(Anti-)dual-BRST symmetries: Abelian 2-form gauge theory

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Abstract – We derive the absolutely anticommuting (anti-)dual-BRST symmetry transformations for the appropriate Lagrangian densities of the (3+1)-dimensional (4D) free Abelian 2-form gauge theory, under which, the total gauge-fixing term remains invariant. These symmetry transformations are the analogue of the co-exterior derivative of differential geometry, in the same sense, as the absolutely anticommuting (anti-)BRST symmetry transformations are that of the exterior derivative. A bosonic symmetry transformation is shown to be the analogue of the Laplacian operator. The algebraic structures of these symmetry transformations are derived and they are demonstrated to be the reminiscent of the algebra obeyed by the de Rham cohomological operators of differential geometry.

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Introduction. – The Becchi-Rouet-Stora-Tyutin (BRST) and anti-BRST symmetry transformations emerge when the “classical” local gauge symmetry transformations of the local gauge theories are elevated to the “quantum” level. The above (anti-)BRST symmetry transformations are found to be nilpotent of order two and they anticommute with each other. These properties are very sacrosanct and they encode i) the fermionic nature of these symmetries, and ii) the linear independence of these transformations (see, e.g., [1]).

In the realm of the 4D Abelian 2-form [2] gauge theories, the known nilpotent (anti-)BRST symmetry transformations are found to be anticommuting only up to the $U(1)$ vector gauge transformations (see, e.g., [3–5]). However, the application of the superfield approach to BRST formalism, in the context of above 2-form theory, requires the above (anti-)BRST symmetry transformations to be absolutely anticommuting [6]. This key requirement is achieved in our earlier works [7,8] where a Curci-Ferrari type of restriction is invoked for the absolute anticommutativity. The above restriction, which happens to be a key signature of the non-Abelian 1-form gauge theory [9], has been shown to have a close connection with the concept of gerbes [7].

It is well-known that the (anti-)BRST transformations are the analogue of the exterior derivative of differential geometry. In our earlier works [4,5], we have attempted to obtain the symmetry transformations (for the Abelian 2-form gauge theory) that correspond to the co-exterior (or dual-exterior) derivative and the Laplacian operator of differential geometry. However, the nilpotent (anti-)dual-BRST symmetry transformations (which are the analogue of the co-exterior derivative) turn out to be anticommuting only up to a $U(1)$ vector gauge transformation. Thus, the absolute anticommutativity between the co-BRST and anti-co-BRST symmetry transformations is absent in the context of the 4D free Abelian 2-form gauge theory [4,5].

In a very recent work [10], we have obtained an appropriate set of coupled (anti-)BRST invariant Lagrangian densities for the 4D Abelian 2-form gauge theory. The central purpose of our present investigation is to generalize the above appropriate Lagrangian densities so as to obtain the absolutely anticommuting (anti-)BRST as well as the (anti-)co-BRST symmetry transformations together for the above Abelian 2-form gauge theory. Furthermore, we obtain a bosonic symmetry transformation (an analogue of the Laplacian operator) that turns out to be the anticommutator of the (anti-)BRST and (anti-)co-BRST symmetry transformations. The algebraic structures of the above symmetry transformations are obtained and they are shown to be the reminiscent of the algebra obeyed by the de Rham cohomological operators.

1These symmetries would be also called as the (anti-)co-BRST symmetries.
The prime factors that have contributed towards our present investigation are as follows. First and foremost, the known (anti-)co-BRST symmetry transformations [4,5] are found to be non-anticommuting. Thus, it is essential to obtain the correct anticommuting (anti-)dual-BRST symmetry transformations. Second, the above symmetry transformations are the essential ingredients of our theoretical approach to provide a convincing proof that the 4D 2-form gauge theory is a tractable model for the Hodge theory. Our present endeavour is a warm-up exercise towards this goal. Finally, our present understanding of the 2-form theory would provide useful insights to go a step further and study the higher p-form (p > 2) gauge theories.

