Approach to Operational Experimental Estimation of Static Stresses of Elements of Mechanical Structures

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Abstract. The evaluation of static stresses and strength of units and components is a crucial task for increasing reliability in the operation of vehicles and equipment, to prevent emergencies, especially in structures made of metal and composite materials. At the stage of creation and commissioning of structures to control the quality of manufacturing of individual elements and components, diagnostic control methods are widely used. They are acoustic, ultrasonic, X-ray, radiation methods and others. The using of these methods to control the residual life and the degree of static stresses of units and parts during operation is fraught with great difficulties both in methodology and in instrumentation. In this paper, the authors propose an effective approach of operative control of the degree of static stresses of units and parts of mechanical structures which are in working condition, based on recording the changing in the surface wave properties of a system consisting of a sensor and a controlled environment (unit, part). The proposed approach of low-frequency diagnostics of static stresses presupposes a new adaptive-spectral analysis of a surface wave created by external action (impact). It is possible to estimate implicit stresses of structures in the experiment due to this approach.

1. Introduction

Various types of diagnostic control methods are widely used at the stages of the creation, commissioning and using of mechanical structures to evaluate the workmanship and the functioning of individual elements and components. A rather complete review of these methods, their merits and demerits is given in [1-5].

There are known works in which attempts were made to determine the stress states of structures from the analysis of responses to serial shock effects [6,7].

The features of the determination of the state (without a defect or with a defect) of a vertically suspended physical model of a periodic welded rod structure were established in [6]. This problem is solved by constructing the Fourier transforms of linear and nonlinear deformation responses to serial shock effects and comparing the phase diagrams of deformation behavior of an intact and damaged model.

The qualitative and quantitative features of the structure staying in the stressed state are experimentally substantiated based on the analysis of the nonlinear and linear regions of the constructed Fourier transforms in [7]. The using of Fourier transforms limited the possibilities for accuracy of these methods, as the possibility of adaptation, self-learning and tuning to a specific type of response functions.

Successful application of these approaches, ensuring the required accuracy, are directly related to the used mathematical methods of processing of the measured response mechanical wave - reactions. Thus,
the most effective approaches of mathematical processing of reactions are traditional spectral analysis [6-10]; the correlation analysis of signals [9-11]; the artificial neural networks [12,13]; statistical approaches and models [14,15,16]; wavelet transform signals [9,15,17]; optimal orthogonal decompositions of signals on the adaptively adjusted basis based on training sample [11].

Exactly the latter approach is a detail considered in the present work as a promising one with a number of advantages. The main advantage is the adaptive adjustment of the orthogonal basis, in accordance with the criteria for the best recognition of the stressed state of the structure.

In the work, the evaluation of the diagnostic observability [18,19] of static stresses of a mechanical structure with the using of the method of low-frequency diagnostics of static stress and recording of the changing in the surface wave properties of a system consisting of a sensor and a controlled environment (unit, part) was made. In this case, a set of diagnostic state variables in the form of the coordinates of the orthogonal diagnostic space was defined.

2. The scheme of the research of the approach to estimation of static stresses

A multifunctional measuring complex and a package of mathematical modeling of mechanical structures, which makes it possible to conduct research, to compare signals and to build spectral characteristics with sensors of various types, were used for the research and evaluation of the proposed approach.

The features of the dynamic process on the surface of the parallelepiped made of aluminum material were researched (see figure 1). Influences forming static stresses in the sample were applied along three perpendicular axes \( x_1, x_2, x_3 \).

The effect was determined by the quantity of shortening (compression) \(-\Delta x_i\) or extension (stretching) \(+\Delta x_i\) of the sample along the axis. The compression and stretching of the sample along the axes were varied. Each of the quantities set its static stress in the sample. The probing surface oscillations in the sample were excited by a shock force action \( F \) of constant type at one of the ends. The occurred oscillations \( f(t) \) for a time interval \( T_H \) sufficient for the arrival of reflected waves from the opposite end of the structure.

A general view of the fixed response functions \( f(t) \) for samples with different stress along the axes \( x_1, x_2, x_3 \) of the structure is shown in figure 2. As follows from figure 2, their shape depends on the presence and magnitude of the stress in the structure and its distribution along the axes \( x_1, x_2, x_3 \).

![Figure 1](image1.png)

![Figure 2](image2.png)

3. Adaptive-spectral analysis of the surface wave in the estimation of static stresses

In the case of a discrete implementation of the recognition of the magnitude of the stress from the
response functions \( f(t) \), the construction of the adaptive feature diagnostic space \( \xi \) is carried out as follows. A similar mathematical apparatus was used for modeling in power engineering [10] and low-frequency nondestructive testing of mechanical structures [17].

