Weyl focusing effects on the image magnification due to randomly distributed isothermal objects

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ABSTRACT
Weyl focusing effects on the image magnifications are investigated by using the multiple gravitational lens theory. We focus on the gravitational lensing effects due to the small scale virialized objects, such as galaxies and clusters of galaxies. We consider a simple model of an inhomogeneous universe. The matter distribution in the universe is modeled by randomly distributed isothermal objects. We found that, for the majority of the random lines of sight, the Weyl focusing has no significant effect and the image magnification of a point like source within redshift of 5 is dominated by the Ricci focusing.

Key words: cosmology – gravitational lensing.

1 INTRODUCTION
Distance-redshift relation plays an important role in the astronomy and observational cosmology. The standard distance, which has been used in the most of previous studies, is based on a postulate that a distribution of matter in the universe is homogeneous (e.g., Weinberg 1972). It has been, however, well recognized that our universe is highly inhomogeneous on small scales. Since the inhomogeneities of the mass distribution focus (defocus) the bundle of light rays (the gravitational lensing effects), the distance in an inhomogeneous universe deviates from that in the homogeneous Friedmann universe. It is, therefore, obvious that a detail understanding of the propagation of light rays in the inhomogeneous universe is necessary for correct studies of objects in a distant universe.

Since the pioneering work by Gunn (1967), there has been a lot of progress in this subject. Babul & Lee (1991), among others, studied the effects of the Ricci focusing by weak inhomogeneities. They found that the dispersion in image magnifications due to large scale ($\sim 0.5h^{-1}$Mpc, where $H_0 = 100h$km/sec/Mpc) structures is negligible even for sources at redshift of 4. They also pointed out that the dispersion is very sensitive to the nature of the matter distribution on small scales. The same result was also obtained by Frieman (1996). He improved Babul & Lee’s study to reflect the recent developments in numerical and observational studies of the large scale structure. In these studies, the effects of the Weyl focusing which induce a shear of light ray bundle were neglected. Nakamura (1997) examined the effects of the shear on the image magnification in the cold dark matter model universe with linear density perturbation. He found that the effect is sufficiently small and concluded that the Weyl focusing can be safely neglected for a light ray passing through a linear density inhomogeneities. Jaroszyński et al. (1990) and Wambsganss et al. (1997) used the multiple gravitational lens theory with a large N-body simulation of the cold dark matter universe. Their result agreed well with analytical studies.

The above studies mainly focused on the large scale inhomogeneities, whereas the effects of small scale objects, such as galaxies and clusters of galaxies, have not been fairly taken into account. Kayser & Refsdal (1988) investigated the gravitational lensing effects due to randomly distributed King model galaxies. They paid a special attention to a high magnification part of the magnification probability distribution. Recently Wambsganss, Cen & Ostriker (1998) studied the gravitational lensing effects by using the large N-body simulation with an effective resolution of comoving $10h^{-1}$kpc. They, first, shoot the light ray through the lens planes by using the multiple gravitational lens equation, then the magnification matrix is determined from the mapping of the light ray positions between the image and source plane. In this procedure, the Ricci and Weyl focusing cannot be treated independently, therefore no discussion is given for the Weyl focusing effect in their paper. However, the magnification factor as a function of position in the source plane and image plane are presented in Figure 4 and 6 of their
paper. Those Figures show that there is quite a large region that is demagnified by a small amount, and a few relatively small spots that are quite highly magnified by the small scale inhomogeneities.

The purpose of this paper is to examine the Weyl focusing effect due to the small scale inhomogeneities, and we are not concerned with the effect due to the large scale structure. The density distribution in the universe is modeled as a randomly distributed isothermal lenses. The isothermal lens is approximated by virialized objects, such as galaxy and cluster of galaxies. This model is similar to the one studied by Kayser & Refsdal (1988). However our emphasis is different from theirs, i.e. we pay special attention to the gravitational lensing effects on a majority of light rays. Although this model is a very simplified and unrealistic one, we believe that the model is good enough to investigate the essential points of the Weyl focusing effect due to the small scale inhomogeneities.

This paper is organized as follows: In section 2, we describe our models and method of simulating the light propagation. The results of the simulation are summarized in section 3. The paper concludes with discussion in section 4.

