Effects of periodically modulated coupling on amplitude death in nonidentical oscillators

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Abstract – The effects of periodically modulated coupling on amplitude death (AD) in two coupled nonidentical oscillators are explored. The AD domain is significantly influenced by tuning the modulation amplitude and frequency of the coupling. There is an optimal value of the modulation amplitude of the coupling with which the largest AD domain is observed in the parameter spaces. The AD domain is enlarged (shrunk) with the decrease of the modulation frequency for a given small (large) modulation amplitude. The mechanism of AD in the presence of periodic modulation in the coupling is investigated based on the local conditional Lyapunov exponent. The stability of the AD state can be availably characterized by the conditional Lyapunov exponents of the coupled system. The transition process from oscillating to the AD state is clearly verified by the fact that the conditional Lyapunov exponent transits from positive to negative. Our results are helpful to many potential applications in neuroscience and dynamical control in engineering.

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Introduction. – By modeling the coupled nonlinear oscillators, a rich source of ideas and insights into understanding the emergence of self-organized behaviors in physics, biology, chemistry and neuroscience has been explored [1–4] during the last few decades. Quenching of oscillation, as one of the basic collective dynamical behaviors, has attracted much attention in various fields of nonlinear science since it has potential application on the control of chaotic oscillations and the stabilization of unstable dynamics in mechanical engineering [5], synthetic genetic networks [6,7], and laser systems [8,9]. There are two main categories of oscillation quenching according to their mechanisms and manifestations, amplitude death (AD) and oscillation death (OD) [10,11]. In AD, oscillations are suppressed to the same original homogeneous steady state (HSS) [10]. It is mainly applied as a control mechanism in physical and engineering systems. In contrast, OD occurs due to a stabilization of a newborn inhomogeneous steady state (IHSS), where the individual units stay in different branches of the IHSS [11]. The main implications of OD are in biological systems, since OD has been interpreted as one of the background mechanisms of cellular differentiation [6,12,13] and related neurological conditions [14,15]. AD is manifested to transit to OD via Turing bifurcation [16], mean-field diffusive coupling, dynamics coupling [17], and time delay coupling in experimental observations [18,19].

Generally, there are three types of main factors which influence the oscillation suppression dynamics, (1) the parameter mismatches between the nodes in coupled oscillators [20–22]; (2) The structures of the interaction networks; (3) The coupling schemes between interacting nodes. In ref. [11], the standard oscillatory solutions are eliminated in a large region of parameter mismatches by establishing the dominance of oscillation death under strong coupling in qualitatively different sets of coupled oscillator models (such as genetic, membrane, Ca metabolism, and chemical oscillators). In our previous work [21], AD is a general regime in a ring of coupled oscillators with parameter mismatches where the spatial distribution of parameter mismatches significantly influences the critical coupling strength needed for AD. Rich dynamics of the oscillation quenching are observed in the coupled oscillators with regular networks such as all-to-all [23], nearest-neighbor networks [24–27], as well as complex networks such as small-world [28], BA networks [29] where the topological property of the networks distinctly influences the AD dynamics. Furthermore, complex spatiotemporal
patterns of the coupled oscillators in the networks were observed, such as the transition from amplitude chimeras to chimera death states [30] where the population of oscillators splits into several coexisting domains of spatially coherent amplitude of oscillation and spatially incoherent amplitude of oscillation. The transition from oscillation to OD can be the continuous one [24,29] or the discontinuous one (named as explosive death) [31–33].

Various kinds of coupling schemes are available for oscillation quenching in the coupled oscillators, such as dynamic coupling [34], conjugate coupling [35], nonlinear coupling [36], gradient coupling [25], mean-field diffusive coupling [37], amplitude-dependent coupling [38], time delay in coupling due to a finite propagation of the signal [39–41], etc. In all these existing studies, the interactions take effects continuously for all the time. However, the continuous interactions do not always remain in many real systems such as the biological signal transmission between synapses and the mechanical control of engineering. The strengths of synapses in neuronal networks are modified according to the external stimuli. The links in metabolic networks are activated only during some specific tasks which lead to nonstatic interactions. On-off coupling, as one manifestation of the discontinuous coupling, has been verified to optimize the synchronization stability and speed [42,43]. Schroder et al. [44] explored a scheme for synchronizing chaotic dynamical systems by transiently uncoupling them and revealed that the discontinuous coupling contributes to realize synchronization in system where the continuous coupling cannot reach a synchronous state. In the sense of control, the synchronous efficiency may be greatly improved by the discontinuous coupling. Periodic coupling, another time-varying coupling scheme, is verified to maximize the network synchronizability with properly selected coupling frequency and amplitude [45].

