Small Elements in Fermion Mass Matrices and Anomalous Dipole Moments

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Abstract

Assuming the small entries in the mass matrices are produced by fermion-scalar loops, we calculate the anomalous dipole moments of the leptons and quarks. The top quark appears in all the loops as the mass seed. When comparing the results with experimental data, including electric and magnetic dipole moments, and radiative transition rates, we obtain the mass limits which are typically larger than .1 TeV for the relevant neutral scalars, and 70 TeV for the relevant lepto-quarks. We then discuss the $P - \bar{P}$ mixing with a toy model. Rates of the known mixings require the masses of some neutral scalars to be large.

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An understanding of the masses and mixing of quarks and leptons (the fermions) is one of the challenges of high energy physics. Thirty years ago, many studies followed an understanding of the spectrum of hadrons. Hopefully, the study of masses and mixing of fermions will similarly open a new world of physics study. Indeed, much effort has been made to solve this puzzle in the past twenty years [1,2,3,4]. There are strong indications, from the previous results, that the mass problem is deeply related to physics at very high scales, the grand unification scale, or even the superstring scale.  

However the possibility of explaining at least part of the mass matrices by low energy physics is still very attractive. As an effort to find a window to a low scale explanation, we try here an approach in which small entries in the mass matrices are assumed to be products of radiative corrections. In other words, the famous Higgs mechanism $m_f = Gv$ (where $G$ is the relevant Yukawa coupling constant and $v$ is the vacuum expectation value of the relevant Higgs field) is no longer responsible for the full texture of the mass matrices. There are some initial texture zeros (ITZs), which according to the previously suggested textures should be non-zeros. Early suggestions of ITZs can be found in [5]. Recently, the author suggested a different pattern of ITZs [6], in which the needed corrections to the ITZs are all less than $.001m_t$, which are comfortable for radiative corrections. Inversely, radiative correction provides a new mechanism for an extra mass hierarchy.

Once the bold assumption is made, the mechanism of mass hierarchy must play an active role in low energy phenomenology. The easiest low energy physics to consider is the anomalous dipole moments corresponding to the radiatively produced mass. The relation between the two important quantities is controlled roughly by only one parameter, the largest mass of the involved scalars. By confronting the resultant dipole moments with experimentation, the scale of new physics, the physics of new scalars, is obtained. This study can be done model independently as shown below. One may go along the model-independent way further (although this is not done in this paper). However, in order to simplify the relation of this mechanism to other aspects of physics, we will discuss a toy model in the later stage (Section IV) of this work. The relevant loop diagrams in this model

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Footnote: For example, the minimal grand unification theory predicts $m_b = m_\tau$ [2]. For recent studies in this direction, see [4].
are obviously convergent.

I. General Considerations

A typical Feynman diagram for a radiatively produced mass matrix element is shown in Fig. 1. Note that since this matrix element, whatever it is, vanishes at the tree level, the loop diagram which produces this element must be convergent, for the sake of the renormalizability of the assumed theory. Therefore the boson line in this loop diagram must be made of two different bosons which mix with each other. If Fig. 1 exists in the assumed theory and is responsible for the corresponding mass matrix, then the Feynman diagrams in Fig. 2 must exist too, where Fig. 2a is only for neutral current couplings and Fig. 2b should be added if charged couplings are involved. A comparison of the two diagrams leads immediately to a relation between a mass matrix element and the corresponding dipole matrix element. This relation actually sets a limit to the masses of the internal bosons, when combined with the relevant experimental data, because the most sensitive parameter in this relation is the heaviest mass in the loop.

As is well known, the loop diagram is proportional to the mass of the internal fermion \( M \), so called the mass seed, with a suppression factor \( \kappa = \frac{1}{16\pi^2} \frac{\delta\mu^2}{\mu^2} g_1 g_2 \) where \( \mu^2 \) is the square of the largest mass among the three masses in the loop. \( \delta\mu^2 \) is the mixing mass between the two bosons, with \( \frac{\delta\mu^2}{\mu^2} \leq \frac{1}{2} \). This suppression factor immediately excludes large matrix elements to be considered as radiatively produced, if new extremely heavy fermions are not introduced to provide the mass seed \( M \). The reason is simple because the largest fermion mass in the standard model (SM) is the mass of the top quark \( m_t \). Of course, it is still possible that some hidden heavy fermion sectors may provide the large mass seed \( M \). Then all the fermion mass matrix elements can be radiatively produced. However, in this article we will examine the most interesting scenario whereby the top mass is the mass seed for radiative production of small masses, although some of our conclusions may also be valid in other situations. This scenario is interesting because it is most restrictive.

