Direct Statistical Constraints on the Natal Kick Velocity of a Black Hole in an X-Ray Quiet Binary

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Abstract

In recent years, a handful of “dark” binaries have been discovered with a nonluminous compact object. Astrometry and radial velocity measurements of the bright companion allow us to measure the post-supernova orbital elements of such a binary. In this paper, we develop a statistical formalism to use such measurements to infer the presupernova orbital elements, and the natal kick imparted by the supernova (SN). We apply this formalism to the recent discovery of an X-ray quiet binary with a black hole, VFTS 243, in the Large Magellanic Cloud. Assuming an isotropic, Maxwellian distribution on natal kicks and using broad agnostic mass priors, we find that kick velocity can be constrained to $V_k < 72 \text{ km s}^{-1}$ at 90% confidence. We find that a Blaauw kick cannot be ruled out, and that at least about 0.6$m_{\odot}$ was lost during the supernova with 90% confidence. The pre-SN orbital separation is found to be robustly constrained to be around 0.3 au.

Unified Astronomy Thesaurus concepts: Black holes (162); Bayesian statistics (1900); Astrophysical black holes (98); Binary stars (154)

1. Introduction

Galactic binaries with a compact object companion provide one of the few ways to study the population of black holes (BHs) in our galaxy. The list of known stellar-mass BHs in our galactic environment is small. About 23 BHs in accreting binary systems luminous in X-rays have been discovered with mass and spin measurements (Corral-Santana et al. 2016; Reynolds 2021). These have been complemented in recent years by a handful of “dark” binaries, where the nonluminous companion’s mass is constrained by measurement of orbital parameters through astrometric or spectroscopic means (Giesers et al. 2018, 2019; Shenar et al. 2022; Chakrabarti et al. 2023; El-Badry et al. 2023a, 2023b). In addition, a free-floating compact object has been recently detected (Lam et al. 2022; Mróz et al. 2022; Sahu et al. 2022; Lam & Lu 2023) through lensing with a mass of around $\sim 7M_{\odot}$.

Recent years have also seen the population of extragalactic stellar-mass black holes (BHs) come into focus, driven primarily by the gravitational-wave (GW) discovery of binary black hole (BBH) mergers by the Laser Interferometer Gravitational-Wave Observatory (LIGO) and Virgo detectors (Abbott et al. 2016, 2021a, 2021b, 2021c). Understanding the multiple evolutionary pathways of BH formation has become an important astrophysical problem. For instance, GWs and X-ray binary observations are starting to show us the mass and spin distributions of BHs that contain signatures of their evolutionary pathways (e.g., Corral-Santana et al. 2016; Qin et al. 2019; Abbott et al. 2021d, 2023; Reynolds 2021; Draghis et al. 2023).

While the coming years promise many BH detections through GWs, these systems are formed at higher redshifts and lower metallicities and are therefore not representative of the Galactic environment. Moreover, it is possible that X-ray binaries and BBHs discovered through GWs represent different, somewhat distinct parts of the population of BHs in binaries (Fishbach & Kalogera 2022; Gallegos-Garcia et al. 2022; Liotine et al. 2023). Gaia, however, is expected to uncover the galactic population of binaries containing a compact object through precision astrometric measurements of objects (e.g., Breivik et al. 2017; Andrews et al. 2019; Chawla et al. 2022; Janssens et al. 2022).

The binary’s orbital dynamics can be impacted by SN physics, in particular through any kicks imparted to the compact object during its birth. In general, these kicks are caused by the recoil on the remnants from the spatially asymmetric momentum loss during the SN explosion (e.g., Janka 2012; Wongwathanarat et al. 2013). Such natal kicks can change the eccentricity and the post-SN orbital separation, and perhaps most importantly can unbind binaries if binaries are wide and kicks are large. Even in the limit of negligible SN asymmetry, in which case the remnant does not receive a recoil kick, the orbital parameters and the center-of-mass velocity of the binary could still change due to mass–energy loss. This is sometimes called a Blaauw kick (Blaauw 1961). Through their impact on the post-SN separation and eccentricity, the kicks also affect the time delay between star formation and binary coalescence through Peters’s formula (Peters 1964; O’Shaughnessy et al. 2010; Mapelli et al. 2017; Fishbach & Kalogera 2021). Natal kicks, in particular in the case of a second SN, can also affect the spin tilts in the field binary formation channel (Kalogera 1996; Farr et al. 2011).

Using proper motions and parallaxes of individual galactic pulsars, several studies (Lyne & Lorimer 1994; Hobbs et al. 2005; Igoshev 2020; Kapil et al. 2023) have attempted to
measure the natal kick distribution of isolated NS. Hobbs et al. (2005) in particular found the dispersion of the kick velocities for isolated galactic NSs to be $\sigma = 265$ km s$^{-1}$ using a Maxwellian model, although there is some debate regarding the nature of the distribution (e.g., Igoshev 2020). Binary physics can also be important (Beniamini & Piran 2016; Vigna-Gómez et al. 2018; Willcox et al. 2021), and kick-velocity distribution of NSs in a binary might be significantly different from isolated NSs. Moreover, NSs can suffer much lower kicks when they originate through an electron-capture supernova (Podsiadlowski et al. 2004) or an ultra-stripped supernova (Tauris et al. 2015), owing to the low degree of asymmetry expected in those explosions. Nevertheless, clearly, a large fraction of NSs receive kicks of $\mathcal{O}(100)$ km s$^{-1}$ or higher.

On the other hand, the magnitude and nature of the natal kicks for BH remnants have remained much more uncertain. Because proper motion measurements are much harder for unbound BHs (however, see Andrews & Kalogera 2022), similar constraints are not readily obtainable. It is expected in general that BH progenitors suffer lower mass loss during an SN, and thereby smaller kicks. Some studies have used measurements of proper motion, masses, and orbital parameters of the X-ray binaries to trace back their evolutionary history, obtaining their natal kick history in the process (Willemse et al. 2005; Fragos et al. 2009; Wong et al. 2012, 2014). In particular, recently Kimball et al. (2022) studied the low-mass X-ray binary MAXI J1305-704, finding that its BH received a natal kick of at least 70 km s$^{-1}$ with 95% confidence. A similar lower limit of 80 km s$^{-1}$ was obtained for XTE J1118 $+$ 480 by Fragos et al. (2009).

