Ferromagnetism in neutron and charge neutral beta equilibrated nuclear matter

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(Dated: May 7, 2014)

Abstract

Ferromagnetism in infinite neutron matter as well as beta equilibrated, charge neutral, dense, and infinite nuclear matter is investigated using a model of interacting baryons and mesons. The standard minimal couplings between the magnetic field and the particle charges as well as the baryon dipole moments are included in the Lagrangian density. Minimizing the energy density with respect to the magnetic field yields a self-consistent expression for the ferromagnetic field. We calculate the phase boundary at a given density by increasing the strength of the baryon dipole moments till the energy density of magnetized matter is lower than that of unmagnetized matter. We find that, depending on the density, it is crossed when the baryon dipole moments are increased by a factor of 35. It is also sensitive to the details of the nuclear matter parameterizations and crossing it induces a magnetic field of $\sim 10^{17}$ gauss.

PACS numbers: 26.60.Kp, 21.30.Fe, 21.60.Jz, 21.65.Cd

Keywords: mean-field approximation, nuclear matter, ferromagnetism, neutron star interior
I. INTRODUCTION

The behaviour of highly magnetized dense matter systems finds application from collider experiments to astrophysical environments. One such astrophysical object is a neutron star. These stars are observed as highly magnetized, rapidly rotating, and radio-emitting compact stellar objects known as pulsars. Since the discovery of pulsars in 1967 the origin of their magnetic fields, as well as its interaction with the matter in the interior of the star, has been a topic of much discussion and research. In 1969 Brownell and Callaway as well as Silverstein proposed that a ferromagnetic phase of nuclear matter in the interior of the neutron star can make a significant contribution to the star’s magnetic field. Since then various authors built on this notion and investigated the magnetization and/or ferromagnetic/antiferromagnetic phase transition in various types of nuclear matter with varied results as summarized recently by Bigdeli.

Strongly magnetized matter can also be found in relativistic heavy-ion collision, with magnetic field strengths of up to almost $10^{19}$ G. There matter is highly thermalized such that baryons might undergo a phase transition to quark-gluon plasma. However, our study is predominately concerned with matter that can be found in the neutron star interior: cold (zero temperature), dense nuclear matter. Due to the short range of the nuclear interaction this type of matter is assumed to be infinite and also charge neutral to balance the Coulomb repulsion between the protons. Infinite neutron matter meets all these criteria, but free neutrons are unstable and beta equilibrium will also be enforced. Hence our study will investigate both infinite neutron matter as well as charge neutral beta equilibrated infinite nuclear matter consisting out of protons, neutrons, electrons and muons. For brevity’s sake the former will be referred to as neutron matter while the latter will be described as neutron star matter, due to the association with the neutron star interior.

We use the description of nuclear matter called Quantum Hadrodynamics, or QHD, where meson exchanges mediate the various aspects of the nuclear interaction. There are various parameterizations of the meson-baryon coupling constants as well as the meson self-coupling constants: we use the QHD1, NL3, FSUGold (in short FSU) and IU-FSU parameter sets. Our work is preceded by similar studies of magnetized matter, the first of
which is a paper by Chakrabarty et al. \[12\].

In \[12\] the authors found that neutron star matter (without muons) can be bound by very strong magnetic fields and that its proton fraction will increase. Furthermore, the Landau quantization of the charged particles softens its equation of state (EoS) which means that the pressure does not increase as rapidly with (energy) density. Broderick et al. \[13\] expanded on \[12\] and included a coupling between the baryonic magnetic dipole moment and the magnetic field in their description. It was done to take into account the higher-order contributions to the baryon dipole moments due to their internal structure. The coupling is referred to as the “anomalous magnetic moment” or “AMM” coupling. They found that if the AMM coupling is included it results in a shift in the particle energy spectrum which overwhelms the softening induced by the Landau levels \[13\]. Mao et al. \[15, 16\] also considered the inclusion of the anomalous contribution to the electron magnetic dipole moment in neutron star matter (also without muons). They concluded that the impact of the electron AMM coupling is negligible.

