ELECTROMAGNETIC RADIATION OF BARYONS
CONTAINING TWO HEAVY QUARKS

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Abstract
The two heavy quarks in a baryon which contains two heavy quarks and a light one, can constitute a scalar or axial vector diquark. We study electromagnetic radiations of such baryons, (i) \(\Xi_{(bc)}^0 \rightarrow \Xi_{(bc)}^0 + \gamma\), (ii) \(\Xi_{(bc)}^1 \rightarrow \Xi_{(bc)}^0 + \gamma\), (iii) \(\Xi_{(bc)}^{**0}(1/2, l = 1) \rightarrow \Xi_{(bc)}^0 + \gamma\), (iv) \(\Xi_{(bc)}^{**0}(3/2, l = 1) \rightarrow \Xi_{(bc)}^0 + \gamma\) and (v) \(\Xi_{(bc)}^{**0}(3/2, l = 2) \rightarrow \Xi_{(bc)}^0 + \gamma\), where \(\Xi_{(bc)}^{**0}(1)\) are S-wave bound states of a heavy scalar or axial vector diquark and a light quark, and \(\Xi_{(bc)}^{**0}(l \geq 1)\) are P- or D-wave bound states of a heavy scalar diquark and a light quark. Analysis indicates that these processes can be attributed into two categories and the physical mechanisms which are responsible for them are completely distinct. Measurements can provide a good judgment for the diquark structure and better understanding of the physical picture.

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I. Introduction

Lack of an effective way to properly handle non-perturbative QCD effects becomes a more and more intriguing problem when one needs to extract information from data. In other words, the hadronic matrix elements cannot be reliably estimated in the present theoretical framework. Thanks to the heavy quark effective theory (HQET) \[ SU(2)_s \otimes SU(2)_f \] greatly simplifies the picture in the heavy flavor involved processes. Developments in this field enable us to more accurately evaluate hadronic transition matrix elements since the number of form factors is reduced in the heavy quark limit \[ \mathbb{E} \].

As many authors suggested, there may exist the diquark structure in baryons \[ \mathbb{E} \]. If it is the real physics, or at least a good approximation, we only need to deal with two-body problems instead of three-body one. Consequently, the number of independent form factors can be remarkably reduced. Especially, when the baryons contain two heavy quarks, it is reasonable to assume that the two heavy quarks constitute a color-anti-triplet boson-like diquark of spin 0 or 1 \[ \mathbb{E} \]. Based on this picture Savage and Wise studied the spectrum of baryons with two heavy quarks \[ \mathbb{E} \] and in the potential model, the spectra have been evaluated \[ \mathbb{E} \].

Although the diquark structure is very likely, the small color-anti-triplet system is not point-like in general. Consequently, we should replace the vertex gained from any fundamental theory such as the Standard Model by an effective vertex. A (or a few) reasonable form factor(s) will be involved in the effective vertex for compensating the non-point-like spatial dispersion of the diquark. The form factor(s) can be derived in many ways, and one of them is the Bethe-Salpeter (B-S) equation. With the effective vertex, we estimated the production and weak decay rates of such baryons \[ \mathbb{E} \] in our previous work based on the superflavor symmetry \[ \mathbb{E}, \mathbb{E} \].

To further investigate the diquark structure and the governing mechanisms inside the diquark, we will study electromagnetic radiations of baryons with two heavy quarks in the present work. Since such processes are cleaner, we may expect to gain more exact knowledge from the
data. In fact, similar electromagnetic radiation processes for baryons containing only one heavy quark have been discussed in literature recently [10].

