Analytical buckling temperature prediction of FG piezoelectric sandwich plates with lightweight core

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Abstract
Buckling temperature prediction of a functionally graded piezoelectric (FGP) sandwich plate with lightweight core is studied in the present work. The composite plate is made of three layers. The upper and lower layers are made of two different piezoelectric materials so that the electrical and mechanical properties are smoothly varied through the thickness based on a power law distribution. Whereas, the lightweight core is considered as hexagonal honeycomb structure or functionally graded porous structure. The porous core contains internal pores with different porosity distributions. According to a modified four–unknown shear deformation plate theory, the displacement field is described. The smart advanced plate is subjected to thermal and humid loadings as well as external electric voltage. The thermal and humid loadings may be uniform, linear or non-linear through the thickness. The stability equations are constructed from the principle of virtual work based on the proposed shear deformation plate theory. The obtained results will be validated by introducing some comparison examples. Finally, the influences of different parameters like the side-to-thickness ratio, aspect ratio, core type, core thickness, power law index, moisture concentration, external applied voltage and boundary conditions on the buckling temperature change of the sandwich plates with honeycomb core or with porous core are discussed. It is found that the buckling temperature of the sandwich plate with honeycomb core is greater than that of the sandwich plate with porous core. Moreover, with increasing the lightweight core thickness, the sandwich plate stiffness decreases leading to a reduction of the buckling temperature.

Nomenclature

| Symbol | Description                              | Units |
|--------|------------------------------------------|-------|
| a      | Length of the sandwich plate             |       |
| b      | Width of the sandwich plate              |       |
| h      | Total thickness of the sandwich plate    |       |
| h_p    | Thickness of the face sheets             |       |
| h_c    | Core thickness                           |       |
| x_1    | x-coordinate                             |       |
| x_2    | y-coordinate                             |       |
| x_3    | z-coordinate                             |       |
| t_1    | Bottom surface coordinate               |       |
| t_2,t_5| Interfaces coordinates                   |       |
| t_4    | Top surface coordinate                   |       |
| E_m    | Metal Young’s modulus                    |       |
| G_m    | Metal shear modulus                      |       |
| ρ_m    | Metal mass density                       |       |
| λ_p    | Piezoelectric coefficients              |       |
| ε_p    | Dielectric coefficients                  |       |
| β_p    | Pyroelectric constants                   |       |
| Ξ      | Electric potential                       |       |
| φ      | Electric potential in the mid-plane of the face sheets |       |
| T      | Temperature                              |       |
| C      | Moisture                                 |       |
| T_0    | Initial temperature (moisture)           |       |
| T_f    | Final temperature (moisture)             |       |
| T_b    | Temperature (moisture) of the bottom surface |       |
| T_s(C) | Temperature (moisture) of the top surface |       |
| ζ      | Hygrothermal exponent                    |       |
| H(x_3) | Shape function                           |       |

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(Continued.)

| Symbol | Description                                      |
|--------|--------------------------------------------------|
| $\alpha_m$ | Metal thermal expansion coefficient              |
| $\beta_m$ | Metal moisture expansion coefficient             |
| $k_v$   | Poisson’s ratio                                  |
| $\nu$   | Stress tensor                                    |
| $\sigma_{ij}$ | Strain components                                |
| $\epsilon_{ij}$ | Elastic coefficients                            |
| $a_{xv}$ | Vertical cell length                             |
| $b_x$   | Inclined cell length                             |
| $t_x$   | Thickness cell rib                               |
| $\chi$  | Cell inclined angle                              |
| $P_{eff}^{(i)}$ | Effective material properties of face     |
| $P_{p1}$ | Material properties of PZT-4                    |
| $u_i$   | Displacements along $x_i$ — axes                |
| $u_x$   | In-plane displacements along $x$ — axes         |
| $w$     | Bending displacement                             |
| $\delta$ | Shear displacement                               |
| $\delta_f$ | Variation of strain energy                      |
| $\delta_i$ | Variation of the work done by the external loads|

1. Introduction

Porous and cellular structures are attracting significant attention as advanced engineering materials in aerospace engineering, automotive industry and civil constructions because of their excellent multi-functionality such as low specific weight, good marine buoyancy capability, reduced thermal and electrical conductivity, acoustic damping, enhanced recyclability, efficient capacity of energy dissipation and machinability. Lightweight porous materials with cellular structures such as metal foams consist of a solid metal containing a large volume fraction of gas-entrapped porosities. Porous and honeycomb cores are now commercially available for manufacturing lightweight sandwich structures. Cellular and porous cores sandwich structures have gained significant interest to study their behaviors. Within the context of Timoshenko beam theory, nonlinear vibrational behavior of sandwich beam with a functionally graded (FG) porous core was illustrated by Chen et al [1] considering different porosity distributions. While, Zeng et al [2] studied the nonlinear vibrational behavior of piezoelectric sandwich nanoplates with FG porous core based on first order shear deformation plate theory and Von Karman’s nonlinear strain theory. Setoodeh et al [3] investigated natural frequency of doubly curved smart sandwich shells with FG porous core and FG carbon nanotube (CNT) reinforced face sheets using the higher-order shear deformation theory. Rostami and Mohammadimehr [4] employed the first-order shear deformation theory of shells to illustrate the vibration of spinning sandwich cylindrical shell with FG porous core, nanocomposite face sheets and FG magnetoelectrical layers. Hamed et al [5] demonstrated the mechanical buckling of three-layered sandwich beam having FG porous core with various porosity distributions and FG face sheets reinforced by the CNTs. Duc et al [6] illustrated the post-buckling behavior of stiffened truncated conical sandwich shells having FG porous core and FG face sheets on elastic foundations. Akbari et al [7] presented the vibrational analysis of sandwich cylindrical shells with homogenous face sheets and FG porous core. Whereas, Alambeigi et al [8] investigated the free and forced vibration of sandwich beams with composite face layers reinforced by shape memory alloy and FG porous core resting on Vlasov’s elastic foundation.

In addition, the mechanical properties of the honeycomb auxetic cells were discussed by Scarpa and Tomlinson [9] employing the cellular material theory. The hygrothermal static analysis of CNT-reinforced plastic sandwich shells with a honeycomb core exposed to hot and moisture conditions was illustrated by Arao et al [10]. Katunin [11] employed wavelet analysis method to study vibration behavior of sandwich structures with honeycomb core. Duc et al [12] utilized the first-order plate theory and Galerkin method to investigate the nonlinear dynamic response and vibration analysis of sandwich cylindrical shells with honeycomb core within the framework of the fourth-order Runge-Kutta method. The vibrational behavior of sandwich plate with honeycomb core was demonstrated by Zhu et al [13] employing Reddy’s third-order shear deformation theory and Galerkin technique. Sobhy [14] analyzed the displacements and stresses in sandwich curved beams with graphene/Al reinforced skins and auxetic honeycomb core utilizing the differential quadrature method. There are several works presented in the literature considering the sandwich structures with cellular cores such as Nguyen et al [15], Khorasani et al [16], Yaghoobi and Taheri [17], Sobhy [18, 19], Abazid et al [20] and Singh and Harsha [21].

Piezoelectric materials have previously motivated a considerable interest for industrial products, sensors and actuators products, medical products, civilian products, military products, etc. The piezoelectric materials are usually designed as single-layered, bilayer or multi-layered structures. Moreover, the concept of FG materials is extended by many authors ([22–30]) to improve the features of the piezoelectric materials. According to the
FGM concept, the material properties are graded smoothly along one or more directions, therefore they avoid delamination and resistant unwanted fatigue and cracks. Piezoelectric materials may be integrated with other structures providing the ability to perform self-diagnosis and adaptation to the environment change of these structures, which have caused to widely increase their applications over the last decade. Dai et al [24] analyzed the bending response of FG piezoelectric sphere and solid cylinder exposed to a magnetic field, electric loading and external pressure. The thermal stress distributions in FG piezoelectric strip containing an embedded crack were studied by Ueda [25]. In the framework of the three-dimensional theory of piezoelectricity, the axisymmetric bending of an FG piezoelectric circular plate was investigated by Wang et al [26] utilizing a semi-inverse method. The hygrothermal bending analysis of FG piezoelectric hollow cylinders exposed to a mechanical load and an electric potential was presented by Zenkour [27, 28]. In Zenkour’s studies, the effective material properties of the cylinder as well as the temperature and moisture are changed in the radial direction. The buckling response of FG piezoelectric plates was illustrated by Abdollahi et al [29] utilizing the higher-order shear deformation theory containing thickness stretching effect. Whereas, Su et al [30] employed the first order shear deformation theory to investigate the transient response and free vibration of FG piezoelectric plates exposed to electric voltage with various boundary conditions. Marzbanrad et al [31] demonstrated the nonlinear free vibration of FGP nanobeam subjected to hygrothermal as well as electro-magnetic loads based on Von Karman geometric nonlinearity. The electro-elastic surface/interface theory and Von-Karman-Donnell-type kinematics of nonlinearity were employed by Fang et al [32] to illustrate the nonlinear buckling and postbuckling behavior of FGP cylindrical nanoshells considering the surface energy effect. While, Gao et al [33] employed Eringen’s nonlocal elasticity theory to investigate the nonlinear bending, buckling temperature and postbuckling of FG piezoelectric nanobeams with immovable clamped ends. Li et al [34] used the classical plate theory and Rayleigh–Ritz method to investigate the active vibration control of FG piezoelectric plates. Zenkour and Aljadani [35] and Zenkour and Hafed [36] illustrated the buckling temperature and bending response of porous FG piezoelectric nanoplates exposed to electric voltages depending on a higher-order shear deformation theory. Wang et al [37] investigated the free and forced vibration of FG piezoelectric plates based on the first-order shear deformation theory and the domain energy decomposition method.

