Neutron Star Vortex Dynamics and Magnetic Field Decay: Implications for High Density Nuclear Matter

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Abstract

We investigate the effect of the density-dependent proton and neutron gaps on vortex dynamics in neutron stars. We argue that the persistence of neutron star magnetic fields on timescales of $10^9$ y suggests a superconducting gap curve with local maximum at intermediate density. We discuss the implications for exotic core phenomena such as pion/kaon condensation or a transition to quark matter.

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In this letter we address the evolution of magnetic fields in neutron stars, in particular the distribution of magnetic vortices inside the star. Residual magnetic fields are believed to persist over very long timescales (\(\sim 10^9\) y) in neutron stars. While naively attributed to the confinement of magnetic flux into vortices (henceforth, flux vortices or FVs) due to proton superconductivity [1], the phenomena is more involved, and may involve the interaction of FVs with neutron superfluid vortices (henceforth, SVs) [2]. Here we will argue that a prerequisite for the persistence of magnetic fields, as well as for the applicability of models in [2], is that the proton gap curve \(\Delta_p\) have a certain shape as a function of density within the neutron star. The point is that the density-dependent proton gap leads to a force which acts on FVs. At low densities (in the outer core), this force will always act to eject vortices into the non-superconducting crust. A simple calculation shows that this proton gap force dominates any vortex buoyancy effects [3], and leads to ejection on timescales of \(\sim 10^6\) y. However, if the proton gap decreases at higher densities after reaching a local maximum at some intermediate density, the sign of the force will reverse and act to anchor vortex segments to the core of the neutron star [4, 5]. We will argue that without this effect, interactions between pinned SVs and FVs are insufficient to prevent FV ejection.

The phenomenology of magnetic fields in neutron stars has long been of interest to those studying pulsar glitches [6], and has recently been given a prominent role in the magnetar model of local gamma ray bursters [7]. Our main interest here will be in the dynamics of fluxoids deep within the core of the neutron star, in particular the forces which act to either anchor or expel them. We will conclude with a discussion of the implications of our work on exotic states of matter in neutron stars.

Below we list some neutron star properties of relevance to our analysis

- Neutron star structure: In the outer layer of thickness \(\sim 1\) km, a lattice of neutron-rich nuclei is surrounded by a neutron superfluid. As the density increases, conversion of protons and captured electrons into neutrons becomes more efficient, and eventually the proton and electron fraction becomes of order a few percent, sufficient to prevent neutron decay by Fermi blocking. The neutron superfluid order parameter (see [8] for recent computations) is initially in the \(^1S_0\) channel, but probably shifts to the \(^3P_2\) channel at higher density, due to the repulsive core of the neutron-neutron potential. The gap size is of order 1 MeV. A proton gap of similar size, leading to superconductivity, is also expected in the core region. Due to uncertainties in the equation of state at high density, the maximum core density is unknown. Various exotic phenomena such as pion [9] or kaon [10] condensation, or even a transition to quark matter [11] may occur deep in the core. We note that in all of these scenarios a superconducting gap which is larger than the proton gap is to be expected. (See [12] for recent progress on the quark color-superconducting gap.)
• Superfluid vortices (SVs) carry the star’s angular momentum in quantized lines parallel to the spin axis. They have an area density

\[ n_{SV} = 2m_n \Omega / \pi \sim 10^4 / P(\text{sec}) \text{ cm}^{-2} . \tag{1} \]

Because of the strong coupling between neutrons and protons, the circulation of neutrons leads in turn to circulation of protons, and the SVs are themselves expected to carry magnetic fields.

• Magnetic flux vortices (FVs) are the result of the type II superconductivity in the inner crust and core region. The magnetic field of the star is confined into individual vortices of flux \( \Phi_0 = \pi / e = 2 \cdot 10^{-7} \) Gauss/cm\(^2\). The number density of such vortices is

\[ n_{FV} = B / \Phi_0 \sim 10^{19} B_{12} \text{ cm}^{-2} , \tag{2} \]

where \( B_{12} = B / 10^{12} \) Gauss. Note that the density of FVs is enormously larger than that of the SVs.

Now let us consider the effect on vortex dynamics of the shapes of the relevant gap curves. Because the string tension (energy per unit length) of a vortex behaves as

\[ \mu = c \Delta^2 , \tag{3} \]

(where \( c \) is a dimensionless constant of order 1) there is a force per unit length exerted on the vortex due to the variation of the gap with radial position (density) within the star:

\[ f_\Delta = 2c \Delta \frac{\partial \Delta}{\partial r} \hat{r} . \tag{4} \]

The magnitude of this force per unit length is of order

\[ f_\Delta \sim \text{MeV}^2 / R , \tag{5} \]

where \( R \) is the characteristic length scale over which the gap varies. For the FV gap \( R \sim R_{NS} \sim 10^4 \) m, while for the \( ^1S_0 \) superfluid gap \( R \sim 10^3 \) m. Comparing with the buoyancy effect of Muslimov and Tsygan [3], we see that this effect is of similar but somewhat larger size.

In the region where \( \Delta \) is increasing with density, the force will act to expel vortices. The characteristic time for this to occur depends on the drag force exerted on the vortex due to interactions with leptons (at high densities there may be muons present). Since the protons and neutrons form a superfluid their contribution to the drag is negligible. The lepton drag force has been considered in some detail by Jones [5], and is of the order

\[ f_{\text{drag}} \sim \text{MeV}^3 v_{\text{vortex}} . \tag{6} \]
Using this result, the terminal velocity can be found and therefore the expulsion time, which is \( \tau_\Delta \sim 10^6 \) y for magnetic vortices. Once a vortex has been expelled into the outer crust, the magnetic field can decay by ohmic dissipation on timescales of \( \tau_\omega \sim r_c^2 \sigma \sim 10^7 \) y, where \( r_c \) is the crust thickness and \( \sigma \) the conductivity. These timescales are inconsistent with the observed persistence of magnetic fields of order \( 10^9 - 10^{10} \) G in millisecond pulsars with ages \( 10^9 \) y.

