Evidence for pronounced quark loop effects in QCD

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We have measured the hadron spectrum in lattice QCD, using staggered fermions, for 0 (the quenched approximation), 2 and 4 light degenerate dynamical quarks. In addition to earlier results involving extrapolations in valence quark masses for fixed dynamical mass, we also report results where we extrapolate in the dynamical mass for 4 flavors. We see a marked difference in the hadron spectrum for 2 and 4 flavors; the hadron spectrum is nearly parity doubled for 4 flavors, indicating smaller effects of chiral symmetry breaking. This pronounced effect in the hadron spectrum cannot be removed by a simple change in scale as the number of light quark flavors is changed. Further simulations at larger volume are needed to rule out finite volume effects.

1. INTRODUCTION

During the past two years, the lattice QCD group at Columbia has been exploring the effects of light (by today’s standards) dynamical quarks on the hadron spectrum in zero temperature simulations of QCD. Light quark effects are readily seen at finite temperature, where studies have shown the order of the QCD phase transition is sensitive to the number of light quarks, as expected from theoretical arguments. Zero temperature simulations to date, including the ones reported here, have shown little difference between 0 and 2 flavors of dynamical quarks. To see if this continues, we have done extensive simulations using 4 flavors ($N_f = 4$).

On our existing 256-node, 16 Gflop peak speed computer, we are essentially limited to lattices of size $16^3 \times N_t$ for light quark mass simulations ($ma \sim 0.01$, where $m$ is the bare staggered quark mass). Using $m_\rho$ to set the scale gives us lattice spacings of $a^{-1} \sim 2$ GeV, for the lighter masses we simulate. (The exact value for the scale depends on whether one uses $m_\rho$ extrapolated to zero quark mass or not.) This also makes our lattice about 1.5 fermi in the spatial directions. Quenched simulations show this introduces errors at the 5-10% level [1] and similar systematic errors can be expected for the full QCD simulations, although complete studies await the arrival of more powerful computers. Since our volume is limited, we cannot extrapolate to the infinite volume limit and also not the continuum limit. Thus, our results are expected to be changed quantitatively as these limits are taken, but there is no compelling reason to expect a qualitative change.

2. SIMULATIONS

Table I lists the parameters for our simulations. It is important to note that the 2 flavor results employ an inexact algorithm [2], with errors proportional to $(\Delta \tau)^2$. Since we have used the same step size, for a given dynamical mass, in both the 2 and 4 flavor calculations and the 4 flavor calculations have a very high acceptance, the finite step size errors for the inexact algorithm are apparently very small. We have used a smaller conjugate gradient stopping condition for the 4 flavor runs, since this enters into the accept/reject step, and we wanted to insure that this step is accurate.

In these simulations we have measured hadron correlators for a variety of different source types and sizes with many different valence quark
Table 1
Run parameters for the simulations whose results are reported here. The 2 flavor, \( m_a = 0.015, 0.02 \) and \( 0.025 \) results are from [3]. The longer thermalization time for the 0.025 run merely reflects measurements not being made from the beginning of the run.

| \( N_f = 2, \, \beta = 5.7 \)                  | \( m_{\text{dyn}}a = 0.01 \) | \( m_{\text{dyn}}a = 0.015 \) | \( m_{\text{dyn}}a = 0.02 \) | \( m_{\text{dyn}}a = 0.025 \) |
|---------------------------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| volume                                      | \( 16^3 \times 40 \)        | \( 16^3 \times 32 \)        | \( 16^3 \times 32 \)        | \( 16^3 \times 32 \)        |
| run length                                  | 4870                        | 3010                        | 1425                        | 2830                        |
| thermalization                              | 250                         | 250                         | 250                         | 530                         |
| step size                                   | 0.0078125                   | 0.0078125                   | 0.01                        | 0.01                        |
| CG stopping condition                       | \( 1.01 \times 10^{-5} \)   | \( 1.13 \times 10^{-5} \)   | \( 1.13 \times 10^{-5} \)   | \( 1.13 \times 10^{-5} \)   |
| \( N_f = 4, \, \beta = 5.4 \)              |                             |                             |                             |                             |
| volume                                      | \( 16^3 \times 32 \)        | \( 16^3 \times 32 \)        |                             |                             |
| run length                                  | 4450                        | 2725                        |                             |                             |
| thermalization                              | 250                         | 250                         |                             |                             |
| step size                                   | 0.0078125                   | 0.01                        |                             |                             |
| CG stopping condition                       | \( 1.13 \times 10^{-6} \)   | \( 1.13 \times 10^{-6} \)   |                             |                             |
| acceptance                                  | 0.95                        | 0.99                        |                             |                             |

masses (the masses which appear in the explicit Green functions for the hadron). This allows us to fit for excited states and check the stability of our effective mass plateaus [4]. We also have measured the chiral condensate for valence quark masses spanning 10 orders of magnitude which is useful as a probe of finite volume effects and chiral symmetry breaking.

