A REVIEW OF ELLIPTICAL AND DISC GALAXY STRUCTURE, AND MODERN SCALING LAWS

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Condensed / Shortened version

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ABSTRACT

A century ago, in 1911 and 1913, Plummer and then Reynolds introduced their models to describe the radial distribution of stars in ‘nebulae’. This article reviews the progress since then, providing both an historical perspective and a contemporary review of the stellar structure of bulges, discs and elliptical galaxies. The quantification of galaxy nuclei, such as central mass deficits and excess nuclear light, plus the structure of dark matter halos and cD galaxy envelopes, are discussed. Issues pertaining to spiral galaxies including dust, bulge-to-disc ratios, bulgeless galaxies, bars and the identification of pseudobulges are also reviewed. An array of modern scaling relations involving sizes, luminosities, surface brightnesses and stellar concentrations are presented, many of which are shown to be curved. These ‘redshift zero’ relations not only quantify the behavior and nature of galaxies in the Universe today, but are the modern benchmark for evolutionary studies of galaxies, whether based on observations, N-body-simulations or semi-analytical modelling. For example, it is shown that some of the recently discovered compact elliptical galaxies at $1.5 < z < 2.5$ may be the bulges of modern disc galaxies.

Subject headings: galaxy structure — galaxy scaling relations — galaxy elliptical — galaxy spiral — galaxy compact — galaxy cD, halos — galaxy nuclei — galaxy central mass deficits — galaxy excess nuclear light — galaxy bulge-disc ratios — galaxy bars — galaxy pseudobulges — galaxy bulgeless — galaxy dust — Sérsic model — core-Sérsic model — Einasto model — dark matter halos.

1. Introduction

For the last century astronomers have been modelling the structure of ‘nebulae’, and here we focus on those external to the Milky Way. A key activity performed by many astronomers, past and present, is the categorisation of these galaxies (Sandage 2005) and the quantification of their physical properties. How big are they? How bright are they? What characteristics distinguish or unite apparent subpopulations? Answers to such questions, and the establishment of “scaling relations” between two or more galactic properties provides valuable insight into the physical mechanisms that have shaped galaxies.

Understanding how galaxies form, increasingly through the use of simulations and semi-analytic modelling (e.g. Cole 1991; White & Frenk 1991; Kauffmann et al. 1993, 2003; Avila-Reese et al. 1998; Cole et al. 2000; de Lucia et al. 2006; Bower et al. 2006; Kauffmann et al. 2004; Di Matteo et al. 2005; Croton et al. 2006; Naab et al. 2006; Nipoti et al. 2006; Covington et al. 2011; Guo et

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as elucidated by Driver (2011). Not surprising, our knowledge of galaxies is best in the nearby Universe — out to distances typically measured in megaparsecs rather than by redshift $z$ — where galaxy structures can be reasonably well resolved. The properties of these galaxies provide the $z = 0$ benchmark used in the calibration of galactic evolutionary studies — both observed and simulated.

Popular scaling relations involving global galaxy parameters such as size, surface brightness, luminosity and concentration are reviewed here. As we shall see, many bivariate distributions, which are frequently assumed to be linear, are often only approximately so over a restricted luminosity range. For example, it may come as a surprise for many too learn that the useful Kormendy (1977b) relation is only the tangent to the bright arm of a continuous but curved effective radius-(surface brightness) relation which unifies dwarf and giant elliptical galaxies (section 3.2.3). Similarly, the Faber-Jackson (1976) relation with a slope of 4 represents the average slope over a restricted luminosity-(velocity dispersion) distribution, in which the slope is 2 rather than 4 at lower luminosities (section 3.3.3). Knowing these trends, the relation may indicate a range of galaxy types (e.g. Penny & Conselice 2008, and references therein).

This article has been structured into four main sections. Section 1 provides this general overview plus a further review and introduction to galaxies on the Hubble-Jeans sequence. Included are diagrams showing the location of dynamically hot stellar systems in the mass-size and mass-density plane, revealing that some high-$z$ galaxies have properties equivalent to the bulges of local disc galaxies. Section 2 provides an historical account of how the radial distribution of stars in elliptical galaxies have been modelled, and the iterative steps leading to the development of the modern core-Sérsic model (section 2.2). Subsections cover the Sérsic model (section 2.1), its relation and applicability to dark matter halos (section 2.3), partially-depleted galaxy cores (section 2.2.4), excess nuclear light (section 2.3) and excess light at large radii in the form of halos or envelopes around giant elliptical galaxies (section 2.4). Section 3 presents and derives a number of elliptical galaxy scaling relations pertaining to the main body of the galaxy. From just two linear relations which unite the faint and bright elliptical galaxy population (section 3.1), a number of curved relations are derived (section 3.2). Several broken relations, at $M_B \approx -20.5$ mag, are additionally presented in section 3.3. For those interested in a broader or different overview of elliptical galaxies, some recent good reviews include Renzini (2006), Cecil & Rose (2007), Ciotti (2009) and Lisker (2009). Finally, the latter third of this paper is tied up in section 4 which contains a discussion of the light profiles of disc galaxies and their bulge-disc decomposition. Also included are subsections pertaining to dust (section 1.2), the difficulties with identifying pseudobulges (section 3.3), potential bulgeless galaxies (section 4.3) and methods to model bars (section 4.5). Throughout the article references to often overlooked discovery or pioneer papers are provided.

### 1.1. Early Beginnings

Looking out into the Milky Way arced across our night sky, the notion that we are residents within a pancake-shaped galaxy seems reasonable to embrace. Indeed, back in 1750 Thomas Wright also conjectured that we reside within a flat layer of stars which is gravitationally bound and rotating about some centre of mass. However, analogous to the rings of Saturn, he entertained the idea that the Milky Way is comprised of a large annulus of stars rotating about a distant centre, or that we are located in a large thin spherical shell rotating about some divine centre (one of the galactic poles). While he had the global geometry wrong, he was perhaps the first to speculate that faint, extended nebulae in the heavens are distant galaxies with their own (divine) centers.

As elucidated by Hoskin (1970), it was Immanuel Kant (1755), aware of the elliptically-shaped nebulae observed by Maupertuis, and
working from an incomplete summary of Wright (1750) that had been published in a Hamburg Journal, who effectively introduced the modern concept of disc-like galactic distributions of stars — mistakenly crediting Wright for the idea.

Using his 1.83 m “Leviathan of Parsonstown” metal reflector telescope in Ireland, Lord William Henry Parsons, the 3rd Earl of Rosse, discovered 226 New General Catalogue (NGC: Dreyer 1888) and 7 Index Catalogue (IC: Dreyer 1895, 1908) objects (Parsons 1878). Important among these was his detection of spiral structure in many galaxies, such as M51 which affectionately became known as the whirlpool galaxy.

Further divisions into disc (spiral) and elliptical galaxy types followed (e.g. Wolf 1908; Knox Shaw 1915; Curtis 1918; Reynolds 1920; Hubble 1926 and Shapley & Swope (1924) and Shapley (1928) successfully identified our own Galaxy’s (gravitational) center towards the constellation Sagittarius (see also Seares 1928).

With the discovery that our Universe contains Doppler shifted ’nebulae’ that are expanding away from us (de Sitter 1917; Slipher 1917; see also Friedmann 1922, Lundmark 1924 and the reviews by Kragh & Smith 2003 and Shaviv 2011), in accord with a redshift-distance relation (Lemaitre 1927, Robertson 1928, Humason 1929, Hubble 1929) — i.e. awareness that some of the “nebulae” are external objects to our galaxy — came increased efforts to categorise and organise these different types of “galaxy”. As noted by Sandage (1924, 1925), Sir James Jeans (1928) was the first to present the (tuning fork)-shaped diagram that encapsulated Hubble’s (1926) early-to-late type galaxy sequence, a sequence which had been in-inspired in part by Jeans (1919) and later popularised by Hubble (1936a; see Block et al. 2004). Quantifying the physical properties of galaxies along this sequence, with increasing accuracy and level of detail, has occupied many astronomers since. Indeed, this review addresses aspects related to the radial concentration of stars in the elliptical and disc galaxies which effectively define the Hubble-Jeans sequence. Irregular galaxies are not discussed here.

1.2. The modern galaxy

For reasons that will become apparent, this review uses the galaxy notation of Alan Sandage and Bruno Binggeli, in which dwarf elliptical (dE) galaxies are the faint extension of ordinary and luminous elliptical (E) galaxies, and the dwarf spheroidal (dSph) galaxies — prevalent in our Local Group (Grebel 2001) — are found at magnitudes fainter than $M_B \approx -13$ to $-14$ mag ($\approx 10^8 M_\odot$ in stellar mass; see Figure 1). Figure 1 reveals a second branch of elliptically-shaped object stretching from the bulges of disc galaxies and compact elliptical (cE) galaxies to ultra compact dwarf (UCD) objects (Hilker et al. 1999; Drinkwater et al. 2000; Norris & Kannappan 2011 and references therein). A possible connection is based upon the stripping of a disc galaxy’s outer disc to form a cE galaxy (Nieto 1990; Bekki et al. 2001b; Graham 2002; Chilingarian et al. 2009) and through greater stripping of the bulge to form a UCD (Zinnecker et al. 1988; Freeman 1990; Bassino et al. 1994; Bekki 2001a). It is thought that nucleated dwarf elliptical galaxies may also experience this stripping process, giving rise to UCDs.

