Descriptions of stress-strain responses of non-linear unloading and closure of stress-strain hysteresis loop based on the Yoshida-Uemori model

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Abstract. This paper proposes models to describe a non-linear unloading-reloading stress-strain response and closure of a cyclic stress-strain hysteresis loop based on the Yoshida-Uemori (Y-U) model. The average stress-strain slope given by the non-linear model is consistent with the Y-U model of Young's modulus degradation. The closure of a cyclic stress-strain curve is described by automatically changing one of the Y-U material parameters. An advantage of the above approaches is that no additional parameters are needed for modeling.

1. Introduction

For accurate numerical simulation of springback, taking in account of two material behaviors, one is ‘the degradation of Young’s modulus’ (defined by unloading stress-strain slope that includes micro plasticity) with increasing plastic strain, and the other is the Bauschinger effect in reverse loading, is of vital importance. The value of Young’s modulus directly influences the amount of springback, and the Bauschinger effect affects it strongly when sheets are subjected to cyclic bending-unbending, e.g. in draw-bending. Especially for advanced high strength sheets (AHSSs), they have very strong natures of the Young’s modulus degradation and the Bauschinger effect. The present author (Yoshida) and his co-workers (Uemori and Fujiwara) [1] proposed a model of the Young’s modulus degradation (so-called model of ‘plastic-strain dependent Young’s modulus’), and nowadays, it is widely used in the springback simulation. In the model, we neglected the non-linear stress-strain response in unloading-reloading, although we pointed out that the stress-strain response includes slight nonlinearity. Recently, some researchers attempted to describe the nonlinear unloading behavior [4-7], however, these models are rather complicated and they need several additional material parameters (or even tensor variables). In the present paper, a simple non-linear elasticity model is presented, which is consistent with the Y-U model of Young’s modulus degradation.

The second topic in this paper is cyclic elasto-plasticity modeling to describe the closure of cyclic stress-strain hysteresis loop. The Y-U kinematic hardening model [2, 3] describes the most of cyclic plasticity behavior well, however, for a special case of the subsequent small-strain-range cycling after a large plastic pre-strain, predicted stress level after a strain cycling is slightly lower than the real material behavior. It would be improved if we would have a cyclic plasticity model that guarantees the closure of cyclic stress-strain hysteresis loop. For its description, one of the approaches is the use of multi-linear-kinematic-hardening (with a stress threshold) components [8], which gives piecewise linear stress-strain responses. The multi-surface modeling [9] is another type of cyclic plasticity model that describes the closure of a cyclic stress-strain hysteresis loop. However, all these multi-component
type models need many additional tensor variables, which may increase the computational cost enormously. In the present paper, an improved version of the Y-U model is presented to describe the closure of the cyclic stress-strain loop. Recently, Sumikawa et al. [10] also proposed a cyclic plasticity model to describe the closure of a stress-strain hysteresis loop based on the Y-U model, however their model requires several additional material parameters. In contrast, in the present model, necessary material parameter changes are given automatically, and thus no additional material parameter is needed.

2. Cyclic stress-strain responses calculated by the Y-U model

2.1. Y-U model

With the assumption of a small elastic and large plastic deformation, the rate of deformation $D$ is decomposed as

$$ D = D' + D^p, $$

where $D'$ and $D^p$ are the elastic and plastic parts of the rate, respectively. The present constitutive model of plasticity has been constructed within the framework of two-surface modeling, wherein the yield surface moves kinematically within a bounding surface. When the yield function at the initial (non-deformed) state, $f_0$, has a general form:

$$ f_0 = \phi(\sigma) - Y = 0, $$

where $\phi$ denotes a function of the Cauchy stress $\sigma$, and $Y$ is the initial yield strength, the subsequent yield function $f$ is given by the equation:

$$ f = \phi(\sigma - \alpha) - Y = 0, $$

where $\alpha$ denote the backstress. The associated flow rule is written as

$$ D^p = \frac{\partial f}{\partial \sigma} \dot{\sigma}, $$

where $\dot{\sigma}$ is the effective plastic strain rate. The bounding surface $F$ is expressed by the equation:

$$ F = \phi(\sigma - \beta) - (B + R) = 0, $$

where $\beta$ denotes the center of the bounding surface, and $B$ and $R$ are its initial size and its isotropic hardening (IH) component. The kinematic hardening of the yield surface describes the transient Bauschinger deformation characterized by early re-yielding and the subsequent rapid change of workhardening rate. The relative kinematic motion of the yield surface with respect to the bounding surface is expressed by

