The electroweak model with rarely interacted neutrinos

N.A. Gromov
Department of Mathematics, Komi Science Center UrD, RAS,
Kommunisticheskaya st. 24, Syktyvkar 167982, Russia
E-mail: gromov@dm.komisc.ru

The electroweak model, which lepton sector correspond to the contracted gauge group \( SU(2;j) \times U(1) \), \( j \to 0 \), whereas boson and quark sectors are standard one, is suggested. This model describe in a natural manner why neutrinos so rarely interact with matter, as well as why neutrinos cross-section increase with the energy. Dimensionfull parameter of the model is interpreted as neutrino energy. Dimensionless contraction parameter \( j \) for low energy is connected with the Fermi constant of weak interactions and is approximated as \( j^2 \approx 10^{-5} \).

Keywords: gauge theory; electroweak model; contraction; neutrino
PACS number: 12.15-y

1 Introduction

The standard electroweak model based on gauge group \( SU(2) \times U(1) \) gives a good description of electroweak processes. Due to this model the W- and Z-bosons was predicted and experimentally observed at the end of the last sentury. Higgs boson is now searched at the modern Large Hadron Collider. At the same time the grave disadvantage of the standard model is the presence about fifteen free parameters. But among these there is not such parameter, which a priori can be regarded as a small one and can be connected with the experimentally observed very rare interaction neutrinos with matter.

The purpose of this paper is to build up the variant of the electroweak model, which naturally describe the vanishingly small, as compared to other particles, interaction neutrinos with anything with the help of the zero tending contraction parameter, as well as to give the physical interpretation of the contraction procedure. The model is suggested where the boson and quark sectors are the same as in the standard electroweak model, but the lepton sector correspond to the gauge group \( SU(2;j) \times U(1) \), where \( j \to 0 \) is dimensionless contraction parameter.

2 Standard electroweak model

The Lagrangian of the standard electroweak model is given by the sum

\[
L = L_B + L_Q + L_L
\]
of the boson $L_B$, of the quark $L_Q$ and of the lepton $L_L$ Lagrangians [1]–[3]. As far as $L_B$ and $L_Q$ are not changed we concentrate our attention on the lepton Lagrangian, which for the first lepton generation is written in the form

$$L_{L,e} = L_l^\dagger i\bar{\tau}_\mu D_\mu L_l + e_r^\dagger i\tau_\mu D_\mu e_r - h_e [e_r^\dagger (\phi^\dagger L_l) + (L_l^\dagger \phi)e_r],$$

(1)

where $L_l = \begin{pmatrix} \nu_l \\ e_l \end{pmatrix}$ is the $SU(2)$-doublet, $e_r$ is the $SU(2)$-singlet, $h_e$ is constant, $\tau_0 = \tilde{\tau}_0 = 1$, $\tilde{\tau}_k = -\tau_k$, $\tau_\mu$ are Pauli matrices, $\phi \in C_2$ are matter fields and $e_r, e_l, \nu_l$ are two component Lorentzian spinors. The covariant derivatives of the lepton fields are given by

$$D_\mu e_r = \partial_\mu e_r + ig' A_\mu e_r \cos \theta_w - ig' Z_\mu \sin \theta_w L_l - ie A_\mu Q L_l,$$

$$D_\mu L_l = \partial_\mu L_l - i \frac{g}{\sqrt{2}} \left( W^+_\mu T_+ + W^-_\mu T_- \right) L_l - i \frac{g}{\cos \theta_w} Z_\mu \left( T_3 - Q \sin^2 \theta_w \right) L_l - ie A_\mu Q L_l,$$

where $T_k = \frac{1}{2} \tau_k, k = 1, 2, 3$ are generators of $SU(2)$, $T_\pm = T_1 \pm iT_2$, $Q = Y + T_3$, $Y = -\frac{i}{2} 1$ is hypercharge of the left leptons, $e = gg'(g^2 + g'^2)^{-\frac{1}{2}}$ is electron charge and $\sin \theta_w = eg^{-1}$. The gauge fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left( A^{1\dagger}_\mu + i A^2_\mu \right), \quad Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left( g A^3_\mu - g' B_\mu \right),$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left( g' A^3_\mu + g B_\mu \right)$$

are expressed through the fields

$$A_\mu(x) = -ig \sum_{k=1}^{3} T_k A^k_{\mu}(x), \quad B_\mu(x) = -ig' B_{\mu}(x),$$

which take their values in the Lie algebras $su(2), u(1)$, respectively.

