Stationary queue length distribution of a continuous-time queueing system with negative arrival

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Abstract. This paper studies a continuous-time single-server infinite capacity queueing system with two types of customer: positive and negative customers. Positive customers are ordinary customers that receive service in the server. A negative customer that arrives to the system according to a Poisson process with rate $\gamma$ will remove one positive customer at the head upon its arrival. By assuming that the interarrival time and service time distributions tend to a constant when time $t$ goes to infinity, a set of equations will be derived by using an alternative approach to find the stationary queue length distribution. Numerical results obtained by the alternative approach will be compared to those obtained by the existing method and verified by the simulation procedure.

1. Introduction

Queueing system with negative customer has been studied extensively in the previous two decades and majority of the published work can be found in Do [1]. It was initiated by Gelenbe [2] in neural networks and used in reducing the congestion condition of an M/M/1 queue in Gelenbe [3]. He then applied the concept in computer and communication system where the arrival of negative customers will cancel the request of positive customers [4]. Typically, there are two types of killing discipline: removal of a customer at the head (RCH) and removal of a customer at the end (RCE). Thereafter, Henderson [5] and Harrison and Pitel [6, 7] extended the research about negative customers by taking into consideration different queueing disciplines. In real life application, negative customers could be used to represent inhibitor in random neural networks, task termination and server break-down [2, 3, 4, 8, 9]. In recent years, Lee [10] considered the case in which the arrival of a negative customer will remove all positive customers from the queue. Soujaynya [11] used negative customers to represent customers who opt for other stores and applied the concept into inventory system. Lately, Fourneau and Gelenbe [12] presents the product form solution of G-network by including a new type of customer called “Adders” that acts to re-route customer traffic while negative customers reject the service request.

Gelenbe [3] had shown that an M/M/1 queue with negative customers is a Markov model. Harrison and Pitel [6] then used the Laplace transform to solve the sojourn times density of the M/M/1 queue with different killing strategies and queueing disciplines. In Harrison and Pitel [7], the exponential assumption of the service time distribution was relaxed and the stationary queue length distribution was derived. In this paper, numerical method proposed in Koh [13] is explored to find the stationary queue
length distribution for queueing system with negative arrivals. Both the interarrival time and service
time distributions are assumed to have constant hazard rate when time $t$ goes to infinity. These
distributions are also known as distributions with constant asymptotic rate (CAR). Many distributions,
for instance, Erlang, hyperexponential, gamma, etc. have this characteristic. This paper aims to provide
a new approach in finding the stationary queue length distribution not only for an M/M/1 queueing
systems with negative arrivals but it can also be further extended to the queue with different interarrival
time and service time distributions.

This paper is organized as follows: Section 2 describes the model studied. Section 3 provides the
derivation of the equations satisfied by the stationary probabilities. The method used to solve for the
equations is discussed in Section 4. Numerical results will be shown in Section 5. Future work will be
discussed in Section 6 and concluding remarks is presented in Section 7.

2. Model description
Consider a single-server, first come first serve queueing system (FCFS) with negative customers
arriving to the system according to a Poisson process with rate $\gamma$. Negative customers do not receive
service and apply the principle of removal customer at the head (RCH). It removes one positive customer
from the head of the queue if the system is not empty. Both the interarrival time and service time of
positive customers are assumed to have distributions with constant asymptotic rate.

3. Equation derivation for stationary probabilities
A set of equations is derived to find the stationary queue length distribution of the system. Firstly, let
$g(t)$ and $h(t)$ be the probability density function (pdf) of the interarrival time and service time of positive
customers, respectively. Denote $\tau_k$ as the interval of $[k\Delta t, (k+1)\Delta t]$ for $k = 1, 2, 3, \ldots$. Let

$$
\lambda_k = \frac{g(k\Delta t)}{\int_{k\Delta t}^{\infty} g(u)du}, \quad 1 \leq k \leq I, \quad \text{and} \quad \mu_k = \frac{h(k\Delta t)}{\int_{k\Delta t}^{\infty} h(u)du}, \quad 1 \leq k \leq J,
$$

where $I$ and $J$ are large enough such that

$$
\lambda_k \approx \lim_{k \to \infty} \lambda_k, \quad \text{and} \quad \mu_k \approx \lim_{k \to \infty} \mu_k.
$$

