Selective electron transfer between the quantum dots under the resonant pulse

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Abstract

The coherent quantum dynamics of an electron in the quantum-dot ring structure under the resonant electromagnetic pulse is studied theoretically. A possibility of the selective electron transfer between any two dots is demonstrated. The transfer probability as a function of the pulse and dot parameters is calculated. It is shown that this probability can be close to unity. The factors lowering the transfer probability in real systems are discussed. The results obtained may be used in the engineering of novel nanoelectronic devices for quantum bits processing.

1 Introduction

Rapid development of nanotechnology and unavoidable progress in miniaturization of the basic elements of contemporaneous microelectronics give rise to the new field of investigations, the physics of low-dimensional structures and nanoelectronics. In recent years, the great advances have been maid in the fabrication of nanostructures and in the study of their properties [1],[2]. Much attention is now paid to the quantum dots (QD) - "artificial atoms"[3] - which combine the properties of real atoms with the properties imposed by the process of fabrication. A possibility to use the QDs for quantum information processing is now extensively discussed [4]-[7]. In these hypothetical devices, known as quantum computers, the quantum information is encoded in ground and/or excited orbital states of electron in the QD or in electron spin degrees of freedom. In principle, it will be possible to realize the quantum algorithms in such systems. To achieve this goal, however, one has to overcome some challenges concerning with initialization, processing, readout and storage of quantum information. One of the main problems here is a coherent control of evolution of electron states under the influence of external fields.

The behavior of low-dimensional objects is governed by quantum effects. Coherent evolution of one-electron states in a double-dot system upon the influence of a resonant laser pulse presents a good example of such phenomena. As was recently shown [8],[9], the pulse parameters (the frequency, duration, and amplitude) can be chosen so as to drive the electron, localized initially in the ground state in one of the dots, to the ground state of another dot via the delocalized state, common for
both dots and used as the "transport state". If the states localized in different dots are viewed as the Boolean states 0 and 1, the electron transfer between them may be considered as the unitary operation NOT \([8]-[10]\).

Recently, an attempt has been made \([11,12]\) to generalize the results obtained for a double-dot system \([8]\) to a chain-like multi-dot system. It was shown that it is rather difficult to implement the selective (addressed) electron transfer between arbitrary two QDs. This is because the probability amplitude to find an electron in a given QD depends strongly on the location of this QD in a linear chain of QDs with free boundary conditions.

In this work, we demonstrate the possibility of selective electron transfer between arbitrary two QDs in the QD’s ring structure under a resonant laser pulse, the local bias voltages being applied to those two dots. We derive an analytic expression for the transfer probability which takes into account the possible deviations of QD’s and/or bias parameters from ideal ones as well as the detuning of the laser pulse from the resonance.

2 The model

We consider a system composed of \(N\) identical QDs arranged in a ring, see Fig.1. We suppose that there are at least two size-quantized levels in each QD. One of them, with the energy \(\varepsilon_1\), corresponds to the ground state \(|1\rangle_n\) localized in a given QD with the number \(n = 1; \ldots; N\). If the value of \(\varepsilon_1\) is close to the minimum of the potential energy of an electron in the QD, and the height and/or width of the potential barriers between QDs are large, then the ground state wave functions of neighboring QDs overlap weakly because of the strong localization of the wave functions \(|r\rangle_1\) in the corresponding QDs. In this case, the ground state of the whole system may be considered as \(N\)-fold degenerate with respect to the electron localization in the QD system. We suppose that the excited levels \(|2\rangle_n\) with the energy \(\varepsilon_2\) (not necessarily second in the level numeration) in each of the QDs lie close to the barrier edge. In this case, the wave functions \(|r\rangle_2\) of the neighboring QDs overlap strongly, resulting in the electron tunneling between QDs and splitting of the excited levels into the subband of \(N\) levels, each being delocalized over the QD system. Since in the following we will use the resonant (with respect to the external time-dependent field) approximation, we neglect all levels whose energies are far from \(\varepsilon_1\) and \(\varepsilon_2\).

