Abstract

We report our calculations on the logarithmic slope of the diffractive structure function $F_D^2$ considering distinct approaches coming from both the pQCD and the Regge descriptions of diffractive DIS. Such a quantity is a potential observable to discriminate between pQCD and Regge domains and allows to disentangle the underlying dynamics.

Key words: Perturbative QCD, Regge formalism, Structure functions

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1 Introduction

The experimental measurements of the logarithmic slope ($Q^2$-slope) of the inclusive structure function $F_2(x, Q^2)$ presented a new challenge for the current small $x$ approaches [1]. The standard DGLAP [2] dynamics was found to fail in describing the low momentum transfer $Q^2 < 1$ GeV$^2$ region in the correlated $(x, Q^2)$ plane, overestimating the data. The QCD expected result is that the slope should be proportional to the nucleon gluon distribution. Thus, new phenomena could be present modifying the gluonic content of the proton or a transition between hard and soft domains would be emerging. Indeed, the earlier observed turn over in the $Q^2$-slope was considered as an evident signal for change of the dynamics (see e.g., Ref. [3]). The most recent non-correlated measurements [4] support a broad DGLAP description using the updated pdf’s. However, the turn over observed when one considers the slope

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at fixed center of mass energy $W$ still deserves criterious work [5]. Therefore, a complete description of the $F_2(x, Q^2)$ logarithmic $Q^2$-slope in the full kinematical range is not known at present and suggests an interplay or a competition between hard and soft regimes.

When we look into diffraction dissociation in deep inelastic scattering, in particular the diffractive structure function $F_2^D$, an interplay between hard and soft domains is more explicit [6]. The soft content in diffractive dissociation is known to be large, justifying the extensive non-perturbative phenomenology used to describe its energy dependence. Regarding this feature, the structure function $F_2^D$ is a good test for Pomeron models, and the question if we have in the real world a perturbative (BFKL) or a soft Pomeron is an open discussion in the community. The main advantage from the Regge inspired models is a quite simple phenomenological description of the process. However, they cannot constrain in a theoretical ground the behavior on $Q^2$ and $\beta$ variables. On the other hand, a great progress has been made in perturbative QCD, setting the correspondent dominant contributions order by order in perturbation theory. The notions of photon wavefunction and dipole cross section are now the basic blocks to describe perturbatively the diffractive events. The main appeal of the pQCD approaches is to produce very clear predictions concerning the $\beta$ spectrum, as well as setting a deeper connection with the machinery builded for the inclusive case (DIS).

In a similar way as the $F_2$ slope case, the logarithmic slope of the diffractive structure function $F_2^D$ should be studied and its role into disentangle dynamics needs investigation. The experimental status concerning $F_2^D$ provides high precision data in a wide kinematical region, possibly allowing to extract the slope with good statistic precision. Bearing in mind these issues, we have proposed [7,8] to calculate this quantity from the available models describing diffractive DIS. In particular, we choose two representative approaches in order to obtain quantitative results. One of them is the pQCD approach [9], where the diffractive process is modeled as the scattering of the photon Fock states with the proton through a gluon ladder exchange (in the proton rest frame). Moreover, we consider a Regge inspired model [10], where the diffractive production is dominated by the soft Pomeron. These approaches differs basically in the Pomeron exchange definition and its coupling with the incident photon.

In Ref. [7] we have calculated the slope for both pQCD and Regge approaches considering a kinematical constraint for the correlated variables $x$ and $Q^2$. In Ref. [8] such constraint was disregarded, and the calculations were also extended to include the analysis of the saturation model [11]. However, here we report basically the conclusions presented in [7]. In a general pointview, the distinct behaviors obtained for the logarithmic slope from different physical approaches could give hints in the underlying dynamics and which can be tested if such observable is to be experimentally measured.
Analytical results for the Regge based model

A few years ago Capella-Kaidalov-Merino-Tran Thanh Van (CKMT) proposed a model for diffractive DIS based on Regge theory and the Ingelman-Schlein ansatz [10]. The Pomeron is considered as a Regge pole with a trajectory $\alpha_{\mathcal{P}}(t) = \alpha_{\mathcal{P}}(0) + \alpha'_{\mathcal{P}} t$ determined from soft processes, in which absorptive corrections (Regge cuts) are taken into account. Explicitly, $\alpha_{\mathcal{P}} = 1.13$ and $\alpha'_{\mathcal{P}} = 0.25$ GeV$^{-2}$. The diffractive contribution to DIS is written in the factorized form:

