Broadband Focusing Acoustic Lens Based on Fractal Metamaterials

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Acoustic metamaterials are artificial structures which can manipulate sound waves through their unconventional effective properties. Different from the locally resonant elements proposed in earlier studies, we propose an alternate route to realize acoustic metamaterials with both low loss and large refractive indices. We describe a new kind of acoustic metamaterial element with the fractal geometry. Due to the self-similar properties of the proposed structure, broadband acoustic responses may arise within a broad frequency range, making it a good candidate for a number of applications, such as super-resolution imaging and acoustic tunneling. A flat acoustic lens is designed and experimentally verified using this approach, showing excellent focusing abilities from 2 kHz and 5 kHz in the measured results.

Results
Design of fractal acoustic metamaterials and retrieved effective parameters. Two-dimensional spatial fractal structures, presented in Fig. 1a–c, illustrate Hilbert FAM elements of 1-order, 2-order, and 3-order, respectively. It is necessary to explain how varying degrees of parameters expected can be engineered through the FAM design. Up to now, coiling-up structures or folded channels have been employed for acoustic wave

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Application: Acoustic focusing lens based on the FAM elements. As a potential application, we set out to design and fabricate an acoustic focusing lens by use of FAM elements. As discussed previously, by gradually increasing the fractal order, a gradient refractive index can be easily obtained as a result. The hyperbolic secant distribution, defined as \( n(y) = n_0 \text{sech}(\alpha y) \), has been proved to reduce the aberration of the focal spot, where \( n_0 \) is the maximal refractive index at the center of the lens\(^{26,29}\). \( \alpha \) denotes the gradient index coefficient with \( \alpha = \frac{1}{h} \cosh^{-1}\left( \frac{h}{h_0} \right) \), where \( h_0 \) refers to the minimal refractive index at the edge of the lens and \( h \) is half-height of the lens.

From Fig. 2, the maximum and minimum refraction index of the FAM lens are 3.5 and 1.1 respectively. The lens can be discretized into 21 segments with the period 0.94 cm (Fig. 3c), covered by the elements F1-F11 with detailed material parameters and geometric dimensions listed in Table 1. To meet the requirement of the refraction index (Fig. 3b) for elements with different orders, the width \( W_j \), \( W_2 \), and \( W_3 \) should be carefully adjusted in the design. For example, with the increase of \( W_j \), the refraction index of the first order FAM elements grow slowly.
from 1 to 1.3 as shown in Fig. 4(a). By looking up Table 1, we can find that the elements F1 and F2 are actually 1st FAM unit with corresponding \( n \) to be 1.21 and 1.37 respectively, whose corresponding element width \( W_1 \) can be easily determined from the red curve in Fig. 4(a). As a result, the element width for other high order elements can also be achieved in a similar way, as indicated by asterisks in Fig. 4(a, b). The designed FAM focusing lens can be found in Fig. 3(c), where an ultrathin acoustic flat lens is obtained due to the high refractive index yielded by the fractal geometry structure. The thin focusing lens is composed by two layers, with the total thickness \( W_f = 1.73 \) cm, and the length of the lens \( L_f = 19.74 \) cm. All the orders of FAM elements have the same periodicity with \( U = 0.94 \) cm, which are aligned along the \( y \) direction.

To validate the performance of the FAM lens, both simulation and experiment have been carried out, as demonstrated in Fig. 3(d, e). During the numerical simulations, the perfect matched layers are used to prevent undesired reflections at the boundaries. The pressure field distributions at 2 kHz and 5 kHz along two orthogonal lines passing through the focus are extracted from the simulation, which is plotted in Fig. 3(d, e). The focal length is found to be 125 mm behind the lens and a narrow focusing spot can be clearly observed, which shows good focusing capability of the proposed FAM lens.

