Effects of Likelihood Functions on Value of Information Analysis in Resource Development

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Adequate information-gathering activities play a crucial role in making a rational decision under uncertainty in resource development. How adequate a particular information-gathering activity is can be quantified by using the concept of the value of information (VOI), which measures the possible increase of monetary value expected from an observation made by the information-gathering activity. In order to evaluate the VOI, one needs to model prior probability distributions for uncertain parameters, a monetary value function for each decision option, and a likelihood function representing the reliability of an observation. Among them, one often has little knowledge about a likelihood function, which is thus difficult to model. In this paper, we study the effects of likelihood functions on the VOI analysis in the context of resource development. Through numerical experiments, we show how the VOI is affected by the choice of a likelihood function. Moreover, we discuss how to include unknown bias of an observation in the definition of the VOI, and show its effect on the VOI analysis.

Keywords
Value of information, Likelihood function, Decision making, Unknown bias, Resource development

1. Introduction

Development of petroleum reservoirs is usually subject to some uncertainty due to their remoteness and broadness, and a rational decision making for designing a proper development scenario is recognized as a challenging problem. Adequate information-gathering activities play a crucial role in reducing some degree of uncertainty as well as determining the range of uncertainty, and as a result, in making a better decision on a development scenario. It is not necessarily the case, however, that either a reduction of uncertainty or an accurate determination of the range of uncertainty is directly connected to an increase of one’s ability to make a proper decision. Even if a certain information-gathering activity can reduce some degree of uncertainty, it may be worthless if it does not have any probability to change one’s decision regardless of its observation result. Thus, from a viewpoint of decision making, how adequate a particular information-gathering activity is should be measured in terms of a possibility to change one’s decision.

The concept of the value of information (VOI) is of use in addressing the above issue. It has been widely studied in the context of decision analysis, and has also been applied to petroleum engineering problems. The VOI measures the possible increase of monetary value expected from an observation made by an information-gathering activity. If an information-gathering activity does not have any probability to change one’s decision, no possible increase of monetary value can be expected, and thus, the VOI is zero. On the other hand, if an information-gathering activity has some probability to change one’s decision, there is some expected possible increase of monetary value, and thus, the VOI becomes positive. In this way, the VOI can be used as a quality criterion of an information-gathering activity.

By definition, in order to evaluate the VOI for a particular information-gathering activity, one needs to model the following three inputs: prior probability distributions for uncertain parameters, a monetary value function for each development scenario (i.e., each decision option), and a likelihood function representing the reliability of an observation. The first two can be modelled somehow from prior knowledge on a target reservoir, while we often have little knowledge on the last one so that its adequate modelling is largely unknown. Since the variance of likelihood distributions has been shown to affect the VOI significantly, an adequate modelling of a likelihood function is of importance in practical applications of the VOI analysis.

In this paper, as a continuation of the papers, we study the effects of likelihood functions on the VOI.
analysis in the context of resource development. Since the effects of the variance of likelihood distributions on the VOI were already examined as mentioned above, our primary interest of this paper is in the effects of higher order statistics of likelihood distributions on the VOI. We are also interested in the effects of first order statistic on the VOI, since first order statistic of likelihood distributions represents bias of an observation in the current context. If one knows bias beforehand, it does not affect the VOI at all as we shall prove later in this paper. In case that one does not know bias and makes a decision as if an observation were unbiased, we need to revisit the definition of the VOI and include unknown bias of an observation in a suitable manner. Then it becomes possible to see the effects of unknown bias on the VOI analysis.

The remainder of this paper is organized as follows. In the next section, we introduce the definition of VOI and its approximate evaluation by numerical integration. In Section 3, we explain about our setting of decision-making problem, which shall be used throughout this paper. Note that we follow the same problem set-up as that in the paper6 but with various choices of likelihood functions. In Section 4, we show the effects of likelihood distributions on the VOI, and moreover, discuss how to include unknown bias of an observation in the definition of the VOI and show its effect on the VOI. Finally we conclude this paper with some remarks.

2. VOI Analysis

In this section, we introduce the definition of VOI, and then discuss its approximate evaluation by numerical integration. Although a decision maker is assumed to be risk neutral throughout this paper, risk attitude can be incorporated by using an appropriate utility function7).

