Research on Multi-Objective Attribute Decision Algorithm Based on Inclusion Degree

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Abstract. By studying the arising problem in the decision-making of multi-objective attributes in regard to interval valued intuitionistic fuzzy soft set of interval-valued intuitionistic fuzzy soft set based on the triangular fuzzy number and the inclusion degree, and discuss their related properties and operations. At the same time, aiming at the problem that the equal inclusion degree value cannot determine in the previous algorithms, a new approach of decision-making of multi-objective attributes based on triangular interval-valued intuitionistic fuzzy soft set inclusion degree is proposed. In this method, we introduce the mean and variance, which can select the sum of the largest inclusion degrees, and choose the smaller variance as the best decision goal. At last, the validity and the correctness will be proved through instances.

1. Introduction
In the real world, there exist a lot of uncertainty and incompleteness. In order to solve the problems encountered in the economics, engineering, environmental science, social science and other disciplines, rough sets theory [8], probability theory [10], interval number theory and etc have been proposed by many scholars. Although these theories greatly promoted the development of mathematics, they still exist some limitations. Molodtsov, Russian scholar, first proposed soft sets [1] in 1999. It was the non-binding in the aspect of its parameters that made it have inherent advantages in dealing with uncertainty and imprecision. In 1965, Zadeh proposed fuzzy sets theory [3], which researched and developed by many scholars, and the fruitful research results were achieved. In 1986, the fuzzy sets theory was further expanded by Atanassov, and proposed the interval-valued intuitionistic fuzzy set [9]. The integrated approach to the interval-valued intuitionistic fuzzy information and its application were also studied by JIANG [11], and proposed the IIFW operator, IIGA operator, IIFOWA operator of the interval-valued intuitionistic fuzzy number. The interval-valued intuitionistic fuzzy soft set was further extended by LIU, using triangular fuzzy numbers as the membership and non-membership, and coming up with the triangular fuzzy number intuitionistic fuzzy soft set. The inclusion degree of fuzzy soft sets was extended to Vague sets by HUANG [13], providing the definition and calculation formula. A variety of computing in the inclusion degree of intuitionistic fuzzy sets was discussed by LU [12]. VIKOR was applied to multiple target attribute decision by GENG [14]. It’s been shown that fuzzy soft sets based on the triangular interval had a great potential for application in decision analysis, pattern recognition and data mining.

Inclusion Degree [5] is an effective measure of the uncertainty relation description to describe the uncertainty relation and generalize the existing uncertainty reasoning methods, such as probability theory reasoning, evidential reasoning approach, fuzzy reasoning method and information reasoning.
method. As a result, it provides a general principle for the uncertainty reasoning. Inclusion degree is not only concise, but also convenient to carry out the information synthesis, communicate and correct information dissemination and correction, penetrating which has penetrated into all branches of artificial intelligence, and played a significant role in expert systems, pattern recognition, fuzzy sets theory and other fields.

In this paper, In the process of decision-making based on multi-objective attributes, inclusion degree is introduced to triangular interval-valued intuitionistic fuzzy soft set, redefine the inclusion degree function between the decision attributes of triangular interval-valued intuitionistic fuzzy soft set, describe the quantitative relationship between decision-targeted of decision goals attribute sets with the inclusion degree theory, improve the decision algorithm in literature [7], and avoid the wrong decision targets caused by the equal inclusion degree due to constructing the ideal decision goal. Through improving the method of multi-objective attribute decision-making, comparing the sum of the inclusion degree with other decision objectives, then bringing in the mean and variance, using the theory that the smaller the variance, the smaller the volatility, the more stable inclusion relationship, further selecting the condition that the largest inclusion degree is equal, the relatively smaller mean is selected as the best decision target. Finally, the validity and the correctness will be proved through instances. At the same time, it provides a new approach for the fuzzy multi-objective decisions.

2. Basic Concept

Definition 1: Given $U$ is a finite non-empty universe of objects and $E$ is a finite non-empty set of parameters, $P(U)$ the power set of $U$. Then we call $(F, E)$ as one soft set of $U$, in which $F: E \to P(U)$ is a mapping, that is, for $\forall e \in E$,

$$F(e) = \{< x, \mu_{F(e)}(x) > : x \in U \} \subseteq P(U), \quad \mu_{F(e)}(x) \in [0,1]$$
represents that element $x \in U$ belongs to characteristic function of set $F(e)$. That is, element $x$ belongs to the satisfaction degree of $F(e)$ in the attribute $e$.

