Genetically nonlinear combined model of pile field under
dynamic impacts

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Abstract. The authors propose a combined approach for definition of the shear rigidity of the multilayered soil which is cut through by a pile. The solution for the vertical direction is presented in the view of an axisymmetric problem. As to the horizontal direction, the solution is presented in view of a beam on elastic subsoil with genetically non-linear transition to equivalent horizontal rigidity of the wide pile field in condition of dynamic forces action. The axisymmetric solution provides visual clarity in the analysis of the stress-strain state of the pile and near-pile soil in comparison with the approved analytical methods. To speed up calculations at the stage of the main combination of constant and long-term impacts, the vertical rigidity of the base under the foot of the pile can be calculated analytically as for a stamp on an elastic-plastic base. The horizontal rigidity is considered as for a discrete single bent pile in the medium of an elastic layered half-space at the stage of formation of the stress-strain state of the system under the main combination of static loads. At the final stage of short-term or special load, a transition to the integral shear rigidity of a pile field is proposed.

1. Introduction
The calculation of a wide pile base under dynamic conditions is one of the most difficult tasks in soil mechanics. On the one hand, there are many fragmentally illuminated aspects of the elementary round-cylindrical cell of a single pile, the rigidity of which is calculated using two fundamentally different mathematical models for the directions of vertical and horizontal forces. On the other hand, there are almost no methods of accounting genetically non-linear transfer system "Pile subsoil – Foundation" from the stress-strain state at the main combinations of permanent and long-term impacts to a new combination of the dynamic impact that is short-term or special. In the first case, it can be wind pulsations, artificial seismic loads from rail transport or the operation of technological equipment. In the case of special impacts, it is necessary to distinguish seismic events or dynamic reaction of load-bearing element elimination in the analysis of resistance to the progressive collapse.

2. Analytical model of a vertically loaded single pile
The mathematical shear two-layer cylindrical model for the settlement of a single pile or a group of up to 25 piles was introduced in 2010 in SP 24.13330.2011 "Pile foundations" (clause 7.4.2) instead of the conditional foundation method. The model is based on a significant amount of experimental data
generalized by the research team of the NIIOSP [1, 2, 3]. However, the mathematical presentation with an excessive number of intermediate coefficients makes it difficult to perceive the method physical meaning.

In the formula (7.39) SP 24.13330, in the condition that there is an influence of stresses from neighboring piles on the settlement of the pile in question, the concept of the maximum radius of mutual influence of piles $a_{lim}$ (1) is used implicitly. This parameter is the radius of the cylindrical geomechanical model, which is equal in height to the length of the pile $L$ with the underlying layer of 0.5 $L$ (figure 1 (a)). The exclusion of the deformability of the concrete pile body and the introduction of the radius of influence simplifies the form of the formulas of SP 24.13330 for a homogeneous subsoil:

$$a_{lim} = k_v \frac{G_L}{2\sigma_2} = (2.82 - 3.78v + 2.18v^2) \frac{G_L}{2\sigma_2}$$

(1)

$$S_i = 0.17 \ln \left( \frac{a_{lim}}{2a_i} \right) \frac{N_i}{G_L}$$

(2)

$$S_{ad,ij} = 0.17 \ln \left( \frac{a_{lim}}{a_j} \right) \frac{N_j}{G_L}$$

(3)

$$S_{i,gr} = S_i + \sum_{j \neq i} S_{ad,ij}$$

(4)

The obvious advantage of the single pile shear model is the ease of application for automating numerical calculations in the finite element structural analysis [4, 5, 6].

**Figure 1.** Diagram of a two-layer cylindrical geomechanical pile model: (a) the stress-strain state of the near-pile soil; (b) an incompressible suspending pile in the absence of resistance in the underlying layer of soil; (c) an incompressible pile-stand with the formation of a compacted core (I) and zones of the limit state (II) under the foot as under a buried stamp.

