Finite temperature phase transition in a cross-dimensional triangular lattice

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Abstract

Atomic many-body phase transitions and quantum criticality have recently attracted much attention in non-standard optical lattices. Here we perform an experimental study of finite temperature superfluid transition of bosonic atoms confined in a three dimensional triangular lattice, whose structure can be continuously deformed to dimensional crossover regions including quasi-one and two dimensions. This non-standard lattice system provides a versatile platform to investigate many-body correlated phases. For the three dimensional case, we find that the finite temperature superfluid transition agrees quantitatively with the Gutzwiller mean field theory prediction, whereas tuning towards reduced dimensional cases, both quantum and thermal fluctuation effects are more dramatic, and the experimental measurement for the critical point becomes strongly deviated from the mean field theory. We characterize the fluctuation effects in the whole dimension crossover process. Our experimental results imply strong many-body correlations in the system beyond mean field description, paving a way to study quantum criticality near Mott-superfluid transition in finite temperature dimension-crossover lattices.

1. Introduction

Phase transition, a ubiquitous concept in many-body physics, has long been a central subject in the study of condensed matter physics, describing a broad range of phenomena from superconductivity, magnetism, to Bose–Einstein condensation. With recent experimental developments, ultracold atoms in optical lattices have become a fascinating platform to explore quantum phase transitions with control and tuning capability unreachable in conventional systems [1, 2]. Both fermionic and bosonic Hubbard models previously proposed as theoretical toy models to study strongly correlated physics in solid state systems, have now been precisely implemented by confining alkali atoms in optical lattices [1–13]. For the former, experimental efforts have been largely focused on the finite-temperature physics for the experimental challenge to reach the zero-temperature quantum ground states [8, 9, 14–19]. For the latter, the ground state superfluid phase and the Mott–superfluid transition, have been accomplished in optical lattices of different dimensionality and geometries [6, 20–27]. It has been found that this phase transition is qualitatively captured by a Gutzwiller–type mean field theory [28–32].

For a finite temperature system near a quantum phase transition point, it is well known that thermal fluctuations play essential roles in characterizing the relevant physical properties, leading to a wide quantum critical region [33, 34] potentially of deep connections to the understanding of high Tc superconductivity [35] mechanism. Finite temperature effects near a Mott–superfluid transition have been explored in theory [3, 36–40], but the experimental studies are relatively scarce [41–44]. The temperature effects are particularly...
intricate for optical lattices undergoing a dimension crossover, where thermal fluctuations intertwine with quantum kinematics. For such systems, renormalization group analysis implies strong fluctuation effects and inapplicability of mean field theories even at a qualitative level\cite{45-47}. Characterizing quantum and thermal fluctuation effects beyond the mean field theory for the phase transition of an optical lattice in the dimension crossover region thus demands experimental studies.

In this paper, we perform experimental studies of the finite temperature superfluid phase transition in a triangular optical lattice whose dimensionality is continuously tuned from quasi-one dimension to two and three dimensions. Atoms in lattices of different dimensionality can be clearly distinguished from the experimentally measured momentum distributions as probed in two orthogonal directions. As shown in figure 1, the lattice contains a triangular lattice in the $xy$-plane and a one-dimensional lattice along $z$-axis. The lattice structure is determined by the two-dimensional triangular lattice depth $V_{xy}$ and the one dimensional lattice depth $V_z$. When $V_{xy} \approx V_z$, the lattice is three dimensional, where we find phase transition properties agree with mean field theory predictions. The system becomes quasi-two and one-dimensional at $V_z \gg V_{xy}$ and $V_z \ll V_{xy}$ for both of which experimental measurements are strongly deviated from mean field theory predictions, yielding strong fluctuation effects in dimension crossover regions. This is attributed to quantum and thermal fluctuations within a single-band or across different orbitals in optical lattices\cite{12}. Our experiment paves a way to study novel many-body physics of dimension crossover lattices, where quantum and thermal fluctuations are both dominant.

2. Experimental system and model description

In our experiment, the triangular lattice is formed by three laser beams with a wavelength $\lambda_{||} = 1064$ nm that intersect at the position of Bose–Einstein condensate (BEC) in $xy$-plane. For the confinement in the third direction, i.e. the $z$ axis, we add a vertical optical lattice formed by the interference of counter-propagating laser beams with a wavelength $\lambda_z = 852$ nm. The optical potential produced in this setup is then given by

$$V(x) = -V_{xy} \sum_{i=1}^{3} 2 \cos \left(\frac{2\pi}{\lambda_{||}} \sqrt{3} \vec{b}_i \cdot \vec{r} + \Delta \phi_i\right) + 2V_z \cos^2 \left(\frac{2\pi}{\lambda_z} z\right),$$

where we have $x = (x, y, z)$, and $\vec{b}_1 = [1, 0, 0]$, $\vec{b}_2 = [-1/2, \sqrt{3}/2, 0]$, and $\vec{b}_3 = [-1/2, -\sqrt{3}/2, 0]$. Here the unit of potential $V_{xy}$ and $V_z$ is $E_R$, which equals to $\frac{2\pi}{k_{||}}$ with $k_{||} = \frac{2\pi}{\lambda_{||}}$, respectively.