Each graph \( f_i(t) \) of the functions \( f(t) \) of the low-frequency response to external impact \( F \) in the diagnostics of structures can be considered as a vector of real values \( \vec{f}_i = [f_{i1}, f_{i2}, f_{i3}, \ldots, f_{in}]^T \in \mathbb{R}^N \) of the change of response in time. The sampling step \( \Delta t \) in time or the number of samples \( N \) in the observation interval is chosen in accordance with the sampling theorem [8-10].

Conditionally, the number of vectors \( \vec{f}_i \) in the sample \( \{\vec{f}_i, \vec{f}_2, \vec{f}_3, \ldots, \vec{f}_n\} \in \mathbb{R}^{N \times n} \) is determined by the required accuracy of stress identification in the structure and is dependent on the stress measurement range. And it is determined, as a rule, by the number \( n = 5 \pm 15 \) of uniformly distributed stresses throughout the measured range in the structure.

Initially, the set of vectors \( \{\vec{f}_1, \vec{f}_2, \vec{f}_3, \ldots, \vec{f}_n\} \) is linearly dependent. This is due to the excessive dimensionality \( N \) of the vectors \( \vec{f}_i \), the proximity of the shapes of the graphs and is confirmed by the value of the rank of the matrix \( \mathbf{f} = [\vec{f}_1, \vec{f}_2, \vec{f}_3, \ldots, \vec{f}_n]^T \in \mathbb{R}^{n \times N} \) for which \( \text{rank}(\mathbf{f})<< N \).

The basis of stress identification in a structure is the finding of such orthogonal transformation \( \mathbf{X} \in \mathbb{R}^{N \times m} \) of a matrix \( \mathbf{f} \in \mathbb{R}^{m \times n} \) into a matrix \( \mathbf{A} \in \mathbb{R}^{m \times m} \), \( \mathbf{A} = \mathbf{fX} \), that excludes an excessive dimensionality \( \mathbf{f} \) associated with poorly informative, often random variations in the graphs. The matrix \( \mathbf{X} = [\xi_1, \xi_2, \xi_3, \ldots, \xi_m] \) in this case defines a linear subspace \( \mathbb{R}^N \).

Moreover, the vectors \( \xi_1, \xi_2, \xi_3, \ldots, \xi_m \) form an orthonormal basis in \( \mathbb{R}^N \), and for the matrix \( \mathbf{X} \), \( \mathbf{X}^T \mathbf{X} = \mathbf{I}_m \) is true. Conditionally, a matrix \( \mathbf{X} \) can be considered as a matrix of orthogonal compressing of a linear space \( \mathbb{R}^N \) into a space \( \mathbb{R}^m \), \( m < N \). Vectors \( \vec{f}_i \in \mathbb{R}^N \) of responses are transformed by means of a matrix \( \mathbf{X} \) into images \( \mathbf{A}_i \in \mathbb{R}^m \), and, thus, the entire matrix of responses \( \mathbf{f} \) is transformed into an image matrix \( \mathbf{A} = [\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \ldots, \mathbf{A}_n]^T \in \mathbb{R}^{m \times n} \).

In the general form, the orthogonal decomposition of the initial vectors \( \vec{f}_i \in \mathbb{R}^N \) in terms of the basis \( \mathbf{X} \) can be represented in the form \( \vec{f}_i = \mathbf{X} \mathbf{A}_i + \mathbf{A}_0 \) where \( \mathbf{A}_0 \) is the constant component of the transformation.

An orthonormal basis \( \xi_1, \xi_2, \xi_3, \ldots, \xi_m \) is adaptive, trained and dependent on a recognizable sample \( \{\vec{f}_1, \vec{f}_2, \vec{f}_3, \ldots, \vec{f}_n\} \in \mathbb{R}^{N \times n} \). In determination of the basis, a complex of optimization problems is solved, in particular:

- providing the best reproducibility: \( \| \mathbf{f} - \mathbf{A} \mathbf{X}^T \mathbf{A}_0 \|_2 \to \min \), where \( \mathbf{A}_0 \in \mathbb{R}^{m \times N} \) - the matrix of constant components of the transformation, consisting of vectors \( \mathbf{A}_0 \).

- ensuring the orthonormality of the basis: \( \| \mathbf{X}^T \mathbf{X} - \mathbf{I}_m \|_2 \to \min \).

- realizing the best discernibility: \( d^2(\mathbf{A}) = \frac{1}{m^2 - m} \sum_{i,j} \| \mathbf{A}_i - \mathbf{A}_j \|_2 \to \max \).

The discrete realization of the method of recognizing the magnitude of the stress from the response functions makes it possible to simplify the algorithmization of the processing of graphs.

For the simplicity of the graphic and physical interpretation, the recognizing of the stresses of the samples will be carried out in the two-dimensional feature space by images \( \mathbf{A}_i = (a_{i1}, a_{i2}) \in \mathbb{R}^2 \). As the
two coordinates of the image, let us choose those that have the maximum varieties of projections of the images.

