Masses and Decay widths of Charmonium states in presence of strong magnetic fields

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Abstract

The masses and decay widths of charmonium states are studied in strong magnetic fields. The mixing between the pseudoscalar and vector charmonium states at rest is observed to lead to appreciable negative (positive) shifts in the masses of the pseudoscalar (longitudinal component of the vector) charmonium states in vacuum/hadronic medium in the presence of strong magnetic fields. The pseudoscalar and vector charmonium masses in the hadronic medium, calculated in an effective chiral model from the medium changes of a scalar dilaton field, have additional significant modifications in the hadronic medium due to the mixing effects. The partial decay widths of the vector charmonium state to $D \bar{D}$ in the presence of strong magnetic fields are investigated, using a field theoretical model for composite hadrons with quark/antiquark constituents. The masses of the $D$ and $\bar{D}$ mesons in the magnetized matter are calculated from their interactions with the nucleons and the scalar mesons within the chiral effective model. The effects of the mixing are observed to lead to significant contributions to the masses of the pseudoscalar and vector charmonium states, and are observed to lead to appreciable increase in the decay width $\psi(3770) \rightarrow D \bar{D}$, at large values of the magnetic fields. These studies should have observable consequences on the dilepton spectra, as well as on the production of the open charm mesons and the charmonium states in ultra relativistic heavy ion collision experiments.

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I. INTRODUCTION

The study of properties of hadrons under extreme conditions, e.g., high temperatures and/or densities is a topic of intense research in strong interaction physics. This is due to its relevance in the ultra relativistic heavy ion collision experiments at various high energy particle accelerators, as well as in the study of the bulk matter of astrophysical objects, e.g., neutron stars. In the recent past, there has been a lot of studies on the in-medium properties of the heavy flavour mesons [1], as these can have observable consequences in high energy heavy ion collision experiments. The heavy quarkonium (charmonium and bottomonium) states have been studied using the potential models [2–6], QCD sum rule approach [7–10], the Quark Meson coupling (QMC) model [11], coupled channel approach [12], and a chiral effective model [13, 14]. The heavy quarkonium (charmonium and bottomonium) state in the presence of a gluon field has been studied in Refs. [15–17]. Assuming the heavy quark ($Q$) and antiquark ($\bar{Q}$) in the quarkonium state to be bound by color Coulomb potential, and the $Q\bar{Q}$ separation to be small compared to the scale of the gluonic fluctuations, the mass shift in the quarkonium state in the leading order is observed to be proportional to the medium modifications of the scalar gluon condensate. Using the leading order formula and the linear density approximation for the gluon condensate in the nuclear medium, the mass modifications of the charmonium states have been studied in Ref. [18]. The in-medium masses of the charmonium states have been computed within a chiral effective model [13, 14], from the medium change of a dilaton scalar field, which simulates the gluon condensate within the hadronic model.

The magnetic fields created in non-central ultra relativistic heavy ion collision experiments, have been estimated to be huge, e.g., $eB \sim 2m_{\pi}^2$ at Relativistic Heavy Ion Collider (RHIC) at BNL and $eB \sim 15m_{\pi}^2$ at Large Hadron Collider (LHC) at CERN [19]. This has led to a lot of work on the study of effects of strong magnetic fields on the properties of the hadrons. The strong magnetic fields produced in the high energy heavy ion collisions rapidly drop after the collision. This leads to induced currents, which slow down the decrease in the magnetic field. The time evolution of the magnetic field [19] is still an open question, and needs the solutions of the magnetohydrodynamic equations, with a proper estimate of the electrical conductivity of the medium. In the near central collisions, the impact parameter is small, the magnetic field produced is weak and the created medium is dense. On the other hand, strong magnetic feilds
are created in the peripheral ultra relativistic heavy ion collisions and the formed medium has low density. The effects of the magnetic fields on the heavy quarkonium (charmonium and bottomonium) states can have observable consequences as these are formed in heavy ion collisions at the early stage, when the magnetic fields can still be large. The open charm mesons \[20–23\] as well as the charmonium states \[24–26\] have been studied in the presence of magnetic fields. There is mixing of the pseudoscalar and vector mesons in the presence of magnetic fields, which modifies the properties of the charmonium states \[25–28\]. The masses of the charmonium states in the presence of magnetic fields have been studied in a consistent manner using QCD sum rule approach, incorporating the mixing of the pseudoscalar and vector charmonium states in the hadron spectral function on the phenomenological side, as well as, including the effects of magnetic field on the OPE (operator product expansion) side \[25, 26\]. The QCD sum rule approach \[25, 26\], as well as, a study of the charm-anticharm bound state described by an effective potential and solving the Schrodinger equation in the presence of an external magnetic field \[27\] show that the charmonium masses have dominant contributions from the mixing effects. A study of the mixing effects on the formation time of the charmonia are observed to lead to delayed (faster) formation time of the vector (pseudoscalar) charmonium states \[28\]. The \(J/\psi – \eta_c\) as well as \(\psi’ – \eta_c’\) mixings and the faster formation time of the pseudoscalar mesons, might show as peaks in the dilepton spectra, due to ‘anomalous’ decay modes, e.g., \(\eta_c, \eta_c’ \rightarrow l^+l^-\) and can act as a probe of the existence of strong magnetic field at the early stage \[28\].

The masses of vector charmonium states in a (magnetized) hadronic medium have been studied using a chiral effective model \[13, 14, 24\]. These are calculated from the medium modification of a scalar dilaton field which mimics the gluon condensates of QCD in the effective hadronic model. Within the chiral effective model, the modifications of the masses of the open charm \((D, \bar{D})\) mesons in the (magnetized) hadronic medium arise from their interactions with the baryons and scalar mesons, and have been studied in Refs. \[13, 14, 23\]. The in-medium decay widths of the vector charmonium states to \(D\bar{D}\) have been computed from the mass modifications of the \(D, \bar{D}\) and charmonium states in Refs. \[13, 14, 29, 30\] using a light quark pair creation model, namely the \(3P_0\) model \[31, 32\], as well as using a field theoretical model with composite hadrons with quark/antiquark constituents \[33\]. In the present work, the masses of
the pseudoscalar \( \eta_c \equiv \eta_c(1S) \) and \( \eta'_c \equiv \eta_c(2S) \) as well as the vector charmonium states \( (J/\psi, \psi(2S) \equiv \psi(3686) \) and \( \psi(1D) \equiv \psi(3770) ) \) are computed from the medium changes of the scalar dilaton field within the chiral effective model, with additional contributions from the mixing of the pseudoscalar and the vector charmonium states in the presence of strong magnetic fields. The mixing is observed to lead to significant modifications to these charmonium masses. The decay widths of the charmonium states to the open charm mesons \( (D\bar{D}) \) are computed using a field theoretical model with composite hadrons with quark/antiquark constituents. 

