The role of leaky plasmon waves in the directive beaming of light through a subwavelength aperture

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Abstract: We show that the enhanced directivity phenomenon for light passing through a subwavelength aperture in a silver film with corrugations on the exit face, is due to a leaky wave that decays exponentially from the aperture. We show quantitatively that the field along the interface of the silver film is dominated by the leaky wave, and that the radiation of the leaky wave, supported by the periodic structure, yields the directive beam. The leaky wave propagation and attenuation constants parameterize the physical radiation mechanism, and provide important design information for optimizing the structure. Maximum directivity occurs when the phase and attenuation constants are approximately equal

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1. Introduction

We investigate here the fundamental physical phenomenon responsible for the directive beaming of light through a single subwavelength slit in a silver film, when the slit is surrounded by an optimized periodic structure [1-4]. The structure under investigation is shown in Fig. 1. It consists of a silver film with a subwavelength slit through it, and a set of one-dimensional (1D) corrugations or grooves on the surface surrounding the aperture on the exit side. As already shown in [1-4], when an optical beam (essentially a plane wave) is incident on the entrance side, a narrow beam of radiation may be created from the aperture radiating into the exit region, due to the presence of the periodic set of grooves. Usually the periodic set of grooves has a periodicity that is chosen to give a beam at broadside, although radiation at other angles is also possible. As shown in [5], this directive beaming effect is related by reciprocity [6] to the enhanced transmission of light through a subwavelength aperture when the aperture is surrounded by a periodic structure on the entrance face [7-11]. Enhanced transmission of light through arrays of subwavelength apertures has also been recently explored [12-15].

![Fig. 1. The structure used to obtain narrow beam radiation at broadside from a subwavelength slit in a silver film, when illuminated on the entrance (bottom) side by a plane wave.](image_url)

Previous work has established that a leaky wave is responsible for the narrow beaming effect when a simplified idealized model is used, in which a subwavelength aperture is surrounded by a 2D periodic structure consisting of a set of perfectly conducting rectangular patches [5]. This idealized structure was very useful for demonstrating the role of leaky waves...
in creating the beaming effect, although the structure itself was hypothetical, since perfect conductors (or even a close approximation to them) do not exist at optical frequencies.

We establish here in a direct way that the narrow beaming effect for the practical 1D structure shown in Fig. 1 is due to a leaky wave, which is a mode that decays exponentially away from the point of excitation (the narrow aperture). We thus establish for the first time that an exponentially-decaying leaky mode is the fundamental physical mechanism responsible for the narrow beaming effect on the realistic corrugated structure. The leaky-wave point of view provides a simple and direct physical explanation of the phenomenon, and gives a direct parameterization of the phenomena that allows for future design optimization. For example, leaky-wave theory yields simple formulas for determining the field along the interface and the far-field radiation pattern (see Eqs. (4) and (6)), and it establishes that an optimum beam at broadside occurs when the phase and attenuation constants of the leaky wave are equal (discussed later). This design rule is explored here for the plasmon structure, and results confirm that this rule does result in the optimum plasmon structure for producing a beam at broadside. Leaky-wave theory also allows for a simple estimation of the radiation efficiency of the structure, as explained later. In addition to directive beaming on plasmonic structures, the leaky-wave point of view also explains why the beaming and enhanced transmission phenomena may be created at microwave frequencies using corrugated metallic surfaces [16], even though plasmonic effects are clearly not involved at these frequencies.

Although the term “leaky wave” has been used before as a general description of radiation effects on corrugated silver film structures at optical frequencies [2], it is clear that in this previous work there was no direct connection between an exponentially-decaying guided mode with a complex wavenumber and the directive beaming effect. This is established here for the first time, and furthermore, the usefulness of the leaky-wave point of view is clearly demonstrated in optimizing the structure and predicting the radiation efficiency of the structure.

2. Background and Discussion of Leaky Waves

The subject of leaky waves is quite well established, particularly in the microwave area, where the design of leaky-wave antennas has been around since the 1950s [17]. An excellent overview of the physics of leaky waves, and the role of leaky waves in producing narrow-beam radiation, may be found in a pair of fundamental papers published by Tamir and Oliner in 1963 [18,19]. The reader is referred to these references for more detailed information about leaky waves. Only a brief overview will be presented here.

A leaky wave is a guided wave that radiates, or leaks, power continuously as it propagates on the guiding structure. Because of the power leakage, the mode attenuates exponentially as it propagates, even if the structure is lossless. A physical leaky wave is one that radiates at a real angle in space, and thus has the property that the phase constant lies within the visible region \((-k_0, k_0)\), where \(k_0 = \omega / c\) is the free-space wavenumber, \(\omega\) is the radian frequency, and \(c\) the speed of light. That is, the wave is a fast wave with respect to free space, having a phase velocity greater than the speed of light.

