Coding Improves the Optimal Delay-Throughput Trade-offs in Mobile Ad-Hoc Networks: Two-Dimensional I.I.D. Mobility Models

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Abstract—In this paper, we investigate the delay-throughput trade-offs in mobile ad-hoc networks under two-dimensional i.i.d. mobility models. We consider two mobility time-scales: (i) Fast mobility where node mobility is at the same time-scale as data transmissions; (ii) Slow mobility where node mobility is assumed to occur at a much slower time-scale than data transmissions. Given a delay constraint $D$, the main results are as follows: (1) For the two-dimensional i.i.d. mobility model with fast mobiles, the maximum throughput per source-destination (S-D) pair is shown to be $O\left(D^2/n\right)$, where $n$ is the number of mobiles. (2) For the two-dimensional i.i.d. mobility model with slow mobiles, the maximum throughput per S-D pair is shown to be $O\left(D^2/n\right)$. (3) For each case, we propose a joint coding-scheduling algorithm to achieve the optimal delay-throughput trade-offs.

I. NOTATIONS

The following notations are used throughout this paper, given non-negative functions $f(n)$ and $g(n)$:

1. $f(n) = O(g(n))$ means there exist positive constants $c$ and $m$ such that $f(n) \leq cg(n)$ for all $n \geq m$.
2. $f(n) = \Omega(g(n))$ means there exist positive constants $c$ and $m$ such that $f(n) \geq cg(n)$ for all $n \geq m$. Namely, $g(n) = O(f(n))$.
3. $f(n) = \Theta(g(n))$ means that both $f(n) = \Omega(g(n))$ and $f(n) = O(g(n))$ hold.
4. $f(n) = o(g(n))$ means that $\lim_{n \to \infty} f(n)/g(n) = 0$.
5. $f(n) = o(g(n))$ means that $\lim_{n \to \infty} g(n)/f(n) = 0$. Namely, $g(n) = o(f(n))$.

II. INTRODUCTION

The throughput of a random wireless network with $n$ static nodes and $n$ random S-D pairs was studied by Gupta and Kumar [10]. They showed that the maximum throughput per S-D pair is $O(1/\sqrt{n})$, and proposed a scheduling scheme achieving a throughput of $\Theta(1/\sqrt{n\log n})$ per S-D pair. The throughput decreases with $n$ because each successful transmission from source to destination needs to take $\sqrt{n/\log n}$ hops. Later Grossglauser and Tse [9] considered mobile ad-hoc networks, and showed that $\Theta(1)$ throughput per S-D pair is achievable. The idea is to deliver a packet to its destination only when it is within distance $\Theta(1/\sqrt{n})$ from the destination. However, packets have to tolerate large delays to achieve this throughput.

We first review the results for i.i.d. mobility models. Neely and Modiano [15] studied the i.i.d. mobility model where the positions of nodes are totally reshuffled from one time slot to another, and showed that the mean delay of Grossglauser and Tse’s algorithm is $\Theta(n)$. In the same paper, they also proposed an algorithm which generates multiple copies of each data packet to reduce the mean delay. Since more transmissions are required when we generate multiple copies, the throughput per S-D decreases with the number of copies per data packet. The delay-throughput trade-off is shown to be $\lambda = \Omega(D/n)$ in [15], where $\lambda$ is the throughput per S-D pair, and $D$ is the number of time slots taken to deliver packets from source to destination.

In [15], fast mobility is assumed. A different time-scale of mobility, slow mobility, was considered by Toumpis and Goldsmith in [20], and Lin and Shroff in [11]. For slow mobiles, node mobility is assumed to be much slower than data transmissions. So the packet size can be scaled down as $n$ increases, and multi-hop transmissions are feasible in single time slot. The delay-throughput trade-off was shown to be $\lambda = \Omega\left(\sqrt{D/n\log n}\right)$ in [20]. A better trade-off was obtained in [11], where the maximum throughput per S-D pair for mean delay $D$ was shown to be $\lambda = O\left(\sqrt{D/n}\log n\right)$, and a scheme was proposed to achieve a trade-off of $\lambda = \Theta\left(\sqrt{D/\left(n\log^{9/2} n\right)}\right)$.

Besides the i.i.d. mobility model, other mobility models have also been studied in the literature. The random walk model was introduced by El Gamal et al in [5], and later studied in [6], [7] and [18]. In [6] and [7], the throughput per S-D pair is shown to be $\Theta(1/\sqrt{n\log n})$ for $D = O(1/\sqrt{n\log n})$, and $\Theta(D/n)$ for $D = \Omega(1/\sqrt{n\log n})$, where [6] focused on the slow mobility and [7] focused on the fast mobility. Other mobility models, like Brownian motion, one dimensional mobility, and...
hybrid random walk models have been studied in [12], [3], [8] and [18].

Although the delay-throughput trade-off has been widely studied for various mobility models, the optimal delay-throughput trade-off has not yet been established except for two cases of mobility models [6], [7], [12]. In this paper, we investigate ad-hoc networks with the two-dimensional i.i.d. mobility. Our main results are as follows:

(1) For the two-dimensional i.i.d. mobility model with fast mobiles, we show that the maximum throughput per S-D pair is $O\left(\sqrt{D/n}\right)$ under a delay constraint $D$. A joint coding-scheduling algorithm is presented to achieve the maximum throughput for $D$ is both $\omega\left(\sqrt{n}\right)$ and $o(n)$.

(2) For the two-dimensional i.i.d. mobility model with slow mobiles, we first prove that the maximum throughput per S-D pair is $O\left(\sqrt{D/n}\right)$ given a delay constraint $D$. Then we propose another joint coding-scheduling algorithm to achieve the maximum throughput for $D$ is both $\omega(1)$ and $o(n)$. In both case (1) and (2), we need a lower bound on delay to ensure decodability of packets with high probability for large $n$.

The above results can be extended to other mobility models as shown in a companion paper [21].

We also would like to mention that there is a recent result by Ouzgur, Leveque, and Tse [16] where they showed a throughput of $O(1)$ per S-D pair is achievable using node cooperation and MIMO communication; see also the earlier paper by Aeron and Saligrama in [1]. These schemes require sophisticated signal processing techniques, not considered in this paper.

The remainder of the paper is organized as follows: In Section III we introduce the communication and mobility model. Main results along with some intuition into them are presented in Section IV. Then we analyze the two-dimensional i.i.d. mobility models with fast mobiles in Section VI and slow mobiles in Section VII. Finally, the conclusions are given in Section VII. In the appendix, we collect some results that are frequently used in the paper.

III. MODEL

In this section, we first present the mobility and wireless interference models used in this paper. Then the definitions of delay and throughput are provided.

**Mobile Ad-Hoc Network Model:** Consider an ad-hoc network where wireless mobile nodes are positioned in a unit square. Assuming the time is slotted, we study the two-dimensional i.i.d. mobility model in this paper, which was introduced in [15] and defined as follows:

(i) There are $n$ wireless mobile nodes positioned on a unit square. At each time slot, the nodes are uniformly, randomly positioned in the unit square.

(ii) The node positions are independent of each other, and independent from time slot to time slot. So the nodes are totally reshuffled at each time slot.

(iii) There are $n$ S-D pairs in the network. Each node is both a source and a destination. Without loss of generality, we assume that the destination of node $i$ is node $i+1$, and the destination of node $n$ is node 1.

