6+ isomers in neutron-rich Sn-isotopes beyond N= 82 and effective interaction

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Recent observation of the 6+ seniority isomers and measurements of the B(E2) values in the Sn isotopes lying close to the neutron drip line have raised some questions about the validity of the presently used effective interactions in the neutron-rich region. Simpson et al. [Phys. Rev. Lett. 113, 132502 (2014)] had to modify the diagonal and non-diagonal νf2/2 two body matrix elements of the “Vlk” interaction by ~ 150 keV in their shell model calculations in order to explain the data of 136Sn. In contrast, we are able to explain the observed energy levels and the B(E2) values after marginal reduction of the same set of matrix elements by 25 keV in the “RCDB” (Renormalized CD-Bonn) interaction. The observed mismatch in reproducing the data of 136Sn is due to the seniority mixing. Further, we do not find it necessary to consider the core excitations, and the “RCDB” interaction seems better suited to explain the data beyond N= 82 magic number.

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I. INTRODUCTION

The seniority isomers 1 in the semi-magic nuclei present a crucial testing ground for the large scale shell model calculations as well as the effective interactions. The Z= 50 Sn-isotopes, known from 100Sn to 134Sn, have been the focus of a large number of recent studies. In this paper, we focus upon the neutron-rich Sn-isotopes beyond the N= 82 closed shell, where new data have recently become available in 136,138Sn 2. More specifically, the 6+ isomers in 134–138Sn-isotopes present an interesting set of data to test the validity of the effective interactions in neutron-rich systems close to the drip line. Some of the earlier works in the Sn-isotopes beyond A= 132 are due to Hoff et al. 3 in 133Sn, Zhang et al. 4, Korgul et al. 5 and Beene et al. 6 in 134Sn. Theoretical works on the effective interactions and the shell model calculations beyond N= 82 have also been reported by Kartamyshev et al. 7, Sarkar and Sarkar 8, and Covello et al. 9.

In a recent experiment, Simpson et al. 2 have populated the 136,138Sn-isotopes by using the RIBF facility at RIKEN. They have reported measurements of the low-lying energy levels as well as the life-times of the 6+ isomers. They have combined the earlier studies on 134Sn, carried out by Zhang et al. 4, Korgul et al. 5 and Covello et al. 6, with their studies, and discussed the properties of the 6+ isomers in the Sn-isotopes beyond the doubly magic 132Sn. They have also reported the results of large scale shell model calculations by using the “OSLO” code 10 along with the “Vlk” interaction 11. The calculations were able to reproduce the measured B(E2) values reasonably well, except for 136Sn, where the calculated B(E2; 6+ → 4+ 1) value between the yrast 6+ and 4+ levels differs from the experimental value by a factor of more than 10. Their calculated B(E2) value for 136Sn is very small and close to the value obtained from the pure ν= 2 seniority scheme. They also found a mismatch between the calculated and the measured level energies of the yrast 2+, 4+ and 6+ states for the 134–138Sn-isotopes. Further, they could not reproduce the measured B(E2; 6+ → 4+ 1) value of 136Sn, even when the core excitations were included.

Simpson et al. 2 were, however, able to explain the measured B(E2) value of 136Sn after reducing the diagonal and non-diagonal νf2/2 matrix elements by ~ 150 keV. This has the effect of lowering the second 4+ state (seniority ν= 4) by 250 keV, bringing it much closer to the yrast 4+ state (seniority ν= 2). As a result, the two 4+ states acquire almost 50% mixing in seniority. This modified interaction also improves the level energies for all the three Sn-isotopes. The authors 2 conclude that further theoretical work is needed for constructing the effective interactions in the neutron-rich region. This has motivated us to test the new experimental data with other effective interactions in this region.

We have, therefore, carried out large scale shell model calculations by using the Nushell code of Brown and Rae 12 along with the “RCDB” interaction 13, sometimes also referred as the “CWG” interaction. We find that we are able to explain the observed energy levels and the B(E2) values quite well after reducing the νf2/2 matrix elements by 25 keV only in the “RCDB” interaction. Our results with the modified interaction may be helpful in further works in this direction. We present the details of the calculations in section II and the results in section III. The last section concludes the present work.

II. CALCULATIONS

We have used the Nushell code 12 along with the “RCDB” interaction 13, which has been modified for the latest neutron single particle energies 14, for calculating the spectroscopic properties of 134–138Sn. This interaction assumes the doubly-closed Z= 50, N= 82, 132Sn as an inert core and has been optimized for Z= 50 – 82 and

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The single particle energies for these neutron orbitals
\[ \nu_{2f} \] and \[ \nu_{N=82}^{-} \] respectively. These Sn-isotopes have 2\( ^{-} \) neutrons so that only the neutron-neutron part of the interaction plays the major role. We have used the effective neutron charge to be 0.65e similar to the value used by Simpson et al. [2].