At the tree level, the \( \gamma \)–emission is a pure electromagnetic process. In this work we study two cases which in fact are determined by completely different mechanisms. First, we consider (i) \( \Xi^{(bc)}_{1} \rightarrow \Xi^{(bc)}_{0} + \gamma \) and (ii) \( \Xi^{* (bc)}_{1} \rightarrow \Xi^{(bc)}_{0} + \gamma \), where \( \Xi^{(bc)}_{1} \) and \( \Xi^{* (bc)}_{1} \) are spin 1/2 and 3/2 baryons (respectively) which consist of a heavy axial vector diquark and a light quark in S-wave, and \( \Xi^{(bc)}_{0} \) is a spin-1/2 baryon which consists of a heavy scalar diquark and a light quark. Then we study (iii) \( \Xi^{** (bc)}_{0}(1/2, l = 1) \rightarrow \Xi^{(bc)}_{0} + \gamma \), (iv) \( \Xi^{** (bc)}_{0}(3/2, l = 1) \rightarrow \Xi^{(bc)}_{0} + \gamma \), and (v) \( \Xi^{** (bc)}_{0}(3/2, l = 2) \rightarrow \Xi^{(bc)}_{0} + \gamma \) where \( \Xi^{** (bc)}_{0}(s, l \geq 1) \) are spin 1/2 \( (s = 1/2) \) and 3/2 \( (s = 3/2) \) baryons (respectively) composed of a heavy scalar diquark and a light quark in higher angular momentum states. It is noted that we study the \( (bc)_{1(0)} \) diquark because only \( (bc) \) can constitute either spin 1 or 0 states with even parity (i.e., the orbital angular momentum between \( Q \) and \( Q' \) is set to be 0 in our discussion).

In the reactions (i) \( \Xi^{(bc)}_{1} \rightarrow \Xi^{(bc)}_{0} + \gamma \) and (ii) \( \Xi^{* (bc)}_{1} \rightarrow \Xi^{(bc)}_{0} + \gamma \), the axial vector \( (bc)_{1} \) transits into a scalar \( (bc)_{0} \) by emitting a photon; whereas in the radiations (iii) \( \Xi^{** (bc)}_{0}(1/2, l = 1) \rightarrow \Xi^{(bc)}_{0} + \gamma \), (iv) \( \Xi^{** (bc)}_{0}(3/2, l = 1) \rightarrow \Xi^{(bc)}_{0} + \gamma \), and (v) \( \Xi^{** (bc)}_{0}(3/2, l = 2) \rightarrow \Xi^{(bc)}_{0} + \gamma \), the diquark \( (bc)_{0} \) remains in the spin-0 state, and the photon is radiated from the light quark hand. The later three reactions are in analog to the radiation of atom where electron transits from a higher (angular and/or radial) exited state into a lower one and emits a photon. In our case, the light quark of \( \Xi^{** (bc)}_{0}(s, l \geq 1) \) in an angular-momentum excited state transits into the ground state \( (l = 0) \) \( \Xi^{(bc)}_{0} \) and emits a photon. Analysis indicates that the possibility of radiating a photon from the spin-0 heavy diquark is very small, exactly as in the case of atoms.

Of course, in general, there may be processes like \( \Xi^{* (bc)}_{0}(3/2) \rightarrow \Xi^{(bc)}_{0}(1/2) + \gamma \). However, since the spin interaction between gluon and heavy diquarks decouples in the heavy quark limit, the mass splitting between \( \Xi^{* (bc)}_{0} \) and \( \Xi^{(bc)}_{0} \) is 0. Consequently, in the heavy quark limit, the radiative transition between these two states is forbidden by the null phase space. So we do not
discuss such processes in this work.

In the next section, we present our formulation for the two different radiation mechanisms and in the third section, we give the numerical results. The last section is devoted to discussion and conclusion and finally in the appendix, we give all the concerned expressions which are omitted in the context.

II. Formulation

In this section, we discuss the two different mechanisms respectively.

(a) Radiation from the heavy diquark hand.

As discussed in the introduction, for the radiation processes \( \Xi_{(bc)} \rightarrow \Xi_{(bc)} + \gamma \) and \( \Xi^{*}_{(bc)} \rightarrow \Xi_{(bc)} + \gamma \), the axial vector diquark transits into a scalar diquark by emitting a photon and the light quark remains as a spectator. In this case, all the non-perturbative effects can be attributed into a form factor at the leading order of expansion with respect to the heavy quark mass. To evaluate the transition matrix elements, we employ the superflavor symmetry \([8, 9]\), which is applicable to this situation.

At the effective vertex \( AS\gamma \), where \( A \) and \( S \) denote axial vector and scalar diquarks (respectively) and \( \gamma \) is the emitted photon, a form factor can be derived in terms of the B-S equation \([7]\). The transition amplitude can be written as

\[
T = \epsilon^*_\alpha \langle J^\alpha \rangle ,
\]

where \( \epsilon^*_\alpha \) is the polarization vector of the axial vector diquark, and \( J^\alpha \) is the effective current at the quark level and \( \langle J^\alpha \rangle \) is the corresponding transition amplitude.

