A demonstration that the observed neutrinos are not Majorana particles

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Abstract

It is shown that Majorana neutrinos cannot couple vectorially to the neutral-current SU(2)$_L \times$ U(1) gauge field of the standard model. Since strong evidence for the existence of such a vector coupling in neutral current reactions has recently been presented by the Charm II collaboration, it is unlikely that the observed neutrinos are predominantly Majorana particles. Theorems on the “reappearance” of vector interactions in neutral current scattering of Majorana neutrinos and the indistinguishability of Majorana and Dirac neutrinos in the massless case are discussed critically.

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1 Introduction

The answer to the question of whether the neutrino is a Dirac or a Majorana particle is considered to be one of the most important clues to physics beyond the standard model. If the neutrino were a “Majorana particle” (i.e. a particle identical to its antiparticle), the so called “see-saw” mechanism could naturally explain the smallness of neutrino masses, which remains puzzling within the standard model (see ref. for a recent review on neutrino masses). The “see-saw” mechanism requires the existence of Higgs-field configurations beyond the one of the standard model, which makes the prospect of experimentally proving the Majorana nature of neutrinos (e.g. via neutrinoless double beta-decay experiments) very attractive.

In this paper I present a contribution to this long-standing problem, concluding that the neutrino species observed up to now cannot be predominantly Majorana particles. If neutrinos have Dirac character, the existing neutral-current scattering data are in complete agreement with the standard model of particle physics. In particular the neutral-current

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vector coupling of neutrino, contained in the standard model, is necessary for a satisfactory
description of the experimental data (section 3 and appendix 4, second part). In section 2 it is shown that for Majorana particles any neutral-current vector coupling is forbidden.
Therefore the experimental data cannot be quantitatively understood in the standard way
under the assumption that neutrinos are Majorana particles. It will probably still be pos-
sible to call in some new physics, which is fine tuned to explain the experimental data
under the assumption that neutrinos are Majorana particles. Such new physics does not
seem to be required in a natural way, though (e.g. by the existence of Higgs-field configu-
rations leading to Majorana masses). Faced with this situation it seems quite likely that
the known neutrino species are Dirac particles. I will confine the demonstration to purely
neutral-current reactions.

The “modern” choice for the metric of the 4-vectors (defined e.g. in the textbooks of
Bjorken and Drell[3] and Mandl and Shaw[4]) is used. If not otherwise noted (e.g. in
eq.(10) I will work in the Majorana representation[1, 4] for the γ matrices in the Dirac
equation (Pauli’s fundamental theorem states that the choice of the representation can have
no influence on any physical result of the theory[4]). The space-time arguments of all fields
are taken as positive. The discussion will be in the q-number formalism throughout (full
second-quantized field theory).

2 The vanishing of the vector coupling in the Dirac equation
for Majorana fields

The general Dirac equation for a complex valued neutrino field operator (operators are
symbolized by the hat ^ ) ^Ψν of arbitrary helicity and rest mass mν in a neutral weak
boson field Zμ, is obtained from the standard model Langrangian[4] via the Euler-Lagrange
equations as:

iγ^μ (ℏ/2e)Zμ(gν'V - gν'Aγ5)) ^Ψν - mνc^Ψν = 0

here e is the positron charge, θW is the Weinberg angle and gV' = gA' = 1/2 are the vector
and axial couplings of the neutrino to the Zμ field. ^Ψν is a Dirac bispinor. The aim is now
to find the corresponding equation of motion for the “abbreviated”[3] case of a Majorana
neutrino.

A Majorana particle (symbolized by the subscript M) is defined by the “supplementary
condition” that the field and its charge conjugate (symbolized by the superscript c) are
identical[1, 6] for all positions x, t in space time:

^ΨM(x, t) = ^ΨM(x, t)

1 see the appendix of Mandl and Shaw’s book[4] of the explicit presentation of the γ matrices as used in
this paper, all five γ matrices are purely imaginary.
It is possible to introduce a purely conventional phase factor in the definition of this condition. I follow a usual practice (and Majorana’s original publication[1]) and set this factor to 1.