The Hamiltonian of an excess electron added to the conduction band (over the filled valence band) of such a QD structure, has the form

\[
\hat{H}_0 = \varepsilon_1 \sum_{n=1}^{N} \hat{a}^+_1,n \hat{a}_1,n + \varepsilon_2 \sum_{n=1}^{N} \hat{a}^+_2,n \hat{a}_2,n - V \sum_{n=1}^{N} (\hat{a}^+_2,n+1 \hat{a}_{2,n} + h.c.),
\]

where \(\hat{a}^+_1,n(\hat{a}_1,n)\) and \(\hat{a}^+_2,n(\hat{a}_2,n)\) are, respectively, the operators of creation (annihilation) of an electron in the ground and excited states of an isolated QD with the number \(n\); \(V\) is the matrix element of electron tunneling between the excited states of neighboring QDs. Note that \(\hat{a}_{2,N+1} = \hat{a}_{2,1}\) since the QD structure has the ring form. We don’t show the spin indexes explicitly in the Hamiltonian \((1)\) since we consider a one-electron problem.

Let an electron be initially localized in the ground state \(|1\rangle_{n_1}\) of a QD with the number \(n_1\). In the absence of an external field, the electron lifetime in the state
\[|1\rangle_{n_1}\] is very long because of weak overlap of the ground state wave functions of neighboring QDs. We assume this time to be longer than all other characteristic times of the problem. We wish to realize the selective electron transfer to the ground state \(|1\rangle_{n_2}\) of a QD with the number \(n_2\), i. e., to change the location of an electron in the QD system.

To realize the selective electron transfer between the QDs with the numbers \(n_1\) and \(n_2\), we apply equal local bias voltages to those QDs, thus shifting the energies of their ground and excited electron states (\(\varepsilon_1 \rightarrow \varepsilon_1 + \tilde{U}, \varepsilon_2 \rightarrow \varepsilon_2 + U\)). The Hamiltonian becomes

\[
\hat{H} = \hat{H}_0 + \hat{U}(\hat{a}^+_1 \hat{a}_{1,n_1} + \hat{a}^+_1 \hat{a}_{1,n_2}) + U(\hat{a}^+_2 \hat{a}_{2,n_1} + \hat{a}^+_2 \hat{a}_{2,n_2}),
\]

where, for the sake of simplicity, we ignore the changes in the matrix elements of electron tunneling from the QDs with the numbers \(n_1\) and \(n_2\) to neighboring QDs. This approximation is valid if the local biases are small compared to \(V\). For definiteness, we will consider \(\tilde{U} < 0\) and \(U < 0\) (in general, \(\tilde{U} \neq U\), although they are of the same order). To diagonalize the Hamiltonian (2), we go to operators \(\hat{a}_k^+ = \sum_{n=1}^{N} C_{k,n} \hat{a}_{2,n}^{+} (k = 1; \ldots; N)\). We have

\[
\hat{H} = \varepsilon_1 \sum_{n=1}^{N} \hat{a}^+_n \hat{a}_{1,n} + \hat{U}(\hat{a}^+_1 \hat{a}_{1,n_1} + \hat{a}^+_1 \hat{a}_{1,n_2}) + \sum_{n=1}^{N} E_k \hat{a}_k^+ \hat{a}_k,
\]

where the energies \(E_k\) of the delocalized levels and the coefficients \(C_{k,n}\) are determined by the set of equations

\[
E_k C_{k,n} = \varepsilon_1 C_{k,n} - V(C_{k,n-1} + C_{k,n+1}) + U C_{k,n}(\delta_{n,n_1} + \delta_{n,n_2}), \quad k = 1; \ldots; N,
\]

where \(C_{k,N+1} = C_{k,1}\). The coefficients \(C_{k,n}\) are the probability amplitudes to find an electron occupying the \(k\)-th state in the excited state \(|2\rangle_n\) centered in a QD with the number \(n\). They satisfy the normalization condition \(\sum_{n=1}^{N} |C_{k,n}|^2 = 1\) for any \(k = 1; \ldots; N\).

