Wireless Energy and Information Transfer Tradeoff for Limited Feedback Multi-Antenna Systems with Energy Beamforming

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Abstract—In this paper, we consider a multi-antenna system where the receiver should harvest energy from the transmitter by wireless energy transfer to support its wireless information transmission. In order to maximize the harvesting energy, we propose to perform adaptive energy beamforming according to the instantaneous channel state information (CSI). To help the transmitter to obtain the CSI for energy beamforming, we further propose a win-win CSI quantization feedback strategy, so as to improve the efficiencies of both power and information transmission. The focus of this paper is on the tradeoff of wireless energy and information transfer by adjusting the transfer duration with a total duration constraint. Through revealing the relationship between transmit power, transfer duration and feedback amount, we derive two wireless energy and information transfer tradeoff schemes by maximizing an upper bound and an approximate lower bound of the average information transmission rate, respectively. Moreover, the impact of imperfect CSI at the receiver is investigated and the corresponding wireless energy and information transfer tradeoff scheme is also given. Finally, numerical results validate the effectiveness of the proposed schemes.

Index Terms—Wireless energy and information transfer, energy beamforming, limited feedback, resource allocation.

I. INTRODUCTION

Recently, wireless energy transfer has aroused general interests in wireless research community, as it can effectively prolong the lifetime of the power-limited node or network in a relative simple way [1]-[3]. As a typical example, in medical area, the implanted equipment in body can be powered through wireless power transfer [4].

In general, wireless energy transfer from a power source to a receiver is implemented through electromagnetic propagation [5]. Since electronic energy is propagated isotropically if the transmit antenna is isotropic, the receiver only harvests a portion of the transmitted energy without specific control, resulting in a low energy transfer efficiency. In order to maximize the harvested energy, it is necessary to coordinate transmit direction to the receiver, namely energy beamforming. The key of the energy beamforming is the achievement of channel state information (CSI) at the power source. Inspired by signal beamforming in multi-antenna systems [6] [7], we also propose to use quantization codebooks of limited size to convey the CSI from the receiver to the power source. Based on the feedback CSI, the power source performs adaptive energy beamforming.

The ultimate goal of wireless energy transfer is to fulfill the need of the receiver for work. As an example, the implanted equipment transmits the medical data to the outside receiver with the harvested energy. However, most of previous analogous works analyze and design the wireless energy transfer without considering the utilization of the harvested energy. There are some works joint considering wireless energy and information transfer in the point-to-point system [8] and the multiuser downlink [9], but the harvested energy is not used for information energy, they only share a common transmitter. In this paper, we consider joint wireless energy and information transfer in a multi-antenna system employing energy beamforming, but the power receiver also plays a role as the information transmitter. Specifically, the power receiver, such as the implanted equipment, transmits the information with the harvested energy. For such a system, since energy harvesting and information transmission are impossible carried out simultaneously, the time slot should be divided into the harvesting and the transmitting components. The larger harvesting duration denotes higher available transmit power, but also shortens the transmitting duration, which may lead to a lower average transmission rate. Thus, for a given duration of one time slot, it is imperative to determine the optimal time allocation between energy and information transfer, namely the wireless energy and information transfer tradeoff. The focus of this paper is on the optimal tradeoff with the goal of maximizing the average information transmission rate.

As mentioned above, energy beamforming has a great impact on wireless power transfer, and thus the ultimate average information transmission rate. On the other hand, the amount of CSI at the power source has a tight connection to the efficiency of power transfer. Therefore, we give a detailed investigation of the impact of the feedback amount on the average information transmission rate and derive the tradeoff between the durations of energy and information...
transfer in terms of feedback amount from the perspectives of maximizing an upper bound and an approximate lower bound on the average information transmission rate, respectively. Moreover, considering the limited channel estimation capacity, there will be estimation error at the power receiver. Under such a condition, we also analyze the impact of estimation error on the average information transmission rate, and obtain the corresponding wireless energy and information transfer tradeoff.

