Resolving Eight-Fold Neutrino Parameter Degeneracy by Two Identical Detectors with Different Baselines

Takaaki Kajita, Hisakazu Minakata, Shoei Nakayama, and Hiroshi Nunokawa

1Research Center for Cosmic Neutrinos, Institute for Cosmic Ray Research, University of Tokyo, Kashiwa, Chiba 277-8582, Japan
2Department of Physics, Tokyo Metropolitan University, Hachioji, Tokyo 192-0397, Japan
3Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro, C. P. 38071, 22452-970, Rio de Janeiro, Brazil

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Abstract

We have shown in a previous paper that two identical detectors with each fiducial mass of 0.27 megaton water, one in Kamioka and the other in Korea, which receive the (anti-) muon neutrino beam of 4 MW power from J-PARC facility have potential of determining the neutrino mass hierarchy and discovering CP violation by resolving the degeneracies associated with them. In this paper, we point out that the same setting has capability of resolving the $\theta_{23}$ octant degeneracy in region where $\sin^2 2\theta_{23} < 0.97$ at 2 standard deviation confidence level even for very small values of $\theta_{13}$. Altogether, it is demonstrated that one can solve all the eight-fold neutrino parameter degeneracies in situ by using the Tokai-to-Kamioka-Korea setting if $\theta_{13}$ is within reach by the next generation superbeam experiments. We also prove the property called “decoupling between the degeneracies”, which is valid to first order in perturbation theory of the earth matter effect, that guarantees approximate independence between analyses to solve any one of the three different type of degeneracies.

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*Electronic address: kajita@icrr.u-tokyo.ac.jp
†Electronic address: minakata@phys.metro-u.ac.jp
‡Electronic address: shoei@suketto.icrr.u-tokyo.ac.jp
§Electronic address: nunokawa@fis.puc-rio.br
I. INTRODUCTION

Physics of neutrinos has entered into a new stage after establishment of the mass-induced neutrino oscillation due to the atmospheric \cite{1}, the accelerator \cite{2,3}, and the reactor neutrino \cite{4} experiments, confirming the earlier discovery \cite{5,6,7} and identifying the nature of the phenomenon. In the new era, the experimental endeavors will be focused on search for the unknowns in neutrino masses and the lepton flavor mixing, $\theta_{13}$, the neutrino mass hierarchy, and CP violation. On the theory side, various approaches toward understanding physics of lepton mixing and the quark-lepton relations are extensively pursued \cite{8}, which then further motivate precision measurement of the lepton mixing parameters. We will use the standard notation \cite{9} of the lepton mixing matrix, the Maki-Nakagawa-Sakata (MNS) matrix \cite{10}, throughout this paper.

It was recognized sometime ago that there exists problem of parameter degeneracy which would act as an obstacle against precision measurement of the lepton mixing parameters. The nature of the degeneracy can be understood as the intrinsic degeneracy \cite{11}, which is duplicated by the unknown sign of atmospheric $\Delta m^2_{21}$ (hereafter, “sign-$\Delta m^2$ degeneracy” for simplicity) and by the possible octant ambiguity of $\theta_{23}$ \cite{13} that exists if $\theta_{23}$ is not maximal. For an overview of the resultant eight-fold degeneracy, see e.g., \cite{14,15}.

In a previous paper \cite{16}, we have shown that the identical two detector setting in Kamioka and in Korea with each fiducial mass of 0.27 Mton water, which receives the identical neutrino beam from the J-PARC facility can be sensitive to the neutrino mass hierarchy and CP violation in a wide range of the lepton mixing parameters, $\theta_{13}$ and the CP phase $\delta$. It is the purpose of this paper to point out that the same setting has capability of resolving the $\theta_{23}$ octant degeneracy to a value of $\theta_{23}$ which is rather close to the maximal, $\sin^2 2\theta_{23} < 0.97 (0.94)$ at 2 (3) standard deviation confidence level (CL). It is achieved by detecting solar-$\Delta m^2$ scale oscillation effect in the Korean detector. Together with the sensitivities to resolution of the degeneracy related to the mass hierarchy and the CP phase discussed in the previous paper, we demonstrate that the Kamioka-Korea two detector setting is capable of solving the total eight-fold parameter degeneracy. We stress that resolving the degeneracy is crucial to precision measurement of the lepton mixing parameters on which we make further comments at appropriate points in the subsequent discussions. We also emphasize that it is highly nontrivial that one can formulate such a global strategy for resolving all the known degeneracies (though in a limited range of the mixing parameters) only with the experimental apparatus using conventional muon neutrino super beam.\(^1\)

In some of the previous analyses including ours \cite{16,18}, people often tried to resolve the degeneracy of a particular type without knowing (or addressing) the solutions of the other types of degeneracies. But, then, the question of consistency of the procedure immediately arises; Can one solve the degeneracy of type A without knowing the solutions of the other degeneracies B and C? Does the obtained solution remain unchanged when the assumed solutions for the other type of degeneracies are changed to the alternative ones?, etc. We answer to these questions in the positive in experimental settings where the earth matter effect can be treated as perturbation. We do so by showing that the resolution of the degeneracy of a particular type decouples from the remaining degeneracies, the property

\(^1\) It may be contrasted to the method for resolving the degeneracy based on neutrino factory examined in \cite{17}; It uses a 40 kton magnetized iron calorimeter and a 4 kton emulsion chamber, and conventional $\nu_{\mu}$ beam watched by a 400 kton water Cherenkov detector.
called the “decoupling between the degeneracies” in this paper.

In Sec. II we present a pedagogical discussion of how the eight-fold degeneracy can be lifted by measurement with the Kamioka-Korea two detector setting. In Sec. III we prove the “decoupling” and make a brief comment on its significance. In Sec. IV we discuss some characteristic features of the $\nu_e$ and $\bar{\nu}_e$ appearance probabilities that allow the Kamioka-Korea identical two detector setting to resolve the $\theta_{23}$ octant degeneracy. In Sec. V the actual analysis procedure and the obtained sensitivities for solving the $\theta_{23}$ degeneracy are described in detail. In Sec. VI we reexamine the sensitivities to the mass hierarchy and CP violating phase by using our new code with disappearance channels and additional systematic errors. In Sec. VII we give a summary and discussions.

II. HOW THE IDENTICAL TWO DETECTOR SYSTEM SOLVES THE EIGHT-FOLD DEGENERACY?

We describe in this section how the eight-fold parameter degeneracy can be resolved by using two identical detectors, one placed at a medium baseline distance of a few times 100 km, and the other at $\sim$1000 km or so. We denote them as the intermediate and the far detectors, respectively, in this paper. Whenever necessary we refer the particular setting of Kamioka-Korea two detector system, but most of the discussions in this and the next sections are valid without the specific setting.

To give the readers a level-one understanding we quote here, ignoring complications, which effect is most important for solving which degeneracy:

- The intrinsic degeneracy; Spectrum information solves the intrinsic degeneracy.
- The sign-$\Delta m^2$ degeneracy; Difference in the earth matter effect between the intermediate and the far detectors solves the sign-$\Delta m^2$ degeneracy.
- The $\theta_{23}$ octant degeneracy; Difference in solar $\Delta m^2$ oscillation effect (which is proportional to $c_{23}^2$) between the intermediate and the far detectors solves the $\theta_{23}$ octant degeneracy.

