One dimensional EFIT modeling and experimental validation of dynamic interfacial bonding

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Abstract
The contact and non-contact behaviors of material interfaces generate harmonics arising from interactions with large amplitude incident waves. In this phenomenon, which is called contact acoustic nonlinearity (CAN), intermittent impacts by an incident wave cause the interface contact phase to alternate between contact and separation. This paper proposes a method for numerical simulation of CAN using an elastodynamic finite integration technique (EFIT). The accuracy of the EFIT simulation was verified through comparison with an analytical solution. In one-dimensional CAN theory, a wave penetrating at an interface demonstrates a sawtooth waveform indicating that the interface is closing with a constant velocity. Simulation results revealed that the closing velocity is determined by the compressive stress of the material, and experimental measurements on a polymethylmethacrylate specimen revealed the harmonics caused by the sawtooth wave.

Key words : Nonlinear ultrasonic wave, Harmonics, Interfacial bonding, Elastodynamic finite integration technique (EFIT), Sawtooth wave

1. Introduction

Ultrasonic testing (UT) utilizes the traveling time and amplitude of a scattered wave to evaluate cracks in a material (Silk, 1982). A distinct scattered wave is obtained from an open crack face. It is difficult, by contrast, to detect signals from closed cracks such as stress corrosion or fatigue cracks using conventional UT, as most of the incident wave penetrates the crack face and there is minimal generation of scattered waves. Recently, nonlinear ultrasonic methods based on contact acoustic nonlinearity (CAN) that use the dynamic behavior of contact and non-contact states at the crack face have been reported. The analytical model for CAN was first developed by Richardson (1979), following which many experimental studies using CAN have been conducted to evaluate closed cracks (Buck, et al., 1978, Nagy, 1998, Solodov, 1998, Cantrell and Yost, 2001, Ohara and Kawashima, 2004). There have also been several numerical modeling studies to investigate CAN. Mendelsohn and Doong (1989) and Hirose (1994) proposed using a boundary element method (BEM) to carry out dynamic contact analyses in two-dimensional (2D) out-of-plane and in-plane wave fields, respectively. A finite difference time domain (FDTD) technique has also been proposed to solve the 2D in-plane problem (Jinno, et al., 2014, Kimoto and Ichikawa, 2015).

In this paper, numerical modeling using the elastodynamic finite integration technique (EFIT) (Schubert, 2004) is applied to express the dynamic state of an interface. The EFIT performs integration over a volume V whose surface is S, assuming that velocity and stress are constant within V and on S. This method results in staggered grids and provides a very stable code, allowing easy and flexible treatment of various boundary conditions. For an infinite homogeneous model, the final grid alignment in the EFIT becomes the same as that in the FDTD. However, the treatment at boundaries
and interfaces in the FDTD might be different from the EFIT (Jinno, et al., 2014), because the EFIT offers the mathematical process based on the volume integral in the cell (Fellinger, et al., 1995). This is useful to model the elastic wave propagation in inhomogeneous material. In the EFIT framework, digital 2D and three-dimensional (3D) images with a unified cell size are used as input data for simulation (Nakahata, et al., 2009); based on this framework, it is possible to model inhomogeneous cracks that demonstrates CAN at a small preprocessing cost. To investigate the accuracy of the EFIT application, the solution of a one-dimensional (1D) EFIT model is compared with results produced using the analytical method (Richardson, 1979). It is known that a sawtooth waveform is observed in the 1D case (Richardson, 1979, Solodov, 1998): when the incident wave arrives at the interface, it initially causes the interface to separate; however, the compressive stress in the material eventually causes the interface to begin closing, and the displacement on the penetrated side produces a sawtooth waveform over the course of this process. Here we detail the characteristics of the sawtooth wave in particular, the closing velocity of the interface. The simulation of the sawtooth wave is validated through an experimental measurement using a laser Doppler vibrometer (LDV) set at the penetrated side of a polymethylmethacrylate (PMMA) specimen. Biwa, et al. (2004) introduced an interfacial stiffness model with pressure-dependent nonlinear stiffness constants. While their model was validated by comparing measured second harmonic amplitudes, in our measurement, the PMMA interface is assumed to be smooth, making the simple CAN concept applicable.

