SOLITONIC BLACK HOLES IN GAUGED N=2 SUPERGRAVITY

V. Alan Kostelecký\textsuperscript{a} and Malcolm J. Perry\textsuperscript{b}

\textsuperscript{a}Physics Department
Indiana University
Bloomington, IN 47405, U.S.A.

\textsuperscript{b}D.A.M.T.P.
University of Cambridge
Silver Street
Cambridge CB3 9EW, England

A sequence of zero-temperature black-hole spacetimes with angular momentum and electric and magnetic charges is shown to exist in gauged $N = 2$ supergravity. Stability of a subset of these spacetimes is demonstrated by saturation of the Bogomol’nyi bound arising from the supersymmetry algebra. The mass of the resulting solitonic black holes is given in terms of the cosmological constant and the angular momentum. We conjecture that at the quantum level these solitons are dyons with angular momentum determined by the electric and magnetic charges.

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1. Introduction

In classical general relativity, black holes are regions of spacetime from which it is impossible to escape, with the event horizon being the boundary between the trapped region and normal spacetime. If the horizon had a future endpoint, the spacetime would contain a naked singularity. Assuming the cosmic-censorship hypothesis [1], it follows that black holes do not disappear after formation. There is substantial evidence in favor of this supposition in the form of the classical stability theorems [2]. The combination of the nontrivial spacetime structure and the stability of black holes at the classical level are features reminiscent of solitons.

The discovery of the Hawking effect rendered classical notions of stability invalid [3]. Black holes are unstable as a result of their nonvanishing Hawking temperature and negative specific heat. For nonrotating electrically neutral holes, the temperature is given by

\[ T = \frac{1}{8\pi M}, \]  

where \( M \) is the mass of the hole. A black hole loses mass at a rate \( dM/dt \sim -M^{-2} \). If its initial mass is \( M_0 \), then it evaporates in a time interval \( \tau \sim M_0^3 \).

Suppose that instead of electrically neutral black holes we consider charged non-rotating black holes. Then, the Hawking temperature is given by

\[ T = \frac{1}{2\pi} \frac{\sqrt{M^2 - Q^2}}{(M + \sqrt{M^2 - Q^2})^2}. \]  

If the charge is large enough, \( Q = \pm M \), then it follows that the Hawking temperature is zero and one might expect these objects to be stable. However, the true situation is more complicated than this simple argument suggests. Vacuum polarization effects cause the black hole to discharge itself rapidly [4]. An electrically charged hole preferentially creates electrons (or positrons), thereby losing its charge.

There are two ways to stabilize the situation. One is to take the charge to be topological in nature, so there are no particles to radiate [5]. The other possibility is to suppose that the lightest charged particle cannot be created by the black hole. A discussion along these lines for baryonic charge can be found in ref. [6]. For example, suppose that the black hole carries magnetic charge instead of electric charge. The
only way for the hole to lose this charge would be via the creation of magnetic monopoles. If the monopoles are heavy enough, however, the probability of decay is heavily suppressed even for hot holes. A variant on this scenario is to suppose that the charge arises as a central charge in a supersymmetry algebra. In either case, there seems to be no obstacle to having a stable black-hole soliton, i.e., a stable object with zero Hawking temperature.

An example of a soliton of this type is the extreme charged Reissner-Nordstrom black hole with $|Q| = M$ for large $M$ in $N = 2$ supergravity [7]. A remarkable property of these holes is that they are supersymmetric. This means that they saturate the Bogomol’nyi bound derived from the supersymmetry algebra, and hence they have Killing spinors. Furthermore, since the soliton is in fact a degenerate black hole, the black-hole spacetime has a global structure [8], unlike the case where $|Q| < M$. These features indicate that solitonic black holes are somewhat different from conventional black holes formed by gravitational collapse. Note that, if $|Q| > M$, then the spacetime becomes a naked singularity.

