The Phase Diagram of a U(1) Higgs-Yukawa Model at Finite $\lambda$

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Abstract

In this paper we investigate how the phase diagram of a U(1) symmetric Higgs-Yukawa system depends on the scalar self coupling $\lambda$. The phase diagram of similar models with continuous symmetry were extensively studied in the infinite scalar self coupling $\lambda = \infty$ limit. Recent analytical and numerical calculations at zero self coupling showed qualitatively different phase diagram, raising the question of the $\lambda$ dependence of the phase diagram. Here we use analytical (large $N_f$, perturbative and mean field) approximations as well as numerical simulations to investigate the system.

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1 Introduction

The non-perturbative studies of the Higgs-Yukawa systems were motivated by the ever increasing top quark mass limit and the triviality problem of the Standard Model. The phase diagram investigations of four dimensional Higgs-Yukawa models produced several surprising non-perturbative results in recent years [1].

In the weak Yukawa coupling region a Higgs-Yukawa model with continuous symmetry can be in a ferromagnetic (FM) phase, a symmetric (SYM) phase or an antiferromagnetic (AFM) phase, depending on the scalar hopping parameter $\kappa$ values. At large positive $\kappa$ values the continuous symmetry is spontaneously broken and the system is in the FM phase. The model has a massive scalar particle, one or more massless Goldstone bosons and fermions with mass generated via the spontaneous symmetry breaking of the scalar field. For small $\kappa$ values the system is in the SYM phase, in which the theory contains degenerate massive scalars and massless fermions (assuming there are no bare fermion mass terms in the action). The FM and SYM phases are separated by a second order phase transition line. The critical behavior of the model along this FM-SYM phase transition line is expected to be governed by the perturbative Gaussian fixed point, where both the scalar and Yukawa couplings are marginally irrelevant. In the infinite cut-off limit the renormalized couplings vanish, $y_R = 0, \lambda_R = 0$. As the pure scalar model is “trivial”, i.e. it has no other fixed point than the perturbative one, similar behavior is expected even for strong scalar coupling as long as the Yukawa interaction is weak. At large negative $\kappa$ values the model is in the AFM phase. It is generally believed that this phase is separated from the symmetric phase by a second order phase transition line the same way as the ferromagnetic and symmetric phases are in the positive hopping parameter region.

For large Yukawa couplings the situation is quite different. Quenched and unquenched Monte Carlo simulations of the model with naive fermions, supported by strong Yukawa coupling expansion, revealed the existence of non-perturbative symmetric and broken phases in the large Yukawa coupling region. These phases are separated by a second order phase transition line where the fermions of the model have large (at the order of the cut-off) mass, the fermions decouple in the continuum limit leaving a non-interacting scalar theory behind.

Several numerical calculations investigated Higgs-Yukawa systems recently [1]. Apart from studies of the $Z_2$ discrete symmetry model [2], all considered the limit of infinite scalar coupling. The simulations for both the $U(1)$ and $SU(2)$ symmetric systems observed the phase diagrams in agreement with the above descriptions. With naive fermions the phase diagrams showed perturbative and non-perturbative (strong Yukawa coupling) SYM, FM and AFM phases. In addition a ferrimagnetic (FI) phase was found numerically at the intermediate Yukawa coupling values, where both the magnetization and staggered magnetization are finite [3]. All phase boundaries were claimed to be second order. There exists a special point in the phase diagram where three phases, symmetric, ferromagnetic and ferrimagnetic coexist. It was speculated that this point could be a non-trivial fixed point where the critical behavior of the system might change. This scenario however could not be confirmed by Monte Carlo simulations [4]. It was found that the critical behavior of the system very close
to this point is still consistent with the perturbative predictions.

The vanishing scalar coupling \( \lambda \) limit received attention last year when it was shown in the large fermion number \((N_f)\) limit that the Higgs-Yukawa model is equivalent (up to inverse cut-off corrections) to the generalized Nambu-Jona-Lasinio type models [4]. The phase diagram was calculated exactly in the large \( N_f \) limit for arbitrary Yukawa and vanishing scalar couplings. Numerical simulations agreed surprisingly well with the large \( N_f \) predictions even for \( N_f = 2 \). However this phase diagram turned out to be very different from the infinite scalar coupling case. A strong first order phase transition line was observed at \( \lambda = 0 \). It was found that the FM to SYM phase transition line, which is relevant to the Standard Model physics, stops at the first order phase transition line. There is no sign for non-trivial critical behavior along this line. All the points on it belong to the attractive domain of the Gaussian fixed point. The FI phase, observed at \( \lambda = \infty \) [3], does not exist at \( \lambda = 0 \).

