Reduction of neutrino - nucleon scattering rate by nucleon - nucleon collisions

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We studied possible modifications of the neutrino – nucleon scattering rate due to the nucleon – nucleon collisions in the hot dense matter which we find in the supernova core. We show that the finite width of the nucleon spectral function induced by the nucleon collisions leads to broadening of the dynamical spin structure function of the nucleon, resulting in the reduction of the rate of neutrino – nucleon scattering via the axial vector current and making the energy exchange between neutrinos and nucleons easier. The reduction rate is relatively large \( \sim 0.6 \) even at density \( \sim 10^{13} \text{g/cm}^3 \) and could have a significant impact on the dynamics of the collapse-driven supernova as well as the cooling of the proto neutron star.

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I. INTRODUCTION

The collapse-driven supernova is supposed to be an outcome of gravitational collapse of a massive star \( \gtrsim 8 \text{M}_\odot \) at the end of its evolution. Although more than 60 years have passed since the original idea by Baade and Zwicky [1,2], and the detection of neutrinos from SN1987A [3,4] confirmed that our scenario is correct on the whole, we have not yet figured out how this phenomenon occurs.

These days many researchers of the supernova consider that neutrinos diffusing out of the proto neutron star play a crucial role in heating the material behind the shock wave and expelling the outer layer of the star [5,6]. It turned out, however, that this mechanism is quite sensitive to the neutrino luminosity [7,8]. In fact, Janka and Müller [8] showed by numerical simulations that only about 20% of increase in neutrino luminosity could lead to a successful explosion of a model which otherwise failed to explode. Hence mechanisms to boost the neutrino luminosity have been quested. Although many authors have been devoted in the study of convection in the core and it has been shown that the convection does help neutrinos heat matter, it is still controversial if the convection alone is sufficient for successful explosion or not [9,10]. It is also found that the sophistication of the numerical treatment of neutrino transport could increase the neutrino luminosity although the quantitative assessment of the difference it makes in the realistic context remains to be done [11,12].

On the other hand, our knowledge of the neutrino reaction rates in the hot dense medium is rather poor. In fact, even in the most elaborate simulations of supernovae it has been assumed that the reactions of neutrinos with nucleons are the same as in vacuum and the effect of surrounding matter is ignored [13,14]. However, the wavelength of a 30MeV neutrino, for example, is longer than the mean separation of nucleons for the density \( \sim 10^{13} \text{g/cm}^3 \), and the time scale corresponding to the same energy is roughly of the same order as the mean free time of nucleon between collisions. Hence we have to study the spatial and temporal correlations of the matter, and it could be possible that the many body effects change the opacity for neutrinos considerably and we get the desired enhancement of neutrino luminosity and/or energy. In fact, efforts to find a possible modification of rates of neutrino reactions with nucleons have been made by several authors [15]. In these studies neutrino - nucleon scatterings are one of the targets, since it is one of the major sources of opacity for neutrinos. Reddy et al. [16] pointed out that taking a correct effective mass of nucleon into account changes the scattering rate in the high density regime \( \gtrsim \rho_0 \) which is the saturation density. Horowitz et al. [17], Burrows et al. [18,19], Reddy et al. [20] and Yamada et al. [21] discussed the correlation effects due to the particle - hole excitation using a random phase approximation (RPA). In these studies the nucleons were assumed to be quasi-particles with the vanishing width of the spectral functions. On the other hand, Raffelt and his collaborators [22] claimed that the neutrino scattering rate could be also reduced by losing the temporal correlations of the spins of nucleons, thus broadening the width of the structure function due to collisions of the nucleon with other nucleons surrounding it (see also [23]). This is a counter part for scattering of the so-called Landau-Pomeranchuk-Migdal effect [24,25] for the bremsstrahlung. It was also pointed out that this