$$F^D_2(x_{\mathcal{P}}, \beta, Q^2, t) = \frac{[g_{pp}^\mathcal{P}(t)]^2}{16\pi x_{\mathcal{P}}^{1-2\alpha_{\mathcal{P}}(t)}} F_{\mathcal{P}}(\beta, Q^2, t),$$ (1)

where the details about the coefficients appearing in the Pomeron flux can be found in Refs. [10]. One of the main points of this model is the dependence on $Q^2$ of the Pomeron intercept [$F_{\mathcal{P}} \propto \beta^{\Delta(Q^2)}$]. For low values of virtuality (large cuts), $\Delta$ is close to the effective value found from analyzes of the hadronic total cross sections ($\Delta \sim 0.08$), while for high values of $Q^2$ (small cuts), $\Delta$ takes the bare Pomeron value, $\Delta \sim 0.2 - 0.25$. The comparison of the CKMT model with data is quite satisfactory, mainly when considering a perturbative evolution of the Pomeron structure function. We notice that here one uses the pure CKMT model rather than including QCD evolution of the initial conditions, which has been considered to improve the model at higher $Q^2$. Such procedure ensures that we take just a strict Regge model, namely avoiding contamination by pQCD phenomenology.

Justifying our choice, the CKMT is a long standing approach describing in a consistent way both inclusive and diffractive deep inelastic based on Regge theory, which is continuously improved considering updated experimental results [12]. A subtle question in CKMT approach is the dependence of the Pomeron intercept on the virtuality for the inclusive case: the smooth interpolation between a soft and a semihard intercept seems to break down the pure reggeonic feature of the model. However, in diffractive DIS the energy dependence (i.e., the Pomeron flux) is driven by a soft Pomeron with a fixed $\alpha_{\mathcal{P}} = 1.13$, properly corrected by absorptive effects. Indeed, this fact is verified in [7], when considering the effective slope $\partial \ln F^D_2 / \partial \ln(1/x_{\mathcal{P}})$. 

Considering all properties of the diffractive structure function in the CKMT model, the calculation of the logarithmic slope is straightforward. The expression reads now:

$$\frac{\partial F^D_2}{\partial \ln Q^2} = N x_{\mathcal{P}}^{1-2\alpha_{\mathcal{P}}(0)} \left[ eA \beta^{-\Delta(Q^2)} (1 - \beta)^n(Q^2) + 2 \left( \frac{Q^2}{Q^2 + a} \right)^{1+\Delta(Q^2)} S_{\mathcal{P}}(Q^2, \beta) \right]$$
\[
+ f B \beta^{1-\alpha_R} (1 - \beta)^n(Q^2) - 2 \left( \frac{Q^2}{Q^2 + b} \right)^\alpha_R S_R(Q^2, \beta) \] ,
\]

where the overall normalization \( N \) comes from the integration over \( t \) of the Pomeron flux, \( n(Q^2) = \frac{3}{2} \left( 1 + \frac{Q^2}{Q^2 + c} \right) \), \( \alpha_R \) is the secondary reggeon intercept (only the f meson). The coefficients and constants are taken from [10], and the factors \( S_{F,R}(Q^2, \beta) \) are defined as

\[
S_{F}(Q^2, \beta) = \Delta(Q^2) \left[ \frac{a}{Q^2 + a} \right] + \frac{3c}{2} \left[ \frac{Q^2}{(Q^2 + c)^2} \right] \ln(1 - \beta) + 2d \Delta_0 \left[ \frac{Q^2}{(Q^2 + a)^2} \right] \ln \left( \frac{Q^2}{\beta(Q^2 + a)} \right) ,
\]

\[
S_{R}(Q^2, \beta) = \alpha_R(0) \left[ \frac{b}{Q^2 + b} \right] + \frac{3c}{2} \left[ \frac{Q^2}{(Q^2 + c)^2} \right] \ln(1 - \beta) .
\]

3 Analytical results for the pQCD based model

Although the diffractive dissociation to be mainly connected to soft processes and thus linked with the Regge theory, the pQCD framework has been recently used to describe quite well the diffractive structure function. In particular, we have considered the sound model in [9]. The physical picture is that, in the proton rest frame, diffractive DIS is described by the interaction of the photon Fock states (\( q \bar{q} \) and \( q \bar{q}g \) configurations) with the proton through a Pomeron exchange, modeled as a two hard gluon exchange. The corresponding structure function contains the contribution of \( q \bar{q} \) production to both the longitudinal and the transverse polarization of the incoming photon and of the production of \( q \bar{q}g \) final states from transverse photons.

