Interacting quintessence and growth of structure

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Abstract. In standard cosmologies, dark energy interacts only gravitationally with dark matter. An extension to this picture is interacting quintessence (IQ) model where scalar field coupled directly to cold dark matter. The percentage deviation is studied in IQ model with respect to $\Lambda$CDM for varied values of the interacting parameter $W$. We investigated the effect of interaction on matter, kaiser and galaxy power spectrums. The deviation in power spectrum increases with interaction on both large and small scales. On small scales, variation is comparatively smaller than on large scale. On large scales, it is due to dark energy perturbation, while it is background evolution that causes a difference on small scale. These variations decreases with increase in redshift. Herein the thawing class of model with linear potential is studied.

Keyword. Cosmology theory—large-scale structure of the Universe—dark energy.

1. Introduction

The late time acceleration in the standard Einstein gravity is propelled by a mysterious energy component which consist of a huge negative pressure that expands the Universe. This is called dark energy (DE) (Riess et al. 1998; Perlmutter et al. 1999; Tonry et al. 2003). Keeping in mind the standard cosmological model, the DE undertakes the simplest form of the cosmological constant $\Lambda$, which has absolutely no spatial fluctuations but a negative pressure and constant energy density which covers the entire expansion history of the Universe. That leads to $\Lambda$CDM model. Cosmic microwave background (CMB) (Ade et al. 2016), supernova type-Ia (SNIa) (Betoule et al. 2014), and baryon acoustic oscillation (BAO) measurements in galaxy surveys (Lauren et al. 2014) can be demonstrated via the $\Lambda$CDM model. However, it gets into serious conceptual problems like fine-tuning problem (Weinberg 1989) and coincidence problem (Huey & Wandelt 2011). Recent observational results also indicate inconsistency with the $\Lambda$CDM model (Heymans et al. 2013).

In order to solve these problems, a number of scalar field models, quintessence (Ratra and Peebles 1988; Peebles & Ratra 1988; Copeland et al. 2006; Padmanabhan 2003; Sahni & Starobinsky 2006; Sami 2009), phantom field (Caldwell 2002; Carroll et al. 2003; Singh et al. 2003), rolling tachyon (Sen 2002; Mazumdar et al. 2001) and other models have been proposed. Another way to look at this problem is through interaction between dark energy and other matter species in the Universe as suggested by Wetterich (1995), Amendola (2000), Farrar and Peebles (2004) and Gubser and Peebles (2004). In relation to this, minimally coupled dynamic scalar field can be extended to interacting quintessence (IQ) models. In these categories of models, DE is coupled to matter (dark matter and baryons), but coupling of DE with baryonic particles will result in time variation of constants of nature and hence is tightly bound by observations (Hagiwara et al. 2002). However, these constraints does not apply to interaction between dark sector. This interaction between DM and DE is still permitted by observations (Damour et al. 1999; Casas et al. 1992). But, just like quintessence, interacting quintessence too has fine-tuning issues. In this work, we will focus on coupling between dark sector only.

Various types of interaction between DE and CDM have been proposed and investigated in the literature, such as linear coupling of scalar field with matter (DM and baryon), suggested by Amendola (2000), coupling with dark matter only (Damour et al. 1990), nonlinear couplings (Billyard & Coley 2000; Bartolo & Pietroni 2000), variational approach (Christian et al. 2015),
coupling to the dark matter via Yukawa-like interaction (Das et al. 2006). In relation to the aforementioned models, the coupling describes the exchange of energy–momentum between dark energy and dark matter.

Interaction between dark sectors could change the expansion history of the Universe. Interaction can change the source term of Poisson equation through scalar field perturbation. Additionally it can produce a fifth force between matter particles which results into stronger clustering of matter (Li & Barrow 2011). All these factors could affect structure formation on both small and large scales. In recent years, the impact of interaction on linear growth of structure (Caldera-Cabral et al. 2009b; Amendola et al. 2008; Koyama et al. 2009; Caldera-Cabral et al. 2009b; Valiviita et al. 2008) and nonlinear structure formation (Macciò et al. 2004) have been studied in detail. Observational data like CMB, BAO, LSS, weak lensing (Valiviita et al. 2010; Timothy et al. 2012) local gravity tests (Kesden & Kamionkowski 2006) and high redshifted intergalactic medium (Baldi & Viel 2010) have constrained the interaction, but none of them have ruled out the interaction.

