Mathematical Modeling of Antenna-Mast Structures With Aerodynamic Effects

A A Loktev¹, V V Korolev¹, O I Poddaeva ², I YU Chernikov³

¹ Russian University of Transport (MIIT), Chasovaya str., 22/2, Moscow, 125190, Russia
² Moscow State (National Research) University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia
³ Engineering center of technical expertise and diagnostics "Expert", street of Designers, 82, Voronezh, 394038, Russia

E-mail: aaloktev@yandex.ru

Abstract. At the present stage of development of the systems for wireless transmission of information, including providing high-quality communication channels of the transport systems, the important issue is to increase the coverage area of the electromagnetic waves emitted by base stations. The area of the region of the coating increases with the increase in installation height of transmitter, and at the same time, communication structures with height more than 75 meters it is necessary to additionally determine the aerodynamic coefficients of the wind load through purging models of buildings in wind tunnels or on the recommendations developed by specialized organizations. Such events can significantly increase the final cost of the project for the construction of antenna-mast structures, so relevant and timely is the development of mathematical models of the behavior of antenna-mast structures under wind influence, which would determine the aerodynamic coefficients at the gathering of loads on the designed or operated facility, and to adequately predict the behavior and condition of the property at different stages of the life cycle. In the work as the equations that define the dynamic behavior of the design used three-dimensional non-stationary nonlinear equations of fluid dynamics considering viscous resistance of the type of Navier-Stokes equations, the solution of which it is proposed to use a numerical analytical method, which allows to represent and calculate the desired functions in the form of discrete values. The results obtained agree well with the results of using verified software and computing systems, as well as with analytical solutions to test problems. At the present day, it is important to study the behavior of traditional constructions under various operating conditions of the rolling stock, including an increase in the frequency of dynamic effects and the superposition of effects from different loads and oscillations to achievement of the high-frequency range. High-frequency vibrations in the elements of the railway track and the rolling stock are more likely to appear during the traffic on artificial structures, such as bridges with different design diagrams and made from both reinforced concrete and steel. In such cases, the forced and free oscillation frequencies of different composite constructions of transport infrastructure are superimposed.

1. Introduction

The problem of providing wireless fast and high-quality channels to subscribers in terms of the location of base transmission stations could conceptually be solved in two ways: by installation of a
relatively small antenna set of transmission stations where the data traffic is heavier or, where there are no subscribers, it is possible to use fiber optic cables that link the clusters of the wireless communication system; another solution is the installation of rather power transmitters of base stations on the rather high antenna-mast structures and a uniform coverage of the whole area with radio channels. In terms of the Russian construction and maintenance standards, the calculations are more actual for communication structures with height more than 75 meters because the communication structures with height from seventy-five to one hundred meters according to design documentation are classified as particularly dangerous and technically complex communication structures [1], if the height is more than one hundred meters than these communication structures are classified as unique structures. For the structures with height more than 75 meters the provisions of domestic construction standards would use the data as well as conduct additional studies that can be quite expensive, especially for manufacturing of construction models and their further wind-tunnel test.

As a basis for the construction under consideration should be taken an antenna-mast structure which is made according to the three-dimensional diagram in the form of the lattice tower with height more than 75 meters. Generally modern design and computing systems with the permission of the formal application in the territory of the Russian Federation are applicable to the analysis of the three-dimensional bar diagram of the ultimate and service limit states and implement the finite element modeling of static and dynamic design diagrams, stability analysis, selection of disadvantaged stress combinations, reinforcement selection for concrete elements and verification of load-carrying ability of steel structures. This analysis is based on the finite element displacement method [2-4], as well as applicable binding reference documents [5-7]. But these software systems do not provide the full analysis of the mathematical model of the behavior of antenna-mast structures under the dynamic influence, as well as all aspects of pulsating wind load are applicable to each particular structural elements.

2. Traditional methods

Traditional methods. The standard strength and stability analysis of a designed or examined antenna-mast structure can be performed using the finite element method [2-3], which is implemented in different software systems such as Scad++, Mirage, ANSYS, etc. [2-4,8] verified and permitted in the territory of the Russian Federation. The structure design should be a set of standard bodies (bars, plates and massive bodies) attached to nodes.

