Proper dissipative torques in antiferromagnetic dynamics

H. Y. Yuan\textsuperscript{1}, Qian Liu\textsuperscript{2,3}, Ke Xia\textsuperscript{2,3}, Zhe Yuan\textsuperscript{3(a)} and X. R. Wang\textsuperscript{4,5(b)}

\textsuperscript{1} Department of Physics, Southern University of Science and Technology - Shenzhen 518055, China
\textsuperscript{2} Institute for Quantum Science and Engineering and Department of Physics, Southern University of Science and Technology - Shenzhen, 518055, China
\textsuperscript{3} The Center for Advanced Quantum Studies and Department of Physics, Beijing Normal University Beijing 100875, China
\textsuperscript{4} Physics Department, The Hong Kong University of Science and Technology - Clear Water Bay, Kowloon, Hong Kong
\textsuperscript{5} HKUST Shenzhen Research Institute - Shenzhen 518057, China

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Abstract – There is little doubt that the magnetization dynamics of ferromagnetic systems is governed by the Landau-Lifshitz-Gilbert equation or its generalization with various spin torques. In contrast, there are several sets of dynamic equations for two-sublattice antiferromagnets (AFMs) in the literature that have different forms of dissipative torques and no proper dynamic equations for multi-sublattice AFMs and ferrimagnets in general. Here we introduce the general Rayleigh dissipation functional into the Lagrange equation and derive the proper form of the dissipative torques in the phenomenological equations for the AFMs with multiple sublattices. A new type of dissipative torque arising from the inter-sublattice drag effect is discovered which has important effects on the magnon lifetime and domain wall motion.

Introduction. – There is a reviving interest in antiferromagnetic physics \cite{1-11} since the discoveries of spin-transfer torque \cite{1,2} and anisotropic magnetoresistance \cite{3} in antiferromagnets (AFMs). These discoveries make AFMs promising spintronics materials for data storage and information processing besides their traditional usages as pinning materials. Compared with ferromagnets, AFMs have significant advantages including the high resonance frequencies of the order of terahertz (THz) \cite{12} and the absence of stray field. Therefore, the devices based on AFMs have very high operation speed and can avoid cross-talking \cite{3}. The future development and application of AFM devices rely on our comprehensive understanding of AFM dynamics, which are fundamentally different from ferromagnetic case not only at the quantum-mechanical level, but also at the classical physics level. At the quantum level, it is impossible to use a unitary transformation to map one into the other. At the classical level, a ferromagnet (FM) can be described by magnetization $\mathbf{m}$ while a two-sublattice AFM should be described by two order parameters: the magnetization of each sublattice $\mathbf{m}_1$ and $\mathbf{m}_2$, which can be equivalently redefined as the Néel order parameter $\mathbf{n} \equiv \mathbf{m}_1 - \mathbf{m}_2$ and the net magnetization $\mathbf{m} \equiv \mathbf{m}_1 + \mathbf{m}_2$. Thus, it is not surprising that the dynamics of FMs and AFMs are different.

The magnetization dynamics of a FM is governed by the Landau-Lifshitz-Gilbert (LLG) equation \cite{13,14},

$$\dot{\mathbf{m}} = -\mathbf{m} \times \mathbf{h} + \alpha \mathbf{m} \times \dot{\mathbf{m}},$$

where $\mathbf{h}$ is the effective field consisting of an exchange field, an anisotropy field and an external field and $\alpha$ is the damping constant. The first term on the right-hand side of eq. (1) describes the precessional motion of magnetization around its effective field. The second term is the Gilbert damping that forces the magnetization to align with the effective field \cite{14}. The correctness of the LLG equation for the magnetization dynamics of FMs was verified by...
the good agreement between experiments [15] and theories [16], such that there is little doubt about the general applicability of the LLG equation for FMs.

