Fire spread estimation on forest wildfire using ensemble kalman filter

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Abstract. Wildfire is one of the most frequent disasters in the world, for example forest wildfire, causing population of forest decrease. Forest wildfire, whether naturally occurring or prescribed, are potential risks for ecosystems and human settlements. These risks can be managed by monitoring the weather, prescribing fires to limit available fuel, and creating firebreaks. With computer simulations we can predict and explore how fires may spread. The model of fire spread on forest wildfire was established to determine the fire properties. The fire spread model is prepared based on the equation of the diffusion reaction model. There are many methods to estimate the spread of fire. The Kalman Filter Ensemble Method is a modified estimation method of the Kalman Filter algorithm that can be used to estimate linear and non-linear system models. In this research will apply Ensemble Kalman Filter (EnKF) method to estimate the spread of fire on forest wildfire. Before applying the EnKF method, the fire spread model will be discreted using finite difference method. At the end, the analysis obtained illustrated by numerical simulation using software. The simulation results show that the Ensemble Kalman Filter method is closer to the system model when the ensemble value is greater, while the covariance value of the system model and the smaller the measurement.

1. Introduction
Fires are one of the most common disasters in the world, for example in densely populated and forested housing. This has resulted in declining forest populations worldwide, especially in Indonesia. Fires not only cause material losses but also cause loss of life and injuries that can not be assessed how great their losses are when compared to material losses[1].

Recently forest fires have become an international concern as environmental and economic issues, particularly after the ENSO 1997/98 disaster that devastated 25 million hectares of forest land worldwide[2]. Fires are considered a potential threat to sustainable development because of their direct effect on the ecosystem, the contribution of carbon emissions and their impact on biodiversity. According to the government, 2.6 million hectares of land and forests have been burning between June and October 2015, equivalent to four and a half times the size of Bali Island. As of October 2015, each of the eight provinces recorded fires with an area exceeding 100,000 hectares[3].

The development of science, especially in the field of mathematics provides an important role in overcoming the spread of fire. Many mathematical models were created to analyze the nature of the fire so as to investigate the spreading area of fire. In research Renggana discussed mathematical model of spreading fire between laboratory scale buildings. Experiments conducted using the scale down model so that the cost and the level of danger can be reduced. This study mambahas value of heat flux
and temperature as the critical value of the spread of fire at a predetermined distance from the results of a survey conducted in 2008.

Andrianus Radipta examines the modeling of fire wildfire distribution. This research designs a Wireless Sensor Network (WSN) system to monitor the occurrence of forest fires wirelessly. The fire spreading model used follows the model of fire spray generated by burners based on the classification of burners in the forest. This study also discusses the calculation of rate of spread and flame length of forest fire causes[4].

In this research writer use fire spread model which have been made by previous researcher by using data assimilation technique. The method applied to this mathematical model uses the Ensemble Kalman Filter method to estimate the spread of fire. Next at the end will be done simulation using software.

2. Reaction-diffusion equation model

The equation of the diffusion reaction is that equation mathematics describing one or more distributed substances in space is altered by the influence of two processes of local chemical reactions in which the substance is converted to another and the diffusion that causes the substance to spread in space. The equation can be represented in a general form[5]:

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + f(T)$$

where k is the coefficient associated with the diffusion properties of T, and f(T) is a function that describes how T increases or decreases depending on the current concentration. A reaction often refers to the interaction between two or more objects. Consider a model consisting of a layer of fuel with concentration \( F(x,y,t) > 0 \text{ (kg/m}^3) \), \( k \text{ (m}^3 \text{ s}^{-1}) \) is the thermal diffusivity, and temperature \( T(x,y,t) \) where x and y are horizontal coordinates (m) on the Earth’s surface and \( t \text{ (s)} \) is the time. The fuel is assumed to burn at the relative rate \( r(T)/I/s \), dependent only on the temperature. In the diffusion reaction model the fire spread in forest fires uses the following model[6]:

$$c \frac{\partial T}{\partial t} = \nabla (k \nabla T) - c(\hat{v} + \gamma \nabla \hat{z}) \cdot \nabla T + f(T)$$

$$= \nabla (k \nabla T) - c(\hat{v} + \gamma \nabla \hat{z}) \cdot \nabla T + \mathcal{A} F \mathcal{D} e^{-\Theta/(T - T_a)} - \hat{C}(T - T_a)$$

