A New Technique for Determining the Properties of a Narrow $s$-channel Resonance at a Muon Collider

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We explore an alternative to the usual procedure of scanning for determining the properties of a narrow $s$-channel resonance. By varying the beam energy resolution while sitting on the resonance peak, the width and branching ratios of the resonance can be determined. The statistical accuracy achieved is superior to that of the usual scan procedure in the case of a light SM-like Higgs boson with $M_H > 130$ GeV or for the lightest pseudogoldstone boson of a strong electroweak breaking model if $M_{\rho} > 150$ GeV.

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The optimal means for studying $s$-channel resonance production at a lepton collider depends critically on the resonance width $\Gamma$ compared to the natural resolution, $\sigma_E$ (the Gaussian width), in $E \equiv \sqrt{s}$. If $\Gamma$ is at least as large as the natural value of $\sigma_E$ at $E = M$, the best procedure is to simply scan the resonance using measurements at $E = M$ and at several locations off the resonance peak. However, either a light Standard Model (SM) (or SM-like) Higgs boson, $H$, or the lightest pseudogoldstone boson of a strong electroweak breaking model, $P^0$, will typically have such a small width that $\Gamma \gtrsim \sigma_M$ can only be achieved by compressing the beams to $R$ values far smaller than the natural value, which can only be accomplished with substantial loss of instantaneous luminosity. Writing $\Delta E_{\text{beam}}/E_{\text{beam}} = 0.01 R$, with $R$ in percent, $\sigma_E = 0.01 R(\%)/\sqrt{2} \sim 2$ MeV $\left(\frac{E}{100 \text{ GeV}}\right) \left(\frac{R}{0.003}\right)$, where $R = 0.003\%$ is the best resolution that can be achieved at a $\sqrt{s} \sim 100$ GeV muon collider. For comparison, $\Gamma_H \sim 1, 2, 4, 16$ MeV at $M_H = 50, 100, 130, 150$ GeV while $\Gamma_{\rho_0} \sim 2, 5, 11, 21$ MeV at $M_{\rho_0} = 50, 100, 150, 200$ GeV (for $N_{\text{TC}} = 4$ and the model parameter choices of Ref. [4]).

At a muon collider, the natural $R$ range yielding maximal luminosity is $R = 0.1\% \div 0.15\%$. $\mathcal{L}$ declines rapidly as $R$ is decreased below this range. For $E = 100$ GeV, one finds \cite{5} (see also Table 5 in \cite{6}) $L_{\text{year}} = 1.2 \text{ fb}^{-1} \left(\frac{R}{0.12\%}\right)^{0.67362}$ for $0.003\% \lesssim R \lesssim 0.12\%$, yielding $L_{\text{year}} = 0.1 \text{ fb}^{-1} (0.22 \text{ fb}^{-1})$ at $R = 0.003\% (0.01\%)$. We will presume that a machine specifically designed for operation at any given energy within a factor of 2 of $E = 100$ GeV will have the same $L_{\text{year}}$. Variations of the luminosity of a machine designed for operation at $E = 100$ GeV, but run at some other energy will not be accounted for.

A scan determination of the properties of a SM-like Higgs boson at a muon collider has been studied in Ref. \cite{6} (see also \cite{7}), where it was shown that the accuracy of the $R = 0.003\%$ measurements might make it possible to distinguish a SM Higgs from the lightest Higgs of the minimal supersymmetric model (MSSM). Precision measurements of the properties of the $P^0$ would also be possible via scanning \cite{6} and very valuable. In this letter, we explore an alternative technique to scanning. The new procedure consists of collecting two sets of data at $E = M$, one while operating with $\Gamma > \sigma_M$ (or at least $\sim \sigma_M$) and one with $\sigma_M > \Gamma$. We demonstrate that this alternative procedure leads to smaller statistical errors for resonance properties than the con-
ventional scanning procedure for some ranges of $M_H$ and $M_{P_{10}}$.

