Impact of Inert Higgsino Dark Matter

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Abstract

We consider a recently proposed supersymmetric radiative seesaw model which is coupled with the minimal supergravity. The conventional R parity and $Z_2$ invariance are imposed, which ensures the existence of a multi-component dark matter system. We assume that the pair of the lightest neutralino $\tilde{\chi}$ and the fermionic component $\tilde{\xi}$ of the inert Higgs supermultiplet is dark matter. If $\tilde{\xi}$ is lighter than $\tilde{\chi}$, and the lightest neutral inert Higgs boson is kinematically forbidden to decay (third dark matter), the allowed region in the $m_0$-$M_{1/2}$ plane increases considerably, where $m_0$ and $M_{1/2}$ are the universal soft-supersymmetry-breaking scalar and gaugino mass, respectively, although the dominant component of the multi-component dark matter system is $\tilde{\chi}$. There is a wide allowed region above the recent LHC limit.

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The minimal supersymmetric model (MSSM) is one of the attractive extensions of the standard model (SM) \[1\]. Especially, if it is coupled with the minimal supergravity (mSUGRA)\[3\] the supersymmetry (SUSY) breaking sector is drastically simplified \[3\], and we have at hand a constrained MSSM (CMSSM) with the universal and flavor-diagonal soft-SUSY-breaking parameters. Because of this simplicity the allowed parameter region in the SUSY breaking sector has become smaller and smaller as more and more experimental data have become available \[4, 5\]. A severe constraint also comes from the relic density of dark matter \[9\] if one assumes that the dark matter candidate is the lightest neutralino \[10, 11\].

If it is displayed in the $m_0 - M_{1/2}$ plane for a given value of \(\tan \beta\), where \(\tan \beta\) is the ratio of the vacuum expectation value of the up-type Higgs field to that of the down-type Higgs field, \(m_0\) and \(M_{1/2}\) are the universal soft scalar and gaugino mass at the unification scale \(M_{\text{GUT}}\), the allowed region is only a narrow strip \[12\].

The constraints on the soft-SUSY-breaking parameters may be relaxed in various ways as reviewed e.g. in \[4, 6\]. Here we will consider a supersymmetric extension \[13, 14\] (see also \[15\]) of the model of \[16\], in which the neutrino mass and mixing are generated in higher orders of perturbation theory \[17\]. In a class of recent radiative seesaw models the tree-level neutrino mass is protected by an extra discrete symmetry \(Z_2\) which, if it is unbroken, ensures the existence of a \(Z_2\) odd stable particle, a potential candidate for dark matter \[16, 18\]. \footnote{A variety of similar models have been recently constructed in \[19–24\]. Leptogenesis \[25\] in radiative seesaw models has been discussed in \[19, 24\]. Baryogenesis \[27\] in radiative seesaw models has been discussed in \[26, 28\].}

It has been shown for the model of \[16\] that the lightest right-handed neutrino can become a realistic dark matter candidate \[29\] (see also \[30\]). A motivating reason to supersymmetrize this type of models is that because of the extended Higgs sector the models are meaningful only up to energies \(\sim \text{few TeV}\) \[31\] and supersymmetry can considerably improve this situation. There will be a set of potential candidates for dark matter in the supersymmetric radiative seesaw models \[13, 14\], because in addition to \(Z_2\), the usual R parity is assumed. To exhaust all such possibilities is certainly interesting, but it will be beyond the scope of the present Letter. Here we will assume that the lightest neutralino \(\tilde{\chi}\) and the lightest inert higgsino \(\tilde{\xi}\) are dark matter candidates. This combination of the dark matter particles has not been considered in the past. The lightest inert Higgs boson \(\xi\), which is heavier than \(\tilde{\chi}\) and \(\tilde{\xi}\) by assumption, can become the third candidate, if its decay is kinematically forbidden. We will show that in the parameter region, where the decay of \(\xi\) is kinematically not allowed, the allowed region in the $m_0 - M_{1/2}$ plane increases considerably, if \(\tilde{\xi}\) is lighter than \(\tilde{\chi}\).

In the supersymmetric extension \[13, 14\] of the model of \[16\], a product of abelian discrete symmetries \(R \times Z_2 \times Z_L^J\) is assumed to be intact. The discrete \(Z_2\) forbids the tree-level neutrino mass, and \(Z_L^J\) is the discrete lepton number, while \(R\) is the usual R parity. The matter content of the model with their quantum numbers is given in Table I. \(L, H^u, H^d\) and \(\eta^u, \eta^d\) stand for \(SU(2)_L\) doublets supermultiplets of the leptons, the MSSM Higgses and the inert Higgses, respectively. The MSSM quarks are \(Q, U^C, D^C\) as usual. Similarly, \(SU(2)_L\) singlet supermultiplets of the charged leptons and right-handed neutrinos are denoted by \(E^C\) and \(N^C\). The gauge singlet supermultiplet \(\phi\) is an additional neutral Higgs

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\footnote{We use the definition given by the Particle Data Group \[2\].}
supermultiplet which is needed to generate neutrino masses radiatively. The superpotential is \( W = W_Y + W_\mu \), where
\[
W_Y = Y_{ij}^u Q_i^C H^u + Y_{ij}^d Q_i^C D_j^C H^d + Y_{ij}^e L_i^C E_j^C H^d + Y_{ij} H_i^C \eta^u + \lambda^u H^d \eta^u \phi + \lambda^d H^d \eta^d \phi ,
\]
\[
W_\mu = -\mu_H H^u H^d + \frac{(M_N)_{ij}}{2} N_i^C N_j^C + \mu_\eta \eta^u \eta^d + \frac{1}{2} \mu_\phi \phi^2 .
\]
The soft-SUSY-breaking Lagrangian is \( \mathcal{L}_{SB} = \mathcal{L}_A + \mathcal{L}_{m^2} + \mathcal{L}_B \), where
\[
\mathcal{L}_A = h_{ij}^u \tilde{Q}_i \tilde{Q}_j H^u + h_{ij}^d \tilde{Q}_i \tilde{D}_j^C H^d + h_{ij}^e \tilde{L}_i \tilde{E}_j^C H^d + h_{ij}^\nu \tilde{L}_i \tilde{\nu}_j \eta^u + h_{\lambda u} H^u \eta^u \phi + h_{\lambda d} \eta^d H^d \phi + h.c. ,
\]
\[
\mathcal{L}_{m^2} = -\left( \tilde{m}_Q^2 \right)_{ij} \tilde{Q}_i \tilde{Q}_j^* - \left( \tilde{m}_D^2 \right)_{ij} \tilde{U}_i \tilde{U}_j^C \phi^* - \left( \tilde{m}_D^2 \right)_{ij} \tilde{D}_i \tilde{D}_j^C \phi^* - \left( \tilde{m}_L^2 \right)_{ij} \tilde{L}_i \tilde{L}_j^* - \left( \tilde{m}_E^2 \right)_{ij} \tilde{E}_i \tilde{E}_j^C \phi^* - \left( \tilde{m}_N^2 \right)_{ij} \tilde{N}_i \tilde{N}_j^C \phi^* - m_{\mu H} H^u H^d + m_{\mu H} H^d H^d + \tilde{m}_{\phi} \phi \phi^* ,
\]
\[
\mathcal{L}_B = -\left( B_H H^u H^d \right) + \frac{(B_N)_{ij}}{2} \tilde{N}_i \tilde{N}_j^C + B_\eta \eta^u \eta^d + \frac{1}{2} B_\phi \phi^2 + h.c. .
\]

