Does accelerating universe indicates Brans-Dicke theory?

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The evolution of universe in Brans-Dicke (BD) theory is discussed in this paper. Considering a parameterized scenario for BD scalar field \( \phi = \phi_0 a^\alpha \) which plays the role of gravitational constant \( G \), we apply the Markov Chain Monte Carlo method to investigate a global constraints on BD theory with a self-interacting potential according to the current observational data: the Union2 dataset of type supernovae Ia (SNIa), the high-redshift Gamma-Ray Bursts (GRBs) data, the observational Hubble data (OHD), the cluster X-ray gas mass fraction, the baryon acoustic oscillation (BAO), and the cosmic microwave background (CMB) data. It is shown that an expanded universe from deceleration to acceleration is given in this theory, and the constraint results of dimensionless matter density \( \Omega_0 \) and parameter \( \alpha \) are, \( \Omega_0 \) = 0.286 \( \pm 0.037 \pm 0.050 \) and \( \alpha = 0.0046 \pm 0.0149 \pm 0.0171 \) which is consistent with the result of current experiment exploration, \( | \alpha | \leq 0.132124 \). In addition, we use the geometrical diagnostic method, jerk parameter \( j \), to distinguish the BD theory and the cosmological constant model in Einstein’s theory of general relativity.

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I. Introduction

The observation of the supernovae of type Ia [1, 2] provides the evidence that the universe is undergoing accelerated expansion. Combining the observations from Cosmic Background Radiation [3] and lager scale structure [4], one concludes that the universe at present is dominated by 70% exotic component. An interpretation to the accelerating universe can just be obtained by introducing an exotic fluid in the theory of general relativity. Considering the standard cosmological model this unknown component is called as dark energy with owning a character of negative pressure to push the universe to an accelerated expansion. And the popular dark energy model is the positive tiny cosmological constant (CC), though it suffers the so-called fine tuning and cosmic coincidence problems. However, in 2\( \sigma \) confidence level, it fits the observations very well [5]. If the cosmological constant is not a real constant but is time variable, the fine tuning and cosmic coincidence problems can be removed. In fact, this possibility was considered in the past years [6–18], such as quintessence, Chaplygin gas, holographic, agegraphic dark energy, etc. Also, the accelerating universe is related to the modification of the gravity theory on large distances, such as \( f(R) \) modified gravity theory [19] and higher dimensional theory [20, 21], etc. Scalar-tensor modified gravity theories have recently

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attracted much attention, in part because they emerge naturally as the low-energy limit of string theory as a result of the dilaton coupling with gravitons \[22, 27\]. In addition, it has been argued that in the early universe, gravity might obey a scalar tensor type theories rather than general relativity (GR) \[25\], and they are also important for cosmological inflation models \[23, 28–30\], since they have effectively solved the problems of inflation \[25\], where the end of inflation can be brought by nucleation without considering the fine-tuning cosmological parameters.

II. Brans-Dicke theory

Brans-Dicke (BD) theory \[31–33\] of gravity is a simple but very important one among the scalar tensor theories, which is apparently compatible with Mach’s principle \[34\]. The generalized BD theory is an extension of the original BD theory by considering coupling parameter \(\omega\) as a function of the scalar field \[35–38\], and an accelerating universe can be obtained when coupling parameter \(\omega\) varies with time \[39, 40\]. In the framework of generalized BD theory, the action is described as (choosing the speed of light \(c = 1\))

\[
S = d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi^2} \phi \phi,\phi - V(\phi) + L_m\right].
\]

where \(L_m\) is the matter Lagrangian, \(\phi\) is the BD scalar field which is non-minimally coupled to gravity, \(V(\phi)\) is the self-interacting potential for the BD scalar field, \(\omega(\phi)\) being a function of \(\phi\) is generalized version of the dimensionless BD coupling parameter \(\omega\), and \(\phi\) plays the role of the gravitational constant \(G\) which is related to the inverse of \(\phi\).

The gravitational field equation derived from above action by varying the action with respect to the metric is

\[
G_{\mu\nu} = \frac{8\pi}{\phi} T^m_{\mu\nu} + \frac{\omega(\phi)}{\phi^2} [\phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu} \phi,\phi,\phi] + \frac{1}{\phi} [\phi_{,\mu;\nu} - g_{\mu\nu} \Box \phi] - \frac{V(\phi)}{2\phi} g_{\mu\nu}
\]

with

\[
\Box \phi = \frac{8\pi T}{3 + 2\omega(\phi)} - \frac{1}{3 + 2\omega(\phi)} [2V(\phi) - \phi \frac{dV(\phi)}{d\phi}] - \frac{\frac{d\omega(\phi)}{d\phi}}{3 + 2\omega(\phi)} \phi,\mu\phi,\nu.
\]

where \(G_{\mu\nu}\) is the Einstein tensor, \(T^m_{\mu\nu}\) denotes the energy-momentum tensor of matter, and \(T = T^m_{\mu\nu} g^{\mu\nu}\). Considering Robertson-Walker universe

\[
ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - k r^2} + r^2(d\theta^2 + \sin^2\theta, d\phi^2)\right],
\]

where \(k\) denotes the spacial curvature with \(k = -1, 0, +1\) corresponding to open, flat and closed universe, respectively. The modified Friedmann equation and wave equation for the generalized BD scalar field \(\phi\) are given as \[41, 43\],

