Proposal and analytical investigation of optical comparator for optical and electrical converted Viterbi-decoding scheme

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A novel optical comparator is proposed for the novel Viterbi-decoding scheme, and the performance of the proposed scheme with the optical comparators is analytically investigated. The net coding gain of the proposed scheme is not degraded compared with conventional scheme, and the proposed scheme could reduce the calculation steps by a factor of \(~4/5\) of the conventional scheme.

Introduction: Optical forward error correction (FEC) technology is one of the promising approaches for cut-through operation in optical nodes in future optical systems owing to the demand to eliminate optical-to-electrical-to-optical converters [1–3]. In particular, optical FEC with convolutional code are of considerable interest because the decoding scheme for convolutional code which is called Viterbi-decoding offers high net coding gain (NCG). However, Viterbi-decoding involves a lot of electrical processing. To reduce the processing, the cascaded processing scheme of optical and electrical parts in the Viterbi-decoding is desirable.

A conceptual image of the conventional Viterbi-decoding scheme is shown in Fig. 1a. The conventional scheme consists of three parts of electrical steps. One of the steps known as ‘branch-metric calculation’ is based on the comparison processing between an expected signal and a received signal. In contrast, the proposed scheme consists of one optical step and two electrical steps, whose ‘branch-metric calculation’ is optically executed by using optical comparators (Fig. 1b).

In this Letter, we propose a novel optical comparator for the proposed Viterbi-decoding scheme to replace a part of electrical processing with the optical processing. The performance of the proposed Viterbi-decoding scheme with the optical comparators is analytically investigated.

Operating principles of optical comparator: The diagram of the novel optical comparator for a quadrature phase shift keying (QPSK)-modulated received signal of three successive symbols is shown in Fig. 2a. It comprises a 1:3 optical serial-to-parallel (SP) converter, phase shifters, and an optical coupler.

Here, we define the optical comparator designed for discriminating a code word ‘xx xx xx’ as the condition that three successive symbols of ‘xx xx xx’ should be converted into ‘00 00 00’. In the QPSK-modulated two-bit sequence of ‘xx’, right-hand side and left-hand side bits contribute to the in-phase and quadrature-phase components. For example, in the case of a comparator for a code word of ‘10 11 00’, we set the phase shifts for the 1st, 2nd, and 3rd symbols to be \(-3\pi/2\), \(-\pi\), and 0, respectively. When the signal of ‘10 11 00’ is received, the signal is converted into the successive symbol of ‘00 00 00’. Consequently, the comparator generates the coupled signal of ‘00 00 00’. Similarly, we could prepare other comparators that would have the phase-shift conditions designed for all code words according to the foregoing design concept.

Fig. 2 Projected operating principles of optical comparators when received signal is equal to ‘10 11 00’

Fig. 3 Diagram of encoder and trellis graph for (7, 5)8 code

Project operations are shown in Fig. 2 for four comparators whose designs are related to the code word of ‘10 11 00’, ‘10 11 11’, ‘10 00 10’, and ‘01 10 01’. In Fig. 2, the received signal that corresponds to ‘10 11 00’ is injected into these comparators. The comparator for ‘10 11 00’ generates a coupled signal of ‘00 00 00’ according to its definition. However, the coupled signals generated from the other comparators are different from ‘00 00 00’, because they are for another code words. In that case, the Hamming distances between the received signal and the code words of four comparators reflect the Euclid distance from the position of ‘00 00 00’ in the complex plane. Consequently, we can achieve the comparison operation optically by using the novel optical comparators.

Analytical methodology: In Fig. 3a, we assume that received signals are encoded by the (7, 5)8 convolutional coding scheme whose generating polynomial is \(G(D) = 1 + D + D^2, 1 + D^2\). The octal numbers 7 and 5 in (7, 5)8 are related to the number of shift register connections in the upper and lower output ports in Fig. 3a, which read in binary (111, 101)2 correspond to (1 + D + D2, 1 + D2). The (7, 5)8 coding scheme mainly comprises two exclusive-OR gates and shift registers, and the coded symbol for the QPSK format is assumed that two-bit QPSK signal is generated from one-bit coded symbol 1 and 2.

Trellis diagram for the (7, 5)8 coding scheme is shown in Fig. 3b, where the two-bit sequence in the \(A_i-D_i\) nodes is the internal state of the shift registers in the encoder and \(i\) is the time corresponding to the symbols of the received signal. As a trellis diagram is a graph whose nodes at each time are connected to one node at an earlier and a later time, a sent signal can be estimated from the state transition of these nodes, which is called the trellis path. The (7, 5)8 coding scheme generates \(2^5 = 32\) code words, because two coded outputs are a function of the previous three input symbols and they have 1/2 redundancy \((2^{2.5} - 2) = 2^5\). About 32 code words are related to the trellis paths. Therefore, we prepare the optical comparators corresponding to 32 possible trellis paths for three successive symbols, which correspond to the six-bit code words with the QPSK format.

