The type II Weyl semimetals at low temperatures: chiral anomaly, elastic deformations, zero sound

M.A. Zubkov and M. Lewkowicz

1 Physics Department, Ariel University, Ariel 40700, Israel

We consider the properties of the type II Weyl semimetals at low temperatures basing on the particular tight-binding model. In the presence of electric field directed along the line connecting the Weyl points of opposite chirality the occupied states flow along this axis giving rise to the creation of electron-hole pairs. The electrons belong to a vicinity of one of the two type II Weyl points while the holes belong to the vicinity of the other. This process may be considered as the manifestation of the chiral anomaly that exists without any external magnetic field. It may be observed experimentally through the measurement of conductivity. Next, we consider the modification of the theory in the presence of elastic deformations. In the domain of the considered model, where it describes the type I Weyl semimetals the elastic deformations lead to the appearance of emergent gravity. In the domain of the type II Weyl semimetals the form of the Fermi surface is changed due to the elastic deformations, and its fluctuations represent the special modes of the zero sound. We find that there is one-to-one correspondence between them and the sound waves of the elasticity theory. Next, we discuss the influence of the elastic deformations on the conductivity. The particularly interesting case is when our model describes the intermediate state between the type I and the type II Weyl semimetal. Then without the elastic deformations there are the Fermi lines instead of the Fermi points/Fermi surface, while the DC conductivity vanishes. However, even small elastic deformations may lead to the appearance of large conductivity.

I. INTRODUCTION

The Dirac semimetals, where the low energy physics is described by Dirac equation, were discovered recently [1–7]. In those materials the Fermi points corresponding to the left-handed and the right-handed fermions coincide. The materials, where in each pair of the left-handed and the right-handed fermions the Weyl points of opposite chirality are separated by the finite distance in momentum space are called Weyl semimetals and were also discovered recently [8].

In [9] it was proposed that there may exist the type of the Weyl fermions, which was later discovered in the materials called the type II Weyl semimetals [10]. The Dirac cone of the quasiparticle dispersion is tilted for the type II Weyl fermions in such a way, that instead of the single Fermi point the two Fermi pockets appear touching each other at the type II Weyl point. Surprisingly the fermions inside the horizon of the equilibrium black holes discussed in [11] reveal the analogy to the type II Weyl semimetals [12]. Even more exotic forms of the Weyl fermions were proposed in [13].

The mentioned above materials are of especial importance because the physics of the low energy fermionic excitations inside them is described by the same equations as the elementary particles of the high energy physics. Although certain difference appears when the interactions are taken into account, those materials are able to simulate within laboratory the effects specific for the high energy physics. To some extent, such materials may be used as analogue computers for the simulation of the elementary particle physics.

Among those effects, in particular, there are the effects of chiral anomaly. In the 3 + 1 d systems with massless fermions (both in high energy physics and in condensed matter physics) the expressions for anomalies in fermion currents in the presence of external fields (without interaction between the fermions) may be derived using the explicit solution of Weyl equation. The spectral flow in the presence of gauge fields and gravity should result in the appearance of anomalies in the particle currents [14]. There are also the other methods developed for the calculation of anomalies in fermion currents (Fujikawa method [15], perturbative calculations, consideration of the 4 + 1 d topological isolators with 3 + 1 d chiral fermions living on their boundaries [16], etc). The derivation that uses the spectral flow is in a certain sense distinguished because it does not refer to the integration measure over fermions in the functional integral (contrary to the Fujikawa method) and it typically does not require any regularization (contrary to all mentioned above alternative methods).

At the present moment the expressions for the anomalies in flat background are well-established and commonly accepted. All mentioned above methods of anomaly calculation give the same result. It is worth mentioning, however,
that the consideration of the case when background gravity is present is more involved than the consideration of the case of flat background. There are contradictions in the expressions for chiral anomaly in this case: different methods of calculation give different expressions (see references [16–22], where those expressions are presented).

Certain observable effects in the condensed matter systems may be related to anomalies - for example, the appearance of Kopnin force acting on vortices in superfluid helium [14], magnetoresistance in Weyl semimetals [16–23, 42], etc. The nature of the chiral anomaly is pumping of particles from vacuum. There are also the cousins of the chiral anomaly - the effects, which are not related directly to pumping of the particles from vacuum, but nevertheless are related intimately to anomaly. Those are the chiral separation effect (CSE) and the chiral vortical effect (CVE) existing both in high energy physics and in condensed matter physics (see, for example, [14, 53–57, 43] and references therein). Actually, it was proposed to derive the expressions for the mentioned effects directly from the expression for the anomaly (see, for example, [32] and references therein).

In the present paper we consider the type II Weyl semimetal in the presence of external electric field directed along the line connecting the Weyl points of opposite chirality. It appears, that this situation is similar to the one of the type I Weyl semimetal in the presence of magnetic field parallel to electric field. The electric field pumps the pairs of electrons and holes from vacuum. Electrons and holes belong to the vicinities of the Weyl points of opposite chiralities. Therefore, we may refer to this phenomenon as to the kind of chiral anomaly specific for the type II Weyl semimetals, which exists without external magnetic field. It may be observed experimentally in the clean enough materials at the sufficiently small temperatures, when the corresponding resistivity exceeds the resistivity existing due to impurities. It is worth mentioning, that in order to observe the mentioned above phenomena we need that the given material does not undergo the phase transition to the superconducting state at the considered values of temperature. For the discussion of the latter possibility see [44].

Another important class of effects appears due to the elastic deformations [45]. In graphene [46–51], in the type I Weyl semimetals and in the Dirac semimetals the elastic deformations lead to the appearance of the emergent gauge field and emergent gravitational field (see, for example, [52–60]). The physics of the interior of the type I Weyl semimetals describes qualitatively the elementary particles of the high energy physics. The elastic deformations allow to simulate elementary particles in the presence of gravity and the gauge field. The latter appears as the variation of the Fermi point position in momentum space while the former is the variation of the slope of the dependence of energy on momentum.

At the same time as it was mentioned above, the type II Weyl semimetals simulate the interior of the black holes if they undergo the transition to the equilibrium state [61]. Therefore, we expect, that the elastic deformations for the type II Weyl semimetals allow to simulate the physics of elementary particles in curved space inside the equilibrium black holes. Contrary to the type II Weyl semimetals the variation of the Fermi surface does not lead to the emergent gauge field. This variation influences the ensemble of fermionic quasiparticles in the very special way. In particular, in the case when the elastic deformations oscillate, the corresponding variation of the Fermi surface becomes the special mode of the zero sound. It is the oscillation of the shape of the Fermi surface that is able to propagate inside the material [14]. We find, that this propagation occurs with the velocity of the sound waves of the elasticity theory. Moreover, there is the one - to one correspondence between those waves and the mentioned zero sound modes. This is necessary to distinguish those zero sound modes from the conventional zero sound. The latter is purely non-equilibrium phenomenon and is not related to any excitations of atoms. The modes of the zero sound investigated here may be considered as the oscillations of the Fermi surface forced by the oscillations of the atoms of the crystal lattice.

In addition, we consider the effect of the elastic deformations on the conductivity of clean enough type II Weyl materials at small temperatures when the corresponding resistivity dominates over the one caused by the impurities. (For the description of some real Weyl materials see [62–64].) In general case the contribution to the resistivity due to the elastic deformations is linear in the deformation tensor. Besides, we consider the special case when the unstrained material is in the intermediate state between the type I and the type II Weyl semimetals. In this state the Fermi lines appear. For the discussion of the cases, when the quantum state appears with the Fermi lines instead of the Fermi points/Fermi surface see [65] and references therein. In this marginal case the DC conductivity is absent. However, even small elastic deformations are able to drive the material to the type II state with the huge conductivity.

The paper is organized as follows. In Section II we remind the reader the general description of the emergent Weyl spinors in the multiferion systems. In Section III we explain when and how the type II Weyl fermions appear. In Section IV we remind the reader the description of the chiral anomaly in the type I Weyl semimetal. In Section V we discuss the chiral anomaly in the particular model of the type II Weyl semimetal that may exist without any magnetic field. In Section VI we demonstrate how this anomaly may be observed experimentally through the resistivity of the sufficiently clean materials at very small temperatures. In Section VII we consider the influence of elastic deformations on the considered toy model of the type I/type II Weyl semimetal and derive the emergent gravitational field for the case when the system is in the type I state. In Section VIII we consider the same model in the type II domain and demonstrate how the modification of the Fermi surface appears due to the elastic deformations. In Section IX...
we establish relation between the sound waves of the elasticity theory and the zero sound modes. In Section X we consider the influence of the elastic deformation on the conductivity. In particular we discuss the marginal case when without the deformation there are the Fermi lines, so that the system is between the type I and the type II states. In Section XI we end with the conclusions.

II. EMERGENT WEYL SPINORS IN THE SYSTEM WITH THE MULTICOMPONENT FERMIONS

In [55] it was proposed that the emergent low dimensional fermions in a vicinity of the topologically protected Fermi points in general, rather complicated, fermion systems are necessarily described by the two component Weyl spinors. The proof of this conjecture may be found, for example, in [9]. Below we briefly remind the reader the corresponding constructions.

Let us consider the \( n \)-component spinors \( \psi \). The partition function has the form:

\[
Z = \int D\psi D\bar{\psi} \exp \left( i \int dt \sum_x \bar{\psi}_x(t)(i\partial_t - \hat{H})\psi_x(t) \right),
\]

where the Hermitian Hamiltonian \( H \) is the function of momentum operator \( \hat{p} = -i\nabla \). We use here the symbol of the summation over the points of coordinate space because first of all we have in mind the tight-binding models of solids. However, the whole formalism works also for the general fermion systems, in which case this symbol is to be understood as the integral over \( d^3x \). The key ingredient of the construction discussed here is the repulsion between the energy levels of \( \hat{H} \). Any small perturbation pushes apart the two crossed branches. Therefore, only the minimal number of branches may cross each other.

