Study on the strong decays of $\phi(2170)$ and a grand expectation for the future charm-tau factory

Hong-Wei Ke$^1$ and Xue-Qian Li$^2$

$^1$ School of Science, Tianjin University, Tianjin 300072, China
$^2$ School of Physics, Nankai University, Tianjin 300071, China

The present data imply that $\phi(2170)$ may not be an excited state of $\phi$, but is a four quark state with $s\bar{s}\bar{s}\bar{s}$ constituents. Furthermore, there are no two mesons of $s\bar{s}$ available to form a molecule which fits the mass spectrum of $\phi(2170)$, thus we suggest it should be an $s\bar{s}\bar{s}\bar{s}$ tetraquark state. In this scenario, we estimate its decay rates through the fall-apart mechanism. Our theoretical estimates indicate that its main decay modes should be $\phi(2170)$ into $f_{0}(980)$, $h_{1}\eta$, $h_{1}\eta'$, $K_{s}(1270)K$ and $K_{s}(1400)K$. Under this hypothesis the modes \( \phi(2170) \rightarrow K^{*}(890)K^{*}(890)\), $K^{+}K^{-}$ and $K_{S}^{0}K_{S}^{0}$ should be relatively suppressed. Since the width of $h_{1}$ is rather large, at present it is hard to gain precise data on $BR(\phi(2170) \rightarrow h_{1}\eta)$ and $BR(\phi(2170) \rightarrow h_{1}\eta')$ whose measurements may be crucial for drawing a definite conclusion about the inner assignment of $\phi(2170)$. We lay our expectation to the proposed charm-tau factory which will have much larger luminosity and better capacities.

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I. INTRODUCTION

Very recently a meson $\phi(2170)$ comes into the view of researchers because it may be a special exotic state. It is observed via its decay into $\phi + f_{0}(980)$ [9] meanwhile some possible final states $K^{0}K^{+}\pi^{0}$ and $K^{*0}K^{*0}$ are not seen. If it were a normal meson i.e. an excited state of $\phi$ the decay portals into $K^{*}K^{-}$ and $K_{S}^{0}K_{S}^{0}$ would be preferred as $\phi$ does. Moreover, even the channel $K^{*0}K^{*0}$ should also be seen since a sufficient phase space is available. Furthermore in Ref.[10] the theoretical evaluation on the total width obviously conflicts to data if $\phi(2170)$ is a normal meson. A reasonable interpretation is needed. It is suggested that the observed $\phi(2170)$ could be a molecular state of $\Lambda\bar{\Lambda}$ or a tetraquark state [11]. In reference [12] the author thinks that $\phi(2170)$ is an excited $q\bar{q}s\bar{s}$ tetraquark ($q = u, d$). But this assignment is questionable because no ground $q\bar{q}s\bar{s}$ tetraquark has ever been observed.

Being hinted by the decay mode $\phi(2170) \rightarrow f_{0}(980)$, a natural conjecture is that $\phi(2170)$ may be a four quark state with $s\bar{s}\bar{s}\bar{s}$ constituents. There are two choices: a molecular state or a tetraquark. However, there are no two mesons with $s\bar{s}$ constituents available to form a molecular hadron which fits the mass spectrum of $\phi(2170)$, thus we turn to suggest that it is an $s\bar{s}\bar{s}\bar{s}$ tetraquark state. This conjecture was also considered by the authors of Ref.[14, 15].

