Gamma-Ray Bursts: the Isotropic-Equivalent-Energy Function and the Cosmic Formation Rate

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ABSTRACT
Gamma-ray bursts (GRBs) are brief but intense emission of soft γ-rays, mostly lasting from a few seconds to a few thousand seconds. For such kind of high energy transients, their isotropic-equivalent-energy ($E_{iso}$) function may be more scientifically meaningful when compared with GRB isotropic-equivalent-luminosity function ($L_{iso}$), as the traditional luminosity function refers to steady emission much longer than a few thousand seconds. In this work we first time construct the isotropic-equivalent-energy function for a sample of 95 bursts with measured redshifts (z) and find an excess of high-z GRBs. Assuming that the excess is caused by a GRB luminosity function evolution in a power-law form, we find a cosmic evolution of $E_{iso} \propto (1 + z)^{1.51 \pm 0.56}$, which is comparable to that between $L_{iso}$ and z, i.e., $L_{iso} \propto (1 + z)^{2.3 \pm 0.35}$ (both 1σ). The evolution-removed isotropic-equivalent-energy function can be reasonably fitted by a broken power-law, in which the dim and bright segments are $\psi(E_{iso}) \propto E_{iso}^{-0.27 \pm 0.01}$ and $\psi(E_{iso}) \propto E_{iso}^{-0.87 \pm 0.07}$, respectively (1σ). For the cosmic GRB formation rate, it increases quickly in the region of $0 \lesssim z \lesssim 1$, and roughly keeps constant for $1 \lesssim z \lesssim 4$, and finally falls with a power index of $-3.80 \pm 2.16$ for $z \gtrsim 4$, in good agreement with the observed cosmic star formation rate so far.

Key words: gamma-ray burst: general-stars: formation

1 INTRODUCTION
Gamma-ray bursts (GRBs) are among the brightest cosmological explosions in the universe, mostly lasting from a few seconds to a few thousand seconds in soft γ-ray. Thanks to quick follow-up observations in optical band, the redshifts of some GRBs have been measured by detecting the absorption lines of their afterglows or the emission lines of their host galaxies. So far the number of Swift-detected GRBs with known redshifts has grown up to about one hundred and thus makes a reliable statistical analysis possible. Among various statistical works, the luminosity function as well as the cosmic formation rate of GRBs are particularly interesting. The luminosity function is a measure of the number of bursts per unit luminosity, which sheds light on the energy release and emission mechanism of GRBs. The cosmic formation rate is a measure of the number of events per comoving volume and time, which can help us understand the production of GRBs in various stages of the universe.

The isotropic-equivalent luminosity ($L_{iso}$) function of GRBs was firstly assumed to be a standard candle (Wijers et al. 1998; Totani 1999) and later more realistic shapes of the luminosity function were derived (Firmani et al. 2004; Guetta & Piran 2005; Guetta, Piran & Waxman et al. 2005; Liang et al. 2007; Virgili, Liang, & Zhang 2009). The cosmic GRB formation rate has also been extensively investigated (e.g., Fenimore & Ramirez Ruiz 2000; Norris et al. 2003; Schaefer et al. 2003; Lloyd-Ronning et al. 2002; Murakami et al. 2003; Yonetoku et al. 2004; Firmani et al. 2004; Wanderman & Piran 2010; Qin et al. 2010). For example, Fenimore & Ramirez Ruiz (2000) found a correlation between the variability degree of the prompt gamma-ray light curve and the luminosity and then adopted it to estimate the luminosities/redshifts of 220 bright long GRBs detected by CGRO/BATSE.

Lloyd-Ronning et al. (2002) used this GRB sample to estimate the luminosity function evolution and the cosmic formation rate of GRBs. Wei & Gao (2003) find that there is a tight correlation between the peak energy of the prompt emission spectrum ($E_{peak}$) and the luminosity. Recently, Zhang et al. (2013) find that both short and long GRBs all comply with this correlation. Using this correlation, Yonetoku et al. (2004) estimated the luminosities/redshifts of the 689 BATSE GRBs and hence derived their luminosity function and the formation rate. Essentially these two works with simulated GRB redshifts reached quite similar results, i.e., the GRB formation
rate increases quickly in the region of $0 \leq z < 1$ and keeps constant up to $z \sim 10$, which is inconsistent with the cosmic star formation rates (SFRs) inferred from UV, optical, and infrared observational data so far (Madau et al. 1996; Lilly et al. 1996; Barger et al. 2000; Stanway et al. 2003).

