Quantum exchange interaction of spherically symmetric plasmoids

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Abstract

We study nano-sized spherically symmetric plasma structures which are radial nonlinear oscillations of electrons in plasma. The effective interaction of these plasmoids via quantum exchange forces between ions is described. We calculate the energy of this interaction for the case of a dense plasma. The conditions when the exchange interaction is attractive are examined and it is shown that separate plasmoids can form a single object. The application of our results to the theoretical description of stable atmospheric plasma structures is considered.

Keywords: exchange interaction; molecular ion; long-lived atmospheric plasma structure

1 Introduction

The exchange interaction between charged particles plays an important role for the dynamics of quantum plasmas. For the first time the exchange interaction for the degenerate electron gas in metals was considered by Wigner (1934). Since then a lot of works on this subject has been published (see, e.g., the recent review by Shukla and Eliasson (2011) and references therein).

The exchange effects are significant for degenerate and dense gases. The degenerate quantum plasmas were discussed by Landau and Lifshitz (1980). It was found that, owing to the smallness of the electron mass, the electron component of plasma should be treated using the quantum mechanics whereas ions still obey a classical dynamics. However, if the temperature of ions is much lower than that of electrons, quantum effects may be important for the ion component of plasma as well. Moreover, when one considers a structured plasma, i.e. when the parameters of such a media are not stochastic, the behavior of charged particles may obey quantum laws.

In the present work we shall discuss the situation when the exchange interaction is significant for the ion component of plasma. Firstly, in Sec. 2 we formulate the model of the effec-
tive interaction of spherically symmetric plasmoids via the quantum exchange forces between ions. Then, in Sec. 3 we present the numerical estimates of the magnitude of this interaction for the realistic case of an atmospheric plasma. In Sec. 4 we consider a possible application of the obtained results to the description of stable atmospheric plasmoids. Finally, in Sec. 5 we briefly summarize our results.

2 Model

In this section we formulate the model for the quantum exchange interaction of two spherically symmetric plasmoids which are radial nonlinear plasma oscillations. We compute the effective energy of this interaction and compare it with the typical electromagnetic energy of each of the plasmoids.

Let us discuss a plasma composed of hot electrons and singly ionized cold ions. We can excite a spherically symmetric oscillation of electrons in this medium. One can parameterize this kind of oscillation using the vector of the electric field,

$$\mathbf{E}(r, t) = \mathbf{E}_1(r)e^{-i(\omega t + \varphi)} + c.c.,$$

where

$$\mathbf{E}_1(r) = nE_1(r),$$

$$E_1(r) = Ar\exp\left(-\frac{r^2}{2\sigma}\right).$$

Here \(\omega\) is the frequency of the plasma oscillation, \(\varphi\) is the possible phase shift of the oscillation, \(n\) is the unit radial vector, \(A\) is the amplitude of the oscillation, and \(\sigma\) is the effective size of the plasmoid. Note that the shape of the envelope of the electric field in Eq. (2) is often used in the parametrization of nonlinear waves (Anderson, 1983).

Owing to various classical (Skorić and ter Haar, 1980) or quantum (Haas and Shukla, 2009) nonlinearities, the plasma structure described by Eqs. (1) and (2) can be stable. However, in our analysis we do not specify what kind of physical processes underlie the plasmoid stability. Thus we shall take \(A\) and \(\sigma\) in Eq. (2) as phenomenological parameters. The magnitude of these parameters will be evaluated below.

Typically the value of the oscillation frequency \(\omega\) does not exceed the Langmuir frequency for electrons, \(\omega_p\). For example, \(\omega\) can be 10% less than \(\omega_p\) for a classical electron nonlinearities treated in frames of the perturbation theory (Skorić and ter Haar, 1980). Thus ions are only slightly involved in the oscillation process since we suggested that ion temperature is much lower than that of electrons. Therefore we shall suppose that ions are practically at rest and are embedded in the negatively charged background of electrons. Therefore the direct Coulomb interaction of ions is quite perfectly screened by electrons.