\[
H^2 + \frac{k}{a^2} = \frac{8\pi \rho_m}{3\phi} - H \frac{\dot{\phi}}{\phi} + \frac{\omega(\phi)}{6} \left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{V(\phi)}{6\phi} \equiv \frac{8\pi}{3} \rho_{\text{eff}},
\]

\[
2\ddot{a} + H^2 + \frac{k}{a^2} = -\frac{8\pi p}{\phi} - \frac{\omega(\phi)}{2} \left(\frac{\dot{\phi}}{\phi}\right)^2 - 2H \frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} + \frac{V(\phi)}{2\phi} \equiv -8\pi p_{\text{eff}},
\]

and

\[
\ddot{\phi} + 3H \dot{\phi} = \frac{8\pi (\rho_m - 3p)}{3 + 2\omega(\phi)} + \frac{1}{3 + 2\omega(\phi)} [2V(\phi) - \phi \frac{dV(\phi)}{d\phi}] - \frac{\frac{d\omega(\phi)}{d\phi}}{3 + 2\omega(\phi)}.
\]
where $\rho_m$ and $p$ denote the energy density and pressure for the matter, $\rho_{eff}$ and $p_{eff}$ are effective energy density and pressure for the combination of matter and BD scalar field. Obviously, the conservation equation of effective fluid is satisfied, $p_{eff} + 3H(\rho_{eff} + 3p_{eff}) = 0$. BD theory as a leading alternative to Einstein’s theory of GR, it can be obtained by considering $\omega(\phi) = \omega = \text{constant}$ in above generalized form. Also, GR as a special case in BD theory, the theory can be approximately reduced to GR when take constant coupling parameter $\omega \to \infty$ and scalar function $\phi = \text{constant}$.

### III. Cosmic constraints on Brans-Dicke theory

It has been shown that the BD theory is effective for interpreting the cosmic acceleration by considering a non-minimal coupling between the gravitational term and the BD scalar field [41–43]. In this work, we investigate the global cosmological constraints on BD theory. In order to solve above equations of motion the scale factor $a = a(t)\beta$ is considered in keeping with the recent observations [44]. Then the BD scalar field $\phi$ is solved as $\phi = \phi_0 t^\gamma$ as a function of time [44]. Here $\beta$ and $\gamma$ are two constant parameters. Also, these two power-law solutions of the scale factor $a$ and the BD scalar field $\phi$ can be found in other references [45–51], which are consistent with the evolution of expanded universe. Inspired by these papers, for calculation we consider a parameterized power-law form of Brans-Dicke scalar field, $\phi = \phi_0 a^n$ which can be easily given by above two solutions of the scale factor $a$ and the scalar field $\phi$. Then the Friedmann equation in BD theory with a flat geometry $k = 0$ is derived as,

$$H^2 = \frac{\rho_m}{3\phi} - \alpha H^2 \frac{\omega}{6}(\phi) \frac{H^2}{2H} - \frac{V(\phi)}{\phi}.$$  \hspace{1cm} (8)

Defining $\frac{2}{6 + 6\omega - \omega \alpha^2} \frac{1}{\phi_0} = \frac{8\pi G_0}{3}$, then one has

$$H^2 = H_0^2 \Omega_{0m} a^{-(3+\alpha)} - \frac{8\pi G_0 \phi_0 V(\phi)}{2\phi}$$

$$= H_0^2 \Omega_{0m} a^{-(3+\alpha)} - \frac{4\pi G_0 a^{-\alpha}V(\phi)}{3},$$  \hspace{1cm} (9)

with $\Omega_{0m} \equiv \frac{8\pi G_0 \rho_{0m}}{3H_0^2}$. In addition, for a flat universe according to Eqs. (5), (6), (7), and using the equation of state (EOS) for matter $w_m = p/\rho_m = 0$ and the conservation equation $\rho_m = \rho_{0m} a^{-3(1+w_m)} = \rho_{0m} a^{-3}$, the Friedmann equation in BD theory can also be expressed as [52]

$$H^2 = A a^{-3-\alpha} + B a^n$$

$$= A a^{-3-\alpha} + B a^n,$$  \hspace{1cm} (10)

with $n = \frac{-2\alpha(1+\omega)\alpha^{-1}}{2+\alpha}$, here $A$, $B$ are constant parameters. Comparing Eqs. (9) and (10), one has $A = H_0^2 \Omega_{0m}$, and the second term plays the role of potential. Thus Friedmann equation is written as,

$$H^2 = H_0^2 \Omega_{0m} a^{-(3+\alpha)} + B a^n.$$  \hspace{1cm} (11)

If interpret the constant parameter $B$ in the second term as the current value of energy density for an ”new” component including the BD potential, the above equation becomes

$$H^2 = H_0^2 \Omega_{0m} a^{-(3+\alpha)} + H_0^2 \Omega_{0x} a^n$$  \hspace{1cm} (12)
with writing \( B = H_0^2 \Omega_{0x} \). According to Eq. (12) and considering the index \( n = -3(1 + w_x) \), it is obtained that the equation of state is expressed as, \( w_x = -1 - \frac{n}{3} \) for this component.