To show how the eight-fold parameter degeneracy can be resolved, we present in Fig 1 a comparison between the sensitivities achieved by the Kamioka only setting and the Kamioka-Korea setting by taking a particular set of true values of the mixing parameters which are quoted in caption of Fig 1. The left four panels of Fig 1 show the expected allowed regions of oscillation parameters in the Tokai-to-Kamioka phase-II (T2K II) setting, while the right four panels show the allowed regions by the Tokai-to-Kamioka-Korea setting. For both settings we assume 4 years of neutrino plus 4 years of anti-neutrino running and the total fiducial volume is kept to be the same, 0.54 Mton. Some more information of the experimental setting and the details of the analysis procedure are described in the caption of Fig 1 and in Sec. VII.

2 It was shown in the previous study [16] that the sensitivity obtained with 2 years of neutrino and 6 years of anti-neutrino running in the T2K II setting is very similar to that of 4 years of neutrino and 4 years of anti-neutrino running.
FIG. 1: The region allowed in $\delta - \sin^2 2\theta_{13}$ and $\sin^2 \theta_{23} - \sin^2 2\theta_{13}$ spaces by T2K II (left four panels) and by the Kamioka-Korea two detector setting (right four panels) in both of which 4 years of neutrino plus 4 years of anti-neutrino running are assumed. The upper (lower) four panels show the allowed region for the positive (negative) sign of $\Delta m^2_{31}$. The detector fiducial volumes of T2K II and Kamioka-Korea settings are assumed to be 0.54 Mton and each 0.27 Mton, respectively, and the beam power of J-PARC is assumed to be 4 MW. The baseline to the Kamioka and Korea detectors are, 295 km and 1050 km, respectively. The true solution is assumed to be located at $\sin^2 2\theta_{13} = 0.01$, $\sin^2 \theta_{23} = 0.60$ and $\delta = \pi/4$ with positive sign of $\Delta m^2_{31} (= +2.5 \times 10^{-3}$ eV$^2$), which is indicated by the green star. The solar mixing parameters are fixed as $\Delta m^2_{21} = 8 \times 10^{-5}$ eV$^2$ and $\sin^2 \theta_{12} = 0.31$. Three contours in each figure correspond to the 68% (blue line), 90% (black line) and 99% (red line) C.L. sensitivities, which are defined as the difference of the $\chi^2$ being 2.30, 4.61 and 9.21, respectively.

Let us first focus on the left four panels of Fig. 1. In the left-most two panels labeled as (aN) and (aI), one observes some left-over degeneracies of the total eight-fold degeneracy; if we plot the result of a rate only analysis without spectrum information we would have seen 8 separate (or overlapped) allowed parameter regions. The $\theta_{23}$ octant degeneracy remains unresolved as seen in panels (bN) and (bI). Note that the overlapping two regions in (aN) and (aI) are nothing but the consequence of unresolved $\theta_{23}$ degeneracy. The intrinsic degeneracy, horizontal pair seen in (aN), is almost resolved apart from 99% CL region at the particular set of values of the mixing parameters indicated above. The corresponding pair in (aI) is missing because the intrinsic degeneracy is completely lifted. Since the matter effect plays minor role in the T2K II setting it is likely that the spectral information is mainly responsible for lifting the intrinsic degeneracy. See Sec. III C for more about it.

Here is a brief comment on the property of the intrinsic and the sign-$\Delta m^2$ degeneracies. Because the degenerate solutions of CP phase $\delta$ satisfy approximately the same relationship $\delta_2 = \pi - \delta_1$ in both the intrinsic and the sign-$\Delta m^2$ degeneracies (see Eqs. (5) and (6) in Sec. III), the would-be four (one missing) regions in the panels (aN) and (aI) in Fig. 1 forms a cross (or X) shape, with crossing connection between a pair of solutions of the sign-$\Delta m^2$ degeneracy.

In the right four panels of Fig. 1 it is exhibited that the intrinsic degeneracy as well as $\theta_{23}$ octant degeneracy are completely resolved by the Kamioka-Korea two-detector setting at
the same values of the mixing parameters, indicating power of the two detector method [20]. Namely, the comparison between the spectral shapes in Kamioka and in Korea located at the first and nearly the second oscillation maxima, respectively, supersedes a single detector measurement in Kamioka with the same total volume despite much less statistics in the Korean detector. We will give a detailed discussion on how \( \theta_{23} \) octant degeneracy can be resolved by the Kamioka-Korea setting in Sec. IV and present the details of the analysis in Sec. V.

It should be noted that the sign-\( \Delta m^2 \) degeneracy is also lifted though incompletely at the particular set of values of the mixing parameters as indicated in the panels (cI) of Fig. 1 where only the 99% CL regions remain. In fact, we have shown in our previous paper that the Kamioka-Korea identical two detector setting is powerful in resolving the sign-\( \Delta m^2 \) degeneracy in a wide range of the mixing parameters [16]. We note that resolution of the degeneracy in turn leads to an enhanced sensitivity to CP violation than that of T2K II setting in a region of relatively large \( \theta_{13} \). See [16] for comparison with T2K II sensitivity. Altogether, we verify that the identical two detector setting in Kamioka-Korea with neutrino beam from J-PARC solves all the eight-fold parameter degeneracy in situ if \( \theta_{13} \) is within reach by the next generation superbeam experiments such as T2K [19] and NO\( \nu \)A [21].

### III. DECOUPLING BETWEEN DEGENERACIES

In this section we discuss the property called the “decoupling between degeneracies” which arises due to the special setting of baselines shorter than \( \sim 1000 \) km. The content of this section is somewhat independent of the main line of the discussion in this paper, and the readers can skip it to go directly to the analysis of \( \theta_{23} \) octant degeneracy in Secs. IV and V. Nonetheless, the property makes the structure of analysis for resolving the eight-fold degeneracy transparent, and therefore it may worth to report.

The problem of decoupling came to our attention via the following path. In most part of the previous paper [16], we have discussed how to solve the sign-\( \Delta m^2 \) degeneracy without worrying about the \( \theta_{23} \) octant degeneracy. Conversely, the authors of [18] analyzed the latter degeneracy without resolving the former one. Are these correct procedure? The answer is yes if the analysis procedure and the results for the \( \theta_{23} \) degeneracy is independent of which solutions we take for the sign-\( \Delta m^2 \) degeneracy, and vice versa. We call this property the “decoupling between the degeneracies” \(^3\). Though discussion on this point was partially given in [18], we present here a complete discussion of the decoupling.