3. PARTIALLY QUENCHED RESULTS

Results from some of these earlier simulations can be found in [4], [5] and [6]. We studied the valence hadron spectrum for 0, 2 and 4 quark flavors with a single fixed dynamical quark mass \( m_{\text{dyn}}a = 0.01 \) for \( N_f = 2 \) and 4. For \( N_f = 2 \) we used \( \beta = 5.7 \) and for \( N_f = 4, \, \beta \) was 5.4. The parameters were chosen so that the rho mass, extrapolated to zero valence quark mass, was equal (we achieved equality at the 2% level) for 0, 2 and 4 flavors. In choosing our parameters for the 4 flavor simulation, we were helped by the earlier 4 flavor work of [4].

Figure 1 shows valence hadron masses for partially quenched 2 and 4 flavor simulations with \( m_{\text{dyn}}a = 0.01 \). The parity partners \( \rho, \, a_1 \) and \( N \), \( N' \) for 4 flavors show less splitting, particularly in the \( m_{\text{val}}a \rightarrow 0 \) limit, than for 2 flavors.

The other major results of this work were (using \( m_\rho \) to set the scale):

1. \( m_N \) and \( m_\pi \) are very similar between 0 and 2 flavors. However, the mass of the parity partners, \( m_{N'} \), \( m_{a_1} \), differs by 2-3 \( \sigma \) between these two cases, with \( m_{N'} \) and \( m_{a_1} \) closer to their parity partners for 2 flavors.

2. For 4 flavors, \( m_N/m_\rho \) extrapolated to zero valence quark mass is 7% larger for 4 flavors than for 2 flavors (about 2\( \sigma \)). The dependence of this quantitative difference on lattice volume and spacing deserves further study.

3. Linear fits of \( m_\pi^2 \) to the valence quark mass do not go to zero as expected for a Goldstone particle. This is consistent with the finite volume effects expected from a cutoff in the Dirac eigenvalue spectrum. In particular, the mass of the Goldstone particle at zero quark masses is predicted to increase as the amount of chiral symmetry breaking decreases and this is seen in the data [4].
Figure 1. Masses for the parity partners, $\rho$ and $a_1$, $N$ and $N'$ for a partially quenched calculation. Unfilled symbols are for 2 flavors, filled circles are for 4 flavors. All simulations had $m_{\text{dyn}}a$ fixed at 0.01.

These results agree with the naive expectation that adding more powers of the determinant (more quark flavors) suppresses the small eigenvalues of the Dirac operator, which decreases the effects of chiral symmetry breaking since the chiral condensate is given directly by the density of eigenvalues at the origin. The size of the effect, for the quark masses we are currently able to simulate with, is certainly larger than anticipated, given the close agreement between the 0 and 2 flavor simulations.

4. FULL QCD RESULTS

Having seen a marked decrease in the splitting between parity partners in the partially quenched hadron spectrum, we have been investigating the hadron spectrum in full QCD as a function of $N_f$. To date, we have results for two dynamical mass values for 4 flavors and we can compare these to four dynamical mass values for 2 flavors. The three heavier masses for 2 flavors are results from [3] while the $m_{\text{dyn}}a = 0.01$ is newer and includes 3 times the data of the 0.01 point in [3]. Table 2 gives some of the hadron masses for our 2 and 4 flavor simulations.

Figure 2 shows the $\pi$, $\rho$ and nucleon masses for 2 and 4 flavors as a function of the dynamical quark mass. The unfilled symbols are the 2 flavor results; the filled symbols are for 4 flavors. The close agreement between the values for $m_\rho$ is due to our choice of parameters. The solid lines are fits to the 2 flavor results and show the extrapolation to zero quark mass. The fits have large $\chi^2$ per degree of freedom (7, 3 and 5 for the $\pi$, $\rho$ and nucleon respectively). The reason for this is unknown (finite volume effects are a likely candidate) and will hopefully be settled by future simulations. The dashed lines extrapolate the two points we have for 4 flavors. They are consistent with the larger value for $m_N/m_\rho$ we found for 4 flavors in the partially quenched hadron spectrum.

