The Formation of Binary Star Cluster in Our Galaxy from Fractal Stellar Distribution

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Abstract. Observations showed that there are star clusters in paired condition, called as binary star cluster. Some statistical studies attempted to identify the existence of binary and multiple clusters in our Galaxy and Magellanic Clouds. MIStiX survey also found that most of embedded younger star clusters (a few Myr) have clumpy structures, indicating star distribution in young clusters cannot be simply described as spherical shape. This structure can be the key-factor of the formation of primordial binary cluster in our Galaxy. In this work, we simulate the formation of binary star cluster from fractal distribution in isolated condition and under Galactic tidal field for 50 Myr. This aims to investigate the most probable condition to let the highly formation of binary star cluster in our Galaxy and the role of Galactic tidal field in that formation. We find that the clumpier structure of star cluster, the number of binary/multiple cluster will increase, and the same condition also occurs for the warmer condition or higher $\alpha_{\text{vir}}$ (the kinetic to potential energy ratio). But this number will decrease at larger time due to the merger processes between the star clusters. By these arguments, we conclude that the most probable condition of star cluster to let the highly formation of binary star cluster is the one with highly sub structured and warm condition. For such condition of star cluster, the fraction of binary/multiple in our Galaxy is $0.87 \pm 0.06$ at time 20 Myr and decreases to be $0.63 \pm 0.09$ at time 50 Myr, with reduction $0.24 \pm 0.08$. As comparison, this reduction is only $0.07 \pm 0.05$ for the same condition in isolated. The higher deflation shows that the Galactic tidal field has important role in the formation of binary star cluster, as well as the initial mass of star cluster. And we suggest the discovery of multiple clusters in this work is interesting to learn more deeply in the future work.

Keywords: Binary Star Cluster and Fractal Stellar

1. Introduction
The most of stars are born in star cluster [1]. Some observations showed that there are star clusters (open clusters) in paired condition [2, 3]. Some of them are gravitationally interacting, known as binary star cluster. Besides that, many statistical studies attempted to identify binary and multiple star clusters in our Galaxy and Magellanic Clouds, see [2, 4, 5, 6, 7, 8]. They found that the number of binary/multiple clusters in our Galaxy is smaller than Large Magellanic cloud due to the difference of star formation rate both in the galaxies. Binary star clusters are systematically younger than single clusters, i.e., $\leq 25$ Myr [2]. Besides that, several previous studies such as [9, 10, 11] showed most of younger star clusters (a few Myr) have clumpy structures. Furthermore, MIStiX survey supports these arguments after observed embedded star clusters in our Galaxy, see [12]. The clumpy structure of star cluster at early age can trigger the formation of binary star cluster, especially primordial binary cluster that formed through simultaneous mechanism [2]. A few numerical studies such as [13] and [14]...
attempted to model the clumpy structure of star cluster with fractal distribution to investigate the formation of binary star cluster. But their simulations were made in isolated condition without the effect of Galactic tidal field. In this work, we simulate the formation of binary star cluster not only in isolated condition, but also under Galactic tidal field. It aims to investigate the most probable condition of star cluster which let the highly formation of binary star cluster in the Milky Way. Furthermore, we also would like to investigate the role of Galactic tidal field in the formation of binary star cluster. In the section 2, we introduce our fractal model of star cluster and Milky Way potential. Our simulations and statistical method are presented in the section 3. Finally, we summarize our results in the section 4.

2. Model of Star Cluster

We follow [15] to model our star cluster from fractal distribution. To model the fractal cluster, we have to define a cube with length of $N_{div}$ and put a parent particle at the center of the cube (first generation parent particle). Then divide the cube into $N_{div}^3$ subcubes and put a child particle at the center of each subcubes. Child particle with probability $N_{div}^{D-3}$ is taken to be the second generation parent particle (survived particle), while others are deleted. That probability depends on fractal dimension ($D = 0 – 3$) which shows the clumpier structure of star cluster, the smaller value of $D$ (see Figure 1). A little noise is added to each position of survived particle, to avoid an obviously gridded structure. This process is repeated recursively until there is a sufficiently large survived particle. We then impose a spherical envelope of radius 2 pc within the cube (as radius of star cluster) to reduce the survived particles that located out of the radius. The remaining survived particles are then culled randomly until the required number is left. That’s why, there should be more survived particles than the required number of stars before imposing the radius.

![Figure 1. Model of star clusters from fractal distribution with $D = 1.6, 2.2$, and 2.9 for $\alpha_{vir} =$](image)

| Parameters                  | Values    | References |
|-----------------------------|-----------|------------|
| Number of stars in cluster ($N$) | 5000      |            |
| Mass of cluster ($M_c$)     | 3000 M$_\odot$ |            |
| Radius of cluster ($R_c$)   | 2 pc      | [14, 15, 16] |
| Kroupa [17] IMF             | 0.1 – 50 M$_\odot$ | [13, 14, 15] |

We set the star velocity dispersion is coherent, as suggested by [15]. Random velocity is given for the first generation parent particle. Then the second generation parent particles get their velocities inherited from the first one, plus a little noise. This process is repeated recursively and the noise is large at the higher levels, and then becomes smaller, in order to scale the velocities so that $\alpha_{vir}$ has prescribed value, see [14]. Here $\alpha_{vir}$ is the kinetic to potential energy ratio of star cluster and this parameter describes how large the star velocity dispersion in star cluster. We set some physical parameters to model our star cluster in Table 1. We follow [14] for the combination of $D$ and $\alpha_{vir}$ to
describe various condition of structure and velocity dispersion of star cluster (see Table 2). For the simulations under Galactic tidal field, we use Milky Way axisymmetric potential as presented in [18].