Expanding \(C_{k,n}\) into a Fourier series, \(C_{k,n} = \frac{1}{N} \sum_{m=1}^{N} C_{k,n} \exp(i2\pi mn/N)\), we obtain from Eq. (4) the following relationships between \(C_{k,n_1}\) and \(C_{k,n_2}\):

\[
C_{k,n_1} = A_k C_{k,n_1} + B_k C_{k,n_2}, \quad C_{k,n_2} = B_k^* C_{k,n_1} + A_k C_{k,n_2},
\]

where

\[
A_k = -\frac{U}{N} \sum_{m=1}^{N} \frac{1}{E_k - \varepsilon_2 + 2V \cos(2\pi m/N)}, \quad B_k = -\frac{U}{N} \sum_{m=1}^{N} \frac{\exp(i2\pi (n_1 - n_2)m/N)}{E_k - \varepsilon_2 + 2V \cos(2\pi m/N)}.
\]

It follows from Eqs. (5) and (6) that \(|C_{k,n_1}| = |C_{k,n_2}|\) for any delocalized level \(k\) from the excited subband. This is important for the following consideration. Note that the coefficients \(C_{k,n_1}\) and \(C_{k,n_2}\) may be chosen real, so that \(C_{k,n_1} = \pm C_{k,n_2}\).

Let the ac electric field \(\mathbf{E}(t) = E_0 \cos(\Omega t)\) be imposed on the QD system. The field frequency \(\Omega\) is close to the difference between the energy of one of the levels from the excited subband, \(E_{tr}\) (in the following called as the ”transport level”), and the ground state energies of an electron in the QDs with the numbers \(n_1\) and \(n_2\) (hereafter we set the Planck constant \(\hbar = 1\)). In the resonant approximation [8,9],...
the Hamiltonian reads

$$
\hat{H}(t) = \sum_{n=1}^{N} \left( \varepsilon_1 + \tilde{U}(\delta_{nm_1} + \delta_{nm_2}) \right) \hat{a}_{1,n}^{\dagger} \hat{a}_{1,n} + E_{tr} \hat{a}_{tr}^{\dagger} \hat{a}_{tr} - \left( e/m^* c \right) A(t) \sum_{n=1}^{N} \left( p_n \hat{a}_{tr}^{\dagger} \hat{a}_{1,n} + h.c. \right) = \\
\sum_{n=1}^{N} \left( \varepsilon_1 + \tilde{U}(\delta_{nm_1} + \delta_{nm_2}) \right) \hat{a}_{1,n}^{\dagger} \hat{a}_{1,n} + E_{tr} \hat{a}_{tr}^{\dagger} \hat{a}_{tr} - \left\{ \frac{1}{2} \exp(-i\Omega t) \sum_{n=1}^{N} \lambda_n \hat{a}_{tr}^{\dagger} \hat{a}_{1,n} + h.c. \right\},
$$

(7)

where \( p_n = \langle tr|\hat{p}|1\rangle_n \) are the matrix elements of the momentum operator; \( A(t) \) is the vector potential (we use the Lorenz gauge with zero scalar potential and neglect the interaction term quadratic in the vector potential), \( m^* \) is the electron effective mass. In Eq. (7), we introduced \( \lambda_n = -\frac{ie}{m^* \Omega} E_0 p_n \) making use of the well-known relationship between the vector potential and the strength of an electric field with the frequency \( \Omega \) and the amplitude \( E_0 \).

Here we point on a relationship between \( \lambda_n \) and the coefficients \( C_{tr,n} \) in the expansion of the delocalized transport state \( |tr\rangle = \sum_{n=1}^{N} C_{tr,n} |2\rangle_n \) in the states \( |2\rangle_n \). From the definitions of \( \lambda_n \) and \( p_n \) one obtains

$$
\lambda_n = -\frac{ie}{m^* \Omega} E_0 \sum_{n'} C_{tr,n'}^{*} \langle n'|2\rangle \langle 2|p|1\rangle_n.
$$

(8)

Since the wave functions \( \langle r|2\rangle_n \) of the excited states of QDs are centered in the vicinity of the corresponding QDs, and the ground state wave functions are localized in the QDs, one may suppose that \( n, n' \langle 2|\hat{p}|1\rangle_n = n \langle 2|\hat{p}|1\rangle_n \delta_{nn'} \), then it follows from Eq. (8) that \( \lambda_n = \lambda C_{tr,n} \), where \( \lambda = -\frac{ie}{m^* \Omega} E_0 p \) and \( p = n \langle 2|\hat{p}|1\rangle_n \).

