BELAVKIN-KOLOKOLTSOV WATCH–DOG EFFECTS IN INTERACTIVELY CONTROLLED STOCHASTIC COMPUTER-GRAPHIC DYNAMIC SYSTEMS. A MATHEMATICAL STUDY\footnote{A preliminary shortened draft of this paper is located in the e–print archive of Los Alamos National Laboratories (USA) on Chaos and Dynamics under the number "chao-dyn/9406013". It may be received via the sending of an empty e-mail to "chao-dyn@xyz.lanl.gov" with the command "get 9406013" in the subject field.}

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Abstract. Stochastic properties of the long time behavior of a continuously observed (and interactively controlled) quantum–field top are investigated mathematically. Applications to interactively controlled stochastic computer-graphic dynamic systems are discussed.
I. INTRODUCTION
(DESCRIPTION OF PROBLEMS, MOTIVATIONS AND GENERAL DISCUSSIONS)

The main difficulty to account the high–frequency eye tremor in mobilevision (MV) (Juriev 1992, 1994a, 1994b) is that in this case a solution of the complete MV evolution equations in real time requests about $10^8$–$10^9$ arithmetical operations per second (moreover, it needs special displays of a high refreshing rate ($\sim 300$–500 frames per second) and a small image inertia). Such account may be performed only on a narrow class of computers for the purposes of scientific experiments on peculiarities of human vision in interactive computer-graphic systems (Juriev 1992, 1994a), but it is very inconvenient for an assimilation of MV as a computer-graphic tool f.e. for an interactive visualization of 2D quantum field theory (Juriev 1994c). So one should to use some stochastic simulation of the interactive processes, i.e. to consider an imitated stochastic process instead of the tremor. Such approach leads to stochastic mobilevision (SMV) (Juriev 1992), which evolution equations have a stochastic Belavkin–type form (Belavkin 1988, Belavkin & Kolokoltsov 1991, Kolokoltsov 1991). It seems that the interactive effects for ordinary MV and SMV are similar in general, because the interactive processes accounting saccads are not stochastized; though it is not an undisputable fact that they are always identical (f.e. in a situation of the so–called ”lateral vision”). The combination of MV with cluster and spline techniques allows to work on computers with $10^5$–$10^6$ arithmetical operations per second (as well as to use simpler devices for eye motion detection and a wider class of displays), whereas all enumerated above circumstances make the tremor accounting in terms of ordinary MV almost unreasonable nowadays.

Nevertheless, all these advantages of SMV are not crucial in view of the permanent progress in the computer hightech (for example, the using of a distributed parallel processing allows to diminish the request for the tremor accounting ordinary MV to $\sim 10^6$ arithmetical operations per second, etc.). A deeper advantage of SMV is more theoretical — it is a presence of the Belavkin–Kolokoltsov watch–dog effects (Kolokoltsov 1993, see also the original papers (Chiu et al 1977, Misra & Sudarshan 1977), where ”watch–dog effects” or a ”quantum Zeno paradox” were put under a consideration, and a recent note (Home & Whitaker 1992) for refer-
ences and a description of a current state of the problem, or the book (Peres 1993) for a general point of view) in SMV in certain rather natural and general cases (i.e. for certain values of internal parameters measuring the degree of localization of interaction) that means an \textit{a priori} finiteness of sizes of stochastic "cores" of an image during observation, moreover they may be diminished to several pixels by a suitable choice of a free controlling parameter (the so–called "accuracy of measurement" (Belavkin 1988, Belavkin & Kolokoltsov 1991, Kolokoltsov 1991, 1993)).

The watch–dog effect may be considered as a weaker but also tamer form of nondemolition than the quasistationarity (Juriev 1992, 1994a): there exists a wide class of models, in which the first is observed whereas the least is broken, one may consider canonical projective $G$–hypermultiplets (Juriev 1994a) (see also Juriev 1994b) as a simple example.

Thus, a transition from MV to SMV partially solves \textit{a priori} the main problem of dynamics in interactive psychoinformation computer-graphic systems (Juriev 1992, 1994a) — a problem of the nondemolition of images by the interactive processes (i.e. their stability under observation). Certainly, SMV does not solve the nondemolition problem completely \textit{a priori}. It only guarantees that the stochastic cores of image have finite sizes during observation, it means that details of image do not diffuse. Nevertheless, they may move, being ruled by the slow eye movements. So though details of image are perserved, the image may be destructed as a whole. It seems that the quasistationarity conditions (Juriev 1992, 1994a) are realistic complements to watch–dog effects and together they provide a complete long–time nondemolition of images.