A two-layer concept for cylindrical pile model was proposed in 1953 [7]. Excluding the low compressibility of the pile body compared to the soil the total pile settlement $S$ is easily decomposed into the settlement along the side surface $S_r$ (figure 1 (b)), which is always equal to the settlement of the sub soil $S_0$ from the penetration of the foot of the pile (figure 1 (c)). Based on this concept several new geomechanical models of the pile was developed, considering the rheological and elastic-plastic processes [8, 9, 10, 11, 12, 13].
3. Numerical axisymmetric model of a vertically loaded pile

The authors propose a further development of the cylindrical model by replacing the analytical description for the area of linear shear deformations along the length of the pile with a numerical solution of the FEM of a multilayer axisymmetric solid body [14].

The method is verified in the Mathcad with a solid model from three-dimensional finite elements (figure 2 (b)) and with an axisymmetric model in SCAD (figure 2 (c)). A fragment of a cell with a height of 0.4 m and a radius of 0.57 m is considered. The pile of concrete B20 and 0.34 m in diameter is surrounded by loam with a deformation modulus of 10 MPa.

The maximum error of the reaction in node 2 in Mathcad was 0.7% (figure 2) and for the vertical deformation in node 7-0.3% compared to the axisymmetric solution in SCAD (figures 3 (a) and 3 (b)). The maximum settlement at the upper edge node in the SCAD spatial problem was 6.28 mm, or 1.5% more than 6.19 mm for axisymmetric solutions in Mathcad and SCAD (figure 3 (d)). The maximum error for the calculated vertical stresses $\sigma_z$ is 8.8% in element No. 7 and it decreases sharply to 0.004% towards the cell periphery as it moves away from the stress concentrators along the axis of rotation (figure 2 (c)), which indicates the rationality of modelling the pile body with two rows of finite elements.

The settlement of a single pile in an elastic axisymmetric setting (figure 3 (a)) with a depth of the underlying layer of 0.5 $L$ is 24% less than the empirical solution according to clause 7.4.2 of SP 24.13330. Increasing the thickness of the elastic underlying layer to the depth of the compressible thickness of the conditional foundation is insufficient (figure 3 (c)). Only at a double depth of
2xHc=22.8 m, the settlement corresponds to the solution according to clause 7.4.2 (figure 3 (d)). This is since zones of compacted core I and limit equilibrium II are formed under the pile foot (figure 1 (c)) [15]. The complete exclusion of the underlying layer leaves only the shear rigidity along the lateral surface $S_\tau=6.41 \text{ mm}$ with an excess of settlement by 4.8%. The solution corresponding to the empirical method of SP 24.13330 can be achieved by replacing the underlying layer [7] with a spring with the rigidity of the subsoil under the round stamp (figure 3 (e)).

The spring parameter under the foot of the pile can be calculated from the criterion of applicability of the formula 7.32 of SP 24.13330, in which the inverse ratio G2d/G1L indicates what proportion of the rigidity under the foot is from the rigidity on the lateral surface, equal in this example to 3.4%. This indicates, that the use of theoretical solutions offered by, allows us to move from an empirical method to an accurate mathematical definition of elastic-plastic deformations under the foot of the pile, as well as to enter into the calculation of the rheological properties of soils. The axisymmetric numerical component of the geomechanical model simplifies the analysis and description of the properties of inhomogeneous soil along the length of the pile.

Figure 3. Analysis and scheme of a multilayer axisymmetric geomechanical model of a pile: (a) settlement of a suspended pile in the axisymmetric model SCAD with a deep underlying layer of 0.5L; (b) settlement with the underlaying layer in the depth of the compressible thickness of the conditional foundation $H_c=11.4 \text{ m}$; (c) settlement with the underlaying layer for the double depth of the compressible thickness $H_c=22.8 \text{ m}$; (d) settlement in the absence of the underlaying layer; (e) the scheme of the numerical-analytical axisymmetric model.

