Low-Energy Cutoffs In Electron Spectra Of Solar Flares: Statistical Survey

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Abstract The Ramaty High Energy Solar Spectroscopic Imager (RHESSI) X-ray data base (February 2002 – May 2006) has been searched to find solar flares with weak thermal components and flat photon spectra. Using a regularised inversion technique, we determine the mean electron flux distribution from count spectra of a selection of events with flat photon spectra in the 15–20 keV energy range. Such spectral behaviour is expected for photon spectra either affected by photospheric albedo or produced by electron spectra with an absence of electrons in a given energy range, e.g. a low-energy cutoff in the mean electron spectra of non-thermal particles. We have found 18 cases which exhibit a statistically significant local minimum (a dip) in the range of 10–20 keV. The positions and spectral indices of events with low-energy cutoff indicate that such features are likely to be the result of photospheric albedo. It is shown that if the isotropic albedo correction was applied, all low-energy cutoffs in the mean electron spectrum were removed and hence the low energy cutoffs in the mean electron spectrum of solar flares above ∼12 keV cannot be viewed as real features in the electron spectrum. If low-energy cutoffs exist in the mean electron spectra, the energy of low energy cutoffs should be less than ∼12 keV.

Keywords: Flares; X-Ray Bursts, Spectrum; Energetic Particles, Electrons

1. Introduction

X-ray observations are often used to infer various properties of energetic electrons accelerated during the solar flares. The spatially integrated X-ray photon spectrum $I(\epsilon)$ (photons cm$^{-2}$ s$^{-1}$ keV$^{-1}$) is related to mean electron flux spectrum $F(E)$ (electrons cm$^{-2}$ s$^{-1}$ keV$^{-1}$) via the rather simple linear integral relation (Brown et al., 2003)

$$I(\epsilon) = \frac{nV}{4\pi R^2} \int_\epsilon^\infty \mathcal{P}(E)Q(\epsilon,E)dE$$

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for source volume $V$, mean plasma density $\bar{n}$, and isotropic bremsstrahlung cross-section per unit photon energy $\epsilon$, $Q(\epsilon, E)$ (Haug, 1997). Although the angular distribution of energetic electrons is generally unknown, the recent observations (Kontar and Brown, 2006) suggest rather close to isotropic distribution of electrons. The exact plasma density distribution and flaring volume are also poorly known and therefore value $\bar{n}V\bar{F}(E)$ is normally inferred. The value $\bar{n}V\bar{F}(E)$ is model independent (Brown et al., 2003), and the detailed energy structure of this is related to electron acceleration and propagation physics. Radio emission spectrum of solar energetic particles, although normally only at above a few hundred keV energies, is an alternative approach to infer electron beam and plasma parameters (Altyntsev et al., 2008).

The spatially integrated spectrum of energetic electrons $\bar{n}V\bar{F}(E)$ is often approximated as a sum of an isothermal Maxwellian distribution and a non-thermal power-law distribution (Holman et al., 2003). The thermal component often dominates the overall spectrum at low energies $\leq 20$ keV and little can be said about the low-energy part of non-thermal distribution. However, the low-energy part of non-thermal spectra plays a crucial role in the solar flare diagnostics. Most of the non-thermal electron energy is concentrated in this part, hence this defines the total energy budget of the flare. In addition, this part of the spectrum is more effectively influenced by various electron propagation effects like collisions (Brown, 1971) or beam-plasma interactions (Melnik et al., 1999), thus playing an important role in the electron transport diagnostics in the solar flares. Various model-based methods to find the value of low energy cutoff have been used. Requiring that the assumed thermal emission dominate over non-thermal emissions Sui et al. (2005) find a low energy cutoff of $\sim 24$ keV should be present. Assuming ”theoretical Neupert effect” to be satisfied Veronig et al. (2005) conclude that the low energy cutoff should be between 10 keV and 30 keV for four flares analysed in the paper. Hannah et al. (2008) have used empirical relationship between the observed parameters of the photon power-law fit and the low-energy cutoff of the electron distribution and have found that the low-energy cutoffs in microflare events could range from 9 to 16 keV with the median being around 12 keV. In this paper we will focus on the model-independent inference of low-energy cutoffs in the mean electron spectra.