“Charge conjugation” is defined as taking the hermitian conjugate of the field operator and multiplying it with a “charge conjugation matrix” $S_C$ which is defined by the condition $S_C^{-1} \gamma^\mu S_C = -\gamma^\mu \gamma^0$ hence:

$$\hat{\Psi}^c(\vec{x}, t) = S_C \hat{\Psi}^T(\vec{x}, t).$$

Here the transpose operation $T$ only brings the bispinor back to a column form and does not otherwise act on the operator[3]. In the Majorana representation $S_C$ is the unit matrix $I$[6, 5], charge conjugation is equivalent to hermitian conjugation and the Majorana field is necessarily[4] real valued[1]. The first appendix gives a more mathematical explanation of this “real valuedness” in the field case.

According to equation (2) one can represent a field fulfilling condition (4) by demanding that it is a superposition of a Dirac field $\hat{\Psi}$ and its charge conjugate for all $\vec{x}, t$[6]:

$$\hat{\Psi}_M(\vec{x}, t) = \frac{1}{\sqrt{2}}(\hat{\Psi}^c(\vec{x}, t) + \hat{\Psi}(\vec{x}, t))$$

In the Majorana representation the equation of motion for the charge conjugate field of the neutrino $\hat{\Psi}_\nu^c$ then simply follows by taking the hermitian conjugate of eq. (1) (as the $\gamma$ matrices are purely imaginary in the Majorana representation they change sign under hermitian conjugation):

$$i\gamma^\mu \left( \frac{\hbar}{2} \frac{\partial}{\partial x^\mu} + \frac{ie}{2 \cos(\theta_w) \sin(\theta_w)c} Z_\mu (g_\nu^V + g_\nu^A \gamma^5) \right) \hat{\Psi}_\nu^c - m_\nu c \hat{\Psi}_\nu^c = 0$$

We obtain the equation of motion for a Majorana neutrino by adding eq.(5) and eq.(6) and identifying $\hat{\Psi}_M$ in the sum according to eq.(4):

$$i\gamma^\mu \left( \frac{\hbar}{2} \frac{\partial}{\partial x^\mu} + \frac{ie}{2 \cos(\theta_w) \sin(\theta_w)c} Z_\mu (g_\nu^V \gamma^5) \right) \hat{\Psi}_M^\nu - m_\nu c \hat{\Psi}_M^\nu = 0$$

This equation of motion is equivalent to the one for a Dirac neutrino (eq.(1)) with $g_\nu^V = 0$ (vanishing of neutrino vector coupling). The fact that “Majorana projections” (eq.(4)) only “persist in time” (i.e. fulfill equation (5) for all t) if they do not couple via vector interactions was already pointed out immediately after Majorana’s original work by Furry[8]. He also noted that scalar interactions are possible for Majorana neutrinos. We now recognize

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\[2\] In order to avoid confusion I use Sakurai’s symbol “$S_C$” for the charge conjugation matrix in (eq.(3)) rather than the more usual “$C$”. Many authors define $C = S_C \gamma^0$[3, 5] while others use the notation $C = S_C[1]$. In spite of these differences in notation the definition of a “Majorana particle” (eq.(3)) is unequivocal.

\[3\] Racah writes (my translation from the Italian)[7]: “The imposition of real valuedness on the neutrino wavefunction ... is a logical consequence of the hypothesized physical identity of neutrinos and antineutrinos.”
that axial coupling (not mentioned by Furry) is also allowed. From the study of the phenomenology of supersymmetric particles it is already known that vector couplings have to be absent in general for all Majorana fields $\lambda$ i.e.:

$$\bar{\lambda} \gamma^\mu \lambda = 0$$

3 The experimental data on neutral-current elastic neutrino-electron scattering

Recent experiments on the neutral-current coupling of neutrinos show that eq. (6) does not properly describe the observed neutrinos. In its experiment on the purely neutral-current scattering of muon neutrinos on electrons the Charm II collaboration found for the effective neutral-current coupling constant [10]:

$$g_{\nu e}^{\nu e} = -0.035 \pm 0.017 \text{ (combined statistical and systematical error)}$$

