Search for the $\Delta^{++}$ component in the $^{12}\text{C}$ ground state using the $^{12}\text{C}(\gamma, \pi^+p)$ reaction

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Abstract

The differential cross section for the $^{12}\text{C}(\gamma, \pi^+p)$ reaction has been measured in the $\Delta^{++}(1232)$ resonance region at high recoil momenta of the residual nuclear system. The data are analyzed under the assumption that the formation of the $\pi^+p$ pairs may be interpreted as a $\gamma\Delta^{++} \to \pi^+p$ process which takes place on a $\Delta^{++}$ preexisting in the target nucleus. Estimates of the $\Delta^{++}$ momentum distribution $\rho_{\Delta^{++}}(\vec{p}) = 0.17 \text{ fm}^3$ for a mean momentum $\vec{p} = 300 \pm 49 \text{ MeV}/c$ as well as the number of $\Delta$ isobars per nucleon $N_{\Delta} = 0.017$ were obtained for $^{12}\text{C}$.

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1 Introduction

The interaction of energetic projectiles with nuclei at high momentum transfer implies probing the nuclear structure at medium and at short internucleon distances. In short range nucleon-nucleon dynamics a wide variety of developments concerns the internal nucleon degrees of freedom. The nontrivial substructure of a nucleon manifests itself in the existence of the internally excited states, i.e. baryon resonances or isobars. One possibility to describe these subnucleonic effects in modern nuclear models is the addition of exotic components, the so-called isobar configurations, to the conventional nuclear wave functions.

Comparing different approaches (for comprehensive reviews see e.g. [1, 2, 3]) it was found that the strongest isobar admixture in nuclei stems from the $\Delta(1232)$ resonance. The exchange of $\pi$ and $\rho$ mesons may result in virtual $N\Delta$ and $\Delta\Delta$ states. Most theoretical and experimental investigations of the $\Delta$ isobar configuration have been done for the few-body systems $^2\text{H}$ and $^3\text{He}$, and to a smaller degree for heavier nuclei. At the same time, the amount of the virtual $\Delta$ isobars in heavier nuclei with their higher density is expected to be more essential than in lighter nuclei. Here we would like to mention the explicit calculation [4] which resulted in a 3.2% probability for $\Delta$ components in the $^{12}\text{C}$ ground state.

As for the experimental search for the isobar admixture in nuclei with $A \geq 4$, the observation of $\Delta$ knock-out from $^9\text{Be}$ [5] and the recent double charge exchange experiment ($\pi^+, \pi^-$) on $^3\text{H}$, $^4\text{He}$, $^6\text{Li}$, $^7\text{Li}$, $^{12,13}\text{C}$, $^{90}\text{Zr}$, and $^{208}\text{Pb}$ [6] are of the greatest interest. The first experiment [5] was carried out at the proton beam of the synchrocyclotron at the Petersburg

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Nuclear Physics Institute where the inclusive differential cross sections for the $^9\text{Be}(p,^8\text{He})$ reaction at the proton energy of 1 GeV have been measured. An enhancement of the $^8\text{He}$ yield was observed for the spectator momentum less than 300 MeV/c, which was connected with the $\Delta^{++}$ knock-out from $^9\text{Be}$. The second experiment [6] was performed using the 500 MeV pion beam at the Clinton P. Anderson Meson Physics Facility. The cross sections for the $(\pi^+,\pi^+p)$ reaction have been measured in quasi-free kinematics. The observed enhancement of the $(\pi^+,\pi^-p)$ cross section was interpreted as resulting from a contribution of a preexisting $\Delta^-$ component of the nuclear wave function. The corresponding data analysis points to the probability of existence of the virtual $\Delta$ isobars in the above mentioned nuclei at a level of $0.5 \div 3\%$.

The photo- and electroproduction processes as a test of $\Delta$ admixture in nuclei were considered in Ref. [7]. In Ref. [8] a combined study of $(e,e'\Delta^{++})$ and $(e,e'\Delta^o)$ reactions on $^3\text{He}$ was proposed. The corresponding investigations are being realized at MAMI (Mainz) [9] and Jefferson Lab. [10]. Results for the reaction $^3\text{He}(e,e'\pi^+)$ obtained at MAMI do not contradict the assumption that a preformed $\Delta^{++}$ contributes to the reaction yield [11]. As for the processes with real photons, only the measurements of $^{12}\text{C}(\gamma,\pi^+p)$ and $^3\text{He}(\gamma,\pi^+p)$ reactions have been reported in Refs. [12] and [13]. The $^{12}\text{C}(\gamma,\pi^+p)$ reaction was explored at MAMI in order to study the mechanisms connected with final state interaction (FSI) [12]. An estimation of the $\Delta$ admixture in $^{12}\text{C}$ from these data is difficult since the experimental cross sections were obtained mainly at small recoil momenta of the residual nucleus where the background effects are large. The probability of the $\Delta NN$ configuration in the $^3\text{He}$ ground state of $2.2 \pm 1.3\%$ was reported in Ref. [13].