Next two lepton generation (muon and muon neutrino, $\tau$-lepton and $\tau$-neutrino) are introduced in a similar way. Full lepton Lagrangian is the sum

$$L_L = L_{L,e} + L_{L,\mu} + L_{L,\tau},$$

where each term has the structure (1) with constants $h_e, h_\mu, h_\tau$, correspondingly.

3 Limiting case of the lepton sector of electroweak model

The contracted group $SU(2; j)$ is defined [4] as the transformation group

$$z'(j) = \begin{pmatrix} jz_1' \\ jz_2' \end{pmatrix} = \begin{pmatrix} \alpha & j\beta \\ -j\beta' & \bar{\alpha} \end{pmatrix} \begin{pmatrix} jz_1 \\ jz_2 \end{pmatrix} = u(j)z(j),$$

2
\[ \det u(j) = |\alpha|^2 + j^2|\beta|^2 = 1, \quad u(j)u^\dagger(j) = 1. \]

of the space \( C_2(j) \), which keep invariant the hermitian form
\[ z^\dagger z(j) = j^2|z_1|^2 + |z_2|^2, \]

where \( z^\dagger = (j\bar{z}_1, \bar{z}_2) \), parameter \( j = 1, \iota \), and \( \iota \) is nilpotent unit \( \iota^2 = 0 \). The equivalent and more traditional in physics way of group contraction [5] is to tend contraction parameter to zero \( j \to 0 \).

The group generators \( T_1(j) = j\frac{1}{2}\tau_1, \ T_2(j) = j\frac{1}{2}\tau_2, \ T_3(j) = \frac{1}{2}\tau_3 \) are subject of commutation relations
\[ [T_1(j), T_2(j)] = -ij^2T_3(j), \quad [T_3(j), T_1(j)] = - iT_2(j), \]
\[ [T_2(j), T_3(j)] = -iT_1(j) \]

and form the Lie algebra \( su(2; j) \). The actions of the unitary group \( U(1) \) and the electromagnetic subgroup \( U(1)_{em} \) in the fibered space \( C_2(\iota) \) with the base \( \{z_2\} \) and the fiber \( \{z_1\} \) are given by the same matrices as on the space \( C_2 \).

In the standard electroweak model the gauge group \( SU(2) \times U(1) \) acts in the boson, lepton and quark sectors, i.e. it is the invariance group of the boson \( L_B \), lepton \( \bar{L}_L \) and quark \( L_Q \) Lagrangians. We consider a model where the group \( SU(2) \times U(1) \) acts only in the boson and quark sectors, whereas in the lepton sector acts contracted group \( SU(2; j) \times U(1) \). In other words, boson and quark Lagrangians remain the same as in the standard model, but lepton Lagrangian is transformed.

The fibered space \( C_2(j) \) of the fundamental representation of \( SU(2; j) \) group can be obtained from \( C_2 \) by substituting \( jz_1 \) instead of \( z_1 \). Substitution \( z_1 \to jz_1 \) induces another ones for Lie algebra generators \( T_1 \to jT_1, \ T_2 \to jT_2, \ T_3 \to T_3 \). As far as the gauge fields take their values in Lie algebra, we can substitute gauge fields instead of transformation of generators, namely
\[ A^1_\mu \to jA^1_\mu, \quad A^2_\mu \to jA^2_\mu, \quad A^3_\mu \to A^3_\mu, \quad B_\mu \to B_\mu. \quad (3) \]

For the gauge fields (2) these substitutions are as follows
\[ W^\pm_\mu \to jW^\pm_\mu, \quad Z_\mu \to Z_\mu, \quad A_\mu \to A_\mu. \quad (4) \]

Let us stress that we can substitute transformation of the generators by the transformation of the gauge fields only in the lepton Lagrangian, which is built with the help of \( SU(2; j) \times U(1) \) group. In the boson and quark Lagrangians gauge fields are not changed. The field \( L_l = \left( \begin{array}{c} \nu_l \\ e_l \end{array} \right) \) is \( SU(2) \)-doublet, so its components are transformed in the similar way as components of vector \( z \), namely
\[ \nu_l \to j\nu_l, \quad e_l \to e_l, \quad e_r \to e_r. \quad (5) \]