The original concept of luminosity function comes from astrophysical objects such as stars and galaxies which are long-lasting and quite stable in releasing their energy. For GRB-like high-energy transients, the total isotropic-equivalent energy ($E_{iso}$) released in the whole duration of one event can be reliably measured, and its function (i.e., the number density of bursts per $E_{iso}$ interval) likely provides an independent or even more representative clue on the underlying physics. That’s why in this work we focus on the so-called “isotopic-equivalent-energy function” rather than the traditional luminosity function.

This paper is arranged as follows. § 2 introduces our sample and data selection. § 3 presents the statistical technique while § 4 shows the results. We adopt a robust, nonparametric statistical technique to derive the isotopic-equivalent-energy function and the cosmic formation rate of GRBs from a $E_{iso} \sim z$ sample. For comparison, the results from a $L_{iso} \sim z$ GRB sample are also presented. In § 5, we discuss the implication of our results and compare the cosmic GRB formation rate with the observational cosmic star formation rate. Throughout the paper, we use the standard Λ cold dark matter cosmology with the typical parameters $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$, and $h = 0.7$.

## 2 DATA ANALYSIS

In this work, two sets of data are analyzed. The $E_{iso} \sim z$ sample comes from Amati et al. (2008, 2009), containing 95 long GRBs and X-Ray Flashes (XRF, i.e. particularly soft bursts). This sample is made up of two parts. The first part consists of 70 long GRBs from Amati et al. (2008) and their redshifts range from 0.033 to 6.3. The $E_{iso}$ values in this work are slightly different from those in Amati et al. (2008) because of the different cosmological parameters adopted in two works. The second part is from Amati et al. (2009) without any modification. All the GRB spectra have been extrapolated and corrected to $[1,10000]$ keV in the cosmological restframe.

The $L_{iso} \sim z$ sample is from Wanderman & Piran (2010) ($L_{iso}$ represents the isotropic peak luminosity, and the time resolution is $1 s$). Due to the Swift/BAT narrow energy band, only a small fraction of bursts have a well determined spectrum. In order to obtain reasonable estimates of $E_{iso}$ for all bursts, Wanderman & Piran (2010) considered the characteristic Band function, i.e.,

$$N(E) = \left\{ \begin{array}{ll} A \left( \frac{E}{E_{peak}} \right)^{\alpha} \exp \left( - \frac{E}{E_0} \right), & \text{for } E \leq (\alpha - \beta) E_0 \\ A \left( \frac{E}{E_{peak}} \right)^{\alpha} \exp \left( - \frac{(\alpha - \beta) E_0}{E} \right), & \text{for } E > (\alpha - \beta) E_0 \end{array} \right. 
(1)$$

where the characteristic parameters ($E_{peak}$, α, β) taken as (511 keV, −1, −2.25), respectively. To account for the so-called k-correction, again all spectra have been extrapolated and corrected to $[1,10000]$ keV in the cosmological restframe. To test whether the above Band function applicable to all bursts, Wanderman & Piran (2010) preformed Monte-Carlo simulation in order to compare GRBs having simulated spectral parameters with GRBs having measured spectral parameters. The result of such a simulation demonstrates robustness of the $L_{iso}$ sample adopted above.