The rest of this paper is organized as follows: Section II gives a brief introduction of the considered wireless energy and information transfer protocol for a limited feedback multi-antenna system with energy beamforming. Section III focuses on the analysis and design of the wireless energy and information transfer tradeoff with limited feedback. The impact of imperfect CSI caused by channel estimation error is investigated in Section IV. Section V presents several numerical results to validate the performance of the proposed algorithms, and finally Section VI concludes the whole paper.

II. SYSTEM MODEL

![Diagram of the multi-antenna system employing wireless energy and information transfer.](Image)

We consider a frequency division duplex (FDD) multi-antenna system employing wireless energy and information transfer, as shown in Fig. 1 where the power source and the information receiver are equipped with $N_t$ antennas to enhance the energy transfer efficiency by energy beamforming and improve the average information transmission rate by receive combining, respectively. In fact, the power source and the information receiver can be integrated into one equipment $T_1$ and the work mode is switched according to the proposed wireless energy and information transfer tradeoff scheme. Similarly, $T_2$ can play the roles as both the power receiver and the information transmitter. Due to space limitation, $T_2$ deploys a single antenna for energy harvesting and information transmitting. $T_2$ only has limited power to maintain the active state, it needs to harvest the enough energy for information transmission from the outside, such as $T_1$. Considering the limited storage, $T_2$ should be charged from $T_1$ slot by slot. We use $\sqrt{\alpha}h$ to denote the $N_t$ dimensional channel vector from $T_1$ to $T_2$, where $\alpha$ represents the path loss and $h$ denotes the channel fast fading component that is an $N_t$ dimensional zero-mean complex Gaussian vector having a real part with variance 1 and an imaginary part with variance 1. According to the law of energy conservation, the harvesting power at $T_2$ from $T_1$ can be expressed as [9]

$$P_{\text{harv}} = \eta \alpha P_1 |h^Hw|^2,$$

where $P_1$ is the transmit power of $T_1$, the constant parameter $0 \leq \eta \leq 1$ is the efficiency ratio at $T_2$ for converting the harvested energy to the electrical energy to be stored. Following [10], we assume $\eta = 0.8$ in this paper. $w$, as the energy beamforming vector with unit norm, is used to adaptively adjust the energy transmit direction according to the instantaneous channel state $h$, so as to maximize the harvesting power at $T_2$. Clearly, the CSI at $T_1$ has a great effect on the performance of energy transfer. As a simple example, if full CSI is available, $w = h/\|h\|$, namely maximum ratio transmission (MRT), can achieve the optimal performance. However, as $T_1$ is at the transmit side of the channel $\sqrt{\alpha}h$, it is difficult to obtain the CSI by itself. Inspired by signal beamforming in multi-antenna systems, we propose to use a quantization codebook $\mathcal{W} = \{w_1, \ldots, w_{2^B}\}$ of size $2^B$ to convey the optimal beam from $T_2$ to $T_1$ for each time slot. The quantization codebook is designed according to the distribution of the channel direction vector $h/\|h\|$ by making use of the vector quantization (VQ) method and stored at both $T_1$ and $T_2$ in advance. $T_2$ selects the optimal quantization codeword, namely the beam, so as to maximize the effective channel gain. Mathematically, the codeword selection criterion can be expressed as

$$i^* = \arg \max_{w_i \in \mathcal{W}} |h^Hw_i|^2.$$  

$T_2$ conveys the optimal codeword index $i^*$ to $T_1$, and $T_1$ recovers the beam from the same codebook, then uses it for energy beamforming.