The contents of our present paper are organized as follows. In the second section, we discuss briefly the off-shell nilpotent (anti-)BRST and (anti-)co-BRST symmetry transformations of our earlier works [4,5] which are found to be non-anticommuting. Our third section is devoted to the derivation of the absolutely anticommuting (anti-)BRST and (anti-)co-BRST symmetry transformations. In the fourth section, we derive a bosonic symmetry transformation. Our fifth section deals with the algebraic structures obeyed by the above symmetry transformations and we also establish their connection with the algebra of the de Rham cohomological operators. Finally, in the last section, we make some concluding remarks.

Preliminaries: non-anticommuting off-shell nilpotent symmetry transformations. – We begin with the generalized version of the Kalb-Ramond Lagrangian density $\mathcal{L}^{(0)} = \frac{1}{12} H_{\mu\nu\kappa} H^{\mu\nu\kappa}$ for the 4D$^2$ free Abelian 2-form gauge theory that respects the off-shell nilpotent (anti-)BRST and (anti-)dual-BRST symmetry transformations [4,5]. This Lagrangian density is

$$\mathcal{L}^{(0)}_B = \frac{1}{2} B \cdot B - B^\mu \left( \frac{1}{3!} \varepsilon_{\mu\nu\kappa\rho} H^{\nu\kappa\rho} - \partial_\rho \phi_2 \right)$$

$$+ B^\mu (\partial^\nu B_{\nu\mu} - \partial_\nu \phi_1) - \frac{1}{2} B \cdot B$$

$$- \partial_\mu \partial_\nu \phi_1 + (\partial_\mu C_{\nu\kappa} - \partial_\nu C_{\mu\kappa}) (\partial^\nu C^\kappa)$$

$$+ \rho (\partial \cdot C + \lambda) (\partial \cdot C + \rho) \lambda,$$  

(1)

where $B_\mu = \partial^\nu B_{\nu\mu}$, $B_\mu = \frac{1}{3!} \varepsilon_{\mu\nu\kappa\rho} H^{\nu\kappa\rho}$, and $\phi_2$ are the Lorentz vector auxiliary fields that have been invoked to linearize the gauge-fixing and kinetic terms, the massless ($\Box \phi_1 = \Box \phi_2 = 0$) scalar fields $\phi_1$ and $\phi_2$ have been introduced for the stage-one reducibility in the theory and the totally antisymmetric curvature tensor $H_{\mu\nu\kappa}$ is constructed with the 2-form antisymmetric gauge field $B_{\mu\nu}$.

The fermionic Lorentz vector (anti-)ghost fields $(C_\mu)C_\mu$ (carrying ghost number $(-1)$) have been introduced to compensate for the above gauge-fixing term and they play important roles in the existence of the (anti-)BRST symmetry transformations for the 2-form gauge potential. The bosonic (anti-)ghost fields $(\beta)\beta$ (carrying ghost numbers $(-2)$) are needed for the requirement of ghost-for-ghost in the theory. The auxiliary ghost fields $\rho = -\frac{1}{4}(\partial \cdot C)$ and $\lambda = -\frac{1}{2}(\partial \cdot C)$ (with ghost numbers $(-1)$) are also present in the theory.

The following off-shell nilpotent $(\delta_{(a)b} = 0)$ (anti-)BRST symmetry transformations $\delta_{(a)b}$ for the fields of the Lagrangian density (1):

$$\delta_{a} B_{\mu\nu} = -\partial_\mu C_{\nu},$$

$$\delta_{a} C_{\mu} = -\partial_\mu \beta,$$

$$\delta_{a} \phi_1 = \rho,$$

$$\delta_{a} \phi_2 = \lambda.$$

It can be readily checked that the Lagrangian density (1) transforms to a total spacetime derivative under the above off-shell nilpotent transformations $\delta_{(a)b}$.

The noteworthy points at this stage are i) under the (anti-)BRST and (anti-)co-BRST transformations, the curvature tensor $H_{\mu\nu\kappa}$ and the gauge-fixing term $(\partial^2 B_{\mu\nu})$ remain invariant, respectively, and ii) the anticommutators of the (anti-)BRST and (anti-)co-BRST transformations produce non-zero results when they act on the...
fermionic (anti-)ghost fields \((\bar{C}_\mu) C_\mu\). Both these observations are very important for our present discussions.

