Bose-Einstein Condensation of Nonequilibrium Magnons in Laterally Confined Systems

Morteza Mohseni, 1,7 Alireza Qaiumzadeh, 2 Alexander A. Serga, 1 Arne Brataas, 2 Burkard Hillebrands 1 and Philipp Pirro 1

1 Fachbereich Physik and Landesforschungszentrum OPTIMAS, Technische Universität Kaiserslautern, 67663 Kaiserslautern, Germany
2 Center for Quantum Spintronics, Department of Physics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway

A high density of weakly interacting quasiparticles can initiate a phase transition into a macroscopic Bose-Einstein condensate (BEC). Theory predicts that magnon condensation is unstable in extended ultrathin insulating magnets of yttrium iron garnet (YIG). Here, we introduce a novel method to overcome this limit and obtain a magnon BEC for a short time in an ultrathin film by using spatial confinement. We predict how dipolar interactions and nonlinear magnon scattering assist in the generation of a metastable magnon BEC in quantized nanoscopic systems. We verify our predictions by using numerical simulations and demonstrate the generation of magnon BEC in nanoscopic YIG microconduits. We directly map out the nonlinear magnon scattering processes behind this phase transition to show how fast quantized thermalization channels allow the BEC formation in confined structures. Our results provide new insight into strongly nonlinear spin dynamics in ultrathin film structures and introduce a mechanism to enhance the stability of magnon BEC in nanoscale systems.

Introduction---Bose-Einstein condensates (BEC) are exotic quantum states of matter [1-2]. The associated phase transition manifests as an abrupt macroscopic growth in the population of particles in the quantum ground state. Similar phenomena occur for bosonic quasiparticles (QPs) which are elementary excitations of a solid-state system, such as photons, excitons, polaritons, and magnons [3-8]. Magnons, the quanta of spin-waves (SWs) carrying a quantized amount of energy and momentum, are the low energy spin excitations of a magnetically ordered system [9].

Starting with the first observation of nonequilibrium magnon BEC in macroscopic Yttrium Iron Garnet (YIG) films at room temperature, almost all experimental studies consider similar system sizes, see e.g., Refs. [6-8, 10-12]. Indeed, YIG films are the most suitable hosts to explore similar phenomena due to their ultra-low magnon relaxation rates [13]. However, the use of relatively thick films does not allow the investigation of this phenomenon in quantized states since the magnon bands in these systems are quasi-continuous and show crossings and hybridization [13-14]. This limitation prevents an explicit understanding of the nonlinear scattering processes (e.g. among modes) that leads to the formation of a magnon BEC. Moreover, the spectral limitations of the experimental techniques hinder a full understanding of the details of the nonlinear scattering processes behind this phenomenon. Furthermore, the presented analytical models use many approximations since the inclusion of all magnon modes and nonlinear interactions is challenging, see e.g., Ref [8, 11-12, 15-16].

To overcome these challenges, downscaling macroscopic systems toward nanometer sizes to enhance the quantization of the magnon energy levels is necessary [17-18]. Downscaling is also important for applications such as data transport via spin superfluidity [19-25]. However, recent works claim that the magnon BEC is unstable in extended nanoscopic YIG films due to the presence of attractive magnon-magnon interactions that causes depletion of the BEC [26-27]. These claims called into question the research in this direction until the recent discovery of a magnon BEC in a confined nanoscopic YIG thin film structure [28]. These findings raised the fundamental question whether the confinement may enhance the magnon BEC stability in nanoscopic systems.

In this Letter, we address this central question and show that the confinement enhances
the stability and increases the lifetime of the nonequilibrium magnon BEC in ultrathin YIG structures. Indeed, such a metastable state is influenced significantly by the dipolar interactions [26]. Therefore, since the effects of dipolar interactions can be more pronounced if the lateral dimensions of a YIG thin film structure are confined, we predict that a magnon BEC can form for a longer time.