Now, if we consider a couple of such plasmoids, then, because of their spherical symmetry, no interaction mediated by classical forces is possible between these structures. However, if we suggest that ions possess nonzero static electric dipole moments (EDM), then instantaneously their spins will be directed along/opposite the oscillating electric field (1). Therefore the spins of ions are structured and we may expect that some nonzero quantum exchange force may exist between these plasmoids.

For simplicity, we may discuss two identical spherically symmetric plasma structures separated by the distance \(R\) (see Fig. 1). The values of the parameters \(A\), \(\sigma\), and \(\omega\) depend on the intrinsic properties of the surrounding me-
Figure 1: The schematic illustration of two interacting plasmoids separated by the distance $R$. The level of saturation of the grey hue corresponds to the strength of the electric field $E_1(r)$ in Eq. (2). The vectors $r_{i,j}$ indicate the positions of two ions connected by the vector $R_{ij}$.

That is why we should assume that they are the same for both plasmoids. In other words, both plasma structures exist owing to the same nonlinear effect. On the contrary, the phase shift in Eq. (1) depends on the initial condition of the plasmoids generation. It means that $\varphi$ may be different for each plasmoid.

The majority of highly abundant neutral atoms and molecules have integer spins. It means that a singly ionized positively charged ion, corresponding to such a neutral particle, will have one uncompensated electron spin. Thus we may expect that ions in our system are half-integer-spin particles. For simplicity we suggest that ion spins are equal to $1/2$ (see also Sec. 3).

The Hamiltonian, describing the quantum exchange interaction of $1/2$-spin particles, is given by Kittel (1996, p. 446),

$$H_{\text{ex}} = -\sum_{i\neq j} J_{ij} (\hat{S}_i \cdot \hat{S}_j), \quad (3)$$

where $J_{ij}$ is the exchange integral and $\hat{S}_i$ is the spin operator of an ion situated at $r = r_i$. The sum in Eq. (3) is taken over the pairs of ions belonging to different plasmoids. It means that we do not account for the exchange interaction inside a plasmoid. Note that in Eq. (3) we keep only the spin dependent term.

When we calculate the averaging over a quantum ensemble of ions, $\langle ... \rangle_q$, we may suppose that the evolution of ions belonging to different plasmoids is uncorrelated, i.e. $\langle (\hat{S}_i \cdot \hat{S}_j) \rangle_q = \langle (\hat{S}_{ij})_q \rangle_q = \langle (\hat{S}_i)_q \rangle_q \langle (\hat{S}_j)_q \rangle_q$. This approximation is valid when the distance between plasmoids is sufficiently great.

The mean local polarization, $P = n_i (\langle p \rangle)$ (where $n_i$ is the ion density and $\langle p \rangle$ is the mean dipole moment of ions) of the $1/2$-spin particles gas can be calculated using the Brillouin function (Kittel, 1996, p. 442),

$$B_J(x) = \frac{2J+1}{2J} \coth \left( \frac{2J+1}{2J} x \right) - \frac{1}{2J} \coth \left( \frac{1}{2J} x \right), \quad (4)$$

where $x = p_0 E(r_i,t)/T$, $p_0$ is the ion EDM, $T$ is the ion temperature, and $J = 1/2$ is the total angular momentum of an ion since we suppose that ions are at the lowest energy state, i.e. the rotational degrees of freedom do not contribute to $J$. Thus we get the ion polarization in the form,

$$P(r_i,t) = g_J n_i p_0 B_{1/2}(x) \frac{r_i}{r_i} \approx 2 \frac{p_0^2}{T} n_i (r_i,t) E(r_i,t), \quad (5)$$

where $g_J = 2$ is the $g$-factor of a $1/2$-spin ion. In Eq. (5) we also take $B_{1/2}(x) \approx x$ in the weak electric field limit. Finally we can find the mean local spin of ions as

$$\langle \hat{S}_i \rangle_q = 2 \frac{p_0^2}{T} E(r_i,t). \quad (6)$$
To derive Eq. (6) we take that 
\[ \langle \hat{S}_{ij} \rangle_q = \langle \hat{p} \rangle_q / p_0. \]