As viewed in the previous studies, several investigations have been performed to analyze the different behaviors of FG piezoelectric plates, beams and shells with neglecting sandwich ones. Furthermore, the previous studies have been especially focused on the structures that have lightweight core and homogeneous

Figure 1. FG piezoelectric sandwich plate with lightweight core.
or nonhomogeneous face sheets without paying any attention to the lightweight core sandwiched by FGP. Therefore, our study is devoted to fill this gap focusing on the buckling temperature of an FGP sandwich plate with lightweight core subjected to thermal load, humid load and external electric voltage. The top and bottom layers are made of FG two piezoelectric materials. The properties of these layers are graded through the thickness according to a power law distribution. The middle layer is composed of FG porous materials or hexagonal honeycomb structures. Three porosity distributions of the porous core are considered. The thermal and humid loadings may be uniform, linear or non-linear through the thickness. The governing equations are derived based on a four-unknown shear deformation plate theory. An analytical solution procedure is employed to solve the stability equations. The accuracy and reliability of the solution are verified by comparing the present buckling temperature with that existed in the open literature. Eventually, a parametric study is introduced to demonstrate the influences of some geometrical and physical parameters on the buckling temperature of the sandwich plates.

2. Plate configuration

Consider a rectangular sandwich plate with length $a$, width $b$ and total thickness $h$ as displayed in figure 1. It is presumed that the lightweight core is glued perfectly with the two FG piezoelectric (FGP) sheets. In addition, the adhesive thickness of the bonding interface is neglected. The thickness of the FG piezoelectric sheets is $h_p$. While, the thickness of the core is $h_c$. The coordinate system $(x_1, x_2, x_3)$ is established in the middle plane of the sandwich plate. The coordinates of the bottom surface, two interfaces between the layers and top surface are defined as: $t_1 = -h/2$, $t_2 = -h_c/2$, $t_3 = h_c/2$, and $t_4 = h/2$. As mentioned above, the lightweight core is composed of FG porous materials or hexagonal honeycomb structures. The mechanical properties of the core layer and face layers will be discussed in the following subsections.

2.1. Porous core of the sandwich plate

The material of the porous core is aluminium foam. The following three different porosity distributions ([2, 17]) are assumed in the present analysis:
Porous-I

In this case, the material properties such as Young’s modulus, shear modulus and mass density are assumed to be constant through the thickness direction of the core (layer 2), see figure 2(a).

\[
E^{(2)}(x_3) = E_m(1 - k_0 \psi), \quad t_2 \leq x_3 \leq t_3,
\]
\[
G^{(2)}(x_3) = G_m(1 - k_0 \psi), \quad t_2 \leq x_3 \leq t_3,
\]
\[
\rho^{(2)}(x_3) = \rho_m(1 - k_0 \psi)^{0.5}, \quad t_2 \leq x_3 \leq t_3,
\]
\[
\alpha^{(2)}(x_3) = \alpha_m(1 - k_0 \psi), \quad t_2 \leq x_3 \leq t_3,
\]
\[
\beta^{(2)}(x_3) = \beta_m(1 - k_0 \psi), \quad t_2 \leq x_3 \leq t_3,
\]

where \(E_m, G_m, \rho_m, \alpha_m\) and \(\beta_m\) are, respectively, the maximum values of Young’s modulus, shear modulus, mass density, thermal expansion coefficient and moisture expansion coefficient of the porous core; \(k_0\) is the porosity coefficient that takes values between 0 and 1, i.e., \(0 \leq k_0 < 1\), and

\[
\psi = \frac{1}{k_0} - \frac{1}{k_0} \left[ \frac{2}{\pi} \sqrt{1 - k_0} - \frac{2}{\pi} + 1 \right]^2.
\]

Porous-II

For Porous-II, the maximum values of the properties will occur at the interfaces, whereas the lowest values of the properties are at the mid-plane of the core, see figure 2(b).

\[
E^{(2)}(x_3) = E_m \left[ 1 - k_0 \cos \left( \frac{\pi x_3}{h_c} \right) \right], \quad t_2 \leq x_3 \leq t_3,
\]
\[
G^{(2)}(x_3) = G_m \left[ 1 - k_0 \cos \left( \frac{\pi x_3}{h_c} \right) \right], \quad t_2 \leq x_3 \leq t_3,
\]
\[
\rho^{(2)}(x_3) = \rho_m \left[ 1 - k_m \cos \left( \frac{\pi x_3}{h_c} \right) \right], \quad t_2 \leq x_3 \leq t_3,
\]
\[
\alpha^{(2)}(x_3) = \alpha_m \left[ 1 - k_0 \cos \left( \frac{\pi x_3}{h_c} \right) \right], \quad t_2 \leq x_3 \leq t_3,
\]
\[
\beta^{(2)}(x_3) = \beta_m \left[ 1 - k_0 \cos \left( \frac{\pi x_3}{h_c} \right) \right], \quad t_2 \leq x_3 \leq t_3,
\]

where

\[
k_m = 1 - \sqrt{1 - k_0}.
\]

Porous-III

As shown in figure 2(c), the properties are graded from the maximum values at the upper interface to the lowest values at the bottom interface.

\[
E^{(2)}(x_3) = E_m \left[ 1 - k_0 \cos \left( \frac{\pi x_3}{2h_c} + \frac{\pi}{4} \right) \right], \quad t_2 \leq x_3 \leq t_3,
\]
\[
G^{(2)}(x_3) = G_m \left[ 1 - k_0 \cos \left( \frac{\pi x_3}{2h_c} + \frac{\pi}{4} \right) \right], \quad t_2 \leq x_3 \leq t_3,
\]
\[
\rho^{(2)}(x_3) = \rho_m \left[ 1 - k_m \cos \left( \frac{\pi x_3}{2h_c} + \frac{\pi}{4} \right) \right], \quad t_2 \leq x_3 \leq t_3,
\]
\[
\alpha^{(2)}(x_3) = \alpha_m \left[ 1 - k_0 \cos \left( \frac{\pi x_3}{2h_c} + \frac{\pi}{4} \right) \right], \quad t_2 \leq x_3 \leq t_3,
\]
\[
\beta^{(2)}(x_3) = \beta_m \left[ 1 - k_0 \cos \left( \frac{\pi x_3}{2h_c} + \frac{\pi}{4} \right) \right], \quad t_2 \leq x_3 \leq t_3.
\]
For the above three cases, Poisson’s ratio of the FG porous core can be estimated as [38]:

\[
\nu^{(2)}(x) = \nu_m(0.342\varphi^2 - 1.21\varphi + 1) + 0.221\varphi, \quad \varphi = 1 - \frac{\rho^{(2)}(x)}{\rho_m}
\]

(6)

