A note on the infrared behavior of the compactified Ginzburg-Landau model in a magnetic field

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Abstract – We consider the Euclidean large-\(N\) Ginzburg-Landau model in \(D\) dimensions, \(d (d \leq D)\) of them being compactified. For \(D=3\), the system can be supposed to describe, in the cases of \(d=1\), \(d=2\), and \(d=3\), respectively, a superconducting material in the form of a film, of an infinitely long wire having a rectangular cross-section and of a brick-shaped grain. We investigate the fixed-point structure of the model, in the presence of an external magnetic field. An infrared-stable fixed points is found, which is independent of the number of compactified dimensions. This generalizes previous work for type-II superconducting films.

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Introduction. – A large amount of work has already been done on the Ginzburg-Landau (GL) model, both in its single component and in the \(N\)-component versions, using the renormalization group approach [1–7]. In particular, an analysis of the renormalization group in finite-size geometries can be found in [8] and a general study of phase transitions in confined systems is in [9]. These studies have been performed to take into account boundary effects on thermodynamical quantities, in particular on the transition temperature. The existence of phase transitions is in this case associated to some spatial parameters related to the breaking of translational invariance, for instance, the distance \(L\) between planes confining the system. Also, in other contexts, the influence of boundaries on the behavior of systems undergoing transitions has been investigated as in, for instance, [10].

We analyze in the present letter effects of boundaries on the transition by considering that such confined systems are modeled by compactifying spatial dimensions [9]. Compactification is engendered as a generalization of the Matsubara (imaginary-time) prescription to account for constraints on the spatial coordinates. In the original Matsubara formalism, time is rotated to the imaginary axis, \(t \rightarrow i\tau\), where \(\tau\) (the Euclidean time) is limited to the interval \(0 \leq \tau \leq \beta\), with \(\beta = 1/T\) standing for the inverse temperature. The fields then fulfill periodic (bosons) or antiperiodic (fermions) boundary conditions and are compactified on the \(\tau\)-axis in an \(S^1\) topology, with circumference of length \(\beta\). Such a formalism leads to the description of a system in thermal equilibrium at the temperature \(\beta^{-1}\). Since in a Euclidean field theory space and time are on the same footing, one can envisage a generalization of the Matsubara approach to any set of spatial coordinates as well [11,12].

The conceptual framework for studying simultaneously finite temperature and spatial constraints has been developed by considering a simply or nonsimply connected \(D\)-dimensional manifold with a topology of the type \(\Gamma_D^{d+1} = R^{D-d-1} \times S_0^d \times S_1^d \times \ldots \times S_d^d\), with \(S_0^d\) corresponding to the compactification of the imaginary time and \(S_1^d,\ldots,S_d^d\) referring to the compactification of \(d\) spatial dimensions [12]. Physical manifestations of this type of topology include, for instance, the vacuum-energy fluctuations giving rise to the Casimir effect (see, for instance, [9] and other references therein). In the study of phase transitions, the dependence of the critical temperature on the compactification parameters is found in several situations of condensed-matter physics [9,13–17]. Also, this kind of formalism has been employed in the investigation of the confining phase transition in effective theories for Quantum Chromodynamics [18–22]. In the \(\Gamma_D^{d+1}\) topology, the Feynman rules are modified by introducing a generalized

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Matsumbara prescription, performing the following multiple replacements (compactification of a \((d+1)\)-dimensional subspace):

\[
\int \frac{dk_0}{2\pi} \to \frac{1}{\beta} \sum_{n_i=\infty}^{+\infty}, \quad \int \frac{dk_i}{2\pi} \to \frac{1}{L_i} \sum_{n_i=\infty}^{+\infty}, \quad k_0 \to \frac{2(n_0 + c)\pi}{\beta}, \quad k_i \to \frac{2(n_i + c)\pi}{L_i},
\]

where for each \(i = 1, 2, \ldots, d\), \(L_i\) is the size of the compactified spatial dimension \(i\) and \(c = 0\) or \(c = 1/2\) for, respectively, bosons and fermions.

The compactification formalism described above has been applied to field-theoretical models in arbitrary dimension with compactification of any subspace [16,17,23]. This formalism has also been developed from a path-integral approach in [24]. This allows to generalize to any subspace previous results in the effective potential framework for finite temperature and spatial boundaries. This mechanism generalizes and unifies results from recent work on the behavior of field theories in the presence of spatial constraints [10,12,23], and previous results in the literature for finite-temperature field theory as, for instance, in [25].