It is unlikely that the bosons in the loop are gauge bosons. It has been noticed that if the gauge bosons are coupled to the fermions with the same chirality, then Fig. 1 will be proportional to the larger of the outside fermion masses, which is too small. Even if the two
gauge bosons are coupled to fermions with different chiralities, the factor $g_1g_2$ will already be at order 0.1. This suppression will further limit the usage of Fig. 1\[1\]. In contrast, the discovery of the heavy top quark implies that the corresponding Yukawa coupling constant is about 1. This provides quite a lot of room for the mechanism of radiative production of masses.

Another line of thinking also leads to the idea of radiative production of part of the fermion mass matrices. Since the observation of the fermion mass hierarchy, many authors have speculated that perhaps the masses of the third family are produced first, and the other small masses are produced later by different mechanisms [8]. A possible candidate of the second mechanism is radiative production\[9\]. Indeed, considering the order of magnitude of the matrix elements, the following patterns for initial mass matrices are recommended [6]

\[
M_i^F = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -b^F \\ 0 & b^F & c^F \end{pmatrix},
\]

with $F = U, D, L$ for up, down and leptonic mass matrices, where $c^U = m_t$, $c^D = m_b$ and $c^L = m_\tau$; and $c^U : b^U \sim c^D : b^D \sim \sqrt{m_t/m_c}$, $c^L : b^L \sim \sqrt{m_\tau/m_\mu}$. The small step hierarchical chain $c^U \rightarrow b^U \rightarrow c^D \rightarrow b^D \rightarrow c^L \rightarrow b^L$ is realized by a combination of sequentially smaller Yukawa coupling constants and vacuum expectation values. Since there is not a principle to require the equalness of Yukawa couplings or VEVS in case of a multi-Higgs contribution to masses, a reasonable small difference cannot be ruled out. In Ref. 6 the (ambiguous) naturalness principle is appealed to explain the sequence of the Yukawa couplings. In other words it was assumed that a smaller Yukawa coupling is originated from a smaller symmetry of the corresponding Yukawa term in the Lagrangian. It was found that the same philosophy cannot go beyond the initial pattern of Eq(1). Therefore a second mechanism for mass hierarchy must be introduced in order to produce the whole mass matrix. By separating the initial elements from the initial texture zeros (ITZs) we can at least expect an extra hierarchy due to the loop suppression factor $\kappa$. Note, to be different, our initial texture is

\[3\]If the hidden heavy fermion sector is the so-called right handed mirror fermions [7], then this mechanism may work, although the model is less phenomenologically restrictive.
not Hermitian. The intended mass matrices produced by radiative corrections are then

\[ M^F_r = \begin{pmatrix} 0 & -x^F & 0 \\ x^F & y^F & 0 \\ 0 & 0 & 0 \end{pmatrix}, \]

(2)

where \( y^D \sim m_s, \ y^U = y^L = 0 \), etc. The scalars which are involved in the leptonic loop diagram to produce the element \( x^L \) from the top mass must be lepto-quarks which carry both the lepton number and the baryon number. Their electric charges could be either -1/3 (if the fermion in the loop is the anti-top quark) or -5/3 (if it is the top quark in the loop). The total mass matrices for each type of the fermions are the sums of the corresponding matrices

\[ M^F = M^F_t + M^F_r \]

which is similar to one of the desired texture patterns suggested in the literature [10].

The corresponding dipole moment matrices are

\[ \mu^F = \begin{pmatrix} 0 & -x'^F & 0 \\ x'^F & y'^F & 0 \\ 0 & 0 & 0 \end{pmatrix}. \]

(3)

Note, in principle, the anomalous dipole matrices need not correspond to the radiative mass matrices, Eq. (2). In particular, for instance, the \((3, 3)\) element can be non-zero. However, its value is unrelated because of uncertainties in the internal parameters. Therefore, we would rather make it vanishingly small. When the mass matrices are diagonalized by two unitary matrices\(^4\), the dipole matrices should be subject to the same rotation to become the dipole matrices in the mass representation,

\[ \mu^m_F = U^F_L \mu^F U^F_R, \]

(4)

where \( U_L, U_R \) are U-matrices such that

\[ U_L \, M U_R \]

is diagonalized with all eigenvalues positive.

