Monthly sunspot number time series analysis and its modeling through autoregressive artificial neural network

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Abstract. This study reports a statistical analysis of monthly sunspot number time series and observes nonhomogeneity and asymmetry within it. Using the Mann-Kendall test a linear trend is revealed. After identifying stationarity within the time series we generate autoregressive AR\((p)\) and autoregressive moving average (ARMA\((p,q)\)). Based on the minimization of AIC we find 3 and 1 as the best values for \(p\) and \(q\), respectively. In the next phase, autoregressive neural network (AR-NN(3)) is generated by training a generalized feedforward neural network (GFNN). Assessing the model performances by means of Willmott’s index of second order and the coefficient of determination, the performance of AR-NN(3) is identified to be better than AR(3) and ARMA(3,1).

1 Introduction

Sunspots are generated by strong magnetic fields that are created in the interior of the Sun. The sunspots are basically visible surfaces within the Sun that are pock-marked with black flecks. Of all solar features, sunspots are the most easily observed and their intermittent appearance has numerous impacts on Earth. Sunspots may appear as single, isolated umbra (the dark central region) surrounded by a symmetric penumbra (a less dark pattern surrounding umbra) or they may appear in groups. The process of creation of sunspots due to the solar magnetic fields is thoroughly discussed in ref. [1]. It has been pointed out long back that the records of sunspot appearances indicate a strong tendency of having an 11-year cycle. The origin of the sunspot cycle has been an area of research since long. Central to the occurrence of the 11-year cycle is the oscillating magnetic dynamo within the Sun. Sunspot cycles are observed to vary both in size and length; therefore, it is difficult to describe the shape of sunspot cycles with a universal function [2]. Plethora of literatures is available where the authors have attempted to describe the cycles as a periodic phenomenon (e.g., refs. [3–8]). Understanding the solar activity cycle remains a key unsolved problem in solar physics (along with, e.g., heating of the solar corona and solar flares). It is not only an outstanding theoretical problem, but also an important practical issue, since the solar activity and the radiation output related to it influences the biosphere, space weather and technology on the Earth [9,10]. In the past, it has been postulated that the irregular dynamics of the solar cycle may embed a low-order chaotic process [11] which, if true, implies that the future behaviour of solar activity should be predictable [12]. A new concept of the solar cycle as a low-dimensional chaotic system was introduced in ref. [13], and since the early 1990s, many authors have considered solar activity as an example of low-dimensional deterministic chaos, described by strange attractor (e.g., ref. [14] and references therein). A dynamic system approach to the prediction of solar cycle was adopted by Orfila et al. [12].

The sunspot number (SN) is the most commonly predicted solar activity index. The rate of solar flares and amount of energy they release are well correlated with the sunspot number, as is the rate of coronal mass ejections. Cosmic rays, whose flux is anticorrelated with the solar cycle, are a significant source of radiation hazard in space. Geomagnetic activity has one component that is proportional to SN and another, which can be a source of significant space weather that resembles the sunspot number but shifted several years forward [15]. A categorized prediction of the solar cycle has been thoroughly reviewed in ref. [15]. The impact of the solar cycle on climate variability has been discussed in ref. [16], which shows from a global climate model including an interactive parameterization of stratospheric chemistry how upper stratospheric ozone changes may amplify the observed 11-year solar cycle irradiance changes to affect climate.

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The effect of the solar cycle on climate has also been reviewed in [17–19]. Various statistical methods of predicting the solar cycle have been reviewed in [20]. Petrovay [21] presented an exhaustive review work on solar cycle prediction. In the review, Petrovay categorized the prediction methods form three main groups, namely, precursor methods, extrapolation methods and model-based predictions. The attractor analysis and the phase space reconstruction methods have been reviewed in ref. [21] and this includes the artificial neural network (ANN) as a nonparametric fitting to the sunspot number time series. Detailed theoretical accounts of ANN are presented in the ref. [22]. Inspired by biological systems, particularly by research into the human brain, ANNs are able to learn from and generalize from experience [23]. An extensive discussion on the suitability of ANN in complex forecasting problems is available in a review by Zhang et al. [23]. Silverman and Dracup [24] summarized the advantages of ANN over conventional statistical methods:

- an a priori knowledge of the underlying process is not required;
- the existing complex relationships among the various aspects of the process under investigation need not be recognized;
- constraints and a priori solution structures are neither assumed nor enforced.

Since Yule [25] first proposed a technique called “autoregressive method”, a great variety of techniques have been proposed to predict the magnitude of sunspot activities (e.g., refs. [6] and [26]). ANN methods, till date, have been used by some authors to predict the sunspot activities [26–29]. Various numerical prediction techniques have been used for the sunspot number time series, e.g., curve fitting, artificial intelligence, neural networks and so on [8].