**Communication Model:** We assume the protocol model introduced in [10] in this paper. Let $\text{dist}(i, j)$ denote the Euclidean distance between node $i$ and node $j$, and $r_i$ to denote the transmission radius of node $i$. A transmission from node $i$ can be successfully received at node $j$ if and only if following two conditions hold:

(i) $\text{dist}(i, j) \leq r_i$;

(ii) dist$(k, j) \geq \left(1 + \Delta\right)\text{dist}(i, j)$ for each node $k \neq i$ which transmits at the same time, where $\Delta$ is a protocol-specified guard-zone to prevent interference.

We further assume that at each time slot, at most $W$ bits can be transmitted in a successful transmission.

**Time-Scale of Mobility:** Two time-scales of mobility are considered in this paper.

(1) Fast mobility: The mobility of nodes is at the same time-scale as the data transmission, so $W$ is a constant independent of $n$ and only one-hop transmissions are feasible in single time slot.

(2) Slow mobility: The mobility of nodes is much slower than the wireless transmission, so $W \gg n$. Under this assumption, the packet size can be scaled as $W/H(n)$ for $H(n) = O(n)$ to guarantee $H(n)$-hop transmissions are feasible in single time slot.

**Delay and Throughput:** We consider hard delay constraints in this paper. Given a delay constraint $D$, a packet is said to be successfully delivered if the destination obtains the packet within $D$ time slots after it is sent out from the source.

Let $\Lambda_i[T]$ denote the number of bits successfully delivered to the destination of node $i$ in time interval $[0, T]$. A throughput of $\lambda$ per S-D pair is said to be feasible under the delay constraint $D$ and loss probability constraint $\varepsilon > 0$ if there exists $n_0$ such that for any $n \geq n_0$, there exists a coding/routing/scheduling algorithm with the property that each bit transmitted by a source is received at its destination with probability at least $1 - \varepsilon$, and

$$
\lim_{T \to \infty} \Pr \left( \frac{\Lambda_i[T]}{T} \geq \lambda, \forall i \right) = 1. 
$$

IV. MAIN RESULTS AND SOME INTUITION

Recall that our objective is to maximize throughput in a wireless network subject to a delay constraint and a wireless interference constraint. More precisely, the constraints can be viewed as follows:

(1) Wireless interference: Throughput is limited due to the fact that transmissions interfere with each other.

(2) Mobility: A packet may not be delivered to its destination before the delay deadline since neither the packet’s source nor any relay node may get close enough to the destination.

In this section, we present some heuristic arguments to obtain an upper bound on the maximum throughput subject to these two constraints and derive the key results of the paper. While the heuristics are far from precise derivations of the optimal delay-throughput trade-offs, they may be useful to the reader in understanding the main results. In addition, the heuristic...
arguments provide the right order for the “hitting distance” (to be defined later) which plays a critical role in the optimal scheme used to achieve the delay-throughput trade-offs.

Consider the two-dimensional i.i.d. mobility model with fast mobiles. We say that a packet hits its destination at time slot $t$ if the distance between the packet and its destination is less than or equal to $L$. Under the two-dimensional i.i.d. mobility model, a packet hits its destination with probability $\pi L^2$ at each time slot. So given a delay constraint $D$, the probability that a packet hits its destination in one of $D$ time slots is

$$1 - (1 - \pi L^2)^D.$$ 

Furthermore under the fast mobility, only one-hop transmissions are feasible at each time slot. So the transmission radius needs to be at least $L$ to deliver packets to the destinations when their distance is $L$. Assume all nodes use a common transmission radius $L$ and that all nodes wish to transmit at each time slot, then each node has $1/(c_1 n L^2)$ fraction of time to transmit, and the throughput per S-D pair is no more than $1/(c_1 n L^2)$ where $c_1$ is a positive constant independent of $n$. Thus the network can be regarded as a system where there are two virtual channels between each S-D pair as in Figure 1. The packets are first sent over the erasure channel with erasure probability

$$P_e = (1 - \pi L^2)^D,$$

and then over the reliable channel with rate

$$R = \frac{1}{c_1 L^2 n}$$

bits per time slot. Each source can transmit at most $W$ bits per time slot on average. So in this virtual system, the maximum throughput of a S-D pair is

$$\lambda = \max_{L} \min_{W} \left\{ \frac{1}{c_1 L^2 n}, \frac{1}{W} \left(1 - (1 - \pi L^2)^D\right) \right\}$$

$$= \sqrt{\frac{\pi WD}{c_1 n}},$$

and the corresponding optimal hitting distance $L^* = b_1/\sqrt{nD}$ where $b_1 = \sqrt{c_1 \pi W}$.

To achieve this throughput, we first need to use the optimal $L$. Furthermore, a coding scheme achieving the capacity of the erasure channel is needed. Since the erasure probability is determined by $L$ and $D$, which are different under different delay constraints, rate-less codes become a reasonable choice. The key idea in this paper is to encode data packets using Raptor codes, which are near optimal rate-less codes with low complexity. We also note that the idea of using coding to improve reliability of packet delivery has also been considered by Shah and Shakkottai in [17] for ad hoc sensor networks in a different context. Our first result is as follows.

**Main Result 1:** Under the two-dimensional i.i.d. mobility model with fast mobiles, the throughput per S-D pair is

$$\lambda = O\left(\sqrt{D/n}\right)$$

given a delay constraint $D$. For $D$ is both $o(\sqrt{n})$ and $o(n)$, this throughput can be achieved using a joint coding-scheduling algorithm.

Note that the heuristic arguments leading up to the above result have many flaws. For example, it suggests that one can wait for the source to hit the destination to deliver the packet. In reality, such a scheme will not work since we deliver only one packet to the destination during each encounter between the S-D pair. Thus other packets at the source which are not delivered may violate their delay constraints. This problem in the heuristic argument is due to the fact that it assumes that we have an independent erasure channel for each packet despite the fact that the transmitting node is the same source. Despite the flaws, the heuristic argument surprisingly captures the delay-throughput trade-off and the optimal hitting distance correctly up to the right order. In practice, the bound is achievable by exploiting the broadcast nature of the wireless channel to transmit each packet to several relay nodes and allowing relay nodes to independently attempt to deliver the packet to the destination.

Next consider the two-dimensional i.i.d. mobility model with slow mobiles. Since multi-hop transmissions are feasible at each time slot, using a precise version of the result [10] which was obtained in [4], the maximum throughput per S-D pair under the slow mobility assumption is

$$\frac{1}{c_2 L \sqrt{n}}$$

where $c_2$ is a positive constant independent of $n$. We provide a crude version of the argument from [10] here for ease of readability. Suppose each node uses a transmission radius $r$ and the distance between a S-D pair is $L$, then each bit has to travel $L/r$ hops. The number of bit-hops needed to satisfy a throughput requirement of $\lambda$ bits/slot/node in $T$ slots is $\lambda LT/r$. Due to the interference model, the number of simultaneous transmissions possible in one time slot is $1/(c_2 r^2)$ for some constant $c_2$. Thus we need

$$\frac{n \lambda LT}{r} \leq \frac{T}{c_2 r^2},$$

or

$$\lambda \leq \frac{1}{c_2 L r n}.$$ 

Intuitively, since the total area is 1 and the number of nodes is $n$, the smallest radius of transmission that can be used while ensuring connectivity is given by $n \pi r^2 = 1$, so

$$\lambda \leq \frac{1}{c_2 L \sqrt{\pi n}}.$$ 

That this is indeed achievable in an order sense is proved in [4], and therefore, we take $\lambda$ to be $1/(c_2 L \sqrt{\pi})$ where $c_2 = \sqrt{\pi c_2}$. Then the virtual channels between a S-D pair are as depicted in Figure 2. In this virtual system, the maximum
throughput of a S-D pair is
\[ \lambda = \max_L \min \left\{ W \left(1 - (1 - \pi L^2)^D\right), \frac{1}{c_2 L \sqrt{n}} \right\} \]
and the optimal hitting distance \( L^* = b_2/\sqrt{nD^2} \) where \( b_2 = \sqrt{c_2 \pi W} \). This throughput can also be achieved using a joint coding-scheduling scheme. The main result is summarized as follows.

