FOLD-SE: Scalable Explainable AI

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Abstract

FOLD-R++ is a highly efficient and explainable rule-based machine learning algorithm for binary classification tasks. It generates a stratified normal logic program as an (explainable) trained model. We present an improvement over the FOLD-R++ algorithm, termed FOLD-SE, that provides scalable explainability (SE) while inheriting all the merits of FOLD-R++. Scalable explainability means that regardless of the size of the dataset, the number of learned rules and learned literals stay small and, hence, understandable by human beings, while maintaining good performance in classification. FOLD-SE is competitive in performance with state-of-the-art algorithms such as XGBoost and Multi-Layer Perceptrons (MLP). However, unlike XGBoost and MLP, the FOLD-SE algorithm generates a model with scalable explainability. The FOLD-SE algorithm outperforms FOLD-R++ and RIPPER algorithms in efficiency, performance, and explainability, especially for large datasets. The FOLD-RM algorithm is an extension of FOLD-R++ for multi-class classification tasks. An improved FOLD-RM algorithm built upon FOLD-SE is also presented.

1 Introduction

Dramatic success of machine learning has led to a torrent of Artificial Intelligence (AI) applications. However, the effectiveness of these systems is limited by the machines’ current inability to explain their decisions and actions to human users. That is mainly because the statistical machine learning methods produce models that are complex algebraic solutions to optimization problems such as risk minimization or data likelihood maximization. Lack of intuitive descriptions makes it hard for users to understand and verify the underlying rules that govern the model. Also, these methods cannot produce a justification for a prediction they compute for a new data sample.

Rule-based machine learning (RBML) algorithms have been devised that learn a set of relational rules that collectively represent the logic of the concept encapsulated in the data. The generated rules are more comprehensible to humans compared to the complicated deep learning model or complicated formulas. Examples of such algorithms include the FOIL (Quinlan 1990) and RIPPER (Cohen 1995). Some of these algorithms allow the knowledge learned to be incrementally extended without retraining the entire model. The learned symbolic rules make it easier for users to understand and verify them.

The RBML problem can be regarded as a search problem for a set of rules from the training examples. Usually, the search for rules can be processed either top-down or bottom-up. A bottom-up approach starts by creating most-specific rules from training examples and exploring the hypothesis space by employing generalization techniques. Bottom-up approaches are not applicable for large datasets. A top-down approach starts by building the most-general rules from training examples and then specializes them into a final rule set. Most of the RBML algorithms are not efficient for large datasets even if they follow a top-down approach. Some of them, e.g., TREPAN (Craven and Shavlik 1995) extract rules from statistical machine learning models, but their performance and efficiency is limited by the target machine learning model. Yet other algorithms train models with a logic program solver, for example, most of the Inductive Logic Programming (ILP) based algorithms (Cropper and Dumancic 2020).

An important aspect of RBML systems is that they should be scalable, meaning that they should work with large datasets, and learn the rules in a reasonable amount of time. For RBML algorithms, the size of the rule-set—represented by the number of rules and number of conditional predicates (features) involved in the rules—has a big impact on human-understanding of the rules (interpretable) and explaining predictions (explainability). The more rules and predicates a rule-set representing a model contains, the harder it is for a human to understand. Generally, as the size of dataset increases, the number of rules and conditional predicate increases. Ideally, we would like for the rule-set size to not increase with dataset size. We call this concept scalable explainability, i.e., the size of the rule-set is a small constant regardless of the dataset size. Thus, even when size of the input training data is very large, the rule-set representing the model should be small enough for a human to comprehend.

The FOIL algorithm by Quinlan (Quinlan 1990) is a popular top-down RBML algorithm. FOIL uses heuristics from information theory called weighted information gain. The use of greedy heuristics allows FOIL to run much faster than bottom-up approaches and scale up much better. The FOLD algorithm by Shakerin (Shakerin, Salazar, and Gupta 2017) is inspired by the FOIL algorithm, it learns a default the-
ory with exceptions represented as a stratified normal logic program. The FOLD algorithm incrementally generates literals for default rules that cover positive examples while avoiding covering negative examples. It then swaps the positive and negative examples and calls itself recursively to learn exceptions to the default when there are still positive examples uncovered. Subsequently, we developed the FOLD-R++ algorithm (Wang and Gupta 2022) on top of the FOLD algorithm that utilizes the prefix sum computation technique with a special comparison operators to speed up literal selection while avoiding one-hot encoding for mixed-type data. FOLD-R++ also introduced a hyperparameter called ratio to speed up training while reducing the number of generated rules (Wang and Gupta 2022).

The FOLD-R++ algorithm is able to generate much fewer rules than the well-known rule-based ML algorithm RIPPER while outperforming it in classification accuracy. For very large datasets, most of the RBML algorithms are not scalable. They fail to finish training in a reasonable time. Even RIPPER and FOLD-R++ algorithms often generate too many rules making them incomprehensible to humans. For example, Rain in Australia is a large dataset with over 140K training examples. With the same target class ‘No’, RIPPER generates 180 rules with over 700 literals that achieve 63% accuracy while FOLD-R++ generates 48 rules with around 120 literals that achieve 79% accuracy. That’s too many rules, arguably, for a human to understand. Another explainability-related problem with these RBML algorithms is that the rules they generate change significantly when a small percentage of training data changes. Thus, the learned rule-set will be different for different splits of the dataset for training and testing. We would like for the rule-set to not change as the training dataset changes.