The constitutive equations of the porous core layer are given as:

\[
\begin{bmatrix}
\sigma_{11}^{(2)} \\
\sigma_{22}^{(2)} \\
\sigma_{12}^{(2)} \\
\sigma_{23}^{(2)} \\
\sigma_{13}^{(2)}
\end{bmatrix} = \begin{bmatrix}
\tilde{\alpha}_1^{(2)} & 0 & 0 & 0 & 0 \\
0 & \tilde{\alpha}_2^{(2)} & 0 & 0 & 0 \\
0 & 0 & \xi_{66}^{(2)} & 0 & 0 \\
0 & 0 & 0 & \xi_{44}^{(2)} & 0 \\
0 & 0 & 0 & 0 & \xi_{55}^{(2)}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11}^{(2)} \\
\varepsilon_{22}^{(2)} \\
\varepsilon_{12}^{(2)} \\
\varepsilon_{23}^{(2)} \\
\varepsilon_{13}^{(2)}
\end{bmatrix} - \begin{bmatrix}
\tilde{\beta}_1^{(2)} \\
\tilde{\beta}_2^{(2)} \\
\tilde{\beta}_3^{(2)} \\
\tilde{\beta}_4^{(2)} \\
\tilde{\beta}_5^{(2)}
\end{bmatrix},
\]

where \(\sigma^{(2)}_i\) are the stress tensor for layer 2, \(\varepsilon^{(2)}_i\) are the strain components, \(\tilde{\alpha}_i^{(2)}\) and \(\tilde{\beta}_i^{(2)}\) are the coefficients, and \(\tilde{\alpha}_i\) and \(\tilde{\beta}_i\) are thermal and moisture moduli that defined as:

\[
\tilde{\alpha}_i^{(2)} = \frac{E^{(2)}}{1 - \nu^{(2)} F_i}, \quad \tilde{\beta}_i^{(2)} = \frac{E^{(2)}}{1 - \nu^{(2)} F_i}, \quad \varepsilon^{(2)}_1 = \varepsilon^{(2)}_6 = \varepsilon^{(2)}_{11} = \varepsilon^{(2)}_{66} = G^{(2)}
\]

(7)

2.2. Honeycomb core of the sandwich plate

The hexagonal honeycomb cells are made of aluminum and their geometry are displayed in figure 3. Based on Gibson model, the properties of the honeycomb core are given as [14, 39]:

\[
\begin{align*}
E_i^{(2)} &= E_m \frac{J^3 \cos \chi}{(\kappa + \sin \chi) \sin^2 \chi}[1 - J^2 \cot^2 \chi], \\
E_{12}^{(2)} &= E_m \frac{J^4 (\kappa + \sin \chi)}{\cos^3 \chi}[1 - J^2 (\kappa \sec^2 \chi + \tan^2 \chi)], \\
\nu_{12}^{(2)} &= \frac{J^4 \chi}{(\kappa + \sin \chi) \sin \chi}[1 - J^2 \csc^2 \chi], \\
\nu_{21}^{(2)} &= \frac{J^4 \chi}{(\kappa + \sin \chi) \sin \chi}[1 - J^2 (1 + \kappa) \sec^2 \chi], \\
G_{12}^{(2)} &= E_m \frac{J^3 (\kappa + \sin \chi)}{\kappa^2 (1 + 2 \kappa) \cos \chi}, \\
\rho^{(2)} &= \rho_m \frac{J (\kappa + 2)}{2 (\kappa + \sin \chi) \cos \chi}, \quad \alpha^{(2)} = \alpha_m \frac{J (\kappa + 2)}{2 (\kappa + \sin \chi) \cos \chi}, \\
\beta^{(2)} &= \beta_m \frac{J (\kappa + 2)}{2 (\kappa + \sin \chi) \cos \chi},
\end{align*}
\]

(9)

where \(a_h, b_h,\) and \(t_h\) denote the vertical cell rib length, inclined cell rib length and thickness cell rib and \(\chi\) is the inclined angle as shown in figure 3. Note that, equation (7) also represents the constitutive equations of the honeycomb core, where the elastic coefficients are given as...
The smart composite layers consist of two different piezoelectric materials PZT-4 (p1) and PZT-5H (p2). According to the piezoelasticity theory, the effective material properties of the face layers \( P_{eff}^{(j)}, j = 1, 3 \) are given as:

\[
P_{eff}^{(1)}(x_3) = P_{p1} + (P_{p2} - P_{p1}) \left( \frac{x_3 - \xi}{t_2 - \eta} \right)^\eta, \quad \xi \leq x_3 \leq t_2,
\]

\[
P_{eff}^{(3)}(x_3) = P_{p3} + (P_{p2} - P_{p3}) \left( \frac{x_3 - t_4}{t_3 - t_4} \right)^\eta, \quad t_3 \leq x_3 \leq t_4,
\]

where \( P_{p1} \) and \( P_{p2} \) stand for the material properties of PZT-4 and PZT-5H, respectively, and \( \eta \) is the power index. According to the piezoelectricity theory, the constitutive equations and electric displacements \( D_{ij}^{(f)} \) of the upper and lower layers can be expressed as:

\[
\begin{align*}
\sigma_{ij}^{(f)} &= \begin{bmatrix}
\tilde{\sigma}_{11} & \tilde{\sigma}_{12} & 0 & 0 & 0 & 0 \\
0 & \tilde{\sigma}_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & \tilde{\epsilon}_{44} & 0 & 0 & 0 \\
0 & 0 & 0 & \tilde{\epsilon}_{55} & 0 & 0
\end{bmatrix} + \begin{bmatrix}
\tilde{\alpha}_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & \tilde{\alpha}_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & \tilde{\beta}_{44} & 0 & 0 & 0 \\
0 & 0 & 0 & \tilde{\beta}_{55} & 0 & 0
\end{bmatrix} \begin{bmatrix}
\tilde{E}_{11}^{(f)} \\
\tilde{E}_{22}^{(f)} \\
\tilde{E}_{44}^{(f)} \\
\tilde{E}_{55}^{(f)}
\end{bmatrix}, \\
D_{ij}^{(f)} &= \begin{bmatrix}
\tilde{D}_{11}^{(f)} \\
\tilde{D}_{22}^{(f)} \\
\tilde{D}_{33}^{(f)}
\end{bmatrix}, \\
D_{ij}^{(f)} &= \begin{bmatrix}
\tilde{D}_{11}^{(f)} \\
\tilde{D}_{22}^{(f)} \\
\tilde{D}_{33}^{(f)}
\end{bmatrix} \begin{bmatrix}
\tilde{E}_{11}^{(f)} \\
\tilde{E}_{22}^{(f)} \\
\tilde{E}_{33}^{(f)}
\end{bmatrix} + \begin{bmatrix}
\tilde{\beta}_1 & 0 & 0 \\
0 & \tilde{\beta}_2 & 0 \\
0 & 0 & \tilde{\beta}_3
\end{bmatrix} \begin{bmatrix}
\tilde{E}_{11}^{(f)} \\
\tilde{E}_{22}^{(f)} \\
\tilde{E}_{33}^{(f)}
\end{bmatrix}.
\end{align*}
\]

where \( f = 1, 3 \), \( \tilde{E}_{ij}^{(f)} \) are the electric field components for layer \( f \), \( \tilde{\lambda}_{ij} \) stand for piezoelectric coefficients, \( \tilde{\epsilon}_i \) stand for dielectric coefficients, and \( \tilde{\beta}_j \) and \( \tilde{\alpha}_j \) are pyroelectric constants that are given as [41–44]:

\[
\begin{align*}
\tilde{z}_{i1}^{(f)}(x_3) &= \tilde{c}_{i1}^{(f)} - \frac{\tilde{c}_{13}^{(f)}}{\tilde{c}_{33}^{(f)}} \tilde{z}_{13}^{(f)}(x_3), \\
\tilde{z}_{i2}^{(f)}(x_3) &= \tilde{c}_{i2}^{(f)} - \frac{\tilde{c}_{i3}^{(f)}}{\tilde{c}_{33}^{(f)}} \tilde{z}_{13}^{(f)}(x_3), \\
\tilde{z}_{i3}^{(f)}(x_3) &= \tilde{c}_{i3}^{(f)} - \frac{\tilde{c}_{13}^{(f)}}{\tilde{c}_{33}^{(f)}} \tilde{z}_{13}^{(f)}(x_3), \\
\tilde{z}_{i4}^{(f)}(x_3) &= \tilde{c}_{i4}^{(f)} - \frac{\tilde{c}_{i3}^{(f)}}{\tilde{c}_{33}^{(f)}} \tilde{z}_{13}^{(f)}(x_3), \\
\tilde{z}_{i5}^{(f)}(x_3) &= \tilde{c}_{i5}^{(f)} - \frac{\tilde{c}_{i3}^{(f)}}{\tilde{c}_{33}^{(f)}} \tilde{z}_{13}^{(f)}(x_3), \\
\tilde{z}_{i6}^{(f)}(x_3) &= \tilde{c}_{i6}^{(f)} - \frac{\tilde{c}_{i3}^{(f)}}{\tilde{c}_{33}^{(f)}} \tilde{z}_{13}^{(f)}(x_3), \\
\tilde{\lambda}_{i3}^{(f)}(x_3) &= \tilde{\lambda}_{i3}^{(f)} - \frac{\tilde{\lambda}_{i3}^{(f)}}{\tilde{\lambda}_{33}^{(f)}} \tilde{\lambda}_{33}^{(f)}(x_3), \\
\tilde{\lambda}_{i5}^{(f)}(x_3) &= \tilde{\lambda}_{i5}^{(f)} - \frac{\tilde{\lambda}_{i3}^{(f)}}{\tilde{\lambda}_{33}^{(f)}} \tilde{\lambda}_{33}^{(f)}(x_3), \\
\tilde{\lambda}_{i6}^{(f)}(x_3) &= \tilde{\lambda}_{i6}^{(f)} - \frac{\tilde{\lambda}_{i3}^{(f)}}{\tilde{\lambda}_{33}^{(f)}} \tilde{\lambda}_{33}^{(f)}(x_3), \\
\tilde{\lambda}_{33}^{(f)}(x_3) &= \tilde{\lambda}_{33}^{(f)} - \frac{\tilde{\lambda}_{i3}^{(f)}}{\tilde{\lambda}_{33}^{(f)}} \tilde{\lambda}_{i3}^{(f)}(x_3), \\
\tilde{\lambda}_{55}^{(f)}(x_3) &= \tilde{\lambda}_{55}^{(f)} - \frac{\tilde{\lambda}_{i3}^{(f)}}{\tilde{\lambda}_{33}^{(f)}} \tilde{\lambda}_{i3}^{(f)}(x_3), \\
\tilde{\lambda}_{66}^{(f)}(x_3) &= \tilde{\lambda}_{66}^{(f)} - \frac{\tilde{\lambda}_{i3}^{(f)}}{\tilde{\lambda}_{33}^{(f)}} \tilde{\lambda}_{i3}^{(f)}(x_3).
\end{align*}
\]
2.3.1. Electric field