In this paper, we show how the measurements of the post-SN orbital parameters of a binary with a BH/NS and a luminous star can be used to constrain natal kick velocity and pre-SN orbital parameters. We develop a Bayesian statistical formalism that can be generally applied as long as the pre-SN eccentricity is negligible and the binary is wide with negligible post-SN interactions.

We apply this method to the recent observation by Shenar et al. (2022) of VFTS 243, an X-ray quiet BH in a binary in the Large Magellanic Cloud (LMC). The luminous companion is reported to be a $\approx 25 M_\odot$ O-type star, while the mass of the black hole is reported to be $\approx 10 M_\odot$. A nondegenerate dim alternative to the BH is excluded at high statistical confidence. Through radial-velocity measurements from the Fiber Large Array Multi Element Spectrograph (FLAMES; Evans et al. 2011), orbital parameters of this system were inferred. Observations of the system using Chandra detected no signs of X-ray luminosity, indicating a quiescent BH with little accretion from its companion over its history.

Stevance et al. (2023) recently analyzed VFTS 243 using the BPASS population synthesis code to study the evolutionary history and progenitors of the system. Comparing the results of their population synthesis models with the post-SN properties of VFTS 243, they find that the SN had a low explosion energy and low recoil kick velocities of less than 33 km s$^{-1}$ at 90% confidence. In this paper, we treat this as an inverse problem and develop a Bayesian statistical formalism for inferring the natal kick velocity directly from observations. Thereby, we develop a way to statistically constrain certain progenitor properties, without using population synthesis.

The rest of this paper is organized as follows. First, in Section 2 we briefly describe the measurements of the masses and the orbital parameters of VFTS 243, as well as the properties that make it a good candidate for the kind of analysis done here. In Section 3, we briefly describe the dynamics of natal kicks and how they impact the orbital parameters. We develop the statistical formalism for inferring natal kicks and pre-SN orbital parameters in Section 4. In Section 5, we describe priors and selection effects, followed by the results of this formalism when applied to VFTS 243 in Section 6. We discuss some implications of this work, followed by a summary, in Section 7.

### 2. Properties of VFTS 243

Shenar et al. (2022) report the discovery of an X-ray quiet dark binary VFTS 243 in the Tarantula nebula of the Large Magellanic Cloud. The system was analyzed using spectra obtained from the FLAMES spectrograph of the European Southern Observatory. The primary star in the binary is an O-type star with a mass of about $25 \pm 2.5 M_\odot$, with the mass of the dark companion estimated to be around $10.1 \pm 2.0 M_\odot$. The minimum mass of the companion is constrained to be at least $8.7 \pm 0.5 M_\odot$ (1$\sigma$ uncertainty). The orbital period and eccentricity are tightly constrained at about $P_f \approx 10$ days and $e \approx 0.017$, respectively.

The values of some important parameters are given in Table 1. Full posterior distributions of these parameters are shown in the Supplementary Materials of Shenar et al. (2022). Through spectral analysis and comparing the data with a mock data set, they rule out a faint nondegenerate companion, concluding the companion is a degenerate star. The minimum mass limit implies that it cannot be a neutron star and has to be a BH.

Shenar et al. (2022) also rule out X-ray emission from the binary, and thereby any significant accretion, through upper limits from Chandra on the X-ray luminosity, $\log L_X < 32.84$ erg s$^{-1}$. The primary was observed to be rapidly rotating with a period that is not synchronized with the orbital period. Therefore, they conclude that tidal effects and accretion after the SN can be ignored, and the orbital parameters after the SN have been maintained. They also point out that the rapid rotation of the primary and the presence of CNO-processed material in the spectrum indicates a period of accretion before the SN, likely when the BH-progenitor was passing through a giant phase. This strongly implies that the pre-SN orbit was circularized.

### 3. Dynamics of a Natal Kick in a Binary System

We use as a starting point the derivation from Kalogera (1996) that relates how the post-SN orbital parameters are related to the pre-SN parameters after a natal kick, but
following more closely the notation of Andrews & Zezas (2019). Some of the relevant details of the derivation are reproduced below. First, following Kalogera (1996), we define the parameters

\[ \alpha = \frac{a_f}{a_i} \quad \text{and} \quad \beta = \frac{M_1 + M_2}{M_1 + M_2}, \]

where \( a_i \) and \( a_f \) are the pre-SN and post-SN orbital separations, respectively. Throughout this paper, we label the star that undergoes the SN as “2” and its (originally less massive) companion as “1.” Therefore, \( M_2 \) and \( M_f \) represent the pre-SN and the post-SN mass, respectively. \( M_1 \) is the mass of the other binary companion, which we assume does not change appreciably during this process. We also define the pre-SN orbital velocity,

\[ V_r = \sqrt{\frac{G(M_1 + M_2)}{a_i}}, \]

where \( G \) is the Newtonian gravitational constant. We will further assume that the pre-SN orbit is circular. For VFTS 243, this follows the argument made in Shenar et al. (2022) that the rapid rotation and signs of pre-SN accretion onto the primary from the BH progenitor show that the orbit was circularized by mass transfer.

We define a coordinate system in which the orbital velocity of star 2 just before the SN is along the \( Y \)-axis, with the \( X \)-axis along the line connecting stars 1 and 2 at the instant of the SN. The \( Z \)-axis then is perpendicular to the pre-SN orbital plane. A schematic of the kick along with all the parameters is provided in Figure 1.