To deal with the above explainability issue on large datasets, this paper presents an improved FOLD-R++ algorithm that employs a newly created heuristic for literal selection that greatly reduces the number of rules and predicates in the generated rule-set. In addition, we also improve explainability by introducing a rule pruning mechanism in the training process. The pruning mechanism ameliorates the long-tail effect, namely, that rules generated later in the learning process cover fewer examples than those generated earlier. Finally, we add two comparison operators that improve the literal selection process for sparse datasets containing many missing values. The improved learning algorithm, that we call FOLD-SE, provides scalable explainability. FOLD-SE generates a rule-set that uses a small number of rules and predicates (features) regardless of the dataset size. Also, the generated rule-set is almost the same regardless of the training/testing split used. Our experimental results indicate that FOLD-SE is competitive in accuracy and efficiency with well-known machine learning algorithms such as XGBoost classifier and Multi-Layer Perceptron (MLP). The FOLD-SE algorithm significantly outperforms FOLD-R++, on which it is based, as well as the RIPPER algorithm. In addition, the FOLD-SE implementation provides justification for a prediction.

2 Background

FOLD-SE represents the rule-set that it learns from a dataset as a default theory that is expressed as a normal logic program, i.e., logic programming with negation. The logic program is stratified in that there are no recursive calls in the rules. We briefly describe default logic below. We assume that the reader is familiar with logic programming (Brazi 2012) as well as classification problems (Bishop 2006).

Default Logic (Reiter 1980) is a non-monotonic logic to formalize commonsense reasoning. A default $D$ is an expression of the form

$$A : MB$$

which states that the conclusion $\Gamma$ can be inferred if prerequisite $A$ holds and $B$ is justified. $MB$ stands for “it is consistent to believe $B$”. Normal logic programs can encode a default theory quite elegantly (Gelfond and Kahl 2014). A default of the form:

$$\gamma := \alpha_1 \land \alpha_2 \land \cdots \land \alpha_n : M \gamma_1 \gamma \gamma_2 \cdots \gamma_m$$

can be formalized as the normal logic programming rule:

$$\gamma := \alpha_1, \alpha_2, \ldots, \alpha_n, not \beta_1, not \beta_2, \ldots, not \beta_m.$$ 

where $\alpha$’s and $\beta$’s are positive predicates and $not$ represents negation-as-failure. We call such rules default rules. Thus, the default $\frac{bird(X):M=penguin(X)}{fly(X)}$ will be represented as the following default rule in normal logic programming:

$$fly(X) := bird(X), not \ penguin(X).$$

We call $bird(X)$, the condition that allows us to jump to the default conclusion that $X$ can fly, the default part of the rule, and $not \ penguin(X)$ the exception part of the rule.

3 FOLD-SE algorithm

3.1 Heuristic for Literal Selection

Most top-down RBML algorithms employ a heuristic to guide the literal selection process; information gain (IG) and its variations are the most popular ones. Every selected literal leads to a split on input examples during training. The heuristic used in split-based classifiers greatly impacts the accuracy and structure of the learned model, whether its rule-based or decision tree-based. Specifically, the heuristic used in literal selection of RBML algorithms impacts the number of generated rules and literals, therefore it has an impact on explainability.

FOLD-SE employs a heuristic that we have newly created called Magic Gini Impurity (MGI). MGI is inspired by Gini Impurity (GI) heuristic to guide the literal selection process. It helps reduce the number of generated rules and literals while maintaining competitive performance compared to using information gain (IG). GI for binary classification is defined as:

$$GI(tp, fn, tn, fp) = \frac{tp \times fp}{(tp + fp)^2} + \frac{tn \times fn}{(tn + fn)^2}. \quad (1)$$

To verify MGI’s effectiveness, a comparison experiment of 4 different heuristics (MGI, information gain, weighted Gini
Index, Chi-Square) with 2-way split decision trees on various datasets is performed. The weighted Gini Impurity is also tested but its result is exactly the same as for weighted Gini Index because they are equivalent in comparison, mathematically. The decision tree employs the literal selection process of FOLD-R++ for splitting nodes. The accuracy, numbers of generated tree nodes, the average depth of leaf nodes, and time consumption of the 10-fold cross-validation test are averaged and reported. This comparison experiment is performed on a small form factor desktop with an Intel i7-8705G CPU and 16 GB RAM. As we can see from the results shown in Table 1 all the heuristics have equivalent splitting performance on decision trees. The numbers of generated tree nodes and the average depth of leaf nodes are also very close.

Next, we use Magic Gini Impurity (MGI) as the heuristic in the FOLD-R++ algorithm and compare it to FOLD-R++ with the Information Gain (IG) heuristic. Interestingly, the number of rules and predicates in the rules is reduced drastically. As we can see in Table 2 the number of generated rules and generated literals is reduced significantly with MGI compared to IG, while performance remains the same. Thus, Magic Gini Impurity significantly improves interpretability and explainability of the FOLD-R++ algorithm while preserving performance.