4. Numerical static model of horizontally loaded pile

The completed numerical-analytical model of a pile in the soil cannot be limited only by the axisymmetric model, which is applicable only to the analysis of the vertical rigidity of the pile. In most cases, piles undergo a complex stress-strain state. Significant transverse forces and moments are observed in the pile clusters of frame buildings and in the edge piles of a wide pile field. Nevertheless, the conditions of dynamic forces on the wide pile field, considered by the authors, correspond mainly to the horizontal force on the pile head. For the applicability of the model to all stages of the calculation of pile bases, a complex numerical model of the pile is considered. Complexity refers to the independent calculation of vertical and horizontal rigidity. The results of calculations can be given both in the intermediate nodes of ordinary rod finite elements of reinforced concrete piles for static
calculations, and in the level of the pile head for performing direct dynamic nonlinear calculations in the temporary area that require significant optimization of the model size.

To increase the accuracy and reduce the computational requirements for solving the problem of a bending pile, which is immersed in an elastic half-space, the authors propose the application of a numerical solution to the differential equation of an elastic beam on an elastic base [16]:

\[ u'''' + \frac{(b_p K_0 x)}{E I} u = \frac{q}{E I} \]  

(5)

where \( u \) is the unknown displacement of piles; \( E I = E (\pi d^4/64) \) – flexural rigidity of circular piles with a diameter \( d \) or \( E I = E (\frac{bh^3}{12}) \) rectangular pile with the sides \( b \) and \( h \) in-plane deformation along the side \( h \); \( b_p \) and \( K_0 \) the width of the conditional section of the pile and linearly increasing \( z \) the depth ratio of the elastic base on the lateral surface of piles in accordance with Appendix C of SP 24.13330; \( \alpha \) – coefficient of reduction of soil rigidity on the lateral surface of piles consisting of horizontally loaded group, according to Appendix C of SP 24.13330, which is equal to 1 for a single pile; \( q \) distributed horizontal load on a small portion of the pile head \( a \), which replaces concentrated force \( H \) missing in the right side of the differential equation (5).

Proposed option of the estimate of horizontal rigidity of the pile in head level (figure 4 (a)) is suitable for wide pile with a flexible pile raft, when the edge piles undergo additional bending moments and make a small contribution to the total shear rigidity. If we need a detailed analysis of the flexible grillage of a pile cluster, in which the piles undergo significant bending moments (figure 4 (b)), we do not need to have specialized finite super-elements, since for a small number of piles in the cluster, it is enough to use classical rod finite elements with the specified coefficients of soil reaction along the lateral surfaces of the piles.

(a)  

(b)  

Figure 4. Finite element models of a discrete horizontally loaded single pile (a) and pile cluster (b).

Figure 5 shows an algorithm for solving piles as beams on an elastic base in the Mathcad mathematical software in the form of a fourth-order differential equation. Because of replacing the concentrated horizontal load \( H \) with an equivalent distributed force \( q \) on a small section of the pile head, zero values of the transverse moments and transverse forces corresponding to the second and third derivatives of the deflection are assumed as the four necessary boundary conditions. The
horizontal force $H$ in the algorithm is taken as a vector, which allows us to consider the additional effective pressure on the side face of the pile from the surface load. In the given example of the algorithm, the initial data, which correspond to the drill pile in the cluster.

Figure 5. Algorithm for numerical model of a horizontally loaded single pile.

The presented numerical model of a horizontally loaded single pile implements the classical solution of this problem and corresponds to the provisions of the methodology in Appendix C of SP 24.13330. However, in calculations of a wide pile field this model may be used only in the initial stages of the formation of genetically nonlinear stress-strain state of the system "Pile Base-Foundation-Structure" in the construction and operation stages under the action of the main normative-long combinations of external forces. The analysis of the system at the following stages, at which there is a dynamic short-term or special impact, should be performed with a fundamentally different model of the pile base.

5. Finite element analysis of a vertically loaded pile cluster

Figure 6 shows an example of a pile cluster calculation for the action of vertical and horizontal loads, which is considered in detail in one of the previous publications [17].