![Diagram of the wireless energy and information transfer protocol.](Image)

The whole system is operated in slotted time of length $T$, as shown in Fig. 2. At the beginning of each time slot, $i^*$ is fed back from $T_2$ to $T_1$. It is assumed that the energy and duration for feedback can be negligible due to the small amount of transmission. Then, $T_1$ performs wireless energy transfer by energy beamforming with the duration $\tau$, and the harvesting energy at $T_2$ is given by

$$Q_{\text{harv}} = \eta \alpha P_1 \|h^Hw_{i^*}\|^2 \tau.$$  

After that, $T_1$ and $T_2$ toggle to the wireless information transfer mode. With the harvesting energy $Q_{\text{harv}}, T_2$ transmits the information to $T_1$ in the rest of the time slot $T - \tau$, and
the receive signal can be expressed as
\[ y = \sqrt{\frac{Q_{\text{harv}}}{T - \tau}} \beta s + n, \quad (4) \]
where \( s \) is the normalized transmit signal, \( y \) is the \( N_t \) dimensional receive signal vector, and \( n \) is the additive Gaussian white noise with zero mean and variance matrix \( \sigma^2 I_{N_t} \). \( \sqrt{\beta g} \) denotes the channel from \( T_2 \) to \( T_1 \), where \( \theta \) is the path loss and \( g \) is the channel fast fading vector, which is independent of \( h \). \( \frac{Q_{\text{harv}}}{T - \tau} \) is the transmit power. Assuming perfect CSI at \( T_1 \), maximum ratio combining (MRC) is performed to maximize the average information transmission rate, which is given by
\[
R(\tau) = E\left[ \log_2 \left( 1 + \frac{Q_{\text{harv}} \beta \|g\|^2}{(T - \tau)\sigma^2} \right) \right]
= \frac{T - \tau}{T} E\left[ \log_2 \left( 1 + \frac{\eta \alpha \theta P_1 \|h\|^2 \|h^H w_\tau\|^2 \|g\|^2 \tau}{(T - \tau)\sigma^2} \right) \right],
\]
where \( h = h/\|h\| \) is the channel direction vector, and \( E[\cdot] \) denotes the expectation operation.

III. WIRELESS ENERGY AND INFORMATION TRANSFER TRADEOFF

In this section, we focus on the wireless energy and information tradeoff to optimize the average information transmission rate in (4). Specifically, we attempt to derive the optimal energy transfer duration \( \tau \) so as to maximize the average information transmission rate. Due to multiple random variables in (4), it is difficult to obtain the closed-form expression of the average information transmission rate. For tractability, we turn to derive the tradeoff from the perspectives of the maximization of an upper and lower bounds of the average information transmission rate, respectively.