These results suggest a possible deep connection between supersymmetry and black-hole physics. In fact, any black hole that is supersymmetric must have zero temperature, although the converse does not necessarily hold. Consider a black-hole spacetime with Hawking temperature $T$. The region close to a Killing horizon can always be complexified, and a space with riemannian signature can be constructed for which the absence of conical singularities on the euclidean horizon is equivalent to requiring that the euclidean time coordinate generated by the Killing vector on the horizon is periodic with period $1/T$. For a spinor field to be nonsingular in such a space requires antiperiodicity under translation through a period. However, supersymmetry implies the existence of a spinor field solving the Killing spinor equation, and this spinor must be periodic to give a regular solution. These two periodicity constraints are compatible only when the period is infinite, or equivalently when the Hawking temperature vanishes [8]. Note that zero Hawking temperature alone is insufficient to ensure the existence of a Killing spinor, as there are other criteria that must be satisfied involving various conditions on the Riemann tensor and possibly other fields.

Based on the coincidence between extreme Reissner-Nordstrom holes and super-
symmetric holes in $N = 2$ supergravity, it has recently been conjectured that there might be a relationship between supersymmetric black holes and cosmic censorship [10]. A similar phenomenon also exists in $N = 4$ supergravity. In its strongest form, we interpret the conjecture as the statement of equivalence of the Bogomol’nyi bound and the zero-temperature condition.

An examination in general relativity of stationary axisymmetric black-hole solutions in asymptotically flat spacetimes reveals a very restricted set of allowed configurations. The only possibilities are the Kerr-Newman sequence of solutions, characterized solely [11] by the mass $M$, angular momentum $J$, and charge $Q$. If $M < \sqrt{Q^2 + J^2 / M^2}$ then the solution is a black hole with nonzero Hawking temperature, while if $M = \sqrt{Q^2 + J^2 / M^2}$ the solution is a zero-temperature black hole. If, however, $M > \sqrt{Q^2 + J^2 / M^2}$ then the solution is a naked singularity. It turns out that the condition for the existence of Killing spinors is [12] $M = |Q|$, which coincides with the zero temperature requirement only when $J = 0$. In fact, for $J = 0$ the Killing spinors are regular on and outside the horizon. For $J \neq 0$, the spacetimes are naked singularities and the Killing spinors diverge at the singularities. This therefore provides a counterexample to the strong form of the conjectured relationship between supersymmetry and cosmic censorship. Examples of spherically symmetric static configurations that obey the supersymmetric bound but violate cosmic censorship are also known [13, 14, 15].

Further examples along these lines would evidently be of interest. One candidate theory that could be examined is gauged $N = 2$ supergravity [16]. In this model, the vacuum state is anti-de Sitter space rather than Minkowski spacetime, and for that reason we might expect the properties of black holes to be somewhat different from the ungauged theory. In this paper, we explore this issue. We present a family of zero-temperature black-hole solutions with nonzero angular momentum and charge. As in the Kerr-Newman case, we find that the zero-temperature condition for the
new black holes is distinct from the equation saturating the Bogomol’nyi bound.

2. Gauged $N = 2$ Supergravity

The supersymmetry algebra of gauged $N = 2$ supergravity is $\text{osp}(4|2)$. Ten of its generators are the bosonic generators $M_{AB}$ for the anti-de Sitter subgroup $\text{SO}(3,2)$, corresponding to the ten Killing vectors of anti-de Sitter space. The supersymmetries are generated by a pair of Majorana fermionic operators $Q_i^\alpha$, where $i = 1, 2$ for the two supersymmetries and $\alpha$ is a Dirac index. There is also an additional bosonic generator that rotates the two supersymmetries into each other $^{[17, 18]}$. The anticommutator of the fermionic generators, from which one can construct explicitly the whole superalgebra, is

$$\{Q_i^\alpha, Q_j^\beta\} = \delta^{ij} \left((\gamma^a M_{a4} + i\sigma^{ab} M_{ab}) C\right)_{\alpha\beta} + i(C_{\alpha\beta} U^{ij} + i(C\gamma^5)_{\alpha\beta} V^{ij}) \ ,$$

where $a, b = 0, 1, 2, 3$, $\sigma^{ab} = i[\gamma^a, \gamma^b]/2$, $C_{\alpha\beta}$ is the charge-conjugation matrix, and $U^{ij} = Q\epsilon^{ij}$ and $V^{ij} = P\epsilon^{ij}$ are possible central charges, with $\epsilon^{ij}$ being the two-dimensional alternating symbol. The term in braces on the right-hand side could be rewritten in $\text{SO}(3,2)$-invariant form as $\sigma^{AB} M_{AB}$.