2. Elastodynamic finite integration technique (EFIT)

Consider a vertical interface with infinite length \((h \to \infty)\) embedded in a 1D linear elastic material. The stress \(\sigma_{11}\) and velocity \(v_1\) are governed by the equation of motion and the constitutive law, which are expressed in integral form as Eqs. (1) and (2), respectively:

\[
\int \rho \frac{\partial v_1(x_1, t)}{\partial t} \, dA = \int \sigma_{11}(x_1, t) \, \delta x_1 \, dL = \int \sigma_{11}(x_1, t) n_1(x_1) \, dL \tag{1}
\]

\[
\int \frac{\partial \sigma_{11}(x_1, t)}{\partial t} \, dA = \int (\lambda + 2\mu) \frac{\partial v_1(x_1, t)}{\partial x_1} \, dA = \int (\lambda + 2\mu) v_1(x_1, t) n_1(x_1) \, dL \tag{2}
\]

where \(\rho\) denotes the mass density, \(n_1\) is the outward normal vector, and \(\lambda\) and \(\mu\) are the Lamé constants. We perform integration over a cell \(A\) whose size is uniformly \(\Delta x \times h\) to enable image-based modeling (Nakahata, et al., 2009). Assuming that \(v_1\) and \(\sigma_{11}\) are constant within \(A\), we have the following forms:

\[
\frac{\rho \Delta t}{2} \, \hat{v}_1 \Delta x = h \left[ (\sigma_{11})^R - (\sigma_{11})^L \right] \tag{3}
\]

\[
\sigma_{11} \, \hat{\Delta x} = h \left[ (\lambda + 2\mu) \left( (v_1)^R - (v_1)^L \right) \right] \tag{4}
\]

where \(\hat{v}_1 = \partial \rho v_1 / \partial t\), and the left and right hand sides of the cell are abbreviated as (L) and (R), respectively. Since the mass density and the Lamé constants are constant in the integral volume of the stress, we can calculate as \(\int \rho \, dA = \rho^{(L)} \, h \Delta x + \rho^{(R)} \, h \Delta x\) in the integration of \(v_1\) in Eq. (1). In the time domain, the stress components \(\sigma_{11}\) are allocated at half-time steps, while the velocities \(v_1\) are allocated at full-time steps. The central difference approximation yields an explicit leap-frog scheme:

\[
(v_1)_i^{k+1} = (v_1)_i^k + \left[ \frac{\Delta t}{\Delta x} \hat{\sigma}_{11} \right]^{k+1/2} - \left[ \frac{\Delta t}{\Delta x} \hat{\sigma}_{11} \right]^{k-1/2} \tag{5}
\]

\[
(\sigma_{11})_i^{k+1/2} = (\sigma_{11})_i^{k-1/2} + \left[ \frac{\Delta t}{\Delta x} (\lambda + 2\mu) \right] \left( (v_1)_i^{k+1/2} - (v_1)_i^{k-1/2} \right) \tag{6}
\]

where \(\Delta t\) is the time interval, and we set \(2\hat{\sigma} = \rho^{(L)} + \rho^{(R)}\). The stress and velocity grids are located as shown in Fig. 1. Equations (5) and (6) are then solved explicitly using the spatial staggered grid expressed by \(i\) and time increment by \(k\). In this simulation, a longitudinal wave with velocity \(c_L = [(\lambda + 2\mu)/\rho]^{1/2}\) incidents perpendicularly to the interface.

2.1. Modeling of the nonlinear interface

In modeling the interacting faces, the state of separation and contact at the interface varies according to the explicit scheme shown in Fig. 2. As shown in Fig. 1, the interface is located on the velocity grid in \(i = n\). When the interface is separated, the node at the interface is split in two. The left and right side nodes at the interface are indicated as \(n^-\) and \(n^+\),
Separation state
\[ \sigma_{11}(L/2, t) = 0, \quad [u_1] > 0. \]  

The velocities at both sides on the interface are then expressed as
\[ (v_1)^{k+1}_{n+} = (v_1)^{k}_{n+} + \frac{\Delta t}{x_h} \left( 2(\sigma_{11})^{k+1}_{n+} \right), \quad (v_1)^{k+1}_{n-} = (v_1)^{k}_{n-} - \frac{\Delta t}{x_h} \left( 2(\sigma_{11})^{k+1}_{n-} \right). \]  

Conversely, the continuity condition
\[ (v_1)^{k+1} = (v_1)^{k}_{n+}, \quad \sigma_{11}(L/2, t) < 0 \]  
is applied when the interface is closed, in which case the split-nodes are bound together. In this model, the velocity and displacement after crack closure can be related as
\[ (v_1)_n = \frac{\rho_n(v_1)_{n+} + \rho_{n-1}(v_1)_{n-}}{\rho_n + \rho_{n-1}}, \quad (u_1)_n = \frac{(u_1)_{n+} + (u_1)_{n-}}{2}. \]  

