Operational vibration shape measurement of piezoceramic disc actuator using digital image correlation vibrometry with a single reference signal

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Abstract. This paper describes the enhancement and application of a digital image correlation (DIC) vibrometry method which avoids the need for high speed cameras. The underlying technique, described earlier by members of the present team (Warburton et al, Exp. Mech. 56(7) 1219–1230), was used to measure the operational vibration shapes of a piezoceramic synthetic jet actuator disc. Following the method previously reported, pairs of images were captured using the Dantec Q-400 hardware at a frequency of 0.25 Hz, with synchronised flash illumination but not synchronised with the excitation signal, which was captured simultaneously with each image pair. The images were processed using the Dantec Istra4D software and were post-processed largely as described by Warburton in order to obtain amplitude and phase maps for each frequency, but instead of using an externally-generated quadrature signal as a reference, an alternative reference was taken from the median displacement over an area of the specimen, in order to resolve the phase position of each image pair. Although the raw measured deflected shapes were generally dominated by the main dishing mode of the actuator, alternative higher order shapes such as asymmetric and trefoil shapes occurred in quadrature with these dishing-mode deflections.

1. Introduction

Earlier work by members of the present team has described an approach to the use of low-speed digital image correlation (DIC) equipment for measurement of operational deflected shapes as close approximations to mode shapes. The approach involves exciting a specimen at resonant frequency and using an asynchronous, free-running DIC system to image the vibration shape. The approach was described in detail by Warburton et al. [1], and has some similarities with (but also major differences from) an approach described by Fruehmann et al. [2] As described in reference [1], the data acquisition unit in the DIC system is used to capture the excitation signal, and in order to resolve the phase ambiguity of that signal a second signal, integrated using an integrating amplifier so that it is approximately in quadrature with the first signal, is used as an additional reference.

The aims of the present paper are twofold: to describe a variant of the approach described in reference [1] which avoids the need for the integrating amplifier, and to apply both the original method and the variant to the practical case study of a piezoelectric actuator disc used within a synthetic jet actuator mechanism for fluid mechanics applications [3], in order to determine the shapes with which it vibrates operationally. Specifically, the vibration modes of a typical commercial (PZT-5A) polycrystalline actuator disc will be explored, and a comparative study will be undertaken regarding a

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PZT disc and a custom-made single-crystal (PMN-PT) disc of the same dimensions, comparing the results of the present study against those in [3].

2. Summary of existing approach

The approach described this paper involves a modification to the apparatus and the processing procedure described by Warburton et al. [1]. It is not the present intention to duplicate the background or description given in that paper but a brief summary will be given before outlining the modification of the approach which is the subject of the present paper.

2.1. Existing apparatus and procedure before incorporation of novel referencing approach

In its original form requires the following apparatus including the underlying commercial system:

- A 3D digital image correlation system, in this case the Dantec Q-400 system equipped with two 5 MP cameras. This incorporated a data acquisition and interfacing unit capable of inputting several analogue signals. It also outputs a TTL (5V logic) digital trigger signal to the cameras, which is also used for triggering the flash.
- A signal generator with (if required) a power amplifier to drive the excitation system. The output from the signal generator is also fed into one of the analogue inputs of the Q-400 system to provide part of the phase position identification for signal capture.
- A means of providing intense illumination, specifically a conventional studio flashlamp system (Bowens ProLite 60)
- A special circuit to trigger the flashlamp system from the TTL camera trigger signal using a MOSFET to emulate the closing of electrical contacts
- Some means of mechanically exciting the vibrations e.g. a shaker coupled to the specimen via a stinger, or a loudspeaker unit acoustically coupled to the specimen; in the present case the actuator itself is the subject of the experiment.
- An integrating amplifier circuit based on a 741 operational amplifier, used to generate a second reference signal approximately in quadrature with the excitation signal and hence provide a means of identifying the phase position within a given cycle.