Anti-de Sitter spacetime is conformal to flat spacetime. This can be seen explicitly from the line element, which can be written as

$$ds^2 = \frac{1}{\alpha^2 \cos^2 \rho} (-d\hat{t}^2 + d\rho^2 + \rho^2 d\Omega^2) \ ,$$

where $\hat{t}$ and $\rho$ are temporal and radial coordinates, $d\Omega = d\theta^2 + \sin^2 \theta d\phi^2$ is the line element on the unit two-sphere, and

$$\alpha = \sqrt{-\frac{\Lambda}{3}} \ ,$$

with $\Lambda < 0$ being the cosmological constant.

One can make a coordinate transformation by

$$t = \frac{\hat{t}}{\alpha} \ , \quad r = \frac{1}{\alpha} \tan \rho \ ,$$

where $r$ is the radial coordinate in flat space.

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which puts the metric into the Schwarzschild-type form

\[ ds^2 = -(1 + \alpha^2 r^2) dt^2 + \frac{dr^2}{1 + \alpha^2 r^2} + r^2 d\Omega^2. \]  

(7)

In these latter coordinates, the Killing vectors for time translation and spatial rotation about the third axis can be expressed as

\[ K_{04} = \frac{1}{\alpha} \frac{\partial}{\partial t}, \quad K_{12} = \frac{\partial}{\partial \phi}. \]  

(8)

The remaining eight Killing vectors can also be given in this coordinate system, but their form is not needed in what follows. The commutation rules for all ten Killing vectors are the same as those of the generators of SO(3,2).

Any object that moves in anti-de Sitter space forms a representation of the anti-de Sitter group. The generators \( M_{AB} \) of O(3,2) can best be represented by a set of real antisymmetric \( 5 \times 5 \) matrices. Since such a matrix has eigenvalues \( 0, \pm i \lambda_1, \text{ and } \pm i \lambda_2 \), the zero eigenvector can be determined algebraically by

\[ V^A = \epsilon^{ABCDE} M_{BC} M_{DE}, \]  

(9)

with \( \epsilon^{ABCDE} \) the five-dimensional alternating symbol. It then follows from the Jacobi identity that

\[ M_{AB} V^A = 0. \]  

(10)

The (nonpositive) norm of \( V \) is given by

\[ ||V||^2 = V^A V^B \eta_{AB}, \]  

(11)

where \( \eta_{AB} \) is the so(3,2)-invariant metric

\[ \eta_{AB} = \text{diag}(- + + + ) \],  

(12)

which can be used for raising and lowering the indices \( A, B, \ldots \).

The group SO(3,2) has a pair of Casimir operators, one that is quadratic in the generators,

\[ C_2 = M_{AB} M^{AB} = 2(\pm \lambda_1^2 \pm \lambda_2^2), \]  

(13)
and one that is quartic,

\[ C_4 = M_B^A M_C^B M_D^C M_A^D = \left(2(\pm \lambda_1^4 \pm \lambda_2^4), \pm 2\lambda_1^4, \pm 2\lambda_2^4, 0\right). \]  

(14)

The norm can be determined as

\[ ||V||^2 = 8C_2^2 - 16C_4 \]  

(15)
in terms of the Casimir operators.

