Logical Reversibility and Physical Reversibility
in Quantum Measurement

Masahito Ueda
Department of Physical Electronics, Hiroshima University,
Higashi-Hiroshima 739, Japan

Abstract
A quantum measurement is logically reversible if the premeasurement density operator of the measured system can be calculated from the postmeasurement density operator and from the outcome of the measurement. This paper analyzes why many quantum measurements are logically irreversible, shows how to make them logically reversible, and discusses reversing measurement that returns the postmeasurement state to the premeasurement state by another measurement (physical reversibility). Reversing measurement and unitarily reversible quantum operation are compared from the viewpoint of error correction in quantum computation.

1. Introduction
In classical mechanics, one can, in principle, measure a quantity of an object very precisely while keeping the backaction of the measurement as small as one wishes. The situation drastically changes in quantum mechanics; the more precisely one wants to measure a quantity of an object, the more strongly one has to couple the measuring apparatus to the object with the greater backaction of the measurement. In addition, the state reduction occurs in a way that depends on a specific outcome of measurement which is unpredictable. A quantum measurement thus introduces an asymmetry in the direction of time. With respect to the past, it confirms the predicted probability distribution of an observable by a number of measurements performed on an ensemble representing the same quantum state. With respect to the future, it produces a new quantum state via state reduction by a single measurement. Such disparate roles played by a quantum measurement with respect to the past and future are the origin of irreversibility in the dynamical evolution of a quantum-mechanical system. I will nevertheless show that a broad class of quantum measurement can be logically reversible in the sense that the premeasurement density operator of the measured system can be calculated from the postmeasurement density operator and from the outcome of measurement [1].

The concept of logical reversibility provides a criterion for deciding whether or not the system’s information is preserved in the course of measurement. A quantum measurement is logically reversible if and only if all pieces of information concerning the measured systems are preserved during the measurement. If some pieces of information are lost, the measurement is logically irreversible.

Recently, there has been considerable interest in restoring ‘lost’ coherence in a device that performs quantum computation because in a quantum computer information is stored in a
superposition of states and therefore decoherence makes long computation impossible. One candidate to recover the ‘lost’ coherence is unitarily reversible quantum operation which restores the original state by a unitary transformation, hence with unit probability \[2\]. I will propose another possibility to resolve this problem, namely, by means of a reversing measurement scheme \[3\] and compare these two methods.

This paper is organized as follows. Section II analyzes why many familiar quantum measurements are logically irreversible and examines conditions that are necessary for a measurement to be logically reversible. Section III mathematically formulates logically reversible measurement and describes some useful theorems. Section IV illustrates examples of logically reversible measurement. Section V discusses the necessary and sufficient condition for a measurement process to be reversed by another measurement (physical reversibility). It will be shown, however, that such a reversing measurement scheme cannot be used to measure the wave function of a single quantum system. Section VI compares a reversing measurement scheme with unitarily reversible quantum operation from the viewpoint of error correction.

2. Sources of Irreversibility in Quantum Measurement

Let us analyze why many familiar quantum measurements are logically irreversible and examine conditions that are necessary for a measurement to be logically reversible.

Consider first a sharp position measurement of a massive particle, where measurements are called sharp if they involve no measurement error, and unsharp otherwise. Since the sharp position measurement belongs to Pauli’s first-kind measurement, the postmeasurement state \(\psi(q, t^+ \pm)\) is uniquely specified by the outcome \(q_0\) of the measurement, i.e., \(\psi(q, t^+ \pm) = \delta(q - q_0)\). This postmeasurement state, however, contains no information about the premeasurement state except that the states before and after the measurement have a nonzero overlap. An exceptional situation is the one in which the premeasurement state is a priori known to be a position eigenstate; then the state is not disturbed by the measurement, and therefore the outcome of the measurement uniquely specifies both the postmeasurement state and the premeasurement state. Excluding such an exception, a sharp position measurement is logically irreversible.

In general, any sharp measurement is logically irreversible. Logically reversible measurement should therefore be unsharp.