We presume that the initial scan required to precisely locate the resonance provides a rough determination of its width. A Breit-Wigner form for the resonance cross section convoluted with a Gaussian energy distribution in $E$ of width $\sigma_M$ centered at $E = M$ yields the effective cross section $\sigma_c$. For a given final state $F$, one finds (see Fig. 2): $\sigma_F^c = \frac{4\pi B_{\pi^+\pi^-}B_F}{M^2} \frac{1}{\sqrt{\Gamma}}$ for $\Gamma \gg \sigma_M$ and $\sigma_F^c = \frac{\pi B_{\pi^+\pi^-}B_F}{2\sqrt{\Gamma}}$ for $\Gamma \ll \sigma_M$. Here $B_{\pi^+\pi^-}$ and $B_F$ are the $\mu^+\mu^-$ and $F$ branching ratios. If we operate the collider at $\sigma_M^\text{min} \ll \Gamma$ and $\sigma_M^\text{max} \gg \Gamma$, we find $\sigma_c (\sigma_M^\text{min})/\sigma_c (\sigma_M^\text{max}) = [2\sqrt{\sigma_M^\text{max}}]/[\sqrt{\Gamma}]$.

Since $\sigma_M$ will be precisely known, $\Gamma$ can be determined from the ratio. The best determination of $\Gamma$ is obtained by combining results for all viable final state channels $F$. Once $\Gamma$ is known, the two measurements of $\sigma_F^c$ determine $B_{\mu^+\mu^-}B_F$ for any $F$. The total width and branching ratios (converted to partial widths using $\Gamma$) are key to understanding the nature of the resonance.

In practice, $\sigma_M^\text{max}/\sigma_M^\text{min}$ will be limited in size. We define $\sigma_M^\text{central} = \sqrt{\sigma_M^\text{max}\sigma_M^\text{min}}$ and compute $r_c \equiv \sigma_c (\sigma_M^\text{min})/\sigma_c (\sigma_M^\text{max})$ (we temporarily drop the final state $F$ label) as a function of $\Gamma$. In Fig. 3 we plot $\Gamma/\sigma_M^\text{central}$ as a function of $r_c$. We denote the magnitude of the slope in the log – log plot by $|s|$. For a known $\sigma_M^\text{central}$, the $|s|$ at any $\Gamma/\sigma_M^\text{central}$ gives the relation $(\Delta \Gamma / \Gamma) = |s| (\Delta r_c / r_c)$, where $\Delta r_c / r_c$ is computed by combining the fractional statistical errors for $\sigma_c (\sigma_M^\text{min})$ and $\sigma_c (\sigma_M^\text{max})$ in quadrature. We observe that $\Gamma/\sigma_M^\text{central} \sim 2$ at $s \sim 1$ is not that much larger. The larger $\sigma_M^\text{max}/\sigma_M^\text{min}$, the smaller $|s|$ at any given $\Gamma/\sigma_M^\text{central}$. For example, for $\Gamma/\sigma_M^\text{central}$ in the range 2 to 3 (near 1), $\sigma_M^\text{max}/\sigma_M^\text{min} = 5,10,20$ gives $|s| \sim 2,1.55,1.3$ $(2.5,1.8,1.44)$; $|s| \rightarrow 1$ for very large $\sigma_M^\text{max}/\sigma_M^\text{min}$.

The $\Delta r_c / r_c$ fractional statistical error depends upon how $L$ behaves as a function of $\sigma_M$. For the $H$ and the $P^0$ it is best to use $\sigma_M^\text{min}$ corresponding to $R = 0.003\%$ and $\sigma_M^\text{max}$ corresponding to $R = 0.03\%$. The variation of $L_{\text{year}}$ given earlier implies $L_{\text{year}} = 0.1 \text{ fb}^{-1}$ $(0.47 \text{ fb}^{-1})$ for $R = 0.003\%$ $(0.03\%)$. If, for example, $\Gamma/\sigma_M^\text{central} = 1$, one finds $\sigma_c (\sigma_M^\text{min})/\sigma_c (\sigma_M^\text{max}) = 4.5$, implying that the signal rate $S(\sigma_M) = L_{\text{year}} (\sigma_M) \sigma_c (\sigma_M)$ is nearly the same for $\sigma_M^\text{max}$ as for $\sigma_M^\text{min}$. However, the background rate $B$ is proportional to $L$ and thus $B/L$ is a factor of 4.7 times larger at $\sigma_M^\text{max}$ than at $\sigma_M^\text{min}$. Consequently, the statistical error in the measurement of $\sigma_c (\sigma_M^\text{max})$ will be worse than for $\sigma_c (\sigma_M^\text{min})$ for the same $S$.