For \( \Xi_{(bc)} \rightarrow \Xi_{(bc)} + \gamma \),

\[
\langle J^\alpha \rangle = \langle \Xi_{(bc)}(v')| J^\alpha |\Xi_{(bc)}(v) \rangle = \xi(v' \cdot v) i f \epsilon^{\alpha\beta\rho\sigma} v_\rho v_\sigma \bar{u}'(v') \gamma_5 \gamma_5 u(v) ,
\]

where \( f \) is the diquark coupling constant, \( \epsilon^{\alpha\beta\rho\sigma} \) is the structure constant of the diquark, \( v_\rho \) and \( v_\sigma \) are the momentum of the diquark and the emitted photon, respectively, and \( \xi(v' \cdot v) \) is the form factor.
and for $\Xi^*_{(bc)} \rightarrow \Xi_{(bc)} + \gamma$,

$$
\langle J^\alpha \rangle = \langle \Xi_{(bc)}(v')|J^\alpha|\Xi^*_{(bc)}(v) \rangle = \xi(v' \cdot v) \gamma^5 \epsilon^{\alpha \rho} \bar{v} u'(v') u_\rho(v),
$$

(3)

where $\xi(v' \cdot v)$ is the Isgur-Wise function, $v$ and $v'$ are the four-velocities of the parent and daughter baryons, respectively, $u$ is the four-component spinor for the parent or produced baryon $\Xi_{(bc)}$, and $u_\rho$ is the Rarita-Schwinger spinor-vector corresponding to $\Xi^*_{(bc)}$ with spin 3/2.

The form factor is evaluated in the B-S equation approach and all the details were given in our previous work [7].

Taking the amplitude square, we have: for $\Xi_{(bc)} \rightarrow \Xi_{(bc)} + \gamma$,

$$
\frac{1}{2} \sum_{\text{all spins}} |T|^2 = \frac{e^2}{36} |\xi(v' \cdot v)|^2 \frac{\epsilon^{\alpha \rho} \epsilon_{\alpha' \nu} \epsilon_{\nu' \rho'} u' \gamma^5 u}  \sum_{\lambda} \epsilon^{\alpha}_{(\lambda)} \epsilon^{\alpha'}_{(\lambda)},
$$

(4)

and for $\Xi^*_{(bc)} \rightarrow \Xi_{(bc)} + \gamma$,

$$
\frac{1}{4} \sum_{\text{all spins}} |T|^2 = \frac{e^2}{36} |\xi(v' \cdot v)|^2 \frac{\epsilon^{\alpha \rho} \epsilon_{\alpha' \nu} \epsilon_{\nu' \rho'} u' \gamma^5 u}  \sum_{\lambda} \epsilon^{\alpha}_{(\lambda)} \epsilon^{\alpha'}_{(\lambda)},
$$

(5)

where

$$
C^{\rho \sigma} = f v^\rho v'^\sigma,
$$

(6)

and numerically

$$
f \sim 1.
$$

In our case, the photon emitted from the heavy diquark only carries very small momentum and energy, thus $v \cdot v'$ would be very close to unity, so

$$
\xi(v \cdot v') \approx 1.
$$

Then we can easily obtain the widths of these radiative decay processes as

$$
\Gamma = \frac{1}{2M} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} \frac{1}{2\omega} \frac{1}{(2\pi)^4} \delta^4(P - p - k) \frac{1}{2s + 1} \sum_{\text{all spins}} |T|^2,
$$

(7)
where $P, p, k$ are the four-momenta of the initial, final baryons and emitted photon, respectively, and $M$ is mass of the initial baryon. Because it is a two-body final state case, the integration is very easy to be carried out.

(b) Radiation from the light quark hand.

In this case, $\Xi_{(bc)0}^{**}(s, l \geq 1)$ are composed of a scalar diquark and a light quark in a higher angular momentum state ($l \geq 1$), thus the radiation is realized via a process that the light quark transits from a higher angular momentum state into the ground state ($l = 0$) via emitting a photon. This process is in analog to the photon radiation of atoms where the electron jumps from an excited state (radial or $l \geq 1$) into the ground state via emitting a photon.