2.1. Definition of VOI

We first introduce the definition of VOI for a risk-neutral decision maker in a general setting of decision making. Let \( A \) be a set of alternative decision options with an individual option represented by \( a \). The task of a decision maker is to decide which option \( a \in A \) is optimal under uncertainty of \( X \), where \( X \) is supposed to be a continuous random variable defined on \( \Omega \). For every option \( a \in A \), a monetary value function \( f_a : \Omega \to \mathbb{R} \) is assigned.

For a risk-neutral decision maker, the optimal option is one which maximizes the expected monetary value

\[
\int_{\Omega} f_a(x) p_X(x) \, dx
\]

where \( p_X : \Omega \to \mathbb{R} \) denotes the probability density function of \( X \). Thus, the expected monetary value (EMV) without information is given by

\[
\text{EMV}_{\text{without}} = \max_{a \in A} \int_{\Omega} f_a(x) p_X(x) \, dx
\]

Now let us consider the situation where the decision maker can decide which option \( a \in A \) is optimal after information \( Y \) is given. The conditional probability density function of \( X \) given \( Y \) is \( p_{XY} (x|y) = \frac{p_{X|Y} (x,y) p_Y (y)}{\text{p}_{Y}} \) for any \( X = x \) and \( Y = y \), where \( p_{X|Y} \) and \( p_Y \) denote the joint probability density function of \( X \) and \( Y \) and the probability density function of \( Y \), respectively. Given \( Y = y \), the optimal option for a risk-neutral decision maker is one which maximizes the conditional expected monetary value

\[
\int_{\Omega} f_a(x) p_{XY} (x|y) \, dx
\]

Thus, the conditional expected monetary value given \( Y = y \) is

\[
g(y) = \max_{a \in A} \int_{\Omega} f_a(x) p_{XY} (x|y) \, dx
\]

Suppose that \( Y \) is a continuous random variable defined on \( \Omega' \). Then the expected monetary value with information \( Y \) is given by

\[
\text{EMV}_{\text{with}} = \int_{\Omega'} g(y) p_Y (y) \, dy = \int_{\Omega'} \left( \max_{a \in A} \int_{\Omega} f_a(x) p_{XY} (x|y) \, dx \right) p_Y (y) \, dy
\]

(1)

By using the chain rule of conditional probability and Bayes’ theorem, the probability density function \( p_Y \) and the conditional probability density function \( p_{XY} \) can be expressed for \( x \in \Omega \) and \( y \in \Omega' \) as

\[
p_Y (y) = \int_{\Omega} p_X (x) p_{XY} (y|x) \, dx
\]

and

\[
p_{XY} (x|y) = \frac{p_{X|Y} (y|x) p_X (x)}{p_Y (y)}
\]

respectively, where \( p_{X|Y} \) denotes the likelihood function which represents the reliability of information \( Y \). Substituting the above relations into Eq. (1), we obtain

\[
\text{EMV}_{\text{with}} = \int_{\Omega'} \left( \max_{a \in A} \int_{\Omega} f_a(x) p_{X|Y} (y|x) p_X (x) \, dx \right) p_Y (y) \, dy
\]

(2)

Finally, the value of information (VOI) for \( Y \) is defined as the possible increase of the expected monetary value, that is,

\[
\text{VOI} := \text{EMV}_{\text{with}} - \text{EMV}_{\text{without}}
\]

From the definitions of \( \text{EMV}_{\text{without}} \) and \( \text{EMV}_{\text{with}} \), one needs to have the following three inputs to evaluate the VOI:

\[
\cdot p_X : \text{a prior probability distribution for } X,
\cdot f_a : \text{a monetary value function for each alternative decision option } a \in A
\cdot p_{X|Y} : \text{a likelihood function of information } Y.
\]

Note that the VOI is ensured to be non-negative from the chain rule of conditional probability. If the VOI is
zero, there is no possibility of changing one’s decision and thus information \( Y \) is worthless. On the other hand, if the VOI is positive, there must be some possibility of changing one’s decision and information \( Y \) is worthwhile.

2.2. Approximate Evaluation of VOI

Since it is often difficult to evaluate the VOI analytically, one resorts to some computational algorithm to approximate the VOI. In the following, we assume that both \( \Omega \) and \( \Omega' \) denote the one-dimensional intervals \([a_x, b_x]\) and \([a_y, b_y]\), respectively, where \(-\infty < a_x < b_x \leq \infty \) and \(-\infty \leq a_y < b_y \leq \infty \).