The small size of the isobar admixture in nuclei hampers the experimental observation of the $\Delta$ component. The main difficulty consists in separating the different background mechanisms leading to the same final state. We have solved this problem by measuring the photoproduction cross section for $\pi^+p$ pairs in the reaction

$$\gamma + ^{12}\text{C} \rightarrow \pi^+p + X.$$  

The measurements were performed in the kinematical region of high momentum transfer to the residual nuclear system $X$, whereas the background processes favour the small momentum transfer range. The use of reactions of the type (1) has two main advantages, outlined also in our previous paper [14]. Firstly, charge conservation prevents the creation of the $\Delta^{++}$ isobar by photons impinging on a single nucleon. Thus, the interpretation of the experimental results in terms of knocked-out $\Delta$ isobars is simplified. Secondly, since the virtual $\Delta$ isobars are produced in NN collisions, it is reasonable to expect that the probability of finding a $\Delta$ isobar in a nucleus is quadratic in the nuclear density. Therefore, the search for the $\Delta$ components in electromagnetic reactions is preferred since the photon beam is able to probe the whole nuclear volume up to the region of the highest densities. Also because of the reduced initial state interaction, a more direct test of isobar configurations can be carried out with electromagnetic probes than with strongly interacting probes.

Our experimental setup is described in section 2. In section 3 we briefly summarize the general formalism of the $\Delta$ knock-out model. Next the estimation of the background contributions as well as the analysis of the data are performed. After the determination of the kinematic region where the background effects are negligible, we analyse the data under the assumption that the $\pi^+p$ pairs found in the detectors are the product of the $\gamma \Delta^{++} \rightarrow \pi^+p$ process which starts from the $\Delta^{++}$ constituent preexisting in a target nucleus. This allows...
us to determine a value of the $\Delta^{++}$ momentum distribution and the number of $\Delta$ isobars in $^{12}\text{C}$.

The preliminary results of the present experiment have been published in Ref. [15].

2 Experiment

Measurements of the differential cross section of the reaction $^{12}\text{C}(\gamma, \pi^+ p)$ were performed at the Tomsk synchrotron at two electron beam energies, $E_e = 420$ and 500 MeV, respectively. The electron beam exhibited a duty cycle of 6% and had an energy spread of 0.2%. The electrons hit a 0.4 mm thick Ta-radiator and produced bremsstrahlung photons. After leaving the vacuum tube of the accelerator, the photon beam was confined by two lead collimators with an aperture of 6 mm and 10 mm in diameter and cleaned in a sweeping magnet (see Fig. 1). At the target position the photon beam had a spot size of 16 mm in diameter and an intensity of about $10^8$ equivalent quanta per second. The shape of the bremsstrahlung spectrum was described by the Schiff distribution [16]. The total energy of the photon flux during the runs was measured by the Gauss-quantameter ($Q$ in Fig. 1) with an accuracy of 3% [17]. The photon beam hit the target at an angle of 45°. A carbon plate of a natural isotopic composition and $4 \times 10^{22}$ nuclei/cm$^2$ in thickness was employed as a target. Its dimensions were 4 cm in height and 6 cm in width.

As is shown in Fig. 1, the experimental setup includes two coplanar arms detecting the positive pion and the proton in coincidence.

A strong focusing magnetic spectrometer [18] was set at an angle $\theta_\pi = 54^\circ$ with respect to the photon beam in order to select $\pi^+$ with mean momentum $p_\pi = 181$ MeV/c. Earlier this spectrometer was used for measurements of a deuteron photodisintegration asymmetry below pion threshold [19] and in measurements of the $^{12}\text{C}(\gamma, \pi^- p)$ reaction cross section in the second resonance region [20]. The angular and the momentum acceptances were determined by a Monte Carlo calculation taking into account the beam size, the angle of the target with respect to the beam, multiple scattering, energy loss, and pion decay. The solid angle $\Delta\Omega_\pi$ covered $3 \cdot 10^{-3}$ sr (azimuthal and polar angular acceptances were $\Delta\varphi_\pi \approx 6.4^\circ$ and $\Delta\theta_\pi \approx 2.0^\circ$, respectively) and the momentum acceptance $\Delta p_\pi/p_\pi$ was 24%. The scintillation hodoscope [21] located in the focal plane of the spectrometer determined the pion momentum with a resolution of $\sigma_{p_\pi} = 1.2 \%$. This value includes the contribution due to those muons from pion decay in flight which reached the telescope.

The proton channel included a $\Delta E - E$ plastic scintillation counter hodoscope and two auxiliary counters $S_{m}$ with absorbers intended for monitoring the proton channel stability. The $E$ detector included three scintillation counters of the dimensions $10 \times 10 \times 50$ cm$^3$, which were located on top of each other at a distance of 64 cm from the target. A single $\Delta E$ counter in front defined the solid angle of the proton channel to $\Delta\Omega_p = 0.26$ sr. The average polar angle of the proton channel with respect to the photon beam and its angular coverage were $\theta_p = (75 \pm 19)^\circ$, respectively. The scintillators of the $E$ counters were read out by two photomultiplier tubes FEU-139 at each end, and the scintillator of the $\Delta E$ counter by two photomultiplier tubes FEU-30. The absolute energy calibration of the proton channel and the measurement of the proton counter responses as functions of the particle coordinate position were performed by using a secondary proton beam selected by the magnetic spectrometer of the pion channel by positioning the $\Delta E - E$ counters behind the magnet. Within the range of the proton energy $T_p = 40 \div 130$ MeV the analysis of the signals from the photomultipliers
of the $\Delta E$ and $E$ counters allowed to determine the polar angle and the energy of the
detected proton with position independent accuracies better than $\sigma_\theta = 3^{\circ}$ and $\sigma_E = 4$ MeV,
respectively. The nuclear interaction of the proton with the scintillators contributed in part
to these values which where determined by the calibration. The accuracy of the azimuthal
angle $\phi$ was determined by the counter dimensions to $\sigma_\phi \sim 2^{\circ}$.

