Variation of the fine structure constant in QSO spectra from coherent dark matter oscillations

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ABSTRACT

We consider the problem of the evolution of the fine structure coefficient α under the assumption that the scalar field coupling to the Maxwell term satisfies the condition mt ≫ 1 for coherent dark matter oscillations.

In this case we find that the coupling scale f in the leading order coupling \(-\phi/4f)F_{\mu\nu}F^{\mu\nu}\) affects the cosmological evolution of \(\alpha\) according to \(\ln(\alpha/\alpha_0) \propto \xi(m_{Pl}/f) \times \ln(\tanh(t/2\tau)/\tanh(t_0/2\tau))\). A fit to the QSO observations by Murphy et al. yields \(f = \xi \times 2.12^{+0.35}_{-0.37} \times 10^9 m_{Pl}\). Here \(m_{Pl} = (8\pi G_N)^{-1/2}\) is the reduced Planck mass, and \(\xi^2 = g_\phi/g_m\) parametrizes the contribution of \(\phi\) to the matter density in the universe.

Key words: atomic data – cosmology: theory – dark matter.

1 INTRODUCTION

The question whether the value of Sommerfeld’s fine structure constant should actually be determined through the dynamics of a scalar field had been addressed already by Fierz (1956) and Jordan (1956), whose investigations were partly motivated by Kaluza–Klein theory and by Dirac’s proposal of a variability of constants over cosmological time scales. Nowadays it is well known that dynamical gauge couplings are predicted by string theory, and the relevant coupling e.g. of the heterotic string dilaton to gauge fields in four dimensions is of gravitational strength \(f^{-1} = (16\pi G_N)^{1/2} = \sqrt{2}m_{Pl}^{-1}\) (Bekenstein 1982). In an independent development Bekenstein (1982) had introduced a class of models for dynamical \(\alpha\) where the evolution of the fine structure constant is driven through couplings to energy densities.

Therefore there was always theoretical interest in dynamical models for \(\alpha\), but in recent years Webb et al. also reported evidence for a variation of the fine structure coefficient over cosmological time scales (Webb et al. 1999; Murphy et al. 2001; Webb et al. 2001; Murphy, Webb & Flambaum 2003). The analysis of 128 quasar absorption lines by Murphy et al. yields \(f = \xi \times 2.12^{+0.35}_{-0.37} \times 10^9 m_{Pl}\). Here \(m_{Pl} = (8\pi G_N)^{-1/2}\) is the reduced Planck mass, and \(\xi^2 = g_\phi/g_m\) parametrizes the contribution of \(\phi\) to the matter density in the universe.

\[\frac{\Delta \alpha}{\alpha} = \frac{\alpha_z - \alpha_0}{\alpha_0} = -0.534 \pm 0.116 \times 10^{-5}, \quad (1)\]

since redshift \(z = 1.67\), i.e. over the last \(\approx 9.6\) billion years. Here \(\alpha_z\) is the fine structure constant at redshift \(z\) and \(\alpha \equiv \alpha_0 \equiv \alpha(t_0)\). In Murphy et al. (2003) results are reported for 143 quasar absorption lines, yielding \(\Delta \alpha/\alpha = -(0.57 \pm 0.11) \times 10^{-5}\) since \(z = 1.75\), but in our present analysis we used the well documented sample from Murphy et al. (2003).

Dynamical gauge couplings can equivalently be expressed as dynamical permeabilities, see e.g. Maguire, Sandvik & Kibble (2001). Suppose \(q\) is a particular fixed value for the dynamical gauge coupling \(Q(x)\) \((q\) will be further specified below). With the transformation \(qA_\mu(x) = Q(x)A_\mu(x)\) the covariant derivatives can be written in terms of a variable or a constant gauge coupling

\[D_\nu(x) = \partial_\nu - iQ(x)A_\nu(x) = \partial_\nu - iA_\nu(x),\]

but in the theory with the manifestly variable gauge coupling \(Q(x)\) the field strength tensor is

\[F_{\mu\nu} = i\frac{Q}{Q} [D_\mu, D_\nu] = \frac{1}{Q} \partial_\nu (Q A_\mu) - \frac{1}{Q} \partial_\mu (Q A_\nu)\]

while the field strength tensor with the constant coupling has the standard form \(F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu = (Q/q)F_{\mu\nu}\).

