Early-Stage Shear Viscosity far from Equilibrium
via Holography

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Abstract

Shear viscosity is a crucial property of QCD matter which determines the collective behavior of the quark-gluon plasma (QGP) in ultrarelativistic heavy-ion collisions. Extending the near-equilibrium, high-precision investigations in theory and experiment, we take into account the fact that, in a collision, the QGP is generated far from equilibrium. We use the AdS/CFT correspondence to study a strongly coupled plasma and find a significant impact on the ratio of shear viscosity to entropy density, $\eta/s$. In particular, we investigate the initial heating phase and find a decrease reaching down to below 60\% followed by an overshoot to 110\% of the near-equilibrium value. This finding might be highly relevant for the extraction of transport coefficients from anisotropic flow measurements at RHIC and LHC.

Keywords: $\eta/s$, holography, far from equilibrium, quark-gluon plasma, shear viscosity, time-dependent transport
1 Introduction

Heavy-ion collisions are violent processes: Two nuclei collide, heat up above the critical temperature \( T_C \approx 155 \text{ MeV} \), and produce a fireball of deconfined quarks and gluons. This quark-gluon plasma (QGP) finally hadronizes and emits particles which can be measured in particle detectors. Despite the fact that the QGP exists only for some \( \text{fm}/c \), advances in theory and experiment allow to study its properties at high precision. In particular, transport coefficients are under scrutiny. A prime example is the shear viscosity, \( \eta \), which is the dominating factor for elliptic flow, \( v_2 \), of charged particles.

It was the AdS/CFT correspondence which predicted a small and constant value of the shear viscosity for a large class of strongly-coupled gauge theories. When expressed in terms of the entropy density, the value reads \( \eta/s = 1/4\pi \) when expressed in natural units \( (c \equiv \hbar \equiv k_B \equiv 1) \) [1]. Hydrodynamic simulations extract similar values from experimental flow data [2–5]. The theoretical calculation of \( \eta/s \) is challenging because the QGP is non-dilute and strongly coupled. Most notable are the temperature-dependent results by lattice QCD [6] and the functional renormalization group (FRG) [7]. However, these approaches are restricted to the near-equilibrium regime. We aim at overcoming this limitation and improving the description of the early collision phase [8] which is far from equilibrium [9]. We apply the AdS/CFT correspondence and focus on the initial heating phase.

2 Holographic Setup

The appeal of the AdS/CFT correspondence in the field of heavy-ion physics is the equivalence of a thermal field theory state to a black brane configuration, a black hole with planar horizon. Technically speaking, the AdS/CFT correspondence identifies certain pairs of strongly coupled gauge theories at large gauge group rank and supergravity on spacetimes with Anti-de Sitter asymptotics. The gravitational spacetime is referred to as “bulk” while the field theory lives on its “boundary”: Introducing the inverse radial direction \( z \), the plasma is located at \( z = 0 \), the black brane horizon lies at \( z_h \), and the singularity is found at \( z \to \infty \), cf. Fig. 1. The thermodynamic state variables of plasma and black brane are closely related. In equilibrium, the field-theory temperature equals the black brane Hawking temperature, and the entropy agrees with the Bekenstein-Hawking entropy.

\[ \begin{align*}
\text{field theory} & \quad \rightarrow \quad \text{black brane} \\
\text{initial equilibrium} & \quad \rightarrow \quad \text{sudden energy deposition} \\
\rightarrow & \quad \rightarrow \\
\text{final equilibrium} & \quad \rightarrow
\end{align*} \]

Figure 1: Sketch of the holographic system during the time evolution through the far from equilibrium regime. The coordinate \( z \) denotes the inverse radial coordinate, the black brane horizon is located at \( z_h \). The rapid energy deposition in the boundary theory corresponds to a rapid mass increase of the black brane.
On the field-theory side, we extend the calculation of $\eta/s$ of a strongly coupled plasma to the far-from-equilibrium regime. We consider a sudden homogeneous energy deposition as a model for the heating phase during a heavy-ion collision. On the gravity side, this is realized by a rapid mass accretion onto a black brane. We use a Reissner-Nordström Vaidya black brane which is a solution of Einstein’s general relativity coupled to Maxwell theory [10–12]. The line element in infalling Eddington-Finkelstein coordinates reads

\[ ds^2 = \frac{1}{z^2} \left( -f(v,z) \, dv^2 - 2 \, dv \, dz + dx^2 + dy^2 \right). \]  