In these processes, the heavy diquark acts as a spectator. Since the reaction happens on the light flavor side, HQET is not applicable in this case. Instead, we use the B-S equation to calculate the transition matrix elements. For consistency, the wavefunctions of $\Xi^{**}_{(bc)0}(1/2(3/2), l = 1, 2)$ and $\Xi_{(bc)0}(1/2, l = 0)$ are also obtained in terms of the B-S equation. The wavefunctions are given in the following:

\begin{align*}
\kappa_{P}^{(1/2,0)}(p) &= (\phi_{1}^{(10)}(p) + \phi_{2}^{(10)}(p)\hat{p}_{t})u(P), \quad (s = \frac{1}{2}, l = 0) \\
\kappa_{P}^{(1/2,1)}(p) &= (\phi_{1}^{(11)}(p) + \phi_{2}^{(11)}(p)\hat{p}_{t})\gamma_{5}u(P), \quad (s = \frac{1}{2}, l = 1) \\
\kappa_{P}^{(3/2,1)}(p) &= (\phi_{1}^{(31)}(p) + \phi_{2}^{(31)}(p)\hat{p}_{t})p_{\mu}u^{\mu}(P), \quad (s = 3/2, l = 1) \\
\kappa_{P}^{(3/2,2)}(p) &= [\phi_{1}^{(32)}(p) + \phi_{2}^{(32)}(p)\hat{p}_{t}]\gamma_{5}p_{\mu}u^{\mu}(P), \quad (s = \frac{3}{2}, l = 2),
\end{align*}

where $u(P)$ is the spinor for the baryon of spin-1/2 and $u^{\mu}(P)$ is the Rarita-Schwinger spinor-vector. Here we use the transverse momentum $p_{t}$ which is defined as

\begin{equation}
p_{t}^{\mu} = p^{\mu} - p_{l}v^{\mu},
\end{equation}

and $v^{\mu}$ is the four-velocity of the concerned baryon, $p_{l} \equiv p \cdot v$ is the longitudinal momentum.

The vertex $\bar{q}q\gamma$ is the typical QED coupling. Taking the loop integration with the obtained B-S wavefunctions we can have the transition amplitude square as the following.
For $\Xi^{***}_{(bc)_0}(1/2, l = 1) \to \Xi_{(bc)_0} + \gamma$, 

$$\frac{1}{2} \sum_{\text{all spins}} \left| \sum_{\text{all spins}} |\bar{u}(v')G^\mu u(v)\epsilon_{\mu}^{(\lambda^*)}|^2 \right|,$$

where 

$$G^\mu \equiv G_1 (2\gamma^\mu \gamma_5 + G_2 \gamma^\mu \gamma^I \gamma_5 + G_3 \gamma^\mu \gamma_5 + G_4 (-2\gamma^\mu \gamma_5 + \gamma^\mu \gamma_5) + G_5 \gamma^\mu \gamma^I \gamma_5).$$

For $\Xi^{***}_{(bc)_0}(3/2, l = 1) \to \Xi_{(bc)_0} + \gamma$, 

$$\frac{1}{4} \sum_{\text{all spins}} \left| \sum_{\text{all spins}} |\bar{u}(v')H^\mu u(v)\epsilon_{\mu}^{(\lambda^*)}|^2 \right|,$$

where 

$$H^\mu \equiv H_1 \gamma^\mu \gamma_5 + H_2 \gamma^\mu \gamma^I \gamma_5 + H_3 \gamma^\mu \gamma_5,$$

For $\Xi^{***}_{(bc)_0}(3/2, l = 2) \to \Xi_{(bc)_0} + \gamma$, 

$$\frac{1}{4} \sum_{\text{all spins}} \left| \sum_{\text{all spins}} |\bar{u}(v')F^\mu u(v)\epsilon_{\mu}^{(\lambda^*)}|^2 \right|,$$

where 

$$F^\mu \equiv F_1 \gamma^\mu \gamma_5 + F_3 \gamma^\mu \gamma^I \gamma_5 + F_4 \gamma^\mu \gamma_5 + F_5 \gamma^\mu \gamma^I \gamma_5.$$

All the coefficients $G_i, H_i$ and $F_i$ in eqs.(14,16,18) are related to the B-S integrals and we give their explicit expressions in the appendix. The derivation in terms of the B-S equation is very tedious but standard.

The partial width is obtained in the same way as in II(a).

III. The numerical results

(a) Radiation from the heavy diquark hand.

