Interference in spin-orbit coupled transverse magnetic focusing; emergent phase due to in-plane magnetic fields

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Spin-orbit (SO) interactions in two dimensional systems split the Fermi surface, and allow for the spatial separation of spin-states via transverse magnetic focusing (TMF). In this work, we consider the case of combined Rashba and Zeeman interactions, which leads to a Fermi surface without cylindrical symmetry. While the classical trajectories are effectively unchanged, we predict an additional contribution to the phase, linear in the applied in-plane magnetic field. We show that this term is unique to TMF, and vanishes for magnetic (Shubnikov de Haas) oscillations. Finally we propose some experimental signatures of this phase.

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I. INTRODUCTION

Transverse magnetic focusing (TMF) has a long history, being employed in metals and semi-conductors, and has been used to investigate the shape of the Fermi surface. A TMF experiment consists of a source and a detector, separated by a distance $l$, with charges focused from the source to the detector via a weak transverse magnetic field. It is the direct translation of charge mass spectroscopy to the solid state. Despite the nearly half century of experimental history, TMF is still producing novel results, with the most recent application in systems with non-quadratic dispersion relations, such as Graphene and two dimensional charge gases with large spin-orbit (SO) interactions. In spin-orbit coupled systems, the spin-split Fermi surfaces result in a “doubled” focusing peak, which provides a novel platform investigations of polarisation effects in the source and detector quantum point contact. The separation of the peaks also allows for the direct determination of the magnitude of the spin-orbit splitting, hence TMF can be used in addition to quantum magnetic oscillations to yield detailed information about spin-orbit coupled electron and hole systems.

Much of the theoretical and experimental work concerning TMF with large spin-orbit splittings has considered a singular dominant SO interaction. This leads to a cylindrically symmetric Fermi surface, and a double peak structure that is, in essence, two copies of the single peak structure. This assumption is well justified for many typical experimental systems grown along high symmetry crystal axes, as classical trajectories are not significantly altered except in the case of extremely large asymmetry. While a sufficiently large secondary SO interaction can lead to magnetic breakdown like behaviour, the requirement for resolution of the double peak structure means that the typical regime is one characterised by the secondary SO interaction being weaker than the primary interaction that yields the double peak structure of spin-split TMF.

Like earlier studies in semiconductors, SO coupled systems have Fermi wavelengths comparable to the feature size making interference an important feature of the magnetic focusing spectrum. With the addition of SO coupling, the interference effects are further enriched, and yield new methods of studying SO interactions. In this paper, we focus on the problem of interference in TMF in systems with non-cylindrically symmetric spin-orbit interactions due to an applied in plane field. Importantly, due to the large pre-factors, proportional to the Fermi momentum and focusing length, relatively small in plane fields can lead to large phase contributions. While the classical trajectories are effectively unchanged, an additional phase term emerges, linear in the applied magnetic field. We show that this additional contribution to the phase can significantly alter the TMF interference spectrum.

Our paper is organised as follows. In Sec. II we present the classical trajectories for magnetic focusing, and introduce the relevant Hamiltonian. Following on from this, in Sec. III we develop a theory of interference in the absence of cylindrical symmetry, building on previous work on interference in TMF with SO coupling. Finally, in Sec. IV we consider some relevant examples, with a minimalistic model of the injector and detector wave functions.

II. SPIN-ORBIT INTERACTIONS AND CLASSICAL TRAJECTORIES

Semiconductor heterostructures allow for a great diversity of SO interactions. While the approach we will detail is general, for specificity we will consider two interactions; the Rashba interaction resulting from a lack of surface inversion symmetry in the sample, and the Zeeman interaction due to an applied in plane magnetic field. These two interactions have the advantage of being tunable. In electron systems, the Rashba interaction has the kinematic structure