Things are very different for AFMs. Despite many attempts, there is no consensus on the dynamic equations of AFMs, where the proper form of the dissipation is particularly unclear. So far, there have been different sets of equations used in studying magnetization dynamics of two-sublattice AFMs. In 1950s, Kittel and coworkers introduced the coupled Landau-Lifshitz equations on each sublattice to describe the antiferromagnetic resonance [17,18]. This set of equations (with Gilbert damping) has also been used recently to study spin-transfer torque, spin wave excitation and domain wall (DW) dynamics [2,12,19,20]. Later, Bar’yakhtar et al. proposed a phenomenological theory to include both the longitudinal and transverse relaxation of magnetic moments based on the assumption of magnetization conservation [21–24]. This approach was further pursued by Gomonay and coworkers [25] to construct the dissipation function in antiferromagnetic dynamics. An alternative set of equations has been recently derived from the Lagrange equation of an AFM, in which the dissipative torques are phenomenologically introduced [4,26]. Specifically, \( \alpha_M \) and \( \alpha_n \) are defined as the damping coefficients for the motion of magnetization \( \mathbf{m} \) and Néel order \( \mathbf{n} \), respectively. The resulting equations of this approach are later used to investigate the AFM dynamics by assuming \( \alpha_M = \alpha_n \) [6,8,9,27] or \( \alpha_M = 0 \) [28]. Recent first-principles calculation [29,30] has demonstrated a finite \( \alpha_n \) associated with the motion of Néel order. In addition, \( \alpha_M \) is found to be three orders of magnitude larger than \( \alpha_n \) due to the inter-sublattice exchange coupling [29].

Furthermore, as was pointed out in recent review articles [19,31], determining the quantitative values of the damping coefficients and their physical mechanisms arising from the exchange interaction or the relativistic origin remains a challenge in the field of magnetism. However, all these problems cannot be solved until the proper dissipative torques are formulated and incorporated into the dynamical equations of AFMs.

In this letter, we consider a general antiferromagnet with \( N \) sublattices that may not be collinear with each other. By introducing the proper Rayleigh dissipation functional into the Lagrange equation, we derive new dissipative torques resulting from the inter-sublattice drag effect. This new torque has the anti-damping characteristic and increases the magnon lifetime in AFMs. Releasing the (improper) constraints used in literature, our new AFM dynamic equations essentially unify all different equations as various special cases of a general theory. In addition, our results can naturally explain the recent first-principles calculation of the damping parameters in AFMs. It is worth noting that the same theoretical framework has been applied in two-sublattice ferrimagnets later [32], which is equivalent to this work at the compensation point. But our theory works for both AFM and ferromagnetic system with any number of sublattices. Furthermore, our approach naturally defines the non-trivial order parameters for multiple-sublattice AFMs.

**General theory.** – We consider an \( N \)-sublattice AFM with sublattice magnetization \( \mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_N \). The Lagrangian functional of the AFM depends on \( \mathbf{m}_i \) and their time derivative \( \dot{\mathbf{m}}_i \equiv \partial_t \mathbf{m}_i \), i.e., \( \mathcal{L} = \mathcal{L}(\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_N, \dot{\mathbf{m}}_1, \dot{\mathbf{m}}_2, \ldots, \dot{\mathbf{m}}_N) \). The dissipation is described by the Rayleigh dissipation functional \( \mathcal{R} = \mathcal{R}(\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_N) \) \(^2\). Then the Lagrange equation with the dissipation term is given by

\[
-\frac{\delta \mathcal{L}}{\delta \dot{\mathbf{m}}_i} + \frac{\partial}{\partial t} \frac{\delta \mathcal{L}}{\delta \mathbf{m}_i} + \frac{\delta \mathcal{R}}{\delta \mathbf{m}_i} = 0. \tag{2}
\]

The Lagrangian, \( \mathcal{L} = T - U \), consists of the kinetic energy density functional \( T \) and the potential energy density functional \( U \). Thus, the Lagrange equation (2) can be recast as

\[
-\frac{\delta T}{\delta \mathbf{m}_i} + \frac{\partial}{\partial t} \frac{\delta T}{\delta \mathbf{m}_i} + \left( -h_i + \frac{\delta \mathcal{R}}{\delta \mathbf{m}_i} \right) = 0, \tag{3}
\]

where \( h_i = -\delta \mathcal{L}/\delta \dot{\mathbf{m}}_i \) is the effective magnetic field acting on the \( i \)-th sublattice. We have used the fact that \( \mathcal{R} \) depends only on the magnetization \( \mathbf{m}_i \) and hence \( \delta \mathcal{L}/\delta \dot{\mathbf{m}}_i = 0 \).