Let us consider an approximation of EVGW first. We denote by \( P_X : [a_x, b_x] \to [0, 1] \) the cumulative distribution function of \( X \) given by

\[
P_X(x) = \int_a^x p_X(z)dz
\]

which satisfies \( P_X(a_x) = 0 \) and \( P_X(b_x) = 1 \). By transforming the variable according to the relation \( z = P_X(x) \), we have

\[
\text{EMV}_{\text{without}} = \max_{a_x} \int_a f_X(z)dz
\]

Numerical integration with \( N \) equi-distributed points gives an approximation

\[
\text{EMV}_{\text{without}} = \max_{a_x} \frac{1}{N} \sum_{n=1}^{N} f_X \left( \frac{n}{N} - \frac{1}{2N} \right)
\]

where \( P_X^{-1} \) denotes the inverse function of \( P_X \). Note that the \( n \)-th point is located at the middle of the interval \([n-1)/N, n/N] \).

Let us move on to an approximation of EMV with, to which we apply the idea of importance sampling. First, we introduce a new probability density function for \( Y \) which we denote by \( q_Y : [a_y, b_y] \to \mathbb{R} \). Note that the choice of \( q_Y \) is arbitrary as long as the Lebesgue measure of the set \( \{ y \in [a_y, b_y] \colon q_Y(y) = 0 \} \) is 0.

Multiplying and dividing the bracket on the right-hand side of Eq. (2) by \( q_Y \), we have

\[
\text{EMV}_{\text{with}} = \int_{\Omega'} \frac{1}{q_Y(y)} \left( \max_{a_x} \int_a f_X(z)p_{\text{GW}}(y|z)p_X(x)dz \right) q_Y(y)dy
\]

We denote by \( Q_Y : [a_y, b_y] \to [0, 1] \) the cumulative distribution function of \( Y \) given by

\[
Q_Y(y) = \int_a^y q_Y(w)dw
\]

which satisfies \( Q_Y(a_y) = 0 \) and \( Q_Y(b_y) = 1 \). By transforming the variables \( x \) and \( y \) according to the relations \( z = P_X(x) \) and \( w = Q_Y(y) \), we have

\[
\text{EMV}_{\text{with}} = \int_{\Omega'} \frac{1}{q_Y(Q_Y(w))} \left( \max_{a_x} \int_a f_X(z)p_{\text{GW}}(y|z)p_X(x)dz \right) dy
\]

In a way similar to an approximation of \( \text{EMV}_{\text{without}} \), numerical integration with \( M \times N \) equi-distributed points gives an approximation

\[
\text{EMV}_{\text{with}} = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{q_Y(W_m)} \left( \max_{a_x} \frac{1}{N} \sum_{n=1}^{N} f_X(Z_n)p_{\text{GW}}(W_m|Z_n) \right)
\]

where we write

\[
W_m = Q_Y^{-1}\left( \frac{m}{M} - \frac{1}{2M} \right) \quad \text{and} \quad Z_n = P_X^{-1}\left( \frac{n}{N} - \frac{1}{2N} \right)
\]

for \( 1 \leq m \leq M \) and \( 1 \leq n \leq N \).

The value of \( CV \) approximates the VOI. Although we shall omit a detailed argument on the choice of \( q_Y \) in the remainder of this paper, we always choose \( q_Y \) carefully depending on the problems at hand.

3. Problem Setting

We now introduce our problem setting by following the exposition in Section 7 of the paper. Let us consider a situation where a decision maker has to decide whether or not to develop a target reservoir under uncertainty about the size of the reservoir. The size of the reservoir is denoted by \( X \) as in the previous section, and \( X \) is assumed to be log-normally distributed as \( \ln X \sim N(\mu, \sigma^2) \), that is,

\[
px(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp \left[ -\frac{(\ln x - \mu)^2}{2\sigma^2} \right]
\]

for \( x > 0 \). Suppose that a decision maker knows the mean \( \mu_X \) and the standard deviation \( \sigma_X \) of \( X \) as prior knowledge. Here we have \( \mu = 2 \ln \mu_X - \ln(\sigma_X^2 + \mu_X^2)/2 \) and \( \sigma^2 = \ln(1 + \sigma_X^2/\mu_X^2) \). In the following argument, we shall normalize the uncertainty of \( X \) by using the coefficient of variation \( CV = \sigma_X/\mu_X \). The value of \( CV \) measures the uncertainty of prior knowledge.

