Full-Duplex Cloud Radio Access Networks: An Information-Theoretic Viewpoint

Osvaldo Simeone, Elza Erkip, and Shlomo Shamai (Shitz)

Abstract—The conventional design of cellular systems prescribes the separation of uplink and downlink transmissions via time-division or frequency-division duplex. Recent advances in analog and digital domain self-interference interference cancellation challenge the need for this arrangement and open up the possibility to operate base stations, especially low-power ones, in a full-duplex mode. As a means to cope with the resulting downlink-to-uplink interference among base stations, this letter investigates the impact of the Cloud Radio Access Network (C-RAN) architecture. The analysis follows an information theoretic approach based on the classical Wyner model. The analytical results herein confirm the significant potential advantages of the C-RAN architecture in the presence of full-duplex base stations, as long as sufficient fronthaul capacity is available and appropriate mobile station scheduling, or successive interference cancellation at the mobile stations, is implemented.

Index terms: Full duplex, cellular wireless systems, Wyner model, Cloud Radio Access Networks (C-RAN), successive interference cancellation.

I. INTRODUCTION

The conventional design of cellular systems prescribes the separation of uplink and downlink transmission via time-division or frequency-division duplex. One of the main reasons for this choice is that operating a base station in both the uplink and the downlink at the same time causes the downlink transmitted signal to interfere with the uplink received signal. This self-interference, if not cancelled, overwhelms the uplink signal and makes the full-duplex operation of the base station impractical. Recent advances in analog and digital domain self-interference cancellation challenge the need for this arrangement and open up the possibility to operate base stations, especially low-power ones, in a full-duplex mode (see the review in [1]).

Full-duplex base stations with effective self-interference cancellation seemingly enable the throughput of a cellular system to be doubled, since the available bandwidth can be shared by the uplink and the downlink. However, this conclusion neglects two additional sources of interference between uplink and downlink transmissions, namely: (i) the downlink-to-uplink (D-U) inter-cell interference that is caused on the uplink signals by the downlink transmissions of neighboring base stations; and (ii) the uplink-to-downlink (U-D) interference that is caused on the downlink signals by the transmission of mobile stations (MSs), both within the same cell and in other cells [2]. The impact of intra-cell U-D interference has been studied in [3] and [4] for a single-cell system with a single-antenna or a multi-antenna base station, respectively. Multi-cell systems, in which D-U interference and also inter-cell U-D interference arise, have been studied in [2] via system simulation and in [5] using stochastic geometry.

The prior work mentioned above focuses on single-cell processing techniques, in which baseband processing is carried out locally at the base stations. Single-cell processing is inherently limited by the D-U interference. With the aim of overcoming this limitation, here we investigate the impact of the Cloud Radio Access Network (C-RAN) architecture on a full-duplex cellular system. In a C-RAN system, the base stations operate solely as radio units (RUs), while the baseband processing is carried out at a central unit (CU) within the operator’s network [6]. This migration of baseband processing is enabled by a network of fronthaul links, such as fiber optics cables or mmwave radio links, that connect each RU to the CU. The centralization of both uplink and downlink baseband processing at the CU allows the CU to perform cancellation of the D-U interference since the downlink signal is known at the CU. In order to further cope also with the U-D interference, we evaluate the advantages of performing successive interference cancellation at the MSs. Accordingly, the strongest intra-cell uplink transmissions are decoded and cancelled before decoding the downlink signals.

The analysis in this letter takes an information theoretic approach that builds on the prior work reviewed in [7]. Specifically, in order to capture the key elements of the problem at hand, with particular emphasis on the various sources of interference, we focus on a modification of the
classical Wyner model. The adoption of this model enables us to derive analytical expressions for the achievable rates under single-cell processing and C-RAN operation assuming either half-duplex or full-duplex base stations. These analytical results provide fundamental insights into the regimes in which full-duplex RUs, particularly when implemented with a C-RAN architecture, are expected to be advantageous.

II. SYSTEM MODEL AND NOTATION

Consider the extended Wyner model depicted in Fig. 1. The model contains one MS per cell that transmits in the uplink and one that receives in the downlink. With conventional half-duplex RUs, the two MSs transmit in different time-frequency resources, while, with full-duplex RUs, uplink and downlink are active at the same time. We describe here the system model for the full-duplex system – the modifications needed to describe the half-duplex system will be apparent.

