Practical corollaries of transverse Ward–Green–Takahashi identities

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** Abstract **

The gauge principle is fundamental in formulating the Standard Model. Fermion–gauge-boson couplings are the inescapable consequence and the primary determining factor for observable phenomena. Vertices describing such couplings are simple in perturbation theory and yet the existence of strong-interaction bound-states guarantees that many phenomena within the Model are nonperturbative. It is therefore crucial to understand how dynamics dresses the vertices and thereby fundamentally alters the appearance of fermion–gauge-boson interactions. We consider the coupling of a dressed-fermion to an Abelian gauge boson, and describe a unified treatment and solution of the familiar longitudinal Ward–Green–Takahashi identity and its less well known transverse counterparts. Novel consequences for the dressed-fermion–gauge-boson vertex are exposed.

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1. Introduction

Identities of the Ward–Green–Takahashi (WGT) type \([1–3]\) have long been known and used in gauge theories. The widely familiar forms provide constraints on the longitudinal part of n-point Schwinger functions; i.e., propagators and vertices. For example, in an Abelian gauge theory the dressed-fermion–gauge-boson vertex, \(\Gamma_\mu^\nu(k, p)\) in Fig. 1, satisfies

\[
q_\mu i\Gamma_\mu^\nu(k, p) = S^{-1}(k) - S^{-1}(p),
\]

where the dressed-fermion propagator may be written

\[
S(p) = 1/[i\gamma \cdot pA(p^2) + B(p^2)].
\]

Eq. (1) is a nonperturbative consequence of gauge invariance in an Abelian theory and, following Ref. [4], it has been used extensively in the construction of models for the dressed-fermion–gauge-boson vertex. (Renormalisation does not affect the form of the identities we consider, so we do not explicitly refer to it. A Euclidean metric is used herein:

\[
\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}; \quad \gamma_\mu = \gamma_\mu^\dagger.
\]

Fig. 1. Dressed-fermion–gauge-boson vertex, \(\Gamma_\mu^\nu(k, p)\), with the momentum flow indicated.

\(\gamma_5 = \gamma_4\gamma_1\gamma_2\gamma_3, \quad \text{tr}\{\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\} = -4\epsilon_{\mu\nu\rho\sigma}; \quad \sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]; \quad a \cdot b = \sum_{i=1}^{4} a_i b_i; \quad \text{and } Q_{\mu} \text{ spacelike} \Rightarrow Q^2 > 0.\)

It is natural to question the need for vertex Ansätze, given that QED has been tested perturbatively to remarkable precision through comparison with experiment [5]. However, the context for utility here is not the QED of interactions between leptons and photons. Instead, one has in mind theories in which the dressed-fermion propagator is strongly modified from its perturbative behaviour and hence so, too, is the vertex.

An obvious example is QCD, in which the dressed-quark two-point function is described by two momentum-dependent functions: a wave-function renormalisation, \(Z(p^2) = 1/A(p^2)\), and mass function, \(M(p^2) = B(p^2)/A(p^2)\), both of which are strongly modified from their perturbative forms for \(p^2 \lesssim (5\Lambda_{\text{QCD}})^2\) [6,7]. This dressing is associated with dynamical chiral symmetry

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breaking (DCSB), a particularly striking feature of the Standard Model that plays an important role in the formation of the visible matter in the Universe [8]. In a massless theory; i.e., in QCD’s chiral limit in the absence of DCSB, \( M(p^2) \equiv 0\). (Notably, the gluon also acquires a momentum-dependent mass function \([9,10]\), \( m_G(k^2)\), which is large at infrared momenta \([11–18]\); \( m_G(0) \approx 0.5 \text{ GeV} \gtrsim M(0)\).)

The electromagnetic vertex associated with such a quark must differ markedly from the perturbative form at infrared momenta. This is obvious from Eq. (1), since

\[ i q \gamma_{\mu} k - i \gamma \cdot k A(k^2) + B(k^2) - i \gamma \cdot p A(p^2) - B(p^2); \]

and the mismatch is largest within that domain upon which \( A, B \) differ conspicuously from their perturbative forms. The importance of this and kindred dressing to the reliable computation of observables involving composite systems was exposed in Refs. [19–22].