Our notation is such that the component fields with a tilde have odd R parity. Since the model is coupled with the mSUGRA, the soft-SUSY-breaking parameters are universal and flavor-diagonal at \( M_{GUT} \) and the underlying parameters are \(^3\)

\[
m_0 , \ M_{1/2} , \ A_0 , \tan \beta , \ \text{sign}(\mu_H) , \ \mu_\eta , \ \mu_\phi , \ MN , \ B_\eta , \ B_\phi , \ BN .
\]

It is essential for the radiative seesaw model that the discrete symmetry \( Z_2 \) is unbroken. Since our model is coupled with the mSUGRA, it is not obvious that \( Z_2 \) is not spontaneously broken at low energy. We assume that there exists a set of the boundary conditions at \( M_{GUT} \) such that \( Z_2 \) remains unbroken. Furthermore, the Yukawa couplings \( Y_{ij}^u \) are the important ones for the radiative generation of the neutrino mass. They need not be necessarily small.

\(^3\) Since the matter content of the present model is different from that of the MSSM, the unified soft parameters \( m_0 \) and \( M_{1/2} \) cannot be directly identified with those of the MSSM. In the renormalization group (RG) running we add a pair of \( d^C \) and \( \tilde{d}^C \) of SU(3)_C) to obtain gauge coupling unification. So, for the gaugino mass we have \( M_{1/2} = M_{1/2}^{MSSM} (\alpha_{GUT}/\alpha_{MSSM}) \simeq 1.2 \ M_{1/2}^{MSSM} \). This change of \( M_{1/2} \) can take into account the dominant change in the RG running of \( m_0 \) so that \( m_0 \) can be approximately identified with \( m_0^{MSSM} \). We assume that \( d^C \) and \( \tilde{d}^C \) are \( Z_2 \) odd, so that a Yukawa coupling of the form \( Q d^C \eta^d \) is possible. This Yukawa coupling will contribute to the RG running. Here we assume it is negligibly small.
Therefore, they drive the soft-SUSY-breaking scalar mass matrix $\tilde{m}_L^2$ to deviate from the universal, flavor-diagonal form so that lepton flavor violations are generated at low energy [32]. Here we assume that we can impose certain constraints on $Y_{ij}^\nu$ to suppress the lepton flavor violations without having contradictions with the observed neutrino mass and mixing.

There are many candidates for the dark matter in this model [13, 14]. The lightest combination of each row in Table II could be a dark matter. In this Letter we assume that the lightest supersymmetric particle (LSP) in the MSSM sector. Since the spectrum in the inert Higgs sector plays a crucial role in evaluating the relic densities, we first discuss it in some detail before we come to the calculation of the relic density $\Omega h^2$ can also vary in the entire interval below the maximal value. Since the spectrum in the inert Higgs sector plays a crucial role in evaluating the relic densities, we first discuss it in some detail before we come to the calculation of the relic density of the dark matter particles.

The inert $SU(2)_L$ doublet Higgs supermultiplets $\{\text{scalar, fermion}\}$ are defined as

$$
\eta^u = \left( \frac{\eta^{u+}}{\eta^{u0}}, \frac{\bar{\eta}^{u+}}{\bar{\eta}^{u0}} \right), \quad \eta^d = \left( \frac{\eta^{d0}}{\eta^{d-}}, \frac{\bar{\eta}^{d0}}{\bar{\eta}^{d-}} \right).
$$

Further we decompose the neutral fields into the real and imaginary parts as $\eta^{u0} = (\eta_R^{u0} + i\eta_I^{u0})/\sqrt{2}$, $\eta^{d0} = (\eta_R^{d0} + i\eta_I^{d0})/\sqrt{2}$ and $\phi = (\phi_R + i\phi_I)/\sqrt{2}$. Then the mass matrix of the CP even neutral inert Higgs fields can be written as

$$
m_{\xi_R}^2 = \begin{pmatrix}
X_{uu} & \epsilon B_\eta + \lambda^u \lambda^d s_\beta \bar{c}_\beta v^2/2 & X_{ud} \\
\epsilon B_\eta + \lambda^u \lambda^d s_\beta \bar{c}_\beta v^2/2 & X_{dd} & X_{d\phi} \\
X_{u\phi} & X_{d\phi} & \mu_\phi^2 + m_\phi^2 + \epsilon B_\phi + (\lambda^u s_\beta)^2 + (\lambda^d c_\beta)^2 v^2/2
\end{pmatrix}
$$

with $\epsilon = 1$ in the $(\eta_R^{u0}, \eta_R^{d0}, \phi_R)$ basis, where

$$
X_{uu} = m_{\eta^u}^2 + \mu_\eta^2 - c_{2\beta} M_Z^2/2 + (\lambda^u c_\beta v)^2/2,
$$

$$
X_{dd} = m_{\eta^d}^2 + \mu_\eta^2 + c_{2\beta} M_Z^2/2 + (\lambda^d s_\beta v)^2/2,
$$

$$
X_{u\phi} = -[(\lambda^d \mu_\eta + \epsilon \lambda^u \mu_H) s_\beta + (\lambda^u \mu_\phi - \epsilon h_{\lambda^u}) c_\beta] v/\sqrt{2},
$$

$$
X_{d\phi} = -[(\lambda^u \mu_\eta + \epsilon \lambda^d \mu_H) c_\beta + (\lambda^d \mu_\phi - \epsilon h_{\lambda^d}) s_\beta] v/\sqrt{2},
$$

| $R \times Z_2 \times Z_2$ | Bosons | Fermions |
|-------------------------|--------|----------|
| $(-,+,+)$              | $h_i^{u0}, h_i^{d0}, \tilde{Z}, \tilde{\gamma}$ |          |
| $(-,-,+)$              | $\bar{h}_i^{u0}, \bar{h}_i^{d0}, \bar{\phi}$ |          |
| $(+,+,+)$              | $\eta^{u0}, \eta^{d0}, \phi$ |          |
| $(+,+-,-)$             | $\tilde{N}_C$ |          |
| $(+,--,+)$             | $\tilde{N}_C$ |          |
| $(-,+,+)$              | $\tilde{\nu}_L$ |          |

