Coherent control of single photons in the cross resonator arrays via the dark state mechanism

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Received 15 October 2012 / Received in final form 6 January 2013
Published online 5 April 2013 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2013

Abstract. We study the single photon transfer in a hybrid system where the normal modes of two coupled resonator arrays interact with two transition arms of a $\Lambda$-type atom localised in the intersectional resonator. It is found that, due to the Fano-Feshbach effect based on the dark state of the $\Lambda$-type atom, the photon transfer in one array can be well controlled by the bound state of the photon in the other array. This conceptual setup could be implemented in some practical cavity QED system to realise a quantum switch for single photon.

1 Introduction

In quantum information physics and technology, photons play an important role since they can robustly transfer information over a long distance as a flying qubit in free space. High-fidelity transfer of an independently prepared quantum state from photons onto an atomic ensemble has been experimentally feasible [1]. Most recently, the study of confined photons in low dimension structures, such as the coupled resonator array (CRA), is attracting more and more attention [2–7]. The nonlinear dispersion relation of CRA systems can result in single photon quasi-bound states [6,8,9], which can be applied to realise information storage and coherent control of single photon transmission in a hybrid system. Some special atomic media enhance the nonlinearity of the resonator hence are capable of demonstrating the photon blockade phenomenon [10–12].

Moreover, a one-dimension wave guide constructed by a CRA with an atom embedded in can realize controllable photon transport. The two-level atom within the wave guide acts as a perfect mirror for the light field at resonance [9]. To realise a better tunable mirror, people use the three-level atom instead of the two-level one, thus the electromagnetically induced transparency (EIT) mechanism can be utilised to control the behaviour of the probe photon by a classical control light beam [8,13]. However, we prefer a full quantum network without introducing any classical element. This consideration motivates us to discuss the Fano-Feshbach resonate in the CRA system, which is an analog of the Fano resonance in the ionisation process of the atomic system [14] or the Feshbach resonate used in the cold atom system for controlling the interaction strength [15,16]. In our consideration, if a photon bound state is formed in one transfer channel, the transmission feature of the photon in another channel is greatly influenced [17].

In this paper, we study the coherent transport of a single photon in two crossed CRAs with a $\Lambda$-type atom embedded in the intersectional resonator. We use the discrete coordinates method [6] to calculate the transmission and reflection coefficients of the incident photon. We find that the photon incidenting in one array is perfectly reflected when it resonates with single photon bound states in the other array, namely, we use the Feshbach resonance mechanism to control the transmission of single photons in this two-channel CRA system. Under the two-photon resonate condition, we find that when the incident photon is perfectly transmitted or reflected, its wave function has the maximum overlap with the dark or bright states in the intersectional resonator. This implies the EIT mechanism [18,19] intrinsically exists in our system.

This paper is organised as follows: in Section 2, we present the model Hamiltonian for a single photon scattered by a $\Lambda$-type atom in two crossed CRA systems. In Section 3, we study the single photon scattering process, and the transmission coefficient is obtained by discrete coordinate scattering equations. In Section 4, three non-trivial cases where the photon is totally reflected or transmitted are well studied. In Section 5, we show the role of the dark state mechanism in the controlling of photon transmission. The conclusion and physical realisation is given in Section 6.

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between $|f\rangle$ and $|e\rangle$, where $J_a$ and $J_b$ are the coupling strengths respectively. Under the rotating wave approximation, the atom-photon interaction is modelled as a Jaynes-Cummings Hamiltonian,

$$H_I = \varepsilon_e \langle e | \langle e | + \varepsilon_f \langle f | \langle f | + J_a \langle a_0 | \langle e | \langle g | \rangle + H.c.$$ 

$$+ J_b \langle b_0 | \langle e | \langle f | \rangle + H.c. \rangle. \quad (2)$$

We choose the energy of the ground state as zero, $\varepsilon_e$ and $\varepsilon_f$ are the energies of the excited state and the metastable state respectively. Thereafter, we set $\hbar = 1$.