When studying the compactification of spatial coordinates, however, it is argued in [9] from topological considerations, that we may have a quite different interpretation of the generalized Matsumbara prescription: it provides a general and practical way to account for systems confined in limited regions of space at finite temperature. Distinctly, we shall be concerned here with a stationary field theory and employ the generalized Matsumbara prescription to study bounded systems by implementing the compactification of spatial coordinates, no imaginary-time compactification will be done. We will consider a topology of the type \(\Gamma^D = \mathbb{R}^{D-d} \times S_1^1 \times \cdots \times S_d^1\), where \(S_1^1, \ldots, S_d^1\) refer to the compactification of \(d\) spatial dimensions.

We consider in the present letter the Euclidean vector \(N\)-component \((\lambda \phi^\lambda)^D\) theory at leading order in \(1/N\), the system being submitted to the constraint of being limited by \(d\) pairs of parallel planes. Each pair is orthogonal to the coordinate axes \(x_1, \ldots, x_d\), respectively, and in each of them the planes are at distances \(L_1, \ldots, L_d\) apart from one another. This may be pictured as a parallelepiped-shaped box embedded in the \(D\)-dimensional space, whose parallel faces are separated by distances \(L_1, L_2, \ldots, L_d\). From a physical point of view, we could take in particular \(D = 3\) and introduce temperature by means of the mass term in the Hamiltonian in the usual Ginzburg-Landau way. These models can then describe a superconducting material in the shapes of a film \((d = 1)\), of a wire \((d = 2)\) and of a grain \((d = 3)\). With geometries such as these, some of us have been able to obtain general formulas for the dependence of the transition temperature and other quantities on the parameters delimiting the spatial region within which the system is confined (see, for instance, [16,17] and other references therein).

We consider the critical behavior of the system under the influence of an external magnetic field. Physically, for \(D = 3\), this corresponds to superconducting films, wires and grains in a magnetic field. In [5], a large-\(N\) theory of a second-order transition for arbitrary dimension \(D\) is presented and the fixed-point effective free energy describing the transition is found. The theory is based on the Ginzburg-Landau model with the coupling of scalar and gauge fields. While ignoring gauge-field fluctuations, the model includes an external magnetic field. The authors in [5] also claim that it is possible that in the physical situation of \(N = 1\), a mechanism of reduction of the lower critical dimension could allow a continuous transition in \(D = 3\). In [7], the possibility of the existence of a phase transition for a superconducting film in the presence of an external magnetic field has been investigated. This has been done in the renormalization-group framework by looking for the existence of infrared-stable fixed points for the \(\beta\)-function.

In this letter, we study, for arbitrary space dimension \(D\) and for any number \(d \leq D\) of compactified dimensions, the fixed-point structure of the model, thus generalizing a previous study for films [7]. We shall neglect the minimal coupling with the vector potential corresponding to the intrinsic gauge fluctuations. Our main concern will be to analyze the model from a field-theoretical point of view. In this sense, the present work may be seen as a further development of previous papers by some of us, as, for instance, [7,12,14,24].

The compactified model in the presence of an external field. – We consider the \(N\)-component vector model described by the Ginzburg-Landau Hamiltonian density

\[
\mathcal{H} = \left[ (\partial_\mu - i e A^{\lambda\mu}_{\text{ext}}) \varphi_{\lambda} \right] \left[ (\partial_\mu - i e A^{\mu\lambda}_{\text{ext}}) \varphi_{\lambda} \right] + m^2 \varphi_{\lambda} \varphi_{\lambda} + u (\varphi_{\lambda} \varphi_{\lambda})^2,
\]