The effective coupling constant is given as [10]:

$$g_{\nu e}^{\nu e} = 2 g_{\nu \mu}^{\nu \mu} \cdot g_{e}^{e}$$

where $g_{e}^{e}$ is the vector-coupling constant of the electron to the $Z_{\mu}$ field. For Majorana neutrinos from eq.(8) and $g_{\nu \mu}^{\nu \mu} = 0$ (eq.(3)) we would expect $g_{\nu e}^{\nu e} = 0$ for Majorana neutrinos which is more than two sigmas away from the measured value. The measured value for $g_{\nu e}^{\nu e}$ is in excellent agreement with the assumption of standard model values for the vector coupling constant of the electron ($g_{e}^{e} = -0.037 \pm 0.0006$ [11]) and a Dirac neutrino ($g_{\nu \mu}^{\nu \mu} = 1/2$ [4]). For the neutral-current effective axial coupling Charm II found $g_{A}^{\nu e} = -0.503 \pm 0.017$ which is consistent with the standard model expectation ($g_{A}^{\nu e} = -0.507 \pm 0.0004$ [11]) for both eq.(4) and eq.(6) [1]. The result for vector coupling disfavors the identification of the muon neutrino (and by analogy also the other neutrino flavors) as a Majorana particle at the > 95 % confidence level.

That the observed neutrinos are not Majorana particles, is not in conflict with previous work on Majorana neutrinos (masses, mixing, see-saw etc.). These ideas could still apply either to a small admixture to the known neutrinos or a new species of neutrino (for example a heavy fourth generation neutrino [12]). Imposing the “Majorana supplementary condition” is quite reasonable and can be physically “explained” e.g. by a “see-saw” mechanism. It necessarily leads to particles with no vector coupling, however. The properties of Majorana neutrinos thus remain a fascinating topic for further research.

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For a more detailed explanation of this equation and the conclusion of $g_{\nu \mu}^{\nu \mu} \neq 0$ from the Charm II data, see the second part of appendix 4.

The method to select the quoted solution for their result used by the Charm collaboration (based on $e^+e^-$ data) has to be disregarded in our case, because it implicitly assumes standard model values for the coupling constants of the neutrino. The other three possible solutions in $g_{\nu e}^{\nu e}$, $g_{A}^{\nu e}$ for the neutrino scattering results found by the collaboration are, however, also in disagreement with the values expected for Majorana neutrinos.
4 Criticism of the Kayser/Shrock argument on the vector coupling in the neutral current

The fact that the vector part of the neutrino current vanishes for Majorana neutrinos, thus leading to a different neutral-current scattering cross section for Majorana as compared to Dirac neutrinos, had already been clearly stated by Kayser and Shrock [13, 14], who drew a different conclusion than the present paper, though. Their argument can be summarized as follows:

“In spite of the absence of vector coupling in the interaction Lagrangian for Majorana neutrinos the vector interaction “reappears” because the “empirically observed” highly relativistic neutrino is a “left-handed” state. The neutrino spinor can thus be multiplied by a “state preparation factor” $P_L = (1-\gamma^5)/2$ without changing it:

$$\hat{\Psi}_{L\nu} = P_L \hat{\Psi}_{L\nu} \quad (9)$$

If one performs this substitution for $\hat{\Psi}_{M\nu}$ in the axial interaction term of eq.(6) the vector part of the interaction is recovered. Therefore Majorana and Dirac neutrinos have the same neutral-current interaction in principle.” (end of my summary of the Kayser/Shrock argument).

Though formally correct, there must be some logical fallacy in this reasoning: one finds that a given special state of the neutrino (namely a chiral left-handed one, i.e. with chirality=$-1$) leads to vector parts in the interaction Lagrangian in direct contradiction with the original eq.(6) and a general theorem of Majorana fermions (eq.(7)). The conclusion can then only be that this state (whether experimentally observed or not) cannot occur for Majorana fermions.

