Initial conditions for vector inflation

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Abstract. Recently, a model of inflation using non-minimally coupled massive vector fields has been proposed. For a particular choice of non-minimal coupling parameter and for a flat Friedmann–Robertson–Walker model, the model is reduced to the model of chaotic inflation with massive scalar field. We study the effect of non-zero curvature of the universe on the onset of vector inflation. We find that in a curved universe the dynamics of vector inflation can be different from the dynamics of chaotic inflation, and the fraction of the initial conditions leading to inflationary solutions is reduced as compared with the chaotic inflation case.

Keywords: inflation, physics of the early universe
1. Introduction

The inflationary universe scenario has now become the standard paradigm for the early universe because of its success in explaining the present state of the Universe. Currently all successful models of inflation are based on weakly interacting scalar field(s) because higher spin fields generically induce a spatial anisotropy and because the effective masses of such fields are usually of the order of the Hubble scale and slow-roll inflation does not occur [1] (see [2] for recent efforts). Therefore, it comes as a surprise that slow-roll inflation can be made possible even for massive vector fields [3] by introducing a non-minimal coupling of the vector fields to curvature (for the density perturbations from vector inflation, see [4]). The most intriguing point of the model proposed in [3] is that non-minimally coupled vector fields appear to behave in precisely the same way as a massive minimally coupled scalar field (chaotic inflation model) in a flat universe. Then, an immediate question is that of to what extent the correspondence is perfect. In this paper, we address the problem from the point of view of the initial conditions for vector inflation as a primary (primordial) inflation, namely, how generic is vector inflation among all possible classical initial conditions? This problem has been extensively studied for chaotic inflation [6–8], and it was found that an initial kinetic energy or a spatial curvature of the universe generally does not prevent the onset of chaotic inflation, and hence the inflation is a general property of the system (see however [9] for the problem of the measure). We shall study the ‘vector counterpart’ of the problem studied for chaotic inflation in [6, 7].

2. Vector inflation

We briefly introduce the model of vector inflation proposed in [3]. The model is a massive vector field (Proca field [10]) $U_\mu$ non-minimally coupled to gravity, and the action is given by

$$S = \int \sqrt{-g} \, d^4 x \left( \frac{1}{16\pi G} R - \frac{1}{4} F_{\mu}^{\nu} F^{\mu\nu} - \frac{1}{2} m^2 U_\mu U^\mu + \frac{1}{2} \xi R U_\mu U^\mu \right),$$

(1)
curvature. Therefore, surprisingly, for $\xi$ a dimensionless parameter for non-minimal coupling, and we adopt the metric signature of $(-+++).$ The correspondence between vector inflation and scalar inflation is given in table 1. Note the difference in choice of sign for $\xi$ between the vector field and scalar field.

The equations of motion are given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G[F_{\mu\alpha}F_{\nu}^{\alpha} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + (m^2 - \xi R)U_{\mu}U_{\nu} - \frac{1}{4}g_{\mu\nu}(m^2 - \xi R)U_{\alpha}U^{\alpha} - \xi R_{\mu\nu}U_{\alpha}U^{\alpha} + \xi(\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box)U_{\alpha}U^{\alpha}],$$

$$\nabla_\nu F^{\mu\nu} - m^2 U^{\mu} + \xi RU^{\mu} = 0.$$  

In particular, the equations of motion for the vector field in a FRW universe model are given by

$$\frac{1}{a^2}\Delta U_0 - \frac{1}{a^2}\partial_i \dot{U}_i - m^2 U_0 + \xi RU_0 = 0,$$

$$\dot{U}_i + \frac{\dot{a}}{a}U_i - \frac{1}{a}(\alpha \partial_i U_0) + \frac{1}{a^2}(\partial_i (\partial_k U_k) - \Delta U_i) + m^2 U_i - \xi RU_i = 0,$$

where $a$ is the scale factor, $\Delta$ is the Laplacian with respect to the spatial metric, the dot denotes the derivative with respect to the cosmic time and the summation over repeated spatial indices is assumed. Thus for the homogeneous vector field, equation (4) implies $U_0 = 0$ and, from equation (5), in terms of $\varphi_i = U_i/a,$ we obtain

$$\ddot{\varphi}_i + 3H\dot{\varphi}_i + \left(m^2 + (1 - 6\xi)(\dot{H} + 2H^2) - \frac{6\xi k}{a^2}\right)\varphi_i = 0,$$

where we have used $R = 6(\dot{H} + 2H^2 + k/a^2)$ with $H = \dot{a}/a,$ $k = (0, \pm 1)$ being the spatial curvature. Therefore, surprisingly, for $\xi = 1/6$ and\(^1\) for a flat universe ($k = 0$), the equation of motion for $\varphi_i$ (the norm of the vector field $U_i$) is reduced to that of minimally coupled massive scalar fields.