While the identification of local spiral galaxies is relatively free from debate, the situation is not so clear in regard to elliptically-shaped galaxies. The discovery of UCDs, which have sizes and fluxes intermediate between those of galaxies and (i) the nuclear star clusters found at the centres of galaxies and (ii) globular clusters (GCs: e.g. Hasan et al. 2005; Brodie & Strader 2006), led Forbes & Kroupa (2011) to try and provide a modern definition for what is a galaxy (see also Tollerud et al. 2011). Only a few years ago there was something of a divide between GCs and UCDs — all of which had sizes less than $\sim 30$ pc — and galaxies with sizes greater than 120 pc (Gilmore et
Fig. 1.— Left panel: The radius containing half of each object’s light, $R_{1/2}$ (as seen in projection on the sky), is plotted against each object’s stellar mass. Open circles: dwarf elliptical (dE) and ordinary elliptical (E) galaxies from Binggeli & Jerjen (1998), Caon et al. (1993), D’Onofrio et al. (1994) and Forbes et al. (2008). Filled circles: Bulges of disc galaxies from Graham & Worley (2008). Shaded regions adapted from Binggeli et al. (1984, their figure 7), Dabringhausen et al. (2008, their figure 2), Forbes et al. (2008, their figure 7), Misgeld & Hilker (2011, their figure 1). The location of the so-called “compact elliptical” (cE) galaxies is shown by the rhombus overlapping with small bulges. The location of dense, compact, $z = 1.5$ galaxies, as indicated by Damjanov et al. (2009, their figure 5), is denoted by the dashed boundary overlapping with luminous bulges. Right panel: Stellar mass density within the volume containing half each object’s light, $\rho_{1/2}$, versus stellar mass. The radius of this volume was taken to equal $4/3 \times R_{1/2}$ (Ciotti 1991; Wolf et al. 2010).
al. 2007). However, as we have steadily increased our celestial inventory, objects of an intermediate nature have been found (e.g. Ma et al. 2007, their Table 3), raising the question asked by Forbes & Kroupa for which, perhaps not surprisingly, no clear answer has yet emerged. While those authors explored the notion of a division by, among other properties, size and luminosity, they did not discuss how the density varies. As an addendum of sorts to Forbes & Kroupa (2011), the density of elliptically-shaped objects is presented here in Figure 1b. This is also done to allow the author to wave the following flag.

Apparent in Figure 1b, but apparently not well recognised within the community, is that the bulges of disc galaxies can be much denser than elliptical galaxies. If the common idea of galaxy growth via the accretion of a disc, perhaps from cold-mode accretion streams, around a pre-existing spheroid is correct (e.g. Navarro & Benz 1991; Steinmetz & Navarro 2002; Birnboim & Dekel 2003; see also Conselice et al. 2011 and Pichon et al. 2011), then one should expect to find dense spheroids at high-$z$ with $10^{10}$–$10^{11} M_{\odot}$ of stellar material, possibly surrounded by a faint (exponential) disc which is under development.

It is noted here that the dense, compact early-type galaxies recently found at redshifts of 1.4–2.5 (Daddi et al. 2005; Trujillo et al. 2006) display substantial overlap with the location of present day bulges in Figure 1b, and that the merger scenarios for converting these compact high-$z$ galaxies into today’s elliptical galaxies are not without problems (e.g. Nipoti et al. 2009; Nair et al. 2011). It is also noted that well-developed discs and disc galaxies are rare at the redshifts where these compact objects have been observed alongside normal-sized elliptical galaxies. Before trying to understand galaxy structure at high-redshift, and galaxy evolution — themes not detailed in this review — it is important to first appreciate galaxy structures at $z = 0$ where observations are easier and local benchmark scaling relations have been established.

2. Elliptical Galaxy Light Profiles

Over the years a number of mathematical functions have been used to represent the radial distribution of stellar light in elliptical galaxies, i.e. their light profiles. Before getting to de Vaucouleurs’ $R^{1/4}$ model in the following paragraph, it seems apt to first quickly mention some early competitors. Although Plummer’s (1911) internal-density model was developed for the nebulae which became known as globular clusters, because of its simplicity it is still used today by some researchers to simulate elliptical galaxies, even though, it should be noted, no modern observers use this model to describe the radial distribution of light in elliptical galaxies. Reynold’s (1913) surface-density model, sometimes referred to as Hubble’s (1930) model or the Reynold-Hubble model, was used to describe the nebula which became known as elliptical galaxies. It has an infinite mass and is also no longer used by observers today. The modified Hubble model (Rood et al. 1972), which also has an infinite mass, is also still sometimes used by simulators, even though, again, observers do not use this model anymore. Oemler’s (1976) exponentially-truncated Hubble model, known as the Oemler-Hubble model, is also not used to represent the observed stellar distribution in elliptical galaxies because it too, like its predecessors, was simply an approximation applicable over a limited radial range, as noted by King (1978). It is interesting to note that up until the 1980s, departures at large radii from the Reynold’s model were attributed to tidal-stripping by external gravitational potentials. That is, for three quarters of a century, Reynold’s model — originally developed from low-quality data for one galaxy — was generally thought to describe the original, undisturbed stellar distribution in elliptical galaxies.

de Vaucouleurs’ (1948, 1953) $R^{1/4}$ surface-density model had traction for many years, in part due to de Vaucouleurs (1959) arguing that it fits better than the Reynold’s model used by Hubble, — a point re-iterated by Kormendy (1977a) and others — and the revelation that it fits the radially-extended data for NGC 3379 exceedingly well (de Vaucouleurs & Capaccioli 1979). Hodge (1961a,b) had however revealed that de Vaucouleurs’ model was inadequate to describe faint elliptical galaxies and Hodge (1963, 1964), in addition to King (1962) noted that the 3-parameter King model, with its flatter inner pro-

\footnote{King (1962) also noted that his model failed to fit the inner region of bright elliptical galaxies.}
file and steeper decline at large radii, did a better job. For a time, King’s (1962, 1966) model became popular for describing the light distribution in faint elliptical galaxies, at least until the exponential model — also used for the discs of spiral galaxies — was noted to provide a good description of some dwarf elliptical galaxies (Hodge 1971; Faber & Lin 1983; Binggeli et al. 1984) and that these galaxies need not have experienced any tidal truncation (a prescription of the King model with its tidal radius parameter). Lauer (1984, 1985) additionally showed that King’s modified isothermal model, with its flat inner core, was inadequate to describe the deconvolved light-profiles of ordinary elliptical galaxies with “cores”, i.e. galaxies whose inner light profile displays a nearly flat core. King’s model does however remain extremely useful for studies of star clusters, globular clusters, dwarf spheroidal galaxies and galactic satellites which, unlike ordinary elliptical galaxies, can have flat cores in their inner surface brightness profile.

Today, the model of choice for describing nearby (and distant) dwarf and ordinary elliptical galaxies is Sérsic’s (1963, 1968) generalisation of de Vaucouleurs’ $R^{1/4}$ model to give the $R^{1/n}$ surface-density model (section 2.1). This model reproduces the exponential model when $n = 1$ and de Vaucouleurs’ model when $n = 4$; it can thus describe the main body of faint and luminous elliptical galaxies. The key advantage that this model has is (i) its ability to describe the observed stellar distributions that have a range of central concentrations (known to exist since at least Reaves 1956) and (ii) it provides a very good description of the data over (almost) the entire radial extent. Indeed, departures in the light profile from a well-fit Sérsic’s model invariably signal the presence of additional features or components, rather than any failing of the model. Expanding upon the Sérsic model, the core-Sérsic model (section 2.2) is nowadays used to quantify those galaxies with “cores”.

Although referring to the King model, the following quote from King (1966) seems particularly insightful “... de Vaucouleurs’ law appears to refer to a particular central concentration and should be appropriate only for galaxy profiles that have that concentration.” While noted by others, such as Oemler (1976), Capaccioli (1985), Michard (1985) and Schombert (1986), some three decades elapsed before the relevance of King’s remark to elliptical galaxies re-surfaced — albeit slowly at first — in the 1990s. Indeed, de Vaucouleurs’ useful, albeit limited, $R^{1/4}$ model was referred to as a “law” for nearly half a century. However we are now more keenly aware that (even normal) elliptical galaxies possess a range of central concentrations: concentrations which are well quantified in terms of the exponent $n$ in Sérsic’s $R^{1/n}$ model (see Trujillo, Graham & Caon 2001; Graham, Trujillo & Caon 2001). Although, it should be confessed that one can still encounter papers which use $R^{1/4}$ model parameters alongside some model-independent measure of galaxy concentration, unaware of the inconsistency arising from the fact that every $R^{1/4}$ model actually has exactly the same level of concentration.

Before introducing the equation for Sérsic’s model in the following section, it is pointed out that in addition to modelling what can be considered the main body of the galaxy, one can also find excess stellar light at (i) small radii in the form of nuclear (i.e. centrally located) discs and dense nuclear star clusters (section 2.3) and also at (ii) large radii in the form of halos or envelopes in cD and central cluster galaxies (section 2.4). As briefly noted above, deficits of stellar flux at the cores of massive galaxies are also observed, and a model to quantify these stellar distributions, relative to the outer non-depleted light profile, is described in section 2.2. While non-symmetrical components in elliptical galaxies can also exist, they are not addressed here given the focus on well-structured systems. Somewhat random, non-symmetrical components may be a sign of a disturbed morphology (see E.Barton’s Chapter in this volume), of on-going non-uniform star formation (see S.Boissier’s Chapter in this volume) or gravitationally-induced tidal features from external forces.

