Reduction in the operating voltage and active power of extended electric power sources

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Abstract. Chemical sources of current are investigated as lines with distributed parameters. Analytical expressions are obtained for the voltage and active power values of the source at different distances from the beginning of the cell as well as dependences of the working voltage and active power on the source length. Effects of a reduction in the operating voltage and active power are due to the flow of electric current along the source during operation. The magnitude of these effects depends not only on the length of the source, but also on the ratio of the characteristic resistance to the load resistance.

1. Introduction
The most widely used for "direct" production of electrical energy are chemical processes [1-3], photoelectric processes in semiconductors [4-6], thermoelectric processes in semiconductors and metals [7-9]. The purpose of the study in this paper is a feature of powerful electric sources operating on the basis of such processes, in particular natural sources having large geometric dimensions. The main objectives of the study are to obtain analytical expressions for the voltage and active power values of the source at different distances from the beginning of the cell as well as dependences of the working voltage and active power on the source length. Significant geometric dimensions have solar batteries [4-6], powerful chemical sources [1-3], especially those operating at high discharge rates, when individual cells are connected in parallel. In this case, the load is attached to the source as a rule locally to the poles of one of many cells. In addition, for example, the construction of lithium primary sources is such that the length of even one cell is very significant [2]. In this case, the formation of the current through the load resistance involves charges that flow throughout the whole source. That is, the operation of a large-sized source with local load turns out to be similar to the operation of a power line with distributed parameters [10]. In [11], a preliminary consideration was taken of this circumstance for solar batteries. Let us consider these effects with the example of chemical sources of current (CSC).

2. CSC with local load
Let us consider the features of the CSC operation taking into account its length in the direction of the x axis. In this case, we assume that the CSC operates in a regime where the effects leading to a decrease in the electromotive force are negligible.
Figure 1. Extended CSC.

Figure 1 shows the scheme of the simplest CSC in a form similar to the scheme of a line with distributed parameters. The electrochemical system consists of an anode (the upper plate in Figure 1), a cathode (the lower plate in Figure 1), and an ionic conductor, an electrolyte between them. The following designations are used in the case of CSC. \( r_0 \), \( g_0 \) are the longitudinal plate resistance and the transverse conductivity of the ion conductor per unit length of the line. Then the resistance of an infinitely small segment of the line length \( dx \) will be \( dr = r_0 dx \), and electrolyte conductivity of this segment \( dg = g_0 dx \), \( l \) – length of the source along the \( x \) direction.

The main difference from the line with distributed parameters lies in the fact that the extended source uses charging currents instead of leakage currents. Let us consider a dependence of the voltage between the upper and lower plates of the CSC \( U \) and current \( I \) on the distance \( x \) from the end of the line (of an extended source). In this case, the voltage and current at the end of the line, \( U_2 \) and \( I_2 \), are considered known. A voltage drop in the \( dx \) element, as in a usual line with distributed parameters [10], becomes:

\[
\frac{dU}{dx} = \frac{1}{r_0} dr = I r_0 dx.
\]  

(1)

A change in the strength of current due to the passage of charging current (unlike the leakage current in a usual line) has the form:

\[
\frac{dI}{dx} = -(E - U) g_0 dx.
\]  

(2)

Here \( E \) is EMF of the source – voltage \( U \) with the open switch prior to load (at idle). From equations (1) and (2) we have:

\[
\frac{dU}{dx} = I r_0,
\]

(3)

\[
\frac{dI}{dx} = -(E - U) g_0.
\]

(4)

We see that the difference from the usual line [10] lies in equation (4).

By differentiating the two sides of equation (3) and replacing \( \frac{dI}{dx} \) from equation (4), we obtain:

\[
\frac{d^2U}{dx^2} = -(E - U) g_0 r_0.
\]

(5)

This inhomogeneous second-order differentiation equation describes a dependence of the voltage \( U \) on the \( x \) distance. Its general solution has the form:

\[
U = E - \left( A_1 e^{\beta x} + A_2 e^{-\beta x}\right),
\]

(6)

\( A_1, A_2 \) are expansion coefficients, \( \beta = \sqrt{r_0 g_0} \) is the analog of the attenuation factor for a long line. From equation (3) we have:

\[
I = \frac{1}{r_0} \frac{dU}{dx} = -A_1 \frac{1}{r_c} e^{\beta x} + A_2 \frac{1}{r_c} e^{-\beta x},
\]

(7)
\[ r_c = \sqrt{\frac{r_0}{g_0}} \]
is a characteristic resistance.

Let when \( x = 0 \) there be \( U = U_2 \) and \( I = I_2 \).

