Blowing-Up the Four-Dimensional $Z_3$ Orientifold

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Abstract

We study the blowing-up of the four-dimensional $Z_3$ orientifold of Angelantonj, Bianchi, Pradisi, Sagnotti and Stanev (ABPSS) by giving nonzero vacuum expectation values (VEV’s) to the twisted sector moduli blowing-up modes. The blowing-up procedure induces a Fayet-Iliopoulos (FI) term for the “anomalous” $U(1)$, whose magnitude depends linearly on the VEV’s of the blowing-up modes. To preserve the $N = 1$ supersymmetry, non-Abelian matter fields are forced to acquire nonzero VEV’s, thus breaking (some of) the non-Abelian gauge structure and decoupling some of the matter fields. We determine the form of the FI term, construct explicit examples of (non-Abelian) $D$ and $F$ flat directions, and determine the surviving gauge groups of the restabilized vacua. We also determine the mass spectra, for which the restabilization reduces the number of families.
I. INTRODUCTION

With the advent of the duality symmetries of M-theory [1] a new domain of string vacua (previously considered as strongly coupled) have become accessible for study. Such dual four-dimensional string vacua with $N = 1$ supersymmetry should provide a fruitful domain for studying novel phenomenological implications of string theory. In particular, the Type I string orientifolds [2,3] provide a promising set of new string vacua, where the techniques of the open-string theory allow for a quantitative study of the gauge structure, mass spectrum and (certain) couplings in the effective theory. The four-dimensional orientifolds [4,5] with $N = 1$ supersymmetry are thus particularly well suited for phenomenological studies.

One of the interesting features of the $N = 1$ orientifolds is that in general they contain a set of “anomalous” $U(1)$’s and the breaking of these $U(1)$’s is inherently related to the blowing-up procedure. The massless chiral superfields, formed from the Neveu-Schwarz-Neveu-Schwarz (NS-NS) and Ramond-Ramond (R-R) fields appearing in the twisted sector of type IIB orientifolds, are blowing-up moduli whose nonzero vacuum expectation values (VEV’s) correspond to the geometric smoothing-out (blowing-up) of the orientifold singularities. These fields play an instrumental role in the cancellation [3,4] of the triangular gauge anomalies via the Green-Schwarz mechanism. In addition, their coupling to the gauge superfields [8] contributes to the $D$ term of the “anomalous” $U(1)$, and its structure is fixed by the the anomaly cancellation constraints. Thus, the Fayet-Iliopoulos (FI) term is induced when the blowing-up modes acquire nonzero VEV’s. To maintain the anomalous $U(1)$ $D$ flatness of the blown-up orientifold, additional matter fields in the theory have to acquire nonzero VEV’s subject to the constraint that the $F$ flatness and the $D$ flatness of the rest of the gauge sector is maintained. The generic effect of the blowing-up procedure and subsequent vacuum restabilization is then the spontaneous breaking of the gauge symmetry as well as the decoupling of a number of matter fields in the effective theory. In particular, the number of families in the effective theory may be reduced.

It is instructive to contrast the blowing-up and the accompanying vacuum restabilization of Type I orientifolds with the blowing-up of perturbative heterotic string orbifolds. There, the blowing-up of the orbifold singularities [9] and the restabilization of vacuum due to the anomalous $U(1)$ [10,13] are somewhat disconnected and the two procedures are different from those on the Type I side. For the $(2,2)$ orbifolds, i.e., those with the spin and gauge connection identified (analogous to constructions on the Type I side are nonexistent), there is no “anomalous” $U(1)$ and the blowing-up modes (twisted sector moduli) are charged under the enhanced gauge symmetry that commutes with the discrete gauge connection [14,15]. (This is in contrast to those of the Type I orientifolds which are total singlets.) Their nonzero VEV’s fully break the enhanced gauge symmetry that at the orbifold limit commutes with the discrete spin connection and decouple some matter states; the procedure geometrically corresponds to blowing-up the orbifold singularities producing a smooth $(2,2)$ Calabi-Yau three-folds [4]. On the other hand (asymmetric) orbifold and free fermionic constructions with only $(0,2)$ worldsheet symmetry in general possess an anomalous $U(1)$. In contrast with the Type I orientifolds such perturbative heterotic string vacua have only one anomalous $U(1)$, whose anomaly cancellation is ensured by an universal Green-Schwarz mechanism, due to an effective Chern-Simons (CS) term, at the genus-one level, of the untwisted sector two-form field to the gauge field strength [10]. The
dual of this antisymmetric field-axion, along with the (untwisted sector) dilaton field, form a scalar component of the chiral superfield, which couples universally to the gauge sector of the theory. By supersymmetry the CS term is accompanied by a FI term (also at the genus-one level), which is proportional to the nonzero VEV of the dilaton field. Since this VEV determines the strength of the gauge coupling and thus it is necessarily nonzero, these vacua necessarily have nonzero FI terms \[10, 13\]. The structure of this FI term is universal; it is completely fixed by the VEV of the dilaton and the trace of the anomalous \(U(1)\) charges. Therefore the existence of “anomalous” \(U(1)\) necessarily triggers the vacuum restabilization. For recent work on the systematic classification of flat directions for a set of perturbative heterotic string vacua see, e.g., Refs. \[16–18\].

The crucial difference in the case of the Type I orientifolds is that the blowing-up procedure and the restabilization of vacuum are now inherently connected. Since in general there are more than one anomalous \(U(1)\) the anomaly associated with each is cancelled by non-universal Green-Schwarz terms, which arise due to the CS coupling of the gauge field strengths to the twisted sector R-R two-form fields (which is dual to the twisted sector R-R scalars -"axions"). Thus each of the anomalous gauge group factors has a cancellation ensured by a specific combination of twisted sector R-R axions. The non-universality of the Green-Schwarz mechanism via twisted R-R sector axions is generic in \(N = 1\) orientifold constructions\[4\]. Due to supersymmetry, these CS terms are accompanied by the corresponding FI terms, which involve a specific combination of the twisted sector NS-NS fields-"blowing-up modes". Consequently, when these twisted sector dilatons acquire nonzero VEV’s, the orbifold singularity, associated with particular fixed points where the \(D\) branes are located, is blown-up. This procedure in turn induces non-universal FI terms, which for each anomalous \(U(1)\) is proportional to a specific combination of the VEV’s of the blowing-up modes. The appearance of FI term then triggers the vacuum restabilization.

The purpose of the present paper is to explore in a concrete way the effects of the blowing-up procedure for four-dimensional \(N = 1\) orientifolds. The explicit vacuum restabilization triggered by the blowing-up has to be carried out, and the surviving gauge structure and light mass spectrum determined before phenomenological implications of the blown-up orientifolds can be addressed. The goal is to carry out this procedure explicitly for specific blown-up orientifolds and to study its consequences. (Unlike the four-dimensional \(N = 1\) orientifolds, the blowing-up procedure of six-dimensional \(N = 1\) orientifolds is better understood and related geometrically to blowing-up of the ADE singularities of \(K3\) surfaces \[8\].)