To see the relevance of this group theory, consider an object in a spacetime that is asymptotically (i.e., at large distances from the object) anti-de Sitter. Suppose further that the spacetime is stationary and antisymmetric. Then, the energy \( E \) and angular momentum \( J \) along the third axis are given by the expressions \[ \]  

\[ E = \frac{1}{8\pi} \int_{\Sigma} \nabla_a \delta K_{04b} dS^{ab}, \]  

(16)

\[ J = \frac{1}{4\pi} \int_{\Sigma} \nabla_a \delta K_{12b} dS^{ab}, \]  

(17)

where the symbol \( \delta X \) indicates the difference between \( X \) in the spacetime in question and \( X \) in anti-de Sitter space, and where the integrals are taken over a celestial sphere \( \Sigma \) at spatial infinity. The fact that the spacetime has these two Killing vectors implies that the values of \( E \) and \( J \) are independent of the particular way in which \( \Sigma \) is chosen. In terms of the Casimir operators in the rest frame of the object immersed in anti-de Sitter space, the energy and angular momentum are \[ \]  

\[ E = M_{04} = \sqrt{\frac{1}{4}C_2^2 + \frac{1}{2}\sqrt{C_4 - \frac{1}{4}C_2^2}}, \]  

(18)

\[ J = M_{13} = \sqrt{\frac{1}{4}C_2^2 - \frac{1}{2}\sqrt{C_4 - \frac{1}{4}C_2^2}}. \]  

(19)

Spacetimes that are asymptotic to anti-de Sitter space have a positive-energy theorem that can be motivated from supersymmetry \[ \]  

Consider the antisymmetric tensor \[ \]  

\[ E^{ab} = \text{Re} \left[ \bar{\epsilon} \gamma^{abc} \nabla_c \epsilon \right], \]  

(20)

where \( \epsilon \) is a spinor field in spacetime, and

\[ \gamma^{abc} = \gamma^{[a} \gamma^{b} \gamma^{c]}. \]  

(21)
In Eq. (20), the supercovariant derivative $\hat{\nabla}_a$ is defined by

$$\hat{\nabla}_a = \nabla_a + \frac{1}{2} i \alpha \gamma_a$$

(22)

with $\nabla_a$ being the usual covariant derivative. If

$$\hat{\nabla}_a \epsilon = 0$$

(23)

then the spacetime has a Killing spinor. This shows that one can carry out a supersymmetry transformation on a purely bosonic background that satisfies the field equations, and hence find an invariance of the system. In other words, the background is supersymmetric. If the background is exactly anti-de Sitter spacetime then there is a pair of Killing spinors, indicating the presence of unbroken $N = 2$ supersymmetry. Otherwise, no solutions to Eq. (23) exist [24].

The idea of a Killing spinor can be generalized to include the U(1) gauge field that occurs in the supersymmetry multiplet [7]. This is natural for $N = 2$ since the vanishing of the supersymmetry transformation of a gravitino is equivalent to the existence of a Killing spinor, and the gravitino carries the U(1) charge of the gauge field. Thus, the modified covariant derivative is now

$$\hat{\nabla}_a = \nabla_a + \frac{1}{2} i \alpha \gamma_a + \frac{1}{4} \gamma_a \sigma^{bc} F_{bc}$$

(24)

where $F_{ab}$ is the field-strength tensor of the U(1) gauge field. The charge on the U(1) gauge field manifests itself as the central charge in the supersymmetry algebra. This means that

$$Q = \frac{1}{4\pi} \int \Sigma F_{ab} dS^{ab}, \quad P = \frac{1}{4\pi} \int \Sigma *F_{ab} dS^{ab}$$

(25)

where $*$ is the spacetime dual, $*F_{ab} = \epsilon_{abcd} F^{cd}/2$. The possibility of having $F_{ab} \neq 0$ means that Killing spinors could exist outside spacetimes that are exactly anti-de Sitter.

