Dynamic reactive power optimization method based on the transformer dummy node model

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Abstract. This paper proposes a dynamic reactive power optimization method based on the transformer dummy node model. Compared to the method based on the traditional equivalent Π circuit, the proposed method adds the voltage at the dummy node as a variable, together with the voltage at the other side of the transformer, to express the voltage change on the two sides of the transformer. So the turn ratio of transformer is removed from variables, and the power flow equations of transformer branches do not contain turn ratio and its square, both the dimension of power flow equations and the number of non-zero elements in the Jacobian and Hessian matrix reduce, which decreases the calculative amount and accelerates the solution process. The test results of IEEE4, 14, 30 bus systems show that the proposed method obtains the same optimal solution with the traditional method but converges more quickly. With higher computational efficiency, the proposed method is suitable for the practical application of power systems.

1. Introduction
Dynamic reactive power optimization (DRPO) is of vital significance for the secure and economic performance of power systems since it could improve voltage quality and reduce network losses effectively. It is used to minimize transmission losses by determining all kinds of controllable variables, such as reactive power output of generators and shunt capacitor banks, taps of on-load tap changers (OLTC), etc, while satisfying some equality and inequality constraints including the power flow equations, upper and lower limits of variables, and the action number limits of controllable equipment. Highly nonlinear of power flow equations, discretization of reactive power compensators and taps of transformers, and strong coupling of multi-period make the DRPO problem a large-scale, multi-period, and strongly coupled mixed-integer nonlinear programming problem that is difficult to solve directly.

One thinking of solving the DRPO problem is to eliminate coupling constraints among periods, so the DRPO problem converts into a series of static reactive power optimization problems. The literature [1] puts forward the concept of dynamic reactive power optimization, it divides the whole optimization time into a handful of long periods, and each long period is divided into several short periods. Discrete variables are fixed value in long periods, which limits the action number of discrete devices by long periods’ division, the continuous variables adjust their values according to load change in short periods. However, the periods’ division is generally dependent on the operating experience of dispatchers, with uncertainty, and the optimization results are often not satisfactory. The paper [2] add the adjusting cost of equipment to the objective function to form a multi-objective optimization problem together with the power losses of the system. Therefore the optimization at different periods is not interrelated each other. However, it is difficult to determine the adjusting cost of equipment. Another way to solve the DRPO problem is to combine various algorithms to form a hybrid algorithm [3]. The DRPO problem is decomposed into two sub-problems: continuous variables optimization and discrete variables optimization. The interior-point method [4, 5] and the intelligent algorithm [6] are
used to iterate alternately to obtain the optimal solution of the problem, making full use of the respective advantages of the two algorithms.

Besides, researches improve solving efficiency by model transforming. In reference [7], the non-convex mixed-integer nonlinear programming for DRPO problem transforms into a convex mixed integer second-order cone programming problem, and dramatically reduces the complexity of the solution. In the literature [8], the DRPO problem is decomposed into three stages: relaxation solution, discrete variables solution and continuous variables solution, which simplifies the model and improves the solving efficiency of the problem.

In the traditional optimal reactive power flow (ORPF) method, the OLTC branches are represented by equivalent π circuit. So the formulated power flow equations contain the variables of transformer ratios and the square of transformer ratios multiplied by the square of the voltage magnitudes, which increases the dimension of power flow equations and difficulty of solving Jacobian and Hessian matrix. At the same time, the equivalent π circuit does not reflect the physical meaning of the OLTC branch and is difficult for people to understand, analyze, and remember.

The paper [9] proposes a rectangular form ORPF method based on the transformer dummy node model (TDNM). The OLTC branch is represented by an ideal transformer and its series impedance with a dummy node located between them. Instead of the transformer ratio variable, the voltage magnitude variable at the dummy node is used to express the transformation function of the ideal transformer. As a result, it simplifies the ORPF model and obtains a better calculation efficiency. However, under the rectangular form, a number, equal to the number of OLTC branches, of quadratic equality constraints were added to ensure that the voltage angles at two sides of the ideal transformer are identical. Meanwhile, the inequality constraints of the limits of transformer ratios become nonlinear from linear.

