Testing Chiral Dynamics in Pionic Atoms

E. Friedman and A. Gal
Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

Abstract

The energy dependence of chirally expanded $\pi N$ isoscalar and isovector amplitudes $b_0(E)$ and $b_1(E)$ respectively, for zero-momentum off shell pions near threshold, is used to impose the minimal substitution requirement $E \to E - V_c$ on the properly constructed pion optical potential within a large-scale fit to 100 pionic-atom data across the periodic table which also include the recently established ‘deeply bound’ pionic atoms of Pb and Sn. This fit cannot be reconciled with the well known free-space values of the $\pi N$ threshold amplitudes. In contrast, introducing the empirically known energy dependence for on-shell pions leads to a better fit and to satisfactory values for the $\pi N$ threshold amplitudes. The difference between these two approaches is briefly discussed.

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Corresponding author: Avraham Gal,
Tel: +972 2 658 4930, FAX: +972 2 5611519,
E mail: avragal@vms.huji.ac.il

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I. INTRODUCTION AND METHODOLOGY

The recent observation of 1s and 2p ‘deeply bound’ $\pi^-$ atomic states in isotopes of Pb [1–3] and very recently also of such 1s states in isotopes of Sn [4] has triggered renewed interest in the issue of partial restoration of chiral symmetry in dense nuclear matter [5–13]. In a nutshell, it was argued that since (i) the pion in deeply bound states with relatively large neutron excess charts a fairly dense portion of the nuclear medium, and since (ii) the most influential term of the optical potential $V_{\text{opt}}$ for this situation is generated by the $s$-wave isovector $\pi N$ threshold amplitude $b_1$, and since (iii) $b_1$ in free-space is well approximated in lowest chiral-expansion order by the Tomozawa-Weinberg expression [14]

$$b_1 = -\frac{\mu_{\pi N}}{8\pi f_\pi^2} = -0.08 \, m_\pi^{-1},$$

then deeply-bound states could yield valuable information on the dependence of $f_\pi$ on the density $\rho$. The pion decay constant $f_\pi$ serves as an order parameter for the spontaneously broken chiral symmetry in hadronic physics, and its free-space value $f_\pi = 92.4$ MeV should go to zero in dense matter if and when chiral symmetry is restored. Indeed, it has been known for quite some time that the renormalized value of $b_1$ required to fit pionic-atom data is about $-0.12 \, m_\pi^{-1}$ [15,16] clearly more repulsive than the free-space value $-0.09 \, m_\pi^{-1}$ [17].

Our most extensive recent work [11] has shown, however, that the deeply bound states by themselves on statistical grounds are insufficient to draw firm conclusions about whether or not $b_1$ is renormalized in dense matter. In fact, contrary to the expectation (i) above, the pion in deeply bound 1s states does not chart higher-density regions of the nucleus than it does so in ‘normal’ 1s states in light nuclei. It was shown in Ref. [11] that only by using a substantially larger data base that includes plenty of normal pionic atom data, and carefully considering uncertainties in the knowledge of neutron density distributions, it becomes possible to make a meaningful statement on the renormalization of the isovector threshold amplitude $b_1$, i.e. $b_1 = -0.108 \pm 0.007 \, m_\pi^{-1}$. It is tempting to ascribe this value for $b_1$, using Eq. (1), to a renormalization of $f_\pi$ in the nuclear medium.

Recently, Kolomeitsev, Kaiser and Weise [12] have suggested that pionic atom data could be reproduced using a pion optical potential underlain by chirally expanded $\pi N$ amplitudes, retaining the energy dependence of the amplitudes $b_0(E)$ and $b_1(E)$ for zero-momentum ($q=0$) pions in nuclear matter in order to impose the minimal substitution requirement $E \rightarrow E - V_c$, where $V_c$ is the Coulomb potential. This has the advantage of enabling one to use a systematic chiral expansion as an input [18], rather than singling out the leading order term Eq. (1) for $b_1$. Kolomeitsev et al. applied this programme to study the shifts and widths of the Pb and Sn pionic deeply bound states and reported substantial improvement in reproducing these data with only minimal phenomenological input, mostly limited to the $p$-wave $\pi N$ interaction to which allegedly these data are insensitive [12,13]. One could argue, however, that the deeply bound states do not offer sufficient variation over the energy range spanned by the bulk data on pionic atoms which include both ‘shallow’ states as well as ‘deep’ states. In the present work we test whether or not the energy dependence of this chiral expansion provides a satisfactory description for the bulk of pionic atom data.

