1. Introduction

Virtual engineering is an increasingly important part of the design process. Computer analyses and numerical simulations are used to test and optimise the product before actually producing it. In the simulation of railway vehicle behaviour the contact between wheel and rail is a crucial factor. Dependent on the type of analyses different mathematical models of the wheel-rail contact can be used. The paper describes the differences in the computation methods of the tangential forces between wheel and rail used in railway vehicle dynamics and in axle drive dynamics.

2. Methods of wheel-rail forces computation in vehicle dynamics

2.1 Survey of methods used in computer simulations

There are various methods for the computation of wheel-rail forces. The best known methods can be divided into four groups:

- exact theory by Kalker (programme CONTACT)
- simplified theory by Kalker (programme FASTSIM)
- look-up tables
- simplified formulae and saturation functions.

The exact theory by Kalker (computer programme CONTACT [5]) has not been used in the simulations because of its very long calculation time.

The simplified theory used in Kalker’s programme FASTSIM [3] is much faster than the exact theory, but the calculation time is still relatively long for use in complicated multi-body systems.
The maximum value of tangential stress as Freibauer [1], obtains the resulted tangential force (without spin) contact area.

Searching for faster methods some authors found approximations based on simple saturation functions (e.g. Vermeulen-Johnson [18], Shen-Hedrick-Elkins [17]). The calculation time using these approximations is short, but there are significant differences to the exact theory especially in the presence of spin. Simple approximations are often used as a fast and less exact alternative to standard methods (e.g. in MEDYNA, VAMPIRE, SIMPACK).

2.2 A time saving method

A very good compromise between the calculation time and the required accuracy allows a fast method for the computation of wheel-rail forces developed by the author [9], [10], [14]. In spite of the simplifications used, spin is taken into consideration. Due to the short calculation time, the method can be used as a substitute for Kalker’s programme FASTSIM to save computation time or instead of approximation functions to improve the accuracy.

The proposed method has been used in the simulations in different programmes since 1990. The experience with use of this method in various simulation tools was published in [15]. The algorithm is implemented in ADAMS/Rail as an alternative parallel to FASTSIM and to the table-book. The calculation time is faster than FASTSIM’s and usually faster than the table-book as well. The proposed algorithm provides a smoothing of the contact forces in comparison with the table-book and there are no convergence problems during the integration. The programme was tested as user routine in programmes: SIMPACK, MEDYNA, SIMFA and various user’s own programmes with very good experience.

The proposed method assumes an ellipsoidal contact area with half-axis \(a\), \(b\) and normal stress distribution according to Hertz. The maximum value of tangential stress \(\tau\) at any arbitrary point is

\[
\tau_{\text{max}} = f \cdot \sigma \tag{1}
\]

where \(f\) – coefficient of friction,
\(\sigma\) – normal stress.

The coefficient of friction \(f\) is assumed constant in the whole contact area.

The solution described in [9], [14], based on the work of Freibauer [1], obtains the resulted tangential force (without spin) as

\[
F = -\frac{2 \cdot Q \cdot f}{\pi} \left( \frac{\epsilon}{1 + \epsilon^2} + \arctan \epsilon \right) \tag{2}
\]

where \(Q\) – wheel load
\(f\) – friction coefficient
\(\epsilon\) – gradient of the tangential stress in the area of adhesion

\[
\epsilon = \frac{2}{3} \frac{C \cdot \pi \cdot a^2 \cdot b}{Q \cdot f} \tag{3}
\]

where \(C\) – proportionality coefficient characterising the contact elasticity (tangential contact stiffness)
\(s\) – creep
\(s_x, s_y\) – creep in x and y directions.

Using the Kalker’s linear theory to express the coefficient \(C\), the equation (3) has then the form (in the case of only longitudinal creep)

\[
\epsilon = \frac{1}{4} \frac{G \cdot \pi \cdot a \cdot b \cdot c_{11}}{Q \cdot f} \cdot s_x \tag{5}
\]

where \(c_{11}\) – Kalker’s coefficient for longitudinal direction [2]
\(G\) – modulus of rigidity.