Most researches are devoted to the effect of time-varying coupling on the synchronizability of the coupled system, that of discontinuous coupling on the AD dynamics is still an open question. Recently, AD was observed theoretically [46] and experimentally [47] in two discontinuously coupled oscillators. Sun et al. [1] went further to explore the on-off coupling schemes and concluded that AD domains are enlarged in the parameter spaces with a proper switching frequency and switching rate of the coupling. However, the discontinuous form of the on-off switch is sharp and difficult to realize physically owing to a finite response time of the switcher. Therefore, it is necessary to reveal the effect of some more flexible discontinuous coupling (i.e., periodically modulated coupling) on the oscillation quenching. The main goal in this work is to investigate the effects of periodically modulated coupling on the emergence of AD in the coupled nonidentical oscillators. In particular, we show that the occurrence of AD in nonidentical oscillators with time-varying coupling can be well characterized by the conditional Lyapunov exponent of the coupled system.

Models. – In this section, a periodically modulated coupling scheme is introduced to the coupled oscillators system as follows:

\[
X_1(t) = f_1(X_1(t)) + \epsilon(t)\Gamma(X_2(t) - X_1(t)), \\
X_2(t) = f_2(X_2(t)) + \epsilon(t)\Gamma(X_1(t) - X_2(t)),
\]

where \(X_i \in \mathbb{R}^n (i = 1, 2)\), \(f: \mathbb{R}^n \rightarrow \mathbb{R}^n\) is nonlinear and capable of exhibiting rich dynamics such as limit cycle or chaos, and \(\Gamma\) is a constant matrix describing the coupling scheme. \(\epsilon(t)\) is a periodically modulated coupling strength and can be described as

\[
\epsilon(t) = \epsilon_0[1 + \alpha \cos(\omega_0 t)],
\]

where \(\epsilon_0, \omega_0\) and \(\alpha \in [0, 2]\) are the average coupling strength, modulation frequency \((\omega_0 = \frac{2\pi}{T})\), and modulation amplitude of the periodically modulated coupling, respectively. The coupling strength varies positively for all the time as \(\alpha \in (0, 1)\) and remains constant if \(\alpha = 0\). Otherwise, it varies between positive and negative as \(\alpha \in (1, 2]\).

Results. –

Coupled Stuart-Landau (SL) oscillators. In order to observe the effects of the periodically modulated coupling on the AD dynamics, we first consider the coupled nonidentical Stuart-Landau oscillators whose dynamics can be described as

\[
\dot{Z}_i(t) = [1 + j\omega_i - |Z_i(t)|^2]Z_i(t),\]

