Analysis of the absolute and the relative transmissibility on the centrifugal pump

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Abstract. In this paper, we proposed a new technique on the relative transmissibility which are generally only known as the absolute transmissibility or transmissibility. Relative transmissibility is ratio of the relative deflection amplitude of the isolator to the displacement amplitude imposed at the foundation. Absolute transmissibility is ratio of the vibration amplitude of the equipment to the vibration amplitude of the foundation. Centrifugal pump system which is supported by foundation in the form of A36 steel hollow rectangular has one degree of freedom. This method lead to calculate absolute and relative transmissibility ratio both normal pump and damage pump with variation of spring stiffness and coefficient damping. Relative transmissibility is significant only in an isolator used to reduce vibration transmitted from a vibrating foundation. It is proven theoretically and experimentally that pump with the A36 steel as the foundation has relative transmissibility less than one, which means that system has a good isolation. The minimum value of spring stiffness which is used of isolation system to have a good isolation is $10^7$ N/m. This research opens opportunities for industry in determining type of material used for machine foundation which is related to the value of spring stiffness.

1. Introduction

Mathematical modelling can be applied in a modeling system in the centrifugal pump. It can be used to detect damaged pump as the rotating machine [1]. The damaged pump is caused by inside damaged pump and it because of other vibration around. Pumps which suffered damage to one of its components will result in performance unbalanced and cause vibration anomalies that would negatively affect to other parts. If the source of the vibration around the larger pump will interfere with the performance of the engine at the pump. In almost literature, it is generally only known absolute transmissibility or transmissibility. There is still other type namely the relative transmissibility. The relative transmissibility is the ratio of the relative deflection amplitude of the isolator to the displacement amplitude imposed at the base, while the absolute transmissibility is the ratio of the vibration amplitude of the equipment to the vibration amplitude of the host structure (base).

Both of those transmissibility principles are almost same to find transmitted energy, but the difference is the kind of vibration being compared. The absolute transmissibility is used to determine reduction of transmitted energy conducted by an isolator, whereas the relative transmissibility is used to determine the clearance that transmitted to an isolator. This relative transmissibility would be significant only when an isolator is used to reduce transmitted vibration of a vibrating foundation. The principle of transmissibility can be used to determine the transmitted energy on a structure model of the building with 3 floors as on a thesis titled "Analysis of Dynamic transmissibility as a Feature for Structural

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Damage Detection" by Thimoty James Johnson, 2002 [2]. Moreover, the principle of transmissibility also can be used to determine the performance characteristics between linear viscous damping system with pure cubic damping [3].

The principle of vibration transmissibility is used to detect damaged pump in the research which conducted by Anisatul Fauziah [1]. The study states that the transmissibility can be detected using the accelerometer array as evidenced by the change in the diagnosis of damage to the pump that known in the spectrum of the Fast Fourier Transform. From these studies, the principle of transmissibility will be used to analyze the absolute and relative transmissibility in centrifugal pumps and also to determine the approximate value of stiffness (spring stiffness) and the damping coefficient (damping coefficient) of the system.

2. Theory

2.1. Centrifugal pump

Centrifugal pump is a machine that converts fluid kinetic energy into potential energy through a rotating impeller in the casing [4]. Centrifugal pump has an impeller to lift liquid from lower to higher place. Excitation power is given by the pump shaft to rotate the impeller in the liquid, so the liquid in the impeller rotates by impulse blades. The liquid flows from the middle of the impeller to the outside through the channels between the blades because of centrifugal pump arising. It causes the liquid head pressure becomes higher, as well as head speed increases because the liquid is accelerating.

There are several types of damage in pump that can be detected by the level of vibration amplitude at a specific frequency. The types of those damaged are bearing fault, unbalance, and misalignment [5].

2.2. Transmissibility

Transmissibility is commonly used in the field of vibration isolation. It concerns to minimize or control unexpected transmission and motion from a source to a machine device or other system [6]. There are several types of transmissibility such as force transmissibility, acceleration transmissibility, velocity transmissibility, and displacement transmissibility. Those kind of transmissibility, have the same general equation of transmissibility at one degree of freedom [2]. At displacement transmissibility is generally divided into two types namely transmissibility absolute and relative transmissibility. The absolute transmissibility is the ratio of the vibration amplitude of the equipment to the vibration amplitude of the host structure (base), which should be as small as possible [3] [4]. The other characteristic is the relative transmissibility which represents the ratio of the relative deflection amplitude of the isolator to the displacement amplitude imposed at the base [3][4].

