Colour Octet Contribution in Exclusive Charmonium Decay into Baryon-Antibaryon

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Abstract

It is argued that colour octet contribution in exclusive charmonium decays is very important even though the same infrared divergence found in P-wave colour singlet inclusive decay is absent. This comes about because of the suppression at the level of the wave-function of P-wave quarkonia. Using the modified hard scattering approach of Brodsky, Lepage, Botts, Li and Sterman, we use charmonium decay into baryon-antibaryon pair as an example and show that the colour singlet contribution alone is clearly insufficient to explain the experimental decay widths.

1 Introduction

It is known that in inclusive decay of P-wave charmonium into light hadrons, there is an infrared (IR) divergence\(^1\) in the reaction \(c\bar{c} \rightarrow gq\bar{q}\) which is the leading inclusive contribution for \(\chi_1\) but next-to-leading for \(\chi_0\) and \(\chi_2\). This IR divergence reveals itself in the limit that the gluon becomes soft and the heavy \(c\bar{c}\) are allowed to go on the mass-shell. In view of the divergence and the \(c\bar{c}\) is a bounded system, one is obliged to keep them off-shell by an amount of the binding energy \(\varepsilon\). The resulting inclusive decay width has a logarithmic dependence on this binding energy \(\Gamma^{\text{incl}} \propto \ln \varepsilon\). However, it was shown in Ref. 2 that, in fact, this infrared divergence is there only due to the neglect of the next higher Fock state of the P-wave charmonium, the so-called colour octet component, which becomes degenerate with the valence \(c\bar{c}\) singlet state in the already mentioned decay process in the soft gluon limit.

In exclusive P-wave charmonium decay, this same IR divergence is not present because it only happens in a small region in momentum space and the decay products are again bound states. Therefore a priori, there is no good reason to introduce the colour octet. Indeed, the exclusive decay widths for \(\chi_J\) have been calculated for a number of decay modes using the colour singlet valence Fock state alone and claims have been made that these calculations can account for the corresponding experimental partial widths. In the following, using the example of \(\chi_J \rightarrow N\bar{N}\), we shall argue that, in fact, there is a very good reason to include the colour octet component in exclusive decay even in the absence of the IR divergence as found in the calculation of the inclusive width and point out a fallacy of some of the previous works. We show that within a self-consistent perturbative scheme, the modified hard scattering approach, colour singlet contribution alone is insufficient to account for the experimental data. Thus, based on the theoretical argument to be presented, colour octet is required.

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2 Colour Singlet vs Colour Octet

The decay of the $c\bar{c}$ system through annihilation is a short distance process with an annihilation size, $L$, of the order of the inverse charmonium mass $L \sim 1/M_{\chi_J} << 1$. So for S-wave decay, one needs essentially $\psi_s(L \sim 0)$, the wavefunction at the origin. For a P-wave system, the wavefunction at the origin is zero, so instead one uses $L\psi_p'(L \sim 0)$, the derivative of the wavefunction at the origin weighed by the annihilation size. In momentum space, the contributions from the wavefunctions alone will provide a P-wave to S-wave ratio of $k\psi_p(k)/M_{\chi_J}\psi_s(k)$ so P-wave is suppressed in comparison to S-wave by $1/M_{\chi_J}$. This suppression is very important, as we shall see presently, for whether the colour octet component should be included.

In the hard scattering picture of Brodsky and Lepage, the decay probability amplitude is given by a convolution of the distribution amplitudes and the hard scattering perturbative part

$$M_{\chi_J \rightarrow N\bar{N}} \sim f_N \phi_N \otimes f_{\bar{N}} \phi_{\bar{N}} \otimes f_{\chi_J} \phi_{\chi_J} \otimes T_H.$$  \hspace{1cm} (1)

The only dimensional quantities in the amplitude are the decay constants and some power of the large scale of the process in question, namely the charmonium mass, hidden in the hard part $T_H$. All these must together make up the right dimension of the probability amplitude which has one mass dimension because it is related to the decay width through

$$\Gamma \sim \frac{1}{M_{\chi_J}} |M|^2.$$  \hspace{1cm} (2)

