Risk trading, network topology, and banking regulation

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Abstract

In the context of understanding the nature of the risk transformation process of the financial system we propose an iterative risk-trading game between several agents who build their trading strategies based on a general utility setting. The game is studied numerically for different network topologies. Consequences of topology are shown for the wealth time-series of agents, for the safety and efficiency of various types of networks. The proposed setup allows an analysis of the effects of different approaches to banking regulation as currently suggested by the Basle Committee of Banking Supervision. We find a phase transition-like phenomenon, where the Basle parameter plays the role of temperature and system safety serves as the order parameter. This result suggests the existence of an optimal regulation parameter. As a consequence a tightening of the current regulatory framework does not necessarily lead to an improvement of the safety of the banking system. Moreover, we show that banking systems with local risk-sharing cooperations have higher global default rates than systems with low cyclicality.

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I. INTRODUCTION

The efficiency and stability of the financial system and its institutions is seen as one of the core elements of modern economy. The regulation of financial intermediaries is thus a central issue which raises a number of questions both for practical implementations and for academic research. During the past two decades an intensive discussion about the regulation of financial markets evolved mostly driven by media attended events like the ”Russian crisis” in 1998 and defaults of big financial institutions like the failure of Barings Bank or, more recently, Long Term Capital Management. This discussion formed the basis for a regulatory innovation process under the lead of the Basle Committee of Banking Supervision which sets the relevant legal regulatory framework for the G-10 economies as well as for the EU. As a consequence of the US Savings & Loans disaster in the mid-eighties the first milestone of this process was the first Capital Accord in 1988 (Basle I) which forced banks to hold a capital cushion of 8 % of total assets to prevent default due to extreme unexpected losses. This was the first step where the activities of financial intermediaries were limited by the risk (measured by total assets) of their positions. Since total assets is an insufficient measure of risk at least for trading positions in securities and derivatives, the 1995 Amendment to the Capital Accord introduced a more risk-sensitive framework for market risk activities. Finally, the second Capital Accord (Basle II) which currently is under consultation is aimed to introduce a risk-sensitive framework for credit risk as well.

Although the discussion about the cornerstones of the new regulatory framework is in its final phase, there remains a number of open questions. Firstly, if the activities of financial intermediaries are limited by the risk of their current positions, how should risk be measured? Secondly, if a capital cushion relative to the risk run by a bank is needed, what is its optimal size? Further, does a specific risk-sensitive framework lead to undesirable pro-cyclical effects in the economy? Finally, is this risk-regulatory framework sufficient to prevent the financial system from a total collapse when catastrophic events like September 11, 2001, occur?

It is the main purpose of this paper to address these questions and to develop a formal framework which allows a stringent analysis of the effects of different approaches of banking regulation. Moreover, the potential impact of catastrophic events on the financial system can be measured within this framework. To achieve this goal we propose an iterative risk-trading game between several agents who build their trading strategies based on a very general utility
setting. The financial system is modeled using different network topologies which enables us to explicitly measure the network-topology dependence of different regulatory actions and various other important factors. This is a major contribution to the existing research because to our knowledge this is one of the first attempts of an analysis of financial markets where dynamic game models and network-topology are combined. Differing effects of single bank defaults and regional banking crises can easily be captured by our model. Additionally, the chosen network topology can be used to mimic specific structures of several national banking systems like "sectors" or risk-sharing cooperations.

This research is related to the discussion about systemic risk in the banking sector, see [1, 2, 3, 4]. The authors of [1] empirically examine systemic risk in the Italian inter-bank clearing network where they explicitly address the potential size of a "domino effect" where the default of a single bank may jeopardize the ability of "neighbor" banks to meet their obligations. A first attempt to model systemic risk in financial networks was undertaken in [2], where the authors show comparative statistics describing the relationship between the clearing vector and the underlying parameters of the financial system which is exposed to external stochastic shocks. However, they model the behavior of financial institutions purely deterministically and they do not examine potential effects of network topology. A third stream of research, e.g. [3], focuses on the default correlation of banks as the central parameter of measuring systemic risk or contagion in the banking sector. The framework suggested in this paper enables us to analyze contagion as well as "domino effects" and provides a profound basis for comparative statics with respect to the architecture of the financial market combined with the regulatory framework. In a recent paper [4] an empirical application of an extended version of [2] is presented employing bank balance sheet data. Their major finding is that contagious default from inter-bank relations plays only a minor role when risk is assessed at the level of the entire banking system. This is an interesting contribution to the discussion about the scope and possible disadvantages of international banking regulation.

This paper is also related to research focused on the implications of the risk sensitive minimum regulatory capital set by the Basle II’ Capital Accord. The role of "buffer capital", i.e., free capital held by banks that exceeds the minimum regulatory level, is addressed in [5]. In a dynamic model with endogenous capital they find that under reasonable assumptions changes in capital requirements have no impact on bank behavior. Studies related to the
possible pro-cyclical effect of risk-sensitive capital regulation like \cite{6, 7} find empirical evidence for pro-cyclicality at least for some widely used credit rating frameworks. From a purely theoretical perspective a macro-economic model recently proposed in \cite{8} demonstrates that in the short-run changes in capital regulation may have perverse effects (increasing supply for riskier borrowers when restrictions are tightened and vice versa). In the long-run, however, these effects are absent and bank capital regulation serves as a perfect tool for monetary policy transmission.