2 MODELS AND METHOD

2.1 Theory of multiple gravitational lensing

We use the multiple lens equations to trace the propagation of infinitesimal bundles of light rays. Schneider, Ehlers & Falco (1992) deals with the theory of multiple gravitational lensing in detail. Here we simply describe only the aspects which are directly relevant to this paper.

As was done in previous studies (see e.g., Blandford & Narayan, 1986 and Kovner 1987), we consider $N$ screens (lens planes) between an observer ($z = 0$) and source ($z = z_s$), located at redshifts $z_i$ with $i$ runs from 1 to $N + 1$ with $z_{N+1} = z_s$. In the following, the quantities on the lens and source planes are described by indices $\{A, B,..\} \equiv \{1, 2\}$.

The position vector of a light ray on the $i$-th lens plane is denoted by $y_A(z_i)$. Let $\hat{\alpha}_A(y(z_i))$ denotes the deflection angle of a light ray at position $y_A(z_i)$ on the $i$-th lens plane. The multiple gravitational lens equation and the evolution equation of the lensing magnification matrix $M_{AB}$ are written as,

$$y_A(z_i) = \frac{D_j}{D_i} y_A(z_i) \sum_{i=1}^{j-1} D_{ij} \hat{\alpha}_A(y(z_i)),$$

$$M_{AB}(y(z_i)) = \delta_{AB} - \sum_{i=1}^{j-1} \frac{D_{ij} D_j}{D_i} \hat{\alpha}_{A,C}(y(z_i)) M_{CB}(y(z_i)),$$

for $2 \leq j \leq N + 1$, where $D_{ij}$ ($D_i$) denotes the standard angular diameter distance between redshifts of $z_i$ and $z_j$ ($0$ and $z_i$) with $i < j$, the comma denotes differentiation with respect to the components of $y_A(z_i)$ and $y(z_i) \equiv y_A(z_i)$. The deflection angle $\hat{\alpha}_A(y)$ is determined by the following equation:

$$\hat{\alpha}_A(y) = \frac{4G}{c^2} \int d^2 y' \frac{y - y'}{|y - y'|^2} (\Sigma(y') - \langle \Sigma \rangle),$$

where $\Sigma(y)$ is the surface mass density and $\langle \Sigma \rangle$ is its average value. For a notation convenience, we introduce the following quantities:

$$\alpha_A(z_i, z_j) = D_{ij} \hat{\alpha}_A(y(z_i)),$$

$$T_{AB}(z_i, z_j) = \frac{D_{ij} D_j}{D_i} \hat{\alpha}_{A,B}(y(z_i)).$$

The optical tidal matrix $T_{AB}(z_i, z_j)$ is decomposed into the Ricci and Weyl focusing terms, respectively:

$$\mathcal{R}(z_i, z_j) = \frac{1}{2} (T_{11}(z_i, z_j) + T_{22}(z_i, z_j)),$$

$$\mathcal{F}(z_i, z_j) = \frac{1}{2} (T_{11}(z_i, z_j) - T_{22}(z_i, z_j)) + i T_{12}(z_i, z_j).$$

In general, equation (3) is not an explicit equation for $M_{AB}$, since the equation involves a summation over $T_{AB}$ evaluated on the light ray path, such that one first has to solve the multiple gravitational lens equation (4). However for the light rays traveling in regions where $\alpha_A$ and $T_{AB} < 1$, one can expand $M_{AB}$ in powers of $\alpha_A$ and $T_{AB}$ about its value when the light ray is unperturbed. We rewrite equation (3) as $y_A(z_i) = y^{(0)}_A(z_i) + y^{(1)}_A(z_i) + O(\alpha^2)$ and equation (4) as $M_{AB}(y(z_i)) = M^{(0)}_{AB}(y^{(0)}(z_i)) + M^{(1)}_{AB}(y^{(0)}(z_i)) + M^{(2)}_{AB}(y^{(0)}(z_i)) + ...$, where $y^{(0)}_A(z_i)$ is the first term of the right hand side of equation (3) and $y^{(1)}_A(z_i)$ is the second term, but the deflection angle is evaluated at the unperturbed light ray position. Expanding equation (4) in terms of $\alpha_A$ and $T_{AB}$, one finds

