Grover’s algorithm and human memory

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Abstract. In this article we consider an experimental study showing the influence of emotion regulation strategies on human memory performance: part of such experimental results are difficult to explain within a classic cognitive allocation model. We provide a first attempt to build a model of human memory processes based on a quantum algorithm, the Grover’s algorithm, which allows to search a particular item within an unsorted set of items more efficiently than any classic search algorithm. Based on Grover’s algorithm paradigm, this new memory model results to have interesting features: it is an iterative process, it uses parallelism and interference effects. Moreover, the strength of such interference effects depends on a parameter of the generalized Grover’s algorithm, called the phase, which admits an interpretation in terms of the emotions involved by the items and by the emotion regulation strategies. Thus we show that a reasonable choice of the phase is able to describe correctly the experimental results we consider.

1. Introduction

In a traditional computer, information is encoded in a series of bits which are manipulated via boolean logic gates arranged in succession to produce the end result. Similarly, a quantum computer manipulates qubits (the quantum analog of bits) by executing a series of quantum gates which form a quantum algorithm. A classical computer is effectively incapable of performing many tasks that a quantum computer could perform with ease. This is consistent with the fact that the simulation of a quantum computer on a classical one is a computationally hard problem [1].

The quantum computation uses two important effects: the parallelism, due to the linearity of the quantum gates, and the interference. The combination of parallelism and interference is what makes quantum computation powerful, and plays an important role in quantum algorithms [2]. The first quantum algorithm which combines interference and parallelism to solve a problem faster than classical computers, was discovered by Deutsch et al. [3]. This algorithm addresses the problem we have encountered before in connection with probabilistic algorithms: distinguish between constant and balanced databases. The quantum algorithm solves this problem exactly, in polynomial cost, while classical computers cannot do this, and must release the restriction of exactness. Shor’s algorithm (1994) is a famous polynomial quantum algorithm for factoring integers, and for finding the logarithm over a finite field [4]. For such problem, the best known classical algorithms are exponential.
In 1995 Grover [5] discovered an algorithm which searches an unsorted database of \( N \) items and finds a specific item in \( \sqrt{N} \) time steps. This result is surprising, because intuitively, one cannot search the database without going through all the items. Grover's solution is quadratically better than any possible classical algorithms. The Grover's algorithm is based on the hypothesis that exists a function \( C(i) \) (with \( i = 1, \ldots, N \)) such that \( C(\tilde{i}) = 1 \) for the marked item \( \tilde{i} \), while \( C(i) = 0 \) for the others. Such function can be applied simultaneously to all the database entries, but the useful information is extracted by repeating a suitable operation which makes use of interference. Some generalizations of Grover's algorithm have been provided, such as [6, 7, 8].

There are many features of quantum algorithms that induces us to use concepts of quantum computation in describing human memory effects. First of all, there is evidence that we retain more than we can retrieve: the effectiveness of retrieval can depend on many factors, such for example emotions [9, 10]. This seems consistent with the generalized Grover’s algorithm, where the effectiveness of quantum search depends on the intensity of the interference effects which extract the information. In other words, such algorithm is consistent with the fact that we are potentially able to recall the information, but we have to extract the correct answer from the other potential outputs by using interference effects. We provide a psychological interpretation of the important quantum gate known as selective phase rotation in terms of emotional influence. To confirm such suggestions, we recall the fact that the forgetting mechanism is also connected to interference effects. Secondly, the process of search in Grover’s algorithm has an optimal number of iterations: a lower or higher number of iterations leads to a lower effectiveness of the search. Such effects, known respectively as undercooking and overcooking, seem to be consistent with the fact that sometimes we do not remember because we have too less or too much time to think.

This article is addressed both to physicists and to psychologists. To this end, we give the mathematical details in the appendix, while in the body of the paper we introduce only the mathematical basic concepts which are essential to the understanding.

