Hinged Quantum Spin-Hall Effect in Antiferromagnetic Topological Insulators

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In this work, we predict a hinged quantum spin-Hall (HQSH) effect featured by a pair of helical hinge modes in antiferromagnetic (AFM) topological insulator (TI) multilayers. This pair of helical hinge modes are localized on the hinges of the top and bottom surfaces of the AFM TI multilayers. Unlike the conventional QSH effect, the HQSH effect survives the breaking of time-reversal symmetry (TRS) and thus represents a different kind of topological phenomenon. The helical hinge modes are sustainable to inelastic scattering and TRS-breaking disorder, which can be observed in macroscopic samples. We show that this HQSH effect can be understood as a three-dimensional generalization of the Su-Schrieffer-Heeger model and its topology is characterized by the spin Chern number. At last, we propose that the HQSH effect can be realized in newly found intrinsic AFM TI materials (MnBi2Te4)m(Bi2Te3)n or magnetic-doped TI multilayers by current experimental setups.

Introduction.—Searching for topological insulators (TIs) has been one of the focus areas in condensed matter physics in the past years [1, 2]. Among them, a quantum spin-Hall (QSH) insulator which supports helical edge modes, is a two-dimensional (2D) $Z_2$ TI and can only survive in the presence of time-reversal symmetry (TRS) [3–11]. In reality, such helical edge modes are generally subjected to inelastic backscattering even without TRS-breaking, which would ruin the ballistic transport mediated by helical edge modes [6, 12, 13]. Therefore, a truly quantized helical-edge conductance was only experimentally observed in mesoscopic samples within several micrometers. On the other hand, the QAH state that breaks TRS is characterized by the Chern number (CN) and can support quantized conductance carried by chiral edge states in millimetre-size samples [14–21]. Moreover, higher-order topological insulators, which generalize the concept of TI, were recently proposed [22–29]. The higher-order topology gives rise to one-dimensional (1D) helical and chiral states localized on the hinges of 3D second-order topological insulators (SOTIs) [30–32].

Recently, there are tremendous progresses in fabricating magnetic TIs experimentally [33–42]. Notably, a breakthrough was made in synthesizing intrinsic 3D antiferromagnetic (AFM) TI material MnBi2Te4, which has van der Waals bonded layer structures. Soon after, both the QAH states and axion insulators were observed in MnBi2Te4 [43–46]. Interestingly, (MnBi2Te4)m(Bi2Te3)n family materials (such as, MnBi4Te7 and MnBi6Te10), constructed by stacking MnBi2Te4 septuplelayers and quintuple Bi2Te3 layers with different ratio m/n, are also realized in experiments [47–52]. Therefore, the (MnBi2Te4)m(Bi2Te3)n materials are highly tunable and become a versatile platform to realized various topological phases.

In this Letter, we propose a TRS-breaking QSH effect characterized by a pair of helical hinge modes (HHMs) in AFM TI multilayers. The HHMs consist of two spatially separated chiral hinge states with opposite chirality, which are localized on the hinges of the top and bottom surfaces of 3D AFM TIs. The QSH effect associated with the HHMs can be regarded as a topological characteristic of an SOTI in the 3D system, and we name this QSH effect as hinged quantum spin-Hall (HQSH) effect. To realize the HQSH effect, we design the AFM TI by alternatively stacking magnetic TIs (e.g. MnBi2Te4 septuplelayers) with opposite magnetization as shown in Fig. 1. Two adjacent magnetic TI layers in opposite magnetization carrying opposite CNs form a composite layer [see Fig. 1(a)]. Here, the intralayer and interlayer couplings can be controlled by introducing spacer layers. When the interlayer coupling dominates over intralayer coupling, each pair of neighboring counter-propagating chiral states from two adjacent composite layers tend to hybridize in pair and develop a gap in the bulk, while the two unhybridized chiral states on the hinges of the top and bottom surfaces remain gapless and form a pair of spatially separated HHMs [Fig. 1(a)]. On the contrary, if the intralayer coupling dominates, the two chiral edge states within individual composite layers tend to hybridize and thus all the edge modes are gapped out in pairs [see Fig. 1(b)], resulting in a trivial insulator.

