Finite Analysis of Support Time for Tunnels in Strain-softening Rock Mass in Consideration of Fictitious Support Pressure

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Abstract. This study considers that the support time is quantitatively determined by the production limit of the displacement reduction factor and the support force under the extrusion conditions of the strain-softening rock mass. Therefore, the two indicators of the downlines under the support time are the displacement reduction factor of the support force and the yield limit. Based on the solution of the fictitious pressure proposed in an existing paper, the finite difference method is adopted to investigate the variations of the support force and displacement reduction factor versus the delayed distance considering different support types, initial stresses, and post-peak behaviours. The results show that on the one hand, the delay distance is suggested within 1\( R_0 \) in most tunnel cases; on the other hand, the factors have greater impact on rock-support interactions are rock mass and in-situ stress. Relatively contrast, softening and expansion behavior was not significant enough. Furthermore, it is also very important in composite support systems to assess the proportion of loads shared with the weakest part.

Keywords: Fictitious pressure, strain-softening rock mass, support time, delayed distance, allowable deformation.

1. Introduction

Rock-support interaction is critical to the assessment of the load imposed on the support during tunneling, which is often analysed with convergence-confinement method [1-5]. Improvement of the convergence constraint method yields a two-stage analysis, a method that divides the tunneling process into two stages [6]. It is first assumed that the fictitious support pressure caused by the tunnel surface effect supports the rock mass during the first stage before supporting the installation. As the tunnel surface gradually moves forward, the fictitious support pressure gradually shifts to the support pressure. Through a two-stage analysis method, Einstein and Schwartz [7,8] presented an analytical solution to compute the stresses and displacements on the support interacting with an elastic rock mass under a non-uniform stress field, and discussed the influence of stress reduction factor on the loads and displacements on the support. Cai et al. [9,10] delivered an analytical model to investigate the interaction between the ground and the rock bolt and considered the released stresses at the installation of the rock bolt. Assuming a stress reduction factor, Carranza-Torres and Diederichs [11] computed the force on the liner and the steel set of a composite support by using the commercial finite difference software. Hoek [12] obtained the fictitious support pressure by substituting the released displacement...
into the strain prediction equation, and evaluated the force in different support types by relieving the fictitious pressure.

However, the fictitious support pressure and the stress release factor in most studies [7-12] are predicted by simplifying the rock mass as elastic, which is not appropriate if more complicated mechanical behaviours of the rock mass are considered. Cui et al. [6] accounted for the strain-softening and dilatant behaviours for rock mass in first stage, but without discussing the rock-support interaction in the second stage. Meanwhile, the support in many studies [7-12] is postulated to be added at the time the tunnel is excavation without any delay. However, effects of the support time on the forces in the support and rock deformation are significant for support safety. Therefore, how to reduce the support load under the extrusion conditions is the top priority. It is recommended to install delay or yield support and permitted rock displacement [12-16]. However, the impact of rock mass mechanical properties is not clear enough, while the delay distance and reserved displacement are difficult to obtain the relevant quantitative analysis.

In this paper, the rock-support interaction is investigated with the two-stage analysis method by incorporating a Finite Difference Method (FDM) solver. It mainly includes the following parts:

1. The fictitious support pressure presented in Ref. [6] is adopted as the initial pressure on the support at the beginning of the second stage.

2. Based on the above theory, a planar model with a circular water purification stress field allows the rock-support interaction by gradually loosening the virtual support pressure outside the tunnel from the initial value to zero.

3. The displacement reduction factor is proposed to evaluate the impact of the support on controlling the rock deformation, yielding the ratio of the supported displacement reduction to the maximum rock deformation.

4. Combined with considerations of different support types, initial stress, and post-peak behavior, the variation of support force and displacement reduction factors with delay distance is investigated. The two indicators of the support installation delay distance and the allowable deformation are also determined by quantification.

2. Problem Statement

2.1. Basic Assumptions

The following assumptions are made in this study:

1. The tunnel opening is circular and excavated under a hydrostatic stress field $\sigma_0$.

2. The tunnel problem is reduced to a plane problem by applying virtual support pressure on the periphery of the tunnel. On the plane, radial stress $\sigma_r$ in the rock body corresponds to smaller main stress $\sigma_3$, while tangential stress $\sigma_\theta$ corresponds to large main stress $\sigma_1$.

