Modeling and Simulation Active Vibration Control of Flexible Solar Wing Support Structure

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Abstract. In this paper, the flexible solar wing support structure is taken as the research object, and the vibration is actively controlled by the piezoelectric material. In this paper, the Euler-Bernoulli beam theory is used to simplify the overall model structure. The principle of the hypothetical modal method is used to derive the dynamic model of the piezoelectric flexible system. The obtained system dynamics model was simulated by PID control, and good results were obtained.

1. Introduction

In recent years, due to the increasing number of human activities in space, the widespread use of various spacecraft has led to higher demands on the solar wing. Because the traditional rigid solar wing has great limitations, in order to meet the needs of space missions, flexible solar wings with small folding envelope, light weight and high specific power are inevitable development trends.

In 2017, the National Aeronautics and Space Administration (NASA) tested the new compact high-power flexible expandable solar array Roll-Out Solar Array (ROSA) on the International Space Station, which can be rolled up to form a compact cylinder for launch. This design greatly reduces mass and volume, saves huge cost and increases power. The University of Surrey in the UK has also made a packaged photovoltaic solar array [1], which uses a closely arranged intermediate truss to convert photovoltaic cells. The module is suspended between the trusses and provides a large power generation area.

Spacecraft for space missions usually cause their own stiffness to be reduced due to their large, lightweight, damped structure. In the mid-to-late 19th century, the 12 space vehicles launched by the United States had 88 failures due to vibration problems; the Hubble Space Telescope launched in 1990 was repaired several times due to the vibration of the sun blades; "Landsat-4 " was also malfunctioning due to the vibration of the solar wing; "Explorer-1" even rolled due to vibration, causing the mission to fail.

Compared with the rigid spacecraft main body, the flexible solar wing has the characteristics of large span, low structural rigidity, large deflection, and small modal damping. It is difficult to make the spacecraft change orbit and the mechanical movement of its internal components. The vibration of the self-attenuating vibration affects the normal operation of the entire spacecraft and even causes structural damage of the spacecraft. Therefore, it is necessary to study the vibration control of the flexible solar wing [2-4]. The structure of the research object is similar to that of the Surrey University's packaged photovoltaic solar cell array. By dynamic modeling and control algorithm simulation, the vibration of the support structure is actively controlled [5-7].
2. Dynamic model of active vibration control system
Since the support structure is a cantilever structure as shown in the figure 1 and 2, the longitudinal direction is set to the x direction, the direction of the bending vibration is set to the y direction, and the piezoelectric actuator and the piezoelectric sensor are attached to the root.

![Carbon fiber bistable rod](image1.png)
![Piezoelectric sensor](image2.png)

![Fixed base](image3.png)

![Piezoelectric actuator](image4.png)

Figure 1. Flexible solar wing support structure model
Figure 2. Mechanical model of the support structure

2.1 Mathematical description of the sensor
The piezoelectric sensor measures the amount of deformation at the sticking point, converts it into a charge change amount, and then amplifies it to obtain a voltage signal proportional to the amount of deformation. The strain in the direction of the sensor is

$$\varepsilon_y = -y_c \frac{\partial^2 y(x,t)}{\partial x^2}$$

(1)

$y_c$ is the distance between the flexible rod and the neutral surface of the sensor, and $y(x, t)$ is the vibration equation of the flexible rod.

The amount of charge inside the sensor is

$$Q(t) = \iint_A D_y dA = b_{pc} d_{c,31} E_{pc} \left( \frac{t_{bc}}{2} + t_{pc} \right) \frac{\partial y(x,t)}{\partial x} \mid_{y_i}$$

(2)

$b_{pc}$ is the width of the sensor, $t_b$ is the thickness of the flexible rod, $t_{pc}$ is the thickness of the sensor, $d_{c,31}$ is the strain constant of the sensor, and $E_{pc}$ is the elastic modulus of the sensor.