in Euclidean \(D\)-dimensional space, where \(u\) is the coupling constant and \(m^2\) is a mass parameter such that \(m^2 = \alpha(T - T_0)\) and \(T_0\) the bulk transition temperature. Summation over repeated indices \(\mu\) and \(\lambda\) is assumed. In the following, we will consider the model described by the Hamiltonian (2) and take the large-\(N\) limit, such that \(u \to 0\), \(N \to \infty\) with \(N u = \lambda\) fixed. For \(D = 3\), from a physical point of view, such Hamiltonian is supposed to describe type-II superconductors. In this case, it has been assumed that the external magnetic field \(H\) is parallel to the \(z\)-axis and the gauge \(A^{\text{ext}} = (0, x H, 0)\) was chosen. In the present \(D\)-dimensional case, we assume analogously a gauge \(A^{\text{ext}} = (0, x_1 H, 0, 0, \ldots, 0)\), with \(x_1 = x_1, x_2, \ldots, x_D\), meaning that the applied external magnetic field lies on a fixed direction along one of the coordinate axes; for simplicity, in the calculations that follow, we have adopted the notation \(x_1 \equiv x\), \(x_2 \equiv y\).
If we consider the system in unlimited space, the field $\varphi$ should be written in terms of the well-known Landau-level basis,

$$\varphi(r) = \sum_{\ell=0}^{\infty} \int \frac{dp_y}{2\pi} \int \frac{d^D-2p}{(2\pi)^{D-2}} \tilde{\varphi}_{\ell,p_y,p} \chi_{\ell,p_y,p}(r),$$  \hspace{1cm} (3)

where $\chi_{\ell,p_y,p}(r)$ are the Landau-level eigenfunctions given in terms of Hermite polynomials $H_{\ell}$ by

$$\chi_{\ell,p_y,p}(r) = \frac{1}{\sqrt{2^{\ell} \ell!}} \left( \frac{\omega}{\pi} \right)^{1/4} e^{i(p \cdot r + p_y y)} e^{-\omega(x - p_y/\omega)^2/2} \times H_{\ell} \left( \sqrt{\omega x - p_y} \sqrt{\omega} \right),$$  \hspace{1cm} (4)

with energy eigenvalues $E_\ell(|p|) = |p|^2 + (2\ell + 1)\omega + m^2$ and $\omega = eH$ is the so-called cyclotron frequency. In the above equation, $p$ and $r$ are $(D-2)$-dimensional vectors. We use Cartesian coordinates $r = (x_1, \ldots, x_d, z)$, where $z$ is a $(D-d)$-dimensional vector, with corresponding momenta $k = (k_1, \ldots, k_d, q)$, $q$ being a $(D-d)$-dimensional vector in momentum space. Under these conditions, the generating function of correlation functions is written as

$$Z = \int D\varphi^* D\varphi \exp \left( - \int d^{D-2}z \int_0^{L_1} dx_1 \cdots \int_0^{L_d} dx_d \right) \left( \int d^{D-d-2}q \mathcal{H}(|\varphi|, |\nabla \varphi|) \right),$$  \hspace{1cm} (5)

the field $\varphi(x_1, \ldots, x_d, z)$ satisfying the condition of confinement inside the box, $\varphi(x_i < 0), \varphi(x_i > L_i), z = \text{const}$. Then the field representation should be modified and have a mixed series-integral Fourier expansion of the form

$$\varphi(x_1, \ldots, x_d, z) = \sum_{\ell=0}^{\infty} \sum_{i=1}^{d} \sum_{n_i=-\infty}^{\infty} c_{\ell,n_i} \int \frac{dp_y}{2\pi} \int d^{D-d-2}q b(q) e^{-i\omega_{n_i} x - i\varphi_{\ell}(\omega_{n_i}, q)},$$  \hspace{1cm} (6)

where, for $i = 1, \ldots, d$, $\omega_{n_i} = 2\pi n_i/L_i$ and the coefficients $c_{\ell,n_i}$ and $b(q)$ correspond, respectively, to the Fourier series representation over the $x_i$ and to the Fourier integral representation over the $(D-d-2)$-dimensional $z$-space. We now apply the Matsubara-like formalism according to eq. (1), remembering that here we have no imaginary-time compactification.

**Infrared behavior and fixed points.** In the following, we consider only the lowest Landau level $\ell = 0$. For $D = 3$, this assumption usually corresponds to the description of superconductors in the extreme type-II limit. Under this assumption, we obtain the effective $|\varphi|^4$ interaction in momentum space and at the critical point, $\lambda(p, D, \{L_i\}; \omega)$, from the four-point function,

$$\Gamma^{(4)}(p, \{L_i\}, m = 0).$$

The four-point function is given by the sum of all chains of one-loop diagrams, which leads to

$$\lambda(p, D, \{L_i\}; \omega) \equiv \lim_{N \to \infty} N \Gamma^{(4)}(p, \{L_i\}, m = 0) = \lambda$$