In order to advance the work by Dinda and Sen (2018) we included coupling between quintessence and CDM and investigate its effect on linear structure formation. We considered a thawing class of model. We include red shift space distortion term and GR effects in power spectrum. We varied the interaction strength to include red shift space distortion term and GR effects in power spectrum. We considered a model which allows coupling between dark matter but not to baryons. Here, we follow prescription discussed many times in literature (Amendola 2004; Amendola et al. 2008; Piloyan et al. 2013; Sumit et al. 2013). We also follow the same formalism here. The important equations are as follows:

\[ \ddot{\phi} + \frac{dV}{d\phi} + 3H \dot{\phi} = C(\phi) \rho_d \]

\[ \dot{\rho}_d + 3H(\rho_d) = -C(\phi) \rho_d \dot{\phi} \]

\[ \dot{\rho}_b + 3H(\rho_b) = 0 \]

\[ H^2 = \frac{\kappa^2}{3} (\rho_b + \rho_d + \rho_\phi) \]

\[ 1 = \frac{\kappa^2}{3 \Omega_\phi^2} \left( \frac{\kappa^2}{3} \rho_d + \frac{\kappa^2}{6} \dot{\phi}^2 + \frac{\kappa^2}{3} V(\phi) \right) \]

Herein, \( C(\phi) \) comprises an interaction between dark sectors. Due to lack of detail of the nature of interaction we consider it to be constant (Amendola 2004). We can even study uncoupled case by putting \( C = 0 \).

Now, we introduce these dimensionless parameters:

\[ x = \frac{\kappa \dot{\phi}}{\sqrt{6} H}, \quad y = \frac{\kappa \sqrt{V(\phi)}}{\sqrt{3} H} \]

\[ s = \frac{\kappa \sqrt{\rho_b}}{\sqrt{3} H}, \quad \lambda = \frac{-1}{\sqrt{\kappa V}} \frac{dV}{d\phi}, \quad \Gamma = \frac{\sqrt{\frac{d^2V}{d\phi^2}}}{\Gamma} \]

Here, the variable \( \Gamma \) shows potential. For linear potential \( \Gamma = 0 \).

In terms of \( x \) and \( y \), \( \Omega_\phi \) and equation of state \( w_\phi \) are:

\[ \Omega_\phi = x^2 + y^2 \]

\[ \gamma = 1 + w_\phi = \frac{2x^2}{x^2 + y^2} \]

Using aforementioned dimensionless variables, Equations (1) and (2) can be converted in following autonomous systems:

\[ \Omega'_\phi = W \sqrt{3}\gamma \Omega_\phi (1 - \Omega_\phi - s^2) + 3\Omega_\phi (1 - \Omega_\phi)(1 - \gamma) \]

\[ y' = W \frac{3\gamma}{\Omega_\phi} (1 - \Omega_\phi - s^2)(2 - \gamma) + \lambda \sqrt{3}\gamma \Omega_\phi (2 - \gamma) \]

\[ s' = -3s^2 \Omega_\phi (1 - \gamma) \]

\[ \lambda' = \sqrt{3}\gamma \Omega_\phi \lambda^2 (1 - \Gamma). \]

Here, \( W = \frac{C}{\epsilon} \).

Different forms for the potential \( V(\phi) \) on phenomenological basis have been considered in various studies. In Panda et al. (1998), a form of the potential has been constructed in string theory using the axion field originated from the Ramond–Ramond sector of the Type-II string theory. For this field the form of this potential is
linear. In our subsequent calculations, we assume this form of the potential as it has some definite origin. But one can always extend the subsequent calculations for other form of the potentials.

Throughout this work we have considered a positive value of the interacting parameter $W$. Positive interaction terms mean transfer of energy is from dark matter to dark energy; if we take it to be negative then it would mean transfer from dark energy to dark matter. But, in the thawing model, initially dark energy is negligible, so it would not be viable to assume energy transfer from dark energy.

2.1 Initial conditions to solve background equations

To solve this autonomous system (7) we would need initial conditions for $(\gamma_i, \Omega_{\phi i}, s_i, \lambda_i)$. We settled our initial conditions at $(z = 1000)$. At $(z = 1000)$ for thawing class of models, scalar field is frozen; thus, $\gamma_i \approx 0$. We have taken initial value of $\lambda_i$ as a model parameter. We settle the initial condition for $\Omega_{\phi i}$ by fine-tuning it so as to obtain appropriate value of $\Omega_{\phi i}$ today at $z = 0$. In the same way, we set initial value of $s_i$ to get appropriate value of the $\Omega_{\phi i}$ today. Subsequently, we set initial condition for IQ model to get same $\Omega_{m0}$ and $H_0$ as in non-interacting case.

2.2 Behaviour of background cosmological parameter

The set of equations (7) will be solved using the earlier-mentioned initial conditions, and the cosmological parameters for different values of interacting parameter $W$ will be studied.