The minimal number of the crossed branches at the given point is two. Near the crossing point the physics is described by the reduced \( 2 \times 2 \) Hamiltonian and the reduced \( 2 \)-component spinors \( \Psi \) (assuming that the Fermi energy is tuned accordingly). As a result, at low energies we may deal with the theory that has the following partition function:

\[
Z = \int D\Psi D\bar{\Psi} \exp \left( i \int dt \sum_x \bar{\Psi}_x(t)(i\partial_t - m \sigma^k - \hat{m}(\hat{p}))\Psi_x(t) \right)
\]

where functions \( m^k, m \) are real-valued. In the presence of the CP symmetry generated by \( CP = -i\sigma^2 \) and followed by the change \( x \rightarrow -x \) \( CP\Psi(x) = -i\sigma^2\bar{\Psi}^T(-x) \) the term with \( m(\hat{p}) \) is forbidden while the Hamiltonian \( \hat{H} \) can be represented as

\[
\hat{H} = \sum_{k=1,2,3} m^k(\hat{p})\sigma^k
\]

It is worth mentioning that because of the CPT theorem actually the CP symmetry coincides with the time reversal symmetry. Therefore, in condensed matter physics the given situation is typically referred to as the time reversal invariant.

The above mentioned nontrivial topology means that the topological invariant

\[
N = \frac{e\epsilon_{ijk}}{8\pi} \int_\sigma dS^i \hat{m}^j \cdot (\frac{\partial \hat{m}^k}{\partial p_j} \times \frac{\partial \hat{m}^k}{\partial p_k}), \quad \hat{m}^j = \frac{m^j}{|m|^j}
\]

is nonzero, where \( \sigma \) is the \( S^2 \) surface around the crossing point of the two branches. If \( N = 1 \) in Eq.(3), then the expansion near the branches crossing point \( p^{(0)}_j \) gives

\[
m^j_k(\hat{p}) = f^j_k(p_j - p^{(0)}_j).
\]

Here by \( f^j_k \) we denote the coefficients in the expansion. Thus we come to

\[
Z = \int D\Psi D\bar{\Psi} \exp \left( i \int dt \sum_x \bar{\Psi}_x(t)(i\partial_t - f^j_k(\hat{p}_j - p^{(0)}_j)\sigma^k)\Psi_x(t) \right)
\]

Now let us suppose, that a weak inhomogeneity is present in the given system. It may be caused, for example, by the elastic deformations, or by other reasons. Anyway, in a vicinity of each point within the material the partition
function is described by Eq. (8), but the coefficients in this expression as well as the position of the branch crossing point vary. This situation has been discussed in some details for graphene \[46--53\], and also for the three-dimensional materials called Dirac and Weyl semimetals \[58,59,58,60\]. Near the branches crossing point we obtain the following fermionic action

$$ S_R = \frac{1}{2} \int d^4x |\bar{\Psi}ie^\alpha_0(x)\sigma^b \mathcal{D}_j \Psi - [\mathcal{D}_j \bar{\Psi}]ie^\alpha_0(x)\sigma^b \Psi | $$

(6)

Here

$$ i\mathcal{D}_\mu = i\partial_\mu + A_\mu(x) $$

(7)

is the covariant derivative. It corresponds to the \(U(1)\) gauge field \(A_\mu\), while \(A_\mu\) represents the position-dependent Fermi point (that is the emergent gauge field). The field \(e^\alpha_0\) may be interpreted as the emergent vierbein field, \(\eta_{ab}\) is metric of Minkowski space. Internal \(SO(3,1)\) indices are denoted by Latin letters \(a, b, c, ...\) while the space-time indices are denoted by Greek letters or Latin letters \(i, j, k, ...\) The inverse vierbein is denoted by \(e^\alpha_a\) (it is assumed, that \(e^\alpha_a e^\alpha_b = \delta^\alpha_j\)). The determinant of \(e^\alpha_a\) is denoted by \(|e|\). By \(\sigma^a\) we denote Pauli matrices, and imply \(\sigma^0 = 1\).

Without elastic deformations the emergent gauge field \(A_\mu\) is the same for both left-handed and right-handed fermions, does not depend on coordinates, and its space-components are given by the position of the unperturbed Dirac point \(K^{(0)}\) (the time component of \(A\) vanishes in this case). The vierbein in the absence of elastic deformations is given by

$$ |e^{(0)}| = v_F, \ e^{(0)i}_a = \hat{\gamma}^i_a, \ e^{(0)i}_0 = 0, \ e^{(0)i}_a = 0, \ e^{(0)i}_0 = \frac{1}{v_F} $$

(8)

where \(a, i, j, k = 1, 2, 3,\) and \(v_F f^i_a\) is the anisotropic Fermi velocity.

Thus Eq. (8) may be considered as the action for the right-handed Weyl fermions. They correspond to the topological invariant of Eq. (8) \(N = 1\). For \(N = -1\) we come to the action for the emergent left-handed Weyl fermions:

$$ S_L = \frac{1}{2} \int d^4x |\bar{\Psi}ie^\alpha_i(x)\bar{\sigma}^b \mathcal{D}_j \Psi - [\mathcal{D}_j \bar{\Psi}]ie^\alpha_i(x)\bar{\sigma}^b \Psi | $$

(9)

Here \(\bar{\sigma}^0 = 1, \bar{\sigma}^a = -\sigma^a\) for \(a = 1, 2, 3\) while

$$ i\mathcal{D}_\mu = i\partial_\mu + A_\mu(x) $$

(10)

The solids, where the above discussed emergent Weyl fermions appear are called the type I Weyl semimetals. According to the Nielsen-Ninomiya theorem the number of the emergent left-handed fermions precisely coincides with the number of the right-handed ones. The materials, in which the positions of the right-handed and the left-handed Weyl fermions coincide are called Dirac semimetals. The typical Weyl and Dirac semimetals \[1,2\] are anisotropic, that results in the anisotropic Fermi velocity corresponding to \(3 \times 3\) matrix \(\hat{f} = \text{diag}(\nu^{-1/3}, \nu^{-1/3}, \nu^{2/3})\) with \(\nu \neq 1\). Notice, that in the presence of elastic deformations, in principle, the vielbeins (as well as the emergent gauge fields) may differ for the (left-handed or the right-handed) fermions incident at the different Fermi points.

### III. THE TYPE II WEYL FERMIONS

In the present section we briefly recall the discussion of \[8\] of the new type of the Weyl fermions, called in \[10\] the type II. In the absence of the mentioned above \(CP\) symmetry (or, equivalently, in the absence of the time reversal symmetry) we have the nonzero function \(m(p)\) that is to be expanded around the branch crossing point \(p^{(0)}\):

$$ m(p) \approx f^i_d(p_j - p^{(0)}_j), \ i, j = 1, 2, 3. $$

Here \(f^i_d\) are the new coefficients (which in the case of elastic deformations will become space-dependent). We come to the expression for the partition function of Eq. (8), where the sum is over \(a = 0, 1, 2, 3,\) and \(k = 1, 2, 3\) while \(\sigma^0 = 1\). The type II Weyl point appears if \(|G^k| = |f^i_d h^{-1} f^j_i| > 1\). (Here \(h^i_j = f^i_j\) for \(i, j = 1, 2, 3\)) Then there is the conical Fermi-surface given by the equation

$$ \text{Det} \sigma^a f^k_a(p_k - p^{(0)}_k) = 0, \ k = 1, 2, 3; \ a = 0, 1, 2, 3 $$

(11)

That is

$$ \pm \sqrt{\sum_{k=1,2,3} f^i_d f^k_d (p_j - p^{(0)}_j)(p_n - p^{(0)}_n) + f^i_d (p_j - p^{(0)}_j)^2} = 0, \ j, k = 1, 2, 3; $$

(12)
Introducing the variable $P_j = f_j^k (p_k - p_k^{(0)})$ we come to the equation
\[ \pm \|P\| + G_j P_j = 0, \quad j = 1, 2, 3; \] (13)
One can see, that this equation indeed has the nontrivial solution of the form of the Fermi surface for $\|G\| > 1$. This is the case of the type II Weyl Fermi point. Notice, that at $\|f_0^k h^{-1}\|_a^2 = 1$ instead of the Fermi point or the Fermi surface there is the Fermi line. There may also be the other marginal case, when $\det f_0^k = 0$. Then the theory may become effectively $2 + 1$ dimensional or even $1 + 1$ dimensional, i.e. there is the reference frame, in which the Hamiltonian does not depend on one or even two coordinates.

For definiteness we may consider the following particular tight-binding lattice model that describes the type II Weyl fermions:
\[ H = \frac{v_F}{a} \sigma^k \sin (p_k a) + \frac{v_F}{a} \sigma^3 \left( \sin (p_3 a) + \sum_{k=1,2} (1 - \cos (p_k a)) \right) - \frac{v}{a} \sin (p_3 a) \] \[ (14) \]
Here $a$ is the lattice spacing, i.e. the distance between the adjacent sites of the tight-binding model while $v_F$ and $v$ are the parameters. The first one has the meaning of the Fermi velocity. The corresponding branches of spectrum of this Hamiltonian are given by
\[ E = \frac{\pm v_F}{a} \sqrt{\sum_{k=1,2} \sin^2 (p_k a) + \left( \sin (p_3 a) + \sum_{k=1,2} (1 - \cos (p_k a)) \right)^2} - \frac{v}{a} \sin (p_3 a) \] \[ (15) \]
For $v < v_F$ there are two Fermi points of opposite chiralities
\[ K^+ = (0, 0, 0), \quad K^- = (0, 0, \pi/a) \]
The Hamiltonians near those two points are:
\[ H_+ = v_F \sum_{k=1,2,3} \sigma^k p_k - v p_3, \quad H_- = v_F \sum_{k=1,2} \sigma^k (p_k - K^-_k) - v_F \sigma^3 (p_3 - K^-_3) + v (p_3 - K^-_3) \] \[ (16) \]
At $v > v_F$ we come to the description of the type II Weyl semimetal with the Fermi pockets incident at the points $K^\pm$. As an illustration we represent in Fig. 1 the dispersion of the quasiparticles in this model for the particular case $v_F = 1, v = 2$.

