Proportional subspaces of spaces with unconditional basis have good volume properties

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Abstract

A generalization of Lozanovskii’s result is proved. Let $E$ be $k$-dimensional subspace of an $n$-dimensional Banach space with unconditional basis. Then there exist $x_1, \ldots, x_k \subset E$ such that $B_E \subset \text{absconv}\{x_1, \ldots, x_k\}$ and

$$\left(\frac{\text{vol}(\text{absconv}\{x_1, \ldots, x_k\})}{\text{vol}(B_E)}\right)^{\frac{1}{2}} \leq \left(e \frac{n}{k}\right)^2.$$

This answers a question of V. Milman which appeared during a GAFA seminar talk about the hyperplane problem. We add logarithmical estimates concerning the hyperplane conjecture for proportional subspaces and quotients of Banach spaces with unconditional basis.

Introduction

An open problem in the theory of convex sets is the following

**Hyperplane problem:** Does there exist a universal constant $c > 0$ such that for all $n \in \mathbb{N}$ and all convex, symmetric bodies $K \subset \mathbb{R}^n$ one has

$$|K|^{\frac{n-1}{n}} \leq c \sup_{H \text{ hyperplane}} |K \cap H| ?$$

For some classes of convex sets there is a positive solution to this problem. For example Bourgain first proved the existence of a constant independent of dimension for the class of convex sets with unconditional basis. This can be formulated as follows

**Theorem 1 (Bourgain)** For all convex, symmetric bodies $K \subset \mathbb{R}^n$ one has

$$|K|^{\frac{n-1}{n}} \leq 2 \sqrt{6} \inf\left\{\left(\frac{|B|}{|K|}\right)^{\frac{1}{n}} : K \subset B \text{ and } B \text{ with unc. basis}\right\} \sup_{H \text{ hyperplane}} |K \cap H|.$$
For further positive solutions and background information we refer to the papers of Ball [BA], Milman/Pajor [MIPA] and the author [JU].

In a seminar talk about the hyperplane problem V. Milman asked whether the unit ball of a proportional subspaces of a Banach space with unconditional basis is well contained (in the volume sense) in a convex body with unconditional basis, more precisely, whether the infimum on the right hand side of Bourgain’s theorem is uniformly bounded for proportional subspaces of Banach spaces with unconditional basis. This can be answered in the positive.

**Theorem 2** Let $X$ be a $n$-dimensional Banach space with unconditional basis and $E$ a $k$-dimensional subspace. Then there exist $x_1, \ldots, x_k \in E$ such that

$$B_E \subset \text{absconv}\{x_1, \ldots, x_k\} \quad \text{and} \quad \left(\frac{|\text{absconv}\{x_1, \ldots, x_k\}|}{|B_E|}\right)^{\frac{1}{k}} \leq \left(\frac{e}{n} \cdot \frac{n}{k}\right)^{2}.$$ 

This theorem is a generalization of Lozanovskii’s result, which corresponds to the case $k = n$. In fact we use his approach. In particular, the above theorem gives a uniform bound for the hyperplane problem in the case of proportional subspaces of a Banach space with unconditional basis. This includes proportional subspaces of $\ell^p_\infty$ which are often used to produce more or less pathological phenomena in the local theory of Banach spaces. For the hyperplane problem the estimates of theorem 2 can even be improved to a logarithmical order.

**Theorem 3** Let $E$ be a $k$-dimensional subspace of a $n$-dimensional Banach space with unconditional basis. Then one has

$$|B_E|^{\frac{k-1}{k}} \leq 2e \sqrt{6 + 3 \ln \frac{n}{k}} \sup_{H\text{ hyperplane}} |B_E \cap H|.$$ 

Apart from the geometric interpretation, a convex polytope with not too many faces nearly satisfies the hyperplane conjecture, theorem 3 destroys the hope of producing counter examples by taking ‘bad’ subspaces of ‘good’ spaces. For convex polytopes with not too many extreme points, we can proof a slightly weaker result. Although in this case the operator ideal theory which is involved in the proof is a little bit harder.

**Theorem 4** Let $E$ be a $k$-dimensional quotient of an $n$-dimensional Banach space with unconditional basis. Then one has

$$|B_E|^{\frac{k-1}{k}} \leq c_0 (1 + \ln n) \sup_{H\text{ hyperplane}} |B_E \cap H|,$$

where $c_0$ is a universal constant.