The data used in the present work consist of 100 strong-interaction shifts and widths, stretching from $^{20}\text{Ne}$ to $^{238}\text{U}$ [11]. The Klein-Gordon equation solved for the pionic-atom eigen energies is given by [11,19]:

$$\text{Eigen energies} = \ldots$$
\[
[\nabla^2 - 2\mu(B + V_c) + (B + V_c)^2 - \Pi(E)\] \psi = 0 \quad (\hbar = c = 1) \tag{2}
\]

where \(\mu\) is the pion-nucleus reduced mass, \(B\) is the complex binding energy and \(V_c\) is the finite-size Coulomb interaction of the pion with the nucleus, including vacuum-polarization terms. The pion-nuclear polarization operator \(\Pi(E)\) is given by the standard Ericson-Ericson form [20]

\[
\Pi = 2\mu V_{opt} = q(r) + \vec{\nabla} \cdot \alpha(r) \vec{\nabla}, \tag{3}
\]

with the s-wave part of \(V_{opt}\)

\[
q(r) = -4\pi(1 + \frac{\mu}{M})\{b_0(r)[\rho_n(r) + \rho_p(r)] + b_1[\rho_n(r) - \rho_p(r)]\}
-4\pi(1 + \frac{\mu}{2M})4B_0\rho_n(r)\rho_p(r). \tag{4}
\]

In these expressions \(\rho_n\) and \(\rho_p\) are the neutron and proton density distributions normalized to the number of neutrons \(N\) and number of protons \(Z\), respectively, and \(M\) is the mass of the nucleon; \(q(r)\) is referred to as the s-wave potential term and \(\alpha(r)\) is referred to as the p-wave potential term. The function \(\bar{b}_0(r)\) in Eq. (4) is given in terms of the local Fermi momentum \(k_F(r)\) corresponding to the isoscalar nucleon density distribution:

\[
\bar{b}_0(r) = b_0 - \frac{3}{2\pi}(b_0^2 + 2b_1^2)k_F(r), \tag{5}
\]

where the quadratic terms in \(b_0\) and \(b_1\) represent double-scattering modifications of \(b_0\). In particular, the \(b_1^2\) term represents a sizable correction to the nearly vanishing linear \(b_0\) term. Similar double-scattering modifications of \(b_1\), as well as other correction terms to \(\Pi(E)\) listed in Refs. [12,18], were found by us to yield negligibly small effects and will not be further discussed below. The complex parameter \(B_0\) is due to s-wave absorption on pairs of nucleons. Its microscopic evaluation is outside the scope of chiral perturbation theory. Finally, the p-wave term \(\alpha(r)\) is a standard one with the same form as in Ref. [11].

The chiral expansion of the \(\pi N\) amplitudes for \(q = 0\) at the two-loop level is well approximated by the following expressions [12,18]:

\[
4\pi \left(1 + \frac{m_\pi}{M}\right) b_0(E) \approx \left(\frac{\sigma - \beta E^2}{f_\pi^2} + \frac{3g_A^2m_\pi^3}{16\pi f_\pi^2}\right), \tag{6}
\]

\[
4\pi \left(1 + \frac{m_\pi}{M}\right) b_1(E) \approx \frac{E}{2f_\pi^2} \left(1 + \frac{\gamma E^2}{(2\pi f_\pi)^2}\right), \tag{7}
\]