Besides the quantum ensemble averaging we should calculate the mean value over the time interval \( \tau = 2\pi / \omega \), since the frequency of the electric field oscillation is typically high. Finally, using Eqs. (3) and (6), we get the mean value of the quantum exchange Hamiltonian as

\[ \langle \hat{H}_{ex} \rangle_{\tau} = -8 \cos \Delta \varphi \left( \frac{p_0 n_0}{T} \right)^2 \times \sum_{i \neq j} J_{ij} (\mathbf{n}_i \cdot \mathbf{n}_j) E_1(r_i) E_1(r_j), \tag{7} \]

where \( \Delta \varphi \) is the phase difference in oscillations of two plasmoids and \( \mathbf{n}_i = \mathbf{r}_i / r_i \).

The form of the exchange integral \( J_{ij} \) in Eqs. (3) and (7) depends on the ion wave functions inside a plasmoid, which are quite difficult to calculate. Nevertheless, we may suggest that ions are in a self-consistent potential formed by nonlinear interactions of the electron gas. In fact, this potential provides the plasmoid stability. Therefore, according to Landau and Lifshitz (1991), we may approximate the exchange integral by

\[ J_{ij} = J_0 e^{-(\kappa_i + \kappa_j) R_{ij}}, \tag{8} \]

where \( J_0 \) is a positive constant, \( R_{ij} = |\mathbf{R}_{ij}| \) is the distance between two ions (see Fig. 1), \( \kappa_i = \sqrt{2M \mathcal{E}_i} / \hbar \), \( \mathcal{E}_i \) is the ion energy, and \( M \) is its mass. In the following we shall assume that ion energies are approximately equal inside the plasmoid, i.e. \( \kappa_i \approx \kappa_j \approx \kappa = \text{const} \). It is a quite rough approximation, but it significantly simplifies the calculations. Anyway, we cannot account for the coordinate dependence of ion energies until the type of a nonlinearity, providing the plasmoid stability, is specified.

Moving to the continuous density distribution and supposing that the ion density is approximately constant and uniform, \( n_i(r, t) = n_0 \), on the basis of Eqs. (7) and (8) we get for the effective potential of the exchange interaction, \( V_{ex} = \langle \hat{H}_{ex} \rangle_{\tau} \), the following expression:

\[ V_{ex} \approx -8 J_0 \cos \Delta \varphi \left( \frac{p_0 n_0}{T} \right)^2 \int d^3 r_1 d^3 r_2 e^{-2\kappa R_{12}} \times E_1(r_1) E_1(r_2) \frac{(r_1 \cdot r_2)}{r_1 r_2}, \tag{9} \]

where \( R_{12} = |\mathbf{R} - \mathbf{r}_1 + \mathbf{r}_2| \).

The sixfold integral in Eq. (9) can be calculated. It is convenient to compare the final result for the effective potential with the total electromagnetic energy of a plasmoid, \( W_{em} = \frac{1}{8\pi} \int \mathbf{E}^2 d^3 r \) (we remind that the magnetic field is identically equal to zero for a spherically symmetric system of charged particles), as

\[ \frac{V_{ex}}{W_{em}} = -6.0 \times 10^3 \times J_0 \sigma^3 \cos \Delta \varphi \times \left( \frac{p_0 n_0}{T} \right)^2 F(a, b), \tag{10} \]

where

\[ F(a, b) = b \exp(-a^2 / 4) \times \left\{ (1 + 4b^2)[\text{erfc}(2b - a / 2) - \text{erfc}(2b + a / 2)] / a - b[\text{erfc}(2b - a / 2) + \text{erfc}(2b + a / 2)] \right\}. \tag{11} \]

Here \( \text{erfc}(x) = \exp(x^2) \text{erfc}(x) \) is a scaled complementary error function as well as \( a = R / \sigma \) and \( b = \kappa \sigma \) are the dimensionless parameters. We remind that \( \sigma \) is the effective plasmoid size defined in Sec. 2 cf. Eq. (2). In the next section we evaluate the magnitude of \( V_{ex} \) in a realistic situation.

