High Energy Electron Proton Scattering in View of Born Approximation

M.T. Hussein and N.M. Hassan

Physics Department, Faculty of Science, Cairo University

The hadronic quark structure is investigated in the frame of high energy electron proton scattering. A phenomenological model based on the Born approximation is used to calculate the transition matrix element for the quark system forming the proton target. A potential of electromagnetic nature is assumed for the calculation of the multiple scattering of electron with the constituent valance quarks of the proton. It is found that the first two terms of the Born approximation are sufficient to describe the experimental data of differential cross section for the electron proton system only at low momentum transfer square. On the other hand, a two body scattering amplitude with a relevant form factor may give proper agreement extended to high momentum transfer square region. A harmonic oscillator wave function is used to reproduce the data of the experiment at low momentum transfer. This satisfies quietly the confinement condition of the quarks in the hadron system. However the data of the experiment at relatively high momentum transfer, show that hadronic quarks behave more freely and may be described by just a Coulomb wave function.
Experiments SLAC-E-140 $Q^2 = 1 - 3(GeV/c)^2$ $E_e = 8GeV$

SLAC-E-136 $Q^2 = 2.9 - 31.3(GeV/c)^2$ $E_e = 5 - 21.5GeV$

SLAC-NH-11 $Q^2 = 1.75 - 8.83(GeV/c)^2$ $E_e = 1.5 - 9.8GeV$

PACS numbers: 13.40.Fn , 25.40.Cm

Typeset Using REVTEX
I. INTRODUCTION

The most recent data of the electron-proton (ep) scattering experiments at SLAC [1-3] have played a significant role in investigating the hadron structure. In the last decade, many trails have been executed to deal with the ep scattering problem. The Glauber multiple scattering approach [4] was used to present an eikonal picture for the ep scattering. The quantum chromodynamics QCD [5,6] was developed as the most convincing present theory for the interactions between those constituent quarks, formulated in terms of the exchange of colored vector gluons. At sufficiently high values of $Q^2$, the running strong coupling constant is expected to become small enough, due to the properly of asymptotic freedom, to allow the use of perturbation theory to simplify QCD. However, there is considerable controversy as to how large a value of $Q^2$ is sufficient for perturbative quantum chromodynamics PQCD [7].

The exclusive processes such as electron proton (e-p) scattering are predicted to have a single dimensional scaling [8] at large $Q^2$. In this case only the valance quark states are important and a rough idea of the $Q^2$ dependence can be gained by simply counting the number of quark-gluon vertices. Elastic form factor, for example, should scale asymptotically as $(Q^2)^{-n+1}$, where $n$ is the number of valance quarks participating in the interaction. For (e-p) scattering $n=3$ thus the structure function behaves as $q^{-4}$. Perturbative QCD [7] predicts calculable logarithmic departures from the $Q^2$ dependence of exclusive amplitudes given by the simple dimensional scaling law. The earliest efforts [9] used unrealistic symmetric distribution amplitudes and required a large multiplicative factor to normalize the results to the data at $Q^2 \approx 10(\text{GeV}/c)^2$. Chernyak and Zhitnitsky [10,11] proposed a model form for the nucleon distribution amplitude which satisfies the sum rules and in which the momentum balance of the valance quarks in the proton is quite asymmetric. The result of this model is justified in the diquark model [12,13]. The nonperturbative calculations succeeded in modeling the region $Q^2 < 20(\text{GeV}/c)^2$ fairly well but some difficulties have been found at large values of $Q^2$. By this article we aim to develop the eikonal optical picture of the (ep) scattering on the bases of the Born multiple scattering of the incident electron with the...
constituent valance quarks of the target hadron. The paper is organized so that in section 2, we present the hypothesis and the mathematical formalism of the model. Results and discussion as well the comparison with experimental data are given in section 3.

II. THE MODEL

During the electron-proton (e-p) scattering, we consider the proton as a bag including three valance quarks. We proceed using the semi-classical Born approximation which is a useful technique when the de Broglie wavelength of the incident particle is sufficiently short compared with the distance in which the potential varies appreciably. In this approximation we consider the multiple scattering series in which the projectile interacts repeatedly with the potential and propagates freely between two such interactions. We expect that the Born series converges if the incident particle is sufficiently fast so that it cannot interact many times with the potential and/or if the potential is weak enough. Starting with a plane wave as a zero order wave function of the incident electron, and by successive iteration, the nth order of the Born scattering amplitude is given by,

\[ f_n = -2\pi^2 < \phi_{k_f} | U | \Psi_{n-1} > \]  

