Instabilities of a Bose-Einstein condensate in a periodic potential: an experimental investigation

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Abstract: By accelerating a Bose-Einstein condensate in a controlled way across the edge of the Brillouin zone of a 1D optical lattice, we investigate the stability of the condensate in the vicinity of the zone edge. Through an analysis of the visibility of the interference pattern after a time-of-flight and the widths of the interference peaks, we characterize the onset of instability as the acceleration of the lattice is decreased. We briefly discuss the significance of our results with respect to recent theoretical work.

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1. Introduction

Bose-Einstein condensates (BECs) trapped in the periodic potentials created by optical lattices have become a thriving field of research in recent years [1, 2, 3]. Since such systems are very similar in their theoretical description to electrons in a solid state crystal, many experiments have been carried out demonstrating phenomena that were, in some cases, well-known from condensed matter physics, e.g. Bloch oscillations and Landau-Zener tunneling [4, 5]. In a recent experiment, a BEC in an optical lattice even made possible the observation of a quantum phase transition that had, up to then, only been theoretically predicted for condensed matter systems [6].

Apart from the latter, most experiments to date have been carried out in the regime of shallow lattice depth, for which the system is well described by the Gross-Pitaevskii equation with a periodic potential. The nonlinearity induced by the mean-field of the condensate is included in this description and has been shown both theoretically and experimentally to give rise to instabilities [2, 7, 8, 9, 10, 11, 12] in certain regions of the Brillouin zone. In this paper we experimentally study these instabilities by accelerating the optical lattice and thus scanning the Brillouin zone in a controlled way.

Nonlinearity-induced instabilities are observed in many different branches of physics, and they provide a dramatic manifestation of strongly nonlinear effects (see review paper [13]). The effects of modulational and transverse instabilities have been observed mostly for continuous media. However, periodic structures can strongly influence such instabilities. The growth of the instability can be controlled in periodic structures where the effective geometric dispersion provides a key physical mechanism for manipulating waves in various physical systems, including Bragg gratings in optical fibers, waveguide arrays and, more recently, optically induced photonic lattices and crystals, as seen in recent observations of the propagation characteristics of a probe laser beam [14, 16, 15].

In this Letter we study, both theoretically and experimentally, matter wave propagation in optically-induced lattices near the condition for Bragg reflection. Formally, these experiments are closely linked to light scattering in periodic structures and can be described by similar mathematical models. In particular, our experiments reveal the role of nonlinear Bloch-wave interactions in periodic media.

This paper is organized as follows. After briefly describing in section 2 our experimental apparatus and the method used for scanning the Brillouin zone in a controlled way, we present the results of our experiments in section 3. A comparison between these results and a 1-D numerical simulation is the subject of section 4, followed by a discussion of our findings and prospects for future investigations (section 5).

2 Experimental setup and procedure

Our experimental apparatus for creating BECs of $^{87}\text{Rb}$ atoms is described in detail in [17]. The main feature of our apparatus relevant for the present work is the triaxial time-averaged orbiting potential (TOP) trap with trapping frequencies $\nu_x : \nu_y : \nu_z$ in...
the ratio $2 : 1 : \sqrt{2}$. Our trap is, therefore, almost isotropic. The consequences for our experiment of this near isotropy will be discussed at the end of this paper.

The optical lattice is created by two counterpropagating laser beams with parallel linear polarizations and wavelength $\lambda$, as described in detail in [4, 5]. The two beams are derived from the first diffraction orders of two acousto-optic modulators that are phase-locked but whose frequencies are independent, allowing us to introduce a frequency difference $\Delta \nu$ between them. The resulting periodic potential has a lattice constant $d = \lambda/2 = 0.39 \mu m$, and the depth of the potential (depending on the laser intensity and detuning from the atomic resonance of the rubidium atoms) can be varied between 0 and $\approx 2E_{\text{rec}}$ ($E_{\text{rec}} = \frac{\hbar^2 \pi^2}{2md^2}$). In addition, by linearly chirping the frequency difference $\Delta \nu$, the lattice can be accelerated with $a = d \frac{d\Delta \nu}{dt}$. In our experiments, we used accelerations ranging from $a = 0.3 \text{ m s}^{-2}$ to $a = 5 \text{ m s}^{-2}$.