## 3 Numerical estimates

In this section we evaluate the quantum exchange interaction of spherically symmetric plas-
moids for the case of a realistic atmospheric plasma. In particular we find the conditions when this interaction is attractive.

First, we notice that the magnitude of the effective interaction (10) depends on EDM of an ion. The most abundant molecules in the Earth’s atmosphere, like N$_2$ and O$_2$, are non-polar. However, a water molecule has a rather big EDM (Clough et al., 1973), $|p_0| = 6.18 \times 10^{-30}$ C · m. Note that the total spins of stable $^1$H and $^{16}$O atoms, which a water molecule is made of, are integers. Thus, we can consider a plasma composed of singly ionized water ions, which should be half-integer-spin particles. We shall suppose that they are in the lowest energy state and have spins equal to $1/2$, for the results of Sec. 2 to be valid.

The main uncertainty of Eq. (10) is in the parameters of the exchange integral: $\kappa$ and $J_0$. To evaluate $\kappa$ we notice that it depends on the binding energy of an ion inside a plasmoid, $E$. However, for a plasmoid to be stable against thermal fluctuations, $|E|$ should be greater than the typical thermal energy of an ion. For a water ion the average thermal energy is equal to $3T$ since it is a triatomic molecule. Thus we may take that $\kappa \sim 10^{11}$ m$^{-1}$ for $T \approx 300$ K (the typical air temperature) and $M \sim 10^{-26}$ kg (the mass of the water molecule). To make a rough estimate one may also assume that $J_0 \sim 3T$.

Dvornikov and Dvornikov (2006); Dvornikov (2012) found that stable quantum plasma structures of the nano-scaled size ($10^{-8} - 10^{-9}$ m can exist in the atmosphere. Let us discuss a pair of such plasmoids with $\sigma \sim 10^{-9}$ m. Taking into account our estimate for $\kappa$ we get that $b \approx 100$ for these plasmoids. The function $F(a,b)$, given in Eq. (11), versus $a$ for $b = 100$ is shown in Fig. 2(a).

One can see in Fig. 2(a) that $F(a,b) \to 0$ at

![Figure 2](image-url)
Thus spins will be mainly parallel at any moment of the effective interaction since there is a shortest segments will give the greatest contribution to other. It is clear that ions which are in these segments will give the greatest contribution to the effective interaction since there is a shortest distance between them, cf. Eq. (3). Indeed, let us discuss the ball segments of plasmoids which are in front of each other. It is clear that ions which are in these segments will give the greatest contribution to the effective interaction since there is a shortest distance between them, cf. Eq. (3). If we discuss the case of the antiphase oscillations, their spins will be mainly parallel at any moment of time. Thus \( \langle \hat{S}_i \cdot \hat{S}_j \rangle_q > 0 \) and, according to Eq. (3), this situation should correspond to the effective attraction. At \( b = 100 \) the minimum of the function \( F_{\text{min}} \approx -2.9 \times 10^{-9} \) is reached at \( a \approx 3.16 \).

The strength of the effective attraction between plasmoids quadratically depends on the number density of water ions. Let us suppose that we study plasma structures in the air having 100% relative humidity at \( T = 3000 \text{ K} \). In this case \( n_0 \approx 2 \times 10^{23} \text{ m}^{-3} \). Using Eq. (9) for these plasma parameters and supposing that \( \cos \Delta \varphi = -1 \) we get that |\( V_{\text{ex}}/W_{\text{em}} \approx 10^{-14} \), i.e. the energy of the quantum exchange energy is very small compared to the total electromagnetic energy of a plasmoid. However, if we assume that \( n_0 \approx 10^{29} \text{ m}^{-3} \), that corresponds to the number density of molecules in liquid water, we get that |\( V_{\text{ex}}/W_{\text{em}} \approx 10^{-2} \). It means that quantum exchange interaction can reach a few percent of the electromagnetic interaction.