The effect of simplification by applying the TDNM to the ORPF method weakens to some extent. A new ORPF method in mixed polar form based on TDNM was proposed in the paper [10]. Compared with the conventional ORPF method based on equivalent π circuit, the power flow equations of the OLTC branches do not contain complicated items multiplied by the squares of variables, and drop from four-dimensional variables of the transformer equivalent π circuit to three-dimensional variables. It simplifies the ORPF model efficiently and has a higher computational efficiency.

The ORPF methods based on TDNM proposed in the above literature improve the solving efficiency but cannot be used to guide the operation of power systems, because they consider taps of transformers as continuous variables. In actual power systems, taps of transformers are discrete variables and commonly has a service life expressed by limits of action number. An excess of actions will not only accelerate the aging of equipment but also increase the work intensity of the operators and running cost. Therefore, this paper proposes a DRPO method based on TDNM, considering taps of transformers as discrete variables and has limits of action number. At the same time, the interior point method, together with the branch and bound method, were employed alternately to solve the problem for continuous variables and discrete variables, respectively. Test results prove that the proposed method is effective.

The rest of this paper is organized as follows. Section 2 describes the DRPO method based on the traditional equivalent π circuit. Section 3 proposes the DRPO method based on the TDNM. Section 4 analyzes the difference between the two DRPO methods. Section 5 discusses the performance of the proposed method, and conclusions drawn in section 6.

2. The DRPO method based on the traditional transformer equivalent π circuit

2.1. The traditional transformer equivalent π circuit

This paper considers the voltage magnitude changes of the OLTC branch, ignoring its phase changes. In the traditional transformer equivalent π circuit (see Figure 1), the OLTC branch is between bus $i$ and bus $j$, and the turn ratio of the OLTC branch is $1:k_j$. The admittance of the OLTC branch is $\frac{1}{\sqrt{V_i V_j}} = G_{i j} + jB_{i j}$, the voltage at bus $i$ and bus $j$ are $V_i = V_i e^{i \delta_i}$ and $V_j = V_j e^{i \delta_j}$, where $G_{i j}$ and $B_{i j}$ are the conductance and susceptance of the OLTC branch between bus $i$ and bus $j$, while $V_i$, $V_j$, and $\delta_i$, $\delta_j$ are the voltage magnitude and angle at bus $i$ and bus $j$. 
According to the power equation and the law of Kirchhoff, the active and reactive power flowing through the OLTC branch is:

\[ P_{ij} = V_i^2 k_{ij}^2 G_{ij} - V_j V_i k_{ij} (G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)) \]

\[ Q_{ij} = -V_i^2 k_{ij}^2 B_{ij} - V_j V_i k_{ij} (G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)) \]

\[ P_{t,ji} = V_j^2 G_{ij} - V_j V_i k_{ij} (G_{ij} \cos(\delta_j - \delta_i) + B_{ij} \sin(\delta_j - \delta_i)) \]

\[ Q_{t,ji} = -V_j^2 B_{ij} - V_j V_i k_{ij} (G_{ij} \sin(\delta_j - \delta_i) - B_{ij} \cos(\delta_j - \delta_i)) \]

2.2. The DRPO method based on the traditional transformer equivalent \( \Pi \) circuit

This paper mainly considers the optimization of the reactive power output of generators and taps of transformers, in which the reactive power output of generators are continuous variables, taps of transformers are discrete variables and have limits of action number. Assuming that \( N \) is the number of buses, \( M \) is the number of generators, \( U \) is the number of OLTC branches, and the optimization time, such as a day, is divided into \( T \) periods.

