Search for a continuum limit of the PMS phase

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Introduction

- Update on continuing study of a four-fermion model with an interesting phase structure.
- PMS phase at strong couplings with massive fermions without any Spontaneous Symmetry Breaking.
- Continuum limit of this PMS phase would be interesting.
- Previous work in 3D, pointed to such a continuum limit.
- Update on study in 4D on small lattices.
Previous studies\textsuperscript{1} of lattice Yukawa models show a very interesting phase structure.

\( g \rightarrow \) Yukawa coupling, \( m \rightarrow \) boson mass

- Massless PMW phase.
- Spontaneously broken FM phase with massive fermions.
- Exotic Paramagnetic PMS phase with massive fermions at strong couplings.
- No bilinear condensates in PMW and PMS phases.

\begin{itemize}
  \item \( m^2 < 0 \) (massive fermions, broken phase)
  \item \( m^2 > 0 \) (massless fermions, symmetric phase)
\end{itemize}

\textsuperscript{1} Hasenfratz, Neuhaus (1989); Lee, Shigemitsu, Shrock (1990); W. Bock, A. K. De, K. Jansen, J. Jersak, T. Neuhaus, and J. Smit (1990).
Four-fermion model

- Equivalent to Yukawa model with fixed $m^2 > 0$
- Easier to study.
- Will also show 3 phase structure.

- Can there be models with a PMW-PMS phase transition?
Our Lattice model

Reduced staggered fermion action for four massless flavors
\[ \psi_{x,1}, \psi_{x,2}, \psi_{x,3}, \psi_{x,4} \]
\[ S = S_0 + S_I \]

\[ \begin{align*}
S_0 &= \sum_{i=1}^4 \sum_{x,y} \{ \psi_{x,i} M_{x,y} \psi_{y,i} \} \\
S_I &= -U \sum_x \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4}
\end{align*} \]

In addition to the usual discrete space-time symmetries\(^1\), the action has a continuous \( SU(4) \) symmetry.

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\(^1\) M. Golterman and J Smit. Selfenergy and Flavor Interpretation of Staggered Fermions. Nucl.Phys., B245:61, 1984.
The Fermion Bag approach \(^1\)

\[
Z = \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} e^{U \sum_x \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4}} \\
= \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} \prod_x e^{U \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4}} \\
= \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} \prod_x (1 + U \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4}) \\
= \sum_{[m_x]} \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} \prod_x (U \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4})^{m_x} \\
\text{Integrate over monomer sites} \\
Z = \sum_{[m_x]} U^k \text{Det}(\tilde{A})^4
\]

Assigning \(m_x = 0\) or 1 to each lattice site

- \(m_x = 0 \equiv \text{free sites}\)
- \(m_x = 1 \equiv \text{monomers}\)

**Fermion Bag** \(\equiv\) Set of connected free sites

where \(k \equiv \text{number of monomers,} \)
\(\tilde{A}\) is a sub-matrix of the staggered matrix \(M\).

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\(^1\) S. Chandrasekharan - The Fermion bag approach to lattice field theories (2010)
Phase diagram: What do we know?

Irrelevant coupling

Massless phase

Correlators decay exponentially

Massive phase & no condensates

PMW

? PMS

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Observables

- Average Monomer density:
  \[ \rho_m = \frac{U}{V} \sum_x \langle \psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4} \rangle \]

- Bosonic correlators:
  \[ C_1(x, y) = \langle \psi_{x,1}\psi_{x,2}\psi_{y,1}\psi_{y,2} \rangle, \]
  \[ C_2(x, y) = \langle \psi_{x,1}\psi_{x,2}\psi_{y,3}\psi_{y,4} \rangle \]

- Susceptibilities
  \[ \chi_1 = \frac{1}{2} \sum_x \langle \psi_{0,1}\psi_{0,2}\psi_{x,1}\psi_{x,2} \rangle \]
  \[ \chi_2 = \frac{1}{2} \sum_x \langle \psi_{0,1}\psi_{0,2}\psi_{x,3}\psi_{x,4} \rangle \]

