Effect of unsteady oscillatory MHD flow through a porous medium in porous vertical channel with chemical reaction and concentration

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Abstract. In this paper, we investigate the effect of chemical reaction on the unsteady oscillatory MHD flow through porous medium in a porous vertical channel in the presence of suction velocity. The flow is assumed to be incompressible electrically conducting and radiating viscoelastic fluid in the presence of uniform magnetic filed applied perpendicular to the plane of the plates of the channel. The closed forms of analytical solution are obtained for the momentum, energy and concentration equation. The effect of various flow parameters like Schmidt number, chemical radiation parameter, Grashof number, solutal Grashof number on velocity profile, temperature, concentration, wall shear stress, and the rate of heat and mass transfer are obtained and their behaviour are discussed graphically.

1. Introduction.
The unsteady convection motion of an oscillatory flow in the presence of heat source through a porous medium has been a subject of interest of many researchers because of its application in geophysics, solid mechanics, ground water, hydrology, oil recovery, thermal insulation, heat storage and in the field of engineering. The study of electrically conductive fluid has many application in engineering problems such as heat transfer in MHD flow of viscous fluids, nuclear reactors, geothermal energy, extraction and Boundary layer in the field of Aerodynamics. The application of free convection and Heat transfer flow through porous medium under the influence of magnetic field has attracted many researchers in this field. Raptis et al 1985[1] studied the unsteady free convective through a porous medium bounded by an infinite vertical plate. O.D.Makinde and P.Y.Mhone 2005 [2] studied the Heat transfer to MHD oscillatory flow in a channel filled with porous medium. Mustafa et al 2008 [3] investigated unsteady MHD memory flow with oscillatory suction, variable free stream and heat sources. S.N.Sahoo 2013[8] studied the Heat and mass transfer effect on MHD flow of a viscoelastic fluid through a porous medium bounded by an oscillating porous plate in slip-flow regime. Rita choundhury et al 2011 [6] studied the heat transfer to MHD oscillatory viscoelastic flow in a channel filled with porous medium. The phenomenon of slip-flow regime has been studied by many researchers due to its wide ranging applications in the modern science, technology and vast ranging industrialization.

M.M.Hamza et al 2011 [7] studied the unsteady Heat transfer to MHD oscillatory flow through a porous medium under slip condition. K.D.singh 2013 studied the effect of slip condition on
viscoelastic MHD oscillatory forced convection flow in a vertical channel with heat radiation. There have been several studies on the convection heat transfer through porous channels due to several other important suction or injection controlled applications. Umavathi et al 2009[4] studied the unsteady flow of viscous fluid through a horizontal composite channel whose half-width is filled with porous medium. Ajibade and Jha 2010[5] presented the effect of suction and injection on hydrodynamics of oscillatory fluid through parallel plates. K.M.Joseph et al 2015 [9] studied the chemically reacting fluid on unsteady MHD oscillatory slip flow in a planer channel with varying temperature and concentration in the presence of suction/injection. More recently J.A.Falade et al 2016 [10] studied the effect of suction or injection on the MHD oscillatory flow through a porous channel saturated with porous medium. In this paper, we studied the effect of unsteady oscillatory hydro magnetic field and radiative heat transfer through a porous medium in porous vertical channel with chemical reaction and concentration in the presence of suction velocity.

2. Mathematical formulation.
Consider the unsteady mixed convective oscillatory MHD, viscoelastic, incompressible and electrically conducting fluid through a porous medium bounded between two infinite vertical porous plates at a distance $d$ with suction velocity. The Cartesian co-ordinate $X^* -$ axis is taken vertically upward of the channel and $Y^*$- axis is taken perpendicular to the plane of the plates. A uniform magnetic field with magnetic flux density vector $B_0$ is applied normal to the direction of the flow. All the physical quantities depend on $y$ and $t$ only because the plate of the channel occupying the plane $y = \pm \frac{d}{2}$ are of infinite extent. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible.