The analytical method for calculating pile bushes with a rigid slab grillage in accordance with the recommendations of SP 24.13330 is convenient for mathematical implementation by the finite element method. In contrast to the calculation of a single pile, for a group of piles or for a pile field, an iterative calculation is required with the specification of the stiffness of single-node springs at the lower ends of the piles or the introduction of additional forces $\Delta Nh$. Both approaches allow reflecting the deformations of the elastic subsoil in the form of a common settlement funnel, considering the mutual influence of neighboring piles. Figures 8e and 8f show the result of pile cluster settlement calculation with single-node springs and additional nodal loads $\Delta Nh$.

The given example of calculating piles in a group in accordance with normative analytical methods can be implemented by a unified algorithm for foundations in the form of a single pile, a group of piles, or a pile field with equivalent stiffness at various stages of the formation of the stress-strain state of structures.

The horizontal influence of in-group piles is based on empirical formulas [3]. The horizontal soil backpressure along the lateral surface of piles within the framework of the study depends on the reduction coefficient $\alpha$ which is calculated with the empirical formula. Then, the coefficient values of soil reaction on the lateral sides $C_z$ are calculated according to the table of stiffness for different types of soils [18]:

$$ C_z = K \cdot z \cdot \alpha, $$

where $K$ is the tabular value of the stiffness of the soil with horizontal back pressure on the lateral sides of the piles in the table. B.1 of SP 24.13330.2011; $z$ is depth, from the surface of the earth, on which lateral reduction is calculated.
Figure 6. The pile cluster calculation example in SCAD FEA software: (a) view of a pile cluster; (b) coefficients of soil reaction along the lateral surface of the pile, kN/m³; (c) vertical and horizontal loading of pile cluster, kN; (d) vertical springs and additional nodal loads ΔNh; (e) deformation of the nodes from the vertical load, mm; (f) deformation from the horizontal load, mm.

Within the second step of the calculation in SCAD the starting boundary conditions in vertical direction are assigned without considering the mutual influence of the pile group. The calculation of the preliminary vertical stiffness of the piles is made in accordance with clause 7.4.2. of SP 24.13330.2011. The shear modulus \( G_1 \) of the soil layers cut by the pile is calculated based on the averaged modulus of deformation \( E_1 \) and the averaged dimensionless Poisson ratio \( \nu_1 \) for the layers cut by the pile. Similarly, the shear modulus \( G_2 \) is calculated for soil layers located under the piles:

\[
G_1 = \frac{E_1}{2(1+\nu_1)} \tag{7}
\]

\[
E_1 = \frac{\sum_{i=1}^{\text{rows}(E)-1} E_i \cdot L_i}{\sum_{i=1}^{\text{rows}(E)-1} L_i} \tag{8}
\]

\[
\nu_1 = \frac{\sum_{i=1}^{\text{rows}(E)-1} \nu_i \cdot L_i}{\sum_{i=1}^{\text{rows}(E)-1} L_i} \tag{9}
\]

\[
G_2 = \frac{E_2}{2(1+\nu_2)} \tag{10}
\]

\[
\nu = \frac{\nu_1 + \nu_2}{2} \tag{11}
\]

\[
k_2 = \frac{g_1 L}{\beta'} \tag{12}
\]

where \( E_1 \) - the modulus of deformation of soil separate layers cut by the pile, MPa; \( L_i \) - the thickness of the soil layers cut by the pile, m; \( \nu_i \) - Poisson’s ratio of soil layers cut by the pile; \( \beta' = 0.17 \ln[(k_y \cdot G_1 \cdot L)/G_2 \cdot d] \) – additional settlement coefficient of the rigid pile; \( k_y = 2.82 - 3.78\nu + 2.18\nu^2 \) – intermediate empiric coefficient.

The modulus of deformation of the soil layers located under the pile, \( E_2 \), is taken as an averaged one within a depth equal to half the pile length 0.5L or equal to 10d of the equivalent pile diameters of
the lower ends of the pile. Poisson's ratio $\nu_2$ is set according to the layer, which is located below the base of the equivalent foundation. Similarly, the shear modulus $G_2$ is calculated for the soil layers, which are located under the lower ends of the piles.