**Proofs**

We will use standard Banach space notation, in particular we denote by $B_X$ the unit ball of a Banach space $X$. In contrast to this $B_\ell^n_p$ is the unit ball of the classical sequence space $\ell^p_\infty$, $1 \leq p \leq \infty$. For the volume of a convex body $B \subset \mathbb{R}^n$ we use $|B|$. The same notation
is used for the lower dimensional volumes of sections of a convex body. A Banach space \( X \) has a \((1-)\) unconditional basis if there exists a basis \((e_i)_{i \in I}\) such that for all signs \((\varepsilon_i)_{i \in I}\) and coefficients \((\alpha_i)_{i \in I}\)

\[
\left\| \sum_{i \in I} \varepsilon_i \alpha_i e_i \right\| \leq \left\| \sum_{i \in I} \alpha_i e_i \right\| .
\]

The following lemma of Lozanovskii [LO] is crucial for the following.

**Lemma 1** Let \( X \) be an \( n \)-dimensional Banach space with unconditional basis \((e_i)_1^n\). Then there exists positive weights \((\lambda_i)_1^n\) such that

\[
\frac{1}{n} \sum_{i=1}^{n} |\alpha_i| \leq \left\| \sum_{i=1}^{n} \alpha_i \lambda_i e_i \right\| \leq \sup_{i=1,\ldots,n} |\alpha_i| .
\]

The next lemma reduces the problem to subspaces of \( \ell_1^n \).

**Lemma 2** Let \( X \) be a \( n \)-dimensional Banach space with unconditional basis. Then there exists an operator \( T : X \to \ell_1^n \) with \( \|T\| \leq 1 \) such that for every \( k \)-dimensional subspace \( E \) one has

\[
\left( \frac{\|T^{-1}(B_1^n) \cap E\|}{|B_E|} \right)^\frac{1}{k} \leq e \frac{n}{k} .
\]

**Proof:** Using the weights from lemma 1 we define

\[
T : X \to \ell_1^n; \quad T(\sum_{i=1}^{n} \alpha_i e_i) := \left( \frac{\alpha_i}{n \lambda_i} \right)_1^n \quad \text{and} \quad S : L_\infty^n \to X; \quad S((\alpha_i)_1^n) := \sum_{i=1}^{n} n \lambda_i \alpha_i e_i .
\]

According to lemma 1 we have \( \|T\| \leq 1 \) and \( \|S\| \leq n \). For the subspace \( H := T(E) \subset \mathbb{R}^n \) we can use Meyer/Pajor’s volume estimate [MEP] to deduce

\[
\left( \frac{\|T^{-1}(B_1^n) \cap E\|}{|B_E|} \right)^\frac{1}{k} = \left( \frac{\|H \cap B_1^k\|}{|T(E)|} \right)^\frac{1}{k} = \left( \frac{\|H \cap B_\infty^k\|}{|T(E)|} \right) \left( \frac{\|H \cap B_1^k\|}{|H \cap B_\infty^k|} \right) \left( \frac{|H \cap B_1^k|}{|T(E)|} \right)^\frac{1}{k} \leq \left( \frac{\|S(H \cap B_\infty^k)\|}{|B_E|} \right)^\frac{1}{k} \left( \frac{|B_1^k|}{|B_\infty^k|} \right)^\frac{1}{k} \leq n (k!)^{-\frac{1}{k}} \leq e \frac{n}{k} .
\]

**Proof of theorem 2:** By lemma 2 we are left to prove the assertion for a \( k \)-dimensional subspaces \( H \) of \( \ell_1^n \). For this let us denote by \( P \) the orthogonal projection from \( \ell_2^n \) onto \( H \). Define \( x_i := P(f_j) \), where \((f_j)_1^n\) denotes the standard unit vector basis in \( \mathbb{R}^n \). The polar of \( H \cap B_1^n \) is a zonotope whose volume can be estimated with a well known determinant formula [MCM], namely

\[
|(B_1^n \cap H)^\circ| = 2^k \sum_{\text{card}(\sigma) = k} |\text{det}_k(x_j)_{j \in \sigma}| \leq \left( \begin{array}{c} n \\ k \end{array} \right) \sup_{\text{card}(\sigma) = k} 2^k |\text{det}_k(x_j)_{j \in \sigma}| .
\]
Now fix a subset $\sigma \subset \{1, \ldots, n\}$ of cardinality $k$ where the supremum is attained (in particular the vectors $(x_j)_{j \in \sigma}$ are independent). Clearly we have for all $x \in H$

$$\|x\|_1 = \sum_{j=1}^n |\langle x, x_j \rangle| \geq \sum_{j \in \sigma} |\langle x, x_j \rangle| =: \|x\|_\sigma.$$ 

