Abstract

We investigate an alternative to the DGLAP evolution of structure functions through the use of Fixed Order Perturbation Theory. Remarkable agreement between the two methods are found in the polarized sector for a wide $x$, $Q^2$ region. However, for the unpolarized sector the agreement is poorer.

In the last few years we have seen an explosion of new and precise data for unpolarized \cite{1} and polarized \cite{2} structure functions. These experimental data cover a wide kinematical range, with $10^{-5} < x < 0.85$ and $0.2 < Q^2 < 5000 GeV^2$ for the unpolarized case and with $0.003 < x < 0.7$ and $1.3 < Q^2 < 58 GeV^2$ for the polarized case. Along with the tremendous experimental advances, we have also seen firm theoretical developments in the sector of perturbative QCD \cite{3,4}. For instance, nowadays we have at our disposition the complete set of the polarized next-to-leading order (NLO) anomalous dimensions \cite{5,6}. These developments, together with the accurate data, have allowed us to perform a number of analysis with NLO precision of the behaviour of the parton distributions and of the structure functions \cite{7,8}. However, for practical applications outside the published parametrizations for the data one needs to write a somewhat long and involved program to solve the DGLAP \cite{11} equations for QCD evolution of the parton distributions. In doing so one meets problems of all sorts, from lengthy expressions for the anomalous dimensions in NLO to worries about analytical continuation, etc. These sorts of technical difficulties make it hard for those who want a straightforward and more direct way to study evolution and are not directly

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involved with these programs. Moreover, the extension from the present day NLO calculations to the full computation of the NNLO anomalous dimensions will be, per se, a giant task. Hence, a more simple approach should be available, at least for the evolution of the structure functions. In this work, we explore an alternative to the evolution of structure functions via a systematic study of what is known as Fixed Order Perturbation Theory (FOPT) [1]. There is also a related interest in FOPT when dealing with heavy quark production [12, 13, 14, 15]. Although we do not treat this problem here, our considerations are relevant because they address exactly the main point of heavy quark production: when to remove the mass-singular logarithms of the coefficient functions? Following this point, we make a brief discussion of the problem of logarithm resummation and its necessity. This is a problem very much in vogue today because of the apparent lack of necessity of resummation of the $1/x$ logarithms, as seen from the HERA data. In our specific case, we will see that these $1/x$ singularities, apparently disconnected from what we are investigating here (the scale logarithms), may play a fundamental role.

Structure functions are cross sections which can be calculated with the help of the factorization theorem:

$$F(x, Q^2) = \sum_{f=q,g} \int_1^x \frac{dz}{z} f\left(\frac{x}{z}, \mu^2\right) \left[ \delta(z - 1)\delta_{fq} + \frac{\alpha_s(\mu^2)}{4\pi} C_f^{(1)}(z, \frac{Q^2}{\mu^2}) \right] + \ldots,$$

where $F(x, Q^2)$ is a general structure function and expression (1) is saying that the full cross section can be separated into a perturbative part, the term in brackets, and a nonperturbative part, the parton distribution $f(x/z)$. The renormalization and mass factorization scales are chosen to be the same and equal to $\mu$. The delta function $\delta_{fq}$ reflects the fact that the gluon contribution starts only at order $\alpha_s$.

Because of the infrared singularity, the coefficients $C^{(1)}$ depend on $\ln(Q^2/\mu^2)$. Once we fix $\mu^2$ we can calculate $F(x, Q^2)$ at any $Q^2$. In this sense, the evolution of $F(x, Q^2)$ is made through the logarithms appearing in $C^{(1)}$. However, this way to evolve the structure function is usually considered to be inadequate because the logarithms diverge when $Q^2 >> \mu^2$. To avoid this divergence, the logarithms are then resummed through the renormalization group equations and $F(x, Q^2)$ is given by:
\[ F(x, Q^2) = \sum_{f=q,g} \int_x^{1} \frac{dz}{z} f \left( \frac{x}{z}, Q^2 \right) \left[ \delta(z - 1) \delta_{fq} + \frac{\alpha_s(Q^2)}{4\pi} C^{(1)}_f(z) + \ldots \right]. \quad (2) \]

The calculation of \( F(x, Q^2) \) through Eq. (1) is what is meant by FOPT evolution in this paper, and it is quite simple to implement this program; a few lines in a Mathematica program will do it. One only needs to calculate the relevant photon-parton cross sections to the required order. On the other hand, the computation of \( F(x, Q^2) \) through Eq. (2) requires the knowledge not only of the coefficient functions \( C^{(1)}_f(z) \) but also the singlet and non-singlet anomalous dimensions. Besides that, one needs to solve the corresponding Altarelli-Parisi evolution equations for the parton distributions \( f(x/z) \), which may turn out to be a nontrivial task [16, 17]. The question is then: which are the limits of applicability, or of equivalence, of equations (1) and (2)?

