Students’ algebraic thinking: a study of mathematical modelling competencies

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Abstract. Several components shows that the process of algebraic thinking on students in solving problems. One of these components is the process of using algebra as a tool for doing mathematical modelling. This study aims to reveal how the students’ competency in making mathematical modelling for solving an algebraic problem. The descriptive qualitative study design was used to obtain an overview of the mathematical modelling competencies of nine students (15-16 years old) in Pekanbaru, Riau Province. Each participant received a test that contains an algebraic problem. The results of this study show that when students do mathematical modelling there are several errors as follows: (1) failure in making suppositions for the problem and simplifying the given situation; (2) failure in recognizing quantities that influence the given situation, to identify the key variable; (3) failure in construct relation between variable; and (4) failure in solving question within mathematical modelling.

1. Introduction
There is a connection between mastery of mathematics and the survival of a civilization. Every individual requires to have mastery of mathematics at a certain level. Basically, the skills required are not in the form of mastery mathematics as a science, but mastery of mathematical skills to support one’s success in daily life. Through mathematics teaching and learning, students are guided to have mathematical skills so they can develop their life skills. [1].

One of the subjects of mathematics learning at the secondary school is algebra. Algebra can be used in daily life problems, so this makes algebra important and must be learned by every student [2, 3]. Besides that, algebra in the world of mathematics education can be interpreted as a way to express things becomes more general, a way to learn how to manipulate symbols and solve equations, learn functions, and a way to make the model from real condition [4]. In order for someone to manipulate these mathematical symbols, the existence of an algebraic thinking process is needed. This is in line with the opinion of Habert and Brown that algebraic thinking is the use of mathematical symbols and tools to analyze different conditions by representing mathematical information in the form of equations, diagrams, tables, words, and graphs, and interpreting; and use mathematical results such as solving unknown values, testing proofs and looking for the relationship of a function [5]. Then, algebraic thinking becomes one of the terms contained in the learning process of mathematics which will prepare students to gain good expertise in the scope of algebra and in other fields [6].

Algebra as a tool for learning functions and mathematical modelling is a component of algebraic thinking proposed by Krieger [6]. The ability to make mathematical modelling itself is referred to as a competency which includes skills and abilities in presenting the modelling process appropriately and is
goal oriented and willing to take action. Furthermore, there are five sub-competencies of mathematical modelling according to Blum and Kaiser. The first competence is student’s ability to comprehend the real problems and to compile a certain model in accordance with the real situation, which requires students to be able to (1) make suppositions for the problem and simplify the given situation; (2) to recognize quantities that influence the given situation, give their names and to recognize the key variables; (3) to arrange relations between the variables; (4) to look for information and distinguish between relevant and irrelevant information. The another competence is student’s ability to solve mathematical questions within this mathematical model, which involve as follows (1) to use heuristic strategies such as dividing the problem into several sections, forming relationships with identical problems or analogous problems, rephrasing the problem, viewing the problem in a different form, varying the quantity or the available data etc.; (2) to use mathematical knowledge to solve the problem [7].

At this time, mathematics teaching and learning in Indonesia is guided by the Curriculum 2013. There are basic competencies that students must master through the learning process. This basic competency includes students’ ability to solve the real problems with the aid of mathematical concepts they have learned [8]. Based on the demands of this curriculum, it is clearly seen that mathematical modeling competence is something that students must have in order to make them easier to achieve the competency mastery criteria. Furthermore, Indonesian students have also participated in international studies relating to the measurement of students’ ability to solve problems from real situations. This study namely PISA (Program for International Student Assessment). PISA study results show that Indonesian students are still at a low level in terms of problem solving based on real situations [9, 10].

In addition, what is being tested in PISA can be interpreted as an individual capability to identify and comprehend the role that mathematics plays in the world for making judgements properly. Therefore, mathematizing real situation as well as interpreting, reflecting and validating mathematical results in real situation are essential processes when solving PISA problems [11]. The proficiency levels identified in the context of PISA mathematics can be interpreted as a first attempt towards such a competence model [12]. In the PISA study, the types of questions related to algebra are referred to as problems with change and relationship content. The aspects that are the focus of these questions are aspects of traditional mathematical content which include functions and algebra, and include algebraic expressions, equations and inequalities, table and graphical representations, which are key centers in describing, modelling and interpreting a phenomenon [13].

Previous study which discussed algebraic thinking in students, focused on how the problem-solving process presented by students. Do students solve the problem by using the process of algebraic thinking or maybe using the process of arithmetic thinking [14]. Whereas, the study that discusses in detail about students' abilities in each component of algebraic thinking has not been found particularly in Indonesia. So, through this study, an analysis will be conducted on how to describe students' ability to do mathematical modelling as a component of algebraic thinking by providing a test question with change and relationship content adopted from the PISA test. Then, the analysis was carried out based on several sub-competencies of the mathematical modelling competencies described by Blum and Kieran.