Under the usual Boussinesq incompressible fluid model the flow is governed by the following equations:

\[
\frac{\partial u^*}{\partial t} + v^* \frac{\partial u^*}{\partial y} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x} + v^* \frac{\partial^2 u^*}{\partial y^2} - \frac{\partial}{K^*} u^* - \frac{\sigma B_0^2}{\rho} u^* + g \beta (T^* - T_1^*) + g \beta (C^* - C_1^*)
\]

(1)

\[
\frac{\partial T^*}{\partial t} + v^* \frac{\partial T^*}{\partial y} = \frac{K^*}{\rho C_p} \frac{\partial^2 C^*}{\partial y^2} - \frac{1}{\rho} \frac{\partial \rho u^*}{\partial y} + \frac{Q (T^* - T_1^*)}{\rho C_p}
\]

(2)

\[
\frac{\partial C^*}{\partial t} + v^* \frac{\partial C^*}{\partial y} = D \frac{\partial^2 C^*}{\partial y^2} - K_r^* (C^* - C_1^*)
\]

(3)

Where $v^* = -v_1 (1 + \epsilon e^{i\omega t})$ is the suction velocity, $u^*$, $v^*$ is the velocity components in the $x^*$, $y^*$ directions respectively, $\theta$ is the kinematics viscosity, $k^*$ is the thermal conductivity, $\beta^*$ is the coefficient of volume expansion due to temperature, $\beta^*$ is the volume expansion due to concentration, $g$ is the acceleration due to gravity, $\rho$ is the density of the fluid, $B_0$ is the magnetic flux density vector, $\sigma$ is the electrical conductivity of the fluid, $T^*$ is the temperature of the fluid $T_1^*$ is the wall temperature of the fluid, $C^*$ is the concentration of the fluid, $C_1^*$ is the wall concentration of the fluid, $D$ is the mass diffusivity, $C_p$ is the specific heat at a constant pressure, $k_r^*$ is the chemical reaction.
The appropriate boundary condition is

\[ u^* = 0, T^* = 0, C^* = 0; y = -\frac{d}{2} \]  \hspace{1cm} (4)

\[ u^* = 0, T^* = 0, C^* = 0; y = \frac{d}{2} \]  \hspace{1cm} (5)

The radiative heat flux is given by

\[ \frac{\partial q^*}{\partial y^*} = 4(T_1^* - T)I^* , \text{ where } I^* = \int_0^\infty K_{\lambda\omega} \frac{\partial e_{b\lambda}}{\partial T} d\lambda \]

Where \( K_{\lambda\omega} \) is the coefficient of radiation absorption at the wall and \( e_{b\lambda} \) is the Planck’s function.

Introducing the following non-dimensional quantities

\[ x = \frac{x^*}{d}, y = \frac{y^*}{d}, p = \frac{p^*}{\mu u_0}, u = \frac{u^*}{u_0}, \theta = \frac{(T^* - T_1^*)}{(T_2 - T_1)}, \phi = \frac{(C^* - C_1)}{(C_2 - C_1)}, t = \frac{t}{d}, S = \frac{u_0 d}{\theta}, \gamma = \frac{k^*}{d}, M = \frac{\sigma B_0^2 d^2}{\mu}, \]

\[ Gr = \frac{g \beta (T_2 - T_1) d^2}{\partial u_0}, Gc = \frac{g \beta (C_2 - C_1) d^2}{\partial u_0}, R = \frac{4I^* d^2}{k}, Pe = \frac{\rho C^* u_0 d}{k}, Sc = \frac{D}{u_0 d}, Kr = \frac{K r^*}{u_0}, K^* = K d^2 \]  \hspace{1cm} (6)

Using the above non–dimensional quantities, equation (1) – (3) can be reduced to the following non-dimensional form

\[ S \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \xi_1 \frac{\partial u}{\partial y} + Gr \theta + Gc \phi - \left( M + \frac{1}{k} \right) u \]  \hspace{1cm} (7)

\[ Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + Pe \xi_2 \frac{\partial u}{\partial y} + (C^* R + \alpha) \theta \]  \hspace{1cm} (8)

\[ \frac{\partial \phi}{\partial t} = Sc \frac{\partial^2 \phi}{\partial y^2} + \xi_1 \frac{\partial \phi}{\partial y} + K \phi \]  \hspace{1cm} (9)