Also it should be marked that such \textit{a priori} nondemolition in SMV confirms a presence of \textit{a posteriori} one in the tremor accounting ordinary MV.

The purpose of this note is to investigate the Belavkin–Kolokoltsov watch–dog effects in SMV mathematically.

Summarizing arguments above one may conclude that such investigations are motivated by the overlapping of two problems:

1) the difficulty to account the high–frequency eye tremor in ordinary mobile–vision, which leads to the necessity to consider tremor’s stochastic simulations;
2) the main problem of dynamics in interactive psychoinformation computer-
graphic systems, i.e. a problem of the nondemolition of images by the
interactive processes; it motivates investigations of long-time properties of
(nonlinear) stochastic dynamics in SMV.

So the first problem explains, why stochastic mobilevision is put under a consider-
ation, the second one explains a choice of questions, which are tried to be solved in
the paper.

II. MATHEMATICAL SET UP
(DEFINITION AND COMMENTS)

First of all, stochastic mobilevision as well as ordinary mobilevision are interac-
tive computer-graphic systems, the evolution of images in which is governed by the
eye movements in accordance to the certain dynamical perspective laws, i.e. dynam-
alical equations, which govern an evolution of image during observation (see Juriev
1994b,c). So their definitions are just the specifications of such laws (it should be
specially stressed that we restrict now our interest in interactive computer-graphic
systems by an intrinsic constructive point of view (cf. Kaneko & Tsuda 1994),
considering them as such but not as descriptive tools of any use for modelling or
visualizing of various physical processes (as in Juriev 1994c), such approach may
be rather narrow but effective and it is reasonable to adopt it for the further dis-
cussion). The laws for MV were written in (Juriev 1992, 1994a,b,c). Stochastic
mobilevision have the slightly different laws. A difference may be briefly sum-
marized in the following terms: (1) the high-frequency eye tremor is decoupled
from the slow eye motions (including saccads), (2) it is stochastized in such a way
that it may be considered as purely internal process in the system so that (3) its
characteristics are not completely determined by the real eye motion and may be
reinforced.

This qualitative description of stochastic mobilevision is sufficient for the un-
derstanding of results as well as their significance for applications but we need in
a more formal definition for their deduction. However, a reader, which is not in-
terested in formal expositions may omit all mathematical constuctions below and
restrict himself to some comments.

Note once more that to define stochastic mobilevision means to specify its dy-
namical perspective laws (dynamical equations, which govern an evolution of image during observation) and we prefer to do it rather formally in purely mathematical terms. Such specification is rather analogous to one for the ordinary MV and is based on concepts of 2D quantum field theory. However, it should be noted that this paper is not a suitable place for a new detailed review of mathematical foundations of MV (and, certainly, of 2D QFPF, the rather systematic exposition of which may be found in the papers (Juriev 1994b,c) (see also original papers (Juriev 1992, 1994a)). So a knowledge of mathematical formalism for ordinary mobilevision is preferred. Certainly, all necessary objects will be formally introduced below, but many motivations for definitions, constructions and notations as well as unavoidable remarks on their interrelations are omitted if they are just the same as for ordinary MV. In particular, all objects of 2D QFT are used without any comments. It is explained by the fact that a detailed presentation of all related (sometimes, rather mathematically technical) material would rather overload an exposition and would not help to the clarification of general ideas and the understanding of results. So it should be emphasized once more that the description of stochastic mobilevision will be rather formal, whereas the interpretations of mathematical results and their significance for applications will be commented in detail throughout the text, in the conclusion and in remarks on applications after it.

However, a brief but as complete as possible account of keypoints of mobilevision is included into the appendix. Certainly, it is written specially for this paper to make it more free–standing and self–consistent. It does not pretend on any more autonomous existence and can not be regarded as a refined extract of the cited papers.