Our calculations are able to reproduce the experimental \( B(E2; 6^{+} \rightarrow 4_{1}^{+}) \) values for \( ^{134}\text{Sn} \) and \( ^{138}\text{Sn} \) reasonably well, but fail for \( ^{136}\text{Sn} \). Our calculated \( B(E2) \) value for \( ^{136}\text{Sn} \) differs from the experimental value by a factor of less than 2. This, however, confirms the claim of ref. 2 that there is indeed a problem in explaining the results of \( ^{136}\text{Sn} \).

Following the prescription of Simpson et al. [2], we also decided to reduce the \( \nu_{f_{7/2}}^{2} \) diagonal and non-diagonal matrix elements. We find that it is possible to obtain very good results for both the level energies as well as the \( B(E2) \) values of \( ^{134-138}\text{Sn} \) after reducing the \( \nu_{f_{7/2}}^{2} \) matrix elements by a small amount of 25 keV only. The calculated \( B(E2) \) values now match extremely well with the experimental data. We denote these calculations with the modified interaction as “RCDBMO”. The agreement of the calculated level schemes with the experimental levels also improves with “RCDBMO” as compared to the unmodified “RCDB” interaction.

### III. RESULTS AND DISCUSSION

We have plotted in Fig. 1 the calculated and the experimental level energies for the yrast 0\( ^{+} \), 2\( ^{+} \), 4\( _{1}^{+} \) and 6\( ^{+} \) states in \( ^{134-138}\text{Sn} \)-isotopes. The second 4\( _{2}^{+} \) state has also been shown only for \( ^{136,138}\text{Sn} \) and lies above the yrast 4\( _{1}^{+} \) state. All the energies are in keV. The \( B(E2) \) values in units of \( e^{2} fm^{4} \), shown in *italics*, have been rounded off.

FIG. 1: (Color online) Energy level schemes for \( ^{134-138}\text{Sn} \). The experimental data shown on the left, are taken from ref. 2 and results of the shell model calculations with “RCDB” interaction and a modified “RCDB” (“RCDBMO”) interaction (see text for details) are shown on the right. The second 4\( _{2}^{+} \) state has been shown only for \( ^{136,138}\text{Sn} \) and lies above the yrast 4\( _{1}^{+} \) state.
We have also plotted in the last columns of Fig. 1 the calculated level energies obtained by using the modified interaction “RCDBMO”, wherein the diagonal and non-diagonal $\nu f_{7/2}$ matrix elements have been reduced by 25 keV. We find that the modified interaction “RCDBMO” significantly improves the agreement between the calculated and the experimental level energies for $^{136,138}$Sn, while it leaves the levels of $^{134}$Sn almost unchanged. The second $4^+_2$ state in $^{136}$Sn comes down in energy by 41 keV and lies below the yrast $6^+$ state. On the other hand, the second $4^+_2$ state in $^{138}$Sn goes up in energy and lies above the yrast $6^+$ state.

We have plotted in Fig. 2 the effective single particle energies (ESPE) of the valence neutron orbitals from valence neutron number 0 to 8 for the “RCDB” interaction. The $f_{7/2}$ orbital is also plotted for the “RCDBMO” interaction. The calculations have been done by using the relation

$$ESPE = E_{jn} + \sum_{jn} E(j_n, j_n)\hat{a}_{jn}$$

where $E_{jn}$ and $\hat{a}_{jn}$ denote the single-particle energies and number of neutrons occupying the orbital $j_n$. The term $E(j_n, j_n)$ represents the monopole corrected interaction energy which has been averaged over the total angular momentum $J$, and is given by:

$$E(j_n, j_n) = \sum_{j} (2J + 1) \frac{<J_n J_n; J|V|J_n J_n; J>}{\sum_{j} (2J + 1)}$$

where $<J_n J_n; J|V|J_n J_n; J>$ stands for a two-body matrix element of the effective interaction. On using the modified interaction “RCDBMO”, the ESPE of the $f_{7/2}$ orbital get reduced by an amount which varies linearly with the particle number, while the ESPE of rest of the orbitals remain the same. The small change in the energy of $f_{7/2}$ orbital plays a key role in deciding the calculated level energies of the yrast states in these nuclei. Even with this small change, our calculated energies come closer to the experimental ones.