A. The Upper Bound Case

Since \( \log_2(1 + x) \) is a concave function with respect to \( x \), according to Jensen’s inequality, we have
\[
R(\tau) \leq \frac{T - \tau}{T} E\left[ \log_2 \left( 1 + \frac{\eta \alpha \theta P_1 \|h\|^2 \|h^H w_\tau\|^2 \|g\|^2 \tau}{(T - \tau)\sigma^2} \right) \right]
= \frac{T - \tau}{T} \log_2 \left( 1 + \frac{\eta \alpha \theta P_1 \|h\|^2 \|h^H w_\tau\|^2 \|g\|^2 \tau}{(T - \tau)\sigma^2} \right)
\]
where \( a = \eta \alpha \theta P_1 / \sigma^2 \), and (6) holds true since \( h \) and \( g \) are independent of each other in such a FDD system by assuming the frequency gap is larger than the correlation bandwidth. Considering both the real and imaginary parts of \( h \) and \( g \) are distributed according to \( CN(0, I_{N_t}) \), so we have \( E[\|h\|^2] = E[\|g\|^2] = 2N_t \). Moreover, for the squared inner product of two \( N_t \) dimensional vectors uniformly distributed on the unit sphere, it is \( \beta(1, N_t - 1) \) distributed. Thus, \( \|h^H w_\tau\|^2 \) is the maximum of \( 2^B \) i.i.d. \( \beta(1, N_t - 1) \) distributed random variables caused by the beam selection in (1), and its probability density function (pdf) can be expressed as
\[
f(x) = 2^B (N_t - 1)(1 - x)^{N_t - 2} (1 - (1 - x)^{N_t - 1})^{2^B - 1}. \quad (7)
\]
Then, the expectation of \( \|h^H w_\tau\|^2 \) can be computed as
\[
E\left[ \|h^H w_\tau\|^2 \right] = \int_0^\infty xf(x)dx
= 1 - 2^B \text{Beta} \left( 2^B, \frac{N_t}{N_t - 1} \right). \quad (8)
\]
where \( \text{Beta}(a, b) \) denotes the beta function. By utilizing the property of beta function, we have the following relationship (9)
\[
1 - 2^{-\frac{N_t}{N_t - 1} - 2} < E\left[ \|h^H w_\tau\|^2 \right] < 1 - \frac{N_t - 1}{N_t} 2^{-\frac{N_t}{N_t - 1}}. \quad (9)
\]
Substituting (9) into (6), an upper bound on the average information transmission rate can be expressed as
\[
R^u(\tau) = \frac{T - \tau}{T} \log_2 \left( 1 + \frac{4a N_t^2 (1 - \frac{N_t - 1}{N_t} 2^{-\frac{N_t}{N_t - 1}}) \tau}{T - \tau} \right). \quad (10)
\]
Hence, the wireless energy and information transfer tradeoff based on the upper bound in (10) is equivalent to the following optimization problem
\[
J_1 : \max_{\tau} \frac{T - \tau}{T} \log_2 \left( 1 + \frac{b \tau}{T - \tau} \right) \quad \text{s.t.} \quad 0 \leq \tau \leq T, \quad (11)
\]
where \( b = 4a N_t^2 (1 - \frac{N_t - 1}{N_t} 2^{-\frac{N_t}{N_t - 1}}) \). \( J_1 \) is an optimization problem with a single continuous variable. Taking the derivative of (11) with respect to \( \tau \), and letting the derivative equal to zero, we have
\[
\frac{b}{(T + (b - 1)\tau) \ln 2} - \frac{1}{T} \log_2 \left( 1 + \frac{b \tau}{T - \tau} \right) = 0. \quad (13)
\]
Let \( \tau_1 = 0, \tau_2 = T \) and \( \tau_3 \) be the solution of equality (13), then the optimal \( \tau \) can be determined according to the following criterion:
\[
\tau^* = \arg\max_{\tau \in \Omega_1} R^u(\tau), \quad (14)
\]
where \( \Omega_1 = \{\tau_1, \tau_2, \tau_3\} \).

B. The Lower Bound Case

For the average information transmission rate in (4), by some mathematical operations, it can be approximated as
\[
R(\tau) = \frac{T - \tau}{T} E\left[ \log_2 \left( 1 + \exp\left( \ln \left( \frac{\eta \alpha \theta P_1 \|h\|^2 \|h^H w_\tau\|^2 \|g\|^2 \tau}{(T - \tau)\sigma^2} \right) \right) \right) \right]
+ \exp\left( \ln \left( \frac{\eta \alpha \theta P_1 \|h\|^2 \|h^H w_\tau\|^2 \|g\|^2 \tau}{(T - \tau)\sigma^2} \right) \right).
\]
\[ \sum_{t=1}^{N} \left( 1 - \frac{\eta}{\rho} - \psi(N_t) \right) = \sum_{t=1}^{N} \psi(N_t) - \frac{\eta}{\rho} N = \log \left( \frac{T - \tau}{T} \right) \log_2 \left( 1 + \frac{c \tau}{T - \tau} \right), \]

which is derived by letting the derivative of the objective function in (22) with respect to \( \tau \) equal to zero.

