Diffractive $\rho$-meson Leptoproduction on Polarized Nucleon

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Abstract

The amplitude of diffractive $\rho$ (and open quark) leptoproduction on a polarized target is calculated in the leading log approximation of pQCD using the hadron-parton duality hypothesis. The spin-spin asymmetry is expressed in terms of the spin dependent gluon and quark structure functions in the small $x$ region. Therefore the $\gamma^* + p \to \rho + p$ reaction provides a promising tool to study the spin dependent gluon distribution $\Delta G(x, q^2)$.

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1 Introduction

In ref. [1] it was demonstrated that based on the hadron-parton duality one can describe rather well the diffractive $\rho$-meson electroproduction on the unpolarized target in the framework of perturbative QCD, not only for the longitudinal part of the cross section but also for the transverse one. At large energies (i.e. in the small $x$ region) the amplitude of this process is proportional to the gluon density and the cross section is proportional to the gluon density squared.

Correspondingly, on the polarized nucleon the spin dependent part of the amplitude should be given in terms of the polarized parton densities $\Delta G$ and $\Delta q$. In the present paper, the same hadron-parton duality hypothesis will be used to calculate the spin dependent part of the amplitude of diffractive $\rho$-leptoproduction within the Leading Log Approximation (LLA) of perturbative QCD.[2]

First we recall the hadron-parton duality hypothesis. Let us consider the amplitude of the open $q\bar{q}$ leptoproduction, project it on the state with the spin $J_{PC} = 1^{-+}$ (isospin $I_{G} = 1^{+}$) and average over a mass interval $\Delta M_{q\bar{q}}$ (typically $\sim 1$ GeV$^2$) in the region of $\rho$ mass. In this domain the more complicated partonic states ($q\bar{q} + g$, $q\bar{q} + 2g$, $q\bar{q} + q\bar{q},...$) are suppressed, while on the hadronic side the $2\pi$ states are known to dominate. Thus for low $M^2$ we mainly have $\gamma^* \rightarrow q\bar{q} \rightarrow 2\pi$ or in other words

$$\sigma(\gamma^* p \rightarrow \rho p) \simeq \sum_{q=u,d} \int_{M_a^2}^{M_b^2} \frac{\sigma(\gamma^* p \rightarrow (q\bar{q})')}{dM^2} dM^2$$

(1)

where the limits $M_a^2$ and $M_b^2$ are chosen so that they appropriately embrace the $\rho$-meson mass region and $(q\bar{q})'$ means the projection of the $q\bar{q}$ system onto the $J_{PC} = 1^{-+}$, $I_{G} = 1^{+}$ state.

The procedure of the projection onto the $J_{PC} = 1^{-+}$ state is described in sect. 2. Then in sect.3 we calculate the spin dependent Born amplitude for $\gamma^* p \rightarrow q\bar{q} p$ process in the small $x = (Q^2 + m_p^2)/s \ll 1$ region; here $Q^2 = -q^2$ is the heavy photon virtuality and $s = (q + p)^2 = W^2$ is the photon-proton energy squared. Taking into account the leading log$Q^2$ corrections within the LLA in sect.4 we express the result in terms of the parton distributions

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1 An analogous calculation for $J/\Psi$ was done in [2] assuming a nonrelativistic $J/\Psi$ meson wave function.
\( \Delta G \) and \( \Delta q \) and compare it with the formulae for the unpolarized case. This way the spin-spin asymmetry for the diffractive open quark production is calculated. Finally in sect. 5 we discuss the asymmetry for \( \rho \)-meson production.

2 Projection onto the \( J^{PC} = 1^{--} \) state

The cross section of diffractive open quark leptoproduction

\[
\frac{d\sigma(\gamma^* p \rightarrow (q\bar{q})p)}{dt dM^2} = 4\pi e_q^2 N_c \sum_\lambda \int d^2k_t dz \frac{1}{16\pi^3} \delta \left( M^2 - \frac{k_t^2 + m_q^2}{z(1-z)} \right) \frac{|M_\lambda|^2}{16\pi s^2}, \tag{2}
\]

where: \( e_q \) is the electric charge of the quark (say, \( e_u = \frac{2}{3}\sqrt{\frac{1}{137}} \)), \( N_c = 3 \) is the number of colours while \( k_t^* \) and \( z \) are the transverse momentum and photon momentum fraction carried by the quark; \( m_q \) is the quark mass, \( \lambda \) is the quark helicity and \( M_\lambda \) is the amplitude of the process.