### 2.2. Initial condition and incident wave

In the initial state, it is assumed that the interface is in the closed condition. The material is subjected to the constant compressive stress \( \sigma_{11}(0, t) = \sigma_{11}^{\mu}(L, t) = -P_0 \) as illustrated in Fig. 1. The velocity and the stress in the initial state are given by
\[ v_1(x_1, 0) = v_1^{\mu}(x_1, 0), \]  
\[ \sigma_{11}(x_1, 0) = \sigma_{11}^{\mu}(x_1, 0) + \sigma_{11}^{\nu}(x_1, 0). \]  

### 2.3. Incident wave

The input wave in terms of the stress field is given by
\[ \sigma_{11}^{\mu}(x_1, t) = \sigma_0 \left( 1 - \frac{x_1}{c_{\mu}} \right) \]  
where \( \sigma_0 \) is the stress amplitude. The incident wave in this simulation uses a tone burst signal with period \( T \). The incident velocity field consistent with the stress field in Eq. (13) is given by
\[ v_1^{\mu}(x_1, t) = -v_0 \left( 1 - \frac{x_1}{c_{\mu}} \right), \quad v_0 = \frac{\sigma_0}{\rho c_{\mu}}. \]
If the incident wave has angular frequency \( \omega (=2\pi/T) \), the displacement is described as

\[
u_1(t) = \frac{\sigma_0}{\rho\omega c_L} f\left(t - \frac{x_1}{c_L}\right)
\]

(15)

where the displacement amplitude is defined as

\[A = \frac{\sigma_0}{\rho\omega c_L}.
\]

(16)

The ratio of the compressive stress and the stress amplitude was derived by Richardson (1979) as \( \eta = \frac{\sigma_0}{\rho\omega A\Delta} \). Using Eq. (17), we can rewrite the parameter \( \eta \) as

\[\eta = \frac{P_0}{\sigma_0}.
\]

(17)

2.4. Accuracy check of the numerical simulation

In order to check the accuracy of the EFIT, the numerical result was compared with the analytical solution (Richardson, 1979). Figure 3 shows three cases for \( \eta = 0.1, 0.5, \) and 0.9. The Courant-Friedrichs-Lewy (CFL) condition in the present numerical analysis is \( c_L\Delta t/\Delta x = 0.8 \), and the numerical parameters \( c_L T/\Delta x = 80 \) were used in the simulation. The resulting numerical solutions were overlaid onto a carbon copy of the analytical solution by Richardson, which revealed that the proposed EFIT modeling results were in close agreement with an analytical solution hidden by the numerical solutions. From this result, it was found that the gap increases as the value of \( \eta \) becomes small; this means that a large incident wave will open the interface wide. Despite the boundary-type nonlinearity introduced in the explicit scheme, the stability of this calculation was fairly high.

![Figure 3](image_url)

Fig. 3 The opening displacement (gap) of the interface for \( \eta = 0.1, 0.5, \) and 0.9. The numerical results were overlaid onto a carbon copy of the analytical solution (Richardson, 1979).

3. Generation of sawtooth wave

When an incident wave arrives at an interface, it causes interface separation. In the process of closing the interface, the displacement on the penetrated side produces a sawtooth waveform. Here, we detail the characteristics of the sawtooth wave, in particular, the closing velocity of the interface. In the simulation, an 8-cycle modulated tone burst wave, as shown in Fig. 4 was transmitted from the left-hand side of the material. We set the CFL condition as \( c_L\Delta t/\Delta x = 0.6825 \), and the ratio of the amplitude of the incident wave to the compressive stress was varied as \( \eta = 0.1, 0.2, 0.4, 0.8, \) and 1.6.

The displacement of the penetrated wave through the interface is plotted as shown in Fig. 5. It is seen that the sawtooth wave becomes large when the amplitude \( \sigma_0 \) of the incident wave becomes large, and it should be noted that the sawtooth waveforms assume a constant tilt angle even if the amplitude of the incident wave increases. The velocity field of the
penetrated wave through the interface is shown in Fig. 6, in which the vertical axis is normalized by $\rho c_L/P_0$. As the tilt angle of the displacement field in Fig. 5 corresponds to the constant negative velocity in Fig. 6, the closing velocity of the interface is determined by the compressive pressure. The closing velocity can be expressed as

