Experimental realization of optimal asymmetric cloning and telecloning via partial teleportation

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We report an experimental realization of both optimal asymmetric cloning and telecloning of single photons by making use of partial teleportation of an unknown state. In the experiment, we demonstrate that, conditioned on the success of partial teleportation of single photons, not only the optimal asymmetric cloning can be accomplished, but also one of two outputs can be transferred to a distant location, realizing the telecloning. The experimental results represent a novel way to achieve the quantum cloning and may have potential applications in the context of quantum communication.

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Quantum cloning is a process to distribute the quantum information in a state onto multiple output states. However, perfect quantum cloning of an unknown state is forbidden due to the restriction of quantum no-cloning theorem [1]. To understand the underlying limits, Bužek and Hillery first proposed a symmetric universal quantum-copying machine (UQCM) [2] that produces two identical outputs whose quality is independent of the input states. The UQCM was later proved to be optimal [3, 4], and constitutes the optimal attack in the six-state protocol of quantum cryptography [3, 4].

Because of its fundamental importance in quantum mechanics and quantum cryptography, various UQCMs have been demonstrated either by constructing the complex quantum network [7, 8], or by exploiting the process of stimulation emission [9, 10, 11, 12] or by photon bunching [13, 14]. However, in all these experiments, only identical outputs have been realized.

To investigate the asymmetric distribution of an unknown quantum state, Cerf [15] first proposed a family of Pauli cloning machines that produced two unnecessarily identical output qubits. The state-independent fidelities \( F_e \) and \( F_d \) of the two copies was bound to a no-cloning inequality [15]

\[
(1 - F_d) (1 - F_e) \geq \frac{1}{2} - (1 - F_d) (1 - F_e)^2.
\]  

(1)

This inequality sets the optimal tradeoff between the quality of the two copies in the sense that for given fidelity \( F_d \) one cannot obtain a better fidelity \( F_e \) [15]. The optimal unbalanced fidelities quantify the novel cloning limit imposed by quantum mechanics, thereby extending the results on the symmetric UQCM.

Moreover, in the context of quantum cloning, it is highly desirable to transfer one of the quantum cloning to a distant location, realizing the so-called telecloning [17, 18]. Although it is, in principle, straightforward to combine quantum teleportation with optimal quantum cloning, it will involve the extra resources and also leads to the extremely low efficiency with the existing technology. Alternatively, the procedure could be reduced to exploit the particular multiparticle entangled states [17], but such crucial resource has not yet been experimentally demonstrated.

In this letter, we report an experimental realization of both asymmetric cloning and telecloning by making use of partial teleportation of an unknown state [19]. In the experiment, we demonstrate that, conditioned on the partial quantum teleportation of single photons, not only asymmetric cloning can be realized, but also one of two outputs can be transferred to a distant location, realizing the telecloning.

Let us first consider the scenario that Alice wants to send an unknown polarization state of a single photon in a mode \( b \) to Bob at a distant location. Eve seeks to

FIG. 1: Scheme of asymmetric cloning and telecloning by making use of partial teleportation.
extract partial (or full) quantum information of the state by using partial quantum teleportation having a pair of entangled photons in modes $c$ and $d$. The procedure is that Eve performs a partial Bell-state measurement on the photons in mode $b$ and $c$, then resends the teleported state to Bob in mode $d$. The pair of entangled photon held by Eve is in the Bell state

$$|\Psi^+\rangle_{cd} = \frac{1}{\sqrt{2}}(|H\rangle_c |V\rangle_d - |V\rangle_c |H\rangle_d),$$

and the partial Bell state measurement is achieved through an unbalanced beam splitter (BS) with a variable reflectivity, $0 \leq R \leq 0.5$.

