Strength of Short Concrete Filled Steel Tube columns of Annular Cross Section

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Abstract. The procedure for calculating the strength of short centrally compressed concrete filled steel tube columns of annular cross-section is presented in the paper. The procedure takes into account the complex stress state of the concrete core and the steel shell as well as the nonuniform distribution of transversal stresses across the cross-section of the calculated element. When there are reinforcement bars in the concrete, the stress in these bars must be calculated taking into account the increased deformability of concrete. The proposed calculation procedure is applicable to structures made of different concrete types and steel grades.

1. Introduction
The design of concrete filled steel tube columns (CFSTC) is one of the few successful examples where concrete and steel mutually increase the bearing capacity of each other and the entire element. CFSTC have a number of advantages of technological, constructive and economic nature in comparison with steel and reinforced concrete structures. All the main advantages of these structures are detailed in the work [1].

In recent years CFSTC with hollow cross-section have been increasingly used in the building industry [1-6]. When the external steel shell is made of round tubes, this section is normally defined as annular (figure 1). In addition, the concrete core may contain longitudinal reinforcement bars.

2. Timeliness
In the literature there are many suggestions regarding the methods of determining the bearing capacity of CFSTC of circular cross-section. The method of breaking stresses [7-15] is most often used for that purpose which allows to obtain an approximate but quite a simple solution. It is presently believed that more reliable calculation results can be obtained on the basis of the deformation model.
This model is intensively developing [13, 16, 17] for evaluating the strength resistance of CFSTC. However, mathematical models based on it have not yet been suggested for the columns of annular cross-section.

This paper presents the problem of determining the strength of a short centrally compressed CFSTC of annular cross-section using the method of breaking stresses.

3. Formulation of the problem

The strength of a short centrally compressed CFSTC with annular cross-section and reinforced concrete core can be determined using the following formula:

$$N = R_{b3} A_b + \sigma_{pz} A_p + \sigma_s A_s,$$

Figure 1. Annular cross-section of concrete filled steel tube columns: a - without reinforcement of concrete core; b - with reinforcement of concrete core.

where $R_{b3}$ is the design value of the three-dimensionally compressed concrete core;

$\sigma_{pz}$ and $\sigma_s$ are the axial compression stresses in the steel shell and the longitudinal reinforcement in the limit state of the CFSTC;

$A_b$, $A_p$ and $A_s$ are the cross-section areas of the concrete core, steel shell and longitudinal reinforcement.

Design value of strength $R_{b3}$ and stresses $\sigma_{pz}$ and $\sigma_s$ are directly dependent on the accepted version of the annular section of the column.

It is known that the strength of the three-dimensionally compressed concrete mainly depends on two factors: the design value of concrete compressive strength $R_b$ and the values of transversal stresses in the concrete arising from the lateral pressure applied by the external steel shell on the concrete at their point of contact. It is widely believed that circumferential stresses $\sigma_t$ and radial stresses $\sigma_r$ are equal to this pressure at every point of the circular cross-section of a centrally compressed CFSTC. Stresses $\sigma_t$ and $\sigma_r$ vary in compressed concrete filled tube elements of annular cross-section (figure 2). According to the well-known Lyame solution, the principle of their variation can be written in the following form:
\begin{equation}
\sigma_r = \sigma_{br}^\text{ext}\left(1 - \frac{r_0^2}{r^2}\right)\left(1 - \frac{r_0^2}{r_b^2}\right)^{-1};
\end{equation}
\begin{equation}
\sigma_t = \sigma_{br}^\text{ext}\left(1 + \frac{r_0^2}{r^2}\right)\left(1 - \frac{r_0^2}{r_b^2}\right)^{-1},
\end{equation}

where \(\sigma_{br}^\text{ext}\) is the pressure applied on the concrete by the steel shell; 
\(r_b\) is the outer radius of the concrete core; 
\(r_0\) is the radius of the hole in the concrete core; 
\(r\) is the current radius.

Since the radial stresses \(\sigma_r\) do not exceed circumferential stresses \(\sigma_t\) at any point it is recommended to obtain the value of \(R_{b3}\) depending on the average cross-section value of radial stress \(\sigma_{brm}\), to simplify the calculation.