![Figure 1. The $E_{iso}$ distribution and the fluence limit.](image)

### 3 STATISTICAL TECHNIQUES

The $E_{iso} \sim z$ sample suffers from various selection effects (Nava et al. 2008; Lu et al. 2011), among which the dominated one is the truncation due to the detection limit of the telescope, as seen in Fig. 1. If this bias not removed, the $E_{iso} \sim z$ correlation would be far from the intrinsic one. A nonparametric $\tau$ statistical technique may be introduced to resolve this problem, which was first put forth by Lynden-Bell (1971) and further developed by Efron & Petrosian (1992) and Lloyd-Ronning et al. (2002) first applied this technique to GRBs with simulated redshifts and later Yonetoku et al. (2004) applied it to a larger GRB sample still with simulated redshifts but up to $z \sim 10$. In this work, we continue to use this technique, but for the first time to two GRB samples with observed/measured redshifts.

In short, this nonparametric $\tau$ statistical technique uses a well-defined truncation criterion to estimate the correlation (if any) between the relevant variables and their underlying parent distributions.

The $E_{iso}$ and $z$ are not independent. Without loss of generality, the total isotropic-equivalent-energy function can be rewritten as $\Phi(E_{iso}, z) \sim \rho(z) \Phi(E_{iso}, g(z))/g(z)$, where $\rho(z)$ is the GRB formation rate at $z$, $\Phi(E_{iso}, g(z))$ is the present-day (i.e., $z = 0$) isotropic-equivalent-energy function, and $g(z)$ counts for the cosmic evolution of $E_{iso}$, that said, $E_{iso} = E_{iso}/g(z)$. In the following analysis, the evolution $g(z)$ will be removed from the $E_{iso}$ sample; after that one obtains the $E_{iso}$ distribution, then the cumulative function $\psi(E_{iso})$, and finally the GRB formation rate $\rho(z)$.

Consider a set of observable $E_{iso}$ and $z_i$, where $i$ indexes the $i$th burst and in our case $i$ runs from 1 to 95. As shown in Fig. 1, for the $i$th sample of $(z_i, E_{iso})$, we consider an associated set of

$$J_i = \{ j | E_{iso,j} > E_{iso,i}, z_j < z_{lim} \}, \quad \text{for } 1 \leq i \leq 95, \quad (2)$$

in which the number of samples in the $J_i$ set is $N_i$. The $z_{lim}$ is the redshift of the crossing point between two lines of $E = E_{iso}$ and the fluence limit corresponding to its “isotopic-equivalent-energy” limit. If $z$ and $E_{iso}$ are independent to each other, one would expect the number of the following sample

$$R_i = \text{Number} \{ j \in J_i | z_j \leq z \} \quad (3)$$

to be uniformly distributed between 1 and $N_i$. To estimate the correlation degree between $E_{iso}$ and $z$, one may introduce the test statistic $\tau$ parameterized as

$$\tau = \frac{\sum_i (R_i - E_i)}{\sqrt{\sum_i V_i}}, \quad (4)$$

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where \( E_i = (N_i + 1)/2 \) and \( V_i = (N_i^2 - 1)/12 \) are the expected mean and the variance of the uniform distribution, respectively.

If \( R_i \) follows an ideal uniform distribution, then the samples of \( R_i < E_i \) and \( R_i > E_i \) should be equal, and thus the statistic parameter \( \tau \) tends to be zero. Note that the \( \tau \) value here has been normalized by the square root of variance, so the correlation degree \( \tau \) and \( E_{iso} \) can be measured in units of standard deviation.

The two variables \( E_{iso} \) and \( z \) are connected through the form of \( E_{iso} = 4\pi FD_{iso}^2(z)/(1 + z) \), where \( F \) is the fluence in 1–10000 keV in the burst restframe. Note that bursts in the sample were actually detected by instruments onboard different satellites. The flux sensitivities of these instruments are as follows: \( \sim 4 \times 10^{-8} \text{erg cm}^{-2} \text{s}^{-1} \) for CGRO/BATSE; \( \sim 10^{-7} \text{erg cm}^{-2} \text{s}^{-1} \) for Konus-Wind, BeppoSAX and Fermi/GBM; \( \sim 3 \times 10^{-8} \text{erg cm}^{-2} \text{s}^{-1} \) for HETE-2; and \( \sim 10^{-9} \text{erg cm}^{-2} \text{s}^{-1} \) for Swift/BAT. Some bursts were detected by more than one satellite. To be safe, we choose the best sensitivity of \( \sim 10^{-9} \text{erg cm}^{-2} \text{s}^{-1} \) as the flux limit of the whole sample.