**Main Result 2:** Under the two-dimensional i.i.d. mobility model with slow mobiles, the throughput per S-D pair is
\[ \lambda = O\left(\sqrt{D/n}\right) \]
given a delay constraint \( D \). This throughput can be achieved using a joint coding-scheduling scheme when \( D \) is both \( \omega(1) \) and \( o(n) \).

![Virtual-channel Representation for the Two-Dimensional I.I.D. Mobility Model with Slow Mobiles](image)

As stated before, the crude virtual channel representation used in this section surprisingly yields the correct results. However, they do not form the basis of the proofs in the rest of the paper. Several assumptions have been made in deriving the virtual channel representation:

(i) The hitting events for various packets are assumed to independent which is difficult to ensure since the same node may act as a relay for multiple packets.

(ii) It assumes a fixed hitting distance which is not reasonable to obtain an upper bound on the throughput. An upper bound must be scheme-independent.

In view of these limitations, we use the virtual channel model to only provide some insight into the results and the hitting distance we should use in the achievable algorithms, but rigorous proofs of the main results are provided in subsequent sections.

V. TWO-DIMENSIONAL I.I.D. MOBILITY MODEL, FAST MOBILES

In this section, we investigate the two-dimensional i.i.d. mobility model with fast mobiles. Assuming that all mobiles have wireless communication and coding capability, we investigate the maximum throughput the network can achieve by using relaying and coding to recover packet loss as discussed in the heuristic arguments. Given a delay constraint \( D \), we will first prove that the maximum throughput per S-D pair which can be supported by the network is \( O\left(\sqrt{D/n}\right) \). Then a joint coding-scheduling scheme will be proposed to achieve the maximum throughput when \( D \) is both \( \omega(\sqrt{n}) \) and \( o(n) \).

A. Upper Bound

In this subsection, we show the maximum throughput the network can support without network coding, i.e., under the following assumption.

**Assumption 1:** Packets destined for different nodes cannot be encoded together. Further, we assume that coding is only used to recover from erasures and not for data compression. Specifically we assume that at least \( k \) coded packets are necessary to recover \( k \) data packets, where all packets (coded or uncoded) are assumed to be of the same size.

Assumption 1 is the only significant restriction imposed on coding/routing/scheduling schemes. We also make the following assumption.

**Assumption 2:** A new coded packet is generated right before the packet is sent out. The node generating the coded packet does not store the packet in its buffer.

Assumption 2 is not restrictive since the information contained in the new packet is already available at the node.

**Assumption 3:** Once a node receives a packet (coded or uncoded), the packet is not discarded by the node till its deadline expires.

Assumption 3 is not restrictive since we are studying an upper bound on the throughput in this section.

Next we introduce following notations which will be used in our proof.

- \( b \): Index of a bit stored in the network. Bit \( b \) could be either a bit of a data packet or a bit of a coded packet. If a node generates a copy of a packet to be stored in another node, then the bits in the copy are given different indices than the bits in the original packet.
- \( d_b \): The destination of bit \( b \).
- \( c_b \): The node storing bit \( b \).
- \( t_b \): The time slot at which bit \( b \) is generated.
- \( S_b \): If bit \( b \) is delivered to its destination, then \( S_b \) is the transmission radius used to deliver \( b \).
- \( R[T] \): The set of all bits stored at relay nodes at time slot \( T \). We do not include bits that are still in their source node in defining \( R[T] \).
- \( \Lambda[T] \): \( \Lambda[T] = \sum_{i=1}^{n} \Lambda_i[T] \).

Assume that the delay constraint is \( D \), and a data packet is processed by the source node at time slot \( t_p \). Then the data packet is said to be active from time slot \( t_p \) to \( t_p + D - 1 \). A bit \( b \) is said to be active if at least one data packet encoded into the packet containing bit \( b \) has not expired. It is easy to see that any bit expires at most \( D \) time slots after the bit is generated. Also a bit is said to be good if it is active when delivered to its destination. Now let \( \tilde{\Lambda}[T] \) denote the number of good bits delivered to destinations in \([0,T]\). Without loss of generality, we assume good bits are indexed from 1 to \( \tilde{\Lambda}[T] \). Note that expired bits might help decode good source bits but would not contribute to the total throughput, so we have

\[ \tilde{\Lambda}[T] \geq \Lambda[T], \]
where \( \Lambda[T] \) is the number of good source bits successfully recovered at destinations.

Next we present three fundamental constraints. In the following lemma, inequalities (1) and (3) hold since the total number of bits transmitted or received in \( T \) time slots cannot exceed \( nWT \). Inequality (5) holds since under the protocol model, discs of radius \( \Delta r_i/2 \) around the receivers should be mutually disjoint from each other.
Lemma 1: For any mobility model, the following inequalities hold,
\[ \tilde{\Lambda}[T] \leq nWT \] (3)
\[ \frac{\tilde{\Lambda}[T]}{|\mathcal{S}[T]|} \leq nWT \] (4)
\[ \sum_{b=1}^{\tilde{\Lambda}[T]} \frac{\Lambda^2}{16} (S_b)^2 \leq \frac{WT}{\pi}, \] (5)
where $|\mathcal{S}[T]|$ is the cardinality of the set $\mathcal{S}[T]$.

Proof: Since each node can transmit at most $W$ bits per time slot, the total number of bits transmitted in $T$ time slots is less than $nWT$ which implies inequalities (3) and (4). Inequality (5) was proved in [2].

We first consider the scenario where packet relaying is not allowed, i.e., packets need to be directly transmitted from sources to destinations. In the following lemma, we show that the throughput in this case is at most $\Theta(1/\sqrt{n})$ even without the delay constraint.

Lemma 2: Consider the two-dimensional i.i.d. mobility model with fast mobiles. Suppose that packets have to be directly transmitted from sources to destinations, then
\[ \frac{8\sqrt{\pi}WT\sqrt{n}}{\Delta} \geq E[|\Lambda[T]|]. \] (6)

Proof: First from the Cauchy-Schwarz inequality and inequality (5), we have
\[ \left( \sum_{b=1}^{\tilde{\Lambda}[T]} \frac{\Lambda^2}{16} (S_b)^2 \right)^{1/2} \leq \left( \sum_{b=1}^{\tilde{\Lambda}[T]} \frac{\Lambda^2}{16} \right)^{1/2} \left( \sum_{b=1}^{\tilde{\Lambda}[T]} (S_b)^2 \right)^{1/2} \]
\[ \leq \tilde{\Lambda}[T] \frac{16WT}{\pi \Delta^2}, \]
which implies
\[ E \left[ \sum_{b=1}^{\tilde{\Lambda}[T]} S_b \right] \leq \left( \frac{16WT}{\pi \Delta^2} \right) E \left[ \sqrt{\tilde{\Lambda}[T]} \right]. \] (7)
This gives an upper-bound on the expected distance travelled. Next we bound the total number of times that each mobile gets within a distance $L$ of its destination for $L \in [0, 1/2]$. From the i.i.d. mobility assumption, we have that for any $i$, $j$ and $t$,
\[ \Pr(\text{dist}(i, j)(t) \leq L) = \pi L^2, \]
which implies
\[ E \left[ \sum_{i=1}^{T} \left( \sum_{n=1}^{n \in \mathcal{D}} \text{dist}(i, ((i+1) \mod n)(t) \leq L) \right) \right] = \pi L^2 nT. \]
Since at most $W$ bits can be transmitted at each time slot, we further have
\[ \sum_{b=1}^{\tilde{\Lambda}[T]} 1_{S_b \leq L} \leq W \sum_{i=1}^{T} \sum_{n=1}^{n \in \mathcal{D}} \text{dist}(i, ((i+1) \mod n)(t) \leq L). \]
Taking expectation on both sides of above inequality, we obtain
\[ E[\tilde{\Lambda}[T]] - E \left[ \sum_{b=1}^{\tilde{\Lambda}[T]} 1_{S_b > L} \right] \leq W \pi L^2 nT. \] (8)