2.1. Sérsic’s model

José Sérsic’s (1963, 1968) $R^{1/n}$ model, which was introduced in Spanish, describes how the projected surface-intensity $I$ varies with the projected...
radius $R_{\text{e}}$, such that
\[
I(R) = I_{\text{e}} \exp \left\{ -b_n \left[ \left( \frac{R}{R_{\text{e}}} \right)^{1/n} - 1 \right] \right\}
\] (1)

and $I_{\text{e}}$ is the intensity at the ‘effective’ radius $R_{\text{e}}$ that encloses half of the total light from the model (Ciotti 1991; Caon et al. 1993). The term $b_n$ ($\approx 1.9992n - 0.3271$ for $0.5 < n < 10$, Capaccioli 1989) is not a parameter but is instead dependent on the third model parameter, $n$, that describes the shape, i.e. the concentration, of the light profile.\(^8\) The exact value of $b_n$ is obtained by solving the equation $\Gamma(2n) = 2\gamma(2n, b_n)$ where $\gamma(2n,x)$ is the incomplete gamma function and $\Gamma$ is the (complete) gamma function (Ciotti 1991). Useful Sérsic related expressions have been presented in Ciotti (1991), Simonneau & Prada (2004) and Ciotti & Bertin (1999), while Graham & Driver (2005) provide a detailed review of Sérsic’s model plus associated quantities and references to pioneers of this model.

The relation between the effective surface brightness ($\mu_e = -2.5 \log I_{\text{e}}$) and the central surface brightness ($\mu_0 = -2.5 \log I_0$) is given by the expression
\[
\mu_e = \mu_0 + 1.086b_n
\] (2)

where we have dropped the subscript $n$ from the term $b_n$ for simplicity, while
\[
\langle \mu_e \rangle = \mu_e - 2.5 \log [e^n \Gamma(2n) / b^{2n}]
\] (3)
gives the difference between the effective surface brightness and the mean effective surface brightness ($\langle \mu_e \rangle$) within $R_{\text{e}}$. Figure 2 shows the behaviour of the Sérisc model.

Uniting CCD data with wide and deep photographic images, Caon et al. (1990, 1993, 1994) revealed that the Sérisc $R^{1/n}$ model provided a remarkably good description to the stellar distribution over a large radial range, down to surface brightnesses of $\sim 28$ $B$-mag arcsec$^{-2}$, for the early-type galaxies brighter than $M_B = -18$ mag in the Virgo cluster. This work was in essence an expansion of de Vaucouleurs & Capaccioli’s (1979) study of NGC 3379 which is very well-fit with $n = 4$. Different galaxies were discovered by Caon et al. (1993) to be equally well fit, but required different values of $n$ (see also Bertin et al. 2002).

Importantly, Caon et al. (1993) additionally showed that a correlation existed between stellar concentration, i.e. the Sérsic index $n$, and (model-independent) galaxy size that was not due to parameter coupling in the Sérisc model (see also Trujillo et al. 2001, their section 2). One of the commonly overlooked implications of this result is that $R^{1/4}$, and similarly Petrosian (1976), magnitudes, sizes and surface brightnesses are systematically in error as a function of galaxy concentration (Graham et al. 2005; Hill et al. 2011). That is, application of a model which fails to adequately capture the range of stellar distributions will result in parameters which are systematically biased as a function of galaxy mass. For example, fitting an $R^{1/4}$ model to elliptical galaxies which are actually described by an $R^{1/n}$ model with $n$ less than and greater than 4 will yield sizes and luminosities which are, respectively, greater than and less than the true value (e.g. Binggeli & Cameron 1991; Trujillo et al. 2001; Brown et al. 2003). Similarly, fitting an exponential model to bulges that are best described by an $R^{1/n}$ model with $n$ less than and greater than 1 will yield sizes and luminosities which are, respectively, greater than and less than the true value (e.g. Graham 2001). Obviously one does not want to fine tune their galaxy simulations to match scaling relations that contain systematic biases due to poor measurements, and observers are therefore busy fitting $R^{1/n}$ models these days.

A good approximation to the internal-density profile associated with Sérisc’s model, i.e. with its deprojection, was introduced by Prugniel & Simien (1997). Useful expressions for the dynamics, gravitational potential and forces of this model have been developed by Trujillo et al. (2002), Terzić & Graham (2005) and Terzić & Sprague (2007). Somewhat more complex than the early light-profile models, such expressions can, importantly, accommodate a range of concentrations, rather than only varying one scale radius and one scale density. Such a model is vital if one wishes to properly simulate and understand the mass spectrum of elliptical galaxies, whose Sérisc index $n$ increases with stellar mass. While Graham & Driver’s (2005) review stated that “No attempt has been made here to show the numerous scientific advances engendered via appli-

\(^8\)Ellipticity gradients result in a different Sérisc index for the major- and minor-axis, as noted by Caon et al. (1993) and later quantified by Ferrari et al. (2004).
cification of the $R^{1/n}$ model”, this article reveals how Sérsic’s model, and the core-Sérsic model 0 (subsection 2.2), have become key in unifying and understanding the galaxies around us.

Like the majority of surface- and internal-density models from the last century, the Sérsic function is an empirical model created to match data rather than developed from theory, and as such we should be cautious before calling it a law. Attempts to find a physical explanation for de Vaucouleurs’ model yielded results which helped to keep it in vogue. Dissipational models have long been touted for producing $R^{1/4}$ profiles (e.g. Larson 1969, 1974), and in the 1980s papers based on dissipationless N-body simulations of a cold clumpy collapse or the merger of disc galaxies also claimed to finally produce $R^{1/4}$ (and also Reynolds) profiles (e.g. van Albada 1982; McGlynn 1984; Carlberg et al. 1986; Barnes 1988). However a closer inspection reveals clear departures from the $R^{1/4}$ profile, with the simulated profiles better described by an $R^{1/n}$ model with $n < 4$. Obviously their inability (or perhaps lack of desire, although see Farouki, Shapiro & Duncan 1983 whose non-homologous merger remnants were initially criticised by $R^{1/4}$ aficionados) to create the range of stellar concentrations now observed in elliptical galaxies highlights a limitation of these early works. Nonetheless, these pioneering studies have led to N-body simulations by Nipoti et al. (2006) and Aceves et al. (2006) — and Farouki et al. (1983), whose results with a smaller force softening appeared years ahead of their time — have now recovered a range of Sérsic profile shapes for gravitational collapses in a dark matter halo and for disc galaxy mergers, respectively.

Given the empirical nature of Sérsic’s $R^{1/n}$ model, Hjorth & Madsen (1995) revealed how dissipationless merging and violent relaxation provided a physical explanation for the departure from the homologous $R^{1/4}$ model. Other works have explained how the quasi-constant specific entropy associated with the post violent-relaxation stage of elliptical galaxies results in the observed mass-dependent range of stellar concentrations in elliptical galaxies (Gerbal et al. 1997; Lima Neto 1999; Márquez et al. 2001).

It does not seem too unreasonable to speculate that elliptical galaxies, whether built by near-monolithic collapse, collisions of disc galaxies, wet or dry mergers, appear to eventually experience the same force(s) of nature that results in their radial stellar distribution depending on the total stellar mass. That is, it may not matter how the mass was accumulated into an elliptical galaxy, once it becomes a dynamically-heated, bound stellar-system, it appears to eventually obey certain universal scaling relations (see section 3).

It is interesting to note that Sérsic actually introduced his model as a way to parameterise disc galaxies which he thought were comprised...
of differing ratios of a disc plus an $R^{1/4}$-bulge. His model was not initially intended to fit elliptical galaxies, and as such it did not immediately threaten de Vaucouleurs’ model. Credit for popularising the use of Sérisc’s $R^{1/n}$ model for approximating not only lenticular bulge+disc galaxies but for describing pure elliptical galaxies resides largely with Massimo Capaccioli (e.g. Capaccioli 1985, 1987, 1989; Caon et al. 1993; D’Onofrio et al. 1994). However, Davies et al. (1988) had also introduced this model for dwarf elliptical galaxies, while Sparks (1988) developed an early Gaussian seeing correction for this model, and Ciotti (1991) developed a number of associated expressions such as the velocity dispersion profile and a distribution function. The important quantification that Capaccioli and others provided is how the radial distribution of stars in elliptical galaxies, i.e. their concentration, varies with the size, luminosity and thus the mass of the elliptical galaxy (see also Cellone, Forte, & Geisler 1994; Vennik & Richter 1994; Young & Currie 1994, 1995; Graham et al. 1996; Karachentseva et al. 1996, Vennik et al. 1996). As we shall see in this article, the implications of this breakthrough have been dramatic, unifying what had previously been considered two distinct species of galaxy, namely dwarf and ordinary elliptical galaxies, previously thought to be described by an exponential and $R^{1/4}$ model, respectively.

### 2.1.1. Dark Matter Halos

This review would be somewhat incomplete without a few words regarding the connection between Sérisc’s model and (simulated) dark matter halos. While modified theories of gravity may yet make dark matter redundant at some level, it is intriguing to note that the Prugniel-Simien (1997) internal-density model, developed to approximate the deprojected form of Sérisc’s $R^{1/n}$ model, additionally provides a very good representation of the internal-density profiles of simulated dark matter halos. Merritt et al. (2006) revealed that it actually provides a better description than not only the Navarro, Frenk & White (1997) model but even a generalised NFW model with an arbitrary inner profile slope $\gamma$.

Sérisc’s former student, Navarro, independently applied Sérisc’s surface-density model to the internal-density profiles of simulated dark matter halos (Navarro et al. 2004; Merritt et al. 2005). Jaan Einasto (1965) had previously developed this same function as Sérisc to describe the internal-density profiles of galaxies. Rather than a universal profile shape, as advocated by Navarro et al. (1997), a range of simulated dark matter density profile shapes is now known to vary with the dark matter halo mass (Avila-Reese et al. 1999; Jing & Suto 2000; Merritt et al. 2005; Del Popolo 2010 and references therein). A number of useful expressions related to this “Einasto model”, which has the same functional form as Sérisc’s model but is applied to the internal rather than projected density profile, can be found in Cardone et al. (2005), Mamom & Lokas (2005) and Graham et al. (2006).