In comparison with the conventional QSH effect, the HQSH effect shows two distinguishing features: (i) It survives the breaking of TRS, while the conventional QSH effect requires the protection by TRS [3–5]. This extends the concept of QSH effect to TRS-breaking (magnetic) materials, which were believed not to support the QSH effect. (ii) Unlike the ordinary helical edge/hinge states in the conventional QSH effect or the previous SOTIs
the interlayer coupling between two adjacent composite layers are hybridized pairwise when the interlayer coupling \( t_2 \) dominates over the intralayer coupling \( t_1 \), leaving two chiral edge modes localized on the top and bottom layers. They are equivalent to a pair of helical edge modes in the QSH effect. On the contrary, (b) all the chiral edge modes within the composite layers hybridize in pairs if \( t_1 \) dominates. (c) Phase diagram of AFM TIs on the plane of \( t_{23} / t_1 \) and \( t_3 / t_1 \), with \( t_{23} \) hopping between two adjacent composite layers. The two red stars on red line \( t_3 = 0 \) indicate the parameter region studied in Fig. 2.

[6, 30–32, 53, 54], the HHMs in the HQSH effect support a quantized conductance in macroscopic samples because they are sustainable to dephasing and magnetic disorder. Thus the HQSH effect, which is protected by spin Chern number, represents a different kind of topological phenomenon.

**Model Hamiltonian.**— The model Hamiltonian \( \mathcal{H} \) of 3D AFM TI multilayers can be obtained by alternatively stacking 2D QAH Hamiltonian with opposite CNs in the \( z \) direction, which can be written as

\[
\mathcal{H} = \sum_{n=1}^{N_z} \Psi_n^\dagger \left( h_D \zeta_z + t_1 \zeta_x \right) \Psi_n + \left| \Psi_n^\dagger \right| \left( t_2 \zeta_x + t_3 \zeta_z \right) \Psi_n + h.c.,
\]

where \( \Psi_n \) is electron creation operator in the \( n \)-th composite layer, and \( h_D = v_f (\sigma_y \sin k_x + \sigma_x \sin k_y) + (\Delta_z + m_k) \sigma_z \) is the QAH insulator Hamiltonian. The Pauli matrices \( \sigma_x, \sigma_y, \sigma_z \) act on the spin and layer spaces, respectively, and \( \zeta_x = (\zeta_x \pm i \zeta_y) / 2 \). The mass term \( m_k = m_0 - m_1 (2 - \cos k_x - \cos k_y) \) with \( m_0 < 0 \) and \( m_1 > 0 \), where \( k_{x,y} \) is the wave vector in the \( x(y) \) direction. \( \Delta_z \) is the strength of exchange field and \( v_f \) is the Fermi velocity. The Chern numbers of the QAH states are \( C_\pm = \pm 1 \) for \( \pm h_D \) when \( \Delta_z > |m_0| \). Note that we fix \( \Delta = 0.7, m_0 = -0.1, m_1 = 1 \) and \( v_f = 1 \) in the following. Each two neighboring QAH with opposite CNs form a composite layer, and \( n_i \) is the composite layer index. The QAH states are coupled by the intralayer hopping \( t_1 \) and interlayer hopping \( t_{23} \) that are determined by the thickness of spacer layers [see Figs. 1(a)-1(b)]. At the beginning, we set \( t_3 = 0 \) and discuss how the topological phase of the system is determined by the intralayer and interlayer hoppings \( t_1 \) and \( t_2 \). In the limited case of \( t_3 \neq 0 \) and \( t_2 = 0 \), the AFM TIs are decoupled into \( N \) isolated composite layers, and the two QAH insulators within a composite layer hybridize to gap out the two chiral states. On the contrary, when \( t_1 = 0 \) and \( t_2 \neq 0 \), each two neighboring QAH insulators from two adjacent composite layers are coupled by \( t_2 \) and there are two isolated QAH insulators localized on the top and bottom composite layers, leaving a pair of HHMs in 3D AFM TIs. We name it the HQSH effect.

