Application of power theory for electricity metering in presence of distortion

A I Bardanov1,*, T V Pudkova1

1 Saint-Petersburg Mining University, 2, 21st Line of Vasilyevsky Island, St. Petersburg 199106, Russian Federation

* E-mail: Bardanov.AI@pers.spmi.ru

Abstract. At enterprises, electricity metering devices are used everywhere, which incorporate various formulas for calculating power. The division of power into active, reactive and full is valid only in the case of sinusoidal currents and voltages. In the case of non-sinusoidal waveforms of current and voltage, which are most often found in industrial networks of enterprises, it is necessary to use additional concepts and correct formulas for calculating powers in order to avoid errors. The article compares the methods of calculating the power in the non-sinusoidal mode, which can be used to more correctly record electricity in industrial networks of enterprises.

Key words: Reactive power metering, distortion power, nonlinear load modeling, power theory.

1. Introduction

The modern power metering devices contain formulas for calculating power, which do not take into account the influence of higher harmonics. This entails overpayments for reactive power, and therefore requires a more thorough assessment of the method of calculating power.

There are many theories of power calculation in non-sinusoidal mode and there are still lively discussions. The first authors who noted the need to clarify the calculation of the power of steel C.P. Steinmetz (1900) and M. Iliovic (1925). Then, a significant contribution to the development of the theory made C.I. Budeanu (1927) and S. Fryze (1932) [1,2]. Further, the theories of authors who improved these two basic theories became the most widely spread: P.S. Filipski (1980,1984,1994)[3]; A.E. Emanuel (1974-2012)[4]; W. Shepherd and P. Zakikhani (1972)[5]; D. Sharon (1973)[6]; W.J.M. Moore and N.L. Kusters (1980); C.H. Page (1980); H. Akagi and other (1984)[7–9]; M. Depenbrock, D.A. Marshal and J.D. van Wyk (1993) [10,11]; E. W. Kimbark (1995); F.D. Yildirim and W. Fuchs (1999); L.S. Czarnecki (1987-2018)[2,11,12], P. Tenti (2003).

Nowadays, the current electricity quality standard (IEEE. Standard 1459-2010. IEEE Power and Energy Society) takes into account the developments of A.E. Emanuel.

2. Method

In all theories of power, total power is decomposed into various components (Table 1). The authors suggest various ways to calculate them. The standard proposes to use the term “nonactive power”, which is the root of the squares of the difference between the apparent and active powers. In some
methods, nonactive power is equal to reactive, in some it is the sum of different types of powers, for example, reactive power and distortion power. It must be noted that all methods presented above deal with steady-state and don’t take into account transients such as voltage dips or overvoltages [13,14].