Generally, a dynamical vector field has a preferred direction, and introducing such a vector field may not be consistent with the isotropy of the universe. In fact, the energy–momentum tensor of the vector field $U_\mu$ (RHS of equation (2)) has anisotropic components. However, the anisotropic part of the energy–momentum tensor can be eliminated by introducing a triplet of mutually orthogonal vector fields [5]. After doing that, we obtain the energy density $\rho$ and the pressure $p$ of the vector fields:

$$\rho = \frac{3}{2}\dot{\varphi}_i^2 + \frac{3}{2}m^2 \varphi_i^2 - \frac{3k}{2a^2} \dot{\varphi}_i^2,$$

$$p = \frac{3}{2}\dot{\varphi}_i^2 - \frac{3}{2}m^2 \varphi_i^2 + \frac{k}{2a^2} \dot{\varphi}_i^2.$$  

\(^1\) Note that the value of $\xi$ for the conformal coupling for a scalar field is $\xi = 1/6$ in the notation given in table 1.

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**Table 1. Non-minimal vector field versus non-minimal scalar field.**

| Lagrangian density | Vector | Scalar |
|-------------------|--------|--------|
| $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 U_{\mu}U^{\mu} + \frac{1}{2}\xi RU_{\mu}U^{\mu}$ | $\xi = 0$ ($m = 0$) | $-\frac{1}{4}\nabla_\mu \phi \nabla^\mu \phi - V(\phi) - \frac{1}{2}\xi \phi^2$ |
| Conformal coupling | $\xi = 0$ ($m = 0$) | $\xi = \frac{1}{6}$ ($V = 0$) |

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Then we finally obtain the basic equations of motion for a general FRW universe:

\[ \ddot{\phi}_i + 3H \dot{\phi}_i + \left( m^2 - \frac{k}{a^2} \right) \phi_i = 0, \tag{9} \]

\[ H^2 + \frac{k}{a^2} = 4\pi G \left( \dot{\phi}_i^2 + m^2 \phi_i^2 - \frac{k}{a^2} \phi_i^2 \right), \tag{10} \]

\[ \dot{H} + H^2 = -4\pi G (2\dot{\phi}_i^2 - m^2 \phi_i^2), \tag{11} \]

\[ \dot{a} = Ha. \tag{12} \]

It is noted that the effective mass squared of the vector field becomes negative and the field becomes tachyonic for a closed universe with \( k/a^2 > m^2 \).

3. Initial conditions for vector inflation

Having introduced the basic equations, we have to specify the initial conditions for \( a, H, \phi_i, \dot{\phi}_i \) in order to solve them. Because of the resemblance to chaotic inflation, as a primordial inflation, the natural choice would be chaotic initial conditions: the energy density of the fields is of the order of the Planck scale and one may choose arbitrary initial values of \( \phi_i \) and \( \dot{\phi}_i \) among all the possible choices. For chaotic inflation it has been shown \([6,7]\) that almost all of the initial conditions lead to inflationary solutions in flat and open universes, and in a closed universe the probability of inflation\(^2\) is about 67% for a realistic value of \( m \) \([7]\). Although the universe might be inhomogeneous at the Planck epoch, as a first step toward more general universes, the classical evolution in a FRW universe is studied.

