Evaluation of the stress-strain relationship in the high strain region of high strength materials by using the shear stretch test

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Abstract. Stress-strain relationships are used in FEM analysis. After uniform deformation, they are simply extended values expected from ones measured without experiment. The purpose of this research was to understand the stress-strain relationship in the high strain region after uniform deformation. We investigated a method for understanding this relationship in high strength materials using a simple shear test. The relationship in this region was calculated by determining a constant parameter κ as the coefficient between equivalent stresses and shear stresses. Using this constant, the relationship after uniform deformation was extended from the result of a stretching test. This method is called the κ method. In the case of high-strength materials, it is difficult to avoid rotating the specimen under a high testing load. Because of this problem, we designed the shape of the specimen so that it had two symmetrical deformation areas. The notch shapes for the deformation areas on the specimen were designed to reduce the load and provide a strong gripping force. In terms of shortening the deformation region, it is difficult to ignore a non-uniform deformation. To reduce the influence of non-uniform deformations, the effect of the R shape on the notch end was evaluated. The uniformity of two specimens with R1.5 mm and R7.0 mm on the notch end was examined by FEM. The values of κ, which were determined from the shear stress and shear strains based on the results of FEM, depended on the equivalent strain. In the case of R7.0 mm, the κ results were almost constant, and this was a suitable result for applying the κ method. The change in κ was affected by the non-proportional stress ratio, the stress error of the observed shear stress, and the strain error of observed shear strain. In the case of R7.0 mm, it was found that there was an advantage for the constant stress error in the higher strain region. Using R7.0 mm and R1.5 mm specimen shapes, we applied a shear test with two kinds of high-tensile steels and aluminum materials. From experimental results, the values for κ on R7.0 mm were also more constant than on R1.5 mm in the higher strain region. A high stress-strain relationship after uniform deformation was obtained in high-tensile steels and aluminum materials. This research could therefore be used to understand the stress-strain relationship in the high strain region of high strength materials.

1. Introduction
In order to reduce the cost of the dies with trial and error, a more accurate prediction for sheet metal forming analysis is required. Constitutive equations for material modeling are important for accurately predicting defects such as cracks and wrinkles during the press forming process. One of the most
important constitutive equations is the work hardening law. In general, this law is determined by using the results of uniaxial tensile tests. However, it is difficult to determine the work hardening law in the high strain region, because of diffusion necking after uniform deformation. During usual sheet metal formation, strain over uniform elongation sometimes occurs. The stress-strain relationship after uniform elongation has to be decided on for metal forming analysis, but it is impossible to use a tensile test alone.

Therefore, instead of a tensile test, a shear test [1, 2] has been proposed as a $\kappa$ method, where the stress-strain relationship in the high strain region can be measured without diffusion necking. Shirakami et al.[1] introduced a conversion method called the $\kappa$ method, which decided the proportional constant between equivalent stress and shear stress by comparing results of a simple shear test with that of a tensile test in the lower strain region.

In this paper, to obtain the stress-strain relation of higher strength materials following their method, an optimum specimen shape with a lower test force and accurate determination of $\kappa$ was investigated both experimentally and numerically.

2. Simple shear test

2.1. Derivation of stress-strain relationship using the $\kappa$ method

In a simple shear test (Fig. 1), shear deformation occurs in the specimen due to the jig moving up and down. Shear strain $\gamma$ is measured from the deviation ($\theta$) of the horizontal straight line at the center of the deformation area. Shear stress $\tau$ is measured from the load $F$ on the longitudinal deformation area $A_0$ [3].

$$\tau = \frac{F}{A_0}, \quad \gamma = \tan \theta$$  \hspace{1cm} (1)

The obtained shear stress-strain relationship is converted into an equivalent stress-strain relationship by the $\kappa$ method. (Lee et al. [4]) The $\kappa$ method decides the proportional constant $\kappa$ between equivalent stress-strain and shear stress-strain by the assumption of plastic work conjugation and isotropic hardening.

$$\bar{\sigma} = \kappa_\sigma \tau, \quad d\bar{\varepsilon}^p = \kappa_\varepsilon d\gamma : \frac{\kappa_\sigma}{\kappa_\varepsilon} = \kappa$$  \hspace{1cm} (2)

$\bar{\sigma}$, $d\bar{\varepsilon}^p$ are the equivalent stress and the equivalent plastic strain increments. The constant $\kappa$ agrees with $\kappa_\sigma$ and $1/\kappa_\varepsilon$ from assuming plastic work conjugation (Equation (2). The constant $\kappa$ is decided using the least-squares method so that shear and tensile deformation agree within the uniform elongation.