Now, let the SN remnant receive a natal kick of magnitude \( V_k \) in the pre-SN rest frame of star 2. Suppose the kick makes an angle \( \theta \) with the \( Y \)-axis and subtends an azimuthal angle \( \phi \) in the \( X-Z \) plane as shown in Figure 1. Then, using a spherical coordinate system aligned with the pre-SN orbital velocity, the pre-SN and post-SN orbital parameters can be related as (Kalogera 1996; Andrews & Zezas 2019)

\[ \alpha = \frac{\beta}{2\beta - v_k^2 - 1 - 2v_k \cos \theta}, \]

\[ 1 - e^2 = \frac{1}{\alpha \beta} \left( 2 \beta - \frac{\beta}{\alpha} - v_k^2 \sin^2 \theta \cos^2 \phi \right), \]

where \( v_k = V_k / V_r \). The center of mass (CM) of the binary also receives a recoil kick \( V_{CM} \) with a magnitude given by (Kalogera 1996; Andrews & Zezas 2019)

\[ v_{CM}^2 = \kappa_1 + \kappa_2 \left( \frac{2\alpha - 1}{\alpha} \right) - \frac{v_k \cos \theta + 1}{\sqrt{\beta}}, \]

where \( v_{CM} = V_{CM} / V_k \), and \( \kappa_1, \kappa_2, \) and \( \kappa_3 \) are

\[ \kappa_1 = \frac{M_2^2}{(M_1 + M_2)^2}, \]

\[ \kappa_2 = \frac{M_2^2}{(M_1 + M_2)(M_1 + M_2)}, \]

\[ \kappa_3 = \sqrt{\frac{1}{4\kappa_1 \kappa_2}}. \]

We note that, through Equations (3), (4), and (5), for a given set of masses, kick magnitude, and direction, the post-SN parameters deterministically depend on the pre-SN parameters.

4. Statistical Formalism for Measuring Natal Kicks

This section will set up a general statistical formalism to infer kick velocities from the orbital parameters of a dark binary. We assume that the post-SN orbital and mass parameters of the system are known, in particular the eccentricity \( e_p \), the orbital period \( P_f \) (from which the post-SN separation \( a_f \) can be calculated) and the masses \( M_1 \) and \( M_2 \). We shall also assume that the post-SN radial CM velocity, \( \Gamma_f \), has...
been measured, although we make no assumptions about constraints on inclination.

Let \(\pi(P_p, e_p, M_1, M_2, \Gamma)\) represent the priors used for the post-SN parameters. Using Bayes’ theorem, we relate the posteriors and priors on the post-SN parameters to the likelihood over the radial-velocity data \(d\),

\[
P(P_j, e_j, M_1, M_2 | d) \propto \mathcal{L}(d | P_j, e_j, M_1, M_2) \times \pi(P_j, e_j, M_1, M_2).
\]  

(7)

We assume that \(P(P_j, e_j, M_1, M_2 | d)\) are known from prior analysis. Our goal is to estimate the pre-SN parameters. Then we can write

\[
P(a_i, v_i, M_2 | d) \propto \mathcal{L}(d | a_i, v_i, M_2) \times \pi(a_i, v_i, M_2).
\]  

(8)

We define a hyperprior \(\pi(P_j, e_j, \Gamma_j, M_2 | a_i, v_i, M_2)\) that describes the distribution of post-SN parameters for a given set of pre-SN parameters. Then we can write

\[
P(a_i, v_i, M_2 | d) \propto \pi(a_i, v_i, M_2) \int dP_j \, d e_j \, d \Gamma_j \, d M_1 \, d M_2 \propto \pi(P_j, e_j, \Gamma_j, M_2 | a_i, v_i, M_2) \pi(M_1).
\]  

(9)

We will also assume that the hyperprior can be factorized so that

\[
\pi(P_j, e_j, \Gamma_j, M_2 | a_i, v_i, M_2) = \pi(P_j, e_j, \Gamma_j | a_i, v_i, M_2) \times \pi(M_2 | M_1).
\]  

(10)

This factorization assumes that mass loss from the SN is only dependent on the pre-SN mass and not on the kick velocity or orbital period. This may not be strictly true in nature, but we take it as a starting assumption here, which can be built upon in future work.

In order to derive \(\pi(P_j, e_j, \Gamma_j | a_i, v_i, M_2)\), we first look at the simplified case where only the orbital parameters are measured, and the radial CM velocity is completely unconstrained. Under the assumption that the natal kicks have no particular directional preference in the pre-SN rest frame of star 2, and marginalizing over those angular variables, an analytical probability distribution for \(\alpha, e\) can be derived (Kalogera & Lorimer 2000; Andrews & Zezas 2019):

\[
\pi(\alpha, e_j | \beta, v_k) = \frac{e_j}{\pi_0 \alpha \sqrt{\gamma}} \sqrt{\frac{\beta^3}{C_1 C_2}}.
\]  

(11)

The terms \(C_1\) and \(C_2\) are functions of \(\alpha, e_j\):

\[
C_1 = 2 - \frac{1}{\alpha} - \alpha (1 - e^2),
\]  

(12)

\[
C_2 = 4v_k^2 - \left( \frac{2 \beta}{\alpha} - v_k^2 - 1 \right)^2 - 4 \beta C_1.
\]  

(13)

We now extend this and generalize to the case where the radial CM velocity is measured as well. We do this by additionally marginalizing over the unknown pre-SN CM velocity \(\Gamma\). This is assumed to be drawn from the environmental distribution of radial velocities, which we model by a normal distribution:

\[
p(\Gamma) = \frac{1}{\sqrt{2\pi \sigma_0^2}} \exp\left\{ - \frac{(\Gamma - \mu_0)^2}{2\sigma_0^2} \right\}.
\]  

(14)

Also marginalizing over pre-SN inclination with a uniform prior, we get (see Appendix A for derivation)

\[
\pi(\alpha, e_j, \Gamma_j | \beta, v_k) = \frac{e_j \beta^{3/2}}{(2\pi)^{5/2} \alpha \sqrt{\sigma_0} \sqrt{C_1 C_2}} \times \left\{ \text{erf} \left( \frac{\Gamma_j - \delta \Gamma_{CM} - \mu_0}{\sqrt{2\sigma_0^2}} \right) - \text{erf} \left( \frac{\Gamma_j - \delta \Gamma_{CM} - \mu_0}{\sqrt{2\sigma_0^2}} \right) \right\}.
\]  

