A Statistical Method for Corrupt Agents Detection

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Abstract. The statistical method is used to identify the hidden leaders of the corruption structure. The method is based on principal component analysis (PCA), linear regression, and Shannon information. It is applied to study the time series data of corrupt financial activity. Shannon’s quantity of information is specified as a function of two arguments: a vector of hidden corruption factors and a subset of corrupt agents. Several optimization problems are solved to determine the contribution of corresponding corrupt agents to the total illegal behavior. An illustrative example is given. A convenient algorithm for computing the covariance matrix with missing data is proposed.

INTRODUCTION

This paper demonstrates new application of principal component analysis (PCA) which was originally proposed by K. Pearson \cite{1}. This method is also known as Karhunen-Loeve Transform (see for example \cite{2, 3}). Transition to the independent variables (principal components) allows to reduce the dimension of matrices associated with the problem and to simplify computations when maintaining sufficient precision and relevancy of the results. This property has made the method particularly popular in time series analysis, where it is known as singular spectrum analysis (ASS). This method has been extensively used in visualization and graphical representation of multidimensional data \cite{4, 5, 6}. In this paper the method is used to formalize a regression model and select relevant data \cite{7, 8}.

The problem of relevant data selection is formalized and solved as an optimization problem of regression analysis. The most well-known criteria of optimality are D-criterion \cite{9, 10}, A-criterion, and G-criterion. A-criterion is iterative one and so quite appealing to the researcher as it allows to easily solve the optimization problem although G. Seber has criticized its usage \cite{11}. In this paper the estimated parameters of regression are principal components that are stochastic in nature. So Shannon’s quantity of information in vector form is used as an optimality criterion. This approach is similar to D-criterion but, unlike it, is iterative, so the computations can be performed as easily as in the case of A-criterion. Other optimization and relevant numerical techniques could also be potentially used \cite{12, 13, 14, 15, 16, 17, 18, 19}.

The problem of corruption has been extensively studied mainly in game-theoretic context \cite{20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}. In this paper the time series of corrupt financial activity data is studied with principal component analysis. The covariant matrix of biased estimators \(C\) is calculated with missing data. The matrix \(C\) is then used to find the matrix \(P\) which is utilized in a linear regression model. Shannon’s quantity of information is written as a function of two arguments: a vector of hidden corruption factors and a subset of corrupt agents. Shannon’s quantity of information is calculated with biased estimators (matrix \(P\)) \cite{31}. Quantity of information maximization problems are formalized for every successive natural number \(1, 2, 3, \ldots, m - 1, m\). Solutions to these optimization problems allow us to estimate contributions of corrupt agents to the total illegal behavior. A numerical illustrative example is given.
FIGURE 1. Increments in Shannon’s quantities of information reflecting contribution of each additional agent to the illegal behavior (hidden corruption factors).

FORMAL MODEL

Let \( M = \{1, 2, \ldots, m\} \) be an ordered set of \( m \) corrupt agents. Divide the observed time interval into equal periods of time of the length \( \tau \). Denote the number of these intervals by \( J \), so that the intervals are numbered accordingly by \( 1, 2, \ldots, J \). Each corrupt agent \( i, i = 1, 2, \ldots, m \), is engaged in some financial transactions at each time interval \( j, j = 1, 2, \ldots, J \). Let \( x_{ij} \) be a total net value of all financial transaction of the agent \( i \) during the time period \( j \). As agents’ illegal and corrupt behavior is studied, assume that \( x_{ij} = 0 \) if no transactions have signs of being illegal. Let \( N \) be an integer matrix such that \( n_{ij} = 1 \) if \( x_{ij} \neq 0 \) and \( n_{ij} = 0 \) otherwise.

Denote by \( Y = \{y_{i,j}\} \) a matrix of deviations from mean values. Let \( n_{i,*} = \sum_{j=1}^{J} n_{i,j} \). If \( n_{i,*} = 0 \) for some \( i \), then \( y_{i,j} = 0 \) for all possible \( j \) and this index \( i \). Both matrices \( Y \) and \( N \) are \( m \times J \) dimensional. Assume that matrices \( N \) and \( Y \) do not contain any zero rows. Then it is possible to compute a \( m \times m \) matrix \( C = c_{ij} \) as follows: if \( [NN^T]_{i,j} \neq 0 \), then

\[
y_{i,j} = x_{i,j} - \frac{\sum_{j=1}^{J} x_{i,j}}{n_{i,*}}.
\]

otherwise \( c_{i,j} = 0 \). Here \( T \) is the transpose operator and \([\cdot]_{i,j} \) denotes the element of the corresponding matrix with indices \( i, j \). The matrix \( C \) obtained in this manner is a matrix of biased covariance estimates of vectors \( x_j = (x_{1,j}, x_{2,j}, \ldots, x_{m,j}), j = 1, 2, \ldots, J \). Let \( R \) be an orthogonal \( m \times m \) matrix such that \( R^T C R = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m) = \Lambda \) and \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m \).