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broadening of the structure function could enhance the inelasticity of the scattering and affect the energy spectra of neutrinos \([35]\), since the width of the structure function is of the order of the matter temperature at a relatively low density \(\sim 10^{13} \text{g/cm}^3\) according to their estimate. In these studies they assumed that the structure function of nucleon takes the Lorentzian form and inferred its width from the typical collision rate of nucleon with the overall normalization determined by the sum rule and the detailed balance imposed by hand. In addition, they only discussed long wave length limits. In this paper we study this collisional effect on the neutrino – nucleon scattering rates on the field theoretical basis. The width of the spectral function of nucleon is evaluated by solving the Bethe-Salpeter equation in medium and calculating the imaginary part of the nucleon self-energy \([36–38]\), and then it is used to calculate the structure functions of nucleon as a function of the transferred four momentum \([39]\). In so doing the separable Yamaguchi potential \([10]\) was used for simplicity.

The paper is organized as follows. In the next section we formulate the basic equations. The results are presented in section 3. Discussion is given in the last section.

II. FORMULATION

A. scattering rates

The scattering rates of neutrinos with nucleons are quite generally formulated as:

\[
R(E^\nu_{\text{in}}, E^\nu_{\text{out}}, \cos \theta) = 4 G_F^2 E^\nu_{\text{in}} E^\nu_{\text{out}} [R_1(k)(1 + \cos \theta) + R_2(k)(3 - \cos \theta) - 2(E^\nu_{\text{in}} + E^\nu_{\text{out}})R_3(k)(1 - \cos \theta)],
\]

(2.1)

where \(E^\nu_{\text{in}}\) and \(E^\nu_{\text{out}}\) are the incident and outgoing neutrino energy, respectively, \(\theta\) is the scattering angle, and \(k\) is the four momentum transferred from neutrino to nucleon. In the right hand side of Eq. (2.1), the third term is usually much smaller than the other two, so that we ignore it in the following. The so-called dynamical structure functions \(R_1\) and \(R_2\) are given in the non-relativistic limit which we assume in the following, as

\[
R_1(k) \approx \frac{h^2}{\nu} \int d^4x \ e^{i k x} \langle \rho_N(x) \rho_N(0) \rangle,
\]

(2.2)

\[
R_2(k) \approx \frac{h^2}{3} \int d^4x \ e^{i k x} \langle \vec{s}_N(x) \cdot \vec{s}_N(0) \rangle.
\]

(2.3)

Here \(h\) and \(\nu\) are the weak coupling constants of nucleons, \(\langle \cdot \cdot \cdot \rangle\) stands for the thermal ensemble average of the argument, and \(\rho_N(x)\) and \(f_N(x)\) are the density and the spin density of nucleon. Thus it is obvious that \(R_1(k)\) and \(R_2(k)\) are nothing but their correlations in the matter. As is clear from the factor of the \(R_2(k)\) in Eq. (2.1), \(R_2(k)\) is more important than \(R_1(k)\). Thus we will discuss in this paper the modification of \(R_2(k)\), or the spin-density correlation function, due to the nucleon – nucleon collisions in the supernova core.

B. temporal spin-density correlation of a nucleon in random walk

Just by the same argument as made by Knoll et al. \([33]\) for the momentum correlation, the spin-density correlation of the nucleon which is successively scattered by other nucleons with a mean collision rate, \(\Gamma\), changing the direction of its spin via spin-dependent interactions, is given by ignoring the spatial non-uniformity as

\[
S^{ik}(\tau) = \langle s^i(\tau) s^k(0) \rangle = e^{-i \Gamma \tau} \sum_{n=0}^{\infty} \frac{(i \Gamma \tau)^n}{n!} \langle s^i_m s^k_{m+n} \rangle_m.
\]

(2.4)