We note that \( p \neq 0 \) (i.e., \( \lambda \neq 0 \)) only if a certain relationship between the symmetries of the wave functions \( \langle r|1\rangle_n \) and \( \langle r|2\rangle_n \) takes place. For example, \( p = 0 \) if both those functions have s-symmetry, while \( p \neq 0 \) if one of them has s-symmetry and another has p-symmetry. Besides, for the value of \( \lambda_n \) to be independent of \( n \) (this is needed to meet the equality \( |\lambda_{n_1}| = |\lambda_{n_2}| \) which follows from the condition \( |C_{tr,n_1}| = |C_{tr,n_2}| \) obtained earlier and to optimize the electron transfer between QDs), the vector \( p \) (not only its absolute value) should be independent of \( n \). This is so if, e.g., the functions \( \langle r|1\rangle_n \) have s-symmetry and the functions \( \langle r|2\rangle_n \) have \( p_z \)-symmetry, where \( z \) axis is perpendicular to the QD ring plane, see Fig.1. In this case, \( E_0 \) should have non-zero component along \( z \) axis in order \( \lambda \neq 0 \).

In the resonant approximation, the evolution of the electron wave function

$$
|\Psi(t)\rangle = \sum_{n=1}^{N} B_n(t) |1\rangle_n \exp[-i(\varepsilon_1 + \tilde{U}(\delta_{nn_1} + \delta_{nn_2}))t] + B_{tr}(t) |tr\rangle \exp(-iE_{tr}t)
$$

(9)

is governed by the non-stationary Schrödinger equation

$$
\frac{i}{\hbar} \frac{\partial |\Psi(t)\rangle}{\partial t} = \hat{H}(t) |\Psi(t)\rangle
$$

(10)

with the Hamiltonian (7) and the initial conditions \( B_n(0) = \delta_{nn_1}, B_{tr}(0) = 0 \). Our goal is to calculate \( B_n(t), B_{tr}(t) \) and thus to find \( |\Psi(t)\rangle \). The probability to find an electron in the ground state of the QD with the number \( n \) at a time \( t \) is equal to \( p_n(t) = |B_n(t)|^2 \).

We choose the lowest level of the excited subband as a transport level. This choice is motivated by the following considerations. First, since the excited states of the QDs are close to the barrier edge, some of the upper levels of the excited subband
may belong to the continuum spectrum. In contrast, the energy of the lowest excited level becomes smaller than \( \varepsilon_2 \), and hence the corresponding wave function remains localized in the QD system (although delocalized between different QDs). Second, the lowest excited level at \( U = 0 \) is non-degenerate for arbitrary \( N \) and remains non-degenerate at \( U \neq 0 \), while other excited levels at \( U = 0 \) constitute the set of the doubly degenerate states (except for the highest level at even \( N \)). At \( U \neq 0 \), this degeneracy is lifted, see Figs. 2 and 3, but the energy separations within the doublets are small compared with the doublet separations themselves, so that choosing any but the lowest of the excited delocalized levels as a transport level makes it difficult to tune the laser to the resonance.

