$B \to K^*\gamma$ decay on APE

As. Abada, Ph. Boucaud, M. Crisafulli, J.P. Leroy, V. Libic, G. Martinelli, F. Rapuano, M. Serone, N. Stella, and A. Bartolini, C. Battista, S. Cabasino, N. Cabibbo, E. Panizzi, P.S. Paolucci, R. Sarno, G. M. Todesco, M. Torelli, P. Vicini.

The APE Collaboration

1. Introduction

The hadronic matrix element which governs the radiative decay $B \to K^*\gamma$ is parametrised in terms of three form factors:

$$
(K^*_+(\eta, k)|J_\mu|B(p)) = C_1^\mu T_1(q^2) + iC_2^\mu T_2(q^2) + iC_3^\mu T_3(q^2),
$$

where

$$
C_1^\mu = 2\epsilon^{\mu\rho\sigma\tau}c_1(k)\eta_\tau(p)\eta_\rho(p),
$$

$$
C_2^\mu = \eta_\tau(k)(M_B^2 - M_K^2) - (\eta_\tau(k)q)(p + k)^\mu,
$$

$$
C_3^\mu = \eta_\tau(k)q(q^2 - \frac{q^2}{M_B^2 - M_K^2})(p + k)^\mu.
$$

and $J_\mu = \bar{s}\sigma_{\mu\nu}\frac{1 + \gamma_5}{2}q^\nu b$; $\eta$ is the polarization vector of the $K^*$ and $q$ the momentum transfer.

When the emitted photon is real, $T_3$ does not contribute to the physical rate and $T_1(0) = T_2(0)$. At $q^2 = 0$, the physics of this decay is thus described by only one form factor, $T_1$. The feasibility of the lattice approach has been demonstrated first by the work of Bernard et al. [1].

2. Scaling laws and $q^2$ dependence of the form factors

In order to obtain the form factors at the physical point, we need to extrapolate both to large meson masses and small values of $q^2$. The final results critically depend on the assumptions made on the $q^2$- and heavy mass-dependence. At fixed $p_K$, with $|p_K| \ll M_B$ in the B-meson rest frame, the following scaling laws can be derived [2]:

$$
\frac{T_1}{\sqrt{M_B}} = \gamma_1 \times \left(1 + \frac{\delta_1}{M_B} + \ldots\right)
$$

$$
T_2\sqrt{M_B} = \gamma_2 \times \left(1 + \frac{\delta_2}{M_B} + \ldots\right)
$$

which are valid up to logarithmic corrections. On the other hand, “scaling” laws for the form factors at $q^2 = 0$ can only be found by using extra assumptions for their $q^2$ dependence. This procedure is acceptable, provided the “scaling” laws derived in this way respect the exact condition $T_1(0) = T_2(0)$. This is a non-trivial constraint: the $q^2$ behaviour of $T_1$ and $T_2$ has to compensate for the different mass dependence of the two form factors near the zero recoil point given in
than the mass obtained from the axial two-point
with the pole mass as a free parameter is larger
the time position of the current in the interval
the Wilson hopping parameter
computed the quark propagators for seven values of
total set. For each configuration we have com-
tistical errors estimated by a jackknife procedure
ample of 170 gauge configurations and the sta-
mation. The results have been obtained from a
SW-Clover action \[3\] in the quenched approxi-
4. Results
From our data, if we assume a pole dominance
behaviour for \(T_2\), the mass extracted from the fits
with the pole mass as a free parameter is larger
than the mass obtained from the axial two-point
correlation functions. As a consequence \(T_2(q^2)\) is
flatter than predicted by pole dominance (see fig.
\[\text{Figure 1. } T_2(q^2) \text{ as a function of } q^2 \text{ for } K_H = .1330. \text{ The curves show the pole dominance with either the lattice axial pole mass (dashed line) or with a free pole mass (full line).} \]

In the case of \(T_1\), the absence of data at \(q^2_{max}\)
and the large errors in the data at high momenta
make it difficult to test directly the validity of
the pole dominance hypothesis. We can use the
value for \(T_1(q^2 = 0)\) obtained from the the con-
tion \(T_1(0) = T_2^{free}(0)\), together with the point
at \(\tilde{p}_{K^*} = 2\pi/(La) (1,0,0)\) in a fit of \(T_1\) to a pole
dominance behaviour; the pole mass determined
along this way for \(T_1\) is compatible, though with
large errors, with the mass of the corresponding
lattice vector meson. More data (i.e. with a
moving \(B\) meson) are needed to test this point
more accurately. As a matter of comparison,
we give in table \[\text{Table 1. } T_1^{pole}(q^2 = 0)\]
\((T_2^{pole}(q^2 = 0))\) obtained under the assumption
of a pole dominance with the lattice vector (axial)
meson mass. Although the quality of our
data is not accurate enough to draw a definite
conclusion, they suggest that assuming \(T_2\) flatter
than pole dominance and \(T_1\) following pole
dominance gives a good description of our data.
We call this option \(m^{-1/2}\)-scaling. In fig. \[\text{Figure 2. } T_2^\prime(q^2) \text{ as a function of } q^2 \text{ for } K_H = .1330. \text{ The curves show the pole dominance with either the lattice axial pole mass (dashed line) or with a free pole mass (full line).} \]