### 3 Single photons scattering

To explore the single photon scattering behaviour in this model, we consider a single photon incident into the vertical resonator array chain $B$ from the resonator at $-\infty$. The total excitation number $N = |e\rangle\langle e| + \sum_m a^\dagger_m a_m + \sum_n b^\dagger_n b_n$ is conserved. The eigen state of the total Hamiltonian $H = H_C + H_I$ can be expressed as:

$$|E\rangle = \sum_m u_g(m) |m,g\rangle + \sum_n u_f(n) |n,f\rangle + u_e |\phi,e\rangle, \quad (3)$$

within the single excitation subspace. Where $|m,g\rangle \equiv |m\rangle \otimes |g\rangle$, $|n,f\rangle \equiv |n\rangle \otimes |f\rangle$ is the state with one photon in the $m$th ($n$th) resonator of the chain $A$ ($B$) while the atom in the ground ( metastable) state, $|\phi,e\rangle$ is the state with no photons in the CRAs and the atom in the excited state. $u_g(m)$, $u_f(n)$, and $u_e$ are the probability amplitudes for the corresponding states. According to the stationary Schrödinger equation $H |E\rangle = E |E\rangle$, we eliminate $u_e$ and obtain the equations for $u_g(m)$ and $u_f(n)$ as:

$$[E - \omega_a + V_a(m)] u_g(m) + V(m) u_f(0) = -\xi_a [u_g(m+1) + u_g(m-1)], \quad (4)$$

$$[E - \omega_b - \varepsilon_f + V_b(n)] u_f(n) + V(n) u_g(0) = -\xi_b [u_f(n+1) + u_f(n-1)]. \quad (5)$$

Here, we define the effective potentials as

$$V_{a(b)}(i) \equiv \frac{J_{a(b)}}{E - \varepsilon_e} \delta_{i,0}, \quad (6)$$

$$V(i) \equiv \frac{J_a J_b}{E - \varepsilon_e} \delta_{i,0}. \quad (7)$$

The strengths of the $\delta$-type potentials $V_{a(b)}$ and $V$ are related to the hopping strength $J_{a(b)}$, the excitation energy $\varepsilon_e$ of the atom, and especially the energy $E$ of the incident photon itself. We assume and the probability amplitudes $u_g(m)$ in chain $A$ and $u_f(n)$ in chain $B$ have the plane-wave solutions,

$$u_g(m) = \begin{cases} A e^{i k m} & m < 0, \\ A e^{-i k m} & m > 0, \end{cases} \quad (8)$$
and

\[ u_f (n) = \begin{cases} e^{ik'n} + r e^{-ik'n} & n < 0, \\ \frac{r}{s} e^{ik'n} & n > 0, \end{cases} \tag{9} \]

where \( A \) is the normalisation constant, \( r \) and \( s \) are reflection and transmission coefficients of the photon in chain \( B \). \( k \) and \( k' \) are the wave vectors of the two chains.

If we substitute equations (8) and (9) into the scattering equations equations (4) and (5), when \( m, n \neq 0 \), we obtain the dispersion relations for two chains as:

\[ E = \omega_a - 2\xi_a \cos k, \tag{10} \]
\[ E = \omega_b + \varepsilon_f - 2\xi_b \cos k'. \tag{11} \]

In the intersectional resonator, the continuous condition \( u_f (0^+) = u_f (0^-) \) leads to

\[ 1 + r = s. \tag{12} \]

We solve the scattering equations (4) and (5) for the intersectional resonator with the help of equations (10)–(12) and obtain the transmission amplitude \( s \) as

\[ s = \frac{\frac{in (E)}{i\kappa (E) + \frac{\sqrt{f (E)}}{J_a^2 (E-\varepsilon_a)^2 + \varepsilon_f^2}} - \frac{\sqrt{f (E)} \varepsilon_f}{J_a^2 (E-\varepsilon_a)^2 + \varepsilon_f^2}} \tag{13} \]

with \( \kappa (E) \) is defined as

\[ \kappa (E) \equiv \sqrt{4\xi_a^2 - (E - \omega_b - \varepsilon_f)^2} \tag{14} \]

and

\[ \zeta (E) = \begin{cases} \sqrt{f (E)}, & E \in [\omega_b + \varepsilon_f - 2\xi_b, \omega_a - 2\xi_a] \\
\frac{i\sqrt{f (E)}}{\sqrt{f (E)}}, & E \in [\omega_a - 2\xi_a, \omega_a + 2\xi_a] \\
-\sqrt{-f (E)}, & E \in [\omega_a + 2\xi_a, \omega_b + \varepsilon_f + 2\xi_b] \end{cases} \tag{15} \]

where we introduce the notation \( f (E) = 4\xi_a^2 - (E - \omega_a)^2 \).

