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Multi-pheromone ant colony optimization for socio-cognitive simulation purposes

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Abstract
We present an application of Ant Colony Optimisation (ACO) to simulate socio-cognitive features of a population. We incorporated perspective taking ability to generate three different proportions of ant colonies: Control Sample, High Altercentricity Sample, and Low Altercentricity Sample. We simulated their performances on the Travelling Salesman Problem and compared them with the classic ACO. Results show that all three 'cognitively enabled' ant colonies require less time than the classic ACO. Also, though the best solution is found by the classic ACO, the Control Sample finds almost as good a solution but much faster. This study is offered as an example to illustrate an easy way of defining inter-individual interactions based on stigmergic features of the environment.

Keywords: Ant Colony Optimization, Metaheuristics, Psychology, Cognition

1 Introduction

Empathy is considered to be the warp across which the fabric of human society is woven [11]. One mechanism underlying empathy is being able to view a situation from another agents perspective, which is termed as perspective taking. If we can model how perspective taking influences individual agents’ behaviours in a society, and how macro-level social phenomena emerge from their interaction, it would help us understand why some societies seem harmonious whereas others are conflict ridden. It can further help us devise strategies to reduce conflicts. These models may also provide us in developing new optimisation strategies for traditional computational problems, based on the assumption, that diversification of computational agents and their behaviour, can lead to better exploration of the search space (similar
to such solutions as multi-deme and coevolutionary algorithms). At the same time, we aim to experiment with different computational models of perspective taking to study what sort of macro-level behaviours emerge from them, and then seek to verify these behaviours through psychological experiments in order to validate the models.

In recent years there has been an increasing synergetic interaction between biological and cognitive systems on one hand and computational systems on the other. In one direction, for instance, ant colonies, bird flocks, bee swarms, hippocampus of a rat, and so on, have inspired innovative computational algorithms. In the other direction, computational techniques have been applied to understand and model how macro phenomena emerge through micro-level interactions of individuals in a large group. The research being carried out by our group straddles both these aspects.

Optimization heuristics, particularly biologically-inspired techniques, have been gaining attention for over 20 years. These approaches are supposed to be universal, though critics point out high computation time and complexity of the algorithms. However, difficult problems posing challenges to deterministic approaches call for alternative strategies of solving [18], and these may justify the above-mentioned costs.

Ant systems have proven to be a popular tool for solving many discrete optimization problems, e.g. TSP (Traveling Salesman Problem), QAD (Quadratic Assignment Problem), VRP (Vehicle Routing Problem), GCP (Graph Coloring Problem) and others [9]. In this paper, we consider the ant system as a way to express socio-cognitive behaviours of a population of ants, differentiating them into species and defining their stigmergic interactions. Our main goal is to simulate perspective taking during decision making, with the decision-making moment in the world of ants being connected to choosing the subsequent path while standing in one of the graph nodes. Thus, the ant system becomes a framework for analysing socio-cognitive interactions by introducing different types of pheromones left by different species, and defining particular behaviours of ants based on their detection of the concentrations of these different types of pheromone on the paths to be selected from. It is to note that besides ACO, other stigmergic or quasi-stigmergic systems as Particle Swarm Optimization can be considered as a starting point for simulations, and the objects of enhancement using proposed socio-cognitive features.

The rest of the paper is organized as follows. First ACO and its selected variants are shortly discussed, then socio-cognitive aspects relevant for simulations are shown. Then their incorporation into ACO system is presented, experimental results are discussed and the paper is concluded.

2 Ant Colony Optimization: Classical and novel approaches

Ant System, introduced in 1991, applied to solve TSP, is considered to be a progenitor of all ant colony optimization (ACO) algorithms [7]. Because the action of a certain ant during one iteration is completely independent of the actions of other ants during any iteration, the sequential ant algorithm can be easily parallelized.