The experimental protocol for ‘moving’ the condensate across the Brillouin zone is as follows. After creating BECs with $\approx 10^4$ atoms, we adiabatically relax the magnetic trap frequency to $\nu_x = 42 \text{ Hz}$. Thereafter, the intensity of the lattice beams is ramped up from 0 to a value corresponding to a lattice depth of $\approx 2E_{\text{rec}}$. The ramping time is of the order of several milliseconds in order to ensure adiabaticity [18]. Once the final lattice depth has been reached, the lattice is accelerated for a time $t$. Finally, both the magnetic trap and the optical lattice are switched off, and the condensate is observed after a time-of-flight of 21 ms by absorption imaging.

![Graphs](attachment:image.png)

Fig. 1. Integrated longitudinal and transverse profiles of the interference pattern of a condensate released from an optical lattice after acceleration to a quasimomentum $\approx 0.9$ and a subsequent time-of-flight of 21 ms. In (a) and (b), the acceleration $a$ was $5 \text{ m s}^{-2}$, whereas in (c) and (d) $a = 0.3 \text{ m s}^{-2}$. In (a) and (c), the horizontal axis has been rescaled in units of recoil momenta. Note the different vertical axis scales (by a factor 4) for the upper and lower graphs. The total number of atoms was measured to be the same in both cases.

3 Results: visibility and radial width

When the lattice is accelerated, the condensate feels a force in the rest frame of the lattice, resulting in a change of quasimomentum of the condensate. In the linear problem, this simply means that when switching off the lattice and magnetic trap at the end of the acceleration process, the instantaneous group velocity of the condensate in the lattice frame is given by the inverse of the curvature of the lowest energy band of the
lattice. The resulting Bloch oscillations have been observed in a previous experiment [4]. In practice, the time-of-flight interference pattern of the condensate released from the lattice then consists of a series of well-defined peaks corresponding to the momentum classes (in multiples of the lattice momentum $2p_{rec} = 2\hbar k_L$ with $k_L = 2\pi/\lambda$), as can be seen in Figs. 1 (a) and (c). The shape of the interference pattern in the transverse direction is shown in Figs. 1 (b) and (d).

In the nonlinear problem, the solutions of the Gross-Pitaevskii equation are predicted to be unstable in the vicinity of the Brillouin zone edge [2, 8, 11, 12]. When the condensate is close to the zone edge, the unstable solutions grow exponentially in time, leading to a loss of phase coherence of the condensate along the direction of the optical lattice. In our experiment, the time the condensate spends in the ‘critical region’ where unstable solutions exists is varied through the lattice acceleration. When the acceleration is small, the condensate moves across the Brillouin zone more slowly and hence the growth of the unstable modes [2] becomes more important. Figures 1 (c) and (d) show typical integrated profiles of the interference pattern for a lattice acceleration $a = 0.3\,\text{m s}^{-2}$. Here, the condensate has reached the same point close to the Brillouin zone edge as in Figs. 1 (a) and (b), but because of the longer time it has spent in the unstable region, the interference pattern is almost completely washed out. It is also evident that the radial expansion of the condensate is considerably enhanced when the Brillouin zone is scanned with a small acceleration.