3.2 Comparison of Feature Values

During the learning process, FOLD-SE has to compare categorical and numerical data values. FOLD-SE employs a carefully designed comparison operator, which is an extension of the comparison operator of FOLD-R++ for comparing categorical and numerical values. This gives FOLD-SE the ability to elegantly handle mixed-type values and, thus, learn from datasets that may have features containing both numerical and categorical values (a missing value is considered as a categorical value). The comparison between two numerical values or two categorical values in FOLD-R++ is straightforward, as commonsense would dictate, i.e., two numerical (resp. categorical) values are equal if they are identical, otherwise they are unequal. The equality between a numerical value and a categorical value is always false, and the inequality between a numerical value and a categorical value is always true. In addition, numerical comparisons (< and >) between a numerical value and a categorical value is always false. However, the numerical comparisons ≤ and ≥ are not complementary to each other with this comparison assumption. For example, x ≤ 4 means that x is a number and x is less than or equal to 4. The opposite of x ≤ 4 should be x is a number greater than 4 or x is not a number. Without the opposite of these two numerical comparisons being used, the literal selection process of FOLD-R++ would be limited. The FOLD-SE algorithm, thus, extends the comparison operators with ≤ and ≥ as the opposites of < and >, respectively. The literals with ≤ and ≥ will be candidate literals in the literal selection process but converted to their opposites, ≤ and ≥, in the final results. An example is shown in Table 3.

Given $E^+ = \{3, 4, 4.5, x, y, y\}$, $E^- = \{1, 1.2, 3, y, y, z\}$, and literal $(i, >, 3)$ in Table 3 the true positive example $E_{tp}$, false negative examples $E_{fn}$, true negative examples $E_{tn}$, and false positive examples $E_{fp}$ implied by the literal are $\{4, 4.5\}$, $\{x, x, y\}$, $\{1, 1.2, 3, y, y, z\}$, $\emptyset$ respectively. Then, the heuristic of literal $(i, >, 3)$ is calculated as $\text{MGI}_{(i, >, 3)}(3, 4.8, 0) = 0.38$ through Formula 3.

### Table 1: Decision tree with different heuristics on various Datasets

| Data Set | Name | Rows | Cols | Magic Gini Impurity | Information Gain | Weighted Gini Index | Chi-Square |
|----------|------|------|------|--------------------|------------------|--------------------|------------|
|         |      |      |      |                    |                  |                    |            |
| acute   | 120  | 7    | 1.0  | 4.0                | 1.3              | 4.0                | 2          |
| wine    | 178  | 14   | 0.97 | 5.8                | 6.7              | 4.2                | 4          |
| heart   | 270  | 14   | 0.73 | 3.8                | 6.1              | 5.3                | 4          |
| ionosphere | 351  | 35   | 0.88 | 9.1                | 6.2              | 4.5                | 4          |
| kidney  | 400  | 25   | 1.0  | 7.1                | 6.4              | 3.1                | 4          |
| voting  | 435  | 17   | 0.94 | 24.2               | 6.9              | 3.2                | 4          |
| credit-a| 690  | 18   | 0.81 | 78.2               | 10.4             | 8.6                | 4          |
| breast-w| 699  | 10   | 0.93 | 35.9               | 10.1             | 8.3                | 4          |
| autism  | 704  | 18   | 0.83 | 46.3               | 7.6              | 5.2                | 4          |
| parkinson| 765  | 754  | 0.83 | 33.7               | 10.4             | 8.2                | 4          |
| diabetes| 768  | 9    | 0.69 | 119.5              | 11.1             | 9.5                | 4          |
| cars    | 1728 | 7    | 1.0  | 47.2               | 9.9              | 5.3                | 4          |
| kr vs. kp| 3196 | 37   | 1.0  | 43.3               | 10.1             | 6.5                | 4          |
| mushroom| 1824 | 23   | 1.0  | 11.0               | 5.1              | 3.3                | 4          |
| churn-model| 10000 | 11 | 0.80 | 260.9              | 17.2             | 4.98               | 4          |
| intention| 12330| 19   | 0.87 | 884.3              | 19.3             | 7.00               | 4          |
| eeg    | 14980| 15   | 0.84 | 1135.7             | 17.6             | 11.64              | 4          |
| credit card | 30000 | 24 | 0.73 | 4011.9             | 17.9             | 6.28               | 4          |
| adult  | 32561 | 15  | 0.82 | 4273.9             | 29.2             | 38.26              | 4          |

### Table 3: Examples of literals with different comparison operators

Given $E^+ = \{3, 4, 4.5, x, y, y\}$, $E^- = \{1, 1.2, 3, y, y, z\}$, and literal $(i, >, 3)$ in Table 3 the true positive example $E_{tp}$, false negative examples $E_{fn}$, true negative examples $E_{tn}$, and false positive examples $E_{fp}$ implied by the literal are $\{4, 4.5\}$, $\{x, x, y\}$, $\{1, 1.2, 3, y, y, z\}$, $\emptyset$ respectively. Then, the heuristic of literal $(i, >, 3)$ is calculated as $\text{MGI}_{(i, >, 3)}(3, 4.8, 0) = 0.38$ through Formula 3.

