Effective Fatigue Evaluation on Rubber Mounts for Rail Vehicles

Robert Keqi Luo

Department of Railway Engineering, School of Civil Engineering, Central South University, Changsha, Hunan, 410075, China

Correspondence should be addressed to Robert Keqi Luo; robert.luo@trelleborg.com

Received 30 January 2021; Accepted 20 April 2021; Published 30 April 2021

Academic Editor: Jiang Jin

Copyright © 2021 Robert Keqi Luo. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The accurate evaluation of fatigue damage is a key issue in designs for rubber antivibration mounts. To find the most cost-effective route for antivibration design, a fatigue criterion (effective stress) was fully (including both magnitude and orientation) applied to the suspension components of rail vehicles, i.e., a longitudinal buffer and sandwich products. Hyperelastic models, widely applicable to industry, were used for load-deflection calculations and validated with the experimental data. The fatigue cracks were located at the points where the effective stress reached its maximum. The orientation prediction correlated with the experimental observations. For the buffer, the predicted crack initiation was approximately 45 k cycles at the top interface and 80 k cycles at the bottom interface, whereas nearly complete debonding in the top interface and ring-shape debonding in the bottom interface were experimentally observed at 200 k cycles. For the sandwich mount, 150 k cycles for crack initiation were predicted against 380 k cycles with an observed crack length measuring approximately 150 mm from the fatigue test. Furthermore, an important aspect was that the orientation of the cracks was defined in analytical functions so that an expensive critical plane search could be evaded, which would save 99% of calculations (144 calculations are needed for three-dimension analysis if the rotation angle is 15°, whereas only 1 calculation is required using the proposed methodology). As limited cases were verified, more engineering cases would be needed to verify this approach further.

1. Introduction

Rubber antivibration products have been widely employed in a complex dynamic environment. They are usually simplified to spring and damping elements for a dynamic analysis, such as for a bogie frame of a rail vehicle [1] and for an off-road vibratory roller [2]. For fatigue evaluation on a rubber suspension mounts of a rail vehicle, however, a real solid model needs to be used instead of using a simplified element. A limited space envelope of a suspension component has led to very high stress with more demands on fatigue service life. This now places additional pressures on designers to focus on suspension spring design in its dynamic environment more closely now than they have in the past. Hence, the accurate evaluation of fatigue damage is a key issue in designs for rubber antivibration mounts. Fatigue damage was evaluated in both uniaxial, e.g., the work of Salma [3] et al., and multiaxial conditions [4–12]. The critical plane search method was frequently used to predict the fatigue life of rubber products in some of the abovementioned references and in the work of Mars et al. [13] and Zhang et al. [14]. Luo et al. [15] proposed an effective stress tensor as a parameter for fatigue damage by taking all three principal stress ranges into consideration. Verron [16] suggested a configurational stress to evaluate fatigue damage. Lüders et al. [17] used a micromechanical approach to investigate numerical parameters of models for the damage behaviour of fibre-reinforced plastics. Shangguan et al. [18, 19] compared the energy, strain, and stress criteria for fatigue damage. They found that stress criteria were the best, which achieved the accuracy within a factor of two. Some other influences have also been studied for rubber fatigue, e.g., temperature [20–23] and strain-induced crystallization [24]. To improve the accuracy of fatigue prediction, a probabilistic methodology [25] and an artificial intelligence method [26] were also studied.

The rubber mounts must be changed as soon as possible when fatigue cracks are observed, e.g., the suspension components in high-speed trains. Hence, the crack initiation (nucleation) approach has usually been adopted in industry.
In practice, a crack with a 1–2 mm length is used as one of the criteria for fatigue. Rubber is a type of high nonlinear materials and subjected to a large deformation. An optimal fatigue damage criterion should be able to predict the critical locations and the orientation of the cracks, as well as a range of the experienced stress. In addition, cost effectiveness is important in engineering design.

In this investigation, a damage criterion for the fatigue crack initiation is applied to two antivibration mounts. The objective is to predict the crack initiation and its orientation without using the expensive critical plane search method so that the most cost-effective route could be used for anti-vibration design.

