The Dark Matter Profile in the Galactic Center

Oleg Y. Gnedin  
*Space Telescope Science Institute  
ognedin@stsci.edu

Joel R. Primack  
*Physics Department, University of California, Santa Cruz  
joel@scipp.ucsc.edu

(Dated: August 27, 2018)

We describe a quasi-equilibrium profile of dark matter particles in the inner parsec of the Galaxy, $\rho_{\text{dm}} \propto r^{-3/2}$. This “mini-cusp” profile is caused by scattering with the dense stellar cluster around the supermassive black hole in Sgr A*. It is independent of the initial conditions. The implications for detection of gamma rays from WIMP dark matter annihilation in the Galactic center are a mild enhancement of the flux, and a characteristic central feature in the angular distribution which could be detectable by high-resolution Atmospheric Cherenkov Telescopes.

PACS numbers: 95.35.+d; 14.80.Ly; 98.35.Jk; 98.70.Rz

The distribution of dark matter in the very center of our Galaxy is important for experimental searches for signatures of possible annihilation of supersymmetric dark matter particles. Such weakly interacting massive particles (WIMPs) have remained favored candidate for the dark matter that represents most of the mass of the universe ever since it was first proposed that the lightest supersymmetric partner particle, stable as a result of R-parity conservation, is the nonbaryonic dark matter [1]. In current standard supersymmetric theories these are expected to be neutralinos $\chi$ [2] which naturally have the required cosmological density $\Omega_\chi \sim 0.25$ [3].

Since dark matter makes a negligible contribution to the dynamical mass in the central parsec, its distribution cannot be probed directly. Instead, it should be inferred considering all relevant physical processes in the Galactic center. Previous studies have reached different, often conflicting conclusions based on various initial assumptions and processes considered. All previous work assumed that dark matter particles are collisionless and therefore conserve their phase-space density.

- On scales above $\sim 1$ kpc, most dissipationless simulations of galaxy formation predict a power-law cusp in the dark matter density, $\rho_{\text{dm}} \propto r^{-\gamma}$ with $\gamma = 1 - 1.5$ [4, 5, 6, 7], or perhaps even $\gamma < 1$ [8].
- In the vicinity of the Galactic center, the supermassive black hole with $M_{\text{bh}} = 3.7 \times 10^6 M_\odot$ dominates the mass in the inner $r < r_{\text{bh}} = 2$ pc [9, 10, 11]. If the central black hole grew adiabatically from a small seed, for example by accretion of gas, stars, and dark matter [12, 13], the dark matter cusp would be enhanced and would form a spike, $\rho_{\text{dm}} \propto r^{-A}$ with $A = (9 - 2\gamma)/(4 - \gamma) \approx 2.3 - 2.4$ [14].
- If instead the black hole appeared instantaneously, being brought in by mergers of progenitor halos, the enhancement is weaker and the spike has a slope $A = 4/3$ [15]. A combination of this and the previous effect, such that the mergers create a seed black hole that later grows by accretion, produces an intermediate slope.

- Possible mergers of black holes in the centers of the progenitor halos [14, 15] may have the opposite effect on the dark matter distribution. Numerical simulations [16] show that the kinetic heating of particles during the merger may reduce their density to a very weak power-law, $\rho_{\text{dm}} \propto r^{-1/2}$. However, the simulations do not extend further than about 1 pc towards the center and thus cannot tell us about the very inner profile.
- The same weak cusp, $A = 1/2$, results if the black hole grows away from the center of the dark matter distribution [17, 18].

Thus there is a considerable ambiguity in what the inner dark matter profile could be, all due to the uncertainty in the history of the central region of our Galaxy.

*Scattering off stars sets a quasi-equilibrium profile.*— The above considerations assumed that the phase-space density of dark matter particles is conserved. However, in addition to the supermassive black hole, the Galactic center harbors a compact cluster of stars, with the density $\rho_* = 8 \times 10^6 M_\odot$ pc$^{-3}$ [19] in the inner 0.1" (0.004 pc at the distance of the Sun of 8 kpc). These stars frequently scatter dark matter particles and cause the distribution function to evolve towards an equilibrium solution. Both stars and dark matter experience two-body relaxation.

The idealized problem of a stellar distribution around a massive black hole in star clusters has been considered in the past (cf. [20] for a review). Stars driven inward towards the black hole by two-body relaxation try to reach thermal equilibrium with the stars in the...
core, but are unable to do so because of tidal disruption or capture by the black hole. Unlike core collapse in self-gravitating star clusters, however, the density of inner stars does not grow toward infinity. A steady-state solution is possible where the energy released by removal of the most bound stars is transported outward by diffusion. Because there is no special scale in the problem, the quasi-equilibrium distribution function is a power-law of energy, \( f(E) \propto E^p \), and the density is a power-law of radius, \( \rho \propto r^{-3/2-p} \). The solution is unique and independent of the initial conditions.