The voltage of the sensor can be expressed as

$$v_a = K_q b_{pc} d_{c,31} E_{pc} \left( \frac{t_{bc}}{2} + t_{pc} \right) \frac{\partial y(x,t)}{\partial x} \mid_{y_i} = K_c \frac{\partial y(x,t)}{\partial x} \mid_{y_i}$$

(3)

$K_q$ is the charge amplification factor of the connected sensor in the circuit,

$$K_c = K_q b_{pc} d_{c,31} E_{pc} \left( \frac{t_{bc}}{2} + t_{pc} \right).$$
2.2 Mathematical description of the actuator

Similarly, the piezoelectric actuator controls the deformation of the support structure at the attachment of the actuator by applying a voltage signal to the piezoelectric actuator. The stress caused by strain in the length direction is:

\[ \sigma = E_{pa} d_{a31} E_3 \]  \hspace{1cm} (4)

\( d_{a31} \) is the strain constant of the sensor, and \( E_{pa} \) is the elastic modulus of the piezoelectric bimorph. Thus, the bending moment generated by the stress is

\[ M_a = \frac{1}{2} E_{pa} b_{pa} d_{a31} (t_{pa} + t_b) U_a = K_a U_a \]  \hspace{1cm} (5)

\( b_{pa}, t_{pa} \) are the width and thickness of the piezoelectric bimorph, and \( U_a \) is the voltage applied to both sides, \( K_a = \frac{1}{2} E_{pa} b_{pa} d_{a31} (t_{pa} + t_b) \).

2.3 Dynamic model of support structure

The support structure is regarded as a flexible continuous cantilever structure, which is simplified by Euler-Bernoulli beam model. The general solution of lateral free vibration is:

\[ y(x,t) = Y(x) \sin(\omega t + \phi) \]  \hspace{1cm} (6)

where \( Y(x) = A \sin(kx) + B \cos(kx) + C \sinh(kx) + D \cosh(kx) \) is the displacement function.

in the formula, \( sh(x) = \frac{e^x - e^{-x}}{2} \), \( ch(x) = \frac{e^x + e^{-x}}{2} \), \( C_i a = A \), \( C_i b = B \), \( C_i c = C \), \( C_i d = D \).

One end is fixed and the other end is free as a boundary condition, so

\[ \begin{cases} y(x,t) = 0, & x = 0 \\ \frac{\partial y(x,t)}{\partial x} = 0, & x = l \end{cases} \]  \hspace{1cm} (7)

Substituting and sorting the above boundary conditions can be obtained:

\[ Y_i(x) = A_i \left[ (\cos(k_i x - chk_i x)) + \gamma_i \left( \sin(k_i x - shk_i x) \right) \right], \quad i = 1, 2, \ldots, n \]  \hspace{1cm} (8)

in the formula, \( \gamma_i = -\frac{\cos(k_i l + chk_i l)}{\sin(k_i l + shk_i l)} \), \( k_i = \sqrt{\omega_i^2 \frac{\rho A}{E J}} \).

2.4 Kinetic energy and potential energy

According to the energy method, the kinetic energy expression of the support structure that ignores the shear deformation can be obtained:

\[ T = \frac{1}{2} \int_0^l \rho \dot{y}^2 dx \]  \hspace{1cm} (9)

Similarly, the potential energy expression can be obtained:

\[ U = \frac{1}{2} \int_0^l E J \left( y^2 \right) dx \]  \hspace{1cm} (10)

2.5 Dynamic model based on Hamilton's principle

The Hamilton's principle is based on variational mechanics and can be expressed as

\[ \int_{t_0}^{t_1} \left( \delta(T - U) + \delta w \right) dt = 0 \]  \hspace{1cm} (11)

\( T \) is kinetic energy and \( U \) is potential energy.