This paper is our first attempt to understand how the phase diagram and the critical properties of the \( U(1) \) invariant Higgs-Yukawa model changes from \( \lambda = 0 \) to the \( \lambda = \infty \) limit. In section 2 we study the Higgs-Yukawa model under various theoretical approximations and a typical phase diagram for small \( \lambda \) is discussed. In section 3 we present our numerical results for \( \lambda \leq 1 \) and compare them with the theoretical expectations. In section 4 we conclude this study and discuss the \( \lambda = \infty \) limit briefly.

# Analytical Considerations

## 2.1 The model

The lattice action for the \( U(1) \) chiral invariant Higgs-Yukawa model with naive fermions is defined as

\[
S = S_f + S_H .
\]

The fermion part of the action \( S_f \) is given by

\[
S_f = \sum_{x,z} \bar{\psi}_i(x) M(x,z) \psi_i(z) \quad , \quad i = 1, 2, ..., N_f/2 .
\]  

In eqn. (2) the fermion matrix may be written as

\[
M(x,z) = \sum_\mu \gamma_\mu [\delta_{x+\mu,z} - \delta_{x-\mu,z}] + y [\phi_1(x) + i\gamma_5\phi_2(x)] \delta_{x,z} ,
\]

where \( \gamma_\mu, \gamma_5 \) are the Hermitian Dirac matrices and \( y \) stands for the Yukawa coupling. The scalar part of the action \( S_H \) in eqn. (1) is given by

\[
S_H = -\kappa \sum_{x,\mu} \phi_a(x) [\phi_a(x + \mu) + \phi_a(x - \mu)] + \sum_x \phi_a^2(x) + \sum_x \lambda \left[ \phi_a^2(x) - 1 \right]^2 , \quad a = 1, 2 .
\]
In the numerical simulations, we use $detM^\dagger M$ for the fermion matrix to keep the partition function positive definite. This is equivalent to including an extra fermion species in the action eqn. (1).

### 2.2 Large $N_f$ limit

The model defined in eqns. (1-4) can be solved in the large fermion number $N_f$ limit. The $\lambda = 0$ case was discussed in Ref [4], and now we include the quartic term in the analysis.

It is convenient to consider a modified form of the scalar action

\[
S_H = -\kappa_N \sum_{x,\mu} \varphi_a(x) [\varphi_a(x + \mu) + \varphi_a(x - \mu)] + \sum_x \varphi_a^2(x) + \lambda_N \left( \varphi_a^2(x) - N_f \right)^2 .
\]

(5)

The usual lattice action eqns. (1-4) is obtained by identifying

\[
\kappa_N = C^2 \kappa , \quad \lambda_N = C^4 \lambda , \quad y_N = Cy ,
\]

where the factor $C$ satisfies the equation

\[
C^4 - (1 - 2\lambda_N N_f)C^2 - 2\lambda_N = 0
\]

(7)

The scalar field $\varphi_a(x)$ is related to the original field $\phi_a(x)$ by

\[
\varphi_a(x) = \phi_a(x)/C .
\]

(8)

In the $1/N_f$ expansion the couplings $\tilde{y}_N = \sqrt{N_f}y_N$, $\tilde{\lambda}_N = N_f\lambda_N$ are kept fixed to be $O(1)$. As $N_f \to \infty$ the relations in eqn. (6) simplify, giving

\[
\kappa = \frac{\kappa_N}{1 - 2\lambda_N} , \quad \lambda N_f = \frac{\tilde{\lambda}_N}{(1 - 2\lambda_N)^2} , \quad y \sqrt{N_f} = \frac{\tilde{y}_N}{\sqrt{1 - 2\lambda_N}} , \quad \tilde{\lambda}_N < \frac{1}{2} .
\]

(9)

In the large $N_f$ limit the constant mode of the scalar field dominates the path integral which suggests the Ansatz

\[
\varphi_1(x) = \sqrt{N_f} (a + (-1)^{x^r}b) , \quad \varphi_2(x) = 0 ,
\]