Since there are no data for the masses of baryons containing two heavy quarks yet, we have to take the theoretically estimated values which are given in literatures. Here we use the results given by Ebert et al. [8] as $M_{\Xi^{*}_{(bc)}} = 7.02$ GeV and $M_{\Xi_{(bc)}} = 6.95$ GeV. We have 

$$\Gamma(\Xi_{(bc)_1} \to \Xi_{(bc)_0} + \gamma) \sim 2.75 \times 10^{-9} \text{ GeV},$$
\[ \Gamma(\Xi_{(bc)}^\ast \rightarrow \Xi_{(bc)} + \gamma) \sim 7.48 \times 10^{-9} \text{ GeV}. \]

Namely, the widths are in order of eV’s.

(b) Radiation from the light quark hand.

For consistency, we have also obtained the binding energies of the concerned baryons in terms of the B-S equation. We have

\[ E_{\Xi_{(bc)}^\ast}^{(3/2, l=2)} = 1.39 \text{ GeV}, \quad E_{\Xi_{(bc)}^\ast}^{(3/2, l=1)} = 0.69 \text{ GeV}, \]
\[ E_{\Xi_{(bc)}^\ast}^{(1/2, l=1)} = 0.66 \text{ GeV}, \quad E_{\Xi_{(bc)}^\ast}^{(1/2, l=0)} = 0.026 \text{ GeV}. \]

In this framework, we have

\[ M_{\Xi_{(bc)}^\ast}^{(s,l)} = m_1 + m_2 + E_{\Xi_{(bc)}^\ast}^{(s,l)}, \]
\[ M_{\Xi_{(bc)}^\ast} = m_1 + m_2 + E_{\Xi_{(bc)}^\ast}, \]

where \( m_1 \) and \( m_2 \) are the masses of the light quark and the heavy scalar diquark, respectively, \( E \) is the binding energy. To evaluate the binding energies, we take the simplest potential form which contains only the Coulomb and linear confinement pieces as the B-S kernel [8].

Numerically, we take

\[ m_1 = 0.33 \text{ GeV (for u – and d – quark), 0.5 GeV (for s – quark)}; \quad m_2 = 6.52 \text{ GeV}, \]

as inputs [8].

We use these values in the numerical evaluations and obtain:

\[ \Gamma(\Xi_{(bc)}^\ast (1/2, l = 1) \rightarrow \Xi_{(bc)} (1/2) + \gamma) \sim 1.5 \times 10^{-4} \text{ GeV}, \]
\[ \Gamma(\Xi_{(bc)}^\ast (3/2, l = 1) \rightarrow \Xi_{(bc)} (1/2) + \gamma) \sim 3.7 \times 10^{-5} \text{ GeV}, \]
\[ \Gamma(\Xi_{(bc)}^\ast (3/2, l = 2) \rightarrow \Xi_{(bc)} (1/2) + \gamma) \sim 6.2 \times 10^{-4} \text{ GeV}. \]
As discussed above, these partial widths are evaluated in terms of the B-S equation. Indeed these reactions are governed by a mechanism different from that in (a), and the methods we use for evaluating the widths are distinct.

In this subsection, we obtain the masses of $\Xi^{**}_{(bc)0}(1/2(3/2), l \geq 1)$ and $\Xi_{(bc)0}(1/2)$ and the transition matrix element $\langle \Xi_{(bc)0}(1/2) | J_\mu | \Xi^{**}_{(bc)0}(1/2(3/2), l \geq 1) \rangle$ in the same framework, i.e. the B-S equation. In fact, there is no any substantial difference from the values we take in subsection (a) for $\Xi_{(bc)1} \to \Xi_{(bc)0} + \gamma$ and $\Xi^*_{(bc)1} \to \Xi_{(bc)0} + \gamma$.

It is noted that the mass difference between the angular-momentum excited state $\Xi^{**}_{(bc)0}(3/2, l \geq 1)$ and the ground S-state $\Xi_{(bc)0}$ is about 0.6~1.4 GeV. It is much larger than that between $\Xi^*_{(bc)1}$ and $\Xi_{(bc)0}$ (0.07 GeV). It is easy to understand: the former one is due to the orbital angular momentum excitation and the later one is due to an energy splitting between axial vector and scalar diquarks, which is caused by the spin-spin interaction. Therefore for $\Xi^{**}_{(bc)0}(s, l \geq 1) \to \Xi_{(bc)0} + \gamma$ the threshold effects are not obvious and the widths are about 4 orders of magnitude larger than $\Gamma(\Xi_{(bc)1}(\Xi^*_{(bc)1}) \to \Xi_{(bc)0}(1/2) + \gamma)$. In other words, the remarkable width difference for the two processes is due to the threshold effects while the matrix elements for both reactions are of the same order of magnitude.