\(^4\)Our symbols in matrices (1) to (3) are intended just to show the pattern. In particular, we do not imply that the matrix elements are real.
II. The Results of the Loop Diagram Calculation

The diagrams in Figs. 1 and 2 are all convergent and are solidly calculated. For a radiative mass we have

\[ m_{12} = \frac{M^* \delta \mu^2}{16\pi^2} \frac{\mu^2 g_1 g_2 C}{\mu^2} \]

(5)

with \( m = |M| \) and

\[ C = \mu^2 \left\{ \frac{m^2}{(m^2 - \mu_1^2)(m^2 - \mu_2^2)} \ln m^2 + \frac{\mu_2^2}{(\mu_2^2 - m^2)(\mu_2^2 - \mu_1^2)} \ln \mu_2^2 + \frac{\mu_1^2}{(\mu_1^2 - m^2)(\mu_1^2 - \mu_2^2)} \ln \mu_1^2 \right\} \]

(6)

where \( \mu \) is the largest among \( m \equiv m_t, \mu_1, \) and \( \mu_2 \). The value of \( C \) is at order 1. When one of the three masses is negligibly small, \( C \) is convergent even if this small mass is set to zero. For example, when \( m = 0 \), we have

\[ C = \frac{\mu^2}{\mu_2^2 - \mu_1^2} \ln \frac{\mu_2^2}{\mu_1^2}. \]

However, setting any two masses vanishing will cause \( C \) to be infrared (logarithmically) divergent. One can, if one likes, express \( m_{12} \) in terms of mass eigenstates of the relevant Higgs particles, instead of the eigenstates of interactions as in Eq. (5). In this case, Fig. 1 will be replaced by two divergent diagrams, with their coupling constants satisfying a GIM [11,12] like unitarity condition which guarantees the convergence of the sum of the two diagrams.\(^5\)

The magnitude of \( m_{12} \) is proportional to the mass of the fermion in the loop, \( m = |M| \). The hierarchy property of the fermion mass spectrum immediately leads to the dominance of the loop diagram whose internal fermion is a top quark. Loops with lighter internal fermions can be completely ignored. This will greatly simplify the calculation and the analysis. It is worth noting that \( m_{12} \) does not decrease with the scale of the loop \( \mu^2 \). Instead, it is proportional to the ratio \( \delta \mu^2/\mu^2 \). This means somehow the radiatively produced mass matrix elements are immune from the “decoupling theorem”, which is typical for physics related to Higgs particles [13]. Since too many parameters appear in (5), other data are needed to extract a specific piece of information. It turns out that the corresponding dipole moment is the most convenient of these.

\(^5\)The author thanks James Liu for providing such a discussion.
For the corresponding anomalous dipole moments, a universal formula can be written as

$$\mu_{12} = e \frac{m_{12}}{\mu^2} \left[ \frac{Q_tC_t + Q_HC_H}{C} \right],$$

(7)

where \(Q_t + Q_H = Q_{\text{out}}\). The second term in the brace appears only when the electric charge of the Higgs is non-zero, \(Q_H \neq 0\). \(\mu\) has been explained before. \(C_t\) and \(C_H\) are, respectively,

$$C_t = \frac{\mu^4}{\mu_2^2 - \mu_1^2} \left[ \frac{\mu_2^2}{(m^2 - \mu_2^2)^2} \ln \frac{m^2}{\mu_2^2} - \frac{\mu_1^2}{(m^2 - \mu_1^2)^2} \ln \frac{m^2}{\mu_1^2} \right] - \frac{\mu_1^4}{(m^2 - \mu_1^2)(m^2 - \mu_2^2)},$$

(8)

$$C_H = \frac{\mu_2^4}{2\mu_1^2\mu_2^2}.$$

(9)

For \(C_H\), we have only calculated a simple case in which \(m^2 \ll \mu_1^2, \mu_2^2\). When masses of the two charged scalars are very different, the \(C_H\) term in (7) will dominate and the dipole moment will have no longer a \(1/\mu^2\) suppression. Now, from Eq. (7) we see that we can indeed extract some specific information about the loop scale once \(m_{12}\) is somehow known.

The uncertainty factor in the squared bracket is a slow varying real function of the ratios of masses in the relevant loops (except when \(C_H\) term dominates). \(\mu_{12}\) and \(m_{12}\) share the same phase up to modular \(\pi\), \(\arg(\mu_{12}) = \arg(m_{12}) + \mod[\pi]\). Furthermore this extraction can be done without digging into the detail of the underlying physics, especially its complicated Higgs sector. Since so far no scalar has ever been found, one may suspect the existence of the scalar particles; on the other hand, the extreme uncertainty leaves room for bold speculations.