Before turning on the optical lattice, we prepare BECs of about $1 \times 10^5 \text{Rb}$ atoms in a harmonic trap and the temperature is 50 nK. We have a harmonic trap with frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi \times (24 \text{ Hz}, 48 \text{ Hz}, 55 \text{ Hz})$ in the three directions, respectively. The procedure details have been provided in our earlier works\cite{48-52}. After the preparation of BECs, we adiabatically ramp on the triangular lattice within 80 ms. Then the vertical lattice $V_z$ is adiabatically turned on within 20 ms. Then we hold the system for 10 ms. The average filling of the lattice is approximately six atoms per site. Finally, all the trap and lattices are turned off and an absorption image is obtained after time-of-flight. In the experiment, we can get the absorption image from $z$-direction or $y$-direction, which is called Probe-Z and Probe-Y, respectively.

![Figure 1. Pictorial illustration of the experimental lattice system. (a) shows the structure of the optical lattice. There are three laser beams in the $xy$-plane with $\lambda_{||} = 1064$ nm (red) forming a triangular pattern, and two laser beams in the $z$ direction with $\lambda_z = 852$ nm (blue) providing an additional one-dimensional confinement. (b)–(d) Schematic diagram of momentum distribution and the spatial distribution of atoms. (b) corresponds to the three dimensional case with $V_{xy} \approx V_z$, (c) corresponds to the quasi-two dimensional case with $V_{xy} \ll V_z$, where the momentum interference peaks are dispersed on $\hat{x} \cdot \hat{y}$ plane. (d), quasi-one dimensional case with $V_{xy} \gg V_z$ where the interference are dispersed in $\hat{z}$ direction. The red and white dashed lines represent the tunnelings $t_x$ and $t_y$, respectively.](image-url)
A single-band lattice Hamiltonian is reached under tight-banding approximation

\[ H = \sum_{\langle r, r' \rangle} \left[ -t_{ij} \hat{b}_{r}^{\dagger} \hat{b}_{r'} + \text{h.c.} \right] + \sum_{r} \left[ -t_{z} \hat{b}_{r}^{\dagger} \hat{b}_{r+\mathbf{e}_{z}} + \text{h.c.} \right] + \frac{U}{2} \sum_{r} \hat{b}_{r}^{\dagger} \hat{b}_{r}^{\dagger} \hat{b}_{r} \hat{b}_{r} - \mu \sum_{r} \hat{b}_{r}^{\dagger} \hat{b}_{r}. \]  

Here \( (r, r') \) represents two neighboring sites in the \( xy \)-plane, \( \hat{b} \) and \( \hat{b}^{\dagger} \) the annihilation and creation operators, \( U \) the interaction strength, \( \mu \) the chemical potential. The tunnelings in \( xy \)-plane and in the \( z \) direction are \( t_{x} \) and \( t_{z} \), respectively (see figure 1). The tunnelings are determined by fitting the tight-binding energy dispersion to the band structure through exact calculations. The model parameters as shown in figure 2 are highly controllable by tuning the lattice depths \( V_{xy} \) and \( V_{z} \) in our experimental setup.

3. Dimensional crossover

Since the lattice depths \( V_{xy} \) and \( V_{z} \) can be separately tuned in our experiment, the dimensionality of the system is controllable. When \( V_{xy} \) and \( V_{z} \) are comparable, the system is a regular three dimensional lattice. In this region, mean field theory is expected to capture the essential physics, because it is close to the upper critical dimension of the \( U(1) \) phase transition [53]. When \( V_{z} \) is much weaker than \( V_{xy} \), atoms are then less confined in the \( z \) direction, and the system should be treated as weakly coupled one dimensional chains. The corresponding theoretical description is coupled Luttinger liquids, the transition temperature is determined by inter-chain couplings [45, 46, 54, 55]. In the opposite limit, when \( V_{xy} \) is much weaker than \( V_{z} \), the system is formed of weakly coupled two dimensional layers, whose physical properties rely on the comparison between Kosterlitz Thouless transition temperature and inter-layer couplings [47]. Then the superfluid transition in the dimension crossover regions should be intrinsically taken as a finite-temperature phase transition rather than a zero temperature Mott-superfluid transition.

