ON A QUANTUM UNIVERSE FILLED
WITH YANG - MILLS RADIATION

Marco Cavaglià\textsuperscript{(a),(c)} and Vittorio de Alfaro\textsuperscript{(b),(c)}

\textsuperscript{(a)} Sissa - Int. School for Advanced Studies Trieste, Italy
\textsuperscript{(b)} Dipartimento di Fisica Teorica dell’Università di Torino
\textsuperscript{(c)} INFN, Sezione di Torino, Italy

ABSTRACT

We investigate the properties of a quantum Robertson - Walker universe described by the Wheeler – DeWitt equation. The universe is filled with a quantum Yang – Mills uniform field. This is then a quantum mini copy of the standard model of our universe. We discuss the interpretation of the Wheeler – DeWitt wave function using the correspondence principle to connect $|\psi|^2$ for large quantum numbers to the classical probability for a radiation dominated universe. This can be done in any temporal gauge. The correspondence principle determines the Schrödinger representation of the momentum associated to the gravitational degree of freedom. We also discuss the measure in the mini–superspace needed to ensure invariance of the quantum description under change of the temporal gauge. Finally, we examine the behaviour of $|\psi|^2$ in inflationary conditions.

Mail Address:
Dipartimento di Fisica Teorica
Via Giuria 1, I-10125 Torino
Electronic mail:VAXTO::VDA or VDA@TO.INFN.IT
1. Introduction.

We may believe that the universe at Planck size for the scale factor is quantistic, but not too much, meaning that an underlying field structure (strings or any sort of quantum field treatment) is not predominant; thus a quantum treatment of a few degrees of freedom could be sufficient. Then it is tempting to investigate the properties of the Wheeler - DeWitt (WDW) equation [1,2] for a mini universe (a single degree of freedom, the scale factor) filled by Yang - Mills (YM) quantized radiation. This is then a quantum version of a radiation dominated RW universe (we shall discuss only in the last section the presence of an effective cosmological constant).

In a reasonable closed quantum universe, when the system is confined to about a Planck length $L_p$ both the YM field and the gravitational degree of freedom have small quantum numbers since the two are connected. In order to get a Standard Model RW universe (confined, if it is the case, to at least some $10^{60} L_p$), enormously large excitation quantum numbers are required, which makes a quantum description irrelevant and at the same time shows once more the unnaturalness of the conditions for the present universe (in absence of inflation of course) as seen from the quantum point of view.

However, this leads us to use the correspondence principle between classical probability and $|\psi|^2$ having imposed square integrability conditions on the wave functions. If we allow the Schrödinger representation of the momentum associated to the scale factor $a$ to depend on the gauge chosen for the classical time, then $|\psi|^2$ satisfies the correspondence principle in any temporal gauge. Then the wave function depends on the time gauge. We will see how to define the quantum mechanical measure so that amplitudes are gauge independent.

Finally we discuss briefly the behaviour of the wave function in inflationary condition. Of course a YM field is of no help in producing inflation, that requires the usual ingredient, some effective cosmological constant whose effect is to stretch enormously the extension of the wave function depressing its amplitude. After inflation and reheating, the radiation has no connection with the original solution and the description of the universe is classical.

2. Classical preliminaries.

We write the gravitational action as

$$A_g = -\int d^4x \sqrt{-g} (R + 2\Lambda) \quad (2.1)$$

The signature of spacetime is $(+,-,-,-)$; definitions as in Landau - Lifshitz. We have put $16\pi G = L_p^2 = M_p^{-2} = 1$ thus measuring all dimensional quantities in these units.
We take the Robertson - Walker metric of topology $R \times S^3$ where $S^3$ is the three-sphere or the euclidean flat space ($k=1,0$).

\[ ds^2 = N^2(\theta)d\theta^2 - a^2(\theta)\omega^p \otimes \omega^p \]  \hspace{1cm} (2.2)

$\omega^p$ are the 1-forms invariant under translations in space and $N(\theta)$ is the lapse function. The cosmic time $t$ corresponds to $N = 1$ and the conformal time $\tau$ to $N = a$. The $\omega^p$'s satisfy the SU(2) Maurer-Cartan structure equation:

\[ d\omega^p = \frac{k}{2} \epsilon_{pqr} \omega^q \wedge \omega^r \]  \hspace{1cm} (2.3)

and thus (2.2) has the SU(2)$_L \times$SU(2)$_R$ group of isometries.

