Consequences of covariant kaon dynamics in heavy ion collisions

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Abstract

The influence of the chiral mean field on the kaon dynamics in heavy ion reactions is investigated. Inside the nuclear medium the kaons are described as dressed quasi-particles carrying effective masses and momenta. A momentum dependent part of the interaction which resembles a Lorentz force originates from spatial components of the vector field and provides an important contribution to the in-medium kaon dynamics. This contribution is found to counterbalance the influence of the vector potential on the $K^+$ in-plane flow to a strong extent. Thus it appears to be difficult to restrict the in-medium potential from the analysis of the corresponding transverse flow.

Keywords: Subthreshold $K^+$ production, kaon flow, kaon mean field, Ni+Ni, E=1.93 GeV/nucleon reaction
25.75.+r
In recent years strong efforts have been made towards a better understanding of the medium properties of kaons in dense hadronic matter. This feature is of particular relevance since the kaon mean field is related to chiral symmetry breaking. The in-medium effects give rise to an attractive scalar potential inside the nuclear medium which is in first order, i.e. in mean field approximation, proportional to the kaon-nucleon Sigma term \( \Sigma_{KN} \). A second part of the mean field originates from the interaction with vector mesons. The vector potential is repulsive for kaons \( K^+ \) and, due to G-parity conservation, attractive for antikaons \( K^- \). A strong attractive potential for antikaons may also favor \( K^- \) condensation at high nuclear densities and thus modifies the properties of neutron stars.

One has extensively searched for signatures of these kaon-nucleus potentials, in particular in heavy ion reactions at intermediate energies. Here the transverse flow of \( K^+ \) mesons has attracted special attention. Due to the lack of reabsorption, \( K^+ \) mesons should, in contrast to \( K^- \) mesons and pions, not exhibit a pronounced shadowing effect but rather reflect the behavior of the primary sources, i.e. the nucleons (or \( \Delta \)-resonances). However, as first proposed by Li and Ko the presence of a \( K^+ \) potential should distort this scaling of the flow. The repulsive vector potential tends to push the \( K^+ \) mesons away from the nucleons leading to an anti-correlated flow. The attractive scalar potential counterbalances this behavior to some extent. Thus the net effect is expected to be a zero flow around midrapidity.

Due to its relativistic origin, the kaon mean field has a typical relativistic scalar–vector type structure. For the nucleons such a structure is well known from Quantum Hadron Dynamics. This decomposition of the mean field is most naturally expressed by an absorption of the scalar and vector parts into effective masses and momenta, respectively, leading to a formalism of quasi-free particles inside the nuclear medium. The application of the quasi-particle picture in heavy ion collisions is well established for nucleons and has extensively been used within the framework of relativistic transport theories.

In the present work we extend this manifestly covariant approach to the treatment of kaon in-medium properties and discuss implications on the \( K^+ \) flow in heavy ion reactions.
Starting from the chiral Lagrangian set up by Kaplan and Nelson \[1\], Li and Ko investigated the kaon properties in mean field approximation \[9\]. Including higher order terms kaon potentials have been derived within chiral perturbation theory by Brown and Rho \[2\] and by Waas, Weise and Kaiser \[14\]. Our starting point will be the mean field approximation \[9\].

From the chiral Lagrangian the field equations for the $K^\pm$-mesons are derived from the Euler-Lagrange equations \[9\]

$$\left[ \partial_\mu \partial^\mu + \frac{3i}{4f_\pi^2} j_\mu \partial^\mu + \left( m_K^2 - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s \right) \right] \phi_{K^\pm}(x) = 0 \quad .$$

(1)

Here the mean field approximation has already been applied. In Eq. (1) $\rho_s$ is the baryon scalar density, $j_\mu$ the baryon four-vector current and $i = \sqrt{-1}$ ensures the hermiticity of the operator. To make Eq. (1) more transparent the kaonic vector potential

$$V_\mu = \frac{3}{8f_\pi^2} j_\mu$$

is introduced and Eq. (1) is rewritten in the form

$$\left[ (\partial_\mu + iV_\mu)^2 + m_{K^\pm}^2 \right] \phi_{K^\pm}(x) = 0$$

(3)

$$\left[ (\partial_\mu - iV_\mu)^2 + m_{K^\pm}^2 \right] \phi_{K^\pm}(x) = 0 \quad .$$