The system has a spring stiffness and damping coefficient with Single Degree of Freedom (SDOF). The formula for the absolute and the relative transmissibility ratio for SDOF shown in equation (1) and (2).

\[
T_a = \frac{\sqrt{1 + 4\xi^2\left(\frac{\omega}{\omega_0}\right)^2}}{\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2 + 4\xi^2\left(\frac{\omega}{\omega_0}\right)^2}} \tag{1}
\]

\[
T_r = \frac{\left(\frac{\omega}{\omega_0}\right)^2}{\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2 + 4\xi^2\left(\frac{\omega}{\omega_0}\right)^2}} \tag{2}
\]

The value of the damping coefficient, natural frequency, damping ratio, and frequency ratio expressed in equation (3), (4), (5) and (6),
\[ c = 2\zeta \sqrt{k m_{\text{total}}} \]  
(3)
\[ \omega_0 = \sqrt{\frac{k}{m_{\text{total}}} \text{ (4)}} \]
\[ \zeta = \frac{c}{2\sqrt{k m_{\text{total}}}} \text{ (5)} \]
\[ r = \frac{\omega}{\omega_0} \text{ (6)} \]

where \( T_a \) is absolute transmissibility ratio (dB), \( T_r \) is relative transmissibility ratio (dB), \( x \) is displacement (m), \( \omega \) is rotational speed (rad/s), \( \omega_0 \) is natural frequency, \( r \) is frequency ratio, \( c \) is damping coefficient (Ns/m), \( k \) is spring stiffness (N/m), \( m_{\text{total}} \) is total mass (kg), and \( \zeta \) is damping ratio.

Isolation is a material used to dampen a vibration. When the value of the frequency ratio is larger than \( \sqrt{2} \) which have absolute transmissibility ratio less than 1, it means that it has a good isolator. When the value of the frequency ratio is less than \( \sqrt{2} \) which have absolute transmissibility ratio more than 1, it means that it has not good isolator. The difference in the value of transmissibility ratio is related to the value of its rotational speed (\( \omega \)). If the value of rotational speed is small, then it will cause the value of stiffness become smaller so that the absolute transmissibility ratio becomes lower [7].

2.3. Double Clamped Beam
Beam is a structural component that used as the foundation. Beam is commonly used to support a vertical load, shear stress, and bending moments. There are several types of beam that classified into several categories. One type of category of the beam is doubly clamped beam which is classified based on the type of supporters. Doubly clamped beam is the beam with both ends attached. It is not free to move [8]. The value of spring stiffness on a doubly clamped beam either in vertical or horizontal axis is shown in the equation (7),
\[ k = \frac{2EI}{L} \text{ (7)} \]
where \( k \) is spring stiffness (N/m), \( E \) is modulus of elasticity (Pa), \( I \) is inertia moment (m\(^4\)), and \( L \) is length of beam (m).

2.4. Mass
Mass is a characteristic of the material that causes inertia. The system that consist of a pump with a beam has the formula as shown in equation (8),
\[ m_{\text{total}} = m_{\text{pump}} + \Omega m_{\text{beam}} \text{ (8)} \]
where \( m_{\text{total}} \) is total mass (kg), \( m_{\text{pump}} \) is mass of each pump (kg), \( m_{\text{beam}} \) is mass of beam (kg), and \( \Omega \) is position of centroid.

