**Abstract**  The different flow regimes of two-phase gas-in-liquid flow—as it occurs in the shallow conduit of basaltic volcanoes—are characterized by distinct frequency-size distributions of the liquid and gaseous slugs. Assuming that the ascent of gaseous bubbles is indicated by seismic events, we explore the possibility of inferring the flow regime from the frequency distributions of magnitudes and interevent times. Our data set consists of 20,000 volcanic seismic events recorded in early March 2012 at Villarrica Volcano (Chile), which are commonly attributed to Strombolian activity. One crucial factor is the completeness of the catalog in terms of detectable amplitudes, which we assess using a stochastic simulation of the network output based on statistical properties of the ambient seismicity. Magnitudes, at which we consider the catalog complete, show an exponential occurrence. Yet, the simulation approach indicates that low magnitude events occur indeed more sparsely than expected for an exponential distribution, and that the magnitude distribution does not obey the Gutenberg-Richter law. Interevent times are log-normally distributed, which implies a preferred recurrence interval. The distributions are consistent with a slug flow regime. They correlate weakly with the preceding magnitude, suggesting that slugs coalesce while ascending.

**Plain Language Summary**  A common type of volcanic seismic events at Villarrica is thought to result from gas bubbles, which ascend through the liquid magma in the conduit and produce mild volcanic explosions at the surface. We investigated the statistical occurrence of the time between these events and of their size. In contrast to normal earthquakes, the events recur more periodically and the number of smaller events does not increase exponentially, hence confirming their different nature. Laboratory experiments showed that bubbles flowing in a liquid develop characteristic patterns of size and distance between them, which depend on the gas supply rate and the flow ability of the liquid. We qualitatively compared the statistical distributions of event sizes to bubble sizes and the time between the events to the distance between gas bubbles. The result suggests a flow pattern where the gas bubbles span the entire width of the conduit and are several times longer than wide. Such batches of gas are known to produce the typical volcanic activity of Villarrica.

1. **Introduction**

Magma flow and degassing in open-vent volcanoes are the result of a complex interaction between molten rock and the gas phase. When magma rises inside a conduit, gas bubbles start to nucleate once the lithostatic pressure is low enough and a separated two-phase flow develops consisting of gas bubbles in liquid magma (Burgisser & Degruyter, 2015; Jaupart & Vergniolle, 1988; Parfitt, 2004; Parfitt & Wilson, 1995; Pering et al., 2017; Seyfried & Freundt, 2000; Shinohara, 2008; Sparks, 1978; Spina et al., 2019). The presence of gas bubbles can be detected seismically, either when they pass an obstacle in the conduit or when they burst at the surface of the magma column, producing Strombolian activity (Chouet & Matoza, 2013; Harris & Ripepe, 2007a; Ishii et al., 2019; James et al., 2004, 2006, 2008; Neuberg et al., 1994; Ripepe et al., 2001; Spina et al., 2019). Due to the complex volcanic medium, however, it is not always possible to reliably distinguish explosion quakes from tectonically driven shear fractures (Chouet & Matoza, 2013; Wassermann, 2012). In this study, we show how statistical analysis of magnitudes and interevent time between seismic events can be used to discriminate fluid-dynamic and tectonic processes.
Laboratory experiments of vertical two-phase flow showed that the size distribution of gas bubbles depends on the flow regime (Barnea & Shémer, 1989; Barnea & Taitel, 1993; Dukler & Fabre, 1994; van Hout et al., 1992, 2001). In these experiments, tiny gas bubbles are released at the bottom of a liquid-filled pipe and their velocity and size during their ascent is measured. Depending upon the gas flow rate and viscosity of the liquid different flow patterns arise.

We use the term “bubble” generically for gas pockets of any size. A “slug” in contrast has approximately the same diameter as the pipe and may be longer than wide, in which case it is sometimes denoted as “Taylor bubble” (James et al., 2004; Llewellyn et al., 2011; Pering & McGonigle, 2018). Sometimes the liquid sections between the gas slugs are referred to as “liquid slugs” (Barnea & Shémer, 1989; Barnea & Taitel, 1993; James et al., 2004).

Figure 1 summarizes characteristics of the different flow regimes. Bubbles in bubbly flow are quite similar in size, resulting in a narrow unimodal distribution. Slug flow exhibits a bimodal distribution. The peak at larger sizes represents the gas slugs while the peak around smaller sizes originates from the small bubbles, which trail in the wake of the slug and are dispersed in the liquid (Barnea & Shémer, 1989). The lengths of the slugs and the liquid packages between them can both be modeled for example, by a log-normal distribution (van Hout et al., 1992, 2001). At the transition to churn flow, the distribution of liquid slug lengths becomes much more right-skewed (van Hout et al., 1992) while the peaks of the gas slug size distribution merge into a broad, unimodal distribution (Barnea & Shémer, 1989). In annular flow, the gas flows continuously through the annularly displaced liquid. The transition between the regimes occurs with increasing viscosity of the liquid, higher superficial gas velocities (volumetric gas flux divided by the cross-sectional area of the pipe) and increasing pipe diameter (Barnea & Shémer, 1989; Ohnuki & Akimoto, 2000; Spina et al., 2019; van Hout et al., 1992).

It is well-established that the different aspects of Strombolian activity are the result of differently sized gas releases (Gaudin, Taddeucci, Scarlato, Bello, et al., 2017; Jaupart & Vergniolle, 1988; Parfitt, 2004; Pering...
et al., 2017; Ripepe & Gordeev, 1999; Seyfried & Freundt, 2000; Spina et al., 2019). Puffing—the intermittent release of small amounts of gas—corresponds to relatively small bubbles, while explosions are produced by larger slugs. Sustained lava fountains result from temporary annular flow. While the dynamic and eruption process of single gas bubbles are reasonably well understood, the connection between the continual eruption process and the gas flow regime in the conduit is subject of current research (Ishii et al., 2019; Pering & McGonigle, 2018; Pioli et al., 2012; Spina et al., 2019). In this work, we characterize the occurrence of seismic transient events at Villarrica Volcano (Chile) in terms of the statistical distribution of interevent times and magnitudes. These transients are attributed to Strombolian activity (Calder et al., 2004; Gurioli et al., 2008; Lehr et al., 2019; Palma et al., 2008; Richardson et al., 2014). We compiled a catalog of these events using an automatic trigger algorithm from 12 days of continuous seismic records from two stations at the crater rim.

