A new test of the FLRW metric using distance sum rule

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We present a new test of the validity of the Friedmann–Lemaître–Robertson–Walker (FLRW) metric, based on comparing the distance from redshift 0 to z1 and from z1 to z2 to the distance from 0 to z2. If the universe is described by the FLRW metric, the comparison provides a model-independent measurement of spatial curvature. The test is kinematic and relies on geometrical optics, it is independent of the matter content of the universe and the applicability of the Einstein equation on cosmological scales. We apply the test to observations, using the Union2.1 compilation of supernova distances and Sloan Lens ACS Survey galaxy strong lensing data. The FLRW metric is consistent with the data, and the spatial curvature parameter is constrained to be $-1.22 < \Omega_{K0} < 0.63$, or $-0.08 < \Omega_{K0} < 0.97$ with a prior from the cosmic microwave background and the local Hubble constant, though modelling of the lenses causes significant systematic uncertainty.

I. INTRODUCTION

Testing the FLRW metric. In addition to providing tight constraints on cosmological parameters in specific models, the increasing precision and breadth of cosmological observations makes it possible to test assumptions behind entire classes of models. A particularly important assumption is that the universe is on average described by the exactly homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) metric. More precisely, we consider the assumption that light propagation over long distances is well described by the FLRW metric, at least for typical light rays. This can be quantitatively tested by kinematic consistency conditions between different observables derived from geometrical optics. Such tests are independent of the matter content of the universe and its relation to spacetime geometry (usually given by the Einstein equation). It has been proposed that the observed late-time accelerated expansion could be related to the failure of the FLRW approximation. Possibilities include extra dimensions [1], violation of statistical homogeneity and isotropy [2], and the effect of deviation from exact homogeneity and isotropy on the average expansion rate, i.e. backreaction [3, 4].

Testing the FLRW metric by comparing observations of the expansion rate and distance was proposed in [5] and implemented in [6–8]. A similar test using parallax distance and angular diameter distance was put forth in [9]. We propose a third consistency test, based on the sum rule of distances along null geodesics of the FLRW metric, and apply it to real data. If the sum rule is violated, the FLRW metric is ruled out. If the data is consistent with the sum rule, the test provides a model-independent measurement of the spatial curvature of the universe, like the tests proposed in [3, 4].

II. FLRW CONSISTENCY CONDITION

Distances. If space is exactly homogeneous and isotropic, spacetime is described by the FLRW metric (we use units in which the speed of light is unity),

$$ds^2 = -dt^2 + \frac{a(t)^2}{1 - K r^2} dr^2 + a(t)^2 r^2 d\Omega^2,$$  \hspace{1cm} (1)

where $K$ is a constant related to the spatial curvature; the Ricci scalar of the hypersurface of constant proper time is $6K/a(t)^2$. (When $K > 0$, the metric [1] covers only half of the spacetime.) The Hubble parameter is $H \equiv \dot{a}/a$, and its present value is denoted by $H_0$. Let $D_A(z_1, z_s)$ be the angular diameter distance of a source at redshift $z_s$ (emission time $t_s$) as seen at redshift $z_1$ (observation time $t_1 > t_s$). From [1], we find that the dimensionless distance $d(z_1, z_s) \equiv (1 + z_s)H_0D_A(z_1, z_s)$ is

$$d(z_1, z_s) = \frac{1}{\sqrt{-k}} \sinh \left( \sqrt{-k} \int_{t_s(z_s)}^{t_1(z_1)} \frac{H_0 dt}{a(t)} \right),$$  \hspace{1cm} (2)

where $k \equiv K/H_0^2$. We denote $d(z) \equiv d(0, z)$.

Distance sum rule. Using [2], $d_{ls} \equiv d(z_1, z_s)$ can be written in terms of $d_1 \equiv d(z_1)$ and $d_s \equiv d(z_s)$ as

$$d_{ls} = \epsilon_1 d_s \sqrt{1 - kd_1^2} - \epsilon_2 d_1 \sqrt{1 - kd_s^2},$$  \hspace{1cm} (3)

where $\epsilon_i = \pm 1$. For $k \leq 0$, $\epsilon_i = 1$. For $k > 0$, the signs depend on which halves of the three-dimensional
hypersphere the source and the lens are located and in which direction the light propagates. If there is a one-to-one correspondence between \( t \) and \( z \) and \( d''(z) > 0 \), then \( \epsilon_i = 1 \). We assume that this is the case, so we have

\[
\frac{d_{ls}}{d_s} = \sqrt{1 - k d_s^2} - \frac{d_l}{d_s} \sqrt{1 - k d_s^2}.
\]

The relation (3) (or 4) is a sum rule for distances in the FLRW universe. In the spatially flat case, the distances are simply added together, whereas for non-zero spatial curvature, the relation is more involved. Using (3) in the case \(|k| \ll 1\) to obtain a model-independent estimate of the spatial curvature was proposed in [10].