In order to characterize our experimental findings more quantitatively, we define two observables for the time-of-flight interference pattern. By integrating the profile in a direction perpendicular to the optical lattice direction, we obtain a two-peaked curve (see Fig. 1 (a) ) for which we can define a visibility (in analogy to spectroscopy) reflecting the phase coherence of the condensate (visibility close to 1 for perfect coherence, visibility $\rightarrow 0$ for an incoherent condensate). In order to avoid large fluctuations of the visibility due to background noise and shot-to-shot variations of the interference pattern, we have found that a useful definition of the visibility is as follows:

$$visibility = \frac{h_{peak} - h_{middle}}{h_{peak} + h_{middle}},$$

where $h_{peak}$ is the mean value of the two peaks (both averaged over 1/10 of their separation symmetrically about the positions of the peaks). By averaging the longitudinal profile over 1/3 of the peak separation symmetrically about the midpoint between the peaks, we obtain $h_{middle}$. For instance, applying this definition to the profiles shown in Figs. 1 (a) and (c), we obtain $visibility = 0.98$ and $visibility = 0.6$, respectively. Owing to fluctuations in the background and hence the definition of the zero point of the longitudinal profile, the visibility thus measured can slightly exceed unity, in which case we define it to be 1. The second observable is the width of a Gaussian fit to the interference pattern integrated along the lattice direction over the extent of one of the peaks (see Fig. 1 (b) and (d)).

The results of our experiment are summarized in Fig. 2. For four different accelerations ($a = 5, 1, 0.5$ and $0.3\,\text{m s}^{-2}$) we measured the visibility and radial width of the interference pattern as a function of the quasimomentum of the condensate. For large accelerations, the quasimomentum can be simply calculated from $a$ and the duration of the lattice acceleration, whereas for small accelerations the restoring force of the magnetic trap has to be taken into account as the spatial motion of the condensate becomes appreciable. In these cases, we derived the quasimomentum reached in the experiment from a numerical integration of the semi-classical equations of motion of the condensate in the presence of the periodic potential (giving rise to Bloch oscillations due to the dispersion relation of the lowest energy band) and of the magnetic trap.
Fig. 2. Visibility and radial width as a function of quasimomentum (in units of \( p_{rec} \)) for different accelerations. As the acceleration is lowered, instabilities close to quasimomentum 1 (corresponding to the edge of the Brillouin zone) lead to a decrease in visibility and increase in radial width. For comparison, in each graph the (linear) fits to the visibility and radial width for the \( a = 5 \text{ m s}^{-2} \) data are included. The error bars on the visibility correspond to an estimated 10% systematic error, whereas the error bars on the radial width are the standard deviations of the Gaussian fits.
Figure 2 shows clearly that for accelerations down to $1 \text{ m s}^{-2}$, both the visibility and the radial width of the interference pattern remain reasonably stable when the edge of the Brillouin zone is crossed. In contrast, for $a = 0.5 \text{ m s}^{-2}$ and $a = 0.3 \text{ m s}^{-2}$ one clearly sees a drastic change in both quantities as the quasimomentum approaches the value 1. For those accelerations, the condensate spends a sufficiently long time in the unstable region of the Brillouin zone and hence loses its phase coherence, resulting in a sharp drop of the visibility. At the same time, the radial width of the interference pattern increases by a factor between 1.5 and 1.7. This increase is evidence for an instability in the transverse direction and may, for instance, be due to solitons in the longitudinal direction decaying into vortices via a snake instability [11]. For $a = 0.5 \text{ m s}^{-2}$ and $a = 0.3 \text{ m s}^{-2}$, the interference patterns for quasimomenta larger than 1 were so diffuse that it was not possible to measure the visibility nor the radial width in a meaningful way.

![Figure 3](image)

**Fig. 3.** Results of a one-dimensional numerical simulation of our experiment for different values of the nonlinear parameter $C$ and acceleration $a = 0.3 \text{ m s}^{-2}$. The open squares, circles and triangles correspond to $C = 0.008$ (the value for our experiment), $C = 0.004$ and $C = 0$, respectively. The closed symbols are the experimental values of the visibility as reported in Fig. 2 for $a = 0.3 \text{ m s}^{-2}$. The dashed lines connect the theoretical points to guide the eye.