For online monitoring of the stability of the proton detector, two narrow auxiliary scin-
tillators $S_m$ with absorbers have been set up behind the $\Delta E - E$ hodoscope (see Fig. 1).
Simultaneously with the accumulation of the $\pi^+ p$ events we measured individual pulse height
spectra of the $\Delta E$ and $E$ scintillators, which were triggered by the coincident pulses from
the $\Delta E, E$- and $S_m$-counters. Two ranges of the spectra corresponding to minimal pion and
minimal proton energies were identified and monitored. These energies were determined by
the absorber thickness and they were chosen within the proton channel operating range in
such a way that light responses in the $E$-counters were corresponding to $T_p \approx 52.5$ MeV and
$T_p \approx 77.2$ MeV, respectively.

To reduce cosmic background, the focal plane detectors of the pion channel were cov-
ered with a big-area scintillation counter $S_a$ working in the anti-coincidence mode. The
final trigger was formed only during the accelerator pulse, which provided a cosmic back-
ground suppression to the level of less than 2%. Further data analysis suppressed the cosmic
background to a negligible fraction.

Background measurements were taken using only the pion channel with mean momentum
of 181 MeV/c. The non-target contribution was measured as a ratio of pion yields with and
without the target. The ratio was found to be less than 0.7%.

The measured reaction yields were corrected for the following effects:

1. The level of the accidental coincidences depended on the proton energy and was esti-
mated from the intensity of the events lying outside the time-correlation peak of the
$\pi^+ p$ coincidences. This level changed from 6% to 1.5% in the region $T_p = 50 ÷ 80$ MeV
and remained less than 1.5% for $T_p > 80$ MeV;

2. A portion of the particles was lost due to nuclear interaction of the protons and pions
in the material of the target and detector. The associated correction was momentum
dependent and reached 6% at maximum;

3. The pion decay correction was found to be 18 %;

4. The non-target contribution (air and target holder) was 0.7%;

5. The dead-time due to data recording was 2%.

In the present experiment fifty-three $\pi^+ p$ events have been detected in total. Fig. 2 shows
the distribution of these events in the proton emission angle and the proton energy for the
two values of electron beam energy.

3 Results and discussion

Before dealing further with the analysis of the obtained results we would like to outline
the general ideas which have governed the calculations for this investigation. Firstly, we
note that the distribution of events shown in Fig. 2 exhibits a rather pronounced structure
formed by the two groups at $T_p < 80$ MeV and $T_p > 80$ MeV. In the kinematic conditions of our experiment the larger proton energy $T_p$ implies a larger momentum transferred to the residual nucleus. Since the isobar configurations are responsible mainly for high momentum components of the nuclear wave function, a higher recoil momentum of the residual nuclear system is favored in this case. Keeping this in mind one may assume that the $\gamma \Delta^{++} \rightarrow \pi^+ p$ reaction on preexisting $\Delta$’s results in the right groups of the events in Fig. 2. As is confirmed by a calculation (described below), the remaining reaction channels forming the background lead to the events localized in the left part of the figure. In accordance with the observed structure of the distribution of events we used two different models for the data analysis. The first one, hereafter called the $\Delta$ knock-out model, assumes the $\pi^+ p$-production to proceed through the decay of the knocked-out $\Delta^{++}$. The second model, also called in the literature the Valencia Model [22, 23, 24], was explored to describe the essential part of background reactions. We outline below the main ingredients of both models and apply them to the analysis of the data.

3.1 The $\Delta$ knock-out model

To interpret the experimental data in the region of high momentum transfer, we used the approach in which the $\pi^+ p$ pair production was considered as a result of the $\gamma \Delta^{++} \rightarrow \pi^+ p$ process on a $\Delta^{++}$ preexisting in the target nucleus. The differential cross section for the reaction $^{12}\text{C}(\gamma, \pi^+ p)$ within the $\Delta$ knock-out model was discussed in our earlier work [14]. The main ideas are presented here for convenience. The differential cross section may be written in the form

$$
d\sigma = \frac{d^3\sigma}{dE_p d\Omega_p d\Omega_\pi} = \frac{E_f E_p p_p p_\pi^2}{4(2\pi)^3 E_\gamma} |T_{\gamma \Delta^{++} \rightarrow \pi^+ p}|^2, \quad (2)$$

where the subscripts $\gamma, \pi, p,$ and $f$ stand for the photon, pion, proton, and the final nucleus, respectively. The total energies and momenta of the participating particles are denoted by $E$ and $p$. The squared modulus of the reaction amplitude $M_{fi}$ in the laboratory system is

$$|M_{fi}|^2 = \rho_{\Delta^{++}}(p) f_{\pi p} |T_{\gamma \Delta^{++} \rightarrow \pi^+ p}|^2. \quad (3)$$

Here, the subscript $\Delta$ stands for the $\Delta$ isobar, $T_{\gamma \Delta^{++} \rightarrow \pi^+ p}$ is the elementary $\gamma \Delta^{++} \rightarrow \pi^+ p$ amplitude. The expression (3) was obtained employing the impulse approximation and using the closure relation when summing over all the states of the undetected residual nuclear system. The function $\rho_{\Delta^{++}}(p)$ in (3) represents the $\Delta^{++}$ isobar momentum distribution in the ground state of the initial nucleus. The factor $f_{\pi p}$ takes into account the absorption of the produced particles $\pi^+$ and $p$ inside the nucleus. It depends on the pion and proton energies and their opening angle.

The momentum distribution $\rho_{\Delta^{++}}(p)$ obeys the following normalization condition

$$\int \rho_{\Delta^{++}}(p) \frac{dp}{(2\pi)^3} = A \frac{N_{\Delta^{++}}}{c}, \quad (4)$$

where $N_{\Delta^{++}}$ is the number of $\Delta^{++}$ isobars per nucleon in the ground state of the $^{12}\text{C}$ nucleus and $A = 12$ is its mass number.

