Розглянуто особливості експлуатації підземних сховищ газу, проведено аналіз стану свердловин підземних сховищ України, надійності експлуатації. Виділено особливості небажаних ситуацій з перетіканням рідини та газу через поверхні сховищ. Запропоновано та реалізовано моделі фільтраційної течії з врахуванням особливостей геометрії об'єктів та граничних умов. Запропоновано методику оцінки втрат углеводородів при їх перетіканні газу через поверхню свердловини.

Ключові слова: підземні сховища, модельювання процесів, експлуатаційні свердловини, перетікання газу, процес фільтрації.

В роботі розглянуті особливості експлуатації підземних сховищ газу, проведено аналіз стану сховищ підземних сховищ України, надійності їх експлуатації. Виділено особливості несподіваних ситуацій з перетіканням рідини та газу через поверхні сховищ. Запропоновано та реалізовано моделі фільтраційної течії з врахуванням особливостей геометрії об'єктів та граничних умов. Предложено методику оцінки втрат углеводородів при їх перетіканні газу через поверхню свердловини.

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1. Introduction

One of the conditions for successful operation of underground storages is the prevention of gas outflow beyond the design outline, and gas cross-flow to other horizons. It is also required to execute clear and permanent control over the content of water, condensate and other components in gas. Technological modes of underground gas storages (UGS) are determined by the volume and duration of processes of gas injection and extraction, periodicity of cycles, and other indicators.

Drilled wells of underground storages are typically operated for several decades. During this time, a deposit undergoes different stages of development, from the initial to the final. Bed pressure in the process of development also decreases, which is why at the next stages it is necessary to extract large volumes of fluid at lower dynamic levels. Some beds are heterogeneous and split into independent streaks. This necessitates separate operation or separate injection of water into different streaks through one and the same well. It does not seem possible to reliably identify conditions of operation of a given well over the whole operation period.

It is necessary to note that gas is injected and extracted from the reservoir through the development wells. On the territory of the storage and beyond its boundaries (outlines), observation, monitoring and piezometric wells are arranged. Their main purpose is to control pressure within the reservoir and in its part outside the boundary, as well as on the horizons, located above the reservoir.

Depending on the capacity of a reservoir, engineering-geological structure and the structure of rocks of UGS, pressure variations at injection and extraction of gas are from 4 to 7 MPa. Pressure fluctuations are cyclical in nature over many years and during spring and autumn.

Therefore, the issue of reliability of wells and reduction of losses in the process of operation is of great importance in order to provide efficient and smooth operation.

Different experimental research methods are typically used to estimate losses of products. Up to now, methods of mathematical modeling of processes of fluids and gases' cross-flow through the boundaries of the examined areas have not been sufficiently studied. This is predetermined by the complexity of geometrical configuration of the examined objects.

It is necessary to establish the extent to which the existing methods of studying filtering processes describe in different approximations the features of gas cross-flow in the studied objects.

2. Literature review and problem statement

In the underground gas storages, 1,316 wells out of 1,515 are involved in the process of injection-extraction [1]. Control over wells of UGS by the Ukrainian Research Institute of Natural Gases (Kharkiv, Ukraine) allowed us to categorize them by the magnitude of inter-string pressures and gas outflow (Fig. 1).

When analyzing the diagram of states of the wells (Fig. 1), one can argue that at tightness of the wells of the general fund at about 50%, the magnitude of gas losses during continuous outflow exceeds 5.5 million m³. At aver-
age monthly gas consumption of 100 m³, such an amount can provide a settlement with more than 200 thousand residents with household gas for a month.

Scientists have been studying the issues of construction, repair and reliable operation of wells from the moment of creation of underground storages. The issue of reliable operation of underground storage of the Ukrainian network was studied and characterized by scientists from different countries [2, 3]. Paper [4] describes techniques of wells’ repairing, their advantages and disadvantages. However, new challenges and tasks emerged during operation period. The problem of reliability and tightness of the walls of underground storages ranks very high with new requirements and standards.

The subject-matter of research remains relevant because the equipment of deposits is operated for a long time under difficult conditions. That is why it is necessary to constantly estimate residual resource of the storages, taking into consideration financial situation of an enterprise that operates them.

