Considerations on friction coefficient in a simple harmonic motion

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Abstract. An experimental device was designed and developed to study the friction coefficient between a steel rod and two drive rollers on which it is supported. The motion of the steel rod is achieved by means of two rods mounted on bearing and driven by two engines. The two rollers have rotational motions with equal and opposite sign angular velocities. These cause a harmonic motion of the horizontal rod. By measuring the oscillation period of the steel rod, the coefficient of sliding friction between the rod and the drive roller is obtained.

1. Introduction
The dry friction laws have been developed over a long period of time and with the contribution of many philosophers and scientists. They have been discovered experimentally and cover a wide range of load conditions, materials and condition of the surfaces in contact.

In ancient Greece, Aristotle (384-322 BC) recognizes the existence of the friction force in his "Questiones Mechanicae" and notes that it is smaller in the case of round objects, Dowson [1].

Leonardo da Vinci (1452-1519) introduces for the first time the concept of coefficient of friction as the ratio of the friction force and the normal load \( \mu = \frac{F}{P} \), by assigning the value of 0.25, Engel [2]. His manuscripts containing important results in the field of friction were unknown for a long time, being published only at the end of the 9th century in the Codex Atlanticus.

At the end of the 17th century by Guillaume Amontons (1663-1705) initiated in France the study of friction. His experiments and interpretations of the results are contained in an article presented at the Royal Academy in December 1699. He said that friction results from the mechanical work developed from slipping rigid or elastic stiffness over one another, Dowson [1].

In 1750, mathematician Leonhard Euler (1707-1783) published two studies on friction at the Berlin Academy of Sciences. It distinguishes between static and kinetic friction and concludes that kinetic friction is less than static friction, [3]. He develops an analytical friction approach, introduces the symbol "\( \mu \)" for the coefficient of friction and proposes the expression \( \mu = \tan \alpha \).

French engineer Charles Augustin Coulomb (1736-1806) is well known for his contributions in friction, the law of friction \( \mu = \frac{F_t}{N} \) bearing its name. He investigated both static and dynamic friction of sliding surfaces, as well as friction in bending of ropes and in rolling, [4]. Coulomb's memories on friction "Théorie des machines simples" receives the Academy of Sciences Award in Paris in 1781.

Following research on friction, Berthier [5], shows that the coefficient of friction is not an intrinsic property of the materials and the result of a friction experiment is influenced by the tribometer used.
Friction is a complex physical phenomenon and little is understood of friction mechanisms that start in a simple mechanism such as a tribometer and the different couplings that link the elements of the tribological triplet: the mechanism, the first bodies and the third body, [6]. One of the main parameter which describes the friction is the coefficient of friction, and this paper presents a method of its evaluation using a harmonic oscillatory motion of a rod.

2. Theoretical aspects
A cylindrical rod of mass M rests on top of two identical rollers of radius r, the distance between the centres of the rollers being 2L. Initially, the rod is held at rest with its centre at distance x from the midpoint of the rollers. At time t=0 it is released on the rollers that are continuously turned rapidly in opposite directions, as shown in Figure 1.

In order to obtain the rod motion equation, it is assumed that this makes contact with the biconical rollers in sections 1 and 2. The two rollers rotate clockwise, so that the tendency for the rod to move under the action of the friction forces is to shift to the right. Each friction rod contact point has a frictional force proportional to normal pressure.

![Figure 1. Free body diagram](image)

The contact point on the roll has speed

\[ v_{c_{1,2}} = \omega_{1,2}r \] (1)

The focal point of the rod has the speed

\[ v_{C_{1,2}} = \dot{x} \] (2)

In each of the two sections the friction forces will act on the rod

\[ F_{f_{1,2}} = 2\mu N_{1,2} \text{sgn}(\omega_{1,2}r - \dot{x}) \] (3)

The centre of mass movement theorem has projections on the axes

\[ M\ddot{x} = F_{f_1} + F_{f_2} \] (4)

\[ -Mg + 2N_1 \sin \beta + 2N_2 \sin \beta = 0 \] (5)