Our work is part of a research topic which can be called "quantum cognition", which studies within the quantum formalism experiments of logical fallacies [11, 12], decision theory [13] and semantic [14].

2. Memory and emotion regulation

Quite recently, researchers in cognitive science have begun to examine how individuals regulate their emotions and to document what consequences such attempts at emotion regulation have [9]. It is quite natural to accept that emotions frequently arise when important goals are at stake and thus when peak cognitive performance is critical. If from a point of view emotion regulation is quite natural, from another it seems to be effortful: this means that emotion regulation can deplete mental resources. For example, emotion-regulation participants were found to solve fewer problems than no-regulation participants [10].
Two emotion-regulation strategies have been identified: the reappraisal (for example, appraising an upcoming task as a challenge rather than a threat, or looking at the situation as an external observer) and the expressive suppression (inhibition of the urge to act on emotional impulses that press for expression). For a list of references about examples where emotion regulation consumes cognitive resources, see [9]. A particular object of study is the regulation of negative emotions, and its influence on subjects’ memory. In the present article we consider an experiment performed by [9] in order to study how the expressive suppression and the reappraisal conditions influence the ability to remember. In such experiment participants watched emotion-eliciting slides (distinguished between high-level or low-level emotion eliciting slides), and answered to questions about verbal and nonverbal memory.

2.1. Description of the experiment

Experiment 2 of [9] attempted to show that experimentally manipulating reappraisal and expressive suppression in a controlled laboratory setting would differentially influence memory for information presented during the induction period. Moreover, it studied whether suppression would lead to poorer memory even when low or high levels of emotion were elicited.

Eighteen slides were presented in two sets of nine slides each on a television monitor. As slides were presented, three data—a name, an occupation, and a cause of injury—were presented using an audio recording. Slides were presented individually for 10 s; slides within each set were separated by 4 s. Some of the slides (9) showed people who appeared healthy because their injuries had happened a long time ago (low-emotion slide set), but other slides (9) showed people who appeared gravely injured because they had been photographed shortly after sustaining their injuries (high-emotion slide set).

Participants were randomly assigned to: the watch condition (40), the expressive suppression condition (20) and the reappraisal condition (22). Participants viewed a first set of nine individually presented slides (low-emotion for example), and after a self-report emotion experience measure, they viewed a second set of slides (high-emotion for example). Finally they performed by a cued-recognition test of the slides and a cued-recall test of the orally presented biographical information.

Cued-recognition test (non-verbal memory): participants were shown 18 photo spreads, one corresponding to each of the 18 slides they saw in the first phase of the experiment. For each photo spread, participants were asked to identify which of four alternatives most closely resembled the slide they had seen earlier. The correct alternative was the same image participants had seen earlier, with the only difference being that it was reduced in size. Incorrect alternatives were generated by modifying particulars of the original image. Participants had 8 s to view each photo spread and to give their answer.

Cued-recall test (verbal memory): after viewing the photo spreads, participants viewed the original slides one more time. This time, they were asked to write down the
information that had been paired with each slide during the initial slide-viewing phase (i.e., name, occupation, injury).

Results for verbal memory: only suppression participants showed a reliable decrease in memory (13%), compared with watch (18%) and reappraise (16%) conditions. Overall, verbal information was remembered less well if it accompanied high-emotion slides.

Results for non-verbal memory: unexpectedly, reappraisal participants were more likely to correctly identify high-emotion slides they had seen earlier than watch participants. Low-emotion case: watch (43%), reappraise (40%), suppress (35%) High-emotion case: watch (37%), reappraise (48%), suppress (40%). In the following, we will focus our attention on the results of non-verbal memory, which are quite surprising from the point of view of a classic allocation model: in fact, even in the high-emotion case the watch condition seems to involve no cognitive costs in emotion regulation., and thus it should evidence the highest success percentages. The quantum Grover’s algorithm provides a completely different model, where the emotions seem to play a key role in manipulating interference effects which drive the memory process.