Then
\[ A_1 = \frac{1}{2} (E - U_2 - r_c I_2), \quad (8) \]
\[ A_2 = \frac{1}{2} (E - U_2 + r_c I_2) \quad (9) \]

Upon substituting equations (8, 9) into (6, 7), we obtain a dependence of the voltage and current in the source on the distance \( x \).
\[
U = E - \left[ \frac{1}{2} (E - U_2 - r_c I_2) e^{\beta x} + \frac{1}{2} (E - U_2 + r_c I_2) e^{-\beta x} \right] \quad (10)
\]
\[
I = -\frac{1}{2} (E - U_2 - r_c I_2) \frac{1}{r_c} e^{\beta x} + \frac{1}{2} (E - U_2 + r_c I_2) \frac{1}{r_c} e^{-\beta x} \quad (11)
\]
The voltage and the current at the edge of the source on the other side of the load is obtained by substituting \( x = l \) into these dependencies.
\[
U_1 = E - \left[ \frac{1}{2} (E - U_2 - r_c I_2) e^{\beta l} + \frac{1}{2} (E - U_2 + r_c I_2) e^{-\beta l} \right] \quad (12)
\]
\[
I_1 = -\frac{1}{2} (E - U_2 - r_c I_2) \frac{1}{r_c} e^{\beta l} + \frac{1}{2} (E - U_2 + r_c I_2) \frac{1}{r_c} e^{-\beta l} \quad (13)
\]
The load resistance \( R_2 \), the voltage \( U_2 \) and the current \( I_2 \) are interrelated by Ohm’s law, that is,
\[
R_2 = \frac{U_2}{I_2} \quad (14)
\]
In the mode of matched load (when \( R_2 = r_c \)), from equations (9, 10) we have:
\[
U' = E(1 - ch \beta x) + U_2 e^{\beta x}, \quad (15)
\]
\[
I' = -\frac{E}{r_c^2} sh \beta x + I_2 e^{\beta x}, \quad (16)
\]
sh and ch denote the hyperbolic sine and cosine.

As in the usual line with distributed parameters, a change in the voltage along the source in the extended source under consideration is caused by a voltage drop on the plate resistance of the source at the passage of current. It may therefore be assumed at \( l \gg d \) that when \( x = l \) we have \( U_1 = E \). From equation (11) we then have:
\[
U_2 = E \cdot f_l, \quad (17)
\]
Here \( f_l \) is a function:
\[
f_l = \frac{ch \beta l}{ch \beta l + \frac{R_2}{r_c}}. \quad (18)
\]
Upon substituting equation (17) into (10), we obtain:
\[
U = E \left[ 1 - \frac{1}{2} \left( 1 - f_l - \frac{r_c}{R_2} f_l \right) e^{\beta x} + \frac{1}{2} \left( 1 - f_l - \frac{r_c}{R_2} f_l \right) e^{-\beta x} \right]. \quad (19)
\]
Under the assumption that \( U = E \) when \( x = l \) from equation (15) we have in the mode of matched load:
\[
U_2' = E \frac{ch \beta l}{e^{\beta l}} = \frac{1}{2} E \left( 1 + \frac{e^{-\beta l}}{e^\beta l} \right) = \frac{1}{2} E \left( 1 + \frac{1}{e^\beta l} \right). \quad (20)
\]
Upon substituting equation (20) into (15, 16), we obtain in the mode of matched load:
\[ U' = E \left[ 1 - \frac{1}{2} ch \beta x + \frac{1}{2} \frac{1}{2e^{2\beta l}} e^{\beta x} \right], \]  
\[ I' = \frac{1}{\tau_0} \frac{du}{dx} = E \frac{\beta}{\tau_0} \left[ \left( \frac{1}{2} + \frac{1}{2e^{2\beta l}} \right) e^{\beta x} - sh \beta x \right]. \]  

3. Results and discussions

Processes in the extended CCS occur almost as well as similar processes in an extended solar battery. So, the dependences of voltage and current on distance \( x \) are practically the same in both cases, although the meaning of the parameters is different. The greatest difference between the parameters of the processes from those considered should occur in the case of natural sources. Therefore, a more detailed analysis of the obtained dependences is of interest.

For practical use, the most interesting is the dependence of the "working" voltage \( U_2 \) on the length of the source (17, 18, 20) since it is easy to derive a dependence of the useful active power on the length of the source from these formulas.

Hence we obtain:

\[ W = \frac{U_2^2}{R_2} = \frac{E^2}{R_2} f_i^2 = \frac{E^2}{R_2} \frac{ch^2 \beta l}{(ch \beta l + \frac{rc}{R_2} sh \beta l)^2}, \]  
\[ W' = \frac{(U_2')^2}{R_2} = \frac{E^2}{R_2} \left( f_i' \right)^2 = \frac{E^2}{R_2} \frac{1}{4} \left( 1 + \frac{1}{e^{2\beta l}} \right)^2, \]  

\( W \) is the useful active power, \( W' \) is this quantity in the matched load.

![Figure 2](image-url)  

Figure 2. \( f_i^2 \) as a function of \( \beta l \) in the formula (23).

Figure 2 illustrates the dependence of the active power (multiplied by \( R_2/E^2 \)) on the length of the
source (on the βl) for various r_c/R_2. It is seen that for small values of r_c/R_2, the active power is practically independent of the source length. For each value of r_c/R_2, there is a limit to which the active power tends with increasing source length. So in the mode of matched load this limit is 0.25 of maximum power. For r_c/R_2 = 0.1, this limit is 0.825 of the maximum power. Saturation (reaching the limit) occurs earlier, the larger r_c/R_2.

4. Conclusions
Thus, in long sources, there are effects of a reduction in the operating voltage and active power due to the flow of electric current along the source during operation. The magnitude of these effects depends not only on the length of the source, but also on the r_c/R_2 ratio. Since the characteristic resistance r_c and the length l are large for natural sources [12], the most significant such effects should primarily be manifested in natural sources. In artificial sources, the marked effects should be noticeable first of all in operating modes close to a short circuit, that is, for small values of R_2.

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