Abstract

The 331 model, an extension of the standard electroweak theory to $SU(3)_L \times U(1)_X$, naturally predicts three families of quarks and leptons via the requirement of anomaly cancellation. This is accomplished by making one of the quark families transform differently from the other two, thus leading to flavor changing neutral currents. Using experimental input on neutral meson mixing, we show that the third family must be the one that is singled out, at least up to small family mixing. We additionally describe a convenient way to parametrize the new mixing matrix that plays a role in the gauge interactions of the ordinary quarks with the new 331 quarks.

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The 331 model is an $SU(3)_L \times U(1)_X$ extension of the standard $SU(2)_L \times U(1)_Y$ electroweak theory \cite{1,2}. This model attempts to answer the family replication question by relating the number of families to the number of colors via the requirement of anomaly cancellation. While anomalies cancel for each individual family in the Standard Model (SM), they only vanish in the 331 model when all three families are included. This novel method of anomaly cancellation requires that at least one family transforms differently from the others, thus breaking generation universality. A result of this is that the 331 model suffers from potentially large flavor changing neutral currents (FCNC). Unlike generic grand unified theories where FCNCs may be suppressed by large masses, there is an upper bound on the scale of 331 physics on the order of a few TeV \cite{2,3}.

The 331 model predicts several new gauge bosons beyond the SM. These are a new neutral gauge boson $Z'$ and a dilepton gauge boson doublet ($Y^{++}, Y^+$). Both the $Z'$ and new neutral scalars may have flavor changing interactions with the quarks. Since the leptons are generation universal, they couple diagonally to the $Z'$ (however tree level dilepton exchange may be lepton flavor violating \cite{4}). Thus $Z'$ FCNC is present only in the hadronic sector of the 331 model.

Previous analyses of $Z'$ FCNC contributions to neutral meson mass splittings have attempted to put a lower bound on the allowed $Z'$ mass \cite{1}. However, it has since been realized that unknown mixing parameters beyond the ordinary CKM matrix prevent one from making quantitative statements about such a lower bound \cite{3,5}. In this paper, we show that while $Z'$ FCNC constraints do not rule out the 331 model, the theoretical upper bound on the $Z'$ mass may instead be used to greatly restrict the unknown mixing parameters. This is essentially the opposite approach from that taken previously \cite{1,3,5}. Additionally, we clarify some of the confusion over whether the first or the third family of quarks must be taken to transform differently.

In order to understand the origin of the $Z'$ FCNC in the 331 model, we begin by describing the fermion representations. While all three lepton families are treated identically, anomaly cancellation requires that one of the three quark families transform differently from the other two \cite{1,2}. In particular, cancelling the pure $SU(3)_L$ anomaly requires that there are the same number of triplets as anti-triplets. Putting the three lepton families in as anti-triplets, and taking into account the three quark colors, we find that two families of quarks must transform as triplets and the third must transform as an anti-triplet.

In terms of weak eigenstates, we do not need to distinguish which family falls in the anti-triplet. However, as we demonstrate later, it is convenient to think of the different family as the third family. We thus denote the first two families as
\[ Q_{1,2} = \begin{pmatrix} u \\ d \\ D \end{pmatrix}_L , \begin{pmatrix} c \\ s \\ S \end{pmatrix}_L , \]

and the third family (anti-triplet) as

\[ Q_3 = \begin{pmatrix} b \\ -t \\ T \end{pmatrix}_L . \]

The sign ensures that the \( SU(2)_L \) quark doublet, when embedded in \( SU(3)_L \), has the conventional form.

Using the standard normalization of non-abelian generators, the hypercharge is embedded in \( SU(3)_L \times U(1)_X \) as \( Y/2 = \sqrt{3}T^8 + X \) where \( X \) is the \( U(1)_X \) charge of the representation (we use the conventions of [4]). The electric charge is then given by \( Q = T^3 + Y/2 \) so that \( Y \) corresponds to twice the average charge of \( SU(2)_L \) representations. From these relations between \( Q, Y \) and \( X \), we find that the electric charge of each component of \( SU(3)_L \) (anti-)triplets changes by exactly one unit and that the \( X \) charge is given by the average of the electric charges (which is just the electric charge of the middle component). Thus the triplets, \( Q_1 \) and \( Q_2 \) above, have \( X = -1/3 \) and the third quark family, \( Q_3 \), has \( X = 2/3 \).