The action space density is

\[ S_g = \int d\theta \ L_g \]  \hspace{1cm} (2.4)

(We do not imply by this that the action is uniform in the 3-D space with local topology $R^3$, only that we consider a region in which eqs. (2.2,3) hold over a domain of the extension of order of $L_p$. This is enough if inflation does the rest. See for instance [3].) $L_g$ is given by

\[ L_g = 6 \left( -\frac{a\dot{a}^2}{N} + kNa \right) - 2\Lambda Na^3 \]  \hspace{1cm} (2.5)

$L_g$ has been obtained from (2.1) by integration by parts in time and neglecting total derivatives. This is enough if we are just interested in the classical equations of motion.

Let us now review briefly how to introduce a YM field configuration with the same SU(2)$_L \times$SU(2)$_R$ symmetry as the metric. We shall use a YM group SU(2)$_L$ for simplicity (the general case has been investigated in ref. [4]). It could be different in different regions of size $L_p$; this choice has little to do with what will be found after filling the universe after inflation when reheating will have created large excitation numbers democratically for the whole set of YM fields.

The most general YM potential invariant under SU(2)$_L \times$SU(2)$_R$ depends essentially only on time.

The YM action density is

\[ A_{YM} = \frac{1}{2} \int F \wedge *F \]  \hspace{1cm} (2.6)

where (we have set $\kappa = 1$ the gauge coupling constant) the field strength 2-form $F = dA + A \wedge A$ is written in terms of the 1-form potential

\[ A = A^a_\mu(x) \frac{i}{2} \sigma_a dx^\mu \]  \hspace{1cm} (2.7)
The most general form of the SU(2) × SU(2) invariant field, written in the vierbein cotangent space, is

\[ A = \frac{i}{2} \xi(\theta) \sigma_p \omega^p \]  

(2.8)

With the definition (2.8) \( A \) is evidently left- invariant; it is also right- invariant up to a gauge transformation [5,6].

It is straightforward to find the field strength \( F \):

\[ F = \frac{i}{2} \sigma_p \dot{\xi} \, d\theta \wedge \omega^p + \frac{i}{4} \sigma_r \epsilon_{rqp} \xi(k - \xi) \omega^q \wedge \omega^r \]  

(2.9)

Thus the physical space action density of the YM field is

\[ S_{YM} = \int d\theta \, \frac{3}{2} \left( N^{-1} a \dot{\xi}^2 - N a^{-1} \xi^2 (1 - \xi)^2 \right) \]  

(2.10)

We give also the form of the energy - momentum tensor:

\[ T_{\theta \theta} = \frac{3}{2} N^2 \left( N^{-2} a^{-2} \dot{\xi}^2 + a^{-4} \xi^2 (1 - \xi)^2 \right) \]

\[ T_{ij} = -\frac{1}{2} \left( N^{-2} a^{-2} \dot{\xi}^2 + a^{-4} \xi^2 (1 - \xi)^2 \right) g_{ij} \]  

(2.11)

To establish the WDW equation for the mini universe [1,7-10] let us introduce the conjugate momenta

\[ p_a = -12 \frac{a \dot{a}}{N} \]

\[ p_\xi = 3 N^{-1} a \dot{\xi} \]  

(2.12)

The Hamiltonian to be used in the WDW equation is

\[ H = \frac{1}{12a} \left[ -\left( \frac{1}{2} p_a^2 + V_a(a) \right) + 4 \left( \frac{1}{2} p_\xi^2 + V_\xi(\xi) \right) \right] \]  