One can use the tensor $E^{ab}$ to find some interesting properties of the spacetime. Suppose the spacetime admits a spacelike surface $\Sigma$ that is complete exterior to a horizon and is asymptotic to anti-de Sitter space. Suppose also that there is a spinor field $\epsilon$ in the spacetime that tends to $\epsilon_0 \neq 0$ on $\Sigma$ and obeys the fall-off condition

$$\hat{\nabla}_a \epsilon = 0 \left( \frac{1}{r^2} \right)$$

(26)
Then, it follows that
\[
\frac{1}{2} \int_{\Sigma} E_{ab} \, dS^{ab} = \bar{\epsilon}_0 \left[ J_{AB} \sigma^{AB} + i(Q + iP) \right] \epsilon_0 .
\]  
(27)

If, in addition, any matter exterior to any horizon obeys the dominant energy condition, and if the Witten equation
\[
(\delta^a_b + t_a t^b) \hat{\nabla}_a \epsilon = 0 ,
\]  
(28)

where \( t^a \) is an arbitrary timelike unit vector, is satisfied on some spatial surface then
\[
\bar{\epsilon}_0 [ J_{AB} \sigma^{AB} + i(Q + iP) ] \epsilon_0 \geq 0 .
\]  
(29)

Equality holds if and only if \( E_{ab} = 0 \), whereupon there must exist a Killing spinor. The saturation of this inequality is equivalent to saying that there is at least one state \(|s\rangle\) in the theory such that \( Q^i_{\alpha} |s\rangle = 0 \). By taking the expectation value of the fermionic anticommutator (3) in a general state, we discover the Bogomol’nyi bound
\[
(M \pm J)^2 \geq Q^2 + P^2 ,
\]  
(30)

which is saturated if \( Q^i_{\alpha} |s\rangle = 0 \). Hence, supersymmetric states in the theory obey
\[
M \pm J = \sqrt{Q^2 + P^2} .
\]  
(31)

The sign choice corresponds to the two ways of breaking \( N = 2 \) supersymmetry to \( N = 1 \).

3. Black Holes

A family of dyonic black-hole spacetimes analogous to the Kerr-Newman sequence but embedded in anti-de Sitter (or de Sitter) spacetime rather than Minkowski spacetime is known [25]. The metric exterior to the horizon in the analogue of Boyer-Lindquist coordinates is
\[
d\tau^2 = \rho^2 \left( \frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_{\theta}} \right) + \sin^2 \theta \frac{\Delta_{\theta}}{\rho^2 \Xi^2} (adt - (r^2 + a^2)d\phi)^2 \\
- \frac{\Delta_r}{\rho^2 \Xi^2} (dt - a \sin^2 \theta d\phi)^2 ,
\]  
(32)
with

\[ \rho^2 = r^2 + a^2 \cos^2 \theta \]  
\[ \Delta_r = (r^2 + a^2)(1 + \alpha^2 r^2) - 2mr + q^2 + p^2 \]  
\[ \Delta_\theta = 1 - \alpha^2 a^2 \cos^2 \theta \] ,

and

\[ \Xi = 1 - \alpha^2 a^2 \] ,

The U(1) gauge potential is a one-form given by

\[ A = q \frac{r}{\rho^2 \Xi} (dt - a \sin^2 \theta d\phi) + p \frac{\cos \theta}{\rho^2 \Xi} \left( adt - (r^2 + a^2) d\phi \right) \] ,

where \( m \) is the mass, \( a \) is the rotation parameter, \( q \) is proportional to the electric charge, and \( p \) is proportional to the magnetic charge.

To relate the constants of integration \( m, a, q, p \) to the physical mass, angular momentum, electric charge, and magnetic charge, we evaluate Eqs. (16), (17), and (25). The results are

\[ M = \frac{m}{\alpha(1 - \alpha^2 a^2)^2} \] ,
\[ J = \frac{am}{(1 - \alpha^2 a^2)^2} \] ,
\[ Q = \frac{q}{\alpha(1 - \alpha^2 a^2)} \] ,
\[ P = \frac{p}{\alpha(1 - \alpha^2 a^2)} \] .

The reader is cautioned that the corresponding results for spacetimes with zero cosmological constant cannot be obtained by the direct substitution \( \alpha = 0 \). We have normalized the Killing vectors so that the associated conserved quantities generate the so(3,2) algebra. To obtain the Poincaré limit of this algebra, a Wigner-Inönü contraction is required. This involves a rescaling of the generators and therefore a corresponding rescaling of the conserved charges. The practical effect of this procedure is to remove the full denominators of the expressions for \( M, J, Q, \) and \( P, \) thereby regaining the usual expressions for zero cosmological constant.