## 3 Results and discussion

We define the resonant frequency and the detuning from the resonance as \( \Omega_r = E_{tr} - (\varepsilon_1 + \bar{U}) \) and \( \delta = \Omega - \Omega_r \), respectively. From Eqs. (7), (9), and (10) we obtain the set of the differential equations for the coefficients \( B_n(t) \) and \( B_{tr}(t) \):

\[
\begin{align*}
\dot{B}_n(t) &= i\frac{1}{2}\lambda_n^*B_{tr}(t)\exp\left[i\left(\delta - \bar{U}(1 - \delta_{nn_1} - \delta_{nn_2})\right)t\right], \\
\dot{B}_{tr}(t) &= i\frac{1}{2}\sum_{n=1}^{N}\lambda_n B_n(t)\exp\left[-i\left(\delta - \bar{U}(1 - \delta_{nn_1} - \delta_{nn_2})\right)t\right]
\end{align*}
\]  
(11)

where we took into account that the states \( |1\rangle_n \) and \( |tr\rangle \) are the eigenstates of the stationary Schrödinger equation with the eigenvalues \( \varepsilon_1 + \bar{U}(\delta_{nn_1} + \delta_{nn_2}) \) and \( E_{tr} \), respectively.

Since the shift of the QD levels \( \bar{U} \) caused by the local bias voltages applied to the QDs is finite and detuning from the resonance is small (ideally, \( \delta = 0 \)), one can take \( |\delta| << |\bar{U}| \). Moreover, we assume that the inequalities \( |\delta| << |\lambda| \) and \( |\lambda| << |\bar{U}| \) are satisfied, so that \( |\delta| << |\lambda| << |\bar{U}| \). Then, as follows from Eq. (11), the characteristic time \( \sim 1/|\lambda| \) it takes for the coefficients \( B_{n_1}(t), B_{n_2}(t), \) and \( B_{tr} \) to vary is much longer than the corresponding time \( \sim 1/|\bar{U}| \) for the coefficients \( B_n(t) \) with \( n \neq n_1, n_2 \), and hence we have \( |B_{n \neq n_1, n_2}| \sim (|\lambda|/|\bar{U}|)|B_{n_1, n_2}| << |B_{n=n_1, n_2}| \). Thus, it is sufficient to restrict ourselves with \( n = n_1 \) and \( n = n_2 \) in the sum (11):

\[
\begin{align*}
\dot{B}_{n_1}(t) &= i\frac{1}{2}\lambda_{n_1}^*B_{tr}(t)\exp(i\delta t), \\
\dot{B}_{n_2}(t) &= i\frac{1}{2}\lambda_{n_2}^*B_{tr}(t)\exp(i\delta t), \\
\dot{B}_{tr}(t) &= i\frac{1}{2}\lambda_{n_1} B_{n_1}(t)\exp(-i\delta t) + i\frac{1}{2}\lambda_{n_2} B_{n_2}(t)\exp(-i\delta t), \\
\dot{B}_{n \neq n_1, n_2}(t) &= i\frac{1}{2}\lambda_n B_{tr}(t)\exp(i(\delta - \bar{U})t).
\end{align*}
\]  
(12)

In this manner, the \((N+1)\)-level problem can be reduced to the 3-level problem since there are only three levels \((|1\rangle_{n_1}, |1\rangle_{n_2}, \) and \(|tr\rangle \) relevant for the quantum dynamics of the system at the resonance or close to it. This problem was recently solved by us for the case \( \delta \neq 0, |\lambda_{n_1}| = |\lambda_{n_2}| \) in Ref. [8] and for the case \( \delta \neq 0, |\lambda_{n_1}| \neq |\lambda_{n_2}| \) in Ref. [9]. The results obtained in Refs. [8,9] for the probabilities \( p_n(t) \) can be directly applied to the problem treated here. One can see [9] that at \( \delta = 0 \), the probability of electron transfer between QDs is

\[
p_{n_2}(t) = \left(\frac{2|\lambda_{n_1}||\lambda_{n_2}|}{|\lambda_{n_1}|^2 + |\lambda_{n_2}|^2}\right)^2 \sin^4(\omega_R t),
\]  
(14)
where $\omega_R = \sqrt{|\lambda_{n_1}|^2 + |\lambda_{n_2}|^2}/4$, and hence the selective electron transfer between QDs takes place in a time $T = \pi/2\omega_R$ if $|\lambda_{n_1}| = |\lambda_{n_2}|$. A deviation of $\delta$ from zero and $|\lambda_{n_1}|$ from $|\lambda_{n_2}|$ causes the value of $p_{n_2}(t)$ to decrease.

