Trajectory Planning and Simulation for Live-Working Robot
Zhao LIU¹, Zhao-ling LIU², Ye LI¹*, Wei GAO², Tian-chun XIANG² and Jing LIU²
¹TsingHua TongChuang Robot Co. Ltd., Tianjin, China
²State Grid Tianjin Electric Power Company, Tianjin, China
*Corresponding author

Keywords: Live-working robot, Kinematics, Collision detection, Trajectory planning.

Abstract. The traditional live-working method is very easy to cause people to die, and the use of live-working robots instead of manual work is an effective solution. A self-developed modular six-degree-of-freedom robot is adopted for live-working robot. Firstly, the expressions of inverse solutions are deduced by inverse transformation method, and the optimal solution is selected according to the weighted “shortest travel” criterion and the collision detection method based on cylinder envelope. Then, the straight-line trajectory planning of Cartesian space is carried out. Finally, the correctness of the trajectory planning algorithm is verified by MATLAB, which provides a theoretical basis for further dynamic research of live robot.

Introduction
With the proposal of “Made in China 2025”, comprehensively improving the quality and level of China’s manufacturing industry has become the goal of scientific research workers. Intelligent manufacturing and high-end equipment manufacturing have become the new commanding heights for industrial giants.

For live-working robots, foreign research in this field is earlier. Japan began the research of high-voltage live-working robots in 1984, and has entered the third generation (Phase III) [1-3]. TRC Corporation of the United States, Haydro-Quebec Institute of Canada, EDF of France and so on also carried out this research in the mid-1980s [4-5]. Research on live-working robots started in China relatively late. In 1999, Shandong Electric Power Group Company set up a project for live-working robots for the first time in China. In 2002, the first autonomous innovative live-working robot was developed and the prototype of robot was completed in 2005 [6].

In the task of high voltage distribution system in our country, the traditional live-working method is to work with wearing insulating gloves manually for a long time in the high altitude, high voltage and strong magnetic environment, which is very easy to cause casualties. With the increasing demand for live work and the continuous development of power industry in China, the research on the new generation of live-working equipment represented by high voltage live-working robot will be of great significance. Usually, the operation environment of high-voltage distribution lines is complex and dangerous, and the arrangement of circuit equipment is compact [7], which leaves less space for robots to work. Therefore, obstacle avoidance path planning of the robot is required.

Common path planning methods include artificial potential field method, genetic algorithm, fuzzy logic algorithm, ant colony algorithm, etc. [7]. But these methods mainly consider whether the end of the robot bypasses obstacles without collision, and don’t consider whether the manipulator will touch obstacles in the environment during operation. Clark proposed bounding box technology in 1976 [8]. Its basic idea is to surround complex geometric objects in the working environment with a simple geometric shape, and then calculate the distance between different bounding boxes to achieve the purpose of collision avoidance detection. At present, this method has been widely used in collision detection. The method based on “cylindrical envelope” is adopted to realize the collision detection between the manipulator and the obstacle in references [9] and [10].

In this paper, the live-working robot adopted a self-developed six-degree-of-freedom robot. The kinematics model is constructed based on D-H matrix method. The inverse transformation method
is used to calculate the inverse solutions and eight sets of analytical solutions for six joints of the robot are obtained. According to the weighted “shortest travel” criterion and the collision detection method based on cylindrical envelope, the optimal solution of the inverse solution is selected. The Cartesian space linear interpolation is applied to complete the trajectory planning of the robot, and the live-working task of the robot is realized. Finally, the correctness of trajectory planning algorithm is verified by MATLAB software simulation, which provides a theoretical basis for the dynamic research of live robot.

Kinematics of the Live-Working Robot

Forward Kinematics

The Live-working robot is mainly used in 6-10 kV transmission lines. As shown in Fig. 1, the robot is fixed on the platform, the tool rack is fixed on one side of the platform, and different kinds of tools are placed on the tool rack. The robot loads and unloads a tool through the quick-change device to operate on three main cables and three lead cables at three operation points.