In the following we apply the Markov Chain Monte Carlo (MCMC) method to investigate the global constraints on above BD scenario. For the used observational data, we consider 557 Union2 dataset of type supernovae Ia (SNIa) [53], 59 high-redshift Gamma-Ray Bursts (GRBs) data [54], observational Hubble data (OHD) [55], X-ray gas mass fraction in cluster [56], baryon acoustic oscillation (BAO) [57], and cosmic microwave background (CMB) data [58], and for the analysis method please see appendix. In our joint analysis, the MCMC code is based on the publicly available CosmoMC package [59] and the modified CosmoMC package [56, 60, 61]. The latter package is about the constraint code of X-ray cluster gas mass fraction. In the calculation the baryon matter density is taken to be varied with a tophat prior: \( \Omega_b h^2 \in [0.005, 0.1] \). In addition, we run 8 independent chains in the MCMC calculation, and to get the converged results we test the convergence of the chains by typically getting \( R - 1 \) to be less than 0.03. The total \( \chi^2 \) is expressed as,

\[
\chi^2_{\text{total}}(p_s) = \chi^2_{\text{SNIa}} + \chi^2_{\text{GRBs}} + \chi^2_{\text{OHD}} + \chi^2_{\text{CBF}} + \chi^2_{\text{BAO}} + \chi^2_{\text{CMB}},
\]

and the parameter vector reads

\[
p_s = \{ \Omega_b h^2, \Omega_c h^2, \alpha, n \}.
\]

Here the expression of \( \chi^2 \) for each observation corresponds to Eqs. (A5), (A6), (A8), (A12), (A20) and (A24), \( \Omega_b \) and \( \Omega_c \) denote dimensionless energy density of baryon matter and dark matter, respectively. Based on the basic cosmological parameters \( p_s \) we can also obtain the derived parameters \( \Omega_{0m} = \Omega_b + \Omega_c \), \( \Omega_{0x} = 1 - \Omega_{0m} \), and the Hubble constant \( H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \). Using the currently observed data with the \( \chi^2_{\text{total}} \) in Eq. (13), Fig. 1 (left) plots the 2-D contours with 1σ, 2σ confidence levels and 1-D distribution of model parameters in the flat BD theory. Solid lines
are mean likelihoods of samples, and dotted lines are marginalized probabilities for 1D distribution (for Gaussian distributions they should be the same). And the calculation results are listed in Table I for the constraint on model parameters. In addition, Table II shows the values for the best-fit sample, and projections of the n-Dimensional 1σ and 2σ confidence regions. The n-D limits give some idea of the range of the posterior, and are much more conservative than the marginalized limits [59]. Often, the best-fit results are recommended. According to the best fit values of model parameter, it is shown the value of equation of state is, \( w_x = -1 - \frac{\alpha}{\omega} \simeq -1.034 \), which is in the phantom region with a small deviation. And according to the best fit values of \( \alpha = 0.0046 \) and \( n = 0.102 \), the best fit value of coupling parameter, \( \omega = \frac{-n(2+\alpha)+2\alpha(1-\alpha)}{2\alpha^2} = -4615.11 \) is calculated, which is consistent with other results, such as \( \omega < -40 \) constrained from the growth rate data [62], and \( \omega < -120 \) or \( \omega > 97.8 \) constrained from the WMAP CMB and the SDSS BAO data [63]. In addition, comparing with the literature [64], where the Brans-Dicke theory is constrained from the recent observational data with introducing the holographic dark energy in universe in order to obtain an accelerating universe, it can be found that our results in this paper have a larger value of parameter \( \alpha \). And for the constraint on the dimensionless matter density \( \Omega_{\text{om}} \) in the BD theory, from Table III one can see that it is consistent with results from other analyses, such as the constraints on several dynamical dark energy models and model independent scenarios by the recently observed data [65, 69], where the best fit value of \( \Omega_{\text{om}} \) is about 0.27. At last, it may be said that the BD theory with a self-potential is slightly preferred by the current observational data because the quantity \( \chi^2_{\text{min}} \) measures the goodness of model fit, i.e. the less value of \( \chi^2_{\text{min}} \), the better model of agreeing with observations. Therefore, it seems that the BD theory with a self-potential is better than the cosmological constant to interpret an accelerating universe, and it is also signficative to discuss the evolution of cosmological quantities in BD theory.

| \( \alpha \) | \( n \) | \( \Omega_{\text{om}} \) | \( H_0 \) |
|---|---|---|---|
| BD | 0.0052(±0.0059) | 0.149(±0.164) | 0.285(±0.015) | 70.028(±1.201) |
| CC | — | — | 0.272(±0.012) | 70.118(±0.913) |

TABLE I: The means, standard deviations (the numerical results in brackets) and the marginalized limits for the model parameters from MCMC calculation, obtained by using SNIa Union2, GRBs, OHD, CBF, BAO, and CMB data. CC denotes the cosmological constant (CC) model in the Einstein’s general relativity.

| \( \chi^2_{\text{min}} \) | \( \chi^2_{\text{min}} \) | \( \Omega_{\text{om}} \) | \( H_0 \) |
|---|---|---|---|
| BD | 619.466 | 0.918 | 0.286 | 70.537 |
| CC | 619.840 | 0.917 | 0.274 | 70.255 |

TABLE II: The maximum likelihood values \( \chi^2_{\text{min}}, \chi^2_{\text{min}}/\text{dof} \), the best fit model parameters, and the limits from the extremal values of N-dimensional distribution from the MCMC calculation by using SNIa Union2, GRBs, CBF, OHD, BAO, and CMB data, where \( \text{dof} \) is degree of freedom of model, and its value equals the number of observational data points minus the number of parameters.