Viterbi-decoding algorithm uses two types of metrics: the branch-metric and the path-metric. The branch-metric is the Hamming
distance between the expected signals and the received signal, and the path-metric is the sum of the branch-metric over the most likely path from the initial state to the current state. The path with the smallest path-metric is selected in the algorithm to minimise the total number of bit errors.

A model of the analytical investigation is shown in Fig. 4. First, a bit sequence based on \((7, 5)\) coding is created from a 253−1 pseudorandom bit sequence (PRBS) bit stream. The coded sequence is modulated with the QPSK format, and additive white Gaussian noise (AWGN) is added to the QPSK signals after modulation, and after that the novel Viterbi-decoding was performed. The details of the algorithm corresponding to #1–#4 in Fig. 4 are as follows:

- The received signal is divided into 32 signals for 32 possible paths, which correspond to the 32 code words of the \((7, 5)\) coding scheme, from \(A_0–D_0\) to \(A_7–D_7\) in the trellis diagram (#1).
- Three successive QPSK symbols are divided into individual symbols with the S/P converter in each optical comparator (#2).
- About 32 coupled signals are generated from three successive symbols by optical couplers after a phase rotation whose phase-shift conditions are designed for 32 code words (#3).
- The comparison results between the received signal and all code words are achieved by a hard decision of the coupled signals in the complex plane, because their Hamming distance accurately corresponds to the Euclidian distance based on the position of ‘00 00 00’ (#4).
- These values are regarded as branch metrics in the next Viterbi-decoding, and the smallest path metrics are selected at each state of \(A_7–D_7\).

![Fig. 4 Sequence of numerical simulation](image)

**Analytical results:** We analytically investigated the performance of the novel Viterbi-decoding scheme. First, an operating example of the optical comparators is confirmed. In Fig. 5, the outputs of 32 optical comparators, which correspond to the 32 paths from \(A_0–D_0\) to \(A_7–D_7\) in the trellis diagram, were plotted in the case that received signal is equal to ‘00 11 01’. The signal-to-noise ratio (SNR) of the added AWGN is set at 10 dB.

![Fig. 5 Simulation result of complex signals of 32 optical comparators that correspond to 32 paths from A0–D0 to A7–D7 in trellis diagram when received signal equal to ‘00 11 01’](image)

In Fig. 5, open circles, triangles, squares, and diamonds are the coupled signals that correspond to the trellis paths from \(A_0–D_0\) to \(A_7–D_7\) in the trellis diagram when received signal equal to ‘00 11 01’. Theoretically, the values of 32 coupled signals from the position of ‘00 00 00’ are the reflected Hamming distances between 32 code words and the received signal, even if the AWGN is added. Therefore, we can estimate the sent code by the novel optical comparators.

Then, the performance of the novel Viterbi-decoding is numerically evaluated with 253−1 PRBS. In Fig. 6, the BER is plotted as a function of the SNR. The BER is calculated by the error count based on optical homodyne detection for the received signal with various AWGNs. Here, the theoretical and numerical results for the BER without FEC are indicated by the dashed line and closed circles, respectively. The theoretical results were obtained by simulating (1)

\[
\text{BER} = \frac{1}{2} \text{erfc} \left( \frac{E_b}{\sqrt{2}N_0} \right)
\]

where \(E_b\) is the signal energy in 1 bit, \(N_0\) is the noise energy at the 1 Hz bandwidth, and erfc( ) is the complementary error function.

![Fig. 6 BER evaluation of decoding performance](image)

In Fig. 6, the open and closed squares correspond to the BER results for the novel Viterbi-decoding and the conventional scheme, respectively. The performance of the novel scheme approximately agrees with that of the conventional scheme. The NCGs for the novel scheme and the conventional scheme are 3.9 and 4.4 dB at a BER = 10−4, respectively.

![Table 1: Number of calculation steps in Viterbi-decoding scheme](image)

| Proposed Viterbi-decoding | Conventional Viterbi-decoding | Ratio |
|---------------------------|-------------------------------|-------|
| 162688116                 | 201326544                      | 0.81  |

**Conclusion:** In this Letter, we proposed a novel optical comparator, which consists of an optical S/P converter, phase shifters, and an optical coupler, for realisation of an optically and electrically converted Viterbi-decoding scheme. The performance of the proposed Viterbi-decoding scheme is analytically investigated for 253−1 PRBS QPSK-coded signals, which are generated by combining two BPSK-modulated outputs with the \((7, 5)\) convolutional coding scheme. The NCG for the proposed and the conventional decoding scheme are ~3.9 and 4.4 dB at BER = 10−4, and the proposed scheme could reduce the calculation steps by about 4/5 in comparison with the conventional scheme.

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Submitted: 5 April 2016 E-first: 19 July 2016 doi: 10.1049/el.2016.1132

One or more of the Figures in this Letter are available in colour online.

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