If we regard the 2D Hamiltonian \( h_D \) as a model parameter, \( \mathcal{H} \) is exactly a 3D generalization of the Rice-Mele model [55, 56], which consists of \( 2N_z \) quasi-2D bands related to \( h_D \). When \( |t_1| < |t_2| \), there exist two quasi-2D bands localized on the top and bottom surfaces, originating from the two end states of the Rice-Mele model [55, 56]. Then we can obtain an effective Hamiltonian by performing a projection \( H_{proj} = \sum \left\{ \hat{P}_- \mathcal{H} \hat{P}_+ \right\} \) onto the occupied states of \( \mathcal{H} \) [57–60].

In virtue of the bulk-boundary correspondence, the effective Hamiltonian of the HHMs can be obtained by a projection similarly. The effective Hamiltonian for two chiral edge states of the neighboring QAH insulators in one composite layer can be written as \( H_{1D} = v_f k_x \zeta_x \). Then the surface states of the AFM TIs on the \( x-z \) plane can be expressed by \( H_{surf} = \sum_{n=1}^{N_z} \psi_n^\dagger \left( v_f k_x \zeta_x + t_1 \zeta_x \right) \psi_n + v_f^2 t_2 \zeta_z \psi_n + h.c. \) with the energy spectrum \( E_k = \pm (v_f^2 k_x^2 + (t_1 + t_2 \cos k_z))^2 + t_2^2 \sin^2 k_z \)}^{1/2}. The gap at \( k_z = 0 \) closes when \( |t_1| = |t_2| \), giving rise to a topologically nontrivial phase with \( |t_1| < |t_2| \) and a trivial phase with \( |t_1| > |t_2| \). In fact, at \( k_z = 0 \), \( H_{surf} \) is exactly reduced to the Su-Schrieffer-Heeger (SSH) model which has two zero-energy states localized on the boundaries for \( |t_1| < |t_2| \), with the wave functions \( \left| \psi_{0z}^\pm \right\rangle \) at the large \( N_z \) limit [55, 56]. Correspondingly, the HHMs on the \( xz \) plane can be captured by the effective Hamiltonian \( \langle \hat{\nu}_{0z}^\pm | H_{surf} | \hat{\nu}_{0z}^\pm \rangle = \pm \hbar v_f k_x \).
AFM TI multilayers with the width $N_x = 16$, thickness $N_z$ and intralayer coupling $t_1$ and interlayer coupling $t_2$. (a) The two chiral edge modes are hybridized and gapped out for $N_x = 1$. (b)-(c) For $t_1 = 0.04 < t_2 = 0.1$, the chiral edge modes in different composite layers are hybridized and gapped out in pairs, leaving two isolated chiral edge modes [see the dashed red and black lines in (b) $N_z = 2$ and (c) $N_z = 6$] localized on the top and bottom layers. (e) Local density of states with the Green’s function $G = \left[G^{\dagger}\right]$ is the transmission coefficient from the lead $p$ to the Green’s function $G^* = \left[G^{\dagger}\right]^\dagger$ and the line width function $\Gamma_q [61]$. Here, TRS-breaking Anderson disorder is included as a random on-site potential $V(r)$ on each orbital independently, which is uniformly distributed in the range of $[-W/2, W/2]$. In the following, we systematically evaluate the longitudinal conductance $G_{12,12}$ and the nonlocal conductance $G_{14,23}$ for the two-terminal device and the $\pi$ device, respectively [53, 62].