Unsharpness is necessary for a measurement process to be logically reversible, but it is not sufficient. To understand this, let us consider photon counting. Photon counting is a typical example of continuous measurement and has played a major role in the theory of measurement. A unique feature of photon counting is that a photodetector does not register all photons at a time, but rather one by one. In principle, an infinitely long observation time is necessary to uniquely determine the number of photons enclosed in a cavity. Therefore, a photon counting process — or, in general, any continuous measurement process — with a finite observation time can be viewed as unsharp measurement.Photon counting consists of a time sequence of two fundamental processes, namely, “one-count” and “no-count” processes \[4\]. The one-count process means that one photon is detected during an infinitesimal time, and the no-count process means that no photon is detected during a time interval.
3. Introduction

In classical mechanics, one can, in principle, measure a quantity of an object very precisely while keeping the backaction of the measurement as small as one wishes. The situation drastically changes in quantum mechanics; the more precisely one wants to measure a quantity of an object, the more strongly one has to couple the measuring apparatus to the object with the greater backaction of the measurement. In addition, the state reduction occurs in a way that depends on a specific outcome of measurement which is unpredictable. A quantum measurement thus introduces an asymmetry in the direction of time. With respect to the past, it confirms the predicted probability distribution of an observable by a number of measurements performed on an ensemble representing the same quantum state. With respect to the future, it produces a new quantum state via state reduction by a single measurement. Such disparate roles played by a quantum measurement with respect to the past and future are the origin of irreversibility in the dynamical evolution of a quantum-mechanical system. I will nevertheless show that a broad class of quantum measurement can be logically reversible in the sense that the premeasurement density operator of the measured system can be calculated from the postmeasurement density operator and from the outcome of measurement [1].

The concept of logical reversibility provides a criterion for deciding whether or not the system’s information is preserved in the course of measurement. A quantum measurement is logically reversible if and only if all pieces of information concerning the measured systems are preserved during the measurement. If some pieces of information are lost, the measurement is logically irreversible.

Recently, there has been considerable interest in restoring ‘lost’ coherence in a device that performs quantum computation because in a quantum computer information is stored in a superposition of states and therefore decoherence makes long computation impossible. One candidate to recover the ‘lost’ coherence is unitarily reversible quantum operation which restores the original state by a unitary transformation, hence with unit probability [2]. I will propose another possibility to resolve this problem, namely, by means of a reversing measurement scheme [3] and compare these two methods.

This paper is organized as follows. Section II analyzes why many familiar quantum measurements are logically irreversible and examines conditions that are necessary for a measurement to be logically reversible. Section III mathematically formulates logically reversible measurement and describes some useful theorems. Section IV illustrates examples of logically reversible measurement. Section V discusses the necessary and sufficient condition for a measurement process to be reversed by another measurement (physical reversibility). It will be shown, however, that such a reversing measurement scheme cannot be used to measure the wave function of a single quantum system. Section VI compares a reversing measurement scheme with unitarily reversible quantum operation from the viewpoint of error correction.

4. Sources of Irreversibility in Quantum Measurement

Let us analyze why many familiar quantum measurements are logically irreversible and examine conditions that are necessary for a measurement to be logically reversible.

Consider first a sharp position measurement of a massive particle, where measurements are called sharp if they involve no measurement error, and unsharp otherwise. Since the sharp position measurement belongs to Pauli’s first-kind measurement, the postmeasurement state
ψ(q, t+) is uniquely specified by the outcome q₀ of the measurement, i.e., \( \psi(q, t^+) = \delta(q - q_0) \).

This postmeasurement state, however, contains no information about the premeasurement state except that the states before and after the measurement have a nonzero overlap. An exceptional situation is the one in which the premeasurement state is a priori known to be a position eigenstate; then the state is not disturbed by the measurement, and therefore the outcome of the measurement uniquely specifies both the postmeasurement state and the premeasurement state. Excluding such an exception, a sharp position measurement is logically irreversible. In general, any sharp measurement is logically irreversible. Logically reversible measurement should therefore be unsharp.