**III. The Anomalous Dipole Moments**

We will not go into the details of how the corresponding relevant parameters fit into the scheme. What makes such work difficult is the lack of information about the concrete form of the mass matrices. What we have measured so far, in the case of weak interactions of the quarks, are the (diagonalized) mass values and their weak mixing (the so called CKM mixing matrix [12], which is the product of two unitary matrices, \(V(CKM) = U_L^U U_L^{D\dagger}\)). To skip these complications, we therefore assume that the following mass matrix is reached [10] as the sum of initial and radiative textures

$$M = \begin{pmatrix} 0 & -x & 0 \\ x & y & -b \\ 0 & b & c \end{pmatrix}.$$
The diagonalized unitary matrices are close to identity matrices (therefore they are called small unitary matrices), up to a diagonal phase matrix $P$. Without loss of generality we choose $U_L$ and $U'$ to be small unitary matrices and let $U_R = U'P$. These small unitary matrices are typically of the form

$\begin{pmatrix}
1 & -\epsilon_1 & -\epsilon_3 + \epsilon_1\epsilon_2 \\
\epsilon_1^* & 1 & -\epsilon_2 \\
\epsilon_3^* & \epsilon_2^* & 1
\end{pmatrix}$ (11)

Applying this form to $MM^\dagger$ to obtain $U_L$ and to $M^\dagger M$ to obtain $U'$, we obtain, to the leading orders of hierarchical quantities, for the left handed unitary matrices:

$\epsilon_{1L} = \frac{x}{y + \epsilon_{2L}b}, \quad \epsilon_{2L} = -\frac{b}{c}, \quad \epsilon_{3L} = 0$, (12)

and the masses are

$m_1 = |\epsilon_{1L}x|, \quad m_2 = |y + \epsilon_{2L}b|, \quad m_3 = |c|$. (13)

The parameters in $U'$ are

$\epsilon'_1 = -\epsilon_{1L}^*, \quad \epsilon'_2 = -\epsilon_{2L}^*, \quad \epsilon'_3 = 0$. (14)

The phase matrix $P$ is

$P = \text{diag}(e^{-i(2\phi_1 - \phi_2)}, \ e^{-i\phi_2}, \ e^{-i\phi_3}) \equiv (P_1, \ P_2, \ P_3)$, (15)

with $\phi_1 = \text{arg}(x), \ \phi_2 = \text{arg}(y + b^2/c), \ \phi_3 = \text{arg}(c)$. These results are seen in the literature [10] in different contexts with some changes. (Mainly because our mass matrices are not Hermitian, in contrast to most of the literature which are Hermitian.) Corresponding to the three mass matrices for the up-, down-, and charged lepton-type of fermions (We do not consider massive neutrinos in this paper.), there are altogether six U-matrices for diagonalization of the respective mass matrices. All of them are physically relevant as will be shown below. Only one combination of the six U-matrices makes the CKM matrix. The other five different combinations represent physics beyond the CKM matrix.

Since the dipole matrices are not proportional to their corresponding mass matrices, miracle enhancement or suppression of the dipole moments which already exist in Eq. (3) is
not expected after diagonalization, except that the zero elements in (3) may become non-zero. Indeed, the dipole moment in the mass representation is

$$\mu^m = \begin{pmatrix} 2\epsilon_1 x' P_1 - \frac{m_1}{m_2} y' P_2 & (-x' + \epsilon_1 y') P_2 & (-\epsilon_2 x' + \epsilon_1 \epsilon_2 y') P_3 \\ (x' - \epsilon_1 y') P_1 & (2\epsilon_1^* x' + y') P_2 & (\epsilon_1^* \epsilon_2 x' + \epsilon_2^* y') P_3 \\ (-\epsilon_2^* x' + \epsilon_1 \epsilon_2^* y') P_1 & (-\epsilon_1^* \epsilon_2^* x' - \epsilon_2^* y') P_2 & -\epsilon_2^* y' P_3 \end{pmatrix}, \quad (16)$$

where $\epsilon_i = \epsilon_{Li}$. The calculation has been consistently done under leading order approximations, based on the hierarchical property of the mass matrices. Of course there are three kinds of such dipole matrix corresponding to flavor changed (and neutral) EM processes among U-type, D-type, and L-type fermions respectively.

