Time-varying reliability modelling and quasi-static accuracy optimization of precision CNC machine tools

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Abstract. Aiming at the influence of time-varying characteristics of geometric error on precision CNC machine tool quasi accuracy retention, a time-varying reliability accuracy maintenance method based on multi-objective reliability and maintenance cost tradeoff is proposed. Firstly, on the basis of geometric error transfer model and Achard adhesive wear theory, a quasi-static accuracy reliability limit state function to meet time-varying characteristics is established and the time-varying accuracy reliability is solved based on the First order second moment method. Secondly, according to the time-varying characteristics of the quasi-static accuracy of machine tools, the maintenance cost model and reliability multi-objective optimization model are established, and the trade-off between cost and reliability function is realized. Finally, the optimization results of three-axis machining center show that the method can realize the optimization of geometric accuracy and determine the quasi-static error compensation value which meets the requirements of reliability accuracy.

1. Introduction

Machining accuracy is an important indicator for evaluating the performance of CNC machine tools, the error sources that cause the accuracy of the machine tool to fall are diverse, and which can be divided into quasi-static errors and dynamic errors. Quasi-static error is relative to the work-piece that changes with time. Among the many factors, about 70% of the error comes from the geometric error of the machine tool [1].

The reliability analysis of geometric error terms is a heated research topic in recent years, due to both implementation of the error compensation and real-time error avoid. The deep-seated reason is that the precision of machine tool can be further enhanced by analyzing time-varying effect of geometric error terms. Zhang et al. [2] conducted system reliability and global sensitivity analyses of three-axis machine tool by multiplicative dimensional reduction method. Yu [3] established geometric error model of grinding machine and the approximate reliability model of machining accuracy based on response surface method. Zhang et al. [4] conducted geometric accuracy evaluation of machine tool by establishing machining accuracy retentivity function model based on high-order moment standardization technique. Huang et al [5] conducted thorough research on reliability theory of mechanical systems, and analyzed various uncertainty factors that influencing the reliability of the spindle of heavy-duty CNC machine tool.

Previous studies mainly focused on geometric error compensation and static reliability analysis. In another respect, accuracy degradation of the machine tool are critical to the machining accuracy which is not been fully considered in the operation and maintenance, therefore the characteristic of time-
varying reliability is caused by geometric error which should be analyzed, and the corresponding conclusions should be integrated in accuracy optimization.

2. Time-varying reliability analysis method
Under the influence of random loads such as friction, wear and temperature, the strength and effective load of parts and systems change with time. At this time, the static failure state equation can be expressed as [6]:

$$g(Y,t) = r(t) - \delta(t)$$  \hspace{1cm} (1)

where $g(Y,t)$ is state function, the state of the part or machine tool feed system can be described as:

$$\begin{cases} g(Y,t) \leq 0, & \text{Failure state} \\ g(Y,t) > 0, & \text{Safe state} \end{cases}$$  \hspace{1cm} (2)

The time-varying reliability index and the time-varying reliability calculation formula of the quasi-static accuracy parameters is:

$$\beta(t) = \frac{\mu_G}{\sigma_G} = \Phi\left[\frac{g(Y,t)}{\sqrt{\text{var} g(Y,t)}}\right]$$  \hspace{1cm} (3)

The time-varying failure rate expression is

$$\lambda(t) = f(t)/\beta(t) = -d\beta(t)/\beta(t)$$  \hspace{1cm} (4)

According to the wear experience formula and theoretical model of the feed system, Holm abrasive wear model, Achard adhesion wear model, Suh's delamination fatigue wear model, which can be expressed as [7]:

$$\frac{dV}{ds} = K \frac{W}{\sigma_s}$$  \hspace{1cm} (5)

In the above formula, $W$ is the load, $\sigma_s$ is the material yield strength, and $K$ is the wear factor of the guide rail pair.

The wear and deformation of the feed system can be expressed as:

$$V = \frac{KWt}{H}$$  \hspace{1cm} (6)

In the above formula, $H$ is the material hardness, $v$ is the feed speed of the guide rail, and $t$ is the movement time of the feed system.

The function relationship of the quasi-static accuracy $\delta(t)$ of the feed system with time $t$ is [6,7]:

$$\delta(t) = 2KWt/[t_{\text{max}} + t_{\text{min}}]Hb$$  \hspace{1cm} (7)

3. Time-varying reliability modelling of quasi-static accuracy of CNC machine tools

3.1. Quasi-static accuracy model of precision machine tools
Multi-axis CNC machine tool can be regarded as a multi-body system. The structure diagram and topology of three-axis precision machine tool are shown in Figs 1 and 2 respectively; the low-order body array with the topology is shown in Table 1. As shown in Figure 1 is a schematic diagram of the machine tool structure, according to the low-order body array method can obtain machine topology structure, as shown in Figure 2.
When the machine tool coordinate system is set as the reference coordinate system, the vector of the spindle coordinate system relative to the reference coordinate system is expressed as $P_1 = [x_1, y_1, z_1, 1]^T$, homogeneous coordinates of the tool forming point in the coordinate system of the work-piece $P_0 = [P_{w_x}, P_{w_y}, P_{w_z}, 1]^T$. The tool forming point is $P_t = [P_{t_x}, P_{t_y}, P_{t_z}, 1]^T$ in the tool coordinate system, and the command positions of the three linear axes relative to the initial position are $x$, $y$, and $z$.