For the \( L_{iso} \) sample, the two variables are connected by \( L_{iso} = 4\pi FD_{iso}^2(z) \) where \( F \) is the flux in 1–10000 keV in the burst restframe. We set the flux limit as \( F = 5 \times 10^{-8} \text{erg cm}^{-2} \text{s}^{-1} \) in view of the fact that the sensitivity of BAT is a few \( \sim 10^{-8} \text{erg cm}^{-2} \text{s}^{-1} \) (Barthelmy et al. 2002). Nevertheless, our results are not sensitive to the limit of \( F \) and/or \( F \).

### 4 RESULTS

Following Maloney & Petrosian (1999) and Lloyd-Ronning et al. (2002), we take the form of \( g(z) = (1 + z)^2 \) in order to separate the isotropic energy evolution \( g(z) \) from the GRB sample. The value \( E'_{iso} = E_{iso}/g(z) \) represents the isotropic energy after removing the evolution effect. When \( \tau \) is not equal to zero, we change the \( \tau \) values until \( \tau = 0 \) with a proper k. Fig. 2 shows the \( \tau \) value as a function of \( k \). The null hypothesis of the evolution is rejected at about 3.5 \( \sigma \) confidence level. The best fit to the \( E_{iso} - z \) data yields that \( E_{iso} = E_{iso}/(1 + z)^{1.80} \), i.e., \( k = 1.80 \).

After converting \( E_{iso} \) into \( E'_{iso} = E_{iso}/(1 + z)^{1.80} \), we can nonparametrically derive the cumulative (local) “isotropic-equivalent-energy function” \( \psi(E'_{iso}) \) with the following equation of Lynden-Bell (1971); Efron & Petrosian (1992); Petrosian (1993); Lloyd-Ronning et al. (2003); Yonetoku et al. (2004).

\[
\ln \psi(E'_{iso}) = \sum_{j=0}^{n} \ln \left( 1 + \frac{1}{N_j} \right) .
\]

As can be seen, the cumulative number at the \( \epsilon \)th point is calculated from \( N_j \) and for each point indexed by \( j \), a truncation parallel to the axes is made and a weight \( 1/N_j \), based on the number of points in the associated set, is assigned to that data point.

Fig. 3 shows the “isotropic-equivalent-energy function” of \( E'_{iso} = E_{iso}/(1 + z)^{1.80} \). The shape of the “isotropic-equivalent-energy function” roughly follows a broken power law, and the dim and bright segments can be parameterized as \( \psi(E_{iso}) \propto E_{iso}^{-0.27 \pm 0.03} \) and \( \psi(E_{iso}) \propto E_{iso}^{0.03 \pm 0.07} \). It corresponds to the isotropic energy distribution at \( z = 0 \) since the evolution effect has been removed. The “isotropic-equivalent-energy function” in the comoving frame is \( \psi(E'_{iso}) \propto (1 + z)^{1.80} \).

To estimate the cosmic GRB formation rate, the cumulative number distribution \( \psi(z) \) as a function of \( z \) is derived using the function analogous to equation (5). In this case, for the \( \epsilon \)th sample the associated set is given by

\[
J^\epsilon_{i,j} = \{ j|z_j < z_i, E_{iso,j} > E_{iso,j,lim} \} ,
\]

where \( E_{iso,j,lim} \) is calculated at the crossing point of the fluence limit and \( z = z_i \). The resulting cumulative GRB formation rate \( \psi(z) \) is shown in Fig. 4.