The unit ball $B_\sigma$ of the norm $\|\cdot\|_\sigma$ is the image of an $\ell^1_k$-ball and contains $B^1_n \cap H$. By the inverse Santaló inequality for zonoids, due to Reisner [RE], we obtain

$$|B_\sigma| |B_\sigma^o| = |B^1_k| |B^\infty_o| \leq |B^1_n \cap H|(B^1_n \cap H)^o| \leq |B^1_n \cap H| \left( \frac{n}{k} \right) |B_\sigma^o|.$$

Therefore we have proved

$$\left( \frac{|B_\sigma|}{|B^1_n \cap H|} \right)^\frac{1}{k} \leq \left( \frac{n}{k} \right)^{\frac{1}{k}} \leq e \frac{n}{k}. \quad \square$$

**Remark 3** By duality we obtain that the unit ball $B$ of a $k$-dimensional quotient of a $n$-dimensional Banach space with unconditional basis contains the affine image of a cube $C$ with

$$\left( \frac{|B|}{|C|} \right)^\frac{1}{k} \leq c_0 \left( \frac{n}{k} \right)^2.$$

For the hyperplane problem let us recall that a symmetric, convex body $K$ is in isotropic position if

i) $|K| = 1,$

ii) $\int_K \langle x, e_j \rangle \langle x, e_i \rangle dx = L_K^2 \delta_{ij}.$

In this case $L_K$ is the constant of isotropy of $K$. Let us note that for every convex, symmetric body there is an affine image which is in isotropic position. With the help of this it’s essentially Hensley’s result [HE], that an upper bound for the constant of isotropy solves the hyperplane problem for any position of $K$. For further information see for instance [MIPA]. In the following we will denote by $E_K$ the Banach space $\mathbb{R}^n$ equipped with the gauge $\|\cdot\|_K$, i.e. $E_K$ is the Banach space whose unit ball is $K$. It was already discovered by K. Ball that the notion of (absolutely) $p$-summing ($1 \leq p < \infty$) is a useful tool for certain estimates of the constant of isotropy. An operator $T : X \to Y$ is $p$-summing if there exists a constant $c \geq 0$ such that for all $n \in \mathbb{N}$, $(x_k)_1^n \subset X$

$$\left( \sum_{k=1}^n \|Tx_k\|^p \right)^{\frac{1}{p}} \leq c \sup_{\|x^*\|_X \leq 1} \left( \sum_{k=1}^n |\langle x_k, x^* \rangle|^p \right)^{\frac{1}{p}}.$$

The best possible constant $c$ will be denoted by $\pi_p(T)$. 

\medskip

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Lemma 4 Let $K \subset \mathbb{R}^n$ be in isotropic position. For the formal identity $\iota : \ell_2^n \to E_K$ one has

$$L_K \pi_1(\iota^*) \leq 2\sqrt{2}.$$  

Proof: As a consequence of C. Borell’s lemma we have for all $\alpha \in \mathbb{R}^n$

$$L_K \|\alpha\|_2 \leq 2\sqrt{2} \int_K |\langle x, \alpha \rangle| \, dx.$$  

(For the precise constant see [MIPA].) Now let $m \in \mathbb{N}$, $(\alpha_j)_1^m \subset \mathbb{R}^n$. Then we have

$$L_K \sum_1^m \|\alpha_j\|_2 \leq \frac{2}{\sqrt{n}} \sum_1^m \int_K |\langle x, \alpha_j \rangle| \, dx$$

$$= \frac{2}{\sqrt{n}} \int_K \sum_1^m \left| \frac{x}{\|x\|_K} , \alpha \right| \|x\|_K \, dx$$

$$\leq \frac{2}{\sqrt{n}} \int_K \|x\|_K \, dx \sup_{\|y\|_K \leq 1} \sum_1^m |\langle y, \alpha_j \rangle|$$

$$\leq \frac{2}{\sqrt{n}} \sup_{\|y\|_K \leq 1} \sum_1^m |\langle y, \alpha_j \rangle|. \quad \square$$