The remainder of the paper is organized as follows. Section 2 describes the model proposed to address the central questions raised in the introductory section. The main results of the numerical analysis are presented in section 3. Section 4 contains a brief discussion of the results and concludes the paper.

II. THE MODEL

A. Model Structure

The aim of this work is to develop a simple model of a network of banks or financial institutions who can trade risk amongst each other. Banks are forced to enter risk through demands of their clients, but try to compensate or share it amongst each other. We introduce an iterative dynamical model where agents (banks) can choose between different trading strategies, depending on their need to reduce individual risk. The need to reduce risk depends on the wealth of a bank and regulatory parameters. Risk trading actions take place between players whose connections are characterized by an exogenously specified graph topology. This topology remains constant over time in the current setting, i.e., the number of banks and the structure of the trading network can change endogenously only due to bank defaults. We introduce measures which characterize the performance of different network topologies with respect to overall fairness and stability of the network, individual safety, i.e., survival probability, and the efficiency of reducing exogenous, i.e., externally enforced, risk. By studying the time evolution of the wealth of interacting players we find remarkable coincidence with realistic financial time-series. Characteristics of those time-series depend on topology and regulation parameters. It is important to mention here that in contrast to some recent approaches in cooperative network- and game theory, see e.g. \cite{9, 10, 11, 12},
our agents do not learn but are purely selfish. In our present setting they do not have any incentive to adapt for cooperation.

The specific model analyzed in this paper restricts the decision of the agents only to depend on their individual risk rather than to their assessment of their counterpartie’s risk. If agents take the risk of their counterparties into account the terms of trade (e.g. bid-ask spreads) of the riskier agents will deteriorate and as a consequence a self-reinforcing dynamics will increase the risks of these agents and their default probabilities. Recent empirical evidence, like the default of Enron, supports this reasoning. The simplified framework of this paper, however, enables us to analyze some realistic situations. Firstly, the risk assessment of counterparties does not change continuously but in discrete time steps, e.g. when a rating change is announced. Thus, for shorter periods of time (particularly during intra-day periods) changes in the risk of an agent’s position will not change its terms of trade immediately. Secondly, the regulatory framework currently in discussion is aimed to create a ”level playing-field” where agents can safely interact without permanent risk assessment of their counterparties. This is exactly related to our simplified framework. We have to emphasize that the inclusion of risk assessment by agents is a valuable extension of the framework presented in this paper but is left for future research.

In the following we deal with a set of \( N \) sites (representing banks) labeled by \( i = 1, \cdots, N \) who receive requests from their \( N \) exterior clients. For simplicity each bank has only one client. Note, that no other interactions of the financial sector (banks) and the real sector of the economy (clients) are endogenously modeled in this framework. The client’s requests (forwards, swaps, weather derivatives, etc.) are modeled by bets which depend on an external random process \( X(t) \), say the weather, and a time of maturity \( t_m \). For the rest of this paper we consider a simple binary random process \( X(t) \in \{-1,1\} \) with equal probabilities \( p(-1) = p(1) = \frac{1}{2} \). Note, that the model can be easily extended to cover non-iid-processes like cyclical processes by changing the specification of the dynamics of \( X(t) \).

For example, a client today bets 20 m USD that on \( t_m = \text{Dec. } 15^{\text{th}} 2004 \) there will be rain. In this model, the bank is forced to accept these external bets and receives a spread or incentive, \( \delta^{exo} \), as a compensation. This incentive may be interpreted as a fixed fee or a customer related bid-ask spread. At each time-step in the game, exterior clients confront their individual banks with external bets with uniformly distributed times of maturity, \( t_m = t + \tau \), \( \tau \in \{0,1 \cdots, \tau_{\text{max}}\} \), where \( t \) denotes present time. For two players only, compare
e.g. with [13, 14]. The associated betting volume offered to bank $i$ is a random number also drawn from a uniform distribution $B_i^{exo}(t + \tau) \in [-D(t) \cdot W_i(t), D(t) \cdot W_i(t)]$, where $W_i(t)$ is the present wealth of bank $i$. This means that the maximum betting volume asked from a bank is proportional to its present wealth (the smaller the banks the smaller their transactions). $D$ is the fraction of external risk a bank is allowed to take within a given time-step, with respect to its own wealth. $D$ could be seen as a regulatory parameter since it imposes an upper limit for a bank’s risk dependent on its wealth (or equity in a wider sense). This parameter does not yet resemble the 8 %-rule currently set by regulators, where some risk measure (e.g., risk-weighted nominal value of positions) must not exceed the 12.5 multiple of a bank’s equity. Rather, $D$ can be understood as a measure that prevents banks to accept large risks within short time scales. The 8 %-rule will be incorporated in the model in the next section. Since in case of no interactions, $D$ would simply be the volatility of wealth processes, in the following we shall call $D$ the exogenous volatility.