At the end of last century a stimulating question was raised: did multiquark states indeed exist in nature, because in the primary paper Gell-Mann predicted them along with the simplest assignments of $q\bar{q}$ for mesons and $qq$ for baryons [16]. The first proposed pentaquark of $qqq\bar{s}$ with unusual $B = 1$ and $S = 1$ quantum numbers would definitely be a multiquark state. In that assignment except $\bar{s}$, all other quarks are light ones (u or d types). The passion of detecting such pentaquarks was very high, however, after hard and desperate search, such pentaquarks were never observed experimentally. The despair discourages researchers who decided to give up. But following conduction of more accurate experiments and innovated skills of analysis, many exotic mesons have been measured. They are proposed to be four-quark states (molecular states or tetraquarks states) [17–26], later two pentaquarks were observed by the LHCb collaboration [27]. It validates the suggestion about existence of multi-quark states. However, we have observed that all the discovered multi-quark states contain at least one heavy quark ($c$ or $b$). This may hint that the existence of heavy quarks in the multi-quark states is fatal [28]. Is that the conclusion of the story? $\phi(2170)$ which comes into our attention recently, could be identified as a four quark state with $s\bar{s}\bar{s}\bar{s}$ constituents. Even though it is true, the early allegation might not be completely subverted because the mass of $s$-quark resides between that of very light quark and the supposed “heavy” charm quark and the rest constituents in the exotic state are all $s$-flavor ($\bar{s}$) with “middle” mass.

A naive analysis may provide us a support about this conjecture. The masses of $\Omega$ and $\phi$ which consist of three $s$ quarks and an $\bar{s}$ quark respectively, are 1672 MeV and 1020 MeV. It implies the $s$-quark mass to be around 500–600 MeV, thus a simple estimate on the mass of $s\bar{s}\bar{s}\bar{s}$ tetraquark state should fall in a region close to the mass of $\phi(2170)$. If the assignment is true $\phi(2170)$ is indeed a tetraquark with a single flavor of strangeness.

No doubt, it is absolutely important to get a better understanding about the inner structure of $\phi(2170)$. Since it only possess $s$ flavor, its decays would be dominated by the modes where the final states mainly contain $s$ flavors. Let us turn to investigate the mechanism which governs the strong decay of $\phi(2170)$. It is the so called “fall-apart” mechanism [29, 30].

In Refs.[29, 30] they suggested that the fall-apart mechanism induces the main decay modes of the tetraquark state. By this mechanism the constituents in a tetraquark are rearranged into two color singular pairs by exchanging soft gluons and then simply fall apart into two mesons. In this work we will employ this mechanism to study the decays of $\phi(2170)$.

This paper is organized as follows: after the introduction, in section II we will explore the decays of $\phi(2170)$. Section III is devoted to our conclusion and discussions.
II. FALL-APART DECAYS OF $\phi(2170)$

1. Since $\phi(2170)$ of $J^P = 1^-$ is supposed to be an $s\bar{s}s\bar{s}$ tetraquark which is in a diquark-antidiquark configuration, its spin state is

$$|J, J_{12}, J_{34}| = |1, 1, 1|,$$

where $J$ is the spin of the tetraquark $s\bar{s}s\bar{s}$, $J_{12}$ is the spin of $s\bar{s}$ and $J_{34}$ is the spin of $s\bar{s}$. The orbital angular momentum between diquark and anti-diquark is 1, i.e. in $p$-wave for guaranteeing the parity to be negative. Moreover, since the $C$-parity of $\phi(2170)$ is odd so the spin configuration of the tetraquark is fully determined as

$$|1, 1| = \frac{1}{\sqrt{2}}(|1, 1, 0, 0⟩ - |1, 0, 1, 1⟩).$$

The color configuration is $[1, 1, 1, 1]$ which can be written as $[30]$:

$$\frac{1}{\sqrt{48}}R_{133f}R_{03f}^{a}\bar{s}^{b}\bar{d}^{c}d^{d}(\bar{s}\bar{s}\bar{s}\bar{s}).$$

Note, the spin configuration of the tetraquark $s\bar{s}s\bar{s}$ is in the diquark and antidiquark spin bases. When it decays via the fall-apart mechanism, one needs to switch a pair quark-anti-quark around and rearrange their spins and colors to make proper combinations for the two mesons in the final state.

2. Now let us study the decay of $\phi(2170)$ via the fall-apart mechanism. Apparently the two-body final states with $s$ wave is preferred if it is allowed. Since the $J^PC$ of $\phi(2170)$ is $1^{--}$ tetraquark $s\bar{s}s\bar{s}$ can fall apart into two mesons with the quantum number assignments as $1^{--}$ and $0^{++}$ or $1^{++]$ and $0^{+-}$

$$|1, J_2⟩ = \frac{1}{\sqrt{2}}(|1, m⟩|0, 0⟩|24⟩ + |0, 0⟩|1, m⟩|24⟩ + |1, m⟩|0, 0⟩|23⟩ + |0, 0⟩|1, m⟩|23⟩),$$

with $J_2 = m$.

