Numerical simulation of water hammer and cavitation phenomena including the convective term in pipeline problems

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Abstract. Water hammer problems occur where restrictions flow suddenly change in pipelines, usually the closing or opening of valves. Presence of a high velocity pressure wave traveling through the fluid could cause damages in the pipeline structures due to the implosion of gas cavities formed by a physical phenomenon called cavitation. On this work, it is studied the water hammer physical problem considering the influence of the convective terms in the momentum and continuity equations, the cavitation problem has been evaluated by the discrete vapour cavity physical model. A MATLAB code is developed to solve the transient problem and find the hydraulic head evolution in some points along the pipeline. The method of characteristics is used to find a numerical solution of the coupled partial differential equation system. Results show good agreement with results presented in literature reviewed, also, it is found that the influence of the convective term is small compared with a simple model where these terms are neglected, the maximum difference value of 2.4x10^{-4} % where found, considering and neglecting the convective term on the physical model.

1. Introduction

When a pipeline full of liquid experience a disturbance or sudden change due to any kind of restrictions in the mean flow, a series of pressure waves appears in the system, this is known as Hammer Phenomenon. The waves propagation creates transient flow and pressure condition that potentially may cause damage on pipes due to overpressure and cavitation, and problems in the normal operative conditions [1]. For instance, when a valve close suddenly a surge of pressure propagation wave is produced upstream. When the pressure of filled liquid pipe reach the vapor pressure, the fluid starts to evaporate and vapor bubbles and cavities suddenly appears. The cavitation occurs when the pressure increases and the formed bubbles collapse. One model used to simulate the cavitation is the discrete vapor cavity model (DVCM) [2,3]. According to [4] this model is recommended for void fraction below 10%. Some parametric studies have been carried out by [5], where the method of characteristic (MOC) is used combined with the general interface vapour cavitation model (GIVCM); they found that larger diameters in the pipe decrease the effect maximum pressure reached during wave propagation in water hammer. The most used cavitation models are the discrete vapor cavity model (DVCM) and the discrete gas cavity model (DGCM). The DVCM assumes there is no free gas in the system, furthermore, in
steady state and when the pressure is above the vaporization pressure, there is no vapor present in the system. The DGCM assumes that there is always a small amount of free gas present, so a new expression for the head must be implemented that takes into account the effect of the gas cavity. The DVCM has the advantage of being the simplest and reproduces many characteristics of the physical events of column separation in pipes, in addition, it can be used for many flow conditions [6,7]. In this work a comparative study is presented to determine the influence of a convective term in the solution of governing equations. Results including the cavitation phenomenon and the comparison between different boundary condition are also presented.

2. Model configuration
In this section a physical model is present to evaluate the water hammer and cavitation physical phenomena inside a pipeline with an instantaneous valve closure. Equations derived from conservative physical principles are presented and the method of characteristics is applied to solve the numerical problem.

2.1. System description
The system is composed of a reservoir-pipe-valve to study the water hammer phenomenon as presented in Figure 1.

![Figure 1. System schematic elements.](image)

where $H_R$ is the upstream total head in a tank reservoir (T) flowing inside the pipe towards a valve (V), $D$ represents a constant pipe diameter, $L$ the pipe length and $\theta$ is inclination angle. Initial flow (at $t = 0$) is steady flow with constant velocity $v_0$, transient flow is generated by instantaneous closure of the downstream valve. System data for process simulation using MATLAB software can be found in Table 1, where the physical variables are given to define the case of study.

| Symbol | Variable name            | Value   | Unit   |
|--------|--------------------------|---------|--------|
| $D$    | Diameter                 | 0.022   | m      |
| $t$    | Thickness                | 0.010   | m      |
| $L$    | Length                   | 37.200  | m      |
| $\theta$ | Angle of inclination   | 3.200   | °      |
| $\omega$ | Wave frequency       | $\Pi$   | 1/s    |
| $u_0$  | Initial flow velocity    | 0.300   | m/s    |
| $a$    | Speed of the pressure wave| 1454.500 | m/s    |
| Re     | Reynolds number          | 6573.700 | -      |

2.2. Mathematical model
The dynamic equation is derived by isolating a small element of fluid of length $dx$ and considering the forces acting on the mass element, it produces the dynamic equation as follow, Equation (1) [2,3].

$$L_2 = g \frac{\partial H}{\partial x} + V \frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} + \frac{\partial |V|}{2D} = 0. \quad (1)$$
With mass conservation in the fluid element, the equation of continuity can be formulated, Equation (2) [2,3].