For a given running time at a given $\sigma_M$, one must compute the channel-by-channel $S$ and $B$ rates, compute the fractional error in $\sigma_c (\sigma_M)$ for each channel, and then combine all channels to get the net $\sigma_c (\sigma_M)$ error. This must be done for $\sigma_M = \sigma_M^\text{min}$ and $\sigma_M = \sigma_M^\text{max}$. One then computes the net $r_c$ and net $\sigma_c$ errors following standard procedures. The error $\Delta \sigma_c / \sigma_c$ is minimized by running only at $\sigma_M^\text{min}$, but $\Delta r_c / r_c$ is typically minimized for $L (\sigma_M^\text{max}) / L (\sigma_M^\text{min}) \lesssim 1$. For the SM Higgs, a good compromise is to take $L (\sigma_M^\text{min}) / L (\sigma_M^\text{max}) = 1$.

SM-Higgs boson - Below 110 GeV, the width of the Higgs increases approximately linearly with the mass (aside from logarithmic effects due to the running of the quark masses) which means that the ratio $\Gamma_H/\sigma_M$ is approximately constant. By choosing $R = 0.003\%$ we get $\Gamma_H/\sigma_M \approx 1$. The analysis at a muon collider done in Ref. 6 gives statistical errors for a three-point scan using scan points at $E = M, E = M \pm 2\sigma_M$ and $R = 0.003\%$, assuming $L = 0.4 \text{ fb}^{-1}$ total accumulated luminosity (corresponding to 4 years of operation), with $L/5$ employed at $E = M, 2L/5$ at $E = M + 2\sigma_M$ and $2L/5$ at $E = M - 2\sigma_M$. The results of that analysis are summarized in Table 6.

Let us now compare to the $r_c$-ratio technique. We have followed the procedure outlined in the previous section. We employ the same total of 4 years of operation as considered for the three-point scan, but always with $E = M_H$. We adopt the compromise choice of devoting two years to running at $R = 0.003\%$, accumulating $L = 0.2 \text{ fb}^{-1}$, and a second two years to running at $R = 0.03\%$, corresponding to $L = 0.94 \text{ fb}^{-1}$ of accumulated luminosity. The resulting statistical errors are summarized in Table 6.

We observe that the ratio technique becomes superior to the scan technique for the larger $M_H$ values. This is correlated with the fact that $\Gamma_H/\sigma_M^\text{min}$ (where $\sigma_M^\text{min}$ is that for $R = 0.003\%$) becomes substantially
larger than 1 for such $M_H$. In particular, for larger $M_H$, $\Gamma_H/\sigma_M^\text{central}$ is in a range such that $|s|$ and, consequently, the error in $\Gamma_H$ will be minimal. Thus, the two techniques are actually quite complementary — by employing the best of the two procedures, a very reasonable determination of $\Gamma_H$ and very precise determinations of the larger channel rates will be possible for all $M_H$ below $2M_N$.

For the larger $M_H$ values such that $\Gamma_H/\sigma_M(R = 0.003\%)$ is substantially above 1, one could ask whether the scan-procedure errors could be reduced by running at larger $R$. In fact, the statistical errors for $\Gamma_H$ are much poorer if a larger value of $R$ is employed; the $R = 0.003\%$ results are the best that can be achieved despite the smaller luminosity at $R = 0.003\%$ as compared to higher $R$ values. For example, the error in $\Gamma_H$ for a given luminosity using $R = 0.01\%$ can be read off from Fig. 13 of [4]. One finds that $L(R = 0.01\%)/L(R = 0.003\%) = 20, 10, 2$ is required in order that the $\Gamma_H$ statistical errors for $R = 0.01\%$ be equal to those for $R = 0.003\%$ at $M_H = 130, 140, 150$ GeV, respectively. Existing machine designs are such that $L_{\text{year}}(R = 0.01\%)/L_{\text{year}}(R = 0.003\%) = 0.22 \text{ fb}^{-1}/0.1 \text{ fb}^{-1} = 2.2$. Thus, increasing $R$ would not improve the scan-procedure errors until $M_H > 150$ GeV.