$\lambda_i$ is a model parameter, and given a form of the potential, it controls the deviation of the background evolution from the $\Lambda$CDM universe. For smaller values of $\lambda_i$, the scalar field behaves very close to the cosmological constant (CC). For larger values of $\lambda_i$, the deviation from CC is larger. We wanted to study the large-scale perturbation in our model for background evolution different from $\Lambda$CDM. For this purpose, we chose $\lambda_i = 0.7$ for our subsequent calculations. For $\lambda_i < 0.7$, the deviation from $\Lambda$CDM will be smaller, whereas for $\lambda_i > 0.7$ the deviation will be larger than what we report in this study. For different values of $\lambda_i$, variation would be different. The value of $\lambda_i$ was chosen as 0.7 just to compare the result with that of Dinda and Sen (2018) in non-interacting case. Without loss of generality, this work can be extended to other values of $\lambda_i$ as well.

Figure 1 displays the variation of equation of state as function of redshift for different values of $W$. Since we are considering thawing model, $w_\phi$ starts from $-1$ at $z = 1000$ for the non-interacting case. On adding interaction, $w_\phi$ increases, which shows strong dark energy effect. There is a jump initially in the $w_\phi$ from this frozen state, as can be seen from Figure 1. This is due to the energy transfer due to the coupling between the axion field and dark matter component. This behaviour is discussed in detail by Kumar et al. (2013).

Figure 2 shows the percentage change in matter density parameter with respect to LCDM. Here percentage change is negative, which implies suppression with respect to LCDM. The background dark matter density ($\Omega_m$) decreases with respect to LCDM. The background dark matter density ($\Omega_m$) decreases with respect to LCDM.

Figure 3 shows the percentage change in the normalized Hubble parameter with respect to LCDM. Here, percentage change is positive, which implies enhancement with respect to LCDM. On increasing interacting parameter $W$, the percentage change increases, which entails stronger dark energy effect. At $z = 0$ all models converge to LCDM due to our normalization; hence, percentage change is 0.
Figure 2. (Left) Percentage diversion of $\Omega_m$ in IQ as compared to $\Lambda CDM$ model for different values of interacting parameter $W$; negative values in y-axis means they are less than that of $\Lambda CDM$. $\Omega_m = 0.28$ and $\lambda = 0.7$ in these plots. Subsequently, $\% \Delta X = (X^{de}/X^{\Lambda} - 1) \times 100$. (Right) Percentage deviation in normalized Hubble $H$ from $\Lambda CDM$ model for different values of interacting parameter $W$.

Figure 3. Conduct of evolution of normalized hubble parameter for IQ model and $\Lambda CDM$ and normalized hubble $H(z)$ data with 1σ error bars: red, $\Lambda CDM$; black, $W = 0$, green, $W = 0.02$; blue, $W = 0.04$.

3. Observational constraints

1σ error bars for normalized Hubble parameter ($H$) have been calculated from the compilation of 29 points of $H(z)$ data using present value of the Hubble parameter $H_0 = (67.8 \pm 0.9)\, km^{-1}Mpc^{-1}$ from plank 2015 result. Errors in $H/H_0$ can be calculated as

$$\sigma = \frac{(\sigma H/H_0)H}{H_0}$$

Figure 3 shows the evolution of normalized Hubble parameter for the IQ model and $\Lambda CDM$ with 1σ error bars, which shows that $H$ for IQ model is well within error bars.

Also, Kumar et al. (2013) have shown using the data from supernova type-Ia, baryon acoustic oscillations, measurements of the Hubble parameter as well as the measurement of the growth, that on introducing the coupling between the axion field and the dark matter, larger deviation from the $W = -1$ behaviour is allowed with increasing strength of the coupling parameter up to $W = 0.06$.

4. Growth of linear perturbation with interacting quintessence

Herein effect of the interaction on matter and scalar field perturbation in linear regime will be reflected upon. Matter includes both dark matter and baryons. Since dark matter perturbation is dominant and baryon follows dark matter perturbation, baryonic perturbations can be excluded from our study, although inclusion of it will not effect our results.

The perturbed (FRW) metric in conformal and Newtonian gauge is given by

$$ds^2 = a^2 [-(1 + 2\Phi) d\tau^2 + (1 - 2\Psi) dx^i dx^j]$$

where, $\Phi$ and $\Psi$ are gravitational potential and $\tau$ is conformal time. Without anisotropic stress the two can be related as $\Phi = \Psi$.