**Definition 1.** Let $H$ be a canonical projective $G$–hypermultiplet (Juriev 1994a,b), $A_t(u, \dot{u})$ – an angular field (obeying the Euler–Arnold equations $\dot{A}_t = \{H, A_t\}$, where the hamiltonian $H \in S'(g)$ ($g$ is the Lie algebra of a Lie group $G$) is a solution of the Virasoro master equation) (or its finite–dimensional lattice approximations (Juriev 1994c)). Let $J(u)$ — an additional $q_R$–affine current (Juriev 1994b,c)(or its finite–dimensional lattice approximation (Juriev 1994c)) commuting with $G$. A stochastic evolution equation

$$d\Phi(t, [\omega]) = A_t(u, \dot{u})\Phi(t, [\omega]) dt + \lambda J(u)\Phi(t, [\omega]) d\omega,$$
where $d\omega$ is the stochastic differential of a Brownian motion (i.e. $\frac{d\omega}{dt}$ is a white noise), will be called the (quantum–field) Euler–Belavkin–Kolokoltsov formulas, the parameter $\lambda$ will be called the accuracy of measurement (cf. Belavkin 1988, Belavkin & Kolokoltsov 1991, Kolokoltsov 1991).

**Remark 1.** These formulas are a reduced version of more general ones

$$d\Phi(t, [\omega]) = \{A_t(u, \dot{u}) + \alpha\lambda^2 : J^2(u) : \} \Phi(t, [\omega]) dt + \lambda J(u) \Phi(t, [\omega]) d\omega,$$

which will be also called the (quantum–field) Euler–Belavkin–Kolokoltsov formulas; $\lambda^2 : J^2(u) :$ is a Belavkin–type quantum–field counterterm (cf. Belavkin 1988, Belavkin & Kolokoltsov 1991, Kolokoltsov 1991), where $: J^2(u) :$ is a spin–2 primary field received from the current $J(u)$ by the truncated Sugawara construction (Juriev 1994a).

Here $u = u(t)$ and $\dot{u} = \dot{u}(t)$ are the slow variables (Juriev 1992) of observation (sight fixing point and its relative velocity), the tremor is simulated by a stochastic differential $d\omega$, $\lambda$ is a free parameter, $\Phi = \Phi(t, [\omega]) \in H$ is a collective notation for a set of all continuously distributed characteristics of image (Juriev 1992, 1994b,c), $q_R$ is a free internal parameter of a model, which measures the degree of localization of interaction (the local case corresponds to $q_R = 0$). The most important case is one of $q_R \ll 1$ and all our results will hold for this region of values of $q_R$. The stochastic Euler–Belavkin–Kolokoltsov formulas coupled with the deterministic Euler–Arnold equations define a dynamics, which may be considered as a candidate for one of a continuously observed (and interactively controlled) quantum–field top (Juriev 1994a).

**Remark 2.** It should be specially emphasized that in stochastic mobilevision $\lambda$ is a free parameter, which may be chosen arbitrary by hands (f.e. as great as it is necessary). It means that slow movements (including saccads) and tremor are decoupled, the firsts are considered such as in an ordinary MV, whereas the least is stochastized in a way that its amplitude may be reinforced.

**Remark 3.** As it was mentioned above the internal parameter $q_R$ measures a degree of localization of a man–machine interaction in MV and SMV. It is natural to suppose that the Belavkin–Kolokoltsov watch–dog effects will appear for sufficiently
small values of $q_R$ and the condition $q_R \to 0$ will produce the diminishing of stochastic cores of image. Indeed, we shall see that sizes of stochastic cores diminish if $q_R$ tends to 0 and $\lambda$ increases.

Below we shall work presumably with finite-dimensional lattice approximations (cf. Kolokoltsov 1993) and the associate evolution equation in $H^*$ (Juriev 1994c), keeping all notations. Also $\Phi$ will be considered as defined on a compact (the screen of a display or a cluster). It should be marked that in this case the Euler–Belavkin–Kolokoltsov formulas are transformed into the ordinary (matrix) stochastic differential equations of diffusion type (Gihman & Skorohod 1979, Skorohod 1982), and hence, $\Phi = \Phi_t = \Phi(t,[\omega])$ is a diffusion Markov process (Dynkin 1965).

**Remark 4.** Lattice approximations of the ordinary (unobserved and non-controlled) quantum–field top (in this case angular fields are reduced to single currents) were actively investigated by St.Petersburg Group directed by Acad.L.D.Faddeev (Alekseev et al 1991, 1992). The main difficulties (technical as well as principal) in their treatments were caused by a locality of ordinary ($q_R = 0$) affine currents. However, $q_R$–affine currents are not local so their discretizing is easily performed (Juriev 1994c). It is very interesting to receive lattice current algebras of (Alekseev et al 1991, 1992) from naturally discretized $q_R$–affine currents by a limit transition $q_R \to 0$, but this problem is a bit out of the line here.

