Long-wave instabilities of two interlaced helical vortices

H U Quaranta$^{1, 2}$, M Brynjell-Rahkola$^3$, T Leweke$^1$ and D S Henningson$^3$

$^1$ IRPHE, Aix Marseille Univ, CNRS, Centrale Marseille, 13384 Marseille, France
$^2$ Aerodynamics Department, AIRBUS HELICOPTERS, 13725 Marignane, France
$^3$ Linné FLOW Centre, KTH Mechanics, Royal Institute of Technology, SE–10044 Stockholm, Sweden

E-mail: leweke@irphe.univ-mrs.fr

Abstract. We present a comparison between experimental observations and theoretical predictions concerning long-wave displacement instabilities of the helical vortices in the wake of a two-bladed rotor. Experiments are performed with a small-scale rotor in a water channel, using a set-up that allows the individual triggering of various instability modes at different azimuthal wave numbers, leading to local or global pairing of successive vortex loops. The initial development of the instability and the measured growth rates are in good agreement with the predictions from linear stability theory, based on an approach where the helical vortex system is represented by filaments. At later times, local pairing develops into large-scale distortions of the vortices, whereas for global pairing the non-linear evolution returns the system almost to its initial geometry.

1. Introduction

The near wake behind multi-bladed rotors consists of several interlaced helical vortices, generated at the tips of the rotor blades. Various instabilities can exist in this system, involving perturbations at different wavelengths: long-wave displacement modes or short-wave vortex core deformations [1]. The linear stability of a single helical vortex was first analyzed by Widnall [2], followed by the work of Gupta and Loewy [3]. These authors determined the stability of long-wave displacement modes by perturbing the helices and deriving an eigenvalue problem considering the induced velocity obtained using the Biot–Savart law and the vorticity transport theorem (see [3] for details). A study aimed at verifying these theoretical results was recently performed by Quaranta and Leweke [4], who carried out detailed experiments with a single-bladed rotor in a water channel. By imposing perturbations with various wave numbers (i.e. number of perturbation wavelengths in one helix turn), they were able to obtain growth rates that closely matched those predicted by Widnall [2] and Gupta and Loewy [3]. It was also shown that the long-wave instability can be linked to the local pairing of successive loops of the helical vortex, and quantitative comparisons were made with the well-known pairing mechanisms of an infinite array of straight vortices [5, 6]. In the present study, the work by Quaranta and Leweke [4] is extended to the configuration of two helical vortices. Similar cases have previously been considered theoretically by Gupta and Loewy [3], and by Okulov and Sørensen [7]. Both analyses predict instability of the system, involving again local pairing at various locations along
the filaments, depending on the perturbation wave number $k$, as well as another mode of global pairing, with an azimuthally uniform ($k = 0$) relative displacement of one helical vortex with respect to the other. The main parameter governing this new instability mode is the pitch of the initial helical system (see [7]). Various modes of pairing in systems of three helical vortices have recently been investigated numerically by Sarmast et al. [8].

2. Experimental details

2.1. Set-up

Experiments were carried out in a recirculating free-surface water channel with a test section of dimensions 38 cm (width) $\times$ 50 cm (height) $\times$ 150 cm (length). Two helical vortices were generated near the test section entry by a two-bladed rotor, mounted on an ogive-tipped shaft (diameter 15 mm) and driven by a computer-controlled stepper motor outside the channel using a belt (figure 1(a)). The rotor blade geometry, shown schematically in figure 1(b), is based on the low-Reynolds number airfoil A18 by Selig et al. [9]. It is designed to operate in the wind turbine regime and produce a constant radial circulation distribution (Joukowsky rotor) over the outer 75% of the span, in order to generate a highly concentrated tip vortex. The rotor has a radius $R_0 = 8.0$ cm and a tip chord 1.0 cm. Details of the design procedure can be found in [10]. For the present experiments, the blade is rotated at a frequency $f = 4$ Hz, and placed in a uniform incoming flow of velocity $U = 37$ cm/s. This results in a tip chord-based Reynolds number of 20100 and a tip speed ratio $2\pi f R_0 / U = 5.4$. The tip vortices of the rotor form a pair of interlaced helical vortex filaments, which are schematically represented in figure 2.

Figure 1. (a) Side view of the water channel test section with the rotor set-up. Flow is from left to right. (b) Schematic of the rotor geometry.