To decompose the cross section onto the definite \( (q\bar{q}) \) spin \( (J^P) \) state one can integrate over \( d^2k_t \) and write

\[
\frac{d\sigma(\gamma^* p \rightarrow (q\bar{q})p)}{dt dM^2} = 4\pi e_q^2 N_c \sum_\lambda \int dz \frac{z(1-z)|M_\lambda|^2}{16\pi s^2} \sum_{Jm} e_q^2 N_c \frac{|C_{Jm}^J|^2}{(16\pi)^2} \tag{3}
\]

with the coefficients \( C_{Jm}^J \) given by the projection of the amplitude with the help of the conventional spin rotation matrices \( d_{Jm,m'}^J(\theta) \)

\[
C_{jm}^J = \int_{-1}^{1} d_{Jm,m'}^J(\theta) \frac{M_\lambda}{s} z(1-z) \sqrt{2J+1} \sqrt{2J+1} d\cos \theta. \tag{4}
\]

The factor \( \sqrt{z(1-z)} \) comes from the denominator in the \( \delta \)-function after the \( dk_t^2 \) integration of (2) while the \( \sqrt{2J+1} \) reflects the normalization of the spin rotation matrices

\[
\int_{-1}^{1} |d_{Jm,m'}^J(\theta)|^2 d\cos \theta = \frac{2}{2J+1},
\]

\( m \) and \( m' \) are the projection of the spin \( J \) onto the initial photon or quark axis respectively; the value of \( m \) is given by the helicity of heavy photon,
while for the quark helicity $\lambda = \pm 1/2$ the value of $m' = (\lambda + \lambda') = \pm 1$ due to helicity conservation of the $\gamma \to q\bar{q}$-vertex; $\theta$ is the quark polar angle which may be expressed in terms of $z$ as $z = (1 + \cos \theta)/2$.

The projection onto the isospin $I^G = 1^+$ state gives just the factor 0.9 in accordance with the well known $\omega : \rho = 1 : 9$ ratio in Vector Dominance Model.

3 Born amplitude for the open quark production

3.1 Gluon exchange

Let us consider for the beginning interaction with the quark target. The Born amplitude of the diffractive reaction $\gamma^* + q \to (q\bar{q}) + q$ is described by the sum of the diagrams shown in fig.1.

At large energies $s = (q + p)^2 \gg |q^2| + m^2$ the main contribution comes from the longitudinal polarizations of t-channel gluons ($l$ and $l + Q_{tr}$ in fig.1), i.e. the spin part of gluon propagator is given by

$$g_{\rho\sigma} = \frac{1}{g_{\rho\sigma}} + \frac{p'_\rho q'_\sigma + q'_\rho p'_\sigma}{(p'q')} \simeq \frac{p'_\rho q'_\sigma}{(p'q')}$$

with

$$q_\mu = q'_\mu + \frac{q^2}{s} p'_\mu; \quad p_\mu = p'_\mu + \frac{m_N^2}{s} q'_\mu \simeq p_\mu; \quad s = 2(p'q'); \quad p'^2 = q'^2 = 0.$$ 

Here $m_N$ is the target (quark or nucleon) mass, $p$, $p''$ and $q$ are the 4-momenta of the target (initial quark or proton), recoil quark (or proton) and photon correspondingly and we assume that the momentum transferred $Q_{tr}^2 = (p - p'')^2$ is small. Indeed, in the forward direction the transverse component $Q_{tr,t} = 0$ and the longitudinal part gives $Q_{tr}^2 = t_{min} = -x^2 m_N^2 \to 0$ for $x \ll 1$.