The ACO algorithm is an iterative process during which certain number of ants (agents) gradually create a solution [8, 9]. The problem being solved is usually depicted as a graph, and the main goal of ants is to cross this graph somehow in an optimal way. Each move of an ant consists in choosing a subsequent component of the solution (graph edge) with certain probability. This decision may be affected by the interaction among the ants based on the
existence of pheromones (according to stigmergy—i.e. communication using the environmental properties for mediation among individuals instead of direct contact—rules proposed in [7]) that may be deposited into the environment (on the edges of the graph) and perceived by the agents. The iteration process is finished when a feasible solution is created by all ants.

However, recently new interesting modifications of ACO-related techniques have been introduced, namely multi-type ACO [13, 17], allowing many species of ants and defining their stigmergic interactions (e.g. based on attraction to the pheromone of the same species and repulsion from the other pheromones). Such algorithms were successfully applied to such problems as edge disjoint path problem [13] or light path protection [17].

There are other modifications of the classic ACO, such as hierarchical ACO, where additional means of control are introduced to manage the output of particular ants (or ant species) [14]. In another approach, ants are endowed with different skills (e.g. sight, speed) in order to realize global path planning for a mobile robot [12]. In a successful approach to solve TSP, the authors propose to use two types of ants: classic and exploratory (creating “short routes”, moving according to some predefined conditions, e.g. near some selected cities, etc.) [10].

In [5], the authors introduce different ant sensitivity to pheromones such that ants with higher sensitivity follow stronger pheromone trails, while ants with lower sensitivity behave more randomly. This model strives to sustain a balance between exploration and exploitation.

Taking inspiration from these approaches, especially the ones proposed by Nowé et al. [13] (many species of ants with detailed stigmergic interactions) and by Chira et al. (different sensitivity of the ants to the pheromones) [5] we propose a novel method of simulation and analysis of socio-cognitive properties of individuals of a certain population. The population of ants (and its application to TSP) is presented only as an example to illustrate an easy way of defining inter-individual interactions based on stigmergic features of the environment.

3 Incorporating social and cognitive aspects

Two factors have contributed most to a rapid growth in this field of social and cognitive simulations in recent years. One is the exponential increase in the computing power, which makes it possible to simulate very large-scale multi-agent systems in reasonable time. The other is the availability of a huge amount of quantitative data that traces the activities of individuals and their interaction patterns in a society [6, 4]. For example, two recent active areas of research are how norms [1] and fairness emerge in a society [15, 16].

We focus here on perspective taking, which refers to the ability of an agent to take another agent’s point of view. Typically, this is taken to be a one-dimensional ability: the degree to which an agent can take another one’s perspective. But some recent research has explored a two-dimensional approach [2], where one distinguishes between the ability of an agent to handle conflict between its own and the other agent’s perspectives, and the relative priority given to these two perspectives. Experimental research has suggested that these two dimensions might be independent; and factors such as guilt or shame affect each of these dimensions individually.

Let us consider the following types of the individuals and their possible interactions:

- Egocentric individuals: they focus on their own perspective and can become creative by finding their own new solutions to a given task. They do not get inspired by others’ actions (or the inspirations do not become a main factor of their work).

- Altercentric individuals: they focus on the perspective of others and thus follow the mass of others. They are less creative but do end up supporting good solutions by simply following them.
• Good-at-conflict-handling individuals: they get inspired in a complex way by the actions
of other individuals by considering the different perspectives and choosing the best.

• Bad-at-conflict-handling individuals: they act purely randomly, following sometimes one
perspective, sometimes another without any inner logic.

The characterisation of the individuals within a population was based on realistic propor-
tions of perspective taking profiles in humans. In recent work, it has been shown that the
proportion of altercentric, egocentric, good and bad perspective conflict handlers can fluctuate
depending on situational factors. We choose three types of proportions found in humans: one
representing the proportion of perspective-taking profiles in a baseline condition (without ma-
nipulation of situational factors) and two types of proportion corresponding the effects of two
situational factors with conflicting effects on perspective taking, i.e. the effects of guilt and
anger, which heightens and lowers, respectively, the proportion of altercentric individuals [3]:

• Control Sample (baseline proportions of different types of perspective takers found in a
human sample): here the good conflict handlers are the major elements with a roughly
similar proportion of the three other types of perspective takers. Note that this is also
the sample with the highest proportion of egocentric individuals.