We verify our prediction by solving the Landau–Lifshitz equation including Gilbert damping and thermal excitations in the geometry of a nanoscopic YIG microconduit. We show that the injection of a sufficiently high density of magnons using parametric pumping leads to a sequence of nonlinear scattering processes that redistribute the energy among magnon states and form a magnon condensate. We map out the sequences of the scattering processes and show that the thermalization time of the magnons in confined systems with dilute spectra is much smaller than in macroscopic samples [8].

We note that the existence of a metastable BEC with attractive interactions between bosonic particles has been confirmed experimentally and theoretically in confined geometries [29-31].

Theory—We consider a magnetic thin film structure with periodic boundary conditions in the $x$-direction, a finite thickness $d$ in the $z$-direction, and a finite width $w$ in the $y$-direction, as shown in the inset of Fig. 1. If the system is magnetized along the $x$-direction and magnons propagate parallel to the magnetization direction, the lowest magnon energy band ($n=0$) reads [17, 32],

$$
\omega_{n=0}(k_x) = \sqrt{\left[\omega_H + \omega_M (\lambda^2 k_x^2 + P_{k_x}^{yy})\right]\left[\omega_H + \omega_M (\lambda^2 k_x^2 + P_{k_x}^{zz})\right]},
$$

(1)

where $k_x$ is the longitudinal component of the wavevector along the magnetization vector $\mathbf{M}$, $\omega_H = \gamma B$, $\omega_M = \gamma \mu_0 M_s$, $\gamma$ is the gyromagnetic ratio, $B$ is the static applied magnetic field along the $x$ direction, $\mu_0$ is the permeability of free space, $K = \sqrt{k_x^2 + k_y^2}$ with $k_y = \pi/w$, and $\lambda = \sqrt{2A_{\text{exch}}/(\mu_0 M_s^2)}$ is the exchange length, with an exchange constant $A_{\text{exch}}$ and a saturation magnetization $M_s$. Here, $P_{k_x}^{yy}$ and $P_{k_x}^{zz}$ are the $y$ and $z$ components of the demagnetization tensors, respectively [17]. In this Letter, we consider a YIG film with a fixed thickness of $d = 85$ nm, $A_{\text{exch}} = 3.5$ pJ/m, $M_s=140$ kA/m and a Gilbert damping of $\alpha = 0.0002$ [17-18].

Figure 1 compares the lowest magnon mode of an infinitely extended film (blue curve) with the dispersion relations of two laterally confined conduits with a width of $w = 5$ µm (red curve) and $w = 1$ µm (black curve). The interplay between the dipolar and exchange interactions leads to the appearance of an energy minimum in the frequency spectrum denoted as $f_{\text{min}}$, at $k_x = \pm Q$, the band bottom which corresponds to the quantum ground state of this system. More interestingly, the lateral confinement leads to an enhanced dipolar interaction along the propagation direction and consequently a deeper band depth. We define the band depth as the difference between the band bottom of the lowest mode and the ferromagnetic resonance (FMR) frequency as $A_{\text{FMR}} = f_{\text{FMR}} - f_{\text{min}}$ shown by the dashed arrows in Fig. 1.

As reflected by the observed finite lifetime of the magnon BEC [8, 28], the magnon BEC is a metastable state with respect to thermal activations or decay to the excited states [26]. This causes lower
and upper limits of the magnon density required to form and destroy a magnon BEC, respectively. It is expected that its lifetime is enhanced in systems with a deeper band depth. This is caused by the fact that in such a system, there is a large energy separation between the condensate state at the band bottom and the excited states (e.g. externally injected magnons or magnons scattered to higher energies than the quantum ground state) that cannot be exceeded by the magnon-magnon scattering within the weak interaction regime once the BEC is established [7-8, 26]. In our case, this is equivalent to systems with narrower lateral dimensions, i.e., a system with enhanced dipolar interactions along the propagation direction. Therefore, we expect that under this condition, the magnon condensate forms during a finite time period which is influenced by the band depth. In the following, we verify this hypothesis.

