1. Introduction

We consider a coupled (hybrid) system, which describes the interaction of a homogeneous viscous incompressible fluid, which occupies a domain $\Omega$ bounded by the (solid) walls of the container $\partial$ and a horizontal boundary $/DEL$ on which a thin (nonlinear) elastic plate is placed. The motion of the fluid is described by the linearized 3D Navier–Stokes equations. To describe the deformations of the plate, we involve a general (full) Kirchhoff–Karman type model (see, e.g., [1–3] and the references therein) with additional hypothesis that the transversal displacements of the plate are negligible relative to in-plane displacements. Thus we only consider longitudinal deformations of the plate and take account of tangential shear forces, which the fluid exerts on the plate. These kinds of models arise in the study of the problem of blood flow in large arteries (see, e.g., [4] and also [5] for a more recent discussion and references).

We note that the mathematical studies of the problem of fluid–structure interaction in the case of viscous fluids and elastic plate/bodies have a long history. We refer to [5–9] and the references therein for the case of plates/membranes, to [10] in the case of moving elastic bodies, and to [11–16] in the case of elastic bodies with a fixed interface; see also the literature cited in these references.

Our mathematical model is formulated as follows:

Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with a sufficiently smooth boundary $\partial \Omega$. We assume that $\partial \Omega = \Omega \cup \partial$, where $\Omega \subset \{x = (x_1; x_2; 0) : (x_1; x_2) \in \mathbb{R}^2\}$ and $\partial$ is a smooth surface, which lies under the plane $x_3 = 0$. The exterior normal on $\partial \Omega$ is denoted by $n$. We have that $n = (0; 0; 1)$ on $\Omega$. We consider the following linear Navier–Stokes equations in $\Omega$ for the fluid velocity field $v = v(x, t) = (v^1(x, t); v^2(x, t); v^3(x, t))$ and for the pressure $p(x, t)$:

\begin{align}
\partial_t v - v \Delta v + \nabla p &= \mathbb{G} & \text{in } & \Omega \times (0, +\infty), \\
\text{div } v &= 0 & \text{in } & \Omega \times (0, +\infty), \\
v(x, 0) &= v_0(x) & \text{in } & \Omega,
\end{align}

Communicated by W. Sprößig

We study asymptotic dynamics of a coupled system consisting of linearized 3D Navier–Stokes equations in a bounded domain and the classical (nonlinear) elastic plate equation for in-plane motions on a flexible flat part of the boundary. The main novelty of the model is the assumption that the transversal displacements of the plate are negligible relative to in-plane displacements. These kinds of models arise in the study of blood flows in large arteries. Under some conditions this attractor is an exponentially attracting single point. We also show that the corresponding linearized system generates an exponentially stable $C_0$-semigroup. We do not assume any kind of mechanical damping in the plate component. Thus our results mean that dissipation of the energy in the fluid because of viscosity is sufficient to stabilize the system. Copyright © 2011 John Wiley & Sons, Ltd.

Keywords: fluid–structure interaction; linearized 3D Navier–Stokes equations; nonlinear plate; finite-dimensional attractor
where \( v > 0 \) is the dynamical viscosity and \( \tilde{G} \) is a volume force. We supplement Equations (1)–(3) with the (non-slip) boundary conditions imposed on the velocity field \( v = v(x, t) \):

\[
v = 0 \text{ on } S_t; \quad v = \left(v^1; v^2; v^3\right) = \left(u^1_0; 0\right) = \left(u^1_1; u^2_0; 0\right) \text{ on } \Omega,
\]

(4)

where \( u = u(x, t) \equiv (u^1(x, t); u^2(x, t)) \) is the in-plane displacement vector of the plate placed on \( \Omega \) satisfying the following equations (see, e.g., [2] and [3]):

\[
\rho u_{tt}^1 - \left(\partial_{x_1} N_{11} + \partial_{x_2} N_{12}\right) + F^1 = 0 \text{ in } \Omega \times (0, \infty),
\]

(5)

and

\[
\rho u_{tt}^2 - \left(\partial_{x_1} N_{21} + \partial_{x_2} N_{22}\right) + F^2 = 0 \text{ in } \Omega \times (0, \infty),
\]

(6)

with

\[
N_{11} = D \left(u^1_1 + \mu u^2_{x_2}\right), \quad N_{22} = D \left(u^2_2 + \mu u^1_{x_1}\right)
\]

and

\[
N_{12} = N_{21} = D \frac{1}{2} (1 - \mu) \left(u^1_1 + u^2_{x_2}\right),
\]

where \( D = Eh / (1 - \mu^2) \), \( E \) is Young’s modulus, \( 0 < \mu < 1/2 \) is Poisson’s ratio, \( h \) is the thickness of the plate, \( \rho \) is the mass density. The external (in-plane) force \( (F^1; F^2) \) in Equations (5) and (6) consists of two parts,

\[
F^i = f^i \left(u^1, u^2\right) + T_i(v), \quad i = 1, 2,
\]

where \( \left(f^1(u^1, u^2); f^2(u^1, u^2)\right) \) is a nonlinear feedback force represented by some potential \( \Phi \) (which we specify below)

\[
f^i \left(u^1, u^2\right) = \frac{\partial \Phi \left(u^1, u^2\right)}{\partial u^i}, \quad i = 1, 2,
\]

and \( (T_1(v); T_2(v)) \) is the viscous shear stress exerted by the fluid on the plate, \( T_i(v) = ((Tn)|_{\Omega}, e_i)_{\mathbb{R}^3} \). Here, \( T = \left\{ T^{ij}_{ij} \right\}_{ij=1}^3 \) is the stress tensor of the fluid, given by

\[
T^{ij} = T_{ij} (v) = v \left(v^i_{x_j} + v^j_{x_i}\right) - \rho \delta^{ij}, \quad i, j = 1, 2, 3,
\]

\( e_1 = (1; 0; 0), e_2 = (0; 1; 0) \) are unit tangential vectors on \( \Omega \subset \partial \Omega \), and \( n = (0; 0; 1) \) is the outer normal vector to \( \partial \Omega \) on \( \Omega \). A simple calculation shows that

\[
T_{ij} (v) = T_{ij} (v) = v \left(v^i_{x_j} + v^j_{x_i}\right) = vv_{x_i x_j}, \quad i = 1, 2
\]

(in the last equality, we use the fact that \( v^3 (x_1; x_2; 0) = 0 \) for \( (x_1; x_2) \in \Omega \) because of the second relation in Equation (4) and hence \( v^3 = 0 \) on \( \Omega, i = 1, 2 \)).