**The lightest PNGB** - The $s$-channel production of the lightest neutral pseudo-Nambu-Goldstone boson (PNGB) ($P^0$), present in models of dynamical breaking of the electroweak symmetry which have a chiral symmetry larger than $SU(2) \times SU(2)$, has recently been explored [1]. The $P^0$ is much lighter than any other state in the models considered in [4] — 10 GeV < $M_{P^0}$ < 200 GeV is expected. The width $\Gamma_{P^0}$ as a function $M_{P^0}$ was summarized earlier. For low (high) $M_{P^0}$ it is somewhat larger (smaller) than that of a SM Higgs boson. Very high $\mu^+\mu^- \rightarrow P^0$ $s$-channel production rates are predicted for typical $\mu^+\mu^- \rightarrow P^0$ coupling strength if one operates the $\mu^+\mu^-$ collider so as to have extremely small beam energy spread, $R = 0.003\%$, for which $\sigma_M < \Gamma_{P^0}$. Once discovered at the LHC (or Tevatron) in the $\gamma \gamma$ mode, the $\mu^+\mu^-$ collider could quickly (in less than a year) scan the mass range indicated by the previous discovery (for the expected uncertainty in the mass determination) and center on $\sqrt{s} \approx M_{P^0}$ to within $< \sigma_M$. Using the optimal three-point scan [4] of the $P^0$ resonance (with measurements at $E = M_{P^0}$ and $E = M_{P^0} \pm 2\sigma_M$ using $R = 0.003\%$) one can determine with high statistical precision all the $\mu^+\mu^- \rightarrow P^0 \rightarrow F$ channel rates and the total width $\Gamma_{P^0}$. For the particular technicolor model parameters analyzed in [4], 4 years of the pessimistic yearly luminosity ($L_{\text{year}} = 0.1 \text{ fb}^{-1}$) devoted to the scan yields the results presented in Fig. 19 of [1]. Sample statistical errors for $\sigma_c B(P^0 \rightarrow all)$ and $\Gamma_{P^0}$ are given in Table [1].

Let us now consider the $r_c$-ratio technique for the $P^0$. We will compare to the scan technique using the choices $R = 0.003\%$ for $\sigma_M^\text{min}$ and $R = 0.03\%$ for $\sigma_M^\text{max}$. This means $\sigma_M^\text{central} \sim 6.3 \text{ MeV} (M_{P^0}/100 \text{ GeV})$, implying that $\Gamma_{P^0}/\sigma_M^\text{central}$ rises from $\sim 0.7$ at $M_{P^0} = 50 \text{ GeV}$ to $\sim 1.6$ at $M_{P^0} = 200 \text{ GeV}$. This region is that for which the slope $|s|$ (see Fig. [6]) is smallest. Consequently, the error in $\Gamma_{P^0}$ will be small if that for $r_c$ is. We follow the procedure outlined earlier for computing $\Delta r_c/\Gamma_{P^0}$. We scale the errors given in Fig. 19 of [1] to $L = 0.2 \text{ fb}^{-1} f$ (corresponding to 2$f$ years of operation at $R = 0.003\%$), where $f$ will be chosen to minimize the error in $r_c$. We also compute $\Delta \sigma_c/\sigma_c$ for $L = 0.94 \text{ fb}^{-1} (2-f)$ devoted to $R = 0.03\%$ running (corresponding to 4$-2f$ years of operation at this latter $R$). The net errors, $\Delta \sigma_c/\sigma_c$ and $\Delta \Gamma_{P^0}/\Gamma_{P^0}$, computed after combining all final state channels, are given in Table [4] for $\sigma_c$, the $r_c$-ratio procedure statistical errors are similar to the 4-year three-point scan statistical errors. The $r_c$-ratio procedure errors for $\Gamma_{P^0}$ are smaller than the scan errors for larger $M_{P^0}$ values where $\Gamma_{P^0}/\sigma_M^\text{central}$ is significantly bigger than unity.

**Summary** - We have compared the statistical accuracy with which the width and cross sections of a very narrow resonance can be determined at a muon collider via the usual scan procedure vs. a technique in which one sits on the resonance peak and takes the ratio of cross sections for two different beam energy resolutions. For the same total machine operation time, the ratio technique gives smaller statistical errors than the scan technique for a SM-like Higgs with $M_H > 130 \text{ GeV}$ or a light pseudogoldstone boson with $M_{P^0} > 120 \text{ GeV}$. Further, systematic errors associated with uncertainty in $\sigma_M$ are smaller for the ratio technique than for the scan technique.
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FIG. 1. $\Gamma/\Gamma_M$ plotted as a function of the cross section ratio $\sigma_e(\sigma_M^{\text{max}})/\sigma_e(\sigma_M^{\text{min}})$ for the indicated values of $\sigma_M^{\text{central}} = \sqrt{\sigma_M^{\text{max}} \sigma_M^{\text{min}}}$ keeping $\sigma_M^{\text{central}}$ fixed.