In the experiment, the two dimensional platform (detail information described in the methods section) is shown in Fig. 3(a), which is used to scan the sound pressure distributions within a plane slightly above the sample under test. The maxim scanning range is 120 mm \( \times \) 180 mm, and the gap between the microphone and the sample is 0.5 mm. The measured data is also presented in Fig. 3(d, e) for comparison, showing excellent agreement with the simulation results. From the simulations data (see solid lines in Fig. 3(d, e)), we can calculate the sound pressure amplitude at the focal point relative to the reference sound pressure as \( 10 \log \left( \frac{P_{\text{ref}}}{P_f} \right) = 11.3 \) dB at 5 kHz and 9.2 dB at 2 kHz, which is easy to understand since the radiation aperture becomes larger with the increase of the operation frequency. From Fig. 3(d, e), good agreement between the simulation and experimental results is observed as expected, verifying the accuracy and reliability of FAM element design.

To check the broadband property of the FAM focusing lens, the simulated and measured field distributions of the focusing lens are presented in Fig. 5 at 2 kHz, 3 kHz, 4 kHz, and 5 kHz respectively. From the simulated pressure distribution in Fig. 5 at different frequencies, the incident sound can be focused efficiently in a wide frequency range, and the focal spot is brighter and more concentrated when the frequency goes up, which is...
Figure 3. Experiment sample, simulated and measured results. (a) The schematic of the experimental setup. A plexiglass plate (not shown in the diagram) is located slightly above the sample. An eight-speaker array is used to generate a plane wave, and a microphone is used to sweep the area under test by a two-dimensional stepping motor and record the distributions of sound pressure around the sample. (b) The ideal refractive index and ideal impedance distributions required by the lens, and the available refractive index and realistic impedance distributions from the FAM elements. F1–F11 represent FAM elements distributions required by the lens. (c) The lens prototype with $L_f = 19.74$ cm, $U = 0.94$ cm, $W_f = 1.73$ cm, and $W_s = 0.5$ mm. (d) The distribution of sound pressure amplitude along a line passing through the focus in the x-direction. (e) The distribution of sound pressure amplitude along a line passing through the focus in the y-direction.

Figure 4. The relationship between the refractive index of FAM elements of different orders and the corresponding element width. The red, black (a) and blue (b) lines stand for the refraction index distributions of the 1st, 2nd, 3rd-order FAM elements at 5 kHz by sweeping the element width $W_1-W_3$. The asterisks stand for the refraction index of the lens unit F1–F11 according to that required in Fig. 3(b).
consistent to the analysis stated above. Due to limitation of our experimental platform, only a small region near the focus of the lens can be scanned, which is indicated by the white dashed lines in Fig. 5(a,c,e,g). The transmitted to incident power ratio is approximate to 5:1 at 5 kHz and 2:1 at 2 kHz from the experimental data. Comparing with the field-mapping results, the simulated results are in accordance with the actual conditions. However, there still exist small discrepancies at those frequencies that can be attributed to two reasons: one is sound leakage around the sample in the platform due to the unavailable small gaps between the sample and the upper plate, and the other is the accuracy of manufacture precision. To minimize the measurement error, a number of methods are employed in the experiment, including the use of elastic sound absorption materials to reduce the sound leakage, and high-precision microphone and transducer for detecting weak signals. The maximum impedance contrast between the lens and air appears at 5 kHz (see Fig. 5(g,h)), while the best impedance matching can be observed at 2 kHz (see Fig. 5(a,b)), since the large refractive index at the center of the lens usually requires larger dimensions of the FAM element at higher frequencies, which may deteriorate the impedance matching and in turn bring larger reflections at the lens surface.

Conclusion

In summary, we propose and demonstrate a new kind of broadband FAM element for acoustic applications. By increasing the order of the fractal structure, it is easy to achieve high refractive index and therefore benefit the design of ultrathin acoustic devices such as lens and absorbers. This element shows non-resonant property with nearly constant bulk modulus and mass density at kilohertz frequencies. By carefully varying the dimensions of FAM element, a gradient refractive index can be achieved as a result. To demonstrate the performance of the resulting FAM elements, an acoustic focusing lens is proposed and experimentally verified. The lens is much thinner than previously reported results in the literature, due to the high refractive index achieved by the FAM element, which is especially useful for the design of novel acoustic devices in the future.

