Circularization and Final Spin in Eccentric Binary Black Hole Inspirals

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We present results from numerical relativity simulations of equal mass, non-spinning binary black hole inspirals and mergers with initial eccentricities \( e \leq 0.8 \) and coordinate separations \( D \geq 12M \) of up to 9 orbits (18 gravitational wave cycles). We extract the mass \( M_f \) and spin \( a_f \) of the final black hole and find, for eccentricities \( e \leq 0.4 \), that \( a_f/M_f \approx 0.69 \) and \( M_f/M_{\text{min}} \approx 0.96 \) are independent of the initial eccentricity, suggesting that the binary has circularized by the merger time. For \( e \gtrsim 0.5 \), the black holes plunge rather than orbit, and we obtain a maximum spin parameter \( a_f/M_f \approx 0.72 \) around \( e = 0.5 \).

The field of numerical relativity (NR) has now entered a stage where binary black hole (BBH) simulations can reliably be used to investigate a vast range of interesting phenomena. Studies have produced gravitational waveforms from binary systems in essentially circular orbits \([1, 2, 3, 4]\), involving spinning black holes as well as unequal mass systems. The level of numerical accuracy achieved by these codes is impressive \([3, 5]\), and in some of these studies, the initial binary separations were such that it was feasible to directly compare with post-Newtonian (PN) waveforms \([2, 3]\). Other examples of exciting new results in NR are investigations of the kick imparted to the final black hole (BH) \([4, 5, 6, 7, 8, 9, 10, 11]\), the spin dynamics of the merging BHs \([12, 13]\), and, of relevance to the present work, the merger threshold between bound and unbound BBHs \([14, 15]\), and studies of the result of a circular inspiral of spinning BHs \([16, 17, 18]\). All of this has been possible since the pioneering work of Brügmann et al. \([19]\), Pretorius \([20]\), Baker et al. \([21]\), Campanelli et al. \([22]\).

It is well known that gravitational radiation leads to circularization of a binary system \([23]\). In this work, we study this circularization in the nonlinear regime. In Ref. \([1]\), it was found that, for equal-mass, non-spinning BHs initially in quasi-circular orbits, the merger produced a BH with spin parameter \( a_f/M_f \approx 0.69 \), which, within the accuracy of the results, was independent of the initial separation.

In this Letter, our main goals are (1) to investigate whether sufficient eccentricity is lost during the late stages of inspiral to circularize the orbit and exhibit the same universality as in the circular case and (2) to extract the spin parameter and mass of the final BH, and compare the values with those from circular inspirals.

Although isolated stellar mass BBHs will have completely circularized by the time they are observable by ground-based interferometers, scenarios have been suggested for which BBHs in eccentric orbits are not only astrophysically interesting but also could be detected by space- or ground-based interferometers \([24, 25]\). For instance, galactic mergers leave behind massive BBHs that likely interact with a gaseous environment. A gaseous-gravitational driven inspiral could yield a BBH arriving at the last few orbits and merger with a non-vanishing eccentricity. An observation of the gravitational waves from an eccentric BBH merger will allow us to determine the amount of angular momentum lost to gas and, in particular, the gravitational torques between the binary and a circumbinary disc that affect the eccentricity of the binary \([26]\).

**Methods:** We construct initial data using the puncture approach \([27]\), which requires specifying the coordinate locations and momenta for the two BHs. For “circular” orbits, we follow \([28]\). For eccentric models, we use the conservative 3PN expressions in Ref. \([29]\). These expressions require the specification of the eccentricity \( e \) and the mean motion \( n = 2\pi/P_r \), where \( P_r \) is the radial (peri-center to peri-center) orbital period. There are three PN eccentricities, which are the same to 1 PN order, and we choose \( e_t \), which appears in the PN Kepler equation, following Ref. \([29]\). It is important to keep in mind that the eccentricities we quote (and we use them also to label the models) are to be taken only as a guide to the eccentricity in the initial data, as the PN expressions used do not include radiation reaction, and the PN parameters are in a different coordinate system to the puncture initial data.

We construct a family of initial data by fixing \( n = 0.01625/M \) (\( P_r \sim 387M \)) and varying \( e \) in the range \( 0.05 - 0.8 \) (note that to 2PN order, this means that the...
systems have the same binding energy and that, at high eccentricities, there are portions of the orbit for which the PN condition \( v/e < 1 \) is no longer valid. The binary separation \( D \) is determined from Eq. (23) in Ref. [29], and the tangential linear momentum, \( P/M \), of each BH at apocenter is obtained from \( J = PD \), where \( J \) is the total angular momentum computed as a PN expansion in \( n \) and \( e \) (Eq. (21) in Ref. [29]). The bare BH masses \( m_{1,2} \) are chosen to make the irreducible BH masses \( M_{1,2} = 0.5 \) (i.e. \( M = m_1 + m_2 = 1 \)). Table I provides the initial data parameters.