The calculation of tangential forces for general case with combination of longitudinal and lateral creep and spin allows the algorithm published in [14].

The proposed method was verified by making a comparison between curving behaviour calculations used for the computation of wheel-rail forces programme FASTSIM and the proposed method, and by comparison with measurements. A model of the SBB 460 locomotive of the Swiss Federal Railways was built by means of the simulation tool ADAMS/Rail. The locomotive design combines very good curving performance with high maximal speed due to the coupling of wheel sets, realised by a mechanism with a torsion shaft assembled to the bogie frame [12], [13]. The model used in simulations consists of 51 rigid bodies and contains 266 degrees of freedom (Fig. 1). The results, using both calcula-

![Fig. 1. ADAMS/Rail model of the SBB 460 locomotive of the Swiss Federal Railways](image-url)
tion methods mentioned, are similar, see Fig. 2. However, there is a significant difference in the calculation time. The results computed using the proposed method show good agreement with the measurements. Especially in the case of the leading wheel sets, they are nearer to the actually measured values than the results obtained in the simulations using FASTSIM.

3. Computation of wheel-rail forces useful for simulation of vehicle dynamics and axle drive dynamics interaction

3.1 Differences between the vehicle dynamics and axle drive dynamics

Depending on the aim of the tests, different measured creep-force functions can be found in literature [7]. Because of the variety of measurements, different models are used for the same physical phenomenon – forces between wheel and rail – in the vehicle dynamics and axle drive dynamics (Fig. 3). How is this possible? The reasons for this are different fields of parameters and different areas of investigations.

In vehicle dynamics small creep values are of main importance. Tangential forces in longitudinal as well as in lateral directions influence the vehicle behaviour; therefore, longitudinal and lateral creep as well as spin should be taken into account. Based on the theory of rolling contact, the creep forces depend on the creep as non-dimensional value. The friction coefficient is assumed to be constant. The difference between dry and wet conditions is

![Fig. 2. Measured lateral wheel-rail forces in a curve with 300 m radius compared with the simulations using the method developed by the author [14] and using FASTSIM](image-url)

![Fig. 3. Differences between the typical creep-force functions used in the vehicle dynamics and in axle drive dynamics](image-url)
usually expressed only with the value of friction coefficient and not with a change of the creep-force function gradient.

In the drive dynamics large values of longitudinal creep influence its behaviour. In the simulated systems usually only the longitudinal direction is taken into account. Based on the experiments, the creep forces are usually assumed as dependent on the slip velocity between wheel and rail. There is a maximum of creep-force function, so called adhesion optimum, and a decreasing section behind this maximum. The gradient and the form of creep-force functions for wet, dry or other conditions are different.

3.2 Wheel-rail contact model applicable for both vehicle and axle drive dynamics

For the complex simulation of dynamic behaviour of locomotive or traction vehicle in connection with drive dynamics and traction control, high longitudinal creep and decreasing section of creep-force function behind the adhesion limit has to be taken into account. The different wheel-rail models described above have to be combined into one model, which is suitable for both vehicle dynamics and drive dynamics simulations.

A creep-force law with a marked adhesion optimum can be modelled using the friction coefficient decreasing with increasing slip velocity between wheel and rail. The dependence of friction on the slip velocity was observed by various authors and is described in [16], [6], [8].

The variable friction coefficient can be expressed by the following equation

\[ f = f_0 \cdot [(1 - A) \cdot e^{-Bw} + A] \]  

where

- \( f_0 \) – maximum friction coefficient
- \( w \) – slip velocity [m/s]
- \( B \) – coefficient of exponential friction decrease in function of slip velocity [s/m]
- \( A \) – ratio of limit friction coefficient \( f_\infty \) at infinity slip velocity to maximum friction coefficient \( f_0 \)

\[ A = \frac{f_\infty}{f_0} \]

From the point of view of tractive effort and traction dynamics the creep law for bad adhesion conditions is of main importance: wet rail and surface pollution e.g. oil, dirt, moisture. Even for dry conditions the gradient of creep-force functions is usually lower than the theoretical value. The reason of this is layer of moisture, which can be taken into consideration in the “stiffness coefficient” of surface soil [4]. In the vehicle dynamics, these real conditions are taken into account simply with a reduction of Kalker’s creep coefficients [2]. This method is used for the linear creep force law but can be used generally as well.