$\phi(2170)$ can also fall apart into two mesons with the quantum numbers $1^{++}$ and $1^{--}$

$$|1, J_2⟩ = \frac{1}{\sqrt{2}}(|1, m_{13}⟩|1, m_{13}⟩|13⟩|1, m_{24}⟩ + |1, m_{14}⟩|0, 0⟩|23⟩ + |0, 0⟩|1, m_{14}⟩|23⟩),$$

with $J_2 = m$.

with $J_{13} = J_1 + J_2$, $J_{24} = J_2 + J_4$, $m_{13}$ and $m_{24}$ are their projections along $Z$-axis, while $J_c = m_{13} + m_{24}$. $C_{m_{13}m_{24}}$ and $C_{m_{14}m_{23}}$ are corresponding C-G coefficients.

One also notices: the $I^G$ of $\phi(2170)$ is $0^-$, for such strong OZI-allowed decays the two final mesons should more favorably be in $J^G = 0^-$ and $0^+$ respectively, of course, the combination of $1^- 1^-$ could also work, but naively may be suppressed (further discussion will be presented in the last section). This analysis advocates the final states $\phi(1020)f_0(980)$, $\phi(1020)f_0(500)$, $\phi(1680)f_0(500)$, $\omega(782)f_0(980)$, $\omega(782)f_0(500)$, $\omega(1420)f_0(500)$, $\omega(1650)f_0(500)$, $h_{1}(1170)\eta$, $h_{1}(1170)^+$, $\omega(782)f_1(1285)$ which satisfy all the constraints from matching concerned quantum numbers.

3. In the simple quark model $(s\bar{s})_{1^-}$ can be decomposed into $c_1\phi(1020) + c_2\omega(782)$ where the values of $c_1 \approx 1$ and $c_2 \approx 0$ are estimated by fitting the decay rates of $\phi(1020)$ $\rightarrow K^+K^-$ and $\phi(1020)$ $\rightarrow \pi^+\pi^-$. In this picture, $\omega$ only contains a very tiny fraction of strange flavor, thus those modes involving $\omega(782)$ in the aforementioned channels would have a very small probability to occur via the fall-apart mechanism directly but the channel $\phi(2170) \rightarrow f_0(980)\omega$ still has a chance to be measured, which we will discuss latter.

If $f_0(980)$ and $f_0(500)$ are two normal mesons $\{32, 30\}$, the decomposition follows $(s\bar{s})_{0^+} = c'_1f_0(980) + c'_2f_0(500)$. Moreover, another relation is $(s\bar{s})_{1^-} = c''_1\eta + c''_2\eta'$. For the $1^-$ quantum system, the only candidate is $(s\bar{s})_{1^-} = h_{1}(1170)$. With those decompositions we may estimate the corresponding decay rates of $\phi(2170)$ into the final products involving those mesons via the fall-apart mechanism.

It is widely accepted that if the fall-apart mechanism exists, the dominant decay processes should be determined via this mechanism. Thus we can estimate the decay rates of $\phi(2170)$ roughly by inputting the coefficients of relevant decompositions and the relations are listed in the following table.

Relevant factors for the decays of $\phi(2170)$ are listed in Tab. 4. There exists an unknown factor $g_{FA}$ which is the parameter corresponding to the fall-apart mechanism, and it should be universal for all the processes.

At present, accurate values of the coefficients $c'_1, c'_2, c''_1$ and $c''_2$ cannot be obtained from data, because so far there are no measurements with sufficient precision on the relevant processes available yet. However, we can make rough estimates using the information we have so far. That is what we are doing below.

4. $\phi \rightarrow f_0(980)\gamma$ exists, but $\phi \rightarrow f_0(500)\gamma$ does not $\{31\}$, the fact implies $c'_1 \approx 1$. Of course, a possibility is that $f_0(500)$ is a rather wide resonance, such a radiative decay would be hard to observe. Anyhow, one can roughly assert that the channel $\phi(2170) \rightarrow f_0(500)$ may be of small probability to be found and then we set $c'_1 = 1$.