Figure 3 shows time-of-flight measurements of the atomic system, which confirms our capability to control dimensionality from three dimensions to dimension crossover regions. With the Probe-Z and -Y, we measured the momentum distribution in the \( xy \)- and \( zx \)- planes. As shown in figure 3(a), with decreasing \( V_{z} \) at a fixed \( V_{xy} \), \( t_{z} \) becomes larger, which drives a phase transition from normal to a superfluid. In the small \( V_{z} \) limit, the system behaves as a coupled array of Luttinger liquids at finite temperature. Figure 3(b) corresponds to varying \( V_{xy} \) with a fixed \( V_{z} \). In this case, the system is formed of weakly coupled two dimensional systems at small \( V_{xy} \). The weakening of phase coherence with larger \( V_{xy} \) is attributed to the decrease in the tunneling \( t_{z} \).

4. Finite temperature mean field theory

To characterize fluctuation and many-body correlation effects beyond mean field theory in the dimension crossover regions, we provide a finite temperature mean field theory to compare with experimental results.
Under the mean field approximation, the density matrix of the system is given by \( \rho = \prod \exp[-\beta H_{\text{field}}(\mathbf{r})] \), with \( \beta \) the inverse temperature, \( H_{\text{field}}(\mathbf{r}) = -t_{\text{eff}}(\varphi^\dagger \varphi^\dagger + \varphi^\dagger \varphi^\dagger) / 2 + U n_\mathbf{r}(n_\mathbf{r} - 1) / 2 - \mu n_\mathbf{r} \), the tunneling parameter \( t_{\text{eff}} = 6t_i + 2t_z \), and the superfluid order parameter \( \varphi = \{ \mathrm{Tr} \exp[-\beta H_{\text{field}}(\mathbf{r})] \} / \{ \mathrm{Tr} \exp[-\beta H_{\text{field}}(\mathbf{r})] \} \), which is self-consistently determined. The mean field phase diagram is solely dependent on two dimensionless parameters, \( \kappa_B T / t_{\text{eff}} \) and \( U / t_{\text{eff}} \), which characterize the strengths of thermal and quantum fluctuations. As shown in figure 2, we can control the \( t_{\text{eff}} \) and \( U \) by tuning \( V_z \) and \( V_{xy} \), and then observe the phase diagram in experiments. At the zero temperature limit, an instability analysis shows Mott-superfluid phase boundary is given by \( t_{\text{eff}}^* = -[\mu + (1 - n)U] \{ [\mu - nU]/[\mu + U] \} \), with \( n \) the filling of the Mott state. Considering a transition with a fixed particle number, the phase boundary is further reduced to \( t_{\text{eff}}^* / U = 2n + 1 - \sqrt{(2n + 1)^2 - 1} \), reproducing the previous ground state analysis for symmetric lattices [3, 4]. At finite temperature, we rely on numerical self-consistent calculations to compare with experimental results.

5. Experimental determination of finite temperature phase diagram

We perform experimental measurements of the superfluid transition in different parameter regions of the system corresponding to three-, quasi-one and two dimensions. Firstly, we increase \( V_z \) at a fixed \( V_{xy} \). As shown in figure 3(a), we change \( V_z \) from zero to \( 9E_R \) with \( V_{xy} \) fixed at \( 6E_R \). The superfluid phase transition is revealed with Probe-Z. There are several methods to get the transition point from the absorption images [6, 56–59].

To characterize the condensate in the lattice, we extract the visibility from the TOF measurements and analyze the presence/absence of superfluid phase coherence following the approach in [56]. The visibility can be extracted from the TOF images as shown in figure 4(a). We count the number of atoms in the red areas, which indicates the atoms in first order momentum states, and the total number of atoms in these six red areas is expressed as \( N_{\text{atoms}} \). For the contrast, we also count the atoms in the yellow areas, which indicates the incoherent atoms at the edge of the first Brillouin zone and expressed as \( N_{\text{background}} \). Then, the visibility is given as:

\[
\text{Visibility} = \frac{N_{\text{atoms}} - N_{\text{background}}}{N_{\text{atoms}} + N_{\text{background}}} \tag{3}
\]

It should be pointed out that the visibility in the main text is normalized. And the bare absolute value of visibility at \( V_z = 0 \) is about 0.5.

The behavior of the visibility across the transition is shown in figure 4(b). We use a horizontal line and an oblique line to fit the visibility trend, and the transition point is the crossover point of two lines. In figure 4(b), the transition point is \( V_z = 4.9E_R \).

According to the finite temperature mean field theory and the parameters in figure 2, we can get the superfluid order parameters in theory, which is shown in figure 4(b) by the blue line. The temperature refers to the temperature of the BEC, and we keep the entropy constant in the calculation process. The theoretical superfluid order parameters reduce to zero at \( V_z = 5.3E_R \). The experimental measurement thus agrees
quantitatively with mean field theory prediction. This implies trap effects are negligible for the determination of the phase boundary in our experiment. Nonetheless, including trap effects by local density approximation or by the characteristic density approach \[60, 61\] is expected to give more precise results.