To answer this question, we start considering the \( F_{2p} \) structure function for four flavours in FOPT:

\[
F_{2p}(x, Q^2) = \frac{5}{18} \int_x^{1} \frac{dz}{z} q^S \left( \frac{x}{z}, \mu^2 \right) C^S_q(z, Q^2/\mu^2) + \frac{1}{6} \int_x^{1} \frac{dz}{z} q^{NS}(x/z, \mu^2) C^{NS}_q(z, Q^2/\mu^2).
\]

The singlet distribution is \( q^S(z) = u(z) + \bar{u}(z) + d(z) + \bar{d}(z) + s(z) + \bar{s}(z) + c(z) + \bar{c}(z) \), and the nonsinglet distribution is \( q^{NS}(z) = u(z) + \bar{u}(z) - d(z) - \bar{d}(z) - s(z) - \bar{s}(z) - \bar{d}(z) + c(z) + \bar{c}(z) \). As we are studying NLO evolution of parton distributions, the coefficient functions are necessary to be known only to order \( \alpha_s \). The non-singlet quark coefficient is [11]

\[
C^{NS}_q(z, Q^2/\mu^2) = \delta(z - 1) + \frac{\alpha_s(\mu^2)}{4\pi} \left\{ C_F \left( \frac{4}{1 - z} \ln(1 - z) - \frac{3}{1 - z} + \frac{4}{1 - z} \ln \left( \frac{Q^2}{\mu^2} \right) \right) + C_F \left[ 2(1 + z)\ln(1 - z) - 2 \frac{1 + z^2}{1 - z} \ln z + 6 + 4z - 2(1 + z)\ln \left( \frac{Q^2}{\mu^2} \right) \right] \right. \\
\left. + C_F \left[ -4\zeta(2) - 9 + 3\ln \left( \frac{Q^2}{\mu^2} \right) \right] \delta(z - 1) \right\}, \quad (4)
\]
and to this order, $C_q^S(z, Q^2/\mu^2) = C_q^{NS}(z, Q^2/\mu^2)$. The gluon coefficient is given by (4):

$$C_g(z, Q^2/\mu^2) = n_f T_f \frac{\alpha_s(\mu^2)}{4\pi} \left\{ 4(1 - 2z + 2z^2) \ln \left( \frac{Q^2}{\mu^2} \right) - 4 + 32z(1 - z) + 4(1 - 2z + 2z^2) \ln \left( \frac{1-z}{z} \right) \right\} . \quad (5)$$

For the quark distributions, we choose the CTEQ3 [10] parametrization, valid at $Q^2 = 2.56 \text{ GeV}^2$. It is then very clear why FOPT is so simple: Eqs. (3) - (5) is all we need to calculate $F_{2p}(x, Q^2)$ for any $x, Q^2$ pair.

We now present the behaviour of $F_{2p}(x, Q^2)$ with $x$ and $Q^2$ for FOPT and DGLAP evolution. We start showing in Figure 1 the non-singlet piece, evolved from $\mu^2 = 2.56 \text{ GeV}^2$ to $Q^2 = 10 \text{ GeV}^2$. The continuous line is the evolution result using FOPT and the dashed line is the result using DGLAP. They totally agree down to $x = 10^{-4}$. Of course, at $10 \text{ GeV}^2$, $\ln(Q^2/\mu^2) = 1.36$, which is in principle a small number. In this case, there would not be necessary any resummation of these logarithms, something which would be relevant only when $Q^2 >> \mu^2$. Thus, the interesting point here is to study the behaviour of $F_{2p}^{NS}(x, Q^2)$ with $Q^2$. That is shown in Figure 2 for two specific points: $x = 0.1$ and $x = 0.001$. We see that even for $Q^2 = 1000 \text{ GeV}^2$, FOPT and DGLAP still give very close results, questioning the necessity of logarithm resummation for the nonsinglet piece of $F_{2p}$.

In the singlet sector the agreement between FOPT and DGLAP is much poorer. As shown in Figure 3, the two methods become inconsistent for $x$ below 0.01. The origin of this discrepancy should not be in the logarithm involving the scale, as it is a small number for the evolution from 2.56 to 10 GeV$^2$. This result together with considerations made throughout this paper implies, contrary to general belief, that the necessity of resummation may not be directly connected to the size of the logarithm. At least in the case of the logarithms involving the scale. A discussion of this point, along with a possible interpretation to it, is made after the polarized evolution is studied.

