One-Loop Electron Mass and Three-Loop Dirac Neutrino Masses

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Abstract

In the context of a left-right extension of the standard model of quarks and leptons with the addition of a gauged $U(1)_D$ dark symmetry, it is shown how the electron may obtain a radiative mass in one loop and two Dirac neutrinos obtain masses in three loops.
Introduction: Neutrino masses are very small. In the context of the minimal standard model (SM) of particle interactions, it has a natural explanation because they must come from a dimension-five operator [1], i.e.

\[ L_5 = \frac{f_{ij}}{2\Lambda} (\nu_i \phi^0 - l_i \phi^+)(\nu_j \phi^0 - l_j \phi^+) \]

(1)

where \((\nu, l)_{1,2,3}\) are the left-handed lepton doublets of the three families, and \((\phi^+, \phi^0) = \Phi\) is the one Higgs scalar doublet. Neutrino masses are thus Majorana and inversely proportional to the large scale \(\Lambda\), hence the name “seesaw”. There are exactly three ways [2] to realize \(L_5\) at tree level, establishing the nomenclature Type I,II,III seesaw, as well as three ways to realize it radiatively in one loop [2]. One particular application is to let dark matter [3] be the origin of radiative neutrino mass, as in the “scotogenic” model [4].

If neutrinos are Dirac particles, thereby requiring the existence of \(\nu_R\) and the maintenance of a conserved lepton number \(L\), the SM is not a natural accommodating framework. First, under \(SU(3)_C \times SU(2)_L \times U(1)_Y\), \(\nu_R\) is trivial so its existence is not very well justified. Second, the \(L\) symmetry must be imposed to forbid the otherwise allowed \(\nu_R\) Majorana mass. Third, the Yukawa coupling linking \(\nu_L\) to \(\nu_R\) through \(\phi^0\) must be chosen to be of order \(10^{-12}\) to account for the data on neutrino masses. To alleviate this last shortcoming, a symmetry is often employed to forbid the offending Yukawa term, say \(Z_2\) under which \(\nu_R\) is odd. At the same time, \(L\) is still assumed to be exact, whereas \(Z_2\) will be broken softly by the addition of other fermions and scalars. This procedure [5] has been studied in a variety of models.

Consider now the left-right extension of the SM. The existence of \(\nu_R\) is required as part of an \(SU(2)_R\) doublet. Neutrinos obtain Dirac masses in the same way as the other fermions through a scalar bidoublet. If \(SU(2)_{L,R}\) doublets \(\Phi_{L,R}\) are also added to break \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) to \(U(1)_Q\), then \(L\) remains a global symmetry. The first and second shortcomings of the SM for Dirac neutrinos no longer apply, but the third remains. A
well-known approach \cite{6} is to keep only $\Phi_{L,R}$ without the scalar bidoublet. Hence all fermion masses are zero at this point. By adding heavy singlet fermions, the known quarks and leptons may now obtain seesaw masses \cite{7}. Neutrinos may also be chosen to have radiative Dirac masses \cite{8,9,10,11}. Recently, it has been shown \cite{12} that all fermion masses could be radiative if the $SU(2)_R$ breaking scale is high enough. In this work, a new and different scenario is envisioned: the emergence of three-loop neutrino masses in the presence of a one-loop electron mass, with the help of a dark gauged $U(1)_D$ symmetry.

**Description of Model**: The particle content of the proposed model is listed in Table 1. The

| fermion/scalar | $SU(3)_C$ | $SU(2)_L$ | $SU(2)_R$ | $U(1)_{B-L}$ | $U(1)_D$ |
|----------------|----------|----------|----------|-------------|----------|
| $(\nu_e, e)_L$ | 1        | 2        | 1        | $-1/2$      | 0        |
| $(\nu_e, e)_R$ | 1        | 1        | 2        | $-1/2$      | 0        |
| $\Phi_L = (\phi_L^+, \phi_L^0)$ | 1 | 2 | 1 | $1/2$ | 0 |
| $\Phi_R = (\phi_R^+, \phi_R^0)$ | 1 | 1 | 2 | $1/2$ | 0 |
| $(N, E)_{L,R}$ | 1 | 2 | 1 | $-1/2$ | 1 |
| $(N', E')_{L,R}$ | 1 | 1 | 2 | $-1/2$ | 1 |
| $S_{L,R}$ | 1 | 1 | 1 | 0 | 1 |
| $\chi^-$ | 1 | 1 | 1 | $-1$ | 1 |
| $\eta^-$ | 1 | 1 | 1 | $-1$ | 2 |
| $\sigma$ | 1 | 1 | 1 | 0 | 2 |

Table 1: Fermion and scalar content of left-right model with $U(1)_D$.

one-loop diagram for the electron mass is given in Fig. 1, with the understanding that there

![Figure 1: Scotogenic electron mass.](image-url)
are two additional insertions linking $N_L$ to $N'_R$ as shown in Fig. 2.

