Polarization States in $B \to \rho K^*$ and New Physics

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Abstract

The standard-model explanations of the anomalously-large transverse polarization fraction $f_T$ in $B \to \phi K^*$ can be tested by measuring the polarizations of the two decays $B^+ \to \rho^+ K^{*0}$ and $B^+ \to \rho^0 K^{*+}$. For the scenario in which the transverse polarizations of both $B \to \rho K^*$ decays are predicted to be large, we derive a simple relation between the $f_T$’s of these decays. If this relation is not confirmed experimentally, this would yield an unambiguous signal for new physics. The new-physics operators which can account for the discrepancy in $B \to \pi K$ decays will also contribute to the polarization states of $B \to \rho K^*$. We compute these contributions and show that there are only two operators which can simultaneously account for the present $B \to \pi K$ and $B \to \rho K^*$ data. If the new physics obeys an approximate $U$-spin symmetry, the $B \to \phi K^*$ measurements can also be explained.
1 Introduction

One class of $B$ decays which is particularly intriguing involves processes whose principal contribution comes from $\bar{b} \to \bar{s}$ penguin amplitudes. The reason is that there are already several results in these processes hinting at the presence of physics beyond the standard model (SM).

First, within the SM, the CP asymmetry in $B_d^0(t) \to J/\psi K_S$ ($\sin 2\beta = 0.725 \pm 0.037$ [1]) should be approximately equal to that in penguin-dominated $\bar{b} \to \bar{s}q\bar{q}$ transitions ($q = u, d, s$). However, on average, these latter measurements yield a smaller value: $\sin 2\beta = 0.43 \pm 0.07$ [2].

Second, within the SM, one expects no triple-product asymmetries in $B \to \phi K^*$ [3]. However both BaBar and Belle have measured such effects, albeit at low statistical significance [4].

Third, the latest data on $B \to \pi K$ branching ratios and CP asymmetries [5] appear to be inconsistent with a SM fit [6, 7]. The model-independent analysis in Ref. [7] has shown that the data can be accommodated with a new-physics (NP) operator in the electroweak penguin sector.

A fourth possible hint of NP occurs in $B \to V_1V_2$ decays, where the $V_i$ are light vector mesons. In such decays the final-state particles can be found with transverse or longitudinal polarization. SM factorizable amplitudes, which are expected to dominate in the heavy $b$-quark limit, result in a dominant longitudinal polarization, with the transversely-polarized amplitudes suppressed by $m_v/m_B$. While this is realized for $B \to \rho \rho$ decays, which receive $\bar{b} \to \bar{d}$ penguin contributions, in $B \to \phi K^*$ decays it is found that the transverse fraction $f_T$ is about equal to the longitudinal fraction $f_L$ [8, 9]. Competing NP [10, 11], and SM [12, 13, 14] explanations have been proposed. $B \to \rho K^*$ decays may offer a way to resolve this discrepancy.

In this paper we will be mainly focussing our attention on the third and fourth points above. In the decay $B \to \rho K^*$, unlike $B \to \phi K^*$, there are two states, distinguished by the charge of the $\rho$ meson: $\rho^+$ or $\rho^0$. Here, the final-state particles are also vector mesons, so that one can measure their polarization states. Now, the polarization states of $B \to \rho K^*$ can be related to those in $B \to \phi K^*$. For this latter decay, it is not clear whether the large observed value of $f_T/f_L$ is accommodated by the SM or best explained with NP. However one can distinguish between a SM and NP explanation by comparing the two charge states. In particular, we show that if one of the SM scenarios proposed in Refs. [12, 13] does explain the large $B \to \phi K^*$ transverse polarization, then the transverse fractions of the two charge states in $B \to \rho K^*$ should satisfy $f_T^+/f_L^0 \simeq 2(BR^0/BR^+)$. Alternatively, if the SM scenario for the $B \to \phi K^*$ modes in Ref. [14] is correct, then the $f_L$ fraction of both charged $B \to \rho K^*$ decays should be greater than 90%. If neither of these two results is observed then non-SM physics is involved in the decays. We derive and discuss these prediction in Sec. 2.

The decay $B \to \rho K^*$ is described at the quark level by $\bar{b} \to \bar{s}q\bar{q}$ ($q = u, d$). This
is the same quark-level decay that contributes to $B \to \pi K$. If there is NP in these latter decays, it will affect $B \to \rho K^*$. Thus, given a $B \to \pi K$ NP scenario, we can examine its effects on the $B \to \rho K^*$ polarizations. We review the data on $B \to \pi K$ decays, as well as the size of NP operators which can account for it, in Sec. 3.

Sec. 4 contains the calculation of the contribution of these NP operators to the polarization states of charged $B \to \rho K^*$ decays. Under some simplifying assumptions we show that only NP operators of the form $\bar{b}\gamma_R s \bar{d}\gamma_R d$ or $\bar{b}\gamma_L s \bar{d}\gamma_L d$ can explain both the $\pi K$ and $\rho K^*$ data. We then discuss ways of testing this scenario.

Finally, in Sec. 5 we examine the consequences of the NP scenario for $B \to \phi K^*$ decays. We show that if the NP respects an approximate U-spin symmetry, it can simultaneously account for the $\pi K$, $\rho K^*$ and $\phi K^*$ data. We conclude in Sec. 6.

2 $B \to \rho K^*$: Standard Model Predictions

Before examining the contributions of new physics to the polarization states in $B \to \rho K^*$ decays, it is first necessary to understand the SM predictions for these states.

In the following, we denote $A_0$ as the longitudinal polarization amplitude for a decay, and $A_{++}$ and $A_{--}$ as the amplitudes with both vector mesons in the right-handed or left-handed helicity state, respectively. The transverse amplitudes are then $A_{\parallel} = (A_{++} + A_{--})/\sqrt{2}$ and $A_{\perp} = (A_{++} - A_{--})/\sqrt{2}$, while the total amplitude squared is $|A_{\text{total}}|^2 = |A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2$. The individual polarization fractions are

$$ f_L = \frac{|A_0|^2}{|A_{\text{total}}|^2}, \quad f_{\parallel} = \frac{|A_{\parallel}|^2}{|A_{\text{total}}|^2}, \quad f_{\perp} = \frac{|A_{\perp}|^2}{|A_{\text{total}}|^2}. \tag{1} $$

For a given decay, the branching ratio is related to the polarization amplitudes by

$$ \text{BR} = \frac{(|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2)}{\text{PS/}\Gamma_{\text{total}}}, \tag{2} $$

where PS is a phase-space factor, and $\Gamma_{\text{total}}$ is the total decay width.

It is useful to express the amplitudes for the various decays in terms of diagrams [15]. These include a “tree” amplitude $T'$, a “color-suppressed” amplitude $C'$, a gluonic “penguin” amplitude $P'$, a color-favored electroweak penguin (EWP) amplitude $P'_{\text{EW}}$ and a color-suppressed EWP amplitude $P'_{\text{C EW}}$. Other diagrams are higher-order in $1/m_B$ and are expected to be smaller. They will be neglected in our calculations. Here the prime on the amplitude stands for a strangeness-changing decay.

