Charm rescattering contribution in rare $B^+_c \to K^+K^−\pi^+$ decay

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Abstract

Following the experimental results from LHCb on the rare decay $B^+_c \to K^-K^+\pi^+$, we investigate the possibility where this process is dominated by a double charm rescattering. The $B_c$ decay to double charm channels have a weak topology that is favoured in comparison with the direct production of $K^-K^+\pi^+$ in the final state, suppressed by quark annihilation. The decay amplitude for $B^+_c \to K^-K^+\pi^+$ with $B_c$ decaying first to double charm channels is described by charm hadronic triangle loops, which reach the final state of interest after $D\bar{D} \to KK$ or $D^+D^− \to π^+K^−$ transitions. We show that these processes give rise to non-resonant amplitudes with a clear signature in the Dalitz plot. In a near future, the new data from LHCb run II will be able to confirm if the main hypotheses of this work is correct and the dominant mechanism to produce $K^+K^+\pi^−$ from the decay of $B^+_c$ is through charm rescattering.

Keywords: rare three-body decays, hadronic loop, rescattering.

1. Introduction

Recently a LHCb experiment [1] reported the data for the $B^+_c \to K^-K^+\pi^+$ decay. Evidence was presented for a 4σ signal for the $B^+_c \to χ_c(\to K^+K^-)π^+$ and an indication of 2.4 σ signal for the $B^+_c \to K^-K^+\pi^+$ decay out of $χ_c0$ mass region, using run I data. This rare decay at quark tree level is produced only by the suppressed W-annihilation topology. However, one can also consider the possibility that this $K^+K^-\pi^+$ final state is produced by a favoured topology, like the allowed double charm $B_c$ decay channel, followed by the rescattering of the charm mesons leading to light pseudoscalar channels. The importance of rescattering in nonleptonic B decays has been discussed in different theoretical approaches [2, 3, 4, 5, 6, 7, 8]. In this context the $B^+_c \to K^-K^+\pi^+$ channel, suppressed at quark level and favoured by double charm final states, is a good candidate to study the importance of hadronic final state interactions (FSI) to form the observed decay channels. In this particular decay the rescattering mechanism is proposed to be driven by a triangle hadronic loop [2, 6].

In a recent work we have also investigated the charm penguin contribution in $B^+ \to K^-K^+K^+$ decay [2] for two different regions of the phase space: the nonperturbative region, described by a hadronic triangle loop, and the quasi perturbative region described by a quark loop. The former includes a $D\bar{D} \to KK$ transition for which we proposed a scattering amplitude inspired by the Regge expansion, considering the damping factor of the S-matrix with off-shell effects. In our studies for $B^+ \to K^-K^+K^+$ decay we showed that both penguin loops, i.e. the double charm quark level loop and the hadronic triangle with charm mesons propagators, will present a particular signature in Dalitz plot.

In another theoretical approach Gronau, London and Rosner [5] proposed a method to estimate the relative importance of the double charm rescattering to light mesons in nonleptonic B decays for weak processes that at the fundamental level generally occur via the suppressed quark annihilation and W exchange diagrams. They size the amplitude of the diagram associated with the dominant topology based on known branching fractions of processes with similar topologies. The authors predicted branching fractions, including the $B^0 \to K^+K^−$, also suppressed at quark level by W-exchange (still unseen at that time), which agrees with recent LHCb results [6]. Other important issue raised by reference [5] was the quantification of hadronic rescattering to different channels including the double charm to light pseudoscalar mesons, such as $B_s \to D^+_s D^−_s \to π^+π^−$.

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In this work, we conjecture that the $B_c^+ \rightarrow K^- K^+ \pi^+$ decay occurs mainly through the production of Cabbibo-favoured double charm meson states followed by the hadronic rescattering to the $K^+ K^- \pi^+$ channel. We propose to investigate the Dalitz plot signatures of these contributions by performing an analysis following closely our previous work [2]. In the near future we can expect that the new results from LHCb run II increase the statistics presenting a better definition of the branching fraction and event distribution in the Dalitz plot for $B_c^+ \rightarrow K^- K^+ \pi^+$, which can confirm or disprove the dominance of the double-charm rescattering to form this particular decay channel.

