Experimental Evidence for Efimov Quantum States

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Abstract. Three interacting particles form a system which is well known for its complex physical behavior. A landmark theoretical result in few-body quantum physics is Efimov’s prediction of a universal set of weakly bound trimer states appearing for three identical bosons with a resonant two-body interaction [1, 2]. Surprisingly, these states even exist in the absence of a corresponding two-body bound state and their precise nature is largely independent of the particular type of the two-body interaction potential. Efimov’s scenario has attracted great interest in many areas of physics; an experimental test however has not been achieved. We report the observation of an Efimov resonance in an ultracold thermal gas of cesium atoms [3]. The resonance occurs in the range of large negative two-body scattering lengths and arises from the coupling of three free atoms to an Efimov trimer. We observe its signature as a giant three-body recombination loss when the strength of the two-body interaction is varied near a Feshbach resonance. This resonance develops into a continuum resonance at non-zero collision energies, and we observe a shift of the resonance position as a function of temperature. We also report on a minimum in the recombination loss for positive scattering lengths, indicating destructive interference of decay pathways. Our results confirm central theoretical predictions of Efimov physics and represent a starting point from which to explore the universal properties of resonantly interacting few-body systems.

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Efimov’s treatment of three identical bosons [1, 2] is closely linked to the concept of universality [4] in systems with a resonant two-body interaction, where the s-wave scattering length $a$ fully characterizes the two-body physics. When $|a|$ greatly exceeds the characteristic range $\ell$ of the two-body interaction potential, details of the short-range interaction become irrelevant because of the long-range nature of the wave function. Universality then leads to a generic behavior in three-body physics, reflected in the energy spectrum of weakly bound Efimov trimer states. Up to now, in spite of their great fundamental importance, these states could not be observed experimentally. An observation in the realm of nuclear physics, as originally proposed by Efimov, is hampered by the presence of the Coulomb interaction, and only two-neutron halo systems with a spinless core are likely to feature Efimov states [5]. In molecular physics, the helium trimer [6] is predicted to have an excited state with Efimov character [7]. The existence of this state could so far not be confirmed [8]. A different approach to experimentally study the physics of Efimov states is based on the unique properties of ultracold atomic
FIGURE 1. Efimov’s scenario: Appearance of an infinite series of weakly bound Efimov trimer states (ET) for resonant two-body interaction. The binding energy is plotted as a function of the inverse two-body scattering length $1/a$. The hatching indicates the scattering continuum for three atoms ($a < 0$) and for an atom and a dimer ($a > 0$). The arrow marks the intersection of the first Efimov trimer with the three-atom threshold. To illustrate the series of Efimov states, we have artificially reduced the universal scaling factor from 22.7 to 2. For comparison, the dashed line (T) indicates the presence of a tightly bound non-Efimov trimer [12] which does not interact with the scattering continuum.

quantum gases. In such systems [9] an unprecedented level of control is possible to study interacting quantum systems. The ultra-low collision energies allow to explore the zero-energy quantum limit and two-body interactions can be precisely tuned based on Feshbach resonances [10, 11].

Efimov’s scenario [1, 2, 4] can be illustrated by the energy spectrum of the three-body system as a function of the inverse scattering length $1/a$ (Fig. 1). Let us first consider the well-known weakly bound dimer state, which only exists for large positive $a$. In the resonance regime, its binding energy is given by the universal expression $E_b = -\hbar^2/(m a^2)$, where $m$ is the atomic mass and $\hbar$ is Planck’s constant divided by $2\pi$. In Fig. 1 where the resonance limit corresponds to $1/a \to 0$, the dimer energy $E_b$ is represented by a parabola for $a > 0$. If we now add one more atom with zero energy, a natural continuum threshold for the bound three-body system (hatching in Fig. 1) is given by the three-atom threshold ($E = 0$) for negative $a$ and by the dimer-atom threshold ($E_b$) for positive $a$. Energy states below the continuum threshold are necessarily three-body bound states. When $1/a$ approaches the resonance from the negative-$a$ side, a first Efimov trimer state appears in a range where a weakly bound two-body state does not exist. When passing through the resonance the state connects to the positive-$a$ side, where it finally intersects with the dimer-atom threshold. An infinite series of such Efimov states is found when scattering lengths are increased and binding energies are decreased in powers of universal scaling factors $e^{\pi/s_0} \approx 22.7$ and $e^{-2\pi/s_0} \approx 1/515$ ($s_0 = 1.00624$), respectively [1, 2, 4].