By accepting a bet (which matures in $\tau$ days from now) from a client, bank $i$ enters risk $R_i^{exo}(t + \tau)$, which is defined as the maximum amount the bank can loose at time $t + \tau$, i.e., $R_i^{exo}(t + \tau) = |B_i^{exo}(t + \tau)|$. This risk measure is related to a great variety of risk measures used by regulators, e.g. the standard deviation of profit and losses (volatility), and the 1 %-quantile of the profit and loss distribution (value-at-risk).

**B. Model Dynamics**

The essence of the model is that this risk ($R_i^{exo}$) can be traded away, if the bank is able to find a neighbor bank, which is willing to enter a betting contract which serves to reduce this risk. In the above example, the bank will either look if an appropriate bet is already offered on the ”market” (issued by a neighbor bank), or it will issue one itself to its neighbor sites, which would read: 20 m USD that on $t_m = Dec. 15^{th}$ 2004 there will be sunshine. All the contracts are kept in the ”betting book” $B_{ij}(t + \tau)$, which contains the betting volume bank $i$ is betting against bank $j$ at time $t + \tau$. Note, that a bet which one bank wins the other loses, i.e., $B_{ij}(t + \tau) = -B_{ji}(t + \tau)$. If bank $i$ offers a bet maturing at $t + \tau$ it does so through an entry in the ”market” book $M_i(t + \tau)$. The numerical value of $M_i$ is the volume. The market book is ”visible” to all neighbor banks of $i$ only. Whenever a bank agrees to a bet, the offering party is obliged to pay a spread, $\delta$, to the accepting bank. In reality $\delta$ is a
dynamical quantity which is determined on the market. Here, for simplicity, we choose \( \delta \) to be 10\% of the bet volume \( B_{ij}(t + \tau) \). After entering a contract, risk of bank \( i \) is defined as the maximum amount he can loose at time \( t + \tau \), and now consists of the external bets and all the bets entered with neighbor sites, i.e.,

\[
R_i(t + \tau) = \left| B_{exo i}^j(t + \tau) + \sum_j B_{ij}(t + \tau) \right|,
\]

(1)

where \( j \) indexes the neighboring banks. To explicitely model the decision making process we assume that each bank \( i \) is equipped with a von Neumann-Morgenstern utility function with risk aversion depending on a parameter \( \alpha_i > 0 \). To model heterogeneity of banks we model the risk aversion parameter as a random variable in our framework. From a regulator’s point of view this makes sense because in reality there is a non-negligible uncertainty about the risk aversion parameters of banks either across the cross-section or across time.

There are two distinct kinds of decisions banks can make. The first situation arises if a bank \( i \) finds a bet (with maturity at \( t + \tau \) on the market \( M_j(t + \tau) \) offered by a neighbor site \( j \)) which serves to reduce its risk, i.e., \( |B_{exo}^i(t + \tau) + \sum_j B_{ij}(t + \tau) + M_j(t + \tau)| < R_i(t + \tau) \), it will obviously accept it and receive the spread for the deal. This we call the passive strategy. On average, the bank will gain the spread for sure and then await the outcome of the bet, where there is a fair 1:1 chance to win. In this situation a bank can increase its expected utility and simultaneously reduce its risk. Thus, this decision does not depend on \( \alpha_i \) and the specific choice of the utility function is irrelevant, as long as positive marginal utility and risk aversion are ensured (which both follow from the von Neumann-Morgenstern property).

In the second situation there is no (risk-reducing) offer in the market book for any given date of maturity \( t + \tau \), and the bank has to decide to either do nothing or to offer a bet itself to its neighbor banks (for that date), which we call the active strategy. The decision now depends on the present wealth \( W_i(t) \), the risk of the bank \( R_i(t + \tau) \), the spread \( \delta \) and the risk aversion parameter \( \alpha \). Since agents have to decide whether to adopt the risk-reducing active strategy for each maturity \( t + \tau \) separately, we are interested in the random outcome of the decision represented by the probability of adopting such a strategy which we denote with \( p_i^{active}(t, \tau) \).

To simplify the decision making process in a numerical framework we avoid the explicit maximization of expected utility and directly model the outcome of the process by assuming
that
\[ p_i^{\text{active}}(t, \tau) = 1 - \exp\left[ -\frac{\alpha_i R_i(t + \tau)}{W_i(t) + \delta} \right] . \] (2)

Note, that this structure is consistent with the general von Neumann-Morgenstern setting, since \( p_i^{\text{active}}(t, \tau) \) is increasing in \( \alpha_i \) and \( R_i(t + \tau) \) and decreasing in \( W_i(t) \) and \( \delta \), respectively.

The interaction between banks and the decision making is thus not fully deterministic. Here we kept the risk aversion factor constant, \( \alpha_i = 5 \), for all banks in all the following computations. In contrast to previous work, e.g. [2], we introduce a noise component to the behavior of agents which replaces the role of the utility function in the decision making process. Whereas from a single-agent perspective this might be regarded as an arbitrary approximation, from a regulator’s perspective, however, this construction is realistic, because the regulator faces a large number of agents with different utility functions which are not known by the regulator. Thus, our decision making rule (2) explicitly models the uncertainty about the single-agent utility functions, or put equivalently, models the stochastic deviations of the decisions of single agents from the representative agent. Note, however, that the introduction of decision making rule (2) serves only as a simplification of a purely utility based framework and does not lead to any loss of generality.