(4)

Thus, the vector field is introduced by minimal coupling into the Klein-Gordon equation. The effective mass $m_{K^*}^e$ of the kaon is then given by

$$m_{K^*}^e = \sqrt{m_K^2 - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s + V_\mu V^\mu} \quad .$$

(5)

Due to the bosonic character, the coupling of the scalar field to the mass term is no longer linear as for the baryons but quadratic and contains an additional contribution originating from the vector field. The effective quasi-particle mass defined by Eq. (5) is a Lorentz scalar and is equal for $K^+$ and $K^-$. It should not be mixed up with the quantity, i.e. kaon energy at zero momentum, which is sometimes denoted as in-medium mass \[12–15\] and which determines the shift of the corresponding production thresholds. The form of Eqs.
(6) is quite general and also holds for the description of other spin-0 mesons, i.e. pions, where the mesons interact with an external vector field, respectively the baryon field, by a derivative coupling in the Lagrangian. Introducing an effective momentum

$$k^*_{\mu} = k_{\mu} \mp V_{\mu}$$

for $K^+(K^-)$, the Klein-Gordon equation (3,4) reads in momentum space

$$\left(k^* \cdot m^*_{K^\pm}(k) = 0 \right)$$

which is just the mass-shell constraint for the quasi-particles inside the nuclear medium. These quasi-particles can now be treated like free particles. Such a description is analogous to the well known quasi-particle picture for nucleons in a relativistic mean field [10] where the in-medium spinors obey an effective Dirac equation $\left[k^* - m^*\right]u^*(k) = 0$. In nuclear matter at rest the spatial components of the vector potential vanish, i.e. $V = 0$, and Eqs. (3,4) reduce to the expression already given in Ref. [9]. However, Eqs. (3,4) generally account for the correct Lorentz properties which are not obvious from the standard treatment of the kaon mean field [9,12,13,4]. In particular the quadratic term in density which enters into the effective mass is a Lorentz scalar determined by the baryon density in the local nuclear matter rest frame (RF), i.e. the frame where the spatial components of the baryon current vanishes,

$$V_{\mu}V^{\mu} = \left(\frac{3}{8f^2_\pi}\right)^2 \left(\rho_B\right)^2$$

To keep the present investigations as general as possible various mean field parameterizations with different strengths are considered. With MF we denote the mean field obtained with a value of $\Sigma_{KN}=350$ MeV originally used in [9]. For comparison we also adopt a mean field suggested by Brown and Rho [2] which has a value of $\Sigma_{KN}=450$ MeV and accounts for the rescaling of the vector part by medium modifications to the pion decay constant, i.e. $f^{s2}_\pi = 0.6f^2_\pi$. The second interaction, denoted as MF2 in the following, results in a much stronger vector part which is nearly twice as large as for MF. The MF2 parameterization has
already been used in our previous investigations based on the non-covariant QMD approach \cite{17,18}.

Higher order corrections to the kaon self-energy have been found, e.g., in Ref. \cite{14} to cancel the attractive part of the scalar self-energy given by \(-\Sigma_{\text{KN}}/f_{\pi}^2\rho_s\) to high extent. In order to complete the systematics the results of Ref. \cite{14} are parameterized on the mean field level. Motivated by Eqs. (3,4) we make the following ansatz for the dispersion relation given in the nuclear matter rest frame

\[
\omega_{K^\pm}(k = 0) = m^*_K \pm V_0, \quad m^*_K = \sqrt{m^2_K - \tilde{\Sigma}^2_S + V_0^2}.
\] (9)

The quantities \(V_0 = \frac{1}{2}(\omega_{K^+} - \omega_{K^-})\) and \(m^*_K = \frac{1}{2}(\omega_{K^-} + \omega_{K^+})\) are determined from the dispersion relations. To account for the correct density dependence scalar and vector parts are represented by non-linear proportionality factors \(V_\mu = g_V(\rho_B) j_\mu\) and \(\tilde{\Sigma}^2_S = g_S(\rho_B) \rho_s\) which defines the coupling functions \(g_V = V_0/\rho_B\) and \(g_S = \tilde{\Sigma}^2_S/\rho_s\). This parameterization reproduces the results of Ref. \cite{14} with high accuracy.