(1)

\( \mathcal{A}(/kg) \) is the temperature rise per second at maximum burn rate with full initial fuel load no cooling, \( c (l/m^2 K) \) is the 2D thermal capacity of the surface layer at constant pressure, \( \gamma \nabla \hat{z} \) is the correction for the gradient of the height of the area \( \hat{z}, \mathcal{D}(K) \) and \( \mathcal{B}(K) \) are the coefficient of proportionality in the modified Arrhenius law, \( \hat{C}(W/m^2 K) \) is the scale coefficient of heat transfer to the environment, \( T_a(K) \) is the ambient temperature, \( \hat{v}(m/s) \) is wind speed, \( V= [(\partial/\partial x), (\partial/\partial y)] \) is a gradient, \( c \frac{\partial T}{\partial t} \) is a heat flow model that is absorbed in the fuel and changes the temperature of the fuel layer, \( \nabla (k \nabla T) \) is a diffusion model of heat transfer by short-range radiation and air mixture which causes the fire to spread between adjacent fuel particles such as branches and tree roots, \( \nabla T \) is the heat flow due to wind, \( T(T)(W/m^2 s) \) is an external heat flow comprising heat flow \( \mathcal{A} F \mathcal{D} e^{-\Theta/(T - T_a)} \) which is generated by burning fuel, reduced by heat flow \( \hat{C}(T - T_a) \) which passes into the atmosphere at ambient temperature \( T_a \) according to Newton’s law of cooling which says that the rate of heat loss of a body is directly proportional to the difference in the temperatures between the body and its surroundings provided the temperature difference is small and the nature of radiating surface remains same.

Equation (1) denotes the spread of combustion waves, at least for coefficients in a given range. At the beginning edge of the combustion wave, the term diffusion diffuses heat to the unfitted fuel and rises in temperature. The heat generated by the reaction causes the temperature to rise rapidly and fuel fused. The fuel then burns out and cools, creating a line of fire propagating on the combustion wave. As a result, combustion waves move on unburned fuels and leave burning fuel for quantities that are no longer enough to sustain the reaction.
However, taking the coefficients from the fuel properties and expecting to get the correct wave speed of combustion is not easy. The reason is the unknown homogeneous shape coefficient of micro data, as well as many important physical processes that are not modeled.

3. Discretization of model

Ensemble kalman filter algorithm can be implemented only for discrete system. Equation (1) is discriminated using a different method to an explicit (Forward Time Center Space) scheme because the time variable \( n + 1 \) is calculated based on the variable at time \( n \). For degrees with degrees one is discriminated using different to advance because it will estimate for one step ahead. As for the derivative with the second degree is discriminated using a different method to the center. For the three variable functions \( x, y, \) and \( t \), the Taylor series is partially derived (partial) to the variable. The model can be written as follows:

\[
c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) - c(\nabla \cdot \gamma N) \cdot \nabla T + \tilde{A} \tilde{T} \tilde{D} e^{-\tilde{B}/(T - T_a)} - \tilde{C}(T - T_a)
\]

\[= k \frac{\partial^2 T}{\partial x^2} + k \frac{\partial^2 T}{\partial y^2} - c \frac{\partial T}{\partial x} - c \gamma \frac{\partial T}{\partial y} - cy \frac{\partial^2 T}{\partial x \partial y} - cy \frac{\partial^2 T}{\partial y \partial x} + \tilde{A} \tilde{T} \tilde{D} e^{-\tilde{B}/(T - T_a)} - \tilde{C}(T - T_a)
\]

(2)