Within the sphere of influence of the central black hole \( r_{bh} \), the distribution functions of both stars and dark matter are determined by two-body scattering and have the above power-law form. The scatterers in both cases are stars, but the distribution of dark matter differs in the exponent \( p \) from that of stars because of the vastly different masses of the two species. The evolution of the dark matter distribution \( f(E,t) \) in a two-component system of dark matter particles of mass \( m_\chi \) and stars of mass \( m_* \) can be described by a collisional Fokker-Planck equation (first derived in this form in \( \text{[24]} \)):

\[
-\frac{\partial q}{\partial t} \frac{\partial f}{\partial E} = A \frac{\partial}{\partial E} \left[ \frac{m_\chi}{m_*} \int_E^{\infty} f_* \frac{\partial q_*}{\partial E_*} dE_* \right] + \frac{\partial f}{\partial E} \left\{ \int_E^{\infty} f_* q_* dE_* + q \int_{-\infty}^{E} f_* dE_* \right\},
\]

where \( E = GM_{bh}/r - \frac{1}{2} v^2 \) is the binding energy per unit mass, \( q(E) = (2^{3/2}/3)\pi^3 G^3 M_{bh}^3 E^{-3/2}, A \equiv 16\pi^2 G^2 m_*^2 \ln \Lambda, \) and \( \ln \Lambda = \ln M_{bh}/m_* \approx 15 \) is the standard Coulomb logarithm. The equilibrium distribution function of stars is \( f_*(E_*, t) \propto E_*^{1/4}, \) i.e. \( p = 1/4 \) \( \text{[22]} \).

For dark matter particles, however, the first term in the square brackets vanishes since the particle mass is negligible compared to stellar mass. An equilibrium solution with no energy flux requires \( \partial f/\partial E = 0 \), or \( p = 0 \). The corresponding density profile is \( \rho_{dm} \propto r^{-3/2} \).

Dark matter particles cannot be tidally disrupted by the black hole, but they will be captured from the loss cylinder where their angular momentum per unit mass is less than \( J_{\text{cap}} = 4GM_{bh}/c \) (in which case their minimum distance to the black hole is within the last stable orbit). This would drive the outward flux of energy that needs to be balanced by the inward flux of particles, just as in the stellar case. As long as the relaxation time is shorter than the Hubble time, the quasi-equilibrium solution should be gradually achieved over a large range of energies. Note that this solution would be broken at the inner boundary where the distribution function would smoothly vanish due to the particle loss. The normalization of the profile may also slowly evolve (see \( \text{[37]} \) for discussion).

The observed stellar distribution around the black hole is roughly consistent with the equilibrium slope: \( \rho_*(r) = 1.2 \times 10^6 (r/0.4 \text{ pc})^{-\alpha} M_\odot \text{ pc}^{-3} \) with \( \alpha = 2.0 \pm 0.1 \) at \( r > 0.4 \text{ pc} \) and \( \alpha = 1.4 \pm 0.1 \) at \( r < 0.4 \text{ pc} \). The inner slope of the stellar profile is somewhat shallower than the predicted \( \alpha = \frac{3}{2} + p = \frac{5}{2} \), but the overall profile is consistent with that of a relaxed cluster. The inner profile is well measured down to \( r = 0.004 \text{ pc} \) from the center, where the local relaxation time is \( \text{[25]} \):

\[
t_{\text{rel}} = 0.065 \frac{v^3}{G^2 m_* \rho_* \ln \Lambda} \approx 2 \times 10^9 \text{ yr},
\]

where \( v(r) \approx (GM_{bh}/r)^{1/2} \) is the rms velocity of stars and dark matter, and \( m_* \approx 1 M_\odot \). The density increase towards the black hole almost exactly balances the increase of the particle velocities, and the relaxation rate is roughly independent of radius. If the \( \alpha = 1.4 \) stellar profile continues all the way to the gravitational radius of the black hole, \( r_g \equiv 2GM_{bh}/c^2 = 4 \times 10^{-7} \text{ pc}, \) the relaxation time is shorter than the Hubble time everywhere.