The numerical simulations and results in this work were obtained with the same infrastructure used in our previous BBH studies (see Ref. [30] for full details). We have evolved the circular model at three different resolutions (finest grid spacings of \( M/38.7, M/51.6 \) and \( M/64.5 \)). We obtain approximately fourth order convergence in the total energy and angular momentum radiated, consistent with the designed 4th order accuracy.

**Results:** In Figs. 1 and 2 we display the gravitational wave strains and coordinate inspiral tracks for \( e = 0.1 \) and \( e = 0.3 \). It is evident that the difference in initial eccentricity has a large effect during the inspiral. Qualitatively, the case with larger eccentricity exhibits a more rapid inspiral [23]. However, at some point both systems enter a “circular” plunge, hinting that circularization may have occurred. We find that the simulations with \( e \geq 0.5 \) show plunge-type rather than orbital-type behavior in the coordinate motion from the very start. Note that the tracks shown in Fig. 2 represent the coordinate positions of the individual BHs, and once a common horizon forms, they are less meaningful.

We now consider the emitted radiation and focus on the dominant \( \ell = 2 \), \( m = 2 \) mode of the complex Newman-Penrose (NP) quantity \( \Psi_4 = A(t) \exp(-i\varphi(t)) \). To compare the orbits, we apply a time shift to \( A \) and \( \varphi \), so that the maximum of \( A \) (i.e. the peak of the amplitude of the gravitational wave) is at \( t/M_f = 0 \) in each simulation. In Fig. 3 we plot the shifted amplitudes and frequencies \( \omega = d\varphi/dt \) extracted at \( r = 70 M \). The cases displayed are those with eccentricities \( e = 0-0.5 \) in steps of 0.1 and \( e = 0.8 \).

In Fig. 3 the oscillations and growth in \( \omega \) at early times (inspiral) can be in general terms understood from simple Newtonian considerations. That is, ignoring radiation reaction, the oscillations (i.e. amplitude and period) in \( \omega \) are a direct consequence of the eccentricity and not present in the \( e = 0 \) case. The period of these oscillations is the period \( P_\omega \) of the extrema in the separation, and the amplitude of the oscillations increases with \( e \). The addition of radiation reaction leads to an overall growth of \( \omega \) with time due to the energy and angular momentum loss, and this is clearly visible in the figure. The amplitude of the oscillations in \( \omega \) should decrease with time, corresponding to a reduction in eccentricity. However, over these timescales, it is difficult to separate this effect from the secular increase in \( \omega \). Also consistent with the predictions in [23], the higher eccentricity evolutions merge more quickly. We note that in the Newtonian case, we would observe that \( P_\omega = P_\varphi \), where \( P_\varphi \) is the time the binary takes to complete one revolution in the angular coordinate \( \phi \). Due to the effects of precession caused by general relativity, the two periods are very different (this

| \( e \) | \( D/M \) | \( P_{1,2}/M \) | \( e \) | \( D/M \) | \( P_{1,2}/M \) |
|---|---|---|---|---|---|
| 0.00 | 12.000 | 0.0850 | 0.40 | 18.459 | 0.0498 |
| 0.05 | 12.832 | 0.0792 | 0.50 | 20.023 | 0.0429 |
| 0.10 | 13.645 | 0.0741 | 0.60 | 21.539 | 0.0361 |
| 0.15 | 14.456 | 0.0695 | 0.70 | 22.955 | 0.0292 |
| 0.20 | 15.264 | 0.0651 | 0.80 | 24.072 | 0.0214 |
| 0.30 | 16.870 | 0.0571 | \( - \) | \( - \) | \( - \) |
During the merger or plunge phase, ω increases dramatically and then levels off, signaling that the BBH has merged. After this point, ω remains constant, a direct consequence of the quasi-normal mode (QNM) ringing of the final BH.

As mentioned before, one of the objectives of this work is to investigate whether a given initial data configuration will circularize before it merges. By this we mean that the radiation from the late stages of the evolution is identical to that from an orbit which started with zero eccentricity. We see in Fig. 3 that the low e evolutions approach the same final state as a circular orbit at the very late stage of inspiral. The ωs from different eccentricities near $t / M_f = 0$ seem to be indistinguishable for low enough eccentricity. In order to investigate this in more detail, in the inset of Fig. 3 we focus on the plunge stage. Here we plot eccentricities $e = 0 - 0.8$ in steps of 0.1. Up until $e = 0.4$ and after $t / M_f \approx -50$, the frequencies ω from each run follow each other. Noticeable differences start showing for $e \geq 0.5$, which is the first configuration to plunge immediately without orbiting first.