The fact that the ordinary quantum–fielded top may be received as a particular case of our construction ($\lambda = 0$, $q_R = 0$, $A(u, \dot{u}) = J(u)\dot{u}$, where $J(u)$ is a current) motivates to consider our object as a continuously observed (and interactively controlled) quantum–field top. Continuous observation means the inclusion of a stochastic term ($\lambda \neq 0$), whereas the interactive controlling means the presence of complete angular fields $A(u, \dot{u}) = \sum_k B_k(u)\dot{u}^k$, where $B_k(u)$ are primary fields of spin $k$, instead of single currents. It seems that these arguments are sufficient for our terminological innovation.

**Remark 5.** The Euler–Belavkin–Kolokoltsov formulas are postulated to be the dynamical perspective laws for stochastic mobilevision. So they are regarded as a mathematical definition of SMV (cf. Juriev 1994b,c). From such point of view a transition from MV to SMV consists in:

1) the decoupling of slow movements (including saccads) and tremor;
2) a stochastization of tremor;
3) the setting the controlling parameter $\lambda$ free, so that its value may be chosen by hands and it is not completely determined by real parameters of the eye motions.

Thus, the main difference between MV and SMV is that tremor in MV is an external process governing an evolution of a computer graphic picture, whereas its stochastization is an internal process (in spirit of endophysics of O.E. Rössler (Rössler 1987)) and its characteristics may be specified by hands.

Let’s summarize the material of this paragraph. Note once more that the ordinary mobilevision is an interactive computer-graphic system, the evolution of images in which is governed by the eye movements in accordance to the certain dynamical perspective laws, which were written in (Juriev 1992, 1994a,b,c). Stochastic mobilevision is an analogous interactive computer-graphic system, but with slightly different dynamical perspective laws. Namely, in the dynamical perspective laws of MV the high-frequency eye tremor is decoupled from the slow eye motions (including saccads), is stochastized in such a way that it may be considered as purely internal process in the system so that its characteristics are not completely determined by eye motions and may be reinforced. So the parameters of an external real process (eye tremor) may be transformed and scaled up to receive ones an internal virtual process (stochastization of tremor). For the understanding of results the explicit form of dynamical perspective laws is not necessary though it is, of course, unavoidable for their deduction, which is presented in the following paragraph, which may be omitted by a reader interested only in applications, who may restrict himself by the comment and remark at its end.

III. MATHEMATICAL ANALYSIS
(THE MAIN STATEMENTS AND DISCUSSIONS)

Let $D_A(\Phi) = \left\langle A^2 - \langle A \rangle^2 \right\rangle_{\Phi}$, $\langle A \rangle_{\Phi} = \frac{\langle A(\Phi, \Phi) \rangle}{\langle \Phi, \Phi \rangle}$ (Kolokoltsov 1993). It should be mentioned that one may consider the Euler–Belavkin–Kolokoltsov formulas with a redefined quantum field $\bar{J}(u) = J(u) - \langle J(u) \rangle$ instead of the $q_R$–affine current $J(u)$ to receive a full likeness to the original Belavkin quantum filtering equation (Belavkin 1988, Belavkin & Kolokoltsov 1991, Kolokoltsov 1991, 1993) if the inner (scalar) product $(\cdot, \cdot)$ is claimed to be translation invariant and scaling homoge-
neous. $E_{\Phi}$ is the mathematical mean with respect to the standard Wiener measure for observation process with initial point $\Phi$ (Kolokoltsov 1993).

**Lemma 1.**

$$\left( \forall \Phi_0 \right) \limsup_{t \to \infty} E_{\Phi_0} D_J (\Phi(t, [\omega])) = K \lambda^{-2} \to \lambda \to \infty 0.$$ 

The l.h.s. expression (multiplied by $\lambda^2$, i.e. just the constant $K$) is called the Kolokoltsov coefficient of quality of measurement (Kolokoltsov 1993).