The diagram $P'$ in fact includes three pieces, corresponding to the internal quarks $u, c$ and $t$.

$$ P' = V_{ub}^* V_{us} P'_{ut} + V_{cb}^* V_{cs} P'_{ct} + V_{tb}^* V_{ts} P'_{ct} = V_{ub}^* V_{us} P'_{ut} + V_{cb}^* V_{cs} P'_{ct}. \tag{3} $$
Here, $P'_q = P'_q - P'_t$ ($q = u, c$). On the right-hand side, the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix has been used to reduce the number of terms. Since $|V_{us}V_{us}^*| \ll |V_{cd}V_{cd}^*|$, only the last term above is important; the first piece can be neglected. In addition, $P'_{EW}$ and $C'$ are expected to be smaller than $P'_{EW}$ and $T'$ [14], and will also be neglected in our calculations. Our amplitudes will therefore be expressed in terms of the diagrams $P'_ct$, $T'$ and $P'_{EW}$.

Furthermore, it has been shown that, to a good approximation, the EW amplitude $P'_{EW}$ can be related to $T'$ [16]:

$$P'_{EW} \simeq \frac{3}{4} \left( \frac{c_9 + c_{10}}{c_1 + c_2} - \frac{c_9 - c_{10}}{c_1 - c_2} \right) \frac{1}{\lambda^2} \frac{\sin(\beta + \gamma)}{\sin \beta} T' \equiv -Z T' \simeq -0.65 T' ,$$  

where $\lambda = 0.22$ is the Cabibbo angle, $\beta$ and $\gamma$ are CP phases (the phase information in the CKM quark mixing matrix is conventionally parametrized in terms of the unitarity triangle, in which the interior (CP-violating) angles are known as $\alpha$, $\beta$ and $\gamma$ [8]), and the $c_i$ are (known) Wilson coefficients [17].

We begin with a study of $B \to \phi K^*$. This is a pure penguin decay whose amplitude can be written

$$A(B \to \phi K^*) \simeq P'_ct - \frac{1}{3} P'_{EW} - \frac{1}{3} P'^C_{EW} .$$  

The penguin operator $P'_ct$ has $(V - A) \times (V - A)$ and $(V - A) \times (V + A)$ pieces while the EW’s have mainly $(V - A) \times (V - A)$ structure. For operators with $(V - A) \times (V \mp A)$ structure, a single spin flip is required to produce the $A_{-\perp}$ amplitude and a double spin flip for the $A_{\parallel \parallel}$ amplitude. Each spin flip leads to a $1/m_B$ suppression, causing the amplitudes $A_{\perp \parallel}$ and $A_{\parallel \parallel}$ to be $1/m_B$ suppressed. Thus, the SM operators naturally contribute mainly to the longitudinal polarization in $B \to \phi K^*$; their transverse polarization contribution is down by at least $O(1/m_B^2)$ relative to the total decay amplitude. The SM predictions for this decay can then be written as

$$f_L = 1 - O(1/m_B^2) \ , \quad f_T = O(1/m_B^2) \ , \quad f_\perp / f_\parallel = 1 + O(1/m_B) .$$  

The large transverse polarization observed in $B \to \phi K^*$ is then a puzzle for the SM.

However, there may be certain sources of large transverse polarization within the SM. Rescattering effects from tree-level $\bar{b} \to \bar{s}c\bar{c}$ operators have been identified as a possible source of large transverse polarization [12]. In Eq. 3 this effect is represented by $P'_c$ and is contained in $P'_ct$. The claim here is then that rescattering effects from $P'_c$ can enhance one or both of the transverse amplitudes associated with $P'_ct$.

Another possible source for the enhancement of the transverse amplitudes is associated with $P'_c$ though annihilation topologies [13]. The dominant contribution
comes from the \((S - P) \times (S + P)\) operators in the effective Hamiltonian, produced by performing a Fierz transformation on the \((V - A) \times (V + A)\) piece of \(P_{ct}'\). Even though formally suppressed in the heavy \(m_b\) limit, these contributions can produce an \(O(1)\) effect on the transverse polarization amplitudes due to large coefficients.

Finally, a third SM explanation for the large transverse polarization in \(B \to \phi K^*\) is proposed in Ref. [14]. Here, the transverse amplitudes are enhanced because the gluon from the \(\bar{b} \to \bar{s}g\) transition hadronizes directly into the \(\phi\), with the exchange of additional gluons to take care of color factors.

We therefore see that it may be possible to account for the large transverse polarization in \(B \to \phi K^*\) through SM effects. Fortunately, it is possible to test these explanations through the measurement of the transverse polarization in \(B \to \rho K^*\) decays. The key point here is that, in contrast to \(B \to \phi K^*\), there are two decays, \(B \to \rho^+ K^*\) and \(B \to \rho^0 K^*\). It is the measurement of the polarization states of both decays which allows us to distinguish the various explanations of the \(B \to \phi K^*\) data. In the following, we concentrate on charged \(B\) decays; the discussion is similar when neutral \(B\)'s are involved. We use the indices \('+\) and \('0\) to indicate the decays \(B^+ \to \rho^+ K^*\) and \(B^+ \to \rho^0 K^{*+}\), respectively.

In the SM, neglecting the small amplitudes, the two \(B^+ \to \rho K^*\) amplitudes are given by

\[
A(B^+ \to \rho^+ K^{*0}) \equiv A^+ = P_{ct}' , \\
\sqrt{2}A(B^+ \to \rho^0 K^{*+}) \equiv \sqrt{2}A^0 = -P_{ct}' - T' e^{i\gamma} - P_{EW}' .
\] (7)

We have explicitly written the dependence on the weak phase \(\gamma\), but the amplitudes contain strong phases. These amplitudes allow us to test the SM explanations of the large transverse polarization in \(B \to \phi K^*\) by comparing the two \(B \to \rho K^*\) decays. In particular, we calculate the transverse polarization pieces of

\[
\frac{|A^+|^2 - 2 |A^0|^2}{|A^+|^2}.
\] (8)

Consider first Ref. [12], which invokes rescattering from tree-level \(\bar{b} \to \bar{s}c\bar{c}\) operators, so that \(P_{ct}'\) is affected. The rescattering represented by \(P_{u}' (\bar{b} \to \bar{s}u\bar{u}\) operators) is small because of CKM suppression, so that the amplitudes \(T'\) and \(P_{EW}'\) are essentially unaffected. Ref. [13] is similar. Here, large annihilation effects modify \(P_{ct}'\); the amplitudes \(T'\) and \(P_{EW}'\) remain effectively unchanged. In both cases, the change in \(P_{ct}'\) persists in \(B \to \rho K^*\) decays, so that a large transverse polarization in these processes is expected. Since both decays are dominated by \(P_{ct}'\), to leading order the numerator of Eq. 8 vanishes, and it is predicted that

\[
f_T^+ = 2f_T^0 \left( \frac{BR^0}{BR^+} \right) .
\] (9)
The systematic error in this relation comes from the contribution of $T'$ to the transverse polarization, which is suppressed by $m_\nu/m_B$:

$$\text{sys} = O \left( \frac{2 T' m_\nu}{P'_\text{ew} m_B} \right) \sim 10\% \ .$$  \hspace{1cm} (10)

We repeat that this systematic error holds only for the case in which the transverse polarization in both $B \to \rho K^*$ decays is large. If it is small, then the systematic error is correspondingly larger.

In the third SM explanation \[14\], the transverse amplitude in $B \to \phi K^*$ is enhanced due to direct gluon hadronization into the $\phi$. Since the gluon has isospin zero, there should be no effect on $B \to \rho K^*$. Thus, in this model the usual SM arguments apply to both decay modes, giving a $f_T$ that is suppressed by $(m_\nu/m_B)^2$.