As a consequence the gauge theory with variable coupling and constant permeability appears as a gauge theory with constant coupling and variable permeability:

\[-\frac{1}{4\mu_0} F_{\mu\nu}(x) F^{\mu\nu}(x) = \frac{-q^2}{4\mu_0 Q^2} F_{\mu\nu}(x) F^{\mu\nu}(x).\]

The dynamical coupling constant for charge \(Ze\) and the dynamical permeability in SI units are

\[Q(x) = Ze(x)/\hbar = 2Z\sqrt{\pi\epsilon_0 c\alpha(x)/\hbar}\]
and
\[ \mu(x) = \mu_0 \alpha(x)/\alpha_0 = \frac{4\pi \hbar}{e c^2 \alpha_0} \alpha(x), \]
respectively. In the sequel we use units with \( \hbar = c = 1 \).

The scalar variable \( Q(x) \) may not have a canonically normalized kinetic term. Therefore a transformation \( Q(x) = Q(\phi(x)) \) may be required if we want in leading order a standard \((\phi^2)^2\) term for the dynamics of \( Q \):
\[ \mathcal{L} = -\frac{\kappa^2}{4Q^2(\phi)} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi). \] (2)

Here we assume that the potential \( V(\phi) \) has a unique minimum \( V(\phi_0) \) at some value \( \phi = \phi_0 \), and we parametrize the scalar field \( \phi \) such that \( \phi_0 = 0 \). Furthermore, any non-vanishing term \( V(0) \) would contribute to the cosmological constant and will not be considered as part of the energy density \( \phi_0 \) stored in the scalar field \( \phi \). The leading order expansion of the potential is then
\[ V(\phi) \simeq \frac{1}{2} m^2 \phi^2, \quad m^2 = \frac{d^2 V(\phi)}{d\phi^2} \bigg|_{\phi=0}. \] (3)

With this proviso it seems prudent to choose \( q = Q(0) \) as the equilibrium value to which the gauge coupling should evolve due to the presence of the Hubble term \( 3H \dot{\phi} \). In leading order this implies the following parametrization for the coupling function \( Q(\phi) \):
\[ Q^2(\phi) \simeq q^2 \left( 1 - \frac{\phi}{f} \right), \] (4)

with the coupling scale defined accordingly
\[ q = \frac{1}{f} \left. \frac{dQ^2(\phi)}{d\phi} \right|_{\phi=0}. \] (5)

Besides the convention \( \phi_0 = 0 \) for the equilibrium position this also implies a sign convention on the field \( \phi \) if we require \( f > 0 \): \( \phi \) is chosen as positive if it reduces the fine structure constant in first order (see also e.g. \( \text{Damour & Nordtvedt} \) 1993), and the observations of Murph et al. then indicate that \( \phi \) is decaying from a positive value towards its equilibrium value \( \phi_0 = 0 \).

Examples of specific coupling functions are provided e.g. by string theory or Kaluza–Klein theories:
\[ Q^2(\phi) = q^2 \exp(-\phi/f), \] (6)
and the Coulomb problem in these theories exhibits an ultra-violet regularization at a scale\(^2\) \( r_f = q/(8\pi f) \) (\( \text{Dick} \) 1997).

For the present investigation we will not specify the coupling function \( Q(\phi) \) any further but only use the linear expansion \( \phi \). \( \text{Landau & Vucetich} \) 2002 reconsidered the original Bekenstein model and concluded that it would not comply with the observations of Murphy et al. In a very interesting extension \( \text{Olive & Pospelov} \) 2002 investigated a model where the dynamics of a massless scalar field \( \phi \) is mostly driven by its couplings to dark matter and the cosmological constant, and they analyzed compatibility of \( \mathcal{E} \) with various constraints on variations of \( \alpha \). Our coupling parameter \( f \) is related to the parameters \( M_*, \xi_\phi \) and \( \omega_\phi \) in \( \text{Olive & Pospelov} \) (2002) through \( f = M_*/\xi_\phi = \sqrt{\omega_\phi M_{pl}/\xi_\phi} = \sqrt{\omega_\phi M_{pl}/\xi_\phi} \), and their results favor \( f > 10^3 M_{pl} \), corresponding to a sub-gravitational coupling strength of \( \phi \) to photons.