• Increased Good Conflict Handling Sample (proportions inspired from a sample of humans
induced to feel anger): here the proportion of good conflict handlers is further increased
compared to the control sample at the expenses of the altercentric and egocentric indi-
viduals whose proportion is now significantly decreased compared to both other samples.

• Increased Altercentricity Sample (proportions inspired from a sample of humans induced
to feel guilt): the proportion of good conflict handlers and egocentric individuals is sig-
ificantly decreased and is compensated by a higher proportion of altercentric individuals
and to a lesser extent a higher proportion of bad conflict handlers.

The samples described do not exhaust all interesting possibilities and should be treated as a
starting point for further research.

4 Socio-cognitive ant colony optimization

Starting from the definition of classic ACO, one should consider optimization of a combinatorial
problem (e.g. to find a Hamiltonian cycle in a graph as in Travelling Salesman Problem). The
method is based on agents, namely ants, that roam along the edges of the graph, finding the
exemplary cycles and leaving trails of pheromones behind them.

4.1 Classic ACO

In the classical ACO algorithm, the ants are deployed in a graph consisting of vertices $V =
i, j, \ldots; i, j \in \mathbb{N}$ and edges $E$, where each edge is associated with a cost of moving through
it. Each ant gets a randomly chosen starting graph node. Beginning from this node, the ant
constructs its cycle in step-by-step manner, by moving from one node to another, choosing the
next one and not coming back. While considering nodes to choose from, an ant has to compute
attractiveness for all possible paths that can be taken from the present node. The attractiveness
\( n_{ij} \) of the edge \( ij \) starting from a node \( i \) ant is standing on is transformed into probability by a simple normalization:

\[
\mu_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}
\]

(1)

where \( j \) is computed only for the nodes that have not yet been visited by the ant.

Note that the exact values of \( n_{ij} \) being the attractivenesses computed for the next edges constituting the constructed path will be given in details in the next paragraphs.

Finally, the ant randomly selects a path based on previously computed probability – paths with higher attractiveness are more likely to be chosen. After visiting all nodes exactly once, the ant finishes its trip and returns the found cycle as a proposed solution, and then retreats depositing certain amount of pheromone on the path belonging to its current cycle. The amount of pheromone deposited in an edge \( e_{ij} \) is denoted by \( \pi_{ij} \) and the deposition algorithm of ant \( a_k \) retreating along cycle \( c_{ak} \) is as follows:

\[
\pi'_{ij} \leftarrow \pi_{ij} + \frac{\pi_d}{\sum_{e \in c_{ak}} \text{cost}(e)}
\]

(2)

where the default pheromone deposit \( \pi_d \) is 1, and \( x' \) means the \( x \) after assigning the new value, \( e_{ij} \) denotes edge in the cycle and \( \text{cost}(e_{ij}) : E \rightarrow \mathbb{R} \) is a certain function assignign the cost to the edges.

However, the pheromone evaporates in each iteration (in each edge of the graph) according to this formula:

\[
\pi_{ij}' = (1 - \pi_e) \cdot \pi_{ij}
\]

(3)

Default pheromone evaporation coefficient \( \pi_e \) is 0.01.

**Classic ants** consider both pheromone and distance while choosing their directions (by computing path attractiveness) in order to complete the cycle. So an ant standing at node \( i \) will choose the next edge with a uniform probability, proportional to the following estimate:

\[
n_{ij} = \left( \frac{\sum_{e_{ik}, k \in V(i) \setminus \{i\}} \text{cost}(e_{ik})^\alpha}{\text{cost}(e_{ij})^\beta} \right)
\]

(4)

Default factors are, pheromone influence \( \alpha = 2.0 \), distance influence \( \beta = 3.0 \), \( V^{(i)} \) does not include the vertices already visited by the ant when reaching the vertex \( i \).

### 4.2 Multi-pheromone ACO

In socio-cognitive ACO, the idea of multiple pheromones is implemented by introducing different ‘species’ of ants and enabling their interactions (similar to the approach presented in e.g. [13]). The interaction is considered as a partial inspiration, or even perspective taking, realized by a particular ant reacting to the decisions taken by the ants belonging to other species. This is made possible by having the ants of different species leave different ‘smells’ (see Fig. 1). Different ants use different rules (consider different path’s properties) for computing attractiveness; and looking for inspirations or perspective taking, they utilize the smells of pheromones left by other species in a predefined way. Therefore different species may be treated as organisms with selective smelling capabilities (subject to different combinations of the present smells).