Statistical analysis of the interevent time distribution of seismic transients and their magnitudes might be a robust way of comparing seismic activity to theoretical flow models. Previous investigations on Strombolian explosions and associated seismic signals found some evidence for periodicity in their occurrence (Bell et al., 2017, 2018; Dominguez et al., 2016; Martino et al., 2012; Pering, Ilanko, & Liu, 2019; Pering et al., 2015; Ripepe & Gordeev, 1999; Ripepe et al., 2001, 2002; Taddeucci et al., 2013; Varley et al., 2006). This would be consistent with the idea of a slug flow regime. However, the validity of these results is sometimes questionable because the events might not have been detected correctly.

The incompleteness of seismic event catalogs toward low magnitudes is a well-known problem in tectonic seismology and a number of methods have been established to determine the magnitude of completeness of a catalog (Mignan & Woessner, 2012). However, most approaches assume that magnitudes follow the Gutenberg-Richter relation, which states that the number of events decreases exponentially with magnitude (Gutenberg & Richter, 1956). While this is a well-established relation for tectonic earthquakes, it might be unsuited to describe events, that are driven by magma dynamics in a volcano. In particular, assuming that the seismic events correspond to gas slugs and their amplitude corresponds to the size of the slugs, a log-normal distribution would be expected. To avoid the Gutenberg-Richter assumption, we applied a Monte-Carlo simulation to estimate the detection limit of our seismic stations (Ringdal, 1975; von Seegern & Blandford, 1976). This approach even allows the estimation of the true distribution of magnitudes.

The study is structured as follows: After introducing the volcano and the seismic network, we explain the event detection process and the determination of magnitudes, which provide the database for our analysis. Next, we estimate the magnitude of completeness. Subsequently, we analyze the statistical distributions of magnitudes and interevent times. The results are discussed in comparison to studies at other volcanoes and in the context of flow regime experiments.

2. Villarrica Volcano

2.1. Volcanology

Villarrica Volcano is a 2,851-m high, open-vent stratovolcano of basaltic to basaltic-andesitic composition in the Southern Chilean Andes. It is one of the most active volcanoes in Chile and one of the most dangerous ones due to its glacier coverage and nearby-living population of several thousand people (Global Volcanism Program, 2013; Ortiz et al., 2003; Parejas et al., 2010). The last major eruption occurred in 2015 and the eruptive phase is still ongoing (Johnson et al., 2018).

The central summit crater hosts an active lava lake of 30–60 m in diameter (Calder et al., 2004; Moussallam et al., 2016), usually with a free magma surface. The depth of the lake varies in irregular cycles between 50 and 250 m below the crater rim (Calder et al., 2004; Johnson et al., 2018; Palma et al., 2008; Richardson et al., 2014). The activity of the lake consists of vigorous magma convection, seething and splashing of magma, bubble bursting and occasionally lava fountains (Gurioli et al., 2008; Moussallam et al., 2016; Palma et al., 2008). Ejection of material is usually confined to the crater although more violent emissions of ash and tephra occur. Gurioli et al. (2008) reported an occurrence of gas release events (bubble bursting with and without material ejections) at an average rate of 9 events/min. The width of thrown magma sheets ranged between 2 and 17 m.
Villarrica is also known for its persistent degassing. SO$_2$ emission rates vary between 3 and 50 kg/s (Brede-meyer & Hansteen, 2014; Liu et al., 2019; Mather et al., 2004; Moussallam et al., 2016; Palma et al., 2008; Shinohara & Witter, 2005; Witter et al., 2004). Given the low ejection rate of magma compared to the degassed mass, a very efficient convection process through the conduit is postulated (Moussallam et al., 2016; Palma et al., 2011; Ripepe et al., 2010).

2.2. Seismic Activity

The seismic activity originates in the crater area and consists of a persistent seismic tremor, which is overlain by transient events at ∼1-min intervals (Calder et al., 2004; Gurioli et al., 2008; Lehr et al., 2019; Ortiz et al., 2003; Palma et al., 2008; Richardson & Waite, 2013; Ripepe et al., 2010). A similarly structured signal is observed in infrasound (Goto & Johnson, 2011; Richardson et al., 2014; Ripepe et al., 2010). Transients can be weak or absent during some periods. They are commonly attributed to the activity in the lava lake such as bubble and slug bursting, although reports are somewhat inconclusive. Palma et al. (2008) reported a good correlation between seismic transients and observed bubble bursting while Goto and Johnson (2011) failed to correlate infrasonic transients with lake activity—possibly due to generally calmer activity during their observation. Gurioli et al. (2008) also indicated a generally good correlation, but noted a lack of an infrasonic or seismic signal during some observed bursts. Using waveform inversion Richardson and Waite (2013) presented a horizontal, E-W aligned single force as source mechanism for a repetitive transient waveform. They interpreted their result as magma dragging at the lake bottom, while filling the void left by an escaping slug. Volcano-tectonic events were detected about 5 km east of the summit using the network of this study (Mora-Stock, 2015).

3. Data

The data were acquired by a dense local seismic network (Figure 2), installed from March 2 to 14, 2012 (Lehr et al., 2019; Mora-Stock, 2015; Rabbel & Thorwart, 2019). The stations were equipped with 3-component or 1-component 4.5-Hz geophones, and DSS-cubes, sampling at 100 Hz. We found that their data can reliably be recovered up to a tenth of the nominal frequency (0.45 Hz) by correcting for the instrument response (Lehr et al., 2019). We used the velocity data of the Z-component, resampled to 50 Hz.

The seismic signal throughout the network was dominated by a persistent unrest, which originated from the crater region (Lehr et al., 2019). The signal at the crater is characterized by transient increases in amplitude in ∼1-min intervals (Figure 2). These transients exhibit a variety of complex waveforms and are attributed to explosions, bubble bursting and seething of magma in the lava lake (Calder et al., 2004; Palma et al., 2008; Richardson & Waite, 2013). They last for a few seconds up to several tens of seconds. Many of them lack a clear onset. Our data generally resemble the samples shown and described by Palma et al. (2008).