In the above equation, \(n\) is the number of the scatterings during the time interval of \(\tau\). \(s^i_m\) is the spin-density at the \(m\)-th collision, and \(\langle \cdot \cdot \cdot \rangle_m\) means taking the average of the argument with respect to the ensemble of the sequence of \(n\) scatterings. Taking the Fourier transform of \(S^{ik}(\tau)\), we obtain

\[
S^{ik}(\omega) = \sum_{n=0}^{\infty} \langle s^i_m s^k_{m+n} \rangle_m \frac{2(\Gamma + i \omega)^{n+1}}{(\omega^2 + \Gamma^2)^{n+1}}.
\]

(2.5)

It was pointed out in the paper by Knoll et al. \([33]\) that the above summation of the Lorentz functions correspond to the summation of the Feynman diagrams depicted in Fig. 1 in the close time path formalism.
If we further assume that the spin-density auto-correlation decreases by a constant rate, $\alpha$, after each scattering, 
\[ \langle s^i_m s^k_{m+n} \rangle_m = \alpha^n \langle s^i_m s^k_m \rangle_m, \]
then we obtain

\begin{equation}
S^{ik}(\omega) = \frac{2 \Gamma'}{\left(\omega^2 + \Gamma'^2\right)} \langle s^i_m s^k_m \rangle_m \tag{2.6}
\end{equation}

\[ \Gamma' = (1 - \alpha) \Gamma. \tag{2.7} \]

This implies that the higher order terms account for the difference between the collision rate, $\Gamma$, and the relaxation time of spin-density, $\Gamma'$. If the spin flip is fast enough, $\alpha \approx 0$, these two rates become almost identical. In this case, the spin correlation function of Eq. (2.8) is approximated by the first diagram in Fig. 1. Note that the propagator in that diagram is not a free propagator but a dressed one in the medium. The self-energy included in the dressed propagator comes from the nucleon collisions with other nucleons. In this paper this self-energy is evaluated by solving the Bethe-Salpeter equation for two nucleons as drawn schematically in Fig. 2 [36–38]. In so doing, we assumed a separable Yamaguchi potential [40] in order to facilitate solving the Bethe-Salpeter equation. Although this is an oversimplification of the nuclear interactions, this is not a bad approximation as long as the low density regimes, $\rho_b \lesssim 10^{14}$ g/cm$^3$ is concerned, which is the case in this study, and gives some insight into what the collisional effects are like and how important they could be. Since the scatterings are suppressed due to the Fermi blocking in the high density regime, it is expected the scattering effects are most important in the low density regime. In addition, the resulting broadening of the structure functions of nucleon will affect the neutrino energy spectra which are formed in this low density regime [35].

Some remarks are necessary. The above approximation is not applicable to the vector current contribution to the neutrino scattering rates, since it does not take into account the conservation law, and, as a result, the long wave length limit is not properly reproduced. In the case of the axial vector current, the spin density is not conserved if
one takes into account the nuclear tensor force. What is more important, the nuclear axial vector current is nearly isovector. Thus unless the matter is entirely composed of neutrons or protons, the axial vector component of the nuclear weak current is not a conserved one as long as the nuclear force is spin-dependent.

C. approximations

Here we summarize the formulation outlined above. The spin correlation functions are given by the dressed Green function as

$$ R_2(k) = \hbar^2 \frac{2}{2M_N} \int \frac{d^3p}{(2\pi)^3} G_N^<(p+k) G_N^<(p), $$

where the nucleon Green functions are defined as

$$ i G_N^<(p) = \rho(p) [1 - f_N(p)] $$

$$ i G_N^>(p) = -\rho(p) f_N(p). $$

$f_N(p)$ is the Fermi-Dirac distribution of nucleon. $\rho(p)$ is the spectral function of nucleon given by the self-energy $\Sigma(p)$ as

$$ \rho(p^0, p) = \frac{Im \Sigma(p)}{\left[ p^0 - \frac{p^2}{2M_N} - Re \Sigma(p) \right]^2 + \left[ \frac{Im \Sigma(p)}{2} \right]^2}. $$

$M_N$ is the nucleon mass. The quasi-particle approximation, which is usually assumed, is obtained as a limit $Im \Sigma(p) \rightarrow 0$. In the Hartree or Hartree-Fock approximation, the self-energy is real and the nucleon can be treated as a quasi-particle. Beyond those approximations, the self-energy has an imaginary part in general. It is obvious that the imaginary part of the self-energy is a width of the spectral function of the transfer energy $p^0$. Hence the spectral function becomes a $\delta$ function in the quasi-particle approximation.