3. Post-peak stages will involve elastic-perfectly-plasticity (EPP), strain-softening (SS), and elastic – brittle-plasticity (EBP) behavioral [17]. Meanwhile, the rock bulk strength mainly follows the Mohr-Coulomb (M-C) and Hoek-Brown (H-B) failure criteria. Thus, the fault standard is set as follows:

$$f(\sigma_\theta, \sigma_r, \eta) = \begin{cases} 
\sigma_\theta - \sigma_r - \sigma_c \left[ m_b(\eta) \sigma_1/\sigma_c + s(\eta) \right]^{\omega(\eta)} \\
\sigma_0 - K_c(\eta) \sigma_r - 2C(\eta) \sqrt{\sigma_0(\eta)} 
\end{cases}$$

(1)

In which, for M-C criterion, $C$ is the cohesion and $K_0$ is the friction coefficient; for the H-B failure criterion, $m_b, s, a$ are the strength constants; the $\omega$ is assumed to be a function of the plastic softening parameter $\eta$. The relation between $\eta$ and $\omega$ can be expressed as:

for EPP rock mass:

$$\omega(\eta) = \omega_0$$

(2)

for SS rock mass:
for the EBP rock mass:

\[
\omega(\eta) = \begin{cases} 
\omega' - (\omega' - \omega^p) \frac{\eta}{\eta^*}, & 0 < \eta < \eta^* \\
\omega', & \eta \geq \eta^*
\end{cases}
\]

(3)

In Equation (2), (3) and (4), \( \eta^* \) is the critical plastic softening parameter; \( \omega^p \) is the peak intensity parameter; and \( \omega' \) is the residual strength parameter. Among them, \( \eta^* \) varies by rock, in the EPP rocks, \( \eta^* \) is \( \infty \); in the EBP rock mass, \( \eta^* \) is 0.

(4) M-C function is selected as the flow potential \( \phi \), which is described as:

\[
\phi = \sigma_x - K_{\psi} \psi
\]

(5a)

\[
K_{\psi} = \frac{(1 + \sin \psi)}{(1 - \sin \psi)}
\]

(5b)

Where \( K_{\psi} \) is the dilatancy coefficient and \( \psi \) is the dilatancy angle.

2.2. Two-stage Analysis Method

As the name suggests, the two-stage analysis method is divided into stage one and two. In the first phase, the rock bunker near the tunnel surface before the scaffold mounting is supported by a fictitious support pressure \( p_f \) mobilized by the tunnel surface effect and evenly distributed around the tunnel periphery. The tunneling process is plotted based on this method, as shown in figure 1. In the initial state, the distance of the cross-section \( A - A' \) to the tunnel face is \( x_{ini} \), the rock displacement at the tunnel periphery is \( u_0,ini \), and the fictitious support pressure is \( p_{f,ini} \). During the second stage, \( p_{f,ini} \) at the cross-section \( A - A' \) is unloaded and transferred to the support as the excavation advances. It is seen that the tunnel surface and support together support the rock mass. The pressure \( p_i \) can be decomposed into the fictitious support pressure \( p_f \) and the support pressure \( p_s \). Therefore, the intermediate and final states of the second phase can be analyzed separately. The case of the factor change in the intermediate state is: \( x_{ini} \) and \( u_0,ini \) increases to \( x_{int} \) and \( u_0,int \) respectively. The \( p_s \) was increased from zero to the \( p_{s,fin} \), the \( p_{f,ini} \) drops to the \( p_{f,int} \). At the final state, the pressure is fully transferred to the support, \( p_{f,int} \) decreases to \( p_{f,fin} = 0 \), and \( p_s \) grows to \( p_{s,fin} \). Given that the study does not consider the rheological behavior of the rock mass, and the bracket installation at \( x_{ini} \), \( p_{s,fin} \) is the final load of each excavation step, and \( u_0,fin \) is the final rock mass displacement at the periphery of the tunnel.

Figure 1. Tunnel excavation process with two-stage analysis method.