(15)

Because we desire a distribution on \(P_f\) rather than \(\alpha\), we perform an additional Jacobian transformation using Kepler’s third law,

\[
\alpha \propto a_f \propto P_f^{2/3} \Rightarrow \frac{\partial \alpha}{\partial P_f} = -\frac{2}{3} P_f^{-1}.
\]  

(16)

Substituting this in Equation (15), we get

\[
\pi(P_j, e_j, \Gamma_j | \beta, v_k) = \frac{2e_j \beta^{3/2}}{(2\pi)^{5/2} \alpha \sqrt{\sigma_0} \sqrt{C_1 C_2}} \times \left\{ \text{erf} \left( \frac{\Gamma_j - \delta \Gamma_{CM} - \mu_0}{\sqrt{2\sigma_0^2}} \right) - \text{erf} \left( \frac{\Gamma_j - \delta \Gamma_{CM} - \mu_0}{\sqrt{2\sigma_0^2}} \right) \right\}.
\]  

(17)

Armed with a mass prior (the choices of which are discussed in more detail in Section 5.1), we can put together the hyperprior \(\pi(P_p, e_p, \Gamma_p, M_2 | a_i, v_i, M_2)\) and calculate posteriors on \(a_i, v_i,\) and \(M_2\). If we already have fair draws from \(P(e_j, a_f, M_1, M_2 | d)\), we can approximate this expression as

\[
P(a_i, v_i, M_2 | d) \propto \pi(a_i, v_i, M_2) \times \sum_n \left[ \pi(P_p^n, e_p^n, M_2^n | a_i, v_i, \beta, M_2) \pi(e_p^n, P_f^n, M_2^n) \right],
\]  

(18)

where \(n\) is an index over the Monte Carlo samples of post-SN parameters. The prior on \(M_1\) is assumed to be unchanged in the hierarchical step—and hence cancels out in this expression.

There are two bounds of validity for Equation (11) that have to be considered. First, from the argument that the post-SN orbit has to contain the position of the star just before the SN, Flannery & van den Heuvel (1975) showed that \((1 + e)^{-1} < \alpha < (1 - e)^{-1}\). This is equivalent to the condition that \(C_1 > 0\). Furthermore, because the probability in Equation (11) has to be real, we also need \(C_2 > 0\). It can be shown that this implies (Andrews & Zezas 2019)

\[
e^2 < 1 - \frac{(\beta + \alpha (v_k^2 - 2 \beta - 1))^2}{4 \beta \alpha^3}.
\]  

(19)

\(5\) This step is necessary because we only care about the radial CM velocity in the particular case of VFTS 243. But it can be straightforwardly extended to include the proper motion as well.
4.1. Assumptions

We list here, explicitly, the assumptions that went into this derivation.

1. The orbital eccentricity before the SN was negligible. This follows from the argument that mass transfer circularized the pre-SN orbit. See Section 2 and Shenar et al. (2022).
2. The natal kick has no a priori directional preference in the pre-SN rest frame of star 2.
3. Any change in $M_1$ from its pre-SN value is negligible. This implies that any accretion onto star 1 during the SN, any mass loss due to wind after the SN, and mass transfer onto the BH after the SN are all small.
4. Orbital parameters from immediately after the SN have been preserved and tidal synchronization can be neglected. These assumptions again follow the argument of Shenar et al. (2022) that there is no evidence of post-SN mass transfer or tidal synchronization.
5. Again following Shenar et al. (2022), the inclination of the system is assumed to be unconstrained. In principle, however, it would be straightforward to also draw from an inclination posterior distribution if it were better constrained.

5. Priors

In this section, we list the priors we use for the analysis. For the pre-SN parameters, we adopt a uniform prior on the pre-SN separation, $a_i \sim U(0.01, 2) \text{ au}$. For the natal kick magnitude, we have two prior models. The first is a uniform prior on the kick magnitude, $V_k \sim U(0, 200) \text{ km s}^{-1}$, and it is isotropic in direction in the pre-SN frame. The second prior assumes that the kick is drawn from a distribution that is Maxwellian in velocity magnitude and isotropic in the pre-SN frame, a choice motivated by observations of isolated galactic pulsars (Hobbs et al. 2005):

$$
\pi(V_k|\sigma_k) = \frac{V_k^2}{\sigma_k^3 \sqrt{2 \pi}} \exp\left(-\frac{V_k^2}{2 \sigma_k^2}\right).
$$

The kick-velocity dispersion $\sigma_k$ is treated as an independent unknown variable that we can estimate from the radial-velocity data through the Bayesian framework developed in this paper. We further use a uniform prior on the dispersion, $\sigma_k \sim U(0.01, 200) \text{ km s}^{-1}$, to match the bounds for the velocity prior in the uniform model.

We note that the velocity priors technically decouple the mass lost during the SN from the recoil kick, which is not entirely realistic. In reality, mass–energy loss and kick are tied together, modulated by the asymmetry of the explosion, as can be understood from simple physical grounds that the kick is caused by the recoil from the SN. Theoretical work also emphasizes a relation between them; Janka (2017), for example, develops scaling relations between the mass of the ejecta and the kick magnitude. Other papers have used SN kick recipes that allow for some dependence between progenitor masses and kick velocity (Bray & Eldridge 2016; Mandel & Müller 2020). However, using such a fit would mean assuming some foreknowledge of the explosion asymmetry, which we wish to avoid due to the paucity of data constraining it for BHs. The simple decoupled priors we use here are agnostic about the asymmetry, and because we consider only one system, they are sufficient. We leave to future work the consideration of a coupled prior that combines mass–energy loss and kick velocities in a more sophisticated way by marginalizing over the uncertainty in asymmetry.

5.1. Mass Priors

We explore two mass prior choices. The first is a broadly agnostic uniform mass prior, where $M_{2i} \sim U(5, 30) M_\odot$ and $\pi(M_{2i}|M_{2s})$ is also uniform but bounded by $M_{2i}, \text{i.e., } M_{2i} \sim U(5 M_\odot, M_{2s})$. The latter ensures that the post-SN mass always is less than the pre-SN mass.