To model the Basle regulatory framework, the probability for adopting an active strategy is always equal to one, whenever the total risk exceeds a certain percentage of wealth, i.e.,
\[ p_i^{\text{active}}(t, \tau) = 1 \text{ if } \sum_{\tau} |R_i(t + \tau)| \geq L_{\text{Basle}} \cdot W_i(t) \] (3)

Presently, the actual Basle parameter \( L_{\text{Basle}} \) is 12.5, but is an open parameter in the model. Note, that the restriction to act if \( \sum_{\tau} |R_i(t + \tau)| \geq L_{\text{Basle}} \cdot W_i(t) \) only enforces the agent to adopt the active strategy, i.e., to place an (for himself risk-reducing) order on the market. It is possible, however, that the order is not executed because there is no counterparty willing or able to accept the bet. Under this scenario the actual risk of the bank will exceed its regulatory boundary. This scenario is thus related to a (at least local) banking crisis where due to a lack of liquidity in the market banks are not able to reduce the risk of their positions even when they are forced to by regulators. This feature of our model allows to capture propagation of illiquidity effects comparable to what happened during the Russian crisis in 1998.

Note, that in the setting described above the spread is exogenous, i.e., the agents are not allowed to adapt the "prices" \( \delta \) of the bets. This feature of the model makes only sense when
examining pronounced market crashes where empirical evidence shows that the adaption of prices has little or no influence on the willingness of agents to trade. However, our framework can be easily extended to a dynamic endogenous determination of the spread variable $\delta$ at each decision stage at the cost of additional computation burden.

Each of the $N$ players participating in risk trading is represented by a node or site. Sites are connected by links, which are non-zero entries of the interaction-graph matrix, $G_{ij}$. A value of $G_{ij} = 1$ means site $i$ has a connection to site $j$, $G_{ij} = 0$ means no connection. Players which are connected by links are called neighbors and can interact with each other. In the following we shall study the particular classes of different network topologies shown in Fig. 1. The connectivity of these graphs, defined as the average number of links per site $C = \langle \#\text{links} / \text{site} \rangle$, models the efficiency of "information flow" in the network, a low connectivity $C$ means high "market friction", i.e., not all the deals which would have been possible and desirable in the network can be executed due to lack of connections.

The actual model dynamics is shown schematically in Fig. 2. At the beginning of each time-step, the external bets are offered to the banks. Each bet carries a time of maturity from now ($t$) till $\tau$ time-steps in the future. As a maximum future date for bets we chose $\tau_{\text{max}} = 10$ time-steps. The total exogenous risk entering at this stage at time $t$ is $R_{\text{exo total}}(t + \tau) = \sum_i B_{i\text{exo}}(t + \tau)$. After this exogenous update the sites are run through in a random asynchronous update. Once a site $i$ is chosen, the routine goes through all of its neighbor sites $j$ (determined by $G_{ij}$), again in randomized order. The actual game between the pair ($i, j$) now takes place:

Bank $i$ checks in the market book if any bets are available from bank $j$ for all times $\tau$ which can reduce $i$'s risk, i.e., $\left| B_{i\text{exo}}(t + \tau) + \sum_j B_{ij}(t + \tau) + M_j(t + \tau) \right| < R_i(t + \tau)$. If that is possible the deal will be accepted and $i$ will receive the spread $\delta$ from $j$. The betting book is now adjusted according to the terms of the deal, i.e., $B_{ij}(t + \tau)$ now becomes $B_{ij}(t + \tau) + M_j(t + \tau)$. If no more passive strategies are possible $i$ decides – according to Eq. (2) – if it will issue an offer itself by placing an entry in the market book of size $M_i(t + \tau) = R_i(t + \tau)$. If an other bank will (later) accept this bet, $i$ will then pay the corresponding spread $\delta$ and be then free of risk. After all sites have been run through, all the bets of today ($t$) will be settled according to the random outcome of $X(t)$. The wealth
update now is nothing but,

\[ W_i(t + 1) = W_i(t) + X_i(t) \cdot \left( B^{\text{exo}}(t) + \sum_j B_{ij}(t) \right), \tag{4} \]

since all spreads have been taken care of along the way.

A bank is said to be defaulted if its wealth falls below zero. In such an event the bank is put into “receivership”. For the remaining duration of the simulation the defaulted bank is not allowed to enter new bets anymore. Positive payoffs of existing bets are collected by the receivership whereas negative payoffs are not paid out. Any open payments of the defaulted bank to external customers will be shared by all the remaining banks. This resembles to some extent the function of a deposit insurance system. We are aware of the approximative nature of treating the defaults. Numerical experiments however, show that the influence of different ways of modeling default events have little impact on the results of this study.

A default of a neighbor site can be disastrous for a bank, since it can become rapidly exposed to external risk, which can be high, and in case of a bad outcome of \( X(t) \) can cause its own default as well. In this context spatial catastrophe-spreading can be studied dynamically.