Assuming a positive customer arrives to the system at $t = 0$, then the probability that a positive arrival
in the interval $\tau_1$ is approximately $\lambda_1\Delta t$ and if given there is no positive arrival before the interval $\tau_k$, the
probability that a positive customer arrives in the interval $\tau_k$ is approximately $\lambda_k\Delta t$, where $k = 2, 3, 4, \ldots$ and $\lambda_k = \lambda_1$ for $k \geq 1$. Similarly, the probability that the positive customer completed the service in
the interval $\tau_1$ is approximately $\mu_1\Delta t$. Given that there is no service completion before the interval $\tau_k$, the
probability that a service completion occurs in the interval $\tau_k$ is approximately $\mu_k\Delta t$, where $k = 2, 3, 4, \ldots$ and $\mu_k = \mu_1 = \mu$ for $k \geq J$.

Let $\tau_0$ be the interval before $\tau_1$ and assume that there is a positive arrival in $\tau_0$. Denote the state
numbers of the positive arrival process, negative arrival process and service process at the end of the
interval $\tau_k$ as $\Lambda_k$, $\Gamma_k$ and $M_k$, respectively. The state numbers $\Lambda_k$, $\Gamma_k$ and $M_k$ are defined as follows:

$$
\Lambda_k = \begin{cases} 
0, & \text{if } k = 0; \\
\min(k, I), & \text{if the next positive customer arrives in } \tau_k, k \geq 1.
\end{cases}
$$

or the next positive customer arrives in $\tau_k, k \geq 1$. (3)
Let $n_k$ be the queue length at the end of the interval $\tau_k$ and the state vector $Z_k = \{n_k, \Lambda_k, \Gamma_k, M_k\}$ denotes the queue length, states of the positive arrival process, negative arrival process and service process at the end of the interval $\tau_k$. Next, $P^{(k)}_{nirj}$ is defined to be the probability that at the end of interval $\tau_k$, where $n$ is the number of positive customers in the system when the positive arrival process, negative arrival process and service process are in state $i$, $r$ and $j$ respectively. Assuming that:

$$P_{nirj} = \lim_{k \to \infty} P^{(k)}_{nirj}. \tag{6}$$

Suppose that at the end of the interval $\tau_{k-1}$, the queue is not empty where $n_{k-1} = n \geq 1$, the positive arrival process is in state $i-1$ where no positive arrival in interval $\tau_{k-1}$. There is no negative arrival and $r = 0$ at the end of the interval $\tau_{k-1}$. No service completion happened in the interval $\tau_{k-1}$ and the state of the service process at the end of the interval $\tau_{k-1}$ is $j-1$. Hence, we have $Z_{k-1} = \{n, i-1, 0, j-1\}$ at the end of the interval $\tau_{k-1}$. Let $i^* = \min(i, J)$ and $j^* = \min(j, J)$, only one of the following events can happen in the next interval $\tau_k$:

1. A positive arrival with the arrival rate $\lambda_i$ and $Z_k = \{n+1, 0, 0, j^*\}$.
2. A negative arrival with the arrival rate $\gamma$ and $Z_k = \{n-1, i^*, 1, 0\}$.
3. A service completion with the service rate $\mu_j$ and $Z_k = \{n-1, i^*, 0, 0\}$.
4. None of the above events happened and $Z_k = \{n, i^*, 0, j^*\}$.

If the system is empty, $n_{k-1} = n = 0$ at the end of the interval $\tau_{k-1}$, then only items 1, 2 and 4 can take place in the next interval $\tau_k$.