The virtual work of external force is
\[ \delta W = \int_0^l M_a'' \delta y dx - \nu_s \int_0^l \dot{y} \delta y dx \] (12)

\( M_a \) is the moment generated by the piezoelectric actuator on the support structure, \( \nu_s \) is the Rayleigh structure damping, the damping ratio of the \( i \)-th order is \( \xi_i = \frac{1}{2} \left( \frac{a}{\omega_i} + \frac{b}{\omega_i} \right) \), \( \omega_i \) is the \( i \)-th order frequency, and \( a \) and \( b \) are the Rayleigh damping coefficients.

Substituting the principle, the dynamic equation of the flexible rod obtained by the functional extreme condition is:

\[ \rho A \ddot{y} + EJy''' - M_a'' + \nu_s \dot{y} = 0 \] (13)

\subsection*{2.6 Hypothetical modal method modeling}

According to the hypothetical modal method, the first \( n \)-order modal expression modes can be intercepted, and the kinetic model is derived accordingly.

\subsubsection*{2.6.1 Intercepting modal function to intercept modal function}

It is obtained by the hypothetical modal method formula:

\[ y(x,t) = \sum_{i=1}^{n} Y_i(x)q_i(t) = Y^T(x)q(t) \] (14)

in the formula, \( Y = \left[ Y_1, Y_2, \ldots, Y_n \right]^T \in \mathbb{R}^{n \times n}, q = \left[ q_1, q_2, \ldots, q_n \right] \in \mathbb{R}^{n \times 1} \).

The basis function \( Y_i \) is a mode function represented by (8); the support structure is a lightly damped structure, mainly a low-order mode, so \( n = 2 \) is taken.

\subsubsection*{2.6.2 Derivation of dynamic model of vibration active control system}

Substituting the kinetic equation obtained in (13) into the vibration mode obtained by (14), after mathematical processing, the orthogonal condition and the mass normalization condition are transformed to obtain:

\[ \rho A \ddot{q}_i(t) + \nu_s \dot{q}_i(t) + \rho A \omega_i^2 q_i(t) = \int_0^l Y_j(x)M_a'' \, dx \] (15)

The bending moment \( M_a \) of the piezoelectric actuator input, the expression is:

\[ M_a = \sum_{i=1}^{m} M_{ai}B(x) \] (16)

in the formula, \( B(x) = H(x-x_1) - H(x-x_2) \), \( H(x) \) is the Heaviside step function.

Substituting and sorting can be obtained:

\[ \int_0^l Y' M_a'' \, dx = \int_0^l \sum_{i=1}^{m} M_{ai} \frac{d^2}{dx^2} \left( H(x-x_1) - H(x-x_2) \right) \, dx \]

\[ = \sum_{i=1}^{m} K_a \left[ Y'(x_2) - Y'(x_1) \right] U_{ai} \] (17)

\( m = 1 \), the control amount is the voltage \( U_a \).

The dynamic equation of the active vibration control system is:

\[ M \ddot{q} + C \dot{q} + Kq = Q \] (18)

the parameters in the equation are: \( Q = \beta U_a \), \( M = \begin{bmatrix} \rho A & 0 \\ 0 & \rho A \end{bmatrix} \), \( C = \begin{bmatrix} \nu_{11} & 0 \\ 0 & \nu_{22} \end{bmatrix} \), \( K = \begin{bmatrix} \rho A \omega_1^2 & 0 \\ 0 & \rho A \omega_2^2 \end{bmatrix} \), \( \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} K_a \left( Y_1'(x_2) - Y_1'(x_1) \right) \\ K_a \left( Y_2'(x_2) - Y_2'(x_1) \right) \end{bmatrix} \).
3. Support structure control algorithm

With the kinetic model obtained above, the PID control algorithm is used to actively control the vibration of the support structure.