There are then three new quarks which we denote \( D \) and \( S \) with electric charge \(-4/3\) and \( T \) with electric charge \( 5/3 \). Note that all fermion states given here are weak interaction eigenstates and must be related to physical (mass) eigenstates by the appropriate unitary transformations. Unlike the left-handed representations, all right handed quarks are incorporated as \( SU(3)_L \) singlets. As a result, the ordinary right handed quarks are generation universal and hence \( Z' \) FCNC is limited to the left handed interactions.

When \( 331 \) is broken to the SM, the neutral gauge bosons \( W^8_\mu \) and \( X_\mu \) mix to give the \( Z'_\mu \) and hypercharge \( B_\mu \) gauge bosons. This mixing may be parametrized by a 331 mixing angle \( \theta_{331} \) (generalizing the Weinberg angle) defined by [4]

\[ g' = \frac{1}{\sqrt{3}} g \cos \theta_{331} = \frac{1}{\sqrt{6}} g_X \sin \theta_{331} , \]

where \( g \) and \( g_X \) are the \( SU(3)_L \) and \( U(1)_X \) coupling constants, and the hypercharge coupling constant \( g' \) is given by \( \tan \theta_W = g' / g \). In terms of \( W^8_\mu \) and \( X_\mu \), the hypercharge and \( Z'_\mu \) gauge bosons are given by a rotation parametrized by \( \theta_{331} \)

\[ \begin{pmatrix} B_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_{331} & \sin \theta_{331} \\ -\sin \theta_{331} & \cos \theta_{331} \end{pmatrix} \begin{pmatrix} W^8_\mu \\ X_\mu \end{pmatrix} . \]

Since the \( Z' \) is a combination of \( W^8 \) and \( X \), it couples to fermions according to
\[ \mathcal{L} = \frac{g}{\sin \theta_{331}} Z'_\mu \bar{\psi} \gamma^\mu [\cos^2 \theta_{331} Y/2 - T^8] \psi. \]

Using \( \cos \theta_{331} = \sqrt{3} \tan \theta_W \), this may be rewritten as

\[ \mathcal{L} = \frac{g}{\cos \theta_W} \frac{1}{2\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}} Z'_\mu J^\mu Z', \]

where \( J^\mu Z' \) is given by

\[ J^\mu Z' = \bar{\psi} \gamma^\mu [3 \sin^2 \theta_W Y - 2\sqrt{3} \cos^2 \theta_W T^8] \psi. \]

Since the value of \( T^8 \) is different for triplets and anti-triplets, the \( Z' \) coupling to left-handed ordinary quarks is different for the third family and thus flavor changing. If we assume \( J^\mu Z' \) has a “standard” form for quark triplets, then the flavor changing interaction occurs for the third (weak-eigenstate) family and may be written as

\[ J^\mu Z'_{(FCNC)} = -2\sqrt{3} \cos^2 \theta_W \bar{q} \gamma_L \gamma^\mu [T^8(3^*) - T^8(3)] q = 2 \cos^2 \theta_W \bar{q} \gamma^\mu \gamma_L q, \]

for both up- and down-type quarks (\( \gamma_L = \frac{1}{2}(1 - \gamma^5) \) is the left-handed projection operator). Other than in the scalar sector, this is the only tree level FCNC interaction present since when 331 is broken to the SM, all three families of ordinary quarks are in usual \( SU(2)_L \) doublets and thus couple in the ordinary manner to the \( Z \) and photon.

The dilepton currents are also sensitive to the \( SU(3)_L \) structure of the quark representations, and hence the difference in the third family. However, with only ordinary external quarks, these dilepton effects first show up at loop level. Since tree level \( Z' \) FCNC presumably dominates over loop processes, a good place to study the effects of dilepton exchange on flavor changing interactions would be in the process \( b \to s \gamma \) which cannot occur at tree level.