Fig.1 Schematic illustration of simple shear test
2.2. Experimental equipment and method

In this paper, the specimen and test jig shown in Fig. 2 were used for the simple shear test. The specimen was made symmetrical with two shear deformation parts on the left and right to prevent rotation of the specimen during the experiment. The test jig has a mechanism that can apply sufficient surface pressure (84 MPa) with bolts and can constrain the specimen in a vertical orientation, thereby suppressing slippage of the test specimen.

The shear strain was measured by digital image correlation (DIC) from the deviation of the horizontal straight line in the centre of the deformation area. Shear stress was measured from the test load obtained from the load cell.

![Specimen and Test Jig](image)

(a) Appearance of specimen  
(b) Appearance of test jig  
Fig. 2 Simple shear test.

2.3. Problems with the shear test method

The deformation area of the shear test becomes inhomogeneous due to the edge face effect. The effect of non-uniform deformation is reduced by increasing the deformation area, but the capability of the test equipment reaches its limit when the material is a high-strength material. Therefore, the shape of a specimen that could reduce the test load and non-uniform deformation was examined by numerical analysis.

3. Examination of shear specimen shapes by FEM analysis

3.1. FEM analysis conditions

Notches are provided along both edges of the deformation area to reduce the test load. The notched R shape was examined by FEM analysis (LS-DYNA9.3.0: solid element). The effects of the notch shape were evaluated from two types of notch shapes (R1.5, R7.0 mm) as shown in Fig. 3. As a material model, the stress-strain relationship was extrapolated from a uniaxial tensile test, and Mises and isotropic hardening were assumed for the yield function.

![Notched Specimens](image)

(a)Notch R1.5mm  
(b)Notch R7.0mm  
Fig. 3 Two specimens with R1.5 mm and R7.0 mm on the notch end.
3.2. Results of the supposition experiment using FEM analysis

The shear stress-strain was measured from FEM analysis, with the same definition as the experiment, and the constants $\kappa$ and $\kappa_\sigma$, $\frac{1}{\kappa_e}$ were determined (Fig. 4).

$\kappa_\sigma$ and $\frac{1}{\kappa_e}$ for both R1.5 mm and R7.0 mm indicated a constant value until uniform elongation ($\varepsilon^p < 0.1$), and R1.5 mm was closer to the ideal $\kappa = 1.73$. However, as deformation progressed, the $\kappa_\sigma$ and $\frac{1}{\kappa_e}$ of R1.5 mm were not constant, and the strain dependence of $\kappa$ was confirmed at R1.5 mm. On the other hand, $\kappa_\sigma$ and $\frac{1}{\kappa_e}$ of R7.0 mm indicated a constant value in the high strain region ($\varepsilon^p = 0.45$), indicating that the strain dependence of $\kappa$ was small. Since the $\kappa$ value in the high strain region cannot be measured experimentally, it must be estimated from the low strain region until uniform elongation is achieved. Therefore, the strain dependence of $\kappa$ in the high strain area is important, and it was found that R7.0 mm, which had a small strain dependence of $\kappa$, was superior.

4. Investigation of the effect of notched R shape on the $\kappa$ value

From the results of FEM analysis with a different Notch R, it was found that there is a difference in the strain dependence of $\kappa$ depending on the R notch shape. Therefore, the difference in the strain dependence of $\kappa$ due to R notch shape can be considered as follows.

