Summary: We study the torus partition functions of free bosonic CFTs in two dimensions. Integrating over Narain moduli defines an ensemble-averaged free CFT. We calculate the averaged partition function and show that it can be reinterpreted as a sum over topologies in three dimensions. This result leads us to conjecture that an averaged free CFT in two dimensions is holographically dual to an exotic theory of three-dimensional gravity with $U(1)^c \times U(1)^c$ symmetry and a composite boundary graviton. Additionally, for small central charge $c$, we obtain general constraints on the spectral gap of free CFTs using the spinning modular bootstrap, construct examples of Narain compactifications with a large gap, and find an analytic bootstrap functional corresponding to a single self-dual boson.

MSC:
83C80 Analogues of general relativity in lower dimensions
83E05 Geometrodynamics and the holographic principle
83C45 Quantization of the gravitational field
81T40 Two-dimensional field theories, conformal field theories, etc. in quantum mechanics

Keywords: AdS-CFT correspondence; conformal field theory

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