![FIG. 1. The dispersion of the quasiparticles (in the units of $1/a$) in the model with the Hamiltonian of Eq. (14) at $v_F = 1, v = 2$.](image)

IV. CHIRAL ANOMALY IN THE TYPE I WEYL SEMIMETALS IN THE PRESENCE OF EXTERNAL MAGNETIC FIELD

Let us start from the consideration of one particular Weyl fermion (either right-handed or left-handed). We consider the particular case, when the fields do not depend on the $z$ coordinate. The one-particle Hamiltonian in the
presence of the electromagnetic field $A_k$ for the right-handed fermions is given by
\[ H^{(R)} = f_a^k \sigma^a (\hat{p}_k - A_k), \tag{17} \]
while for the left-handed fermions:
\[ H^{(L)} = - f_a^k \sigma^a (\hat{p}_k - A_k), \tag{18} \]
where
\[ f_a^k = \frac{e_a^k}{e^0_\parallel} = v_F f_a^k, \quad a, b, k = 1, 2, 3. \tag{19} \]

Let us consider the case, when the electromagnetic potential $A$ corresponds to the constant magnetic field $B$ directed along the z axis. We suppose that the magnetic field is nonzero within the cylinder of finite radius, and denote the magnetic flux through the surface $S_\perp$ orthogonal to the cylinder axis
\[ \hat{\Phi} = \frac{S_\perp}{2\pi} B^3, \tag{20} \]
The lowest Landau level modes correspond to the branches of spectrum with the dispersion
\[ \mathcal{E}^{(R)} \approx v_F \nu^{2/3} \text{sign}(\hat{\Phi}) p_3, \tag{21} \]
and
\[ \mathcal{E}^{(L)} \approx - v_F \nu^{2/3} \text{sign}(\hat{\Phi}) p_3. \tag{22} \]

![FIG. 2. Schematic pattern of the lowest Landau level in the presence of external electric field $E$ and magnetic field. We represent the given branches of spectrum as functions of $p_3$, where $p_3$ is the third component of the total momentum. It is supposed, that the third axis is directed along $\mathbf{K}$. Crossing of the branches occurs at the two Weyl points of opposite chirality $\pm |\mathbf{K}|$. The left-handed branch at $-\mathbf{K}$ is at the same time the right-handed branch at $\mathbf{K}$. (The lines are closed through the boundary of the Brillouin zone.) The values of energy carried by the occupied states at $t \neq 0$ are shifted by $E t$ (solid lines).](image)

In the presence of an external electric field $\mathbf{E}$ directed along the z axis the states that correspond to those branches flow in the correspondence with the following equation:
\[ \langle \dot{p}_3 \rangle = E_3. \tag{23} \]
These branches of spectrum have the definite value of the spin projection to axis z: $s = \frac{1}{2} \text{sign} \hat{\Phi}$ and are given, respectively, by the following dispersion relation (see, for example, [38, 56] and references therein):
\[ \mathcal{E}_{R/L}(p_3) = \pm 2sv_F \nu^{2/3} p_3, \]
its degeneracy is:
\[ N_{R/L} = |\hat{\Phi}| + 1 \tag{24} \]
Let us consider for definiteness the right-handed fermions. Then in vacuum the states with \( p_z \text{sign} \hat{\Phi}(p_z) < 0 \) are occupied. Those states flow according to Eq. (23). As a result the right-handed quasiparticles within the unit distance along the \( z \) axis are pumped from vacuum with the rate

\[
\dot{\rho}_R = \frac{E_z}{2\pi S_\perp} \text{sign} \hat{\Phi} N_{R/L},
\]

(Here \( \rho_R \) is the density of the right-handed quasiparticles while \( S_\perp \) is the area of the sample in the \( xy \) plane.) Correspondingly, the left handed quasiparticles are pumped from vacuum with the rate

\[
\dot{\rho}_L = -\frac{E_z}{2\pi S_\perp} \text{sign} \hat{\Phi} N_{R/L},
\]

This pattern is illustrated by Fig. 2. For the case \( N_{R/L} \gg 1 \) the conventional expression for the anomaly appears:

\[
\dot{\rho}_{R/L} = \pm \frac{1}{4\pi^2} EB
\]

It results in the appearance of charge carriers that consist of pairs of left-handed electrons and right-handed holes or vice versa [38, 56]. Those quasiparticles give the contribution to conductivity that is not caused by the thermal excitations (see, for example, [37, 39–42]).

V. CHIRAL ANOMALY IN THE TYPE II WEYL SEMIMETALS WITHOUT EXTERNAL MAGNETIC FIELD

In this section we consider for definiteness the type II Weyl semimetal with the Hamiltonian of Eq. (14). In the presence of the electric field \( E_z \) directed along the line connecting the Weyl points the states flow according to

\[
\langle \dot{p}_3 \rangle = E_3.
\]

For \( \nu < 1 \) one of the Weyl points is left-handed while another one is the right-handed. One may extend this assignment of chirality to the case of the type II Weyl points. However, in the presence of the Fermi sphere the relevant degrees of freedom may belong to the region of momentum space distant from the Weyl points. We extend the notion of chirality to those states as well. For the simple model with only two Weyl points \( K^\pm \) the state with momentum \( p \) is the left-handed if \( \|K^- - p\| < \|K^+ - p\| \) and it is right-handed if \( \|K^- - p\| > \|K^+ - p\| \). Instead of Eqs. (25) and (26) we have:

\[
\dot{\rho}_R = \frac{E_z}{2\pi S_\perp} N_R,
\]

and

\[
\dot{\rho}_L = -\frac{E_z}{2\pi S_\perp} N_L,
\]

Here

\[
N_{R/L} = \frac{S_\perp}{2} \int \frac{dp_x dp_y}{(2\pi)^2}
\]

is the integral over the whole Fermi surface.

The process of the pair creation in the presence of electric field is counterbalanced in practise by the annihilation of the electrons and holes. Assuming that the relaxation time is equal to \( \tau \), we may estimate the number of pairs

\[
\rho_5 = \frac{E_z}{4\pi^2} \int \frac{dp_x dp_y}{(2\pi)^2} \tau
\]

The Fermi surface (see Fig. 3) consists of the two closed pieces having two common points (that are the type II Weyl points). Those pieces have correspondingly the spherical topology and the topology of the Riemann surface with two holes. In the integral of the above equation it is assumed that the elementary area \( dp_x dp_y \) is always positive.
VI. CONDUCTIVITY OF SUFFICIENTLY CLEAN TYPE II WEYL SEMIMETALS AT VERY SMALL TEMPERATURE

Assuming that the given type II semimetal does not become the superconductor, we may estimate roughly the electric conductivity caused by the given process at \( T = 0 \) modeling the considered steady state by the collection of the quasiparticles that occupy the states from the position of the Fermi surface up to the extent of the third component of momentum \( \Delta p_z = E_z \tau \). We may simply assume that for some reason for any point \( (p_x, p_y, p_z) \) on the Fermi surface, the states between \( (p_x, p_y, p_z) \) and \( (p_x, p_y, p_z + E_z \tau) \) are occupied. Since \( \partial \mathcal{E} / \partial p_z \) is the velocity of the quasiparticle, this gives the electric current

\[
j_z = N_D \frac{E_z}{2\pi} \int \frac{dp_x dp_y}{(2\pi)^2} \left| \frac{\partial \mathcal{E}}{\partial p_z} \right| \tau \tag{30}\]

Here \( N_D = 1 \) is the number of the spectrum branches that participate in the process at any particular point in momentum space. Here the integral is over the Fermi surface (which has the form presented in Fig. 4). Here

\[
\frac{\partial \mathcal{E}}{\partial p_3} = v_F \cos (p_3 a) \left( \frac{\sin (p_3 a) + \sum_{k=1,2} (1 - \cos (p_k a))}{\sqrt{\sum_{k=1,2} \sin^2 (p_k a) + (\sin (p_3 a) + \sum_{k=1,2} (1 - \cos (p_k a)))^2}} - \frac{v}{v_F} \right) \tag{31}\]

The Fermi surface is the solution of the following equation:

\[
0 = \pm \frac{v_F}{a} \sqrt{\sum_{k=1,2} \sin^2 (p_k a) + \left( \sin (p_3 a) + \sum_{k=1,2} (1 - \cos (p_k a)) \right)^2} - \frac{v}{a} \sin (p_3 a) \tag{31}\]

It gives

\[
\frac{\partial \mathcal{E}}{\partial p_3} = \frac{v_F^2}{v} \cos (p_3 a) \left( \frac{\sum_{k=1,2} (1 - \cos (p_k a))}{\sin (p_3 a)} + 1 - \frac{v^2}{v_F^2} \right) \tag{31}\]

Notice that the product of the relaxation time \( \tau \) and the Fermi velocity \( \left| \frac{\partial \mathcal{E}}{\partial p} \right| \sim v_F \) on the Fermi surface may be considered as the mean free path \( l \) of the quasiparticle. In turn, it may be estimated as

\[
l \sim v_F \tau \approx \frac{1}{\rho_\sigma \sigma_t} \tag{31}\]

where \( \sigma_t \) is the average transport cross - section of the electron - hole annihilation. This gives

\[
\tau \approx \left[ \frac{\sigma_t v_F E_z}{4\pi} \int \frac{dp_x dp_y}{(2\pi)^2} \right]^{-1/2} \tag{32}\]

The expression for the electric current receives the form

\[
j_z = \sqrt{\frac{2E_z}{v_F \sigma_t}} \left[ \frac{\int dp_x dp_y \left| \frac{\partial \mathcal{E}}{\partial p_z} \right|}{\left[ \int \frac{dp_x dp_y}{(2\pi)^2} \right]^{1/2}} \right] \tag{32}\]

The integrals of Eq. [32] have been calculated numerically for the case \( v_F = 1 \) and the result is represented in Fig. 3. One can see that the current vanishes when the system drops from the type II phase to the type I phase.