IV. THE IMPACT OF IMPERFECT CSI

In the last section, we derived the wireless energy and information transfer tradeoffs in limited feedback multi-antenna systems from the perspectives of the maximization of an upper bound and an approximate lower bound on the average information transmission rate respectively in the case of perfect CSI at \( T_2 \). However, in practical systems, due to limited channel estimation capacity, there is more or less estimation error at \( T_2 \). A commonly used CSI error model is given by [13]

\[ h = \rho h + \sqrt{1 - \rho^2} n, \]

where \( h, h_e \) are the real and estimated CSI, respectively. \( n_e \) is the estimation error noise distributed as \( \mathcal{C}\mathcal{N}(0, 1) \), and is independent of \( h_e \). \( \rho \) is the correlation coefficient between \( h \) and \( h_e \), and larger \( \rho \) means higher estimation precision. As seen in (1), the energy beamforming vector is selected based on the estimated CSI, so CSI mismatch may result in SNR loss. Moreover, we assume \( T_1 \) has the perfect CSI of \( g \) Under this condition, the harvesting power and the average information transmission rate are given by

\[ Q_{h} = \eta a \rho \| h^H w^e \|^2 \]

\[ = \eta a \rho (\rho^2 \| h^H w^e \|^2 + (1 - \rho^2)) \]

and

\[ R_e(\tau) = \frac{T - \tau}{T} \left[ \log_2 \left( 1 + \frac{a \rho^2 \| h \|^2 \| h^H w^e \|^2}{T - \tau} \right) + \frac{a(1 - \rho^2) \| h^H w^e \|^2 \| g \|^2}{T - \tau} \right. \]

\[ + 2a \rho \sqrt{1 - \rho^2} \mathcal{R}(w^H h_e, n_e, w^e) \| g \|^2 \right], \]

respectively, \( \mathcal{R}(x) \) denotes the real part of complex value \( x \). Thus, we can derive an upper bound on the average information transmission rate in presence of imperfect CSI as

\[ R_{e}^u(\tau) = \frac{T - \tau}{T} \log_2 \left( 1 + \frac{4a \rho^2 N_t^2 \left( 1 - \frac{N_t - 1}{N_t} 2^{-\frac{\eta}{\rho}} \frac{1}{\tau} \right)}{T - \tau} \right) \]

\[ + \frac{2a(1 - \rho^2) N_t \tau}{T - \tau}. \]

Compare (29) with (10), it is found that the impact of imperfect CSI at \( T_2 \) is to replace the term \( 4a \rho^2 N_t^2 \left( 1 - \frac{N_t - 1}{N_t} 2^{-\frac{\eta}{\rho}} \frac{1}{\tau} \right) \) with

\[ \left( 4a \rho^2 N_t^2 \left( 1 - \frac{N_t - 1}{N_t} 2^{-\frac{\eta}{\rho}} \frac{1}{\tau} \right) + 2a(1 - \rho^2) N_t \right) \tau \]

Notice that only when \( 4a \rho^2 N_t^2 \left( 1 - \frac{N_t - 1}{N_t} 2^{-\frac{\eta}{\rho}} \frac{1}{\tau} \right) + 2a(1 - \rho^2) N_t \) is small, namely

\[ B < -(N_t - 1) \log_2 (1 + \frac{4a \rho^2 N_t^2 \left( 1 - \frac{N_t - 1}{N_t} 2^{-\frac{\eta}{\rho}} \frac{1}{\tau} \right)}{T - \tau} < 0, \]

the performance in presence of imperfect CSI is better than that under ideal condition based on the same wireless energy and information transfer tradeoff. However, \( B \) is impossible to be smaller.
than zero, so imperfect CSI will lead to the performance loss inevitably. Furthermore, replacing $b$ in the optimization problem $J_1$ with $a\rho^2 N_t^2 \left( 1 - N_t^{-1} \left( \frac{\rho}{N_t} - 1 \right) + a(1 - \rho^2) N_t \right)$, we can derive the corresponding optimal tradeoff.