However, the situation becomes ambiguous for vector inflation. It is not clear to what energy density we should assign the initial conditions. Of course one choice would be \( \rho \), equation (7), derived from the energy–momentum tensor which, together with \( p \), satisfies energy–momentum conservation: \( \dot{\rho} + 3H(\rho + p) = 0 \), and one may specify \( \rho_{\text{init}} \sim M_{\text{pl}}^4 \) with \( M_{\text{pl}} = G^{-1/2} \) (case (1)). However, it contains a curvature induced term, and such a geometric part may alternatively be included in the definition of the effective Planck mass, \( M_{\text{pl}}^2 \). Then another possible choice would be \( \rho_\varphi = (3/2)(\dot{\phi}_i^2 + m^2 \phi_i^2) \) and taking \( \rho_{\varphi\text{init}} \sim M_{\text{pl}}^4 \) (case (2)). However, it is noted that \( \rho_\varphi \) together with \( p_\varphi = (3/2)(\dot{\phi}_i^2 - m^2 \phi_i^2) \) no longer satisfies the conservation law. As a more geometrical choice of the initial conditions, one may also consider the case of \( R \sim M_{\text{pl}}^2 \) (case (3)).

One immediately finds that for the first choice (case (1)) the surface of the initial conditions in the \((\phi_i, \dot{\phi}_i)\) plane can become hyperbolic for a closed universe with \( k/a^2 > m^2 \),\(^3\) while it is elliptic both in the latter case and in the case of chaotic inflation. Then the possible initial values of the vector fields are unbounded and the initial kinetic energy of \( \phi_i \) exceeds the Planck scale: \( \phi_i^2 \gtrsim M_{\text{pl}}^4 \), which suggests that inflation may not take place.

\(^2\) There is caveat here. In \([6,7]\) the uniform measure in \((\phi, \dot{\phi})\) space is assumed. However, it is not a dynamically invariant measure. The natural measure contains the factor \( a^3 \), which leads to the opposite conclusion that the probability of inflation is reduced by \( \exp(-3N) \) with \( N \) being the e-folding number \([9]\). We will not pursue this problem here. Rather we compute the probability of vector inflation in comparison with chaotic inflation.

\(^3\) Note that the effective mass of the vector field is tachyonic for \( k/a^2 > m^2 \).
or simply implies that such initial conditions should be eliminated from chaotic initial conditions since the kinetic energy exceeds the Planck scale and the analysis cannot be treated classically. Restricting to the elliptic initial data for a closed universe ($k/a^2 < m^2$), then from equation (10) the allowed initial value of the expansion rate is restricted to being within the very narrow range $M_{pl}^2 \gtrsim H_{\text{init}}^2 \gtrsim M_{pl}^2 - m^2$. Thus from the comparison with chaotic inflation, we expect that the probability of vector inflation in a closed universe will be reduced by a factor of $m^2/M_{pl}^2$. Moreover, for an open universe, the surface of the initial conditions is shrunk in the $\varphi_i$-direction: $\varphi_i \lesssim M_{pl}^2/\sqrt{m^2 - k/a^2}$, and the slow-roll condition ($\varphi_i \gtrsim M_{pl}$) can be violated from the beginning if $-k/a^2 \gtrsim M_{pl}^2 - m^2$.

On the other hand, for the second choice (case (2)), the surface of the initial conditions in the ($\varphi_i, \dot{\varphi}_i$) plane becomes hyperbolic irrespective of the spatial curvature of the universe, and the kinetic energy is bounded from below: $\dot{\varphi}_i^2 \gtrsim M_{pl}^4$, which suggests that inflation does not occur.

Finally, the third choice (case (3)), using the Einstein equation, corresponds to $8\pi G(\rho - 3p) \sim M_{pl}^2$, which implies, in terms of $\varphi_i$ and $\dot{\varphi}_i$, $-\ddot{\varphi}_i^2 + (2m^2 - k/a^2)\dot{\varphi}_i^2 \sim M_{pl}^4$. Again the surface of the initial conditions in the ($\varphi_i, \dot{\varphi}_i$) plane becomes hyperbolic irrespective of the spatial curvature of the universe and this time the potential energy has a minimum, $\varphi_i^2 \gtrsim M_{pl}^2/(2m^2 - k/a^2)$, and generically dominates over the kinetic energy, which suggests that inflation may occur generically. Note that this applies also to chaotic inflation. However, for vector inflation, a closed universe with a large spatial curvature of $k/a^2 > 2m^2$ is forbidden as an initial condition. Moreover, for an open universe, the minimum of $\varphi_i$ can be smaller than $M_{pl}$ if $-k/a^2 \gtrsim M_{pl}^2 - 2m^2$ and consequently some of the initial conditions violate the slow-roll condition from the beginning.