In the territory of Ukraine, over recent years, the problem of underground gas storage is studied around the world. The subject-matter of research remains relevant because the equipment of deposits is operated for a long time under difficult conditions. That is why it is necessary to constantly estimate residual resource of the storages, taking into consideration financial situation of an enterprise that operates them.

In paper [6], it is proven that an increase in the diameter of flow pipes makes it possible to enhance performance of extraction, to decrease hydraulic resistance in flow pipes and to increase pressure at the inlet of BCS. A decrease in the consumption of fuel gas occurs due to a decrease in the duration of compressor extraction.

The characteristic of problems that arise during operation of underground gas storage is studied around the world. In the territory of Ukraine, over recent years, the problem was most completely described in [7], and the problem of typical storages in other countries was addressed in [8].

However, it is necessary to consider the problem of studying losses using the modeling of processes, specifically, exploring processes of mutual penetration of fluid through the wall of a well.

The network of underground gas storages in Ukraine was created to provide reliability of export gas supplies. Dashava underground storage was created on the base of exhausted deposits. Typical underground storages were built and are operated around the world (in countries of Western Europe, Russia, the United States, and Canada).

Thus, world tendencies in the development of underground gas storage present a range of tasks for science, which are typical for Ukraine. The issue of solving them reliably depends on the combined efforts in addressing problems that arise.

3. The aim and objectives of the study

The aim of present research is to create a mathematical model of the process of fluid filtration through the medium with a certain level of permeability. This will make it possible to study the peculiarities of filtering process and estimate dimensions of the areas of fluid propagation in the medium.

To accomplish the set goal, the following tasks have been solved:

– to propose and implement a model of the filtering flow taking into consideration features of geometry of the area and boundary conditions;

– in order to study the features of filtering process, as well as to assess dimensions of fluid propagation in the medium, to create a mathematical model of fluid filtration through the medium with a certain level of permeability;

– based on the analysis of the state of wells, to draw conclusions on the possible losses of gas and to specify the state of the wells of a given storage.

4. Materials and methods of research

It should be noted that operation of UGS is allowed only with the use of lift pipes. It is not allowed to operate wells by flow string. Since putting an underground storage facility in commercial operation, it is necessary to arrange measurement and accounting of the quantity of gas that is used to create a storage. In the process of operation of a storage, they keep records of injected and extracted gas. This makes it possible to determine costs of technological operations, and account for all gas losses [4].

Control over operation of UGS, determined by a technological scheme, is carried out at two stages:

– during experimental-industrial operation;

– during cyclic operation.

During experimental-industrial operation, tightness of the storage, the state of wells, are controlled, the rates of filling, formation of the deposit’s contour, and pressure changes are specified.

In the period of cyclic operation, the state of wells, changes in the boundary of gas-saturated zone, and tightness of the storage continue to be monitored. Gas survey of the storage area is carried out [9].

Dashava UGS was established based on the exhausted deposits of E and G of the gas deposit of the same name, as the peak one, in order to compensate for seasonal fluctuations in gas consumption, as well as to ensure reliability of natural gas supply abroad, carried out since 1973. Over the entire period of operation of the underground storage, it has been characterized by high efficiency of its operation on the whole, which involves providing projected amounts of gas injection and extraction.

However, current market conditions require improvement of product quality and efficient use of resources.
Enhancement of efficiency of the underground gas storage is possible in the case gas losses are reduced. An important issue is also the characteristics and the quality of the wells, because within the operation time of Dashava gas deposit and creation of the underground gas storage, 232 wells were drilled in its area, of which 96 were abandoned for various reasons. In addition, there are 43 wells with depth of up to 50 m in the UGS territory, designed for the anode protection of underground communications [10].

In accordance with technological schemes of creation and operation of UGS, operational fund of the wells was to include 106 wells. There were also 7 wells of the old fund, but during creation of UGS, most of them were used as development-injection wells.

During 1975–1978, in deposits “G” and “E+G”, the first series of the new 16 wells was created, and the second series of 82 wells was created over the following 10 years.

For UGS functioning, 98 development-injection wells were drilled. At the beginning of 2010, the fund of wells already totaled 136 units (99 – development-injection, 7 – observation, 10 – monitor, 17 – monitor-relief wells). The fund was increased at the expense of 109 wells of the old fund that discovered deposit “G”. In the territory of the underground storage, there are 43 wells with depth of up to 50 m, which are intended for the anode protection of underground communications.