where \(Z_i(t) = x_i(t) + jy_i(t), i = 1, 2, j = \sqrt{-1}, \omega_i\) is the intrinsic frequency of the oscillator \(i\). Without coupling \((\epsilon_0 = 0)\), each oscillator has an unstable focus at the origin \(|Z_i| = 0\) and an attracting limit cycle with an oscillating frequency \(\omega_i\). Considering the coupling scheme \(\Gamma = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\), AD can be realized in the two coupled oscillators (eq. (1)) with frequency mismatches \(\Delta \omega = |\omega_2 - \omega_1|\) for constant coupling strength \((\alpha = 0)\) and \(\omega_1 = 2\). The results [20] indicate that the AD domain is bounded in V shape in the parameter spaces \(\epsilon \sim \Delta \omega\) as shown in fig. 2(a). Since the modulated coupling strength has three control parameters, \(i.e.,\) average coupling strength \(\epsilon_0\), modulation frequency \(\omega_0\), and modulation amplitude \(\alpha\), we limit the exploration of the modulation amplitude on AD with the fixed modulation frequency \(\omega_0 = 4\), and the average coupling strength \(\epsilon_0 = 7.0\). Figures 1(a)–(d) present the bifurcation diagram of \(x_1\) for \(\alpha = 0.0, 0.8, 1.0, 1.8\), respectively. For \(\alpha = 0\), the coupling strength is constant, and the coupled system transits to AD from the oscillating state when the frequency mismatch increases from zero to the value larger than \(\Delta \omega_c = 7.3\). One example of periodic oscillation state is presented in form of time series for \(\Delta \omega = 2.0\) as shown in the inset of fig. 1(a). As \(\alpha = 0.8\), the critical value of \(\Delta \omega_c\) for AD becomes 5.6 which is less than that of the constant coupling strength. The AD state is displayed by the time series of \(x_1(t)\) in the insets of fig. 1(b) for \(\Delta \omega = 6\). The coupled system has two intervals of AD domain \(\Delta \omega \in [4.6, 7.1]\).
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and $\Delta \omega \in [9.85, 20]$ as $\alpha = 1.0$. Then three disconnected intervals of the AD domain along the direction of $\Delta \omega$ are observed as $\Delta \omega \in [1.1, 1.2]$, $\Delta \omega \in [15.1, 16.0]$, $\Delta \omega \in [18.3, 20]$ as $\alpha = 1.8$. The oscillating state is in periodic 2 for $\Delta \omega = 8$ and $\alpha = 1.0$ and in multi-period state for $\Delta \omega = 5$ and $\alpha = 1.8$ as shown in the insets of figs. 1(c), (d), respectively.

To integrally figure out how the periodically modulated coupling influences the AD domain of the coupled oscillators, it is natural to reveal the mechanisms of the stabilities of AD under the periodically modulated coupling. Generally the stability of the AD in coupled oscillators is obtained from the linear stability analysis of eq. (1) around $Z_0 = 0$ as the coupling strength is constant. The characteristic eigenvalues are [20]

$$\lambda_{1,2,3,4} = 1 - \epsilon \pm \sqrt{\frac{(\omega_2^2 - \omega_1^2)^2}{4} \pm j \frac{(\omega_1 + \omega_2)}{2}},$$

then the AD domain is determined by the real part of the eigenvalues $\text{Re} (\lambda) < 0$, that is, $1 \leq \epsilon \leq \Delta \omega/2$ and $\Delta \omega/2 < \epsilon < 1/2 + \Delta \omega^2/8$, which are right the boundary lines of the AD domain as shown in fig. 2(a). However, when $\epsilon$ is varying with time, the linear stability analysis around the original fixed points is not available any more and needs further studies.

We numerically record the AD domain in the parameter space $\Delta \omega$ vs. $\epsilon_0$ (both are in the range of [0, 20]) for a given $\omega_0 = 4.0$ and $\alpha = 0, 0.5, 1.0, 1.1, 1.4, 1.8$ as shown in figs. 2(a)–(f), respectively. When the parameters of the coupled oscillators are in the blue (red) area, the oscillators are in AD (oscillating) states. With the increment of the modulation amplitude $\alpha$, the AD domain firstly expands by decreasing the critical frequency mismatches which are the lower boundaries of the AD domain for all the given average coupling strengths. Then the AD domain shrinks with the increment of $\alpha$ when $\alpha$ is larger than a critical value $\alpha_c$. Moreover, the AD domains split into two parts when $\alpha > 1$ (it is repulsively coupled in some interval of each period of the time-varying coupling) which leads to a kind of ragged AD along the direction of the frequency mismatch $\Delta \omega$. It should be emphasized that the AD is firstly observed to be ragged in the direction of the parameter space $\Delta \omega$; as a comparison, the OD domain is ragged in the direction of the parameter space $\epsilon$ in the coupled system with a certain spatial frequency distribution [48]. There are two segments of AD domains in the parameter space of $\Delta \omega$ vs. $\epsilon_0$; as an example, the AD state occurs in the intervals of $\Delta \omega \in [3.6, 5.4]$ and $\Delta \omega \in [9.3, 20]$ as $\alpha = 1.1$ and $\epsilon_0 = 5$ (the vertical line in fig. 2(d)). The two ragged AD domains keep shrinking and leaving away from each other with the increment of the modulation amplitude $\alpha$. Moreover, for a given $\alpha = 1.5$, we explore the effects of the modulation frequency $\omega_0$ by numerically plotting the phase diagram of the parameter spaces $\Delta \omega$ vs. $\epsilon_0$ for $\omega_0 = 1.5, 10, 13, 16, 19$ in figs. 3(a)–(f), respectively. The results show that the increment of the modulation frequency $\omega_0$ firstly splits the AD domain into two parts with the upper one larger than the lower one, then the lower one expands while the upper one shrinks. Finally, the two parts merged into one large AD domain again.