The rest of this article is outlined as follows. Experimental results for a rail vehicle buffer with the stiffness and fatigue measurement are described. Next, the hyperelastic approach and the damage criterion are presented, followed by the detailed prediction with validation. To verify this criterion further, the second case is offered using a sandwich mount. Finally, findings from this study are summarised.

2. Experiment

A suspension component, known as a longitudinal buffer, used in a bogie of a rail vehicle, was selected for a fatigue test. This product is shown in Figure 1(a). The rubber part was bonded with the top and the bottom plate. The buffer was 115 mm long, 70 mm wide, and 70 mm in height. The hardness of the filled natural rubber was 50 IRHD (International Rubber Hardness Degree). A load-deflection curve and duty cycles in service were required from a customer. In experiments, the bottom plate of the buffer was clamped into a frame and a vertical load 60 kN was applied on the top plate. The environmental temperature was 23°C, and the loading rate of the stiffness measurement was set to 10 mm/min. Figure 1(b) shows the load-deflection curve in compression along the vertical direction. The fatigue test was performed in a loading range from 0.5 kN to 55 kN along the vertical direction. The fatigue test was stopped at circa 200 k cycles due to severe damage. Figure 1(c) shows the failed mount in which two failed locations were identified. The first one was debonding over the interface between the rubber and the top plate so that the whole interface was almost completely destroyed. The second one was a bulged ring shape along the low circumference indicating underneath debonding between the rubber and the bottom plate. The following sections will present the detailed investigation on the failed mount.

3. Hyperelastic Model for Rubber Material

Hyperelastic models have been widely used in industry for engineering design and applications [27–30]. The approach is based on the strain energy density. The main equations are given as follows:

\[
W = W_I(T) + W_J(J),
\]

where \(W\) is the strain energy density, \(W_I\) is the deviatoric part of \(W\), and \(W_J\) is the volumetric part of \(W\). \(T\) is a set of the invariants of the left Cauchy–Green deformation tensor \(B\) which can be expressed using the deformation gradient tensor \(F\).

\[
F = \begin{bmatrix}
1 + \frac{\partial u_1}{\partial X_1} & \frac{\partial u_1}{\partial X_2} & \frac{\partial u_1}{\partial X_3} \\
\frac{\partial u_2}{\partial X_1} & 1 + \frac{\partial u_2}{\partial X_2} & \frac{\partial u_2}{\partial X_3} \\
\frac{\partial u_3}{\partial X_1} & \frac{\partial u_3}{\partial X_2} & 1 + \frac{\partial u_3}{\partial X_3}
\end{bmatrix},
\]

\[
B = F \cdot F^T, \quad (B_{ij} = F_{ij}F_{jk}).
\]

The first part of \(T\)

\[
T_1 = \frac{I_1}{J^{(2/3)}},
\]

\[
I_1 = \text{trace}(B).
\]

\(J\) is the Jacobian of the deformation gradient:

\[
J = \sqrt{\text{det}(B)} = \text{det}(F).
\]

The second part of \(T\)

\[
T_2 = \frac{I_2}{J^{(4/3)}},
\]

\[
I_2 = \frac{1}{2}(I_1^2 - B \cdot B).
\]

The expression for the stress is

\[
\sigma_{ij} = \frac{2}{J} \left[ \frac{1}{J^{(2/3)}} \left( \frac{\partial W_I(T)}{\partial T_1} + T_1 \frac{\partial W_J(T)}{\partial T_2} \right) B_{ij} + \frac{\partial W_J(T)}{T_1} + 2T_2 \frac{\partial W_J(T)}{T_2} \right] \delta_{ij} + \frac{\partial W_I(J)}{J} \delta_{ij},
\]

where \(\delta_{ij}\) is the Kronecker delta.

A classic polynomial form \((N = 1)\) is used for the rubber part of the longitudinal buffer.

\[
W = C_{10}(T_1 - 3) + C_{01}(T_2 - 3) + \frac{1}{D}(J - 1)^2,
\]

where \(C_{ij}\) and \(D\) are constants.