with $Nu = \lambda$ fixed and where the single 1-loop bubble $\Pi(p, D, \{L_i\}, m = 0; \omega)$ is given by

$$ \Pi(p, D, \{L_i\}, m = 0) = \frac{1}{L_1 \cdots L_d} \sum_{i=1}^{d} \sum_{n_i = -\infty}^{\infty} \int_0^{1} \int \frac{d^{D-d-2}q}{(2\pi)^{D-d-2}} \times \left[ \frac{1}{q^2 + \omega_{n_i}^2 + \cdots + \omega_{n_d}^2 + p^2 x(1 - x)} \right].$$  \hspace{1cm} (8)

This is the same kind of expression that is encountered in [17], with the only modification that $D \to D - 2$ and the role of the mass is played by the quantity $\sqrt{p^2 x(1 - x)}$. Also, one should be reminded that $p$ is a $(D-2)$-dimensional vector. The analysis is then performed along the same lines as in [17] and we obtain, analogously,

$$ \Pi(p, D, \{L_i\}, m = 0) = (2\pi)^{1-D/2} \left( \frac{2^{1-D/2} \Gamma(\frac{D}{2}-3)}{(2\pi)^{D/2-3}} \right) \times \Gamma \left( 3 - \frac{D}{2} \right) \left( \frac{p^2}{2} \right)^{(D-2)/2}$$

$$+ \int_0^1 dx \left[ \sum_{i=1}^{d} \sum_{n_i = 1}^{\infty} \left( \frac{\sqrt{p^2 x(1 - x)}}{2\pi L_i n_i} \right) \times K_{(D-2)/2-2} \left( \frac{1}{2\pi} \sqrt{p^2 x(1 - x) L_i n_i} \right) \right. $$

$$+ \sum_{i<j=1}^{d} \sum_{n_i, n_j = 1}^{\infty} \left( \frac{\sqrt{p^2 x(1 - x)}}{2\pi L_i n_i^2 \pm L_j n_j^2} \right) \times K_{(D-2)/2-2} \left( \frac{1}{2\pi} \sqrt{p^2 x(1 - x) \pm L_i n_i^2 \pm L_j n_j^2} \right) \right.$$}

$$+ \left. \sum_{n_1, \ldots, n_d = 1}^{\infty} \left( \frac{\sqrt{p^2 x(1 - x)}}{2\pi L_i n_i^2 \pm \cdots \pm L_d n_d^2} \right) \times K_{(D-2)/2-2} \left( \frac{1}{2\pi} \sqrt{p^2 x(1 - x) \pm L_i n_i^2 \pm \cdots \pm L_d n_d^2} \right) \right]\right],$$  \hspace{1cm} (9)

where

$$c(D) = \int_0^1 dx (x(1 - x))^{D/2-3} = 2^{5-D} \sqrt{\pi} \left( \frac{D-2}{D-3} \right)^{(D-2)/2}. $$  \hspace{1cm} (10)

As for the infrared behavior of the $\beta$-function, it suffices to study it in the neighborhood of $|p| = 0$, so that we can
use the asymptotic formula in the $|p| \approx 0$ limit, for small values of the argument of the modified Bessel functions,

$$K_v(z) \approx \frac{1}{2} \Gamma(v) \left( \frac{z}{2} \right)^{-v} (z \sim 0). \quad (11)$$

From eq. (9), it turns out that in the $|p| \approx 0$ limit, the bubble II is written in the form

$$\Pi(|p| \approx 0, D, \{L_i\}, m = 0) = A(D) |p|^{D-0} + C_d(D, \{L_i\}), \quad (12)$$

with

$$A(D) = (2\pi)^{-D/2-1} 2^{1-D/2} c(D) \Gamma \left( 3 - \frac{D}{2} \right), \quad (13)$$