Figure 3 shows the $\pi$, $\rho$ and nucleon masses for 2 and 4 flavors as a function of the dynamical quark mass. Again, the unfilled symbols are for 2 flavors and the filled for 4. The 2 flavor fits have good $\chi^2$ values for the $a_1$ and $N'$. It is clear from the figure that the parity partners are not degenerate.
Table 2
Hadron masses for 2 and 4 flavor QCD.

|                     | $m_{\text{dyn}a} = 0.01$ | $m_{\text{dyn}a} = 0.015$ | $m_{\text{dyn}a} = 0.02$ | $m_{\text{dyn}a} = 0.025$ |
|---------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $N_f = 2$, $\beta = 5.7$ |                          |                          |                          |                          |
| $\pi$               | 0.249(2)                 | 0.293(2)                 | 0.349(2)                 | 0.388(1)                 |
| $\rho$              | 0.435(5)                 | 0.455(8)                 | 0.501(7)                 | 0.551(4)                 |
| $a_1$               | 0.564(13)                | 0.594(22)                | 0.638(18)                | 0.755(32)                |
| $N$                 | 0.663(8)                 | 0.685(10)                | 0.781(10)                | 0.839(6)                 |
| $N'$                | 0.760(14)                | 0.833(38)                | 0.905(46)                | 1.022(41)                |
| $N_f = 4$, $\beta = 5.4$ |                          |                          |                          |                          |
| $\pi$               | 0.292(5)                 |                          |                          |                          |
| $\rho$              | 0.438(8)                 |                          |                          |                          |
| $a_1$               | 0.493(7)                 |                          |                          |                          |
| $N$                 | 0.690(21)                |                          |                          |                          |
| $N'$                | 0.731(16)                |                          |                          |                          |

in the $m_{\text{dyn}a} \to 0$ limit. The dashed lines extrapolate the 4 flavor results to zero quark mass and one sees the almost complete degeneracy in this limit.

Figure 3. Masses for the parity partners, $\rho$ and $a_1$, $N$ and $N'$. Unfilled symbols are for 2 flavors, filled circles are for 4 flavors.

This apparent degeneracy leads to the question of whether these lattices are still in a confining, chirally asymmetric phase. Figure 4 shows the staggered fermion chiral condensate, $\langle \bar{\chi}\chi \rangle$, for 0, 2 and 4 flavors as a function of the quark mass (the dynamical mass for 2 and 4 flavors). The solid lines are fits for 0 and 2 flavors; the dashed line is an extrapolation for 4 flavors. The 4 flavor value at zero quark mass, 0.0023, is almost 4 times smaller than the 2 flavor value, 0.00854(17). Using the Gell-Mann–Oakes–Renner formula for $f_\pi$ gives (using $m_\rho$ at zero quark mass)

$$f_\pi/m_\rho(0) = 0.127$$ 2 flavors \hspace{1cm} (1)

$$f_\pi/m_\rho(0) = 0.061$$ 4 flavors \hspace{1cm} (2)

The error on the 2 flavor result is 0.005, but this involves using fits of poor $\chi^2$ and hence the error is likely unreliable. However, the difference between the 2 and 4 flavor results is large on the scale of the error.

5. CONCLUSIONS

We have presented data showing a strong $N_f$ dependence in the hadron spectrum for full QCD for fixed lattice volumes and spacings (in units of $m_\rho$). The results are consistent with the naive expectation that increasing the number of light quarks suppresses the part of the Dirac eigenvalue spectrum responsible for chiral symmetry...
It should be noted that extrapolating our two values for $m_{\pi}^2$ for 4 flavors does not give $m_{\pi}^2 \to 0$ as the dynamical quark mass goes to zero. We have seen this effect in the partially quenched simulations, where it is consistent with finite volume effects. It deserves more study for the full simulations. In addition, it would be interesting to see if partially quenched Wilson spectrum measurements also showed the effects seen for staggered fermions.

The author would like to thank Jim Sexton, Tony Kennedy and Robert Edwards for useful discussions and Catalin Malareanu for measuring the Wilson lines on these lattices.

REFERENCES

1. S. Gottlieb, Nucl. Phys. B (Proc. Suppl.) 53 (1997), 155.
2. S. Gottlieb, et. al., Phys. Rev. D 35 (1987), 2531.
3. Frank R. Brown, et. al. Phys. Rev. Lett. 67 (1991), 1062.
4. Dong Chen, Nucl. Phys. B (Proc. Suppl.) 42 (1995), 312; Nucl. Phys. B (Proc. Suppl.) 47 (1996), 382.
5. Dong Chen and Robert D. Mawhinney, Nucl. Phys. B (Proc. Suppl.) 53 (1997), 216.
6. Robert D. Mawhinney, Lattice QCD with 0, 2 and 4 Quark Flavors, RHIC Summer Study 1996, ed. D.E. Kahana and Y. Pang, Brookhaven National Laboratory-52514, CU–TP–802.
7. R. Altmeyer, et. al., Nucl. Phys. B 389 (1993) 445.
8. Frank R. Brown, et. al., Phys. Rev. D46 (1992) 5655.