After \( SU(2)_L \times U(1)_Y \) breaking, the weak eigenstate \( Z \) and \( Z' \) may mix, forming mass eigenstates \( Z_1 \) and \( Z_2 \). This mixing of the neutral gauge bosons may be parametrized by a mixing angle \( \phi \) so that

\[ \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix}. \]

A fit to precision electroweak observables gives a limit on the mixing angle of \(-0.0006 < \phi < 0.0042 \) and a lower bound on the mass of the heavy \( Z_2 \) of \( M_{Z_2} > 490 \text{GeV} \) (both at 90% C.L.) [3]. While this mass limit is not as strong as the indirect limit \( M_{Z_2} > 1.4 \text{TeV} \) given by the dilepton mass bound and the \( M_{Z_2} - M_Y \) relation of the minimal Higgs sector [4], it is
sensitive to the choice of Higgs representations and provides an independent lower bound on \( M_{Z_2} \).

Due to the mixing, the mass eigenstate \( Z_1 \) now picks up flavor changing couplings proportional to \( \sin \phi \). In particular, using (8) and (9), we may write

\[
\mathcal{L}_{\text{FCNC}} = \frac{g}{\cos \theta_W} \frac{1}{2 \sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}} (\sin \phi Z_{1\mu} + \cos \phi Z_{2\mu}) J_{Z(\text{FCNC})}^\mu .
\]

(10)

For sufficiently large mixing, the flavor changing \( Z_1 \) decays may be observable. However, because \( Z - Z' \) mixing is constrained to be very small, evidence of 331 FCNC can only be probed indirectly at present via the \( Z_2 \) couplings.

In order to examine the flavor changing \( Z' \) interaction given in (8), we need to relate weak and mass eigenstate quarks. Symmetry breaking and mass generation in the minimal 331 model is accomplished by four Higgs multiplets — the three triplets

\[
\Phi = \left( \Phi_Y \, \phi^0 \right) , \quad \phi = \left( \Phi_1 \, \Delta^- \right) \quad \phi' = \left( \Phi_{2} \, \rho^- \right) ,
\]

(11)

with \( X \) charges 1, 0, and \(-1\) respectively and a sextet \( H \) with \( X = 0 \). We have written the triplets in terms of \( SU(2)_L \) component fields: \( \Phi_Y = (\Phi_Y^+, \Phi_Y^0)^T \), the Goldstone boson doublet corresponding to the massive dileptons and \( \Phi_i = (\phi_i^+, \phi_i^0)^T \), which are SM Higgs doublets where \( \bar{\Phi}_i = i \tau^2 \Phi_i^* \). A third SM doublet arises from the sextet \( H \), but plays no role in generating quark masses.

The vacuum expectation value \( \langle \Phi \rangle \) breaks 331 and gives masses to the new quarks \( D, S, T \). The remaining scalars implement \( SU(2)_L \times U(1)_Y \) breaking and gives masses to the remaining fermions. In particular, the most general gauge invariant Yukawa couplings of the above scalars to the quarks may be written

\[
- \mathcal{L} = \overline{Q}_{Li} h_d^{ik} d_{Rk}^i \Phi + \overline{Q}_{Li} h_d^{3k} d_{Rk}^i \Phi'^* \\
+ \overline{Q}_{Lj} h_u^{ik} u_{Rk}^j \phi - \overline{Q}_{L3} h_u^{3k} u_{Rk}^j \phi'^* \\
+ \overline{Q}_{Li} h_D^{ij} D_{Rj}^* \Phi + \overline{Q}_{L3} h_T T_R \Phi^* + \text{h.c.} ,
\]

(12)

where \( i, j = 1, 2 \) runs through the first two families only and \( k = 1, 2, 3 \). As usual, the primes denote weak eigenstates. Since \( T \) is the only charge 5/3 quark, it is a simultaneous gauge and mass eigenstate.

When 331 is reduced to the SM, the Yukawa interactions may be written in terms of ordinary left-handed quark doublets \( q_{Li} = (u_i, d_i)^T_L \) and singlets. We separate \( \mathcal{L} \) into two pieces, \( \mathcal{L}_0 \) which contains only lepton number \( L = 0 \) scalars and \( \mathcal{L}_2 \) which has \( |L| = 2 \) scalars that change ordinary and new quarks into each other. We find...
Because the third family of quarks is treated differently, it has different couplings to scalars as well as the $Z'$. Thus natural flavor conservation [9] is necessarily violated in the 331 model, leading to potentially large flavor changing neutral Higgs (FCNH) processes in addition to $Z'$ FCNC.