Thus, we arrive at the following equations for the in-plane displacement \( u = (u^1; u^2) \) of the plate (below for some notational simplifications we assume that \( \rho h = 1 \) and \( D(1 - \mu)/2 = 1 \)):

\[
u^i_{tt} - \Delta u^i - \lambda \partial_{x_i} \left[\text{div } u\right] + vv^i_{x_3} |_{x_3} = 0 + f^i (u) = 0, \quad i = 1, 2,
\]

(7)

where \( \lambda = (1 + \mu) (1 - \mu)^{-1} \) is a nonnegative parameter. For the displacement \( u = (u^1; u^2) \) we impose clamped boundary conditions on \( \Gamma = \partial \Omega \):

\[
u^i = 0 \text{ on } \Gamma, \quad i = 1, 2.
\]

(8)

Our main point of interest is the well-posedness and long-time dynamics of solutions to the coupled problem in Equations (1)–(4), (7), and (8) for the velocity \( v \) and the displacement \( u = (u^1; u^2) \) with the initial data

\[
v|_{t=0} = v_0, \quad u|_{t=0} = u_0, \quad u_t|_{t=0} = u_1.
\]

(9)

This problem in the case when \( \tilde{G} \equiv 0, \lambda = 0, \) and \( f^i (u) \equiv 0 \) was considered in [8] (see also [5, 7]) with an additional locally distributed strong (Kelvin–Voight type) damping force applied to the interior of the plate. These papers deal with the existence and asymptotic stability of the corresponding semigroup. In contrast with [5, 7, 8], we do not assume the presence of mechanical damping terms in the plate component of the system but consider nonlinearly forced model.
Our main result (see Theorem 4.1) states that under some natural conditions concerning the potential $\Phi(u)$ of feedback forces the system (1)–(4), (7)–(9) possesses a compact global attractor of finite fractal dimension, which also has some additional regularity properties. If the force potential $\Phi(u)$ possesses some monotonicity properties and the volume force $G$ in the fluid is absent, then this attractor is an exponentially attracting single point (see Proposition 4.3). Our argumentation involves a recently developed approach based on quasi-stability properties and stabilizability estimates (see, e.g., [17–19] for the second order in time evolution equations and also [20, 21] for thermoelastic problems).

We also improve substantially the stability results presented in [5, 7, 8]. Namely, as a consequence of the dissipativity property of the nonlinear model we prove (see Corollary 4.4) that the linear part of Equations (1)–(4) and Equations (7)–(9) generates a uniformly exponentially stable $C_0$-semigroup of contractions without any dissipation mechanisms (except the fluid viscosity) in the system.

We note that a linear model related to Equations (1)–(4) and Equations (7)–(9) was considered in [22, Section 3.15] from the point of view of boundary controllability. In [22], the authors deal with the so-called linearized model of well/reservoir coupling with monophasic flow. This is a scalar linear model represented by the diffusion (heat) equation in $O$ coupled with the wave equation on $\Omega$ with an interface condition like in Equation (4). The scalar structure of the model makes it possible to prove [22, Proposition 3.15.5] that this model generates a contractive analytic and thus exponentially stable semigroup. We do not know whether the result on analyticity remains true for the hydrodynamical situation we consider. We also note that in fact the long time dynamics in problem (7) with a given stationary smooth velocity field $v$ was studied in [23] and [18, Chapter 7].

The paper is organized as follows: in Section 2, we discuss an auxiliary Stokes problem and rewrite problem (1)–(4), (7)–(9) as a single first order equation in some extended space $\mathcal{H}$. Then we prove that the linear version of the problem generates a strongly continuous contractive semigroup in $\mathcal{H}$. In Section 3, we prove the existence of strong and generalized solutions to the original nonlinear problem and establish an energy balance identity. For this, we use ideas presented in [24]. Section 4 contains our main results on long-time dynamics of system (1)–(4), (7)–(9). Our approach is based mainly on the idea of quasi-stability (see, e.g., [18] and [19, Section 7.9]).

## 2. Preliminaries

In this section, we first provide some results for the Stokes problem and then consider the abstract form of the system. In the succeeding texts, we denote by $H^s(D)$ the Sobolev space of the order $s$ on the set $D$, by $H^s_0(D)$ the closure of $C_0^\infty(D)$ in $H^s(D)$ and by $H^s(D)/\mathbb{R}$, the factor–space with the naturally induced norm. To describe traces of the fluid velocity field on $\Omega$, we use the interpolation space $[L_2(\Omega), H^s_0(\Omega)]_{1/2} = H_{00}^{1/2}(\Omega)$ and its adjoint $H_{00}^{-1/2}(\Omega)$ (for more details concerning these spaces, we refer to [25]).

### 2.1. Stokes problem

In our further argumentation, we need some regularity properties of the terms responsible for fluid–plate interaction. To achieve this, we consider the following Stokes problem:

\[
\begin{align*}
-v\Delta v + \nabla p &= g, & \text{div } v &= 0 & \text{in } O; \\
v &= 0 & \text{on } \Sigma; \\
 v &= (\psi; 0) & \text{on } \Omega,
\end{align*}
\]

where $g \in [L_2(O)]^3$ and $\psi = (\psi^1; \psi^2) \in [L^2(\Omega)]^2$ are given. This type of boundary value problems for the Stokes equation was studied by many authors. We refer to a discussion and references in the classical monographs [26, 27] (see also a more recent publication [28] and the references therein). We collect some properties of solutions of Equation (10) in the following assertion:

**Proposition 2.1**

Let $g \in [L^2(O)]^3$ and $\psi \in [H_0^1(\Omega)]^2$. Then

1. Problem (10) has a unique solution $\{v, p\} \in [H^{3/2}(O)]^3 \times [H^{1/2}(O)]^3$ such that

\[
\|\nabla\psi\|_{[H^{1/2}(O)]^3} + \|p\|_{[H^{1/2}(O)]^3} \leq C_0 \left(\|g\|_{[H^{1/2}(O)]^3} + \|\psi\|_{[H^{1/2}(O)]^3}\right) \quad \text{for every } 0 < s \leq 1/2.
\]

2. We have $\gamma_{\Omega} v \in [H_{00}^{1/2}(\Omega)]^2$ for the trace operator $\gamma_{\Omega}$ defined (on smooth functions) by the formula

\[
\gamma_{\Omega} v = v \left( v^1, v^2 \right) \quad \text{for } v = (v^1; v^2; v^3) \in [H^2(O)]^3.
\]

3. If $g = 0$, we also have

\[
\|v\|_{[L_2(O)]^3} \leq C_0 \|\psi\|_{[H^{1/2}(\Omega)]^3} \quad \text{for every } \delta > 0,
\]

thus we can define a bounded linear operator $N_0 : \left[H^{1/2+\delta}(\Omega)\right]^2 \mapsto [L_2(O)]^3$ by the formula

Copyright © 2011 John Wiley & Sons, Ltd. Math. Meth. Appl. Sci. 2011, 34 1801–1812
\[ N_0 \psi = w \text{ iff } \begin{cases} -\nu \Delta w + \nabla p = 0, & \text{div} \ w = 0 \text{ in } \Omega; \\ w = 0 \text{ on } \Sigma; & \ w = (\psi; 0) \text{ on } \Omega, \end{cases} \] (14)

for \( \psi = (\psi_1; \psi_2) \in \left[H^{-1/2+\delta}(\Omega)\right]^2 \) \( (N_0 \psi \text{ solves Equation (10) with } g = 0). \)

**Proof**

1. The existence and uniqueness of solutions along with the bound in Equation (11) follow from Proposition 2.3 and Remark 2.6 on interpolation inequalities for Sobolev norms in [27, Chapter 1]. We also use the fact that the extension by zero gives us a continuous embedding of \([H^s_0(\Omega)]^2\) into \([H^{s+1/2}(\partial \Omega)]^2\) for every \( s > 0, s \neq 1, 2, \ldots \) (see [25]).