**TABLE II:** The dark matter candidates. $\eta^{u0}$ and $\eta^{d0}$ are the neutral scalar components of $\eta^u$ and $\eta^d$, respectively. The $(+,+,--)$ candidates are dropped, because they are the left-handed neutrinos.
and \( s_\theta(c_\theta) \) represents \( \sin(\theta(\cos \theta)) \). In deriving the mass matrix (8) we have assumed that all the parameters appearing in (14) are real. The eigenvalues of \( m_\xi^2 \) in ascending order are denoted by \( m_{\xi_i}^2 \) \((i = 1, 2, 3)\), and the eigenstates are denoted by \( \xi_i \):

\[
\begin{pmatrix}
\eta^0_R \\
\eta^0_R \\
\phi_R
\end{pmatrix} = \begin{pmatrix}
C^R_{ij} & \xi^0_R \\
S^R_{ij} & S^R_{ij} \\
C^R_{ij} & S^R_{ij}
\end{pmatrix}.
\]

(13)

The mass matrix \( m_\xi^2 \) for the CP odd components can be obtained from (8) with \( \epsilon = -1 \), where we denote their eigenstates by \( \xi_i^0 \). The mass matrix for the charged inert Higgs fields is given by

\[
m_{\xi_i^+}^2 = \begin{pmatrix}
m_{\eta_i^u}^2 + \mu_\eta^2 - c_{2\beta}(1 - 2c_{1W}^2)M_2^2/2 \\
-B_\eta \\
m_{\eta_i^d}^2 + \mu_\eta^2 + c_{2\beta}(1 - 2c_{1W}^2)M_2^2/2
\end{pmatrix}
\]

(14)

in the basis of \( (\eta^{u+}, \eta^{d-})^* \). The eigenvalues of (14) are \( m_{\xi_i^+}^2 \) and \( m_{\xi_i^-}^2 \) with \( m_{\xi_i^+}^2 < m_{\xi_i^-}^2 \), and the eigenstates are

\[
\begin{pmatrix}
\eta^{u+} \\
(\eta^{d-})^*
\end{pmatrix} = \begin{pmatrix}
C_{kl} \\
S_{kl}
\end{pmatrix}
\]

(15)

Because of the boundary conditions \( m_{\eta_i^u}^2 = m_{\eta_i^d}^2 = m_\phi^2 = m_\tilde{\eta}^2 \) at the unification scale \( M_{\text{GUT}} \), the soft scalar masses are constrained. At low energy we expect that \( m_\phi^2 < m_{\eta_i^u}^2 < m_{\eta_i^d}^2 \), because \( \phi \) is gauge singlet, and

\[
\frac{d(m_{\eta_i^d}^2 - m_{\eta_i^u}^2)}{d\mu} \sim -\frac{6}{16\pi^2}m_0^2 \text{Tr}[Y^\nu(Y^\nu)^T] < 0.
\]

Further, neglecting the D-term contributions along with the assumption of small \( \lambda_{u,d} \) and \( h_{\lambda_{u,d}} \) in (8), we find that the upper bound of the smallest eigenvalues of \( m_{\xi_i^0}^2 \) and \( m_{\xi_i^+}^2 \) (\( m_{\xi_i^0}^2 \) and \( m_{\xi_i^+}^2 \)) can be written as

\[
m_{\xi_i^0}^2 \simeq \frac{1}{2}(m_{\eta_i^u}^2 + m_{\eta_i^d}^2) + \mu_\eta^2 - |B_\eta|.
\]

(17)

Since the soft mass \( B_\eta \) is a free parameter, we regard \( m_{\xi_i^0}^2 \) \((\simeq m_{\xi_i^0}^2 \simeq m_{\xi_i^+}^2)\) as a free parameter. Under the assumption mentioned above, we may approximately write the lightest mass eigenstates as

\[
\xi_{R1}^0 \simeq \frac{1}{\sqrt{2}}(\eta_R^0 - \eta_R^0) \quad \xi_{i1}^+ \simeq \frac{1}{\sqrt{2}}(\eta_i^{u+} + (\eta_i^{d-})^*)
\]

(18)

and similarly for \( \xi_{i1}^0 \). In Table III we summarize the approximate mass eigenstates with their approximate mass eigenvalues squared.

We next come to the inert higgsino sector. The charged inert higgsinos \( \eta_i^{u+} \) and \( \eta_i^{d-} \) form a Dirac spinor \( \xi^+ \) with the mass \( M_{\tilde{\eta}^+} = \mu_\eta \). The mass matrix of the neutral inert higgsinos is given by

\[
M_{\tilde{\eta}^0} = \begin{pmatrix}
0 & \mu_\eta & -\lambda_c \beta v/\sqrt{2} \\
\mu_\eta & 0 & -\lambda_d s_\beta v/\sqrt{2} \\
-\lambda_c \beta v/\sqrt{2} & -\lambda_d s_\beta v/\sqrt{2} & 2\mu_\phi
\end{pmatrix}.
\]

(19)
in the \((\tilde{\eta}^{u0}, \tilde{\eta}^{d0}, \tilde{\phi})\) basis. The mass eigenstates \(\tilde{\xi}_i^0\) with the mass \(M_{\tilde{\xi}_i^0} (M_\xi \equiv M_{\tilde{\xi}_i^0})\) are defined as

\[
\begin{pmatrix}
\tilde{\eta}^{u0} \\
\tilde{\eta}^{d0} \\
\tilde{\phi}
\end{pmatrix}
= C_{ij}^{0} \begin{pmatrix}
\tilde{\xi}_i^0 \\
\tilde{\xi}_j^0 \\
\tilde{\xi}_k^0
\end{pmatrix}.
\]

The lightest one \(\tilde{\xi} = \tilde{\xi}_1^0\) is the dark matter candidate and is a linear combination of the form

\[
\tilde{\xi} = \tilde{\xi}_1^0 = (C_{11}^{0})^* \tilde{\eta}^{u0} + (C_{21}^{0})^* \tilde{\eta}^{d0} + (C_{31}^{0})^* \tilde{\phi}.
\]

One can show from \cite{19} that the charged inert higgsino is always heavier than the dark matter \(\tilde{\xi}\).

The annihilation rate of \(\tilde{\xi}\) depends on the mixing parameters \(C_{ij}^{0}\) in \cite{21}, because the dominant contributions to the rate are due to gauge interactions and \(\tilde{\phi}\) has no gauge coupling. If \(\xi\) is \(\tilde{\eta}\)-like, the \(\xi\) behaves like a higgsino-like \(\tilde{\chi}\) of the MSSM so that the annihilation cross section tends to be large. It decreases as \(\xi\) contains more \(\tilde{\phi}\). So, the annihilation cross section can be controlled by \(C_{ij}^{0}\). Therefore, the relic density \(\Omega_\chi h^2\) can vary from a small to the observed value \(\simeq 0.11\) \cite{9}. That is, \(\Omega_\chi h^2\), too, may assume a value \(\lesssim 0.11\). In Fig. 11 we plot the allowed region (green) in the \(m_0 - M_{1/2}\) plane. The red area is the region for \(\Omega_\chi h^2 = 0.1126 \pm 0.0036\) \cite{9}. As we see from Fig. 11 the allowed region in the \(m_0 - M_{1/2}\) plane expands only slightly, if the annihilation cross section of \(\tilde{\chi}\) into inert Higgs bosons and inert higgsinos is sufficiently suppressed.