An apparent “bulge-halo conspiracy” between the radial distribution of stellar mass and dark matter (after modification by baryons) has arisen in recent years, such that elliptical galaxies reportedly have total internal-density profiles $\rho(r)$ described by power-laws (Bertin & Stiavelli 1993; Kochanek 1995; Gavazzi et al. 2007; Buote & Humphrey 2011). These power-laws were originally claimed to be close to isothermal, such that $\rho(r) \propto r^{-2}$ (Koopmans et al. 2006, 2009; Gavazzi et al. 2007). Recent work now emphasises that only the sample average profile slope is close to $-2$, and that a trend in slope with galaxy size exists (Humphrey & Boute 2010; Auger et al. 2010). This is a developing field and it is worth noting that the analyses have been confined to massive galaxies with velocity dispersions greater than $\sim 175 \text{ km s}^{-1}$, and thus with Sérisc indices $n \gtrsim 4$ (Graham, Trujillo & Caon 2001). While the light profile shape changes dramatically as the Sérisc index $n$ increases from $\sim 1$ to $\sim 4$, there is not such an obvious change in light profile shape from $\sim 4$ to higher values of $n$ (see Figure 2). The apparent isothermal profiles of elliptical galaxies, and the alleged “bulge-halo conspiracy”, may turn out to be a by-product of sample selection (i.e. choosing galaxies which have approximately the same structure). It would be interesting to expand the Sloan Lens ACS (SLACS) Survey (Bolton et al. 2006) to

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9This term was introduced by Knox Shaw (1915) and Reynolds (1920) in their galaxy classification scheme.

10It is worth noting that D’Onofrio (2001) re-modelled the Virgo and Fornax 2-component lenticular galaxies with an $R^{1/n}$-bulge plus an exponential disc (see section 4).
a greater range than only bright early-type galaxies that are approximately well fit with an $R^{1/4}$ model, and to go beyond the use of simple power-laws to describe the total mass density profile once the data allows this. Two component mass models (Prugniel-Simien+Einasto) to a range of galaxy masses await. Claims that “early-type galaxies are structurally close to homologous” may therefore be premature, as was the case for the distribution of stellar light in elliptical galaxies while the $R^{1/4}$ model was thought to be a law.

2.2. The core-Sérsic model

The centres of luminous galaxies have long been known to possess “cores”, such that the surface-density profile flattens at the center (e.g. King & Minkowski 1966), and King & Minkowski (1972) remarked on the inability of the Reynold’s and de Vaucouleurs’ model to match these flattened cores in giant galaxies. Although King (1978) identified a number of galaxies thought to be well described by his model, using seeing-deconvolved ground-based images, Lauer (1983, 1984, 1985) analysed 14 galaxies with ‘cores’, ranging from 1.5–5.0 arc-seconds in radius, revealing that they, like M87 (Young et al. 1978; Duncan & Wheeler 1980; Binney & Mamon 1982), were not exactly described by the King model which had a completely flat core. Similar conclusions, that cores existed but that they do not have flat inner surface brightness profiles, were also reported by Kormendy (1982, 1985a), creating the need for a new model to describe the stellar distribution in galaxies.

Nearly a decade later, the Hubble Space Telescope was flying and offered factors of a few improvement over the best image resolution achievable from the ground at that time. Not surprisingly, astronomers explored the centres of galaxies. In an effort to quantify these nuclear regions, after the abandonment of the King model and the lack of a flattened core in the $R^{1/4}$ model, Crane et al. (1993), Ferrarese et al. (1994), Forbes et al. (1994) and Jaffe et al. (1994) used a double power-law model to describe the inner light profiles of large galaxies. Grillmair et al. (1994), Kormendy et al. (1994) and Lauer et al. (1995) also adopted a double-power-law model but one with an additional, fifth, parameter to control the sharpness of the transition. Their model, which they dubbed the “Nuker law” for describing the nuclear regions of galaxies (after excluding any apparent excess light), has the same functional form as the double power-law model presented by Hernquist (1990, his equation 43) to describe the internal-density of galaxies (Zhao 1996).

However, as noted by the above authors, these double power-law models were never intended to describe the entire radial extent of a galaxy’s stellar distribution, and they provided no connection with the outer ($R^{1/4}$-like) radial profile. This disconnection turned out to be their downfall. Due to the curved nature of the outer light profiles beyond the core, which were being fitted by the double power-law model’s outer power-law, the five parameters of the Nuker model systematically changed as the fitted radial extent changed. This was first illustrated in a number of diagrams by Graham et al. (2003) who revealed that none of the Nuker model parameters were robust, and as such they could not provide meaningful physical quantities. For example, Trujillo et al. (2004) reported that the Nuker-derived core-radii were typically double, and up to a factor of five times larger, than the radius where the inner power-law core broke away from the outer $R^{1/4}$-like profile — a result reiterated by Dullo & Graham (2011, in prep.). An additional problem was that these “break radii” were being identified in the so-called “power-law” galaxies that showed no evidence of a downward departure and flattening from the inward extrapolation of the outer $R^{1/4}$-like profile. This situation arose because of the curved nature of what were actually Sérsic profiles. That is, the so-called “power-law” galaxies not only had no distinct “core” like the “core galaxies” do, but confusingly they do not even have power-law light profiles.

Given that Caon et al. (1993) and D’Onofrio et al. (1994) had established that the Sérsic function fits the brightness profiles of elliptical galaxies remarkably well over a large dynamic range (see the figures in Bertin et al. 2002), it is possible to confidently identify departures from these profiles that are diagnostic of galaxy formation. While in this and the following section we deal with partially-depleted cores — also referred to as “missing light” — in luminous galaxies (thought to be built from dissipationless mergers), the ensuing section addresses extra central light above the inward extrapolation of the outer Sérsic profile.
The core-Sérsic model is further discussed and used by Ferrarese et al. (2006a,b), Côté et al. (2006, 2007), Kawata, Cen & Ho (2007) and Cioffi (2009).

2.2.1. Central Mass Deficits

The collisional construction of galaxies from the merger of lesser galaxies is thought to be a common occurrence in the Universe. Coupled with the presence of a supermassive black hole (SMBH) at the heart of most galaxies (Wolfe & Burbidge 1970; Magorrian et al. 1998), dissipationless mergers were proposed by Begelman, Blandford & Rees (1980; see also Ebisuzaki, Makino, & Okumura 1991) to explain the depleted nuclei, i.e. the cores, observed in giant elliptical galaxies (e.g. King 1978, and references therein). It is thought that core-depletion is primarily due to the gravitational slingshot (Saslaw, Valtonen, & Aarseth 1974) effect that the coalescing SMBHs — from the pre-merged galaxies — have on stars while they themselves sink to the bottom of the potential well of the newly wed galaxy.

Theory predicts that the central mass deficit $M_{\text{def}}$ should scale with $0.5 N M_{\text{bh}}$, where $M_{\text{bh}}$ is the final (merged) black hole mass and $N$ the number of major “dry” (i.e. gas free, dissipationless) mergers (Milosavljević & Merritt 2001; Merritt & Milosavljević 2005; Merritt 2006a,b). Graham (2004) used the core-Sérsic model to quantify the central deficit of stars relative to the inward extrapolation of the outer Sérsic profile. Figure 4 suggests that the luminous elliptical galaxies sampled have experienced an average of 1 or 2 major dry (i.e. dissipationless) mergers, a conclusion in agreement with select ΛCDM models of galaxy formation (Haehnelt & Kauffmann 2002; Volonteri et al. 2003).

Quantification of the central stellar deficit relative to the inward extrapolation of the outer Sérsic profile has also been applied to bright Virgo clus-

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11The core-Sérsic, and also Sérsic, model provide robust parameters beyond the core if one has sufficient data to sample the curvature in the light profile, in practice this requires radial data out to $\sim 1 R_e$.

12The complex nature of this model has resulted in the appearance of alternate expressions, e.g. Spergel (2010, see his figure 3).
Fig. 3.— Left: Core-Sérsic model (solid line) fit to the major-axis $R$-band light profile of NGC 3348 (dots), with the dashed line showing the associated Sérsic component. The inner depleted zone corresponds to a stellar mass deficit of $3 \times 10^8 M_\odot$ (see Graham 2004). Right: NGC 5831 has, in contrast, no (obvious) partially depleted core and is well described by the Sérsic’s model alone. The rms scatter is shown in the lower panels. Figure taken from Graham et al. (2003).

Fig. 4.— Central mass deficit for seven core galaxies derived using the core-Sérsic model (circles) and the Nuker model (stars) plotted against each galaxy’s predicted central supermassive black hole mass. The solid and dashed line shows $M_{\text{def}}$ equal 1 and 2 $M_{\text{bh}}$, respectively. Figure adapted from Graham (2004).
et al. (1984), van den Bergh (1986) and Binggeli & Cameron (1991, 1993) in a number of dwarf elliptical galaxies. As far back as Larson (1975) it was known that simulations containing gas can account for these dense nuclear star clusters; clusters which became easier to detect with the Hubble Space Telescope (e.g. Carollo, Stiavelli & Mack 1998).

