Limits of computational white-light holography

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Abstract. Recently, computational holograms are being used in applications, where previously conventional holograms were applied. Compared to conventional holography, computational holography is based on imaging of virtual objects instead of real objects, which renders them somewhat more flexibility. Here, computational holograms are calculated based on the superposition of point sources, which are placed at the mesh vertices of arbitrary 3D models. The computed holograms have full parallax and exhibit a problem in viewing that we have called "ghosting", which is linked to the viewing of computational holograms based on 3D models close to the image plane. Experimental white-light reconstruction of these holograms showed significant blurring, which is explained here based on simulations of the lateral as well as the axial resolution of a point image with respect to the source spectrum and image distance. In accordance with these simulations, an upper limit of the distance to the image plane is determined, which ensures high quality imaging.

1. Introduction

Conventional holography has been applied in various forms such as off-axis holography, image plane holography and rainbow holography [1]. Each of these techniques requires rather sophisticated equipment and skilled experimentalists to achieve high quality results. All these methods require further "real" objects from which the hologram will be made. These objects can be simple binary masks, gray scale masks and/or even complex 3D models.

In this work we focus on computational holograms, which can in principle replace all hologram types mentioned above. The main advantage is that one is not limited by the real objects available to the experimentalists, but only to the imagination of the hologram designer. Any 2D object, e.g. a gray scale mask or a virtual 3D object might be suitable, but we focus here on holograms based on virtual 3D mesh models, which can be reconstructed with white light. The computation is carried out on a standard desktop computer, while the writing of the hologram as a phase profile is done by a laser writer or a spatial light modulator SLM of the type Holoeye Pluto-VIS.

The main difference in terms of reconstruction quality of these writing devices is the minimum writable pixel size. The limited spatial resolution of the holograms places restrictions on the object size, the object placement, the field of view and the imaging resolution. The latter is numerically analyzed in section 4 in great detail, while in section 3 an effect is described, which we have called "ghosting". The cause of the effect is described and two suggestions to reduce it are proposed. Before commenting however on the limiting factors of computational holography, we would like to give a brief description of the point source method, which is used to compute all the holograms in this work.
2. The point source method
Several complex methods to create computer-generated holograms have been proposed in literature [2,3], but we limited ourselves to an idea in close analogy to the one presented by Yoshikawa [4]. The virtual object is assumed to be represented by a point cloud, which has been created in this work based on the vertex coordinates of a 3D mesh model, see figure 1.

Each point of the virtual object is assumed to emit a spherical wave of unit amplitude,

\[ U_i(x_h, y_h) = \exp(ikr)/r, \quad r = ((x_h - x_i)^2 + (y_h - y_i)^2 + z_i^2)^{-1/2}, \]

(1)

where \(x_h\) and \(y_h\) denote the spatial coordinates in the hologram plane \((z_h = 0)\) and \(x_i, y_i, z_i\) denote the coordinates of the \(i\)-th model point. The spatial extent of the field in the hologram plane is limited by the following aliasing criteria:

\[
\begin{align*}
    x_h &\in \{x_i - |z_i| \cdot \tan(\eta_{\text{max}}), x_i + |z_i| \cdot \tan(\eta_{\text{max}})\}, \eta_{\text{max}} = \sin^{-1}(\lambda/2\Delta_x), \\
    y_h &\in \{y_i - |z_i| \cdot \tan(\theta_{\text{max}}), y_i + |z_i| \cdot \tan(\theta_{\text{max}})\}, \theta_{\text{max}} = \sin^{-1}(\lambda/2\Delta_y),
\end{align*}
\]

(2)

where \(\eta_{\text{max}}\) and \(\theta_{\text{max}}\) are the maximum first-order diffraction angles and \(\Delta_x\) and \(\Delta_y\) are the pixel sizes of the hologram in \(x\)- and \(y\)-direction, respectively. Obeying the aliasing criteria ensures that no spatially shifted copies of the object are reconstructed since this would degrade the image visibility and the image perception. Thus one should only calculate the field values within the inner square drawn in figure 2 for each point of the model. Please note however that obeying the aliasing criteria also leads to the fact that the aperture size of each point depends on its distance to the hologram plane. Thus, points closer to the hologram plane may be reconstructed less brightly than the ones further away from it due to the difference in aperture size.

