Cavity geometry in real conditions

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Abstract. The paper presents the formulation of the problems and calculation method of the cavity geometry in real conditions. The calculating procedure used here is based on the method of small perturbations of the thin body. The calculation results and their comparison with the experimental data for the following cases are presented: cavity axis deformation under the effect of gravity forces, cavity behind elliptical cavitator, cavity behind cavitator at nonzero attack angle, the pressure pulsations in the cavity, the motion near boundaries and others.

1. Introduction and method description

In real condition the cavity is not axisymmetric because it is effected by different disturbing factors. These factors are design features of the moving object, effect of gravity forces, presence of solid boundary, motion parameters, and others. And they can effect simultaneously. Thus, there is the problem of developing the calculating methods that allows to define different deformations of liquid boundaries of the cavity.

The method described in the works [1] and [2] meet these requirements. The method is based on the theory of small perturbation of the known thin axisymmetric cavity ($\frac{R_0}{L_0} \approx 1$) in an infinite ideal liquid (see figure 1).

We consider that the cavity shape deformations $\eta$ are small compared to its radius $R_0$ (see Figure 2).

Figure 1 Thin cavity and coordinate system

Figure 2 Cavity small deformations
\[ R = R_0 + \eta , \quad \eta = d \ll 1, \quad \delta = \epsilon , \delta \ll 1. \]

These assumptions allow to consider the evolution of the every cavity cross-section independent from the other cross-sections. In this formulation, the calculation of cavity deformations \( \eta \) can be made only in the linear approximation. The attempts to increase the accuracy of deformation calculation as proposed in [4] are ineligible, as this will lead to the need of considering the effect of longitudinal flow in the adjacent cross-sections of the cavity.

The water flow is considered as potential. In the set of equations of flow continuity and motion the terms of order of greater than \( \epsilon^2 \delta \) are neglected. Small cavity shape deformations \( \eta \) are decomposed in the Fourier series.

\[ \eta = \sum_0^{\infty} \eta_j(t) \cos j\theta \]

As the result we get a set of 2nd order linear differential equations for the Fourier decomposition.

\[
\begin{cases}
2 \frac{d^2 R_0 \eta_j}{dt^2} = R_0 \frac{\partial \psi_0}{\partial r} + R_0 \frac{\partial^2 \psi_0}{\partial \theta \partial r} - \frac{\partial \psi_0}{\partial \theta} - U_0 - \frac{\Delta \sigma_0}{2}

ij R_0 + 2\eta R_0 - (j - 1)\eta R_0 = (j + 1)R_0 \frac{\partial \psi_j}{\partial r} + R_0 \frac{\partial^2 \psi_j}{\partial \theta \partial r} + j \frac{\partial \psi_j}{\partial \theta} - jU_j - \frac{\Delta \sigma_j}{2}, \ j > 0
\end{cases}
\]

Dots indicate the time derivatives. Disturbing factors that lead to deformations of the cavity cross-section are:

- the initial conditions of the fluid flow from the deformed cavitator edges:
  \[ \eta(0) = \eta_0 = \sum_0^{\infty} \eta_j \cos j\theta \]

- cavitation number changes: \( \Delta \sigma = \sum_0^{\infty} \Delta \sigma_j \cos j\theta \)
- external fluid flow with the velocity potential: \( \psi = \sum_0^{\infty} \psi_j \cos j\theta \)
- potential function of gravity forces \( U = \sum j U_j \cos j\theta \)

In the view of the linear formulation we can superpose small deformations generated by a variety of factors. The examples of cavity shape deformations under effect of different factors are presented below.

2. Examples

2.1. Effect of gravity forces

The gravity forces, depending on its direction relative to the vector of cavitator movement, differently effect the cavity shape. Assume that the cavitator moves at a constant speed in a straight line inclined at \( \theta \) angle to the horizontal. Then the potential of fluid gravity forces is:

\[
U_j = \begin{cases} \frac{\text{Fr}^2 (X - x) \sin \theta}{\text{Fr}^2 R_0 \cos \theta}, & j = 0 \\ \frac{\text{Fr}^2 R_0 \cos \theta}{0}, & j = 1 \\ \frac{\text{Fr}^2 R_0 \cos \theta}{0}, & j > 1 \end{cases}
\]

(2)
Here $Fr^2 = \frac{V_o^2}{gL_c}$ is the Froude number. The solution of (1) with (2) allows finding a thin cavity axis curvature $\eta$. The results of cavity axis curvature calculations and the comparison with the experimental data are presented in Figure 3.

![Figure 3](image1.png)  Figure 3 Cavity axis curvature under influence of gravity forces at $\vartheta = 0$

![Figure 4](image2.png)  Figure 4 The cavity axis curvature when cavitator moving at nonzero attack angle

2.2. The influence of the cavitator angle of attack

The first time the problem of the cavity axis deformation when cavitator has nonzero attack angle was solved G.V. Logvinovich [3] by considering the transverse motion of the cavity circular cross-section. The system of equations (1) also allows solving this problem. Assume that the potential of the external flow in (1) is given by the expression:

$$\psi_j = \begin{cases} \alpha(x) \cdot r & j = 1 \\ 0 & j \neq 1 \end{cases}$$

This means that in each cross section transverse speed $\frac{\partial \psi_j}{\partial r} = \alpha(x)$ disturbing the main flow exists.

By substituting it into the (1) we can obtain small cavity deformations:

$$\eta = \alpha(x)t - \alpha(x)R^2 \int_0^t R_0^{-2} dt$$

Figure 4 shows the theoretical and experimental dependences of the relative cavity shape deformation when cavitator moving at attack angle $\alpha$.

2.3. Cavity shape behind non-circular cavitator

System (1) allows defining cavity shape behind non-circular cavitator. The factors disturbing the main flow are cavitator deformation in the plane of flow separation. From a mathematical point of view, these are initial conditions $\eta_j^*$ and $\eta_j^\alpha$ for (1).

2.4. The cavity shape at motion near a plane boundary

For solving the problems of the flow in the presence of planar boundaries it is required to find a potential harmonic function $\Phi$, which in addition to the usual conditions on the cavity surface, satisfies the conditions at a plane boundary: $\frac{\partial \Phi}{\partial n} = 0$ for solid boundary and $\Phi = 0$ for liquid free surface. It is known that the mirroring method, in which the features are located on the other side of the border and the flow is considered as unlimited, is widely used to solve such a problem. The
potential function is \( \Psi = \pm \Lambda (x, r_1, \theta) \). Where sign (+) is for solid boundary, sign (-) is for free surface.

2.5. Impact of gravity waves on the cavity shape
The disturbing factor is the potential of wave flow
\[
\psi = ae^{-H \cos \theta} \sin(kx - \omega t)
\]
\( \eta = \eta_0 + \eta_1 \)
Substituting of (4) into (1) allows to define the cavity shape.

2.6. Effect of static pressure fluctuations inside the cavity
The solution of (1) allows defining cavity shape when the pressure inside it pulses harmonically
\[
\Delta \sigma = \frac{\Delta P}{q} = A \sin \omega t
\]
(see Figure 5). The Figure 6 shows a comparison of the calculated and experimental data.

3. Conclusions
In this paper we developed a theoretical model that allows to determine the three-dimensional form of a thin cavity under the influence of various factors such as the effects of gravity, the attack angle of cavitator etc. The model contains a relatively simple system of differential equations that does not require a lot of computational resources. Experimental studies have confirmed the sufficient accuracy of this approach.

References
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