In this work, we account for a possible differences in the voltage bias applied to two selected QDs. We take $U_{n_1} \neq U_{n_2}$ and $\bar{U}_{n_1} \neq \bar{U}_{n_2}$, then Eqs. (12) and (13) become

\[
\begin{align*}
\dot{B}_{n_1}(t) &= i\frac{\lambda_{n_1}}{2}B_{tr}(t) \cdot \exp(i\delta t), \\
\dot{B}_{n_2}(t) &= i\frac{\lambda_{n_2}}{2}B_{tr}(t) \cdot \exp(i(\delta - \Delta\varepsilon)t), \\
\dot{B}_{tr}(t) &= i\frac{\lambda_{n_1}}{2}B_{n_1}(t) \cdot \exp(-i\delta t) + i\frac{\lambda_{n_2}}{2}B_{n_2}(t) \cdot \exp(-i(\delta - \Delta\varepsilon)t),
\end{align*}
\]

(15)

\[
\dot{B}_{n \neq n_1,n_2}(t) = i\frac{\lambda_n}{2}B_{tr}(t) \exp(i(\delta - \bar{U}_{n_1})t),
\]

(16)

where now $\Omega = E_{tr} - (\varepsilon_1 + \bar{U}_{n_1})$ and we designated $\Delta\varepsilon = \bar{U}_{n_1} - \bar{U}_{n_2}$.

Using the following substitutions,

\[
\begin{align*}
B_{n_1}(t) &= \tilde{B}_{n_1}(t) \cdot \exp\left(i\frac{\Delta\varepsilon}{2}t\right), \\
B_{n_2}(t) &= \tilde{B}_{n_2}(t) \cdot \exp\left(-i\frac{\Delta\varepsilon}{2}t\right), \\
B_{tr}(t) &= \tilde{B}_{tr}(t) \cdot \exp\left(-i\left(\delta - \frac{\Delta\varepsilon}{2}\right)t\right),
\end{align*}
\]

(17)

we have from Eq. (15):

\[
\begin{align*}
\dot{\tilde{B}}_{n_1}(t) + i\frac{\Delta\varepsilon}{2}\tilde{B}_{n_1}(t) &= i\frac{\lambda_{n_1}}{2}\tilde{B}_{tr}(t), \\
\dot{\tilde{B}}_{n_2}(t) - i\frac{\Delta\varepsilon}{2}\tilde{B}_{n_2}(t) &= i\frac{\lambda_{n_2}}{2}\tilde{B}_{tr}(t), \\
\dot{\tilde{B}}_{tr}(t) - i(\delta - \Delta\varepsilon/2)\tilde{B}_{tr}(t) &= i\frac{\lambda_{n_1}}{2}\tilde{B}_{n_1}(t) + i\frac{\lambda_{n_2}}{2}\tilde{B}_{n_2}(t).
\end{align*}
\]

(18)

Next, we express $\tilde{B}_{n_1}(t)$ in terms of $\tilde{B}_{n_2}(t)$, $\tilde{B}_{tr}(t)$, and $\tilde{B}_{tr}(t)$ and $\tilde{B}_{n_2}(t)$ in terms of $\tilde{B}_{n_2}(t)$ and $\tilde{B}_{n_2}(t)$ from the third and second equations of the set (18), respectively. Inserting these expressions into the first equation of the set (18), we obtain the equation for the coefficient $\tilde{B}_{n_2}(t)$:

\[
\ddot{\tilde{B}}_{n_2}(t) - i(\delta - \Delta\varepsilon/2)\dot{\tilde{B}}_{n_2}(t) + \left[\frac{\lambda_{n_1}^2 + \lambda_{n_2}^2}{4} + \left(\frac{\Delta\varepsilon}{2}\right)^2\right]\tilde{B}_{n_2}(t) - \\
- \frac{\Delta\varepsilon}{2}\left[\frac{\lambda_{n_1}^2 - \lambda_{n_2}^2}{4} + \frac{(\delta - \Delta\varepsilon/2)\Delta\varepsilon}{2}\right]\tilde{B}_{n_2}(t) = 0.
\]

(19)

Taking into account that $\tilde{B}_{n_1}(0) = 1$ and $\tilde{B}_{n_2}(0) = B_{tr}(0) = 0$, we have the following initial conditions: $\tilde{B}_{n_2}(0) = \dot{\tilde{B}}_{n_2}(0) = 0$, $\dot{\tilde{B}}_{n_2}(0) = -\lambda_{n_1}\lambda_{n_2}^*/4$.

