Study on River Water Environmental Capacity under Unascertained Information

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Abstract. Based on lack of hydrological and water quality data, incomplete information in many rivers, the unascertained mathematics is used to calculate river water environmental capacity. This model can provide not only the interval value of river water environmental capacity and its corresponding confidence level, but also the certain value under unascertained information. It presents more useful and scientific information for water pollution prevention. This case study shows that it is feasible, simple, reliable and of good popularization value.

1. Introduction

The calculation of river water environmental capacity is an important work to implement the water resources management system. Water environmental capacity in the current Code of Practice for Computation on Allowable Permitted Assimilative Capacity of Water Bodies (GB/T 25173-2010) is defined as the maximum quantity of a pollutant to meet the requirements of water quality target under the designed hydrological conditions. The water environmental capacity defined in this way is a certain value corresponding to the designed hydrological condition, not the actual water pollution carrying capacity. There are some uncertainties in the information of discharge, velocity, pollutant concentration and degrading coefficient of pollutant. These uncertainties include objective factors such as incomplete data, and subjective factors such as incomplete understanding of their internal laws. Because we cannot fully grasp the real state of things cause purely subjective uncertainty known as "unascertained information" [1,2]. At present, most of the calculation of river water environmental capacity is under certain information [3-6], and there are few studies on the uncertainty of information [7]. In this paper, the unascertained mathematics operation is used to deal with the unascertained information, which provides a reference for the calculation of the river water environmental capacity.

2. Unascertained Number Theory

2.1. Definition of Unascertained Number

Suppose \( A = \{(x_1, x_2), \emptyset(x)\} \) denotes an unascertained number, where

\[
\emptyset(x) = \begin{cases} 
\alpha_i, & x = x_i (i = 1, 2, \ldots, n) \\
0, & \text{otherwise}
\end{cases}
\]

and \( 0 < \sum_{i=1}^{n} \alpha_i = \alpha \leq 1, 0 < \alpha_i \leq 1, i = 1, 2, \ldots, n \)
If \( F(x) = \begin{cases} 0, & x < x_1 \\ \alpha_1 + \alpha_2 + \cdots + \alpha_n, & x_1 \leq x \leq x_{i+1}(i = 1, 2, \ldots, n-1) \end{cases} \), then it is called an interval \( \langle x_1, x_n \rangle \) distribution type unascertain number with function \( F(x) \). Where, \( \alpha \) is the overall confidence level, \( \langle x_1, x_n \rangle \) is the distribution interval, and \( F(x) \) is the reliability distribution function [8].

### 2.2. Unascertained Number Operation

Suppose \( \{x_i\}(i = 1, 2, \ldots, n) \), \( \{y_j\}(j = 1, 2, \ldots, m) \) are the sequence of possible values of unascertain number A and B respectively, and \( f(x_i)(i = 1, 2, \ldots, n) \), \( g(y_j)(j = 1, 2, \ldots, m) \) are the sequence of confidence level of A and B respectively. Then the matrix

\[
\begin{bmatrix}
{x_1 + y_1} & {x_1 + y_2} & \cdots & {x_1 + y_m} \\
{x_2 + y_1} & {x_2 + y_2} & \cdots & {x_2 + y_m} \\
\vdots & \vdots & \ddots & \vdots \\
{x_n + y_1} & {x_n + y_2} & \cdots & {x_n + y_m}
\end{bmatrix}
\]

is called the possible sum matrix A and B. And the matrix

\[
\begin{bmatrix}
f(x_1)g(y_1) & f(x_1)g(y_2) & \cdots & f(x_1)g(y_m) \\
f(x_2)g(y_1) & f(x_2)g(y_2) & \cdots & f(x_2)g(y_m) \\
\vdots & \vdots & \ddots & \vdots \\
f(x_n)g(y_1) & f(x_n)g(y_2) & \cdots & f(x_n)g(y_m)
\end{bmatrix}
\]

is called the confidence level multiplication matrix A and B. Where, \( (x_i + y_j) \) in the possible sum matrix A and B corresponds to \( (f(x_i)g(y_j)) \) in the confidence level multiplication matrix A and B. The elements in the possible sum matrix A and B are arranged in order from smallest to largest \( z_1, z_2, \ldots, z_k \), and one for the same element. The corresponding element of \( z_i(1, 2, \ldots, k) \) in the confidence level multiplication matrix A and B is \( r_1, r_2, \ldots, r_k \). If \( z_i \) has \( l \) different positions in the possible sum matrix A and B, then \( r_1 \) is the sum of the corresponding elements of \( l \) identical elements.

Suppose \( \varnothing(z) = \begin{cases} r_i, & z = z_i(i = 1, 2, \ldots, k) \\ 0, & \text{others} \end{cases} \), then the unascertained number \( [(z_1, z_i), \varnothing(z)] \) is the sum of A and B, which is recorded as A+B.