However the longitudinal t-channel (Coulomb-like) gluon looses the information about the polarization of the target. Thus at least one gluon must have transverse polarization. Indeed for the longitudinally polarized target with spin vector $s_\mu \parallel p_\mu$ the spin dependent part of the trace in the bottom of
the diagram fig.1 (the target loop) looks like

\[ B_{\sigma\sigma'} = \frac{1}{2} \text{Tr}[\hat{\gamma}_5 \gamma_\sigma (\hat{p} + \hat{l}) \gamma_{\sigma'}] = -2i \epsilon^{\sigma\sigma'\alpha\beta} p_\alpha p_\beta \]  

where \( \epsilon^{\sigma\sigma'\alpha\beta} \) is the antisymmetric tensor and \( \sigma (\sigma') \) corresponds to the polarization of the left (right) t-channel gluon in fig.1. (Considering the forward amplitude we put the momentum transfer \( Q_{\text{tr}} \approx 0 \).)

As \( p_\mu \) is the longitudinal vector while the \( l_\mu \approx l_{t\mu} \), two other indices \( \sigma, \sigma' \) should be the transverse (say, \( g_{\rho\sigma} = g_{\rho\sigma}^\perp \)) and the longitudinal \( g_{\rho\sigma'} \) ones. So the spin dependent contribution of the lower part of the graph takes the form

\[ g_{\rho\sigma} B_{\sigma\sigma'} g_{\rho\sigma'} = B_{\rho'\rho} \approx 2(e_{\rho}^\perp p_{\rho'}^\perp - p_{\rho}^\perp e_{\rho'}^\perp)/l^4. \]

Here the gluon polarization vector \( e_{\nu}^\perp = i \epsilon_{\nu\mu}^\perp l^\mu \) (see Eq.(5)); \( \epsilon_{\nu\mu}^\perp \) is the two-dimensional antisymmetric tensor acting in the transverse plane. The denominator \( l^4 \) comes from the two gluon \( (l + Q_{\text{tr}}) \) propagators (recall that in the forward direction \( Q_{\text{tr}} \rightarrow 0 \)).

To find the contribution corresponding to the upper quark loop we use the formalism of ref. [3, 4] and calculate first the matrix element of the \( \gamma \rightarrow q\bar{q} \) transition putting both the quarks (with helicities \( \lambda \) and \( \lambda' = \pm 1/2 \)) on mass shell.

For the transversely polarized photon the matrix element reads

\[ \Psi_{\lambda\lambda'}(k'_t, z) = \frac{1}{2\sqrt{z(1-z)}} \bar{u}_\lambda(\gamma_\mu \cdot E^\pm) v_{\lambda'} = \frac{\delta_{\lambda\lambda'}}{2z(1-z)} (E^\pm k'_t) [2\lambda(1 - 2z) \mp 1], \]

where \( E^\pm = \frac{1}{\sqrt{2}} (0, 1, \pm i, 0) \) is the photon polarization vector; the first factor \( 1/2\sqrt{z(1-z)} \) reflects the normalization \( (1/\sqrt{2E_j}; \ j = 1, 2) \) of the quark fields.

The quark-gluon vertex conserves the helicity of the high energy quark and the momentum fraction \( z \) is not changed either. Therefore only the transverse momentum of the quark \( k'_t \) in Eq.(7) may be different from the final quark momentum \( k_{jt} \). In order to simplify the projection onto the \( J^P = 1^- \) state we use the momentum \( k_t \) in the rest frame of the \( q\bar{q} \)-system (the \( z \) axis is directed along the target proton momentum) and put the light quark mass \( m_q = 0 \). So the value of \( k_{jt} = (M/2) \sin \theta \) (where \( M = M_{q\bar{q}} \) is the mass of the \( q\bar{q} \) pair and \( \theta \) is the quark polar angle).
For the case of the longitudinal polarizations \((e^\parallel = p'_\parallel)\) of both t-channel gluons the quark-gluon vertex \(\bar{u}((k'_\mu)\gamma_{\mu}\cdot p'_{\mu})u(k'_\mu-l) = ((2k'_\mu-l)\cdot p')\) looks as the emission of a soft \((z_g \ll 1)\) gluon by a classical colour charge (i.e. by the \(\text{‘current’} \ j_\mu = 2k_{j\mu}\)) giving the factor \(2(p' \cdot k'_j) = z_j s\) for each vertex. This factor \(z_j\) cancels the normalization factors \((1/\sqrt{z_j})\) of the quark fields. So the contribution of the upper part of Feynman diagram shown in fig.1a takes the form \[\Phi_\lambda = \frac{\delta_{\lambda,\lambda'}}{Q^2 + k'^2_\perp}(E^\pm k'_\perp)[2\lambda(1 - 2z) \mp 1].\] (8)

