Absolute rotation detection by Coriolis force measurement using optomechanics

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Abstract

In this article, we present an application of the optomechanical cavities for absolute rotation detection. Two optomechanical cavities, one in each arm, are placed in a Michelson interferometer. The interferometer is placed on a rotating table and is moved with a uniform velocity of $\dot{y}$ with respect to the rotating table. The Coriolis force acting on the interferometer changes the length of the optomechanical cavity in one arm, while the length of the optomechanical cavity in the other arm is not changed. The phase shift corresponding to the change in the optomechanical cavity length is measured at the interferometer output to estimate the angular velocity of the absolute rotation. An analytic expression for the minimum detectable rotation rate corresponding to the standard quantum limit of measurable Coriolis force in the interferometer is derived. Squeezing technique is discussed to improve the rotation detection sensitivity by a factor of $\sqrt{\gamma_m/\omega_m}$ at 0 K temperature, where $\gamma_m$ and $\omega_m$ are the damping rate and angular frequency of the mechanical oscillator. The temperature dependence of the rotation detection sensitivity is studied.

1. Introduction

Detection of absolute rotation has significant importance in fundamental physics for testing gravitation theories [1] and in practical applications for improving navigation systems. The Sagnac effect [2] is one of the well known phenomena used for the detection of rotation. In the Sagnac effect, absolute rotation introduces a time delay [2–4] between two counter propagating laser beams placed on a rotating table. The time delay is manifested in the form of phase difference in a fiber optic laser gyroscope [5, 6], and as frequency beat signal in an active laser gyroscope [7]. The Sagnac effect is also extended to atomic interferometry [8, 9] for improved rotation detection. A review of rotation detection schemes can be found in [10]. Correlated spontaneous emission theory is developed in [11] to improve the rotation detection by reducing phase noise. The Sagnac effect is generalized to linear motion in [12]. Another prominent method for detecting rotations is by measuring the effects of rotation induced pseudo forces like centrifugal force and Coriolis force. Hence, a very sensitive force detector can potentially be used as a rotation detector. In our previous work [13], Coriolis force induced displacements are measured using enhanced longitudinal Fizeau drag effect [14] to detect the rotation. In this paper, we propose a rotation detector based on the effects of the Coriolis force on the optomechanical cavity placed on a rotating table. We also discuss the application of squeezed light for improving the rotation detection sensitivity. Temperature dependence of the rotation detection sensitivity is also discussed.

Optomechanical cavities are based on the principles of radiation pressure force, but these systems are very sensitive to any external forces acting on them. Hence they were extensively studied for gravitational wave detection [15–18]. In general, an optomechanical cavity consists of a Fabry–Perot cavity with one of its two cavity mirrors freely oscillating. When such a cavity is driven by an external laser field, the freely oscillating...
mirror of the cavity will be displaced by the radiation pressure force [19–21] of the intra cavity field. This changes the length of the cavity and thus the optical response of the cavity itself. Optomechanical cavities are found to be extremely useful as displacement sensors [22–25] and as weak force sensors [26–32]. Recently, optomechanical systems have become a focus for a broad range of research activities [20, 21] and several interesting phenomena like phase conjugation [33], squeezing [34], super-radiance [35], laser cooling [36–38], optomechanically induced transparency [39–43], etc. have been predicted in optomechanics.

In the first part of this paper, we derive the Hamiltonian for an optomechanical cavity placed on a rotating table [44]. Then we derive the equations of motion for a mechanical oscillator in the presence of non-inertial forces arising due to rotation. Then we derive analytic expression for the minimum detectable rotation rate by neglecting the thermal effects. In the second part, we show that the main contribution to the output noise comes from the vacuum fluctuations entering through the empty port of a beam splitter. Then we show that squeezing the input vacuum fluctuations can improve the rotation detection sensitivity by a factor of $\sqrt{\gamma_m/\omega_m}$, where $\gamma_m$ and $\omega_m$ are the damping and angular frequency of the mechanical oscillator, at 0 K temperature. In the last part we discuss the effects of temperature in our system.

2. Optomechanical oscillator in rotating frame

Consider a perfectly reflecting optomechanical mirror which is moving with velocity $$(\dot{x}, \dot{y})$$ on a rotating table. $$(x, y)$$ represent the instantaneous position co-ordinates of the mechanical oscillator in a reference frame co-rotating with the rotating table. The velocity of the optomechanical mirror in a non-rotating fixed lab frame is given as

$$\vec{v} = \frac{d}{dt} (x^\hat{i} + y^\hat{j}) = \left( \frac{dx}{dt} - y\dot{\theta}\right)^\hat{i} + \left( \frac{dy}{dt} + x\dot{\theta}\right)^\hat{j},$$

(1)

where $v$ is velocity of the mechanical mirror observed from the lab frame, $^\hat{i}$ and $^\hat{j}$ are the unit vectors in the frame of reference co-rotating with rotating table. The Lagrangian of the optomechanical mirror in the lab frame is given as

$$L = \frac{1}{2}m \left\{ \left( \frac{dx}{dt} - y\dot{\theta}\right)^2 + \left( \frac{dy}{dt} + x\dot{\theta}\right)^2 \right\} - \frac{1}{2}m\omega_m^2 (x - a_x)^2 - \frac{1}{2}m\omega_m^2 (y - a_y)^2,$$

(2)

where $m$ and $w_m$ are the mass and angular frequency of the optomechanical mirror. The co-ordinates $$(a_x, a_y)$$ represent the equilibrium position in the frame of reference rotating with the rotating table of the mechanical mirror when the table is not rotating. The angular velocity of the rotating table is given by $\dot{\theta}$. The Lagrangian conjugate variables corresponding to the generalized co-ordinates $x$ and $y$ are given as

$$p_x = \frac{\partial L}{\partial \dot{x}} = m(x - y\dot{\theta}); \quad p_y = \frac{\partial L}{\partial \dot{y}} = m(y + x\dot{\theta}).$$

(3)

Using equations (2) and (3), the classical Hamiltonian for the optomechanical mirror [44] in the lab frame is given as

$$H = \frac{p_x^2 + p_y^2}{2m} + (p_x y \dot{\theta} - p_y x \dot{\theta}) + \frac{1}{2}m\omega_m^2 (x - a_x)^2 + \frac{1}{2}m\omega_m^2 (y - a_y)^2.$$

