Gauge mediated supersymmetry breaking without exotics in orbifold compactification

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Abstract

We suggest SU(5)$'$ in the hidden sector toward a possible gauge mediated supersymmetry breaking scenario for removing the SUSY flavor problem, with an example constructed in $\mathbb{Z}_{12-I}$ with three families. The example we present has the Pati-Salam type classification of particles in the observable sector and has no exotics at low energy. We point out that six or seven very light pairs of $5'$ and $\bar{5}'$ out of ten vectorlike $5'$ and $\bar{5}'$ pairs of SU(5)$'$ is achievable, leading to a possibility of an unstable supersymmetry breaking vacuum. The possibility of different compactification radii of three two tori toward achieving the needed coupling strength is also suggested.

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I. INTRODUCTION

The gauge mediated supersymmetry breaking (GMSB) has been proposed toward removing the SUSY flavor problem [1]. However, there has not appeared yet any satisfactory GMSB model from superstring compactification, satisfying all phenomenological constraints.

The GMSB relies on dynamical supersymmetry breaking [2]. The well-known GMSB models are an SO(10)’ model with 16’ or 16’ + 10’ [3], and an SU(5)’ model with 10’ + 5’ [4]. If we consider a metastable vacuum also, a SUSY QCD type is possible in SU(5)’ with six or seven flavors, satisfying $N_c + 1 \leq N_f < \frac{3}{2}N_c$ [5]. Three family standard models (SMs) with this kind of hidden sector are rare. In this regard, we note that the flipped SU(5) model of Ref. [6] has one 16’ and one 10’ of SO(10)’, which therefore can lead to a GMSB model. But as it stands, the confining scale of SO(10)’ is near the GUT scale and one has to break the group SO(10)’ by vacuum expectation values of 10’ and/or 16’. Then, we do not obtain the spectrum needed for a GMSB scenario and go back to the gaugino condensation idea. If the hidden sector gauge group is smaller than SU(5)’, then it is not known which representation necessarily leads to SUSY breaking. The main problem in realizing a GMSB model is the difficulty of obtaining the supersymmetry (SUSY) breaking confining group with appropriate representations in the hidden sector while obtaining a supersymmetric standard model (SSM) with at least three families of the SM in the observable sector.

In this paper, we would like to address the GMSB in the orbifold compactification of the $E_8 \times E_8'$ heterotic string with three families at low energy. A typical recent example for the GMSB is

$$W = m\overline{Q}Q + \frac{\lambda}{M_{Pl}} Q\overline{Q}f\bar{f} + Mf\bar{f}$$

where $Q$ is a hidden sector quark and $f$ is a messenger. Before Intriligator, Seiberg and Shih (ISS) [5], the GMSB problem has been studied in string models [7]. After [5] due to opening of new possibilities, the GMSB study has exploded considerably and it is known that the above idea is easily implementable in the ISS type models [8]. Here, we will pay attention to the SUSY breaking sector, not discussing the messenger sector explicitly. The messenger sector $\{f, \cdots\}$ can be usually incorporated, using some recent ideas of [8], since there appear many heavy charged particles at the GUT scale from string compactifications. The three family condition works as a strong constraint in the search of the hidden sector representations.
In addition, the GUT scale problem that the GUT scale is somewhat lower than the string scale is analyzed in connection with the GMSB. Toward the GUT scale problem, we attempt to introduce \textit{two scales of compactification in the orbifold geometry}. In this setup, we discuss physics related to the hidden sector, in particular the hidden sector confining scale related to the GMSB. If the GMSB scale is of order $10^{13}$ GeV, then the SUSY breaking contributions from the gravity mediation and gauge mediation are of the same order and the SUSY flavor problem remains unsolved. To solve the SUSY flavor problem by the GMSB, we require two conditions: one is the \textit{relatively low hidden sector confining scale} ($< 10^{12}$ GeV) and the other is the \textit{matter spectrum allowing SUSY breaking}.

Toward this kind of GMSB, at the GUT scale we naively expect a \textit{smaller coupling constant for a relatively big hidden sector nonabelian gauge group} (such as $SU(5)'$ or $SO(10)'$) than the coupling constant of the observable sector. But this may not be needed always.

The radii of three two tori can be different in principle as depicted in Fig. 1. For simplicity, we assume the same radius $r$ for (12)- and (56)- tori. A much larger radius $R$ is assumed for the second (34)-torus. For the scale much larger than $R$, we have a 4D theory. In this case, we have four distance scales, $R, r, \alpha' = M_s^{-2}$, and $\kappa = M_P^{-1}$, where $\alpha'$ is the string tension and $M_P$ is the reduced Planck mass. The Planck mass is related to the compactification scales by $M_P^2 \propto M_s^8 r^4 R^2$. Assuming that strings are placed in the compactified volume, we have a hierarchy $\frac{1}{R} < \frac{1}{r} < M_s < M_P$. The customary definition of the GUT scale, $M_{\text{GUT}}$, is the unification scale of the QCD and electroweak couplings.