Considering the value of parameter \( \alpha \) is not arbitrary, we calculate its values by using other experiment exploration on Newton’s gravitational ”constant” \( G \). According to the independent observations of Hulse-Taylor binary pulsar B1913 + 16 [68, 69], and asteroesimcosmological data from the pulsating white dwarf star G117-B15A [70, 71], it is
indicated by the current constraints on the variation of $G$,

$$|\frac{\dot{G}}{G}| \leq 10^{-11} \text{yr}^{-1}. \quad (15)$$

For BD theory and the parameterized scenario $\phi = \phi_0 a^\alpha$, it corresponds to

$$|\frac{\dot{\phi}}{\phi}| = |\alpha H| \leq 10^{-11} \text{yr}^{-1}, \quad (16)$$

i.e.,

$$|\alpha| \leq \frac{1}{H} \times 10^{-11} \text{yr}^{-1}. \quad (17)$$

Considering the current value of Hubble constant $h = 0.742^{+0.036}_{-0.036}$ \cite{72} and taking the best fit value of $H_0 = 74.2 \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} = 2.40 \times 10^{-18} \text{s}^{-1} = 7.57 \times 10^{-11} \text{yr}^{-1}$, one obtains the bounds on $\alpha$,

$$|\alpha| \leq 0.132124. \quad (18)$$

It is easy to see that the value of constraint on parameter $\alpha$ with $2\sigma$ confidence level for BD theory is under this bound, i.e., it is lied in a physical significative region.

![FIG. 2: The evolution of $q(z)$ for BD and cosmological constant (CC) model.](image)

| $z_T$ (1\sigma) | $q_0$ (1\sigma) |
|-----------------|----------------|
| BD 0.695^{+0.031}_{-0.028} | -0.603^{+0.051}_{-0.050} |
| CC 0.744^{+0.007}_{-0.006} | -0.589^{+0.003}_{-0.004} |

**TABLE III:** The values of transition redshift $z_T$, and current deceleration parameter $q_0$ from MCMC calculation, obtained by using SNIa Union2, GRBs, CBF, OHD, BAO, and CMB data.

Furthermore, we investigate the evolution of deceleration parameter $q(z) = -\frac{\ddot{a}}{a H^2} = (1+z)\frac{dH}{dz} - 1$ in this scenario. Considering the propagation of the errors for $q(z)$ by the Fisher matrix analysis, the errors are evaluated by using the covariance matrix $C_{ij}$ of the fitting parameters \cite{73}, which is the inverse of the Fisher matrix and given by

$$(C_{ij}^{-1}) = \frac{\partial^2 \ln L}{\partial p_i \partial p_j} = \frac{1}{2} \frac{\partial^2 \chi^2(p_s)}{\partial p_i \partial p_j}, \quad (19)$$
where $p_s$ is a set of parameters, and $\ln L$ is the logarithmic likelihood function. The errors on a function $f = f(p_s)$ in terms of the variables $p_s$ are calculated by

$$
\sigma^2 = \sum_{i=1}^{m} \left( \frac{\partial f}{\partial p_{s_i}} \right)^2 C_{ii} + 2 \sum_{i=1}^{m} \sum_{j=i+1}^{m} \left( \frac{\partial f}{\partial p_{s_i}} \right) \left( \frac{\partial f}{\partial p_{s_j}} \right) C_{ij},
$$

(20)

where $m$ is the number of parameters, and $f$ will be deceleration parameter $q(z; p_{s_i})$. The parameters $p_{s_i}$ respectively represent $(\Omega_b h^2, \Omega_c h^2, \alpha, n)$. In Fig. 2 (left) we plot the evolutions of $q(z)$ with errors by

$$
q_{1\sigma}(z) = q(z) \bigg|_{p_s = \bar{p}_s \pm \sigma_q},
$$

(21)

here $\bar{p}_s$ are the best fit values of the constraint parameters. From this figure it can be seen that a universe from decelerated expansion to accelerated expansion is obtained. And the values of transition redshift $z_T$ and current decelerating parameter $q_0$ with 1$\sigma$ confidence level are, $z_T = 0.692^{+0.028}_{-0.024}$ and $q_0 = -0.594^{+0.034}_{-0.035}$, as shown in table III.

For comparison we also consider the observed constraints on popular cosmological constant (CC), i.e. $\Lambda$CDM model in the Einstein’s theory of GR (or dubbed as standard cosmology). The Friedmann equation for this case is expressed as

$$
H^2 = H_0^2 \Omega_m a^{-3} + H_0^2 \Omega_\Lambda.
$$

(22)

The calculation results on cosmological parameters are listed in table I and II for this model. In addition, for the evolution of deceleration parameter $q(z)$ it is plotted in Fig. 2 (right), and the values of $z_T$ and $q_0$ are listed in table III.