\( \text{Gardner} \) 2003 has recently discussed the implications of a mass term on the evolution of the fine-structure constant, and reported it to be consistent with mass values for the scalar field \( \phi \) around \( m \approx H_0 \approx 10^{-33} \) eV. Three crucial assumptions in Gardner's work are that the contribution of \( \phi \) to the dark matter density is negligible, that \( f \leq m_{pl} \), and that \( 0 < (\zeta_m m^2_{pl}) ^2 < 10^{-5} \), where \( \zeta_m \) is the coupling of \( \phi \) to matter. A low mass value \( m \sim H_0 \) was also preferred in the recent work by \( \text{Anchordoqui & Goldberg} \) 2003, who identified \( \phi \) with the quintessence field, and contrary to Gardner also assumed \( f > 10^3 M_{pl} \) in accordance with \( \text{Olive & Pospelov} \) 2002.

\( \text{Copeland, Nunes & Pospelov} \) 2003 also identified \( \phi \) with the quintessence and concluded that \( f \sim 10^3 M_{pl} \) to fit the QSO data. However, this result did not comply with the Oklo constraint, and Copeland et al. proposed that a photon momentum dependence of \( f \) around 10 MeV may suppress the effects of dynamical \( \alpha \) in nuclear reactions.

In the present paper we propose yet another analysis of the implications of the results of Murphy et al. for a dynamical fine structure constant. In particular we assume \( m > 10^{-28} \) eV for the mass of the scalar field \( \phi \) governing the evolution of \( \alpha \). Under this assumption a very weakly coupled field behaves like pressureless dust ever since dust domination, even though it may not satisfy the usual thermal dust condition\(^3\) \( m \gg T \). The virtue of \( m > 10^{-28} \) eV for our present analysis is that under this condition we can use the late time behavior of \( \phi(t) \) for \( t \gg m^{-1} \) to characterize the evolution of \( \phi \) ever since radiation–dust equality. Furthermore, our ignorance about evolution of \( \phi \) during radiation domination can be collected in a single parameter\(^4\) \( \xi = (\phi_0/\zeta_m)^{1/2} \), and we perform a least squares fit of the time evolution of \( \alpha \) in our model to the \( \alpha_0 \) values reported by \( \text{Murphy, Webb & Flambaum} \) 2003.

This explores a completely different mass range than \( \text{Gardner} \) 2003. For \( mt \gg 1 \) the mass term generates temporal and spatial fluctuations of \( \phi \) at scales \( m^{-1} \), but the Hubble expansion damps these oscillations \( \propto t^{-1} \), such that the amplitude of these oscillations is well below current observational limits from laboratory based search experiments for variable \( \alpha \). Furthermore, with \( mt \gg 1 \) we will be able to use a virial theorem to eliminate \( m \) from the long term variation of \( \phi \). The fit of the long term behavior of \( \alpha(t) \) de-

\(^2\) Subsequently the abelian and non-abelian Coulomb problem was also found to be exactly solvable for other functions \( Q(\phi) \) (\( \text{Dick} \) 1999, \( \text{Chabab, Markazi & Said} \) 2003, \( \text{Sluarczyk & Wereszczyński} \) 2001, 2003), and with mass couplings of \( \phi \) (\( \text{Dick} \) 1997b, \( \text{Bekenstein} \) 2002).

\(^3\) This general result that non-thermal coherent oscillations behave like cold dark matter was observed for the first time in axion physics (\( \text{Abbott & Sikivie} \) 1983, \( \text{Dine & Fischler} \) 1983, \( \text{Preskill, Witten & Wilczek} \) 1983).