The outline of the paper is as follows. In section II, we describe briefly the chiral effective model used to compute the masses of the (pseudoscalar and vector) charmonium states as well as the open charm meson masses in the magnetized matter. In the presence of a magnetic field, the mixings between the pseudoscalar and the vector mesons are taken into account through an effective hadronic interaction. In section III, we describe briefly the field theoretical model with composite hadrons with quark/antiquark constituents, used in the present work to compute the partial decay widths of the charmonium states to \( D\bar{D} \) in magnetized hadronic matter. In section IV, we discuss the results obtained in the present investigation of the charmonium masses as well as charmonium decay widths, accounting for the mixing of the pseudoscalar and vector charmonium states in the presence of strong magnetic fields. In section V, we summarize the findings of the present study.

II. CHARMONIUM MASSES IN STRONG MAGNETIC FIELDS

In this section, we investigate the mass modifications of the vector and pseudoscalar charmonium masses in the presence of strong magnetic fields. The masses of these heavy quark-antiquark bound states are calculated in the magnetized nuclear matter from the in-medium gluon condensates, which are simulated by a scalar dilaton field within a chiral effective model. The mixing of the pseudoscalar and vector charmonium states in the presence of an external magnetic field is studied using an effective Lagrangian interaction term and is observed to be the dominant contribution to the mass shifts for these charmonium states.

The Lagrangian density of the chiral effective model, in the presence of magnetic field, is
given as \[34\]
\[
\mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_W \mathcal{L}_{BW} + \mathcal{L}_{\text{vec}} + \mathcal{L}_0 + \mathcal{L}_{\text{scalebreak}} + \mathcal{L}_{\text{SB}} + \mathcal{L}_{\text{mag}}^B, 
\]
where, \(\mathcal{L}_{\text{kin}}\) corresponds to the kinetic energy terms of the baryons and the mesons, \(\mathcal{L}_{BW}\) contains the interactions of the baryons with the meson, \(W\) (scalar, pseudoscalar, vector, axialvector meson), \(\mathcal{L}_{\text{vec}}\) describes the dynamical mass generation of the vector mesons via couplings to the scalar fields and contains additionally quartic self-interactions of the vector fields, \(\mathcal{L}_0\) contains the meson-meson interaction terms, \(\mathcal{L}_{\text{scalebreak}}\) is a scale invariance breaking logarithmic potential given in terms of a scalar dilaton field, \(\chi\) and \(\mathcal{L}_{\text{SB}}\) describes the explicit chiral symmetry breaking. The term \(\mathcal{L}_{\text{mag}}^B\), describes the interaction of the baryons with the electromagnetic field, which includes a tensorial interaction \(\sim \bar{\psi}_i \sigma^{\mu\nu} F_{\mu\nu} \psi_i\), whose coefficients account for the anomalous magnetic moments of the baryons \[23\].

The trace of the energy momentum tensor of QCD is equated to that of the chiral effective model to obtain a relation between the scalar gluon condensate, \(\langle \frac{a_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle\) and the dilaton field, \(\chi\) of the the scale breaking term \(\mathcal{L}_{\text{scalebreak}}\). The in-medium masses of the charmonium states are hence obtained from the medium changes of the dilaton field. The dilaton field, \(\chi\) is solved along with the scalar fields (non-strange scalar-isoscalar, field \(\sigma\), non-strange scalar isovector, \(\delta\), the strange field \(\zeta\)) in the magnetized hadronic matter, from the coupled equations of motion of these fields. The values of the scalar fields are used to obtain the in-medium masses of the \(D\) and \(\bar{D}\) mesons as well as the charmonium states \[23, 24, 30\].

The mass shifts of the (pseudoscalar and vector) charmonium states, computed from the medium change of the dilaton field calculated within the chiral effective model are given as \[13, 14\]
\[
\Delta m_{P,V} = \frac{4}{81} (1 - d) \int dk^2 \langle |\frac{\partial \psi(\vec{k})}{\partial \vec{k}}|^2 \rangle \frac{k}{k^2/m_c + \epsilon} \left( \chi^4 - \chi_0^4 \right),
\]
where
\[
\langle |\frac{\partial \psi(\vec{k})}{\partial \vec{k}}|^2 \rangle = \frac{1}{4\pi} \int |\frac{\partial \psi(\vec{k})}{\partial \vec{k}}|^2 d\Omega,
\]
In the above, \(\psi(\vec{k})\) are the wave functions of the charmonium states, assumed to be harmonic oscillator wave functions \[13, 14, 18\] in the present study.

The mixings of the pseudoscalar \((P \equiv \eta_c(1S), \eta_c(2S))\) and vector \((V \equiv J/\psi, \psi(2S), \psi(1D))\)
charmonium states are taken into account through the interaction \[26\]

\[
\mathcal{L}_{PV\gamma} = \frac{g_{PV}}{m_{av}} e \tilde{F}_{\mu\nu} (\partial^\mu P^\nu),
\]

(4)

where \( m_{av} = (m_V + m_P)/2 \), \( m_P \) and \( m_V \) are the masses for the pseudoscalar and vector charmonium states. In the hadronic medium, these masses are calculated from the medium modification of the dilaton field, using equation (2) within the chiral effective model. In equation (4), the coupling parameter \( g_{PV} \) is fitted from the observed radiative decay width \( \Gamma(V \rightarrow P + \gamma) \), given as

\[
\Gamma(V \rightarrow P\gamma) = \frac{e^2 g_{PV}^2 p_{cm}^3}{12 \pi m_{av}^2}.
\]