$$M^{(0)}_{AB}(z_i) = \delta_{AB},$$
of the mean separation length between the lens objects defined by
\[ m = \left| \det \mathcal{M}_{AB} \right|^{-1}, \]
for \( 3 \leq j \leq N + 1 \). In the above expressions, \( T_{AB} \) and \( \alpha_A \) are evaluated at the unperturbed light ray position. The image magnification factor of a point like source is given by the inverse of the determinant of the magnification matrix, i.e. \( \mu = \left| \det \mathcal{M}_{AB} \right|^{-1} \). Up to the order of \( \mathcal{M}_{AB}^{(2)} \), the determinant is
\[
\det \mathcal{M}_{AB}(z_j) = 1 - 2 \sum_{i=1}^{j-1} R(z_i, z_j) + \left( \sum_{i=1}^{j-1} R(z_i, z_j) \right)^2 - \sum_{i=1}^{j-1} \mathcal{F}(z_i, z_j)^2
+ 2 \sum_{i=2}^{j-1} \sum_{k=1}^{i-1} \{ R(z_i, z_j) R(z_k, z_i) + \text{Re}[\mathcal{F}'(z_i, z_j) \mathcal{F}(z_k, z_i)] + R(z_i, z_j) \alpha_A(z_k, z_i) \},
\]
for \( 3 \leq j \leq N + 1 \). This is our principal equation. In general, the effects of Ricci and Weyl focusing on image magnification are coupled. However up to this order, they are not coupled. We call the terms in the equation (11) which involve Ricci focusing terms as “Ricci contribution” and that involve Weyl focusing terms as “Weyl contribution”.

### 2.2 Truncated singular isothermal sphere lens model

We adopt the truncated singular isothermal sphere as the lens model. Its surface mass density is written as
\[
\Sigma(R) = \frac{\sigma_v^2}{2GR} \left( 1 + \frac{R}{R_G} \right)^{-2},
\]
where \( \sigma_v \) is the one-dimensional velocity dispersion and \( R_G \) is the half mass radius (Pei, 1993). The corresponding 3-dimensional mass density runs as \( r^{-2} (r \ll R_G) \) and as \( r^{-4} (r \gg R_G) \), and the mass within the radius \( R \) is
\[
m(\leq R) = \frac{\pi \sigma_v^2 R_G}{G} \frac{R}{R_G + R},
\]
then the total mass is \( m_{\text{tot}} = \pi \sigma_v^2 R_G/G \). This models should provide a fair approximation to virialized objects, such as a galaxy and cluster of galaxies with isothermal dark halos.

For the distribution of matter in the universe, we assume that the isothermal lenses are randomly distributed with the average mass density \( \rho_L(z) \) and the rest of the matter has the uniform distribution. We also assume that the comoving density of lenses is constant in time, thus \( \rho_L(z) = (1 + z)^3 \rho_L(0) \). Furthermore, we approximate the lensing effects of the isothermal objects, except the nearest one, to a form of uniform surface mass density.

Under the above assumptions and the circularly symmetric mass distribution of the lens model, the deflection angle is given by
\[
\alpha(z_i, z_j) = \sqrt{\alpha_1^2(z_i, z_j) + \alpha_2^2(z_i, z_j)}
= 4\pi \left( \frac{\sigma_v}{c} \right)^2 D_{ij} \left[ \frac{R_G}{R_G + R} + \frac{R_G^2 R}{R_G^2 (R_G + R)} - \frac{R_G R}{R_G^2} \right]
= a_{cr}(z_i, z_j) \left[ \frac{R_G}{R_G + R} - \frac{R_G R}{R_G (R_G + R)} \right],
\]
with
\[
a_{cr}(z_i, z_j) \equiv 4\pi \left( \frac{\sigma_v}{c} \right)^2 D_{ij} D_i,\]
where \( R \) is the distance between a light ray position and center of the nearest lens object in the \( i \)-th lens plane. \( R_c \) is a half of the mean separation length between the lens objects defined by
\[
R_c = \left( \frac{m_{\text{tot}}}{\pi \Sigma_L^2} \right)^{\frac{1}{2}}
= \sqrt{a_{cr}(z_i, z_j) R_G} \left[ \frac{3}{2} \Omega_L(0)(1 + z_i)^3 \left( \frac{H_0}{c} \right)^2 \frac{D_{ij} D_i}{D_j} \frac{cdt}{dz} \delta z \right]^{-\frac{1}{2}},
\]
where \( \Sigma_L \) is the average surface mass density of the lens objects in the \( i \)-th lens plane, and \( \delta z \) is the redshift interval between the \( (i - 1) \)-th and \( i \)-th lens planes, and \( \Omega_L(0) \equiv \rho_L(0)/\rho_c(0) = \rho_L(0)(8\pi G/3H_0^2) \) is the density parameter of lens objects.
The first term in square bracket of the second line in equation (13) describes the deflection due to the nearest lens object, the second term is due to the others, and the last term is due to the background average mass density of lens objects. From the deflection angle (13), the Ricci and Weyl focusing terms are immediately given by