Baryons are composite particles and at high densities its internal dynamics may be altered. In turn this could change the baryon’s dipole moment, the coupling strength of its AMM coupling. This possible influence of the baryon substructure on its dipole moment has been investigated by Ryu et al. \[17, 18\] using the quark-meson coupling (QMC) model. They found that the baryon dipole moments depend on the magnetic field, density \[17\], as well as the size of the MIT-bag in the QMC model \[18\]. Furthermore, the density-dependent dipole moments of nucleons (in particular the proton) are more enhanced than those of hyperons. This causes the proton fraction to increase and the formation of hyperons to be suppressed.

With this in mind the focus of this paper falls on the possible onset of a ferromagnetic phase transition in neutron and neutron star matter if it is assumed that the proton and neutron dipole moments increase with baryon density. Experimental investigations (summarized in \[19\]) of the dipole moment of copper isotopes with an even number of valence neutrons shown an increase of about 50% over a mass number range of 10 \[20\]. Copper has one proton outside the Z=28 proton shell. Since the neutrons are paired the change in the
dipole moment can be interpreted as the medium effects on the single proton’s magnetic dipole moment. We also assume that protons and neutrons are the appropriate baryonic degrees of freedom for the considered density range and that the equilibrium value of the ferromagnetic field will always be such that the energy density is at a minimum.

Under these assumptions we calculated the energy density of magnetized neutron and neutron star matter using the various QHD parameter sets. We employed the mean-field (MF) approximation where the meson field operators are replaced by their ground state expectation values \[8\]. The MF approximation assumes that the meson interaction length is much larger than the distance between the baryons. However, at the densities at which the approximation is made, these length scales are comparable. Hence the MF approximation is at best a phenomenological model of nuclear matter \[6\], but it has been used with great success \[11\].

The AMM coupling was included for the baryons with the generic coupling strength \(g_b\). For a relative weak magnetic field the energy density of unmagnetized and magnetized matter were compared. The phase transition takes place at the value of \(g_b\) where the energy density of the magnetized matter is just lower than that of unmagnetized matter. The phase boundary is mapped by repeating the calculation at various densities.

II. FORMALISM

The interacting part of the QHD MF Lagrangian for magnetized neutron star matter and free electromagnetic component is \[21\]

\[
\mathcal{L}_{int} = \bar{\psi} \left[ g_s \phi_0 - \gamma^\mu \left( g_b \frac{1 + \tau_3 A_\mu}{2} + g_v V_0 + \frac{g_\rho}{2} \tau_3 b_0 \right) \right] \psi - \frac{\kappa}{3!} (g_s \phi_0)^3 - \frac{\lambda_\phi}{4!} (g_s \phi_0)^4 + \frac{\zeta}{4!} (g_v V_0)^4

+ \Lambda_v (g_v V_0)^2 (g_\rho b_0)^2 - \frac{\bar{\psi} g_b}{2} F^{\mu\nu} \sigma_{\mu\nu} \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \sum_l q_l \bar{\psi}_l \gamma^\mu A_\mu \psi_l, \tag{1}
\]

where \(\psi = \begin{bmatrix} \psi_p \\ \psi_n \end{bmatrix}\) is the isodoublet baryon field operator where the subscript \(p\) and \(n\) indicates the proton and neutron components. The leptons (electrons and muons) are identified by \(l = e, \mu\) \[22\]. The MF values of the scalar, vector, and isovector mesons are indicated by \(\phi_0, V_0,\) and \(b_0\) respectively. They couple to the baryons via \(g_s, g_v,\) and \(g_\rho\) while \(\kappa, \lambda_\phi, \zeta,\)
Table I lists the values of the various masses and coupling constants of the QHD parameter sets. The coupling between the baryons and $A^\mu$ goes like the baryon charge $q_b = \begin{bmatrix} q_p & 0 \\ 0 & q_n \end{bmatrix}$, while $g_b = \begin{bmatrix} g_p & 0 \\ 0 & g_n \end{bmatrix}$ is once again the strength of the coupling between the magnetic field and the baryon dipole moments.