One can also notice in Fig. 2(a) that, decreasing the distance between plasmoids, the function \( F(a,b) \) becomes positive. Formally the situation when \( a = 0 \) corresponds to completely overlapped plasmoids. In this case, provided that the oscillations are in phase, the spins of ions will be always parallel. Again using Eq. (3) with |\( \Delta \varphi \) < \( \pi/2 \) we get that one has an effective attraction between plasmoids. However we cannot discuss small values of \( a < 2 \) since in this case there will be a strong correlation between spins of different plasmoids which will distort the dynamics of the system making our analysis invalid. Nevertheless, at \( 2 < a < a_0 \), where \( a_0 \approx 2.45 \) is the root of the function \( F(a,b = 100) \), our results are likely to be valid. Physically this situation corresponds to slightly touching plasma structures without a significant overlapping. For the values used above, \( T = 3000 \text{ K} \) and \( n_0 \approx 10^{23} \text{ m}^{-3} \), we again obtain that |\( V_{\text{ex}}/W_{\text{em}} \approx 10^{-14} \), provided that \( \cos \Delta \varphi = 1 \). If we increase the water ions density up to \( 10^{29} \text{ m}^{-3} \), we get that |\( V_{\text{ex}}/W_{\text{em}} \approx 10^{-2} \).

We obtained that, for realistic conditions, corresponding to a typical atmospheric plasma which contains saturated water vapor, the energy of the quantum exchange attraction is only a small fraction of the total electromagnetic energy of a spherical plasma structure. Thus the exchange interaction cannot significantly influence the dynamics of oscillations inside a plasmoid. The exchange interaction can reach a few percent of the electromagnetic interaction if we study plasmoids in denser media. Hence one may expect the coagulation of separate nano-sized plasma structures excited, e.g., in liquid water.

We also notice that, if we lower the ion temperature, the quantum exchange interaction should reveal itself more intensively. The function \( F(a,b) \) for \( b = 10 \), that corresponds to a temperature of a few Kelvin degrees, is shown in Fig. 2(b). One can see that, in this case, the
quantum exchange interaction is several orders of magnitude stronger, as it should be, since quantum effects are more essential at low temperatures. However this case is unlikely to be implemented in an atmospheric plasma.

At the end of this section we mention that the assumption that mean ion spins are always parallel the electric field was made in Sec. 2, cf. Eq. (6). We remind that the electric field inside a plasmoid is rapidly oscillating, see Eq. (1). Thus, for Eq. (6) to be valid, the typical time of the ion depolarization in an oscillating electric field, i.e. the time of the spin-flip, $\tau_d$, should be much shorter than $1/\omega$. The spin-flip time can be estimated as $\tau_d \sim I/\hbar \approx 10^{-12}$ s, where $I \sim 10^{-46}$ kg $\cdot$ m$^2$ is the moment of inertia of a water ion. The oscillations frequency for the observed plasmoids does not exceed several MHz [Klimov and Kutlaliev, 2010; Bychkov et al., 2010, p. 211]. Therefore Eq. (6) is always valid.

4 Application

In this section we discuss the application of the results, obtained in Secs. 2 and 3, to the description of stable atmospheric plasma structures. We show that the quantum exchange interaction of plasmoids can result in the formation of a composite object.

A long-lived atmospheric plasmoid, called a ball lightning (BL), is a luminous object, having the form of a sphere of $(0.1 - 0.2)$ m in diameter. Numerous trustworthy reports on such an object, as well as its photo and video recordings, weigh heavily to the reality of this natural phenomenon. Nonetheless the existence of BL is still a puzzle for the modern physics because of its extraordinary properties. The numerous theoretical models of BL, including very exotic ones, are listed in the recent review by Bychkov et al. (2010).