The elementary $\gamma \Delta^{++} \rightarrow \pi^+ p$ amplitude $T_{\gamma \Delta^{++} \rightarrow \pi^+ p}$ was obtained within the diagrammatic approach. The model was based on the coupling of photons to pions, nucleons and $\Delta$ isobars.
using effective Lagrangians. This leads to a set of the Feynman diagrams at the tree level shown in Fig. 3 (a-e). In the nonrelativistic limit up to the order of \((p/M)^2\) the elementary amplitude can be written as (for more details see Ref. [14])

\[
T_{\gamma\Delta \rightarrow \pi p} = i \sqrt{2} f_{\pi N\Delta} \frac{e}{2 M_p} \left\{ 2 S^+ \cdot p_{\pi p} \frac{2 p_\Delta \cdot \varepsilon_\lambda + i \frac{\mu_{\Delta^{++}}}{3} \cdot \sigma_\Delta \cdot [p_\gamma \times \varepsilon_\lambda]}{E_\Delta + E_\gamma - E'_\Delta + i \Gamma_\Delta/2} \right.
\]

\[\left. + i \frac{f_{\pi \Delta}}{f_{\pi N\Delta}} \frac{3 \mu_{N\Delta} S^+ \cdot [p_\gamma \times \varepsilon_\lambda]}{E_\Delta - E_\pi - E'_\pi} \sigma_\Delta \cdot p_\pi + 2 \frac{p_\rho \cdot \varepsilon_\lambda + i \frac{\mu_{\rho}}{2} \cdot \sigma_\rho \cdot [p_\gamma \times \varepsilon_\lambda]}{E_\Delta - E_\pi - E'_\pi} S^+ \cdot p_{\pi p} \right. \]

\[\left. - \frac{4 M_\rho}{t - m_\pi^2} S^+ \cdot (p_\pi - p_\gamma) p_\pi \cdot \varepsilon_\lambda - 2 M_p S^+ \cdot \varepsilon_\lambda \right\} \]  \(5\)

Here \(\lambda = \pm 1\) and \(\varepsilon_\lambda\) are the index and vector of the photon polarization. The variables

\[E'_p = M_p + (p_\Delta - p_\pi)^2/2M_p,\]

\[E''_\Delta = M_\Delta + (p_\Delta - p_\pi)^2/2M_\Delta\]

and

\[E'_\Delta = M_\Delta + (p_\Delta + p_\gamma)^2/2M_\Delta\]

are the energies of the intermediate nucleon and the \(\Delta\). The vectors

\[p_{\pi p} = (p_\pi M_p - p_\rho E_\pi)/(M_p + E_\pi)\]

and

\[p_\gamma = (p_\gamma M_\Delta - (p_\Delta - p_\pi) E_\gamma)/(M_\Delta + E_\gamma)\]

are the relative momenta in the \(\pi p\) and \(\gamma\Delta\) systems. In \(\text{(5)}\), \(S^+\) is the transition operator between the states with the spin 1/2 and 3/2 and \(\sigma_\Delta\) is the analog of the Pauli spin matrix for the spin \(\frac{3}{2}\) object (full expressions of \(S\) and \(\sigma_\Delta\) are given e.g. in Ref. [4]). For the hadronic coupling constants we employ \(f_{\pi N\Delta}^2/4\pi = 0.37\) from the decay \(\Delta \rightarrow \pi N\) and \(f_{\pi \Delta} = 4/5f_{\pi NN}\), as predicted by the trivial quark model. The magnetic moments used in this calculation are \(\mu_p = 2.79, \mu_{\Delta^{++}} = 4.52\) [25] and \(\mu_{N\Delta} = 3.24\) [26] in terms of nuclear magnetons.

### 3.2 The Valencia Model

The Valencia model [22, 23, 24] is well suited to estimate the background contributions to the direct \(\Delta\) knock-out reaction under investigation.

It has already been applied in several studies of similar reactions [12, 23, 27, 28]. This model incorporates all major photoabsorption mechanisms as one-, two- and three-nucleon absorption as well as single-pion production. These photoabsorption mechanisms are treated microscopically in a quantum-mechanically correct way taking into account the relevant photon-nucleon interaction amplitudes. Within the local density approximation the nuclear matter results are transferred to finite nuclei. The nuclear many-body problem is simplified by using a Fermi gas model. The particles which are produced in the photoabsorption process are then propagating independently through the target nucleus. They are traced in a phenomenological way considering the most important N-N and \(\pi-N\) reaction channels until
the primary and/or secondary hadrons leave the target nucleus. One of the main assumptions of this model is the treatment of the knocked out particles on their path through the medium as on-shell particles. The angular distributions are included in a phenomenological manner. With these ingredients for each event, the final momentum of each hadron is the result of a Monte Carlo type calculation.

The Valencia model describes total cross sections or dominant reaction channels in the Δ resonance region, e.g., as \((\gamma,\text{np})\) or \((\gamma,\pi,n)\), quite well (Refs. [27, 28]). However, for some of the less important reaction channels sizeable deviations can be found as these are sensitive mostly to final state interactions. For the particular kinematics of Ref. [28] a 50% overestimation of the \((\gamma,p\pi^\pm)\) cross sections was found for the photon energy range of \(E_\gamma = 400 \div 600\) MeV.

