Controlled Secure Direct Communication by Using GHZ Entangled State

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We present a controlled secure direct communication protocol by using Greenberger-Horne-Zeilinger (GHZ) entangled state via swapping quantum entanglement and local unitary operations. Since messages transferred only by using local operations and a public channel after entangled states are successfully distributed, this protocol can protect the communication against a destroying-travel-qubit-type attack. This scheme can also be generalized to a multi-party control system.

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The quantum key distribution (QKD) is an ingenious application of quantum mechanics, in which two remote legitimate users (Alice and Bob) establish a shared secret key through the transmission of quantum signals. Since the pioneering work of Bennett and Brassard was published in 1984 [1], different quantum key distribution protocols have been presented [2, 3, 4, 5, 6]. Different from the key distribution protocol, some quantum secure direct communication (QSDC) protocols, which permit important messages to be communicated...
directly without first establishing a random key to encrypt them, have been shown recently [7, 8, 9, 10].

Beige et al. [2] have proposed that messages can be read out only after the transmission of an additional piece of classical information for each qubit. Deng et al. [8] have proposed a two-step quantum direct communication protocol using Einstein-Podolsky-Rosen (EPR) pair. It was shown to be probably secure. Nguyen [9] has proposed an entanglement-based protocol for two people to simultaneously exchange their messages. However, in all these secure direct communication schemes, it is necessary to send the qubits carrying secret messages in a public channel. Therefore, Eve (eavesdropper) can attack the qubits in transmission.

Very recently, Man et al. [10] have proposed a new deterministic secure direct communication protocol by using swapping quantum entanglement [11, 12, 13, 14] and local unitary operations. This communication protocol can be used to transmit securely either a secret key or a plain text messages. In this paper, we present a controlled secure direct communication by using GHZ entangled state via swapping quantum entanglement and local unitary operations. We show that the communication is secure under some eavesdropping attacks [15, 16] and that two users can transmit their secret messages securely.

Let us first describe the quantum entanglement swapping simply. Let $|0\rangle$ and $|1\rangle$ be the horizontal and the vertical polarization states of a photon, respectively. Then, the four Bell states $|\psi^\pm \rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ and $|\phi^\pm \rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ are maximally entangled states in the two-photon Hilbert space. Let the initial state of two-photon pairs be the product of any two of the four Bell states, such as $|\psi_{12}^+\rangle$ and $|\psi_{34}^+\rangle$. Then, after Bell measurements on the photons 1 and 3 pair and the photons 2 and 4 pair, since the equations

$$|\psi_{12}^+\rangle \otimes |\psi_{34}^+\rangle = \frac{1}{2}(|\psi_{13}^+\rangle|\psi_{24}^+\rangle - |\psi_{13}^-\rangle|\psi_{24}^-\rangle + |\phi_{13}^+\rangle|\phi_{24}^-\rangle - |\phi_{13}^-\rangle|\phi_{24}^+\rangle),$$

$$|\psi_{12}^+\rangle \otimes |\psi_{34}^-\rangle = \frac{1}{2}(|\psi_{13}^+\rangle|\psi_{24}^-\rangle - |\psi_{13}^-\rangle|\psi_{24}^+\rangle - |\phi_{13}^+\rangle|\phi_{24}^+\rangle + |\phi_{13}^-\rangle|\phi_{24}^-\rangle),$$

$$|\psi_{12}^+\rangle \otimes |\phi_{34}^+\rangle = \frac{1}{2}(|\psi_{13}^+\rangle|\phi_{24}^+\rangle - |\psi_{13}^-\rangle|\phi_{24}^-\rangle + |\phi_{13}^+\rangle|\psi_{24}^-\rangle - |\phi_{13}^-\rangle|\psi_{24}^+\rangle),$$

$$|\psi_{12}^+\rangle \otimes |\phi_{34}^-\rangle = \frac{1}{2}(|\psi_{13}^+\rangle|\phi_{24}^-\rangle - |\psi_{13}^-\rangle|\phi_{24}^+\rangle - |\phi_{13}^+\rangle|\psi_{24}^+\rangle + |\phi_{13}^-\rangle|\psi_{24}^-\rangle),$$