The present paper is aimed at predicting the monthly sunspot number (SN) in an univariate manner. The paper is organized as follows: First, we have examined the time series for the presence of any homogeneity and linear trend by means of Pettitt’s test and the Mann-Kendall trend analysis, respectively. Then, the monthly SN time series has been examined for symmetry by means of the Kolmogorov-Smirnov test. The tests stated above have been aimed at putting some light into the intrinsic behaviour of the monthly SN time series. Subsequently, the time series have been investigated to look for the presence of any nonstationarity by means of the fitting autoregressive model, and a comparative study by means of the minimization Akaike Information Criterion (AIC) has been carried out among the usual autoregressive AR(p) and autoregressive moving average (ARMA(p, q)) methods. The order of autoregression has been identified in this manner and, based on this autoregressive neural network (AR-NN), has been trained to investigate its suitability in predicting monthly sunspot numbers.

2 Methodology

Methodology of this work consists of

- Pettitt’s test for homogeneity;
- Mann-Kendall trend analysis;
- Kolmogorov-Smirnov test for normal distribution;
- autoregressive modeling;
- development of ANN;
- skill assessment of the prediction model.

2.1 Pettitt’s test for homogeneity

Pettitt’s test [30–34] is used to investigate the homogeneity of a long time series. In the geophysical time series analysis, this technique has been adopted by several authors. The test statistic $X_k$ is defined by [34]

$$X_k = 2R_k - k(n + 1),$$

$$R_k = \sum_{i=1}^{k} r_i,$$

where $r_i$ is the rank of the $i$-th element in the complete series of $n$ elements. If shifts are absent in the series, i.e. under the null hypothesis of randomness, the expectation value of $X_k$ is 0.

2.2 Mann-Kendall test for trend

The Mann-Kendall (MK) test is a very popular tool for identifying the existence of increasing or decreasing trend within a time series. The MK test is the rank-based nonparametric test for assessing the significance of a trend, and has been widely used in climatological trend detection studies. Examples include the studies in [35–38]. The null hypothesis $H_0$ is that a sample of data $\{Y_t : t = 1, 2, \ldots, n\}$ is independent and identically distributed. The alternative hypothesis, $H_1$, is that a monotonic trend exists in $\{Y_t\}$. Each pair of observed values $(y_i, y_j)$, where $i > j$ is inspected
to find out $Y_i > Y_j$ (first type) or $Y_i < Y_j$ (second type). There is a correction for the case $Y_i = Y_j$. If the numbers of the first- and second-type observations are $P$ and $M$, respectively, then the statistic $S$ is defined as $S = P - M$. A standard normal variate $Z$ is now constructed following ref. [39]. In a two-sided test for the trend, the null hypothesis of no trend is rejected if $|Z| > Z_{\alpha/2}$, where $\alpha$ is the significance level.

### 2.3 Kolmogorov-Smirnov test for normal distribution

The Kolmogorov-Smirnov (KS) statistic provides a means for testing whether a set of observations are from some completely specified continuous distribution. The KS test has two major advantages over the chi-square test that is

1. It can be used with small sample sizes, where the validity of the chi-square test would be questionable.
2. Often it appears to be a more powerful test than the chi-square test for any sample size.

The KS test for normal distribution has been discussed thoroughly in [40]. The KS test has been carried out for climatological studies by several authors. Examples include [41–43].

The KS statistic, given by $D = \max\{|F(x_{(i)}) - \hat{F}(\hat{\theta})|, i = 1, \ldots, n\}$, is based on the largest vertical difference between the empirical distribution function $F_n(x_{(i)})$ and the hypothesized distribution function $F(x_{(i)}, \hat{\theta})$. The computational formula is given by

$$
D^+ = \max_i \{i/n - Z_{(i)}\},
D^- = \max_i \{Z_{(i)} - (i - 1)/n\},
D = \max(D^+, D^-).
$$

Here, $Z_{(i)}$ represents an ordered dataset with $F(x_{(i)}, \hat{\theta})$. In this statistical test, the null hypothesis is that the observed data are drawn from the chosen theoretical distribution. If the value of the KS statistic is excessively large, then the null hypothesis is rejected. A rejection would imply that the distribution parameters are not doing an adequate job of modeling the empirical distribution of rainfall at a location. The acceptable KS value for the rejection depends on the number of points in the empirical distribution being used to test the theoretical distribution, and the rejection level chosen for the study (Husak et al., 2007 [43]).