(2.13)

where

\[ V_a(a) = 72 \left( k a^2 - \frac{\Lambda}{a^4} \right) \]

\[ V_\xi(\xi) = \frac{9}{2} \xi^2 (1 - \xi)^2 \]  

(2.14)

The classical Friedmann - Einstein (FE) equation of motion is

\[ H = 0 \]  

(2.15)

From the YM equation follows [5]

\[ (N^{-1} a \dot{\xi})^2 + \xi^2 (1 - \xi)^2 = K^2 \]  

(2.16)
where $K$ is independent of $\theta$. Using (2.16), eq. (2.13) becomes

$$\frac{a^2 \dot{a}^2}{N^2} + \left( k a^2 - \frac{\Lambda}{3} a^4 \right) = \frac{1}{4} K^2$$

(2.17)

In the cosmic gauge $N = 1$ this reads ($\dot{a} \equiv da/dt$)

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{\Lambda}{3} + \frac{1}{4} K^2$$

(2.18)

We see that, as due to radiation, the energy density scales as $a^{-4}$. The meaning of $K^2$ is apparent:

$$K^2 = \frac{2}{3} \rho(a = 1)$$

(2.19)

$K^2$ is the energy density of the radiation in a RW universe with scale factor $a$ of one Planck length. Now suppose for a moment we take our present classical universe and run it back by the Standard Model, no inflation, up to the time at which $a \simeq L_p$ ($\leq 10^{-61} a_0$). Now, in the Standard Model the Planck density, $\rho = 1$, is reached when $a \simeq 10^{-31} a_0 \simeq 10^{30} L_p$. Then, going further to $a = L_p$ the density must be sort of $10^{120} \rho_p$. $K^2$ would be a very strange number indeed, in order to obtain our present universe by the sole power of the Standard Model + YM radiation. This is only too well known, it is just the old observation of mismatch between size and density. We shall now see the quantum counterpart.

3. The WDW equation.

The WDW equation for a YM filled RW universe has been first discussed in [11]. We will emphasize some different aspects, mainly the correspondence principle and the connection between eigenvalues of the gravitational and the YM sectors.

The WDW equation is given by

$$H \Psi(a, \xi) = 0$$

(3.1)

where $H$ is given by (2.13) and we deal now with quantum operators:

$$[a, p_a] = i; \quad [\xi, p_\xi] = i$$

(3.2)

In the quantum version of the Hamiltonian (2.13) there is an ordering problem between the term $1/a$ and $p_a^2$. This may be connected to the Schrödinger representation of the momentum through the correspondence principle and will be discussed later. For the moment we simply drop that overall factor $1/a$ from the Schrödinger equation $H \psi = 0$.

It is a beautiful property of the radiation field [11] that in the quantum equation there is no direct coupling between the YM field and the gravitational degree of freedom (similar to the case of a conformal scalar field discussed in [7],
but physically much more interesting). This is the quantum form of the request that the density scales like $a^{-4}$. The two fields are in reality essentially coupled, since it is the presence of the YM part that allows the gravitational degree of freedom to have a quantum solution. The state of the quantum mini universe is determined by the quantum configuration of the YM field: the presence of a non vanishing eigenvalue of the YM sector supports a non trivial gravitational configuration.

Thus, the WDW equation separates. We write

$$\Psi(a, \xi) = \psi(a) \eta(\xi)$$  \hspace{1cm} (3.3)

and look for eigenvalues of the YM part:

$$\left[ \frac{1}{2} p_{\xi}^2 + V_\xi(\xi) \right] \eta_n(\xi) = E_n^{YM} \eta_n(\xi)$$  \hspace{1cm} (3.4)

Now about the boundary conditions for the YM wave function. It is natural to ask that the wave function tends to zero for large $|\xi|$. Then eigenvalues are quantized. $n$ counts the number of oscillations. It is easy to derive, and can be controlled by the WKB expansion \cite{12}, that the asymptotic behaviour of $E_n$ for large $n$ is $E_n \sim n^{4/3}$.