That model discretized using forward finite difference method[7]:

\[
\frac{\Delta T(x_{i+1, 1}, T_n)}{\Delta x} \approx \frac{T(x_{i+1, 1}, T_n) - T(x_{i, 1}, T_n)}{\Delta x}, \quad \frac{\Delta T(x_{i, 1}, T_n)}{\Delta y} \approx \frac{T(x_{i+1, 1}, T_n) - T(x_{i, 1}, T_n)}{\Delta y}
\]

\[
\frac{\Delta z(x_{i, 1}, T_n)}{\Delta y} \approx \frac{z(x_{i+1, 1}, T_n) - z(x_{i, 1}, T_n)}{\Delta y}
\]

and using centered finite difference method:

\[
\frac{\Delta T(x_{i, 1}, T_n)}{\Delta y} \approx \frac{T(x_{i+1, 1}, T_n) - 2T(x_{i, 1}, T_n) + T(x_{i-1, 1}, T_n)}{\Delta y^2}
\]

\[
\frac{\Delta T(x_{i, 1}, T_n)}{\Delta y} \approx \frac{T(x_{i+1, 1}, T_n) - 2T(x_{i, 1}, T_n) + T(x_{i-1, 1}, T_n)}{\Delta y^2}
\]

So the Equation (2) becomes:

\[
T_{i,j}^{n+1} = \frac{k\Delta t}{c\Delta x^2} T_{i-1,j}^n + \frac{k\Delta t}{c\Delta y^2} T_{i+1,j}^n + \left( 1 - 2 \frac{k\Delta t}{c\Delta x^2} - 2 \frac{k\Delta t}{c\Delta y^2} + \frac{\theta \Delta t}{\Delta x} + \frac{\theta \Delta t}{\Delta y} + \gamma \frac{\Delta t}{\Delta x^2} (z_{i+1,j}^n - z_{i,j}^n) + \gamma \frac{\Delta t}{\Delta y^2} (z_{i+1,j}^n - z_{i,j}^n) \right) T_{i,j}^n + \left( \frac{k\Delta t}{c\Delta x^2} - \frac{\theta \Delta t}{\Delta x} - \gamma \frac{\Delta t}{\Delta x^2} (z_{i+1,j}^n - z_{i,j}^n) \right) T_{i,j}^n
\]

(3)

for example \( \tilde{A} = 1 - 2 \frac{k\Delta t}{c\Delta x^2} - 2 \frac{k\Delta t}{c\Delta y^2} + \frac{\theta \Delta t}{\Delta x} + \frac{\theta \Delta t}{\Delta y} + \gamma \frac{\Delta t}{\Delta x^2} (\tilde{z}) + \gamma \frac{\Delta t}{\Delta y^2} (\tilde{z}), \quad \tilde{B} = \frac{k\Delta t}{c\Delta y^2} - \frac{\theta \Delta t}{\Delta y} - \gamma \frac{\Delta t}{\Delta y^2} (\tilde{z}), \quad \tilde{C} = \frac{k\Delta t}{c\Delta x^2} - \frac{\theta \Delta t}{\Delta x} - \frac{\theta \Delta t}{\Delta x} (\tilde{z}), \quad \tilde{D} = \frac{k\Delta t}{c\Delta y^2} - \frac{\theta \Delta t}{\Delta y} - \gamma \frac{\Delta t}{\Delta y^2} (\tilde{z}), \quad \tilde{E} = \frac{k\Delta t}{c\Delta x^2} - \frac{\theta \Delta t}{\Delta x} - \gamma \frac{\Delta t}{\Delta x^2} (\tilde{z}), \quad \tilde{F} = \frac{k\Delta t}{c\Delta y^2} - \frac{\theta \Delta t}{\Delta y} - \gamma \frac{\Delta t}{\Delta y^2} (\tilde{z}) \).
The model in Equations (1) used to understand the balance of heat in the fuel layer on forest wildfire. In fact, there is temperature, fuel, and amount of forest wildfire that didn’t match with the model, which was called by noise. Noise caused deterministic model turn into stochastic model. The Equation (4) can be written in the form

\[ x_{k+1} = f(x_k, u_k) + w_k \]

\[ z_k = H_k x_k + v_k \]

The system noise, \( w_k \), and noise measurements, \( v_k \), are generated through the computer and are generally taken normally distributed and have mean value zero, the system noise variation is expressed by \( Q_k \), and the measurement noise variation is expressed by \( R_k \). Both depend on time and the value assumed to be constant.