3. The generalized Grover’s algorithm

Grover’s algorithm, invented by Lov Grover in 1996 [5], is a quantum algorithm for searching an unsorted database. Classically, searching an unsorted database with \( N \) items requires a linear search. This means that the time we need is in mean proportional to the number of items \( N \). On the contrary, Grover’s algorithm requires a number of steps proportional to \( \sqrt{N} \), from which it is easy to note that such quantum algorithm is more efficient than the classical one. A simple example, cited in [5], is a phone directory containing \( N \) names arranged in completely random order. If we want to find someone’s phone number with a probability of 1/2, any classical algorithm (deterministic or probabilistic) will need to look at a minimum of \( N/2 \) names.

We give now a simple description of a particular generalization of Grover’s algorithm which is useful for our purposes [6, 7], evidencing three main features:

1) The preparation of the quantum register: given the \( N \) items, we attribute to each item \( i = 1, \ldots, N \) the same quantum amplitude

\[
\psi(i) = \frac{1}{\sqrt{N}}. \tag{1}
\]

We recall that the quantum amplitude \( \psi(i) \) is a generalization of the classic probability weight. The probability to measure any item \( P(i) \) is the square modulus of the amplitude \( P(i) = |\psi(i)|^2 \) and thus \( P(i) = 1/N \). This means that each item is supposed to have initially the same probability to be measured. We note that the quantum formalism allows for more general evenly distributed superpositions. For example, the amplitude associated to a particular item \( i \) could be a complex number \( e^{i\phi}/\sqrt{N} \). Since the probability to measure such item is the square modulus of the amplitude, we would have \( |e^{i\phi}/\sqrt{N}|^2 = 1/N \). The complex factor, called the phase, has an important role in
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interference effects.

2) The function of the marked item $C(i)$ such that:

$$C(\bar{i}) = 1, \quad C(i) = 0 \forall i \neq \bar{i}$$  \hspace{1cm} (2)

In other words, the Grover’s algorithm hypothesizes the existence of a function which is true for the marked item and false for the other items.

3) The iterative search engine, representing the quantum gate which uses the function $C(i)$ and the interference effect to make the probability relevant to the marked item $P(\bar{i})$ near to 1. The search engine is characterized by a specific parameter, the phase $\phi$. The probability to find the marked item $P(\bar{i})$ after $J$ iterations and with the characteristic phase $\phi$ is

$$P(\bar{i}) = \sin \left\{ [2J \sin(\phi/2) + 1] \arcsin \left( \frac{1}{\sqrt{N}} \right) \right\}^2$$  \hspace{1cm} (3)

We can see from this formula that the probability to find the marked item is a periodic function of the number of iterations $J$. To obtain $P(\bar{i}) \simeq 1$, the quantum search engine has to be repeated $J_{\text{opt}}$ times, where $J_{\text{opt}}$ is the optimal number of iterations. In the high $N$ case we have:

$$J_{\text{opt}} \simeq \frac{\pi \sqrt{N}/4 - 1/2}{\sin(\phi/2)}.$$  \hspace{1cm} (4)

In the high $N$ condition, a number of iterations lower than $J_{\text{opt}}$ leads to a probability to find the marked item lower than 1: this fact is known as undercooking. Also a number of iterations higher than $J_{\text{opt}}$ leads to a probability to find the marked item lower than 1: this fact is known as overcooking.

3.1. A quantum-like model for memory

In this section a quantum-like model for human memory is proposed, based on a generalization of Grover’s algorithm of Long et al. [6, 7]. Such model does not assume that the human mind has a quantum nature, although some evidences and models have been provided by Vitiello et al. [15]; it simply defines a formal mathematical model, based on a quantum algorithm, which is able to give a good description of human memory experiments.