$$\mathcal{H}_{R,v} = i\gamma_1 \frac{p}{2} \sigma_+ + \text{h.c.}$$

$$\sigma_\pm = \sigma_x \pm i\sigma_y \quad p_\pm = p_x \pm ip_y$$

where $\gamma_1$ is material parameter dependent on the electric field perpendicular to the two dimensional plane. The
Pauli matrices $\sigma$ correspond to electron spin $s = 1/2$, and
the selection rule for $\sigma_\pm$ is $\Delta s = \pm 1$. Spin splitting in
the magnetic focusing spectrum was recently observed in
InGaAs quantum wells\cite{Ishiguro06}. In GaAs heterostructures, the
spin-orbit interaction is typically not large enough to ob-
tain a spin-split magnetic focusing spectrum. Heavy hole
gases can also be engineered to have a Rashba spin orbit interaction\cite{Ishiguro06}. Due to the heavy holes having angular moment-
num $J_z = \pm 3/2$, the Rashba interaction arises from the
combined action of both the Luttinger, $(p \cdot J)^2$, and
Rashba terms, $p \cdot (J \times z)$, with $\mathcal{H}_R \propto (J \cdot p)^2(p \cdot J \times z)$. Typically, the
light holes $J_z = \pm 1/2$, have significantly higher energy,
and it is more convenient to work in the
subspace spanned by the Pauli matrices, with $J_z^2 \to \sigma_z$.

The selection rule is $\Delta J_z = \pm 3$. In this subspace the
kinematic structure is\cite{Ishiguro06}
\[
\mathcal{H}_{R,h} = i\frac{\gamma_3}{2}p_3 \sigma_+ + \text{h.c.}
\] (2)
where $\gamma_3$ is a material parameter analogous to $\gamma_1$.

To induce an asymmetry in the spin-split Fermi surface,
we consider an applied in plane magnetic field. For
electrons, this results in the usual Zeeman interaction,
\[
\mathcal{H}_{Z,e} = \frac{g}{2}\mu_B B_\perp \sigma_+ + \text{h.c.}
\] (3)
where $g$ is the electron $g$ factor, and $B_\perp = B_x \pm iB_y$.
There is no equivalent expression for heavy holes, as
$J_z = \pm 3/2$ cannot be coupled directly, but requires the
combined action of Zeeman, $J \cdot B$ and Luttinger, $(p \cdot J)^2$, with
$\mathcal{H}_{Z,h} \propto (J \cdot p)^2(J \cdot B)$. The kinematic structure is
\[
\mathcal{H}_{Z,h} = \frac{g_1}{2}\mu_B p^2 \sigma_+ + \text{h.c.}
\] (4)
where we use the aforementioned subspace of heavy
holes\cite{Ishiguro06}.

We use a dimensionless form of the coefficients $\gamma_3$ in
Eq. (2) and $\gamma_1$ in Eq. (1),
\[
\gamma_1 = \tilde{\gamma}_1 \frac{\epsilon_F}{k_F}
\] (5)
\[
\gamma_3 = \tilde{\gamma}_3 \frac{\epsilon_F}{k_F}
\] (6)
\[
k_F = \sqrt{2m\epsilon_F}
\] (7)
where $\epsilon_F$ is the Fermi energy (chemical potential). The
dimensionless coefficient $\tilde{\gamma}_1, \tilde{\gamma}_3$ represents the value of the
SO interaction at $p = k_F$ in units of the Fermi energy.
This can be directly related to the splitting the “double”
TMF peaks. For the heavy holes Rashba interaction, $\tilde{\gamma}_3$ can
as large as $\tilde{\gamma}_1 \sim 0.1 - 0.2$, in GaAs depending on the
$z$ confinement\cite{Ishiguro06}. For the electron Rashba interaction,
in InGaAs quantum wells, $\tilde{\gamma}_1 \sim 0.3$\cite{Ishiguro06}. For the Zeeman
interaction in holes, we consider the dimensionless coeffi-
cient, $\tilde{g}_1$,
\[
\tilde{g}_1 = g_1 k_F^2
\] (8)
For GaAs heavy hole quantum wells, $\tilde{g}_1 \sim 1.1$\cite{Ishiguro06}. The
electron $g$ factor in InGaAs quantum wells is $g \sim -9$\cite{Ishiguro06}.

