Analysis and Imaging in Magnetic Induction Tomography using the Impedance Method

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Abstract. This article discusses the utilization of the impedance method in computation of the forward problem in magnetic induction tomography (MIT). The algorithms for the inverse problem were also developed. The new approach for solving the resulting ill-conditioned linear system of the inverse problem is proposed and the quality of images obtained is discussed based on a quality index proposed in the literature. The results show the prevalence of TAS in relation to linear correlation between the real image and obtained image. With respect to contrast TRT prevalece in relation TAS. The indices of average luminance presents similarity for the both methods. TAS prevails for smaller objects and TRT for larger objects, showing the greater robustness of TAS.

Keywords – Eddy currents, tomography, impedance methods, numerical modelling.

1. Introduction

Magnetic Induction Tomography (MIT) is a non-invasive technique for obtaining low-resolution images for the visualization of electromagnetic properties of an object using eddy currents. MIT is used for medical applications, such as checking the condition of biological tissues [1], and in industrial applications such as a non-destructive testing in conductive materials [2].

MIT is a system consisting of several sources and magnetic field sensors, formed by coils distributed around the object of interest. When the object is excited with an alternating magnetic field, eddy currents circulate inside and produce a secondary magnetic field. The magnitude and direction of the eddy currents and the field produced by them depend on the electromagnetic properties of the material medium of which the object of interest is formed. MIT is based on the possibility of obtaining images of the distribution of one or more electromagnetic properties within the object, from the measurement of the resultant magnetic field which is distributed around the object.

This study uses the impedance method [3],[4],[5] for computing eddy currents and the Biot-Savart law for calculating magnetic fields. These methods use quasi-static approximations for Maxwell equations, which are valid if the distances involved are small compared to the wavelength of the field.

In these imaging techniques, the process for the computation of eddy currents and the resulting magnetic field is known as the forward problem. And the process for achievement of electrical conductivity distribution inside the object is named inverse problem.
2. Tomograph Model

The presentation of numeric methods used in MIT forward and inverse problems will be done in Figure 1. Here, there is the schematic representation of a tomograph with 16 coils. In this illustration, the shaded circles are radially distributed in relation to the geometric center of the tomograph and represents, for example, 12 sources and 4 field sensors.

![Tomograph Model](image)

Figure 1 – Geometry of the tomograph used in the numerical model.

3. Calculation of Magnetic Inductions

Biot-Savart’s Law is used for computing primary distributions of magnetic induction $B$ also known as primary fields, from numeric integration along the length of generation field coils. Figure 2 shows the geometric details of this calculation. Discretizing the contour of the helicoidal curve of the coil in $N_c$ divisions and forgetting the transversal section of wire, we obtain the expression for the calculation of the primary field $B_p$ in a space position $r$:

$$B_p(r) = \frac{\mu_o I_s}{4\pi} \sum_{n=1}^{N_c} \frac{\Delta L_n \times (r - s_n)}{|r - s_n|^3},$$

(1)

where $\mu_o$ is a magnetic permeability in vacuum, $I_s$ is the peak value of the sinusoidal current flowing in the solenoid, $\Delta L_n$ is the elementary displacement along the solenoid, $s_n$ is the vector that expresses the position of the element $\Delta L_n$, and $B_p$ is the magnetic induction produced by the action of the solenoid at the point $P$. 

![Calculation of Magnetic Inductions](image)
4. Calculation of Eddy Currents

In order to obtain the values of the secondary field, you must first calculate the induced currents. The impedance method was used by dividing the region of interest in a regular grid of \( N_x \times N_y \times N_z \) cubic volume elements, considered homogeneous, connected to their neighbors by impedance dependent on the size of the element and their electrical characteristics. The resulting circuit is shown in Figure 3.