First of all, we give some remarks about cued-recognition task and cued-recall task: in the first the subject is presented some alternatives, and the searched item is part of them. On the contrary, in the second the subjects have to give the searched item without having any alternatives, but only with the aid of a cue. However, in the cued-recall task the alternatives have been previously presented during the first part of the experiment. It follows that in both cases we will assume the existence of a quantum register, as described before.

Thus, given a memory task over $N$ alternative items (recall or recognition), the main hypotheses of our quantum-like model of memory are:

1) subjects first assign equal amplitudes $\psi(i) = 1/\sqrt{N}$ to the possible items, as described
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2) subjects are provided a function of the marked item as evidenced by equation (2). In other words, an unconscious knowledge of the marked item is supposed;

3) subjects perform an iterative process, by repeating for a number of times $J$ the same searching engine, characterized by a specific phase $\phi$, as described by equation (3), which evidences the probability to find the marked item after $J$ iterations.

4) the characteristic phase $\phi$ of a search engine is determined by two factors: a) it increases with the level of emotion elicited by the items or by the cues, and b) it decreases with the emotion regulation strategy. Thus, given the characteristic phases in watch strategy for low emotion $\phi(W_1)$ and high emotion $\phi(W_2)$, reappraisal strategy $\phi(R_1)$, $\phi(R_2)$ and suppression $\phi(S_1)$, $\phi(S_2)$, we impose:

$$\phi(W_1) > \phi(R_1) > \phi(S_1)$$

$$\phi(W_2) > \phi(R_2) > \phi(S_2).$$

Given a fixed number of iterations $J$ and a fixed number of items $N$, the probability to find the marked item is given by formula (3) and depends only on the parameter $\phi$. If we associate a unit of time to each iteration, the fixed $J$ condition is equivalent to a fixed-time memory task.

We consider the test of non-verbal memory of experiment 2 in [9]. The number of items is given not only by the total number of images, but also by the details which have been modified in each spread. We estimate a value of $N = 80$. In figure 1 we show the probability to find the marked item as a function of $\phi$ for different number of iterations $J$. We can see that in the first panel the probability 1 to remember the market item is
never reached $P(\tilde{t}) < 1$: this entails a condition of undercooking, since the number of iterations is not sufficient. The second panel shows a probability near to 1 at $\phi = \pi$, which is consistent by the original Grover’s algorithm; for values different from $\phi = \pi$, we have undercooking. The third panel evidences a central region where $P < 1$ between two peaks of value 1: such region describes the overcooking condition.

Since in experiment 2 of [9] the optimal situation of probability 1 is never reached, this seems to imply that we are in a condition similar to the left panel of figure 1, where $J$ is fixed to 3. Thus the number of iterations is not sufficient to completely remember the items. In left panel of figure 1, we have evidenced the points corresponding to the experimental situation. In particular, we call $W_1$, $R_1$ and $S_1$ the situations of watch, reappraisal and suppression respectively for the low emotion case, while we call $W_2$, $R_2$ and $S_2$ the situations of watch, reappraisal and suppression respectively for the high emotion case. We attempted to evidence such phases on the left panel of figure 1: it is important to note that we have not at the moment a precise mathematical law to calculate them, we only have conditions (5,6). The situation $W_2$ results to have a characteristic phase higher that $\pi$, and thus a sub-optimal recognition condition. In the table below we summarize such data as the corresponding characteristic phases.

| Level of emotion | Watch       | Reappraise   | Suppress   |
|------------------|-------------|--------------|------------|
| Low              | $W_1$ situation | $R_1$ situation | $S_1$ situation |
|                  | $P(\tilde{t}) = 0.43$ | $P(\tilde{t}) = 0.40$ | $P(\tilde{t}) = 0.35$ |
|                  | $\phi \simeq 2.8$ | $\phi \simeq 2.5$ | $\phi \simeq 2.0$ |
| High             | $W_2$ situation | $R_2$ situation | $S_2$ situation |
|                  | $P(\tilde{t}) = 0.37$ | $P(\tilde{t}) = 0.48$ | $P(\tilde{t}) = 0.40$ |
|                  | $\phi \simeq 4.3$ | $\phi \simeq 3.6$ | $\phi \simeq 2.5$ |

We stress that the numerical values of the phase, as well as the number of items $N$ are only reasonable values, based on the experimental set and on the results.