To present a detailed insight into the effects of $\alpha$ on the AD domain, the normalized ratio factor $R = S(\alpha)/S(\alpha = 0)$ is defined to qualify the change of AD domains under designated regions ($\epsilon_0 \in [0, 20]$ and $\delta \omega \in [0, 20]$) in fig. 4(a) where the modulation frequency $\omega_0 = 3.5, 5.0, 15, 20$, respectively. $S(\alpha)$ is the area of the AD domains for a given $\alpha$ while $S(\alpha = 0)$ represents the area of AD domains for $\alpha = 0$ (i.e., the red domains in fig. 2(a)). It is obvious that the ratio $R$ firstly increases slightly (expansion of the AD domain) and then decreases sharply to a small value (reduction of the AD domain) as $\alpha$ increases from 0 to 2 for all modulation frequencies $\omega_0$. There is a critical value $\alpha_c$ with which the AD domain gets to the largest value for each given modulation frequency $\omega_0$. There is an optimal modulation amplitude of the coupling strength with which the coupled system has the largest AD domain. When $\alpha < \alpha_c$, the increment of $\alpha$ tends to enlarge the area of the AD domain by shrinking that of the oscillating domain, otherwise, the AD domain is torn into
multi-domains by the birth of an oscillating domain and shrinks with the increment of the modulation amplitude $\alpha$. Interestingly, the optimal value of $\alpha_0$ increases with the increment of the modulation frequency $\omega$ as shown in the inset in fig. 4(a). The larger the modulation frequency $\omega$, the larger the modulation amplitude needed to maximize the AD domain.

Now let us focus on the effects of the modulation frequency $\omega_0$ on the AD domain. Define the proportion of the AD domain on the designated regions $\epsilon_0 \in [0, 20]$ and $\delta \omega \in [0, 20]$ as $P(\omega_0) = S(\omega_0)/S_{\text{tot}}$ for a given $\omega_0$, where $S(\omega_0)$ is the area of the AD domain for given $\omega_0$, and $S_{\text{tot}}$ is the area of the designated region. Then, $P(\omega_0)$ vs. the modulation frequency $\omega_0$ can be numerically presented for an arbitrarily given modulation amplitude $\alpha$. $P(\omega_0)$ linearly decreases with a slow speed for $\alpha = 0.6$ while obviously increases for $\alpha = 1.8$ as $\omega_0$ increases from 7 to 20 as shown in fig. 4(b). Therefore, the modulation frequency $\omega_0$ is beneficial to shrink the AD domain for small modulation amplitude $\alpha$, otherwise, the AD domain expands quickly with the increment of the modulation frequency $\omega_0$ as $\alpha$ is larger than the maximal $\alpha_0$.