The right electron field \( e_r \) is \( SU(2) \)-singlet and therefore is not transformed.
The substitutions \((4),(5)\) in \((1)\) give rise to the lepton Lagrangian

\[
L_{\text{L,e}}(j) = \frac{e_i^j i \bar{\tau}_\mu \partial_\mu e_l + e_i^j i \tau_\mu \partial_\mu e_r - e e_i^j \bar{\tau}_\mu A_\mu e_l + \frac{g \cos 2\theta_w}{2 \cos \theta_w} e_i^j \bar{\tau}_\mu Z_\mu e_l + g' \cos \theta_w e_i^j \bar{\tau}_\mu A_\mu e_r + g' \sin \theta_w e_i^j \tau_\mu Z_\mu e_r - m_e [e_i^j e_l + \bar{e}_i^j e_r] + j^2 \left\{ \nu_i^l i \bar{\tau}_\mu \partial_\mu \nu_l + \frac{g}{2 \cos \theta_w} \bar{\nu}_i^l \bar{\tau}_\mu \nu_l + \frac{g}{\sqrt{2}} \left[ \nu_i^l \bar{\tau}_\mu W_\mu^+ e_l + \bar{e}_i^l \bar{\nu}_l \right] \right\} = L_{\text{e,b}} + j^2 L_{\text{e,f}}. \tag{6}
\]

We put \(\phi = \phi_{\text{vac}} = \begin{pmatrix} 0 \\ \nu \sqrt{2} \end{pmatrix} \) in \((1)\) and denote electron mass as \(m_e = h_e v / \sqrt{2}\). Next lepton generations fields are transformed like \((5)\)

\[
\nu_{\mu,l} \rightarrow j \nu_{\mu,l}, \quad \nu_{\tau,l} \rightarrow j \nu_{\tau,l}, \quad \mu_l \rightarrow \mu_l, \quad \tau_l \rightarrow \tau_l, \quad \mu_r \rightarrow \mu_r, \quad \tau_r \rightarrow \tau_r.
\]

The full lepton Lagrangian is the sum

\[
L_{\text{L}}(j) = L_{\text{L,e}}(j) + L_{\text{L,\mu}}(j) + L_{\text{L,\tau}}(j) = L_{\text{L,b}} + j^2 L_{\text{L,f}}. \tag{7}
\]

where each term has the structure \((6)\) with the mass \(m_q = h_q v / \sqrt{2}, q = e, \mu, \tau\).

Let contraction parameter tends to zero \(j^2 \rightarrow 0\), then the contribution of electron, muon and tau neutrinos as well as their interactions with others fields to the Lagrangian \((7)\) will be vanishingly small in comparison with electron, muon, tau-lepton and gauge boson fields.

An ideal mathematical constructions are physically realized approximately with some errors. When contraction parameter \(j\) is small, but different from zero, the full Lagrangian of the model

\[
L(j) = L_B + L_Q + L_{\text{L}}(j) = L_r + j^2 L_\nu \tag{8}
\]

is splitted on two parts: the Lagrangian \(L_\nu\), which include neutrino fields along with their interactions with gauge and lepton fields and Lagrangian \(L_r\), which include all other fields. The neutrino fields part \(L_\nu\) turn out very small with respect to all other fields \(L_r\) due to the small contraction parameter \(j\). So Lagrangian \((8)\) describe very rare interaction neutrino fields with matter.

In the mathematical language the fields space of the standard electroweak model is fibered after contraction in such a way that neutrino fields are in the fiber, whereas all other fields are in the base. In order to avoid terminological misunderstanding let us stress that we have in view locally trivial fibering, which is defined by the projection in the field space. This fibering is understood in the context of semi-Riemannian geometry \([6]–[8]\) and has nothing to do with the principal fiber bundle. The simple and best known example of such fiber space is the nonrelativistic spacetime with one dimensional base, which is interpreted as time, and three dimensional
fiber, which is interpreted as proper space. It is well known, that in nonrelativistic physics the time is absolutely, while the space properties can be changed in time. The space-time of the special relativity is transformed to the nonrelativistic space-time when dimensionful contraction parameter — velocity of light $c$ — tends to infinity and dimensionless parameter $\frac{v}{c} \to 0$.

Weak interactions for low energies are characterized by the Fermi constant $G_F$. This constant is determined by experimental measurements and turn out to be very small $G_F = 10^{-5} \frac{1}{m^2_p} = 1.17 \cdot 10^{-5} \text{GeV}^{-2}$. Fermi constant is expressed by the parameters of the standard electroweak model as follows

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{2m^2_W}.$$  

Since $m^2_W = g^2v^2/4$, one obtain $v \approx 246 \text{ GeV}$ [2]. This ”large” dimensionfull parameter enters in Lagrangian of electroweak model through the mass terms $m_q = h_qv/\sqrt{2}$ of quarks, electron, muon, $\tau$-lepton, where it is multiplied on the free parameters $h_q$. Therefore a priori we can not say are these terms ”small” or ”large”.