If one obeys the aliasing criteria, the reconstruction shows a single object point, while ignoring it causes the formation of additional and spatially shifted image points as shown in figure 3. The top row of the figure shows two holograms of an object point placed at \(z_i = 90\)mm. The left one was created according to the aliasing criteria (2), while the right one was not. Below each hologram, one finds the corresponding numerical reconstruction at \(z_i = 85\)mm. The distance 85mm was chosen, since a
reconstruction at 90mm leads to point images with such a small extent that they a barely noticeable in the full field plot of the numerical reconstruction.

![Image](image1.png)

**Figure 3.** Holograms of an object point at $z_i = 90$mm without aliasing (top left) and with aliasing (top right) and the corresponding numerical reconstructions at 85mm (bottom row)

The total field in the hologram plane is obtained by superimposing the fields of all $N_p$ points of the virtual object and multiplying the sum by a reference field $U_r$,

$$U_h(x_h, y_h) = U_r^*(x_h, y_h) \sum_{i=1}^{N_p} U_i(x_h, y_h).$$  \hfill (3)

The field in the hologram is sampled at a resolution depending on the reconstruction device, e.g. at 8 micron if the SLM Holoeye Pluto-VIS is used for reconstruction, and the reference field may be given by a plane wave impinging under an angle $\theta$ with respect to the $x_h$-z-plane

$$U_r^*(x_h, y_h) = \exp[iky_h\sin(\theta)].$$  \hfill (4)

The angle $\theta$ should be chosen to be the opposite of the angle used during reconstruction, so that the reconstruction with a plane wave under this angle will lead to a cancelling of the reference field term and only the desired field that reconstructs the point sources will propagate. The reconstruction devices used in this work are only able to modify the phase of an impinging light wave and thus only the phase information $\phi_h$ can be explored

$$\phi_h(x_h, y_h) = \text{angle}[U_h(x_h, y_h)] = \tan^{-1}\left(\frac{\text{Re}[U_h(x_h, y_h)]}{\text{Im}[U_h(x_h, y_h)]}\right)$$  \hfill (5)

and the amplitude information at the hologram plane is ignored from this point on. Furthermore, the phase $\phi_h$ is approximated by a finite numbers of phase levels $p_{\text{lev}}$ in the range $[0,2\pi(1-1/p_{\text{lev}})]$. Thus the discrete phase values $\phi_{\text{ch}}$ may be computed by

$$\phi_{\text{ch}}(x_h, y_h) = \text{floor}(p_{\text{lev}} - 1) \cdot \phi_h(x_h, y_h)/2\pi.$$  \hfill (6)

The values of $\phi_{\text{ch}}$ are for example all integer values in the range $[0,255]$ for $p_{\text{lev}} = 256$, which can then be stored as a grey scale bitmap with 8-bit depth. This format is compatible to the used SLM as well as to the laser writer which was used. Equation (6) thus represents the final form of the computer-generated holograms and we can now turn our attention to a particular problem due to calculation and writing of $\phi_{\text{ch}}$ on a regular grid.
3. Self-ghosting due to higher order images

The cause of this problem will be explained using figure 4 (top), where the propagation of the holograms "zero-order field" created by +/-1st- order diffraction of a single image point is indicated by two solid arrows. The angle of each line with respect to the optical axis is given by \( \theta_{\text{max}} \) since the aliasing condition (2) is assumed to be obeyed.

If one observes the image point at a distance of 250mm, the linear field of view is 19.3mm wide for the particular point shown in figure 6 (top) and \( \Delta_x = 8\mu m \). In addition to the zero-order field, the holograms "+1st-order field" is indicated by two dashed arrows. The angles of these arrows are calculated by taking the sum of \( \pm \theta_{\text{max}} \) and the +1st-order diffraction angle of the hologram, which is 3.81° for \( \Delta_x = 8\mu m \) and \( \lambda = 532\text{nm} \). The hologram thus acts as a grating since it is written on a regular pitch \( \Delta_x \). Please note that the diffraction angle of the hologram is twice as large as the maximum angle \( \theta_{\text{max}} \approx 1.91° \) of the intended +/-1st-order diffraction for these parameters.