2. To prove Statement 2, we use the same idea as in [12], which involves the boundary properties of harmonic functions (see [29]).

We can represent \( \psi \) in the form \( \psi = \tilde{\psi} + \psi^* \), where \( \tilde{\psi} \) solves Equation (10) with \( g = 0 \) and \( \psi^* \) satisfies Equation (10) with \( g = 0 \). By Proposition 2.3 [27, Chapter 1], we have \( \tilde{\psi} \in \left[H^2(\Omega)\right]^2 \) and thus by the standard trace theorem, there exists \( \partial_n \tilde{\psi}|_{\partial \Omega} \in \left[H^{1/2}(\partial \Omega)\right]^2 \). Consequently \( \gamma_{\Omega} \psi \in \left[H^{1/2}(\Omega)\right]^2 \subset \left[H^{-1/2}(\Omega)\right]^2 \).

Thus we need to establish Statement 2 in the case \( g = 0 \) only. In this case, the pressure \( p \) is a harmonic function in \( \Omega \), which belongs \( L_2(\Omega) \). Thus by Theorem 3.8.1 [29], we can assign meaning to \( p|_{\Sigma} \) in \( H^{-1/2}(\partial \Omega) \). Now using the Gauss–Ostrogradskii formula, one can see that \( \nabla p \in ((H^1(\Omega))^3) \). Therefore from Equation (10) with \( g = 0 \), we have \( \Delta \psi \in ((H^1(\Omega))^3) \). Using the Green formula

\[
\int_{\Omega} \partial_n \psi \, dS = \int_{\Omega} \Delta \psi \, dx - \int_{\partial \Omega} \nabla \psi \cdot n \, dS, \quad \forall \psi \in H^1(\Omega),
\]

for every velocity component \( \psi \), we conclude that \( \partial_t \psi \in H^{-1/2}(\partial \Omega) \) and thus \( \gamma_{\Omega} \psi \in \left[H^{-1/2}(\Omega)\right]^2 \).

3. The relation in Equation (13) is a special case of Theorem 3 in [28]. To apply this theorem, we first need to extend \( \psi \) by zero from \( \Omega \) to \( \partial \Omega \). This is why the positive parameter \( \delta \) appears in Equation (13).

**2.2. Abstract representation of the problem**

We introduce the following spaces:

\[
X = \left\{ \psi \in [L_2(\Omega)]^3 : \text{div} \ \psi = 0; \ \gamma_\Sigma \psi = (\psi, n) = 0 \text{ on } \partial \Omega \right\};
\]

and

\[
V = \left\{ \psi \in [H^1(\Omega)]^3 : \text{div} \ \psi = 0; \ \psi = 0 \text{ on } \Sigma; \ \gamma_\Sigma \psi = 0 \text{ on } \partial \Omega \right\}.
\]

We equip \( X \) with \( L_2 \)-type norm \( \| \cdot \|_X \) and denote by \( (\cdot, \cdot)_X \) the corresponding inner product. The space \( V \) is endowed with the norm \( \| \cdot \|_V = \| \nabla \cdot \|_0 \).

Let \( \mathcal{P}_l \) be the Leray projector, which maps the space \([L_2(\Omega)]^3\) onto \( X \). With this projector, we rewrite Equations (1)–(4) in \( X \) as follows:

\[
\partial_t v + A_0 (v - N_0 u_\Sigma) = G, \quad t > 0, \quad v|_{t=0} = v_0,
\]

where \( G = \mathcal{P}_l \xi \in X, N_0 \) is defined by Equations (14), and \( A_0 = -\nu \mathcal{P}_l \Delta \) is a positive self-adjoint operator with the domain

\[
\mathcal{D}(A_0) = \left\{ \psi \in [H^2(\Omega)]^3 : \text{div} \ \psi = 0; \ \psi = 0 \text{ on } \partial \Omega \right\}.
\]

For the plate component we use the spaces

\[
Y = [L_2(\Omega)]^2 \times L_2(\Sigma) \quad \text{and} \quad W = \left[H^2_0(\Omega)\right]^2.
\]

We denote by \( \| \cdot \|_Y \) and \( (\cdot, \cdot)_Y \) the norm and the inner product in \( Y \).

In the space \( Y \) for \( \lambda > 0 \), we introduce the operator

\[
A = -\begin{bmatrix}
(1 + \lambda) \partial^2_{x_1} + \partial^2_{x_2} & \lambda \partial^2_{x_1 x_2} \\
\lambda \partial^2_{x_1 x_2} & \partial^2_{x_1} + (1 + \lambda) \partial^2_{x_2}
\end{bmatrix}
\]

with the domain \( \mathcal{D}(A) = ((H^2 \cap H^1_0)(\Omega))^2 \subset W \). One can see that \( A \) is a positive operator in \( Y \) generated by the form

\[
\sigma(u, \ddot{u}) = \sum_{i=1}^{2} \int_{\Omega} \nabla u^i \nabla \ddot{u}^i \, d\Omega + \lambda \int_{\Omega} \text{div} \ u \cdot \text{div} \ \ddot{u} \, d\Omega = (A^{1/2} u, A^{1/2} \ddot{u})_\Omega,
\]
where \( u = (u^1; u^2) \) and \( \dot{u} = (\dot{u}^1; \dot{u}^2) \) are from \( W \). With this operator \( A \) problems (7) and (8) can be written in the space \( Y \) as

\[
\begin{align*}
\text{for } t > 0, \quad u|_{t=0} = u_0, \quad u|_{t=0} = u_1.
\end{align*}
\]

where \( u = (u^1; u^2) \), the operator \( A \) is defined in Equation (16), \( \gamma \Omega \) is given by Equation (12), and \( f(u) = (f^1(u); f^2(u)) \).

Now, we consider the phase space \( \mathcal{H} = X \times W \times Y \) with the inner product

\[
(U, U^*)_\mathcal{H} = (v, v^*)_X + \left( A^{1/2} u_0, A^{1/2} u_0^* \right)_\Omega + (u_1, u_1^*)_\Omega,
\]

where \( U = (v, u_0, u_1) \) and \( U^* = (v^*, u_0^*, u_1^*) \) are elements of \( \mathcal{H} \).

We rewrite problems (15) and (17) as a first order equation for the phase variable \( U = (v, u_0, u_1) \in \mathcal{H} \) of the form

\[
\frac{dU}{dt} + AU + F(U) = 0, \quad t > 0, \quad U|_{t=0} = U_0,
\]

where \( F(U) = (0; 0; f(u)) \) and

\[
A = \begin{bmatrix}
-\nu P_1 \Delta & 0 & 0 \\
0 & 0 & -I \\
\gamma \Omega & A & 0
\end{bmatrix} = \begin{bmatrix}
A_0 & 0 & -A_0 N_0 \\
0 & 0 & -I \\
\gamma \Omega & A & 0
\end{bmatrix}
\]

with the domain

\[
\mathcal{D}(A) = \left\{ U = \begin{bmatrix} v \\ u_0 \\ u_1 \end{bmatrix} \in \mathcal{H} \mid A_0 (v - N_0 u_1) = -\nu P_1 \Delta v \in X, \ u_1 \in W, \ A u_0 + \gamma \Omega v \in Y, \ v|_{\Omega} = (u_1; 0) \right\}
\]

To solve Equation (18), we use methods from [24]. We first prove the following assertion:

**Proposition 2.2**

The operator \( A \) is a maximal accretive operator in \( \mathcal{H} \). Moreover, \( R(A) = \mathcal{H} \) and thus \( A \) is invertible and by the Lumer–Phillips theorem (see [24, p.14]) generates a \( C_0 \)-semigroup of contractions in \( \mathcal{H} \).