These qualitative arguments can be made quantitative. We note that the amplitudes given in Eq. (7) apply to the longitudinal and transverse polarizations individually. Thus, the transverse pieces ($T = \perp, \parallel$) of the two amplitudes are related as

$$\sqrt{2}(A^0) = -(A^+) e^{i \Delta_T} \ .$$  \hspace{1cm} (11)

with

$$x_T e^{i \Delta_T} = \frac{T'_\text{ew} e^{i \gamma} + P'_\text{ew} T}{P'_T} = \frac{T'_\text{ew} (e^{i \gamma} - Z)}{P'_T} \ .$$  \hspace{1cm} (12)

Now, because QCD respects isospin symmetry, the phase factors in Eq. (2) for both $B^+ \to \rho^+ K^*$ and $B^0 \to \rho^0 K^*$ are equal to within a few percent. Thus, a prediction of the SM using Eq. (11) is that the transverse polarizations in both charge states of $B \to \rho K^*$ should be related. At leading order, $\sqrt{2}(A^0) = -(A^+) e^{i \Delta_T}$, so that

$$E_T = \frac{f^+ T^+ - 2 f^0 T^0}{f^+ T^+} \approx 0 \ .$$  \hspace{1cm} (13)

The systematic error in this relation, $\Delta E_T$, can be estimated by keeping terms linear in $x_T$:

$$\Delta E_T \approx -2 x_T \cos \Delta_T \ , \quad x_T \approx \left| \frac{T'_T}{P'_L} \right| (1 + Z^2 - 2Z \cos \gamma)^{1/2} \sqrt{\frac{f^+}{f^+ T^+}} \ .$$  \hspace{1cm} (14)

where $P'_L$ is the longitudinal contribution from $P'$. Using $|T'_T| \sim (m_{K^*}/m_B)|T'_L|$ and taking $|T'_L/P'_L| \sim 0.4$ \[3\], we find

$$|\Delta E_T| \lesssim 10\% \sqrt{\frac{f^+}{f^T}} \ .$$  \hspace{1cm} (15)

From this expression, we see that a large value of $\sqrt{f^+ / f^T}$ would result in a smaller systematic error in Eq. (13). Thus, this relation is most useful if a large transverse polarization is observed in the $\rho^+ K^*$ mode.
Table 1: Branching ratios and polarization fractions for the two $B^+ \to \rho K^*$ decays. Data comes from Ref. [18]; averages are taken from Ref. [19].

|                | $B^+ \to \rho^+ K^{*0}$ | $B^+ \to \rho^0 K^{*+}$ |
|----------------|--------------------------|--------------------------|
| $BR[10^{-6}]$  |                          |                          |
| Belle          | $8.9 \pm 1.7 \pm 1.2$    | $10.6^{+3.0}_{-2.6} \pm 2.4$ |
| BaBar          | $17.0 \pm 2.9^{+2.0}_{-2.8}$ | $10.6^{+3.8}_{-3.5}$     |
| average        | $10.6 \pm 1.9$           | $10.6^{+3.8}_{-3.5}$     |
| $f_L$          |                          |                          |
| Belle          | $0.43 \pm 0.11^{+0.05}_{-0.02}$ | $0.96^{+0.04}_{-0.15} \pm 0.04$ |
| BaBar          | $0.79 \pm 0.08 \pm 0.04$  | $0.96^{+0.06}_{-0.15}$   |
| average        | $0.66 \pm 0.07$          | $0.96^{+0.06}_{-0.15}$   |

Relations involving the longitudinal polarizations will have errors of the order of $x_L \sim (m_B/m_{K^*}) x_T$, which can be significant. Additional measurements, such as direct CP asymmetries and triple-product asymmetries in both $\rho K^*$ modes would provide important constraints on the various amplitudes and their phases, thereby providing strong tests of the SM.

The above SM predictions can now be compared with the present $B \to \rho K^*$ data, shown in Table 1. Using the central values, and using the SM relation $A_{\perp} \approx A_{\parallel}$, we find $E_{\perp} \approx E_{\parallel} \approx 77\%$. This is very far from the expected value of zero, so that one might be tempted to claim the presence of new physics. However, even though the systematic error $\Delta E_{\perp} \approx \Delta E_{\parallel}$ is relatively small, $\sim 20\%$, the statistical error is enormous, $\pm 129\%$. Thus, the errors are still much too large to claim any discrepancy with the SM. However, this does demonstrate the importance of more precise measurements of the polarizations in $B \to \rho K^*$ decays.

While the predictions of Refs. [12, 13] are not invalidated, the same is not true for Ref. [14]. In this scenario, the $f_L$ fraction of both charged $B \to \rho K^*$ decays is predicted to be greater than 90%. However, the data in Table 1 show that this clearly does not hold for $B^+ \to \rho^+ K^{*0}$, ruling out this SM explanation at the $3.5\sigma$ level.

Finally, we note that in the pQCD approach, even with annihilation and nonfactorizable effects, the large transverse polarization in $B \to \phi K^*$ cannot be explained [20]. In Ref. [21], it is argued that one of the $B \to K^*$ form factors must be reduced to explain the $B \to \phi K^*$ polarization. It is not clear whether this can be done, but the prediction of this scenario is then that the $B \to \phi K^*$ longitudinal polarization is smaller than that of both the $B^+ \to \rho^+ K^*$ and $B^+ \to \rho^0 K^{*+}$ modes. The careful measurement of the polarization fractions in the $B \to \rho K^*$ modes will test this scenario.
3 \( B \rightarrow \pi K \) Decays

There are four \( B \rightarrow \pi K \) decays. In the SM, neglecting small diagrams as usual, their amplitudes are given by

\[
\begin{align*}
A(B^+ \rightarrow \pi^+ K^0) &\equiv A^{+0} = P'_c, \\
\sqrt{2} A(B^+ \rightarrow \pi^0 K^+) &\equiv \sqrt{2} A^{0+} = -T'_c e^{i\gamma} - P'_c - P'_{EW}, \\
A(B^0 \rightarrow \pi^- K^+) &\equiv A^{--} = -T'_c e^{i\gamma} - P'_c, \\
\sqrt{2} A(B^0 \rightarrow \pi^0 K^0) &\equiv \sqrt{2} A^{00} = P'_c - P'_{EW},
\end{align*}
\]

(Isospin implies the relation \( A^{+0} + \sqrt{2} A^{0+} = A^{--} + \sqrt{2} A^{00} \)). It is difficult to explain the present data (branching ratios, CP asymmetries) using only this parametrization [7].

We therefore consider the addition of new \( \bar{b} \rightarrow s q \bar{q} \) \((q = u, d)\) operators. One can show that the strong phase of any NP operator is much smaller than that of the SM [22]. In this case, for a given type of transition, all NP matrix elements can now be combined into a single effective NP amplitude, with a single weak phase:

\[
\sum \langle \pi K | \mathcal{O}'_{NP} | B \rangle = A^q e^{i\Phi_q},
\]

in which the symbols \( \mathcal{A} \) and \( \Phi \) denote the NP amplitudes and weak phases, respectively. In \( B \rightarrow \pi K \) decays, there are four classes of NP operators, differing in their color structure: \( \tilde{b}_a \Gamma_i s_\alpha \tilde{q}_j q_\beta \) and \( \tilde{b}_a \Gamma_i s_\beta \tilde{q}_j q_\alpha \) \((q = u, d)\). The matrix elements of these operators can be combined into single NP amplitudes, denoted \( \mathcal{A}'\mathcal{A} e^{i\Phi'_q} \) and \( \mathcal{A}'\mathcal{A} e^{i\Phi'_q} \), respectively [23]. Each of these contributes differently to the various \( B \rightarrow \pi K \) decays. (Note that, despite the color-suppressed index \( C \), the matrix elements \( \mathcal{A}'\mathcal{A} e^{i\Phi'_q} \) are not necessarily smaller than the \( \mathcal{A}'\mathcal{A} e^{i\Phi'_q} \).)