![Figure 4. Scheme to visualize self-ghosting of virtual image point](image)

One can see in figure 6 (top) that both fields intersect each other 7mm above the optical axis and the size of the overlap is approximately 2.7mm. The observer thus sees the holograms zero- and the first-order field simultaneously, an effect we have called "self-ghosting". Figure 6 (bottom) shows that the effect becomes less dominant with decreasing distance of the virtual image to the hologram plane (overlap of only 0.7mm), but due to the finite size of the observers pupil it is still visible. For real object points, the fields do not overlap, but again the effect might be observed due to observers finite pupil size. The effect can however be reduced by limiting the field of view or avoided by writing the hologram on a non-periodic grid. Further details and the description of two point ghosting are given in [5].

4. Imaging limitations of computational white-light holograms

The issue of imaging resolution will be discussed in the following based on a point image. In order to determine the resolution, one may first distinguish between lateral and axial resolution of an image. The latter will be related to the focal tolerance, but first the lateral resolution of a point image will be considered.

4.1. Lateral resolution

Kozacki treated in [6] the resolution question in the context of the Wigner domain and he presented a rather simple graph that showed the relation between lateral resolution, imaging distance and point position \( x_i \) in the image plane. Figure 5 was created in close analogy to figure four of his paper and it shows that the image of a point placed within the green cone will be reconstructed with full lateral resolution, while resolution and maximum intensity of a point placed outside the cone decreases linearly till the intersection of the image plane with the upper or lower arrow. The area enclosed by the arrows is the maximum range in which the object can be placed if resolution is of no importance. Our
holograms explore however the full resolution at the design wavelength $\lambda$ and thus the virtual objects are always placed within the green area of full resolution. This holds true for real and virtual imaging points. To determine the resolution of our holograms we used the $z$-position $z_{\text{max}}$ of the last point, which can be imaged with the holograms full aperture of width $2a$, which is given by

$$z_{\text{max}}(\Delta_x, a, \lambda) = a/\tan\eta_{\text{max}} \equiv 2a\Delta_x/\lambda,$$

for small ratios of $\lambda/A_{\text{min}} = \lambda/(2\Delta_x)$. It is the only point that is imaged with full resolution and full aperture of the hologram and this special point is selected for all results described below.

In agreement with [6] the resolution of a phase-only hologram of a point image is given in Fresnel approximation at the design wavelength by

$$I_1(x, z_1) = (2a/\lambda z_1)^2\text{sinc}^2(2ax_1/\lambda z_1).$$

In our case the full aperture is contributing to the image formation process and thus the relation $z_1 = z_{\text{max}}$ holds true, where $z_{\text{max}}$ is given by (7), which leads to

$$I_1(x_1, z_{\text{max}}) \approx (B_T)^2\text{sinc}^2(B_Tx_1), \quad B_T = 1/\Delta_x,$$

Equation (9) shows two interesting properties of the point image. First, the lateral resolution may be defined as $2\Delta_x$, if the distance between the two first zeros of the sinc function is used to quantify resolution (other definitions see table 1) and second, the resolution does not depend on the wavelength. However, the distance $z_{\text{max}}$ given by (7) varies with wavelength and thus also the size of the cone of full resolution. The cone is smaller for longer wavelengths and vice versa. Please note further that the hologram is designed for a specific wavelength $\lambda$ and the analytic results above only hold true for this particular wavelength.

