Decoherence-free propagation and ramification of a solitary pulse in a superconducting circuit

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Using a microscopic master equation to account for the environmental effects, we compute the decoherence culminated during the propagation of a microwave pulse of arbitrary shape through a superconducting qubit. It is shown that the qubit decoherence vanishes and the pulse shape remains absorption-free when the latter adopts a soliton shape with \( n\pi \) area. Otherwise, the environmental feedback decelerates the velocity of the soliton envelop and induces an monotonic increase of phase in the microwave. A pulse of non-\( n\pi \) area thus ramifies into a transparent part that travels decoherence-free at incident velocity and a slowing part that decays through space. The ramification explains the environmental origin of pulse splitting observed in self-induced transparency.

I. INTRODUCTION

Superconducting circuit systems comprising one or a few qubits are controlled by microwave pulses. When coupled with a cavity field residing in a coplanar stripline resonator, these qubits form a pulse-controlled circuit quantum electrodynamic (cQED) system [1, 2], which serves as the foundation of solid-state entanglement generation [3, 4] and quantum computing [5, 6]. For examples, resonant and dispersive square pulses with appropriate lengths are transmitted to read out qubit states [7]; Gaussian pulses are used to perform \( x- \) and \( y- \) rotations to achieve GHZ states [8]. In general, the enveloping shapes of the microwave pulses are proven to be pivotal to the desired operations on the qubits [8, 10].

A natural question to ask then is whether microwave pulses can be used to eliminate decoherence. There had been multiple approaches that targets qubit decoherence. First, the problem is algebraically approached through decoherence-free subspaces [11], which is implemented in a superconducting circuit using Purcell effect [12]. The second approach is device-based, where the anharmonicity of the qubit is fine tuned through, for instance, the introduction of transmon [13]. The latter can prolong the decoherence times to over 2\( \mu \)s [14]. In this article, we take a third quantum-optical approach to determine the circumstance when the decoherence of a qubit can be removed through the interaction of a solitary microwave pulse with the qubit in a superconducting circuit.

Since a superconducting qubit when biased to an optimal operation point (e.g. reduced flux \( f = 1/2 \)) is essentially a two-level atom, many optical effects induced by atom-field interactions, such as electromagnetically induced transparency [15] and parametric amplification [16], also apply to qubits on a superconducting circuit [17]. The closest analogous study to the scenario considered here that occurs in natural atomic systems is the propagation of narrow pulses in resonant atomic media [18], for which the effect of self-induced transparency (SIT) is a prime manifestation [19, 20]. Nevertheless, the relative scale of a microwave pulse with respect to a superconducting qubit is incommensurable to that of an optical pulse with respect to an atomic medium. For example, SIT regards an optical narrow pulse as those widths at half height being much shorter than the length of the atomic medium (e.g. 7 nsec pulses in a 1 mm Rb-sample at density \( n = 10^{11} \text{cm}^{-3} \) [22]). In contrast, a microwave pulse at nanosecond range [7] translates to a length of centimeter range when propagating in a silicon-substrated circuit and facing a qubit typically measures at only 300\( \mu \)m [16]. The scenario is illustrated in Fig. [1].

Therefore, the scenarios of SIT and the microwave propagation through a qubit are analogous although the roles of the field and the medium are switched due to their relative length scales. At resonance, the traveling pulse is similarly described by an inhomogeneous Maxwell equation, with the inhomogeneous term contributed by the polarization of the medium, i.e. the density matrix of the qubit. Nevertheless, we introduce a microscopic adiabatic master equation approach to depict the time evolution of the qubit, taking all decoherence effects into account, in contrast to past analyses where \( T_1 \) and \( T_2 \) relaxation times are assumed infinite to simplify calculations [14, 15]. We analytically solve the master equation to trace the qubit decay through a complex decoherence factor, whose real and imaginary parts correspond to the dephasing and the longitudinal relaxation susceptibility of the qubit. A hypersecant solution is found to associate with a microwave propagation of no absorption by the qubit. More importantly, during the transparent propagation, the culminated longitudinal relaxation of the qubit also vanishes while the transverse relaxation is determined by an integral formula. Zero-dephasing evolution during propagation is theoretically obtainable when the spectral distribution of the reservoir becomes orthogonal to the sinusoids of the pulse phase. The absorption-free propagation is similar to SIT but the decoherence-free propagation has not been discovered before.