3. Finite Difference Analysis

3.1. Model Configuration

A series of FDM analyses simulating the tunnel excavation process are conducted using a commercial finite difference software [18]. 1 / 4 of the correlation regions is modeled based on the symmetry of
tunneling through the tunnel to the vertical and horizontal planes (Figure 2 (a) – (b)). A problem slice with a thickness of 0.5m was considered in the out-of-plane direction of the structure. Meanwhile, the displacement in this direction was fixed to follow the planar strain condition. Furthermore, The model size under plane strain conditions was 100m in the vertical and horizontal directions, respectively.

Figure 2. FDM model for the rock-support interaction: (a) Boundary conditions that belong to the model; (b) Meshes around circular openings.

The support is discretised with structural shell elements. This predicts a high interface strength between the support and the rock mass, assuming a rigid contact with the periphery of the tunnel. Three types of supports (I, II, III) are adopted. Support types I and II are comprised of shotcretes with an equivalent thickness $t$ of 0.21 but different elastic moduli $E_s$, the former is 20 GPa, and the latter is 30 GPa. Support type III can be referred to Ref. [11]. Its components include shotcrete and two steel sections along the width of the unit (Type W6 × 25). The elastic modulus $E_s$ of the support type III is derived as 37.69 GPa from the equivalent section approach in Ref. [11].

The data in Table 1 show the properties of the rock mass. From A1 to A3, B1 to B6, C1 to C5, D1 to D4, the study considered different GSI$_{max}$, $\eta^*$, $\psi$ and $\sigma_0$ within these intervals. EPP rock mass is assumed in the cases A1 to A3, C1 to C5 and D1 to D4. The peak of the geological strength index is indicated by GSI$_{max}$, which also means rock mass. Meanwhile, the numerical procedure proposed in Ref. [6] can solve the virtual support pressure $p_{f,ini}$. Values of $p_{f,ini} / \sigma_0$ versus a normalised initial distance to the tunnel face $X^*_ini$ with each case are presented in the Appendix A. $X^*_ini$ is the ratio of $x_{ini}$ to the radius of the circular opening $R_0$. The mechanical properties of rock masses from A1 to A3 are displayed in table 1. From cases B1 to B6, and C1 to C5, the basic parameters are: $\varphi$ is 32.47°, $C$ is 3.08 MPa, $\varphi^*$ is 24.41°, $E_s$ is 9000 MPa, $R_0$ is 5 m, $\sigma_0$ is 35 MPa. For cases B1 to B6, $\eta^*$ is 0, 0.005, 0.01, 0.05, 0.5, $\infty$. For cases C1 to C5, $\psi$ is 0°, 8.12°, 16.24°, 24.36°, 32.47°. The basic parameters for cases D1 to D4 are the same to Case A3. For cases D1 to D4, $\sigma_0$ is 5 MPa, 25 MPa, 35 MPa, 50 MPa.

Table 1. Data on rock mass mechanical properties from cases A1 to A3

| GSI$_{max}$ | $m_0$ | $s$ | $\alpha$ | $E_s$/ MPa | $p_{f,ini}$/ MPa | $\mu$ | $R_0$/ m | $\sigma_0$/ MPa | $\sigma_0$/ MPa | $\eta^*$ | $\psi$ /
|------------|-------|-----|---------|------------|--------------|-----|--------|--------------|-------------|--------|---------|
| A1         | 75    | 6.35| 0.0509  | 0.5        | 30           | 0   | 0.25   | 5            | 110         | 35     | 0       |
| A2         | 50    | 1.71| 0.0026  | 0.5        | 9000         | 0   | 0.25   | 5            | 80          | 35     | 0       |
| A3         | 40    | 0.99| 0.0008  | 0.5        | 3000         | 0   | 0.25   | 5            | 30          | 35     | 0       |

3.2. Numerical Procedures
Implementing the FDM analysis can be done by gradually relaxing the virtual support pressure outside the tunnel from the in-situ value to zero. The numerical procedures to implement the FDM analysis are as follows:
(i) Exert $p_{f,ini}$ on the tunnel periphery in the model, and iterate to reach the equilibrium state and obtain $u_{0,ini}$.
(ii) Add shell elements on the tunnel periphery; at excavation step $i$, $x_{ini}$ increases to $x_{i,ini}$, obtain $p_{f,i}$ at $x_{i,ini}$ and reduce $p_{f,ini}$ to $p_{f,i}$ to mimic the excavation advance; iterate to reach the equilibrium state.
(iii) Check whether \( p_{f(i)} \) decreases to zero or not; if yes, cease the rock-support interaction and obtain \( u_{0,\text{fin}} \) and the final thrust force in the shell element \( N_s \), otherwise, set \( i = i + 1 \) and repeat the iteration (ii).