Choosing different \(V_{xy}\), we can get critical strengths of \(V_z\) for the superfluid transition as a function of \(V_{xy}\). Then we get a finite temperature phase diagram as shown in figure 4(c), where the measured transition points are represented by the red squares. According to the finite temperature mean field theory and the parameters obtained in figure 2, we can get the phase diagram in theory. As shown in figure 5, the systematic increase of fluctuation effects as we go from the three to one or two dimensions are clearly revealed.

In the above discussion, the temperature of the quantum gas is 50 nK, which refers to the temperature of the BECs before turning on the optical lattice. We get the temperature in experiments by a bimodal fitting to the time-of-flight images. We also systematically study the temperature effect on the superfluid transition phase boundary. The lines in figure 5 show the transition points for different temperatures, 50, 80, and 110 nK. For the
three temperatures, the number density of the atomic gas remains unchanged. For these different temperatures, we use the same method as in figure 4 to find the transition points. For instance, if we fix $V_{xy} = 6E_R$, the transition points are $V_z = 4.9E_R$, $4.1E_R$, and $2.5E_R$ corresponding to the temperature 50 nK, 80 nK and 110 nK, respectively. The theoretical results are shown in figure 5. At these different temperatures, we still see the experimental agreement (disagreement) in the three dimensional (quasi-two and one dimensional) case. The major observed effect of increasing temperature is the decrease in the critical lattice potential. The significance of strong quantum and thermal fluctuations are revealed in the experiment, which implies the dimension crossover lattices provide a natural platform to study quantum critical behaviors, of fundamental interest to the understanding of high Tc superconductivity in Cuprates.

In figure 5, we find that when $V_z$ is very small, the mean-field transition points are no longer reduced as the depth $V_{xy}$ becomes larger. The reason is because the mean-field effective tunneling $t_{eff} = t_1 + t_2$ is dominated by $t_2$ in that limit, and remains the same as $V_{xy}$ increases (see figure 6). Meanwhile, with a larger $V_{xy}$, the chemical potential $\mu$ and interaction $U$ are increasing together, with their ratio roughly unaffected. As a consequence, for small $V_z$, the mean-field superfluid order parameter will not reduce to zero with increased $V_{xy}$. For large $V_z$, the order parameter will drop to zero at large $V_{xy}$ (see figure 6). We calculate how the band gap varies in the 1D or 2D limits. As shown in figure 7, it can be seen that there is a significant parameter region ($2 < V_{xy}/E_R < 4$ or $2 < V_z/E_R < 5$) where the band gap is much larger than the tunneling and the interaction strength $U$ and at the same time, the experimental observations do not agree with the single-band Bose–Hubbard model description. In this parameter region, the single band Bose–Hubbard model is expected to be a valid model describing the

Figure 6. The parameters in Bose–Hubbard model for $V_z = 3E_R$ and $4.5E_R$. (a) The dashed lines, dash–dot lines and solid lines represent the chemical potential $\mu$, hopping amplitude $t_{eff}$ and on-site interaction $U$ in the unit of $E_R$, respectively. (b) The ratio of chemical potential $\mu$ to interaction $U$ and the mean-field superfluid order parameter at $V_z = 3E_R$ and $4.5E_R$. In the mean-field theory calculation, we use a temperature $T = 110$ nK.

Figure 7. The band gap in the 1D or 2D limits. (a) The parameters $T$, $U$, energy gaps in 2D lattice. (b) The parameters in 1D lattice. The red solid lines and blue dashed lines represent effective tunneling $t_{eff}$ and the interaction energy $U$. The black solid lines represent the energy gap between the lowest bands in zero quasi-momentum. The dotted lines represent the smallest gap between the lowest two bands.
experimental system. We mention here that when the lattice confinement in a certain direction is extremely small, the single-band model becomes invalid, and field theories incorporating the continuous degrees of freedom should be constructed. Modeling the strong fluctuation effects revealed by our experiment requires more accurate theoretical methods such as quantum Monte-Carlo, which are left for future study.

6. Conclusions

To conclude, we studied the finite temperature superfluid transition in a three dimensional triangular lattice, continuously tuned from three to quasi-one and two dimensions. For the three dimensional case, the experimentally measured superfluid transition point is found to agree with the Gutzwiller mean field theory prediction, whereas it strongly deviates from the mean field theory in the reduced dimensional cases, revealing strong many-body correlation effects in this optical lattice system. The strong quantum and thermal fluctuation effects established in the dimension crossover regions of our triangular optical lattice, suggest rich quantum critical behavior worth further theoretical and experimental exploration.

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