Spin-hybrid-phonon resonance in anisotropic quantum dots

V A Margulis and A V Shorokhov
Institute of Physics and Chemistry, Mordovian State University, 430000 Saransk, Russia
E-mail: shorokhovav@math.mrsu.ru

Abstract. We have studied the absorption of electromagnetic radiation of an anisotropic quantum dot taking into account the spin-flip processes that is associated with the interaction of the electrons with optical phonons. It is shown that these processes lead to the resonance absorption. Explicit formula is derived for the absorption coefficient. The positions of the resonances peaks are found.

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1. Introduction

Quantum dots support three main types of intraband resonances associated with absorption of high frequency electromagnetic radiation by electrons: (a) transitions between two energy states caused by photon absorption; (b) transitions between two energy states accompanied by simultaneous absorption and emission of a phonon; (c) transitions between two energy states accompanied by simultaneous scattering by impurities. The above-mentioned processes were studied by the authors for the case of anisotropic parabolic quantum dot \[1, 2, 3\]. The intraband resonances are of particular interest for its possible applications because quantitative understanding of optical properties due to electron-photon, electron-phonon and electron-impurity interactions is important for a successful construction of optical devices based on quantum nanostructures \[4, 5\]. In particular, the semiconductors quantum lasers based on the quantum dots is the most perspective for using as active medium for the new generation of injection lasers \[6\] and for using in infrared detectors. The external magnetic field lets us to control the working frequency of these devices and the magnitude of absorption.

In the case (b), the absorption of a quantum $\hbar \omega$ of the high-frequency field is accompanied by the absorption or emission of an optical phonon. However the interaction of electrons with phonons can lead to additional resonances due to spin-flip processes \[7\]. As a rule this effect can arise in semiconductors with a strong spin-orbital interaction, in particular, in III-V compounds. These spin-flip processes can be considered in the second-order perturbation theory in electron-photon and electron-impurities perturbations using the method developed in \[8\].

In this paper we consider an anisotropic quantum dot located in a 2D layer. We model the confining potential in the direction perpendicular to the 2D layer using $\delta$-function potential. The confining potential in the plane we model using parabolic potential with characteristic frequencies $\Omega_x$ and $\Omega_z$. The magnetic field $B$ is directed along $y$-axes to be perpendicular to the 2D layer.

Note that these resonance can be observed only if all levels (including spin sublevels) are well-resolved and the photon frequency is sufficiently monochromatic. Hence in what follows we assume that the photon frequency is high ($\omega \tau \gg 1$, $\tau$ is the relaxation time), the hybrid confinement is sufficiently strong $\Omega_i \tau \gg 1$, quantizing $\hbar \Omega_i \gg T$ ($i = x, z$) and magnetic field is sufficiently strong $\omega_c \tau \gg 1$ ($\omega_c = eB/m^*c$ is the cyclotron frequency, $m^*$ is the effective mass). In this case the transitions occur between levels of the discrete spectrum.

Using the method suggested in Refs.\[7, 8\], we find the absorption coefficient by applying ordinary perturbation theory for the interactions of electrons with the high-frequency field $H_R$ and the lattice $H_L^{pp}$, which are switched on simultaneously.

$$\Gamma(\omega) = \frac{2\pi \sqrt{\varepsilon(\omega)}}{\hbar n_f} \left[ 1 - \exp \left( -\frac{\hbar \omega}{T} \right) \right]$$
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\[
\times \sum_{n,m,n',m'} f_0(\varepsilon_{nm}) \left| \langle n'm'|\vec{H}|nm \rangle \right|^2 \delta (\varepsilon_{n'm'} - \varepsilon_{nm} \mp \hbar \omega_q \mp g\mu_0 B - \hbar \omega) \tag{1}
\]

Here \( \varepsilon_{nm} = \hbar \omega_1 (n + 1/2) + \hbar \omega_3 (m + 1/2) - m \alpha^2 / \hbar^2, (n, m = 0, 1, 2 \ldots), \alpha \) is the strength of delta-potential, \( g \) is \( g \)-factor, \( \mu_0 = e\hbar / 2m_0 c \) is the Bohr magneton, \( \varepsilon(\omega) \) is the real part of the dielectric constant, \( \mathbf{f} \) is the wave vector of photons, \( f_0(nm) \) is the electron distribution function normalized to unity, \( N_\mathbf{f} \) is the number of initial-state photons, \( \omega_q \) is the phonon frequency, the factor \( (1 - \exp(-\hbar \omega / T)) \) takes into account spontaneous transitions, and \( \omega_i \) are the hybrid frequencies

\[
\omega_{1,3} = \frac{1}{2} \left[ \sqrt{(\Omega_x + \Omega_z)^2 + \omega_c^2} \pm \sqrt{(\Omega_x - \Omega_z)^2 + \omega_c^2} \right] \tag{2}
\]

The matrix elements of the operator \( \vec{H} \) that is responsible for spin-flip processes is determined by the formula

\[
\langle n'm'|\vec{H}|nm \rangle = \sum_{n''m''} \frac{\langle n'm'|H_R|n''m'' \rangle \langle n''m''|H_{sp}^L|nm \rangle}{\varepsilon_{n'm'} - \varepsilon_{n''m''} - \hbar \omega} + \sum_{n''m''} \frac{\langle n'm'|H_{sp}^R|n''m'' \rangle \langle n''m''|H_R|nm \rangle}{\varepsilon_{n'm'} - \varepsilon_{n''m''} + \hbar \omega}. \tag{3}
\]

In (3), the first term describes processes involving, first, emission of a phonon and then, absorption of a photon; and the second term accounts for the processes involving, first, absorption of a phonon and, then, emission of a photon.

To calculate the matrix elements of operators electron-phonon and electron-photons interactions we use the method of a linear canonical transformation of the phase space of the system [9, 10, 11] because in this case we can analytically calculate matrix elements of the corresponding operators.

The matrix elements of the electron-phonon interaction operator was calculated in [10] and have the form

\[
\langle n''m''|H_R|nm \rangle = \frac{e \varepsilon_{\omega} \sqrt{\hbar}}{\sqrt{2m^* \omega}} \left[ X_1 \sqrt{\frac{n'' + 1}{2}} \delta_{n',n''+1} \delta_{m',m''} + X_3 \sqrt{\frac{m'' + 1}{2}} \delta_{n',n''} \delta_{m',m''+1} \right]. \tag{4}
\]

Here, \( \varepsilon_{\omega} \) is the amplitude of the electromagnetic wave polarized along the \( Oz \) axis and the coefficients \( X_i \) are given by

\[
X_i = \frac{\Omega_z^2 \omega_c}{\sqrt{\omega_i \sqrt{\left( \Omega_z^2 - \omega_i^2 \right)^2 + \Omega_z^2 \omega_c^2}}}, \quad i = 1, 3. \tag{5}
\]

The part of operator of electron-phonon interaction that is responsible for spin-flip processes has the form [7, 12]

\[
H_{sp}^L = \sum_q d \left( \frac{1}{2NM \omega_0 \hbar} \right)^{1/2} \begin{bmatrix} 0 & [\mathbf{h}_- \times \mathbf{e}] \\ [\mathbf{h}_+ \times \mathbf{e}] & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p} + \frac{e}{c} \mathbf{A} + \frac{\hbar \mathbf{q}}{2} \\ \mathbf{p} + \frac{e}{c} \mathbf{A} - \frac{\hbar \mathbf{q}}{2} \end{bmatrix}.
\tag{6}
\]
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Here $\mathbf{h}_\pm = l_x \pm i l_y, \mathbf{1}_x, \mathbf{1}_y$ are the unit vectors of axes $Ox$ and $Oy$, $\mathbf{q}$ is the polarization vector of an optical phonon, $M = M_1 M_2/(M_1 + M_2)$ is the reduced mass of the lattice cell of III-V compounds, $d$ is electron-phonon coupling constant, $\mathbf{q}$ is the phonon wave vector, $N$ is the number of cells in the crystal, $\mathbf{A} = (Bz/2, 0, -Bx/2)$ is the vector potential of a magnetic field $\mathbf{B}$.

Let us consider the transitions from the state with $s = -1$ to the state with $s = 1$ ($s = \pm 1$ is the spin quantum number). Denoting $D_\mathbf{q} = d/\sqrt{2N M \omega_0 k}$, we get that the part $H_L$ of the $H^{\text{ph}}_p$ operated on the coordinate part of wave functions is

$$H_L = \sum_\mathbf{q} D_\mathbf{q} (ie_{||} - ie_{\perp}) \left[ e^{-i\mathbf{q} \mathbf{r}} b^+_\mathbf{q} \left( \mathbf{p} + \frac{e}{c} \mathbf{A} - \frac{\hbar \mathbf{q}}{2} \right) + \text{c.c.} \right],$$

where the phonon wave vector is written in cylindrical coordinates $e_z = e_{||}, e_y = e_{\perp} \sin \varphi, e_x = e_{\perp} \cos \varphi$.

After some cumbersome algebra, using the method of canonical transformation of the phase space we get the following formulae for the matrix elements of $H_L$

$$\langle n'', m'' | H_L | n, m \rangle = \sum_\mathbf{q} D_\mathbf{q} \sqrt{N_0 + \frac{1}{2} \pm \frac{1}{2}} \left[ C_1 J(n'', m'', n, m) + C_2 \sqrt{\frac{n}{2}} J(n'', m'', n - 1, m) 
+ C_3 \sqrt{\frac{n + 1}{2}} J(n'', m'', n + 1, m) + C_4 \sqrt{\frac{m}{2}} J(n'', m'', n, m - 1) 
+ C_5 \sqrt{\frac{m + 1}{2}} J(n'', m'', n, m + 1) \right]$$

(8)

Here

$$J(n'', m'', n, m) = \frac{n!m!}{n''!m''!} (-1)^{n''-n} (-1)^{m''-m} e^{-g^2/2} e^{(\kappa_1 \lambda_1 + \kappa_3 \lambda_3) i/2} e^{-i\varphi_1 (n''-n)} e^{-i\varphi_3 (m''-m)} \times L_{n''}^{n-n} (g_1^2) L_{m''-m}^{m-m} (g_2^2) g_3^{n''-n} g_4^{m''-m},$$

(9)

$L_{n''}^{n-n}$ are generalized Laguerre polynomials, $\kappa_i, \lambda_i, \varphi_i, g_i$ are the functions of the characteristic frequencies of the parabolic potential, phonon wave vector and magnetic field (see [10]). The constants $C_1$ are determined by the following expressions

$$C_1 = -\frac{1}{2} i \hbar q_x e_{||}^2 + \frac{1}{2} i \hbar q_x e_{\perp} - \hbar q_y \frac{\mu}{\kappa_0} e_{||},$$

(10)

$$C_2 = i \mu a_{13} e_{||} - \hbar \mu a_{21} l_1 e^{-i\varphi} e_{\perp} + \frac{1}{2} i m^* \omega_c \mu a_{43} e_{||} + \frac{1}{2} \hbar m^* \omega_c \mu a_{31} l_1 e^{-i\varphi} e_{\perp},$$

(11)

$$C_3 = i \mu a_{13} e_{||} + \hbar \mu a_{21} l_1 e^{-i\varphi} e_{\perp} + \frac{1}{2} i m^* \omega_c \mu a_{43} e_{||} - \frac{1}{2} \hbar m^* \omega_c \mu a_{31} l_1 e^{-i\varphi} e_{\perp},$$

(12)

$$C_4 = i \mu a_{14} e_{||} - i \hbar \mu a_{22} l_1 e^{-i\varphi} e_{\perp} + \frac{1}{2} i m^* \omega_c \mu a_{44} e_{||} + \frac{1}{2} \hbar m^* \omega_c \mu a_{32} l_1 e^{-i\varphi} e_{\perp},$$

(13)

$$C_5 = i \mu a_{14} e_{||} - i \hbar \mu a_{22} l_1 e^{-i\varphi} e_{\perp} + \frac{1}{2} i m^* \omega_c \mu a_{44} e_{||} - \frac{1}{2} \hbar m^* \omega_c \mu a_{32} l_1 e^{-i\varphi} e_{\perp}.$$
Let us consider in (13) processes with emission of a phonon and absorption of a photon. In this case we can write the absorption coefficient as a sum of partial absorptions

$$\Gamma(\omega) = \sum_{n'm', n'm} \Gamma(n', m', n, m),$$  \hspace{1cm} (15)

where

$$\Gamma(n', m', n, m) = \frac{e\varepsilon\omega}{2\sqrt{m^*}cN_\omega} \left[ 1 - \exp \left( -\frac{\hbar\omega}{T} \right) \right] f_0(\varepsilon_{nm}) \times \sum_q \left| D_q \sqrt{N_q} + 1 \left\{ \frac{X_1}{\omega_1 - \omega} \left[ \sqrt{n'C}_1J(n', 1, m', n, m) + \sqrt{nC}_2J(n' - 1, m', n - 1, m) + \sqrt{n + 1C}_3J(n' - 1, m', n + 1, m) + \sqrt{nC}_2J(n', m' - 1, n - 1, m) + \sqrt{n + 1C}_3J(n', m' - 1, n + 1, m) + \sqrt{mC}_4J(n', m' - 1, n, m - 1) + \sqrt{m + 1C}_5J(n', m' - 1, n, m + 1) \right] \right\} \right|^2 \delta(\Delta\omega),$$  \hspace{1cm} (16)

where $\Delta\omega = \omega_1(n' - n) + \omega_3(m' - m) + \omega \pm g\mu_0B - \omega_q$ is the resonance detuning.

In the case of transitions which originate from the states (0, 0) into (0, 0) because these transitions give the main contribution in the absorption. In this case we get

$$\Gamma(0, 0, 0, 0) = \frac{e\varepsilon\omega}{2\sqrt{m^*}cN_\omega} \left[ 1 - \exp \left( -\frac{\hbar\omega}{T} \right) \right] f_0(\varepsilon_{00}) \times \sum_q \left| D_q \sqrt{N_q} + 1 \left( C_3 + C_5 \right) \left\{ \frac{X_1}{\omega_1 - \omega} + \frac{X_3}{\omega_3 - \omega} \right\} \right|^2 \delta(\Delta\omega),$$  \hspace{1cm} (17)

Replacing the sum by the integral and assuming a parabolic dispersion law for long-wave phonons $\omega_q = \omega_0(1 - \omega_0^2V_sq^2)$, where $\omega_0$ is the optical-phonon threshold frequency and $V_s$ is the speed of sound, one can easily evaluate the integral with respect to $|q|$ (converting to spherical coordinates) thanks to the presence of a delta function.

In conclusion, we have investigated theoretically the spin-hybrid-phonon resonance in anisotropic quantum dots in the presence of a magnetic field. If we ignore optical phonon dispersion, the partial absorption peaks have a delta-function singularity at the points where $\Delta\omega = 0$. Hence, in this case arise additional resonances in the small vicinity of the peaks of the hybrid-phonon resonances [2] due to spin-flip processes. These peaks are symmetrically positioned to the left and right to the points of hybrid-phonon resonances. The width and the position of the resonance peaks depend strongly on the magnetic field and the characteristic frequencies of the parabolic confinement.

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