Therefore, we divide the set of ants into following sub-species...
\textbf{Multi-pheromone ACO setting}

- \textit{EC} – egocentric ants that are supposed to be creative in finding new solutions,
- \textit{AC} – altercentric ants that simply follow the mass of other agents,
- \textit{GC} – good-at-conflict-handling ants that get inspired by the actions of other ants,
- \textit{BC} – bad-at-conflict-handling ants, that act randomly.

Moreover, different ant species leave pheromones that ‘smell’ different, so the pheromone left at a particular edge is described as a sum of the following components:

\[
\pi_{ij} = \pi_{ij}^{(EC)} + \pi_{ij}^{(AC)} + \pi_{ij}^{(GC)} + \pi_{ij}^{(BC)}
\]  

Therefore other ants may react to different combinations of all the pheromones. Of course, more species (and more pheromones) may be introduced into the system.

Now, based on this framework, details of the actions undertaken by various ant species are formulated below.

\textbf{Egocentric ants} (elements of the \textit{EC} set), are creative in trying to find a new solution and finding their own way. They are less caring for others and for pheromone trail. Thus, they focus mostly on the distance as a way to determine their next directions. So an ant standing at node \(i\) will choose the next edge with an uniform probability, proportionally to the following fraction:

\[
\frac{1}{\text{cost}(e_{ij})^\beta}
\]

Default distance influence \(\beta = 3.0\), again.

\textbf{Altercentric ants} (elements of the \textit{AC} set), follow the mass of other ants (thus they focus on the pheromone, not caring for the distance). So an ant standing at node \(i\) will choose the next edge with a uniform probability, proportionally to the following expression:

\[
\pi_{ij}^\alpha
\]

Default pheromone influence \(\alpha = 2.0\).
**Good-at-conflict-handling ants** (elements of the GC set) will wait and observe the others. Thus, they care for all existing pheromones (the particular weights are to be determined experimentally). So an ant standing at node \( i \) will choose the next edge with a uniform probability, proportionally to the following expression:

\[
\left( 14 \cdot \pi^{(EC)}_{ij} + 2 \cdot \pi^{(AC)}_{ij} + 2.5 \cdot \pi^{(GC)}_{ij} + 0.5 \cdot \pi^{(BC)}_{ij} \right)^{\alpha}
\]

Default pheromone influence \( \alpha = 2.0 \).

**Bad-at-conflict-handling ants** behave impulsively (in effect randomly), irrespective of the pheromone or the distance. So an ant at node \( i \) will choose the next edge with a uniform probability, proportionally to the following expression:

\[
\frac{1}{\sum_{e \in V \setminus \{i\}}}
\]

5 Experimental results

The experimental results were obtained based on a dedicated software developed in Python\(^1\), run on a typical desktop PC. We considered the Travelling Salesman Problem: based on finding a Hamiltonian in a graph defined by a network of cities, the goal is to look for a cycle with minimum cost (distance). The particular instance tackled was taken from TSPLIB library\(^2\).

During the experiments, the following compositions of the simulated populations regarding the involvement of particular ants' species were considered:

- **Classic Ant Population**: only ants acting as in classic ACO.
- **Human-inspired sample populations**:
  - **Control Sample Population**: 22% egocentric, 15% altercentric, 45% good at conflict handling, 18% bad at conflict handling.
  - **Increased Altercentricity Sample Population**: 3% egocentric, 46% altercentric, 23% good at conflict handling, 28% bad at conflict handling.
  - **Increased Good Conflict Handling Sample Population**: 6% egocentric, 6% altercentric, 63% good at conflict handling, 25% bad at conflict handling.

These proportions were inspired by psycho-cognitive features observed in human populations. In the future we plan also to identify optimal set of parameters for solving the given problem (e.g. TSP).