In the actual forming movement, due to the existence of various errors, the actual position of the tool forming point will inevitably deviate from the ideal position, resulting in spatial position error and tool attitude deviation. The comprehensive spatial error of the tool forming point is:

$$E = a T_t P_t - a T_w P_w$$

(8)
where
\[ \begin{align*} aT_w &= T_{101}T_{011}T_{010}T_{101m}T_{123}T_{123m}T_{122m}T_{121m}T_{121e} \quad (9) \\
\alpha T_I &= T_{045}T_{045e}T_{045m}T_{455}T_{455e}T_{455m}T_{566}T_{566e}T_{566m}T_{566e} \quad (10) \\
\end{align*} \]

Combine equations (8) to (10)
\[ \begin{align*}
E &= \begin{bmatrix} E_x & E_y & E_z \end{bmatrix}^T = \alpha T_I P - \alpha T_w P_w \\
i T_0 P_0 - i T_0 P_I = 0 \\
\end{align*} \quad (11) \]

In the above formula, \( E_x, E_y, \) and \( E_z \) are vectors in three directions of X, Y, and Z axes, respectively.

According to the above analysis and the requirements of work-piece machining accuracy, the quasi-static accuracy reliability limit state function model is derived:
\[ G(X, Y(d, t), I) = |\Delta E| - I = \left(\bar{E}_x^2 + \bar{E}_y^2 + \bar{E}_z^2\right) - I \quad (12) \]

Where \( X = (\delta, \delta, \ldots, \alpha; \beta; \gamma) \), \( Y(d, t) \) is the wear distribution of the quasi-static error term at time \( t \), \( I \) is the maximum allowable quasi-static error.

4. Precision optimization of precision machine tools based on time-varying reliability

Maintenance cost is one of the important factors that affect the total cost of mechanical products. In order to balance the relationship between geometric errors and maintenance costs, a reasonable functional relationship model between maintenance costs and geometric errors needs to be established. According to the relationship between the different features of the work-piece and the cost function, the cost decreases with the increase of the geometric error, so this paper uses a power exponential function to characterize the relationship between geometric error and cost:
\[ C(x_i) = a + b/x_i^e \quad (13) \]

Where \( a \) and \( b \) are cost factors.

Reliability is a key factor in evaluating the accuracy of machine tools. This article focuses on the impact of geometric errors between CNC machine tools and accuracy-related functional components on machine tool reliability. Therefore, the reliability indexes of machine tools with regard to geometric errors can be constructed as:
\[ \begin{align*}
\mu_s &= \sum_{i=1}^{n} K\mu_{si} \\
\sigma_s &= \sqrt{\sum_{i=1}^{n} K\sigma_{si}^2} \\
E &= f\left(\mu_{s1}, \mu_{s2}, \ldots, \mu_{sm}\right) + \frac{1}{2} \sum_{i=1}^{n} \sigma_{si}^2 \frac{\partial^2 f(S)}{\partial \mu_i^2} \left|_{\mu=\mu_i}\right. \\
D &= \sum_{i=1}^{n} \left(\frac{\partial f(S)}{\partial \mu_i} \right|_{\mu=\mu_i} \sigma_{si}^2 \\
R &= P\left(\epsilon_{min} < \Lambda < \epsilon_{max}\right) = P\left(\Lambda < \epsilon_{max}\right) - P\left(\Lambda < \epsilon_{min}\right) = \Phi\left(\frac{\epsilon_{max} - \mu}{\sigma}\right) - \Phi\left(\frac{\epsilon_{min} - \mu}{\sigma}\right) \quad (17) \]

Before conducting the reliability optimization of the NC machine tool, each geometric error terms needs to be determined firstly.
The objective functions are $E_x$, $E_y$ and $E_z$, in order to unify the optimization goals, the mathematical model of multi-objective optimization of CNC machine tools with respect to accuracy is expressed as:

$$
\begin{align*}
\min f(x) &= (f_1(x_1), f_2(x_2)) \\
f_1(x_1) &= \sum_{i=1}^{n} a_i f\left(\frac{a + b_i x_i}{\sigma}\right) \\
f_2(x_2) &= \Phi\left(\frac{\epsilon_{max} - \mu}{\sigma}\right) - \Phi\left(\frac{\epsilon_{min} - \mu}{\sigma}\right)
\end{align*}
$$

(18)

s.t. $x_i \in X$, $\sum_{i=1}^{n} a_i = 1$

Where $X^*$ is the tolerance value range of the geometric error variable.