Table 2. The combination of $D$ and $\alpha_{vir}$ to describe various conditions of structure and velocity dispersion of star cluster, adopted from [14]. Highly substructured ($D = 1.6$), moderate substructure ($D = 2.2$), and smooth structure ($D = 2.9$) are denoted by letter H, M, and S. Cool ($\alpha_{vir} = 0.3$), virialised ($\alpha_{vir} = 0.5$), and warm ($\alpha_{vir} = 0.7$) regions are denoted by letter C, V, and W.

| $D$  | $\alpha_{vir}$ | 0.3 | 0.5 | 0.7 |
|------|---------------|-----|-----|-----|
| 1.6  | HC            | HV  |   HW|
| 2.2  | MC            | MV  |   MW|
| 2.9  | SC            | SV  |   SW|

3. Simulation and Identification of Binary Star Cluster

3.1. N-Body Simulations

We use AMUSE Framework to make our N-body simulations [19]. We assume that all gases are expelled from the cluster environment. Furthermore, stellar evolution and binary fraction are also not considered in the simulations. In every case, we run 30 simulations for each initial condition as set in Table 2 for 50 Myr with time step 0.1 Myr. For isolated condition, we use BHTree code [20] to calculate the gravitational interactions between stars in the cluster. For the condition under Galactic tidal field, we use Bridge code [21] to couple the BHTree code with the calculation of gravitational interactions between stars and Milky Way. There is no different distribution of stars’ position, velocity, and mass in the cluster for simulations in isolated and under Galactic tidal field. We put the position of star cluster at $(x, y, z) = (8.5, 0.0, 0.02)$ kpc in the Milky Way, adopted from [22]. Then the velocity $(u, v, w) = (11.1, 12.4 + v_{\text{vir}}, 7.25)$ km/s, adopted from [23]. Here $v_{\text{vir}} = 220$ km/s which is corresponding to our Milky Way model. By the physical parameters from Table 1 and the position of star cluster in the Milky Way, we calculate the initial tidal radius of star cluster at time $0$ Myr is $8.5$ pc. It is 4.25 times radius of the cluster.

3.2. Identification of Binary Star Cluster

We introduce weighted K-Means algorithm to identify binary star cluster at time 20 Myr and 50 Myr. K data points are chosen from dataset as initial centroids using K-Means++ algorithm. Here we use the position of stars as dataset. By calculating weighted euclidean distance between data points and every centroid, all those are closest to a centroid will create a cluster (clustering process). We use mass of stars as weight in this calculation. Now we have K clusters that need new centroids by calculating mean of all members’ position in a cluster (moving centroids process). Repeat clustering and moving centroids processes until new centroids are no longer similar with previous ones.

K-Means++ algorithm is used in order to choose K initial centroids. In this algorithm, firstly we choose one data point from dataset randomly as an initial centroid. Then calculate squared weighted euclidean distance between each data point and the initial centroid, denoted by $d_i^2$ where $i$ is each data point. A data point will be an initial centroid when its probability is proportional to $\frac{d_i^2}{\sum_i d_i^2}$. Repeat this process until K initial centroids have been chosen. In this work, we set $K = 1, 2, \ldots, 10$ as guessed number of centroids and then implement Bayesian Information Criterion (BIC) and angle-based method in order to optimize the best K centroids for each dataset. By using K-Means algorithm, we calculate the fraction of single, binary/multiple, and merger. The term of single refers to the star cluster that formed as a single star cluster. Binary/multiple refers to the star cluster that formed as binary/multiple cluster, both of the one that orbiting each other or separated in different directions. When the formed binary/multiple cluster then merges into a single star cluster, it is categorized as
merger. We calculate these fractions at time 20 Myr and 50 Myr to investigate the fate of dynamical evolution of star cluster at different times.

4. Results and Discussion

We present the fractions of single, binary/multiple, and merger at 20 Myr and 50 Myr for isolated condition in Figure 2. At time 20 Myr, we can see that the clumpier structure of star clusters, the fractions of binary/multiple increase. The highest fraction of binary/multiple occurs in the H conditions ($D = 1.6$), where the star clusters are highly substructured. Besides that, the velocity dispersion of stars in a cluster has also important role to form binary star cluster. From the same figure, we can see the W conditions ($\alpha_{vir} = 0.7$) have the highest fractions of binary/multiple. The higher value of $\alpha_{vir}$, the higher star velocity dispersion in a cluster. That’s why the warm condition tends to form agglomerative hierarchical clustering. We can conclude that the most probable condition to let the highly formation of binary star cluster is the one with highly substructured and warm condition (HW condition).