Consider two components, a scalar field and cold dark matter, described by the energy–momentum tensors $T_{\mu\nu}(\phi)$ and $T_{\mu\nu}(d)$. Conservation equation with interacting terms for scalar field and cold dark matter as discussed by Amendola (2000) are:

$$\nabla_\mu T^{\mu}_{(\phi)\nu} = CT_d \nabla_\nu \phi$$

$$\nabla_\mu T^{\mu}_{(c)\nu} = -CT_d \nabla_\nu \phi$$
Here, $C$ defines interaction between scalar field and dark matter. We assume phenomenologically $C$ to be constant.

For perturbation equations we follow the set up as provided by (Dinda and Sen 2018). We generalize their result for the interacting scenario. We mention only relevant equations here; for detailed calculation, see Dinda and Sen (2018).

The relativistic poisson equation is

$$\nabla^2 \Phi - 3 \mathcal{H} (\Phi' + \mathcal{H} \Phi) = 4 \pi G a^2 \Sigma \delta \rho_i$$

(12)

The scalar field influences a change upon source term of the Poisson equation (Li & Barrow 2011). Here, prime represents the derivative with respect to conformal time.

The perturbation equations for scalar field coupled to dark matter are

$$\delta \Phi' + 3 \mathcal{H} \delta \phi - 4 \Phi \delta \phi + 2 \Phi V_{\phi}$$

$$+ V_{\phi \Phi} \delta \phi - \frac{1}{a^2} \nabla^2 \delta \phi = C (\delta \rho_\phi + 2 \Phi \rho_\phi),$$

(13)

where $V_{\phi} = \frac{dV}{d\phi}$, $V_{\phi \Phi} = \frac{d^2V}{d\phi^2}$ and $\delta \phi$ are scalar field fluctuations.

Then we construct following dimensionless variables:

$$g = -\frac{V_{\phi}}{H \Phi}$$

(14)

$$q = \frac{\delta \phi H}{\Phi}$$

(15)

$$B = 3 + \frac{H}{\dot{H}}$$

(16)

$$B_{\Phi} = 6g - \frac{1}{H} \frac{dH}{dt} - 2Bg + \frac{k^2}{a^2 H^2}$$

(17)

$$W = C.$$  

(18)

In terms of these dimensionless parameters, Equations (11) and (12) can be written as

$$\frac{d^2 \Phi}{dN^2} + (1 + B) \frac{d\Phi}{dN}$$

$$+ (2B - 3 + 3x^2) \Phi$$

$$= 3x^2 \left[ \frac{dq}{dN} + \frac{3W \Omega_d q}{\sqrt{6x}} + (2g - B)q \right]$$

(19)

$$\frac{d^2 q}{dN^2} + \left( \frac{\sqrt{6} \Omega_d}{x} + 2g - B \right) \frac{dq}{dN}$$

$$+ \left( \frac{9W \Omega_d}{\sqrt{6x}} - 3 \Omega_d W^2 + B_q \right) q$$

$$= \frac{4 \Phi}{dN} + 2g + \frac{3W \Omega_d}{\sqrt{6x}} (\delta_d + 2 \Phi)$$

(20)

Matter density contrast is given by

$$\delta_d = -\frac{2}{\Omega_d} \left[ \frac{d\Phi}{dN} + \left( 1 - x^2 + \frac{k^2}{3 \mathcal{H}^2} \right) \Phi \right.$$}

$$\left. + x^2 \left( \frac{dq}{dN} - B_q + \frac{3W \Omega_d q}{\sqrt{6x}} \right) \right]$$

(21)

where $N = \ln a$.

Here, comoving density contrast

$$\Delta_d = \delta_d + y_d$$

(22)

where

$$y_d = 3 \mathcal{H} v_d = \frac{2}{\Omega_d} \left( \frac{d\Phi}{dN} + \Phi - 3x^2 q \right)$$

(23)

On sub-horizon scale $\Delta_d \approx \delta_d$, which is usually used in study of small-scale structure (Newtonian limit). On large scales, $\Delta_d$ should be used instead of $\delta_d$ (Duniya et al. 2013).

For growth function we used the equation given in Dinda and Sen 2018. In the following equations $\Phi_d$ represents gravitational potential at decoupling. Gravitational potential growth function $D_\Phi$ is defined by

$$D_\Phi(k, z) = \frac{1}{(1 + z)} \Phi_d(k)$$

(24)

$D_d$ is the growth function of the comoving matter overdensity.

$$D_d(k, z) = -\frac{3 \Delta_d(k, a) \Omega_d \Omega_{do} H_0^2}{2k^2 \Phi_d(k)}$$

(25)

Matter velocity growth function:

$$Dv_d(k, z) = -\frac{3 v_d(k, z) \Omega_d H_0^2}{2 \Phi_d(k)}$$

(26)

Dark energy velocity growth function $D_\Phi(k, z)$ is associated to that of dark matter by

$$\frac{Dv_\Phi(k, z)}{Dv_d(k, z)} = -\frac{\Omega_d}{(1 - \Omega_d)(1 + w)} \left( \frac{H D_\Phi'}{Dv_d} + 1 \right)$$

(27)

The ratio of comoving matter density $\Delta_d$ and gravitational potential $\Phi(0, k)$ can be defined as

$$D_d \Phi(z, k) = \frac{\Delta_d(z, k)}{\Phi(0, k)}$$

(28)

Here, prime is derivative with respect to redshift $z$.