For the 4D calculation of the unification of gauge couplings to make sense, we assume that the GUT scale is below the compactification scale $\frac{1}{R}$, leading to the following hierarchy

$$M_{\text{GUT}} \leq \frac{1}{R} \leq \frac{1}{r} < M_s, M_P$$

where we have not specified the hierarchy between $M_s$ and $M_P$.

In Sec. II, we discuss phenomenological requirements in the GMSB scenario toward the
SUSY flavor problem. In Sec. III we present a $\mathbb{Z}_{12-I}$ example. In Sec. IV we discuss the hidden sector gauge group $SU(5)'$ where a GMSB spectrum is possible.

II. SUSY FCNC CONDITIONS AND GAUGE MEDIATION

The MSSM spectrum between the SUSY breaking and GUT scales fixes the unification coupling constant $\alpha_{\text{GUT}}$ of the observable sector at around $\frac{1}{25}$. If a complete $SU(5)$ multiplet in the observable sector is added, the unification is still achieved but the unification coupling constant will become larger. Here, we choose the unification coupling constant in the range $\alpha_{\text{GUT}} \sim \frac{1}{30} - \frac{1}{20}$.

The GMSB scenario has been adopted to hide the gravity mediation below the GMSB effects so that SUSY breaking need not introduce large flavor changing neutral currents (FCNC) [1]:

$$\frac{\Lambda_h^2}{M_P^2} \leq 10^{-3} \text{ TeV} \Rightarrow \Lambda_h \leq 2 \times 10^{12} \text{ GeV}$$

(2)

$$\frac{(\xi\Lambda_h)^2}{M_X} \sim 10^3 \text{ GeV}$$

(3)

where $M_P$ is the reduced Planck mass $2.44 \times 10^{18} \text{ GeV}$, $M_X$ is the effective messenger scale (including coupling constants) in the GMSB scenario, $M_X \geq \frac{1}{2} \times 10^6 \text{ GeV}$ for acceptable FCNC effects, and $\xi$ measures the hidden sector squark condensation scale compared to the hidden sector confining scale. So, a possible range of $\Lambda_h$ is $\Lambda_h = [0.7 \times 10^5 \xi^{-1} \text{ GeV}, 2 \times 10^{12} \text{ GeV}]$. Because of the SUSY breaking scale fixed at TeV, the messenger scale $M_X$ is a function of $\Lambda_h$. These conditions on the confining scale of the hidden sector fix the strength of the hidden sector unification coupling constant $\alpha_{\text{GUT}}^h$. The GUT scale coupling constant is related to the coupling at scale $\mu$, at one loop order, by

$$\frac{1}{\alpha_{\text{GUT}}^h} = \frac{1}{\alpha_j^h(\mu)} + \frac{-b_j}{2\pi} \ln \left| \frac{M_{\text{GUT}}^h}{\mu} \right|.$$  

(4)

Now the expression (4) is used to give constraint on $\alpha_{\text{GUT}}^h$. Defining the inverse of unification coupling constants as

$$A = \frac{1}{\alpha_{\text{GUT}}}, \ A' = \frac{1}{\alpha_{\text{GUT}}^h},$$

(5)
FIG. 2: Constraints on $A'$. The confining scale is defined as the scale $\mu$ where $\alpha_h^b(\mu) = 1$. Using $\xi = 0.1, M_X = 2 \times 10^{16}$ GeV in the upper bound region and $\xi = 0.1, M_X = \frac{1}{2} \times 10^6$ GeV in the lower bound region, we obtain the region bounded by dashed vertical lines. Thick dash curves are for $-b_h^j = 5$ and 9.

we express $A'$ in terms of the scale $\Lambda_h$ as\(^1\)

$$A' - 1 = -\frac{b_h^j}{2\pi} \ln \left( \frac{M_{\text{GUT}}^h}{\Lambda_h} \right). \quad (6)$$

If $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV and $\Lambda_h \simeq 2 \times 10^{10}$ GeV, we obtain $A'$ in terms of $-b_h^j$ as shown in Eq. (7).

$$
\begin{array}{cccccccc}
-b_h^j & A' & -b_h^j & A' & -b_h^j & A' & -b_h^j & A' \\
2 & 5.4 & 4 & 9.8 & 6 & 14.2 & 8 & 18.6 & 10 & 23.0 & 12 & 27.4 & 14 & 31.8 & 16 & 36.2 & 18 & 40.6 & 20 & 45.0
\end{array} \quad (7)
$$

In Fig. 2 we present figures of $A'$ versus $\Lambda_h$ for several values of $-b_h^j$.

