Monte Carlo event generator for black hole production and decay in proton-proton collisions
– QBH version 1.03

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Abstract
We describe the Monte Carlo event generator for black hole production and decay in proton-proton collisions – QBH version 1.03. The generator implements a model for quantum black hole production and decay based on the conservation of local gauge symmetries and democratic decays. The code is written entirely in C++ and interfaces to the PYTHIA 8 Monte Carlo code for fragmentation and decays.

Keywords: Extra dimensions, Black holes, Hardon colliders, Monte Carlo methods

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1. Description of Generator

Black holes are produced in proton-proton collisions and decay to two-particle final states [1]. Electric charge, QCD colour, and spin are conserved, while baryon and lepton number need not be conserved. PYTHIA 8 is used for the initial- and final-state radiation, multiple interactions, final state parton showers, fragmentation, and decay processes [2, 3]. The QBH generator interfaces to PYTHIA 8 using the Les Houches accord (LHA) [4].

The first black hole generators were TRUENOIR written by Dimopoulos and Landsberg [5], and an unnamed generator used in the first black hole study specific to ATLAS by Tanaka et al. [6]. These generators were made quick and efficient for specific studies by averaging over some dynamical quantities, rather than generating them according to probabilistic distributions. The first two true Monte Carlo event generators for black hole production and decay were CHARYBDIS [7] and CATFISH [8]. A comparison between CHARYBDIS and CATFISH can be found in Ref. [9].

The most recent generators CATFISH [8], Charybdis 2 [10], and BlackMax [11] all include the facility to generate black holes decaying to two-particle final states. It is possible to generate two-particle final states with CATFISH but there are some caveats. One sets the final N-body decay to two particles and raises the black hole mass threshold at which Hawking evaporation ends $Q_{\text{min}}$ to be above the minimum black hole mass and Planck scale.

\footnote{http://hep.brown.edu/users/Greg/TrueNoir/}
If $Q_{\text{min}}$ is set too low, one will occasionally produce black holes above $Q_{\text{min}}$ resulting in more than two particles in the final state. If $Q_{\text{min}}$ is set too high, one will occasionally produce black holes with high mass that only decay to two-particle final states rather than the anticipated Hawking evaporation. BlackMax implements the suggestions of Meade and Randall verbatim [12]. The decay species are chosen according to thermal greybody-modified distributions, which is perhaps not applicable to black holes near the Planck scale [1]. Charybdis 2 allows the user to set parameters so that effectively a black hole decaying to two-particle final states is generated. CATFISH and Charybdis 2 do not take colour explicitly into account when determining the two-particle final state. In all three cases, the generators do not allow for the ability to generate a particular black hole state, which is a representation of $SU(2)$ colour, charge, and spin. They thus act as inclusive black hole generators and sum over all possible partons giving rise to black holes. In addition, these generators do not allow the ability to easily generate jet-lepton or jet-boson topologies. Although baryon number can be violation by these generators when run in standalone mode, the partons in the output record can not be hadronized by programs that are typically used to provide the hardonization, fragmentation, and decay process. QBH can be used to select the specific black hole state to generate and includes all possible decay modes properly weighted by conserving $SU(3)_c$ and $U(1)_{em}$. In the following, we refer to these different black holes states as quantum black holes, with the understanding that we are considering a model which is an extrapolation from the classical regime [1]. All the QBH code, including PYTHIA 8, is written in C++.

2. How to use the Generator

The generator consists of four C++ classes, the main class QuantumBlackHole, a derived LHdUp user class dLHdUp, and two utility classes Decay and Random. The user normally only needs to interface to the QuantumBlackHole class, and only for initialization.

The package is typically used as two files: the header file qbh.h containing class definitions and the file qbh.cc containing the bodies of the member functions.

