Stress drop, earthquake aftershocks and regional stress relation based on synthetic static Coulomb failure stress model

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Abstract. Coulomb failure criterion has been applied widely in the scope of earthquake science to explain earthquake interactions base on stress change, with the well-known method named Coulomb failure stress change (ΔCFS). Preceding studies have showed: increase ΔCFS, depicted as positive stress lobes, has correlation with occurrence of following events. However in the calculation process, ratio between regional stress and earthquake stress drop would affect stress distribution. Based on preceding researches, earthquake stress drop with similar magnitude to regional stress, would give results positive stress lobes along and at the base of the fault. Those stress distribution, could explain events interaction and mechanism of earthquake. This work carries out synthetic modeling of static ΔCFS upon varying earthquake stress drop and regional stress using COULOMB3.3. In accord with preceding studies, the results show positive ΔCFS along the fault when stress drop is comparable to regional stress. And yet, positive ΔCFS would take place at the top and at the base of the fault, expanding to the center of the fault -where the hypocenter is assumed- as the stress drop reaching regional stress in magnitude. This could explain the separated clusters of aftershock in depth observed in some earthquakes.

Keywords: mainshock-aftershock interaction, static Coulomb failure stress change, stress drop, synthetic Coulomb failure stress model.

1. Introduction
In general, the aftershock events are not uniformly distributed around the mainshock. Some studies have been performed to find the correlation between the distributions of aftershocks with the pattern of stress release. Mendoza and Hartzell [1] compared several aftershock patterns with coseismic slip area. They found that aftershocks mostly occur outside or near the edges of main shock slip area, and they explained them with pure shear model by a single crack [1]. Moreover, Mendoza and Hartzell interpreted that the aforementioned phenomenon is consistent with hypothesis of secondary stress redistribution following primary failure on earthquake fault.

The stress redistribution concept was modified into Coulomb failure concept, whose results are similar to pure shear model [2, 3, 4]. Those models both explain the relation between aftershock...
distributions with the stress pattern that comes up by main shock faulting. Yet, the normal stress distribution and effective friction are taken into account for Coulomb failure model whereas only shear stress distribution for the pure shear model [1].

King et al. [4] applied static Coulomb failure stress change (ΔCFS) model to Homestead Valley 1979 strike slip earthquake. They found that aftershocks mostly occur on positive delta stress lobes or stress increase area. However, the stress lobes near the fault are affected by the regional stress magnitude, specifically the ratio of earthquake stress drop and regional stress magnitude [4, 5].

The seismicity around Sumatra is high, which is showed from previous studies on hypocenter relocation in Sumatra region [6], Lake Toba area [7]; and tomography studies in north [8] and southern Sumatra [9] region. Consequently, earthquake stress transfer studies are important in these area.

Preliminary static Coulomb failure stress change (ΔCFS) analysis was done for the recently Pidie Jaya Aceh 2016 Earthquake [10]. Although the position of the aftershock pattern and the stress lobes might show discrepancy, region of no seismicity appeared in Figure 1 (b) section A1-B1, which suggest stress increase on the edges of the fault might be the caused as shown in Figure 1 (b) section A2-B2 [10].

Theoretically, the occurrence of aftershock clusters is related with the positive stress lobes. However for Pidie Jaya preliminary static ΔCFS model [10], the separation of positive stress lobes on the fault tips is wider than the aftershock clusters. It might be due to the uncertainty of the input parameters; i.e. aftershocks location, stress magnitude and fault geometry. The stress heterogeneity along the great Sumatran fault might also affect the stress changes in Pidie Jaya [11]. But at near the fault, ratio of regional stress magnitude and earthquake stress drop plays a significant role in affecting the distribution of the stress transfer [4, 5].

![Figure 1](image)

**Figure 1.** (a) Map view of static Coulomb stress change of Pidie Jaya M 6.5, calculated at 17.5 km. Main shock from GCMT. Black dash rectangle outlined the all-depth-aftershock area. Blue dash dot lines depicted cross section lines; (continued)
This work carries out synthetic modelling of static ΔCFS to assess the effect of earthquake stress drop and regional stress ratio, upon varying the regional stress at fixed depth. The fault plane model used in this work is from ISC Pidie Jaya M6.5 earthquake focal parameter (isc.ac.uk/iscbulletin/search/catalogue/) and the workflow is modified previous research [10].