Quantity $f$ which is related to velocity perturbation and hence redshift-space distortion term can be defined as

$$f = \frac{Dv_d}{\mathcal{H} D_d}$$

(29)

which reduces to the growth rate of dark matter in standard uncoupled DE models.
Figure 4. Percentage diversion of $\Phi$ in IQ model as compared to $\Lambda$CDM model for different values of interacting parameter $W$.

Standard matter power spectrum

$$P(z, k) = A k^{n_s - 4} T^2(k)(1 + z)^2 \frac{\Delta_d(z, k)}{\Phi(0, k)}^2 \tag{30}$$

Here $A$ represents the normalization constant the value of this is determined by $\sigma_8$ normalization, whereas $n_s$ is defined as spectral index, $T(k)$ as given by Einstein and Hu is the transfer function (Eisenstein & Hu 1998).

4.1 Initial conditions

To solve perturbation equations (18) and (19), we need initial conditions for $(\Phi, \frac{d\Phi}{dN}, q, \frac{dq}{dN})$. At the time when matter dominated the Universe there was negligible dark energy contribution. We have set our initial condition at decoupling ($z = 1000$). Due to negligible dark energy, the initial conditions for interacting and non-interacting model are the same. Thus, we have used initial conditions discussed by Dinda and Sen (2018) for the non-interacting model.

There is no contribution from DE at $z = 1000$; hence scalar field perturbation is insignificant and $q = \frac{dq}{dN} = 0$.

Since $\Phi$ is constant during matter domination, hence $\frac{d\Phi}{dN} = 0$. Using Poisson equation (9) and the fact that $\Delta_m \sim a$, its initial condition is

$$\Phi_i = -\frac{3 \mathcal{H}_i^2}{2 k^2} d_{in} \tag{31}$$

4.2 Behaviour of cosmological parameters

Using aforementioned initial conditions we solved perturbed equation (18) and (19) and studied various perturbation parameters, for different sets of interacting parameter $W$.

Figure 4 shows deviation in gravitational potential from LCDM model for different values of interacting parameter $W$. For low redshift on sub-Hubble scale there is a suppression from LCDM, which is due to different background evolutions as there is no contribution from DE perturbation at smaller scales. However, on larger scale, the enhancement in $\Phi$ is due to contribution from dark energy perturbation. On increasing the value of interacting parameter $W$, there is overall enhancement on sub-Hubble and super-Hubble scales, respectively. With increase in redshift this variation becomes scale independent. This is due to transfer of energy and momentum from DM to DE in both background as well as perturbed Universe.

Figure 5 shows variation in comoving matter density contrast $\Delta_m$. For non-interacting case variation is very small and is almost scale independent. At redshift $z = 0, 0.5, 1$ and on large scales, an increase in interaction $W$ results in an enhancement in $\Delta_m$ with respect to LCDM (0–8.5%), which decrease with redshift. But on smaller scales, $\Delta_m$ is slightly suppressed, less than 1%.
Figure 5. Percentage diversion of comoving density contrast $\Delta_m$ in IQ model as compared to LCDM model.

Figure 6. Ratio of $D_{d\Phi}(k, z)$ IQ model and LCDM defined in Equation (27).

At redshift $z = 3.5$ on large scale there is slight suppression with respect to LCDM less than 1% and on small scale slight enhancement less than 1%.

Figure 6 displays variation of $f$ defined in Equation (30) which is related to velocity perturbation and hence redshift space distortion. For non-interacting case, just like $\Delta_m$, variation is small and scale independent. On adding interaction its behaviour is exactly opposite to that of matter density contrast $\Delta_m$. On large scale, with increasing interaction, a suppression can be seen at redshift 0,0.5 and 1 (4–8%). But at higher redshift, $z = 3.5$, there is slight enhancement less than 1%.

Hence interaction affects growth of structure at all scales. But this effect is smaller at higher redshift. Figure 7 shows the redshift space distortion term.