$$ \alpha_\ast = \alpha - \beta. $$

For the evolution of $\alpha_\ast$, we assume

$$ \dot{\alpha}_\ast = C \left[ \left( \frac{\alpha}{Y} \right) (\sigma - \alpha) - \sqrt{\frac{\alpha}{\alpha - \alpha_\ast}} \right] \dot{\sigma}, \quad \bar{\alpha}_\ast = \phi(\alpha_\ast), \quad a = B + R - Y $$

where $C$ is a material parameter that controls the rate of the kinematic hardening. Here, $(\dot{\circ})$ stands for the objective rate. For the evolution equations of $\beta$ and $R$, refer to articles [2, 3].

In the present model, the size of yield surface $Y$ is fixed constant. However, if we carefully observe the stress-strain response during unloading after plastic deformation, we will find out that the stress-strain curve is no longer linear but slightly curved due to very early re-yielding and the Bauschinger effect. In order to describe the Young’s modulus degradation, in the model, the following equation was presented [1]:

$$ E_{\ast\ast} = E_\ast - (E_\ast - E_a) \left[ 1 - \exp(-\xi \rho) \right], $$

where $E_\ast$ is the initial Young's modulus.
where \( E_0 \) and \( E_* \) stand for Young’s modulus for virgin and infinitely large prestrained materials, respectively, and \( \xi \) is a material constant.

2.2. \( Y-U \) calculations to be improved
Although the \( Y-U \) model is able to express cyclic plasticity behavior mostly well, the model should be improved so as to describe the following two stress-strain responses:
- non-linear unloading/reloading stress-strain responses,
- complete closure of cyclic stress-strain hysteresis loop.

Figure 1(a) shows the comparison of experimentally observed cyclic stress-strain response on a 590 MPa high strength steel (HSS) sheet (it was taken from a paper by Sumikawa et al. [10]) from the corresponding \( Y-U \) model calculation for a case of the subsequent small-strain-range cycling after a large plastic re-strain. In the experiment, the stress-strain point after a full cycle reaches just the same point as that of unloading start. However, the \( Y-U \) model predicts slightly lower stress after a full cycling compared to the unloading-start stress. This is because the backstress does not return to the unloading-start point after a cycle, as shown in Fig. 1(b). This is a common problem appearing in most of non-linear kinematic hardening models (e.g., Armstrong-Frederick model [11]). Note that this problem appears only for a case of small-strain-range cycling after a large plastic prestrain.

![Stress-strain responses](image1.png)

(a) Cyclic stress-strain responses
(b) Backstress evolution in cyclic loading

Fig. 1 Stress-strain responses during subsequent small-strain-range cycling after a large pre-strain.

3. Non-linear unloading/reloading stress-strain curve
Figure 2 shows a model of stress-dependent Young’s modulus when a stress point exists (at point B) in a yield surface. We assume that Young’s modulus \( E \) is determined as a function of stress-point position measured by the length of an extrapolated stress path (B-C in Fig. 2), \( \rho \), as

\[
E = E_* + (E_0 - E_*) \left( \frac{\rho}{Y} \right), \quad \rho = 2(Y - Z)
\]  

(9)

Under the uniaxial stress-state, Young’s modulus at unloading-start point (A in Fig. 2) is always equal to \( E_0 \), and \( E_* \) at the final elastic unloading point (C in Fig. 2). Here, \( E_* \) is given by the following equation:

\[
E_* = 2E_{0v} - E_0
\]  

(10)
where \( E_{av} \) is a decreasing function of plastic strain (see Eq. (7)). In Fig. 2, point B is a current stress state, at which Young’s modulus is \( E \), and an arrow schematically shows the direction of unloading. The stress-strain response during elastic unloading and the subsequent reloading, calculated by this model, is shown in Fig. 3. Here, non-linear stress-strain curves are successfully described. Note that this model is consistent with the Y-U model of Young’s modulus degradation with increasing plastic strain, where Eq. (9) uses only initial Young’s modulus \( E_0 \), yield strength \( Y \), and \( E_* \). This model can be applied to multi-axial stress state, as schematically shown in Fig. 2 (see the definition of \( Z \) in the multi-axial (non-proportional) stress path \( A \rightarrow B' \)). Figure 3 shows a stress-strain response during an unloading-reloading process, calculated by the present model, and the corresponding experimental data on DP980AHSS.