In the language of the cohomological operators, the curvature tensor \(H_{\mu\nu\kappa}\) owes its origin to the exterior derivative \(d = dx^\mu \partial_\mu\) (with \(d^2 = 0\)) because the 3-form \(H^{(3)} = \frac{1}{3!}(dx^\mu \wedge dx^\nu \wedge dx^\kappa)\) \(H_{\mu\nu\kappa}\) defines it through \(H^{(3)} = dB^{(2)}\), where \(B^{(2)} = \frac{1}{2}(dx^\mu \wedge dx^\nu)B_{\mu\nu}\) introduces the gauge potential \(B_{\mu\nu}\). The operation of the co-exterior derivative \(\delta = - d^* \) (with \(\delta^2 = 0\)) on the 2-form produces the gauge-fixing term (i.e. \(\delta B^{(2)} = dx^\mu(\partial_\mu B_{\nu\kappa})\)). Here \(\ast\) is the Hodge duality operation on the 4D spacetime manifold. Thus, the nilpotent (anti-)BRST and (anti-)co-BRST symmetry transformations owe their origin to \(d\) and \(\delta\), respectively (because \(\mu_{\nu\kappa}\) and \((\partial^\mu B_{\nu\kappa})\) remain invariant under them).

It can be checked that \((\delta B, \delta B)\) \(C_\mu = \partial_\mu \lambda\) and \((\delta B, \delta B)\) \(\lambda = - \lambda\). Similarly, the anticommutators \(\{\delta s, s\} C_\mu = \{\delta s, s\} \lambda = \partial_\mu \lambda\) and \(\{\delta s, s\} \lambda = 0\) are not equal to zero. The above anticommutators for the rest of the fields, however, turn out to be absolutely anticommuting. Thus, we note that the (anti-)BRST and (anti-)co-BRST symmetry transformations are anticommuting only up to the \(U(1)\) vector gauge transformations. They are not absolutely anticommuting in nature.

**Anticommuting off-shell nilpotent symmetry transformations.** — We begin with the appropriate BRST \((\mathcal{L}_B)\) and anti-BRST \((\bar{\mathcal{L}}_B)\) invariant Lagrangian densities that have been proposed in our very recent work [10] connected with the 4D free Abelian 2-form gauge theory. These are\(^4\) [10]

\[
\mathcal{L}_B = \frac{1}{6} H^{\mu\nu\kappa} H_{\mu\nu\kappa} + B^\mu (\partial^\nu B_{\nu\kappa} - \delta_\mu \phi_1)
\]

\[
+ B \cdot B + \partial_\mu \bar{\partial} \partial^\mu + (\partial_\mu \bar{C}_\nu - \delta_\mu \phi_2) (\partial^\nu C^\kappa) + (\partial^\mu - \delta_\mu) \lambda.
\]

\[
\bar{\mathcal{L}}_B = \mathcal{L}_B^* = \frac{1}{6} H^{\mu\nu\kappa} H_{\mu\nu\kappa} + B^\mu (\partial^\nu B_{\nu\kappa} + \mu_\phi_1)
\]

\[
+ B \cdot B + \partial_\mu \bar{\partial} \partial^\mu + (\partial_\mu \bar{C}_\nu + \delta_\mu \phi_2) (\partial^\nu C^\kappa) + (\partial^\mu + \delta_\mu) \lambda.
\]  

We focus on these Lagrangian densities for the discussion of all the underlying symmetries of the 4D free Abelian 2-form gauge theory. The kinetic term \(\frac{1}{2} (H^{\mu\nu\kappa} H_{\mu\nu\kappa})\) can be linearized by introducing the auxiliary Lorentz vector fields \(B_{\mu}, \bar{B}_{\mu}\) and massless \(|\phi_2| = 0\) scalar field \(\phi_2\) as

\[
\mathcal{L}_{(B, \bar{B})} = B \cdot B - B^\mu (\bar{C}_\mu) - B^\mu (\partial^\nu B_{\nu\kappa}) - \bar{B} \cdot \bar{B}
\]

\[
+ B^\mu (\partial^\nu B_{\nu\kappa} - \delta_\mu \phi_1) + B \cdot B
\]