(10)

where $\sqrt{N_f}a$ and $\sqrt{N_f}b$ correspond to the magnetization and staggered magnetization, respectively. The effective potential at leading order is

\[
\frac{1}{N_f} V_{eff}(a, b) = -8\kappa_N(a^2 - b^2) + (a^2 + b^2) + \tilde{\lambda}_N \left( (a^2 + b^2 - 1)^2 + 4a^2b^2 \right)
\]

\[
- 2 \int \frac{d^4k}{(2\pi)^4} \log \left[ \sum_{\mu} \sin^2 k_\mu + \tilde{y}_N^2(a^2 - b^2) \right] .
\]

(11)
Minimizing $V_{eff}(a,b)$ with respect to $a, b$ can give in general four types of solutions:

(1) Symmetric (SYM) solution: $a = 0$, $b = 0$.

(2) Ferromagnetic (FM) solution: $a \neq 0$, $b = 0$. The magnetization $a$ is determined by the equation of state

$$\frac{1}{N_f} \frac{\partial V_{eff}}{\partial a} \bigg|_{b=0} = 2a(1 - 8\kappa_N + 2(a^2 - 1)\lambda_N - 2\tilde{y}_N^2 I(\tilde{y}_N^2 a^2)) = 0 , \quad (12)$$

where the integral $I$ is defined by

$$I(x) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{\sum_{\mu} \sin^2 k_{\mu} + x} . \quad (13)$$

(3) Antiferromagnetic (AFM) solution: $a = 0$, $b \neq 0$. The staggered magnetization $b$ is given by

$$\frac{1}{N_f} \frac{\partial V_{eff}}{\partial b} \bigg|_{a=0} = 2b(1 + 8\kappa_N + 2(b^2 - 1)\lambda_N + 2\tilde{y}_N^2 I(-\tilde{y}_N^2 b^2)) = 0 . \quad (14)$$

(4) Ferrimagnetic (FI) solution: $a \neq 0, b \neq 0$. It is straightforward to show that this solution can exist only when $\lambda_N > 1/2$. However, the relation given in equation (13) becomes invalid for $\lambda_N > 1/2$. Actually it is easy to find out from equation (13) that $\lambda_N > 1/2$ corresponds to the intermediate and strong $\lambda$ region for the original lattice action eqn. (1), which we will investigate in a future study. Thus the large $N_f$ calculation indicates that at least there is no FI phase in the weak $\lambda$ region.

In some parameter range several solutions may coexist. It is a simple numerical exercise to evaluate the effective potential and find the solution that gives the absolute minimum.

Although the above approach is exact in the limit $N_f \to \infty$ with fixed $\lambda_N$ and $\tilde{y}_N$, it only describes the small $\lambda(= O(1/N_f))$ and $y(= O(1/\sqrt{N_f}))$ region of the original model eqn. (1). For large $y$ values another type of large $N_f$ expansion is possible [5]. We start with the action eqns. (3) and (5), only now assuming that $y_N \sim O(1)$ and $\kappa_N \sim O(1/N_f)$. After integrating out the fermions, we get an effective action for the scalar variables

$$S_{eff} = S_H - \frac{N_f}{2} tr \log MM^\dagger , \quad (15)$$

where $S_H$ and $M$ are given by eqns. (3) and (5), respectively. One may expand the fermion determinant in powers of $1/N_f$. The leading term in $1/N_f$ fixes the amplitude of $\varphi_a(x)$

$$\varphi_a(x) = \varphi_0 \sigma_a(x) , \quad 1 + 2\lambda_N \left( \frac{\varphi_0^2}{N_f} - 1 \right) - \frac{2N_f}{\varphi_0^2} = 0 , \quad (16)$$

where $\sigma_a(x)$ is a two-component field with unit length. At next to leading order in $1/N_f$, the effective action eqn. (15) becomes, up to an additive constant, an effective 4-dimensional XY-model

$$S_{eff} = -\kappa_{eff} \sum_{x,\mu} \sigma_a(x) \left[ \sigma_a(x + \mu) + \sigma_a(x - \mu) \right] . \quad (17)$$
The effective hopping parameter $\kappa_{\text{eff}}$ is given by

$$\kappa_{\text{eff}} = \kappa_N \phi_0^2 + \frac{N_f}{2y_N^2 \phi_0^2}.$$  \hspace{1cm} (18)

The XY-model is known to have second order phase transitions at $\kappa_{\text{eff}} \approx 0.15$ (FM-SYM) and $\kappa_{\text{eff}} \approx -0.15$ (AFM-SYM). Thus eqn. (18) predicts the existence of two second order phase transition lines for large $y_N$.