The second kind of prior is fixed-$\beta$, i.e., a delta function in $\beta$. To convert this to a prior on $M_{2i}$, we first compute the Jacobian:

$$
\pi_\beta(M_{2i}) = \pi(\beta) \left| \frac{d\beta}{dM_{2i}} \right|.
$$

Setting $\pi(\beta)$ to be a delta function, we get

$$
\pi_\beta(M_{2i}) = \frac{\beta^2}{(M_1 + M_{2i})}.
$$

The fixed-$\beta$ priors are not expected to be realistic priors. But because the dependence of the post-SN orbital parameters on the masses is primarily through $\beta$ (see Equations (3) and (4)), these priors can help us gain intuition on the effect of the SN mass loss on the analysis.

5.1.1. Astrophysical Selection Effects

When measuring $\sigma_k$, we also have to account for selection effects arising from the fact that large natal kicks—which are more likely to occur when the underlying dispersion $\sigma_k$ is large—can unbind a system. In other words, in the presence of large $\sigma_k$, we are more likely to see binaries that suffered comparatively smaller natal kicks. To account for these selection effects, we follow the statistical prescription from Mandel et al. (2019). We first define $p_{\text{surv}}(\alpha, e, V_k)$ as the fraction of systems that survive the kick, marginalized over the kick direction (Andrews & Zezas 2019):

$$
p_{\text{surv}}(\alpha, e, V_k) = \frac{2 \beta - (V_k - 1)^2}{4 V_k}.
$$

The selection function is then just the probability that a system survives the natal kick, marginalized over the distribution of pre-SN parameters, i.e.,

$$
\alpha_{\text{surv}}(M_{2i}, \alpha_i, \sigma_k) = \int dV_k \left| d\beta \right| p_{\text{surv}}(\alpha, e, V_k) \frac{p(V_k|\sigma_k)}{\pi(\beta|M_1, M_{2i})} P(M_i),
$$

where $p(V_k|\sigma_k)$ is given by Equation (20), and $\pi(\beta|M_1, M_{2i})$ is the distribution of $\beta$ under the mass priors described in Section 5.

We estimate the integral Equation (24) stochastically, drawing from the prior probabilities of the parameters. Once drawn, we then correct for the systematic effects as

$$
P_{\text{corr}}(\alpha_i, V_k, M_{2i}|d) = \frac{P(\alpha_i, V_k, M_{2i}|d)}{\alpha_{\text{surv}}(M_{2i}, \alpha_i, \sigma_k)}.
$$

While we have not included any observational selection effects for this analysis of one system, a population level analysis should include them.
6. Results and Discussion

We now apply this formalism to the results from Shenar et al. (2022) for VFTS 243, directly resampling their posteriors on the post-SN parameters $\pi(P_f, e_f, M_1, M_2)$ using the importance-sampling expression in Equation (18). We use the nested sampler DYNESTY (Skilling 2006; Speagle 2020) for sampling over the pre-SN parameters using Equation (18). Shenar et al. (2022) use the following priors for the post-SN parameters:

- $P_f \sim U(9.9, 10.9)$ days,
- $e_f \sim U(0, 0.2)$,
- $\Gamma_f \sim U(240, 280)$ km s$^{-1}$,

while $M_2$ is drawn from a flat prior with a mass function of 0.581. $M_1$ was assumed to be drawn from a Gaussian $M_1 \sim N(25.02M_\odot, 2.32M_\odot)$, and we use these for the denominator in Equation (18).

Figure 2 shows the constraints on kick velocity and pre-SN separation for both uniform and Maxwellian priors on velocity, with broad uniform priors on mass. In both cases, the kick-velocity posteriors are fully consistent with $V_k = 0$. While we do not measure the kick velocity well, it is constrained to less than 72 and 97 km s$^{-1}$ at 90% confidence for the Maxwellian and uniform velocity priors, respectively. The probabilities, $P(V_k \leq 72 \text{ km s}^{-1})$ and $P(V_k \leq 97 \text{ km s}^{-1})$ for a Hobbs distribution with $\sigma_k = 265$ km s$^{-1}$ are only 0.5% and 1.3%, respectively. Therefore, we conclude that the kick velocity received by the BH in VFTS 243 was likely smaller than what is plausible for the $\sigma_k = 265$ km s$^{-1}$ Hobbs distribution observed for neutron stars (Hobbs et al. 2005). This is in line with the physical expectation of relatively low mass loss during BH formation compared to a neutron star, thereby yielding small kicks.\footnotemark

\footnotetext{Although possible theoretical mechanisms have been proposed by which a BH can get a large natal kick; see, e.g., Janka (2013).}

We note that, while the 90% kick-velocity limits are smaller for the Maxwellian prior, it has a much longer tail that extends up to $\sim 500$ km s$^{-1}$. This is likely due to the selection effects, which make higher velocity dispersion harder to rule out.

Figure 3 plots the posterior distribution of the dispersion, $\sigma_k$, for a uniform mass prior and three different fixed-$\beta$ priors. We note that values of $\beta = 0.95, 0.9$, and 0.8 correspond to $M_2 \approx 12M_\odot, 14M_\odot$, and $19M_\odot$, respectively. While the tails of these distributions go past 150 km s$^{-1}$, there is little support past $\approx 80$ km s$^{-1}$, implying again that $\sigma_k$ is small. For instance, the uniform mass prior yields a 90% upper limit of 68 km s$^{-1}$ for $\sigma_k$, further showing that this system is incompatible with a Hobbs-like kick distribution.

It is important to put these limits in context. Because the results in this paper are obtained only from this one system, and not a population of BHs, they are not very informative about...
the underlying kick distribution of the population of BHs as a whole. Therefore, the claim that a Hobbs kick is disfavored should be understood to only mean it is unlikely that this particular system suffered a kick compatible with that distribution. It is entirely possible there exists a population of BHs that suffer Hobbs kicks during their SN; our results are agnostic about such a possibility. Similar caution should be exercised with the constraints on \( \beta \) fixed-priors.