We can consider the SO interaction as a momentum
dependent effective Zeeman magnetic field, $B(p)$. Hence
the Hamiltonian is
\[
\mathcal{H} = \frac{p^2}{2m} + B(p) \cdot \sigma
\] (9)
\[
\sigma \cdot B(p) = \mathcal{H}_R + \mathcal{H}_Z
\] (10)
with the application of a transverse magnetic field, $B_z$,
$p \rightarrow \pi = p - eA$, with the vector potential chosen in an
appropriate gauge, $A = B_z(x, -z, 0)$. The semi-classical

dynamics of the charge carriers are characterised by
cyclotron orbit\cite{Ishiguro06}, with a cyclotron radius, $r_c = k_F/eB_z$,
and a cyclotron frequency, $\omega_c = eB_z/m$. Due to the cur-
vature of the trajectories, the effective magnetic field, $B$ evolve
in time. Since TMF experiments are typically performed at relatively small transverse magnetic fields, $B_z \leq 0.1T$, the spin adiabatically follows the effective magnetic
field, $\mathcal{B}$. Provided $|\mathcal{B}| \gg \omega_c$, there is no tun-
nelling between the two spin states. We note that this is
also a condition for a “double” focusing peak.

We are now in a position to explore the semiclassical
dynamics. The Hamiltonian, with an applied magnetic
field, $\mathcal{B}$ is
\[
\mathcal{H} = \frac{p^2}{2m} + \sigma \cdot \mathcal{B}
\] (11)
\[
\mathcal{B} = |B||\cos \varphi, B||\sin \varphi, B_z|
\] (12)
\[
\pi = p - eA
\] (13)
where $\varphi$ is the field angle, $B||$ is the in plane magnetic
field, and $B_z$ is the (weak) transverse focusing field. We
stress that typically $B_z \sim 0.1T$, while $B||$ can be a few
Teslas for heavy hole quantum well in GaAs\cite{Ishiguro06}. For
electrons in InGaAs, $B|| \sim 1T$ due to the much large $g$ factor
in these systems. If the spin follows the effective field
adiabatically, $\sigma \to s\mathcal{B}/|\mathcal{B}|$, where $s$ is a pseudo scalar, and
describes the two possible spin states. The resulting
adiabatic Hamiltonian is
\[
\mathcal{H}_{el} = \frac{\pi^2}{2m} + s|\mathcal{B}|
\] (14)

The semiclassical dynamics of this hamiltonian has been
found with expansion in powers of $|\mathcal{B}|/\epsilon_F$\cite{Ishiguro06}. This is valid
in the regime $\omega_c \ll |\mathcal{B}| \ll \epsilon_F$. The effective magnetic
field, $\mathcal{B}$, is
\[
|\mathcal{B}| = \epsilon_F |\gamma_3| b(\theta)
\] (15)
\[
b(\theta) = \rightarrow
\] (16)
\[
= \sqrt{1 + 2(\tilde{g}_1 \mu_B / |\gamma_3| \epsilon_F)B|| \cos \theta - \varphi + (\tilde{g}_1 \mu_B / |\gamma_3| \epsilon_F)^2 B_{||}^2}
\] (17)

For holes, and
\[
|\mathcal{B}| = \epsilon_F |\tilde{\gamma}_1| b(\theta)
\] (18)
\[
b(\theta) = \rightarrow
\] (19)
\[
= \sqrt{1 + 2(\mu_B / |\tilde{\gamma}_1| \epsilon_F)B|| \cos \theta - \varphi + (\mu_B / |\tilde{\gamma}_1| \epsilon_F)^2 B_{||}^2}
\] (20)

for electrons. Evidently, these effective magnetic fields
are identical, and the dynamics of electrons and holes
are the same in this adiabatic semiclassical approach, despite the kinematic structure of the spin-orbit interactions, Eqs. (2) and (1) being markedly different. For clarity, in the following calculations, we will exclusively refer to holes.

The equations of motion of this classical hamiltonian are

\[
\begin{align*}
\dot{v}_+ &= \frac{\partial H_{el}}{\partial \pi_+} = \frac{\pi_+}{m} - \frac{s}{|B|} \frac{\partial B^2}{\partial \pi_-} \\
\dot{\pi}_+ &= i\omega_m v_+ ,
\end{align*}
\]