4. Fatigue Criterion

A rubber damage criterion is usually based on the crack initiation in antivibration design, i.e., a crack in a length range of 1–2 mm. Rubber mounts are subjected to multiaxial stress conditions due to complex geometries and dynamic environment. To convert a multiaxial state to a much simpler situation, such as a condition of a uniaxial fatigue loading, a damage
criterion has been developed using the effective stress tensor. The principle is that the fatigue resistance of mount B should be the same as that of mount A if both have been subjected to the same cycle number with the same effective stress \( \sigma_f \).

\( \sigma_f \) is defined as a function of the principal Cauchy stress ranges \( \sigma_{f1}, \sigma_{f2}, \) and \( \sigma_{f3} \):

\[
\sigma_f = \sqrt{\sigma_{f1}^2 + \sigma_{f2}^2 + \sigma_{f3}^2}.
\]  

(8)

If the maximum principle stress range only is considered, equation (8) is simplified to

\[
\sigma_f = \sigma_{f1},
\]  

(9)

which is the maximum stress criterion.

\( \sigma_{fi} \) needs to be compared with the real stresses along its direction. There is no fatigue damage generated if a stress range \( \sigma_{fi} \) is entirely in compression.

If any \( \sigma_{fi} \) consists of a tension portion \( \sigma_{fit} \) and a compression portion \( \sigma_{fic} \), then

\[
\sigma_{fi} = \sigma_{fit} + |\sigma_{fic}|, \quad \text{if } \sigma_{fit} \geq \sigma_{fic},
\]  

(10)

\[
\sigma_{fi} = 2\sigma_{fit}, \quad \text{if } \sigma_{fit} < \sigma_{fic}.
\]  

(11)

This damage criterion is a stress tensor whose orientation can be calculated using the following analytical equations so that a critical plane search can be evaded.

The orientation of the effective stress can be calculated as follows.

Let the three principal stress ranges be

\[
\sigma_{f1} = [\sigma_{f11}, \sigma_{f1j}, \sigma_{f1k}],
\]

\[
\sigma_{f2} = [\sigma_{f2i}, \sigma_{f2j}, \sigma_{f2k}],
\]  

(12)

\[
\sigma_{f3} = [\sigma_{f3i}, \sigma_{f3j}, \sigma_{f3k}].
\]

Then, the direction of the effective stress tensor

![Figure 1: The rail vehicle buffer and its experimental results. (a) The photo of the rail vehicle buffer. (b) The load-deflection curve of the buffer in compression (along the vertical direction) from the test. (c) The failed buffer at circa 200 k cycles.](image-url)
\[ d = \begin{bmatrix} \sigma_{f_1} & \sigma_{f_1} & \sigma_{f_1k} \end{bmatrix}, \quad \text{when } \sigma_f = \sigma_f \leq 0, \]  
\[ d = \begin{bmatrix} d_i, d_j, d_k \end{bmatrix}, \quad \text{when } \sigma_f = \sigma_f > 0, \]  
where

\[ d_i = \frac{\sigma_{f_{1i}} + \sigma_{f_{2i}} + \sigma_{f_{3i}}}{M}, \]  
\[ d_j = \frac{\sigma_{f_{1j}} + \sigma_{f_{2j}} + \sigma_{f_{3j}}}{M}, \]  
\[ d_k = \frac{\sigma_{f_{1k}} + \sigma_{f_{2k}} + \sigma_{f_{3k}}}{M}, \]  
\[ M = \sqrt{\left(\sigma_{f_{1i}} + \sigma_{f_{2i}} + \sigma_{f_{3i}}\right)^2 + \left(\sigma_{f_{1j}} + \sigma_{f_{2j}} + \sigma_{f_{3j}}\right)^2 + \left(\sigma_{f_{1k}} + \sigma_{f_{2k}} + \sigma_{f_{3k}}\right)^2}. \]  

Equation (8) defines an ellipsoidal failure envelope (valid for \( \sigma_f > 0 \)), as illustrated in Figure 2. Any point on the envelope has the same fatigue damage caused by cyclic loading events.