Definition 2 [2]: Given $U$ is a non-empty finite set, $X \subseteq U$, and then we called $A=\{< x, \mu_u(x), v_u(x) > : x \in X \}$ as the intuitionistic fuzzy set. Wherein, $\mu_u(x)$ and $v_u(x)$ respectively represent in $x$ element $x$ belongs to the member ship and the non-membership of $A$, that is $\mu_u(x): X \to [0,1], v_u(x): X \to [0,1]$, and $0 \leq \mu_u(x) + v_u(x) \leq 1, \forall x \in X$ Given $IF(U)$ represents the sets group of all the intuitionistic fuzzy sets on $U$.

Definition 3 [3]: Given $U$ is a non-empty finite set, then we call $(F, E)$ as intuitionistic fuzzy soft set on $U$. Wherein, $F$ is the mapping of $E$ to $IF(U)$, and for every parameter $e \in E$, $F(e)$ is intuitionistic fuzzy soft set about parameter $e$ on $U$, $F(e)=\{< x, \mu_{F(e)}(x), v_{F(e)}(x) > : x \in U \}$, wherein $\mu_{F(e)}: U \to [0,1], v_{F(e)}: U \to [0,1]$ and satisfy $0 \leq \mu_{F(e)}(x) + v_{F(e)}(x) \leq 1$.

Definition 4 [4]: Given $U$ is a non-empty finite set, then we call $A=\{< x, \mu_u(x), v_u(x) > : x \in U \}$ as the triangular interval-valued intuitionistic fuzzy soft set, wherein, $\mu_u(x)=[t_u(x), i_u(x), c_u(x)]$ and $v_u(x)=[t'_u(x), i'_u(x), c'_u(x)]$ are all triangular fuzzy numbers on $[0,1]$ , and they respectively represent that in $U$ element $x$ belongs to the member ship and the non-membership of $A$, and satisfy $t_u(x)+c_u(x)+t'_u(x)+c'_u(x) \leq 1$. $t_u(x) \geq 0$, $t'_u(x) \geq 0$. $i_u(x) \leq t_u(x) \leq i'_u(x)$. $f_u(x) \leq f'_u(x) \leq f_u(x)$. We record all the triangular interval-valued intuitionistic fuzzy soft sets of $U$ as $TIFS(U)$.

Definition 5: Given $A=\{x \in U \mid ([t_u(x), i_u(x), c_u(x)], [f_u(x), f'_u(x), f_u(x)]) \}$ and $B=\{x \in U \mid ([t'_u(x), i'_u(x), c'_u(x)], [f'_u(x), f_u(x), f'_u(x)]) \}$, are two triangular interval-valued fuzzy soft sets on $U$, then the provided operations are as follows:

1. $A \subseteq B \iff t'_u(x) \leq t_u(x) \& i'_u(x) \leq i_u(x) \& c'_u(x) \leq c_u(x) \& f_u(x) \leq f'_u(x) \& f'_u(x) \leq f_u(x) \& f_u(x) \geq f'_u(x) \& f_u(x) \geq f_u(x)$.
2. $A = B \iff A \subseteq B \& B \subseteq A$.
3. $A^c = \{x \in U \mid ([f'_u(x), f_u(x), f'_u(x)], [t_u(x), i_u(x), c'_u(x)]) \}$.


3. The Inclusion Degree of the Triangular Interval-Valued Intuitionistic Fuzzy Soft Set

Definition 6 [5]: Given universe $U$ is the finite non-empty universe, $p(U)$ represent all the classic subsets on universe $U$, $TIFS(U)$ represent all the fuzzy subsets on universe $U$. Given $TIFS_\theta(U) \subseteq TIFS(U)$, $\forall A, B, C \in TIFS(U)$, if the real number $D(A, B)$ satisfies:

1. $0 \leq D(A, B) \leq 1$
2. $A \subseteq B \Rightarrow D(A, B) = 1$
3. $A \subseteq B \subseteq C \Rightarrow D(C, A) \leq D(B, A) \& D(C, A) \leq D(C, B)$
4. $D(X, \emptyset) = 0$

Then we call $D(A, B)$ as inclusion degree on $FS_\theta(U)$, and $D$ as the inclusion degree function. That is, $D(A, B)$ describe the inclusion extent of target $A$ in the target $B$.