Fig. 2 Schematic illustration of the variation of Young’s modulus at a stress point in the yield surface

![Figure 2](image)

Fig. 3 Non-linear unloading-reloading stress-strain responses in elastic region calculated by the present model (comparison of calculation (-) with experiment (●) on DP980 AHSS)

![Figure 3](image)

4. Closure of cyclic stress-strain hysteresis loop
Under the uniaxial cyclic stress state, as schematically shown in Fig. 4, the evolution equations of the backstress are expressed as follows, e.g.,
- for $a \rightarrow b$, $d\alpha_a = -Ca\left(1 + \sqrt{\alpha_a}/a\right) dp$

then $p_{ab} = \frac{2}{C} \left[ \sqrt{\alpha_{\text{max}}}/a - \ln\left(1 + \sqrt{\alpha_{\text{max}}}/a\right) \right] = \frac{2}{C} \left[ t_{\text{max}} - \ln(1 + t_{\text{max}}) \right]$, \quad (11)

where $t_{\text{max}} = \sqrt{\alpha_{\text{max}}}/a$.

- for $b \rightarrow c$, $d\alpha_b = -Ca\left(1 - \sqrt{\alpha_b}/a\right) dp$

then $p_{bc} = -\frac{2}{C} \left[ \sqrt{\alpha_{\text{min}}}/a + \ln\left(1 - \sqrt{\alpha_{\text{min}}}/a\right) \right] = -\frac{2}{C} \left[ t_{\text{min}} + \ln(1 - t_{\text{min}}) \right]$, \quad (12)

where $t_{\text{min}} = \sqrt{\alpha_{\text{min}}}/a$.

Thus the plastic strain in the process $a \rightarrow b \rightarrow c$ is

\[ p_{ac} = p_{ab} + p_{bc} = \frac{2}{C} \left[ t_{\text{max}} - t_{\text{min}} - \ln(1 - t_{\text{min}})(1 + t_{\text{max}}) \right]. \quad (13) \]

In the similar way, the process $c \rightarrow d \rightarrow a$ are analysed by using the slightly different value of $C'$, as

\[ p_{ca} = p_{cd} + p_{da} = \frac{2}{C'} \left[ t_{\text{min}} - t_{\text{max}} - \ln(1 - t_{\text{max}})(1 + t_{\text{min}}) \right]. \quad (14) \]

From the condition of closure of hysteresis loop, $a \rightarrow b \rightarrow c \rightarrow d \rightarrow a$, i.e., $p_{ac} = p_{ca}$, the value of $C'$ is explicitly determined as follows:

\[ C' = C' \frac{t_{\text{min}} - t_{\text{max}} - \ln(1 - t_{\text{max}})(1 + t_{\text{min}})}{t_{\text{max}} - t_{\text{min}} - \ln(1 - t_{\text{min}})(1 + t_{\text{max}})}. \quad (15) \]

Figure 5 shows the cyclic stress-strain curves calculated by the present model, which successfully describes the complete closure of cyclic hysteresis loop, where the experimental data plots on 590 MPa HSS were taken from Sumikawa et al. [10].

This uniaxial-stress model is extended to the multi-axial-stress model by using following definitions of $\alpha_{\text{max}}$ and $\alpha_{\text{min}}$ (see schematic illustration, Fig. 6):
\[
|\alpha| = \bar{\alpha} = \phi(\alpha), \\
|\alpha_{\max}| = \bar{\alpha}_{\max} = \phi(\alpha_{\max}^+), \\
|\alpha_{\min}| = \phi(\alpha_{\max}^+ - \alpha)_{\max} - \phi(\alpha_{\max}^+). \\
\]

(16)

Fig. 6 Definitions of \(|\alpha_{\max}|\) and \(|\alpha_{\min}|\) for the multi-axial stress state, where \(\alpha_{\max} : d\alpha \geq 0\)

It should be noted that the above-mentioned complete closure of cyclic stress-strain loop is a material-dependent characteristic. For some materials, stress level after a strain cycle is slightly lower than that of unloading-start point (refer to [1]). This model is easily extended to describe such a variety of cyclic plasticity behavior by determining various \(C'\) values (details will be presented at the conference).

5. Concluding remarks

The new version of the Y-U model is able to describe the nonlinear unloading-reloading hysteresis loop by introducing the nonlinear elasticity model, as well as the closure of cyclic stress-strain loop by using automatically changing \(C\)-parameter in the Y-U model. A great advantage of the above approaches is that no additional material parameters are needed for modeling.

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