It should be pointed out that the critical plane search method can obtain both damage and a failed plane by rotating a plane at a fixed angle. This method is frequently used for the stiffness calculation, which reflects accuracy of both the finite element model and the elastic constants. The comparison of the load-deflection curves between the simulation and the experiment is demonstrated in Figure 3. There was an excellent agreement between the two curves, which provided a solid base for the implementation of the fatigue damage criterion in the next section.

6. Implementation of the Fatigue Criterion and Validation

6.1. The Buffer. The fatigue prediction was performed in accordance with the experimental procedure and the proposed damage criterion. The hyperelastic parameters were input as the same as that used in the load-deflection calculation. The criterion’s components were calculated using equation (10 and 11), and its magnitude was derived from equation (8).

Hot spots were located at the top interface in the form of a rad belt shape, shown in Figure 4(a). The maximum value of the effective stress was 3.03 MPa, derived from its components: \( \sigma_{f_1} = 3.01 \) MPa, \( \sigma_{f_2} = 0.31 \) MPa, and \( \sigma_{f_3} = -1.72 \) MPa. Direction cosines of the hot spot were \((-0.469, -0.119, 0.875)\). Hence, angles in the coordinate system were \(-7.7^\circ\) in the YOZ plane; \(82.9^\circ\) in the XOY plane, and \(-61.8^\circ\) in the XOZ plane. Figure 4(b)–4(d) illustrate both the hot spot and the corresponding angles (the red dot represents the hot spot, and the arrow is for the effective stress). The direction of the effective stress was nearly within the top interface, which was consistent with the experimental observation. The hot spot was located around the boundary of two surfaces of the top interface where the distance was 31.6 mm from the left edge and 45.8 mm from the bottom edge, as shown in Figures 4(c), and 4(e) and 4(f) demonstrate effective stress distribution and failed profile of the top interface of the whole buffer. The effective stress caused initial debonding that led to the nearly total separation between the rubber and the top plate.

Figure 5(a) and 5(b) show the effective stress profile at the bottom interface of the buffer. The hot spot was in the symmetrical plane and was approximately 2 mm away from
the interface. The value was 2.21 MPa, and its components were $\sigma_{f1} = 2.21$ MPa, $\sigma_{f2} = -1.54$ MPa, and $\sigma_{f3} = -3.10$ MPa. The direction cosines of the hot spot were $(-0.005, -0.283, 0.959)$, which led to the angles $-16.4^\circ$ from Z in the YOZ plane; $88.9^\circ$ from X in the XOY plane, and $-89.7^\circ$ from X in the XOZ plane. The hot spot (red point) and the angles are shown in Figure 5(c) to 5(e). Similar to the top interface, the directions of the effective stress were nearly within the bottom interface and caused the debonding. Figure 5(f) and 5(g) demonstrate the effective stress profile in the bottom interface and the experimentally observed ring shape caused by the debonding underneath.

The abovementioned description has validated the locations of the crack initiation and the crack orientation. The hot spots were located at the places where the effective stress was the maximum. Furthermore, the fatigue life needs to be
obtained quantitatively and validated. Figure 6 illustrates an S-N curve for the rubber compound used on the buffer. The predicted cycle number, at the location of the crack initiation, was approximately 45k cycles (the effective stress value was 3.03 MPa), and the cycle number of the crack initiation for the bottom interface was circa 80k cycles (2.21 MPa). These numbers were agreed reasonably against the observation of the failed buffer at circa 200k cycles, nearly total debonding in the top interface and ring-shape debonding in the bottom interface.