The new factor \(1/(\tilde{Q}^2 + k'^2_\perp)\) \((\tilde{Q}^2 = z(1 - z)Q^2)\) comes from the energy denominator \(\Delta E = E_{q\bar{q}} - E_{\gamma*} = \frac{z(1-z)Q^2 + k'^2_\perp}{z(1-z)}\) and corresponds to the propagator of quark \(k'\).

However in our case one of the t-channel gluons has the transverse polarization \(e^\perp_\mu = i\epsilon^\perp_{\mu\nu\lambda\mu}(\text{see Eq.(2)})\). When this gluon couples to the quark line it gives the factor \(2(e^\perp \cdot k'_j)\) instead of the old one \(2(p' \cdot k') = z s\) for a longitudinal gluon polarization vector \(e^\parallel_\mu = p'_\parallel\). Note that, as it was confirmed by the explicit calculation of the Trace, within the LLA the quark-gluon vertex may be written as the emission of a soft \((z_g \ll 1)\) gluon by a classical colour charge (i.e. by the \(\text{‘current’} \ j_\mu = 2k_{j\mu}\)) for the polarized case also.

Thus for the spin dependent part of the amplitude fig.1a in comparison with Eq.(8) one looses the factor \(z_s\) but gets the \(4(e^\perp \cdot k'_j)\); an extra factor of two reflects the contributions of two terms in the expression (6) (due to the permutation of gluon polarizations).

To select the leading logarithm in the \(dl^2\) integral (coming from the region of \(l^2 \ll \tilde{Q}^2\)), we have to separate in the numerator the term proportional to \(l^2\). There is no such term in the fig.1a contribution, but in the case of fig.1b, where \(k' = k_1 + l \equiv k + l\), one can take the product \((E^\pm \cdot l_i)(e^\perp \cdot k_i)\) and put \(k' \simeq k\) in the denominator.

So the amplitude corresponding to the fig.1b takes the form

\[M^b_\lambda = \frac{2}{9}4\pi s \int \frac{dl^2}{l^4} \alpha_s^2 \frac{[2\lambda(1 - 2z) \mp 1]}{Q^2 + k'^2_\perp} \frac{2(E^\pm l_i)(e^\perp \cdot k_i)}{z s}.\] (9)

\(^2\) The \((k_i \cdot l_i)\) term coming from the expansion of the denominator \(Q^2 + (k + l)^2\) does not contribute as, after the averaging over the \(l_i\) direction (in the azimuthal plane), the product \((e^\perp \cdot k'_j)(k_i \cdot l_i)\) gives zero. Indeed, the integral over the azimuthal angle \(\phi\) reads \(\int (k_\mu e_{\mu\perp} l_\nu)(k_\beta l_\beta) d\phi = 0.\)
where $\frac{2}{9}$ is the colour coefficient.

Adding the contribution of another graph where the left t-channel gluon couples to the antiquark with $z_2 = 1 - z$ (instead of $z_1 = z$ for the quark) and neglecting the $k_t^2 < M^2/4 \ll Q^2$ in our small ($M^2 \sim m_q^2$) mass region one finally gets

$$M_{\lambda} = \frac{2}{9} \int \frac{dl_t^2}{l_t^2} \alpha_s^2 \frac{2\lambda(1 - 2z)}{Q^4} \mp \frac{1}{x}(E_\mu^\pm \cdot \epsilon_\perp k_t)$$

Here we average over the direction of vector $l_t$ in azimuthal plane and the equalities $1/z + 1/(1 - z) = 1/z(1 - z)$ and $\frac{1}{z(1 - z)x} = \frac{Q^2}{xQ^2} = x/Q^2$ (with $x = Q^2/s$) is used.