**Micromagnetic Simulations**-- We use MuMax 3.0 open source GPU-based software to numerically solve the LLG equation with thermal noise [33-34]. The system is a YIG microconduit with a thickness of $d = 85$ nm and a lateral width of $w = 1$ µm as shown in Fig. 2a. A microwave pumping pulse with a duration of $t_{\text{pump}} = 50$ ns with a carrier frequency of $f_{\text{p}} = 4.2$ GHz is applied to the simulated microwave stripline (placed on top of the conduit) whose dynamic Oersted fields leads to parametric generation of the magnons at $f_{\text{p}}/2 = 2.1$ GHz that is slightly below the FMR frequency [35, 36-37]. This process has been studied experimentally and numerically in Ref. [37]. After the pumping pulse, we let the system relax for $t_{\text{relax}} = 50$ ns. We analyze the data during the pumping and relaxation periods separately. More details about the simulations can be found in [34, 37].

We first set the microwave current to $i_{\text{rf}} = 12$ mA, which produces a pumping field with an amplitude of $\mu_0 h_{\text{p}} = 8.9$ mT that is well above the threshold of the parametric generation of magnons in the system [37]. In Fig. 2c, the frequency spectrum directly shows the parametrically generated magnons at $f_{\text{p}}/2$. The magnon band structure under this pumping field as displayed in Fig. 2d shows that these magnons predominantly

---

**FIG. 2.** (a) Schematic view of the sample under study. (b) Magnon band structure near the band bottom obtained from numerical simulations (color plot) and analytical calculations (dotted lines). The first three width modes ($n = 0, 1, 2$) are present in the band structure. (c-d), (e-f): Frequency spectra and the magnon band structures during pumping when microwave currents of $i_{\text{rf}} = 12$ mA and $i_{\text{rf}} = 15$ mA are injected into the stripline, respectively. Thermal magnons are distinguished via gray color at (c-e). (g-h): Magnon band structure during pumping and relaxation, when the microwave current is $i_{\text{rf}} = 20$ mA. Magnon condensation at the band bottom is visible during relaxation.
populate the dipolar branch of the first mode, due to its large ellipticity and highest coupling to the pumping field [37-38].

Next, we increase the microwave current to $i_{rf} = 15 \text{ mA}$ ($\mu_0 h_p = 11.2 \text{ mT}$) in order to generate a higher density of magnons. Indeed, if the induced magnon density is high enough, nonlinear four-magnon scattering becomes pronounced [6-8]. Such nonlinear scattering processes, indicated by blue arrows in Fig. 2e, obey energy and momentum conservation laws [7-8, 35, 39],

$$\sum_i \hbar \omega_i = \sum_j \hbar \omega_j,$$

$$\sum_i \hbar k_i = \sum_j \hbar k_j. \quad (2)$$

where two incoming magnons with frequencies $\omega_i$ and momenta $\hbar k_i$ scatter into two outgoing magnons with frequencies $\omega_j$ and momenta $\hbar k_j$. However, the total number of magnons during this process is constant. We can distinguish two types of four-magnon scattering processes in the system as presented in Fig. 2f: i) four-magnon scattering processes in which two incoming magnons with frequency $f_p/2$ and with opposite momenta are scattered into two outgoing magnons at the same frequency but with different momenta. This frequency-conserving scattering process only occurs for parametrically injected magnons as indicated by light orange arrows in Fig. 2f; and ii) four-magnon scattering processes in which two magnons with a frequency of $f_p/2$ and different momenta scatter to two outgoing magnons with different frequencies and momenta. This frequency-nonconserving magnon scattering process involves all magnons in the system, i.e., both parametrically injected and scattered magnons, as indicated by red arrows in Fig. 2f.

Such four-magnon scattering processes are essential for the thermalization of the magnon gas and formation of a magnon condensate at the bottom of the band [7-8]. Here, we should emphasize that we do not observe a high density of magnons at the band bottom during the pumping, see Fig. 2f. Furthermore, we note that since the lowest frequency $f_{\text{min}}$ in our system fulfills $2f_{\text{min}} > f_p/2$, three-magnon scattering processes to the band bottom (where the condensation is expected) are prohibited.