The typical value of the lattice spacing (interatomic distance) is \( a \sim 0.25 \text{ nm} \sim 10^{-5} \text{eV}^{-1} \). Let us suppose, that the typical electric field strength (in usual units) is of the order of \( 1 \text{ V/cm} \). In order to obtain the value of \( E_z \) we need to transfer this value into relativistic units. The length unit \( \text{cm} \) should be expressed in \( \text{eV}^{-1} \) based on the relation \( [200 \text{ MeV}]^{-1} \approx 1 \text{ fm} = 10^{-13} \text{ cm} \), i.e. \( 1 \text{ cm} \) corresponds to \( 5 \times 10^4 \text{eV}^{-1} \). At the same time \( 1 \text{ V} \) transforms to \( 1 \text{ eV} \). The resulting value of \( E_z \) is

\[
E_z \approx 2 \times 10^{-5} \text{eV}^2 \tag{33}\]
FIG. 3. The ratio $\frac{\sigma}{v_F E_z/\sigma_t}$ as a function of $v$ for the model with the Hamiltonian of Eq. (14).

The typical value of $v_F$ is $\sim 1/100$. The cross-section $\sigma_t$ is given by the inverse effective mass squared of the quasiparticle. The effective mass of the quasiparticle near the Fermi surface is $m^* \sim |p/v_F| \sim \frac{1}{av_F}$. Therefore, $\sigma_t \sim \left(\frac{\alpha}{v_F \mu m}\right)^2 \sim a^2$ (here the effective coupling constant $\alpha$ is of the order of unity). As a result at $E_z = 1$ V/cm the conductivity $\partial j/\partial E_z$ is

$$\sigma \sim \sqrt{\frac{v_F}{a^2 E_z \sigma_t}} \sim 10^8 eV$$

The typical inter-collision time is

$$\tau \sim \sqrt{\frac{a^2}{v_F E_z \sigma_t}} \sim 10^{-3} eV^{-1} \sim 10^{-12} s \sim 10^{-2} cm/c$$

and the corresponding mean free path is $l \sim 10^{-4}$ cm $\sim 1000\mu$m.

The typical value of the conventional conductivity in semimetals at room temperature [62] is $\sigma^{(0)} \sim 3 \times 10^4/(Ohm \times cm) = 3 \times 10^4 A/(V \times cm)$. One Ampere corresponds to the flow of $1.6 \times 10^{-19}$ electric charges. One second corresponds to the distance of $3 \times 10^{10} cm$, that is $\sim 1.5 \times 10^{15}eV^{-1}$. Recall that we absorb the elementary charge into the definition of electric field, and the electric current is defined as the current of the electrons rather than the current of electric charge, that is our current is equal to the conventional electric current divided by the charge of electron. Thus in relativistic units we have

$$\sigma^{(0)} \sim 2 \times 10^3 eV$$

One can see, that the calculated above conductivity is

$$\sigma \sim 10^5 \sigma^{(0)} \sqrt{\frac{1 V/cm}{E_z}} \sim 3 \times \sqrt{\frac{1 V/cm}{E_z}} \times 10^3/(Ohm \times cm)$$

This value of the conductivity is of the order of the known experimental values of the conductivity for certain real metals at temperatures of the order of several Kelvin. The values of the conductivity for clean enough Au and Cu at $T \sim 1$ K may be of the same order of magnitude. The above value remains finite and is smaller than the formally infinite value predicted by the kinetic theory that takes into account the interaction of the quasiparticles with phonons. For example, the version of the theory presented in the classical textbook [66] gives $\sigma^{(T)} \sim \text{const} (\Theta/T)^{\delta}$ (here $\Theta$ is the Debye temperature). Some other theories predict the dependence $\sim T^n$ with $1 < n \leq 5$. In practise, different values of $n$ correspond to different materials.

The method used above for the calculation of the conductivity may be applied for the clean enough samples of the semimetal if the temperature is much smaller than $v_F \delta p_z = v_F E_T$:

$$T \ll v_F E_z \tau \sim \sqrt{\frac{v_F E_z a^2}{\sigma_t}} \sim \sqrt{v_F E_z} \sim 0.001 \sqrt{\frac{E_z}{1 V/cm}} eV \sim 10 \sqrt{\frac{E_z}{1 V/cm}} K$$
We also need, that the obtained value of the resistivity $1/\sigma$ is essentially larger than the resistivity $R^{(0)}$ caused by the interaction with phonons and impurities to be calculated using the more conventional methods. In practise, for the temperatures smaller than $\sim 10$ K the contribution of phonons is already not relevant. As for the impurities, their contribution to the resistivity may be estimated as

$$1/\sigma_i \sim \frac{3 (2\pi)^3}{2 |S_F| l_i}$$

where $|S_F|$ is the area of the Fermi surface while $l_i$ is the mean path corresponding to the scattering on impurities.

For large Fermi surfaces $S_F \sim 4\pi (\pi/a)^2$ and $\sigma_i \sim \frac{l_i}{a} \sim 10^5 \text{ eV} \sim \frac{l_i}{a} 10^3/(\text{Ohm} \times \text{cm})$. One can see, that we need rather clean materials (or rather large electric fields) with $l_i/a \gg \sqrt{\frac{1}{V/cm} E_z} \times 10^5$

Notice, that the density of impurities is related to the value of $l_i$ as $\rho_i \sim \frac{1}{l_i \sigma_i}$, where $\sigma_i$ is the transport cross section for the scattering of the quasiparticles on the impurity. We come to the condition

$$\left( \frac{1}{a \sigma_i (\rho_i)} \right)^{1/3} \gg \left( \frac{1 \text{V/cm}}{E_z} \right)^{1/6} \times 50$$

The detailed consideration of this condition depends on the type of the material and the type of the impurity. Assuming as an example, that the impurity does not disturb strongly the effective tight - binding model, so that the cross section $\sigma_i$ corresponds to the effective distance of the order of the interatomic distance, we come to the requirement, that there is less than one atom of impurities within each cube of the original lattice of the size (in lattice units) $100 \times 100 \times 100$.

Under these conditions the considered above resistivity as well as the non - linear dependence of electric current on the electric field may be observed experimentally at the temperature of the order of several Kelvin. Recall also, that the type I Weyl semimetal has the vanishing value of conductivity at zero temperature.

VII. THE TYPE I SEMIMETAL IN THE PRESENCE OF ELASTIC DEFORMATIONS

The low energy of the type I Weyl semimetal is described by the actions Eqs. (6) and (9). The vierbein $e^k_i$ deviates from the expression given by Eq. (8), while the positions of the Weyl points $A^{\mu}$ deviate from their unperturbed values. As a result the fermionic quasiparticles experience the emergent gauge field $A^{\mu}(x)$ and the emergent gravity given by $e^k_i(x)$. Both fields depend on the tensor of elastic deformations. In the presence of elastic deformations, in principle, the vierbeins (as well as the emergent gauge fields) may differ for the (left - handed or the right - handed) fermions incident at the different Fermi points. Let us introduce the tensor of elastic deformations $u_{ij} = \frac{1}{2} \left( \partial_i u^j + \partial_j u^i \right)$, where $u^i$ is the displacement vector. In the general case the emergent vierbein is expressed up to the terms linear in displacement as follows:

$$e^i_0 = \frac{1}{v_F} (1 + \frac{1}{3} \gamma_{kij} u^{ij}) - \hat{f}_u^0 \gamma_{i}^{jk} u^{jk}$$

$$e^0_a = -\frac{1}{v_F} \gamma_{i}^{jk} u^{jk}, \quad e^0_a = 0$$

$$e^0_0 = \frac{1}{v_F} (1 + \frac{1}{3} \gamma_{kij} u^{ij})$$

$$|e| = v_F (1 - \frac{1}{3} \gamma_{kij} u^{ij})$$

$$a, i, j, k, n = 1, 2, 3$$

The emergent gauge field is given by

$$A_i \approx \frac{1}{a} \beta_{ijk} u^{jk}, \quad A_0 = 0, \quad i, j, k = 1, 2, 3$$
Here tensors $\beta$ and $\gamma$ may also, in principle, be different for the right-handed and the left-handed fermions. Following [56] we may assume, that the values of these parameters are of the order of unity.

