Higgs Pair-Production in the Standard Model at Next Generation Linear $e^+e^-$ Colliders

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Abstract

We study the Higgs pair-production in the Standard Model of the strong and electroweak interactions at future $e^+e^-$ collider energies, with the reaction $e^+e^- \rightarrow t\bar{t}HH$. We evaluated the total cross section of $t\bar{t}HH$ and calculate the number total of events considering the complete set of Feynman diagrams at tree-level. The numerical computation is done for the energy which is expected to be available at a possible Next Linear $e^+e^-$ Collider: with center-of-mass energy 800, 1600 $GeV$ and luminosity 1000 $fb^{-1}$.

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I. INTRODUCTION

In the Standard Model (SM) [1] of particle physics, there are three types of interactions of fundamental particles: gauge interactions, Yukawa interactions and the Higgs boson self-interaction. The Higgs boson [2] plays an important role in the SM; it is responsible for generating the masses of all the elementary particles (leptons, quarks, and gauge bosons). However, the Higgs-boson sector is the least tested one in the SM, in particular the Higgs boson self-interaction. In the SM, the profile of the Higgs particle is uniquely determined once its mass $M_H$ is fixed. The decay width, the branching ratios and the production cross sections are given by the strength of the Yukawa couplings to fermions and gauge bosons, the scale of which is set by the masses of these particles. Unfortunately, the mass Higgs boson is a free parameter.

The only available information on $M_H$ is the lower limit $M_H \geq 114.1 \text{ GeV}$ established at LEP2 [3]. The collaborations have also reported a $2.1\sigma$ excess of events beyond the expected SM backgrounds consistent with a SM like Higgs boson with a mass $M_H = 115^{+1.3}_{-0.9} \text{ GeV}$ [3]. Furthermore, the accuracy of the electroweak data measured at LEP, SLC, and the Tevatron provides sensitivity to $M_H$: the Higgs boson contributes logarithmically, $\propto \log(M_H/M_W)$, to the radiative corrections to the $W/Z$ boson propagators. A recent analysis yields the value $M_H = 88^{+60}_{-37} \text{ GeV}$ corresponding to 95% C.L.

The search for Higgs boson is one of the main missions of present and future high-energy colliders. The observation of this particle is of major importance for the present understanding of the interactions of the fundamental particles.

The trilinear Higgs self-coupling can be measured directly in pair-production of Higgs particles at hadron and high-energy $e^+e^-$ linear colliders. Higgs pairs can be produced through double Higgs-strahlung of $W$ or $Z$ bosons [4–7], $WW$ or $ZZ$ fusion [5,8–11]; moreover through gluon-gluon fusion in $pp$ collisions [12–14] and high-energy $\gamma\gamma$ fusion [5,8,15] at photon colliders. The two main processes at $e^+e^-$ colliders are double Higgs-strahlung and $WW$ fusion:
double Higgs-strahlung: $e^+e^- \rightarrow ZHH$

$WW$ double-Higgs fusion: $e^+e^- \rightarrow \nu_e\bar{\nu}_eHH$. \hfill (1)

The $ZZ$ fusion process of Higgs pairs is suppressed by an order of magnitude since the electron-$Z$ coupling is small. However, the process $e^+e^- \rightarrow t\bar{t}H$, has been extensively studied. This three-body process is important because it is sensitive to Yukawa couplings. The inclusion of four-body processes with heavy fermions $f$, $e^+e^- \rightarrow f\bar{f}HH$ \hfill (5) in which the SM Higgs boson is radiated by a $t(\bar{t})$ quark, at future $e^+e^-$ colliders \hfill (16–18) with a c.m. energy in the range of 500 to 1600 GeV, such as the TESLA machine \hfill (19) is necessary in order to know its impact on the three-body mode processes and also to search for new relations that could have a clear signature of the Higgs boson production.

Moreover, this process depends on the Higgs boson triple self-coupling, which could lead us to obtain the first non-trivial information on the Higgs potential. We are interested in finding regions that could allow the observation of the process $t\bar{t}HH$ at the next generation of high energy $e^+e^-$ linear colliders. We consider the complete set of Feynman diagrams at tree-level (Fig.1) and used the CalcHep \hfill (20) packages for the evaluation of the amplitudes and of the cross section.