States of chirality=$-1$ are indeed forbidden for Majorana neutrinos: charge conjugation as defined by eq.(3) turns chiral left-handed states into right-handed ones, which is in contradiction with the mathematical identity required by eq.(7) for Majorana fermions (see appendix 2 for a more detailed discussion).

States of helicity=$-1$ are not necessarily in contradiction with eq.(3) (appendix 3). It is therefore not possible to exclude an identification of the observed neutrinos as Majorana fermions merely by way of their empirically proven “left-handedness”. However, Majorana states with helicity=$-1$ cannot fulfill eq.(9) (appendix 3), as erroneously assumed in the argument of Ref.[13].

5 The distinction between Lee-Yang and Majorana fields for vanishing rest-mass

There is a widely held conviction that the Lee-Yang two-component neutrino theory is equivalent to the Majorana abbreviation for the case of $m_\nu=0$ ("Dirac-Majorana Confusion
Theorem” \cite{13, 14}. I disagree in the following sense: the Lee-Yang neutrino \cite{16} (i.e. a massless Dirac neutrino interacting via V-A coupling) and the Majorana neutrino are both “two-component neutrinos”. In spite of this fact these cases are physically distinguishable because eq.\cite{11} and eq.\cite{12} remain different also for the case $m_\nu=0$, due to the presence of vector coupling in eq.\cite{11}. These two possibilities for “two-component” neutrinos are now examined in further detail.

- In the Weyl representation (denoted by the superscript “W”) eq.\cite{11} can be written as the following system of two equations \cite{5}:

$$i\hbar \left( \frac{\partial}{\partial x^0} + \frac{ik(g_\nu^V - g_\Lambda^A)}{\hbar c} Z_0 - \vec{\sigma} \cdot \nabla - \frac{ik(g_\nu^V - g_\Lambda^A)}{\hbar c} \vec{\sigma} \cdot \vec{Z} \right) \Psi^W_R - m_\nu c \Psi^W_L = 0$$

$$i\hbar \left( \frac{\partial}{\partial x^0} - \frac{ik(g_\nu^V + g_\Lambda^A)}{\hbar c} Z_0 + \vec{\sigma} \cdot \nabla + \frac{ik(g_\nu^V + g_\Lambda^A)}{\hbar c} \vec{\sigma} \cdot \vec{Z} \right) \Psi^W_L - m_\nu c \Psi^W_R = 0$$

where $k=e/(2 \cos(\theta_w) \sin(\theta_w))$ and $\vec{\sigma}$ is the 3-vector of the Pauli matrices in standard form. The Lee-Yang neutrino (a special case of Weyl’s massless two-component fermion \cite{17}), can be described by the equations \cite{11} for the case of vanishing rest mass $m_\nu$. In this case the two equations decouple and the observed neutrinos can be fully described by the chiral left handed field $\Psi^W_L$ fulfilling the upper equation of \cite{11}. $\Psi^W_L$ is a complex valued two-component spinor. $\Psi^W_R$ does not interact in the standard model because $g_\nu^V = g_\Lambda^A$. As already noted $\Psi^W_L$ cannot describe a Majorana particle because it is distinguishable from its charge conjugate \cite{6}. This neutrino can obviously couple vectorially without becoming a four-component neutrino, but a finite rest mass makes such a description unavoidable.

- Using the Majorana representation we can write in general the real and imaginary components of the Dirac equation separately, in a way analogous to eq.\cite{11}:

$$i\gamma^\mu \left( \hbar \frac{\partial}{\partial x^\mu} + \frac{ik g_\nu^V}{c} Z_\mu \gamma^5 \right) \Psi^R + \frac{k g_\nu^V}{c} Z_\mu \Psi^I = m_\nu c \Psi^R$$

$$i\gamma^\mu \left( \hbar \frac{\partial}{\partial x^\mu} + \frac{ik g_\nu^V}{c} Z_\mu \gamma^5 \right) \Psi^I - \frac{k g_\nu^V}{c} Z_\mu \Psi^R = m_\nu c \Psi^I$$