### III. RESULTS

We implemented the above model for \( N = 36 \) sites to stay within reasonable computing times. For simulations of regulation effects we used also \( N = 100 \) banks for networks with low connectivity \( C \). The actual topologies we used for simulations are schematically shown in Fig. 1. The external spread was set \( \delta^{\text{exo}} = 0 \), in order to keep things as transparent as possible and not be disturbed by externally enforced drifts in the wealth processes. As the initial conditions we set \( W_i(0) = 1 \), and \( R_i(0 + \tau) = B_{ij}(0 + \tau) = M_i(0 + \tau) = 0 \) for all \( i \) and \( \tau \).

#### A. Network Effects

In order to estimate the effects of different network topologies on the game, in a first set of simulations we set the Basle parameter \( L_{\text{Basle}} \) to infinity, i.e., we simulate an unregulated economy where \( L_{\text{Basle}} \) does not play any role. We first analyze the time-series of the wealth
processes. In Fig. 3(a) and (b) we show the wealth trajectories of the 36 sites over 200 time-steps for an exogenous volatility $D = 0.07$. The latter was chosen to avoid defaults during this run. The $C = 2$ lattice (left row) is compared to the fully connected graph $C = 35$ (right). The difference due to network topology is visible by plain eye. The fully connected graph appears to keep the sites at more or less the same level of wealth, while for the badly connected topology some sites become ”rich” and some are at the edge of defaulting. Note, that total wealth in the system is not conserved and constant but fluctuates due to the payoffs of the exogenous bets of the clients. The fully connected graph provides a ”fair” basis for all sites. In Fig. 3(c) and (d) the log-return of the wealth process, $r_i(t) = \log W_i(t) - \log W_i(t - 1)$, is shown, again for all sites. It can be seen that for the fully connected net the average spread in returns is significantly smaller, the market is less volatile. We now turn to finite values of $L_{\text{Basle}}$.

B. Regulation Parameter Effects

The log-return is frequently used in financial time-series, and – for efficient and complete markets (which is unrealistic) – is often assumed to be Gaussian \[15\]. Log-return distributions for two different topologies ($C = 2$ and $C = 35$) are shown in Fig. 4 (top) for $L_{\text{Basle}} = 0.05$, which corresponds to a highly regulated economy. Probabilities (normalized histograms) for large events (high risk) are larger for the $C = 2$ network. The log-return distribution of the fully connected network clearly does not extend as far as the $C = 2$. In Fig. 4 (bottom) the situation for the $C = 35$ graph is depicted for two different regulation regimes, $L_{\text{Basle}} = 0.05$ (highly regulated), and $L_{\text{Basle}} = 5$ (practically unregulated). As expected, the probabilities for large log-return is higher for the unregulated scenario. Common to all the distributions is that they are compatible with power law tails. To become more quantitative we characterize the distributions by power-fits to their tails, the value of the probability-density at $r = 1$, and the maximum of $r$, which occurred in simulations. The data is collected in Table 1. Fat tails are a well known phenomenon in financial time-series \[16, 17, 18, 19\], which has been addressed in a number of physical models recently, see e.g. \[20, 21, 22, 23, 24\].

In Fig. 5a we present the number of defaults as a function of connectivity, occurred during a simulation with exogenous volatility $D = 0.15$. All the data and errors shown in
the figure are mean values and standard deviations from 100 independent simulation runs, each a 200 time-steps long. In the highly regulated scheme \( (L_{\text{Basle}} = 0.05) \) the topology effect is small, in the unregulated world, however, a large \( C \) has a mayor impact on reducing the number of defaults.

It seems reasonable to define an index of network efficiency which relates the total exoge-
nously imposed risk \( R_{\text{exo}}^{\text{total}}(t) \) (at the beginning of a trading day) to the total risk the banks are left with after risk-trading (at the end of a trading day):

\[
E(t) = \frac{R_{\text{exo}}^{\text{total}}(t) - \sum_{i,\tau}^{N,\tau_{\text{max}}} R_i(t + \tau, \text{ after trades})}{R_{\text{exo}}^{\text{total}}(t)}.
\] (5)

If all the risk is compensated \( E = 1 \), if trading did not have any risk-reducing effect, \( E = 0 \). The result as a function of \( C \) is shown in Fig. 5 (b). For both regulation schemes there is about a 30 percent increase in \( E \) from \( C = 2 \) to \( C = 35 \). System volatility is defined as

\[
V = \frac{1}{N} \sum_{i,t} |r_i(t)|,
\] (6)

and is a measure of trading activity in markets. In the regulated scenario volatility is always below the unregulated (c). The spread \( S \), defined as the variance of wealth of the \( N \) sites at the latest time in the simulation (200 time-steps), is intended as an indicator of system "un-fairness", and is shown in Fig. 5 (d).

In Fig. 5 (e) we show the ratio of active to passive strategies as adopted by the players during the run. Here "act" means the number of active offers placed on the market, which were not enforced by the regulation threshold. It is seen that for the poorly connected topologies sites are more active since they face high risk/wealth ratios. The ratio is smaller for the highly regulated scheme \( L_{\text{Basle}} = 0.05 \). "act" is not to be confused with "\# enforced" which is the number of enforced trades due to the Basle threshold, i.e., through Eq. 3. The situation for the two values of \( L_{\text{Basle}} \) are shown in Fig. 5 (f). For the unregulated scheme there are no such offers, whereas in the regulated world the number of enforced offers is high and declines somewhat for higher connectivities.