**Proof**

One can see that

\[
(AU, U^*)_\mathcal{H} = (v (\nabla v, \nabla v^*)_X - (Au_0, u_0^*))_\Omega + (Au_1, u_1^*)_\Omega,
\]

where \( U = (v, u_0, u_1) \) and \( U^* = (v^*, u_0^*, u_1^*) \) are elements of \( \mathcal{D}(A) \). This implies that \( (AU, U^*)_\mathcal{H} = v ||\nabla v||_Y^2 \geq 0 \) and thus \( A \) is accretive.

To prove maximality, it suffices to show that \( R(A) = \mathcal{H} \); that is, to solve the equation of the form \( AU = F \) for \( F \equiv (g; h_0; h_1) \in \mathcal{H} \). We obviously have \( U = (v; u; -h_0) \), where \( v \in V \) solves

\[
-\nu \Delta v + \nabla p = g, \quad \text{div } v = 0 \quad \text{in } \mathcal{O};
\]

\[
v = 0 \text{ on } \partial \mathcal{O}; \quad v = (-h_0; 0) \text{ on } \Omega,
\]

with \( h_0 \in W = [H^2_0(\Omega)]^2 \), and \( u \in W \) satisfies the equation

\[
Au = -\gamma \Omega v + h_1, \quad h_1 \in Y = [L_2(\Omega)]^2.
\]

It follows from Proposition 2.1 that there exists \( v \in [H^{3/2}(\mathcal{O})]^3 \) satisfying Equation (21) such that \( \gamma \Omega v \in [H^{-1/2}_0(\Omega)]^2 = [\mathcal{D}(A^{1/4})]' \).

Now, we can solve Equation (22) with respect to \( u \). Thus \( R(A) = \mathcal{H} \).

**Remark 2.3**

The previous argument shows that

\[
\mathcal{D}(A) \subset \left\{ U = \begin{bmatrix} v \\ u_0 \\ u_1 \end{bmatrix} \in \mathcal{H} \mid v \in [H^{3/2}(\mathcal{O})]^3, \ \gamma \Omega v \in [H^{-1/2}_0(\Omega)]^2, \ v|_{\partial \mathcal{O}} = 0, \ v|_{\Omega} = (u_1; 0), \ u_0 \in [(H^{3/2} \cap H^1_0)(\Omega)]^2, \ u_1 \in [H^1_0(\Omega)]^2 \right\}.
\]

Copyright © 2011 John Wiley & Sons, Ltd.  
Math. Meth. Appl. Sci. **2011**, 34 1801–1812
3. Well-posedness

We assume that the plate force potential \( \Phi(u) \) possesses the properties

- \( \Phi(u) \in C^2(\mathbb{R}^2) \) is a non-negative polynomially bounded function, that is,
  \[
  \left| \frac{\partial^2 \Phi(u)}{\partial u_i \partial u_j} \right| \leq C (1 + |u|^p), \quad i, j = 1, 2, \quad u = (u^1, u^2) \in \mathbb{R}^2, \tag{23}
  \]
  for some \( C, p \geq 0 \).
- The following dissipativity condition holds: for any \( \delta > 0 \) there exists \( c_1(\delta) > 0 \) and \( c_2(\delta) \geq 0 \) such that
  \[
  \sum_{i=1,2} u^i f'(u) - c_1(\delta)\Phi(u) + \delta |u|^2 \geq -c_2(\delta) \quad \text{with} \quad f'(u) = \frac{\partial \Phi(u)}{\partial u_i}. \tag{24}
  \]

As an example of a such potential \( \Phi(u) \) we can consider

\[
\Phi(u) = \psi_0(|u^1|^2 + |u^2|^2) \text{ or } \Phi(u) = \psi_1(u^1) + \psi_2(u^2),
\]

where \( \psi_i(s) \) are non-negative functions in \( C^2(\mathbb{R}) \) such that

(a) \( |\psi_i''(s)| \leq C (1 + |s|^q) \) for some \( C, q \geq 0 \);
(b) \( s \psi_i'(s) - c_0 \psi_i(s) \geq -c_1 \) for some \( c_0 > 0 \) and \( c_1 \geq 0 \).

For instance, \( \psi_i(s) \) can be polynomials of even degree with a positive leading coefficient and and with a sufficiently large free term (the value of this free term has no importance because force potentials are defined up to arbitrary constants).

We also note that under the assumption in Equation (23) the nonlinear force

\[
f(u) = (f^1(u); f^2(u)) = \left( \frac{\partial \Phi(u)}{\partial u^1}, \frac{\partial \Phi(u)}{\partial u^2} \right)
\]

in Equation (17) satisfies the following local Lipschitz property:

\[
\|f(u) - f(\tilde{u})\| \leq C \|u - \tilde{u}\|, \quad u, \tilde{u} \in W,
\]

for every \( 0 < \sigma < 1 \) and \( u, \tilde{u} \in W \), where \( \| \cdot \|_\sigma, \Omega \) denotes the norm in the space \( H^\sigma(\Omega)^2 \). To see this, we note that Equation (23) implies the estimate

\[
\|f(u) - f(\tilde{u})\| \leq C (1 + |u|^p + |\tilde{u}|^p) |u - \tilde{u}|, \quad u, \tilde{u} \in \mathbb{R}^2.
\]

Therefore the Hölder inequality and the embedding \( H^\sigma(\Omega) \subset L_{2/(1-\sigma)}(\Omega) \) for \( 0 < \sigma < 1 \) imply Equation (26).

Following the standard semigroup approach (see, e.g., [24] or [30]) we give the following definition.