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Substituting into the expression for the Bogomol’nyi bound, we find that it is saturated if
\[
m = \sqrt{q^2 + p^2} \left(1 + a\alpha\right).
\] (42)
In this form, the bound can be compared with the zero-temperature condition to be obtained next.

The black hole has a horizon determined by the condition
\[
\Delta_r = 0.
\] (43)
This horizon has zero Hawking temperature if the equation
\[
\left.\frac{d\Delta_r}{dr}\right|_{\Delta_r=0} = 0
\] (44)
is simultaneously satisfied. If \(m > 0\), \(\Delta_r\) has at most two real zeros because the sum of the roots must be zero since the coefficient of \(r^3\) vanishes in Eq. (34). If \(\Delta_r\) does have two real zeros or a coincident pair of real zeros, then the solution is a black hole. If \(\Delta_r\) has no real zeros, then the spacetime is a naked singularity. A key point is that imposing the Bogomol’nyi bound (42) differs here from the requirement that \(\Delta_r\) has two real zeros. This indicates that supersymmetry is different from cosmic censorship for these spacetimes.

An interesting issue is whether there exist dyonic black holes at zero temperature that satisfy the Bogomol’nyi bound. Combining the conditions (43) and (44) leads after some algebra to the equation
\[
a^{10} \alpha^8 + a^8 \left(-4 \alpha^6 + \alpha^8 \left(p^2 + q^2\right)\right) + a^6 \left(6 \alpha^4 - \alpha^6 m^2 - 12 \alpha^6 \left(p^2 + q^2\right)\right) + a^4 \left(-4 \alpha^2 + 33 \alpha^4 m^2 + 22 \alpha^4 \left(p^2 + q^2\right) - 8 \alpha^6 \left(p^2 + q^2\right)^2\right) + a^2 \left(1 + 33 \alpha^2 m^2 - 12 \alpha^2 \left(p^2 + q^2\right) + 36 \alpha^4 m^2 \left(p^2 + q^2\right) + 32 \alpha^4 \left(p^2 + q^2\right)^2\right) - m^2 - 27 \alpha^2 m^4 + p^2 + q^2 + 36 \alpha^2 m^2 \left(p^2 + q^2\right) - 8 \alpha^2 \left(p^2 + q^2\right)^2 + 16 \alpha^4 \left(p^2 + q^2\right)^3 = 0.
\] (45)
Substitution of the Bogomol’nyi bound (42) into this condition produces an equation having the form of a vanishing product of two factors, one of which is always positive.
Setting the remaining factor to zero generates the equation

\[ \alpha m^2 = a(1 + \alpha a)^4. \tag{46} \]

This and Eq. (42) are the conditions for existence of a zero-temperature supersymmetric black-hole spacetime. In terms of the physical mass and angular momentum, Eq. (46) reads

\[ \pm JM = \alpha^4(M \mp J)^4. \tag{47} \]

Note that this equation reduces correctly in the limit \( \alpha \to 0 \) to the usual condition \( J = 0 \) for the solitonic Kerr-Newman black hole. For nonzero cosmological constant, Eq. (46) implies that solitonic black holes must be rotating. This is a consequence of the anti-de Sitter background. We see that rotating black holes in gauged \( N = 2 \) supergravity are the analogue of the extreme Reissner-Nordstrom black hole in ordinary general relativity.

4. Discussion

In this paper, we have shown the existence in the context of gauged \( N = 2 \) supergravity of a new sequence of stable zero-temperature black-hole spacetimes with nonzero angular momentum and electric and magnetic charges. The associated cosmological constant, mass, and angular momentum satisfy the relations (42) and (46). These solitonic dyonic black holes belong to a larger family of zero-temperature black-hole spacetimes determined by Eq. (45). The Bogomol'nyi bound arising from the supersymmetry algebra is saturated under the distinct condition (42).