C. Global Safety

As a measure for overall system safety in the various networks we looked at the mean first-default-times in separate runs for various values of the exogenous volatility \( D \). The
mean first-default-times are averages over the time-steps in the simulation until the fist 
default occurs. 1000 independent runs have been performed; random graphs have been 
additionally averaged over 10 different random topologies. For little external risk (small 
$D$) an approximate power law is seen whereas higher external risk levels seem to indicate a 
 faster increase of safety with $C$, Fig. 6.

For a fixed value of $D$ we run simulations for different values of $L_{\text{Basle}}$. To study the 
effects of network type we perform runs for all connectivities in a $N = 36$ bank world, and 
for the $C = 2$ and $C = 4$ cases for $N = 100$. For 100 banks higher connectivity was beyond 
reasonable computation time. Results are shown in Fig. 7 for $N = 36$ (top) and $N = 100$ (bottom).

As expected, for all connectivities network safety becomes higher as $L_{\text{Basle}}$ lowers. We find 
an existence of 2 safety-plateaus, which are separated by a phase transition like drop. This 
suggests a strong connection to statistical physics: The regulation parameter $L_{\text{Basle}}$ plays 
the role of a temperature, the mean first-default-time becomes the order parameter. In the 
small $L_{\text{Basle}}$ limit all banks are forced to be in the active state and the system approaches an 
"ordered" phase in terms of strategies. The origin of the plateaus is not entirely surprising: 
$L_{\text{Basle}}$ enters the system as a threshold upon which trading is enforced. As this threshold is 
lowered the number of forced offers ("#forced") rises. This is demonstrated in Fig. 8(a). For 
$C = 35$ (630 links) the absolute number of forced offers is still less than for $C = 2$ (36 links) 
for all $L_{\text{Basle}}$. The number of active offers "#act" (not enforced by $R_i(t+\tau)/W_i(t) \geq L_{\text{Basle}}$) 
is more or less independent of $L_{\text{Basle}}$ for $C = 2$, and increases slightly for $C = 35$ converging 
to the level of $C = 2$ in the unregulated limit (b). The number of passive trades (c) follows 
the number of offers ( #forced + #act). It is seen that in the highly regulated regime there 
is always more offers than passive trades, i.e., an oversupply of offers. Trading activity 
saturates at this point since no more passive trades are needed to compensate external risk 
and to reach locally optimal risk levels. This explains the existence of the left plateau. The 
right plateau in the unregulated regime occurs when the number of offers and passive trades 
approach an equilibrium (same levels). The situation looks similar for both lattice sizes. 
Naively comparing figures 7 (top) and (bottom) for $C = 2$ and $C = 4$ suggests that the 
smaller network is slightly safer. This is of course not so if one considers the individual 
risk of default within those networks. A further observation is that in the small $L_{\text{Basle}}$ limit 
safety levels are separated equidistantly (in log units) for the equidistantly separated (in log

13
units) connectivities. This indicates a power law of safety as a function of $C$ in this regime. Finally, for completeness we give the ratio of active offers to passive trades in Fig. 8 (d).

The strong decay from one plateau to the other is often related to a phase transition in physics. This means the existence of two stable regimes (where regulatory activity has no effect) which can change from one state to the other within a relatively small critical region. Whenever a phase transition is present powerful mathematical tools from statistical physics – such as the existence of universality classes – can be applied and used to describe, and more importantly, relate models to real-world systems.

D. Effects of Cyclicality

We study how cyclicality in inter-bank connections effect default probabilities or global safety. For this end we fix the number of nodes and links, i.e. the connectivity $C$, and vary the number of cycles of given lengths. We focus especially on small cycles composed of 3 links (3-cycles) which can be used to define the clustering coefficient $CC = \frac{3 \times (\#3\text{-cycles})}{\#\text{connected node--triples}}$ of networks [25]. The studied graphs are shown in Fig. 9. The graphs differ in the spectrum of cycles. The number of 3-cycles decreases from 36, 18, 9, and 0 in Figs. 9 a, b, c, and d, respectively. The mean values and standard deviations of first default times of these networks based on 100 independent simulations per graph are contained in Tab. 11. For the regulated region ($L = 0.05$) an increase in safety as a consequence of less 3-cycles can be inferred, for the unregulated ($L = 5$) case such a trend is not visible, given the size of errors. This indicates that the influence of cyclicality increases with the degree of regulation. This result implies that in regulated regimes banking systems with risk-sharing cooperations, schematically represented by Fig. 9 a, have higher global risk than systems with lower cyclicality.

E. Contagion Effects

In a set of simulations we observed contagion effects. Contagion means that the default of a given bank significantly increases the default probability of its neighbors. In many scenarios the default of neighbors is realized and local ”banking crises” can be observed. To model effects like September 11 we artificially removed a single large (wealthy) bank from
the network at a given simulation time-step. The spread of the crisis depends heavily on the network topology, and ranges from small, locally bounded events to the total collapse of the network. Since these results are hard to visualize over the course of time we do not present them in this version of the paper.