In the presence of these NP matrix elements, the \( B \rightarrow \pi K \) amplitudes take the form [16 [23]]:

\[
\begin{align*}
A^{+0} &= P'_c + \mathcal{A}'\mathcal{A} e^{i\Phi'_q}, \\
\sqrt{2} A^{0+} &= -P'_c - T'_c e^{i\gamma} - P'_{EW} + \mathcal{A}'\mathcal{A} e^{i\Phi'_q} - \mathcal{A}'\mathcal{A} e^{i\Phi'_q}, \\
A^{--} &= -P'_c - T'_c e^{i\gamma} - \mathcal{A}'\mathcal{A} e^{i\Phi'_q}, \\
\sqrt{2} A^{00} &= P'_c - P'_{EW} + \mathcal{A}'\mathcal{A} e^{i\Phi'_q} + \mathcal{A}'\mathcal{A} e^{i\Phi'_q},
\end{align*}
\]

where \( \mathcal{A}'\mathcal{A} e^{i\Phi'_q} \equiv -\mathcal{A}'\mathcal{A} e^{i\Phi'_q} + \mathcal{A}'\mathcal{A} e^{i\Phi'_q} \).

Even taking into account the fact that \( P'_{EW} \) and \( T' \) are related [16], there are too many theoretical parameters to perform a fit. For this reason, the authors of Ref. [7] assumed that a single NP amplitude dominates. They considered four possibilities: (i) only \( \mathcal{A}'\mathcal{A} e^{i\Phi'_q} \neq 0 \), (ii) only \( \mathcal{A}'\mathcal{A} e^{i\Phi'_q} \neq 0 \), (iii) only \( \mathcal{A}'\mathcal{A} e^{i\Phi'_q} \neq 0 \), (iv) \( \mathcal{A}'\mathcal{A} e^{i\Phi'_q} = \mathcal{A}'\mathcal{A} e^{i\Phi'_q} \), \( \mathcal{A}'\mathcal{A} e^{i\Phi'_q} = 0 \) (isospin-conserving NP). Of these, only choice...
(i) gave a good fit; the others produced poor or very poor fits. The good fit found best-fit values of $|A_t^{comb}/P'| = 0.36$ and $|T'/P'| = 0.22$. Thus, the NP parameter was found to be larger than the tree amplitude, with $|A_t^{comb}/T'| = 1.64$.

In what follows, we assume that NP of type (i) is present in $B \to \pi K$ decays. This same NP will affect $B \to \rho K^*$ decays. In order to calculate the effect on the $B \to \rho K^*$ polarization states, we must assume a particular form for $A_t^{comb}$. There are many NP operators which can contribute to $A_t^{comb}$. They are

$$4G_F\sqrt{2} \sum_{A,B=L,R} \left\{ f_q^{AB} \bar{b} \gamma_A s \bar{q} \gamma_B q + g_q^{AB} \bar{b} \gamma^\mu \gamma_A s \bar{q} \gamma_\mu \gamma_B q \right\}. \quad (19)$$

There are a total of 16 contributing operators ($A, B = L, R, q = u, d$); tensor operators do not contribute to $B \to \pi K$. For simplicity, we assume that a single operator contributes to $A_t^{comb}$, and we analyze their effects one by one.

Note that all operators contribute directly to $\pi K$ final states involving a $\pi^0$. They can also contribute to states involving a $\pi^+$ if one performs Fierz transformations of the fermions and colors. However, the effects on $\pi^+ K^0$ are all suppressed by at least $1/N_c$, so that the contributions to $\pi^0 K^+$ are larger. This is approximately consistent with the hypothesis of including only $A_t^{comb}$.

We begin by considering the operators whose coefficients are $f_q^{AB}$ [Eq. (19)]. Using $|A_t^{comb}/T'| = 1.64$ and

$$T' = \frac{G_F}{\sqrt{2}} V^*_{ub} V_{us} \left( c_1 + \frac{c_2}{N_c} \right) \left\langle \pi^0 | K^+ | \bar{u} \gamma^\mu (1 - \gamma_5) s \bar{b} \gamma_\mu (1 - \gamma_5) u | B^+ \right\rangle, \quad (20)$$

where $c_1 = 1.081$ and $c_2 = -0.190$ are the Wilson coefficients characterizing $T'$. We can estimate the size of the NP coefficients. To do this, we use naive factorization. This is reasonable since we are interested only in estimates. More accurate calculations can use a more precise formalism, e.g. Ref. [21].

We then have:

$$\left| \frac{4f_q^{AB} \left\langle \pi^0 | \bar{q} \gamma_B q | 0 \right\rangle \left\langle K^+ | \bar{b} \gamma_A s | B^+ \right\rangle}{V^*_{ub} V_{us} \left\langle \pi^0 | \bar{b} \gamma_\mu (1 - \gamma_5) u | B^+ \right\rangle \left\langle K^+ | \bar{u} \gamma^\mu (1 - \gamma_5) s | 0 \right\rangle} \right| = 1.64. \quad (21)$$

Using the matrix elements given in the Appendix, we find

$$f_q^{AB} = \frac{f_K (m_b^2 - m_s^2) F_0^\pi / \sqrt{2}}{[(m_b^2 - m_s^2)/(m_b - m_s)] F_0^\pi (m_b^2/2m_q) f_\pi / \sqrt{2}} 1.64 \left( c_1 + \frac{c_2}{N_c} \right) |V^*_{ub} V_{us}|. \quad (22)$$

Note that the poor fit gave a discrepancy of only about $2\sigma$ with the SM, so that, strictly speaking, it cannot be ruled out. However, in what follows, we concentrate on the good fit.
With the procedure for computing the SM or NP contributions to polarization amplitudes described above, and calculate the effect on \( B \to \rho K \) and will, in general, contribute to \( B \to \rho K^* \) decays. In this section, we proceed as above, and calculate the effect on \( B \to \rho K^* \) of each of the operators in Eq. (19).

We begin with some general statements. The amplitude for an arbitrary \( B \to \rho K \) decay can be written as (for example, see Ref. [3])

\[
\mathcal{M} = a \epsilon_1^* \cdot \epsilon_2 + \frac{b}{m_B^2} (\epsilon_1^* \cdot p_2) (\epsilon_2^* \cdot p_1) - 2i \frac{c}{m_B^2} \epsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu \epsilon_1^\alpha \epsilon_2^\beta ,
\]

(25)

with

\[
A_\parallel = \sqrt{2}a , \quad A_0 = -ax - \frac{m_1 m_2}{m_B^2} b(x^2 - 1) , \quad A_\perp = 2\sqrt{2} \frac{m_1 m_2}{m_B^2} c\sqrt{x^2 - 1} ,
\]

(26)

where \( x = p_1 \cdot p_2 / (m_1 m_2) \). Here we are considering \( B \to V_1 V_2 \) decays in which the final vector mesons are light: \( m_{1,2} \ll m_B \). Neglecting terms of \( O(m_{1,2}^2/m_B^2) \), we can then approximate \( E_1 \sim E_2 \sim |k| = E = m_B/2 \). Then, using Eq. (26), we have for the various linear polarization amplitudes

\[
A_0 \approx -(2a + b) \frac{E^2}{m_1 m_2} , \quad A_\parallel \approx \sqrt{2}a , \quad A_\perp \approx \sqrt{2}c .
\]