In the implementation, the interval between \( x(i) \) and \( x(i+1) \) denotes the excavation step size. As listed in Table A-1, a majority of the step sizes are equal to \( R_0 \), whereas several step sizes are smaller than 0.5\( R_0 \). As in Ref. [19], when the step size is lower than or equal to \( R_0 \), the LDP remains basically constant. This means various step sizes in Table A-1 exert negligible impact on the FDM analysis, and the analysis is applied to the tunnelling with all the excavation step size smaller than or equal to \( R_0 \).

3.3. Delayed Distance and Allowable Deformation

Allowable rock formations are indicated by \( u_{0,\text{allow}} \). \( X^*_{\text{del}} \) is reserved for support installation, which is the ratio of the delayed distance \( x_{\text{del}} \) to the radius of the circular opening \( R_0 \). In the LDP, \( u_{0,\text{allow}} \) and \( X^*_{\text{del}} \) represent a one-to-one communication. In addition to \( X^*_{\text{del}} \), \( X^*_{\text{ini}} \) also indicates the normalized distance of the bracket mounting to the tunnel surface. Furthermore, \( u_{0,\text{ini}} \) and \( u_{0,\text{allow}} \) indicate the initial rock mass displacement at \( X^*_{\text{ini}} \). However, it need to note the purpose of estimating \( X^*_{\text{del}} \) and \( u_{0,\text{allow}} \), ensuring the safety of support and the effectiveness of controlling rock displacement. \( X^*_{\text{del}} \) and \( u_{0,\text{allow}} \) represent relatively narrow ranges of \( X^*_{\text{ini}} \) and \( u_{0,\text{ini}} \), whereas \( X^*_{\text{ini}} \) and \( u_{0,\text{ini}} \) represent random distance and initial displacement, respectively. The thrust of the concrete in the final state \( N_c \) can be used to determine the range of the \( u_{0,\text{allow}} \) and \( X^*_{\text{del}} \) used to support the installation, and the displacement reduction factor \( f_0 \). \( f_0 \) denotes the reduction ratio of rock deformation in terms of the support installation at \( X^*_{\text{ini}} \), which is defined as:

\[
 f_0 = \frac{u_{0,\text{max}} - u_{0,\text{fin}}}{u_{0,\text{max}}} \tag{6}
\]

In formula, \( u_{0,\text{max}} \) represents the maximum rock displacement at the periphery of the tunnel, the parameter can be obtained in the FDM model by applying \( p_i = 0 \) to the tunnel periphery; figure 3 is the flowchart for the methodology, the changes of \( f_0 \) and \( N_s \) are negatively correlated with the \( X^*_{\text{ini}} \), they increase with decreasing \( X^*_{\text{ini}} \), and the upper and lower bounds of \( X^*_{\text{del}} \) and \( u_{0,\text{allow}} \) can be obtained by the following steps:

(i) Solve \( u_{0,\text{max}} \) and the variations of \( u_{0,\text{ini}}, u_{0,\text{fin}} \) and \( N_c \) versus \( X^*_{\text{ini}} \) by implementing FDM analysis.
(ii) Obtain the variation of \( f_0 \) by implementing \( u_{0,\text{fin}} \) and \( u_{0,\text{max}} \) into Eq. (6).
(iii) Solve the lower bound of \( X^*_{\text{del}} \) by the variation of \( N_c \) versus \( X^*_{\text{ini}} \) corresponding to the yield limit of \( N_c \); solve the lower bound of \( u_{0,\text{allow}} \) by the variation of \( u_{0,\text{ini}} \) versus \( X^*_{\text{ini}} \) at the lower bound of \( X^*_{\text{del}} \).
(iv) Solve the upper bound of \( X^*_{\text{del}} \) by the variation of \( f_0 \) versus \( X^*_{\text{ini}} \) corresponding to the required \( f_0 \); solve the upper bound of \( u_{0,\text{allow}} \) by the variation of \( u_{0,\text{ini}} \) versus \( X^*_{\text{ini}} \) at the upper bound of \( X^*_{\text{del}} \).