(5)

In the above, \( p_{cm} \) is the magnitude of the center of mass momentum in the final state given as \( p_{cm} = (m_V^2 - m_P^2)/(2m_V) \). The masses of the pseudoscalar and the longitudinal component of the vector mesons including the mixing effects are given by

\[
m_{V||,P}^{(PV)} = \frac{1}{2} \left( M_+^2 + \frac{c_{PV}^2}{m_{av}^2} \pm \sqrt{M_+^4 + \frac{2c_{PV}^2 M_+^2}{m_{av}^2} + \frac{c_{PV}^4}{m_{av}^4}} \right),
\]

(6)

where \( M_+^2 = m_P^2 + m_V^2 \), \( M_-^2 = m_V^2 - m_P^2 \) and \( c_{PV} = g_{PV} eB \). The effective Lagrangian term given by equation (4) has been observed to lead to the mass modifications of the longitudinal \( J/\psi \) and \( \eta_c \) due to magnetic field, which agree remarkably with the study of these charmonium states using QCD sum rule approach incorporating the mixing effects \[25, 26\].

We shall investigate the decay widths of the vector charmonium states to \( D \bar{D} \) in the presence of magnetic fields, from the mass modifications of these charmonium states and the open charm mesons in the magnetized nuclear matter. The \( D \) and \( \bar{D} \) meson masses are calculated within the chiral effective model from their interactions with the nucleons and scalar mesons in the magnetized nuclear matter. The lowest Landau level contributions are retained for the charged \( D \) and \( \bar{D} \) mesons in the presence of the external magnetic field. These decay widths are calculated using a field theoretical model of composite hadrons with quark/antiquark constituents as described in the following section.
III. DECAY WIDTHS OF CHARMONIUM STATES TO $D\bar{D}$ WITHIN A MODEL FOR COMPOSITE HADRONS

We investigate the charmonium decay widths to the open charm ($D$ and $\bar{D}$) mesons in nuclear matter in the presence of strong magnetic fields using a field theoretical model with composite hadrons [35–37]. The model describes the hadrons comprising of quark and antiquark constituents. The constituent quark field operators of the hadron in motion are constructed from the constituent quark field operators of the hadron at rest, by a Lorentz boosting. Similar to the MIT bag model [38], where the quarks (antiquarks) occupy specific energy levels inside the hadron, it is assumed in the present model for the composite hadrons that the quark/antiquark constituents carry fractions of the mass (energy) of the hadron at rest (in motion) [35, 36].

With explicit constructions of the charmonium state and the open charm mesons, the decay width is calculated using the light quark antiquark pair creation term of the free Dirac Hamiltonian for constituent quark field [33]. The relevant part of the quark pair creation term is through the $d\bar{d}(u\bar{u})$ creation for decay of the charmonium state, $\Psi$, to the final state $D^+D^- (D^0\bar{D}^0)$. For $\Psi \rightarrow D(p)\bar{D}(p')$, this pair creation term is given as

$$H_{q\bar{q}}(x, t = 0) = Q_{q}^{(p)}(\vec{x})^{\dagger}(-i\alpha \cdot \nabla + \beta M_{q})\bar{q}_{q}^{(p')}(\vec{x})$$  \hspace{1cm} (7)

where, $M_{q}$ is the constituent mass of the light quark, $q = (u, d)$. The subscript $q$ of the field operators in equation (7) refers to the fact that the $\bar{q}$ and $q$ are the constituents of the $D$ and $\bar{D}$ mesons with momenta $p$ and $p'$ respectively in the final state of the decay of the charmonium state, $\Psi$.

The charmonium state, $\Psi$ with spin projection $m$, at rest is written as

$$|\Psi_{m}(0)\rangle = \int d\mathbf{k}c_{r}\text{i}^{i}(\mathbf{k})^{\dagger}u_{r}a_{m}(\Psi, \mathbf{k})\bar{c}_{s}\text{i}^{i}(-\mathbf{k})v_{s}|\text{vac}\rangle,$$ \hspace{1cm} (8)

where, $i$ is the color index of the charm quark/antiquark operators, $u_{r}$ and $v_{s}$ are the two component spinors for the quark and antiquark. The expressions for $a_{m}(\Psi, \mathbf{k})$ are given in terms of the wave functions (assumed to be harmonic oscillator type) for the charmonium states [33, 39].

The $D(D^+, D^0)$ and $\bar{D}(D^-, \bar{D}^0)$ states, with finite momenta are contructed in terms of the constituent quark field operators obtained from the quark field operators of these mesons at rest.
through a Lorentz boosting \[37\]. These states, assuming harmonic oscillator wave functions, are explicitly given as

\[
|D(p)\rangle = \frac{1}{\sqrt{6}} \left(\frac{R_D^2}{\pi}\right)^{3/4} \int dk \exp\left(-\frac{R_D^2 k^2}{2}\right)c_i^*(\mathbf{k} + \lambda_2 \mathbf{p})^\dagger u_s^\dagger \tilde{q}_s^i(-\mathbf{k} + \lambda_1 \mathbf{p})v_s dk, \quad (9)
\]

\[
|\bar{D}(p')\rangle = \frac{1}{\sqrt{6}} \left(\frac{R_D^2}{\pi}\right)^{3/4} \int dk \exp\left(-\frac{R_D^2 k^2}{2}\right)q_i^*(\mathbf{k} + \lambda_1 \mathbf{p'})^\dagger u_s^\dagger \tilde{c}_s^i(-\mathbf{k} + \lambda_2 \mathbf{p'})v_s dk, \quad (10)
\]

where, \( q = (d, u) \) for \( (D^+, \bar{D}^-) \) and \( (\bar{D}^0, D^0) \) respectively.