Unsharpness is necessary for a measurement process to be logically reversible, but it is not sufficient. To understand this, let us consider photon counting. Photon counting is a typical example of continuous measurement and has played a major role in the theory of measurement. A unique feature of photon counting is that a photodetector does not register all photons at a time, but rather one by one. In principle, an infinitely long observation time is necessary to uniquely determine the number of photons enclosed in a cavity. Therefore, a photon counting process — or, in general, any continuous measurement process — with a finite observation time can be viewed as unsharp measurement. Photon counting consists of a time sequence of two fundamental processes, namely, “one-count” and “no-count” processes [4]. The one-count process means that one photon is detected during an infinitesimal time, and the no-count process means that no photon is detected during a time interval.

Let us first analyze the state evolution in the no-count process. The density operator of the photon field immediately after the no-count process \( \hat{\rho}(t + \tau) \) is related to that \( \hat{\rho}(t) \) before it by [4] [5]

\[
\hat{\rho}(t + \tau) = \exp\left(-\frac{\lambda}{2} \hat{a}^\dagger \hat{a} \tau\right) \hat{\rho}(t) \exp\left(-\frac{\lambda}{2} \hat{a} \hat{a}^\dagger \tau\right) \frac{\text{Tr}[\hat{\rho}(t) \exp(-\lambda \hat{a}^\dagger \hat{a} \tau)]}{\text{Tr}[\hat{\rho}(t + \tau) \exp(\lambda \hat{a}^\dagger \hat{a} \tau)]},
\]

(1)

where \( \hat{a}^\dagger \) and \( \hat{a} \) are the creation and annihilation operators of the photon field, and \( \lambda \) is a coupling constant between the photon field and the photodetector. By inspection we find that Eq. (1) can be inverted, giving

\[
\hat{\rho}(t) = \exp\left(\frac{\lambda}{2} \hat{a}^\dagger \hat{a} \tau\right) \hat{\rho}(t + \tau) \exp\left(\frac{\lambda}{2} \hat{a} \hat{a}^\dagger \tau\right) \frac{\text{Tr}[\hat{\rho}(t + \tau) \exp(\lambda \hat{a}^\dagger \hat{a} \tau)]}{\text{Tr}[\hat{\rho}(t) \exp(-\lambda \hat{a}^\dagger \hat{a} \tau)]}.
\]

(2)

This formula gives the premeasurement density operator \( \hat{\rho}(t) \) in terms of the postmeasurement density operator \( \hat{\rho}(t + \tau) \) and the readout information that no photon has been detected during time \( \tau \). The no-count process is therefore logically reversible, though the state evolution is nonunitary.

The density operator \( \hat{\rho}(t^+) \) of the photon field immediately after the one-count process is related to that \( \hat{\rho}(t) \) before it by [4] [3]

\[
\hat{\rho}(t^+) = \frac{\hat{a} \hat{\rho}(t) \hat{a}^\dagger}{\text{Tr}[\hat{\rho}(t) \hat{a} \hat{a}^\dagger]}.
\]

(3)

This formula cannot be inverted for \( \hat{\rho}(t) \) because information about the vacuum state vanishes upon operation of \( \hat{a} \) (or \( \hat{a}^\dagger \)) on \( \hat{\rho}(t) \) from the left (or from the right). Physically, this means that the conventional photodetector does not respond to vacuum field fluctuations and therefore \( \hat{\rho}(t^+) \) does not contain any information concerning the premeasurement vacuum state. The one-count process is therefore logically irreversible. From this example we find that sensitivity of the detector to the vacuum state or, in general, sensitivity of the detector to all states in the Hilbert space is essential for a measurement process to be logically reversible.
5. Mathematical Formulation of Logically Reversible Measurement

The concept of logical reversibility in quantum measurement can be formulated on a firm mathematical basis.