There are \( N \) cells and inter-cell interference takes place only between adjacent cells as shown in Fig. 1. In order to avoid border effects, as it is customary, we take \( N \) to be very large. Due to the limited span of the interference, results in the regime of \( N \to \infty \) are known to be accurate also for small values of \( N \) (see [7]). In the uplink, the MS active in the \( k \)th cell transmits a signal \( x_{u,k} \) with power \( p_u = E[|x_{u,k}|^2] \leq P_u \), with \( P_u \) being the power constraint. Similarly, in the downlink, each \( k \)th RU transmits a signal \( x_{d,k} \) with power \( p_d = E[|x_{d,k}|^2] \leq P_d \). The baseband signal received in uplink by the \( k \)th RU is given as

\[
y_{u,k} = h_k \ast x_{u,k} + h_{du,k} \ast x_{d,k} + z_{d,k},
\]

where \( \ast \) denotes the convolution; \( h_k = \delta_k + \alpha \delta_{k-1} + \alpha \delta_{k+1} \), where \( \delta_k \) is the Kronecker delta function, accounts for the direct channel, which has unit power gain, and for the inter-cell interference, which is characterized by the inter-cell interference power gain \( \alpha^2 \); \( h_{du,k} = \beta_{du} \delta_{k-1} + \gamma_{du} \delta_k + \beta_{du} \delta_{k+1} \) models the D-U interference with inter-cell power gain \( \beta^2_{du} \) and self-interference power gain \( \gamma^2_{du} \); and \( z_{d,k} \) is white Gaussian noise with unit power.

In the downlink, the signal received by the MS in the \( k \)th cell can be written as

\[
y_{d,k} = h_k \ast x_{d,k} + h_{ud,k} \ast x_{u,k} + z_{d,k},
\]

where \( h_{ud,k} = \beta_{ud} \delta_{k-1} + \gamma_{ud} \delta_{k-1} + \beta_{ud} \delta_{k+1} \) describes the U-D interference, which has inter-cell power gain \( \beta^2_{ud} \) and intra-cell power gain \( \gamma^2_{ud} \); and \( z_{d,k} \) is white Gaussian noise with unit power. As depicted in Fig. 1 the parameter \( \gamma^2_{ud} \) accounts for the power received by the MS active in the downlink from the MS active in the uplink within the same cell.

Each RU is connected to the CU with a fronthaul link of capacity \( C_u \) in the uplink and \( C_d \) in the downlink. These capacities are measured in bits/s/Hz, where the normalization is with respect to the bandwidth shared by the uplink and downlink channels.

We assume full channel state information at the CU for both uplink and downlink. Define as \( R_d \) and \( R_u \) the per-cell rates, measured in bits/s/Hz, achievable in uplink and downlink, respectively, by a particular scheme. The equal per-cell rate is now defined as \( R_{eq} = \min \{ R_u, R_d \} \).

**Notation:** For convenience of notation, we define the Shannon capacity \( C(S) = \log_2(1 + S) \) and the function \( q(a, b, c) = \min(a, \max(b, c)) \).

III. HALF-DUPLEX OPERATION

In this section, we review the performance in the presence of the conventional half-duplex constraint on the RUs. In this case, a fraction \( f \in [0, 1] \) of the time-frequency resources are devoted to the uplink and the remaining fraction \( 1 - f \) to the downlink.

A. Single-Cell Processing

With single-cell processing, each RU encodes in downlink and decodes in uplink with no cooperation from the other RUs. The fronthaul links are used to convey the downlink information streams to the CU to the RUs and to transport the decoded uplink data streams from the RUs to the CU.

In the uplink, the inter-cell interference, which has power \( 2\alpha^2 P_u \), is treated as noise. As a result, the achievable rate per cell is given as

\[
R_u = \min \left\{ C\left(\frac{P_u}{1 + 2\alpha^2 P_u}\right), C_u\right\}.
\]  

In (5), the second term accounts for the limitations imposed by the fronthaul links for the transmission of the decoded data streams to the CU. Similarly, by treating inter-cell interference as noise in the downlink, we obtain the per-cell achievable rate \( R_d = \min \left\{ C\left(\frac{P_d}{1 + 2\alpha^2 P_d}\right), C_d\right\} \). We note that full power is used in both uplink and downlink with no loss of optimality. Finally, the equal per-cell rate is obtained by optimizing over the fraction \( f \) as \( R_{eq} = \max \min \{ f R_u, (1 - f) R_d \} \), which yields

\[
R_{eq} = R_d R_u / (R_d + R_u).
\]  

B. C-RAN

With C-RAN operation, baseband processing is carried out at the CU, while the RUs act solely as downconverters in the uplink and upconverters in the downlink. Unlike the single-cell processing case, the fronthaul links here carry compressed baseband information.