In order to observe a notable amount of condensed magnon at the global energy minima $f_{\text{min}}$ at $k_x = \pm Q$, we increase the microwave pumping current further to $i_{rf} = 20 \text{ mA}$. We plot the band structure of the magnons during the pumping (Fig. 2g) and relaxation (Fig. 2h). Comparing Fig. 2g and Fig. 2h reveals how the energy redistribution occurs in the frequency-momentum space and how the parametrically injected magnons condense to the band bottom once the pumping is switched off.
Figure 2g demonstrates that the system under pumping possesses a strong redistribution of the energy to the entire spectrum, as discussed above. Once the pumping is switched off (Fig. 2h), most of the magnons start to condense at the two global minima of the magnon dispersion at \( k_x = \pm Q \), as will be discussed in the following.

To elaborate on how this cascade of nonlinear four-magnon scattering processes occur in the absence of pumping, we present the frequency spectrum of the microconduit in Fig. 3a corresponding to Fig. 2h. The condensation at the band bottom is evident as a peak with the highest amplitude. Additionally, the peaks at higher frequencies indicate avalanche-like multi-level scattering processes.

To further illustrate the details of these scattering mechanisms, in Fig. 3b we present the levels of the four-magnon scattering processes in which the numbers correspond to the frequencies of magnon peaks as displayed in the frequency spectrum in Fig. 3a. Although higher-level scattering processes also occur, for simplicity, we only discuss the first, second and one of the third levels of scattering which are indicated by blue, red and green arrows, respectively.

First, the parametrically generated magnons undergo a multi-level set of four-magnon scattering processes (see blue arrows). Next, each scattered pair of magnons undergoes a second four-magnon scattering process individually as illustrated by the red arrows. This sequence continues to higher levels. For example, the third order as displayed by green arrows, shows scattering towards both higher frequencies and the band bottom. Finally, the population of the magnons at the band bottom increases significantly, and due to their long lifetime, a condensate at the band bottom forms.

In order to investigate the speed of magnon thermalization upon switching off the pumping, we present in Fig. 3c the temporal evolution of the amplitude of the magnons at \( f_p/2 \) (blue curve) and condensed magnons (red curve). We now let the system relax for \( t_{\text{relax}} = 200 \text{ ns} \). This figure demonstrates that during the pumping, the number of parametrically injected magnons increases rapidly, whereas the number of magnons at the band bottom increases slowly. Once the pumping is switched off, the parametrically injected magnons decay rapidly within only \( t_{\text{decay}} = 5 \text{ ns} \). However, in an opposite manner, the number of magnons at the band bottom increases dramatically within \( t_{\text{rise}} \sim 5 \text{ ns} \) after the pumping has been switched off and only starts to decay afterwards. The thermalization time of approximately 5 ns that we observe is much faster than previous reports on bulk samples [8, 10]. This can be explained by the quantized spectrum of the system. In addition, the decay time of the condensed magnons at the band bottom is \( t_{\text{decay}} = 94 \text{ ns} \), which is shorter than the analytically calculated lifetime for linear (low amplitude) magnons at this spectral position (\( t_{\text{lifetime}} \sim 220 \text{ ns} \)). We believe that such a difference is caused by the three-magnon confluence process of the condensed magnons as shown by \( 2f_{\text{min}} \) in Fig. 3a, in addition to nonlinear scattering processes near the band bottom as discussed above, which can open extra channels for magnon dissipation [8, 25, 36].

It is important to mention that the condensate has a shorter lifetime and requires a longer thermalization time to form in a conduit with a larger width, see supplemental materials [34]. This is caused by the fact that the laterally larger conduit has a shallower band depth (smaller \( \Delta f \)) and, consequently, a smaller energy distance between the condensed states at the bottom and the excited states [34], in agreement with our initial prediction.