here $\Psi = \Psi^R + i\Psi^I$. $\Psi^R$, $\Psi^I$ are independent hermitian operators (see appendix 1). The Majorana neutrino is described by equations \cite{11} for the case of vanishing vector coupling $g_\nu^V$. Only in this case (and not for $m_\nu=0$) the two equations decouple, and neutrinos can be fully described by the real part of the field $\Psi^R$ fulfilling the upper equation of \cite{11}. $\Psi^R$ is a bispinor with four real valued components, which is equivalent in number of independent components to the two complex components of the Lee-Yang case (this is the sense in which it is also a “two-component” neutrino). This real valued field can obviously have a non-vanishing rest mass $m_\nu$ (then called “Majorana mass”) without becoming a four-component neutrino. This Majorana mass might well be very different from the mass in the lower eq.\cite{11}.

\footnote{This fact is clearly stated in the original Lee and Yang paper on their two-component neutrino\cite{14}: “In this theory it is clear that the neutrino state and the antineutrino state cannot be the same. A Majorana theory for such a neutrino is therefore impossible.”}
A description with Weyl spinors (eq.(10)) is indeed physically equivalent to a description in the Majorana representation (eq.(11)) according to Pauli’s fundamental theorem. However it is only the “choice” of the upper equation in eq.(10) which defines the Lee-Yang neutrino. This requirement that $\hat{\Psi}_L$ and its charge conjugate alone describe the observed neutrinos is incompatible with a description as a Majorana neutrino also when $m_\nu=0$ [18] (see also the appendix 2).

This paper solves a problem in neutrino physics but the solution deepens the puzzle of the small neutrino masses.

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6 Appendices

6.1 The mathematical characterization of Majorana fields as real valued fields

Let us clarify the exact mathematical meaning of the well known “real-valuedness of the wavefunction” [7] as a defining property for Majorana particles in a field theoretical context. The understanding of the Majorana field as a field which is hermitian in the Majorana representation is crucial for the understanding of the fundamental difference between Lee-Yang and Majorana particles (i.e. the difference between eq.(10) and eq.(11)).

The most general solution of the Dirac equation in the Majorana representation can be written as a complete set of plane-wave states (see [4] eq.(4.51)):

$$\hat{\Psi}(\vec{x}, t) = \sum_{\vec{k}} \frac{m}{E(2\pi)^3} \left( \hat{b}(\vec{k}) u_r(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} + \hat{d}^\dagger(\vec{k}) u^*_r(\vec{k}) e^{i\vec{k}\cdot\vec{x}} \right)$$  \hspace{1cm} (12)

Here $\hat{b}$ and $\hat{d}$ are particle and antiparticle creation operator and are given as [3]:

$$\hat{b} = \frac{1}{\sqrt{2}}(\hat{a}_1 + i\hat{a}_2) \hspace{1cm} \hat{d} = \frac{1}{\sqrt{2}}(\hat{a}_1 - i\hat{a}_2)$$  \hspace{1cm} (13)

$\hat{a}_1$ and $\hat{a}_2$ are the annihilation operators for the Hermitian fields $\hat{\Psi}_1$ and $\hat{\Psi}_2$ (called $\hat{\Psi}_{Re}$ and $\hat{\Psi}_{Im}$ in eq.(11)) which are combined as $\hat{\Psi}_1 + i \hat{\Psi}_2$ to obtain the most general non-Hermitian field $\hat{\Psi}$. The bispinor $u_r$ with the 2 spin components $r$ is the usual positive energy solution of the Dirac equation. (the superscript W is a reminder that they are given in the Weyl representation). $\vec{k}$ is the four momentum, $m$ and $E$ the particle mass and energy respectively. The “supplementary condition” for a Majorana particle (i.e. eq.(2), which in the Majorana representation becomes : $\hat{\Psi}_M=\hat{\Psi}_M^T$) requires $\hat{b} = \hat{d}$. This means $a_2=0$, i.e. $\hat{\Psi}_M$ is hermitian and the annihilation and creation operators are real.
(but not hermitian!). The most general Majorana state can be written as:

\[
\hat{\Psi}(\vec{x}, t) = \sum_{r} \sqrt{\frac{m}{E(2\pi)^3}} \left( \hat{a}_1(k) u_r^M(k) e^{-ikx} + \hat{a}^\dagger_1(k) u_r^M(k) e^{ikx} \right)
\]

This is the exact sense of the “reality of the wavefunction” in the field case, in the c-number limit this leads to purely real wavefunctions.