Let us discuss the particular tight-binding model of Eq. (13). We introduce into this model the hopping parameters $t$, $f$, and $r$ and consider the corresponding tight-binding Hamiltonian:

$$ H_0 = \frac{1}{2} \sum_{x,l,j=1,2,3} \bar{\psi}(x+l_j) i t \sigma^j \psi(x) - \frac{r}{2} \sum_{x,l,j=1,2} \bar{\psi}(x+l_j) \sigma^3 \psi(x) + r \sum_x \bar{\psi}(x) \sigma^3 \psi(x) - \frac{i f}{2} \sum_x \bar{\psi}(x+1_3) \psi(x) + (h.c.) $$

(39)

Here the sum is over the positions $x$ of the 3D cubic lattice and over $j = 1, 2, 3$. $\gamma^i$ are the Dirac matrices in chiral representation. Vectors $1_j$ connect the nearest neighbor sites of the lattice. It is assumed that the values of the parameters are chosen in such a way, that the system remains in the domain of the type I Weyl semimetal. The conditions to be fulfilled in order to drop into the type II phase will be considered below in Sect. X.

Let us consider the Hamiltonian in momentum representation:

$$ H = t \sum_{p,j=1,2,3} \bar{\psi}(p) \sin(p l_j) \sigma^j \psi(p) + r \sum_{p,j=1,2} \bar{\psi}(p) (1 - \cos(p l_j)) \sigma^3 \psi(p) - f \sum_p \bar{\psi}(p) \sin(p l_3) \psi(p) $$

(40)

Here $p$ is momentum of the quasiparticle. Next, let us define

$$ \hat{p}_i = \frac{\sin(p_i a)}{a}, \quad k_i = p_i a $$

(41)

(where $a = |l_i|$ is the lattice spacing). This gives the one-particle Hamiltonian

$$ H = v_F \sum_{k=1,2,3} \sigma^k \hat{p}_k + r \sigma^3 \left( \sum_{k=1,2} (1 - \cos(p k a)) \right) - v \hat{p}_3 $$

(42)

We introduce the dimensionless parameters

$$ v_F = t a, \quad v = f a $$

(43)

$v_F$ has the meaning of Fermi velocity.

In the presence of elastic deformations [53] the hopping parameters depend on direction and on the position in space. First let us consider the simplest model that relates hopping parameters with the tensor of elastic deformations. In this model the hopping parameter corresponding to the jump between the two sites $x$ and $x + l_j$ depends only on the real distance between these two sites given by $r(x,l_j) = |l_j + u(x + l_j) - u(x)|$, where $u$ is the displacement vector.

Therefore, we substitute the hopping parameter at link ($x,j$) with (summation over $k$ and $m$ is assumed):

$$ t \rightarrow t (1 - \beta_t \ell_j^k l_m^m u_{km}) = t (1 - \beta_t u_{jj}) $$

(44)

and

$$ r \rightarrow r (1 - \beta_r \ell_j^k l_m^m u_{km}) = r (1 - \beta_r u_{jj}) $$

(45)

and

$$ f \rightarrow f (1 - \beta_f \ell_3^k l_3^m u_{km}) = v (1 - \beta_f u_{33}) $$

(46)

$\beta_r$ and $\beta_t$ are the material parameters.

One might also consider the following complication. Let us assume, that the parameter $t$ receives the extra correction due to the non-diagonal elements of the deformation tensor [53]:

$$ t \sigma^j \rightarrow t \sigma^j (1 - \beta_t \ell_j^k l_m^m u_{km}) + t \beta_t' \sum_{n \neq j} \ell_j^k l_m^m u_{kn} \sigma^n = t \sigma^j (1 - \beta_t u_{jj}) + t \beta_t' \sum_{n \neq j} u_{jn} \sigma^n $$

(47)

with the new material parameter $\beta_t'$. The modified tight-binding model corresponds to the Hamiltonian

$$ H = \frac{1}{2} \sum_{x,l,j=1,2,3} \bar{\psi}(x+l_j) \left( i t (1 - \beta_t u_{jj}) \sigma^j + it \beta_t' \sum_{n \neq j} u_{jn} \sigma^n \right) \psi(x) $$

$$ + \frac{1}{2} \sum_{x,l,j=1,2} \bar{\psi}(x+l_j) \left( - r (1 - \beta_r u_{jj}) \sigma^3 \right) \psi(x) $$

$$ + \sum_x \bar{\psi}(x) \left( r (1 - \frac{\beta_r}{2} u^{(2)}) \sigma^3 \right) \psi(x) - \frac{1}{2} \sum_x \bar{\psi}(x+1_3) \left( i f (1 - \beta_f u_{33}) \right) \psi(x) + (h.c.) $$

(48)
We denote
\[ u^{(2)} = \sum_{j=1,2} u_{jj}, \quad u = \sum_{j=1,2,3} u_{ij} \] (49)

Notice, that the term \( r (1 - \frac{\beta_t}{3} u^{(2)}) \sigma^3 \) is needed in order to save the Fermi points. Without this coherent modification of the hopping parameter \( r \) the fermions become gapped. We arrive at the one-particle Hamiltonian
\[
H = v_F \sum_{i=1,2,3} \sigma^i \left( \hat{p}_i (1 - \beta_f u_{ii}) + \beta'_l \sum_{n \neq i} u_{in} \hat{p}_n \right) \\
+ r \sum_{i=1,2} (1 - \beta_r u_{ii}) \left[ 1 - \cos \mathbf{p} \mathbf{l}_i \right] \sigma^3 - v (1 - \beta_f u_{33}) \hat{p}_3
\] (50)

Here the product of momentum operator \( \hat{p}_k \) and the coordinate dependent function \( u_{ij}(x) \) is defined as \( \frac{1}{2} (\hat{p}_k u_{ij}(x) + u_{ij}(x) \hat{p}_k) \). The same symmetric rule is applied to the product of \( \cos (p_k a) \) and \( u_{ij}(x) \).

One can see, that for sufficiently small values of \( v \), when the system remains in the type I domain the positions of the Weyl points are not changed and are given by
\[
\mathbf{K}^+ = (0,0,0), \quad \mathbf{K}^- = (0,0,\pi/a)
\] (51)

That means, that the emergent gauge field does not appear. In the small vicinity of \( \mathbf{K}^+ \) the Hamiltonian receives the form
\[
\mathcal{H}^{(R)} = \sigma^a f^+_a (p_k - K^+_k), \quad (a = 0,1,2,3)
\]

while in the vicinity of \( \mathbf{K}^- \):
\[
\mathcal{H}^{(L)} = -\sigma^3 f_a^- k \sigma^a (p_k - K^-_k) \sigma^3,
\]
where
\[
f^\pm_{4\times3} = v_F \begin{pmatrix} 0 & 0 & \mp \frac{r}{v_F} (1 - \beta_f u_{33}) \\ 1 - \beta_l u_{11} & \beta'_l u_{12} & \beta'_l u_{13} \\ \beta'_l u_{21} & 1 - \beta_l u_{22} & \beta'_l u_{23} \\ \beta'_l u_{31} & \beta'_l u_{32} & 1 - \beta_l u_{33} \end{pmatrix}
\] (52)

The vierbein \( e^k_a \) is related to tensor \( f \) as follows:
\[
1 = |\det_{4\times4} e|^{-1} e^0_0, \quad |\det_{4\times4} e|^{-1} e^k_a = f^k_a
\] (53)

Here
\[
\det_{4\times4} e = e^0_0 \det_{3\times3} e
\]

Therefore, \( \det_{3\times3} e = 1 \) and
\[
e^0_0 = \det^{-1/3} f_{3\times3}, \quad e^k_a = \frac{f^k_a}{\det^{1/3} f_{3\times3}}, \quad a = 0,1,2,3, \quad k = 1,2,3
\]

Therefore, the \( 4 \times 4 \) matrix of the vierbein is
\[
e^k = \begin{pmatrix} \frac{1}{v_F} (1 + \frac{\beta_t}{3} u) & 0 & 0 & \mp \frac{r}{v_F} (1 + \frac{\beta_t}{3} u - \beta_f u_{33}) \\ 0 & 1 + \beta'_l u - \beta_l u_{11} & \beta'_l u_{12} & \beta'_l u_{13} \\ 0 & \beta'_l u_{21} & 1 + \beta'_l u - \beta_l u_{22} & \beta'_l u_{23} \\ 0 & \beta'_l u_{31} & \beta'_l u_{32} & 1 + \beta'_l u - \beta_l u_{33} \end{pmatrix}
\] (54)
VIII. THE TYPE II WEYL SEMIMETAL IN THE PRESENCE OF ELASTIC DEFORMATIONS

For the type II Weyl semimetals the low energy theory describes excitations that reside near the whole Fermi surface, not only near the type II Weyl points. Therefore we cannot restrict ourselves by the consideration of the vicinities of the Weyl points, and are to consider the whole momentum space. In the absence of elastic deformations the type II semimetal appears for \( v > v_F \). In the presence of elastic deformations the corresponding condition is

\[
\begin{vmatrix}
1 - \beta_t u_{11} & \beta'_t u_{12} & \beta'_t u_{13} \\
-\beta_t u_{21} & 1 - \beta_t u_{22} & \beta'_t u_{23} \\
-\beta_t u_{31} & -\beta_t u_{32} & 1 - \beta_t u_{33}
\end{vmatrix}^{-1}
\begin{pmatrix}
0 \\
0 \\
\mp \frac{v}{v_F} (1 - \beta_t u_{33})
\end{pmatrix} > 1
\]

that is (up to the terms linear in the deformation tensor)

\[
v |(1 + \beta_t u_{33} - \beta_F u_{33})| > v_F
\]

The dispersion of the quasiparticles is given by

\[
\mathcal{E} = \pm \frac{v_F}{a} \left( \sum_{k=1,2} \left( (1 - \beta_t u_{kk}) \sin (p_k a) + \beta'_t \sum_{n \neq k} u_{kn} \sin (p_n a) \right)^2 \\
+ \left( (1 - \beta_t u_{33}) \sin (p_3 a) + \beta'_t \sum_{n \neq 3} u_{3n} \sin (p_n a) + \gamma \sum_{k=1,2} (1 - \beta_r u_{kk})(1 - \cos (p_k a)) \right)^2 \right)^{1/2}
\]

\[
-\frac{v}{a} (1 - \beta_F u_{33}) \sin (p_3 a)
\]

where

\[
\gamma = \frac{r a}{v_F}
\]