### 3.2 Quark exchange

Note that for the unpolarized case the amplitude given by the gluon exchange was proportional to the gluon density $xG(x, Q^2)$ and in the Born approximation it tends to $\text{const}$ as $x \to 0$. The quark exchange corresponds (in the same Born approximation) to the quark structure function which was negligible ($xq(x, Q^2) \to 0$) at small $x \to 0$.

In contrast, the spin dependent part of the amplitude (10) is proportional to $x$, reflecting the fact that the polarized gluon distribution $x\Delta G(x, Q^2) \propto x$ (modular the $\alpha_s$ corrections) in the small $x$ region. The quark distribution $x\Delta q(x, Q^2) \propto x$ possesses the same property. Indeed, in the polarized DGLAP evolution the splitting kernels $\Delta P_{G\bar{G}}(z)$, $\Delta P_{Gq}(z)$ and $\Delta P_{qq}(z)$ have no $1/z$ singularity (such a singular term in $P_{G\bar{G}}(z) \approx 2N_c/z$ provides the $1/x$ behaviour of the unpolarized gluons $G(x, Q^2) \propto 1/x$).

Therefore now we can not omit the quark exchange amplitude. In other words after the $\gamma \to q\bar{q}$ transition one has consider not only the quark-quark scattering (shown in fig.1) but the antiquark-quark annihilation (see fig.2) as well. Thanks to the helicity conservation, in the forward direction ($Q_{t\bar{t}} \equiv 0$) the amplitude shown in fig.2 conserves the helicity of initial quark line $[3, 4]$; it contains the factor $\delta_{\lambda_i, \lambda_f} \delta_{\lambda_i, \lambda_f}$, where: $\lambda_i = \lambda'$ and $\lambda_f$ are the helicities of the initial and final antiquark whilst $\lambda_t$ denotes the helicity of target quark. The momentum fraction $z$ of the fast antiquark is also not changed. From this point of view both the gluon and the quark exchange act in the same way. The only difference is the precise form of the quark-quark amplitude.
In the small $x$ region, the leading logarithmic contribution of the quark-antiquark annihilation amplitude is well known. The expression for $T$ is given by:

$$T = 2\pi \frac{C_F^2}{N_c} \int \frac{d^2 l^2}{l^2} \alpha_s^2 \left( \frac{C_F \alpha_s}{\pi} \right) \frac{2\lambda(1-2z) \mp 1}{Q^2} \frac{(E^\pm \cdot k_t)}{(1-z)s}.$$  

(with $C_F = \frac{N_c^2-1}{2N_c}$).

Thus the contribution of the Feynman diagram shown in fig.2 (fast antiquark with the momentum fraction $z_2 = 1 - z$ and $k_{2t} = -k_t$ annihilates with the target quark) reads:

$$\frac{M^q_\lambda}{s} = -\delta_{\lambda,\lambda'} \delta_{\lambda',\lambda''} \frac{8}{9} \frac{1}{\pi^2} \int \frac{d^2 l^2}{l^2} \alpha_s \left( \frac{C_F \alpha_s}{\pi} \right) \frac{2\lambda(1-2z) \mp 1}{Q^2} \frac{(E^\pm \cdot k_t)}{(1-z)s}.$$  

With the help of the relation $1/((1-z)s) = zx/Q^2$ it may be written as:

$$\frac{M^q_\lambda}{s} = -\delta_{\lambda,\lambda'} \delta_{\lambda',\lambda''} \frac{16}{9} \frac{1}{\pi^2} \int \frac{d^2 l^2}{l^2} \alpha_s \left( \frac{C_F \alpha_s}{2\pi} \right) \frac{2\lambda(1-2z) \mp 1}{Q^4} zx(E^\pm \cdot k_t).$$

4 Spin-spin asymmetry

4.1 Leading logarithmic corrections

In order to make the (Born) calculation more realistic we have to include the 'ladder evolution' gluons (shown symbolically by the dashed lines in fig.1,2) and to consider the process at the proton, rather than the quark, level; i.e. the heavy photon diffractive dissociation on the proton target. This is achieved by the replacements:

$$C_F \int \frac{\alpha_s d^2 l^2}{\pi l^2} \rightarrow \Delta G(x, Q^2),$$

for the gluon exchange amplitude and

$$C_F \int \frac{\alpha_s d^2 l^2}{2\pi l^2} \rightarrow \Delta q(x, Q^2)$$

for the antiquark-target quark annihilation.