$$R(z_i, z_j) = a_{cr}(z_i, z_j) \frac{1}{2} \frac{R_G^2}{R(R_G + R)^2} - \frac{R_G}{R(R_G + R)} \right],$$

(16)

$$|F(z_i, z_j)| = a_{cr}(z_i, z_j) \left[ \frac{1}{2} \frac{R_G(R_G + 2R)}{R(R_G + R)^2} \right].$$

(17)

We introduce a compactness parameter $\nu$ as follows;

$$\nu = 4\pi \left( \frac{\sigma_v}{c} \right)^2 \frac{1}{R_G} \frac{c}{H_0} \approx 1.68 \times \left( \frac{\sigma_v}{200 \text{km/sec}} \right)^2 \times \left( \frac{10^{16} \text{pc}}{R_G} \right) h^{-1}.$$  

(18)

This parameter measures the effectiveness of an isothermal object as a gravitational lens. As was pointed out by Kayser &Refsdal (1988), the model with randomly distributed isothermal lens objects is completely described by two parameters. The parameters they used depend on the distance between source and observer. On the other hand, we characterize the model by the compactness parameter $\nu$ and density parameter of lens object $\Omega_L(0)$ (hereafter we shall simply denote it as $\Omega_L$) which are independent of redshifts of the source and lenses.

We should note that the optical tidal matrix $T_{AB}$ becomes larger than unity at the very central region of the lens object (the inner region of $\sim 0.5 \times$ Einstein radius), therefore the validity of assumptions used in deriving the equation (14) breaks down for light rays passing through that region. We now estimate the strong lensing effect on our study in two ways.

The first is based on an order-of-magnitude estimate (section 14 of Peebles, 1993, and also Futamase & Sasaki, 1989), we examine a magnitude of the tidal matrix $T_{AB}$. Suppose the lensing objects are randomly distributed and each with mass $M = 2\sigma_v^2 l/G$, where $l$ is a characteristic comoving size of a lens object and is of order $R_G$. Hence the mean comoving number density of the lens objects is $n_L = \Omega_L(3H_0^2/16\pi)\sigma_v^{-2}l^{-1}$, so the mean comoving separation distance is $r_0 = \Omega_L^{-1/3}(3H_0^2/16\pi)^{-1/3}\sigma_v^{-2/3}l^{-1/3}$. Then for a geodesic affine comoving distance of $\lambda$, the light ray gravitationally encounters such objects $N_\theta = \lambda/r_0$ times in average. At each encounter, the contribution to the tidal matrix is $\delta T = 4\pi(\sigma_v^2/c^2)(r_0/h^2)(\hat{D}_\theta \hat{D}_\theta \hat{D}_s \hat{D}_s) \sim 4\pi(\sigma_v^2/c^2)\rho^{-1}(\hat{D}_\theta \hat{D}_\theta \hat{D}_s \hat{D}_s)$, where $\hat{D}_\theta$ is a comoving angular diameter distance with the subscript $d$ stands for a lens (source), $b$ is the comoving impact parameter, and we have assumed that the mean comoving impact parameter is of order $r_0$. Since the sign of each contribution will be random, the total contribution to the optical tidal matrix will be

$$\delta T \sqrt{N_\theta} \sim 3/4\sqrt{\Omega_L} \left[ 4\pi(\sigma_v^2/c^2)\frac{1}{T H_0} \right]^{1/2} \left( \frac{H_0}{c} \frac{\hat{D}_\theta \hat{D}_\theta \hat{D}_s \hat{D}_s}{\hat{D}_s} \right) \left( \frac{H_0}{c} \right)^{1/2}$$