The kinetic energy of a spin comes from the Berry phase caused by spin motion [34], i.e., \( T(\mathbf{m}, \dot{\mathbf{m}}) \equiv i(\mathbf{A}(\mathbf{m}) \cdot \dot{\mathbf{m}}) \). For a FM, the kinetic energy can be rewritten in a coordinate invariant form of \( T(\mathbf{m}, \mathbf{n}) = \mathbf{A}(\mathbf{m}) \cdot \mathbf{n} \), where the magnetic potential \( \mathbf{A} \) is determined by \( \nabla \times \mathbf{A}(\mathbf{m}) = \mathbf{m} \) (see footnote \(^3\)). As a natural extension to the \( N \)-sublattice AFM, \( T(\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_N) = \sum_{i=1}^{N} \mathbf{A}(\mathbf{m}_i) \cdot \mathbf{m}_i \). This is because the Berry phase induced by the variation of the magnetization is additive for multiple sublattices \( \mathbf{m}_i \) (see Supplementary Material Supplementarymaterial.pdf (SM) for details). Substituting the kinetic energy term into eq. (3), we obtain (see SM for details)

\[
\dot{\mathbf{m}}_i = -\mathbf{m}_i \times \left( h_i - \frac{\delta \mathcal{R}}{\delta \mathbf{m}_i} \right). \tag{4}
\]

Here the dissipation term is essentially a “damping field” \(-\delta \mathcal{R}/\delta \dot{\mathbf{m}}_i \) in addition to the effective magnetic field \( h_i \).

\(^1\)References [21–24] use the same theoretical approach to consider the dissipation in magnetization dynamics for ferromagnets (ref. [21]), and ferrimagnets (refs. [22] and [23]). Reference [24] is a review article on the subject.

\(^2\)In an AFM with non-collinear magnetization, the dissipation depends on both the magnetization \( \mathbf{m}_i \) and its gradient \( \nabla \mathbf{m}_i \). The influence of the magnetization gradient is not the focus of the present manuscript. This effect does not change the physics presented in our work, but renormalizes the damping parameters in the dynamic equations.

\(^3\)See appendix A of the SM for the detailed derivations of the kinetic energy term and eq. (4), the cases of AFMs with three and four sublattices, the discussion on the incompleteness of the Bar’yakhtar approach, and the DW velocity driven by the spin-orbit torques.
The Rayleigh dissipation functional $\mathcal{R}$ is a quadratic functional of the dynamic variables $\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_N$ \cite{14,33}, i.e., $\mathcal{R} = (\mathbf{v} \cdot \mathbf{R} \cdot \mathbf{v})/2$, where $\mathbf{v} = (\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_N)$ and $\mathbf{R}$ is the so-called dissipation matrix.

Thus, eq. (4) in terms of the dissipation matrix $\mathbf{R}$ becomes

$$\dot{\mathbf{m}}_i = -\mathbf{m}_i \times \mathbf{h}_i + \mathbf{m}_i \times \left( \sum_{j=1}^{N} R_{ij} \dot{\mathbf{m}}_j \right), \quad (5)$$

which governs the AFM dynamics.

Following the standard Lagrange mechanics, the energy dissipation rate due to magnetization motion is

$$\dot{E} = -2\mathcal{R}(\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_N) = -\sum_{i,j} R_{ij} \dot{\mathbf{m}}_i \cdot \dot{\mathbf{m}}_j. \quad (6)$$

Before proceeding, we discuss the mathematical properties of the dissipation matrix $\mathbf{R}$ and corresponding physical meanings. Firstly, for a particular motion of $\mathbf{m}_i$, the energy dissipation rate, which is a physically observable quantity, must be unique, indicating the uniqueness of every matrix element $R_{ij}$. Secondly, according to the second law of thermodynamics, the energy of a system without any energy source must always decrease. In another word, $\dot{E}$ is always negative for an arbitrary motion $\mathbf{m}_i$, indicating that all the elements of the dissipation matrix $\mathbf{R}$ must be real and positive. Thirdly, if an AFM is symmetric under sublattice permutation, one has the identical diagonal matrix elements, $R_{ii} = R_{jj}$ for arbitrary $i$ and $j$. Lastly, both sublattice permutation symmetry and the action-reaction law require the dissipation matrix to be symmetric, i.e., $R_{ij} = R_{ji}$. Furthermore, the real symmetric matrix is also consistent with the requirement of real eigenvalues of $\mathbf{R}$.