Both kick models give nearly similar estimates for the mass lost, \( \Delta M = M_{\text{2i}} - M_{\text{2f}} \), during the SN. With the uniform mass prior, the Maxwellian model gives a median value of \( \Delta M = 3.6 \pm 2.9 M_{\odot} \), while the uniform velocity model gives \( \Delta M = 4.0 \pm 3.2 M_{\odot} \). The 90\% lower limits on the mass lost during the SN are 0.69\( M_{\odot} \) and 0.77\( M_{\odot} \), respectively, for the two models.

### 6.1. Pre-SN Orbital Separation

We see that the pre-SN orbital separation is constrained quite well at \( a_i \approx 0.3 \) au in both the uniform and Maxwellian velocity models in Figure 2. We further explore any dependency of this on mass models in Figure 4 by including several fixed-\( \beta \) priors. We also include an analysis with an \( a_{\text{max}} = 1 \) au instead of the fiducial 2 au bound used in other runs. The constraint on \( a_i \) is seen to be quite robust and not strongly affected by prior assumptions on mass or velocity. For reference, we also plot in Figure 4 the post-SN separation. The pre-SN and post-SN orbital separations are consistent with small symmetric mass loss, which is what we would expect from this system (see also Section 6.2).

### 6.2. Blaauw Kick

Under the limit that the natal kick velocity \( V_K \rightarrow 0 \), the binary receives only a CM kick from mass loss that is sometimes called the Blaauw kick (Blaauw 1961). While the Maxwellian model rules out only a Blaauw kick for VFTS 243, this is somewhat artificial because the prior \( \pi(V_k = 0 | \sigma) = 0 \) (see Equation (20)). On the other hand, \( V_k = 0 \) is entirely consistent with the posterior for the kick velocity in the uniform velocity model, as seen in Figure 2. This implies that a Blaauw kick cannot be ruled out for VFTS 243. If the SN indeed imparted no recoil kick, this places a strong constraint on the mass lost of \( \Delta M \approx 0.60 M_{\odot} \), consistent with the lower limits estimated in Section 6.

The symmetric mass loss in a Blaauw kick can also cause a change in binary orbital parameters. In particular (Blaauw 1961),

\[
\alpha = \frac{\beta}{2\beta - 1}
\]

and

\[
e_f = \frac{1 - \beta}{\beta}.
\]

Combining these two expressions, we get \( a_i = a_i(1 - e_f) \approx 0.3 \) au, which is consistent with the posteriors from Figure 2, further lending credence to low kick velocities.

The structure of the covariance between the posteriors of \( M_{\text{2i}} \) and \( V_k \) in Figure 2 can also be explained by considering the interplay between the effects of simple mass loss and the recoil from the natal kick. At higher \( M_{\text{2i}} \), i.e., lower \( \beta \) values, mass loss itself can cause a large change in orbital parameters as seen from Equations (26) and (27). In particular, as \( \beta \) approaches 0.5, the post-SN eccentricity induced by mass loss can become quite high. A higher natal recoil would then be needed as a counterbalance to maintain the small eccentricity value that is physically observed as seen in Figure 2.

### 6.3. Scaled Hobbs Kick

We further explore a scaled Hobbs distribution for BH natal kick velocities. In this proposed distribution, the dispersion of natal kicks for BH-generating supernovae is given by

\[
\sigma_{k,BH} = \frac{M_{\text{NS}}}{M_{\text{BH}}} \times \sigma_{k,NS},
\]

where we use a fiducial neutron star mass of \( M_{\text{NS}} = 1.4 M_{\odot} \). Adopting \( \sigma_{k,NS} = 265 \) km s\(^{-1}\) from Hobbs et al. (2005) and \( M_{\text{BH}} \approx 10 M_{\odot} \), we perform a run fixing \( \sigma_{k,BH} = 37.1 \) km s\(^{-1}\) with the uniform mass prior. Contrasting this number with Figure 3,
we see that, while it is not in the bulk of the posterior, it cannot be ruled out either. Indeed, unlike Stevance et al. (2023), we find that the scaled Hobbs prior is not heavily disfavored compared to a uniform velocity draw, with the Bayes factor between the two models being $B^{\text{Hobbs}}_{\text{uniform}} = 1.37$. However, it is disfavored compared to the Maxwellian model, where $\sigma_k$ is allowed to vary with $B^{\text{Maxwell}}_{\text{Hobbs}} = 4.41$. While these specific numbers depend on several modeling choices and are only based on one one system, they imply that the scaled Hobbs model of BH natal kicks cannot be ruled out for VFTS 243.

7. Discussion and Conclusion

In this paper, we have developed a statistical framework for inferring black hole natal kicks from binaries with a black hole and a detached luminous companion, and we have applied it to the system VFTS 243. We find that the black hole in VFTS 243 received a kick less than 72 km s$^{-1}$, assuming a Maxwellian prior over the kick-velocity magnitude and a broad uninformative mass prior. This finding is consistent with a previous estimate from Stevance et al. (2023), but is more conservative and data-driven, utilizing only the mass and orbital dynamics measurements obtained from radial-velocity data by Shenar et al. (2022) and the orbital physics of a binary. Both these analyses find low kick values and that a Blaauw kick cannot be ruled out. However, we do find that a scaled Hobbs prior also cannot be ruled out.

These results for VFTS 243 are also in line with most other BH natal kick measurements, for example, from Willems et al. (2005), Wong et al. (2012, 2014), Atri et al. (2019), Sánchez et al. (2021), and other potential measurements (Andrews & Kalogera 2022), even in cases where a nonzero kick velocity is measured, such as in Fragos et al. (2009) and Kimball et al. (2022). This adds to evidence that black hole natal kicks are usually small compared to those of neutron stars.