2. Experimental Method
This study uses qualitative methods with descriptive design. The study participants consisted of nine students (15-16 years old). Determination of the participant of this study was carried out with the assistance from a teacher who taught at a public school in Pekanbaru, Riau Province. This study was conducted in order to obtain an overview of students' mathematical modelling competencies in solving problems adopted from the PISA test in 2012 [15]. Thus, each participant is given a question as shown in Figure 1 and asked to complete within 20 minutes. Furthermore, based on the students' answers, the researcher analyzes what kinds of errors occur according to the sub-competencies of mathematical modelling competencies.
3. Result and Discussion

This study discusses the description of students' mathematical modelling competencies depend on students' answers to the questions presented in Figure 1. Based on the results of the participant's answers, it appears that there are several types of errors made by students when solving problems that require mathematical modelling competencies. Figure 2 presents the type of student errors in making suppositions for the problem and simplifying the given situation.
Student answers as shown in Figure 2 indicate that there is an error in making an assumption for each variables that will be used to obtain an equation that makes the problem seem simpler. The variable \( z \) should be interpreted as "the amount of income earned every week" not "per week". Question (a) can be answered by using a two variable equation. Students should be able to identify that \( x \) is always equal to 240. Thus, the error of the assumptions presented in Figure 2 shows the equation with three variables. This is because the number of newspapers sold is assumed by using two variables. Student has not succeeded in making the problem situation seem simpler.

Figure 3 shows the failure in recognizing quantities that influence the situation. Through the answers presented by the participants, it was seen that there were students making failures in understanding "the additional salary of 1,000 rupiah for each additional newspaper sold". The word "addition" has disturbed the students' understanding of this problem so that they write the equation as follows:

\[
\begin{align*}
S(x) &= 500(240 + x) + 1000(x) \\
S(x) &= 120,000 + 500x
\end{align*}
\]

**Figure 3. Students’ failure in recognize quantities that influence the given situation**

Furthermore, Figure 4 below presents the student's failure in constructing the relation between variables. Students do not start by writing clearly the meaning of each variable that he uses. Thus, there is a misunderstanding when he makes a requirement for the variable \( y \) which should be meant as the number of additional newspapers and has a relationship with the variable \( x \). In this answer, student makes the relationship among variable \( y \) and \( n \). In the end, students also do not produce an equation that describes the amount of salary per week that John will get correctly. Because, previously there was no variable that became a representative to state the salary amount.

**Figure 4. Students’ failure in construct relation between variables**

Other types of errors made by students in solving problems in Figure 1 are the failure in solving question within mathematical modelling. In the answers presented in Figure 5, it can be seen that in the beginning the mathematical modelling produced by students to answer Question a) and b) is still not correct. There is no step in making assumptions for each variable that will be used and there is a misunderstanding of the question, making the equation produced by students is incorrect. Furthermore, students also make failures in completing questions c by utilizing the mathematical modelling that they should have by answering Questions a) and Questions b).
There are two sub-competencies that can describe the ability to solve problems with mathematical modelling. The sub-competencies are (1) to use heuristic strategies such as dividing the problem into several sections, forming relationships with identical problems or analogous problems, rephrasing the problem, viewing the problem in a different form, varying the quantity or the available data etc.; (2) to use mathematical knowledge to solve the problem. Figure 5 indicates that students do not have these two sub-competencies in solving problems using mathematical modelling. Because, students do not carry out any heuristic strategies or show the use of any mathematical knowledge in determining which company John should apply for.

Based on students' answers, there are several types of errors in solving problems related to algebraic material that demand mathematical modelling competencies. Of course, this indicates that students' mathematical modelling competencies have not been well developed. Teachers are advised to more routinely provide practice questions that require competence in mathematical modelling in order to develop students' mathematical modelling abilities. Furthermore, the teacher must also be more careful in examining student answers so that they can identify what sub-competencies students have or have not mastered.

4. Conclusion
The findings in this study show that the ability of students to do mathematical modelling which is one indicator of thinking algebra is at different levels. Students still make several failures with the following categories: (1) failure in making suppositions for the problem and simplifying the given situation; (2) failure that recognizes quantities that influence of the given situation, to identify the key variable; (3) failure in construct relation between variable; and (4) failure in solving question within mathematical modelling. Some descriptions of students' failures in doing mathematical modelling are expected to be able to add to the reference of mathematics instructors and make this a focus in mathematics learning. There is a limitation in this study since there is several mathematical modelling sub-competencies according to Blum and Kaiser which has not been discussed in this study. Therefore, further research is needed to identify students' mathematical modelling competencies based on other sub-competency criteria.

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**Acknowledgement**
The author would like to thank teacher and students who were willing to be the participants in this study.