$$v = -\frac{P_0}{\rho c_L}. \quad (18)$$

The Fourier spectrum of the velocity waveform is shown in Fig. 7, in which the values are normalized by the maximum in the result $\eta = 1.6$. The amplitude of the fundamental frequency in the case $\eta = 0.8$ is about twice as high as in the case $\eta = 1.6$, indicating the occurrence of nonlinearity and thus the lack of a clear second harmonic at this amplitude. In the cases in which $\eta$ is below 0.4, however, the amplitude of the fundamental frequency does not increase in proportion to the voltage, although the amplitude of the second harmonic increases with the amplitude of the incident wave. It is thus seen that the appearance of the constant negative velocity is related to the significant generation of harmonics. Note that, the appearance of frequency component around $f = 0.2/T$ is due to the baseline shift of the waveform to the negative side in Fig. 6 and the use of the discrete Fast Fourier Transform of the shifted waveforms.

4. Validation of the 1D EFIT by experimental measurement

4.1. Measurement setup

The purpose of the measurements was to compare the numerical results with results experimentally obtained by observing the dynamic state of the interface. To do this, we used two blocks composed of PMMA, in which the longitudinal wave velocity is 2,778 m/s. Each PMMA plate was initially 5 mm thick. The measurement setup is shown in Fig. 8. In the measurement, the surface of the PMMA was polished while keeping dry. The two PMMAs were set up in parallel on the stable table. Because of the mirror polishing, the interface of the PMMA was stuck by the intermolecular attraction. Here we used a rubber band to keep them stable, however did not apply a strong tension. A contact-transducer (Panametrics) was attached to one side of the plates, and the velocity / displacement on the opposite side was measured using a LDV device (OPTOMET Vector Series). A 5-cycle, 1 MHz tone burst signal was generated using a high power ultrasonic generator (RITEC Advanced Measurement System RAM-5000). During the measurements, we increased the input voltage of the generator as 350, 750, 1,190, and 1,530 peak-to-peak voltage (Vpp). Figure 9 (a) and (b) show waveforms of the velocity and displacement at the surface of a single PMMA specimen with 10 mm thick (no interface), respectively. The Fourier spectrum of the velocity waveform was shown in Fig. 10. These spectrums were normalized by the maximum value in the spectrum at 350 Vpp. Since the components of the fundamental frequency increased as the input voltage became high, it was understood that influence of nonlinearity by the voltage increase was small.

4.2. Results and discussion

Figure 11 shows the velocity of the penetrated wave as measured by the LDV. Although the velocity is not constant during the first or second cycles of the tone burst wave, during the third and fourth cycles a constant velocity (approximately $v = -0.1$ mm/s) emerges, indicating that the interfacial separation occurs during this time. The compressive stress $P_0$ can be calculated using Eq. (18), and the stress amplitude of the incident wave can be calculated using Eq. (14), as $\sigma_0 = \nu_0 c_L$. The results obtained from these measurements are summarized in Table 1.

The frequency spectrum of the measured velocity is shown in Fig. 12. All the spectrums were normalized by the maximum value in the spectrum at 350 Vpp. The amplitude of the fundamental frequency at 750 Vpp is about twice as high as at 350 Vpp, and harmonics do not clearly appear at the latter voltage. As the excitation voltage exceeds 1,190 Vpp, the amplitude of the fundamental frequency ceases to increase in proportion to the voltage; however, the amplitude of the second harmonic becomes large. In these measurements, the ratio of the amplitude of the fundamental frequency to that of the second harmonic differed between simulation and measurement; this is attributable to nonlinear interfacial stiffness caused by surface roughness (Biwa, et al., 2004) and an interface friction caused by the sliding in the tangential direction (Solodov, et al., 2011), which were not incorporated into the CAN framework.

5. Conclusions

A 1D EFIT simulation was proposed as a means for modeling the interfacial dynamic behavior of contact and separation. The simulation was developed with the view toward application in the ultrasonic testing of closed cracks. The EFIT utilizes a split node at an interface at which contact and separation vary depending on the stress and gapping of the interface. The numerical results were in close agreement with an analytical solution. The study focused on the
**Fig. 4** The modulated incident wave.

**Fig. 5** Displacement of penetrated wave (EFIT simulation).

**Fig. 6** Velocity of penetrated wave (EFIT simulation).

**Fig. 7** Frequency spectrum of penetrated wave.
investigation of sawtooth waves, which are produced in the 1D CAN framework, as the velocity of the interfacial closure can be found from the tilt angle of the sawtooth wave. It was found that the velocity corresponds to the compressive stress applied on the interface. Sawtooth waves were then observed in the measurements using PMMA specimens, which increasing the voltage produced the sawtooth patterns and increased the Fourier spectrum of the second harmonic.