4. Conclusions

We have introduced an early simple quantum model of human memory based on the generalized Grover’s algorithm. The output of the model is a probability to find the searched object, called the "marked item". Es evidenced by Figure 1, such probability to find the marked item depends on the following parameters: the number of items $N$, the time to remember (function of the number of iterations $J$) and the characteristic phase $\phi$. From another point of view, $\phi$ can be considered a parameter describing, for fixed $N$ and $J$, the personal emotivity involved in the process. Thus each subject could in principle use a different phase. However, in our model we hypothesized for simplicity that all the subjects in the same strategy and emotion condition are described by the same characteristic phase of the search engine.

The use of Grover’s algorithm to describe memory processes has some interesting features: 1) it involves a clear and well-known mathematical formalism, 2) it
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hypothesizes the unconscious knowledge of the marked item, 3) has an iterative nature and 4) the efficiency of the memory process depends on a simple parameter, the phase, which seems to admit a simple interpretation in terms of emotivity involved by the items and or by the emotion regulation strategy.

However, we can evidence the following open problems: a) the relation between the time of the memory task and the number of iterations \(J\) is not known at the moment; b) a precise mathematical relation between the phases \(\phi\) and the emotion regulation strategies is not known, but only conditions (5,6); c) it is difficult to estimate the total number of items \(N\) in non-verbal memory tasks.

We conclude by noting that at the moment the use of Grover’s algorithm is only an attempt, and the study of several memory experiments will allow us to decide whether or not the Grover’s algorithm is a good model for human memory.

APPENDIX: Mathematical description of Grover’s algorithm

Given a database with \(N\) items, we build a complex Hilbert space \(H\) with dimension \(N\); the items are labelled with the integer \(i = 1,\ldots,N\), and each item \(i\) of the database corresponds to the vector \(|i\rangle\). The set of vectors \(|i\rangle\) defines an orthonormal basis of the Hilbert space \(H\), which is called the computational basis. The algorithm requires as the initial state a superposition of the vectors \(|i\rangle\) with equal weights \(1/\sqrt{N}\), consistently with equation (1):

\[
|s\rangle = \sum_{i=1}^{N} \frac{1}{\sqrt{N}} |i\rangle.
\]  
(7)

Moreover, let there be a unique item \(\overline{i}\), that satisfies the condition \(C(\overline{i}) = 1\), whereas for all other items \(C(i) = 0\) (assume that for any state \(i\), the condition \(C(i)\) can be evaluated in unit time). The goal of the quantum algorithm is to identify the item for which \(C(\overline{i}) = 1\), which is performed by repeating \(O(\sqrt{N})\) times the following operation

\[
-WI_0WI_\overline{s},
\]  
(8)

where (a) \(I_\overline{s} = I - 2|\overline{i}\rangle\langle\overline{i}|\) is the selective phase inversion (a selective phase rotation of \(\pi\) amplitude), which simply changes the sign to the component \(\overline{i}\) and leaves unchanged all the other components otherwise (where \(I\) is the identity matrix), (b) \(W\) is the Walsh-Hadamard transform defined as \(W_{i,j} = (-1)^{i\cdot j}\) where \(i, j \in [0,N]\) and \(\cdot\) is the bitwise dot product, (c) \(I_0 = I - 2|0\rangle\langle0|\) is a selective phase inversion (selective phase rotation of \(\pi\) amplitude), which simply changes the sign to the vector \(|0\rangle\) of the computational basis and leaves unchanged all the other components \(i \neq 0\).