If $D$ is satisfied (1), (2), (3) and the following (5):

5. $A \subseteq B \Rightarrow D(A, C) \leq D(B, C)$

Then we call $D(A, B)$ as the strong inclusion degree of $FS_\theta(U)$.

If $D(A, B)$ is satisfied (1), (3) and the following (2):

2. For $\forall A, B \in FS_\theta(U) \cap p(U), A \subseteq B \Rightarrow D(A, B) = 1$

Then we call $D(A, B)$ as the weak inclusion degree of $FS_\theta(U)$.

Actually, inclusion degree is the characterization by a number of the inclusion relationship degree between the set and set. In the above definitions, (1) is the normalization of inclusion degree, and (2) represents the coordination of inclusion degree and the classic inclusion, (3) and (4) is the monotonicity of inclusion relationship.

Definition 7 [6]: We call $\theta$ as the normal fuzzy implication operator, which is abbreviated as the normal implication. If the mapping $\theta: [0,1]^2 \rightarrow [0,1]$ satisfies the following conditions:

1. $\theta(1,0) = 0$; 2. $\theta(0,0) = \theta(0,1) = \theta(1,1) = 1$

Theory 1: Given universe $U$ is a non-empty finite set, $\theta$ is a normal implication. If $\theta$ satisfies:

1. $\forall \mu, \nu \in [0,1]$, and $\mu \leq \nu \Rightarrow \partial(\mu, \nu) = 1$; 2. $\partial(\mu, \nu)$ is the non-increasing function about $\mu$, $\partial(\mu, \nu)$ is the non-decreasing function about $\nu$.

In this paper, we take normal implication $\theta$ as Lukasiewica implication, that is, for $\forall a, b \in [0,1]$, it has $a \theta b = \min[1-a+b, 1]$.

Definition 8: The definition of inclusion degree of the triangular interval-valued intuitionistic fuzzy soft set is as follows: Given universe $U$ is a non-empty set. $X \subseteq U$. $A, B \in TIFS(U)$, $\theta$ is a normal implication, the inclusion degree function of the triangular interval-valued intuitionistic fuzzy soft set is:

$$D(A, B) = \frac{1}{2} \sum_{x \in X} \left[ \lambda_1 \theta(t'_{A}(x), t'_{B}(x)) + \lambda_2 \theta(t_{A}(x), t_{B}(x)) + \lambda_3 \theta(t'_{A}(x), x) + \lambda_4 \theta(f'_{A}(x), f'_{B}(x)) + \lambda_5 \theta(f_{A}(x), f_{B}(x)) + \lambda_6 \theta(f_{A}(x), f'_{B}(x)) \right]$$

Wherein, $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = 1$. $|X|$ represents the whole number of all decision-making goals.

Next, let me prove the inclusion degree function satisfies the definition 6:

Proof:
(1) Can be easily verified by the definition of the inclusion degree.
(2)\(A \subseteq B \Rightarrow \text{satisfy inequality} f_A(x) \leq f_B(x)\), then we can know from definition 7:
\[
\theta(t_A(x),t_B(x)) = 1, \quad \theta(t_A(x),t_A(x)) = 1, \quad \theta(f_A(x),f_A(x)) = 1, \quad \theta(f_A(x),f_B(x)) = 1, \quad \theta(f_B(x),f_A(x)) = 1.
\]
Therefore, \(D(A,B) = \frac{1}{|\mathcal{X}|} \sum_{x \in \mathcal{X}} \theta(t_A(x),t_B(x)) = 1\).

(3)\(A \subseteq C \Rightarrow \text{satisfy inequality} f_A(x) \leq f_C(x)\), and for each decision goal, we can calculate the inclusion degree of each decision goal.

And for \(\forall x \in U\), we can know from Theory 1, \(\theta(\mu, v)\) is decreasing about \(u\), \(\theta(u, v)\) is increasing about \(v\). Thus,
\[
\theta(t_1(x),t_1(x)) \leq \theta(t_1(x),t_1(x)), \quad \theta(t_1(x),t_2(x)) \leq \theta(t_1(x),t_1(x)), \quad \theta(t_1(x),t_1(x)) \leq \theta(t_1(x),t_1(x)).
\]
We can obtain:
\[
D(C,A) = \frac{1}{|\mathcal{X}|} \sum_{x \in \mathcal{X}} \theta(t_1(x),t_1(x)) = D(B,A).
\]

(4) \(D(X,D) = \frac{1}{|\mathcal{X}|} \sum_{x \in \mathcal{X}} [\lambda_1 \theta(1,0) + \lambda_2 \theta(0,1) + \lambda_3 \theta(1,0) + \lambda_4 \theta(0,1) + \lambda_5 \theta(1,0) + \lambda_6 \theta(0,1)] = 0\). Therefore, the inclusion degree function proposed by definition 8 satisfies the conditions of the inclusion degree definition.