The combination of

- the friction coefficient as a function of slip velocity between wheel and rail and
- the reduction of Kalker’s creep coefficients allows
- on the one hand to achieve the form of creep-force function with an adhesion optimum as known from measurements and as necessary for traction investigations
- and on the other hand to keep the principles of creep forces computation in dependence on longitudinal and lateral creep and spin.

In this manner, the creep-force functions can be adapted for various conditions of wheel-rail contact, see Fig 4.

At the same time, using this principle, the adhesion maximum decreases and parallel to this creep value at this maximum decreases with increasing of vehicle speed, which is a phenomenon well known from measurements. It can be observed on Fig. 5, which
shows examples of creep-force functions modelled using the method according to Polach [14] with the extension described above. The same principles can be used to extend the model of wheel-rail forces based on FASTSIM or any other method of creep force computation.

### 3.3 Extension of the time-saving method for use in both vehicle and axle drive dynamics

Although the method described above allows reproducing the basic tendencies of measured creep-force functions at high creep values, there are some limitations. To achieve the adhesion optimum at high creep values, a significant reduction of Kalker’s coefficients would be necessary (less than 0.1). But the measurements of the creep-force function gradient do not usually show so low values.

The explanation may lie in the combination of dry and wet friction. For small creep values, the area of adhesion extends to the greater part of the contact area. The conditions are similar to dry friction. For large creep values, there is slip in the main part of the contact area. The layer of water or pollution influences the resulted force. The ‘stiffness coefficient’ of surface soil decreases and, as a result of this, the creep-force function changes significantly its gradient.

In spite of complexity of this phenomenon, a good possibility was found to model the wheel-rail contact forces in good agreement with the measured functions using a simple extension of the time saving method proposed by the author [14].

In this method the tangential force (equation (2)) is expressed as a function of the gradient $\epsilon$ of the tangential stress (equation (5)). Using a reduction of Kalker’s coefficient with a factor $k$, the equation (5) has the form

$$\epsilon_k = \frac{1}{4} \frac{G \cdot \pi \cdot a \cdot b \cdot k \cdot c_{11}}{Q \cdot f} s_x = k \cdot \epsilon$$

(7)

In (2) there are two terms: one of them connected to the area of adhesion, another one to the area of slip. Using different reduction factors $k_A$ in the area of adhesion and $k_S$ in the area of slip, the equation (2) has the form

$$F = -\frac{2 \cdot Q \cdot f}{\pi} \left( \frac{k_A \cdot \epsilon}{1 + (k_A \cdot \epsilon)^2} + \arctan(k_S \cdot \epsilon) \right)$$

(8)

Using $k_A > k_S$, there is nearly no reduction of the creep-function gradient at small creepages but a significant reduction of the gradient near saturation. The gradient of the creep-force function at the co-ordinate origin corresponds to the reduction of Kalker’s coefficient

$$k = \frac{k_A + k_S}{2}$$

(9)

The form of the creep-force functions is more similar to the form of measured functions than with the reduction factor $k$, see Fig. 6. The diagram shows creep-force functions for longitudinal direction in non-dimensional co-ordinates

- non-dimensional wheel-rail force

$$f_s = \frac{F_s}{Q \cdot f}$$

(10)

- non-dimensional longitudinal creep

$$\theta_s = \frac{G \cdot a \cdot b \cdot c_{11} \cdot s_x}{Q \cdot f}$$

(11)

Using the proposed method various measured creep-force functions were modelled. As an example, Fig. 7 shows measurements with the Adtranz SBB 460 locomotive [11]. There are results of seven measurements with speed of 40 km/h on wet rail. The measured points were classified and average values, maximal and minimal values were calculated. Using the model described, parameters were found which provide a creep-force model with a very good agreement with the measurements.