The solution of Eq. (19) can be found exactly. It is, however, too cumbersome, so here we restrict ourselves to the approximate solution for the case that the deviations from the ideal case are small, i.e., $|\delta|/|\lambda|$, $|\Delta\varepsilon|/|\lambda|$, $|\Delta\lambda|/|\lambda| << 1$, where $\Delta\lambda = |\lambda_{n_1}| - |\lambda_{n_2}|$:

\[
\tilde{B}_{n_2}(t) = -\frac{\lambda_{n_1}\lambda_{n_2}^*}{\lambda_{n_1}^2 + \lambda_{n_2}^2 + (\Delta\varepsilon)^2} \exp\left(-i\frac{\Delta\varepsilon}{|\lambda_{n_1}|^2 + |\lambda_{n_2}|^2}t\right) \cdot \\
\cdot \left\{1 - \left(\cos(2\omega_R t) - i\frac{\delta}{4\omega_R} \sin(2\omega_R t)\right) \exp\left(i\frac{\delta t}{2}\right)\right\},
\]

(20)
where \( \bar{\delta} = \delta - \frac{\Delta \varepsilon}{2} + \frac{3}{2} \Delta \varepsilon \left| \frac{\lambda_{n_2} - \lambda_{n_1}}{\lambda_{n_1}} \right|^2 \) and \( \omega_R = \frac{1}{4} \sqrt{|\lambda_{n_1}|^2 + |\lambda_{n_2}|^2 + \delta^2 + (\Delta \varepsilon)^2} \). Then we have for the probability of the electron transfer:

\[
p_{n_2}(t) = |B_{n_2}(t)|^2 = \left( \frac{2|\lambda_{n_1}| |\lambda_{n_2}|}{|\lambda_{n_1}|^2 + |\lambda_{n_2}|^2 + (\Delta \varepsilon)^2} \right)^2 \cdot \\
\sin^4(\omega_R t) + 2 \sin^2(\omega_R t) \sin(2\omega_R t) + \frac{\beta^2}{8\omega_R^2} \sin^2(2\omega_R t) - \frac{\bar{\delta}}{8\omega_R} \sin \left( \frac{\delta t}{2} \right) \sin(2\omega_R t).
\]

For \( |\delta|/|\lambda|, |\Delta \varepsilon|/|\lambda|, |\Delta \lambda|/|\lambda| << 1 \), the maximum value of \( p_{n_2} \) is

\[
p_{n_2}^{\text{max}} \approx 1 - \left[ (\Delta \lambda)^2 + (\Delta \varepsilon)^2 + (\pi^2/8) (\delta - \Delta \varepsilon/2)^2 \right]/|\lambda|^2
\]

at \( T = \pi/2\omega_R \), in agreement with Ref. [9].

If at \( t = 0 \) an electron is placed in the superposition of states \( |1\rangle_{n_1} \) and \( |1\rangle_{n_2} \), i.e., \( B_{n_1}(0) = \alpha \) and \( B_{n_2}(0) = \beta \) (so that \( |\alpha|^2 + |\beta|^2 = 1 \), i.e., \( B_{n \neq n_1,n_2}(0) = B_{tr}(0) = 0 \)), then for the ideal structure \( (\Delta \varepsilon = \Delta \lambda = 0) \) and at exact resonance \( (\delta = 0) \) one obtains \( B_{n_1}(T) = -\beta \) and \( B_{n_2}(T) = -\alpha \). Thus, if the electron states \( |1\rangle_{n_1} \) and \( |1\rangle_{n_2} \) localized in different QDs are considered as the Boolean states \( |0\rangle \) and \( |1\rangle \), respectively, so that their linear combination is viewed as a quantum bit (qubit), then the resonance pulse with duration \( T \) is equivalent to the unitary quantum operation NOT combined with the simultaneous change of the qubit phase by \( \pi \):