The user must supply his own main program to initialize the package and generate events. The procedure is very similar to writing a standalone
PYTHIA 8 main program. Above the main program, the following lines must appear in this order.

```cpp
#include "Pythia.h"
using namespace Pythia8;
#include "qbh.h"
using namespace QBH;
```

Near the top of the main program, the three objects `QuantumBlackHole`, `dLHAup`, and `Pythia` need to be allocated. The following is an example of creating them dynamically.

```cpp
Pythia* pythia = new Pythia();
dLHAup* lhaPtr = new dLHAup();
QuantumBlackHole* qbh = new QuantumBlackHole(pythia,true);
```

The second argument to the `QuantumBlackHole` constructor needs to be `true` for initialization. After initialization, the `QuantumBlackHole` object may be deleted.

The user is free to further initialize any QBH or Pythia variables before the final initialization of the generator. The final initialization of the generator is made by passing a pointer to the derived class `dLHAup` to PYTHIA as follows.

```cpp
bool status = pythia->init(lhaPtr);
```

After this, the main program looks like any other PYTHIA main program. The user is free to generate events and study these events using all the machinery of the PYTHIA classes.

Les Houches accord event files may be written using the derived class `dLHAup` member functions `openLHEF`, `initLHEF`, `eventLHEF`, and `closeLHEF`.

2.1. User Initialization

The QBH package initializes automatically with a sensible set of parameters. But typically the user will want to change a few parameters to set up the run of interest. The parameters are available as static global variables of the `QuantumBlackHole` class. The first time the `QuantumBlackHole` constructor is call with a boolean argument that is true, a more detailed initialization
occurs. Any further instantiations of the class, with or without arguments, does no further initialization. However, the user is free to initialize any of the parameters using a collection of “set” methods.

The constructor sets the lower bound on the quantum black hole mass to the Planck scale and the upper bound on the mass to three times the Planck scale or the proton-proton centre of mass energy, whichever is less. The constructor also initializes the LHAPDF parton distribution functions, which are the default for QBH, and calculates the branching ratios. Lower bounds on the cross section using the calculations of Yoshino and Rychkov [14] may be used. In this case, the constructor reads in the appropriate data file.

Normally the user uses the “set” methods to set the quantum black hole state, Planck scale, number of dimensions, etc. For example, the quantum black hole state is set using the member functions setQstate, setIstate, and the Planck scale is set using setMplanck. Table I shows a complete set of “set” methods available to the user.

The method setMaxmass, along with setMinmass, allow the user to set a range of black hole masses. This could be useful for restricting the mass range for quantum black holes and thus cutting out semiclassical black hole, sampling the differential cross section by using narrow mass bins, or for user debugging purposes. If the user sets the maximum black hole mass above the centre of mass energy, set by method setEcm, the program at initialization will reset the maximum black hole mass to be the centre of mass energy.

There are a corresponding set of methods which return the values of the parameters: ecm(), mplanck(), minmass(), maxmass(), qstate(), istate(), totdim(), planckdef(), lhaglue(), qscale(), yrform(), trap(), poisson(), RS1(), sm(), chiral(), majorana(), higgs(), and graviton().

It is necessary to define some of the parameter here so their meaning is clear to the user. A detailed description of the model can be found in Ref. [1]. The mass scale used by the Randall-Sundrum black hole (setPlanckdef(1)) is defined as

\[ \tilde{M} \equiv \frac{m_1}{x_1 c^{2/3}} \]  

(1)

where \( m_1 \) is the first KK graviton mass, \( x_1 = 3.83 \), and \( c \equiv k/M_{Pl} \), where \( k \) is the ADS curvature and \( M_{Pl} \) is the reduced Planck scale. In ADD, the fundamental Planck scale in the Dimopoulos-Landsberg, PDG, and Giddings-Thomas definitions are, respectively,
Table 1: QuantumBlackHole class public member functions for setting parameters. The default values are shown as (D=...).