The specimen, in this case one side of the actuator disc, is prepared with a speckle pattern using (for example) carefully-applied aerosol spray. The 3D DIC system is calibrated in the usual way using a chequerboard target following the automated procedures provided within the Q-400 system. The procedure for each resonant frequency involves manually tuning the frequency of the signal generator until maximum amplitude of vibration is observed (either by ear or by observing a “dancing bead” on the specimen). The Q-400 system is then set to capture a large number of image pairs (typically 60) in free-running mode i.e. not synchronised to the excitation signal, at a frequency e.g. 0.25 Hz which allows time for the flash system to re-charge between images. Indeed, it is important that excitation signal is not an exact multiple of the capture frequency, though no special care is normally needed to achieve this. Each capture event involves the recording of two images at very short exposure (1/10000 s) illuminated by the flash system, along with the two analogue signals representing the directly measured and integrated excitation signals. A diagram of the apparatus, including the items omitted when using the variant of the method to be described, is given in figure 1.

The images are processed in the standard manner using the ISTRA4D software to obtain measured displacement fields in the local coordinate system of the specimen for each capture event. These are then post-processed, again as described by Warburton [1], firstly to find the phase relationship of the directly-captured and integrated excitation signals (by fitting an elliptical Lissajous figure to the points obtained by plotting the integrated signal against the directly captured one), and hence to find the phase position or circular time (ωt) within each cycle of excitation. The displacement vector at any point on the specimen is fitted to a sinusoidal function of ωt to recover the amplitude and phase angle of the vibration response at each point. Finally, the displaced shapes can be recovered at an arbitrary number of phase positions within each cycle (e.g. every 10°) can be reconstructed.
While the above procedure is effective, a practical issue is that the gain of the integrating amplifier needs to be adjusted to match both the frequency and amplitude of the input signal to avoid either insufficient amplitude or clipping, noting that the gain of an integrating amplifier is inversely proportional to frequency.

![Figure 1: Apparatus for DIC vibrometry on synthetic disc actuator, including items omitted when using internal reference signal (modified from reference [1]).](image)

2.2. Modified approach avoiding need for second reference signal
The present variant of the method arose because of the non-availability of the integrating amplifier used in the original implementation of the process at a time when indicative measurements were needed on a sample of the piezoelectric actuator, and relies upon the fact that not every displacement on the specimen is in phase with the excitation signal. The principle is that an area of the specimen is chosen and the median displacement over this area is calculated for each of the time points. This value is then used in place of the integrated signal in plotting the Lissajous figure, and hence used for disambiguating the phase information from the excitation signal. Clearly this approach will work best if there is significant phase shift between the normal mode vibrations of a structure and the excitation signal, or if there is significant phase difference between the vibrations at different points on the structure i.e. if the structure is exhibiting complex modes and there is a region with significant phase difference. Conversely, if all the points on the structure are vibrating approximately in phase with (or in phase opposition to) the excitation signal the Lissajous figure will be close to a straight line, and the inevitable noise on the results will make it very difficult to fit an ellipse to the results in a way which successfully resolves the phase ambiguity of a given point. As well as avoiding the need for an integrating amplifier, this new variant of the process avoids the issues caused by the frequency-dependent gain of the integrating amplifier.

3. Apparatus and procedure for present experiments.
Two sets of experiments were undertaken. The first was an initial trial used to explore the problem and understand the typical vibration modes, for which the above method was used to obtain results in the absence of an available integrating amplifier. The specimen was a commercial (APC International) 50 mm diameter PZT disc of total thickness 0.23 mm, clamped within the actuator cavity, and it was prepared in the usual manner with a speckle spray pattern. The apparatus was set up as shown in figure 1 with the integrating amplifier and oscilloscope omitted, and the frequency tuned manually to detect seven different resonant frequencies.
The experiment was re-run at a later date with a replacement integrating amplifier. Two piezoelectric actuator discs were tested: a commercial PZT disc smaller than that used in the familiarisation study (27 mm diameter, 0.45 mm thickness), and a single-crystal PMN-PT disc of the type intended for driving the air jet manufactured to the same dimensions. For each disc, two frequencies were excited, each at two different voltage levels, which before amplification were approximately 70 and 140 mV (rms) respectively, in order to give peak amplitudes after amplification of 10 V and 20 V respectively to match reference [3]. Two different frequencies were used: one (1.8 kHz) corresponds to the Helmholtz frequency of the cavity into which the disc is inserted, and the other (2.8 kHz) is close to the mechanical/structural resonant frequency of the PMN-PT disc itself. Graphs of air velocity and disc displacement vs. frequency for both kinds of actuators are presented in reference [3].