The consistency condition. The sum rule (3) is specific to the FLRW metric. We get a consistency condition by solving for \( k \) to obtain (regardless of the values of \( \epsilon_i \))

\[
k_S = \frac{d_1^2 + d_2^2 - 2d_1 d_2 - 2d_1^2 d_s^2 - 2d_2^2 d_s^2}{4d_1^2 d_2^2 d_s^2},
\]

where we have added the subscript \( S \) to indicate that \( k \) has been solved from the sum rule for distances. We now drop the assumption that the universe is described by the FLRW metric and take (5) as the definition of a function \( k_S(z_1, z_2) \) in any spacetime (neglecting angular dependence). If the universe is described by the FLRW metric, \( k \) is constant and equal to \(-\Omega_K\), the present value of the spatial curvature density parameter. Conversely, if it is observationally found that \( k_S \) is different for any two pairs \((z_1, z_2)\), then light propagation on large scales is not described by the FLRW metric.

The consistency condition (5) provides a powerful test. In principle, the FLRW metric can be falsified by measuring the three quantities \((d_1, d_s, d_{ls})\) for two different values of \((z_1, z_2)\). The test is very general because it is purely kinematic. The derivation assumes only geometrical optics and that light propagation can be described with the FLRW metric. Unlike the condition between distance and expansion [3], the consistency condition (5) does not involve derivatives of the distance. Unlike the condition between angular diameter and parallax distances [3], there are already measurements of the distances involved, \( d \) and \( d_{ls} \), on cosmological scales.

III. DETERMINING \( d \) AND \( d_{ls} \)

The distance \( d \). We determine \( d \) from luminosities of type Ib supernovae (SNe). We use the Union2.1 compilation [11], which provides luminosity distances \( D_L \) to 580 SNe, with arbitrary overall normalisation. The highest redshift in the compilation is 1.4. Normalising by \( H_0 \), we obtain \( d_L = H_0 D_L = (1 + z)d \), where the last relation holds in any spacetime [12]. Our test involves only ratios of distances, so it does not depend on the value of \( H_0 \).

In the Union2.1 analysis, the empirical parameters that describe SN light curves are fitted at the same time as the cosmological parameters, and it is assumed that the universe is described by the spatially flat FLRW model with dust and vacuum energy, so the resulting distances are model-dependent [13]. There are also significant differences between light curve fitters [14]. However, the model-dependence of the SN analysis is likely subdominant to the uncertainties in the modelling of the strong lensing systems that we use to determine \( d_{ls} \). We therefore simply use the distances to individual SN given in the Union2.1 compilation, with the reported statistical and systematic errors. We determine \( d(z) \) model-independently, with the above caveats, by fitting a polynomial to the Union2.1 data, as described in section IV.

The distance \( d_{ls} \). Angular separation between strongly lensed images of the same source depends on \( d_{ls}/d_s \) in a way that depends on the structure of the lens. We assume that general relativity holds on the scale of the lensing system. If the lens can be approximated as a singular isothermal ellipsoid (SIE), we have [15]

\[
\frac{d_{ls}}{d_s} = \frac{\theta_E}{4\pi f^2 \sigma^2},
\]

where \( \theta_E \) is the Einstein radius (in radians), \( \sigma \) is the velocity dispersion of the lens and \( f \) is a phenomenological coefficient that parametrises the uncertainty due to differences between the velocity dispersion of the observed stars and the underlying dark matter, and other systematic effects. Observations suggest the range \( 0.8 < f^2 < 1.2 \) [16, 17].

We consider two different treatments of (6), which we call models Ia and Ib. In model Ia, we take \( f = 1 \). In model Ib, we model \( f \) by assigning an extra Gaussian error of 20\% on \( d_{ls}/d_s \). Leaving \( f \) as a free parameter would significantly degrade the constraints owing to a degeneracy between \( f \) and \( k \) for \(-k \gg 1\), due to limited redshift coverage and small number of lensing data points.