These dipole matrices need immediately to be compared with experiments. The most sensitive are the electrical dipole moments of the lepton and the neutron. Note since $y' = 0$ for leptons, the electrical dipole moment of the electron is zero, compatible with the experiments [15,16]. The electrical dipole moment of the neutron comes from that of the u-quark ($D_u^u$) and that from the d-quark ($D_d^d$). We express this as $D_n = D_u^u + D_d^d = \frac{4}{3} d_d - \frac{1}{3} d_u$, with $d_u$, and $d_d$, the E-dipoles of the u-quark and the d-quark respectively. Since $y'^U = 0$, $d_u = 0$.

For the d-quark,

$$D_d^d \sim \frac{4}{3} e \frac{m_d}{\mu^2} \left( \frac{2C_t - C_H}{C} \right) \arg(m_u - \epsilon_2^* D_d^D). \quad (17)$$

$\mu$, the mass of one of the charged scalars in the $y^D$ loop, has to be 20 TeV, in order to obtain the needed suppression [17] (the angle in the formula is taken as 0.1.) The experimental and theoretical uncertainties of the magnetic moments of the electron and the muon are both $10^{-22}$ e·cm [18,19]. These are comfortable with the scale 1.4 TeV for the lepto-quarks in the relevant loop. However, the process $\mu \rightarrow e + \gamma$ puts a stronger limit to the same lepto-quark mass. The results on dipole moments are summarized in Table 1. In this table, experimental data are compared with the minimal standard model predictions. The scales of the dipole loops (Fig. 2) are obtained based on the experimental data. For the names of the scalars which appear in the table, see the next section. From this table we can see that the restrictions from dipole moment experiments to the masses of the neutral scalars, which couple to $t\bar{c}$ or $t\bar{u}$, may not exist. Therefore, the possibility of a top to on-shell scalar decay is not ruled out, because a light scalar may exist.

However, when we come to a specific model, this possibility needs to be reexamined.
In particular, we will be able to calculate the $P - \bar{P}$ mixing due to scalar mediated flavor changed neutral currents. The mixing mass here will be proportional to $\mu^{-2}$ of the relevant scalar. It is also very sensitive to the relevant Yukawa couplings. The Yukawa couplings for each of the scalars involved are originally given in the interaction representation of the fermions. When we discuss physics in the mass representation of the fermions, we also need to transfer these couplings into the mass representation. This will be exemplified in the following section.

IV. Physics with an ITZ Toy Model

Our previous discussion on the dipole moments is model independent. However, we would like to proceed further, in particular to understand the implications of the obtained mass limits in Table 1 and compare these limits with the limits obtained elsewhere. To work with a specific model will make such a discussion much easier, and much more specific. Therefore, we will introduce a toy model which will be able to produce the desired ITZs in (1) and the radiative corrections in (2). This model enjoys less symmetries in the Yukawa terms compared with that in Ref. [6]. However this model provides less stringent limits for the masses of the relevant scalars. This by no means that the limits obtained here are the lowest possible. On the other hand, one may hold a different philosophy so to keep the terms as symmetric as possible. Then the mass scales of the new scalars will be much higher, most of them in the beginning of the desert of GUT.

The gauge sector and the fermion gauge interaction sector of this model is completely standard, however we have a complicated Higgs sector and a complicated Yukawa sector. Let us plainly write down the whole Yukawa interactions, since our main concern here is the Yukawa sector:
$L_Y = G_1 \left[ \bar{\psi}_L^3 \Phi_{33} U_R^3 + \sqrt{2} \bar{\psi}_L^2 \Phi_{23} U_R^3 + \sqrt{2} \bar{\psi}_L^1 \Phi_{13} U_R^3 + \sqrt{2} \bar{\psi}_L^3 \Phi_{23} U_R^2 \right] + h.c. \\
+ G_2 \left[ \bar{\psi}_L^3 \xi_1 U_R^2 - \bar{\psi}_L^2 \xi_1 U_R^3 - \bar{\psi}_L^3 \xi_2 U_R^1 \right] + h.c. \\
+ G_3 \left[ \bar{\psi}_L^3 \phi^{33} D_R^3 + \frac{1}{2} \bar{\psi}_L^3 \phi^{23} D_R^2 \right] + h.c. \\
+ G_4 \left[ \bar{\psi}_L^3 \eta_1 D_R^2 - \bar{\psi}_L^3 \eta_2 D_R^1 - \bar{\psi}_L^3 \eta_1 D_R^3 \right] + h.c. \\
+ \text{leptonic part} + \text{lepto - quark part}.$

