Nonperturbatively Improved Hadron Spectroscopy Near the Continuum Limit

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We report the results of our quenched lattice simulations of the Wilson action with a nonperturbatively determined clover term at $\beta = 6.2$ and compare them with those of the standard Wilson action at the same $\beta$ value.

Simulations in full QCD are extremely slow, and are still carried out far from the continuum limit. A possible solution for this problem is the use of improved actions, for which the simulation gives the continuum limit at higher values of the cutoff $a$, reducing computational costs. An important step in the improvement program was taken recently by the Alpha collaboration, with the nonperturbative determination of the coefficient $c_{SW}$ in the clover action $[1,2]$. We present here the results of our study of hadron spectroscopy using the clover-improved Wilson action (see $[1]$ for notation)

$$S = S_G + S_W + c_{SW} a^5 \sum_x \overline{\Psi(x)} \sigma_{\mu\nu} \hat{F}_{\mu\nu} \Psi(x).$$

The nonperturbative expression of $c_{SW}$ is given in terms of the coupling $\beta$ in $[2]$. We consider lattice volume $24^3 \times 48$ and coupling $\beta = 6.2$. We thus take $c_{SW} = 1.61375065$. We also consider the unimproved case ($c_{SW} = 0$) for comparison. For the improved case we consider the following values for the hopping parameter $\kappa$: 0.1240, 0.1275, 0.1310, 0.1340, 0.1345, 0.1350, 0.1352. For the $c_{SW} = 0$ case we consider: 0.1350, 0.1400, 0.1450, 0.1506, 0.1510, 0.1517, 0.1526. The simulations were carried out on the APE100 computer at Tor Vergata.

Our statistics come from 50 quenched gauge configurations, generated by a hybrid overrelaxation algorithm with each update corresponding to a heat-bath sweep followed by 3 overrelaxation sweeps. The configurations are separated by 1000 updates.

The inversion of the fermion matrix is performed using the stabilized biconjugate gradient algorithm $[3]$. We restart the inversion from the current solution every 100 iterations, in order to reduce accumulation of roundoff errors $[4]$. We employ point-like sources.

We sum fermion propagators over the space directions $x, y, z$ for sites within blocks of side 3, and then store the result. We then form hadron correlations from these “packed” propagators. This procedure becomes exact in the limit of an infinite number of configurations, since it corresponds to including gauge-noninvariant terms in the computation of hadron correlations (in our case we have checked that the errors thus introduced are negligible). This corresponds to gaining a factor $3^3$ in storage, and has enabled us to have all quark propagators stored simultaneously. In this way we can form hadron correlations from nondegenerate combinations of quark flavors, as done in $[3]$.

Within the improvement program one can determine the critical value $\kappa_c$ using the unrenormalized current quark mass or “Ward identity” mass, defined through the improved PCAC relation as $[3]$

$$m_{WI} \equiv \frac{<\partial_\mu \{A_\mu^{(bare)} + c_A a \partial_\mu P^{(bare)}\} O>}{2 <P^{(bare)} O>}. $$

Linear extrapolation to the limit of $m_{WI} = 0$ provides a much more stable fit for the determi-
nation of $\kappa_c$ than the conventional fit of pseudoscalar meson masses to the limit of zero pion mass. Our results for $\kappa_c$ are given below.

**improved case:**
- from $m_{W_I} = 0$ $\kappa_c = 0.135802(6)$
- from $M_{PS}^2 = 0$ $\kappa_c = 0.135861(19)$

**c_{SW} = 0 case:**
- from $M_{PS}^2 = 0$ $\kappa_c = 0.153307(19)$

Once the value for $\kappa_c$ has been determined, one can obtain chiral extrapolations from plots of hadron masses as linear functions of the bare quark mass, defined as $m_q(\kappa) \equiv (1/\kappa - 1/\kappa_c)/2$, or as linear functions of the measured squared pseudoscalar mass $M_{PS}^2(\kappa)$. In the improved case one can define the modified or “improved” bare quark mass $\tilde{m}_q$ by $\tilde{m}_q(\kappa) \equiv m_q(\kappa) [1 + b_m m_q(\kappa)]$ (Note that $\tilde{m}_q$ is the renormalized mass with $Z_m = 1$) The improvement coefficient $b_m$ has been determined nonperturbatively from $b_m = -0.60$. Note that since we have nondegenerate flavor combinations we use averages of the masses defined above, e.g. for a meson corresponding to the flavors $\kappa_1$ and $\kappa_2$ we define $m_q(\kappa_1, \kappa_2) \equiv [m_q(\kappa_1) + m_q(\kappa_2)]/2$.

| $a^{-1}$ from | IMPROVED | $c_{SW} = 0$ |
|----------------|----------|--------------|
| $M_V$         | 2475(231) | 2759(223)    |
| $M_K^{(I)}$   | 2526(122) | 2839(115)    |
| $M_K^{(II)}$  | 2575(42)  | 2915(39)     |
| $M_\phi^{(I)}$| 2624(54)  | 2945(44)     |

Table 1: Values of $a^{-1}$ in MeV.

| $J_{K^*}$ | IMPROVED | $c_{SW} = 0$ |
|-----------|----------|--------------|
| 0.487     | 0.40(14) | 0.40(9)      |
| $J_\phi$ | 0.557    | 0.44(10)     | 0.45(10)     |

Table 2: Values of the quantity $J$ and comparison with experiment.

