A Boundary Integral Method for 3-D Nonuniform Dielectric Waveguide Problems via the Windowed Green Function

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Abstract—This communication proposes an efficient boundary-integral-based “windowed Green function” (WGF) methodology for the numerical solution of 3-D general electromagnetic problems containing dielectric waveguides. The approach, which generalizes a recently introduced 2-D version of the method, provides a highly effective solver for such problems. In particular, using an auxiliary integral representation, the proposed method is able to accurately model incident mode excitation. On the basis of a smooth window function, the integral operators along the infinite waveguide boundaries are smoothly truncated, resulting in errors that decay faster than any negative power of the window size.

Index Terms—Chebyshev approximation, integral equations, numerical analysis, optical waveguides.

I. INTRODUCTION

In view of their ability to efficiently guide electromagnetic energy, dielectric (open) waveguides play central roles in many engineered electromagnetic systems, including antenna feeds, coaxial cable transmission lines, optical fibers, and nanophotonic devices, among many others [1], [2], [3], [4], [5]. As discussed in what follows, however, the computational simulation of electromagnetic waveguides in three dimensions (3-D) has posed a number of significant challenges—mostly concerning the accuracy and computational cost. Seeking to tackle this difficulty, this communication introduces an efficient boundary integral equation (BIE) Green-function-based methodology for this problem. This method incorporates, as a central enabling element, a novel “windowed Green function” (WGF) strategy—previously demonstrated in the context of layered media in two-dimensions and 3-D, and for waveguide structures in two dimensions [6], [7], [8], [9]—to handle the truncation of BIE integration domains for fully vectorial problems involving 3-D dielectric waveguides with arbitrary shapes and configurations.

Most of the existing approaches for waveguide simulation rely on volumetric discretizations and approximation of the differential form of Maxwell’s equations [10], incorporating an absorbing condition, such as a perfectly matched layer (PML) [11], [12], to truncate the computational domain and, in particular, all of the semiinfinite waveguides (SIWGs) present in the structure. The approach, thus, requires the use of large 3-D computational domains, which may amount to tens or even hundreds of wavelengths in relevant applications, can lead to significant dispersion errors by accumulation, over many wavelengths, of the errors inherent in the derivative approximations used in finite difference and finite element methods at each discretization point. The time-domain versions of these methods that are often employed additionally suffer from dispersion and accuracy loss in the time variable. As a result, these approaches may require extremely fine meshes and associated high computing costs to maintain accuracy.

On the basis of the electromagnetic Green’s function, on the other hand, boundary integral methods (BIE) can lead to significantly reduced numbers of unknowns, since they only require discretization of the interfaces between different materials. Additionally, BIE methods are virtually free of numerical dispersion, in view of Green’s function’s ability to analytically propagate oscillatory fields to arbitrary distances. And, although in their straightforward implementations, they lead to computing costs that grow quadratically with the discretization sizes (at least if iterative solvers, such as generalized minimum residual method (GMRES) [13], are used), BIE methods can be accelerated by a variety of techniques [14], [15], [16], [17]. Unfortunately, the application of BIE methods has almost exclusively been restricted to systems with bounded scatterers, in view of the lack of a suitable condition for the termination of the infinite computational domain arising from infinite boundary interfaces.

Several effective approaches have recently emerged for the BIE waveguide-truncation problem, however. These include the WGF method [6], [7], [8], [9], the closely related integral PML method [18], and a physically motivated “surface conductive absorber” (SCA) approach [19], [20]. Other methods, based on the use of adiabatic absorbers in conjunction with volumetric integral formulations, have also been demonstrated [21], [22]. The WGF method relies on a reformulation of the waveguide BIE equations, which results in Green’s function being smoothly windowed to compact support via multiplication by a “slow-rise” window function in the integration variable. This procedure, which only requires multiplication of Green’s function by an adequate smooth window function, effectively screens out infinite boundary portions without introducing nonphysical reflection points on the boundary of the truncation domain, and it, thus, yields super-algebraic convergence—that is, asymptotically, faster convergence than any negative power of the window size. (A detailed and motivated description of the WGF method, including demonstrations of its accuracy and a strategy for the selection of algorithmic parameters, can be found in [6]). The integral PML contribution [18] operates on the basis of the same principle as the WGF method: in the integral PML case the windowing effect is achieved by complexifying the spatial variables starting at a certain
distance along the waveguide, which results in decaying exponentials in Green’s function, and thus, provide the desired reflection-screening effect. The SCA approach [19], finally, incorporating a surface conductivity on the material interfaces sufficiently far along the waveguides, demonstrates decay rates of the order of $1/x^p$ with the distance $x$ along the waveguide, with, e.g., $4 \leq p \leq 8$ for screening of waveguide modes and with $p = 1$ for screening of reactive fields; see [19, Figs. 9 and 10].