where \( f_k \), is the zero order plane wave and,

\[ \Psi_n = \phi_{k_f}(\vec{r}) + \int G_0(\vec{r}, \vec{r}')U(\vec{r})\Psi_{n-1}(\vec{r}')d\vec{r}' \]  

and

\[ G_0(\vec{r}, \vec{r}') = \frac{exp(ik|r - r'|)}{|r - r'|} \]  

is the free Green’s function. Let us denote by \( \vec{r}_o, \vec{r}_1, \vec{r}_2 \) and \( \vec{r}_3 \) the coordinates of the incident electron and the constituent valance quarks of the target proton. According to the first Born approximation, the scattering amplitude is given by:

\[ f_1 = -(2\pi)^2 \int e^{i\vec{q} \cdot \vec{r}_o} A(r_o)d\vec{r}_o \]  

\[ f_1 = -(2\pi)^2 \frac{4\pi}{q} \int \sin(qr_o)A(r_o) d\vec{r}_o \] (5)

This term represents the two-body scattering amplitude or the impulse approximation. The second Born approximation is,

\[ f_2 = -(2\pi)^2 \int < k_f |A| \kappa > \frac{1}{\kappa^2 - k^2 - i\epsilon} < \kappa |A| k_i > d\kappa \] (6)

which represents the double scattering term in the scattering amplitude. \( A(r_o) \) is the average potential acting on the incident electron due to the target valance quarks \( r_i \) with initial and final states \( \Psi_a \) and \( \Psi_b \) so that,

\[ A(r_o) = \int \Psi_b^*({\{r_{1}\}})V({\{r_{1}\}})\Psi_a({\{r_{1}\}}){\{d\vec{r}_{1}\}} \] (7)

In the present situation we consider that the electron-quark (eq) potential to be a pure Coulomb with the form,

\[ V({\{r_{1}\}}) = \frac{1}{r_{o1}} - \frac{1}{r_{o2}} - \frac{1}{r_{o3}} \] (8)

and \( r_{oi} \) is the relative coordinate \( r_{oi} = r_o - r_i \) The term \( 1/r_{oi} \) may be expanded in terms of the spherical harmonics \( Y_{l,m} \) as

\[ \frac{1}{r_{oi}} = \sum_{l=0}^{l} \sum_{m=-l}^{l} \frac{4\pi}{2l + 1} \frac{(r_{<})^l}{(r_{>})^{l+1}}Y_{l,m}^\ast(\hat{r}_o)Y_{l,m}(\hat{r}_i) \] (9)

where \( r_{>}(r_{<}) \) is the greater (the lesser) of \( r_o \) and \( r_i \). The radial integral of Eq.(3) may be divided into two parts:

\[ \int_0^\infty = \int_0^{r_o} + \int_{r_o}^\infty \] (10)

(10) and substitute for \( r_{>}(r_{<}) \) by \( r_i(r_o) \) in the first integral and for \( r_{>}(r_{<}) \) by \( r_o(r_i) \) in the second integral. Then writing

\[ A(r_o) = \sum_{i=1}^{3} A(r_i) \] (11)

where \( A(r_i) \) is the average potential due to the \( i'th \) quark, then
\[ A_i(r_o) = \int \Psi_b^*(\{r_i\}) \frac{1}{r_{oi}} \Psi_a(\{r_i\}) \{d\vec{r}_i\} \]  
(12)

\[ A_i(r_o) = \int \Psi_b^*(r_i) \frac{1}{r_{oi}} \Psi_a(r_i) d\vec{r}_i \]

\[ = 4\pi \int_0^{r_o} |\Psi(r_i)|^2 \frac{1}{r_{oi}} r_i^2 dr_i + 4\pi \int_{r_o}^{\infty} |\Psi(r_i)|^2 r_i dr_i \]  
(13)

The evaluation of \( A_i(r_o) \), \( \{i = 1, 2 and 3\} \) for the three quark system differs only in their signs due to the charge situation. On the other and the differential cross section is calculated either in terms of the multiple scattering Born approximation amplitude,

\[ d\sigma/d\Omega = |f_1 + f_2 + ...|^2 \]  
(14)

or in terms of the two body scattering amplitude with a proton form factor correction.

\[ d\sigma/d\Omega = |f_o|^2 F^2(q) \]  
(15)

here, \( f_o \) is the scattering amplitude of the electron scattering by a point-like proton. The proton form factor \( F(q) \) is also an important physical quantity which reflects information about the particle structure. It is defined as:

\[ F(q) = < O' | e^{i\vec{q}\cdot\vec{r}} | O > \]

\[ F(q) = 2\pi \int \Psi'(\vec{r}) e^{iqr\cos(\theta)} \Psi(\vec{r}) r^2 dc\cos(\theta) dr \]  
(16)

as the model is a phenomenological type, it is then branches in two ways,

I) The first one assumes that quarks are confined inside the bag by an extremely deep potential, and the quark states are reasonably represented by a harmonic oscillator wave function.