Under normal conditions the proton and neutron dipole moments are, in units of the nuclear magneton $\mu_N$, $2.793 \mu_N$ and $-1.913 \mu_N$ respectively [23]. In [24] it is shown that to reproduce these values of the dipole moments $g_n$ and $g_p$ must be equal to

$$g_p = -\frac{0.793}{2} \mu_N = g_p^{(0)}$$

and

$$g_n = \frac{1.913}{2} \mu_N = g_n^{(0)}.$$ 

(2)

The apparent discrepancy [25] between these expressions stems from the origin of the magnetic dipole moment. For the proton $2 \mu_N$ comes from its charge (see [24] for the full calculation). The rest, as well as the full strength of the neutron dipole moment, stems from the baryons’ finite size and internal charge distributions and currents. Since the baryons’ charges are fixed, any increase in its dipole moments must be caused by a change in its

| Model   | $g_s^2$ | $g_v^2$ | $g_\rho^2$ | $\kappa$ | $\lambda_\phi$ | $\zeta$ | $\Lambda_v$ |
|---------|---------|---------|------------|----------|----------------|--------|-------------|
| QHD1    | 109.6   | 190.4   | 0.0        | 0.0      | 0.0            | 0.0    | 0.0         |
| NL3     | 104.3871| 165.5854| 79.6000    | 3.8599   | -0.015905      | 0.00   | 0.00        |
| FSU     | 112.1996| 204.5469| 138.4701   | 1.4203   | +0.023762      | 0.06   | 0.0300      |
| IU-FSU  | 99.4266 | 169.8349| 184.6877   | 3.3808   | +0.000296      | 0.03   | 0.0460      |

TABLE I. Coupling constants of different QHD parameter sets from [8] and [11]. All coupling constants are dimensionless, except for $\kappa$ which is given in MeV. The baryon mass $m$ is taken as 939 MeV. For the QHD1 parameter set $m_s$ and $m_\omega$ are taken as 520 MeV and 783 MeV. For all the other parameter sets $m_s$, $m_\omega$ and $m_\rho$ are 491.5, 782.5, and 763 MeV, except for $m_s$ of NL3 which is 508.194 MeV.

and $\Lambda_v$ are the meson self-coupling strengths. Furthermore, $\tau_3$ is the isospin operator, $A^\mu = (0, 0, Bx, 0)$ where $B = |B|$, and $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$ are the generators of the Lorentz group.
internal dynamics. In our formalism the strength of the baryon dipole moments can be adjusted by a factor of $x$ by changing $g_n$ and $g_p$ to 

$$g_n = x \frac{1.913}{2} \mu_N = x g_n^{(0)}$$

and

$$g_p = -\frac{2.793 x - 2}{2} \mu_N = x g_p^{(0)}.$$  \hspace{1cm} (3)

In addition to its density dependence, the possible isospin dependence of the baryon magnetic dipole is also not known. Since the baryons have the same three quarks substructure, we will make the simplest assumption to investigate the phase boundary; namely that both baryon dipole moments will change by the same factor. However, we do not expect the existence of the phase transition and the qualitative features of the neutron star matter equation of state to depend sensitively on the isospin dependence of the magnetic dipole moment. We should point out that by increasing the dipole moments symmetrically using (3) the 2 : 3 ratio of the dipole moment strengths is also preserved.

From [24] the energy density of magnetized neutron star matter, which is dependent on $B$ and the total baryon density $\rho_b$, is

$$\epsilon(\rho_b, B) = \sum_{\lambda,n} \frac{|q_p B|}{4\pi^2} \int E_p(k_z, \lambda, n) \Theta[\mu_p - e_p] dk_z + \sum_{l,\lambda,n} \frac{|q_l B|}{4\pi^2} \int e_l(k_z, \lambda, n) \Theta[\mu_l - e_l] dk_z$$

$$+ \sum_{\lambda} \int \frac{d\mathbf{k}}{(2\pi)^3} E_n(\mathbf{k}, \lambda) \Theta[\mu_n - e_n] + \frac{1}{2} m^2 \phi_0^2 + \frac{\kappa}{3!} (g_s \phi_0)^3 + \frac{\lambda_s}{4!} (g_s \phi_0)^4$$

$$+ g_v V_0 (\rho_p + \rho_n) - \frac{1}{2} m^2 V_0^2 - \frac{\zeta}{4!} (g_v V_0)^4 + \frac{g_v b_0}{2} (\rho_p - \rho_n) - \frac{1}{2} m^2 b_0^2$$