Proof of theorem 3: Let $E$ be a $k$-dimensional subspace of a $n$-dimensional Banach space $X$ with unconditional basis. We can find an isotropic position for the unit ball of $E$, i.e. there exists a linear map $T : \mathbb{R}^k \to X$ such that $E = T(\mathbb{R}^k)$ and $K = T^{-1}(B_X)$ is in isotropic position. Let us define $S := T \iota : \ell_2^k \to X$. By lemma 4 we have

$$L_K \pi_1(S^*) \leq 2\sqrt{2}.$$  

Since $X$ has an unconditional basis the same is true for $X^*$ and therefore $S^*$ well-factors through $\ell_1^n$ [PS, Lemma 8.15]. By duality there exist $W : \ell_2^k \to \ell_\infty^n$, $\|W\| \leq 1$ and $V : \ell_\infty^n \to X$ such that $S = VW$ and

$$\|V\| \leq 2\sqrt{2} L_K^{-1}.$$  

Let $B := W^{-1}(B_\infty^n)$. From $Im(VW) = E$ we deduce

$$S(B) = V(B_\infty^n \cap W(\ell_2^k)) \subset \|V\| \, B_E.$$  

and therefore $B \subset \|V\| \, K$. Gluskin’s theorem together with $\|W\| \leq 1$ implies a lower estimate for the volume ob $B$. Using the constant from [BAPA] we obtain

$$2\sqrt{2} = 2\sqrt{2} \left( \frac{|K|}{|B|} \right)^{\frac{1}{k}} \left( \frac{|B|}{|K|} \right)^{\frac{1}{k}}$$

$$\leq 2\sqrt{2} \frac{1}{|B|} \frac{1}{|K|} \|V\|$$

$$\leq e \sqrt{2 + \ln \frac{n}{k}} \, 2\sqrt{2} L_K^{-1}.$$  

This means $L_K \leq e \sqrt{2 + \ln \frac{n}{k}}$. Hensley’s theorem [HEN] yields the assertion. \hfill \square
The logarithmic estimate of the hyperplane constant for quotient spaces is based on the use of C. Borell’s lemma in a similar setting as in lemma [4].

**Lemma 5** Let $K \subset \mathbb{R}^k$ be in isotropic position and $T : E^*_K \to Y$ an isometric embedding of $E^*_K$ in a $n$-dimensional Banach space $Y$. Then there exists an extension $S : Y \to \ell^k_2$ of the formal identity $\iota^* E^*_K \to \ell^k_2$ with $ST = \iota^*_K$ and

$$L_K \pi_1(S) \leq c_0 \ (1 + \ln n) .$$

**Proof:** Let $K$ be in isotropic position and denote by $\mu$ the Lebesgue measure restricted on $K$. Choosing $p = 2 + \ln n \geq 2$ we want to construct a suitable factorization of $L_K \iota^*$. For this consider $J : E^*_K \to L_\infty(K, \mu), \alpha \mapsto (x \mapsto \langle x, \alpha \rangle)$. Clearly $\|J\| \leq 1$. Since $L_\infty(K, \mu)$ has the extension property, see [PI2], there is an operator $L : Y \to L_\infty(K, \mu)$ with $LT = J$ and $\|L\| \leq 1$. Furthermore, we define $I : L_\infty(K, \mu) \to L_{p'}(K, \mu)$ the formal identity, $p'$ the conjugate index to $p$, and $P : L_{p'}(K, \mu) \to \ell^k_2$ by

$$P(f) := \left( \int_K \frac{\langle x, e_j \rangle}{L_K} \ d\mu(x) \right)^k_1 .$$

It is easy to see that $L_K \iota^*_K = PIJ$ and $S := PIL$ is an appropriate extension. For the norm of $P$ we deduce from C. Borell’s lemma, see [MIS, Appendix], and the isotropic position of $K$

$$\|P\| = \|P^*\| = \sup_{\|\beta\|_2 \leq 1} \left( \int_K \frac{\langle \beta, x \rangle}{L_K} \ dx \right)^{\frac{1}{p'}} \leq c_0 p \ \sup_{\|\beta\|_2 \leq 1} \left( \int_K \left| \frac{\beta}{L_K}, x \right|^2 \ dx \right)^{\frac{1}{2}} \leq c_0 p .$$