The matrix of square-shaped system noise variations corresponds to the size of the error covariance of the estimates. While the variance matrix size of the measurement noise is square-shaped in accordance with the number of rows from the observed \( H \)-vector.

The measurement vector \( H \) is determined from the state variable that is used as the measurement variable. In this case, it is assessed by the measurement variable that there are state variables \( T \) and \( R \) so that the measurement vector \( (H) \) is

\[
H = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

4. Assimilation method

In the Ensemble Kalman Filter method, the estimation process is done by three stages: initialization stage by generating a number of \( N_e \) ensembles, the time of the update, and the measurement update ensemble[8].

a) Initialization stage

Resurrected a number \( N_e \) ensemble with mean 0 and covariance \( P \) according to the initial guess \( \bar{x}_0 \) with \( N_e \) is the number of ensembles that are raised. Let's say \( l = N_e \) ensemble the resurrected is:
\[
x_{0,l} = [x_{0,1}, x_{0,2}, x_{0,3}, \ldots, x_{0,N_e-1}, x_{0,N_e}]
\]

the resulting ensemble is randomly distributed normally. Next set the initial value

\[
\hat{x}_0 = \frac{1}{N_e} \sum_{l=1}^{N_e} x_{0,l}
\]

b) Prediction stage

At the prediction stage, the estimation of the ensemble, mean and covariance estimation error is determined. Further ensemble estimation is determined as follows:

\[
\hat{x}_{n+1,l} = A\hat{x}_{n,l} +Bu_n + w_{n,l}
\]

with \( w_{n,l} \sim N(0, Q) \)

So from the estimation equation \( N_e \) The ensemble, we get the estimation equation with \( l = N_e \) ensemble and \( n \) iteration as follows:

\[
\hat{x}_{n+1,l} = [A\hat{x}_{n,l} + Bu_n + w_{n,1} \ A\hat{x}_{n,l} + Bu_n + w_{n,2} \ \ldots \ \ A\hat{x}_{n,l} + Bu_n + w_{n,N_e}]
\]

So the state of estimation becomes:

\[
\hat{x}_{n+1}^- = \frac{1}{N_e} \sum_{l=1}^{N_e} \hat{x}_{n+1,l}
\]

From the above equation obtained the average of ensemble number \( N_e \) and \( n = 1 \) as follows:

\[
\hat{x}_1^- = \frac{1}{N_e} \sum_{l=1}^{N_e} \hat{x}_{1,l}
\]

with the following error covariance:

\[
P_1^- = \frac{1}{N_e - 1} \sum_{l=1}^{N_e} (\hat{x}_{1,l}^- - \hat{x}_1^-)(\hat{x}_{1,l}^- - \hat{x}_1^-)^T
\]

Furthermore, the estimated value and the covariance error that has been obtained will be corrected for accuracy in estimating the temperature change and parameter values.

c) Correction stage

In the correction phase begins by replicating the measurement data into a sized matrix \( m \times N_e \) with \( m \) is the number of measurement variables.

\[
z_{n,l} = z_n + v_{n,l}
\]

\[
 v_{n,l} \sim N(0, R)
\]

for \( n = 1 \) obtained

\[
z_{1,l} = z_1 + v_{1,l}
\]

for \( l = 1, 2, 3, \ldots, N_e \)

Then we get the Kalman Gain matrix as follows:

\[
K_1 = P_1^- H^T (HP_1^- H^T + R)^{-1}
\]

The Kalman Gain matrix is used to determine the ensemble estimate as follows:

\[
\hat{x}_{1,l} = \hat{x}_{1,l}^- + K_1 (z_{1,l} - H\hat{x}_{1,l}^-)
\]

From the equation, the mean estimation is obtained:

\[
\hat{x}_1 = \frac{1}{N_e} \sum_{l=1}^{N_e} \hat{x}_{1,l}
\]

with the covariant error is:

\[
P_1 = [I - K_1 H]P_1^-
\]

5. Simulation of the model

In this section, the model is simulated with given noise value in state space \( (Q) \) and noise measurement in state space \( (R) \). With the parameter values used are as follows:

\[
\Delta t = 0.001 \quad \Delta x = 0.1 \quad \Delta y = 0.1
\]
The concentration of fuel is 0.1 kg/m², the thermal diffusivity is 0.2 m² s⁻¹, the temperature of fuel is 250°C, wind speed is 10 m/s[9].