Both $\Gamma(D_s \rightarrow \eta \pi^+)$ and $\Gamma(D_s \rightarrow \eta \pi^+)$ have been measured, and one can obtain the ratio of the rates of the two channel as $\Gamma(D_s \rightarrow \eta \pi^+)/\Gamma(D_s \rightarrow \eta \pi^+) = 2.32$. Taking account the phase space difference $0.82$, we obtain the ratio $c''_2/c''_1 = 1.68$. Then we can use the ratio and the required normalization condition $|c''_1|^2 + |c''_2|^2 = 1$ to determines the modules of coefficients $c''_1$ and $c''_2$. The advantage of using...
Easily integrate the width over the phase space factor relative phase between achieved. However, in this scenario, we cannot determine the billetes into u. We can use the data of experimental errors. Finally, the ratio instead of the widths enables us to avoid some experimental errors. Finally \( |c_2^*| = 0.89 \) and \( |c_2^*| = 0.51 \) are achieved. However, in this scenario, we cannot determine the relative phase between \( c_1^* \) and \( c_2^* \). Using these values, an estimate on the ratios is made as: 
\[
\Gamma(\phi(2170) \to \phi f_0(980)) = \Xi(\phi(2170) \to h_1 \eta) = 1 : 0.4 : 0.4.
\]
We suggest to experimentally search the two channels \( \phi(2170) \to h_1 \eta \) and \( \phi(2170) \to h_1 \eta \) because they do have substantial branching ratios and should be “seen” according to our prediction.

Even though \( \omega f_0(980) \) cannot be produced via the final-state interaction, namely the charm-quark turns into \( s \) and \( u \), and the spectator \( s \) joins the produced \( s \) quark, thus the \( s \) quark annihilates into \( u \) or \( d \). Due to the similarity, phenomenologically, we can use the data of \( D_s \to \omega \pi^+ \) and \( D_s \to \omega \pi^+ \) to predict the width of \( \phi(2170) \to \omega f_0(980) \). Using the ratio 
\[
\Gamma(D_s \to \omega \pi^+) / \Gamma(D_s \to \phi \pi^+) = 0.053 \text{ and taking the corresponding phase factors into account, we have } 
\]
\[
\Gamma(\phi(2170) \to \omega f_0(980)) = \Xi(\phi(2170) \to \phi f_0(980)) \approx 0.068 : 1.
\]

Along the same line, since there are no valence \( u \) or \( d \) components in the tetraquark \( s \bar{s}s \), \( \psi(2170) \) cannot fall apart into \( K(1270)K \) or \( K(1400)K \). In order to produce \( K(1270)K \) or \( K(1400)K \) an \( s \bar{s} \) pair in tetraquark annihilates into \( u \bar{u} \) or \( d \bar{d} \). The leading Feynman diagram is Fig. 2. The color and spin factors are presented in Tab. The production process is somehow similar to \( \phi(2170) \to \phi f_0(980) \), but is suppressed by \( \alpha_s \). For \( c \bar{c} \) system \( \alpha_s \) is about 0.39, whereas for the \( s \bar{s} \) case \( \alpha_s \) may be slightly larger, but the suppression exists. Moreover, there are twice-color matching (at initial and final sides), thus an extra factor \( g_{FA} \) is introduced.

7. If we set \( \alpha_s \sim 0.5 \) and \( g_{FA} \sim 1 \) we expect \( \Gamma(\phi(2170) \to K(1270)K) \). In terms of \( \Gamma(\chi(4S) \to \gamma(1S) \pi \pi) \), we estimate \( \Gamma(\phi(2170) \to K(1270)K) \) to be close to \( \Gamma(\phi(2170) \to K(1400)K) \). We would ask whether \( K^* (890)^0 K^* (890)^0 \) can be experimentally measured? Since the relative orbital angular momentum between the daughter mesons is \( l = 1 \), then since the reactions occur near the threshold, the 3-momentum is small, thus the \( p \)-wave suppression would remarkably reduce the production rate, comparing to \( s \)-wave case. A rough estimate of the suppression factor is \( \frac{p^3}{4m} \sim 0.08 \). Moreover, to take into account additional factors which may affect evaluation, we adopt the suppression factor for the \( p \)-wave using the data \( \Gamma(\chi(2S) \to \eta \gamma) \) and \( \Gamma(\chi(2S) \to \gamma(1S) \pi \pi) \) and where the three-momentum of finial mesons is close to that in \( \phi(2170) \to K^* (890)^0 K^* (890)^0 \).