Consequently, solving for the envelop of the pulse shows that a non-transparent pulse experiences a reduced propagation velocity while its energy is absorbed. Hence, an arbitrary solitary pulse is ramified into a hypersecant part which travels at the incident velocity and a remainder part whose travel is dragged by environmental feedbacks. Therefore, the microscopic approach here points out the thermal environmental
origin of pulse splitting observed in SIT [21, 22] and provides clues for decoherence control using microwave pulses in superconducting circuits. We begin the study by describing the qubit-pulse interaction model in Sec. III and follow with the derivation of the decoherence factor in Sec. III. The full solutions of the pulse envelop and phase are presented in Sec. IV along with the discussion of pulse ramifications, before a conclusion is given in Sec. V.

II. QUBIT-PULSE INTERACTION IN THERMAL ENVIRONMENT

We begin the derivation by assuming the electric field of an incident microwave pulse take the form

$$E(x, t) = E(x, t) \cos [\varphi(x, t) - kx + \omega t]$$  \hspace{1cm} (1)

where $E(x, t)$ and $\varphi(x, t)$ denote, respectively, its envelop and phase during its traveling along a waveguide. $\omega$ denotes the frequency and $k$ the associated wavenumber of the carrier wave. In Eq. (1), $x$ and $t$ denote laboratory spatial and time coordinates but they are customarily compressed [18, 23] into the single variable $\tau = t - x/v$ of local time under the reference frame that travels along with the wavefront. Since dispersive effects are not considered, the velocity $v$ of the wavefront is assumed constant $v = \omega/k$ throughout the propagation and the electric field becomes a single-variable function $E(\tau) = E(\tau) \cos [\varphi(\tau) + \omega \tau]$ under the traveling reference frame. The system, illustrated in Fig. 1, is described by the time-dependent Hamiltonian ($\hbar = 1$)

$$H_S(\tau) = \frac{\omega_0}{2} \sigma_z + \mu E(\tau) [\sigma_+ + \sigma_-]$$  \hspace{1cm} (2)

where $\omega_0$ and $\mu$ are the transition frequency and the effective dipole moment, respectively, of the qubit. In other words, $E(\tau)$ is non-zero during the propagation of the pulse through the qubit; otherwise, $E(\tau)$ vanishes, letting the qubit evolve freely and the pulse travel freely.

This semiclassical Hamiltonian is diagonalizable through the dressed states

$$|\nu_+(\tau)\rangle = e^{-i\varphi(\tau)} \cos \theta(\tau) |e\rangle - i \sin \theta(\tau) |g\rangle,$$  \hspace{1cm} (3)

$$|\nu_-(\tau)\rangle = e^{-i\varphi(\tau)} \sin \theta(\tau) |e\rangle + \cos \theta(\tau) |g\rangle,$$  \hspace{1cm} (4)

for the eigenvalues $\Omega_{\pm} = \pm \sqrt{\delta^2 + (\mu E)^2}$ in the rotating frame $e^{i\omega \tau}$ of the microwave carrier. The transformation angle

$$\theta(\tau) = \frac{1}{2} \tan^{-1} \frac{\mu E}{\delta}$$  \hspace{1cm} (5)

depends on the qubit-field detuning $\delta = \omega_0 - \omega$. When the pulse is not overlapping with the qubit, the two dressed states assume the asymptotic $|e\rangle$ or $|g\rangle$ of the bare states.

The environmental effects to the qubit are modeled on a multi-mode-resonator bath with free Hamiltonian

$$H_B = \sum_j \omega_j a_j^\dagger a_j.$$

paired with the diagonalized $H_S = \Omega_+ |\nu_+\rangle \langle \nu_+| - \Omega_- |\nu_-\rangle \langle \nu_-|$, the universe has the Hamiltonian $H = H_S + H_B + H_I$, where the system-bath coupling is the tensor product

$$H_I = h_S \otimes H_B = |e\rangle \langle g| + |g\rangle \langle e| \otimes \sum_j g_j (a_j + a_j^\dagger).$$  \hspace{1cm} (6)

Since the system eigenstates does not remain static but rather follow the change in the amplitude of the microwave pulse, the basis for system-bath coupling should align with the time-dependent basis in Eqs. (3)-(4) to take into account of dressed states.

To express the total phase, i.e. both the dynamic (the first term) and the geometric phase (the second term), culminated in each dressed state.