where \(\sigma\) is the \(\pi N\) sigma term, \(\sigma \approx 50\text{ MeV}[21]\), \(g_A\) is the nucleon axial-vector coupling constant, \(g_A = 1.27\), \(\beta\) and \(\gamma\) are tuned to reproduce the threshold values \(b_0(m_\pi) \approx 0\) and \(b_1(m_\pi) = -0.0885^{+0.0010}_{-0.0021} m_\pi^{-1}\) [17] respectively. For \(b_0\), in view of the accidental cancellations that lead to its near vanishing we limit our discussion to the \(f_\pi^{-2}\) term in Eq. (6), therefore choosing \(\beta = \sigma m_\pi^{-2}\). The next, \(f_\pi^{-4}\) term is much bigger than the scale of variation from zero expected for the threshold value and its inclusion here would appear somewhat dubious; if included, it would increase the energy dependence from the conservative estimate adopted
by us. Implementing the minimal substitution requirement in the calculation of pionic atom observables, the constant parameters $b_{0,1}$ of the conventionally energy independent optical potential have been replaced in our calculation by

$$b_{0,1}(r) = b_{0,1} - \delta_{0,1}(\text{Re}B + V_c(r)),$$

where $\delta_{0,1} = \partial b_{0,1}(E)/\partial E$ is the appropriate slope parameter at threshold, $\text{Re}B$ is the (real) binding energy of the corresponding pionic atom state and $V_c(r)$ is the Coulomb potential. The constant fit parameters $b_{0,1}$ are then expected to agree with the corresponding free $\pi N$ threshold amplitudes if the energy dependence is indeed responsible for the renormalized values found in conventional analyses. The added piece proportional to $\delta$ in Eq. (8) is dominated by the attractive $V_c(r)$. Since the slope parameters $\delta$ from Eqs. (6,7) are negative, this added piece is always repulsive, in agreement with Refs. [12,22].

Before testing the above `chiral' energy dependence for off-shell $q = 0$ pions we present results for the empirically known on-shell $\pi N$ amplitudes, when the pion energy $E$ and its three-momentum $q$ are related by $E^2 = m_\pi^2 + q^2$. This choice corresponds to the original suggestion by Ericson and Tauscher [22] to consider the effect of energy dependence in pionic atoms. Ericson subsequently [23] pointed out that, for strongly repulsive short-range $NN$ correlations, the on-shell requirement follows naturally from the Agassi-Gal theorem [24] for scattering off non-overlapping nucleons. The corresponding $\pi N$ amplitudes will be denoted below as ‘empirical’. Figure 1 shows the energy dependence of the empirical $b_0(E)$ and $b_1(E)$ on-shell amplitudes as derived from the SAID data base [25]. The value of $b_0(E)$ at threshold is very close to zero and the empirical slope $\delta_0$, which corresponds to adding repulsion in Eq. (8), is quite well-determined over the whole relevant energy range, having changed little since the classical KH80 analysis [26] of the pre-pion-factories data to the most recent analyses of modern data. We note that the slope of the $q = 0$ chiral $b_0(E)$ amplitude of Eq. (6) is larger than the on-shell empirical slope by about 60%. For the empirical $b_1(E)$, its value at threshold has also changed little since KH80 to the present day analysis, and the slope of the empirical amplitude is essentially zero, in contrast to the fairly large slope of the $q = 0$ chiral amplitude of Eq. (7).

II. RESULTS

The present analysis is based on the ‘global 3’ data set of Ref. [11] consisting of 100 data points from $^{20}\text{Ne}$ to $^{238}\text{U}$. For the nuclear density distributions $\rho_p$ and $\rho_n$ we adopt the procedure of Ref. [11] where $\rho_p$ is obtained from the experimental charge distribution by unfolding the finite size of the charge of the proton, and where simple but physical parameterizations are used for $\rho_n$. A key quantity in this context is the difference $r_n - r_p$ between the root-mean-square radii. Relativistic mean field (RMF) calculations [27] yield to a good approximation [11]

$$r_n - r_p = \alpha \frac{N - Z}{A} + \eta,$$

with the values $\alpha = 1.51 \pm 0.07$ fm, $\eta = -0.03 \pm 0.01$ fm. A similar expression, but with $\alpha = 1.0$, was obtained by analyzing strong interaction effects in antiprotonic atoms [28]. Owing to
the strong correlation between the values assumed for \( r_n - r_p \) and the values of \( b_1 \) derived from \( \chi^2 \) fits to pionic atom data, we have varied the neutron-excess parameter \( \alpha \) over a wide range, with the expectation that a value in the range of 1.0 to 1.5 will represent on the average the 41 nuclei in the present data base.