The spatial distribution of the magnetic potential vector \( A \) is calculated from the contributions of the elementary field sources \( I_s \Delta L_n \). In others words,

\[
A(r) = \frac{\mu_0 I_s}{4\pi} \sum_{n=1}^{N} \frac{\Delta L_n}{|r - s_n|}
\]  

(2)
Once the discrete distribution of the magnetic potential vector $\mathbf{A}$ in the various vertices of the three-dimensional grid is known, it is possible to calculate the induced voltage in the primary field at each grid vertice using the equations (3), (4) and (5), that are based on Faraday's Law:

$$V_x(i, j, k) = j\omega \left( [A_y(i, j, k + 1) - A_y(i, j, k)]\Delta y + [A_z(i, j, k) - A_z(i, j, k + 1)]\Delta z \right);$$

$$V_y(i, j, k) = j\omega \left( [A_x(i, j, k) - A_x(i, j, k + 1)]\Delta x + [A_z(i + 1, j, k) - A_z(i, j, k)]\Delta z \right);$$

$$V_z(i, j, k) = j\omega \left( [A_x(i, j, k) - A_x(i + 1, j, k)]\Delta x + [A_y(i, j + 1, k) - A_y(i, j, k)]\Delta y \right),$$

where $\Delta x$, $\Delta y$ and $\Delta z$ are, respectively, the length in directions $x$, $y$ and $z$ of the volume elements. $A_x$, $A_y$ and $A_z$ are the components $x$, $y$ and $z$ of the magnetic potential vector calculated by Equation (2).

Eddy currents can be calculated using traditional mesh analysis. Each branch of the equivalent circuit is shared by four meshes in two perpendicular planes, and four mesh currents must be algebraically added to each branch.

Assuming that in each volume element, the value of the parameters $\sigma$ and $\varepsilon$ are constant. However, these parameters may be different in adjacent elements, which enables modelling heterogeneous media and anisotropic shape.

As each volume element is considered homogeneous, the value of the impedances can be calculated by Equations (6) to (8).

$$Z_x(i, j, k) = \Delta x \left( \left[ \sigma_x(i, j, k) + j\omega \varepsilon_x(i, j, k) \right] \Delta y \Delta z \right)^{-1}$$

$$Z_y(i, j, k) = \Delta y \left( \left[ \sigma_y(i, j, k) + j\omega \varepsilon_y(i, j, k) \right] \Delta x \Delta z \right)^{-1}$$

$$Z_z(i, j, k) = \Delta z \left( \left[ \sigma_z(i, j, k) + j\omega \varepsilon_z(i, j, k) \right] \Delta x \Delta y \right)^{-1}$$

where $\sigma$ is the conductivity, $\varepsilon$ is the dielectric constant and $\varepsilon_0$ is the vacuum permittivity.
Applying Kirchhoff's Voltage Law to the element meshes involving \((i, j, k)\) results in Equations (9) to (11).

\[
Z_x(i, j, k)[I_x(i, j, k) - I_x(i, j, k - 1) + I_x(i - 1, j, k) - I_x(i, j, k)] + \\
Z_y(i, j, k)[I_y(i, j, k) - I_y(i - 1, j, k) + I_y(i, j, k - 1) - I_y(i, j, k)] + \\
Z_z(i, j, k)[I_z(i, j, k) - I_z(i, j, k + 1) + I_z(i, j, k - 1) - I_z(i, j, k + 1) + I_z(i, j, k + 1) - I_z(i - 1, j, k) - I_z(i - 1, j, k - 1) + I_z(i - 1, j, k)]
\]

\[= V_x(i, j, k) \quad \text{(9)}\]

\[
Z_x(i, j, k)[I_x(i, j, k) - I_x(i, j, k - 1) + I_x(i - 1, j, k) - I_x(i, j, k)] + \\
Z_y(i, j, k)[I_y(i, j, k) - I_y(i - 1, j, k) + I_y(i, j, k - 1) - I_y(i, j, k)] + \\
Z_z(i, j, k)[I_z(i, j, k) - I_z(i, j, k + 1) + I_z(i, j, k - 1) - I_z(i, j, k + 1) + I_z(i, j, k + 1) - I_z(i - 1, j, k) - I_z(i - 1, j, k - 1) + I_z(i - 1, j, k)]
\]