In order to investigate the sensitivity within the present kinematics some parameters have been varied. In Fig. 4 we present the upper and lower boundaries for the cross section given by the Valencia Model at the photon energy \(E_\gamma = 450\) MeV. In particular, to account for nucleon-nucleon correlations the nucleon wave functions are modified by a Jastrow type correlation \(j_0(q,x)\). This has a direct effect on the momentum distributions and thus at the relative strength of competing reaction channels. The many-body cut-off momentum \(q_c\), whose standard value involved in the model is 783 MeV/c, has been varied in the region 650 ÷ 900 MeV/c.

The important observation of these trial calculations is the proton energy spectrum which changes shape and strength, however, above \(T_p = 80\) MeV the background contribution remains very small.

### 3.3 Data analysis for cross sections

With the aim of determining the kinematic region where the reaction mechanisms caused by non-nucleon degrees of freedom may dominate, the analysis was carried out for the contributions from the background \(\pi^+p\) pair production mechanisms. For this purpose we employed the Valencia model.

The calculations were performed for three photon energies, namely \(E_\gamma = 350, 400\) and 450 MeV. For the cut-off momentum we have used \(q_c = 783\) MeV/c adopted as a standard value of this parameter in this model. The differential cross sections as a function of the proton kinetic energy \(T_p\) for \(E_\gamma = 400\) and 450 MeV have been averaged over the acceptances of the other observables and are shown in the histograms of Fig. 5. No result for the lowest photon energy is presented, because the corresponding cross section is negligible. According to the Valencia model, the \(\pi^+p\) yield at our experimental conditions results mainly from the sequential processes: \(\pi^0p\) and \(\pi^+n\) pair production followed by charge-exchange rescattering of the \(\pi^0\)-mesons to \(\pi^+\)-mesons and the neutrons to protons. As one can see in Fig. 5, the maximum of the background reactions is located at the small proton energies. This is due to the fact that the quasi-free \(\pi^+n\) and \(\pi^0p\) production dominates in the region of small momenta of the residual nuclear system, which in the kinematic conditions of our experiment corresponds to small proton energies. This is corroborated by Fig. 6, where the residual nucleus momentum \(p_f\) is presented as a function of the proton energy \(T_p\) for various values of the proton angle \(\theta_p\).

Note that all estimates for the competing reaction mechanisms predict contributions dominantly for the proton energies less than 80 MeV. The estimate of the background reactions according to Valencia model is 4% for \(T_p > 80\) MeV. Also we would like to note that this conclusion is independent from the values chosen for the parameters of the Valencia.
model. Fig. 4 shows that the contribution from the background reactions remains rather insensitive to their variation in the region $T_p > 80$ MeV.

To analyze the data for $E_e = 500$ MeV, we took into account the background from the double pion production in the $^{12}\text{C}(\gamma, p\pi^+\pi^-)^{11}\text{B}$ reaction, which was absent at $E_e = 420$ MeV. This photoabsorption mechanism is not included in the Valencia model. An estimate of the double pion production background has been done within the framework of the quasi-free approximation, whose formalism is presented in detail in Ref. [29]. For proton energies $T_p > 80$ MeV the calculation shows that the net effect from this process is less than 3% at $E_e = 500$ MeV (see Fig. 5).

As can be seen from Fig. 2, a considerable reaction yield is observed in the range $T_p \geq 80$ MeV. Therefore, we interpret these events to be caused by the binary process $\gamma \Delta^{++} \rightarrow \pi^+p$ on a preexisting $\Delta^{++}$.

A further support of this idea is provided by the analysis of the $\pi^+p$ invariant mass distribution presented in Fig. 7. Despite the statistics one sees that the events from the region $T_p \geq 80$ MeV fall within the range of the $\Delta(1232)$, which confirms once again our estimate about the reaction mechanism in this energy region. Due to these reasons, we have selected for further analysis only those events, which lie in the range $T_p \geq 80$ MeV (right of the dashed line in Fig. 2). Even when combining both runs there are only 13 events left.

As a result, we determined the differential yield $d^4Y/dE_\pi d\Omega_\pi dE_p d\Omega_p$, which is related to the measured quantities and to the differential cross section by the equations

$$\frac{d^4Y}{dE_\pi d\Omega_\pi dE_p d\Omega_p} = \frac{N_{\pi p}E_{\gamma max}}{\Delta E_\pi \Delta \Omega_\pi \Delta E_p \Delta \Omega_p W_\gamma t},$$

$$\frac{d^4Y}{dE_\pi d\Omega_\pi dE_p d\Omega_p} = \frac{d^3\sigma}{dE_p d\Omega_p d\Omega_\pi} f(E_\gamma) \left| \frac{\partial E_\gamma}{\partial E_\pi} \right|,$$

where $N_{\pi p}$ is the number of events in the phase space determined by the intervals $\Delta E_\pi$, $\Delta E_p$, $\Delta \Omega_\pi$, and $\Delta \Omega_p$. The target thickness is denoted by $t$ (in nuclei/cm$^2$). The Jacobian

$$\frac{\partial E_\gamma}{\partial E_\pi} = \frac{E_f p_\pi^2 - E_\pi p_\pi p_f}{E_\gamma E_f - p_\pi p_f} \frac{E_\gamma}{p_\pi^2}$$

connects the intervals $\Delta E_\pi$ and $\Delta E_\gamma$. In equation (8) $W_\gamma$ is the total energy of the photon flux, and $E_{\gamma max} = E_e - m_e c^2$ stands for the endpoint energy of the bremsstrahlung spectrum $f(E_\gamma)$, which is normalized as

$$\int_0^{E_{\gamma max}} f(E_\gamma) E_\gamma dE_\gamma = E_{\gamma max}.$$

The number of photons in the interval $\Delta E_\gamma$ is determined by the relation

$$N_{\gamma}(E_\gamma) \Delta E_\gamma = \frac{W_\gamma}{E_{\gamma max}} f(E_\gamma) \Delta E_\gamma.$$