Now let us turn to the gravitational degree of freedom. We drop the cosmological term and discuss the case $k=1$ (see also \cite{13} where the WDW equation in presence of classical matter is discussed too as a quantum bound state). We have

$$\left[ \frac{1}{2} p_a^2 + V_a(a) \right] \psi_n(a) = E_n^g \psi_n(a)$$  \hspace{1cm} (3.5)

The gravitational degree of freedom is given by a harmonic oscillator. It is natural to set the boundary condition at $a \to \infty$ by asking the square integrability of the wave function (for this suggestion see also \cite{14}). With this criterion one obtains the correspondence with the classical gravitational motion for large oscillator quantum numbers as we shall see. About the condition at $a = 0$, if we ask that $\psi \to 0$, then $p_\xi$ (and thus $H_{YM}$) is hermitean (this does not mean however that $p_\xi$ is observable, as its eigenfunctions do not fulfil the boundary condition). Let us adhere to these boundary conditions. Then the quantum numbers $n_g$ are odd. We have

$$E_n^g = 12 \left( \frac{1}{2} + n_g \right)$$  \hspace{1cm} (3.6)

The eigenvalues of the two eigenequations are connected:

$$E_n^g = 4E_n^{YM}$$  \hspace{1cm} (3.7)

In this equation two constants are actually present: the Newton constant and the YM coupling. It may look that this equality could hold only for a few states;
however in order to have a mini universe one just needs it to be valid for one state. The passage to macroscopic configurations requires inflation.

If for a moment we use the numbers quoted before for our classical universe (run back to $a \sim L_p$ by the Standard Model without inflation), we would need $E_n^g \sim n_g \sim 10^{120}$, and $n_{YM} \sim 10^{90}$ (see [1]). With such artificial quantum numbers both degrees of freedom should be described classically. This displays how unnatural is our universe without inflation in the frame of the WDW equation for YM fields.

In this context it is interesting to notice that one should be able to recover some information about the quantum wave function and its meaning using the principle of correspondence between classical and quantum systems for large $n$ (as suggested originally in [1]).

4. The wave function and the correspondence principle.

Let us examine with some care the gravitational wave function $\psi (a)$. We are discussing a closed RW universe filled with radiation. The point is, in eq. (3.5) which is the Schrödinger representation of $p_a$? is it the naive one, namely

$$p_a \rightarrow -i \frac{d}{da} \ ? \ (4.1)$$

It is tempting to interpret $|\psi|^2$ as the probability density for the value $a$ for the scale factor [8], to be compared through the correspondence principle to the classical probability density. This implies that in some way $|\psi|^2$ depends on $N(a)$ (we consider $N$ as function of the mini-superspace variable $a$, as for instance in the case of the conformal gauge). We will see how that can be implemented. If one takes this attitude, then the correspondence principle is the guidance to solve this problem. The idea is that the Schrödinger representation of $p_a$ is to be determined by the request that for large quantum numbers $|\psi|^2$ approaches, in average, the classical probability distribution of the physical quantity $a$ for an ensemble of trajectories. Let us first discuss this point with some care. The WDW equation is independent of time. This is because time is just a degree of freedom in the universe, being a measure of correlation of positions of physical objects [1,15]. Now in our WDW equation the universe has been reduced to just two degrees of freedom; we have no hands nor clocks in the hamiltonian (2.13). So, to compare the quantum system to a classical one we may consider the classical motion for an ensemble forgetting their starting time. We are taking a snapshot at the scale factor $a$ in the classical system and do not know how much time has lapsed since its classical beginning. We only ask for the probability (density) of finding the classical system at a certain $a$. This is inversely proportional to the speed of $a$ in time. Thus (probability not normalized)

$$P_{cl}^\theta = \frac{1}{da/d\theta} \ (4.2)$$

$$6$$
From the classical equation of motion (2.17) for \( k = 1, \Lambda = 0 \) and \( K^2/4 \equiv a_M^2 \)
we get

\[
P_{\theta} = \frac{a}{N(a)} \frac{1}{\sqrt{a_M^2 - a^2}} \tag{4.3}
\]

where \( N(a) = 1 \) for the cosmic time and \( N(a) = a \) for the conformal time. Now if this probability has to be compared to \( |\psi|^2 \) by the correspondence principle, we must admit that \( \psi \) is not invariant under change of the lapse function \( N(a) \), in spite of the fact that the WDW equation is independent of \( N \) (see also [14]).