6.2 Detailed analysis of the Kayser/Shrock argument on the neutral-current vector coupling

Here I present a detailed proof that chiral left-handed states of a quantum field (i.e. states of chirality=-1) necessarily violate the Majorana “supplementary condition” (eq.(2)). Therefore Majorana particles cannot fulfill the defining condition for negative chirality states (eq.(9)), thus withdrawing the basis from the Kayser/Shrock argument[13] about the “reappearance” of vector interactions in Majorana neutrino - electron scattering.

Expanding in plane waves like in eq.(12) we can write the most general state of negative chirality in the Weyl representation:

\[
\hat{\Psi}_W^L(\vec{x}, t) = \sum_k \sqrt{\frac{m}{E(2\pi)^3}} \left( \hat{b}(k) u^W_L(k) e^{-ikx} + \hat{d}^\dagger(k) v^W_L(k) e^{ikx} \right)
\]

here \(u^W_r\) and \(v^W_r\) are the usual positive and negative energy bispinors (as always the superscript W is a reminder that they are given in the Weyl representation). The bispinor \(u^W_L\) can by symbolized as \(\begin{pmatrix} 0 \\ \phi_L \end{pmatrix}\), using the Pauli two-component spinor \(\phi_L\). \(S_C\) as defined in eq.(3) is given as \(\begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}\) in the Weyl representation. Here \(\sigma_2\) is the usual Pauli matrix. A multiplication of \(\begin{pmatrix} 0 \\ \phi_L \end{pmatrix}\) with this matrix leads to \(\begin{pmatrix} i\sigma_2\phi_L \\ 0 \end{pmatrix}\) which is a chiral right-handed spinor. It can be shown[3]:

\[
S_C u^T_L = v_R; \quad S_C v^T_L = u_R
\]

Using eq.(16) to obtain \(\hat{\Psi}_W^L\) from eq.(15) it can be seen that \(\hat{\Psi}_L(\vec{x}, t) \neq \hat{\Psi}_W^L(\vec{x}, t)\) independent of the form of \(\hat{b}\), for each combination of individual k-components. This means that any field with purely negative chirality violates the Majorana condition eq.(2), or:

Majorana neutrinos cannot be in a state of pure chirality.

6.3 On the helicity of Majorana neutrinos

The result of the previous section does not mean that Majorana fermions cannot have a definite (e.g. left-handed) helicity! Remember that antiparticle states with chirality=1
have helicity=-1 [3]. Consider e.g. the following state which has a helicity of -1 in the ultra-relativistic limit (i.e. it is “left-handed”):

\[
\hat{\Psi}_{h=-1} = \sum_k \sqrt{\frac{m}{E(2\pi)^3}} \left( \hat{a}_1(k)u_L(k)e^{-ikx} + \hat{a}_1^\dagger(k)v_R(k)e^{ikx} \right)
\]

This state fulfills the Majorana condition eq.(13). It describes “a particle identical to itself” with left handed helicity, and has all the properties that are attributed to Majorana neutrinos in the standard textbooks[14]. It has no negative chirality however because:

\[
\frac{(1 - \gamma^5)}{2} \hat{\Psi}_{h=-1} = \sum_k \sqrt{\frac{m}{E(2\pi)^3}} \left( \hat{b}(k)u_L(k)e^{-ikx} \right) \neq \hat{\Psi}_{h=-1}
\]

Kayser and Shrock overlooked this possibility, and erroneously concluded the general validity of eq.(11) merely from the fact that a state has helicity=-1.

The importance of the Charm II result is, that by proving that the neutrino-electron interaction has properties which are directly incompatible with the Majorana nature of the neutrino field, it provides firm evidence that the original Lee-Yang theory, rather than some slight modification like eq.(17) describes the physical muon neutrino.