### 2.4 Autoregressive modelling

Autoregressive models have long been used in astronomical data analysis. Vio et al. [44] reviewed various potentialities and limitations of the astronomical time series data analysis where the autoregressive model is discussed as a discrete approach. Buffa and Porceddu [45] developed an autoregressive neural network for temperature forecast and dome seeing minimization. In a more recent work, Brajˇsa et al. [9] described the autoregressive moving average (ARMA) for solar cycle predictions and reconstructions. The set of adjustable parameters $\phi_1, \phi_2, \ldots, \phi_p$ of an autoregressive process of order $p$, i.e. AR($p$) process [46],

$$
\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \ldots + \phi_p \tilde{z}_{t-p} + \epsilon_t,
$$

satisfies certain conditions for the process to be stationary. Here, $\bar{z}_t = z_t - \mu$. The parameter $\phi_1$ of an AR(1) process must satisfy the condition $|\phi_1| < 1$ for the time series to be stationary. It can be shown that the autocorrelation function satisfies the equation

$$
r_k = \phi_1 r_{k-1} + \phi_2 r_{k-2} + \ldots + \phi_p r_{k-p}.
$$

Substituting $k = 1, 2, \ldots, p$ in the above equation we get the system of Yule-Walker equations [46]

$$
\rho_1 = \phi_1 + \phi_2 \rho_1 + \ldots + \phi_p \rho_{p-1},
\rho_2 = \phi_1 \rho_1 + \phi_2 + \ldots + \phi_p \rho_{p-2},
\vdots
\rho_p = \phi_1 \rho_{p-1} + \phi_2 \rho_{p-2} + \ldots + \phi_p.
$$

The Yule-Walker estimates of the autoregressive parameters $\phi_1, \phi_2, \ldots, \phi_p$ are obtained by replacing the theoretical autocorrelation $\rho_k$ by the estimated autocorrelation $r_k$. Thus, the matrix notation, the autoregression parameters can be written as

$$
\Phi = R^{-1} r.
$$

The $p$-th–order autoregressive process may be written as

$$
\phi(B) \tilde{z}_t = e_t,
$$

where $B$ is the backshift operator and $\phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p$ is the characteristic polynomial of the process.
where, $e_t$ follows the $q$-th–order moving average process

$$e_t = \theta(B)a_t.$$ 

Now, an ARMA($p, q$) process is presented as

$$\phi(B)\tilde{z}_t = \theta(B)a_t,$$

where $\phi(B)$ and $\theta(B)$ are polynomials of degree $p$ and $q$, respectively, and $B$ is the backward shift operator. The ARMA process is stationary if the roots of $\phi(B) = 0$ lie outside the unit circle and it exhibits explosive nonstationary behaviour if they lie inside the unit circle.

### 2.5 Development of the artificial neural network (ANN)

The method of choosing the predictors in the univariate problem under consideration shall be based on the outcomes of the experiments based on the methodologies explained above. A generalized feed-forward neural network (GFNN) shall be adopted with two hidden layers. The number of nodes in both of the hidden layers is 4. In both of the hidden layers the sigmoid nonlinearity is used. It should be stated that before applying the ANN methodology we have removed the seasonal and trend components from the data. Trend and seasonality removal processes are referred to as pre-whitening methods [47]. The training and test dataset ration is 3:1. The GFNNs are a generalization of the conventional multilayer perceptron such that connections can jump over one or more layers [48, 49]. Theoretical and mathematical details are available in [22]. The GFNN model is trained in batch mode up to 3000 epochs and run thrice. Minimization of the mean squared error is considered as the stopping criterion. The implementation details are described in the subsequent section.

### 3 Discussion

In this work, the time series of monthly SN for the years 1992–2008 have been analyzed for various statistical aspects. First of all, homogeneity within the data set has been investigated through Pettitt’s test, where the null hypothesis has been assumed that the data are homogeneous. The two tailed $p$-value computed using Monte Carlo simulations show that the $p$-value is less than 0.0001. Since the computed $p$-value is found to be lower than the significance level $\alpha(= 0.05)$ the null hypothesis is rejected and hence it may be concluded that the data are not homogeneous. After identification of inhomogeneity within the data set, we examine the data set for symmetry and it is found that the time series of monthly SN is positively skewed. For further confirmation of asymmetry we carry out the Kolmogorov-Smirnov test to examine a null hypothesis that assumes that the data are following a normal distribution. However, being the computed $p$-value 0.005 ($<0.05$), it is concluded that the time series under consideration does not follow any normal distribution.

Subsequently, we apply the Box-Cox transformation for smoothing of the time series. The $\lambda$ parameter for this transformation is optimizing to 0.345. The data series after the Box-Cox transformation is presented in the second panel of fig. 1 which shows the reduced variability within the time series plotted in the first panel of fig. 1.
In the following step, we explore the transformed time series for the existence of any linear trend within it for this purpose. We carry out the MK test (two-tailed) based on the null hypothesis of no trend within the time series. Kendall $\tau$ is found to be $-0.205$ with $S = -4169$. The two-tailed $p$-value being found to be less than 0.05, the null hypothesis is rejected against the alternative hypothesis assuming the existence of linear trend. Hence, it is concluded that the time series is characterized by a linear trend within it. Therefore, the statistical analysis carried out so far identifies the following:

- The monthly SN time series is inhomogeneous
- The time series is not symmetric and positively skewed
- There is significantly variability within the time series
- The time series is characterized by linear trend.