Under the approximation of lowest nontrivial order in matter effect, we prove that the decoupling holds between the above two degeneracies, and furthermore that it can be generalized, though approximately, to the relation between any two pair of degeneracies among the three types of degeneracies. To our knowledge, leading order in matter perturbation theory appears to be the only known circumstance that the argument goes through. Fortunately, the approximation is valid for the setting used in this paper with baseline up to \( \sim 1000 \) km, in particular in the Kamioka-Korea setting. In the following treatment we make a further approximation that the degenerate solutions are determined primarily by the mea-

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\(^3\) Here is a concrete example for which the decoupling does not work; In the method of comparison between \( \nu_\mu \) and \( \bar{\nu}_\mu \) disappearance measurement for lifting \( \theta_{23} \) octant degeneracy [22] one in fact determines the combined sign of \( \cos 2\theta_{23} \times \Delta m^2_{31} \) as noticed in [18], and hence no decoupling.
measurement at the intermediate detector. It is a sensible approximation because the statistics is about 10 times higher at the intermediate detector, and its validity is explicitly verified in the analysis performed in \[14\].

\[\text{A. Approximate analytic treatment of the parameter degeneracy}\]

To make the discussion self-contained, we start from the derivation of the degenerate solutions by using the matter perturbation theory \[23\], in which the matter effect is kept to its lowest nontrivial order. Namely, the matter effect can be ignored in leading order in the disappearance channel whose oscillation probability is order of unity. Then, the disappearance probability \(P(\nu_\mu \to \nu_\mu)\) can be given by the vacuum oscillation approximation with leading order in \(s^2_{13}\) and the solar \(\Delta m^2_{21}\) corrections as

\[
1 - P(\nu_\mu \to \nu_\mu) = \left[\sin^2 2\theta_{23} + 4s^2_{13}s^2_{23}(2s^2_{23} - 1)\right] \sin^2 \left(\frac{\Delta m^2_{31}L}{4E}\right) - c^2_{12} \sin^2 2\theta_{23} \left(\frac{\Delta m^2_{21}L}{2E}\right) \sin \left(\frac{\Delta m^2_{31}L}{2E}\right). \tag{1}\]

The probability for anti-neutrino channel is the same as that for neutrino one in this approximation. One can show \[23\] that the other solar \(\Delta m^2_{21}\) corrections are suppressed further by either a small \(\sin^2 2\theta_{13} \lessapprox 0.1\), or the Jarlskog factor \(J \equiv c_{12}s_{13}\bar{c}_{13}s_{23}\bar{s}_{23}\sin\delta \lessapprox 0.04\). In fact, the validity of the approximation is explicitly verified in \[18\] where the matter effect terms are shown to be of the order of \(10^{-3}\) even at \(L = 1000\) km. A disappearance measurement, therefore, determines \(s_{23}^2\) to first order in \(s_{13}^2\) as

\[
(s^2_{23})^{(1)} = (s^2_{23})^{(0)}(1 + s^2_{13}), \tag{2}\]

where \((s^2_{23})^{(0)}\) is the solution obtained by ignoring \(s^2_{13}\). From the first term in Eq. \(1\), the two solutions of \((s^2_{23})^{(0)}\) is determined as \((s^2_{23})^{(0)} = \frac{1}{2} \left[1 \pm \sqrt{1 - \sin^2 2\theta_{23}} \right]\), the simplest form of the \(\theta_{23}\) octant degeneracy. For example, \((s^2_{23})^{(0)} = 0.4\) or 0.6 (0.45 or 0.55) for \(\sin^2 2\theta_{23} = 0.96\) (0.99).

For the appearance channel, we use the \(\nu_\mu(\bar{\nu}_\mu) \to \nu_e(\bar{\nu}_e)\) oscillation probability with first-order matter effect \[23\]

\[
P[\nu_\mu(\bar{\nu}_\mu) \to \nu_e(\bar{\nu}_e)] = c^2_{23} \sin^2 2\theta_{12} \left(\frac{\Delta m^2_{21}L}{4E}\right)^2 + \sin^2 2\theta_{13} s^2_{23} \left[\sin^2 \left(\frac{\Delta m^2_{31}L}{4E}\right) - \frac{1}{2} s^2_{12} \left(\frac{\Delta m^2_{21}L}{2E}\right) \sin \left(\frac{\Delta m^2_{31}L}{2E}\right) \pm \left(\frac{4Ea}{\Delta m^2_{31}}\right) \sin^2 \left(\frac{\Delta m^2_{31}L}{4E}\right) + \frac{aL}{2} \sin \left(\frac{\Delta m^2_{31}L}{2E}\right)\right] + 2J_r \left(\frac{\Delta m^2_{21}L}{2E}\right) \left[\cos \delta \sin \left(\frac{\Delta m^2_{31}L}{2E}\right) \mp 2 \sin \delta \sin^2 \left(\frac{\Delta m^2_{31}L}{4E}\right)\right], \tag{3}\]

where the terms of order \(s_{13} \left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right)^2\) and \(aLs_{13} \left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right)\) are neglected. In Eq. \(3\), \(a \equiv \sqrt{2G_FN_e} \[24\]\ where \(G_F\) is the Fermi constant, \(N_e\) denotes the averaged electron number.
density along the neutrino trajectory in the earth, \( J_r \) (\( \equiv c_{12}s_{12}c_{13}s_{23}s_{a3} \)) denotes the reduced Jarlskog factor, and the upper and the lower sign \( \pm \) correspond to the neutrino and anti-neutrino channels, respectively. We take constant matter density approximation in this paper. The first term of Eq. (3) is due to the oscillation driven by the solar \( \Delta m^2_{21} \), which is essentially negligible in the intermediate detector but not at the far detector and is of key importance to resolve the \( \theta_{23} \) octant degeneracy.

We make an approximation of ignoring terms of order \( (\Delta m^2_{21}/\Delta m^2_{31})J_r \cos 2\theta_{23} \) in Eq. (3). Note that keeping only the leading order in this quantity is reasonable because \( J_r \lesssim 0.04 \), \( |\Delta m^2_{21}/\Delta m^2_{31}| \approx 1/30 \), and \( \cos 2\theta_{23} = \pm 0.2 \) for \( \sin^2 2\theta_{23} = 0.96 \). Then, the two degenerate solutions obey an approximate relationship

\[
\left( \sin^2 2\theta_{13}s_{23}^2 \right)_{1st} = \left( \sin^2 2\theta_{13}s_{23}^2 \right)_{2nd},
\]

or, \( s_{13}^{1st} = s_{23}^{2nd} \), ignoring higher order terms in \( s_{13} \). We can neglect the leading order correction in \( s_{13}^2 \) to \( s_{23}^2 \) in these relations because it gives \( O(s_{13}^4) \) terms.

Analytic treatment of the intrinsic and the sign-\( \Delta m^2 \) degeneracies is given in \([11]\). In an environment where the vacuum oscillation approximation applies the solutions corresponding to the intrinsic degeneracy are given by \([11]\)

\[
\theta_{13}^{(2)} = \theta_{13}^{(1)}, \quad \delta^{(2)} = \pi - \delta^{(1)},
\]

where the superscripts (1) and (2) label the solutions due to the intrinsic degeneracy. Under the same approximation the solutions corresponding to the sign-\( \Delta m^2 \) degeneracy are given by \([12]\)

\[
\theta_{13}^{\text{norm}} = \theta_{13}^{\text{inv}}, \quad \delta^{\text{norm}} = \pi - \delta^{\text{inv}}, \quad (\Delta m^2_{31})^{\text{norm}} = -(\Delta m^2_{31})^{\text{inv}},
\]

where the superscripts “norm” and “inv” label the solutions with the positive and the negative sign of \( \Delta m^2_{31} \). The degeneracy stems from the approximate symmetry under the exchange of these two solutions through which the degeneracy is uncovered \([12]\). The validity of these approximate relationships in the actual experimental setup in the T2K II measurement is explicitly verified in \([16]\). It should be noticed that even if sizable matter effect is present the relation (5) holds at the energy corresponding to the vacuum oscillation maximum, or more precisely, the shrunk ellipse limit \([25]\).