The electric field applied to the face sheets is expressed as \([41, 42]\):

\[
\mathcal{E}_i^{(r)} = -\Xi, \quad i = 1, 2, 3, \tag{15}
\]

where \(X = \partial X/\partial x_i\) and \(\Xi(x_1, x_2, x_3)\) denotes the electric potential of the smart composite plate that is defined as \([41, 42, 45]\):

\[
\Xi(x_1, x_2, x_3) = -\phi(x_1, x_2)\cos \left( \frac{\pi x^{(r)}}{h_p} \right) + 2V_0 \frac{x^{(r)}}{h_p}, \quad r = 1, 3, \tag{16}
\]

where \(\phi(x_1, x_2)\) is the electric potential in the mid-plane of the face sheets and

\[
x^{(1)} = x_3 + \frac{h_c}{2} + \frac{h_p}{2}, \quad x^{(3)} = x_3 - \frac{h_c}{2} - \frac{h_p}{2}. \tag{17}
\]

Incorporating equation (16) into equation (15) leads to the electric field as:

\[
\begin{pmatrix}
\mathcal{E}_1^{(r)} \\
\mathcal{E}_2^{(r)} \\
\mathcal{E}_3^{(r)}
\end{pmatrix} =
\begin{pmatrix}
\phi_1 \cos \left( \frac{\pi x^{(r)}}{h_p} \right) \\
\phi_2 \cos \left( \frac{\pi x^{(r)}}{h_p} \right) \\
-\frac{\pi}{h_p} \phi \sin \left( \frac{\pi x^{(r)}}{h_p} \right)
\end{pmatrix} - \begin{pmatrix}
0 \\
0 \\
2V_0
\end{pmatrix}, \quad r = 1, 3. \tag{18}
\]

2.3.2. Hygrothermal field

For precise description of the temperature and moisture influences, different temperature and moisture distributions through-the-thickness are taken into account in the present analysis as follows:

**Uniform hygrothermal rise**

The temperature and moisture are expressed as

\[
T(x_3) = \Delta T_T = T_f - T_0, \quad C(x_3) = \Delta C_C = C_f - C_0, \tag{19}
\]

where \(T_0\) and \(C_0\) are the initial temperature and moisture, respectively. The temperature and moisture are uniformly raised to the final values \(T_f\) and \(C_f\), respectively.

**Linear hygrothermal rise**

In this case, it is assumed that the temperature and moisture are linearly increased from \(T_b\) and \(C_b\) at the bottom surface to \(T_t\) and \(C_t\) at the top surface. Accordingly, the temperature and moisture are defined as:

\[
\begin{align*}
T(x_3) &= \Delta T_T \left( x_3 + \frac{1}{2} \right) + T_0, \quad \Delta T_T = T_f - T_0, \\
C(x_3) &= \Delta C_C \left( x_3 + \frac{1}{2} \right) + C_0, \quad \Delta C_C = C_f - C_0. \tag{20}
\end{align*}
\]

**Non-linear hygrothermal rise**

The temperature and moisture are distributed through-the-thickness according to the following two cases:

1. The temperature and moisture of the bottom and top surfaces are \(\Theta_b\) and \(\Theta_t\) (\(\Theta = T, C\)), respectively, and they are considered to vary from \(\Theta_b\) to \(\Theta_t\) according to the power law variation through-the-thickness as \([18, 46]\):

\[
\begin{align*}
T(x_3) &= \Delta T_T \left( x_3 + \frac{1}{2} \right)^\zeta + T_0, \quad \Delta T_T = T_f - T_0, \\
C(x_3) &= \Delta C_C \left( x_3 + \frac{1}{2} \right)^\zeta + C_0, \quad \Delta C_C = C_f - C_0. \tag{21}
\end{align*}
\]

where \(\zeta\) is the hygrothermal exponent, \(0 < \zeta < \infty\) and \(\zeta \neq 1\).

2. In this case, the variations of the temperature and moisture through the thickness direction follow a sinusoidal law as \([47]\):

\[
\text{Mater. Res. Express 8 (2021) 095704} \quad M \text{ Sobhy}
\]
4. Stability equations

The principle of virtual work containing the variation of strain energy \( \delta J_s \) and the variation of the work done by the external loads \( \delta J_f \) is given as:

\[
\delta J_s - \delta J_f = 0,
\]

where

\[
\delta J_s = \int_A \int_0^1 \sigma_{ij}^{(i)} \delta \varepsilon_{ij} + D_{ij}^{(i)} \delta \varepsilon_{ij}^{(i)} \, dx^3 \, dA + \int_A \int_0^1 \sigma_{ij}^{(2)} \delta \varepsilon_{ij} \, dx^3 \, dA
\]

\[
+ \int_A \int_0^1 \sigma_{ij}^{(3)} \delta \varepsilon_{ij} - D_{ij}^{(3)} \delta \varepsilon_{ij}^{(3)} \, dx^3 \, dA,
\]

\[
i, j = 1, 2, 3.
\]

Inserting equations (18) and (25) into (28) leads to

\[
\delta J_s = \int_A (\mathcal{N}_1 \delta \varepsilon_{11}^{(0)} + \mathcal{M}_1 \delta \varepsilon_{11}^{(1)} + \mathcal{R}_1 \delta \varepsilon_{12}^{(1)} + N_{12} \delta \varepsilon_{12}^{(2)} + \mathcal{N}_{12} \delta \varepsilon_{12}^{(3)} + R_{12} \delta \varepsilon_{12}^{(3)} + S_{13} \delta \varepsilon_{13}^{(2)} + S_{23} \delta \varepsilon_{23}^{(2)} + K_1 \delta \phi_1 + K_2 \delta \phi_2 + K_3 \delta \phi_3) \, dA,
\]

where

\[
T(x_3) = \Delta T \left( 1 - \cos \left( \frac{\pi x_3}{h} + \frac{1}{2} \right) \right) + T_0, \quad \Delta T = T_i - T_0,
\]

\[
C(x_3) = \Delta C \left( 1 - \cos \left( \frac{\pi x_3}{h} + \frac{1}{2} \right) \right) + C_0, \quad \Delta C = C_i - C_0.
\]
where

\[
\begin{align*}
\{N_{jk}, M_{jk}, R_{jk}\} &= \sum_{i=1}^{n+1} \frac{\sigma_{ij}^{(i)}[1, x_j, H]}{h_{p}} \, dx_3, \\
S_{ij} &= \sum_{i=1}^{n+1} \frac{\sigma_{ij}^{(i)}H}{h_{p}} \, dx_3, \\
K_j &= \int_{t_1}^{t_2} D_j^{(i)} \cos \left( \frac{\pi x^{(i)}}{h_{p}} \right) \, dx_3 + \int_{t_1}^{t_2} D_j^{(i)} \cos \left( \frac{\pi x^{(i)}}{h_{p}} \right) \, dx_3 + \int_{t_1}^{t_2} D_j^{(i)} \sin \left( \frac{\pi x^{(i)}}{h_{p}} \right) \, dx_3, \\
K_3 &= \int_{t_1}^{t_2} D_j^{(i)} \sin \left( \frac{\pi x^{(i)}}{h_{p}} \right) \, dx_3 + \int_{t_1}^{t_2} D_j^{(i)} \sin \left( \frac{\pi x^{(i)}}{h_{p}} \right) \, dx_3, \\
&\quad j, k = 1, 2. (30)
\end{align*}
\]