![Diagram](image)

**Figure 2:** Additional insertions to electron mass.

Assuming that $N'$ is the heaviest particle in the loop, the electron mass is then approximately given by

$$m_e \sim \frac{f^4 v_L v_R}{16\pi^2 m_{N'}}.$$  \hspace{1cm} (2)

where $f$ is a typical Yukawa coupling and $v_{L,R}/\sqrt{2}$ are the vacuum expectation values of $\phi_{L,R}^0$. Note that lepton number $L$ may be defined to be transmitted by $\chi^-$ across the loop. Note also that neutrinos remain massless in one loop because the corresponding transmitter $\chi^0$ with $D = 1$ is absent. With the help of a second set of $(N, E)_{L,R}, S_{L,R}$, and $(N', E')_{L,R}$ fermions, as well as a charged scalar $\eta^-$ with $D = 2$, two neutrinos will acquire three-loop Dirac masses as shown in Fig. 3, again with two additional insertions.

![Diagram](image)

**Figure 3:** Scotogenic Dirac neutrino mass.

A very rough estimate of the neutrino mass is then

$$m_\nu \sim \frac{\lambda f^6 v_L v_R}{(16\pi^2)^3 m_{N'}}.$$  \hspace{1cm} (3)
where $\lambda$ is the quartic coupling of $(\chi^+\chi^-)(\eta^+\eta^-)$. Hence $m_\nu/m_e \sim \lambda f^2/(16\pi^2)^2$ which is naturally of order $10^{-7}$ or so, in agreement with data. Since there are two copies each of $(N,E), S_{L,R}$, and $(N',E')$, two neutrinos will get mass. The third neutrino gets a negligible mass from $W_{L,R}$ exchange, in analogy to the $2-W$ exchange \cite{13} in the SM for Majorana neutrinos. Other fermions may acquire seesaw Dirac masses \cite{7} or radiatively \cite{11, 12}.

Whereas Fig. 3 represents the realization of the dimension-five operator

$$L_5^\nu = \frac{f_\nu}{2\Lambda} (\nu_{iL} \phi^0_L - l_{iL} \phi^+_L) (\bar{\nu}_{jR} \phi^0_R + \bar{l}_{jR} \phi^-_R),$$

(4)

Fig. 1 corresponds to

$$L_5^e = \frac{f_e}{2\Lambda} (\nu_{iL} \phi^-_L + e_{iL} \phi^0_L) (\bar{\nu}_{jR} \phi^0_R - \bar{e}_{jR} \phi^0_R).$$

(5)

These are the left-right analogs of Eq. (1). In previous applications, the electron mass often comes from a tree-level realization of $L_5^e$, and only $L_5^\nu$ is radiative in origin. Recently \cite{11, 12}, both operators are derived in one loop, in which case there is no real understanding for why the Dirac neutrino masses are so much smaller than the electron mass. Here the first example of one-loop electron mass and three-loop Dirac neutrino masses is presented. Note also that Fig. 3 is a close analog to that \cite{14} for a Majorana neutrino, known already many years ago.

**Higgs and Gauge Sectors**: The Higgs sector is identical to that of Ref. \cite{12}, consisting of scalars $\Phi_{L,R}$ and $\sigma$. Although $\sigma$ now has $D = 2$ instead of $D = 3$, it does not affect the resulting Higgs potential, i.e.

$$V = -\mu^2_L \Phi^\dagger_L \Phi_L - \mu^2_R \Phi^\dagger_R \Phi_R - \mu^2_\sigma \sigma^* \sigma + \frac{1}{2} \lambda_L (\Phi^\dagger_L \Phi_L)^2 + \frac{1}{2} \lambda_R (\Phi^\dagger_R \Phi_R)^2 + \frac{1}{2} \lambda_\sigma (\sigma^* \sigma)^2 + \lambda_{LR} (\Phi^\dagger_L \Phi_L)(\Phi^\dagger_R \Phi_R) + \lambda_{L\sigma} (\Phi^\dagger_L \Phi_L)(\sigma^* \sigma) + \lambda_{R\sigma} (\Phi^\dagger_R \Phi_R)(\sigma^* \sigma).$$

(6)

After the spontaneous breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_D$, the only physical scalars left are the real parts of $\phi^0_{L,R}$ and $\sigma$. Let

$$\Phi_L = \begin{pmatrix} 0 \\ (v_L + h_L)/\sqrt{2} \end{pmatrix}, \quad \Phi_R = \begin{pmatrix} 0 \\ (v_R + h_R)/\sqrt{2} \end{pmatrix}, \quad \sigma = \frac{1}{\sqrt{2}} (v_D + h_D).$$

(7)
then the $3 \times 3$ mass-squared matrix spanning $(h_L, h_R, h_D)$ is

\[
\mathcal{M}_h^2 = \begin{pmatrix}
\lambda_L v_L^2 & \lambda_{LR} v_L v_R & \lambda_{L\sigma} v_L v_D \\
\lambda_{LR} v_L v_R & \lambda_R v_R^2 & \lambda_{R\sigma} v_R v_D \\
\lambda_{L\sigma} v_L v_D & \lambda_{R\sigma} v_R v_D & \lambda_\sigma v_D^2
\end{pmatrix}.
\] (8)