Now the boundary condition becomes

\[ u = 0, \theta = 0, \phi = 0; y = -\frac{1}{2} \]  \hspace{1cm} (10)
\[ u = 0, \theta = 0, \phi = 0; \ y = \frac{1}{2} \]  

Where \( S \) is the suction/injection parameter, \( M \) is the magnetic parameter, \( k \) is the permeability parameter, \( pe \) is the Peclet number, \( Sc \) is the Schmidt number, \( R \) is the thermal radiation parameter, \( \xi_1, \xi_2, \xi_3 \) are the real constants, \( Gr \) is the thermal Grashof number and \( Gc \) is the solutal Grashof number, \( K_r \) is the chemical reaction parameter.

3. METHOD OF SOLUTION:

In order to solve the equation with respect to the boundary conditions (10) and (11) for oscillatory flow, let us take

\[-\frac{\partial p}{\partial x} = \lambda e^{i\omega x} \]  

\[ u(y,t) = u_0(y)e^{i\omega t} \]  

\[ \theta(y,t) = \theta_0(y)e^{i\omega t} \]  

\[ \phi(y,t) = \phi_0(y)e^{i\omega t} \]  

Where \( A \) is any positive constant and \( \omega \) is the frequency of oscillation.

Substituting the equations (12) to (15) in equations (7) to (9), we get

\[ u_0''(y) + b_1u_0'(y) - b_2u_0(y) = -(\lambda + Gr\theta_0 + Gc\phi_0) \]  

\[ \theta_0''(y) + b_3\theta_0'(y) + b_4\theta_0(y) = 0 \]  

\[ \phi_0''(y) + b_5\phi_0'(y) - b_6\phi_0(y) = 0 \]  

The corresponding boundary conditions are

\[ y = -\frac{1}{2}, \ u_0 = 0, \ \theta_0 = 0, \ \phi_0 = 0 \]  

\[ y = \frac{1}{2}, \ u_0 = 0, \ \theta_0 = 1, \ \phi_0 = 1 \]  

Solving equation (16) using the boundary condition (20), we obtain the velocity profile

\[ u_0(y,t) = A_1e^{m_1y} + A_2e^{m_2y} + D_0 + D_1e^{m_1y} + D_2e^{m_2y} + D_3e^{m_3y} + D_4e^{m_4y} \]  

Solving equation (17) using the boundary condition (20), we obtain the temperature distribution

\[ \theta_0(y,t) = A_5e^{m_1y} + A_6e^{m_2y} \]  

Solving equation (18) using the boundary condition (20), we obtain the concentration distribution
The rate of heat transfer (Nu) at the walls \( y = -\frac{1}{2} \) and \( y = \frac{1}{2} \) are given by

\[
Nu_1 = \left[ \frac{\partial \theta_0}{\partial y} \right]_{y=\frac{1}{2}} \\
Nu_1 = \frac{\partial \theta}{\partial y} = \left( A_5 e^{-\frac{m_5}{2}} + A_6 e^{-\frac{m_6}{2}} \right) e^{i\omega t} \tag{24}
\]

\[
Nu_2 = \left[ \frac{\partial \theta_0}{\partial y} \right]_{y=-\frac{1}{2}} \\
Nu_2 = \frac{\partial \theta}{\partial y} = \left( A_5 e^{\frac{m_5}{2}} + A_6 e^{\frac{m_6}{2}} \right) e^{i\omega t} \tag{25}
\]

The rate of mass transfer (Sh) at the walls \( y = -\frac{1}{2} \) and \( y = \frac{1}{2} \) are given by

\[
Sh_1 = \left[ \frac{\partial \phi_0}{\partial y} \right]_{y=\frac{1}{2}} \\
Sh_1 = \frac{\partial \phi}{\partial y} = \left( A_5 e^{-\frac{m_5}{2}} + A_6 e^{-\frac{m_6}{2}} \right) e^{i\omega t} \tag{26}
\]

\[
Sh_2 = \left[ \frac{\partial \phi_0}{\partial y} \right]_{y=-\frac{1}{2}} \\
Sh_2 = \frac{\partial \phi}{\partial y} = \left( A_5 e^{\frac{m_5}{2}} + A_6 e^{\frac{m_6}{2}} \right) e^{i\omega t} \tag{27}
\]