For many years it was common practice to simply exclude these additional nuclear components from the analysis of the galaxy light profile (e.g. Lauer et al. 1995; Byun et al. 1996; Rest et al. 2001; Ravindranath et al. 2001). Using HST data, Graham & Guzmán (2003) simultaneously modelled the host galaxy and the additional nuclear component with the combination of a Sérac function for the host galaxy plus a Gaussian function for the nuclear star cluster. As we will see in section 4 they also showed that the lenticular galaxies in their early-type galaxy sample could be modelled via a Sérac-bulge plus an exponential-disc plus a Gaussian-(star cluster) decomposition of their light profile — as done by Wadadekar et al. (1999) with ground-based images. Many other studies have since modeled the nuclear star clusters seen in HST images, see Figure 5 with the combination of a nuclear component plus a Sérac host galaxy (e.g. Grant et al. 2005; Côté et al. 2006; Ferrarese et al. 2006a; Graham & Spitler 2009).

While Graham & Guzmán (2003) and Côté et al. (2006) found that some nuclear star clusters could actually be resolved, it is not yet established what mathematical function best describes their radial distribution of stars. The closest example we have to study is of course the 30 million solar mass nuclear star cluster at the centre of the Milky Way (Launhardt et al. 2002). Graham & Spitler (2009) provided what may have been the first ever analysis after allowing for the significant contamination from bulge stars. Although they found that the cluster could be well described by a Sérac index with \( n = 3 \) (see Figure 2.3) it remains to be tested how well a King model can describe the data, or at least the old stellar population known to have a core (e.g. Genzel et al. 1996). The excess nuclear light in M32 has also been well fit with an \( n = 2.3 \) Sérac function (Graham & Spitler) but this too may yet be better described by a King model. What is apparent is that neither the underlying bulge nor the nuclear component are described by a power-law. Theories which form or assume such power-law cusps appear to be at odds with current observations.

2.4. Excess halo light

This section shall be somewhat cursory given R.Bower’s detailed chapter on clusters and the intracluster medium in this volume.

Brightest cluster galaxies (BCGs), residing close to or at the centres of large galaxy clusters, have long been recognised as different from less luminous elliptical galaxies: their light profiles appear to have excess flux at large radii (e.g. Oemler 1976; Carter 1977; van den Bergh 1977; Lugar 1984; Schomber et al. 1986). However, before exploring this phenomenon, it is important to recall that the light profiles of large galaxies have Sérac indices \( n \) greater than 4 (e.g. Caon et al. 1993; Graham et al. 1996)\(^\text{13}\). Subsequently, at large radii in big elliptical galaxies, there will be excess flux above that of an \( R^{1/4} \) model fit to some limited inner radial range.

An initially puzzling result from the Sloan Digital Sky Survey (SDSS; York et al. 2000) survey was the lack of light profiles with Sérac \( n \geq 5–6 \) (Blanton et al. 2005a). This was however soon resolved when Blanton et al. (2005b), Mandelbaum et al. (2005) and Lisker (2005, 2006b, 2007) pointed out a serious sky-subtraction problem with the early SDSS Photometric Pipeline (photo: Ivezić et al. 2004). The sky-value to be subtracted from each galaxy had been measured to close to the galaxy in question, and because galaxies with large Sérac indices possess rather extended light-profiles, their outer galaxy light was actually measured and then subtracted from these galaxies. As a result, the high Sérac index light-profiles were erroneously erased and missed from the SDSS. This resulted in the \( R^{1/4} \) model appearing to provide good fits to bright elliptical galaxies, and consequently a series of papers based on structurally biased radii, magnitudes and surface brightnesses.

Bearing in mind that large elliptical galaxies have high Sérac indices, it is important to distinguish between (i) an inadequacy of the \( R^{1/4} \) model to describe what is actually a single \( R^{1/4} \)

\^13\text{As } n \to \infty, \text{the Sérac model can be approximated by a power-law (see Graham & Driver 2005).}
Fig. 5.— Left: 5-parameter Nuker model fit to the $V$-band light-profile of the nucleated galaxy NGC 596 after excluding the inner three data points (Lauer et al. 2005). Right: 3-parameter Sérsic model plus 2-parameter point-source (Gaussian) fit to the same light profile of NGC 596. With the same number of parameters, this model fits both the inner, intermediate, and outer light-profile. *Figure from Dullo & Graham (2011, in prep.)*

Fig. 6.— Uncalibrated, 2MASS, $K_s$-band intensity profile from the centre of the Milky Way, taken from Schödel et al. (2009, their Figure 2). The nuclear star cluster is modelled with a Sérsic function with $n = 3$ (dotted curve) and the underlying host bulge with an exponential function (Kent et al. 1991) that has an effective half-light radius of $\sim 4.5$ degrees (e.g. Graham & Driver 2007) and is therefore basically a horizontal line. One parsec equals 25 arcseconds. *Figure from Graham & Spitter (2009).*

profile, and (ii) a distinct physical component such as an envelope of diffuse halo light surrounding a central galaxy. Early quantitative photometry of cD galaxies (supergiant D galaxies; e.g. Matthews et al. 1964; Morgan & Lesh 1965) suggested the presence of an inner $R^{1/4}$ spheroid plus an outer exponential corona (de Vaucouleurs 1969; de Vaucouleurs & de Vaucouleurs 1970). One should however question if this outer corona is a distinct entity or not. To answer this in the affirmative, astronomers can point to how the light profiles can display inflections marking the transition from BCG light to intracluster light. Gonzalez et al. (2005) additionally showed that the inflection in the light profiles of many BCGs was also associated with an a change in the ellipticity profile, signalling the switch from BCG light to intracluster light.

Gonzalez et al. (2005) and Zibetti et al. (2005) chose to model their BCG sample using an $R^{1/4} + R^{1/4}$ model to describe the inner galaxy plus the outer halo light. However, given that elliptical galaxies are better described by the $R^{1/n}$ model, and the desire to measure the actual concentration of halos rather than assign a fixed $R^{1/4}$ profile, Seigar et al. (2007) fitted an $R^{1/n} + R^{1/n}$ model to
the light profiles of five BCGs. Not surprisingly, they found that the $R^{1/4} + R^{1/4}$ model was not the optimal representation. An $R^{1/n}$-galaxy plus an exponential-halo model was found to provide the optimal fit in three instances, with an additional galaxy having no halo detected. The associated galaxy-to-halo luminosity ratios can be found there. This re-revelation of an exponential model, rather than an $R^{1/4}$ model, describing the halo has since been confirmed by Pierini et al. (2008). Intriguing is that the halo does not trace the NFW-like dark-matter halo density profiles produced in ΛCDM simulations (section 2.1.1). Stellar halos around non-BCG galaxies have also now been reported to display an exponential radial distribution (e.g. Gadotti 2011; Tal & van Dokkum 2011).

3. Structure related scaling relations

While it is common practice, and somewhat helpful, to call elliptical galaxies fainter than about $M_B = -18$ mag by the term “dwarf elliptical” (Sandage & Binggeli 1984), it should be noted, and this section will reveal, that on all measures they appear to be the low-mass end of a continuous sequence which unifies dwarf and normal/luminous elliptical galaxies. Not only is this true in terms of their structural properties (e.g. Binggeli et al. 1984; Graham & Guzmán 2003; Gavazzi et al. 2005; Ferrarese et al. 2006a; Côté et al. 2006, 2007, 2008; Misgeld et al. 2008, 2009; Janz & Lisker 2009; Graham 2010; Chen et al. 2010; Glass et al. 2011) but even the degree of kinematically distinct components is similar (Chilingarian 2009).

There are many important relations between stellar luminosity, colour, metallicity, age, and dynamics that reveal a continuous and linear behaviour uniting dwarf and giant elliptical galaxies (e.g. Caldwell 1983; Davies et al. 1983; Binggeli et al. 1984; Bothun et al. 1986; Geha et al. 2003; Lisker & Han 2008). However, Kormendy et al. (2009, their section 8) dismiss all of these apparently unifying relations by claiming that they must not be sensitive to different physical processes which they believe have produced a dichotomy between dwarf and ordinary elliptical galaxies. They claim that it is only relations which show an apparent different behavior at the faint and bright end that are sensitive to the formation physics and the remainder are not relevant. This section explains why such non-linear relations are actually a consequence of the (dismissed) linear relations, and as such these non-linear relations actually support a continuum between dwarf and ordinary elliptical galaxies.

To begin, it should be reiterated that (dwarf and ordinary) elliptical galaxies — and the bulges of disc galaxies (section 2.1) — do not have structural homology. Instead, they have a continuous range of stellar concentrations — quantified by the Sérsic index $n$ (Davies et al. 1988; Caon et al. 1993; D'Onofrio et al. 1994; Young & Currie 1994, 1995; Andredakis et al. 1995) — that varies linearly with both stellar mass and central surface brightness (after correcting for central deficits or excess light). A frequently unappreciated consequence of these two linear relations is that relations involving either the effective half-light radius ($R_e$) or the effective surface brightness ($μ_e$), or the mean surface brightness within $R_e$ ($⟨μ_e⟩$), will be non-linear. Such curved relations have often been heralded as evidence that two different physical processes must be operating because the relation is not linear and has a different slope at either end. To further complicate matters, sample selection which includes faint and bright elliptical galaxies, but excludes the intermediate-luminosity population, can effectively break such continuously curved relations into two apparently disconnected relations, as can selective colour-coding.

There are three distinct types of (two-parameter) relations involving the properties of elliptical galaxies: (i) linear relations which are taken to reveal the unified nature of dEs and Es; (ii) curved relations revealing a continuity that had in the past been mis-interpreted to indicate that distinct formation process must be operating; and (iii) broken relations which imply that two physical mechanisms are operating. In the following sections we shall learn how the linear relations result in the existence of curved relations when using effective radii and surface brightnesses. We shall also see that the transition in the broken relations occurs at $M_B ≈ -20.5$ mag and thus has nothing to do with the previously held belief that dEs and Es are two distinct species separated at $M_B = -18$ mag (Wirth & Gallager 1984; Kormendy 1985b, 2009).
3.1. Linear relations

This section introduces two key relations, from which a third can be derived, involving structural parameters. They are the luminosity-concentration \((L-n)\) relation and the luminosity-(central density) \((L-\mu_0)\) relation. We shall use the Sérsic shape parameter \(n\) to quantify the central concentration of the radial light profile, and use the projected central surface brightness \(\mu_0\) as a measure of the central density\(^{14}\).