(19)

The contribution from the direct encounters can be similarly estimated by noting that the average number of encounters is $N_d = l^4/\lambda r_0^3 = \lambda\Omega_L(3H_0^2/16\pi)\sigma_v^{-2}l^3$, with each encounter contributing $\delta T_d = 4\pi(\sigma_v^2/c^2)l^{-1}(\hat{D}_\theta \hat{D}_\theta \hat{D}_s \hat{D}_s)$ with random sign. The result turns out to be the same as that of gravitational distant encounters, equation (14). In the case of Einstein-de Sitter universe model, the comoving distance $\lambda$ becomes $c/H_0$ at the source redshift $z_s = 3$ and the averaged value of distance combination over the lens redshifts is $\langle (H_0/c)(\hat{D}_\theta \hat{D}_\theta \hat{D}_s \hat{D}_s) \rangle = 1/6$ for any redshifts of source. Accordingly, we find that the magnitude of the optical tidal matrix scales as $\sim 0.2\sqrt{\Omega_L} \sqrt{\nu}$ for the source redshift of 3. In the following we only consider lens models with $\Omega_L \leq 1$ and $\nu \leq 1$, therefore a typical value of the optical tidal matrix can be expected to be of order $\mathcal{O}(0.1)$ or lower. Thus, we can conclude that $T_{AB}$ is less than unity for a majority of random lines of sight.

Next we examine the lensing optical depth defined by Turner, Ostriker & Gott, (1984) to quantify the probability of the light rays being affected by strong lensing. The differential optical depth for the truncated singular isothermal sphere model is given by

$$d\tau = \frac{3}{2} \Omega_L E(1 + z_d)^3 \left( \frac{H_0}{c} \right)^2 \frac{\hat{D}_\theta \hat{D}_\theta \hat{D}_s \hat{D}_s}{\hat{D}_s} \frac{cdt}{dz_d} dz_d.$$  

(20)

where

$$E = \frac{1}{4} \frac{R_G}{a_{cr}} \left[ \left( 1 + \frac{z_s}{a_{cr}} \right) \frac{4}{R_G(R_G + R_c)} \right]^{1/2} - 1 \right]^2.$$  

(21)

We numerically integrate the last equation for cases of the lens models with $(\Omega_L, \nu) = (1, 0.1), (1, 1), (0.2, 0.1)$ and $(0.2, 1)$, and for the Einstein-de Sitter universe model. The results are presented in Table 1. From the Table 1, it can be found that the
Table 1. The lensing optical depth.

| \((\Omega_L, \nu)\) | \((1, 1)\) | \((0.1, 1)\) | \((0.2, 0.1)\) | \((0.2, 1)\) |
|-----------------|---------|---------|---------|---------|
| \(z_s = 1\)    | \(9.8 \times 10^{-4}\) | \(8.5 \times 10^{-3}\) | \(2.0 \times 10^{-4}\) | \(1.7 \times 10^{-3}\) |
| \(2\)           | \(2.9 \times 10^{-3}\) | \(2.5 \times 10^{-2}\) | \(5.9 \times 10^{-4}\) | \(5.0 \times 10^{-3}\) |
| \(3\)           | \(4.9 \times 10^{-3}\) | \(4.1 \times 10^{-2}\) | \(9.8 \times 10^{-4}\) | \(8.1 \times 10^{-3}\) |
| \(4\)           | \(6.6 \times 10^{-3}\) | \(5.5 \times 10^{-2}\) | \(1.3 \times 10^{-3}\) | \(1.1 \times 10^{-2}\) |
| \(5\)           | \(8.1 \times 10^{-3}\) | \(6.7 \times 10^{-2}\) | \(1.6 \times 10^{-3}\) | \(1.3 \times 10^{-2}\) |

Probabilities of strong lensing events are very small except for an extreme model with \((\Omega_L, \nu) = (1, 1)\). Even for the extreme model, the probability is not significantly large. We thus conclude that the strong lensing effects do not significantly alter our results. It can be said from the above two estimations that we can safely use the perturbative equation (11).