We may identify $\Phi_{ij}$ (and $\phi^{ij}$) ($i, j = 1, 2, 3$) as a set of $SU(3)$ sextet (and anti-sextet). However, part of their components is missing in the Yukawa sector. $\eta_i$ (and $\xi^i$) are triplet (and anti-triplet) Higgs fields in the same sense. These Higgs fields are also all $SU(2)_L$ doublets. The $SU(3)$ here is a global symmetry group. All left handed fermions are in triplet (3) of $SU(3)$ and right-handed fermions are in $3^*$. There is also a global $U(1)$ symmetry. The $U(1)$ charges follow the formula $\xi = I - L/3$, where $I$ is the $SU(3)$ index of the multiplet. For the Higgs potential and how Higgs develop VEVs, see Ref. [20].

The ratios of the Yukawa coupling constants are assumed to be

$$G_1 : G_2 \sim 3, \quad G_2 : G_3 \sim 2, \quad G_3 : G_4 \sim 5.$$  \hspace{1cm} (19)

Therefore the Yukawa coupling constants in this model are all compatible. Such small differences of Yukawa couplings will, combined with the differences in VEVs of the four Higgs fields $\Phi_{33}$, $\xi^1$, $\phi^{33}$, $\eta_1$, produce the needed small step hierarchy in the initial pattern of the mass matrices as that in (1). Therefore after SSB we will have the following initial mass matrices:

$$M_i^U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & G_2 V' \\ 0 & -G_2 V' & G_1 V \end{pmatrix},$$  \hspace{1cm} (20)

$$M_i^D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & G_4 v' \\ 0 & -G_4 v' & G_3 v \end{pmatrix},$$  \hspace{1cm} (21)
\[ M^L_i = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & G_4' v' \\ 0 & -G_4' v' & G_4' v \end{pmatrix}, \] (22)

There are five initial texture zeros (ITZs) in each mass matrix. We assume

\[ V : V', \sim 3.6, \ V' : v \sim 1.7, \ v : v' \sim 2.1. \]

Of course, \( \sum \text{VEV}^2 = 245^2 \text{GeV}^2 \). Note that the initial mass hierarchy in this toy model is a combined effect of sequentially smaller coupling constants and the VEVs. All ITZs in these matrices are protected by the symmetry property of the Yukawa sector. Actually all naive corrections to ITZs vanish, unless Higgs mixing is introduced, as is in Fig. 1.

Eq. (5) can be applied to the specific Yukawa interactions, with \( \mu_1, \mu_2 \) masses of the specific Higgs particles which appear in the specific diagrams, and \( \delta \mu^2 \) their respective mixing masses squared. We assume that the factor \( g_1 g_2 M^* \) in (5) are replaced respectively by the specific coupling constants in the model as in the following

\[ x^U \propto |G_1|^2 G_2 V*/\sqrt{2}, \ y^D \propto |G_1|^2 G_3 V*/2, \ x^D \propto |G_1|^2 G_4 V*/\sqrt{2}, \ x^L \propto \lambda_1 \lambda_2 G^* V*. \] (23)

One can easily recognize, for instance, in order to produce the (2,1) element \( x^U \), the mixing between \( \Phi^0_{23} \) and \( \xi^2 \) must be introduced, and to produce the (1,2) element, that between \( \Phi^0_{13} \) and \( \xi^1 \) must be introduced. In general, these two elements may be different, however, for simplicity, we will only analyze the case when they follow the pattern in (2). A general analysis is not difficult to work out. Once these small elements are obtained, all the analyses summarized in Table 1 will follow. It is worth pointing out that electric dipole moment of the d-quark is generally nonzero in this model. Indeed, the angle in (17) is decided by the phase of

\[ \frac{(b^D)^2}{c^D} \frac{1}{y'^D} \propto \frac{(G_4 v')^2}{G_3 V} \frac{1}{G_3 V^*}. \]

This quantity is rephasing invariant, unless there is an exact symmetry which allows separate changes of the phases of different vacuum expectation values.