4 Fano-Feshbach reso
tate effect

In the above section we have obtained the single photon transmission amplitude in chain \( B \). From equations (10) and (11), the energy spectra for single photons in chain \( A \) and chain \( B \) have band structures. Due to the interaction with the \( A \)-type atom, there are also two isolated bound state levels outside the band. In chain \( A \), if the wave vector \( k \) has a negative imaginary part the photon wave function will decay with the distance from the intersection resonator. We call this state a single photon bound state. The complex wave vector \( k \) with negative imaginary part corresponds to the photon bound states. The single photon bound state in one-dimension CRA is discussed in the Appendix.

In Figure 2, we plot the photon transmission rate \( T = |s|^2 \), reflection rate \( R = |r|^2 \) and \( |s|^2 + |r|^2 \) as a function of photon energy \( E \), from which we find the perfect transmission and reflection occur at certain resonant points. Firstly, the transmission generally vanishes at the energy band boundaries of chain \( B \) with \( k = 0, \pi \). It follows from equation (13) that these energies are zeros of \( s \). Secondly, the photon is totally reflected when the incident energy equals to the energy band boundaries of chain \( A \) which correspond to \( k' = 0, \pi \). Thirdly, the energies of single photon bound states in chain \( A \) can be obtained by solving the transcendental equation

\[ E = \varepsilon_e \pm J_a^2 / \sqrt{(E - \omega_a)^2 - 4\xi_a^2} \tag{16} \]

which gives \( E = 0.68 \) and \( E = 1.3 \) by choosing parameters as \( \varepsilon_e = 0.95, \omega_a = 1, J_a = 0.15, \) and \( \xi_a = 0.15 \). When the photon incidents with the energy accidentally resonating with the bound state energies, the perfect reflection takes place. This is just the Fano-Feshbach resonate phenomenon. In addition, when energy of the photon is in the range \( \omega_a - 2\xi_a < E < \omega_a + 2\xi_a \), the summation of the photon transmission rate and reflection rate is less than 1 as shown in Figure 2 since the incident photon in chain \( B \) can be scattered into chain \( A \). In Figure 3, we plot the energy spectrum sketch map of our system and display three kinds of nontrivial cases for \( R = 1 \) and \( T = 1 \).

Considering the improvement of experimental techniques in solid state systems, our model can be realised in several physical systems of superconducting qubits interacting with microwave stripline resonators [20], quantum dot with photonic crystal defects [21], as well as a nature atom with monolithic microresonator [22]. Especially it is reported that in a single quantum dot and semiconductor microcavity system, the coupling strength is about 70 µeV [23] and the coupling strength between cavities can achieve 0.8 THz [24]. These parameters are suitable for our consideration in the above discussion.
5 Dark state mechanism

In the intersectional resonator, the $A$-type atom couples to two field modes of the two CRAs, so that we can realise the EIT effect by tuning one of the field modes. In the present section we will explore the coherent control for the single photon transfer in our system.

The effective Hamiltonian in the interaction picture is obtained as

$$H'_I = e^{iH_0't}H_I e^{-iH_0't} - H'_0$$
$$= -\Delta_1 |e\rangle \langle e| - \Delta_2 |f\rangle \langle f|$$
$$+ |[J_a |e\rangle \langle g| a + J_b |e\rangle \langle f| b] + h.c.]. \quad (17)$$

We omit the subscript 0 of the photon annihilation and creation operators in the intersectional resonator. Here we choose

$$H'_0 = \varepsilon_f |f\rangle \langle f| + \varepsilon_e |e\rangle \langle e| + \omega_b a^\dagger a + \omega_b b^\dagger b$$
$$+ \Delta_1 (J_a |e\rangle \langle g| a + J_b |e\rangle \langle f| b) - \Delta_2 |f\rangle \langle f| \quad (18)$$

with $\Delta_1 = \varepsilon_e - \omega_a$ and $\Delta_2 = \varepsilon_e - \varepsilon_f - \omega_b$ are detunings.