Changing “+” in A+B to “-”, “×” and “÷” in the operation can be changed into possible value subtraction matrix, possible value multiplication matrix and possible value division matrix, and everything else remains the same. For “÷”, \( y_j \) \( (j = 1, 2, \ldots, m) \) cannot be 0 [9,10].

### 3. The Establishment of Unascertained Mathematical Model of River Water Environmental Capacity

In this study, a one-dimensional river model with uniform mixing of pollutants in cross section is taken as an example. The formula for calculating river water environmental capacity can be denoted as follows:

\[
M = [C_s - C_{s0}\exp(-K\frac{L}{u})]Q
\]

where, \( M \) is the river water environmental capacity, \( g/s \); \( C_s \) is the water quality object, \( mg/L \); \( C_{s0} \) is the pollutant concentration of initial section, \( mg/L \); \( K \) is the degrading coefficient of pollutant, \( 1/s \); \( L \) is the longitudinal distance along the river, \( m \); \( Q \) is the inflow of initial section, \( m^3/s \); \( u \) is the average velocity of river section under design discharge.

Suppose \( Q \) and \( u \) denote unascertained numbers, according to the definition of unascertain number, formula (1) can be expressed as:
where, \( C_0, K, L \) are real numbers.

Because of the introduction of unascertained number, the \( M \) is no longer a certain value. Here, formula (2) is the unascertained mathematical model of one-dimensional river water environmental capacity. In the same way, this method not only can be extended to other parameters of one-dimensional mathematical model of river water environmental capacity, such as \( C_0 \) and \( K \), but also can be extended to the zero-dimensional model and two-dimensional model commonly used in the calculation of river water environmental capacity.

The unascertained number \( M = [(x_1, x_n), \varphi(x)] \) of river water environmental capacity can be obtained by unascertained number operation. Where, \( \varphi(x) = \{ \alpha_i, x = x_i (i = 1, 2, \ldots, n) \}, \)

\[ 0 < \sum_{i=1}^{n} \alpha_i = a \leq 1, \quad 0 < \alpha_i \leq 1, \quad i = 1, 2, \ldots, n, \]

and the reliability distribution function \( F(x) \).

If the certain value of river water environmental capacity under unascertained information is required, the following formula can be considered:

\[
\bar{M} = \frac{1}{a} \sum_{i=1}^{n} \alpha_i x_i
\]

### 4. Applications

#### 4.1. Basic Data of River

A river \( L = 10 \text{km}, C_s = 20 \text{mg/L}, C_c = 10 \text{mg/L}, K = 1.0 \times 10^{-6}/\text{s} \). Through the hydrologic data statistics of this river, the unascertained information of \( L \) and \( C \) as follows:

\( Q = \{(40, 150), \varphi(x)\} \), where, \( Q(x) = \begin{cases} 0, & \text{if } Q < 40 \\ 0.2, & \text{if } 40 \leq Q < 80 \\ 0.8, & \text{if } 80 \leq Q < 150 \\ 1, & \text{if } Q \geq 150 \end{cases} \)

\( u = \{(0.1, 0.4), \varphi(x)\} \), where, \( u(x) = \begin{cases} 0, & \text{if } u < 0.1 \\ 0.3, & \text{if } 0.1 \leq u < 0.2 \\ 0.9, & \text{if } 0.2 \leq u < 0.4 \\ 1, & \text{if } u \geq 0.4 \end{cases} \)

#### 4.2. Calculation of River Water Environmental Capacity and Corresponding Reliability

The formula for calculating river water environmental capacity can be denoted as follows:

\[
M = \left[ \frac{C_s - C_0 \exp \left(-K \frac{L}{u}\right)}{u} \right] Q = QC_s - QC_0 \exp \left(-K \frac{L}{u}\right)
\]

The formula (4) can be formally changed as follows:

\( M = A \times B \) where, \( A = C_s - C_0 \exp \left(-K \frac{L}{u}\right) \), \( B = Q \). Or \( M = C - D \) where, \( C = QC_s \), \( D = QC_0 \exp \left(-K \frac{L}{u}\right) \).

If \( A, B, C \) and \( D \) are all real numbers, there is no problem. When \( A, B, C \) and \( D \) are unascertained numbers, the results calculated by the two methods are different. The reason is that both \( C \) and \( D \) contain unascertained number \( Q \), that is, \( C \) and \( D \) are related and can not be regarded as independent unascertained numbers.