Figure 1. The first simulation using the value of N is 100 and the value of Q and R are 0.1. Simulation results of system model and Ensemble Kalman Filter (a), Error value Ensemble Kalman Filter (b), Simulation results of system model in the contour form (c), and Ensemble Kalman Filter simulation results in contour form (d).
In the first simulation, figure 1 with the number of iteration times 25 at the last iteration date at point 1,1 is the hotspot with the initial temperature of 28°C then after a rise in temperature around 250°C. The high fire temperature occurs at the closest points of the point 1,1 is called point 1,2, point 1,3, point 2,1, and point 3,1. The error value is obtained from the difference between the real system and the estimated results of the Kalman Filter Ensemble Kalman Filter method. The highest error value is 0.47. The fire spread on the real system can be seen after the 25th iteration time at the starting point ie at index i,j=1,1 fire temperature between 245°C -250°C. High fire temperature at points around point 1,1, such as point 1,2 is 249.98°C, point 1,3 is 249.97°C, point 2,1 is 249.99°C, and point 3,1 is 249.98°C. The fire temperature that occurs at point 10,10 is 35.69°C. From this explanation it can be said that the fire point at point 1,1 spreads to the points to the midpoint of 10,10 at the time of the sixteenth iteration with the final temperature at the 25th iteration time of 35.69°C. The fire spread on Ensemble Kalman Filter considerations can be seen after the 25th iteration time at the starting point ie at index i,j=1,1 fire temperature ranges between 249.97°C. High fire temperature at points around point 1,1, such as point 1,2 is 249.96°C, point 1,3 is 249.95°C, point 2,1 is 249.98°C, and point 3,1 is 249.97°C. The smallest fire temperature that occurs at point 10,10 is 35.67°C. From this explanation it can be said that the fire point at point 1,1 spreads to the points to the midpoint of 10,10 at the time of the sixteenth iteration with the final temperature at the 25th iteration time of 35.67°C.
The second simulation using the value of N is 200 and the value of Q and R are 0,1. Simulation results of system model and Ensemble Kalman Filter (a), Error value Ensemble Kalman Filter (b), Simulation results of system model in the contour form (c), and Ensemble Kalman Filter simulation results in contour form (d).

In the first simulation, figure 2 with the number of iteration times 25 at the last iteration date at point 1,1 is the hotspot with the initial temperature of 28°C then after a rise in temperature around 248°C -250°C. The high fire temperature occurs at the closest points of the point 1,1 is called point 1,2, point 1,3, point 2,1, and point 3,1. The error value is obtained from the difference between the real system and the estimated results of the Kalman Filter Ensemble Kalman Filter method. The highest error value is 0,44. The fire spread on the real system can be seen after the 25th iteration time at the starting point ie at index i,j=1,1 fire temperature between 250°C -250,05°C. High fire temperature at points around point 1,1, example point 1,2 is 249,97°C, point 1,3 is 249,97°C, point 2,1 is 250,01°C, and point 3,1 is 249,98°C. The fire temperature that occurs at point 10,10 is 35,695°C. From this explanation it can be said that the fire point at point 1,1 spreads to the points to the midpoint of 10,10 at the time of the sixteenth iteration with the final temperature at the 25th iteration time of 35,695°C.