\[
L_1 = \frac{\partial^2 V}{\partial x^2} + V \frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} - V \sin \theta = 0,
\]

where \( g \) is the gravitational acceleration, \( H \) is the piezometric head, \( t \) is time, \( A \) is the cross-sectional area of the pipe, \( Q \) is the volumetric flow rate, \( x \) is position in the axial direction, \( f \) is the friction term, and \( \theta \) is the angle from horizontal line. There are several models to define the friction term such as the traditional Steady and Quasi-Steady model [3], and the unstable models such as Brunone [8], Zielke [9], Vardy & Brown [10,11] and Zarzycki [12]. The term Quasi-Steady was chosen to model the friction term because the unsteady term increases the numerical complexity and computational cost [13]. The momentum equation (Equation (1)) and continuity equation (Equation (2)) describe the transient flows in a closed pipe. Usually the convective terms \( V \frac{\partial V}{\partial x} \) and \( V \frac{\partial H}{\partial x} \) are very small relative to other parts of the equations (due to the wave speed \( a \) are much larger than \( V \)), and those terms sometimes can be neglected. Equations \( L_1 \) and \( L_2 \) are combined linearly \((L_1 + \lambda L_2)\) using an unknown parameter \( \lambda \).

Though MOC method, it is eliminated the independent variable \( x \) and \( t \) partial differential problem get converted into ordinary differential equations (ODE) in the variable \( t \) (Equation (3), Equation (4), Equation (5), and Equation (6)). The method of characteristics is used to discretize Equation (1) and Equation (2), Figure 2 shows a spatial-time mesh used in the MOC to evaluate the time evolution of pressure head and flow of water inside the pipe (discretized into a total of \( NS \) number of nodes). The curved lines connecting the dots inside the mesh are the characteristic lines.

\[
\frac{dH}{dt} + \frac{a}{g} \frac{dV}{dt} + V \sin \theta + \frac{a|\sin \theta|}{2gd} = 0,
\]

\[
\frac{dx}{dt} = V + a,
\]

\[
\frac{dH}{dt} - \frac{a}{g} \frac{dV}{dt} + V \sin \theta - \frac{a|\sin \theta|}{2gd} = 0,
\]

\[
\frac{dx}{dt} = V - a.
\]

In this study, the convective term will not be neglected to evaluate their effect on the pressure wave. From Equation (3) to Equation (6) have restrictions respect where the new ODEs are valid, the equations \( L_1 \) and \( L_2 \) are valid in every point in plan \( x-t \) shown in Figure 2, but the ODE Equation (3) and Equation (5) are only valid over the characteristics lines \( C^+ \) and \( C^- \) described by the Equation (4) and Equation (6).
formation, and all the free gas is presented in a single package at the node where the pressure drops.

Hydraulic grade line across the valve and $\tau$ is the dimensionless valve opening function, as shown in [3].

For the upstream end, the boundary conditions will be Equation (11). At the end of a single pipe, one of the compatibility equations is available in the two variables.

Figure 2, shows that the end points (nodes 1 and NS) of the system requires boundary conditions to be evaluated. At the end of a single pipe, one of the compatibility equations is available in the two variables. For the upstream end, the $C^+$ characteristic line is set and for the downstream boundary the characteristic line $C^-$ is available. An auxiliary equation is needed in each case, and those are the boundary conditions:

2.3.1. Reservoir at upstream end with known elevations. At a large upstream reservoir, the elevation of the hydraulic grade line ($H_R$) normally can be assumed constant during short duration transient, because it is assumed that the reservoir is large enough and the elevation changes during operation can be neglected. But, if the reservoir level changes in a known function of time during the transient phenomena [3], the boundary conditions will be Equation (11).

$$H_{P_{(i,j)}} = H_R + \Delta H \sin \omega t,$$

where $\Delta H$ is the static head variation amplitude and $\omega$ is the sine frequency; $Q_P$ can be determined using Equation (10).

2.3.2. Valve at downstream. In general, where $\Delta H$ is the instantaneous drop of hydraulic grade line across the valve, the flow though the valve ($Q_P$) can be estimated by Equation (12); where $Q_0$ is the steady-state flow, $H_0$ the steady-state head loss across the valve, $\Delta H$ is the instantaneous drop in hydraulic grade line across the valve and $\tau$ is the dimensionless valve opening function, as shown in [3].

$$Q_P = \frac{Q_0}{\sqrt{H_0}} \tau \sqrt{\Delta H}.$$  

2.4. Discrete vapor cavity model

This model is presented by [7] and it assumes that the wave speed velocity, is not affect by the bubble formation, and all the free gas is presented in a single package at the node where the pressure drops.
under the vapor pressure. The change of gas volume in time can be modelled considering the differences between the volumetric flow rate outlet the node and the volumetric flow rate and integrating from \( t - 2\Delta t \) and \( t \) inlet according to Equation (13) and Equation (14).