In regions where the gap decreases with increasing density, the force acts to pull the vortex deeper into the star. For example, in the case of a superfluid vortex, the \( ^1S_0 \) gap falls off after reaching its maximum at a Fermi momentum \( p_F \sim 150 \) MeV. In this region an SV is pulled toward the center of the star, until the sign of the gradient switches again. The case of superfluid vortices is complicated, because the superfluid order parameter switches from \( ^1S_0 \) to \( ^3P_2 \) at \( p_F \sim 300 \) MeV. In addition, because SVs also carry magnetic fields, they are also affected by the proton gap force gradient. In figure 1 we show the likely behavior of the neutron and proton gap functions. The leftmost curve shows the likely behavior of the \( ^1S_0 \) superfluid gap, while the two curves on the right display possibilities for the superconducting gap. We will refer to the upper curve, which increases monotonically with density, as curve 1, and the lower curve as curve 2. Superfluid vortices can minimize their energy in the region where the superfluid and superconducting gap curves intersect. The evolution of FVs depends crucially on the shape of the \( \Delta_p \) curve. If there is no local maximum (as shown in curve 1), then all FVs will eventually be ejected from the star. Alternatively, if curve 2 is correct then FVs with sufficient length in the attractive core will be anchored against ejection (see figure 2). Some sub-population of the FVs could presumably remain indefinitely.

One might think that the interactions between FVs and SVs, or their respective pinning to the crust, might be enough to prevent FV ejection even in the case of curve 1. However, since the number of FVs is so much greater than the number SVs, they will either carry the SVs along in their motion, or cut through them on their way to the surface. (Note that intercommutation of vortices is highly efficient \cite{13}, so if a vortex line is cut through it will almost always reconnect with itself afterwards.) The crustal pinning force on an SV is less than of order MeV\(^2\), so it is easily overcome by the combined force exerted by \( f_\Delta \) through \( \sim n_{FV}/n_{SV} \) flux vortices, each of order \( R_{NS} \) in length. The total force exerted on a single SV is

\[
F_\Delta \sim \frac{n_{FV}}{n_{SV}} \text{MeV}^2,
\]

which completely dominates any restraining effects on the SV.

\footnote{Some calculations, such as that of Sang and Chanmugam \cite{14}, have obtained timescales for ohmic decay which are larger than the usual estimate. However, it is important to note that the mechanism described here ejects the magnetic fields into the outer crust (\( \rho < 10^{12} \text{g/cm}^3 \)), where the conductivity is lower and where even the calculations of \cite{14} yield decay timescales of order \( 10^7 \) yr.}
The general form of curve 2 in figure 1 is to be expected from standard calculations, given that pp interactions are attractive at long distances and have a repulsive core. Of course, medium effects due to the large density of neutrons will be important and are difficult to account for. The particular values of curve 2 were obtained using the Fermi surface effective field theory technique of [8], using experimentally determined pp phase shifts and the beta-stability condition to determine the proton density relative to the neutron density. The result should be accurate at lower densities, but the eventual behavior of the curve (i.e. curve 1 vs curve 2) is subject to large uncertainties. We have argued that the long time persistence of pulsar magnetic fields favors case 2.

As previously mentioned, many of the exotic possibilities for the inner core behavior (pion or kaon condensation, quark matter) imply superconducting gaps larger than of order 1 MeV, due to condensation of electrically charged degrees of freedom: $\pi^\pm$, $K^+$ or a diquark pair, at densities of several times $\rho_0 = 2 \cdot 10^{14} \text{g/cm}^3$. In the case of quark matter [12], the gap size is expected to be at least 10 MeV, and perhaps as large as 50-100 MeV. This would be hard to reconcile with curve 2. The transition from normal matter to exotic phase would have to occur at sufficiently high density (i.e. at the far right of figure 1) to allow for a
region in the star which remains attractive to FVs. The maximum of the proton gap curve in figure 1 is already at a density of $\approx 2\rho_0$ (and density increases with the cube of Fermi momentum), so this at most leaves room for a thin shell of attractive volume. We conclude that exotic phases (if they occur at all) (1) can only occur at very high density ($> (\text{few})\rho_0$) and (2) will occupy at most only a small fraction of the volume of the star.

Figure 2: Two magnetic vortices, one destined to be expelled, the other attached to the core.

In summary, we have argued that the proton gap curve is likely to exhibit a local maximum at intermediate density, implying a region at higher density which traps flux vortices and disfavoring an exotic phase at the core. Vortices which are formed with insufficient length in this region will be ejected on timescales of order $\tau_\Delta \sim 10^6$ y, and decay in the outer crust. As mentioned, the asymptotic values of neutron star magnetic fields are estimated to be less than of order $10^{10}$ G, compared to $10^{12}$ G or more at formation. It is not known whether the decay of the magnetic field is due to accretion or flux decay. If the cause is flux decay, it would imply that in any (young) neutron star the ejection process is under way, with some FVs being pushed into the crust at all times. It is not clear what the phenomenological implications of this are, although the presence of large magnetic fields confined to the outer crust presumably leads to significant crustal stresses and perhaps starquake activity.

Another issue worth considering is the fate of SVs if they are carried along in the expulsion of FVs to the surface of the star. This may lead to spin down which is correlated to the decay of the magnetic fields. While the causality is different, the phenomenology might resemble that of models in which magnetic field decay is caused by the flow of SVs during spin down [6].
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