In the following we consider each possibility in detail by numerically solving the equations of motion. For later convenience, we introduce the following dimensionless variables:

\begin{align*}
\varphi &= \frac{M_{pl}}{\sqrt{4\pi}} x, \\
\dot{\varphi} &= \frac{m M_{pl}}{\sqrt{4\pi}} y, \\
H &= mz, \\
t &= \frac{\tau}{m},
\end{align*}

where $M_{pl} = G^{-1/2}$. In terms of these variables, the basic equations (9)–(12) are rewritten in the form

\begin{align*}
x' &= y, \\
y' &= -3yz - x + \frac{k}{a^2 m^2} x, \\
z' &= x^2 - 2y^2 - z^2, \\
x^2 \left( 1 - \frac{k}{a^2 m^2} \right) + y^2 - z^2 &= \frac{k}{a^2 m^2}, \\
a' &= za,
\end{align*}

where the prime denotes the derivative with respect to $\tau$. The evolution of the universe is described by the trajectories in the three-dimensional phase space $(x, y, z)$.

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3.1. Case (1)

As a possible choice of the initial conditions for $\varphi$ and $\dot{\varphi}$, we consider

$$\rho = \frac{3}{2} \left( \dot{\varphi}_i^2 + m^2 \varphi_i^2 - \frac{k}{a^2} \varphi_i^2 \right) \sim M_{\text{pl}}^4.$$  \hfill (19)

In terms of dimensionless variable $x, y$, we set the initial conditions as

$$x_{\text{init}}^2 \left( 1 - \frac{k}{a^2 m^2} \right) + y_{\text{init}}^2 = \left( \frac{M_{\text{pl}}}{m} \right)^2.$$  \hfill (20)

Given the initial values of $x_{\text{init}}$ and $y_{\text{init}}$, from equation (17) the initial $z_{\text{init}}$ is restricted as $z_{\text{init}} > M_{\text{pl}}/m$ for $k < 0$ or $z_{\text{init}} < M_{\text{pl}}/m$ for $k > 0$. Given $z_{\text{init}}$, the scale factor $a$ is determined from the constraint equation (17).

In an open universe ($k < 0$), from equation (20) the surface of the initial conditions in the $(x, y)$ plane is an ellipse and the trajectories are inside a cone, $z^2 > x^2 + y^2$. The typical trajectories projected onto the $(x, y)$ plane are shown in figure 1. We take $m = 0.1M_{\text{pl}}$ for clarity of the figures. The results of chaotic inflation are also shown there for comparison. The surface of the initial conditions is a circle then. The trajectories of the vector field are shrunk in the $x$-direction. Hence in the $(x, y)$ plane, the fraction of the initial conditions leading to inflationary solutions is slightly reduced. Moreover, since the slow-roll condition is violated for $\varphi_i \lesssim M_{\text{pl}}$ (or $x \lesssim 1$), from equation (20) vector inflation does not take place even in an open universe for $-k/a^2 \gtrsim M_{\text{pl}}^2 - m^2$. Therefore, from equations (17) and (20), the allowed range of $z_{\text{init}}$ is bounded:

$$z_{\text{init}} \lesssim \sqrt{2 \left( \frac{M_{\text{pl}}}{m} \right)^2 - 1}.$$  \hfill (21)
which is in sharp contrast with chaotic inflation where the range of $z_{\text{init}}$ for an open universe is unbounded.

In a closed universe ($k > 0$), from equation (20), the surface of the initial conditions in the $(x, y)$ plane is an ellipse for $k/a^2 < m^2$, while it becomes a hyperbola for $k/a^2 > m^2$. Thus for $k/a^2 > m^2$ the allowed values of $x$ and $y$ are unbounded, and moreover the initial kinetic energy already exceeds the Planck density, $y^2 > (M_{\text{pl}}/m)^2$, which suggests that inflation does not occur. On the other hand, for $k/a^2 < m^2$, from equations (17) and (20), the allowed range of $z_{\text{init}}$ is very narrow:

$$\sqrt{\left(\frac{M_{\text{pl}}}{m}\right)^2 - 1} < z_{\text{init}} < \frac{M_{\text{pl}}}{m}. \quad (22)$$