4.1. Effects of non-proportional loading

In this paper, plastic work conjugation assumes two things: uniaxial tensile test and pure shear test. However, pure shear tests are difficult, we used a simple shear test in which shear deformation and tensile deformation are mixed. We therefore discuss the effect of mixing between shear and tensile deformation. Even in simple shear deformation, if the shear stress and tensile stress and the shear strain and tensile strain pass through the proportional loading path, the value of $\kappa$ can be considered constant, though different from the ideal value. Therefore, the $\kappa$ value was evaluated when all stress and strain states could be measured with this experimental method.

The equivalent stress $\bar{\sigma}$ and the equivalent plastic strain increment $d\varepsilon^P$ at the center of the deformation were obtained from the FEM analysis results, and $\kappa_\sigma$, $\frac{1}{\kappa_e}$ were determined (Fig. 5). To calculate the equivalent stress $\bar{\sigma}$ and the equivalent plastic strain increment $d\varepsilon^P$, the shear stress and strain ($\tau_0$, $\gamma_0$), the stress and strain in the X direction ($\sigma_x$, $\varepsilon_x$), and the stress and strain in the Y direction ($\sigma_y$, $\varepsilon_y$) were used.
\[
\bar{\sigma} = \frac{1}{\sqrt{2}} \sqrt{2\sigma_x^2 + 2\sigma_y^2 - 2\sigma_x\sigma_y + 6\tau_0^2},
\]
\[
d\bar{\varepsilon} = \frac{\sqrt{2}}{3} \sqrt{2d\varepsilon_x^2 + 2d\varepsilon_y^2 - 2d\varepsilon_x \times d\varepsilon_y + 6d\gamma_0^2}
\]

The \(\kappa_\sigma\) and \(1/\kappa_\varepsilon\) for R1.5 mm were constant in the high strain region, and the strain dependence of the \(\kappa\) was small. On the other hand, \(\kappa_\sigma\) and \(1/\kappa_\varepsilon\) of R7.0 mm were not constant due to deformation, indicating that the strain dependence of the \(\kappa\) value was larger. Therefore, when the stress and strain can be measured at the center of the deformation area, the strain dependence of the \(\kappa\) value is smaller for R1.5 mm, and it was found that R1.5 mm formed a proportional load path showing a constant \(\kappa\) value in the high strain area.

![Shear stress diagram](image)

Fig. 5 Changes in the \(\kappa\) value calculated from stress-strain at the center of the deformation area.

### 4.2. Effects of stress and strain measurement errors

The stress in the experiment was measured by the force acting on the jig in the deformation area (Fig. 6). However, during the shear deformation, non-uniform stress-strain distribution occurs in the deformation area, so that the measured stress in the experiment is the average value in the deformation area (Fig. 6). Therefore, it is considered that this measurement error affects the conversion accuracy of the \(\kappa\) value.

The shear stress \(\tau_0\) at the center of the deformation area and the shear stress \(\bar{\tau}\) which was calculated from the load were obtained by the results of the FEM analysis. The stress error rate of the shear stress was calculated from equation (4) (Fig. 7 (a)). For the shear strain, shear strain \(\bar{\gamma}\) was calculated from the deviation of the horizontal straight line at the center of the deformation area, and the shear strain \(\gamma_0\) at the center of the deformation area was obtained by the results of the FEM analysis. The strain error rate was calculated from equation (5) (Fig. 7 (b)). When both the stress error rate \(\alpha\) and the strain error rate \(\beta\) are close to 0.0, the measured stress and strain are close to the exact values on the center of the deformation area.

\[
\text{Stress error } \alpha(\%) = \left( \frac{\tau_0 - \bar{\tau}}{\tau_0} \right) \times 100
\]

\[
\text{Strain error } \beta(\%) = \left( \frac{\gamma_0 - \bar{\gamma}}{\gamma_0} \right) \times 100
\]

The stress error rate is closer to 0.0 for R1.5 mm, but \(\alpha\) changed with deformation. On the other hand, at R7.0 mm, the stress error rate is larger, but \(\alpha\) didn’t change during the deformation. It was found that
the shear strain error rate changed during the deformation at both notches R. Also, the difference between the two notches was small.

