Three-channel Synergetic State Observer for Data Transmission System with Chaotic Dynamics

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Abstract. The paper presents the first developed procedure of state variables three-channel observer design for a nonlinear dynamic system. The state observer allows to estimate the values of parameters that are not available for direct measurement. Here, the observer is used to build a data secure communication system with a chaotic carrier bearing signal. As a chaotic carrier generator, we use the model of the novel chaotic attractor. The data transmission system is implemented by nonlinear mixing of useful signals into parameters of the chaotic generator model on the transmitter side and by reconstructing the useful signals on the receiver side with the designed state observer. The introduction of the state observer into chaotic generator data transmission system provides the system implementation without the use of chaotic synchronization. We provide step-by-step general description for a synergetic state observer design as well as example of the chaotic carrier bearing signal observer design procedure along with closed-loop system computer simulation results.

Keywords: State observer; Dynamic chaos; Data transmission; A novel chaotic attractor; Chaotic carrier; Synergetics; Nonlinear dynamics.

1. Introduction

State observers provide evaluation of dynamic system state variables values that are out of the reach of direct measurement. Observers are widely used in control systems for indirect estimation of unmeasurable parameters by virtue of mathematical model of the control object. An observer is built on the basis of a mathematical model of the initial object. The observer's equations can be joined to the original system, forming a closed-loop system. With the help of state observers, control systems are able to estimate and take into account some parameters of external disturbances, if disturbance model represents the worst case of possible disturbances in the system [1], [2].

Basically, state observers are built for linear systems and linearized objects. This is due to the fact that the best known methods for observer design, for example Luenberger observer [3], Kalman filter [4], and more, require linearization of the original nonlinear system. Application of other modern methods of observer’s design, i.e. Adaptive observer [5], [6], SMC observer [7], Reduced order observer [8], [9], is limited for nonlinear objects.

In the paper a synergetic state observer is used for nonlinear dynamic system state observer design; the synergetic state observer belongs to methods of Synergetic Control Theory (SCT). SCT was proposed and developed by Professor A.A. Kolesnikov [10], [11], an application of synergetic state observers to nonlinear objects is shown in his works and the works of his scientific school [1], [2], [12], [13], [14], and more.

A synergetic observer of state variables has the following advantages and outstanding feature in comparison with other modern techniques for nonlinear system observer design procedures [12]:

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(i) classical observers have a rigidly defined structure. In this case, the links between observer variables are setting up as given, but in synergetic observer they are formed directly via design procedure with taking into account all the features of the explored nonlinear system; (ii) since the structure of the synergetic observer is determined as a result of the design procedure, it must be performed every time for every new explored system, thereby it means achieving of great flexibility in observer design; (iii) the structure of a synergetic observer includes the right-hand sides of the equations of an object nonlinear mathematical model, which allows to take into account the nonlinear properties of the explored system.

Nowadays many scientists explore data transmission systems where the bearing oscillations generator is not a generator of regular van der Pol oscillations, but a nonlinear system with chaotic behavior and ability to generate some chaotic oscillations at certain combination of its parameters. In this case, the transmitted signal can be masked by chaotic oscillations and to be hidden for an outside viewer, thereby ensuring protection of the transmitted information.

There are different ways to embed and recover a useful signal in data transmission systems with chaotic carrier. The most well-known ways of embedding a useful signal into a chaotic one are additive chaos modulation (ACM) and multiplicative chaos modulation (MCM) [15]. With ACM, the function reflecting the changes in the useful signal is introduced into the equation of the chaotic generator as a term, and when using MCM is introduced as a factor. ACM could not provide a tamper-resistant data transmission system as was proved in [16]. Therefore, for modulation of useful signals it is necessary to use MCM [12]. To restore the useful signal, the method of synchronizing of identical chaotic generators introduced into both the transmitter and the receiver is very practical.

Synchronization can be performed on the basis of back-stepping [17], the ADAR method [18], a sliding control system (SMC), as well as a sliding control observer [19], [20], [21]. However, the use of chaotic synchronization leads to a number of problems, e.g. we need to determine the Lyapunov function in design procedure.