In this situation, the number of alternative options is \( |\mathcal{A}| = 2 \): one is to develop the reservoir \( (a = 1) \), the other one is not to develop the reservoir \( (a = 2) \). If a decision maker chooses the option \( a = 2 \) any monetary value cannot be gained, so that \( f_2 \equiv 0 \). On the other hand, if a decision maker chooses the option \( a = 1 \), some monetary value can be gained by developing the reservoir. We assume that \( f_1 \) depends only on \( X \) and is given by \( f_1(x) = 0.01x^2 - 100 \). For \( x < 100 \), the reservoir is too small to cover investment costs, so that the monetary value becomes negative. For \( x > 100 \), on the other hand, we can expect more and the monetary value becomes positive.

Now a decision maker may conduct some test that estimates the reservoir extent. Let \( Y \) denote an observation made by this information-gathering activity. In the papers, the reliability of \( Y \), i.e., the likelihood

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function, is given by a symmetric triangular distribution
\[
p_{Y|X}(y|x) = \begin{cases} 
\frac{2(y-x) - d_{l,\text{min}}}{d_{l,\text{max}} - d_{l,\text{min}}} & \text{if } y - x \leq d_{l,\text{min}} \\
\frac{2(y-x)}{d_{l,\text{max}} - d_{l,\text{min}}} & \text{if } d_{l,\text{min}} < y - x < d_{l,\text{mode}} \\
\frac{2(y-x)}{d_{l,\text{max}} - d_{l,\text{min}}} & \text{if } d_{l,\text{mode}} \leq y - x < d_{l,\text{max}} \\
0 & \text{otherwise}
\end{cases}
\]  
for some \(d_l > 0\). Given \(X = x\), the mean and the variance of \(Y\) are \(x\) and \(d_l^2/6\), respectively. Obviously, \(d_l\) small means that the information accuracy is high. When \(d_l = 0\), information is perfect. For normalization, the parameter \(b = d_l/\mu_X\) was introduced\(^{6}\), and shall be used also in this paper.

In our numerical experiments below, we shall consider various types of likelihood functions and see the effects of likelihood functions in the VOI in the present problem. Since the effect of the standard deviation has been already studied in the papers\(^{5,6}\), our primary interest is to see the effects of higher order statistics of likelihood functions. The third order statistics, or the skewness, of likelihood functions can be controlled by imposing the conditions that the mean equals \(X\) and the variance equals \(d_l^2/6\), respectively. Thus for the cases \(X = 120\), \(b = 0.5\), and different values of skewness \(\gamma\) from \(-10\) to 10.

The VOI profiles as a function of \(CV\) for \(b = 0.5\), different values of \(\mu_X\) and \(\gamma\) are compared in Fig. 2. The VOI is almost 0 up to around \(CV = 0.05\) and \(CV = 0.15\) for the cases \(\mu_X = 95\) and \(\mu_X = 120\), respectively. Beyond those values of \(CV\), the VOI increases monotonically as the uncertainty increases for both cases. For the case \(\mu_X = 95\), the VOI becomes larger when \(\gamma\) is

![Asymmetric Triangular Distributions for Different Values of \(\gamma\)](image)

![VOI Profiles as a Function of CV for Different Values of \(\mu_X\) and \(\gamma\)](image)
negative, whereas the VOI becomes larger when $\gamma$ is positive for the case $\mu_X = 120$. For both cases, the difference of the VOI for various values of $\gamma$ gets smaller as the uncertainty increases. Thus this means that the skewness $\gamma$ has little impact on the VOI when the uncertainty of prior knowledge is high.

The insensitivity of $\gamma$ on the VOI can be also observed in Figs. 3 and 4, where the VOI profiles as a function of $\gamma$ with $b = 0.5$ are shown for the cases $\mu_X = 95$ and $\mu_X = 120$, respectively. For both cases, the VOI profiles are almost parallel to the horizontal axis, meaning that the VOI is not significantly affected by the value of $\gamma$. Note that when CV is small, the VOI slightly decreases as $\gamma$ increases for the case $\mu_X = 95$, whereas the VOI slightly increases for the case $\mu_X = 120$. Although we do not show the results for other values of $b$ here, we could see the similar effect of $\gamma$ on the VOI as observed above.