Now using Jensen’s inequality and inequalities (7) and (8), we can conclude that
\[ \sqrt{\frac{16WT}{\pi \Delta^2}} E[\tilde{\Lambda}[T]] \geq \left( \frac{16WT}{\pi \Delta^2} \right) E \left[ \sqrt{\tilde{\Lambda}[T]} \right] \geq E \left[ \sum_{b=1}^{\tilde{\Lambda}[T]} S_b \right] \geq L \left( E[\tilde{\Lambda}[T]] - W \pi L^2 nT \right). \] (9)
Note that inequality (9) holds for any $L \in [0, 1/2]$. We choose $L^* = \sqrt{\frac{E[\tilde{\Lambda}[T]]}{2\pi nWT}}$, which is less than $1/2$ since $\tilde{\Lambda}[T] \leq nWT$. Substituting $L^*$ into inequality (9), we have
\[ \sqrt{\frac{16WT}{\pi \Delta^2}} E[\tilde{\Lambda}[T]] \geq \frac{1}{2} L^* E[\tilde{\Lambda}[T]], \]
which implies that
\[ \frac{8\sqrt{\pi}WT\sqrt{n}}{\Delta} \geq E[|\Lambda[T]|]. \] (10)
The lemma then follows from inequality (6).

Next we investigate the maximum throughput the network can support using coding/routing/scheduling schemes. We have obtained an upper bound on the number of bits directly transmitted from sources to destinations in Lemma 2. To bound the maximum throughput with relaying, we will calculate the number of bits transmitted from relays to destinations in the following analysis.

Theorem 3: Consider the two-dimensional i.i.d. mobility model with fast mobiles, and assume that Assumption 1-3 hold. Then given a delay constraint $D$, we have that
\[ \frac{8\sqrt{\pi}WT\sqrt{n}}{\Delta} \sqrt{n} \left( \sqrt{D} + 1 \right) \geq E[|\Lambda[T]|]. \] (10)

Proof: In the proof of the theorem, we treat active bits at relays and active bits at sources differently since we can bound the number of active bits at relays using inequality (4), while the number of active bits at sources could be larger. Let $\tilde{\Lambda}^r[T]$ denote the number of good bits delivered directly from relays to destinations in $[0, T]$. Without loss of generality, we assume these good bits are indexed from 1 to $\tilde{\Lambda}^r[T]$. Similar to inequality (7), we first have
\[ E \left[ \sum_{b=1}^{\tilde{\Lambda}^r[T]} S_b \right] \leq \left( \frac{16WT}{\pi \Delta^2} \right) E \left[ \sqrt{\tilde{\Lambda}^r[T]} \right]. \] (11)
Let $\tilde{L}_b$ denote the minimum distance between node $d_b$ and node $c_b$ from time slot $t_b$ to time slot $t_b + D - 1$, i.e.,
\[ \tilde{L} = \min_{b} \text{dist}(d_b, c_b)(t). \]
Then for any $L \in [0, 1/2]$ and any bit $b \in \mathcal{S}[T]$, we have
\[ \Pr(\tilde{L}_b \leq L) = 1 - (1 - \pi L^2)^D \leq \pi L^2 D, \]
which implies
\[ E \left[ \sum_{b \in S[T]} 1_{L_b < L} \right] \leq nWT \pi L^2 D. \]

Furthermore, we have
\[ \sum_{b = 1}^{N[T]} 1_{S_b < L} \leq \sum_{b \in S[T]} 1_{L_b < L}, \]
which implies that
\[ E \left[ \sum_{b = 1}^{N[T]} S_b \right] \leq nWT \pi L^2 D. \]

Thus we can conclude that
\[ E \left[ \sum_{b = 1}^{N[T]} S_b \right] \geq LE \left[ \sum_{b = 1}^{N[T]} 1_{S_b > L} \right] \geq L \left( E[\hat{N}[T]] - E \left[ \sum_{b = 1}^{N[T]} 1_{S_b < L} \right] \right) \geq LE[\hat{N}[T]] - nWT \pi L^3 D, \]
where the last inequality follows from inequality (12).

Now using Jensen’s inequality and inequalities (11) and (13), we have that for any \( L \in [0, 1/2) \),
\[ \sqrt{\frac{16WT}{\pi \Delta}} E[\hat{N}[T]] \geq LE[\hat{N}[T]] - nWT \pi L^3 D. \]

Substituting
\[ L^* = \sqrt{\frac{E[\hat{N}[T]]}{2nWT \pi D}}, \]
into inequality (13), we can conclude that
\[ \frac{2\sqrt{2WT}}{\Delta} \sqrt{nD} \geq E[\hat{N}[T]]. \]

The theorem follows from inequalities (3), (15) and (2).

From Theorem 3 we can conclude that the throughput per S-D is \( O(\sqrt{D/n}) \) given a delay constraint \( D \).

B. Joint Coding-Scheduling Algorithm

In Section 1V, we motivated the need to first encode data packets. In this subsection, we use Raptor codes and propose a joint coding-scheduling scheme to achieve the maximum throughput obtained in Theorem 3.

Motivated by the heuristic argument in Section 1V, we divide the unit square into square cells with each side of length equal to \( 1/\sqrt{nD} \), which is of the same order as the optimal hitting distance. In our scheme, we will allow final delivery of a packet to its destination only when a relay carrying the packet is in the same cell as the destination. Thus, a packet is delivered only when the relay and destination are within a distance of \( \sqrt{2}/\sqrt{nD} \), which is also the same as the hitting distance calculated in Section 1V except for a constant factor which does not play a role in the order calculations. The mean number of nodes in each cell will be denoted by \( M \) and is equal to \( \sqrt{n/D} \). The transmission radius of each node is chosen to be \( \sqrt{2}/\sqrt{nD} \) so that any two nodes within a cell can communicate with each other. This means that, given the interference constraint, two nodes in a cell can communicate if all nodes in cells within a fixed distance from the given cell stay silent. Each time slot is further divided into \( C \) mini-slots and each cell is guaranteed to be active in at least one mini-slot within each time slot. Assume \( C = 9 \). The reason we use nine mini-slots is that if a node in a cell is active, then no other nodes in any of its neighboring eight cells can be active, but nodes outside this neighborhood can be active. Further, we denote the packet size to be \( W/(2C) \) so that two packets can be transmitted in each mini-slot.

A cell is said to be a good cell at time \( t \) if the number of nodes in the cell is between \( 9M/10 + 1 \) and \( 11M/10 \). We also define and categorize packets into four different types.

- Data packets: There are the uncoded data packets that have to be transmitted by the sources and received by the destinations.
- Coded packets: Packets generated by Raptor codes. We let \( (i, k) \) denote the \( k \)th coded packet of node \( i \).
- Duplicate packets: Each coded packet could be broadcast to other nodes to generate multiple copies, called duplicate packets. We let \( (i, k, j) \) denote a copy of \( (i, k) \) carried by node \( j \), and \( (i, k, J) \) to denote the set of all copies of coded packet \( (i, k) \).
- Deliverable packets: Duplicate packets that happen to be within distance \( L \) from their destinations.