Our considerations begin with the Liouville equation for the system density matrix $\rho'$ in the Schrödinger picture, which reads

$$\frac{d\rho'}{d\tau} = -i [H_I(\tau), \rho'(\tau) \otimes \rho]$$  \hspace{1cm} (9)

where the integral corresponds to the first nontrivial term in the perturbative expansion of time-ordered evolution of the universe. The density matrix $\rho$ of the bath will be partial-traced out when taking the ensemble average. This integral
represents the memory effect of the bath onto the system during the propagating of the pulse through the qubit under the Born-Markov approximation. Expanding the double commutator will give four terms involving both \( \tau \) and \( \tau' \). Those related to the bath part only involves double-time correlations when taking the trace and read [25]

\[
\langle h_B(\tau) h_B(\tau - \tau') \rangle =
\]

\[
\langle h_B(\tau) h_B(\tau - \tau') \rangle^* = \sum_j g_j^2 e^{-i\omega_j \tau'}.
\] (10)

Those related to the system can be considered separately. Following the method [26], the memory effect can be recorded by reversing the direction of time \((\tau' \to \tau - \tau')\) such that \( U_{sd}(\tau - \tau') = \exp \{ it' H_S(\tau) \} U_{sd}(\tau) \).

### III. DECOHERENCE FACTOR

Expanding the commutators and tracing out the bath operators in the Liouville equation yields the microscopic master equation in the Lindblad form

\[
\frac{d\rho}{dt} = -i [H_S, \rho] + \gamma(\Omega) \sin^2 \varphi \times \left[ \hat{\sigma}_- \hat{\rho} \hat{\sigma}_+ - \frac{1}{2} \{ \hat{\sigma}_+ \hat{\sigma}_-, \rho \} \right]
\] (11)

after converting to the Schroedinger picture. The Pauli matrices are hatted to indicate that the dressed basis is assumed, e.g. \( \hat{\sigma}_x = |\nu_+(\tau)\rangle \langle \nu_-(\tau)| + |\nu_-(\tau)\rangle \langle \nu_+(\tau)| \).

\[
\gamma(\Omega) = 2\pi \sum_j g_j^2 \delta(\omega_j - \Omega)
\] (12)

denotes the spectral density distribution of the bath stemming from the integration, which is essentially the Fourier transform of Eq. (11). The detailed derivations of Eq. (11) is given in Appendix A. The effect of an incident pulse on the qubit is reflected in its polarization \( P(\tau) = \mu \text{tr} \{ (\hat{\sigma}_+ \rho(\tau) \} \) as a time-dependent response to the pulse, where the trace is taken over the dressed system basis. From Eq. (11), one can derive that \( P(\tau) = \mu F \exp(i(\varphi + \omega \tau))/2 + h.c. \) where \( F \) indicates a complex factor with the real and imaginary parts

\[
\Re\{F\} = 1 - e^{-\Gamma(\tau)},
\]
\[
\Im\{F\} = -e^{-\Gamma(\tau)/2} \sin \int_{\tau_0}^\tau \Omega(s)ds.
\] (13) (14)

In the two parts of the factor,

\[
\Gamma(\tau) = \int_{\tau_0}^\tau ds \gamma(\Omega) \sin^2 \varphi
\] (15)

converts the spectral function \( \gamma(\Omega) \) into the time domain to determine the effective decay in the response of the polarization. It can be regarded as the bath-spectrum-weighted transform of Eq. (12) and thus a decay factor corresponding to the bath correlations prescribed in Eq. (10).

Equipped with the expression of \( P(\tau) = P(t - x/v) \), we can determine how the microwave pulse responds to the qubit and when its propagation can be decoherence-free. Consider the standard Maxwell equation

\[
\frac{\partial^2 E}{\partial t^2} + \kappa \frac{\partial E}{\partial t} - c^2 \frac{\partial^2 E}{\partial x^2} = -\frac{1}{\epsilon_0} \frac{\partial^2 P}{\partial t^2}
\] (16)

where \( \kappa \) is the classical decay factor of the electric field. Since \( E(t) \) assumes the form of Eq. (1), in which the envelop \( E(t) \) and the phase \( \varphi(t) \) are the slow variables compared to \( \omega \), and the precession of \( P(t) \) follows \( F(t) \), which is also slow compared to \( \omega \), the terms not on the order of \( \omega \) can be ignored [21, 27]. After substituting the expressions of \( E(t) \) and \( P(t) \) into the derivatives and Eq. (16) can be linearized. Comparing the coefficients of the carrier \( e^{i(\omega t + \pi/2)} \) and its conjugate, we obtain the coupled equations

\[
\mathcal{E}\left(\frac{1}{v} \frac{d\varphi}{dt} \right) = \frac{\mu k}{2\epsilon_0} \Re\{F\},
\]
\[
\mathcal{E}\left(\frac{1}{v} \frac{d\varphi}{dt} \right) = -\frac{\mu k}{2\epsilon_0} \Im\{F\},
\] (17) (18)

about the envelop and the phase, respectively, under the local time frame \( \tau = t - x/v \). In the equations, \( v \) is the velocity of the envelop wavefront and not necessarily equal to the phase velocity \( c \). The detailed derivations are given in Appendix B.