The angular momentum of dark matter particles grows on the average as \((GM_{bh}r)^{1/2}\) and the radius at which it equals \( J_{\text{cap}} = L_{\text{cap}} = 8 r_g \approx 3 \times 10^{-6} \text{ pc} \). However, the extrapolated density at \( L_{\text{cap}} \) is so high that dark matter particles can annihilate faster than they are scattered in. For a typical value of the annihilation cross section \( \sigma_{\text{ann}} \), \( v \sim 10^{-26} \text{ cm}^3 \text{ s}^{-1} \) (c.f. Fig. \( \text{[1]} \)), \( \rho_0 = 100 M_\odot \text{ pc}^{-3} \) (see below), and \( m_\chi \approx 100 \text{ GeV} \text{ c}^{-2} \), the annihilation time equals the relaxation time at \( L \approx 10^{-5} \text{ pc} \). We therefore take this value of \( L \) as the smallest radius at which we can trust our dark matter profile.

Note also that stellar-mass black holes resulting from stellar evolution of ordinary stars would sink towards the center from the inner 5 pc and would form a tight cluster of their own \( \text{[20]} \). This cluster scatters dark matter particles as well and additionally contributes to the relaxation.

**Implications for dark matter searches.**—The dark matter density in the central region of the Galaxy is thus given by

\[
\rho_{dm}(r) = \begin{cases} 
\rho_0 (r/r_{bh})^{-3/2} & L < r \leq r_{bh}, \\
\rho_0 (r/r_{bh})^{-\alpha} & r_{bh} < r,
\end{cases}
\]

where \( L = 10^{-5} \text{ pc} \), and we expect that \( 0 < \alpha < 1.5 \). In order to calculate the flux of gamma rays due to WIMP annihilation near the Galactic center, we need to know the dark matter density \( \rho_0 \) at radius \( r_{bh} = 2 \text{ pc} \). This radius is at least two orders of magnitude smaller than the best currently available dissipationless simulations have reached. Unfortunately, the normalization of the dark matter profile in the central few pc is quite uncertain, since it is affected by various phenomena associated with the baryons which are the dominant mass component interior to the solar radius, \( d_\odot \). A starting assumption is that \( \rho_0 \) is given by extrapolating inward an NFW profile \( \text{[2]} \). For example, the fit of \( \text{[8]} \), corresponding to
\[ \rho_{\text{dm}}(d_{\odot}) = 0.46 \text{ GeV}^2 \text{c}^{-4} \text{ cm}^{-3} \text{ with NFW scale radius parameter } r_s = 27 \text{ kpc}, \text{ implies } \rho_{\text{dm}}(r_{\text{bh}}) \approx 90 \ M_{\odot} \text{ pc}^{-3}. \]

Adiabatic compression of the dark matter due to baryonic infall is likely to result in an increased central density of dark matter. For example, model A1 of the Galaxy in [27], including the effects of baryonic compression, has \( \rho_{\text{dm}}(100 \text{ pc}) = 10 \ M_{\odot} \text{ pc}^{-3} \), with the density increasing inward as \( r^{-1.4} \); even if it scaled from 100 pc to \( r_{\text{bh}} \) only as \( r^{-1} \), this would give \( \rho_{\text{dm}}(r_{\text{bh}}) \approx 560 \ M_{\odot} \text{ pc}^{-3} \).

On the other hand, in model B1 from [27], in which half of the angular momentum of the baryons is assumed to be transferred to the dark matter, \( \rho_{\text{dm}}(100 \text{ pc}) \) is approximately 5 times lower than in model A1. In addition, if the black hole formed via mergers of the lower-mass black holes during the early stages of galaxy formation, the density may be reduced further [18]. For definiteness, we take \( \rho_0 = \rho_{\text{dm}}(r_{\text{bh}}) = 100 \ M_{\odot} \text{ pc}^{-3} \) as our fiducial value, although it is likely that baryonic infall increased this.

The flux at the earth from a region of angular radius \( \sigma_0 = 0.05^\circ \), subtending solid angle \( \Delta \Omega = \pi \sigma_0^2 \), centered on the black hole at the center of the Galaxy is then

\[
\text{Flux} = \frac{N_\gamma (\sigma_{\text{ann}} v) I}{2d_{\odot}^2 m^2_{\chi}}.
\]

Here the number of gamma rays above the threshold energy \( E_{\text{th}} \) of the detector is approximately [30]

\[
N_\gamma = \frac{5}{6} x^{3/2} - \frac{10}{3} x + 5x^{1/2} + \frac{5}{6} x^{-1/2} - \frac{10}{3},
\]

where \( x \equiv E_{\text{th}}/m_{\chi} c^2 \) and \( m_{\chi} \) is the WIMP mass. Assuming \( E_{\text{th}} = 50 \text{ GeV} \) and \( m_{\chi} = 100 \text{ GeV} \), \( x = 0.5 \) and \( N_\gamma = 0.0087 \). Taking the distance of the sun from the center of the Galaxy to be \( d_{\odot} = 8 \text{ kpc} \) [31] and assuming \( \alpha = 1 \) for definiteness,

\[
\frac{I}{d_{\odot}^2} = \int_{L}^{\sigma_0 d_{\odot}} \rho_{\text{dm}}^2 \left( \frac{r}{d_{\odot}} \right)^2 dr = \rho_0^2 d_{\odot}^2 \left[ r_{\text{bh}} \ln \frac{r_{\text{bh}}}{L} + (\sigma_0 d_{\odot} - r_{\text{bh}}) \right] = 0.73[24.4 + 5.2] \text{pc} \left( \frac{\rho_0}{100M_{\odot}\text{pc}^{-3}} \right)^2 \text{ GeV}^2 \text{c}^{-4}\text{cm}^{-6}.
\]