Wells 66, 72 and 75 of the old fund have a single-casing structure. They have been used for a long time for the development of Dashava gas deposit. At the beginning of creation of UGS, these wells worked as development-injecting and eventually were abandoned and transferred to monitor-geophysical ones [10].

Wellheads of development wells are equipped with string heads OKK-1-210-168×245,324 and OKK-2-210-168×245,324 and flowing fittings AFK-2-65×210 and AFK-3-65×210. Fittings IAF-65 Kr×140 were mounted on 15 wells. Pipes with diameter of 219 mm connect wells with UKPG. 47 wells have individual pipes, brought together in bunches, and 52 wells are brought together in 26 pairs (two wells operate in the same pipeline).

Using monitor wells 83, 107, 165, 212, 222, 223, 405, the main sites of UGS were discovered – horizons ND-8 and ND-9. To monitor the UGS operation and to control its tightness, monitor wells 46, 47, 70, 74, 81, 82, 105, 186, 211, and 302 are used. They discovered horizons that lie above and below the operating facilities of UGS. To control gas pollution of the UGS territory and to eliminate gas pollution of the anode protection of underground communications, there are 43 wells with depth of up to 50 m, which are intended for the anode protection of industrial equipment and pipelines.

In all observation, monitor and relief wells, regular measurements of pressure are carried out, and gas and water are sampled for current analysis. The wellheads are equipped with flowing fittings AFK2. On the heads of relief wells 231, 236–240, 243, 244 tees and snaps of local design are mounted.

In addition, there are 96 abandoned wells on the territory of Dashava UGS. The intervals of perforation of exhausted productive horizons are located by 40–50 m higher than the interval of perforation. Wellbores are filled with clay solution of density of 1200–1260 kg/m³ [10]. Depending on design, wellhead setup can be performed by one of the schematics (Fig. 2, 3).

![Fig. 2. Typical scheme of well setup (string head GMK-125-146-245: 1 – pump-compressor pipes (73 mm, 89 mm), 2 – flow string (168 mm)), 3 – technical string (245 mm), 4 – conductor (324 mm), 5 – cement ring, 6 – perforation zone, 7 – casing of string, 8 – suspension of flow string, 9 – tee, 10 – buffer bolt, 12 – annulus space, 13 – faceplate, 14 – nut of quick-connect with plug, 15 – cross-piece of flow fittings, 16 – flanged connection, 17 – clutch for suspension of flow string, 18 – semiring, 19 – rubber rings, 20 – metal cuffs, 21 – crane valve, 22 – inter-string duct between “6×9” strings, 23 – well pipe)](image1)

![Fig. 3. Typical scheme of piping of wells (string head OKK-1-210×168×245: 1 – pump-compressor pipes (114 mm, 89 mm), 2 – flow string (168 mm), 3 – technical string (245 mm), 4 – conductor (324 mm), 5 – cement ring, 6 – perforation zone, 7 – casing of the string head OKK-1-210×168×245, 8 – wedge, 9 – packer, 10 – tight ring, 11 – inter-string duct between “6×9” strings, 12 – cross-piece of fountain fittings, 13 – faceplate, 14 – tee, 15 – root valve, 16 – over-root valve (flow), 17 – annulus space, 18 – pipe space, 19 – buffer bolt, 20 – bolt for pressing the well, 21 – gauge for pressure measurement, 22 – well pipe, 23 – quick-connect nut)](image2)
The upper part of the flow string is covered with a cement cup of 10.0 m or by a cement stopper of 2.0 m. The wellhead is closed by blind flange, welded to the string, or by the plug with the pipe and the valve for pressure measurement. Most of the wells have concrete piers of 1×1×1 mm mounted on the wellhead. Bench marks, where the numbers of the wells and their abandonment dates are written, are mounted on the heads of all abandoned wells.

Within the limits of the contour, there are also wells, which did not discover UGS sites. The depth of their bottom hole ranges from 200 m to 652 m and the number of abandoned wells is 24. Due to the fact that the wells did not discover UGS sites, and above lying gas-bearing horizons are basically exhausted, these wells may not be sources of uncontrolled significant gas crop out to the day. In exceptional cases, gas can arrive by the untightened borehole of individual wells if secondary gas is accumulated in depleted deposits. Given these circumstances, wells are divided into two categories.