Twelve days of data were recorded at the crater rim by two stations (KRA1, KRA3), which were used for event detection. A third station (KRA2) operated only during the first half of the campaign and was used to choose the frequency range for the analysis (Figure 2c). The frequency content was generally stable throughout the measurement (see Figure S1 for spectrograms). Near the vent, the frequency range extended up to 16 Hz and can be divided roughly into three bands: 0.5–5.0, 5.0–7.5, and 7.5–16 Hz. Frequencies beyond 7.5 Hz even dominated the signal at stations KRA1 and KRA3, but showed very different patterns at the two stations, while at KRA2, they were reduced. Furthermore, the correlation of the signal between the stations was poorer at the high frequency band than at the lowest one. Due to these strong differences, we concluded that the frequencies beyond 7.5 Hz were too much influenced by effects from the local structure around the station, which altered and overprinted the common properties of the signal determined by the source. Therefore, we selected the lowest frequency band of 0.5–5.0 Hz for our analysis. Beyond a radius of 1 km the spectral content concentrated below 5 Hz.
4. Event Detection

4.1. STA/LTA Network Trigger

Transient events were detected using a short-term-average/long-term-average (STA/LTA) trigger (Withers et al., 1998) on stations KRA1 and KRA3. Both stations needed to trigger to declare an event (network coincidence trigger). The trigger function at each station consisted of the ratio between a short-term and a long-term moving mean of the squared amplitude. The ratio at each sample was computed from the LTA window terminating and the STA window starting at that sample to ensure statistical independence between the two windows.
The window lengths were chosen based on the recommendations by Amadej (2009). The STA window should be long enough to capture a few periods. We chose 4 s which yields two periods for the lowest frequency of 0.5 Hz. The LTA window determines whether the trigger is more sensitive to emergent or sharp onsets. The longer the LTA window, the more sensitive the trigger is to emergent onsets. The seismicity contained events with both a very sharp onset and short duration as well as longer lasting, emergent events. To capture this variety, we used multiple LTA window lengths of 10, 12, 16, 24, 32, 48, 64 s and combined the catalogs afterward. This was done by a modified algorithm of the network coincidence trigger, in which each LTA-catalog was treated as a station. We required a detection for at least three LTA window sizes to include the event in the final catalog (see Figure S2 for data sample). The trigger and detrigger thresholds were set to 2.0 and 1.0, respectively.

Six regional tectonic events visibly affected the data of the network. Therefore, any transient events occurring during these earthquakes were removed from the catalog. The final catalog contained 23,505 events.

4.2. Amplitude and Magnitude

For the final catalog, peak-to-peak velocity amplitudes of the Z-component were extracted for each event at both stations. Generally, magnitude scales link the amplitude \( A \), measured at a station, and the distance \( r \) to the source as
\[
M_L = \log(A) + a \log(r) + b \log(c)
\]

(Lehmann et al, 2010). Typically, the half peak-to-peak amplitude \( A^{pp} \) is used. Rather than displacement though, we used amplitudes of particle velocity as \( A^{pp} \). Factors \( a \), \( b \), \( c \) are specific to each station and can be determined by least-squares inversion. In our case, we assumed that the events all originated from the same location, namely the lava lake (Lehr et al., 2019). Therefore, the distance-related terms could be omitted and a makeshift local magnitude was determined as
\[
m_j = \log\left(\frac{1}{2} A^{pp} \right) - c_i
\]

We inverted:
\[
\log\left(\frac{1}{2} A^{pp} \right) = m_j + c_i
\]

To limit the computational cost, \( m_j \) and \( c \) were determined from randomly selected 1,500 events. This was repeated 50 times. If an event was selected multiple times the mean over all available values was taken as the “inverted magnitude.” The site parameters \( c \) were determined from the mean over all runs. Eventually, we computed the magnitudes of all events as the mean of the converted amplitudes over \( N \) stations:
\[
m_j = \frac{1}{N} \sum^N_{i=1} \left( \log(A^{pp}_i) - c_i \right) \quad \text{with} \quad c_1 = -10.29 \text{ for station KRA1 and } c_2 = -10.35 \text{ for KRA3.}
\]

The event database, including start and endtimes, magnitudes and intermediate processing steps, is provided as Data Sets S1 and S2.

5. Magnitude of Completeness

No statistical analysis of event amplitudes is complete without knowing the magnitude of completeness (MoC) of the catalog, which is the lowest magnitude, at which all events were detected. Many methods of determining the MoC work solely on the data in the catalog and rely on the assumption of exponentially distributed magnitudes. However, we preferred to not exclude other possibilities at this stage. Instead, we resolved to a stochastic simulation of the detection process. The result was corroborated by comparing the magnitude distribution of events at different levels of background activity.

5.1. Monte-Carlo Simulation of Trigger

Ringdal (1975) and von Seggern and Blandford (1976) suggested the use of stochastic simulation in order to determine the detection capabilities of a network, given the statistical properties of the data and a magnitude distribution model. Many practical details are explained in von Seggern (2004). The stochastic properties of STA and LTA and their relation between each other and to the magnitude of an event were determined empirically from the real data. The STA/LTA trigger was then simulated by creating random
pairs of LTA and STA values for a random set of magnitudes for each station. The network was simulated
from a logic combination of the stations. The detection probability of any given magnitude resulted from
the ratio of detected to simulated events of that magnitude.

This approach allows the representation of relatively complicated networks and statistical parameters. For
event detection, we combined the catalogs of different LTA windows, with each catalog being the result of
a physical 2-station network. For the simulation, we treated this situation as a network of virtual stations,
that represent a combination of a real station with an LTA window length.

5.1.1. LTA Samples

A representative sample of the entire observation period is needed to derive a statistical description of the
LTA amplitudes (“noise level”). For each LTA window size, 5-min long samples were extracted at the be-

ginning of every hour from the LTA-processed data (i.e., mean-squared amplitudes over the LTA window).
For each LTA window, the sample was approximately log-normally distributed with parameters of the dis-

tributions defined by the mean and standard deviation of the logarithmic data. We note though that the
Kolmogorov-Smirnov test for similarity failed. The log-normal model is, however, frequently used because
it allows for correlation. The events were detected at different stations and at different LTA window lengths,

between which the noise level obviously correlates. Correlated log-normal data can easily be generated from
correlated normal data by logarithmic transformation.