As shown in Fig. 2, the live-working robot includes seven parts: the base, the waist, the upper arm, the lower arm, the wrist head, the wrist end and the end flange, which are connected by rotating joints \( A_i (i=1 \sim 6) \) in turn. The link coordinate systems \( O_j-x_jy_jz_j \) \((j=0 \sim 6)\) are connected with seven parts separately and are established by D-H method. The D-H parameters are shown in Table 1. \( \alpha_{i-1} \) and \( a_{i-1} \) represent the twist angle and length of the links, respectively. \( \theta_i \) and \( d_i \) represent rotation angle of each joint and offset between adjacent joints. \( a_2=712.9\)mm, \( a_3=671.6\)mm, \( d_1=128\)mm, \( d_2=-176\)mm, \( d_3=-127.8\)mm, \( d_4=115.7\)mm, \( d_5=115.7\)mm.

![Figure 1. Living working system.](image1)

![Figure 2. Connecting rob coordinate systems.](image2)

The homogeneous transformation matrix between the connecting rob \( i \) relative to the connecting rob \( i-1 \) is expressed as Eq. 1, where \( c\theta_i=\cos(\theta_i) \), \( s\theta_i=\sin(\theta_i) \), \( c\alpha_{i-1}=\cos(\alpha_{i-1}) \), \( s\alpha_{i-1}=\sin(\alpha_{i-1}) \).

| \( i \) | \( \alpha_{i-1} \) | \( a_{i-1} \) | \( \theta_i \) | \( d_i \) |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | \( a_1 \) | 0 | \( \theta_2 \) | \( d_2 \) |
| 3 | 0 | \( a_2 \) | \( \theta_3 \) | \( d_3 \) |
| 4 | 0 | \( a_3 \) | \( \theta_4 \) | \( d_4 \) |
| 5 | \( a_4 \) | 0 | \( \theta_5 \) | \( d_5 \) |
| 6 | \( a_5 \) | 0 | \( \theta_6 \) | 0 |
\[ i^{-1}T = R_{x_i}(\alpha_{i-1})T_{x_i}(a_{i-1})R_{z_i}(\theta_i)T_{z_i}(d_i) \]

\[
= \begin{bmatrix}
  c\theta_i & -s\theta_i & 0 & a_{i-1} \\
  c\alpha_{i-1}s\theta_i & c\alpha_{i-1}c\theta_i & -s\alpha_{i-1} & -c\alpha_{i-1}d_i \\
  s\alpha_{i-1}s\theta_i & s\alpha_{i-1}c\theta_i & c\alpha_{i-1} & c\alpha_{i-1}d_i \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

(1)

Therefore, the transformation matrix of the robot end coordinate system \(O_6-x_6y_6z_6\) relative to the base coordinate system \(O_0-x_0y_0z_0\) can be expressed as Eq. 2. \(n, o, a\) are the pose vectors and \(p\) is the position vector of the robot end in the base coordinates, respectively, as shown in Eq. 3.

\[
^0T = ^0T_2^T T_3^T T_4^T T_5^T T_6^T = \begin{bmatrix}
n_x & o_x & a_x & p_x \\
o_y & o_y & a_y & p_y \\
a_z & e & f & g \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(2)

\[
\begin{aligned}
n_x &= c_5s_6 + c_1(c_{234}s_6 - s_{234}s_6), \\
o_y &= c_2c_5s_6 + c_1(c_{234}s_6 - s_{234}s_6), \\
o_z &= -c_6s_{234} - c_{234}s_6, \\
a_x &= -c_5s_6s_{234} + c_6s_6 + c_5s_6, \\
a_y &= -c_5s_6s_{234} + c_6s_6 + c_5s_6, \\
a_z &= -c_4c_5, \\
p_x &= -s_5k_1 + c_1(s_2a_2 + c_2a_3 + c_3d_5), \\
p_y &= c_1k_1 + s_1(s_2a_2 + c_2a_3 + c_3d_5), \\
p_z &= c_2a_2 - s_3d_3 + d_1 - s_{234}d_5
\end{aligned}
\]

(3)