To be scientifically useful, one needs to convert the cumulative formation rate into the differential form (e.g., to compare with the cosmic star formation rate). The conversion is given by

\[
\rho(z) = \frac{d\psi(z)}{dz} (1 + z)^{1 - \frac{dV(z)/dz}{dV}} .
\]

where \( (1 + z) \) comes from the cosmological time dilation, and \( dV(z)\) is a differential comoving volume described by

\[
dV = 4\pi \left( \frac{c}{H_0} \right)^2 \int_z^\infty \frac{dz}{\sqrt{\Omega_M + \Omega_k}} (1 + z)^2 \times \frac{1}{\sqrt{\Omega_M + \Omega_k}} .
\]

Fig. 5 shows the resulting differential GRB formation rate.
The best-fit power-laws for different segments are

\[ \rho(z) \propto \begin{cases} 
(1 + z)^{2.24 \pm 4.48} & \text{for } z < 1 \\
(1 + z)^{0.54 \pm 0.64} & \text{for } 1 < z < 3.5 \\
(1 + z)^{3.80 \pm 2.16} & \text{for } z > 3.5
\end{cases} \]

with 95% confidence bounds.

The \( L_{\text{iso}} - z \) sample was treated in the same way. For this sample, we found \( k = 2.30^{+0.56}_{-0.51} \), which is close to the value from the \( E_{\text{iso}} - z \) sample. The corresponding luminosity function, cumulative GRB formation rate and differential form of the GRB formation rate have been reported in Figs. 6-8.

We use the best-fit isotropic-equivalent-energy function and GRB formation rate to calculate log N-log S distributions and compare them with the observed log N-log S distributions of BATSE and Swift bursts. The results are shown in Fig. 9. From this figure, we find that the results obtained from our model can well reproduce the observation (The probabilities of the K-S test are \( P=0.95 \) and \( P=0.74 \) for the BATSE and Swift GRB samples, respectively).

Figure 4. Cumulative GRB formation rate \( \psi(z) \) as a function of \( z \), which is also normalized to unity at the highest point.

Figure 5. Relative GRB formation rate, which is normalized to unity at the first point. The error bar represents the 1 \( \sigma \) statistical uncertainty of each point.

Figure 6. Cumulative luminosity function \( \psi(L'_{\text{iso}}) \) of \( L'_{\text{iso}} = L_{\text{iso}}/(1 + z)^{2.30} \), which is normalized to unity at the dimmest point. The evolution effect is removed.

Figure 7. Cumulative GRB formation rate of \( L_{\text{iso}} \) sample as a function of \( z \), which is normalized to unity at the highest point.

Figure 8. Relative GRB formation rate of \( L_{\text{iso}} \) sample, which is normalized to unity at the first point. The err bar represents the 1 \( \sigma \) statistical uncertainty of each point.
to the chosen value of fluence
from 1.80 to 2.05, changing the flux limit from 5
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5 CONCLUSION AND DISCUSSION
GRBs are brief but intense emissions of soft γ-ray, lasting from a
few seconds to a few thousand seconds. For GRB-like high energy
transients, the total isotropic-equivalent energy, \(E_{\text{iso}}\), can be reli-
ably measured, and the number density of bursts per \(E_{\text{iso}}\) interval
may provide an independent or even more representative clue on
the underlying physics of GRBs. In this work, using a sample con-
taining 95 bursts with measured redshifts, we for the first time con-
structed the isotropic-equivalent-energy function and then adopted
it to estimate the GRB formation rate. The fluence-truncation ef-
fect has been properly addressed by adopting a \(\tau\) statistical tech-
nique. We find there exists cosmic evolution between \(E_{\text{iso}}\) and \(z\),
i.e., \(E_{\text{iso}} \propto g(z) = (1 + z)^{1.84 \pm 0.38}\) (see Fig. 2), which is comparable
with that between \(L_{\text{iso}}\) and \(z\), i.e., \(L_{\text{iso}} \propto (1 + z)^{2.30 \pm 0.11}\). Our finding
of the evolution is largely consistent with previous findings using
GRB samples with simulated redshifts (Lloyd-Ronning et al. 2002;
Wei & Gao 2003; Yonetoku et al. 2004).