Let \( R \) be the matrix containing all pairwise correlations \( r_{ij} \). The correlated normal samples \( Y \) can be gener-
ated using the transformation:

\[
y = Ax
\]

with \( x \) being an uncorrelated multivariate normal distribution and \( AA^T = R \). \( A \) can be determined from the
Cholesky decomposition of \( R \).

The actual, log-normal LTA values were then obtained by transforming

\[
y_i = \exp(\sigma_i z_i + \mu_i)
\]

5.1.2. STA Samples

The magnitude of an event converts to an STA value at each of the stations. We found that the magnitudes
correlated approximately linear with the logarithmic maximum STA value \( a_{ij} \) at any given station during
the event. Thus, the magnitude determines the observed STA as

\[
a_{ij}^{\text{max}} = m_i q_j + b_i
\]

5.1.3. Detection Curves

The detection capability of the network was tested on two different sets of simulated events with exponen-
tially and uniformly distributed magnitudes, respectively. Each set consisted of 100,000 events (Figure
3). For the determination of the threshold, we applied a logistic regression to obtain a detection curve, that
gives the probability of detecting an event of magnitude \( m \) as

\[
p(m) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 m)}}
\]

Using the definition of the odds \( O = \frac{p}{1-p} \), the inverse of the above equation gives the magnitude, at which
a certain level of completeness is reached

\[
m(p) = \frac{\log(O(p)) - \beta_0}{\beta_1}
\]

Each model was repeated 100 times resulting in a mean and standard deviation of the logistic parameters
as: \( \beta_0 = 1.194 \pm 0.010 \) and \( \beta_1 = 5.532 \pm 0.026 \) for the uniform magnitude model and \( \beta_0 = 1.197 \pm 0.013 \) and
\( \beta_1 = 5.535 \pm 0.025 \) for the exponential model. Since the standard deviations overlap significantly, we con-
considered the parameters for the two models to be the same. Using the values of the exponential model, one
obtains threshold magnitudes of \( m = 0.181 \) and \( m = 0.614 \) for 90% and 99% completeness of the catalog.
5.2. Sections of Similar Amplitude

We also derived an MoC directly from the catalog (Figure 4). To this purpose, we identified sections of similar amplitude levels using the root-mean-square amplitude in 60 s intervals shifted by 30 s. At first, a Decision Tree Regression (scikit-learn) was used to find discrete sections of constant amplitude. The resulting finite set of unique amplitudes was then further simplified by finding similar amplitudes using a KMeans-algorithm.

6. Statistical Analysis of Event Catalog

We investigated mainly two parameters of the event catalog for their statistical distribution: the magnitude of the events and the difference between the start times of two consecutive events—the interevent time.

The coefficient of variation is a measure of the dispersion of a distribution, given by the standard deviation of the data divided by its mean: $C_v = \sigma / \mu$. Applied to the interevent times, it is a common measure in earthquake statistics to discriminate between periodic ($C_v < 1$), clustered ($C_v > 1$), or completely random ($C_v = 1$) processes (Bell et al., 2017; Bottiglieri et al., 2005; De Lauro et al., 2009; Martino et al., 2012). The latter is known as Poisson process. We computed $C_v$ of the interevent time between events within 12-h long, adjacent time frames. Using the same binning, we also determined the number of events and the mean interevent time, the inverse of which is the rate.

6.1. Time Series

The occurrence of higher magnitudes is reflected in the RMS amplitude at the crater stations (Figures 5f and 5g). $C_v$ for events above the 90%-completeness magnitude oscillates around 1 with a mean of 1.03 and a standard deviation of 0.17, suggesting a random occurrence of higher-magnitude events. Extreme values occur solely in the clustered regime. High $C_v$ are associated with interevent times close to or larger than 2,000 s. Their occurrences are not associated with one of the removed regional earthquake, which could possibly limit event detection due to their coda. In contrast to events above the MoC, $C_v$ for the unfiltered catalog varies around 0.56 ± 0.04, which indicates a periodic process.
The first two and last three days seem to have seen fewer high-magnitude events than the days in between, resulting in larger interevent times toward the start and end of the observation period (Figures 5a, 5d and 5e). Correspondingly, the level of seismic amplitude is on average higher during the center period and notably on March 5–7 (Figure 4). The \( v_{EC} \) and the interevent times between events in the unfiltered catalog, however, seem to be unaffected.

6.2. Magnitudes

The magnitudes of the transient events at Villarrica range from −1.5 to 2.0 (Figure 6). The distribution resembles those of tectonic earthquakes with an almost horizontal part toward small magnitudes, a linear slope toward higher magnitudes and the roll-off where the two parts connect. For tectonic earthquakes, the roll-off and flat part result from the incompleteness of the catalog at low magnitudes. The slope of the linear part corresponds to the actual exponential distribution of the magnitudes (Gutenberg-Richter law; Gutenberg & Richter, 1956) and in fact continues at small magnitudes. The completeness threshold of the catalog lies at higher magnitudes than the roll-off and beyond the mode, respectively. Hence, for the

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**Figure 4.** Magnitude-frequency distributions for events in sections of similar amplitude. (Top) The thick white line marks segments of constant RMS amplitude (black) found by Decision Tree Regression. Colored lines represent clusters of similar amplitudes. (Bottom) Magnitude-frequency relation for events in sections of similar amplitude with more than 1,000 events. Colors correspond to those in top panel. Counts are normalized to number of events in \( m = [0.6,0.7] \). Dotted lines indicate threshold magnitudes from MC simulation. The distribution at the highest amplitude level (dark gray) branches off around the 90%-completeness magnitude.
The complete catalog, the magnitude seems to be exponentially distributed. The catalog contains 5,697 events above \( m = 0.181 \) (90% completeness) and 643 events above \( m = 0.615 \) (99% completeness). The histogram indicates a unimodal distribution, reflecting the lack of events at small magnitudes and fewer events at high magnitudes.