Let us consider how the elastic deformations affect the Fermi surface for the case

\[
\beta_t = \beta_r = \beta, \quad \beta'_t = \beta_f = 0, \quad \gamma = 1
\]

The small variations of the positions of the point on the Fermi surface \( \delta p_i, i = 1, 2, 3 \) are related to the small values of the deformation tensor as follows:

\[
0 = \left( (1 - v^2 / v_F^2) \cos p_3 a \sin p_3 a + \cos p_3 (2 - \cos p_1 a - \cos p_2 a) \right) \delta p_3 \\
+ \left( \sin p_1 a (\sin p_3 a + 2 - \cos p_2 a) \right) \delta p_1
\]
FIG. 5. The form of the Fermi surface for the type II Weyl semimetal described by Eq. 57 (elastic deformations neglected) when the system is close to the transition to the type I phase. The chosen parameters are $v_F = 1, v = 1.02$. Here the axes correspond to the values $k_x = p_x a, k_y = p_y a, k_z = p_z a$. One can see, that when $v \rightarrow v_F + 0$ the Fermi surface is reduced to the four straight Fermi lines that cross each other in a single point. When the value of $v$ becomes smaller than $v_F$ the Fermi lines disappear and the two Fermi points remain.

$$+ \left( \sin p_3 a \left( \sin p_3 a + 2 - \cos p_1 a \right) \right) \delta p_2$$
$$- \frac{\beta}{a} \left( \sin^2 p_1 a u_{11} + \sin^2 p_2 a u_{22} \right)$$
$$+ \left( \sin p_3 a + 2 - \cos p_1 a - \cos p_2 a \right) (u_{33} \sin p_3 a + u_{11} (1 - \cos p_1 a) + u_{22} (1 - \cos p_2 a))$$

(58)

Shift of the Fermi surface is given by vector (normal to its original form)

$$\begin{pmatrix} \delta p_3 \\ \delta p_1 \\ \delta p_2 \end{pmatrix} = \begin{pmatrix} (1 - v^2 / v_F^2) \cos p_3 + \cos p_3 a (2 - \cos p_1 a - \cos p_2 a) \\ \sin p_1 a \left( \sin p_3 a + 2 - \cos p_2 a \right) \\ \sin p_2 a \left( \sin p_3 a + 2 - \cos p_1 a \right) \end{pmatrix} \delta \xi$$

(59)

where

$$\delta \xi |_{u_{11}, u_{22}, u_{33}} = \frac{\beta}{a} \left[ \left( 1 - v^2 / v_F^2 \right) \cos p_3 a \sin p_3 a + \sin p_3 a \left( 2 - \cos p_1 a - \cos p_2 a \right) \right]^2$$
$$+ \left( \sin p_1 a \left( \sin p_3 a + 2 - \cos p_2 a \right) \right)^2$$
$$+ \left( \sin p_2 a \left( \sin p_3 a + 2 - \cos p_1 a \right) \right)^2 \times$$
$$\left[ \sin^2 p_1 a u_{11} + \sin^2 p_2 a u_{22} \right.$$ \begin{align*}
&+ (\sin p_3 a + 2 - \cos p_1 a - \cos p_2 a) (u_{33} \sin p_3 a + u_{11} (1 - \cos p_1 a) + u_{22} (1 - \cos p_2 a)) \end{align*}

(60)

If the free energy that is responsible for elastic deformations is isotropic, it is given by

$$F = \frac{1}{2} \int d^3x \left( \lambda u^2 + 2\mu u_{ik} u_{ik} \right)$$

(61)

where $\lambda$ and $\mu$ are the Lame coefficients. Minimum of this functional gives the elasticity equations for the static deformations. In order to describe the deformations depending on time we should consider the action of the form

$$S = \frac{1}{2} \int d^3x dt \left( \rho \left( \partial_t u_i \right)^2 - \lambda u_t^2 - 2\mu u_{ik} u_{ik} \right)$$

(62)

where $\rho$ is density.
In order to build the effective theory for the deviation of the Fermi surface from its original form we describe this deviation by the above introduced parameter $\delta \xi$. It becomes the effective scalar field $z$ (depending on time) defined on the Fermi surface parametrized by the two coordinates $s_k$, $k = 1, 2$. The corresponding partition function is

$$Z = \int Dz(s_1, s_2, t) \delta \left( z - \delta \xi[u] \right) Du D\bar{\psi} D\psi e^{iS[u] + iS_f[\bar{\psi}, \psi, u]}$$

(63)

where $S_f$ is the fermion action. The effective action is

$$e^{iS_0[z]} \equiv \int \delta \left( z - \delta \xi[u] \right) Du D\bar{\psi} D\psi e^{iS[u] + iS_f[\bar{\psi}, \psi, u]}$$

(64)

We may assume here, that the integral over the fermions in the leading order contributes the effective action through the modification of the parameters of Eq. (62). Then we have

$$e^{iS_0[z]} = \int \delta \left( z - \delta \xi[u] \right) Du e^{iS[u]}$$

(65)

So far we discussed how the elastic deformation alters the position of the Fermi surface. However, we did not discuss the change in the occupation of the states inside the modified Fermi surface. There are the two distinct limiting regimes. In the first case there is enough time for the fermions to rearrange themselves and occupy the states with the negative energy bounded by the modified Fermi surface. This occurs if the relaxation to thermal equilibrium occurs much faster than the variation of the Fermi surface shape. In this case if the elastic deformation has the form of the propagating wave we may speak of the special mode of the zero sound, which is the propagating vibration of the Fermi surface. Unlike the conventional zero sound it is semi - equilibrium phenomenon from the point of view of the ensemble of the fermions. Recall that the conventional zero sound occurs due to the oscillation of the shape of the region of the occupied states in momentum space around equilibrium. It is, therefore, the completely non - equilibrium phenomenon that is described by kinetic theory (see, for example, chapter 4 of [67]). The conventional zero sound velocity cannot exceed the Fermi velocity (otherwise, the wave disappears fast due to the dissipation). However, there is no such bound on the special zero sound modes discussed here.

The condition for the realization of this regime is that the modification of the Fermi surface occurs more slowly than the relaxation of the ensemble of the fermionic quasiparticles to their equilibrium. That means that $\tau \frac{d}{dt} \delta p$ is much smaller than $\delta p$ (here $\delta p$ is the shift of the Fermi surface, $\tau$ is the typical inter - collision time). The latter may be estimated through the mean free path

$$l \sim v_F \tau \approx \frac{1}{\rho \sigma_t}$$

where $\rho$ is the density of the quasiparticles outside of the Fermi surface. It may be estimated as

$$\rho \sim \frac{4}{3} \frac{\delta p}{\pi} \int dp_x dp_y \sim \frac{4}{3} \frac{\delta p}{\pi a^2 r}$$

which gives

$$\tau \sim \frac{1}{v_F \frac{4}{3} \pi \delta p}$$

In turn, for the rate of the modification of the Fermi surface caused by the sound wave $u \sim U \cos(\omega t - qx)$ we have

$$\delta p \sim \frac{1}{a} U |q|, \quad \frac{d}{dt} \delta p \sim \frac{\omega}{a} U |q|$$

Here $\omega \sim v_{\text{sound}} |q|$ where $v_{\text{sound}}$ is the speed of sound while $q$ is the wave vector. The sound wavelength is $\lambda_{\text{sound}} \sim \frac{1}{|q|}$. We come to the condition: $\sqrt{\frac{\omega}{v_{\text{sound}}} U |q|} \ll \frac{\delta p}{a} U |q|$ that is

$$\frac{v_{\text{sound}}}{v_F} a \ll U$$

(66)

Here $U$ is the typical displacement of atoms due to the elastic deformations in the sound wave. The speed of sound is typically of the order of $3 \times 10^3$ m/s. At the same time the typical value of the Fermi velocity in Weyl semimetal is rather high - of the order of $10^{-2} c \sim 3 \times 10^6$ m/s. The elasticity theory requires $U \ll \lambda_{\text{sound}}$, and we need

$$10^{-3} a \ll U \ll \lambda_{\text{sound}}$$

(67)
In the opposite limiting case strictly speaking we already cannot speak of the zero sound because the modification of the (would be) Fermi surface occurs much faster than the relaxation to the thermal equilibrium. Obviously, this occurs only for the sound waves with very small amplitude \( \frac{v_{\text{sound}}}{v} \gg U \). For such vibrations of atoms almost all originally occupied states remain occupied. However, the wave of the vibration of atoms is able to excite the fermionic quasiparticles due to the interaction between them.

Quantization of the sound waves gives the typical value of the amplitude in the wave corresponding to one phonon that is much smaller than the above discussed bound \( \frac{v_{\text{sound}}}{v} \). Therefore, on the quantum level the single phonons do not produce the zero sound wave of the type discussed here. Those waves correspond to the classical sound wave, which contains the huge number of single phonons.