In Fig. 1 the effective quasi-particle masses \(m^*_K\) obtained in the mean field (MF, MF2) approach, Eq. (5), and from chiral perturbation theory (ChPT) Ref. \cite{14}, Eq. (9), are compared. We want to stress again that the quasi-particle mass \(m^*_K\) is equal for \(K^+\) and \(K^-\). The effective mass \(m^*_K\) is generally reduced inside the nuclear medium due to the attractive scalar potential. However, in the approach of Ref. \cite{14} (ChPT) this reduction is much weaker than in the simple MF parameterization where the scalar field is in first order proportional to the scalar nucleon density. Interestingly enough, the effective mass in the MF2 parameterization behaves similar to MF which is due to the fact that the significantly higher value of \(\Sigma_{\text{KN}}\) is counterbalanced by the stronger quadratic vector term.

The covariant equations of motion are obtained in the classical (testparticle) limit from the relativistic transport equation for the kaons which can be derived from Eqs. (3,4). They are analogous to the corresponding relativistic equations for baryons and read

\[
\frac{dq^\mu}{d\tau} = \frac{k^*^\mu}{m^*_K}, \quad \frac{dk^*^\mu}{d\tau} = \frac{k^*^\mu}{m^*_K}F^\mu_\nu + \partial^\mu m^*_K.
\] (10)
Here \( q^\mu = (t, \mathbf{q}) \) are the coordinates in Minkowski space and \( F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu \) is the field strength tensor for \( K^+ \). For \( K^- \) where the vector field changes sign the equation of motion are identical, however, \( F^{\mu\nu} \) has to be replaced by \(-F^{\mu\nu}\). The structure of Eqs. (10) may become more transparent considering only the spatial components

\[
\frac{dk^*}{dt} = -\frac{m_k^*}{E^*} \frac{\partial m_k^*}{\partial \mathbf{q}} \mp \frac{\partial V^0}{\partial \mathbf{q}} \pm \frac{k^*}{E^*} \times \left( \frac{\partial}{\partial \mathbf{q}} \times \mathbf{V} \right)
\]

where the upper (lower) signs refer to \( K^+ \) (\( K^- \)). The term proportional to the spatial component of the vector potential gives rise to a momentum dependence which can be attributed to a Lorentz force, i.e. the last term in Eq. (11). Such a velocity dependent (\( v = k^*/E^* \)) Lorentz force is a genuine feature of relativistic dynamics as soon as a vector field is involved. Concerning the nucleons such terms are generally included in relativistic transport approaches \[11,13,15\] and have been found to provide an essential contribution to the dynamics \[11,16\].

If the equations of motion are, however, derived from a static potential

\[
U(k, \rho) = \omega(k, \rho) - \omega_0(k) = \sqrt{k^2 + m_K^2 + \frac{\Sigma_{KN}}{f^2}} \rho_s + V_0 \pm V_0 - \sqrt{k^2 + m_K^2}
\]

as, e.g. in Refs. \[4,9,12,15\], the Lorentz-force like contribution is missing. The same holds for non-relativistic approaches \[17,18\] where the Lorentz force has also not yet been taken into account. Such non-covariant treatments are formulated in terms of canonical momenta \( k \) instead of kinetic momenta \( k^* \) and then the equations of motion (11) read

\[
\frac{dk}{dt} = -\frac{m_k^*}{E^*} \frac{\partial m_k^*}{\partial \mathbf{q}} \mp \frac{\partial V^0}{\partial \mathbf{q}} \pm v_i \frac{\partial \mathbf{V}_i}{\partial \mathbf{q}},
\]

with \( v = k^*/E^* \) the kaon velocity.