Consider, for example, one vertically polarized photon is sent by Alice to Bob. Then, in the above eavesdropping protocol, the evolution of the initial state $|V\rangle_b \otimes |\Psi^+\rangle_{cd}$ is determined by the evolution of photons in modes $b$ and $c$,

$$b \rightarrow (irf + te), \ c \rightarrow (tf + ire),$$

where $R = r^2$, $1 - R = t^2$. If we restrict ourself to the cases where both photons leave the beam splitter separately, then we obtain the following state (unnormalized)

$$(t^2 |V\rangle_c |H\rangle_f - r^2 |H\rangle_c |V\rangle_f) |V\rangle_d - (t^2 - r^2) |V\rangle_c |V\rangle_f |H\rangle_d.$$  

Thus, tracing over photons in modes $d$ and $f$ and measuring the probability of the output to be vertically polarized as the input photon, Eve could obtain the local cloning in mode $e$ with a fidelity:

$$F_e (R) = \frac{1}{2P(R)} \left[ (1 - 2R)^2 + (1 - R)^2\right]$$

and similarly Bob would obtain the telecloning state in mode $d$ with a fidelity:

$$F_d (R) = \frac{1}{2P(R)} \left[ R^2 + (1 - R)^2\right]$$

where $P(R) = 1 - 3R + 3R^2$, corresponds to the probability that the two photons exit Eve’s beamsplitter separately. Although the above scheme only succeed probabilistically, it is sufficient to provide a proof-in-principle demonstration of both the optimal asymmetric cloning and telecloning.

It follows, that the fidelities $F_e$ and $F_d$ satisfy the no-cloning inequality (1) and represent the optimal asymmetric distribution of the initial quantum information between the local clone and the distant clone. For the reflectivity of $R = 1/3$ the protocol was reduced to the symmetric distribution with fidelities $F_e = F_d = 5/6$, but one of the two clones was distributed to a distant location, realizing the telecloning. The foregoing analysis are justified for any input polarization owing to the rotational invariance of the Bell state in Eq. (1).

A schematic of our experimental apparatus is shown in Fig. 2. We first generate two pairs of entangled photons in the maximally entangled state $|\Psi^-\rangle$ by type II down-conversion from an ultraviolet (UV) pulsed laser in a BBO crystal. The UV pulse passing through the crystal twice creates two pair of entangled photons in modes $a-b$ and $c-d$. The UV pulsed laser with a central wavelength of 394nm has a pulse duration of 200fs, a repetition rate of 76MHz, and an average power of 450mW. Photons in modes $b$ and $c$ are first overlapped at the BS$_1$ and then recombined at the BS$_2$ where the path lengths of the two photons have been adjusted so that they arrive at two beam splitters simultaneously. Through spectral filtering ($\Delta \lambda_{FWHM} = 3 \text{nm}$) and fiber-coupled single-photon detectors, we can ensure that all the four photons are in the perfect temporal and spatial mode overlap.

In the experiment, the crucial requirement is to overlap two photons in the Mach-Zehnder (M-Z) interferometer, with which the controllable phase difference will lead to the desired variable reflectivity. However, when
two photons with different polarizations pass through the same two arms of the interferometer, they will usually experience unbalanced phase differences due to the birefringent effect of the BS$_1$, the BS$_2$, and prisms (DL$_2$ and its counterpart in Fig. 2). To overcome this difficulty, we incorporate a 1.2mm type I LBO crystal as a compensator (C in Fig. 2) to vary the phase shift in one arm of the interferometer so that the identical phase difference could be reached by tilting the compensator.

To show that the crucial M-Z interferometer works properly, two photons with orthogonal polarizations in modes $b$ and $c$ are steered to the interferometer. We predefine two photons in the vertical and horizontal polarizations by performing the polarization measurements on the photons in modes $a$ and $d$ behind 0$^\circ$ and 90$^\circ$ polarizers respectively. With these settings, we first measure the twofold coincidence between the output modes $a$ and $d$ to verify the polarization-independent reflectivity. We slightly tilt the compensator inside the M-Z interferometer with a step size of 0.36µm around the centre of the envelope and simultaneously observe the two independent twofold coincidences between the output modes $a$-$c$ and modes $d$-$f$ in order to verify the polarization-independent reflectivity. We slightly tilt the compensator inside the M-Z inter-

FIG. 3: Experimental results showing that the two-photon Mach-Zehnder interferometer works properly and exhibits the polarization-independent reflectivity. In (a) and (b), we measure the twofold coincidence between the modes $c$ and $a$ behind 0$^\circ$ polarizer. In (c), we measure the twofold coincidence between the modes $f$ and $d$ behind 90$^\circ$ polarizer. (a), the envelope of the observed twofold coincidences demonstrates that the single photon passing through the interferometer interferes with itself. (b) and (c), the two twofold coincidences exhibit a synchronized variation. This demonstrates that the two photons with orthogonal polarizations have undergone the identical phase difference, which consequently leads to the polarization-independent reflectivity.