Inverse Kinematics

The inverse kinematics of robot can be described as solving all possible solutions of the joint angles of robot by the known position and orientation of the end link of robot. By the inverse transformation analytic method, the inverse solution expression of each joint angle can be obtained as shown in Eq. 4.

\[
\begin{aligned}
\theta_1 &= \text{atan2}(p_y, p_x) - \text{atan2}(k_1, \pm \sqrt{\rho_1^2 - k_1^2}), \\
\theta_2 &= -\text{atan2}\left(\frac{k_{23}}{k_{21}}\right) + \text{atan2}\left(\frac{k_{23}}{k_{21}}\right) + \text{atan2}\left(\frac{k_{23}}{k_{21}}\right) - \theta_2 = \text{atan2}\left(-\frac{c_1a_x + s_1a_y}{c_5}, \frac{a_z}{c_5}\right) - \theta_2 + \theta_3 \\
\theta_3 &= \text{atan2}\left(-\frac{s_1a_x + c_1a_y}{c_5}\right), \theta_4 = \text{atan2}\left(-\frac{s_1a_x - c_1a_y}{c_5}\right), \theta_5 = \text{atan2}\left(-\frac{s_1a_x + c_1a_y}{c_5}\right), \theta_6 = \text{atan2}\left(-\frac{s_1a_x - c_1a_y}{c_5}\right)
\end{aligned}
\]

(4)

where

\[
\rho_1 = \sqrt{\rho_1^2 + \rho_2^2}, \quad \rho_2 = \sqrt{(2a_2k_{21})^2 + (2a_2k_{21})^2}, \quad k_1 = d_2 + d_4 + d_4, \quad k_2 = k_{21} + k_{22} + a_2^2 - a_3^2, \\
k_{21} = c_1p_x + s_1p_y - c_{234}d_5, \quad k_{22} = -d_1 + p_z + s_{234}d_5, \quad c_5 = \pm \sqrt{(-s_1a_x + c_1a_y)^2 + (-s_1a_x + c_1a_y)^2}.
\]

There are eight sets of possible inverse solutions for the live-working robot. However, due to structural constraints, some solutions cannot be achieved. Therefore, it is necessary to select the optimal solution to meet the needs of live operation in the trajectory planning.

Collision Detection Based on Cylindrical Envelop

The collision detection method based on cylindrical envelope refers to a method of enveloping a robot link or an obstacle with a cylinder of a suitable diameter and height. Therefore, the collision problem between the robot links and the obstacles can be converted into a problem of determining...
the relative positional relationship between the cylinders enveloping the robot links and the obstacles.

As shown in Fig. 3, assume that a robot link and an obstacle are enveloped with the cylinder $i$ and the cylinder $j$, respectively. The position coordinates of the head and end of the cylinder $i$ are $P_i(\theta_i) = (x_i, y_i, z_i)$ and $P_{i+1}(\theta_i) = (x_{i+1}, y_{i+1}, z_{i+1})$, and the radius of the cylinder $i$ is $r_i$. The position coordinates of the head and end of the cylinder $j$ are $P_j = (x_j, y_j, z_j)$ and $P_{j+1} = (x_{j+1}, y_{j+1}, z_{j+1})$, and the radius of the cylinder $j$ is $r_j$.

The direction vectors of the centerlines of the two cylinders are calculated as $L_i(\theta_i) = P_{i+1}(\theta_i) - P_i(\theta_i)$ for cylinder $i$, and $L_j = P_{j+1} - P_j$ for cylinder $j$.

If the centerline of the cylinder $i$ is parallel to that of the cylinder $j$, the shortest distance between the two centerlines can be expressed as:

$$d_{ij}(\theta_i) = \frac{|L_i \times L_j|}{|L_i|}$$

If the centerline of the cylinder $i$ intersects that of the obstacle $j$, the common perpendicular vector of the two centerlines can be expressed as $n_{ij} = L_i \times L_j$, and the shortest distance between the two centerlines can be calculated as:

$$d_{ij}(\theta_i) = \frac{|n_{ij} \cdot L_j|}{|n_{ij}|}$$

When $d_{ij}(\theta_i) > r_i + r_j$, the two cylinders do not interfere, that is, the robot link does not collide with the obstacle; when $d_{ij}(\theta_i) \leq r_i + r_j$, the two cylinders interfere.

