Remarkable effects of dirty limit on superconducting condensate

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Abstract. Using heterostructures that combine a two superconductor ( Nb-Pb). We demonstrate the modulation of the superconducting condensate at the nanoscale via variation of mean-free path. The modulation of superconductivity can be obtained not only for choosing smaller superconducting lengths comparing with bulk superconducting length or considering several geometric shapes, but also whether strong local doping effect can be produced over the superficial area of the superconductor. Through this mechanism, a nanoscale pattern of two condensates regions can be created in the superconductor. This yields a magnetization curves that has no counterpart in the literature. We show that this form of modulation based on the possibility of change mean-free path represent a groundbreaking prospects in the study of the effects that might exploit unique superconducting properties, due to allows the manipulation of magnetic flux quanta.

1. Introduction

The study of the superconducting mesoscopic samples received recently a lot of attention due to results obtained in the analysis of the properties of the superconductors and therefore their applications, which are determined by critical parameters, i.e. the critical fields \(H_{c1}, H_{c2}, H_{c3}\), the critical current \(j_c\) and the critical temperature \(T_c\), those can be basically tuned by their intrinsics characteristic length scales: the coherence length \(\xi\), the length scale of the Cooper-paired electrons, and the London penetration length \(\lambda\), the length scale of magnetic field penetration into the superconductor. For application purposes is necessary to control these parameters. Previous works focused on the effect of superconducting condensate confined strongly in small elements as a procedure to improve the critical properties, researchers has found novels effects on confinement in mesoscopics disks [1, 2, 3, 4], and the contribution of symetry [5] in comparing disks, squares [6], and triangles [7]. The individual vortices pierce the sample in direction of applied magnetic field forming vortex clusters in those mesoscopic systems, both theoretical and experimental, revealed multivortex Abrikosov-like states that tending to mimic the symmetry of the sample due to the repulsion with the surrounding screening (Meissner) currents, as well as depending on the details of the condensate confinement because edge currents may even compress the vortex lines in to a single bundle, often called gigant vortex [9, 10, 11, 12, 13, 14, 15].

Recently, there has been growing interest in investigating nanostructured superconductivity that include topological superconductors which are analogs of the topological insulators. In
these systems, the main proposals are focused on combining in proximity with a conventional superconductor [16] to use the transport properties, observed experimentally, in transport measurements [17, 18] and scanning tunneling spectroscopy [19, 20]. This systems has been studied by using half-Heusler materials to offer multifunctional topological devices [21], due to its superconducting properties with low carrier concentration in electron transport measurements [22]. These measurements demonstrate that is possible to control the energy gaps, critical temperature and critical magnetic field in selected region as a result of creating of tailored nanostructured superconductors with complex superconductor materials quantum technology applications. This developments has reached new advances including the superconducting proximity effect in epitaxial graphene induced by a graphene-superconductor interface [23, 24].

From the point of view of numerical calculations have shown a lot of effects usually observed in nanostructured superconductors, resulting in complex vortex patterns when barrier or defects are included [25, 26, 27, 28, 29].

Similar behavior has also been simulated including anisotropy in the superconducting sample, through variation of $T_c$ in different layer of the sample that leads to distinct vortex states and their free energy [30, 31, 32, 33]. Another possibility for the nonconventional vortex structure can be obtained from multiband superconductivity, since high temperature superconductor $MgB_2$ was observed in the experiments [34, 35, 36]. These multiband systems are provided with two or more electronic condensates from Cooper paring in different bands of the material, and exhibit a variety of new and interesting phenomena with no counterpart in conventional single-component superconductors [36, 37, 38, 39, 40]. One of most intriguing of these phenomena are related to the exotic vortex structures that can emerge in a two-component superconductor, because of the different length scales $\xi$, at which the Cooper-pair density varies in each component [41, 42, 43].

2. Physical parameters two-component superconductors in Ginzburg-Landau Equations

In the analysis of the superconducting state of two-component superconductors, we have to consider the relation between the parameters in each materials, in order to describe correctly important physical quantities used to describe the vortex state such as the Gibbs free energy, magnetization, density of order parameter, superconducting current density and phase of order parameter. In what follows we will derive an expression for each kind of parameter used in superconductivity.

(1) The coherence length.

In $S_1$, $\xi^2_1(0) = -\hbar^2/m^*_1|\alpha_1(0)|$ whereas $S_2$, $\xi^2_2(0) = -\hbar^2/m^*_2|\alpha_2(0)|$

\[ \frac{\xi^2_2(0)}{\xi^2_1(0)} = \frac{m^*_1|\alpha_1(0)|}{m^*_2|\alpha_2(0)|} \]  

(2) The penetration length.