In equations (9) and (10), \( \lambda_1 \) and \( \lambda_2 \) are the fractions of the mass (energy) of the \( D(\bar{D}) \) meson at rest (in motion), carried by the constituent light \( (d, u) \) antiquark (quark) and the constituent heavy charm quark (antiquark), with \( \lambda_1 + \lambda_2 = 1 \). The values of \( \lambda_1 \) and \( \lambda_2 \) are calculated by assuming the binding energy of the hadron as shared by the quark (antiquark) to be inversely proportional to the quark (antiquark) mass \[36\]. The energies of the the light antiquark (quark) and heavy charm quark (antiquark), \( \omega_i = \lambda_i m_D(i = 1, 2) \), are assumed to be \[33, 36\]

\[
\omega_1 = M_q + \frac{\mu}{M_q} \times BE, \quad \omega_2 = M_c + \frac{\mu}{M_c} \times BE, \quad (11)
\]

where \( BE = (m_D - M_c - M_q) \) is the binding energy of \( D(\bar{D}) \) meson, with \( M_c \) and \( M_q \) as the masses of the constituent charm and light quark (antiquark), and, \( \mu \) is the reduced mass of the heavy-light quark-antiquark system (the \( D(\bar{D}) \) meson), defined by \( 1/\mu = 1/M_q + 1/M_c \).

The reason for making this assumption comes from the example of hydrogen atom, which is the bound state of the proton and the electron. As the mass of proton is much larger as compared to the mass of the electron, the binding energy contribution from the electron is \( \frac{\mu}{m_e} \times BE \simeq BE \) of hydrogen atom, and the contribution from the proton is \( \frac{\mu}{m_p} \times BE \), which is negligible as compared to the total binding energy of hydrogen atom, since \( m_p \gg m_e \). With this assumption, the binding energies of the heavy-light mesons, e.g., \( D \) and \( \bar{D} \) mesons \[33\] and, \( B \bar{B} \) mesons \[40\], mostly arise from the contribution from the light quark (antiquark).

To compute the decay width of the charmonium state, \( \Psi \) to \( D\bar{D} \), we evaluate the matrix element of the light quark-antiquark pair creation part of the Hamiltonian, between the initial charmonium state and the final state for the reaction \( \Psi \to D(p) + \bar{D}(p') \) as given by

\[
\langle D(p)||\bar{D}(p')\rangle \int \mathcal{H}_{d\bar{d}}(x, t = 0) dx |\Psi_m(\vec{0})\rangle = \delta(\mathbf{p} + \mathbf{p'})A^\Psi(||\mathbf{p}||)\rho_m, \quad (12)
\]
where,

\[
A^\Psi(|p|) = 6c_\Psi \exp\left[(a_\Psi b_\Psi^2 - R_D^2 \Lambda_2^2)p^2\right] \cdot \left(\frac{\pi}{a_\Psi}\right)^{3/2} \left[F_0^\Psi + F_1^\Psi \frac{3}{2a_\Psi} + F_2^\Psi \frac{15}{4a_\Psi^2}\right].
\]  \tag{13}

In the above, the parameters \(a_\Psi, b_\Psi\) and \(c_\Psi\) are given in terms of \(R_D\) and \(R_\Psi\), which are the strengths of the harmonic oscillator wave functions for the \(D(\bar{D})\) and the charmonium states, and \(F_i^\Psi(i = 0, 1, 2)\) are polynomials in \(|p|\), the magnitude of the momentum of the outgoing \(D(\bar{D})\) meson \[33\]. With \(\langle f|S|i\rangle = \delta_4(P_f - P_i)M_{fi}\), we have

\[
M_{fi} = 2\pi(-iA^\Psi(|p|))p_m.
\]  \tag{14}

The expression of the decay width of the charmonium state, \(\Psi\) to \(D\bar{D}\), as calculated in the present model for composite hadrons, without accounting for the mixing effects, is given by

\[
\Gamma(\Psi \rightarrow D(p)\bar{D}(-p)) = \frac{\gamma_\Psi}{2\pi} \int \delta(m_\Psi - p_D^2 - \frac{p_{\bar{D}}^2}{m_\Psi}|M_{f\bar{i}}|^2 \cdot 4\pi|p_D|^2 d|p_D|
\]

\[
= \frac{\gamma_\Psi}{2} \frac{8\pi^2}{3} |p|^{3} \overline{\frac{p_D^0(|p|)}{m_\Psi}} \frac{p_{\bar{D}}^0(|p|)}{A^\Psi(|p|)^2}
\]  \tag{15}

In the above, \(p_D^0(|p|) = (m_D^2 + p^2)^{1/2}\), and, \(|p|\) is the magnitude of the momentum of the outgoing \(D(\bar{D})\) meson, and the expression for \(A^\Psi(|p|)\) is given by equation (13). The parameter, \(\gamma_\Psi\), in the expression for the charmonium decay width, is a measure of the coupling strength for the creation of the light quark antiquark pair, to produce the \(D\bar{D}\) final state. This parameter is adjusted to reproduce the vacuum decay widths of \(\psi(3770)\) to \(D^+D^-\) and \(D^0\bar{D}^0\) \[33\]. The decay width of the charmonium state is observed to have the dependence on the magnitude of the 3-momentum of the produced \(D(\bar{D})\) meson, \(|p|\), as a polynomial part multiplied by an exponential term. The medium modification of the charmonium decay width is studied due to the mass modifications of the charmonium state, the \(D\) and \(\bar{D}\) mesons through \(|p|\), which is given as,

\[
|p| = \left(\frac{m_\psi^2}{4} - \frac{m_D^2 + m_{\bar{D}}^2}{2} + \frac{(m_D^2 - m_{\bar{D}}^2)^2}{4m_\psi^2}\right)^{1/2}.
\]  \tag{16}

The expression for the charmonium decay width given by equation (15) is for the case when the mixing of the pseudoscalar and the longitudinal component of the vector mesons is not taken into account, and in equation (16), the masses of the charmonium and open charm mesons are the effective masses in the hadronic matter in the presence of a magnetic field.
When we include the mixing effect, the expression for the decay width is given as

$$\Gamma_{PV}^{\psi}(\psi \rightarrow D(p) \bar{D}(-p)) = \frac{\gamma^2}{3} \left[ \frac{2}{3} |p|^3 p_D^0 (|p|) p_{\bar{D}}^0 (|\bar{p}|) A_\psi (|p|^2) \right] + \left( \frac{1}{3} |p|^3 p_D^0 (|p|) p_{\bar{D}}^0 (|\bar{p}|) A_\psi (|p|^2) \right) \left( |p| \rightarrow |p| (m_\psi = m_{PV}) \right). \quad (17)$$

In the above, the first term corresponds to the transverse polarizations for the charmonium state, \( \psi \), whose masses remain unaffected by the mixing of the pseudoscalar and vector charmonium states. The second term in (17) corresponds to the longitudinal component of the charmonium state whose mass is modified due to mixing with the pseudoscalar meson in the presence of the magnetic field, as given by equation (6).