Finally, determining the average value of the Poisson’s ratio, it will be possible to calculate the initial vertical stiffness $k_z$ (kN/m) of one-node elastic springs. This one-node spring are assigned to all nodes of the lower ends of single piles in cluster to simulate the interaction of foundation structures with the surrounding soil within the finite element method without considering the vertical mutual influence of the pile group and is determined with already known variables.

The significant excess of the initial value of the vertical stiffness $k_z$ compared to the values $C_1$ in the Winkler model is explained by the fact that the final stiffness $k_z$ decreases due to iterative improvement during the next stages.

$$k_z = \frac{C_1 L}{2G_2 a_2}$$

$$a_2 = \begin{bmatrix}
0 & 0 & 1.0 & 0 & 1.273 & 2.012 & 1.0 & 2.012 & 2.546 \\
0.9 & 0.9 & 0.9 & 1.273 & 0.9 & 1.273 & 2.012 & 1.8 & 2.012 \\
1.8 & 0.9 & 0.9 & 2.012 & 1.273 & 0.9 & 2.546 & 2.012 & 1.8 \\
0.9 & 1.273 & 2.012 & 0 & 0.9 & 0.9 & 1.273 & 2.012 \\
1.273 & 0.9 & 1.273 & 0.9 & 0.9 & 0.9 & 1.273 & 0.9 & 1.273 \\
2.012 & 1.273 & 0.9 & 1.8 & 0.9 & 0 & 2.012 & 1.273 & 0.9 \\
2.012 & 1.8 & 2.012 & 1.273 & 0.9 & 0 & 2.546 & 2.012 & 1.8 \\
2.012 & 2.012 & 1.273 & 0.9 & 1.273 & 0.9 & 1.8 & 0.9 \\
2.546 & 2.012 & 1.8 & 2.012 & 1.273 & 0.9 & 1.8 & 0.9
\end{bmatrix}$$

$$D_j = \begin{bmatrix}
0 & 0.189 & 0.081 & 0.189 & 0.14 & 0.062 & 0.081 & 0.062 & 0.022 \\
0.189 & 0 & 0.198 & 0.14 & 0.188 & 0.14 & 0.062 & 0.081 & 0.062 \\
0.081 & 0.189 & 0 & 0.062 & 0.14 & 0.188 & 0.062 & 0.081 & 0.062 \\
0.189 & 0.14 & 0.062 & 0 & 0.189 & 0.081 & 0.189 & 0.14 & 0.14 \\
0.062 & 0.14 & 0.188 & 0 & 0.062 & 0.188 & 0 & 0.189 & 0.14 \\
0.081 & 0.062 & 0.081 & 0.188 & 0 & 0.189 & 0.14 & 0.189 & 0.14 \\
0.022 & 0.062 & 0.081 & 0.062 & 0.14 & 0.188 & 0.14 & 0.189 & 0.198 \\
0.189 & 0 & 0.198 & 0 & 0.189 & 0 & 0.198 & 0 & 0.198 \\
0.062 & 0.14 & 0.188 & 0 & 0.062 & 0.188 & 0 & 0.189 & 0.14 \\
0.081 & 0.062 & 0.081 & 0.188 & 0 & 0.189 & 0.14 & 0.189 & 0.14
\end{bmatrix}$$

Figure 7. Fragments of the algorithm for the mutual influence of piles in a cluster: (a) matrix-columns with numerical data for the names of piles and coordinates in plan in X and Y; (b) calculation of the matrix $a2$ of the relative position of the piles; (c) matrix of mutual influence of piles in group $\delta2$; (d) an algorithm for calculating the resulting forces $\Delta Nh$; (e) columnar matrices with numerical data, lower node forces $Nh$, and additional forces $\Delta Nh$.