**B. Decoupling between degeneracies**

Resolution of the degeneracy can be done when a measurement distinguishes between the values of the oscillation probabilities with the two different solutions corresponding to a degeneracy. Therefore, we define the probability difference

\[
\Delta P^{ab}(\nu_\alpha \rightarrow \nu_\beta) \\
\equiv P\left(\nu_\alpha \rightarrow \nu_\beta; \theta_{23}^{(a)}, \theta_{13}^{(a)}, \delta^{(a)}, (\Delta m^2_{31})^{(a)}\right) - P\left(\nu_\alpha \rightarrow \nu_\beta; \theta_{23}^{(b)}, \theta_{13}^{(b)}, \delta^{(b)}, (\Delta m^2_{31})^{(b)}\right)
\]

where the superscripts \( a \) and \( b \) label the degenerate solutions. Suppose that we are discussing the degeneracy A. The decoupling between the degeneracies A and B holds if \( \Delta P^{ab} \) defined in \([11]\) for the degeneracy A is invariant under the replacement of the mixing parameters corresponding to the degeneracy B, and vice versa.
The best example of the decoupling is given by the one between the $\theta_{23}$ octant and the sign-$\Delta m^2_{31}$ degeneracies. By noting that 

$$J^1 - J^2 = \cos 2\theta^1_{23} J^1_{1st}$$

in leading order in $\cos 2\theta_{23}$, the difference between probabilities with the first and the second octant solutions can be given by

$$\Delta P^{1st - 2nd} = \cos 2\theta^1_{23} \sin^2 2\theta_{12} \left( \frac{\Delta m^2_{21}}{4E} \right)^2$$

$$+ 2J^1_{1st} \cos 2\theta^1_{23} \left( \frac{\Delta m^2_{31}}{2E} \right) \left[ \cos \delta \sin \left( \frac{\Delta m^2_{31}}{2E} \right) \pm 2\sin \delta \sin^2 \left( \frac{\Delta m^2_{31}}{4E} \right) \right].$$

(8)

The remarkable feature of (8) is that the leading-order matter effect terms drop out completely. Therefore, our approximated treatment remains valid until the second order matter effect starts to become sizable in the appearance oscillation probability. More importantly, $\Delta P^{1st - 2nd}$, being composed only of the vacuum oscillation terms, is obviously invariant under the replacement $normal \leftrightarrow inverted$ solutions with different signs of $\Delta m^2_{31}$ given in Eq. (6). Therefore, the resolution of the $\theta_{23}$ octant degeneracy can be carried out without worrying about the presence of the sign-$\Delta m^2_{31}$ degeneracy.

Next, we examine the inverse problem; Does the determination of mass hierarchy decouple with the resolution of the $\theta_{23}$ degeneracy? One can show, by using Eq. (3), that the similar probability difference between the solutions in Eq. (6) with the normal and the inverted hierarchies is given by

$$\Delta P^{norm - inv} = \sin^2 2\theta^norm_{13} \left( \frac{s^norm_{23}}{2} \right) \left[ -s_{12}^2 \left( \frac{\Delta m^2_{31}}{2E} \right) \sin \left( \frac{\Delta m^2_{31}}{2E} \right) \right]$$

$$\pm 2(aE) \left\{ \left( \frac{4E}{\Delta m^2_{31}} \right)^2 \sin^2 \left( \frac{\Delta m^2_{31}}{4E} \right) \left[ \frac{\Delta m^2_{31}}{2E} \sin \left( \frac{\Delta m^2_{31}}{2E} \right) \right] - \frac{1}{2} \sin \left( \frac{\Delta m^2_{31}}{2E} \right) \right\}.$$

(9)

where the superscripts “norm” and “inv” can be exchanged if one want to start from the inverted hierarchy. We notice that most of the vacuum oscillation terms, including the solar term, drop out because of the invariance under $\delta \rightarrow \pi - \delta$ and $\Delta m^2_{31} \rightarrow -\Delta m^2_{31}$. Now, we observe that $\Delta P^{norm - inv}$ is invariant under the transformation $\theta^1_{23} \leftrightarrow \theta^2_{23}$ and $\theta^1_{13} \leftrightarrow \theta^2_{13}$, because $\Delta P^{norm - inv}$ depends upon $\theta_{13}$ and $\theta_{23}$ only through the combination $\sin^2 2\theta_{13} \sin^2 2\theta_{23}$ within our approximation. Therefore, the sign-$\Delta m^2_{31}$ and the $\theta_{23}$ degeneracies decouple with each other.

In our previous paper, we have shown that the sign-$\Delta m^2_{31}$ degeneracy can be lifted by the Kamioka-Korea two detector setting. The above argument for decoupling guarantees that our treatment is valid irrespective of the solutions assumed for $\theta_{23}$ degeneracy.

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4 We remark that in most part of [16], we have assumed that $\theta_{23} = \pi/4$ so that this problem itself does not exist. The above discussion implies that even in the case of non-maximal value of $\theta_{23}$ the similar analysis as in [16] must go through without knowing in which octant $\theta_{23}$ lives. The resultant sensitivity for resolving the mass hierarchy will change as $\theta_{23}$ goes away from $\pi/4$ but only slightly as will be shown in Sec. [16].
C. Including the intrinsic degeneracy

Now we turn to the intrinsic degeneracy for which the situation is somewhat different. First of all, in our setting, the intrinsic degeneracy is already resolved by spectrum informations at the intermediate detector if \( \theta_{13} \) is relatively large, \( \sin^2 2\theta_{13} \gtrsim 0.02 \) as illustrated in Fig. 1 before the information from the far detector is utilized. It means that there is no intrinsic degeneracy from the beginning in the analysis with spectrum informations. Because of this feature, the intrinsic degeneracy decouple from the beginning from the task of resolving the eight-fold degeneracy in our setting for the relatively large values of \( \theta_{13} \). Because of a further enhanced sensitivity, the intrinsic degeneracy may be resolved to a smaller values of \( \theta_{13} \) in the Kamioka-Korea setting.

In fact, powerfulness of the spectral information for resolving the intrinsic degeneracy can be understood easily by noting that \( \Delta P^{ab} \) defined in (7) is given by using the intrinsic degeneracy solution (5) as

\[
\Delta P_{12}(\nu_\mu \to \nu_e) = 4J_r \left( \frac{\Delta m_{21}^2 L}{2E} \right) \cos \delta \sin \left( \frac{\Delta m_{31}^2 L}{2E} \right),
\]

with notable feature that the matter effect cancels out. Notice that even if the matter effect cannot be negligible the solution (5) holds for measurement at energies around the vacuum oscillation maximum. The right-hand side of Eq. (10) is proportional to \( E^{-2} \) at energies near the vacuum oscillation maximum, \( \frac{\Delta m_{21}^2 L}{2E} = \pi \), and the steep energy dependence can be used to lift the degeneracy. Hence, the spectrum analysis is a powerful tool for resolving the intrinsic degeneracy.