The kinetic moment has projection only on the axis perpendicular to the plane of motion.
\[
\frac{(F_{f_1} + F_{f_2})d}{2} + 2N_1(L - x) \sin \beta - 2N_2(L + x) \sin \beta = 0
\]  

(6)

The system of equations 2, 3 and 4 has the unknowns $\dot{x}$, $N_{1,2}$, $F_{f_{1,2}}$. The solution to this system is

\[
\dot{x} = -2\mu g \frac{(L + x) \text{sgn}(\omega r - \dot{x}) - (L - x) \text{sgn}(\omega r - \dot{x})}{\mu d [\text{sgn}(\omega r - \dot{x}) - \text{sgn}(\omega r - \dot{x})] - 4L \sin \beta}
\]

(7)

\[
N_1 = \frac{1}{2} Mg \frac{\mu d \text{sgn}(\omega r - \dot{x}) - 2(L + x) \sin \beta}{\mu d [\text{sgn}(\omega r - \dot{x}) - \text{sgn}(\omega r - \dot{x})] \sin \beta - 4L \sin^2 \beta}
\]

(8)

\[
N_2 = -\frac{1}{2} Mg \frac{\mu d \text{sgn}(\omega r - \dot{x}) + 2(L - x) \sin \beta}{\mu d [\text{sgn}(\omega r - \dot{x}) - \text{sgn}(\omega r - \dot{x})] \sin \beta - 4L \sin^2 \beta}
\]

(9)

\[
F_{f_{1,2}} = 2\mu N_{1,2} \text{sgn}(\omega r - \dot{x})
\]

(10)

The equation (7) that describes the movement of the rod is a nonlinear differential equation to be integrated numerically.

The integration of the equation is done using a 4th order Runge Kutta algorithm for the following parameter values

\[
\mu = 0.15, \ d = 0.01 m, \ L = 0.15 m \ \omega_1 = -\omega_2 = -30 \text{rad/sec}
\]

(11)

\[\text{Figure 2. Numerical solution of the differential equation of motion}
\]

For roller speeds greater than a certain critical value, the movement of the rod is an undamped harmonic motion, as Figure 2 shows.

If the angular velocity values of the rollers are greater than a certain value, the sgn (x) functions appearing in the motion equation keep a constant sign so that the expression (7) can be simplified

\[
\dot{x} = -\frac{\mu g}{L \sin \beta} x
\]

(12)

Equation (12) is a linear, homogeneous differential equation of the order of 2 with constant coefficients. The proportionality of the acceleration with the displacement and the opposite signs give the movement of the bar in form of harmonic motion with pulsation

\[
\omega^2 = \frac{\mu g}{L \sin \beta}
\]

(13)

From the relation (13), experimentally determining the oscillation period of the movement of the bar, we can find the coefficient of sliding friction with relation
\[ \mu = \frac{4\pi^2 L}{T^2 g} \sin \beta \] 

(14)

3. Experimental device
In order to validate theoretical results presented above, the device from Figure 3 was built, [7].

The stand consists of two parallel rods placed on a support plate. At one end of the rods there are the two engines, and at the other end there are two sets of drive rollers, which support the rod whose motion is studied. The drive rollers of different materials are shown in Figure 4.

A set of rollers is made of steel and a set is made of textolite to allow different pair of materials during the experiment. On the outside, the rollers practiced a V-shaped groove to allow the cylindrical rod to be correctly supported. Support horizontal axis on which are mounted rollers is made with a necklace bearing. At the opposite end, the engine is mounted in a reducing sleeve that connects the horizontal rod, locks being kept by a clamp bearing.

4. Experimental results

IV.1 Determination of the coefficient of friction using the harmonic oscillation of the rod
According to the previous model, the rod will have a harmonic oscillatory motion with pulsation

\[ \omega = \sqrt{\frac{\mu g}{L \sin \beta}} \] 

(15)
where
\( \mu \) is the coefficient of friction to be determined, \\
g is the gravitational acceleration, \( g=9.81 \text{ m/s}^2 \) \\
L is half the distance between the axis of the two drive rollers. \\
\( \beta=60^\circ \) is the half angle of biconical rollers.