The constraint function $D_1$ of collision detection is defined as the reciprocal sum of the distance between the cylindrical envelope centerline of each connecting rod and that of each obstacle. The expression is:

$$D_1 = \frac{1}{\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}(\theta_i)}$$

**Optimal Selection of Inverse Solutions**

In order to select the optimal inverse solution which meets the structural characteristics of the robot and the actual operation requirements, the weighted “shortest travel” criterion [11] is adopted. The selection process of the optimal inverse solution is shown in Fig. 4. $\theta_i (i=1,2,\ldots,6)$ represents the six current joint angles of the robot. $\theta_k (k=1,2,\ldots,8)$ represents the eight sets of inverse solutions calculated by inverse kinematics. $\omega_i$ is the weighting factor. The weighted Euclidean distance
between the current joint angles of the robot and each group of the calculated inverse solutions is expressed as Eq. 8.

$$D_2 = \sqrt{\sum_{i=1}^{6} \omega_i (\theta_k - \theta_i)^2} \quad (k = 1, 2, \ldots, 8)$$  \hspace{1cm} (8)

The three links at the end of the robot are regarded as small links and the rest as large rods. In the actual motion of the robot, it is expected that the robot will mainly move the small links to reduce power consumption and the space movement range of the robot. Therefore, the ratio of weighting factors of each joint is set to $\omega_1: \omega_2: \omega_3: \omega_4: \omega_5: \omega_6 = 40:20:20:10:5:5$. The minimum value $\min D_2$ of the weighted Euclidean distance is taken as the second constraint condition for the selection of the optimal inverse solution.

![Diagram](image)

**Figure 4. Selection process of the optimal inverse solution.**

**Trajectory Planning of the Live-Working Robot**

In order to ensure the motion accuracy of the live-working robot, Cartesian space linear interpolation is used to realize path planning of position and direction. Assuming that the number of interpolation points is $N$, the starting point is $A (x_A, y_A, z_A)$ and the ending point is $B (x_B, y_B, z_B)$. The coordinates $(x_i, y_i, z_i)$ of the interpolation points in the middle are expressed as Eq. 9. The trajectory planning process is shown in Fig 5.

$$\begin{align*}
x_i &= (1-t)x_A + tx_B \\
y_i &= (1-t)y_A + ty_B, \quad \left( t = \frac{0}{N}, \frac{1}{N}, \ldots, \frac{N-1}{N}, \frac{N}{N} \right) \\
z_i &= (1-t)z_A + tz_B
\end{align*}$$  \hspace{1cm} (9)
Simulation Verification

The trajectory flow of live-working robot is “grasping tool - live-working at three operating points - releasing tool”. The joint angles of the robot corresponding to the position of the tool rack and the three working points and the position coordinates of the head and end of the three main cables are listed in Table 2. The radius of the cylinder enveloping the three main cables is 25 mm. The radiuses of the cylinder enveloping the upper arm and the lower arm are 90 mm and 70 mm, separately.

Table 2. Value corresponding to key location points.

| Key Location Points | Value corresponding to key location points |
|---------------------|------------------------------------------|
| Tool rack           | \( \theta_i = (27.2^\circ, -20.1^\circ, 133.2^\circ, 26.7^\circ, 62.7^\circ, 180.0^\circ) \) \( (i=1,2,...,6) \) |
| Operation point 1   | \( \theta_i = (47.4^\circ, -8.1^\circ, 68.7^\circ, 103.2^\circ, 42.6^\circ, 180.0^\circ) \) \( (i=1,2,...,6) \) |
| Operation point 2   | \( \theta_i = (84.9^\circ, -14.5^\circ, 32.4^\circ, 133.1^\circ, 5.4^\circ, 180.0^\circ) \) \( (i=1,2,...,6) \) |
| Operation point 3   | \( \theta_i = (120.9^\circ, -21.9^\circ, 50.9^\circ, 107.1^\circ, -30.9^\circ, 180.0^\circ) \) \( (i=1,2,...,6) \) |
| Main cable 1        | \( P_{1s} = (-2900 \text{mm}, 500 \text{mm}, 1000 \text{mm}) \); \( P_{1e} = (2100 \text{mm}, 500 \text{mm}, 1000 \text{mm}) \) |
| Main cable 2        | \( P_{2s} = (-2900 \text{mm}, 0, 1290 \text{mm}) \); \( P_{2e} = (2100 \text{mm}, 0, 1290 \text{mm}) \) |
| Main cable 3        | \( P_{3s} = (-2900 \text{mm}, -500 \text{mm}, 1000 \text{mm}) \); \( P_{3e} = (2100 \text{mm}, -500 \text{mm}, 1000 \text{mm}) \) |