IV. RESULTS AND DISCUSSIONS

In the present work, the masses and the partial decay widths of the charmonium states to \( D \bar{D} \) are investigated in the presence of strong magnetic fields. These are studied taking the effects of the mixing of the pseudoscalar and vector mesons into consideration, in the presence of magnetic fields. The created magnetic fields in the peripheral ultra-relativistic heavy ion collision experiments, e.g., at RHIC and LHC, are huge, and the matter resulting from the high energy collision is of (extremely) low density. In the present work, the effects of magnetic field on the charmonium states are studied at zero density as well as at the nuclear matter saturation density in the magnetized nuclear matter. In the nuclear medium, the masses of the charmonium states and the open charm mesons in the presence of a magnetic field are calculated using a chiral effective model. The in-medium charmonium masses are studied in the magnetized nuclear matter within the chiral effective model, due to the medium changes in the scalar dilaton field, \( \chi \), which mimics the gluon condensates of QCD [24]. The masses of the open charm mesons in the magnetized nuclear matter are modified due to their interactions with the nucleons as well as the scalar mesons, \( \sigma \), \( \zeta \), and \( \delta \) [23]. These scalar fields and the dilaton field \( \chi \) are solved from their coupled equations of motion. In the present work, the charmonium and open charm meson masses are obtained from the changes in the scalar fields in the isospin asymmetric nuclear matter (with isospin asymmetry parameter, \( \eta \) defined as \( \eta = (\rho_n - \rho_0)/(2\rho_B) \)), in the presence of magnetic fields using the chiral effective model [30].
FIG. 1: (Color online) The masses of the vector charmonium states \((J/\psi, \psi(3686), \psi(3770))\), plotted as functions of \(eB/m_c^2\). The effects of the mixing with the pseudoscalar mesons, \(\eta_c(1S)\) and \(\eta_c(2S)\) in the presence of strong magnetic fields on the masses of the charmonium states are shown for the case of \(\rho_B = 0\), as well as for \(\rho_B = \rho_0\) in symmetric \((\eta=0)\) nuclear matter.

The effects of the mixing of the pseudoscalar and longitudinal vector mesons are taken into account, which are observed to lead to significant contributions to the charmonium masses. These, as we shall see, have appreciable effects on the partial decay width of the \(\psi(3770)\) to \(D\bar{D}\) at high magnetic fields.

The mass modifications of the pseudoscalar \((\eta_c \equiv \eta_c(1S)\) and \(\eta_c' \equiv \eta_c(2S)\)) and the vector \((J/\psi, \psi(2S) \equiv \psi(3686), \psi(1D) \equiv \psi(3770))\) charmonium states, are investigated in the presence of magnetic fields. These masses are studied accounting for the the mixing of the pseudoscalar and vector mesons in the presence of magnetic fields, described by the effective interaction Lagrangian term given by equation (4). For the charmonium states at rest, the mixing effect leads to a drop (increase) in the mass of the pseudoscalar meson (the longitudinal component of the vector charmonium state) as given by equation (6), and these are observed to be the
dominant contributions to the charmonium masses.

The in-medium masses of the vector charmonium states \((J/\psi, \psi(2S), \psi(1D))\) have been studied from the medium change of the gluon condensate, using the leading order mass formula in Ref. [18] and using the chiral effective model in Refs. [13, 14, 24] in (magnetized) hadronic matter. The mass modifications of the charmonium states within the chiral effective model arise from the modifications of a scalar dilaton field which simulates the gluon condensates of QCD. These in-medium masses were studied assuming the harmonic oscillator wave functions for the charm-anticharm bound states. The values of the strength parameter, \(\beta\), of the charmonium wave functions were fitted to the rms radii of \(J/\psi\), \(\psi(2S)\) and \(\psi(1D)\) to be \((0.47 \text{ fm})^2\), \((0.96 \text{ fm})^2\) and \(1 \text{ fm}^2\) respectively [18], yielding their values as 513 MeV, 384 MeV and 368 MeV [13, 14]. In the present work, the mass modifications for the pseudoscalar states, \(\eta_c \equiv \eta_c(1S)\) and \(\eta'_c \equiv \eta_c(2S)\) in the nuclear matter are investigated within the chiral effective model. The values of the harmonic oscillator strengths for \(\eta_c\) and \(\eta'_c\) are obtained as 535 MeV and 394.6 MeV respectively, assuming these states to be lying in a straight line with the states \(J/\psi\) and

FIG. 2: (Color online) Same as fig. 1 for asymmetric nuclear matter with \(\eta=0.5\).
FIG. 3: (Color online) The masses of $D$ mesons ($D^+$ and $D^0$) are plotted as functions of $eB/m_\pi^2$. These masses are shown for the case of $\rho_B = 0$, as well as for $\rho_B = \rho_0$ in symmetric ($\eta=0$) asymmetric (with $\eta=0.5$) nuclear matter.

$\psi(2S)$ in the mass versus $\beta$ graph. In the absence of a magnetic field, the masses for the pseudoscalar mesons $\eta_c$ and $\eta_c'$ are modified to 2977.26 (2977.47) and 3548.55 (3551.46) MeV at $\rho_B = \rho_0$ for symmetric (asymmetric with $\eta=0.5$) nuclear matter from their vacuum masses of 2983.9 MeV and 3637.5 MeV respectively. The mass shift of $\eta_c$ in symmetric nuclear matter of around 6.6 MeV may be compared to the values of 5 MeV in Ref. [9], 5.69 MeV in Ref. [10] using the QCD sum rule approach, and of 3 MeV calculated from $\eta_c$-nucleon scattering length [41].