\[
\frac{dV_g}{dt} = Q_{out} - Q_{in}, \quad (13)
\]

\[
V_{gP(i,j)} = V_{gP(i,j-2)} + 2\Delta t \left( \psi \left( Q_{P(i,j)} - Q_{uP(i,j)} \right) + \left( 1 - \psi \right) \left( Q_{P(i,j-2)} - Q_{uP(i,j-2)} \right) \right), \quad (14)
\]

where \( V_{gP(i,j)} \) is the volume of the gas pocket at point \( i \) at any time \( j > 2 \), \( Q_{P(i,j)} \) is the volumetric flow rate at time \( i \), subscripts \( P(i,j-2) \) indicate points at time \( t - 2\Delta t \), the subscript \( u \) refers to the volumetric flow rate entering the node, and \( \psi \) is a weighting factor which control the weight values at \( t \) and \( t - 2\Delta t \). Numerical oscillation could be expected when gas cavity sizes are small, in order to decrease the numerical oscillations, the value of \( \psi \) can be increased towards unity. It is recommended to select a value of \( \psi \) as close to 0.5 as possible while experiencing minimal oscillation [3]. Using the multiphase model, at steady state it is assumed there is no vapor in the system, and the flow is treated as a single-phase system. When interior pressure drops lower or equal to the vapor pressure the nodes are treated as boundary nodes with a fixed pressure known \( H_p \). The volumetric flows \( (Q_{P(i,j)}, Q_{uP(i,j)}) \) and the vapor volume \( V_{gP(i,j)} \) can be calculated according to [7].

### 3. Results and discussion

Several simulations were performed using a code programmed in MATLAB for various cases for a total of 25 nodes with a time interval of 0 to 2 seconds. In the first case, a simulation was carried out without the phenomenon of cavitation (as a single phase system), the convective term is considered and two cases of boundary conditions at the reservoir are compared: constant value \( (H_0 = 22m) \) and variable reservoir with an oscillator perturb \( (H_0 = (22 + 3 \sin(\pi t))m) \). To study the total head 25 nodes are considered. In Figure 3 is presented the total pressure at two different nodes over the pipeline: node 6 (near the reservoir) corresponds to the blue lines and 19 (near to the valve) corresponds to the red lines, the solid lines are the results considering a constant boundary condition at the reservoir and the dashed lines reports the solution with the sine perturbation. A detailed view is presented to compare the difference between the pressure profile with and without an oscillate boundary condition at the reservoir. It is notice that when the pressure at the inlet increase with the positive side of the sine wave, the pressure profile in all nodes are higher than the pressure values of the same node with a constant head at the reservoir. Otherwise, when the sine function takes negative values, the pressure profile decrease respect to the constant boundary condition case.

In the second case analysed it is also considered a single-phase fluid without cavitation and constant boundary condition at reservoir; the impact of the convective term is evaluated on the pressure profile. Figure 4 shows the pressure values in time for node 13 located at the middle of the line; it is found that there are no significant differences between the pressure values for all nodes considering and neglecting the convective term in Equation (1) and Equation (2). Figure 5 present the percentage difference between both results, with a maximum difference value of 2.4x10^{-4}\% at some peaks, those peaks values increase in time due to a numerical propagation error in time; nevertheless, the difference between those two cases remains extremely small. Those results can be explained examined Equation (1) and Equation (2), and the value of \( \alpha \) in Table 1, showing that the wave propagation velocity is higher than the fluid velocity; therefore, the convective term will not affect significatively the results as it is shown in Figure 5.
Figure 3. Wave behavior according to the sine wave without the cavitation phenomenon. (a) Pressure profile with positive sine wave side; (b) Pressure profile with negative sine wave side.

Figure 4. Pressure profile for a middle node considering (solid line) and neglecting (circular dots) the convective term in a single-phase fluid.

Figure 5. Percentage difference between the pressure profile for a middle node considering and neglecting the convective term in a single-phase fluid.

The last case studied consider the cavitation with a DVCM model and considering a variable reservoir pressure as a boundary condition $H_0 = (22 + 3 \sin(\pi t)) \text{m}$. Result are presented in Figure 6 for node 13 located at the middle of the line. There are also no significant differences on the pressure values in time with the two situations compared. The cavitation phenomenon can be observed at three-time instants: about 0.8s, 0.13s and 0.21s when pressure suddenly falls and remains in a low constant value during a small period of time. There are some abnormal pressure peaks as shown in the circle markers of Figure 6, those peaks are characteristics of the MOC and not represents the actual pressure of the node at that specific time; similar results are reported by other authors like [14], they also found that when the number of nodes in the method increase higher values peaks are reached.
4. Conclusions
In this study, water hammer and transmitted pressure in an inclined pipe were examined implementing a characteristic method to solve a couple of partial differential equation in transient cavitation and column separation phenomena, taking in to account the convective term to evaluate their effect on the pressure wave. Three cases were evaluated, where the first and second cases are considered a single – phase system, considering the convective term in partial differential equations; also, the variable reservoir with an oscillator sinusoidal pressure variable and constant boundary condition were considered on the first two cases. For the last case, the cavitation phenomena were studied with a DVCM model and considering both constant and variable reservoir pressure as a boundary condition. The numerical solution is in agree with the presented results in literature reviewed, which can be a good tool for predicting cavitation in pipelines with low computational cost. Additionally, it is found that the influence of the convective term is small (2.4x10^{-4}%) compared with a simple model results where those terms are neglected.

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