5. Influence of interaction on power spectrums

The growth of large-scale structure in the Universe is ascertained by matter power spectrum. Forthcoming surveys of galaxies can probe distribution of dark matter on large scales. These surveys can provide strong bound on dark energy models including interaction in dark sector. We must incorporate observed galaxy distribution
effects like redshift space distortion, GR effects like weak lensing convergence, SW, ISW and time delay effect in our analysis to recognize the potential of these surveys (Yoo 2010; Challinor & Lewis 2011; Jeong et al. 2012). Certain astrophysical processes such as gas cooling, star formation and feedback from supernovae, in conjunction with the gravitational effect of the dark matter, has a bearing upon formation of galaxies. This can further cause a contrast between the spatial distribution of baryons and dark matter. The association between the spatial distribution of galaxies and ubiquitous dark matter must be understood to employ galaxies as cosmological probes known as galaxy bias. Matter power spectrum can be related to galaxies distribution through bias $b$ defined as (Challinor & Lewis 2011)

$$\Delta_g(k, z) = b(z) \Delta_d(k, z).$$

(32)

To study effect of interaction on matter and galaxy power spectrum we use prescription discussed by Duniya et al. in Didam et al. (2015).

In a galaxy redshift survey, the observers measure the number of galaxies in direction $n$ at redshift $z$. The number overdensity of galaxy ($\Delta^{obs}$) is shown as follows

$$\Delta^{obs} = \left[ b + f\mu^2 + A\left(\frac{H'}{k}\right)^2 + i\mu B\left(\frac{H'}{k}\right) \right] \Delta_m,$$

(33)

Here $\mu = -\frac{n \cdot \hat{k}}{k}$, $\hat{n}$ expressing the direction of observation, $f$ signifies the redshift space distortion and $b$ stands for galaxy bias. Variables $A$ and $B$ which are considered in connection to GR corrections are delineated as follows:

$$A = (3-b_e) f - \frac{3\Omega_{0d}H_0^2}{2H^2D_d} (4Q - b_e - 1)$$

$$-(1+z)\frac{\mathcal{H}'}{\mathcal{H}} + 2 \frac{(1-Q)}{r\mathcal{H}}$$

$$+ \frac{1}{D\Phi(1+z)^2} (-1+z)^2$$

$$\left(D\Phi + (1+z)D'\Phi\right)D\Phi(1+z)$$

(34)

$$B = [b_e - 2Q + (1+z)\frac{\mathcal{H}'}{\mathcal{H}} - \frac{2(1-Q)}{r\mathcal{H}}$$

$$-w\sqrt{6x} \left(1 - \frac{D\Phi}{D\nu_d}\right) f$$

(35)

Full general relativistic power spectrum includes all GR effects (Didam et al. 2015).

$$P_g^{obs}(k, z) = \left( (b + f\mu^2)^2 + 2(b + f\mu^2)\frac{AH'^2}{k^2}$$

$$+ A^2\frac{H'^4}{k^4} + \mu^2 B^2\frac{H'^4}{k^4} \right) P(k, z)$$

(36)

Matter power spectrum with kaiser term is given by

$$P_k(z, k) = \left[ b(z) + f(z, k)\mu^2 \right] P(z, k)$$

(37)

Here, prime is the derivative with respect to redshift. A constant comoving galaxy number density is presumed, hence $b_e = 0$, galaxy bias $b = 1$ and the magnification bias $Q = 1$. Here $x$ is defined in Equation (4). $A$ is connected with with peculiar velocity potential and the gravitation potential, on the other hand $B$ is connected to the Doppler effect. The latter comprises an
Figure 8. (Left) Standard matter power spectrum at $z = 0$ for different values of interacting parameter $W$. (Right) Standard matter power spectrum on large scale $k = 0.0001$ to $k = 0.0005$.

Figure 9. Percentage of diversion in $P(k)$ in IQ model as compared to $\Lambda$CDM model for different values of interacting parameter $W$ as a function of $K$ for different redshifts. First column is standard matter power spectra $P$ which is specified by Equation (29), second column present Kaiser power spectra $P_k$ specified by Equation (36), and third column portrays galaxy power spectrum specified by Equation (35).

interaction explicitly. However, no momentum is transported in the dark energy rest frame which leads to last term in Equation (34) being zero (Didam et al. 2015).

Figure 8 shows standard matter power spectrum at $z = 0$ for different values of interacting parameter $W$. Even on large scales, percentage deviation is small, which be seen in plot on the right. To show even small deviation with respect to $\Lambda$CDM we have plotted their percentage change with respect to $\Lambda$CDM in Figure 9.