Figure 2 shows results for the ‘conventional’ model (left) for which \( \delta_{0,1} = 0 \) and for the ‘empirical’ model (right) as function of the neutron-excess parameter \( \alpha \) in Eq. (9) for two shapes of neutron densities. The dependence of the quality of fits on the shape of the neutron density distribution and the vanishingly small sensitivity of the derived values of \( b_1 \) to this shape are demonstrated by using either the ‘skin’ or the ‘halo’ shape, as discussed in Ref. [11]. The results for the conventional (energy independent) model are practically the same as in Ref. [11] in spite of adopting now the ‘current’ SAID values [25] for the parameters. This slight change was made for consistency, since the slope parameters \( \delta \) for the empirical model were taken from the ‘current’ SAID analysis. In fact, we also incorporated in the right hand side (rhs) of the figure the SAID weak energy dependence of the amplitudes, the minimum in the \( \chi^2 \) curve has shifted slightly towards the acceptable region of \( \alpha = 1.0 \) to 1.5, and the value of \( b_1 \) for that minimum is in agreement with the free \( \pi N \) value marked by ‘exp.’ in the upper panels of the figure. It is self-evident that the ‘halo’ shape for the neutron density distributions cannot be reconciled with the data. Finally, we add that the resulting values of \( b_0 \) for both the conventional and empirical models are close to zero, well within the experimental error [17].

A comment on the ‘anomalous s-wave repulsion’ in pionic atoms is here in order. The net effect of the nearly vanishing parameter \( b_0 \), of the repulsive \( b_1 \) and of the phenomenological parameter \( \text{Re} B_0 \) has been known [19] to produce a repulsive potential inside nuclei which is twice as large as expected. This is due to the combined action of the too repulsive \( b_1 \) and of \( \text{Re} B_0 \) which turns out to be too repulsive compared to the expectations that \(|\text{Re} B_0| < \text{Im} B_0 \) (see also Ref. [7]). For the fits mentioned above we obtain for the conventional potential \( \text{Im} B_0 = 0.053 \pm 0.002 \, m_\pi^{-4} \) and \( \text{Re} B_0 = -0.10 \pm 0.03 \, m_\pi^{-4} \). Although the latter is determined to a moderate accuracy, we note that setting its value to zero while repeating the fits leads to a significant increase in the resulting \( \chi^2 \) value and to a value for \( b_0 \) which is incompatible with experiment (cf. Table 4 of Ref. [11]). Using the empirical \( b_{0,1}(E) \) we find \( \text{Re} B_0 = -0.07 \pm 0.03 \, m_\pi^{-4} \). We conclude that using the empirical energy dependence the anomaly in \( \text{Re} B_0 \) is reduced, whereas there is essentially no anomaly in the parameter \( b_1 \).

Figure 3 shows results for the ‘chiral’ model, when either \( b_0 \) (left) or \( b_1 \) (right) is made energy dependent according to Eqs. (6) and (7), respectively. The left hand side of the figure shows that the quality of the best fit, upon incorporating only the energy dependence of the chiral \( b_0 \) amplitude, is significantly inferior to the corresponding best fit obtained using the conventional, energy independent model (shown in Fig. 2). Furthermore, the value of \( \alpha \) at the \( \chi^2 \) minimum is unacceptably large and the corresponding value of \( b_1 \) is in sharp disagreement with the experimental free \( \pi N \) threshold value. The rhs of the figure shows good fits with almost acceptable values for \( \alpha \) and for \( b_1 \), upon incorporating only the energy dependence of the chiral \( b_1 \) amplitude, again for the ‘skin’ shape of the neutron density. However, incorporating the energy dependence of the chiral \( b_0(E) \) (even within the limited
scope of using only the $f_\pi^{-2}$ term on the rhs of Eq. (6) on top of that for $b_1(E)$, leads to substantial disagreement between the resulting best-fit value for $b_1$ and the threshold value $b_1(m_\pi)$ which is marked by ‘exp.’ in Fig. 3. We conclude that, at present, the energy dependence generated by the chirally expanded $s$-wave $\pi N$ amplitudes of Eqs. (6,7) fails badly in reproducing consistently the bulk of pionic atom data.