In this paper, following Alm et al. [37], we evaluate the imaginary part of the nucleon self-energy from the two-particle $T$-matrix which is given as a solution of the Bethe-Salpeter equation in the ladder approximation,

$$ T(p_1, p_2, p_1', p_2', z) = V(p_1, p_2, p_1', p_2') + \int \frac{d^3p_3}{(2\pi)^3} \frac{d^3p_4}{(2\pi)^3} \frac{d^3p_5}{(2\pi)^3} \frac{d^3p_6}{(2\pi)^3} V(p_1, p_2, p_3, p_4) G_0^<(p_3, p_4, p_5, p_6, z) T(p_5, p_6, p_1', p_2', z). $$

Here $G_0^<(p_1, p_2, p_1', p_2', z)$ is defined as the product of two single-particle Green functions and calculated in the quasi-particle approximation as

$$ G_0^<(p_1, p_2, p_1', p_2', z) = \frac{1 - f_N(p_1) - f_N(p_2)}{z - E(p_1) - E(p_2)} \delta^3(p_1 - p_1') \delta^3(p_2 - p_2'). $$

$E(p)$ is the on-shell energy determined from the real part of the denominator of Eq. (2.11) and approximated as explained later. Eq. (2.12) is solved for the separable Yamaguchi potential, $V_\alpha(p, p') = w_\alpha(p)\lambda_\alpha w_\alpha(p')$, with $w_\alpha(p) = \frac{\lambda}{p^2 + \gamma^2}$, where the coupling constant and the effective range are $\lambda_\alpha = -12.3178$ (MeV fm$^{-1}$)$^{1/2}$ for $\alpha = 1S_0$ and $\lambda_\alpha = -14.6988$ (MeV fm$^{-1}$)$^{1/2}$ for $\alpha = 3S_1$ and $\gamma = 1.4488$ fm$^{-1}$, respectively. Then the imaginary part of the self-energy is given by

$$ Im \Sigma(p, \omega + i\varepsilon) = \int \frac{d^3p'}{(2\pi)^3} \left[ f_N(E(p')) + g_B(E(p') + \omega) \right] Im T_{ex}(p, p', p, p', E(p') + \omega + i\varepsilon). $$

Here again the quasi-particle approximation is used for a single-particle Green function. $g_B(E) = \{\exp[(E - 2\mu)/T] - 1\}^{-1}$ is a Bose-Einstein distribution function with the chemical potential of the nucleon pair. The subscript $ex$ denotes the inclusion of the exchange term. The real part of the self-energy is also derived from the $T$-matrix in a similar way. Since it depends on the on-shell energy which is in turn determined by the real part of the self-energy, iterations are required to obtain the self-consistent value of the on-shell energy. For simplification we abandon this consistency in this paper and set the on-shell energy as $E(p) = p^2 / 2M_N + U$, where the effective mass $M_N$ and the potential $U$ are taken from the relativistic mean field theory [11,12]. Using this on-shell energy and the corresponding imaginary part of the self-energy calculated by Eq. (2.14), we approximate Eq. (2.11) as

$$ \rho(p) = Im \Sigma(p)/\{[p^0 - E(p)]^2 + [Im \Sigma(p)/2]^2\}. $$

Finally we evaluate Eq. (2.8) with this approximate spectral function of nucleon.
III. RESULTS