D. The case that the intrinsic degeneracy is not solved

Even if \( \theta_{13} \) is too small, or if the energy resolution is too poor for the spectrum analysis to resolve the intrinsic degeneracy, we can show that the intrinsic degeneracy approximately decouples from the other degeneracies. \( \Delta P^{12} \) in Eq. (10) is not exactly but approximately invariant under the transformation first-octant \( \leftrightarrow \) second-octant solutions. The difference between \( \Delta P^{12}(1st) \) and \( \Delta P^{12}(2nd) \) is of order \( \cos 2\theta_{23}J_r\Delta m_{21}^2/\Delta m_{31}^2 \simeq 3 \times 10^{-4} \) for \( s_{23}^2 = 0.4 \) and \( \sin^2 2\theta_{13} = 0.1 \) apart from the further suppression by \( \sin \left( \frac{\Delta m_{21}^2 L}{2E} \right) \) at around the oscillation maximum. Being the vacuum oscillation term \( \Delta P^{12} \) is obviously invariant under the replacement normal \( \leftrightarrow \) inverted solutions with different signs of \( \Delta m_{31}^2 \). Therefore, resolution of the intrinsic degeneracy can be done, to a good approximation, independent of the presence of the sign-\( \Delta m_{31}^2 \) and the \( \theta_{23} \) octant degeneracies.

The remaining problem we need to address is the inverse problem, whether the resolution of the sign-\( \Delta m_{31}^2 \) and the \( \theta_{23} \) octant degeneracies can be carried out without knowing solutions of the intrinsic degeneracy. The sign-\( \Delta m_{31}^2 \) degeneracy decouples from the intrinsic one because \( \Delta P^{\text{norm inv}} \) in (9) is invariant under the exchange of two intrinsic degeneracy solutions. The \( \theta_{23} \) octant degeneracy also approximately decouples from the intrinsic one. \( \Delta P^{1st~2nd} \) in (5) changes under the interchange of two intrinsic degeneracy solutions only by the same amount as the difference between \( \Delta P^{12} \) of the first and the octant \( \theta_{23} \) solutions, \( \cos 2\theta_{23}J_r\Delta m_{21}^2/\Delta m_{31}^2 \simeq 3 \times 10^{-4} \) (for \( s_{23}^2 = 0.4 \) and \( \sin^2 2\theta_{13} = 0.1 \)).

Here is a clarifying comment on what the decoupling really means; Because of the cross-shaped structure of the degenerate solutions of the intrinsic and the sign-\( \Delta m_{31}^2 \) degeneracies
(as was shown in Sec. II) the decoupling of the former from the latter does not imply that the correct value of $\delta$ can be extracted from the measurement without knowing the correct sign of $\Delta m^2_{31}$. It means that the elimination of one of the “intrinsic” degenerate pair solutions related by $\delta \leftrightarrow \pi - \delta$ for a given sign of $\Delta m^2_{31}$ can be done without knowing the mass hierarchy, the true sign of $\Delta m^2_{31}$. Therefore, the situation that the intrinsic degeneracy is always resolved by the spectrum analysis in region of not too small $\theta_{13}$, as is the case in our setting, is particularly transparent one from this viewpoint.

To sum up, we have shown that to leading order in the matter effect the intrinsic, the sign-$\Delta m^2_{31}$, and the $\theta_{23}$ octant degeneracies decouples with each other. They do so exactly except for between the intrinsic and the $\theta_{23}$ octant degeneracies for which the decoupling is approximate but sufficiently good to allow one-by-one resolution of all the three types of degeneracies. The decoupling implies that in analysis for lifting the eight-fold degeneracy the structure of the $\chi^2$ minimum is very simple in multi-dimensional parameter space, and it may be of use in discussions of how to solve the degeneracy in much wider context than that discussed in this paper.

IV. HOW IDENTICAL TWO DETECTOR SETTING SOLVES $\theta_{23}$ OCTANT DEGENERACY?

Now, we turn to the problem of how the identical two detector setting can resolve the $\theta_{23}$ degeneracy, the unique missing link in a program of resolving eight-fold parameter degeneracy in the Kamioka-Korea two detector setting. The solar $\Delta m^2$ oscillation term, the first term in Eq. (3) with the coefficient of $c_{23}^2$, may be of key importance to do the job. While it was argued on very general ground \[18\] that the $\theta_{23}$ degeneracy is hard to resolve only by accelerator experiments with baseline of $\lesssim 1000$ km or so, the argument can be circumvented if the solar term can be isolated. We emphasize that the accuracy of the determination of $\theta_{23}$ is severely limited by the octant degeneracy, as discussed in detail in \[20\].

Therefore, the question we must address first is the relative importance of the solar term to the remaining terms in $\Delta P^{1\text{st} 2\text{nd}}$ in Eq. (8). We note that the ratio of the solar term to the $\delta$-dependent solar-atmospheric interference term in $\Delta P^{1\text{st} 2\text{nd}}$ is given by $\sin^2 2\theta_{12} (\Delta m^2_{21} L/4E)/4J_r$, assuming the square parenthesis in (8) is of order unity. The ratio is roughly given by $\approx 3(1/30)(\pi/2)0.86(1/4J_r) \approx 0.9 (0.16/s_{13})$ with beam energy having the first oscillation maximum in Kamioka. Therefore, the solar term is indeed comparable or larger for smaller $\theta_{13}$ in size with the interference terms in $\Delta P^{1\text{st} 2\text{nd}}$ at the far detector. Obviously, the solar term is independent of $\theta_{13}$, which suggests that the sensitivity to resolve the $\theta_{23}$ degeneracy is almost independent of $\theta_{13}$, as will be demonstrated in Sec. V. We note that while the solar term is the key to resolve the $\theta_{23}$ degeneracy, the interference terms also contributes to lift the degeneracy. In particular, as shown in (8), the sin $\delta$ term has opposite sign when the polarity of the beam is switched from the neutrino to the anti-neutrino runs.

The next question we must address is how the solar term can be separated from the other terms to have enhanced sensitivity to the $\theta_{23}$ degeneracy. To understand the behavior of the solar term and its difference from that of the atmospheric terms in the oscillation probability, we plot in Fig. 2 a comparison between them in Kamioka (left panels) and in

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\footnote{Notice, however, that it does not obscure the CP violation, because the ambiguity is only two-fold; $\delta \leftrightarrow \pi - \delta$.}
Normal Hierarchy, $\sin^2 2\theta_{13} = 0.05$, $\sin^2 \theta_{23} = 0.5$

Kamioka

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig2a.pdf}
\caption{The energy dependence of the solar term (red solid line) is contrasted with the ones of atmospheric plus interference terms in the $\nu_e$ appearance oscillation probabilities with various values of CP phase $\delta$; $\delta = 0$ (dotted line), $\delta = \pi/2$ (dashed line), $\delta = \pi$ (dash-dotted line), and $\delta = 3\pi/2$ (double-dash-dotted line). For this plot, we used analytic expression in Eq. (3); $P_{\text{solar}}$ is defined to be the first term in Eq. (3) whereas $P_{\text{atm}}$ is defined to be the rest in Eq. (3). $\bar{P}_{\text{solar}}$ and $\bar{P}_{\text{atm}}$ refer to the corresponding terms for anti-neutrinos.

Korea (right panels) for various values of $\delta$. As one can observe in the right panels, the energy dependence of the solar oscillation term, a monotonically decreasing (approximately $1/E^2$) behavior with increasing energy, is quite different from the oscillating behavior of the atmospheric ones. It is also notable that the ratio of the solar term to the atmospheric-solar interference term is quite different between the intermediate and the far detectors. Due to the differing relative importance of the solar term in the two detectors and the clear difference in the energy dependences between the solar and the atmospheric terms, the spectrum analysis, the powerful method for resolving the intrinsic degeneracy, must be able to isolate the solar term from the remaining ones. This will be demonstrated in the quantitative analysis in the next section.