In Table 3 we give our values of the inverse lattice spacing $a^{-1}$ coming from ratios of the vector meson mass $M_V$ over its experimental value at different flavor combinations. We determine the strange-quark mass $m_s$ in lattice units from two methods: a fit of $(M_{PS}/M_V)^2$ (method I) and a fit of $(M_{PS}/M_V)^2$ (method II) as functions of the quark mass, interpolating to the experimental values of these ratios. (Here $M_{PS}^2$ denotes the chiral extrapolation of $M_V$.) For the improved case we use $\tilde{m}_q$ as the bare quark mass, while in the unimproved case we use $m_q$. In Table 2 are the values of the quantity $J \equiv M_V (dM_V/dM_{PS}^2)$ obtained from $(M_K/M_{K^*})^2$ vs. $M_{PS}^2$.

All our baryon mass values for the improved case come from linear fits of the masses as functions of the improved quark mass $\tilde{m}_q$, except for the $\Sigma - \Lambda$ mass splitting, where we used a quadratic fit. (Similarly for the $c_{SW} = 0$ case, using as variable $m_q$.)

Note that we have defined the quantity $M_{Oct} \equiv$
\((\mathcal{M}_N + \mathcal{M}_\Lambda)/2\) (this combination was chosen so that the resulting mass is flavor-symmetric). For this quantity the advantage of using the improved quark mass is clearest, see Fig. 1. (A similar improvement is observed also with respect to plots using the unimproved bare quark mass \(m_q\).)

We show an APE plot in Fig. 2 for the improved case. Note that we divide at each point by \(M_V\) interpolated to the strange-quark mass, corresponding to \(M_\Phi\) (i.e. a constant value). Experimental points in these figures correspond to \(M_N\), \(M_{\Sigma}\) and \(M_{\Xi}\) and appropriate meson masses. In Table 3 we give our mass values in MeV. Table 4 contains data in lattice units for the improved case (diagonal flavor combinations only).

| Particle | Exp. value | \(c_{SW} = 0\) | IMPROVED |
|----------|------------|----------------|----------|
| \(M_N\) | 939        | 977 (115)      | 953 (112) |
| \(M_\Lambda\) | 1115.7 | 1142 (113)      | 1127 (115) |
| \(M_\Delta\) | 1232 | 1382 (172) | 1307 (160) |
| \(M_{\Delta^+}\) | 1132.35 | 1138 (113) | 1141 (113) |
| \(M_{\Delta^-}\) | 293 | 381 (90) | 366 (86) |
| \(M_{\Sigma^+}\) | 232.45 | 320 (70) | 295 (66) |
| \(M_{\Sigma^-}\) | 73.7 | 50 (39) | 59 (25) |

Table 3: Baryon masses in MeV and comparison with experiment.

| \(\kappa\) | \(M_{PG}\) | \(M_V\) | \(M_N\) |
|------------|-----------|--------|--------|
| 0.1240     | 1.15951   | 1.1044 (18) | 1.7046 (54) |
| 0.1275     | 0.73101   | 0.8922 (19) | 1.3821 (50) |
| 0.1310     | 0.367794  | 0.6595 (22) | 1.0220 (55) |
| 0.1340     | 0.117867  | 0.4379 (42) | 0.6588 (91) |
| 0.1345     | 0.083452  | 0.3995 (59) | 0.5929 (100) |
| 0.1350     | 0.051696  | 0.3660 (133) | 0.5169 (141) |
| 0.1352     | 0.039728  | 0.3543 (225) | 0.4859 (177) |

Table 4: Masses in lattice units.

With our present statistics the main effect of the nonperturbative improvement observed on hadron masses is a smaller spread in the value of the lattice spacing extracted from mesons with and without strange quarks. We have also shown how the use of the improved quark mass turns the rough behavior of the dependence of the octet baryon mass upon quark masses into a smooth one.

We are currently increasing our statistics and we plan to extend our simulations to the case with bermions, an approximate method that allows for an estimate of dynamic quark effects.

REFERENCES

1. M. Lüscher, S. Sint, R. Sommer and P. Weisz, Nucl. Phys. B478, 365 (1996).
2. M. Lüscher, S. Sint, R. Sommer, P. Weisz and U. Wolff, Nucl. Phys. B491, 323 (1997).
3. A. Frommer, V. Hannemann, B. Nockel, T. Lippert and K. Schilling, Int. J. Mod. Phys. C5, 1073 (1994).
4. G. Cella, A. Hoferichter, V.K. Mitrjushkin, M. Müller-Preussker and A. Vicere, Int. J. Mod. Phys. C7, 787 (1996).
5. T. Bhattacharyya, R. Gupta, G. Kilcup and S. Sharpe, Phys. Rev. D53, 6486 (1996).
6. G.M. de Divitiis, talk given at this conference.
7. A.Cucchieri et al, in preparation.
8. G.M. de Divitiis, R. Frezzotti, M. Guagnelli, M. Masetti, and R. Petronzio, Phys. Lett. B387, 829 (1996).

Figure 2. APE plot for the nucleon mass.