The GMSB relies on dynamical supersymmetry breaking (DSB) [2]. The well-known DSB models are an SO(10)$'$ model with $16'$ or $16' + 10'$, and an SU(5)$'$ model with $10' + 5'$. If we consider a metastable vacuum, a SUSY QCD type is possible in SU(5)$'$ with six or

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\(^1\) One can determine $\Lambda_h$ where $\alpha_h = \infty$ for which near $\Lambda_h$ the one loop estimation is not valid. So we estimate $\Lambda_h$ at $\alpha_h = 1$. 

several flavors, $6 (5' + 5')$ or $7 (5' + 5')$. The reason that we have this narrow band of $N_f$ is that the theory must be infrared free in a controllable way in the magnetic phase. Three family models with $\alpha' < \frac{1}{25}$ are very rare, and we may allow at most up to 20% deviation from $\alpha_{GUT}$ value, i.e. $\alpha' > \frac{1}{30}$. Then, from Fig. 2 we note that it is almost impossible to have an SO(10)$'$ model from superstring toward the GMSB. The reason is that SO(10)$'$ matter representations from superstring are not big and hence $-b_j = 24 - \sum_i l(R_i)$ seems very large. The flipped SU(5) model of Ref. 6 has one 16$'$ and one 10$'$ of SO(10)$'$ with $-b_{SO(10)}^h = 21$, which can lead to a GMSB if the hidden sector coupling at the GUT scale is very small, $\alpha_{GUT}^h < \frac{1}{33}$. On the other hand, SU(5) models can have many possibilities with $-b_{SU(5)}^h = 15 - N_f$. The SU(5) model with seven flavors gives $-b_{SU(5)}^h = 8$, which allows a wide range of $\Lambda_h$. It is even possible to have $\alpha_{GUT}^h = \alpha_{GUT} \simeq \frac{1}{25}$ for $\Lambda_h \sim 3 \times 10^7$ GeV with the messenger scale $M_X$ around $10^{12}$ GeV. Bigger SU($N$)$'$ groups with $N > 5$ are also possible for the ISS scenario, but it is difficult to obtain many flavors of SU($N$)$'$ in orbifold compactification. Most orbifold models have chiral fields at the order of 200 fields (among which many are singlets) and if we go to large SU($N$)$'$ groups it is more difficult to obtain a large number of SU($N$)$'$ flavors with the required three families of quarks and leptons.

The ISS type models are possible for SO($N_c$) and Sp($N_c$) groups also 5. In this paper, however we restrict our study to the SU(5)$'$ hidden sector only. We just point out that SO($N_c$) groups, with the infrared free condition in the magnetic phase for $N_f < \frac{5}{2}(N_c - 2)$, are also very interesting toward the unstable vacua, but the study of the phase structure here is more involved. On the other hand, we do not obtain Sp($N_c$) groups from orbifold compactification of the hidden sector $E_8$.

III. A $Z_{12-1}$ MODEL

We illustrate an SSM from $Z_{12-1}$. The twist vector in the six dimensional (6d) internal space is

$$Z_{12-1} \text{ shift : } \phi = (\frac{5}{12}, \frac{4}{12}, \frac{1}{12}).$$

The compactification radius of (12)- and (56)-tori is $r$ and the compactification radius of (34)-torus is $R$, with a hierarchy of radii $r \ll R$.

We obtain the 4D gauge group by considering massless conditions satisfying $P \cdot V = 0$
and \( P \cdot a_3 = 0 \) in the untwisted sector \([9]\). This gauge group is also obtained by considering the common intersection of gauge groups obtained at each fixed point.

We embed the discrete action \( \mathbf{Z}_{12-I} \) in the \( E_8 \times E'_8 \) space in terms of the shift vector \( V \) and the Wilson line \( a_3 \) as

\[
V = \frac{1}{{\tau}} (2 2 2 4 4 1 3 6)(3 3 3 3 3 1 1 1)'
\]
\[
a_3 = \frac{1}{{q}} (0 0 0 0 0 0 0 0)(0 0 0 0 0 2 -1 -1)'.
\]

(a) Gauge group: The 4D gauge groups are obtained by \( P^2 = 2 \) vectors satisfying \( P \cdot V = 0 \) and \( P \cdot a_3 = 0 \) mod integer,

\[
SU(4) \times SU(2)_W \times SU(2)_V \times SU(2)_n \times U(1)_a \times U(1)_b
\]
\[
\times [SU(5) \times SU(3) \times U(1)^2]'.
\]