Combining the above two large $N_f$ expansion results, we plot the phase diagram for fixed $\tilde{\lambda}_N (= 0.1)$ in Fig. 1. We find the following:

1. An FM-SYM second order phase transition line (AB). It is described by eqn. (12) in the limit $a \rightarrow 0$.

2. An AFM-SYM second order phase transition line (CD). It is given by eqn. (14) in the limit $b \rightarrow 0$.

3. A first order phase transition line (EDBF). On this line the effective potential has an AFM minimum in coexistence with either another AFM, SYM, or FM minimum. The staggered magnetization is discontinuous along the line DE but it is finite on both sides of the phase transition line. The discontinuity decreases from D to E and becomes zero at the point E. Therefore the AFM phases in the small and large $y_N$ regions are analytically connected. For increasing $\lambda_N$ values the position of the DE segment moves to larger $y_N$ and the end point E moves to more negative $\kappa_N$ value. As $\lambda_N \rightarrow 0$ the line CD disappears and the point E coincides with the point C as given in Ref [5].

4. A second order FM-SYM phase transition line (GH) given by eqn. (18) setting $\kappa_{\text{eff}} = 0.15$.

5. A second order AFM-SYM phase transition line (IJ) given by eqn. (18) setting $\kappa_{\text{eff}} = -0.15$.

The region between F,K,I,G is outside the validity of both large $N_f$ expansions. The order of phase transitions for line FK,KI, and KG can be only determined by numerical simulations. The second order FM-SYM transition line AB ends on a first order phase transition line but the first order line does not get critical at this point. The whole AB line, including the endpoint B, is in the domain of attraction of the Gaussian fixed point. No new non-trivial fixed point is found in this region.

Although the above phase diagram is obtained in the large $N_f$ expansions, the bare perturbation calculation in the small $y$ region predicts the same structure for finite $N_f$ as shown in Fig. 2a.
2.3 Mean field calculation

When $\lambda$ is relatively large, both large $N_f$ expansions and the bare perturbation calculation become invalid. For a qualitative phase diagram, one may use mean field calculations. Here we follow the well known [7, 8, 6, 2] saddle point type mean field approximation.

Because of the $U(1)$ chiral symmetry, one may choose the Ansatz for the saddle point

$$
\phi_1(x) = a + (-1)^{x^a} b, \quad \phi_2(x) = 0 ,
\quad h_1(x) = h + (-1)^{x^a} h_{st}, \quad h_2(x) = 0 ,
$$

where $h_1(x), h_2(x)$ are the auxiliary fields introduced in the mean field calculation. The saddle point conditions define $h$ and $h_{st}$ as implicit functions of the magnetization $a$ and the staggered magnetization $b$

$$
a + \frac{1}{2} u'(h + h_{st}) + \frac{1}{2} u'(h - h_{st}) = 0 ,
$$

$$
b + \frac{1}{2} u'(h + h_{st}) - \frac{1}{2} u'(h - h_{st}) = 0 .
$$

where $u(x)$ is a function defined by

$$
\exp\{u(x)\} = \int d\rho d\theta \exp \{-V(\rho) - \rho x \cos \theta\} ,
$$

and $V(\rho) = \rho^2 + \lambda(\rho^2 - 1)^2$.