Since the dissipation matrix $\mathbf{R}$ is real and symmetric, one can always find an orthogonal matrix $\mathbf{U}$ to diagonalize $\mathbf{R}$, i.e., $\mathbf{U}^T \mathbf{R} \mathbf{U} = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N)$. Thus, eq. (6) can be rewritten as

$$\dot{E} = -\sum_{i=1}^{N} \alpha_i \dot{n}_i^2, \quad (7)$$

where $\mathbf{n}_i = \sum_{j=1}^{N} U_{ji} \mathbf{m}_j$ is the linear combination of $\mathbf{m}_j$. Since all the diagonal elements $\alpha_i$ must be real and positive, the dissipation matrix $\mathbf{R}$ is positive-definite, and $\mathbf{n}_i$ (i = 1, 2, ..., $N$) are the natural order parameters of an $N$-sublattice AFM\footnote{For the case in which the eigenvalues are degenerate, we need to make a linear superposition of the eigenvectors to make them orthogonal.}. Then the generalized dynamic equation (5) can be reformulated in terms of $\mathbf{n}_i$ via an orthogonal transformation,

$$\dot{\mathbf{n}}_i = -\sum_{j,k=1}^{n} U_{ji} U_{jk} \mathbf{n}_k \times \mathbf{h}_j + \sum_{j,k,p=1}^{N} \alpha_p U_{ji} U_{jk} U_{jp} \mathbf{n}_k \times \dot{\mathbf{n}}_p. \quad (8)$$

This set of equations are universal to describe the dynamics of an $N$-sublattice AFM.

We first compare our result, eq. (5) and eq. (8), with the present theories in the literature with $N = 1$ and $N = 2$. For an FM with its magnetization as the only order parameter, eq. (5) with $N = 1$ recovers the LLG equation for a FM. For $N = 2$, $\mathbf{R}$ is a $2 \times 2$ matrix defined by two real positive numbers, $\alpha$ and $\alpha_c$ for the diagonal and off-diagonal elements,

$$\mathbf{R} = \left( \begin{array}{cc} \alpha & \alpha_c \\ \alpha_c & \alpha \end{array} \right). \quad (9)$$

The matrix defines two order parameters $(\mathbf{m}_1 + \mathbf{m}_2)/\sqrt{2}$ and $(\mathbf{m}_1 - \mathbf{m}_2)/\sqrt{2}$, which are the well-known net magnetization and Néel order parameter. Two corresponding eigenvalues $\alpha \pm \alpha_c$ are the damping coefficients associated with the motion of the two order parameters. Because the dissipation of the magnetic system should always be positive without external pumping according to the second law of thermodynamics, the eigenvalues of dissipation matrix $(\alpha \pm \alpha_c)$ that mathematically quantify the strength of dissipation are positive, i.e., $\alpha > \alpha_c > 0$. Kittel’s AFM theory \cite{17,18}, and its extension to include the dissipative torque correspond to $\alpha \neq 0$ and $\alpha_c = 0$. The Bar’yakhtar approach is the special case of $\alpha = \alpha_c$ \cite{23,24}, which is not true in general (see SM for details). The ad hoc damping terms added into the dynamic equations by Hals et al. \cite{4} are justified by our results with the correspondence $\alpha_m = (\alpha + \alpha_c)/2$ and $\alpha_n = (\alpha - \alpha_c)/2$, respectively. Therefore, our result, eq. (5) essentially unifies the existing phenomenological theories in the literature for the two-sublattice AFMs.