Figure 7 shows a fragment of the algorithm for considering the mutual influence of piles in a cluster at the final fourth stage of the calculation. The problem is solved using linear programming
tools. The array with the imported pile node coordinates is transformed into two numerical series with the X and Y coordinates. The next step is to form a common matrix $a2$ of the relative position of the piles in the cluster in the form of the calculated distances between the piles. The size of the square matrix corresponds to the number of piles in the foundation.

Based on the matrix of the relative position of the piles $\delta 2$, the matrix $\Delta Nh$ of the vertical mutual influence of the piles in the cluster is calculated according to the theory of elastic half-space in the form of additional efforts (figure 3 and 4) or in the form of coefficients of lowering the initial stiffness. This is ensured by performing multiple calculations of each member of the matrix in accordance with the formulas of SP 24.13330.2011 clause 7.4.4, which provide for the zeroing of the coefficient of mutual influence of one pile on another when a certain distance between them is exceeded. The limiting radius of influence, beyond which there is no mutual influence on the piles, can be expressed as the formula:

$$R_v = \frac{k_2 G_1 L}{2 G_2}$$ (13)

The last step is the calculation of additional efforts $\Delta Nh$, which are the sum of the initial vertical reactions $Nh$ in closely spaced piles, considering the coefficient of mutual influence $\delta$. These efforts are necessary in the design model for additional deformations in the lower node of each pile to model a common settlement funnel.

6. Numerical dynamic model of horizontally loaded pile fields

When performing dynamic calculations, the discrete model of a pile cluster is the most studied and widely presented in regulatory documents. Accounting for the reduction in the load-bearing capacity of piles for vertical loads is carried out by introducing reducing coefficients in accordance with SP 24.13330.2011. The calculation of the zone of partial loss of contact with the soil in the upper area of the pile along its lateral surface was described in [19] and later introduced in SP 24.13330.2011.

At the same time, in the current regulatory and technical documents of the Russian Federation, there is no methodology for considering the dynamic rigidity of a wide pile field in the conditions of seismic impacts. It should be noted that the single formalized methodology is of JSC "Atomenergoproekt" (Moscow), which was developed within the framework of the project of the «Bashkir» Nuclear Power Plant, but it was not tested due to the conservation of the construction site [20]. In addition, this technique is based on specialized software tools, such as SASSI, which limits its application.

Difficulties in the development of such a technique are associated with a number of features of the pile base, arranged on weak soils [19]. First, it is necessary to re-perform geophysical surveys within the framework of seismic zoning of the territory. After driving the piles, the arrangement of bored piles or soil-cement piles, the dynamic properties of the base significantly change in comparison with the natural base previously studied at the survey stage. At the same time, on the scale of the pile field, the dimensions of the pile sections are so small, and their number is so large, that the applicability of discrete models with the rod analogy of piles is not feasible in practice.

The increased flexural rigidity of the piles compared to the original soil mass in its natural state, as well as the possibility of losing contact with the soil along the lateral surface of the piles, does not allow us to consider the pile field as a layered half-space with anisotropic properties.

A new approach adopts a methodology for an impedance numerical-analytical model in the SASSI software [21] for use in direct physical models of the pile and reinforced base, which can be implemented in the widespread SCAD Office software or in the complex numerical-analytical model of the pile field developed by the authors [22].

The method is based on the use of dynamic characteristics of soils (elastic modulus, shear modulus, Poisson's ratio), studied by seismic and microseismic methods before and after the construction of the pile field. The results of the seismic method on the velocities of longitudinal and transverse waves allow us to identify soil layers with characteristic acoustic rigidity in the upper part of the geological
section, as well as to check their damping properties. If not only weak, but also strong seismic movements of the soil are possible on the site, then further data processing for a given intensity of seismicity of the construction site is performed by proven methods for hydrotechnical construction objects.

The change the integral rigidity seismological models of the natural base and pile base is analyzed by the method of seismic rigidity, which is compared with the results of the second micro seismic studies to assess the change in the intensity of seismic impacts before and after of the arrangement of pile base according to Nakamura method. The micro seismic method also makes it possible to determine in a direct form the amplitude-frequency characteristics of the entire soil mass before and after reinforcement by piles in order to identify numerical and numerical-analytical models. Additional quality control of the piles is performed by ultrasonic and acoustic methods.