The use of synergetic observer in data transmission with a chaotic carrier makes it possible to build a useful signal reconstructor without solving the problem of chaotic synchronization [12]. Examples of single-channel and two-channel observers design procedures for data transmission systems with a chaotic carrier were given in our earlier papers [13], [14] and [12]. Here, in this paper, we demonstrate for the first time an extended procedure for state observer design for a nonlinear system with chaotic dynamics, and that is possible to recover three useful signals on the receiver side simultaneously.

2. Statement of the Problem

In the paper we state the problem to design a chaotic carrier data transmission system based on a synergetic state observer. In contrast to the above-mentioned examples of designing systems based on the phenomena of chaotic synchronization and SMC, in this work we will use the design of nonlinear synergetic state observer that will "reflects" the model of a chaotic generator on the receiver side. This scheme, with taking into account the assumptions made below, will make it possible to recover three original useful signals transmitted simultaneously over one chaotic carrier.

As a generator of chaotic oscillations, we explore chaotic system model of the novel chaotic attractor [22].

\[
\begin{align*}
x' &= y - ax + yz; \\
y' &= by - xz; \\
z' &= cyx - dz - hx^2.
\end{align*}
\]

(1)

To confirm the presence of chaotic phenomena in equation (1), we will plot its phase portrait (see figure 1).
Figure 1. The phase portrait of equations (1) system at $a = 2.9$, $b = 3$, $c = 8$, $d = 11$, $h = 0.5$, $x(0) = (-0.5; 1.5; 0.3)$. Here: (a) is 3-dimensional projection; (b) is 2-dimensional projection illustrating the model symmetry property.

Information signals will be introduced into the system through multiplicative mixing to the parameters $a$, $b$ and $d$: $$\dot{a} = a \mu_1; \quad \dot{b} = \dot{a} \mu_2; \quad \dot{d} = d \mu_3,$$
where $\mu_1$, $\mu_2$ and $\mu_3$ are useful signals transmitted via the system. As a data transmission system should have steganographic properties, it is required to maintain it in a chaotic state. So, we introduce the required variation ranges for system parameters as follows:

$$2.7 < \dot{a} < 6.85; \quad 3 < \dot{b} < 7; \quad c = 8; \quad 11 < \dot{d} < 16; \quad h = 0.5.$$ 

The structure of the future system should have the form like figure 2.

Figure 2. The structure of three-channel chaotic carrier data transmission system.

3. Synergetic Observer Design Methodology

In the observer design problem [1], [12], [13] we explore the following system:

$$\dot{y} = g(y, v, u); \quad \dot{v} = h(y, v, u),$$

where $y$ and $v$ are the components of the state vector, $\text{dim } y = n$ and $\text{dim } y = u$; $u$ is control vector; $g(\cdot)$ and $h(\cdot)$ are continuous nonlinear functions. Vector $y$ is assumed to be observable, and the vector $v$ to be unobservable. Let us introduce $m$-vector $\psi(t)$ defined by the formula:

$$\psi(t) = \varphi(y, v) - \dot{\varphi}(t),$$

where functions $\varphi(y, v)$ and $\dot{\varphi}(t)$ satisfy the following conditions: (i) Functions are continuous and differentiable in their arguments; (ii) a solution of the equation $\varphi(y, v) - \dot{\varphi}(t) = 0$ with respect to $v$ exists and is unique for all $y \in \mathbb{R}^n$. We require that vector (2) satisfy the homogeneous differential equation (3):

$$\dot{\psi}(t) = L(y)\psi,$$
where \( m \times m \) matrix \( L(y) \) is such that the trivial solution \( \psi = 0 \) be asymptotically stable on the whole. In the simplest case \( L(y) \) is numeric stable matrix. Taking into account equation (2) and equation (3), equation (4) takes the form:

\[
\frac{\partial \psi}{\partial y} g(y, v, u) + \frac{\partial \psi}{\partial v} h(y, v, u) - \dot{\psi} = L(y)\psi(y, v) - L(y)\dot{\psi}.
\]