The decay constants in Eq. (1) all have mass dimension two for both colour singlet and octet contribution to the width because both the wavefunctions of the baryon and the colour octet component are three-particle wavefunctions while the colour singlet component is a P-wave two-particle wavefunction. Using the right power of the charmonium mass from the hard part $T_H$ to make up for the right mass dimension of the amplitude gives

$$M^{(1)}_{\chi_J \rightarrow N\bar{N}} \sim M_{\chi_J} \left( \frac{f_B}{M_{\chi_J}^2} \right)^2 \left( \frac{f_{\chi_J}^{(1)}}{M_{\chi_J}^2} \right) \sim \frac{1}{M_{\chi_J}^5}.$$  \hspace{1cm} (3)

$$M^{(8)}_{\chi_J \rightarrow N\bar{N}} \sim M_{\chi_J} \left( \frac{f_B}{M_{\chi_J}^2} \right)^2 \left( \frac{f_{\chi_J}^{(8)}}{M_{\chi_J}^2} \right) \sim \frac{1}{M_{\chi_J}^5}.$$  \hspace{1cm} (4)

As one can see, both the octet and singlet contribution depend on the same power of $M_{\chi_J}$. The octet contribution is not negligible as far as the large scale $M_{\chi_J}$ is concerned. This is the case only because, as we saw above, P-wave charmonium is weighed down by the annihilation size $L \sim 1/M_{\chi_J}$ in the amplitude. Had we been dealing with the S-wave charmonium $J/\psi$ instead, the singlet amplitude would go as $1/M_{\chi_J}^4$ and the colour octet can be neglected in comparison with the colour singlet valence component. Returning to the $\chi_J$ decay, possible suppression from the $\alpha_s$ dependence is absence. The singlet and octet contribution are only differed by $\sqrt{\alpha_s}$ for typical graphs which is not small. Therefore there is no good reason to neglect the colour octet from exclusive decay.
3 Colour Singlet Contribution within the Modified Hard Scattering Approach

In the modified hard scattering picture, the decay probability amplitude is given by

\[ M_{\chi J \rightarrow N\bar{N}} \sim \psi_B \otimes \psi_B \otimes \psi_{\chi J} \otimes T_H \otimes \exp(-S). \tag{5} \]

a convolution of the wavefunctions, hard part and Sudakov factor. In the valence colour singlet contribution, the part of the convolution that can potentially render the result ambiguous is the nucleon wavefunction. The reason being that there are many nucleon model wavefunctions available that one can use, for example see Ref. 3, for this calculation. However, none of them can explain data on nucleon form factor at low \( Q^2 < 50 \text{ GeV}^2 \), which is the relevant energy range of interest in our present problem. As shown in Ref. 4, these models yield nucleon form factor results that are well below the experimental data. As a result, one cannot possibly hope to use these models to calculate the \( \chi_J \) partial decay width. In view of the smallness of the perturbative contribution, one is forced to leave the perturbative region and find non-perturbative contribution which can account for the data. The full expression for the nucleon form factor can be written as

\[ F_1(Q^2) = \sum_{n=3}^{\infty} \int [dx][d^2k_\perp] \sum_{j \in \{q, \bar{q}\}} e_j \psi^{(n)}(x, k_\perp) \psi^{(n)*}(x, k'_\perp) \] \tag{6} \]

where \( k'_\perp = k_\perp + (1 - x_i)q \) for the struck quark and \( k'_\perp = k_\perp - x_jq \) for the spectator quarks, the second sum is over the parton index of the quark and antiquark constituents and we have only put a representative \( x \) and \( k_\perp \) to stand for all the momentum fractions and internal momenta of the constituents in the argument of the \( n \)-constituent nucleon wavefunction \( \psi^{(n)} \). When hit by a highly virtual photon, one knows that the perturbative contribution comes from the region where the internal momenta of either one of the wavefunction in Eq. (6) are hard while those in the other remain soft. Since this is not the dominant contribution, one looks at the region where the internal momenta in both wavefunctions are soft. The reason being that bounded hadrons tend to have small internal momentum so if the incoming photon momentum is not so hard, as in the low \( Q^2 \) but \( Q^2 >> \text{MeV}^2 \) region, the overlap of both wavefunctions with small internal momentum should dominate. This can happen only if the struck quark takes up almost all the momentum of the nucleon so that \( (1 - x_i)q \) and \( x_jq \) all become soft. This is most unlikely if many constituent partons are there to share the nucleon momentum. So one can take only the valence Fock state as the dominant contribution