With Eqs. (17) and (18), it becomes clear that the factor \( F \) affects the envelop \( \mathcal{E}(t) \) only through its imaginary part and the phase \( \varphi(t) \) through both its real and imaginary parts. For \( \mathcal{E}(t) \), the effect from \( F \) is the decoherence imposed by the environment through the factor \( e^{-\Gamma(t)/2} \). With the precedent sinusoidal factor, decoherence would vanish when the integral \( \int \Omega(\tau) d\tau \) is an integer multiple of \( \pi \). Note that at qubit-pulse resonance, \( \Omega(t) \) reduces to \( \mu \mathcal{E}(t) \). That means, omitting the classical decay occurring in the waveguide owing to \( \kappa \), the envelop would retain its shape during the propagation as long as the enveloping area of the pulse is an integer multiple of \( \pi \). This phenomenon exactly coincides with the observations of self-induced transparency. In addition, \( \mathcal{E}(t) \) not only preserves its energy after traveling through the qubit, but makes the qubit immune from the environment. That is, the decoherence factor vanishes along with the sinusoidal factor, showing an \( n\pi \)-pulse would propagate absorption-free and decoherence-free simultaneously. Consequently, the master-Maxwell equation pair given by Eqs. (11) and (16) serves as a microscopic foundation of SIT effective for one single artificial atom and inclusive of environmental effects.

### IV. DECOHERENCE-FREE PROPAGATION AND RAMIFICATION

To find a general solution to Eq. (18) for \( \mathcal{E}(t) \) with arbitrary initial area, we converts the integro-differential equation of \( \mathcal{E} \) into the second-order differential equation

\[
\dot{A} = M^2 e^{-\Gamma/2} \sin A
\] (19)
of the enveloped area $\mathcal{A}(\tau) = \mu \int_{0}^{\tau} ds \mathcal{E}(s)$ up to the wavefront, which is a pendulum equation augmented with a decay factor. We have used $M = \sqrt{\mu^2 \kappa^2/2c_0 (c - v)}$ to abbreviate the equation. Note that when the spectral function $\gamma(\Omega)$ happens to be orthogonal to $\Gamma(\Omega)$, the diagonal elements of $\rho(t)$ determined by the master equation (11) would only retain terms proportional to $e^{-\Gamma t/4}$. Therefore, we see that nonvanishing $\Gamma$ leads to finite dephasing to the qubit as a dipole moment.

To find the analytic expression for $A$, the pendulum equation is first reduced to the first-order equation: $\dot{A} = 2Me^{-\Gamma t/4} \tilde{A} \sin(\tilde{A}/2)$, showing that $2\pi\tau$-pulses experience transparent transmission in addition to being free from decoherence. For pulses of arbitrary enveloping area, we retain the phase variable $\phi$ in the expression of $\Gamma(\tau)$ and solve Eq. (19) formally as a pendulum equation. The envelop $\mathcal{E}(\tau)$ as a time derivative of $A$ reads

$$\mathcal{E}(\tau) = \frac{4M}{\mu} e^{-\Gamma(\tau)/4} \text{sech}(\int_{\tau_0}^{\tau} ds e^{-\Gamma(s)/4} + \tau_D),$$

(20)

where $\tau_D$ is a delay time. Note that Eq. (20) retains the characteristic hypersecant hump of a soliton. The environment culminates an attenuation on the pulse peak amplitude and a variable time duration in the temporal argument, the latter of which affects the traveling speed of pulses of different areas. The derivation process of Eq. (20) is given in Appendix C.

The manifestation of the environmental effects depends on the knowledge of the spectral density $\gamma(\Omega)$ to determine the decay factor $\Gamma(\tau)$. Without knowing its exact expression, we give a numerical simulation of the $\mathcal{E}(\tau)$ evolution during the first moments of qubit-pulse interaction by making the following assumptions: (i) change of $\Omega(\tau)$ during the pulse-qubit interaction is small relative to the bare qubit level spacing, thus adiabatic approximation is valid; and (ii) the change of phase $\phi(\tau)$ is small over the course of interaction. The latter assumption is correlated to the former, it will be evidence by the simulations to be given. It follows that the integrand of $\Gamma(\tau)$ in Eq. (15) can be regarded as constant $4C_0$ under the slow variation, which allows the approximation $\Gamma(\tau) \approx 4C_0(\tau - \tau_0)$ and leads to an exponential decay of the pulse peak in Eq. (20) (the scale factor 4 is added to simplify expressions below).