### 3.3 Literal Selection

The FOLD-R++ algorithm starts the learning process with the candidate rule $p(...): - \text{true}$, where $p(...)$ is the target predicate to learn. It specializes the rule by adding literals to its body during the training process. It adds a lit-
Table 2: Comparison of Magic Gini impurity (MGI) and information gain (IG) in FOLD-R++

| Data Set       | FOLD-R++ | FOLD-R++ with MGI |
|----------------|----------|------------------|
| Name           | Acc Prec Rec F1 t(ms) | Acc Prec Rec F1 t(ms) |
| acute          | 0.99 1.0 0.99 0.99 2 2.7 | 3.0 | 0.99 1.0 0.99 0.99 1 2.7 |
| heart          | 0.77 0.80 0.80 0.79 38 | 15.9 32.2 | 0.74 0.77 0.78 0.77 11 | 4.3 8.9 |
| ionosphere     | 0.90 0.92 0.93 0.92 275 | 12.4 19.7 | 0.91 0.89 0.98 0.93 94 | 5.1 7.1 |
| kidney         | 0.99 1.0 0.98 0.99 16 | 4.9 5.9 | 0.98 1.0 0.97 0.98 25 | 5.2 6.3 |
| voting         | 0.94 0.92 0.95 0.92 23 | 10.0 27.2 | 0.95 0.92 0.96 0.94 25 | 7.8 20.2 |
| credit-a       | 0.63 0.90 0.78 0.83 84 | 10.3 23.3 | 0.85 0.93 0.78 0.85 39 | 2.2 3.8 |
| breast-w       | 0.95 0.97 0.93 0.96 34 | 10.5 18.6 | 0.95 0.98 0.94 0.94 32 | 8.1 12.9 |
| autism         | 0.93 0.95 0.95 0.95 62 | 25.4 54.8 | 0.92 0.95 0.94 0.94 41 | 19.4 43.4 |
| parkinson      | 0.82 0.83 0.93 0.89 10.75 | 13.7 21.2 | 0.81 0.82 0.96 0.89 1,746 | 7.4 13.2 |
| diabetes       | 0.74 0.79 0.83 0.80 66 | 8.3 19.4 | 0.74 0.78 0.84 0.84 48 | 4.8 11.6 |
| cars           | 0.96 1.0 0.95 0.97 31 | 12.3 29.8 | 0.96 1.0 0.95 0.97 35 | 8.8 17.5 |
| kr vs. kp      | 0.99 1.0 0.99 0.99 226 | 19.3 46.7 | 0.97 0.97 0.97 0.97 170 | 11.4 27.6 |
| mushroom       | 1.0 1.0 1.0 1.0 281 | 7.9 11.9 | 1.0 1.0 1.0 1.0 257 | 7.6 13.4 |
| intention      | 1.0 1.0 1.0 1.0 1.0 1.0 | 1.0 1.0 1.0 1.0 | 0.99 1.0 1.0 1.0 0.94 | 16.7 40.8 |
| ceg            | 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 | 1.0 1.0 1.0 1.0 | 0.99 1.0 1.0 1.0 0.94 | 16.7 40.8 |
| credit card    | 0.72 0.72 0.72 0.72 2.735 | 69.1 152.6 | 0.68 0.75 0.64 0.69 1.353 | 18.7 40.8 |
| adult          | 0.82 0.81 0.96 0.89 5.95 | 19.1 48.8 | 0.82 0.83 0.96 0.89 1,827 | 10.9 25.5 |
| rain in aus     | 0.75 0.86 0.95 0.90 2.508 | 16.8 46.7 | 0.84 0.86 0.95 0.90 1,414 | 6.8 13.2 |

Table 3: Comparing numerical and categorical values

| comparison | evaluation | comparison | evaluation |
|------------|------------|------------|------------|
| 10 = 'cat' | False      | 10 ≠ 'cat' | True       |
| 10 ≤ 'cat' | False      | 10 > 'cat' | False      |
| 10 ≥ 'cat' | True       | 10 ≠ 'cat' | True       |

Table 4: Evaluation and count for literal(i, >, 3).