Next we discuss the case that the fermions in the inert Higgs sector are lighter than \(\tilde{\chi}\), and make therefore the following assumption on the mass hierarchy:

\[
M_{\tilde{\xi}_i^0}, \ M_{\tilde{\xi}_j^0} < M_{\tilde{\xi}_k^0}, \ M_{\tilde{\xi}_k^+}.
\]
FIG. 1: The allowed region in the \(m_0 - M_{1/2}\) plane. For the green area the relic density \(\Omega h^2\) is assumed to vary from zero to 0.11. The red area corresponds to \(\Omega h^2 = 0.1126 \pm 0.0036\) \cite{11}. We have computed \(\Omega h^2\) using the package “micrOMEGAs” \cite{33} with a set of the input parameters: \(A_0 = 0\), \(\tan \beta = 10\) and \(\text{sign}(\mu_H) = +\). Included are the constraints coming from the stau LSP, the electroweak symmetry breaking and the LEP chargino mass limit. (See the comment of footnote 3.)

The annihilation cross section \(\sigma_\tilde{\chi}\) of \(\tilde{\xi} (= \tilde{\xi}_0)\) can be obtained from that of the higgsino-like \(\tilde{\chi}\) of the MSSM. The diagrams are shown in Fig. 2. As in the case of the MSSM \(\sigma_\tilde{\chi}\) tends to be large if \(\tilde{\xi}\) is \(\tilde{\eta}\)-like, where the diagrams (a), (b), (c) and (d) in Fig. 2 are dominant, if the scalar partner \(\tilde{\eta}_C\) of the right-handed neutrino is much heavier than \(Z\). Under this assumption the only possibility to suppress \(\sigma_\tilde{\chi}\) is to increase the \(\tilde{\phi}\) content in \(\tilde{\xi}\) as we have discussed above. Note that the relic density \(\Omega h^2\) is inversely proportional to \((|\tilde{C}_{11}^0|^2 + |\tilde{C}_{21}^0|^2)^2 = (1 - |\tilde{C}_{31}^0|^2)^2\) (if we neglect the contribution (e) in Fig. 2), where \(\tilde{C}_{ij}^0\) are defined in (20). We have computed \(\Omega h^2\) for \(\tilde{C}_{31}^0 = 0\) with a representative set of the fixed values of the parameters as \(\tan \beta = 10\), \(M_\tilde{\xi} = 120\) GeV, and found \(\Omega h^2 \sim 10^{-3}\), where \(\Omega h^2\) depends only very weakly on \(m_0\) and \(M_{1/2}\). So, for a wide range of mixing of \(\tilde{\eta}_{u, d}^0\) and \(\tilde{\phi}\) the relic density \(\Omega h^2\) is much smaller than the observed value.

If \(M_\tilde{\xi} + M_\chi > m_\xi\) is satisfied, where \(m_\xi\) is the mass of the lightest inert Higgs boson \(\xi\) (either \(\xi_{R1}^0\) or \(\xi_{I1}^0\)), then it is stable despite (22): Its decay is kinematically forbidden, and so it can be a dark matter particle (third dark matter), too \footnote{4 Multi-component dark matter has been discussed e.g. in \cite{35}.}. In Refs. \cite{34, 36} the feature of the inert Higgs boson dark matter was studied in detail for non-SUSY models. It turned out \cite{36} that to obtain a realistic relic density its mass has to be either very small (\(\lesssim 80\) GeV) or very large (\(\gtrsim 500\) GeV) and the relic density is smaller than 0.02 between 100 GeV and 300 GeV \footnote{5 The extra SUSY contributions, \(\xi + \xi \rightarrow \tilde{\chi} + \tilde{\chi}, \tilde{\xi} + \tilde{\xi}\) etc., even decrease the relic density.} (see also \cite{37}). As we will see, this range of \(m_\xi\) is particularly interesting, because we can expand the allowed range in the \(m_0 - M_{1/2}\) plane considerably.

Keeping this in mind we turn to the relic density of \(\tilde{\chi}\). As the assumption (22) indicates \(\tilde{\chi}\) dark matter particles can annihilate into \(\tilde{\xi}_i^0 \tilde{\xi}_j^0\) and \(\xi^+ \xi^+\), as shown in Fig. 3 which contribute...
to the relic density $\Omega_{\tilde{\chi}} h^2$. The MSSM part is the same as in the MSSM, so that for the case of the higgsino-like $\tilde{\chi}$ the dark matter mass $M_{\tilde{\chi}}$ should be $O(1)$ TeV to obtain $\Omega_{\tilde{\chi}} h^2 = 0.11$. For a bino-like $\tilde{\chi}$, the contributions from the MSSM sector to the annihilation cross section $\sigma_{\tilde{\chi} \tilde{\chi}}$ is indeed too small. But $\sigma_{\tilde{\chi} \tilde{\chi}}$ can increase through the diagrams of Fig. 3 (b) in the inert Higgs sector. This indicates that the parameter space of the MSSM sector in this region can be relaxed. This is what we would like to see below. To this end we make further approximation: As we see from Table III we can always choose $B_\eta$ such that the contributions to the annihilation mediated by the exchange of the heavier ones are negligibly small compared with those mediated by the exchange of the lighter ones. Furthermore, $\xi_{R,I}^0 (\simeq \phi_{R,I})$ is not coupled with a gaugino-like $\tilde{\chi}$. Therefore, the relevant part of the Lagrangian can be written as

$$
\mathcal{L}_{\text{eff}} = \frac{g}{2} \left\{ (N_{12} + t_W N_{11}) \bar{\chi} \tilde{\chi}^+ (\xi_{R}^+)^* + (-N_{12} + t_W N_{11}) \bar{\chi} \{ (\tilde{C}_{1i}^0)^* P_L + \tilde{C}_{2i}^0 P_R \} \xi_{R}^0 \xi_{I}^0 + h.c. \right\} 
$$

$$
- \frac{g}{2 c_W} \left\{ (1 - 2 s_W^2)\bar{\xi}^+ \gamma^\mu \tilde{\xi}^+ - \frac{1}{2}[(\tilde{C}_{1i}^0)^* \tilde{C}_{1j}^0 - (\tilde{C}_{2i}^0)^* \tilde{C}_{2j}^0] \xi_{R}^0 \gamma^\mu P_L \tilde{\xi}_j^0 
$$

$$
+ \frac{1}{2} [\tilde{C}_{1i}^0 (\tilde{C}_{1j}^0)^* - \tilde{C}_{2i}^0 (\tilde{C}_{2j}^0)^*] \xi_{R}^0 \gamma^\mu P_R \xi_{I}^0 \right\} Z_\mu ,
$$

where $P_R = (1 \pm \gamma_5)/2$, $\xi_{R}^0 = (\xi_{R1}^0 + i \xi_{I1}^0)/\sqrt{2}$, $t_W = \tan \theta_W$, $\tilde{C}_{ij}^0$ are defined in (20), and the mixing parameters $N_{11}(N_{12})$ is the bino (wino) content in $\tilde{\chi}$, respectively. To proceed we make use of the fact that the relic density is inversely proportional to the annihilation cross section, and approximate the inverse of $\Omega_{\tilde{\chi}} h^2$ as

$$
\frac{1}{\Omega_{\tilde{\chi}} h^2} \simeq \frac{1}{\Omega_{\text{MSSM}} h^2} + \frac{1}{\Omega_{\text{EXTRA}} h^2} ,
$$

where $\Omega_{\text{MSSM}} h^2$ is the relic density of $\tilde{\chi}$ computed without including the diagrams of Fig. 3 while $\Omega_{\text{EXTRA}} h^2$ is computed only with the annihilation processes of Fig. 3. The approximation becomes better if one of $\Omega h^2$’s is dominated. We compute $\Omega_{\text{MSSM}} h^2$ using the package