Our result for isolated condition at time 20 Myr is in a good agreement with [14], although it seems that our more massive star cluster models evolve faster than [14] which makes our fractions of binary/multiple are higher and the fractions of single are smaller. This shows that the initial mass of star cluster has important role in the formation of both binary and multiple cluster. If we compare the fractions at time 20 Myr to 50 Myr, we get there are decreasing fractions of binary/multiple for all conditions in Figure 2 due to merger processes between star clusters. It means that the older age of star cluster, the fractions of binary/multiple decrease and followed by the increasing fractions of merger for the same lifetime.

Figure 2. The fractions of single (green), binary/multiple (red), and merger (orange) for isolated condition at 20 Myr (top panel) and 50 Myr (bottom panel).

Figure 3. The fractions of single, binary/multiple, and merger for condition under Galactic tidal field at 20 Myr (top panel) and 50 Myr (bottom panel). The colors as explained in Figure 2.
We get the trends of fractions in condition under Galactic tidal field are similar with isolated. But the fractions of binary/multiple are a bit smaller and the fractions of single are a bit higher than isolated at time 20 Myr. For HW condition under Galactic tidal field, we get the fraction of binary/multiple is $0.87 \pm 0.06$ at time 20 Myr and then decreases to be $0.63 \pm 0.09$, with deflation $0.24 \pm 0.08$. While the HW isolated condition, the deflation is only $0.07 \pm 0.05$. The higher deflation we get in the condition under Galactic tidal field shows that our Galaxy has important role on the dynamical process of the formation of binary star cluster. The existence of multiple clusters that identified in our simulations are interesting to investigate more deeply how this systems can be formed. Besides that, the fractions of binary/multiple are still high enough at time 50 Myr and suggesting that binary and multiple clusters could have lifetime more than 50 Myr. It is interesting to learn in the future work of how do these binary and multiple clusters end their life after 50 Myr with considering the stellar evolution.

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References
[1] Lada C J, and Lada E A 2003 Annu. Rev. Astron. Astrophys. 41 57
[2] de la Fuente Marcos R, and de la Fuente Marcos C 2009 Astron. Astrophys. 500 L13
[3] de la Fuente Marcos R, and de la Fuente Marcos C 2010 Astron. Astrophys. 500 L13
[4] Bhatia R K, and Hadzidimitriou D 1988 Mon. Not. R. Astron. Soc. 230 215
[5] Subramaniam, A, Gorti U, Sagar R, and Bhatt H C 1995 Astron. Astrophys. 302 86
[6] Pietrzynski G, and Udalski A 2000 Acta. Astron. 50 355
[7] Pietrzynski G, Udalski A, Kubiak M, Szymanski M, Wozniak P, and Zebrun K 1999 Acta. Astron. 49 521
[8] Dieball A, Muller H, and Grebel E K 2002 Astron. Astrophys. 391 547
[9] Elmegreen B G, Efremov Y, Pudritz R E, and Zinnecker H 2000 in Protostars and Planets IV, ed. V. Mannings, A. P. Boss, & S. S. Russell (University of Arizona Press) 151
[10] Williams J P, Blitz L, and McKee C F 2000 in Protostars and Planets IV, ed. V. Mannings, A. P. Boss, & S. S. Russell (Tucson, AZ: Univ. Arizona Press) 97
[11] de la Fuente Marcos R, and de la Fuente Marcos C 2006 Astron. Astrophys. 452 163
[12] Kuhn M A, Feigelson E D, Getman K V, Baddeley A J, Broos P S, Sills A, Bate M R, Povich M S, Luhrman K L, Busk H A, Naylor T, and King R R 2014 Astrophys. J. 787 107
[13] Yu J L, De Grijs R, and Chen L 2011 Astrophys. J. 732 16
[14] Arnold B, Goodwin S P, Griffiths D W, and Parker R J 2017 Mon. Not. R. Astron. Soc. 471 2498
[15] Goodwin S P, and Whitworth A P 2004 Astron. Astrophys. 413 929
[16] Priyatikanto R, Kouwenhoven M B N, Arifyanto M I, Wulandari H R T, and Siregar S 2016 Mon. Not. R. Astron. Soc. 457 1339
[17] Kroupa P 2002 Science 295 82
[18] Haghi H, Zonoozi A H, and Taghavi S 2015 Mon. Not. R. Astron. Soc. 315 2153
[19] Portegies-Zwart S F, McMillan S L W, van Elteren A, Pelupessy F I, and de Vries N 2013 Comput. Phys. Commun. 183 456
[20] Barnes J, and Hutt P 1986 Nature 324 446
[21] Fujii M, Iwasawa M, Funato Y, and Makino J 2007 Publ. Astron. Soc. Japan 59 1095
[22] Martinez-Barbosa C A, Jilkova L, Portegies-Zwart S, and Brown A G A 2016, Mon. Not. R. Astron. Soc. 457 1062
[23] Schonrich R, Binney J, and Dehnen W 2010 Mon. Not. R. Astron. Soc. 403 1829