**Sketch of the proof.** Indeed

$$\lambda^2 \limsup_{t \to \infty} E_{\Phi_0} D_J (\Phi(t, [\omega])) = \limsup_{t \to \infty} E_{\Phi_0} D_{\lambda J} (\Phi(t, [\omega])) = \limsup_{t \to \infty} E_{\tilde{\Phi}_0} D_J (\tilde{\Phi}(t, [\omega])),$$

where $\tilde{\Phi}$ is a solution of the Euler–Belavkin–Kolokoltsov formulas with $\lambda = 1$ and with the initial data $\tilde{\Phi}_0$ being equal to $\Phi_0$ scaled in $\lambda$ times (the least equality follows from the scaling homogenity of the Euler–Belavkin–Kolkoł’tsov formulas). As a sequence of results of (Kolokoltsov 1993) (the dependence of the $q_R$–affine current $J$ on $u$ is not essential in view of the translation invariance) the expression

$$\limsup_{t \to \infty} E_{\tilde{\Phi}_0} D_J (\tilde{\Phi}(t, [\omega])),$$

being the Kolokoltsov coefficient $\kappa(A_t, J)$ for the pair $(A_t, J)$, does not depend on $\tilde{\Phi}_0$, and hence, it is certainly independent on $\lambda$.

**Remark 6.** The sketch of the proof is rather instructive itself. Instead of difficult calculations of the stationary probability measure (cf. Kolokoltsov 1993, see also Huang et al 1983) and a complicated estimation of its $\lambda$–behaviour (that is non–trivial to perform rather in the simplest 2–dimensional case considered in (Kolokoltsov 1993)) we use general group–theoretical properties (the translation invariance and the scaling homogenity) of the Euler–Belavkin–Kolokoltsov formulas, combining them with the strong results of (Kolokoltsov 1993) on an existence of the Kolokoltsov coefficient $K = \kappa(A_t, J)$ and its independence on the initial data.

**Comments on the proof.** Concerning the sketch of the proof two remarks on some details should be made. First, in view of the dependence of the angular field $A_t (u, \dot{u})$ on the controlling parameters the unique stationary probability measure does not exist; however, we consider all controlling parameters as slow ones so one may assume that there exists the slowly evolving stationary probability measure,
which form depends only on the current values of controlling parameters (of course, it is clear that such assumption is natural from mathematical physics point of view, however, it means a certain ”gap” in the rigorous proof from pure mathematics one; but here any ”purification” will be out of place). Such parameters varies through a compact set (in the continuous version, or may have only finite number of values in the lattice version), so one can define the Kolokoltsov coefficient as the supremum of such coefficients calculated for the measures from the compact (or finite) set (just this circumstance causes the appearing of ”lim sup” in Lemma 1). However, second, now one may use the scaling rigorously only for infinite regions, whereas we have to deal with finite ones (the screen of a display or clusters); however, the transition to the compact regions may only cause that the Kolokoltsov coefficient $K$ being a function of $\lambda$ decreases if $\lambda$ tends to infinity.

Let’s $Q$ be the coordinate operator $Qf(x) = xf(x)$; $J^o$ be a singular part of the current $J$ (Juriev 1992, 1994a), i.e. $J^o(u) = (Q - u)^{-1}$.

**Lemma 2.**

$$E_{\Phi_0}(D_J(\Phi(t, [\omega])) - D_{J^o}(\Phi(t, [\omega]))) \to 0 \quad \text{if} \quad q_R \to 0.$$ 

It should be marked that the statement of the lemma naïvely holds only in the continuous version; after a finite–dimensional approximation the expression ”$\to 0$” should be understand as the l.h.s. becomes uniformly less than a sufficiently small constant $\epsilon$ (which depends on the chosen approximation), when $q_R$ tends to zero.

**Hint to the proof.** The lemma follows from the explicit computations of eigenfunctions of a $q_R$–conformal current $J(u)$.

**Main Theorem.**

$$(\forall \Phi_0) \lim_{\lambda \to \infty, q_R \to 0} \limsup_{t \to \infty} E_{\Phi_0} D_Q(\Phi(t, [\omega])) = 0.$$ 

The statement of the theorem is a natural sequence of two lemmas above; it remains true in the multi–user mode (Juriev 1994d) also. Certainly, the statement of the theorem naïvely holds only in the continuous version (cf. Lemma 2); after a finite–dimensional approximation the equality of the limit to 0 should mean that this limit is less than a sufficiently small constant $\epsilon$, which depends on the chosen approximation.
Comment. Thus, we received that the Belavkin–Kolokoltsov watch–dog effects in stochastic mobilevision appear for all values of the accuracy of measurement $\lambda$ for sufficiently small values of parameter $q_R$. Moreover, if $\lambda$ increases and $q_R$ tends to 0 the stochastic cores may be diminished to several pixels.