The mathematical model of the DRPO method is made up of the objective function, power flow equality constraints, and variables inequality constraints. The DRPO problem generally takes the minimum active power losses of the power system for a whole day as the objective function:

\[
\min \sum_{t=1}^{T} P_{loss} = \sum_{i=1}^{T} \sum_{i=1}^{N} P_{Gi} - \sum_{i=1}^{T} \sum_{i=1}^{N} P_{Di}^t
\]

Where \( P_{loss} \) is the active power losses of the system at the period of \( t \); \( P_{Gi}^t \) is the active power output of generator at bus \( i \) at the period of \( t \); \( P_{Di} \) is the active load at bus \( i \) at the period of \( t \).

The power flow equality constraints can be expressed as:

\[
P_{Gi} - P_{Di} - V_i^t \sum_{j \in S_i} V_j^t (G_{ij} \cos \delta_j^t + B_{ij} \sin \delta_j^t) + (V_i^t)^2 \sum_{j \in S_i} (k_j^t)^2 G_{ij} = 0
\]

\[
Q_{Gi} - Q_{Di} - V_i^t \sum_{j \in S_i} V_j^t (G_{ij} \sin \delta_j^t - B_{ij} \cos \delta_j^t) - (V_i^t)^2 \sum_{j \in S_i} (k_j^t)^2 B_{ij} = 0
\]

Where \( Q_{Gi} \) and \( Q_{Di} \) are the reactive power output of generator and the reactive load at bus \( i \) at the period of \( t \); \( V_i^t, V_j^t, \delta_i^t, \delta_j^t \) are the voltage magnitude and angle of bus \( i \) and bus \( j \) at the period of \( t \); \( G_{ij} \) and \( B_{ij} \) are the conductance and susceptance of the transmission line between bus \( i \) and bus \( j \); \( k_j^t \) is the turn ratio of the OLTC branch at the period of \( t \); \( S_i \) is the set of transmission lines; \( S_{ij} \) is the set of buses at the \( k_j \) side; \( S_{ji} \) is the set of buses at the other side of the OLTC branch; \( \delta_j^t \) is the phase difference between bus \( i \) and bus \( j \) at the period of \( t \); \( \delta_i^* = \delta_i^t - \delta_j^t \).

The upper and lower inequality constraints of the variables are as follows:
is a positive integer, represents the tap \( \delta \) in all phases. Thus the mode, the voltage magnitude at bus \( i \), and bus \( j \). The action number limit of the transformer tap between bus \( i \) and bus \( j \) satisfy the following inequality constraints:

\[
\sum_{i=1}^{T-1} |M^t_{ij} - M^{t-1}_{ij}| \leq k^\text{max}_y
\]

Where: \( k^\text{max}_y \) is the maximum action number of the transformer tap between bus \( i \) and bus \( j \) in all day.

3. The DRPO method based on the transformer dummy node model

3.1. The transformer dummy node model

The OLTC branch represented by an ideal transformer and its series admittance with a dummy node located between them, as shown in Figure 2. The ratio of the voltage magnitude at bus \( i \) to that at bus \( m \) is \( 1:k_y \), and the voltage phases at two sides of the ideal transformer are identical, that is \( V_m = k_y V_i \) and \( \delta_m = \delta_i \). In the following text of this paper, the voltage phase at bus \( m \) is replaced by the voltage phase at bus \( i \). Thus the model is simplified.

Because there are no power losses consumed on the ideal transformer, the power at two sides of the ideal transformer is equal. According to the power equation and the law of Kirchhoff, the active and reactive power flowing through the OLTC branch can be expressed as:

\[
P_{T_{ij}} = P_{T_{mj}} = V^2_m G_{T_{ij}} - V_m V_i G_{T_{ij}} \cos(\delta_i - \delta_j) + B_{T_{ij}} \sin(\delta_i - \delta_j)
\]