We can easily know by definition 8, \(D(A,A) = 1\).

4. The Group Decision Making Based on the Triangular Interval-Valued Intuitionistic Fuzzy Soft Set
Because in the past, the group decision making of inclusion degree is based on the construction of the ideal decision goal, and compare the inclusion degree of the goals in the ideal decision goal, and select the decision goal of the maximal inclusion degree as the best decision goal. However, if the maximum inclusion degree of the decision goal appears to equal, then this method cannot determine the best decision goal.

Given the decision goal set is \(A = \{A_1, A_2, \ldots, A_n\}\), constraint condition set is \(C = \{C_1, C_2, \ldots, C_m\}\), and given the following decision table, we elaborate multi-objective decision algorithm in this paper by the decision table.

| Table 1. Decision Table |
|-------------------------|
| \(A_1\) | \(A_1\) | \(A_1\) | \(A_1\) | \(A_1\) |
| \(A_2\) | \(A_2\) | \(A_2\) | \(A_2\) | \(A_2\) |
| \(A_3\) | \(A_3\) | \(A_3\) | \(A_3\) | \(A_3\) |
| \(A_4\) | \(A_4\) | \(A_4\) | \(A_4\) | \(A_4\) |

Wherein, \(A_i = ([t_{A_i}(x), t_{A_i}(x), t_{A_i}(x)], [f_{A_i}(x), f_{A_i}(x), f_{A_i}(x)])\).

According to the given decision table, we can calculate the inclusion degree of each decision goal by inclusion degree function and obtain the following inclusion degree table:
We can construct the decision goal algorithm according to the inclusion degree table:

**Definition 9:** Given the decision goal set is \( A = \{A_1, A_2, \ldots, A_m\} \), constraint condition set is \( C = \{C_1, C_2, \ldots, C_n\} \), \( D(A, A_i) \) is the inclusion degree of decision goal \( A_i \) in \( A \), \( i, j = 1, 2, \ldots, m \), then, the sum of the inclusion degree of \( A_i \) in each decision goal is:

\[
S_i = \sum_{j=1}^{m} D(A_i, A_j) \quad (2)
\]

By compassion the sum of the inclusion degree of each decision goal, we select the maximums \( S_i \) as the best decision goal. However, this decision algorithm will fail because there exists the same situation that the sum of maximum decision goal appears to equal and cannot determine the best decision goal. Thus, this paper introduces variance. Since variance can be used to vividly describe the decision goal of the inclusion degree fluctuations in other decision goals. That is, the larger the variance, the greater gap on inclusion degree of the decision goal in other decision goals, and more unstable decision goal performance. The smaller the variance, the smaller gap on inclusion degree of the decision goal in other decision goals, and more stable decision goal performance. By using the variance to filter the decision goal, we select the smaller variance as the best decision goal.

**Definition 10:** Given the decision goal set is \( A = \{A_1, A_2, \ldots, A_m\} \), constraint condition set is \( C = \{C_1, C_2, \ldots, C_n\} \), \( D(A, A_i) \) is the inclusion degree of the decision goal \( A_i \) and \( A_j \), \( i, j = 1, 2, \ldots, m \). \( E_A \) represents the mean value of the inclusion degree of \( A_i \) in other decision goals. \( D_A \) represents the variance of \( A_i \), that is,

\[
E_A = \frac{1}{m} \left( \sum_{j=1}^{m} D(A_i, A_j) \right) \quad (3)
\]

\[
D_A = \frac{1}{2} \left( \sum_{j=1}^{m} (D(A_i, A_j) - E_A)^2 \right) \quad (4)
\]

From the above, we can obtain the decision based on the triangular interval-valued intuitionistic fuzzy soft set, the specific algorithm steps are as follows:

**Step1:** Given the decision goal set is \( A = \{A_1, A_2, \ldots, A_m\} \), constraint condition set is \( C = \{C_1, C_2, \ldots, C_n\} \), \( i, j = 1, 2, \ldots, m \), and successively calculate the inclusion degree \( D(A, A_i) \) of decision goal \( A_i \) in other decision goal \( A_j \).