![Figure 3. The flowchart for the methodology.](image)

3.4. Verification for FDM Model

\( u_{0,\text{max}} \) by FDM model and by the numerical approach proposed in Ref. [23] are compared. In Ref. [23], the surrounding rock mass was divided into finite annuli with a constant radial stress increment, and the stress and strain components were solved with the stress equilibrium and displacement

\[
\text{Implement FDM analysis}
\]

Solve variations of \( u_{0,\text{max}}, u_{0,\text{fin}}, N_c, \) and \( u_{0,\text{ini}} \)

- By \( u_{0,\text{max}} \) and \( u_{0,\text{fin}} \)
  - Obtain variation of \( f_0 \)
    - By \( f_0 \)
      - Obtain upper bound of \( X^*_{\text{del}} \)
      - By \( u_{0,\text{ini}} \) and upper bound of \( X^*_{\text{del}} \)
    - Obtain upper bound of \( u_{0,\text{allow}} \)

- By \( N_c \)
  - Obtain lower bound of \( X^*_{\text{del}} \)
  - By \( u_{0,\text{ini}} \) and lower bound of \( X^*_{\text{del}} \)
  - Obtain lower bound of \( u_{0,\text{allow}} \)
compatibility equations by the FDM method. The number of finite annuli is about 5000. It has been compared to a numerical procedure in Ref. [24], the numerical error of the maximum rock deformation is less than 1%. The comparison of $u_{0,\text{max}}$ by the FDM analyses and Ref. [23] for cases A1 to A3, B1 to B6, C1 to C5, D1 to D4 are shown in figure 4. As shown in figure 4, in most cases the difference between the two methods is less than 3%. This implies a satisfying agreement between the two methods. The proposed method is acceptable.

![Image](image.png)

**Figure 4.** Comparison of $u_{0,\text{max}}$ by FDM model and a numerical approach proposed in Ref. [23].
(a) Cases A1 to A3; (b) Cases B1 to B6; (c) Cases C1 to C5; (d) Cases D1 to D4

4. Results and Discussion

4.1. Deformation Reduction Ratio and Thrust force

Figures 5 - 8 plot variation laws of $N_s$ and $f_0$ with cases A1 to A3, cases D1 to D4, cases B1 to B6 and cases C1 to C5. Various values of GSI max, $\sigma_0$, $\eta^*$ and $\psi$ are assigned to the above cases, respectively (see table 2). For cases B1 to B6 with different $\eta^*$, and cases C1 to C5 with different $\psi$, support type I is adopted. For cases A1 to A3 with different GSI max, and cases D1 to D4 with different $\sigma_0$, support types II and III are adopted. Compared with the steel sets, the shotcrete is easier to yield while the support is loaded. Hence, for the support type III, $N_s$ shared in the shotcrete alone is discussed. As $N_s$ calculated by the shell element in the FDM model denotes the composite force in the two materials, i.e., concrete and steel sets, $N_s$ shared by the shotcrete should be obtained by the equivalent section approach proposed in Ref. [6].

As observed in figures 5 to 8, with the support type II, $N_s$ and $f_0$ for the case D4 with $\sigma_0 = 50$ MPa are 9 times and 1.5 times as higher as those for case D1 when $\sigma_0$ is 5 MPa; $N_s$ and $f_0$ for the case A3 with GSI max = 40 are 3.67 times and 5 times as higher as those for the case A1 with GSI max = 75. In contrast, with the influences of $\psi$ and $\eta^*$, the ranges of $N_s$ and $f_0$ is relatively small. Therefore, the influences of the rock mass quality and the in-situ stress on the support force and support effect are more remarkable than post-peak behavior such as the softening and dilatancy.
As shown from figure 5 and 6, $N_s$ is negatively correlated with $\text{GSI}_{\text{max}}$, and it declined as $\text{GSI}_{\text{max}}$ grows. $f_0$ was positively associated with $\sigma_0$, and it decreased with $\sigma_0$. The results show that both $N_s$ and $f_0$ have large values under weak rock mass conditions with high initial stress. The larger $f_0$ is beneficial to the
safety of the tunnel, because the support energy can effectively control the rock mass displacement and make the shotcrete cracks. Therefore, this demonstrates that the yield support is installed when the condition is a weak rock mass with a high initial stress, while the allowed deformation is also retained, the purpose of which is to substantially reduce \( N_s \). Figures 5 - 6 indicates that \( N_s \) and \( f_0 \) decline with \( X_{\text{ini}}^* \). The above shows that the supporting effect limiting the rock displacement will be obvious when the delay distance or the deformation is large. This means that the scaffold mount is needed to be investigated for the optimal allowed deformation or delay distance, thereby guaranteeing a smaller \( N_s \) and a larger \( f_0 \).