Indeed, the logarithmic $d^2 l^2/l^2$ integration in Eq. (10) is nothing else but the first step of the DGLAP evolution of the spin dependent gluon distribution $\Delta G(x, \bar{q}^2)$ with the splitting function $\Delta P_{Gq} = C_F \frac{1-(1-z)^2}{z} \sim 2C_F$ for
Correspondingly, in the quark case of Eq. (13) this is the first step of the DGLAP evolution of the spin dependent quark distribution $\Delta q(x, \bar{q}^2)$ with the splitting function $\Delta P_{qq} = C_F \frac{1+z^2}{1-z} \simeq C_F$ for $z \ll 1$.

Strictly speaking, even at zero transverse momentum $Q_{tr,t} = 0$ one does not obtain the exact gluon structure function, as a non-zero component of the longitudinal momentum is transferred through the two-gluon (two-quark) ladder. However, in the region of interest, $x \ll 1$, the value of $|t_{min}| = m_N^2 x^2$ is so small that we may safely put $t \equiv Q_{tr,t}^2 = 0$ and identify the ladder coupling to the proton with $\Delta G(x, \bar{Q}^2)$ (or with $\Delta q(x, \bar{Q}^2)$ in the quark case); see [9] for more details. The arguments of the parton structure function should be $x = (|Q^2| + M^2)/s$ and $\bar{Q}^2$.

Now in the leading log($Q^2$) approach the spin dependent parts of the amplitudes (10,13) read

$$\frac{M^G_\lambda}{s} = \frac{2}{3} \pi^2 \alpha_s x \Delta G(x, \bar{Q}^2) \frac{2\lambda(1-2z) \mp 1}{Q^4} (E^\pm \cdot i \epsilon_{\mu \nu} k_\nu),$$

$$\frac{M^q_\lambda}{s} = -\frac{16}{9} \pi^2 \alpha_s x q_\lambda(x, \bar{Q}^2) \frac{2\lambda(1-2z) \mp 1}{Q^4} z(E^\pm \cdot k_t).$$

and

$$\frac{M^{\bar{q}}_\lambda}{s} = 16 \pi^2 \alpha_s x \bar{q}_\lambda(x, \bar{Q}^2) \frac{2\lambda(1-2z) \mp 1}{Q^4} (1-z)(E^\pm \cdot k_t)$$

for the antiquark component of the target wave function.

In the last two equations (17,18) the helicity of fast quark (antiquark) is strongly correlated with the target helicity ($\lambda = \lambda_{target}$ see Eqs. (12,13)) and the polarized quark distribution $\Delta q = g_{\lambda=1/2} - g_{\lambda=-1/2}$. On the other hand in Eq. (16) there is no such a correlation and calculating the cross section with fixed helicities of the initial photon and proton one has to sum two contributions ( with $\lambda = 1/2$ and $\lambda = 1/2$).

### 4.2 Asymmetry for diffractive open quark production

In the same notation the forward amplitude for the unpolarized case (and a transversely polarized photon) looks like:

$$\frac{M^{sym}_\lambda}{s} = 4 \frac{2}{3} \pi^2 \alpha_s x G(x, \bar{Q}^2) \frac{2\lambda(1-2z) \mp 1}{Q^4} (E^\pm k_t).$$
In other words the whole amplitude is proportional to

\[ M_\lambda \propto i \left( G(x, \bar{Q}^2) E^\pm_\mu g^\perp_{\mu\nu} + \frac{1}{4} \Delta G(x, \bar{Q}^2) \cdot E^\pm_\mu i\epsilon^\perp_{\mu\nu} \right) - z \frac{2}{3} g_\lambda(x, \bar{Q}^2) E^\pm_\mu g^\perp_{\mu\nu} + (1 - z) \frac{2}{3} \bar{g}_\lambda(x, \bar{Q}^2) E^\pm_\mu g^\perp_{\mu\nu} \right) k_{\nu} \cdot \] (20)