\[
+ \partial_\mu \bar{\partial} \partial^\mu + (\partial_\mu \bar{C}_\nu - \delta_\mu C_\kappa) (\partial^\nu C^\kappa) + (\partial^\mu - \delta_\mu) \lambda.
\]

\[
\mathcal{L}_{(B, \bar{B})} = B \cdot B - B^\mu (\partial^\nu B_{\nu\kappa} + \partial_\mu \phi_2)
\]

\[
+ B^\mu (\partial^\nu B_{\nu\kappa} + \partial_\mu \phi_1) + B \cdot B
\]

\[
+ \partial_\mu \bar{\partial} \partial^\mu + (\partial_\mu \bar{C}_\nu + \delta_\mu \phi_2) (\partial^\nu C^\kappa) + (\partial^\mu + \delta_\mu) \lambda.
\]

\[
(5)
\]

\[
(6)
\]

Here all the mathematical symbols denote their usual meanings (cf. the previous section).

The following off-shell nilpotent \((s^2_{(a)} = 0)\) and anticommuting \((s_a s_b + s_b s_a = 0)\) (anti-)BRST transformations on the fields of (5) and (6), namely;

\[
s_a B_{\mu} = - (\partial_\mu C_\nu - \partial_\nu C_\mu),
\]

\[
s_a C_\mu = - \partial_\mu \beta,
\]

\[
s_b \phi_1 = \lambda,
\]

\[
s_b \beta = - \rho,
\]

\[
s_{[\lambda, \rho, \beta, \phi_2, B, B, H_{\mu\nu\kappa}] = 0},
\]

\[
s_a B_{\mu} = - (\partial_\mu C_\nu - \delta_\mu C_\kappa),
\]

\[
s_a \beta = - \mu \beta,
\]

\[
s_a C_\mu = + B_{\mu},
\]

\[
s_b \phi_1 = \rho,
\]

\[
s_b \beta = - \lambda,
\]

\[
s_{[\lambda, \rho, \beta, \phi_2, B, B, H_{\mu\nu\kappa}] = 0},
\]

are the symmetry transformations for (5) and (6) because

\[
s_a \mathcal{L}_{(B, \bar{B})} = - \partial_\mu \left[(\partial^\mu C_\nu - \partial^\nu C_\mu) B_\nu + \lambda B_\nu + \rho \partial^\mu \beta\right],
\]

\[
s_{[\lambda, \rho, \beta, \phi_2, B, B, H_{\mu\nu\kappa}] = 0},
\]

\[
(7)
\]

Thus, the Lagrangian densities change to the spacetime total derivatives.

Similarly, the following off-shell nilpotent \((s^2_{(a)} = 0)\) and absolutely anticommuting \((s_a s_b + s_b s_a = 0)\) (anti-)co-BRST transformations

\[
s_a B_{\mu} = - \frac{1}{2} \varepsilon_{\mu\nu\kappa} \partial^\nu C^\kappa,
\]

\[
s_a C_\mu = - \partial_\mu \beta,
\]

\[
s_a \phi_1 = - \rho,
\]

\[
s_a \beta = \lambda,
\]

\[
s_{[\rho, \lambda, \beta, B, B, B, B, H_{\mu\nu\kappa}] = 0},
\]

\[
s_a B_{\mu} = - \frac{1}{2} \varepsilon_{\mu\nu\kappa} \partial^\nu C^\kappa,
\]

\[
s_a C_\mu = + \partial_\mu \beta,
\]

\[
s_a \phi_1 = - \lambda,
\]

\[
s_a \beta = \rho,
\]

\[
s_{[\rho, \lambda, \beta, B, B, B, B, H_{\mu\nu\kappa}] = 0},
\]

leave the Lagrangian densities (5) and (6) quasi-invariant because

\[
s_a \mathcal{L}_{(B, \bar{B})} = - \partial_\mu \left[(\partial^\mu C_\nu - \partial^\nu C_\mu) B_\nu + \lambda B_\nu + \rho \partial^\mu \beta\right],
\]

\[
s_{[\lambda, \rho, \beta, \phi_2, B, B, H_{\mu\nu\kappa}] = 0},
\]

\[
(10)
\]

The absolutely anticommuting (cf. the section “Algebraic structures...” below) and nilpotent (anti-)BRST as well

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as (anti-)co-BRST transformations are the symmetry transformations of the equivalent and coupled Lagrangian densities (5) and (6).