\[= V_y(i, j, k) \quad \text{(10)}\]

\[
Z_x(i, j, k)[I_x(i, j, k) - I_x(i, j, k - 1) + I_x(i - 1, j, k) - I_x(i, j, k)] + \\
Z_y(i, j, k)[I_y(i, j, k) - I_y(i - 1, j, k) + I_y(i, j, k - 1) - I_y(i, j, k)] + \\
Z_z(i, j, k)[I_z(i, j, k) - I_z(i, j, k + 1) + I_z(i, j, k - 1) - I_z(i, j, k + 1) + I_z(i, j, k + 1) - I_z(i - 1, j, k) - I_z(i - 1, j, k - 1) + I_z(i - 1, j, k)]
\]

\[= V_z(i, j, k) \quad \text{(11)}\]

To specify the boundary conditions, all currents out of the mesh discretization are nil. Here, \(I_x, I_y\) and \(I_z\) are the induced currents in the mesh. The set of equations \(3N\), where \(N\) is the number of cubic elements, must be resolved to obtain the mesh currents in the equivalent circuit. For the secondary field produced by currents induced in the object, you must obtain the branch currents using Equations (12) to (14).

\[
I_{hx}(i, j, k) = I_x(i, j, k) - I_x(i, j, k - 1) - I_y(i, j, k) + I_y(i, j, k - 1); \quad \text{(12)}
\]

\[
I_{hy}(i, j, k) = I_x(i, j, k) - I_x(i, j, k - 1) - I_z(i, j, k) + I_z(i, j, k - 1); \quad \text{(13)}
\]

\[
I_{hz}(i, j, k) = I_x(i, j, k) - I_x(i, j, k - 1) - I_y(i, j, k) + I_y(i, j, k - 1); \quad \text{(14)}
\]

Thus, the secondary field is calculated using the Biot-Savart Law:

\[
B_r = \frac{\mu_0 h}{4\pi} \sum_{n=1}^{N} \left( I_{hx} u_x + I_{hy} u_y + I_{hz} u_z \right) \times \frac{r - r_n}{|r - r_n|},
\]

where the mesh parameter \(h = Ax = Ay = Az\), \(N = N_xN_yN_z\) is equal to the number of volume elements used in the discretizing, \(u_x, u_y\) and \(u_z\) are the unit vector in directions \(x, y\) and \(z\), and \(r_n\) is the position vector of element \(n\).
5. Inverse Problem

5.1.1. Sensitivity Computing

With the topology used, as can be seen in Figure 2b, the sensor coils are excited by the radial component of the magnetic field resulting in the radial direction, which is calculated using Equation (16).

\[
B_r = B_y \sin \theta + B_z \cos \theta,
\]

where \( \theta \) is the angle measured from the positive direction of the axis \( z \) in direction of the axis \( y \) in the positive direction. When a conductive object is introduced into the tomograph, it induces a secondary field at the time quadrature with respect to the primary field, the signal induced in the sensors is shifted in phase by the following quantity:

\[
\Delta \phi_{m,n} = \tan^{-1} \left( \frac{B_{3,m,n}}{B_{p,m,n}} \right) - \frac{B_{3,m,n}}{B_{p,m,n}},
\]

where \( m \) denotes the generation position and \( n \) the measuring position (Figure 1). The approach used is valid because the secondary field has much smaller amplitude than the primary field. As the phase shift is directly related to the sensitivity to the conductivity, it is possible to obtain the distribution of conductivity in the region of interest from this measurement.