We now vary the pumping field amplitude to study systematically the threshold dependence of the dynamics in the system. Figure 3d represents the amplitude of parametrically generated magnons during pumping (red circles), the nonlinear frequency-nonconserving four magnon scattering processes during the pumping (blue triangles), and the amplitude of condensed magnons during relaxation (green triangles). At \( i_{\text{rf}} = 10 \text{ mA} \), the amplitude of the generated magnons at \( f_p/2 \) starts to grow exponentially with the microwave field amplitude. This point is the onset of the parametric magnon generation instability [37, 38]. Increasing the microwave current increases the amplitude of
the parametrically generated magnons. However, this trend changes at $i_{rf} = 11.5$ mA since a kink occurs in the growth rate of the parametrically generated magnons and their growth rate gets decreased by further increasing the current from this point, indicated by the black arrow [38-39]. This kink is caused by the onset of the frequency-nonconserving four magnon scattering processes which start to grow starting at this microwave current. More interestingly, the condensed magnon state also possesses a clear threshold, $i_{rf} = 13.5$ mA, which is above the threshold of the explained four-magnon scattering processes.

It is important to note that a higher density of the injected magnons eventually destroys the condensed state, since the amplitude of the magnon-magnon interactions will be large enough to overcome the distance between magnons in the condensed state to the excited modes, in agreement with the initial prediction, see supplemental materials [34].

In conclusion, we have proposed a new way to enhance the lifetime of nonequilibrium magnon condensates in magnetic thin films by lateral confinement. We revealed the role of dipolar interactions in the generation of a magnon BEC as a metastable state in YIG ultrathin film structures. We showed that increasing the density of the parametrically injected magnons leads to the formation of a gaseous state involving weakly interacting magnons. Such a highly excited system establishes a magnon condensation at the bottom of the lowest magnon mode (quantum ground state) during relaxation. Our results corroborate that the magnon condensation forms and its lifetime is enhanced if the two following criteria are addressed: $i)$ the density of the magnons is high enough to allow the onset of frequency-nonconserving four-magnon scattering processes and formation a magnon gas, and $ii)$ the band depth, i.e. the distance between the global minimum and the ferromagnetic resonance should be enhanced by lateral confinement. Our results determine that the thermalization time in a system with a diluted spectrum is much faster than in bulk samples with hybridized magnon mods, in agreement with recent observations [28]. Our study provides a new way to obtain magnon condensation phenomena in nanoscopic confined geometries, and will motivate further experimental efforts to observe this effect.

Acknowledgement--- This project is funded by the European Research Council within the Advanced Grant No. 694709 “SuperMagnonics”, the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) - TRR 173 - 268565370 (“Spin+X”, Project B01), and the Nachwuchsrising of the TU Kaiserslauern. A. Q. and A. B. are supported by the European Research Council via Advanced Grant No. 669442, “Insulatronics” and by the Research Council of Norway through its Centers of Excellence funding scheme, Project No. 262633, “QuSpin”. Fruitful discussions with Q. Wang and H. Yu. Musienko-Shmarova are appreciated.

Correspondence to: *mohseni@rhrk.uni-kl.de*

References

[1] K. B. Davis, M. O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Bose-Einstein Condensation in a Gas of Sodium Atoms, Phys. Rev. Lett. 75, 3969 (1995)

[2] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, E. A. Cornell, Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor, Science 269, 5221 (1995)

[3] L. V. Butov, A. L. Ivanov, A. Imamoglu, P. B. Littlewood, A. A. Shashkin, V. T. Dolgopolov, K. L. Campman, and A. C. Gossard, Stimulated scattering of indirect excitons in coupled quantum wells: signature of a degenerate Bose-gas of excitons. Phys. Rev. Lett. 86, 5608–5611 (2001).

[4] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szymańska, R. André, J. L. Staehli, V. Savona, P. B. Littlewood, B. Deveaud & Le Si Dang, Bose–Einstein condensation of exciton polaritons. Nature 443, 409–414 (2006).