The new dissipative torques in eq. (5), $R_{ij} \mathbf{m}_i \times \dot{\mathbf{m}}_j$ (i $\neq j$), can also be viewed as an effective torque on spin $i$ dragged by the motion of spin $j$. This is analogous to the motion of a particle in a fluid where a force can be exerted by a neighbouring moving particle. One can also interpret this torque as the inter-sublattice spin pumping effect \cite{29}: the motion of $\mathbf{m}_j$ pumps a spin current of $\alpha_{sp} \mathbf{m}_j \times \dot{\mathbf{m}}_j$, which is absorbed by $\mathbf{m}_i$ and results in an effective damping torque on $\mathbf{m}_i$ in the form of $\alpha_{sp} \mathbf{m}_i \times [\mathbf{m}_i \times (\mathbf{m}_j \times \dot{\mathbf{m}}_j)] \approx \alpha_{sp} \mathbf{m}_i \times \dot{\mathbf{m}}_j$, $\alpha_{sp}(>0)$ measures the magnitude of the spin pumping. Meanwhile, spin pumping from $\mathbf{m}_i$ enhances its own damping to be $(\alpha_0 + \alpha_{sp}) \mathbf{m}_i \times \dot{\mathbf{m}}_i$, where $\alpha_0(>0)$ is the intrinsic Gilbert damping of the $i$-th sublattice via, e.g., spin-orbit interaction. This consideration leads to $R_{ii} = \alpha_0 + \alpha_{sp}$ and $R_{ij} = \alpha_{sp}$, and further implies that the diagonal elements of the dissipation matrix should be larger than the off-diagonal elements, i.e., $R_{ii} > R_{ij} > 0$ (i $\neq j$). For Mn-based metallic AFMs, recent first-principles calculations show that the magnitude of the diagonal and off-diagonal dissipation matrix elements are very close to each other. This implies that the inter-sublattice spin pumping is the dominant mechanism of damping in bulk metallic AFMs \cite{29,30}. It is also interesting to note that the new dissipative torques $R_{ij} \mathbf{m}_i \times \dot{\mathbf{m}}_j$ (i $\neq j$) play an important role in the interfacial spin pumping \cite{35}.  

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In the following, we choose two non-trivial cases, \( N = 3 \) and \( N = 4 \), as examples to illustrate the applicability of our theory for an arbitrary number of sublattices in AFMs. For \( N = 3 \), the dissipation matrix reads

\[
R = \begin{pmatrix}
\alpha & \alpha_c & \alpha_c \\
\alpha_c & \alpha & \alpha_c \\
\alpha_c & \alpha_c & \alpha
\end{pmatrix}.
\] (10)

The eigenvalues and the corresponding eigenvectors are

\[
\begin{align*}
\alpha_1 &= \alpha - \alpha_c, & v_1 &= \frac{1}{\sqrt{2}}(-1, 1, 0), \\
\alpha_2 &= \alpha - \alpha_c, & v_2 &= \frac{1}{\sqrt{2}}(-1, 0, 1), \\
\alpha_3 &= \alpha + 2\alpha_c, & v_3 &= \frac{1}{\sqrt{3}}(1, 1, 1).
\end{align*}
\] (11)

Therefore, the dissipation matrix \( R \) can be diagonalized as \( \Lambda = \text{diag}(\alpha_1, \alpha_2, \alpha_3) \) via the transformation \( U^T RU \) with \( U = (v_1^T, v_2^T, v_3^T) \). Consequently, the order parameters of a 3-sublattice AFM can be naturally defined by

\[
\begin{align*}
n_1 &= \frac{1}{\sqrt{2}}(-m_1 + m_2), \\
n_2 &= \frac{1}{\sqrt{2}}(-m_1 + m_3), \\
n_3 &= \frac{1}{\sqrt{3}}(m_1 + m_2 + m_3).
\end{align*}
\] (12)

For \( N = 4 \), the dissipation matrix reads

\[
R = \begin{pmatrix}
\alpha & \alpha_c & \alpha_c & \alpha_c \\
\alpha_c & \alpha & \alpha_c & \alpha_c \\
\alpha_c & \alpha_c & \alpha & \alpha_c \\
\alpha_c & \alpha_c & \alpha_c & \alpha
\end{pmatrix},
\] (13)

with the eigenvalues and eigenvectors,

\[
\begin{align*}
\alpha_1 &= \alpha - \alpha_c, & v_1 &= \frac{1}{2}(-1, 1, -1, 1), \\
\alpha_2 &= \alpha - \alpha_c, & v_2 &= \frac{1}{2}(-1, -1, 1, 1), \\
\alpha_3 &= \alpha - \alpha_c, & v_3 &= \frac{1}{2}(-1, 1, 1, -1), \\
\alpha_4 &= \alpha + 3\alpha_c, & v_4 &= \frac{1}{2}(1, 1, 1, 1).
\end{align*}
\] (14)