(27)

The procedure for computing the SM or NP contributions to polarization amplitudes is then clear: we first express the amplitude for a particular \( B \to V_1 V_2 \) decay as

\[
|f_d^{LL}| = |f_d^{RR}| = |f_d^{LR}| = \left\{ \begin{array}{l}
0.069 |V_{ts}^* V_{ts}| \\
0.138 |V_{tb}^* V_{ts}| \end{array} \right. m_d = 4 \text{ MeV},
\]

\[
|f_u^{LL}| = |f_u^{RR}| = |f_u^{LR}| = \left\{ \begin{array}{l}
0.026 |V_{tb}^* V_{ts}| \\
0.069 |V_{tb}^* V_{ts}| \end{array} \right. m_u = 1.5 \text{ MeV},
\]

(23)

The operators associated with the parameters \( g_q^{AB} \) [Eq. (19)] can be analyzed similarly. The sizes of the NP coefficients are

\[
|g_q^{LL}| = |g_q^{RR}| = |g_q^{LR}| = 0.035 |V_{ts}^* V_{ts}| , \quad q = u, d .
\]

(24)

We remind the reader that we have assumed that a single NP operator contributes to \( A_{\text{comb}} \). For each operator, we have calculated the size of the coefficient which reproduces the \( B \to \pi K \) data. These same operators will affect the \( B \to \rho K^* \) polarization states. We compute these effects in the next section.

4 \( B \to \rho K^* \): New-Physics Contributions

4.1 For each operator, we have calculated the size of the coefficient which reproduces the \( B \to \pi K \) data. These same operators will affect the \( B \to \rho K^* \) polarization states. We compute these effects in the next section.
in Eq. (25) and then use the above relations to obtain $A_0$, $A||$ and $A\perp$. For the SM, in which all operators have $(V - A) \times (V \mp A)$ structure, one can show that $2a + b \sim m_V/m_B$, so that the polarization fractions are predicted to be as in Eq. (2). (We will see this explicitly below for $B^+ \to \rho^+ K^{*0}$.)

The present data is consistent with the SM expectations for $B^{+} \to \rho^{0} K^{*+}$, but suggests that there may be new physics in $B^{+} \to \rho^{+} K^{*0}$. For this reason, we concentrate on this latter decay in what follows.

Using factorization, the SM amplitude for the decay $B^{+} \to \rho^{+} K^{*0}$ is given by

$$A[B^{+} \to \rho^{+} K^{*0}] = \frac{G_F}{\sqrt{2}} X_{\rho} P_{K^{*}}^{0},$$

(28)

with

$$X_{\rho} = - \sum_{q=u,c,t} V_{qB} V_{qs}^* \left( a_1^q - \frac{1}{2} a_1^{q0} \right),$$

$$P_{K^{*}}^{0} = m_{K^{*}} g_{K^{*}} \varepsilon_{K^{*}}^{\mu} \langle \rho^{+} | d^{\mu}(1 - \gamma_5) b | B^{+} \rangle,$$

(29)

The above amplitude depends on combinations of Wilson coefficients, $a_i$, where $a_i = c_i + c_{i+1}/N_c$ for $i$ odd and $a_i = c_i + c_{i-1}/N_c$ for $i$ even. The terms described by the various $a_i$’s can be associated with the different decay topologies introduced earlier. The term proportional to $a_4$ is the color-allowed penguin amplitude, $P'$. The dominant electroweak penguin $P^{\text{ew}}_W$ is represented by term proportional to $a_9$, $P^{\text{ew}}_C$ is $a_{10}$, and $a_7$ and $a_8$ are additional small EWP amplitudes. (If there were terms proportional to $a_1$ and $a_2$, they would represent the color-allowed and color-suppressed tree amplitudes $T'$ and $C'$, respectively.) The values of the Wilson coefficients can be found in Ref [17].

Using the matrix elements found in the Appendix, this amplitude can be put in the form of Eq. (25). The polarization amplitudes are then given by

$$A_0 \approx \frac{G_F}{\sqrt{2}} 2m_B m_{K^{*}} g_{K^{*}} X_{\rho} \left[ (A_1^0 - A_2^0) + \frac{m_{\rho}}{m_B} (A_1^0 + A_2^0) \right] \frac{m_B^2}{4m_{\rho} m_{K^{*}}},$$

$$A|| \approx - \frac{G_F}{\sqrt{2}} 2m_B \left[ m_{K^{*}} g_{K^{*}} \left( 1 + \frac{m_{\rho}}{m_B} \right) A_1^0 (m_{K^{*}}^2) X \right],$$

$$A\perp \approx - \frac{G_F}{\sqrt{2}} 2m_B \left[ m_{K^{*}} g_{K^{*}} \left( 1 - \frac{m_{\rho}}{m_B} \right) V^\rho (m_{K^{*}}^2) X \right].$$

(30)

In the large-energy limit, the form factors are related [25]:

$$A_1 = A_2 + O(m_{\rho}/m_B), \quad V = A_1 + O(m_{\rho}/m_B).$$

(31)

We therefore find the same suppression of the $A||, \perp$ amplitudes relative to $A_0$ as was found from helicity arguments [Eq. (2)]. We therefore see that the SM naturally
predicts the longitudinal polarization for the decay $B^+ \to \rho^+ K^{*0}$ to be enhanced by $O(m_B/m_V)$.

In our simplified approach we will assume the form-factor relations above and ignore possible power-suppressed and $\alpha_s$ corrections to them. We then have

\[
A^\rho_1 \approx \zeta_\perp (1 - \frac{m_{\rho}}{m_B}) \\
A^\rho_2 \approx \zeta_\perp (1 + \frac{m_{\rho}}{m_B}) - 2m_{\rho}\zeta_\parallel \\
V^\rho_1 \approx \zeta_\perp (1 + \frac{m_{\rho}}{m_B}).
\]

Choosing $\zeta_\perp \approx \zeta_\parallel$ gives $A^\rho_1 \approx A^\rho_2$, and hence the SM prediction is that

\[
A^{SM}_0 \approx \frac{G_F}{\sqrt{2}} \frac{f_{RR}}{m_B^2} X \zeta_\parallel, \\
A^{SM}_\parallel \approx -G_F g_{K*} m_K m_B \cdot X \zeta_\parallel, \\
A^{SM}_\perp \approx -G_F g_{K*} m_K m_B \cdot X \zeta_\parallel,
\]

where $X \approx -a_4^t |V_{tb}V_{ts}| = 0.035|V_{tb}V_{ts}|$.

We now turn to the new-physics contributions. As mentioned earlier, there are 16 possible NP operators. We present the calculations in some detail for two of them; the results for the others are included in tables. We begin with the operator whose coefficient is $f_{RR}^d$ [Eq. (19)]:

\[
\frac{4G_F}{\sqrt{2}} f_{RR}^d \bar{b}\gamma_s \bar{d}\gamma_s \bar{d}. \tag{34}
\]

Because this is a scalar/pseudoscalar operator, within factorization it does not contribute to $B^+ \to \rho^0 K^{*+}$. However, it can affect $B^+ \to \rho^+ K^{*0}$. To see this, we perform a Fierz transformation of this operator (both fermions and colors):

\[
-\frac{4}{N_c} \frac{G_F}{\sqrt{2}} f_{RR}^d \left[ \frac{1}{2} \bar{b}\gamma_R d \bar{d}\gamma_R s + \frac{1}{8} \bar{b}\sigma_{\mu\nu}\gamma_R d \bar{d}\sigma_{\mu\nu}\gamma_R s \right]. \tag{35}
\]

It is the second term which is important (in contrast to $B \to \pi K$), as it contributes to $B^+ \to \rho^+ K^{*0}$.