Now, the stage is set to comment on the anticommutativity properties of the off-shell nilpotent (anti-)BRST and (anti-)co-BRST symmetry transformations. The equations of motion, that emerge from (5) and (6), are

\[ B_\mu = -\frac{1}{2} (\partial^\nu B_{\nu\mu} - \partial_\mu \phi_1), \quad \bar{B}_\mu = -\frac{1}{2} (\partial^\nu B_{\nu\mu} + \partial_\mu \phi_1) , \]

\[ B_\mu = \frac{1}{2} (\epsilon_{\mu\nu\rho\eta} \partial^\nu B^{\eta\rho} - \partial_\mu \phi_2), \]

\[ \bar{B}_\mu = \frac{1}{2} (\epsilon_{\mu\nu\rho\eta} \partial^\nu B^{\eta\rho} + \partial_\mu \phi_2). \]  

(11)

The above relations imply: \( \Box \phi_1 = \Box \phi_2 = 0 \) and \( \partial \cdot B = \partial \cdot \bar{B} = \partial \cdot B = \partial \cdot \bar{B} = 0 \). Furthermore, we obtain the Curci-Ferrari type of restrictions

\[ B_\mu - \bar{B}_\mu - \partial_\mu \phi_1 = 0, \quad B_\mu - \bar{B}_\mu + \partial_\mu \phi_2 = 0, \]  

(12)

which enable us to prove that \( \{s_\mu s_{\nu\rho\eta}, \bar{s}_\mu \bar{s}_{\nu\rho\eta}\} = 0 \) and \( \{s_\delta s_{\alpha\beta}, \bar{s}_\delta \bar{s}_{\alpha\beta}\} = 0 \). In particular, it can be checked that \( \{s_\mu, s_\nu\} B_{\mu\nu} = 0 \) and \( \{s_\mu, s_\nu\} B_{\mu\nu} = 0 \) only if we exploit the Curci-Ferrari–type restrictions of (12) (cf. also the fifth section).

Using nilpotent transformations (7) and (9), it can be explicitly checked that the Curci-Ferrari–type restrictions (12) are the (anti-)BRST as well as (anti-)co-BRST invariant quantities (see, the fifth section below, for more details). As a consequence, in some sense, they are very much “physical” in nature. Thus, the imposition of these restrictions, in the proof of anticommutativity of the (anti-)BRST and (anti-)co-BRST symmetry transformations, is physically not unsolicited. Furthermore, it has been shown in our earlier work [7] that one of the above restrictions (i.e. \( B_\mu - \bar{B}_\mu - \partial_\mu \phi_1 = 0 \)) is connected with the geometrical objects called gerbes. Thus, relations (12) are interesting.

**Bosonic symmetry transformations.** – It is crystal clear that the coupled and equivalent Lagrangian densities (5) and (6) respect four nilpotent symmetry transformations. In particular, the Lagrangian density (5) is endowed with the BRST and dual-BRST symmetry transformations and the symmetry transformations for the Lagrangian density (6) are the anti-BRST and anti-dual-BRST transformations. It is very natural to expect the existence of a bosonic symmetry transformation \( s_w = \{s_b, s_d\} \) (with \( s_w^2 \neq 0 \)) for the Lagrangian density (5) and \( s_w = \{s_b, s_{\alpha\beta}\} \) (with \( s_w^2 \neq 0 \)) for that of the Lagrangian density (6).

The following local and infinitesimal bosonic transformations:

\[ s_w B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + \frac{1}{2} \epsilon_{\mu\nu\rho\eta} \partial^\rho B^{\eta\sigma}, \quad s_w C_\mu = -\partial_\mu \lambda, \]

\[ s_w \bar{C}_\mu = -\partial_\mu \rho, \quad s_w [\rho, \lambda, \phi_1, \phi_2, \beta, \bar{\beta}, B_\mu, \bar{B}_\mu] = 0, \]  

(13)

are the symmetry transformations for the Lagrangian density (5) because

\[ s_w \mathcal{L}(B, \bar{B}) = \partial_\mu \left[ \lambda (\partial^\mu \rho) - (\partial^\mu \lambda) \rho + B^\mu \partial^\mu B_\kappa \right. \]

\[ - B^w \partial^w B_\kappa + B^{\mu} (\partial \cdot B) - B^\mu (\partial \cdot \bar{B}) \].  