obviously hold, one can see that there is an explicit correspondence between a known initial state of two qubit pairs and its swapped measurement outcomes. In addition, it is easily
verified that the four Bell states can be transformed into each other by some unitary operations. Then, encodings of secret messages can be imposed on the Bell states by using the local unitary operations, i.e., \( \hat{I} |\psi^+_{34}\rangle = |\psi^+_{34}\rangle \), \( \hat{\sigma}_x |\psi^+_{34}\rangle = |\phi^+_{34}\rangle \), \(-i\hat{\sigma}_y |\psi^+_{34}\rangle = |\phi^-_{34}\rangle \), and \( \hat{\sigma}_z |\psi^+_{34}\rangle = |\psi^-_{34}\rangle \). We assume that each of the above four unitary operations corresponds to a two-bit encoding, i.e., \( \hat{I} \) to 00, \( \hat{\sigma}_x \) to 01, \(-i\hat{\sigma}_y \) to 10, and \( \hat{\sigma}_z \) to 11. Taking advantage of the quantum entanglement swapping and the assumption of the two-bit codings, we can propose a controlled secure direct communication by GHZ entangled state protocol.

Let us turn to depicting our controlled secure direct communication protocol. First, we write the eight GHZ entangled state basis in two different bases as follows:

\[
|\psi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{htc}
\]

\[
= \frac{1}{2}[[+)_h(|+\rangle_t|+\rangle_c + |−\rangle_t|−\rangle_c) + |−\rangle_h(|+\rangle_t|−\rangle_c + |−\rangle_t|+\rangle_c]], \quad (5)
\]

\[
|\psi_2\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)_{htc}
\]

\[
= \frac{1}{2}[[+)_h(|−\rangle_t|+\rangle_c + |+\rangle_t|−\rangle_c) + |−\rangle_h(|+\rangle_t|+\rangle_c + |−\rangle_t|−\rangle_c)], \quad (6)
\]

\[
|\psi_3\rangle = \frac{1}{\sqrt{2}}(|100\rangle + |001\rangle)_{htc}
\]

\[
= \frac{1}{2}[[+)_h(|+\rangle_t|+\rangle_c + |−\rangle_t|−\rangle_c) + |−\rangle_h(|+\rangle_t|−\rangle_c + |−\rangle_t|+\rangle_c)], \quad (7)
\]

\[
|\psi_4\rangle = \frac{1}{\sqrt{2}}(|100\rangle - |001\rangle)_{htc}
\]

\[
= \frac{1}{2}[[+)_h(|−\rangle_t|−\rangle_c + |+\rangle_t|−\rangle_c) + |−\rangle_h(|+\rangle_t|+\rangle_c + |−\rangle_t|+\rangle_c)], \quad (8)
\]

\[
|\psi_5\rangle = \frac{1}{\sqrt{2}}(|010\rangle + |101\rangle)_{htc}
\]

\[
= \frac{1}{2}[[+)_h(|+\rangle_t|+\rangle_c + |−\rangle_t|−\rangle_c) + |−\rangle_h(|+\rangle_t|−\rangle_c + |−\rangle_t|+\rangle_c)], \quad (9)
\]

\[
|\psi_6\rangle = \frac{1}{\sqrt{2}}(|010\rangle - |101\rangle)_{htc}
\]
\[
= \frac{1}{2} \left[ |+\rangle_h (|+\rangle_t |c \rangle - |c \rangle |+\rangle_t) + |c \rangle (|+\rangle_t |+\rangle_c - |+\rangle_t |c \rangle \right],
\]
(10)

\[
|\psi_7\rangle = \frac{1}{\sqrt{2}} (|110\rangle + |001\rangle)_{htc}
\]
\[
= \frac{1}{2} \left[ |+\rangle_h (|+\rangle_t |c \rangle - |c \rangle |+\rangle_t) + |c \rangle (|+\rangle_t |c \rangle - |c \rangle |+\rangle_t) \right],
\]
(11)

\[
|\psi_8\rangle = \frac{1}{\sqrt{2}} (|110\rangle - |001\rangle)_{htc}
\]
\[
= \frac{1}{2} \left[ |+\rangle_h (|+\rangle_t |c \rangle - |c \rangle |+\rangle_t) + |c \rangle (|+\rangle_t |c \rangle - |c \rangle |+\rangle_t) \right],
\]
(12)

where
\[
|+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle),
\]
(13)
\[
|c \rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle),
\]
(14)

\[h\] stands for “home,” \[t\] stands for “travel,” and \[c\] stands for “control.”