A theory of measurement should describe both the probability for each outcome of measurement and the corresponding postmeasurement state. For many cases of interest, these two roles are described by a family of linear operators \( \{ \hat{A}_\nu \} \). The probability for outcome \( \nu \) is given by

\[
\text{Tr}[\hat{\rho} \hat{A}_\nu^\dagger \hat{A}_\nu],
\]

where \( \hat{\rho} \) is the premeasurement density operator of the measured system, and the postmeasurement state \( \hat{\rho}'_\nu \) is given by

\[
\hat{\rho}'_\nu = \frac{\hat{A}_\nu \hat{\rho} \hat{A}_\nu^\dagger}{\text{Tr}[\hat{\rho} \hat{A}_\nu^\dagger \hat{A}_\nu]}.
\]

Equation (5) indicates that \( \hat{A}_\nu \) plays a role of a generalized projection operator. The state change from the premeasurement density operator \( \hat{\rho} \) to the postmeasurement density operator \( \hat{\rho}'_\nu \) may be viewed as a mapping. Let this mapping be denoted by \( \Gamma_\nu : \hat{\rho}'_\nu = \Gamma_\nu(\hat{\rho}) \). In terms of \( \Gamma_\nu \), the concept of logical reversibility may be defined in the following way [3]:

Definition. If \( \Gamma_\nu \) is a one-to-one mapping, a measurement process described by an operator \( \hat{A}_\nu \) is logically reversible. If \( \Gamma_\nu \) is a one-to-one mapping for every possible outcome \( \nu \), the entire measurement \( \{ \hat{A}_\nu \} \) is logically reversible.

If \( \Gamma_\nu \) is a one-to-one mapping, \( \hat{A}_\nu \) has a left inverse, and vice versa. A measurement process described by an operator \( \hat{A}_\nu \) is therefore logically reversible if and only if \( \hat{A}_\nu \) has a left inverse. We can prove two useful theorems [3]. The first one is [3]

**Theorem 1.** The necessary and sufficient condition for a measurement process with outcome \( \nu \) to be logically reversible is that \( \hat{A}_\nu |\psi\rangle \) does not vanish for any nonzero vector \( |\psi\rangle \) in the Hilbert space.

This theorem implies that the outcome \( \nu \) does not exclude the possibility of any state for the premeasurement state. To put it another way, the detector responds to all states in the Hilbert space. The second theorem is

**Theorem 2.** For any sharp measurement \( \{ \hat{A}_\nu \} \), there exists a logically reversible measurement \( \{ \hat{A}_\nu(\epsilon) \} \) that becomes arbitrarily close to \( \{ \hat{A}_\nu \} \) as the measurement error, which is characterized by \( \epsilon \), approaches zero.

This theorem implies that a broad class of sharp (and hence logically irreversible) measurements can be made logically reversible by making them unsharp in such a manner that \( \hat{A}_\nu(\epsilon) \) has a left inverse and that a functional dependence of \( \hat{A}_\nu(\epsilon) \) on \( \epsilon \) is known. We will illustrate this theorem in the following section.
6. Examples of Logically Reversible Measurement

Until now, four different kinds of measurement have been shown to be logically reversible. The first example is the so-called quantum counter [1]. The quantum counter was proposed by Bloembergen as an infrared photon detector [7]. This device was later recognized by Mandel to be sensitive to vacuum field fluctuations [8]. The quantum counter is thus a vacuum-field-sensitive photon counter, and is shown to perform a logically reversible measurement of photon number [1].

Subsequently, Imamoglu showed that a three-state Λ atomic system, which has been used to study electromagnetically induced transparency, can be used for a logically reversible quantum nondemolition (QND) measurement of photon number [9].

Royer then pointed out that an unsharp measurement of a spin-1/2 can be reversed by another measurement with a nonzero probability of success [10]. This affords an example of physical reversibility in the sense that the premeasurement state can be recovered from the postmeasurement state by means of a physical process. We will return to this subject later.

Most recently, we have shown that an unsharp Kerr QND measurement of photon number is logically reversible [3]. The basic idea of this measurement is as follows. Suppose that we measure the photon number of the signal light in a nondestructive way. For this purpose, we pass the signal light and the probe light through a nonlinear medium called Kerr medium. In this medium, information concerning the signal photon number is transferred to the change in phase of the probe light. Measuring this change by the homodyne detection, we can estimate the signal photon number in a nondestructive way.