The first-category wells were drilled in the pre-war and war years (17, 19, 22, 24, 27, 30, 34, 42, 43a, 54a, 54b and 74a). Intermediate and flow strings in them were filled with rubble instead of being cemented. Concrete stones were mounted on the heads of wells 17, 30, 34, 42, 54a, plugs were mounted on the heads of wells 19, 22 and 43a, a concrete slab was mounted on the head of well 24. Wells 19 and 34 are at the distance of less than 50 m from the industrial facilities [10].

The second-category wells (13, 21, 25, 35, 37, 40, 46a, 46b, 57a and 151) are distinguished by existence of liners, absence of cement beyond strings, or cement lift lower than 300 m from the wellhead, as well as by absence of casing pipes (well 151). Concrete stones were mounted on the heads of wells 35, 57a during abandonment, plugs were mounted at the heads of the wells 13, 21, 37 and 46b, and a concrete slab was mounted at wellhead 151. Wells 13, 35 are located at the distance of less than 50 m from industrial facilities. There is no driveway to well 40 and the wellhead is not equipped.

Wells 15, 44, 51, 56, 59, 73, 73a, 85, 95, 100 and 110 discovered horizons ND-8 and ND-9 beyond the gas-bearing contour, so they cannot be the source of gas motion in strings and beds. Wells 2, 6, 7, 8, 11, 18, 20, 41, 61, 101 discovered horizons ND-8 within deposit “D-D” wells 52, 58a, 66a, 72a and 75a – within the limits of deposit “E”, wells 28, 29 – within the limits of deposit “Zh+B”. Depending on technical condition, the wells are divided into two categories:

– the first category (18, 20, 41, 52, 58a, 61, 66a, 72a, 75a, and 102) includes the wells, in which intermediate and flow strings were run in and cemented to the wellhead. When abandoned, the wellheads were equipped with a cement stopper of 2.0 m, concrete stone or a cap;
– the second category (2, 6, 7, 8, 11) includes the wells, in which intermediate and flow strings were not cemented, there is no information on cementing in regard to wells 28, 29. When abandoned, concrete stones or plugs were mounted on the wellheads, equipped with concrete stoppers [10].

The wells, included in the group, discovered PSG sites and low-lying horizons ND-8 and ND-9, as well as AGT. The majority of wells were drilled in the period from 1924 to 1947. Many wells were drilled by the shock method without cementing of casing strings. In this connection, these wells will require special attention, studying of the technical state and controlling possible inter-bed cross-flows and the ways of uncontrolled gas outflow from UGS onto the surface.

According to the geological characteristics, the wells are divided into two categories:

– the first-category wells: 43, 48, 49, 57, 62, 65, 72, 76, 80, 101, 200, 205 (discovered AGT and horizons ND-9 and ND-8), 38, 45, 53, 54, 58, 195 (horizons ND-9, ND-8), 26, 50, 75, 108 (horizon ND-9). In the wells, intermediate and flow strings were run in, cemented or filled with rubble up to the wellhead, or cement was raised up to 200–300 m from the head. When abandoned, cement stoppers of 2.0 m, concrete stones or plugs were mounted on the wellheads;

– the second-category wells: 36, 39, 47a, 106, 170, 180, 1-Verchany (discovered AGT and horizons of ND-8 and ND-9), 32, 33, 90, 90a, 103 (horizons ND-9 and ND-8), 1, 3, 4, 5, 9, 10, 12, 14, 16, 23, 31, 66 (horizon ND-9). Well 180 was not cased, in wells 1, 3, 14, 16, intermediate and flow strings were not cemented, in wells 10, 90, 90a and 170, cement beyond strings is 300–700 m below the wellhead, drilling tools were left in the string, in wells 9, 31, 32, 33, 39, 106, 1-Verchany, there is no technical documentation on the structure of the wells. When abandoned, cement cups (10.0 m) or cement stoppers wells mounted on the wellhead. The heads of wells 1, 4, 5, and 50 are closer than 50 m from residential premises.

As the analysis shows, there is now an urgent need to improve technical and economic indicators of UGS operation. To do this, it is necessary to provide rational operation modes of wells, to increase their daily flow rate and to enhance reliability of operation. Normal operation of UGS is related to regular research and study of the state of wells and analysis of their work.