Leaky waves may arise in two different ways. First, a guiding structure may exist that supports a fast wave, and if this structure is open so that radiation can escape, the guided mode will be a leaky wave. A hollow rectangular waveguide with a long narrow slit in one of the walls, running parallel to the waveguide axis, is an early example of this [17].

The second way in which a leaky wave can arise is by the placement of a periodic perturbation on a slow-wave structure, which is open so that radiation can escape. The periodic perturbation causes the overall mode that propagates on the structure to now be composed of an infinite set of space harmonics (or Floquet waves). If \(x\) is the direction of propagation, the wavenumber \(k_{x,n}\) of the \(n\)th space harmonic is

\[
k_{x,n} = k_{x0} + \frac{2\pi n}{d} = \beta_n + i\alpha,
\]
where $\beta_\nu = \text{Re}(k_{x,\nu}) = \beta + 2\pi n/d$ (the fundamental phase constant $\beta$ is defined to be $\beta_0$) and $d$ is the period. (An $\exp(-i\omega t)$ time dependence is assumed and suppressed.) If one of the space harmonics (usually $n = -1$) is a fast wave (i.e., $-k_0 < \beta_\nu < k_0$), then the overall mode will radiate and hence be a leaky wave, with complex wavenumbers $k_{x,\nu}$. This in turn implies that the fundamental wavenumber of the mode, namely $k_{x,0} = \beta + i\alpha$, will be complex, with $\alpha > 0$. Since $\text{Im}(k_{x,\nu}) = \alpha$, all of the space harmonics attenuate along the interface with the same attenuation constant $\alpha$. The addition of a periodic structure can thus cause a guided mode to transform from a non-radiating slow wave with a real-valued wavenumber on the unperturbed structure, to a leaky mode with a complex fundamental wavenumber $k_{x,0}$. Such a mode will always decay exponentially along the structure from the point of excitation, even for a lossless structure.

It is our thesis that for the periodically-corrugated silver film structure shown in Fig. 1, it is a leaky plasmon wave that is responsible for the narrow-beam radiation effect. This will be demonstrated below. Although previous works [1, 2] have already demonstrated that the narrow-beaming effect is due to plasmonic effects, the new contribution here is the conclusive evidence that an exponentially-decaying leaky plasmon wave is responsible for the effect.

Before the periodic corrugations are added, the silver film supports a guided mode that is a slow wave, namely, a surface plasmon. This mode is polarized with a magnetic field that is transverse to the slow wave, namely, a surface plasmon. This mode can propagate at the interface between a half-space of material that has a permittivity with a positive real part (such as air) and one that has a permittivity with a negative real part (such as silver at optical frequencies). The wavenumber of the unperturbed surface plasmon (denoted as $k_{\text{plasmon}}$) for the interface (half-space) problem is given exactly by

$$k_{\text{plasmon}} = k_0 \sqrt{\varepsilon_r \over \varepsilon_r + 1},$$  \hspace{1cm} (2)

where $\varepsilon_r$ is the complex relative permittivity of the silver. For a finite thickness film as in Fig. 1, the mode decays exponentially away from the interface, so the half-space approximation is usually a good one. The introduction of the periodic corrugations results in a shift in the phase constant (the phase constant $\beta = \text{Re}(k_{x,0})$ of the leaky wave is slightly different than the phase constant $\beta_{\text{plasmon}} = \text{Re}(k_{\text{plasmon}})$ of the unperturbed plasmon mode). There is also a nonzero attenuation constant $\alpha = \text{Im}(k_{x,0})$ due to the leakage.

### 3. Analysis

According to the equivalence principle, the aperture in Fig. 1 may be closed and replaced by transverse (to $z$) equivalent magnetic and electric currents [6]. If the silver film is a reasonably good conductor, the equivalent magnetic currents will be dominant. For an electric field that is polarized in the $x$ direction across the aperture, the magnetic current will be $y$ directed. Hence, a uniform (in $y$) magnetic line current serves as a good model for the aperture. The model used for analysis here therefore consists of the corrugated silver film excited by a uniform $y$-directed magnetic line current located at the site of the original aperture, as shown in Fig. 2. The line source excites a leaky wave (a guided mode) and also radiates into space (including along the interface) in the form of direct source radiation [20, 21]. Along the interface the leaky mode decays exponentially with an attenuation constant $\alpha$, while the direct source radiation decays algebraically for large $|x|$ as $1/|x|^{3/2}$ [20]. The wavenumber of the leaky wave does not depend on the location or amplitude of the line source, but the amplitude of it does. The amplitude of the magnetic current is taken as unity for simplicity.