We have also calculated the B($E2$) values for the transition between the yrast ($6^+ \rightarrow 4^+_1$) states for $^{134-138}$Sn isotopes and the same are plotted in Fig. 3. The B($E2$’s for all the transitions between the yrast states have also been calculated and the rounded off values are shown in italics in Fig. 1. As already pointed out, our calculated B($E2$) values with the “RCDB” interaction reproduce the measured ones except for $^{136}$Sn isotope, where a deviation by a factor $< 2$ is noticed. We have also calculated the B($E2$; $6^+ \rightarrow 4^+_1$) value in $^{136}$Sn-isotope with both the “RCDB” and “RCDBMO” interactions, and the rounded off values have been included in Fig. 1.

Semi-magic nuclei have been shown to be good candidates for the seniority isomerism because the $E2$ transition probability between the same seniority states become very small or vanish when the valence shell is close to half-filled. This is due to the fact that the matrix elements of even tensor operators between states with the same seniority vanish at the mid-shell. Therefore, the B($E2$; $6^+ \rightarrow 4^+_1$) between the yrast $6^+$ and $4^+_1$ states, and B($E2$; $4^+_1 \rightarrow 2^+$) between the yrast $4^+_1$ and $2^+$ states should diminish for $^{136}$Sn, a mid-shell nucleus, if these states have pure seniority $v = 2$. Our calculated nonzero B($E2$) values for both the decays confirm that a mixing of $v = 2$ and $v = 4$ seniority is already present in $^{136}$Sn even with the unmodified “RCDB” interaction. In comparison, these states appear to have a pure $v = 2$ seniority in the work of Simpson et al. As a result, our calculated B($E2$) value in $^{136}$Sn is off by a factor $< 2$, while Simpson et al. find it off by a factor $> 10$.

![FIG. 2: (Color online) Evolution of effective single particle energies of different orbitals with increasing neutron number for the “RCDB” interaction. The $f_{7/2}$ orbital is also plotted for the “RCDBMO” interaction. $^{132}$Sn is taken as the core.](image-url)
error bars are taken from \[2, 4\]. Calculated values for both “RCDB” and “RCDBMO” interactions are also shown.

The structure of yrast states in \(^{134}\text{Sn}\). The other \(B(E2)\) values can be explained reasonably well by the “RCDBMO” interaction. The calculated \(B(E2; 6^+ \rightarrow 4^+_1)\) and \(B(E2; 2^+ \rightarrow 0^+)\) values agree with the experimental data as shown in Fig. 1. This implies that no core excitations are required to explain the structure of \(^{134}\text{Sn}\). This also supports the earlier claims that \(N=82\) is a robust shell closure \(17^\text{th}\).

Fig. 1 shows the average occupancy of the \(f_{7/2}\) and \(p_{3/2}\) orbitals for the yrast \(4^+_1\) and \(6^+\) states along with the \(4^+_2\) and \(6^+\) states in \(^{134}-^{138}\text{Sn}\) isotopes. The yrast \(4^+_1\) and \(6^+\) states dominantly consist of the \(f_{7/2}\) orbital in all the isotopes. However, the \(4^+_2\) state dominantly consists of the \(f_{7/2}\) orbital in the \(^{136}\text{Sn}\) isotope, while it has a significant mixing of the \(p_{3/2}\) orbital with the \(f_{7/2}\) orbital in the \(^{134,138}\text{Sn}\) isotopes.

Also, the \(4^+_2\) state lies below to the \(6^+\) isomeric state in the \(^{136}\text{Sn}\) isotope with both the “RCDB” as well as the “RCDBMO” interactions. The occurrence of the \(4^+_2\) state, therefore, can strongly affect the \(B(E2; 6^+ \rightarrow 4^+_1)\) value in the \(^{136}\text{Sn}\) isotope. We note that the two \(4^+\) states coming from \(f_{7/2}\) orbital can not have the same seniority. The yrast \(4^+_1\) state is most likely to be a \(v=2\) seniority state while the second \(4^+_2\) state is most likely to be a \(v=4\) seniority state, and the two can mix with each other. The seniority mixing seems to be responsible for a nonzero \(B(E2; 6^+ \rightarrow 4^+_1)\) value in the \(^{136}\text{Sn}\) isotope, as already discussed. The \(B(E2; 6^+ \rightarrow 4^+_1)\) value increases from 14.5 \(e^2fm^4\) to 22.0 \(e^2fm^4\), while the \(B(E2; 6^+ \rightarrow 4^+_2)\) value decreases from 53.1 \(e^2fm^4\) to 46.4 \(e^2fm^4\), on modifying the interaction (See Fig. 1). The relative energy gap between the two \(4^+\) states also slightly decreases after modification of the interaction. It seems that the seniority mixing in both the \(4^+\) states increases by reducing the \(f_{7/2}\) matrix elements, as they come closer in energy with almost similar average occupancies.