The state evolution in the Kerr medium is given by the unitary operator \( \hat{U} = \exp(i\kappa \hat{n}_s \hat{n}_p) \), where \( \hat{n}_s \) and \( \hat{n}_p \) are the photon number operator of the signal light and that of the probe light, respectively, and \( \kappa \) is an effective coupling constant between the signal light and the probe light which is proportional to the third nonlinear susceptibility and to the length of the medium. When the probe light is initially in a coherent state with amplitude \( \alpha \), and when readout \( \beta_2 \) of the homodyne detection is obtained, the generalized projection operator \( \hat{A} \) which describes the state change of the signal light according to Eq. (5) is calculated to be [3]

\[
\hat{A}(\nu, \epsilon) = \sum_{n=0}^{\infty} \sqrt{C_n(\nu, \epsilon)} |n\rangle \langle n|,
\]

where \( |n\rangle \) is the Fock state (i.e., the eigenstate of the photon number), \( \nu \equiv \beta_2/|\alpha|\kappa \) gives an estimated photon number and \( \epsilon = 1/(2|\alpha|\kappa) \) gives the measurement error. As long as the coupling constant \( \kappa \) is finite, \( \hat{A} \) is not a sharp projection operator of the photon number but is distributed around an estimated photon number \( \nu \) according to the Gaussian distribution with mean \( \nu \) and width \( \epsilon \):

\[
C_n(\nu, \epsilon) = \frac{1}{\sqrt{2 \pi \epsilon^2}} \exp \left[ -\frac{(n - \nu)^2}{2\epsilon^2} \right].
\]

As the intensity \( |\alpha|^2 \) of the probe light becomes infinite, the measurement error \( \epsilon \) becomes zero and therefore a measurement process described by \( \hat{A} \) approaches a sharp measurement of the signal photon number:

\[
\lim_{\epsilon \to 0} C_n(\nu, \epsilon) = \delta(n - \nu).
\]

As long as the measurement error \( \epsilon \) is nonzero, \( C_n(\nu, \epsilon) \) is always positive, so that \( \hat{A}(\nu, \epsilon) \) has a left inverse. Therefore, an unsharp Kerr QND measurement is logically reversible.
7. Reversing Measurement and Impossibility of Measuring the Wave Function of a Single Quantum System

Logical reversibility implies an information preserving measurement. It is not clear, however, if the premeasurement state can be recovered from the postmeasurement state by another measurement which we will call reversing measurement[3]. Let us discuss the necessary and sufficient condition for a measurement process to have a reversing measurement and its implications.

Suppose that we perform a measurement \( \{\hat{A}_\nu\} \) on a state represented by density operator \( \hat{\rho} \), and that an outcome \( \nu \) is obtained. The postmeasurement density operator \( \hat{\rho}'_{\nu} \) is given by Eq. (5). Let the reversing measurement be \( \{\hat{R}_{\nu}^{(\mu)}\} \), where \( \mu \) represents possible outcomes of this measurement, and let \( \mu = 0 \) be the “successful outcome”, that is, if the outcome \( \mu = 0 \) is obtained, the postmeasurement state \( \hat{\rho}''_{\nu} \) of this measurement is given by the initial state \( \hat{\rho} \). The necessary and sufficient condition for such a successful reversal to occur is that \( \hat{R}_{\nu}^{(\mu)} \) satisfies

\[
\hat{R}_{\nu}^{(\mu)} \hat{A}_\nu = c^{(\nu)} \hat{1}
\]

where \( c^{(\nu)} \) is a nonzero and finite c-number and \( \hat{1} \) is the identity operator. Because \( \hat{R}_{\mu=0}^{(\nu)} \) should give the probability of successful reversal according to Eq. (4), it has to be bounded. If Eq. (4) holds, the postmeasurement density operator \( \hat{\rho}''_{\nu} \) corresponding to the outcome \( \mu = 0 \) can easily be shown to be equal to the initial state \( \hat{\rho} \):

\[
\hat{\rho}''_{\nu} = \frac{\hat{R}_{\nu}^{(\mu)} \hat{\rho} \hat{R}_{\nu}^{(\mu)*}}{\text{Tr}[\hat{\rho}''_{\nu} \hat{R}_{\nu}^{(\mu)*} \hat{R}_{\nu}^{(\mu)}]} = \hat{\rho}.
\]