In the gauge sector, the $Z_D$ boson gets a mass equal to $2g_D v_D$. The charged $W_{L,R}^\pm$ masses are $g_L v_L$ and $g_R v_R$. The $Z, Z'$ mass-squared matrix is

\[
\mathcal{M}_{Z,Z'}^2 = e^2 \left( \begin{array}{cc}
v_L^2 / x (1-x) & v_L^2 / (1-x) \sqrt{1-2x} \\
v_L^2 / (1-x) \sqrt{1-2x} & (1-x) v_R^2 / x (1-2x) + x v_L^2 / (1-x) (1-2x) \end{array} \right),
\] (9)

where $e^{-2} = g_L^{-2} + g_R^{-2} + g_B^{-2}$, and $g_L = g_R$ with $x = \sin^2 \theta_W$. The $Z-Z'$ mixing is then about $x \sqrt{1-2x} v_L^2 / (1-x)^2 v_R^2$, which is constrained by the experimental bound of about $10^{-4}$ [15].

The dark $U(1)_D$ gauge symmetry is broken by two units through the complex singlet scalar $\sigma$, which couples to $S_L S_L$ and $S_R S_R$. This means that gauged $U(1)_D$ breaks to $Z_2$ dark parity $(-1)^D$, which is exactly conserved, allowing thus the lighter of the four Majorana fermions from the $(S_L, S_R)$ sector to become dark matter.

**Dark Sector**: The dark fermions $(N, E)$ and $(N', E')$ are assumed heavier than $S$. Since $N$ and $N'$ mix with $S$ through $\phi_{0_{L,R}}$, they decay to $S h_{L,R}$ and $E, E'$ decay to $S W^-_{L,R}$. The dark scalar $\chi^-$ decays to $eN$ or $\nu E$, whereas $\eta^-$ decays to $E_1 N_2$ or $N_1 E_2$. The $2 \times 2$ mass matrix spanning $(\bar{S}_L, S_R)$ of the lighter of the two sets of $S_{L,R}$ is of the form

\[
\mathcal{M}_S = \begin{pmatrix}
f_L v_D & m_S \\
m_S & f_R v_D
\end{pmatrix}.
\] (10)

Assuming $f_L = f_R = f$ and neglecting the $N - S$ and $N' - S$ mixings for now, this yields two Majorana fermions of masses $|m_S \pm f v_D|$, each coupling to $h_D$. Let $S_1$ be the lighter, i.e. the dark-matter candidate, with mass $m_{S_1} = |m_S - f v_D|$. Let $m_{h_D} < m_{S_1}$, then $S_1$ will annihilate to $h_D$, as shown in Fig. 4. The first diagram is also accompanied by its $u-$channel counterpart, which has the same amplitude in the limit that $S_1$ is at rest. Let $x = m_{h_D} / m_{S_1}$ and using $m_{h_D}^2 = \lambda_\sigma v_D^2$, this cross section at rest multiplied by relative velocity is

\[
\sigma_{\text{ann}} \times v_{\text{rel}} = \frac{\sqrt{1-x^2}}{128 \pi} \left| \frac{2f^2}{m_{S_1} (1+x^2)} - \frac{3fx^2}{v_D (4-x^2)} \right|^2.
\] (11)
As an example, let $m_{S_1} = v_D = 1 \text{ TeV}$, $m_{h_D} = 500 \text{ GeV}$, then the canonical value $3 \times 10^{-26}$ cm$^3$/s for the correct relic abundance of dark matter in the Universe is obtained if $f = 0.89$. This in turn implies $m_S = 1.89 \text{ TeV}$.

As for direct detection, $S_1$ couples to the SM Higgs $h_L$ through mixing with $h_D$ which is $\lambda_{L\sigma} v_L v_D/m_{h_D}^2$. For $m_{S_1} = 1 \text{ TeV}$, the spin-independent cross section of dark matter scattering off a xenon nucleus is bounded by $10^{-45}$ cm$^2$. This puts a limit of $4 \times 10^{-4}$ on $\lambda_{L\sigma}$ for $v_D = 1 \text{ TeV}$ and $m_{h_D} = 500 \text{ GeV}$. With this value of $\lambda_{L\sigma}$, the decay $h_D \rightarrow h_L h_L$ has a lifetime of $2.4 \times 10^{-19}$ s, certainly short enough to allow it to be in thermal equilibrium with the SM particles. Since $S_1$ mixes with $N$ through $\phi_L^0$ and $N'$ through $\phi_R^0$, it couples to the vector gauge bosons $Z$ and $Z'$, which interact with quarks. However, $S_1$ is a Majorana fermion, so this cross section is zero in the limit that it is at rest.

**Conclusion**: In the conventional left-right gauge extension of the standard model with just one $SU(2)_L$ scalar doublet and one $SU(2)_R$ scalar doublet, it is shown how a radiative electron mass arises in one loop, as well as related Dirac neutrino masses in three loops. The particles in the loop belong to a dark sector with $U(1)_D$ gauge symmetry. The spontaneous breaking of $U(1)_D$ leads to a viable Majorana fermion dark-matter candidate.

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