2. Static Coulomb Stress Transfer

Earthquake interaction might be explained by static Coulomb stress transfer [12]. The deformation resulted from the earthquake slip will cause stress redistribution around the fault plane. If the stress change is increasing, then the area is brought closer to failure; or in other words, the probability of aftershock occurrence is higher on that area. On the other hand, if the stress change is decreasing, then it will inhibit failure [4, 5, 12]. Static Coulomb failure stress change (ΔCFS) calculation is based on the following equation [12].

\[
ΔCFS = Δ|τ_p| + μ' Δσ_p
\]

The Δ|τ_p| is for shear stress change on plane P and Δσ_p is for normal stress change on plane P. While μ’ is the apparent friction coefficient equal to (μ (1 - β’)), with β’ similar to the Skempton’s coefficient β for soils [12].

The regional stress magnitude, beside its orientation, is expected to have an effect to the stress lobes [4, 5]. King et al. [4] have showed that if the ratio of the earthquake stress drop (Δτ) and the regional stress (σ_R^β) is high enough, then positive stress change will extend over the fault plane. While if the earthquake stress drop is negligible to the regional stress, positive stress change will only occur
on the fault tips separated by a large negative stress change along the fault plane. The earthquake stress drop itself is relatively small compared to the regional stress magnitude; that is around 1 to 10 MPa [13]. For the static cracks, strike slip earthquake stress drop ($\Delta \tau$) follows equation (2) below [14], where $\mu$ is the rigidity coefficient equals to $3.3 \times 10^{10}$ Pa, D is the average slip and W is the fault plane width.

$$\Delta \tau = 2 \mu D (\pi W)^{1}$$

(2)

3. Methodology

In this work we assessed the effect of regional stress magnitude and earthquake stress drop ratio by running static $\Delta$CFS synthetic models. We used the Pidie Jaya M6.5 ISC (isc.ac.uk/iscbulletin/search/catalogue/) focal solution (Table 1) with NE – SW orientation [10], as the basis earthquake synthetic model. Then the original magnitude from the basis model is varied into two values: M6.4 and M6.6; in order to obtain different stress drop values. The remains of ISC focal parameters are fixed. The variation in magnitude will also generate different fault plane sizes and different slip values, as they are derived from the Wells and Coppersmith [15] empirical relation. The earthquake models, that would be run, are listed in Table 2.

Table 1. ISC (isc.ac.uk/iscbulletin/search/catalogue/) focal solution for Pidie Jaya M6.5 Earthquake.

| No. | Mw | Length (km) | Width (km) | Average displacement (m) | Max. displacement (m) | Stress drop (bar) | Note |
|-----|----|-------------|------------|--------------------------|-----------------------|------------------|------|
| 1   | 6.4 | 24.5        | 10.09      | 0.1909                   | 0.2980                | 3.977            | Synthetic |
| 2   | 6.5 | 28.6        | 11.03      | 0.2473                   | 0.4006                | 4.71            | ISC   |
| 3   | 6.6 | 33.38       | 12.05      | 0.3203                   | 0.5379                | 5.587            | Synthetic |

Table 2. Fault plane parameters of the earthquake models. Length, width, average and maximum displacement are derived from Wells and Coppersmith [15] empirical relation; whereas the stress drop is obtained by Equation 1 [14].

Table 3. List of synthetic models for $\Delta$CFS calculation. Regional stress magnitude is calculated and varied at 13 km depth. Three earthquake models are differ by the stress drop values. The code next to the checklist indicates figure number.

| $\Delta \tau$ (bar) | $\sigma_{i1}$ 4420 | $\sigma_{i2}$ 2210 | $\sigma_{i3}$ 1105 | $\sigma_{i4}$ 552.5 | $\sigma_{j1}$ 276.25 | $\sigma_{j2}$ 138.125 | $\sigma_{j3}$ 97.5 |
|-------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 3.977 (M6.4)      | $\checkmark$ (3a)| $\checkmark$ (3b)| $\checkmark$ (3c)| $\checkmark$ (3d) | $\checkmark$ (3e)| $\checkmark$ (3f) |
| 4.713 (M6.5)      | $\checkmark$ (4a)| $\checkmark$ (4b)| $\checkmark$ (4c)| $\checkmark$ (4d) | $\checkmark$ (4e)| $\checkmark$ (4f) |
| 5.587 (M6.6)      | $\checkmark$ (5a)| $\checkmark$ (5b)| $\checkmark$ (5c)| $\checkmark$ (5d) | $\checkmark$ (5e)| $\checkmark$ (5f) |

The regional stress magnitude is referred from in-situ well stress profile in Suban field [16], i.e. 34 MPa/km for horizontal maximum stress, 24 MPa/km for vertical stress and 18 MPa/km for horizontal minimum stress. We take these values as the maximum and then vary by dividing them in sequence by a factor of two. The horizontal maximum stress orientation is taken to be N39°E, referring from Central Sumatra region [17].