It is assumed that for the given functions \( g(y, v, u) \) and \( h(y, v, u) \) there is a vector \( \gamma(y, u) \) and matrix \( \Gamma(y) \) independent of \( v \), which turns out to be true:

\[
\frac{\partial \psi}{\partial y} g(y, v, u) + \frac{\partial \psi}{\partial v} h(y, v, u) - L(y)\psi(y, v) = \Gamma(y)\psi(y, v, u) + \gamma(y, u).
\]

Then equation (5) with taking into account the first equation of system (2) will take the form:

\[
\Gamma(y)\dot{y} - \dot{\psi} + \gamma(y, u) + L(y)\dot{\psi} = 0,
\]

or

\[
\dot{z} = L(y)z - L(y) \int_0^y \Gamma(y)dy - \gamma(y, u),
\]

where

\[
z = \int_0^y \Gamma(y)dy - \dot{\psi}.
\]

According to equation (3) and equation (9) the vector

\[
\varphi(y, \dot{y}) = \int_0^y \Gamma(y)dy - z.
\]

The problem of observer design by this means to be reduced to finding the functions \( \varphi(y, v) \), \( L(y) \), \( \Gamma(y) \), \( \gamma(y, u) \) satisfying equation (6). Moreover, the function \( \varphi(y, v) \) must meet the above requirements (i) and (ii), and the matrix \( L(y) \) must ensure the asymptotic stability of system (4) [11].

4. Example of Three-Channel Observer Design Procedure

Here and below, parameters \( a, b \) and \( d \) from equation (1) will be rewrote as \( w_1, w_2 \) and \( w_2 \). According to the procedure described by equations (2) - (10), we write an extended model of the system for the design problem:

\[
\dot{x} = y - w_1 x + yz; \quad \dot{y} = w_2 y - xy; \quad \dot{z} = cxy - w_3 z - hx^2,
\]

\[
\dot{w}_1 = 0; \quad \dot{w}_2 = 0; \quad \dot{w}_3 = 0.
\]

And let's introduce the following macro-variable:

\[
\Psi = \begin{bmatrix} w_1 - \dot{w}_1 \\ w_2 - \dot{w}_2 \\ w_3 - \dot{w}_3 \end{bmatrix}
\]

Then, let us write down the reduction equations:

\[
\dot{\Psi} = Q_1 + v_1; \quad \dot{\Psi} = Q_2 + v_2; \quad \dot{\Psi} = Q_3 + v_3.
\]

The time derivatives for the reduction equations are:

\[
\frac{d\dot{w}_1}{dt} = \frac{\partial Q_1}{\partial x}(x) + \frac{\partial Q_1}{\partial y}(y) + \frac{\partial Q_1}{\partial z}(z) + \frac{dv_1}{dt},
\]

\[
\frac{d\dot{w}_2}{dt} = \frac{\partial Q_2}{\partial x}(x) + \frac{\partial Q_2}{\partial y}(y) + \frac{\partial Q_2}{\partial z}(z) + \frac{dv_2}{dt},
\]

\[
\frac{d\dot{w}_3}{dt} = \frac{\partial Q_3}{\partial x}(x) + \frac{\partial Q_3}{\partial y}(y) + \frac{\partial Q_3}{\partial z}(z) + \frac{dv_3}{dt},
\]

The \( L \) coefficient matrix takes the following form:
Following the above method, we have the expressions:

\[
\begin{align*}
\frac{dv_1}{dt} &= -\frac{\partial Q_1}{\partial x}(y - w_1x + yz) - \frac{\partial Q_1}{\partial y}(w_2y - xz) - \\
&\quad \frac{\partial Q_1}{\partial z}(cxy - w_3z - hx^2) + L_{11}(w_1 - \tilde{w}_1) + L_{12}(w_2 - \tilde{w}_2) + L_{13}(w_3 - \tilde{w}_3), \\
\frac{dv_2}{dt} &= -\frac{\partial Q_2}{\partial x}(y - w_1x + yz) - \frac{\partial Q_2}{\partial y}(w_2y - xz) - \\
&\quad \frac{\partial Q_2}{\partial z}(cxy - w_3z - hx^2) + L_{21}(w_1 - \tilde{w}_1) + L_{22}(w_2 - \tilde{w}_2) + L_{23}(w_3 - \tilde{w}_3), \\
\frac{dv_3}{dt} &= -\frac{\partial Q_3}{\partial x}(y - w_1x + yz) - \frac{\partial Q_3}{\partial y}(w_2y - xz) - \\
&\quad \frac{\partial Q_3}{\partial z}(cxy - w_3z - hx^2) + L_{31}(w_1 - \tilde{w}_1) + L_{32}(w_2 - \tilde{w}_2) + L_{33}(w_3 - \tilde{w}_3).
\end{align*}
\]

Let’s write out all the terms containing unobservable variables:

\[
\begin{align*}
\frac{\partial Q_1}{\partial x}x + L_{11} &= 0; \quad -\frac{\partial Q_1}{\partial y}y + L_{12} = 0; \quad \frac{\partial Q_1}{\partial z}z + L_{13} = 0; \quad \frac{\partial Q_2}{\partial x}x + L_{21} = 0; \quad -\frac{\partial Q_2}{\partial y}y + L_{22} = 0; \\
\frac{\partial Q_2}{\partial z}z + L_{23} &= 0; \quad \frac{\partial Q_3}{\partial x}x + L_{31} = 0; \quad -\frac{\partial Q_3}{\partial y}y + L_{32} = 0; \quad \frac{\partial Q_3}{\partial z}z + L_{33} &= 0.
\end{align*}
\]

Based on the stability condition, we choose the values of L as follows:

\[
L = \begin{bmatrix}
\alpha_1 x^2 & 0 & 0 \\
0 & \alpha_2 y^2 & 0 \\
0 & 0 & \alpha_3 z^2
\end{bmatrix}.
\]

As a result, we have the following observer equations and parameter estimates:

\[
\begin{align*}
\frac{dv_1}{dt} &= -a_1x^2\tilde{w}_1 + a_1xyz + \alpha_1 x^2; \\
\frac{dv_2}{dt} &= a_2xyz - a_2 y^2 \tilde{w}_2; \\
\frac{dv_3}{dt} &= a_3 cxy - a_3 hx^2z - a_3 z^2 \tilde{w}_3; \\
\tilde{w}_1 &= -\frac{a_1 x^2}{2} + v_1; \\
\tilde{w}_2 &= \frac{a_2 y^2}{2} + v_2; \\
\tilde{w}_3 &= -\frac{a_3 z^2}{2} + v_3.
\end{align*}
\]

5. Simulation Results

Let’s simulate a closed-loop system that describes the structure in figure 2. The simulation results are presented in figures 3-5. Figure 3 shows plots of the original test signals before they enter the chaotic generator. Figure 4 shows the signals at the output of the receiver, that were reconstructed with the synergetic observer. Figure 5 shows the plots of errors in recovering the transmitted useful signal by a synergetic state observer.

Figure 3. Initial test signals transmitted over the communication channel before entering the chaotic generator. (a) useful signal $a$; (b) useful signal $b$; (c) useful signal $d$. 
6. Conclusion

In this work, the procedure for a three-channel synergetic state observer for the nonlinear dynamic system is obtained and presented for the first time. The procedure is based on methods and approaches of Synergetic Control Theory (SCT). The problem is solved in application to three-channel data transmission system with a chaotic carrier. The resulted system distinctive features are simultaneous transmission of three useful signals over one communication channel and no need for chaotic synchronization between the transmitter and receiver, which facilitates the implementation of such systems. In further studies, we will apply the three-channel state observer design procedure to other chaotic systems to demonstrate the practical applicability and flexibility of the proposed method. In addition, the observer obtained in the work will be used for the software and hardware implementation of a three-channel data transmission system with a chaotic carrier.

Acknowledgments

The reported study was funded by RFBR according to the research project No. 19-08-00366.

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