If it were not for the third family, the ordinary quarks in $\mathcal{L}_0$ would have a normal two Higgs doublet coupling with separate Higgs couplings to up- and down-type quarks (usually referred to as model II). The third family, however, has a “flipped” coupling, with $\Phi_1$ and $\Phi_2$ exchanging roles. Including the $SU(3)_L$ sextet Higgs which give masses to the leptons, we end up with a three-Higgs doublet model, albeit with unusual Yukawa couplings dictated by the underlying $SU(3)_L$ theory.

Since the $Z'$ couples differently to the third weak-eigenstate family, $Z'$ FCNC occurs through a mismatch between weak and mass eigenstates. Since there are more states than in the SM, this mixing is described by more than just the CKM matrix. The charge 2/3, $-1/3$ and $-4/3$ mass matrices are diagonalized by three independent bi-unitary transformations which we denote by the $3 \times 3$ unitary matrices $U_{L,R}$ and $V_{L,R}$ and the $2 \times 2$ unitary matrices $W_{L,R}$ respectively. In the standard fashion, the ordinary CKM matrix is given by $V_{CKM} = U_L^\dagger V_L$. The new mixing shows up in both dilepton currents and the FCNC part of the $Z'$ interaction.

Because the first two families are generation symmetric, we may make a convenient choice of letting $D$ and $S$ be simultaneous gauge and mass eigenstates. This replaces the standard choice of using up-type quarks in this fashion which is no longer possible in this case. As a result, the charged currents in the quark sector and the $Z'$ FCNC interaction may be written

$$
-J^\mu_{W+} = \overline{u} \gamma^\mu \gamma_L V_{CKM} d
$$

$$
-J^\mu_{Y+} = \overline{d} \gamma^\mu \gamma_L V_L^D \left( \begin{array}{cc}
1 & 0 \\
0 & 1 \\
0 & 0
\end{array} \right) D + \overline{T} \gamma^\mu \gamma_L \left( \begin{array}{cc}
0 & 0 \\
0 & 1
\end{array} \right) U_L u
$$

\[13\]
in a matrix notation where $D = (D, S)^T$. If we had not initially picked $D$ to be generation diagonal, we could simply have absorbed the unitary matrix $W_L$ into a redefinition of $U_L$ and $V_L$.

Unlike the SM where only $V_{CKM}$ is physical, there is additional freedom in the mixing present above [3]. Although we have introduced three matrices in (14), they are not independent but are related by $V_{CKM} = U_L^\dagger V_L$. Since flavor changing interactions involving down-type quarks have been studied the most extensively, we find it convenient to specify the two unitary matrices $V_{CKM}$ and $V_L$. As usual, $V_{CKM}$ contains three angles and one complex phase. $V_L$ is specified by three angles and three phases since we may remove three phases from the general unitary matrix by appropriately transforming the three new quarks.

In the absence of CP violating phases, the three angles of $V_L$ have a simple interpretation. We may use a CKM like parametrization

$$V_L = \left(\begin{array}{ccc} v_{1d} & v_{1s} & v_{1b} \\ v_{2d} & v_{2s} & v_{2b} \\ v_{3d} & v_{3s} & v_{3b} \end{array}\right) = \left(\begin{array}{ccc} c_{12}c_{13} & -s_{12}c_{23} & c_1c_2s_{13} \\ s_{12}c_{13} & c_{12}c_{23} & -c_1c_2s_{13} \\ s_{13} & s_{23}c_{13} & c_{23}c_{13} \end{array}\right), \quad (15)$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. Since the third row corresponds to the anti-triplet weak eigenstate, $\theta_{13}$ and $\theta_{23}$ specify which down-type quark is in the anti-triplet and, orthogonal to that, $\theta_{12}$ specifies the mixing between the first two triplets (i.e. $D$ and $S$).

Previous examinations of the $Z'$ in the 331 model have concentrated on putting lower bounds on $M_{Z_2}$ [1,2] to prevent excessive tree-level FCNC. The drawback to this approach is that the new mixing specified by $V_L$ is in principle unknown and has to be estimated. Here, we instead use the upper limit $M_{Z_2} < 2.2 \text{GeV}$ [3] to place restrictions on $V_L$.