3.1 Hypothetical modal method modeling

Deform the kinetic model obtained in the previous chapter into the form of the state space equation, taking the state variable \( X(t) = [q(t) \quad \dot{q}(t)]^T = [q_1(t) \quad q_2(t) \quad \dot{q}_1(t) \quad \dot{q}_2(t)]^T \), control variables \( U = [u(t)] \), output variables \( Y = [v_a] \).

A dynamic model of a piezoelectric flexible rod system in the form of a state space matrix is obtained:

\[
\begin{align*}
\dot{X} &= AX + BU \\
Y &= CX + DU
\end{align*}
\]

(19)

the parameters in the equation are: \( A = \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ -M^{-1}K & -M^{-1}C_a \end{bmatrix} \), \( B = \begin{bmatrix} 0_{2 \times 1} \\ M^{-1} \beta \end{bmatrix} \), \( C = [c \quad 0_{2 \times 2}] \),

\( c = [c_1 \quad c_2] = [K_i \quad Y'_i(x_i) - Y'_i(x_i)] \quad K_c \quad [Y'_c(x_i) - Y'_c(x_i)] \), \( K_c = -K_p d_3 E_p b_p (t_c/2 + t_p) \), \( D = [0] \).

3.2 PID controller design

The representation method of the PID controller is

\[
u(t) = K_p \left[ e(t) + \frac{1}{T_i} \int_0^t e(t) dt + T_d \frac{de(t)}{dt} \right]
\]

(20)

\( u(t) \) is the output signal of the regulator, \( e(t)=r(t)-y(t) \) is the deviation amount, \( K_p \) is the proportional coefficient, \( T_i \) is the integration time, \( T_d \) is the differential time, \( r(t) \) is the expected value, \( y(t) \) is the output value.

Since the integral control is likely to cause instability of the control system to some extent, only the PD controller is used here.

Since in practice the ideal state does not want the support structure to vibrate, so \( r(t)=0 \), and the actual vibration signal of the beam is \( y(t)=v_a \), the actual amount of control of the piezoelectric actuator can be obtained as

\[
u(t) = -K_p v_a - K_d \dot{v}_a
\]

(21)

\( K_p>0, K_d>0 \) is the ratio and differential coefficient of the piezoelectric sensor’s control amount to the piezoelectric actuator.

Finishing is available:

\[
U = [u(t)] = -K_c X(t)
\]

(22)

In the formula, \( K_c = [K_p c_1 \quad K_p c_2 \quad K_d c_1 \quad K_d c_2] \).

The closed-loop control equation for substituting the above formula into the available support structure system is

\[
\dot{X}(t) = (A - BK_c) X(t) \quad \text{or} \quad X(t) = e^{(A - BK_c)t} X(0)
\]

(23)

It can be seen that when \( t \to \infty \), there is

\[
X(t) \to 0 = [0 \quad 0 \quad 0]^T
\]

(24)

That is, the control algorithm can make the final state of the vibration amount of the flexible rod zero.
3.3 Simulation study of PD controller

The following control simulation studies can be performed using the relationships obtained above.

In the simulation, the initial amount of the system is \( x(0) = [0 \ 0 \ 0.25 \ -0.3]^T \), the final expected state of the system is \( x(t) = [0 \ 0 \ 0]^T \), the control parameters \( K_p = 1.8 \), \( K_i = 0.016 \) are selected.

![Figure 3. Piezoelectric sensor voltage](image1)

![Figure 4. First order modal position](image2)

It can be seen from Figure 3 that the piezoelectric sensor signal characterizing the vibration intensity has a higher attenuation rate of the vibration signal than the non-control attenuation rate after applying the PD control, indicating that the PD control has a significant effect on the signal attenuation. It can be seen from Figure 4 that the application of this PD control has an inhibitory effect on the low-order mode.

4. Conclusion

In this paper, the dynamics of the flexible solar wing support structure is modeled by the mechanical method. The obtained dynamic model is transformed into the state space equation. The PD algorithm is used to construct the control loop. The software simulation has achieved good results.

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