We note that several alternative methods are proposed to resolve the $\theta_{23}$ degeneracy.
They include: the atmospheric neutrino method \([27, 28, 29]\), and the reactor accelerator combined method \([18, 30]\), the atmospheric accelerator combined method \([31]\). The atmospheric neutrino method discussed in \([27, 28]\) is closest to ours in physics principle of utilizing the solar mass scale oscillation effect. Possible advantage of the present method may be in a clean detection of the solar term by the intermediate-versus-far two detector comparison.

V. SENSITIVITY FOR RESOLVING \(\theta_{23}\) OCTANT DEGENERACY

In this section, we describe details of our analysis for resolving the \(\theta_{23}\) octant degeneracy. They include treatment of experimental errors, treatment of background, and the statistical procedure which is used to investigate the sensitivity of the experiment. Then, the results of our analysis are presented.

A. Assumptions and the definition of \(\chi^2\)

In order to understand the sensitivity of the experiment with the two detector system at 295 km (Kamioka) and 1050 km (Korea), we carry out a detailed \(\chi^2\) analysis. To address the \(\theta_{23}\) octant degeneracy, it is of course necessary to include \(\nu_\mu\) and \(\bar{\nu}_\mu\) disappearance channels in addition to the appearance ones in our treatment. In short, the definition of the statistical procedure is similar to the one used in Ref. \([16]\) with necessary extension for including muon events. The assumption on the experimental setting is also identical to that of the best performance setting identified in Ref. \([16]\). Namely, 0.27 Mton fiducial masses for the intermediate site (Kamioka, 295 km) and the far site (Korea, 1050 km). The neutrino beam is assumed to be 2.5 degree off-axis one produced by the upgraded J-PARC 4 MW proton beam. It is assumed that the experiment will continue for 8 years with 4 years of neutrino and 4 years of anti-neutrino runs.

We use various numbers and distributions available from references related to T2K \([32]\), in which many of the numbers are updated after the original proposal \([19]\). Here, we summarize the main assumptions and the methods used in the \(\chi^2\) analysis. We use the reconstructed neutrino energy for single-Cherenkov-ring electron and muon events. The resolution in the reconstructed neutrino energy is 80 MeV for quasi-elastic events. We assume that \(|\Delta m^2_{31}|\) should be known precisely by the time when the experiment we consider in this report will be carried out. We take \(\Delta m^2_{31} = \pm 2.5 \times 10^{-3}\text{eV}^2\). Hence, we assume that the energy spectrum of the beam is the one expected by the 2.5 degree off-axis-beam in T2K. The shape of the energy spectrum for the anti-neutrino beam is assumed to be identical to that of the neutrino beam. The event rate for the anti-neutrino beam in the absence of neutrino oscillations is smaller by a factor of 3.4 due mostly to the lower neutrino interaction cross sections and partly to the slightly lower flux. The signal to noise ratio is worse for the anti-neutrino beam than that for the neutrino beam by a factor of about 2.

28 background electron events are expected for the reconstructed neutrino energies between 350 and 850 MeV for \((0.75 \times 0.0225 \times 5)\text{MW} \cdot \text{Mton} \cdot \text{yr}\) measurement with the neutrino beam. The energy dependence of the background rate and the rate itself are taken from \([32]\). The background rate is expected to be higher in the lower neutrino energies. The expected number of electron events is assumed to be 122 for \(\sin^2 2\theta_{13} = 0.1\) with the same detector exposure and beam, assuming the normal mass hierarchy and \(\delta = 0\).

We assume that the experiment is equipped with a near detector which measures the
rate and the energy dependence of the background for electron events, un-oscillated muon spectrum, and the signal detection efficiency. These measurements are assumed to be carried out within the uncertainty of 5%. We already demonstrated that the dependence on the assumed value of the experimental systematic errors is rather weak \[16\]. We stress that in the present setting the detectors located in Kamioka and in Korea are not only identical but also receive neutrino beams with essentially the same energy distribution (due to the same off-axis angle of 2.5 degree) in the absence of oscillations. However, it was realized recently that, due to a non-circular shape of the decay pipe of the J-PARC neutrino beam line, the flux energy spectra viewed at detectors in Kamioka and in Korea are expected to be slightly different even at the same off-axis angle, especially in the high-energy tail of the spectrum \[33\]. The possible difference between fluxes in the intermediate and the far detectors is newly taken into account as a systematic error in the present analysis.

We compute neutrino oscillation probabilities by numerically integrating neutrino evolution equation under the constant density approximation. The average density is assumed to be 2.3 and 2.8 g/cm\(^3\) for the matter along the beam line between the production target and Kamioka and between the target and Korea, respectively \[16\]. We assume that the number of electron with respect to that of nucleons to be 0.5 to convert the matter density to the electron number density. In our \(\chi^2\) analysis, we fix the absolute value of \(|\Delta m^2_{23}|\) to be 2.5 \(\times\) \(10^{-3}\) eV\(^2\), and fix solar parameters as \(\Delta m^2_{23} = 8 \times 10^{-5}\) eV\(^2\) and \(\sin^2 \theta_{12} = 0.31\).

Fig. 3 shows an example of the energy spectrum of electron and muon events to be observed in Kamioka and Korea for 4 years of neutrino beam plus 4 years of anti-neutrino beam. The two sets of parameters give very similar spectrum for both the electron and muon events at the Kamioka detector and the muon events at the Korea detector. However, due to the long baseline distance, the solar term plays some role in the \(\nu_e \rightarrow \nu_\mu\) oscillation probability at the Korean detector. Therefore, the two sets of parameters give slightly different oscillation probabilities in Korea. Since the solar term is proportional to \(c^2_{23}\) we use this feature to obtain information on \(\sin^2 \theta_{23}\).

The statistical significance of the measurement considered in this paper was estimated by using the following definition of \(\chi^2\):

\[
\chi^2 = \sum_{k=1}^{4} \left( \sum_{i=1}^{5} \left( \frac{(N(e)_i^{\text{obs}} - N(e)_i^{\text{exp}})^2}{\sigma_i^2} \right) + \sum_{i=1}^{20} \left( \frac{(N(\mu)_i^{\text{obs}} - N(\mu)_i^{\text{exp}})^2}{\sigma_i^2} \right) \right) + \sum_{j=1}^{7} \left( \frac{\epsilon_j}{\sigma_j} \right)^2 , \tag{11}
\]

where

\[
N(e)_i^{\text{exp}} = N_i^{\text{BG}} \cdot (1 + \sum_{j=1,2,7} f(e)_j \cdot \epsilon_j) + N_i^{\text{signal}} \cdot (1 + \sum_{j=3,7} f(e)_j \cdot \epsilon_j) , \tag{12}
\]

\[
N(\mu)_i^{\text{exp}} = N_i^{\text{non-QE}} \cdot (1 + \sum_{j=4,6,7} f(\mu)_j \cdot \epsilon_j) + N_i^{\text{QE}} \cdot (1 + \sum_{j=4,5,7} f(\mu)_j \cdot \epsilon_j) . \tag{13}
\]