This paper is organized as follows: In Sec. II we present the total cross section for the process $e^+e^- \rightarrow t\bar{t}HH$ at next generation linear $e^+e^-$ colliders, and in Sec. III, we give our conclusions.

II. CROSS SECTION OF THE HIGGS PAIRS PRODUCTION IN THE SM AT NEXT GENERATION LINEAR POSITRON-ELECTRON COLLIDERS

In this paper, we evaluate the total cross section of the Higgs pair-production in the SM at next generation linear $e^+e^-$ colliders.

For the SM parameters, we have adopted the following: the angle of Weinber $\sin^2\theta_W = 0.232$, the mass ($m_t = 175$ GeV) of the top, the mass ($m_{Z^0} = 91.2$ GeV) of the $Z^0$, with the mass $M_H$ of the Higgs boson having been taken as inputs \hfill (21).
We have considered the high energy stage of a possible Next Linear $e^+e^-$ Collider with $\sqrt{s} = 800, 1600$ GeV and design luminosity $1000 \, fb^{-1}$. In the evaluation of the amplitudes and of the cross section, we used the CalcHep [20] packages.

In order to illustrate our results of the production of Higgs pairs in the SM, we present in Fig. 2 a plot for the total cross section as a function of Higgs boson mass $M_H$. We observe in this figure that the total cross section for the double Higgs production is of the order of $0.02 \, fb$ for Higgs masses in the lower part of the intermediate range. The cross sections are at the level of a fraction of femtobarn, and they quickly drop as they approach the kinematic limit. In these conditions, it would be very difficult to extract any useful information about the Higgs self-coupling from the studied process except that the $e^+e^-$ machine works with very high luminosity.

Fig. 3 shows the total cross section as a function of the center-of-mass energy $\sqrt{s}$ for two representative values of the Higgs mass $M_H = 110, 130$ GeV. We observe that the cross section is very sensitive to the Higgs boson mass and decreases when $M_H$ increases. Our conclusion is that for an intermediate Higgs boson, a visible number of events would be produced, as is illustrated in Table I.

For center-of-mass energies of 800-1600 GeV and high luminosity, the possibility of observing the process $t\bar{t}HH$ is promising as shown in Table I.

| Total Production of Higgs Pairs | $t\bar{t}HH$ |
|-------------------------------|--------------|
| $M_H (GeV)$                   | $\sqrt{s} = 800 \, GeV$ | $\sqrt{s} = 1600 \, GeV$ |
| 110                           | 14           | 19           |
| 130                           | 6            | 15           |
| 150                           | 2            | 11           |
| 170                           | 1            | 9            |
| 190                           | 7            |              |

Table I. Total production of Higgs pairs in the SM for $L = 1000 \, fb^{-1}$ and $m_t = 175$ GeV.
We also include in Fig. 4 a contours plot for the number of events of the studied process, as a function of $M_H$ and $\sqrt{s}$.

III. CONCLUSIONS

In conclusion, the double Higgs production in association with $t(\bar{t})$ quarks ($e^+e^- \to t\bar{t}HH$) will be observable at the Next Generation Linear $e^+e^-$ Colliders. The study of this process is important in order to know their impact on the 3-body process and it could be useful to probe anomalous $HHH$ coupling given the following conditions: very high luminosity, center-of-mass large energy and intermediate range Higgs mass.

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**FIGURE CAPTIONS**

**Fig. 1** Feynman Diagrams at tree-level for $e^+e^- \rightarrow t\bar{t}HH$.

**Fig. 2** Total cross section of the Higgs pairs production $e^+e^- \rightarrow t\bar{t}HH$ as function of the Higgs mass $M_H$ for $\sqrt{s} = 800, 1600$ GeV with $m_t = 175$ GeV.

**Fig. 3** Total cross section of the Higgs pairs production $e^+e^- \rightarrow t\bar{t}HH$ as function of the center-of-mass energy $\sqrt{s}$ for two representative values of the Higgs mass $M_H = 110, 130$ GeV with $m_t = 175$ GeV.

**Fig. 4** Contours plot for the number of events as a function of $M_H$ and $\sqrt{s}$. 
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