### 6.2.1. Discarding the Exponential Model

The complete part of the catalog suggests an exponential distribution of magnitudes. We tested this hypothesis by deriving the exponential distribution that fits the complete part of the catalog and using it in the detection simulator to see whether the decline in detected events could be reproduced. If the exponential distribution continued at low magnitudes, the simulated detected events should correspond to the observed distribution. That is, of course, under the premise that the simulation adequately represents reality. The rate of the exponential distribution was determined using the maximum-likelihood estimate (MLE) \( \lambda = \frac{1}{N} \sum (m - m_0) = 5.507 \) with \( m_0 = 0.615 \) being the completeness magnitude and \( N \) being the number of events with \( m \geq m_0 \). Since the number of events that needed to be simulated to match the

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**Figure 5.** Time series of magnitude, interevent time, and related parameters. Interevent times were computed between events in the unfiltered catalog (blue) and above the 90%-completeness magnitude 0.181 (orange). Parameters in panels (c)-(e) were computed in 12-h bins. The RMS amplitude in panel (g) was computed every 2 s in 4-s windows. It is clipped at 49 \( \mu \)m/s to highlight the similarity with the magnitudes. The dotted line in panel (c) marks \( C_1 = 1 \), which indicates a Poisson process.
observed complete part was too large to be run at once, the simulation was run repeatedly until the number of simulated events with $m > M_0$ was larger than that of the observed ones. The simulation clearly detected more events than were observed (Figure 7) suggesting that the true magnitude-frequency distribution of the transients tends to smaller values at the low magnitude end than the exponential Gutenberg-Richter distribution.

6.2.2. Deriving a Magnitude Model

The detection curve $h(x) = 1/(1 + \exp(-kx - x_0))$ tells us how many events were missed at any given magnitude by the automatic detection method. Hence, we can use it also to estimate the true distribution of magnitudes based on the model of the observed magnitudes $m_{\text{obs}}(x)$ as $m_{\text{true}}(x) = m_{\text{obs}}(x)/h(x)$.

To find the latter, a log-normal, log-logistic, generalized-gamma, gamma, normal, and Weibull distribution were fitted to the observed magnitude distribution using maximum-likelihood estimation (using scipy.stats). The quality of the models was assessed using the Akaike Information Criterion (AIC, Akaike, 1974), which gives the relative goodness-of-fit, and the Kolmogorov-Smirnov test (KS-Test), which provides an absolute fit. The KS-Test tests for the null-hypothesis that two samples come from the same distribution.

The best model was clearly the log-logistic distribution

$$f(x(x_0), \beta, \alpha) = \frac{\beta}{\alpha} \frac{x^{-\beta-1}}{1 + \frac{x}{\alpha}^{-\beta}}$$

with shape $\beta = 35.552054$, scale $\alpha = 6.120555$ and location $x_0 = -6.142437$. It yielded the lowest AIC and was the only model that yielded a $p$-value above 0.1 (0.42) in the KS-Test, thus did not reject the null-hypothesis (Figure S8 and Table S3).

The combination of the log-logistic distribution function and the detection curve gives
This function still needs to be normalized in order to serve as a probability density function. Since the integral could not be found analytically, we integrated the function numerically within the interval \([-2, 2]\) and divided it by the found value \(n = 2.806\). So the probability density of the “true” magnitude distribution is given as:

\[
f(x) = \frac{1}{n} q(x) = 0.356 \frac{\beta \left( \frac{x}{\alpha} \right)^{\beta - 1}}{1 + \left( \frac{x}{\alpha} \right)^{\beta}} (1 + \exp(-kx - x_0))
\]

Figure 7. Histogram (left) and empirical survival function (right) for simulated event detection. Black solid lines represent observed data, dashed lines indicate completeness magnitude. (a and b) Events with exponentially distributed magnitudes derived from the complete catalog. (c and d) Magnitudes distributed according to “true” model (Equation 1).
Running the detection simulation with this distribution resulted in the observed distribution (Figure 7). The magnitude distribution of events observed during low-noise levels continue the one of simulated detections from the “true” distribution much better than the one from the exponential model (Figure 8). In addition, the distribution of low-noise events is identical to the “true” distribution down to magnitudes well below 0. Since the determination of sections of similar amplitude was independent from the MC simulation, we see this as confirmation that the simulation provides adequate information on the detection capabilities of low magnitude events.

Since the definition of the “true” distribution in Equation 1 is somewhat artificial, we fitted a variety of standard models to the simulated magnitudes and assessed their relative fit using the AIC. The KS-Test always rejected the null-hypothesis of similarity which was however expected, given the high number of samples and the fact that they were derived from a different probability density function. The Weibull distribution performed best followed by the Gumbel and Gompertz distribution. The Gumbel distribution of magnitudes is interesting in as much as it indicates an exponential distribution of amplitudes. The Weibull distribution is part of Equation 1 if \( k = \beta \) and, given its success in the fitting, may be the dominating term. Interestingly, the log-logistic distribution performed worst. The generalized-gamma, normal, and inverse Gaussian were mediocre.

### 6.3. Interevent Times

#### 6.3.1. Dependence on Threshold Magnitude

Figure 9 shows the distributions of interevent times between events of magnitudes above the 90%-completeness and 99%-completeness threshold found by the MC simulation. The 90% completeness provides a more robust estimate due to the almost 10-fold increase in the number of events. We used the data from the entire observation period. For better comparison of the distributions, the interevent time is normalized by its mean. For the (almost) complete part of the catalog, the distributions are strongly left-skewed with a mode close to 0. For the higher threshold magnitude, the mode shifts closer to 0. The right tail is well represented by an exponential distribution. Hence, the occurrence of events in time is approximately a Poisson process, which is corroborated by the mean coefficient of variation of \( \approx 1 \).
To demonstrate how strongly the interevent time distribution depends on the completeness of the catalog, we also plotted the distribution for the entire catalog (i.e., all detected events). It clearly has a mode $E > E_0$, indicating a preferred and thus more regular recurrence time (Table 1).

### 6.3.2. Interevent Time Models

In order to find a model for the occurrence of events in time, we fitted different probability density functions to the interevent time distributions of events above different threshold magnitudes. Fitting of parameters and model selection was done in the same way as for the magnitudes (Section 6.2.2) except that the location parameter was fixed to 0. We tested the log-normal, log-logistic, generalized-gamma, gamma, normal, Weibull, and exponential distribution on the data of the whole period. The log-normal and generalized-gamma (GG) distributions yielded the lowest AIC for all subsets. In all cases, the distributions decline toward 0, so all models indicate a preferred recurrence time, which suggests a slight periodicity in the temporal occurrence of the transients. However, based on the $p$-values of the KS-Test and using a significance level of 1%, only some models for the 99%-complete catalog ($M > 0.615$) were acceptable.