Resonant scattering phenomena arise as a natural consequence of Efimov’s scenario [13]. When an Efimov state intersects with the continuum threshold at negative scattering lengths $a$, three free atoms in the ultracold limit resonantly couple to a trimer. This results in a triatomic Efimov resonance [14, 15]. At finite collision energies, the
phenomenon evolves into a triatomic continuum resonance [16]. Another type of Efimov resonance [17] is found at positive values of $a$ for collisions between a free atom and a dimer, when Efimov states intersect with the dimer-atom threshold. While the latter type of Efimov resonance corresponds to Feshbach resonances in collisions between atoms and dimers [17], triatomic Efimov resonances can be interpreted as a three-body generalization to Feshbach resonances [18].

Striking manifestations of Efimov physics have been predicted for three-body recombination processes in ultracold gases with tunable two-body interactions [4, 14, 15, 19, 20, 21]. Three-body recombination leads to losses from a trapped gas with a rate proportional to the third power of the atomic number density. These losses are commonly described [22] in terms of a loss rate coefficient $L_3$. In the resonant case ($|a| \gg \ell$), it is convenient to express this coefficient in the form $L_3 = 3C(a)\hbara^4/m$, separating a general $a^4$-scaling [22, 23] from an additional dependence [14, 15, 20] $C(a)$. Efimov physics is reflected in a logarithmically periodic behavior $C(22.7a) = C(a)$, corresponding to the scaling of the infinite series of weakly bound trimer states. For negative scattering lengths, the resonant coupling of three atoms to an Efimov state opens up fast decay channels into deeply bound dimer states plus a free atom. Triatomic Efimov resonances thus show up in giant recombination loss. This striking phenomenon was first identified in numerical solutions to the adiabatic hyperspherical approximation of the three-body Schrödinger equation assuming simple model potentials and interpreted in terms of tunneling through a potential barrier in the three-body entrance channel [14]. A different theoretical approach [4, 15], based on effective field theory, provides the analytic expression $C(a) = 4590\sinh(2\eta_-)/(\sin^2[\ln(|a|/a_-)] + \sinh^2\eta_-)$. The free parameter $a_-$ for the resonance positions at $a_-, 22.7a_-, \ldots$ depends on the short-range part of the three-body interaction and is thus not determined in the frame of the universal long-range theory. As a second free parameter, the dimensionless quantity $\eta_-$ describes the unknown decay rate of Efimov states into deeply bound dimer states plus a free atom. It thus characterizes the resonance width.

Our measurements are based on the magnetically tunable interaction properties of Cs atoms [24] in the lowest internal state (quantum numbers $F = 3$ for the total spin and $m_F = 3$ for its projection). By applying fields between 0 and 150 G, we can vary the s-wave scattering length $a$ in a range between $-2500a_0$ to $1600a_0$, where $a_0$ is Bohr’s radius. The dependence can in general be well approximated by the fit formula