$$- \Lambda_v (g_v V_0)^2 (g_v b_0)^2 + \frac{1}{2} B^2, \hspace{1cm} (4)$$

where $\Theta$ is the Heaviside step function, $\mu_i$ ($i = p, n, e, \mu$) the particles’ Fermi energies,

$$E_p(k_z, \lambda, n) = \sqrt{k_z^2 + \left( \sqrt{m^* + 2 |q_p B| n + \lambda g_p B} \right)^2}, \hspace{1cm} (5a)$$

$$E_n(\mathbf{k}, \lambda) = \sqrt{k_z^2 + \left( \sqrt{k^2 + m^* + \lambda g_n B} \right)^2}, \hspace{1cm} (5b)$$

$$e_l(k_z, \lambda, n) = \sqrt{k_z^2 + m^2 + 2 |q_l B| n}, \hspace{1cm} (5c)$$

$m^* = m - g_s \phi_0$ the reduced mass. $E_n$ and $E_p$ are the baryon energies without the contribution of the vector mesons: $e_b$ being the full energy. Hence

$$e_p = E_p + g_v V_0 + \frac{g_v b_0}{2}$$

and

$$e_n = E_n + g_v V_0 - \frac{g_v b_0}{2}. \hspace{1cm} (6)$$
and correspondingly $e_l$ indicates the lepton energies.

For neutrons $k_{\perp}^2 = k_x^2 + k_y^2$ since the presence of the magnetic field breaks the spherical symmetry of the Fermi surface. For both baryons $\lambda = \pm1$ refers to the orientation of the magnetic dipole moment with respect to $B$ (since the magnetized Hamiltonian does not commute with the spin operator spin is not a good quantum number).

$E_p$ and $e_l$ are the spectrum of the three dimensional relativistic Landau problem. For particles with a charge $q$, $n$ is an integer referring to the number of occupied Landau levels where $n = (n' + \frac{1}{2} - \alpha \frac{1}{2})$ with $n' = 0, 1, 2, 3...$ and $\alpha = \text{sgn}(qB)$. The Landau levels are degenerate [26] and it is accounted for by the factor of $|qB|/4\pi^2$ in the densities. This prefactor stems from a comparison of the fundamental magnetic flux per particle to the total magnetic flux through the level [26]. For more detail please see [24] and references therein.

The densities and Fermi energies of the various particles are established by imposing the condition of charge neutrality and beta equilibrium on the system. These are $\rho_p = \rho_e + \rho_\mu$ and $\mu_n = \mu_p + \mu_e$ respectively. Muons are assumed to populate the system when $\mu_e > m_\mu$ and then the condition $\mu_\mu = \mu_e$ is also imposed.

For the system to undergo a ferromagnetic phase transition $\epsilon(\rho_b, B)$ must be less than $\epsilon(\rho_b, 0)$. Minimizing the energy density with regards to $B$, i.e. setting $\frac{d}{dB}\epsilon(\rho_b, B) = 0$ for $B \neq 0$, yields a self-consistent equation for the ferromagnetic field. Including the various equilibrium conditions it simplifies to [24]

$$B = \left(\mu_n - g_b V_0 - \frac{g_p b_0}{2}\right) \frac{\rho_p}{B} - \sum_l \frac{\epsilon_l}{B} - \frac{\epsilon_p}{B} - \sum_\lambda \int \frac{dk}{(2\pi)^3} \frac{\partial E_n}{\partial B} \Theta[\mu_n - e_n]$$

$$- \sum_{\lambda, n} \left|\frac{q_p B}{4\pi^2}\right| \int \frac{\partial E_p}{\partial B} \Theta[\mu_p - e_p] dk_z - \sum_{l, \lambda, n} \left|\frac{q_l B}{4\pi^2}\right| \int \frac{\partial e_l}{\partial B} \Theta[\mu_l - e_l] dk_z.$$

The phase boundary was calculated by adjusting $g_b$ using (3) till $\epsilon(\rho_b, B) < \epsilon(\rho_b, 0)$ for $B \approx 10^{16}$ G which might seem like a large magnetic field, but compared to the nuclear energies it is quite small.
FIG. 1. In the top row, the ferromagnetic phase diagram for neutron matter. Plotted on the x-axis is the neutron density normalized to the density of saturated nuclear matter, $\rho_0 = 0.153$ fm$^{-3}$. On the y-axis the $g_n$, normalized to $g_n^{(0)}$, is plotted. In the bottom row the neutron matter scalar meson field for the various QHD parameters sets.