The solution to these classical equations of motion has been found with expansion in powers of \(|B|/\epsilon_F|^{11}\). The particle trajectories are given by

\[
\begin{align*}
\theta_0 &= \omega_c \epsilon \\
\theta &= \theta_0 - s \frac{3|\gamma_3|}{2} \int_{\theta_0}^{\theta} \frac{a(\theta')}{b(\theta')} d\theta' \\
x + iy &= \frac{k_F}{m\omega_c} \left\{ i(e^{i\beta} - e^{i\beta'}) + s \frac{|\gamma_3|}{2} \int_{\theta_0}^{\theta} e^{i\theta'} \left[ b(\theta') + 3i c(\theta') \right] d\theta' \right\} ,
\end{align*}
\]

where \(k_F = \sqrt{2m\epsilon_F}\) is the Fermi momentum. We have introduced the initial angle \(\theta_0\). The condition for the adiabatic evolution of the spin implies that \(b(\theta)\) does not vanish, \(|\gamma_3| b(\theta) \gg \omega_c/\epsilon_F\). The functions \(c(\theta)\) and \(a(\theta)\) are given by

\[
\begin{align*}
\dot{a}(\theta) &= \omega_c \left[ 1 - s \frac{3|\gamma_3| a(\theta)}{b(\theta)} \right] \\
a(\theta) &= 1 + (5/3)(\tilde{g}_1 \mu_B |B||/\gamma_3 \epsilon_F) \cos(\theta - \varphi) \\
&+ (2/3)(\tilde{g}_1 \mu_B |B||/\gamma_3 \epsilon_F)^2 \\
c(\theta) &= \frac{1}{3} (\tilde{g}_1 \mu_B |B||/\gamma_3 \epsilon_F) \sin(\theta - \varphi) .
\end{align*}
\]

These solve the problem of the classical motion. We have presented illustrative trajectories in Fig. 2. The classical trajectories are essentially unchanged, even up to several Tesla.

A peculiar feature to note is that \(\theta_i = 0\) does not correspond to the classical trajectory, since \(\theta_i = 0\) has non-zero \(v_y\). We define the physical injection angle, \(\beta\) such that the classical trajectories shown in Fig. 2 correspond to injection with \(\beta = 0\). To relate this to \(\theta_i\), we differentiate Eq. (13) to obtain \(v_y\) at the source

\[
\begin{align*}
v_y &\approx \frac{k_F}{m} \left( \sin \theta_i + \frac{\tilde{g}_1 \mu_B |B||}{2 \epsilon_F} \sin(2\theta_i + \varphi) \right) \\
k_{F,s} &= \frac{k_F}{m} \left( 1 + \frac{\tilde{g}_1 \mu_B |B||}{2 \epsilon_F} \right) ,
\end{align*}
\]

Setting \(v_y = 0\) and solving, we obtain \(\theta_i \approx \tilde{g}_1 \mu_B |B||/\gamma_3 \epsilon_F \sin \varphi/2 \epsilon_F\). In general, \(\beta\) is related to \(\theta_i\) by

\[
\theta_i = \beta + \frac{\tilde{g}_1 \mu_B |B||}{2 \epsilon_F} \sin \varphi
\]

We stress again that the method used here, and following in Sec. [III](#sec:interference) can equally be applied to electron systems with a Rashba SOI and an applied in-plane magnetic field.

### III. Interference

The problem of interference in systems with large SOIs has been treated in detail for cylindrically symmetric systems\[^{13}\]. Like any interference problem, there are two trajectories (see Fig. 1), connecting the source located at the origin, \((0,0)\), to a detector located at \((0,l)\). These
two paths are defined by injection angles $\pm \beta$, with
\[
\cos \beta = \frac{1}{\sqrt{c^2 + v^2}} \quad (17)
\]
\[
r_{c,s} = k_{F,s}/eB_z
\]
Interference arises from the difference between the phases of the two trajectories, with the semiclassical propagator defined as the sum over the two classically allowed paths,
\[
K(\beta) \sim e^{iS(\beta)} + e^{iS(-\beta)} \quad (18)
\]
with the phase $S \propto \int p \cdot dl$, where $dl$ is integrated along the path of the trajectories. In a typical TMF setup, the source and detector are of some finite aperture, with the Huygens Kernel, Eq. (18) averaged over this aperture\textsuperscript{13}.