The simple roots of \( SU(4) \), \( SU(2)_W \), \( SU(2)_V \), and \( SU(2)_n \) are

\[
SU(4) : \begin{cases} 
\alpha_1 = (0 1 -1 0 0 ; 0 0 0) \\
\alpha_2 = (\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} ; \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}) \\
\alpha_3 = (\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} ; \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2})
\end{cases}
\]
\[
SU(2)_W : \alpha_W = (0 0 0 1 -1; 0 0 0)
\]
\[
SU(2)_V : \alpha_V = (\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} ; \frac{1}{2} \frac{1}{2} \frac{1}{2})
\]
\[
SU(2)_n : \alpha_n = (\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} ; \frac{1}{2} \frac{1}{2} \frac{1}{2})
\]

The \( SU(2)_V \) is like \( SU(2)_R \) in the Pati-Salam (PS) model \([11]\). The gauge group \( SU(4) \) will be broken by the vacuum expectation value (VEV) of the neutral singlet in the PS model. In the PS model, the hypercharge direction is

\[
Y = \tau_3 + Y_4 + Y'
\]

where \( \tau_3 \) is the third \( SU(2)_V \) generator, \( Y_4 \) is an \( SU(4) \) generator, e.g. for \( \mathbf{4} \),

\[
Y_4 = \text{diag.}(\frac{1}{6} \frac{1}{6} \frac{1}{6} -\frac{1}{2}),
\]

\[\text{(17)}\]

\[\text{Another interesting standard model from } \mathbf{Z}_{12-I} \text{ can be found in } [10].\]

\[\text{We will use the representations } \mathbf{4}, \overline{\mathbf{4}} \text{ and } \mathbf{6} \text{ of } SU(4) \text{ as the complex conjugated ones obtained from Eq. (12) but still keep the } U(1) \text{ charges so that } t, b, e, \text{ etc. are shown instead of } t^c, b^c, e^c, \text{ etc.}\]
and $Y'$ is a hidden-sector $E'_8$ generator. We find that exotics cannot be made vectorlike if we do not include $Y'$. We succeed in making the model exotics-free by choosing $Y'$ as

$$Y' = (0^8)(\begin{array}{cccccc} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{array})'. \tag{18}$$

Note that $SU(2)_V$ doublet components have the unit hypercharge difference. Two $U(1)$ charges of $E_8$ are obtained by taking scalar products with

$$Q_a \rightarrow (0 \ 0 \ 0 \ 0 \ 1 \ -1 \ 0) \tag{19}$$
$$Q_b \rightarrow (1 \ 1 \ 1 \ -1 \ -1 \ 1 \ -3). \tag{20}$$

(b) Matter representations: Now there is a standard method to obtain the massless spectrum in $Z_{12-I}$ orbifold models. The spectra in the untwisted sectors $U_1, U_2,$ and $U_3$, and twisted sectors, $T_{10}, T_{20}, T_{30}, T_{40}, T_{50},$ and $T_{60}$, are easily obtained \[10\]. The representations are denoted as

$$[SU(4), SU(2)_W, SU(2)_V; SU(2)_n; SU(5)', SU(3)'], \tag{21}$$

and for obvious cases we use the standard PS notation

$$(SU(4), SU(2)_W, SU(2)_V)_{Y'}. \tag{22}$$
We list all matter fields below,

\[ U_1 : \begin{pmatrix} 4,2,1 \end{pmatrix}_0, \ 2(6,1,1)_0 \]
\[ U_2 : \begin{pmatrix} 4,1,2 \end{pmatrix}_0, \ (6,1,1)_0 \]
\[ U_3 : \begin{pmatrix} 4,1,2 \end{pmatrix}_0, \ 2(1,2,2)_0, \ (1,1,1;2;1,1)_0 \]
\[ T_{10} : \begin{pmatrix} 4,1,1 \end{pmatrix}_{1/2}, \ (1,2,1)_{1/2}, \ (1,1,2)_{1/2} \]
\[ T_{1_+} : (1,2,1)_{-1/2}, \ (1,1,2)_{-1/2} \]
\[ T_{1_-} : (1,1,2;1;5';1)_{-1/10} \]
\[ T_2_0 : (6,1,1)_0, \ 2'_{n}, \ 1_0 \]
\[ T_{2\pm} : 5'_{2/5}, \ 3'_{0}, \ (1,2,1)_{0}, \ 2'_{0}, \ 2 \cdot 1_0 \]
\[ T_3 : \begin{pmatrix} 4,1,1 \end{pmatrix}_{1/2}, \ (4,1,1)_{-1/2}, \ (4,1,1)_{1/2}, \ 2(4,1,1)_{-1/2}, \ 3(1,2,1)_{1/2}, \ 2(1,2,1)_{-1/2}, \ 2(1,1,2;2;1;1)_{1/2}, \ (1,1,2;2;1;1)_{-1/2}, \ (1,2,1;5';1)_{-1/10}, \ 2 \cdot (1,2,1;5';1)_{1/10} \]
\[ T_{4_0} : 2(1,1,1;2;1;3')_0, \ 2 \cdot 3'_0 \]
\[ T_{4_+} : 2(4,1,2)_0, \ 2(4,1,2)_0, \ 2(6,1,1)_0, \ 7 \cdot 2'_{n}, \ 9 \cdot 1_0 \]
\[ T_{4_-} : 2(1,1,1;2;1;3')_0, \ 2 \cdot 3'_0 \]
\[ T_{7_+} : \begin{pmatrix} 4,1,1 \end{pmatrix}_{1/2}, \ (1,1,2)_{1/2} \]
\[ T_{7_-} : \begin{pmatrix} 4,1,1 \end{pmatrix}_{-1/2}, \ (1,1,2;2;1;1)_{-1/2}, \ (1,1,2)_{-1/2} \]
\[ T_6 : 6 \cdot 3'_{-2/5}, \ 5 \cdot 5'_{2/5}, \]