Substituting equations (7), (12) and (13) into equation (30) gives

\[
\begin{align*}
\left\{N_{11}, N_{21}, M_{11}, M_{21}, R_{11}, R_{21}, K_1\right\} &= \left\{N_{11}^T, N_{21}^T, M_{11}^T, M_{21}^T, R_{11}^T, R_{21}^T, K_1^T\right\}, \\
\left\{N_{12}, M_{12}, R_{12}\right\} &= \left\{N_{12}^T, M_{12}^T, R_{12}^T\right\}, \\
\left\{S_{13}, S_{23}\right\} &= \left\{S_{13}^T, S_{23}^T\right\}.
\end{align*}
\]

where

\[
\begin{align*}
\{b_{jk}, s_{jk}, r_{jk}\} &= \sum_{i=1}^{n+1} \int_{t_i}^{t_{i+1}} e_{jk}^{(i)} \left[1, x_j, H\right] \, dx_3, \\
\{g_{jk}, d_{jk}, f_{jk}\} &= \sum_{i=1}^{n+1} \int_{t_i}^{t_{i+1}} e_{jk}^{(i)} \left[x_j^2, x_j H, H^2\right] \, dx_3, \\
\tilde{f}_{44} &= \sum_{i=1}^{n+1} \int_{t_i}^{t_{i+1}} e_{jk}^{(i)} H^2 \, dx_3, \\
\{a_{11}, a_{12}, a_{13}\} &= \int_{t_1}^{t_2} \frac{\pi}{h_{p}} \alpha_{11} \sin \left( \frac{\pi x^{(i)}}{h_{p}} \right) \, dx_3 + \int_{t_1}^{t_2} \frac{\pi}{h_{p}} \alpha_{13} \sin \left( \frac{\pi x^{(i)}}{h_{p}} \right) \, dx_3, \\
\{a_{33}\} &= -\int_{t_1}^{t_2} \frac{\pi^2}{h_{p}} \epsilon_{33} \sin^2 \left( \frac{\pi x^{(i)}}{h_{p}} \right) \, dx_3 + \int_{t_1}^{t_2} \frac{\pi^2}{h_{p}} \epsilon_{33} \sin^2 \left( \frac{\pi x^{(i)}}{h_{p}} \right) \, dx_3, \\
a_{44} &= \int_{t_1}^{t_2} \frac{\pi}{h_{p}} \alpha_{44} \cos \left( \frac{\pi x^{(i)}}{h_{p}} \right) H' \, dx_3 + \int_{t_1}^{t_2} \frac{\pi}{h_{p}} \alpha_{44} \cos \left( \frac{\pi x^{(i)}}{h_{p}} \right) H' \, dx_3, \\
\tilde{a}_{44} &= \int_{t_1}^{t_2} \frac{\pi}{h_{p}} \alpha_{44} \cos \left( \frac{\pi x^{(i)}}{h_{p}} \right) H' \, dx_3 + \int_{t_1}^{t_2} \frac{\pi}{h_{p}} \alpha_{44} \cos \left( \frac{\pi x^{(i)}}{h_{p}} \right) H' \, dx_3.
\end{align*}
\]
\[
\{ \mathcal{N}^E_{11}, \mathcal{M}^E_{11}, \mathcal{R}^E_{11} \} = \{ \mathcal{N}^E_{22}, \mathcal{M}^E_{22}, \mathcal{R}^E_{22} \} = \frac{2V_0}{h_p} \left[ \int_{x_1}^{x_3} \chi^{(1)}_{31} \{ 1, x_3, \overline{H} \} \, dx_3 + \int_{x_1}^{x_3} \chi^{(3)}_{31} \{ 1, x_3, \overline{H} \} \, dx_3 \right]
\]

\[
K^E_{3} = - \frac{2V_0}{h_p} \frac{\pi}{h_p} \int_{x_1}^{x_3} \phi_1 \, dx_3 + \int_{x_1}^{x_3} \phi_3 \, dx_3,
\]

\[
\{ \mathcal{N}^T_{11}, \mathcal{M}^T_{11}, \mathcal{R}^T_{11} \} = \sum_{i=1}^{3} \int_{x_1}^{x_3} \phi_{i1}^{(1)} \{ 1, x_3, \overline{H} \} \, dx_3,
\]

\[
\{ \mathcal{N}^C_{11}, \mathcal{M}^C_{11}, \mathcal{R}^C_{11} \} = \sum_{i=1}^{3} \int_{x_1}^{x_3} \phi_{i1}^{(3)} \{ 1, x_3, \overline{H} \} \, dx_3,
\]

\[
K^C_{3} = - \frac{\pi}{h_p} \int_{x_1}^{x_3} \phi_3^{(1)} \, dx_3 + \int_{x_1}^{x_3} \phi_3^{(3)} \, dx_3.
\]

Now, the variation of the work done by the temperature and humid loads as well as the external electric voltage \(\delta f\) is expressed as [42]:

\[
\delta f = \int_A \left[ F_1(w_{0,11} + w_{x,11}) + F_2(w_{0,22} + w_{x,22}) \right] \delta(w_0 + w) \, dA,
\]

where

\[
F_j = \mathcal{N}^E_j + \mathcal{N}^T_j + \mathcal{N}^C_j, \quad j = 1, 2,
\]

in which \(\mathcal{N}^E_j, \mathcal{N}^T_j\) and \(\mathcal{N}^C_j\) are the in-plane force due to the external electric voltage, temperature and humid loads, respectively, that are defined in equation (35).

The concept of virtual displacement concept ([18, 46, 56–59]) is employed to present the stability equations at a neighboring stable state. The displacements of equilibrium state are defined as \((u_1^0, u_2^0, w_0^0, w)^0\), while the virtual displacements of a neighboring stable state are \((u_1^1, u_2^1, w_0^1, w)^1\). The total displacement components of a neighboring state are given by:

\[
u_1 = u_1^0 + u_1^1, \quad u_2 = u_2^0 + u_2^1, \quad w_0 = w_0^0 + w_0^1, \quad w = w_0^0 + w_1.
\]

By substituting equations (29) and (36) into equation (27) considering equation (38), one obtains the stability equations as:

\[
\mathcal{N}^{1,1}_{11} + \mathcal{N}^{1,2}_{12,2} = 0,
\]

\[
\mathcal{N}^{1,1}_{12,1} + \mathcal{N}^{1,2}_{22,2} = 0,
\]

\[
\mathcal{M}^{1,1}_{11,11} + 2\mathcal{M}^{1,1}_{12,12} + \mathcal{M}^{1,1}_{22,22} + F_1(w_{0,11} + w_{x,11}) + F_2(w_{0,22} + w_{x,22}) = 0,
\]

\[
\mathcal{R}^{1,1}_{11,11} + 2\mathcal{R}^{1,1}_{12,12} + \mathcal{R}^{1,1}_{22,22} + S^{1,1}_{13,1} + S^{1,1}_{23,2} + F_1(w_{0,11} + w_{x,11}) + F_2(w_{0,22} + w_{x,22}) = 0,
\]

\[
\mathcal{K}^{1,1}_{11} + \mathcal{K}^{1,1}_{22,2} + \mathcal{K}^{1,1}_3 = 0.
\]

where the superscript 1 indicates the neighboring stable state.

5. Solution procedure

In this section, the above stability equations will be analytically solved for simply supported and clamped boundary conditions that given as:

**simply supported (S)**

\[
u_1 = w_1^0 = w_1^1 = w_{x,11}^0 = w_{x,11}^1 = \mathcal{N}^{1,1}_{11} = \mathcal{M}^{1,1}_{11} = \mathcal{R}^{1,1}_{11} = 0, \quad \text{at} \quad x_1 = 0, a,
\]

\[
u_1 = w_1^0 = w_1^1 = w_{x,11}^0 = w_{x,11}^1 = \mathcal{N}^{1,1}_{22} = \mathcal{M}^{1,1}_{22} = \mathcal{R}^{1,1}_{22} = 0, \quad \text{at} \quad x_2 = 0, b.
\]
Table 1. The admissible functions $\Lambda_j(x_k)$ and $\tilde{\Lambda}_i(x_k)$.

| B.C. | The functions |
|---|---|
| $x_1 = 0$ | $x_1 = a$ | $x_2 = 0$ | $x_2 = b$ | $\Lambda_j(x_k)$ | $\tilde{\Lambda}_i(x_k)$ |
| S | S | S | S | $\sin(\mu x_1)$ | $\sin(\xi x_1)$ |
| C | S | S | S | $\sin(\mu x_1)[1 - \cos(\mu x_1)]$ | $\sin(\xi x_1)$ |
| C | C | S | S | $\sin^2(\mu x_1)$ | $\sin^2(\xi x_1)$ |
| C | C | C | S | $\sin(\mu x_1)[1 - \cos(\mu x_1)]$ | $\sin^2(\xi x_1)$ |
| C | C | C | C | $\sin^2(\mu x_1)$ | $\sin^2(\xi x_1)$ |

