Apparent Acceleration through Large-scale Inhomogeneities

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We re-analyze the observed magnitude-redshift relation of type Ia supernovae (SNe Ia) and examine the possibility that the apparent acceleration of the cosmic expansion is not caused by dark energy but is instead a consequence of the large-scale inhomogeneities in the universe. We propose a method to phenomenologically describe the effects of the large-scale inhomogeneities without relying on the specific toy models of the inhomogeneous universe. This method clearly illustrates how the post-Friedmannian effects of inhomogeneities, i.e., the effects due to the deviation from a perfectly homogeneous and isotropic model, act as an effective cosmological constant in the magnitude-redshift relation of SNe Ia.

§1. Introduction

The Cosmological Principle, which states that our universe is described by the homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) model, in some averaged sense, is a working hypothesis which has been widely accepted among cosmologists. The present, past, and future evolution of the FLRW model is determined by a few constant parameters, such as the Hubble parameter, \( H_0 \), the matter density parameter, \( \Omega_m \), and the cosmological constant, \( \Lambda \) (or the normalized parameter \( \Omega_A = \Lambda/(3H_0^2) \)). The determination of the cosmological parameters is one of the main purposes of observational cosmology.

The recent observations of type Ia supernovae (SNe Ia) now strongly suggest the acceleration of the cosmic expansion. As long as we employ perfectly homogeneous and isotropic FLRW models, this requires dark energy, an exotic energy component which accelerates the cosmic expansion with its negative pressure.

Instead of introducing such a mysterious energy component, there have been attempts to explain the apparent accelerated expansion of the universe resulting from the large-scale inhomogeneities in the universe. For example, Tomita using his local void model, and Iguchi et al. using the Lemaître-Tolman-Bondi (LTB) model, studied the possibility of explaining the observed magnitude-redshift \((m-z)\) relation of SNe Ia. Moreover, recently Alnes et al. have concluded that not only the \(m-z\) relation of SNe Ia but also the position of the first peak in the cosmic microwave background (CMB) anisotropy can be explained by the inhomogeneity in the LTB model. However, these works depend specifically on simplified toy models. Therefore, due to the lack of strong support for such toy models as providing realistic descriptions of our universe, the study of inhomogeneous effects in the universe is not the mainstream of research in cosmology.
In this article, we re-analyze the observed \( m-z \) relation of SNe Ia and point out some theoretical possibilities of the inhomogeneity interpretation to explain the apparent acceleration. Then, we propose a method to phenomenologically describe the effects of the large-scale inhomogeneities in the universe, without relying on specific toy models. This method clearly illustrates how the post-Friedmannian effects of inhomogeneities, \textit{i.e.} the effects due to the deviation from a perfectly homogeneous and isotropic FLRW model, act effectively as a cosmological constant in the magnitude-redshift relation of SNe Ia.

\section{The magnitude-redshift relation of SNe Ia}

The apparent magnitude \( m \) of a SN Ia of absolute magnitude \( M \), at redshift \( z \), is
\begin{equation}
    m = M + 5 \log_{10} \frac{D_L(z)}{10 \text{ (pc)}},
\end{equation}
where \( D_L(z) \) is the luminosity distance in units of parsecs. The luminosity distance is obtained by solving the propagation of light ray bundles through space-time. In the FLRW universe, it is written in the form
\begin{equation}
    D_L(z) = \frac{c}{H_0 \sqrt{1 - \Omega_m - \Omega_\Lambda}} \times \sinh \left( \frac{z}{\sqrt{1 - \Omega_m - \Omega_\Lambda}} \int_0^z \frac{dz'}{\sqrt{(1 + \Omega_m z')(1 + z')^2 - z'(2 + z') \Omega_\Lambda}} \right).
\end{equation}
The luminosity distance \( D_L(z) \) is a slightly complicated function of \( z \) with three constant parameters, \( H_0, \Omega_m, \) and \( \Omega_\Lambda \).