While by now a large class of \(N = 1\) orientifolds have been constructed \[3,4\], only specific models (usually with additional Wilson lines included) contain matter fields which are non-Abelian singlets. These singlets could be candidates for the restabilization of the blown-up orientifold vacua, since in such cases it should be possible to achieve a systematic classification of flat directions, very much along the lines developed for vacuum restabilization of the perturbative heterotic string vacua via non-Abelian singlets \[16,17\].

On the other hand, a large class of (simpler) Type I orientifold constructions have no

\[1\]It is confirmed that at the orientifold limit there is no genus-one correction to the FI term, calculated for the ABPSS orientifold model in Ref. \[20\].
non-Abelian gauge singlets, and thus the vacuum restabilization of the blown-up orientifold should necessarily proceed by giving VEV’s to non-Abelian matter fields. In this case the classification of $D$ flat directions is complicated by non-Abelian $D$ flatness constraints, and a systematic approach to the study of vacuum restabilization is lacking. Nevertheless, the powerful connection between the holomorphic gauge-invariant monomials and the $D$ flat directions [21] (which generalizes in the non-Abelian case to polynomials) facilitates the construction of non-Abelian $D$ flat directions. Applications of this approach to the vacuum restabilization of the blown-up Type I orientifolds (without gauge singlets) will be the focal point of this paper.

One of the immediate consequences of the vacuum restabilization with non-Abelian fields is the breaking of the large non-Abelian gauge groups down to smaller ones. From the phenomenological point of view, the vacuum restabilization of the blown-up orientifold is one of a few ways to achieve smaller gauge groups with reduced massless particle content. In that sense, it can be viewed as complementary to other “stringy” methods in orientifold construction, which involve splitting the 32 nine- (or five-) branes among different fixed points of the orbifold, turning on the background NS-NS antisymmetric $B$-field, adding discrete Wilson lines (see e.g., [4]), etc.

The paper is organized in the following way. In section II we describe the spectra and the superpotential of the first four-dimensional orientifold with $N = 1$ supersymmetry, constructed by Angelantonj, Bianchi, Pradisi, Sagnotti and Stanev (ABPSS orientifold) [4]. We discuss the general procedure of anomaly cancellation and the generation of FI terms and explicitly write down the FI term for the anomalous $U(1)$ of the ABPSS model. In section III, we discuss the flatness conditions of the model in the restabilized vacuum and the method of classifying the $D$ flat directions. In section IV, we present classes of $D$ flat directions that are also $F$ flat to all orders, and discuss the consequences of the restabilized vacuum. In section V, we present the conclusions.

II. ABPSS MODEL AND ANOMALY CANCELLATION

A. Massless spectrum and superpotential couplings of the ABPSS orientifold

We choose to analyze the ABPSS model. This is a $Z_3$ orientifold with the gauge structure:

$$SO(8) \times SU(12) \times U(1)$$

and a matter content of three copies of

$$\psi^\alpha = (8, \overline{12})_{-1}, \quad \chi^\alpha = (1, 66)_{+2} \quad \alpha = 1, 2, 3,$$

which arise from the open-string sector, due to the strings stretching between the nine-branes located at the orientifold singularities.

In the Type IIB orientifold (closed string) sector, in the NS-NS sector there is the gravity supermultiplet and the 36 (chiral) supermultiplets corresponding to the 9 untwisted (“toroidal”) and 27 twisted (blowing-up) sector moduli. The moduli are total gauge singlets (unlike the twisted sector moduli of the perturbative heterotic orbifolds), whose real and
imaginary components arise from the NS-NS and R-R sector, respectively. In particular, the ABPSS orientifold has the untwisted sector dilaton $S$, moduli singlets $T_i$ ($i = 1, \cdots, 9$) and 27 twisted sector supermultiplets, out of which only one participates in the blowing-up procedure, since all the nine-branes are sitting at the fixed point of the $Z_3$ orbifold at the origin. The twisted sectors $(k = 1, 2)$ give the NS-NS fields $\Phi_k$ and the R-R two-form fields $C_{k}^{(2)}$ (which by duality are related to the twisted sector R-R axions $\Psi_k$). They are constrained by the reality condition $\Phi_1 = \Phi_2^\ast$, $C_1^{(2)} = C_2^{(2)\ast}$. In addition, the orientifold projection removes [22] the real components of $\Phi_k$ and $C_k^{(2)} (\Psi_k)$.

The renormalizable superpotential is of the form

$$W_3 = y \epsilon_{\alpha\beta\gamma} \psi_i^{\alpha} \psi_i^{\beta} \chi_{[a,b]}^\gamma,$$

where $y$ is a constant; $\alpha, \beta, \gamma$ are family indices; $i$ is an $SO(8)$ index; and $a, b$ are $SU(12)$ indices.

In the following we shall address the nature of Chern-Simons (CS) terms in orientifold models and derive the explicit expressions for the case of the ABPSS orientifold.

**B. Chern-Simons terms, Fayet-Iliopoulos terms, and anomaly cancellation**

The $U(1)$ triangular gauge anomalies are cancelled via the Green-Schwarz mechanism involving the exchange of twisted sector R-R fields $\Psi_k$ (twisted axions) due to the CS couplings [7,8]. For the four-dimensional Type I orientifold the coupling takes the form:

$$I_{CS} = \sum_k \int d^4x \ C_k \wedge e^F = \sum_k \int d^4x \ C_k^{(2)} \wedge Tr(\gamma_{\theta^k})F + \ldots,$$

where $C_k^{(2)}$ is the R-R 2-form in the $k$th twisted sector; their duals are the scalar fields $\Psi_k$. $F$ schematically represents the gauge field strength of the anomalous $U(1)$, associated with the $D$ brane located at the orientifold singularity. The matrix $\gamma_{\theta^k}$ describes the action of the orbifold group on the Chan-Paton (CP) factors in the $k$th twisted sector. For the $Z_N$ orientifold, it takes the form $\gamma_{\theta^k} = e^{-i2\pi kV_I H_I}$ in the Cartan-Weyl basis, where $V$ is a 16-dimensional real vector and $H_I$, $(I = 1, \ldots, 16)$ are the Cartan generators of $SO(32)$ represented by tensor products of $2 \times 2 \sigma_3$ submatrices. For the sake of simplicity we confine ourselves to the case of 32 branes of the same type located at a single fixed point.

The supersymmetric completion of the first term in eqn. (2) gives the Fayet-Iliopoulos (FI) contribution to the action:

$$I_{FI} = \sum_k \int d^4x \ \Phi_k Tr(\gamma_{\theta^k})D,$$

where $D$ is the auxiliary component of the vector multiplet which contains the gauge field $A_\mu$ of the anomalous $U(1)$. Hence, the FI term is given by

$$\xi_{FI} = \sum_k Tr(\gamma_{\theta^k} \lambda)\Phi_k,$$
where the sum is over twisted sectors, $\lambda$ is the Chan-Paton matrix associated with the gauge boson of the anomalous $U(1)$. In the Cartan-Weyl basis, it takes the form $\lambda_i = Q_i \cdot H$, where $Q_i$ is a 16-dimensional real vector. For the $Z_N$ orientifold, the spectrum of the $k^{th}$-twisted sector and the $(N - k)^{th}$ twisted sector satisfy reality conditions, e.g., $\Phi_k = \Phi^*_{(N-k)}$ and $C_k^{(p)} = C_{N-k}^{(p)*}$. Furthermore, the orientifold projection projects out the real components of $\Phi_k$ and $C_k^{(p)}$ (see e.g. [22]). In addition, the action of the orbifold group on the Chan-Paton indices has the following property:

$$\text{Tr}(\gamma \theta^k \lambda) = [\text{Tr}(\gamma \theta^{(N-k)} \lambda)]^*.$$  \hspace{1cm} (5)

Then the FI term takes the form:

$$\xi_{FI} = 2 \sum_{k=1}^{[\frac{N-1}{2}]} \text{Re}(\text{Tr}(\gamma \theta^k \lambda)\Phi_k) = (-2) \sum_{k=1}^{[\frac{N-1}{2}]} \text{Im}[\text{Tr}(\gamma \theta^k \lambda)]\text{Im}(\Phi_k).$$  \hspace{1cm} (6)

Note that due to the reality constraint, the sum is only over the first half of the twisted sectors.