| Function            | Default Value | Description                                                                 |
|---------------------|---------------|-----------------------------------------------------------------------------|
| setQscale(bool)     | (D=true)      | Definition of QCD scale for PDFs.                                            |
|                     |               | = false: quantum black hole mass,                                            |
|                     |               | = true: inverse gravitational radius.                                       |
| setYRform(bool)     | (D=false)     | Use Yoshino-Rychkov factors [14].                                           |
| setTrap(bool)       | (D=false)     | Use Yoshino-Rychkov trapped surface calculation [14].                       |
| setRS1(bool)        | (D=false)     | RS or ADD black hole.                                                        |
|                     |               | = false: ADD black hole,                                                    |
|                     |               | = true: Randall-Sundrum type-1 black hole.                                  |
| setSM(bool)         | (D=true)      | Conserve global symmetries.                                                 |
| setMajorana(bool)   | (D=false)     | Neutrinos are Majorana or Dirac.                                            |
|                     |               | = false: Dirac,                                                             |
|                     |               | = true: Majorana.                                                          |
| setChiral(bool)     | (D=true)      | Handedness of neutrinos.                                                    |
|                     |               | = false: neutrinos are left-handed only,                                    |
|                     |               | = true: neutrinos left-handed and right-handed.                            |
| setHiggs(bool)      | (D=true)      | Include Higgs as allowed particle.                                          |
| setGraviton(bool)   | (D=true)      | Include graviton as allowed particle.                                       |
| setTotdim(int)      | (D=10)       | Total number of spacetime dimensions.                                       |
|                     |               | = 5 \text{–} 11 allowed.                                                   |
| setPlanckdef(int)   | (D=3)         | Definition of Planck scale [15].                                            |
|                     |               | = 1: Randall-Sundrum,                                                      |
|                     |               | = 2: Dimopoulos-Landsberg definition,                                       |
|                     |               | = 3: PDG definition,                                                       |
|                     |               | = else: Giddings-Thomas definition.                                         |
| setLHAglue(int)     | (D=10042)    | LHA glue code [16],                                                        |
|                     |               | = 10042: LHA cteq6ll.LHpdf,                                                |
|                     |               | = 19050: LHA cteq5m.LHgrid,                                                |
|                     |               | = 29041: LHA MRST98l.LHgrid,                                               |
|                     |               | = else: CTEQ5L (internal PYTHIA 8).                                         |
| setQstate(int)      | (D=9)         | Three times electric charge.                                                |
|                     |               | = 4, 3, 2, 1, 0, −1, −2, −3, −4 allowed,                                   |
|                     |               | = 9 used for all partons.                                                  |
| setIstate(int)      | (D=3)         | Initial state. = 0: q\bar{q}, = 1: q\bar{g}, = 2: g\bar{g}, = 3: all.     |
| setMplanck(double)  | (D=1000.0 GeV)| Fundamental Planck scale.                                                  |
| setMinmass(double)  | (D=1000.0 GeV)| Minimum quantum black hole mass.                                           |
| setMaxmass(double)  | (D=14000.0 GeV)| Maximum quantum black hole mass.                                           |
| setEcm(double)      | (D=14000.0 GeV)| Proton-proton centre of mass energy.                                       |
\[ M_{DL}^{D-2} \equiv \frac{(2\pi)^{D-4}}{8\pi G_D}, \quad M_{D}^{D-2} \equiv \frac{1}{G_D}, \quad M_{GT}^{D-2} \equiv \frac{(2\pi)^{D-4}}{4\pi G_D}, \] (2)

where \( G_D \) is the \( D \)-dimensional Newton constant in the Myers-Perry metric.

2.2. User Process Run Information

The PYTHIA initialization call \texttt{pythia->init(lhaPtr)} calls the virtual function \texttt{setInit()} of the derived class \texttt{dLHAup} to initialize the physics process in the \texttt{QuantumBlackHole} class. Virtual function \texttt{setInit()} calls \texttt{QuantumBlackHole::init()} to calculate the total proton-proton cross section, and then \texttt{QuantumBlackHole::banner(lhaPrt)} to write a banner to the output. The banner lists all the parameters, the beam, and quantum black hole configuration, as well as, the cross section and estimated statistical error. Some simple consistency checks are made, and the user may be warned or asked to supply different parameters.