\(^4\) \( \xi \approx 1 \) would imply that \( \phi \) is a dominant cold dark matter component. This possibility was pointed out for heavy dilatons (\( m \gg T \), dilaton WIMPS) by (\( \text{Gasperini & Veneziano} \) 1994) and by (\( \text{Damour & Vilenkin} \) 1996), and for oscillations of light dilatons (\( T \gg m \gg t^{-1} \)) by (\( \text{Dick} \) 1997c).
riever to the data of Murphy et al. then allows us to estimate the parameter \(f/\xi\).

For cosmological parameters we rely on the recent evaluation from the Wilkinson Microwave Anisotropy Probe (WMAP) and the Sloan Digital Sky Survey (SDSS) (Spergel et al. 2003; Tegmark et al. 2003). We use in particular the values from the “vanilla lite” model (Tegmark et al. 2003):

\[
\Omega_\Lambda = 0.707^{+0.031}_{-0.039}, \quad h = 0.708^{+0.023}_{-0.024},
\]

(7)

\(t_0 = 13.40^{+0.12}_{-0.12}\) Gyr.

We included the errors given by Tegmark et al. (2003) for illustration, but do not use them for error propagation. They are negligible compared to the uncertainties in the \(\alpha\) values for the QSO absorption systems, which generate a 1\(\sigma\) uncertainty of about 22\%, see Eq. (20) below.

The dependence of the WMAP CMB results on \(\alpha\) is relatively weak in that it complies with 0.95 < \(\alpha_{\text{dec}}/\alpha_{\text{obs}}\) < 1.02 (Rocha et al. 2003). The dynamical evolution of \(\alpha\) calculated below implies that at the time of decoupling \(0 > (\alpha_{\text{dec}} - \alpha) \alpha > -1 \times 10^{-4}\), such that at this stage we can safely use WMAP results on cosmological parameters for the determination of \(f/\xi\).

Sec. 4 recalls the relevant features of the dynamical evolution of a scalar field in an expanding universe and includes a virial theorem that will be useful in the analysis of dynamical gauge couplings.

Our main result in Sec. 4 is an equation for the evolution of \(\alpha(t)\) from the \(\phi-\gamma\) coupling, and the fit to the results of Murphy et al. in Sec. 4 yields \(f/\xi\). In Sec. 5 we will compare the time evolution of \(\alpha\) in our model with the Oklo and meteorite constraints, and Sec. 6 contains our conclusions.

2 THE COSMOLOGICAL EVOLUTION OF \(\phi\)

With \(mt \gg 1\) the mass parameter induces spatial and temporal fluctuations of \(\phi\), and therefore of \(\alpha\). One might hope to use this to determine \(m\) from a Fourier decomposition of observations of \(\alpha\) over cosmological distances. Our primary interest here is the coupling scale \(f\) of the scalar field to photons, and the strategy is to use a fit of the long term evolution of \(\phi\) in the expanding universe to the time variation of \(\alpha\) reported by Murphy et al. At this stage this allows us to infer a value for \(f/\xi\).

\(\phi\) is usually assumed to have at most extremely weak matter couplings, and the long term evolution of very weakly coupled helicity states follows

\[
\dot{\phi}(t) + 3H(t)\dot{\phi}(t) + m^2\phi(t) = 0,
\]

(8)

with a corresponding evolution of the comoving energy density

\[
\sqrt{-g}\ddot{\phi} = a^3(\dot{\phi}^2 + m^2\phi^2)/2.
\]

(9)

Note that in Eq. 8 we used the convention \(\phi_0 = 0\), cf. 4.

The solution of Eq. 8 for \(m = 0\),

\[
\dot{\phi}(t) \propto a^{-3}(t)
\]

implies \(\phi\propto a^{-6}(t)\), as appropriate for the ultrahard fluid component generated by massless weakly coupled helicity states (see e.g. Dick 2003).

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The solution of Eq. 8 for \(m > 0\) and \(a \sim t^{2/3}\)

\[
\phi(t) = \frac{1}{\sqrt{t}} \left( AJ_\frac{1}{2}(mt) + BY_\frac{1}{2}(mt) \right)
\]

has asymptotics for \(mt \gg 1\):

\[
\phi(t) \propto \frac{1}{t} \cos(mt + \varphi),
\]

\[
\phi_\alpha \propto t^{-2} \propto a^{-3}.
\]

(10)

This means that at late times \(p_\alpha \simeq 0\) since

\[
\frac{d}{dt} \left( g_\alpha a^3 \right) \simeq 0,
\]

just as for thermalized non-relativistic matter, but here even for \(m \leq T\).