### 6.4. Interdependence of Magnitude and Interevent Times

To illuminate the relationship between the magnitudes of consecutive events and the interevent time between them, we computed 2D-histograms of the corresponding pairings of variables (Figure 10).
Table 1
Tested Distributions as Interevent Time Models for Entire Observation Period 2012-03-02T00:00:00 to 2012-03-14T00:00:00 for Different Threshold Magnitudes

| Threshold Magnitude | AIC     | KS_pval  | Loc | Scale | Shape_0 | Shape_1 |
|---------------------|---------|----------|-----|-------|---------|---------|
| M > −∞              |         |          |     |       |         |         |
| Lognorm             | 16203.6 | 9.14 × 10^-24 | 0.0 | 0.86 | 0.56 | –       |
| Loglogistic         | 16380.8 | 1.67 × 10^-13 | 0.0 | 0.88 | 3.13 | –       |
| Gengamma            | 16086.6 | 6.28 × 10^-90 | 0.0 | 0.01 | 12.24 | 0.52 |
| Gamma               | 16264.9 | 5.59 × 10^-05 | 0.0 | 0.29 | 3.49 | –       |
| Weibull             | 17288.9 | 1.03 × 10^-35 | 0.0 | 1.13 | 1.87 | –       |
| Expon               | 23510.0 | 0.00 × 10^-00 | 0.0 | 1.00 | –       | –       |
| M > 0.181           |         |          |     |       |         |         |
| Lognorm             | 5171.0  | 7.67 × 10^-04 | 0.0 | 0.63 | 0.94 | –       |
| Loglogistic         | 5272.2  | 2.63 × 10^-05 | 0.0 | 0.62 | 1.83 | –       |
| Gengamma            | 5246.4  | 5.11 × 10^-08 | 0.0 | 0.00 | 11.15 | 0.31 |
| Gamma               | 5624.6  | 3.84 × 10^-33 | 0.0 | 0.81 | 1.24 | –       |
| Weibull             | 5688.8  | 9.17 × 10^-26 | 0.0 | 1.02 | 1.05 | –       |
| Expon               | 5702.0  | 1.79 × 10^-37 | 0.0 | 1.00 | –       | –       |
| M > 0.615           |         |          |     |       |         |         |
| Lognorm             | 606.9   | 8.86 × 10^-02 | 0.0 | 0.47 | 1.30 | –       |
| Loglogistic         | 613.4   | 3.88 × 10^-01 | 0.0 | 0.49 | 1.35 | –       |
| Gengamma            | 603.1   | 6.12 × 10^-01 | 0.0 | 0.00 | 7.87 | 0.28 |
| Gamma               | 636.0   | 1.63 × 10^-04 | 0.0 | 1.26 | 0.80 | –       |
| Weibull             | 626.1   | 1.24 × 10^-02 | 0.0 | 0.89 | 0.83 | –       |
| Expon               | 648.0   | 8.46 × 10^-08 | 0.0 | 1.00 | –       | –       |

Note. Last four columns give the parameters of the pdf determined by maximum-likelihood estimate. First two columns give the criteria for model selection. A lower Akaike Information Criterium (AIC) indicates a better relative fit. Models yielding the lowest AIC are highlighted in bold. If the p-value of the KS-Test is above the chosen significance level of 0.01 the data and the model are considered as similar.

Pearson correlation coefficient was computed to quantify the amount of linear correlation between the variables. We used the data from all detected events because we think that it gives a more realistic representation of the relationships between adjacent events despite the uneven detection probability. If a high threshold magnitude is selected, interjacent smaller events are almost certainly missed, which leads to clearly false interevent times. Using the entire data increases the chance to analyze intact sequences of events. Nevertheless, the following results are possibly affected by the decreasing detection rate toward lower magnitudes.

The magnitudes of two consecutive events were uncorrelated. Similarly, no correlation was found between the interevent time and the magnitude of the event after the interevent time. In contrast, the magnitude appears to be linearly correlated to the logarithm of the subsequent interevent time, although the correlation coefficient is only ≈0.3.

7. Discussion

We investigated frequency distributions of magnitudes and interevent times for transient seismic events at Villarrica, which are commonly attributed to Strombolian explosions. Based on the statistical properties of the seismic noise we derived a detection curve for our network, which indicates the probability to detect a given magnitude and the magnitude of completeness for our event catalog. Additionally, we could reconstruct a possible distribution of magnitudes, which we refer to as the “true” distribution. In the following, we compare the distributions to tectonic earthquakes and to results from other volcanoes. We discuss possible implications for the flow regime, assuming that the events reflect slugs or bubbles of gas in the magma column.

7.1. Magnitudes

The distributions of both the magnitudes and the interevent times turned out to be clearly unimodal, which sets these events apart from normal shear fracture earthquakes. Even though the magnitude-frequency relation seems to be exponential for larger magnitudes, we could demonstrate that the occurrence of small magnitudes decreases again, given their observed frequency and their detection probability in our network. In other words, the Gutenberg-Richter law (Gutenberg & Richter, 1956) does not apply here. It predicts a power-law distribution of seismic amplitudes and therefore an exponential distribution for the whole range of magnitudes with magnitudes being proportional to the logarithm of the amplitude.

Deviances from the GR-law were already observed for Strombolian activity in other basaltic systems (Cauchie et al., 2015; Nishimura et al., 2016; Pering et al., 2015). Nishimura et al. (2016) proposed an exponential decay of amplitudes (not magnitudes!) based on seismic data. If the amplitudes were exponentially distributed, the magnitudes would follow a Gumbel distribution, which was among the better performing models for the simulated “true” distribution. Therefore, we do not want to exclude the possibility of exponentially distributed amplitudes.