**IX. RELATION BETWEEN THE SOUND WAVES OF ELASTICITY AND THE SPECIAL MODES OF THE ZERO SOUND**

The action Eq. (52) describes the two types of the sound waves - the longitudinal ones with velocity \( v_\parallel = \sqrt{\frac{2\mu+\lambda}{\rho}} \) and the transversal ones with velocity \( v_\perp = \sqrt{\frac{\lambda}{\rho}} \). On the quantum level those waves become the two types of phonon excitations with the linear dispersion and the same values of velocity. If the condition of Eq. (47) is satisfied and the wavelength is sufficiently large, then each sound wave produces the vibration of the Fermi surface that propagates with the same velocity. It is described by the variation of the form of the Fermi surface given by Eqs. (59), (60), where we should substitute the deformation tensor \( u_{ij} \) corresponding to the three modes of the sound waves. Those modes appear from the Fourier transformation of \( u \):

\[
\mathbf{u}(t, \mathbf{x}) = \sqrt{\frac{V}{T}} \int \frac{d^3q dE}{(2\pi)^3} \left( U^q_{\parallel} \mathbf{q} |\mathbf{q}| \exp\left(-iEt+iq \cdot \mathbf{x}\right) + \sum_{a=1,2} U^q_{\perp,a} \hat{m}^a_q \exp\left(-iEt+iq \cdot \mathbf{x}\right) \right)
\]  

(68)

Here \( \hat{m}^a_q \) for \( a = 1, 2 \) are the two unity vectors orthogonal to \( \mathbf{q} \). We assume, that \( \hat{m}_q^q = -\hat{m}_{-q} \) and find that \( U^q_{\parallel} = -[U^q_{\perp} - E]^+ \), which guarantees that \( \mathbf{u}(t, \mathbf{x}) \) is real. \( V \) is the overall volume while \( T \) is temperature. The effective action for the variations of the Fermi surface (the zero sound) receives the form

\[
e^{iS_0[z]} = \int \delta(z - \delta(u)\Pi_\parallel \Pi_\perp dU^q_{\parallel} dU^q_{\perp} \exp\left[\frac{iV}{2T} \int \frac{d^3q dE}{(2\pi)^3} \rho \sum_{a=1,2} [U^q_{\perp,a}]^+(E^2 - \omega^2_\parallel(q))U^q_{\perp,a}\right]
\]

(69)

where \( \omega_\parallel(q) = v_\perp |\mathbf{q}| \) and \( \omega_\parallel(q) = v_\parallel |\mathbf{q}| \).

One may represent

\[
\delta(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; t, \mathbf{x}) = K^{ij}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \partial_j u_i(t, \mathbf{x}) = \sqrt{\frac{V}{T}} \int \frac{d^3q dE}{(2\pi)^3} \delta_\xi q^{ij} \exp\left(-iEt+iq \cdot \mathbf{x}\right)
\]

(70)

where \( K^{ij}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \partial_j \) is the linear differential operator defined by Eq. (44). For the Fourier modes \( \delta_\xi q^{ij} = \delta(\xi^{ij} - E^+ - q)^+ \) we have

\[
\delta_\xi q^{ij}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = iK^{ij}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) q_j \left( U^q_{\parallel} \mathbf{q} |\mathbf{q}| + \sum_{a=1,2} U^q_{\perp,a} \hat{m}^a_q \right)
\]

where

\[
K^{ij}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \frac{\beta}{a} \left[ \left( (1 - v^2/v^2_p) \cos p_3a \sin p_3a + \cos p_3a (2 - \cos p_1a - \cos p_2a) \right)^2 + \left( \sin p_1a \left( \sin p_3a + 2 - \cos p_2a \right) \right)^2 + \left( \sin p_2a \left( \sin p_3a + 2 - \cos p_1a \right) \right)^2 \right]^{-1} \times
\]

\[
\left[ \sin^2 p_1a \delta_{1\delta_1} \delta_{2\delta_2} + \sin^2 p_2a \delta_{1\delta_2} \delta_{2\delta_2} + (\sin p_3a + 2 - \cos p_1a - \cos p_2a)(\delta_{13\delta_3} \sin p_3a + \delta_{11\delta_1} (1 - \cos p_1a) + \delta_{12\delta_2} (1 - \cos p_2a)) \right]
\]

(71)
We also introduce
\[
    z(p_1, p_2, p_3; t, x) = \sqrt{\frac{V}{T}} \int \frac{d^3q dE}{(2\pi)^4} z^{q,E}(p_1, p_2, p_3) \exp \left( -iEt + i\mathbf{q} \cdot \mathbf{x} \right)
\]  

(72)

For the Fourier components of \( z \) we have the following effective action
\[
e^{iS_0[z]} = \int \Pi_{p_1, p_2, p_3} \Pi_q \Pi_E \delta \left( z^{q,E}(p_1, p_2, p_3) - iK^{ij}(p_1, p_2, p_3)q_i \left( U_{\parallel}^{q,E} \frac{q_i}{|\mathbf{q}|} + \sum_{a=1,2} U_{\perp,a}^{q,E} \hat{n}_{i,a} \right) \right)
\]
\[
dU_{\perp}^{q,E} dU_{\parallel}^{q,E} \exp \left[ i\frac{V}{2T} \int \frac{d^3q dE}{(2\pi)^4} \rho \left[ U_{\perp}^{q,E} \right] + (E^2 - \omega_{\perp}^2(q)) U_{\perp}^{q,E} \right]
\]
\[
+ i\frac{V}{2T} \int \frac{d^3q dE}{(2\pi)^4} \rho \left[ U_{\parallel}^{q,E} \right] + (E^2 - \omega_{\parallel}^2(q)) U_{\parallel}^{q,E} \right]
\]

(73)

One can see, that each sound wave given by the wave vector \( q \) and the complex numbers \( U_{\parallel}^{q,E}, U_{\perp,a}^{q,E} \) corresponds to the particular configuration of the Fourier transform of \( z(p_1, p_2, p_3; t, x) \) with respect to \( t, x \). This is the propagating mode of the zero sound. Its components proportional to \( U_{\parallel}^{q,E}, U_{\perp,a}^{q,E} \) propagate correspondingly with the velocities of sound \( v_{\parallel} \) and \( v_{\perp} \).

For each wave vector \( q \) there are the three modes of sound (one longitudinal mode and two transverse modes):
\[
u_{\parallel}^{q,E}(t, x) = \text{Re} \frac{U_{\parallel}^{q,E} q_i}{|\mathbf{q}|} \exp \left( -i\omega_{\parallel}(q)t + i\mathbf{q} \cdot \mathbf{x} \right)
\]
\[
u_{\perp,a}^{q,E}(t, x) = \text{Re} \frac{U_{\perp,a}^{q,E} \mathbf{m}_a}{|\mathbf{q}|} \exp \left( -i\omega_{\perp}(q)t + i\mathbf{q} \cdot \mathbf{x} \right)
\]

(74)

Each mode produces the vibration of the Fermi surface with the particular shape given by Eqs. (59), (60) (we should substitute into those equations the corresponding expression from Eq. (74)). If the temperature is sufficiently small
\[
T \ll v_F E_z \tau \sim 10 \sqrt{\frac{E_z}{1 \text{ V/cm} K}}
\]

then the fermionic quasiparticles feel the discussed here zero sound waves as the waves of the deformation of the Fermi surface.

X. THE TYPE II WEYL SEMIMETAL IN THE PRESENCE OF ELASTIC DEFORMATIONS AND THE CONDUCTIVITY

![FIG. 6. The typical form of the slice of the Fermi surface at \( p_z = -\pi/2 \) for \( v_F = v \), small but positive \( u_{33} \) with the remaining components of the deformation tensor that are equal to zero.](image)
The above expressions allow to estimate the contribution to the electric current of Eq. (32) due to the elastic deformations:

\[ j_z = \sqrt{\frac{2E_z}{v_F r_1}} \int \frac{dp_x dp_y}{(2\pi)^2} \frac{\partial \xi}{\partial p_z} \left[ \frac{dF}{dp_z} \right]^{1/2} \]  

(75)

where we should substitute the expression of Eq. (57) for \( E \) and evaluate it in a vicinity of the Fermi surface:

\[ \frac{\partial \xi}{\partial p_z} \approx v_F \frac{v_F}{v} (1 - \beta_t u_{33}) \cos p_3 a \left( \frac{1 - \beta_t u_{33}}{1 - \beta_F u_{33}} \right) \sin p_3 a + \beta_t' \sum_{n \neq 3} u_{3n} \sin p_n a + \gamma \sum_{k=1,2} (1 - \beta_t u_{kk})(1 - \cos p_k a) \]

(76)

\[ \frac{1}{(1 - \beta_F u_{33}) \sin p_3 a} \]

In addition we should take the integrals in Eq. (75) over the Fermi surface of the form changed due to the elastic deformations. The change of the Fermi surface form is given by Eqs. (59), (60). The expressions for the electric current contain the terms proportional to the deformation tensor with some coefficients. In general case the expressions for those coefficients are rather complicated and we do not represent them here.

Let us notice the case, when our expressions are simplified. This is the degenerate case, when the system without the elastic deformation is at the position of the transition between the type II and the type I Weyl semimetals (i.e. \( v = v_F \)).

In this intermediate state instead of the Fermi surface there are the Fermi lines, and the value of electric current at zero temperature vanishes. However, the elastic deformations may drive further the system towards the type II phase, and then the electric current at zero temperature appears given by Eq. (75), which is simplified considerably. Namely, in the leading order there is no at all the contribution from the terms containing \( u_{ij} \) in Eq. (76). The only contribution remains from the change of the form of the Fermi surface.

First of all, in the intermediate state in the absence of elastic deformations we have the three Fermi lines that intersect each other at the point \((0, 0, -\pi/2)\):

\[ (III) : p_1 = p_2 = 0; \quad (II) : p_3 = -\pi/2, p_1 = 0; \quad (I) : p_3 = -\pi/2, p_2 = 0 \]  

(77)

Correspondingly, the velocity of the quasiparticles that reside along the Fermi lines is given by:

\[ (III) : \frac{\partial \xi}{\partial p_z} = v_F \frac{v_F}{v} \cos(p_3 a) \left( 1 - \frac{v^2}{v_F^2} - \frac{2\beta_t}{v_F^2} \right) u_{33} \]

\[ = -2 \frac{v_F}{a} \cos(p_3 a) \left( \beta_t - \beta_F u_{33} \right) \]

(78)

\[ (II) : \frac{\partial \xi}{\partial p_z} = 0 \]

\[ (I) : \frac{\partial \xi}{\partial p_z} = 0 \]

Therefore, we will be interested in the modification of the form of the Fermi surface part reduced to the line (III) without elastic deformations.