**Coupled Rossler oscillators.** AD is a common regime in coupled chaotic oscillators with continuous coupling [8], or on-off coupling [1]. It is natural to explore the effects of the periodically modulated coupling on the AD domain in coupled chaotic nonidentical oscillators. Let us consider the coupled chaotic Rossler oscillators with two different time scales in the form of $\dot{x}_i(t) = -\omega_i(y_i(t) + z_i(t)), \dot{y}_i(t) = \omega_i(x_i(t) + ay_i(t)), \dot{z}_i(t) = \omega_i(b + z_i(t)(x_i(t) - c)$, where $\omega_i$ rescales the rolling speed of a single chaotic oscillator. The single uncoupled oscillators is in a chaotic regime for the given parameters $a = 0.15, b = 0.4, c = 8.5$, and has an unstable fixed point $(-ay^*, -z^*, z^*)$ with $z^* = (c - \sqrt{c^2 - 4ab})/(2a)$. The frequency mismatch $\Delta \omega = |\omega_i - \omega_j|$ by arbitrarily set $\omega_i = 2$. The periodically modulated coupling term $\epsilon(t)$ is the same as eq. (2) with the modulation frequency $\omega_0 = 1$. The coupling scheme is set as $\Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, (i.e., the interacting variable is $x(t)$).

Then the AD domain with different periodic coupling strength can be conveniently observed by presenting the phase diagram of parameters $\Delta \omega$ vs. $\epsilon_0$ for $\alpha = 0, 0.5, 1.0, 1.2, 1.4, 1.6$ as shown in figs. 5(a)–(f), respectively. With the constant coupling strength ($\alpha = 0$), the coupled Rossler oscillators have three states: the AD state (V-shaped blue domain), the oscillating state (red domain), and the blowup to infinite state (green domain). As $\alpha = 0.5$ the AD domain is enlarged while the domain of the blowup state is shrunk. Then the AD domain is ragged into two parts as $\alpha$ is larger than 1.0. Finally, the AD domain shrinks while the domain of the blowup state occurs again and keeps enlarging as $\alpha$ increases from 1.2 to 2.0.

Figure 6(a) exhibits the change of the areas of the AD domain by the ratio factor $R$ vs. the modulation amplitude $\alpha$ for a given modulation frequency $\omega_0 = 1, 3, 5, 10, 15, 20$, respectively, where the ratio factor $R$ is defined as the same as the above one with the designated region of $(\epsilon, \Delta \omega)$-space in the range of $\epsilon \in [0, 8]$ and $\Delta \omega \in [0, 8]$. The effects of the modulation frequency and modulation amplitude of the coupling strength on AD domains in coupled Rossler oscillators are similar to those in the coupled Stuart-Landau oscillators. The increment of the modulation amplitude tends to enlarge the AD domain first then shrinks the AD domain for a given modulation frequency $\omega_0$. There is also a critical value $\alpha_c$ with which the coupled Rossler oscillators have the largest AD domain. Similarly, we may define the proportion of blowup domain as $P(\alpha) = S(\alpha)/S_{\text{tot}}$ for a given $\alpha$, where $S(\alpha)$ is the area of the parameter space of the blowup domain and $S_{\text{tot}}$ is the area of domain in the designated area of $\epsilon \in [0, 8]$ and $\Delta \omega \in [0, 8]$. Then the effects of $\alpha$ on the blowup can be indicated by $P(\alpha)$ vs. $\alpha$ as shown in fig. 6(b) for the given $\omega_0 = 1, 3, 5, 8, 10$, respectively. It is obvious that $P(\alpha)$ is small and approaches to zero for small $\alpha$ and then grows up again when $\alpha$ is larger than a critical value.
Mechanism analysis. – Since the modulated coupling strength varies with time, the linear stability analysis near the fixed points is not available to predict the dynamics regimes. Since the conditional Lyapunov exponent [49] is a valid tool to determine the generalized synchronization, it is expected to determine the stability of the AD state in the coupled nonidentical oscillators. Let $\delta z_i = z_i - z_i^*$, $(i = 1, 2)$ be an infinitesimal perturbation added to oscillator $i$, then whether the perturbed trajectories of eq. (1) could be converged to the fixed point $z^*$ is mainly determined by the set of the variational equations

\[
\begin{bmatrix}
\delta z_1(t) \\
\delta z_2(t)
\end{bmatrix} = D F_1 \begin{bmatrix}
z_1^* \\
z_2^*
\end{bmatrix} \begin{bmatrix}
\delta z_1(t) \\
\delta z_2(t)
\end{bmatrix}
+ \epsilon(t) A \begin{bmatrix}
\delta z_1(t) \\
\delta z_2(t)
\end{bmatrix},
\]