where \( 1 = (1,1,1;1;1,1), 2' = (1,1,1;2;1,1), 3' = (1,1,1;1;1;3') \) and \( 3' = (1,1,1;1;1;3') \). In the model, there does not appear any exotics. All SU(5)' singlet fields carry the standard charges, i.e. quarks with \( Q_{em} = \frac{2}{3}, -\frac{1}{3} \) and leptons and Higgs with \( Q_{em} = 0, \pm 1 \). The real representation \( 6 \) of SU(4) carries \( Q_{em} = -\frac{1}{3} \) for \( 3 \) and \( Q_{em} = \frac{1}{3} \) for \( 3' \).

Thus, this model is exotics free. The classification of the particles is along Pati-Salam, but it is not the Pati-Salam model since it is not symmetric under SU(2)_W \leftrightarrow SU(2)_Y. In addition, the hypercharge \( Y' \) belongs to \( E_6 \) and hence SU(4) × SU(2)_W × SU(2)_Y × U(1)_Y cannot belong to an SO(10). The SU(5)' singlet fields do not have any SU(3)_c × SU(2)_W × U(1)_Y gauge anomaly. For example, six lepton doublets \( \tilde{l}_{1/2} \) from \( U_1, T_3 \) and \( T_{4_+} \) and three anti-doublets \( \tilde{l}_{-1/2} \) from \( T_{1_+} \) and \( T_3 \), lead to lepton doublets of three families. The charge \( \pm 1 \) leptons \( (e^\pm) \) appear as twelve \( e^- \) from \( 2U_2, 1U_3, 1T_{1_+}, 3T_3, 2T_{4_+}, 3T_{5_-} \) and nine \( e^+ \) from

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4 We found another exotics free model by including \( Y' \) in the hypercharge \( Y' \).
| $P + [4V + 4a]$ | $\chi$ | No.×(Repts.)$_{Y,Q_1,Q_2}$ | PS rep. | Label |
|----------------|--------|-------------------------|--------|-------|
| $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)_{U_1}$ | $L$ | $(\bar{3}, 2, 1; 1, 1, 1)_{L_{-1/6,1,-2}}^L$ | $(4, 2, 1)_0$ | $\tilde{q}_3$ |
| $0 0 0 0 0 0 0 -1)_{U_2}$ | $L$ | $(1, 2, 1; 1, 1, 1)_{L_1/2,0,4}$ | $(\bar{3}, 2, 1)_0$ | $\tilde{t}_3$ |
| $(-1)_{U_2}$ | $L$ | $(3, 1, 1; 1, 1, 1)_{L_{2/3,0,2}}^L$ | $(4, 1, 2)_0$ | $t$ |
| $(-1)_{U_2}$ | $L$ | $(1, 1, 1; 1, 1, 1)_{L_{1,-1/2}}^L$ | $(4, 1, 2)_0$ | $b$ |
| $(0 0 0 0 0 1 0)_{U_2}$ | $L$ | $(1, 1, 1; 1, 1, 1)_{L_{0,0,2}}^L$ | $(4, 1, 2)_0$ | $\tau$ |
| $(0 0 0 0 0 0 0)_{U_2}$ | $L$ | $(3, 1, 1; 1, 1, 1)_{L_{2/3,0,2}}^L$ | $(4, 1, 2)_0$ | $\nu_0$ |
| $(-1)_{U_2}$ | $L$ | $(3, 1, 1; 1, 1, 1)_{L_{1,-1/2}}^L$ | $(4, 1, 2)_0$ | $(c)$ |
| $(0 0 0 0 0 1 0)_{U_2}$ | $L$ | $(1, 1, 1; 1, 1, 1)_{L_{1,0,2}}^L$ | $(4, 1, 2)_0$ | $(\mu)$ |
| $(0 0 0 0 1 0 0)_{U_3}$ | $L$ | $(1, 2, 1; 1, 1, 1)_{L_{1,0,-1}}^L$ | $(1, 2, 2)_0$ | $H_u$ |
| $(0 0 0 0 0 1 0 0)_{U_3}$ | $L$ | $(1, 2, 1; 1, 1, 1)_{L_{1,-2,-1}}^L$ | $(1, 2, 2)_0$ | $H_d$ |
| $(-1)_{U_3}$ | $L$ | $(3, 1, 1; 1, 1, 1)_{L_{1,0,-1}}^L$ | $(1, 2, 2)_0$ | $H_u$ |
| $(0 0 0 0 1 0 0)_{U_3}$ | $L$ | $(1, 2, 1; 1, 1, 1)_{L_{1,-2,-1}}^L$ | $(1, 2, 2)_0$ | $H_d$ |
| $(0 0 0 0 0 0 0 0)_{U_4}$ | $L$ | $(3, 1, 1; 1, 1, 1)_{L_{1,0,-1}}^L$ | $(1, 2, 2)_0$ | $H_u$ |
| $(0 0 0 0 0 0 0 0)_{U_4}$ | $L$ | $(1, 2, 1; 1, 1, 1)_{L_{1,-2,-1}}^L$ | $(1, 2, 2)_0$ | $H_d$ |