**Table 1. Power calculation methods**

| No | Theory authors | Apparent power | Other types of power |
|----|-----------------|----------------|---------------------|
| 1  | C.I. Budeanu    | $S^2 = P^2 + F^2$ | $P = \sum_n U_n I_n \cos \phi_n; Q_B = \sum_n U_n I_n \sin \phi_n$ |
|    |                 | $S^2 = P^2 + Q_B^2 + D^2$ | $D = \sqrt{S^2 - P^2 - Q_B^2}; F = \sqrt{S^2 - P^2} = \sqrt{Q_B^2 + D^2}$ |
|    |                 | $F$ - fictitious/ complementary power; | $D$ - distortion power. |
| 2  | S. Fryze        | $S^2 = P^2 + Q_f^2$ | $P = U I_a = \frac{1}{T} \int_0^T \! \! pdt$ |
|    |                 | $S^2 = (U I_a)^2 + (U I_f)^2$ | $S = U I_a; Q_F = U I_f = U I_F$ |
|    |                 | $Q_{F} = \sqrt{S^2 - P^2} = \sqrt{(U I_a)^2 + (U I_f)^2}$ | $Q_{F} = \sqrt{S^2 - P^2} = \sqrt{(U I_a)^2 + (U I_f)^2}$ |
| 3  | W. Shepherd & P. Zakikhani | $S^2 = S_R^2 + S_Q^2 + S_D^2$ | $S_R$ - apparent active power; |
|    |                 | $S_R = \sum_{h=1}^{n} \sum_{k=1}^{m} u^2_s l^2_z \cdot \cos^2 \phi_n$ | $S_D$ - apparent distortion power. |
|    |                 | $S_Q = \sqrt{ \frac{1}{n} \sum l^2_z \cdot \sin^2 \phi_n}$ | $S_Q = \sqrt{ \frac{1}{n} \sum l^2_z \cdot \sin^2 \phi_n}$ |
|    |                 | $S_D = \sum_{h=1}^{n} \sum_{k=1}^{m} u^2_s l^2_z + \sum_{l=1}^{u} \sum_{k=1}^{m} u^2_s (\sum_{h=1}^{n} l^2_z + \sum_{k=1}^{m} l^2_z)$ | $S_D = \sum_{h=1}^{n} \sum_{k=1}^{m} u^2_s l^2_z + \sum_{l=1}^{u} \sum_{k=1}^{m} u^2_s (\sum_{h=1}^{n} l^2_z + \sum_{k=1}^{m} l^2_z)$ |
| 4  | N.L. Kusters & W.J.W. Moore | $S^2 = P^2 + Q^2$ | $P = U I_p = \frac{1}{T} \int_0^T \! \! pdt$ |
|    |                 | $S^2 = P^2 + Q^2 + Q_{ef}$ | $Q_{ef} = \sum_{h=1}^{n} \sum_{k=1}^{m} u^2_s l^2_z \cdot \cos^2 \phi_n$ |
|    |                 | $S^2 = P^2 + Q^2 + Q_{er}$ | $Q_{er} = \sum_{h=1}^{n} \sum_{k=1}^{m} u^2_s l^2_z \cdot \sin^2 \phi_n$ |
|    |                 | $Q_{C} = U I_{qC} = \frac{U}{U_{der} T} \int_{T}^{T} u_{der} \! \! idt$ | $Q_{C} = U I_{qC} = \frac{U}{U_{der} T} \int_{T}^{T} u_{der} \! \! idt$ |
|    |                 | $Q_{L} = U I_{qL} = \frac{U}{U_{int} T} \int_{T}^{T} u_{int} \! \! idt$ | $Q_{L} = U I_{qL} = \frac{U}{U_{int} T} \int_{T}^{T} u_{int} \! \! idt$ |
|    |                 | $Q_{CR} = \sqrt{S^2 - P^2 - Q_C^2}; Q_{LR} = \sqrt{S^2 - P^2 - Q_L^2}$ | $Q_{CR} = \sqrt{S^2 - P^2 - Q_C^2}; Q_{LR} = \sqrt{S^2 - P^2 - Q_L^2}$ |
|    |                 | $Q = \sum_{n} U_n I_n \sin \phi_n \cdot \sqrt{ \frac{\sum_{n} U_n^2 / \sum_{n} (U_n^2/n^2)}{\sum_{n} U_n^2 / \sum_{n} (U_n^2/n^2)}}$ | $Q = \sum_{n} U_n I_n \sin \phi_n \cdot \sqrt{ \frac{\sum_{n} U_n^2 / \sum_{n} (U_n^2/n^2)}{\sum_{n} U_n^2 / \sum_{n} (U_n^2/n^2)}}$ |
| 5  | D. Sharon       | $S^2 = P^2 + S_Q^2 + S_C^2$ | $P = \frac{1}{T} \int_0^T \! \! pdt$ |
|    |                 | $S_Q = U_n \sum_{h=1}^{n} \sum_{k=1}^{m} u^2_s l^2_z \sin \phi_n; \quad S_C = \sqrt{S^2 - P^2 - S_Q^2}$ | $S_Q = U_n \sum_{h=1}^{n} \sum_{k=1}^{m} u^2_s l^2_z \sin \phi_n; \quad S_C = \sqrt{S^2 - P^2 - S_Q^2}$ |
3. **Mathematical model**

One of the tasks in the construction of mathematical models was to provide automatic change of distributed devices of linear and non-linear loads on switchgear. Simulink MATLAB used two main methods for modeling nonlinear loads: modeling nonlinear loads with a set of current sources and modeling nonlinear loads with a six-pulse diode bridge rectifier with intensity and resistance on the DC side.

For simplicity, the percentage of nonlinear $S_{nl}$ and linear load $S_{ll}$ is calculated so that the following conditions are met:

\[
\begin{align*}
    k_{nl} + k_{ll} &= 1; \\
    S_{nl} &= S \cdot k_{nl}; \\
    S_{ll} &= S \cdot k_{ll},
\end{align*}
\]

Where $D_H$ – distortion power $D_H = \sqrt{S_H^2 + P_h^2}$

\[
\begin{align*}
    P_h &= U_0 I_0 + \sum_{n \neq 1} U_{n} I_{n} \cos \varphi_n = P - P_1
\end{align*}
\]

These calculations were provided in MATLAB Simulink functional block.