8. For other \( p \)-wave decays of \( \phi(2170) \) into \( h_1 \eta \) and \( h_1 \eta^* \) incorporating the phase factors we estimate \( \Gamma(\phi(2170) \to h_1 \eta) \): 
\[
\Gamma(\phi(2170) \to h_1 \eta^*) = 0.015 : 1.
\]

9. \( f_0(980) \) may also considered as a molecular state of a tetraquark, if so, the picture would be slightly different and the relevant Feynman diagram is shown in the following figure. The ratio \( \Gamma(\phi(2170) \to f_0(980)) \) is suppressed by \( \alpha_s \) comparing to the aforementioned case where \( f_0(980) \) is supposed to be a normal meson. Now the ratio \( \Gamma(\phi(2170) \to f_0(980)) = \frac{\Gamma(\phi(2170) \to h_1 \eta)}{\Gamma(\phi(2170) \to h_1 \eta)} \) would be close to 1:1.6:1.6. However, as well understood, \( f_0(980) \) may be a mixture of \( s \bar{s} \) state and a multi-quark state, thus according to our estimate, one can roughly evaluate the fraction of each constituent, and it would answer a long standing question about the identity of \( f_0(980) \). Obviously precise measurement on \( \phi(2170) \to f_0(980) \) would be very helpful.

10. At last we can estimate the results if \( \phi(2170) \) is an excited \( q \bar{q}s \) tetraquark, \( q = u, d \) as suggested. Naturally, the following decay portals would dominate the total width of

| Table I: Some factors for the decay \( \phi(2170) \to \) two mesons |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| decay mode      | \( \phi f_0(980) \) | \( \phi f_0(500) \) | \( \h_1 \eta \) | \( h_1 \eta^* \) | \( \phi \eta \) | \( \phi \eta^* \) | \( \omega f_0(980) \) | \( K_1(1270)K \) | \( K_1(1400)K \) |
| color factor    | \( \sqrt{2} \)   | \( \sqrt{2} \)   | \( \sqrt{2} \)   | \( \sqrt{2} \)   | \( \sqrt{2} \)   | \( \sqrt{2} \)   | \( \sqrt{2} \)   | \( \sqrt{2} \)   | \( \sqrt{2} \)   |
| spin factor     | \( c_1' \)       | \( c_1' \)       | \( c_1'' \)      | \( c_1'' \)      | \( c_1'' \)      | \( c_1'' \)      | \( c_1'' \)      | \( c_1'' \)      | \( c_1'' \)      |
| flavor space factor \( ^{a} \) | 0.0036          | 0.0056          | 0.0054          | 0.0018          | 0.0062          | 0.0038          | 0.0053          | 0.0050          | 0.0041          |

\( ^{a} \) The partial decay width is \( \Gamma = \frac{\sqrt{2}}{2\pi M^2} |c|^2 \), where \( M \) is the hadronic transition amplitude. Supposing it is irrelevant to the solid angle, one can easily integrate the width over the phase space factor \( \frac{1}{4\pi} \).
the fall apart mechanism. In this case we expect \( \phi(2170) \) to be a four-quark state and needs further verification by both of theoretical calculations and experiment, we suggested that a destructive interference between the molecular state and tetraquark suppressed the decay \( \phi(2170) \) to \( K_1(1270)K \) only receive a p-wave suppression, but not color rearrangement suppressions, different from the aforementioned case.