2. Rescattering contribution to rare $B$ decay

Hadronic decays of the $B_c$ meson, the heaviest one established so far, are particularly intriguing. The dominant decay channels are composed of two heavy mesons in the final state, while decays directly producing light hadrons in final states at the fundamental level are suppressed by the weak annihilation topology of the quark processes. Another important characteristic of $B_c$ decays to light mesons is the huge phase space available, which allows contributions from charm and bottom mesons as intermediate states, as well as many rescattering combinations from double charm or exotics.

In the case of $B_c^+ \rightarrow K^- K^+ \pi^+$ one can see in Fig. 1 different possibilities of producing $KK\pi$ in the final state: a) directly from a tree level diagram, and b) and c) double charm production hadronizing to a pair of charmed mesons, which radiates a light meson and rescatters to a pair of light mesons, creating the final three-body decay channel via a hadronic process, as illustrated in Fig. 2. All diagrams are proportional to the same CKM elements, but in one side the direct process is known to be strongly suppressed by helicity conservation, whereas the rescattering contributions introduce a damping factor from the loop itself, as one could see from Eq. 1.

As discussed by Gronau, London and Rosner [5], the hadronic rescattering from topologies that are favoured, even when considering the suppression introduced by the rescattering amplitude, can give a significant contribution to the non-favoured light meson three-body decay channel, which might leave a signature in the data. In order to identify this signature we first have to build the three-body decay amplitude, starting from the favoured charmed meson two-body decay channels.

There are a number of double charm meson intermediate states that can contribute to the $K^- K^+ \pi^-$ final state of the $B_c$ decay. Following reference [5] we select only processes with large branching fractions to be considered as the dominant ones. In the $KK$ channel we consider the $B_c^+ \rightarrow K^- K^+ \pi^+$ decay to be mediated by $D^{*+}(2010)\bar{D}^0$, represented by the hadronic triangle loop shown in Fig. 2 (left). The branching fraction for $B_c^+ \rightarrow D^{*+}(2010)\bar{D}^0$ was measured to be of the order of $10^{-3}$ [10]: whereas $D^{*+}(2010) \rightarrow D^0 \pi^+$ accounts for $\approx 67\%$ of $D^{*+}(2010)$ width [10]. A similar double charm rescattering contribution is expected in the $K^- \pi^+$ channel, where $B_c^+ \rightarrow D^{0}\bar{D}^+$ is followed by the virtual decay $D^{0}\rightarrow D_s^+ K^+$ and the rescattering $D^+ D_s^- \rightarrow \pi^+ K^-$ as illustrated in Fig. 2 (right). Although the relative branching ratios are unknown, we expect them to be at same order of the previous one.

To fully calculate the two hadronic heavy loops presented in Fig. 2 we need the rescattering amplitudes $D^+ D_s^- \rightarrow \pi^+ K^-$ and $D^0 D^0 \rightarrow K^+ K^-$. The latter was developed in detail in our previous study of $B^+ \rightarrow K^- K^+ K^+$ decay [2] and can be adapted to describe the first one.
Figure 2: Charm hadronic loop contribution to \( B^+_c \rightarrow K^- K^+ \pi^+ \) decay. The final state \( K^- K^+ \pi^+ \) is reached after \( \bar{D}^0 D^0 \rightarrow K^+ K^- \) or \( D^+ D^- \rightarrow \pi^+ K^- \) rescattering. Left diagram: pion emission by the \( D^+ \) followed by \( D^0 D^0 \rightarrow K^+ K^- \) rescattering. Right diagram: kaon emission by the \( D^+ \) followed by \( D^+ D^- \rightarrow \pi^+ K^- \) rescattering.