III. RESULTS

To achieve a ferromagnetic state two things must actually happen: firstly $\epsilon(\rho_b, B) < \epsilon(\rho_b, 0)$ and secondly the induced field must be parallel to the magnetic field inducing the magnetization. From (5a) and (5b) for $B > 0$ the lowest energy magnetized baryon state
have $\lambda g_b < 0$. From the sign convention used in (1), as well as taking the derivatives of the baryon energies in (7) into account, we deduce that the baryon dipole contribution to the magnetization goes like $-\lambda g_b$. Hence the induced ferromagnetic field will also be positive, independent of isospin composition.

Qualitatively the origin of the possible ferromagnetic phase can be explained as follows: Any magnetic field induces a magnetization in the system due to the asymmetric filling of the energy levels corresponding to different orientations of the dipole moments (which we denote by $\lambda$). From (5b) $m^* \pm g_n B$ are the lowest energy neutron states. Since the baryon Fermi energies are independent of $\lambda$, the difference of $2|g_n B|$ results in a mismatch in the number of filled $\lambda$ states which in turn causes the magnetization. However, $2|g_n B|$ needs to be large enough so that the induced magnetization can sustain the ferromagnetic field. In addition it has to cancel the increase in $\epsilon(\rho_b, B)$ due to the $B^2$ term in (4) so that $\epsilon(\rho_b, B) < \epsilon(\rho_b, 0)$.

The values of $g_n$ needed to achieve this in neutron matter is shown in figure 1 as a function of density. It is plotted for a non-interacting Fermi gas of neutrons (denoted by “Bare”) as well as the QHD1, NL3, FSU, and IU-FSU parameter sets. To understand the qualitative behavior of the phase boundary the differences between the parameter sets need to be considered.

The EoS can also be related to the system’s increase of the energy density with density. Thus for a stiffer EoS the energy per particle is higher than for a softer EoS. Hence the softest EoS can accommodate the most particles in the energy difference $2|g_n B|$. This means that $\epsilon(\rho_b, B)$ is more effectively lowered and since more dipole moments are unpaired and the magnetization is the strongest. Correspondingly for a soft EoS the phase transition occurs at lower values of $g_n$.

Since a non-interacting neutron gas has the softest EoS, its phase boundary occurs at the smallest values of $g_n$. For interacting neutron systems both the vector meson, $V_0$, and the isovector rho meson, $b_0$, fields are proportional to the neutron density $\rho_n$ (see [24] for the calculation). For neutron matter the ratio of the two $\lambda$ neutron densities differ but $\rho_n$
(their sum) is constant. From (3) $g_n V_0$ and $g_p b_0$ simply shifts the energy of both $\lambda$ neutrons
and their inclusion does not influence the phase boundary. Therefore any difference in the
neutron matter phase boundaries must stem from the parameterization of the scalar me-
son. The scalar meson $\phi_0$ supplies the long-range attractive part of the nuclear interaction
through $m^*$ [8]. Since the nuclear interaction saturates at higher densities the strength of
$\phi_0$ must also taper off. As each parameterization is unique and the behaviour of each $g_s \phi_0$
is different, also shown in figure [1]. Comparing these plots we deduce that the relative dif-
ference in the individual parameter sets’ phase is the result of $\phi_0$ and its influence on the EoS.

| $\frac{g_p}{g_n}$ | $\rho_0$ | $2\rho_0$ | $3\rho_0$ | $4\rho_0$ |
|------------------|---------|---------|---------|---------|
| 0.5              | 41.7    | 43.2    | 44.0    | 43.7    |
| 0.75             | 37.9    | 38.9    | 39.5    | 39.3    |
| 1                | 34.0    | 34.5    | 35.0    | 34.9    |
| 1.25             | 30.5    | 30.7    | 31.0    | 30.9    |
| 1.5              | 27.3    | 27.3    | 27.6    | 27.5    |

TABLE II. The value of $g_n$ (from [3] in units of $g_n^{(0)}$) at the phase boundary for neutron
star matter in the IU-FSU parameter set with an induced isospin dependence. The
rows correspond to different ratios of the baryon magnetic dipole moment (in units of
$g_p^{(0)}/g_n^{(0)}$) to simulate its isospin dependence at various baryon densities: symmetrically
adjusted matter corresponds to a ratio of 1.