The internal processing of the run information is described next. Although the user normally does not touch this part of the class, it is informative to know how the Les Houches Accord is implemented. The generator assumes both beams are protons to use the symmetry of the collision to gain some efficiency. The LHA weighting strategy is declared by setting the master switch \texttt{IDWTUP} to +1. The generator supplies positive weighted events, and PYTHIA returns events with weight 1 by using an accept or reject algorithm. Only one physics process is declared in the interface to PYTHIA. The selection of which quantum black hole to generate is determined by \texttt{QBH} according to the user settings. The user may select any of the 14 quantum black holes states for generation, or a mixture of all the states weighted by their relative cross section is possible (see Table 2).

2.3. User Process Event Information

The user generates an event by calling \texttt{pythia->next()} which calls the virtual function \texttt{setEvent()} of the derived class \texttt{dLHAup} to generate one event with the \texttt{QuantumBlackHole} class. Virtual function \texttt{setEvent()} calls \texttt{QuantumBlackHole::event()} to generate an event.

The differential cross section is integrated and sampled using importance sampling. The integration and importance sampling is very fast since the cross section is only a function of two variables and transformations are applied to flatten the function that is sampled.
Table 2: \textbf{Qstate} and \textbf{Istate} codes for the 14 quantum black hole states.

\begin{table}[h]
\centering
\begin{tabular}{lcc}
\hline
State & Qstate & Istate \\
\hline
QBH_{uu}^{4/3} & 4 & 0 \\
QBH_{ud}^{1} & 3 & 0 \\
QBH_{ug}^{2/3} & 2 & 1 \\
QBH_{dd}^{2/3} & 2 & 0 \\
QBH_{dg}^{1/3} & 1 & 1 \\
QBH_{ud}^{1/3} & 1 & 0 \\
QBH_{qq}^{0} & 0 & 0 \\
QBH_{gg}^{0} & 0 & 2 \\
QBH_{ud}^{-1/3} & -1 & 0 \\
QBH_{dg}^{-1/3} & -1 & 1 \\
QBH_{dd}^{-2/3} & -2 & 0 \\
QBH_{ug}^{-2/3} & -2 & 1 \\
QBH_{du}^{-1} & -3 & 0 \\
QBH_{ud}^{4/3} & -4 & 0 \\
all & N/A & 3 \\
\hline
\end{tabular}
\end{table}

Sometimes the sampled cross section is vanishingly small. This can occur near the edges of the parameters space or if the trapped surface cross section is being calculated. In these cases, the partons are both set to up-quarks and \texttt{setEvent()} still returns \texttt{true}. If \texttt{false} was returned, the cross section calculated by PYTHIA would not be the same as that calculated by QBH.

3. QuantumBlackHole Class Member Functions

Figure 1 shows the program flow through the QuantumBlackHole member functions. Each public member function is described next.

\begin{verbatim}
bool dLHAup::setInit(void)
\end{verbatim}
Figure 1: Program flow through the member functions. `xpart`, `pdf`, `xsec`, and `sample` are private member functions.

**Purpose:** Les Houches initialization virtual function.

**Objects declared** QuantumBlackHole.

**Functions called:** `LHAup::setBeamA`, `setBeamB`, `setStrategy`, `addProcess`, `QuantumBlackHole::init`, `banner`.

**Return:** always true.

`setInit` sets the beams characteristics, declares the generation strategy, and defines a single physics process. It also calculates the total proton-proton cross section and prints a program banner on the standard output.

```cpp
bool dLHAup::setEvent(int idProcess)
```
Purpose: Les Houches event virtual function.

Objects declared: QuantumBlackHole.

Functions called: LHAup::setProcess, setPdf, addParticle,
QuantumBlackHole::event, decprd, flow, deckin.

Return: always true.

**setEvent** sets the weight for each event, sets some PDF information, and adds the beam and generated hard scattering particles to the event record. It also adds the quantum black hole state to the event record for documentation and process identification purposes. It generates the event by calculating a weight for the event and determines random $Q$, $x$ and beam particle IDs. It determines the quantum black hole decay products, connects the colour flow, and determines the mothers of the partons. The kinematics of the decay are calculated.