On the contrary, in the full Lagrangian $L(j)$ [8] different order terms are appeared due to the small contraction parameter $j^2$ therefore neutrino fields and their interactions are small with respect to all other fields. Probability amplitude for weak current interactions which include two neutrinos is multiplied by $j^2$ when $SU(2)$ group is replaced by $SU(2; j)$. Therefore Fermi constant, which is the factor of such amplitude, is just the dimensionfull limit parameter of the model, which describe the rarely interacting neutrinos at low energies. If one introduce the dimensionfull constant $G_0 = \frac{1}{m^2_p}$, then one can approximate dimensionless contraction parameter $j^2 \approx 10^{-5}$.

It is well known that interaction cross-section for neutrinos increase with energy [2, 3]. For energies greater than $1 \text{ GeV}$ this dependence is linear. The interaction cross-sections are proportional to $G^2_F$, i.e. to $j^4$ for dimensionless parameter. This leads to the physical interpretation of the contraction procedure as the decreasing of the interaction cross-section for neutrinos with nucleons when energy decrease. The dimensionfull limit parameter $j^2G_0$ is interpreted as energy square in that case.

### 4 Conclusion

We explain the rarely neutrinos-matter interaction with the help of contraction the gauge group of the lepton sector of the standard electroweak model, leaving the invariance group of the boson and quark sectors untouched. The mathematical contraction procedure is connected with the energy dependence of the interaction cross-section for neutrinos. The dimensionfull limit parameter $j\sqrt{G_0}$ is physically interpreted as neutrino energy and the dimensionless contraction parameter at low energies is approximated as $j \approx 10^{-3}$.  

5
Limiting case of boson sector of the electroweak model for contracted gauge group was discussed in [9]. The electroweak model based on the gauge group $SU(2; j) \times U(1)$ in boson and lepton sectors have been considered in [10]. The fiber of this model consist of neutrino fields and gauge $W^\pm$-boson fields. If the quark sector is also invariant with respect $SU(2; j) \times U(1)$ group, then the $u$-, $c$- and $t$-quark fields are in the fiber too. The definite choice between these versions of the electroweak model can be done in the presence of detailed information on interactions of elementary fields. The preliminary version of the suggested model was discussed in [11].

This work is supported by the program ”‘Fundamental problems of nonlinear dynamics’” of Russian Academy of Sciences.

References

[1] V.A. Rubakov, Classical Gauge Fields (Moscow, Editorial URSS 1999) (in Russian).

[2] L.B. Okun’, Leptons and Quarks (Moscow, Editorial URSS 2005) (in Russian).

[3] M.E. Peskin and D.V. Schroeder, An Introduction to Quantum Field Theory (Addison-Wesley 1995).

[4] N.A. Gromov, Possible contractions of $SU(2)$ group, Proc. Komi SC UrB RAS, 2010, issue 1, 5–10 (in Russian).

[5] Inonu E., Wigner E.P., On the contraction of groups and their representations, Proc. Nat. Acad. Sci. USA, 1953, v. 39, 510–524.

[6] R. I. Pimenov, To the definition of semi-Riemannian spaces, Vestnik Leningrad Univ., 1965, 1, 137 (in Russian).

[7] R. I. Pimenov, Semi-Riemannian geometry, Proc. Sem. Vect. Tens. Anal. MSU, 1968, issue 14, 154–173 (in Russian).

[8] N.A. Gromov, The R.I. Pimenov unified gravitation and electromagnetism field theory as semi-Riemannian geometry, Phys. At. Nucl., 2009, v. 72, N. 5, 794–800; arXiv:08100349v1 [gr-qc].

[9] N.A. Gromov, Analog of electroweak model for contracted gauge group, Phys. At. Nucl., 2010, v. 73, N. 2, 326–330; arXiv:0811.4701v1 [math-ph].

[10] N.A. Gromov, Neutrino and contraction of electroweak model, arXiv:1010.5512v1 [physics.gen-ph].

[11] N.A. Gromov, Contraction of electroweak model and neutrino, Syktyvkar, 2010 (Scientific reports / Komi Science Centre, Ural Division, Russian Academy of Sciences; Issue 512) (in Russian).