FIG. 2: Annihilation diagrams of $\tilde{\chi}$. 
“micrOMEGAs” [33], and \( \Omega_{\text{EXTRA}} h^2 \) analytically using the approximate formula given in [10].

![Diagram](image)

FIG. 3: Annihilation diagrams of \( \tilde{\chi} \tilde{\chi} \to \tilde{\xi}^+ \tilde{\xi}^+ \), \( \tilde{\xi}_i^0 \tilde{\xi}_j^0 \).

According to [10] we expand the relativistic cross section \( \sigma(\tilde{\chi} \tilde{\chi} \to \tilde{\xi}^+ \tilde{\xi}^+ \), \( \tilde{\xi}_i^0 \tilde{\xi}_j^0 \)) in powers of their relative velocity \( v \), \( \sigma(\tilde{\chi} \tilde{\chi} \to \tilde{\xi}^+ \tilde{\xi}^+ \), \( \tilde{\xi}_i^0 \tilde{\xi}_j^0 \))v = a_{\text{EXTRA}} + b_{\text{EXTRA}} v^2. \) We then use [10]

\[
\Omega_{\text{EXTRA}} h^2 \simeq 2.82 \times 10^8 \left( \frac{M_\chi}{\text{GeV}} \right) Y_{\infty},
\]

where

\[
Y_{\infty}^{-1} = 0.264 M_{\text{PL}} M_\chi g_*^{1/2}(a_{\text{EXTRA}}/x_f + 3b_{\text{EXTRA}}/x_f^2),
\]

\( M_{\text{PL}} = 1.22 \times 10^{19} \text{ GeV} \) is the Planck mass, \( g_* \simeq 90 \) and \( x_f = M_\chi/T \) is the inverse dimensionless freeze-out temperature. The quantities \( a_{\text{EXTRA}} \) and \( b_{\text{EXTRA}} \) are found to be

\[
a_{\text{EXTRA}} = \sum_{\tilde{\epsilon}^+ \tilde{\epsilon}^+, \tilde{\xi}_i^0 \tilde{\xi}_j^0} \frac{4 G_F^2 m_W^4 \sqrt{1 - M_\xi^2/M_\chi^2}}{\pi (m_\xi^2 - M_\xi^2 + M_\chi^2)^2} \left( M_\xi^2 (w_1 + w_4) + w_2 M_\chi^2 + w_3 M_\xi M_\chi \right),
\]

\[
b_{\text{EXTRA}} = \sum_{\tilde{\epsilon}^+ \tilde{\epsilon}^+, \tilde{\xi}_i^0 \tilde{\xi}_j^0} \left\{ \frac{G_F^2 m_W^4 \sqrt{1 - M_\xi^2/M_\chi^2}}{6 \pi (m_\xi^2 - M_\xi^2) (m_\xi^2 - M_\chi^2 + M_\chi^2)^4} \sum_{i=1}^4 w_i B_i ight. \\
+ \frac{2 G_F^2 m_Z^4 \sqrt{1 - M_\xi^2/M_\chi^2} (N_{\tilde{\chi}^0}^2 - N_{\tilde{\chi}^+}^2)^2 (M_\chi^2 + 2 M_\xi^2)}{3 \pi (16 M_\chi^2 - 8 M_\chi^2 m_Z^2 + m_\chi^2 + \Gamma_Z^2 m_Z^2)} w_5 \right\},
\]

\[
B_1 = (M_\xi^2 - M_\chi^2)^2 (13 M_\chi^4 - 10 M_\chi^2 M_\xi^2 + 16 M_\chi^4) + m_\chi^4 (13 M_\chi^4 - 26 M_\chi^2 M_\xi^2 + 16 M_\chi^4) \\
+ m_\chi^2 (-26 M_\chi^6 + 70 M_\chi^4 M_\xi^4 - 44 M_\chi^2 M_\xi^6),
\]

\[
B_2 = (M_\xi^2 - M_\chi^2)^2 (6 M_\chi^4 + 17 M_\chi^2 M_\xi^2 - 4 M_\chi^4) + 3 m_\chi^2 (2 M_\chi^4 - 5 M_\chi^2 M_\xi^2 + 4 M_\chi^4) \\
- 2 m_\chi^2 (6 M_\chi^6 - 13 M_\chi^4 M_\xi^2 - M_\chi^2 M_\xi^6 + 8 M_\chi^6),
\]

\[
B_3 = M_\chi M_\xi \left[ 3 m_\chi^4 M_\xi^2 + 19 M_\chi^2 (M_\xi^2 - M_\chi^2)^2 + m_\chi^2 (-22 M_\chi^4 + 62 M_\chi^2 M_\xi^2 - 40 M_\chi^4) \right],
\]

\[
B_4 = M_\chi^2 \left[ (21 M_\chi^2 - 2 M_\chi^4) (M_\xi^2 - M_\chi^2)^2 - 3 m_\chi^4 (6 M_\chi^4 - 7 M_\chi^2) \\
- 6 m_\chi^2 (7 M_\chi^4 - 17 M_\chi^2 M_\xi^2 + 10 M_\chi^4) \right],
\]

where for the annihilation processes \( \tilde{\chi} \tilde{\chi} \to \tilde{\xi}^+ \tilde{\xi}^+ \)

\[
w_1 = \frac{1}{32} (N_{12} + t_W N_{11})^4 = \frac{w_2}{2} = \frac{w_3}{4} = w_4, \ w_5 = \frac{1}{16} (1 - 2 s_W^2)^4,
\]

\( w_1, w_2, w_3, w_4, w_5 \) are found to be

\[
\begin{align*}
w_1 &= \frac{1}{32} (N_{12} + t_W N_{11})^4, \\
w_2 &= \frac{w_3}{4}, \\
w_3 &= \frac{1}{16} (1 - 2 s_W^2)^4,
\end{align*}
\]
and for $\tilde{\chi}\tilde{\chi} \rightarrow \tilde{\xi}^0 \xi^0_j$

$$w_1 = \frac{1}{32} \left(-N_{12} + t_w N_{11}\right)^4 = \frac{w_2}{2} = \frac{w_3}{4} = w_4, \quad w_5 = \frac{1}{16},$$

(34)

To simplify the situation we have assumed that all the inert higgsinos have the same mass as $\tilde{\xi}$ (i.e. $M_{\tilde{\xi}} = M_{\tilde{\xi}^0_1} \simeq M_{\tilde{\xi}^0_2} \simeq M_{\tilde{\xi}^0_3} \simeq M_{\tilde{\xi}^+}$) and the light inert Higgs bosons also have the common mass $m_{\xi}$ (i.e. $m_{\xi} = m_{\xi^0_{\tilde{r}1}} \simeq m_{\xi^0_{\tilde{r}2}} \simeq m_{\xi^+}$). In this limit the mixing parameters can be approximated as $\tilde{C}_{0i}^0 = (i, 1, 0)/\sqrt{2}, \quad \tilde{C}_{2i}^0 = (-i, 1, 0)/\sqrt{2}$.