We now discuss $M_f$ and $a_t$, computed using two independent methods. In one method, they are obtained from the radiated energy and angular momentum using $M_f = M_{adm} - E_{rad}$ and $a_t / M_f = (J_{adm} - J_{rad}) / M_f^2$. In the second method, $M_f$ and $a_t$ are computed from the QNM frequencies [31] emitted by the final BH, extracted using least squares fitting. As a cross-check, for some of the models we also determine $a_t / M_f$ using an approximate technique derived from the isolated horizon formalism [15, 32]. Table II gives the energy $E_{rad}$ and angular momentum $J_{rad}$ radiated as well as the final mass $M_f$ and spin $a_t$ that the three methods give within the estimated error bars. The final mass and spin also agree well in the circular case with the values obtained in Ref. [33]. The error bars for the QNM-derived quantities are dominated by the uncertainties in the fitting procedure, which are estimated as the variations of the fit parameters over a range of fitting windows. The errors on the radiation-balance quantities are dominated by the finite differenting error of the simulations. Due to excessive computational expense, we have not run very high resolution versions of the eccentric simulations, and so use the errors from the corresponding low resolution circular orbit as a rough guide to the errors in the eccentric cases.

Given an initial eccentricity, it is possible to choose a large enough semimajor axis or orbital period for which the binary circularizes before it arrives at the merger. Our family of initial configurations was designed to investigate, for a fixed initial orbital period, how much initial eccentricity a binary is able to have and still enter the merger with essentially vanishing eccentricity. Since we do not have a good measure of eccentricity applicable prior to the merger, we focus on the end state, namely $M_f$ and $a_t$ of the final BH. We see from Fig. 4 that $a_t / M_f \approx 0.69$ for $e \lesssim 0.4$ and $M_f / M_{adm} \approx 0.96$ for $e \lesssim 0.5$, both values of $M_f$ and $a_t$ in agreement with the circular result. We note that the remaining orbits, the ones which do not circularize, are all configurations which seem to plunge immediately rather than entering an orbital phase. We conclude that for the systems we studied with approximately constant initial orbital period, within our error bars, orbits with $e \lesssim 0.4$ essentially circularize before they merge, and orbits with $e \gtrsim 0.5$ plunge.

We also observe for $e \gtrsim 0.4$ that rather than $a_t$ decreasing monotonically, a maximum spin parameter $a_t / M_f \approx 0.72$ is obtained around $e = 0.5$. Given the size of our uncertainties and that the maximum is found in the three independent methods used to calculate the spin, we are
confident that this maximum is real for our family of initial data. At about $e = 0.6$, $\alpha_t$ starts decreasing monotonically. We are currently considering larger, but still computationally feasible, initial separations to investigate if there is any bound orbital (rather than plunge) configuration that does not circularize.

As $e \to 1$, corresponding to vanishing linear momenta (i.e. a head-on collision from rest), we find that $\alpha_t/M_t \to 0$, in line with the symmetry of the head-on collision, and $M_f \sim M_{\text{adm}}$, as expected, since NR simulations of a head-on collision have shown that $M_f \sim (1 - 0.001) M_{\text{adm}}^{34}$. Note that the ringdown result for $e = 0.8$ gives $M_f > M_{\text{adm}}$ which is clearly unphysical, but the error bars account for this.

Conclusions: We have carried out a series of eccentric orbit simulations of BBH systems in full nonlinear general relativity to investigate the merger regime and final BH. The family of simulations consisted of binaries with approximately constant initial orbital period and varying initial eccentricity. We found that for initial $e \leq 0.4$, the final BH parameters are $M_f/M_{\text{adm}} \approx 0.96$ and $\alpha_t/M_t \approx 0.69$, the same as in the circular case. As a consequence of this, we also found that for $e < 0.4$ the binary begins to enter a universal plunge at $t \sim 50 M_t$ before the amplitude of the gravitational radiation reaches its peak.

While preparing the manuscript of this work, a study by Sperhake et al. [33] appeared with both similar and complementary conclusions to those in our present work. This work was supported in part by NSF grants PHY-0354821, PHY-0653443, PHY-0244788, PHY-0555436, PHY-0114375 (CGWP) and computer allocation TG-PHY060013N. The authors thank M. Ansorg, E. Bengivegna, T. Bode, A. Knapp, R. Matzner and E. Schnetter for contributions and helpful discussions, and E. Berti for the data tables used in the quasinormal fitting.

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