The finite element type is determined by its geometric form and the rules that determine the dependence between the displacements of nodes of the finite element and system nodes, as well as by physical law that determine the dependence between the internal force and internal displacements, and also a set of parameters (rigidities) which are included in the description of this law. But this approach does not allow to accurately determine the effect of nonstationary processes of elastic wave propagation, aerodynamic influence of wind and other quick change types of loading on displacements of nodes and elements, as well as on forces in them, since according to the ideology of the finite element method, the true shape of the displacement field within the element is approximately presented by various simplified dependencies.

3. Constitutive equations

Constitutive equations. In order to determine the aerodynamic coefficients of each element or the whole design it is proposed that modeling of the dynamic behavior of an antenna-mast structure is considered with the three-dimensional non-stationary nonlinear Navier-Stokes equations for fluid dynamics, taking into account the viscous environmental resistance [9-10]:

\[
\begin{align*}
\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \rho \mathbf{w} \cdot \nabla \mathbf{u} & = -\nabla p + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right], \\
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{v} + \rho \mathbf{w} \cdot \nabla \mathbf{v} & = -\nabla p + \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right], \quad (1)
\end{align*}
\]
The solution of the constitutive equations (1) shall be obtained considering the ratio of continuity and state \[10,11\]:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right).
\]

The solution of the constitutive equations (1) shall be obtained considering the ratio of continuity and state \[10,11\]:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0,
\]

\[
p = \rho RT.
\]

The following notations are used in the relations (1) – (3): \(u, v, w\) – are the sought components of the velocity vector (around \(x, y, z\) axes), \(p\) is the pressure, \(t\) is the time, \(\mu\) is the dynamic coefficient of viscous resistance for air, \(\rho\) is the density, \(R\) is the absolute gas constant, \(T\) is the temperature.

In order to decrease the modeling complexity without any serious damage for the solution accuracy of engineering problems, the wind flows are supposed to be incompressible (\(\rho = \text{const}\)) and isothermal. The mass forces shall not be taken into account. The accurate analytical solution of the Navier-Stokes equations is only possible for several sufficiently simple problems without any application in the construction industry \[9-11\].

In foreign studies \[9-13\], the direct numerical simulation is often used for the solution of the Navier-Stokes equations which consists in the solution of the full three-dimensional non-stationary Navier-Stokes equations without the use of special turbulence models. But even with modern computer capabilities, this approach applies to the low flow velocity, so the researchers have to use different turbulence models that reduce the computational complexity of the problem by the introduction of some simplifying assumptions. An alternative approach applies to the large eddy simulation. For this purpose special filtering is used in deriving the equations for resolvable scales \[12,13\]. The important issue is that the eddies are small near plate design elements, so the order of energy-intensive and dissipated eddies coincides. This fact creates severe Reynolds number limitations for possible use of such models in engineering problems.

4. Solution method

In this study, it is proposed to use an approach based on the aggregation of the semi-empirical approach, which was based on the velocity solution into time-average and pulsation components \(v_i^t = \bar{v}_i + v_i^i\) \[6\], and on the use of the numerical-analytic method proposed in \[14-16\] and received approval in test problems \[16-18\] as well as on the equation representation (1) by the Reynolds averaged Navier-Stokes method \[15,17\]:

\[
\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial \rho \bar{v}_i}{\partial x_i} \rho \bar{v}_i \cdot \nabla \bar{v}_i = - \frac{\partial \rho}{\partial x_i} + \frac{\partial \rho}{\partial x_j} \left( \frac{\partial \bar{v}_i}{\partial x_i} + \frac{\partial \bar{v}_i}{\partial x_j} \right) \rho \nabla \bar{v}_i,
\]

\[
\frac{\partial \bar{v}_i}{\partial x_i} = 0
\]

where is the average pressure, the indices \(i=1,2,3\) and \(j=1,2,3\) correspond to the coordinates \(x, y, z\), \(\rho \nabla \bar{v}_i\) are the Reynolds stresses that are represented as six unknowns (in addition to averaged \(\bar{v}_i\) and \(\bar{v}\)) and are approximated by the Boussinesq hypothesis \[14,16\]:

\[
\rho \nabla \bar{v}_i = -\mu_i \left( \frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i} \right) + \frac{2}{3} \rho k \delta_i,
\]

where \(\mu_i\) is the additional viscosity caused by pulsations; \(k\) is the average energy of the turbulent pulsations.