We also consider a more complicated treatment of the lens, introduced in [18], where (6) is replaced by

\[
\frac{d_{ls}}{d_s} = N(\alpha, \beta, \delta) \frac{\theta_E}{\sigma},
\]

with \( \alpha, \beta \) and \( \delta \) being the slope of the density, anisotropy of the velocity dispersion and the luminosity, respectively. We call this model II. Following [18], we treat \( \alpha \) and \( \beta \) as universal parameters with a Gaussian distribution with fixed mean and variance. For \( \delta \) we use values reported for each individual lens. These depend on the aperture. We treat this variation as a lens-specific error on \( \delta \), assumed to be Gaussian, with the 1\sigma range given by the difference between the maximum and minimum values. The average mean value is \( \delta = 2.39 \) and the average 1\sigma error is 0.05. The values given in [18] are not centered around the SIE model, due to non-zero mean anisotropy in the velocity dispersion and the different slopes of the density and luminosity. For the mean values of \( \alpha \) and \( \beta \) and the value of \( \delta = 2.4 \) used in [18], \( d_{ls}/d_s \) is 12\% lower than in the SIE case [6]. However, in our best-fits, the mean value of \( d_{ls}/d_s \) is the same in models Ia, Ib and II to 2\%.
Lensing data. We select strong lensing systems for which there is a well-measured value for lens and source redshift, velocity dispersion and Einstein radius. We require the lens to be either an elliptical or a lenticular galaxy, so that it can be modelled as a SIE. We also require that there is either an Einstein ring or arcs, not just multiple images, because without individual spectra, we cannot be sure that separated images are from the same source. These criteria leave us with 30 lenses, listed in table I. We have checked that the lenses are isolated from other galaxies and clusters. The data is mostly from the Sloan Lens ACS Survey [19], with additional data from the SIMBAD database [20] and [21]. The maximum source redshift is $z_s = 0.98$, well below the maximum redshift of 1.4 of the Union2.1 SN compilation. Following [19], we assign an error of 2% on $\theta_E$ and a minimum error of 5% on $\sigma$.

### IV. DATAFIT AND RESULTS

Fitting function. In principle, the function $k_S(z_l, z_s)$ defined in [5] can be reconstructed from observations, and if it is not constant, the FLRW metric is ruled out. Such a construction technique has been applied to $k$ defined with the expansion rate and distance $H(z)$ in [6, 8]. However, [5] gives a biased estimate of $k$. If we insert values of $d_l$, $d_s$ and $d_{ls}$ with errors into [5], the result will not be centred on the real value of $k$. In any case, given the small number of $d_{ls}/d_s$ datapoints, we do not try to reconstruct $k$ as a function of redshift. Instead, we fit a constant $k$ to the data and consider the goodness of fit. Large values of $\chi^2/d$ ofr. would be evidence against the FLRW model, or for unaccounted systematic errors. If the FLRW hypothesis is not rejected, the $\chi^2$ values give the probability distribution of $k$.

As mentioned in section III we obtain $d(z)$ by fitting a function to the Union2.1 datapoints. We have compared polynomials of different order, as well as splines, rational functions and Bézier curves by fitting to mock datasets of FLRW models with zero, positive or negative spatial curvature, as well as the real data. We find that with current data, it doesn’t make much difference which function we use, as long as it is more flexible than a second-order polynomial. Note that, in contrast to attempts to reconstruct the deceleration parameter or the equation of state [22], we don’t need derivatives of the distance. We present the results for a fourth order polynomial.

Upper limit on $k$ from CMB and $H_0$. On a hypersphere, the comoving angular diameter distance is bounded from above by $r_K \equiv 1/\sqrt{K}$, so $k \leq 1/d(z)^2$ for all $z$, as we see from [3]. Given $d' > 0$, the strongest constraint comes from the largest value of $z$. We adopt the model-independent distance $D_A(0, 1090) = 12.8 \pm 0.07$ Mpc from the cosmic microwave background (CMB) [23] and the locally measured Hubble parameter $H_0 = 72.5 \pm 2.5$ km/s/Mpc [24]. (We give error bars as 68% limits and ranges as 95% limits.) These values do not depend on the assumption that the universe at late times is well-described by the FLRW metric on large scales. Taking the $2\sigma$ lower bound for both quantities, we have $d > 3.1$, which implies $k < 0.10$. (In fact, the conservative bounds $D_A > 12$ Mpc and $H_0 > 60$ km/s/Mpc would be sufficient for $k < 0.1$.)