| №  | Theory authors | Apparent power | Other types of power |
|----|----------------|----------------|---------------------|
| 6  | E. Kimbark     | $S^2 = P^2 + Q_K^2 + D_K^2$ | $P = \frac{1}{T} \int_0^T p dt; Q_K = U_1 I_1 \sin \varphi_1$ |
|    |                | $Q_K^2 + D_K^2$ – deactive power; $Q_K$ – reactive power; $D_K$ – distortion power. | $D_K = \sqrt{S^2 - P^2 - Q_K^2}$ |
| 7  | IEEE 1459-2010 | $S^2 = S_{ll}^2 + S_{nl}^2$ | $S_{ll} = U_0 I_0; S_{nl} = U_{1} I_{1} \cos \varphi_1; Q_1 = U_{1} I_{1} \sin \varphi_1$ |
|    |                | $S^2 = P^2 + N^2$ | $P_1 = U_{1} I_{1} \cos \varphi_1; Q_1 = U_{1} I_{1} \sin \varphi_1$ |
|    |                | $S_{ll}$ – fundamental apparent power; $S_{nl}$ – non-fundamental apparent power; $S_N$ – harmonic apparent power; | $D_I = U_{I} S_{THD} I; D_U = U_{I} S_{THD} U$ |
|    |                | $N$ – non-active power. | $S_H = S_{ll} + S_{nl} + S_{N} = U_{I} S_{THD} U + S_{N}$ |
|    |                | Where $D_H$ – distortion power $D_H = \sqrt{S_H^2 + P_H^2}$ | $N = \sqrt{S^2 - P^2}; P = P_1 + P_h$ |

In the case of using current sources as a source of distortion, the amplitudes of higher harmonics were calculated by the inverse proportion of the harmonic number. In the case of modeling a nonlinear load with a diode rectifier with active resistance, it was required to adequately select the value of this resistance. For this, it was assumed that the rectifier is not underloaded and not overloaded and the current spectrum of the rectifier contains only odd harmonics with the exception of a multiple of three (Figure 1). The capacitors of the capacitors in the DC links were chosen from the same condition. Such a spectrum of currents is typical for any electrical systems, which include a variable frequency drive [15–17].
The operation of the diode rectifier was modeled at different loads in a given range. According to the simulation results of the active power of the rectifier, the current value of the current was set in accordance with (Figure 2). It was found that the current is related to the power of the rectifier by a second-order polynomial. The coefficients of the polynomial are also shown in Figure 2.

Thus, by means of the revealed dependence, it was possible to relate the magnitude of the effective value of the current consumed by the active rectifier and the magnitude of the active resistance connected to it, which made it possible to automate the work of the model. The mathematical description of such dependencies is widely used in scientific practice [18,19].

The active and reactive impedances of power transmission lines and transformer parameters are calculated for a sinusoidal mode. The measurement unit is installed on the input of the enterprise. All parameters were also chosen for a sinusoidal symmetric mode. The model of a three-phase distribution network of the enterprise is presented in Figure 3.
The simulation was performed for a different share of non-linear load, from 0 to 100% in 10% increments. The chosen methods for representing non-linear loads allowed us to fully automate the process of calculating model parameters.

4. Simulation results
The simulation showed that the authors' input powers, such as nonactive power and distortion power, are basically the same. In some methods (Sharon, Fryze, Shepard and Zakikhani) nonactive power completely coincides with reactive power.

In the methods that provide for distortion power, as the load non-linearity increases, the distortion power increases in direct proportion, which adequately satisfies the modeling conditions.

Using the example of reactive power, one can also see the similarity of a number of methods. Thus, for example, a noticeable failure at average values of the relative nonlinearity is observed both in the Fryze method and in the methods of Sharon, Shepard and Zakihani (first group of lines on Figure 4). The direct dependence of the reactive power on the non-linearity of the load is observed in Budeanu, Kimbark, Kasters and Moore, including in the method proposed by IEEE standard (second group of lines on Figure 4).
Nonactive power has an inverse linear dependence on relative non-linearity in the Custers and Moore and Kimbark method. In other methods of calculation, nonactive power has a pronounced minimum (Figure 5).

![Figure 5. Nonactive powers.](image)

The total power of all methods is calculated in the same way, but its value is somewhat lower for Caster's and Moore's than in other cases.

It can also be noted that the complementary reactive power $S_c$ (Sharon) has no analogues in the IEEE standard.

**Conclusion**

Summarizing the above, when commercially accounting for electricity, it makes sense to abandon the concept of reactive power and use the concept of nonactive power. This means to deal with currents orthogonal components RMS values that are necessary connected with power loses.

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