We now describe our coding/scheduling algorithm.

**Joint Coding-Scheduling Scheme I:** We group every 6D time slots into a super time slot. At each super time slot, the nodes transmit packets as follows.

1. **Raptor Encoding:** Each source takes \( 6D/(25M) \) data packets, and uses Raptor codes to generate \( D/M \) coded packets.

2. **Broadcasting:** This step consists of \( D \) time slots. At each time slot, the nodes executes the following tasks:
   
   (i) In each good cell, one node is randomly selected. If the selected node has not already transmitted all of its \( D/M \) coded packets, then it broadcasts a coded packet that was not previously transmitted to \( 9M/10 \) other nodes in the cell during the mini-slot allocated to that cell. Recall that our choice of packet size allows one node in every good cell to transmit during every time slot.
   
   (ii) All nodes check the duplicate packets they have. If more than one duplicate packets have the same destination, select one at random to keep and drop the others.

3. **Receiving:** This step consists of \( 5D \) time slots. At each time slot, if a cell contains no more than two deliverable packets, the deliverable packets are delivered to their destinations using one-hop transmissions during the mini-slot allocated to that cell. At the end of this step, all undelivered packets are dropped. The destinations decode the received coded packets using Raptor
decoding.

Note that in describing the algorithm, we did not account for the delays in Raptor encoding and decoding. However, Raptor codes have linear encoding and decoding complexity. Hence, even if these delays are taken into account, our order results will not change.

**Theorem 4:** Consider Joint Coding-Scheduling Algorithm I. Suppose \( D \) is both \( o(n^{1/3}) \) and \( o(n) \), and the delay constraint is \( 6D \). Then given any \( \epsilon > 0 \), there exists \( n_0 \) such that for any \( n \geq n_0 \), every data packet sent out can be recovered at the destination with high probability, where a coded packet is said to be source are successfully duplicated after the broadcasting step happen with high probability.

Combining inequalities (17)-(19), we can conclude that for any \( n \geq n_1 \),

\[
\operatorname{Pr}\left( Y \geq \frac{6D}{25} \right) \geq 1 - 3e^{-\frac{D}{11\epsilon}},
\]

which implies that every data packet sent out can be recovered with probability at least \( 1 - \epsilon \). Since \( 1 - \epsilon \geq 18/19 \), from the Chernoff bound (see Lemma [11] provided in the Appendix C for convenience), we can conclude that for \( n \geq n_0 \),

\[
\operatorname{Pr}\left( \sum_{i=1}^{T} 1_{\delta_i[1]} \geq \frac{9}{10} T_i \right) \geq 1 - e^{-\frac{9D}{1000\epsilon}},
\]

where we choose \( \delta = 1/20 \) in Lemma [12]. Note that \( \sum_{i=1}^{T} 1_{\delta_i[1]} \geq \frac{9}{10} T_i \) implies at least

\[
\frac{9}{10} T_i \times \frac{6D}{25M} \times \frac{W}{2C} = \frac{27W}{250C} DT_i \frac{D}{n} = \frac{27W}{250C} \sqrt{\frac{D}{n}}
\]

bits are successfully transmitted from node \( i \) to node \( i+1 \) in \( T_i \) super time slots. Since each super time slot consists of \( 6D \) time slots, we can conclude that for \( n \geq n_0 \),

\[
\operatorname{Pr}\left( A_{i}[6DT_i] \geq \frac{27W}{250C} DT_i \sqrt{\frac{D}{n}} \right) \geq 1 - ne^{-\frac{D}{11\epsilon}},
\]

which implies that, for a fixed \( n \geq n_0 \),

\[
\operatorname{Pr}\left( \sum_{i=1}^{D} 1_{\beta_i[1]} \geq \frac{4D}{5M} \right) \geq 1 - e^{-\frac{D}{11\epsilon}}, (20)
\]

for \( n \geq n_1 \). Thus, with a high probability, more than \( 4D/(5M) \) coded packets are broadcast, and each broadcast generates \( 9M/10 \) copies.

Duplicate packets might be dropped at step (ii) of the broadcasting step. We next calculate the number of duplicate packets of node \( i \) left after the broadcasting step. Assume node \( i \) broadcasts \( \tilde{D}_i \) coded packets, so \( \tilde{D}_i \leq D/M \). Then the number of duplicate packets left after the broadcasting step is the same as the number of nonempty bins of following balls-and-bins problem, where the bins represent the mobile nodes other than node \( i \), and the balls represent the duplicate packets broadcast from node \( i \).

**Balls-and-Bins Problem:** Assume we have \( (n-1) \) bins. At each time slot, we select \( 9M/10 \) bins and drop one ball in each of them. Repeat this \( \tilde{D}_i \) times.

Using \( N_1 \) to denote this number, from Lemma [12] in Appendix C, we have

\[
\operatorname{Pr}(N_1 \geq (1 - \delta)(n-1)\tilde{p}_1) \geq 1 - 2e^{-\delta^2(n-1)\tilde{p}_1/3},
\]

where

\[
\tilde{p}_1 = \left( 1 - e^{-\frac{9D}{1000\epsilon}} \right).
\]
Using the fact $1 - e^{-x} \geq x - x^2/2$ for any $x \geq 0$, we get
\[
(n-1)\bar{p}_1 = (n-1) \left(1 - e^{-\frac{9\bar{D}M}{100n-100}}\right) \\
\geq \frac{9\bar{D}M}{10} - \frac{81\bar{D}^2 M^2}{100n-100} \\
\geq \frac{44}{49}\bar{D}M,
\]
where the last inequality holds for $n \geq \bar{n}_2$ for some $\bar{n}_2$ since $\bar{D}M \leq D = o(n)$. Thus choose $\delta = 1/50$ and we can conclude for $n \geq \bar{n}_2$,
\[
\Pr\left(N_1 \geq \frac{22}{25}\bar{D}M \sum_{i=1}^D 1_{[\rho_i = \bar{D}_i]} = \bar{D}_i\right) \geq 1 - 2e^{-\frac{\bar{D}M}{10000}}. \tag{22}
\]
Recall a coded packet is said to be successfully duplicated if it has at least $4M/5$ copies at the end of the broadcasting step. Inequality (22) implies for $n \geq \bar{n}_2$,
\[
\Pr\left(A_i \geq \frac{4}{5}\bar{D}_i \sum_{i=1}^D 1_{[\rho_i = \bar{D}_i]} = \bar{D}_i\right) \geq 1 - 2e^{-\frac{\bar{D}M}{10000}},
\]
since otherwise, less than $22\bar{D}M/25$ duplicate packets are left in the network. Thus we can conclude that for $n \geq \bar{n}_2$,
\[
\Pr\left(A_i \geq \frac{16}{25}D \sum_{i=1}^D 1_{[\rho_i = \bar{D}_i]} \geq \frac{4}{5}D \bar{M}\right) \geq 1 - 2e^{-\frac{3D}{1000}}. \tag{23}
\]
Letting $n_1 = \max\{\bar{n}_1, \bar{n}_2\}$, inequality (23) follows from inequalities (21) and (23) for $n \geq n_1$.

**Analysis of receiving:** Assume coded packets $\{(1, i), \ldots, (k, 16D/(25M))\}$ are successfully duplicated. We let $\mathcal{D}(i,k)[t]$ denote the event that coded packet $(i,k)$ is delivered at time slot $t$. Then $\mathcal{D}(i,k)[t]$ will definitely occur if both the following conditions hold:

(i) One and only one duplicate packet of $(i,k)$ becomes a deliverable packet. Let $\mathcal{D}^1(i,k)[t]$ denote this event. Assume the duplicate packet is $(i,k,j)$, i.e., node $j$ contains packet $(i,k)$.