ceral that maximizes information gain. FOLD-EXT extends the literal selection process of FOLD-R++ by employing MGI as a heuristic instead of IG. In addition, candidate literals of the form \( m \not< n \) and \( m \not> n \) are also considered. The literal selection process of FOLD-SE is summarized in Algorithm 1. In line 2, \( cnt^+ \) and \( cnt^- \) are dictionaries that hold, respectively, the numbers of positive and negative examples of each unique value. In line 3, \( set_+, set_- \) are sets that hold, respectively, the unique numerical and categorical values. In line 4, \( tot_+^+ \) and \( tot_-^+ \) are the total number of, respectively, positive and negative examples with numerical values; \( tot_+^- \) and \( tot_-^- \) are the total number of, respectively, positive and negative examples with categorical values. In line 6, the prefix sums of numerical values have been computed as preparation for calculating heuristics of candidate literals. After the prefix sum calculation process, \( cnt^+ [x] \) and \( cnt^- [x] \) represents the number of positive examples and negative examples that have a value less than or equal to \( x \). Preparing parameters correctly is essential to calculating MGI values for candidate literals. In line 11, the MGI value for literal \((i, \leq, x)\) is computed by taking parameters \( cnt^+ [x] \) as the number of true positive examples, \( tot_+^+ - cnt^+ [x] + tot_+^- \) as the number of false positive examples, \( tot_-^+ - cnt^- [x] + tot_-^- \) as the number of true negative examples, and \( cnt^- [x] \) as the number of false positive examples. The reason for this is as follows: for the literal \((i, \leq, x)\), only numerical values that are less than or equal to \( x \) can be evaluated as positive, otherwise negative. \( tot_+^+ - cnt^+ [x] + tot_+^- \) represents the number of positive examples that have a value greater than \( x \) plus the total number of positive examples with categorical values. \( tot_-^+ - cnt^- [x] + tot_-^- \) represents the number of negative examples that have a value greater than \( x \) plus the total number of negative examples with categorical values. \( cnt^- [x] \) represents the number of negative examples that have a value less than or equal to \( x \). The heuristic calculation for other candidate literals also follows the same comparison regime mentioned above. Finally, the best literal on attr function returns the best heuristic score and the corresponding literal except the literals that have been used in current rule-learning process.

Example 1 Given positive and negative examples in Table 3 \( E^+, E^- \), with mixed type of values on \( i^{th} \) feature, the target is to find the literal with the best information gain on the given feature. There are 7 positive examples, their values on \( i^{th} \) feature are \([3, 4, 5, x, x, y, y]\), and the values on \( i^{th} \) feature of the 8 negative examples are \([1, 1, 1, 2, 3, y, y, z]\).

With the given examples and specified feature, the number of positive examples and negative examples for each unique value are counted first, which are shown as \( count^+ \), \( count^- \) in Table 5. Then, the prefix sum arrays are calculated for computing heuristic as \( sum_{pfx}^+, sum_{pfx}^- \). Table 6 shows the MGI heuristic for each candidate literal and the literal \((i, \not<, 2)\) gets selected as it has the highest score.
Algorithm 1 FOLD-SE Algorithm, Find Best Literal function

Input: $E^+$: positive examples, $E^-$: negative examples, used: used literals

Output: literal: the best literal that has the best heuristic score

1: function BEST_literal_onATTR($E^+$, $E^-$, i, used)
2:   cnt$, cnt$- \leftarrow \text{count_class}(E^+, E^-, i)$
3:   set$p$, set$- \leftarrow \text{unique_values}(E^+, E^-)$
4:   tot$_+$, tot$-_n$, tot$_+$, tot$-_n \leftarrow \text{count_total}(E^+, E^-, i)$
5:   $num \leftarrow \text{counting_sort}(set_n)$
6:   for $j \leftarrow 1$ to $\text{size}(num)$ do $\triangleright$ compute prefix sum
7:     cnt$+_{num\_j} \leftarrow \text{cnt$+_{num\_j} + cnt^+_{num\_j-1}$}$
8:     cnt$-_{num\_j} \leftarrow \text{cnt$-_{num\_j + cnt^-_{num\_j-1}$}$
9:   end for
10:  for $x \in \text{set}_n$ do $\triangleright$ H function computes MGI
11:    score$(i, \leq x) \leftarrow H(\text{cnt$+_{[x]}$, tot$-_n - cnt^{-}_{[x]} + tot^+_n, cnt^-_{[x]} + tot^-_n})$
12:    score$(i, > x) \leftarrow H(\text{tot$+_n - cnt^{-}_{[x]} + cnt^+_{[x]} + tot^-_n, cnt^-_{[x]} + tot^-_n})$
13:    score$(i, \neq x) \leftarrow \text{H(tot$^+_n - cnt^{-}_{[x]} - cnt^+_{[x]} + tot^-_n, cnt^-_{[x]} - tot^-_n})$
14:  end for
15: end function

3.4 Rule Pruning

The FOLD-R++ algorithm (Wang and Gupta 2022) is a recent rule-based ML algorithm for binary classification that generates a normal logic program in which all the default rules have the same rule head (target predicate). An example is covered means that it is predicted as positive. An example covered by any default rule in the set would imply the rule head is true. The FOLD-R++ algorithm generates a model by learning one rule at a time. After learning a rule, the already covered examples would be ruled out for better literal selection of remaining examples. If the ratio of false positive examples to true positive examples drops below the preset threshold, it would next learn exceptions by swapping remaining positive and negative examples then calling itself recursively. The ratio stands for the upper bound on the number of true positive examples to the number of false positive examples implied by the default part of a rule. It helps speed up the training process and reduces the number of rules learned. The training process of FOLD-R++ is also a process of ruling out already covered examples. Later generated rules cover fewer examples than the early generated ones. In other words, FOLD-R++ suffers from long-tail effect. Here is an example:

**Example 2** The “Adult Census Income” is a classic classification task that contains 32,561 records. We treat 80% of the data as training examples and 20% as testing examples. The task is to learn the income status of individuals (more/less than 50K/year) based on features such as gender, age, education, marital status, etc. FOLD-R++ generates the following program that contains 9 rules:

1. (1) income(X,'<50K') :-
2. (2) income(X, '<50K') :-
3. (3) income(X,'<=50K') :-
4. (4) ab1(X,'True') :-
5. (5) ab2(X,'True') :-
6. (6) ab3(X,'True') :-
7. (7) ab4(X,'True') :-
8. (8) ab5(X,'True') :-
9. (9) marital_status(X,'Married-civ-spouse'),
10. education_num(X,N1), N1<12.0, capital_gain(X,N2), N2<5013.0, not ab5(X,'True'), not ab6(X,'True').