(4)

In the lab frame of reference, the corresponding quantum mechanical Hamiltonian for the mirror oscillator is given as

$$\hat{H} = \hbar\omega_m \left( \hat{b}_x^\dagger \hat{b}_x + \frac{1}{2} \right) + \hbar\omega_m \left( \hat{b}_y^\dagger \hat{b}_y + \frac{1}{2} \right) + \frac{\chi_p \chi_q}{i\hbar} \left( \hat{b}_x \hat{b}_y^\dagger - \hat{b}_y \hat{b}_x^\dagger \right) \dot{\theta} + \frac{\chi_p \chi_q}{i\hbar} \left( \hat{b}_x^\dagger \hat{b}_y - \hat{b}_y^\dagger \hat{b}_x \right) \dot{\theta}$$

$$- \frac{\chi_p \chi_q}{i\hbar} \left( \hat{b}_y^\dagger - \hat{b}_x \right) \dot{\theta},$$

where $\chi_p = \sqrt{2\hbar m\omega_m}$, $\chi_q = \sqrt{\hbar \omega_m}$, $\hat{b}_x (\hat{b}_x^\dagger)$ and $\hat{b}_y (\hat{b}_y^\dagger)$ are the annihilation (creation) operators for the simple harmonic motion along $x$-axis and $y$-axis, respectively. The non-zero commutation relations are given as $[\hat{b}_x, \hat{b}_x^\dagger] = 1$, $[\hat{b}_y, \hat{b}_y^\dagger] = 1$.

3. Rotation detection

Two optomechanical cavities, cavity-1 and cavity-2, are placed along the arm-1 and arm-2 of a Michelson interferometer as shown in figure 1(b). The interferometer is placed on the rotating table in such a way that arm-1 coincides with the $y$-axis, and the arm-2 is parallel to the $x$-axis, of the rotating table (shown in figure 1(a)).
laser source with amplitude \( \hat{E} \) and with angular frequency \( \omega_d \) drives both the optomechanical cavities. For simplicity, we assume that the input and output fields for the cavity-1 and cavity-2 are along \( y \)-axis and \( x \)-axis, respectively. Using equation (5) and the optomechanical Hamiltonian description in [45], the Hamiltonian for the interferometer placed on the rotating table, in the lab frame of reference, is given as

\[
\hat{H} = \sum_{\alpha=1}^{2} \left( \hbar \omega_c \left( \hat{c}_\alpha^\dagger \hat{c}_\alpha + \frac{1}{2} \right) + \frac{\chi_p \chi_q}{\hbar} \left( \hat{b}_\alpha \hat{c}_\alpha^\dagger - \hat{b}_\alpha^\dagger \hat{c}_\alpha \right) \hat{\theta} + \hbar \omega_m \left( \hat{b}_\alpha \hat{b}_\alpha^\dagger + \frac{1}{2} \right) \right) + \frac{\chi_p \chi_q}{\hbar} \left( \hat{b}_2 \hat{c}_2^\dagger + \hat{b}_2^\dagger \hat{c}_2 \right) + \frac{\chi_p \chi_q}{\hbar} \left( \hat{b}_1 \hat{c}_1^\dagger + \hat{b}_1^\dagger \hat{c}_1 \right) + \chi_p \chi_q \left( \hat{b}_2 \hat{c}_2^\dagger + \hat{b}_2^\dagger \hat{c}_2 \right) + \chi_p \chi_q \left( \hat{b}_1 \hat{c}_1^\dagger + \hat{b}_1^\dagger \hat{c}_1 \right) \right) \hat{\theta} \]

where \( \chi_\alpha = -\hbar \omega_m / l \), \( l \) is the length of the optomechanical cavities when the optomechanical mirrors are in equilibrium position. The annihilation(creation) operator for cavity filed in cavity-\( \alpha \) is \( \hat{c}_\alpha \) (\( \hat{c}_\alpha^\dagger \)). \( \hat{b}_\alpha \) and \( \hat{b}_\alpha^\dagger \) are the annihilation operators for the optomechanical mirror oscillation along the \( x \)-axis and \( y \)-axis, respectively, in cavity-\( \alpha \). The equations of motion for the cavity fields after making rotating wave approximation at input laser frequency \( \omega_d \) are given as

\[
\hat{\dot{c}}_1 = \left( i \omega_d - \omega_b - \frac{\chi_p}{\lambda_p} \right) \hat{c}_1 + \sqrt{2 \zeta} \hat{E}_1, \quad \hat{\dot{c}}_2 = \left( i \omega_d - \omega_b - \frac{\chi_p}{\lambda_p} \right) \hat{c}_2 + \sqrt{2 \zeta} \hat{E}_2, \]

where \( \zeta \) and \( \hat{E}_i \) are the decay rate and input for the cavity field oscillator in cavity-\( \alpha \), respectively. The equations of motion for the \( \hat{b}_2 \) and \( \hat{b}_1 \) are given as

\[
\hat{\dot{b}}_1 = -i \omega_m \hat{b}_1 + \frac{\chi_p \chi_q}{2\hbar} \hat{b}_1 \hat{\dot{\theta}} - \frac{\chi_p \chi_q}{2\hbar} \hat{\dot{\theta}} \hat{b}_1 + \chi_q \hat{c}_1^\dagger \hat{\dot{c}}_1, \quad \hat{\dot{b}}_2 = -i \omega_m \hat{b}_2 + \frac{\chi_p \chi_q}{2\hbar} \hat{b}_2 \hat{\dot{\theta}} + \frac{\chi_p \chi_q}{2\hbar} \hat{\dot{\theta}} \hat{b}_2 + \chi_q \hat{c}_2^\dagger \hat{\dot{c}}_2. \]