**Step2:** Calculate the sum \( S_i \) of the inclusion degree of decision goal in other decision goals according to the formula (2).

**Step3:** Sequence \( S_i \), and check the maximum \( S_i \) whether there exists an equal situation. If the maximum \( S_i \) is unique, then turn to Step 4. If the maximum \( S_i \) exists two or more than two equivalence, then turn to Step 5.

**Step4:** Select the corresponding decision goal of the maximum \( S_i \) as the best decision goal.

**Step5:** If the maximum \( S_i \) exists two or more than two equivalence, then respectively calculate the mean values of their corresponding inclusion degree by using the formula (3), and then calculate their corresponding variances by using the formula (4).
Step 6: Because the larger the variance, the greater gap on inclusion degree of the decision goal in other decision goals, and more unstable decision goal performance. The smaller the variance, the smaller gap on inclusion degree of the decision goal in other decision goals, and more stable decision goal performance. Thus, we select the smaller variance as the best decision goal.

5. Case Analyses

To further verify the validity of the inclusion degree decision algorithm based on the triangular interval-valued intuitionistic fuzzy soft set, the following are combined with the selection and decision problems of the core enterprise strategic partners in supply chain to analyze examples. The supply chain management emphasizes enterprises in the supply chain to establish strategic partnership. It aims by improving the level of information sharing, to reduce inventory of the entire supply chain products, reduce costs and improve operational performance of the entire supply chain. However, the factors that affect the supply chain collaboration are multifaceted. Given the decision goal set is $A = \{A_1, A_2, A_3, A_4, A_5\}$, constraint condition set is $C = \{C_1, C_2, C_3\}$, the corresponding decision matrix is as follows:

**Table 3. Decision Matrix**

|        | $C_1$                  | $C_2$                  | $C_3$                  |
|--------|------------------------|------------------------|------------------------|
| $A_1$  | ([0.5,0.6,0.7],[0.2,0.2,0.3]) | ([0.6,0.7,0.8],[0.1,0.1,0.2]) | ([0.4,0.5,0.6],[0.3,0.3,0.4]) |
| $A_2$  | ([0.4,0.5,0.6],[0.1,0.2,0.3]) | ([0.5,0.6,0.7],[0.1,0.2,0.3]) | ([0.5,0.5,0.6],[0.2,0.3,0.3]) |
| $A_3$  | ([0.7,0.7,0.8],[0.1,0.1,0.2]) | ([0.5,0.6,0.7],[0.1,0.1,0.2]) | ([0.6,0.7,0.8],[0.1,0.1,0.2]) |
| $A_4$  | ([0.5,0.6,0.7],[0.1,0.2,0.2]) | ([0.3,0.3,0.4],[0.1,0.2,0.2]) | ([0.4,0.5,0.6],[0.2,0.3,0.4]) |
| $A_5$  | ([0.6,0.6,0.7],[0.1,0.2,0.3]) | ([0.6,0.7,0.8],[0.1,0.1,0.2]) | ([0.4,0.5,0.6],[0.2,0.3,0.4]) |

The corresponding decision table is in Table 4 below:

**Table 4. Decision Table**

|        | $C_1$                  | $C_2$                  | $C_3$                  |
|--------|------------------------|------------------------|------------------------|
| $A_1$  | ([0.5,0.6,0.7],[0.7,0.8,0.8]) | ([0.6,0.7,0.8],[0.8,0.9,0.9]) | ([0.4,0.5,0.6],[0.6,0.7,0.7]) |
| $A_2$  | ([0.4,0.5,0.6],[0.7,0.8,0.9]) | ([0.5,0.6,0.7],[0.7,0.8,0.9]) | ([0.5,0.5,0.6],[0.7,0.8,0.8]) |
| $A_3$  | ([0.7,0.7,0.8],[0.8,0.9,0.9]) | ([0.5,0.6,0.7],[0.8,0.9,0.9]) | ([0.6,0.7,0.8],[0.8,0.9,0.9]) |
| $A_4$  | ([0.5,0.6,0.7],[0.8,0.8,0.9]) | ([0.3,0.3,0.4],[0.8,0.9,0.9]) | ([0.4,0.5,0.6],[0.6,0.7,0.8]) |
| $A_5$  | ([0.6,0.6,0.7],[0.7,0.8,0.8]) | ([0.6,0.7,0.8],[0.8,0.9,0.9]) | ([0.4,0.5,0.6],[0.6,0.7,0.8]) |

According to Step 1, we draw the inclusion degree table as the following table 5.