The present contribution generalizes the 2-D WGF method for waveguides [7] to the fully vectorial 3-D counterpart. It shows that the approach can handle both illuminating beams and point sources, as well as direct mode illumination. This communication is organized as follows. Section II describes the integral representation of the fields and the associated integral equations over the unbounded waveguide boundaries, making a distinction between two types of illuminating fields, namely, beams and point sources on one hand, and direct mode sourcing, on the other. Section III describes the ideas underlying the windowing of integral operators, and it presents our implementations for the two types of incidence considered in this communication. Finally, Section IV presents a variety of numerical examples that demonstrate the effectiveness and flexibility of the WGF method for geometrically complex waveguide structures.

II. BOUNDARY INTEGRAL FORMULATION FOR 3-DIELECTRIC WAVEGUIDES

Our approach is based on Müller’s frequency-domain integral formulation [23], [24] at a given temporal frequency $\omega$, which follows from the use of Green’s representation theorems [25]. (A time dependence of the type $\exp(-i\omega t)$ is assumed and suppressed throughout.) For clarity, in our description, we consider a waveguide structure consisting of a single-core (interior) region $\Omega_i$ (containing material of permittivity $\varepsilon_i$, magnetic permeability $\mu_i$, and spatial wavenumber $k_i$), and a single-cladding (exterior) region $\Omega_e$ (containing material of permittivity $\varepsilon_e$, magnetic permeability $\mu_e$, and spatial wavenumber $k_e$), as depicted in Fig. 1. The unbounded interface between $\Omega_i$ and $\Omega_e$, and the corresponding normal pointing from the former to the latter are denoted by $\Gamma$ and $n$, respectively.

The wavenumber is related to the frequency and material properties by the relation $k = \omega/\sqrt{\mu\varepsilon}$. The generalization of these methods to structures including an arbitrary number of waveguides and dielectric materials does not pose major difficulties.

In this communication, we consider two types of incident excitation, namely, type I excitation by fields $(E^{inc}, E^{inc\prime})$ (beams or point source incidence), for which the scattered fields $E_i$ and $E_e$ in the decomposition

$$E = E^{inc} + E_i \quad \text{in } \Omega_i, \quad E^{inc\prime} + E_e \quad \text{in } \Omega_e$$

of the total electric field $E$ are used as unknowns; and type II excitation (bound-mode incident field), for which the total electric fields

$$E = E_i \quad \text{in } \Omega_i, \quad E_e \quad \text{in } \Omega_e$$

themselves are used as unknowns.

Remark 1: For both type I and type II problems, the electromagnetic field can be expressed in terms of Green’s function-based integral representations. The main difference in our treatment of these two cases is that for type II problems, we use an integral representation of the total field (including the incident bound mode), whereas for type I problems we use an integral representation of the scattered field.

In order to introduce the aforementioned integral representations, we consider the following vector potentials acting on a surface tangential density $a(r')$:

$$\mathcal{B}_i[a](r) \equiv \nabla \times \int_{\Gamma} G_i(r, r')a(r')d\sigma(r')$$

$$\mathcal{B}_e[a](r) \equiv \nabla \times \nabla \times \int_{\Gamma} G_i(r, r')a(r')d\sigma(r').$$

Here, $G_i(r, r') = \exp(i k_i |r - r'|)/4\pi |r - r'|$ denotes the free-space Helmholtz Green’s function with wavenumber $k_i$. Subscript values $\ell = i$ and $\ell = e$ indicate quantities corresponding to the interior and exterior domains, respectively.