In the case of absolute rotation detection, the detector, the laser source, and the entire experiment is on the rotating table. Hence, it is more logical to transform the equations of motion (equations (7)–(9)) from the lab frame of reference to the frame of reference co-rotating with the rotating table. This can be achieved by writing the annihilation and creation operators as following:

\[
\hat{\dot{b}}_2 = \frac{\hat{b}_2}{\chi_q} + \frac{i m \xi_2}{\chi_p} \hat{\dot{\theta}} - \frac{m \xi_2}{\chi_p} \hat{\dot{\theta}}, \quad \hat{\dot{b}}_1 = \frac{\hat{b}_1}{\chi_q} + \frac{i m \xi_1}{\chi_p} \hat{\dot{\theta}} - \frac{m \xi_1}{\chi_p} \hat{\dot{\theta}}.
\]
\[ \hat{b}_{y2} = \frac{\hat{y}_2 - a_{y2}}{\chi_q} + \frac{i m \hat{y}_2}{\lambda_p} + \frac{i m \hat{\theta}}{\lambda_p}, \quad (12) \]

\[ \hat{b}_{x1} = \frac{\hat{x}_1 - a_{x1}}{\chi_q} + \frac{i m \hat{x}_1}{\lambda_p} - \frac{i m \hat{\theta}}{\lambda_p}, \quad (13) \]

\[ \hat{b}_{y1} = \frac{\hat{y}_1 - a_{y1}}{\chi_q} + \frac{i m \hat{y}_1}{\lambda_p} + \frac{i m \hat{\theta}}{\lambda_p} = \hat{b}_1 + i \frac{m \hat{\theta}}{\lambda_p}, \quad (14) \]

where \( \hat{b}_1 = \frac{\hat{y}_1 - a_{y1}}{\chi_q} + \frac{i m \hat{y}_1}{\lambda_p} \) and \( \hat{b}_2 = \frac{\hat{x}_1 - a_{x1}}{\chi_q} + \frac{i m \hat{x}_1}{\lambda_p} \). Substituting equations (11–14) into equations (8–10) the equations of motion for the mechanical oscillator and cavity field in the frame of reference co-rotating with rotating table are given as

\[ \hat{b}_1 = (-i \omega_m - \gamma_m) \hat{b}_1 - \frac{i m}{\lambda_p} (2 \hat{\theta} \hat{\theta} - \hat{y}_1 \hat{\theta} + \hat{x}_1 \hat{\theta}) - i \frac{\chi_q}{2 \hbar} \hat{z}_1^\dagger \hat{z}_1 + \sqrt{2 \gamma_m} \hat{s}_1, \quad (15) \]

\[ \hat{b}_2 = (-i \omega_m - \gamma_m) \hat{b}_2 + \frac{i m}{\lambda_p} (2 \hat{\theta} \hat{\theta} + \hat{y}_2 \hat{\theta}^2 + \hat{y}_1 \hat{\theta}) - i \frac{\chi_q}{2 \hbar} \hat{z}_2^\dagger \hat{z}_2 + \sqrt{2 \gamma_m} \hat{s}_2, \quad (16) \]

\[ \hat{\epsilon}_a = \left( \left[ \omega_d - \omega_o - \frac{\chi_q}{\lambda_p} (\hat{b}_1 + \hat{b}_2) \right] - \zeta \right) \hat{\epsilon}_a + \sqrt{2 \gamma} \hat{E}_{a1}, \quad (17) \]

where \( \zeta \) and \( \gamma_m \) are the phenomenological damping rate for the cavity field oscillator and mirror oscillator, respectively. \( \hat{E}_{a1} \) and \( \hat{s}_a \) are the inputs for the cavity field oscillator and mirror oscillator. The second term on the RHS of the equations (15) and (16) indicates the presence of non-inertial force due to the rotation of the table. In equations (15) and (16), the \( \hat{\theta} \cdot \hat{\theta} \) term represents the centrifugal force, \( \hat{\theta} \cdot \hat{\theta} \) term represents the Coriolis force and \( \hat{\theta} \cdot \hat{\theta} \) term represents the force due to the angular acceleration of the rotating table. We set \( \hat{\theta} = 0 \), by assuming that the table is rotating with constant angular velocity. We are interested in detecting the small rotation rates, hence the contribution from the centrifugal force term, which has \( \hat{\theta} \cdot \hat{\theta} \) dependence, is negligible in comparison with the Coriolis force term which has \( \hat{\theta} \cdot \hat{\theta} \) dependence. The mechanical oscillator in cavity-1 oscillates only along the y-axis of the rotating table, hence \( \hat{x}_1 = 0 \) in equation (15). The mechanical oscillator in cavity-2 oscillates only along the x-axis of the rotating table. Hence \( \hat{y}_2 = \hat{y}_1 \), where \( \hat{y}_1 \) is the velocity with which the interferometer is moving along the y-axis of the rotating table, in equation (16). We want to investigate the effect of classical Coriolis force, hence we replace \( \hat{\theta} \) in equations (15) and (16) with its classical mean value \( \bar{\theta} \). Using the above arguments, the equations (15) and (16) can be simplified as

\[ \hat{b}_1 = (-i \omega_m - \gamma_m) \hat{b}_1 - \frac{i \chi_q}{2 \hbar} \hat{z}_1^\dagger \hat{z}_1 + \sqrt{2 \gamma_m} \hat{s}_1, \quad (18) \]

\[ \hat{b}_2 = (-i \omega_m - \gamma_m) \hat{b}_2 + \frac{i \chi_q}{2 \hbar} \hat{z}_2^\dagger \hat{z}_2 + \sqrt{2 \gamma_m} \hat{s}_2, \quad (19) \]

Linearize the operator equations in equations (7, 8, 18, 19) around their steady state classical mean values by taking \( \hat{\bar{b}}_1 = \bar{b}_1 + \bar{e}_{b1}, \hat{\bar{b}}_2 = \bar{b}_2 + \bar{e}_{b2}, \hat{\bar{\epsilon}}_a = \bar{\epsilon}_a + \bar{\epsilon}_{a1}, \hat{\bar{E}}_a = \bar{E}_a + \bar{E}_{a1}, \hat{\bar{s}}_a = \bar{s}_a + \bar{s}_{a1} \). The \( \bar{e}_{b1}, \bar{e}_{b2}, \bar{\epsilon}_{a1}, \bar{s}_{a1} \) represents the fluctuation in mirror oscillator, cavity field oscillator, cavity field oscillator input and mirror oscillator input for the cavity-\( \alpha \), respectively. \( \bar{b}_{\alpha1}, \bar{\epsilon}_{a1}, \bar{E}_{a1}, \bar{s}_{a1} \) represents the steady state classical values for mechanical oscillator, cavity field oscillator, cavity field oscillator input and mechanical oscillator input for the cavity-\( \alpha \), respectively. We take \( \bar{s}_{a1} = 0 \).

