Extremely Asymmetrical Acoustic Metasurface Mirror at the Exceptional Point

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(Received 19 March 2019; published 22 November 2019)

Previous research has attempted to minimize the influence of loss in reflection- and transmission-type acoustic metasurfaces. This Letter shows that, by treating the acoustic metasurface as a non-Hermitian system and by harnessing loss, unconventional wave behaviors that do not exist in lossless metasurfaces can be uncovered. Specifically, we theoretically and experimentally demonstrate a non-Hermitian acoustic metasurface mirror featuring extremely asymmetrical reflection at the exception point. As an example, the metasurface mirror is designed to have high-efficiency retroreflection when the wave comes from one side and near-perfect absorption when the wave comes from the opposite side. This work marries conventional gradient index metasurfaces with the exceptional point from non-Hermitian systems, and it paves the way for identifying new mechanisms and functionalities for wave manipulation.

DOI: 10.1103/PhysRevLett.123.214302

Molding the flow of acoustic energy using functional materials is a research area that has recently generated a proliferation of work [1–6]. As a member of functional acoustic materials, acoustic metasurfaces stand out as a distinct choice for wave manipulation owing to their advanced capabilities on sound control as well as their vanishing size [6–10]. Conventional transmission- or reflection-type acoustic metasurfaces operate by modulating the real part of their effective refractive indices [7–13] and are typically treated as lossless systems. However, due to the existence of resonance or narrow regions in the deep-subwavelength units, losses are naturally present [14,15], rendering a nonzero imaginary part of the refractive index. The intrinsic loss could compromise the performance of acoustic metasurfaces, and the conventional wisdom is that their effects should be minimized. The emergence of non-Hermitian physics [16], however, offers a brand new prospective on the role of loss. Instead of minimizing the loss in functional materials, recent research suggests that losses can be harnessed to engender highly unusual phenomena [17].

The publication of the seminal paper by Bender and Boettcher [16] immediately spurred an intense interest in quantum mechanics on the non-Hermitian Hamiltonian. Their theory describes a new family of systems that, although they violate time-reversal (T) symmetry, they retain the combined parity-time (PT) symmetry. Such a system possesses entirely real-valued energy spectra below the PT symmetry breaking threshold—the exception point (EP), where the associated eigenvalues and the corresponding eigenvectors coalesce [18,19]. Introducing PT symmetry into the classical optic and mechanical wave systems has paved the way for identifying new mechanisms to control light and sound [20–24]. Given that gain is more challenging to achieve than loss in practical systems, it has been proposed to relax the restriction on exact gain-loss modulation, giving rise to the so-called passive non-Hermitian PT-symmetric systems—systems with only loss [25–27]. In fact, besides PT-symmetric systems, EPs can also be observed in other non-Hermitian systems [28,29]. In these passive systems, intriguing features such as unidirectional near-zero reflection and unidirectional focusing have been observed at the EP [25–28].

Previous works, however, are largely based on the archetype [20–22,25,26,28] or modification (i.e., by curving or extending) [24,27] of the 1D waveguide model, which is a 1D scattering system where the wave propagates along modulated potentials. Studies on higher dimensions are scarce and mostly theoretical [30,31], and this is true for studies on EPs in metasurfaces [32,33]. In this Letter, by marrying acoustic metasurfaces and the concept of the EP, we theoretically and experimentally demonstrate a non-Hermitian metasurface mirror in 2D space. In this system, the elegantly engineered loss provides an additional degree of freedom, from which extremely asymmetrical yet arbitrarily tailored reflection can be observed at the EP.

Additionally, a crux in designing passive, unidirectional reflectionless systems [25–27,29] is that, although it is relatively easy to have the reflection from one side diminished, it is extremely challenging to, at the same time, have high reflectivity from the opposite side. For instance, a recent work has demonstrated asymmetrical reflection for electromagnetic waves [34]. To realize extreme asymmetry (reflectivity from one side is over 90%, whereas that from the opposite side is close to zero) though, more than 100 subunits need to be included in one period, and all subunits must be
elaborately modulated in index and loss [34]. In contrast, by leveraging the EP, our metasurface works effectively with sparse subunits and, remarkably, only one subunit is required to be lossy. As will be shown in the following, such a simple design principle could in theory achieve over 97% reflectivity from one side and zero reflectivity from the other side.

A gradient index acoustic metasurface, as depicted in Fig. 1, is studied. This metasurface consists of $N$ subunits in one period, with each having an effective refractive index $n_i$ ($i = 1, 2, \ldots, N$). The reflected waves are constituted by different diffraction modes for which the angles satisfy [9,17,35]

$$k_0(\sin \theta_r - \sin \theta_i) = \xi + mG = (m + 1)G, \quad (1)$$

where $k_0$ is the wave number, $\theta_i(\theta_r)$ represents the angle of incidence (reflection), $\xi$ is the phase gradient induced by the metasurface ($\xi = dq/dx$, where $q$ is the phase shift and $x$ is the coordinate along the surface), $G$ is the amplitude of the reciprocal lattice vector ($G = 2\pi/D$, where $D$ is the period of the metasurface), and $m$ is the diffraction order. It can be easily shown that $\xi = G$ for a gradient index metasurface.