The most sophisticated Ansätze currently available are detailed in Refs. [23,24] but they follow upon a great deal of work, which may be traced from Ref. [4].

2. Transverse identities

The transverse WGT identities \([25–29]\) are less familiar and, \textit{prima facie}, less useful, too. In (colour-singlet) vector and axial-vector channels connected with a fermion of mass \( m \), these identities read (\( t = k + p \)):

\[
q_{\mu} \Gamma_{V}(k, p) - q_{\mu} \Gamma_{A}(k, p) = S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k)
+ 2im\Gamma_{\mu\nu}(k, p) + t; \varepsilon_{\mu\nu\rho\sigma} \Gamma_{\rho\sigma}(k, p)
+ A_{\mu}(k, p),
\]

\[
q_{\mu} \Gamma_{V}(k, p) - q_{\mu} \Gamma_{A}(k, p) = S^{-1}(p)\sigma_{\mu\nu} - \sigma_{\mu\nu} S^{-1}(k)
+ t; \varepsilon_{\mu\nu\rho\sigma} \Gamma_{\rho\sigma}(k, p)
+ V_{\mu}(k, p),
\]

where \( \sigma_{\mu\nu} = \gamma_5 \sigma_{\mu\nu} \) and \( \Gamma_{A}(k, p) \) is an inhomogeneous tensor vertex. Whereas the longitudinal WGT identity expresses properties of the divergence of the vertex, the transverse identities relate to its \textit{curl} (as Faraday’s law of induction involves an electric field).

The last two terms in Eq. (4) arise in computing the momentum space expression of a nonlocal axial-vector vertex, whose definition involves a gauge-field-dependent line integral \([29]\); and the last two terms in Eq. (5) arise from similar manipulations of an analogous nonlocal vertex. Note that, like Eq. (1), the transverse identities are valid in any covariant gauge, which is the class we focus upon, and do not explicitly display dependence on the gauge-fixing parameter. (Whilst an anomaly term can be included in Eq. (5) \([30]\), we concern ourselves with flavoured vertices; i.e., those free from such amendment.)

It is the presence of the unfamiliar quantities \( A_{\mu}(k, p) \), \( V_{\mu}(k, p) \) in the transverse identities that lends them an appearance of impracticality, since even at one-loop order the expressions for \( A_{\mu}(k, p) \), \( V_{\mu}(k, p) \) are complicated \([30–32]\) and, moreover, they lead to a coupling between the vector and axial-vector identities. We cannot overcome the complexity but something can be done about the induced coupling between the identities.

Before doing so, it is worthwhile explaining that, in general, twelve independent tensor structures are required to specify a fermion–vector–boson vertex: given the matrix-valued vectors \( \gamma_{\mu}, k_{\mu} I_0 \), \( p_{\mu} I_0 \), where \( I_0 \) is the \( 4 \times 4 \) identity in Dirac space, one can construct twelve independent quantities that behave as a vector under Poincaré transformations.

We make our conventions explicit by writing

\[
\Gamma_{\mu}(k, p) = \Gamma_{\mu}^0(k, p) + \Gamma_{\mu}^T(k, p),
\]

\[
\Gamma_{\mu}^0(k, p) = \sum_{j=1}^{4} \lambda_j(k, p) L_{\mu j}(k, p),
\]

\[
\Gamma_{\mu}^T(k, p) = \sum_{j=1}^{8} \tau_j(k, p) T_{\mu j}(k, p),
\]

where the matrix-valued tensors \( \{ L_{\mu j}(k, p), j = 1, \ldots, 4 \} \) and \( \{ T_{\mu j}(k, p), j = 1, \ldots, 8 \} \) are given, respectively, in Eqs. (A.1) and (A.2). Following inspection of Eqs. (A.2), it becomes clear that \( q_{\mu} \Gamma_{\mu}^T(k, p) \equiv 0 \).