As an illustration, we use the observed SNe Ia data presented in the paper of Perlmutter et al.\cite{2} In total, 60 SNe Ia with redshift in the range \( 0.014 \leq z \leq 0.830 \) are listed in Tables 1 and 2 of Ref. \cite{2}. Because all of the SNe Ia have redshifts satisfying \( z < 1 \), the luminosity distance \( D_L(z) \) may be most usefully expressed as a power series,
\begin{equation}
    D_L(z) = \frac{c}{H_0} \left( z + d_2 z^2 + d_3 z^3 + \cdots \right),
\end{equation}
where the expansion coefficients \( d_2 \) and \( d_3 \) are given by\cite{17}
\begin{align}
    d_2 &= \frac{1}{4} (2 - \Omega_m + 2 \Omega_\Lambda), \\
    d_3 &= \frac{1}{8} \left( \Omega_m^2 + 4 \Omega_\Lambda^2 - 4 \Omega_m \Omega_\Lambda - 2 \Omega_m - 4 \Omega_\Lambda \right).
\end{align}
Substituting Eq. (2.3) into the \( m-z \) relation Eq. (2.1) gives
\begin{equation}
    m = M - 5 + 5 \log_{10} D_L(z) = \mathcal{M} + 5 \log_{10} \left( z + d_2 z^2 + d_3 z^3 \right),
\end{equation}
where \( \mathcal{M} \equiv M - 5 + 5 \log_{10} c/H_0 \). This quantity is often called as the “Hubble-constant-free absolute magnitude”\cite{2} or the “magnitude zero-point”\cite{17}. Note that
Taylor expansion up to at least $O(z^3)$ is necessary to determine the three parameters $H_0$, $\Omega_m$, and $\Omega_A$ by fitting to the data. Once the best fit coefficients $d_2$ and $d_3$ are obtained from such a fitting, we can calculate the cosmological parameters $\Omega_m$ and $\Omega_A$ as follows:

$$\Omega_m = 2 \left(1 - d_2\right) (1 - 2d_2) - 2d_3,$$

(2.7)

$$\Omega_A = d_2 (2d_2 - 1) - d_3.$$

(2.8)

Fig. 1. Hubble diagram for all SNe Ia. These data are taken from Tables 1 and 2 of Perlmutter et al. The solid curve represents the best fit $m$-$z$ relation for a flat cosmology given by Perlmutter et al. with the parameter values $\Omega_m = 0.28$ and $\Omega_A = 0.72$.

In Fig. 1, we plot the Hubble diagram for all 60 SNe Ia. Also plotted there is the best fit $m$-$z$ curve for a flat cosmology with the parameter values $\Omega_m = 0.28$ and $\Omega_A = 0.72$, which was determined by Perlmutter et al.

§3. Inhomogeneous interpretation?

In order to formulate an alternative interpretation of the $m$-$z$ relation without dark energy, it is instructive to divide the whole SNe Ia data set into two parts. For the sake of convenience, we define SNe Ia with redshifts satisfying $z < 0.2$ as low-redshift (low-$z$) SNe Ia, and those with redshifts satisfying $z > 0.3$ as high-redshift (high-$z$) SNe Ia. Then, the low-$z$ data set consists of 20 SNe Ia with redshifts in the range $0.014 < z < 0.18$, and the high-$z$ data set consists of 40 SNe Ia with redshifts in the range $0.320 < z < 0.830$. No SNe Ia are in the range $0.2 \leq z \leq 0.3$. 
Fig. 2. Hubble diagram for the 20 low-redshift ($z < 0.2$) SNe Ia data set and the best fit $m$-$z$ curve of a zero-$\Lambda$ cosmology with the parameter values $M = 24.01$ and $\Omega_m = 0.26$.

Fig. 3. Hubble diagram for the 40 high-redshift ($z > 0.3$) SNe Ia data set and the best fit $m$-$z$ curve for a zero-$\Lambda$ cosmology with the parameter values $M = 24.27$ and $\Omega_m = 0.35$. 
In this paper, we use an implementation of the nonlinear least-square methods of the Levenberg-Marquardt algorithm\textsuperscript{13,19} to fit the $m$-$z$ relation Eq. (2.6) to each data set.

Figure 2 displays only the low-$z$ SNe Ia data set, along with the best fit $m$-$z$ curve of a zero-$\Lambda$ cosmology with the parameter values $M = 24.01$ and $\Omega_m = 0.26$. Since the cosmological constant $\Lambda$ does not play an important role for $z \ll 1$, it is evident that the zero-$\Lambda$ cosmology can fit the low-$z$ data set fairly well.