A similar argument applies to the coupling between the twisted sector R-R scalar field $\Psi_k$ and the gauge field $A_\mu$, thus yielding the CS coupling of the type:

$$(-2) \int d^4x \sum_{k=1}^{[\frac{N-1}{2}]} \text{Im}[\text{Tr}(\gamma \theta^k \lambda)]\partial_\mu \text{Im}(\Psi_k)A^\mu.$$  \hspace{1cm} (7)

Thus the imaginary components of $\Phi_k$ and $\Psi_k$ can be combined into the physical moduli $R_k$ of the $Z_N$, $R_k = \text{Im}(\Phi_k) + i\text{Im}(\Psi_k)$ in which $k$ goes from 1 to $[\frac{N-1}{2}]$.

For $Z_3$ orientifold models, which have two twisted sectors, the reality condition on the twisted sector NS-NS scalars reduces to $\Phi_1 = \Phi_2^*$ and the FI term (6) then takes the form

$$\xi_{FI} = -2\text{Im}[\text{Tr}(\gamma \theta^1 \lambda)]\text{Im}(\Phi_1).$$  \hspace{1cm} (8)

The physical moduli $R$ of the $Z_3$ of the single twisted sector is $R = \text{Im}(\Phi_1) + i\text{Im}(\Psi_1)$.

A few comments are in order regarding the units appearing in front of the CS term. The nine-brane CS term in 10 dimensions has dimension one; it is proportional to $\sqrt{\kappa_{10}/g_{10}}$ [23]. Dimensionally reducing such a term to four dimensional effective theory, the prefactor gains a volume factor $V_6^{-1/4} \sqrt{\kappa_4/g_4}$. With the convention of assigning VEV’s of $\text{Re}(R)$ in terms of dimensionless quantities (just as the convention for the dilaton field $S$), the prefactor of the FI term (14) is of dimension 2. However, since the CS-type couplings arise from the untwisted sector, they are absent in the four-dimensional theory of the $N = 1$ orientifolds. Nevertheless we would like to argue that the dimensionful parameters of the CS term associated with the twisted sector are of the same structure. Such a term should be calculated in perturbative open-string theory by evaluating the disk diagram for two matter fields at the boundary and a twisted field $C_k$ integrated over the bulk. While this calculation is technically involved due to subtleties of the twisted sector fields$^2$, the dimensionful parameter of the resulting term

$^2$A related calculation was given for six-dimensional untwisted sector fields in the Appendix of [8].
is expected to have the same structure as that obtained by a naive dimensional reduction.\footnote{In compactifications from $D = 10$ to $D = 4$, the case in which the gauge groups arises from the five-brane world-volume theory, this relationship is modified by ratios of the compactified five-brane world-volume and the volume of the bulk. However in the case of the nine-branes (which fill up the full nine-dimensional spatial part of the ten-dimensional theory), the result depends only on the volume of the six-dimensional space.}

**Gauge coupling corrections.** Analogously, one would like to determine the correction of the twisted sector blowing-up modes to the gauge function:

$$f = S + \delta f(R).$$

(9)

Here $S$ is the (untwisted sector) dilaton for the case of the nine-brane sector (it is the untwisted toroidal modulus $T$ in the case of the five-brane sector.) The coupling of $\text{Im}\delta f(R)F\tilde{F}$ could in principle appear as the second order expansion of the Chern-Simons term \cite{2}. In the case of $Z_N$ orientifold such a term takes the form:

$$\sum_k \int d^4x \, Tr(\gamma_\theta \lambda^2)\Psi_k F\tilde{F},$$

(10)

Thus when summed over the twisted sectors only the real component of $\Psi_k$ survives. However, it is projected out by the orientifold projection. Hence, the term $\text{Im}\delta f(R)F\tilde{F}$ seems to be absent,\footnote{One resolution to this problem may have to do with modifying the prescription that the sum is done only over the first $[(N-1)/2]$ twisted sectors of the $Z_N$ orientifold. However, this would have to be confirmed by explicit string calculations. We thank A. Uranga for communication on that point.} indicating that $\delta f(R) = 0$, i.e., for $Z_N$ Type I orientifolds, there seems to be no gauge coupling correction due to the twisted sector moduli. Note that in contrast to Type I orientifold, Type II $Z_N$ orbifolds allow such terms since the real components of the twisted sector R-R fields are not projected out.

**Anomaly cancellation.** The symmetry factors in the string amplitudes for a disc that involves three gauge fields as external legs can be determined by identifying the effective coupling of the twisted-sector R-R fields to these gauge field strengths as arising from the CS couplings of the type \cite{2} (such a string diagram can then be viewed as dominated by the exchange of RR fields \cite{4}). In principle, the prefactors of the amplitudes can be fixed by requiring the exact cancellation of the gauge anomalies. In particular, the amplitude of the scattering of a $U(1)_i$ gauge boson and two non-Abelian $G_j$ gauge bosons with tree level exchange of a R-R scalar is \cite{5}.
where $|P|$ is the order of the orientifold group and $k$ runs over the twisted sectors. For a particular of the $Z_N$ orbifold, $C_k$ are given by \[ C_k = \prod_{a=1}^3 2 \sin \pi k v_a, \]

where $v_a$ is the compact space twist vector. $A_{ij}$ cancels the usual field theory triangular anomalies of the model \[.\]

C. Fayet-Iliopoulos term and anomaly cancellation of the ABPSS model

In the ABPSS model, the CP matrix in the Cartan-Weyl basis for the anomalous $U(1)$ gauge field is $\lambda = \text{diag}\{I_{12}, -I_{12}, 0 \times I_8\}$, in which $I_n$ is the $n \times n$ identity matrix. The $\gamma$ matrices are given by \[ \gamma_{\theta k} = \text{diag}\{e^{ik\theta} I_{12}, e^{-ik\theta} I_{12}, I_8\}, \]

where, $\theta = \frac{2\pi}{3}$.

The FI term of the anomalous $U(1)$, which arises from the supersymmetric completion of the Chern-Simons couplings between the twisted sector R-R fields and the gauge fields \[,\] is given by eqn. (4) with $\lambda$ and $\gamma_{\theta k}$ of the ABPSS model:

\[ \xi_{FI} = -2 \times 12 \times 2 \sin \left(\frac{2\pi}{3}\right) \text{Re}(R) = -24\sqrt{3} \text{Re}(R), \]

where $R$ is the twist sector moduli field. $\xi_{FI}$ modifies the usual $U(1)$ $D$ term as \[ D \rightarrow D + \xi_{FI}. \]

Therefore, the FI term is proportional to the VEV of the real component of the twisted moduli $R$.