6.2. Sandwich Mount. This type of sandwich mounts has 4 layers of rubber bounded with metal plates through a moulding process. The products are usually used as primary suspension systems for a rail vehicle where the components are employed in pairs, fitted at an angle to the vertical axis. This component is shown in Figure 7(a). The dimension of the component was 245 mm height, 135 mm thickness, and 295 mm (end plate) width. In service and laboratory tests, the parts were arranged in pairs with 13° inclination against the vertical axis. A load-deflection test was performed.
Effective stress
(avg: 100%)

Figure 5: Continued.
Figure 5: The profile and the orientation of the effective stress with the failed profile at the bottom interface of the whole buffer. (a) The effective stress profile of the bottom interface of the buffer. (b) The zoomed area of the hot spot at the bottom interface of the buffer. (c) The effective stress direction at the hot spot of the bottom interface in the YOZ plane (−16.4° from Z). (d) The effective stress direction at the hot spot of the bottom interface in the XOY plane (88.9° from X). (e) The effective stress direction at the hot spot of the bottom interface in the XOZ plane (−89.7° from X). (f) The hot spots at the bottom interface of the whole buffer. (g) The failed bottom interface of the whole buffer (debonding underneath).

![Figure 6: The S-N curve.](image)

Figure 6: The S-N curve.

![Figure 7: The sandwich mount and its experimental results.](image)

Figure 7: The sandwich mount and its experimental results. (a) The photo of the sandwich mount. (b) The load-deflection curve of the sandwich mount in the vertical direction from the test. (c) The failed sandwich spring at approximately 380 K cycles: the hot spot was around the second layer of rubber and close to the interleaf.
according to the customer’s specification. Figure 7(b) plots the load-deflection response up to 60 kN. A fatigue test was performed after the stiffness requirement was achieved. Figure 7(c) shows the failed sandwich mount at approximately 380 k cycles. The hot spot was in the second rubber layer and close to the interleaf. The bulging length shown was approximately 150 mm, indicating cracks underneath.

A finite-element model of the sandwich mount was generated, as displayed in Figure 8(a). The model had approximately 62,000 elements. Hyperelastic parameters were the same as those used in the rail vehicle buffer. A deformed profile at a load 60 kN is shown in Figure 8(b). All four layers of the rubber were compressed with the interleaves. The comparison of the load-deflection curves between the simulation and the experiment is plotted in Figure 8(c). The comparison indicated good agreement.

Following the same procedure on the rail vehicle buffer, the location and the orientation of the crack with the stress range need to be obtained and verified. Figures 9(a) and 9(b) illustrate the effective stress profile and a zoomed critical area. The hot spot was located at the same place observed from the fatigue test (shown in Figure 7(c)). The value of the effective stress was 1.77 MPa. The calculated stress components were $\sigma_{f1} = 1.77$ MPa, $\sigma_{f2} = -1.16$ MPa, and $\sigma_{f3} = -2.29$ MPa. The direction cosines were (0.0001, −0.870, −0.493) from the calculation. Hence, the angles in the coordinate system were 60.5° in the YOZ plane; −90° in the XOY plane, and −90° in the XOZ plane. Both the hot spot (red point) and the angles (arrow) are demonstrated in Figures 9(c)–9(e). The orientation of the crack initiation was in agreement with the observed bulging locations (Figure 7(c)).
Figure 9: Continued.
As the same rubber compound was used for both the buffer and the sandwich mount, the same S-N curve should be used. Therefore, the S-N curve shown in Figure 6 was employed to predict the fatigue life of the sandwich mount. The predicted crack initiation (1–2 mm) could occur after approximately 150 k cycles. This prediction was consistent with the experimental observation: 380 k cycles with approximately 150 mm length.

7. Conclusions

The proposed fatigue criterion has been applied to the suspension mounts of rail vehicles, i.e., the longitudinal buffer and the sandwich mounts. The damage criterion was fully (both damage magnitude and orientation) applied to the antivibration mounts of rail vehicles. The main findings are as follows:

(i) The fatigue cracks were located at the points where the effective stress reached its maximum
(ii) The orientation prediction agreed with the experimental observations
(iii) For the buffer, the predicted crack initiation was approximately 45 k cycles in the top interface and 80 k cycles in the bottom interface, whereas nearly complete debonding in the top interface and ring-shape deboning in the bottom interface were experimentally observed at 200 k cycles
(iv) For the sandwich mount, 150 k cycles for crack initiation were predicted against 380 k cycles with an observed crack length measuring approximately 150 mm from the fatigue test
(v) An important aspect was that the orientation of the cracks was defined in analytical functions so that an expensive critical plane search could be evaded, which would save significant CPU time using the proposed methodology
(vi) As all three principal stress ranges were included, the criterion could be applied to multiaxial fatigue predictions
(vii) By using a system dynamic method, fatigue life would be improved for a bogie frame of a rail vehicle [31, 32]

This methodology could be useful for design engineers and analysts. As limited cases were verified, more engineering cases would be needed to verify this method further.