With this Ansatz, the effective potential of the system has the form

$$
\begin{align*}
-V_{MF}(a,b) &= 8\kappa(a^2 - b^2) + 2N_f \int \frac{d^4k}{(2\pi)^4} \ln \left[ \sum_{\mu} \sin^2 k_{\mu} + y^2(a^2 - b^2) \right] \\
&+ ha + h_{st}b + \frac{1}{2} u(h + h_{st}) + \frac{1}{2} u(h - h_{st}) .
\end{align*}
$$

Minimizing $V_{MF}(a,b)$ with respect to $a,b$ gives three possible phases

1. SYM phase: $a = 0, h = 0, b = 0, h_{st} = 0$.

2. FM phase: $a \neq 0, h \neq 0, b = 0, h_{st} = 0$. $a$ and $h$ satisfy the equations

$$
a + u'(h) = 0 ,
$$

$$
16\kappa a + 4N_f y^2 a \int \frac{d^4k}{(2\pi)^4} \frac{1}{\sum_{\mu} \sin^2 k_{\mu} + y^2a^2} + h = 0 .
$$

3. AFM phase: $a = 0, h = 0, b \neq 0, h_{st} \neq 0$. $b$ and $h_{st}$ satisfy the equations

$$
b + u'(h_{st}) = 0 ,
$$
\[ -16\kappa b - 4N_f y^2 b \int \frac{d^4k}{(2\pi)^4} \frac{1}{\sum_{\mu} \sin^2 k_{\mu} - y^2 b^2} + h_{st} = 0. \] (27)

Using the fact that \( u'(x) \) is an odd and monotonically increasing function of \( x \), one can show that the FI \((a \neq 0, b \neq 0)\) solution is excluded in the mean field approximation.

Similar to the large \( N_f \) calculation, in some parameter range several solutions may coexist. Again one needs to evaluate the effective potential and find the solution that gives the absolute minimum.

The above mean field calculation breaks down in the large \( y \) region due to the following simple reasoning: When \( y \) is large, the fermion determinant can be expanded and to leading order in \( 1/y^2 \) we have an effective action

\[
S[\phi] = -\kappa \sum_{x,\mu} [\rho(x)\rho(x + \mu) \sigma_a(x)\sigma_a(x + \mu) + (\mu \rightarrow -\mu)]
+ \sum_x \left\{ \rho(x)^2 + \lambda[\rho(x)^2 - 1]^2 - 2N_f \log \rho^2(x) \right\},
\]

where the radial and angular notation for the \( \phi \) field is used

\[
\phi_1(x) = \rho(x)\sigma_1(x), \quad \sigma_1(x) = \cos(\theta(x)),
\]
\[
\phi_2(x) = \rho(x)\sigma_2(x), \quad \sigma_2(x) = \sin(\theta(x)).
\]

If we stay on the \( \kappa = 0 \) axis, the system becomes a collection of uncorrelated rotators and the U(1) chiral symmetry will be unbroken. An expansion around the \( \kappa = 0 \) axis will have a finite radius of convergence. Thus by analytical continuation we expect the system to be in the symmetric phase in the large \( y \) region around \( \kappa = 0 \). However, this symmetric phase in the large \( y \) region is not predicted by the mean field calculation.

We comment here that for the Higgs-Yukawa model the saddle point type mean field approximation is not equivalent to the variational type mean field approximation. With the saddle point type approximation, one is not guaranteed to get an upper bound on the free energy. If the fluctuation around the saddle point is large, the true free energy may be completely different from the saddle point estimate. In contrast, the variational type mean field approximation gives the rigorous upper bound of the free energy. Unfortunately, the variational calculation can not be completed without further approximation (small or large \( y \) expansion [9]) for the Higgs-Yukawa model.

A different type of mean field approximation can be performed in the large \( y \) region. If we expand the fermion determinant to next leading order in \( 1/y^2 \), the action becomes

\[
S = -\sum_{x,\mu} \left\{ \left[ \kappa \rho(x)\rho(x + \mu) + \frac{N_f}{2y^2 \rho(x)\rho(x + \mu)} \right] \sigma_a(x)\sigma_a(x + \mu) + (\mu \rightarrow -\mu) \right\}
+ \sum_x \left\{ \rho(x)^2 + \lambda[\rho(x)^2 - 1]^2 - 2N_f \log \rho^2(x) \right\}.
\]