In Fig. 3, we show the imaginary part of the neutron self-energy as a function of the transfer momentum and energy for the density \( \rho_0 = 3 \times 10^{13} \text{g/cm}^3 \) and temperature \( T = 10 \text{MeV} \) and electron fraction \( Y_e = 0.4 \). The on-shell momenta roughly correspond to the ridge of the hill in this figure. The imaginary part of the self-energy is essentially a scattering rate of a neutron with another nucleon. It is evident from the figure that this is energy- and momentum-dependent and becomes as large as the average kinetic energy of the nucleon \( \sim 3T \) for small momentum transfers. \( \text{Im}\Sigma \) decreases as one goes to higher energies along the on-shell, since the scattering rate becomes smaller for higher energy incident nucleons. The self-energy of proton is almost the same as that of neutron in this case. For more asymmetric matter with smaller \( Y_e \), \( \text{Im}\Sigma_p \) becomes larger while \( \text{Im}\Sigma_n \) gets smaller as seen in Fig. 3. This is simply because the scattering rate for the neutron – proton pair is larger than that for the neutron – neutron or the proton – proton pair. The greater degeneracy for neutron also contributes to the reduction of the neutron self-energy. The density-, temperature- and \( Y_e \)-dependences of \( \text{Im}\Sigma \) can be understood essentially in the same way. Since we solved the Bethe-Salpeter equation, the contribution from the deuteron pole, if any, is automatically taken into account. In fact, the parameter values of the Yamaguchi potential used in this paper are chosen so that the deuteron binding energy is reproduced in the vacuum. For the above cases, we confirmed no bound state can exist due to the medium effect as shown by Alm et al. [37]. For the lower density \( \rho_0 = 10^{13} \text{g/cm}^3 \), however, the deuteron pole does exist and give tiny contributions to the imaginary part of the self-energy as shown in Fig. 2, where only the contribution from the deuteron pole was shown. The existence of the bound state is, thus, not relevant for the density, temperature and \( Y_e \) of our interest.

The spectral function is shown in Fig. 4 as a function of the transfer energy for different transfer momenta. The on-shell momentum is located again at the peak of the spectrum. As mentioned above, this function would be a \( \delta \)-function of the transfer energy if there were no interactions between nucleons. With collisions of two nucleons, the spectral function is broadened, as expected, and the width of the spectral function is roughly the scattering rate for the given transfer momentum. It is seen that the width of the spectral function or the scattering rate of nucleon is dependent on the transfer momentum and the spectral function becomes more narrow-peaked as the transfer momentum gets greater, reflecting the decrease of scattering rates. It should be again emphasized that the width of the spectral function is of the same order for the low momentum transfer as the average kinetic energy of nucleon. Hence it is expected that the kinetics of neutrino – nucleon scattering will be also affected. The density-, temperature- and \( Y_e \)-dependences of the spectral function can be inferred from that of the imaginary part of the nucleon self-energy. As the density increases, the scattering rate of nucleon with another one is increased, thus leading to larger widths of the spectral function. Note that the greater degeneracy of nucleon counteracts via the Fermi blocking to reduce the scattering rate. The scattering rate is determined by this competition. As the temperature increases, the blocking effect is diminished. This enhances the scattering rate. However, the larger kinetic energy of nucleon lowers the scattering rates. Again the width is determined by the balance of these factors. As the matter becomes neutron rich, the width of the spectral function for neutron is decreased partly because the scattering rate for neutron – neutron pair is smaller than that for neutron – proton and partly because the neutron becomes more degenerate and the Fermi blocking reduces the scattering rate more effectively.