Probability distribution for $k$. Our fitting model consists of a fourth order polynomial for $d(z)$ and $d_{ls}/d_s$ given by [4] with a constant $k$. The resulting $\chi^2$ for the supernovae is the same for all three models, but the $\chi^2$ for the fit to the lensing data is 78, 20 and 54 for models Ia, Ib and II, respectively. Given that we have 30 lensing datapoints, these values indicate that either model Ia underestimates the statistical errors, or there are systematic problems with lens data. The increased errors of model Ib overcompensate, whereas the more complicated model II does not result in a reasonable goodness-of-fit. A look at outliers indicates that the issue is probably systematic. For models Ia and II there are 7 and 5 lenses, respectively, that are outliers at more than $2\sigma$. (There are no outliers for model Ib.) We produce a conservative truncated list of lenses by removing the most extreme outlier, refitting and iterating until all lenses are within $2\sigma$. For model II, one initial outlier is no longer an outlier and one initial non-outlier becomes an outlier. This leaves us with 23 and 25 lenses for models Ia and II, respectively. For model Ib we use all 30 lenses. The lensing dataset is summarised in table I.

We marginalise over the three polynomial coefficients to obtain the probability distribution $P(k)$. The results are shown in figure I. Even without a prior on $k$, the probability distributions are not Gaussian, and have a tail at negative values of $k$. The 95% ranges, mean and best-fit values for $k$ as well as the goodness-of-fit values are given in table 5. Our studies of mock datasets show that for current data the typical offset of the mean and the best-fit from the real underlying values is much smaller than the error bars. With the outliers removed, the $\chi^2$ of models Ia and II is reasonable. Imposing the prior on $k$ leads to an increase in $\chi^2$ for model Ia, indicating tension between the lensing and CMB data. There

| Lens model | Best-fit Mean | 95% range | $\chi^2_{SN}$ | $\chi^2_{L}$ | $\chi^2_{tot}$ | Prior on $k$ |
|-----------|--------------|-----------|---------------|--------------|---------------|--------------|
| Ia        | 0.55         | 0.38      | [-0.63, 1.22] | 545          | 23            | 568          | None         |
| Ib        | 0.76         | 0.37      | [-1.42, 1.54] | 545          | 20            | 565          | None         |
| II        | -1.21        | -2.37     | [-7.54, 0.96] | 545          | 20            | 565          | None         |
| Ia        | 0.09         | -0.25     | [-0.97, 0.08] | 545          | 28            | 573          | $k < 0.1$    |
| Ib        | 0.09         | -0.53     | [-1.92, 0.07] | 545          | 21            | 566          | $k < 0.1$    |
| II        | -1.21        | -2.87     | [-8.16, -0.13]| 545          | 20            | 565          | $k < 0.1$    |

TABLE I. Results for $k$, with and without the prior from CMB and $H_0$. In $\chi^2$, the subscript SN refers to SNe, L refers to lenses and tot refers to their sum.
is no such issue in model II, but given its large number of parameters, and remaining possible systematic issues with the lenses, it would be premature to conclude that model II is more realistic.

For model Ia we have $-0.63 < k < 1.22$, or $-0.97 < k < 0.08$ with the prior. For model Ib, the range increases by a factor of 1.6. For model II, there is a long negative tail, due to degeneracy between decreasing $N$ and increasing $-k > 0$. There is no evidence for deviations from the FLRW model, nor for spatial curvature.

For comparison, fitting a FLRW model with dust and vacuum energy (and assuming that the Einstein equation holds) and keeping all lenses gives $-0.56 < k < 0.53$ for models Ia and Ib and $-0.51 < k < 0.51$ for model II. In this case, the constraints on $k = -\Omega_K$ are dominated by the SN data, the lensing data is unimportant.

V. CONCLUSIONS

Results and comparison to previous work. The $k_S$ test based on the distance sum rule \( 3 \) for the FLRW metric is independent of the matter content of the universe and its relation to spacetime geometry on cosmological scales, though general relativity has been assumed to be valid when determining $d_\text{ls}/d_s$ from astrophysical systems. We find that the data is consistent with the FLRW metric. For the treatment of lenses as SIE with the published errors, the spatial curvature parameter $\Omega_K = -k$ is determined to be $-1.22 < \Omega_K < 0.63$ from SN and lensing data, and $-0.08 < \Omega_K < 0.97$ with a prior from CMB and $H_0$. These numbers are sensitive to lens modelling.