In the following the influence of covariant dynamics is examined for \( K^+ \). Since in particular the in-plane flow turned out to react sensitively on the kaon mean field \[8,12,17\] we will focus on this observable. We consider the reaction Ni on Ni at 1.93 A.GeV incident energy which has been studied extensively from the theoretical \[12,13,17\] as well as from the experimental \[3,8\] side. The \( K^+ \) creation mechanism is thereby treated as described in
Ref. [19]. However, in contrast to Ref. [19], we use the improved cross section of Ref. [20] for the baryon induced $K^+$ creation channels; for the pion induced channels we adopt the cross sections given in Refs. [21]. The baryon dynamics are treated within the framework of Relativistic Quantum Molecular Dynamics (RQMD). Here the extension of this approach to covariant scalar–vector mean fields [11] is used. For the nucleon mean field we adopt the non-linear NL2 version of the $\sigma\omega$ model which is able to reasonably describe the dynamical features of heavy ion collisions in the energy range considered [11,12].

In Fig. 2 exemplarily a semi-central ($b=3$ fm) reaction is considered. For better transparency of the results the rescattering of $K^+$ mesons with baryons is not included in these calculations since it may wash out the mean field effects to some extent [22]. The standard treatment (upper panel) where kaons are propagated in the static potential, Eq. (12), is compared to the covariant approach (lower panel) given by Eq. (11), respectively Eq. (11). Without any potential a clear flow signal is observed which reflects the transverse flow of the primary sources of the $K^+$ production. The dominantly repulsive character of the in-medium potential (12) tends to push the kaons away from the spectator matter which leads to a zero flow around midrapidity for the ChPT and also a nearly vanishing flow for the MF mean field. This deviation can be understood from the fact that the magnitude of the vector field is stronger using the ChPT parameterization ($V_0 = 79$ MeV at a nuclear density of $0.16 \text{ fm}^{-3}$) compared to MF ($V_0 = 57$ MeV). The effective mass, Fig. 1, starts, however, to differ significantly only at higher densities. Using the more repulsive MF2 parameterization the vector potential is so strong ($V_0 = 95$ MeV) that it produces an anti-correlated flow signal which is in good agreement with our previous investigation using the QMD model [17] and also with the results of Ref. [12].

The situation changes, however, dramatically when the full Lorentz structure of the mean field is taken into account. The influence of the repulsive part of the potential, i.e. the time-like component, on the in-plane flow is almost completely counterbalanced by the velocity dependent part of the interaction. Hence, no net effect of the potential is any more visible. This feature is rather independent on the actual strength of the potential, i.e. it
is the same for all potentials considered. Although the Lorentz force vanishes in nuclear matter at rest, it is clear that this force generally contributes in heavy ion collisions. Kaons are produced in the early phase of the reaction where the relative velocity of projectile and target matter is large. Thus the kaons feel a non-vanishing baryon current in the spectator region, in particular in non-central collisions.

In Fig. 3 the calculations are finally compared to the FOPI data [3]. The results are obtained for impact parameters $b \leq 4$ fm and with a transverse momentum cut $P_T/m_K > 0.5$. Here the rescattering of the $K^+$ mesons with the baryons and the influence of the Coulomb force is taken into account. The electromagnetic interaction is treated analogously to the strong interaction, i.e. by adding $F_{\text{el}}^{\mu \nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$ given by the electromagnetic vector potential to Eq. (10). It again turns out that the influence of the covariant in-medium dynamics on the $K^+$ in-plane flow is hardly visible. The calculation including the mean field (MF) yields a $K^+$ flow which is very close to the result obtained without any potential. The latter is in good agreement with the calculation of Ref. [12] for the free case. However, comparing to the experiment it seems that in the rapidity range $|Y_{c.m.}/Y_{proj}| \sim 0.5 - 1$ the flow is slightly overpredicted with respect to the data although the calculations lie inside the error limits. The more sophisticated ChPT potential leads to a slight reduction of the flow and thus to an improved agreement with the data which is, however, not satisfying. This is in particular the case if one has the preliminary and yet unpublished new flow data with smaller error bars in mind.