where $z_1^* = (0, 0)$, $z_2^* = (0, 0)$ are the original fixed points of the single oscillators. $D F_1$ and $D F_2$ are the Jacobian matrices of the two coupled oscillators. $\Gamma$ is the coupling scheme ($\Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for coupled LS oscillators) and $A = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$ is the link matrix whose eigenvalue is $\lambda_1 = 0$ and $\lambda_2 = -2$. Solving eq. (4) numerically for $\lambda_2 = -2$, we are able to obtain the conditional Lyapunov exponent $\lambda_\epsilon$ with respect to the parameters of the coupling strength and frequency mismatches, based on which the stable domain of AD, i.e., the domain with $\lambda_\epsilon < 0$, can be identified. In figs. 7(a)–(d), we plot the conditional Lyapunov exponent $\lambda_\epsilon$ as a function of $\Delta \omega$ for $\epsilon_0 = 7$, and $\alpha = 0.0, 0.8, 1.0, 1.8$, respectively. The conditional Lyapunov exponents transit to negative from positive when the coupled system transits from the oscillating state to the AD state which matches well with the bifurcation results shown in grey dots.
finite preciseness of the computer, AD can be observed in the numerical results when the distance between the oscillator and the original fixed point is smaller than the computer’s preciseness. In fig. 8(b), the oscillator may also diverge from (stage DE) and approach to (stage EF) the original fixed point in each period of the modulated coupling. Noting that the speed of approaching to the fixed point is larger than that of diverging from the fixed point, however, the time of the former one is shorter than the latter one. As a result, the coupled oscillator forms an oscillating state. The speed of approaching to or leaving away from the fixed point is related to the value of the local conditional Lyapunov exponent [50] as shown in figs. 8(c), (d), respectively. The positive local conditional Lyapunov exponent makes the oscillator leave away from the fixed point while the negative one drives the coupled system to converge to the fixed point. The speed of approaching to or diverging from the fixed point is related to the absolute value of the local conditional Lyapunov exponent. Comparing the results in fig. 8(c) and fig. 8(d), the final fate of the coupled oscillator is completely determined by the average value of the local conditional Lyapunov exponent in each modulation period. The coupled system is in the AD state if the average value of the local conditional Lyapunov exponent is negative, otherwise it is in the oscillating state. The conditional Lyapunov exponent is also available to predict the AD dynamics in the coupled Rossler oscillators. Figures 9(a)–(d) present the conditional Lyapunov exponent together with the bifurcation diagram on variable $x_1$ vs. $\Delta \omega$. The conditional Lyapunov exponent gets to a negative value when the coupled Rossler oscillators experience AD which also agrees well with the bifurcation diagram.

**Conclusions.**—Totally, both the modulation frequency and the modulation amplitude of the periodically modulated coupling strength significantly influence the dynamics of the coupled limit cycles or chaotic oscillators. The increment of the modulation amplitude firstly increases then decreases the AD domain as it is larger than a critical value which is related to the modulation frequency. That is to say, the small modulation amplitude of the coupling strength is helpful to enlarge the AD domain of the coupled nonidentical oscillators. However, when the coupling term is varying between repulsive (negative) and attractive (positive), the AD domains may be shrunk and ragged to several parts by the occurrence of the oscillating domain. Moreover, the increment of the modulation frequency of the periodic coupling tends to slightly decrease the AD domains for small modulation amplitude while it dramatically increases the AD domains for large modulation amplitude. According to the local conditional Lyapunov exponent of the periodically coupled oscillators, one may find that the stability of the AD states is varying with the coupling strength. Whether the coupled oscillators can converge to the AD state or not is completely determined by the sign of the averaged conditional Lyapunov exponent. The periodically modulated coupling is beneficial to realize AD and is easier to the physical realization than the on-off coupling which is hard to apply owing to the limit response speed of the switchers, therefore, it has potential application in the dynamical control in engineering.

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