The coupling of $\text{Im}\delta f(R) F \tilde{F}$ in the case of ABPSS orientifold could take the form:

\[ \sum_{k=1,2} \int d^4x \, Tr(\gamma_{\theta k} \lambda^2) \Psi_k F \tilde{F} = 2 \times 12 \times \cos \frac{2\pi}{3} \times 2 \times \text{Re}(\Psi_1). \]
Since the real component of $\Psi_1$ is projected out by the orientifold projection, the term $\text{Im}\delta f(R)F\tilde{F}$ seems to be absent (as discussed on general grounds in the previous subsections).

In the ABPSS model, the $U(1)^3$, $U(1) \times SO(8)^2$, and $U(1) \times SU(12)^2$ anomalies are

$$ (432, -36, 18), \quad (17) $$

respectively.

Since the compact space twist vector for $Z_3$ is $v = \frac{1}{3}(1, 1, -2)$, one obtains $C_1 = -C_2 = -3\sqrt{3}$. Thus, the scattering amplitudes $A_{ij}$, with the $\gamma$ and $\lambda$ matrices presented previously, are

$$ A_{U(1), (U(1), SO(8), SU(12))} = \frac{1}{3}2(-3\sqrt{3})12\frac{\sqrt{3}}{2}(24\frac{1}{2}, -1, \frac{1}{2}) = (-432, 36, -18), \quad (18) $$

which cancel the $U(1)$ anomalies in eqn.(17).

**III. ANOMALOUS U(1) AND VACUUM RESTABILIZATION**

The appearance of the FI term for the anomalous $U(1)$ due to the blowing-up procedure requires the well known vacuum restablization procedure to preserve supersymmetry at the string scale. Certain fields that are charged under the anomalous $U(1)$ are triggered to acquire nonzero vacuum expectation values (VEV’s) that cancel the FI $D$ term, subject to the constraints that they are both $D$ flat with respect to the other gauge groups and $F$ flat, leading to a consistent “restabilized” string vacuum. As a consequence, some fields become massive (depending on the size of the FI term, which generally sets the scale of the VEV’s, they will either decouple or remain in the low energy theory). In addition, the rank of the gauge group is usually reduced as well as the number of families.

In previous work [16], techniques were developed to construct the moduli space of the flat directions for models with an anomalous $U(1)$ systematically. The method utilizes the one to one correspondence of $D$ flat directions (under both the non-anomalous Abelian gauge groups and the non-Abelian gauge groups) with holomorphic gauge-invariant polynomials built out of the chiral fields in the model. For simplicity, the flat direction analysis in [16] considered only the non-Abelian singlet fields in the model, in which case the flat directions correspond to monomials (HIM’s). In particular, the superbasis, which is the set of the one-dimensional (i.e., that depend on one free VEV) HIM’s of the model, can be constructed. Every $D$ flat direction can be expressed as a product of the elements in the superbasis, such that the positivity of the VEV-squares of the fields can be satisfied automatically. For example, if the $l^{th}$ HIM for the flat direction is $P_l = \prod_p \Phi_p^{n_p^l}$, then the fields $\Phi_p$ have VEV’s

$$ |\langle \Phi_p \rangle|^2 = \sum_l n_p^l |v_l|^2, \quad (19) $$

where $v_l$ is the VEV corresponding to $P_l$. The phase of the $|\langle \Phi_p \rangle|$ can be chosen for convenience.
The HIM’s of the superbasis are then classified according to the sign of their contribution to $D_A$, the $D$ term of the anomalous $U(1)$. Since the FI term for the anomalous $U(1)$ has to be cancelled by the VEV’s of certain fields in the model, the sign of the FI term is crucial. To ensure the $D_A$ flatness constraint, the required HIM’s should necessarily contain one or more elements that are opposite in sign to that of $\xi_{FI}$.

The constraints of $F$ flatness require that $\langle \partial W/\partial \Phi_p \rangle = 0$ and $\langle W \rangle = 0$ for all of the massless superfields $\Phi_p$ in the model. Further consideration of these conditions demonstrates that there are two types of dangerous terms which can lift a given $D$ flat direction. The first class of terms, which we denote as the $W_A$ terms, are formed solely of the fields that are in the $D$ flat direction. Gauge invariance dictates that if such a term can be constructed, it can appear in the superpotential raised to any positive power. In this case, for the $D$ flat direction to remain $F$ flat to all orders in the superpotential, string selection rules must conspire to forbid the infinite number of $W_A$ terms, which can be difficult to prove in general. We choose to adopt a conservative strategy and do not consider $D$ flat directions for which $W_A$ terms can appear (with the recognition that in doing so, we may be neglecting possible flat directions which are in fact $F$ flat to all orders).

The other type of dangerous terms, which we denote as the $W_B$ terms, are linear in an additional massless superfield which is not in the flat direction (i.e., which has zero VEV), such that $\langle W \rangle = 0$ but $\langle \partial W/\partial \Psi \rangle$ may be nonzero. In this case, gauge invariance constrains the number of $W_B$ terms to be finite, and an explicit string calculation can be performed to determine if such terms are in fact present in the superpotential. Thus, the flat direction can be proven to be $F$ flat to all orders if all such $W_B$ terms vanish. It is also possible that in certain cases the contributions to the $F$ term $\partial W/\partial \Psi$ from different $W_B$ terms linear in the same field $\Psi$ (which is not in the flat direction) could be arranged to cancel for appropriate magnitudes and signs of the VEV’s of the fields involved.

In the present model, the $D$ flat directions necessarily involve non-Abelian fields due to the matter content. In principle, the problem could be simplified significantly if only one component for each superfield which is charged under $SU(12)$ and/or $SO(8)$ is nonzero, such that only diagonal generators would be involved in $D$ terms for the non-Abelian gauge groups. In this case, the problem is similar to that of the Abelian case (Abelian-like), and the techniques developed in [14] and described above can be directly applied.

The one to one correspondence between holomorphic gauge invariant polynomials (HIP’s) and non-anomalous $D$ flat directions [21] provides a powerful way of searching for a more general class of $D$ flat directions with non-Abelian fields. We first construct a gauge invariant polynomial from the non-Abelian fields, which is a sum of monomials involving the components of the fields. Then one monomial term defines a $D$ flat direction. Each field in the monomial will have the same magnitude of the VEV (or times $\sqrt{n_p}$ if the component field is raised to the $n_p$ power). The $D$ flatness constraints for both diagonal and off-diagonal generators of the non-Abelian gauge group are automatically satisfied. Other flat directions are gauge rotations of the direction corresponding to a single monomial. They are equivalent to a product of monomials from the same HIP. Each monomial introduces the same magnitude of the VEV for each component present (or $\sqrt{n_p}$), but the phases of the VEV’s are dictated by the gauge rotation.