If we now turn on $t_3$, the phase diagram [Fig. 1(c)] of the AFM TIs can be determined by the gap closing points of the surface state spectrum $E_k = \pm\{v_f^2 k_x^2 + [t_1 + (t_2 + t_3) \cos k_z]^2 + (t_2 - t_3)^2 \sin^2 k_z\}^{1/2}$. Generally, the HQSH and normal insulator phases are separated by $|t_1| = |t_2 + t_3|$, furthermore the HQSH are divided into two parts (HQSH I and HQSH II) by $t_2 = t_3$. Note that the HHMs in the HQSH I and HQSH II has opposite chirality. That’s because the two QAH insulators and thus the chiral edge states in the composite layer switch positions if we exchange $t_2$ and $t_3$ in the Hamiltonian $\mathcal{H}$. In the following, we focus on the HQSH I in the phase diagram and fix $t_3 = 0$ unless otherwise specified.

To demonstrate the emergence of the HHMs in the AFM TI multilayers, we numerically investigate the energy band structure of the Hamiltonian $\mathcal{H}$ as varying the composite layer number $N_z$ in Fig. 2. In general, the energy spectra are two-fold degenerate, because $\mathcal{H}$ respects a combined symmetry of TRS and inversion symmetry. Figure 2(a) shows two chiral edge modes with opposite chirality are coupled and gapped out when $N_z = 1$. For $t_2 = 0.1 > t_1 = 0.04$, when $N_z > 1$, the chiral edge modes from two adjacent composite layers tend to hybridized in pairs, leaving two sets of isolated chiral edge modes [see the dashed red and black lines in Fig. 2(b) $N_z = 2$ and Fig. 2(c) $N_z = 6$] localized on the top and bottom layers. Note that the system has a small hybridization gap for $N_z = 2$ due to the finite-size effect, which is closed for a larger size $N_z = 6$. Furthermore, under the open boundary condition along all the three directions, we show the local density of states (DOS) in Fig. 2(e) at $E = 0.01$ and find that the midgap states are localized on the hinges of the sample surfaces. On the contrary, fixing $N_z = 6$, when $t_2 = 0.04 < t_1 = 0.08$, the chiral edge modes in the same composite layers are hybridized in pairs, leaving an energy gap in the surface band spectrum as shown in Fig. 2(d). Therefore, the HQSH characterized by a pair of HHMs is realized in AFM TIs when the interlayer coupling $t_2$ dominates over the intralayer one.

To explore robustness of the HQSH effect, we study the conductance in the presence of TRS-breaking disorder by using the Landauer-Buttiker formula. The current in the lead $p$ can be expressed as: $I_p = e^2 / h \sum_{q \neq \pi} T_{pq} (E_F) (V_p - V_q)$ where $V_p$ is the bias in the lead $p$ and $T_{pq} (E_F) = \text{Tr}[\Gamma_q G^* G p^\dagger]$ is the transmission coefficient from the lead q to p with the Green’s function $G' = \left[G^{\dagger}\right]^\dagger$ and the line width function $\Gamma_q [61]$. Here, TRS-breaking Anderson disorder is included as a random on-site potential $V(r)$ on each orbital independently, which is uniformly distributed in the range of $[-W/2, W/2]$. In the following, we systematically evaluate the longitudinal conductance $G_{12,12}$ and the nonlocal conductance $G_{14,23}$ for the two-terminal device and the $\pi$ device, respectively [53, 62].

![Figure 2](attachment:image.png)

**FIG. 2:** (Color online). The energy band structure of AFM TI multilayers with the width $N_y = 16$, thickness $N_z$ and intralayer coupling $t_1$ and interlayer coupling $t_2$. (a) The two chiral edge modes are hybridized and gapped out for $N_z = 1$. (b)-(c) For $t_1 = 0.04 < t_2 = 0.1$, the chiral edge modes in different composite layers are hybridized and gapped out in pairs, leaving two isolated chiral edge modes [see the dashed red and black lines in (b) $N_z = 2$ and (c) $N_z = 6$] localized on the top and bottom layers. (e) Local density of states with the Green’s function $G = \left[G^{\dagger}\right]$ is the transmission coefficient from the lead $p$ to the Green’s function $G^* = \left[G^{\dagger}\right]^\dagger$ and the line width function $\Gamma_q [61]$. Here, TRS-breaking Anderson disorder is included as a random on-site potential $V(r)$ on each orbital independently, which is uniformly distributed in the range of $[-W/2, W/2]$. In the following, we systematically evaluate the longitudinal conductance $G_{12,12}$ and the nonlocal conductance $G_{14,23}$ for the two-terminal device and the $\pi$ device, respectively [53, 62].