**Definition 3.1**

The function \( U(t) = (v(t); u(t); u_1(t)) \in C(0, T; \mathcal{H}) \) such that \( U(0) = U_0 = (v; u_0; u_1) \) is said to be

- a strong solution to problem (18) on an interval \( [0, T] \) if (i) \( U(t) \in \mathcal{D}(A) \) for almost all \( t \in (0, T) \); (ii) \( U_1 \in L_2(0, T; \mathcal{H}) \); and (iii) (18) is satisfied as an equality in \( \mathcal{H} \) for almost all \( t \in (0, T] \);
- a generalized solution to problem (18) if there exist a sequence of initial data \( U_0^n \) and the corresponding strong solutions \( U^n(t) \) such that

\[
\lim_{n \to \infty} \max_{t \in [0, T]} \| U^n(t) - U(t) \|_{\mathcal{H}} = 0.
\]

**Theorem 3.2**

Let \( U_0 \in \mathcal{H} \). Then for any interval \( [0, T] \) there exists a unique generalized solution \( U(t) = (v(t); u(t); u_1(t)) \) such that \( v \in L_2(0, T; V) \) and the energy balance equality

\[
\mathcal{E}(v(t), u(t), u_1(t)) + \nu \int_0^t \| \nabla v \|^2_{\Omega} \, dt = \mathcal{E}(v_0, u_0, u_1) + \int_0^t (G, v)_{\Omega} \, dt \tag{27}
\]

holds for \( t > 0 \), where

\[
\mathcal{E}(v(t), u(t), u_1(t)) = \frac{1}{2} \| v(t) \|^2_{\Omega} + E(u(t), u_1(t)) \tag{28}
\]

with the energy \( E(u, u_1) \) of the plate given by

\[
E(u, u_1) = \frac{1}{2} \| u_1 \|^2_{\Omega} + \| A^{1/2} u \|^2_{\Omega} + \int_\Omega \Phi(u(x)) \, d\Omega.
\]
Moreover,

- Any generalized solution is also mild, that is,
  \[ U(t) = e^{-tA}U_0 + \int_0^t e^{-(t-\tau)A}F(U(\tau))d\tau, \quad t > 0. \]  
  (29)

- There exists a constant \( a_{R,T} > 0 \) such that for any couple of solutions, \( U(t) = (v(t); u(t); u_t(t)) \) and \( \hat{U}(t) = (\hat{v}(t); \hat{u}(t); \hat{u}_t(t)) \) with the initial data possessing the property \( \| U_0 \| _{\mathcal{H}}, \| \hat{U}_0 \| _{\mathcal{H}} \leq R \), we have
  \[ \| U(t) - \hat{U}(t) \| _{\mathcal{H}}^2 + \int_0^t \| \nabla (v - \hat{v}) \| _{C_0}^2 d\tau \leq a_{R,T} \| U_0 - \hat{U}_0 \| _{\mathcal{H}}^2 \]  
  (30)

  for every \( t \in [0,T] \).

- The solution \( U(t) \) is strong if \( U_0 \in \mathcal{D}(A) \).

Proof

By Proposition 2.2 the linear part of Equation (18) generates a strongly continuous semigroup. By Equation (26), we also have that the nonlinear part in Equation (18) is locally Lipschitz on \( \mathcal{H} \). Therefore the existence of (local) strong and generalized solutions follows from Theorem 6.1.6 [24] and from the argument provided in the proof of Theorem 6.1.4 [24]. The latter theorem also means that generalized solutions are mild on the existence interval.

Now, we consider strong solutions on the existence interval and establish the energy relation. A simple calculation gives

\[ \frac{1}{2} \frac{d}{dt} \| v(t) \| _{C_0}^2 + \| \nabla v(t) \| _{C_0}^2 = v \int_{\Omega} \left[ \frac{\partial v}{\partial n} u_1^2 + \frac{\partial v}{\partial n} u_2^2 \right] d\Omega + \int_{\partial \Omega} vGd\tau. \]  

(31)

It is also clear that for plate we have that

\[ \frac{d}{dt} E(u(t), u_t(t)) + v \int_{\Omega} \left[ \frac{\partial v}{\partial n} u_1^2 + \frac{\partial v}{\partial n} u_2^2 \right] d\Omega = 0 \]  

(32)

The sum of equalities (31) and (32) along with the relation \( n = (0; 0; 1) \) on \( \Omega \) after integration gives the energy equality in Equation (27) for strong solutions on the existence interval.

It follows from Equations (27) and (23) that

\[ \| U(t) \| _{\mathcal{H}}^2 \leq C_R + \int_0^t \| G(\tau) \| _{C_0}^2 d\tau, \quad \| U_0 \| _{\mathcal{H}} \leq R, \]  

(33)

on the existence interval. This implies (see, e.g., Theorem 6.1.4 [24]) that both strong and generalized solutions cannot blow up, and therefore they can be extended on \( \mathbb{R}_+ \).

By the same argument as in the proof of the energy equality, using relations (26) and (33), we can prove that

\[ \| U(t) - \hat{U}(t) \| _{\mathcal{H}}^2 + 2v \int_0^t \| \nabla (v - \hat{v}) \| _{C_0}^2 d\tau \leq \| U_0 - \hat{U}_0 \| _{\mathcal{H}}^2 + \int_0^t \| U(t) - \hat{U}(t) \| _{\mathcal{H}} \| u_t - \hat{u}_t \| _{\Omega} d\tau, \quad t \in [0,T]. \]

Applying the standard trace theorem to the boundary condition in Equation (4), we have that \( \| u_t - \hat{u}_t \| _{\Omega} \leq C \| \nabla (v - \hat{v}) \| _{C_0} \). Therefore applying a Gronwall type argument we obtain Equation (30).

The relation in Equation (30) allows us to make limit transition in energy relation (27) from strong to generalized solutions and to conclude the proof of Theorem 3.2. \( \square \)

Theorem 3.2 makes it possible to define a dynamical system \( (\mathcal{H}, S_t) \) with the phase space \( \mathcal{H} = X \times W \times Y \) and the evolution operator \( S_t \) defined by

\[ S_t U_0 = U(t) = (v(t); u(t); u_t(t)), \]

where \( U(t) \) is a generalized solution to Equation (18) with initial data \( U_0 \in \mathcal{H} \).
4. Main result

Our main result is the following theorem:

**Theorem 4.1**

The dynamical system $(\mathcal{H}, S_t)$ generated by Equation (18) possesses a compact global attractor $\mathcal{A}$ of finite fractal dimension. Moreover, for any trajectory $\{U(t) = (v(t); u(t); u_t(t)), t \in \mathbb{R}\}$ belonging to the attractor $\mathcal{A}$, we have $U(t) \in C(\mathbb{R}; D(A)) \cap C^1(\mathbb{R}; \mathcal{H})$ and

$$
\sup_{t \in \mathbb{R}} \left\{ \|v(t)\|_\mathcal{O} + \|P_L \Delta v(t)\|_\mathcal{O} + \|A^{1/2} u(t)\|_\Omega + \|u_t(t)\|_\Omega + \|Au(t) + \gamma_0 v(t)\|_\Omega \right\} \leq C_{\mathcal{A}}.
$$

(34)

We recall (see, e.g., [32–34]) that a *global attractor* of a dynamical system $(\mathcal{H}, S_t)$ is a bounded closed set $\mathcal{A} \subseteq \mathcal{H}$, which is invariant (i.e., $S_t \mathcal{A} = \mathcal{A}$) and uniformly attracts all other bounded sets, that is,

$$
\lim_{t \to \infty} \sup \{\text{dist}(S_t U, \mathcal{A}) : U \in B\} = 0 \quad \text{for any bounded set} \ B \text{ in} \ \mathcal{H}.
$$

The fractal dimension $\dim^H \mathcal{A}$ of a set $\mathcal{A}$ in the space $\mathcal{H}$ is defined as

$$
\dim^H \mathcal{A} = \sup_{\varepsilon \to 0} \frac{\ln N(\mathcal{A}, \varepsilon)}{\ln(1/\varepsilon)},
$$

where $N(\mathcal{A}, \varepsilon)$ is the minimal number of closed sets in $\mathcal{H}$ of diameter $2\varepsilon$ needed to cover the set $\mathcal{A}$.