The basic elements of this approach are the photon light-cone wavefunction and the unintegrated gluon distribution (or dipole cross section in the dipole formalism). For elementary quark-antiquark final state, the wavefunctions depend on the helicities of the photon and of the (anti)quark. For the \( q \bar{q}g \) system one considers a gluon dipole, where the \( q \bar{q} \) pair forms an effective gluon state associated in color to the emitted gluon and only the transverse photon polarization is important. The interaction with the proton target is modeled by two gluon exchange, where they couple in all possible combinations to the dipole. In a comparison with data, the transverse \( q \bar{q} \), \( q \bar{q}g \) production and the longitudinal \( q \bar{q} \) production dominate in distinct regions in \( \beta \), namely medium, small and large \( \beta \) respectively [9]. The \( \beta \) spectrum and the \( Q^2 \)-scaling behavior follow from the evolution of the final state partons, and they are derived from the light-cone wavefunctions of the incoming photon, decoupling from the dy-
The energy dependence and the normalizations are free parameters.

The calculations for the $F_2^D$-logarithmic slope are simple to perform, considering each contribution coming from the different configurations of the photon Fock state. Moreover, this approach allows to obtain parameter free predictions for the logarithmic slope, since all coefficients have been obtained in comparison with the H1 and ZEUS data. We justify our choice due to the simple analytic expressions for the diffractive structure function regarding each Fock state configuration, which turns the analyzes clearest and avoid cumbersome numeric calculations. The expressions for the distinct contributions to $F_2^{D(3)}$ as well as the remaining parameters can be found in Ref. [9]. Thus, the diffractive logarithmic slope is written as

$$
\frac{d F_2^{D(3), q\bar{q}T}(x_p, \beta, Q^2)}{d \ln Q^2} = \frac{n_1^1}{(\ln \frac{Q^2}{Q_0^2} + 1)} \frac{F_2^{D(3), q\bar{q}T}(x_p, \beta, Q^2)}{F_2^{D(3), q\bar{q}T}(x_p, \beta, Q^2)} ,
$$

$$
\frac{d F_2^{D(3), q\bar{q}G}(x_p, \beta, Q^2)}{d \ln Q^2} = \frac{1}{(\ln \frac{Q^2}{Q_0^2} + 1)} \left[ n_2^1 + \frac{Q^2}{Q^2 + Q_0^2} \right] \frac{F_2^{D(3), q\bar{q}G}(x_p, \beta, Q^2)}{F_2^{D(3), q\bar{q}G}(x_p, \beta, Q^2)} ,
$$

$$
\frac{d F_2^{D(3), q\bar{q}L}(x_p, \beta, Q^2)}{d \ln Q^2} = \left[ \frac{n_4^1}{(\ln \frac{Q^2}{Q_0^2} + 1)} + \frac{Q^2 - 7 \beta Q_0^2}{Q^2 + 7 \beta Q_0^2} \right] \frac{F_2^{D(3), q\bar{q}L}(x_p, \beta, Q^2)}{F_2^{D(3), q\bar{q}L}(x_p, \beta, Q^2)} .
$$

Fig. 1. The $Q^2$-slope versus $\beta$ for the pQCD (on the left) and CKMT (on the right) models.
4 Results and Discussions

In Figs. (1)-(2) we present a comparison between the models mentioned above. In Fig. (1) one shows the $\beta$-dependence for typical values of $x_P$ and $Q^2$, where the momentum transfer is ranging from 1 up to 100 GeV$^2$. The CKMT model predicts a flat behavior on the whole $\beta$ range. Particularly, at $Q^2 = 1$ GeV$^2$ there is a strong decreasing of the slope at large $\beta$. This is due to the presence of factors $\ln(1 - \beta)$ in the second terms of Eqs. (3)-(4). For larger $Q^2$ values, the logarithm on $Q^2$ appearing at the third term of Eq. (3), compensates the decreasing. At 100 GeV$^2$ of momentum transfer, the CKMT predicts a flat behavior of the slope for the whole $\beta$ spectrum. The pQCD model produces an increasing of the slope in both small and large values of $\beta$, while presents a flat behavior at the medium one. This increasing of the slope is due to the enhancements in the $qqG$ (dominant at low $\beta$) and $qqL$ (leading at high $\beta$) contributions from the $Q^2$-factors appearing in Eqs. (5), respectively. A steep $Q^2$-slope decreasing into negative values is also present at the virtuality $Q^2 = 1$ GeV$^2$ for large $\beta$, in a similar way to the CKMT model. However, beyond the contribution of the (dominant) longitudinally polarized $qq$ pair configuration, this region also receives contributions associated to the $qqG$ configuration, as discussed in [8].