The steady state classical values for the cavity field can be obtained by solving equations ((7), (18), (19)) with their time derivatives set to zero.

\[ \bar{\epsilon}_2 = -\sqrt{2 \gamma} \frac{\bar{E}_2}{i \Delta - \zeta}, \quad \bar{\epsilon}_1 = \sqrt{2} \frac{\bar{E}_1}{\zeta}, \quad (20) \]

where \( \Delta = -\frac{\chi_q}{\lambda_p} \frac{4 m \hbar \omega_m}{\lambda_p^2}, \bar{\gamma}_m \approx \frac{\chi_q}{\lambda_p} \frac{4 m \hbar \omega_m}{\lambda_p^2} \) represents the frequency shift in the cavity field oscillator because of the action of Coriolis force on the mechanical oscillator. When there is no rotation, the Coriolis force is zero and hence \( \Delta \) is zero. We adjust the drive field detuning such that \( \omega_d - \omega_o + \frac{\chi_q}{\lambda_p} \frac{4 m \hbar \omega_m}{\lambda_p^2} = 0 \) in equations (7) and (8). The fluctuations in the cavity field oscillator and mechanical oscillator are solved using the Fourier transform function.
\[ F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt. \]  

(21)

The solution to the fluctuations, in frequency domain, for the cavity-\( \alpha \) is given as

\[
\begin{bmatrix}
\delta_1(\omega) \\
\delta_1(\omega) \\
\delta_{11}(\omega)
\end{bmatrix} = M \begin{bmatrix}
\sqrt{2\zeta}\delta_{E1}(\omega) \\
\sqrt{2\zeta}\delta_{E1}(\omega) \\
\sqrt{2\eta}\delta_{E1}(\omega)
\end{bmatrix}, \quad \begin{bmatrix}
\delta_{2}(\omega) \\
\delta_{2}(\omega) \\
\delta_{22}(\omega)
\end{bmatrix} = N \begin{bmatrix}
\sqrt{2\zeta}\delta_{E2}(\omega) \\
\sqrt{2\zeta}\delta_{E2}(\omega) \\
\sqrt{2\eta}\delta_{E2}(\omega)
\end{bmatrix},
\]

where

\[
N = \begin{bmatrix}
\Gamma_1\Gamma_3\Gamma_4 + \frac{\chi_2^2}{\lambda_p^2}|\bar{g}|^2(\Gamma_3 - \Gamma_4) - \frac{\chi_2^2}{\lambda_p^2}\bar{g}^2(\Gamma_4 - \Gamma_3) - \frac{i\chi_2}{\lambda_p}\varepsilon_2\Gamma_1\Gamma_4 - \frac{i\chi_2}{\lambda_p}\varepsilon_2\Gamma_1\Gamma_3 \\
- \frac{\chi_2^2}{\lambda_p^2}\bar{g}^2(\Gamma_3 - \Gamma_4) + \Gamma_1\Gamma_3\Gamma_4 + \frac{\chi_2^2}{\lambda_p^2}|\bar{g}|^2(\Gamma_4 - \Gamma_3) + \frac{i\chi_2}{\lambda_p}\varepsilon_2\Gamma_1\Gamma_4 + \frac{i\chi_2}{\lambda_p}\varepsilon_2\Gamma_1\Gamma_3 \\
- \frac{i\chi_2}{\lambda_p}\varepsilon_1\Gamma_2\Gamma_4 - \frac{i\chi_2}{\lambda_p}\varepsilon_1\Gamma_2\Gamma_3 - \frac{i2\Delta}{\lambda_p}\frac{\chi_2^2}{\lambda_p^2}|\bar{g}|^2 - \bar{g}^2 - \Gamma_1\Gamma_2\Gamma_4 - \Gamma_1\Gamma_2\Gamma_3
\end{bmatrix} \frac{1}{D}
\]

with \( \Gamma_1 = (\zeta - i\omega - i\Delta), \Gamma_2 = (\zeta - i\omega + i\Delta), \Gamma_3 = \gamma_m + i(\omega_m - \omega), \Gamma_4 = \gamma_m - i(\omega_m + \omega) \) and \( D = \Gamma_1\Gamma_2\Gamma_3\Gamma_4 + 4\frac{\chi_2^2}{\lambda_p^2}|\bar{g}|^2\omega_m\Delta \). The matrix \( M \) can be obtained by setting \( \Delta = 0 \) and replacing \( \varepsilon_1 \) with \( \varepsilon_1 \) in the matrix \( N \). Using input–output formalism [46, 47], the fluctuations in the output cavity field oscillator, represented by \( \delta_\alpha \), are given by the relation \( \delta_\alpha = \delta_{E\alpha i} - \sqrt{2\zeta}\delta_{E\alpha} \). Hence the fluctuations in the output field from cavity-\( \alpha \) are given as

\[
\delta_1(\omega) = (1 - 2\zeta m_{21})\delta_{E1}(\omega) - 2\zeta m_{12}\dot{\delta}_{E1}(\omega) - 2\sqrt{\zeta\eta}(m_{13}\delta_{E1}(\omega) + m_{14}\dot{\delta}_{E1}(\omega)),
\]

(24)

\[
\delta_2(\omega) = (1 - 2\zeta m_{21})\delta_{E2}(\omega) - 2\zeta m_{12}\dot{\delta}_{E2}(\omega) - 2\sqrt{\zeta\eta}(m_{13}\delta_{E2}(\omega) + m_{14}\dot{\delta}_{E2}(\omega)),
\]