To solve the equations (1) – (3) rewritten in the form (4), the boundary condition for the air velocity components \(v_i^i, i = 1,2,3\) shall be added to the constitutive relations. The air velocity components are velocity vector projections on the axis of the Cartesian coordinate system \((x,y,z)\) and pressure \(p\).
\[ v'_{\mid_{\Omega}} = 0, \quad i = 1, 2, 3, \quad v'_{\mid_{\partial\Omega}} = v'_{\infty}, \quad i = 1, 2, 3, \quad p_{\mid_{\partial\Omega}} = 0, \]  

where \( \Omega \) is the body of rotation about the \( z \)-axis; \( \partial\Omega \) is its boundary; \( v'_{\infty}(i = 1, 2, 3) \) is the free-stream velocity of a viscous fluid.

The system of the constitutive equations is proposed to represent in the curvilinear coordinate system \((r, \theta, \phi)\) \[11, 19, 20\] which makes it possible to simplify the final relations in some way. The original Cartesian coordinates can be written as follows:

\[ x = V(r, \theta) \cos \phi, \quad y = V(r, \theta) \sin \phi, \quad z = U(r, \theta). \]

In this case, the geometric functions \( U \) and \( V \) can be chosen on the basis of the fulfillment of the following conditions:

\[ \psi = U + iV, \quad z = r \exp (i\theta). \]

As we can see, the relations (7) define geometrically the mapping of a unit radius sphere onto the surface of the design element \( \Omega \). The analytical form of the functions \( U \) and \( V \) is presented in \[12, 15\].

The feature of the function representation \( V \) is its ambiguity with equality to zero because in this case the angle \( \phi \) can take any value. In order to avoid it, the velocity vector projections \( v'_{i}(i = 1, 2, 3) \) on the axis of the Cartesian coordinate system are taken as required functions. The partial derivatives in the coordinates \( x, y, z \) can be represented in terms of the coordinates \( r, \theta, \phi \) by means of the generalized function \( \Phi(x, y, z) = \Phi(V \cos \phi, V \sin \phi, U) \).

The constitutive equations (1) can be rewritten according to (2) in the new coordinate basis:

\[ (u' \alpha \cos \phi + u'^2 \alpha \sin \phi + u'^3 \frac{rV'}{w^2}) \frac{\partial u'}{\partial r} + (u' \beta \cos \phi + u'^2 \beta \sin \phi - u'^3 \frac{rV'}{w^2}) \frac{\partial u'}{\partial \theta} + \]  

\[ (- \frac{1}{V} \sin \phi u' + \frac{1}{V} \cos \phi u'^2) \frac{\partial u'}{\partial \phi} + (\alpha \cos \phi \frac{\partial p}{\partial r} + \beta \cos \phi \frac{\partial p}{\partial \theta} - \frac{1}{V} \sin \phi \frac{\partial p}{\partial \phi}) = \frac{1}{\text{Re}} V^2 u'; \]

\[ (u' \alpha \cos \phi + u'^2 \alpha \sin \phi + u'^3 \frac{rV'}{w^2}) \frac{\partial u'}{\partial r} + (u' \beta \cos \phi + u'^2 \beta \sin \phi - u'^3 \frac{rV'}{w^2}) \frac{\partial u'}{\partial \theta} + \]  

\[ (- \frac{1}{V} \sin \phi u' + \frac{1}{V} \cos \phi u'^2) \frac{\partial u'}{\partial \phi} + (\alpha \sin \phi \frac{\partial p}{\partial r} + \beta \sin \phi \frac{\partial p}{\partial \theta} + \frac{1}{V} \cos \phi \frac{\partial p}{\partial \phi}) = \frac{1}{\text{Re}} V^2 u; \]