**TABLE I**: Some conventionally charged massless states in $U$ and $T_{4+}$. Out of four $Q_{em} = \frac{2}{3}$ quarks (and $-\frac{1}{3}$ quarks and $-1$ leptons) of this table, only three combinations form families, i.e. one combination from bracketed ones. The VEVs of $\nu_0$s break SU(4) down to SU(3)$_c$.

$2T_{10}, 5T_3, 2T_{5+}$, and three $e^-$s are left. Thus, these leptons do not have the SM gauge anomaly. If composite leptons are made from $5'$ and $\tilde{5}'$, they must be anomaly free by themselves.

As shown in Table II, the model has three families of the SSM, one in the untwisted sector and two in the twisted sector. Breaking of SU(4) down to SU(3)$_c$ is achieved by VEVs of neutral components in $(4, 1, 1)_{1/2} \equiv V_1, (4, 1, 2)_0 \equiv V_2, (4, 1, 1)_{-1/2} \equiv \overline{V}_1, (1, 1, 2)_{1/2} \equiv v$ and $(1, 1, 2)_{-1/2} \equiv \overline{v}$. A SUSY $D$-flat direction at the GUT scale requires $V_1^2 + V_2^2 = \overline{V}_1^2, v^2 =$
\( V_2^2 + \overline{v}^2, \) and \( V_1^2 + v^2 = \overline{V}_1^2 + \bar{v}^2. \) Certainly, these conditions can be satisfied. At this point, we are content merely with having three SSM families without exotics, and let us proceed to discuss SUSY breaking via the GMSB scenario, using the hidden sector SU(5)'.

IV. HIDDEN SECTOR SU(5)'

As shown in Table II, there are ten \( 5' \)'s and ten \( \overline{5}' \)'s. But some of these obtain masses by Yukawa couplings. The H-momenta of the fields from the sector \( s \) are \([10, 12, 13]\)

\[
U_1 : (-1, 0, 0), \quad U_2 : (0, 1, 0), \quad U_3 : (0, 0, 1), \\
T_1 : (-\frac{1}{12}, \frac{4}{12}, \frac{1}{12}), \quad T_2 : (\frac{1}{6}, \frac{4}{6}, \frac{1}{6}), \quad T_3 : (-\frac{3}{4}, 0, \frac{1}{4}), \\
T_4 : (-\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), \quad \{ T_5 : (\frac{1}{12}, \frac{4}{12}, \frac{7}{12}) \}, \quad T_6 : (-\frac{1}{2}, 0, \frac{1}{2}), \\
T_7 : (-\frac{1}{12}, \frac{4}{12}, \frac{7}{12}), \quad T_9 : (-\frac{1}{3}, 0, \frac{2}{3}).
\] (24)

Therefore, from the H-momentum rule alone, the cubic Yukawa couplings \( T_3 T_9 U_2 \) and \( T_6 T_9 U_2 \) are expected for \( 5' \)'s and \( \overline{5}' \)'s appearing in \( T_3, T_9 \), and \( T_6 \), if they make the total H-momentum \((-1, 1, 1) \mod (12, 3, 12)\). However, the gauge symmetry forbids them at the cubic level. But we expect that the Yukawa couplings appear at higher orders. For example, to make \( H = (-1, 1, 1) \) we can multiply \( T_3 T_9 \) or \( T_6 T_9 \) times

\[
(4, 1, 2)^{(T_2)}_0 (\overline{4}, 1, 1)^{(T_7)}_{-1/2} (1, 1, 2)^{(T_{10})}_{1/2} (T_4^0 T_4^0 T_4^0)^{11} \quad (25)
\]

where \( T_{4^+} = 1_0 \) and \( T_{4^0} = \overline{3}_0 \) and \( T_{4^0} T_{4^0} T_{4^0} = \epsilon^{\alpha \beta \gamma} \overline{3}_0 \times \overline{3}_0 \times \overline{3}_0 \). Every field in the above has neutral components which can develop a large VEV.