Clamped (C)

$$u_1^i = u_2^i = w_1^i = w_2^i = \phi_1^i = w_{b,i} = w_{s,i} = \phi^1 = 0,$$  

at \( x_1 = 0, a, \quad x_2 = 0, b, \quad i = 1, 2. \)  

The solutions of the stability equations (39), which satisfy the conditions (40) and (41), are presumed as:

$$u_1^i = \sum_{j} \sum_{k} U_{jk} \Lambda_{j,1}(x_1) \tilde{\Lambda}_k(x_2),$$

$$u_2^i = \sum_{j} \sum_{k} U_{jk} \Lambda_{j,1}(x_1) \tilde{\Lambda}_k(x_2),$$

$$[w_1^i, w_2^i, \phi^1] = \sum_{j} \sum_{k} [W_{jk}, W_{jk}, \Upsilon_{jk}] \Lambda_j(x_1) \tilde{\Lambda}_k(x_2),$$

where \( U_{jk}, \ U_{jk}, \ W_{jk}, \ W_{jk} \) and \( \Upsilon_{jk} \) are constant coefficients and the admissible functions $\Lambda_j(x_1)$ and $\tilde{\Lambda}_k(x_2)$ are defined in table 1 noting that $\mu = \pi/\alpha$ and $\xi = 2\pi/b$.

Inserting equations (31)–(33) into 39 subject to equations (26), (38) and (42) gives the stability equations as:

$$\begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\
A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\
A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\
A_{51} & A_{52} & A_{53} & A_{54} & A_{55}
\end{bmatrix}\begin{bmatrix}
U_{jk} \\
U_{jk} \\
W_{jk} \\
W_{jk} \\
\Upsilon_{jk}
\end{bmatrix} = 0,$$

where the elements $A_{ij}$ are given as:

$$A_{11} = \chi_{11}(12) + \chi_{13}(10) b_{11}, \quad A_{12} = \chi_{13}(12) (b_{12} + b_{66}), \quad A_{13} = -\chi_{13}(12) (s_{12} + 2 s_{66}) - \chi_{13}(10) r_{12}, \quad A_{14} = -\chi_{13}(12) (s_{12} + 2 s_{66}) - \chi_{13}(10) r_{12},$$

$$A_{15} = \chi_{13}(10) a_{11}, \quad A_{21} = \chi_{22}(21) (b_{12} + b_{66}), \quad A_{22} = \chi_{22}(12) b_{22} + \chi_{22}(21) b_{66},$$

$$A_{23} = -\chi_{22}(21) (s_{12} + 2 s_{66}) - \chi_{22}(23) s_{22}, \quad A_{24} = -\chi_{22}(21) (s_{12} + 2 s_{66}) - \chi_{22}(23) s_{22},$$

$$A_{31} = \chi_{22}(21) (s_{12} + 2 s_{66}) + \chi_{22}(23) r_{12}, \quad A_{32} = \chi_{22}(21) (s_{12} + 2 s_{66}) + \chi_{22}(23) s_{22},$$

$$A_{33} = -\chi_{22}(21) (s_{12} + 2 s_{66}) - \chi_{22}(23) g_{12} + \chi_{22}(21) g_{66} + \chi_{22}(23) (N_{11} + N_{11} + N_{11} + b_{11} + b_{11} + b_{11} + b_{11} + b_{11}),$$

$$A_{34} = -\chi_{22}(23) (d_{12} + 2 d_{66}) - \chi_{22}(23) g_{12} + \chi_{22}(23) g_{66} + \chi_{22}(23) (N_{11} + N_{11} + b_{11} + b_{11} + b_{11} + b_{11} + b_{11} + b_{11} + b_{11}),$$

$$A_{35} = (\chi_{22}(21) + \chi_{22}(23)) a_{22}, \quad A_{41} = \chi_{22}(21) (s_{12} + 2 s_{66}) + \chi_{22}(23) r_{12},$$

$$A_{42} = \chi_{22}(21) (s_{12} + 2 s_{66}) + \chi_{22}(23) s_{22}, \quad A_{43} = -\chi_{22}(21) (s_{12} + 2 s_{66}) - \chi_{22}(23) (d_{12} + 2 d_{66}) - \chi_{22}(23) (d_{12} + 2 d_{66}) + \chi_{22}(23) g_{12} + \chi_{22}(23) g_{66} + \chi_{22}(23) (N_{11} + N_{11} + b_{11} + b_{11} + b_{11} + b_{11} + b_{11} + b_{11}),$$

$$A_{44} = -\chi_{22}(23) (d_{12} + 2 d_{66}) - \chi_{22}(23) g_{12} + \chi_{22}(23) g_{66} + \chi_{22}(23) (N_{11} + N_{11} + b_{11} + b_{11} + b_{11} + b_{11} + b_{11} + b_{11}),$$

$$A_{45} = (\chi_{22}(21) + \chi_{22}(23)) a_{22}, \quad A_{51} = \chi_{22}(21) (s_{12} + 2 s_{66}) + \chi_{22}(23) r_{12},$$

$$A_{52} = \chi_{22}(21) (s_{12} + 2 s_{66}) + \chi_{22}(23) (d_{12} + 2 d_{66}) - \chi_{22}(23) g_{12} + \chi_{22}(23) g_{66} + \chi_{22}(23) (N_{11} + N_{11} + b_{11} + b_{11} + b_{11} + b_{11} + b_{11} + b_{11}),$$

$$A_{53} = (\chi_{22}(21) + \chi_{22}(23)) a_{22}, \quad A_{54} = \chi_{22}(21) (s_{12} + 2 s_{66}) + \chi_{22}(23) s_{22},$$

$$A_{55} = \chi_{22}(21) (s_{12} + 2 s_{66}) + \chi_{22}(23) s_{22},\quad (44)$$
Table 2. The properties of the piezoelectric materials PZT-4 and the PZT-5H [44, 60].

| Properties       | PZT-4 (p1) | PZT-5H (p2) |
|------------------|------------|-------------|
| $c_{11}$ (GPa)   | 139        | 126         |
| $c_{12}$ (GPa)   | 77.8       | 79.1        |
| $c_{13}$ (GPa)   | 74         | 83.9        |
| $c_{44}$ (GPa)   | 115        | 117         |
| $c_{66}$ (GPa)   | 25.6       | 23          |
| $\lambda_f$ (C m$^{-2}$) | 30.6  | 23.5        |
| $\lambda_f$ (C m$^{-2}$) | -5.2  | -6.5        |
| $\lambda_f$ (C m$^{-2}$) | 15.1  | 23          |
| $\lambda_f$ (C m$^{-2}$) | 12.7  | 17          |
| $\epsilon_f$ ($10^{-9}$F m$^{-1}$) | 6.46  | 15.05       |
| $\epsilon_f$ ($10^{-9}$F m$^{-1}$) | 5.62  | 13.02       |
| $\alpha_f$ ($10^6$Pa K$^{-1}$) | 4.738 | 3.468       |
| $\alpha_f$ ($10^6$Pa K$^{-1}$) | 4.529 | 3.468       |
| $\beta_f$ (GPa wt%H$_2$O) | 94.219 | 93.636      |
| $\beta_f$ (GPa wt%H$_2$O) | 85.212 | 92.275      |

Table 3. Comparison of critical buckling temperature $T_{cr}$ of FGM sandwich square plates subjected to uniform temperature rise for different values of the power-law index $g$ and core thickness.