**FIG. 4:** The allowed region in the $m_0$-$M_{1/2}$ plane for $m_{\xi} > M_{\tilde{\xi}} > M_{\tilde{\xi}} = 120$ GeV with $A_0 = 0$, $\tan\beta = 10$ and $\text{sign}(\mu_H) = +$, where we have used: $\sin^2\theta_W = 0.23, \quad m_W = 80.4$ GeV, $G_F = 1.17 \times 10^{-5}$ GeV$^{-2}$. In the actual calculation of $\Omega_{\tilde{\xi}} h^2$, we have approximated that all the inert higgsinos have the same mass as $\tilde{\xi}$ (i.e. $M_{\tilde{\xi}} = M_{\tilde{\xi}^0_1} \simeq M_{\tilde{\xi}^0_2} \simeq M_{\tilde{\xi}^0_3} \simeq M_{\tilde{\xi}^+}$) and the light inert Higgs bosons also have the common mass $m_{\xi}$ (i.e. $m_{\xi} = m_{\xi^0_{\tilde{r}1}} \simeq m_{\xi^0_{\tilde{r}2}} \simeq m_{\xi^+}$). The constraints coming from the stau LSP, the electro-weak symmetry breaking and the LEP chargino mass limit are included, and the dashed line is the recent LHC limit. (See the comment of footnote 3.)

Now we present the calculation of $\Omega_{\tilde{\xi}} h^2$. In Fig. 4 we plot the allowed region in the $m_0$-$M_{1/2}$ plane. We have obtained the allowed region for $m_{\xi} > M_{\tilde{\xi}} > M_{\tilde{\xi}} = 120$ GeV with $\tan\beta = 10$. The allowed region in Fig. 4 should be compared with that in Fig. 1 which is obtained under the assumption that the annihilation cross section of $\tilde{\chi}$ into the inert Higgs sector is sufficiently suppressed. The mass values of $m_{\xi}$ and $M_{\tilde{\xi}}$ giving the green area of Fig. 4 except the area along the left and right border lines, are shown in Fig. 5 (where $M_{\tilde{\xi}}$ is fixed at 120 GeV). We see that $m_{\xi}$ is smaller than $M_{\tilde{\chi}} + M_{\tilde{\xi}}$ in this area so that the lightest neutral inert Higgs boson is a dark matter particle, too. As we mentioned, the feature of the inert Higgs boson dark matter was studied in Refs. 34, 36. In our case $m_{\xi}$ varies from 160 to 220 GeV (see Fig. 5). Using their results, we find that the contribution of the inert Higgs boson
FIG. 5: The region in the $M_\chi - m_\xi$ plane for $M_\xi = 120$ GeV, which gives the green area of Fig. 4 except the area along the left and right border lines.

dark matter to the relic density 0.11 is at most 15%, where the extra SUSY contributions, $\xi + \xi \rightarrow \tilde{\chi} + \tilde{\chi}$, $\tilde{\xi} + \tilde{\xi}$ etc., are not taken into account. These extra contributions can be as large as that without them. We roughly estimate that dark matter consists more than 90% of $\tilde{\chi}$ in the most of the green area of Fig. 4. The area along the left and right border lines is the area very closed to the red line of Fig. 4. In this area $m_\xi > M_\chi + M_\xi$ is satisfied, and the annihilation cross section for $\tilde{\chi}\tilde{\chi} \rightarrow \xi^+\xi^-$, $\tilde{\xi}^0\tilde{\xi}^0$ is very small, implying that dark matter consists almost 100 percent of $\tilde{\chi}$ in this area.

If the masses $m_\xi$, $M_\chi$ and $M_\xi$ are very close, there can be co-annihilations among the dark matter particles. We see from Fig. 4 that there exists a small region where $M_\chi \simeq M_\xi$ and $m_\xi \simeq M_\chi$ are satisfied, respectively. For these regions, co-annihilation processes such as $\tilde{\chi} + \tilde{\xi} \rightarrow \xi_1^0 + Z$, $\tilde{\chi} + \xi_1^0 \rightarrow \tilde{\xi} + Z$ may become possible. Indeed, $m_\xi$ and $M_\xi$ are degenerate in the area close to the upper borderline of the green region in Fig. 4 while $M_\xi$ and $M_\chi$ are degenerate in the area close to the lower borderline. So, one should take into account the effects of the co-annihilation processes. However, we have ignored them in Fig. 4 because these effects will change only the narrow area close to the borderlines and not the gross structure of the allowed region in Fig. 4.

There will be some differences in direct and indirect searches of dark matter. Let us make a few comments on this, where the details will be published elsewhere. We recall that the direct rate is proportional to the relic density, while the indirect rate is proportional to the square of the relic density. Therefore, indirect search of the inert Higgs boson dark matter suffers from a suppression factor of at least $0.1^2 (\sim 10^{-6}$ in the case of the inert higgsino dark matter). At first sight, indirect detection rate of the dark matter $\tilde{\chi}$ seems to be suppressed compared with the case of the CMSSM, because the higgsino portion of $\tilde{\chi}$ is very small in the green area of Fig. 4 except on the left and right border lines. However, annihilation not only into the neutral $\xi$'s, but also into the charged $\xi^+$'s is possible, and this rate is large. So, indirect detection of $\tilde{\chi}$ has to be carefully studied.

Direct detection of $\xi$ suffers from a suppression factor of at least 0.1. Using the result of
we may conclude that direct detection of $\xi$ in the mass range in question, i.e. $160 \text{ GeV} < m_\xi \sim 220 \text{ GeV}$, does not need to be discussed. As for direct detection of $\tilde{\chi}$ the spin-dependent cross section with the nuclei is much smaller than in the case of the CMSSM, because the higgsino portion of $\tilde{\chi}$ is small in the most of area of the green region of Fig. 4. Instead, the spin-independent cross section with the nuclei is of the same order as in the case of the MSSM. The relatively large coupling of $\tilde{\chi}$ to the inert Higgs sector does not change the cross section with the nuclei.