Some exotic processes are allowed in this model, if suitable scalar mixing is introduced. For example, \( t \rightarrow c (u) s \bar{b} \) is allowed if the mixing between the neutral components \( \Phi^0_{23} (\Phi^0_{13}) \) and \( \phi^{23,0} \) exists; and \( t \rightarrow c b b \) exists if \( \Phi^0_{23} - \phi^{33,0} \) mixing exists etc. However since
these mixings are not used in our small mass matrix element calculations, the magnitude of mixing here can be any small, therefore the width of these processes can be any small, unless $\Phi_{23}^0$ ($\Phi_{13}^0$) is lighter than the top quark. The possibility of a light $\Phi_{23}^0$ ($\Phi_{13}^0$) is not ruled out by the dipole experiments. However, in this specific model, it could be ruled out because it also mediates flavor changed neutral currents which may cause the $K - \bar{K}$ like mixing. Existing data on the mixing of such systems are very stringent. But before going into a detailed analysis, let us first find out the Yukawa couplings in the mass representation.

The quark neutral current Yukawa interactions in (18) are typically of the pattern

$$
\begin{pmatrix}
0 & 0 & a \\
0 & b & a' \\
a' & b' & c
\end{pmatrix}.
$$

When the quarks are transformed into the eigenstates of masses, the corresponding Yukawa coupling becomes (The phase factor $P$ is neglected here.)

$$
\begin{pmatrix}
\epsilon[a + a' + \epsilon_1 \Delta b + \epsilon c] & -\epsilon_2[a - |\epsilon_1|^2 a' + \epsilon_1 \Delta b + \epsilon c] & a + \epsilon_1(b + |\epsilon_2|^2 b') + \epsilon c \\
\epsilon_2[a' + |\epsilon_1|^2 a + \epsilon_1 \Delta b + \epsilon c] & \epsilon_2[\epsilon_1^*(a' - a) - \Delta b - \epsilon_2 c] & -\epsilon_1^* a + b + |\epsilon_2|^2 b' + \epsilon_2 c \\
(a' - \epsilon_1 b' - \epsilon_1 |\epsilon_2|^2 b + \epsilon c) & -\epsilon_1^* a' + b' + |\epsilon_2|^2 b - \epsilon_2 c & -\Delta b + c
\end{pmatrix},
$$

(25)

where $\epsilon = \epsilon_1 \epsilon_2$, $\Delta b = b - b'$. Applying this form to $G_1$ couplings, we find that, for example, the contribution of $\Phi_{23}^0$ to $D - \bar{D}$ mixing vanishes. This is an accidental fact of this specific model. Consequently, the mass of $\Phi_{23}^0$ is not limited by any existing $P - \bar{P}$ mixing data. However the contribution of $\Phi_{13}^0$ to $D - \bar{D}$ mixing is proportional to $(\epsilon_1 U_1 \epsilon_2 U_2)^2$, which is not big enough as a suppression. Therefore the mass of $\Phi_{13}^0$ needs to be larger than 2.1 TeV. Such a heavy $\Phi_{13}^0$ is an indication that the $SU(3)$ symmetry is explicitly badly broken also in the Higgs sector, because otherwise this particle would be a light pseudo-Nambu-Goldstone particle. The mass limits for other neutral scalars from similar considerations are summarized in Table 2. We can see that in this specific model, the bounds obtained from the $B_d - \bar{B}_d$ mixing are all overwritten by those obtained from the $K - \bar{K}$ mixing. Comparing Table 1 and Table 2, we still find attractive scenarios for the decay of top to an on shell neutral scalar$^6$, $t \rightarrow c + \Phi_{23}^0$. The life time of $\Phi_{23}^0$ will be relatively long because both of the smallness of the relevant mixing and of the heaviness of $\phi^{23,0} > 4.2$ TeV, if it decays into $\bar{b}s$) and $\phi^{33,0} > .53$ TeV, if it decays into $\bar{b}b$.

$^6$Besides the simple $t \rightarrow c + \Phi_{23}^0$ mode, such a decay through a mixing between $\Phi_{23}^0$ (in case it is too heavy to be on shell in, for example, some alternative models) and a light scalar is discussed in [6].
V. Concluding Remarks

The puzzle of the pattern of masses and mixing of quarks and leptons is believed to be deeply related to physics both at high scales and low scales. Here we have made an effort to connect the mass matrices of the fermion with other low energy physics phenomena. Our approach is based on an almost model-independent relation between a radiatively produced mass and an anomalous dipole moment, and an assumption on the initial texture zeros in the mass matrices.

By doing so we find that if the desired small elements in the fermion mass matrices are radiatively produced from a top mass seed, then masses of the new scalar bosons which are needed for this mechanism to work are above .1 TeV for neutral scalars and up to 70 TeV for charged scalars, in order to fit the known data on electric and magnetic dipoles of the fermions. The highest scale for this mechanism to work is two orders of magnitude higher than the weak scale, however it is well below the scale of grand unification. Our study with a toy model in section IV has enhanced the scale of some neutral scalars up to 4 TeV, although the model, in particular its parameters, are subject to optimization.