Figure 9 (left column) shows percentage change in standard matter power spectrum with respect to $\Lambda$CDM. It depends on the ratio $\frac{\Delta \phi}{\phi}$ which is shown in Figure 6. At $z = 0$ and on large scale, percentage change for non-interacting case ($W = 0$) is negative, which shows suppression in power. But on adding interaction (2–8%), enhancement is observed. This shows that transfer of energy from scalar field to dark matter: rate of transfer increase with increase in interaction parameter $W$. On smaller scales, all models converge to $\Lambda$CDM due to
our normalization. On larger scales and higher redshift, dark energy gives negative contribution as compared to \( z = 0 \). Thus, suppression is seen with respect to \( \Lambda \)CDM. This suppression contribution increases with redshift. But on smaller scales, slight enhancement of power can be seen which is due to difference in background evolution.

Figure 9 (middle column) displays percentage change in matter power spectrum with kaiser term with respect to \( \Lambda \)CDM. At \( z = 0 \) on large scales, enhancement in kaiser spectrum is 0–2%, which is less than standard matter power spectrum discussed earlier, which is due to contribution from kaiser term (Equation 36) that gives negative contribution to power spectrum. It depends on parameter \( f \) in Equation (32). Behaviour of \( f \) is shown shown in Figure 7 which is suppressed with respect to \( \Lambda \)CDM. On higher redshift, suppression is more as compared to standard power spectrum. But on smaller scale all models converge to \( \Lambda \)CDM for all redshifts.

Figure 9 (right column) displays percentage change in galaxy power spectrum with respect to \( \Lambda \)CDM. At \( z = 0 \) for large scales, percentage change is negative (12–23%), which shows suppression with respect to \( \Lambda \)CDM. On increasing interaction this suppression increases and it decreases with redshift. The reason for suppression is general relativistic terms \( A \) and \( B \) in Equation (33), which give negative contribution. On small scales and low redshift, percentage change is small, but on high redshift \( z = 3.5 \), percentage change is positive (2–6%). The reason for this is weaker dark energy effect at higher \( z \) but GR effects are comparative stronger even on small scales.

6. Conclusion

We have generalised non-interacting quintessence model as discussed by Dinda and Sen (2018) for the interacting scenario.

We studied effect of interaction between quintessence and dark matter on both background and perturbed Universe. At the background level, interaction affects background energy density and Hubble parameter. Suppression in dark matter density \( \Omega_\text{d} \) with respect to \( \Lambda \)CDM is 0–8%, while enhancement in Hubble is 0–5%. From this it can be concluded that at background level interaction makes the dark energy effect stronger. Using 1\( \sigma \) error bars for Hubble data we show that value of \( H \) is well within 1\( \sigma \) error bars.

A careful analysis of growth of structure characterises that interaction affects matter, kaiser and galaxy power spectrum on sub- and super-horizon scales. The former is because of dark energy perturbation and GR effect and the latter is the result of background evolution.

We find that standard matter power spectrum at \( z = 0 \) is enhanced with respect to \( \Lambda \)CDM on large scales, and on small scales, there is no deviation due to our normalization. This enhancement on large scales increases with increase in interaction. At higher redshift, matter power spectrum suppressed with respect to \( \Lambda \)CDM. This suppression also increases with interaction. On adding kaiser redshift distortion term, enhancement is less as compared to standard matter power spectrum. Thus, it can be concluded that that kaiser term gives negative contribution to power spectrum, which suppress power on large scaled. On higher redshift, it is further suppressed.

We also found that galaxy power spectrum is suppressed (12–24%) with respect to \( \Lambda \)CDM on large scales. This suppression increases with interaction. But on higher redshift, effect of dark energy is weaker, and hence suppression decreases with increase in \( z \). On small scales enhancement is observed (2–6%) which is due to difference in background evolution and GR effects. Thus, on higher redshifts, interaction affects galaxy power spectrum even on smaller scales. This deviation can be probed by future surveys like SKA.