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TABLE I. Percentage errors ($1\sigma$) for $\Gamma_H$ and $\sigma_e H \rightarrow b\bar{b}, WW^*, ZZ^* \rightarrow H$. For the scan procedure [5] we use $R = 0.003$% and 4-year luminosity of $L = 0.4$ fb$^{-1}$, using $L/5$ at $E = M_H$, 2$L/5$ at $E = M_H + 2\sigma_M$ and 2$L/5$ at $E = M_H - 2\sigma_M$. For the $r_c$-ratio procedure, we assume $E = M_H$ and accumulate $L = 0.2$ fb$^{-1}$ at $R = 0.003$% and $L = 0.94$ fb$^{-1}$ at $R = 0.03$%, corresponding to two years of running at each $R$. For efficiencies and cuts, see [5].

| Quantity | Errors for the scan procedure |
|----------|-------------------------------|
| Mass (GeV) | 100 110 120 130 140 150 |
| $\sigma_e(\sigma_M^{\text{max}})$ | 4% 4% 3% 3% 5% 5% 9% 28% |
| $\sigma_e(\sigma_M^{\text{min}})$ | 32% 15% 10% 8% 7% 7% 9% |
| $\sigma_e(\sigma_M^{\text{central}})$ | 190% 50% 30% 26% 26% 34% |
| $\Gamma_H$ | 30% 16% 16% 18% 29% 165% |

| Quantity | Errors for the $r_c$-ratio procedure |
|----------|-------------------------------|
| Mass (GeV) | 100 110 120 130 140 150 |
| $\sigma_e(\sigma_M^{\text{max}})$ | 3.8% 2.8% 2.8% 4.4% 7.6% 21% |
| $\sigma_e(\sigma_M^{\text{min}})$ | 26% 12% 7.7% 5.7% 5.0% 5.0% 5.6% |
| $\sigma_e(\sigma_M^{\text{central}})$ | 190% 46% 25% 29% 22% |
| $\Gamma_H$ | 45% 25% 20% 19% 17% 18% |

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TABLE II. Fractional statistical errors ($1\sigma$) for $\sigma_e H \rightarrow P^0$ (combining $bb$, $\tau^+\tau^-$, $c\bar{c}$ and $gg$ tagged-channel rates) and $\Gamma_P$ for $\mu^+\mu^- \rightarrow P^0$. The $R = 0.003$% three-point scan with total $L = 0.4$ fb$^{-1}$ ($L/5$ at $E = M_P$, 2$L/5$ at $E = M_P + 2\sigma_M$ and 2$L/5$ at $E = M_P - 2\sigma_M$) is compared to the $r_c$-ratio technique with $E = M_P$ luminosities of $L = 0.2$ fb$^{-1}$ at $R = 0.003$% and $L = 0.94$ fb$^{-1}$ (2$f$) at $R = 0.03$% (corresponding to 2$f$ and 4 $-2f$ years of running, respectively). $f$ (tabulated below) is chosen to minimize the error in $\Gamma_P$. Efficiencies, cuts and tagging procedures are from [5].

| Quantity | Errors for the scan procedure |
|----------|-------------------------------|
| Mass | 60 80 $M_P$ $M_P$ $M_P$ $M_P$ $M_P$ |
| $\sigma_e(\sigma_M^{\text{max}})$ | 0.0029 0.0054 0.043 0.0003 0.012 0.018 |
| $\Gamma_P$ | 0.014 0.029 0.25 0.042 0.052 0.10 |

| Quantity | Errors for the $r_c$-ratio procedure |
|----------|-------------------------------|
| Mass | 60 80 $M_P$ $M_P$ $M_P$ $M_P$ $M_P$ |
| $f$ | 0.8 0.7 0.6 0.8 0.9 1.0 |
| $\sigma_e(\sigma_M^{\text{max}})$ | 0.0029 0.0062 0.055 0.010 0.011 0.016 |
| $\Gamma_P$ | 0.014 0.028 0.24 0.041 0.039 0.053 |