- Focus on $SU(4)$ transformations.
- Corresponding order parameter $\psi_{x,a}\psi_{x,b}$.
- Condensate $\Phi = \langle \psi_{0,1}\psi_{0,2} \rangle$.
- $\lim_{L \to \infty} C_{1,2}(0, L) \sim \Phi^2$
- $\lim_{L \to \infty} \chi_{1,2} \sim \Phi^2 L^D$
Single 2nd order phase transition in 3D

Near a 2nd order critical point,
\[ R_1 = L^{-(1+\eta)} f \left[ (U - U_c) L^{\frac{1}{\nu}} \right]. \]

- No intermediate FM phase.
- PMW-PMS transition is 2\textsuperscript{nd} order.
- Critical exponents w/o corrections to scaling:
  \( \eta = 1.05(5), \nu = 1.30(7), \)
  \( U_c = 0.943(2) \)
- With corrections to scaling, cannot rule out large \( N \) exponents
  \( \eta = 1.0, \nu = 1.0, U_c = 0.95. \)

1 Ayyar, Chandrasekharan PRD 91, 2015.
2 Ayyar, Chandrasekharan PRD(RC) 93, 2016.
4D Results:

\[ \rho_m = \frac{U}{V} \sum_x \langle \psi_x, 1 \psi_x, 2 \psi_x, 3 \psi_x, 4 \rangle \]

- Lattices up to \( L = 12 \).
- Average monomer density rises sharply around \( U = 1.75 \) without any discontinuity.

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1. Ayyar, Chandrasekharan arxiv:1606.06312, 2016.

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Susceptibility $\chi_1$ vs $U$

- Bosonic Susceptibilities:
  \[
  \chi_1 = \frac{1}{2} \sum_x \langle \psi_{0,1} \psi_{0,2} \psi_x, \psi_{1,2} \rangle \\
  \chi_2 = \frac{1}{2} \sum_x \langle \psi_{0,1} \psi_{0,2} \psi_x, \psi_{3,4} \rangle
  \]

- Condensate given by:
  \[
  \chi \sim \Phi^2 L^4
  \]

- $\chi_1$ reaches a maximum for intermediate $U$.

- Sharp rise.
Evidence for a condensate:

- Condensate implies $\chi_1 \sim L^4$.

- $\chi_1/L^4$ vs $L$ seems to saturate at large $L$.

Obtain condensate $\Phi$ upon fit to

$$\chi_1 = \frac{1}{4} \Phi^2 L^4 + b_1 L^2$$
Evidence for a 3 phase structure.

At a 2\textsuperscript{nd} order critical point, we expect $\chi/L^{2-\eta} \sim \text{const}$

$\implies \chi/L^{2-\eta}$ vs $U$ curves for different $L$’s must intersect.

- Plot $\chi/L^{2-\eta}$ vs $U$ using mean field exponents $\eta = 0$, $\nu = 0.5$,  
- Curves intersect at 2 points $\Rightarrow$ two phase transitions.
- Critical couplings:
  
  \[ U_{c1} = 1.60, \quad U_{c2} = 1.80 \]
Phase diagram in 3D and 4D.

3D

- Single 2nd order phase transition.
- Weak and strong coupling phases do not show any SSB $^1, ^2, ^3$.

4D

- 3 phase structure.
- FM phase is found to be narrow.

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$^1$ K. Slagle, Y.-Z. You, C. Xu, Phys. Rev. B 91, 115121 (2015)

$^2$ S. Catterall, JHEP 01, 121 (2016)

$^3$ Y.-Y. He, H.-Q. Wu, Y.-Z. You, C. Xu, Z. Y. Meng, and Z.-Y. Lu, (2016), arXiv:1603.08376
Conclusions

- Massive fermions without fermion bilinear condensates (PMS phase) in a simple lattice four-fermion model in 3D and 4D.
- PMW-PMS transition is 2nd order in 3D ⇒ PMS phase can help define an interesting 3D continuum field theory.
- Conjecture: Mass could arise via formation of a 3 fermion bound state\(^1,\)\(^2\).
- Evidence for a narrow intermediate FM phase in 4D, with two \(^2nd\) order phase transitions with mean field exponents.
- Suggests the presence of a critical point in enhanced coupling space.

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\(^1\) E. Eichten and J. Preskill, (1986)
\(^2\) Golterman et al., (1993)
THANK YOU