To prove the existence of a compact global attractor for $(\mathcal{H}, S_t)$, it is sufficient (see e.g., [31, 32]) to show that the system $(\mathcal{H}, S_t)$ is dissipative and asymptotically smooth. We recall (see [32–34]) that the system is dissipative if there exists a bounded absorbing set $B_0$ in $\mathcal{H}$. A set $B_0$ is said to be absorbing for $(\mathcal{H}, S_t)$ if for any bounded set $B \subseteq \mathcal{H}$, there exists time $t_B$ such that $S_t B \subseteq B_0$ for all $t \geq t_B$. A system $(\mathcal{H}, S_t)$ is said to be asymptotically smooth if for any closed bounded forward invariant set $B \subseteq \mathcal{H}$ there exists a compact set $\mathcal{K} = \mathcal{K}(B)$, which uniformly attracts $B$:

$$
\lim_{t \to \infty} \sup \{\text{dist}(S_t U, \mathcal{K}) : U \in B\} = 0.
$$

The dissipativity property will be proved by using an appropriate Lyapunov function. As for asymptotic smoothness of the system, we rely on a recently developed approach based on stabilizability estimates (see [18, 19] and the references therein).

4.1. Dissipativity

**Proposition 4.2**

The system $(\mathcal{H}, S_t)$ is dissipative. Moreover, there exists a bounded forward invariant absorbing set.

**Proof**

Let $S_t U_0 = (v(t); u(t); u_t(t))$. We consider the following Lyapunov type function

$$
\mathcal{W}(v, u, u_t) = \mathcal{E}(v, u, u_t) + \eta \left[(u, u_t)_{\Omega} + (v, N_0 u)_{\mathcal{O}}\right],
$$

where the energy $\mathcal{E}$ is defined by Equation (28) and the operator $N_0$ is given by Equation (14). The parameter $\eta$ will be chosen later.

In the further calculations, we deal with strong solutions.

The trace theorem and the boundary condition on $\Omega$ given in Equation (4) imply that $\|u_t\|_{L^2}^2 \leq C \|\nabla v\|_{L^2}^2$, and therefore it follows from energy relation (27) that

$$
\frac{d}{dt} \mathcal{E}(v(t), u(t), u_t(t)) + c_0 \left(\|u_t\|_{L^2}^2 + \|\nabla v\|_{L^2}^2\right) \leq c_1 \|G\|_{L^2}^2
$$

(35)

with some positive constants $c_0$. Using Equations (15) and (17), we also have that

$$
\frac{d}{dt} (u, u_{\Omega}) + (v, N_0 u)_{\mathcal{O}} = (u, u_{\Omega}) + (v, N_0 u)_{\mathcal{O}} + (v, N_0 u_t)_{\mathcal{O}}
$$

$$
= (u, u_{\Omega}) - A^{1/2} u_{\Omega}^2 + (f(u), u_{\Omega}) + (\gamma_0 v, u)_{\Omega}
$$

$$
+ (v, \Delta v + G, N_0 u)_{\mathcal{O}} + (v, N_0 u_t)_{\mathcal{O}}
$$

$$
= (u, u_{\Omega}) - A^{1/2} u_{\Omega}^2 + (f(u), u_{\Omega})
$$

$$
- v(\nabla v, \nabla N_0 u)_{\mathcal{O}} + (G, N_0 u)_{\mathcal{O}} + (v, N_0 u_t)_{\mathcal{O}}.
$$

Previously, we also use the fact that $A_0(v - N_0 u) = -P_L \Delta v$ and the Green formula:

$$
v(\Delta v, N_0 u)_{\mathcal{O}} = -v(\nabla v, \nabla N_0 u)_{\mathcal{O}} + (\gamma_0 v, u)_{\Omega}.$$
Therefore, using Proposition 2.1, we obtain
\[
\frac{d}{dt} \left[ (u, u_t) + (v, N_0 u, 0) \right] \leq 2 \left\| u_t \right\|_{\Omega}^2 - (1 - \delta) A^{1/2} u_t \right\|_{\Omega}^2 - (f(u), u) + c_0 \left( \left\| \nabla v \right\|_{\Omega}^2 + \left\| G \right\|_{\Omega}^2 \right) \tag{36}
\]
for every \( \delta > 0 \). By our hypotheses (see Equation (24)), there is \( c^*_f > 0 \) and \( c_f \geq 0 \) such that
\[
(f(u), u) + c_f^{*} \int_{\Omega} \Phi(u) \, d\Omega + \frac{1}{4} A^{1/2} u_t \right\|_{\Omega}^2 \geq -c_f^t. \tag{37}
\]
Therefore, from Equation (36), we have that
\[
\frac{d}{dt} \left[ (u, u_t) + (v, N_0 u, 0) \right] \leq 2 \left\| u_t \right\|_{\Omega}^2 - \frac{1}{4} A^{1/2} u_t \right\|_{\Omega}^2 - c^*_f \int_{\Omega} \Phi(u) \, d\Omega + c_f + \left( \left\| \nabla v \right\|_{\Omega}^2 + \left\| G \right\|_{\Omega}^2 \right) + c_f. \tag{38}
\]
After selecting \( \eta > 0 \) that is small enough, this relation along with Equation (35) implies that
\[
\frac{d}{dt} \mathcal{W}(v(t), u(t), u_t(t)) + C_0 \mathcal{W}(v(t), u(t), u_t(t)) \leq C_1 \left( c_f + \left\| G \right\|_{\Omega}^2 \right) \tag{39}
\]
with some \( C_0, C_1 > 0 \). This yields
\[
\mathcal{W}(v(t), u(t), u_t(t)) \leq \mathcal{W}(v_0, u_0, u_t) e^{-C_0 t} + \frac{C_1}{C_0} \left( c_f + \left\| G \right\|_{\Omega}^2 \right) \left[ 1 - e^{-C_0 t} \right]. \tag{40}
\]
For \( \eta > 0 \) that is small enough, we evidently have
\[
\frac{1}{2} \mathcal{E}(v, u, u_t) \leq \mathcal{W}(v, u, u_t) \leq 2 \mathcal{E}(v, u, u_t). \tag{41}
\]
Therefore, the standard argument (see, e.g., [32–34]) yields dissipativity with a forward invariant bounded absorbing set. \( \square \)

The argument given previously allows us to obtain the following assertion on asymptotic stability.

**Proposition 4.3**

If \( G = 0 \) and relation (24) holds with \( c_2(\delta) = 0 \), then there exist \( c_f > 0 \) and \( \alpha > 0 \) such that
\[
\| S_t U \|_{\mathcal{H}} \leq c_f e^{-\alpha t} \text{ for any } U \in \mathcal{H} \text{ such that } \| U \|_{\mathcal{H}} \leq R. \tag{41}
\]
This means that in this case the global attractor \( \mathcal{A} \) consists of a single point, \( \mathcal{A} = \{ (0, 0, 0) \} \), which is exponentially attractive.