This range is two orders of magnitude wider than the one quoted from the latest CMB plus baryon acoustic oscillation data, $-0.007 < \Omega_K < 0.006$ \( 25 \). That assumes that the universe is described by a FLRW model whose late-time matter content is dust and vacuum energy, and that the Einstein equation is valid on cosmological scales. However, tight constraints, $-0.007 < \Omega_K < 0.01$, are also obtained in an analysis with loose priors on dark energy, combining WMAP7 CMB, Union2 supernova and Big Bang Nucleosynthesis data as well as local $H_0$ measurements \( 28 \). Without the $H_0$ value, which is debated \( 24 \), \( 26 \), \( 27 \), the constraint is $-0.12 < \Omega_K < 0.01$. The sensitivity is due to two distinct effects. First, the overall angular scale of the CMB anisotropy pattern provides a measurement of the angular diameter distance to $z = 1090$, which depends strongly on the spatial curvature via the hyperbolic sine in \( 2 \) \( 29 \). (Note that the labels for the two curvature parameters in table 1 of \( 29 \) should be swapped.) However, if the universe is not well described by an FLRW model, it is possible that spatial curvature evolves so that it is only significant at late times, and is not strongly constrained by high-redshift probes \( 4 \). Second, the large-angle anisotropy of the CMB is sensitive to spatial curvature via the late Integrated Sachs-Wolfe (ISW) effect, which is particularly important in \( 28 \). However, analysis of the ISW effect depends on assumptions about evolution of dark energy perturbations, which are rather speculative, particularly if the equation of state crosses $-1$.

Model-independent constraints based only on geometrical optics, such as the ones provided here or obtained from comparison of distance with expansion rate \( 6 \) \( 8 \) \( 27 \) or cosmic parallax \( 9 \), are thus complementary to model-specific analyses, which involve more assumptions about the matter content and the theory of gravity.