$S_1$, $\lambda^2_1(0) = \frac{m^*_1e^2\beta_1}{4\pi(e^*)^2|\alpha_1(0)|}$ and $S_2$, $\lambda^2_2(0) = \frac{m^*_2e^2\beta_2}{4\pi(e^*)^2|\alpha_2(0)|}$

\[ \frac{\lambda^2_2(0)}{\lambda^2_1(0)} = \frac{m^*_2|\alpha_2(0)|}{m^*_1|\beta_1|\alpha_1(0)|} \]  

We can also to obtain additional relation between coherence length and penetration length,

\[ \frac{\lambda^2_2(0)}{\lambda^2_1(0)} = \left( \frac{m^*_2}{m^*_1} \right)^2 \left( \frac{\beta_2}{\beta_1} \right) \left( \frac{\xi^2_2(0)}{\xi^2_1(0)} \right) \]  

(3) Order parameter.
Using the definition of the order parameter, we obtain:

\[ \frac{\psi_{\infty,2}^2}{\psi_{\infty,1}^2} = \frac{m_2^* \lambda_2^2(0)}{m_1^* \lambda_1^2(0)} \]  

(4)

Additional relation for the order parameter is found in terms of \( c_\lambda \), \( c_\xi \) and \( c_m \), as well as the Ginzburg-Landau parameters \( \beta \) and \( \alpha \):

\[ \frac{\psi_{\infty,2}^2}{\psi_{\infty,1}^2} = \frac{1}{c_\lambda c_m} \]  

(5)

\[ \frac{\beta_1}{\beta_2} = \frac{c_\xi}{c_\lambda} \frac{1}{c_m} \]  

(6)

\[ \frac{\alpha_2}{\alpha_1} = \frac{c_m}{c_\xi} \]  

(7)

3. Results

In this thesis we will denote the two superconductors inside of the sample as \( S_1 \) and \( S_2 \), the quantities are scaled to the units that depend on the parameters of \( S_1 \). In what follows, we first consider a superconducting film of size \( a = 2000 \text{nm} \) and width of size \( d = 5.5 \text{nm} \) in the sample that is formed by niobium (Nb) and lead (Pb) in each sides of the sample. The coherence length and the penetration depth for such a thin and diffusive superconductor take the effective values:

\[ \xi_{\text{eff}} \approx 0.85 \sqrt{\frac{\xi_0 l}{1-T/T_c}} \] and \[ \lambda_{\text{eff}} \approx 0.65 \lambda_0 \sqrt{\frac{\xi_0}{l(1-T/T_c)}} \] where the \( \xi_0 = 80 \text{nm} \) and \( \lambda_0 = 50 \text{nm} \) are, respectively, the coherence length and the penetration depth in the bulk for Pb, while for Nb the characteristics lengths are, \( \xi_0 = 160 \text{nm} \) and \( \lambda_0 = 38 \text{nm} \). The conditions chosen for the hybrid superconducting system allow to find in the sample the circulation of two screening currents in the interface that can be modified considering variations of the effective characteristic lengths \( (\xi_{\text{eff}}, \lambda_{\text{eff}}) \) including changes in the mean free path \( l \), which can be modified in the sample by ion bombardment to reduce the electron mean-free path value. To begin with, we discuss variation of the magnetization in the sample due to vortex entrance in both superconductor components, as illustrated in Fig.1 (a), it presents a series of discontinuities, in which each jump signals the entrance of more vortices into the sample. This picture represents a profile which is not typical, as we know. In this work, the diamagnetic response of the sample in the Meissner state manifest as the magnetization of the sample, increasing with the applied magnetic field. When the vortices enter the sample, the magnetization decrease showing jumps, how it is expected in samples with only one component, but the results in two-component show that after one jump in which the magnetization decrease, in the next jump can increase.

In the case of Fig.1 (a), this first jump in the magnetization, owing to the transition from the Meissner state to the mixed state, occurs through of the component (S2) in which the partial flux penetrate due to the diamagnetic energy cost of holding the field out is less. The magnetization reaches a value higher in the component (S2) in which after the first peak in the magnetization it is growing again [see Fig.1 (a)red curve], we can see that the value of the magnetization on the second peak exceeds the first one implying that the diamagnetism is growing in the other component and the cost of the diamagnetic energy to maintain the magnetic field out is higher in S1 than S2.

