Combining Supernovae and LSS Information with the CMB

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ABSTRACT Observations of the Cosmic Microwave Background (CMB), large scale structure (LSS) and standard candles such as Type 1a Supernovae (SN) each place different constraints on the values of cosmological parameters. We assume an inflationary Cold Dark Matter model with a cosmological constant, in which the initial density perturbations in the universe are adiabatic. We discuss the parameter degeneracies inherent in interpreting CMB or SN data, and derive their orthogonal nature. We then present our preliminary results of combining CMB and SN likelihood functions. The results of combining the CMB and IRAS 1.2 Jy survey information are given, with marginalised confidence regions in the $H_0$, $\Omega_m$, $b_{\text{IRAS}}$ and $Q_{\text{rms-}ps}$ directions assuming $n = 1$, $\Omega_\Lambda + \Omega_m = 1$ and $\Omega_b h^2 = 0.024$. Finally we combine all three likelihood functions and find that the three data sets are consistent and suitably orthogonal, leading to tight constraints on $H_0$, $\Omega_m$, $b_{\text{IRAS}}$ and $Q_{\text{rms-}ps}$, given our assumptions.

KEYWORDS: CMB, large scale structure, supernovae, cosmology

1. INTRODUCTION

By comparing the observed CMB power spectrum with predictions from cosmological models one can estimate cosmological parameters. This has become an area of great current interest, with many groups carrying out the analyses for a range of assumed models (e.g. Hancock et al. 1998, Lineweaver et al. 1997, Bond & Jaffe1998). Generally speaking, the results of using CMB data alone to do this are broadly consistent with the expected range of cosmological parameters, though perhaps with a tendency for $H_0$ to come out rather low (assuming spatially flat models). In an independent manner, similar predictions can be made by comparing Large Scale Structure (LSS) surveys with cosmological models (Willick et al. 1997, Fisher & Nusser 1996, Heavens & Taylor 1995). Also, when distant Type 1A supernovae are used as a standard candle, one can assess the probability as a function of $\Omega_m$ and $\Omega_\Lambda$. We demonstrate that the results from the different data sets are compatible. However using each data set alone it is only possible to constrain combinations of the cosmological parameters. We show how these combinations of parameters are different for each data set, therefore when we combine the data sets we obtain tight constraints.
2. THE COMPLEMENTARY NATURE OF SUPERNOVAE AND CMB DATA

There has recently been great interest in combining Type Ia supernovae (SN) data with results from the CMB (e.g. Lineweaver 1998, Tegmark 1998). It is instructive to see how the complementarity between the supernovae and CMB data arises. The key quantity for this discussion is \( R_0 S(\chi) \), which occurs in the definitions of Luminosity Distance:

\[
d_L = R_0 S(\chi)(1 + z),
\]

and Angular Diameter Distance:

\[
d_\theta = R_0 S(\chi)/(1 + z).
\]

Here \( R_0 \) is the current scale factor of the Universe, \( \chi \) is a comoving coordinate, and \( S(\chi) \) is \( \sinh(\chi) \), \( \chi \) or \( \sin(\chi) \) depending on whether the universe is open, flat or closed respectively. For a general Friedmann-Lemaître model, one finds that

\[
R_0 S(\chi) \propto \frac{1}{|\Omega_k|^{1/2}} \sin(h) \left\{ |\Omega_k|^{1/2} \int_0^z \frac{dz'}{H(z')} \right\}
\]

where

\[
\Omega_k = 1 - (\Omega_m + \Omega_\Lambda),
\]

\[
H^2(z) = H_0^2 \left( (1 + \Omega_m z)(1 + z)^2 - \Omega_\Lambda z(2 + z) \right).
\]

For small \( z \), it is easy to show that

\[
d_L \propto z + \frac{1}{2}(1 - 2q_0)z^2,
\]

where \( q_0 = \frac{1}{2}(\Omega_m - 2\Omega_\Lambda) \) is the usual deceleration parameter.
Therefore, for small $z$, SN results are degenerate along a line of constant $q_0$. However, the contours of equal $R_0 S(\chi)$ shift around as $z$ increases and for $z \gtrsim 100$ the contours are approximately orthogonal to those corresponding to $q_0$ constant. This is the reason why CMB and SN results are ideally complementary. The current microwave background data is mainly significant in delimiting the left/right position of the first Doppler peak in the power spectrum, and this depends on the cosmology via the angular diameter distance formula, evaluated at $z \sim 1000$. Thus the CMB results will tend to be degenerate along lines roughly perpendicular to those for the supernovae in the $(\Omega_m, \Omega_A)$ plane.