The assumptions on the \(q^2\)-dependence of the
form factors can be tested directly on the numerical
results, although only in a small domain of
momenta.

3. Lattice set-up
The numerical simulation was performed on the
6.4 Gigaflops version of the APE machine,
at \(\beta = 6.0\), on a \(18^3 \times 64\) lattice, using the
SW-Clover action \[3\] in the quenched approxima-
tion. The results have been obtained from a
sample of 170 gauge configurations and the sta-
tistical errors estimated by a jackknife procedure
with a decimation of 10 configurations from the
total set. For each configuration we have com-
puted the quark propagators for seven values of
the Wilson hopping parameter \(K_W\), corresponding
to “heavy” quarks, \(K_H = 0.1150\), 0.1200,
0.1250, 0.1330, and “light” quarks, \(K_L = 0.1425,
0.1432\) and 0.1440. Due to memory limitations,
the propagators are “thinned”. The matrix ele-
ments have been computed for an initial meson
at rest and a final vector meson with momentum
\(\tilde{p}_{K^*}\). We have taken \(\tilde{p}_{K^*} = 2\pi/(La) (0,0,0),
(1,0,0), (1,1,0), (1,1,1), and (2,0,0), where \(La\)
is the spatial extension of the lattice. The initial
(final) meson was created (annihilated) by using
a pseudoscalar (local vector) density inserted at
a time \(t_B/a = 28 (t_{K^*} = 0)\), and we have varied
the time position of the current in the interval
\(t_B/a = 10 - 14\). Two procedures, denoted by “ra-
tio” and “analytic” in the tables, have been used
to extract the plateaux: the three point functions
are divided either by the numerical two point
functions (“ratio”) or by an analytical expression
(“analytic”). Details can be found in ref \[3\].

4. Results

would follow from a dipolar $q^2$-dependence for $T_1$. We take the two possibilities, $m^{-1/2}$- and $m^{-3/2}$-scaling, as representatives of a whole class of possible “scaling” laws.

Table 1

| Form factors at $q^2 = 0$ extrapolated to the strange quark, assuming independence on the spectator quark. “ratio” and “analytic” are explained in the text. $T_1^{pole}$ and $T_2^{pole}$ are computed with the appropriate lattice meson mass for the pole dominance, $T_2^{free}$ with the pole mass as a free parameter. |
|-----------------|-----------------|-----------------|-----------------|
| $T_1^{pole}(0) \kappa_h = .1150$ | $T_2^{pole}(0) \kappa_h = .1150$ | $T_2^{free}(0) \kappa_h = .1150$ |
| $T_1^{pole}(0) \kappa_h = .1200$ | $T_2^{pole}(0) \kappa_h = .1200$ | $T_2^{pole}(0) \kappa_h = .1200$ |
| $T_1^{pole}(0) \kappa_h = .1250$ | $T_2^{pole}(0) \kappa_h = .1250$ | $T_2^{pole}(0) \kappa_h = .1250$ |
| $T_1^{pole}(0) \kappa_h = .1330$ | $T_2^{pole}(0) \kappa_h = .1330$ | $T_2^{pole}(0) \kappa_h = .1330$ |
|                    | ratio     | analytic     | ratio     | analytic     |
| $T_1^{pole}(0)\kappa_h$ | .286(35) | .297(34) | .286(35) | .297(34) |
| $T_2^{pole}(0)\kappa_h$ | .280(52) | .301(56) | .280(52) | .301(56) |
| $T_2^{free}(0)\kappa_h$ | .238(17) | .242(17) | .238(17) | .242(17) |
| $T_1^{pole}(0)\kappa_h$ | .298(33) | .309(37) | .298(33) | .309(37) |
| $T_2^{pole}(0)\kappa_h$ | .293(40) | .309(40) | .293(40) | .309(40) |
| $T_2^{pole}(0)\kappa_h$ | .262(16) | .265(17) | .262(16) | .265(17) |
| $T_1^{pole}(0)\kappa_h$ | .311(32) | .322(31) | .311(32) | .322(31) |
| $T_2^{pole}(0)\kappa_h$ | .310(30) | .320(28) | .310(30) | .320(28) |
| $T_2^{pole}(0)\kappa_h$ | .288(17) | .292(17) | .288(17) | .292(17) |
| $T_1^{pole}(0)\kappa_h$ | .331(31) | .339(30) | .331(31) | .339(30) |
| $T_2^{pole}(0)\kappa_h$ | .345(19) | .348(18) | .345(19) | .348(18) |
| $T_2^{pole}(0)\kappa_h$ | .340(19) | .343(19) | .340(19) | .343(19) |