A way out is the suggestion that the rule for the representation of \( p_a \) is

\[
p_a \rightarrow \frac{a}{N(a)} \left( -i \frac{d}{da} \right) \sqrt{N(a)} a \tag{4.4}
\]

Indeed, with \( \psi_n = \sqrt{a/N(a)} \chi_n \) the eigenvalue equation reads

\[
\left( -\frac{1}{2} \frac{d^2}{da^2} + V_a(a) \right) \chi_n(a) = E_n^2 \chi_n(a) \tag{4.5}
\]

Then the correspondence principle works properly:

\[
\Sigma_{av} |\psi_n(a)|^2 \rightarrow \frac{a}{N(a)} \frac{1}{\sqrt{a_M^2 - a^2}} \tag{4.6}
\]

coincident with the classical probability (4.3). The representation (4.4) gives the required result. Thus the correspondence principle suggests that the Schrödinger representation of \( p_a \), and thus of \( \psi(a) \), depends on the gauge chosen for the time.

Let us check how these ideas work for the case of a flat RW universe, \( k = 0 \).

In this case, in the cosmic gauge the classical equation of motion (2.18) is

\[
a\dot{a} = \text{constant} \tag{4.7}
\]

Thus

\[
P_{cl} \propto a \tag{4.8}
\]

Now the corresponding quantum equation is

\[
-\frac{1}{2} \frac{d^2}{da^2} \frac{\psi(a)}{\sqrt{a}} = E \psi(a) \tag{4.9}
\]

The solution is \( \psi(a) = C \sqrt{a} e^{\pm ia\sqrt{2E}} \) whose probability distribution gives back (4.8) (we have nothing new to say about the problem of the boundary conditions in this case).

It is interesting to observe that in the conformal gauge \( \Lambda(a) = a \) the representation for \( p_a \) is just the naive one. The conformal gauge is privileged in this respect. We will see the reason.
The choice (4.4) implies that the integrals representing scalar products in the Hilbert space have the measure

\[ (\Psi_1, O(p_a, a; p_\xi, \xi)\Psi_2) = \int \Psi_1^* O(p_a, a; p_\xi, \xi)\Psi_2 \frac{N(a)}{a} da \ d\xi \]  
(4.10)

First of all notice that any matrix element of the form (4.10) is lapse independent. Indeed we have

\[ (\psi_1, O(p_a, a)\psi_2) = \int \chi_1^* O(-id/da, a)\chi_2 \ da \]  
(4.11)

Nor the equation for the \( \chi \)'s nor the boundary conditions depend on \( N(a) \) (in particular \( H \) is hermitian if \( \chi(0) = 0 \)). One sees how the measure compensates the dependence of the wave function on the lapse function required by the correspondence principle.

Thus, the matrix elements that have the fundamental quantum role are given by (4.10). In particular, the invariant probability with the usual quantum properties is \( dP = |\psi|^2(N(a)/a) da \). It is curious but true that the squared modulus of the wave function obeys the correspondence principle as we discussed, but is not a true probability density since (as the corresponding classical density) it is not invariant under changes of the time gauge.

Let us observe that our procedure agrees with the Halliwell discussion [16] of the rescaling of the lapse function. The difference is that we allow for the lapse dependence of the representation of the wave function and this guarantees the invariance of matrix elements under changes of \( N(a) \). The conformal time gauge has a special role since when \( N = a \) the measure in (4.10) is the elementary one.

Let us note that in our simplified quantum frame a quantity that could be interpreted a posteriori, at least for large quantum numbers, as a substitute for a classical measure of “time interval” is

\[ \Delta \theta = \int_{a_1}^{a_2} |\psi|^2 \ da \]  
(4.12)

as suggested by the classical interpretation. It is hard to see how this could be generalized.