Methods

Numerical simulations. Numerical simulations based on finite element method (FEM) are carried out by COMSOL Multiphysics throughout this paper. In the numerical simulations, the frequency domain Helmholtz equation for sound pressure is solved in the air region, while in the photopolymer resin region the equation of structural mechanics for harmonic stresses and strains is solved. The acoustic waves are incident from the left side along x direction shown in Fig. 3(a), with the amplitude of sound pressure 1 Pa. The material parameters used in
the numerical simulations are mass density of air $\rho_0 = 1.29 \text{ kg/m}^3$, sound velocity in air $c_0 = 343 \text{ m/s}$, mass density of photopolymer resin $\rho = 1300 \text{ kg/m}^3$, sound velocity in photopolymer resin $c = 716 \text{ m/s}$, bulk modulus of photopolymer resin $B = 666.47 \text{ MPa}$, and Poisson's ratio of photopolymer resin $\sigma = 0.41$.

**Field mapping measurement.** The field mapping measurement is performed in a two-dimensional acoustic scanning platform. A linear 8-speaker array is used to generate a plane wave, and a microphone is used to scan the area under test and record the sound pressure distributions based on a 2D stepping motor controlled by a computer. The FAM elements are made of photopolymer resin thermal plasmas and fabricated via 3D printing to meet the requirement of the model.

