BATSE GRB Location Errors

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Abstract. We characterize the error distribution of BATSE GRB locations by modeling the distribution of separations between BATSE locations and IPN annuli. We determine error model parameters by maximizing likelihood and rank the models by their Bayesian odds ratios. The best models have several systematic error terms. The simplest good model has a 1.9 degree systematic error with probability 73% and 5.4 degrees with probability 27%. A more complex model adds a dependence on the datatype used to derive the location.

ANALYSIS METHOD

The purpose of this paper is to develop better models of the error distribution of burst locations in the 3B [5] and 4B [6] catalogs, all of which were obtained with the same algorithm. A more accurate error distribution will aid counterpart searches and improve the accuracy of limits on burst repetition and clustering.

Measuring the actual location errors requires a comparison dataset of sufficient accuracy and size—the most suitable set is the locations of the Interplanetary Network (IPN) [4]. The advantages of this dataset are its large size and small errors. The disadvantages are that most of the error boxes are single annuli and that the sample is biased towards bright GRBs.

After removing 11 wide annuli (3σ error > 0.8°), the IPN supplement [4] to the 4B catalog consists of 442 GRBs. Of these, 30 have intersecting annuli that yield essentially ‘point’ locations and thus provide measurements of the angles γ between the true locations and the BATSE locations. For the remaining events with only single annuli we can determine only the closest approach angles ρ between the annuli and the corresponding BATSE locations.

Our method of analyzing this set of γ and ρ measurements is similar to that of Graziani & Lamb [3]. We assume various models for the systematic error σ_{sys} and obtain the total error σ_{tot} as the rms sum of the statistical error σ_{stat} listed in the catalogs and σ_{sys}. For the probability distribution P(γ) we assume the Fisher distribution, which is generally considered to be the equivalent of the Gaussian
distribution on the sphere [2]. For those GRBs with single annuli for which we know \( \rho \) but not \( \gamma \), we derive the exact distribution \( P(\rho) \) from \( P(\gamma) \) [1].

The model parameters are obtained by maximizing the likelihood, which is the product of the probability, according to the model, of the observations:

\[
L = \prod_i P(\gamma_i) \prod_j P(\rho_j),
\]

Once the likelihood is optimized for each model, the models are compared by their Bayesian odds ratio [8]:

\[
O_{B/A} = \frac{P(B)}{P(A)} = \frac{P_{\text{prior}}(B) \times L(B) \times \text{Occam}(B)}{P_{\text{prior}}(A) \times L(A) \times \text{Occam}(A)}
\]

We set the first factor, representing our prior preferences for the models, to unity. The second factor, the likelihood ratio, indicates how strongly the data favor one model over another. The “Occam’s factor” penalizes a model to the extent that the better fit is obtained by additional parameters. For the data and models presented below, the factor per parameter by which a model is penalized ranges from about 5 to 20 so that the dominant factors in distinguishing between the models are the likelihood ratios.

**RESULTS**

We first try the minimal model (Model 0), which has \( \sigma_{\text{sys}} = 1.6^\circ \) for all GRBs. The value 1.6\(^\circ\) was obtained using 50 BATSE locations and the corresponding WATCH, COMPTEL and IPN locations as the rms difference between the actual separations \( \gamma \) and the errors estimated from the errors of the other instrument and the BATSE statistical errors [5,7]. Figure 1 shows that the agreement of the minimal model with the data is very poor. Model 1 generalizes Model 0 by making \( \sigma_{\text{sys}} \) a free parameter with the same value for all GRBs. While a vast improvement over Model 0 (see Table 1), the agreement between Model 1 and the data is still quite poor (histogram not shown).

While the minimal model is the previously published quantitative model, the poor agreement was expected by the BATSE Team, e.g., the 3B catalog [5] notes “However, the various tests yield only an estimate of the average systematic error. A small fraction of the locations could be substantially worse than the average, i.e., the location error distribution may have a non-Gaussian tail.” We now have enough accurate comparison locations to test models with additional parameters.

Model 2 uses the sum of two Fisher distributions with differing values of \( \sigma_{\text{sys}} \) to implement a Gaussian distribution with an extended tail. It is a very major improvement over the simpler models (Table 1). Model 2 is an uncorrelated model—it does not use any datum to determine the value of \( \sigma_{\text{sys}} \) for a location and is best thought of as a Gaussian with wings rather than a two-component model because BATSE data does not assign a burst to a component.
FIGURE 1. Histograms of the data and models. Left: 30 events for which two annuli are available. Right: 412 events for which only single annuli are available. The data and both models are binned in units of $\sigma_{\text{tot,0}}$, as calculated with the minimal model (Model 0) of $\sigma_{\text{sys}} = 1.6^\circ$.

The solid lines show the data, the dotted lines the minimal model, and the dashed lines show Model 12. The histogram for Model 2 (not shown) is very similar to the histogram of Model 12.