IV. jerk parameter geometrical diagnostic for Brans-Dicke theory

In the following we use a new geometrical diagnostic method, jerk parameter $j$, to investigate discriminations between BD and CC model. The jerk parameter is defined by scale factor $a$ and its third derivative

$$
\dot{j} \equiv - \frac{1}{H^3} \left( \frac{\dot{a}}{a} \right) = - \frac{1}{2} (1 + z)^2 \frac{\dot{H}(z)^2}{H(z)^2} - (1 + z) \left[ \frac{H(z)^2}{H(z)^2} + 1 \right].
$$

(23)

The use of the cosmic jerk parameter provides more parameter space for geometrical studies, and transitions between phases of different cosmic acceleration are more naturally described by models incorporating a cosmic jerk. In addition, for a dark energy model Eq. (23) can be derived as

$$
\dot{j} = -1 - \frac{9}{2} w_{de} \dot{H} = \frac{3}{2} \Omega_{de} \dot{w}_{de},
$$

(24)

where $\Omega_{de}$ denotes dimensionless energy density for dark energy. From Eq. (24) it is easy to see that, for flat $\Lambda$CDM model ($w_{de}(z) = -1$), it has a constant jerk with $j(z) = -1$. Thus, it can provide us with a simple, convenient approach to distinguish and search departures for both cosmological dynamic and kinematical models from the cosmic concordance model, CC. Using Eq. (23), we plot the evolution of jerk parameter $\dot{j}(z)$ for BD model in Fig. 3 and compare it with cosmological constant. According to this figure we get the current values of jerk parameter, $j_0 = -1.002^{+0.003}_{-0.004}$. And it is shown that at 1$\sigma$ confidence level the BD theory can not be distinguished with cosmological constant model in standard cosmology according to the jerk parameter geometrical diagnostic.
V. Conclusions

Many observations and phenomena indicate that the gravitational theory of general relativity should be modified. Scalar tensor (ST) theory as a modified theory of gravity has been widely studied [79–88]. Considering the ST theory can be well used to interpret the inflation problem, in this paper Brans-Dicke gravitational theory as an important one in ST theories is studied to interpret the late accelerating universe. In BD theory gravitational "constant" $G$ is described by a function of scalar field which couples to the Ricci scalar $R$, so that the effective $G$ generally varies in time. Concretely, with a parameterized scenario for BD scalar field $\phi$ which plays the role of the variable gravitational constant $G$, we use the MCMC analysis method and the current observed data, including 557 Union2 SNIa, 59 GRBs, OHD, cluster X-ray gas mass fraction, BAO and CMB data, to constrain the BD theory with a self-interacting potential. According to the constraint results on deceleration parameter $q(z)$, the late accelerating universe can be obtained in this theory. And for the parameter $\alpha$ that is corresponded to the variable gravitational "constant" $G$ in physics, its constraint value is consistent with the result of current experiment exploration, $|\alpha| \leq 0.132124$. Considering the application of BD theory in cosmology, according to Eq. (12) since the both energy densities are dynamical, not a constant as in cosmological constant model, it will not suffer the so-called cosmic coincidence problem. In addition, relative to other geometrical diagnostic methods, such as $\Omega_m$ and statefinder parameters $\{r,s\}$ which have been widely discussed in many models [89–95], in this paper we use a new geometrical diagnostic method—-jerk parameter to Brans-Dicke theory, and it is shown that it can not be distinguished with cosmological constant model. Obviously, the perturbation theory could distinguish this modified gravity theory of BD with the CC dark energy scenario in Einstein’s theory of general relativity, which deserves to be studied in the future.

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Appendix A: Observational data and cosmological constraint methods

In this part we introduce the cosmological constraints methods and the current observed data used in this paper.

1. Type Ia supernovae

We use the 557 SNIa Union2 dataset, which includes 557 SNIa \[^53\]. Following \[^96–99\], one can obtain the corresponding constraints by fitting the distance modulus $\mu(z)$ as

$$
\mu_{th}(z) = 5 \log_{10}[D_L(z)] + \frac{15}{4} \log_{10} \frac{G}{G_0} + \mu_0, \tag{A1}
$$

where $G_0$ is the current value of Newton’s constant $G$. In this expression $D_L(z)$ is the Hubble-free luminosity distance $H_0 d_L(z)/c$, with $H_0$ the Hubble constant, defined through the re-normalized quantity $h$ as $H_0 = 100 h$ km $s^{-1} Mpc^{-1}$, and

$$
d_L(z) = \frac{c(1+z)}{\sqrt{|\Omega_k|}} \sinh(\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{H(z')}),
$$

$$
\mu_0 = 5 \log_{10} \left( \frac{H_0^{-1}}{Mpc} \right) + 25 = 42.38 - 5 \log_{10} h.
$$

where $\sinh(\sqrt{|\Omega_k|} x)$ respectively denotes $\sin(\sqrt{|\Omega_k|} x)$, $\sqrt{|\Omega_k|} x$, $\sin(\sqrt{|\Omega_k|} x)$ for $\Omega_k < 0$, $\Omega_k = 0$ and $\Omega_k > 0$. Additionally, the observed distance moduli $\mu_{obs}(z_i)$ of SNIa at $z_i$ is

$$
\mu_{obs}(z_i) = m_{obs}(z_i) - M, \tag{A2}
$$

where $M$ is their absolute magnitudes.