(14)

Thus, we note that the anticommutator \( \{s_w, s_d\} \) does generate a bosonic symmetry transformation for the Lagrangian density (5) which happens to be the analogue of the Laplacian operator of differential geometry.

Similarly, we obtain the following bosonic transformations:

\[ s_w B_{\mu\nu} = - (\partial_\mu B_\nu - \partial_\nu B_\mu + \frac{1}{2} \epsilon_{\mu\nu\rho\eta} \partial^\rho B^{\eta\sigma}), \]

\[ s_w C_\mu = -\partial_\mu \lambda, \quad s_w \bar{C}_\mu = -\partial_\mu \rho, \]

\[ s_w [\rho, \lambda, \phi_1, \phi_2, \beta, \bar{\beta}, B_\mu, \bar{B}_\mu] = 0, \]

(15)

from the anticommutator \( \{s_b, s_{\alpha\beta}\} \). This bosonic transformation is a symmetry transformation for the Lagrangian density (6) because

\[ s_w \mathcal{L}(B, \bar{B}) = \partial_\mu \left[ \lambda (\partial^\mu \rho) - (\partial^\mu \lambda) \rho - B^\mu \partial^\mu B_\kappa \right. \]

\[ + B^w \partial^w B_\kappa - B^{\mu} (\partial \cdot B) + B^\mu (\partial \cdot \bar{B}) \].  

(16)

Thus, we have obtained a couple of bosonic symmetry transformations \( s_w \) and \( s_\omega \) (cf. (13) and (15)) for the Lagrangian densities (5) and (6) which are derived from the basic nilpotent symmetry transformations of the theory.

**Algebraic structures of the symmetry transformations and their relevance.** – The appropriate Lagrangian densities of our present 4D Abelian 2-form gauge theory are the ones given in (5) and (6). It has been demonstrated that the Lagrangian density (5) is endowed with the BRST \( (s_b) \), co-BRST \( (s_d) \) and a bosonic \( (s_w = \{s_b, s_d\}) \) symmetry transformations. The operator form of these symmetry transformations is as follows:

\[ s_w^2 = 0, \quad s_d^2 = \frac{1}{2} \{s_d, s_d\} = 0, \quad s_w = \{s_b, s_d\}, \]

\[ [s_w, s_b] = 0, \quad [s_w, s_d] = 0, \quad s_w = (s_b + s_d)^2, \]  

(17)

where it is understood that these algebraic relations act on any arbitrary (i.e. the generic) field of the Lagrangian density (5). For instance, the operator relation \( s_w = \{s_b, s_d\} \) implies that \( s_w \Omega_1 = \{s_b, s_d\} \Omega_1 \), where \( \Omega_1 = (B_{\mu\nu}, B_\mu, B_\mu, \phi_1, \phi_2, \beta, \bar{\beta}, C_\mu, \bar{C}_\mu, \rho, \lambda) \) is the generic field of (5).

Exactly in a similar manner, we note that the anti-BRST \( \{s_b, s_d\}, \) anti-co-BRST \( \{s_b, s_{\alpha\beta}\} \) and a bosonic \( (s_w = \{s_{ab}, s_{\alpha\beta}\}) \) symmetry transformations are respected by the Lagrangian density (6) of our present 2-form gauge theory. It can be checked explicitly that these transformations, in
where it should be kept in mind that the above operator form of transformations act on the generic field Ω(=Bµν, Bμ, Bµ, ϕ1, ϕ2, β, β', Cµ, Cν, ρ, λ) of the Lagrangian density (6) of our present 2-form gauge theory.

The algebraic structures of (17) and (18) are exactly the same as the algebra obeyed by the de Rham cohomological operators of differential geometry [11,12]. The following algebra of these celebrated operators

\[ d^2 = 0, \quad \delta^2 = 0, \quad \Delta = \{d, \delta\}, \]

\[ [\Delta, d] = 0, \quad [\Delta, \delta] = 0, \quad \Delta = (d + \delta)^2, \]

(19)
captures the relationships amongst the exterior derivative \(d = dx^\mu \partial_\mu\), the co-exterior derivative \(\delta = \pm d^*\) and the Laplacian operator \(\Delta = d^2 + \delta^2\) which constitute the de Rham cohomological operators. Here * is the Hodge duality operation on a spacetime manifold without a boundary.

The anticommutativity of the following basic nilpotent transformations

\[ \{s_b, s_{ab}\} = 0, \quad \{s_b, s_{ad}\} = 0, \]

\[ \{s_d, s_{ab}\} = 0, \quad \{s_d, s_{ad}\} = 0, \]

(20)
is guaranteed only when the Curci-Ferrari type of restrictions (12) are exploited. Furthermore, in addition to the transformations (7) and (9), the following off-shell nilpotent transformations on the auxiliary fields

\[ s_b B_\mu = 0, \quad s_d B_\mu = 0, \]

\[ s_{ad} B_\mu = 0, \quad s_d B_\mu = -\partial_\mu \rho, \]

\[ s_b B_\mu = -\partial_\mu \lambda, \quad s_{ab} B_\mu = \partial_\mu \rho, \]

\[ s_{ad} B_\mu = \partial_\mu \lambda, \quad s_{ab} B_\mu = 0, \]

(21)
are to be taken into consideration for the full proof of the anticommutativity. We re-emphasize that the restrictions (12) are (anti-)BRST as well as (anti-)co-BRST invariant quantities as can be checked by using (7), (9) and (21).

Conclusions. – The appropriate set of coupled and equivalent Lagrangian densities (cf. (4)) for the 4D free Abelian 2-form gauge theory were proposed in [10] which were endowed with the off-shell nilpotent and anticommuting (anti-)BRST symmetry transformations. The central theme of our present investigation was to generalize the Lagrangian densities of [10] so as to obtain off-shell nilpotent (anti-)co-BRST transformations together with the (anti-)BRST symmetry transformations of [10]. We have achieved this goal in the form of the Lagrangian densities (5) and (6) of our present investigation.

To accomplish the above objective, we have introduced a pair of Lorentz vector auxiliary fields (i.e. \(B_\mu, \bar{B}_\mu\)) and a massless (\(\Box \phi_2 = 0\)) scalar field \(\phi_2\) to generalize the Lagrangian densities (cf. eq. (4)) of our earlier work [10]. It turns out that a pair of Curci-Ferrari type of restrictions (cf. (12)) are required to obtain the absolute anticommutativity of the (anti-)BRST as well as (anti-)co-BRST symmetry transformations together for the Abelian 2-form gauge theory. These restrictions are found to be invariant under the (anti-)BRST as well as (anti-)co-BRST symmetry transformations as can be checked by exploiting the transformations (7), (9) and (21).

The above four basic nilpotent transformations are the symmetry transformations for the Lagrangian densities (5) and (6) of our present 2-form theory and they correspond to the exterior and co-exterior derivatives of the differential geometry. The anticommutator of the BRST and co-BRST transformations produces a bosonic symmetry transformation which is the analogue of the Laplacian operator for the Lagrangian density (5). In a similar fashion, the anticommutator of the anti-BRST and anti-co-BRST transformations leads to the derivation of a bosonic symmetry transformation that corresponds to the Laplacian operator for the Lagrangian density (6).

One of the Curci-Ferrari type of restriction in (12) has been shown, in our work [7], to have a deep connection with the concept of gerbes which have become very active area of research in theoretical high energy physics (see, e.g. [13,14]). It is a challenging problem for us to find out the meaning of the other restriction (i.e. \(B_\mu - \bar{B}_\mu + \partial_\mu \phi_2 = 0\)) in the language of gerbes.

We have discussed, in our present endeavour, only the continuous symmetry transformations that imbibe the algebraic structure of the de Rham cohomological operators. This exercise is our preparation to accomplish our main goal of proving the present Abelian 2-form gauge theory to be a field theoretic model for the Hodge theory. We have achieved this goal in our recent paper [15]. Right now, the Hamiltonian analysis of our present model is being investigated intensively. Our results would be reported elsewhere [16].

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