It is noted that one can expect a certain level of scatter in the \(L-n\) and \(L-\mu_0\) diagrams because both the central density and the radial concentration of stars that one observes depends upon the chance orientation of one’s triaxial galaxy (Binney 1978). This is of course also true for measurements of effective surface brightness, half-light radii, velocity dispersions, etc. To have the best relations, it is important that we use Sérsic parameters from elliptical galaxies rather than parameters from single-Sérsic fits to samples of elliptical and lenticular galaxies given the two-component (2D-disc plus 3D-bulge) nature of the lenticular galaxies. Given the offset nature of bulges and elliptical galaxies in the \(L-n\) diagram (e.g. Graham 2001; see also Möllenhoff & Heidt 2001) it is also important that bulges not be combined in this section’s analysis of elliptical galaxy scaling relations.

3.1.1. Luminosity-(central surface brightness) relation

Caldwell (1983; his Figure 6) and Bothun et al. (1986, their figure 7) revealed that, fainter than \(M_B \approx -20.5\) mag, there is a continuous linear relation between luminosity and central surface brightness. Furthermore, Binggeli et al. (1984, their figure 11) and Binggeli & Cameron (1991, their figure 9 and 18) revealed that, when using the inward extrapolation of King models, the \(L-\mu_0\) relation is continuous and roughly linear from \(-12 > M_B > -23\) mag. This same general result was also highlighted by Jerjen & Binggeli (1997) and Graham & Guzmán (2003) when using the inward extrapolation of the Sérsic model. The benefit of this approach is that one’s central surface brightness is not biased by the presence of a depleted core or any additional nuclear components within the host galaxy. Figure 7 displays the elliptical galaxy \((M_B, \mu_0)\) data set from Graham & Guzmán (2003) fit by the expression

\[
M_B = 0.67\mu_{0,B} - 29.5. \tag{6}
\]

The actual central surface brightness of the luminous “core galaxies” is shown in Figure 7, rather than the value obtained from the inward extrapolation of their outer Sérsic profile. As such these “core galaxies” were excluded from the fit, but see the discussion in section 3.3. As an aside, if the central supermassive black hole mass \(M_{bh}\) in elliptical galaxies is directly related to the central stellar density (see Graham & Driver 2007), then the connections between \(M_{bh}\) and the global galaxy properties, such as total mass and velocity dispersion, may be secondary.

3.1.2. Luminosity-concentration relation

The linear relation between luminosity and Sérsic index, or strictly speaking the logarithm of these quantities, has been shown many times (e.g. Young & Currie 1994; Jerjen & Binggeli 1997; Graham & Guzmán 2003; Ferrarese et al. 2006a). This continuous relation between magnitude and concentration\(^{15}\) for elliptical galaxies had of course been recognised before (e.g. Ichikawa et al. 1986, their figure 11). The following \(M_B-n\) expression is shown in Figure 7, again matched to the sample of elliptical galaxies compiled by Graham & Guzmán (2003).

\[
M_B = -9.4\log(n) - 14.3. \tag{7}
\]

Graham & Guzmán (2003) excluded two-component lenticular galaxies fit by others with a single-component Sérsic model. It may be prudent to continue to exclude these galaxies even after a Sérsic-bulge plus exponential disc fit because the \(M_B-n\) relation defined by bulges, at least in spiral galaxies, is different to that defined by elliptical galaxies (Graham 2001, his figure 14).\(^{16}\)

\(^{14}\) Here the “central density” refers to the density prior to core depletion in giant elliptical galaxies or the growth of additional nuclear components in smaller elliptical galaxies.

\(^{15}\) To convert from the surface density to the internal density, one can use equation 4 from Terzić & Graham (2005).

\(^{16}\) Graham et al. (2001) contains a review of various concentration indices used over the decades, while Trujillo et al. (2001) was the first to quantify the monotonic relation between Sérsic index and concentration.
3.1.3. Concentration-(central surface brightness) relation

Combining the above two equations provides an expression between central surface brightness and Sérsic index such that

$$\mu_0 = 22.8 - 14.1 \log(n),$$

which is shown in Figure 7b, where it can be seen to be roughly applicable for values of $n \gtrsim 1$.

3.2. Curved relations

This section explains why, as a direct result of the above linear relations — which unite dwarf and giant elliptical galaxies — expressions involving either the effective half-light radius $R_e$, the associated effective surface brightness $\mu_e$ at this radius, or the mean surface brightness $\langle \mu \rangle_e$ enclosed within this radius, are curved.

3.2.1. Luminosity-(effective surface brightness) relation

The following analysis is from Graham & Guzmán (2003).

Given the empirical $M_B-n$ relation (equation 7), one knows what the expected value of $n$ is for some value of $M_B$. One can then convert the empirical $M_B-\mu_0$ relation (equation 6) into an $M_B-\mu_e$ relation using the exact relation between $\mu_0$ and $\mu_e$ which depends only on the value of $n$ (equation 2). Doing so, one obtains the expression

$$\mu_e = 1.5M_B + 44.25 + 1.086b,$$

$$= 1.5[M_B + 14.3] + 22.8 + 1.086b,$$

where $b \approx 1.9992n - 0.3271$ and equation 7 is used to replace $n$ in terms of $M_B$, such that $n = 10^{-(14.3+M_B)/9.4}$.

One can similarly convert the empirical $M_B-\mu_0$ relation (equation 6) into an $M_B-\langle \mu \rangle_e$ relation using the exact relation between $\mu_0$ and $\langle \mu \rangle_e$ which also depends only on the value of $n$ (equation 3). Doing this, one obtains the expression

$$\langle \mu \rangle_e = 1.5M_B + 44.25 + 1.086b - 2.5 \log \left[ \frac{e^b n \Gamma(2n)}{b^2 n} \right],$$

where again $b \approx 1.9992n - 0.3271$ and equation 7 is used to replace $n$ in terms of $M_B$. These curves
can be seen in Figure 8 (adapted from Graham & Guzmán 2004).

Binggeli et al. (1984, their figure 8) and Capaccioli & Caon (1991) previously showed with empirical data that the $M_B - \langle \mu \rangle_e$ relation is curved. What was new in Graham & Guzmán (2003) was the explanation. In the past, evidence of non-linear relations involving parameters from dwarf and giant elliptical galaxies were heralded as evidence of a dichotomy. Coupled with galaxy sample selection that excluded the intermediate population, and therefore resulted in two apparently disconnected relations, acted to further convince some that they were dealing with two classes of object.

### 3.2.2. Size-Luminosity relation

Now that we know how to play this game, one can additionally make predictions for relations involving the effective radius $R_e$, because we know that the luminosity $L = 2\pi (I_e)_{e} R_e^2$, with $\langle \mu \rangle_e = -2.5 \log (I_e)$. As explained in Graham & Worley (2008, their section 5.3.1), one can derive the size-luminosity relation such that

$$\log R_e [\text{kpc}] = \frac{M_B}{10} + 1.066 + 0.434n + 0.5 \log \left[ \frac{b^{2n}}{e^{2nI_e}(2n)} \right]$$

for $0.5 < n < 10$, with $b \approx 1.9992n - 0.3271$ and where equation 11 is used to replace $n$ in terms of $M_B$. This size-luminosity relation for elliptical galaxies is shown in Figure 8: along with real galaxy data.

Binggeli et al. (1984, their figure 7; cf. Misgeld & Hilker 2011, their figure 1) also demonstrated, with empirical data, that the $L - R_e$ relation for dwarf and giant elliptical galaxies is curved. Their diagram, in addition to Figure 8, see here, reveals why studies which only sample bright elliptical galaxies are often contempt to simply fit a straight line (e.g. Kormendy 1977). The explanation for why this happens is of course akin to approximating the Earth as flat when one (forgivably) does not sample enough of what is actually a curved profile. As Graham et al. (2006, their figure 1b) re-revealed recently, and as reiterated by Bernardi et al. (2007), a sample of massive elliptical galaxies will have a steeper size-luminosity relation than a sample of ordinary elliptical galaxies, which will in turn have a steeper size-luminosity relation than a sample of dwarf elliptical galaxies because the size-luminosity relation is curved. Graham & Worley (2008) explains why the $L - R_e$ relation given by equation 11 based on two linear relations and the functional form of Sérsic’s model, is curved.

Interestingly, due to the linear relations between magnitude, central surface brightness and the logarithm of the Sérsic exponent $n$, the use of faint isophotal radii results in what is roughly a linear size-luminosity relation (e.g. Oemler 1976; Strom & Strom 1978; Forbes et al. 2008; van den Bergh 2008; Nair et al. 2011), with the bright-end slope dependent on the adopted isophotal limit or Petrosian radius used. The implications of this important observation shall be detailed elsewhere.

Helping to propagate the belief that dwarf and ordinary elliptical galaxies are distinct species, Dabringhausen et al. (2008) and Lisker (2009) fit a double power-law to their curved size-luminosity relation for dwarf and ordinary elliptical galaxies, thus yielding distinct slopes at the faint and bright end. In addition, the interesting study by Janz & Lisker (2008) reported small deviations from the predicted curved relation. However, galaxies that are well described by Sérsic’s function and which follow linear $M - n$ and $M - \mu_0$ relations must follow a single curved $M - R_e$ relation. The deviations that they found are therefore mirroring a) the inadequacy of the fitted linear relations to the $M - n$ and $M - \mu_0$ distribution (a point noted in the caption of Figure 8 and/or b) poor fitting Sérsic models to their sample of elliptical and disc galaxies. Adding uncertainties to the linear relations in section 3.1 and propagating those through to the predicted $M - R_e$ relation is required before we can claim evidence of significant deviations. However, searching for such second order effects may indeed be interesting given the different types of dwarf galaxies that are emerging (Lisker et al. 2007).