The strongest constraints on tree level $Z'$ FCNC come from neutral meson mixing. For the neutral kaon system, the tree level $\Delta S = 2$ interaction is given from (8) and (14) by

$$H_{\text{eff}}^{\Delta S=2} = \frac{g^2 c^2}{12(1 - 4s^2)} (v_{3d} v_{3d})^2 \left( \frac{\cos^2 \phi}{M_{Z_2}^2} + \frac{\sin^2 \phi}{M_{Z_1}^2} \right) [\bar{\pi} \gamma^\mu \gamma_L d][\bar{\pi} \gamma^\mu \gamma_L d] \quad (16)$$

where $s = \sin \theta_W$ and $c = \cos \theta_W$. In addition to the SM box diagram and possible long distance effects, this contributes a term...
∆m_K = \frac{2\sqrt{2}G_F}{9} \frac{e^4}{(1 - 4s^2)} |v_{3s}^* v_{3d}|^2 \left( \eta_{Z_2} \cos^2 \phi \frac{M_{Z_2}}{M_{Z_1}} + \eta_{Z_1} \sin^2 \phi \right) B_K f_K^2 m_K \ , \quad (17)

to the K^0–\bar{K}^0 mass difference [10]. We have included the leading order QCD corrections through the parameters \( \eta_{Z_2} \approx 0.55 \) and \( \eta_{Z_1} \approx 0.61 \). \( B_K \) and \( f_K \) are the bag parameter and decay constant of the kaon. Similar equations hold for \( D^0–\bar{D}^0 \) and \( B^0–\bar{B}^0 \) mixing.

Because the \( Z–Z' \) mixing angle \( \phi \) is very small, the first term in the parentheses dominates and \( \sin^2 \phi \) may safely be neglected.

The present limits on neutral meson mixing are given by [12]

\[
\begin{align*}
K^0–\bar{K}^0 & \quad \Delta m = (3.522 \pm 0.016) \times 10^{-12} \text{MeV} \\
D^0–\bar{D}^0 & \quad < 1.3 \times 10^{-10} \text{MeV} \\
B^0–\bar{B}^0 & \quad = (3.6 \pm 0.7) \times 10^{-10} \text{MeV} .
\end{align*}
\] (18)

Although there is considerable uncertainty in the heavy meson decay constants, this has little effect on the results. We use

\[
\begin{align*}
\sqrt{B_K f_K} & = 135 \pm 19 \text{MeV} \\
\sqrt{B_D f_D} & = 187 \pm 38 \text{MeV} \\
\sqrt{B_B f_B} & = 208 \pm 38 \text{MeV} .
\end{align*}
\] (19)

The kaon quantity comes from \( f_K = 161 \text{MeV} \) and \( B_K = 0.7 \pm 0.2 \). The heavy meson decay constants are taken from a lattice calculation, Ref. [13], where all reported errors are added in quadrature and \( B_D = B_B = 1 \).

Because there are various sources that may contribute to the mass difference, \( \Delta m \), it is impossible to disentangle the tree level \( Z' \) contribution from other effects. However, barring any unexpected cancellations, it is reasonable to expect that \( Z' \) exchange contributes a \( \Delta m \) no larger than the observed values. Using the upper limit, \( M_{Z_2} < 2.2 \text{TeV} \), we find, from the \( K^0, D^0 \) and \( B^0 \) system, respectively

\[
\begin{align*}
|v_{3s}^* v_{3d}| & < 5.0 \times 10^{-3} \\
|u_{3c}^* u_{3u}| & < 10.8 \times 10^{-3} \\
|v_{3b}^* v_{3d}| & < 8.7 \times 10^{-3} ,
\end{align*}
\] (20)

(at 90\%C.L.). \( u_{3i} \) are components of the third row of \( U_L \), the rotation matrix in the up-quark sector, and are given by \( u_{3i} = v_{3j} V_{CKM}^{*ij} \).

It should now be apparent why we have chosen to parametrize the new mixing by \( V_L \). In this case, we make it easy to describe FCNC in the more interesting \( K^0 \) and \( B^0 \) systems.
at the expense of the $D^0$. In the parametrization of $V_L$ given by Eq. (15), $v_{3i}$ is determined (in magnitude) by two angles, $\theta_{13}$ and $\theta_{23}$. Since $|v^*_{3i}v_{3i}| = \frac{1}{2} |\sin \theta_{23} \sin 2\theta_{13}|$ and $|v^*_{3d}v_{3d}| = \frac{1}{2} |\cos \theta_{23} \sin 2\theta_{13}|$, we immediately determine from (20) that $|\sin 2\theta_{13}| < 0.020$, giving $|\theta_{13} + n\pi/2| < 0.010$. This allows two types of solutions, either $|v_{3d}| \approx 0$ (the second or third family is the anti-triplet) or $|v_{3d}| \approx 1$ (the first family is the anti-triplet).