(25)

where \( m_{jk} \) (\( j, k = 1, 2, 3, 4 \)), and \( n_{jk} \) represent the matrix element in the \( j \)th row and \( k \)th column of the matrices \( M \) and \( N \), respectively. Let \( \tilde{\varepsilon}_\alpha \) be the annihilation operator for the output cavity field oscillator from cavity-\( \alpha \). We can separate the classical steady state response from the fluctuations in the output cavity field oscillator by writing \( \tilde{\varepsilon}_\alpha = \tilde{\varepsilon}_\alpha^c + \tilde{\varepsilon}_\alpha^f \). Using input–output formalism [46, 47], the steady state classical output from the cavity-\( \alpha \) \( (\tilde{\varepsilon}_\alpha^c) \) can be obtained from equation (20) as following

\[
\tilde{\varepsilon}_\alpha^c = \frac{i\Delta}{i\Delta - \zeta} \tilde{E}_\alpha, \quad \tilde{\varepsilon}_\alpha^f = -\tilde{E}_\alpha.
\]

(26)

A \( \pi/2 \) degree phase is added to the output from cavity-2 before reaching the beam splitter. Hence, the intensity of the optical field reaching both the detectors is equal when the table is not rotating. The difference in the photo detector readings is given as

\[
I_i = I_1 = \tilde{E}_2^+ \tilde{E}_1^-, \quad I_2 = \tilde{E}_2^+ \tilde{E}_1^-, \quad \tilde{E}_2^+ \tilde{E}_1^-,
\]

(27)

where \( I_1 \) and \( I_2 \) represent the number of photons per unit time at the photo detectors in arm-1 and arm-2, respectively. Substituting \( c_\alpha = \tilde{\varepsilon}_\alpha + \delta_\alpha \) in equation (27), the steady state signal is given as

\[
\langle I_2 - I_1 \rangle = \tilde{\varepsilon}_2^+ \tilde{\varepsilon}_1^- + \tilde{\varepsilon}_1^+ \tilde{\varepsilon}_2^-,
\]

(28)

and the fluctuations in the signal are given as

\[
\delta I_2 - \delta I_1 = \delta \tilde{\varepsilon}_2^+ \tilde{\varepsilon}_1^- + \delta \tilde{\varepsilon}_1^+ \tilde{\varepsilon}_2^- + \tilde{\varepsilon}_2^+ \delta \tilde{\varepsilon}_1^- + \tilde{\varepsilon}_1^+ \delta \tilde{\varepsilon}_2^-.
\]

(29)

The inputs for the cavity field oscillators are taken as the coherent laser field with amplitude \( \tilde{E} \) and the vacuum field entering through the empty port of the beam splitter. For a lossless 50:50 beam splitter, the amplitude of fields entering the cavity-\( \alpha \) is given as

\[
\tilde{E}_\alpha = \tilde{E} + i\tilde{E}_\nu, \quad \tilde{E}_\nu = \frac{\tilde{E} + \tilde{E}_\nu}{\sqrt{2}},
\]

(30)

where \( \tilde{E}_\nu \) is the amplitude of the vacuum field. Linearizing equation (30) gives the steady state classical values for the cavity field oscillator’s inputs as \( \tilde{E}_1 = \tilde{E}/\sqrt{2} \) and \( \tilde{E}_2 = i\tilde{E}/\sqrt{2} \). Assuming that the input laser field \( \tilde{E} \) is real, and using equation (26), the equation (28) is given as
\[ \langle I_2 - I_i \rangle = \frac{4\Delta \zeta}{\Delta^2 + \zeta^2} \frac{|\bar{E}|^2}{2} \approx \frac{8\omega_c I}{\zeta^2 \omega_m^2} I, \]  

(31)

where \( 2\bar{I} = |\bar{E}|^2 \) is the input laser intensity. The input power, represented by \( P \), corresponding to the input intensity is given as \( P = 2\bar{I}/h\omega_c \). We assume that \( \Delta \ll \zeta \) and set \( \Delta^2 + \zeta^2 \approx \zeta^2 \) in equation (31). From equation (30), the amplitude of fluctuations in the input fields are given as

\[ \delta E_1 = \frac{\delta E}{\sqrt{2}} \quad \delta E_2 = \frac{i\delta E}{\sqrt{2}}. \]  

(32)

Assuming that the input vacuum and laser fields are uncorrelated at all times, the only non-zero correlations terms we have are \( \langle \delta E_B (\omega) \delta E_B (\omega') \rangle = \langle \delta E_B (\omega) \delta E_B (\omega') \rangle = \delta (\omega - \omega') \). We also neglect the thermal noise by setting \( \langle \delta E_T (\omega) \delta E_T (\omega') \rangle = \langle \delta E_T (\omega) \delta E_T (\omega') \rangle = \delta (\omega - \omega') \). Using these correlation functions and equation (22), the quantum noise spectrum can be calculated from the variance of \( \delta E_1 - \delta E_2 \),

\[ \langle (\delta E_1 - \delta E_2)(\omega), (\delta E_1 - \delta E_2)(\omega') \rangle = V\delta (\omega - \omega'), \]  

(33)

where

\[ V = I \left( 1 - 2\zeta \frac{\Gamma_1 \Gamma_4 + \frac{\chi_p^2}{\chi_p}(|\bar{E}|^2 - \bar{E}_m^2)(\Gamma_4 - \Gamma_1)}{\Gamma_1 \Gamma_4 + \frac{4\chi_p^2}{\chi_p}I \omega_m \Delta} \right)^2 + 4\zeta \gamma_o I \left( \frac{i\chi_p \Gamma_1 (\bar{E}_m^2 - \bar{E}_m^2) - \bar{E}_m^2 (\Gamma_4 - \Gamma_1)}{\Gamma_1 \Gamma_4 + \frac{4\chi_p^2}{\chi_p}I \omega_m \Delta} \right)^2 \]

\[ + \frac{\Gamma_1 \Gamma_4 \bar{E}_m^2}{\Gamma^2 \Gamma_4} \left( \frac{16\chi_p I \omega_m^2}{\chi_p} \right)^2 + 32\gamma_o I \omega_m \frac{2}{\zeta} \left( \frac{\chi_p^2 \omega_m^2 + 8\gamma_o I \omega_m \Delta}{\chi_p^2 \omega_m^2 + 8\gamma_o I \omega_m \Delta} \right)^2, \]  

\[ \nu_o = \sqrt{\frac{2\bar{I} + 2\bar{I}}{\zeta^2 \omega_m^2 \left( \frac{\omega_m^2}{\chi_p^2} \right) + \frac{32\gamma_o I \omega_m^2}{\zeta}}} = \sqrt{\frac{2\bar{I} + 2\bar{I}}{8\gamma_o I \omega_m^2 \Delta}}. \]  

(34)

(35)

where \( \Gamma = i\omega - \zeta \).