Under such premise, the integral in Eq. (20) is computed numerically, giving rise to the propagation of a decaying pulse as illustrated in Fig. 2. Note that $\mathcal{E}(\tau)$ under the local time frame $\tau$ would appear simply as a decaying envelop peaking at $\tau = 0$. To appreciate the propagation process in the figure, we have returned the reference frame to the separate laboratory axes $x/v$ and $t$, where parameters are set to values accessible by typical qubits in superconducting circuits: $\omega = 5\text{GHz}$ and a Q-factor of $10^8$ [16, 28]. Since $\tau_0$ is an arbitrary initial time point, it is set to zero to simplify the analysis.

We observe that a pulse of arbitrary enveloping area ramifies into two: one of area a multiple of $2\pi$ travels freely, shown as one ridge that converges to a constant height, and one of non-integral area attenuates over time, shown as the other ridge in Fig. 2. Following the wavefronts of the two peaks, one observe that the slopes of their projections onto the $x$-$t$ plane differ. The one traveling decoherence-free pertains to a constant slope and therefore travels at a constant velocity of light while the other has a curving slope. The separation of wavefronts increases monotonically over time, showing that the attenuating pulse is decelerating. This can be proved analytically by taking the $\tau$-derivative of the argument of the hyper-secant function in Eq. (20), giving the velocity $v \exp\{-C_0(t - \tau_0)\}$.

The approximations taken in giving Fig. 2 is essentially a first-order perturbative expansion of $\mathcal{E}(\tau)$, from which the en-

FIG. 2. Plot of the pulse envelop $\mathcal{E}(\tau)$ scale to arbitrary unit as a function of local time $\tau = t - x/v$, illustrating the scenario of pulse ramification. The single pulse at the initial moment $\tau = 0$ splits into one $2\pi$-pulse traveling decoherence- and absorption-free and one non-$\pi$-pulse attenuating over time. System parameters are taken from experiments of superconducting qubit circuits.

FIG. 3. The effect of the variation of pulse phase $\phi(t)$ over time is shown by the carrier wave under the typical hypersecant envelop of a soliton pulse. The blue curve for a pulse with finite $\phi(t)$ is set against a yellow curve for a constant phase and shows that its oscillations are completed in advance to pulses free from environmental effects.
FIG. 4. The pulse envelop area $A(\tau)$ (solid curves and scaled on the left axis) and the pulse carrier phase $\varphi(\tau)$ (dashed curves scaled on the right axis) are plotted as functions of local time $\tau$ over the same duration for the spectral densities $\gamma = 4C_0 = 5\text{MHz}$ (blue curves), $50\text{MHz}$ (yellow curves), $100\text{MHz}$ (red curves), and $150\text{MHz}$ (black curves).

The envelop area and phase governed by Eqs. (17)–(13) are decoupled. Consequently, the dynamic phase accumulated when propagating through the qubit is computed by integrating Eq. (17), which reads

$$
\varphi(\tau) = \varphi_0 + M \int_{\tau_0}^{\tau} ds \ e^{-2C_0(s-\tau_0)} \sinh \{2C_0(s-\tau_0)\} \\
\times \cosh \left\{ -M \left( e^{-2C_0(s-\tau_0)} - C_0\tau D - 1 \right) \right\} \quad (21)
$$

The second term contributed by the environment feedback generates an advancement to the phase. Using the same system parameters as in Fig. 2, the integrals of Eq. (21) can be numerically computed, giving rise to the typical carrier wave oscillations as plotted in Fig. 3, where the phase advancement over a duration of 20 periods is shown against a carrier of no phase variation.

Since the three factors in the integrands of Eq. (21) are either exponential or variants of exponential functions, the phase culminated on a pulse is a monotonically increasing function of time. This is evidenced by the plots of $\varphi(\tau)$ given as dashed curves in Fig. 4 for four different constant spectral densities $\gamma$. The rate of phase accumulation increases along with the increment of $\gamma$ when $\Gamma$, regarded as a measure of rate of feedback from the environment is accordingly increased. The envelop area $A(\tau)$ computed from the integral of $\mathcal{E}(\tau)$ in Eq. (20) is plotted in the same figure, showing that the area variation accentuates on the range of time where the phase variation is minimal, thereby ratifying the assumptions we took above when arriving at the explicit solution of $\mathcal{E}(\tau)$.

V. CONCLUSION

To conclude, we have taken an adiabatic master equation approach to analyze the propagation of a pulse through an environmentally coupled qubit. The qubit would be free from decoherence when the pulse area is $\pi n$. Further, when an orthogonal condition between the pulse phase and the bath spectral density is satisfied, the dephasing vanishes and thus relaxation times $T_1$ and $T_2$ become essentially infinite. It is also proved that when not vanishing the thermal environment is responsible for inducing pulse ramifications which are frequently observed in self-induced transparency experiments. The approach has also enabled us for the first time to compute analytically the phase variation in the pulse during its propagation through a two-level system. Modeled on superconducting qubit circuits, the exact knowledge on pulse-qubit interactions would benefit the designs of more sophisticated microwave control pulses for quantum information processing.