The wall shear (\( \tau \)) at the walls \( y = -\frac{1}{2} \) and \( y = \frac{1}{2} \) are given by

\[
\tau_1 = \left[ -\mu \frac{\partial u_0}{\partial y} \right]_{y=\frac{1}{2}} \\
\tau_1 = -\mu \left( A_5 m_5 e^{-\frac{m_5}{2}} + A_6 m_6 e^{-\frac{m_6}{2}} + D_1 m_5 e^{-\frac{m_5}{2}} + D_2 m_6 e^{-\frac{m_6}{2}} + D_3 m_5 e^{-\frac{m_5}{2}} + D_4 m_6 e^{-\frac{m_6}{2}} \right) e^{i\omega t} \tag{28}
\]
\[
\tau_2 = \left[ -\mu \frac{\partial u_0}{\partial y} \right]_{y=\frac{1}{2}}
\]

\[
\tau_2 = -\mu \left( A_1 m_1 e^{-\frac{m_0}{2}} + A_2 m_2 e^{-\frac{m_0}{2}} + D_1 m_1 e^{-\frac{m_0}{2}} + D_2 m_2 e^{-\frac{m_0}{2}} + D_3 m_3 e^{-\frac{m_0}{2}} + D_4 m_4 e^{-\frac{m_0}{2}} \right) e^{i\omega t}
\]  

(29)

4. Results and discussions.

In this paper, we studied the effect of chemical reaction on the unsteady oscillatory MHD flow through porous medium in a porous vertical channel in the presence of suction velocity. The flow is assumed to be incompressible electrically conducting and radiating viscoelastic fluid in the presence of uniform magnetic field applied perpendicular to the plane of the plates of the channel.

Figure (1), shows the fluid velocity for different values of magnetic parameter (M). It is clear from the figure that an increase in the magnetic parameter decreases the fluid velocity. This effect is because of the rise of resistive type force called Lorentz force (similar to drag force) produced by the transverse magnetic field on an electrically conducting fluid and increasing the values of M increases the drag force which has the tendency to slow down the fluid motion. Figure (2), shows the fluid temperature for different values of thermal radiation parameter (R). It is clear from the figure that an increase in the thermal radiation parameter increases the fluid temperature. Figure (3), shows the fluid temperature increases for the different value of Peclet number. Figure (4), shows the fluid concentration increases for the different value of chemical reaction parameter (Kr). Figure (5), shows Nusselt number or the rate of heat transfer decreases for the different value of chemical parameter (Kr). Finally the figure (6), shows the Sherwood number (Sh) or the rate of mass transfer increases for the different value of Schmidt number (SC).

**Fig -1**: Variation of velocity for different values of magnetic parameter (M) for fixed \( \lambda = 1 \), \( \text{Sc} = 2 \), \( \text{Kr} = 0.3 \)
Fig - 2: Variation of temperature for different values of thermal radiation parameter (R) for fixed $\text{Pe}=4$
$\lambda=1, \text{Sc}=2, \text{Kr}=0.3$.

Fig - 3: Variation of temperature for different values of Peclet number (Pe) for fixed $\text{Kr}=0.4$
$\lambda=1, \text{Sc}=2, \text{Kr}=0.3$

Fig - 4: Variation of concentration for different values of chemical reaction parameter (Kr) for fixed $\lambda=1, \text{Sc}=2, \text{kr}=0.2$. 
Fig - 5: Variation of Nusselt number for different values of chemical reaction parameter (Kr) for fixed $Pe=2.5$, $\lambda=1$, $Sc=2$.

Fig - 6: Variation of Sherwood number (Sh) for different values of Schmidt number (Sc).

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