\[ F_1(Q^2) \simeq \int [dx][d^2k_\perp] \sum_{j=1}^{3} e_j \psi^{(3)}(x, k_\perp) \psi^{(3)*}(x, k'_\perp). \tag{7} \]

This simplification enables one to fit this to the \( F_1 \) data, GRV valence quark distribution and other data like \( \Gamma(J/\psi \rightarrow N\bar{N}) \) to obtain a model wavefunction for the nucleon\(^5\) valid in the low \( Q^2 \) range of interest. The other remaining colour singlet \( \chi_J \) wavefunction is more or less standard (see for example Ref. 6). The hard part \( T_H \) in the MHSP comes from 4 graphs up to permutation of the light quark-antiquark lines\(^7\). Here we do not give the full expression but just to point out that in the MHSP, the probability amplitude has the following dependence on \( \alpha_s \)

\[ M_{\chi \rightarrow N\bar{N}}^{(1)} \sim \alpha_s(t_1) \alpha_s(t_2) \alpha_s(t_3) \] \tag{8} \]
Table 1: Colour singlet contributions\(^7\) to \(\chi_J\) decay into \(p\bar{p}\).

| \(J\) | \(\Gamma^{(1)}(\chi_J \rightarrow p\bar{p})\) [eV] | PDG\(^8\) [eV] |
|------|----------------|----------|
| 1    | 7.49           | 75.68    |
| 2    | 46.97          | 200.00   |

where the renormalization scales \(t_i, i = 1, 2, 3\) are determined by the largest scale in the neighbourhood of the vertices that carry the particular \(\alpha_s\). The scales could be the virtualities of the neighbouring propagators or the inverse of the smallest transverse size squared between the quarks (antiquarks) in the baryon (antibaryon). So the renormalization scales are determined dynamically by the process which is a typical hallmark of the MHSP. The product of \(\alpha_s\)'s are therefore part of the integrand of the convolution. One cannot, as done in some earlier work, take this product outside the integrand and assign some constant effective value for the coupling. In our opinion, this procedure is rather unconvincing because \(\mathcal{M}^{(1)} \sim \alpha_s^3\) and therefore \(\Gamma^{(1)} \sim \alpha_s^6\). Changing \(\alpha_s\) from a value of 0.3 to 0.5 will make a factor of 20 difference in the width so provided the singlet contribution is of reasonable size, one can always choose a value such that the colour singlet alone can account for the experimental width.

As we argued in the previous section, colour octet is not suppressed for P-wave decay with respect to the singlet contribution therefore both should be needed to account for the experimental data. In table 1, we listed the singlet contributions to \(\chi_J \rightarrow p\bar{p}\) partial width. \(\chi_0\) is forbidden in this decay mode by angular momentum conservation. One can see that the singlet contribution can only account for 10% of \(\chi_1\) and 24% of \(\chi_2\) of the experimental partial width. So in agreement with our theoretical argument presented in the previous section, colour octet is needed in exclusive P-wave charmonium decay even in the absence of the same infrared divergence as found in the colour singlet inclusive decay width, to cancel which the colour octet was introduced.

Our theoretical argument and conclusion are however much more general than just for charmonium decay alone. One could apply these equally to bottomium as well as to P-wave quarkonium production. In these processes, colour octet should be required. One can however venture even further by making the suggestion that whenever a P-wave heavy quark-antiquark pair, which could be a part of the constituents of some component of a hadronic wavefunction, is needed, one will also have the need of the component of the corresponding colour octet accompanying the same set of other constituents as well.

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