![Figure 3](attachment:image.png)

**FIG. 3:** (Color online). Plots of (a-c) the two-terminal conductance $G_{12,12}$ and (d) nonlocal conductance $G_{14,23}$ versus the Fermi energy $E$ for different thickness (a) $N_z = 1$, (b) $2$, and [(c),(d)] 6 at various disorder strength $W$. The error bar denotes conductance fluctuation. The three sets of model parameters are the same as those in Figs. 2(a)-2(c), respectively, with the sample size $N_x \times N_y \times N_z = 64 \times 16 \times N_z$ in Figs. 3(a)-(c) and $96 \times 16 \times 6$ in Fig. 3(d). The insets of (b) and (d) show a two-terminal device and a $\pi$-bar device, respectively.

In Figs. 3(a)-3(b), we calculate the longitudinal conductance $G_{12,12}$ in the two-terminal device [see the inset...
of Fig. 3(b)] as a function of the Fermi energy $E$ for different thickness $N_z$ at various disorder strengths $W$. We set the system in the topologically nontrivial region with $t_1 = 0.04 < t_2 = 0.1$. For $N_z = 1$ in Fig. 3(a), we find an insulating gap with $G_{12,12} = 0$ between a quantized conductance plateau $G_{12,12} = 2e^2/h$ at $W = 0$, which loses quantization in the presence of disorder. That’s because the two chiral edge modes are coupled and thus not topologically protected as shown Fig. 2(a). In Fig. 3(b), there are four chiral edge modes with $G_{12,12} = 4e^2/h$ for $N_z = 2$ and the finite-size effect gap with $G_{12,12} = 0$ decreases.

By increasing the thickness to $N_z = 6$ in Fig. 3(c), it is found that a quantized conductance plateau $G_{12,12} = 2e^2/h$ shows up near the band center $E = 0$, because the finite-size gap becomes negligible [see Fig. 2(c)]. Notably, the plateau $G_{12,12} = 2e^2/h$ is topologically protected and remains stable under disorder with strength larger than the band gap, while the higher-order plateaus $2ve^2/h$ ($v = 2, 3, 4, \ldots$) lose quantization in the presence of disorder. Therefore, we conclude that the HHMs in the HQSH are robust to TRS-breaking disorder, distinct from helical edge modes in the conventional QSH effect. At last, we propose the HHMs can be detected by a non-local conductance $G_{14,23}$ in a $\pi$ device consisting of a sample attached to four leads [inset of Fig. 3(d)]. Notably, the first conductance plateau $G_{14,23} = 4e^2/h$ that indicates the existence of the HHMs, is robust to disorder.

**FIG. 4:** (Color online). (a) Two-terminal conductance $G_{12,12}$ of the HQSH versus the Fermi energy $E$ for various dephasing strength $\Gamma$. (b) Phase coherent length $L_\phi$ as a function of $\Gamma$. (c) $G_{12,12}$ of the conventional QSH in the Bernevig-Hughes-Zhang (BHZ) model versus $E$ for various $\Gamma$. (d) $G_{12,12}$ versus $\Gamma$ for the HQSH and the conventional QSH at $E = 0.02$. The parameters of the Dirac Hamiltonian in the BHZ model are the same of the $h_D$ in the HQSH model. The sample size is $N_z \times N_y \times N_x = 32 \times 16 \times 6$.

Now let us come to investigate the inelastic scattering in the system by simulating the dephasing effects on the HQSH [63, 64]. In a realistic sample, there are plenty of possible dephasing processes because of electron-electron or electron-phonon interactions. We introduce one virtual lead with the linewidth $\Gamma$ to each site to simulate the dephasing effect using Landauer-Buttiker formula [63, 64]. $\Gamma$ is dephasing strength and directly related to the phase coherence length $L_\phi$ [12, 65], which is an experimentally measurable parameter. Figure 4(b) shows the relation of the phase coherent length $L_\phi$ versus the dephasing strength $\Gamma$. With increasing $\Gamma$, $L_\phi$ decreases rapidly and monotonically.