The tangential values of the field (3) for $r \in \Gamma$ are given by the boundary integral operators

$$S_i[a](r) \equiv -n(r) \times \int_{\Gamma} G_i(r, r')a(r')d\sigma(r')$$

$$R_i[a](r) \equiv -n(r) \times \nabla \times \int_{\Gamma} G_i(r, r')a(r')d\sigma(r')$$

$$T_i[a](r) \equiv -n(r) \times \nabla \int_{\Gamma} G_i(r, r') \mathbf{a}(r')d\sigma(r')$$

which are defined for $r \in \Gamma$. For conciseness, in what follows, we also use the weakly singular operators

$$R^w_i \equiv \frac{2}{\alpha_i + \alpha_e} [\mathbf{a}(r) \cdot \mathbf{n}_e - \mathbf{a}(r) \cdot \mathbf{n}_i]$$

$$K^w_i \equiv \frac{2 \ell_i}{\omega(\alpha_i + \alpha_e)} \left[ (\mathbf{r}_i - \mathbf{r}_e) \times (\mathbf{k}_i^2 \mathbf{S}_i - \mathbf{k}_e^2 \mathbf{S}_e) \right]$$

where the subindex $\alpha$ stands either for the dielectric constant symbol, $\alpha = \varepsilon$, or the magnetic permeability symbol, $\alpha = \mu$. Finally, we call

$$\mathbf{m}_i \equiv -n \times \mathbf{E}_i \quad \text{and} \quad \mathbf{j}_i \equiv -n \times \mathbf{H}_i \quad \text{on } \Gamma$$

$$\mathbf{m}_e \equiv +n \times \mathbf{E}_e \quad \text{and} \quad \mathbf{j}_e \equiv +n \times \mathbf{H}_e \quad \text{on } \Gamma$$

the tangential magnetic and electric surface currents $\mathbf{m}_i$ and $\mathbf{j}_i$ on the interior ($\ell = i$) and exterior ($\ell = e$) sides of $\Gamma$.

Using these notations, the direct integral representations [25, Th. 5.5.1] (cf. the Stratton–Chu formulas [26, Th. 6.7]) for the exterior and interior fields are given by

$$\mathcal{S}_i[\mathbf{m}_i](r) + \frac{i}{\omega \mu_i} \mathcal{B}_i[\mathbf{j}_i](r) = \begin{cases} 
\mathbf{E}_i(r), & r \in \Omega_i \\
0, & r \not\in \Omega_i
\end{cases}$$

$$\mathcal{S}_e[\mathbf{m}_e](r) - \frac{i}{\omega \varepsilon_e} \mathcal{B}_e[\mathbf{j}_i](r) = \begin{cases} 
\mathbf{H}_e(r), & r \in \Omega_e \\
0, & r \not\in \Omega_e
\end{cases}$$

with $\ell = e$ and $\ell = i$, respectively. The Müller system of BIEs results as a linear combination of the tangential limiting values of the fields in (7) as $r \to \Gamma$ from the exterior and interior domains [24], [23], [27]. The linear combination is selected in such a way that all the resulting integral operators are weakly singular.
In the case of type I excitation, the continuity of the tangential components of the electromagnetic fields tells us that the unknown density currents satisfy the relations

\[ \begin{align*}
    m &\equiv m_\ell = -m_\ell + n \times (E^{inc}_\ell - E^{inc}_\ell) \\
    j &\equiv j_\ell = -j_\ell + n \times (H^{inc}_\ell - H^{inc}_\ell).
\end{align*} \tag{8a} \tag{8b} \]

The resulting Müller system for the type I unknowns \( m \) and \( j \) reads

\[ \begin{bmatrix}
    1 + R^\Delta \hspace{1em} K^\Delta \\
    -K^\Delta \hspace{1em} 1 + R^\Delta
\end{bmatrix}
\begin{bmatrix}
    m \\
    j
\end{bmatrix} =
\begin{bmatrix}
    2(\epsilon_\ell + S_\ell)(\epsilon_\ell - S_\ell)E^{inc}_\ell \times n \\
    2(\mu_\ell + S_\ell)(\mu_\ell - S_\ell)H^{inc}_\ell \times n
\end{bmatrix}. \tag{9} \]

It is easy to check that this system involves only weakly singular kernels; see [9, Ch. 5], [24], [27].