We also find that the evolution power index \(k\) is not sensitive
to the chosen value of fluence/flux as long as it is around the instru-
mental sensitivity. Changing the fluence limit from \(8 \times 10^{-7}\) \(\text{erg cm}^{-2}\)
to \(5 \times 10^{-9}\) \(\text{erg cm}^{-2}\), the best-fit index \(k\) of \(E_{\text{iso}}\) sample only varies
from 1.80 to 2.05, changing the flux limit from \(5 \times 10^{-8}\) \(\text{erg s}^{-1}\) \(\text{cm}^{-2}\)
to \(1 \times 10^{-9}\) \(\text{erg s}^{-1}\) \(\text{cm}^{-2}\), the best-fit index \(k\) of \(L_{\text{iso}}\) sample only varies
from 2.30 to 2.65, all well within the 1\(\sigma\) uncertainty.

As can be seen, the evolution of \(E_{\text{iso}}\) is comparable with that of \(L_{\text{iso}}\). This might indicate that the durations of GRBs do
not evolve significantly with redshift and that the existence of
the isotropic-equivalent-energy evolution may reflect the evolution of
typical physical parameters of GRB progenitors.

After removing the redshift dependence, the isotropic-
equivalent-energy function can be reasonably fitted by a broken
power-law. For the dim and bright segments, we have \(\psi(E_{\text{iso}}) \propto E_{\text{iso}}^{-0.27 \pm 0.03}\)
and \(\psi(E_{\text{iso}}) \propto E_{\text{iso}}^{-0.37 \pm 0.07}\), respectively (see Fig. 3). The
shape of the energy function (see Fig. 3) is similar to that of
the luminosity function (see Fig. 6). Moreover, our indices of
\((-0.27, -0.87)\) are comparable to the indices of \((-0.29, -1.02)\)
reported by Yonetoku et al. (2004).

The GRB formation rate as a function of redshift is also calculated. The connection between (long-duration) GRBs
with broad-lined Type Ic supernovae (e.g., Stanek et al. 2003;
Hjorth et al. 2003; Woosley & Bloom 2006; Fan et al. 2011) sug-
jects that GRB progenitors are very massive, short-lived stars,
leading to expectation that the cosmic GRB formation rate would
nicely follow the cosmic star formation history (Totani 1997;
Wijers et al. 1998; Lamb & Reichard 2000; Blain & Natarajan
2003; Porciani & Madau 2001). So far, the observed star formation
rate follows \(\rho(z) \propto (1 + z)^{1.84 - 0.38}\) for \( z < 1 \), \(\propto (1 + z)^{0.30}\) for \( 1 < z < 4 \)
and \(\propto (1 + z)^{-1.37}\) for \( z > 4 \) (Kistler et al. 2008; Yüksel et al. 2008)
(see Fig. 10). Our results (see Fig. 10) show that the GRB forma-
tion rate increases quickly for \( 1 \leq z \leq 4 \), then roughly keeps constant
for \( 1 \leq z \leq 4 \), and finally decreases at higher redshift with a power
index of \(-3.8\) (i.e., \(\propto (1 + z)^{-3.24}\)), in good agreement with
the star formation rate. In future a larger GRB sample with measured
redshifts would better address this topic and make progress in the
fields of massive star formation and GRB physics in the early uni-
verse.

Our results are based on the assumption of a power-law red-
shift evolution of the \(E_{\text{iso}}\). Besides introducing the evolution of the
isotropic-equivalent-energy function, other models can also repro-
duce the observed data, such as one could enhance high-\(z\) GRB rate
by introducing a metallicity preference of GRBs or increase the to-

Figure 9. Cumulative bursts distribution as a function of the fluence \((S)\). Panel (a): \(\log N - \log S\) distribution for BATSE bursts (Solid line) and the distribution predicted by our model (Dotted line). Panel (b): \(\log N - \log S\) distribution for Swift bursts (Solid line) and the distribution predicted by our model (Dotted line).

Figure 10. GRB rate comparing with SFR. The data of SFR come from Hopkins & Beacom (2006), Kistler et al. (2008), Yüksel et al. (2008). We normalized the first point of GRB rate and SFR for convenience.
tual number of GRBs (e.g., Virgili et al. 2011). These models might be distinguished by the future observations.

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