One can also consider higher dimensional $D$ flat directions (with more than one independent VEV), formed as products of other HIP’s. The flat directions correspond to products
of monomials from each of the HIP’s, each with its own VEV. Such products often have a reduced surviving gauge symmetry and massless particle content. They are sometimes flat (due to cancellations) for specific ratios of the VEV’s and choices of phases, even though the directions corresponding to a single HIP are not. For overlapping polynomials, which are products involving common multiplets, there is a flat direction in which the common multiplets have the same nonzero component (or involve components not connected by any single gauge generator) for each of the monomial factors, which avoids nonzero $D$ terms for off-diagonal generators. The VEV’s of the nonzero components in a product are given by an expression analogous to (19). For products of non-overlapping HIP’s (i.e., with no multiplets in common), the corresponding monomials may have different orientations in the internal symmetry space.

IV. FLAT DIRECTIONS

The $F$ flatness conditions of the ABPSS model are

$$\epsilon_{\alpha\beta\gamma} \psi_i^{\alpha\alpha} \psi_i^{\beta\beta} = 0;$$

$$\epsilon_{\alpha\beta\gamma} \psi_i^{\beta\beta} \chi_{[a,b]} = 0.\quad(20)$$

The $D$ flatness condition for $SO(8)$ is

$$D^I = \sum_{\alpha,a} \sum_{i,j} (\psi_i^{\alpha\alpha\dagger} T_{ij}^{I} \psi_j^{\beta\beta}) = 0,$$

where $T^I$ are generators of the vector representation of $SO(8)$ and $I = 1, \ldots, 28$. For $SU(12)$,

$$D^J = \sum_{\alpha,i,a,b} \psi_i^{\alpha\alpha\dagger} \hat{T}_{ab}^J \psi_i^{\beta\beta} + \sum_{\alpha,a,b,c} \left( \chi_{[a,c]}^\alpha \right)^\dagger T_{ab}^J \chi_{[b,c]}^\alpha,\quad(23)$$

where $T^J (\hat{T}^J \equiv -T^{JT})$ are the generator matrices for the fundamental (anti-fundamental) representation of $SU(12)$ and $J = 1, \ldots, 143$. The $D$ flatness condition for the anomalous $U(1)$ is given by

$$D_A = -\sum_{\alpha,i,a} \psi_i^{\alpha\alpha\dagger} \psi_i^{\alpha\alpha} + 2 \sum_{\alpha,a,b} \chi_{[a,b]}^\alpha \chi_{[a,b]}^\alpha + \xi_{FI}.\quad(24)$$

Since there are no non-Abelian singlet fields in the model, the $D$ flat directions are necessarily formed of superfields which transform nontrivially under the non-Abelian gauge groups. Due to the form of the superpotential and the number of families, we have not been able to construct a $D$ flat direction involving only one component per multiplet, namely, the Abelian-like solution with only the diagonal generators of $SU(12)$ involved.

$^6$Gauge or family rotations of that flat direction may have additional nonzero components of the same multiplets. The $D$ flatness of the off-diagonal generators then occurs by cancellations.
We thus concentrate on the holomorphic gauge invariant polynomial method as a more general and powerful tool. We construct gauge invariant (under $SO(8) \times SU(12)$) combinations of fields $\psi_i^{\alpha a}$ and/or $\chi_{[a,b]}^\gamma$. Any monomial from the HIP involving particular components of the fields $\psi$ and/or $\chi$ is a one-dimensional $D$ flat direction. Other $D$ flat directions can be constructed from products of these one-dimensional directions. We then check the $F$ flatness constraints.

We have found classes of flat directions involving $\chi$ only, exploiting the fact that the totally antisymmetric product $\chi^6$ is an $SU(12)$ singlet. Different combinations of the family indices for the six $\chi$ fields, or products of such polynomials, correspond to different residual symmetries and spectra after vacuum restabilization. The $\chi$ fields have $U(1)_A$ charge +2, so that these directions have $D_A = 0$ for $\xi_{FI} < 0$, i.e., for positive $Re(R)$ in (14). The VEV’s of the components of the fields are proportional to $\xi_{FI}$, so they interpolate smoothly to the limit $\xi_{FI} = 0$.

The one-dimensional flat directions take the form (up to a gauge rotation)

$$\chi^6 = \chi_{[1,2]}^{\alpha_1} \chi_{[3,4]}^{\alpha_2} \chi_{[5,6]}^{\alpha_3} \chi_{[7,8]}^{\alpha_4} \chi_{[9,10]}^{\alpha_5} \chi_{[11,12]}^{\alpha_6},$$

(25)

where the family indices $\alpha_i$ take the values 1, 2, or 3. This monomial is a singlet under $SO(8)$, with anomalous $U(1)$ charge +12.

In addition to $SO(8)$, which obviously remains unbroken by this class of flat directions, the remaining unbroken gauge symmetries from $SU(12)$ clearly contain $SU(2)^6$, where each $SU(2)$ corresponds to the indices of one of the $\chi$ fields in the flat direction. However, since there are only three possible values of the six $\alpha_i$, these are not all the unbroken generators of the original $SU(12)$. There are additional off-diagonal generators which remain unbroken such that the remaining gauge symmetry is $Sp(2l) \times Sp(2m) \times Sp(2n)$, where $l$, $m$, and $n$ are the number of occurrences of the direction 1, 2, and 3, respectively, and $l + m + n = 6$. (Of course, $Sp(2) \sim SU(2)$ and the $Sp(2k)$ factor is absent for $k = 0$, where $k$ is $l$, $m$ or $n$.)

In Appendix A, an example with two $\chi$ fields is given to illustrate the survival of the $Sp(4)$ gauge group.

There are also flat directions which are arbitrary superpositions of directions with different family indices but the same $SU(12)$ indices. These are equivalent to each of the six factors having an independent direction in family space; i.e., $\alpha_i$ is promoted to a vector in the three-dimensional family space, with each component having an arbitrary phase. Thus, $\chi_{[a_i,b_i]}^{\alpha_i} \rightarrow \chi_{[a_i,b_i]}^{\alpha_i} \chi_{[a_i,b_i]}^{\alpha_i} \chi_{[a_i,b_i]}^{\alpha_i}$, where $[a_i,b_i] = [1,2], [3,4], [5,6], [7,8], [9,10], [11,12]$, and the VEV’s of three components in the family space satisfy $|v_{[a_i,b_i]}^1|^2 + |v_{[a_i,b_i]}^2|^2 + |v_{[a_i,b_i]}^3|^2 = |v|^2$, where $v$ is the VEV determined from $\xi_{FI}$. The generic residual symmetry is $SU(2)^6$, except for the special directions for which $k$ of the $\alpha_i$ are aligned, in which case $SU(2)^k \rightarrow Sp(2k)$.

The space of the flat directions of this class is a subspace of the $3 \times 66$ dimensional complex space of $\chi_{[a,b]}^{\alpha}$. At a generic point (the vectors $\alpha_i$ in the family space are different from each other), other flat directions can be constructed by family and phase transformations classified.