Figure 2. Schematic of two interlaced helical vortices, including the relevant parameters: pitch $2h$, radius $R$, core size $a$ and circulation $\Gamma$. 
2.2. Measurement methods

The helical vortex structure was visualised using fluorescent dyes (fluorescein and rhodamine), illuminated in volume by the light of an argon ion laser, coupled to an optical fibre with a spherical lens. The rotor set-up could be tilted to be able to apply the fluorescent paint to the blades tips outside the water. Using different dyes made it possible to identify each one of the two vortices more easily. When the spinning rotor was lowered into channel test section, with the water flowing, the dye was washed off and clearly marked the tip vortices. The helical wake established itself after a short transient, and the wake structure could be observed for about 30 seconds (120 blade rotations) before the dye disappeared. Images and video sequences were recorded with a standard digital camera in high-speed mode at 100 images per second. The visualisations were also used to obtain quantitative information about the vortex geometry (radius, pitch) and to measure the instability growth rates, as explained in section 3.

Velocity fields and vorticity distributions were obtained from Particle Image Velocimetry (PIV). A Nd-YAG pulsed laser was positioned underneath the test section to generate a vertical light sheet illuminating the centre plane of the rotor flow (only half of the plane was illuminated behind the rotor, since the shaft blocked the light). The flow was seeded with 10 $\mu$m plastic particles, and images were recorded with a resolution of $4016 \times 2670$ pixels using a Roper Redlake digital camera. PIV measurements were used to determine the base flow properties of the helical rotor wake (circulation, core size, etc.). Further details about the experimental set-up and procedures are given in [4].

3. Results and discussion

3.1. Unperturbed vortices

A typical dye visualisation of the unperturbed helical vortices, with the rotor operating at a constant rotation rate, is shown in figure 3(a). A regular helical shape, which is close to the theoretical structure of two infinite interlaced helices (figure 3b), is observed over a distance of more than three rotor diameters.

The unperturbed configuration in figure 3(a) is the reference case of our study. The phase-averaged azimuthal vorticity field in the near-wake centre plane of the rotor, obtained from 100 instantaneous PIV fields, is presented in figure 4. The cross sections of the tip vortices are clearly visible. One also recognizes the signature of the blades’ root vortices of opposite-signed vorticity at $r/R_0 \approx 0.4$, as well as the boundary layer on the shaft.

The visualisations and PIV measurements were used to evaluate the geometrical parameters and the vortex properties of the base flow, such as the vortex circulation $\Gamma$ and core radius $a$. Asymptotic values of these parameters are reached, after a short transient phase, between 1 and 2 diameters downstream of the rotor. The radius $R$ and separation distance $h$ between
successive helix loop were measured directly from the visualisations, by locating the centres of the dye loops at the top and bottom of each helix. Using a method similar to the one described in [4], an equivalent Rankine core radius $a_c$ is derived from the profiles of swirl and axial velocity components of the vortices. This allows a comparison with previous theoretical studies [2, 3, 7], which all considered helical vortices of the Rankine type.

The measured properties are summarized in table 1, together with the resulting non-dimensional parameters. The uncertainties result from the scatter of individual measurements and from estimates of the reliability of the employed procedures.

### 3.2. Local pairing

The low turbulence intensity in the channel made it possible to impose controlled displacement perturbations of the helices, by introducing suitable periodic modulations of the initially constant blade rotation frequency. Varying the rotation speed results in a shift of the tip vortex in the axial direction, with respect to its unperturbed position. These perturbations are characterized by their wave number $k$, which is the number of perturbation wavelengths in one turn of the rotor. Details of the experimental procedure can be found in [10] and [4].

When such perturbations are triggered on a rigid rotor, they are in phase on all vortices. Local pairing is expected to occur when the displacements on neighboring helix loops are out of phase. For a rigid rotor, this is the case when $k = N(i + 1/2)$, where $N$ is the number of blades and $i$ an integer [8].

Figure 5(a) shows a visualization of the two-bladed ($N = 2$) rotor wake subjected to a perturbation with $k = 1$. The perturbation is rapidly amplified as the vortices move downstream.
Figure 5. Visualization of the two-bladed rotor wake perturbed with a mode of wave number $k = 1$. (a) Experiment, (b) theory.

Its spatial structure corresponds well to the one predicted theoretically, which is shown in figure 5(b). The theoretical mode shape is obtained by superposing a perturbation on the helix structure shown in figure 3(b), with a given wave number and a spatial growth that is obtained from the temporal growth using the convection velocity of the vortices. Local pairing is found at the top and bottom in this side view, eventually leading to a swapping of the relative positions of the two helices with respect to the horizontal axis. Towards the end of the observation zone, non-linear effects lead to a complicated distortion of the initial double-helix structure, but the two vortices nevertheless appear to remain intact.