and where the quantity $C_d(D, \{L_i\})$ is

$$C_d(D, \{L_i\}) = (2\pi)^{-D/2} \int_0^1 dx \left[ \sum_{i=1}^d \sum_{n=1}^\infty \left( \frac{\sqrt{p^2 x(1-x)}}{2\pi L_i n_i} \right)^{(D-2)/2} \right. \times K_{(D-2)/2-2} \left[ \frac{1}{2\pi} \sqrt{p^2 x(1-x)} L_i n_i \right] \left. + 2 \sum_{i<j=1}^d \sum_{n_i, n_j=1}^\infty \left( \frac{\sqrt{p^2 x(1-x)}}{2\pi L_i^2 n_i^2 + L_j^2 n_j^2} \right)^{(D-2)/2} \right. \times K_{(D-2)/2-2} \left[ \frac{1}{2\pi} \sqrt{p^2 x(1-x)} (L_i^2 n_i^2 + L_j^2 n_j^2) \right] \left. + 2^{d-1} \sum_{n_1, \ldots, n_d=1}^\infty \left( \frac{\sqrt{p^2 x(1-x)}}{2\pi L_1^2 n_1^2 + \cdots + L_d^2 n_d^2} \right)^{(D-2)/2} \right. \times K_{(D-2)/2-2} \left[ \frac{1}{2\pi} \sqrt{p^2 x(1-x)} (L_1^2 n_1^2 + \cdots + L_d^2 n_d^2) \right] \right]. \quad (14)$$

If an infrared-stable fixed point exists for any of the models with $d$ confining dimensions, it is determined by a study of the infrared behavior of the Callan-Symanzik $\beta$-function, i.e., in the neighborhood of $|p| = 0$. Therefore, we should investigate the above equations for $|p| \approx 0$. In this case, we consider a typical term in eq. (14), which has the form

$$\sum_{n_1, \ldots, n_q=1}^\infty \left( \frac{\sqrt{p^2 x(1-x)}}{2\pi \sqrt{L_i^2 n_i^2 + \cdots + L_q^2 n_q^2}} \right)^{(D-2)/2-s} \times K_{(D-2)/2-s} \left[ \frac{1}{2\pi} \sqrt{p^2 x(1-x)} (L_i^2 n_i^2 + \cdots + L_q^2 n_q^2) \right], \quad (15)$$

with $s = 2$ and $q = 1, 2, \ldots, d$. In the limit $|p| \approx 0$, using eq. (11), eq. (15) reduces to

$$\frac{1}{2} \nu \left( \frac{D}{2} - s \right) E_q \left( \frac{D}{2} - s; L_1, \ldots, L_q \right). \quad (16)$$

Equation (16) is expressed in terms of one of the multidimensional Epstein zeta functions $E_q \left( \frac{D}{2} - s; L_1, \ldots, L_q \right)$, for $q = 1, 2, \ldots, d$, which are defined by [26-29]

$$E_q \left( \nu; \sigma_1, \ldots, \sigma_q \right) = \sum_{n_1, \ldots, n_q=1}^\infty \left[ \sigma_1^2 n_1^2 + \cdots + \sigma_q^2 n_q^2 \right]^{-\nu}. \quad (17)$$

Notice that, for $q = 1$, $E_q$ reduces to the Riemann zeta function $\zeta(z) = \sum_{n=1}^\infty n^{-z}$. One can also construct analytical continuations and recurrence relations for the multidimensional Epstein functions, which permit to write them in terms of modified Bessel and Riemann zeta functions [12,27-29]. One gets

$$E_q \left( \nu; L_1, \ldots, L_q \right) = -\frac{1}{2q} \sum_{i=1}^q E_{q-1} \left( \nu; L_i, \ldots, \hat{L}_i, \ldots \right)$$

$$+ \frac{\sqrt{\pi}}{2d \Gamma(\nu)} \Gamma \left( \nu - \frac{1}{2} \right) \sum_{i=1}^q \prod_{j \neq i} \Gamma \left( \nu - \frac{1}{2} \right) W_q \left( \nu - \frac{1}{2}; L_1, \ldots, L_q \right), \quad (18)$$

where the hat over the parameter $L_i$ in the functions $E_{q-1}$ means that it is excluded from the set $\{L_1, \ldots, L_q\}$ (the others being the $q - 1$ parameters of $E_{q-1}$), and

$$W_q \left( \nu; L_1, \ldots, L_q \right) = \sum_{i=1}^q \prod_{n_1, \ldots, n_q=1}^\infty \left( \frac{\pi n_i}{L_i \sqrt{\cdots + L_i n_i^2 + \cdots}} \right)^\nu \times K_{\nu} \left( \frac{2\pi n_i}{L_i} \sqrt{\cdots + L_i n_i^2 + \cdots} \right), \quad (19)$$

with $\cdots + L_i n_i^2 + \cdots$ representing the sum $\sum_{j=1}^q L_j^2 n_j^2 = L_i^2 n_i^2$.