To put it another way, a specific outcome \( \nu \) of the measurement \( \{\hat{A}_\nu\} \) has a reversing measurement \( \{\hat{R}_{\nu}^{(\mu)}\} \) if and only if \( \hat{A}_\nu \) has a bounded left inverse.

The existence of reversing measurement, prima facie, seems to imply that the wave function of a single quantum system can be measured. The argument goes as follows. As a result of a sequence of two measurements we have two outcomes \( \nu \) and \( \mu = 0 \), and the system has returned to its initial state [see Eq. (10)]. By repeating the same sequence of measurements many times, one might think that it is possible to measure the wave function of a single quantum system. If it were true, several consequences would result such as cloning of a single quantum system and superluminal communication that should be impossible.

To resolve this apparent paradox, consider the joint probability that the first measurement yields an outcome \( \nu \) and that the successful reversal \( \mu = 0 \) occurs at the second measurement. The joint probability of such successive measurements is given from Eqs. (4) and (9) by

\[
\text{Tr}[\hat{\rho}(\hat{R}_{\mu=0}^{(\nu)} \hat{A}_\nu)^\dagger (\hat{R}_{\mu=0}^{(\nu)} \hat{A}_\nu)] = |c^{(\nu)}|^2.
\]

The crucial observation is that the right-hand side is independent of the state of the measured system \( \hat{\rho} \). This means that whenever a successful reversal occurs, we cannot obtain any information about the initial state except for the information that the initial state has a nonzero overlap with the state corresponding to the outcome \( \nu \). Because the existence of reversing measurement \( \{\hat{R}_{\nu}^{(\mu)}\} \) automatically guarantees that the first measurement \( \{\hat{A}_\nu\} \) is logically
reversible, we should obtain all possible outcomes \( \nu_1, \nu_2, \nu_3, \ldots \) by repeating the first measurement \( \{ \hat{A}_\nu \} \) followed by the reversing measurement \( \{ \hat{R}_\mu^{(\nu)} \} \) many times; but we cannot obtain the probability distribution for these outcomes because, although the probability distribution of each outcome \( \nu \) or \( \mu = 0 \) depends on the state of the measured system, their joint probability distribution doesn’t as seen from Eq. (11).

When the dimension of the underlying Hilbert space is finite, a linear operator is always bounded. Logical reversibility therefore guarantees the existence of reversing measurement. A simple example is an unsharp measurement of spin systems; the simplest case of a spin-1/2 system was analyzed in Ref. [10].

When the dimension of the underlying Hilbert space is infinite, a logically reversible measurement may not have a reversing measurement. If so, we may introduce an approximate reversing measurement by truncating the Hilbert space. For example, consider an unsharp Kerr QND measurement of photon number. As we discussed in Sec. IV, as long as there is a nonzero measurement error \( \epsilon \), \( C_n(\nu, \epsilon) > 0 \) for all outcomes \( \nu \) and therefore the generalized projection operator \( \hat{A}(\nu, \epsilon) \) in Eq. (6) has a left inverse \( \hat{B}(\nu, \epsilon) \):

\[
\hat{B}(\nu, \epsilon) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{C_n(\nu, \epsilon)}} |n\rangle \langle n|.
\]  