| $g$ | $h_c/h$ | Source | $a/h = 5$ | $a/h = 10$ | $a/h = 15$ | $a/h = 25$ | $a/h = 50$ |
|-----|---------|--------|-----------|-----------|-----------|-----------|-----------|
| 0.5 | 0       | SPT [46]| 2.87276   | 0.80328   | 0.36504   | 0.13294   | 0.03340   |
|     |         | TPT [46]| 2.87073   | 0.80313   | 0.36501   | 0.13294   | 0.03340   |
|     |         | Present | 2.87219   | 0.80324   | 0.36503   | 0.13295   | 0.03340   |
| 1/5 |         | SPT [46]| 2.83194   | 0.79232   | 0.36010   | 0.13116   | 0.03295   |
|     |         | TPT [46]| 2.83029   | 0.79220   | 0.36007   | 0.13115   | 0.03295   |
|     |         | Present | 2.83144   | 0.79229   | 0.36010   | 0.13116   | 0.03295   |
| 1/3 |         | SPT [46]| 2.83331   | 0.79463   | 0.36134   | 0.13164   | 0.03308   |
|     |         | TPT [46]| 2.83224   | 0.79456   | 0.36132   | 0.13164   | 0.03308   |
|     |         | Present | 2.83292   | 0.79461   | 0.36134   | 0.13164   | 0.03308   |
| 1/2 |         | SPT [46]| 2.86992   | 0.80925   | 0.36841   | 0.13430   | 0.03376   |
|     |         | TPT [46]| 2.86971   | 0.80925   | 0.36841   | 0.13430   | 0.03376   |
|     |         | Present | 2.86971   | 0.80924   | 0.36841   | 0.13430   | 0.03376   |
| 2   | 0       | SPT [46]| 2.63459   | 0.71815   | 0.32462   | 0.11789   | 0.02958   |
|     |         | TPT [46]| 2.63018   | 0.71783   | 0.32455   | 0.11788   | 0.02958   |
|     |         | Present | 2.63356   | 0.71808   | 0.32461   | 0.11789   | 0.02958   |
| 1/5 |         | SPT [46]| 2.39953   | 0.65098   | 0.29396   | 0.10671   | 0.02677   |
|     |         | TPT [46]| 2.39637   | 0.65075   | 0.29392   | 0.10670   | 0.02676   |
|     |         | Present | 2.39875   | 0.65092   | 0.29396   | 0.10670   | 0.02677   |
| 1/3 |         | SPT [46]| 2.36195   | 0.64253   | 0.29031   | 0.10541   | 0.02600   |
|     |         | TPT [46]| 2.35999   | 0.64238   | 0.29028   | 0.10540   | 0.02645   |
|     |         | Present | 2.36138   | 0.64249   | 0.29031   | 0.10541   | 0.02645   |
| 1/2 |         | SPT [46]| 2.42899   | 0.66689   | 0.30189   | 0.10972   | 0.02754   |
|     |         | TPT [46]| 2.42873   | 0.66687   | 0.30189   | 0.10972   | 0.02754   |
|     |         | Present | 2.42875   | 0.66687   | 0.30189   | 0.10972   | 0.02754   |
Table 4. Critical buckling temperature $T_{cr}$ of a SSSS FG piezoelectric sandwich plate with honeycomb core ($\eta = 1$, $V^e_0 = 0.05$, $\zeta = 1$, $a/b = 1$, $\Delta C = 0.01\%$, $k_0 = 0.2$, $a/h = 10$).

| B.C. | $h_c/h$ | TPT       | SPT       | EPT       | Present       |
|------|---------|-----------|-----------|-----------|---------------|
| SSSS | 0.1     | 207.24952 | 207.16773 | 207.18876 |               |
|      | 0.2     | 171.88266 | 171.48845 | 171.01645 | 171.58533     |
|      | 0.3     | 129.09010 | 128.26631 | 128.37072 | 128.47543     |
|      | 0.4     | 80.09555  | 79.03130  | 77.78808  | 79.30473      |
|      | 0.5     | 30.70511  | 30.03296  | 29.30613  | 30.20382      |
| CSSS | 0.1     | 219.96472 | 220.04615 | 220.02919 |               |
|      | 0.2     | 187.49955 | 187.38087 | 187.21279 | 187.41277     |
|      | 0.3     | 148.24129 | 147.85835 | 147.38929 | 147.96024     |
|      | 0.4     | 102.68992 | 102.15889 | 101.54059 | 102.30750     |
|      | 0.5     | 54.59835  | 54.32201  | 54.01127  | 54.40765      |
| CCSS | 0.1     | 227.19277 | 227.36196 | 227.32411 |               |
|      | 0.2     | 196.26020 | 196.27902 | 196.26352 | 196.27373     |
|      | 0.3     | 158.83400 | 158.66034 | 158.42174 | 158.71011     |
|      | 0.4     | 113.05745 | 114.77374 | 114.42820 | 114.85610     |
|      | 0.5     | 67.60000  | 67.57259  | 67.44208  | 67.60781      |
| CCSS | 0.1     | 234.79479 | 235.04237 | 234.98310 |               |
|      | 0.2     | 205.37249 | 205.52150 | 205.65132 | 205.48719     |
|      | 0.3     | 169.74377 | 169.75511 | 169.72441 | 169.75646     |
|      | 0.4     | 127.69030 | 127.62288 | 127.51343 | 127.64594     |
|      | 0.5     | 81.04568  | 81.07633  | 81.09397  | 81.06794      |
| CCCC | 0.1     | 241.23864 | 241.56827 | 241.98310 |               |
|      | 0.2     | 213.05364 | 213.30216 | 213.54388 | 213.24212     |
|      | 0.3     | 178.85901 | 179.00099 | 179.12114 | 178.96598     |
|      | 0.4     | 138.17139 | 138.30216 | 138.30216 | 138.22857     |
|      | 0.5     | 92.12633  | 92.24497  | 92.35880  | 92.20648      |

Table 5. Critical buckling temperature $T_{cr}$ of a SSSS FG piezoelectric sandwich plate for different core types ($\eta = 1$, $V^e_0 = 0.05$, $\zeta = 1$, $a/b = 1$, $\Delta C = 0.01\%$, $k_0 = 0.2$, $a/h = 10$).

| B.C. | $h_c/h$ | Honeycomb | Porous I | Porous II | Porous III |
|------|---------|-----------|----------|-----------|-----------|
| SSSS | 0.1     | 207.18876 | 87.01350 | 86.41127  | 85.99058  |
|      | 0.2     | 171.58533 | 41.80985 | 41.43391  | 40.90958  |
|      | 0.3     | 128.47543 | 20.56936 | 20.36762  | 19.93560  |
|      | 0.4     | 79.30473  | 8.97292  | 8.88775   | 8.61373   |
|      | 0.5     | 30.23082  | 2.53676  | 2.52662   | 2.40855   |
| CSSS | 0.1     | 220.02919 | 92.41792 | 91.77685  | 91.33173  |
|      | 0.2     | 187.41277 | 45.55405 | 45.14573  | 44.57368  |
|      | 0.3     | 147.96024 | 23.42077 | 23.19434  | 22.70043  |
|      | 0.4     | 102.30750 | 11.20642 | 11.10515  | 10.76061  |
|      | 0.5     | 54.40765  | 4.27342  | 4.25555   | 4.06410   |
| CCSS | 0.1     | 227.32411 | 95.52727 | 94.86675  | 94.40477  |
|      | 0.2     | 196.27737 | 47.71147 | 47.28446  | 46.68503  |
|      | 0.3     | 158.71011 | 25.06756 | 24.82660  | 24.29726  |
|      | 0.4     | 114.85610 | 12.50039 | 12.38924  | 12.00438  |
|      | 0.5     | 67.60781  | 6.38237  | 6.32604   | 6.02699   |
| CCSS | 0.1     | 234.98310 | 98.82257 | 98.13958  | 97.66165  |
|      | 0.2     | 205.48719 | 50.00053 | 49.55366  | 48.92528  |
|      | 0.3     | 169.75646 | 26.81780 | 26.56124  | 25.99444  |
|      | 0.4     | 127.64594 | 13.87878 | 13.75676  | 13.32928  |
|      | 0.5     | 81.06794  | 6.36268  | 6.33357   | 6.05557   |
| CCCC | 0.1     | 241.49121 | 101.64418| 100.94198 | 100.43041 |
|      | 0.2     | 213.24212 | 51.96264 | 51.49871  | 50.84560  |
|      | 0.3     | 178.96598 | 28.30256 | 28.05040  | 27.45166  |
|      | 0.4     | 138.22857 | 15.06500 | 14.93322  | 14.46945  |
|      | 0.5     | 92.20648  | 7.29413  | 7.25892   | 6.94332   |
Figure 4. Influences of the external electric voltage $V_0^*$ on the critical buckling temperature $T_{cr}$ of SSSS FGP sandwich plates with (a) honeycomb core, (b) porous core (porous-I), (c) porous core (porous-II) and (d) porous core (porous-III) ($\eta = 1$, $h_c/h = 0.4$, $\zeta = 1$, $a/b = 1$, $\Delta C = 0.01\%$, $k_t = 0.2$).