\[
Q_{T_{ij}} = Q_{T_{mj}} = -V^2_m B_{T_{ij}} - V_m V_i G_{T_{ij}} \sin(\delta_i - \delta_j) - B_{T_{ij}} \cos(\delta_i - \delta_j)
\]

\[
P_{T_{ji}} = P_{T_{jm}} = V^2_j G_{T_{ji}} - V_j V_i G_{T_{ji}} \cos(\delta_i - \delta_j) + B_{T_{ji}} \sin(\delta_i - \delta_j)
\]

\[
Q_{T_{ji}} = Q_{T_{jm}} = -V^2_j B_{T_{ji}} - V_j V_i G_{T_{ji}} \sin(\delta_i - \delta_j) - B_{T_{ji}} \cos(\delta_i - \delta_j)
\]

3.2. The DRPO method based on the transformer dummy node model

The objective function of this method is also equation (5). The constraints include power flow equality equations (18) - (19), upper and lower limits of variables inequality equations (8) - (10), and the inequality constraints (20) of the voltage magnitude at the dummy node, the equality constraints (21) about the relationship between turn ratios of adjustable transformers and the voltage magnitude at both sides of the ideal transformer, and the action number limits equation (13) of the transformer taps. The power flow equations are as follows:
\[ P_i^m - P_i^{m*} - V_i' \sum_{j \in S_i} V_j' (G_{ij} \cos \delta_j + B_{ij} \sin \delta_j) + (V_i^m)^2 \sum_{j \in S_i} G_{ij} = 0 \]}

\[ Q_i^m - Q_i^{m*} - V_i' \sum_{j \in S_i} V_j' (G_{ij} \sin \delta_j + B_{ij} \cos \delta_j) - (V_i^m)^2 \sum_{j \in S_i} B_{ij} = 0 \]

Here \( V_i^m \) is the voltage magnitude at the dummy node \( m \) at the period \( t \).

The upper and lower limits of the voltage amplitude at the dummy node \( m \) is as follows:

\[ \frac{k_v}{V_i^m} \leq V_i' \leq \frac{k_o}{V_i^m} \]

The relationship between turn ratios of adjustable transformers and the voltage magnitude at both sides of the ideal transformer is as follows:

\[ V_i^m / V_i' = \frac{k_v}{k_o} + M_j k_{step} \]

4. Comparison of the two DRPO methods based on different transformer models

Compare the two DRPO methods based on different transformer models, the numbers of inequality constraints of the two methods are equal, and all constraints are linear, which has little impact on the solution efficiency. The difference in equality constraints is the main factor that affects the solving speed of the two methods. The equality constraints of the DRPO method based on transformer equivalent \( \Pi \) circuit (referred as 'the traditional method') include equations (6)-(7) and (12), the equality constraints of the DRPO method based on TDNM (referred as 'the proposed method') include equations (18)-(19) and (21). According to \( k_{ij}^* = V_i^m / V_i' \), Equations (6)-(7) and (12) can deduce equations (18)-(19) and (21).

So the two methods are mathematically equivalent.

**Table 1.** The expressions of active power flowing through the OLTC branch under the two methods

|                         | The first item | The second items | Number of variables/ Dimension of equation |
|-------------------------|----------------|------------------|--------------------------------------------|
| The traditional method: Equation (6) | \((V_i')^2(k_{ij}^*)^2G_{ij}\) | \(-V_i'V_j'(G_{ij}\cos \delta_j + B_{ij}\sin \delta_j)\) | 5/4 |
| The proposed method: Equation (18) | \((V_i')^2G_{ij}\) | \(-V_i'V_j'(G_{ij}\cos \delta_j + B_{ij}\sin \delta_j)\) | 4/3 |

Power flow equations (18)-(19) have fewer variables and lower dimension than equations (6)-(7). The difference of power flow equations between the two methods reflects in the expressions of power flowing through the OLTC branches. If the branch between bus \( i \) and bus \( j \) is an OLTC branch, the expressions of active power flowing through the OLTC branch under the two methods are shown in Table 1. The proposed method has four variables, one less than five variables of the traditional method. The dimension of equation (18) is three, one less than four, the dimension of equation (6). In the same way, the active power flow equation at bus \( j \) in the proposed method has fewer variables and lower dimension than the traditional method. Similarly, the reactive power flow equation (19) has fewer variables and lower dimension than equation (7). If there is a total number of \( U \) on-load tap changers, and total \( T \) periods, the proposed method could reduce 2UT variables than the traditional method.