(S1) Alice prepares a series \((N)\) of GHZ entangled states in
\[
|\psi_{1n}\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)_{htc},
\]
(15)

where \(n \in \{0, N\}\). She takes one photon from each GHZ entangled state, say the photons \(t_1, t_2, t_3, t_4, \text{ etc.}\) and forms a string of photons in a regular sequence, say the ordered string of photons is \(t_1 t_2 t_3 t_4 \cdots\). She sends Bob the ordered photon string (called the T sequence hereafter). In accordance with the ordering of the travel photons, Alice sends Charlie the photons \(c_n\) (called the C sequence hereafter) and stores the photons \(h_n\) (called the H sequence hereafter) by way of two photons as a group, i.e., photons \(h_1\) and \(h_2\) as group 1, photons \(h_3\) and \(h_4\) as group 2, etc.

(S2) Bob and Charlie confirm that they have received the travel photons. Also, in a regular sequence, Bob stores the arrived photons by way of two photons as a group in terms of their order of arrival: that is, photons \(t_1\) and \(t_2\) as group 1, photons \(t_3\) and \(t_4\) as group 2, etc, and Charlie stores the arrived photons by way of two photons as a group in terms of their order of arrival: that is, photons \(c_1\) and \(c_2\) as group 1, photons \(c_3\) and \(c_4\) as group 2, etc.
(S3) If Bob wants to transmit messages to Alice, he chooses randomly some photon groups as encoding-decoding groups (say, group 1 and 2, etc.) for his later two-bit encodings via local unitary operations $U$ ($U_1 = \hat{I}, U_2 = \hat{\sigma}_x, U_3 = -i\hat{\sigma}_y, U_4 = \hat{\sigma}_z$). The remaining photon groups are taken as checking groups. He publicly tells his choices to Alice and Charlie.

(S4) For each photon in each checking group, Bob randomly chooses one of two sets of measuring basis (MB), $\{|0\rangle, |1\rangle\}$ or $\{|+, -\rangle\}$, to measure his photons. Then, he publicly announces the order of the photons, the MB, and the measurement outcomes. Then, Alice and Charlie perform their measurements, under the same MB as that chosen by Bob, on the corresponding photons in their checking groups. The intercept-and-resend attack, as well as the entangle-and-measure attack, can be detected efficiently by this method. If their measurement outcomes coincide when they use the same basis in the same order according to Eq. (5)−(12), there are no Eves on the line. In this case, they continue to communication. Otherwise, they have to discard their transmission and abort the communication.

(S5) Bob asks Charlie to carry out a Hadamard operation on each $c_n$ in order. The Hadamard operation has the form

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad (16)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), \quad (17)$$

which will transform the state shown in Eq. (15) into

$$|\psi_1\rangle_n = \frac{1}{2}[(|00\rangle + |11\rangle)_{h_n t_n} \otimes |0\rangle_{c_n} + (|00\rangle - |11\rangle)_{h_n t_n} \otimes |1\rangle_{c_n}]. \quad (18)$$

Charlie measures photon $c_n$. If he obtains the outcome $|0\rangle_{c_n}$, photons $(h_n, t_n)$ will collapse into the state

$$|\eta^+_1\rangle_n = \frac{1}{2}(|00\rangle + |11\rangle)_{h_n t_n}; \quad (19)$$

otherwise, the state of photons $(h_n, t_n)$ will be

$$|\eta^-_1\rangle_n = \frac{1}{2}(|00\rangle - |11\rangle)_{h_n t_n}. \quad (20)$$

(S6) After the above operations, in accord with the encoding-decoding group ordering, Charlie informs Alice and Bob of his measurement results via a classical communication.

(S7) In accord with the encoding-decoding groups ordering, Bob performs his two-bit encodings via local unitary $U$ operations on the encoding-decoding groups according to his
bit strings (say, 0001· · ·) needing to be transmitted this time: for instance, a $U_1$ operation on one photon of group 1 to encoding 00, a $U_2$ operation on one photon of group 2 to encode 01, etc. After the unitary $U$ operations, Bob performs the Bell state measurements on all the encoding-decoding groups.