\[ (u' \alpha \cos \phi + u'^2 \alpha \sin \phi + u'^3 \frac{rV'}{w^2}) \frac{\partial u'}{\partial r} + (u' \beta \cos \phi + u'^2 \beta \sin \phi - u'^3 \frac{rV'}{w^2}) \frac{\partial u'}{\partial \theta} + \]  

\[ (- \frac{1}{V} \sin \phi u' + \frac{1}{V} \cos \phi u'^2) \frac{\partial u'}{\partial \phi} + (\beta \cos \phi \frac{\partial u'}{\partial r} + \frac{rV'}{w^2} \frac{\partial u'}{\partial \phi} - \frac{rV'}{w^2} \frac{\partial u'}{\partial \theta}) = \frac{1}{\text{Re}} V^2 u'; \]

\[ 1 \cos \phi \frac{\partial u'}{\partial r} + \beta \cos \phi \frac{\partial u'}{\partial \theta} - \frac{1}{V} \sin \phi \frac{\partial u'}{\partial \phi} + \alpha \sin \phi \frac{\partial u'}{\partial r} + \beta \sin \phi \frac{\partial u'}{\partial \theta} + \]  

\[ \frac{1}{V} \cos \phi \frac{\partial u'}{\partial r} + \frac{rV'}{w^2} \frac{\partial u'}{\partial r} - \frac{rV'}{w^2} \frac{\partial u'}{\partial \theta} = 0, \]

where \( \text{Re} \) is the Reynolds number, \( \alpha(r, \theta) = -rU_0' / w^2, \quad (w^2 = U_0'^2 + V_0'^2); \quad \beta(r, \theta) = (1 + rU_0'V' / w^2) / V_0'. \)

To solve a system of constitutive equations, according to the reference documents \[5-7\], the wind load should be represented by the following expression

\[ w = w_m + w_p, \]

where \( w_m \) and \( w_p \) are the average and pulsation component of wind load, respectively.

The normative value of the average wind load component at a height \( z \) overland should be determined \[6\] according to the formula:

\[ w_m = w_0 \cdot k(Z_e) \cdot c, \]

where \( w_0 \) is the normative value of wind pressure for a given wind region; \( k(Z_e) \) is the coefficient, taking into account the height change of wind pressure for a particular type of area; \( c \) is the aerodynamic coefficient.
It is proposed that the solution of the equation system (1) – (3) written by means of the above transforms and conditions (6), is to be found in discrete form by the linearization method of the sought functions. In this case, the derivatives are replaced by the discrete derivatives found by the differentiation of corresponding interpolation formulas [11,14]. The Laplace operator of the functions \( v^i (i = 1,2,3) \) is represented as a discrete form for the variables \( r, \theta, \phi \):

\[
\Delta \tilde{u}^i = \frac{r}{Vw^2} \left[ \frac{\partial}{\partial r} \left( r \frac{\partial \tilde{u}^i}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( V \frac{\partial \tilde{u}^i}{\partial \theta} \right) \right] + \frac{1}{V^2} \frac{\partial^2 \tilde{u}^i}{\partial \phi^2}.
\]  

(14)

After the discretization of the Navier-Stokes constitutive equations, they include the function values \( v^i, \tilde{v}^i, \tilde{v}^i \) in the grid nodes \((\theta_i, r, \phi_k)\), \( v = 1,2,...,n; k = 0,1,...,2l \), the whole system obtained will have \(4mnL \) \((L = 2l + 1)\) of non-linear equations.

5. Modeling results

The equations obtained were further linearized based on the assumption that at a small integration step unknown functions behave linearly on each interval \( n - 1 \leq t \leq nT \), i.e., the derivative of an unknown function is calculated as the relation [21-23]:

\[
f \ nT = f_n - f_{n-1} \ \tau,
\]

where \( f \) is the unknown time function to be determined.

Plugging this relation in a constitutive equation, we obtain the recurrent expression [19,22] that allows to determine the value of the unknown function at every instant [24-26]. The results of the solution of the constitutive equations are shown in Table 1.