**Proof**

In the case considered, we have \( c_f = 0 \) in Equation (37) and also \( G = 0 \). Thus Equation (39) has the form
\[
\mathcal{W}(v(t), u(t), u_t(t)) \leq \mathcal{W}(v_0, u_0, u_t) e^{-C_0 t}, \quad t > 0.
\]
Hence, Equation (40) implies Equation (41). \( \square \)

We note that the hypotheses of Proposition 4.3 holds true if \( G = 0 \) and relation (25) is valid provided \( \psi_i \in C^2(\mathbb{R}) \) satisfied the additional properties: \( \psi_i(s) \geq \psi_i(0) = 0 \) and \( \psi_i'(s) \) is a non-decreasing function.

As a consequence of Proposition 4.3 for \( f(u) \equiv 0 \) we have the following assertion, which complements Proposition 2.2.

**Corollary 4.4**

The operator \( \mathcal{A} \) given by Equations (19) and (20) generates an exponentially stable \( C_0 \)-semigroup of contractions \( e^{-t A} \) in \( \mathcal{H} \). In particular, there exists positive \( C, \alpha \) such that
\[
\| e^{-t A} \|_{\mathcal{H}} \leq C e^{-\alpha t} \text{ for all } t > 0. \tag{42}
\]
This corollary improves the result in [8], which states the strong stability only.

### 4.2. Quasi-stability

We use the method developed in [17] (see also [18, 19] and the references therein) to obtain asymptotic smoothness and finiteness of fractal dimension of the attractor. This method is based on quasi-stability properties of the system and involves the so-called stabilizability estimate given in the following proposition.

**Proposition 4.5** (Stabilizability estimate)

Let
\[
S_t U_0 = U(t) = (v(t); u(t); u_t(t)) \text{ and } S_t U_0^* = U^*(t) = (v^*(t); u^*(t); u_t^*(t)).
\]
be two semi-trajectories such that \( \| S_t U_0 \|_{\mathcal{H}} \leq R \) for all \( t > 0 \) and for some \( R > 0 \). Then there exist positive constants \( c_0, \alpha, \) and \( c_R \) such that

\[
\| S_t U_0 - S_t U_0^s \|_{\mathcal{H}} \leq c_0 e^{-\alpha t} \| U_0 - U_0^s \|_{\mathcal{H}} + c_R \int_0^t e^{-\alpha (t-s)} \| u(s) - u^s(s) \|_{\mathcal{H}} \Omega \, ds
\]

(43)

for all \( t > 0 \).

Proof

Using the representation (29), the exponential stability of the linear semigroup \( e^{-tA} \) given by Equation (42), and also the local Lipschitz property in Equation (26), we obtain

\[
\| S_t U_0 - S_t U_0^s \|_{\mathcal{H}} \leq C e^{-\alpha t} \| U_0 - U_0^s \|_{\mathcal{H}} + C_R \int_0^t e^{-\alpha (t-s)} \| z(s) \|_{\mathcal{H}} \Omega \, ds
\]

(44)

for all \( t > 0 \), where \( z(t) = u(t) - u^s(t) \) and \( 0 < \alpha < 1 \). By interpolation,

\[
\| z \|_{\mathcal{H}} \leq \delta \| A^{1/2} z \|_{\mathcal{H}} + C_\delta \| z \|_{\mathcal{H}} \leq \delta \| S_t U_0 - S_t U_0^s \|_{\mathcal{H}} + C_\delta \| z \|_{\mathcal{H}}
\]

for every \( \delta > 0 \). Substituting this relation in Equation (44) and applying a Gronwall type argument with an appropriate choice of \( \delta \), we obtain Equation (43).

We note that the property stated in Proposition 4.5 means that the system \( (\mathcal{H}, S_t) \) is quasi-stable (in the sense of [19, Definition 7.9.2]). This observation make it possible to obtain several important dynamical properties at the abstract level.

4.3. Completion of Theorem 4.1

One can see that

\[
\varphi_T(u, u^s) = c_R \int_0^T e^{-\alpha (t-s)} \| u(s) - u^s(s) \|_{\mathcal{H}} \Omega \, ds
\]

is a compact pseudometric on the space

\[
W_T = \left\{ u \in L^2(0, T; \mathcal{H}) : u \|_{L^2(0, T; \mathcal{H})} \right\}
\]

in the sense that any bounded sequence of elements of \( W_T \) contains a subsequence, which is Cauchy with respect to the pseudometric \( \varphi_T \). Therefore, using the stabilizability estimate in Equation (43) via the Ceron–Lopes type criteria (see [18, Corollary 2.7]), we can easily prove that \( (\mathcal{H}, S_t) \) is asymptotically smooth (for some details of similar systems see, e.g., [18, 21] and also [19, Section 7.9] at the abstract level). Thus there exists a compact global attractor \( \mathcal{M} \) for \( (\mathcal{H}, S_t) \).

This attractor has a finite fractal dimension. To see this one should apply the same argument as in [18, Theorem 4.3], see also the argument given in the proof of Theorem 4.1 in [21] for the case of thermoelastic plate models. We can also apply general Theorem 7.9.6 [19] on finiteness of fractal dimension of quasi-stable systems.

The regularity property in Equation (34) follows by the same argument as in the proof of [18, Theorem 4.17], see also the proof of Theorem 6.2 [21] and the argument given in [19, Section 7.9.2] for abstract quasi-stable systems. The main idea of the argument is to consider a full trajectory \( \{U(t) = (\nu(t); u(t); u_1(t)) : t \in \mathbb{R}\} \) belonging to the attractor and to apply Proposition 4.5 with \( U_0 = U(s + \sigma) \), \( U_0^s = U(s) \) on the interval \( [s, t] \) in place of \( [0, t] \), where \( -1 < \sigma < 1 \) is arbitrary. It follows from Equation (43) that

\[
\| U(t + \sigma) - U(t) \|_{\mathcal{H}} \leq c_0 e^{-\alpha (s-t)} \| U(s + \sigma) - U(s) \|_{\mathcal{H}} + c_R \max_{\tau \in [s,t]} \| u(\tau + \sigma) - u(\tau) \|_{\mathcal{H}}
\]

(45)

for any \( t, s \in \mathbb{R} \) such that \( s \leq t \) and for any \( \sigma \) with \( |\sigma| < 1 \). Letting \( s \to -\infty \), (45) gives

\[
\| U(t + \sigma) - U(t) \|_{\mathcal{H}} \leq c_R \sup_{\tau \in (-\infty, t]} \| u(\tau + \sigma) - u(\tau) \|_{\mathcal{H}} \quad \text{for any } t \in \mathbb{R} \text{ and } |\sigma| < 1.
\]

On the other hand, on the attractor, we have

\[
\frac{1}{\sigma} \| u(t + \sigma) - u(t) \|_{\mathcal{H}} \leq \frac{1}{\sigma} \int_0^\sigma \| u(t + \sigma) - u(t) \|_{\mathcal{H}} \, d\tau \leq C, \quad t \in \mathbb{R}.
\]

This implies that

\[
\max_{\sigma \in \mathbb{R}} \left\| \frac{U(t + \sigma) - U(t)}{\sigma} \right\|_{\mathcal{H}} \leq C \quad \text{for every } |\sigma| < 1.
\]
Therefore, in the limit $\sigma \to 0$, we obtain that
\[
\|v(t)\|_\Omega + \|A^{1/2} u(t)\|_\Omega + \|u_\theta(t)\|_\Omega \leq C
\]
for all $t \in \mathbb{R}$ and for any trajectory $\{U(t) = (v(t); u(t); u_\theta(t)) : t \in \mathbb{R}\}$ trajectory belonging to the attractor. Now, using Equations (15) and (17), we obtain Equation (34).