For using SNIa data, theoretical model parameters $p_s$ can be determined by a likelihood analysis, based on the calculation of

$$
\chi^2(p_s, M') \equiv \sum_{SNIa} \frac{(\mu_{obs}(z_i) - \mu_{th}(p_s, z_i))^2}{\sigma_i^2} = \sum_{SNIa} \frac{(5 \log_{10}[D_L(p_s, z_i)] - m_{obs}(z_i) + M')^2}{\sigma_i^2}, \tag{A3}
$$

where $M' = \mu_0 + M$ is a nuisance parameter which includes the absolute magnitude and the parameter $h$. The nuisance parameter $M'$ can be marginalized over analytically \[^100–106\] as

$$
\bar{\chi}^2(p_s) = -2 \ln \int_{-\infty}^{+\infty} \exp \left[ -\frac{1}{2} \chi^2(p_s, M') \right] dM',
$$

resulting to

$$
\bar{\chi}^2 = A - \frac{B^2}{C} + \ln \left( \frac{C}{2\pi} \right), \tag{A4}
$$

with

$$
A = \sum_{SNIa} \frac{(5 \log_{10}[D_L(p_s, z_i)] - m_{obs}(z_i))^2}{\sigma_i^2},
$$

$$
B = \sum_{SNIa} \frac{5 \log_{10}[D_L(p_s, z_i)] - m_{obs}(z_i)}{\sigma_i^2},
$$

$$
C = \sum_{SNIa} \frac{1}{\sigma_i^2}.
$$
Relation (A3) has a minimum at the nuisance parameter value $M' = B/C$, which contains information of the values of $h$ and $M$. Therefore, one can extract the values of $h$ and $M$ provided the knowledge of one of them. Finally, note that the expression

$$\chi_{SNIa}^2(p_s) = A - (B^2/C),$$

which coincides to (A4) up to a constant, is often used in the likelihood analysis [96, 97, 100–103], and thus in this case the results will not be affected by a flat $M'$ distribution. For minimizing $\chi_{SNIa}^2(p_s)$ to perform a constraint on cosmological parameters, it is equivalent to maximizing the likelihood

$$L(p_s) \propto \exp\left[-\frac{\chi^2(p_s)}{2}\right].$$

(A5)

2. high-redshift Gamma-Ray Bursts data

The GRBs data can be observed at higher redshift than SNIa. The currently observed redshift range of GRBs is at $0.1 \lesssim z \lesssim 9$. Therefore, the GRBs data can be viewed as an excellent complement to SNIa data and would provide more information at high redshift. When several empirical relations of the GRBs are proposed, these indicators have motivated the authors make use of the GRBs as cosmological standard candles at high redshift. However, the fact that there are not sufficient low redshift GRBs leads that the calibration of GRB relations is dependent on the cosmological model, namely, the circularity problem. One of methods to solve the circularity problem is the calibration of GRB relations are performed by the use of a sample of SNIa at low redshift in the cosmology-independent way [107]. Here, the GRBs data we used consists of 59 GRB samples with a redshift range of $1.4 \lesssim z \lesssim 9$ obtained in [54]. These 59 GRBs are calibrated by utilizing the newly released 557 Union2 SNIa and the isotropic energy-peak spectral energy ($E_{iso} - E_{p,i}$) relation (i.e. Amati relation) [108].

The $\chi^2_{GRBs}$ takes the same form as $\chi^2_{SNIa}$

$$\chi^2_{GRBs}(p_s, \mu_0) = \sum_{i=1}^{59} \frac{[\mu_{obs}(z_i - \mu_{th}(z_i; p_s, \mu_0)]^2}{\sigma_i^2}.$$  

(A6)

The same method are used to deal with the nuisance parameter $\mu_0$ as shown in the description of $\chi^2_{SNIa}$ above.

3. Observational Hubble data

The observational Hubble data [109] are based on differential ages of the galaxies. In [110], Jimenez et al. obtained an independent estimate for the Hubble parameter using the method developed in [111], and used it to constrain the cosmological models. The Hubble parameter depending on the differential ages as a function of redshift $z$ can be written in the form of

$$H(z) = -\frac{1}{1 + z} \frac{dz}{dt}.$$  

(A7)

So, once $dz/dt$ is known, $H(z)$ is obtained directly. By using the differential ages of passively-evolving galaxies from the Gemini Deep Deep Survey (GDDS) [112] and archival data [113–118], Simon et al. obtained several values of $H(z)$ at different redshift [55]. The twelve observational Hubble data (redshift interval $0 \lesssim z \lesssim 1.8$) from [72, 119, 120] are list
TABLE IV: The observational $H(z)$ data.

| $z$ | 0   | 0.1 | 0.17 | 0.27 | 0.4  | 0.48 | 0.88 | 0.9  | 1.30 | 1.43 | 1.53 | 1.75 |
|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|
| $H(z)$ (km s$^{-1}$ Mpc$^{-1}$) | 74.2 | 69  | 83   | 77   | 95   | 97   | 117  | 168  | 177  | 140  | 202  |
| 1σ uncertainty | ±3.6 | ±12 | ±8   | ±14  | ±17  | ±60  | ±40  | ±23  | ±17  | ±18  | ±14  | ±40  |

in Table IV. In addition, in [121] the authors take the BAO scale as a standard ruler in the radial direction, and obtain three more additional data: $H(z = 0.24) = 79.69 \pm 2.32$, $H(z = 0.34) = 83.8 \pm 2.96$, and $H(z = 0.43) = 86.45 \pm 3.27$.

