Erratum: Optimal linear drift for the speed of convergence of an hypoelliptic diffusion*†

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Abstract

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The authors correct the two following mistakes:

1. At page 5, line -20, it is proved in [13, Corollary 12] that the entropy converges at rate $2\rho(A)$

$$\text{Ent}_{\psi_{\infty}}\left(e^{tL_{A,D}h}\right) \leq ce^{-2\rho(A)t}\text{Ent}_{\psi_{\infty}}(h),$$

and not simply $\rho(A)$ as it has been written.

2. At page 9, line 8, $C$ should be replaced by $C^T$:

$$\partial_t \left(\alpha''(h_t)(\nabla h_t)^T M \nabla h_t\right) \leq 2\alpha''(h_t)(\nabla h_t)^T MC^T \nabla h_t$$

Indeed, the Jacobian Matrix of the function $b(x) =Cx$ is $C^T$ and not $C$. This initial mistake has the following chain of consequences:

• At page 9, from line 9 to 15, $S_{\frac{1}{2}}$ should be systematically replaced by $S^{-\frac{1}{2}}$. For the computations to hold, the matrix $\tilde{J}$ should be taken equal to its opposite, meaning that at page 8, the line -5 should be

$$\left(\tilde{J}\right)_{k,l} = \frac{\nu_k + \nu_l}{\nu_k - \nu_l}.$$

• At page 9, the computation from line -6 to line -3 should be replaced by

$$\text{Ent}_{\psi_{\infty}}(h_t) \leq \frac{1}{2} \int \frac{(\nabla h_t)^T S^{-1} \nabla h_t}{h_t} d\psi_{\infty}.$$

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\[ \frac{1}{2\nu_1} \int \left| \frac{Q^s S^{-\frac{1}{2}} \nabla h_t}{h_t} \right|^2 d\psi_\infty \leq e^{-2\lambda(t-s)} \frac{\nu_N}{2\nu_1} \int \left| \frac{S^{-\frac{1}{2}} \nabla h_s}{h_s} \right|^2 d\psi_\infty, \]

\[ \leq e^{-2\lambda(t-s)} \frac{\nu_N}{2\nu_1 \min \sigma(S)} \int \left| \nabla h_s \right|^2 h_s d\psi_\infty. \]

Note that an annoying factor \( \frac{\max \sigma(S)}{\min \sigma(S)} \) has disappeared.

As a consequence of both these corrections, the main result is improved to the following correct statement:

**Theorem 2.** For any \( C > 1 \) we can construct \((A, D) \in I(S)\) such that for all \( h > 0 \), with finite entropy, and for all \( t, t_0 > 0 \) with \( t \geq t_0 \),

\[
\text{Ent}_{\psi_\infty} \left( e^{(t-t_0) L_{\lambda, D}^*} e^{t_0 L_{-, S, I_N}} h \right) \leq C \frac{1}{2t_0 \min \sigma(S)} e^{-2(\max \sigma(S))(t-t_0)} \text{Ent}_{\psi_\infty} (h).
\]

Moreover it is possible to construct \((A, D) \in I(S)\) with \( \|A\|_F \leq 4N^2 \sqrt{\frac{(\max \sigma(S))}{\min \sigma(S)}} \) (where \( \|A\|_F = \sqrt{\text{Tr}(A^T A)} \) is the Frobenius norm) such that for all \( h > 0 \), with finite entropy, and for all \( t \geq t_0 > 0 \)

\[
\text{Ent}_{\psi_\infty} \left( e^{(t-t_0) L_{\lambda, D}^*} e^{t_0 L_{-, S, I_N}} h \right) \leq \frac{1}{t_0 \min \sigma(S)} e^{-2(\max \sigma(S))(t-t_0)} \text{Ent}_{\psi_\infty} (h).
\]