In this paper, the problem is decomposed into two sub-problems: continuous variables optimization and discrete variables optimization. The interior-point method is widely preferred to solve continuous subproblem. The number of non-zero elements in Jacobian and Hessian matrices of the expressions of power flowing through OLTC branches in the proposed method is less than that in the traditional method, so its calculation amount decreases. Take the losses expressions consumed on the OLTC branch in the power flow equations of the IEEE4 bus system as an example, (see Figure 3 and Figure 4), the number
of non-zero elements in Jacobian and Hessian matrixes of the proposed method is four and nine less than the traditional method respectively.

| \( \delta V \) | \( \delta V_1 \) | \( \delta V_2 \) | \( \delta V_3 \) | \( \delta V_4 \) | \( K \) |
|----------------|----------------|----------------|----------------|----------------|-------------|
| \( \delta Q_1 / \delta x \) | J | J | J | J | J |
| \( \delta Q_2 / \delta x \) | J | J | J | J | J |
| \( \delta P_1 / \delta x \) | J | J | J | J | J |
| \( \delta Q_1 / \delta x \) | J | J | J | J | J |

(a)

| \( \delta V \) | \( \delta V_1 \) | \( \delta V_2 \) | \( \delta V_3 \) | \( \delta V_4 \) | \( \delta V_{\text{tn}} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \delta P_1 / \delta x \) | J | J | J | J | J |
| \( \delta Q_2 / \delta x \) | J | J | J | J | J |
| \( \delta P_2 / \delta x \) | J | J | J | J | J |
| \( \delta Q_2 / \delta x \) | J | J | J | J | J |

(b)

Figure 3. For IEEE4 bus system, the Jacobian matrix of expressions of active power flowing through the OLTC branches of the two DRPO methods. (a) The traditional method. (b) The proposed method.

| \( \delta V \) | \( \delta V_1 \) | \( \delta V_2 \) | \( \delta V_3 \) | \( \delta V_4 \) | \( K \) |
|----------------|----------------|----------------|----------------|----------------|-------------|
| \( \delta_1 \) | H | H | H | H | H |
| \( V_1 \) | H | H | H | H | H |
| \( \delta_2 \) | H | H | H | H | H |
| \( V_2 \) | H | H | H | H | H |
| \( \delta_3 \) | H | H | H | H | H |
| \( V_3 \) | H | H | H | H | H |
| \( \delta_4 \) | H | H | H | H | H |
| \( V_4 \) | H | H | H | H | H |

(a)

| \( \delta V \) | \( \delta V_1 \) | \( \delta V_2 \) | \( \delta V_3 \) | \( \delta V_4 \) | \( \delta V_{\text{tn}} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \delta_1 \) | H | H | H | H | H |
| \( V_1 \) | H | H | H | H | H |
| \( \delta_2 \) | H | H | H | H | H |
| \( V_2 \) | H | H | H | H | H |
| \( \delta_3 \) | H | H | H | H | H |
| \( V_3 \) | H | H | H | H | H |
| \( \delta_4 \) | H | H | H | H | H |
| \( V_4 \) | H | H | H | H | H |

(b)

Figure 4. For IEEE4 bus system, the Hessian matrix of expressions of active power flowing through the OLTC branches of the two DRPO methods. (a) The traditional method. (b) The proposed method.