Likewise, $P^{(k)}_{nirj}$ can be found by combining $Z_{k-1}$ and the events that could occur in the interval $\tau_k$. When $k \to \infty$, the following equations are obtained with $I = 2$ and $J = 2$:

$$P_{n000} = \sum_{r=0}^{I} \sum_{j=0}^{J} P_{n0r0} (1-\lambda_i \Delta t)(1-\gamma \Delta t)(1-\mu_i \Delta t) + \sum_{j=0}^{J} P_{(n+1)00j} (1-\lambda_i \Delta t)(1-\gamma \Delta t)(\mu_{\min_{j+1, J}} \Delta t) \tag{7}$$

$$+ \sum_{j=0}^{J} P_{(n+1)00j} (1-\lambda_i \Delta t)(1-\gamma \Delta t)(\mu_{\min_{j+1, J}} \Delta t) + \sum_{j=0}^{J} P_{(n+1)10j} (1-\lambda_i \Delta t)(1-\gamma \Delta t)(\mu_i \Delta t), \quad n = 0.$$  

$$P_{n100} = \sum_{r=0}^{I} \sum_{j=0}^{J} P_{n1r0} (1-\lambda_i \Delta t)(\gamma \Delta t) + \sum_{j=0}^{J} P_{(n+1)10j} (1-\lambda_i \Delta t)(\gamma \Delta t) + \sum_{j=0}^{J} P_{(n+1)(l-1)0j} (1-\lambda_i \Delta t)(\gamma \Delta t) \tag{8}$$

$$+ \sum_{j=0}^{J} P_{(n+1)10j} (1-\lambda_i \Delta t)(\gamma \Delta t), \quad n = 0.$$
\[ P_{n10} \approx P_{100}(1 - \lambda_1 \Delta t)(\gamma \Delta t), \quad n = 0. \]

\[ P_{n100} \approx P_{(n+1)00}(1 - \lambda_1 \Delta t)(1 - \gamma \Delta t)(\mu_2 \Delta t), \quad n = 0. \]

\[ P_{n10,J} \approx P_{n00(J-1)}(1 - \lambda_1 \Delta t)(1 - \gamma \Delta t)(1 - \mu_J \Delta t), \quad n = 1. \]

\[ \sum_{j=1}^{J \leq 1} \sum_{i=1}^{I \leq 1} P_{n0,J}(1 - \lambda_i \Delta t)(1 - \gamma \Delta t)(1 - \mu_J \Delta t), \quad n \geq 1. \]

\[ P_{n101} \approx \sum_{i=1}^{J \leq 1} (P_{n00} + P_{n10})(1 - \lambda_i \Delta t)(1 - \gamma \Delta t)(1 - \mu_i \Delta t), \quad n \geq 1. \]

\[ P_{n001} \approx \sum_{j=1}^{J \leq 1} (P_{(n+1)00} + P_{(n+1)i0})(\lambda_{\min(i+1,j)} \Delta t), \quad n \geq 1. \]

\[ P_{n100} \approx \sum_{j=0}^{J \leq 1} P_{(n+1)00j}(1 - \lambda_1 \Delta t)(1 - \gamma \Delta t)(\mu_{\min(j+1,1J)} \Delta t), \quad n \geq 1. \]

\[ P_{n10} \approx \sum_{j=0}^{J \leq 1} P_{(n+1)J0}(1 - \lambda_J \Delta t)(\gamma \Delta t) + \sum_{i=1}^{J \leq 1} P_{i10}(1 - \lambda_i \Delta t)(\gamma \Delta t) + \sum_{j=0}^{J \leq 1} P_{(n+1)(j-1)0}(1 - \lambda_j \Delta t)(\gamma \Delta t), \quad n \geq 1. \]

\[ P_{n00} \approx \sum_{j=0}^{J \leq 1} P_{(n+1)(j-1)0}(1 - \lambda_J \Delta t)(1 - \gamma \Delta t)(\mu_{min(j+1,1J)} \Delta t) + \sum_{i=1}^{J \leq 1} P_{i10}(1 - \lambda_i \Delta t)(1 - \gamma \Delta t)(\mu_i \Delta t) \]