IV. Conclusion and discussion

HQET is proved to be effective in many processes where heavy flavors are involved. In most cases, the light flavors in the hadrons just behave as spectators for the reactions and these degrees of freedom manifest in the hadronization processes, and therefore determine the form factors such as the Isgur-Wise function. However, in some cases, the light flavors may participate in reactions and sometimes can play a crucial role. As we know, when the quark level final state interaction is involved, the W-annihilation and especially the Pauli Interference can be very important in the inclusive B meson decays\cite{12}, then the contribution from the light flavor could
be as important as that from the heavy one.

In this work, we choose two different kinds of processes where the heavy and light flavors are active respectively. $\Xi_{(bc)_1}$ and $\Xi^*_{(bc)_1}$ consist of an axial vector diquark and a light quark. When they transit into $\Xi_{(bc)_0}$ by radiating a photon, the axial vector diquark turns into a scalar one, and the light quark serves as a spectator in this process. On the contrary, $\Xi^*_{(bc)_0}(1/2(3/2), l \geq 1)$ consists of a scalar heavy diquark, and a light quark at angular-momentum excited states ($l = 1, 2$ in this work). Thus when it transits into $\Xi_{(bc)_0}$, the heavy diquark stands as a spectator and the light quark jumps from a higher-excited state into the ground state while radiating a photon. For the former one, HQET definitely applies and by the superflavor symmetry, we can expect to obtain a more accurate result of the decay width. Once the doubly heavy baryon masses are measured, we can immediately have the final numbers with our formula for the partial width. As long as HQET works, the result should be close to data. Of course, there is also an uncertain factor, it is the form factor at the effective vertex of $SA\gamma$. We obtain it in terms of the B-S equation, where the potential kernel would bring up some uncertainty. However, in this case, the diquark is composed of two heavy quarks, so the non-relativistic Cornell potential works well as understood. Moreover, careful studies indicate that for so small recoil situation, $(v \cdot v') \sim 1$, the form factor $f$ is close to 1. Therefore, we can expect that the relative errors for the partial widths of $\Xi_{(bc)_1} \to \Xi_{(bc)_0} + \gamma$ and $\Xi^*_{(bc)_1} \to \Xi_{(bc)_0} + \gamma$ are quite small. The widths are of order of eV’s and similar to that for atomic radiation. The smallness is easy to understand. Let us use $\Xi^*_{(bc)_1} \to \Xi_{(bc)_0} + \gamma$ as an example. From eq.(3), we have

$$\frac{1}{4} \sum_{all \ spins} |T|^2 = \frac{4e^2}{27} f^2 M_{1/2} M'_{3/2} (v \cdot v' - 1)(1 + v' \cdot v)^2,$$

(19)

where $M_{1/2}$ and $M'_{3/2}$ are the masses of $\Xi_{(bc)}(1/2)$ and $\Xi^*_{(bc)}(3/2)$ respectively. In this case, $v \cdot v' - 1$ is close to zero and it is nothing but the threshold effect. With this expression, we can easily obtain the partial width of this radiative decay as

$$\Gamma = \frac{\alpha}{216} f^2 \left(\frac{M'_{3/2}^2 - M_{1/2}^2}{M'_{3/2} M_{1/2}}\right)^3 \left(M'_{3/2}^2 + M_{1/2}^2\right).$$

(20)
It is noted that the width is proportional to \((M'_{3/2} - M_{1/2})^3/M_{3/2}^{5/3}\), hence for small difference between the masses of the parent and daughter baryons, the threshold effects are very obvious. One can expect this threshold effects to strongly suppress the width.

As a matter of fact, these radiative decay processes where the heavy axial vector diquark emits a photon and transits into a scalar one, are in analog to the radiative decay \(J/\psi \rightarrow \eta_c + \gamma\) whose partial width is about 1.13 KeV [14]. But there are several suppression factors in the doubly heavy baryon case. First in \(J/\psi\) \(c\) and \(\bar{c}\) reside in a color singlet, but in the diquark, \(b\) and \(c\) quarks are in a color \(\bar{3}\) state, there should be a factor 1/8 suppression for the diquark transition. From the formula (21), one has a factor \((M'_{3/2} - M_{1/2})/M_{3/2}^{5/3}\), so totally there could be a suppression of about \(5 \times 10^{-3}\) compared to the \(J/\psi\) radiative decay. The net result is of eV order.