\[
\hat{U}_{\text{ideal}}(T) |\Psi(0)\rangle = \hat{U}_{\text{ideal}}(T) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = |\Psi(T)\rangle = \begin{pmatrix} \beta \\ \alpha \end{pmatrix} \exp(i\pi).
\]

At non-zero values of \( \delta, \Delta \varepsilon, \) and \( \Delta \lambda \), the fidelity of this operation, \( F = |\langle \hat{U}_{\text{ideal}}(T) \Psi(0) \mid \hat{U}(T) \Psi(0) \rangle|^2 = \left| -\beta^* B_{n_1}(T) - \alpha^* B_{n_2}(T) \right|^2 \), differs from unity.

For the state-of-the art nanotechnology, it seems very difficult to fabricate an ordered nanostructure composed of a great number of nearly identical QDs. It is worth to mention another physical system, for which the realization of the proposed scheme may be possible, an array of phosphorus donor atoms embedded in a silicon host [13], [14]. The modern techniques of the controlled implantation of the phosphorous atoms into a silicon substrate [14] allows, in principle, to fabricate the structures where the donors serve as the centers of electron localization. Unlike an "artificial" (QD) atoms, all "natural" atoms are identical, while possible differences caused by implantation defects can be minimized, by appropriate annealing. If all but one of \( N \) donors are ionized, the one-electron model studied in this paper may be used to describe the evolution of an electron state.

Finally, since an electron resides in a solid rather than in a free space, its unavoidable interactions with other degrees of freedom can destroy the unitary electron evolution under external pulse. In particular, the processes of electron relaxation and dephasing result to the decoherence. This imposes some technological restrictions on the structure and material parameters [15]. A detailed discussion of decoherence effects in such structures will be presented elsewhere.

### 4 Conclusions

In this work, we have studied the unitary evolution of one-electron states in an array of coupled QDs. We have shown that the selective electron transfer between two
arbitrary QDs under the influence of the resonant pulse is possible. The probabilities to find an electron in the ground states of those QDs as functions of time and the structure and pulse parameters were calculated. We have shown that the probability of electron transfer to the ground state of another QD may be very close to unity. If we consider the states localized in two QDs as logical variables ”0” and ”1”, then the resonant pulse applied to an arbitrary linear superposition of those states (qubit) implements the quantum NOT operation with the simultaneous change of the qubit phase by \( \pi \).

We believe that since the selective electron transfer by virtue of the ”transport” state delocalized over all QDs may be performed as a ”one-step” operation, it has an advantage over the scheme based on sequential ”step-by-step” electron jumps between neighboring QDs, as was recently suggested in Ref. [16]. The results obtained in our work can be directly extended to other nano structures composed of a large number of centers of electron localization, e. g., the phosphorous donor in a silicon host, the atoms absorbed on a surface, etc.

We are grateful to K.A. Valiev for his interest in this study and to S.A. Dubovis who participated this work at its early stage.

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Fig. 1. The quasi-one-dimensional nanostructure composed of identical QDs arranged in a ring. The equal bias voltages are applied to the QDs with the numbers $n_1$ and $n_2$. The selective electron transfer between those QDs is performed under a resonant laser pulse.
Fig. 2. The energy diagram of the $N$-dot structure shown in Fig. 1 shows the energies $E_k$ of the delocalized states in the excited subband versus the local bias voltages $U$ applied to the QDs with the numbers $n_1$ and $n_2$. $N = 9, n_1 = 1, n_2 = 4$. The solid and dash lines correspond to the non-degenerate and degenerate at $U = 0$ states, respectively.
Fig. 3. The same as in Fig. 2 for $N = 10, n_1 = 1, n_2 = 5$. 