II) The second way assumes that the constituent quarks behave as almost free particles as is suggested by experiments of deep inelastic scattering [1,14]. However, in all cases the \((e^-p)\) scattering is treated as the collision between the high energy incident electron with
a composite proton system. Proceeding with the first assumption (I) and assuming a
harmonic oscillator wave function for the valance quarks,

\[ \Psi_a = \Psi_b = \left( \frac{2}{\pi a^2} \right)^{3/4} e^{-r^2/a^2} \]  

(17)

Then the average scattering potential is;

\[ A(r_o) = 4\pi \left[ \frac{(2\pi)^{3/2}}{a e^{2r^2/a^2}} + \frac{\Gamma(3/2, 0, r^2/a^2)}{2\pi^{3/2} r} \right] \]  

(18)

where \( \Gamma(3/2, 0, r^2/a^2) \) is the incomplete Gamma function of order 3/2. The scattering am-
plitude and the differential cross section are to be calculated numerically. Moreover, the
form factor is calculated for the proton system as,

\[ F_{h,sc}(q) = \frac{1}{\exp\left(a^2 q^2 / 8\right)} \]  

(19)

Considering now the second case (II) of elastic scattering and that all quarks are in the 1S
ground state of a Coulomb wave function,

\[ \Psi_a = \Psi_b = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \]  

(20)

'a' is the proton radius , then

\[ A(r_o) = \frac{1}{r_o} + \frac{a + 2r_o}{a^2 e^{-2r_o/a}} - \frac{a + 2r_o + 2r_o^2}{a^2 e^{-2r_o/a}} \]  

(21)

Consequently,

\[ f_1(q) = \left[ \frac{\pi}{2q} - \frac{a}{4 + a^2 q^2} - \frac{1}{q} \text{atan}(a q/2) \right] \]  

(22)

\[ \frac{d\sigma}{d\Omega} = 4\left[ \frac{\pi}{2q} - \frac{a}{4 + a^2 q^2} - \frac{1}{q} \text{atan}(a q/2) \right]^2 \]  

(23)

And the form factor is then,

\[ F_{\text{Coul}}(q) = 16 / (4 + a^2 q^2)^2 \]  

(24)
III. RESULTS AND DISCUSSION

Data used in this article are those from the experiments coded SLAC-E-140 [1] at low momentum transfer extended from 1 to 3 \((GeV/c)^2\) using electron beam of energy 8 GeV, and SLAC-E-136 [2] at a wide range extended from 2.9 to 31.3 \((GeV/c)^2\). The later was conducted using accelerated electrons with energies from 5 to 21.5 GeV were elastically scattered by protons in a liquid-hydrogen target at Stanford Linear Accelerator Center. The center of mass energy of the reaction is found in terms of the energies \((E_e & E_p)\) of the scattered electron and proton respectively and their opening angle \(\theta\).

\[
\sqrt{s} = \sqrt{m_e^2 + m_p^2 + 4E_eE_p\sin^2(\theta/2)}
\]  

(25)

The differential cross section is calculated in the frame of multiple scattering using Eq.(14) for both the experiments SLAC-E-136 and SLAC-E-140. The result of calculation as well as the experimental data are displayed in Figs.(1) and (2) respectively. It is found that only the first two terms of the Born series give appreciable contribution to the differential cross section. The second term contributes for not more than 10% in all cases. The calculations are carried out using harmonic oscillator wave function (solid lines) and Coulomb wave function (dashed lines) for the valance quarks forming the proton system. The experimental data are reproduced to a limited extend by the harmonic oscillator wave function only at low momentum transfer region. On the other hand the use of the Coulomb wave function shows better representation of the data at high momentum transfer. This result reflects the fact that quarks inside the hadron are quite confined at low momentum transfer region while at high momentum transfer, quarks behave more freely and the confinement condition becomes no longer necessary. However, the comparison with the experiment shows also that the multiple scattering approach couldn’t give satisfactory agreement particularly at high momentum transfer regions. The differential cross section is recalculated in terms of the proton form factor according to Eqs.(15-16). Proceeding the same analogy, so that the proton form factor is calculated twice using the harmonic oscillator and the Coulomb
wave functions. The result are displayed in Fig.(3) compared with data of the experiment SLAC-NH-11 [3] as well as the prediction of the simple dipole model of the following form,

\[ F_{SD} = \frac{1}{(1 + q^2/0.71)^2} \]  \hspace{1cm} (26)