Within factorization, the contribution to $B^+ \to \rho^+ K^{*0}$ is given by

\[
-\frac{1}{2N_c} \frac{G_F}{\sqrt{2}} f_{RR}^d \left( K^{*0} \right) \tilde{d}\sigma_{\mu\nu}\gamma_R s |0\rangle \langle \rho^+ | \bar{b}\sigma_{\mu\nu}\gamma_R d |B^+\rangle. \tag{36}
\]

Using the matrix elements given in the Appendix, this gives

\[
Z_d^{RR} \left\{ 2T_2 \left(1 - \frac{m_{K*}^2}{m_B^2}\right) \left(\epsilon_\rho^* \cdot \epsilon_K^*\right) - \frac{4}{m_B^2} \left(T_2 + T_3 \frac{m_{K*}^2}{m_B^2}\right) \left(\epsilon_\rho^* \cdot p_K^*\right) (\epsilon_K^* \cdot \epsilon_\rho^*) \right\},
\]

\[
-\frac{4i}{m_B^2} T_1 \epsilon_\rho^{\mu\nu\alpha\beta} \tilde{p}_\mu \tilde{p}_\nu \epsilon_\alpha K^* \epsilon_\beta \epsilon_\rho^* \epsilon_K^* \right\}, \tag{37}
\]
where the $T_i$ are form factors and

$$Z^{RR}_d \equiv \frac{1}{4N_c} \frac{G_F}{\sqrt{2}} f^{RR}_d g^{K^*}_{TM} m_B^2.$$  \hfil (38)

We again use the form factor relations \[25\]

$$T_1(q^2) \approx \zeta_\perp,$$

$$T_2(q^2) \approx \zeta_\perp \left(1 - \frac{q^2}{m_B^2 - m_c^2}\right),$$

$$T_3(q^2) \approx \zeta_\perp - \frac{2m_V}{m_B} \zeta_\parallel.$$  \hfil (39)

Comparing the above expression for the NP amplitude with the formula in Eqs. \[25\], we see that the NP operator whose coefficient is $f^{RR}_d$ predicts

$$A_0 = -2\zeta_\parallel \frac{m_{K^*}}{m_B} Z^{RR}_d, \quad A_\parallel = 2\sqrt{2}\zeta_\perp Z^{RR}_d, \quad A_\perp = 2\sqrt{2}\zeta_\perp Z^{RR}_d.$$  \hfil (40)

(We note that $A_0$ above is subleading in $1/m_B$ and so we have used the general expressions in Eq. \[26\] instead of Eq. \[27\] to calculate the longitudinal polarization amplitude.) We therefore see that this operator contributes significantly to transverse polarization states of $\rho^- K^{*0}$. The longitudinal polarization is suppressed by $O(m_V/m_B)$ as expected.

We can now calculate the ratio of transverse and longitudinal polarizations, including the SM contribution [Eq. \[33\]]. Assuming $g_{K^*} \approx g_{K^*}^T$ and taking the value of the NP coefficient from $B \to \pi K$ [Eq. \[23\]], we have with $T = \perp, \parallel$

$$\frac{f_T}{f_L} = 2 \frac{|f^{RR}_d/(2N_c)|^2}{|X|^2} = \begin{cases} 0.22, & m_d = 4 \text{ MeV}, \\ 0.86, & m_d = 8 \text{ MeV}. \end{cases}$$  \hfil (41)

We therefore see that this NP operator can generate a large transverse polarization in $B^+ \to \rho^+ K^{*0}$.

Note that we also predict for this NP operator (as well as the operator associated with $f^{LL}$)

$$\frac{f_\perp}{f_\parallel} \approx 1 + O(m_V/m_B)$$  \hfil (42)

which is the same as the SM prediction.

The second NP operator for which we explicitly present calculations is the one whose coefficient is $g^{LR}_u$ [Eq. \[19\]]:

$$\frac{4G_F}{\sqrt{2}} g^{LR}_u \bar{b} \gamma_\mu \gamma_L s \bar{u} \gamma_\mu \gamma_R u.$$  \hfil (43)
Table 2: Contributions to the polarization states of $B^+ \to \rho^0 K^{*+}$ from the various NP operators. Operators which are not shown do not contribute. The various $Z$’s and $X$’s are defined analogously to Eqs. (38) and (44). We take $\zeta_\perp \approx \zeta_{||}$.

|                | $A_0$                          | $A_{||}$                    | $A_{\perp}$                 |
|----------------|--------------------------------|-----------------------------|-----------------------------|
| $f_u^{RR}$     | $O(m_v/m_b)$                   | $2\zeta_{||}\rho Z_{u}^{RR}$| $2\zeta_{\perp}\rho Z_{u}^{RR}$|
| $f_u^{LL}$     | $O(m_v/m_b)$                   | $-2\zeta_{||}\rho Z_{u}^{LL}$| $2\zeta_{\perp}\rho Z_{u}^{LL}$|
| $f_u^{RL}$     | $-\sqrt{2}\zeta_{||}(g_{K^*}/g_{K^*})Z_{u}^{RL}$| $O(m_v/m_b)$ | $O(m_v/m_b)$ |
| $f_u^{LR}$     | $\sqrt{2}\zeta_{||}(g_{K^*}/g_{K^*})Z_{u}^{LR}$| $O(m_v/m_b)$ | $O(m_v/m_b)$ |
| $g_u^{RR}$     | $-\frac{1}{\sqrt{2}}(\zeta_{||}X_{u}^{RR})$| $O(m_v/m_b)$ | $O(m_v/m_b)$ |
| $g_u^{LL}$     | $-\frac{1}{\sqrt{2}}(\zeta_{||}X_{u}^{LL})$| $O(m_v/m_b)$ | $O(m_v/m_b)$ |
| $g_u^{RL}$     | $-\frac{1}{\sqrt{2}}(\zeta_{||}X_{u}^{RL})$| $O(m_v/m_b)$ | $O(m_v/m_b)$ |
| $g_u^{LR}$     | $\frac{1}{\sqrt{2}}(\zeta_{||}X_{u}^{LR})$| $O(m_v/m_b)$ | $O(m_v/m_b)$ |
| $g_d^{RR}$     | $-\frac{1}{\sqrt{2}}(\zeta_{||}X_{d}^{RR})$| $O(m_v/m_b)$ | $O(m_v/m_b)$ |
| $g_d^{LL}$     | $-\frac{1}{\sqrt{2}}(\zeta_{||}X_{d}^{LL})$| $O(m_v/m_b)$ | $O(m_v/m_b)$ |
| $g_d^{RL}$     | $-\frac{1}{\sqrt{2}}(\zeta_{||}X_{d}^{RL})$| $O(m_v/m_b)$ | $O(m_v/m_b)$ |
| $g_d^{LR}$     | $\frac{1}{\sqrt{2}}(\zeta_{||}X_{d}^{LR})$| $O(m_v/m_b)$ | $O(m_v/m_b)$ |}