\footnote{Note that in the orientifold limit, appearance of $Sp(2k)$ groups is usually associated with existence of five-branes. Interestingly, the ABPSS orientifold has only nine-branes, yet the blowing-up procedure introduces $Sp(2k)$ groups.}
by $U(3)/U(2)$ for each of the six factors $\chi^{\alpha_{i,b}}$. Of the five generators of $U(3)/U(2)$, one is from $SU(12) \times U(1)_A$ and the other four correspond to moduli. Hence, the generic points with 6 arbitrary $\alpha_i$ form a 12-dimensional complex moduli space. The points in moduli space that involve permutating the $\alpha_i$ and associated phases are equivalent by discrete gauge transformations, such as the one which maps $\chi^{\alpha_{1,2}}\chi^{\alpha_{3,4}}$ to $\chi^{\alpha_{1,4}}\chi^{\alpha_{2,3}}$. The $\chi$ spectrum for a generic point includes 126 massive states associated with the spontaneous breakdown of $SU(12) \times U(1)_A$ to $SU(2)^6$ (the imaginary parts are the absorbed Goldstone bosons and the real parts are the massive scalar partners). In addition, there are $3 \times 66 - 126 = 72$ massless complex $\chi$ states, 12 of which are associated with the moduli. At the special points in the moduli space where the vectors in the family space $\alpha_i$ are aligned, the gauge symmetries are enhanced. There are correspondingly more massless states and fewer associated with symmetry breaking.

To determine the spectrum for the $\psi$ fields, consider any of the six factors of $\chi^6$. Without loss of generality we can choose axes in family space such that $(\chi^{\alpha_{a,b}}) = v$ for a specific $\alpha = 1, 2, 3$, with the other VEV’s vanishing. From the superpotential

$$W_3 \rightarrow 2yv(\psi_i^{\beta a}\psi_i^{\gamma b} - \psi_i^{\gamma a}\psi_i^{\beta b}),$$

(26)

(where $\beta \neq \gamma \neq \alpha$) one has that $\psi_i^{\beta a}, \psi_i^{\gamma b}, \psi_i^{\gamma a}$ and $\psi_i^{\beta b}$ become massive ($\sim 2y|v|$), while $\psi_i^{\alpha a/\beta}$ remain massless. Similarly, two of the three families become massive for each of the 12 values of the $SU(12)$ index. ($SO(8)$ remains unbroken.) In the special case of $l = m = 0, n = 6$, for example, $\psi_i^{3a}, (a = 1..12)$ remain massless in the restabilized vacuum. Hence, three apparent families are reduced to one. This suggests that it may be worthwhile to consider models with more than three apparent families.

The $\chi^6$ flat directions are $F$ flat to all orders, since $SO(8) \times U(1)_A$ invariance requires at least two factors of $\psi$ for each term in the superpotential. The form of the effective superpotential (after restabilization) for the massless states depends on the specific flat direction. $U(1)_A$ invariance implies that $n_\psi = 2n_\chi$ for an allowed term in the original superpotential, where $n_\psi$ and $n_\chi$ are respectively the number of factors of $\psi$ and $\chi$. Effective cubic terms must therefore result from surviving terms in the original superpotential $W_3$ in (26). Such terms survive for points in the moduli space except for the maximal symmetry points like $(l, m, n) = (0, 0, 6)$. There could conceivably be four-dimensional effective couplings from original non-renormalizable couplings of the type $\psi^4(\chi^2)$. However, these would have to be of different form than $W_3^2$.

The generic $\chi^6$ flat directions that we have discussed so far are technically of the overlapping type; i.e., they involve products of monomials in which more than one component of the same field is allowed to have a VEV, as occurs, for example, if $\alpha_1$ and $\alpha_2$, associated with $\chi^{\alpha_{1,2}}$ and $\chi^{\alpha_{3,4}}$ are not orthogonal. For this class of flat direction this presents no difficulty: no $SU(12)$ generator connects $\chi^{1}_{[1,2]}$ with $\chi^{1}_{[3,4]}$, for example, and no off-diagonal $D$ terms are induced. For this to work, it is necessary for the $SU(12)$ indices to be the same in each monomial, i.e., each has the same $SU(12)$ orientation.

There is another class of $\chi^6$ flat directions involving non-overlapping polynomials, such as $(\chi^1)^6(\chi^2)^6$, or $(\chi^3)^6(\chi^2)^6(\chi^3)^6$. Since they are non-overlapping, there is no need for the $SU(12)$ indices to be the same in each factor. These directions therefore allow even more breaking of the $SU(12)$, although to maintain the non-overlapping character there is much
less freedom for rotations in the family indices\textsuperscript{8}. There are many possibilities for the relative $SU(12)$ orientations of the $(\chi^\alpha)^6$ factors, with different implications for the physics of the associated restabilized vacua. We will simply use two examples given in Appendix B to illustrate the complexity of the physics associated with this class of flat directions.

There are other classes of directions which are $D$ flat with respect to $SO(8) \times SU(12)$, of the generic form $\psi^{12}$ or $(\psi^{12}) \cdots (\psi^{12})$. Such directions have negative $U(1)_A$ charges, and would yield $D_A = 0$ for blown-up constructions with $\xi_{FI} > 0$. These would correspond to $Re(R) < 0$ in (14), and would not have a canonical geometric interpretation. For such directions, each $(\psi^{12})$ factor is totally antisymmetric in the $SU(12)$ indices, while the $SO(8)$ indices are contracted in pairs. The latter can be within the same $(\psi^{12})$ factor, in which case they must have different family indices, or can connect different $(\psi^{12})$ factors. However, we have not found any examples of this class which are also $F$ flat\textsuperscript{9}.

Similarly, we have not found any non-trivial solutions involving both the $\psi$ and $\chi$ fields in the flat direction. Clearly $\psi^{\alpha a}_i \psi^{\beta b}_i \chi^{\gamma [a,b]}$ is not $F$ flat, and we have not found combinations involving different families, etc., which are both $F$ and $D$ flat\textsuperscript{10}. (Of course, the orientifold point with $Re(R) = 0$ has $\xi_{FI} = 0$, and the trivial solution with no nonzero VEV is $D$ and $F$ flat.)

\section*{V. CONCLUSIONS}

The aim of this paper has been to address explicitly the blowing-up procedure of the four-dimensional Type I orientifolds with $N = 1$ supersymmetry. The specific analysis was done for the Type I $Z_3$ orientifold constructed by Angelantonj, Bianchi, Pradisi, Sagnotti and Stanev (ABPSS orientifold)\textsuperscript{4}. We chose it in part due to its simplicity (it involves only nine-branes), and in part due to its potential phenomenological implications since the model contains three families. The goals were two-fold: (I) we identified the fields participating in the blowing-up procedure, and determined the structure of the induced Fayet-Iliopoulos (FI) term, and (II) we provided a detailed analysis of the flat directions, surviving gauge symmetry, and light particle spectra after the subsequent vacuum restabilization of the blown-up orientifold.

\textsuperscript{8}There are also hybrid flat directions, involving family rotations for some of the $\chi$ factors and $SU(12)$ rotations for others.

\textsuperscript{9}We have also not found any $F$ flat examples for generalizations involving overlapping polynomials in which the monomials have different combinations of $SU(12)$, $SO(8)$, and family indices. These generically induce $D$ terms for the off-diagonal generators, but these can often be cancelled by appropriate choices of relative signs for the fields in a monomial.