In figure 1, the effects of the magnetic field on the masses of the charmonium states are shown for the zero baryon density as well as for $\rho_B = \rho_0$ in symmetric ($\eta=0$) nuclear matter. In panel (a), the masses of the pseudoscalar charmonium state, $\eta_c$ and the longitudinal component of $J/\psi$ as modified due to the effect of mixing of these states, are shown as functions of $eB/m_\pi^2$. These masses are calculated by using the equation (6). The parameter $g_{PV} \equiv g_{\eta_cJ/\psi}$ is evaluated
to be 2.094 from the observed radiative decay width $\Gamma(J/\psi \to \eta_c \gamma)$ in vacuum of 92.9 keV, using equation (5). The dotted lines correspond to the case, when these mixing effects are not taken into account and the masses of the charmonium states for $\rho_B=0$ are unaffected by the magnetic field, and remain at the vacuum values of $m_{J/\psi}$ and $m_{\eta_c}$ as 3097 and 2983.9 MeV. The mixing effect is observed to lead to appreciable modifications to their masses at high magnetic fields, with the masses (in MeV) of $J/\psi$ and $\eta_c$ at $eB = 5(10) m^2_\pi$ obtained as 3106.2 (3128.5) and 2975 (2953.9) respectively. In panel (b), the masses of $J/\psi$ and $\eta_c$ are shown for $\rho_B = \rho_0$ in symmetric nuclear matter. In the absence of mixing effects, the masses (in MeV) are observed to vary marginally with magnetic field. The mass modifications are observed to be similar to the $\rho_B=0$ case. Panel (c) shows the masses of $\psi(2S)$ and $\eta_c' \equiv \eta_c(2S)$ for $\rho_B=0$, including the effects of these states in the presence of a magnetic field. The value for $g_{PV} \equiv g_{\eta_c'\psi(2S)}$ is obtained as 3.18 from the observed vacuum value of the decay width $\Gamma(\psi(2S) \to \eta_c' \gamma)$ of 0.2058 keV. For $\rho_B = \rho_0$ as shown in panel (d), the mixing effect is observed to be much more prominent as compared to the zero density case. This is due to the reason that the mass

FIG. 4: (Color online) Same as fig. 3 for $D^-$ and $\bar{D}^0$. 
FIG. 5: (Color online) Decay widths of $\psi(3770)$ to (I) $D^+D^-$, (II) $D^0\bar{D}^0$, and the total of these two channels (I+II), are plotted as functions of $eB/m_{\pi}^2$. These are shown for the case of $\rho_B = 0$, as well as, for $\rho_B = \rho_0$, in symmetric ($\eta = 0$) and asymmetric (with $\eta = 0.5$) nuclear matter. The panels (a), (c) and (e) show the results when the mixing with $\eta'_c$ is not taken into account, and (b), (d) and (f) correspond to the results including this effect.

modification due to mixing is larger, for a smaller difference in the masses of the pseudoscalar and vector mesons. This is due to the fact that the mass squared for the pseudoscalar meson (longitudinal component of the vector meson), obtained from equation (6), is given as

$$m_{P,V}^2 = m_{P,V}^2 \pm \frac{(g_{PV}eB)^2}{m_V^2 - m_P^2},$$

by retaining terms up to the second order in $eB$ and the leading order in $(m_V - m_P)/(m_V + m_P)$. As can be seen from equation (18), the shifts in the masses of the pseudoscalar and vector mesons are inversely proportional to $(m_V^2 - m_P^2)$, and the mass modifications of $\eta'_c$ and $\psi(2S)$ from the mixing effect are thus larger at $\rho_B = \rho_0$ as compared to zero density, due to the smaller mass difference between the pseudoscalar and vector mesons. The vacuum masses of $\eta'_c$ and $\psi(2S)$
of 3637.5 and 3686 MeV, are modified to 3548.6 and 3575.7 MeV at \( \rho_B = \rho_0 \) in symmetric nuclear matter at zero magnetic field. The smaller difference in the masses of \( \psi(2S) \) and \( \eta_c' \) is observed as larger effect from the mixing on these masses at \( \rho_0 \) as compared to at zero baryon density, as can be seen from panels (c) and (d). The contributions from the mixing effects to the masses of \( \eta_c' \) and \( \psi(1D) \equiv \psi(3770) \) are plotted for zero and nuclear matter saturation density in symmetric nuclear matter in panels (e) and (f) respectively. The value for \( g_{PV} \equiv g_{\eta_c' \psi(1D)} \) is obtained as 7.66 from the observed vacuum value of the decay width \( \Gamma(\psi(2S) \rightarrow \eta_c' \gamma) \) of 24.48 keV. Due to the smaller mass difference in the masses of the \( \eta_c' \) and \( \psi(3770) \) in the magnetized nuclear matter, the mixing effects are observed to lead to larger shifts in the masses of these mesons at \( \rho_0 \) as compared to zero density. This is similar to the larger mass shifts of \( \eta_c' \) and \( \psi(3686) \) due to the mixing effects, at \( \rho_B = \rho_0 \) as compared to zero density. The mass of \( \psi(3770) \) has dominant contribution at high magnetic fields due to the mixing effects, leading to its mass to be around 3724.7 and 3808.8 MeV for \( eB \) equal to 6 and 10 \( m_\pi^2 \). These may be compared to the zero magnetic field of 3635.7 MeV at \( \rho_B = \rho_0 \) and \( \eta=0 \). This large shift in the mass of \( \psi(2770) \) leads to significant modification of the partial decay width of \( \psi(3770) \) to \( D\bar{D} \) at high magnetic fields, as we shall discuss later.

The charmonium masses are plotted for asymmetric nuclear matter with \( \eta=0.5 \) in figure 2. The effects of the isospin asymmetry on the charmonium masses are observed to be small at the nuclear matter saturation density, as calculated within the chiral effective model. This, in turn, leads to small modifications to the charmonium masses in the isospin asymmetric matter (with \( \eta=0.5 \)) as compared to the symmetric nuclear matter, in the presence of mixing effects. For example, the masses of \( \psi(3686) \) and \( \psi(3770) \) at the nuclear matter saturation density, and for \( eB = 10 m_\pi^2 \), are observed to be modified from 3650.57 and 3808.45 for symmetric nuclear matter to the values 3662.2 and 3821.4 for asymmetric nuclear matter with \( \eta=0.5 \).