Table 5: Top: Examples and values on $i^{th}$ feature. Bottom: positive/negative count and prefix sum on each value

| $i^{th}$ feature values | $E^+$ | 3 | 4 | 5 | x | y | z |
|-------------------------|------|---|---|---|---|---|---|
| $E^-$                   | 1    | 1 | 1 | 2 | 3 | y | z |

Table 6: The heuristic on $i^{th}$ feature with given examples

| value | $< value$ | $\leq value$ | $> value$ | $\geq value$ | $\neq value$ | $\approx value$ |
|-------|-----------|-------------|-----------|-------------|-------------|-------------|
| 1     | -∞        | -∞         | -∞        | -∞          | NA          | NA          |
| 2     | 0.35      | 0.49       | 0.50      | 0.30        | NA          | NA          |
| 3     | 0.49      | 0.50       | 0.50      | 0.30        | NA          | NA          |
| 4     | -∞        | -∞         | -∞        | -∞          | NA          | -∞          |
| 5     | NA        | NA         | NA        | NA          | -0.42       | -∞          |
| 6     | NA        | NA         | NA        | NA          | -0.49       | -0.47       |

(1) income(X,'<50K') :-
(2) income(X, '<50K') :-
(3) income(X,'<=50K') :-
(4) ab1(X,'True') :-
(5) ab2(X,'True') :-
(6) ab3(X,'True') :-
(7) ab4(X,'True') :-
(8) ab5(X,'True') :-
(9) marital_status(X,'Married-civ-spouse'),
(10) education_num(X,N1), N1<12.0, capital_gain(X,N2), N2<5013.0, not ab5(X,'True'), not ab6(X,'True').
The above generated rules achieve 0.85 accuracy and 0.90 F₁ score. The first rule covers 3428 test examples and the second rule covers 1999 test examples. Subsequent rules only cover small number of test examples. This long-tail effect is due to the overfitting on the training data. FOLD-SE introduces a hyper-parameter tail to limit the minimum number/percentage of training examples that a rule can cover. It helps reduce the number of generated rules and generated literals by reducing overfitting of outliers. This rule pruning is not a post-process after training, rather rules are pruned during the training process itself which helps speed-up training. With the tail parameter, FOLD-SE can be easily tuned to obtain a trade-off between accuracy and explainability. The FOLD-SE algorithm is summarized in Algorithm 2. The added rule pruning process is carried out in Line 32–37 of Algorithm 2. When a learned rule cannot cover enough training examples, the learn_rule function returns. Except for the input parameter tail and the rule pruning process, FOLD-R++ and FOLD-SE have the same algorithmic framework. Of course, the heuristic used in FOLD-SE is MGI, while FOLD-R++ uses IG.

3.5 Complexity Analysis

If M is the number of training examples and N is the number of features that have been included in the training, the time complexity of finding the best literal of a feature is \( O(M) \), assuming that counting sort is used at line 5 in Algorithm 1. Therefore, the complexity of finding the best literal of all features is \( O(MN) \). The worst training case is that each generated rule only covers one training example and each literal only help exclude one example. In this case, \( O(M^2) \) literals would be selected in total. Hence, the worst case time complexity of FOLD-SE is \( O(M^2N) \). Additionally, it is easy to prove that the FOLD-SE algorithm always terminates (proof is omitted due to lack of space).

4 Experimental Results

We next present our experiments on UCI benchmarks and Kaggle datasets. The XGBoost Classifier is a well-known classification model and used as a baseline model in our experiments. Multi-Layer Perceptron (MLP) is another widely-used model that is able to deal with generic classification tasks. The settings used for XGBoost and MLP models is kept simple without limiting their performance. However, both XGBoost and MLP models cannot directly perform training on mixed-type (numerical and categorical values) in a row or a column) data. For mixed-type data, one-hot encoding has been used for data preparation because label encoding would add non-existing numerical relation to categorical values. RIPPER system is another rule-induction algorithm that generates formulas in conjunctive normal form as an explainable model. FOLD-R++ is the foundation of our new FOLD-SE algorithm. Both RIPPER and FOLD-R++ are capable of dealing with mixed-type data and are used as baseline to compare explainability. For binary classification tasks, accuracy, precision, recall, and F₁ score have been used as evaluation metrics of classification performance. For multi-category classification, accuracy and weighted F₁ score have been reported. The number of generated rules and the number of generated literals (predicates) have been used as evaluation metrics of explainability.