For this purpose, it is necessary to consider the problem of modeling of processes that take place during operation of deposits and gas storage in order to identify characteristic signs of the processes of assessment of their impact on parameters of the studied objects [11]. This task includes the need to study the processes of mutual penetration of fluid through the wall of the well or the bed, since considerable losses of product can be observed as a result of this work. One of the main processes that allow correct mathematical statement of the problem is the process of filtering of substances through the walls of the studied objects. Equation of filtering [12] is written down for stationary and non-stationary processes. The most common way of studying filtering processes includes the methods of modeling based on the use of semi-empirical system of Darcy equations [13, 14]. In this case, we accept the hypothesis that velocity of fluid motion is such that magnitude $v$ can be neglected compared to magnitude $e$, where $v$ is the characteristic velocity of fluid motion. It is necessary to explore the problem of to what extent the mentioned hypothesis is true in processes of injection and extraction of gas, to develop mathematical models of filtering processes [15, 16] for fluids which do not satisfy Darcy hypothesis. It is also required to learn the basic regularities of the process of fluids’ propagation in porous medium taking into account features of the geometry of the studied areas.

5. Results of research into modeling of filtering process

The specific feature of modeling for this type of problems is the fact that the studied areas have a complex spatial configuration. Existing approaches [17] allow describing
the system of equations in the coordinate system, linked with the surface of the well. In particular, if \( x, y \) are the coordinates of the line, and \( x = \text{const} \) are the normals to the surface, \( k - k(x) \) is the curve of the axis of a well, for Navier-Stokes equations in the flat area, is the following form of representation is written down:

\[
\frac{\partial u}{\partial t} + \frac{1}{1+ky} \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{k}{(1+ky)^2} \frac{\partial k}{\partial x} = 0;
\]

\[
\frac{\partial v}{\partial t} + \frac{1}{1+ky} \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{k}{(1+ky)^2} \frac{\partial k}{\partial y} = 0.
\]

(1)

It is obvious that at \( k = 0 \), system (1) takes the classic condition that outside the studied area, pressure may be unequal to atmospheric \([18]\).

When constructing a mathematical model, the following assumptions are accepted:

- flow of fluid in a cylindrical form with the radius of the base \( R \) and height \( H \) is symmetric relative to the vertical axis. This allows us to decrease dimensionality of the problem;
- dynamic viscosity \( \mu \) and density \( \rho \) of fluid are considered constant;
- temperature of the medium and its permeability are considered constant, which allows us to disregard corresponding temperature gradients in the model;
- pressure at the points of the area is determined by atmospheric pressure and pressure of the fluid string inside and outside of the studied area, since it is possible to assume that outside the studied area, pressure may be unequal to atmospheric \([18]\).

The problem of assessment of filtering rate in the environment with resistance in a certain rectangular area on condition that \( v \gg \nu \), where \( v \) is the module of vector of rate \( \nabla \) \((u, v)\), is reduced to the solution of a system of the Darcy equations:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0; \\
u &= \frac{kv}{\mu}; \\
v &= \frac{kv}{\mu} + \frac{k}{\mu}g. 
\end{align*}
\]

(2)

where \( u, v \) are the components of the vector of filtering rate, \( p \) is the fluid pressure.

In the case when condition \( v \gg \nu \) is not met, system (2) is represented in a modified form using the Forchheimer law of filtration \([15]\):

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0; \\
\frac{d}{dt} \left( 1 + ky \right) \frac{\partial u}{\partial x} + \frac{k}{1 + ky} \frac{\partial k}{\partial x} &= 0; \\
-\frac{d}{dt} \left( 1 + ky \right) \frac{\partial v}{\partial y} + \frac{k}{1 + ky} \frac{\partial k}{\partial y} &= 0.
\end{align*}
\]

(3)

where \( \beta \) is the dimensionless coefficient, which depends on the structure of porous medium of the reservoir, \( \beta = 1; k \) is the permeability coefficient; \( v \) is the characteristic value of filtering rate.