Data Availability

The raw/processed data required to reproduce these findings cannot be shared at this time due to legal or ethical reasons.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The technical support from Mr. Conrad Hextall, Mr. Mariano Giovannoni, and Mr. Jonathan Foster in Trelleborg AVS (U.K.) and the theoretical discussions with colleagues at the Department of Railway Engineering in Central South University are greatly appreciated.
References

[1] R. K. Luo, B. L. Gabbitas, and B. V. Brickle, “Fatigue life evaluation of a railway vehicle bogie using an integrated dynamic simulation,” *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 208, no. 2, pp. 123–132, 1994.

[2] V. Nguyen, J. Zhang, V. Le, and R. Jiao, “Vibration analysis and modeling of an off-road vibratory roller equipped with three different cab’s isolation mounts,” *Shock and Vibration*, vol. 2018, Article ID 8527574, 17 pages, 2018.

[3] B. Salma, H. Adel, and F. Raouf, “Strain-based criterion for uniaxial fatigue life prediction for an SBR rubber: comparative study and development,” *Journal of Materials: Design and Applications*, vol. 234, no. 7, pp. 897–909, 2020.

[4] W. V. Mars, “Multiaxial fatigue crack initiation in rubber,” *Tire Science and Technology*, vol. 29, no. 6, pp. 171–185, 2001.

[5] R. K. Luo and W. X. Wu, “Fatigue failure analysis of anti-vibration rubber spring,” *Engineering Failure Analysis*, vol. 13, no. 1, pp. 110–116, 2006.

[6] W. X. Wu, P. W. Cook, R. K. Luo et al., “Fatigue life investigation in the design process of metacone rubber springs,” *Elastomers and Components: Service Life Prediction-Progress and Challenges*, vol. 14, pp. 195–207, 2006.

[7] W. V. Mars and A. Fatemi, “Nucleation and growth of small fatigue cracks in filled natural rubber under multiaxial loading,” *Journal of Materials Science*, vol. 41, no. 22, pp. 7324–7332, 2006.

[8] R. K. Luo, “Effective stress criterion for rubber multiaxial fatigue under both proportional and non-proportional loadings,” *Engineering Failure Analysis*, vol. 121, Article ID 105172, 2021.

[9] W. Mars, “Identifying the damaging events in a multiaxial duty cycle,” in *Proceedings of the 6th European Conference on the Constitutive Models for Rubber*, G. Heinrich, M. Kaliske, A. Lion, and S. Reese, Eds., pp. 261–267, Taylor and Francis, London, UK, September 2010.

[10] R. K. Luo, “Multiaxial fatigue prediction on crack initiation for rubber antidamping design—location and orientation with stress ranges,” in *Proceedings of the IMechE Part L: Journal of Materials: Design and Applications*, vol. 235, 2020.

[11] J. B. Cam, B. Huneau, and E. Verron, “Fatigue damage in carbon black filled natural rubber under uniaxial and multiaxial loading conditions,” *International Journal of Fatigue*, vol. 52, pp. 82–94, 2013.

[12] Y. Zhou, S. Jerrams, and L. Chen, “Multi-axial fatigue in magnetorheological elastomers using bubble inflation,” *Materials & Design*, vol. 50, pp. 68–71, 2013.

[13] W. V. Mars, J. G. R. Kingston, A. Muhr et al., in *Proceedings of the 4th European Conference on the Constitutive Models for Rubber*, pp. 23–29, Taylor and Francis, London, UK, September 2005.

[14] J. Zhang, F. Xue, Y. Wang, X. Zhang, and S. Han, “Strain energy-based rubber fatigue life prediction under the influence of temperature,” *Royal Society Open Science*, vol. 5, no. 10, Article ID 180951, 2018.