For \(\Xi^{**}_{(bc)0}(1/2(3/2), l \geq 1) \rightarrow \Xi_{(bc)0} + \gamma\), HQET does not apply and we need to employ the B-S equation method to evaluate the transition matrix elements. In the calculations, the B-S wavefunctions of the initial and final states are needed. Since in such radiative decays, the recoil energy-momentum of the final baryon is very small compared to the involved energy scales, we expect the theoretical predictions are quite reliable.

The numerical results show that for \(\Xi_{(bc)1} \rightarrow \Xi_{(bc)0} + \gamma\) and \(\Xi^{*}_{(bc)1} \rightarrow \Xi_{(bc)0} + \gamma\), the partial width is in order of eV, and for \(\Xi^{**}_{(bc)0}(3/2(1/2), l = 1(2)) \rightarrow \Xi_{(bc)0}(1/2) + \gamma\), it is of 10~100 KeV. The difference is due to the threshold effects.

Besides the study on the reaction mechanisms, this work also concerns testifying the diquark structure in baryons. It is believed that the two heavy quarks inside a baryon can constitute a diquark of scalar or axial vector which is a relatively stable physical subject [7]. Our calculations are based on such a physical picture and the future experiments should test it. Lack of data on baryons which consist of two heavy quarks so far makes drawing a definite conclusion difficult. But it is possible that the data can be accumulated in near future experiments. Once we have the data on the masses, we can re-evaluate the numbers of decay widths easily. Then comparing
the calculated results with data, we can determine the validity of the diquark structure and the
reaction mechanisms. No doubt, the experiments for the electromagnetic radiation are difficult,
but as suggested [13], the radiative decay may be measurable soon, and the background in this
case is clean. We believe that the results can enrich our knowledge on baryons, so is worth of
careful investigations.

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Here we present the explicit expressions of the form factors $G_i$, $H_i$ and $F_i$ in eqs.(14,16,18).

\[ G_i \equiv \int \frac{d\vec{p}_t}{2\pi} g_i; \quad H_i \equiv \int \frac{d\vec{p}_t}{2\pi} h_i; \quad F_i \equiv \int \frac{d\vec{p}_t}{2\pi} f_i. \quad (21) \]

\[
\begin{align*}
  g_1 &= \int \frac{d^3p_t}{(2\pi)^3} ac'; \\
  g_2 &= \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \int \frac{d^3p_t}{(2\pi)^3} ad' |\vec{p}_t| \cos \theta; \\
  g_3 &= \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \int \frac{d^3p_t}{(2\pi)^3} bc' |\vec{p}_t| \cos \theta;
\end{align*}
\]
$g_4 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3 bd' |\mathbf{p}_t|^2} (1 - \cos^2 \theta);$

$g_5 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3 bd' |\mathbf{p}_t|^2} (1 - 3\cos^2 \theta);$

$h_1 = \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \alpha c'' |\mathbf{p}_t| \cos \theta;$

$h_2 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta c'' |\mathbf{p}_t|^2 (1 - \cos^2 \theta);$

$h_3 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta c'' |\mathbf{p}_t|^2 (1 - 3\cos^2 \theta);$

$h_4 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta c'' |\mathbf{p}_t|^2 (1 - \cos^2 \theta);$

$h_5 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta c'' |\mathbf{p}_t|^2 (1 - 3\cos^2 \theta);$

$h_6 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta c'' |\mathbf{p}_t|^2 (1 - \cos^2 \theta);$

$h_7 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta c'' |\mathbf{p}_t|^2 (1 - 3\cos^2 \theta);$

$f_1 = \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \alpha c'' |\mathbf{p}_t| \cos \theta;$

$f_2 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta d |\mathbf{p}_t|^2 (1 - \cos^2 \theta);$

$f_3 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta d |\mathbf{p}_t|^2 (1 - 3\cos^2 \theta);$

$f_4 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta c |\mathbf{p}_t|^2 (1 - \cos^2 \theta);$

$f_5 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta c |\mathbf{p}_t|^2 (1 - 3\cos^2 \theta);$