2.3 Ray shooting

Since we have assumed the random distribution for lens objects, the probability of finding lenses in some region on a lens plane is described by Poisson distribution. If one sets the redshift interval \(\Delta z\) to be sufficiently small, the surface number density of lenses becomes small. In this case, the lensing effects are mainly due to the nearest lens and are well approximated by the equations (13), (14) and (17). Then the all necessary information about evaluating the magnification factor (11) is obtained by randomly determining the relative position between a light ray and the nearest lens object in each lens plane. We perform Monte-Carlo simulations to trace the propagation of light rays. The procedure is described in the following:

(i) First of all, we determine the redshift intervals of lens planes to satisfy the condition that \(R_G/R_c\) is sufficiently small. We set \(R_G/R_c < 0.1\).

(ii) The relative positions between a light ray and a center of lens object are randomly determined in each lens plane.

(iii) Summations in equation (11) are performed for each term, and the results are stored in a file.

Steps (ii) and (iii) are repeated for each light ray.

3 RESULTS

For the background universe, we only consider the Einstein-de Sitter universe model, i.e., \(\Omega_{\text{de}} = 1\) and \(\lambda_0 = 0\). For the lens models, we choose the compactness parameter \(\nu = 1\) and 0.1 which roughly correspond to a galaxy scale inhomogeneity and the scale of a cluster of galaxies respectively. The density parameter of the lens objects are set to be \(\Omega_L = 1\) and 0.2. We choose the source redshifts \(z_s = 1, 2, 3, 4\) and 5. From the condition \(R_G/R_c < 0.1\), the redshift intervals between lens planes are typically set to be \(\sim 10^{-2}\).

\(10^6\) runs are performed for each model. For each run (light ray), we then obtain the Ricci and Weyl contribution and identically the image magnification factor. This immediately gives distribution functions of runs on the Ricci-Weyl contribution plane for all the source redshifts. We calculate number densities of results of runs in Ricci-Weyl contribution plane. The peaks of the number density and the isodensity contours which enclose 68\% of all runs are presented in Figure 1. The probability distributions of Ricci and Weyl contribution for the case of the source redshift of \(z_s = 3\) in \((\Omega_L, \nu) = (1, 1)\) model are shown in Figure 2. As is clearly shown in the optical scalar equation (see e.g., Schneider et al. 1992), the Weyl contribution is always negative. Figure 1 reveals that the Weyl contributions in the majority of light rays are rather small except in the case where the source redshifts \(z_s \geq 3\) in \((\Omega_L, \nu) = (1, 1)\) model. Alternatively, this point can also be shown in Figure 2. The probability distribution of the Weyl contribution has a narrow peak centered at very small values, in marked contrast with that of the Ricci contribution which has a broad distribution. Since we have assumed no evolution for lens objects, the dispersion keeps on spreading in Ricci-Weyl contribution plane even for high redshift.

In order to examine the effects of the Weyl focusing on the image magnification quantitatively, we calculate the magnification factors evaluated without the Weyl contribution (denoted by \(\mu_R\)). Then we calculate the following quantity which measures the influence of Weyl contribution on the image magnifications:

\[
\Delta \mu = \left( \frac{1}{N} \sum_{i=1}^{N} \left( 1 - \frac{\mu_R}{\mu} \right)^2 \right)^{1/2},
\]

where the summation is taken only over rays with \(\det M_{AB} > 0\). The rays with \(\det M_{AB} < 0\) belong to a multiple image system, therefore the percentage of runs which are excluded in the above evaluation roughly exhibits the probability of the strong lensing events among random lines of sight. At the same time, the above condition also excludes the rays which pass through the high \(T_{AB}\) region. The results are presented in Table 2 with the percentages of the excluded runs (in parentheses).

Observationally, the strong lensing events among the random lines of sight are very rare. For example, the probability of

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Figure 1. Isodensity contours which enclose 68% of all runs and peaks of surface number density of runs. The peaks are denoted by pluses. The solid lines are for a source redshift \( z_s = 1 \), dashed lines for \( z_s = 3 \) and long-dashed lines for \( z_s = 5 \). The lens model parameters \((\Omega_L; \nu)\) are denoted in each figure.