\[ + \sum_{j=0}^{J \leq 1} P_{(n+1)0j}(1 - \lambda_J \Delta t)(1 - \gamma \Delta t)(\mu_{min(j+1,1J)} \Delta t), \quad n \geq 1. \]

\[ P_{n00,J} \approx \sum_{i=j}^{J - 1} P_{(n-1)0(j-i-1)}(\lambda_{\min(i+1,1J)} \Delta t) + \sum_{i=j}^{J - 1} P_{(n-1)0i}(\lambda_{\min(i+1,1J)} \Delta t), \quad n = 2. \]

\[ P_{n10,J} \approx \sum_{j=J-1}^{J} P_{n00J}(1 - \lambda_1 \Delta t)(1 - \gamma \Delta t)(1 - \mu_J \Delta t), \quad n \geq 2. \]

\[ P_{n00,J} \approx \sum_{i=0}^{J - 1} P_{(n-1)0(i+1)}(\lambda_{\min(i+1,1J)} \Delta t) + \sum_{i=0}^{J - 1} P_{(n-1)0i}(\lambda_{\min(i+1,1J)} \Delta t), \quad n \geq 3. \]

4. Stationary queue length distribution

The method used in Koh [13] is applied to solve equations (7) to (21) to find the stationary queue length distribution. By letting \( g(t) \) and \( h(t) \) be the pdf of exponential distribution, it can be easily seen that \( \lambda_k = \lambda \) and \( \mu_k = \mu \) for all \( k \). Initially, we introduce the following notation:

- \( \mathbf{P}_n \): Column vector that represents all the stationary probabilities \( P_{nirj} \) when the queue size is \( n \).
**Q_{n,m}**: Coefficient matrix that multiply with \( P_m^* \) and corresponds to \( P_n^* \) on the left hand side (LHS) of the system of equations in Section 3.

**I_n**: Identity matrix of the same size as the number of columns for \( P_n^* \).

By using the above notations, the set of equations when queue length is \( n = 0 \) can be written in matrix form as follows:

\[
P_0^* = Q_{0,0} P_0^* + Q_{0,1} P_1^*,
\]

\[
(I_0 - Q_{0,0}) P_0^* = Q_{0,1} P_1^*.
\]

Similarly, the set of equations when \( n = 1 \) can be expressed as:

\[
P_1^* = Q_{1,0} P_0^* + Q_{1,1} P_1^* + Q_{1,2} P_2^*,
\]

\[
(I_1 - Q_{1,1}) P_1^* = Q_{1,0} P_0^* + Q_{1,2} P_2^*.
\]

Solving for \( P_1^* \), equation (25) becomes

\[
P_1^* = A_{1,0} P_0^* + A_{1,2} P_2^*,
\]

where \( A_{n,m} \) is a coefficient matrix that relates \( P_m^* \) with \( P_n^* \) after some matrix operations. In equation (26), \( A_{1,0} = (I_1 - Q_{1,1})^{-1} Q_{1,0} \) and \( A_{1,2} = (I_1 - Q_{1,1})^{-1} Q_{1,2} \).

Equations (12) to (20) for the case when \( n = 2 \) can be expressed as:

\[
P_2^* = Q_{2,1} P_1^* + Q_{2,2} P_2^* + Q_{2,3} P_3^*.
\]

Substituting \( P_1^* \) in equation (26) into equation (27) and solving for \( P_2^* \), we get

\[
P_2^* = A_{2,0} P_0^* + A_{2,3} P_3^*.
\]

For \( n \geq 3 \), the set of equations can be expressed in the following general form:

\[
P_n^* = Q_{n,n-1} P_{n-1}^* + Q_{n,n} P_n^* + Q_{n,n+1} P_{n+1}^*.
\]