The FOLD-SE algorithm does not need any data encoding for training, a feature that it inherits from FOLD-R++. After specifying the numerical features, both FOLD-R++ and FOLD-SE can deal with mixed-type data directly. Even missing values are handled and do not need to be provided. FOLD-SE has been implemented in Python. We have downloaded FOLD-R++, also coded in Python, from GitHub. The hyper-parameter ratio of these two algorithms is simply set to default value of 0.5 for all experiments. The hyper-parameter tail of the FOLD-SE algorithms is set to default percentage 0.5% of training data size. All the training pro-
cesses have been performed on a small form factor desktop with Intel i7-8705G and 16 GB RAM. To have good performance test, we performed 10-fold cross-validation test on each dataset. We report average classification metrics and execution times.

The experimental results shown in Table 7 indicate that the FOLD-R++ and FOLD-SE outperform RIPPER algorithm in accuracy and explainability (the numbers of generated rules and literals/predicates). The FOLD-SE algorithm outperforms FOLD-R++ in explainability while maintaining comparable performance in classification, especially for large datasets. With enough data, the FOLD-SE algorithm can generate really concise rules that can capture patterns in datasets. The most dramatic result is that FOLD-SE generates a model with 2.5 rules on average with 6.1 literals in these rules on average with average accuracy of 0.82, while RIPPER and FOLD-R++ report much higher values for number of rules and literals (180.1 rules and 776.4 literals for RIPPER and 48.2 rules and 115.8 literals for FOLD-R++) and lower value for accuracy (0.63 for RIPPER and 0.79 for FOLD-R++).

The experimental results comparing FOLD-SE with XGBoost and MLP are listed in Table 8. FOLD-SE always takes less time to train compared to XGBoost and MLP, espe-

### Table 7: Comparison of RIPPER, FOLD-R++, and FOLD-SE on various Datasets

| Data Set | RIPPER | FOLD-R++ | FOLD-SE |
|----------|--------|----------|---------|
| Name     | T(ms)  | T(ms)    | T(ms)   |
| acute    | 120    | 7        | 0.93    |
| heart    | 270    | 14       | 0.76    |
| ionosphere | 351  | 35       | 0.72    |
| kidney   | 400    | 25       | 0.98    |
| voting   | 435    | 17       | 0.95    |
| credit-a | 690    | 16       | 0.89    |
| breast-w | 699    | 10       | 0.93    |
| autism   | 704    | 18       | 0.93    |
| parkinson| 765    | 754      | 0.70    |
| diabetes | 768    | 9        | 0.66    |
| cars     | 1728   | 77       | 0.54    |
| kr vs. kp| 3196   | 37       | 0.99    |
| mushroom | 8124   | 23       | 1.0     |
| churn-model | 10000 | 11       | 0.84    |
| eeg      | 14980  | 15       | 0.85    |
| credit card | 30000 | 24       | 0.76    |
| adult    | 32561  | 15       | 0.71    |
| rain in aus | 145460| 24      |

### Table 8: Comparison of XGBoost, MLP, and FOLD-SE on various Datasets

| Data Set | XGBoost | MLP | FOLD-SE |
|----------|---------|-----|---------|
| Name     | T(ms)   | T(ms) | T(ms)  |
| acute    | 120     | 7    | 0.93   |
| heart    | 270     | 14   | 0.76   |
| ionosphere | 351   | 35   | 0.72   |
| kidney   | 400     | 25   | 0.98   |
| voting   | 435     | 17   | 0.95   |
| credit-a | 690     | 16   | 0.89   |
| breast-w | 699     | 10   | 0.93   |
| autism   | 704     | 18   | 0.93   |
| parkinson| 765     | 754  | 0.70   |
| diabetes | 768     | 9    | 0.66   |
| cars     | 1728    | 77   | 0.54   |
| kr vs. kp| 3196    | 37   | 0.99   |
| mushroom | 8124    | 23   | 1.0    |
| churn-model | 10000 | 11   | 0.84   |
| eeg      | 14980   | 15   | 0.85   |
| credit card | 30000 | 24   | 0.76   |
| adult    | 32561   | 15   | 0.71   |
| rain in aus | 145460| 24   |

The experimental results comparing FOLD-SE with XGBoost and MLP are listed in Table 8. FOLD-SE always takes less time to train compared to XGBoost and MLP, espe-
| Data Set       | FOLD-RM | FOLD-SE-M |
|---------------|---------|-----------|
| Name          | Rows/Cols | Acc | F1 | Rules | Preds | Acc | F1 | Rules | Preds |
| wine          | 176/14   | 0.93 | 0.94 | 7.5 | 7.5 | 0.95 | 0.95 | 6.5 | 7.6 |
| ecoli         | 336/9    | 0.79 | 0.80 | 43.9 | 60.0 | 0.80 | 0.79 | 24.0 | 45.2 |
| weight-lift   | 4024/155 | 1.0  | 1.0 | 14.7 | 16.9 | 1.0 | 1.0 | 7.0 | 10.6 |
| wall-robot    | 3656/25  | 0.99 | 0.99 | 29.6 | 41.6 | 0.99 | 0.99 | 7.1 | 15.8 |
| page-blocks   | 547/11   | 0.97 | 0.97 | 17.8 | 16.1 | 0.90 | 0.99 | 8.5 | 15.0 |
| nursery       | 1296/11  | 0.96 | 0.96 | 29.5 | 19.2 | 0.92 | 0.93 | 18.4 | 19.7 |
| dry-bean      | 1511/11  | 0.93 | 0.93 | 13.9 | 13.5 | 0.90 | 0.98 | 15.4 | 31.4 |
| shuttle       | 58000/10 | 1.0  | 1.0 | 27.2 | 35.4 | 1.0 | 0.99 | 4.0 | 5.0 |