In order to restrict these cases further, we must relate $u_{3i}$ to $v_{3i}$ and take the limit on $D^0$ mixing into account. This requires knowledge of $V_{CKM}$ and possible new CP violating phases as well. We find that in order to satisfy all three conditions of (20) simultaneously, only $|v_{3d}| \approx 0$ is allowed. Restricted to the first quadrant, the limits on $\theta_{ij}$ are

$$\theta_{13} < 0.010, \quad \theta_{23} < 0.26,$$

which means $|v_{3b}| \approx 1$ and hence that the third family must be the anti-triplet (up to small mixing).

There has been some confusion over the issue of whether the first family or the third family must be treated differently in order to sufficiently suppress the $Z'$ FCNC $^{1,3,5}$. Obviously, in terms of weak eigenstates, it makes no difference which family is assigned to the anti-triplet. In terms of mass eigenstates, the anti-triplet has been unitarily transformed into some combination of all three families. However, physically, the almost-diagonal CKM matrix tells us that it makes sense to group mass eigenstates into families. It is in this manner that we may say the third family must be the one that is different. The reason this choice is forced on us is because the Cabibbo angle, $\sin \theta_C \approx 0.22$, is the largest off-diagonal element of $V_{CKM}$, and hence the $\Delta S = 2$ and $\Delta C = 2$ FCNC limits cannot be simultaneously satisfied unless the anti-triplet is in the third family.

When $B_s^0$ mixing is measured, it will put further stronger restrictions on $\theta_{23}$. In the SM, $\Delta m_{B_d}/\Delta m_{B_s} \sim |V_{CKMtd}/V_{CKMts}|^2$ so $B_s^0$ mixing is expected to be large. Although this box diagram contribution is still present in the 331 case, if we assume that the tree level process dominates, we find instead $\Delta m_{B_d}/\Delta m_{B_s} \sim |v_{3d}/v_{3s}|^2 = |\tan \theta_{13}/\sin \theta_{23}|^2$. Depending on the new mixing angles, the $Z'$ contribution to $B_s^0$ mixing may be large or small. Even if this mixing turns out to be unexpectedly small, it will not rule out the 331 model. Because of the additional freedom present in $V_L$, there is a possibility that tree level $Z'$ exchange has the opposite phase as the SM box diagram, and hence would suppress the large SM contribution to $\Delta m_{B_s}$. This intriguing possibility of small $B_s^0$ mixing would present clear evidence of physics beyond the SM, including possible support for the 331 model.

Tree level $Z'$ exchange also contributes to $\Delta S = 1$ FCNC processes such as $K \to \pi \nu \overline{\nu}$. We find
\[
\frac{\text{BR}(K^+ \to \pi^+ \nu \overline{\nu})}{\text{BR}(K^+ \to \pi^0 e^+ \nu_e)} = \frac{c^4}{3} \frac{|v_{3s}^* v_{3d}|^2}{|V_{CKM \text{ us}}|^2} \left( \cos^2 \phi \frac{M_{Z_2}^2}{M_{Z_2}^2 + \sin^2 \phi} \right)^2 .
\] (22)

Since \(\text{BR}(K^+ \to \pi^+ \nu \overline{\nu}) < 1.7 \times 10^{-8}\), we use the upper bound on \(M_{Z_2}\) to find \(|v_{3s}^* v_{3d}| < 0.18\) which is a weaker limit than that from \(K^0 - \overline{K^0}\) mixing, Eq. (21). Similar considerations hold for the rare decay \(K^0_L \to \mu^+ \mu^-\). However, it is theoretically harder to treat because of long-distance contributions. The reason such semi-leptonic decays do not give strong mixing constraints is that the \(Z'\) is only weakly coupled to the leptons.