| Model | Description           | Log$_{10}$ Likelihood | Parameter Values | Odds Ratios     |
|-------|----------------------|------------------------|------------------|-----------------|
| 0     | Minimal              | 140.8                  | $\sigma_{\text{sys}} = 1.6^\circ$ fixed |                 |
| 1     | One $\sigma_{\text{sys}}$ | 215.0                  | $\sigma_{\text{sys}} = 3.07^\circ \pm 0.12^\circ$ | $O_{1/0} \approx 8 \times 10^{72}$ |
| 4     | $\sigma_{\text{sys}} = A(\sigma_{\text{stat}}/1^\circ)^\alpha$ | 217.4                  | A = $3.22^\circ \pm 0.13^\circ$ \[\alpha = 0.18 \pm 0.05\] | $O_{4/1} \approx 25$ $O_{4/0} \approx 2 \times 10^{74}$ |
| 2     | Core plus Tail       | 226.7                  | $f_1 = 0.73 \pm 0.08$ | $O_{2/4} \approx 6 \times 10^6$ $O_{2/1} \approx 2 \times 10^{10}$ $O_{2/0} \approx 1 \times 10^{83}$ |
| 12    | Datatype Dependence  | 229.9                  | $f_{\text{CONT}}^1 = 0.76 \pm 0.07$ \[\sigma_{\text{CONT}}^1 = 1.6^\circ \pm 0.2^\circ$, $\sigma_{\text{CONT}}^2 = 5.4^\circ \pm 0.8^\circ$, $\sigma_{\text{other}} = 3.7^\circ \pm 0.3^\circ$] | $O_{12/2} \approx 100$ $O_{12/4} \approx 6 \times 10^{10}$ $O_{12/1} \approx 2 \times 10^{12}$ $O_{12/0} \approx 1 \times 10^{85}$ |
So far, the only significant correlation we have found between location errors and other location properties is with the datatype used to locate the event. Fitting Model 1, one value of $\sigma_{sys}$ to locations obtained with the 16-channel CONT datatype, the best fit value of $\sigma_{sys}$ is $2.81^\circ \pm 0.14^\circ$, while for the other datatypes, the best value is $3.69^\circ \pm 0.29^\circ$, clearly showing that locations obtained using 16-channel data are more accurate than those obtained with 4- or 1-channel data.

Based upon this correlation, Model 12 was constructed: events located with CONT data have a core-plus-tail distribution similar to that of Model 2, while events located using other datatypes have a single value for $\sigma_{sys}$ (Table 1). The data and Model 12 agree well (Fig. 1). The evidence for an extended tail for the “other” locations is marginal (odds ratio of 4). So far extensions of Model 12, such as adding intensity dependences, have at best marginal odds ratio improvements ($\leq 10$) and frequently unconvincing parameter values.

We find only weak evidence for a direct dependence of $\sigma_{sys}$ on the intensity of a burst, as measured either by $\sigma_{stat}$ or by fluence. Model 4, with $\sigma_{sys}$ a power law of $\sigma_{stat}$ [3], is a definite improvement over Model 1. However, an uncorrelated model is yet superior, indicating that the intensity correlation of Model 4 is not supported by the current data. Additionally, a histogram (not shown) shows Model 4 to be a poor fit to the data. Consequently, the extrapolation of Model 4 to faint bursts (e.g., $\sigma_{sys} = 8^\circ$ for $\sigma_{stat} = 10^\circ$) [3], and the consequent conclusion that the 3B catalog has larger systematic errors for faint bursts than the 1B catalog [3] are not justified.

While we do not find an intrinsic correlation of $\sigma_{sys}$ with burst intensity, the fraction of events located with CONT data decreases with decreasing intensity, thereby causing an indirect intensity correlation via the datatype correlation. The fraction of CONT locations falls from 70% for events with $\sigma_{stat} < 3.7^\circ$ to 40% for the remaining events. However, this indirect correlation does not cause $\sigma_{sys}$ to scale with intensity, instead it causes the fraction of events with $\sigma_{sys} = 3.7^\circ$ to be higher for sets of fainter bursts.

While the odds ratio favors Model 12 over Model 2, Model 2 is probably sufficient for most purposes. Here we give a detailed specification of Model 2 so that others can implement these two models.

Model 2 consists of two terms:

$$P = f_1 P_1 + (1 - f_1) P_2$$

(3)

Each term is a Fisher probability distribution so that the probability of the true location lying in a ring of $\gamma_1$ to $\gamma_2$ about the BATSE location is:

$$P_i = \frac{\kappa_i}{2\pi (e^{\kappa_i} - e^{-\kappa_i})} \int_{\Omega} d\Omega \ e^{\kappa_i \cos \gamma} = \frac{\kappa_i}{e^{\kappa_i} - e^{-\kappa_i}} \int_{\gamma_1}^{\gamma_2} d\gamma \sin \gamma \ e^{\kappa_i \cos \gamma}$$

(4)

By definition, the radius $\sigma_{tot,i}$ contains 68% of the probability. Setting $P_1 = 0.68$, $\gamma_1 = 0$, and $\gamma_2 = \sigma_{tot,i}$, we obtain (with $\sigma_{tot,i}$ in radians):
For each burst, the values of the total location errors $\sigma_{\text{tot},i}$ are calculated from the statistical error $\sigma_{\text{stat}}$ listed in the catalog and the model errors $\sigma_{\text{sys},i}$:

$$\sigma_{\text{tot},i} = [\sigma_{\text{stat}}^2 + \sigma_{\text{sys},i}^2]^{0.5}$$

**CONCLUSIONS**

The best models are 2 and 12, which are implementations of the BATSE Team’s expectation that the location error distribution is Gaussian modified with an extended tail. Model 12 incorporates the only significant correlation that we have identified, basing the error distribution on the datatype used to derive the location.

While the fraction of locations belonging to the larger systematic error term is 27% according to Model 2, this does not mean that 27% of the locations have large errors—the large systematic error term has a significant probability of a small location error. For events with $\sigma_{\text{stat}} \ll \sigma_{\text{sys},1}$, the fraction of locations with an error past $2\sigma_{\text{sys},1} = 3.8^\circ$ is 16%.

The results herein are upper-limits on BATSE systematic errors since they are based on the assumption that there are no errors in the comparison dataset. As part of this effort, most of the outliers were re-examined and eight significant revisions were made to the preliminary IPN catalog. Even a single outlier past $4\sigma$ can significantly effect the error model parameter values.

We continue to test additional models. Post-4B improvements in the location algorithm will lead to revised error model parameters.

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