Figures 5 to 8 indicates that \( f_0 \) increases with the decreases of GSI\(_{\text{max}}\) and \( \eta^* \), and decreases with the decreases of \( \sigma_0 \) and \( \psi \). The reason for presenting such a result is that \( u_{0,\text{max}} \) will have a major effect on \( f_0 \), and it increases as \( u_{0,\text{max}} \) grows (see Eq. (6)). Apparently, lower GSI\(_{\text{max}}\) and \( \eta^* \), greater \( \sigma_0 \) and \( \psi \) give rise to a greater \( u_{0,\text{max}} \). Hence, the trends of \( f_0 \) with variations in \( \eta^* \), \( \sigma_0 \), GSI\(_{\text{max}}\), and \( \psi \) coincides with those of \( u_{0,\text{max}} \). In contrast, variations of \( N_s \) affected by GSI\(_{\text{max}}\), \( \eta^* \), \( \sigma_0 \) and \( \psi \) obviously differentiate from those of \( f_0 \). It will decline, but then rise with \( \eta^* \). Upon analysis, \( u_{0,\text{max}} \) and \( p_{\text{f,ini}} \) ( the fictitious support pressure) will have an impact on \( N_s \), and resulting in such changes.

### 4.2. Lower and Upper Bounds of Delayed Distance and Allowable Deformation

Assessing the yielding limits of \( N_s \), for the concrete in the supports, it can carry out from three aspects of safety factor FS, support area \( A \) and concrete compressive strength \( \sigma_c \) [11]:

\[
N_s = \frac{\sigma_c A}{FS}
\]

(7)

FS of 0.67, 1 and 1.5 are utilised in Ref. [11]; therefore, the yield limits of \( N_s \) are 2.8 MPa, 4.2 MPa, and 5.6 MPa by Eq. (7), its use is to evaluate three different lower bounds of \( X_{\text{del}}^* \) and \( u_{0,\text{allow}} \). From figures 5 to 8 , the values of \( X_{\text{del}}^* \) and \( u_{0,\text{allow}} \) with FS of 0.67, 1.0 and 1.5, and the specific results are shown in Table 2. The displacement reduction factor \( f_0 \) determines the upper bound of \( X_{\text{del}}^* \) and \( u_{0,\text{allow}} \). \( f_0 \) of 30%, 40%, 50% and 60% are presented, different data describe the effects of supporting processes from weak to strong on the control of rock displacement. Furthermore, the blank representative support in Table 2 is very secure. Similarly, as shown from figures 5 to 8, Table 3 shows the value of \( X_{\text{del}}^* \) and \( u_{0,\text{allow}} \) when \( f_0 \) reaches 30%, 40%, 50% and 60%.

#### Table 2. Lower bounds of \( X_{\text{del}}^* \) and \( u_{0,\text{allow}} \) (a) \( X_{\text{del}}^* \) (b) \( u_{0,\text{allow}} \) (unit: cm)

| FS | Support type III | Support type II |
|----|-----------------|-----------------
|    | \( \sigma_0 = 5\text{MPa} \) | \( \sigma_0 = 20\text{MPa} \) | \( \sigma_0 = 35\text{MPa} \) | \( \sigma_0 = 50\text{MPa} \) |
| 1.5 | 0.546 | 0.042 | 0.725 | 0.472 |
| 1.0 | 0.327 | 0.060 | 0.474 | 0.240 |
| 0.67 | 0.407 | 1.406 | 0.937 | 0.590 |