In the next step, it is necessary to identify a representative statistical model for said inhomogeneous time series, as we are working in a univariate environment, an autoregressive approach may be adopted to generate a representative statistical model for this time series. An AR equation is basically a linear nonhomogeneous recurrence relation. For the present problem a fitted AR(1) model is given by $X_{t+1} = 0.931X_t + 54.449$, whose characteristics root lies within the unit circle. Hence, the sunspot number time series under consideration is stationary. Subsequently we carried out AR models for higher orders and based on the minimization of the AIC, the AR(3) is considered to be better than other orders. To further improve the model we adopt ARMA models with autoregression order 3 and variable orders of moving averages. The minimization of AIC indicates ARMA(3,1) to be better than others, that is, increasing the order of the moving average is not increasing the prediction performance of the model. The AICs are presented as a bar diagram in fig. 1.

In the next step, our target is to examine whether the prediction performances of AR(3) and ARMA(3,1) may be enhanced by means of AR-NN(3), i.e. autoregressive neural network with three predictors. The predictors are $X_{t-3}, X_{t-2}, X_{t-1}$ for the predictand $X_t$, where the suffix indicates the time of observation. Hence, the previous three consecutive values of the same SN time series are being used to predict the current value of the same time series by means of neural network. That is why it is being dubbed as “autoregressive neural network” with three predictors and being abbreviated as AR-NN(3). The entire dataset consisting of 204 monthly SN data, the input matrix is of order $(201 \times 3)$ and the output matrix is of order $(201 \times 1)$. The entire dataset is divided into training and test cases in the ratio 3:1. Hence, the number of test cases is 51. The model is tested over these 51 test cases.

To assess the model performance we consider Willmott’s index advocated by Willmott [50] as an index to measure the degree of agreement between actual and predicted values. This is given as

$$d^2 = 1 - \left[ \sum_i |P_i - O_i|^\alpha \right] \left[ \sum_i (|P_i - \bar{O}| + |O_i - \bar{O}|) \right]^{-1}.$$ 

![Fig. 2. The values of Willmott’s index.](image)
Fig. 3. Schematic showing the scatterplots for different models along with the linear trend equation and coefficient of determination $R^2$.

Here, $P$ implies the predicted value and $O$ implies the observed value for the $i$-th data point. For good predictive models, $d^2$ is close to 1. It is observed that the AR(3) and ARMA(3,1) are producing values far away from 1. However, AR-NN(3) is producing $d^2$ above 0.8. This indicates that the AR-NN(3) is significantly better than the other two models in its ability to predict the monthly SN time series. The values of $d^2$ are presented in fig. 2. To further visualize the prediction performance, we produce scatterplots in fig. 3 for three of the univariate models under consideration. From the scatterplots it is found that majority of the points are clustered around the linear trend line for AR-NN(3). Whereas, for the other two models, the points are deviated far away from the linear trend line. Furthermore, the AR-NN(3) produces highest value of the coefficient of determination ($> 0.6$). Finally, the supremacy of AR-NN(3) over AR(3) and ARMA(3,1) is established. For a visual presentation of the performance of the AR-NN(3) we present a line diagram (fig. 4) exhibiting the observed SN for the test cases as well as those predicted by AR-NN(3).
4 Concluding remarks

In this work we have reported a statistical analysis of monthly sunspot number time series and observed nonhomogeneity and asymmetry within it. Also, based on the Mann-Kendall test a linear trend has been identified. Subsequently, predictive models have been generated in an univariate manner. After identifying stationarity within the time series we have generated AR(p) and ARMA(p,q). Based on the minimization of AIC, the best values of p and q have been found to be 3 and 1, respectively. In the next phase, AR-NN(3) has been generated by training a generalized feedforward neural network (GFNN) with two hidden layers and sigmoid nonlinearity. The learning procedure is adopted as Levenberg-Marquardt. Assessing the model performances by means of Willmott’s index of second order and coefficient of determination, the performance of AR-NN(3) is identified to be better than AR(3) and ARMA(3,1). This study, therefore, establishes the potential of the artificial neural network in modeling monthly sunspot number time series and hence identified as a powerful methodology to study the solar activity in a statistical manner.

As a future work, we propose to extend the present study to a larger period and to experiment with other neural network methodologies to generate stronger representative models for sunspot numbers.

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