\[
\begin{align*}
\chi_{10} &= \int_0^a A_1 A_{1,xx} \, dx, & \chi_{11} &= \int_0^a A_{1,xx} \, dx, & \chi_{13} &= \int_0^a A_{1,111} \, dx; \\
\chi_{20} &= \int_0^a A_2^2 \, dx, & \chi_{21} &= \int_0^a A_1 A_{2,xx} \, dx, & \chi_{34} &= \int_0^a A_1 A_{1,111} \, dx; \\
\gamma_{10} &= \int_0^b \lambda_2 \, dx, & \gamma_{12} &= \int_0^b \lambda_k \lambda_{k,xx} \, dx, & \gamma_{20} &= \int_0^b \lambda_k \lambda_{k,xx} \, dx; \\
\gamma_{21} &= \int_0^b \lambda_{k,xx} \, dx, & \gamma_{23} &= \int_0^b \lambda_k \lambda_{k,111} \, dx, & \gamma_{34} &= \int_0^b \lambda_k \lambda_{k,111} \, dx;
\end{align*}
\]

\begin{align}
&b_{j1}^{T \text{ uniform}} = \frac{1}{2}, & b_{j2}^{T \text{ uniform}} = \frac{1}{2}, & b_{j1}^{T \text{ linear}} = \sum_{i=1}^{3} \int_{t_i}^{t_{i+1}} \hat{\alpha}_{j}^{(i)} \, dx_3, & b_{j2}^{T \text{ linear}} = \sum_{i=1}^{3} \int_{t_i}^{t_{i+1}} \hat{\alpha}_{j}^{(i)} \left( \frac{x_3}{h} + \frac{1}{2} \right) \, dx_3, \\
b_{j1}^{T \text{ nonlinear}, 1} = b_{j1}^{T \text{ nonlinear}, 2} = b_{j1}^{T \text{ linear}}, & b_{j2}^{T \text{ nonlinear}, 1} = \sum_{i=1}^{3} \int_{t_i}^{t_{i+1}} \hat{\alpha}_{j}^{(i)} \left( \frac{x_3}{h} + \frac{1}{2} \right) \, dx_3, & b_{j2}^{T \text{ nonlinear}, 2} = \sum_{i=1}^{3} \int_{t_i}^{t_{i+1}} \hat{\alpha}_{j}^{(i)} \left( 1 - \cos \left( \frac{\pi}{2} \left( \frac{x_3}{h} + \frac{1}{2} \right) \right) \right) \, dx_3, & j = 1, 2.
\end{align}

(45)
For nontrivial solution of equation (43), the determinant $|A|$ equals zero. Solving equation $|A| = 0$ gives the buckling temperature change $\Delta T$ of the FG piezoelectric sandwich plates with lightweight core under the effects of the moisture conditions and external electric voltage.

6. Numerical results

6.1. Verification
The accuracy of the present results for the buckling temperature $T_{cr} = 10^{-3} \Delta T$ of simply supported FGM sandwich square plates subjected to uniform temperature rise obtained by the present refined shear deformation theory has been examined by comparing the obtained results with those presented by Zenkour and Sobhy [46] based on the sinusoidal shear deformation plate theory (SPT) and third-order theory (TPT) as shown in table 3. It is shown from this comparison that the present results are in a good agreement with the results of [46]. Table 4 shows the comparison between the results obtained by the current theory and those obtained by the TPT [53], SPT [54] and EPT [55] for different boundary conditions and core thickness. It can be noted for all boundary conditions and core thickness values that the present theory predicts results very close to those of other theories.

![Figure 5. Influences of the moisture change $\Delta C$ on the critical buckling temperature $T_{cr}$ of SSSS FGP sandwich plates with (a) honeycomb core, (b) porous core (porous-I), (c) porous core (porous-II) and (d) porous core (porous-III) ($\eta = 1, h_c/h = 0.4$, $\zeta = 1, a/b = 1, V_k = 0.05\%$, $k_0 = 0.2$).]
6.2. Buckling results of sandwich plates with lightweight core

In this subsection, the numerical results of buckling temperature of the FG piezoelectric sandwich plates with lightweight core are presented. The mechanical and electrical properties of the piezoelectric materials are defined in Table 2.

The material properties of the lightweight core are: \( E_m = 70 \) GPa, \( \nu_m = 0.3 \), \( \rho_m = 2700 \) kg/m\(^3\), \( \alpha_m = 23 \times 10^{-6} \) K\(^{-1}\), \( \beta_m = 0.44 \) (wt%H\(_2\)O\(^-1\)). The following data are used in the present analysis: \( J = 0.0138571 \), \( \kappa = 2 \), \( \chi = -45 \), \( h = 3 \) mm, \( T_b = 100 \) K, \( C_b = 0 \). Table 5 lists the critical buckling temperature of FG piezoelectric sandwich plates with honeycomb core and porous core for different boundary conditions and core thickness. It can be noticed that the thermal conductivity of the honeycomb core is less than that of porous core, therefore the sandwich plate with honeycomb core needs more temperature to buckle. Care must be taken to note that the clamped condition enhances the plate stiffness. Accordingly, the buckling temperature of clamped sandwich plate is greater than that of the simply supported one. Moreover, increasing the lightweight core thickness weakens the strength of the plate leading to a reduction of the buckling temperature. Figures 4, 5 and 6
investigate the influences of the external electric voltage \((V_0^* = \frac{V_0}{k_0})\), moisture change \(\Delta C\) and core thickness, respectively, on the critical buckling temperature \(T_{cr}\) of SSSS FGP sandwich plates with honeycomb core and porous core (porous-I, porous-II and porous-III). As mentioned above, the critical buckling temperature \(T_{cr}\) of the sandwich plates with honeycomb core is greater than that of the sandwich plates with porous core. It is also noted that the increase of the side-to-thickness ratio, electric voltage \(V_0^*\), moisture change \(\Delta C\) and core thickness leads to a severe reduction in the buckling \(T_{cr}\). Moreover, the effects of the electric voltage \(V_0^*\) and moisture change \(\Delta C\) are more pronounced for thin plates. Since the increase of the lightweight core thickness weakens the plate strength, the sandwich plate needs a little heat to buckle.

Influences of the power law index \(\eta\) and plate aspect ratio \(b/a\) on the critical buckling temperature \(T_{cr}\) of SSSS FGP sandwich plates with honeycomb core and porous core (porous-II) are discussed in figure 7. It is noted that the buckling temperature \(T_{cr}\) gradually decreases as the ratio \(b/a\) increases. While, it is no longer decreasing as the power law index \(\eta\) increases.

Critical buckling temperature \(T_{cr}\) of SSSS FGP sandwich plates with honeycomb core and porous core (porous-III) under uniform, linear and nonlinear temperature rise through the thickness is depicted in figure 8. It can be seen that the uniform temperature load leads to a minimum buckling temperature while the nonlinear one leads to a maximum buckling temperature. However, the intermediate buckling temperature occurs with the linear temperature rise. Figure 9 displays the impacts of the porosity factor \(k_0\) on the critical buckling temperature \(T_{cr}\) of SSSS FGP sandwich plates for different porous types (porous-I, porous-II and porous-III). It is noted from this figure that the critical buckling temperature \(T_{cr}\) increases as the porosity factor increases because the thermal conductivity decreases with increasing the pores in the core layer.

7. Conclusions

Thermal buckling load of an FGP sandwich plate with lightweight core under various boundary conditions and subjected to elevated temperature and humid conditions as well as external electric voltage is studied based on a modified four-unknown shear deformation plate theory. The temperature and moisture conditions are uniformly, linearly or nonlinearly varied through the thickness of the sandwich plate. The upper and lower face layers are made of FG piezoelectric materials. While, the lightweight core is considered as hexagonal honeycomb structure or functionally graded porous structure with different porosity distributions. The principle of virtual work is utilized to establish the stability equations involving the thermal, humid and electric resultant forces. The obtained results for FGM sandwich plates are compared with those available in the literature. In addition, the impacts of several parameters such as the plate geometry parameters, core type, power law index, moisture concentration, external applied voltage and boundary conditions on the buckling temperature of the sandwich plates with lightweight core are discussed. The numerical results support the ensuing conclusions:
The buckling temperature of clamped sandwich plate is greater than that of the simply supported one because the clamped condition enhances the plate stiffness.

Increasing the lightweight core thickness, external electric voltage and moisture concentration weakens the strength of the sandwich plate leading to a reduction of the buckling temperature.

Since increasing the porosity factor reduces the thermal conductivity of the core layer, the buckling temperature increases.

The results of the current study may help in designing and manufacturing more applicable structures that can be used in aerospace, automotive and high speed trains industry.

For more generality, the nonlinear temperature buckling of the FGP sandwich plate with lightweight core will be considered in a future work.

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**Figure 9.** Influences of the porosity factor $k_0$ on the critical buckling temperature $T_{cr}$ of SSSS FGP sandwich plates for differen porous types (a) porous-I, (b) porous-II and (c) porous-III ($t_h = 0.4k$, $\Delta C = 0.01$, $\zeta = \eta = 1$, $a/b = 1$, $V_{c0}^* = 0.05\%$).
Data availability statement

No new data were created or analysed in this study.

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