Color flux profiles in SU(2) lattice gauge theory

C. Schlichter\textsuperscript{a}, G.S. Bali\textsuperscript{a} and K. Schilling\textsuperscript{b, a}

\textsuperscript{a}Fachbereich Physik, Bergische Universität, D-42097 Wuppertal, Germany
\textsuperscript{b}HLRZ c/o Forschungszentrum Jülich, D-52425 Jülich, Germany

Results of a high statistics study of chromo field distributions around static sources in pure SU(2) gauge theory on lattices of volumes $16^4$, $32^4$, and $48^3 \times 64$ at $\beta = 2.5$, 2.635, and 2.74 are presented. We establish string formation up to physical distances as large as 2 fm.

1. INTRODUCTION

Various effective models\textsuperscript{b} originated from the type II superconductivity scenario of confinement\textsuperscript{c}. Lattice gauge theory, in principle, offers the laboratory to test such ideas, as it allows for \textit{ab initio} studies from the QCD Lagrangian. Quenched calculations have reached the accuracy necessary for quantitative studies of the chromo field distributions. These are not easily accessible, since (a) the energy density carries dimension $a^{-4}$ and therefore imposes a lower limit onto the lattice spacing $a$, due to statistical noise, and (b) one is forced to work on rather large lattice volumes to attain source separations sufficient for applicability of the string picture. On top of this one is faced with the ubiquitous problem of filtering ground state signals out of an excited state background.

We have carried out a comprehensive investigation of flux tube profiles between static $q\bar{q}$ pairs. Exploitation of state-of-the-art lattice techniques for statistical noise reduction and ground state enhancement enables us to observe string formation up to physical distances of 2 fm, corresponding to more than 30 lattice sites at $\beta = 2.635$.

2. SIMULATION

We study lattice volumes of $16^4$ and $32^4$ at $\beta = 2.5$ ($a \approx 0.083$ fm), $48^3 \times 64$ at $\beta = 2.635$ ($a \approx 0.054$ fm) and $32^4$ at $\beta = 2.74$ ($a \approx 0.041$ fm). The scale has been computed from the string tension value $\sqrt{\kappa}a = 440$ MeV. A hybrid of Fabricius-Haan heatbath and overrelaxation has been applied for the gauge field update.

The central observables in our present investigation, the action and energy densities in presence of two static quark sources,

$$\begin{align*}
\epsilon_R(n) = & \quad \frac{1}{2} \left( \langle W^2 R(n) \rangle - \langle W \rangle \langle R \rangle \right) \\
\sigma_R(n) = & \quad 4 \left( \langle B^2 R(n) \rangle - \langle B \rangle \langle R \rangle \right)
\end{align*}$$

are calculated by taking the correlation between smeared Wilson loops $W$ and plaquettes $\square$,

$$\langle \square \rangle_W = \left( \frac{\langle W \rangle}{\langle W \rangle} - \langle \square \rangle \right).$$

The flux-tube profiles are probed by varying the spatial position of the plaquette which acts like a chromo electric (magnetic) Hall detector. To minimize contaminations from excited states, the temporal position is chosen as close as possible to the center of the Wilson loop. By averaging two (four) adjacent electric (magnetic) plaquettes, we arrive at operator insertions that are symmetric in respect to a given (spatial) lattice site $n$.\hfill

\footnotesize
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For measurement of the colour field distributions we have restricted ourselves to on-axis separations of the two sources. All even distances $R = 2, 4, \ldots, R_{\text{max}}$ with $R_{\text{max}} = 8, 24, 36$ for $L_S = 16, 32, 48$, respectively, have been realized. In order to identify the asymptotic plateau, $T$ was varied up to $T = 6$. The colour field distributions have been measured up to a transverse distance $n_{\perp} = 2$ along the entire $\bar{Q}Q$ axis. In between the two sources and up to 2 lattice spacings outside the sources, the transverse distance was increased to $n_{\perp,\text{max}} = 6, 10, 15$ for the three lattice extents $L_S = 16, 32, 48$, respectively. In addition to "on-axis" positions, $n = (n_1, n_2, 0)$, we chose plane-diagonal points $n = (n_1, n_2, n_2)$ with $n_2 < n_{\perp,\text{max}}/\sqrt{2}$. We averaged over various coordinates $n$, exploiting the cylindrical and reflection symmetry of the problem. The field measurements have been taken every 100 sweeps at $\beta = 2.5, 2.74$ and every 200 sweeps at $\beta = 2.635$. At $(\beta, V) = (2.5, 16^3), (2.5, 32^3), (2.635, 48^3 \times 64), (2.74, 32^4), 8680, 2046, 248, 1480$ measurements have been taken respectively.