In the absence of a consistent quantum theory incorporating gravity, it is evidently difficult to make definitive statements about quantum aspects of black holes. However, general considerations can provide some insight. Consider first the usual Kerr-Newman sequence in the context of general relativity. The asymptotic spacetime symmetry group is the product of the Poincaré group and a U(1) factor. At the quantum level, the Kerr-Newman spacetimes must therefore lie in one or more discrete-series representations of this group. Such representations are characterized by three Casimir operators, which specify the mass, the angular momentum, and
the charge. Of these, only the angular momentum is quantized. Since the zero-
temperature condition for Kerr-Newman spacetimes can be written as

$$J^2 = M^2 (M^2 - Q^2),$$  \hspace{1cm} (48)

we can infer that in the quantum limit the mass and charge of zero-temperature Kerr-
Newman black holes are constrained to lie along a curve in the $M$-$Q$ plane. However,
these arguments are irrelevant for solitonic black holes, since these must satisfy the
Bogomol’nyi limit $M = |Q|$. Only $J = 0$ is allowed, even classically.

The above remarks must be modified in anti-de Sitter space because the asymp-
totic spacetime symmetry group is SO(3,2) instead of the Poincaré group. The repre-
sentations are again labeled by Casimir operators determining the mass and angular
momentum, along with the cosmological constant characterizing the scale. However,
introduction at the quantum level of the discrete-series representations of SO(3,2) now
means that both $M$ and $J$ are quantized. This can have consequences for the quan-
tization of other physical operators. For example, the zero-temperature condition for
the nonrotating black hole in anti-de Sitter space can be written as

$$M = \sqrt{\frac{2(Q^2 + P^2)}{3} - \frac{1}{54\alpha^4} \left( 1 \pm \frac{12\alpha^4(Q^2 + P^2)}{1 + [1 + 12\alpha^4(Q^2 + P^2)]^{\frac{3}{2}}} \right)^\frac{3}{2}}.$$  \hspace{1cm} (49)

For a spacetime with given cosmological constant, the quantization of $M$ means that
the charge $Q^2 + P^2$ is also fixed. The supersymmetry condition in this case is just

$$M = \sqrt{Q^2 + P^2},$$  \hspace{1cm} (50)

which also indicates a discrete value of the charge $Q^2 + P^2$. Note that the two
conditions (49) and (50) are compatible only for $Q^2 + P^2 = 0$.

For the new sequence of black holes presented here, the asymptotic symmetry
group is the superalgebra osp(4|2). The Lie subgroup of the associated supergroup
is SO(3,2)$\times$U(1). The mass and angular momentum therefore take discrete values
at the quantum level. For fixed cosmological constant, it follows both from the zero-
temperature condition (II) and from the supersymmetry condition (I) that the
charge is constrained in this sequence too. However, unlike Reissner-Nordstrom holes
in anti-de Sitter spacetime, Eqs. (I) and (II) are compatible for arbitrary charge.
Taken at face value, the condition (47) therefore becomes a Diophantine-type equation constraining \( M \) and \( J \). The solutions to such equations are typically sparse, which would seem to raise the interesting possibility that the new spacetimes could be incompatible with quantum mechanics except perhaps for special values of the cosmological constant. However, Eq. (47) was derived from classical considerations, which neglects quantum corrections that could potentially be important in this context. One such modification is the replacement \( J^2 \rightarrow J(J + 1) \).

Another interesting issue with bearing on quantum effects stems from the duality of \( Q \) and \( P \). At the quantum level, a charge \( Q \) moving in the field of an object with monopole charge \( P \) has an anomalous contribution \( PQ \) to its angular momentum \([26]\). Since there is no other source of angular momentum, it would appear that consistency requires the identification \( J \equiv PQ \) up to a possible term arising from the anti-de Sitter background. If this additional quantum constraint is correct, then in gauged \( N = 2 \) supergravity the solitons are dyons with angular momentum determined in terms of the charges. It is plausible that this anomalous angular momentum could be interpreted as the gravitational analogue of the \( \theta \) angle in electrodynamics \([27]\), where dyons with electric charge \( Q \) have magnetic charge \( P \) given by \( P = Q\theta/2\pi \).

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