5. Case study
According to the 9-lines method [8], positioning accuracy of 21 geometric errors are determined. Experiment setup of laser interference on three-axis machine tool is shown in Fig. 3.

![Figure 3. Experiment setup of geometric error measurement](image)

The specific data of positioning accuracy is listed in Table 3.

| Error terms | Deviation | Cost weight | Error terms | Deviation | Cost weight |
|-------------|-----------|-------------|-------------|-----------|-------------|
| $\delta_x$  | 0.025mm   | 0.042       | $\epsilon_y$ | 63µrad   | 0.048       |
| $\delta_x$  | 0.023mm   | 0.042       | $\epsilon_y$ | 66µrad   | 0.048       |
| $\delta_x$  | 0.023mm   | 0.042       | $\epsilon_y$ | 72µrad   | 0.048       |
| $\epsilon_x$ | 63µrad   | 0.046       | $\delta_z$  | 0.026mm | 0.052       |
| $\epsilon_x$ | 66µrad   | 0.046       | $\delta_z$  | 0.033mm | 0.052       |
| $\epsilon_x$ | 70µrad   | 0.046       | $\delta_z$  | 0.037mm | 0.052       |
| $\delta_y$  | 0.024mm   | 0.054       | $\epsilon_z$ | 66µrad   | 0.048       |
| $\delta_y$  | 0.026mm   | 0.054       | $\epsilon_z$ | 65µrad   | 0.048       |
| $\delta_y$  | 0.024mm   | 0.054       | $\epsilon_z$ | 60µrad   | 0.048       |

Identification values of squareness error $\alpha_{yz}$, $\beta_{xz}$ and $\gamma_{xy}$ is 45µrad, 58µrad and 56µrad, and the cost weight is 0.054, 0.066 and 0.064 respectively.

For ease of calculation in the multi-objective optimization, the fuzzy cost coefficient $a_i$ is 0, the linear error and angular error $b_i$ are 1 and 0.0008, respectively, and the index $e$ is a constant. The linear error reliability state coefficient $C_i$ is 1, and the angular error $C_i$ is 0.05. Based on the particle swarm optimization algorithm for quasi-static error optimization, the population size is set to 200, the evolution iteration maximum algebra is 20, crossover rate and marginal interest rate are 0.8 and 0.2, respectively.

After the particle swarm optimization method is used to optimize the quasi-static accuracy values, the change curve of objective function during the optimization iteration process is shown in Figure 4.
Figure 4. Reliability-Cost optimization results

The objective function maintenance cost of the optimized design model continuously decreases with the increase of the number of iterations, and finally converges to obtain the best process parameter value. According to the quasi-static precision optimization flow, the optimal design model is solved, and the optimal objective value shown in Table 3, it can be found that the declining trend of the quasi-static accuracy reliability after optimization is less than that before the optimization. When manufacturing with optimized quasi-static accuracy parameters, the quasi-static accuracy reliability is 79.90%, the accuracy reliability is improved by 70.81% compared with before optimization. Figure 4 shows the manufacturing cost change curve before and after the optimization design. When the geometric error parameter before optimization is used for manufacturing, the maintenance cost is 1934.71 yuan, and the optimized maintenance cost is 933.93 yuan. The machine tool manufacturing cost is reduced by 51.95%.

Based on the research result of geometric error compensation by our research group [9], by using the algebraic values of the quasi-static error items before and after optimization, the positioning accuracy and repeated positioning accuracy for each axes can be measured, as shown in Table 4.

| Measurement terms | With optimization (mm) | Without optimization (mm) |
|-------------------|------------------------|---------------------------|
| Error vector $E_x$ | 0.015                  | 0.024                     |
| Error vector $E_y$ | 0.012                  | 0.024                     |
| Error vector $E_z$ | 0.016                  | 0.026                     |

The results of measurement comparison before and after optimization show that the positioning accuracy and repeatability positioning accuracy of the machine tool are greatly improved.

6. Conclusion

(1) For multi-axis precision machine tools that consider the wear of the feed system, the wear of the feed system is considered to the gradual reliability model, and the mathematical model of the reliability state equation with time is obtained, which is the precision of the quasi-static accuracy of precision machine tools. Analysis and manufacturing cost process optimization design laid the foundation.

(2) The geometric error model of the precision CNC machine tool and the quasi-static accuracy time-varying performance evaluation method based on the reliability theory are established. The effect of the geometric error over time on the static accuracy is theoretically studied to reflect the multi-axis CNC objectively. The quasi-static time-varying characteristics of machine tools provide a theoretical basis.

(3) The precision optimization design model of precision machine tool with the minimum space position vector as the target, quasi-static accuracy time-varying reliability and machine tool manufacturing cost as constraints is established, and the sequential quadratic programming method is
used to solve the optimization design process. The calculation example shows that the optimized quasi-static accuracy reliability reaches 79.9%, and the maintenance cost is decreased from 1934.71 yuan to 933.93 yuan, which provides an effective method for the selection of process parameters of precision CNC machine tools.

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