The TSP instance considered was *berlin52* (52 cities, best known solution: 7542). Each simulation was repeated 12 times and average results with standard deviation were presented. All computations were realized for 100 iterations of the algorithm using default input parameters (described in section 4.2).

Fig. 2 shows the average fitness (with standard deviations) obtained in subsequent iterations of the algorithm for different population configurations considered (TSPLIB means the

\(^1\)www.python.org  
\(^2\)http://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/
best known solution of the problem). It is easy to see that though the classic ACO produces the best solution, the Control Sample populations gives almost as good a result and much earlier (even after about 10 iterations). The other interesting observation is the apparent premature convergence of the other populations considered: both Conflict Handling and Altercentric conditioned populations are unable to move from some local extremum found during first several iterations. On some reflection, this is be expected, because in the Control Sample setting, the major forces are good-at-conflict-handling ants and egocentric ants. The former ones take the perspective of the latter ones, while the latter ones are specialized in solving the problem, thereby leading to the observed result.

The results shown in Fig. 2 are summarized in Table 1 for different numbers of ants considered. An interesting feature is that when the number of ants in a population is low (like 20), again the Control Sample population becomes the best. When there are more ants, other populations prevail (mostly classic ACO).

Table 1: The best distances found by different populations, berlin52, 100 iterations

| Population                  | Path distance |
|-----------------------------|---------------|
|                             | 20 ants       | 50 ants       | 100 ants       | Best known |
| Classic Ants                | 8803.57       | 7902.11       | 7628.06        | 7542        |
| Control Sample              | **7749.21**   | 7958.52       | 7805.49        |             |
| Increased Conflict Handling | 9090.96       | 9361.58       | 9390.11        |             |
| Increased Altercentricity   | 9271.01       | 9934.18       | 9901.01        |             |

In Table 2 a summary of the simulation times are given. It is easy to see that all ‘cognitive enabled’ populations require less computing time than classic ACO. The reason for this is the inclusion into these populations, ants that rely on perceiving other ants (e.g. altercentric ants that compute their attractivenes very easily by following other ants, or bad-at-conflict-handling ants that act purely randomly).

Some final observations can be made looking at Fig. 3 that shows the best fitnesses averaged for Control Sample population. Bad-at-conflict-handling ants (acting purely randomly) turn out to be the worst at solving of the problem; the egocentric ants are significantly better; but good-at-conflict-handling ants are similar in their efficiency as the altercentric ones.
Table 2: Summary of computation times or different populations, berlin52, 100 iterations

| Population                   | Best   | Avg   | Stdev | Best   | Avg   | Stdev | Best   | Avg   | Std |
|------------------------------|--------|-------|-------|--------|-------|-------|--------|-------|-----|
| Classic Ants                 | 51.85  | 50.88 | 0.64  | 120.07 | 123.09| 1.03  | 232.64 | 248.73| 2.  |
| Control Sample               | 41.49  | 41.98 | 0.35  | 101.07 | 103.98| 0.61  | 196.00 | 204.69| 1.  |
| Increase Conflict Handling   | 42.50  | 42.91 | 0.42  | 95.03  | 97.18 | 0.44  | 189.22 | 192.00| 1.  |
| Increased Altercentricity    | 39.40  | 39.96 | 0.40  | 89.61  | 93.08 | 0.54  | 178.43 | 184.20| 1.  |

Figure 3: Average fitness for different ant species in Control Sample

6 Conclusion

Difficult problems require novel metaheuristics, so the search for new inspirations continues. Effective methods of computation may be conceived by observing socio-cognitive relations among individuals: this was the inspiration for the research presented here. Surprisingly, our first research goal, namely the simulation of socio-cognitive phenomena in a population of computing ants, turned out to have an interesting side-effect, namely efficient handling of the tackled problem in certain configurations. Our main result, namely prevailing of the Control Sample population, can be considered as somewhat expected from the socio-cognitive point of view, as in this setting the good-at-conflict-handling individuals mostly take the perspective of the egocentric ants (which are quite good in solving the problem as it was perceived when observing the efficiency of the work of different species of ants) to get to the optimal solution. In the future, we plan to expand the experiments by exploring different parameters configurations, keeping in mind how these parameters relate with the real-world socio-cognitive phenomena. We plan to make the constructed environment publicly available, and further extend it to focus on a more detailed simulation of perspective taking among agents.