This operator contributes directly to $B^+ \to \rho^0 K^{*+}$. Its Fierz transformation has the form $(S - P) \times (S + P)$ and, being a scalar/pseudoscalar operator, does not contribute to $B^- \to \rho^- K^{*0}$ within factorization. In this case, the situation is much like the SM, and using the matrix elements found in the Appendix, the amplitude corresponding to this operator for $B^+ \to \rho^0 K^{*+}$ is dominantly longitudinal, with

$$A_0 \approx \frac{1}{\sqrt{2}} X_u^{LR} \zeta_{||}, \quad X_u^{LR} \equiv \frac{G_F}{\sqrt{2}} g_u^{LR} \rho m_B^2. \quad (44)$$

The contributions of all 16 new-physics operators to the $B \to \rho K^*$ polarization states are shown in Tables 2 and 3. Here we present only the dominant contributions to $f_L$ and $f_T$; terms of $O(m_v/m_b)$ are subdominant and contribute to $f_{L,T}$ only at the $O(m^2_v/m^2_B)$ ~ 5% level. Of all the operators, there are only two which reproduce the data of Table 1 i.e. they contribute significantly to the transverse polarization of $B^+ \to \rho^+ K^{*0}$ while leaving $B^+ \to \rho^0 K^{*+}$ essentially longitudinal. They have the coefficients $f_d^{RR}$ and $f_d^{LL}$. These are the only two NP operators which successfully explain both the $B \to \pi K$ and $B^+ \to \rho K^*$ data.

This explanation of the $B \to \rho K^*$ data can be tested. In the SM, there is essentially only one dynamical decay amplitude. Because of this, one expects the CP-violating triple-product correlation (TP) in these decays to be very small [3]. However, this can change with the addition of a second NP amplitude. A nonzero
value of the \( f_d^{RR} \) or \( f_d^{LL} \) amplitude will lead to a nonzero TP. Furthermore, one expects such a TP only in \( B^+ \to \rho^+ K^{*0} \); the TP should remain tiny in \( B^+ \to \rho^0 K^{*+} \).

We can estimate the expected size of the TP in \( B^+ \to \rho^+ K^{*0} \). In Ref. 3, the following measures of the triple-product correlations were defined:

\[
A_T^{(1)} \equiv \frac{\text{Im}(A_\perp A_0^*)}{A_0^2 + A_\parallel^2 + A_\perp^2}, \quad A_T^{(2)} \equiv \frac{\text{Im}(A_\perp A_\parallel^*)}{A_0^2 + A_\parallel^2 + A_\perp^2}.
\]

The corresponding quantities for the charge-conjugate process, \( \bar{A}_T^{(1)} \) and \( \bar{A}_T^{(2)} \), are defined similarly. The comparison of the TP asymmetries in a decay and in the corresponding CP-conjugate process will give a measure of the true T-odd, CP-violating asymmetry for that decay. The TP is therefore due to the interference between the \( A_\perp \) and \( A_0 \) or \( A_\parallel \) amplitudes, and requires that the two interfering amplitude have different weak phases. Recall that it was found in Ref. 7 that, to explain the \( B \to \pi K \) data, a NP weak phase \( \phi_{NP} \approx 100^\circ \) was needed.

Now, at leading order, the SM yields only \( A_0 \); large transverse amplitudes can arise only if NP is included. However, the only way to obtain a nonzero \( A_T^{(2)} \) is through SM–NP interference. We observe from Table 3 that the NP operators associated with the coefficients \( f_d^{RR} \) and \( f_d^{LL} \) yield large values for \( A_\perp \) or \( A_\parallel \). On the other hand, the transverse SM amplitudes are all \( O(1/m_B) \). Thus, the SM–NP interference gives an \( A_T^{(2)} \) of \( O(1/m_B) \). Note that a measurement of the sign of \( A_T^{(2)} \), if possible, can be used to distinguish between the two NP operators.

In contrast, the TP asymmetry \( A_T^{(1)} \) can be sizeable. It can arise due to the interference of the \( A_0 \) SM amplitude and the \( A_\perp \) NP amplitude. As above, this latter amplitude can be big for those NP operators whose coefficients are \( f_d^{RR} \) or \( f_d^{LL} \). For these operators, we can estimate the maximum magnitude of \( A_T^{(1)} \). We first take the strong-phase difference between \( A_0 \) and \( A_\perp \) to be zero (or \( \pi \)). In this case,
is by itself a measure of T-odd CP violation and we can write
\[ |A_T^{(1)}| \leq \frac{\sqrt{f_L/f_L}}{1 + 2f_L/f_L} \sin \phi_{NP} . \] (46)

Using \( \phi_{NP} \sim 100^\circ \) and Eq. (41) we find \( |A_T^{(1)}| \leq 32 \text{-} 34\% \) for \( m_d = 4 \text{-} 8 \text{ MeV} \). Hence we see that a sizeable TP is possible in the decay \( B^+ \rightarrow \rho^0 K^{*+} \).

5 \( B \rightarrow \phi K^* \)

As noted earlier, a sizeable value of \( f_T/f_L \) is observed in \( B \rightarrow \phi K^* \), contrary to expectations. There are different SM explanations, but they all predict either that (i) the transverse polarization fractions are large in both \( B^+ \rightarrow \rho^+ K^{*0} \) and \( B^+ \rightarrow \rho^0 K^{*+} \), with the \( f_T \)'s respecting Eq. (9), or (ii) \( f_T \) is small in both \( B \rightarrow \rho K^* \) decays. If either of these is not seen, new physics is needed.

There are already several non-SM explanations of the \( \phi K^* \) data \([10, 11]\), but one can now ask the question: can one explain the \( \pi K \), \( \rho K^* \) and \( \phi K^* \) observations simultaneously? The answer is yes. One can reproduce the \( \phi K^* \) data with the addition of NP operators of the form \( \bar{b} \gamma_R s \bar{s} \gamma_R s \) or \( \bar{b} \gamma_L s \bar{s} \gamma_L s \) \([11]\). Above, we have shown that NP operators such as \( \bar{b} \gamma_R s \bar{d} \gamma_R d \) or \( \bar{b} \gamma_L s \bar{d} \gamma_L d \) can account for the observations in the \( \pi K \) and \( \rho K^* \) systems. Thus, if the NP obeys an approximate U-spin symmetry, which relates \( d \)- and \( s \)-quarks, one can simultaneously explain the \( \pi K \), \( \rho K^* \) and \( \phi K^* \) observations. (A model which does this will be described in Ref. \([20]\).)

6 Conclusions

At present, there are several discrepancies with the predictions of the standard model (SM), in \( B \rightarrow \phi K \), \( B \rightarrow \phi K^* \) and \( B \rightarrow \pi K \) decays. We must stress that these discrepancies are (almost) all in the 1–2\( \sigma \) range and as such are not yet statistically significant. That is, the existence of physics beyond the SM is not certain. However, if these hints are taken together, the statistical significance increases. Furthermore, they are intriguing since they all point to new physics (NP) in \( \bar{b} \rightarrow \bar{s} \) transitions. For these reasons, it is worthwhile considering the effects of NP on various \( B \) decays.