(1)

The coordinate $v$ denotes the ingoing null time and agrees with the ordinary field-theory time $t$ at the boundary, $z = 0$. The blackening factor $f(v,z) = 1 - 2G_N M(v) \, z^3 + G_N Q^2(v) \, z^4$ depends on the black brane’s mass and charge parameters, $M(v)$ and $Q(v)$, and determines the horizon position. The rapid mass infall is given as

\[ M(v) = m + m_s \frac{(1 + \tanh(v/\Delta t))}{2}. \]  

(2)

The extension of the thermodynamic variables to the time-dependent regime is non-trivial. We use the grand potential, provided by the AdS/CFT correspondence, to define the field-theory temperature and entropy density, $T(t)$ and $s(t)$, in terms of gravitational degrees of freedom.

3 Spacetime Perturbations and $\eta/s$

According to the AdS/CFT correspondence, the bulk metric is dual to the energy-momentum tensor of the field-theory plasma. In particular, the ring-down of a geometry perturbation $h_{mn}$ yields the evolution of the expectation value $\langle T^{\mu\nu}(t) \rangle_h$. Applying linear response theory, we obtain the retarded Green’s function, $G^{xy,xy}_R(t_p, t_2)$, from the 1-point function $\langle T^{xy}(t_2) \rangle_h$ if the perturbation is localized at a time $t_p$:

\[ \langle T^{xy}(t_2) \rangle_h = \int d\tau G^{xy,xy}_R(\tau, t_2) \frac{h_{xy}(0)}{\delta(\tau-t_p)} \propto G^{xy,xy}_R(t_p, t_2). \]  

(3)

The solution to the evolution of the metric fluctuation is found numerically [13, 14]. A pseudospectral method is used on the spatial part, while a fourth-order Runge-Kutta scheme evolves the system in time. Three samples are presented in Fig. 2.

We arrive at a momentum-space version of the retarded Green’s function, $\tilde{G}^{xy,xy}_R(t_{avg}, \omega)$, by a Wigner transformation. It depends on the frequency $\omega$, which is the momentum-space conjugate of the relative time, $t_p - t_2$, and on the average time, $t_{avg} = (t_p + t_2)/2$ [15]. The time-dependent shear viscosity is defined by the time-dependent Kubo formula,

\[ \eta(t_{avg}) = -\lim_{\omega \to 0} \frac{1}{\omega} \Im \tilde{G}^{xy,xy}_R(t_{avg}, \omega). \]  

(4)

At RHIC, a temperature increase to 310 MeV within 0.3 fm is typical for the early phase of a collision at $\sqrt{s_{NN}} = 200$ GeV [16]. Figure 3 presents the corresponding evolution of $\eta/s$. Asymptotically, the curve takes the near-equilibrium value which amounts to $1/4\pi$ in our case 4. During the far-from-equilibrium period, however, there are significant corrections: The value of $\eta/s$ reaches down to below 60% and increases to 110% of the near-equilibrium result.

4 A common misunderstanding should be pointed out: The number $1/4\pi$ is not a universal lower bound, neither for all quantum field theories with a gravity dual, nor for every quantum field theory at strong coupling. However, every near-equilibrium holographic model studied thus far has a lower bound, e.g. Ref. [17].
4 Conclusions and Outlook

We presented the first holographic non-equilibrium calculation of $\eta/s$ via the retarded Green’s function. Far from equilibrium, the viscosity-to-entropy ratio changes drastically. We expect comparable corrections to apply to the near-equilibrium results of FRG and lattice QCD. Our findings directly impact hydrodynamic simulations and the extraction of viscosity from experimental data.
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