In figures 3 and 4, the masses of the \( D(D^+, D^0) \) mesons and \( (D^-, D^0) \) meson masses, are plotted as functions of \( eB/m_\pi^2 \), for \( \rho_B = 0 \) as well as for \( \rho_B = \rho_0 \) with \( \eta=0 \) and \( \eta = 0.5 \). In the nuclear matter in the presence of a magnetic field, these masses are calculated within the chiral effective model, from their interactions with the nucleons and scalar mesons. The charged open charm mesons \( (D^+ \text{ and } D^-) \) have positive shifts in their masses due to contributions from the Landau level. This leads to an increase in their masses with increase in magnetic field. On the
other hand, the neutral $D^0$ and $\bar{D}^0$ mesons are observed to have very small changes in their masses with the magnetic field.

The medium modifications of the charmonium decay widths are computed from the changes in the masses of the charmonium state, $\Psi$, and the open charm mesons, due to the effects of magnetic fields, calculated within a chiral effective model \[23, 24, 30\]. Furthermore, the mass of the longitudinal component of the vector charmonium state has a positive shift due to the pseudoscalar vector meson mixing in the presence of magnetic field. The lowest charmonium state decaying to $D\bar{D}$ is $\psi(3770)$ at $\rho_B = \rho_0$ as well as at zero baryon density. As has already been mentioned, the charmonium decay width, $\Gamma(\psi(3770) \rightarrow D\bar{D})$ is calculated using the light quark pair creation term of the free Dirac Hamiltonian expressed in terms of the constituent quark operators, using explicit constructions for the charmonium and the open charm ($D,\bar{D}$) mesons. The matrix element for the calculation of the decay width is multiplied with a factor $\gamma_\psi$, which gives the strength of the light quark pair creation leading to the decay of the charmonium state to $D\bar{D}$ in the magnetized hadronic medium \[33\]. The value of $\gamma_\psi$ to be 1.35, is chosen so as to reproduce the decay widths of $\psi(3770) \rightarrow D^+D^-$ and $\psi(3770) \rightarrow D^0\bar{D}^0$ in vacuum, to be around 12 MeV and 16 MeV respectively \[33\]. The constituent quark masses for the light quarks ($u$ and $d$) are taken to be 330 MeV and for the charm quark, the value is taken to be $M_c = 1600$ MeV \[33\].

The partial decay widths of $\psi(3770)$ to $D\bar{D}$, is calculated using a field theoretic model for composite hadrons as described in section III. These are plotted as functions of $eB/m_\pi^2$ in figure 3 for $\rho_B = \rho_0$ as well as zero density. In panel (a), for $\rho_B = 0$, the partial decay widths of $\psi(3770)$ to (I) $D^+D^-$, (II) $D^0\bar{D}^0$, and the total of these two channels (I+II), without taking into account the pseudoscalar–vector meson mixing effects. The decay to the charged $D\bar{D}$ is observed to drop with increase in the magnetic field up to $eB$ equal to $3m_\pi^2$ and vanishes at larger values of the magnetic field. This is due to the reason that the charged $D$ and $\bar{D}$ mesons have higher values for their masses in the presence of a magnetic field, due to positive contributions to their masses from the lowest Landau level. On the other hand, the decay width to the neutral $D\bar{D}$ pair is observed to be unaffected by the magnetic field. Panel (b) shows the partial decay widths when the mass modification of the longitudinal component of $\psi(3770)$ is taken into account due to mixing with $\eta'_c$ in the presence of a magnetic field. This leads to an
appreciable increase in the decay to neutral $D^0 \bar{D}^0$ with rise in the magnetic field, whereas the
decay to the $D^+D^-$ is observed to have a moderate dependence on the magnetic field. Panels
c and d show the results for density $\rho_B = \rho_0$ in symmetric nuclear matter ($\eta=0$), without
and with the mixing effects. In the presence of the mixing effects, the contribution to the
partial decay width to the neutral open charm meson pair is observed to be quite significant.
Panels (e) and (f) show the results for the decay widths for asymmetric nuclear matter with
$\eta=0.5$ for $\rho_B = \rho_0$, which show the decay to the neutral $D\bar{D}$ to be the dominant contribution
to the decay of $\psi(3770) \to D\bar{D}$.

V. SUMMARY

In the present work, the charmonium masses as well as the decay widths of the vector
charmonium states to $D\bar{D}$ are investigated in the presence of strong magnetic fields. The study
can be of relevance to the observables in ultra-relativistic heavy ion collision experiments. The
estimated magnetic fields created in the peripheral heavy ion collisions, e.g., at RHIC and at
LHC, are huge, where the formed matter is (extremely) low in density. In the present work, we
study the effects of the magnetic fields on the charmonium states at zero as well as at nuclear
matter saturation density.

The mixing of the pseudoscalar mesons and the vector mesons in the presence of strong
magnetic fields are studied using an effective Lagrangian interaction and are observed to have
dominant contributions to the masses of the charmonium states. In the nuclear medium,
the masses of the charmonium as well as the open charm mesons are computed using a chiral
effective model and the charmonium masses, are additionally modified due to the mixing effects.
The mixing of the pseudoscalar mesons, $\eta_c$ and $\eta'_c$ with the vector charmonium states, might
show as peaks in the dilepton spectra due to ‘anomalous’ decay modes, e.g., $\eta_c, \eta'_c \to l^+l^-$.

The partial decay width of charmonium state to $D\bar{D}$ is computed using a field theoretical
model for composite hadrons with quark (antiquark) constituents. The matrix element for the
calculation of the decay width is calculated from the free Dirac Hamiltonian of the constituent
quarks, using the explicit constructions of the charmonium state, the $D$ and the $\bar{D}$ mesons. At
$\rho_B=0$ as well as at nuclear matter saturation density, the lowest charmonium state which can
decay to $D\bar{D}$ is $\psi(3770)$. Due to mixing with $\eta'_c$ in the presence of strong magnetic fields, the
mass of longitudinal $\psi(3770)$ has appreciable contribution, which is observed to have significant modification to the partial decay width, $\Gamma(\psi(3770) \rightarrow D\bar{D})$. The decay of $\psi(3770)$ to $D^+D^-$ are observed to be quite suppressed as compared to decay to $D^0\bar{D}^0$, in the presence of strong magnetic fields, due to the higher masses of the charged mesons ($D^+, D^-$) which have a positive shift due to the Landau levels. The present study of the charmonium states in strong magnetic fields should have observable consequences in the dilepton spectra, and on the production of the charmonium states as well as open charm mesons in ultra relativistic heavy ion collision experiments, e.g., at RHIC and LHC.