High resolution spectra observed by RHESSI (Lin et al., 2002) allows us to infer detailed structure of electron distribution often never seen before. Piana et al. (2003) have demonstrated that the mean electron spectrum $\bar{n}V\bar{F}(E)$ has a statistically significant local minimum at approximately 50 keV in the electron spectrum of GOES X-class July 23, 2002 solar flare, although this feature is likely to be an instrumental effect caused by a pulse pile-up. Kontar and Brown (2006) show that some electron spectra inferred from RHESSI X-ray spectra free from pile-up issues seem inconsistent with a simple collisional thick-target model (Brown, 1971). However, the photon spectra of these events should be corrected for albedo - Compton back-scattered X-rays (Kontar et al., 2006). Kašparová et al. (2005) have shown that the spectrum of the August 20, 2002 event with a puzzlingly large value of the low-energy cutoff $\sim 30$ keV can be understood in terms of the photospheric albedo.
Flares showing a weak thermal component allow us to scrutinize the low-energy part of the non-thermal distribution of electrons. The analysis can be done either by assuming a functional form of the electron spectrum (Holman et al., 2003) or by using the regularised inversion techniques (Kontar et al., 2004). It is known that flat X-ray spectra (low value of photon spectral index) can require low-energy cutoffs in the power-law distributions when a functional form is assumed (Kašparová et al., 2005; Sui et al., 2007), whereas the model independent approach, via the regularised inversion technique, (Piña et al., 2003) may show a dip or a gap in the electron distribution (Kontar et al., 2006; Kašparová et al., 2007).

In this paper we present the results of a systematic search for dips in the mean electron flux distribution using the RHESSI solar flare database for the period of Feb 2002 - May 2006. Section 2 describes the selection criteria for the flare photon spectra and the application of the regularised inversion method for the determination of the corresponding mean electron flux spectra. Section 3 discusses energies and depths of the obtained statistically significant dips and their relation to the photospheric albedo. The analysis confirms previous suggestions that the isotropic albedo correction is capable of removing all statistically significant dips in the mean electron flux distribution. The obtained results are summarised in Section 4.

2. Data analysis

As a basis, we used the list of 398 flares with weak thermal component previously determined by Kašparová et al. (2007). Although this has limited the total number events for our analysis, it has helped us to avoid various effects, such as pulse pile-up and particle contamination, complicating the spectral analysis (Schwartz et al., 2002). Next, we chose the 177 events with the smallest values of spectral index $\gamma_0 \leq 4.0$, where $\gamma_0$ was measured in the range between 15 and 20 keV - see Kašparová et al. (2007).

For each flare, the spectra were accumulated over the duration of the impulsive phase, i.e. in the interval when counts at energies above 50 keV were above background (Figure 1). The spectra were generated in the energy range from 3 to 100 keV with 1 keV resolution avoiding detectors 2 and 7 due to their low resolution (Smith et al., 2002). The background counts were removed in a standard way (Schwartz et al., 2002).

To obtain a starting point for the regularised inversion, spectra were forward fitted assuming an isothermal plus a non-thermal double power-law distribution of $\tilde{F}(E)$, for example Holman et al. (2003). Spectra were then inverted within OSPEX\textsuperscript{1} using the regularised inversion routines\textsuperscript{2} (Kontar et al., 2004) minimizing the functional (Tikhonov, 1963)

\[
\mathcal{L}(\tilde{F}) \equiv \|A\tilde{F} - C\|^2 + \lambda\|L\tilde{F}\|^2 = \text{min}
\]

\textsuperscript{1}http://hesperia.gsfc.nasa.gov/ssw/packages/spex/doc/ospex\_explanation.htm
\textsuperscript{2}http://www.astro.gla.ac.uk/users/eduard/rhessi/inversion/
where

$$A = RB$$

$$B_{ij} = \frac{\pi V}{4\pi R^2} Q((\epsilon_{i+1} + \epsilon_i)/2, (E_{j+1} + E_j)/2) \Delta E_j$$

where $R$ is a spectral response matrix of RHESSI converting photons to photon counts, $B$ is a matrix representation of our linear integral (1), $L$ is the matrix representation of the additional constraint, $C$ is data vector of background-subtracted count spectrum (counts cm$^{-2}$ s$^{-1}$ keV$^{-1}$), and $F$ is the vector of unknown density-weighted mean electron spectrum $\bar{n}V \bar{F}$.

The equation 2 can be solved analytically using Generalised Singular Value Decomposition. Regularisation parameter, $\lambda$, is determined from the analysis of normalized residuals, $r_k = ((A F_\lambda - C) / \delta C_k$, where $\delta C_k$ are the uncertainties of the count spectrum. Then the deviation weighted by the error

$$\|(A F_\lambda - C)(\delta C)^{-1}\|^2 = \alpha$$

accounts quite accurately for point-to-point error variation. Indeed $\lambda$ defined by Equation (5) has accounted for detailed structure of errors. Parameter $\alpha$ is chosen to make the residuals $r_k$ to be close to gaussian (Kontar et al., 2004).

Using first order regularisation, i.e. operator $L$ being a finite difference representation of a derivative operator, we found mean electron flux spectrum $\bar{n}V \bar{F}(E)$ for all 177 flares with spectral index $\gamma_0$ less than 4.