For the phase transition in neutron star matter charged particles also have to be included,
most notably protons. The isospin dependence of the baryon dipole moments is not known,
but table II shows that the phase transition occurs independently of isospin dependence: it
will only influence where it occurs. Therefore we chose to calculate the phase boundary with
symmetrically adjusted baryon dipole moments according to (3). The results are shown in
figure [2].

Compared to neutron matter the neutron star matter phase boundaries are crossed at
smaller values since the inclusion of more particles softens the EoS. Despite the “Bare”
interaction not favouring a large proton fraction, as can be seen in figure [3] the softening of
FIG. 2. Ferromagnetic phase boundary for neutron star matter as a function of normalized baryon density and coupling constant $g_b$. The different graphs refers to a charge neutral, beta equilibrated gas of baryons and leptons (“Bare”), as well as a charge neutral, beta equilibrated neutron star matter, calculated in the different QHD parameter sets.

the EoS is evident in the phase boundary that is slightly lower.

In figure 3 the $g_s\phi_0$, $g_vV_0$, $g_\rho b_0$ of the interacting parameter sets are also shown. In this case the QHD1 phase boundary is dependent on both $g_s\phi_0$ and the proton fraction $\rho_p/\rho_n$. Initially the QHD1 neutron and neutron star matter phase boundaries are very similar, but at higher densities the baryon Fermi energies have to increase as $g_s\phi_0$ (and $m^*$) tapers off, stiffening the EoS. However, in neutron star matter the degenerate proton Landau levels can more easily absorb baryons than the neutron energy levels: as $g_s\phi_0$ tapers off $\rho_p/\rho_n$ increases by absorbing beta decaying neutrons. This results in the QHS1 neutron star matter phase boundary showing much less variation.

The NL3 parameter set includes $\phi_0$ self-couplings which slightly modifies its $g_s\phi_0$. The largest modification to the NL3 phase boundary comes from including an isospin dependence through the rho meson field. For the NL3 parameter set $g_\rho b_0$ couples directly to the isospin density $\rho_p - \rho_n$ and the large negative value of $g_\rho b_0$ corresponds to the increased proton fraction. At about $\rho_0$ the increase in $g_\rho b_0$ decreases. This stiffens the EoS and
FIG. 3. Various properties of neutron star matter at the ferromagnetic phase boundary: (a) shows the scalar meson field, (b) the expectation values of the omega meson field, (c) the proton density as a percentage of the neutron density, and finally (d) the expectation values of the rho meson field.

correspondingly the phase boundary increases. Eventually this stiffening is overcome by the increasing $\rho_p/\rho_n$ and the phase boundary decreases.

Compared to NL3, the FSU parameter set contains an additional coupling between $g_\rho b_0$ and $g_\omega V_0$ and consequently both $g_\omega V_0$ and $g_\rho b_0$ are damped. Furthermore, FSU has a smaller $\rho_p/\rho_n$ and $g_s \phi_0$ making it the softest EoS of the interacting parameter sets.

Even though IU-FSU and FSU are closely related, different high density behaviour is to be expected from IU-FSU since it is also constrained to satisfy certain neutron star properties \[11\]. Its parameterization acts to further reduce $\rho_p/\rho_n$ and thus $g_\rho b_0$ is even more damped. However, since both $g_\omega V_0$ and $g_s \phi_0$ are increased, the EoS is stiffer and the system magnetizes at slightly higher $g_\rho$ values.
IV. DISCUSSION

Due to high values of the magnetic dipole moment at which the phase boundary occurs it is clear that the ferromagnetic phase transition is a high energy effect, as one would expect. Its occurrence is dependent on whether the system can more effectively lower its energy via the ferromagnetic phase transition. This phase transition is not the only way the system can lower its energy: in this context other means include whether muons populate the system in favour of relativistic electrons and/or the beta decay of baryons. However, since charge equilibrium is enforced the lepton densities are directly coupled to that of the proton density. Since anomalous contribution to the lepton dipole moment stems from higher order couplings to the photon field and not its internal structure, like the baryons, the dipole moment of the leptons are assumed to stay more or less constant. Hence the leptons make a small contribution to the magnetization of the system. Therefore the ferromagnetic phase transition is intimately related to the ratio of baryons and the stiffness of the EoS.