For systems (2) and (3), it is necessary to derive boundary conditions in the form of:

\[
\begin{align*}
p|_{\partial G} &= f(x, y), \\
p|_{\partial G} &= g(x, y),
\end{align*}
\]

(4)

where \( f(x, y) \) is the continuous function in area \( G \). In this case, it is believed that the walls or boundaries of area \( G \) are freely permeable to fluid, the motion of which can be performed through the boundary in both directions \([19]\). If these boundary conditions are set as follows:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0; \\
u &= -\frac{\partial p}{\partial x}; \\
v &= -\frac{\partial p}{\partial y} + c_i g,
\end{align*}
\]

(5)

where \( c \) and \( c_i \) are constants that depend on the choice for building models (2) and (3). Differentiating the second of the equations (6) by \( x \) and the third equation by \( y \), taking into consideration the first equation (6) we acquire:

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0.
\]

(7)

To solve this problem, conditions (4) or (5) provide for correct problem setting. Schematically, it is possible to show the model of a well of the underground storage, using boundary conditions \([20]\) in the following form (which corresponds to (5)) (Fig. 4).

It is also necessary to note that the model, shown in Fig. 4, can be used to describe the process of gas outflow from a well, depending on technological pressure. But in this case, we set the condition that permeability \( \kappa \), viscosity \( \mu^* \) and density \( \rho^* \) of the medium, from which gas flows out, are assigned.
To sum up, it should be noted that for solving system (1), it is necessary to assign the curvature of the axis of the storage well. To do this, it is necessary to know the shape of the well's axis \( y = \varphi(x) \). Then \( \kappa(x) \) can be determined for the axis, which is slightly different from the straight one, from formula \( \kappa(x) = \varphi''(x) \).

If an axis is a straight line, the feature of the proposed model is recording the boundary conditions in the form, presented in Fig. 4. It also creates the opportunity of assigning values \( k', \mu', \rho' \) as the function of coordinates. In the model case, these values are constant. However, if there is some additional information about their distribution around the volume of the area, they must be variables.

6. Discussion of results of modelling the filtration process

In Fig. 4, it is fixed that densities of fluids \( \rho \) and \( \rho_1 \) can be different, so in the process of calculations, it is possible to obtain both fluid inflow in the area and fluid outflow from it (depending on \( \rho \) and \( \rho_1 \)). For numerical solution of the Dirichlet problem (7), (4) or (7), (5), the over-relaxation method by rows is used [21]. The mathematical features of solution of the problem by the over-relaxation method was studied in detail in paper [21, 22], dependence of values of relaxation parameters on convergence of the iteration process was established. Specifically, in article [21], the entire software complex for the implementation of models (2) and (3) with conditions (4) or (5) was developed.

It should be noted that the results, obtained above, can be also applied for the case of arbitrary orthogonal coordinate system [23]. In this case, the Laplace equation is written in the form:

\[
\frac{1}{2} \left( \begin{array}{ccc}
\frac{\partial q_{11} \varphi q_{22}}{\partial x_1} & \frac{\partial q_{11} \varphi q_{22}}{\partial x_2} & \frac{\partial q_{12} \varphi q_{22}}{\partial x_3} \\
\frac{\partial q_{11} \varphi q_{22}}{\partial x_1} & \frac{\partial q_{22} \varphi q_{22}}{\partial x_2} & \frac{\partial q_{12} \varphi q_{22}}{\partial x_3} \\
\frac{\partial q_{12} \varphi q_{22}}{\partial x_1} & \frac{\partial q_{12} \varphi q_{22}}{\partial x_2} & \frac{\partial q_{22} \varphi q_{22}}{\partial x_3}
\end{array} \right) - \frac{\partial P}{\partial x} = 0
\]  

where \( q_\varphi \) is the components of metric tensor [24]; \( P \) is the pressure; \( q \) is the determinant of metric tensor \( q = q_{11}q_{22}q_{33} \).

Equation (8) is recorded not only for the explored, but also for arbitrary, associated with the geometric configuration of the objects coordinate systems.

Based on research results, it was found that when implementing model (3) on conditions of (4) or (5), all basic results, obtained for model (2), with regard to convergence and optimization of the iteration process and described in [24], are true for model (3). In this case, the type of boundary conditions (Fig. 4), proposed in the work, as well as considered possibilities of product inflowing and outflowing through the wall, are taken into consideration. In models (2) and (3), parameters \( k, \mu, \rho \) change. Such kind of models has not been examined for underground storages before.