Figure 3 displays only the high-$z$ SNe Ia data set, along with the best fit $m$-$z$ curve of a zero-$\Lambda$ cosmology with the parameter values $M = 24.27$ and $\Omega_m = 0.35$. It is interesting that a zero-$\Lambda$ cosmology can also accurately fit the high-$z$ SNe Ia data set, although the values of the cosmological parameters $M$ and $\Omega_m$ in that case are different from the best fit values for the low-$z$ SNe Ia. It should also be noted that the high-$z$ best fit $m$-$z$ curve ($M = 24.27, \Omega_m = 0.35, \Omega_\Lambda = 0$) behaves very similarly to that of the flat cosmology with $\Omega_m = 0.28$ and $\Omega_\Lambda = 0.72$ obtained by Perlmutter et al\textsuperscript{2} in the range $0.3 < z < 1$.

Figure 4 summarizes all of the above results. There it is seen that an open zero-$\Lambda$ FLRW model with the parameter values $M = 24.01$ and $\Omega_m = 0.26$ can fit the low-$z$ SNe Ia data, whereas another open zero-$\Lambda$ FLRW model with $M = 24.27$ and $\Omega_m = 0.35$ can fit the high-$z$ data. No cosmological constant nor dark energy is necessary to fit either part. However, a positive cosmological constant ($\Omega_\Lambda = 0.72$)
is necessary to fit the entire SNe Ia data set with a single flat FLRW cosmology. It is evident from Fig. 4 that this conclusion simply comes from the asymptotic behavior of the theoretical curve with $\Omega_m = 0.28$ and $\Omega_A = 0.72$ and does not depend on the particular choices of the SNe Ia data sets.

Because $\mathcal{M} = M - 5 + 5 \log_{10} c/H_0$, where the Hubble distance $c/H_0$ is in units of parsecs, the result that different values of $\mathcal{M}$ fit different redshift data sets implies the following possibilities:

1. The absolute magnitude $M$ of the high-$z$ SNe Ia is systematically different from that of the low-$z$ SNe Ia.
2. The speed of light $c$ is different in different redshift regions.
3. $H_0$ in the high-$z$ region is slightly different from that in the low-$z$ region.

Let us examine the third possibility, namely, the inhomogeneity interpretation. If the difference between $\mathcal{M}(\text{low}-z) = 24.01$ and $\mathcal{M}(\text{high}-z) = 24.27$ is due to the inhomogeneity of the Hubble parameter $H_0$, we can estimate the following ratio representing the inhomogeneity in $H_0$ between the two different redshift regions as

\[
24.01 - 24.27 = 5 \log_{10} \frac{H_0(\text{high}-z)}{H_0(\text{low}-z)}.
\]

This yields $H_0(\text{high}-z) = 0.89 H_0(\text{low}-z)$. An inhomogeneity in which the value of $H_0$ in the high-$z$ region is 11% smaller than that in the low-$z$ region may be sufficient to explain the observed $m$-$z$ relation for SNe Ia, without the need to introduce a cosmological constant or dark energy.

\section*{§4. Effects of large-scale inhomogeneities on the luminosity distance}

In the previous section, we pointed out the interesting possibility that it may be possible to account for the observed $m$-$z$ relation for SNe Ia by the large-scale inhomogeneities in the universe, without introducing a cosmological constant or dark energy. In order to examine this possibility further, we need to study the propagation of light ray bundles through the inhomogeneous universe and obtain the luminosity distance $D_L(z)$ as a function of the redshift $z$. Since the actual space-time inhomogeneities of the present universe are not known in detail, the usual approach employs simplified toy models of the inhomogeneous universe. For example, Tomita\textsuperscript{5, 6, 7, 8, 9} used a local void model, and Iguchi et al.\textsuperscript{10} used the LTB model.

We take another approach. In this section, we propose a method to phenomenologically describe the effects of the large-scale inhomogeneities on the luminosity distance, without relying on specific toy models of the inhomogeneous universe. This method clearly illustrates how the “post-Friedmannian” effects of inhomogeneities, i.e. the effects due to the deviation from a perfectly homogeneous and isotropic FLRW model, act effectively as a cosmological constant in the magnitude-redshift relation of SNe Ia.