Since \( \rho_p/\rho_n \) is influenced by the meson parameterization there is some variation in the predicted behaviour of the phase boundary. However, the phase boundary of the two parameterizations constrained for dense matter applications (FSU and IU-FSU) have quite similar behaviour. The general trend of the high density behaviour of the different parameter sets, as discussed in [21] and references therein, also manifest itself in the phase boundary. The FSU parameter set has the lowest values of \( g_b \), which reflects it having the softest EoS of all the (interacting) parameter sets: Soft equations of state can accommodate a larger number of fermions with a comparatively smaller increase in energy density/pressure than the other parameter sets. Therefore the conditions favourable to ferromagnetization are reached at lower values of \( g_b \) for softer equations of state. Correspondingly IU-FSU, which has a slightly stiffer equation of state, has a phase boundaries just above that of FSU. It is followed by NL3 and QHD1, which have stiffer equations of state. Hence it would probably be possible to establish the most appropriate meson parameterizations of dense matter if the ferromagnetic phase is observed.

Comparing the phase transition in neutron matter to that of neutron star matter we deduce that neutron matter represents the upper limit of the phase transition since it has
the stiffest EoS: Adding protons softens the EoS and lowers the phase boundary. Furthermore, as shown in Table [II], when an isospin preference is included in the variation of the dipole moments in favour of the protons, the phase boundary occurs at even lower values.

That being said, the question remains whether the baryon dipole moment can possibly increase by the factor needed to undergo the phase transition. To estimate the upper limit for the possible increase in the baryon magnetic dipole moment we perform a very naive calculation: since the baryon’s magnetic dipole moment results from the internal quark degrees of freedom, the magnetic field effectively couples to these internal baryonic degrees of freedom. Due to asymptotic freedom of QCD one therefore expects that the baryonic dipole moment at high density can be approximated by the sum of the dipole moments of the constituent free quarks. Thus it can be estimated using the formula for the nuclear magneton and substituting the free quark masses and charges for the baryonic ones. This yields an effective baryon dipole moment about three orders of magnitude larger than the observed baryon dipole moment [24] and thus much larger than the increase of 30 - 40 times needed to cross the phase boundary.

Since our calculation is not concerned with the density dependence of the baryon dipole moment it cannot determine whether a ferromagnetic phase transition will take place, but can only explore the characteristic of such a system. One way to better estimate how the baryon’s properties might change with density could be to calculate it using chiral soliton models [27]. Such a calculation is one of our future aims.

What we can report that if the ferromagnetic phase boundary is crossed the resulting magnetic field is of the order of $10^{17}$ gauss. These field strengths are comparable to those inferred to be present in the interior of highly magnetized neutron stars known as “magnetars” [28, 29]. Based on our results the presence of a ferromagnetic phase will certainly indicate the preference for a softer EoS. The recent discovery of a neutron star whose mass is more than double that of our sun would appear to rule out soft equations of state. However, this star is not classified as a magnetar [30]. Magnetars are characterized by their X- and/or γ-ray emissions which might not only be indicative of their strong magnetic fields, but also of a different type of EoS than that of other neutron stars. Unfortunately our calculation
is not sophisticated enough to be applied directly to the magnetar interior. In order to do that the boundary conditions as well as possible current flowing there has to be considered. Furthermore, mechanisms to generate the long-range correlation between dipole moments necessary for a global ferromagnetic phase also have to be included. This is the focus of our future research.

V. CONCLUSION

We showed the ferromagnetic phase diagram for neutron and neutron star matter as a function of the strength of the baryon dipole moment and the total baryon density. We correlated the behaviour of the phase boundaries to that of the various equations of state as well as their meson field expectation values and baryon ratios. Based on this study we cannot say whether or not the desired increase in the baryon dipole moment will be achieved in dense nuclear matter systems. However, relating the properties of ferromagnetized neutron star matter equation of state to magnetar interior and calculating its observational impact could provide insight into its possible occurrence.

VI. ACKNOWLEDGEMENTS

This research is supported by the South African SKA project as well as the National Research Foundation of South Africa.

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