FIG. 4: Experimental results demonstrating both the asymmetrical cloning and telecloning of single photons in three complementary polarizations of 0$^\circ$ (a), +45$^\circ$ (b) and L (c) with the various reflectivity beam splitter. The experimental results are in well agreement with the theoretical prediction (the solid plot) of Eq. (5) and (6).
ferometer until the two twofold coincidences exhibit a synchronized variation (Fig. 3b and 3c). This demonstrates that the two photons of the orthogonal polarizations passing through the interferometer have undergone the identical phase difference, which consequently leads to the polarization-independent reflectivity. Further results show that our compensation method also works for two photons in the general polarizations.

Specifically, we obtained the reflectivity of $R=1/2$ when the phase difference was set to be $\pi/2$. By scanning the DL$_1$, the perfect temporal overlap was verified through a successful teleportation of photons polarized at $+45^\circ$ with a visibility of $0.75 \pm 0.05$ at zero delay [21].

To further demonstrate our scheme, we varied the phase difference to achieve various reflectivities of 0.1, 0.3, 0.5, 0.7, 0.9. The input photon (to be cloned) was prepared in three complementary polarizations of $0^\circ$, $+45^\circ$ and circular left-handed, $L$ by triggering the polarization measurement on the photon in mode $a$. Then the clones were further verified by performing the conditional projection measurement on photons in modes $e$ and $d$. For example, conditioned on the detection of photon in mode $a$ as well as the trigger measurements in modes $d$ and $f$, the local clones were left in mode $e$, which were confirmed by performing the polarization projection measurements. The fidelities were accordingly obtained by measuring the probability of the output states to be in the input states. Similarly, we could obtain the fidelities of the telecloning in mode $d$.

In our experiment, the integration time for each cloning measurement is 5 minutes while each reflectivity can vary less than 0.025. All these results are shown in Fig. 4. From the figures, it is evident that the experimental results are in well agreement with the theoretical prediction of Eq. (5) and (6), while only $0 \leq R \leq 0.5$ represents the optimal asymmetrical cloning and telecloning. The imperfect fidelities are mainly due to the instability of the interferometer as well as the imperfections of the down-conversion source, and the mode overlap of the photons inside the interferometer.

The experimental realization of both the optimal asymmetrical cloning and telecloning deserves some further comments. First, in the experiment, an unknown state was encoded into two qubit with ancilla pair of entangled photon via partial quantum teleportation, which is significantly different from the previous implementations [5, 8, 9, 10, 11, 12, 13, 14], and thus represents a novel way to realize the quantum cloning. Second, both cloning and teleportation are intrinsically integrated together so that it no longer requires the extra resource to achieve the telecloning and will not lead to any depressed efficiency, comparing with the Innsbruck experiment [21]. Third, although our present experimental demonstration required the coincidence detection of all four photons, the telecloning could be freely transferred to a distant location by using the nonpostselection teleportation technique [25]. For example, we could attenuate the photon intensity in mode $b$ to suppress those spurious $e\cdot f$ coincidence events, (i.e. those events contributed by double pair emission either in modes $a$ and $b$ or $c$ and $d$), the threefold coincidence among the modes $a$, $c$ and $f$ would then be sufficient to guarantee the success of partial teleportation. Thus, the telecloning can be successfully achieved without the need to destructively detect it.

In summary, we have for the first time presented an experimental realization of both optimal asymmetrical cloning and telecloning via conditional partial teleportation of an unknown state $|\Psi\rangle$. The experimental results represent a novel way to achieve the quantum cloning and may have potential applications in the context of future quantum communication.

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