### 4.2. Effect of Even Higher Order Statistics

In order to see the effect of even higher order statistics of likelihood functions, here we consider three types of distribution functions: a symmetric triangular distribution (Eq. (3)), an uniform distribution

$$p_{trf}(y|x) = \begin{cases} \frac{1}{2d_u} & \text{if } |y - x| \leq d_u \\ 0 & \text{otherwise} \end{cases}$$

for some $d_u > 0$, and a normal distribution

$$p_{nrf}(y|x) = \frac{1}{\sqrt{2\pi d_n}} \exp \left( -\frac{(y - x)^2}{2d_n^2} \right)$$

for some $d_n > 0$. For any distribution, the mean of $Y$ equals $x$ and the skewness equals 0. In order to keep the variance unchanged, we always set $d_u^2/b = d_u^2/3 = d_n^2$.

Then the difference between three types of distribution functions only stems from higher order statistics beyond the third order. Figure 5 shows symmetric triangular, uniform and normal distributions for $\mu_X = 120$, $b = 0.5$.

The VOI profiles as a function of CV for $b = 0.5$, different values of $\mu_X$ and different distribution functions are compared in Fig. 6. Again, the VOI is almost 0 up to around CV = 0.05 and CV = 0.15 for the cases $\mu_X = 95$ and $\mu_X = 120$, respectively, beyond which the VOI increases monotonically as the uncertainty increases for both cases. Among three different distributions, the
VOI for the uniform distribution is largest when CV is small, whereas the VOI for the triangular distribution becomes largest when CV is large. However, the difference of the VOI for three distributions is small enough for the whole range of CV to claim that the VOI is insensitive to the choice of a distribution function which yields the different higher order statistics beyond the third order. The same claim can be also applied to the cases with other values of $b$, although we do not show the results here.

4.3. Effect of Bias

4.3.1. The case of Known Bias

We denote by $\Delta \in \mathbb{R}$ the knowledge of an observation and denote by $p^0_{y|x}$ and $p^\Delta_{y|x}$ the likelihood functions without and with bias, respectively. That is, given $X = x$, the means of $Y$ for these distribution functions are $x$ and $x + \Delta$, respectively. For instance, $p^0_{y|x}$ is given by Eq. (3) and $p^\Delta_{y|x}$ by

$$p^\Delta_{y|x}(y|x) = \left\{ \begin{array}{ll} 0 & \text{if } y - (x + \Delta) \leq d_i \\ 1/d_i & \text{otherwise} \end{array} \right. \quad (5)$$

Since $p^\Delta_{y|x}$ is obtained by shifting $p^0_{y|x}$ by $\Delta$, we have

$$p^\Delta_{y|x}(y|x) = p^0_{y|x}(y - \Delta|x) \quad (6)$$

If a decision maker knows $\Delta$ beforehand, EMV with for him/her is given by definition as

$$\text{EMV}_{\text{with}} = \int_\Omega \max_{a \in A} \int_\Omega f_{A}(y) p^\Delta_{y|x}(y|x) p_x(x) \, dx \, dy$$

Using Eq. (6) and by transforming the variable $y$ into $z = y - \Delta$, we have

$$\text{EMV}_{\text{with}} = \int_\Omega \int_\Omega f_{A}(y) p^0_{y|x}(y - \Delta|x) p_x(x) \, dx \, dy \quad (7)$$

As a result, the expected monetary value with information $Y$ containing unknown bias $\Delta$ is given by

$$\text{EMV}_{\text{with}}^\Delta = \int_\Omega \max_{a \in A} \int_\Omega f_{A}(y) p^\Delta_{y|x}(y|x) p_x(x) \, dx \, dy \quad (8)$$

Our new definition reduces to the original definition as in Eq. (1) when $\Delta = 0$.

We now prove that $\Delta = 0$ is a maximizer of $\text{EMV}_{\text{with}}^\Delta$, which implies that our new definition no longer ensures the non-negativity of the VOI unless $\Delta = 0$. By substituting Eq. (7) into Eq. (8), using Eq. (6) and transforming the variable $y$ into $z = y - \Delta$, we have

$$\text{EMV}_{\text{with}}^\Delta = \int_\Omega \int_\Omega f_{A}(y) p^\Delta_{y|x}(y|x) p_x(x) \, dx \, dy \quad (9)$$

where $\Omega'$ is given by shifting the interval $\Omega''$ by $\Delta$. For $\Omega' = [-\infty, \infty]$ as in the present problem, we have $\Omega'' = \Omega'$. Thus the last expression of $\text{EMV}_{\text{with}}^\Delta$ is the same as Eq. (2) with $p_{y|x} = p^\Delta_{y|x}$, which implies that the presence of bias $\Delta$ does not affect the VOI at all.