References

Ade P. A. R. et al. [Planck Collaboration] 2016, A&A, 594, A13
Amendola L. 2000, Phys. Rev. D, 62, 043511
Amendola L. 2004, Phys. Rev. D, 69, 103524
Amendola L., Baldi M., Wetterich C. 2008, Phys. Rev. D, 78, 023015
Baldi M, Viel M. 2010, Mon. Not. Roy. Astron. Soc., 409, L89-L93
Bartolo N., Pietroni M. 2000, Phys. Rev. D, 61, 023518
Betoule M. et al. [SDSS Collaboration] 2014, Astron. Astrophys., 568, A22
Billyard A. P., Coley A. A. 2000, Phys. Rev. D, 61, 083503
Caldera-Cabral G., Maartens R., Urena-Lopez L. A. 2009a, Phys. Rev. D, 79, 063518
Caldera-Cabral G., Maartens R., Schaefer B. M. 2009b, JCAP, 0907, 027
Caldwell R. R. 2002, Phys. Lett. B, 545, 23
Carroll S. M., Hoffman M., Trodden M. 2003, Phys. Rev. D, 68, 023509
Casas J. A, Garcia-Bellido J., Quiros M. 1992, Class. Quant. Grav., 9, 1371
Copeland E. J., Sami M., Tsujikawa S. 2006, Int. J. Mod. Phys. D, 15, 1753
Christian B. G., Tamanini N., Wright M. 2015, Phys. Rev. D, 91, 123002
Challinor A., Lewis A. 2011, Phys. Rev. D, 84, 043516
Damour T., Gibbons G. W., Gundlach C. 1990, Phys. Rev. Lett., 64, 123
Das S., Stefano P., Khoury, J. 2006, Phys. Rev. D, 73, 083509
de Valentino E. et al., arXiv:1704.00762
Didam D. G. A., Bertacca D., Maartens R. 2015, Phys. Rev. D, 91, 063530
Dinda B. R., Sen A. A. 2018, Phys. Rev. D, 97, 083506
Duniya D., Bertacca D., Maartens R. 2013, JCAP 15, 1310, arXiv:1305.4509
Duniya D., arXiv:1606.00712 [hep-th]
Eisenstein D. J., Hu W. 1998, ApJ, 496, 605
Farrar G. R., Peebles P. J. E. 2004, ApJ, 604, 1
Gubser S. S., Peebles P. J. E. 2004, Phys. Rev. D, 70, 12, 123511
Hagiwara K. et al. 2002, Phys. Rev. D, 66, 010001
Heymans C. et al. 2013, Mon. Not. Roy. Astron. Soc., 432, 2433
Huey G., Wandelt B. D. 2011, Phys. Rev. D, 74, 083506
Jeong D., Schmidt F., Hirata C. M. 2012, Phys. Rev. D, 85, 023504, arXiv:1107.5427
Koyama K., Maartens R., Song Y.-S., 2009, JCAP, 0910, 017
Kesden M., Kamionkowski M., 2006, Phys. Rev. D, 74, 083007
Kumar S. et al. 2013, Class. Quant. Grav., 30, 155011
Lauren A. et al. 2014, Mon. Not. Roy. Astron. Soc., 441, 24
Li B., Barrow J. D. 2011, Phys. Rev. D, 83, 024007
Mazumdar A., Panda S., Perez-Lorenzana A. 2001, Nucl. Phys. B, 614, 101
Macchiò A. V., Quercellini C., Mainini R., Amendola L., Bonometto S. A. 2004, Phys. Rev. D, 69, 123516
Padmanabhan T. 2003, Phys. Rep., 380, 235
Panda S., Sumitomo Y., Trivedi S. P. 2011, Phys. Rev. D, 83, 083506
Peebles P. J. E., Ratra B. 1988, Astrophys. J., 325, L17
Perlmutter S. et al. 1999, Astrophys. J., 517, 565
Pilyan A., Marra V., Baldi M., Amendola L., 2013, arXiv:1305.3106 [astro-ph]
Riess A. G. et al. 1998, Astrophys. J., 116, 1009
Riess A. et al. 2016, Astrophys. J., 826, 56
Ratra B., Peebles P. J. E. 1988, Phys. Rev. D, 37, 3406
Sahni V., Starobinsky A. 2006, Int. J. Mod. Phys. D, 15, 2105
Sahni V., arXiv:astro-ph/0202076, arXiv:astro-ph/0502032
Sami M., arXiv:0901.0756 [hep-th]
Sami M. 2009, Curr. Sci., 97, 887
Sami M., Myrzakulov R., arXiv:1309.4188
Sen A. 2002, JHEP, 0204, 048
Singh P., Sami M., Dadhich N. 2003, Phys. Rev. D, 68, 023522
Timothy C., Sami M., Dadhich N. 2003, Phys. Rev. D, 68, 023522
Toner J. L. et al. [Supernova Search Team Collaboration] 2003, Astrophys. J., 594, 1
Valiviita J., Majerotto E., Maartens R. 2008, JCAP, 0807, 020
Valiviita J., Maartens R., Majerotto E. 2010, Mon. Not. Roy. Astron. Soc., 402, 2555
Weinberg S. 1989, Rev. Mod. Phys., 61, 1
Wetterich C. 1995, Astron. Astrophys., 301, 321
Yoo J. 2010, Phys. Rev. D, 82, 083508, arXiv:1009.3021