We may calculate the likelihood of a given set of cosmological parameters for CMB data alone using the bandpower approach described in e.g. Hancock et al. 1998. We use Seljak and Zaldariagga’s CMBFAST code to calculate scalar mode CMB
power spectra for an adiabatic inflationary Cold Dark Matter universe and use the CMB data points described in Webster et al. 1998. In this preliminary analysis we fix $\Omega_c h^2 = 0.145$ and $\Omega_b h^2 = 0.0125$ (where $h = H_0/100$ and $\Omega_c = \Omega_m - \Omega_b$). We marginalise over the initial power spectrum index, $n$, and CMB power spectrum normalisation, $C_2$. The result is shown in the top panel of Figure 2. Contours are at changes in $-\log(\text{Likelihood})$ of 0.5, 2, 4.5, 8 and 12.5 from the minimum value. As expected, there is a degeneracy in a similar direction to the last plot in Figure 1. The SN likelihoods we use are based on data in Perlmutter et al. 1998 and are plotted in the second panel of Figure 2. Again, the direction of degeneracy is along the lines of the first two plots of Figure 1.

The lines of degeneracy are orthogonal and overlap, leaving a small patch of parameter space that fits both data sets. However we may be more quantitative than this. The probability of the Universe having a particular $\Omega_m, \Omega_\Lambda$ given both the CMB and SN data sets is simply found by multiplying together the probabilities of those $\Omega_m, \Omega_\Lambda$ for each data set. We thereby obtain the final plot in Figure 2. The preferred universe is close to flat with a low $\Omega_m$. However, we have applied very limiting assumptions. Detailed likelihood calculations covering a much wider range of assumptions using both supernovae and CMB data are currently being carried out by Efstathiou et al., and should be submitted shortly.

3. COMBINING CMB AND LSS

Recently, Webster et al. have combined the CMB likelihoods with IRAS likelihoods obtained from the 1.2Jy galaxy redshift survey following the spherical harmonic approach of Fisher, Scharf & Lahav. They assume a Harrison-Zel’dovich primordial scalar power spectrum ($n_s = 1$) and the nucleosynthesis constraint $\Omega_b h^2 = 0.024$ (Tytler, Fan & Burles 1996) in a flat universe with a cosmological constant, although clearly it would be interesting to relax these constraints. This approach is complementary to that of Gawiser & Silk, who used a compilation of large scale structure and CMB data to assess the goodness of fit of a wide variety of cosmological models.

Because the CMB and LSS predictions are degenerate with respect to different parameters (roughly: $\Omega_m$ vs $\Omega_\Lambda$ for CMB; $H_0$ and $\Omega_m$ vs $b_{\text{iras}}$ for LSS), the combined data likelihood analysis allows the authors to break these degeneracies, giving new parameter constraints. The different degeneracy directions in the $\Omega_m$, $h$ plane are shown in the top two panels of Figure 4. Note that the two data sets agree well in the region where the lines of degeneracy cross. Figure 3 shows the final 1-dimensional probability distributions for the main cosmological parameters after marginalizing over each of the others. The vertical dashed lines denote the 68% confidence limits and the horizontal plot limits are at the 99% confidence limits.

The best fit results from the joint analysis of the two data sets on all the free parameters are shown in Table 1, for which we can derive $\Omega_b = 0.085$, $\sigma_8 = 0.67$ and shape parameter (Sugiyama 1995, Efstathiou, Bond & White 1992) $\Gamma = 0.15$. A detailed discussion of these estimates and comparison with other results is contained
FIGURE 3 The probability distributions, using CMB and IRAS data, after marginalising over the other free parameters.

in Webster et al. 1998, but in broad terms it is clear that fairly sensible values have resulted, which is encouraging for future prospects within this area.

Table 1. — Parameter values at the joint optimum. The 68% confidence limits are shown, calculated for each parameter by marginalising the likelihood over the other variables.

| Parameter | Value | 68% Confidence Limits |
|-----------|-------|-----------------------|
| $\Omega_m$ | 0.39  | $0.29 < \Omega_m < 0.53$ |
| $h$       | 0.53  | $0.39 < h < 0.58$      |
| $Q$ (µK)  | 16.95 | $15.34 < Q < 17.60$    |
| $b_{\text{IRAS}}$ | 1.21 | $0.98 < b_{\text{IRAS}} < 1.56$ |

For a spatially flat model, the age of the universe is given by:

$$t = \frac{2}{3H_0} \tanh^{-1} \sqrt{\Omega_\Lambda}$$

which evaluates to 16.5 Gyr in the current case, again compatible with previous estimates.

4. COMBINING CMB, SUPERNOVAE AND LSS DATA

Finally, we combine all three data sets under the assumption applied in the above CMB+IRAS analysis and find that the data sets agree well and tighten the
constraints on the four parameters investigated. We find that $Q_{\text{rms-ps}} = 17.5 \pm 1 \mu K$, $H_0 = 59 \pm 8 \text{ kms}^{-1}\text{Mpc}^{-1}$, $\Omega_m = 0.33 \pm 0.07$ and $b_{\text{IRAS}} = 1.05 \pm 0.2$, which may be compared to the results in Table 1. For a flat Universe, as considered here, the SN results shift $h$ up and $\Omega_m$ down slightly compared to the results found using CMB and IRAS data alone.

5. CONCLUSIONS
We have combined CMB, LSS and SN data sets and found good agreement between them, despite the restrictive nature of our assumptions. Due to the complementary nature of the data sets the error bars on the cosmological parameters are dramatically reduced by making such combinations.

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