The wheel-rail models with parameters presented above were tested for various speeds and for range of longitudinal creep from...
very small to high creep values. Even in this large range the results are plausible and the model is not limited to one speed or to a small creep range. Of course, change of conditions in contact of wheel and rail as well as other effects; e.g. the cleaning effect due to large creep (so called rail conditioning) will cause change of wheel-rail model parameters.

The examples confirm that the method gives a very good possibility to model the functionality of wheel-rail contact necessary for the investigation of axle drive systems. Parallel to this the method allows to keep the principles of creep forces computation in dependence on longitudinal and lateral creep and spin, which is necessary for vehicle dynamics investigations.

As application examples of the proposed wheel-rail model, the quasi-static and dynamic investigations of the influence of locomotive traction effort on the wheel-rail forces and curving behaviour will be presented.

4. Investigation of the influence of locomotive tractive effort on its curving behaviour using the proposed wheel-rail contact model

In order to verify the method of wheel-rail force computation, the ADAMS/Rail model of the SBB 460 locomotive was extended by a full model of the driving system (Fig. 8), and runs through a curve on wet rails were simulated.

Fig. 9 shows the distribution of longitudinal and lateral quasi-static wheel-rail forces in a curve of 300 m radius, running at a speed of 87.7 km/h (lateral acceleration $a_{lat} = 1.0 \text{ m/s}^2$). The wheel-rail forces change with increasing tractive effort. Without tractive effort, there are opposite longitudinal forces on the left and right wheel. With increasing tractive effort the longitudinal forces on the outer wheel of the first wheel set and on the inner wheel of the second wheel set increase. An even distribution of the longitudinal forces will be achieved first on the adhesion limit. The angle between the wheel sets in the horizontal plane (so called steering angle) depends on the tractive effort as well. Without tractive effort, the angle between the wheel sets achieves the maximal...
value due to the steering moment of longitudinal forces. With increasing tractive effort the distribution of longitudinal forces changes as explained before, the steering moment acting on the wheel sets decreases and the steering angle between the wheel sets decreases as well. In spite of this the experience with these locomotives confirms that there is no negative influence on the wear of wheels and rails because of the statistically rare occurrence of maximal tractive effort. On the Gotthard-route the locomotives achieve three to four times longer intervals between the renewals of wheel profiles than the previous ones [19].

In order to test the possibility of simulating the dynamic change of traction torque, the adhesion tests [11] of the locomotive SBB 460 with test composition shown on Fig. 10 were simulated. In the course of curving simulation, the traction torque was increased from zero to the adhesion limit, in a similar way as during the adhesion test measurements. In this manner, a run on the unstable (decreasing) section of creep-force function was simulated. The influence of increased tractive effort on longitudinal and lateral wheel-rail forces and the steering angle between the wheel sets of one bogie were investigated. The simulation results show the same behaviour as observed during the tests. The time plots of traction torque, longitudinal wheel-rail forces, lateral displacements of the wheel sets relative to track and steering angle between wheel sets for the speed of 70 km/h are shown in Fig. 11. With increasing tractive effort, the wheel set steering ability decreases. Simultaneously, the first wheel set of the bogie moves to the inner rail. The calculation shows the same dynamic behaviour tendency due to the change of the tractive effort as observed during the tests. Thereby the proposed method was proven as suitable for the investigation of problems regarding the interaction of traction dynamics and vehicle behaviour.

5. Conclusions

A fast method for the computation of wheel-rail forces developed by the author allows saving calculation time. With the presented extension of this model, various wheel-rail contact conditions can be modelled. The proposed method is suitable for the investigations of problems regarding the drive dynamics and their interaction with the traction control and vehicle dynamic behaviour. The results calculated using the proposed method show good agreement with measurements and confirm this as a possible way for the simulation of complex mechatronic systems of railway vehicles.

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Fig. 11. Time plots of adhesion test simulation (curve $R = 400$ m, $V = 70$ km/h)
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