In Fig. (2) one presents the $x_P$ dependence at medium and large $\beta$. The low $\beta$ region was disregarded in order to avoid to deal with subleading reggeonic contributions, important in this kinematical domain. We observe again a smooth behavior for the pQCD predictions, with a positive slope in almost the whole range (negative values are present at both large $\beta$ and $Q^2 = 1$ GeV$^2$, in agree-
ment with the discussion in the previous paragraph). In Ref. [8] we have found that the saturation model produces a transition between positive and negative slope values at low $\beta = 0.04$, while shows a positive slope for medium and large $\beta$, differing from the results for the pQCD model discussed above. Since the diffractive cross section is strongly sensitive to the infrared cutoff, one of the main differences between these two models is the assumption related to the small $Q^2$ region. Returning, the CKMT model predicts predominantly negative slope values in this kinematical domain, converging to a flat value for larger $x_F$. A strict difference between the predictions of the two models is the change of signal of the slope with the $Q^2$ evolution at medium $\beta$, present in the CKMT results.

In Fig. (3) we show the results for $d \ln F_2^D/d \ln(1/x_F)$ (or shortly, $x_F$-slope) as a function of the photon virtuality $Q^2$. Indeed, this quantity gives the Pomeron intercept and its behavior describes the energy dependence of the diffractive structure function. While the CKMT model predicts a constant value, without dependence on $\beta$, the pQCD model is $\beta$-dependent. This feature is associated to the distinct energy dependence of each term in Eqs. (5), which dominate at specific regions of the phase space. For instance, a hard intercept is observed from the slope at large $\beta = 0.9$. In fact, a dependence on $\beta$ for the Pomeron intercept is expected as shown in Ref. [9]. We notice that the pQCD model is only valid above the starting point $Q^2_0 = 1 \text{ GeV}^2$ and our extrapolation for lower values is for sake of comparison. For completeness we include the soft Pomeron intercept in the plot. We verify, therefore, the evident distinction between the predictions from the CKMT and pQCD based approaches.

In summary, the results above allow to discriminate the behaviors predicted from the different approaches, namely perturbative QCD (hard physics) and non-perturbative (soft) physics. Theoretically, the difference between comes from the ansatz for the photon-proton interaction (hard or soft) and also
from the relation of the Pomeron structure function with the proton inclusive one, for instance considered in the CKMT model. Such a relation implies the inclusion at most of the $q\bar{q}_T$ configuration in the photon wavefunction in the CKMT estimates. On the other hand, the pQCD model analyzed here includes the contribution of gluon emission in the photon wavefunction, thus taking into account the leading twist contributions $q\bar{q}_T$ and $(q\bar{g})_T$. Moreover, the higher twist piece, $q\bar{q}_L$ is also included in the description. Therefore, the analyzes of the $\beta$ spectrum of the $F_2^P$-slope would be important in an experimental study. Moreover, concerning the $x_F$ spectrum, the signal of the slope at medium values of $Q^2$ ($Q^2 \approx 10 \text{ GeV}^2$) is a good source of information about the dynamics.

References

[1] A. Caldwell, Invited talk, DESY Theory Workshop. DESY, Hamburg (Germany) October 1997; J. Breitweg et al., Eur. Phys J. C7, 609 (1999).

[2] Yu. L. Dokshitzer, Sov. Phys. JETP 46, 641 (1977); G. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977); V. N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys 28, 822 (1978).

[3] M.B. Gay Ducati, V.P. Gonçalves, Phys. Lett. B487, 110 (2000).

[4] H1 Collaboration, Eur. Phys. J. C21, 33 (2001).

[5] R. Thorne, in Workshop on Low x Physics 2001, June 27 - 30, Krakow, Poland (2001).

[6] A.H. Mueller, Eur. Phys. J. A1, 19 (1999).

[7] M.B. Gay Ducati, V.P. Gonçalves, M.V.T. Machado, Phys. Lett. B506, 52 (2001).

[8] M.B. Gay Ducati, V.P. Gonçalves, M.V.T. Machado, Nucl. Phys. A (in press) [hep-ph/0103245].

[9] J. Bartels, M. Wüsthoff, J. Phys. G: Nucl. Part. Phys. 22, 929 (1996); J. Bartels, J. Ellis, H. Kowalski, M. Wüsthoff, Eur. Phys. J. C7, 443 (1999); J. Bartels, C. Royon, Mod. Phys. Lett. A14, 1583 (1999).

[10] A. Capella et al., Phys. Lett. B337, 358 (1994); Phys. Lett. B343, 403 (1995); Phys. Rev. D53, 2309 (1996).

[11] K. Golec-Biernat, M. Wüsthoff, Phys. Rev. D59, 014017 (1999); Phys. Rev. D60, 114023 (1999).

[12] A.B. Kaidalov, C. Merino, D. Pertermann, Eur. Phys. J. C20, 301 (2001).