Taking into account the fact that in the majority of cases BL appears mainly during a thunderstorm, it is natural to suggest that it is a plasma based phenomenon. However the reported life-time of BL, which can be as long as several minutes, is in contradiction with the typical existence time of an unstructured atmospheric plasma, which is about several ms. Dvornikov and Dvornikov (2006); Dvornikov (2012) constructed a BL model based on nonlinear quantum oscillations of electron gas in plasma. For quantum effects to come into play, the typical size of a plasmoid should be tiny. The detailed analysis of Dvornikov and Dvornikov (2006); Dvornikov (2012) showed that the typical size of a plasmoid should be about $(10^{-8} - 10^{-9})$ m for the existence of stable spherically symmetric quantum oscillations of electrons. Note that the BL model based on radial plasma oscillations was also discussed by Fedele (1999); Shmatov (2003); Tennakone (2011).

The nano-scaled size of a plasmoid predicted by Dvornikov and Dvornikov (2006); Dvornikov (2012) can explain that sometimes BL can easily pass through tiny holes. Despite the recent experimental achievements in the generation of nano-sized BL-like structures reported by Mitchell et al. (2008); Klimov and Kutlaliev (2010); Ito and Cappelli (2012), it is still difficult to reconcile the visible dimensions of a natural BL with the results of Dvornikov and Dvornikov (2006); Dvornikov (2012). Nevertheless, we may suggest that the core of BL, which still can be small, is a coagulate of separate kernels, each of them being a spherically symmetric quantum oscillation of electrons. The mechanism, which
holds these kernels together, may be the quantum exchange interaction between ions described in Secs. 2 and 3.

In our opinion, separate plasma oscillation kernels are created with arbitrarily distributed oscillations phases. Then, owing to the quantum exchange attraction, the parts of a future BL self-organize and form a natural plasmoid. A composite BL model is confirmed by the observational facts that a natural plasmoid often divides into several independent objects (Bychkov et al., 2010, p. 206) and that the motion of smaller parts inside BL is sometimes visible (Bychkov et al., 2010, p. 233). A complex plasmoid consisting of several hot kernels was recently generated in a laboratory by Oreshko and Mavlyudov (2011). Note that a model of BL consisting of separate parts was also considered by Nikitin (2006).

Using the estimates of Sec. 3 one finds that the quantum exchange interaction can be responsible to the coagulation of separate plasmoids only if the density of water ions is rather high, \( n_0 \sim 10^{29} \text{ m}^{-3} \), which corresponds to the case of liquid water. Saturated water vapor, having \( n_0 \approx 2 \times 10^{23} \text{ m}^{-3} \), is not sufficiently dense to provide the required attractive force between plasma structures. It means that the described mechanism may be important at the stage of the formation of a natural BL in case it appears inside a drop of rain water. We also mention that the results of our work may be applied for the analysis of the experiments made by Shabanov (2002); Versteegh et al. (2008), who generated laboratory plasmoids in electric discharges in water.

We should mention that, for the first time, the idea that the exchange interaction can be important for the BL stability was discussed by Neugebauer (1937), who suggested that electrons in a dense and relatively cold plasma can be held together by this kind of interaction providing the plasma cohesion. However, as it was later shown by Landau and Lifshitz (1980), accounting for the exchange effects in quantum plasma just slightly enhances its pressure, making impossible the plasma confinement.

The hypothesis that exchange interaction between electrons is responsible to the cohesion of plasma of a natural plasmoid was later revisited by Kulakov and Rumyantsev (1991). Unlike Neugebauer (1937), who studied the exchange interaction between free electrons, Kulakov and Rumyantsev (1991) considered the situation when an electron in plasma is in a bound state with an ion. In this case, provided that spins of the majority of electrons are equally directed, cf. Eq. (3), there can be an attractive interaction resulting in the formation of a quantum liquid. However, it is unclear how to structure electron spins in a quite hot plasma, discussed by Kulakov and Rumyantsev (1991), at the absence of an external field. Note that electrons can be polarized only by a magnetic field since electron’s EDM has not been discovered yet (Hudson et al., 2011).