### 6.4 Reply to Comments on a previous version of the present paper

Finally I answer to two comments [19, 20] on a previous version of the present paper. Hannestad[19] accepts my argument against Majorana neutrinos of pure chirality for “fields”. He then argues however that “states”, which are defined by the action of a creation operator \(a^\dagger\) on the vacuum, can be of pure chirality. This is impossible as I now show. A chiral left-handed state can be created from the vacuum state \(|0\rangle\) via:

\[
\hat{\Psi}^W_L |0\rangle = \sum_k \sqrt{\frac{m}{E(2\pi)^3}} d^\dagger v^W_L |0\rangle = |1\rangle_L
\]

For the charge conjugated state we have:

\[
|1\rangle = (\hat{\Psi}^W_L |0\rangle)^c = \hat{\Psi}^W_{Lc} |0\rangle = \sum_k \sqrt{\frac{m}{E(2\pi)^3}} b^\dagger v^W_R |0\rangle \neq |1\rangle_L
\]

The last inequality holds also for the case of a neutral particle with \(b=d\). I made the reasonable assumption that \(|0\rangle = |0\rangle^c\) (to drop this assumption would not invalidate the conclusion). Hannestad’s further discussion is similar to the one of Kayser and Shrock. In fairness I have to say that in the previous version of this paper which Hannestad criticises I stated “the Majorana neutrino has to be unpolarized”, rather than the present more concise statement “the Majorana neutrino cannot have a definite chirality”.

Kayser’s recent report [20] mainly repeats his arguments from Ref.[13, 14] in a slightly different form (as he acknowledges in his Ref.[3]). E.g. the transition like the one from his Eq.(1) to eqs.(2) is clearly only possible under the assumption of eq.(1) in my manuscript,
Further Kayser points out correctly that the Charm II collaboration did not attempt to evaluate including neutrinos has a vector and axial coupling constant $g$ Since a fit to the Charm II data to eq.(22) leads to the significant conclusion of the Charm II collaboration on neutral current reactions. This equation is indeed not a general theoretical relation but is justified in the context of the Charm II experiment. In the usual form of the Lagrangian for the standard model each fermion field $\Psi_i$ (i.e. including neutrinos) has a vector and axial coupling constant $g_V^i$ and $g_A^i$ (see e.g. eq.(10.1) in Ref.[11]). For an incident neutrino energy $E_\nu \gg m_e$ the neutral-current cross section for elastic scattering of muon neutrinos on electrons can then be determined from the standard model Lagrangian for the special case of “four-fermion” problems at center of mass energies far below the W,Z masses, as:

$$\frac{d\sigma}{dy_{\nu,\bar{\nu}}} = \frac{G_F^2 m_e E_\nu}{\pi} \left( \left(g_V^{\nu e} + g_A^{\nu e}\right)^2 - \left(g_V^\mu + g_A^\mu\right)^2 \right)^2 \pm 4 g_V^{\nu e} g_A^{\nu e} g_V^\mu g_A^\mu \left(1 - (1 - y)^2\right)$$

Here and in the following equation the upper sign is valid for the neutrino, and the lower sign for the antineutrino cross section. $y \equiv \frac{E_y}{E_\nu}$ is the ratio of the kinetic energy of the recoil electron and the incident $\nu$ or $\bar{\nu}$ energy. Eq.(21) is similar and closely related to the expressions for the forward-backward asymmetry in the reaction $e^+e^- \to \ell^+\ell^-$ (eq. (10.26) of Ref.[11]). The Charm II collaboration used a simplified expression (eq.(10.17) of Ref.[11]) to fit their data, which can be written in the following form:

$$\frac{d\sigma}{dy_{\nu,\bar{\nu}}} = \frac{G_F^2 m_e E_\nu}{2\pi} \left( (g_V^{\nu e2} + g_A^{\nu e2})(1 + (1 - y)^2) \pm 2 g_V^{\nu e} g_V^\mu \right)$$

Here “$g_V^{\nu e}$” and “$g_A^{\nu e}$” are understood as coefficients of effective four-fermion operators. Eq.(22) follows from eq.(21) if $g_V^{\nu e} = 2g_V^{\nu e}g_V^\mu$ and $g_A^{\nu e} = 2g_A^{\nu e}g_A^\mu$ with $g_V^{\nu e}=g_A^{\nu e}=1/2$. Since a fit to the Charm II data to eq.(22) leads to the significant conclusion $g_V^{\nu e} \neq 0$ and $g_A^{\nu e} \neq 0$(see section 3), it follows from eq.(22) that $g_V^{\nu e} \neq 0$.