The voltage levels used here were somewhat lower than in the preliminary experiment, to avoid any risk of fracturing the single crystal actuator which is far more costly than the polycrystalline ones.

![Figure 2](image-url)  
**Figure 2:** Lissajous figures showing relationship between median displacement over an area, and excitation voltage for (a) 347 Hz and (b) 1940 Hz. The outliers were manually excluded from the fitting of the ellipses shown.

4. Results

The results of the familiarisation experiment were processed using the modified approach described in Section 2.2. The use of an averaged displacement as the second reference signal is seen (for example, in figure 2) to give a rather scattered Lissajous figure at each of the frequencies though the elliptical trend is very clear in most cases; in some cases the trend was less clear with the figure being more of an elliptical cloud. Some clear outliers are visible in that figure; this is believed to be due to physical disturbance of the apparatus (due to floor deflections) at the start of each experiment as the operator starts the capture process before leaving the area. These outliers were omitted manually from the ellipse-fitting process; outliers are automatically eliminated from the sinusoidal fitting of displacement to \(\omega t\) as described in [1]. Despite this relatively poor fit of the ellipse in some cases, the use of the resulting values of phase position \(\omega t\) (termed here “circular time”) resulted in clear sinusoidal trends in the displacement vs. circular time, and the clearly-defined operational deflected shapes in figure 3 give confidence in the results.

The results of the second set of experiments were processed as described by Warburton et al. without the modification described in section 2.2. Unlike in the preliminary experiment, the range of frequencies investigated was limited to two frequencies of interest rather than covering a wider range of possible resonances. No obvious complex modes are exhibited and the main mode of deformation appears to be the dishing mode in each case. The results for the polycrystalline (PZT) disc are given in figure 4 and those for the single crystal (PMN-PT) disc are given in figure 5.
Figure 3: (a-g) Typical raw maps of deflection from DIC system; (h-n) main deflected shapes measured using DIC vibrometry with internal reference signal; (o-u) higher order deflected shapes in quadrature with main modes.
Figure 4: Results for polycrystalline (PZT) actuator: (a-d) typical raw deflected shapes from DIC system; (e-h) deflected shapes from DIC vibrometry using external reference signal. Note that no higher-order shapes were detected in quadrature with these deflections.

5. Discussion
The results of the preliminary experiment, relating to the 50 mm polycrystalline actuator, show seven resonances up to around the lower frequency investigated in the main experiments. Minor resonances covering the same kind of frequency range were reported for the polycrystalline actuator tested in [3], though it should be noted that the disc used in this test was of different dimensions from that in [3]. It should be noted that the detection of these resonances was carried out by ear and was not dependent on the DIC system. For example, resonances noted here included 347 Hz, 854 Hz, 1149 Hz, 1224 Hz and 1723 Hz; COMSOL finite element predictions for this 50 mm actuator gave frequencies for mode shapes (not shown here) corresponding to the present quadrature displacement shapes at 312 Hz, 651 Hz, 1068 Hz, 1218 Hz and 1563 Hz (alternatively, the last could plausibly relate to the 1424 Hz resonance).

The results from the preliminary experiment are not directly comparable with [3] as the specimen dimensions and voltages are different. However, they appear to provide a rational explanation for the numerous small amplitude peaks in [3] for the polycrystalline actuator, in the form of higher-order deformation modes superimposed upon (and in quadrature with) the dishing mode desired for actuator operation. Specifically, in most of the cases here (the exception being 1723 Hz) there is a dishing mode, sometimes axisymmetric and sometimes less so, which is the desired deformation mode for the intended diaphragm-like air actuation. In quadrature with each of these is a higher order mode, typically with some form of rotational symmetry, for example a having two, four six or eight antinodes, though these are not always well defined.
The displacement results from the second set of experiments are, however, more directly comparable with those from reference [3] and show some agreement and some variation. Those for the polycrystalline (PZT) actuator are generally in good agreement, for example the peak-peak displacement at 1.8 kHz is measured here as 0.007 mm (0.0035 mm × 2) which is very close to that given in [3]. At 2.8 kHz the peak-peak displacement of the actuator centre relative to the edge appears to be around 0.009 mm, compared with 0.018 mm from [3], perhaps suggesting that the 2.8 kHz resonance is not so pronounced in the present experiment or that sensitivity of the system is reduced at higher frequencies.