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Appendix A: Adiabatic quantum master equation

To construct a master equation for the system when taking into account the geometric phases, we consider customarily a quantum heat bath consisting of a multimode resonator $H_B = \sum_j \omega_j a_j^\dagger a_j$ having dipole-field interaction $H_I = \sum_j g_j \left( a_j^\dagger + a_j \right) \sigma_x$ with the qubit.

Since the incident pulse with envelop $\mathcal{E}(x, t)$ is a weak driving field to the qubit, the qubit evolution under the coupling strength $\mu \mathcal{E}$ could be regarded as an adiabatic process under Born-Oppenheimer approximation when compared to the thermal decoherence process under couplings $\{g_j\}$. Under the total Hamiltonian

$$
H(\tau) = H_B^\prime(\tau) + H_B + H_I \quad (A1)
$$

with $H_B^\prime(\tau) + H_B$ regarded as the free energy, the adiabatic process is described by the master equation for the universe $\mathcal{U}$:

$$
\frac{d\mathcal{U}}{d\tau} = -\int_{\tau_0}^{\tau} ds \ \{[H_I(\tau), [H_I(\tau-s), \rho'(\tau) \otimes \hat{\rho}]]\} \quad (A2)
$$

in interaction picture. $\hat{\rho}$ stands for the density matrix for the thermal bath. The system-bath interaction $H_I(\tau) = U_{ad}(\tau) H_I U_{ad}(\tau)$ has been transformed to the adiabatic evolution frame with the double-time transformation $U_{ad}(\tau-s) = e^{i s H_B^\prime(\tau)} U_{ad}(\tau)$ being used for the historic Hamiltonian $H_I(\tau-s)$ following the convention $[25, 26]$.

As given in Eq. (5) in the main text, the unitary transformations involve the dressed basis defined at both the initial
moment \( \tau_0 \) and the current moment \( \tau \) or the historic moment \( \tau - s \). In practice, we unify the basis reference to time \( \tau \) by inverting eigenstate definitions of Eqs. (3)-(4), finding Eq. (7), i.e.

\[
U_{ad}(\tau) = |\nu_+(\tau)\rangle\langle\nu_+(\tau_0)| e^{-i\phi_+(\tau)} + |\nu_-(\tau)\rangle\langle\nu_-(\tau_0)| e^{-i\phi_-(\tau)}, \quad (A3)
\]

based on which we can also derive

\[
U_{ad}(\tau - s) = e^{i\Omega s}(|\nu_+(\tau)\rangle\langle\nu_+(\tau_0)| e^{-i\phi_+} + e^{i\Omega s}(|\nu_-(\tau)\rangle\langle\nu_-(\tau_0)| e^{-i\phi_-}). \quad (A4)
\]

During the adiabatic process, the bath variables \( a_j \) and \( a_j^\dagger \) stay relatively static while the operator \( \sigma_x \) relevant to the system is rotated. Therefore, the static \( \sigma_x \) in the dressed basis reads

\[
\sigma_x = |e\rangle\langle g| + |g\rangle\langle e| = -\cos \varphi(\tau)|\nu_+(\tau)\rangle\langle\nu_+| + i \sin \varphi(\tau)|\nu_+(\tau)\rangle\langle\nu_-| - i \sin \varphi(\tau)|\nu_-\rangle\langle\nu_+| + \cos \varphi(\tau)|\nu_-\rangle\langle\nu_-| - \text{h.c.} \quad (A5)
\]

which includes the special case with \( s = 0 \).

The derivation of the microscopic master equations begins with tracing out the bath variables in the Liouville equation (A2) of the universe, giving

\[
\frac{d\rho'}{dt} = -\gamma(\Omega) \sin^2 \varphi U_{ad}^\dagger \hat{\sigma}_- U_{ad} \rho' + \text{h.c.} \quad (A6)
\]

with the first term, we consider

\[
\int_0^\infty ds \text{tr}_B \{ H_1(\tau) H_1(\tau - s) \rho'(\tau) \otimes \hat{\rho} \}
\]

\[
= \int ds \hat{\sigma}_x(\tau - s) \rho'(\tau) \text{tr}_B \{ h_B(\tau) h_B(\tau - s) \hat{\rho} \}
\]

\[
= \int ds \hat{\sigma}_x(\tau - s) \rho'(\tau) \sum_j g_j^2 e^{-i\omega_j s}. \quad (A9)
\]