$$a(B)/a_0 = (1722 + 1.52B/G)\left(1 - \frac{28.72}{B/G + 11.74}\right),$$

except for a few narrow Feshbach resonances [24]. The smooth variation of the scattering length in the low-field region results from a broad Feshbach resonance centered at about $-12$ G (equivalent to $+12$ G in the state $F = 3$, $m_F = -3$). In all our measurements we excluded the magnetic field regions where the narrow Feshbach resonances influence the scattering behavior through coupling to other molecular potentials. Accurate three-body loss measurements are facilitated by the fact that inelastic two-body loss is energetically forbidden [22]. The characteristic range of the two-body potential is given by the van der Waals length [25], which for Cs is $\ell \approx 100a_0$. This leaves us with enough room to study the universal regime requiring $|a| \gg \ell$. For negative $a$, a maxi-
mum value of 25 is attainable for $|a|/\ell$. Efimov’s estimate $\frac{1}{\pi} \ln(|a|/\ell)$ for the number of weakly bound trimer states [2] suggests the presence of one triatomic Efimov resonance in the accessible range of negative scattering lengths.

All measurements were performed with trapped thermal samples of Cs atoms at temperatures $T$ ranging from 10 to 250 nK. We used two different experimental setups, which have been described elsewhere [26, 27, 28]. In setup A we first produced an essentially pure Bose-Einstein condensate (BEC) with up to 250,000 atoms in a far-detuned crossed optical dipole trap generated by two 1060-nm Yb-doped fiber laser beams [27]. We then ramped the magnetic field to 16.2 G where the scattering length is negative with a value of $-50 a_0$, thus inducing a collapse of the condensate [29]. After an equilibration time of 1 s we were left with a thermal sample at typically $T = 10$ nK containing up to 20,000 atoms at peak densities ranging from $n_0 = 3 \times 10^{11} \text{cm}^{-3}$ to $3 \times 10^{12} \text{cm}^{-3}$. Alternatively, we interrupted the evaporation process before condensation to produce thermal samples at $T \approx 200$ nK in a crossed dipole trap generated by one of the 1060-nm beams and a 10.6-µm CO$_2$-laser beam. After recompression of the trap this produced typical densities of $n_0 = 5 \times 10^{13} \text{cm}^{-3}$. To determine the three-body loss rates in this setup we recorded the time evolution of the atom number $N$ and the temperature $T$. A detailed description of this procedure has been given in Ref. [22]. In brief, the process of three-body recombination not only leads to a loss of atoms, but also induces “anti-evaporation” and recombination heating. The first effect is present at any value of the scattering length $a$. The second effect occurs for positive values of $a$ when the recombination products remain trapped. Atom loss and temperature rise are modelled by a set of two coupled non-linear differential equations. We used numerical solutions to this set of equations to fit our experimental data. From these fits together with measurements of the trapping parameters we obtained the rate coefficient $L_3$. Alternatively, we simply measured the loss fraction after some fixed storage time as a function of magnetic field, i.e. atomic scattering length. For this the storage time was 200 ms at initial densities of $n_0 = 6 \times 10^{13} \text{cm}^{-3}$. In setup B we used an optical surface trap [28] in which we prepared a thermal sample of 10,000 atoms at $T \approx 250$ nK via forced evaporation at a density of $n_0 = 1.0 \times 10^{12} \text{cm}^{-3}$. The dipole trap was formed by a repulsive evanescent laser wave on top of a horizontal glass prism in combination with a single horizontally confining 1060-nm laser beam propagating along the vertical direction. In this setup we measured three-body loss rates by recording the loss as a function of time at sufficiently short decay times for which heating is negligible. To determine the position of maximum three-body loss we again chose the simpler procedure of measuring the loss fraction after some fixed storage time as a function of magnetic field.