Remark 7. Note that the Belavkin–Kolokoltsov watch–dog effects appear only in the models of SMV with sufficiently small values of the internal parameter $q_R$, which measures the localization of interaction ($q_R = 0$ means the local case). However, $q_R$, being an internal parameter, may be chosen in arbitrary way, so the condition $q_R \ll 1$ may be always provided.

IV. CONCLUSIONS
(SUMMARY OF RESULTS)

Thus, the results may be briefly summarized.

First, let’s emphasize once more that the main difference of SMV from the ordinary MV is that the stochastization of eye tremor in the first is considered as an internal process, so its amplitude characteristics may be reinforced. Second, for all values of $\lambda$ (a free parameter of such stochastization, which measures the reinforcing of the amplitude of tremor — the so-called accuracy of measurement) the Belavkin–Kolkoltsov watch–dog effects for stochastic dynamics of image in SMV are observed (it means that stochastic cores of image have finite sizes for all times) for sufficiently small values of an additional internal parameter $q_R$; it confirms the presence of watch–dog effects also in the models of ordinary MV with the same $q_R$. Moreover, third, if the value of $\lambda$ is great enough, whereas $q_R \ll 1$ than the stochastic scores of SMV image may be diminish to several pixels. Such effect, which is produced by the reinforcing of $\lambda$, may be effectively used in practical computer–graphics for various purposes as it was marked in the introduction. Some further discussions of significance of the obtained results for other applications may be found in the next paragraph.

V. REMARKS ON APPLICATIONS, THEIR RELATIONS TO OBTAINED RESULTS AND GENERALIZATIONS

Remarks on applications. Besides theoretical importance for the interactive visualization of 2D quantum field theory the results of the paper seems to be useful for
applications to (1) the elaboration of computer-graphic interactive systems for psychophysiological self-regulation and cognitive stimulation (Juriev 1994b,c), (2) the interactive computer-graphic modelling of a “quantum computer” (Juriev 1994c) (see (Deutsch 1985, Josza 1991, Deutsch & Josza 1992) for a general discussion on “quantum computers” and their use for rapid computations as well as (Unruh 1994) on fundamental difficulties to construct the “physical” non-interactive “quantum computer”), which may be used for an actual problem of the accelerated processing of the complex sensorial data in the “virtual reality” (visual–sensorial) networks, (3) the creation of computer graphic networks of teleaesthetic communication (Juriev 1994c).

Let’s discuss a significance of obtained results for these applications.

Comment: Obtained results and applications.

(2) is directly related to our results because the maintaining of the coherence is the main problem for ”quantum computers”. As it was mentioned earlier (Juriev 1994c) MV may be regarded as an interactive computer-graphic simulation of a ”quantum computer” behavior. The presence of free parameters (such as \( \lambda \)) in SMV allows to maintain the coherence for long times with an arbitrary precision in the interactive mode.

Moreover, such interactive computer-graphic simulations may be more useful than the original ”quantum computers” for the ”virtual reality” problems in view of the implicit presence of graphical data in the interactive mode. A reorganization of these data by the secondary image synthesis (Juriev 1994e) and their representation via MV or SMV may allow an accelerated parallel processing of the complex sensorial data in such systems.

(1) and (3) are indirectly related to our results because they depend on a solution of the main problem of dynamics in interactive psychoinformation computer-graphic systems (a problem of the nondemolition of images). For (3) its solution allows to transmit the graphically organized information without a dissipation and additional errors. For (1) its solution allows to consider a long–time self–organizing interactive processes, which play a crucial role in systems for psychophysiological self–regulation and cognitive stimulation.

So it should be stressed that the obtained results are essential for the prescribed
applications.

Now let’s discuss the possible generalizations.

*Remarks on generalizations and perspectives.* Really one consider a random (discrete) simulation of the continuous Brownian motion and stochastic differentials. It may be rather interesting to replace it by any their perturbation (f.e. by some version of the weakly self–avoiding or self–attracting walks, especially by their finite memory approximations).