A similar though more exaggerated comparison is observed for the single crystal (PMN-PT) actuator: at 1.8 kHz a peak-peak displacement of 0.02 mm is measured here, compared with 0.012 mm in [3]. However, the very large peak-peak displacement of 0.087 mm in [3] is not encountered here, with the value measured here being only 0.030 mm. It is noted that the resonance shown in [3] is quite sharp so any departure from resonant frequency, or additional damping, would greatly affect the measured amplitude. It should also be noted that repeated measurements were undertaken on the second set of experiments; for space reasons only one set is given here though the results were generally repeatable except in cases of failed experiments; for example the peak-peak displacement at 2.8 kHz on the single crystal model was alternatively measured at around 0.028 mm.

One reason why the amplitudes at 2.8 kHz are lower than expected is likely to be because the electronic shutter time, set to 1/10000 s, covers over a quarter of the vibration cycle period, leading to smearing of the measured displacements and consequent loss of apparent amplitude (this may be shown...

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**Figure 5:** Results for single crystal (PMN-PT) actuator: (a-d) typical raw deflected shapes from DIC system; (e-h) deflected shapes from DIC vibrometry using external reference signal. Again, no higher-order shapes were detected in quadrature with these deflections.
to give around 13% reduction). Further work will therefore need to explore the use of shorter exposure times to ensure that this effect does not occur, provided that such shorter times can be used without loss of image quality.

In terms of measured operational deflected shapes, it is reassuring that all those measured at 1.8 kHz and 2.8 kHz are of the desired dishing mode required for operation as a synthetic jet actuator.

More generally, the results of the DIC vibrometry show that, even when a quadrature reference signal is not available, useful data can be recovered, indeed with little or no apparent loss of fidelity. A more thorough investigation will need be undertaken on the results obtained from the second set of experiments, to determine the loss of accuracy which results from using the internal referencing approach rather than the external quadrature reference signal.

Examination of the preliminary results might indicate that, for the main vibration modes at least, little is gained by performing the sinusoidal fitting etc. described by Warburton [1] rather than merely observing the displacement field from the DIC data. However, recovery of the higher-order mode shapes is not possible just by observation but becomes very clear once the DIC displacement are post-processed. Moreover, the raw DIC displacement results from the second set of experiments generally show very little by way of vibration mode shape information due to static deflections and noise, and it is only when these results are post-processed that the vibration mode shapes become clear. It is very encouraging that operational vibration mode shapes with amplitudes as little as a few microns are successfully recovered; only the polycrystalline results for 1.8 kHz and 0.71 mV, with an amplitude of around 0.001 mm, were too noisy to be meaningful.

Close examination of some of the displacement maps shows the presence of fringe-like features which arise within the raw DIC displacement fields rather than as a result of the post-processing of vibration data; as explored in [1], this is understood to be an artefact arising from the presence of very small speckles, and later versions of the DIC system are understood to overcome this problem using smoothing of the raw images.

6. Conclusions
The DIC vibrometry approach described by Warburton [1] has been successfully applied to a case study consisting of the forced vibration analysis of a synthetic jet actuator, even though (for initial experiments) the equipment needed to generate a quadrature reference signal was not available. Operational vibration mode shapes, including high-order modes superimposed upon, and in quadrature with, the main dishing modes of the actuator, have successfully been recovered in all cases except those where the vibration amplitude is very small (less than 0.002 mm). Displacement amplitudes are broadly in agreement with those obtained from laser vibrometry on different instances of the same types of discs though there were some differences in values relating to resonances. It may be concluded that, with or without an external quadrature reference signal, the low speed DIC vibrometry approach provides a useful alternative to scanning laser vibrometry as a means of measuring operational vibration shapes.

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References
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