So our conclusion is that the allowed region in the $m_0 - M_{1/2}$ plane is considerably enlarged, if the inert higgsinos are lighter than $\tilde{\chi}$, although the dominant component ($\gtrsim 90\%$) of dark matter is $\tilde{\chi}$. There is a wide allowed region even above the LHC limit (dashed line in Fig. 4). Direct and indirect searches of dark matter are slightly different compared with the cases of the CMSSM. We will leave the analysis for our future project.

At last we would like to emphasize that the radiative seesaw model of Ma [16] is a two-Higgs-doublet model with a specific structure of the Higgs sector (see e.g. [34, 36, 38] and also [39]), which is intimately related to the neutrino mass and mixing and indirectly to the lepton flavor violations. Furthermore, in supersymmetric case, the Yukawa terms $\lambda^{u(d)} H^{(d(u))} \eta^{u(d)} \phi$ along with the corresponding A-terms will contribute to the radiative correction to the Higgs mass [40] so that the upper bound on the lightest Higgs mass will change. In the light of the recent LHC results [41] the computation of the upper bound is important for the model, although it is not directly related to the problem of dark matter. Therefore, more detailed investigations of the Higgs sector of the present model along this line will be included to our future study.

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[1] H. E. Haber, G. L. Kane, Phys. Rept. 117 (1985) 75-263; S. P. Martin, arXiv:hep-ph/9709356
[2] K. Nakamura et al. (Particle Data Group), Journal of Physics G37, 075021 (2010) and 2011 partial update for the 2012 edition.
[3] H. P. Nilles, Phys. Rept. 110 (1984) 1-162; P. Nath, R. L. Arnowitt and A. H. Chamseddine, “APPLIED N=1 SUPERGRAVITY,” World Scientific, Singapore, 1984; D. J. H. Chung, L. L. Everett, G. L. Kane, S. F. King, J. D. Lykken and L. T. Wang, Phys. Rept. 407 (2005) 1 [arXiv:hep-ph/0312378].
[4] O. Buchmueller, R. Cavanaugh, D. Colling, A. De Roeck, M. J. Dolan, J. R. Ellis, H. Flacher, S. Heinemeyer et al., Eur. Phys. J. C71 (2011) 1722 arXiv:1106.2529 [hep-ph]].
[5] S. Cassel, D. M. Ghilencea, S. Kraml, A. Lessa, G. G. Ross, JHEP 1105 (2011) 120 [arXiv:1101.4664 [hep-ph]].
[6] S. AbdusSalam, B. C. Allanach, H. K. Dreiner, J. Ellis, U. Ellwanger, J. Gunion, S. Heinemeyer, M. Kraemer et al., arXiv:1109.3859 [hep-ph].
[7] G. Aad et al. [ATLAS Collaboration], arXiv:1109.6572 [hep-ex]; A. Collaboration, arXiv:1110.2299 [hep-ex]; ATLAS Collaboration, “ATLAS Supersymmetry (SUSY) searches”, https://twiki.cern.ch/twiki/bin/view/AtlasPublic/SupersymmetryPublicResults.
[8] S. Chatrchyan et al. [ CMS Collaboration ], arXiv:1107.1279 [hep-ex]; CMS Collaboration,
“CMS Supersymmetry Physics Results”,
https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSUS#Publications.

[9] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 192 (2011) 18 [arXiv:1001.4538 [astro-ph.CO]].

[10] K. Griest, Phys. Rev. D 38 (1988) 2357 [Erratum-ibid. D 39 (1989) 3802]; K. Griest, M. Kamionkowski and M. S. Turner, Phys. Rev. D 41 (1990) 3565; K. Griest, D. Seckel, Phys. Rev. D43 (1991) 3191-3203; G. B. Gelmini, P. Gondolo, E. Roulet, Nucl. Phys. B351 (1991) 623-644; M. Drees, M. M. Nojiri, Phys. Rev. D47 (1993) 376-408 [hep-ph/9207234].

[11] G. Jungman, M. Kamionkowski, K. Griest, Phys. Rept. 267 (1996) 195-373 [hep-ph/9506380].

[12] H. Baer, M. Brhlik, Phys. Rev. D53 (1996) 597-605 [hep-ph/9508321]; V. D. Barger, C. Kao, Phys. Rev. D57 (1998) 181-213 [hep-ph/9704403]; J. R. Ellis, T. Falk, K. A. Olive, Phys. Rev. D43 (1991) 3565; K. Griest, D. Seckel, Phys. Rev. D43 (1991) 3191-3203; G. B. Gelmini, P. Gondolo, E. Roulet, Nucl. Phys. B351 (1991) 623-644; M. Drees, M. M. Nojiri, Phys. Rev. D47 (1993) 376-408 [hep-ph/9207234].

[13] E. Ma, Annales Fond. Broglie 31 (2006) 285 [arXiv:hep-ph/0607142].

[14] H. Fukuoka, J. Kubo and D. Suematsu, Phys. Lett. B 678 (2009) 401 [arXiv:0905.2847 [hep-ph]].

[15] E. Ma, Mod. Phys. Lett. A23 (2008) 721 [arXiv:0801.2545 [hep-ph]]; D. Suematsu, T. Toma, Nucl. Phys. B847 (2011) 567 [arXiv:1011.2839 [hep-ph]].

[16] E. Ma, Phys. Rev. D 73 (2006) 073001 [arXiv:hep-ph/0603331]; A. Zee, Phys. Lett. B 565 (2003) 176 [arXiv:hep-ph/0303043]; H. Baer, C. Balazs, A. Belyaev, JHEP 0203 (2002) 042 [hep-ph/0202076]; J. R. Ellis, K. A. Olive, Y. Santos and V. C. Spanos, Phys. Lett. B 565 (2003) 176 [arXiv:hep-ph/0303043]; H. Baer, C. Balazs, JCAP 0305 (2003) 006 [hep-ph/0303114]; A. B. Lahanas, D. V. Nanopoulos, Phys. Lett. B568 (2003) 55-62 [hep-ph/0303130]; U. Chattopadhyay, A. Corsetti, P. Nath, Phys. Rev. D68 (2003) 035005 [hep-ph/0303201].

[17] E. Ma, Phys. Rev. D 73 (2006) 073001 [arXiv:hep-ph/0601225].

[18] L. M. Krauss, S. Nasri, M. Trodden, Phys. Rev. D67 (2003) 085002 [hep-ph/0210389].

[19] T. Hambye, K. Kannike, E. Ma, M. Raidal, Phys. Rev. D75 (2007) 095003 [hep-ph/0609228]; J. Kubo, D. Suematsu, Phys. Lett. B643 (2006) 336-341 [hep-ph/0610006]; Y. Kajiyama, J. Kubo, H. Okada, Phys. Rev. D75 (2007) 033001 [hep-ph/0610072].

[20] E. Ma, U. Sarkar, Phys. Lett. B653 (2007) 288 [arXiv:0705.0074 [hep-ph]]; D. Suematsu, Eur. Phys. J. C56 (2008) 379 [arXiv:0706.2401 [hep-ph]]; E. Ma, Phys. Lett. B662 (2008) 49 [arXiv:0708.3371 [hep-ph]]; K. S. Babu, E. Ma, Int. J. Mod. Phys. A23 (2008) 1813 [arXiv:0708.3790 [hep-ph]]; E. Ma, Phys. Lett. B659 (2008) 885 [arXiv:0710.2325 [hep-ph]].