On the other hand, the relative gap of the two \(4^+\) states increases in the \(^{138}\text{Sn}\) isotope so that the \(4^+_2\) state lies above the yrast \(6^+\) isomeric state on using the “RCDBMO” interaction. There is a slight decrement in the average occupancy of the \(f_{7/2}\) orbital, and increment in the average occupancy of the \(p_{3/2}\) orbital for the \(4^+_2\) state with the modified interaction, while the opposite happens in the yrast \(4^+_1\) and \(6^+\) states (See Fig. 3). The \(B(E2; 6^+ \rightarrow 4^+_1)\) value, therefore, increases and comes close to the experimental value, because of the less mixing of the two \(4^+\) states. The \(4^+_1\) state lies far from the yrast \(4^+_1\) state in \(^{134}\text{Sn}\), making the mixing of two \(4^+\) states very unlikely, as already discussed above with the applicable \(v=2\) pure-seniority scheme for the yrast \(4^+_1\) and \(6^+\) states.

FIG. 3: (Color online) Reduced transition rates for the yrast \(6^+ \rightarrow 4^+_1\) transitions in \(^{134-138}\text{Sn}\). Experimental data and error bars are taken from \[2, 4\]. Calculated values for both “RCDB” and “RCDBMO” interactions are also shown.

FIG. 4: Average occupancy (in percentage) of the \(f_{7/2}\) and \(p_{3/2}\) orbitals for the \(4^+_1\), \(4^+_2\) and \(6^+\) states in \(^{134-138}\text{Sn}\) isotopes.

16(4) \(e^2fm^4\). The \(4^+_2\) state, which was 7 keV below the yrast \(6^+\) state, now lies 94 keV above it, making the mixing of two \(4^+\) states less likely. This suggests that the \(6^+\) isomeric is following almost pure seniority \(v=2\) scheme in \(^{138}\text{Sn}\). The other \(B(E2; 4^+_1 \rightarrow 2^+)\) and \(B(E2; 2^+ \rightarrow 0^+)\) values also change from 0.5 to 17.9 \(e^2fm^4\) and 188.7 to 163.3 \(e^2fm^4\), respectively. No experimental data for these \(B(E2)\) values are available for comparison.

The structure of yrast states in \(^{134}\text{Sn}\) turns out to be much simpler. Only two valence neutrons are available which occupy the \(f_{7/2}\) orbital. We note that a pure \(v=2\) seniority scheme is applicable for the yrast \(2^+, 4^+_1\) and \(6^+\) states in \(^{134}\text{Sn}\). The level energies and the \(B(E2)\) values can be explained reasonably well by the “RCDBMO” interaction, which remain nearly unchanged even after using the modified “RCDBMO” interaction. The calculated \(B(E2; 6^+ \rightarrow 4^+_1)\) and \(B(E2; 2^+ \rightarrow 0^+)\) values agree with the experimental data as shown in Fig. 1.
IV. CONCLUSIONS

We conclude that the “RCDB” interaction appears to work reasonably well for the highly neutron-rich \(^{134-138}\text{Sn}\) isotopes lying beyond the doubly closed \(^{132}\text{Sn}\) with a small reduction of 25 keV in the \(\nu f_{7/2}^2\) matrix elements. This is in contrast to the large reduction of 150 keV required in the calculations by Simpson et al. \[2\]. Further, we do not find any need to include the contribution from the core excitations even in \(^{134}\text{Sn}\). We also conclude that a pure \(v=2\) seniority scheme is more applicable in \(^{134}\text{Sn}\) and \(^{138}\text{Sn}\) while a seniority mixing is seen in \(^{136}\text{Sn}\); this leads to the observed mismatch in the \(B(E2; 6^+ \rightarrow 4^+_1)\) value in \(^{136}\text{Sn}\). A modification in the interaction leads to a larger \(B(E2; 6^+ \rightarrow 4^+_1)\) value because of increased seniority mixing in the \(4^+_1\) and \(4^+_2\) states of \(^{136}\text{Sn}\).

We also note that further refinement in our calculated results may be possible if we decide to reduce the diagonal and off-diagonal matrix elements differently. We, however, confirm that a small modification in the interaction beyond the \(N=82\) magic number is indeed required. We may also conclude that the “RCDB” interaction seems better suited to explain the data of the neutron rich systems in the \(N=82-126\) region. It would be very useful to measure the remaining \(B(E2)\) values for the transitions in \(^{134-138}\text{Sn}\)-isotopes in order to gain further understanding of the effective interactions in the neutron-rich region.

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