Table 9: Comparison of FOLD-RM, and FOLD-SE-M

The FOLD-RM algorithm (Wang, Shakerin, and Gupta 2022) is the FOLD-R++ algorithm with the “M” extension for multi-class classification tasks. The “M” extension can also work with the FOLD-SE algorithm, we only need to add another parameter tail. The extended FOLD-SE is called FOLD-SE-M. Table 9 compares only FOLD-RM and FOLD-SE-M algorithms (we didn’t find open source RIPPER implementation for FOLD-SE algorithm). The extended FOLD-SE is called FOLD-SE-M. Table 9 compares only FOLD-RM and FOLD-SE-M algorithms. The hyper-parameter is also set as 0.5% of training data size. The FOLD-SE algorithm is more efficient and light-weight.

5 Explainability

With the new heuristic, extended comparison operator, and rule pruning, the FOLD-SE algorithm pushes interpretability and explainability to a higher level. For Example 2 (UCI Adult Dataset), it generates the following logic program with only two rules:

(1) income(X,‘<=50K’) :-
   income(X,‘<=50K’), not marital_status(X,‘Married-civ-spouse’),
   capital_gain(X,N1), N1=<6849.0.

(2) income(X,‘<=50K’) :-
   marital_status(X,‘Married-civ-spouse’),
   capital_gain(X,N1), N1=<5013.0,
   education_num(X,N2), N2=<12.0.

The above rules achieve 0.85% accuracy, 0.86% precision, 0.95% recall, and 0.91% F1 score, the first rule covers 3457 test examples and the second rule covers 1998 test examples. The generated rule set can be understood easily due to the symbolic representation: Who makes less than 50K dollars a year: (1) unmarried people with capital gain less than $6,849; (2) married people with capital gain less than $5,013 and education level not over 12. The generated rule-set for this dataset in the 10-fold cross validation test are almost all identical, only the values in the literals change slightly.

Example 3 The “Rain in Australia” is another classification task that contains 145,460 records with 24 features. We treat 80% of the data as training examples and 20% as testing examples. The task is to find out if it is not rainy tomorrow, the FOLD-SE generates following rules:

(1) raintomorrow(X,’No’) :- humidity3pm(X,N1), N1=<64.0, rainfall(X,N2), N2=<182.6.
(2) raintomorrow(X,’No’) :- rainfall(X,N2), N2=<2.2, humidity3pm(X,N1), not(N1=<64.0), not(N1>=81.0).

The generated rules achieve 0.83% accuracy, 0.85% precision, 0.94% recall, and 0.89% F1 score.

6 Related Work and Conclusions

Rule-base Machine Learning is a long-standing interest of the community. Some RBML algorithms perform training directly on the input data: ALEPH (Srinivasan 2001) is a well-known Inductive Logic Programming algorithm that induces rules by using bottom-up approach, it cannot handle numerical features; those have to be handled manually. ILASP (Law 2018) system is another ILP algorithm that is able to generate normal logic program, it needs to work with a solver and requires a rule set to describe the hypothesis space. Some other RBML algorithms rely on statistical machine learning models: SVM+ProtoTypes (Núñez, Angulo, and Catalá 2002) extracts rule from Support Vector Machine (SVM) models by using K-Means clustering algorithm. RuleFit (Friedman, Popescu, and others 2008) algorithm learns weighted rules from ensemble models of shallow decision trees. TREPAN (Craven and Shavlik 1995) produces a decision tree from trained Neural Networks by querying. Support Vector ILP (Muggleton et al. 2005) uses ILP-learned clauses as the kernel in dual form of SVM, nFOIL (Landwehr, Kersting, and Raedt 2005) system employs the naive Bayes criterion to guide its rule induction. The kFOIL (Landwehr et al. 2006) algorithm integrates the FOIL system with kernel methods. Compared to the above systems, our approach is more efficient and scalable due to being top-down and using prefix-sum technique for literal selection. Thus, the rule-set learned in our approach is more concise because of the use of default rules with exceptions. Finally, our approach is able to provide scalable explainability which, to the best of our knowledge, no other RBML algorithm achieves.

In this paper, we presented a highly efficient rule-based ML algorithm with scalable explainability, called FOLD-SE, to generate a normal logic program for classification tasks. It is built upon the FOLD-R++ algorithm with a newly created heuristic for literal selection and a rule pruning mechanism. Our experimental results show that the generated logic rule-set provides performance comparable to XGBoost and MLP but better training efficiency, interpretability, and explainability. Unlike its predecessor FOLD-R++ and other RBML algorithms, the explainability of FOLD-SE is scalable which means the number of generated logic rules and generated literals stay small regardless of the size of the training data. The tail hyper-parameter added to FOLD-SE can be easily adjusted to obtain a trade-off between accu-
racy and explainability. We also presented an extension of
the FOLD-SE algorithm that can, similarly, handle multi-
class classification tasks.

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