The growth rate ($\alpha$) of a given perturbation was determined from video recordings of visualizations such as in figure 5(a), by measuring the displacement of the perturbed vortices with respect to the initial base configuration (shown in figure 3a) as function of time. This method was found to be a robust way to extract the growth rate from the experiments [4]. Results were obtained for 9 different wave numbers in the interval $0.25 \leq k \leq 3.0$, they are plotted in figure 6(a). The measurements are compared to the theoretical prediction corresponding to the experimental configuration with the parameters given in table 1. Good agreement is found, not only for local maxima at $k = 1$ or 3, but also for intermediate wave numbers. When $k$ is small, the maximum growth rate is very close to $\pi/2$, which is the value for the pairing instability of an array of point vortices, separated by $h$, in two dimensions. The similarity of the long-wave instability of a helical vortex filament and the pairing mechanism in an infinite array of straight
vortices was discussed in [4].

As mentioned before, perturbations introduced by a variation of the rotor motion always have the same phase on both vortices. If this constraint is relaxed, additional unstable modes appear for a system of two interlaced helical vortices. Figure 6(b) shows (in red) the instability growth rate computed with the formalism of Gupta and Loewy [3] for the experimental configuration. Additional maxima appear near even integer values of $k$. In particular, a perturbation with $k = 0$ appears to have the highest growth rate. It corresponds to a global offset of each of the two helices with respect to their base position, which cannot be obtained by a variation of the rotor frequency in the experiments. Therefore a different method had to be used to study this instability mode.

3.3. Global pairing

In the experiments, a perturbation with wave number $k = 0$ could be obtained by slightly offsetting the rotor, in the radial direction, from its symmetric position. This results in a slight difference (typically of a few percent) between the radii of the two helices, which was sufficient to trigger the desired unstable mode.

Figure 7(a) shows the (spatial) growth of the $k = 0$ mode for $h/R = 0.41$ and $a_e/R = 0.038$. It leads to an azimuthally uniform global pairing of the two helices. The radius of the initially larger helix continues to increase, whereas the one of the other helix decreases, until a general swapping of their relative axial positions (or “leapfrogging”) occurs at some downstream location. The initial development is again very close to the prediction that can be derived from the theoretical linear stability analysis, and which is shown in figure 7(b). The growth rate can be determined in the same manner as before; for the case shown in figure 7(a), it was found to be $\alpha(2h^2/\Gamma) = 1.57$, which is again very close to $\pi/2$, as expected for low pitch [7]. After the swapping, the radii of both helices return almost to their initial values, so that at the end of the observation interval one recovers almost the initial double-helix configuration (this non-linear evolution cannot be captured by the ‘linear’ prediction in figure 7b). Similar observations were recently made in the numerical simulations by Selçuk [11].
Figure 7. Uniform pairing mode \((k = 0)\) of two helical vortices. (a) Experimental dye visualization of the wake of a slightly asymmetric rotor. (b) Prediction of the spatial growth, based on the linear instability mode.

For a double-helix configuration similar to the experimental one of the present study, he observed a series of successive leapfrogging events, before the two helical vortices eventually merged. In the experiments, the observation interval was not long enough to be able to detect a second leapfrogging.

4. Conclusion
The stability of a system of two interlaced helical vortices was studied experimentally using a two-bladed rotor in a water channel. The system was found to be highly sensitive to small perturbations introduced in the rotor plane, leading to long-wave displacements and pairing of the vortices. Periodic perturbations, with non-zero wave-number \(k\) and the same phase on both vortices, were generated by a modulation of the rotor frequency. They lead to local pairing at various locations around the azimuth of the helical wake. Uniform perturbations with \(k = 0\) could be imposed by slightly off-centering the rotor. In this case, global pairing is observed, involving variations of the radius of each helix as a whole. The growth rates of local and global pairing were determined from experimental visualizations, and good agreement was found with the corresponding theoretical predictions, based on a filament approach. As for a single helix, the long-wave instability of the system of two helical vortices can be understood by considering the pairing mechanism of two-dimensional point vortex arrays. At larger times, local pairing leads to complicated distortions and eventually to a breakdown of the vortex system, whereas for global pairing a leap-frogging phenomenon is found, which returns the system to almost its initial geometry.
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