One hint of NP occurs in the decays \( B \rightarrow \phi K^* \). The SM naively predicts that the transverse polarization fraction of the final-state particles, \( f_T \), should be much smaller \([O(m_d^2/m_b^2)]\) than that of the longitudinal polarization, \( f_L \). However, it is observed that \( f_T \approx f_L \). There are several SM explanations, all of which go beyond the naive expectations. However, all make predictions for the polarization in \( B \rightarrow \rho K^* \) decays. The key point is that there are two such decays, \( B^+ \rightarrow \rho^+ K^{*0} \) and \( B^+ \rightarrow \rho^0 K^{*+} \) (and similarly for neutral \( B \) decays). By measuring the polarizations in both decays, one can test the SM explanations of the \( B \rightarrow \phi K^* \) measurements.
In one scenario \cite{12,13}, it is predicted that $f_T$ should be large in both $B \to \rho K^*$ decays. We have shown that the values of $f_T$ in both decays should obey Eq. (9). If this relation is not respected, then this scenario is ruled out, yielding a clear signal of new physics. Using present $B \to \rho K^*$ data, the central values violate this relation. However, the errors are still extremely large, so that no firm conclusions can be drawn. This emphasizes the importance of more precise measurements of these decays.

In the second scenario \cite{14}, the transverse polarizations in both $B \to \rho K^*$ decays are predicted to be small, i.e. $f_L$ is close to 1. However, in $B^+ \to \rho^+ K^{*0}$ decays, it is found that $f_L^+ = 0.66 \pm 0.07$ (Table I), ruling out this scenario at the 3.5\(\sigma\) level.

The discrepancy in $B \to \pi K$ decays can be explained by the addition of new-physics operators of the form $\bar{b} \to \bar{s}q\bar{q} (q = u, d)$ \cite{6,7}. There are 16 such operators, all of which will contribute to $B \to \rho K^*$ decays. Assuming that NP is present, we have calculated the effect on the polarization states of $B \to \rho K^*$ of each of these operators (Tables 2 and 3). Of these, there are only two which reproduce the data of Table I, i.e. they contribute significantly to the transverse polarization of $B^+ \to \rho^+ K^{*0}$ while leaving $B^+ \to \rho^0 K^{*+}$ essentially longitudinal. They are $f_\text{rr}^{uR} \bar{b} \gamma_R s \bar{d} \gamma_R d$ and $f_\text{ll}^{uR} \bar{b} \gamma_L s \bar{d} \gamma_L d$. If the $B \to \pi K$ measurements turn out to show statistically-significant evidence of new physics, and if the $B \to \rho K^*$ data remain as in Table I these are the only two NP operators which can explain both sets of observations.

Finally, it is natural to assume that the same type of new physics which accounts for the $B \to \pi K$ and $B \to \rho K^*$ measurements also affects $B \to \phi K^*$ decays and can explain the observed value of $f_T/f_L$. This is possible if the NP obeys an approximate U-spin symmetry. In this case, there are also NP operators of the form $\bar{b} \gamma_R s \bar{s} \gamma_R s$ or $\bar{b} \gamma_L s \bar{s} \gamma_L s$, which can reproduce the $\phi K^*$ data \cite{11}. This type of NP can therefore simultaneously account for the $\pi K$, $\rho K^*$ and $\phi K^*$ data.

Note that it is quite possible that, with more data, the experimental measurements will change, leading to a different pattern of new-physics signals. In this case, the conclusions presented in this paper will have to be modified. However, we must stress that this type of analysis will ultimately be necessary. Rather than look for NP solutions to each individual discrepancy with the SM, it will be far more compelling to search for a single solution to all NP signals. Thus, an analysis of the type presented in this paper will have to be carried out.

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Appendix

The matrix elements used in the paper: we have

\[
\begin{align*}
\langle \pi^0 | \bar{q}(1 \pm \gamma_5)q | 0 \rangle &= \pm i \frac{f_\pi}{\sqrt{2}} \frac{m_\pi^2}{2m_q}, \\
\langle \pi^0 | \bar{q} \gamma^\mu(1 \pm \gamma_5)q | 0 \rangle &= \mp i f_K p_\pi^\mu, \\
\langle K^+ | \bar{u} \gamma^\mu(1 - \gamma_5)s | 0 \rangle &= i f_K p_K^\mu, \\
\langle K^+ | \bar{b}(1 \pm \gamma_5)s | B^+ \rangle &= \frac{m_B^2 - m_K^2}{m_s - m_b} F^K_0, \\
\langle K^+ | \bar{b} \gamma_\mu(1 \pm \gamma_5)s | B^+ \rangle &= \left[ (p_B + p_K)_\mu - \frac{m_B^2 - m_K^2}{q^2} q_\mu \right] F^K_1 \\
&\quad + \frac{m_B^2 - m_K^2}{q^2} q_\mu F^K_2, \quad q_\mu \equiv (p_B - p_K)_\mu, \\
\langle K^* | \bar{q} \gamma^\mu s | 0 \rangle &= g_{K^*} m_K^* \epsilon_\mu^{K^*}, \\
\langle \rho | \bar{b} \gamma_\mu(1 \pm \gamma_5)q | B \rangle \epsilon_\mu^{\star K^*} &= \frac{2i}{m_B + m_\rho} V_\rho \epsilon^{\mu \alpha \beta} p_\mu^\rho p_\nu^\alpha \epsilon_\beta^{\star K^*} \pm (m_B + m_\rho) A_1^\rho \epsilon^{\rho \cdot \epsilon^{\star K^*}} \\
&\quad + A_2^\rho \frac{2}{m_B + m_\rho} \left( \epsilon^{\rho \cdot \epsilon^{\star K^*}} \right) \left( \epsilon^{\star K^*} \cdot \epsilon^{\rho} \right), \\
\langle K^* | \bar{q} \sigma^{\mu \nu} s | 0 \rangle &= -i g_T^K \left( \epsilon^{\mu \nu}_K p_\nu^K - \epsilon^{\mu \nu}_K p^K_\nu \right), \\
\langle \rho | \bar{b} \sigma^{\mu \nu} q | B \rangle \rho_\mu^{K^*} &= -2T_1 \epsilon^{\mu \alpha \beta \gamma} \rho_\nu^{K^*} \rho_\rho^{\alpha \beta} \epsilon_\gamma^{\star K^*}, \\
\langle \rho | \bar{b} \sigma^{\mu \nu} \gamma_5 q | B \rangle \rho_\mu^{K^*} &= -i T_2 \left[ (m_B^2 - m_\rho^2) \epsilon^{\mu \nu}_\rho - (\epsilon^{\rho \cdot \rho}_K \cdot p_\nu^{K^*}) \left( \rho_\nu^{K^*} + \rho_\rho^{\mu} \right) \right] \\
&\quad - i T_3 \left( \epsilon^{\rho \cdot \rho}_K \cdot p_\nu^{K^*} \right) \left[ \rho_\nu^{K^*} - \frac{m_K^2}{m_B^2 - m_\rho^2} \left( \rho_\rho^{\mu} + \rho_\mu^{\rho} \right) \right], \\
\langle \rho | \bar{u} \gamma^\mu u | 0 \rangle &= \frac{1}{\sqrt{2}} g_{\rho} m_\rho \epsilon^{\mu \rho}_\rho, \\
\langle K^* | \bar{b} \gamma_\mu(1 \pm \gamma_5)s | B \rangle \epsilon_\mu^{\star K^*} &= \frac{2i}{m_B + m_K^*} V^{K^*} \epsilon^{\mu \alpha \beta} p_\mu^{K^*} \epsilon_\alpha^{\rho} \epsilon_\beta^{\star K^*} \pm (m_B + m_K^*) A_1^{K^*} \epsilon^{\rho \cdot \epsilon^{\star K^*}} \\
&\quad + A_2^{K^*} \frac{2}{m_B + m_K^*} \left( \epsilon^{\rho \cdot \epsilon^{\star K^*}} \right) \left( \epsilon^{K^*} \cdot \epsilon^{\rho} \right),
\end{align*}
\]
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