Similarly, the order parameters of a 4-sublattice AFM should be

\[
\begin{align*}
n_1 &= \frac{1}{2}(-m_1 + m_2 - m_3 + m_4), \\
n_2 &= \frac{1}{2}(-m_1 - m_2 + m_3 + m_4), \\
n_3 &= \frac{1}{2}(-m_1 + m_2 + m_3 - m_4), \\
n_4 &= \frac{1}{2}(m_1 + m_2 + m_3 + m_4).
\end{align*}
\] (15)

**Magnon lifetime.** – We consider the magnon lifetime and current-driven DW velocity in two-sublattice AFMs to highlight the important consequence of the new dissipative \( \alpha_c \)-torque. According to eq. (5) and eq. (9), the dynamic equations of a two-sublattice AFM are

\[
\begin{align*}
m_1 &= -m_1 \times h_1 + m_1 \times (\alpha \dot{m}_1 + \alpha_c \dot{m}_2), \\
\dot{m}_2 &= -m_2 \times h_2 + m_2 \times (\alpha \dot{m}_2 + \alpha_c \dot{m}_1). 
\end{align*}
\] (16)

In order to see the influence of the \( \alpha_c \)-torque on the magnon lifetime, we consider a spin wave of the wave vector \( \mathbf{k} \) and frequency \( \omega \) such that \( \mathbf{m}_1 = \mathbf{m}_1^0 + \delta \mathbf{m}_1 \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t), \mathbf{m}_2 = \mathbf{m}_2^0 + \delta \mathbf{m}_2 \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t), \) where \( \mathbf{m}_1^0 \) is the magnetic moment of substrate \( i \) in the ground state and \( \delta \mathbf{m}_i \) is a small deviation perpendicular to \( \mathbf{m}_i^0 \). Following the standard Kittel approach [17,18], we determine the spin wave dispersion using the linearized equations of eqs. (16) for \( \delta \mathbf{m}_1 \) and \( \delta \mathbf{m}_2 \). The magnon frequency is obtained by solving the secular equation

\[
\omega^2 \pm (a_{11} + a_{22}) \omega + a_{11}a_{22} - a_{12}a_{21} = 0,
\] (17)

where \( a_{11} = H_0 + H_{nn} m_1^0 + i H_{nn} m_1^0, a_{22} = H_0 - H_{nn} m_1^0 + i H_{nn} m_1^0, a_{12} = (H_E - i\omega \alpha_c) m_1^0 \). Here we explicitly write the effective fields as the sum of the external field \( H_0 \) along the easy axis, the exchange field \( H_E \) and the anisotropy field \( H_{an} \). The two solutions of eq. (17) correspond to the acoustic and optical modes of magnon excitation, respectively. The magnon lifetime \( \tau = -1/\Im(\omega) \) is plotted in fig. 1 as a function of \( \alpha_c \). The lifetimes of both optical and acoustic magnons increase dramatically with increasing \( \alpha_c \) and the enhancement is particularly large for small \( \alpha_c \). This is because the new \( \alpha_c \)-torque, whose direction is opposite to the conventional damping torque (the \( \alpha \)-term), drags the magnetization away from its equilibrium state as schematically illustrated in fig. 1(c) and (d).

**Domain wall velocity.** – To see the influence of \( \alpha_c \) on the DW motion, we rewrite eq. (16) in terms of net magnetization \( \mathbf{m} = \mathbf{m}_1 + \mathbf{m}_2 \) and Néel order \( \mathbf{n} = \mathbf{m}_1 - \mathbf{m}_2 \) that are the natural order parameters as we discussed after...
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The system. For small deviation from the equilibrium, the Neél field and the magnon velocity, and the effective damping is \( \alpha \) (see SM for details). Specifically, one has

\[
\dot{\mathbf{m}} = -\mathbf{n} \times (\mathbf{h}_m - \alpha_m \mathbf{n}) - \mathbf{n} \times (\mathbf{h}_n - \alpha_n \mathbf{n}), \\
\dot{\mathbf{n}} = -\mathbf{m} \times (\mathbf{h}_n - \alpha_n \mathbf{n}) - \mathbf{n} \times (\mathbf{h}_m - \alpha_m \mathbf{m}), 
\]