(ii) There are no other deliverable packets in the cell containing node $j$ except packet $(i,k,j)$ and one possible duplicate packet to node $j$ carried by node $i+1$. Let $\mathcal{D}^2(i,k)[t]$ denote this event.

Note that duplicate packets of node $i$ are carried by different nodes, and their mobilities are independent. Now assume there are $\bar{M}(i,k)$ copies of $(i,k)$ in the entire network, then
\[
\Pr\left(\mathcal{D}^1(i,k)[t]\right) = \frac{\bar{M}(i,k)M}{n} \left(1 - \frac{M}{n}\right)^{\bar{M}(i,k)-1}.
\]
Note that $(1-M/n)^{\bar{M}(i,k)-1} \geq 1 - (n\bar{M}(i,k) - 1)M/n,$ and $\bar{M}(i,k) \geq 4M/\bar{D}$ if $(i,k)$ is successfully duplicated. So for a successfully duplicated packet, there exists $\bar{n}_3$ such that for any $n \geq \bar{n}_3$,
\[
\Pr\left(\mathcal{D}^1(i,k)[t]\right) \geq \frac{7M^2}{10n}.
\]
Suppose we have $\bar{M}$ nodes in the cell containing node $j$, from the Chernoff bound, we have
\[
\Pr\left(\bar{M} \leq \frac{11}{10}M\right) \geq 1 - e^{-\frac{M}{200}}.
\]
Note that condition (ii) is equivalent to the following event: Given node $j$ and node $i+1$ in the cell, no more deliverable packets appear when we put another $M-2$ nodes into the cell. Now given $K$ nodes in the cell, the probability that no more deliverable appears when we put another node is at least
\[
\left(1 - \frac{2KD}{n-K}\right)\tag{24}
\]
This holds due to the following two facts:

(a) The new node should not be the destination of any duplicate packets already in the cell (there are at most $KD$ duplicate packets already in the cell).

(b) The duplicate packets carried by the new node are not destined for any of the existing $K$ nodes. Note that each source has no more than $D$ duplicate packets, so there are at most $KD$ nodes which carry the duplicate packet towards the $K$ existing nodes.

Note that $\lim_{n \to \infty} M = \infty$, so there exists $\bar{n}_4$ such that for any $n \geq \bar{n}_4$,
\[
\Pr\left(\mathcal{D}^2(i,k)[t]\right) \geq \frac{21M^2}{110n} = \frac{21}{110D}.
\]
which implies at each time slot, a successfully duplicated packet $(i,k)$ will be delivered with probability at least $21/(110D)$. Note at each time slot, only one coded packet can be delivered to the destination of node $i$. So the number of distinct coded packets delivered to the destination of node $i$ is the same as the number of nonempty bins of following balls-and-bins problem, where the bins represent the distinct coded packets, the balls represent successful deliveries, and a ball is dropped in a specific bin means the corresponding coded packet is delivered to the destination.

**Balls-and-bins Problem:** Suppose we have $16D/(25M)$ bins and one trash can. At each time slot, we drop a ball. Each bin receives the ball with probability $21/(110D)$, and the trash can receives the ball with probability $1 - p$, where
\[
p = \frac{21}{110D} \times \frac{16D}{25M} = \frac{168}{1375} \approx 0.123.
\]
Repeat this $SD$ times, i.e., $5D$ balls are dropped.

Let $N_2$ denote nonempty bins of the above balls-and-bins problem and choose $\delta = 1/6$. From Lemma 12 in Appendix C, we have
\[
\Pr\left(N_2 \geq \frac{7D}{25M}\right) \geq 1 - 2e^{-\frac{7D}{25M}},
\]
and inequality (13) holds for $n \geq n_2$, where $n_2 = \max\{\bar{n}_3, \bar{n}_4\}$.

**Analysis of decoding:** Inequality (19) follows from Lemma 9 on the error probability of Raptor codes provided in Appendix A.
VI. TWO-DIMENSIONAL I.I.D. MOBILITY MODEL, SLOW MOBILES

In this section, we investigate the two-dimensional i.i.d. mobility model with slow mobiles. Given a delay constraint $D$, we first prove the maximum throughput per S-D pair which can be supported by the network is $O\left(\sqrt[3]{D/n}\right)$. Then a joint coding-scheduling scheme is proposed to achieve the maximum throughput.

A. Upper Bound

Let $\hat{t}_b$ denote the time slot in which bit $b$ is delivered to its destination. Under slow mobility, the delivery in $\hat{t}_b$ ties hold, which implies

$$\lambda[T] \sum_{b=1}^{H_b} 1 \leq nWT$$

Lemma 5: For any mobility model, the following inequalities hold,

$$\lambda[T] \sum_{b=1}^{H_b} 1 \leq nWT$$

$$\lambda[T] \sum_{b=1}^{H_b} \frac{\Delta^2}{16} (S_k^b)^2 \leq \frac{WT}{\pi}.$$  

Similar to Lemma 1, we have the following results.

Lemma 6: Consider the two-dimensional i.i.d. mobility model with slow mobiles. Suppose that packets have to be directly transmitted to destinations from sources, then

$$\frac{4\sqrt{2}WT}{\sqrt[3]{D}} \sqrt[3]{\frac{n}{\Delta}} \geq E[A[T]]$$

Proof: First from the Cauchy-Schwartz inequality and Lemma 5 we have that

$$\left(\lambda[T] \sum_{b=1}^{H_b} S_k^b\right)^2 \leq \left(\sum_{b=1}^{H_b} 1\right) \left(\sum_{b=1}^{H_b} (S_k^b)^2\right)$$

$$\leq \frac{WT \sum_{b=1}^{H_b} (S_k^b)^2}{\pi \Delta^2}$$

which implies

$$\frac{4WT \sqrt{n}}{\Delta \sqrt[3]{\pi}} \geq \sum_{b=1}^{H_b} L_b$$

since $\sum_{b=1}^{H_b} S_k^b \geq L_b$. The rest of the proof is same as the proof of Lemma 2.

$$\Delta$$

Theorem 7: Consider the two-dimensional i.i.d. mobility model with slow mobiles, and assume that Assumption 1-3 holds. Then given a delay constraint $D$, we have

$$\frac{4\sqrt{2}WT}{\sqrt[3]{D}} \sqrt[3]{\frac{n}{\Delta}} \left(\sqrt[3]{D} + 1\right) \geq E[A[T]].$$  

Proof: Similar to the proof of Theorem 3.

From Theorem 2 we can conclude that the throughput per S-D is $O(\sqrt[3]{D/n})$ given a delay constraint $D$.

B. Joint Coding-Scheduling Algorithm

In this subsection, we propose a joint coding-scheduling scheme to achieve the throughput suggested in Theorem 2. In the receiving step, we divide the unit square into square cells with each side of length equal to $1/\sqrt{n}$, which is of the same order as the optimal hitting distance obtained in Section IV. The mean number of nodes in each cell will be denoted by $M_2$ and is equal to $3/\sqrt{n}D$. The packet size is chosen to be

$$\frac{10W}{11c_1cM_2} = \frac{10W}{11c_1CM_1},$$

so that at each time slot, all nodes in a good cell can transmit one packet to some other node in the same cell by using the highway algorithm proposed in [4] (see in Appendix B), where $c_1$ is a constant independent of $n$. In the broadcasting step, the unit square is divided into square cells with each side of length equal to $1/\sqrt{n}D$. The mean number of nodes in each will be denoted by $M_1$ and is equal to $\sqrt{n}/D$. In the broadcasting step, the transmission radius of each nodes is chosen to be $\sqrt[3]{2nD}$. Note the packet size is

$$\frac{10W}{11c_1cM_2} = \frac{10W}{11c_1CM_1}.$$ 

So in the broadcasting step, all nodes in a good cell could be scheduled to broadcast one coded packet at one min-slot. Also note that $M_1M_2D/n = 1$.