### 3.2.3. Size-concentration relation

One can additionally derive an expression relating $R_e$ and $n$ by substituting the magnitude from 19A simple one-dimensional example of sample bias would be a survey of the average physical properties, such as size or mass, of people at a primary school. One would measure the properties of children and adults, but miss the bridging population which reveals a continuity and thus unification of the species.
the empirical $M_B-n$ relation, expressed in terms of $n$ (equation 7), into the size-luminosity relation (equation 11) to give

$$\log R_e [\text{kpc}] = 0.434n - 0.364 - 0.94 \log(n) + 0.5 \log \left( \frac{b^n}{e^n \Gamma(2n)} \right).$$  \hspace{0.5cm} (12)

While the $M_B-n$ relation is linear, the $R_e-n$ relation is curved, as can be seen in Figure 9.

In passing it is noted that the form of this relation (equation 12) matches the bulge data from Fisher & Drory (2010, their Figure 13). They interpret the departure of the low-$n$ bulges ($n < 2$) from the approximately linear relation defined by the high-$n$ bulges ($n > 2$) to indicate that a different formation process is operating to produce the less concentrated “pseudobulges”. However, based upon linear unifying relations that span the artificial $n = 2$ divide, we know that this $R_e-n$ relation must be curved. Without an understanding of this relation, and other curved relations (e.g. Greene, Ho & Barth 2008), they have at times been misinterpreted and used to claim the existence of different physical processes (see section 4.3 for a discussion of pseudobulges: Hohl 1975 and references therein).

It may be worth better defining the behavior of the $R_e-n$ relation at small sizes in Figure 9. The data from Davies et al. (1988) suggests when $n = 0.5$, values of $R_e$ may range from 1 kpc down to 0.2 kpc (Caon et al. 1993, their figure 5). Such a reduction to the flattening of the $R_e-n$ distribution, below $n \approx 1$, may in part arise from the inclusion of dwarf spheroidal galaxies (see Misgeld & Hilker 2011, their figure 1).

### 3.2.4 Size-(effective surface brightness) relation

As discussed in Graham (2010), the first two linear relations in Figure 7 naturally explain the curved $\langle \mu \rangle_e - R_e$ relation in Figure 10. From the empirical $\mu_0-n$ relation (equation 8, Figure 7b), one can convert $\mu_0$ into $\langle \mu \rangle_e$ using equations 2 and 3. The effective radius $R_e$ is acquired by matching the empirical $M_B-\mu_0$ relation (equation 6, Figure 7a) with the absolute magnitude formula

$$M = \langle \mu \rangle_e - 2.5 \log(2\pi R_e^2) - 36.57, \hspace{0.5cm} (13)$$

(see Graham & Driver 2005, their equation 12).

Eliminating the absolute magnitude gives the expression

$$\log R_e = \frac{1}{5} \left\{ \frac{\langle \mu \rangle_e}{3} - 9.07 + 0.72b - 1.67 \log \left( \frac{ne^b \Gamma(2n)}{b^2n} \right) \right\},$$  \hspace{0.5cm} (14)

in which we already know the value of $n$ associated with each value of $\langle \mu \rangle_e$. This is achieved by (again) using the empirical $\mu_0-n$ relation (equa-
\langle \mu_e \rangle = 22.8 + 1.086b - 14.1 \log(n) - 2.5 \log \left( \frac{ne^b \Gamma(2n)}{b^2n} \right) \tag{15}

and \( b \approx 1.9992n - 0.3271 \). Equation 14 obtained from two linear relations involving S\'ersic parameters, is a curved relation that is shown in Figure 10. Overplotted this predicted relation are data points from real galaxies.

For those who may have the S\'ersic parameter set \((R_e, \mu_e, n)\), one can use equation 8 to convert \( \mu_e \) into \( \langle \mu \rangle_e \) if one wishes to compare with the relation given by equation 14. For those who may have the parameter set \((R_e, \mu_e)\), perhaps obtained with no recourse to the S\'ersic model, equation 14 can easily be adjusted using equation 8 to give a relation between \( R_e \) and \( \mu_e \) such that

\[
\log R_e = \frac{\mu_e}{15} - 1.81 - 0.5 \log \left( \frac{ne^b \Gamma(2n)}{10^{0.296b} b^2n} \right), \tag{16}
\]

where the value of \( n \) associated with the value of \( \mu_e \) is given by

\[
\mu_e = 22.8 + 1.086b - 14.1 \log(n). \tag{17}
\]

To summarise, due to the linear relations in Figure 7 which connect dwarf and ordinary elliptical galaxies across the alleged divide at \( M_B = -18 \) mag (Kormendy 1985), or at \( n = 2 \) (Kormendy & Kennicutt 2004), coupled with the smoothly varying change in light profile shape as a function of absolute magnitude, the \( \langle \mu \rangle_e - R_e \) and \( \mu_e - R_e \) relations are expected to be curved (Figure 10), as previously shown with empirical data by, for example, Capaccioli & Caon (1991). This also explains why the fitting of a linear relation to \((R_e, \mu_e)\) data by Hoessel et al. (1987) resulted in slopes that depended on their galaxy sample magnitude.

The Kormendy relation is a tangent to the bright arm of what is actually a curved distribution defined by the relation given by equation 14 that is taken from Graham (2010). The apparent deviant nature of the dwarf elliptical galaxies from the approximately linear section of the bright-end of the \( \langle \mu \rangle_e - R_e \) distribution does not necessitate that a different physical process be operating. Moreover, as noted by Graham & Guzmán (2004) and Graham (2005), galaxies which appear to branch off at the faint end of the Fundamental Plane (Djorgovski & Davis 1987) — the flat
portion at the bright end of a curved hypersurface — also need not have formed from different physical mechanisms. Simulations that assume or reproduce a linear $\mu_c - R_e$ or $\mu_e - R_e$ relation, across too great a magnitude range, have failed to mimic the continuous curved distribution defined by real elliptical galaxies. The same remark is true for simulations of the ‘Fundamental Plane’.

3.3. Broken relations

3.3.1. Luminosity-(central surface brightness) relation

While the relation between a galaxy’s absolute magnitude and extrapolated central surface brightness is remarkably linear (section 3.1.1), there is a clear break in this relation when using the actual central surface brightness at the luminous end of the distribution (Figure 7a). This departure from the $M_B - \mu_0$ relation by elliptical galaxies brighter than $M_B \approx -20.5$ mag ($M > 0.5 - 1 \times 10^{11} M_\odot$) was addressed by Graham & Guzmán (2003) in terms of partially depleted cores relative to the outer Sérsic profile (see also Graham 2004; Trujillo et al. 2004; Merritt & Milosavljević 2005; Ferrarese et al. 2006a; Côté et al. 2007). This transition has nothing to do with the alleged divide between dwarf and giant elliptical galaxies at around $M_B = -18$ mag, but is instead directly related with the Sérsic versus core-Sérsic transition at around $M_B = -20.5$ mag.

As noted in section 2.2.1, such partially depleted cores in luminous core-Sérsic galaxies are thought to have formed from dry, dissipationless galaxy merger events involving the central coalescence of supermassive black holes (Begelman, Blandford, & Rees 1980; Ebisuzaki, Makino, & Okumura 1991; but see footnote [B]) and resulted in Trujillo et al. (2004) advocating a “new elliptical galaxy paradigm” based on the presence of a central stellar deficit versus either none or an excess of light, an approach embraced by Ferrarese et al. (2006a), Côté et al. (2007) and others.

Further evidence for a division at $M_B = -20.5$ mag comes from the tendency for the brighter galaxies to be anisotropic, pressure supported elliptical galaxies having boxy isophotes, while the less luminous early-type galaxies may have discy isophotes and often contain a rotating disc (e.g. Carter 1978, 1987; Davies et al. 1983; Bender et al. 1988; Peletier et al. 1990; Jaffe et al. 1994). Core galaxies also tend to be more radio loud and have a greater soft X-ray flux (e.g. Ellis & O’Sullivan 2006; Pellegrini 2010; Richings, Uttley & Körding 2011, and references therein).

It was, in part, from a diagram of central surface brightness versus magnitude that led Kormendy (1985b, his figure 3) to advocate a separation of dwarf and normal elliptical galaxies at $M_B = -18$ mag. However, as noted by Graham & Guzmán (2003), his sample was missing the bridging population near $M_B = -18 \pm 1$ mag. Excluding galaxies of this magnitude from Figure 7a would also result in two apparently disjoint relations nearly at right angles to each other. It is therefore easy to understand how one may quickly reach the wrong conclusion from an incomplete diagram. Although, Strom & Strom (1978, their figure 8; see also Binggeli et al. 1984) had already revealed that a linear relation exists between magnitude and central surface brightness from $-18.4 < M_V < -21.6$ mag, spanning the magnitude gap in Kormendy (1985b). Nonetheless, Faber & Lin (1983) had just observed that three of their six dwarf elliptical galaxies had near-exponential light profiles, leading them to speculate that dEs are more closely related to “exponential systems” than (tidally truncated) elliptical galaxies, and Wirth & Gallagher (1984, see also Michard 1979) had already advocated a division between exponential-dwarf and $R^{1/4}$-giant elliptical galaxies.