Under the two-photon resonant condition $\Delta_1 = \Delta_2 = \Delta$, the interaction Hamiltonian is

$$H'_I = -\Delta |e\rangle \langle e| + |[J_a |e\rangle \langle g| a + J_b |e\rangle \langle f| b] + h.c.]. \quad (19)$$

The eigenvalues of the above Hamiltonian are

$$E_\pm = \frac{\Delta \pm J'}{2},$$
$$E_0 = 0. \quad (20)$$

With the corresponding eigenstates:

$$|B_\pm\rangle = \frac{1}{\chi} \left( |[J' + \Delta] |\phi, e\rangle \pm 2 (J_a |0, g\rangle + J_b |0, f\rangle) \right) \quad (21)$$

$$|D\rangle = \frac{1}{J} (J_a |0, f\rangle - J_b |0, g\rangle),$$

where $J = \sqrt{J_a^2 + J_b^2}$, $J' = \sqrt{4(J_a^2 + J_b^2) + \Delta^2}$, and $\chi = \sqrt{(J' - \Delta)^2 + 4J^2}$.

The state $|D\rangle$ with vanishing eigen-energy is called dark state because it does not evolve with time and does not transmit to the excited state. The other two states $|B_\pm\rangle$ are called bright states.

We rewrite the total Hamiltonian $H$ in the basis of the state $|B_\pm\rangle$, $|D\rangle$, $|m, g\rangle$, $|n, f\rangle$ to include the free Hamiltonian

$$H_{free} = \left( \frac{4J^2 - \Delta^2}{2J} + \omega_a + \omega_b \right) |B_+\rangle \langle B_+|$$
$$+ \left( \omega_a + \omega_b - \frac{1}{2} \right) |B_-\rangle \langle B_-| + \omega_n |D\rangle \langle D|$$
$$+ (\omega_b + \varepsilon_f) \left( \sum_{m \neq 0} a_m^\dagger a_m + \sum_{n \neq 0} b_n^\dagger b_n \right) \quad (23)$$

and the coupling terms among these states

$$H_{coup} = \frac{2\Delta J}{J} (|B_+\rangle \langle B_-| + h.c.)$$
$$+ \left\{ |\{1, f\} + |1, g\} - |1, g\} - |1, g\} \right\} \times \chi(\frac{J' - \Delta}{J}) (J_a |\xi_a \langle B_+| + J_b |\xi_b \langle B_-|$$
$$+ \frac{1}{J} (J_a |1, g\} + |1, g\} - |1, g\}) \langle D|$$
$$- J_a |\xi_a \langle 1, f\} + |1, f\}) \langle D| + h.c.)$$
$$- \xi_a \sum_{m} (a_m^\dagger a_{m+1} + h.c.)$$
$$- \xi_b \sum_{n} (b_n^\dagger b_{n+1} + h.c.). \quad (24)$$

In the intersectional resonator, the couplings between the single excitation states are displayed in Figure 4a. Diagrams illustrating the coherent interactions mediated by the dark state and two bright states are also shown in Figure 4b.

In order to study the effect of the dark state and bright state channels in single photon transmissions, we adjust
Fig. 4. (Color online) (a) Level coupling scheme under the original presentation. (b) Level coupling scheme under the dark state presentation.

The system parameters to satisfy the two photon resonate condition \( \omega_a = \omega_b + \epsilon_f \). It can be seen from equation (23) that the expectation value of the Hamiltonian for the dark state is \( \omega_a \), which is equal to that of other single excitation states outside the resonators. It means that photon transmission through the dark state channel is easier than the other two bright channels. To explore the different roles of the dark state channel and two bright state channels for the photon transmission, we calculate the overlap between the single photon energy eigenstate and these three states, which are expressed as

\[
\langle E | D \rangle = \frac{1}{\chi} \left[ J_a u_f^*(0) - J_b u_g^*(0) \right],
\]

\[
\langle E | B_+ \rangle = \frac{1}{\chi} \left[ 2 J_a u_g^*(0) + 2 J_b u_f^*(0) + (J' - \Delta) u_e^* \right],
\]

\[
\langle E | B_- \rangle = -\frac{1}{\eta} \left[ 2 J_a u_g^*(0) + 2 J_b u_f^*(0) - (\Delta + J') u_e^* \right],
\]

with

\[
\eta = \sqrt{(\Delta + J')^2 + 4J^2}.
\]

Here \( u_g(0) \) and \( u_f(0) \) are defined in equations (8) and (9).