The best fit values of the model parameters from observational Hubble data are determined by minimizing [122–124]

$$
\chi^2_{OHD}(H_0, p_s) = \sum_{i=1}^{15} \frac{[H_{th}(H_0, p_s; z_i) - H_{obs}(z_i)]^2}{\sigma^2(z_i)},
$$

where $H_{th}$ is the predicted value for the Hubble parameter, $H_{obs}$ is the observed value, $\sigma(z_i)$ is the standard deviation measurement uncertainty, and the summation is over the 15 observational Hubble data points at redshifts $z_i$.

4. The X-ray gas mass fraction

The X-ray gas mass fraction, $f_{gas}$, is defined as the ratio of the X-ray gas mass to the total mass of a cluster, which is approximately independent on the redshift for the hot ($kT \gtrsim 5keV$), dynamically relaxed clusters at the radii larger than the innermost core $r_{2500}$. As inspected in [56], the ΛCDM model is very favored and has been chosen as the reference cosmology. The model fitted to the reference ΛCDM data is presented as [56]

$$f_{gas}^{ΛCDM}(z) = \frac{KAγb(z)}{1 + s(z)} \left[ \frac{Ω_b}{Ω_m} \right] ^{[D_{ΛCDM}^A(z)]^{1.5}},$$

where $D_{ΛCDM}^A(z)$ and $D_A(z)$ denote respectively the proper angular diameter distance in the ΛCDM reference cosmology and the current constraint model. $A$ is the angular correction factor, which is caused by the change in angle for the current test model $θ_{2500}$ in comparison with that of the reference cosmology $θ_{2500}^{ΛCDM}$:

$$A = \left( \frac{θ_{2500}^{ΛCDM}}{θ_{2500}} \right)^{η} \approx \left( \frac{H(z)D_A(z)}{[H(z)D_A(z)]^{ΛCDM}} \right)^{η},$$

here, the index $η$ is the slope of the $f_{gas}(r/r_{2500})$ data within the radius $r_{2500}$, with the best-fit average value $η = 0.214 \pm 0.022$ [56]. And the proper (not comoving) angular diameter distance is given by

$$D_A(z) = \frac{c}{(1 + z)\sqrt{Ω_k}} \sinh[\sqrt{Ω_k}] \int_0^z \frac{dz'}{H(z')},$$

It is clear that this quantity is related with $d_L(z)$ by

$$D_A(z) = \frac{d_L(z)}{(1 + z)^2}.$$
residual uncertainties, such as the instrumental calibration and certain X-ray modelling issues, and a Gaussian prior for the 'calibration' factor is considered by $K = 1.0 \pm 0.1$ \[56\].

Following the method in Ref. \[56, 125\] and adopting the updated 42 observational $f_{\text{gas}}$ data in Ref. \[56\], the best fit values of the model parameters for the X-ray gas mass fraction analysis are determined by minimizing,

$$
\chi^2_{CMB} = \sum_i^{N} \frac{[f_{\text{CM}DM}(z_i) - f_{\text{gas}}(z_i)]^2}{\sigma_{f_{\text{gas}}}(z_i)} + \frac{(s_0 - 0.16)^2}{0.0016^2} + \frac{(K - 1.0)^2}{0.01^2} + \frac{(\eta - 0.214)^2}{0.022^2},
$$

(A12)

where $\sigma_{f_{\text{gas}}}(z_i)$ is the statistical uncertainties (Table 3 of \[56\]). As pointed out in \[56\], the acquiescent systematic uncertainties have been considered according to the parameters i.e. $\eta$, $b(z)$, $s(z)$ and $K$.

5. Baryon acoustic oscillation

The baryon acoustic oscillations are detected in the clustering of the combined 2dFGRS and SDSS main galaxy samples, which measure the distance-redshift relation at $z_{\text{BAO}} = 0.2$ and $z_{\text{BAO}} = 0.35$. The observed scale of the BAO calculated from these samples, are analyzed using estimates of the correlated errors to constrain the form of the distance measure $D_V(z)$ \[57, 126\],

$$
D_V(z) = [(1 + z)^2 D_A^2 (z) \frac{c z}{H(z; p_s)}]^{1/3} = H_0 [\frac{z}{E(z; p_s)} (\int_0^z \frac{d z'}{E(z'; p_s)})^2]^{1/2},
$$

(A13)

In this expression $E(z; p_s) = H(z; p_s)/H_0$, $D_A(z)$ is the proper (not comoving) angular diameter distance, which has the following relation with $d_L(z)$

$$
D_A(z) = \frac{d_L(z)}{(1 + z)^2}.
$$

The peak positions of the BAO depend on the ratio of $D_V(z)$ to the sound horizon size at the drag epoch (where baryons were released from photons) $z_d$, which can be obtained by using a fitting formula \[129\]:

$$
z_d = \frac{1291(\Omega_m h^2)^{-0.419}}{1 + 0.659(\Omega_m h^2)^{0.828}} [1 + b_1 (\Omega_b h^2)^{b_2}],
$$

