observation is made by configuring different number of RBs at macro BS (Fig 6). When we change the number of RBs from 150 to 250 at macro BS, it is clear to see that the total rate obtained by LDD is not sensitive to the variation of the resource hold by macro BS. This result is also related to the solving process in which two phases are needed when employing a LDD based algorithm. As we have discussed in the Introduction section, a solution which can satisfy the basic QoS requirement will be accepted by the LDD based algorithm. It finally affects the allocation of left resource at macro BS. As a result, the algorithm easily falls into the local optima. This problem, to some degree, can be overcome by the ECAV model since there is only one phase in the model. With the ECAV model, a constraint satisfied problem is transformed into a constraint optimization problem. And the advantage of DCOP is successfully applied into solving user association problem.

VI. CONCLUSION

An important breakthrough in this paper is that we take the DCOP into the application of HetNet. More precisely, we propose an ECAV model along with a parameter η to reduce the number of nodes and constraints in the model. In addition, a markov based algorithm is applied to balance the quality of solution and the time consumed. From experimental results, we can draw a conclusion that the quality of the solution obtained by the ECAV-3 model solved with the MC based algorithm is better than the centralized algorithm, Max-SINR and distributed one LDD, especially when the number of users increases but they are limited to the available RBs. In future work, we will extend our research to the following two aspects:

In some algorithms, like K-opt [29] and ADOPT [30] for DCOP, there are already a theoretical analysis on the completeness of solution. However, it is still a challenge job in most research of DCOP algorithm, like the MC based algorithm proposed in this paper. Thus, we will explore the quality of the solution associated with different values of η.

In practice, the BSs in small cells (like pico/femto BSs) have properties of plug-and-play. They are generally deployed in a home or small business where the environment is dynamic. In this way, we should design a DCOP model which is fit for the variations in the environment such as the mobility of users and different states (active or sleep) of BSs. To this end, a stochastic DCOP model can be considered like the one in .

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