The calculation of sensitivity is straightforward using the Biot-Savart Law and the impedance method. Employing \( N_f \) sources and \( N_s \) field sensors, it is possible to obtain \( N_f N_s \) independent measurements of phase, and with this to obtain the conductivity values in \( N_f N_s \) positions. In this work, the interest is to obtain the image of the transversal section of the conductivity. In the idealized image, it is expected that the conductive region will appear white in color, while the region containing air will appear to be black. For this reason, the area of interest is divided into a two-dimensional regular grid of area elements and a system of equations is defined relating the measured phase of the signal in the set of sensors with the distribution of conductivity in the area of the discretized object:

\[
[S]_{N_f \times N_s} \cdot [\sigma]_{N_f \times N_s} = [\Delta \phi]_{N_f \times N_s},
\]

where \( [\Delta \phi] \) is the vector of phase differences defined in equation (19). The sensitivity matrix or Jacobian matrix, denoted by \( [S] \) relates the vector of electrical conductivity \( [\sigma] \) in the object with the vector phase difference \( [\Delta \phi] \) that will be generated when excited by field sources:

\[
[\Delta \phi] = \begin{bmatrix} \Delta \phi_{0,0} & \Delta \phi_{0,1} & \cdots & \Delta \phi_{0,N_x-1} & \cdots & \Delta \phi_{N_y-1,N_x-1} \end{bmatrix}^T.
\]

We can define the sensitivity matrix so that \( S_{ij} \) is the phase difference to the source-sensor combination \( i \) per unit of electrical conductivity in the area element \( j \) of the image area and provides a column of the matrix of sensitivity. The only parameter which changes from one to another simulation is the conductivity of the mesh so that the conductivity is one for all volume elements contained in the region perpendicularly above or below the element area \( j \). The remaining elements of volume conductivity are zero. The process of calculating the matrix of sensitivity would appear time-consuming, but as it is known \textit{a priori} that eddy currents are zero throughout the volume where the conductivity is zero, the system to be solved for calculating the current has reduced their
order. In addition, once the electrical potentials in the loop for all the field sources are calculated, they do not change when the conductivity of the fabric is altered.

5.2. Regularization of the sensitivity matrix

Once the matrix \( S \) is calculated, it is possible to obtain the distribution of conductivity of a given object as the test vector phase difference \( \Delta \phi \) accordingly. However, the matrix sensitivity is extremely ill-conditioned due to the symmetry in this system, a fact which makes the traditional methods of resolution of linear systems inadequate. A widely used method in these cases is the Tikhonov regularization technique that, for a linear system of the form:

\[
Ax = b
\]

is to obtain the value \( x \) such that:

\[
x \to \min \left\{ \|Ax-b\|_2^2 + \lambda^2 \|Lx\|_2^2 \right\}
\]

(21)

where \( L \) is the matrix regularization, usually equal to the identity matrix, and \( \lambda \) is the regularization parameter. There are some ways to get the regularization optimal parameter \( \lambda \). One is the curve \( L \) which is to find the inflection point of the parametric curve in relation to \( \lambda \):

\[
L: C_\lambda \to (\log \|Ax-b\|_2^2, \log \|Lx\|_2^2).
\]

(22)

Good results were obtained using the technique of Tikhonov regularization, however, due to the need to find the optimal parameter \( \lambda \), the process becomes slow. To solve this problem we developed a new relaxation technique for solving linear systems called the technique of successive approximations, briefly described in the Appendix.

6. Results

The imaging algorithm was tested for various configurations of the test object. All results presented are for the following series discretization with a grid with an edge of 24 cm and 48 divisions in each coordinate axis. In the simulation, 16 sources and 16 field sensors were modeled, which totals 256 area elements of the image obtained. Figure 4 shows the calculation area. These are distributed in a regular two-dimensional 16x16 element (pixel) grid. After the image is obtained, it is interpolated for a 48x48 pixel grid which is then filtered to remove high frequency components of the signal.

Figure 4 – Top view of the calculation area; \( R_i = 17.81 \) cm is the internal radius of the tomograph; \( L = 24 \) cm is the length of the discretized field for implementing the impedance method.
The figures illustrations relating to the simulated images are presented in the following. They describe the problems of testing and then display the results, which are images.