(S8) Bob publicly announces his Bell measurement result and the encoding-decoding group’s order for each encoding-decoding group.

(S9) Alice measures her unmeasured photon groups in Bell states after Bob’s public announcement in (S8). After she knows each of Charlie and Bob’s Bell measurement results with the group order and her Bell measurement results with group orders, she can identify the exact unitary $U$ operations performed by Bob on each encoding-decoding group. She can read the two-bit encodings. For example, we suppose that Charlie’s measurement results are $|0\rangle_{c_1}$ and $|0\rangle_{c_2}$. Then, $|\psi_1\rangle_1$ and $|\psi_1\rangle_2$ are collapsed into the states

$$|\eta_1^+\rangle_1 = \frac{1}{2}(|00\rangle + |11\rangle)_{h_1t_1},$$

$$|\eta_1^+\rangle_2 = \frac{1}{2}(|00\rangle + |11\rangle)_{h_2t_2}.$$

If Bob performs $U_1$ operation on one photon of group 1, then the state of the whole system composed of photons ($h_1$, $t_1$; $h_2$, $t_2$) is

$$U_1|\eta_1^+\rangle_1 \otimes |\eta_1^+\rangle_2 = \frac{1}{4}(|00\rangle + |11\rangle)_{h_1t_1} \otimes (|00\rangle + |11\rangle)_{h_2t_2}$$

$$= \frac{1}{4\sqrt{2}}|\psi^+\rangle_{t_1t_2}(|01\rangle + |10\rangle)_{h_1h_2} + \frac{1}{4\sqrt{2}}|\psi^+\rangle_{t_1t_2}(|01\rangle - |10\rangle)_{h_1h_2}$$

$$+ \frac{1}{4\sqrt{2}}|\phi^+\rangle_{t_1t_2}(|00\rangle + |11\rangle)_{h_1h_2} + \frac{1}{4\sqrt{2}}|\phi^+\rangle_{t_1t_2}(|00\rangle - |11\rangle)_{h_1h_2}$$

$$= \frac{1}{4}[|\psi^+\rangle_{t_1t_2}|\psi^+\rangle_{h_1h_2} + |\psi^+\rangle_{t_1t_2}|\psi^+\rangle_{h_1h_2}$$

$$+ |\phi^+\rangle_{t_1t_2}|\phi^+\rangle_{h_1h_2} + |\phi^+\rangle_{t_1t_2}|\phi^+\rangle_{h_1h_2}].$$

If Bob publicly announces his Bell measurement result is $|\phi^+\rangle_{t_1t_2}$ with a probability of 1/16, then Alice’s measurement result is $|\phi^+\rangle_{h_1h_2}$. Thus, she can conclude that Bob performed a $U_1$ operation on one photon of group 1 and, therefore, extract the bits (00), and she can read the two-bit encodings (see Table 1). Similarly, in accordance with the photon group’s ordering, she can obtain the bit string (0001· · ·).
TABLE I: Corresponding relations among the unitary $U$ operation (i.e., the encoding bits), the initial states, and Bob’s and Alice’s Bell measurement results ($|\eta_1^{\pm}\rangle_n = |\eta_{n}^{\pm}\rangle_{h_n t_n} = \frac{1}{2}(|00\rangle \pm |11\rangle)_{h_n t_n}$, $|\mu_{1}^{\pm}\rangle_n = |\mu_{n}^{\pm}\rangle_{h_n t_n} = \frac{1}{2}(|01\rangle \pm |10\rangle)_{h_n t_n}$).