$f_7 = \frac{-1}{2} \int \frac{d^3 \mathbf{p}_t}{(2\pi)^3} \beta d (\lambda_2 M_{(3/2, t=1)} + p_1) |\mathbf{p}_t|^2 (1 - 3\cos^2 \theta),$

where $\theta$ is the angle between $\mathbf{p}_t$ and $v_t$, 

\[
\begin{align*}
a &= \left[ \frac{2\omega_{p_1}^2 (\omega_{p_1}^2 - m_1 - E_{(1/2, t=0)})}{E_{(1/2, t=0)} - p_1^2 + i\epsilon} \right] \tilde{\Phi}_{2}^{(10)}; \\
b &= \left[ \frac{2\omega_{p_1}^2 (\omega_{p_1}^2 - m_1 - E_{(1/2, t=0)})}{E_{(1/2, t=0)} - p_1^2 + i\epsilon} \right] \tilde{\Phi}_{2}^{(10)};
\end{align*}
\]
\[
c = -i \left[ \frac{2\omega_p (\omega_p - m_1 - E_{(3/2,t=2)})}{E_{(3/2,t=2)} - pt} + i\epsilon \right] \frac{-pt}{(pt + m_1)^2 - \omega_{pt}^2 + i\epsilon} \\
\times (2m_2 (pt - E_{(3/2,t=1)})) \tilde{\Phi}_2^{(3)2}; \\
d = -i \left[ \frac{2\omega_p (\omega_p - m_1 - E_{(3/2,t=2)})}{(E_{(3/2,t=2)} - pt) + i\epsilon} \right] \frac{1}{(pt + m_1)^2 - \omega_{pt}^2 + i\epsilon} \\
\times (2m_2 (pt - E_{(3/2,t=2)})) \tilde{\Phi}_2^{(3)2}; \\
c' = -i \left[ \frac{2\omega_p (\omega_p - m_1 - E_{(1/2,t=1)})}{E_{(1/2,t=1)} - pt} + i\epsilon \right] \frac{-pt}{(pt + m_1)^2 - \omega_{pt}^2 + i\epsilon} \\
\times (2m_2 (pt - E_{(1/2,t=1)})) \tilde{\Phi}_2^{(1)1}; \\
d' = -i \left[ \frac{2\omega_p (\omega_p - m_1 - E_{(1/2,t=1)})}{E_{(1/2,t=1)} - pt} + i\epsilon \right] \frac{1}{(pt + m_1)^2 - \omega_{pt}^2 + i\epsilon} \\
\times (2m_2 (pt - E_{(1/2,t=1)})) \tilde{\Phi}_2^{(1)1}; \\
c'' = -i \left[ \frac{2\omega_p (\omega_p - m_1 - E_{(3/2,t=1)})}{E_{(3/2,t=1)} - pt} + i\epsilon \right] \frac{2m + pt}{(pt + m_1)^2 - \omega_{pt}^2 + i\epsilon} \\
\times (2m_2 (pt - E_{(3/2,t=1)})) \tilde{\Phi}_2^{(3)1}; \\
d'' = -i \left[ \frac{2\omega_p (\omega_p - m_1 - E_{(3/2,t=1)})}{E_{(3/2,t=1)} - pt} + i\epsilon \right] \frac{1}{(pt + m_1)^2 - \omega_{pt}^2 + i\epsilon} \\
\times (2m_2 (pt - E_{(3/2,t=1)})) \tilde{\Phi}_2^{(3)1},
\]  

(23)

where \( \tilde{\Phi}_i^{(s,t)} \) are the B-S wavefunctions after integrated over \( pt \),

\[
\tilde{\Phi}_i^{(s,t)} \equiv \int \frac{dp_t}{2\pi} \phi_i^{(s,t)}(pt, pt^2),
\]

\( \omega_{pt} = \sqrt{|pt|^2 + m_1^2} \), and we have defined

\[
\lambda_2 = \frac{m_2}{m_1 + m_2},
\]

with \( m_1 \) being the light quark mass and \( m_2 \) the heavy diquark mass \((m_1 \ll m_2)\). \( E_{(1/2,t)} \) and \( E_{(3/2,t)} \) are binding energies in the corresponding baryons.

All the functions are obtained by carrying out the B-S integrations which are very tedious, but straightforward (see ref. [6]).