Next, we present our numerical results of the dephasing effect on the two-terminal conductance $G_{12,12}$. Figure 4(a) plots $G_{12,12}$ versus the Fermi energy $E$ for various $\Gamma$. It is clear that the quantized plateau at $G_{12,12} = 2e^2/h$ due to the HHMs in the HQSH is hardly affected by the dephasing. By contrast, we simulate the conventional QSH using the Bernevig-Hughes-Zhang model [5] in Fig. 4(c) and find the conductance plateau at $G_{12,12} = 2e^2/h$ loses quantization dramatically in the presence of the (spin) dephasing [6, 12, 13]. To be specifically, as shown in Fig. 4(d), we find that the quantized conductance $G_{12,12} = 2e^2/h$ at $E = 0.02$ decreases by less than 5% at dephasing strength $\Gamma = 0.07$, even when the system size $L = 32$ far exceeds the phase coherent length $L_\phi \approx 10$. That’s because the HHMs in HQSH are spatially separated as discussed above, which thus behave as the chiral edge channel in the quantum Hall effect that is hardly affected the dephasing. In contrast, the conductance plateau $G_{12,12} = 2e^2/h$ in the conventional QSH at $E = 0.02$ decreases by 7% even for $\Gamma = 0.005$ and more than 52% for $\Gamma = 0.07$ [see Fig. 4(d)]. Therefore, we conclude that the HHMs in the HQSH is robust to the dephasing (inelastic scattering), which can be observed in macroscopic samples.

**Experimental Materials.**—The proposed HQSH effect in 3D AFM TI multilayers can be readily realized in realistic materials. Recently, the intrinsic AFM TIs family materials (MnBi$_2$Te$_4$)$_m$(Bi$_2$Te$_3$)$_n$ are fabricated experimentally [47–52], which can be regarded as stacked MnBi$_2$Te$_4$ septuple layers (denoted as “A”) and Bi$_2$Te$_3$ quintuple (denoted as “B”) layers. If we introduce Bi$_2$Te$_3$ quintuple layers to intrinsic 3D AFM TI MnBi$_2$Te$_4$, the new system forms a superlattice structure with different interlayer and intralayer couplings. For example, in “ABBA...” superlattices, the interlayer coupling (between two neighbouring MnBi$_2$Te$_4$ septuple layers) in “AA” is dominant over the intralayer coupling in “ABBA” due to $B$ layers. Moreover, the magnetically doped (Bi,Sb)$_2$Te$_3$ TI and CdSe normal insulator layers in the superlattice structure has been successfully grown by molecular beam epitaxy [33]. It has been shown that the coercive field of magnetic TI in each layer can be controlled by mixing the dopants Cr and V at varied ratios [66]. In fact, a magnetic TI bilayer with different dopants ratios was driven to the opposite magnetization by sweeping the magnetic field in a recent experiment [67]. Therefore, it is natural to expect that
HQSH effect in 3D AFM TIs can be realized in the 
$\text{(MnBi}_2\text{Te}_3)_m \text{(Bi}_2\text{Te}_3)_n$ family materials as well as mag-
netic doped TIs multilayers.

Summary.— In conclusion, we propose to realize the
HQSH effect with a pair of HHMs in 3D AFM TIs,
which can be fabricated by current experimental setups.
These HHMs can be identified by a nonlocal conductance
plateau of $4e^2/h$ in $\pi$-bar measurement. In contrast to
the conventional QSH effect, the HQSH effect has a very
different topological origin and thus can survive without
TRS, where the HHMs therefore is robust to magnetic
impurities as well as spin dephasing.

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Note added.— Recently, we became aware of a com-
plementary study [68], which treats a similar situation but
focuses on a different aspect.