For type II excitation (that is, waveguide-mode incident fields), we utilize integral representations for the total fields, which incorporate integral expressions for the incident bound mode as well as the radiation conditions for waveguide problems—thus taking into account the directionality of incoming and outgoing modes [28].

Following [7], we assume that the waveguide structure includes one or more SIWGs. An SIWG is one-half of an infinite waveguide (of arbitrary cross section), as is obtained by cutting an ordinary infinite waveguide (of arbitrary cross section) along the waveguide and radiating modes. An SIWG is one-half of an infinite waveguide (of arbitrary cross section), as is obtained by cutting an ordinary infinite waveguide (of arbitrary cross section) along the waveguide and radiating modes. An SIWG is one-half of an infinite waveguide (of arbitrary cross section), as is obtained by cutting an ordinary infinite waveguide (of arbitrary cross section) along the waveguide and radiating modes. An SIWG is one-half of an infinite waveguide (of arbitrary cross section), as is obtained by cutting an ordinary infinite waveguide (of arbitrary cross section) along the waveguide and radiating modes. An SIWG is one-half of an infinite waveguide (of arbitrary cross section), as is obtained by cutting an ordinary infinite waveguide (of arbitrary cross section) along the waveguide and radiating modes.

In view of these considerations, for the present incident-mode problems, we define \( \Omega^{inc} \) as the union of all SIWGs on which the incoming bound modes are given. Then, for both \( \ell = i \) and \( \ell = e \), the electric fields are characterized by the decomposition

\[ E_\ell = \begin{cases}
    E^{stat}_\ell + E^{inc}_\ell, & \text{in } \Omega_i \cap \Omega^{inc} \\
    E^{stat}_\ell, & \text{in } \mathbb{R}^3 \setminus (\Omega_i \cup \Omega^{inc})
\end{cases} \tag{10} \]

with similar expressions for the magnetic fields \( H_\ell \). The integral representation in (7) is valid with the surface-current expressions in (6), but, in the present incidence-mode case, \( E_\ell \) (\( \ell = i, e \)) represents the total field, not just the scattered component of the field. We can then reexpress the unknown surface currents in terms of the scattered fields, by utilizing the known incident densities

\[ \begin{align*}
    m &\equiv -n \times E^{inc}_\ell \\
    j &\equiv -n \times H^{inc}_\ell
\end{align*} \tag{11a} \tag{11b} \]

Just as in the case of type I sources, we must enforce the continuity of the tangential components of the fields, which implies that \( m_e = -(m_i + m^{stat}_i) \) and \( j_e = -(j_i + j^{stat}_i) \). Using the representation formula in (7) and with the same derivation as for type I problems, we obtain the Müller system for type II incidence

\[ \begin{bmatrix}
    1 + R^\Delta \hspace{1em} K^\Delta \\
    -K^\Delta \hspace{1em} 1 + R^\Delta
\end{bmatrix}
\begin{bmatrix}
    m \\
    j
\end{bmatrix} =
\begin{bmatrix}
    1 + R^\Delta \hspace{1em} K^\Delta \\
    -K^\Delta \hspace{1em} 1 + R^\Delta
\end{bmatrix}
\begin{bmatrix}
    m^{inc} \\
    j^{inc}
\end{bmatrix}. \tag{12} \]

In order to produce a physical solution (e.g., to avoid the trivial solution \( [m, j] = [m^{inc}, j^{inc}] \)), the scattered currents \( m \) and \( j \) must satisfy the waveguide radiation conditions [28, eq. (36)]. Briefly, these radiation conditions, which are a generalization of the Silver–Müller condition [23], state that the scattered solution must only consist of outgoing bound modes along the waveguide and radiating fields away from the core region. The validity of both of these conditions in the WGF formulation results directly from its use of outgoing Green functions.