Using the spectral functions obtained above, we calculated the nucleon structure functions with Eq. 2.8. The typical result is shown in Fig. 4 for \( R_2(k, \omega) \) as a function of the transfer energy \( \omega \) for three different transfer momenta \( k \). In this figure, three cases are compared. The thin dotted lines represent the structure functions for the free neutron, while the thin solid lines present the case for which only the effective mass of nucleon was taken into account in the spectral function, Eq. (2.11). The smaller effective mass lowers the peak of the spectral function as pointed out by some authors [29,30], although this effect is small for this low density \( (M_N^* = 890 \text{MeV}) \). It also tends to broaden the structure functions, since the recoil of the nucleon becomes larger as the effective mass of nucleon gets smaller. Again this is negligible in the low density regime considered here. On the other hand, the structure function is considerably broadened when the collisions among nucleons were taken into account, as shown with the thick solid lines in Fig. 4. The width is essentially given by the scattering rate of nucleons and taken over from the corresponding spectral function. The widths are not different for this low momentum transfers, which is qualitatively different from the results for the free nucleons and those for the quasi-particle approximation. As the density increases, the width becomes larger in general. As the temperature decreases, the part of the negative energy transfer is reduced, since the extraction of energy from the medium becomes more and more difficult as the nucleons become more degenerate. This is nothing but the detailed balance relation \( R(k) = e^{\beta\omega} R(-k) \) which is automatically satisfied in the formulation used in this paper. The width of the structure function becomes smaller for neutron and greater for proton as \( Y_e \) is decreased, as expected from the above discussions for the spectral function.
The imaginary part of the self-energy for neutron as a function of the transfer energy \( E \) and the absolute value of the transfer three momentum. The baryonic density is \( \rho_b = 3 \times 10^{13} \text{g/cm}^3 \), the temperature is \( T = 10 \text{MeV} \) and the electron fraction is \( Y_e = 0.4 \).

The total scattering rate of the neutrino with the incident energy \( E_{\nu}^{\text{in}} \) is given by the integration of the structure function with respect to the transferred momentum \( k \) and energy \( \omega \):

\[
R_{\text{tot}}(E_{\nu}^{\text{in}}) = \frac{1}{(2\pi)^3} \int 2\pi k \, dk \int_{-k}^{\omega_{\text{max}}} d\omega \frac{E_{\nu}^{\text{in}} - \omega}{2E_{\nu}^{\text{in}} 2E_{\nu}^{\text{out}}} \frac{1}{E_{\nu}^{\text{out}}} R(E_{\nu}^{\text{in}}, E_{\nu}^{\text{out}}, \cos \theta) \left[ 1 - f_{\nu}(E_{\nu}^{\text{out}}) \right],
\]

with \( \omega_{\text{max}} = \min(k, 2E_{\nu}^{\text{in}} - k) \). The scattering angle \( \theta \) and the energy of the outgoing neutrino \( E_{\nu}^{\text{out}} \) are functions of \( E_{\nu}^{\text{in}} \) and \( k \). The collisions of nucleons reduce in two ways the above scattering rates from the quasi-particle approximation. Not only the amplitude of the structure function is lowered, but the broadening of the structure function tends to put some fraction of the structure function outside the integration region. Ignoring the Fermi blocking of the outgoing neutrino in Eq. (3.1), we integrated the axial vector part \( R_{2\nu}(E_{\nu}^{\text{in}}) \) for \( E_{\nu}^{\text{in}} = 3T \) and compared the result with the collision taken into account and the result for the free nucleon. For the model shown in Fig. 7, for example, we found that the scattering rate is suppressed by the factor of 0.41 for neutron and 0.38 for proton, respectively. Note that the contribution of the effective mass is only a few percent in this case. For \( \rho_b = 10^{13} \text{g/cm}^3 \), \( T = 10 \text{MeV} \) and \( Y_e = 0.4 \), we still found the reduction of \( \sim 0.6 \).