3. Hadronic loop

Hadronic triangle loops were investigated previously in a number of processes in \( D \) and \( B \) three-meson decays [2, 7, 11, 13]. The presence of these particular modes of three-body final state interaction were reported to play an important role also in kinematical regions of the phase-space different from the one where the momentum is shared between all the particles in the final state. In the case of \( B^+_c \rightarrow K^- K^+ \pi^+ \), given the suppression of the process represented by the diagram a) of Fig. 1, we assume that there is no tree level contribution from the meson formation directly after the partonic process. Then, the decay amplitude is assumed to be described mainly by the loop diagrams in Fig. 2. These triangle loops are very similar but contribute to different channels: \( KK \) and \( \pi K \). Therefore, in what follows we detail only the calculation which includes the double charm rescattering to the \( KK \) channel.

Following the formalism developed in [2, 7, 11, 13] we obtain the amplitude for \( B^+_c \rightarrow K^- K^+ \pi^+ \) decay from the left diagram in Fig. 2 as a function of the center mass momentum above the threshold and \( m^2_{KK} = s_{23} \), which can be written as:

\[
A^{KK}_{B_c} (s_{23}) = \int \frac{d^4 l}{(2\pi)^4} \frac{T_{B_c \rightarrow D^+ D^-}(s)}{\Delta_{D^+} \Delta_{D^-} \Delta_{s_{23}}},
\]

where \( |\Delta_{D^+}|^{-1} \) are the charm propagators inside the loop:

\[
\Delta_{D^+} = (l - p_B)^2 - M^2_{D^+}, \quad \Delta_{D^-} = (l - P_B)^2 - M^2_{D^-}, \quad \Delta_{s_{23}} = l^2 - \Theta_{D^+},
\]

with \( \Theta_{D^+} = M^2_{D^+} - i\Gamma_{D^+}/2 \). The amplitude \( T_{B_c \rightarrow D^+ D^-}(s) \) accounts for the weak couplings: \( B_c \rightarrow W\bar{D}^0 \) and \( W \rightarrow D^+ \), with the former described by form factors with a single pole approximation with a nonzero width:

\[
F_{B_c \rightarrow D^0}(s) = F_{0} \frac{m^2_{D^0} - i\Gamma_{D^0}}{\Delta_{s_{23}}(s)}, \quad \Delta_{s_{23}}(s) = l^2 - (m^2_{D^0} - i\Gamma_{D^0}) .
\]

\( T_{D^0 D^0 \rightarrow KK} \) is the rescattering amplitude for \( D^0 D^0 \rightarrow K^+ K^- \) described by a phenomenological model taking into account the S-matrix unitarity [2] (for details see Appendix 2 of Ref. [2]). The model also takes into account the off-shell contribution below the double charm threshold \( s_{th DD} \):

\[
T_{D^0 D^0 \rightarrow KK}(s) = \frac{g^2}{s_{th DD}} \frac{2\kappa_2}{\sqrt{s_{th DD}}} \left( \frac{s_{th DD}}{s + s_{QCD}} \right)^{\xi + \alpha} \left[ \frac{c + bk^2_{K^0} - ik^1_{K^0}}{c + bk^2_{K^0} + ik^1_{K^0}} \right]^2 \left( \frac{1 + \kappa_2}{1 - \kappa_2} \right)^\frac{1}{2}, \quad s < s_{th DD},
\]

\[
= \frac{2\kappa_2}{s_{th DD}} \left( \frac{s_{th DD}}{s + s_{QCD}} \right)^{\xi} \left( \frac{m_0}{s - m_0} \right)^\beta \left[ \frac{c + bk^2_{K^0} - ik^1_{K^0}}{c + bk^2_{K^0} + ik^1_{K^0}} \right]^2 \left( \frac{1 - ik_2}{1 + ik_2} \right)^\frac{1}{2}, \quad s \geq s_{th DD},
\]

where \( \kappa_1 = \frac{1}{2} \sqrt{s - s_{th KK}} \) and \( \kappa_2 = \frac{1}{2} \sqrt{s - s_{th DD}} \) are the center mass momentum above the threshold and \( \kappa_1 = \frac{1}{2} \sqrt{s - s_{th KK}} \) and \( s \) is the rest frame double charm momentum below the threshold. The model parameters are given in Table 1.