Table 1. The results of the determination of aerodynamic coefficients by three methods in three directions of the wind load.

| Elevation Point | Calculation \( (\gamma f=1.4) \) average wind load component on separate bars \( γw·k·ka·k(ze)·w0·\) | Calculation \( (\gamma f=1.4) \) average wind load component on separate bars \( γw·k·ka·k(ze)·w0·\) | Calculation \( (\gamma f=1.4) \) average wind load component on separate bars \( γw·k·ka·k(ze)·w0·\) |
|-----------------|---------------------------------------------|---------------------------------------------|---------------------------------------------|
| Direction of OY | Direction of OX | Direction of OY | Direction of OX | Direction of OY | Direction of OX | Direction of OY | Direction of OX |
| Ze, m | \( k_1 \) | \( k_1 \) | \( k_1 \) | \( k_1 \) | \( k_1 \) | \( k_1 \) | \( k_1 \) | \( k_1 \) |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1 | 85.00 | 1.455 | 1.464 | 1.792 | 1.450 | 1.472 | 1.794 | 1.440 | 1.494 | 1.798 |
| 2 | 75.50 | 1.424 | 1.444 | 1.764 | 1.421 | 1.438 | 1.761 | 1.411 | 1.465 | 1.767 |
| 2 | 60.50 | 1.082 | 1.079 | 1.318 | 1.082 | 1.084 | 1.316 | 1.075 | 1.092 | 1.319 |
| 3 | 50.00 | 1.048 | 1.068 | 1.268 | 1.046 | 1.062 | 1.269 | 1.054 | 1.062 | 1.274 |
| 3 | 25.00 | 0.989 | 0.994 | 1.202 | 0.990 | 0.996 | 1.197 | 0.997 | 0.993 | 1.207 |
| 3 | 15.00 | 0.908 | 0.912 | 1.110 | 0.911 | 0.914 | 1.107 | 0.915 | 0.915 | 1.101 |

6. Conclusions

The study proposes a method for analysis of high-rise antenna-mast structures for wind force adapted to the practical application, according to the requirements of the legal reference documents for...
construction and maintenance of such facilities that are applicable in the territory of Russia. In order to
determine the distribution of aerodynamic coefficients of external pressure on the surface of the
object, a number of studies have been made. Mathematical modeling has resulted in the determination
of aerodynamic coefficients of external pressure at the control points on the surface of the structure at
heights of 15.0, 25.0, 50.0, 60.5, 75.5, 85.0, which conform with the fixing points of mast sections.
The comparison of the aerodynamic coefficient values obtained by the proposed method (columns 3,4,5 of Table 1), the values obtained by numerical modeling using the verified laborious method and
expensive software (columns 6,7,8 of Table 1), as well as the values obtained by the multilevel linear
interpolation of the data given in reference documents (columns 9,10,11 of Table 1) shows a close
agreement of the data obtained by the developed method and results of the computer system ANSYS.
Thus, the studies have shown that this method can be recommended in determining the wind load
on antenna-mast supports in design and construction work for the structures with height more than 75
meters and gives exact values of the functional coefficients.