The line source on the corrugated structure launches a bi-directional leaky wave that propagates from the aperture equally in both directions, as shown in the figure. Because the problem is invariant in the $y$ direction, the only component of magnetic field will be $H_y$. If we...
consider this component of the magnetic field of the leaky wave along the surface at $z = W$, the field has the form

\[
H_y(x) = \sum_{n=-\infty}^{\infty} A_n e^{i k_n x}.
\]  

(3)

The normalized leaky-wave field $H_y$ on the interface (normalized to be unity at $x = 0$), denoted as $\psi(x)$, varies from one unit cell $q$ to the next as

\[
\psi_q = \psi\left((q-1/2)d\right) = e^{i k_{q0}(qd-d/2)}, \quad q > 0
\]

\[
\psi_q = \psi\left((q+1/2)d\right) = e^{-i k_{q0}(qd+d/2)}, \quad q < 0,
\]

(4)

where the center of the $q$th groove is at $x = qd$ and the center of the $q$th unit cell is defined to be at $x = (q\pm1/2)d$, with the minus sign for $q > 0$ and the plus sign for $q < 0$. The above result follows from Eq. (3) and the fact that all space harmonics change from one cell to the next by a factor of $\exp(\pm i k_{q0} d)$, since they all have the same attenuation constant $\alpha$ as well as the same phase shift when the value of $x$ changes by the period $d$. Summing the contribution from each unit cell, the far-field radiation is primarily determined by an array factor $AF$, defined as

$$ AF(\theta) = \sum_{q=1}^{\infty} \psi_q e^{-i k_{q0} \sin(\theta)(qd-d/2)} + \sum_{q=-1}^{\infty} \psi_q e^{-i k_{q0} \sin(\theta)(qd+d/2)}.$$

(5)

The array factor treats the radiating field on the interface at $z = W$ as a discrete summation of radiating sources, with the $q$th source located at the center of the $q$th cell, and accounts for the phase delay in the far-field radiation from each cell in accordance with the path length out to the far-field observation point. Inserting the normalized field values from Eq. (4) into Eq. (5), the array factor for the leaky-wave interface field may be evaluated in closed form as

$$ AF(\theta) = \frac{\zeta_1^{1/2}}{1 - \zeta_1} + \frac{\zeta_2^{1/2}}{1 - \zeta_2},$$

(6)

where

$$ z_1 = e^{-i(d \zeta_1)} \quad \text{and} \quad z_2 = e^{i(d \zeta_2)}, \quad \text{with} \quad \zeta_1 = k_0 \sin \theta - k_{x0} \quad \text{and} \quad \zeta_2 = k_0 \sin \theta + k_{x0}.$$
The leaky wave parameterization is important for determining a variety of radiative performances. For example, as shown in this paper, the leaky wave is responsible for the narrow beam radiation. In this case, the angle $\theta_{3dB}$ from broadside where the radiation pattern is 3dB less than the peak value at broadside can be determined from the far-field array factor pattern by using a procedure similar to that in [23]. The result is

$$\sin \theta_{3dB} = \sqrt{2} \frac{\alpha}{k_0}$$

In order to verify that a leaky wave is responsible for the field along the interface, and to determine the complex fundamental wavenumber $k_{0,0}$ of the leaky wave, numerical simulations are performed for the structure shown in Fig. 2. The ASM-FDTD method (Array Scanning Method used together with the Finite-Difference Time Domain simulation method) is used, which is an efficient implementation of the FDTD method for analyzing infinite periodic structures that are excited by a single (non-periodic) source [24]. This method is also used to calculate the numerically exact far-field radiation patterns from the structure in Fig. 2.

In the simulation, a Lorenz-Drude model is used to model the dispersive and lossy properties of the silver film [25], while a simple Drude model is used to model a hypothetical lossless film. Although a lossless film is not achievable in practice, it is interesting to investigate this case since the existence of an exponential decay in the propagating plasmon mode along the interface ($z = W$) must then be due only to leakage loss, and not material loss, thereby confirming that the mode is indeed a leaky mode. The dimensions were as follows (see Fig. 1): $d = 650$ nm, $W = 350$ nm, $a = 40$ nm, and $h = \text{either} 30$ or $40$ nm. The parameters of the Lorenz-Drude and Drude models were taken from [25].