The temporal parts of the Wilson loops, appearing in the colour field correlator, have been link integrated while the spatial parts have been smeared. The former procedure is applied to improve the signal to noise ratio, the latter is essential for getting early (in $T$) ground state dominance. A more detailed description of the lattice methods, analysis procedure and results can be found in Ref. [4]. (A database of colour images methods, analysis procedure and results can be found in the directory pub/colorflux.)

3. RESULTS

Here, we restrict ourselves to some selected topics. In Fig. 1 the longitudinal action density profile for a quark separation $r \approx 1.6$ fm is displayed. The string, connecting the sources, is (almost) homogeneous within a region of extent 1 fm.

The field distributions within the center plane of the flux tube have been studied in detail (see Fig. 2). For small source separations perturbation theory should apply and one might expect the distribution to agree with a dipole shape (up to lattice artefacts):

$$f_d(n_{\perp}) \propto \frac{1}{(4\pi)(\delta^2 + n_{\perp}^2)^3}$$

with $\delta = R/2$. $n_{\perp}$ denotes the distance from the $q\bar{q}$ axis. Fits to the above parametrization have been performed, in which $\delta$ has been treated as a free parameter. At large (compared to the width of the flux tube) source separation, the string picture should apply and we expect a Gaussian shape, at least for small $n_{\perp}$:

$$f_g(n_{\perp}) \propto \frac{1}{2\pi \delta^4} \exp \left( -\frac{n_{\perp}^2}{\delta^2} \right)$$.

Due to cancellations of electric and magnetic components, precise data for the energy density have only been obtained for $r \leq 0.75$ fm. Within this region, where the string is not yet developed the data is well described by leading order lattice perturbation theory with an effective coupling, increasing with $R$ in agreement with asymptotic freedom. The action density data allow for fits up to distances of 2 fm. Ansätze eqn. (3) and (4) describe the data equally well as can be seen from Fig. 2.

The root mean square width, $\delta$, of the flux tube is physically more interesting than the half width which can be read off from plots like Fig. 2.

![Figure 1. Longitudinal action density profile in lattice units at $\beta = 2.635$, $r = 30a \approx 1.6$ fm.](image)
4. CONCLUSIONS

We have demonstrated that Wilson loop-plaquette correlations offer a viable access to a lattice study of the flux tube problem on the required length scale of one to two fm. The crucial ingredient of our method is the smearing of the parallel transporter within the bilocal $Q\bar{Q}$ creation operator. This secures a controlled ground state preparation of long flux tubes within few lattice time slices.

As a result, we can observe flux formation of the action density over source separations as large as 2 fm on lattices with resolution .05 fm and extent 2.7 fm. A logarithmic increase of the width as suggested by string pictures for the “asymptotic” $R$ region is consistent with our data, suggesting a surprisingly large ultra violet cut-off on inverse wavelengths in effective string models, $r_0^{-1} \gg 1$ GeV.

In principle, $\delta$ can be extracted from fits but strongly depends on the underlying parametrization. As we are mainly interested in the $R$ dependence, especially in probing the predicted logarithmic asymptotic increase of $\delta(R)$ [1], we use a geometric definition of the width [2] that avoids large errors on ratios of $\delta$s, obtained at different quark separations. It is based on the assumption that the transverse shape of the distribution is independent of $R$ which holds true to the right of the vertical line in Fig. 3. The model dependence of $\delta^2$ has been cast into a (multiplicative) geometry factor $\gamma$ which is of order one.

The width increases rapidly until it saturates at $r \approx 1$ fm at a value $0.5$ fm $\leq \delta a \leq 0.75$ fm where the error is mainly due to the uncertainty in $\gamma$. Combining the string expectation, $\delta^2 = \delta_0^2 \ln(R/R_0)$, with our data, yields an inverse cut-off wavelength $R_0^{-1} \approx 30\sqrt{K}$ (full curve) when $\delta_0$ is constrained to the universal value $\delta_0 = 1/(\pi\sqrt{K})$ [3]. Without restrictions on $\delta_0$ we find the limits $\delta_0 \leq 2.2/(\pi\sqrt{K})$ and $R_0^{-1} \geq 3\sqrt{K}$, respectively (dashed curve). In case of the simpler case of 3d $Z_2$ gauge theory, Gliozzi has observed a more pronounced logarithmic increase of $\delta(R)$ [4].

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