Based on MATLAB, according to forward and inverse kinematics algorithm, collision detection algorithm and Cartesian space linear interpolation algorithm, the trajectory planning of the live-working robot is simulated. The simulation result of the robot motion process is shown in Fig. 6. The curves of six joint angles of the robot are obtained by simulation as shown in Fig. 7.

As can be seen from Fig. 6, the trajectory planning algorithm of live-working robot is realized by simulation. From Fig. 7, it can be seen that the six joint angles of the robot change smoothly during the operation, and there is no sudden change of one joint angle. The simulation verifies the correctness of the trajectory planning algorithm.
Conclusion

The live-working robot is a self-developed six-degree-of-freedom robot. The trajectory planning of the live-working process is studied. The main conclusions are as follows:

(1) Based on D-H method, the forward kinematics of live-working robot is analyzed. The expressions of the inverse solutions of each joint angle of the robot are deduced by using the inverse transformation method.

(2) According to the weighted “shortest travel” criterion and the collision detection algorithm based on cylindrical envelope, the optimal solution of the inverse solutions is selected, and the collision-free path planning is realized.

(3) Cartesian linear interpolation method is used to complete the trajectory planning of the position and attitude of the robot end, which ensures the motion accuracy of the robot end and enables the robot to complete the task smoothly.

(4) The trajectory planning algorithm is compiled by MATLAB, and the simulation of the trajectory planning of the live-working robot is realized. The correctness of the algorithm is verified, which provides a theoretical basis for the dynamic research of the live-working robot.

References

[1] M. Nakashima, S. Harada, K. Yano, et al. Development of a robot language for hot-line work allowing practical use of the hot-line work robot system “Phase II”, Advanced Robotics, 1995, 10(4), pp. 355-375.

[2] Maruyama, Yoshinaga. Robotic applications for hot-line maintenance. Industrial Robot: An International Journal, 2000, 27(5), pp. 357-365.

[3] K. Takaoka, K. Yokoyama, H. Wakisako, et al. Development of the fully-automatic live-line maintenance robot-Phase III//Proceedings of the 4th IEEE International Symposium on Assembly and Task Planning. Washington: IEEE Press, 2001, pp. 423-428.

[4] P. Greg. Robots repair and examine live lines in severe conditions. Electrical World, 1989(5), pp. 71-72.

[5] M. Boyer. Teletobotics for maintenance of distribution lines. Conference on Line Maintenance, ICOLIM, 1994.

[6] T.Y. Li. Development and prospect of the live-line working robot in distribution network. Distribution & Utilization, 2016, 33(11), pp. 43-48.
[7] Y. Liu. Motion planning and experimental research of distribution line maintenance robot. Nanjing University of Science and Technology, 2017, pp. 1-2.

[8] N. Webster. Webster’s new universal unabridged dictionary. Journal of Rehabilitation, 1983.

[9] Ch.X. Zong. Research on obstacle avoidance motion planning method for spatial multi DOF serial manipulator. Hefei University of Technology, 2017, pp. 21-23.

[10] Z.G. Wang. Research on obstacle avoidance path planning for 6-DOF manipulator. Southwest Jiaotong University, 2018, pp. 21-23.

[11] G.D. Wang, etc. Inverse kinematics of robot based on weighted optimization. Modular Machine Tool & Automatic Manufacturing Technique, 2016(5), pp. 1-3.