In order to introduce the EP, the metasurface is designed as a two-port system so that only two diffraction modes can propagate while others are evanescent. When the acoustic wave at the designated angle of incidence impinges upon the metasurface, it splits into two parts: an extraordinary reflection (we choose retroreflection in this Letter as an example) and an ordinary one (specular reflection).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Schematic of the non-Hermitian metasurface showing extremely asymmetrical reflections at the EP.}
\end{figure}

\begin{equation}
S = \begin{bmatrix}
r^+_{0} & r^-_{0} \\
r^-_{2} & r^+_{1}
\end{bmatrix},
\end{equation}

Here, $r$ is the reflection coefficient of the corresponding diffraction mode. The subscript indicates the mode order $m$, and the superscript $(\pm)$ indicates the left- (right-)side incidence. In this Letter, by using the coupled-mode theory (see Sec. B of Supplemental Material [36]), we can obtain the complete reflectivity spectra of the metasurface mirror.

Owing to reciprocity, specular reflections from both sides are identical, i.e., $r^-_{1} = r^+_{1}$. The left-side extraordinary reflection $r^-_{2}$, however, can be different from that of the opposite side, $r^+_{0}$. Here, the designed metasurface consists of six subunits per period ($N = 6$). Loss is introduced to subunit 1 (can be any other subunit), whereas other units are assumed lossless. This means that only subunit 1 has a complex refractive index ($n^2 = n'_1 + in''_1$ and $n''_1 < 0$). The asymmetry of the metasurface mirror, therefore, can be explored by varying the loss of subunit 1, i.e., $n''_1$. It should be pointed out that, in order to maximize the retroreflection efficiency for the right-side incidence, the refractive indices are fine-tuned around the exact values determined by the index gradient (see Sec. C of Supplemental Material [36] for the two-step method for designing the metasurface mirror). This is a common practice in optimizing the performance of metasurfaces [37].

The eigenvalues $\lambda_{\pm} = r^+_{1} \pm \sqrt{r^+_{2} r^-_{0}}$ of the scattering matrix are calculated using the coupled-mode theory. Their trajectories [38] vary with the loss $n''_1$, as shown in Fig. 2(a); more importantly, they cross each other at the EP. In contrast to a conventional $\mathcal{PT}$-symmetric system (balanced gain and loss modulation) where the EP is a threshold below which the eigenvalues coalesce into a real number value, here, the EP is a crossing point of two eigenvalues,
which is similar to what was observed in 1D passive PT-symmetric systems [26–28]. At the EP where $n_1^i$ is at $-0.313\beta$ ($\beta = \lambda_0/2Nd$, and $d$ is the metasurface thickness), the two eigenvalues $\lambda_{\pm}$ coalesce together with the corresponding eigenvectors $v_{\pm} = [1 \pm \sqrt{r_{-2}^+ / r_0^-}]^T$. In other words, these originally orthogonal eigenvectors become parallel. This is a direct manifestation of the EP from the non-Hermiticity of the system [39]. The theoretically calculated amplitudes and phases of these two extraordinary reflection coefficients ($r_0^-$ and $r_{-2}^+$) are shown in Figs. 2(b) and 2(c), respectively. The extraordinary reflections from opposite sides respond very differently to the varying loss. When the loss of subunit 1 is zero, near-unitary extraordinary reflections arise from both sides in a symmetric fashion. As the loss is initially increased, the left-side retroreflection ($r_{-2}^+$) has a declined amplitude. A peculiar case where the left-side retroreflection vanishes can be observed at the EP when $n_1^i$ reaches $-0.313\beta$. Meanwhile, the right-side retroreflection is very strong ($|r_0^-| = 0.974$). As shown by Fig. 2(c), the phase of the left-side retroreflection ($r_{-2}^+$) experiences an abrupt change of $\pi$ at the EP.

Interestingly, $|r_{-2}^+|$ is found to increase with the loss beyond the EP. This unconventional phenomenon, dubbed loss enhanced reflection, is very similar to the loss enhanced transmission reported in Ref. [40]. On the other hand, the right-side retroreflection is very stable against the varying loss, maintaining a very high efficiency ($|r_0^-| > 0.97$) and a virtually constant phase shift [$\arg(r_0^-) = \pi$].

We theoretically calculate the acoustic pressure field of the metasurface mirror at the EP. The top (bottom) figures in Fig. 3 illustrate the left- (right-)side incidence case. From left to right, they show the incident field, the total scattered field contributed by all diffraction modes, the component of retroreflection contributed by $r_{-2}^+ / r_0^-$, and the component of specular reflection contributed by $r_{-1}^- / r_{-1}^-$, respectively. Figures in the second column manifest the asymmetrical response of the metasurface to opposite incidences. Compared to the left-side incidence that generates strong evanescent waves, the waves reflect straight back under the

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**FIG. 2.** (a) Trajectories of eigenvalues $\lambda_{\pm}$ with the evolution of the loss of unit 1 ($n_1^i$). (b) Amplitude and (c) phase of extraordinary reflection coefficients $r_0^- / r_{-2}^+$ by varying $n_1^i$.