### 4. Results

Table 1 summarizes the four cases that were investigated, with lossy and lossless results obtained for two different groove depths. For each case, the optimized wavelength is reported, for which the power density radiated at broadside (found from the numerically-exact far field in the simulation) was a maximum. Other results are presented below for both the field at the interface ($z = W$) and the far-field radiation pattern for the structure shown in Fig. 2.

| $h$ (nm) | Lossy | Model       | Optimized wavelength (nm) | Phase and Attenuation Constants $(\times 10^{-3})$ |
|---------|-------|-------------|---------------------------|-----------------------------------|
| 40      | Yes   | Lorenz-Drude | 698.9                     | $\beta_1 / k_0 = 18.6$, $\alpha / k_0 = 15.0$ |
| 40      | No    | Drude       | 697.2                     | $\beta_1 / k_0 = 7.94$, $\alpha / k_0 = 7.03$ |
| 30      | Yes   | Lorenz-Drude | 686.1                     | $\beta_1 / k_0 = 10.1$, $\alpha / k_0 = 8.05$ |
| 30      | No    | Drude       | 677.5                     | $\beta_1 / k_0 = 2.93$, $\alpha / k_0 = 2.97$ |

#### 4.1 Field at the Interface

The far-field pattern for the structure in Fig. 2 is determined by the field at the interface, according to the array factor in Eq. (5). Therefore, an investigation into the interface field is useful for establishing the physical mechanism of radiation. Figure 3 shows the interface field $H_y$ calculated at the center of the unit cells for the 40 nm lossless and lossy cases. The field is plotted as $20 \log_{10} |H_y|$ in order to reveal the exponential shape. The plots exhibit an *almost perfect exponential decay*, as evidenced by the close match with the exponential curve that is obtained by best-fitting a curve of the form $|H_y| = A_0 \exp(-\alpha x)$ in the region $x > 0$. The agreement is excellent out to more than 70 unit cells away from the source in the lossless case, and out to 60 unit cells for the lossy case. The excellent agreement in the lossless case is direct numerical evidence that a leaky wave is responsible for the interface field produced by the radiating aperture (modeled as a unit-amplitude magnetic line source), since there are no
material losses that would cause an exponential decay in the interface field. For the lossy case the interface field still exhibits an almost perfect exponential decay, although the attenuation constant is significantly larger now, as expected, since the attenuation is due to both material loss and leakage. The attenuation rate for the lossy case is approximately double that for the lossless case, indicating that the attenuations due to leakage and material loss are roughly equal. This translates into a radiation efficiency (power radiated / power launched) of about 50% for the leaky wave, for these parameter values.

Figure 4 shows similar results for the case of a 30 nm groove depth. Comparing Figs. 3 and 4, it is seen that the leakage rate is less for the shallower grooves. This is expected, since having shallower grooves corresponds to less of a perturbation due to the periodic structure, and hence less radiation from each unit cell (since radiation comes only by virtue of the perturbations, with the unperturbed plasmon mode being a nonradiating mode). Comparing the lossless and lossy cases in Fig. 4, it is seen that the leakage rate due to material loss is now roughly three times larger than that due to leakage. This translates into a radiation efficiency of about 25%.

The numerically extracted phase and attenuation constants for the $n = -1$ space harmonic of the leaky wave for the four cases, obtained from a best-fit curve fitting procedure, are shown in Table 1. (This space harmonic is selected since it is the one that is radiating, i.e. $-k_0 < \beta_1 < k_0$.) These values are obtained by assuming that the field at the interface varies from cell to cell as in Eq. (4), and then fitting the logarithm of the (complex-valued) normalized interface field values $\psi_q$ (from the field values at the center of the cells) with a best-fit linear function (having a complex slope). This yields the value of the complex wavenumber $k_{x,0}$, and from this the value of $k_{x,1}$ is immediately determined.

It is interesting to note that the phase and attenuation constants of the radiating space harmonic are approximately equal for these four cases (i.e., $\beta_1 = \alpha$), for the frequencies corresponding to optimized radiation at broadside. This is especially true for the lossless cases. This result is expected based on leaky-wave radiation principles, since it has been established that optimum radiation at broadside for a periodic leaky-wave antenna occurs when the phase and attenuation constants are equal [26].
4.2 Far-Field Radiation Pattern

Figure 5 shows a polar plot (in dB) of the far-field pattern for $H_y$ (i.e., $20 \log_{10} |H_y|$), for the 40 nm lossy case (labeled “With Corrugations”), along with the pattern of the line source on a smooth silver film without corrugations (labeled “Without Corrugations”). It is seen that with corrugations the beam is highly directive near broadside due to the presence of the corrugations, as already recognized in [1-3]. This directive beam is due to the leaky-wave radiation. Also seen is a broad part of the pattern due to direct radiation from the source that dominates the pattern away from broadside. This direct-source radiation corresponds to the radiation from the aperture that does not get channeled into the leaky wave. The figure also shows that the level of this direct-source radiation is higher when the corrugated structure is present. Furthermore, without corrugations there is no leaky-wave radiation. Hence the pattern without corrugations consists entirely of the direct-source radiation, and therefore does not exhibit any narrow beam.