\[ h_{st} = 0. \]
The radial interaction part may be approximated by its mean field value
\[< \rho > = \frac{\int_0^\infty d\rho \rho^2 e^{-[\rho^2+\lambda(\rho^2-1)^2-2N_f \log \rho^2]}}{\int_0^\infty d\rho e^{-[\rho^2+\lambda(\rho^2-1)^2-2N_f \log \rho^2]}}, \quad (31)\]
and the action eqn. (30) becomes an effective action for the XY-model
\[S_{\text{eff}} = -\kappa_{\text{eff}} \sum_{x,\mu} \sigma_a(x) [\sigma_a(x+\mu) + \sigma_a(x-\mu)], \quad (32)\]
where the effective hopping parameter \(\kappa_{\text{eff}}\) is given by
\[\kappa_{\text{eff}} = \kappa < \rho >^2 + \frac{N_f}{2y^2} < \frac{1}{\rho} >^2. \quad (33)\]
This approximation can be justified as a leading order \(1/y^2\) expansion with the assumption that \(\kappa \sim O(1/y^2)\). The XY-model has second order phase transitions at \(\kappa_{\text{eff}} \approx 0.15\) (FM-SYM) and \(\kappa_{\text{eff}} \approx -0.15\) (AFM-SYM). Thus for the original Higgs-Yukawa model we get the second order phase transition lines at
\[\kappa < \rho >^2 + \frac{N_f}{2y^2} < \frac{1}{\rho} >^2 \approx \pm 0.15. \quad (34)\]

The results of this mean field calculation are plotted in Fig. 2a and 2b in the large \(y\) region. We want to mention that for small \(\lambda\) and \(y\) the mean field results are compatible with the bare perturbation calculations which are plotted in Fig. 2a.

At \(\lambda = 1\) both large \(N_f\) approximation and perturbation theory break down. Fig. 2b shows the phase diagram predicted by the weak and strong \(y\) mean field calculation at \(\lambda = 1\).

### 3 Numerical results

We have performed numerical simulations at \(\lambda = 0.0156, N_f = 2, 10\) and \(\lambda = 0.1, 1.0, N_f = 2\). The Hybrid Monte Carlo method [10] was used for the dynamical fermion simulations. Each molecular dynamics trajectory consists of 10 steps with step size chosen such that the acceptance rate is around 80\%. To decide the order of the phase transition, we looked for hysteresis effects in the thermocycles. For each data point in the thermocycle about 20 trajectories are used as warmup and 100-200 trajectories are used in the measurement.

The magnetization \(v\) is defined as
\[v = < \sqrt{\bar{\phi}_a^2} >, \quad \bar{\phi}_a = \frac{1}{L^4} \sum_x \phi_a(x), \quad (35)\]
and the staggered magnetization \(v_{st}\) is
\[v_{st} = < \sqrt{\bar{\phi}_{st,a}^2} >, \quad \bar{\phi}_{st,a} = \frac{1}{L^4} \sum_x (-1)^{\sum_{\mu} x_{\mu}} \phi_a(x), \quad (36)\]
where $L$ is the linear size of the lattice. $v$ and $v_{st}$ are measured and used as the order parameters to determine the phase. The measurements are done on $4^4$ lattices. A $4^4$ lattice is certainly not sufficient to distinguish second order and weakly first order transitions. However, the combination and the agreement of the numerical and analytical results allow us to determine the order of the phase transitions reliably.

At $\lambda = 0.0156$, we have simulation results for both $N_f = 2$ and 10. For $N_f = 10$, the data can be compared directly with the large $N_f$ calculation. We find complete agreement with the theoretical predictions as shown in Fig. 1. Along the line of the first order phase transition where the hysteresis effects are very strong, it is not easy to determine exactly the position of the phase transition. But the amplitude of $v$ and $v_{st}$ agree very well with the large $N_f$ prediction. Thus the phase transition line predicted by the large $N_f$ expansion should be reliable. For large $y$, the SYM region is quite narrow for $N_f = 10$, making it difficult to establish this region numerically. However, the existing results [5] at $\lambda = 0$ and our simulation results presented in the next paragraph at $N_f = 2$ should be sufficiently convincing that the SYM phase indeed exist in this region.

The phase diagram for $\lambda = 0.0156, N_f = 2$ is plotted in Fig. 2a. The transition points agree well with both the bare perturbation calculation and the large $N_f$ expansion in the small $y$ region. The agreement with the large $N_f$ calculation is probably due to the fact that the effective fermion flavor number around second order phase transition lines is 32 because of the lattice doubling effect. Up to $\lambda = 1$ the picture remains qualitatively the same. The phase diagram for $\lambda = 1, N_f = 2$ is shown in Fig. 2b. As $\lambda = 1$ is already a coupling of intermediate strength, neither bare perturbation theory nor the large $N_f$ approximations agree with the data points. However, the mean field approximation - from which the curves are plotted in the figure - shows reasonable agreement with the simulation result. In particular, there is no indication for an FI phase or a non-trivial fixed point at any of these coupling values.