QuantumBlackHole:QuantumBlackHole(Pythia*,bool initialize)

Purpose: QuantumBlackHole constructor.

Functions called: Many set methods.

Return: All user parameters, initializes PDFs, and calculates branching ratios.

The constructor sets information about the quantum black hole and extra dimensions. These parameters should be fixed for the entire execution of the program.

void QuantumBlackHole::init(double& dxsec)

Purpose: Quantum black differential cross section calculation.

Functions called: QuantumBlackHole::xpart, pdf, xsec.

Return: proton cross section (pb) at random $x$ values.

**init** calculates the parton differential cross section at a random $x_{\text{min}}$. It then determines two random $x$ values and stores the PDFs. It calculates the proton-proton differential cross section at these $x$ values.

void QuantumBlackHole::banner(dLHAup* inf)
Purpose: Output initialization information.
Functions called: none.
Return: none.

**banner** outputs initialization information in a banner.

```cpp
bool QuantumBlackHole::event(double& wxsec, double& Q, double* xx, int* idn)
```

Purpose: Quantum black hole event generation.
Functions called: QuantumBlackHole::xpart, pdf, xsec, sample.
Return: always true.

**event** calculates the parton cross section at a random $x_{\text{min}}$ value and returns the resulting differential cross section and Q value. It then determines two $x$ values and uses these to store the PDFs. The proton-proton cross section is sampled to determine an event weight and the two partons that caused the process are returned.

```cpp
void QuantumBlackHole::decprd(int *idn)
```

Purpose: Determine decay products identifiers.
Functions called: ParticleDataTable::chargeType.
Return: PDG ID codes of decay particles.

**decprd** determines the decay products based on predetermined branching ratios.

```cpp
void QuantumBlackHole::flow(int *idn, int moth[][2], int icol[][2])
```

Purpose: Determine the mothers and colour connections.
Functions called: none.
Return: Mother and colour connections.

**flow** sets the mothers and colour connections for each final-state particle. In most cases, the colour flow is straightforward. Figure 2 shows the set of colour flow diagrams for most of the quantum black hole decay processes.

Figure 3 shows the two diagrams in which the colour flow can not be implemented naturally, and we have used a non-standard procedure. The technique is as follows. We replace one of the incoming quarks or antiquarks
Figure 2: Colour flow diagrams for most the quantum black hole decays. The symbol $X$ represents a colourless particle.

by an antiquark or quark, respectively, of the same flavour. The flavour is changed just before the particle is added to the event record. Thus the particle kinematics and weight of the event are unaffected. Next, an additional outgoing quark or antiquark is added so that electric charge is conserved. The colour connections can then be made between the coloured particles. In this way we can conserve electric charge and allow colour to flow. The additional outgoing particle belongs to the first quark generation and is given zero momentum-energy. The extra zero momentum-energy particle adds little to the event. Typically a colour string will form between it and a beam-remnant diquark. String fragmentation will give a few extra baryons and mesons.
Because of the vanishing momentum-energy, the diquark will dominate the kinematics and typically the extra particles will travel down the beam pipe undetected. Such an approach has been used in previous generators [17, 18].

\[ q \to X \bar{q} \]

Figure 3: Diagrams for which a nonstandard colour flow technique has been used. The symbol $X$ represents a colourless particle.

PYTHIA 8 provides a junction class to deal with baryon violating decays. Currently it handles only the case of one initial state particle. The facility is not readily available to the users via the Les Houches Accord interface.

```cpp
void QuantumBlackHole::deckin(int *idn,dLHAup* lha,double *p1,double *p2)

Purpose: Calculate kinematics of decay.

Functions called: Decay::twobody.

Return: four-vectors of black hole and final-state particles.

deckin calculates kinematics of the decay.

The following four member functions are private.

```cpp
void QuantumBlackHole::xpart(double& xmin,double& Q,double& sighat)

Purpose: Calculate parton-parton cross section (pb).

Functions called: none.