4. Theoretical part

Figure 2 shows that the value of the radial stress varies from maximum \(\sigma_{br}^\text{ext}\) in the zone of contact with the steel shell to zero at the hole. Therefore, the average radial stress \(\sigma_{brm}\) can be obtained by solving the following equation:

\begin{equation}
\sigma_{brm} = \frac{1}{\pi} \int_{r_c}^{r_b} \sigma_r 2\pi r dr.
\end{equation}

Taking the previously conducted researches [19] into account the pressure \(\sigma_{br}^\text{ext}\) can be approximately calculated using the following formula:

\begin{equation}
\sigma_{br}^\text{ext} = \sigma_{br} \left(\frac{r_b - r_0}{r_b}\right)^{\frac{1}{2}},
\end{equation}

where \(\sigma_{br}\) is the lateral pressure of a similar CFSTC with circular cross-section and defined using the previously obtained formula [19]:

![Figure 2. Distribution of transversal stresses across the section of concrete core under the axial compression of CFSTC of annular cross-section.](image-url)
\[ \sigma_{br} = 0.48e^{-(e+b)} \rho^{0.8} R_b. \]  \hfill (6)

In the formula (6) \( \rho \) is the constructional coefficient calculated using the formula:

\[ \rho = \frac{R_p A_p}{R_b A}, \]  \hfill (7)

where \( R_p \) is the design value of tensile strength of the CFST external steel shell; \( A \) is the area of concrete in the cross-section of the column, without including the hole.

The following formula can be obtained after making appropriate transformations:

\[ \sigma_{bem} = \sigma_{br} \frac{(1-\beta)^{1/2}}{1-\beta^2} \left( 1 + \frac{2\beta^2}{1-\beta^2} \ln \beta \right), \]  \hfill (8)

where \( \beta = h_0/h_b \).

Design strength of the three-dimensionally compressed concrete \( R_{b3} \) and the stress \( \sigma_{pz} \) in the steel shell are calculated by the following formulas obtained in the paper [19]:

\[ R_{b3} = R_b \left[ 1 + 0.5\sigma_m + \frac{\bar{\sigma}_m - 2}{4} + \sqrt{\left(\frac{\bar{\sigma}_m - 2}{4}\right)^2 + \frac{\bar{\sigma}_m}{b}} \right]; \]  \hfill (9)

\[ \sigma_{pz} = R_b \left( \sqrt{\rho^2 - 3\bar{\sigma}_m - \bar{\sigma}_m} \right) \frac{A}{A_p}, \]  \hfill (10)

where \( \bar{\sigma}_m = \sigma_{bem}/R_b \).

The third term of formula (1) takes into account the contribution of the compression stress of the longitudinal reinforcement (if any) to the CFSTC strength. This stress is determined from the condition of combined deformation of reinforcement with concrete.

Relative deformations of shortening at the vertex of the deformation diagram of the three-dimensionally compressed concrete core \( \varepsilon_{b00} \) can be calculated using the formula proposed in the paper [20]:

\[ \varepsilon_{b00} = \varepsilon_{b0} \alpha_b^{2.5} - \frac{R_b}{E_b} \left( \alpha_b^{2.5} - \alpha_b \right), \]  \hfill (11)

where \( \varepsilon_{b0} \) is the concrete deformation under axial compression at the vertex of the corresponding diagram; \( E_b \) is the tangent modulus of elasticity of concrete; \( \alpha_b = R_{b3}/R_b \).

Then, when using reinforcement with physical yield point, the compression stress in it can be determined from the formula:

\[ \sigma_{sc} = \varepsilon_{b00} E_s \leq R_s. \]  \hfill (12)

\( E_s \) is the elasticity modulus of longitudinal reinforcement.

Due to the higher deformability of the CFSTC concrete core, there is a possibility of effective use of high-strength reinforcement in these structures. When the concrete core is reinforced with high-tensile steel bars, the compression stress in the bars can be obtained using the formula:
\[ \sigma_s = \left( 0.9 + 0.1 \frac{\varepsilon_s - \varepsilon_{s1}}{\varepsilon_{s0} - \varepsilon_{s1}} \right) R_s \leq R_s^e, \quad (13) \]

where the values of relative deformation are \( \varepsilon_s = \varepsilon_{b00} \) and \( \varepsilon_{s1} = 0.9 R_s^e / E_s \), and the value of deformation is \( \varepsilon_{s0} = R_s / E_s + 0.002 \).

Therefore, it is possible to calculate all the terms of formula (1) determining the strength of a short centrally compressed CFSTC of annular cross-section.

5. Conclusions

A simplified procedure for calculating the strength of short centrally compressed CFSTC of annular cross-section is presented in the paper. The procedure is based on theoretical positions of solid mechanics and implements the method of breaking stresses. It takes into account the features of stress-strain state of concrete and steel shell of columns of annular cross-section as well as the increased deformability of the reinforced concrete core.

Since the proposed calculation procedure does not use empirical formals, it can be regarded as universal and acceptable for CFSTC made of different concrete types and steel grades.

6. References

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