In fact we have proved $\iota_{p'}(S) \leq c_0 p$, where $\iota_{p'}$ denotes the $p'$-integral norm. By the choice of $p$ the proof of the lemma will be completed if we can show

$$(*) \ \ \ \ \ \pi_1(S) \leq n^{\frac{1}{p'}} \ i_{p'}(S) .$$

Given a sequence $(y_j)_1^m \subset Y$ with $\sup_{y^* \in B_{Y^*}, \sum_1^m \langle y_j, y^* \rangle} \leq 1$ we define the operator $R : \ell^m_\infty \to Y, R(\beta)_1^m = \sum_1^m \beta_j y_j$ whose norm is less than 1. In this situation we can use an interpolation formula [GOS] for the $p$-summing norm to deduce

$$\pi_p(R) \leq \pi_2(R)^\frac{2}{p} \|R\|^{1-\frac{2}{p}} \leq n^{\frac{1}{p'}} .$$

Here we have used the well-known fact $\pi_2(R) \leq \sqrt{n} \|R\|$ for any operator of rank at most $n$, see for example [PT2]. Now we can find $(\alpha_j)_1^m \subset B^k_2$ with $\|S(y_j)\| = \langle S(y_j), \alpha_j \rangle$. Clearly the operator $V : \ell^k_2 \to \ell^m_\infty, V(x) := \langle \langle x, \alpha_j \rangle \rangle_1^m$ has also of norm at most 1 and trace duality (see for example [PT1]) implies

$$\sum_1^m \|S(y_j)\| = tr(VSR) \leq \iota_{p'}(VS) \pi_p(R) \leq \iota_{p'}(S)n^{\frac{1}{p'}} .$$

By the definition of the $\pi_1$-summing norm we have proved $(*)$. □
Remark 6 In the proof above an isometric embedding is not really needed. The \( \pi_1 \)-summing norm of an extension can be chosen according to the minimal distance of \( E_K^* \) to a \( k \)-dimensional subspace of \( Y \).

Given lemma 6 the proof of theorem 4 of the introduction follows the same pattern as the proof of theorem 3.

Proof of theorem 4: Let \( X \) be a \( n \) dimensional Banach space with unconditional basis. For a \( k \)-dimensional quotient space \( E \) of \( X \) with quotient map \( Q : X \to E \) we can find an isomorphism \( I : E \to \mathbb{R}^k \), such that \( K = I(B_E) \) is in isotropic position. In this case \( T := Q^* I^* : E_K^* \to X^* \) defines an isometric embedding. Applying lemma 6 there is an extension \( S : X^* \to \ell_2^n \) with \( \pi_1(S) \leq c_0 (1 + \ln n) \). Since \( X^* \) also an unconditional basis \( S \) factors through \( \ell_1^n \). More precisely, there are \( W : X^* \to \ell_1^n \) with \( \|W\| \leq 1 \) and \( V : \ell_1^n \to \ell_2^n \) with \( S = VW \) and

\[
\|V\| \leq \pi_1(S) \leq c_0 (1 + \ln n) .
\]

Now we consider the \( k \)-dimensional subspace \( F := WT(E^*) \subset \ell_1^n \). Instead of Gluskin’s estimate we can use a dual volume estimate first essentially proved by Figiel and Johnson [FIJ].

\[
\sqrt{k} \left( \frac{|V(B_F)|}{|B_{\ell_2^k}|} \right)^{\frac{1}{k}} \leq c_1 \|V\| .
\]

(Indeed, \( \ell_1 \) is of cotype 2 and therefore every subspace has bounded volume ratio. The inequality follows from this if we note that by Grothendieck’s theorem \( V \) is 2-summing.) Since \( STi^* = L_Kid_{\mathbb{R}^k} \) we conclude with the inverse of Santaló’s inequality [BM]

\[
L_K \leq L_K \sqrt{k} \left( \frac{|B_{\ell_2^k}|}{|K|} \right)^{\frac{1}{k}} \leq c_2 \sqrt{k} \left( \frac{|K|}{|B_{\ell_2^k}|} \right)^{\frac{1}{k}}
\]

\[
= c_2 \left( \frac{|W(B_{E^*})|}{|B_F|} \right)^{\frac{1}{k}} \sqrt{k} \left( \frac{|V(B_F)|}{|B_{\ell_2^k}|} \right)^{\frac{1}{k}}
\]

\[
\leq c_2 \|W\| c_1 \|V\| \leq c_0 c_1 c_2 (1 + \ln n) .
\]

Hensley’s theorem implies the assertion, see [HEN] and [JU].