However, many other studies suggest a rather unimodal (i.e., having a mode and a decline toward low sizes) distribution of volcanic explosion sizes measured as seismic amplitude, energy, gas mass, or jet height (Cauchie et al., 2015; Martino et al., 2012; Pering et al., 2015; Tamburello et al., 2012; Zobin, 2017). In those cases, in which the distributions were fitted by probability density functions, log-normal (Martino et al., 2012) or Weibull (Taddeucci et al., 2013) distributions were found.
7.2. Interevent Times

Unlike the magnitudes, the interevent times between undetected events cannot easily be reconstructed from the detection curve. The interevent times between the events in the 90%-complete and the unfiltered catalog ($m > -\infty$) were best described by a log-normal and generalized-gamma distribution, respectively. In both cases, the unimodal shape discards in particular the simple Poisson process model, which is defined by exponentially distributed interevent times and represents a completely random occurrence. Instead, the unimodal shape suggests a preferred recurrence time of the events. Although, if only very large events were taken into account, the distribution of interevent times might become exponential.

Since interevent times can only be computed between known events in a catalog, missing events lead to wrong, larger interevent times. Consequently, the number of large intervals will always be overestimated while the true interevent times remain unknown. In this regard, the values given in Figure 9a are upper values. Interevent times between events above the MoC are the only population that could be fully observed and thus provide an important empirical benchmark for possible models. In contrast, the full, unfiltered catalog might contain sequences, during which all events were detected, and thus might provide more realistic interevent times. Nevertheless, this distribution is still distorted toward large interevent times and deprived of small ones due to the missing events.

A generally non-Poissonian occurrence of explosions was already noted for Strombolian and Vulcanian activity at other volcanoes. Varley et al. (2006) found distributions of monotonically decaying, as well as unimodal shape, although eruption sequences classified as Strombolian only showed the latter. Dominguez

![Figure 10](image-url)
et al. (2016), on the other hand, found only unimodal shapes for both types of activity. All were reasonably well fitted by log-logistic distributions. Other studies proposed Weibull (Cauchie et al., 2015) or Gamma (Bell et al., 2017) distributions. Preferred recurrence times of Strombolian events were not only found in seismic data, but also using thermal or SO$_2$ imaging techniques or infrasound (Gaudin, Taddeucci, Scarlato, Harris, et al., 2017; Harris & Ripepe, 2007b; Pering et al., 2015; Ripepe et al., 2002; Taddeucci et al., 2013).

While most studies of explosion sequences used relatively short periods of observations (especially those relying on video footage only comprise only a few hours at most), Martino et al. (2012) presented interevent time distributions of three 0.5–1 year long sequences from Stromboli. They found periodic recurrence of explosions only during the comparatively short periods of increased activity while quiet periods showed nearly Poissonian behavior. Our data stem from a relatively short observation period, during which the volcano was considered very active by the local monitoring agency OVDAS. The lava lake was visible—indicating that the magma was close to the crater rim—and displayed Strombolian activity. Overall, we think that the volcanic and seismic activity largely corresponds to those described by Palma et al. (2008) for 2004/2005.

The mean interevent times in the 90%-complete catalog were between 100 and 400 s. Even for the unfiltered catalog, the mean time between events was around 40 s. In contrast, observations of bubble bursting activity by camera on basaltic systems (Stromboli, Etna) revealed median interevent times of <10 s (Domínguez et al., 2016; Pering et al., 2015). For Villarrica, Gurioli et al. (2008) reported an average spacing of 6–7 s between all types of degassing events. The average time between more violent events (which involve mass ejection) was around 18 s. At Masaya Volcano, Pering, Ilanko, Wilkes, et al. (2019) counted 5–25 bubbles within 5 s at the surface of the lava lake. Using only seismic observations, it is difficult, or even impossible, to achieve such a detailed resolution because the noise level inhibits the detection of small events, an extended coda masks the following event, and simultaneous occurring events simply appear as a single waveform. Nevertheless, we want to point out that the number of simulated “true” events (based on the magnitude distribution) yields a mean spacing of 13.5 s, which is in the range of the values reported by Gurioli et al. (2008).

### 7.3. Interdependence

Our results on the relationship between magnitudes and interevent times resemble the findings by Pering et al. (2015) for degassing events at Mt. Etna. They found a linear correlation between the logarithms of erupted slug mass and time to the next event, but no correlation between mass and time to the preceding event. Video footage of slug trains in subsequent lab experiments by Pering et al. (2017) suggest that this “repose gap” forms because of coalescence of closely spaced slugs. During coalescence, the trailing slug is accelerated toward the leading one, resulting in a larger slug with a larger gap toward the following slug.

Given the incompleteness of the data, we cannot tell whether the observed relations would hold if we knew all events. Yet, we see the slight linear correlation between interevent time and preceding magnitude as an indication that slug interaction could play a role in the degassing mechanism of Villarrica. This would imply that slugs occur in a densely packed train rather than as single, individual gas pockets.

### 7.4. Flow Regime

In terms of flow regimes, the magnitude of events corresponds to the size of the slugs, that is, notably their length. With $V = CL$ being the volume of a slug with cross-sectional area $C$ and length $L$, from the seismic amplitude $A \propto V$ one obtains $M \propto \log A \propto \log V = \log (CL)$ for the magnitude. The interevent time is proportional to the length of the liquid bridge between the gaseous slugs. The distributions of magnitudes and interevent times both display a relatively concise peak. The distribution of interevent times is approximately log-normal, similar to the distribution of the liquid bridge lengths in slug flow. The estimated “true” distribution of magnitudes may be interpreted as normal, in which case slug lengths would be log-normally distributed. The transients at Villarrica are commonly attributed to Strombolian explosions (Calder et al., 2004; Gurioli et al., 2008; Palma et al., 2008), which in turn are commonly associated with bursting gas slugs (Gaudin, Taddeucci, Scarlato, Bello, et al., 2017; Ishii et al., 2019; James et al., 2004, 2006, 2008; Neuberg et al., 1994; Parfitt, 2004; Ripepe et al., 2001; Vergniolle et al., 1996). Our results support this
assumption, since statistically the occurrence and magnitude of the transients are consistent with size distributions in a slug flow regime.