3. Local minima (dips) in the mean electron flux spectrum

With the mean electron flux determined, the spectrum was examined for local minima or so-called dips. These dips were analyzed to infer the dip (local minimum) parameters: the energy $E_d$ at which the dip minima occurs and the depth of the dip $d$ in terms of $\sigma$, where $\sigma$ is the statistical uncertainty on the inferred mean electron spectrum $\bar{n}V \bar{F}(E)$. This depth was calculated by dividing the difference between the minimum and the maximum above the dip in units of electron spectra uncertainty at the minimum (Figure 2). We have found 18 events with a dip depth deeper than 1 $\sigma$ in the electron distribution function. The details of these events are presented in Table 1. Some of the events presented in the Table 1 have been found using thick-target model fit with a single power-law and low energy cutoff (Sui et al., 2007).

The local minima in the mean electron spectra are 6-10 keV wide and hence cover a few statistically independent energy points. For example, if a dip is three points wide at 1$\sigma$ level in each point, the probability to find three consecutive points outside 1$\sigma$ interval is $(1 - 0.68)^{3} = 0.03$ and the corresponding statistical significance of the minimum is $1 - 0.03 = 0.97$. In general, given that the errors have normal distribution the statistical significance of the local minimum is $1 - \Pi_{i=1}^{N}(1 - \text{erf}(d_i/\sqrt{2}))$, where $N$ is the total number of statistically independent points in a dip (local minimum) and $d_i$ is the depth of each point in units of the corresponding $\sigma_i$ uncertainties. The sizes of statistically independent energy
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Table 1. Events with a local minimum (dip) in the mean electron spectrum dipper than 1σ; \(d_i\) is the depth of a dip in \(\sigma\); \(E_d\) is the energy of the local minimum; \(\mu = \cos(\theta)\) is the cosine of flare heliocentric angle; \(\gamma_0\) is the photon spectral index measured in the range 15 - 20 keV, with typical uncertainty \(\pm 0.2\).

| Flare Date | Time     | \(d_i\) (\(\sigma\)) | \(E_d\) (keV) | \(\mu\) | \(\gamma_0\) |
|------------|----------|-----------------------|---------------|--------|-------------|
| 11-Apr-2002| 03:06:08.00 | 1.7                   | 15.5          | 0.96   | 1.6         |
| 25-Apr-2002| 05:55:12.00 | 2.5                   | 16.5          | 0.96   | 1.7         |
| 29-Jun-2002| 09:29:40.00 | 2.0                   | 15.5          | 0.16   | 2.7         |
| 30-Jul-2002| 17:37:36.00 | 1.9                   | 18.5          | 0.97   | 2.1         |
| 17-Sep-2002| 05:51:12.00 | 2.7                   | 16.5          | 0.74   | 1.7         |
| 24-Oct-2002| 00:09:24.00 | 2.0                   | 15.5          | 0.94   | 2.2         |
| 22-Nov-2002| 13:29:36.00 | 2.8                   | 17.5          | 0.93   | 2.5         |
| 10-Mar-2003| 10:02:56.00 | 1.2                   | 13.5          | 0.72   | 2.9         |
| 20-Nov-2003| 05:10:36.00 | 1.3                   | 12.5          | 0.93   | 2.9         |
| 1-Apr-2004 | 23:00:32.00 | 2.9                   | 15.5          | 0.88   | 2.6         |
| 20-May-2004| 17:16:12.00 | 1.4                   | 15.5          | 0.19   | 2.9         |
| 19-Jul-2004| 20:56:52.00 | 1.9                   | 16.5          | 0.73   | 2.0         |
| 14-Aug-2004| 08:15:30.00 | 1.9                   | 18.5          | 0.82   | 1.6         |
| 28-Oct-2004| 12:13:32.00 | 1.5                   | 16.5          | 0.30   | 3.1         |
| 9-Nov-2004 | 15:10:08.00 | 1.1                   | 15.5          | 0.66   | 3.6         |
| 30-Nov-2004| 03:56:12.00 | 1.2                   | 14.5          | 0.97   | 2.7         |
| 21-Jan-2005| 06:32:20.00 | 1.0                   | 15.5          | 0.29   | 2.5         |
| 5-Apr-2006 | 22:45:28.00 | 2.5                   | 17.5          | 0.62   | 2.2         |

bins can be estimated from horizontal errors (Figure 2). Thus, the local minimum (dip) shown in Figure 2 has statistical significance \(\sim 1 - [1 - \text{erf}(2.9/\sqrt{2})][1 - \text{erf}(1.2/\sqrt{2})] \approx 99.9\%\).

The dips are located between the thermal and non-thermal component and appear approximately at the same energy, in the range between 13 and 19 keV. The dip energies \(E_d\) are given in Table 1 as the bin centre energy. There is no preferential energy in this range (Figure 3 - right panel.)