References
[1] "The town-planning code of the Russian Federation" ot 29.12.2004 N 190-FZ (red. ot
29.07.2017) (s izm. i dop., vstup. v silu s 30.09.2017)
[2] Software complex "MIRAGE" for structural calculations on your PC. User manual 1995 (Kiev:
NIIASS) p 420.
[3] Zenkevich O 1975 Finite element method in engineering (Izdatel’stvo "Mir", Moskva) p 468.
[4] Gorodetskii A S and others 2005 Metod konechnih elementov v proektirovanii transportnih
sooruzhenii (Izdatel’stvo "Fakt", Kiev) p 342.
[5] GOST 27751-2014. Nadezhnost’ stroitel’nih konstruktsii i osnovani
[6] SP 16.13330.2011. Svod pravil. Stal’nie konstruktsii. Aktualizirovannaya redaktsiya SNiP II-
23-81*.
[7] SP 20.13330.2016. Svod pravil. Nagruzki i vozdeistviya. Aktualizirovannaya redaktsiya SNiP
2.01.07 85*.
[8] Koreneva B G, Rabinovich I M 1984 Dinamicheskii raschet zdanii i sooruzhenii (Spravochnik
proekhtirovchika) (M.: Stroiizdat) p 303.
[9] Finn R 1959 On the steady state solutions of the Navier – Stokes partial differential equations
(Arch. Rat. Mech. Anal. v 3) pp 381–396.
[10] Fujita H 1961 On the existence and regulaty of th steady-state solution of the Navier-Stokes
equation (J. Fac. Sci. Univ. of Tokyo. v 9) pp 59–102.
[11] Finn R 1959 Estimates at infinity for stationary solutions of the Navier – Stokes equations (Bull.
Math, de la Soc. Sc, Math.Phys. de la RPR. v. 51), № 3 pp 387–418.
[12] Finn R 1965 On exterior stationary problem for the Navier – Stokes equation and associated
perturbation problems (Arch. Rat. Mech. Anal. v. 19) pp 363–406.
[13] Clar D 1971 The vorticity at infinity for solutions of the stationary Navier – Stokes equations in
exterior domains (Indiana Math. J. v. 20) № 7 pp 633-654.
[14] Babekko K I 2002 Osnovi chislennogo analiza (M.: Nauka, 1986. p 744; Izdanie vtoroe, ispravlennoe i dopolnennoe / Pod red. A D Bryuno. Moskva-Izhevsk: RHD) p 847.
[15] Algzin S D 2002 Chislennie algoritmi bez nasisheniya v klassicheskikh zadachah
matematicheskoi fiziki (M.: Nauchnii mir) p 155.
[16] Algzin S D 2007 Chislennoe issledovanie uravnenii Nav’e-Stoksa (Zhurnal prikladnoi
mehaniki i tehnicheskoi fiziki. v. 48), № 5 pp 43–52.
[17] Meller N L, Pal’tsev B V and Hlyupina E G 1999 O konechno-elementnih realizatsiyah
iterationsnih metodov s rasschepleniem granichnih uslovii dlya system Stoksa i tipa Stoksa v
sharovom sloe. Osesimmetrichnih sluchai ( Zhurnal vichislitel’noi matematiki i matematicheskoi
fiziki. v. 39) № 1 pp 98-123.
[18] Pal’tsev B V, Chechel’ I I 2000 O tochnih otsenkah skorosti shodimosti iteratsionnih metodov s rasschepleniem granichnih uslovi dlya sistem tipa Stoksa v sloe s usloviem periodichnosti (Zhurnal vichislitel’noi matematiki i matematicheskoi fiziki. v. 40) № 12 pp 1823-1837.

[19] Loktev A A 2007 Udar vyazkouprugogo tela po uprugoi izotropnoi plastinke (Mehanika kompozitsionnih materialov i konstruktsii. v. 13) № 3 pp 417-425.

[20] Loktev A A 2007 Uprugoplasticheskaya model’ vzaimodeistviya tsilindricheskogo udarnika i plastinki (Pis’ma v zhurnal tehnicheskoi fiziki. v. 33) № 16 pp 72-77.

[21] Loktev A A, Zaletdinov A V 2010 Opredelenie tochek vzaimodeistviya pryamih i otrazhennih voln v plastinke (Vestnik MGSU) № 4-3 pp 303-308.

[22] Loktev D A, Loktev A A 2015 Determination of object location by analyzing the image blur (Contemporary Engineering Sciences. v. 8) № 9 pp 467-475.

[23] Loktev A, Sychev V, Gluzberg B, Gridasova E Modeling the dynamic behavior of railway track taking into account the occurrence of defects in the system wheel-rail (MATEC Web of Conferences 117, 00108 (2017). XXVI R-S-P Seminar 2017, Theoretical Foundation of Civil Engineering) pp 1-6.

[24] Loktev A, Poddaeva O, Fedosova A, Churin P 2017 An Experimental Study of the Effects of Wind on a Metal Bridge Crossing with Two Independent Parallel Spans (Nonlinearity. Problems, Solutions and Applications. V.1. Theoretical and Applied Mathematics) pp 291-307.

[25] ANSYS CFX 14.5. User’s Guide 2012 (Canonsburg: ANSYS Inc.)

[26] Egorychev O O, Churin P S, Poddaeva O I 2015 Experimental study of aerodynamic loads on high-rise buildings (Advanced Materials Research. v. 1082) p 250.