We calculate the evolution equations (14)–(18) numerically and determine whether inflation with enough e-foldings (>60) occurs or not. The results are given in figure 2 for the case of $k/a^2 > m^2$ ($z_{\text{init}} = 0.9M_{\text{pl}}/m$) and figure 3 for the case of $k/a^2 < m^2$ ($z_{\text{init}} = 1 - 0.1(m/M_{\text{pl}})^2(M_{\text{pl}}/m)$). We find that vector inflation does not occur for $k/a^2 < m^2$ (or $z_{\text{init}} < \sqrt{(M_{\text{pl}}/m)^2 - 1}$). Therefore the initial condition for $z$ for vector inflation is the requirement of satisfying equation (22), and the probability of vector inflation is much reduced: $\sim(m/M_{\text{pl}})^2 \sim 10^{-12}$ instead of 0.67 for $m = 10^{-6}M_{\text{pl}}$. 

Figure 2. The result of evolution from the initial conditions (case (1)) in a closed universe with $z = 0.9M_{\text{pl}}/m$. The circles correspond to the initial conditions which result in inflation, the triangles are for e-folds less than 60, the crosses for recollapse. $m = 0.1M_{\text{pl}}$. The upper (lower) panel is for vector (chaotic) inflation. The dotted curve is the surface of the initial conditions (quantum boundary) for chaotic inflation.
3.2. Case (2)

As another choice for the initial condition, we consider

\[ \rho_\varphi = \frac{3}{2} \left( \dot{\varphi}_i^2 + m^2 \varphi_i^2 \right) \sim M_{\text{pl eff}}^4 = \frac{M_{\text{pl}}^4}{1 + 4 \pi \left( \frac{\varphi_i^2}{M_{\text{pl}}^2} \right)^2}. \]  

(23)

Although this definition of \( \rho_\varphi \) is not derived from a covariantly conserved energy–momentum tensor, we consider this possibility briefly just for reference. In terms of dimensionless variables \( x, y \), we set the initial conditions as

\[ x_{\text{init}}^2 + y_{\text{init}}^2 = \left( \frac{M_{\text{pl}}}{m} \right)^2 (1 + x_{\text{init}}^2)^2. \]  

(24)

Given the initial values of \( x_{\text{init}} \) and \( y_{\text{init}} \), from equation (17) the initial \( z_{\text{init}} \) is restricted as \( z_{\text{init}} > \frac{M_{\text{pl}}}{m}(1 + x_{\text{init}}^2) \) for \( k < 0 \) or \( z_{\text{init}} < \frac{M_{\text{pl}}}{m}(1 + x_{\text{init}}^2) \) for \( k > 0 \). Given \( z_{\text{init}} \), the scale factor \( a \) is determined from the constraint equation (17). In this case, the surface of the initial conditions in the \( (x, y) \) plane becomes hyperbolic irrespective of \( k \), and the initial kinetic energy exceeds the Planck density, \( y^2 > \left( \frac{M_{\text{pl}}}{m} \right)^2 \). In fact, from the numerical calculations, we find that vector inflation does not occur at all for this choice of the initial conditions. In figure 4, the results of the numerical integrations of the evolution equations are shown for a closed universe with \( z = 0.9 M_{\text{pl}}/m \) for comparison with figure 2. Hence, we conclude that this choice of initial conditions is not appropriate for vector inflation.
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Figure 4. The result of evolution from the initial conditions (case (2)) in a closed universe with $z = 0.9 M_{\text{pl}}/m$. $m = 0.1 M_{\text{pl}}$. The upper (lower) panel is for vector (chaotic) inflation. The dotted curve is the quantum boundary for chaotic inflation.

3.3. Case (3)

As a geometrical choice of the initial condition, we consider

$$R = 8\pi G(\rho - 3p) = 24\pi G \left(-\dot{\varphi}_i^2 + 2m^2 \varphi_i^2 - \frac{k}{a^2} \varphi_i^2\right) \sim M_{\text{pl}}^2. \quad (25)$$

In terms of dimensionless variable $x, y$, we set the initial conditions as

$$x_{\text{init}}^2 \left(2 - \frac{k}{a^2 m^2}\right) - y_{\text{init}}^2 = \left(\frac{M_{\text{pl}}}{m}\right)^2. \quad (26)$$

Unlike for the previous two cases, we first specify the initial scale factor $a$ and then specify $x_{\text{init}}$ and $y_{\text{init}}$ according to equation (26) since we cannot take a $z_{\text{init}}$ which is uniform about $x_{\text{init}}$ and $y_{\text{init}}$. Given $a, x_{\text{init}}$ and $y_{\text{init}}, z_{\text{init}}$ is determined from the constraint equation (17). One immediately finds that for a closed universe the spatial curvature is restricted to the range $k/a^2 < 2m^2$. The surface of the initial conditions in the $(x, y)$ plane then becomes hyperbolic and the initial potential energy instead has a minimum, $x^2 > (M_{\text{pl}}/m)^2/(2 - k/a^2 m^2)$; this also applies to chaotic inflation.