4.3.2. The Case of Unknown Bias

Here we discuss what happens if a decision maker does not know bias of an observation and makes a decision as if an observation were unbiased. Given $Y = y$, due to his/her unawareness of bias, the optimal option is one which maximizes the conditional expected monetary value with respect to $p^\Delta_{y|x}$,

$$\int_\Omega f_{A}(x) p^\Delta_{y|x}(x|y) \, dx$$

where we have

$$p^\Delta_{y|x}(y|x) = \int_\Omega p^\Delta_{y|x}(y|x) p_x(x) \, dx$$

and

$$p^0_{y|x}(y|x) = \int_\Omega p^0_{y|x}(y|x) p_x(x) \, dx$$

We now introduce a decision function $\mathcal{A} : \Omega' \to A$ as

$$\mathcal{A}(y) = \arg \max_{a \in A} \int_\Omega f_{A}(x) p^\Delta_{y|x}(x|y) \, dx$$

By definition, this function returns the optimal option $a \in A$ for a decision maker as a function of $Y = y$.

Suppose that an observation is biased indeed. Given $Y = y$, the monetary value which a decision maker can actually gain becomes

$$\int_\Omega f_{A}(x) p^\Delta_{y|x}(x|y) \, dx$$

where similarly to $p^\Delta_{y|x}$ we have

$$p^\Delta_{y|x}(y|x) = \int_\Omega p^\Delta_{y|x}(y|x) p_x(x) \, dx$$

and

$$p^0_{y|x}(y|x) = \int_\Omega p^0_{y|x}(y|x) p_x(x) \, dx$$

As a result, the expected monetary value with information $Y$ containing unknown bias $\Delta$ is given by

$$\text{EMV}_{\text{with}}^\Delta = \int_\Omega \max_{a \in A} \int_\Omega f_{A}(y) p^\Delta_{y|x}(y|x) p_x(x) \, dx \, dy$$

Our new definition reduces to the original definition as in Eq. (1) when $\Delta = 0$.

We now prove that $\Delta = 0$ is a maximizer of $\text{EMV}_{\text{with}}^\Delta$, which implies that our new definition no longer ensures the non-negativity of the VOI unless $\Delta = 0$. By substituting Eq. (7) into Eq. (8), using Eq. (6) and transforming the variable $y$ into $z = y - \Delta$, we have

$$\text{EMV}_{\text{with}}^\Delta = \int_\Omega \int_\Omega f_{A}(y) p^\Delta_{y|x}(y|x) p_x(x) \, dx \, dy$$

where $\Omega''$ is given by shifting the interval $\Omega'$ by $\Delta$, and the last inequality stems from the definition of $\mathcal{A}$. This result is quite intuitive since a biased observation seems not to be informative as compared to an unbiased observation.

We note that approximate evaluation in Subsection 2.2. is still applicable with a slight modification. In the following, we shall normalize $\Delta$ by introducing $\delta = \Delta/\mu_x$ and use a symmetric triangular distribution Eq. (5) as a likelihood function $p^\delta_{y|x}$ with unknown bias $\Delta$.

The VOI profiles as a function of CV for $b = 0.5$, different values of $\mu_x$ and $\delta$ are compared in Fig. 7. As
Although we do not show the difference of the VOI for different values of skewness only when a prior uncertainty is already low. Hence, we need to be cautious about the value of the skewness to zero in a situation where a prior uncertainty is high. For instance, if there is available data beforehand, it does not affect the VOI at all.

The skewness $\gamma$ of a likelihood function has little impact on the VOI when a prior uncertainty is high. Hence, we gave a modification of the definition in this paper. The existence of unknown bias significantly affects the VOI particularly when the bias is positive.