The magnetization curves show a more pronounced or step-like jumps at the lower values of the applied magnetic field. This behavior of the transitions can be explained using the Ginzburg-Landau equations and physical parameters described above in which the ratio of Cooper-pair density between S2 and S1, that depend of the parameters used in each component. Therefore,
the transitions with large variations in the magnetization curve are produced in S1, whereas the small variations to S2. One consequence of simulating a superconducting sample with two components is that we obtain two upper critical magnetic fields. Thus S1 reach its normal state at a value of the applied field less than S2, therefore S1 takes properties as a metal and establish a Dirichlet boundary condition on this side of the sample meanwhile the others sides remain with Newman boundary conditions. This is reflected in the behavior of the magnetization curve [Fig.1 (a)] which normally drop with approximately same slop but in some point it change and drop in a higher value of $H$. In this point the superconductor sample reach a rectangular shape due to only in one have remain in superconducting state. In Fig. 1 (a) shown the influence of the electron mean free path $l$ on the magnetic behavior of the sample. The level of magnetization is strongly influenced by the variations of the $l$, which turns out smaller with the reduction of $l$. This follows from the fact that, the magnetization drop with $l$, and the condensate on each half of the sample change providing poor and better screening of the magnetic field depending of the selected conditions in the superconductors ($S_1$ and $S_2$), and when one of the condensates ceases, the first condensate experiences a large difference in the felt magnetic field, which in turn allows for a larger flux penetration and thus a lower diamagnetic response of the sample. However, the free energy is lower influenced by changes in the electron mean free path [See Fig. 1 (b)] Figs. 1 (c-d) show the Cooper-pair density and the magnetic field $h_0$ as a function of the position.
the applied magnetic field. It is possible to observe that exist a position near to the interface in the sample at which the variation of the \( l \) does not affect the value of the condensates neither on the Nb nor on the Pb [see curves at Fig. 1 (c)], that implies new characteristics that can be found in this kind of samples taking into account that \( l \) modified always the diffusion on every materials. The magnetic field distribution show peaks at the edge as it is expected for a square superconducting sample but inside additional peaks field the interface at \( L \approx 62 \xi_1(0) \) decrease selecting lower values of \( l \) [see Fig. 1 (d)], i.e. Inset of the same figure is possible to see the difference between the field distribution in both components (\( S_1 \) and \( S_2 \)). In Fig. 2 shows obtained vortex configuration for several values of \( l \) at the same value of \( H_0 = 0.0817H_{c2,1} \), in which is observed a huge differences between the condensates on each half. As we know, the coherence length is the characteristic length scale over which the order parameter changes and it is therefore related with the size of vortex core. Therefore, it is intuitive that size of vortices will change when it penetrate from one superconductor to another, but additionally, the mean free path make a contribution in this sense due to it is related with the characteristics length of a superconductor. In consequence, we can see that the reduction of \( l \) can preserve the condensate for higher values of applied magnetic field [see Fig. 2 (a)] while for the same \( H_0 \) the superconductivity is quickly depreciated [see Fig. 2 (e)], which also produce a reduction of the vortex size as the \( l \) reduced. Finally, the hybrid superconductor can be considered a novel kind of superconducting system in which the main characteristic can be tuned to obtain the desired properties for any kind of use in electronic devices. In Figs. 3 show the behavior of the coherence length \( \xi \) and penetration depth \( \lambda \) for niobium and led whereas the electron mean free path is changed as well as between them. In thin films in the dirty limit \( \xi_0 > l \) in which \( \lambda_0 \)
Figure 3: (Color online) Characteristics lengths of a hybrid superconductor formed by niobium and lead as a function of electron mean free path.

and $\xi_0$ become dependent on $l$. The theory predicts that the coherence length is proportional to a square root of the electron mean-free path, $\xi \propto l^{1/2}$, as opposed to the increase of the penetration depth with decreasing $\lambda \propto l^{-1/2}$. However, in thin films, the effective penetration depth $(\lambda_{\text{eff}})$ might even be much larger.

4. Conclusions
In the framework of the models discussed in this work, we studied the superconducting state of two-component mesoscopic square sample, where novel and rich magnetization curves, free energy and vortex configurations are obtained simulating two kind of ultra thin samples using niobium (Nb) and lead (Pb). There has been much interest in such superconducting systems in which the number of degrees of freedom of the wave-function allows for emergent quantum effects that is otherwise unattainable in single-component superconductor. In this work, the two superconductors are simulated including anisotropies into the ultra thin sample which modifies the behavior of the carriers involved in the formation of the superconducting condensate which can be changed locally. The anisotropy in our first kind of two component superconducting samples studied in this work, is considered including two superconductors on each half of the sample, a hybrid superconductor, which depends on the ratio of the sample parameters such as critical temperature $c_T$, penetration depth $c_\lambda$, coherence length $c_\xi$, electron mass $c_m$ and the order parameters in each component which is tunable by ratio of this parameters that interact only in the interface between superconductors, and special boundary conditions is not needed.

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