3. Research Methodology

3.1. Taking Data of Mini Plant
We take the data of centrifugal pump mini plant which type Panasonic GP-129JXK in Vibration and Acoustics Laboratory of Physics Engineering ITS. Those data are masses data from each kind of pump. This centrifugal pump has a value of rotational speed of 3000 rpm (314 rad / s) [9]. Each pump is located on the same foundation that is A36 steel which has a modulus of elasticity \( E = 200 \) GPa [10]. This steel is covered by wood. But in this case, wood is ignored. The value of the total mass of the pump foundation is 62.29 kg.
3.2. Plant Modelling.
Plant modelling of centrifugal pumps are determining the type of pump used in normal pump, pump with 6 grams of damage unbalance, bearing fault, and misalignment of 3 mm. We determine the value of spring stiffness of the beam as the pump foundation that is A36 steel and double clamped beam type using equation (7), the value of the damping coefficient of the pump foundation on the vertical and horizontal axis using equation (3), the total mass of each pump using equation (8), the absolute and relative transmissibility ratio on the vertical and horizontal axis using equation (1) and (2).

![Figure 1. Beam of pump in outer part](image1.png)
![Figure 2. Beam of pump in inner part](image2.png)

| No | Pump Damage       | Vertical Ta (dB) | Horizontal Ta (dB) | Axial Ta (dB) |
|----|-------------------|-----------------|--------------------|---------------|
| 1  | Unbalance 6 gram.cm | 4.4637          | 6.7306             | 1.6573        |
|    | Bearing Fault     | 0.9713          | 1.6744             | -2.5261       |
| 2  | Unbalance 27 gram.cm | 0.52213        | -1.7310            | -3.2755       |
|    | Misalignment 1 mm  | 9.339           | -0.41645           | 0.81151       |
| 3  | Bearing Fault     | 3.0104          | -4.9846            | -1.1          |
|    | Misalignment 3 mm  | 0.63865         | -8.0388            | 2.6465        |

3.3. Model variation and validation
We do the model variations such as the variation of the damping coefficient with constant spring stiffness and the variation of spring stiffness with constant damping coefficient. Model validation is used to make sure the model representative of the actual plant. Validation of this model used the data measurement that has been done by A Fauziah on research of multi-detection of damaged pump using the accelerometer array [1].

4. Data Analysis

4.1. Variation of Damping Coefficient with Constan Spring Stiffness
In the figure 3 and 4, when the value of the damping ratio is less than 1 so that it has the form of a graph which has an amplitude and overshoot as shown in the blue line, red and orange. This system belongs to the type of underdamped. On the other hand, when the value of the damping ratio is greater than one, then the shape of the graph does not have the amplitude and overshoot because this type is the type of overdamped as shown in green and purple lines.

All of the absolute transmissibility ratio is more than one and the relative transmissibility ratio is less than one. The absolute transmissibility ratio value is more than one which means the system does not have good insulation system due to spring stiffness does not function as an insulator because the value of spring stiffness both on the vertical and horizontal axis effect on the value of its natural frequency. The value of its natural frequency is greater than the value of the rotational speed (314 rad/s), so the value of displacement that occurs in the system is transmitted to its mass. In other words, $k$ is not
functioning as an insulator. But in this case, the relative transmissibility ratio is less than one which means a distance of relative deflection to the isolator quite good. It because of rectangular hollow steel that serves as an insulator.

On each pump with the variation of the value of damping coefficient, misalignment 3 mm pump has the greatest values of the absolute and relative transmissibility ratio because misalignment 3 mm pump has the greatest mass pumps among others. The value of this mass is proportional to the transmissibility ratio.

Table 2. Total mass in each pump

| Types of pump   | Mass (kg) | Vertical axis | Horizontal axis |
|-----------------|-----------|---------------|-----------------|
| Normal          | 5.9       | 18.66945      | 40.7824         |
| Unbalance 6 gram| 5.906     | 18.67545      | 40.7884         |
| Bearing Fault   | 5.9       | 18.66945      | 40.7824         |
| Misalignment 3 mm| 6.098     | 18.86745      | 40.9804         |