Out of ten SU(5)’ quarks, there may result any number of very light ones according to the choice of the vacuum. A complete study is very complicated and here we just mention that it is possible to have six or seven light SU(5)’ quarks out of ten. The point is that we have enough SU(5)’ quarks. For example, one may choose the \( T_3 T_9 \) coupling such that one pair of SU(2)_W doublets (two SU(5)’ quarks) becomes heavy with a mass scale of \( m_1 \). For the sake of a concrete discussion, presumably by fine-tuning at the moment, one may

\footnote{Details of the rules for \( Z_{12-I} \) are given in [10, 10].}
TABLE II: Hidden sector SU(5)’ representations. We picked up the left-handed chirality only from $T_1$ to $T_{11}$ representations.

consider the $T_6 T_6$ coupling such that the following $5’ \cdot \Phi$ mass matrix form

$$
\begin{pmatrix}
    m_1 & m_1 & 0 & 0 & 0 \\
    m_1 & m_1 & 0 & 0 & 0 \\
    0 & 0 & m_2 & m_2 & m_3 \\
    0 & 0 & m_2 & m_2 & m_3 \\
    0 & 0 & m_2 & m_2 & m_3 \\
\end{pmatrix}
$$

(26)

where 0 entries are due to the $U(1)_a$ charge consideration. If so, out of five $5’$s and six $\Phi$’s from $T_6$ three $5’$s and four $\Phi$’s remain massless, one pair of $5’$ and $\Phi$ obtain mass $2m_1$ and another pair obtain mass $3m_2$ if $m_3 = 0$. Thus, the mass pattern of the total ten flavors of SU(5)’ hidden sector quarks of Table II will be six light SU(5)’ quarks and four massive SU(5)’ quarks. Choosing a different vacuum, another set of massless SU(5)’ quarks would be obtained. In this consideration, the location of fields at fixed points and the permutation symmetries must be considered. For example, the $T_6$ sector being basically $Z_2$ in the (12)- and (56)-tori has four fixed points in the (12)- and (56)-tori. These may be classified by the permutation symmetry $S_4$. The $S_4$ representations are $1, 1’, 2, 3$ and $3’$. The four fixed points can be split into $3 + 1$ or to $2 + 1 + 1’$. The combination of (12)- and (56)-tori can have $3 \otimes 3 = 3 \oplus 3’ \oplus 2 \oplus 1$. Thus, the $T_6$ sectors can contain $1, 2, 3$, and $(3 + 1)$ representations. The lower right block of Eq. (26) indicates $3$ representation
for $5'$ and $3 + 1$ representation for $\bar{5}'$. Assuming an $S_4$ singlet vacuum for Eq. (25), we have nonvanishing $m_2$ terms but vanishing $m_3$. Anyway, this illustrates that the number of light SU(5)$'$ quarks are determined by the choice of the vacuum. Thus, it is possible to find a six or seven flavor model of $[5]$. The magnetic phase of the six flavor model does not have a magnetic gauge group and we must consider Yukawa couplings only which lead to an infrared free theory. The magnetic phase of the seven flavor model has the SU(2) magnetic gauge group but its beta function is positive and the magnetic phase is again infrared free. Thus, the conclusion on SUSY breaking studied in the magnetic phase is the desired low energy phenomenon. In this sense, our model has an ingredient for the GMSB. Suppose, we have the mass pattern of (26). If $m_{1,2}$ is near the SU(5)$'$ confining scale, we consider a ten flavor model down to near the SU(5)$'$ confining scale. So if $m_{1,2}$ are near the SU(5)$'$ confining scale, some heavy flavors are effectively removed to be close to a six or seven flavor model and a SUSY breaking unstable minimum might be a possibility. So we speculate that in the region $m_{1,2} > \Lambda_h$ an unstable minimum is a possibility. At the unstable minimum, SU(2)$_W$ is not broken by hidden sector squark condensates because their values are vanishing [5]. For $m_{1,2} \ll \Lambda_h$, an unstable minimum is not obtained [5]. Note that the unification of $\alpha_c$ and $\alpha_W$ is not automatically achieved as in GUTs because light $(1, 2, 1; 1; \bar{5}', 1)_{1/10}$ quarks do not form a complete representation of a GUT group such as SU(5). Unification condition must be achieved by mass parameters of the fields surviving below the GUT scale, and the condition depicted in Fig. 2 must be changed accordingly. But we use Fig. 2 below just for an illustration.