The fire spread on Ensemble Kalman Filter considerations can be seen after the 25th iteration time at the starting point ie at index i,j=1,1 fire temperature ranges between 249,98°C. High fire temperature at points around point 1,1, such as point 1,2 is 250°C, point 1,3 is 249,98°C, point 2,1 is 250°C, and point 3,1 is 249,975°C. The smallest fire temperature that occurs at point 10,10 is 35,69°C. From this explanation it can be said that the fire point at point 1,1 spreads to the points to the midpoint of 10,10 at the time of the sixteenth iteration with the final temperature at the 25th iteration time of 35,69°C.
Figure 3. The fifth simulation using the value of N is 300 and the value of Q and R are 0, 1. Simulation results of system model and Ensemble Kalman Filter (a), Error value Ensemble Kalman Filter (b), Simulation results of system model in the contour form (c), and Ensemble Kalman Filter simulation results in contour form (d).

In the first simulation, Figure 3 with the number of iteration times 25 at the last iteration date at point 1,1 is the hotspot with the initial temperature of 28°C then after a rise in temperature around 250°C. The high fire temperature occurs at the closest points of the point 1,1 is called point 1,2, point 1,3, point 2,1, and point 3,1. The error value is obtained from the difference between the real system
and the estimated results of the Kalman Filter Ensemble Kalman Filter method. The highest error value is 0.129. The fire spread on the real system can be seen after the 25th iteration time at the starting point i.e at index i,j=1,1 fire temperature between 250°C -250.3°C. High fire temperature at points around point 1,1, example point 1,2 is 250°C, point 1,3 is 249.98°C, point 2,1 is 249.98°C, and point 3,1 is 249.97°C. The fire temperature that occurs at point 10,10 is 35.69°C. From this explanation it can be said that the fire point at point 1,1 spreads to the points to the midpoint of 10,10 at the time of the sixteenth iteration with the final temperature at the 25th iteration time of 35.69°C. The fire spread on Ensemble Kalman Filter considerations can be seen after the 25th iteration time at the starting point i.e at index i,j=1,1 fire temperature ranges between 250,5°C. High fire temperature at points around point 1,1, such as point 1,2 is 250°C, point 1,3 is 249.98°C, point 2,1 is 249.99°C, and point 3,1 is 249.98°C. The smallest fire temperature that occurs at point 10,10 is 35.692°C. From this explanation it can be said that the fire point at point 1,1 spreads to the points to the midpoint of 10,10 at the time of the sixteenth iteration with the final temperature at the 25th iteration time of 35.692°C.

In this simulation experiment was also conducted as much as 10 times to see the average error value of Ensemble Kalman Filter method. As in Table 1 discussed the average RMSE value of 10 experiments that have been done.

| Table 1. Average RMSE Ensemble Kalman Filter Comparison |
|---------------------------------------------------------|
| Simulation | 100 of ensemble | 200 of ensemble | 300 of ensemble |
|------------|-----------------|----------------|-----------------|
| 1          | 0.5629          | 0.53311        | 0.5559          |
| 2          | 0.5555          | 0.49214        | 0.47845         |
| 3          | 0.54713         | 0.54637        | 0.50312         |
| 4          | 0.57728         | 0.52836        | 0.50014         |
| 5          | 0.53152         | 0.54032        | 0.47642         |
| 6          | 0.70025         | 0.47933        | 0.41237         |
| 7          | 0.63195         | 0.38991        | 0.52197         |
| 8          | 0.54526         | 0.5812         | 0.40445         |
| 9          | 0.55473         | 0.54805        | 0.48721         |
| 10         | 0.51832         | 0.46225        | 0.53872         |
| Mean       | 0.572484        | 0.510104       | 0.487844        |

It can be seen from Table 1 that after performing ten simulations, the RMSE value of the Ensemble Kalman Filter will be smaller if the ensemble value is greater. In addition, the covariance value of the system model and the measurement model also affects the size of the RMSE value. This causes the Ensemble Kalman Filter method to approach the system model to estimate the occurrence of fire spread.

6. Conclusion
The result of simulation analysis of Ensemble Kalman Filter method will be closer to real system when real system covariance value and covariance value of measurement system is smaller while ensemble value is higher. The average RMSE value in the Ensemble Kalman Filter method simulation results will be smaller when the real system covariance value and the covariance value of the measurement system are smaller while the ensemble value is greater.

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