First, these generalizations are motivated by the fact that Brownian motion may be not the best stochastization of the eye tremor. Really, it may be considered only as a first approximation for tremor, whereas the more complicated models will be preferable. However, it seems that the watch–dog effects are conserved by any form of the weakly self–attracting perturbations, which are the most realistic candidates for tremor.

Second, it seems to be rather interesting to use the decoupling of high–frequency tremor from slow eye movements (including saccads) and an internal character of its stochastic simulations for the organization of various ”intelligent” forms of man–machine interaction (the so–called ”semi–artificial intelligence”). In such approach the stochastized tremor plays a role of an internal observer (cf. Rössler 1987), which presence is crucial for a self–organization of graphical data in systems of the semi–artificial intelligence (Kaneko & Tsuda 1994). But this topic (though being related to (1) above) seems to be too manysided and too intriguing that this paper is not a suitable place to discuss it further.

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APPENDIX. KEYPOINTS OF MOBILEVISION

This appendix contains keypoints of mobilevision: a definition of MV in computer-graphic terms and its translation into mathematical language, i.e. a derivation of all mathematical machinery from the first principles.

1. First, let’s explain once more what mobilevision is. An answer on the question is not, however, unique and depends on the used system of terms. Certainly, it presupposes an existence of some intuitive meaning, but its verbal expression is determined by an external ”coordinate system” of thinking. Thus, mobilevision may be defined in one of the following ways:

— mobilevision is an intentional anomalous virtual reality, which naturalizes the quantum projective field theory (Juriev 1992, 1994b,c);
— mobilevision is an artificial computer-graphic interactive psychoinformation system with a projective invariant feedback determined by eye motions of observer.

The first definition is more abstract whereas the second is more technical (in some sense both approaches are complementary to each other: they interpret ”to define smth” as ”to explain what it is” or ”to explain how to make it”, respectively). The first approach was developed in (Juriev 1992, 1994b,c). Here the second approach is preferable.

Definition 2. Mobilevision is an artificial computer-graphic interactive psychoinformation system with a projective invariant feedback determined by eye motions of observer.

Let’s discuss this definition.

First, mobilevision is an artificial interactive information system (this point corresponds to term ”virtual” in the first form of the definition). Principles of its construction are self–consistent and do not copy automatically any natural laws just like principles of airplane’s construction differs from ones of bird’s physiology (this point corresponds to the term ”anomalous”). So MV tries, first of all, to be a useful informatic construction, but not a model of any (may be rather important) natural phenomena.

Second, mobilevision is a computer–graphic information system, so an information stream from a computer to a human is mounted in a form of images on the
screen; also it is a dynamical interactive system, i.e. the computer changes geometric data on the screen by a certain algorithm, and such changes depend on a behaviour of observer. Mobilevision is a psychoinformation interactive system, i.e. characteristics of human behaviour, which are available to the computer, have subconscious character.

Mobilevision is a very special psychoinformation system, a core of the subconscious information stream from a human to a computer is geometric, namely, consists of geometric data on eye motions. Such data may be reduced to the coordinates of a sight point on the screen and its velocity.

Because both information streams in the mobilevision interactive system are essentially geometric, there is postulated a geometric correlation between them. Such correlation is encapsulated in dynamical laws of images, realized by a certain algorithm. These laws should be projectively invariant with respect to simultaneous projective transformations of image and sight geometric data.

However, a self-evident claim of projective invariance does not specify the dynamical laws completely. Another invariance of dynamical laws is related to symmetries of a color space. At the first approximation one has (due to Maxwell, Helmholtz and Young) a SU(3) color symmetry, which is really, however, broken. Nevertheless, an approximate SU(3)–symmetry is rather natural mathematical startpoint. Thus, one claims the dynamical laws of MV to be SU(3)–invariant.

2. The described suppositions are sufficient for a mathematization of mobilevision, i.e. for a derivation of the using of all necessary mathematical requisites from the first principles of MV.

First, let’s represent all geometric continuously distributed data of image by certain quantities \( f_i(x, y) \), where \((x, y)\) are coordinates on the screen. It is convenient to use their chiral factorisation \( f_i(x, y) = \sum_{j,k} a_{ijk} \phi_j(z) \phi_k(\bar{z}) \), where \( \phi_j(z) \) are holomorphic functions of a complex variable \( z \). The projective group PSL(2, \( \mathbb{C} \)) (or, at least, its Lie algebra sl(2, \( \mathbb{C} \)) acts on the quantities \( \phi_j(z) \) (”fields”) as on holomorphic \( \lambda \)–differentials. The color group SU(3) also acts on them globally, i.e. transforms them by a rule independent on a point. Actions of sl(2, \( \mathbb{C} \)) and SU(3) commute.