The larger width of the structure function implies that the neutrino can exchange energy with nucleons more efficiently. In the supernova simulations performed so far, the neutrino – nucleon scattering was commonly approximated as the iso-energetic scattering, which means that this process does not contribute to the thermalization of the neutrino spectra. The above results stand by the claim by Raffelt and his company [24–26,35] that this may not be the case. In fact, the mean square root of energy transfer increased from \( \sim 5 \text{MeV} \) to \( \sim 15 \text{MeV} \) for the model of Fig. 7. Although this is still not a large value, it should be born in my mind that this process has about ten times larger cross section than the scattering with electrons. Hence it is expected the energy spectra of neutrinos, particularly muon- and tau-neutrinos, will be affected by this effect. Thus we need to somehow implement this effect in a numerical code and to study quantitatively the difference it will make in the realistic settings.
FIG. 4. The imaginary part of the self-energy for neutron (left) and proton (right) as a function of the transfer energy $E$ and the absolute value of the transfer three momentum. The baryonic density is $\rho_b = 3 \times 10^{13} \text{g/cm}^3$, the temperature is $T = 10\text{MeV}$ and the electron fraction is $Y_e = 0.1$.

FIG. 5. The deuteron contribution to the imaginary part of the self-energy for neutron as a function of the transfer energy $E$ and the absolute value of the transfer three momentum. The baryonic density is $\rho_b = 10^{13} \text{g/cm}^3$, the temperature is $T = 10\text{MeV}$ and the electron fraction is $Y_e = 0.4$. 
IV. CONCLUSION

We studied in this paper the possible effects of collisions among nucleons on the reaction rates of the neutrino–nucleon scatterings. By solving the Bethe-Salpeter equation for two nucleons, we estimated the imaginary parts of the nucleon self-energy, which in turn give the spectral function of the nucleon in medium. Using this spectral function, we calculated the structure functions of nucleon, which are directly related to the neutrino–nucleon scattering rates. In the limit of no collision, the spectral function reduces to the delta function of the transfer energy and the structure function gives the reaction rates which have been commonly used in the literature thus far. It was shown that the collisional broadening of the structure functions might be the main factor in the low density regime considered in this paper to modify not only the neutrino reaction rates with the nucleon but also the neutrino spectra.

It is obvious that the further study is needed on this issue. Although we used Yamaguchi potential to facilitate calculations, this is clearly an oversimplification of the dynamics of two nucleons. The self-consistency was also sacrificed for simplicity. Thus our results should be regarded as of qualitative nature. More importantly, we have to assess the corrections of the terms we ignored in estimating the the structure functions, Eq. (2.5). The in-medium vertex corrections should be also taken into account and are inferred from the Fermi liquid theory, for example [43,44]. Though we neglected in this paper these corrections in order to make clear the effect of the width of the spectral function of nucleon, they could be as important and should be studied on the same basis in the future work. It should be also mentioned that the formulation in this paper is not conserving [31,45,46] and cannot be applied to the vector current, whose long wave length limit is dictated by the baryon number conservation. The accomodation of this limit should be another subject of the future work.

At present the neutrino transport is thought to play a crucial role in the collapse-driven supernovae. It is, thus, important to study the microphysics involved and, at the same time, to investigate its consequences to the supernova simulations, which we are planning to do with the numerical code recently developed by us to solve the Boltzmann equations for neutrinos [14].

FIG. 6. The spectral function for neutron as a function of the transfer energy $E$ for various absolute values of the transfer three momentum. The baryonic density is $\rho_b = 3 \times 10^{14}$ g/cm$^3$, the temperature is $T = 10$ MeV and the electron fraction is $Y_e = 0.4$. 

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FIG. 7. The dynamical structure function $R_2(k, \omega)$ for neutron as a function of the transfer energy $\omega$ for various absolute values of the transfer three momentum, $k$. The baryonic density is $\rho_b = 3 \times 10^{13}$ g/cm$^3$, the temperature is $T = 10$ MeV and the electron fraction is $Y_e = 0.4$. The thin dotted lines represent the structure functions for the free neutron, while the thin solid lines present the case for which only the effective mass of nucleon was taken into account in the spectral function, Eq. (2.1). The thick solid lines show the results with the collisional effects taken into account for $k = 20, 30, 50$ MeV/c from left to right.

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