V. NUMERICAL RESULTS

To examine the effectiveness of the proposed algorithms for the wireless energy and information transfer tradeoff in limited feedback multi-antenna systems with energy beamforming, we present several numerical results in different scenarios. For convenience, we set $N_t = 4$, $\eta = 0.8$, $\sigma^2 = -125$dBm, $T = 5$ms and $\alpha = \theta = 10^{-2} d^{-\nu}$ for all simulation scenarios, where $d = 10$ms is the distance between $T_1$ and $T_2$ and $\nu = 4$ is the path loss exponent. Note that in the above path loss model, a 20dB path loss is assumed at a reference distance of 1m.

In the following, we use UA and LA to denote the proposed algorithms for the wireless energy and information tradeoff based on an upper bound and an approximate lower bound of the average information transmission rate, respectively. For comparison, we use EA and OA to denote the fixed equal duration allocation algorithm, namely $\tau = T/2 = 2.5$ms, and the optimal algorithm by numerically searching the optimal $\tau$ in (5), respectively.

![Fig. 3. Performance comparison of different tradeoff algorithms](image1)

Fig. 3 compares the average information transmission rates $R$ of the four wireless energy and information tradeoff algorithms, namely UA, LA, EA and OA, when $B = 4$. It is found that the proposed UA can nearly achieve the same performance as the OA in the whole transmit power region. Although the EA has the low complexity, the proposed UA and LA perform better than the EA in the moderate and high transmit power regions. Especially for the UA, it provides more than 5dBm gains at $R = 5$ b/s/Hz with respect to the EA. As the transmit power increases, the performance gain becomes larger. Therefore, the proposed UA is more appealing in the practical systems from the perspectives of both the performance and the complexity.

Next, we show the benefit of the proposed energy beamforming from the perspective of feedback amount. In this scenario, we only use the proposed UA due to its good performance as seen in Fig. 3. It is known that the larger feedback means more CSI at $T_1$, so the efficiency of wireless energy transfer or energy beamforming is higher due to transmission direction alignment, and thus the average information transmission rate is larger. As seen in Fig. 4, compared with no energy beamforming ($B = 0$), energy beamforming even with a very small codebook ($B = 2$) can obtain an obvious performance gain. For example, at $P_1 = 1$dB, there is more than 1 b/s/Hz gain. Moreover, with the increase of transmit power, the performance gain becomes larger. On the other hand, for energy beamforming, the performance gain by adding from $B = 2$ to 4, and from $B = 4$ to that with full CSI, namely $B = \infty$, is not as large as $B = 0$ to $B = 2$. Nonetheless, the CSI feedback plays an important role and we can control the feedback amount or the codebook size to balance the performance and feedback overhead.

![Fig. 4. Performance comparison of UA algorithm with different codebook sizes](image2)

![Fig. 5. Performance comparison of UA algorithm with different correlation coefficients](image3)

Finally, we investigate the impact of imperfect CSI at $T_2$ on the average information transmission rate based on the proposed UA with $B = 4$. As analyzed earlier, we use correlation coefficient $\rho$ to denote the precision of CSI at $T_2$. 

Smaller $\rho$ means severe CSI mismatch. It is shown in Fig. 5 that when CSI estimation is relatively accurate, i.e $\rho = 0.9$, the performance loss is very small. With the decrease of $\rho$, there is a gap with respect to the case of $\rho = 1$, namely perfect CSI at $T_2$.

VI. Conclusion

A major contribution of this paper is the introduction of the wireless energy and information transfer tradeoff, so as to maximize the average information transmission rate. Moreover, we also propose to perform energy beamforming in the multiantenna wireless energy and information transfer system to optimize the efficiency of energy transfer, via the CSI feedback based on a quantization codebook. For a given codebook size, we derive an upper bound and an approximate lower bound of the average information transmission rate. By maximizing the two bounds, we obtain two tradeoff algorithms. Furthermore, we investigate the impact of imperfect CSI on the average information transmission rate, and present the corresponding performance.

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