Since the scale of concern in this approach is low, other low-energy physical processes should also be examined, except for the dipoles and the $P - \bar{P}$ mixing discussed here. Exotic processes, especially exotic top decays, which do not exist in the minimal standard model are generally expected in this approach. In addition, more accurate data, such as $B_{c}(D \rightarrow \rho \gamma)$, will help further clarify the scale of radiative corrections. The improved measurement of $\Delta m$ for the $D^{0} - \bar{D}^{0}$ system will be crucial for the scale of the toy model discussed in section IV. Since we allow some scalar bosons to be as light as .1 TeV, their effects in some other loops will be also measurable and are therefore worth calculating.

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which obtained $\mu_{12} \propto m_{12}$, similar to (7) here.
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### Table 1. Dipole Moments of the Fermions

| dipole  | exper. | MSM | ITZ scalar mass (TeV) |
|---------|--------|-----|-----------------------|
| $d_e$   | $< 10^{-26} \text{e} \cdot \text{cm}[15]$ | 0   | no bound              |
| $d_\mu$ | $< 10^{-19} \text{e} \cdot \text{cm}[16]$ | 0   | no bound              |
| $\Delta \mu_e \& \Delta \mu_\mu$ | $< 10^{-22} \text{e} \cdot \text{cm}[18, 19]$ | compatible | $\mu(\text{lep} - \text{q}r\text{k}) > 4.5$ |
| $D_n$   | $< 10^{-25} \text{e} \cdot \text{cm}[17]$ | $10^{-31} \text{e} \cdot \text{cm}$ | $\mu_{\Phi_{23}}, \mu_{\phi^{21}} > 20$ |
| $\mu \rightarrow e\gamma$ | $Br < 5 \times 10^{-11}[21]$ | $Br = 0$ | $\mu(\text{lep} - \text{q}r\text{k}) > 70$ |
| $B \rightarrow K^*\gamma$ | $Br = 5 \times 10^{-5}[22]$ | compatible[23] | $\mu_{\phi^{23}} > 0.25$ |
| $B \rightarrow \rho\gamma$ | $Br < 2 \times 10^{-5}[24]$ | compatible | no bound |
| $D \rightarrow \rho\gamma$ | $Br < 10^{-1}[25]$ | $Br < 10^{-6}$ | no bound |

### Table 2. Bounds from Data on $P - \bar{P}$ Mixing Systems

| $P - \bar{P}$ | $\Delta m_{\text{exp}}$ (eV) | MSM (eV) | ITZ scalar mass (TeV) |
|----------------|-----------------------------|-----------|-----------------------|
| $D - \bar{D}$ | $< 1.4 \times 10^{-4}[26]$ | $3 \times 10^{-7}$ | $\mu_{\eta_0^0} > .42$, $\mu_{\phi^{23} \ 0} > .14$ |
|                |                             |           | $\mu_{\eta_1^0} > .42$, $\mu_{\phi^{23} \ 0} > .14$ |
|                |                             |           | $\mu_{\Phi_{23}}, \mu_{\phi^{21}} > 2.1$, $\xi_2^0 > 1.0$ |
| $B_d - \bar{B}_d$ | $3.5 \times 10^{-4}[27]$ | compatible | $\mu_{\eta_0^0} > .42$, $\mu_{\phi^{23} \ 0} > .14$ |
|                |                             |           | $\mu_{\eta_1^0} > .42$, $\mu_{\phi^{23} \ 0} > .14$ |
|                |                             |           | $\mu_{\Phi_{23}}, \mu_{\phi^{21}} > 2.1$, $\xi_2^0 > 1.0$ |
| $K - \bar{K}$ | $3.5 \times 10^{-6}[28]$ | compatible | $\mu_{\eta_0^0} > .42$, $\mu_{\phi^{23} \ 0} > .14$ |
|                |                             |           | $\mu_{\eta_1^0} > .42$, $\mu_{\phi^{23} \ 0} > .14$ |
|                |                             |           | $\mu_{\Phi_{23}}, \mu_{\phi^{21}} > 2.1$, $\xi_2^0 > 1.0$ |
| $B_s - \bar{B}_s$ | ?                             | 0.007     | $\Delta m \sim .007 \text{ eV}$ |
|                |                             |           | (if $\mu_{\phi^{23} \ 0} \sim 4.2$) |