Note from the expression of Eq. (A8) the double-time operator product would contribute multiple terms in the integral, but only those with the factor \( e^{i(\Omega - \omega_j)s} \) would remain since the other terms containing the fast-oscillating exponential factors would vanish after the integration over long period. Consequently, since only this exponential factor involves in the integration over the variable \( s \), all other factors about time \( \tau \) do not participate in the integration and

\[
\sum_j g_j^2 \left[ \int ds e^{i(\Omega - \omega_j)s} \right] = \pi \sum_j g_j^2 \delta(\Omega - \omega_j) = \frac{1}{2} \gamma(\Omega) \quad (A10)
\]

where the last equation employs the definition of Eq. (12). The integral would become

\[
-\frac{1}{2} \gamma(\Omega) \sin^2 \varphi U_{ad}^\dagger \hat{\sigma}_- U_{ad} \rho' \quad (A11)
\]

When applying the same arguments to the other three terms in the expansion of the RHS of Eq. (A7), we arrives at

\[
\frac{d\rho'}{dt} = -\gamma(\Omega) \sin^2 \varphi \left\{ U_{ad}^\dagger \hat{\sigma}_+ U_{ad}, \rho' \right\} - U_{ad}^\dagger \hat{\sigma}_- U_{ad} \rho' U_{ad}^\dagger \hat{\sigma}_+ U_{ad} \quad (A12)
\]

Finally, seeing that \( \rho' = U_{ad}^\dagger \rho U_{ad} \) is given in the interaction picture of the adiabatic evolution, we note that for the conversion back to Schroedinger picture,

\[
\frac{d\rho}{dt} = \frac{d}{d\tau} \left\{ U_{ad} \rho' U_{ad}^\dagger \right\}
\]

\[
= U_{ad} \frac{d\rho'}{d\tau} U_{ad}^\dagger - i H_S U_{ad} \rho' U_{ad}^\dagger + i U_{ad} \rho' U_{ad}^\dagger H_S
\]

\[
= -i [H_S, \rho] - \gamma(\Omega) \sin^2 \varphi \left\{ \frac{1}{2} \{ \hat{\sigma}_+, \hat{\sigma}_-, \rho \} - \hat{\sigma}_- \rho \hat{\sigma}_+ \right\}, \quad (A13)
\]

which is the Lindblad form of the master equation as in Eq. (11).

Appendix B: Equations of envelope and phase

From the original Maxwell equation (16), the loss \( \kappa \) in the waveguide is assumed negligible. Then the so-called slowly-varying envelope approximation (23), that is \( \partial E/\partial t \ll \omega E, \partial E/\partial x \ll kE, \partial \varphi/\partial t \ll \omega, \) and \( \partial \varphi/\partial x \ll k \) for the electric field \( E(t) \); \( \partial F/\partial t \ll \omega F \) and \( \partial F/\partial x \ll kF \) for the
polarization \( P(t) \) can be taken. Thus when substituting the expressions of \( E(t) \) and \( P(t) \) (i.e. in the form of Eq. (1)), the second-order partial derivatives with respect to both time and space are regarded as negligible terms in comparison to the linear terms \( \partial E / \partial t \), etc. The Maxwell equation is essentially reduced to a first-order PDE.

When comparing the real and the imaginary parts of both sides of this PDE, one arrives at the coupled equations

\[
\frac{\partial E}{\partial x} + \frac{c}{c} \frac{\partial E}{\partial x} = -\frac{k_{\mu}}{e_{0}} \Im \{ F \}, \tag{B1}
\]

\[
\frac{\partial \varphi}{\partial x} + \frac{1}{c} \frac{\partial \varphi}{\partial x} = -\frac{k_{\mu}}{e_{0}} \Re \{ F \}, \tag{B2}
\]

for the envelope variable and the phase variable. For the local time \( \tau = t - x / v \), the two derivative operators on the left hand side can be combined, i.e.

\[
\frac{\partial}{\partial t} + c \frac{\partial}{\partial x} = \frac{\partial}{\partial \tau} + \frac{1}{c} \frac{\partial}{\partial \tau}, \tag{B3}
\]

under which Eqs. (B1)-(B2) become ODEs of one variable:

\[
\frac{dE}{d\tau} = \frac{\omega}{2 \left(1 - \frac{\tau}{\gamma} \right) e_{0}} \Im \{ F \}, \tag{B4}
\]

\[
\frac{d\varphi}{d\tau} = -\frac{\omega}{2 \left(1 - \frac{\tau}{\gamma} \right) e_{0}} \Re \{ F \}, \tag{B5}
\]

where the equation of \( E(\tau) \) is decoupled from that of \( \varphi(\tau) \). Rearranging the factors on two sides lead to Eqs. (17)-(18).