The quantities not measured in the reaction, $E_\gamma$ and $p_f$, were determined by solving a set of kinematic equations

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

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$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

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$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

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$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$

$$E_\gamma + M_{C^{12}} = E_\pi + E_p + E_f$$

$$p_\gamma = p_\pi + p_p + p_f$$
under the assumption that the residual nucleus $^{11}$Be is in its ground state. In order to estimate the error connected with this assumption, we have repeated the calculation considering the residual nucleus to be excited with the energy $E_x = 20$ MeV. The excitation energy was taken as an average of the excitation spectrum obtained from the $^{12}$C($e, e'p$) data \[30\]. This assumption leads to the difference in $N_\gamma(E_\gamma) \Delta E_\gamma$ and correspondingly in the value of the cross section of about 5% as compared with the choice $E_x = 0$ MeV. This value was also taken into account in the systematical experimental error.

The cross section, obtained from the reaction yields at the two electron energies $E_e = 420$ and $E_e = 500$ MeV, was averaged over the intervals:

$$
T_p = 80 \div 120 \text{ MeV};
$$

$$
T_\pi = 71.5 \div 106.5 \text{ MeV};
$$

$$
\theta_p = 56 \div 94^\circ.
$$

The average photon energy for the events lying in the kinematic region under consideration was 355 MeV.

Within the procedure described above, we have obtained the following results for the runs with electron beam energy $E_e = 500$ MeV and $E_e = 420$ MeV,

$$
\frac{d^3\sigma}{dE_\gamma d\Omega_\gamma d\Omega_\pi} \left[ \frac{\text{nb}}{\text{MeV sr}^2} \right] = \begin{cases} 
7.5 \pm 3.4 \pm 0.7 \pm 0.4 & \text{run at } E_e = 500 \text{ MeV} \\
10.7 \pm 3.8 \pm 0.8 \pm 0.5 & \text{run at } E_e = 420 \text{ MeV} \\
8.9 \pm 2.6 \pm 0.8 \pm 0.5 & \text{sum,}
\end{cases}
$$

where statistical and systematical errors are given. The third value in (13) is the weighted average of the individual results. The systematical error includes the uncertainties of the corrections mentioned in the listing of section 2 and the measurement of the photon flux energy.

The experimental cross section is shown in Fig. 5 by the single point where the vertical bar represents the total uncertainty as quadratic combination of both errors. The horizontal bar is the standard deviation obtained for $T_p$ under the assumption that the events are uniformly distributed over the averaging interval (see Fig. 2).

### 3.4 The $\Delta$ momentum distribution

The knock-out reactions are well known to be a powerful tool to study the momentum distribution of nucleons in nuclei. If we treat the $\Delta$ isobar essentially as an ordinary particle carrying its energy and momentum, it is reasonable to speculate about the $\Delta$ momentum distribution inside the nucleus. The basis of our $\Delta$ knock-out model supports this suggestion as can be seen from expressions (2) to (4) where the $\Delta^{++}$ momentum distribution $\rho_{\Delta^{++}}(p)$ in $^{12}$C is used. One of the aims of the present experiment was to measure the function $\rho_{\Delta^{++}}$ within a certain range of $p$. The average of the $\Delta^{++}$ momentum in the present experiment amounts to $\overline{p} = 300 \text{ MeV/c}$, and the standard deviation of the momentum distribution is 49 MeV/c. This value $\overline{p}$ was determined in the frame of the quasi-free approximation with

$$
p = -p_f,
$$

(14)
where \( \mathbf{p} \) is the momentum of the \( \Delta^{++} \) isobar and \( \mathbf{p}_f \) is the momentum of the residual nucleus, estimated on the basis of the experimental data. Using the expressions (2) to (4) and averaging the cross section over the intervals defined by (12), the value \( \rho_{\Delta^{++}}(\mathbf{p}) \) was determined by fitting our calculation to the weighted average of the data obtained in the two runs (last value in (13)). This approach yields for the mean momentum \( \mathbf{p} = 300 \text{ MeV/c} \)

\[
\rho_{\Delta^{++}}(\mathbf{p}) = 0.17 \pm 0.05 \pm 0.02 \text{ fm}^3,
\]

where the latter two numbers show the statistical and systematic uncertainties retained from the cross sections. In addition, the systematic uncertainty includes the estimate of the background reactions according to the Valencia model and double pion photoproduction.

In the calculation of (15) the suppression factor \( f_{\pi p} \) in (3) was calculated within the optical model using the eikonal approximation. We employed the proton-nucleus optical potential obtained in Ref. [31] from a phenomenological study of proton scattering off nuclei. This optical potential has allowed to explain an effect of interaction of the proton with the residual nucleus in the reaction \( ^{12}\text{C}(\gamma, \pi^- p) \) in the range of the \( \Delta(1232) \) [32]. The pion wave function was distorted by an optical potential calculated in Ref. [33] according to Ref. [34]. The authors [33] have described satisfactorily the inclusive charged pion spectra, produced on carbon and copper in the kinematical region of quasi-free pion photoproduction [35, 36]. The overall quality of these calculations permits an estimate of the precision of \( f_{\pi p} \) to 26%.

It should be kept in mind that our quantitative conclusion also depends on the calculation methods of some nuclear effects and on the constants used in the \( \Delta \) knock-out model. One of the uncertainties is connected with the \( \Delta^{++} \) magnetic moment. We have chosen \( \mu_{\Delta^{++}} = 4.52 \) nuclear magnetons according to the experimental value given in Ref. [25]. It is necessary to remark that many modern theoretical calculations give the \( \Delta^{++} \) magnetic moment close to this experimental value (see Table 1). We attach a model uncertainty of 18% with this factor.