The first and second terms in Eq. (11) are for the number of observed single-ring electron and muon events, respectively. \(N(e \text{ or } \mu)_i^{\text{obs}}\) is the number of events to be observed for the given oscillation parameter set, and \(N(e \text{ or } \mu)_i^{\text{exp}}\) is the expected number of events for the assumed oscillation parameters in the \(\chi^2\) analysis. \(k = 1, 2, 3, 4\) correspond to the four combinations of the detectors in Kamioka and in Korea with the neutrino and anti-neutrino beams, respectively. The index \(i\) represents the reconstructed neutrino energy bin for both electrons and muons. For electron events, both \(N(e)_i^{\text{obs}}\) and \(N(e)_i^{\text{exp}}\) include background
FIG. 3: Examples of electron and muon events to be observed in Kamioka and Korea for 4 years of neutrino plus 4 years of anti-neutrino running are presented as a function of reconstructed neutrino energy. The fiducial masses are taken to be 0.27 Mton for both the detectors in Kamioka and Korea. The dashed histograms for electron events show the background events. The open circles show the expected energy spectrum of signal events with $\sin^2 \theta_{23} = 0.40$ and $\sin^2 2\theta_{13} = 0.01$. The solid circles show the expected energy spectrum of signal events with $\sin^2 \theta_{23} = 0.60$ and $\sin^2 2\theta_{13} = 0.0067$. In both cases, $\delta = 3\pi/4$ and normal mass hierarchy are assumed in simulating the events.

The energy ranges of the five energy bins for electron events are respectively 400-500 MeV, 500-600 MeV, 600-700 MeV, 700-800 MeV and 800-1200 MeV. The energy range for the muon events covers from 200 to 1200 MeV. Each energy bin has 50 MeV width. $\sigma_i$ denotes the statistical uncertainties in the expected data. The third term in the $\chi^2$ definition collects the contributions from variables which parameterize the systematic uncertainties in the expected number of signal and background events.

$N_{i}^{\text{BG}}$ is the number of background events for the $i^{\text{th}}$ bin for electrons. $N_{i}^{\text{signal}}$ is the number of electron appearance events that are observed, and depends on neutrino oscillation parameters. The uncertainties in $N_{i}^{\text{BG}}$ and $N_{i}^{\text{signal}}$ are represented by 4 parameters $\epsilon_j$ ($j = 1$ to 3 and 7). Similarly, $N_{i}^{\text{non-QE}}$ are the number of non-quasi-elastic events for the $i^{\text{th}}$
bin for muons. \(N_{i}^{\text{QE}}\) are the number of quasi-elastic muon events. We treat the non-
quasi-elastic and quasi-elastic muon events separately, since the neutrino energy cannot be
properly reconstructed for non-quasi-elastic events. Both \(N_{i}^{\text{non-QE}}\) and \(N_{i}^{\text{QE}}\) depend on
neutrino oscillation parameters. The uncertainties in \(N_{i}^{\text{non-QE}}\) and \(N_{i}^{\text{QE}}\) are represented by
4 parameters \(\epsilon_{j}\) (\(j = 4\) to 7).

During the fit, the values of \(N(e \text{ or } \mu)^{\text{exp}}_{i}\) are recalculated for each choice of the oscillation
parameters which are varied freely to minimize \(\chi^{2}\), and so are the systematic error parameters \(\epsilon_{j}\). The parameter \(f(e \text{ or } \mu)_{j}^{i}\) represents the fractional change in the predicted event rate
in the \(i^{th}\) bin due to a variation of the parameter \(\epsilon_{j}\). The overall background normalization
for electron events is assumed to be uncertain by \(\pm 5\% \ (\sigma_{1}=0.05)\). It is also assumed that the
background events for electron events have an energy dependent uncertainty with the functional form of
\(f(e \text{ or } \mu)_{j}^{i} = ((E_{\nu}(\text{rec}) - 800 \text{ MeV})/400 \text{ MeV})\). \(5\%\) is assumed to be the uncertainty in
\(\epsilon_{2} \ (\sigma_{2}=0.05)\). The functional form of \(f(\mu)_{j}^{i} = (E_{\nu}(\text{rec}) - 800 \text{ MeV})/800 \text{ MeV}\) is used to
define the uncertainty in the spectrum shape for muon events \(\ (\sigma_{j}=0.05)\). The uncertainties in
the signal detection efficiency are assumed to be \(5\%\) for both electron and muon events
\(\ (\sigma_{3} = \sigma_{5}=0.05)\). The uncertainty in the separation of quasi-elastic and non-quasi-elastic
interactions in the muon events is assumed to be \(20\% \ (\sigma_{6}=0.20)\). These systematic errors
are assumed to be not correlated between the electron and muon events. In addition, for
the number of events in Korea, the possible flux difference between Kamioka and Korea is
taken into account in \(f(e \text{ or } \mu)_{j}^{i}\). The predicted flux difference \(33\) is simply assumed to be the \(1 \sigma\) uncertainty in the flux difference \(\sigma_{7}\).

### B. Sensitivity with two-detector complex

Now we present the results of the sensitivity analysis for the \(\theta_{23}\) octant degeneracy. The results for the mass hierarchy as well as CP violation sensitivities will be discussed in the next section. Fig. 4 shows the sensitivity to the \(\theta_{23}\) octant determination as a function of \(\sin^{2}2\theta_{13}\) and \(\sin^{2}\theta_{23}\). The areas shaded with light (dark) gray of this figure indicate the regions of parameters where the octant of \(\theta_{23}\) can be determined at 2 (3) standard deviation confidence level, which is determined by the condition \(\chi_{\min}^{2}(\text{wrong octant}) - \chi_{\min}^{2}(\text{true octant}) > 4\) \(9\). The upper (lower) panels correspond to the case where the true hierarchy is normal (inverted). Note, however, that the fit was performed without assuming the mass hierarchy. Since the sensitivity mildly depends on the CP phase \(\delta\), we define the sensitivity to resolving the octant degeneracy in two ways: the left (right) panels correspond to the case where the sensitivity is defined such that the octant is determined for any value of delta (half of the \(\delta\) space). From this figure, we conclude that the experiment we consider here is able to solve the octant ambiguity, if \(\sin^{2}\theta_{23} < 0.38 (0.42)\) or > \(0.62 (0.58)\) at 3 (2) standard deviation confidence level. This conclusion depends weakly on the value of \(\sin^{2}2\theta_{13}\), as well as the value of the CP phase \(\delta\) and the mass hierarchy.

The sensitivity of lifting the octant degeneracy by this setting is quite high even for rather
small values of \(\theta_{13}\) to \(\sin^{2}2\theta_{13} \sim 10^{-3}\) where the mass hierarchy is not determined, a possible
consequence of the decoupling. See Figs. 5 in the next section. The sensitivity depends very
weakly on \(\theta_{13}\) in relatively small values of \(\sin^{2}2\theta_{13}\) where the dominant atmospheric terms are small. The feature of almost independence of the sensitivity to \(\theta_{13}\) should be contrasted with that of the accelerator-reactor combined method in which a strong dependence on \(\theta_{13}\) is expected \(18\). Very roughly speaking the sensitivity by the present method is better than
FIG. 4: 2 (light gray area) and 3 (dark gray area) standard deviation sensitivities to the $\theta_{23}$ octant degeneracy for 0.27 Mton detectors both in Kamioka and Korea. 4 years running with neutrino beam and another 4 years with anti-neutrino beam are assumed. In (a), the sensitivity is defined so that the experiment is able to identify the octant of $\theta_{23}$ for any values of the CP phase $\delta$. In (b), it is defined so that the experiment is able to identify the octant of $\theta_{23}$ for half of the CP $\delta$ phase space.