While the above processes occur at tree level via \(Z'\) exchange, the rare decay \(b \to s \gamma\) must still proceed at one-loop. In the 331 model, in addition to the SM \(W\) penguin, this may occur via \(Z'\) and \(Y\) penguins. Although the SM contribution is GIM suppressed, this is no longer the case for both 331 contributions. One might worry that this would lead to too large a rate for \(b \to s \gamma\). However, the non-GIM suppressed contributions are proportional to new mixing given by \(v_{3b}^* v_{3s}\), which may be sufficiently small to prevent conflict with experiment. This is currently under investigation.

In conclusion, FCNC occurs at tree level in the 331 model because of the \(Z'\), which couples differently to triplets and anti-triplets. In order to describe the flavor changing \(Z'\) interaction, we need to understand family mixing in the quark sector, which is complicated by the presence of the new quarks. In addition to the ordinary CKM matrix, three more angles and three new phases are required to describe the mixing between ordinary and new quarks. Although we have not focused on the three new CP violating phases, they may lead to striking predictions beyond the SM and deserve further investigation.

We find that the only way to satisfy the experimental constraints on FCNC is to make the third family transform differently from the other two (up to small mixing). The reason behind singling out the third family is that it has the smallest couplings to the other two families — the Cabibbo angle mixing is sufficiently large that it forces the first two families to be treated identically. Because of the almost diagonal family structure, it makes physical sense to group either weak or mass eigenstate quarks into corresponding families. This is why it is convenient to think of the third family as unique, even in terms of weak eigenstates, although technically it makes no difference.

Going back to the quark Yukawa couplings, (13), we note that since the Higgs couplings to the third family are different, FCNH will occur in the scalar sector. However, the \(Z'\) FCNC constraint, (22), will simultaneously suppress FCNH by restricting the third family to be almost diagonal. Thus the SM Yukawa interactions are similar to that of the two-Higgs doublet model II with the exception that \(t\) and \(b\) get their masses from the opposite Higgs doublet as for the first two families.
Because of the unique feature that there is an upper bound on the unification scale, the 331 model is highly predictive. It is remarkable that in this model, there is just enough freedom to eliminate large FCNC, and the result of this is to constrain the third family to be the one that is different. In turn, this may give us some indication of why the top quark is so heavy and may present a new approach to the question of fermion mass generation.

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REFERENCES

[1] F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992).

[2] P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992).

[3] D. Ng, *The Electroweak Theory of SU(3) × U(1)*, TRIUMF preprint TRI-PP-92-125, hep-ph/9212284 (revised, August 1993).

[4] J. T. Liu and D. Ng, *Lepton flavor changing processes and CP violation in the 331 model*, Texas A&M University preprint CTP-TAMU-77/93 (December 1993).

[5] F. Pisano and V. Pleitez, *Flavor changing neutral currents in SU(3) ⊗ U(1) models*, preprint hep-ph/9307265.

[6] J. T. Liu and D. Ng, *Z–Z’ mixing and oblique corrections in an SU(3) × U(1) model*, preprint hep-ph/9302271, to appear in Z. Phys. C (June 1993).

[7] P. H. Frampton, J. T. Liu, D. Ng and B. C. Rasco, *Phenomenology of the SU(3)c × SU(3)L × U(1)X Model of Flavor*, University of North Carolina preprint IFP-466-UNC, hep-ph/9304294 (August 1993).

[8] R. Foot, O. F. Hernández, F. Pisano and V. Pleitez, Phys. Rev. D 47, 4158 (1993).

[9] S. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977).

[10] For this discussion we ignore CP violating effects, but note that \( H^{\Delta S=2}_{\text{eff}} \) may also give rise to a large \( \epsilon \) parameter, depending on unknown phases.

[11] F. J. Gilman and M. B. Wise, Phys. Rev. D 27, 1128 (1983).

[12] Particle Data Group, *Review of Particle Properties*, Phys. Rev. D 45, part 2, pp. II.10–17 (1992).

[13] C. W. Bernard, J. N. Labrenz and A. Soni, *A Lattice Computation of the Decay Constants of B and D Mesons*, preprint hep-lat/9306009 (June 1993).

[14] M. S. Atiya et al., Phys. Rev. D 48, R1 (1993).

[15] R. Ammar et al. (CLEO Collaboration), Phys. Rev. Lett. 71, 674 (1993).

[16] J. Agrawal, P. H. Frampton and J. T. Liu, work in progress.