That the Charm II neutral-current data imply the existence of neutrino vector coupling can be seen in a very direct way from eq.(21): Charm II found a small (3.6 %) but significant (2.1 $\sigma$) difference between the total elastic scattering cross section in the $\nu_\mu$-e and $\bar{\nu}_\mu$-e case. According to eq.(21) this is only possible if $g_V^{\nu e} \neq 0$.

Further Kayser points out correctly that the Charm II collaboration did not attempt to evaluate $g_V^{\nu e}$ and $g_A^{\nu e}$ individually in Ref.[21]. If one takes the values for $g_V^e$ and $g_A^e$ e.g. from $e^+ - e^-$ experiments, it is clearly possible in principle to obtain experimental values for $g_V^{\nu e}$ and $g_A^{\nu e}$ individually from neutral-current scattering data, using eq.(21) (except for a sign and exchange ambiguity which already occurs in the $g_V^{\nu e}$ case). However, taking into account the limited precision of the Charm II data it was a reasonable strategy to set experimental limits only on a “global” neutrino coupling $g$ (which assumes $g_V^{\nu e}=g_A^{\nu e}$, but no specific absolute value) rather than then vector and axial constants individually.
References

[1] E.Majorana, Nuovo Cimento, Ser.8 14,171 (1937).

[2] S.M.Bilenky, in Proceedings of the II$^{nd}$ Rencontres de Vietnam, “At the Frontiers of the Standard Model”, October 1995, lanl server hep-ph/9601266 (1996).

[3] J.D.Bjorken and S.Drell, Relativistic quantum fields (Mc Graw-Hill, New York, 1965).

[4] F.Mandl and G.Shaw, Quantum Field Theory (John Wiley, Chichester, 1984).

[5] J.J.Sakurai, Advanced Quantum Mechanics (Addison-Wesley, Reading, 1967) appendix C (Pauli’s fundamental theorem), p.140-143(charge conjugation), p.134 (spin,momentum and velocity), p.174(CPT).

[6] W.Pauli, Rev.Mod.Phys. 13,203 (1941).

[7] G.Racah, Nuovo Cimento, Ser.8 14,322 (1937).

[8] W.H.Furry, Phys.Rev. 54,56 (1938).

[9] H.E.Haber and G.L.Kane, Phys.Rep.117,75 (1985); appendix D.

[10] P.Vilain et al. (Charm II collaboration), Phys.Lett.B335,246 (1994).

[11] P.M.Barnett et al., Review of Particle Physics, Phys.Rev.D54,1 (1996).

[12] E.W.Kolb and K.A.Olive, Phys.Rev. D33, 1202 (1986).

[13] B.Kayser and R.E.Shrock, Phys.Lett.112B,137 (1982).

[14] B.Kayser,F.Gibrat-Debu and F.Perrier, The Physics of Massive Neutrinos (World Scientific, Singapore, 1989).

[15] J.A.McLennan, Phys.Rev.106,821 (1957); K.M.Case, Phys.Rev.107,307 (1957).

[16] T.D.Lee and C.N.Yang, Phys.Rev.105,1671 (1957).

[17] H.Weyl, ZS.f.Phys.56,330 (1929).

[18] L.A.Radicati and B.Touschek, Nuovo Cimento, Ser.10 5,1693 (1957).

[19] S.Hannestad, Report no. hep-ph/9701216.

[20] B.Kayser, Report no. hep-ph/9703294, NSF-PT-97-1.

[21] P.Vilain et al. (Charm II collaboration), Phys.Lett. B 320,203 (1994).