To further confound matters, Kormendy (1985b) had the slope wrong for the distribution of dwarf elliptical galaxies in his $M - \mu_0$ diagram, which had two consequences. First, the bright end of his dwarf elliptical galaxy distribution did not point towards the faint-end of his luminous elliptical galaxy distribution, and thus there was no suggestion of a connection. This misrepresentation is unfortunately still propagated today (e.g. Tolstoy et al. 2009, their figure 1), although Kormendy et al. (2009) have now corrected this. Second, two points representing flattened disc galaxies were added at the bright end of the mis-aligned dwarf elliptical galaxy sequence by Kormendy (1985b), implying a connection between dwarf elliptical galaxies and disc galaxies rather than ordinary elliptical galaxies.

A decade later, the Astronomy and Astro-
physics Review paper by Ferguson & Binggeli (1994; their Figure 3) had a big question mark as to “how” and indeed “if” dwarf and ordinary elliptical galaxies might connect in this diagram. When galaxies spanning the gap in Kormendy’s (1985b) analysis were included by Faber et al. (1997, their figure 4c), and shown to follow a trend consistent with the relation from Strom & Strom (1978) and Binggeli et al. (1984), in which the central surface brightness became fainter with decreasing galaxy luminosity Faber et al. (1997) suggested that this behavior in their data was spurious and due to limited resolution — such was the belief in a discontinuity separating dwarf and ordinary elliptical galaxies. At the same time, Jerjen & Binggeli (1997) argued exactly the opposite, suggesting that it was instead the “core” galaxies which had been displaced in the $M-\mu_0$ diagram from a linear $M-\mu_0$ relation, rather than wrong central surface brightness measurements for the faint (non-dwarf) elliptical galaxies. As Graham & Guzmán (2003) and Graham (2004) later explained, in terms of a $\sim 0.1$ percent central mass deficit relative to the outer Sérsic profile in galaxies brighter than $M_B \approx -20.5$ mag, Jerjen & Binggeli (1997) were right, supporting the views expressed by Binggeli et al. (1984) on a continuity between dwarf elliptical and ordinary elliptical galaxies across the alleged divide at $M_B \approx -18$ mag.

3.3.2. Luminosity-colour relation

Additional support for the dry merging scenario at the high-mass end is the flattening of the colour-magnitude relation above $0.5-1\times 10^{11} M_\odot$. While low luminosity, low Sérsic index, elliptical galaxies are bluer than bright elliptical galaxies (e.g. de Vaucouleurs 1961; Webb 1964; Sandage 1972; Caldwell & Bothun 1987), the brightest galaxies have the same colour as each other. This flattening in the colour-magnitude relation was noted by Tremonti et al. (2004) and is evident in Baldry et al. (2004, their Figure 9), Ferrarese et al. (2006a, their Figure 123), Boselli et al. (2008, their Figure 7) and even Metcalfe, Godwin & Peach (1994). These observations help alleviate past tension with semi-analytic models that had predicted a relatively flat colour-magnitude relation for bright elliptical galaxies (e.g. Cole et al. 2000). Previously, based on what was thought to be a linear colour-magnitude relation, Bernardi et al. (2007) had written that “if BCGs formed from dry mergers, then BCG progenitors must have been red for their magnitudes, suggesting that they hosted older stellar populations than is typical for their luminosities”. However, the flattening in the colour-magnitude relation has since been recognised in yet more data sets (e.g. Skelton, Bell & Somerville 2009; Jiménez et al. 2011) although it should perhaps be noted that Skelton et al. reported the transition at $M_R = -21$ mag, i.e. $\sim 1$ mag fainter.

In passing it is noted that the relation between luminosity and supermassive black hole mass (Marconi & Hunt 2003; McLure & Dunlop 2004) was found to be, after several refinements by Graham (2007), a linear one-to-one relation for black hole masses predominantly greater than $10^8 M_\odot$ — consistent with the concept of dry galaxy merging at this high-mass end.

3.3.3. Dynamics

From a sample of 13 early-type galaxies, plus one spiral galaxy, Minkowski (1962) noted that a “correlation between velocity dispersion and [luminosity] exists, but it is poor”. He wrote that “it seems important to extend the observations to more objects, especially at low and medium absolute magnitudes”. This was done by Morton & Chevalier (1973) who noted the same “continuous distribution of dispersions from 60 km/s for M32 to 490 km/s for M87” but also did not attempt to quantify this trend. It was Faber & Jackson (1976) who, with improved data and a larger sample of 25 galaxies, were the first to quantify Minkowski’s relation and discovered that $L \propto \sigma^4$ for their data set. This result has proved extremely popular and is known as the Faber-Jackson relation. Not long after this, Schechter (1980) and Malumuth & Kirsner (1981) revealed that the luminous elliptical galaxies followed a relation with an exponent of $\sim 5$ rather than 4. At the same time, Tonry (1981) revealed that expanding the sample to include more faint elliptical galaxies results in an exponent of $\sim 3$. This led Binney (1982) to write that “probably the correlation cannot be adequately fitted by a single power law over the full range of absolute magnitudes” and Farouki et al. (1983) wrote that “the data suggests the presence of curvature in the $L-\sigma$ relation”. Davies et al. (1983), and later Held et al. (1992), revealed that the dwarf ellipti-
cal galaxies followed a relation with an exponent of \( \sim 2 \), which explains why Tonry (1981) had found a slope of \( \sim 3 \) when including dwarf and ordinary elliptical galaxies. The relation found by Davies et al., with a slope of \( \sim 2 \), has recently been observed by de Rijcke et al. (2005) and the curved or possibly broken \( L - \sigma \) distribution has been interpreted by Matković & Guzmán (2005) as a change in slope at \( -20.5 \) B-mag (see also Evstigneeva et al. 2007) in agreement with Davies et al. (1983).

In spite of all the above work, there is a huge body of literature today which appears unaware that the \( L - \sigma \) relation is curved or broken. Simulations of galaxies which succeed in producing the linear Faber-Jackson relation, \( L \propto \sigma^4 \), have actually failed to produce the full distribution of dynamics seen in real elliptical galaxies as a function of magnitude.

4. Disc Galaxy Light Profiles

Due to contractual agreements with Springer Publishing, the full version of this paper can not be posted to astro-ph. All of Section 4 plus the full version of section 3.3.1 will be available from the Springer Publication.

4.1. The Bulge-Disc Decomposition

4.2. Dust and inclination corrections

4.3. Pseudobulges

4.3.1. Sérôsic index

4.3.2. Rotation

4.3.3. Ages

4.3.4. Scaling relations

4.4. Bulgeless galaxies

4.5. Barred galaxies

5. Summary

We have reviewed the progress over the last century in modelling the distribution of stars in elliptical galaxies, plus the bulges of lenticular and spiral galaxies and their surrounding discs. A number of nearly forgotten or poorly recognised references have been identified. The universality, or at least versatility, of Sérôsic’s \( R^{1/n} \) model to describe bulges (section 4.1) and elliptical galaxies (section 2.1) extends to the stellar halos of cD galaxies (section 2.3) and simulated dark matter halos (section 2.1.1).

Dwarf and ordinary elliptical galaxies were shown in section 5.1 to be united by two continuous linear relations between absolute magnitude and a) the stellar concentration quantified through Sérôsic’s (1963) \( R^{1/n} \) shape parameter (section 3.1.2), and b) the central surface brightness, which is also related to the central density (section 3.1.1). As discussed in section 3.3, a break in the latter relation at \( M_B \approx -20.5 \) mag signals the onset of partially depleted cores relative to the outer Sérôsic profile in luminous elliptical galaxies. Additional scaling relations are also noted to show a change in character at this magnitude, which may denote the onset of dry galaxy merging.

The identification of depleted galaxy cores and excess nuclear light relative to the outer Sérôsic profile was discussed in sections 2.2, 2.2.1 and 2.3. After accounting for these features, it was revealed how the above two linear relations result in curved scaling relations involving effective half light radii and effective surface brightness (section 3.2). Specifically, the \( M-\Re, M-\mu_e, M-\langle \mu \rangle_e, \mu_e-\Re, \langle \mu \rangle_e-\Re \) and \( n-\Re \) relations are non-linear. These continuous curved relations exist because elliptical galaxies do not have a universal profile shape, such as an \( R^{1/4} \) profile, but instead a range of profile shapes that vary smoothly with absolute magnitude. Without an appreciation of the origin of these curved relations, they had in the past been heralded as evidence for a dichotomy between faint and bright elliptical galaxies. Numerical simulations and semi-numerical models which try to reproduce the full elliptical galaxy sequence must be able to reproduce these non-linear relations. This will likely require physical processes which work in tandem, albeit to different degrees over different mass ranges, to produce a continuum of galaxy properties that scale with mass while adhering to the linear \( M-n \) and \( M-\mu_0 \) relations (subject to core-formation).

[snip]

The upcoming 2.6 m VLT Survey Telescope (VST, Arnaboldi et al. 1998; Capaccioli et al. 2005), plus the 4×1.8 m Pan-STARRS array (Kaiser et al. 2002), the 4 m Visible and Infrared
Survey Telescope for Astronomy (VISTA, Emerson et al. 2004) and the 8.4 m Large Synoptic Survey Telescope (LSST, Tyson 2001) are expected to deliver sub-arcsecond, deep and wide field-of-view imaging covering thousands of resolvable galaxies. By pushing down the luminosity function into the dwarf galaxy regime, and through the application of improved galaxy parameterisation methods which allow for structural non-homology and the 2- or 3-component nature of disc galaxies, both statistical and systematic errors will be reduced. This will undoubtedly provide improved constraints on galaxy scaling relations and, in turn, a fuller understanding of galaxy evolution.

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