[5] J. Klaers, J. Schmitt, F. Vewinger & M. Weitz, Bose–Einstein condensation of photons in an optical microcavity. Nature 468, 545–548 (2010).
[6] T. Giamarchi, C. Rüegg & O. Tchernyshyov, Bose–Einstein condensation in magnetic insulators, Nat. Phys. 4, 198–204 (2008)

[7] S. O. Demokritov, V. E. Demidov, O. Dzyapko, G. A. Melkov, A. A. Serga, B. Hillebrands & A. N. Slavin, Bose–Einstein condensation of quasi-equilibrium magnons at room temperature under pumping. Nature 443, 430–433 (2006)

[8] A. A. Serga, V. S. Tiberkevich, C. W. Sandweg, V. I. Vasyuchka, D. A. Bozhko, A. V. Chumak, T. Neumann, B. Obry, G. A. Melkov, A. N. Slavin & B. Hillebrands, Bose–Einstein condensation in an ultra-hot gas of pumped magnons, Nat. Comm. 5, 3452 (2014)

[9] A. V. Chumak, V. I. Vasyuchka, A. A. Serga & B. Hillebrands. Magnon spintronics. Nat. Phys. 11, 453–461 (2015)

[10] V. E. Demidov, O. Dzyapko, S. O. Demokritov, G. A. Melkov, and A. N. Slavin. Thermalization of a parametrically driven magnon gas leading to Bose–Einstein condensation. Phys. Rev. Lett. 99, 037205 (2007).

[11] S. M. Rezende, Theory of coherence in Bose-Einstein condensation phenomena in a microwave-driven interacting magnon gas, Phys. Rev. B 79, 174411 (2009)

[12] A. Rückriegel and P. Kopietz, Rayleigh-Jeans Condensation of Pumped Magnons in Thin-Film Ferromagnets, Phys. Rev. Lett. 115, 157203 (2015)

[13] A. A. Serga, A. V. Chumak, & B. Hillebrands, YIG magnonics. J. Phys. D 43, 264002 (2010).

[14] A. A. Serga, C. W. Sandweg, V. I. Vasyuchka, M. B. Jungfleisch, B. Hillebrands, A. Kreisel, P. Kopietz, and M. P. Kostylev, Brillouin light scattering spectroscopy of parametrically excited dipole-exchange magnons. Phys. Rev. B 86, 134403 (2012).

[15] S. M. Rezende, Wave function of a microwave-driven Bose-Einstein magnon condensate, Phys. Rev. B 81, 020414(R) (2010)

[16] T. Kloss, A. Kreisel, and P. Kopietz, Parametric pumping and kinetics of magnons in dipolar ferromagnets, Phys. Rev. B 81, 104308 (2010)

[17] Q. Wang, B. Heinz, R. Verba, M. Kewenig, P. Pirro, M. Schneider, T. Meyer, B. Lägel, C. Dubs, T. Brächer, and A. V. Chumak. Spin Pinning and Spin-Wave Dispersion in Nanoscopic Ferromagnetic Waveguides. Phys. Rev. Lett. 122, 247202 (2019)

[18] M. Mohseni, R. Verba, T. Brächer, Q. Wang, D. A. Bozhko, B. Hillebrands, and P. Pirro. Backscattering Immunity of Dipole-Exchange Magnonostatic Surface Spin Waves. Phys. Rev. Lett. 122, 197201 (2019)

[19] D. A. Bozhko, A. A. Serga, P. Clausen, V. I. Vasyuchka, F. Heusser, G. A. Melkov, A. Pomyalov, V. S. L’vov, B. Hillebrands, Supercurrent in a room-temperature Bose-Einstein magnon condensate. Nat. Phys. 12, 1027 (2016).

[20] D. A. Bozhko, A. J. E. Kreil, H. Y. Musienko-Shmarova, A. A. Serga, A. Pomyalov, V. S. L’vov & B. Hillebrands, Bogoliubov waves and distant transport of magnon condensate at room temperature, Nat. Comm. 10, 2460 (2019)

[21] Yu. M. Bunkov, E. M. Alakshin, R. R. Gazizulin, A. V. Klochkov, V. V. Kuzmin, V. S. L’vov, and M. S. Tagirov. High-Tc Spin Superfluidity in Antiferromagnets. Phys. Rev. Lett. 108, 177002 (2013)

[22] L. J. Cornelissen, K. J. H. Peters, G. E. W. Bauer, R. A. Duine, and B. J. van Wees. Magnon spin transport driven by the magnon chemical potential in a magnetic insulator. Phys. Rev. B 94, 014412 (2016)