From the second conclusion above, one can focus on, for instance, an asymmetric triangular distribution as a likelihood function. As key parameters which affect the VOI, the first three order statistics (mean, variance, and skewness) of a likelihood function need to be evaluated with care in practical applications of the VOI analysis. From the first conclusion above, one can set the skewness to zero in a situation where a prior uncertainty is high. For instance, if there is available data on the measurement error, i.e., the level of information

![Fig. 7 VOI Profiles as a Function of CV for Different Values of $\mu_x$ and $\Delta$](image1)

![Fig. 8 VOI Profiles as a Function of $\delta$ for $\mu_x = 95$ and Different Values of CV](image2)

![Fig. 9 VOI Profiles as a Function of $\delta$ for $\mu_x = 120$ and Different Values of CV](image3)

can be seen later in Figs. 8 and 9 that the VOI is not significantly affected by $\delta$ when $\delta$ is negative, we only show the VOI profiles for non-negative values of $\delta$. It is obvious that the VOI heavily depends on the value of $\delta$, and in particular, decreases as $\delta$ increases. When $\mu_x = 95$, for instance, the VOI is almost 0 up to around CV = 0.05 for the case $\delta = 0$, whereas it is up to around CV = 0.15 for the case $\delta = 0.5$. The VOI for the case CV = 0.5, $\delta = 0.5$ roughly equals 20, which is about two-thirds of the VOI for the case CV = 0.5, $\delta = 0$. The difference of the VOI for different values of $\delta$ can be also clear when $\mu_x = 120$. Note that the VOI is negative for the whole range of CV for the case $\mu_x = 120$, $\delta = 0.5$.

The sensitivity of $\delta$ on the VOI can be also observed in Figs. 8 and 9, where the VOI profiles as a function of $\delta$ with $b = 0.5$ are shown for the cases $\mu_x = 95$ and $\mu_x = 120$, respectively. Interestingly, the VOI profiles are almost parallel to the horizontal axis up to $\delta = 0$ for both cases, meaning that the VOI is not significantly affected by the value of $\delta$ if $\delta$ is negative. Looking closer at the results, we can see that the VOI increases very slightly up to $\delta = 0$, beyond which the VOI declines rapidly as $\delta$ increases. Thus, the VOI reaches its peak at $\delta = 0$, which supports the above proof that $\delta = 0$ is a maximizer of $EMV_\Delta^{\mu_x}$. Although we do not show the results for other values of $b$ here, we could see the similar effect of $\delta$ on the VOI as observed above.

5. Concluding Remarks

In order to see the effects of likelihood functions on the VOI analysis in the context of resource development, we conducted a series of numerical experiments from which we have arrived at the following conclusions:

- The skewness $\gamma$ of a likelihood function has little impact on the VOI when a prior uncertainty is high. Hence, we need to be cautious about the value of the skewness only when a prior uncertainty is already low.
- Higher order statistics beyond the third order have much less impact on the VOI for a wide range of a prior uncertainty. In other words, the VOI is robust for different types of likelihood functions.
- If a decision maker knows bias of information beforehand, it does not affect the VOI at all.
- If not, on the other hand, the original definition of the VOI is no longer an adequate quality criterion of information, so that we gave a modification of the definition in this paper. The existence of unknown bias significantly affects the VOI particularly when the bias is positive.

From the second conclusion above, one can focus on, for instance, an asymmetric triangular distribution as a likelihood function. As key parameters which affect the VOI, the first three order statistics (mean, variance, and skewness) of a likelihood function need to be evaluated with care in practical applications of the VOI analysis. From the first conclusion above, one can set the skewness to zero in a situation where a prior uncertainty is high. For instance, if there is available data on the measurement error, i.e., the level of information...
reliability, collected from previous similar projects, maximum likelihood estimation becomes an effective tool in evaluating the mean and the variance of a likelihood function.

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要 旨
資源開発における情報の価値分析への尤度関関数の影響
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資源開発では不確実性下において合理的な意思決定を行うために適切な情報収集が必要である。ある情報収集がどの程度適切なのかを定量的に測る指標として情報の価値 (VOI: value of information) 分析という概念がある。情報収集で得られる観測の結果に応じて意思決定がより合理化され、それによって期待される利益の増大量がVOIである。VOIを評価するには、不確実なパラメーターに対する事前確率分布、意思決定で取りえる選択肢それぞれに対する利潤関数、観測の信頼性を表現する尤度関数の三つを設定する必要がある。これらのうち、特に尤度関数に関する事前の知識を得ることは困難であり、モデル化は難しいと考えられる。そこで、本研究では資源開発において尤度関数がVOI分析に及ぼす影響について検討する。具体的には、数値実験を通過して、尤度関数の選定がVOIの値にどの程度影響するのかを示す。さらに、観測が未知のバイアスを有している場合のVOIの定義について議論し、その影響を見る。