The obtained experimental data are used to direct physical or numerical-analytical model base, built on similar principles of averaging of the pile base to the anisotropic properties of volumetric finite elements, or the coefficients of soil reaction on the base of the foundation for vertical and horizontal component of numerical models.

In accordance with the requirements of SP 14.13330, only the integral shear rigidity of concrete piles without soil rigidity is considered for the calculated depth, to which the soil resistance on the side surface is not considered. The shear rigidity of the layer is defined as the sum of the moments of inertia of a single flexible piles without considering the contribution of the flexural component by Steiner, which is equal to the product of the moment intertie of the single pile section on the ratio of the cross-sectional area of the pile to the square of the pile elementary cell. In this case, in the plane of the pile grillage, to take into account the effect of the separation of piles in the plan, an additional brace of the final torsional rigidity in two vertical planes is added at the head of each pile node:

$$C_p = \frac{E(I_1-I_2)}{L}$$

where $E$ is the elastic modulus of the reinforced concrete pile; $L$ - the width of the pile cell; $I_1$ and $I_2$ - the moments of inertia of the pile or group of piles, considering the Steiner component and without it.

The following layers of the soil mass reinforced by piles along the perimeter of the grillage are considered as reduced rigidity in accordance with the lithological structure of the base to the level of the bottom of the piles. The last underlying layer to a depth, which is equal to half the length of the piles by analogy with the shear round-cylindrical model according to SP 24.13330, is considered as a soil with compaction determined by seismic methods.

It should be noted that resource-intensive direct physical models of the pile base are not necessary in all cases. The question of the criterion for the presence of the reverse effect of vibrations of the structure on the movement of the base is covered in an early publication of the authors [23, 24]. If the analysis still indicates the need for a joint calculation of the "Soil - Foundation" system, then the reader can find the rules for constructing direct physical models of the subsoil in the corresponding publication [25, 26].

The proposed numerical-analytical approach to modeling the pile base does not have high requirements for the speed of calculations, so it can be used without the need to estimate the presence of the reverse influence of the structure on the base. The results of the experimental verification of the proposed approach to the construction of a pile base model using acoustic and microseismic methods and the discussion of practical aspects of software and hardware for this type of survey are the subject of the next publication.

7. Conclusions

The considered stages of forming a complex numerically analytical model of a pile foundation allow us to implement a universal finite super-element that changes its properties depending on the size of the foundation and on the type of impact.
The analytical two-layer model of a vertically loaded pile cell is applicable for a single pile or as part of a cluster of up to 25 piles. The proposed axisymmetric model of the solid body rotation allows us to display detailed picture of the stress-strain state of the pile as a single, in a small group, or as part of the piles or combined pile-raft foundation. Axisymmetric setting allows us to consider an arbitrary number of inhomogeneous soil layers cut through by the pile. The result of the calculation in either of the two methods is the equivalent springs under the foot of the discrete piles or at the level of its head.

Under static conditions, the rigidity of a horizontally loaded pile in any type of foundation can be represented as a beam on an elastic base. The solution of the differential equation can be performed both by the numerical method and by the finite element method, with the determination of the equivalent rigidity at the level of the pile head or at the corresponding nodes along the length of the shaft axis of the discrete pile.

Dynamic impacts always affect the foundation, which is already undergoing deformations from permanently acting loads. The proposed approach allows the universal numerical-analytical model piles genetically to consider the nonlinear terms of the formation of the stress-strain state of a pile foundation. After the perception of the horizontal static horizontal loads of the piles in the form of a beam on an elastic base, we go to the integral dynamic rigidity of the pile field, which is determined based on engineering and geophysical studies by seismic methods. This can be accomplished either by uniformly reducing the horizontal coefficients of soil reaction along the lateral surface of discrete piles by changing the coefficient $\alpha$, or by lowering the equivalent braces of the final rigidity at the level of the pile heads.

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