Our experimental results (Fig. 2) indeed show a giant loss feature marking the expected resonance. The Efimov resonance is centered at 7.5 G. We present our data in terms of a recombination length [14] $\rho_3 = [2m/(\sqrt{3} \hbar)] L_3^{1/4}$, which leads to the simple relation $\rho_3/a = 1.36 C^{1/4}$. Note that the general $a^4$-scaling corresponds to a linear behavior in $\rho_3(a)$ (straight lines in Fig. 2). A fit of the analytic theory [4, 15] to our experimental data taken for negative $a$ at temperatures $T \approx 10$ nK shows a remarkable agreement and determines the resonance position to $a_- = -850(20) a_0$ and the decay parameter to $\eta_- = 0.06(1)$. The pronounced resonance behavior with a small value for the
FIGURE 2. Observation of the Efimov resonance in measurements of three-body recombination. The recombination length $\rho_3 \propto L_3^{-1/4}$ is plotted as a function of the scattering length $a$. The dots and the filled triangles show the experimental data from setup A for initial temperatures around 10 nK and 200 nK, respectively. The open diamonds are from setup B at temperatures of 250 nK. The open squares are previous data [22] at initial temperatures between 250 and 450 nK. The solid curve represents the analytic model from effective field theory [4] with $a_- = -850a_0$, $a_+ = 1060a_0$, and $\eta_- = \eta_+ = 0.06$. The straight lines result from setting the $\sin^2$ and $\cos^2$-terms in the analytic theory to 1, which gives a lower recombination limit for $a < 0$ and an upper limit for $a > 0$. The inset shows an expanded view for small positive scattering lengths with a minimum for $C(a) \propto (\rho_3/a)^{4}$ near $210a_0$. The displayed error bars refer to statistical uncertainties only. Uncertainties in the determination of the atomic number densities may lead to additional calibration errors for $\rho_3$ of up to 20%.

decay parameter ($\eta_- \ll 1$) demonstrates a sufficiently long lifetime of Efimov trimers to allow their observation as distinct quantum states.

All the results discussed so far are valid in the zero-energy collision limit of sufficiently low temperatures. For ultralow but non-zero temperatures the recombination length is unitarity limited [21] to $5.2 \hbar (mk_BT)^{-1/2}$. For $T = 10$ nK this limit corresponds to about 60,000$a_0$ and our sample is thus cold enough to justify the zero-temperature limit. For 250 nK, however, unitarity limits the recombination length to about 12,000$a_0$. The Efimov resonance is still visible at temperatures of 200 and 250 nK (filled triangles and open diamonds in Fig. 2). The slight shift to lower values of $|a|$ suggests the evolution of the zero-energy Efimov resonance into a triatomic continuum resonance [16]. In further experiments at higher temperatures (data not shown) we observed the resonance to disappear above $\sim 500$ nK.

To quantify the shift of the resonance position [30] we have determined the magnetic field value of maximum loss for temperatures in the range from 500 nK down to 44 nK in setup B. We observe the peak position to shift from 7.7 G at high temperatures to 7.4 G for the lowest temperature. Fig. 3 presents the data after conversion of the magnetic field values to the corresponding scattering lengths. The loss peak shifts from $-825 a_0$ at 500 nK down to $-872 a_0$ for the coldest samples. Two data points have been measured after deep evaporation followed by a recompression ramp that increased the trapping potential by a factor of three within 900 ms. The results are consistent with the measurements in uncompressed traps and show that plain evaporation is absent. The vertical error bars indicate the statistical uncertainty from the parabolic fitting
For positive scattering lengths, we found three-body losses to be typically much weaker than for negative values. Our measurements are consistent with a maximum recombination loss of \( C(a) \approx 70 \), or equivalently \( \rho_3 \approx 3.9a \), as predicted by different theories \([14, 19, 20]\) (straight line for \( a > 0 \) in Fig. 2). For \( a \) below 600\( a_0 \) the measured recombination length significantly drops below this upper limit (inset in Fig. 2). The analytic expression from effective field theory \([4, 20]\) for \( a > 0 \) reads \( C(a) = 67.1 e^{-2\eta_+} \left( \cos^2 \left[ s_0 \ln(a/a_+) \right] + \sinh^2 \eta_+ \right) + 16.8 (1 - e^{-4\eta_+}) \) with the two free parameters \( a_+ \) and \( \eta_+ \). The first term describes recombination into the weakly bound dimer state with an oscillatory behavior due to an interference effect between two different pathways \([14, 19]\). The second term results from decay into deeply bound states. We use this expression to fit our data points with \( a > 5 \ell \approx 500a_0 \). This somewhat arbitrary condition is introduced as a reasonable choice to satisfy \( a \gg \ell \) for the validity of the universal theory. The fit is quite insensitive to the value of the decay parameter and yields \( \eta_+ < 0.2 \). This result is consistent with the theoretical assumption \([15]\) of the same value for the decay parameter for positive and negative \( a \), which in our case is \( \eta_+ = \eta_- = 0.06 \). For maximum loss, we obtain \( a_+ = 1060(70)a_0 \). According to theory \([4]\), the trimer state hits the dimer-atom threshold at \( a = 1.1a_+ \approx 1170a_0 \). The logarithmic periodicity of the Efimov scenario suggests adjacent loss minima to occur at \( \sqrt{22.7} \times 1060a_0 \approx 5000a_0 \) and at \( 1060a_0 / \sqrt{22.7} \approx 220a_0 \). While the former value is out of our accessible range, the latter value (\( a \approx 2\ell \)) is too small to strictly justify universal behavior in the resonance limit (\( a \gg \ell \)). Nevertheless, our experimental results (inset to Fig. 2) indicate a minimum at \( a \approx 210a_0 \) and the analytic expression for \( C(a) \) is found to describe our data quite well down to this minimum.