The minimum detectable WIMP annihilation cross section times velocity is then

\[
\langle \sigma_{\text{ann}} v \rangle_{\text{min}} = \frac{2 M_s m_{\chi}^2 N_{\text{fcg}}^{1/2}}{N_s A_{\text{eff}} \theta d_{\odot}^2}.
\]

For an ACT with \( E_{\text{th}} = 50 \text{ GeV} \), \( N_{\text{fcg}} = (3.7 + 7.9) \times 10^{-6} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \ A_{\text{eff}} t \Delta \Omega \), where the first term in the parenthesis is the electron-induced background and the second is the hadronic background. Since it is expected that the latter can be significantly reduced for the new generation of ACTs [3, 32], we only include the electron-induced background. Then for a detection of statistical significance \( M_s \sigma \) (taking \( M_s = 3 \) as our fiducial value),

\[
\langle \sigma_{\text{ann}} v \rangle_{\text{min}} = 3.2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \left( \frac{M_s}{3} \right) \left( \frac{m_{\chi}}{100 \text{ GeV} \ c^{-2}} \right)^2 \left( \frac{0.0087}{N_\gamma} \right) \left( \frac{A_{\text{eff}}}{10^5 \text{cm}^2} \right)^{-1/2} \left( \frac{t}{250 \text{ hr}} \right)^{-1/2} \left( \frac{\rho_0}{100 M_{\odot} \text{pc}^{-3}} \right)^{-2}.
\]

The solid curve in Figure 1 shows this detection threshold as a function of the WIMP mass \( m_{\chi} \) for our fiducial assumptions. The dependence on \( m_{\chi} \) here comes entirely from \( m_{\chi}^2 / N_{\gamma} \), but a more realistic calculation would take into account the increasing \( A_{\text{eff}} \) for higher gamma ray energy. In addition, the significance of the detection would be increased by including the gamma rays from the full field of view of the ACT, which is \( 5^\circ \) for the H.E.S.S. ACT array in Namibia [33], where the Galactic center passes nearly overhead. The dashed curve shows an order of magnitude improved sensitivity from baryonic compression increasing \( \rho_0 \) by a modest factor of 10^{1/2}. Since at least that much baryonic compression is rather likely [35], it is quite plausible that annihilation from the Galactic center will be detected by new generation ACTs (H.E.S.S., CANGAROO III, MAGIC, and VERITAS) if the dark matter is actually WIMPs of mass \( m_{\chi} \gtrsim 100 \text{ GeV} \).
The best strategy for detecting WIMPs of lower mass probably involves using the all-sky map that will be produced by the Gamma Ray Large Area Space Telescope (GLAST) satellite to look for the wide angle annihilation flux away from the Galactic center, which can be reliably calculated using dissipationless simulations since it is hardly affected by uncertain physics such as baryonic compression. The recent reanalysis of the EGRET data toward the Galactic center using a gamma ray energy dependent point spread function already constrains some SUSY models with low-mass WIMPs, subject to the uncertainties we have discussed about the dark matter density toward the Galactic center.

To summarize, if the WIMP mass $m_\chi \gtrsim 100 \text{ Gev } e^{-2}$ and the central density is only a little higher than our fiducial value $\rho_0 = 100 \ M_\odot \text{ pc}^{-3}$ due to baryonic compression, then the high angular resolution of the new ACTs such as the H.E.S.S. array should permit detection of the $\rho_{\text{dyn}} \propto r^{-3/2}$ central cusp that is inevitably associated with the dense star cluster around the central black hole.

We would like to thank A. Kravtsov for organizing the workshop at the Center for Cosmological Physics in Chicago in August of 2002 which motivated this work. OYG is supported by STScI Institute Fellowship. JRP is supported by grants NASA-NAG5-12326 and NSF AST-0205944 at UCSC, and he also thanks L. Stodolsky and M. White for hospitality in Munich. We enjoyed discussions with S. White, A. Klypin, F. Prada, and F. Stoehr, and we especially thank Stoehr for giving us the output from his run of DarkSUSY used in Figure 1. We thank the referee D. Merritt for comments that helped us improve the paper.