Static Coulomb failure stress ($\Delta$CFS) calculation is computed from slip on the fault plane. $\Delta$CFS is obtained from each model in Table 2. The slip is assumed to be homogeneous over the fault plane area, with the average displacement as the value. The center of fault plane is positioned on the focal’s centroid location. The $\Delta$CFS computation is carried out using COULOMB3.3, an open-source deformation and stress change software [18, 19]. Although the $\Delta$CFS can be computed from 0 km
depth until 30 km depth, the stress drop and regional stress magnitude ratio \( (\Delta \tau/ \sigma_R) \) would be assessed only at the fault center’s depth, that is 13 km. Regional stress magnitude variation at depth 13 km, and the earthquake synthetic models are listed in Table 3. \( \Delta \text{CFS} \) calculation procedure [20] is depicted in Figure 2.

![Diagram](image)

**Figure 2.** \( \Delta \text{CFS} \) calculation flowchart. Flow for the first earthquake model (M6.4) with the regional stress variation (looping) is shown in solid line. Dashed line is showing looping for the second (M6.5) and third (M6.6) earthquake model.

### 4. Results and Discussion

The magnitude ratios of the stress drop and the regional stress \( (\Delta \tau/ \sigma_R) \) for each model in Table 3 are shown in Table 4. Following King and Deves [5], we used deviatoric stress of maximum horizontal stress as the ratio denominator. The vertical section of \( \Delta \text{CFS} \) results are shown in Figure 3, 4 and 5. The cross sections in those figures are corresponding to cross section A2-B2 in Figure 1. Here, the discussion is focused on the vertical \( \Delta \text{CFS} \) section; hence we do not show the horizontal section.

King et al. [4] showed that if the ratio \( (\Delta \tau/ \sigma_R) \) equal to one, the positive stress is along the fault. Then failure will occur on the optimally oriented planes near the causative fault, due to the stress change from fault’s slip is comparable to the regional stress. Whereas if the ratio \( (\Delta \tau/ \sigma_R) \) equal to 0.1, the positive stress is only on the fault tips due to the optimally oriented planes are fixed by the regional stress. Here we found that the positive stress change above and below the fault plane started to expand to the center of the fault when the ratio \( (\Delta \tau/ \sigma_R) \) falls between \( 0.005 < \text{ratio} < 0.030 \), i.e. begins on Figure 3(d), 4(c) and 5(a).

| Dev. \( \sigma^e \) (bar) | 1126.67 | 563.33 | 281.67 | 140.83 | 70.42 | 35.21 |
|--------------------------|---------|--------|--------|--------|-------|-------|
| \( \Delta \tau \) (bar)  |         |        |        |        |       |       |
| 3.977 (M6.4)             | 0.00353 | 0.00706| 0.01412| 0.02824| 0.05648| 0.11296|
| 4.713 (M6.5)             | 0.00418 | 0.00837| 0.01673| 0.03347| 0.06693| 0.13386|
| 5.587 (M6.6)             | 0.00496 | 0.00992| 0.01984| 0.03967| 0.07934| 0.15868|

**Table 4.** Synthetic models for \( \Delta \text{CFS} \) calculation with the corresponding \( \Delta \tau/ \sigma_R \) ratio. Deviatoric stress of maximum horizontal stress (\( \sigma_{00} \)) is used as ratio denominator.
Figure 3. Vertical ΔCFS section of M6.4 fault plane model with different values of regional stress (bars) and ratio: (a) $\sigma_H 4420, \sigma_v 3120, \sigma_h 2340$ and ratio 0.00353; (b) $\sigma_H 2210, \sigma_v 1560, \sigma_h 1170$ and ratio 0.00706; (c) $\sigma_H 1105, \sigma_v 780, \sigma_h 585$ and ratio 0.01412; (d) $\sigma_H 552.5, \sigma_v 390, \sigma_h 292.5$ and ratio 0.02824; (e) $\sigma_H 276.25, \sigma_v 195, \sigma_h 146.25$ and ratio 0.05648; (f) $\sigma_H 138.125, \sigma_v 97.5, \sigma_h 73.125$ and ratio 0.11296. Color scale is in bar.

We also found that the positive stress change is along the fault plane’ dip when the ratio $\approx 0.1$. This number is contrast to King et al. [4] result. It might be due to different stress drop approximation being used. In this work we used geometry approximation [14] for calculating stress drop. Stress drop can also be approached by far field seismic wave model, namely Brune model [21]. However, in general both results agree, in the sense that: the larger the ratio $((\Delta \tau / \sigma^R))$, the smaller the separation of positive stress lobes. Or, the area of positive lobes at the edges of the fault will expand into the center of the fault plane when the ratio $((\Delta \tau / \sigma^R))$ grows larger.