Figure 6(a) shows an expanded view of the far-field pattern for the 40 nm lossless case, comparing the numerically exact far-field pattern with that obtained analytically from the leaky wave (Eq. (6)). It is seen that there is a good match between the exact and leaky-wave patterns near the peak of the beam at broadside. For the lossy 40 nm case shown in Fig. 6(b) the agreement is worse, but still reasonable. The higher attenuation rate of the leaky wave in the lossy case means that the interface field is less dominated by the field of the leaky wave, and more so from the field of the direct source radiation. This means that the leaky-wave optimization condition $\beta = \alpha$ is a less accurate predictor of the optimum frequency for broadside radiation in the lossy case. Furthermore, the material loss increases the attenuation constant $\alpha$ of the leaky mode, which has the effect of broadening the beam.

Figure 7 shows the corresponding results for the 30 nm cases. For the lossless case the agreement is excellent down to 20 dB from the peak of the beam. For the lossy case the agreement is once again worse, but still reasonable. As described above, the angle $\theta_{3dB}$ where the radiation pattern magnitude is 3dB less than the peak value at broadside is given by Eq. (7). For the case in Fig. 7(a), where $\alpha / k_0 = 2.97 \times 10^{-3}$, the formula yields $\theta_{3dB} = 0.24^\circ$, in excellent agreement with the plot.

Fig. 4. Same as in Fig. 3 except that $h = 30$ nm for the groove depth.
Fig. 5. Far-field radiation patterns (in dB) for the field $H_y$, comparing the far-field pattern of the magnetic line source on the corrugated structure with that of the same source on a smooth silver film (without corrugations).

(a)

$h = 40\text{nm}
\text{no losses}$
Fig. 6. Comparison of the normalized total and leaky-wave far-field patterns for $H_y$ near broadside, for $h = 40$ nm. (a) lossless case; (b) lossy case.

(a) $h = 30$nm
no losses

(b) $h = 40$nm
losses
Summary and Conclusions

It has been established here that the physical mechanism that is responsible for the narrow beaming of light through a single subwavelength slit in a silver film that is surrounded by a periodic set of corrugations is that of radiation from a leaky plasmon wave that decays exponentially along the interface away from the aperture. There are three specific contributions in this paper that demonstrate the importance and usefulness of the leaky-wave viewpoint. The first contribution is that we have shown that the physical mechanism for the narrow radiated beam is that of radiation from the \( n = -1 \) space harmonic of the leaky plasmon wave. The leaky wave has a set of complex wavenumbers \( k_{x,n} = \beta_n + i\alpha \) that attenuate as \( \exp(-\alpha |x|) \) due to radiation (and also material loss if present). The leaky wave is the surface plasmon mode of the silver film that becomes leaky due to the periodic set of corrugations, which causes the \( n = -1 \) space harmonic of the mode to radiate (since \(-k_0 < \beta_1 < k_0\)). The dominance of the leaky wave has been established in two ways: first, by examining the field along the interface and showing that it decays exponentially, even for a lossless structure, and second, by comparing the exact far-field radiation pattern with that of the leaky wave alone.

The leaky-wave viewpoint provides a useful and physically satisfying explanation for the radiation mechanism, and offers new insights into the optimization of the structure to achieve maximum radiation at broadside. In particular, the second contribution of this paper has been to establish that the optimum beam at broadside is achieved when the fundamental condition \( \beta_1 = \alpha \) is satisfied, as predicted by leaky-wave theory. The third contribution of this paper is that the leaky-wave point of view has enabled an approximate calculation of the radiation efficiency of the structure, something that is easily obtainable from the wavenumbers of the leaky wave on the lossy and lossless structures.

Leaky-wave theory also provides guidance into how to taper or truncate the structure to achieve a specified beam shape or efficiency, since the radiation phenomenon is characterized by a complex propagation wavenumber (which in principle could be varied along the length of the structure). These issues will be explored in the future.

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