**Appendix C: Solving for envelope and phase**

Noticing that the sinusoidal factor in Eq. (14) is simply \( \sin A \), given the definition of the envelope area \( A \), we can write Eq. (19) as

\[
\frac{dE}{d\tau} = -\frac{\mu c k}{2 (c / v - 1) e_{0}} e^{-\Gamma / 2} \sin A. \tag{C1}
\]

Recognizing \( dE / d\tau = \frac{d^2 A}{d\tau^2} \) and using the abbreviation \( M = \sqrt{\mu^2 k c v / 2 e_0 (c - v)} \), we arrive at the pendulum equation

\[\hat{A} = M^2 e^{-\Gamma / 2} \sin A. \tag{C2}\]

Since \( \hat{A} = dA / d\tau \), the equation can be rewritten as

\[\hat{A} dA = M^2 e^{-\Gamma / 2} \sin A dA. \tag{C3}\]

Formally integrating on both sides while regarding the decay factor \( \Gamma(\tau) \) as a slow-varying variable compared to \( A(\tau) \) under a Born-Oppenheimer approximation, one gets

\[\hat{A}^2 = 2M^2 (1 - \cos A) e^{-\Gamma / 2} = 4M^2 e^{-\Gamma / 2} \sin^2 \frac{A}{2}. \tag{C4}\]

Then taking the square root, one arrives at a first-order equation

\[\frac{dA}{d\tau} = 2Me^{-\Gamma / 4} \sin \frac{A}{2}, \tag{C5}\]

whereby the \( \sin A / 2 \) factor can be moved to LHS and we can formally solve for \( A \):

\[\ln \tan \frac{A}{4} = M \int_{\tau_0}^{\tau} e^{-\Gamma / 4} d\tau + C \tag{C6}\]

where \( \tau_0 \) is an arbitrary initial time of integration and \( C \) is the integration constant. Hence,

\[\tan \frac{A}{4} = C \exp \left\{ M \int_{\tau_0}^{\tau} e^{-\Gamma / 4} d\tau \right\} \tag{C7}\]

and letting \( A_0 = A(\tau_0) \) shows \( C = \tan A_0 / 4 \). Absorbing \( C \) into the exponential, we can simplify the above into

\[\tan \frac{A}{4} = \exp \left\{ M \int_{\tau_0}^{\tau} e^{-\Gamma / 4} d\tau + \tau_D \right\}, \tag{C8}\]

where \( \tau_D = \frac{1}{4} \ln \tan A_0 / 4 \) can be regarded as the delay time. Then, using the identity \( \sin 2\theta = 2 \tan \theta + (\tan \theta)^{-1} \), Eq. (C5) can be written as

\[\frac{dA}{d\tau} = 4Me^{-\Gamma / 4} \sech \left\{ M \int_{\tau_0}^{\tau} e^{-\Gamma / 4} d\tau + \tau_D \right\}, \tag{C9}\]

which gives Eq. (20).

For the phase \( \varphi(\tau) \), we substitute the decay factor Eq. (13) into Eq. (17) to get

\[\frac{d\varphi}{d\tau} = \frac{\mu c k}{2e(\varepsilon_0 - 1)} (1 - e^{-\Gamma}) = \frac{M^2}{\mu \varepsilon} (1 - e^{-\Gamma}), \tag{C10}\]

which is an integro-differential equation, considering the expression of \( \varepsilon \) in Eq. (20). Like described in the text, we simplify the consideration by reducing \( \Gamma(\tau) \) to a linear dependence on time, writing \( \Gamma(\tau) = 4C_0(\tau - \tau_0) \) and hence

\[\int_{\tau_0}^{\tau} e^{-\Gamma(s)/4} ds = \int_{\tau_0}^{\tau} e^{-C_0(s - \tau_0)} ds = -\frac{1}{C_0} \left[ e^{-C_0(\tau_0 - \tau_0)} - 1 \right]. \tag{C11}\]

Therefore, Eq. (C10) reads

\[\frac{d\varphi}{d\tau} = \frac{M}{4} e^{\Gamma / 4} - e^{-3\Gamma / 4} \sech \left\{ \frac{M}{4} \int_{\tau_0}^{\tau} ds e^{-\Gamma(s)/4} + \tau_D \right\} \]

\[= \frac{M}{4} \left[ e^{C_0(\tau - \tau_0)} - e^{-3C_0(\tau - \tau_0)} \right] \times \sech \left\{ \frac{-M}{C_0} \left( e^{-C_0(\tau_0 - \tau_0)} - C_0 \tau_D - 1 \right) \right\}. \tag{C12}\]

Then integrating both sides with respect to the local time \( \tau \) yields Eq. (21).
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