| FS | Support type III | Support type II |
|----|-----------------|-----------------
|    | \( \sigma_0 = 5\text{MPa} \) | \( \sigma_0 = 20\text{MPa} \) | \( \sigma_0 = 35\text{MPa} \) | \( \sigma_0 = 50\text{MPa} \) |
| 1.5 | 1.082 | 1.079 | 1.079 | 3.849 |
| 1.0 | 0.685 | 0.683 | 0.683 | 3.223 |
| 0.67 | 0.407 | 0.407 | 0.407 | 0.359 |

| FS | Support type III | Support type II |
|----|-----------------|-----------------
|    | \( \sigma_0 = 5\text{MPa} \) | \( \sigma_0 = 20\text{MPa} \) | \( \sigma_0 = 35\text{MPa} \) | \( \sigma_0 = 50\text{MPa} \) |
| 1.5 | 0.159 | 0.079 | 0.079 | 0.079 |
| 1.0 | 1.171 | 1.171 | 1.171 | 1.171 |
| 0.67 | 0.854 | 0.854 | 0.854 | 0.854 |

| FS | Support type III | Support type II |
|----|-----------------|-----------------
|    | \( \sigma_0 = 5\text{MPa} \) | \( \sigma_0 = 20\text{MPa} \) | \( \sigma_0 = 35\text{MPa} \) | \( \sigma_0 = 50\text{MPa} \) |
| 1.5 | 0.350 | 0.350 | 0.350 | 0.350 |
| 1.0 | 0.250 | 0.250 | 0.250 | 0.250 |
| 0.67 | 0.150 | 0.150 | 0.150 | 0.150 |
As shown from Table 3, $\sigma_0 = 5$ MPa or GSI\textsubscript{max} = 75, even if the $X'_{\text{del}}$ is reduced to zero, cannot bring the $N_s$ in the support types II and III to the yield limits of $FS = 1.0$ and 0.67. Among these, when GSI\textsubscript{max} = 75, in the support type III, the $N_s$ is about 3.5 MPa; when $\sigma_0 = 5$ MPa, in the support types III, $N_s$ is about 2.3 MPa, and at the $X'_{\text{ini}}$ equals 0 for the support type II, the $N_s$ is also about 2.3 MPa (see figures 5 and 6).

It is shown that the support force is fairly small as GSI\textsubscript{max} is large but $\sigma_0$ is insignificant. Simple tunnel supports such as rockbolts and concrete are expected. Furthermore, since deformation reduction factor is decreased by increasing of the distance to the tunnel face, for smaller initial stress or strong rock mass to effectively control the deformation of rock, the support should be installed immediately. At a particular $f_0$, the upper bounds of $X'_{\text{del}}$ and $u_{0,\text{allow}}$ are almost equivalent for support types II (see table 3). As shown in figures 5 and 6, $f_0$ versus $X'_{\text{del}}$ change almost consistently in support type III and support type II. The only difference existing between the support types III and II is that the former adds steel groups. Considering the above results, the effect of steel group on limiting rock displacement is not obvious.

Meanwhile, when by the support type II and support type III, The lower bound of the $X'_{\text{del}}$ and $u_{0,\text{allow}}$ of the former is significantly larger than the latter. At the tunnel face, $N_s$ of the support type II is 21.79 % higher than that of the support type III for the case A3 when $\sigma_0$ is 35 MPa (see figure 6). Thus, the more flexible support suffers larger pressure and can be delayed installed. This is contrary to Convergence Confinement Method. By this method, the stronger support is subjected to heavier load while installing close to the tunnel face. The is due to the fact that: the load on support type III is partially bore by steel set, hence, $N_s$ of concrete in type III is smaller than $N_s$ by the type II with a same concrete type. This follows that it is important to assess the proportion of loads shared to the weakest part in the type III acting as a composite support system.