Understanding the distribution of kick velocities has important implications for compact binary formation. Large kick velocities can unbind binaries that form through isolated evolution. Even in dynamical formation channels like a globular cluster, large kicks can eject a BH out of the cluster (see, for example, Antonini & Rasio 2016; Gerosa & Fishbach 2021; Fragione et al. 2022). A precise measurement of the kick distribution is thereby one of the factors that influence the calculation of merger rates from different binary formation channels, and the formation of higher-mass black holes through repeated mergers.

In our formalism, we have also shown how the post-SN radial CM velocity $\Gamma_r$ can be folded in to measure orbital parameters. But because this system’s $\Gamma_r$ is consistent with the environmental dispersion, and because of the small eccentricity, this is probably not very informative. In systems where these conditions are not true, such as Gaia BH1 and BH2 (El-Badry et al. 2023a, 2023b) with $e \sim 0.5$, folding in the radial velocity can be potentially very informative.

While some of the results in this paper are perhaps expected, given the low eccentricity of the post-SN orbit, the general formalism developed here can be readily applied to more wide binaries as they are discovered in the coming years. Importantly, this Bayesian formalism also lends itself well to population analysis to measure, for example, a common velocity dispersion from wide binaries. Binary synthesis models predict that a large number of wide mixed binaries with a BH, potentially detectable by Gaia, exist in our galaxy (Breivik et al. 2017; Chawla et al. 2022). The techniques developed in this paper present an important step toward constraining the distribution of BH kicks and their dependence on remnant and progenitor masses, binary companion parameters, and other details, helping us to ultimately understand the physics of supernova better for BH remnants. Moreover, we are already seeing hints that some of these binaries potentially could have had wide eccentric orbits before SN (El-Badry et al. 2023b). Future work will therefore focus on generalizing the formalism developed here to relax the assumption that the pre-SN orbit was circular.

An important assumption made in this paper is that the natal kick has no directional preference. There is some evidence to suggest that the kick direction of a neutron star might be preferentially aligned with the spin direction, both observationally (e.g., Ng & Romani 2004; Noutsos et al. 2012; Yao et al. 2021) and from simulations (e.g., Ng & Romani 2007; Janka et al. 2022). And because we expect the spin in isolated binaries to be preferentially aligned with the orbital angular momentum, this can change the distribution of orbital parameters in Equation (11). Moreover, as shown in Mandel & Igoshev (2023), any correlation between spin and kick direction could introduce additional inclination-dependent selection effects when measuring radial or transverse velocities. With a larger set of wide binaries in the future, it might be worth revisiting this assumption and exploring the existence of such correlations for BH kicks.

There are some other fundamental assumptions that were made here, which future work could loosen. Perhaps most importantly, we have assumed the pre-SN orbital eccentricity was negligible and that the post-SN orbital parameters are not affected by further binary interactions. Additionally, we have only included selection effects due to binaries becoming unbound from kicks, but have ignored observational selection effects, which will also become important as we get more astrometric binaries from Gaia. With the statistical framework we have developed herein, we will be prepared to analyze more of these BH and luminous companion binaries from Gaia and other missions, ultimately paving the way for a full understanding of how black holes get their natal kicks.

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8 We assume the same prior probabilities for all three models.
Appendix A

Jointly Constraining Orbital Parameters and Systemic Velocity

Let the pre-SN CM radial velocity of the binary be \( \Gamma_i \). The exact velocity is unknown but drawn from a normal distribution with \( \mu_0 = 271.6 \) and \( \sigma_0 = 12.2 \) km s\(^{-1}\) from spectroscopic measurements of nearby B-type stars (Evans et al. 2015):

\[
p(\Gamma_i) = \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left[-\frac{(\Gamma_i - \mu_0)^2}{2\sigma_0^2}\right]. \tag{A1}
\]

The post-SN center of mass radial velocity is related to the pre-SN one as

\[
\Gamma_f = \Gamma_i + \delta V_{\text{CM}}. \tag{A2}
\]

Figure 1 depicts the angle between the line of sight and the systemic velocity kick, \( V_{\text{CM}} \). Clearly, \( \delta V_{\text{CM}} = \delta V_{\text{CM}} \cos \iota_v \). The angle \( \iota_v \) physically has no bearing on the kick direction, and neither should it depend on the pre-SN CM velocity of the system. A natural prior on it is uniformly isotropic. Now, to use the joint constraints from the systemic velocity and the orbital parameters, we first marginalize over the kick directions, \( \iota_v \) and \( \Gamma_v \).

\[
\pi(\alpha', e', \Gamma'_f | \iota_v) = \int d(\cos \theta) d\phi' d(\cos \iota_v) \pi
\times (\alpha', e', \Gamma'_f | \iota_v, \phi, \Gamma_i, V_i, \cos \iota_v)
\times \pi(\phi) \pi(\cos \theta) \pi(\Gamma_i) \pi(\cos \iota_v). \tag{A3}
\]

The angular priors are all isotropic, i.e., \( \pi(\phi) \pi(\cos \theta) = 1/4\pi \) and \( \pi(\cos \iota_v) = \frac{1}{2} \). All probabilities here also depend on the masses through \( \beta \), but we suppress that explicitly for the purpose of clarity. Now, consider the conditional probability distribution. It can be split up as

\[
\pi(\alpha', e', \Gamma'_f | \phi, \cos \theta, \Gamma_i, V_i, \cos \iota_v)
= \pi(\alpha', e', \Gamma'_f | \phi, \cos \theta, V_i) \times \pi
\times (\Gamma'_f | \alpha', e', \phi, \cos \theta, \Gamma_i, V_i, \cos \iota_v), \tag{A4}
\]

where the orbital parameters \( \alpha \) and \( e \) are deterministically known for a given kick direction and velocity:

\[
\alpha = \frac{\beta}{2 - \beta - v_y^2 - (v_y + 1)^2 - v_z^2},
1 - e^2 = (v_y + 1)^2 + v_z^2)
= \frac{2(\beta - v_y^2 - (v_y + 1)^2 - v_z^2)}{\beta^2}. \tag{A5}
\]

Similarly, \( \Gamma_f \) is also deterministically known:

\[
\Gamma_f = \Gamma_i + \delta V_{\text{CM}} \cos \iota_v. \tag{A6}
\]

We assert that, even though the CM velocity is completely determined by the kick velocity, masses, and orbital parameters, \( \iota_v \) is an independent variable that is more akin to the inclination angle, since it is measured with respect to the line of sight.