Figure 5 shows the real images reconstructed using the Tikhonov regularization technique (TRT), and using the technique of successive approximations (TSA) for a cylindrical object of radius 4 cm, 24 cm of height centered on the position \( y = 7 \) cm and \( z = -7 \) cm relative to the center of the tomograph. In Figure 7 the images for two cylindrical objects are presented, one identical to the previous (Fig. 6) and the other of radius 7.5 cm, 24 cm of height and center in \( y = 3 \) cm and \( z = -3 \) cm. Figure 8 shows images for a circular ring centered with the inner radius of 4.5 cm, 9 cm of outer radius. For analysis of the quality of the images obtained, indices defined by Wang and Bovik were used [9]. These indices measure the similarities between the images with respect to average luminance \([0, 1]\), the linear correlation \([-1, 1]\) and contrast \([0, 1]\). Figures in brackets indicate the limits for each index, where the lower limit indicates the maximum dissimilarity and the upper limit indicates the maximum similarity between the images. The image quality is calculated by multiplying the four previous indices. Table 2 presents these indices for each of the objects.

**TABLE I – Mean squared errors.**

| Field                  | Error (%) |
|------------------------|-----------|
| Primary radial \( B_{pr} \) | 0.11      |
| Primary polar \( B_{p\theta} \) | 2.39      |
| Secondary radial \( B_{sr} \)  | 3.17      |
| Secondary polar \( B_{s\theta} \) | 8.67      |

**TABLE II – Indices of image quality.**

| Index                          | Object 1 | Object 2 | Object 3 |
|--------------------------------|----------|----------|----------|
|                                | TRT      | TAS      | TRT      | TAS      | TRT      | TAS      |
| Average luminance              | 0.998    | 0.850    | 0.990    | 0.908    | 0.996    | 0.917    |
| Linear Correlation contrast    | 0.550    | 0.781    | 0.442    | 0.632    | 0.126    | 0.241    |
| Image Quality Contrast         | 0.209    | 0.157    | 0.604    | 0.516    | 0.659    | 0.585    |
| Image Quality                  | 0.115    | 0.104    | 0.264    | 0.297    | 0.0824   | 0.129    |

Figures 8 to 10 show the comparison between the ratios for the images obtained using the Tikhonov and successive approximations techniques for cylindrical objects 24 cm in height, varying their radius \( \Delta x \) to \( \frac{1}{2}Nx\Delta x \) where the radius value is normalized to the number of area elements (pixels) of the image.
7. Conclusion

It can be observed in images reconstructed in the figures (Figs. 5-7) that the technique of Tikhonov regularization (TRT) leads to lower quality images compared to images obtained using the technique of successive approximations (TAS). This fact is confirmed by the levels of image quality shown in Table 2. These results show the prevalence of TAS in relation to linear correlation between the real image and obtained image but also indicate the prevalence of TRT with respect to contrast and similarity between the indices of average luminance for the two methods. These quality characteristics are also shown in Figures 8 to 11. However, they show that TAS prevails for smaller objects and TRT for larger objects, showing the greater robustness of TAS. It also shows the limit of detectability of the techniques relative to the size of the test object which is about 3 pixels radius for a cylindrical object.

7.1 Obtained images
Figure 5 – Real and obtained images by Tikhonov and successive approximations techniques for a cylindrical object of radius 4 cm, 24 cm of height with center at coordinates (0.07, -0.07).
Figure 6 – Real and obtained images by Tikhonov and successive approximations techniques for two cylindrical objects, the first identical to Figure 5 and the second of 7.5 cm of radius, 24 cm of height with center at coordinates (-0.03, 0.03).
Figure 7 – Real and obtained images by the Tikhonov and successive approximations techniques for a circular ring with 4.5 cm of inside radius, 9 cm of external radius and 24 cm of height, concentric to tomograph.
Figure 8 – Comparison between the linear correlation coefficients for Tikhonov technique (•) and successive approximations (○) according to the number of pixels of the object beam.

Figure 9 – Comparison between the average luminance ratios for Tikhonov technique (•) and successive approximations (○) and contrast for Tikhonov technique (⋆) and successive approximations (△) according to the number of pixels of the object beam.
Figure 10 – Comparison between the image quality scores for Tikhonov technique (•) and successive approximations (○) according to the number of pixels of the object beam.