Let \( u \) be a complex coordinate of a sight point, \( \dot{u} \) be its velocity. It is rather natural to suppose that the dynamical laws are differential and that they express
the first time–derivatives of ”fields” as linear operators of ”fields” themselves with coefficients depending on $u$ and $\dot{u}$. The general form of such laws was written in (Juriev 1992). The differential equations were interpreted as quantum–field analogs of the Euler formulas. It should be marked that a quantum–field meaning was given to these formulas by their interpretation and was not derived from general invariance principles. However, such interpretation is a useful source to pick out the most important cases of the dynamical laws. However, one may avoid it and to have deal with operators in dynamical laws in purely mathematical fashion as with the vertex operator fields for the Lie algebra $\mathfrak{sl}(2, \mathbb{C})$. Such vertex operator fields form a certain algebraic structure (QPFT–operator algebra) described in details in (Juriev 1994a). However, the dynamical differential equations possess also SU(3) color symmetry, it manifests itself also as a symmetry of the related QPFT–operator algebra. QPFT–operator algebras with additional SU(3)–symmetries were described in (Juriev 1994a) under the title of projective SU(3)–hypermultiplets. The most natural class of projective SU(3)–hypermultiplets (the canonical projective $G$–hypermultiplets, SU(3) $\subset G$) was considered. Note that the canonical projective $G$–hypermultiplets are parametrized by a real number $q_R$, to which we are essentially addressed in the main text of the paper.

However, solitary Euler formulas are not SU(3)-invariant, so they should be completed by any other formulas. The most natural way to complete classical Euler formulas is to consider the Euler–Arnold equations. In our ”quantum–field” case it means to consider the operators of dynamical laws (of the ”quantum–field” Euler formulas) to be explicitly depending on a time, and to postulate their evolution to be governed by the Euler–Arnold equations (Juriev 1994a). The least have a hamiltonian form, and if a hamiltonian is SU(3)–invariant then the complete dynamical laws will be also SU(3)–invariant.

So the basic dynamical laws of mobilevision in a form of the ”quantum–field” Euler formulas coupled with the Euler–Arnold equations are derived from the first principles. Note that ”quantum–field” Euler formulas are fixed uniquely by the claim of projective invariance whereas the Euler–Arnold formulas may be replaced by any other ones, which will also provide the dynamical laws by SU(3)–invariance. Nevertheless, the Euler–Arnold formulas are, indeed, the most natural ”anzatz”.

Let’s now comment an appendix to (Juriev 1992), where stochastic Euler formu-
Note that the eye motions are not homogeneous. One may extract three different parts from them, namely, slow movements, saccads and tremor. The least may be naturally stochastized, i.e. be simulated by a certain stochastic process. It is resulted in an additional stochastic term in the Euler formulas. However, one may consider Euler formulas with an additional term from the beginning. In this case the dynamical laws are described by a stochastic linear differential equation of the form $\dot{\Phi} = A(u, \dot{u})\Phi \, dt + B(u, \dot{u})\Phi \, d\omega$, where operator fields $A$ and $B$ are independent (certainly, such equations are coupled with the deterministic Euler–Arnold equations on $A$ to provide $\text{SU}(3)$–invariance). To maintain the $\text{SU}(3)$–invariance one should claim $B$ to be $\text{SU}(3)$–invariant. Therefore, the most natural anzatz is to relate $B$ to a $\text{SU}(3)$–invariant spin–1 vertex operator field (current) in the projective $\text{SU}(3)$–hypermultiplet. The resulted stochastic equations are formally a certain ”quantum–field” analog of a form of Belavkin equations but without Belavkin counterterm, which provides exceptional nondemolition properties for solutions of Belavkin equations. Because this is just the effect, which we need for our purposes (see the main text), we shall include a ”quantum–field” analog of Belavkin counterterm (determined by a spin–2 $\text{SU}(3)$–invariant vertex operator field) in our equations by hands. However, one may consider such operation as deus ex machina, but such ”deus” has a very natural character.

Thus, we partially derived and partially motivated a transition from the deterministic dynamical laws to their stochastic analogs adopted in the main text.

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