Evaluation of the phase is treated analogously to the cylindrically symmetric case. The canonical is related to the kinematic momentum and the vector potential by $p = mv + eA$, and the action is
\[
S(\beta) = \int_{\delta S} (mv + eA) \cdot dl \quad (19)
\]
where $v$, $A$, $dl$ and the path, $\delta S$, are dependent on $g_{1\mu B} B_{||}$. Using the previously determined equations of motion, Eqs. (13) and (14), the phase integral, Eq. (19) can be converted into an integral over the running angle,
\[
S(\beta) = eB_z \int_{\theta_i}^{\theta} \left[ \left( \frac{dx}{d\theta'} \right)^2 + \left( \frac{dy}{d\theta'} \right)^2 - x \frac{dy}{d\theta'} \right] d\theta' \quad (20)
\]
The relationship between the physical injection angle $\beta$ and $\theta_i$ is presented in Eq. (16). We must also determine $\theta$ in terms of $\beta$.

The trajectory from the source to the detector is, in terms of the running angle, from $\theta_i$ to $\theta$. This corresponds to the spatial positions $(0,0)$ and $(0,l)$ respectively. From Eq. (13), we have
\[
x = 0 = \frac{k_s}{\omega_m} \left\{ (\sin \theta_i + \sin \theta) + s \frac{g_{1\mu B} B_{||}}{4} \left[ \sin(2\theta - \varphi) - \sin(2\theta_i - \varphi) \right] \right\} \quad (21)
\]
We have restricted ourselves to a first order expansion in $g_{1\mu B} B_{||}$ when performing the integration of Eq. (13). The trajectory deviates only minimally from the arc of a circle (see Fig. 2), and we can reasonably employ the approximation $\theta \approx \pi - \theta_i$ for the $\tilde{g}_{1\mu B} B_{||}$ dependent terms. With this approximation, solving Eq. (22) we obtain
\[
\theta \approx \pi - \theta_i - s\tilde{g}_{1\mu B} B_{||} \sin \theta_i \cos \varphi \quad (22)
\]
Finally, this can be expressed in terms of the injection angle, $\beta$ using Eq. (16), to obtain the integration limits for Eq. (20) in terms of $\beta$.

Using these integration limits, integration of Eq. (20) yields
\[
S(\beta) = \frac{k^2}{2eB} \left\{ \pi - 2\beta + \pi \frac{3}{2} \right\} - s\tilde{g}_{1\mu B} B_{||} \sin \beta(1 - \cos 2\beta) \cos \varphi \quad (23)
\]
\[
\zeta = \tilde{g}_{1\mu B} B_{||} \left\{ \sin \varphi - \cos 2\beta \sin \varphi + \sin \varphi(\cos \beta + \frac{1}{3} \cos^3 \beta) \cos \varphi(\cos \beta + \frac{1}{3} \cos^3 \beta) \right\}
\]

For $-\beta$ injection angles, we take $\beta \rightarrow -\beta$. We have introduced here $\zeta$ which contains the phase terms that do not contribute any net phase difference, that are symmetric for $\beta \rightarrow -\beta$. According to Eq. (18), we then have
\[
K(\beta) \sim e^{iS(\beta)} + e^{iS(-\beta)} \quad (24)
\]
\[
\sim \sin \left[ \frac{k^2}{2eB} \left( 2\beta - \sin 2\beta \right) + \frac{\pi}{4} \right]
\]

The additional factor of $\pi/4$ arises due to the caustic for the $-\beta$ path\textsuperscript{13}. The third line of Eq. (18), which is linear in $\tilde{g}_{1\mu B} B_{||}$, represents the “emergent phase contribution”, and is the first major result of this work. This term is particularly remarkable, since the classical trajectories have no first order dependence on $\tilde{g}_{1\mu B} B_{||}$. For quantum (Shubnikov de Haas) oscillations, the integral is over the entire Fermi surface, and this term vanishes. Thus it is peculiar to the particular geometry of TMF, which defines the angle, $\varphi$ between the in plane magnetic field and the injector.