(A14)

with

$$
b_1 = 0.313(\Omega_m h^2)^{-0.419}[1 + 0.607(\Omega_m h^2)^{0.674}],
$$

(A15)

$$
b_2 = 0.238(\Omega_m h^2)^{0.223}.
$$

(A16)

In this paper, we use the data of $r_s(z_d)/D_V(z)$ extracted from the Sloan Digital Sky Survey (SDSS) and the Two Degree Field Galaxy Redshift Survey (2dFGRS) \[126, 128\], which are listed in Table V where $r_s(z)$ is the comoving sound horizon size

$$
r_s(z) = c \int_0^t \frac{c_s dt}{a} = c \int_0^a \frac{c_s da}{a^2 H} = c \int_z^\infty \frac{dz}{H(z)} \frac{c_s}{H(z)} = \frac{c}{\sqrt{3}} \int_0^{1/(1 + z)} \frac{da}{a^2 H(a)} \sqrt{1 + \frac{3 \Omega_b}{4 \Omega_r} a },
$$

(A17)

where $c_s$ is the sound speed of the photon–baryon fluid \[130, 131\]:

$$
c_s^{-2} = 3 + \frac{4}{3} \times \frac{\rho_b(z)}{\rho_\gamma(z)} = 3 + \frac{4}{3} \times \frac{\Omega_b}{\Omega_\gamma} a ,
$$

(A18)
TABLE V: The observational \( r_s(z_d)/D_V(z) \) data [57].

| \( z \) | \( r_s(z_d)/D_V(z) \) |
|---|---|
| 0.2 | 0.1905 ± 0.0061 |
| 0.35 | 0.1097 ± 0.0036 |

and here \( \Omega_\gamma = 2.469 \times 10^{-5} h^{-2} \) for \( T_{CMB} = 2.725 \text{K} \).

Using the data of BAO in Table V and the inverse covariance matrix \( V^{-1} \) in [57]:

\[
V^{-1} = \begin{pmatrix}
30124.1 & -17226.9 \\
-17226.9 & 86976.6
\end{pmatrix},
\]

(A19)

the \( \chi^2_{BAO}(p_s) \) is given as

\[
\chi^2_{BAO}(p_s) = X^t V^{-1} X,
\]

(A20)

where \( X \) is a column vector formed from the values of theory minus the corresponding observational data, with

\[
X = \begin{pmatrix}
r_s(z_d)/D_V(0.2) - 0.1905 \\
r_s(z_d)/D_V(0.35) - 0.1097
\end{pmatrix},
\]

(A21)

and \( X^t \) denotes its transpose.

6. Cosmic microwave background

The CMB shift parameter \( R \) is provided by [132]

\[
R = \sqrt{\Omega_m H_0^2 (1 + z_\star) D_A(z_\star)/c} = \sqrt{\Omega_m} \int_0^{z_\star} \frac{H_0 dz'}{H(z'; p_s)},
\]

(A22)

here, the redshift \( z_\star \) (the decoupling epoch of photons) is obtained using the fitting function [133]

\[
z_\star = 1048 \left[ 1 + 0.00124(\Omega_b h^2)^{-0.738} \right] \left[ 1 + g_1(\Omega_m h^2)^{g_2} \right],
\]

where the functions \( g_1 \) and \( g_2 \) read

\[
g_1 = 0.0783(\Omega_b h^2)^{-0.238}(1 + 39.5(\Omega_b h^2)^{0.763})^{-1},
g_2 = 0.560 \left( 1 + 21.1(\Omega_b h^2)^{1.81} \right)^{-1}.
\]

In addition, the acoustic scale is related to a distance ratio, \( D_A(z)/r_s(z) \), and at decoupling epoch it is defined as

\[
l_A \equiv (1 + z_\star) \frac{\pi D_A(z_\star)}{r_s(z_\star)},
\]

(A23)

where Eq. (A23) arises a factor \( 1 + z_\star \), because \( D_A(z) \) is the proper (physical) angular diameter distance, whereas \( r_s(z_\star) \) is the comoving sound horizon. Using the data of \( l_A, R, z_\star \) in [58] and their covariance matrix of \( [l_A(z_\star), R(z_\star), z_\star] \) (please see table VI and VII), we can calculate the likelihood \( L \) as \( \chi^2_{CMB} = -2\ln L \):

\[
\chi^2_{CMB} = \Delta d_i [\text{Cov}^{-1}(d_i, d_j)[\Delta d_i]^j],
\]

(A24)
| 7-year maximum likelihood error, $\sigma$ |
|-------------------------------------------|
| $l_A(z_*)$                               |
| 302.09                                   |
| $R(z_*)$                                 |
| 1.725                                    |
| $z_*$                                    |
| 1091.3                                   |

TABLE VI: The values of $l_A(z_*)$, $R(z_*)$, and $z_*$ from 7-year WMAP data.

| $l_A(z_*)$ | $R(z_*)$ | $z_*$   |
|------------|----------|---------|
| 2.305      | 29.698   | -1.333  |
| $R(z_*)$   | 6825.270 | -113.180|
| $z_*$      | 3.414    |         |

TABLE VII: The inverse covariance matrix of $l_A(z_*)$, $R(z_*)$, and $z_*$ from 7-year WMAP data.

where $\Delta d_i = d_i - d_i^{\text{data}}$ is a row vector, and $d_i = (l_A, R, z_*)$.

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