4. Results and discussion

The quantum noise spectrum at the interferometer output is given by \( \sqrt{V} \). The \( \nu_o \) in equation (35) gives the quantum noise at the signal. In the RHS of equation (35), the first term represents shot noise due to the discrete nature of photons in both the arms, the second term represents the back action noise due to the radiation pressure fluctuations [48, 49] in both the cavities, the third term represents the mirror noise originating from the mechanical oscillator input in both the cavities.

With thermal noise neglected, the significant contribution to the noise comes from the shot noise and from the radiation pressure noise. The shot noise can not be manipulated by tuning the optomechanical cavity parameters. On the other hand, the radiation pressure noise can approximately be set to the shot noise level when the following condition is satisfied.

\[ |\zeta^2 \omega_m^2|^2 \geq 256 \frac{\lambda_p^4}{\chi_p^2} I \omega_m^2 \Rightarrow \zeta^2 \omega_m^2 I \omega_m \geq I. \]  

(36)

Signal in equation (31) requires \( I \) to be large, but equation (36) requires the \( I \) to be small. Hence, there exists an optimum intensity [50], say \( I_{\text{opt}} \), at which we obtain the best signal to noise ratio. \( I_{\text{opt}} \) is the maximum value \( I \) can take without violating the condition in equation (36). Hence, we can write

\[ I_{\text{opt}} = \frac{\zeta^2 \omega_m^2 I \omega_m}{8\gamma_o I \omega_m^2 \Delta}. \]  

(37)

For \( I = I_{\text{opt}} \), by direct substitution of equation (37) in equation (35), the quantum noise at signal frequency is given as

\[ \nu_o = \sqrt{\frac{2I_{\text{opt}} + 2I_{\text{opt}} I \omega_m}{I_{\text{opt}} + \Delta/2\zeta} + \frac{4I_{\text{opt}} \gamma_o I \omega_m}{I_{\text{opt}} + \Delta/2\zeta}}. \]  

(38)

Note that, for \( \Delta \ll \zeta \), the contribution from the shot noise and the radiation pressure noise in equation (38) are equal [31]. By equating the signal from equation (31) with noise in equation (38), the minimum detectable angular velocity is given as
\[ \dot{\theta}_m = \frac{\zeta \omega_m^2}{4 \omega_0 \gamma \sqrt{\ln(t_m)}} \sqrt{\frac{1}{2} + \frac{1/2}{1 + \frac{\Delta}{2\gamma}}} + \frac{\gamma_m/\omega_m}{1 + \frac{\Delta}{2\gamma}} \approx \frac{\zeta \omega_m^2}{4 \omega_0 \gamma \sqrt{\ln(t_m)}}, \quad (39) \]

where \( t_m \) is the time of measurement and \( \dot{\theta}_m \) is the minimum detectable rotation rate. By substituting \( \ln(t_m) \) from equation (37) in equation (39), the magnitude of Coriolis force corresponding to the angular velocity \( \dot{\theta}_m \) is given as \( 2m \dot{\theta}_m \sqrt{2m \omega_m^2 \theta_m^{-1}} \), which agrees with the SQL [18, 51] of the minimum measurable force in an interferometer [52–54].

### 4.1. Squeezing

When thermal noise is neglected, the main contribution to the noise comes from the shot noise and the radiation pressure noise. To understand the origin of shot noise and radiation pressure noise, we substitute equations (32) and (26) in equation (33). Then the relation between the noise at the signal in equation (35) and the input noise operators \( \delta \) and \( E_o \) is given as

\[
\nu_s^2 (\omega + \omega') \approx \frac{|E|^2}{4} \left[ \langle E'_s(\omega)E_s(\omega') \rangle \left( 4 + 4 \left( 2 \gamma - \gamma \frac{\Delta}{\zeta} \right)^2 \right) + \langle E'_s(\omega)E_s(\omega') \rangle \left( 4 + 4 \left( 2 \gamma - \gamma \frac{\Delta}{\zeta} \right)^2 \right) + \langle E'_s(\omega)E_s(\omega') \rangle \left( -2 - i2 \left( 2 \gamma - \gamma \frac{\Delta}{\zeta} \right) \right)^2 \right] + \langle E'_s(\omega)E_s(\omega') \rangle \left( -2 - i2 \left( 2 \gamma - \gamma \frac{\Delta}{\zeta} \right) \right)^2, \quad (40) \]

where \( \gamma \approx \frac{16 \sqrt{2} T_{\omega_m}}{\chi_p \zeta \omega_m} \). We neglected the higher order terms of \( \Delta/\zeta \) in equation (40). Clearly the output noise is dominated by the vacuum fluctuations entering the interferometer through the empty port of the beam splitter. Hence, squeezing [56] the input vacuum field will reduce the noise at the output of the interferometer. The squeezing can be accomplished by a squeeze operator \( s(\xi) \) which is related to the annihilation and the creation operators of the vacuum field, and to the squeezed state \( |\xi\rangle \) as

\[
s(\xi) = e^{i(\xi^2/2-|\xi|^2)}, \quad |\xi\rangle = s(\xi)|0\rangle, \quad (41)\]

where \( \xi = re^{i\phi} \) and \( |\xi\rangle \) is the squeezed state. \( r \) and \( \phi \) are amplitude and phase squeezing parameters. Using the equation (41), we can write

\[
\langle E'_s(\omega)E_s(\omega') \rangle = \sinh^2(r) \delta(\omega + \omega'), \quad \langle E(\omega)E'_s(\omega') \rangle = (1 + \sinh^2(r) \delta(\omega + \omega'),
\]

\[
\langle E_s(\omega)E'_s(\omega') \rangle = -\frac{e^{i\phi}}{2} \sinh(2r) \delta(\omega + \omega'), \quad \langle E'_s(\omega)E'_s(\omega') \rangle = -\frac{e^{-i\phi}}{2} \sinh(2r) \delta(\omega + \omega'). \quad (42) \]