A differentiation from bubble or churn flow based on qualitative comparison of size-frequency distributions alone is difficult. Transition criteria are commonly defined by the volumetric gas flow rate or superficial gas velocity, which, however, are unknown in our case. Bubbles in bubbly flow are probably too small and their number to high to produce distinct seismic signals. Instead, it might produce a tremor-like signal (Mudde, 2005; Ripepe et al., 2010) and could constitute the background seismicity. Churn flow on the other hand indicates a more unstable and turbulent system with bubble-rich sections between the gas pockets. This reflects perhaps better the vigorous, boiling appearance of the lava lake surface. If the regime is still reasonably close to slug flow (slug-churn flow), the interevent time and magnitude distributions are probably indistinguishable from slug flow.

In the light of the experiments by Pering et al. (2017) on the ascent of slug trains, the (albeit weak) correlation between preceding magnitude and interevent time suggests, that slugs interact with each other in the Villarrica conduit. On the same grounds, we might carefully rule out slug formation due to gas accumulation at a barrier such as the classic “collapsing-foam” (or “forced coalescence”; Ripepe et al., 2001) model (Jaupart & Vergniolle, 1989; Parfitt, 2004). In this model, slugs form due to partial collapse of accumulated bubbles (foam) at a barrier in the conduit system. More recently, Barth et al. (2019) proposed that a mush of crystallized and liquid magma acts as a valve, which opens if enough gas has accumulated. For such load-and-discharge mechanisms, one would expect a correlation between interevent time and the following, rather than the preceding, magnitude (Pering et al., 2015). In contrast, the “rise-speed dependent” model suggests that slugs form by free coalescence of small bubbles in slowly ascending magma (Parfitt, 2004; Parfitt & Wilson, 1995). This scenario is closer to the conditions in the laboratory experiments, in which slugs also form freely from small bubbles and for which the log-normal size distributions were observed.

For this study, we assumed that the data were stationary, that is, that their statistical properties did not change with time. We think that this is justified, since the seismicity seems to be largely stable over the observation period. Although the RMS seismic amplitude varies visibly (Figure 4), there are no obvious trends, periodicities or extreme changes in it nor in the interevent times and magnitudes (Figure 5). Cyclic, sawtooth-like variations in seismic amplitude have been reported for Villarrica with periods of several hours in 2004/2005 (Palma et al., 2008) and days to weeks in 2010 (Richardson et al., 2014). If such a longer-period variation was present, the available time series would have been too short to unambiguously identify it. Shorter cycles might be present, but no regularity was found. Variations in the data could indicate an additional process that affects event generation. Such processes could be, for example, changes in magma composition or gas content, which, in turn, would affect the density, temperature, and viscosity of the magma and therefore the flow regime and slug formation. If these changes could be identified and were taken into account, different distributions of interevent times and magnitudes might arise. Possibly, the distributions might become more indicative of one or the other flow regime and changes between regimes could be detected.

A strict comparison with the distributions from the flow experiments requires that each slug produces exactly one event only. Field observations and lab experiments, however, suggest that seismic events may be generated by bursting of a slug at the surface or deeper inside the conduit (Spina et al., 2019). The former additionally generates an acoustic signal while the latter does not, leading to a higher number of seismic than acoustic events. Hence, one slug possibly causes multiple seismic signals.

In this context, we envision one particular scenario for Villarrica. The difference between the diameter of ~12 m, calculated for the lower conduit from degassing rates (Palma et al., 2011), and the width of the upper lake reservoir of 20–30 m (Moussallam et al., 2016) requires a substantial widening of the conduit somewhere. Richardson and Waite (2013) interpreted the source force of a frequent, repetitive transient waveform as a drag force at the nearly horizontal lake bottom and depicted a step-like termination of the conduit to the lake reservoir. James et al. (2006) showed in lab experiments that the passage of a slug through a tube widening leads to pressure and displacement signals, which bear strong resemblance with the transient waveforms. Larger slugs may break up into several daughter bubbles, each of which could produce an additional signal when finally bursting at the free magma surface. The reported mismatches
between visible bubble bursting and seismic/acoustic signals (Goto & Johnson, 2011; Gurioli et al., 2008; Ripepe et al., 2010) at Villarrica at least suggest that the relation between single slugs and seismic events is not straightforward. However, a better knowledge about the source mechanisms and origin depths of the events would be required to further explore this aspect.

8. Conclusion

We investigated the statistical distributions of magnitudes and interevent times of transient seismic events from the lava lake of Villarrica Volcano, which are commonly associated with Strombolian activity. By taking into account the detection limits of our seismic network, we could demonstrate that the amplitudes do not follow the power-law of tectonic earthquakes. Instead, the distribution of magnitudes declined again toward small values. The distribution of interevent times was strongly influenced by the selection of the threshold magnitudes with means of 44 and 182 s for all detected events and those above the 90%-complete-ness magnitude, respectively. The corresponding modes were 32 and 115 s. In any case, the distributions were approximately log-normal, suggesting a preferred recurrence time, that is, a weak periodicity. Our results are largely consistent with the size distribution of gaseous slugs and the liquid bridges in between in a slug flow regime. We thus conclude that the fluid-dynamic in the upper conduit of Villarrica in early March 2012 was dominated by a slug flow regime. The time between two events appeared to correlate linearly with the magnitude of the first event. We therefore suspect that coalescence of slugs during the ascent might play a significant role. In contrast, a mechanism, where gas accumulates at depth before release, seems rather unlikely.

The statistical properties, and thus the flow regime, appeared to be largely stable over the observation period. However, a more detailed picture might arise, if the presented approach was applied to expediently selected time sections, which could reveal subtle changes in the magma dynamics.

We see potential that the statistical analysis of the occurrence of seismic events can help in monitoring the flow regime in volcanic conduits over longer time periods. Valuable complementary information could be gained by adding infrasound and degassing rate monitoring. However, clearly more experimental data on the relation between gas flow regime and seismic and acoustic signals is needed to provide a solid reference for the identification of flow patterns.

Data Availability Statement

The raw seismic data are available through the Experiment Database of the Geophysical Instrument Pool Potsdam (GIPP) under the name VITO (doi:10.5880/GIPP.201202.1). All analysis was implemented in Python 3.7 using packages from Jupyter, Obspy, Scipy, Numpy, Matplotlib, Pandas, and Scikit-learn.

Acknowledgments

The authors thank T. Pering and an anonymous reviewer for their helpful and encouraging remarks. Open access funding enabled and organized by Projekt DEAL.

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