Joint Coding-Scheduling Algorithm II: We group every $16D$ time slots into a super time slot. At each super time slot, the nodes transmit packets as follows:

1. **Raptor Encoding**: Each source takes $2D/5$ data packets, and uses Raptor codes to generate $D$ coded packets.

2. **Broadcasting**: The unit square is divided into a regular lattice with $n/M_1$ cells. This step consists of $D$ time slots. At each time slot, the nodes execute the following tasks:

   (i) In each good cell, the nodes take their turns to broadcast a coded packet to $9M_1/10$ other nodes in the cell. We use the same definition of a good cell as in Algorithm I, i.e., the number of nodes in a good cell should be with a factor of the mean, where the factor is required to lie in the interval $[0.9, 1.1]$.  

   (ii) All nodes check the duplicate packets they have. If more than one duplicate packet is destined to a same destination, randomly keep one and drop the others.
II. Suppose \( \Delta^2 \geq \text{destination} \) with probability at least \( 1 \). With high probability, i.e.,

\[
\text{Receiving: \ where a coded packet is said to be successfully duplicated if}
\]

\[
\text{Broadcasting:} \quad \left(\text{node } i, \text{node } j\right) \quad \text{for nodes } i \text{ and } j.
\]

At the end of this step, all undelivered packets are dropped. Destinations use Raptor decoding to obtain the corresponding destination. So

\[
\text{Theorem 8:} \quad \text{Consider Joint Coding-Scheduling Algorithm II. Suppose } D = \Theta(1) \text{ and the delay constraint is } 16D. \text{ Then given any } \epsilon, \text{ there exists } n_0 \text{ such that for any } n \geq n_0, \text{ every data packet sent out can be recovered at the destination with probability at least } 1 - \epsilon, \text{ and furthermore}
\]

\[
\lim_{n \rightarrow \infty} \frac{A_i}{T} \geq \frac{9W}{440c_iC} \frac{D}{n} \quad \forall i, \quad \text{where a coded packet is said to be successfully duplicated if}
\]

\[
\text{Pr} \left( A_i \geq \frac{4D}{5} \right) \geq 1 - 3e^{-\frac{D}{20n}},
\]

where a coded packet is said to be successfully duplicated if the packet is in \( 4M_1/5 \) distinct relay nodes.

\[
\text{Receiving: \ At least } D/2 \text{ distinct coded packets from a source are delivered to its destination after the receiving step with high probability, i.e.,}
\]

\[
\text{Pr} \left( B_j \geq \frac{D}{2} \right) \geq 4 \frac{D}{5} \geq 1 - e^{-\frac{D}{20n}},
\]

After obtaining inequality (29) and (30), the theorem can be proved by following the argument in Theorem 3.

\[
\text{Analysis of broadcasting: \ Similar to the analysis of inequality (17).}
\]

\[
\text{Analysis of receiving: \ Assume that coded packets } \left(i, 1\right), \ldots, \left(i, 4D/5\right) \text{ are successfully duplicated. Note that duplicate packets from a common source are carried by different nodes; and the mobilities of those nodes are independent. So } \left(i, k\right) \text{ will be definitely delivered to its destination if both the following conditions hold:}
\]

\[
\text{(i) A copy of } \left(i, k\right) \text{ is the only deliverable packet for destination } i + 1 \text{ in time slot } t. \text{ Let } \mathcal{D}_{i,k}[t] \text{ denote this event; and assume that the duplicate packet is in node } j.
\]

\[
\text{(ii) Node } j \text{ has no other deliverable packet, and the cell containing node } j \text{ is good. Let } \mathcal{D}^2_{i,k}[t] \text{ denote this event.}
\]

Let \( M_{i,k} \) denote the number of copies of \( (i,k) \). Since each source has at most \( 9M_1/10 \) packets in the network, we have

\[
\text{Pr} \left( \mathcal{D}^2_{i,k}[t] \right) \geq \frac{M_{i,k}M_2}{n} \left( 1 - \frac{M_2}{n} \right)^{9M_1/10}.
\]

It is easy to verify that

\[
\lim_{n \rightarrow \infty} \left( 1 - \frac{M_2}{n} \right)^{9M_1/10} = e^{-0.9}.
\]

Thus, for successfully duplicated packet \( (i,k) \), i.e., \( M_{i,k} \geq 4M_1/5 \), we can conclude that

\[
\text{Pr} \left( \mathcal{D}^1_{j,k}[t] \right) \geq \left( 1 - \frac{M_2}{n} \right)^{9M_1/10} = e^{-0.9}, \quad (31)
\]

where \( 1 - \frac{M_2}{n} \) is the lower bound on the probability that all packets in node \( j \) except \( (i,k) \) are undeliverable, and

\[
1 - 2e^{-\frac{D}{20n}} \text{ is the probability that the cell is good.}
\]

From inequalities (31) and (32), we can conclude that for sufficiently large \( n \),

\[
\text{Pr} \left( \mathcal{D}^2_{i,k}[t] \right) \geq \left( 1 - \frac{M_2}{n} \right)^{DM_1/10} - 2e^{-\frac{D}{20n}}, \quad (32)
\]

Inequality (30) can be proved by the balls-and-bins argument used to show inequality (18).}

VII. Conclusion

In this paper, we investigated the optimal delay-throughput trade-off in ad-hoc networks with two-dimensional i.i.d. mobility models. For the two-dimensional i.i.d. mobility model with fast mobiles, the optimal trade-off was shown to be

\[
\lambda = \Theta \left( \sqrt{D/n} \right)
\]

where \( D \) is the delay constraint. For the two-dimensional i.i.d. mobility model with slow mobiles, the optimal trade-off was shown to be

\[
\lambda = \Theta \left( \sqrt[2]{D/n} \right)
\]

when \( D \) is both \( \omega(1) \) and \( o(n) \).

We now briefly comment on the conditions that we have imposed on the delay requirement to obtain the optimal delay-throughput tradeoffs. In the slow mobility case, we have assumed that the required delay has to be \( o(1) \). This condition on the delay is used to allow the decoding error probability to go to zero as \( n \to \infty \). If we allow a small probability of loss (it can be arbitrarily small), then one can allow the delay to be \( o(1) \). We have also assumed that the delay is \( o(n) \). This is not really a restriction since it is easy to see from prior work...
that the best achievable throughput of $\Theta(1)$ is obtained when the delay is $o(n)$ [9]. The $o(n)$ condition on delay is used in our paper only to ensure that our cell partitioning, scheduling and coding strategy works.

In the case of fast mobiles, when $D$ is $O(\sqrt{n})$, then the number of packets that can be transmitted in $D$ time slots is a constant and hence one cannot use coding to ensure that the probability of packet loss is arbitrarily small. In this case, one can obtain a bound that is a logarithmic factor smaller than the upper bound using packet replication techniques as has been done in [11] for the slow-mobile case. However, the best achievable lower bound is unknown. Again the $o(n)$ requirement is not significant since a throughput of $\Theta(1)$ can be achieved if the delay requirement is larger [9].

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APPENDIX A: RAPTOR CODES

A. Raptor Codes

Raptor codes are low-complexity, near-optimal rate-less codes for erasure channels. It was proposed by Shokrollahi in [19], and the following result was presented in [19].