III. DISCUSSION AND CONCLUSIONS

In the present work we have demonstrated that the consistency between pionic-atom data and the free $\pi N$ threshold amplitudes is greatly improved by using just the on-shell energy dependence of the $\pi N$ $s$-wave amplitudes, in accordance with the original suggestion made by Ericson and Tauscher [22]. The idea behind using this empirical energy dependence is the same one as used for constructing the multiple-scattering series for short-ranged $\pi N$ interactions occurring within an assembly of largely non-overlapping nucleons [20,23]. Multiple scattering is naturally described in this idealized limit as occurring on shell. Whereas using off-shell $q=0$ pions in chiral expansions is motivated by the ground-state wavefunction description of pions in nuclear matter, applying this limitation to the construction of the pion-nuclear optical potential that generates pionic-atom wavefunctions is questionable.

We have also shown that the energy-dependent chiral amplitudes given by Eqs. (6,7) for $q=0$ off-shell pions do not produce consistent or good global fits to pionic atom data. This conclusion is not at odds with the observation made by Kolomeitsev et al. [12] that the $q=0$ off-shell chiral amplitudes work well, and with no need for a dispersive term Re$B_0$ for the few deeply bound states available at present, since partial data sets of this kind do not have sufficient statistical significance to decide one way or another on this issue [10,11]. In fact, as good average reproduction of these deeply-bound data is reached within a wide class of optical potentials, including our ‘empirical’ energy-dependent potential of the present study. We defer this and other ramifications of the present analysis for a forthcoming detailed publication. Given the fact that the on-shell $\pi N$ amplitudes provide by far a better description of pionic atom data than the extremely off-shell $q=0$ chiral amplitudes do, we conclude that chiral dynamics is not yet at a stage of being tested in pionic atoms.

Finally, it should be emphasized that we have strictly adhered in the present calculation to imposing minimal substitution, $E \rightarrow E - V_c$, on the pion-nuclear polarization operator $\Pi(E)$ within the Klein-Gordon equation (2). Nowhere have we renormalized the threshold value of the $\pi N$ isovector amplitude $b_1$ of Eq. (1) by renormalizing the pion decay constant $f_\pi \rightarrow f_\pi(\rho)$ in dense matter [5]. This latter prescription which appears to be rooted in the underlying chiral symmetry has been discussed extensively in the context of pionic atoms [6–9,12,13], but according to Refs. [12,13] it need not be applied once the full energy dependence of $\Pi(E)$ is incorporated.

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FIG. 1. $\pi N$ empirical $s$-wave scattering amplitudes as function of laboratory energy for on-shell pions from the SAID data base [25].
FIG. 2. Pionic-atom fits using ‘conventional’, energy independent $\pi N$ amplitudes (left panels) and ‘empirical’, energy dependent SAID amplitudes (right panels) as function of the neutron-excess parameter $\alpha$, Eq.(9), for two shapes of neutron densities. Lower part: values of $\chi^2$ for 100 data points from $^{20}\text{Ne}$ to $^{238}\text{U}$. Upper part: best-fit values of $b_1$ vs. the free $\pi N$ threshold value [17] marked by ‘exp.’ within the dotted horizontal lines.
FIG. 3. Same as Fig. 2 but for the energy dependent ‘chiral’ amplitudes $b_0$ (left) and $b_1$ (right).