In the perfectly homogeneous FLRW models, $H_0$ denotes the expansion rate at the present time, $t_0$, and it is constant over the entire $t = t_0$ hypersurface, due to the perfect spatial homogeneity. In inhomogeneous universes, however, the expansion
rate is naturally dependent on the spatial positions. Therefore, $H_0$ is not constant and may depend on $z$:

$$H_0 \Rightarrow H_0(z). \quad (4.1)$$

Analogously, the density parameter $\Omega_m$ may also depend on $z$:

$$\Omega_m \Rightarrow \Omega_m(z). \quad (4.2)$$

In general inhomogeneous universes, these cosmological parameters may also depend on the angular direction due to spatial anisotropies. Already in 1966, Kristian and Sachs\textsuperscript{20} emphasized the importance of observing angular variations in the various cosmological effects. Kasai and Sasaki\textsuperscript{21} and Kasai\textsuperscript{22} derived formulae for cosmological observations in a linearly perturbed FLRW model in gauge-invariant manner and found the existence of a quadrupole anisotropy of the Hubble parameter $H_0$, which is directly proportional to the gauge-invariant scalar potential. They found that “the perturbed space-time behaves as a Friedmann-like universe with the direction-dependent $H_0$ and $q_0$”.\textsuperscript{22} In this paper, however, we concentrate on the $z$ dependence of the cosmological observables. Future investigations will include consideration of the angular dependences of $H_0$ and $\Omega_m$.

In the region $z < 1$, the cosmological parameters can be expressed in the power series forms

$$H_0(z) = \bar{H}_0 \left(1 + h_1 z + h_2 z^2 + \cdots\right), \quad (4.3)$$

$$\Omega_m(z) = \bar{\Omega}_m \left(1 + \omega_1 z + \omega_2 z^2 + \cdots\right), \quad (4.4)$$

where $\bar{H}_0 = H_0(z = 0)$ and $\bar{\Omega}_m = \Omega_m(z = 0)$, and the expansion coefficients $h_1, h_2, \ldots, \omega_1, \omega_2, \ldots$ represent the “post-Friedmannian” corrections due to spatial inhomogeneities. The models reduce to the FLRW if and only if all of these coefficients vanish. Under the assumption that $H_0(z)$ and $\Omega_m(z)$ are slowly varying functions of $z$, we substitute them for $H_0$ and $\Omega_m$ in Eq. (2.2) and obtain the following power series formula:

$$D_L(z) = \frac{c}{\bar{H}_0} \left(z + \tilde{d}_2 z^2 + \tilde{d}_3 z^3 + \cdots\right), \quad (4.5)$$

where the expansion coefficients are now

$$\tilde{d}_2 = \frac{1}{4} \left(2 - \bar{\Omega}_m + 2 \Omega_A\right) - h_1, \quad (4.6)$$

$$\tilde{d}_3 = \frac{1}{8} \left(\bar{\Omega}_m^2 + 4 \Omega_A^2 - 4 \bar{\Omega}_m \Omega_A - 2 \bar{\Omega}_m - 4 \Omega_A\right)$$

$$+ \frac{1}{4} \left\{4(h_1)^2 - 4h_1 - 4h_2 - h_1 (2\Omega_A - \bar{\Omega}_m - 2) - \frac{2}{3} \omega_1 \bar{\Omega}_m\right\}. \quad (4.7)$$

Now we can illustrate how the “post-Friedmannian” corrections of the spatial inhomogeneities act effectively as a cosmological constant. Suppose that astronomers obtain the best fit parameters $\tilde{d}_2$ and $\tilde{d}_3$ by fitting the luminosity distance formula
Eq. (4.5) to the observed $m$-$z$ data of SNe Ia. If they assume that the universe is homogeneous and isotropic, they will simply use Eqs. (2.7) and (2.8) to calculate the cosmological parameters. Then, even if the true value of the cosmological constant is zero, they will obtain the following effective value for the cosmological constant from Eq. (2.8):

$$
\Omega_{\text{eff}}^{\Lambda} \equiv \tilde{d}_2 \left( 2\tilde{d}_2 - 1 \right) - \tilde{d}_3
$$

$$
= \frac{1}{4} \left\{ 3h_1\bar{\Omega}_m + \frac{2}{3}\omega_1\bar{\Omega}_m - 2h_1 + 4(h_1)^2 + 4h_2 \right\} \quad (4.8)
$$

This clearly shows that the “post-Friedmannian” correction terms $h_1, h_2$, and $\omega_1$ together act effectively as a cosmological constant. The effective value $\Omega_{\text{eff}}^{\Lambda}$ is unrelated to the true value of the cosmological constant. It simply results from the erroneous assumption that the universe is perfectly homogeneous and that $H_0$ and $\bar{\Omega}_m$ are constant on the $t = t_0$ hypersurface.