It does not contain turn ratio and the square of turn ratio in the proposed method, which is more concise than the traditional method. Moreover, the voltage magnitude at the two sides of the ideal transformer reflects directly the physical significance of the transformer, which makes the proposed method easier to understand, analyze, and remember.

5. Numerical test and discussion

5.1. Test systems and environment

This paper assumes that there are 24 periods in one day, and load at per period follows the load curve shown in Figure 5. Take IEEE 4, 14, 30 bus systems [11] as examples to discuss the performance of the two methods. All the values of test systems calculate in the form of per-unit. The voltage magnitude of all buses are between 0.9 and 1.1, and the turn ratios in per-unit of the OLTC branches are all discrete values, such as 0.9, 0.95, 1.0, 1.05, 1.1.

All the test systems simulate in an HP PC-compatible computer, whose CPU is an Intel Core i5-6500, 3.20GHz, four cores, and its RAM is 8GB. All methods were modeled using the commercial software GAMS (General Algebraic Modeling System). The two DRPO methods were modeled as mixed-integer nonlinear programming problems while interior point method solver and SBB solver were respectively employed to solve the problem for continuous variables and discrete variables. The convergence Gap of integer optimization is 0.01.
Figure 5. The curve of load in per-unit at 24 periods.

5.2. Effectiveness analysis of the proposed method

Table 2 lists the results of IEEE4, 14, and 30 bus systems under the two DRPO methods. The optimal value of active power losses calculated by the two methods is the same with each other, which proves that the two methods are equivalent. Compared with the traditional method, the proposed method has a smaller iteration number, shorter calculation time.

|        | The traditional method | The proposed method |
|--------|------------------------|---------------------|
|        | Optimal value(per-unit)|Calculation time(s) | Iteration number |
| IEEE4  | 0.2301                 | 0.984               | 518              |
| IEEE14 | 1.0948                 | 5.343               | 1088             |
| IEEE30 | 0.2251                 | 18.469              | 1995             |

Table 3. The reduction of active power losses for IEEE30 bus system by using the proposed method.

| The fixed turn ratio value | Active power losses under fixed turn ratio | Percents of active power losses reduction by using the proposed method |
|---------------------------|------------------------------------------|---------------------------------------------------------------------|
| 0.90                      | 0.4312                                   | 47.8%                                                               |
| 0.95                      | 0.2833                                   | 20.5%                                                               |
| 1.00                      | 0.2261                                   | 0.44%                                                               |
| 1.05                      | 0.2443                                   | 7.86%                                                               |
| 1.10                      | 0.3562                                   | 36.8%                                                               |

Figure 6. Taps positions of transformers for the IEEE30 bus system.

After optimization by using the proposed method, the active power losses reduce than before optimization. Taking IEEE30 bus system as an example (see Table 3), assuming that all turn ratios are set 0.9 before optimization, then the active power losses in per-unit of the system are 0.4312, while the
active power loss in per-unit by using the proposed method is 0.2251, decreases by 47.8 percents than before optimization. In the same way, the proposed method can also reduce the active power losses of the system when the transformer ratio is other fixed values, such as 0.95, 1.0, 1.05, and 1.1.

Figure 6 shows the tap positions of each transformer of the IEEE30 bus system after optimization using the proposed method. Transformer taps adjust their positions according to changes of load, and all transformer taps satisfy limit of action number.

6. Conclusion
This paper proposes a dynamic reactive power optimization method based on transformer dummy node model, considering taps of transformers as discrete variables and have limit of action number. Compared with the traditional method, the proposed method has fewer variables and lower dimension of equations. Besides, the number of non-zero elements in Jacobian and Hessian matrices of the expressions of power flowing through OLTC branches in the proposed method is less than that in the traditional method, which decreases the calculation amount. The test results of IEEE4, 14, 30 bus system show that the proposed method is mathematically equivalent to the traditional method, and has higher calculation efficiency. The proposed method can effectively optimize the reactive power distribution of power systems, improve the safe and economic operation level of the power grid.

7. References
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