### Table 1. Resolution values depending on determination criteria

| Criteria | FWHM   | $1/e^2$ | distance between first two zeros |
|----------|--------|---------|----------------------------------|
| Resolution $[\Delta_x]$ | 0.8859 | 1.4000  | 2.0000                           |

4.2. Lateral extent of a point image by white-light reconstruction

In the following it is assumed that the spatial resolution is limited, while the phase resolution is still unlimited as in the analytical results above. The phase values are usually sampled with 8bit resolution (256 values) in the range $[0, 2\pi]$ and thus the errors introduced by the finite phase resolution are assumed to be fully negligible here. The transfer function of the computational hologram $T_{\text{ch}}$ [7] is thus given by

$$T_{\text{ch}}(x_h, z_{\text{max}}, a) = \text{rect}(x_h/2a)\exp[i\phi_{\text{ch}}(x_h, z_{\text{max}})], \quad \phi_{\text{ch}}(x_h, z_{\text{max}}) = \text{mod}(-kx_h^2/2z_{\text{max}}, 2\pi)$$
where \( z_{\text{max}}(\Delta_y, \alpha) = \alpha / \tan \eta_{\text{max}} \) as defined above. The phase values of (10) are sampled with the resolution of the hologram \( \Delta_x \) and calculated at the center of each pixel and the reconstruction is performed numerically based on the angular spectrum decomposition method. One of the properties of this propagation method is that the image has the same spatial resolution as the hologram. As the simulations are performed for \( \Delta_x = 1, 2, \) and 8\( \mu \text{m} \) the spatial resolution of the image might be insufficient to determine the resolution. Each phase value is thus resampled on \( N \) points within one pixel without interpolation to achieve a resolution better or equal to 1\( \mu \text{m} \). The resampling is visualized in figure 6, where the phase value of the first pixel is resampled \( N \) times.

Figure 6. Scheme of the phase resampling

Figure 7. Measured spectrum of a halogen spot lamp

To model the decrease in resolution, it is assumed that the light source emits plane wavefronts with various wavelengths, which are totally independent of each other and normally incident on the hologram. Consequently, the intensity distribution \( I_{\text{phys}}(x_i, z_i, \Delta\lambda) \) at each imaging distance \( z_i \) is the incoherent sum of the wavelength-dependent intensity distributions \( I_i \) multiplied by the wavelength-dependent emission strength \( E_{\text{rel}}(\lambda) \) of the light source

\[
I_{\text{phys}}(x_i, z_i, \Delta\lambda) = \sum_{\lambda} E_{\text{rel}}(\lambda) \cdot I_i(x_i, z_i, \lambda).
\]  

(11)

The modeling is based on two different spectra, a simple uniform spectrum and the spectrum of a halogen spot lamp shown in figure 7. The halogen spot lamp was used in various experiments and its spectrum was measured using the spectrometer Thorlabs CCS100 with a resolution better than 0.5nm FWHM @ 435nm. Since halogen spot lamps are quite commonly used light sources in holography and the spectrum of the sun is in "first approximation" uniform, both cases are of practical interest.

Equation (11) takes into account the wavelength-dependent emission strength of the sources, which leads to the "physical" light distribution in space \( I_{\text{phys}} \); but to determine the observers imaging resolution, one must also take into account the wavelength-dependent sensitivity \( S_{\text{rel}}(\lambda) \) of the observer itself

\[
I_{\text{human}}(x_i, z_i, \Delta\lambda) = \sum_{\lambda} S_{\text{rel}}(\lambda) \cdot E_{\text{rel}}(\lambda) \cdot I_i(x_i, z_i, \lambda).
\]  

(12)

Figure 8 shows the relative sensitivity of the human eye, where the crosses mark the values given in [8] and the dots the linearly interpolated values in between them. One can infer from the plot that the human eye is most sensitive around 555nm, which is very close to the typical design wavelength \( \lambda = 532\text{nm} \), which was also used here together with \( \Delta_x = 1, 2 \) or 8\( \mu \text{m} \) and \( N = 1, 3 \) or 9. The reconstruction wavelength was varied within the range [400nm, 700nm] in steps of 1.0nm and the reconstruction distances were varied in 10 micron steps around \( z_{\text{max}} \).
Figure 8. Relative spectral sensitivity of the human eye

Figure 9. PSF @ \( z_{\text{max}} \) = 3.78mm

Figure 9 shows the result for \( z_{\text{max}} = 4 \)mm based on \( h_{\text{human}} \) evaluated at \( z_{\text{max}} = 3.78 \)mm. The choice of this particular imaging distance will become clear in the next section. One can infer from figure 9 that the point spread function does not decrease as rapidly to zero as the sinc\(^2\)-function in (9) and thus the FWHM and the \( 1/e^2 \)-criteria are used to determine the lateral resolution depending on the hologram resolution and imaging distance \( z_{\text{max}} \), see figures 10 and 11. The \( 1/e^2 \)-criteria is more conservative and probably also more accurate to describe the hologram’s imaging resolution since it agrees well with experimental observations.