Note that \( E^\pm_\mu \epsilon^\perp_{\mu\nu} = \pm E^\pm_\nu \). Therefore the \( \Delta G(x, \bar{Q}^2) \) term changes the sign when one reverses the helicity of incoming photon. For the quark case the amplitudes (17,18) change the sign either due to the sign of \( \Delta q = q^+ - q^- \) or due to the second \((\mp1)\) term in the numerator corresponding to the photon helicity.

In terms of the spin-spin asymmetry \( A = A_{LL} \), eq.(20) means that for the heavy photon "elastic" diffractive dissociation \( \gamma^* + p \rightarrow (q\bar{q}) + p \) the asymmetry is given by

\[ A = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\downarrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\downarrow\uparrow}} = 2 \frac{\frac{1}{2} \Delta G(x, \bar{Q}^2)G(x, \bar{Q}^2)}{(G(x, \bar{Q}^2))^2 + (\frac{1}{4} \Delta G(x, \bar{Q}^2))^2} \approx \frac{\Delta G(x, \bar{Q}^2)}{2G(x, \bar{Q}^2)} \] (21)

The arrows indicate the helicities of the incoming photon and the target nucleon and for the moment we omit the quark contribution in Eq. (21).

As in the case of \( J/\Psi \) lepton production an extra factor 2 in Eq. (21) comes in because the cross section of the diffractive process is proportional to the parton (gluon) density squared. Note, however, that due to the 'Fermi motion' which washes out the effect, the effective analyzing power of the \( \rho \)-meson production turns out to be 4 times smaller in comparison with the nonrelativistic \( J/\Psi \)-meson (see [2]).

The whole expression in the limit of \( G(x, \bar{Q}^2) \gg \Delta G(x, \bar{Q}^2) \) and \( G(x, Q^2) \gg \Delta q(x, \bar{Q}^2) \), \( \Delta \bar{q}(x, \bar{Q}^2) \) (which is reasonable at small \( x \)) takes the form

\[ A_{\gamma^* \rightarrow q\bar{q}} \simeq \frac{\Delta G(x, \bar{Q}^2)}{2G(x, \bar{Q}^2)} + \frac{2[z^2 - (1 - z)^2]}{3[z^2 + (1 - z)^2]} \frac{(1 - z) \Delta \bar{q}(x, \bar{Q}^2) - z \Delta q(x, \bar{Q}^2)}{G(x, \bar{Q}^2)} \] (22)

\(^3\)On the other hand changing the helicity of the target we get \(-i\epsilon^\perp_{\mu\nu}\) instead of \(+i\epsilon^\perp_{\mu\nu}\) (see Eq. (5))

\(^4\)The polarization of the photon emitted by the 100% polarized initial lepton in DIS is \( P_{\gamma^*} = \frac{1 - (1 - y)^2}{1 + (1 + y)^2} \), where \( y \) is the lepton momentum fraction carried by the photon in the target proton rest frame.
It is anticipated that the main part of the corrections (NLO contributions) are cancelled when calculating the asymmetry by Eqs. (21,22). For example, the uncertainty coming from the value of the QCD coupling constant (more exactly from the scale at which $\alpha_s$ is evaluated) do cancel in the ratio Eq.(22). Thus the accuracy of the expressions (21,22) for the asymmetry is expected to be even better than that of the unpolarized diffractive amplitude.

5 Discussion

In order to obtain the spin-spin asymmetry for the case of diffractive $\rho$-meson production we have to project the amplitudes (16-18) onto the spin $J^P = 1^-$ state as it was discribed in sect.2 (see Eq. (4)).