III. WINDOWING OF INTEGRAL OPERATORS

The numerical solution of (9) over the unbounded waveguide boundary \( \Gamma \) requires careful consideration, in view of the slow decay rate—\( O(1/r) \)—of the integrands in the associated integral operators. As discussed in this section, to do this, following [7], we:

1) localize the scattering problem to a bounded region encompassing the nonuniform parts of the waveguide structure;
2) use a window function to evaluate the slowly decaying, oscillatory integrals in the operators involving scattered currents; and
3) for type II problems (bound mode excitation), use the representation formula for the modes to evaluate the incident currents from the boundary mode.

The integrands in the integral operators (4), which are defined over the unbounded boundary \( \Gamma \), decay not only as \( O(1/r) \) as \( r \to \infty \) but they are also oscillatory, which makes their integrals finite. A direct truncation of the boundary \( \Gamma \) leads to extremely poor convergence—only \( O(1/\sqrt{A}) \), where \( A \) denotes the SIWG-truncation length [7, 29]. However, super-algebraic convergence—\( O(A^{-p}) \) for any positive integer \( p \)—can be regained on the basis of the same truncation size \( A \), simply by multiplying the relevant integrands by a compactly supported, smooth “window” function.

For strict super-algebraic convergence, the window function \( w_A(d) \) must satisfy certain smoothness and “slow-rise” conditions [7, Sec. III-B], namely, that the windowing function is smooth and that it decays from 1 to 0 over the region \( d \in (\alpha A, A) \) for some \( \alpha \in (0, 1) \). A suitable choice of window function that satisfies the conditions for super-algebraic convergence is given by the expression

\[ w_A(d) = \begin{cases}
    1, & s(d) < 0 \\
    \exp \left( -2 \frac{1}{1 - s(d)} \right), & 0 \leq s(d) \leq 1 \\
    0, & s(d) > 1
\end{cases} \tag{13} \]

where \( s(d) = (|d| - \alpha A)/(A - \alpha A) \); throughout this communication, we utilize this window function with \( \alpha = 0.5 \) and \( A = 10 \). High-order convergence can also be achieved using other suitable window functions, such as the error function, that do not have strict compact support but tend to zero exponentially fast at infinity.

As shown in [7], using a window function \( w_A \) can greatly improve the rate of convergence—so that oscillatory integrals along infinite domains can be accurately truncated by multiplying the integrand by \( w_A \). On the other hand, although the convergence rate is “super-algebraic” (faster than any power of \( A \)), the integrands are not sufficiently oscillatory, then large values of \( A \) may be necessary to achieve even modest accuracy. Hence, when the integrands might have vanishing oscillations—as is the case of the right-hand side of (12)—a special treatment is required; see the third paragraph in [7, Sec. III(d)].

In the context of the 3-D waveguide problems considered in this communication, we can construct a suitable window function, denoted by \( W_A(r) \) for \( r \in \Gamma \), on the basis of the 1-D window function (13). Indeed, we first set \( W_A(r) \) to equal one when \( r \) is not in any of the SIWGs. Then, for \( r \) in an SIWG, we set \( W_A \) equal to \( w_A(d) \), where \( d \) denotes the distance from \( r \) to the plane perpendicular to the optical axis that determines the end of the SIWG. To define this mathematically, let \( M \) denote the total number of SIWGs, and let \( \Omega_S^{SWG} \) denote the domain that encompasses the qth SIWG with its origin located at \( a_q \) and the optical axis (pointing toward the infinite portion of the SIWG) along the \( \hat{e}_q \) direction. Then, we have

\[ W_A(r) = \begin{cases}
    w_A(\hat{e}_q \cdot (r - a_q)), & r \in \Gamma \cap \Omega_S^{SWG} \\
    1, & \text{otherwise}
\end{cases} \tag{14} \]
This definition of the window function for our waveguide problems leads in a natural way to a truncated domain for which the window is nonzero, i.e., \( \Gamma^w = \Gamma \cap \{ r : W_A(r) > 0 \} \). (For a different window function that does not have strict compact support, this definition of the truncated domain can be easily modified by taking the set for which \( W_A(r) > \eta \), for some small tolerance \( \eta > 0 \).