The latter method in a region $\sin^2 2\theta_{13} \lesssim 0.05 - 0.06$ according to the result given in Fig. 8 of [18]. The sensitivity of our method is also at least comparable to that could be achieved by the high statistics observation of atmospheric neutrinos [27, 28, 29].

VI. REEXAMINATION OF SENSITIVITIES TO NEUTRINO MASS HIERARCHY AND CP VIOLATION

In this section we reexamine the problem of sensitivities to the neutrino mass hierarchy and CP violation achievable by the Kamioka-Korea identical two detector complex. We want to verify that the sensitivities do not depend on which octant $\theta_{23}$ lives, as indicated by our discussion of the decoupling given in Sec. III. It is also interesting to examine how the sensitivities depend upon $\sin^2 2\theta_{23}$. Furthermore, the inclusion of the new systematic error which accounts for difference in the spectral shapes of the neutrino beam between the intermediate and the far detectors makes the reexamination worth to do.
FIG. 5: 2(thin lines) and 3(thick lines) standard deviation sensitivities to the mass hierarchy determination for several values of $\sin^2 \theta_{23}$ (red, yellow, black, green and blue lines show the results for $\sin^2 \theta_{23} = 0.40, 0.45, 0.50, 0.55$ and $0.60$, respectively). The sensitivity is defined in the plane of $\sin^2 2\theta_{13}$ versus CP phase $\delta$. The top and bottom panels show the cases for positive and negative mass hierarchies, respectively. The experimental setting is identical to that in Fig.4.

In Figs. 5 and 6 the regions sensitive to the mass hierarchy and CP violation, respectively, are presented. In both figures, the thin-lines and the thick-lines indicate the sensitivity region at 2 and 3 standard deviations, respectively. As in the previous work [16], 2 (3) standard deviation sensitivity regions are defined by the conditions, $\chi^2_{\text{min}}(\text{wrong hierarchy}) - \chi^2_{\text{min}}(\text{true hierarchy}) > 4$ (9) and $\chi^2_{\text{min}}(\delta = 0 \text{ or } \pi) - \chi^2_{\text{min}}(\text{true value of } \delta) > 4$ (9) for the mass hierarchy and CP violation, respectively.

The sensitivities to the mass hierarchy and CP violation at $\sin^2 \theta_{23} = 0.5$ are almost identical to those obtained in [16]. It is evident that the sensitivities do not depend strongly on $\sin^2 \theta_{23}$ as far as the value is between 0.40 and 0.60. In fact, the mass hierarchy can be determined even if the $\theta_{23}$ octant degeneracy is not resolved. But, the sensitivity to mass hierarchy resolution gradually improves as $\sin^2 \theta_{23}$ becomes larger, as seen in Fig. 5.

It is natural because $\Delta P^{\text{norm inv}}$ in (9), or the appearance probability itself is proportional...
FIG. 6: Sensitivities to the CP violation, $\sin \delta \neq 0$. The meaning of the lines and colors are identical to that in Fig. 5.

to $\sin^2 \theta_{23}$. An alternative way of presenting the same result is to use $s_{23}^2 \sin^2 2\theta_{13}$ for the ordinate. An approximate scaling behavior is observed as expected by $\Delta P_{\text{norm inv}}$ in (9).

VII. SUMMARY AND DISCUSSION

In this paper, we have shown that a setting with two identical water Cherenkov detectors of 0.27 Mton fiducial mass, one in Kamioka and the other in Korea, which receive almost the same neutrino beam from J-PARC has capability of resolving the $\theta_{23}$ octant degeneracy in situ by observing difference of the solar oscillation term between both detectors. The feature of the sensitivity region indicates that the present method is quite complementary to the reactor-accelerator combined method explored in [18]. Together with the potential for resolution of the intrinsic and the sign-$\Delta m^2_{31}$ degeneracies previously reported in [16] (with confirmation in Sec. [19] by an improved treatment), we have demonstrated that the Kamioka-Korea two detector complex can resolve all the eight-fold neutrino parameter degeneracy
under the assumption that $\theta_{13}$ is within reach by the next generation accelerator experiments and $\theta_{23}$ is not too close to $\pi/4$.

As an outcome of these studies, the strategy toward determination of the remaining unknowns in the lepton flavor mixing can be discussed. It is nice to see that such program can be defined only with the single experiment based on the conventional superbeam technology which does not require long-term R&D efforts, and the well established detector technology. It opens the possibility of accurate determination of the neutrino mixing parameters, $\theta_{23}$, $\theta_{13}$, $\delta$, as well as the neutrino mass hierarchy, by lifting all the eight-fold degeneracy which should merit our understanding of physics of lepton sector.

Our treatment in this paper includes a new systematic error which accounts for possible difference in spectral shape of the neutrino beam received by the two detectors in Kamioka and in Korea. We have shown that, despite the existence of such new uncertainty which might hurt the principle of near-far cancellation of the systematic errors, the capability of determining neutrino mass hierarchy and sensitivity to CP violation are kept intact.

We have also reported a progress in understanding the theoretical aspect of the problem of how to solve the parameter degeneracy. Because of the property phrased as “decoupling between degeneracies” which is shown to hold in a setting that allows perturbative treatment of matter effect, one can try to solve a particular degeneracy without worrying about the presence of other degeneracies. This feature may be contrasted to those of the very long baseline approaches, such as the neutrino factory, in which one would not expect the discussion in this paper to hold.

An alternative but closely related approach toward determination of the global structure of lepton flavor mixing in a single experiment is to utilize an on-axis wide band neutrino beam to explore the multiple oscillation maxima, which may be called the “BNL strategy” \cite{34,35}. This strategy can be applied to the far detector in Korea, as examined by several authors \cite{36,37,38}.\footnote{Very roughly speaking ignoring the issue of backgrounds and assuming the same baseline length, one would expect that wide band beam option is better in sensitivity to the neutrino mass hierarchy, while the same off-axis angle option studied in this paper is advantageous to resolve the $\theta_{23}$ octant degeneracy for which low energy bins are essential.} In this case, however, one needs to understand the energy dependence of the background and the signal efficiency as well as the neutrino interaction cross section precisely for both the intermediate and the far detectors. In particular, since the low energy bins are enriched with neutral current background contamination that comes from events with higher neutrino energies \cite{37}, the cancellation of the systematic errors between the two detectors, which is the key ingredient in our analysis, does not hold. Nonetheless, we emphasize that the potentially powerful method is worth to examine further with realistic estimate of the detector performance.

Finally, we remark that the J-PARC 2.5 degree off-axis beam with the baseline length of 1,000 to 1,250 km should be available in the Korean Peninsula. Therefore, it may be possible to further enhance the sensitivity to the $\theta_{23}$ octant by taking a longer baseline length for the Korean detector. The best baseline length and the detector location should be decided so that the experiment has the best sensitivities to the oscillation parameters, especially to the CP phase $\delta$, mass hierarchy and the octant of $\theta_{23}$.
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