The extrapolation to the physical region (i.e. the $B$ mass) is performed following these two hypothesis, $m^{-1/2}$- and $m^{-3/2}$-scaling with linear and quadratic fits. The results are presented in table 2. We give also $\delta_1$, the coefficient of the $1/M$ corrections in the linear fit. It should be noted that in the $m^{-1/2}$-scaling case, the $1/M$ corrections are smaller and the extrapolated value is less affected by the adjunction of a quadratic term than in the $m^{-3/2}$-scaling hypothesis.

| Form factors at $q^2 = 0$ extrapolated to the physical $B$ mass. $\delta_1$ is the coefficient of the $1/M$ correction. |
|-----------------|-----------------|-----------------|-----------------|
| $T_1^{pole}(0)$ fit $m^{-\frac{1}{2}}$ lin. | $T_1^{pole}(0)$ fit $m^{-\frac{1}{2}}$ quad. | $\delta_1$ (MeV) |
| $T_1^{pole}(0)$ fit $m^{-\frac{3}{2}}$ lin. | $T_1^{pole}(0)$ fit $m^{-\frac{3}{2}}$ quad. | $\delta_1$ (MeV) |
| $T_2^{pole}(0)$ fit $m^{-\frac{1}{2}}$ lin. | $T_2^{pole}(0)$ fit $m^{-\frac{1}{2}}$ quad. | $\delta_1$ (MeV) |
| $T_2^{pole}(0)$ fit $m^{-\frac{3}{2}}$ lin. | $T_2^{pole}(0)$ fit $m^{-\frac{3}{2}}$ quad. | $\delta_1$ (MeV) |
|                    | ratio     | analytic     | ratio     | analytic     |
| $T_1^{pole}(0)$ fit $m^{-\frac{1}{2}}$ lin. | .203(28) | .213(27) | .203(28) | .213(27) |
| $T_1^{pole}(0)$ fit $m^{-\frac{3}{2}}$ lin. | .191(40) | .200(40) | .191(40) | .200(40) |
| $T_1^{pole}(0)$ fit $m^{-\frac{1}{2}}$ quad. | 310(109) | 339(97) | 310(109) | 339(97) |
| $T_1^{pole}(0)$ fit $m^{-\frac{3}{2}}$ quad. | 339(109) | 339(109) | 339(109) | 339(109) |
| $T_2^{pole}(0)$ fit $m^{-\frac{1}{2}}$ lin. | .102(11) | .106(12) | .102(11) | .106(12) |
| $T_2^{pole}(0)$ fit $m^{-\frac{1}{2}}$ quad. | .135(20) | .140(21) | .135(20) | .140(21) |
| $T_2^{pole}(0)$ fit $m^{-\frac{3}{2}}$ lin. | 871(34) | 879(34) | 871(34) | 879(34) |
| $T_2^{pole}(0)$ fit $m^{-\frac{3}{2}}$ quad. | .091(12) | .092(13) | .091(12) | .092(13) |
| $\delta_1$ (MeV) | 735(51) | 737(54) | 735(51) | 737(54) |

The same game can be played with $T_2(q^2_{max})$; the results in this case are $T_2(q^2_{max},B) = 217(15)$ and $T_2(q^2 = 0,B) = .090(6)$ for a value of $M_A$ $\sim$ 5.7 GeV for the axial pole mass; this is in agreement with the results in table 2 for the $m^{-3/2}$-scaling.

From table 2, we quote:

- $T_1^{pole}(0) = .196(45)$ ($m^{-1/2}$ scaling)
- $T_2^{pole}(0) = .090(15)$ ($m^{-3/2}$ scaling)

Clearly, the final result depends crucially on the assumption made for the $q^2$-dependence. Given the statistical errors, the systematic uncertainty in the extraction of the form factors, the effects of $O(a)$ terms and the limited range in $q^2$ and masses, the study of the $q^2$- and mass-dependence of the form factors, remains a crucial challenge for lattice calculations.

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