There is a clear pattern in the results: flares with dips tend to occur at locations with large \(\mu = \cos(\theta)\), where \(\theta\) denotes the flare heliocentric angle. Only 4 events are located close to the solar limb \(\mu < 0.5\) while 14 are near the disk centre \(\mu < 0.5\) - see left panel in Figure 3. There is also no strong evidence for the dip energy being dependent on the flare location or on the dip depth - see left and right panel in Figure 4, respectively.

3.1. Correction for X-ray Compton scattered photons

Previous works (Kontar et al., 2004; Kašparová et al., 2007) have shown that a feature such as a dip can be a signature of distortion by albedo contribution. Figure 5 (right panel) shows that larger dips appear for flatter X-ray spectra. Furthermore, events with large depths tend to appear close to the disc centre, see Figure 4. This is consistent with the albedo model (Kontar et al., 2006)
which predicts larger albedo contribution for flat spectra and disc centre events. It is noteworthy that the albedo contribution to the observed photon spectrum is still noticeable even for the flares at heliocentric angles larger than 50°. The albedo-corrected mean electron spectra for flares at $\mu < 0.4$ show no dips larger than 1$\sigma$.

The isotropic albedo correction (Kontar et al., 2006) was applied to all the events with a dip in Table 1 and new $\tilde{n}V\tilde{F}(E)$, i.e. corresponding to the primary photon spectra, were derived. Such albedo corrected mean electron spectra does not reveal any significant dip, i.e. with depth $\geq 1\sigma$. 

Figure 1. Example of a solar flare with flat photon spectrum. Upper panel: RHESSI light curves; The vertical lines show the accumulation time interval for spectroscopic analysis. Lower panel: Photon spectrum and forward fit (solid line), isothermal component (dashed line), nonthermal component (dotted line).
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Figure 2. Mean electron distribution spectrum for April 1, 2004 ~ 23 : 00 UT solar flare. The observed electron spectrum (solid line) and electron spectrum after isotropic albedo correction (dashed line) are given with 1σ error bars. The dip depth, d, is shown.

Figure 3. Histograms of 18 events with clear dip: Left panel: Number of events as a function of cosine of heliocentric angle; Right panel: Number of events as a function of dip energy $E_d$ in keV.

Figure 4. Left panel: Dip energy versus cosine of heliocentric angle $\mu$; Right panel: Dip depth versus dip energy.
4. Summary and Discussion

Our analysis shows that the clear dips are rare, only 18 of 177 events demonstrate a clear dip. The small number of events with a clear dip or low energy cutoff can be explained by a variety of reasons. Firstly, it suggests that the number of very flat primary spectra is rather small and that the vast majority of flares have primary spectral index larger than 2. Indeed, although the total number of events with a dip is small (left panel in Figure 5) the fraction of events could be as high as 60\% for small spectral indices (right panel in Figure 5). This can be viewed as a lower limit on spectral indices of accelerated electrons in solar flares.

In the case of a thick-target model, the spectral index of accelerated electrons should be larger than 3. Secondly, the small number of events with a dip or low energy cutoff suggests that the thermal component substantially influences the spectrum in the range of above 10 keV for the majority of flares. This conclusion is partially supported by Kašparová et al. (2007), who have found a large number of events with very soft spectra with spectral indices $\gamma_0$ which are larger than 5.

However, when dips occur in the mean electron spectrum, the local minima in the electron flux spectrum is consistent with albedo model (Kontar et al., 2006). In the standard solar flare model, the electrons are believed to propagate downwards and hence the reflected flux from the photosphere should be larger. In this work the albedo was assumed to be isotropic and this can be viewed as a lower limit on albedo contribution. Therefore the explanation that albedo might be overestimated seems unlikely. As can be seen in Figure 5 flares with a low value of $\gamma_0$ are very likely to exhibit a local minimum in the mean electron flux spectrum, therefore the small number of flares with flat spectra results in the the low number of flares with dips. In addition, the energies of the dip minima are concentrated near 15 keV, the energy which is expected from isotropic albedo model (see Figure 1 in Kašparová et al. (2007)). We also note that earlier observation of flat X-ray spectrum are consistent with the albedo model. The flares suggesting high value of low energy observed by Nitta et al. (1990), Farnik et al.
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(1995) had flat X-ray spectra and were disk centre events confirming conclusions of this work.

The low-energy cutoff is often introduced to limit the total number of non-thermal electrons in solar flares. Since all dips found in the electron spectra can be easily "removed" by applying albedo correction, our results allow to conclude that if low-energy cutoff exists in solar flare spectra it should be below \( \sim 12 \) keV. This value puts an upper limit on the low energy cutoffs and is somewhat less than the values published in the literature. In addition, since the total number of electrons accelerated in solar flares is dependent on the low-energy cutoff the lower value of low-energy cutoff makes the electron number problem even more severe.

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