Eq. 8 can be used to express the difference of the comoving kinetic and potential energy densities as a time derivative

\[
2a^3\mathcal{H}_{\text{kin}} - 2a^3\mathcal{H}_{\text{pot}} = a^3\ddot{\phi}^2 - m^2\dot{\phi}^2 = \frac{d}{dt}(a^3\dot{\phi}).
\]

(11)

This implies for the time limit \(\mathcal{H} = \lim_{t \to \infty} \frac{1}{\tau} \int_0^{\tau} dt \mathcal{H}(t)\) a virial theorem

\[
a^3\mathcal{H}_{\text{kin}} = a^3\mathcal{H}_{\text{pot}},
\]

\[
a^2\dot{\phi} \equiv a^3\mathcal{H} = m^2\dot{\phi}^2 + a^3\ddot{\phi}.
\]

(12)

However, note that at late times \(a^3\dot{\phi} \propto t^0\), and therefore Eq. 11 also yields

\[
\mathcal{H}_{\text{kin}} \simeq \mathcal{H}_{\text{pot}},
\]

i.e.

\[
|\dot{\phi}| \simeq m|\phi|,
\]

(14)

\[
\mathcal{H} \simeq |\dot{\phi}|.
\]

(15)

This relation can be used to trade the mass dependent \(|\dot{\phi}|\) for the energy density \(g_\phi\) in the late time evolution \((t \gg m^{-1})\) of \(\alpha\).

3 THE COSMOLOGICAL EVOLUTION OF \(\alpha\)

From Eq. 11 we have with \(\alpha \equiv \alpha_0 \equiv \alpha(t_0)\):

\[
\frac{\alpha(t)}{\alpha = \frac{e^{2\phi(t)}}{e^2} \simeq 1 - \frac{\dot{\phi}}{f},
\]

and with 15

\[
\left|\frac{\dot{\alpha}(t)}{\alpha}\right| \simeq \left|\frac{\dot{\phi}}{f}\right| \simeq \frac{\sqrt{g_\phi}}{f}.
\]

(16)

We know from Eq. 11 and Eq. 15 in the Appendix that

\[
\dot{\phi}_\alpha(t) = g_{\phi,0} \left( \frac{a_0}{a(t)} \right)^3 g_{m,0} \dot{\phi}_m(t) = g_{\phi,0} \frac{\Lambda}{g_{m,0} \sinh^2(t/\tau)}.
\]

We denote the contribution from the dynamical gauge coupling to the matter density by \(\xi^2 = g_{\phi,0}/g_{m,0}\), and find for the rate of change of the fine structure constant

\[
\left|\frac{\dot{\alpha}(t)}{\alpha}\right| \simeq \xi \frac{\sqrt{\Lambda}}{f \sinh(t/\tau)}.
\]

(17)
4 THE COUPLING SCALE

The parameter \( f/\xi \) was determined from a fit of Eqs. (18, 20) to the \( \alpha(z) \) values reported by Murphy, Webb & Flambaum (2003). We set

\[
y_i = \ln \left( \frac{\alpha(t_i)}{\alpha(t_0)} \right),
\]

\[
x_i = \ln \left( \frac{\tanh(t_i/2\tau)}{\tanh(t_0/2\tau)} \right),
\]

and the minimal variance

\[
\chi^2 = \sum_i \left( \frac{y_i - s x_i}{\delta y_i} \right)^2
\]

in the fit of \( \chi^2 \) occurs for a slope

\[
s = \xi \frac{2m_{Pl}}{\sqrt{3}f} = \frac{\sum_i x_i y_i/\delta y_i^2}{\sum_j x_j^2/\delta y_j}
\]

with variance

\[
\sigma_s^2 = \sum_i \delta y_i^2 (\partial s/\partial y_i)_y = \left( \sum_i x_i^2/\delta y_i^2 \right)^{-1}.
\]

The errors in \( \ln(\alpha_z/\alpha_0) \) are related to the errors in \( \Delta \alpha/\alpha_0 \) through

\[
\delta y_i = \frac{\delta \Delta \alpha}{\alpha_0 + \Delta \alpha}
\]

This method yields a slope

\[
s = (5.443 \pm 1.174) \times 10^{-6}
\]

corresponding to a coupling parameter

\[
\frac{f}{\xi} = 2.12^{+0.58}_{-0.37} \times 10^3 m_{Pl}.
\]

The variance per degree of freedom is \( \chi^2_{\text{dof}} = 1.067 \).