When SU(5)$'$ confines, there would appear SU(5)$'$ singlet superfields, satisfying the global (including gauge) symmetries. Since the remaining six light pairs of $5'$ and $\bar{5}'$ with the pattern (26) carry SU(2)$_W$, SU(2)$_V$ and $Y$ quantum numbers, the composites are formed such that the anomalies of SU(2)$_W \times$SU(2)$_V \times U(1)_Y$ cancel because we know already that SU(5)$'$ singlet fields of Eq. (23) do not carry the SM gauge group anomalies. The remaining six light pairs of $5'$ and $\bar{5}'$ fields are symmetric under the interchange SU(2)$_W \leftrightarrow$ SU(2)$_V$, and certainly the composite leptons will satisfy this symmetry property. Thus, there is no SM gauge anomaly. In addition, the composite leptons are standard, i.e. they do not carry exotic

But our model is not free from SU(2)$_W \times U(1)_Y$ breaking by $F$-terms of squark condensates and baryons of the hidden sector. For a more satisfactory model, it is better to find a SUSY breaking sector being neutral in the SM gauge group.

6 But our model is not free from SU(2)$_W \times U(1)_Y$ breaking by $F$-terms of squark condensates and baryons of the hidden sector. For a more satisfactory model, it is better to find a SUSY breaking sector being neutral in the SM gauge group.
FIG. 3: The 6d internal space of $T_{1,2,4,7}$ sectors: two pencil topologies and one triangular ravioli topology. In the $(34)$-torus, untwisted string $\ell_0$ and twisted string $\ell_1$ are also shown.

charges since the composites are formed with $(1, 2, 1; 1; 5', 1)_{-1/10}$, $5'_{2/5}$, $(1, 1, 2; 1; 5', 1)_{1/10}$, and $5'_{-2/5}$.

If $m_{1,2}$ are near the GUT scale, we have a six flavor model, and the upper dashed line with $-b_j = 9$ gives $\alpha_h \simeq \frac{1}{15}$ for $\Lambda_h = 10^{12}$ GeV. If $m_{1,2} \simeq \Lambda_h$, referring to the lower bold dashed-line of Fig. 2 we have $\alpha_h \simeq \frac{1}{5}$ for $\Lambda_h = 10^{12}$ GeV. These values are large. To introduce this kind of a large value for the hidden sector coupling constant, we can introduce different radii for the three tori. In this way, a relatively small scale, $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV compared to the string scale, can be introduced also via geometry through the ratio $r/R$. Let the first and third tori are small compared to the second tori as depicted in Fig. 3.

If the radius $R$ of the second torus becomes infinite, we treat the second torus as if it is a fixed torus. Then, one might expect a 6D spacetime, expanding our 4D spacetime by including the large $(34)$-torus. One may guess that the spectrum in $T_1, T_2, T_4$, and $T_7$ sectors would be three times what we would obtain in $T_{io}(i = 1, 2, 4, 7)$. For $T_3$ and $T_6$, the spectrum would be the same since they are not affected by the Wilson line from the beginning. But this naive consideration does not work, which can be checked from the spectrum we presented. If the size of the second torus becomes infinite, we are effectively dealing with 4d internal space, and hence we must consider an appropriate 4d internal space compactification toward a full 6D Minkowski spacetime spectrum. This needs another set of twisted sector vacuum energies and the spectrum is not what we commented above. A more careful study is necessary to fit the hidden sector coupling constant to the needed value.

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7 A naive expectation of the hidden sector coupling, toward lowering the hidden sector confining scale, is a smaller $\alpha_{\text{GUT}}^h$ compared to $\frac{1}{25}$. Because of many flavors, $\alpha_{\text{GUT}}^h$ turns out to be large.
Here we just comment that in our example SU(5)′ is not enhanced further by neglecting the Wilson line. Even though SU(5)′ is not enhanced between the scales 1/r and 1/R, the SU(5)′ gauge coupling can run to become bigger than the observable sector coupling at the GUT scale since in our case the bigger group SU(5)′, compared to our observable sector SU(4) group even without the Wilson line, results between the scales 1/r and 1/R.

The example presented in this paper suggest a possibility that the GMSB with an appropriate hidden sector scale toward a solution of the SUSY flavor problem is realizable in heterotic strings with three families.

V. CONCLUSION

Toward the SUSY flavor solution, the GMSB from string compactification is looked for. We pointed out that the GMSB is possible within a bounded region of the hidden sector gauge coupling. We find that the hidden sector SU(5)′ is the handiest group toward this direction, by studying the gauge coupling running. We have presented an example in Z_{12-I} orbifold construction where there exist enough number of SU(5)′ flavors satisfying the most needed SM conditions: three observable sector families without exotics. Toward achieving the needed coupling strength of the hidden sector at the GUT scale, we have suggested different compactification radii for the three tori.

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