Table 1: Comparison of the magnetic moment \( \mu_{\Delta^{++}} \) predicted by some resent developments.

| \( \mu_{\Delta^{++}} \) in nuclear magnetons | Comments                                      |
|--------------------------------------------|-----------------------------------------------|
| 4.52 ± 0.5 ± 0.45                         | SIN data [25].                                |
| 4.13 ± 1.3                                | QCD sum rules [37].                           |
| 4.4 ± 0.8                                 | light cone QCD sum rules [38].                |
| 4.91 ± 0.6                                | lattice QCD [39].                             |
| 4.0 ± 0.4                                 | chiral perturbation theory [40].              |
| 4.73                                      | chiral quark-soliton model [41].              |

Our estimates show that the model error introduced by the uncertainty in magnetic moment of the \( \Delta^{++} \) isobar and in the final state interaction totals to 0.06 fm\(^3\).

Unfortunately, we cannot compare our result neither with theory nor with experiment because of the lack of literature data on the momentum distribution \( \rho_{\Delta^{++}} \) of the \( ^{12}\text{C} \) nucleus. We hope that results of our experiment will be a stimulus for further detailed studies of the isobar configurations in nuclei.
3.5 The number of preformed $\Delta$’s

In those works, where the isobar configurations in nuclei are studied, the main attention is concentrated on the estimate of the full number $N_\Delta$ of the $\Delta$ isobars per nucleon in the ground state of the nucleus. Since only a small region of the momentum $p$ is covered in our experiment, in order to estimate $N_\Delta$, it is necessary to know the form of the function $\rho_{\Delta^{++}}(p)$. Unfortunately, in the literature the functions $\rho_{\Delta^{++}}(p)$ are given only for the lightest nuclei such as deuteron [3] and $^3$He [2] or for infinite nuclear matter [13]. In the present paper we use the results of the work [13], where the $\Delta$ momentum distribution in the nuclear matter was evaluated within Random Phase Approximation. The free parameters used in the model [13] are the cut-off parameter $\Lambda$, the coupling constant $f_{\pi N\Delta}$, the Landau-Migdal parameter $g'$, and the correlation parameter $q_c$. Reasonable variations in these parameters lead to rather substantial changes of $N^m_\Delta$ in the range from 5.66% up to 15.89%. In the present calculation we have used the occupation number $n^c_\Delta(p)$ predicted by the choice $\Lambda = 1300$ MeV/c, $f^2_{\pi N\Delta}/4\pi = 0.32$, $g' = 0.7$, $q_c = 800$ MeV/c recommended in Ref. [13] as the most reasonable one. It gives $N^m_\Delta(p) = 6.66\%$, which is in close agreement with the result obtained in Ref. [3].

Before making quantitative conclusions we would like to demonstrate that our idea about the reaction mechanism, i.e. the knocking-out of the preformed isobars, agrees qualitatively with the results of measurements in the energy region considered. The $(\pi^+p)$ events from the reaction $^{12}$C$(\gamma, \pi^+p)$ are distributed in six-dimension phase space. From the possible distributions the invariant mass distribution may intuitively be assumed to be the most sensitive to the reaction dynamics. Keeping this in mind, we present in Fig. 7 (solid line) the $\pi^+p$ invariant mass distribution, predicted by the $\Delta$ knock-out model with the $\Delta$ momentum distribution taken from Ref. [13]. One can see that the computational results do not contradict the experimental data. We consider this fact as indirect evidence of the validity of our theoretical basis.

Turning to the estimation of the number $N_\Delta$, we used the following method. The momentum distribution $\rho_{\Delta^{++}}$ is connected with the occupation number $n^c_{\Delta^{++}}(p)$ of $^{12}$C by the relation

$$\rho_{\Delta^{++}}(p) = n_S \frac{4}{3} \pi R^3 n^c_{\Delta^{++}}(p),$$

where $R = 3.2$ fm is the square-well radius for $^{12}$C. The factor $n_S = 4$ is the number of the $\Delta^{++}$ spin states. We assume that the momentum distribution $\rho_{\Delta^{++}}(p)$ of the $\Delta(1232)$ isobars in $^{12}$C is proportional to that for nuclear matter. From the normalization condition (4) and the analogous condition for nuclear matter we obtain

$$n^c_{\Delta^{++}}(p) = \frac{\rho^c_N}{\rho^c_N} \frac{N^c_{\Delta^{++}}}{N^m_{\Delta^{++}}} n^m_\Delta(p).$$

Here $\rho^c_N = 0.087$ fm$^{-3}$ and $\rho^m_N = 0.17$ fm$^{-3}$ are the nucleon densities in $^{12}$C and in nuclear matter, respectively [4]; $N^m_{\Delta^{++}}$ represents the number of $\Delta^{++}$ isobars per nucleon in nuclear matter.