When varying the scale, there is no significant change for points where the agreement between DGLAP and FOPT was already good at 10 GeV$^2$.\footnote{In an earlier work [2], it was concluded that FOPT and DGLAP evolution were equivalent down to $x = 0.01$. However, those authors did not extended their analysis to lower values of $x$.}
Figure 1: The non-singlet part of $F_{2p}$. The evolution is from 2.56 to 10 GeV$^2$. The continuous and dashed lines are, respectively, the FOPT and DGLAP results at 10 GeV$^2$.

However, as can be seen in Figure 4 for points like $x = 10^{-3}$ there is a strong $Q^2$ dependence for the DGLAP evolution while only a very weak dependence for FOPT. In effect, already at 100 GeV$^2$ the two methods give results which differ by 100%.

Our next step is to study the compatibility between FOPT and DGLAP evolution in the polarized sector. The polarized structure function $g_{1p}(x, Q^2)$ is given in FOPT by:

$$g_{1p}(x, Q^2) = \frac{5}{36} \int_x^1 \frac{dz}{z} [\Delta q^S(x/z, \mu^2) \Delta C_q^S(z, Q^2/\mu^2) + \Delta g(x/z, \mu^2) \Delta C_g(z, Q^2/\mu^2)] + \frac{1}{12} \int_x^1 \frac{dz}{z} \Delta q^{NS}(x/z, \mu^2) \Delta C_q^{NS}(z, Q^2/\mu^2).$$

\[ (6) \]
Figure 2: The $Q^2$ dependence of $F_{2p}^{NS}$, for two specific values of $x$. The initial scale for evolution is 2.56 GeV$^2$.

The photon-parton cross sections have to be calculated to order $\alpha_s$, as we are again considering only $\alpha_s$ corrections to the structure functions. Hence, the non-singlet quark coefficient is [6]:

\[
\Delta \mathcal{C}^{NS}_q(z, Q^2/\mu^2) = \delta(z - 1) + \frac{\alpha_s(\mu^2)}{4\pi} \left\{ C_F \left( \frac{4}{1 - z} \ln(1 - z) - \frac{3}{1 - z} + \frac{4}{1 - z} \ln\left( \frac{Q^2}{\mu^2} \right) \right) + C_F \left[ 2(1 + z) \ln(1 - z) - 2 \frac{1 + z^2}{1 - z} \ln z + 4 + 2z - 2(1 + z) \ln\left( \frac{Q^2}{\mu^2} \right) \right] - C_F \left[ -4\zeta(2) - 9 + 3\ln\left( \frac{Q^2}{\mu^2} \right) \right] \right\} \delta(z - 1). \tag{7}
\]

To the order considered, $C^S_q(z, Q^2/\mu^2) = C^{NS}_q(z, Q^2/\mu^2)$ and the gluon coe-
Figure 3: The singlet part of $F_{2p}$. The evolution is from 2.56 to 10 GeV$^2$. The continuous and dashed lines are, respectively, the FOPT and DGLAP results at 10 GeV$^2$. The dotted line is the input distribution at 2.56 GeV$^2$.

The coefficient is given by [3]:

$$
\Delta C_g(z, Q^2/\mu^2) = n_f T_f \frac{\alpha_s(\mu^2)}{4\pi} \left\{ 4(2z-1)ln \left( \frac{Q^2}{\mu^2} \right) + 4(31 - 4z) \\
+ 4(2z-1)ln \left( \frac{1-z}{z} \right) \right\}.
$$

(8)

We use as input data the same parametrizations used in the unpolarized case, i.e., $\Delta q^{NS}(x) = q^{NS}(x)$, $\Delta q^{S}(x) = q^{S}(x)$ and $\Delta g(x) = g(x)$. The only difference in the evolution of the polarized and of the unpolarized structure functions is in the coefficient functions. We are completely free to do this choice here as we are not aiming at a description of the experimental data.
Figure 4: The $Q^2$ dependence of $F_{2p}^S$, for two specific values of $x$. The initial scale for evolution is $2.56\text{ GeV}^2$.

but only trying to determine the range of compatibility between FOPT and DGLAP evolution.