A. Type I Incidence: Direct Windowing

For the sake of simplicity, we assume incident fields for type I problems to have wavevectors with positive projections onto the optical axis \( \hat{e}_p \) of all component SIWGs. This condition guarantees that the net oscillation of the integrands in the operators of (9)—taking into account both the oscillation of the kernels and the solution currents—are bounded-away from zero, so that we can rely on windowing along the infinite boundary \( \Gamma \) to accurately truncate the simulation boundaries onto \( \Gamma^w \). In the case, that this condition is not met, such vanishing oscillations can be treated by means of an approach similar to the one used in [6] for layered media, or, simply, by rescaling the infinite boundaries in terms of the net integrand wavenumber (net number of oscillations).

With the aforementioned assumption on the illuminating fields, we can directly window the integral operators on the left-hand side of (9) to obtain

\[
\begin{align*}
I + R^w_{\ell} W_A & \quad K^\Delta_{\ell} W_A \quad \int m \\
- K^\Delta_{\ell} W_A & \quad I + R^w_{\ell} W_A \quad \int j
\end{align*}
\]

\[= \left[ \begin{array}{c}
2(\varepsilon_r + \varepsilon_i) (e_r E_{\text{inc}}^{\epsilon} - e_i E_{\text{inc}}^{\mu}) \times n \\
2(\mu_r + \mu_i)(\mu_r H_{\text{inc}}^{\mu} - \mu_i H_{\text{inc}}^{\epsilon}) \times n
\end{array} \right].
\]

This system of integral equations over the bounded boundary \( \Gamma^w \) provides a super-algebraic approximation (w.r.t. the window size \( \ell \)) of the original, unbounded problem in the region of interest, i.e., where the nonuniform part of the structure is present. The system in (15) can now be solved numerically using any bounded obstacle numerical method for integral equations.

Remark 2: In view of the definitions of the currents \( j \) and \( m \) in terms of the interior scattered fields in (8), together with the waveguide radiation conditions discussed in Section II, the currents behave asymptotically as the tangential components of a superposition of outgoing bound modes. The \( m \)th mode contribution along the \( q \)th SIWG contains a factor of \( \exp(i k_q \cdot r') \) (where \( k_q \) denotes the propagation constant of the \( m \)th mode). Since the integral kernels oscillate with a factor of \( \exp(i k_q \cdot r - r') \), and since both \( k_q \) and \( k_p \) are positive, the product of the kernels and the currents result in nonvanishing oscillations as \( r_{1r} \to \infty \).

B. Type II Incidence: Accurate Evaluation of Incident Modes

This section proposes an integral equation methodology to accurately simulate the scattering of incident-bound modes. To do this, windowing the integration domain in the same way as in Section III-A, we split the integrals on the right-hand side of (12) as the sum of two integrals—one over the bounded domain \( \Gamma^w \) and another over the unbounded domain \( \Gamma^\infty \) (noting that \( \Gamma = \Gamma^w \cup \Gamma^\infty \)). Let \( R_{\Delta r}^{w} \) and \( R_{\Delta r}^{\infty} \) denote the operators in (5) but with the integration domains in (4) substituted by \( \Gamma^w \) and \( \Gamma^\infty \), respectively. Using similar definitions for the \( K \) operator, we see that the right-hand side operators for the first equation in (12) are given by

\[
R_{\Delta r}^{w}[m^{\text{inc}}](r) + K_{\Delta r}^{\Delta}[j^{\text{inc}}](r) = \left( R_{\Delta r}^{w} + R_{\Delta r}^{\infty} \right)[m^{\text{inc}}](r) + \left( K_{\Delta r}^{w} + K_{\Delta r}^{\infty} \right)[j^{\text{inc}}](r).
\]

In this expression, the terms involving integrals over \( \Gamma^w \) can be easily computed, since the integration domain is bounded. On the other hand, the integrands decay slowly over the infinite surface \( \Gamma^\infty \) for target points \( r \) in the region of interest (\( r \in \Gamma^w \)) and may have vanishing oscillations since the direction of the incoming mode is opposite to that of the oscillations from the integral kernels.