F. Small World Effects?

We moreover looked for topology effects other than connectivity and cyclicality alone. To this end we started from regular networks with a given \( C \) and let links “diffuse” in a way that kept the graphs connected. We could such ”heat” the graphs from regular towards random structure with the same \( C \) in the spirit of small-world networks \[26\]. Results from these studies showed little influence on ”heating” time and it is save to conclude that for the presented measures the relevant topology parameter is \( C \).

IV. DISCUSSION

To summarize, we have introduced a relatively simple model of interacting agents who change their modes of interaction (trading strategy) according to their state of being, i.e., their need to reduce risk. The basic model is inspired from the structure of the well-known iterated prisoner’s dilemma \[27, 28\], where two players are equipped with two possible actions, see also \[29\]. In our case the pay-off matrix additionally depends on an external process, and the size of bets, which in our model are dynamical variables.

The present model contains three relevant parameters \( C, D, \) and \( L_{\text{Basle}} \). We have checked that the remaining parameters, the average riskiness of agents \( \alpha \), and the maximum time horizon of financial instruments \( \tau^{\text{max}} \), are comparably irrelevant. These parameters can be used for scaling the model to realistic data. In reality the bid-ask-spread \( \delta \) is a quantity which crucially depends on the supply and demand of bets. If banks need to reduce risk and can not find counterparties who accept their offers for a certain \( \delta \), they will have to increase the incentive \( \delta \). We have so far run the simulations for two additional values of \( \delta \) (5 % and 20 %), fixed for all bets. With lower values of \( \delta \) we see a tiny increase in trading activity. This can be understood since active strategies become more attractive, see Eq. \[2\], which then also implies slightly more passive trades. This underpins the importance to incorporate
some sort dynamic price formation in the model, i.e., to make $\delta$ a dynamical variable. We plan to include this feature in future work.

We have performed several runs with various sizes of $N$ to check for finite size effects. We found that for $N$ smaller than about 20, scaling starts to vanish. Our main results are that in a number of crucial measures there exists a very sensitive dependence on the connectivity (which models the market structure in the real world). We find that in well connected networks little spread of wealth is able to develop and that topology alone can lead to considerably "fairer" networks. Highly connected networks show significantly less large moves in wealth changes (less volatility), the market becomes less hectic. Distributions of the log-returns are less fat tailed, but still show realistic fat tails, which have realistic power-law behavior. Well connected networks are more efficient in reducing global risk, and show significantly fewer defaults. The average first default time (safety) increases with connectivity.

We finally remark that in reality the parameters $C$, $D$, and particularly $L_{\text{Basle}}$ can be controlled by central banks and governments to regulate risk [30]. The most interesting aspect of this paper is the existence of two plateaus, one for low and one for high values of $L_{\text{Basle}}$. This is independent of the network structure. It is a major finding that the requirement of a capital cushion in the form of the Basle multiplier $L_{\text{Basle}}$ as currently used by regulators may have unexpected adverse effects. We found that for some $L_{\text{Basle}}$ a seemingly strong reduction of the regulatory parameter has vanishing or even zero effect on the safety of the system. This may lead to unjustified overconfidence in the regulatory action. It may also lead to unnecessary restrictions to economic activity, because a relaxation of the Basle multiplier would not lower the safety of the system. Moreover, we show that in regulated regimes banking systems with risk-sharing cooperations have higher global risk than systems with lower cyclicality.

The next stage of the analysis should focus on a more detailed inspection of the inter-connection of the model parameters and on the calibration of the model to real world data. A further promising extension bringing the model closer to reality will be the inclusion the risk assessment of agents. The model is structured in such a way that such an inclusion will involve just to make the information of the betting books available to all the banks.

Even though our model is phrased in terms of a interactions between banks who insure themselves by trading financial assets the core of the model should be very general and can
be seen as a starting point for a dynamical theory of insurance.

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TABLE I: Characterization of the log-return distributions for various values of $N$, $C$, and $L_{\text{Basle}}$, with a given exogenous volatility $D = 0.15$. The tail exponent is the power exponent to the tails of the distributions. max$(r)$ and $p(\cdot)$ correspond to simulations with 720,000 (2m) changes in wealth levels, for the $N = 36$ (100) networks.

| $N$ | $L$ | $C$ | tail exponent | max$(r)$ | $p(1)$ / $10^{-5}$ |
|-----|-----|-----|---------------|----------|------------------|
| 36  | 0.05| 2   | -3.87         | 5.74     | 3.0556           |
| 36  | 0.05| 35  | -3.79         | 4.27     | 0.8333           |
| 36  | 5   | 2   | -3.71         | 6.53     | 3.4722           |
| 36  | 5   | 35  | -3.30         | 4.99     | 1.9445           |
| 100 | 0.05| 2   | -3.87         | 8.10     | 2.7501           |
| 100 | 5   | 2   | -3.62         | 5.54     | 2.9001           |