Figure 10. Resolution based on FWHM criteria

Figure 11. Resolution based on \( 1/e^2 \) criteria

Both figures show that there is a monotonic decrease in resolution with increasing imaging distance \( z_{\text{max}} \) for human as well as physical observation. Compared to the theoretical resolution value of monochromatic reconstruction \( 2\Delta x \), the resolution of the reconstructions with white-light is always lower. The results show further that doubling the resolution of the hologram leads to a decrease in image resolution of about \( \sqrt{2} \) for white-light illumination. High resolution imaging (<70µm or >360dpi) is thus only possible for distances <3mm (image plane holograms), if the hologram's resolution is better or equal 2µm. Please note that in case of the uniform spectrum the resolution is about 10% less in the case of the \( 1/e^2 \)-criteria, but the trends are almost identical.

4.3. Axial extent of a point image by white-light illumination

According to Born and Wolf [8] the intensity distribution along the optical axis \( I(z) \) is proportional to

\[
I(z) = \text{sinc}^2(\frac{a^2 z}{2\lambda z_{\text{max}}}).
\]  

(13)

It is said in [8] that a decrease of 20% from the maximum intensity is permissible to determine the focal tolerance \( \Delta z \) and one may thus measures the axial resolution by defining the axial extent of the focal spot \( \Delta z' = 2\Delta z \). An estimation of the focal tolerance \( \Delta z \) is according to [8] given by

\[
\Delta z = \pm \lambda / 2(z_{\text{max}} / a)^2 \approx \pm 2\Delta x^2 / \lambda,
\]

(14)
if the relation $z_{\text{max}}(\Delta_x, a) \equiv 2a\Delta_x/\lambda$ is used as given by (7), which should hold true for monochromatic illumination at the design wavelength. To determine the decrease in axial resolution due to white-light illumination, the simulations of the previous subsection were used. Figure 12 shows the evaluation of the maximum intensity $I_{\text{max}}$ depending on the imaging distance $z_i$ using again the parameters of figure 9. One can see that the peak intensity is not located at $z_{\text{max}}$, but at a different position $z_{\text{max}}^{\text{max}}$ due to the white-light illumination. The amount depends on the source spectrum and can be easily corrected in the design process for a particular source since the shift is in good approximation linear with imaging distance as shown in figure 13. It also shows that the discrepancy between $z_{\text{max}}$ and $z_{\text{max}}^{\text{max}}$ as well as the focal spots axial extent increase linearly with imaging distance.

Please note that in the case of physical observation the discrepancy and the axial focal spot extent are approximately three times larger compared to the results shown in figure 13.

5. Conclusions

Holograms created by the point source method exhibit a problem in viewing due to writing on a regular grid, which we have termed “ghosting”. The effect can be reduced by limiting the field of view or avoided by writing on a non-periodic grid. Experimental hologram reconstructions with white-light showed significant blurring, which can be explained based on image point simulations.

Numerical results based on a light source with uniform spectrum and the one of a halogen spot lamp showed that the lateral resolution is close to theoretical values if the FWHM criteria is used to determine resolution. The more conservative $1/e^2$-criteria differs significantly from theory, but is in better agreement with experimental observations. The lateral extent of the image point increases in both cases linearly with distance and thus the resolution in dpi decreases proportional to $1/x$. Doubling the holograms resolution leads to a factor $\sqrt{2}$ reduction of the imaging resolution for white-light illumination.

The focal spot’s axial extent is related to the focal tolerance and increases linearly with distance. The position of maximum intensity differs from the design position, but the difference increases again linear with distance and thus corrected in the holograms design process for a particular light source.

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