Table 2. \( \Delta \mu \), the effect of the Weyl contribution on the image magnification with the percentage of runs excluded in this evaluation (in parentheses).

| \( z_s \) | \( (\Omega_L, \nu) = (1, 0.1) \) | \( (1, 1) \) | \( (0.2, 0.1) \) | \( (0.2, 1) \) |
|----------|----------------|----------------|----------------|----------------|
| 1        | \( 1.8 \times 10^{-2} \) (0.16) | \( 5.7 \times 10^{-2} \) (1.0) | \( 7.6 \times 10^{-3} \) (0.024) | \( 2.6 \times 10^{-2} \) (0.18) |
| 2        | \( 3.1 \times 10^{-2} \) (0.55) | \( 1.0 \times 10^{-1} \) (3.3) | \( 1.4 \times 10^{-2} \) (0.083) | \( 4.3 \times 10^{-2} \) (0.56) |
| 3        | \( 4.0 \times 10^{-2} \) (0.96) | \( 1.3 \times 10^{-1} \) (5.6) | \( 1.8 \times 10^{-2} \) (0.15) | \( 5.5 \times 10^{-2} \) (0.95) |
| 4        | \( 4.8 \times 10^{-2} \) (1.34) | \( 1.5 \times 10^{-1} \) (7.5) | \( 2.1 \times 10^{-2} \) (0.21) | \( 6.5 \times 10^{-2} \) (1.3) |
| 5        | \( 5.5 \times 10^{-2} \) (1.7) | \( 1.7 \times 10^{-1} \) (9.2) | \( 2.3 \times 10^{-2} \) (0.27) | \( 7.2 \times 10^{-2} \) (1.6) |

the multiply imaged quasars in a quasar sample is at most \( 10^{-2} \) (e.g., Claeskens, Jaunsen & Surdej, 1996). Combining this fact with our results summarized in Table 2, it may be reasonably concluded that, as far as our simple matter distribution model is concerned, the Weyl focusing has a negligible effect on the majority of light rays even for sources at the redshift of 5. This result can be naturally explained by the following two reasons: First, the Weyl focusing is a second order effect on the image magnification, therefore it becomes important only for the light rays passing through a very high non-linear (relatively rare) region. Secondly, since we have assumed the random distribution for the isothermal lenses, the rays coherently affected by the Weyl focusing are very rare, consequently, the majority of light rays are only weakly influenced by the Weyl focusing.

4 DISCUSSIONS

In this paper, we restricted our study to the gravitational lensing effects due to the randomly distributed isothermal lenses. We found that the Weyl focusing effect is small, \( \Delta \mu \lesssim 0.1 \), for a majority of light rays.

Lee & Paczyński (1990) examined the gravitational lensing effects in randomly distributed clumps with Gaussian surface
mass density profile. They found that the image magnifications are dominated by Ricci focusing and the Weyl focusing has no significant effect. They only considered a case of a source redshift of \( z_s = 1.631 \), in an \( \Omega_0 = 1 \) and \( \lambda_0 = 0 \) universe. However, we have found that, as far as the random distribution of lens objects is concerned, the Weyl focusing is rather small even for higher redshift.

In this study, a correlation of lens objects and large scale structure are not taken into account. The study of the influences of the correlation on the Weyl focusing lies outside the scope of this paper, and will be examined in future works. On the other hand, the Weyl focusing effect due to the large scale structures is investigated by using N-body simulation (Jaroszyński et al., 1990) and analytically (Nakamura, 1997). These two studies show that, although there is an uncertainty of a normalization of the density power spectrum, the magnitude of Weyl focusing due to the large scale structure is of the order \( 10^{-2} \sim 10^{-1} \) for the source redshifts of \( 1 < z_s < 5 \) (Figure 3 of Jaroszyński et al., 1990 and Figure 3 of Nakamura, 1997). Consequently it can be said that, comparing the results of the above mentioned studies with our results, the Weyl focusing effect due to the large scale structures is comparable with or larger than that due to the small scale inhomogeneities.

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