Now consider the matrix-valued tensors

\[
T_{\mu j}^1 = \frac{1}{2} \epsilon_{\alpha\mu\nu\rho} q_{\alpha} \epsilon_{\beta} I_0, \quad T_{\mu j}^2 = \frac{1}{2} \epsilon_{\alpha\mu\nu\rho} q_{\alpha} \epsilon_{\beta}.
\]

Contracting the left-hand-side of Eq. (5) with these tensors produces zero. Operating then with the right-hand-sides equated to zero, and using

\[
T_{\mu j}^1 \varepsilon_{\alpha\mu\nu\rho} GR_{\rho}(k, p) = t^2 q \cdot G - \epsilon_{\alpha\nu\beta} \epsilon_{\beta} \cdot (k, p),
\]

\[
T_{\mu j}^2 \varepsilon_{\alpha\mu\nu\rho} GR_{\rho}(k, p) = \epsilon_{\alpha\nu\beta} \epsilon_{\beta} \cdot (k, p),
\]

one finds

\[
q \cdot \epsilon_{\alpha\nu\beta} \epsilon_{\beta} \cdot (k, p) = T_{\mu j}^1 \varepsilon_{\alpha\mu\nu\rho} S^{-1}(p)\sigma_{\nu\rho} - \sigma_{\nu\rho} S^{-1}(k)
+ \epsilon_{\alpha\nu\beta} \epsilon_{\beta} \cdot \Gamma_{\mu}(k, p) + t \varepsilon_{\alpha\nu\beta} \Gamma_{\nu\rho}(k, p)
+ T_{\mu j}^2 \varepsilon_{\alpha\mu\nu\rho} V_{\nu\rho}(k, p).
\]

This series of identities involves only the vector vertex. We have thus uncoupled the equations.

It is worth remarking that the vector and axial-vector vertices together are required in order to understand electroweak interactions within the Standard Model. One may obtain information about the axial-vector vertex through an analogous procedure involving Eq. (4).

The complicated elements in Eqs. (12), (13) are \( T_{\mu j}^1 V_{\mu j}^A(k, p) \). Whilst these quantities cannot readily be computed, they are, nevertheless, merely matrix-valued scalar amplitudes and hence can be expressed succinctly:

\[
iT_{\mu j}^1 V_{\mu j}^A(k, p) = i_0 X_1(k, p) + \gamma \cdot q X_2(k, p)
+ \gamma \cdot q X_3(k, p) + \gamma \cdot t X_4(k, p),
\]

\[
iT_{\mu j}^2 V_{\mu j}^A(k, p) = i_0 X_5(k, p) + \gamma \cdot q X_6(k, p)
+ \gamma \cdot q X_7(k, p) + \gamma \cdot t X_8(k, p).
\]
3. Solution of the coupled identities

One may now use any reliable means to solve the system of coupled linear equations. Irrespective of the presence and form of the functions \( \{ \xi_i, i = 1, \ldots, 8 \} \), part of the complete solution has

\[
\begin{align*}
\lambda_1(k, p) &= \Sigma_A(k^2, p^2), \\
\lambda_2(k, p) &= \Delta_A(k^2, p^2), \\
\lambda_3(k, p) &= \Delta_B(k^2, p^2), \\
\lambda_4(k, p) &= 0,
\end{align*}
\]

where \( (x = k^2, y = p^2) \) and

\[
\sum_\phi(x, y) = \frac{1}{2} [\phi(x) + \phi(y)], \quad \Delta_\phi(x, y) = \frac{\phi(x) - \phi(y)}{x - y}.
\]

Namely, a necessary consequence of solving Eqs. (1), (12), (13), is the identification of \( \Gamma_{ij}^\mu(k, p) \) with the result derived in Ref. [4]; i.e., the Ball–Chiu Ansatz. The system of equations is linear, so the solution for \( \Gamma_{ij}^\mu(k, p) \) is unique. Note that we made no attempt to impose a particular kinematic structure on the solution. Irrespective of the tensor basis chosen, and we used a variety of forms, not just those in Eqs. (A.1)–(A.9), this part of the solution is free of kinematic singularities. The functional form of \( \lambda_3(k, p) \) signals that the coupling of a dressed-fermion to a gauge boson is necessarily influenced heavily by DCBS.