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[1] A. Hosaka, T. Hyodo, K. Sudoh, Y. Yamaguchi, S. Yasui, Prog. Part. Nucl. Phys. 96, 88 (2017).
[2] E.Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. D 17, 3090 (1978).
[3] E.Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. D 21, 203 (1980).
[4] L. Kluberg and H. Satz, in Relativistic Heavy Ion Physics, edited by R. Stock, Landolt-Börnstein - Group I Elementary Particles, Nuclei and Atoms Vol. 23 (Springer, Berlin, Heidelberg, 2010).
[5] A. Mocsy, P. Petreczky and M. Strickland, Int. Jour. Mod. Phys. A 28, 1340012 (2013).
[6] S.F. Radford and W W. Repko, Phys.Rev D 75, 074031 (2007).
[7] Arata Hayashigaki, Phys. Lett. B 487, 96 (2000); T. Hilger, R. Thomas and B. Kämpfer, Phys. Rev. C bf 79, 025202 (2009); T. Hilger, B. Kämpfer and S. Leupold, Phys. Rev. C 84, 045202 (2011); S. Zschocke, T. Hilger and B. Kämpfer, Eur. Phys. J. A 47 151 (2011)
[8] Sugsk Kim, Su Hounge Lee, Nucl. Phys. A 679, 517 (2001).
[9] F. Klingl, S. Kim, S. H. Lee, P. Morath and W. Weise, Phys. Rev. Lett. 82, 3396 (1999).
[10] Arvind Kumar and Amruta Mishra, Phys. Rev. C 95, 065206 (2010)).
[11] G. Krein, A.W. Thomas, K. Tsushima, Prog. Part. Nucl. Phys. 100, 161 (2018).
[12] R. Molina, D. Gamermann, E. Oset, and L. Tolos, Eur. Phys. J A 42, 31 (2009); L. Tolos, R. Molina, D. Gamermann, and E. Oset, Nucl. Phys. A 827 249c (2009).
[13] Arvind Kumar and Amruta Mishra, Phys. Rev. C 81, 065204 (2010).
[14] Arvind Kumar and Amruta Mishra, Eur. Phys. A 47, 164 (2011).
[15] M.E. Peskin, Nucl. Phys. B156, 365 (1979).
[16] G. Bhanot and M.E. Peskin, Nucl. Phys. B156, 391 (1979).
[17] M.B.Voloshin, Nucl. Phys. B 154, 365 (1979).
[18] Su Houng Lee and Che Ming Ko, Phys. Rev. C 67, 038202 (2003).
[19] K. Tuchin, Adv. High Energy Physics 2013, 490495 (2013).
[20] P. Gubler, K. Hattori, S. H. Lee, M. Oka, S. Ozaki and K. Suzuki, Phys. Rev. D 93, 054026 (2016).
[21] C. S. Machado, F. S. Navarra, E. G. de Oliveira and J. Noronha, Phys. Rev. D 88, 034009 (2013).
[22] C. S. Machado, R.D. Matheus, S.I. Finazzo and J. Noronha, Phys. Rev. D 89, 074027 (2014).
[23] Sushruth Reddy P, Amal Jahan CS, Nikhil Dhale, Amruta Mishra, J. Schaffner-Bielich, Phys. Rev. C 97, 065208 (2018).
[24] Amal Jahan CS, Nikhil Dhale, Sushruth Reddy P, Shivam Kesarwani, Amruta Mishra, Phys. Rev. C 98, 065202 (2018).
[25] S. Cho, K. Hattori, S. H. Lee, K. Morita and S. Ozaki, Phys. Rev. Lett. 113, 122301 (2014).
[26] S. Cho, K. Hattori, S. H. Lee, K. Morita and S. Ozaki, Phys. Rev. D 91, 045025 (2015).
[27] J. Alford and M. Strickland, Phys. Rev. D 88, 105017 (2013).
[28] K. Suzuki and S. H. Lee, Phys. Rev. C 96, 035203 (2017).
[29] B. Friman, S. H. Lee and T. Song, Phys. Lett. B 548, 153 (2002).
[30] Amruta Mishra, Amal Jahan C.S., Shivam Kesarwani, Haresh Raval, Shashank Kumar, Jitendra Meena, Eur. Phys. J. A 55, 99 (2019).
[31] E.S.Ackleh, T. Barnes and E. S. Swanson, Phys. Rev. D 54, 6811, 1996.
[32] T. Barnes, F. E. Close, P. R. Page and E. S. Swanson, Phys. Rev. D 55, 4157 (1997).
[33] Amruta Mishra, S. P. Misra and W. Greiner, Int. J. Mod. Phys. E 24, 155053 (2015).
[34] P. Papazoglou, D. Zschiesche, S. Schramm, J. Schaffner-Bielich, H. Stöcker, and W. Greiner, Phys. Rev. C 59, 411 (1999).
[35] S. P. Misra, Phys. Rev. D 18, 1661 (1978).
[36] S. P. Misra, Phys. Rev. D 18, 1673 (1978).
[37] S. P. Misra and L. Maharana, Phys. Rev. D 18, 4103 (1978).
[38] A. Chodos, R. L. Jaffe, K. Johnson and C. B. Thorn, Phys. Rev. D 10, 2599 (1974).

[39] S.P. Misra, K. Biswal and B. K. Parida, Phys. Rev. D 21, 2029 (1980).

[40] Amruta Mishra and S. P. Misra, Ph. Rev. C 95, 065206 (2017).

[41] A. B. Kaidalov, P.E. Volkovitsky, Phys. Rev. Lett. 69, 3155 (1992).