Of course at the begining the photon produces the $q\bar{q}$ pair in a pure $J^P = 1^-$ state with the $\theta$ distribution $(1 \pm \cos \theta)/2 = d_{1, \pm 1}(\theta)$. Indeed, the $\theta$ distribution is given by the factor $[2\lambda(1-2z) \mp 1]$ in the matrix element (7) and for, say, photon helicity $+1$ ($E^+$) and $\lambda = 1/2$ it is equal to $-2z = -(1 + \cos \theta)$. However, interaction with the target distorts the $q\bar{q}$ state due to the $Q^2$ dependence of the structure functions and an extra factor $\tilde{Q}^2 = z(1-z)Q^2 = \frac{\sin^2 \theta}{4}Q^2$ in the denominators of Eqs. (16-18).

Strictly speaking the integral over $\cos \theta$ (4) should be done numerically with the concrete parton distributions (in the amplitudes $M_{\lambda}$). Instead of this let us discuss the singularities and the main structure of the integral (4).

In the scaling limit (parton densities have no $Q^2$ dependence) the singularities at $\cos \theta = \pm 1$ (i.e. $z \to 0$ or $z \to 1$) comes from the $z^2(1-z)^2$ factor in the denominators $\bar{Q}^4$ of Eqs. (16-18). Note that at fixed mass $M_{\bar{q}q} = M$ the value of transverse momentum $k_t = \frac{M}{2} \sin \theta = \sqrt{z(1-z)}M$ and after the $J^P = 1^-$ projection the integral (4) has only the logarithmic singularity (in the small $Q^2 = z(1-z)Q^2$ region) which is usual for conventional deep inelastic process.

Any small anomalous dimension $\gamma > 0$ which reflects the $q^2$ behaviour of the partons (say, $G(x, q^2) \sim (q^2)^\gamma$) is enough to provide the convergence of the integral.

If, for simplicity, we assume that in the whole essential region of $q^2$ the small $x$ parton distribution may be parametrized as $f(x, q^2) = f(x, q^2_0)(q^2/q^2_0)^\gamma$ with the fixed value of $\gamma = const$, it is easy to calculate the coefficients $C_{jm}^I$. 

11
using the identity
\[
\int_0^\pi \sin^\mu \theta \, d\theta = \sqrt{\pi} \frac{\Gamma\left(\frac{3}{2} + \frac{1}{2}\mu\right)}{\Gamma(1 + \frac{1}{2}\mu)} \tag{23}
\]
or for the case of small \(\gamma \ll 1\) just to put
\[
\int_0^1 \frac{dz}{z^{1-\gamma}} = \frac{1}{\gamma}
\]
Note that due to the factors \(z\) or \(1 - z\) in the quark exchange amplitudes (17,18), we have only one singular point: \(z \to 1\) at \(\lambda = \frac{1}{2} \lambda_{\text{photon}}\) for the case of fast antiquark-quark annihilation (17) and \(z \to 1\) at \(\lambda = -\frac{1}{2} \lambda_{\text{photon}}\) for the case of Eq. (18). It leads to a numerical suppression of the quark exchange contribution in comparison with the gluon exchange one where for any \(\lambda = \pm 1/2\) one has the singularity at both points \(z \to 1\) and \(z \to 0\).

Thus in the small \(\gamma \ll 1\) limit the asymmetry for the diffractive \(\rho\) production reads
\[
A_{\gamma^* \to \rho} \simeq \gamma(G)\Delta G(x, \bar{Q}^2) - \frac{\gamma(q)\Delta q(x, \bar{Q}^2)}{2\gamma(\Delta q)} - \frac{\gamma(\bar{q})\Delta \bar{q}(x, \bar{Q}^2)}{6G(x, \bar{Q}^2)} \tag{24}
\]
where \(\gamma(f)\) denotes the anomalous dimension of the structure function \(f = G, \Delta G, q, \bar{q}, \Delta q, \Delta \bar{q}\).

Hopefully the expression (24) gives us some impression about the expected value of asymmetry \(A_{\gamma^* \to \rho}\) and may be used to estimate the effect in the future experiments.

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**Figure Captions**

**Fig. 1** Diffractive open \(q\bar{q}\) production in high energy \(\gamma^* p\) collisions via the two gluon exchange.
Fig. 2 Diffractive open $q\bar{q}$ production in high energy $\gamma^*p$ collisions via the quark exchange.

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Fig. 2