Lemma 9: The receiver can correctly decode the $M$ data packets with probability at least $1 - 1/(M^{2\epsilon(\epsilon)})$ for some $a(\epsilon) > 0$ after it obtains $(1 + \epsilon)M$ coded packets generated by Raptor codes. The number of operations used for encoding and decoding is $O(M)$.

APPENDIX B: THROUGHPUT OF STATIC WIRELESS NETWORKS

The throughput of a random wireless network with $n$ static nodes and $n$ random S-D pairs is introduced by Gupta and Kumar [10]. They showed that the maximum throughput per S-D pair is $O(1/\sqrt{n})$, and proposed a scheduling scheme achieving a throughput of $\Theta(1/\sqrt{n\log n})$ per S-D pair. This $\log n$ gap was later closed by Franceschetti et al in [4] where they showed a throughput of $\Theta(1/\sqrt{n})$ per S-D pair is achievable. The result is obtained under the physical interference models. However, it can be easily extended to the protocol model by using the same algorithm.

Lemma 10: In a random wireless network with $n$ static nodes and $n$ S-D pairs, a throughput of

$$\lambda = \frac{W}{c_s \sqrt{n}}$$

bits/time-slot per S-D pair is achievable, where $c_s$ is a positive constant independent of $n$.

Suppose the nodes use a common transmission radius $r = \Theta(1/n)$. The key idea of [4] is to construct $\Theta(n)$ disjoint paths traversing the network vertically and horizontally. These paths are called highways in [4], and a throughput of $\Theta(1/\sqrt{n})$ per S-D pair is achievable by transmitting data throughput these highways. We call this algorithm a highway algorithm in this paper.

APPENDIX C: PROBABILITY RESULTS

In this appendix, we present some standard results in probability for the reader’s convenience. In addition, we also present some variations of standard results which do not seem to be available in any book to best of our knowledge.

The following lemma is a standard result in probability, which we provide here for convenience.
Lemma 11: Let $X_1, \ldots, X_n$ be independent $0-1$ random variables such that $\sum_i X_i = \mu$. Then, the following Chernoff bounds hold

\[
\Pr \left( \sum_{i=1}^{n} X_i < (1-\delta)\mu \right) \leq e^{-\delta^2 \mu / 2}, \quad (33)
\]

\[
\Pr \left( \sum_{i=1}^{n} X_i > (1+\delta)\mu \right) \leq e^{-\delta^2 \mu / 3}. \quad (34)
\]

Proof: A detailed proof can be found in [14].

The next lemmas are variations of standard balls-and-bins problems. However, we have not seen the results for the particular variation that we need in this paper. So we present the lemmas along with brief proofs below.

Lemma 12: Assume we have $m$ bins. At each time, choose $h$ bins and drop one ball in each of them. Repeat this $n$ times. Using $N_1$ to denote the number of bins containing at least one ball, the following inequality holds for sufficiently large $n$.

\[
\Pr (N_1 \leq (1-\delta)m\tilde{p}_1) \leq 2e^{-\delta^2 m\tilde{p}_1/3}. \quad (35)
\]

where $\tilde{p}_1 = 1 - e^{-\frac{h}{m}}$.

Proof: At each time, bin $i$ receives a ball with probability $h/m$. We let $\kappa_i$ denote the number of balls in bin $i$. Now consider a related balls-and-bins problem where the ball dropping procedure is replaced by a certain number of trials as dictated by a Poisson random variable. Specifically, define $\bar{n}$ to be a Poisson random variable with mean $n$, and repeat the ball dropping procedure $\bar{n}$ times. Let $\bar{\kappa}_i$ denote the number of balls in bin $i$ in this case. It is easy to see that $\{\bar{\kappa}_i\}$ are i.i.d. Poisson random variables with mean $nh/m$. So we can conclude

\[
\Pr (N_1 \leq (1-\delta)m\tilde{p}_1) = \Pr \left( \sum_{i=1}^{m} \bar{\kappa}_i \geq 1 \leq (1-\delta)m\tilde{p}_1 \right)
\]

\[
\leq \Pr \left( \sum_{i=1}^{m} \bar{\kappa}_i \geq 1 \leq (1-\delta)m\tilde{p}_1 \bar{n} \geq n \right)
\]

\[
\leq \Pr \left( \sum_{i=1}^{m} \bar{\kappa}_i \geq 1 \leq (1-\delta)m\tilde{p}_1 \bar{n} \geq n \right) = 2\Pr \left( \sum_{i=1}^{m} \bar{\kappa}_i \geq 1 \leq (1-\delta)m\tilde{p}_1 \right).
\]

Since

\[
Pr (1_{\kappa_i \geq 1} = 1) = Pr (\bar{\kappa}_i \geq 1) = 1 - e^{-\frac{h}{m}} = \tilde{p}_1,
\]

from Lemma [11] we have

\[
Pr (N_1 \leq (1-\delta)m\tilde{p}_1) \leq 2e^{-\delta^2 m\tilde{p}_1/3}.
\]

The above idea of using a Poisson number of trials to bound the probability of the occurrence of an event in a fixed number of trials is called the Poisson heuristic in [14].

Lemma 13: Suppose $n$ balls are independently dropped into $m$ bins and one trash can. After a ball is dropped, the probability in the trash can is $1 - p$, and the probability in a specific bin is $p/m$. Using $N_2$ to denote the number of bins containing at least 1 ball, the following inequality holds for sufficiently large $n$.

\[
Pr (N_2 \leq (1-\delta)m\tilde{p}_2) \leq 2e^{-\delta^2 m\tilde{p}_2/3}; \quad (36)
\]

where $\tilde{p}_2 = 1 - e^{-\frac{h}{m}}$.

Proof: Let $\kappa_i$ denote the number of balls in bin $i$. Next define $\bar{n}$ to be a poisson random variable with mean $n$. We consider the case such that $\bar{n}$ balls are independently dropped in $m$ bins. Using $\bar{\kappa}_i$ to be number of balls in bin $i$ in this case, it is easy to see that $\{\bar{\kappa}_i\}$ are i.i.d. poisson random variables with mean $\frac{\bar{n}}{m}$.

Now given $\bar{n}$, we first have

\[
Pr (N_2 \leq (1-\delta)m\tilde{p}_2) = Pr \left( \sum_{i=1}^{m} 1_{\bar{\kappa}_i \geq 1} \leq (1-\delta)m\tilde{p}_2 \right)
\]

\[
= Pr \left( \sum_{i=1}^{m} 1_{\bar{\kappa}_i \geq 1} \bar{n} \geq n \right) \leq \frac{Pr \left( \sum_{i=1}^{m} 1_{\bar{\kappa}_i \geq 1} \leq (1-\delta)m\tilde{p}_2 \bar{n} \geq n \right)}{Pr(\bar{n} \geq n)}.
\]

Since

\[
Pr (1_{\bar{\kappa}_i \geq 1} = 1) = Pr (\bar{\kappa}_i \geq 1) = 1 - e^{-\frac{h}{m}} = \tilde{p}_2,
\]

from Lemma [11] we have

\[
Pr \left( \sum_{i=1}^{m} 1_{\bar{\kappa}_i \geq 1} \leq (1-\delta)m\tilde{p}_2 \right) \leq e^{-\delta^2 m\tilde{p}_2/3}.
\]

which implies for sufficiently large $n$,

\[
Pr (N_2 \leq (1-\delta)m\tilde{p}_2) \leq \frac{\sqrt{3\pi}ne^{-\delta^2 m\tilde{p}_2/3}}{2e^{-\delta^2 m\tilde{p}_2/3}}.
\]

\]