4. Experimental research of approach of estimation of the static stress

The research showed that a two-dimensional feature space is sufficient for qualitative estimation of stressed states of structures. Different in magnitude and sign stresses causing displacement of the sample along three mutually perpendicular axes $x_1, x_2, x_3$ were considered (see figure 1). The values of the displacements were specified from the series $0.5 \cdot 10^{-3}; 1.0 \cdot 10^{-3}; 1.5 \cdot 10^{-3}$ m. The shock impact was formed and the emerging surface waves were recorded as a response function. The set of 19 response functions was received:

\[
\{ f(t) \} = \begin{bmatrix}
    f_{bd}(t), f_{1/5}(t), f_{1/0}(t), f_{0/5}(t), f_{1/5}(t), \\
    f_{1/0}(t), f_{0/5}(t), f_{1/5}(t), f_{1/0}(t), f_{0/5}(t), \\
    f_{1/5}(t), f_{1/10}(t), f_{0/5}(t), f_{1/15}(t), f_{1/10}(t),
\end{bmatrix},
\]

where the designations $bd$ - without stress; $s$, $ss$, $sss$ – the compression relative to the axes $x_1, x_2, x_3$ accordingly; $r$, $rr$, $rrr$ - the stretching relative to the axes $x_1, x_2, x_3$ accordingly; 1.5; 1.0 and 0.5 - the values of the displacements along these axis.

The result of the construction of a two-dimensional feature diagnostic space and the location of the images for the specified samples of the response functions is shown in figure 3. An important fact is that all given images corresponding to different stresses along each of the axes $x_1, x_2, x_3$ are located on their ideal parabolas:

\[
a_{2,x_1} = \alpha_1 a_1^2 + \beta_1 a_1 + \gamma_1; \quad a_{2,x_2} = \alpha_2 a_2^2 + \beta_2 a_2 + \gamma_2; \quad a_{2,x_3} = \alpha_3 a_3^2 + \beta_3 a_3 + \gamma_3.
\]

This localization of images greatly simplifies the identification of the current value of the stress in the structure by the location of the image. The parabolic shape of the image localization curve is determined by the shape of the experimental structure and the features of the construction of the diagnostic space. The images of the compression $f_{1/5}(t)$ and stretching $f_{1/5}(t)$ graphs along the axis $x_1$ are removed from figure 3 to increase the scale and to improve detailing. These images $s1.5$ and $r1.5$ are also located higher on the axis parabola $x_1$.

\[\text{Figure 3. Two-dimensional feature diagnostic space and images of response functions corresponding to compression and stretching of the structure along the axes } x_1, x_2, x_3.\]
Each parabola can be viewed as a separate scalable stress transformation along each of the three axes onto the main axis $x_1$. The axis $x_1$ is the main one because it is the shortest path between the probing impact point $F$ and the location of the sensor $S$. This axis covers the longest part of the structure and the signal propagated along this axis is the main measuring one containing the greatest information about the stresses; therefore, the axis will be called the main projection axis.

The fact of the inversion of the arrangement of the images on the parabola takes place for the axes $x_2$ and $x_3$ in relation to the images obtained for the axis $x_1$. By inversion the authors mean the change in the arrangement along the branches of the parabola of the stretching and compression patterns. This can be explained by the scheme of the changing of the structure at the deformation for the axes $x_1$ and $x_2$, considered in figure 4. This inversion can also be interpreted by the projection of stresses along the axis $x_2$ (or $x_3$) onto the main projection axis $x_1$.

![Figure 4](image)

**Figure 4.** The relationship between compression and stretching of the structure along perpendicular axes $x_1, x_2$.

The following notation are used in figure 4: a) - the initial state of the structure; b) - an axial $x_1$ stretching $R$, leading to an axial $x_1$ compression $-\Delta x_1$; c) - the compression $\bar{S}$ along the axis $x_2$ leading to axial $x_1$ stretching $+\Delta x_1$. The presence of two parabolas in figure 3 for the axes $x_1$ and $x_2$ is explained by the indicated nonlinear character of the interrelation between compressions and stretching along perpendicular axes. The relationship between compression and stretching between the axes $x_1$ and $x_3$ can be explained similarly.

5. Conclusions

A new adaptive-spectral approach to the operative experimental estimation of static stresses in mechanical structures is proposed. It is based on the analysis of the response functions to the impact low-frequency nondestructive action, which allows estimating both the magnitude of the stresses and the direction of the action.

The principle of constructing spaces for the analysis of response signals in the operative experimental estimation of static stress based on an adaptive orthogonal basis is proposed in accordance with the criteria for the best recognition of the stressed state of a structure.

The property of the parabolic distribution of images of the response functions for structures is revealed at various stresses caused by the stretching or compressing structure along one or another axis. The property of fluctuations of the parabolas of the image distribution is revealed depending on the direction of the applied action causing the stress.
The proposed method and the principle of constructing diagnostic spaces can be the basis for building trained systems of operational control of stress for various types of structures.

6. References

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