Therefore, the distribution of these quantities is given by delta functions:

\[
\pi(\alpha', e', \Gamma'_f | \phi, \cos \theta, \Gamma_i, V_i, \cos \iota_v)
= \delta(\alpha - \alpha') \delta(e_f - e'_f) \delta(\Gamma_f - \Gamma'_f). \tag{A7}
\]

We now perform a transformation of variables \( \{ \phi, \cos \theta, \Gamma_i \} \rightarrow \{ \alpha, e_f, \Gamma_f \} \). The Jacobian is given by

\[
\mathcal{J}(\alpha, e_f, \Gamma_f) = \begin{vmatrix}
\frac{\partial \phi}{\partial \alpha} & \frac{\partial \phi}{\partial e_f} & \frac{\partial \phi}{\partial \Gamma_f} \\
\frac{\partial \cos \theta}{\partial \alpha} & \frac{\partial \cos \theta}{\partial e_f} & \frac{\partial \cos \theta}{\partial \Gamma_f} \\
\frac{\partial \Gamma_i}{\partial \alpha} & \frac{\partial \Gamma_i}{\partial e_f} & \frac{\partial \Gamma_i}{\partial \Gamma_f}
\end{vmatrix}. \tag{A8}
\]

But physically, the only thing that can depend on the initial CM velocity is the final velocity, so that \( \frac{\partial \cos \Gamma_i}{\partial \alpha} = 0 \) and \( \frac{\partial \cos \Gamma_i}{\partial e_f} = 0 \).

We also see that \( \frac{\partial \Gamma_i}{\partial \Gamma_f} = 1 \) from Equation (A7). This implies

\[
\mathcal{J}(\alpha, e_f, \Gamma_f) = \left| \frac{\partial \phi}{\partial \Gamma_f} \right| \left| \frac{\partial \cos \theta}{\partial \Gamma_f} \right| \left| \frac{\partial \Gamma_i}{\partial \Gamma_f} \right|. \tag{A9}
\]

This \( 2 \times 2 \) determinant has been solved by Andrews & Zezas (2019). Therefore, we get

\[
\mathcal{J}(\alpha, e_f, \Gamma_f) = \frac{4eV_i}{\pi \sqrt{C(\alpha, e)C_2(\alpha, e)}}. \tag{A10}
\]

Putting everything together,

\[
\pi(\alpha', e', \Gamma'_f | V_i) = \frac{V_i \beta^{3/2}}{2\pi^2} \int d\alpha \int dV_i \int d(\cos \iota_v)
\times \frac{e}{\alpha V_i \sqrt{C(\alpha, e)C_2(\alpha, e)}} \delta(\alpha - \alpha')
\times \delta(e_f - e'_f) \delta(\Gamma_f - \Gamma'_f), \tag{A11}
\]

and working through the Dirac-delta functions:

\[
\pi(\alpha', e', \Gamma'_f | V_i) = \frac{V_i \beta^{3/2}}{2\pi^2} \int_{-1}^{1} d(\cos \iota_v)
\times \frac{e_f^2 \pi(\Gamma_f') \delta(\Gamma_f - \Gamma_f')}{\alpha' V_i \sqrt{C(\alpha', e')C_2(\alpha', e')}}. \tag{A12}
\]

We now use Equation (A1) to get

\[
\pi(\alpha', e', \Gamma'_f | V_i) = \frac{V_e \beta^{3/2}}{2\pi^2 \sqrt{\gamma_{\text{CM}}}} \int_{-1}^{1} d \iota_v \exp\left\{ -\frac{(\iota_v - \mu_0)^2}{2\sigma_0^2}\right\}. \tag{A13}
\]
This finally yields

\[
\pi(\alpha', \epsilon_f', \Gamma_f' | V_k) = \frac{V_v \epsilon_f' \beta^{3/2}}{(2\pi)^{3/2} \sigma_0 \epsilon_f' V_k \sqrt{C_1 C_2}} \times \left( \text{erf} \left( \frac{\Gamma_f + \delta V_{CM} - \mu_0}{\sqrt{2} \sigma_0^2} \right) - \text{erf} \left( \frac{\Gamma_f - \delta V_{CM} - \mu_0}{\sqrt{2} \sigma_0^2} \right) \right). \tag{A14}
\]

We still need to figure out what \(\delta V_{CM}\) is, as a function of \(\alpha, \epsilon_f\), since we have marginalized over the kick angles. Following Andrews & Zezas (2019), we have

\[
\frac{V_{CM}^2}{V_r^2} = k_1 + k_2 \frac{2\alpha - 1}{\alpha} - k_3 \frac{V_r \cos \theta + 1}{\sqrt{\beta}}. \tag{A15}
\]

Let us rewrite \(\cos \theta\) in terms of \(\alpha, \beta,\) and \(V_k:\)

\[
\cos \theta = \frac{\beta}{2V_k} \left( 2 - \frac{1}{\alpha} \right) - \frac{V_k^2 + 1}{2V_k}. \tag{A16}
\]

Therefore,

\[
\frac{V_{CM}^2(\alpha, \beta, V_k)}{V_r^2} = k_1 + k_2 \frac{2\alpha - 1}{\alpha} - k_3 \frac{V_k^2 + 1}{\sqrt{V_k}} \frac{2}{\sqrt{\beta}} \left( 2 - \frac{1}{\alpha} \right) + k_3 \frac{2}{\sqrt{\beta}} \left( 2 - \frac{1}{\alpha} \right) + k_3 \frac{k_3}{\sqrt{\beta}} \left( 2 - \frac{1}{\alpha} \right) - k_3 \frac{k_3}{\sqrt{\beta}} \left( 2 - \frac{1}{\alpha} \right). \tag{A17}
\]

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