Choose \( k \leq m \) values \( \lambda_i \). Assume that the relative dispersion \( \frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{m} \lambda_i} \) is large. Then matrix \( P \) is \( m \times k \) matrix such that \( P^T C P = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_k) = \Lambda_k \). The matrix \( P \) contains the first \( k \) columns of \( R \). Let \( y \) be some column of the matrix \( Y \). Consider a simple regression model

\[
y = P \cdot z + \varepsilon
\]

where \( z \) is a \( k \)-dimensional vector and \( \varepsilon \) is a \( m \)-dimensional vector of regression errors. The vector \( z \) is a well-known vector of principal components. In our case it can be interpreted as a vector of hidden corruption factors. Assume that components of \( \varepsilon \) do not correlate and the covariance matrix of the vector \( \varepsilon \) is \( \sigma^2 I \), where \( I \) is the identity matrix.
Note however that biased estimators should be used to compute Shannon’s quantity of information. So deviation is calculated as: $c^2 = \frac{\sum_{i=1}^{m} (y_i - \bar{y})^2}{m}$. Let $q \subseteq \{1, 2, \ldots, m\}$. Denote by $y_q$ a $|q|$-dimensional vector such that $y_q = (y_i)_{i \in q}$, where $y_i$ is a component of the vector $y$ with index $i$; the order of components is preserved. Let $P_q$ be a $|q| \times m$-dimensional matrix that is obtained from the matrix $P$ by deleting all rows whose indices are not in $q$. The quantity of information the vector $y_q$ contains relative to the vector $z$ can be written as:

$$I(y_q; z) = 0.5 \cdot \log_2(det(I + \sigma^{-2} P_q P_q^T)).$$

(4)

Matrix $\sigma^{-2} P_q P_q^T$ is a Fisher information matrix. So $I(y_q; z)$ can be considered as a function of only one argument $q$. Select several subsets $q$ of vector $y$ components (elements with numbers $\{1, 2, 3, \ldots, m\}$) by incrementally increasing $|q|$ from 0 to $m$. Maximize $I(y_q; z)$ for each such subset. The result is a sequence of values $I(y_q; z)$, i.e. $I_1 \leq I_2 \leq \cdots \leq I_m$. Values $\Delta I_1 = I_1, \Delta I_2 = I_2 - I_1, \ldots, \Delta I_m = I_m - I_{m-1}$ are increments in Shannon’s quantity of information. Note that $\Delta I_1 \geq \Delta I_2 \geq \cdots \geq \Delta I_m$. The logarithm base $e$ in (4) can be chosen arbitrarily if relative values $\frac{P}{I_1}$ and $\frac{\Delta I_1}{I_1}$ are considered. Here $I_m = I(y, z)$ is the full quantity of information in the model (3).

If $k << m$ and $j = j$, is relatively small, then the fraction $\frac{I_j}{I_m}$ can become quite large (for example 0.8–0.9) or the value $\frac{\Delta I_j}{I_m}$ can become quite small (for example 0.1). The subset $q$ with the corresponding $j = |q|$ is the subset of corrupt agents that constitute the core of the corruption structure. The choice of the subset $q$ is equivalent to the choice of agents that should be closely monitored for possible corrupt behavior.

**Remark.** The requirement of absence of zero rows in matrices $Y$ and $N$ can be relaxed. Presence of zero rows in these matrices leads to the presence of zero rows in $P$, but the indices of these rows and the corresponding agents are the last in the successive data selection process.

### NUMERICAL EXAMPLE

Consider an illustrative example. Table 1 contains observed values $(x_{ij})$. The number of agents $m = 33$, the number of time periods $J = 7$. By replacing all non-zero values with 1 we get the matrix $N$. Then matrices $Y$ and $C$ can be found using (1) and (2). Note that $\sum_{i=1}^{m} A_i = tr(C)$. So the relative dispersion of the $k$ components is $\frac{\sum_{i=1}^{m} l_i}{\sum_{i=1}^{m} A_i}$. We use von Mises iteration to calculate the values $l_i$ and columns of the matrix $P$. For a example, if $k = 3$, then $\sum_{i=1}^{3} A_i / \sum_{i=1}^{m} A_i = 0.98$ and $tr(C)^{-1} A_i = diag(0.65, 0.21, 0.12)$. So the matrix $P$ is of size $33 \times 3$.

The figure 1 contains the sequence of values $\frac{P}{I}$. The first third of all agents gives almost 90% of all information about hidden corruption factors. Indices of those agents are 30, 31, 33, 22, 6, 23, 18, 4, 28, 21, 27. Unexpectedly, agent 30 contributes the most to the corrupt behavior. This is completely counter intuitive if only raw data in the table 1 is considered.
**CONCLUSION**

Method of statistical analysis is applied to the problem of corrupt behavior detection. Principal component analysis, linear regression, Shannon’s information are used to identify the hidden leaders of the corruption structure. Shannon’s quantities of information are maximized to determine the contribution of corrupt agents to the total illegal behavior. A numerical example is provided. A convenient algorithm for computation of covariant matrix is proposed.

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