8. Appendix

The technique consists basically in carrying out successive increments to each position of the vector \( x \) so that the Euclidean norm of the residual vector \( r \) is minimized. The first method assumes that, given an initial approximation \( x_0 \) to \( x \), it is possible to obtain an increased \( \delta x \) that minimizes the norm of residual vector:

\[
\eta = b - Ax, \tag{23}
\]

where the vector \( x_i \) is the vector \( x_0 \) after the increment in the element \( i \). It is easily verifiable that the residual vector \( r_0 \) for the initial approximation \( x_0 \) and the vector \( r_1 \) for the vector \( x_1 \) are related as follows:

\[
\eta = r_0 - \left[ a_{1,j} \delta x_j \ a_{2,j} \delta x_j \ \cdots \ a_{n,j} \delta x_j \right]^T, \tag{24}
\]

where \( a_{ij} \) denotes the element in row \( i \) and column \( j \) of the matrix \( A \). Thus, it is possible to obtain more accurate solutions by carrying out the increments \( \delta x \) successively in each of the \( n \) elements of \( x_n \), where \( x_n \) is the vector of responses of the iteration \( n \).

In order to obtain the optimum value of \( \delta x \), consider the expression for the norm of the residual vector for the iteration \( m+1 \), depending on the residual vector of the iteration \( m \):

\[
\text{res}_{m+1}(\delta x_i) = \left\{ \sum_{k=1}^{n} \left[ r_m(k) - a_{k,i} \delta x_j \right]^2 \right\}^{1/2}. \tag{25}
\]

We are interested in the minimum point of the function. At this point the first derivative of \( \text{res}_{m+1}(\delta x) \) is zero. Thus, the critical points are obtained from the following equation:

\[
\frac{d \text{res}_{m+1}(\delta x_i)}{d \delta x_i} = \frac{1}{2} \left\{ \sum_{k=1}^{n} \left[ r_m(k) - a_{k,i} \delta x_j \right]^2 \right\}^{-1/2} \left( \sum_{k=1}^{n} a_{k,i} \left( r_m(k) - a_{k,i} \delta x_j \right) \right) = 0. \tag{26}
\]

As the highlighted word in the last equation is non-zero, it must be the case that:

\[
\sum_{k=1}^{n} a_{k,i} \left( r_m(k) - a_{k,i} \delta x_j \right) = 0. \tag{27}
\]
Thus, we obtain the critical value of $\delta x_i$, as:

$$\delta x_i = \frac{\sum_{k=1}^{n} a_{k,i} r_m(k)}{\sum_{k=1}^{n} a_{k,i}^2}.$$  

(28)

So that the value obtained by equation (28) is a minimum point, it is necessary that:

$$\lim_{\delta x_i \to \delta x_i'} \frac{d \text{res}_{m+1}(\delta x_i)}{d \delta x_i} < 0;$$

(29)

$$\lim_{\delta x_i \to \delta x_i'} \frac{d \text{res}_{m+1}(\delta x_i)}{d \delta x_i} > 0.$$  

(30)

As the function $\text{res}_{m+1}(\delta x_i)$ has only one critical point, you can use limits $\delta x_i \to -\infty$ and $\delta x_i \to +\infty$ in the above limits. Substituting Equation (26) for (30), and considering that the term in focus is always positive and nonzero, we obtain:

$$\lim_{\delta x_i \to -\infty} \left\{ \delta x_i \sum_{k=1}^{n} a_{k,i}^2 - \sum_{k=1}^{n} a_{k,i} r_m(k) \right\} < 0;$$

(31)

$$\lim_{\delta x_i \to +\infty} \left\{ \delta x_i \sum_{k=1}^{n} a_{k,i}^2 - \sum_{k=1}^{n} a_{k,i} r_m(k) \right\} > 0.$$

It appears that the conditions imposed in (31) are satisfied for any values of the matrix $A$ and vector $r_m$. Thus it is concluded that the point $\delta x_i$ given by equation (28) is always a minimum as desired.

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