Developing this simulation tool for nonlinear ultrasonic testing should lead to methods for the reliable evaluation of closed cracks. As a next step, we will extend the 1D simulation code to 2D and 3D simulations, and we plan to introduce the effects of interfacial roughness and friction into our simulation.

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References

Biwa, S., Nakajima, S. and Ohno, N., On the acoustic nonlinearity of solid-solid contact with pressure-dependent interface stiffness, Journal of Applied Mechanics, Vol.71, No.4 (2004), pp.508-515.
Buck, O., Morris, W. L. and Richardson, J. M., Acoustic harmonic generation at unbonded interfaces and fatigue cracks, Applied Physics Letters, Vol.33, No.5 (1978), pp.371-373.
Cantrell, J. H. and Yost, W. T., Nonlinear ultrasonic characterization of fatigue microstructures, International Journal of Fatigue, Vol.23 (2001), pp.487-490.
Fellinger, P., Marklein, R., Langenberg, K. J. and Klaholz, S., Numerical modeling of elastic wave propagation and scattering with EFIT elastodynamic finite integration technique, Wave Motion, Vol.21, No.1 (1995), pp.47-66.
Hirose, S., 2-D scattering by a crack with contact-boundary conditions, Wave Motion, Vol.19, No.1 (1994), pp.37-49.
Jinno, K., Sugawara, A., Ohara, Y. and Yamanaka, K., Analysis on nonlinear ultrasonic images of vertical closed cracks by damped double node model, Materials Transactions, Vol.55, No.7 (2014), pp.1017-1023.
Kimoto, K. and Ichikawa, Y., A finite difference method for elastic wave scattering by a planar crack with contacting faces, Wave Motion, Vol.52 (2015), pp.120-137.
Mendelsohn, D. A. and Doong, J. M., Transient dynamic elastic frictional contact: a general 2D boundary element formulation with examples of SH motion, Wave Motion, Vol.11, No.1 (1989), pp.1-21.
Nagy, P. B., Fatigue damage assessment by nonlinear ultrasonic materials characterization, Ultrasonics, Vol.36, No.1 (1998), pp.375-381.
Nakahata, K., Hirose, S., Schubert, F. and Köechler, B., Image based EFIT simulation for nondestructive ultrasonic testing of austenitic steel, Journal of Solid Mechanics and Materials Engineering, Vol.3, No.12 (2009), pp.1256-1262.
Ohara, Y. and Kawashima, K., Detection of Internal micro defects by nonlinear resonant ultrasonic method using water immersion, Japanese Journal of Applied Physics, Vol.43, No.5B (2004), pp.3119-3120.
Fig. 9 Waveform of (a) velocity and (b) displacement at the surface of a single PMMA specimen.

Fig. 10 Frequency spectrum of the velocity waveform at the surface of a single PMMA.

Fig. 11 Velocity of penetrated wave measured using the LDV.

Fig. 12 Frequency spectrum of the measured waveform of velocity.
Table 1  Dimensionless parameters for the measured data

| Input voltage (Vpp) | v₀ (mm/s) | P₀ (kPa) | σ₀ (kPa) | η  |
|---------------------|-----------|----------|----------|----|
| 1,190               | 0.55      | 0.328    | 1.8      | 0.18 |
| 1,530               | 0.7       | 0.328    | 2.3      | 0.14 |

Richardson, J. M., Harmonic generation at an unbonded interface-I. planar interface between semi-infinite elastic media, International Journal of Engineering Science, Vol.17, No.1 (1979), pp.73-85.
Schubert, F., Numerical time-domain modeling of linear and nonlinear ultrasonic wave propagation using finite integration techniques-theory and applications, Ultrasonics, Vol.42, No.1 (2004), pp.221-229.
Silk, M. G., Defect detection and sizing in metals using ultrasound, International Metals Reviews, No.1 (1982), pp.28-50.
Solodov, I. Y., Ultrasonics of non-linear contacts: propagation, reflection and NDE-applications, Ultrasonics, Vol.36, No.1 (1998), pp.383-390.
Solodov, I. Y., Döring, D. and Busse, G., New opportunities for NDT using non-linear interaction of elastic waves with defects, Strojniški vestnik-Journal of Mechanical Engineering, Vol.57, No.3 (2011), pp.169-182.