By repeating the process of substituting \( P_{n-1}^* \) into equation (29) and solving for \( P_n^* \), a general equation of the form similar to equations (26) and (28) is obtained:

\[
P_n^* = A_{n,0} P_0^* + A_{n,n+1} P_{n+1}^*, \quad n \geq 1.
\]

When \( N = n + 1 \) is large enough such that all \( P_{N(n+1)} \approx 0 \), all the elements in \( P_{n+1}^* \) in equation (30) are approximate zero and hence produce

\[
P_n^* = A_{n,0} P_0^*.
\]

When the queue length is \( n - 1 \), equation (30) is written as:

\[
P_{n-1}^* = A_{n-1,0} P_0^* + A_{n-1,n} P_n^*.
\]
Substituting $P^*_n$ in equation (31) into equation (32) and solving for $P^*_{n-1}$, we obtain

$$
P^*_{n-1} = A_{n-1,0}P^*_0 + A_{n-1,n}A_{n,0}P^*_n,
$$

(33)

where $B_{n,0}$ is another coefficient matrix that relates $P^*_n$ with $P^*_n$ after some basic matrix operations processes. In equation (34), $B_{n-1,0} = A_{n-1,0} + A_{n-1,n}A_{n,0}$. Subsequently, by repeating the process of backward substituting $P^*_{n+1}$ into equation (30) and solving for $P^*_n$, when $n = 1$, we obtain

$$
P^*_1 = B_{1,0}P^*_0.
$$

(35)

Finally, substituting $P^*_1$ in equation (35) into equation (23) yields:

$$
(I_0 - Q_{0,0})P^*_0 = Q_{0,1}B_{1,0}P^*_0,
$$

(36)

$$
B_{0,0}P^*_0 = 0.
$$

(37)

An inspection of equation (37) reveals that there are two equations that are linearly dependent. Hence, another linearly independent equation is added to find a unique solution which is:

$$
\sum_{n=1}^{N} \sum_i \sum_r \sum_j P_{nirj} \cong 1.
$$

(38)

Substituting one of the equations in equation (37) with equation (38) and solve for the system of equations, the values for $P_{0ijr}$ is obtained. By substituting these values of $P_{0ijr}$ into equation (31), all the numerical values of $P_{nirj}$ will be obtained and the probability that the queue length is $n$ is obtained by:

$$
P_n = \sum_i \sum_r \sum_j P_{nirj}.
$$

(39)

5. Numerical results

Tables 1 and 2 show the numerical results for the stationary queue length distribution of a queueing system without negative customers (A) and queueing system with negative customers (B). Different utilization factors are taken into consideration. The results obtained by the alternative approach for (A) is compared with Gross [14] and the simulation software “QtsPlus” in [14]. Whereas for (B), the proposed method is compared to Harrison and Pitel [6] and verified by the simulation procedure. For (A), $\gamma = 0$ while for (B), $\gamma = 0.5$ is used.

Tables 1 and 2 show that the results found by the alternative approach are very close to those obtained by methods introduced in Gross [14] and Harrison and Pitel [6]. From the results, it can be shown that negative customers are helpful in reducing the mean queue size of the systems. Its existence in the queue reduces the system utilization factor, $\rho$, by about 25% from 0.9375 to 0.6818 in Table 1 and 10% from 0.2000 to 0.1000 in Table 2, where $\rho$ can be calculated using $1 - P_0$. The fraction of time of the busy server is reduced with the present of negative arrivals. All the results have been verified by the simulation procedure. It can be seen that in Table 1 even with very high utilization factor $\rho = 0.9375$, the numerical results obtained by the alternative approach are still close to those computed by the existing methods.
Table 1. Comparison of stationary queue length distribution computed by the alternative method and those obtained by the methods introduced in Gross [14], Harrison and Pitel [6], software “QtsPlus” and 100 runs of simulations. The parameters are:

\[\Delta t = 0.1^{-7}, \lambda_k = 1.25, \mu_k = \frac{4}{3}, I = 2, J = 2, N_A = 200, N_B = 50\].