As in the unpolarized case, the polarized non-singlet part of $g_{1p}(x, Q^2)$ does not distinguish between FOPT and DGLAP evolution, as can be seen from Figure 5. This feature persists when $Q^2$ is allowed to take values as high as $1000\text{ GeV}^2$, in a pattern very close to that of $F_{2p}^{NS}$. Hence, in practical terms, no problem is detected with the large logarithms involved: there is no practical necessity for their resummation.

Surprisingly, in the singlet sector we still see the same sort of behaviour. Figure 6 shows the $x$ dependence of $g_{1p}^{S}(x, Q^2)$ calculated at $10\text{ GeV}^2$, and it is clear that the two methods of calculating the evolution of $g_{1p}(x, Q^2)$ give identical answers. More astonishing, there is no substantial variation with $Q^2$ even for small $x$ values as we can see in Figure 7, in a behaviour completely
distinct from the unpolarized case.

A possible interpretation of this result goes as follows. In the unpolarized sector, there is a strong singularity in the gluon splitting function which governs the singlet DGLAP equations for small enough $x$. The fact that FOPT can not match DGLAP in the evolution of $F_{2p}^S(x, Q^2)$ means that the singularities appearing in the Wilson coefficients Eqs. 4 and 5 are not strong enough to match the singularities in $P_{gg}$. Thus the difference between FOPT and DGLAP singlet evolution appears, even when the logarithms in the scale are small. On the other hand, the nonsinglet sector has no strong small $x$ singularity in the splitting functions, implying in the agreement between FOPT and DGLAP in the whole $x$ region. Moreover, we observe that the difference between FOPT and DGLAP evolution grows mainly in a relatively
Figure 6: The singlet part of $g_{1p}$. The evolution is from 2.56 to 10 $GeV^2$. The continuous and dashed lines are, respectively, the FOPT and DGLAP results at 10 $GeV^2$. The dotted line is the input distribution at 2.56 $GeV^2$.

small $Q^2$ region, smaller than 100 $GeV^2$. Passing this region, both evolutions become weakly dependent on $Q^2$. That would imply that the “need” for resummation of the $ln(Q^2/\mu^2)$ terms is more connected to the $x$ singularities in the splitting functions than to the actual size of these scale logarithms.

For the polarized sector, the DGLAP evolution of the singlet distributions is governed not only by singularities in the gluon splitting function but also in the quark splitting function. In fact, the polarized quark singlet and the polarized gluon distributions evolve with different signs in the small $x$ region \([17]\). The agreement between FOPT and DGLAP singlet evolution is, in this sense, an indication that these singularities approximately cancel when doing the DGLAP evolution. This mechanism would allow FOPT evolution to follow DGLAP evolution closely, and it also gives a fair description
or justification of why the unpolarized evolution is faster than its polarized counterpart: there is a competition between the quark and gluon singularities, making the final result for polarized structure function evolution slower. Using a more physical language, the last statement should be synonymous with saying that not all of the quark-antiquark pairs created by gluons in the small $x$ region will be polarized\[3\]. Hence unpolarized distributions would grow faster than the polarized ones. In any case, a more detailed study on this subject should be done.

It may look contradictory that in an early work \[14\] we found that FOPT and DGLAP evolution gave quite different answers in the small $x$ region of the singlet piece of $g_{1p}(x, Q^2)$. However, in that work we were considering

\[3\] I thank Wally Melnitchouk for discussions on this point.
heavy quark evolution using a zero heavy quark distribution at \( \mu^2 \). Thus, the evolution was dominated by the gluon sector and the balance between the singularities in the quark and in the gluon splitting functions, as discussed here, is no longer valid. As a simple check, we performed FOPT and DGLAP evolution of \( g_{1p}^q(x, Q^2) \) using the same gluon distribution used for the evolution shown in Figure 6 but using a zero for the quark singlet distribution. As in Ref. [14], we find again a discrepancy between the two methods, corroborating the scheme of canceling singularities. We may then say that the reason we need to remove the mass singular terms from the heavy quark coefficient functions is in the absence of a heavy quark distribution at \( \mu^2 \), and not in the presence of mass singular logarithms. At least for polarized heavy quark electro production.

In summary, we have compared two methods of evolution for the structure functions: FOPT and DGLAP evolution. We found that for the polarized structure function \( g_{1p}(x, Q^2) \), the two methods give identical answers, even if the logarithms in the scale are large. For the unpolarized case, the two methods are not equivalent in the small \( x \) region of the singlet piece of \( F_{2p}(x, Q^2) \). For the non-singlet piece, however, there is no distinction between FOPT and DGLAP evolution, even for very large values of \( Q^2 \). The results of this work question the connection between large logarithms in \( Q^2 \) and the need for their resummation.

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