Return: xmin, Q, sighat.

xpart generates a random $x_{\text{min}}$ and uses this to calculate the parton-parton differential cross section. It also returns $Q$ which is only available after the gravitational radius and mass are calculated.

```cpp
void QuantumBlackHole::pdf(const double* xx,const double Q)

Purpose: Get parton distributions via PYTHIA PDF class.

Functions called: PDF::xf.

Return: fills disf[2][13] array.
```
pdf gets the parton distribution functions via the PYTHIA PDF class and stores them as a private data member of the class. The PDFs are available to this function via a stored private pointer in the class.

```cpp
void QuantumBlackHole::xsec(const double sighat, double& dxsec)
    Purpose: Calculate the proton-proton cross section (pb).
    Functions called: disf[2][13].
    Return: Differential cross section in pb
```

xsec uses the parton cross section and an array of PDFs to calculate the proton-proton cross section.

```cpp
void QuantumBlackHole::sample(const double sighat, const double dxsec, int* idn)
    Purpose: Sample cross section to select beam partons.
    Functions called: disf[2][13].
    Return: weight, beam parton IDs.
```

sample samples the cross section to determine the beam partons and returns a weight.

### 3.1. Utility Classes Random and Decay

The totally static Random class takes care of all the random distributions and random selection utilities, such as selecting a random charged lepton, for example. The class is driven by the uniform random number generator used by PYTHIA. Thus PYTHIA may be used to initialize and manage the seeds of the Random class.

The Decay class handles the two-particle phase space decay. It has one public member to perform the decays. A set of private member functions perform all the required Lorentz transformations.

### 4. Installation and Availability

The QBH generator is written entirely in C++. It interfaces only to PYTHIA 8. If a lower-bound on the cross section is being calculated by using the trapped surface option, then the appropriate file is read in. These small text files must exist in the same directory as the executable image. If the LHAPDF parton densities are required, the LHAPDF library must be linked.
Figure 4: Total cross section versus number of loop iterations used in the cross section calculation.

The generator has been tested with PYTHIA version 8.150 and LHAPDF version 5.8.0. Version 1.02 and earlier of the generator should be used with version 8.130 of PYTHIA. Both Linux and Darwin-gcc4 architectures with the slc4_ia32_gcc34 platform have been tested.

The source code can be obtained from the HepForge site http://projects.hepforge.org/qbh/.

5. Performance

The performance of the event generator has been measured. Figure 4 shows the cross section versus number of loop iterations used to calculate the cross section for three different runs at each value of the number loop iterations. The spread in the three cross section values at each number of loop iterations is consistent with the cross section statistical error bars. The cross section calculation is stable and converges nicely. For a statistical accuracy of 0.2%, $10^5$ iterations are required, while for an accuracy of 0.07%, $10^6$ iterations are required. The totally inelastic cross sections have been used for the performance studies. One might expect the trapped surface cross sections to take a little longer.
As a further check of the cross section calculation, we compared the cross section calculated by QBH using $10^5$ loop iterations with that calculated by PYTHIA for $10^5$ generated events. The percentage difference was less than 0.5%.

Figure 5 shows the processor time versus number of loop iterations in the cross section calculation. To obtain a 0.2% accuracy requires 2 s, while for a 0.07% accuracy 14 s is needed. However the processor used was arbitrary so the time axis should be viewed as a relative time. By default, the generator used $10^5$ loop iterations to calculate the cross section.

Figure 6 shows the processor time versus number of generated events. To decouple the generator from PYTHIA 8, the “hard process” times have fragmentation, hadronization, particle decays, initial- and final-state radiation, and multiple interactions switched off. About $6 \times 10^3$ QBH events can be generated per second. If fragmentation, hadronization, and decays are turned on, about $1 \times 10^3$ events can be generated per second. Turning on initial- and final-state radiation allows 30 events per second to be generated. Finally,
Figure 6: Processor time versus number of events generated. The timing resolutions is 1 s. The processing tasks are cumulative. For example, multiple interactions also includes all the other processing.

turning on multiple interactions (everything) allows 8 events per second to be generated.

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