The eight functions in Eq. (8) are also completely specified. Their form depends on \( \{ \xi_i, i = 1, \ldots, 8 \} \); e.g., the simplest is

\[
\tau_1(k, p) = \frac{1}{2} \frac{X_1(k, p)}{k^2 - p^2},
\]

The expressions for \( \{ \tau_j, j = 2, 4, 6, 7 \} \) are more complicated but, in common with \( \tau_1 \), they do not explicitly involve the scalar functions \( \{ A, B \} \), which define the dressed-fermion propagator. This is the material point. It means that any and all effects of \( \{ A, B \} \) in \( \{ \tau_j, j = 2, 4, 6, 7 \} \) are only expressed implicitly through a solution of the vertex Bethe–Salpeter equation. (N.B. Our subsequent discussion is independent of all other details about the forms of \( \{ \tau_j, j = 2, 4, 6, 7 \} \).)

In contrast, the expressions for \( \{ \tau_j, j = 3, 5, 8 \} \) explicitly involve combinations of \( A(k^2) \), \( A(p^2) \), \( B(k^2) \), \( B(p^2) \) and \( \{ \xi_i, i = 1, \ldots, 8 \} \). If one supposes that \( \{ \xi_i \equiv 0, i = 1, \ldots, 8 \} \), then simple results are obtained:

\[
\begin{align*}
\tau_2(k, p) &= \Delta_A(k^2, p^2), \\
\tau_3(k, p) &= -\Delta_B(k^2, p^2), \\
\tau_5(k, p) &= -\Delta_B(k^2, p^2).
\end{align*}
\]

The following features of the transverse part of the solution to Eqs. (1), (12), (13) are particularly noteworthy. A \( \Gamma_{ij}^\mu(k, p) \) term generally appears in the solution and, with \( \{ \xi_i \equiv 0, i = 1, \ldots, 8 \} \), its coefficient is \( (1/2) \Sigma_A(k^2, p^2) \). Eq. (19). The functional form is a prediction of the transverse WGT identity because, apart from our choice of tensor bases in Eqs. (A.1)–(A.9), we implemented no other constraints. Based upon considerations of multiplicative renormalisability and one-loop perturbation theory, a vertex Ansatz was proposed in Ref. [24]. It involves a \( \Gamma_{ij}^\mu(k, p) \) term whose coefficient is \( a_3 \Delta_A(k^2, p^2) \), with \( a_3 = a_5 = 1/2 \). The choice \( a_3 = 0, a_5 = 1/2 \) produces the Curtis–Pennington Ansatz [33]. In contrast with the coefficients’ functional forms is remarkable. The choice \( \tau_2 = \Delta_A(k^2, p^2) \) in Eq. (8). The agreement between the coefficients’ functional forms is remarkable. The choice \( \tau_2 = \Delta_A(k^2, p^2) \) in Eq. (8). The agreement between the coefficients’ functional forms is remarkable. The choice \( \tau_2 = \Delta_A(k^2, p^2) \) in Eq. (8). The agreement between the coefficients’ functional forms is remarkable. The choice \( \tau_2 = \Delta_A(k^2, p^2) \) in Eq. (8). The agreement between the coefficients’ functional forms is remarkable. The choice \( \tau_2 = \Delta_A(k^2, p^2) \) in Eq. (8). The agreement between the coefficients’ functional forms is remarkable. The choice \( \tau_2 = \Delta_A(k^2, p^2) \) in Eq. (8). The agreement between the coefficients’ functional forms is remarkable. The choice \( \tau_2 = \Delta_A(k^2, p^2) \) in Eq. (8). The agreement between the coefficients’ functional forms is remarkable. The choice \( \tau_2 = \Delta_A(k^2, p^2) \) in Eq. (8).
which yields a Euclidean constituent-quark mass $M = 0.38$ GeV, we find $\kappa = 1.14$. The anomalous moment is positive, as it must be for an Abelian interaction, and commensurate with the value computed using the Ansatz in Ref. [24]; viz., $\kappa = 1.6$. The computed value of $\kappa$ is large but, like the Euclidean constituent-quark mass, this is just one (infrared) value on a curve that describes the anomalous magnetic moment distribution of a dressed-quark [37]; averaged over a nonperturbative domain $p^2 \in [0, 2]$ GeV$^2$, $\bar{\kappa} = 0.49$.