**References**
1. Li, J. & Chan, C. T. Double-negative acoustic metamaterial. Phys. Rev. E 70, 055602 (2004).
2. Cummer, S. A., Christensen, J. & Alù, A. Controlling sound with acoustic metamaterials. Nature Rev. Mater., 1, 7958–7965 (2016).
3. Fang, N. et al. Ultrasonic metamaterials with negative modulus. Nat. Mater. 5, 452–456 (2006).
4. Ding, Y. Q., Liu, Z. Y., Qiu, C. Y. & Shi, J. Metamaterial with simultaneously negative bulk modulus and mass density. Phys. Rev. Lett. 99, 093904 (2007).
5. Lu, M. et al. Negative birefringence of acoustic waves in a sonic crystal. Nature Mater. 6, 744–748 (2007).
6. Graziá-Salgado, R., García-Chocano, V. M., Torrent, D. & Sánchez-Dehesa, J. Negative mass density and $\nu$-near-zero quasi-two-dimensional metamaterials: design and applications. Phys. Rev. B 88, 224305 (2013).
7. García-Chocano, V. M., Christensen, J. & Sánchez-Dehesa, J. Negative refraction and energy funnelling by hyperbolic materials: an experimental demonstration in acoustics. Phys. Rev. Lett. 112, 144301 (2014).
8. Liu, F. & Liu, Z. Elastic waves scattering without conversion in metamaterials with simultaneous zero indices for longitudinal and transverse waves. Phys. Rev. Lett. 113, 175502 (2015).
9. Li, J., Fok, L., Yin, X., Bartal, G. & Zhang, X. Experimental demonstration of an acoustic magnifying hyperlens. Nat. Mater. 8, 931–934 (2009).
10. Kaina, N., Lemoult, F., Fink, M. & Lerosey, G. Negative refractive index and acoustic superlens from multiple scattering in single negative metamaterials. Nature 525, 77–81 (2015).
11. Li, J. & Pendry, J. B. Hiding under the carpet: a new strategy for cloaking. Phys. Rev. Lett. 101, 203901 (2008).
12. Torrent, D. & Sánchez-Dehesa, J. Acoustic cloaking in two dimensions: a feasible approach. New J. Phys. 10, 063015 (2008).
13. Yang, Y., Wang, H., Yu, F., Xu, Z. & Chen, H. A metasurface carpet cloak for electromagnetic, acoustic and water waves. Sci. Rep. 6, 20219 (2016).
14. Chen, H. & Chan, C. T. Acoustic cloaking and transformation acoustics. J. Phys. D: Appl. Phys. 43, 113001 (2010).
15. Baz, A. Active acoustic metamaterials. J. Acoust. Soc. Am. 128, 2428 (2010).
16. Ma, G., Yang, M., Xiao, S., Yang, Z. & Sheng, P. Acoustic metasurface with hybrid resonances. Nat. Mater. 13, 873–878 (2014).
17. Cheng, Y., Zhou, C., Yuan, B., Wu, D., Wei, Q. & Liu, X. Ultra-sparse metasurface for high reflection of low-frequency sound based on artificial Mie resonances. Nat. Mater. 14, 1013–1019 (2015).
18. Wang, P., Casadei, F., Shan, S., Weaver, J. & Bertoldi, K. Harnessing buckling to design tunable locally resonant acoustic metamaterials. Phys. Rev. Lett. 113, 014301 (2014).
19. Lee, S., Park, C., Seo, Y., Wang, Z. & Kim, C. Composite acoustic medium with simultaneously negative density and modulus. Phys. Rev. Lett. 104, 054301 (2010).
20. Liang, Z. & Li, J. Extreme acoustic metamaterial by coiling up space. Phys. Rev. Lett. 108, 114301 (2012).
21. Li, Y., Liang, B., Zou, X. Y. & Cheng, J. C. Extraordinary acoustic transmission through ultra-thin acoustic metamaterials by coiling up space. Appl. Phys. Lett. 103, 063509 (2013).
22. Xie, Y., Popa, B. I., Zigoneanu, L. & Cummer, S. A. Measurement of a broadband negative index with space-coiling acoustic metamaterials. Phys. Rev. Lett. 110, 175501 (2013).
23. Xu, H., Wang, G., Tao, Z. & Cui, T. High-directivity emissions with flexible beam numbers and beam directions using gradient-refractive-index fractal metamaterial. Sci. Rep. 4, 5744 (2014).
24. Freznel, T., Brehm, J. D., Buckmann, T., Schittny, R., Kadic, M. & Wegener, M. Three-dimensional labyrinthine acoustic metamaterials. Appl. Phys. Lett. 103, 061907 (2013).
25. Zhu, X., Li, K., Zhang, P., Zhu, J., Zhang, J., Tian, C. & Liu, S. Implementation of dispersion-free slow acoustic wave propagation and phase engineering with helical-structured metamaterials. Nat. comm. 7, 11731 (2016).
26. Chen, X., Grzegorczyk, T. M., Wu, B.-I., Pacheco, J. & Kong, J. A. Robust method to retrieve the constitutive effective parameters of acoustic metamaterials. Phys. Rev. B 70, 014308 (2004).
27. Fokin, V., Ambati, M., Sun, C. & Zhang, X. Method for retrieving effective properties of locally resonant acoustic metamaterials. Phys. Rev. B 76, 144302 (2007).
28. Zigoneanu, L., Popa, B.-I. & Cummer, S. A. Design and measurements of a broadband two-dimensional acoustic lens. Phys. Rev. B 84, 024305 (2011).
29. Climente, A., Torrent, D. & Sanchez-Dehesa, J. Sound focusing by gradient index sonic lenses. Appl. Phys. Lett. 97, 104103 (2010).

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**Author Contributions**
G.Y.S., Q.C. and T.J.C. conceived the idea. G.Y.S. designed the sample and did the simulation. G.Y.S. and B.H. performed the experiment. H.Y.D. developed the theoretical explanations. G.Y.S., Q.C. and T.J.C. wrote the manuscript based on the input from all authors. All authors contributed to the discussion.

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