5. Inflation.

Let us discuss the behaviour of the wave function and control the correspondence principle in an inflationary region. For these limited purposes in our quantum mini universe we stimulate inflation through the introduction of an effective cosmological parameter \( \Lambda_c(a) \) which is constant for small \( a \) and soon or later tends to 0. This expedient may simulate both the case of chaotic inflation and of the quenched \( \Lambda \) cases.
Let us turn our attention to the classical equation of motion (2.17) for the gravitational degree of freedom. (we use now the cosmic gauge for definiteness and $k = 0$ for simplicity, to avoid discussing tunnelling):

$$a^2 \dot{a}^2 - \frac{\Lambda}{3} a^4 = \frac{E}{72} \tag{5.1}$$

Thus

$$P_{cl}^t = \frac{1}{\dot{a}} = \frac{12a}{[2(E_n + 24\Lambda a^4)]^{1/2}} \tag{5.2}$$

Notice that for large $a$ $P_{cl} \sim 1/a$. The physical interpretation is that during the de Sitter phase the scale factor $a$ changes exponentially in time, $\dot{a} \sim a$, thus the time the classical system spends in a neighbourhood of any given value is inversely proportional to the scale factor.

The scale at which this behaviour sets in is of course $(E_n/24\Lambda)^{1/4}$. We see that the effect of the YM eigenvalue $E_n$ is to delay inflation: the larger is the energy, the higher the value of $a$ at which inflation starts.

Now we turn to the WDW equation (in the cosmic gauge, and $k = 0)$:

$$\left(-\frac{d^2}{da^2} - 48\Lambda a^4\right)\chi_n(a) = 2E_n \chi_n(a) \tag{5.3}$$

The WKB solution (we are far from turning points; if $\Lambda > 0$ there are none for $E_n > 0$) is given by (we are very interested in the pre-exponential factor and not in the outgoing or incoming boundary conditions)

$$\chi_n(a) = \frac{C}{\sqrt{p_n(a)}} e^{\pm i \int_a^p p_n(a) da} \tag{5.4}$$

where

$$p_n(a) = [2(E_n + 24\Lambda a^4)]^{1/2} \tag{5.5}$$

Remembering now that in this gauge $\psi_n(a) = \sqrt{a} \chi_n(a)$ we obtain for the quantum probability distribution the same formula as for the classical one. In particular for sufficiently large $a$ the WKB wave function has pure de Sitter behaviour:

$$|\psi(a)|^2 \sim \frac{1}{a} \tag{5.6}$$

The inflationary stretching of the wave function from the Planck domain into large domains depresses its amplitude.

There is no way in this quantum context to tell how big is the size of the de Sitter region in $a$. Adhering to the chaotic inflation scenario, $\Lambda \sim V(\phi)$, this depends on the parameters of the potential. One knows that one needs a huge
factor, certainly larger than $10^{31}$. This is of course unnatural in terms of the Planck wave function, but, that is the way it goes with inflation. requires an inflationary model classical behaviour of $a$ in time. Quantum mechanics alone cannot help us particularly. Note that the integral of the squared wave function over the de Sitter region is of the order of the classical inflation time:

$$\int_{a_1}^{a_2} |\psi|^2 \, da \sim \Lambda^{-1/2} \ln(a_2/a_1) \quad (5.7)$$

To end with the case of a cosmological constant, let us remark that if $\Lambda$ is negative and $k=0$ the hamiltonian of the two degrees of freedom has the same form (apart from the sign). The condition that both are e.g. in the lowest quantum eigenstate requires that $|\Lambda|$ and the squared gauge coupling constant are equal. This is the quantum analogue of the classical solutions discussed long ago [17].

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After the submission of this paper we have realized that our suggestion (4.12) coincides with the “probabilistic time” extensively explored by M.A. Castagnino and F. Lombardo, *Phys. Rev.* **D48**, 1722 (Aug. 15, 1993).
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