In an open universe, the minimum of $x$ can be smaller than unity and the slow-roll condition is violated from the beginning if $-k/a^2 \gtrsim M_{\text{pl}}^2 - 2m^2$. Then some of the initial conditions do not lead to inflationary solutions. In the upper half of figure 5, the results of the numerical integration of the evolution equations are shown for an open universe with $-k/a^2 = M_{\text{pl}}^2$, the case which violates the slow-roll conditions from the beginning. We find that some of the initial data do not lead to inflationary solutions with enough e-folds. Thus in an open universe, the initial spatial curvature is restricted, as $-k/a^2 \lesssim M_{\text{pl}}^2 - 2m^2$. 

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Figure 5. Upper panel: the result of evolution from the initial conditions (case (3)) in an open universe with \(-k/a^2 = M_{\text{pl}}^2\). Lower panel: the result of evolution in a closed universe with \(k/a^2 = m^2\). \(m = 0.1M_{\text{pl}}\). Only those regions with \(y > 0\) are shown. The dashed curve is the surface of the same initial conditions for chaotic inflation. The dotted curve is the quantum boundary for chaotic inflation.

In a closed universe, on the other hand, not all the initial data satisfying equation (26) satisfy the constraint equation (17). From the positivity of \(z^2\), \(x_{\text{init}}^2\) and \(y_{\text{init}}^2\) must satisfy

\[
x_{\text{init}}^2 \left(1 - \frac{k}{a^2 m^2}\right) + y_{\text{init}}^2 > \frac{k}{a^2 m^2}.
\]

The two conditions in equations (26) and (27) are compatible if \(k/a^2 < 3m^2/2\). Thus for a closed universe the initial spatial curvature is further constrained, as \(k/a^2 < 3m^2/2\). Then in a closed universe, the initial potential energy dominates over the kinetic energy and exceeds the Planck density from the beginning, \(x_{\text{init}}^2 > (M_{\text{pl}}/m)^2/2\), which suggests that inflation always occurs. In fact, from the numerical calculations, we find that vector inflation does indeed occur as long as \(k/a^2 < 3m^2/2\). In the lower half of figure 5, the results of the numerical integrations of the evolution equations are shown for a closed universe with \(k/a^2 = m^2\).

4. Summary

The vector inflation model proposed in [3] is very similar to that of chaotic inflation. However, in a curved universe, the dynamics of vector inflation can be different from the dynamics of chaotic inflation. In particular, in an open universe, the allowed range of the initial Hubble parameter (for case (1)) is bounded. Moreover, in a closed universe, as compared with the chaotic inflation case, the fraction of the initial conditions (for case (1)) which lead to inflationary solutions is much reduced, by \((m/M_{\text{pl}})^2\). We also find
that for a naive choice of the initial conditions (case (2)) vector inflation does not occur, irrespective of the spatial curvature of the universe. Further we find that if the initial spacetime curvature is taken to be Planckian (case (3)), there are some restrictions on the initial spatial curvature, both for an open universe and for a closed universe, which are absent for chaotic inflation and that the fraction of the initial conditions leading to inflationary solutions is reduced as compared with the chaotic inflation case. Since the correspondence with chaotic inflation is realized for a flat FRW universe and the correspondence does not hold for a general spacetime, it would also be interesting to study the effect of anisotropies and/or inhomogeneities on the onset of vector inflation along the lines of [11, 8, 12]. As a final remark, we note that although the dynamics of vector inflation as a primary (primordial) inflation is different from the dynamics of chaotic inflation, vector inflation as a secondary inflation (or inflation after tunneling) is no different from chaotic inflation since the universe is already large enough, and vector inflation takes place without much fine-tuning of the initial conditions.

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