The occurrence of the interference minimum in three-body loss is demonstrated more clearly in another set of experiments where we simply measured the loss of atoms after a fixed storage time in the optical trap (Fig. 4). This minimum is located at \( a = 210(10)a_0 \) in addition to a second minimum close to zero scattering length. We point out that the existence of the minimum at 210\( a_0 \) is very advantageous for efficient...
FIGURE 4. Atom loss for small scattering lengths as measured in setup A. Besides a minimum near zero scattering length, we identify a minimum of recombination loss at $\sim 210 a_0$, which can be attributed to a predicted destructive interference effect [14, 19, 20].

The evaporative cooling of Cs as it combines a large scattering cross section with very low loss. Inadvertently, we have already benefited of this loss minimum for the optimized production of a Bose-Einstein condensate of Cs [27].

The comparison of our experimental results to available three-body theory shows remarkable agreement, although the collision physics of Cs is in general a very complicated multi-channel scattering problem. We believe that the particular nature of the broad, “open-channel dominated” Feshbach resonance [31] that underlies the tunability of our system plays a crucial role. For such a resonance, the two-body scattering problem can be reduced to an effective single-channel model. It is a very interesting question to what degree this great simplification of the two-body physics extends to the three-body problem. In particular, we raise the question how the regions of positive and negative scattering lengths are connected in our experiment, where $a$ is changed through a zero crossing, i.e. through a non-universal region, and not across the universal resonance region. In our case, there is no obvious connection between the Efimov state that leads to the observed resonance for $a < 0$ and the states responsible for the behavior for $a > 0$. In our analysis of the experimental data, we have thus independently fitted the data sets for negative and positive $a$. Nevertheless, the resulting values for the two independent fit parameters $a_-$ and $a_+$ suggest a connection: For the ratio $a_+/|a_-|$ our experiment yields 1.25(9), whereas universal theory [4] predicts 0.96(3). These numbers are quite close in view of the Efimov factor of 22.7. If not an accidental coincidence, we speculate that the apparent relation between $a_+$ and $a_-$ may be a further consequence of universality in a system where the resonant two-body interaction can be modelled in terms of a single scattering channel. In general, the multi-channel nature of three-body collisions near Feshbach resonances [32, 33] leads to further interesting questions, like e.g. possible resonance effects beyond the Efimov scenario. Advances in three-body theory are necessary to answer these questions and to provide a complete interpretation of our present observations.

In the past few years, applications of Feshbach resonances in ultracold gases and the resulting possibility to create dimer states have set the stage for many new developments in matter-wave quantum physics. The observation of an Efimov resonance now confirms the existence of weakly bound trimer states and opens up new vistas [18, 34].
to experimentally explore the intriguing physics of few-body quantum systems.

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