The cancelation effects on the flow can be understood from Eq. (13). The vector field is generally proportional to the baryon current $j_{\mu} = (\rho_B, u\rho_B)$ where $u$ denotes the streaming velocity of the surrounding nucleons. Let us for the moment assume that $u$ is locally constant, then the total contribution of the vector field in Eq. (13) can be written as $\pm \frac{3}{8 f_{\pi}^2} (1 - |v||u| \cos \Theta) \frac{\partial \rho_B}{\partial q}$. Now the angle $\Theta$ between the kaon and the baryon streaming velocities determines the influence of the Lorentz force. Since in our case and also in the calculations of Refs. [3,12] the $K^+$'s initially follow the primordial flow of the nucleons we have $\cos \Theta \sim 1$ which gives rise to the cancelation. However, the value of $\Theta$ and also the
magnitude of the kaon velocity are also related to the rescattering of the $K^+$ mesons with the nucleons. An enhanced rescattering as well as a different primordial $K^+$ flow might reduce the cancelation effects from the Lorentz force. Hence, the complete description of the in-plane $K^+$ flow is still an open question and further theoretical studies seem to be necessary.

In the present work the influence of the chiral mean field on the kaon dynamics in heavy ion reactions has been investigated. Three types of in-medium potentials with different strength, i.e. simple mean field parameterizations and a more sophisticated potential derived from chiral perturbation theory have been applied. The kaons are thereby described as quasi-particles carrying effective masses and momenta. This accounts for the correct mass-shell properties of the particles inside the nuclear medium. Due to the bosonic character of the kaons the corresponding mass-shell condition is slightly different to that of the nucleons. The most striking consequence of covariant kaon dynamics is, however, the appearance of a momentum dependent force proportional to the spatial components of the vector field which resembles the Lorentz force in electrodynamics. Although such Lorentz forces vanish in equilibrated nuclear matter they provide an essential contribution to the dynamics in the case of energetic heavy ion collisions. To summarize, the influence of in-medium effects on the in-plane flow is counterbalanced to large extent by this additional contribution and the improved agreement with the present flow data is destroyed. Concerning the flow of $K^-$ mesons the influence of the momentum dependent Lorentz force which in principle also counteracts against the static time-like component might be changed due to the dominance of reabsorption processes. The restriction of the kaon in-medium properties by the measurement of the in-plane flow appears to be difficult, as well for $K^+$ as for $K^-$. A way out of this problem may be the study of radial flow phenomena. The radial flow can be expected to be less distorted by the Lorentz-force terms since it mainly occurs in the fireball region. However, the present results indicate that a realistic description of the kaon dynamics requires to go beyond the – only density dependent – mean field approximation. E.g., the explicit momentum dependence of the kaon-nucleus potential which arises from
p-wave contributions in the KN interaction should be included in future investigations.
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FIG. 1. Effective kaon (quasi-particle) mass in nuclear matter. The mean field parameterizations MF (long-dashed) and MF2 (dashed) are compared to the result from chiral perturbation theory [14] ChPT (dotted).
FIG. 2. Influence of a covariant description on the transverse $K^+$ flow in a semi-central ($b=3$ fm) Ni + Ni collision at 1.93 A.GeV incident energy. The calculations are performed without (solid lines) and including different $K^+$ mean fields MF (long-dashed) and MF2 (dashed) as well as the respective one derived from chiral perturbation theory (ChPT, dotted). The full covariant dynamics including Lorentz forces (bottom) are compared to the description with a static potential (top).
FIG. 3. Average $K^+$ in-plane flow in a Ni + Ni collision ($b \leq 4$ fm) at 1.93 A.GeV incident energy. The calculations are performed without (solid lines) and including a $K^+$ mean field (MF, long-dashed) and the respective one derived from chiral perturbation theory (ChPT, dotted). A transverse momentum cut $P_T/m_K > 0.5$ has been applied. The FOPI data are shown by circles.