Again, we restrict ourselves by the case

\[ \beta_t = \beta_r = \beta, \quad \beta_t' = \beta_F = 0, \quad \gamma = 1 \]

In the presence of elastic deformations, outside of the vicinity of the point \( p_3 = -\pi/(2a) \), the Fermi surface has the form of a small tube surrounding the \( p_3 \) axis. It is given by equation (see the discussion in the preceding Section)

\[ 0 = \left( (1 - \beta u_{11})^2 + \sin(p_3 a)(1 - \beta u_{11})(1 - \beta u_{33}) \right) p_1^2 + \left( (1 - \beta u_{22})^2 + \sin(p_3 a)(1 - \beta u_{22})(1 - \beta u_{33}) \right) p_2^2 - 2\sin^2(p_3 a) \beta u_{33}/a^2. \]  

(79)
We keep the terms linear in elastic deformations and obtain

\[ 2 \sin^2(p_3a) \beta u_{33}/a^2 = \left( 1 + \sin(p_3a) - (2 + \sin(p_3a)) \beta u_{11} - \sin(p_3a) \beta u_{33} \right) p_1^2 + \left( 1 + \sin(p_3a) - (2 + \sin(p_3a)) \beta u_{22} - \sin(p_3a) \beta u_{33} \right) p_2^2 \]  

(80)

One can see, that for \( u_{33} > 0 \) the Fermi surface appears while for \( u_{33} < 0 \) the system drops into the type I domain. In the former case each slice of the Fermi surface, at a particular value of \( p_1 \), has the form of a circle (for values of \( p_3 \) outside the vicinity of \( -\pi/(2a) \)). Moreover, in the leading order in elastic deformations we may consider it as a circle with radius \( \sim \sqrt{u_{33}} \). In the following we restrict ourselves to the case when the only nonvanishing component of the deformation tensor is \( u_{33} \).

Near the point \( p_z = -\pi/(2a) \) the form of the Fermi surface at nonzero \( u_{33} \) is changed. It surrounds the cross connecting the points \( (p_1, p_2, p_3) = (\pm \pi/(2a), 0, -\pi/(2a)) \) and \( (0, \pm \pi/(2a), -\pi/(2a)) \). When \( u_{33} \) is small, the area of this surface is small too. In Fig. 6 we represent the typical form of the slice of the Fermi surface at \( p_z = -\pi/2 \) for \( v_F = v \), small but positive \( u_{33} \) with the remaining components of the deformation tensor that are equal to zero.

Both integrals in Eq. (73) are non - analytical functions of \( \beta u_{33} \). It is possible to estimate these functions with the following asymptotic expression

\[ \left[ \int dp_x dp_y \right]^{1/2} \approx \frac{1}{a} \sqrt{\frac{8v_F E_z}{(2\pi)^3 \beta u_{33}}} \left( \cos(p_3a) \right) \left[ \int dp_x dp_y \right]^{1/2} \sim \frac{1}{a} \sqrt{\frac{8v_F E_z}{(2\pi)^3 \beta u_{33}}} 2 \pi \beta u_{33} 2 \pi (8 \sqrt{u_{33}})^{1/2} \log^{1/2} \frac{1}{\beta u_{33}} \]  

(81)

Here we used the estimate

\[ \left( \cos(p_3a) \right) \approx \frac{2}{\pi} \int_0^{\pi/2} \cos(x) dx = \frac{2}{\pi} \]

while the integral \( \int dp_x dp_y \) may be fitted by the following asymptotic expression

\[ \int dp_x dp_y \approx \text{const} \times (8 \beta u_{33})^{1/2} \log \frac{1}{\beta u_{33}} \]  

(82)

where \( \text{const} \) is of the order of unity. The final order of magnitude estimate has the form

\[ j_z \sim \frac{4}{a \pi^2} \sqrt{\frac{2v_F E_z}{\pi \beta u_{33}}} (\beta u_{33})^{5/4} \log^{1/2} \frac{1}{\beta u_{33}} \]  

(83)

This expression demonstrates, that the system with the Fermi lines, where the DC conductivity vanishes at zero temperature may acquire the huge conductivity in the presence of elastic deformations because it is driven by them to the type II state. Namely, we may estimate this conductivity \( (\beta u_{33} \ll 1) \) as follows (compare with Eqs. 331 and 30):

\[ \sigma \sim \sqrt{\frac{1V/cm}{E_z}} \times 10^9 \times \theta(u_{33})(\beta u_{33})^{5/4} \log^{1/2} \frac{1}{\beta u_{33}} \times 1/(Ohm \times cm) \]

XI. CONCLUSIONS

In the present paper we systematically investigate the type II Weyl semimetals at small temperatures assuming that they do not drop into the superconducting state. Several phenomena specific for those materials are considered here based on the particular tight - binding model. Although this model is too simple to describe quantitatively the real existing materials we expect that qualitatively the physics of the considered phenomena may be understood properly using this model.

1. First of all we discuss the chiral anomaly in the type II Weyl semimetals that may exist without any external magnetic field. In the type I Weyl semimetals the chiral anomaly appears only in the presence of external
magnetic field. In the presence of magnetic field there is only one lowest Landau level and as a result (in the case presented on Fig. 2) close to the left - handed Weyl point the left - moving states remain, while close to the right - handed Weyl point there are only the right - moving states. The external electric field applied along the line connecting the Weyl points gives rise to the drift of the occupied states inside momentum space. As a result the pairs appear to consist of the right - handed particle and the left - handed hole. This is the ordinary chiral anomaly.

In the type II Weyl materials the Weyl points also always come in pairs. On Fig. 1 we represent the typical pattern of the type II Weyl semimetal. Close to one of the two type II-Weyl points all states are right - moving, while close to the other one all states are left - moving. As a result, in the presence of the external electric field directed along the line connecting the Weyl points the states flow in momentum space and as a result pairs appear consisting of the particles that reside in the vicinity of one of the type II-Weyl points and the holes from the vicinity of the other one.

This phenomenon may be considered as the chiral anomaly without any external magnetic field. It may be observed experimentally through the measurement of conductivity. We find that at sufficiently small temperatures and for sufficiently clean materials Ohm’s law is broken and the electric current that appears due to the mentioned above type of the chiral anomaly, is proportional to the square root of electric field. As a result the resistivity depends on the electric field as \( \sim \sqrt{E} \). Its absolute value may exceed the resistivity caused by impurities only for very clean materials and at very small temperatures.

It is worth mentioning that qualitatively the same behavior of conductivity and the breakdown of Ohm’s law at very small temperatures takes place for clean ordinary metals (see the estimates in Sect. [V]).

2. Next, we consider the effect of the elastic deformations on the considered tight - binding model. The specific feature of this model is that the emergent gauge field does not appear when the model is in the type I domain. However, the emergent gravitational field appears, and was calculated.

In the type II domain in our model the elastic deformations affect the form of the Fermi surface. This deformation of the Fermi surface may propagate thus giving rise to the special modes of the zero sound. For each wave - vector there are three sound waves of the elasticity theory. Each of them gives rise to a propagating variation of the shape of the Fermi surface. This establishes a one - to - one correspondence between the discussed modes of the zero sound and the modes of the classical sound waves of the elasticity theory.

Note that the zero sound modes discussed here are different from the conventional zero sound waves. The latter are completely non - equilibrium phenomena and are propagating oscillations of the regions of the occupied fermionic states (in momentum space). Those oscillations occur without any relation to the oscillations of the atoms of the crystal lattice. They are described often by the collisionless Boltzmann kinetic equation (see [14]).

The zero sound modes considered here are the equilibrium phenomena from the point of view of the fermionic ensemble. They are forced by the vibrations of the crystal lattice existing in the form of the sound waves. Correspondingly, they have the same velocity. We consider them using the path integral formalism. Although the sound waves giving rise to the zero sound modes are classical (i.e. contain a huge number of phonons), the dynamics of the fermions is quantum, and therefore our method is more appropriate for the description of their dynamics rather than the formalism of the Boltzmann equation, which by definition, treats the fermionic quasiparticles semiclassically. In spite of the essential difference between the conventional zero sound modes and the zero sound modes discussed in the present paper, it is appropriate to proceed calling them zero sound, thus accepting the definition given in chapter 8.1.5 of [14] rather than the definition of [67] and [68]. The Fermi surface itself is not necessarily the notion that occurs in the noninteracting fermionic system, but rather the surface in momentum space, where the two point fermionic Green function is singular. This surface is able to oscillate, and such oscillations are able to propagate. In [14] the zero sound is considered as any propagating oscillation of this surface without any relation to its origin.

Elastic deformations cause a contribution to electric conductivity. We see that in the general case such a contribution is proportional to the deformation tensor. The especially interesting case is when the system without strain is in the intermediate state between the type II and the type I phases. In this state there are Fermi lines instead of Fermi points or Fermi surface, and the DC conductivity vanishes at zero temperature. However, even small elastic deformations may drive the system to the type II state. Thus a huge conductivity appears proportional to a non - analytical function of the appropriate component of the deformation tensor.

To conclude, we discussed the properties specific for the type II Weyl semimetals that manifest themselves at small temperatures. Some of these properties may be observed experimentally through the measurement of electric conductivity.
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