TABLE II: Influence of cycle spectra on the first default times on graphs with fixed number of nodes ($N = 36$) and links (63), i.e., fixed global connectivity ($C = 1.75$). The corresponding graphs are shown in Fig. 9. Data corresponds to mean values and standard deviations of 100 independent simulations per graph. For $L = 0.05$ an increase in safety as a consequence of less 3-cycles can be inferred. For the $L = 5$ case such a trend is not visible, given the size of errors.

| $L$ | FDT (Fig. 9 a) | FDT (Fig. 9 b) | FDT (Fig. 9 c) | FDT (Fig. 9 d) |
|-----|----------------|----------------|----------------|----------------|
| 0.05| 128.31(47.46)  | 145.76(65.50)  | 161.28(52.52)  | 165.99(65.21)  |
| 5   | 47.26(19.96)   | 42.24(16.93)   | 41.33(19.59)   | 41.97(19.14)   |
FIG. 1: Schematic plot of different graph topologies. (a) One site is connected to all the other \( N - 1 \) sites (monopoly). (b) Each site is connected to two neighboring sites \( C = 2 \) (1 dim. circle, with periodic boundary) (c) Each site is connected to four neighboring sites \( C = 2 \) (regular 2 dim. lattice, with p.b.) (d) Random lattice: Each site is connected to random other sites, the average number of links per site is fixed. "Random 0.222" means that on average 22.2 \% of all possible \((N \times N)\) links are present. (e) Same as before with "Random 0.444", i.e., 44.4 \% of \( N \times N \) possible links are realized on average. These numbers are chosen to provide a connectivity of \( C = 8(16) \) links/site on graphs with \( N = 36 \) or \( N = 100 \) sites. For the random graphs we checked that they were complete, i.e., every site could be reached from every other site. (f) Each site is connected to all the other sites \( C = 35 \) (fully connected).

FIG. 2: Flow diagram for the model. The dynamics during a single time-step \( t \) is shown schematically.

FIG. 3: Comparison between the \( C = 2 \) (left) and the fully connected \( C = 35 \) (right) topologies. Top: Wealth trajectories for the 36 sites. Bottom: Log-returns \( r_i(t) \) of the sites plotted on top of each other. The volatility for the fully connected graph is clearly less.

FIG. 4: Distribution functions derived from histograms of \( r_i(t) \) for comparison of different connectivities (top) and regulation schemes (bottom). Power fits (in regions indicated by the lines) to the tails are gathered in Table 1.

FIG. 5: Performance measures as a function of the different topologies at two regulation extremes, \( L_{\text{Basle}} = 0.05 \) (highly regulated) and \( L_{\text{Basle}} = 5 \) (practically no regulation). (a) Number of defaults occurred within the first 200 time-steps. (b) Efficiency parameter \( E \). (c) Volatility estimate \( V \). (d) Spread \( S \), a measure for "un-fairness" of the system. (e) Ratio of active offers (without Basle, Eq. (3)) to passive trades. (f) Number of forced active offers (by the Basle condition, Eq. (3))

FIG. 6: Mean first-defaults-time (a measure of system safety) for several values of \( D \) for a practically unregulated economy \( (L_{\text{Basle}} = 5) \). Errors are less than double symbolsize.
FIG. 7: Mean first-default-time (a measure of system safety) as a function of the regulation parameter $L_{\text{Basle}}$ for various values of $C$ on a $N = 36$ (top) and $N = 100$ (bottom) lattice at a fixed value of $D = 0.15$. Errorbars are about twice symbolsize and are omitted for clarity.

FIG. 8: Statistics of the same runs as in the previous plot: (a) number of enforced offers (by the Basle condition) as a function of $L_{\text{Basle}}$. (b) and (c) show the numbers of active offers (without Basle enforcement) and passive trades respectively. Finally, in (d) the ratio of active offers compared to passive trades is shown.

FIG. 9: Connection topology for the study of cycle effects. In these graphs the number of nodes and links are fixed. Different connection topology alters the number of cycles of given length in the total graph. The spectrum of cycle size shifts towards larger cycles from (a) to (d). The number of shortest cycles (length 3) is 36, 18, 9, and 0 for Figs. (a), (b), (c), and (d), respectively. The number of cycles involving four links increase, 9, 18, 18, and 27 for (a), (b), (c), and (d). Effects of the different cycle spectra are collected in Table II.
FIG. 1
FIG. 2

1. Pick bank \( i \) randomly.
2. Add external risk \( \beta^\text{exo}(t+\tau) \) for a random \( \tau \).
3. Is there a bet on the market that reduces risk \( R \) for all \( \tau \)?
   - Yes: Accept bet and earn incentive.
   - No: Pick next bank.
4. If \( R/W > L \):
   - Yes: Issue bet and put it on the market \( M_i \).
   - No: Pick next bank.
5. External process: \( X(t) = \pm 1 \).
6. Wealth update for all banks \( i \):
   \[ W_i(t+1) = W_i(t) + X(t) \left[ \beta^\text{exo}_i + \sum_j \beta_{ij} \right] \]
7. Time \( t+1 \).
FIG. 3
FIG. 4
FIG. 5
FIG. 6
FIG. 7
FIG. 8