| Queue Size, n | Without Negative Customer (A) | With Negative Customer (B) |
|---------------|-------------------------------|-----------------------------|
|               | Numerical Method Gross [14]   | Simulation (QtsPlus)        | Numerical Method Harrison and Pitel [6] | Simulation |
| 0             | 0.06250                       | 0.06249                     | 0.31818                        | 0.31818    | 0.31798    |
| 1             | 0.05859                       | 0.05867                     | 0.21694                        | 0.21694    | 0.21676    |
| 2             | 0.05493                       | 0.05499                     | 0.14791                        | 0.14791    | 0.14780    |
| 3             | 0.05149                       | 0.05143                     | 0.10085                        | 0.10085    | 0.10080    |
| 4             | 0.04828                       | 0.04825                     | 0.06876                        | 0.06876    | 0.06876    |
|               |                               |                             |                               |            |            |
| 200           | 9.68E-9                       | 1.54E-7                     | 0                             | 0          | 0          |

Mean Queue Size

14.99708 14.9995 15.85645 2.14286 2.14286 2.15183

Table 2. Comparison of stationary queue length distribution computed by the alternative method and those obtained by the methods introduced in Gross [14], Harrison and Pitel [6], software “QtsPlus” and 100 runs of simulations. The parameters are:

\[\Delta t = 0.1^{-7}, \lambda_k = 0.1, \mu_k = 0.5, I = 2, J = 2, N_A = 200, N_B = 15\].

| Queue Size, n | Without Negative Customer (A) | With Negative Customer (B) |
|---------------|-------------------------------|-----------------------------|
|               | Numerical Method Gross [14]   | Simulation (QtsPlus)        | Numerical Method Harrison and Pitel [6] | Simulation |
| 0             | 0.80000                       | 0.79998                     | 0.90000                        | 0.90000    | 0.90002    |
| 1             | 0.16000                       | 0.16001                     | 0.09000                        | 0.09000    | 0.08997    |
| 2             | 0.03200                       | 0.03201                     | 0.00900                        | 0.00900    | 0.00901    |
| 3             | 0.00640                       | 0.00640                     | 0.00090                        | 0.00090    | 0.00090    |
| 4             | 0.00128                       | 0.00128                     | 9.00E-5                        | 9.00E-5    | 8.86E-5    |
|               |                               |                             |                               |            |            |
| 15            | 2.6E-11                       | 2.6E-11                     | 0                             | 0          | 0          |

Mean Queue Size

0.25000 0.25000 0.25006 0.11111 0.11111 0.11109

In Table 1, the mean queue size is reduced to 2 with the present of negative customers, which is 7 times less congested compared to the one without negative customers. The probability that the system is empty is increased from 6.2% to 31.8% when there are negative arrivals. Whereas in Table 2, when the system utilization factor is lower, the mean queue size is reduced by half with the present of negative customers. While the probability that the server is empty is increased by 10% from 0.8 to 0.9. From these comparisons, we can observe that negative customers have larger impact to queue with higher utilization factor. In term of service quality when there are negative arrivals, service to some of the
positive customers are disrupted. However, in exchange, other positive customers may experience shorter waiting time.

6. Conclusion
In this study, negative customers are a useful component for congested queueing systems. By assuming that both the interarrival time and service time distributions have constant asymptotic rate, a set of equations to find the stationary queue length distribution is derived. Taking together the results shown in this paper and in Koh [13], there is promising evidence that the alternative approach used in this paper can be applied to find the stationary queue length distribution of a queueing system with negative arrivals and with more general interarrival and/or service time distributions.

7. Future work
It has been shown that the alternative approach can be successfully applied to find the stationary queue length distribution for queueing model with negative arrivals. In this paper, only exponentially distributed interarrival time and service time are considered. Although the same results can be found by using the formula derived in Harrison and Pitel [6], the alternative approach with strong evidence introduced in Koh [13] can yield similar results for queueing system with more general distribution and comparable to the one in Harrison and Pitel [7] in the future.

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