![Shear stress distribution in deformation area](image)

**Fig.6** Shear stress distribution in deformation area.

![Error rate of shear stress and shear strain during the deformation with different notch shape](image)

**Fig.7** Error rate of shear stress and shear strain during the deformation with different notch shape.

### 4.3 Influence of factors on the $\kappa$ value calculated by experimental method

It was found that the notch shape changed the strain dependence of $\kappa$ and the error rate during measurement. Therefore, it is necessary to select an R notch shape that is less affected by the strain dependence of $\kappa$ and measurement errors. In this section, we investigated the relationship between the change in the $\kappa$ value when calculating the shear stress-strain by the same method as in the experiment and the strain dependence of $\kappa$, the measurement error. A regression equation was used for evaluation.

(Equation 6)

$$
\Delta \kappa = 1.0 \times \Delta \kappa_e + 1.726 \times \Delta \alpha - 0.1709 \times \Delta \beta
$$

$\Delta \kappa$ is the change in the $\kappa$ value calculated by the same method as in the experiment, and $\Delta \kappa_e$ is the change in the $\kappa$ value calculated at the centre of the deformation area. The amount of change of both $\kappa$ values was calculated from the reference value of equivalent strain $\varepsilon^p = 0.1$.

The regression equation accurately reproduced the change in the $\kappa$ value calculated by the experimental method (Fig. 8). The stress error rate was the largest factor among these coefficients.
Therefore, the stress error rate is important with respect to the change in the \( \kappa \) value due to the shear stress-strain calculated in the same way as the experiment, and R7.0 was a better notch shape.

\[ R^2 = 0.9972 \]
\[ R^2 = 0.9869 \]

\[ \text{Change in } \kappa \text{ value calculated by regression equation} \]

\[ \text{Change of } \kappa \text{ value by experimental method} \]

\[ \text{Fig. 8 Accuracy of } \kappa \text{ value calculated by regression equation.} \]

5. Experimental results

Based on the results from the previous section, two different notches were used to test different materials (Table 1). Fig. 9 shows the constants \( \kappa, \kappa_\sigma, \) and \( \frac{1}{\kappa_e} \) for each material. In order to examine the strain dependence of \( \kappa, \) the increment of \( \kappa_\sigma \) for the equivalent plastic strain was calculated from equation (7) as an average slope of 15 points (Fig. 10).

\[
\text{Incremental of } \kappa_\sigma = \frac{\sum(\bar{e}^p - \bar{e}^p_{\text{average}})\sum(\kappa - \kappa_{\text{average}})}{\sum(\bar{e}^p - \bar{e}^p_{\text{average}})^2}
\]  

(7)

In all materials, the increase of \( \kappa_\sigma \) in the high strain region was smaller at R7.0 mm. Obtaining similar results of the FEM analysis, it can be said that the strain dependence of \( \kappa \) is small and \( \kappa \) can be estimated stably at the notch tip R7.0 mm.

Fig. 11 shows the stress-strain relationship for each material. The points up to uniform elongation in the tensile test are also shown for reference. For all materials, stress-strain curves beyond the uniform elongation were obtained.

| Materials       | \( t_0 \) (mm) | YS (MPa) | TS (MPa) | U-EL (%) |
|-----------------|---------------|----------|----------|----------|
| 6000-series Al  | 1.2           | 134      | 250      | 20.2     |
| 590 grade steel | 1.4           | 361      | 608      | 15.7     |
| 980 grade steel | 1.4           | 598      | 971      | 8.84     |
Fig. 9 Changes in $\kappa$, $\kappa_\sigma$, $1/\kappa_\varepsilon$ for each material with different R shapes.

Fig.10 Changes in increment of $\kappa_\sigma$ for each material with different R shapes.
6. Conclusion
1) The appropriate specimen shape for obtaining the stress-strain curve of a high-strength material from a shear test was investigated by using the $\kappa$ method. These specimens were used to obtain stress-strain curves for various materials in the high-strain region.
2) The behavior of $\kappa$ in the $\kappa$ method changes depending on the notch R shape. As a result of FEM analysis, it was found that the influence factors were non-proportional loading and the accuracy of stress and strain measurements.
3) Due to these effects, $\kappa$ changes with deformation depending on the material, but by adopting an appropriate R shape, it was found that $\kappa$ is almost constant in the high strain region.

References
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