In Fig. 3 we give examples for thermocycles along the first order phase transition line EDBF in Fig. 2 (see Fig. 1 for notations) for $\lambda = 0.0156, N_f = 2$. The data is compared to the bare perturbation theory (dotted lines). Fig. 3a corresponds to an AFM-AFM transition, Fig. 3b to a SYM-AFM and Fig. 3c to an FM-AFM transition. In all cases $v_{st}$ is plotted. All thermocycles show hysteresis effects in agreement with our theoretical expectations. There is no sign of the first order line becoming critical at the end point B of the FM-SYM second order phase transition line.

4 Conclusion and discussions

In this paper we have investigated the phase diagram of a $U(1) \otimes U(1)$ Higgs-Yukawa model with fluctuating length Higgs field. The results obtained by various analytical methods and numerical simulations show, up to the Higgs self coupling $\lambda = 1$, a phase diagram as

\footnote{Note that this figure also contains the continuous phase transition from the symmetric to the antiferromagnetic phase at $\kappa \approx -0.16$}
summarized in Fig. 1. This picture is quite different from the published results at $\lambda = \infty$. We observe strong first order phase transitions from the antiferromagnetic phase to the ferromagnetic, symmetric and antiferromagnetic phases. The second order line between the perturbative ferromagnetic and the symmetric phases, that can be relevant for the Standard Model, ends on this first order phase transition line. At this end point the first order line does not become critical, no new non-trivial critical behavior is expected, the whole second order line is in the domain of attraction of the Gaussian fixed point. In addition, we do not find a ferrimagnetic phase up to $\lambda = 1$.

In order to understand the critical behavior of the Higgs-Yukawa model one has to find out if the above picture for the phase diagram remains the same as one increases $\lambda$ further. If this is indeed the case, it would provide a natural explanation of why the critical indices do not change along the second order line at $\lambda = \infty$ [6]. Another possibility would be that the phase diagram changes qualitatively at some strong $\lambda$ value and eventually merges into the published result [3] at $\lambda = \infty$. The mean field picture given in section 2 remains unchanged up to $\lambda = \infty$. However, it is unclear how reliable these approximations really are. To determine the critical properties of the large and infinite $\lambda$ systems requires much more computer time, or different analytical approaches. This work is in progress and the result will be reported in a future publication.

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Figure 1: The phase diagram in the large $N_f$ limit at $\tilde{\lambda}_N = 0.1$ and $N_f = 10$. The MC data are indicated by circles where the solid symbol denotes second and the open symbols first order phase transitions. The solid and dashed lines are the results from the $1/N_f$ expansions, where the solid lines represent second order and the dashed line first order phase transitions. In the middle of the phase diagram, where the $1/N_f$ expansions break down, the dotted lines indicate how the phase transition might continue.

Figure 2: Phase diagram at $\lambda = 0.0156$ (a) and $\lambda = 1.0$ (b), both with $N_f = 2$. Here as in Fig.1 solid symbols denote second and open symbols first order phase transitions. In the small $y$ region the solid and dashed lines are obtained by (a) bare perturbation calculation; (b) mean field calculation. In both (a) and (b) the lines in the large $y$ region are obtained from the mean field theory given in the second part of the section 2.3. Solid lines represent second order and dashed lines first order phase transitions. The dotted lines only indicate a possibility how the phase transition lines may continue.

Figure 3: Hysteresis effects for the staggered magnetization $v_{st}$ are shown along the first order phase transition line EDBF (see Fig.1). The data are taken at $\lambda = 0.0156$ and $N_f = 2$. The solid symbols represent the first half of the thermocycle and the open ones the way back. The solid lines are only connecting the data points to guide the eye. The dotted lines are the results from perturbation theory. (a) a point taken between D and E (AFM-AFM) (Note that in addition to the AFM-AFM phase transition also the SYM-AFM phase transition at $\kappa \approx -0.16$ is shown.) (b) a point between D and B (SYM-AFM). (c) a point taken between B and F (FM-AFM). The discontinuity of the hysteresis loop becomes smaller closer to point E indicating a weaker first order phase transition.