Substituting equation (42) in equation (40), the square of the quantum noise at signal is given as

\[

\nu_s^2 = \nu_s^2 \left[ \cosh(2r) - \cos \phi \sinh(2r) + \frac{256 \gamma^3 \omega_m^2 (\cosh(2r) + \cos \phi \sinh(2r))}{4 \chi_p \zeta \omega_m^2 (\zeta \omega_m^2 + 8 \gamma^2 \omega_m^2 \Delta/\zeta)} \right] + \frac{32 \gamma^2 \omega_m (\sin \phi \sinh(2r))}{\sqrt{\zeta \omega_m^2 (\zeta \omega_m^2 + 8 \gamma^2 \omega_m^2 \Delta/\zeta)}} + \frac{2 \gamma_m \omega_m}{\chi_p \zeta \omega_m^2 + 8 \gamma^2 \omega_m^2 \Delta/\zeta}. \quad (43) \]

The shot noise and the radiation pressure noise in equation (43) can be adjusted by the squeeze parameters \( \phi \) and \( r \). Minimizing equation (43) with respect to \( \phi \) gives the optimum phase squeeze parameter \( \phi_o \), which is given as

\[
\tan \phi_o = -\frac{4 \left( 1 - \frac{\gamma \Delta}{2\gamma} \right)}{4 \left( \gamma - \gamma \frac{\Delta}{2\gamma} \right)^2}. \quad (44) \]
At $\phi = \phi_{fr}$, equation (43) is given as

$$v_o^2 = 2I \left( e^{-2r} + \frac{256r^4 I^2 \omega_m^2 e^{-2r}}{\zeta \omega_m^2 (\zeta^2 \omega_m^2 + 8\omega_m^2 \Delta / \zeta)} \right) + \frac{2\gamma_m^2 I^2 \omega_m^2}{\chi_p^2} \left( \frac{2}{\chi_p} + \frac{\omega_m^2 \Delta / \zeta}{\zeta^2 \omega_m^2 + 8\omega_m^2 \Delta / \zeta} \right). \tag{45}$$

In equation (45), the shot noise and the radiation pressure noise goes as $e^{-2r}$ [57], while the mirror noise is not affected by the squeezing. Hence, for large enough $r$, the total noise in equation (45) can be set to the level of mirror noise. At $I = I_{opt}$, equation (44) can be approximated as

$$v_o^2 = 2I_{opt} \left( e^{-2r} + \frac{e^{-2r}}{1 + \frac{\Delta}{2\zeta}} + \frac{2\gamma_m}{\omega_m} \right). \tag{46}$$

By equating the signal from equation (31) with equation (46), the minimum detectable angular velocity with the coherent laser and the squeezed vacuum as input is given as

$$\theta_L = \frac{\gamma_m \omega_m}{4\omega_p \sqrt{I_{opt} \omega_m}} \sqrt{\frac{e^{-2r} + e^{-2r}/2 + \gamma_m/\omega_m}{2 + \frac{\Delta}{2\zeta} + \frac{\gamma_m}{\omega_m}}} \approx \frac{\gamma_m \omega_m}{4\omega_p \sqrt{I_{opt} \omega_m}} \sqrt{\frac{\gamma_m}{\omega_m}}. \tag{47} \quad \text{(For } r \gg 1)$$

The $\theta_L$ represents the minimum detectable angular velocity when the vacuum field entering through the empty port of the beam splitter is squeezed. The first term on the RHS of equation (47) comes from the shot noise, the second term comes from the radiation pressure noise and the third term comes from the mirror noise. The shot noise and the radiation pressure noise terms can be set to zero [38] for $r \gg 1$, but the mirror noise cannot be reduced by the squeezing. Hence the mirror noise sets the lower limit on $\theta_L$ value. Since we are working with optomechanical mirrors with high quality factor, $\gamma_m/\omega_m$ is much smaller than one. For sufficiently large $r$, $e^{-2r} \approx 0$ and the rotation detection sensitivity is improved by a factor of $\sqrt{\gamma_m/\omega_m}$ as given in equation (47).

It is important to note that squeezing can enhance sensitivity only if the mirror noise is less the shot noise and radiation pressure noise. In practical situations, like at room temperature, noise from the mechanical oscillator can be much larger than the shot noise and the radiation pressure noise. In such cases, squeezing the input vacuum field can not enhance the rotation detection sensitivity. The temperature dependence of minimum measurable angular velocity is discussed in the next section.

4.2. Temperature dependence

Equations (39) and (47) estimate the minimum measurable angular velocity at 0 K temperature. In practical situations, especially when $\omega_m \ll \omega_p$, the thermal noise can not be neglected as it can be much larger than the shot noise and the radiation pressure noise. The thermal noise can be estimated by using the correlation function $\langle \dot{v}^2(\omega) \dot{v}(\omega') \rangle \approx k_B T / (\hbar \omega_m) \delta (\omega + \omega')$. $k_B$ and $T$ are the Boltzmann constant and temperature, respectively.

At temperature $T$, for $I = I_{opt}$, the temperature dependence of equation (38) is given as

$$v_o \approx \sqrt{4I_{opt} \left( 1 + \frac{\gamma_m}{\omega_m} \left( 1 + 2\frac{k_B T}{\hbar \omega_m} \right) \right)} \tag{48}.$$  

According to equation (39), the angular frequency of the mechanical oscillator should be small to improve the rotation detection sensitivity. But a smaller angular frequency for the mechanical oscillator increases the thermal noise as shown by the $k_B T / (\hbar \omega_m)$ term in equation (48). By equating equation (31) with equation (48), the minimum detectable angular velocity at temperature $T$ is given as

$$\theta_T = \frac{\gamma_m \omega_m^2}{4\omega_p \sqrt{I_{opt} \omega_m}} \sqrt{\frac{1 + \gamma_m}{\omega_m} \left( 1 + 2\frac{k_B T}{\hbar \omega_m} \right)} \tag{49}.$$  