While implementing correspondent models, the numerical method for over-relaxation for Laplace equation taking into account peculiarities of the model (variable coefficients and special type of boundary conditions) was used. Subsequent research can be linked to the research of the specified model for case \( \mu = \mu(x, y) \), \( k = k(x, y) \). In this case, problems (2) and (4) are reduced to necessity of solving the Poisson problem for fluid pressure with conditions of type (4), (5).

The study of the horizontal component of velocity on the height of the area at \( x = R \) was carried out. It was found that regardless of the model of filtering flow (2) or (3) and the number of zones of fluid penetration through the boundary of the area, this impact of existence of these zones is tangible only in the small circle of these zones. This is proved by results of conducted calculations by numerous algorithms of implementation of the appropriate mathematical model. That is, existence of outflows on the well's area almost does not affect fluid parameters at the bottom of this area, the difference in the results of calculation is less than 0.5%.

This enables us to conclude that detection of the outflow coordinate, as well as the fact of its existence, is impossible within the Darcy (2), and Forchheimer (3) models. Features of distribution of the horizontal velocity component are presented in Fig. 5, 6.
corresponds to the upper part of the object, and \( x=0 \) corresponds to the lower part.

Finding fluid loss through the extraction area \( [y_i, y_{i+1}] \), \( i=1, \ldots, L \), where \( L \) is the number of extraction areas, and comparing with the stream of fluid that arrives in the area, it is possible to estimate the rate of fluid accumulation in the area: let fluid consumption through extraction areas be \( Q_i \), \( Q_0 = \rho \cdot V \cdot S \), \( V \) is the horizontal velocity, \( \rho \) is the fluid density; \( S \) is the area of extraction zones, for example, for the cylindrical area \( S_i = 2\pi \cdot R \cdot L \), where \( L \) is the height of the i-th extraction zone.

\[
S = \sum_{i=1}^{L} S_i
\]

and intensity of water arrival \( Q_{in} \); in this case, it should be noted that dimensionality

\[
[Q_i] = [Q_{in}] = \frac{kg}{s}
\]

Finding the value

\[
Q = Q_i - Q_{in}, \quad Q > 0
\]  

(9)

and accepting that

\[
Q = \frac{V_s \cdot \rho}{t}, \quad V_s = S_{area} \cdot h,
\]

(10)

where \( V_s \) is the volume, taken by the indicated weight of gas and fluid, \( S_{area} \) is the area of the base of the region, \( t \) is the time, and taking into account that for a cylindrical area

\[
V_0 = \pi \cdot R^2 \cdot h.
\]

(11)

We obtain:

\[
h = \frac{Q \cdot t}{\pi \cdot R^2 \cdot \rho},
\]

(12)

where \( h \) is the fluid level after time \( t \) at the assigned intensity of fluid arrival. Using this method, it is possible to evaluate both fluid outflow from the object and fluid arrival to the area from outside.

The benefit of the conducted research is establishment of quantitative characteristics of the zones of fluid and gas outflow through the surface of the storage. The shortcomings include complexity of practical implementation of the specified technique for underground storages. This is explained by the fact that their geometry is considerably different from the cylindrical or spherical configurations. Results of the present research can be applied to assess losses of products at their cross-flow through the wall of the storage. These results continue the study of filtering processes taking into account the features of geometry and other characteristics of storages, in particular the authors propose new parameters for boundary conditions.

## 7. Conclusions

1. Created mathematical models of the process of fluid filtration through the medium with a certain level of permeability allows the study into specific features of filtering process (pressure distribution and filtering rate), as well as estimation of dimensions of the areas of fluid propagation in the medium.

2. Based on analysis of the state of the wells of Dashava underground storage, using the created model that takes into consideration the configuration peculiarities of the particular underground storage, a special type of practical conditions, characteristic only for underground storage and variables on the coordinates of physical and mechanical parameters of the model, it is possible to make conclusions about possible losses of gas, specify the need for changes, repairs or sealing of wells. Currently, there are many different options and techniques. Sealing accounts for 50 % of success. In addition to probable economic benefits due to a decrease in gas losses in the operation of gas underground storages, this problem is relevant for workers of the gas transportation system. Its improved state will make it possible to consolidate Ukraine as a reliable partner, which provides gas transit under any conditions.

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