From the video of oscillatory motion performed by the rod, the oscillation period can be measured as the time divided by the number of complete oscillations:

\[
T = \frac{t}{n} \tag{16}
\]

The period according to the data extracted from the shooting was calculated in Mathcad, [8], as shown in Figure 6.

Frame no. | Period for the first six oscillations
--- | ---
0 | 1.9
1 | 1.9
2 | 1.9
3 | 1.9
4 | 1.9
5 | 1.933

Figure 6. Mathcad sequence

Obtained period is: \( T=1.93 \text{ s} \). Knowing that:

\[
\omega = \frac{2 \cdot \pi}{T} \quad \frac{2\pi}{T} = \sqrt{\frac{\mu g}{L \sin \beta}} \tag{17}
\]

The friction coefficient is:

\[
\mu = \frac{4 \pi^2 \frac{L}{T^2} \sin \beta}{g} \tag{18}
\]

The distance between the roller axes is 0.3m, therefore \( L=0.15\text{m} \).
Introducing in the equation (18) the period $T$ calculated above, the friction coefficient between the steel roller and the rod is obtained:

$$\mu = \frac{4 \pi^2 L}{T^2 g} \sin \beta = 0.14$$

(19)

IV.2 Determination of friction coefficient using the force sensor
To validate the first result for friction coefficient, a Multilog Pro data acquisition and a force sensor were used.

The force sensor uses strain gauge technology to measure force, based on the bending of a beam. The strain gauges attached at both ends of the beam change slowly their strength with the beam bending. These resistors are built in such a way that the change of resistance causes a small change in voltage, that voltage having a linear force variation. An amplifier circuit inside the sensor intensifies this voltage so it can be measured through the laboratory interface, [9].

A thread was attached to the horizontal rod and at the other end was connected to the force sensor hook, Figure 7.

Figure 7. Measurement with force sensor

During the movement, the sensor measured the total friction force, transmitting the experimental data to the computer.

The measured data is processed using the specialized Multilab software that displays the time-plot of the friction force function, Figure 8.

Figure 8. Variation of the friction force in time
The frictional force was measured after starting the two engines that train the rollers. A total frictional force of 0.799N was obtained. The friction force measured by the sensor is given by, [10]:

\[ F_f = F_f^{1} - F_f^{2} = \mu(N_1 - N_2) = -\frac{\mu M g x}{L} \]  

(20)

hence the value of the coefficient of friction

\[ \mu = \frac{F_f L}{M g x} = 0.16 \]  

(21)

**IV.3 Motion model with the Mathcad software**

A program has been developed in Mathcad, which calculates the kinematic elements of the oscillatory movement performed by the rod: velocity and acceleration.

We observe the characteristic elements of the harmonic oscillatory movement performed by the rod under the particular kinematic conditions provided by the construction of the experimental device, Figure 9.

![Figure 9. Velocity and acceleration graphs](image)

The motion period measured on the graph is \( T = 1.9 \) s. The period obtained from the video of oscillatory motion was 1.93s.

**5. Conclusion**

The paper presents a method for evaluating the friction coefficient of a rod supported on two rollers. By imposing to rollers a rotational motion with equal and opposite angular velocities, particular kinematic conditions are obtained that cause a harmonic oscillatory motion of the rod.

In order to calculate the friction coefficient rod-roller, the oscillation period was measured and used on relations deduced from the theoretical model. It is compared with the coefficient of friction obtained by measuring friction force with Multilog Pro data acquisition device and a remote sensor. The values obtained in the two situations have a relative error of only 3%.

**6. References**

[1] Dowson D 1979 *History of tribology* Longman Group Limited London
[2] Engel, P A 1976 *Impact wear of materials*, Elsevier Scientific Publishing Company, Amsterdam.
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