The \( B^+_c \rightarrow K^- K^+ \pi^+ \) decay amplitude is then given by:

\[
A^{KK}_{B^+_c} (s_{23}) = iC \ m^2_{\pi^+} \int \frac{d^4 l}{(2\pi)^4} \frac{T_{D^0 D^0 \rightarrow KK}(s_{23}) \ - 2 p'_{2} \cdot (p'_{2} - p_1)}{\Delta_{D^+} \Delta_{D^-} \Delta_{s_{23}}},
\]

where \( C \) is the decay constant.
Table 1: Parameters used in the phenomenological scattering amplitude given by Eq. (3) [2]. The values for the $D^-D^+_s \to K^-\pi^+$ scattering amplitude are the same as $D^0\bar{D}^0 \to K^+K^-$ except for the low and high energy thresholds.

| $\alpha$ | $\xi$ | $c$ | $b$ | $a$ | $s_{QCD}$ | $m_0$ | $\beta$ | $s_{th\,KK}$ | $s_{th\,D\bar{D}}$ | $s_{th\,K\pi}$ | $s_{th\,DD_s}$ |
|----------|-------|-----|-----|-----|-----------|-----|-------|--------------|----------------|----------------|--------------|
| 3        | 2.5   | 0.2 | 1   | $-\infty$ | 0.2 GeV$^2$ | 8   | 2     | 0.97 GeV$^2$ | 13.85 GeV$^2$ | 0.40 GeV$^2$ | 14.72 GeV$^2$ |

where $C$ includes the weak vertex and unknown coupling constants, and $p'_\mu$ is the momentum of a given $D$ meson inside the loop. The scalar product in Eq. (4) can be written in terms of the meson propagators and the loop integral momentum $l$ as

$$-2 p'_3 \cdot (p'_2 - p_1) = \frac{[\Delta_{D_0} + \Delta_{\bar{D}_0} - 2 s_{23} + M_\pi^2 + 2 M_{D_0}^2 - l^2]}{2 M_{D_0} \Delta_{D_0} \Delta_{\bar{D}_0} \Delta_a} \quad (5)$$

and finally

$$A^K_{\bar{K}_c}(s_{23}) = i C m_a^2 \int \frac{d^4 l}{(2\pi)^4} \left[ T_{DD\to KK}(s_{23}) \right] \frac{(\Delta_{D_0} + \Delta_{\bar{D}_0} - 2 s_{23} + 2 M_{D_0}^2 + M_\pi^2 - l^2)}{2 M_{D_0} \Delta_{D_0} \Delta_{\bar{D}_0} \Delta_a} . \quad (6)$$

The calculation of the integral in Eq. (6) follows the technique proposed in [13]. We got a similar amplitude for the $K\pi$ channel and the total decay amplitude,

$$A_B(s_{23}, s_{13}) = A^K_{\bar{K}_c}(s_{23}) + A^K_{K\pi}(s_{13}) ,$$

becomes a function of both invariant masses, and provides a characteristic interference pattern in the Dalitz plot, as will be shown.

![Toy MC Dalitz plot $B^+\to K^-K^+\pi^+$](image)

**Figure 3**: Dalitz plot for $B^+_c \to K^-K^+\pi^+$ differential decay rates mediated by $D^0\bar{D}^0 \to K^+K^-$ and $D^-\bar{D}^+_s \to K^-\pi^+$. Left frame: same normalization for $A^K_{\bar{K}_c}(s_{13})$ and $A^K_{K\pi}(s_{23})$. Right frame: $A^K_{\bar{K}_c}(s_{13})$ normalized to 20% of $A^K_{\bar{K}_c}(s_{23})$ (see text).