To evaluate the integrals over \( \Gamma^\infty \) with minimal error, we introduce an auxiliary representation for the incident modes. Let \( \Gamma_{\Delta r} \) denote the orthogonal plane that cuts the incident SIWG at the end of \( \Gamma^w \), and let \( \Omega_{\Delta r}^{\Delta r} \) denote the portion of \( \Omega_{\Delta r} \) that goes from \( \Gamma_{\Delta r} \) toward the infinite side of the SIWG (see Fig. 1). Using the representation theorems for the electromagnetic fields [25] we obtain

\[
\left( R_{\Delta r}^{w} + R_{\Delta r}^{\infty} \right)[m^{\text{inc}}](r) + \left( K_{\Delta r}^{w} + K_{\Delta r}^{\infty} \right)[j^{\text{inc}}](r) = \left( R_{\Delta r}^{w}(r), \quad R_{\Delta r}^{\infty}(r) \right) \Omega_{\Delta r}^{\Delta r}, \quad r \in \Omega_{\Delta r}^{\Delta r} \Omega_{\Delta r}^{\Delta r}. \quad (17)
\]

For points \( r \in \Gamma^w \) (all of which are outside of \( \Omega_{\Delta r}^{\Delta r} \)), we then have the relation

\[
\left( R_{\Delta r}^{w}[m^{\text{inc}}](r) + \left( K_{\Delta r}^{w} + K_{\Delta r}^{\infty} \right)[j^{\text{inc}}](r) = \left( -R_{\Delta r}^{w}[m^{\text{inc}}](r) - K_{\Delta r}^{w}[j^{\text{inc}}](r), \quad r \in \Omega_{\Delta r}^{\Delta r} \Omega_{\Delta r}^{\Delta r}. \quad (18)
\]

with \( R_{\Delta r}^{w} \) which can be used to evaluate the right-hand side of (12)

\[
\left( R_{\Delta r}^{w}[m^{\text{inc}}](r) + \left( K_{\Delta r}^{w} + K_{\Delta r}^{\infty} \right)[j^{\text{inc}}](r) = \left( -R_{\Delta r}^{w}[m^{\text{inc}}](r) - K_{\Delta r}^{w}[j^{\text{inc}}](r), \quad r \in \Omega_{\Delta r}^{\Delta r} \Omega_{\Delta r}^{\Delta r}. \quad (19)
\]

Using a representation for the field \( H \) similar to the one used in (17) for the field \( E \) we can obtain the right-hand side (12) in terms of integrals along the bounded surface \( \Gamma^w \) and the unbounded boundary \( \Gamma_{\Delta r} \). The resulting WGF version of (12) reads

\[
I + R^w_{\ell} W_A \quad K^\Delta_{\ell} W_A \quad \int m \\
- K^\Delta_{\ell} W_A \quad I + R^w_{\ell} W_A \quad \int j
\]

\[= \left[ \begin{array}{c}
\left( R_{\Delta r}^{w} - R_{\Delta r}^{\infty} \right)(K_{\Delta r}^{w} - K_{\Delta r}^{\infty})[m^{\text{inc}}](r) \\
\left( -K_{\Delta r}^{w} - K_{\Delta r}^{\infty} \right) + \left( R_{\Delta r}^{w} - R_{\Delta r}^{\infty} \right) [j^{\text{inc}}](r)
\end{array} \right].
\]

Remark 3: The operators on the right-hand side of (20) involve integrals that can be accurately computed. The integrals over the bounded surface \( \Gamma^w \) can be treated as in the bounded obstacle case. On the other hand, the integrands over \( \Gamma_{\Delta r} \), namely, the incident bound modes, decay exponentially as the distance to the interface tends to infinity—along \( \Gamma_{\Delta r} \). This exponential decay allows us to evaluate integrals along \( \Gamma_{\Delta r} \) by simple truncation while incurring only exponentially small truncation errors.

IV. Numerical Examples

Any numerical discretization for boundary integral methods can be used to discretize the WGF integral equations: in particular, Galerkin, Nyström, or the collocation method can be utilized. The illustrations presented in this communication were produced by means of the discretization strategy presented in [27], [30], and [31]. This approach relies on the discretization of boundaries by a set of nonoverlapping quadrilateral curvilinear patches that yields high-order convergence and can conveniently be applied to general computer-aided design (CAD) surface descriptions. The unknown current densities are discretized via Chebyshev polynomials on each surface patch. The far interactions are treated via Fejér’s quadrature, and the near-singular and singular interactions are precomputed using a high-order quadrature rule for the weakly singular integrals. Then, the system of integral equations is solved iteratively via GMRES.