| $U_1(00)$ | $U_2(01)$ | $U_3(10)$ | $U_4(11)$ |
|----------------|----------------|----------------|----------------|
| $|\eta_1^{+}\rangle_{h_1 t_1} \otimes |\eta_1^{+}\rangle_{h_2 t_2}$ | $|\mu_1^{+}\rangle_{h_1 t_1} \otimes |\eta_1^{+}\rangle_{h_2 t_2}$ | $|\mu_1^{-}\rangle_{h_1 t_1} \otimes |\eta_1^{+}\rangle_{h_2 t_2}$ | $|\eta_1^{-}\rangle_{h_1 t_1} \otimes |\eta_1^{+}\rangle_{h_2 t_2}$ |
| $|\phi^{+}\rangle_{t_1 t_2}, |\phi^{+}\rangle_{h_1 h_2}$ | $|\phi^{+}\rangle_{t_1 t_2}, |\phi^{+}\rangle_{h_1 h_2}$ | $|\phi^{-}\rangle_{t_1 t_2}, |\phi^{-}\rangle_{h_1 h_2}$ | $|\phi^{-}\rangle_{t_1 t_2}, |\phi^{-}\rangle_{h_1 h_2}$ |
| $|\phi^{-}\rangle_{t_1 t_2}, |\phi^{-}\rangle_{h_1 h_2}$ | $|\phi^{-}\rangle_{t_1 t_2}, |\phi^{-}\rangle_{h_1 h_2}$ | $|\phi^{+}\rangle_{t_1 t_2}, |\phi^{+}\rangle_{h_1 h_2}$ | $|\phi^{+}\rangle_{t_1 t_2}, |\phi^{+}\rangle_{h_1 h_2}$ |
| $|\psi^{+}\rangle_{t_1 t_2}, |\psi^{+}\rangle_{h_1 h_2}$ | $|\psi^{+}\rangle_{t_1 t_2}, |\psi^{+}\rangle_{h_1 h_2}$ | $|\psi^{-}\rangle_{t_1 t_2}, |\psi^{-}\rangle_{h_1 h_2}$ | $|\psi^{-}\rangle_{t_1 t_2}, |\psi^{-}\rangle_{h_1 h_2}$ |
| $|\psi^{-}\rangle_{t_1 t_2}, |\psi^{-}\rangle_{h_1 h_2}$ | $|\psi^{-}\rangle_{t_1 t_2}, |\psi^{-}\rangle_{h_1 h_2}$ | $|\psi^{+}\rangle_{t_1 t_2}, |\psi^{+}\rangle_{h_1 h_2}$ | $|\psi^{+}\rangle_{t_1 t_2}, |\psi^{+}\rangle_{h_1 h_2}$ |

(S10) Similarly, if Charlie wants secure direct communication with Alice, he can do the same thing as Bob does.

(S11) The controlled secure direct communication has been successfully completed.

In the present protocol, we use the property of quantum entanglement swapping instead of the property of quantum entanglement. If the sender (Bob) wants to transmit messages to the receiver (Alice), Charlie must take an Hadamard operation on each of his photons, measure it, and tell the results to Bob and Alice in order. When GHZ entangled state is successfully shared, no qubit has to be exchanged in a quantum channel. This leads to four important advantages. Firstly, If no eavesdropping is found in the checking procedure, the secret messages can be transmitted successfully. Because there is not a transmission of the qubit that carries the secret messages between Alice and Bob in a public channel, it is completely secure for controlled secure direct communication if a perfect quantum channel is used. That is, after the checking procedure, there is no longer any chance for Eve to attack the secret messages. Secondly, the degradation of the entanglement of the photon pair will not decrease due to the travel of the photon in the H sequence in a real quantum channel. Thirdly, after insuring the security of the quantum channels, if Charlie would like to help Alice and Bob to communicate, Alice and Bob can communicate secret messages directly under the control of the third side Charlie. Only with the help of controller Charlie, the sender and the receiver can implement secure direct communication successfully. Fourthly, this protocol can also be generalized to a multi-party control system in which $N$ parties share a large number of $N$-particle GHZ entangled states. Party $M$ ($M \in N$) as receiver,
party $Q$ ($Q \in N, Q \neq M$) as sender, and $N - 2$ parties, except for party $M$ and party $Q$, as controllers. The multi-party controlled secure direct communication can also succeed.

In summary, we have proposed a controlled secure direct communication protocol by using a large number of GHZ entangled states via entanglement swapping and local unitary operations. When GHZ entangled state is successful shared, no qubit has to be exchanged in a quantum channel. Different from Man's scheme [10], in this protocol, anyone of the multi-partners can send messages to any receiver secretly with the help of controllers by using a local operation and a reliable public channel. The result shows that for such a protocol, can realize successful controlled secure direct communication between two users. Since the message transferred only by using local operations and public channels after entanglement was successfully distributed, this protocol can protect the communication against the destroying-travel-qubit-type attack. This scheme can also be generalized to a multi-party control system.

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