For $\sqrt{k_B T \gamma_m/\hbar} \gg \omega_m$, substituting $I_{opt}$ from equation (37) in equation (38) gives

$$\theta_T = \frac{k_B T \gamma_m}{\sqrt{\hbar^2 m \omega_m}} \tag{50},$$

where $\theta_T$ is the minimum detectable angular velocity at temperature $T$. For $\omega_m \ll \sqrt{k_B T \gamma_m/\hbar}$, when the input intensity is $I_{opt}$ the noise from the mechanical mirror is more dominant than the shot noise and radiation pressure noise. Hence, the $\theta_T$ is determined by thermal noise from the mechanical oscillator.
When the squeezed vacuum is sent through the empty port of the beam splitter, the thermal dependence of $\dot{q}_T$ is given as

$$\dot{q}_T \approx \frac{\gamma_m^2}{4\omega_m^2} \sqrt{I_{\text{opt}}} \left\{ \frac{e^{-2r}}{2} + \frac{e^{-2r}}{2} + \frac{\gamma_m}{\omega_m} \left( 1 + 2 \frac{k_B T}{\hbar \omega_m} \right) \right\}^{1/2},$$

(51)

where $\dot{q}_T$ is the minimum detectable angular velocity at temperature $T$ when the vacuum entering through the empty port of the beam splitter is squeezed. The effect of temperature with squeezed vacuum is discussed below in case (i) and case (ii).

Case (i): for $\left( 1 + 2 \frac{k_B T}{\hbar \omega_m} \right) \frac{\gamma_m}{\omega_m} < 1$: At $r = 0$, the mechanical oscillator noise in equation (51) is smaller than the shot noise and the radiation pressure noise and hence the total noise is determined by the shot noise and radiation pressure noise. By squeezing the input vacuum such that $\phi = \phi_0$ and $r > 1$, the shot noise and the radiation pressure can be approximately set to zero. Then the total noise decreases to the level of the mirror oscillator noise, and hence the rotation detection sensitivity can be improved beyond equation (39) by using squeezed vacuum.

Case (ii): for $\left( 1 + 2 \frac{k_B T}{\hbar \omega_m} \right) \frac{\gamma_m}{\omega_m} > 1$: At $r = 0$, the mechanical oscillator noise in equation (51) is larger than the radiation pressure noise and shot noise. Hence the total noise is determined by the mechanical oscillator noise. Since squeezing the input vacuum can not reduce the mechanical oscillator noise, the rotation detection sensitivity can not be improved. At temperature $T$, the squeezing technique described in this work can improve the rotation detection sensitivity only if $\omega_m > \sqrt{2 \gamma_m k_B T / \hbar}$.

For $\omega_m \ll \sqrt{2 \gamma_m k_B T / \hbar}$, by substituting equation (37) in to equation (51), the minimum detectable angular velocity is given as

$$\dot{q}_T = \frac{k_B T \gamma_m}{\sqrt{2^2 m \omega_m}}.$$

(52)

From equations (50) and (52), we can conclude that the rotation detection sensitivity can not be improved by using the squeezed vacuum technique when the mirror oscillator noise is much larger than the shot noise and radiation pressure noise. However, the rotation detection sensitivity can be improved by increasing the mass of the mechanical oscillator. Increasing the mass of the mechanical oscillator also increases the optimum intensity required to achieve the best sensitivity. According to equation (37), the increase in $I_{\text{opt}}$ value due to an increase in $m$ can be compensated by using a cavity with large $\omega_0 / \zeta$ value.

4.3. Simulation

For simulation purpose, the optomechanical cavity parameters are taken from [55]: $m = 69 \times 10^{-3}$ Kg, $\omega_0 = 18 \times 10^{14}$ Hz, $\omega_m = 532.5$ rad s$^{-1}$, $\zeta = 7.7 \times 10^{6}$ Hz, $\gamma_m = 0.95 \times 10^{-3}$ Hz, $l = 12.3 \times 10^{-3}$ m. For these optomechanical cavity parameters, using equation (37), the optimum power is estimated as $P_{\text{opt}} = \omega_0 I_{\text{opt}} = 0.24$ mWatt. Assuming $\dot{\phi} = 0.1$ m s$^{-1}$, $t_m = 1$ s, at $I = I_{\text{opt}}$, the minimum detectable angular velocity for different cases are given below.

$$\frac{\dot{q}_m}{2\pi} = 2.3 \times 10^{-13} \text{ Hz} \quad \text{ Laser and vacuum fields as input}$$

$$\frac{\dot{q}_l}{2\pi} = 3.1 \times 10^{-16} \text{ Hz} \quad \text{ Laser field and squeezed vacuum as input}$$

$$\frac{\dot{q}_T}{2\pi} = 1.2 \times 10^{-10} \text{ Hz} \quad \text{ at room temperature}$$

Note that for the optomechanical cavity parameters used in this simulation, $\omega_m \ll \sqrt{2 \gamma_m k_B T / \hbar}$ at room temperature. Hence, the mechanical oscillator noise at room temperature is much larger than the shot noise and radiation pressure noise. In this case, the squeezed vacuum technique can not enhance the sensitivity at room temperature.

5. Conclusion

Application of optomechanical cavities for rotation detection via measurement of Coriolis force induced effects is discussed in this paper. Analytic expressions to estimate the optimum power and minimum measurable rotation rate are derived. The implementation of squeezed vacuum technique for improving the rotation detection sensitivity discussed. We showed that at zero Kelvin temperature, using the squeezing technique improves the rotation detection sensitivity by a factor of $\sqrt{\gamma_m / \omega_m}$. Temperature dependence of the minimum measurable angular velocity is studied. We showed that for $\omega_m \ll \sqrt{2 \gamma_m k_B T / \hbar}$, the minimum detectable
rotation rate is given by \( \sqrt{\frac{k_B T m^2 f^2}{m_B}} \). We further showed that squeezing technique can improve the rotation detection sensitivity only if \( \omega_m > \sqrt{2 \gamma_m k_B T / \hbar} \). For \( \omega_m \ll \sqrt{2 \gamma_m k_B T / \hbar} \), the noise from the mechanical oscillator is larger than the shot noise and the radiation pressure noise, hence squeezing the input vacuum does not improve the sensitivity in this case.

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