[21] M. Aoki, S. Kanemura, O. Seto, Phys. Rev. Lett. 102 (2009) 051805 [arXiv:0807.0361 [hep-ph]]; Phys. Rev. D80 (2009) 033007 [arXiv:0904.3829 [hep-ph]].

[22] E. Ma, D. Suematsu, Mod. Phys. Lett. A24 (2009) 583 [arXiv:0809.0442 [hep-ph]]; Q. -H. Cao, E. Ma, G. Shaughnessy, Phys. Lett. B673 (2009) 152-155 [arXiv:0901.1334 [hep-ph]]; S. Andreas, M. H. G. Tytgat, Q. Swillens, JCAP 0904 (2009) 004 [arXiv:0901.1750 [hep-ph]]; X. -J. Bi, P. -H. Gu, T. Li, X. Zhang, JHEP 0904 (2009) 103 [arXiv:0901.0176 [hep-ph]]; D. Suematsu, T. Tama, T. Yoshida, Phys. Rev. D79 (2009) 093004 [arXiv:0903.0287 [hep-ph]]; E. Ma, Phys. Rev. D80 (2009) 013013 [arXiv:0904.4450 [hep-ph]]; Y. Farzan, Phys.
Rev. D80 (2009) 073009 [arXiv:0908.3729 [hep-ph]].

[23] D. Suematsu, T. Toma, T. Yoshida, Phys. Rev. D82 (2010) 013012 [arXiv:1002.3225 [hep-ph]]; W. Chao, Phys. Lett. B695 (2011) 157 [arXiv:1005.1024 [hep-ph]]; M. Aoki, S. Kanemura, T. Shindou, K. Yagyu, JHEP 1007 (2010) 084 [arXiv:1005.5159 [hep-ph]]; S. Kanemura, T. Ota, Phys. Lett. B694 (2010) 233 [arXiv:1009.3845 [hep-ph]].

[24] N. Haba, T. Shindou, Phys. Lett. B701 (2011) 229 [arXiv:1102.3472 [hep-ph]]; T. Araki, Phys. Lett. B704 (2011) 166 [arXiv:1104.1689 [hep-ph]]; W. -F. Chang, C. -F. Wong, arXiv:1104.3934 [hep-ph]; S. Kanemura, T. Nabeshima, H. Sugiyama, Phys. Lett. B703 (2011) 66 [arXiv:1106.2480 [hep-ph]]; M. K. Parida, Phys. Lett. B704 (2011) 206 [arXiv:1108.2753 [hep-ph]]; Y. Kajiyama, H. Okada and T. Toma, arXiv:1109.2722 [hep-ph].

[25] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.

[26] H. Higashi, T. Ishima and D. Suematsu, Int. J. Mod. Phys. A 26 (2011) 995 [arXiv:1101.2704 [hep-ph]]; D. Suematsu, arXiv:1103.0857 [hep-ph].

[27] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155 (1985) 36.

[28] E. Ma, Mod. Phys. Lett. A 21 (2006) 1777 [arXiv:hep-ph/0605180]; K. S. Babu and E. Ma, Int. J. Mod. Phys. A 23 (2008) 1813 [arXiv:0708.3790 [hep-ph]].

[29] J. Kubo, E. Ma and D. Suematsu, Phys. Lett. B 642 (2006) 18 [arXiv:hep-ph/0604114].

[30] D. Aristizabal Sierra, J. Kubo, D. Restrepo, D. Suematsu, O. Zapata, Phys. Rev. D79 (2009) 035001 [arXiv:0808.3340 [hep-ph]]; G. B. Gelmini, E. Osoba, S. Palomares-Ruiz, Phys. Rev. D81 (2010) 063529 [arXiv:0912.2478 [hep-ph]].

[31] M. Aoki, S. Kanemura, K. Yagyu, Phys. Rev. D83 (2011) 075016 [arXiv:1102.3412 [hep-ph]].

[32] R. Barbieri, L. J. Hall and A. Strumia, Nucl. Phys. B 445 (1995) 219 [arXiv:hep-ph/9501334]; J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B 357 (1995) 579 [arXiv:hep-ph/9501407].

[33] G. Belanger, F. Boudjema, P. Brun, A. Pukhov, S. Rosier-Lees, P. Salati and A. Semenov, Comput. Phys. Commun. 182 (2011) 842 [arXiv:1004.1092 [hep-ph]].

[34] R. Barbieri, L. J. Hall and V. S. Rychkov, Phys. Rev. D 74 (2006) 015007 [arXiv:hep-ph/0603188].

[35] C. Boehm, P. Fayet, J. Silk, Phys. Rev. D69 (2004) 101302 [hep-ph/0311143]; Q. H. Cao, E. Ma, J. Wudka and C. P. Yuan, arXiv:0711.3881 [hep-ph]; J. -H. Huh, J. E. Kim, B. Kyae, Phys. Rev. D79 (2009) 063529 [arXiv:0809.2601 [hep-ph]]; K. M. Zurek, Phys. Rev. D79 (2009) 115002 [arXiv:0811.4429 [hep-ph]]; D. Feldman, Z. Liu, P. Nath and G. Peim, Phys. Rev. D 81 (2010) 095017 [arXiv:1004.0649 [hep-ph]].

[36] L. Lopez Honorez, E. Nezri, J. F. Oliver, M. H. G. Tytgat, JCAP 0702 (2007) 028 [hep-ph/0612275]; E. M. Dolle, S. Su, Phys. Rev. D80 (2009) 055012 [arXiv:0906.1609 [hep-ph]].

[37] T. Araki, C. Q. Geng, K. I. Nagao, Phys. Rev. D83 (2011) 075014 [arXiv:1102.4906 [hep-ph]].

[38] Q. H. Cao, E. Ma and G. Rajasekaran, Phys. Rev. D 76 (2007) 095011 [arXiv:0708.2939 [hep-ph]]; E. Lundstrom, M. Gustafsson, J. Edsjo, Phys. Rev. D79 (2009) 035013 [arXiv:0810.3921 [hep-ph]]; E. Dolle, X. Miao, S. Su, B. Thomas, Phys. Rev. D81 (2010) 035003 [arXiv:0909.3044 [hep-ph]].

[39] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, arXiv:1106.0081 [hep-ph].

[40] Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. 85 (1991) 1; H. E. Haber and
R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815; J. R. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B 257 (1991) 83.

[41] CMS Collaboration, “CMS Higgs Physics Results”,
https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsHIG;
ATLAS Collaboration, “Search for neutral MSSM Higgs bosons decaying to tau+ tau- pairs in proton-proton collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector”, ATLAS-CONF-2011-132.
https://twiki.cern.ch/twiki/bin/view/AtlasPublic/HiggsPublicResults.