\textsuperscript{10}In [4], a form of directions involving both $\psi$ and $\chi$ which satisfied $F$ flatness was given. However, no examples which were $D$ flat were presented, nor was their existence proved. The examples presented in this paper are both $D$ and $F$ flat, and are outside of their class.
(I) For the $Z_3$ ABPSS orientifold, only one twisted sector blowing-up mode $R$ participates in the blowing-up procedure and triggers the subsequent vacuum restabilization. It is associated with one out of the 27 fixed points, chosen at the origin where the nine-branes are located. We showed that the real part $Re(R) = \phi$ of this blowing-up mode (which arises in the NS-NS twisted Type IIB sector) contributes to the FI term. The nonzero vacuum expectation value (VEV) of this field, which is based on the geometrical interpretation of the blowing-up procedure should be taken to have positive sign, fixes the magnitude and the sign of the FI term. On the other hand, the Chern-Simons (CS) term, which plays an instrumental role in the anomaly cancellation procedure and is related to the FI term by supersymmetry, is proportional to the imaginary part $Im(R) = \psi$ of the blowing-up mode (which arises from the R-R twisted sector). We also noted that the second order expansion of this CS term, which contributes to the imaginary part of the gauge coupling corrections, does not seem to be present (the part of the R-R sector two-form field that would contribute to this coupling is projected out by the orientifold projection.)

(II) Due to the absence of non-Abelian singlet fields in the spectrum, the subsequent vacuum restabilization necessarily involves non-Abelian fields, thus complicating the analysis. We employed the powerful connection between holomorphic gauge invariant polynomials and flat directions [21], which enabled us to classify the $D$ and $F$ flat directions of the blown-up orientifold in detail. We found that the $D$ and $F$ flat directions of this model are associated with the general set of $\sim \chi^6$ monomials, in which $\chi$ transforms as $(1, 66)_{+2}$ under $SO(8) \times SU(12) \times U(1)$. We were unable to find $D$ and $F$ flat directions for the gauge invariant monomials of the type $\sim \psi^{12}$, in which $\psi$ transforms as $(8, 12)^{-1}$ under $SO(8) \times SU(12) \times U(1)$. We were also unable to find hybrid flat directions with both the $\psi$ and $\chi$ fields involved. Incidentally, only the $\sim \chi^6$ $D$ flat directions of this model have the correct sign of the $U(1)$ charges to cancel the FI term with the positive value of the blowing-up mode.

The generic point in the moduli space of $D$ and $F$ flat directions associated with the $\sim \chi^6$ monomials is specified by 12 complex moduli (fields that can acquire free VEV’s), breaks the gauge group down to $SU(2)^6 \times SO(8)$, and leaves only one family of the $\psi$ multiplets massless. At special points of moduli space, for example where $k$ of the family indices $\alpha$ in the monomials are aligned, the unbroken gauge group $SU(2)^k$ is enhanced to $Sp(2k)$.

We conclude with a number of remarks that in view of the analysis presented in this paper may be more general and could apply to a larger class of Type I blown-up orientifolds. In particular, a preliminary investigation of other orientifold models with non-Abelian singlets, indicates that the vacuum restabilization procedure still necessarily involves non-Abelian fields [19]. A large set of $Z_N \times Z_M$ orientifolds (e.g., [3]) have only the fundamental and antisymmetric tensor representations of the gauge group in the light particle spectrum (i.e., those of the $\psi$- and $\chi$-type, respectively). It is conceivable that in general the $\chi$ type fields play an instrumental role in the vacuum restabilization of this class of blown-up orientifolds.

Our analysis demonstrates that the particular blown-up orientifold addressed in this paper is unlikely to have interesting phenomenological implications, since neither the surviving gauge group, generically $SU(2)^6 \times SO(8)$, nor the particle content, with generically only one family remaining light, are phenomenologically viable. Nevertheless, the approach sets the stage for further systematic analysis of other blown-up orientifolds, which may uncover potentially phenomenologically interesting Type I string vacua.
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APPENDIX A: ENHANCED SYMMETRIES.

As an example, take two \( \chi \) fields from the same family, and assume the components \( \chi^{3}_{[1,2]} \) and \( \chi^{3}_{[3,4]} \) have nonzero and equal VEV’s. Label the \( SU(12) \) generators by \( T_{b}^{a} \), where

\[
T_{b}^{a} \chi_{[c,d]} = \delta_{c}^{a} \chi_{[b,d]} + \delta_{d}^{a} \chi_{[c,b]}.
\]

Then, the unbroken generators in the \( SU(4) \) subgroup are \((T_{1}^{2}, T_{2}^{2}, T_{1}^{1} - T_{2}^{2})\) and \((T_{3}^{4}, T_{4}^{3}, T_{3}^{3} - T_{4}^{4})\), which correspond to \( SU(2)^{2} \), as well as \( T_{3}^{3} - T_{4}^{4}, T_{1}^{4} + T_{2}^{3}, T_{1}^{5} - T_{2}^{4}, T_{1}^{3} + T_{2}^{3} \). It is straightforward to show that the remaining gauge group is \( Sp(4) \). Generalizing to the case with \( k \) factors of \( \chi \) fields aligned in their family indices, \( 3k \) generators remain unbroken from the obvious \( SU(2)^{k} \) subgroup. In addition, there are 4 unbroken generators for each pair of \( \chi \) fields. Therefore, the total number of unbroken generators is \( 3k + 4k(k - 1)/2 = k(2k + 1) \), and the unbroken gauge group can be shown to be \( Sp(2k) \). There are six broken diagonal generators, which are the five \( SU(12) \) generators not in \( SU(2)^{6} \) and that of the anomalous \( U(1) \). The off-diagonal generators which connect factors from different families are also broken, as are the linear combinations of the off-diagonal generators that are orthogonal to the generators in the extensions of \( SU(2)^{k} \) to \( Sp(2k) \).

APPENDIX B: DIRECTIONS INVOLVING NON-OVERLAPPING POLYNOMIALS.

We will simply illustrate with two examples involving an \( SU(4) \) subgroup of \( SU(12) \) and two \( \chi \) fields.

As we have seen in Appendix A, the single monomial \( \chi^{1}_{[1,2]} \chi^{1}_{[3,4]} \) breaks \( SU(4) \) to \( Sp(4) \), while \( \chi^{1}_{[1,2]} \chi^{2}_{[3,4]} \) leaves \( SU(2)^{2} \) unbroken. As an example of a non-overlapping polynomial direction, consider \( (\chi^{1}_{[1,2]} \chi^{1}_{[3,4]}) (\chi^{2}_{[1,4]} \chi^{2}_{[2,3]}) \), where the first (second) pair of fields have VEVs \( v_{1} \) (\( v_{2} \)). Although this direction breaks each individual \( SU(4) \) generator, there are six unbroken linear combinations. These are \( t_{1} = T_{2}^{1} - T_{4}^{4} \), \( t_{2} = T_{4}^{1} + T_{3}^{3} \), and their Hermitian conjugates, as well as the Hermitian generators \( h_{1} = T_{1}^{1} - T_{2}^{2} + T_{3}^{3} - T_{4}^{4} \) and \( h_{2} = i(T_{1}^{2} + T_{3}^{3} - T_{1}^{1} - T_{2}^{2}) \). \( h_{1} \) and \( h_{2} \) commute with each other and the \( t_{i} \). The six surviving generators correspond to an unbroken \( SU(2)^{2} \), with the canonical \( SU(2) \) generators given by linear combinations of the \( t_{i} \) and the \( h_{i} \). This class of directions involves 5 real moduli. There are 8 massless complex \( \chi \) fields associated with the \( SU(4) \) subgroup. Two families of \( \psi \) states become massive and one remains massless.