In Figure 5, we plot the norm of the overlap between the scattering state \( |E\rangle \) and the dark state \( |D\rangle \). The dashed (red) line represents the transmission rate of the incident photon.

In Figures 6 and 7, we plot the norm of the overlap between the single excitation eigenstate and two bright states in the intersectional resonator respectively. If the incident energy is resonate with single photon bound state energies in chain \( A \), \( |\langle E | B_+ \rangle|^2 \) and \( |\langle E | B_- \rangle|^2 \) reach their maximum values, where the incident photon is totally reflected. Therefore, we conclude that the bright state channels open when the Fano-Feshbach resonate effect happens in our system.
Fig. 7. (Color online) Norm of the overlap between the scatter state $|E\rangle$ and the bright state $|B_-\rangle$. The dashed (red) line represents the transmission rate of the incident photon.

6 Conclusions and remarks

In this paper we have studied the single photon coherent transfer in the cross resonator arrays with a $A$-type atom which is localised in the intersectional resonator. The coherent control of photon transfer can be realised via the dark state mechanism with a fully quantum mechanism where no classical field induces the EIT. It is shown that perfect reflection and transmission can be realised when the photon incident energy resonates with continuous energy spectrum boundaries in the two chains. There also exist Fano-Feshbach resonance effects between the two resonator arrays. The dark state mechanism in our system is also explored by considering the condition to form EIT.

In this system we can coherently control single photon transmission by using these properties.

This work was supported by National Natural Science Foundation of China under Grant Nos. 11121403, 10935010, 11074261 and 11175044.

Appendix: Single photon bound state in the chain $A$

In this appendix we derive the single photon bound state energy in chain $A$. Here we consider a two-level atom put into a central resonator of a coupled resonator array. The atom has ground state $|g\rangle$ and excited state $|e\rangle$. We take the central resonator as the origin. The atom interacts with the resonator field mode under the rotating wave approximation. The Hamiltonian of the system reads,

$$H = H_c + H_I,$$

$$H_c = \omega_0 \sum_j a_j^\dagger a_j - \xi_0 \sum_j (a_j^\dagger a_{j+1} + h.c.),$$

$$H_I = \varepsilon |e\rangle \langle e| + J_a (|g\rangle \langle 0| + |0\rangle \langle g| + h.c.),$$

where $a_j^\dagger$ and $a_j$ are the creation and annihilation operators of the photon mode in the $j$th resonator with frequency $\omega_0$. $\xi_0$ is the hopping energy between nearest-neighbour resonators of the field mode. We assume the energy of the atomic ground state is zero, and $\varepsilon$ is the energy corresponding to the excited state. $J_a$ is the coupling strength between the 0th resonator field mode and the atom.

The stationary eigenstate of single excitation can be expressed as

$$|E\rangle = \sum_j u_j (j) |1_j, g\rangle + u_e |\phi, e\rangle.$$  \hspace{1cm} (A.4)

Herein the state $|1_j, g\rangle$ corresponds to one photon in the $j$th resonator and the atom in its ground state, $|\phi, e\rangle$ corresponds to no photon in the resonator arrays and the atom in its excited state.

The eigen equation $H |E\rangle = E |E\rangle$ results in the discrete stationary eigen equations

$$(E - \omega_0) u_j (j) = -\xi_0 [u_j (j + 1) + u_j (j - 1)] + \frac{J_a^2 u_j (0)}{E - \varepsilon} \delta_{j,0}.$$  \hspace{1cm} (A.5)

The wave function of the bound state can be written as

$$u_j (j) = \begin{cases} A e^{ikj}, & j < 0 \\ A e^{-ikj}, & j > 0. \end{cases}$$  \hspace{1cm} (A.6)

Here $A$ is the normalised constant and $k$ is a complex number with negative imaginary part.

At $j = 0$, we can obtain the dispersion relation as

$$E = \omega_0 - 2\xi_0 \cos k.$$  \hspace{1cm} (A.7)

At $j \neq 0$,

$$E = \omega_0 - 2\xi_0 e^{-ik} + \frac{J_a^2}{E - \varepsilon}.$$  \hspace{1cm} (A.8)

Then we can obtain the equation of the bound state energy $E$ as

$$E = \varepsilon \pm \frac{J_a^2}{\sqrt{(E - \omega_0)^2 - 4\xi_0^2}}.$$  \hspace{1cm} (A.9)

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