Therefore, \( M = A \times B \) where, \( A = C_s - C_0 \exp \left(-K \frac{L}{u}\right) \), \( B = Q \).
\[ \exp\left(-\frac{0.01}{u}\right) = \exp\left(-\frac{0.01}{\ln[10.04]u(x)}\right) = \{(0.9048, 0.9753), G(y)\}, \quad \text{where} \]

\[ G(y) = \begin{cases} 
0, & y < 0.9048 \\
0.3, & 0.9048 \leq y < 0.9512 \\
0.9, & 0.9512 \leq y < 0.9753 \\
1, & y \geq 0.9753 
\end{cases} \]

\[ A = \{(10.247, 10.488), f(x)\}, \quad \text{where} \quad f(x) = \begin{cases} 
0, & x < 10.247 \\
0.3, & 10.247 \leq x < 10.488 \\
0.9, & 10.488 \leq x < 10.952 \\
1, & x \geq 10.952 
\end{cases} \]

Sum matrix of possible value and product matrix of possible value of A and B are shown in table 1 and table 2.

**Table 1.** Sum matrix of possible value of A and B.

|          | 10.247 | 10.488 | 10.952 | 10.247 | 10.488 | 10.952 |
|----------|--------|--------|--------|--------|--------|--------|
| 40       | 40     | 40     | 40     | 1537.05| 1573.2 | 1642.8 |

**Table 2.** Product matrix of possible value of A and B.

|          | 0.3   | 0.6   | 1      | 0.1   | 0.2   | 0.6   | 2      |
|----------|-------|-------|--------|-------|-------|-------|--------|
| 0.06     | 0.06  | 0.06  | 0.02   | 0.06  | 0.06  | 0.02  | 0.2    |

The elements in the possible sum matrix A and B are arranged in order from smallest to largest and its corresponding element in the confidence level multiplication matrix A and B are obtained \{y_{1}, y_{2}, \ldots, y_{n}\}.

\[ A \times B = \{(409.88, 1642.8), \Phi(z)\} = \{(409.88, 1642.8), F(z)\} \]

The calculated values of \(\Phi(z)\) and \(F(z)\) are shown in table 3. The possible values of this river water environmental capacity and the corresponding credibility distributions are obtained. The certain value of this river water environmental capacity under unascertained information: \(M = \frac{1}{\alpha} \sum_{i=1}^{n} \alpha_{i} x_{i} = 899.74\) (g/s).

**Table 3.** Possible value and its corresponding faith degree of water environmental capacity.

| Range of river water environmental capacity (g/s) | Confidence interval density \(\Phi(z)\) | Credibility distribution \(F(z)\) |
|-------------------------------------------------|----------------------------------------|----------------------------------|
| \(Z<409.88\)                                   | 0                                      | 0                                |
| \(409.88 \leq z < 419.52\)                    | 0.06                                   | 0.06                             |
| \(419.52 \leq z < 438.08\)                    | 0.12                                   | 0.18                             |
| \(438.08 \leq z < 819.76\)                    | 0.02                                   | 0.20                             |
| \(819.76 \leq z < 839.04\)                    | 0.18                                   | 0.38                             |
| \(839.04 \leq z < 876.16\)                    | 0.36                                   | 0.74                             |
| \(876.16 \leq z < 1537.05\)                   | 0.06                                   | 0.80                             |
| \(1537.05 \leq z < 1573.2\)                   | 0.06                                   | 0.86                             |
| \(1573.2 \leq z < 1642.8\)                    | 0.12                                   | 0.98                             |
| \(Z \geq 1642.8\)                             | 0.02                                   | 1                                |
4.3. Result Analysis

As can be seen from table 3, the calculated values of this river water environmental capacity mainly appear in the range [409.88, 1642.8]. The subjective probability is the highest in the range [839.04, 876.16], and its corresponding confidence level of the range is 36%. The next is in the range [819.76, 839.04], and its corresponding confidence level of the range is 18%. It can be seen that the unascertained number can not only get the interval value of river water environmental capacity, but also get the corresponding subjective credibility. At the same time, the certain value of this river water environmental capacity under unascertained information can also be obtained. This can provide decision makers with richer and more accurate information.

At present, many rivers in China have less hydrological and water quality data and incomplete information, which provides a broad development space for the application of unascertained number in this field.

5. Conclusion

(1) This case study shows that it is feasible in theory and reliable in result to calculate river water environmental capacity under unascertained information. It provides a new way for calculating river water environmental capacity.

(2) This model can provide not only the interval value of river water environmental capacity and its corresponding confidence level, but also the certain value under unascertained information. It presents more useful and scientific information for water pollution prevention.

(3) In the calculation of river water environmental capacity under unascertained information, the independence of unascertained number should be paid attention to avoid being related to the same unascertained number.

(4) The unascertained mathematical model of river water environmental capacity is clear in theory, convenient in operation and simple in calculation. It has practical value and wide application prospect in water environment protection.

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