(18)

where \( \alpha_m = (\alpha + \alpha_z)/2 \), \( \alpha_n = (\alpha - \alpha_z)/2 \), \( \mathbf{h}_m = -\delta \mathbf{H}/\delta \mathbf{m} \), \( \mathbf{h}_n = -\delta \mathbf{H}/\delta \mathbf{n} \) are, respectively, the effective magnetization field and the Neél field and \( \mathbf{H} \) is the free energy of the system. For small deviation \( \mathbf{m}_1 \approx -\mathbf{m}_2 \), \( |\mathbf{m}| \ll |\mathbf{n}| \) and the magnitude of \( \mathbf{m} \) is nearly conserved. We keep only the terms that preserve \( |\mathbf{n}| \) and that are linear in \( \mathbf{m} \), the dynamic equations can be further simplified as

\[
\dot{\mathbf{m}} = -\mathbf{n} \times (\mathbf{h}_m - \alpha_m \mathbf{n}) - \mathbf{m} \times (\mathbf{h}_m - \alpha_m \mathbf{m}), \\
\dot{\mathbf{n}} = -\mathbf{n} \times (\mathbf{h}_m - \alpha_m \mathbf{m})). 
\]

(19)

Note that \( \mathbf{h}_m \) is of the same order of \( \mathbf{m} \) [27]. It is worth mentioning that eq. (19) is the same as the equation used by Hals et al. [4].

For a uniaxial 1D AFM, the free energy density functional is \( \mathcal{H} = H_E \mathbf{m}^2/2 + A(\partial \mathbf{n})^2/2 - H_m \mathbf{n}^2/2 - \mathbf{b} \cdot \mathbf{n} \), where \( A \) is the spin stiffness and \( \mathbf{b} \) is the Neél field generated by an electric current through spin-orbit interaction [10,36]. By eliminating \( \mathbf{m} \) from eq. (19), the decoupled dynamic equation of \( \mathbf{n} \) is [4]

\[
\mathbf{n} \times (-\dot{\mathbf{n}} + \alpha_m \mathbf{h}_n + H_E \mathbf{h}_m - \alpha_n H_E \mathbf{n}) = 0. 
\]

(20)

For the steady motion of a rigid DW that obeys \( \mathbf{n}(z) = \mathbf{n}(z - vt) \), the DW velocity can be analytically obtained (see SM for details). Specifically, one has

\[
v = \frac{bc \Delta_0}{\sqrt{(\alpha_E)^2 c^2 + (b \Delta_0)^2}}, 
\]

(21)

where \( \Delta_0 = \sqrt{A/H_{\text{an}}} \) is the static DW width, \( c = \sqrt{AH_E} \) is the magnon velocity, and the effective damping is \( \alpha_E = \alpha_n + \alpha_m H_{\text{an}}/(3H_E) \). Recent first-principles calculations show that \( \alpha_n \) is one to three orders of magnitude larger than \( \alpha_m \) for Mn-based metallic AFMs [29,30]. Thus, the \( \alpha_n \)-term, or equivalently the resulting \( \alpha_m \)-term, greatly enhances the effective damping \( \alpha_E \) and slows down DW’s motion for the case of \( H_{\text{an}} \geq 10^{-3}H_E \). This effect may be observed by varying the magnetic anisotropy energy of the AFM materials via tuning the strain [37].

Conclusions. – In conclusion, a proper set of dynamic equations for AFMs is derived from the Lagrange equation with the Rayleigh dissipation functional. Our phenomenological theory unifies all the existing AFM dynamic equations in the literature and proposes a universal way to construct the order parameters of antiferromagnetic systems with multiple sublattices. We discover a new anti-damping–like torque that significantly influences the magnon lifetime and DW velocity. This new torque naturally explains the findings that the damping coefficient associated with the motion of net magnetization is much larger than that associated with the motion of Neél order by recent first-principles calculations. The essential physics can be easily generalized to multi-sublattice ferrimagnet/ferromagnet by properly defining the order parameters.

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