In the same way, we also obtain the following effective value for the density parameter from Eq. (2.7):

$$
\Omega_{\text{eff}}^{\rho_m} \equiv 2 \left( 1 - \tilde{d}_2 \right) \left( 1 - 2\tilde{d}_2 \right) - 2\tilde{d}_3
$$

$$
= \left( 1 + \frac{3}{2}h_1 + \frac{1}{3}\omega_1 \right) \bar{\Omega}_m + 3h_1 + 2(h_1)^2 + 2h_2. \quad (4.9)
$$

Following the procedure described in §2., we obtain the best fit values for the cosmological parameters as $\Omega_{\text{eff}}^{\rho_m} = 0.28$ and $\Omega_{\text{eff}}^{\Lambda} = 0.72$. Unfortunately, however, these do not completely determine the inhomogeneities of the actual universe. This is simply because the two cosmological parameters $\Omega_{\text{eff}}^{\rho_m}$ and $\Omega_{\text{eff}}^{\Lambda}$ are functions of four parameters $\bar{\Omega}_m, h_1, h_2$, and $\omega_1$ (or possibly five, including $\Omega_{\Lambda}$). The data fitting of the $m$-$z$ relation itself only yields constraints on some sets of the post-Friedmannian parameters, but it does not determine completely the values of each parameter independently. For this reason, it is highly desirable to incorporate other independent observations, such as CMB data, gravitational lensing data, and so on, in order to determine the extent to which our universe is homogeneous or inhomogeneous.

§5. Summary

We have re-analyzed the observed $m$-$z$ relation of SNe Ia proposed by Perlmutter et al. and have examined the possibility that the apparent acceleration of the cosmic expansion is a consequence of large-scale inhomogeneities in the universe. As previously found by Perlmutter et al., a positive cosmological constant is necessary to fit the whole data set, consisting of 60 SNe Ia in the redshift range $0.014 \leq z \leq 0.830$, with a single FLRW model. They obtained the best fit values $\Omega_m = 0.28$ and $\Omega_{\Lambda} = 0.72$ for a flat cosmology.

In order to examine the feasibility of the inhomogeneity interpretation, we divided the SNe Ia data into two parts, low-$z$ and high-$z$ data sets. The low-$z$ ($z < 0.2$) data set consists of 20 SNe Ia in the redshift range $0.014 \leq z \leq 0.18$, and the high-$z$ ($z > 0.3$) data set consists of 40 SNe Ia in the range $0.320 \leq z \leq 0.830$. We
were able to fit the low-\(z\) and high-\(z\) data sets, respectively, with two zero-\(\Lambda\) FLRW cosmologies. The best fit parameters are \(M = 24.01\) and \(\Omega_m = 0.26\) for the low-\(z\) data set and \(M = 24.27\) \(\Omega_m = 0.35\) for the high-\(z\) data set. The difference between the values of \(M\) implies that the Hubble parameter \(H_0\) in the high-\(z\) region is 11% smaller than that in the low-\(z\) region. This indicates the possibility of the cosmological parameters \(H_0\) and \(\Omega_m\) being dependent on the redshift \(z\). It also suggests that the nearby low-\(z\) region is a local void, i.e. a less dense and more rapidly expanding than the outer high-\(z\) region. This local void interpretation is consistent with the results of previous works employing inhomogeneous toy models\(^5, 6, 7, 8, 9, 10, 14\).

Inspired by the above results from the data fittings, we proposed a method to phenomenologically describe the effects of the large-scale inhomogeneities, without relying on specific toy models of the inhomogeneous universe. In general inhomogeneous universes, the Hubble expansion rate \(H_0\) and the density parameter \(\Omega_m\) are naturally dependent on the spatial coordinates, and therefore the redshift \(z\). We expanded \(H_0(z)\) and \(\Omega_m(z)\) into power series with respect to \(z\) and obtained a luminosity distance formula with corrections due to the large-scale inhomogeneities. This distance formula clearly illustrates how the corrections resulting from the large-scale inhomogeneities act effectively as a cosmological constant.

In this paper, we re-analyzed only the SNe Ia data presented by Perlmutter et al.\(^2\) Other data sets, including those with \(z > 1\) SNe Ia, should also be examined. It is also noted that the \(m-z\) relation of SNe Ia itself does not completely determine the inhomogeneities of the actual universe. The data fitting of the \(m-z\) relation itself only gives constraints on some sets of the post-Friedmannian parameters. It does not completely determine the values of each parameter independently. For this reason, it is highly desirable to incorporate other independent observations, such as CMB data, gravitational lensing data, and so on, in order to determine the extent to which our universe is homogeneous or inhomogeneous.

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