[15] R. K. Luo, W. X. Wu, P. W. Cook, and W. J. Mortel, “An approach to evaluate the service life of rubber springs used in rail vehicle suspensions,” *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 218, no. 2, pp. 173–177, 2004.

[16] E. Verron, “Prediction of fatigue crack initiation in rubber with the help of configurational mechanics,” in *Proceedings of the 4th European Conference on the Constitutive Models for Rubber*, pp. 3–8, Taylor and Francis, London, UK, June 2005.

[17] C. Lüders, M. Sinapius, and D. Krause, “Adaptive cycle jump and limits of degradation in micromechanical fatigue simulations of fibre-reinforced plastics,” *International Journal of Damage Mechanics*, vol. 28, no. 10, pp. 1523–1555, 2019.

[18] W.-B. Shangguan, T.-K. Liu, X.-L. Wang, C. Xu, and B. Yu, “A method for modelling of fatigue life for rubbers and rubber isolators,” *Fatigue & Fracture of Engineering Materials & Structures*, vol. 37, no. 6, pp. 623–636, 2014.

[19] W.-B. Shangguan, G.-F. Zheng, T.-K. Liu, X.-C. Yuan, and S. Rakheja, “Prediction of fatigue life of rubber mounts using stress-based damage indexes,” in *Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications*, vol. 231, no. 8, pp. 657–673, 2017.

[20] S. Asare and J. J. C. Busfield, “Fatigue life prediction of bonded rubber components at elevated temperature,” *Plastics, Rubber and Composites*, vol. 40, no. 4, pp. 194–200, 2011.

[21] S. Mortazavian, A. Fatemi, S. R. Mellott, and A. Khosrovanesh, “Effect of cycling frequency and self-heating on fatigue behaviour of reinforced and unreinforced thermoplastic polymers,” *Polymer Engineering & Science*, vol. 55, no. 10, pp. 2355–2367, 2015.

[22] X. Duan, W.-B. Shangguan, M. Li, and S. Rakheja, “Measurement and modelling of the fatigue life of rubber mounts for an automotive powertrain at high temperatures,” in *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 230, no. 7, pp. 942–954, 2016.

[23] C. Neuhaus, A. Lion, M. Johlitz, P. Heuler, M. Barkhoff, and F. Duisen, “Fatigue behaviour of an elastomer under consideration of ageing effects,” *International Journal of Fatigue*, vol. 104, pp. 72–80, 2017.

[24] B. Ruellan, J.-B. Le Cam, I. Jeanneau, F. Canévet, F. Mortier, and E. Robin, “Fatigue of natural rubber under different temperatures,” *International Journal of Fatigue*, vol. 124, pp. 544–557, 2019.

[25] M. Wang, X. Liu, X. Wang, and Y. Wang, “Probabilistic modeling of unified S-N curves for mechanical parts,” *International Journal of Damage Mechanics*, vol. 27, no. 7, pp. 979–999, 2018.

[26] Q. Liu, W. Shi, and Z. Chen, “Fatigue life prediction for vibration isolation rubber based on parameter-optimized support vector machine model,” *Fatigue & Fracture of Engineering Materials & Structures*, vol. 42, no. 3, pp. 710–718, 2019.

[27] R. W. Ogden, *Non-linear Elastic Deformations*, Ellis Horwood, Chichester, UK, 1984.

[28] A. Bower, *Applied Mechanics of Solids*, CRC Press, Boca Raton, FL, USA, 2010.

[29] R. K. Luo, *Simulation Methods for Rubber Antivibration Systems*, World Scientific Publishing Co Pte Ltd, Singapore, 2020.

[30] S. Rakheja, *Dassault Systems*, Abaqus, Providence, RI, USA, 2019.

[31] A. Bower, *Applied Mechanics of Solids*, CRC Press, Boca Raton, FL, USA, 2010.

[32] R. K. Luo, B. L. Gabbitas, and B. V. Brickle, “Dynamic stress analysis of an open-shaped railway bogie frame,” *Engineering Failure Analysis*, vol. 1, no. 3, pp. 53–64, 1996.