However, since \( 1/\sqrt{C_n(\nu, \epsilon)} \) grows exponentially with \( n \) [see Eq. (7)], \( \hat{B}(\nu, \epsilon) \) is not bounded. In practice, however, we may truncate the Hilbert space at a finite value \( N \) of the photon number, and the operator thus defined

\[
\hat{B}'(\nu, \epsilon) = \sum_{n=0}^{N} \frac{1}{\sqrt{C_n(\nu, \epsilon)}} |n\rangle \langle n|
\]  

is bounded, so that we may introduce an approximate reversing measurement \( \{ \gamma \hat{B}', \sqrt{1 - |\gamma|^2} \hat{B}'\dagger \hat{B}' \} \), where a nonzero c-number \( \gamma \) is introduced to satisfy the condition that the trace of \( |\gamma|^2 \hat{B}'\dagger \hat{B}' \), which is the probability for successful reversal, is equal to or less than unity.

### 8. Restoring Lost Coherence in Quantum Computation

In a quantum computer, information is stored in a superposition of states. It is therefore crucial to protect the system against decoherence in order to perform long computation. Because the system is always subject to contact with its environment, the system easily decoheres via, e.g., spontaneous emission. Shor proposed to using redundant coding to overcome this problem [11].

A scheme of reversing measurement could provide another means to overcome the problem of decoherence. A key idea is that if a decoherence process may be regarded as logically reversible measurement, we may construct (at least an approximate) reversing measurement (see Sec. V) to restore the initial state. A problem of this scheme is how to make the probability of successful reversal close to unity.

If the system’s initial states is known to lie within a special subspace of the entire Hilbert space, we may restore the initial state by a unitary transformation — hence with unit probability [2]. Unitary transformation implies that there should be no information readout. In fact, the subspace must be chosen such that within this subspace the probability for each outcome is independent of the initial state and thereby no information can be extracted about the initial
state. This scheme of restoring the initial state is called unitarily reversible quantum operation [12]. The problem of this scheme is that it is not always easy to prepare an initial state within such a restricted Hilbert space.

The two schemes — reversing measurement and unitarily reversible quantum operation — have one common requirement. That is, the decoherence process must occur in a logically reversible manner; otherwise, some pieces of information about the initial state of the system would be lost in that process and there would be no means to restore the initial state. If the initial state and the decohered state are known to be bounded as in the case of unitarily reversible measurement, we can construct a number of reversing measurements with varying probabilities of successful reversal. The author conjectures that unitarily reversible quantum operation may be viewed as a special case of reversing measurement at the limit of the probability of successful reversal being equal to unity.

Acknowledgments

The author would like to thank N. Imoto and H. Nagaoka for fruitful collaboration yielding Ref. [3] on which parts of Sec. III-V are based. This work was supported by the Core Research for Evolutional Science and Technology (CREST) of the Japan Science and Technology Corporation (JST).

References

[1] M. Ueda and M. Kitagawa, Phys. Rev. Lett. 68 (1992) 3424.
[2] H. Mabuchi and P. Zoller, Phys. Rev. Lett. 76 (1996) 3108.
[3] M. Ueda, N. Imoto, and H. Nagaoka, Phys. Rev. A 53 (1996) 3808.
[4] M. D. Srinivas and E. B. Davies, Opt. Acta 28 (1981) 981.
[5] M. Ueda, Quantum Opt. 1 (1989) 131; Phys. Rev. A 41 (1990) 3875; M. Ueda, N. Imoto, and T. Ogawa, Phys. Rev. A 41 (1990) 3891; N. Imoto, M. Ueda, and T. Ogawa, Phys. Rev. A 41 (1990) 4127.
[6] E. B. Davies and J. T. Lewis, Commun. Math. Phys. 17 (1971) 239.
[7] N. Bloembergen, Phys. Rev. Lett 2 (1959) 84.
[8] L. Mandel, Phys. Rev. 152 (1966) 438.
[9] A. Imamoğlu, Phys. Rev. A 47 (1993) R4577.
[10] A. Royer, Phys. Rev. Lett. 74 (1995) 1040; 74 (1995) 1040(E).
[11] P. W. Shor, Phys. Rev. A 52 (1995) R2493.
[12] M. A. Nielsen and C. M. Caves, Phys. Rev. A 55 (1997) 2547.