In the present work we consider the effective interaction of separate nano-sized plasmoids using the concept of quantum exchange forces. However, contrary to Neugebauer (1937); Kulakov and Rumyantsev (1991), here we suggest that ions, rather than electrons, are involved in this kind of interaction. If an ion has a great EDM (like H\(_2\)O\(^+\)), one may expect that ion spins will be structured in an oscillating electric field of a plasmoid, cf. Eq. (1). The detailed analysis of Secs. 2 and 3 shows that a small attraction between plasmoids is possible if there is a certain phase shift in oscillating structures created in a dense plasma.
The magnitude of the quantum exchange interaction strongly depends on ion’s EDM. Water, which is highly abundant in the Earth’s atmosphere, has a molecule with a great EDM. It can explain the fact why BL appears mainly during or after a thunderstorm, when air is enriched with water vapors (Bychkov et al., 2010, p. 246). It is necessary to mention that the importance of EDM of charged particles to the stability of BL was previously considered by Bergström (1973); Stakhanov (1973); Mesenyashin (1991); Gilman (2003). Contrary to the studies of Bergström (1973); Stakhanov (1973); Mesenyashin (1991); Gilman (2003), in the present work it is suggested that the existence of a plasmoid is provided by a nonlinear interaction of electrons. The interaction of EDM of ions is accounted for to explain the coagulation of separate nano-sized plasmoids into a single object, which can be a natural BL.

At the end of this section we mention that BL observations reveal the variety of characteristics of this natural phenomenon. All the reported properties of BL does not seem to be reconciled in frames of a single model. Thus, perhaps several types of glowing objects are taken as BLs. In this work we construct a theoretical model of a low energy BL which has an internal structure. The explanation of possibly existing atmospheric plasma structures which possess a huge energy, up to several MJ (Bychkov et al., 2010, p. 230), cannot be explained within our model.

5 Conclusion

In conclusion we mention that in the present work we have discussed the effective interaction of spherically symmetric plasma structures via the quantum exchange forces. Each of the plasmoids is taken to be a nonlinear radial oscillation of electrons. In Sec. 2 it has been obtained that a nonzero interaction can exist in such a system. This interaction can be attractive or repulsive depending on the phase shift in plasmoids oscillations. At big distances between plasmoids there is an effective attraction if oscillations are in antiphase. If oscillations are in phase, an effective attraction is also possible, provided the distance between plasmoids is relatively small. However the latter case requires an additional special analysis since some unaccounted spin correlation effects can significantly influence the dynamics of the system.

In Sec. 3 we have made numerical estimates of the magnitude of the quantum exchange interaction. We have discussed a dense plasma having a significant fraction of water ions. It has been found that mutual effective attraction of plasmoids can reach a few percent of the total electromagnetic energy of a plasmoid in cases of both short and great distances between plasma structures if the density of ions is high, \( n_0 \sim 10^{29} \text{ m}^{-3} \). If one studies the attraction of plasma structures existing in a medium containing saturated water vapor with \( n_0 \approx 2 \times 10^{23} \text{ m}^{-3} \), the quantum exchange interaction is insufficiently strong to provide the coagulation of plasmoids. Taking into account that no other types of interaction between spherical plasmoids are allowed, we may conclude that separate plasma structures can be held together by the exchange interaction, in case they exist in a dense medium like liquid water.

In Sec. 4 we have discussed the application of the obtained results to the construction of a theoretical model of a spherical atmospheric plasmoid, BL. Dvornikov and Dvornikov (2006); Dvornikov (2012) demonstrated that a stable nano-sized plasma structure, based on nonlinear
quantum oscillations of electrons, can exist in the atmosphere. In the present work we have shown that separate kernels, with independent radial plasma oscillations, can form a single structure owing to the exchange interaction of ions. This phenomenon may happen at the initial stages of the BL evolution, when plasmoids appear in a drop of rain water. Thus we have developed a composite BL model which is not excluded by the observational data.

Acknowledgments

I am grateful to S. I. Dvornikov, S. Lundeen, A. I. Nikitin, E. V. Orlenko, and V. B. Semikoz for helpful discussions as well as to FAPESP (Brazil) for a grant. A special thank is addressed to A. V. Nemkin for his help on the illustration preparation.

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