**FIG. 3.** Upper figures show theoretical predicted pressure fields with left-side incidence ($\theta_i = 45^\circ$): from left to right, incident field, total scattered field (contributed by all diffraction modes), retroreflection (contributed by diffraction order of $-2$), and specular reflection (contributed by diffraction order of $-1$). Lower figures show theoretical predicted pressure fields with right-side incidence ($\theta_i = -45^\circ$): from left to right, incident field, total scattered field (contributed by all diffraction modes), retroreflection (contributed by diffraction order of 0), and specular reflection (contributed by diffraction order of $-1$).
right-side incidence. Due to reciprocity, specular reflections show a symmetrical pattern, as indicated by the last column of Fig. 3. Here, \( |r_1^+| = |r_2^-| = 0.048 \). This part of the energy flux with an intrinsically symmetric behavior is strongly suppressed so that the overall asymmetrical behavior can be enhanced. The figures in the third column further demonstrate the EP of the system, where no retroreflection is found under the left-side incidence (\( |r_2^+| = 0 \)) and a strong retroreflection is present under the right-side incidence (\( |r_0^-| = 0.974 \)).

To validate the theory, corresponding experiments and numerical simulations have been carried out. The background medium is air. Inspired by the work of [41], the metasurface mirror is constructed by periodically arranged grooves; one of which is opened with a slit (in order to introduce loss) and is covered by a layer of absorptive material at the end [Fig. 4(a)]. Here, the viscous and thermal losses [42–44] of the narrow slit are considered (see Sec. D of Supplemental Material [36]). By tuning the depth of the groove (\( d_j \)) and the width (\( w \)) of the slit, this structure (albeit very elementary) provides fine yet arbitrary manipulation on the real and imaginary parts of the effective refractive index (see Sec. D of Supplemental Material [36]). The operating frequency is 3430 Hz, and the period of the metasurface is \( D = 7.07 \text{ cm} (\lambda_0/\sqrt{2}) \). In each period, six grooves are used with the following depths (\( d_j \)): 2.19, 2.56, 3.85, 4.60, 0.59, and 1.57 cm. All units have the same groove width (\( a = 9.79 \text{ mm} \)) and the same wall thickness (\( t = 1 \text{ mm} \)), and only unit 1 has a slit at its end with a slit width of \( w = 1.34 \text{ mm} \).

The performance of the metasurface is first investigated by COMSOL MULTIPHYSICS. For the measurement, a two-dimensional waveguide with a height of \( h = 4 \text{ cm} \) is built where the acoustic field can be scanned. To scan the field, a microphone moves at a step size of 1 cm in the two regions marked by red boxes in Fig. 4(b) (which identify the retroreflection and specular reflection regions). The length of the sample is 1.06 m (15 periods). The scattered field is obtained by subtracting the incident field (scanned without the metasurface) from the total field (scanned with the metasurface). See Secs. D and E of the Supplemental Material [36] for the details of the sample and the experimental setup. The normalized simulated and measured results are presented in Fig. 4(c). Good agreement can be found between the simulated and measured results, with both suggesting a strongly asymmetrical wave behavior. For the right-side incidence (figure on the left), retroreflection is clearly observable, and the estimated pressure reflection coefficient reaches around 0.87 (simulation) and 0.83 (measurement), respectively. Both reflectivities are lower than predicted. This is because we assumed a perfect plane wave incidence in theory; however,
in the simulation, in order to be consistent with the experimental setup using a loudspeaker array, we used a finite-width plane wave (a quasi-plane-wave beam). Under the opposite incidence, such a reflection is strongly suppressed at the EP due to the judiciously designed loss. It should be pointed out that, although the EP is achieved at 45° for the current design, strongly asymmetrical behavior is in fact observable under a wide range of angles of incidence (see Sec. E of Supplemental Material [36]). However, due to the fact that the leaky loss and phase gradient are dispersive for the given structure, as well as due to the sensitivity of the EP, achieving extreme asymmetry within a broadband frequency range is still a challenge.

To conclude, we have theoretically studied and experimentally validated a non-Hermitian acoustic metasurface mirror. We show that the 2D extreme asymmetry in terms of the extraordinary reflection can be interpreted by the EP of the non-Hermitian system. This peculiar behavior, which is not found in conventional metasurfaces, hinges on elegantly engineered losses, which provide an additional degree of freedom for wave manipulation. In Sec. G of the Supplemental Material [36], we further compare our design with a geometrical-based design and propose a scheme to further reduce the footprint of our metasurface. Our findings expand the exploration of acoustic metasurfaces into the complex domain, and they may open a new route for wave manipulation with new functionalities.

This work was supported by the National Natural Science Foundation of China (Grants No. 11704284 and No. 11774265), the Young Elite Scientists Sponsorship by CAST (Grant No. 2018QNR001), the Shanghai Science and Technology Committee (Grant No. 18JC1410900).

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