The upper and lower bounds of $X'_{\text{del}}$ and $u_{0,\text{allow}}$ decrease with the increase in GSI\textsubscript{max} and the decrease in $\sigma_0$ (see table 3). This indicates that as the rock mass is more stable, the delayed distance and the allowable deformation should be diminished. In addition, the lower bounds of $X'_{\text{del}}$ and $u_{0,\text{allow}}$ grow with the increase of FS. The upper bounds of $X'_{\text{del}}$ and $u_{0,\text{allow}}$ grow with the decrease of $f_0$. Hence, the ranges of $X'_{\text{del}}$ and $u_{0,\text{allow}}$ are correlative with the support strength capacity, and the required displacement reduction factor. Furthermore, with a stress level under 50 MPa, the upper bound of $X'_{\text{del}}$ with a larger $f_0$ (i.e. 50% ~ 60%) is below 1.0 (see table 3). Hence, larger $f_0$ will be achieved when $X'_{\text{del}}$ is less than 1. On the other hand, while $X'_{\text{del}}$ is 1, $p_f/\sigma_0$ for each case is less than 10%. This means the stress relief factor reaches more than 90% when $X'_{\text{del}}$ is 1. Therefore, in order to restrict the rock mass displacement, the delayed distance for support installation is suggested to be within $R_0$ for most tunnels. It is noted that the blank space indicates that $f_0$ is smaller than 30%, 40%, 50% or 60% at the tunnel face.

| Table 3. Upper bounds of $X'_{\text{del}}$ and $u_{0,\text{allow}}$ (a) $X'_{\text{del}}$; (b) $u_{0,\text{allow}}$ (unit: cm) (a) |
|----------------|---------------------------------|----------------|---------------------------------|
| Support type III | Support type II | Support type III | Support type II |
| $\sigma_0 = 5$ MPa | $f_0 = 30\%$ | $f_0 = 40\%$ | $f_0 = 50\%$ | $f_0 = 60\%$ | $f_0 = 30\%$ | $f_0 = 40\%$ | $f_0 = 50\%$ | $f_0 = 60\%$ |
| 0.630 | 0.312 | 0.019 | — | 0.590 | 0.256 | — | — |
| $\sigma_0 = 20$ MPa | 1.475 | 0.961 | 0.606 | 0.298 | 1.463 | 0.948 | 0.590 | 0.276 |
| $\sigma_0 = 35$ MPa | 2.259 | 1.601 | 1.038 | 0.620 | 2.246 | 1.595 | 1.030 | 0.610 |
| $\sigma_0 = 50$ MPa | 2.946 | 2.240 | 1.563 | 0.990 | 2.939 | 2.235 | 1.563 | 0.981 |
| GSI\textsubscript{max} = 40 | 2.259 | 1.601 | 1.038 | 0.620 | 2.246 | 1.595 | 1.030 | 0.610 |
| GSI\textsubscript{max} = 50 | 0.948 | 0.538 | 0.175 | — | 0.894 | 0.469 | 0.090 | — |
5. Conclusion

Due to the delayed effect of support installation, given this, commercial finite difference software is used for two-stage analysis. A displacement reduction factor is mainly introduced, defined as the ratio of displacement reduction to maximum rock mass displacement. Based on the numerical simulations, variations of support force and displacement reduction factor versus the delayed distance considering different support types, the initial stresses, and the post-peak behaviours are investigated. Two indicators of support time refer to the allowable deformation and delay distance of the support installation, and they are both quantitative. The main conclusions are summarised as follows:

1. The influence of the rock mass quality and the in-situ stresses on the support force and support effect is more remarkable than the post-peak behaviours. 
2. The displacement reduction factor depends primarily on the deformation property of rock mass. As the rock mass quality is weakened, the critical plastic softening parameter decreases, the initial stress and the dilatancy angle grow, the displacement reduction factor becomes greater. Determination of the final force caused by the virtual support pressure and rock deformation characteristics at the main delay distance. The support force changes as follows: it decreases with the expansion angle and then increases with the critical plastic softening parameter.
3. Steel set has little effect on restraining rock mass displacement, but it plays a key role in bearing support load. When the installation spacing of supports is the same, the load applied by the steel sets inserted to the shotcrete is smaller than that applied by the shotcrete alone, so that the steel sets has a protective effect on the shotcrete.
4. The upper and lower bounds of the delayed distance and allowable deformation rely heavily on the initial stress, mechanical properties of the rock mass, and rigidity and capacity of the support with a specific case. The delayed distance and allowable deformation diminish as the rock mass quality becomes better and the initial stress decreases. The lower bound grows with the increase in the safety factor of the support; the upper bound grows with a lower displacement reduction factor. In most cases, the delayed distance for the support installation is suggested to be within $R_0$.

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