4. Discussion

The total decay amplitude of the process $B^+_c \to K^-K^+\pi^+$, $A_B(s_{23}, s_{13})$, was simulated in a Monte Carlo generator with 10,000 events using LAURA++ software [14]. The resulting Dalitz plot is shown in Fig. 3 for two different normalizations between the channels, whereas its projection in the different channels are shown in Fig. 4.

In the phenomenological scattering amplitudes $D^0\bar{D}^0 \to K^-K^+$ and $D^+\bar{D}^-_s \to \pi^+K^-$, except for the thresholds ($s_{th\,PP}$) all the parameters in Eq. (3) should be fixed by a fit to the data. In our toy amplitude the parameters were chosen based on our previous study [2] and given in Table 1.

The interference pattern presented in the Dalitz plot of Fig. 3 is a result of the sum of the two amplitudes represented diagrammatically in Fig. 2. We assume the same branching fraction for the two charm decay channels, that leads to the interference pattern shown in the left frame of the figure. However, in principle they could have different production rates favouring one channel with respect to the other one, which would change the Dalitz plot side bands, but not the minimum position associated with the double charm thresholds. This is illustrated in the right frame of the figure, where we arbitrarily normalize $A^K_{\bar{K}_c}(s_{13})$ to 20% of $A^K_{\bar{K}_c}(s_{23})$. 


The projections of the differential decay rate in the $KK$ and $K\pi$ channels are given respectively by the integration on the crossed-channels:

$$\int ds_{K\pi} |A_{B_c}(s_{KK}, s_{K\pi})|^2 \quad \text{and} \quad \int ds_{KK} |A_{B_c}(s_{KK}, s_{K\pi})|^2,$$

and presented in Fig. 4 in the left and right frames. For this illustration we use equal normalizations of $A_{B_c}^{KK}(s_{23})$ and $A_{B_c}^{K\pi}(s_{13})$. In the figure one can see that the minima between the two bumps are in different positions in the left and right frames. The position of these minima correspond to the opening of distinct double charm final states, that have different thresholds and therefore they are displaced from one another. It is worthwhile to mention that the unequal mass parameters in the triangle and double charm scattering amplitudes affect the height of the peaks at the right and left of the minima. A detailed study of these asymmetries can shed light on the parametrization of the transition amplitudes between the different two-body channels.

$$\begin{array}{ll}
\text{Projection } m_{KK}^2 & \text{Projection } m_{K\pi}^2 \\
\end{array}$$

Figure 4: $B_c^+ \to K^- K^+ \pi^+$ decay rate projections in the final states channels $K^+K^-(s_{23})$ (left panel) and $K^-\pi^+(s_{13})$ (right panel).

In the real data, the amplitudes of Fig. 2 should also interfere with other sources of non-resonant and resonant interactions. This will not change the characteristic signature of the minimum found between the two bumps, which are associated with the opening of different double charm thresholds as shown in Fig. 4. If the $KK\pi$ final state is produced directly in the primary vertex, illustrated by the diagram a) of Fig. 1 for the suppressed annihilation topology, one should expect, in analogy of what we know from the charmless B decays, that the final state interaction between the light mesons, will produce resonances in the low mass region in both $K^+K^-$ and $K^-\pi^+$ channels. However this is not what one could see from the actual data [1] consistent with the present proposal.

In summary, our study shows that the hadronic charm triangle amplitude with rescattering can play a role in $B_c^+ \to K^- K^+ \pi^+$ decay, producing a non-resonant process that leads to a signature in the middle of the Dalitz plot with a minimum close to $D\bar{D}$ threshold. We expect that the new data from LHCb run II, will be able to confirm if the dominant mechanism to produce $K^+K^+\pi^-$ through charm rescattering in the $B_c^+$ decay is supported by experiment.

ACKNOWLEDGMENTS

This work was supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). TF acknowledge the support from Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP grant # 17/05660-0) and Project INCT-FNA Proc. No. 464898/2014-5. PCM acknowledge the support from TUM Physics faculty award for the promotion of gender equality in science.

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