The Grover’s algorithm is often described as a first selective inversion \(I_\overline{s}\), followed by the operation \(-WI_0W\), called the diffusion transform, which admits the simple interpretation of inversion about the average:

\[
D = -I + 2P, \quad P_{ij} = \frac{1}{N}
\]  
(9)
We now briefly show how the amplitudes change after $J$ iterations of operation (14): first we note that the initial superposition can be written as

$$|s\rangle = \sin(\beta)|\bar{i}\rangle + \cos(\beta)|c\rangle,$$

where $|c\rangle = (N-1)^{-1/2} \sum_{i \neq \bar{i}} |i\rangle$ and $\beta = \arcsin(1/\sqrt{N})$. In the Hilbert space spanned by $|\bar{i}\rangle$ and $|c\rangle$, we can write the Grover’s operator (14) in the matrix form

$$
\begin{bmatrix}
\cos(2\beta) & \sin(2\beta) \\
-\sin(2\beta) & \cos(2\beta)
\end{bmatrix}
$$

(11)

After $J$ iterations, the amplitude relevant to vector $|\bar{i}\rangle$ is $\sin[(2J + 1)\beta]$. The maximal probability to obtain $\bar{i}$ is thus obtained when $J = \pi/4\beta - 1/2$, which is almost equal to $\pi\sqrt{N}/4$ for large $N$.

An important feature of the Grover’s algorithm is that a number of iterations lower or higher than the optimal number leads to a lower probability to find the marked item. In the case of lower iterations, we have the undercooking, while in the higher case the overcooking.

A simple example: $N=4$

We consider the following vector $1/2(1, 1, 1, 1)$ in the standard basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$: it represents the initial state on which the algorithm acts. Let us suppose for example that the state we want to find is $|11\rangle$. Thus we apply a conditional phase shift, obtaining the vector $1/2(1, 1, 1, -1)$, which can be written as

$$\frac{1}{2}[(|0\rangle(|0\rangle + |1\rangle) + |1\rangle(|0\rangle - |1\rangle)]$$

and evidently represents an entangled state of two qubits (since the conditional phase shift is a nonlocal operation).

Now we apply the diffusion operator of equation $D$, which is the inversion about the average: since the average of the previous state is $1/4$, the inversion about the average leads to 0 for all $i \neq \bar{i}$, and 1 otherwise. Thus we obtain the state $|11\rangle$ after only one iteration, which is a peculiarity of the $N = 4$ case.

Generalized Grover’s algorithm

A generalized version of Grover’s algorithm, introduced by Long et al. [376], replaces the inversion of the marked $|\bar{i}\rangle$ state by an arbitrary phase rotation $\theta$ and the inversion of the state $|0\rangle$ by an arbitrary phase rotation $\phi$. The operator used within formula becomes now $-WR_\theta W R_\phi$, where we used the selective rotations

$$R_\bar{i} = I - (1 - e^{i\theta})|\bar{i}\rangle\langle\bar{i}|$$
$$R_0 = I - (1 - e^{i\phi})|0\rangle\langle0|$$

(12)

(13)

It has been shown that in such case different values of $\phi$ and $\theta$ leads to destroy the quantum search. On the contrary, the phase matching condition $\phi = \theta$ leads to an
efficient quantum algorithm. In fact, the application $J$ times of the generalized Grover’s operator $-WR_0WR_1$ can be written as the matrix

$$
\begin{bmatrix}
\cos[2J\sin(\theta/2)\beta] & \sin[2J\sin(\theta/2)\beta] \\
-sin[2J\sin(\theta/2)\beta] & \cos[2J\sin(\theta/2)\beta]
\end{bmatrix}.
$$

(14)

Thus the amplitude relevant to the marked item after $J$ iterations is $\sin[(2J + 1)\sin(\theta/2)\beta]$, which for $\theta = \pi$ is identical to the standard Grover’s algorithm. This means that values of $\theta$ strictly lower or higher than $\pi$ entail a lower probability to find the marked item at $J$ fixed, or a higher computational time.

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