Final remark 7 For the proofs of theorem 3, 4 we have used operator ideal techniques. This allows us to formulate the results in a little bit stronger form which is similar to the formulation of Bourgain’s theorem in the introduction. Let \( K \subset \mathbb{R}^k \) then we have

\[
|K|^{\frac{k-1}{k}} \leq \inf \left\{ \left( \frac{|B_E|}{|K|} \right)^{\frac{1}{k}} \Big| E \subset X \text{ with unc. basis and } \dim X = n \right\}
\]

\[
\times 2e \sqrt{6 + 3 \ln \frac{n}{k}} \sup_{H \text{ hyperplane}} |K \cap H|
\]
and

$$|K|^{k-1} \leq \inf \left\{ \left( \frac{|B_E|}{|K|} \right)^{\frac{1}{2}} \right\} E \text{ quotient of } X \text{ with unc. basis and } \dim X = n \}
\times c_0 (1 + \ln n) \sup_{H \text{ hyperplane}} |K \cap H| .$$

References

[BA] K. M. Ball: Normed spaces with a weak-Gordon-Lewis property; Proc. of Funct. Anal., University of Texas and Austin 1987-1989, Springer Lect. Notes 1470, 36-47.

[BAPA] K. Ball and A. Pajor: Convex bodies with few faces; preprint.

[BM] J. Bourgain and V. D. Milman: New volume ratio properties for convex symmetric bodies in $\mathbb{R}^n$; Inv. Math. 88 (9187), 319-340.

[FIJ] T. Figiel and W.B. Johnson: Large subspaces of $\ell^n_\infty$ and estimates of the Gordon-Lewis constants, Isr. J. of Math. 37 (1980), 92-112.

[GL] Gluskin; Extremal properties of rectangular parallelepips and their application to Banach spaces; Math. Sbornik 136 (178)(1988), 85-95.

[GOS] Y. Gordon and P. Saphar: Ideal norms on $E \otimes L_p$, Illinois J. of Math. 21 (1979), 266-285.

[HEN] D. Hensley: Slicing convex bodies-bounds of slice area in terms of the body’s covariance; Proc. of AMS 79 (1980), 619-625.

[JU] M. Junge: Hyperplane conjecture for spaces of $\ell_p$; preprint.

[LO] G. Ya. Lozanovskii: On some Banach lattices; Siberian Math. J. 10 (1969), 419-431.

[MCM] P. McMullen: Volume of projections of unit cubes; Bull. London Math. Soc. 16 (1984), 278-280.

[MEP] M. Meyer and A. Pajor: Sections of the unit ball of $\ell^n_p$; J. of Funct. Anal. 80 (1988),109-123.

[MIPA] V. D. Milman and A. Pajor: Isotropic position, inertia ellipsoids and zonoid of the unit ball of a normed n-dimensional space; GAFA Seminar ’87-89, Springer Lect. Notes in Math. 1376 (1989), 64-104.

[MIS] V. D. Milman and G. Schechtman: Asymptotic theory of finite dimensional normed spaces; Springer Lect. Notes in Math. 1200 (1986).
[RE] S. Reisner: *Random Polytopes and the volume product of symmetric convex bodies*; Math. Scand. 57 (1985), 386-392.

[PS] G. Pisier: *Factorization of linear operators and Geometry of Banach spaces*; CBMS Regional Conference Series no 60, AMS 1986.

[PI1] A. Pietsch: Operator ideals; VEB Berlin 1979 and North Holland 1980.

[PI2] A. Pietsch: *Operator Ideals*; Deutscher Verlag Wiss., Berlin 1978 and North Holland, Amsterdam-New York-Oxford 1980. Cambridge University Press, 1987.

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