The resulting asymptotic equilibrium value of the fine structure constant is

\[
\lim_{t \to \infty} \frac{\alpha(t)}{\alpha(t_0)} = \tanh(t_0/2\tau)^{-s} \approx 1 + (3.3 \pm 0.7) \times 10^{-6}.
\]

The current rate of change of \( \alpha \) from Eqs. (18, 20)

\[
\frac{\dot{\alpha}}{\alpha} = \frac{s}{4\tau \sinh(t_0/\tau)} = (4.0 \pm 0.9) \times 10^{-17} \text{ yr}^{-1}
\]

is within the bounds from current atomic clock experiments (Marion et al. 2003):

\[-2.0 \times 10^{-16} \text{ yr}^{-1} \leq \frac{\dot{\alpha}}{\alpha} \leq 1.2 \times 10^{-16} \text{ yr}^{-1}.
\]

5 COMPARISON WITH THE OKLO AND METEORITE CONSTRAINTS

With the cosmological parameters \( \Omega_i \) and Eq. (18) the closest QSO absorption system used in Murphy, Webb & Flambaum (2003) corresponds to a distance of about 2.7 billion light years. A well known more recent constraint on variations of \( \alpha \) over cosmological time scales comes from isotope abundances in the natural Oklo reactor, which had been active about 1.8 billion years ago (Shlyakhter 1976; Damour & Dyson 1996; Fujii 2003).

The recent evaluation by Fujii (2003) yields a bound

\[
\frac{\alpha_{\text{Oklo}} - \alpha}{\alpha} = -(0.8 \pm 1.0) \times 10^{-8},
\]

whereas insertion of (21) into (18) yields

\[
\frac{\alpha_{\text{Oklo}} - \alpha}{\alpha} = -(6.38^{+1.37}_{-1.58}) \times 10^{-7}.
\]

The situation appears to be different with the \( ^{187}\text{Re} \) constraints, which limit the evolution of \( \alpha \) over the last 4.6 Gyr (Peebles & Dicke 1962; Dyson 1972; Olive et al. 2003). The most recent constraint from Olive et al. (2003) is

\[
\alpha_{4.6} - \alpha = -(0.8 \pm 0.8) \times 10^{-6},
\]

while Eqs. (21, 18) yield

\[
\alpha_{4.6} - \alpha = -(1.95 \pm 0.42) \times 10^{-6}.
\]

Olive & Pospelov (2002) and Gardner (2003) could fit their externally driven models for the evolution of \( \alpha \) to both the QSO data and the Oklo constraint, whereas Anchordoqui & Goldberg (2003) and Copeland, Nunes & Pospelov (2003) and we find a lower value of \( \alpha \). However, Mota & Barrow (2003) have recently pointed out that the local variation of \( \alpha \) in virialized overdensities like our own can relax the Oklo constraint by a factor 10-100, because on the one hand a dynamical \( \alpha \) would be expected to have a higher value in overdensities, while on the other hand virialization of overdensities slows down the evolution of \( \alpha \). The qualitative picture emerging from this is that in overdensities \( \alpha \) evolves from a higher initial value after virialization, but at slower pace, whence it approaches again the value in the low-density background universe. This can explain discrepancies between astrophysical and geochemical observations.

6 CONCLUSIONS

We have found the equation (18) for the dynamical time evolution of \( \alpha \) due to the coupling \(-\phi/4f\sqrt{F_{\mu\nu}F_{\nu\mu}}\) of electromagnetic fields to a very weakly coupled massive scalar field
with $mt \gg 1$. A fit of this equation to the quasar absorption data reported by [Murphy, Webb & Flambaum 2003] yields the value 24, where $\xi = \sqrt{\phi_0 / \rho_m}$ parametrizes the contribution of the scalar field $\phi$ to the matter density.