As with all nonperturbative quantities in QCD, the dynamical component of the $\kappa$ distribution vanishes as a power-law for $p^2 \gtrsim (\Lambda_{QCD})^2$. Thus, and for example, the dynamically generated anomalous magnetic moment influences the long-wavelength behaviour of electromagnetic form factors but is not discernible on the domain within which perturbative analyses are valid [40].

4. Epilogue

We elucidated novel consequences of Lagrangian-based symmetries for the fermion–gauge-boson vertex, therewith confirming numerous features of the Ansatz described in Ref. [24] and thus placing it both in a broader context and on firmer ground. Our proposal, Eq. (22), is simpler, however. It is thus easier to use, e.g., in Poincaré–covariant symmetry-preserving studies of hadron electromagnetic form factors of the type described in Refs. [41,42]; and might serve readily as a prototype in the construction of electromagnetic currents for use in few-nucleon physics [43].

Well-motivated, realistic Ansätze for the dressed-quark-gluon vertex are also needed because this vertex is a critical part of all gap and Bethe–Salpeter equation studies of hadron spectra and interactions, and yet information available from continuum or lattice methods is limited [37,38,44–46]. In this connection, Eq. (22) may be compared with the vertex in Ref. [38], which is associated with the most realistic Poincaré–covariant, continuum description of the light-quark meson spectrum that is currently available. While the $\tau_{5,8}$ structures, crucial to the expression of DCSB in the spectrum, are included therein, the $\tau_3$ term is omitted. This suggests both: that the Ansatz in Ref. [38] can be improved; and a simple way by which that may be accomplished. This phenomenological extension of the model-independent results described herein is underway.

In another direction, given that the Abelian transverse Ward–Greens–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47], in the hope that from Green–Takahashi identities may now be judged useful, it is worth revisiting their non-Abelian analogues [47].

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Acknowledgements

We thank A. Bashir, C. Chen, L.-j. Jiang, M. Pitschmann, J. Segovia and X.-y. Xin for valuable comments; and S.-x. Qin is grateful for encouragement from D.H. Rischke. Work supported by: Alexander von Humboldt Foundation Postdoctoral Research Fellowship; Forschungszentrum Jüllich GmbH; National Natural Science Foundation of China, contract Nos. 10935001, 11070502 and 11175004; National Key Basic Research Program of China, contract No. 2013CB834400; and US Department of Energy, Office of Nuclear Physics, contract No. DE-AC02-06CH11357.

Appendix A

Here we list the tensors used in Eqs. (7), (8):

\[ T^i_{\mu}(k, p) = i[p_\mu (k \cdot q) - k_\mu (p \cdot q)], \]

\[ T_{\mu}(k, p) = -iT^1_{\mu}(\gamma' \cdot k + \gamma \cdot p), \]

\[ T_{\mu}(k, p) = i[p_\mu (k \cdot q - \gamma_\mu \gamma' \cdot q) + \gamma_\mu \gamma^\mu], \]

\[ T^i_{\mu}(k, p) = i[p_\mu (k \cdot q) - k_\mu (p \cdot q)], \]

\[ T_{\mu}(k, p) = -i\gamma_\mu (k^2 - p^2) + (k + p)_\mu \gamma \cdot q, \]

\[ T_{\mu}(k, p) = \frac{i}{2}[(k^2 - p^2)[\gamma_\mu (\gamma' \cdot k + \gamma \cdot p) - (k + p)_\mu] + (k + p)_\mu p_\nu k_\rho \sigma_{\nu\rho}], \]

\[ T_{\mu}(k, p) = -i\gamma_\mu \gamma \cdot p - p_\mu \gamma \cdot k - i\gamma_\mu p_\nu k_\rho \sigma_{\nu\rho}. \]
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