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References

[1] G. Andrighetto and R. Conte. Cognitive dynamics of norm compliance. from norm adoption to flexible automated conformity. Artificial Intelligence and Law, 20(4):359–381, 2012.

[2] H.B. Bukowski. What Influences Perspective Taking. PhD thesis, Catholic University of Louvain, 2014.

[3] H.B. Bukowski and D. Samson. Can emotions affect level 1 visual perspective taking?

[4] L.-E. Cederman, R. Conte, D. Helbing, A. Nowak, F. Schweitzer, and A. Vespignani. Exploratory of society. The European Physical Journal Special Topics, 214(1):347–360, 2012.

[5] C. Chira, D. Dumitrescu, and C.M. Pintea. Heterogeneous sensitive ant model for combinatorial optimization. In Proceedings of the 10th Annual Conference on Genetic and Evolutionary Computation, GECCO ’08, pages 163–164, New York, NY, USA, 2008. ACM.

[6] R. Conte, N. Gilbert, G. Bonelli, C. Cioffi-Revilla, G. Deffuant, J. Kertesz, V. Loreto, S. Moat, J.-P. Nadal, A. Sanchez, A. Nowak, A. Flache, M. San Miguel, and D. Helbing. Manifesto of computational social science. The European Physical Journal Special Topics, 214(1):325–346, 2012.

[7] M. Dorigo and T. Stützle. Ant Colony Optimization. Bradford Books, 2004.

[8] G. Di Caro Dorigo M. The ant colony optimization meta-heuristic. In D. Corne, M. Dorigo, F. Glover, editors, New Ideas in Optimization, McGraw-Hill, 11-32., 1999.

[9] G. Di Caro & L. M. Gambardella Dorigo M. Ant algorithms for discrete optimization. Technical report, IRIDIA/98-10, Universit Libre de Bruxelles, Belgium, 1999.

[10] A. Hara, S. Matsushima, T. Ichimura, and T. Takahama. Ant colony optimization using exploratory ants for constructing partial solutions. In Evolutionary Computation (CEC), 2010 IEEE Congress on, pages 1–7, July 2010.

[11] M. Hoffmann. Empathy and moral development: The implications for caring and justice. Cambridge University Press, 2000.

[12] J.-W. Lee and Ju-Jang Lee. Novel ant colony optimization algorithm with path crossover and heterogeneous ants for path planning. In Industrial Technology (ICIT), 2010 IEEE International Conference on, pages 559–564, March 2010.

[13] A. Nowé, K. Verbeeck, and P. Vranx. Multi-type ant colony: The edge disjoint paths problem. In Marco et al. Dorigo, editor, Ant Colony Optimization and Swarm Intelligence, pages 202–213. Springer, 2004.

[14] M. Rusin and E. Zaitseva. Hierarchical heterogeneous ant colony optimization. In Proc. of Federated Conference on Computer Science and Information Systems. 2012.

[15] S. Van Segbroeck, J.M. Pacheco, T. Lenaerts, and F.C. Santos. Emergence of fairness in repeated group interactions. Physical Review Letters, 108.

[16] S. Van Segbroeck, J.M. Pacheco, T. Lenaerts, and F.C. Santos. Evolution of fairness and conditional cooperation in public goods dilemmas. In T. et al. Gilbert, editor, Pro. of the European Conference on Complex Systems 2012, pages 827–830. Springer International Publishing, 2013.

[17] Peter Vranx, Ann Now, and Kris Steenhaut. Multi-type aco for light path protection. In Karl Tuyls, Pieter Jant Hoen, Katja Verbeeck, and Sandip Sen, editors, Learning and Adaption in Multi-Agent Systems, volume 3898 of Lecture Notes in Computer Science, pages 207–215. Springer Berlin Heidelberg, 2006.

[18] David H. Wolpert and William G. Macready. No free lunch theorems for optimization. IEEE Transactions on Evolutionary Computation, 1(1):67–82, 1997.