Table 3. The value of $\zeta$, $Ta$, and $Tr$ misalignment 3 mm pump with variation of damping coefficient in vertical and horizontal axis

| $c$ (Ns/m) | $\zeta$ | $Ta$ (dB) | $Tr$ (dB) | $c$ (Ns/m) | $\zeta$ | $Ta$ (dB) | $Tr$ (dB) |
|------------|---------|-----------|-----------|------------|---------|-----------|-----------|
| 9.57x10^6 | 11.261  | 1.000176  | 0.000185  | 9.57x10^7 | 76.408  | 1.000039  | 0.000128  |
| 9.57x10^5 | 11.261  | 1.000194  | 0.000194  | 9.57x10^6 | 76.408  | 1.000384  | 0.000403  |
| 9.57x10^4 | 0.11261 | 1.000194  | 0.000194  | 9.57x10^5 | 0.76408 | 1.000422  | 0.000422  |
| 9.57x10^3 | 0.011261| 1.000194  | 0.000194  | 9.57x10^4 | 0.076408| 1.000422  | 0.000422  |
| 9.57x10^2 | 0.0011261| 1.000194  | 0.000194  | 9.57x10^3 | 0.0076408| 1.000422  | 0.000422  |

4.2. Variation of Spring Stiffness with Constant Damping Coefficient

On the vertical axis, the value of the damping coefficient is between 1 to $10^9$ Ns/m and the spring stiffness value is less than or equal to $10^5$ N/m to get the value of the frequency ratio $r > \sqrt{2}$. So, the absolute transmissibility ratio value ($Ta$) is less than 1. When the value of $Ta$ is less than 1, it indicates the system has good insulation. In addition, the value of spring stiffness is greater than or equal to $10^6$ N/m to get the value of the frequency ratio $r < \sqrt{2}$ so that the absolute transmissibility ratio value is more than 1. When the value of $Ta$ is more than 1, it indicates that insulation used in the system is not good. On the horizontal axis, the value of the damping coefficient is between 1 to $10^9$ Ns/m and the spring stiffness...
stiffness value is less than or equal to $10^6$ N/m to get the value of the frequency ratio $r > \sqrt{2}$ so the absolute transmissibility ratio value ($Ta$) is less than 1. In addition, the value of spring stiffness is greater than or equal to $10^7$ N/m to get the value of the frequency ratio $r < \sqrt{2}$ so that the absolute transmissibility ratio value is more than 1. The relative transmissibility in the vertical and horizontal axis will be less than one if it has a minimum value of the spring stiffness of $10^7$ N/m. This means that the deflection distance relative to its isolator is quite good. This is because of A36 steel rectangular hollow that serves as an insulator to reduce the vibrations that transmitted to the foundation. In the spring stiffness variation, the greater the value of its mass, the absolute value and relative transmissibility ratio will be smaller. This occurs because the value of their natural frequencies become smaller and affected on transmissibility ratio.

Table 4. The value of $\xi$, $Ta$, and $Tr$ misalignment 3 mm pump with variation of spring stiffness in vertical and horizontal axis

| Vertical       | Horizontal     |
|----------------|----------------|
| $k$ (N/m)      | $k$ (N/m)      |
| $\zeta$        | $\Xi$         |
| $Ta$ (dB)      | $Ta$ (dB)      |
| $Tr$ (dB)      | $Tr$ (dB)      |
| $10^7$         | $10^8$         |
| 0.00364        | 0.00078        |
| 1.22854        | 1.042106       |
| 0.22854        | 0.042106       |
| $10^6$         | $10^7$         |
| 0.01151        | 0.00247        |
| 1.16224        | 1.677979       |
| 2.16101        | 0.677985       |
| $10^5$         | $10^6$         |
| 0.03640        | 0.00781        |
| 0.05954        | 0.329037       |
| 1.05664        | 1.328822       |
| $10^4$         | $10^5$         |
| 0.11511        | 0.02470        |
| 0.01781        | 0.026598       |
| 1.00526        | 1.025345       |
| $10^3$         | $10^4$         |
| 0.36401        | 0.07811        |
| 0.01689        | 0.008176       |
| 1.00040        | 1.002451       |

5. Conclusion

Based on the theoretical and experimental results, this can be concluded several things:

- If the value of the relative transmissibility ratio is less than one, then the required minimum value of spring stiffness is $10^7$ N/m.
- If the absolute value of transmissibility ratio is less than one, the damping coefficient value between 1-10 Ns/m, then the maximum spring stiffness values is $10^5$ N/m on the vertical axis. On the horizontal axis, the value of the damping coefficient between 10-10$^9$ Ns/m, the maximum spring stiffness values is $10^8$ N/m.

6. References

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