[23] A. Rückriegel and P. Kopietz, Spin currents, spin torques, and the concept of spin superfluidity, Phys. Rev. B 95 104436 (2017)

[24] C. Ulloa, A. Tomadin, J. Shan, M. Polini, B. J. van Wees, and R. A. Duine. Nonlocal Spin Transport as a Probe of Viscous Magnon Fluids, Phys. Rev. Lett. 123, 117203 (2019)

[25] A. Qaiumzadeh, H. Skarsvåg, C. Holmqvist, and A. Brataas, Spin Superfluidity in Biaxial Antiferromagnetic Insulators, Phys. Rev. Lett. 118, 137201 (2017)

[26] I. S. Tupitsyn, P. C. E. Stamp, and A. L. Burin. Stability of Bose-Einstein Condensates of Hot Magnons in Yttrium Iron Garnet Films. Phys. Rev. Lett. 100, 257202 (2008)

[27] F. L., Wayne M. Saslow & V. L. Pokrovsky, Phase Diagram for Magnon Condensate in Yttrium Iron Garnet Film. Sci. Rep. 3, 1372 (2013)

[28] M. Schneider, T. Brächer, V. Lauer, P. Pirro, D. A. Bozhko, A. A. Serga, H. Yu. Musienko-Shmarova, B. Heinz, Q. Wang, T. Meyer, F. Heusser, S. Keller, E. Th. Papaioannou, B. Lägel, T. Löber, V. S. Tiberkevich, A. N. Slavin, C. Dubs, B. Hillebrands, and A.V. Chumak, Bose-Einstein condensation of quasi-particles by rapid cooling, arXiv:1612.07305 (2019)

[29] C. C. Bradley, C. A. Sackett, and R. G. Hulet, Bose-Einstein Condensation of Lithium: Observation of
Limited Condensate Number, Phys. Rev. Lett. 78, 985 (1997)

[30] H. Shi and W. M. Zheng, Bose-Einstein condensation in an atomic gas with attractive interactions, Phys. Rev. A 55, 2930 (1997)

[31] M. Ueda and A. J. Leggett, Macroscopic Quantum Tunneling of a Bose-Einstein Condensate with Attractive Interaction, Phys. Rev. Lett. 80, 1576 (1998)

[32] B. A. Kalinikos, A. N. Slavin. Theory of dipole-exchange spin wave spectrum for ferromagnetic films with mixed exchange boundary conditions. Journal of Physics C: Solid State Physics 19 (35), 7013 (1986)

[33] A. Vansteenkiste, J. Leliaert, M. Dvornik, M. Helsen, F. Garcia-Sanchez, and B. V. Waeyenberge, The design and verification of mumax3, AIP Advances 4, 107133 (2014).

[34] See supplemental materials

[35] L’vov, V. S. Wave Turbulence under Parametric Excitation Springer (1994).

[36] P. Clausen, D. A. Bozhko, V. I. Vasyuchka, B. Hillebrands, G. A. Melkov, and A. A. Serga, Stimulated thermalization of a parametrically driven magnon gas as a prerequisite for Bose-Einstein magnon condensation, Phys. Rev. B 91, 220402(R) (2015)

[37] M. Mohseni, M. Kewenig, R. Verba, Q. Wang, M. Schneider, B. Heinz, C. Dubs, A. A. Serga, B. Hillebrands, A. V. Chumak, P. Pirro, Parametric generation of propagating spin-waves in ultrathin yttrium iron garnet waveguides, Phys. Stat. Soli. RRL, 2000011, (2020)

[38] T. Brächer, P. Pirro, B. Hillebrands. Parallel pumping for magnon spintronics: Amplification and manipulation of magnon spin currents on the micronscale, Physics Reports 699, (2017)

[39] P. Pirro, T. Sebastian, T. Brächer, A. A. Serga, T. Kubota, H. Naganuma, M. Oogane, Y. Ando, and B. Hillebrands, Non-Gilbert-damping Mechanism in a Ferromagnetic Heusler Compound Probed by Nonlinear Spin Dynamics, Phys. Rev. Lett. 113, 227601 (2014)