Within this model the evolution of $\alpha$ reported by Murphy et al. appears to be slow due to a small coefficient $s = (5.443 \pm 1.174) \times 10^{-6}$ in Eq. (15): The fine-structure constant varied so little since $z = 3.66$ because the $\phi$ abundance is small and the $\phi - \gamma$ coupling is very weak and presumably of subgravitational strength, in agreement with the analyses of [Olive & Pospelov 2003; Anchordoqui & Goldberg 2003 and Copeland, Nunes & Pospelov 2003].

Our coherent oscillation model for dynamical $\alpha$ (and other self-driven models of dynamical $\alpha$ [Anchordoqui & Goldberg 2003; Copeland, Nunes & Pospelov 2003]) still seems to predict a too small value of $\alpha$ at the time when the Oklo natural reactor was active. However, as Mota & Barrow 2003 have pointed out, at low redshift predictions of faster evolution of $\alpha$ from astrophysical observations are to be expected due to spatial variations in the presence of local overdensities. Note that this does not affect the numerical results 20,21,22 (or the corresponding results of Anchordoqui & Goldberg 2003 and Copeland, Nunes & Pospelov 2003), since the resulting discrepancy of $\alpha_0$ on Earth and far away from virialized objects is smaller than the number of significant figures reported5.

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5 We should emphasize that the spatial variation effect discussed by [Mota & Barrow 2003] cannot be used as an argument against the models of [Olive & Pospelov 2002] and [Gardner 2003], since externally driven models also rather tend to predict a smaller $\alpha$ than the value inferred from the Oklo data. Externally driven models only seem to provide more leeway to fit time evolutions of $\alpha$.

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APPENDIX A: THE SCALE FACTOR IN THE $\Lambda$CDM UNIVERSE

The evolution of the scale factor in a spatially flat $\Lambda$CDM universe follows from direct integration of the corresponding Friedmann equation

$$\frac{a^2}{a^2_0} = \frac{\varrho_m + \Lambda}{3m^2_{Pl}} \tag{A1}$$

after insertion of

$$\varrho_m(t) = \varrho_{m,0} \left( \frac{a_0}{a(t)} \right)^3.$$  

Integration from $t_0$ to $t$ yields

$$\frac{\sqrt{3\Lambda}}{2m_{Pl}} (t - t_0) = \ln \left( \frac{\sqrt{\Lambda a^3 + \sqrt{\Lambda a^3 + \varrho_{m,0} a^3_0}}}{\sqrt{\Lambda a^3_0} + \sqrt{\Lambda + \varrho_{m,0} a^3_0}} \right).$$

Solving for the scale factor yields

$$\left( \frac{a}{a_0} \right)^{3/2} = \exp \left( \frac{t - t_0}{\tau} \right) + \frac{\varrho_{m,0}}{\Lambda + \sqrt{\Lambda^2 + \Lambda \varrho_{m,0}}} \sinh \left( \frac{t - t_0}{\tau} \right), \tag{A2}$$

with the time constant

$$\tau = \frac{2m_{Pl}}{\sqrt{3\Lambda}} \tag{A3}$$

We can simplify our result [A2] because the highest redshift $z = 3.66$ used in the analysis still corresponds to an age $t(z) \simeq 1.7$ Gyr $\gg t_{eq}$ much larger than the time $t_{eq} \simeq 1.3 \times 10^9$ yr of matter radiation equality. At times $\gg t_{eq}$ the modification of the time evolution of the scale factor during the very early radiation dominated era can be neglected, and one can integrate Eq. [A1] from $t_1 = 0$ and still get an extremely good approximation for $t \gg t_{eq}$. This yields

$$\frac{a}{a_0} = \left( \frac{\sinh(t/\tau)}{\sinh(t_0/\tau)} \right)^{2/3} \tag{A4}$$

and

$$\varrho_m = \frac{\Lambda}{\sinh^2(t/\tau)}. \tag{A5}$$

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