Future constraints. In addition to strong lensing image deformation by galaxies, existing observations of time delays and both strong and weak lensing by galaxy groups and clusters can be used to improve the constraints. Strong lens by clusters may be promising, because individual lenses have several sources and some of the lenses are tightly modelled. On the other hand, many source redshifts are higher than current independent measurements of $d_s$. The Euclid satellite, set to launch in 2020, is expected to observe $10^5$ strong lensing systems \( 30 \). The usefulness of these systems for the test discussed here depends on follow-up observations to determine velocity dispersion and other lens properties. Better understanding of the systematics of modelling lensing systems will be crucial. Given such progress, we can expect constraints on deviations from the FLRW metric, and on the spatial curvature of the FLRW universe, to significantly improve in the near future from the proof of concept we have presented here. Assuming lens model Ia and a spatially flat FLRW model with dust and vacuum energy, $10^4$ SNe \( 31 \) and $10^4$ lensing data points with current errors give the constraint $-0.03 < \Omega_K < 0.04$, within a factor of a few of the current model-dependent range.
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| Name           | $z_1$  | $z_a$ | $\sigma$ [km/s] | $\Delta \sigma$ [km/s] | $\theta_K$ [as] | $\sigma_{atm}$ | $\delta(3.0\ as)$ | $\delta(3.6\ as)$ | $\delta(4.2\ as)$ | Outlier in model Ia | Outlier in Model II |
|----------------|--------|-------|-----------------|-------------------------|----------------|-----------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| J0029–0055     | 0.2270 | 0.9313| 229             | 18                      | 0.92           | 1.84000        | 2.355             | 2.391             | 2.422             | No                | No                |
| J0252+0039     | 0.2803 | 0.9818| 164             | 12                      | 1.04           | 2.07250        | 2.426             | 2.547             | 2.644             | Yes               | Yes               |
| J0405–0455     | 0.0753 | 0.8098| 160             | 8                       | 0.80           | 1.80527        | 2.484             | 2.502             | 2.522             | No                | No                |
| J0728+3835     | 0.2058 | 0.6877| 214             | 11                      | 1.25           | 1.80527        | 2.397             | 2.442             | 2.485             | Yes               | Yes               |
| J0737+3216     | 0.3223 | 0.5812| 338             | 17                      | 1.00           | 2.32818        | 2.278             | 2.328             | 2.373             | Yes               | No                |
| J0822+2652     | 0.2141 | 0.5941| 259             | 15                      | 1.17           | 1.93500        | 2.391             | 2.432             | 2.462             | No                | No                |
| J0912+0029     | 0.1642 | 0.3239| 326             | 16                      | 1.63           | 2.80200        | 2.162             | 2.228             | 2.289             | No                | No                |
| J0936+0913     | 0.1897 | 0.5880| 243             | 12                      | 1.09           | 1.43750        | 2.319             | 2.349             | 2.392             | No                | No                |
| J0946+1006     | 0.2219 | 0.6085| 263             | 21                      | 1.38           | 1.23667        | 2.321             | 2.386             | 2.437             | No                | No                |
| J0956+5100     | 0.2405 | 0.4699| 334             | 17                      | 1.33           | 1.65333        | 2.318             | 2.380             | 2.445             | No                | No                |
| J1023+4230     | 0.1912 | 0.6960| 242             | 15                      | 1.41           | 1.85000        | 2.367             | 2.428             | 2.477             | No                | No                |
| J1106+5228     | 0.0955 | 0.4069| 262             | 13                      | 1.23           | 1.94000        | 2.435             | 2.466             | 2.495             | Yes               | Yes               |
| J1153+4612     | 0.1797 | 0.8751| 226             | 15                      | 1.05           | 1.60667        | 2.502             | 2.545             | 2.580             | No                | No                |
| J1204+0358     | 0.1644 | 0.6307| 267             | 17                      | 1.31           | 1.41000        | 2.410             | 2.440             | 2.474             | No                | No                |
| J1205+4910     | 0.2150 | 0.4808| 281             | 14                      | 1.22           | 2.26800        | 2.316             | 2.345             | 2.381             | No                | No                |
| J1250+0523     | 0.2318 | 0.7953| 252             | 14                      | 1.13           | 2.10333        | 2.309             | 2.384             | 2.438             | No                | No                |
| J1402+6321     | 0.2046 | 0.4814| 267             | 17                      | 1.35           | 2.37000        | 2.319             | 2.358             | 2.393             | No                | No                |
| J1403+0006     | 0.1888 | 0.4730| 213             | 17                      | 0.83           | 1.77000        | 2.399             | 2.440             | 2.496             | No                | No                |
| J1420+6019     | 0.0629 | 0.5351| 205             | 10                      | 1.04           | 2.08333        | 2.391             | 2.443             | 2.487             | No                | No                |
| J1430+4105     | 0.2850 | 0.5753| 322             | 32                      | 1.52           | 1.58333        | 2.229             | 2.288             | 2.360             | No                | No                |
| J1531–0105     | 0.1596 | 0.7439| 279             | 14                      | 1.71           | 1.83250        | 2.342             | 2.378             | 2.426             | No                | No                |
| J1538+5817     | 0.1428 | 0.5312| 189             | 12                      | 1.00           | 1.46000        | 2.386             | 2.448             | 2.507             | Yes               | No                |
| J1629–0053     | 0.2076 | 0.5241| 290             | 15                      | 1.23           | 1.85000        | 2.367             | 2.411             | 2.451             | No                | No                |
| J1630+4520     | 0.2479 | 0.7933| 276             | 16                      | 1.78           | 1.54333        | 2.360             | 2.416             | 2.469             | No                | No                |
| J1636+4707     | 0.2282 | 0.6745| 231             | 15                      | 1.09           | 1.35333        | 2.446             | 2.476             | 2.505             | No                | No                |
| J2238–0754     | 0.1371 | 0.7126| 198             | 11                      | 1.27           | 1.80527        | 2.324             | 2.374             | 2.418             | Yes               | Yes               |
| J2300+0022     | 0.2285 | 0.4635| 279             | 17                      | 1.24           | 1.93333        | 2.390             | 2.444             | 2.489             | No                | No                |
| J2303+1422     | 0.1553 | 0.5170| 255             | 16                      | 1.62           | 1.48333        | 2.214             | 2.272             | 2.330             | No                | No                |
| J2321–0939     | 0.0819 | 0.5324| 249             | 12                      | 1.60           | 1.58000        | 2.191             | 2.229             | 2.264             | No                | No                |
| J2341+0000     | 0.1860 | 0.8070| 207             | 13                      | 1.44           | 1.41667        | 2.140             | 2.183             | 2.239             | Yes               | Yes               |

TABLE II. Data for the lensing systems. The quantity $\Delta \sigma$ is the 1$\sigma$ error of the velocity dispersion $\sigma$, $\sigma_{atm}$ is the seeing value, $\delta(n\ \text{as})$ is the value of $\delta$ estimated for an aperture of $n\ \text{as}$. Values are from [19], except for the seeing, which is from [20] and for the values of delta, which are from [21]. For the three systems J0405–0455, J0728+3835 and J2238–0754, no seeing value is given in [20], so we use the average of the seeing values of the other 27 systems.