An extension of this example utilizes all three families, i.e., \( (\chi^{1}_{[1,2]} \chi^{1}_{[3,4]}) (\chi^{2}_{[1,4]} \chi^{2}_{[2,3]}) (\chi^{3}_{[1,3]} \chi^{3}_{[4,2]}) \), with VEVs \( v_{i} \), \( i = 1, 2, 3 \) for the three pairs. In this case, there are three Hermitian generators, \( h_{2}, i(t_{1} - t_{1}^{0}), \) and \( i(t_{2} - t_{2}^{0}) \), of a surviving \( SU(2) \). There are 7 real moduli for these examples, and 5 massless complex \( \chi \) fields. All three families of \( \psi \)’s become massive.
REFERENCES

[1] C. Hull and P. Townsend, Nucl. Phys. B438 (1995) 109; E. Witten, Nucl. Phys. B443 (1995) 85.
[2] E. Gimon and J. Polchinski, Nucl. Phys. B477 (1996) 207, hep-th/9602030; Nucl. Phys. B477 (1996) 701, hep-th/9604178; Phys. Lett. B394 (1997) 302, hep-th/9607041; E. Gimon and C. Johnson, Nucl. Phys. B477 (1996) 715, hep-th/9604129; J. Blum and A. Zaffaroni, Phys. Lett. B387 (1996) 71, hep-th/9607019; J. Blum, Nucl. Phys. B486 (1997) 34, hep-th/9608053; Z. Kakushadze, G. Shiu and S.-H. Tye, Phys. Rev. D58 (1998) 086001, hep-th/9803141.
[3] A. Dabholkar and J. Park, Nucl. Phys. B477 (1996) 207, hep-th/9602030; Nucl. Phys. B477 (1996) 701, hep-th/9604178; Phys. Lett. B394 (1997) 302, hep-th/9607041; E. Gimon and C. Johnson, Nucl. Phys. B477 (1996) 715, hep-th/9604129; J. Blum and A. Zaffaroni, Phys. Lett. B387 (1996) 71, hep-th/9607019; J. Blum, Nucl. Phys. B486 (1997) 34, hep-th/9608053; Z. Kakushadze, G. Shiu and S.-H. Tye, Phys. Rev. D58 (1998) 086001, hep-th/9803141.
[4] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev, Phys. Lett. B385 (1996) 96, hep-th/9606169.
[5] M. Berkooz, R.G. Leigh, Nucl. Phys. B483 (1997) 187, hep-th/9605049; G. Zwart, Nucl. Phys. B526 (1998) 378, hep-th/9708040; Z. Kakushadze, Nucl. Phys. B512 (1998) 221, hep-th/9704059; Z. Kakushadze and G. Shiu, Phys. Rev. D56 (1997) 3686, hep-th/9705163; Nucl. Phys. B520 (1998) 75, hep-th/9706051; L.E. Ibanez, hep-th/9802103; D. O’Driscoll, hep-th/9801114.
[6] L.E. Ibanez, R. Rabadan and A.M. Uranga, hep-th/9808139.
[7] M. Cvetiˇ c and L. Dixon, unpublished, M. Cvetiˇ c, in Proceedings of Superstrings, Cos- mology and Composite Structures, College Park, Maryland, March 1987, S.J. Gates and R. Mohapatra, eds. (World Scientific, Singapore, 1987) and Phys. Rev. Lett. 59 (1987) 2829.
[8] M. Dine, N. Seiberg, and E. Witten, Nucl. Phys. B289 (1987) 585.
[9] J. Atick, L. Dixon and A. Sen, Nucl. Phys. B292 (1987) 109; M. Dine, I. Ichinose, and N. Seiberg, Nucl. Phys. B293 (1987) 253; M. Dine and C. Lee, Nucl. Phys. B336 (1990) 317.
[10] L. Dixon and V. Kaplunovsky, unpublished.
[11] J. Atick and A. Sen, Nucl. Phys. B296 (1988) 157.
[12] L. Dixon, J.A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B261 (1985) 678 and Nucl. Phys. B274 (1986) 285; L. Dixon, D. Friedan, E. Martinec and S. Shenker, Nucl. Phys. B282 (1987) 13.
[13] L. Ibáñez, J.E. Kim, H.P. Nilles and F. Quevedo, Phys. Lett. B191 (1987) 282; J.A. Casas and C. Muñoz, Phys. Lett. B209 (1988) 214 and B214 (1988) 157; J.A. Casas, E. Katehou and C. Muñoz, Nucl. Phys. B317 (1989) 171; A. Font, L. Ibáñez, H.P. Nilles and F. Quevedo, Phys. Lett. B210 (1988) 101; A. Chameddine and M. Quirós, Phys. Lett. B212 (1988) 343, Nucl. Phys. B316 (1989) 101; A. Font, L. Ibáñez, F. Quevedo and A. Sierra, Nucl. Phys. B331 (1990) 421.
[14] G. Cleaver, M. Cvetić, J.R. Espinosa, L. Everett, and P. Langacker, Nucl. Phys. B525 (1998) 3.
[15] G. Cleaver, M. Cvetić, J.R. Espinosa, L. Everett, and P. Langacker, hep-th/9805133, to appear in Nucl. Phys. B.
[18] G. Cleaver, M. Cvetič, J.R. Espinosa, L. Everett, P. Langacker, and J. Wang, \textit{Phys. Rev.} \textbf{D59} (1999) 055005, \texttt{hep-ph/9807479, hep-ph/9811355}, to appear in \textit{Phys. Rev. D}.

[19] M. Cvetič, L. Everett, P. Langacker, M. Plümacher and J. Wang, work in progress.

[20] E. Poppitz, \texttt{hep-th/9810011}.

[21] F. Buccella, J.-P. Derendinger, C. Savoy and S. Ferrara, \textit{Phys. Lett.} \textbf{B115} (1982) 375; I. Affeck, M. Dine and N. Seiberg, \textit{Nucl. Phys.} \textbf{B241} (1984) 493, \textit{Nucl. Phys.} \textbf{B256} (1985) 557; M.A. Luty and W. Taylor IV, \textit{Phys. Rev.} \textbf{D53} (1996) 3399 and references therein; T. Gherghetta, C. Kolda and S. Martin, \textit{Nucl. Phys.} \textbf{B468} (1996) 37; P. Binétruy, N. Irges, S. Lavignac and P. Ramond, \textit{Phys. Lett.} \textbf{B403} (1997) 38.

[22] M.R. Douglas, B.R. Greene and D.R. Morrison, \textit{Nucl. Phys.} \textbf{B506} (1997) 84, \texttt{hep-th/9704151}.

[23] J. Polchinski, \textit{String Theory}; V. II, (Cambridge University Press, Cambridge, 1988).