Getting back to the infrared behavior, we see from (16) that in the limit $|p| \approx 0$, the $p^2$-dependence of the modified Bessel functions exactly compensates the one coming from the accompanying factors. Thus, the remaining $p^2$-dependence is only that of the first term of (12), which is the same for all number of compactified dimensions $d$. It is worth mentioning also that the simultaneous use of the Matsubara prescription and the Feynman parametrization leads to analyticity problems. This fact has been already investigated, for instance, in refs. [30,31]. In our case, we see from eqs. (10) and (13) that, due to the singularities of the gamma functions, $\Pi(|p| \approx 0, D, \{L_i\}, m = 0)$ in eq. (12) is well behaved in the range of dimensions $4 < D < 6$. We will thus study our system for dimensions in this range. We emphasize that this range of dimensions, $4 < D < 6$, is the same that is compatible with the existence of a second-order phase transition for the system in bulk form in previous publications [5,32,33].
Fixed points. For all $d \leq D$, within the domain of validity of $D$, we have, by inserting (12) in eq. (7), the running coupling constant

$$\lambda(p| \approx 0, D, \{L_i\}) \approx \frac{\lambda}{1 + \lambda w e^{-(1/2) \omega_0(p^2 + p^2) \left| A(D) \right| p^{D-6} + C_d(D, \{L_i\})}}.$$  

(20)

Let us take $|p|$ as a running scale, and define the dimensionless coupling

$$g = \omega \lambda(p_1 = p_2 = 0, D, \{L_i\}) |p|^{D-6},$$  

(21)

where we remember that in this context $p$ is a $(D-2)$-dimensional vector.

The $\beta$-function controls the rate of the renormalization-group flow of the running coupling constant and a (nontrivial) fixed point of this flow is given by a (nontrivial) zero of the $\beta$-function. For $|p| \approx 0$, we obtain straightforwardly from eq. (21)

$$\beta(g) = |p| \frac{\partial g}{\partial |p|} \approx (D - 6)|g - A(D)g^2|.$$  

(22)

From eqs. (13) and (10) we see that we have an infrared-stable fixed point, $g_s(D)$, for dimensions $D$ such that $4 < D < 6$,

$$g_s(D) = \frac{1}{A(D)}.$$  

(23)

We see that the $\{L_i\}$-dependent $C_d$-part of the subdiagram II does not play any role in this expression and, since $A(D)$ is the same for all number of compactified dimensions, so is $g_s$, only dependent on the space dimension.

Concluding remarks. – In this letter, we have discussed the infrared behavior and the fixed-point structure of the $N$-component Ginzburg-Landau model in the large-$N$ limit, the system being confined in a $d$-dimensional box with edges of length $L_i$, $i = 1, 2, \ldots, d$ (compactification in a $d$-dimensional subspace). We have studied the case in which the system is submitted to the action of an applied external magnetic field. In this case, we get the result that the existence of an infrared-stable fixed point depends only on the space dimension $D$; it does not depend on the number of compactified dimensions.

In the case of the system in the presence of an external magnetic field, it is interesting to compare our results with those obtained for type-II materials in bulk form. For instance, a large-$N$ analysis and a functional renormalization-group study performed in refs. [5,32,33] conclude for a second-order transition in dimensions $4 < D < 6$. The same conclusion is obtained in ref. [6]. The authors of ref. [32] claim, moreover, that the inclusion of fluctuations does not alter significantly the main characteristic of the system, that is, the existence of a continuous transition into a spatially homogeneous condensate. For the system under the action of an external magnetic field, the existence of a fixed point for $4 < D < 6$ should be taken as an indication, not as a demonstration, of the existence of a continuous transition. As already discussed in [32,33], in this case, even if infrared fixed points exist, none of them can be completely attractive. The existence of an infrared fixed point in the presence of a magnetic field, as found in this paper, does not assure the (formal) existence of a second-order transition. Anyway, we conclude that, for materials in the form of films, wires and grains under the action of an external magnetic field, as is also the case for materials in bulk form, if there exists a phase transition for $D < 4$, in particular in $D = 3$, it should not be a second-order one. Moreover, the fixed point is independent of the size of the system or, in other words, the nature of the transition in the presence of a magnetic field is insensitive to the confining geometry.

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