Regional stress magnitude which we used in the ΔCFS synthetic modeling might be too small in the real case, especially for model (d)-(e) at each fault plane model. Nevertheless, the regional stress magnitude could be little lower than the referenced value [16] if we take into account the pressure within the porous rock.
Figure 4. Vertical $\Delta$CFS section of M6.5 fault plane model with different values of regional stress (bars) and ratio: (a) $\sigma_H$ 4420, $\sigma_\pi$ 3120, $\sigma_h$ 2340 and ratio 0.00418; (b) $\sigma_H$ 2210, $\sigma_\pi$ 1560, $\sigma_h$ 1170 and ratio 0.00837; (c) $\sigma_H$ 1105, $\sigma_\pi$ 780, $\sigma_h$ 585 and ratio 0.01673; (d) $\sigma_H$ 552.5, $\sigma_\pi$ 390, $\sigma_h$ 292.5 and ratio 0.03347; (e) $\sigma_H$ 276.25, $\sigma_\pi$ 195, $\sigma_h$ 146.25 and ratio 0.06693; (f) $\sigma_H$ 138.125, $\sigma_\pi$ 97.5, $\sigma_h$ 73.125 and ratio 0.13386. Color scale is in bar.

Back to Pidie Jaya M6.5 Earthquake, previous study showed that the separation of the two aftershock clusters is around 7 km wide [10]. However, for the regional stress magnitude as in Figure 4 (a), which corresponds to the smallest ratio ($\Delta\tau/\sigma^8$) in M6.5 model, shows the separation of positive stress lobes is around 10 km wide. The separation distance of around 7 km was achieved when the stress ratio is increased to 0.04. In other word, a lower regional stress or a higher stress drop magnitude is required to match the observation in the previous study.

This discrepancy might come from several factors. Assuming that the hypocenter locations are accurate, one of possible factors is the higher pore pressure in this area, than the one at the reference field where the regional stress magnitude is taken. Higher pore pressure will decrease the principal stress magnitude. Besides, the fault plane geometry used in this study is inferred from the empiric equation of Wells & Coppersmith [15]. This might also be a problem, as for the event with magnitude greater than six, the slight changes in magnitude will affect significantly the calculated fault geometry.
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Figure 5. Vertical ΔCFS section of M6.6 fault plane model with different values of regional stress (bars) and ratio: (a) $\sigma_{\parallel}$ 4420, $\sigma_{\perp}$ 3120, $\sigma_{h}$ 2340 and ratio 0.00496; (b) $\sigma_{\parallel}$ 2210, $\sigma_{\perp}$ 1560, $\sigma_{h}$ 1170 and ratio 0.00992; (c) $\sigma_{\parallel}$ 1105, $\sigma_{\perp}$ 780, $\sigma_{h}$ 585 and ratio 0.01984; (d) $\sigma_{\parallel}$ 552.5, $\sigma_{\perp}$ 390, $\sigma_{h}$ 292.5 and ratio 0.03967; (e) $\sigma_{\parallel}$ 276.25, $\sigma_{\perp}$ 195, $\sigma_{h}$ 146.25 and ratio 0.07934; (f) $\sigma_{\parallel}$ 138.125, $\sigma_{\perp}$ 97.5, $\sigma_{h}$ 73.125 and ratio 0.15868. Color scale is in bar.

5. Conclusion

From the results of this study, we concluded that the positive stress arises at the edges of the fault, above and below the fault, and reaching to the fault center, when the ratio ($\Delta\tau/\sigma^R$) increases. The positive stress change above and below the fault plane started to expand to the center of the fault when the ratio ($\Delta\tau/\sigma^R$) falls between $0.005 < \text{ratio} < 0.030$. Moreover, positive stress change is along the fault plane’s dip when the ratio $\approx 0.1$, which is smaller than the ratio King et al. [4] stated. This might be due to different stress drop approximation being used.

In the case of Pidie Jaya M6.5 Coulomb stress change, the discrepancy between the seismic gap obtained in this study (10 km) and the previous study (7 km) [10] might be due to the fact that we did not take into account yet the porosity of the rock. Higher pore pressure will surely decrease the principal stress magnitude, thus increasing ($\Delta\tau/\sigma^R$) ratio. As a result, positive stress change will expand to the fault’s center, and, therefore, minimizing the seismic gap distance (the positive stress change area is the probable seismicity area). Also, it might be due to uncertainty in the fault plane parameter inferred from the Wells & Coppersmith [15] relation. A further study of assessing the sensitivity of each input parameter is required to improve the stress modeling results.
Acknowledgement
Authors thank Institut Teknologi Bandung, for Program Penelitian, Pengabdian kepada Masyarakat, dan Inovasi (P3MI) Kelompok Keahlian Geofisika Global ITB support.

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