The prediction of Eq. (15) is displayed with the experimental data in Figs. (4) and (5) for the experiments E-136 and E-140 respectively. It is clear now that the two body scattering amplitude corrected with a relevant form factor may reproduce the experimental data quite well, assuming harmonic oscillator wave function for the constituent quarks. In the above calculations, we use the units where \( \hbar = c = 1 \), hence the momentum in GeV/c may be represented in units of \( fm^{-1} \) with a conversion factor 0.2 In all the above cases the potential parameters are determined by the Chi- Square fitting method. A root mean square radius of the proton is found to be 0.8 and 1.0 fm on using the harmonic oscillator and the Coulomb wave functions respectively. The distribution of the hadronic system produced in e-p scattering is measured by ZEUS collaboration [15] at center of mass energy of 296 GeV. Comparison of the results with the QCD radiation has a strong influence on the characteristics of the final state. The data are reasonably reproduced by the Lund model based on a matrix element calculated in the first order of \( \alpha \), followed by a appropriate parton shower, as well as by the color dipole model. The HERWING parton shower model also gives a reasonable representation of the data. Neither the first order matrix elements alone nor the Lund parton shower model, without the matrix element calculation.
REFERENCES

[1] A.F. Sill et.al., Phys. Rev. D48, 29 (1993).
[2] P. Bosted et al., Phys. Rev. Lett. 68, 3841 (1992).
[3] R.C. Walker, Phys. Lett. 224B, 353 (1989).
[4] M.T. Hussein, 24th International Cosmic Ray Conference ICRC, Roma, HE, 135 (1995).
[5] N.G. Stefanis, Phys. Rev. D40, 2305 (1989).
[6] C.E. Carlson, M. Gari and N.G. Stefanis, Phys. Rev. Lett. 58, 1308 (1987).
[7] S.J. Brodsky and G.P. Lepage, Phys. Rev. D22, 2157 (1980).
   - G.P. Lepage and S.J. Brodsky, Phys. Rev. Lett. 43, 545 (1979).
   - A.V. Radyushkin, Nucl. Phys. A532, 141c (1991).
[8] S.J. Brodsky and G. Farrar, Phys. Rev. D11, 1309 (1975).
[9] A. Duncan and A.H. Muller, Phys. Lett. 90B, 159 (1980).
[10] V.L. Chernyak and I.R. Zhitnitsky, Nucl. Phys. B246, 52 (1984).
[11] M. Gari and N.G. Stefanis, Phys. Lett. B175, 462 (1986).
[12] Z. Dziembowski and J. Frankil, Phys. Rev. D42, 905 (1990).
[13] P. Kroll, M. Schurmann and W. Schweiger, Z. Phys. A338, 339 (1991).
[14] R. G. Arnold et.al., Phys. Rev. Lett. 57, 174 (1986).
[15] ZEUS Collaboration, Z. Phys. C59, 231 (1993).
FIGURES

FIG. 1. The differential cross section of ep scattering as calculated by the multiple scattering approach using for the proton valance quarks a harmonic oscillator wave function (solid line), and the Coulomb wave function (dashed line). The experimental data of SLAC-E-136 are represented by the cross signs (x).

FIG. 2. The differential cross section of ep scattering as calculated by the multiple scattering approach using for the proton valance quarks a harmonic oscillator wave function (solid line), and the Coulomb wave function (dashed line). The experimental data of SLAC-E-140 are represented by the cross signs (x).

FIG. 3. The proton form factor calculated using for the proton valance quarks a harmonic oscillator wave function (solid line), and the Coulomb wave function (dashed line). The prediction of the simple dipole model is represented by (dotted line). The experimental data of SLAC-NE-11 are represented by the cross signs (x).

FIG. 4. The differential cross section of ep scattering calculated as two body scattering amplitude corrected with a relevant form factor using for the proton valance quarks a harmonic oscillator wave function (solid line), and the Coulomb wave function (dashed line). The experimental data of SLAC-E-136 are represented by the cross signs (x).

FIG. 5. The differential cross section of ep scattering calculated as two body scattering amplitude corrected with a relevant form factor using for the proton valance quarks a harmonic oscillator wave function (solid line), and the Coulomb wave function (dashed line). The experimental data of SLAC-E-140 are represented by the cross signs (x).
Differential Cross Section vs. $q^2$ (GeV/c)$^2$