This section presents several numerical examples that illustrate the accuracy and applicability of the proposed WGF approach.
Our first example concerns a uniform circular waveguide, which allows us to provide comparisons with the analytical solution of a mode propagating unperturbed along the waveguide. In our second example, we then consider a circular waveguide with a 90° bend. In a third example, we consider a dielectric antenna—e.g., an open waveguide with a termination—demonstrating the applicability of the method for this class of devices. The last example, finally, concerns a complex nanophotonic structure with multiple output waveguides.

Our first example concerns a uniform circular waveguide, which allows us to provide comparisons with the analytical solution of a mode propagating unperturbed along the waveguide. In our second example, we then consider a circular waveguide with a 90° bend. In a third example, we consider a dielectric antenna—e.g., an open waveguide with a termination—demonstrating the applicability of the method for this class of devices. The last example, finally, concerns a complex nanophotonic structure with multiple output waveguides. For all problems, in this section, we have assumed an exterior refractive index of 1.0 and an interior refractive index of 1.47 (SiO₂), with the exception of the last one for which the core is silicon (n₁ = 3.47) and the cladding is SiO₂ (n₂ = 1.44).

Fig. 2 presents, in purple color, the errors in the modal fields produced by the proposed window-based solver for a perfectly uniform waveguide with a circular cross section, along the line at the center of the waveguide. For this problem, the modal fields may be expressed in terms of Bessel functions [5], [32], and the errors shown were obtained by comparison with this exact solution. Errors resulting from solutions without the use of windowing but on the basis of the same spatial discretization are also presented in Fig. 2. The color scale for the magnitude |S| goes from A = 5 × 10⁻⁶ to B = 3 × 10⁻¹.

Fig. 3 presents simulation results for a circular waveguide with a 90° bend, illuminated by a bound mode. Fig. 3 presents the x-component of the electric field and shows that the mode is mostly preserved across the curved region: the difference in the character of the fields shown in the vertical and horizontal sections reflects the fact that the x-component of the field is the tangential component in the vertical section, but it is the normal component in the horizontal section.

Fig. 4 displays the fields resulting in a terminated open waveguide illuminated by a bound mode, i.e., a dielectric antenna. Fig. 4(a) presents the discretization used as well as the auxiliary circular surface Γ⁺ utilized in this case to evaluate the incident mode contribution. (b) Real part of E₉, displaying, in particular, the near-field pattern and the energy radiation away from the waveguide termination region. Fig. 4(c) plots the far-field radar cross section (RCS) produced by the proposed method as well as the one obtained via a well-known commercial FDTD solver. The proposed approach produced the solution with a far-field error of the order 1 × 10⁻³ in a computing time of 21 s on 36 cores Xeon Gold 6154 and one NVIDIA Titan V GPU. The FDTD solution, in turn, which does not support GPU acceleration, required 9.82 min on the same machine.
patches shown in Fig. 5(a) and (b). The waveguide structure is illuminated by a Gaussian beam at the input port, and the real part of the magnetic current density is shown in Fig. 5(c).

V. CONCLUSION

We have presented a fully vectorial numerical method for the solution of complex 3-D electromagnetic problems including waveguide structures on the basis of a windowed Green function boundary integral method. In this approach, the relevant integral operators—which are initially posed over the infinite boundaries of the waveguides—can be accurately evaluated, with super-algebraically small errors, by multiplying the integrands by a smooth window function and truncating the integration domain to the region, where the window function does not vanish. In particular, we showed that incident mode excitation can be accurately incorporated by means of an auxiliary representation, which transforms challenging integrals along the infinite boundary of the waveguide carrying the incident mode, onto integrals with exponentially decaying integrands. The ideas presented here are independent of the numerical discretization of the integral operators and can indeed be used, in particular, in conjunction with any Nyström or Galerkin discretization, including the well-known method-of-moments approach.

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