### 4 Comparison with a 1-D numerical simulation

As pointed out in the introduction, instabilities in nonlinear periodic structures are a topic of intense theoretical and experimental interest and have been discussed in many publications to date. In particular, the exact nature of the instabilities, in particular in 3-D, is still a matter of controversy. In this work, we limit ourselves to comparing our experimental results to a simple 1-D numerical simulation. A more in-depth discussion of 3-D simulations and analytical calculations will be presented elsewhere.

Figure 3 shows the results of a numerical integration of the one-dimensional Gross-Pitaevskii equation with the parameters of our experiment. The visibility was calculated in the same way as was done for the experimental interference patterns. It is clear from this simulation that it is, indeed, the nonlinearity that is responsible for the instability at the edge of the Brillouin zone. The nonlinearity can be characterized through the parameter [19]

$$C = \frac{\pi n_0 a_s}{k_L^2},$$

(2)

where $n_0$ is the density of the condensate and $a_s = 5.4 \text{ nm}$ is the $s$-wave scattering length of $^{87}\text{Rb}$. For the experimental parameters used in the experiment described here, the value of $C$ was $\approx 0.008$. 
When $C$ is set to 0 in the numerical simulation, the visibility remains unaltered when the zone edge is crossed, whereas for finite values of $C$ the visibility decreases as a quasimomentum of 1 is approached. Furthermore, the larger the value of $C$, the more pronounced the decrease in visibility near the band edge. For $C = 0.008$, corresponding to the value realized in our experiment, the onset of the instability is located just below a quasimomentum of 0.8. Experimentally, we find that the visibility starts decreasing consistently beyond a quasimomentum of $\approx 0.6 - 0.7$, agreeing reasonably well with the results of the simulation.

5 Discussion and outlook

The experimental results presented in this work demonstrate that at the edge of the Brillouin zone, a BEC in an optical lattice exhibits unstable behaviour. These instabilities are reproduced in a 1-D numerical simulation, and our experimental findings also agree qualitatively with recent 3-D simulations [11]. The fact that we observe a significant effect of the instabilities below accelerations of $\approx 0.5 \, \text{m s}^{-2}$, for which the condensate spends more than 2 ms in the critical region around the edge of the Brillouin zone (having an extension of around 1/10 of the BZ [2]), indicates that the growth rate of the instability should be of the order of $500 \, \text{s}^{-1}$. This agrees reasonably well with a rough estimate of $\approx 300 \, \text{s}^{-1}$ derived from a recent work by Wu and Niu [2].

Clearly, our approach has a number of shortcomings that prevent us from achieving a more quantitative agreement with theory and a better characterization of the instabilities. Firstly, in our experiment the condensate exhibits a residual sloshing motion, meaning that its velocity is not always exactly zero when we load it into the optical lattice. This results in an uncertainty in the quasimomentum which we estimate to be of the order of 10 – 15% and leads to a considerable scatter in the experimental data both of the visibility and the radial width, possibly masking a sharper transition between the stable and unstable regions. Also, the fact that our trap is almost isotropic makes it difficult for us to increase the nonlinear parameter $C$ through the condensate density by increasing the trap frequency, as this would result in an even stronger restoring force along the lattice direction, making it impossible to cross the BZ edge with a constant (and small) acceleration. In order to overcome this problem, it would be advantageous to use, for instance, a dipole trap which allows one to increase the radial trapping frequency whilst still maintaining a small longitudinal frequency. Another interesting question to address is whether the fact that the direction of our optical lattice is at a small angle (a few degrees) to the axis of the magnetic trap might lead to chaotic motion, contributing to the increase in radial width of the interference pattern we observe [20].

In summary, the present experimental observations confirm that Bose-Einstein condensates may be used to simulate a variety of nonlinear physics configurations that are also of interest to other areas of physics, such as solid state physics. Bose-Einstein condensates offer the added advantage of a large flexibility in the parameters and in the exploration of phenomena occurring on a millisecond time scale.

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