Thus the proof of Theorem 4.1 is complete.

Remark 4.6
As in [18] and [19, Section 7.9], we can use the stabilizability estimate to construct an exponential fractal attractor (whose dimension is finite in some extended space) and prove the existence of a finite number of determining functionals supported by the displacement component of the plate. We also note that using the same approach as in [18, Theorem 4.23] (see also [19, Section 8.7]) and assuming additional smoothness of the potential $\Phi$, we can obtain a higher regularity of time derivatives of the trajectories belonging to the attractor.

Acknowledgement
In conclusion, we would like to thank an anonymous referee for careful reading of the manuscript and valuable comments, which made it possible to improve the presentation.

References
1. Lagnese J. Boundary Stabilization of Thin Plates. SIAM: Philadelphia, 1989.
2. Lagnese J. Modeling and stabilization of nonlinear plates. International Series of Numerical Mathematics 1991; 100:247–264.
3. Lagnese J, Lions J. Modeling, Analysis and Control of Thin Plates. Masson: Paris, 1988.
4. Pedley T. The Fluid Mechanics of Large Blood Vessels. Cambridge University Press: Cambridge, 1980.
5. Grobbelaar-Van Dalsen M. On a fluid-structure model in which the dynamics of the structure involves the shear stress due to the fluid. Journal of Mathematical Fluid Mechanics 2008; 10:388–401.
6. Chambolle A, Desjardins B, Esteban M, Grandmont C. Existence of weak solutions for the unsteady interaction of a viscous fluid with an elastic plate. Journal of Mathematical Fluid Mechanics 2005; 7:368–404.
7. Grobbelaar-Van Dalsen M. A new approach to the stabilization of a fluid-structure interaction model. Applicable Analysis 2009; 88:1053–1065.
8. Grobbelaar-Van Dalsen M. Strong stability for a fluid-structure model. Mathematical Methods in the Applied Sciences 2009; 32:1452–1466.
9. Kopachevski N, Pashkova JS. Small oscillations of a viscous fluid in a vessel bounded by an elastic membrane. Russian Journal of Mathematical Physics 1998; 5(4):459–472.
10. Coutand D, Shkoller S. Motion of an elastic solid inside an incompressible viscous fluid. Archive for Rational Mechanics and Analysis 2005; 176:25–102.
11. Du Q, Gunzburger M, Hou L, Lee J. Analysis of a linear fluid-structure interaction problem. Discrete and Continuous Dynamical Systems, Series A 2003; 9:633–650.
12. Avalos G. The strong stability and instability of a fluid-structure semigroup. Applied Mathematics and Optimization 2007; 55:63–184.
13. Avalos G, Triggiani R. The coupled PDE system arising in fluid-structure interaction I. Explicit semigroup generator and its spectral properties. In Fluids and Waves, Contemporary Mathematics, Vol. 440. AMS: Providence, RI, 2007; 15–54.
14. Avalos G, Triggiani R. Semigroup well-posedness in the energy space of a parabolic\$U\$ hyperbolic coupled Stokes–Lamé PDE system of fluid-structure interaction. Discrete and Continuous Dynamical Systems, Series S 2009; 2:417–447.
15. Barbù V, Grujić Z, Lasiecka I, Tuffaha A. Existence of the energy-level weak solutions for a nonlinear fluid–structure interaction model. Fluids and Waves, Contemporary Mathematics, Vol. 440. AMS, Providence, RI, 2007; 55–82.
16. Barbù V, Grujić Z, Lasiecka I, Tuffaha A. Smoothness of weak solutions to a nonlinear fluid–structure interaction model. Indiana University Mathematical Journal 2008; 57:1173–207.
17. Chueshov I, Lasiecka I. Attractors for second order evolution equations. Journal of Dynamics and Differential Equations 2004; 16:469–512.
18. Chueshov I, Lasiecka I. Long-Time Behavior of Second Order Evolution Equations with Nonlinear Damping. Memoirs of AMS, Vol. 195, no.912. AMS: Providence, RI, 2008.
19. Chueshov I, Lasiecka I. Von Karman Evolution Equations. Springer: New York, 2010.
20. Bucci F, Chueshov I. Long-time dynamics of a coupled system of nonlinear wave and thermoelastic plate equations. Discrete and Continuous Dynamical Systems, Series A 2008; 22:557–586.
21. Chueshov I, Lasiecka I. Long-time behaviour of von Karman thermoelastic plates. Applied Mathematics and Optimization 2008; 58:195–241.
22. Lasiecka I, Triggiani R. Control Theory for Partial Differential Equations: Continuous and Approximation Theories. Cambridge University Press: Cambridge, 2000.
23. Chueshov I, Lasiecka I. Global attractors for Mindlin–Timoshenko plates and for their Kirchhoff limits. Milan Journal of Mathematics 2006; 74:117–138.
24. Pazy A. Semigroups of Linear Operators and Applications to Partial Differential Equations. Springer: New York, 1986.
25. Lions J, Magenes E. Problèmes aux limites non homogènes et applications, Vol. 1. Dunod: Paris, 1968.
26. Ladyzhenskaya O. Mathematical Theory of Viscous Incompressible Flow. GIFML: Moscow, 1961 (1st Russian edition); Nauka, Moscow, 1970 (2nd Russian edition); Gordon and Breach: New York, 1969 and 1969 (English translations of the 1st Russian edition).
27. Temam R. Navier-Stokes Equations: Theory and Numerical Analysis, Reprint of the 1984 edition. AMS Chelsea Publishing: Providence, RI, 2001.
28. Galgeli G, Smidt C, Söhr H. A class of solutions to stationary Navier and Navier-Stokes equations with boundary data in $W^{\frac{1}{2},q}$. Mathematische Annalen 2005; 331:41–74.
29. Kellogg B. Properties of solutions of elliptic boundary value problems. In The Mathematical Foundations of the Finite Element Method with Applications to Partial Differential Equations, Aziz AK (ed.). Academic Press: New York, 1972; 47–81.
30. Showalter R. Monotone Operators in Banach Spaces and Nonlinear Partial Differential Equations. AMS: Providence, RI, 1997.
31. Hale J. *Asymptotic Behavior of Dissipative Systems*. AMS, Providence, RI, 1988.
32. Temam R. *Infinite-Dimensional Dynamical Systems in Mechanics and Physics*. Springer: New York, 1988.
33. Babin A, Vishik M. *Attractors of Evolution Equations*. North-Holland, Amsterdam, 1992.
34. Chueshov I. Introduction to the Theory of Infinite-Dimensional Dissipative Systems. Acta: Kharkov, 1999 (in Russian); English translation: Acta: Kharkov, 2002 http://www.emis.de/monographs/Chueshov/.