It is instructive to examine the variation in the interference fringe separation due to the application of the in magnetic plane field, expanding for small $\beta$. For small $\beta$, according to Eq. (18),
\[
\beta \approx \sqrt{\frac{2r_{c,s} - l}{r_{c,s}}} = \sqrt{\frac{y}{r_{c,s}}} \quad (25)
\]
Here $y = 2r_{c,s} - l$ is the detuning from the classically forbidden region. For small $\beta$, Eq. (24) becomes
\[
S \approx \frac{2}{3}\nu_s \left( \frac{y}{r_c} \right)^{\frac{3}{2}} \left( 1 + s \frac{3}{2}g_{1\mu B} B_{||} \cos \varphi \right) \quad (26)
\]
And from Eq. (25), we find a characteristic spacing of the interference fringes to be
\[
\frac{\delta B}{B} \approx \frac{2.2}{2\nu_s} \left( 1 - s\tilde{g}_{1\mu B} B_{||} \cos \varphi \right) \quad (27)
\]
where $\delta B$ is the fringe spacing. This provides a method of determining the strength of the secondary spin orbit interaction. As was detailed in Fig. 4 even for the first interference fringe, there is a measurable shift. While there is no direct enhancement, the strength of the $\tilde{g}_1\mu_BB_{||} \sim 0.1eF$ at fields of a few Tesla in hole systems. Recent TMF experiments have resolved a single interference fringe for the low field peak, which would be sufficient for the determination of $\tilde{g}_1\mu_BB_{||}$.

The remaining elements of the Huygen’s kernel are unchanged cylindrically symmetric case. As was detailed in Ref. [13], the asymptotic form of the Huygen’s kernel can be related to the Airy function. Employing the same reasoning, from Eqs. (25) and (26), we obtain,

$$K_s = e^{i\frac{\pi}{2}(\nu_3 - 1 - \alpha)} \frac{\nu_3^{2/3}}{2\sqrt{2}r_{cs}} \times \left[ (\sigma_s - i\sigma_s)Ai(\gamma_y) + \frac{n}{\nu_3^{1/3}}Ai'(\gamma_y) \right].$$

Here $\gamma = y^{2/3}(1 + s\tilde{g}_1\mu_BB_{||}\cos\varphi)/r_c$. We present plots of the resulting interference spectrum in Fig. 4 with point-like sources and detectors, for both the classical form of the Huygen’s kernel, and Eq. (28). We stress that this semi-classical approach employed is only valid if $\nu_s > 1$. For typical experimental systems, $\nu_s > 30$.

IV. DISCUSSION

In real systems, the source and detector have finite size and can influence the observed interference pattern. Typically experimental devices use quantum point contacts, which consist of a narrow channel connecting a reservoir to the 2DHG. These have some characteristic width, $w$, and can be modelled by standing waves in the $y$ direction,

$$\psi_s \propto \chi_s \sin \left( \frac{\pi y}{w} \right) \quad 0 < y < w$$

$$\psi_d \propto \chi_d \sin \left( \frac{\pi(y - L)}{w} \right) \quad L < y < L + w. \quad (29)$$

where $w$ is the width of the channel, and $\chi_s$ and $\chi_d$ are the eigenspinors at the source and detector respectively. The exit width, $w_2$, is imposed by the lithographic geometry of the QPC however can vary depending on the conductance. We consider a hole gas with a density, $n = 1.85 \times 10^{11} cm^{-2}$, and corresponding Fermi momentum, $k_F = 0.10\pi nm^{-1}$. With a Rashba splitting of $\tilde{\gamma}_3 = 0.2eF$, the smaller spin-split Fermi momentum will be $k_- = 0.096nm^{-1}$ ($\lambda \approx 66nm$) and the larger Fermi momentum $k_+ = 0.118nm^{-1}$ ($\lambda \approx 53nm$). The distance between the source and the detector is $L = 1500nm$. The corresponding magnetic field at the classical edge of the bright region for $k_-$ is $B_- = 77mT$, while for $k_+$, $B_+ = 94mT$. We will start by considering a QPC of width $w = 150nm$, which we note corresponds to the lithographic width of the device of Ref. [22]. The resulting focusing spectrum is presented in Fig. 5. While interference fringes are still visible, they are suppressed, and the additional phase contribution manifests as a suppression or enhancement of the spin-split focusing peaks. We note that this experimental signature is similar to that typically attributed to polarisation in the QPCs.

In summary, we have employed Huygen’s principle to determine quantum interference for systems with asymmetrical Fermi surfaces. While in this work we focus on a specific case of an in-plane magnetic field in combination with a Rashba spin-orbit interaction, the method employed is general. We have predicted an emergent phase contribution, linear in the applied in plane magnetic field, despite there being no first order changes to the classical trajectories. This emergent phase term significantly alters the interference spectrum of TMF. We propose that this could be used to measure the in plane $g$ factor.
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