**Table 5. Inclusion Degree Table**

|        | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|--------|-------|-------|-------|-------|-------|
| $A_1$  | $1$   | $\frac{16.8}{18}$ | $\frac{17.2}{18}$ | $\frac{16.1}{18}$ | $\frac{17.8}{18}$ |
| $A_2$  | $\frac{17.5}{18}$ | $1$   | $\frac{17.4}{18}$ | $\frac{16.9}{18}$ | $\frac{17.5}{18}$ |
| $A_3$  | $\frac{17.5}{18}$ | $\frac{17.0}{18}$ | $1$   | $\frac{16.4}{18}$ | $\frac{17.6}{18}$ |
| $A_4$  | $\frac{17.3}{18}$ | $\frac{17.4}{18}$ | $\frac{17.3}{18}$ | $1$   | $\frac{17.3}{18}$ |
| $A_5$  | $\frac{17.9}{18}$ | $\frac{16.9}{18}$ | $\frac{17.4}{18}$ | $\frac{16.2}{18}$ | $1$   |
According to Step 2, we draw the sum of the inclusion degree of each decision goal.

\[ S_1 = 4 + \frac{15.3}{18} \cdot 4, S_2 = 4 + \frac{15.5}{18} \cdot 4, S_3 = 4 + \frac{14.3}{18} \cdot 4, S_4 = 4 + \frac{15.3}{18} \cdot 4, S_5 = 4 + \frac{14.5}{18} \cdot 4. \]

Thus, sort as follows: \( S_5 > S_1 > S_4 > S_3 > S_2 \).

Obviously, the maximum \( s_i \) exists two or more than two equivalence, so turn to Step 5, then calculate the corresponding inclusion degree mean values of \( E_{A_i} \) and \( E_{A_j} \) in other decision goals:

\[ E_{A_1} = E_{A_2} = \frac{87.3}{90}. \]

Then calculate the each corresponding variance of \( E_{A_i} \) and \( E_{A_j} \).

\[ D_{A_i} = \frac{1}{5} \left( \sum_{j=4}^{5} (D(A_i, A_j) - E_{A_i})^2 \right) = \frac{153.4}{4000}, D_{A_j} = \frac{1}{5} \left( \sum_{j=4}^{5} (D(A_i, A_j) - E_{A_j})^2 \right) = \frac{93.3}{4000}. \]

Obviously, \( D_{A_i} > D_{A_j} \). Therefore, we select \( A_i \) as the best decision goal.

The results obtained in this paper are compared with the results of literature [7].

### Table 6. Result Analysis

| Literature [7] | This paper |
|----------------|------------|
| Decision Results | \( A_i \) | \( A_i \) |

Result Analysis:

Literature [7] is based on the structure of the ideal decision goal, and through the comparison of the decision goal of the inclusion degree size in the ideal decision goal, and selects the maximum inclusion degree as the best decision goal. However, if the maximum inclusion degree exists two or more than two equal situation, then this method is not able to determine which one is the best decision goal. This paper is based on the improved inclusion degree calculation formula, and calculates the sum \( s_i \) of each decision goal in other decision goals, and filter the best decision goal by the size of the inclusion degree sum \( s_i \). Specially, we introduce the mean value and the variance to process the maximum \( s_i \) exists two or more than two equivalence, and select the decision goal of the smaller variance as the best decision goal. Finally, the decision results in Literature [7] and this paper are the same, which shows that the multi-objective decision algorithm in this paper is correct and effective.

### 6. Conclusion

By combining the triangular fuzzy number with the intuitionistic fuzzy soft set, and expand into the triangular interval-valued intuitionistic fuzzy soft set, represent the membership and non-membership by triangular fuzzy numbers. It makes up the defects that the membership and the non-membership degree are lack of focus which is represented by interval number in interval-valued fuzzy soft set. Through the extension of the original inclusion degree function, it can be applied to the multi-objective attribute decision-making system of the triangular interval intuitionistic fuzzy soft set, which provides a new way of thinking and method for the multi-objective attribute decision.

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