III. DISCUSSIONS AND CONCLUSION

With the study on the multi-quark structures stepping deeper and deeper, many unanswered puzzles in this stimulating field have emerged, namely sharp contradiction between theoretical prediction and experimental observation reminds us that our understanding of the exotic hadrons is far away from satisfaction. For example, many theoretical models confirm existence of \( X(5568) \), however, all experimental collaborations offered negative reports [40–43] except the D0 collaboration [44]. To compromise the contradiction between theory and experiment, we suggested that a destructive interference between the molecular state and tetraquark suppressed the concerned decay portals [45]. Indeed, it is a bold conjecture and needs further verification by both of theoretical calculations and more accurate experimental measurements.

The decay modes of \( \phi(2170) \) imply that the assignment of being an excited state of \( \phi \) is disfavored. Some authors suggested that it should be an exotic state. More concretely, its mass and decay behaviors hint that it may be an \( s\bar{s}s\bar{s} \) tetraquark state. Such a structure is special because it may decay via the so-called fall-apart mechanism into hadrons which possess dominantly strange constituents. Employing the fall apart mechanism we estimate the decay modes of \( \phi(2170) \) which are supposed to be its dominant portals. If \( f_0(980) \) is a simple meson with \( s\bar{s} \) structure, our estimate show that \( \Gamma(\phi(2170) \to f_0(980)) \approx 0.31 : 1 \). In this case \( \Gamma(\phi(2170) \to K^+(890)^0\bar{K}^-(890)^0) \), \( \Gamma(\phi(2170) \to K^+K^-) \) and \( \Gamma(\phi(2170) \to K_1^0\bar{K}_2^0) \) are suppressed by about two orders comparing with \( \Gamma(\phi(2170) \to f_0(980)) \).

If \( f_0(980) \) is a four-quark state, the decay \( \phi(2170) \to f_0(980) \) is suppressed and the ratio \( \Gamma(\phi(2170) \to f_0(980)) : \Gamma(\phi(2170) \to \phi(2170) \to h_1\eta) : \Gamma(\phi(2170) \to h_1\eta') \approx 1:1.6:1.6 \).

Supposing \( \phi(2170) \) is an excited \( q\bar{q}ss \) tetraquark (\( q = u, d \)) \( \phi(2170) \to f_0(500), \phi(2170) \to K_1(1270)K \) and \( \phi(2170) \to K_1(1400)\bar{K} \) is expected to be the main decay channels. Even though \( \phi(2170) \to K^+(890)^0\bar{K}^-(890)^0 \), \( \phi(2170) \to K^+K^- \) and \( \phi(2170) \to K_1^0\bar{K}_2^0 \) are p-wave suppressed modes, since the three-momenta for these channels are not too small, they should be observed experimentally.

Along with all other subjects in the hadron physics, a better understanding of the exotic state structure and their production and decay mechanisms are badly needed. We all know that the fundamental theory of strong interaction is QCD, however, the non-perturbative QCD which governs the hadron physics is still not understood yet, so that various reasonable phenomenological models are adopted by researchers. The study on exotic state may help us to gain more information about quark model and non-perturbative QCD. As discussed in the text, \( \phi(2170) \) is a special case worth of concern.

We suggest to measure all decay modes of \( \phi(2170) \) because the data will inform us of its assignment. If the data can decide it to be an \( s\bar{s}s\bar{s} \) tetraquark, just as we mentioned in the introduction, existence of multi-quark states with only \( s \)-flavor which is not very heavy is confirmed, and our scope would be widened.

From our discussion, one can notice that to gain more solid knowledge on the structure of exotic states and concerned dynamics is not easy because many inputs adopted in the computations possess large errors. It means that accurate data are the precondition for drawing definite conclusions. So far, the available facilities cannot offer data with satisfactory accuracy in the energy range of charm, thus we lay hope on the future charm-tau factory which is planned to be built in China. Since the luminosity of the new facility would be enhanced by several orders than that of BEPC II, and some new detection technique will be used, we can be optimistic that the quality of data will be much improved and the statistics can reach a very high level. Then, we may renew our computations based on the more accurate data and draw definite conclusion not only about \( \psi(2170) \) but also many four-quark states and pentaquarks.

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