Within $SU(2)$-symmetry the total number of all $\Delta$ isobars per nucleon in nuclear matter $N^m_\Delta$ is given by

$$N^m_\Delta = 4 N^m_{\Delta^{++}}.$$  

The corresponding relation for the total number of the $\Delta$ isobars per nucleon in $^{12}$C can be
written as

\[ N^\Delta_\Delta = \frac{64}{15} N^{\Delta++}. \]  

(19)

We have extracted \( N^\Delta_\Delta \) according to formulae (2) to (4) from the experimental cross section (13) taken as a weighted average of the results obtained in two runs (latter value in (13)). Using also \( N^{\Delta+}_m = 6.66\% \) from Ref. [13] and expressions for \( \rho^{\Delta++}(p) \) (16) to (19), we have obtained

\[ N^\Delta_\Delta = 0.017 \pm 0.005 \pm 0.002 \Delta \text{ isobars per nucleon}, \]  

(20)

where the errors were estimated by the same way as in expression (13). This value is slightly smaller than the result obtained by other authors [6]. The \( \Delta \) knock-out model error, estimated as described in section 3.4, translates to the uncertainty in \( N^\Delta_\Delta \) of about 0.005 \( \Delta \) isobars per nucleon.

Finally, we would like to make a comment about the present experimental study of the \( \Delta \)-isobars in nuclei. Using the above parametrization of the momentum distribution \( \rho^{\Delta++}(p) \) with the value \( N^\Delta_\Delta \) (20), we have calculated the cross section (4) as a function of the proton kinetic energy \( T_p \). In Fig. 5 we compare the \( \Delta \) knock-out cross section with that for the background reactions and with our data. All results are averaged over the intervals given by (12). One can see that the maximum of the \( \Delta \) knock-out cross section is strongly shifted to higher proton energies and, hence, is well-defined kinematically. The experimental cross section at the proton kinetic energy \( T_p > 80 \text{ MeV} \) is almost two orders of magnitude larger than the theoretical estimates obtained within the models which allows for only the nucleon degrees of freedom in the \( ^{12}\text{C} \) nucleus. This fact, being particularly important for low-intensity signals, is one of the main advantages of the method used in this work.

4 Conclusions

The differential cross section for the \( ^{12}\text{C}(\gamma, \pi^+ p) \) reaction has been measured in the \( \Delta(1232) \) resonance region at high recoil momenta of the residual nuclear system. By using the \( \Delta \) knock-out model and the Valencia model we have determined the kinematical region where the background reactions are of minor importance, and the selected \( \pi^+ p \) pairs are mainly due to the direct interaction of the photons with the preexisting \( \Delta \) isobars. Therefore we interpret the events, which lie in this region, as direct evidence for the \( \Delta^{++} \) constituent in the target nucleus. Within the \( \Delta \) knock-out model the estimate for the \( \Delta^{++} \) momentum distribution \( \rho_{\Delta^{++}}(\mathbf{p}) = 0.17 \text{ fm}^3 \) for the average momentum \( \mathbf{p} = 300 \text{ MeV/c} \) and the number of \( \Delta \) isobars in \( ^{12}\text{C} \) \( N^\Delta = 0.017 \) were obtained. The derived value for \( N^\Delta \) is in general agreement with the results obtained by other authors for p-shell nuclei which are mainly in the range of 0.5−3.0\%.

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Figure captions

Fig. 1. Layout of the experimental setup: (R) radiator, (C) lead collimators, (SM) sweeping magnet, (VT) photon beam vacuum tube, (T) target, (Q) Gauss quantameter, (M) analyzing magnet, (S) scintillation counters, (H) hodoscope, (A) absorbers.

Fig. 2. The proton emission angle $\theta_p$ is plotted versus the proton kinetic energy $T_p$ for the final $53\, \pi^+ + p$ coincident events separately for the two electron beam energies. The vertical dashed line at $T_p = 80$ MeV separates the accepted events (solid squares) from those which are mainly due to background (crosses). The arrow at $T_p = 50$ MeV indicates the detection threshold. The horizontal dashed lines show the angular range of proton detection. The vertical dashed line is the cut applied to the proton energy.

Fig. 3. Diagrams for the $\gamma \Delta^{++} \to \pi^+ p$ amplitude used in the present calculation: (a) s-channel term, (b) and (c) u-channel terms, (d) pion pole term, (e) seagull term.

Fig. 4. Valencia Model calculation for the differential cross section of the reaction $^{12}\text{C}(\gamma, \pi^+ p)$ at $E_\gamma = 450$ MeV. The lines present the boundaries of the results obtained with different values of the model parameters. The cut-off momentum $q_c$ has been varied in the range $q = 650 \div 900$ MeV/c.

Fig. 5. Differential cross section for the $^{12}\text{C}(\gamma, \pi^+ p)$ reaction averaged over the intervals $\theta_p = (56 \div 94)^\circ$ and $T_\pi = (71.5 \div 106.5)$ MeV as a function of the kinetic energy $T_p$: the results of the Valencia model for single pion production at $E_\gamma = 400$ and 450 MeV are shown by the solid and dotted histograms, respectively; the dot-and-dash curve corresponds to the quasi-free double pion photoproduction; the solid line corresponds to the $\Delta$ knock-out model; and the dot represents the experimental cross section with its combined statistical and systematic uncertainties.

Fig. 6. Momentum $p_f$ of the residual nucleus as a function of the kinetic energy $T_p$ for $\theta_\pi = 54^\circ$ and $T_\pi = 90$ MeV.

Fig. 7. Invariant mass distribution of the $\pi^+ p$ events for proton energies $T_p = 80 \div 120$ MeV. The solid curve is the result of the $\Delta$ knock-out model calculation.
Figure 1:
\[
\gamma (E_\gamma, \mathbf{p}_\gamma) \quad \pi^+ (E_\pi, \mathbf{p}_\pi) \\
\Delta^{++} (E_\Delta, \mathbf{p}_\Delta) \quad p (E_p, \mathbf{p}_p)
\]

\[= \]

(a) +

(b) +

(c) +

(d) +

(e)

Figure 3:
Figure 4:
Figure 5:
Figure 6:
Figure 7: