Normalized passivity control for robust tuning in real-time hybrid tests

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Abstract
Real-time hybrid testing involves the separation of a system into an experimental component and a numerically simulated substructure which are coupled and run together. The coupling between substructures is achieved using actuators and force sensors which comprise the transfer system. Close synchronization is required between substructures for reliable hybrid testing. However, actuator lag may cause tracking errors and instability in hybrid tests. Existing lag compensation schemes require identification of the coupled dynamics of the transfer system and experimental component and can be sensitive to changes in these components. Passivity control is a technique intended to maintain stability without the need for system identification or assumptions about the actuators or test specimens. Yet, the tuning of existing passivity controllers is sensitive to both the system being tested and the amplitude and frequency range excited. This paper presents a new, normalized passivity controller which behaves well across a much broader range of operating conditions once tuned for a single-test scenario. The proposed approach uses a virtual damping element on the numerical substructure to dissipate spurious power injected by the actuator into the system, based on the ratio of net power output to mean power throughput. The scheme has been shown to result in identical performance for a linear hybrid test with a range of step excitations from 0.5 mm up to 500 mm. The proposed method can be used to improve test stability and fidelity in isolation or alongside other compensation schemes to further improve performance.

KEYWORDS
mechanical systems, model-in-the-loop-testing, passivity, real-time hybrid test, vibration and dynamics

1 | INTRODUCTION

Real-time hybrid testing (RtHT) involves separating a system into a physical substructure tested experimentally, and a simulated numerical substructure. These two substructures are coupled and run in parallel using actuators and
sensors to transfer forces and motion between substructures in real-time. This method seeks to combine experimentation and simulation to overcome inherent limitations in both and exploit their respective strengths. Early incarnations of the method originated in the civil engineering field, where the combination of large structures and demanding dynamic load requirements drove the development of new hybrid laboratory testing techniques.\(^1\)\(^2\) The transfer system which links the numerical and physical substructures consists of a force sensor and an actuator with an inner-loop feedback controller. In the ensuing decades the techniques garnered popularity and the development of what is commonly referred to as real-time dynamic substructuring is well reported by Williams and Blakeborough\(^3\) and Carrion and Spencer.\(^4\) In more recent years, the methods have been embraced in a growing number of fields, notably in the automotive field where Fathy et al.\(^5\) review the evolution of hardware-in-the-loop from its roots, testing purely electrical components,\(^6\) through the inclusion of chassis dynamics,\(^7\) drivetrains,\(^8\) and other mechanical components all the way up to full vehicle tests with only the environment in simulation.\(^9\) The diversity of more recent applications is illustrated by examples in air-to-air refueling,\(^10\) space robotics,\(^11\) aircraft powerplant,\(^12\) rotorcraft dynamics,\(^13\) power electronics,\(^14\) electric drives,\(^15\) and wind energy.\(^16\) The latter elegantly highlights the benefits of hybrid methods by providing a solution to the inherent Froude–Reynolds scaling conflict.

Examples of the advantages of RtHT are abound in the literature cited above. There are significantly fewer hardware requirements with RtHT when compared to a full experimental test, as the complete system does not have to be set up. This leads to substantial cost savings while enabling rapid prototyping and allows for component testing at an early stage before the complete system has been manufactured. Destructive events such as vehicle accidents can be simulated without damaging real hardware. Advantages over fully simulated tests are clear when evaluating physical components whose dynamics are not fully understood or modeled in advance. Hybrid testing enables extreme environmental conditions to be simulated, such that components can be tested under a range of operating conditions that cannot be physically emulated at full scale in a laboratory. Moreover, parameters of the numerical substructure are easily adjustable enabling fast evaluation of components in a wide range of scenarios.

A key limitation of RtHT lies in the dynamics of the controllers and actuators used to synchronize the motion of the physical substructure with that of the numerical substructure. Actuator lag results in tracking errors, leading to an inaccurate emulation of the true system and, of particular interest here, this can lead to instability. Contemporary methods of mitigating actuator lag include the identification of a linear model of the actuator dynamics which is then inverted and applied as a feed-forward or open-loop outer controller in the real-time hybrid test. For example, Wallace et al.\(^13\) identify actuator dynamics as a first-order linear transfer function which is inverted to compensate lag in a real-time hybrid test of a rotor blade coupled to a lag damper. Carrion et al.\(^17\) use a third-order linear transfer function inversion of the plant to mitigate the transfer dynamics in a hybrid test of a building structure employing an MR damper. Delay compensation techniques are also common. Forward predictive algorithms such as those proposed by Horiiuchi et al.\(^18\) Darby et al.,\(^19\) and Ahmadizadeh et al.\(^20\) are used in RtHT. Tang et al.\(^21\) review the performance of these schemes in hybrid tests actuated by a shaker table, illustrating the development of delay compensation schemes in hybrid testing over the years. The authors conclude that performance gains with delay compensation are confined to a narrow low frequency band where transfer system amplitude ratio deviations are small, with stability and accuracy experiencing deteriorations as frequencies rise. In Reference 22, first- and second-order transfer functions are inverted with and without delay compensation, finding that the inversion of a first-order lag combined with delay compensation gives marked performance improvements over either in isolation, with more marginal improvements available with the move to the higher-order transfer function. Other notable approaches include the use of minimal control synthesis,\(^23\) a model reference adaptive control approach requiring no system identification, where the numerical substructure is used as the reference model, and model predictive control,\(^24\) used to compensate for actuator saturation problems inherent to simpler model-inversion-based schemes.

In many hybrid tests, actuator dynamics may be difficult to characterize using a linear model. This could be due to the actuator behavior being dependant on the physical substructure (whose dynamics are unknown), or due to inherent nonlinearity or complexity in the actuator itself. Adaptive schemes may be relatively slow to respond and difficulties can be encountered with guaranteeing stability. Sometimes transient events such as impacts, discontinuities or abrupt parametric changes in the physical system require a control scheme capable of managing test stability in a robust manner, without prior understanding of the actuator or test specimen dynamics.

In recent work,\(^25\) the present authors have investigated a novel approach to the problem of reconciling the motions at the interface between a physical and numerical substructure: passivity-based control. Passivity control was first developed in the teleoperation field to maintain stability between master and slave manipulators. In a passive system, the energy supplied always exceeds the energy output from the system, with the difference being accounted for by the energy dissipated in the system.\(^26\) In Coelho et al.,\(^27\) the passivity controller is described as measuring the energy flow in the system
and adaptively dissipating energy by acting as a damper to ensure passivity in the communications networks coupling the dynamics at either end. Chen et al.\textsuperscript{28} apply passivity control to a multilateral teleoperation system consisting of two masters and slaves. They utilize a passivity observer which monitors the net energy flow into the system which activates the passivity controller, a time-varying damping element, to dissipate excess energy when the energy flow becomes negative. They conclude that tracking performance on the slave side is not guaranteed; however, it is shown that the slave is able to follow the demands of the master manipulator even in the presence of delays as high as 0.8 s.

In RtHT, the passivity control is applied to the transfer system (actuators, controllers, and sensors) to ensure these remain passive and do not contribute spurious energy which can lead to instability. Energy-based techniques have been used before in RtHT: work by Mosqueda et al.\textsuperscript{29,30} has used energy measures as indicators of tracking errors, and Chen and Ricles\textsuperscript{31} have applied these measures in adaptive inverse compensation schemes. The difference in passivity control is that the energy discrepancy is tackled directly, by removing excess energy rapidly as it is detected, thereby providing a more reliable assurance of stability. The objective of passivity control is, first and foremost, to ensure stability. In Reference 25 the authors demonstrated that passivity-based control can be applied effectively in real-time hybrid tests to produce marked stability improvements, while also producing more modest tracking performance improvements. The approach is attractive because (a) in contrast to model-based approaches it does not require prior knowledge of the test specimen or actuator hardware, and (b) it can be used on its own for simplicity or in parallel with another controller to provide high fidelity performance alongside robust stability. This latter approach has been demonstrated in Reference 32 where passivity control is combined with an adaptive feedforward filter to provide good tracking performance and robust stability in the presence of transient events, yielding better performance than could be achieved with either controller in isolation.

One of the drawbacks of passivity-based control when applied to RtHT is that the tuning of the controller is heavily dependent on the absolute magnitudes of the power flow between the substructures, and thus needs retuning for different motion envelopes. A tuning that will work well for low-response amplitudes may produce overzealous damping at higher amplitudes, while one that works well at high amplitudes may allow the development of spurious limit cycles at lower amplitudes that mask the relevant system response. Often the response amplitudes are not known in advance, and commonly in industry a wide range of amplitudes must be tested. The need to retune for each test thus presents a disincentive.

In this paper, a novel approach to passivity control is presented, using a normalized measure of power instead of the conventional absolute measure, thus permitting a given tuning to be used across a broad spectrum of test envelopes. Simulations and experiments are used to investigate the performance of this new configuration. Section 2 describes the structure of the passivity controller and the method of implementation. Section 3 presents simulated hybrid test results where the benefits of normalized passivity control over conventional passivity control for hybrid testing are illustrated, and the mechanisms underpinning the technique examined. Experimental results are presented in Section 4, where the performance in the presence of nonlinearity is also examined, and conclusions are given in Section 5.

2 | THEORY AND METHOD

2.1 | Theory

Real-time hybrid tests comprise of three main subsystems, namely, the numerical and physical substructures and the transfer system. The numerical and physical substructures reflect the true/emulated system while the transfer system is a foreign subsystem included solely to link the numerical and physical substructures of the test. The transfer system should be designed to influence the dynamics of the system as little as possible. This is because spurious power contributed or dissipated by the actuators result in deviations in dynamics from the true/emulated system and critically, can result in instability. The passivity controller acts on the transfer system to dissipate surplus power to maintain stability. A passive transfer system is one which does not output more energy into the physical substructure than the energy supplied by the numerical substructure or vice versa. Achieving this is the main purpose of the passivity controller. Figure 1 illustrates a generic hybrid test, driven for example by a position-controlled actuator, with a resulting velocity imposed on the physical substructure. The measured force and velocity at the interface with the physical substructure give a measure of the power being transmitted to it. This can be compared to the power leaving the numerical substructure, and any discrepancies compensated using the variable rate virtual damper in the passivity controller to dissipate spurious energy in the system.
Power in the hybrid test is evaluated using the product of force and velocity. The power $p_P$ flowing into the physical substructure from the transfer system is thus expressed as

$$p_P = f_P \dot{x}_P,$$  \hspace{1cm} (1)

where $f_P$ and $\dot{x}_P$ are the measured physical substructure force and velocity, respectively. The positive sign convention is used to indicate the power flow into a substructure from the transfer system while negative powers represent power outputs from the physical/numerical substructures and into the transfer system. The power flow $p_N$ flowing into the numerical substructure and out of the transfer system is thus

$$p_N = -f_N \dot{x}_N,$$  \hspace{1cm} (2)

where the numerical substructure force $f_N$ is effectively the reaction force corresponding with $f_P$, acting in the opposite direction, and $\dot{x}_N$ is the velocity of the numerical substructure degree of freedom (DOF) with the same sign convention as the physical substructure. The proposed new passivity controller uses two power quantities as its basis: (i) the power flow $p_E$ from the transfer system into the emulated system, and (ii) the power flow $p_T$ through the transfer system. The former represents the spurious energy contribution to the simulation and is commonly used in passivity-based control. The latter is a measure of the energy transfer between the physical and numerical substructures, comprised of both spurious and authentic components, and can be used as a normalizing quantity to determine the relative influence of the spurious contribution.

The net power flow from the transfer system into the physical and numerical substructures is given by

$$p_E = p_P + p_N.$$  \hspace{1cm} (3)

In a perfect simulation this quantity would be zero, indicating a passive transfer system. Any deviation from zero corresponds to a lack in test fidelity, but most importantly a positive value of $p_E$ can erode stability margins leading to dangerous
test conditions. A common passivity control configuration is to use the quantity $p_E$ directly to determine the damping coefficient of the virtual damper. This arrangement has the disadvantage of being over-zealous at high response amplitudes and ineffective at low amplitudes. While both these scenarios are better than an unbounded instability, they cause unnecessary loss in fidelity in the form of over-damped response at high amplitude and spurious limit cycles at low amplitudes. To mitigate these effects, a normalized measure of power flow is proposed here, with respect to the mean power $p_T$ flowing through the transfer system between the physical and numerical substructures:

$$p_T = \frac{|p_P - p_N|}{2}.$$  

The factor of 2 can be omitted in the normalization as it can be accounted for in the passivity controller gain introduced below in Equation (7). The normalizing quantity $p_S$ is then

$$p_S = |p_P - p_N|.$$  \hspace{1cm} (4)$$

Using the instantaneous values of $p_E$ and $p_S$ to control the damper coefficient can lead to wild fluctuations, in particular where the normalizing quantity $p_S$ is close to zero. In any case, the desired normalizing quantity is not the instantaneous power throughput but something more akin to the RMS value. To achieve this effect, the power measurements are subjected to low pass filtering:

$$T_E(s) = \frac{1}{t_Es + 1},$$  \hspace{1cm} (5)$$

$$T_S(s) = \frac{1}{t_Ss + 1},$$  \hspace{1cm} (6)$$

where $t_E$ and $t_S$ are the time constants of the filters, the effects of which will be investigated in Section 3. The passivity controller gain, given by $b$, is a positive real number set by the user, which scales the filtered, normalized net power to output the passivity damper rate applied at the numerical substructure by the virtual damper. Thus, the virtual damper rate is finally defined in proportion to the normalized power output from the transfer system, as

$$c_D = \begin{cases} 
  b \left( \frac{T_E(s)p_E}{T_S(s)p_N} \right), & p_E > 0 \\
  0, & p_E \leq 0 
\end{cases}.$$  \hspace{1cm} (7)$$

A saturation limit is used to prevent negative virtual damper rates from being demanded so as to ensure net passivity of the transfer system.

### 2.2 Method

The passivity controller in effect will be acting on the numerical substructure based on the response of the actuation hardware. Therefore, it must be programmed in the virtual environment where the numerical simulation is set up. To begin setting up the controller, the user requires information of various states in the hybrid test to quantify surplus power added by the transfer system. These quantities are the numerical substructure force, the numerical substructure velocity, the physical substructure force, and the physical substructure velocity. The former of the four signals are readily obtainable via the numerical simulation in the virtual test environment. The latter two signals will require sensing as they are quantities of the experimental component of the test. The physical substructure force can be measured using a load cell connecting the actuator and physical substructure. Fortunately, this is already available in all hybrid tests as the load cell acts as an important element of the transfer system in communicating data between the numerical and physical substructures.

The physical substructure velocity can be obtained by measuring the velocity of the actuator which moves the physical substructure. The actuator used for the experiments has a built-in quadrature encoder for position measurements. The position measurements are sampled at 10 kHz, with a low pass filter cut-off frequency of 500 Hz. The velocity is
obtained by numerical differentiation of the position measurements at the 10 kHz sampling rate: three orders of magnitude above the experimental excitation frequencies. A better solution where available is direct velocity measurement with, for example, an inductive sensor. With all four system states, the net power flow and mean power flow can be obtained as described by Equations (3) and (4). The passivity controller can then be programmed as described in Section 2.1. The output of the passivity controller can be restricted by a saturation limit to ensure all passivity damper rates demanded will be nonnegative.

3 | SIMULATION

3.1 | Normalized passivity control versus conventional passivity control for hybrid tests

In this section, the performance of the passivity controller in Reference 25 is compared with that of the new, normalized passivity controller. The hybrid test simulation employed in this section consists of a linear, one DOF mass-spring-damper numerical substructure and a physical substructure modeled as a linear spring. The actuator is modeled as a pure delay, which encompasses the delays due to digital communication, controller time steps, control lags, and the dynamics of the physical system. Sensor dynamics and communication on the return path also play a role which is typically dwarfed by that of the actuator dynamics, and for a linear system will have an identical effect to delays in the actuation path. The hybrid test is simulated in the MATLAB/Simulink software environment with a variable step solver (Dormand-Prince 45) with a maximum computation time step of 0.001 s. Table 1 illustrates the parameters of the hybrid test.

Figure 2 illustrates the numerical and physical substructure components of the system and the excitation force given by $f$, while Figure 3 presents a block diagram of the hybrid test.

| TABLE 1 | Hybrid test parameters |
|-----------------|------------------------|
| **Numerical substructure** | **Physical substructure** | **Actuator** |
| 1 degree of freedom lumped mass-spring-damper system: | Stiffness $k = 2$ kN/m | Pure delay of 3 ms |
| - Mass $m_n = 1$ kg | | |
| - Damper rate $c_n = 10$ Ns/m | | |
| - Stiffness $k_n = 1$ kN/m | | |
The transfer functions relating the numerical substructure force \( f_N \) to the numerical substructure displacement \( x_N \) is described by the transfer function denoted \( N(s) \) as follows

\[
x_N \cdot f_N = N(s) = \frac{1}{m_n s^2 + c_n s + k_n}.
\]  

(8)

\( T(s) \) denotes the transfer function of the actuator with delay \( \tau \) and can be expressed as follows.

\[
T(s) = \frac{x_D}{x_N} = e^{-\tau s}.
\]  

(9)

In the simulation this delay is computed using Simulink’s transport delay block, which stores the time history and interpolates between them to approximate the appropriate value. The transfer function describing the physical substructure displacement \( x_p \) to the physical substructure force \( f_P \) is given by given by Equation (10) as follows.

\[
\frac{f_P}{x_P} = P(s) = k.
\]  

(10)

Setting \( T(s) = 1 \), Equations (8), (9), and (10) can be combined to describe the closed loop transfer function of the emulated system in equation as shown below.

\[
\frac{f_P}{f} = \frac{1}{\frac{1}{N(s)P(s)T(s)} + 1} = \frac{k}{m_n s^2 + c_n s + k_n + k}
\]  

(11)

If the physical substructure displacement is the measured output, the closed loop transfer function of the emulated system can be expressed as follows by combining Equations (10) and (11)

\[
\frac{x_P}{f} = \frac{1}{m_n s^2 + c_n s + k_n + k}
\]  

(12)

This hybrid test is used to compare the performance of the normalized passivity controller with that of the passivity controller introduced in Reference 25. The system was given a step excitation force of magnitude 35 N and the physical substructure output position response with normalized passivity control and then conventional passivity control are shown in Figure 4. The passivity damper rates (given by Equation 7 for the normalized controller) and the spurious power injection (given by Equation 3) are also shown. The passivity controller gains and filter coefficients used are described in Table 2.

Both the controllers are seen to dampen out oscillatory behavior matching the position response of the emulated system with great accuracy. However, the normalized passivity controller achieves this using large damping rates over short time intervals while the conventional passivity controller relies on a smaller, more uniform damping rate. The advantage of the new controller in this instance is that it only acts when there is a discrepancy in synchronization between the two systems and does not affect the system dynamics unnecessarily outside of these periods. The disadvantage is that it introduces a greater degree of distortion into the response.

There are two key differences between the normalized controller, and the conventional controller from Reference 25. Firstly, the virtual damper rate of the conventional passivity controller is proportional to the integral of the substructure power error. As such, it acts on not only the present state of the system but also the response history. The new passivity controller designed in this publication on the other hand, acts on the low pass filtered power error so that the sensitivity to the response history can be tuned using the parameter \( T_E \). The second and more significant change is that while the conventional controller acts on the absolute value of the integrated power error, the new controller acts on the normalized power error defined above.

Moreover, in hybrid testing a major disadvantage of passivity control without normalization as used in Reference 25 is the dependency of the virtual damper rate on the absolute value of the substructure power error. With such a scheme, greater damping can be expected when larger excitation signals are used as this is associated with greater kinetic
energy. This in turn would cause the response of the system to vary with the size and type of the excitation thereby requiring controller tunings unique to specific operating conditions. With the normalized control variable in the new passivity controller, the dependency of the virtual damper rate on the excitation signal is alleviated as shown in the following test.

To assess the response of the hybrid test with passivity control over a range of frequencies and amplitudes, frequency responses are employed. The system was excited by force sweeps from 0.1 to 25 Hz over a period of 25 s and the physical substructure position output was used to create frequency responses of amplitude ratio and phase using the ratio of the cross-spectral density of the input and output to the power spectral density of the input. Figure 5 illustrates the frequency responses of the hybrid test with normalized passivity control and conventional passivity control, with fixed controller gains, swept over the above-mentioned frequency spectrum at different excitation force amplitudes. The response of the emulated system is also shown for comparison. The passivity controllers were tuned such that the amplitude ratio of the hybrid tests near resonance closely match that of the emulated system for the 50-N force input. As such, the normalized passivity controller employed a gain of 1 kNs/m while the conventional passivity controller used a gain of 100 Ns/Jm for all tests shown. The units of these two quantities reflect the key distinction between conventional passivity-based control methods and those proposed in this paper: that while a conventional controller needs its gain tuning with respect to the energy levels in the system, the normalized controller proposed here operates independently of the energy levels in the

**FIGURE 4** Hybrid test step responses

**TABLE 2** Passivity controller parameters

| Variable                                                      | Value                                           |
|---------------------------------------------------------------|-------------------------------------------------|
| Passivity controller gain \( b \) for normalized passivity controller | 1 kNs/m                                         |
| Passivity controller gain for conventional passivity controller | 100 Ns/Jm                                       |
| \( t_f \) filter coefficient for normalized passivity controlled | 0.0116 s (a tenth of the emulated system period) |
| \( t_s \) filter coefficient for normalized passivity controlled | 0.116 s (equal to the emulated system period)    |
system and the gain parameter does not need to change for different operating regimes. For the filters of the normalized passivity control test, $t_E$ was set at 0.005 s, that is, a tenth of the maximum excitation period and $t_S$ was set at 0.05 s, that is, equal to the maximum excitation period.

From Figure 5A, it is evident that the response of the hybrid test with passivity control from Reference 25 varies depending on the amplitude of the excitation signals used. In Figure 5B, however, the normalized passivity controller hybrid test response is seen to be nearly identical at all amplitudes with the single controller gain used. Figure 5B also presents zoomed in windows of the hybrid test responses around 8 Hz. This saves considerable time and effort in controller tuning, alleviating the need to retune the controller each time operating conditions are changed. Furthermore, if validation is achieved at one operating condition it can provide confidence in results across a range of operating conditions, thus offering a route to meaningful validation against a full system while retaining many of the benefits of hybrid testing. The limits of this conclusion in real-world operating conditions are further explored in Sections 3.2.3, 3.2.4, and 4. As with conventional passivity control, the phase lag of the hybrid test at high frequency can be improved but not eliminated, and the method will always benefit from supplementary control techniques to further improve phase response.

### 3.2 Controller parameters

In this section, effects of changing the controller gain and power filter time constants will be assessed. To analyze the effects of the controller gain, step excitation signals are used and the natural frequency, damping ratio and total harmonic distortion of the physical substructure position output are estimated. These quantities allow comparison with the expected behavior of the emulated system. The natural frequency and damping ratio should match the emulated system, while the total harmonic distortion measures the deviation from the expected single-frequency harmonic response of the linear emulated system: any harmonics measured are indicative of a distortion of the desired response. Due to the nonlinear damping nature of the passivity controller, the physical substructure position output will be a nonlinear response. The dominant natural frequency of the output is obtained by evaluating the periodogram of the response and measuring the
frequency of the dominant mode. The damping ratio is obtained using the logarithmic decrement applied to the first four peaks of the response. These quantities are normalized with respect to the natural frequency and damping ratio of the emulated system which are obtained using the same methods applied to the emulated system simulated response.

3.2.1 Parametric study of the normalized passivity controller gain

The normalized damping ratio and natural frequency of the hybrid test with passivity control has been evaluated for a range of controller gains and is plotted in Figure 6 together with the total harmonic distortion of the output. The total harmonic distortion of the response in comparison to the fundamental mode is obtained using the method outlined in Reference 33. As such, a system with a normalized natural frequency and damping ratio of 1 represents a hybrid test with the same natural frequency and damping ratio as the emulated system. The net power filter time constant $t_E$ is arbitrarily set to a tenth of the system period to allow quick adaption of the passivity damper rate while the mean power filter time constant $t_S$ is set to match the period of the system to allow the history of a single cycle to be used in the normalization of the power error. Effects of varying filter time constants will be further investigated in the next section.

It is evident that as the controller gain is increased, the damping ratio of the system increases. It is interesting that even though the virtual damper only acts to correct for spurious excess energy in the system, with high enough gains it can evidently overcompensate and exceed the energy dissipation rates of the emulated system. The total harmonic distortion is also seen to increase with the passivity controller gain since higher gains lead to rapid corrections and more pronounced changes in the damper rate, thus leading to more pronounced departures from the linear response of the system. This distortion in the response is reflective of poor tracking. The natural frequency of the system is seen to decrease with increasing controller gain as a natural consequence of the increased damping in the system. Both the frequency response and the distortion of the response are seen to depart from the desired response of the emulated system as the controller gain increases. As such, there is a trade-off in terms of changes in the system's natural frequency, damping ratio and distortion with respect to the emulated system. For the hybrid system tested, Figure 6 illustrates that a normalized damping ratio of 1, a natural frequency 2% lower than that of the emulated system and total harmonic distortion of
−33.68 dB is obtained when a gain of 544 Ns/Jm. The most suitable trade-off will depend on specific test requirements. Moreover, the performance parameters are once again seen to be independent of the step excitation amplitude.

### 3.2.2 Parametric study of the normalized passivity controller filter coefficients

Having investigated the effect of the controller gain on the hybrid test response, it is important to assess how the system response is affected by the power output and power throughout filters defined earlier as $T_E$ and $T_S$. To do this, step responses will be used, and the position output of the physical substructure will be measured and observed over a range of filter time constants. To begin with, the effect of the power output filter time constant $t_E$ will be studied. The aforementioned hybrid test was excited by a 50-N force step at the numerical substructure. The power throughput filter time constant was arbitrarily set to match the period of the emulated system such that the history of a single past cycle will be used in power normalization. As earlier, the period of the emulated system was identified by applying a periodogram to its step response to measure the frequency of the dominant vibration mode. The controller gain was set to 544 Ns/Jm as this was found to result in a satisfactory response in Figure 6 for a sensible choice of filter coefficients.

Figure 7 illustrates the hybrid test response against that of the emulated system for a range of different $t_E$ values. The response without passivity control is seen in Figure 7A to exhibit oscillations much larger than that of the emulated system. With passivity control active however, all tested $t_E$ values result in an improved response largely following that of the emulated system. The response with $t_E$ set to 1e-3 s is seen to result in the best response for this system following that of the emulated system with greatest accuracy. Higher values of $t_E$ such as 1e-2 s are seen to result in greater phase lag. The reason for this is evident when observing Figure 7B, as larger filter coefficients result in slower changes in the damper rate causing notable damper rates to be active for more time. However, a benefit of doing this is that the maximum damper rate required is small and thus distortion of the response will be low. On the other hand, although setting a small value of $t_E$ such as 1e-3 s was found to result in a good response in Figure 7A, Figure 7B indicates that it is achieved with damper rates as high as 418 Ns/m which highlights a limitation of fast-acting passivity controllers, that they may result in greater nonlinear distortion of the output due to the high and volatile damping. Thus, there is a trade-off that must

![Figure 7](image_url)

**Figure 7** Hybrid test response to varying power output filter period $t_E$. (A) Physical substructure position, (B) Passivity damper rate
be made based on the requirement of the application. Solutions requiring low-phase lag will benefit from rapid passivity controllers while those that prioritize low distortion over phase lag would be suitable with a slower power output filter. It is interesting to note that excessively large $t_E$ values such as $t_E = 1\, \text{s}$ result in more oscillatory performance with less-added phase lag as seen in Figure 7A. This is due to the very low damper rates realized as seen in Figure 7B which results in less passivity control action being applied when required. The damper rate curve is seen to grow overtime as the power error accumulates over the test, bringing the solution closer to that introduced in Reference 25 where the power error is integrated to determine the passivity damper rate.

In the next test, the effect of varying the power throughput filter time constant $t_S$ on the hybrid test response will be analyzed. As earlier, a 50-N step excitation force at the numerical substructure was applied with the same controller gain of 544 Ns/Jm as used in the previous test. The power output filter $t_E$ was fixed to one-tenth of the system period to allow for a reasonably fast-acting controller. Figure 8 illustrates the response of the passivity-controlled hybrid test against that of the emulated system. The throughput filter time constant is set as multiples of the system period so that the normalization can be quantified in terms of numbers of previous cycles.

Figure 8A indicates that excessive oscillation is present in the response without passivity control as seen earlier, while the application of passivity control leads to an improved response more representative of the emulated system. The response matching that of the emulated system most closely is seen when the $t_S$ is set to match the period of the emulated system such that one preceding cycle is used for normalization. With the $t_S$ value set too high or too low, the response is seen to exhibit greater phase lag and nonlinear distortion. The reason for this can be observed in Figure 8B where it is seen that lower $t_S$ values, such as a fiftieth of the system period, result in high and volatile damper rates. When $t_S$ is small, $T_S(s)$ tends to 1 in Equation (6), and thus the virtual damper rate is normalized over the mean power variable $p_S$ in Equation (7). In such a configuration, the virtual damper rate will be large when $p_S$ is small giving rise to greater phase lag while rapid changes in $p_S$ will give rise to notable nonlinear distortion in the response. Similarly, when larger $t_S$ values are used, the values of $T_S(s) p_S$ in Equation (7) will be smaller giving rise to greater, more volatile virtual damping as would be seen when the controller gain $b$ is increased. As such, the most suitable setting for $t_S$ will depend on the controller gain and the levels of distortion and extra phase lag that can be accepted for a given application.

![Figure 8](image)

**Figure 8** Hybrid test response to varying power throughput filter period $t_S$. (A) Physical substructure position, (B) Passivity damper rate
3.2.3 Robustness considerations

This section aims to determine the sensitivity of the hybrid test performance to changes in the parameters of the numerical substructure, in the presence of a fixed passivity controller gain. To do this, the natural frequency of the emulated system is systematically increased while keeping the damping ratio and steady state gain constant. Equation (11) derived in Section 3.1, can be rearranged to find expressions for numerical substructure mass, stiffness, and damper rate. Rearranging Equation (11) allows the emulated system transfer function to be expressed in the standard 2nd order lag form as shown below

$$\frac{k/m_n}{s^2 + \zeta \omega_n s + \frac{k + k_m}{m_n}} \equiv \frac{K \omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2}.$$  \hspace{1cm} (13)

Thus, expressions for numerical substructure mass, stiffness and damping ratio can be written as follows

$$m_n = \frac{k}{K \omega_n^2}. \hspace{1cm} (14)$$

$$c_n = 2 \zeta \omega_n m_n. \hspace{1cm} (15)$$

$$k_n = m_n \omega_n^2 - k. \hspace{1cm} (16)$$

The physical substructure stiffness $k$ was reduced to 1 kN/m to allow a greater range of stability to be studied across the spectrum of natural frequencies tested. The steady-state gain and damping ratio $K$ and $\zeta$ were fixed at 0.5 Ns²/(kgmrad²) and 0.1, respectively. The actuator is modeled as a pure delay of 3 ms as earlier. To assess the versatility of a single controller gain in the face of changing system parameters, the normalized natural frequency, damping ratio and total harmonic distortion of the hybrid test response to a 300-N force step in the numerical substructure is evaluated and plotted in Figure 9 against the emulated system natural frequency. The power output filter time constant is set to a tenth of the emulated system period and the power throughput filter time constant is set to match the emulated system period for each frequency tested.
As the natural frequency of the system rises in the presence of a fixed actuator delay, the stability margins of the system erode. With a passivity control gain of zero, the damping ratios even become negative (unstable) at the top end of the frequency range tested. At these higher frequencies, the need for stability augmentation from the passivity control is evident, with the same trade-off observed previously between the natural frequency, damping ratio, and distortion. For the range of natural frequencies tested, a gain of 100 Ns/m is seen to provide a good response at all frequencies, maintaining a normalized damping ratio close to 1. Thus, it appears that a single control gain may be suitable across a broad range of system parameters, albeit with filter coefficients tuned to the specific choice of parameters.

### 3.2.4 Actuator uncertainty

This section investigates the sensitivity of the proposed system to variation in the actuator dynamics. The hybrid test described in Table 1 of Section 3.2.1 is employed in this analysis. The controller parameters are the same as those used in the test illustrated in Figure 5 (i.e., $b = 1$ kNs/m, $t_E = 0.005$ s, $t_S = 0.05$ s). A chirp excitation force of amplitude 5 N and frequency ramping linearly from 0.1 to 25 Hz over a period of 25 s is applied.

Figure 10 illustrates the response of the uncompensated hybrid test against that of the emulated system and the passivity-controlled hybrid test with a 0%, 20%, and 40% increase in the actuator delay. It is evident that in each case, increases in delay have resulted in only minor changes in performance and stability is maintained throughout. This highlights the key benefit of passivity control over conventional model-based schemes in that it does not rely on accurate modeling of the transfer system, and it is robust to changes in the transfer system.

### 3.2.5 Multiple DOF systems

A key result derived from the foregoing single DOF simulations is the universality of the novel passivity controller for a wide range of operating regimes, unlike the conventional passivity controller which requires retuning each time operating conditions shift. In this section the investigation is extended to verify that this result holds for a simple multiple-DOF (MDOF) system. The 2DOF system shown in Figure 11 is adopted as the emulated system.

Table 3 describes the system and simulation parameters used while Figure 12 illustrates the response of the 2DOF system for a range of excitation amplitudes given the single tuning configuration described in Table 3. A force chirp excitation signal from 0.1 to 15 Hz applied to mass $m_1$ of the numerical substructure over a period of 15 s, is used with amplitudes from 0.1 to 50 N. The frequency response shown is obtained by evaluating the ratio of the cross power spectral density between the input and output to the power spectral density of the chirp input.

![Passivity controlled hybrid test with actuator uncertainty](image-url)
As with the SDOF systems, a single controller tuning for the MDOF system provides a very similar response spectrum for a wide range of operating conditions. Thus, in contrast to a conventional passivity controller, once the new controller is tuned the system can be tested in a wide range of operating conditions using the same controller parameters.

### EXPERIMENTATION

This section presents experimental hybrid test results using the normalized passivity controller. The numerical substructure of the hybrid test is the same as that described in Table 1. The physical substructure is represented by an approximately cubic stiffness:

\[ k = 0.0042x_P^2 + 2.13, \quad (17) \]

where \( x_P \) is the displacement of the physical substructure in mm. This stiffness profile is realized by connecting two linear springs perpendicular to the direction of motion as shown in Figure 13, which illustrates the test rig comprising...
the actuator and the physical substructure. The actuator used is a Copley STA2508S electromagnetic actuator running in position control mode via a proportional position controller with velocity feed-forward nested outside cascaded velocity and current proportional integral controllers. The actuator response is third order in nature and its structure is shown in Figure 14. Due to nontrivial friction acting on the armature, a Coulomb friction compensation scheme as proposed by Eamcharoenying et al.\textsuperscript{34} is applied to the actuator in all tests.

The response of the numerical and physical substructure positions without passivity control are shown in Figure 15A. The system is excited by a force input to the numerical substructure with amplitude 10 N at a frequency of 10 Hz. An unstable response is seen as oscillations grow in magnitude. The displacement of the physical substructure in this unstable test is limited by spatial constraints in the controller indicated by saturation limits in the plot. The saturation limits are
Figure 14  Actuator control system structure

Figure 15  Hybrid test response (A) without passivity control, (B) with passivity control, (C) passivity damper rates required for stabilization
programmed into the actuator position controller with leeway to prevent damage due to overshoot. With the application of passivity control however, the response is seen to be stabilized and like that of the emulated system as seen in Figure 15B. Figure 15C illustrates the damper rate for the virtual damper which indicates that most damping is required close to the turning points of the system which agrees with the simulation result seen in Figure 4 described earlier.

Figure 16 illustrates experimental step responses of the nonlinear hybrid test with different passivity controller gains. The response without passivity control is seen to have far too little damping, with oscillations reaching a steady amplitude of 3.75 mm in Figure 16A. This limit cycle is caused by the increase in the actuator’s phase lag at low velocities due to friction as described in Reference 25. The actuator exhibits greater phase lag at low velocities, making the controller

**FIGURE 16** Hybrid test response with different passivity controller gains (A) physical substructure position, (B) passivity damper rate

**FIGURE 17** Performance characteristics of experimental results shown in Figure 16A
unstable in this regime and preventing a stable equilibrium. Although friction compensation was shown to reduce this behavior in Reference, it is unable to fully compensate the effects. Passivity control is seen to bring the decay rate of the system back in line with that of the emulated system, but with clearly discernible changes to the natural frequency and harmonic distortion, increasing with the controller gain. Figure 16B illustrates that high damper rates are instigated even when the system settles, in order to suppress the limit cycle oscillation. This is interesting because with a more conventional passivity controller, the passivity control would not operate in this low-amplitude regime. It is the innovation introduced in this paper, in normalizing the power measurement, that improves the response here.

Figure 17 illustrates the natural frequencies and damping ratios normalized with respect to the emulated system and the total harmonic distortions of the responses shown in Figure 16A. As done in Section 3, the natural frequency of the output is obtained by evaluating the periodogram of the response and measuring the frequency of the dominant mode. The damping ratio is obtained using the logarithmic decrement applied to the first three peaks of the response. Values of unity indicate close alignment with the correct system dynamics. As seen in simulation, improved stability and damping performance comes at the expense of accurate frequency content, and a trade-off must be reached. The primary goals of passivity control are to improve stability. In simpler industrial applications passivity control may contribute adequate tracking performance, but where high-fidelity representation of the frequency content is required then this controller can be used alongside a controller designed specifically to improve tracking performance. The combination is expected to improve on the performance that can be achieved by either, in isolation.

5 CONCLUSION

The primary purpose of passivity control is to ensure stability when coupling dynamic systems over communication networks. A new algorithm for passivity control has been proposed, using normalized measures of power transfer in place of the conventional absolute power measurements. Its use has been demonstrated in real-time hybrid test simulations and experiments. The result is a controller that is markedly less sensitive to the amplitude of the system response than conventional passivity-based controllers. Where conventional controllers would need their gains tuning each time a new test regime is applied, it is shown using the proposed scheme that a single tuning may be adequate for a wide range of test conditions. It is further shown that a correctly configured controller can be robust to variation and uncertainty in the numerical and physical substructures and the actuation dynamics, as well as nonlinear behavior in test specimens. For RtHT, the controller's insensitivity to the response amplitude is important because it offers the possibility of performing a full validation against a complete system for one test regime to provide confidence in the results of tests across a wide range of other test regimes. This tackles one of the most difficult questions in hybrid testing, that of validation, while still retaining many of the time and cost advantages of hybrid tests. Although the primary purpose of the controller is to ensure stability, it has been shown in many cases to contribute improvements to tracking performance as well. It can be implemented alongside a wide range of other control schemes to achieve further improvements in tracking while retaining robust stability. In general, passivity controllers are easy to implement with little understanding of the dynamics of the system being tested, and only minimal tuning required. The new algorithm proposed here simplifies the tuning further while simultaneously improving the robustness of the method, promising more widespread adoption of the technique in industrial applications.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

NOMENCLATURE

| Symbol | Description          |
|--------|----------------------|
| $b$    | Passivity controller gain |
| $c_D$  | Passivity damper rate  |
| $c_n$  | Numerical substructure damper rate |
| $f$    | Hybrid test excitation force |
| $f_N$  | Numerical substructure force |
\( f_P \) Physical substructure force
\( k \) Physical substructure stiffness
\( K \) Steady state gain
\( k_n \) Numerical substructure stiffness
\( m_n \) Numerical substructure mass
\( N(s) \) Numerical substructure transfer function
\( p_E \) Transfer system spurious power injection
\( p_N \) Numerical substructure power
\( p_P \) Physical substructure power
\( p_S \) Mean substructure power variable
\( p_T \) Mean power flow through transfer system
\( P(s) \) Physical substructure transfer function
\( s \) Laplace operator
\( t_E \) Net power filter time constant
\( t_S \) Mean power filter time constant
\( T(s) \) Actuator transfer function
\( T_E(s) \) Power output filter
\( T_S(s) \) Power throughput filter
\( x_P \) Physical substructure/actuator displacement
\( \dot{x}_P \) Physical substructure/actuator velocity
\( \dot{x}_N \) Numerical substructure velocity
\( \tau \) Actuator delay
\( \omega_n \) Emulated system natural frequency
\( \zeta \) Emulated system damping ratio

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