1) Uplink: In the uplink, the signals received by the RUs are compressed and forwarded to the CU, which then performs joint decoding. To elaborate, each \( i \)th RU produces the compressed version of the received signal

\[
\hat{y}_{u,i} = y_{u,i} + q_{u,i},
\]

where \( q_{u,i} \sim CN(0, \sigma^2_u) \) is the quantization noise, which is white and independent of all other variables. Using standard results in rate-distortion theory (see, e.g., [8] Sec. 3.6), assuming separate decompression at the CU for each fronthaul link, the quantization noise power is obtained by imposing the equality \( C_u = I(y_{u,i}; \hat{y}_{u,i}) \), which yields

\[
\sigma^2_u = \frac{(1 + (1 + 2\alpha^2) P_u)}{(2^{C_u} - 1)}.
\]
Based the received signals (5), the CU performs joint decoding. The corresponding achievable rate per cell can be written as \( R_u = \lim_{N \to \infty} I(x_u^N; y_u^N)/N \), where we have defined \( x_u = [x_{u,1}, \ldots, x_{u,1}]^T \) and similarly for \( y_u^N \). The limit at hand can be calculated as (see, e.g., [7] Sec. 3.1.2)

\[
R_u = \int_0^1 C(P_u H(f)^2/(1 + \sigma_u^2)) df,
\]

where \( H(f) = 1 + 2\alpha \cos(2\pi f) \) is the Fourier transform of \( h_k \). Note that in (7) the quantization noise affects the noise level at the decoder.

2) Downlink: For the downlink, the CU performs channel coding and precoding and then compresses the resulting baseband signals prior to transmission on the fronthaul links to the RUs. The RUs then simply upconverts the baseband signal and transmits it to the MSs. We assume that channel encoding is performed separately on each data stream producing the independent signal \( s_k \sim CN(0, P_k) \) for each \( k \)th cell. The selection of the power \( P_k \) will be discussed below. Linear precoding is then applied at the CU, so that the precoded signal reads \( \tilde{x}_{d,k} = g_k \ast s_k \) for a given precoding vector \( g = \{g_k\}_{k=-\infty}^{\infty} \). Without loss of generality, we impose the constraint \( \|g\|^2 = 1 \). One can also assume that, by the symmetry of the problem, the filter \( g \) is real and symmetric around \( k = 0 \), i.e., \( g_k = g_{-k} \). The precoded signal is quantized producing the quantized baseband signal

\[
x_{d,k} = \tilde{x}_{d,k} + q_{d,k}
\]

with quantization noise \( q_{d,i} \sim CN(0, \sigma_d^2) \), which is white and independent of all other variables. The quantization noise is related to the fronthaul capacity \( C_d \) by imposing the equality \( C_d = I(\tilde{x}_{d,k}; x_{d,k}) \), which yields \( \sigma_d^2 = P_s/(2^{C_d} - 1) \).

The achievable per-cell rate is

\[
R_d = C\left(\frac{P_s h_k^2}{1 + 2P_s \sum_{k>0} h_k^2 + \sigma_d^2(1 + 2\alpha^2)}\right).
\]

with \( \tilde{h}_k = h_k \ast g_k \). Note that in (9) the sources of noise are the interference from the undesired downlink signal streams \((2P_s \sum_{k>0} h_k^2)\) and the quantization noise \(\sigma_d^2(1 + 2\alpha^2))\). The power \( P_s \) is obtained by enforcing the power constraint, namely \( P_d = E[|x_{d,k}|^2] = P_s + \sigma_d^2 \), which leads to \( P_s = P_d(1 - 2^{-C_d}) \). We observe that, in the special case in which zero-forcing (ZF) linear precoding is adopted, we have \( h_k^2 = 0 \) for \( k \neq 0 \) and \( h_0^2 = \int_0^\infty H(f)^2 df = 1 - (4\alpha^2)^{3/2} \) in (3) (see [7] Sec. 4.2.3). In summary, for any given precoding filter \( g \), the per-cell equal rate is equal to (4) with \( R_u \) in (7) and \( R_d \) in (9).

IV. FULL-DUPLEX OPERATION

In this section, we consider the performance with full-duplex RU operation. As in [3]-[5], we assume that the cancellation of known D-U interference signals is ideal in order to focus on the potential advantages of full-duplex.

A. Single-Cell Processing

With single-cell processing, each RU is able to cancel its self-interference D-U signal. As a result, the achievable uplink per-cell rate is obtained, similar to (3), as

\[
R_u = \min \left\{ C \left(\frac{p_u(1 + 2\alpha^2 P_u + 2\beta_d^2 P_d)}{C_u}\right), C_u \right\},
\]

where the additional term \( 2\beta_d^2 P_d \) at the denominator accounts for the D-U interference. Note that we have allowed for a transmit power \( P_u = E[|x_{u,k}|^2] \) \( \leq P_u \), since with full-duplex, unlike the case of half-duplex operation, it can be advantageous not to use the full available power. In an analogous fashion, the achievable rate for the downlink is

\[
R_d = \min \left\{ C \left(\frac{(p_d(1 + 2\alpha^2 P_d + (2\beta_d^2 + \gamma_{ud}) P_u)}{C_d}\right), C_d \right\},
\]

where the additional term \( (2\beta_d^2 + \gamma_{ud}) P_u \) is the power of the U-D interference.

The intra-cell interference, with power \( \gamma_{ud}^2 P_u \), is caused by a MS in the same cell and therefore it is expected to be very relevant. In order to mitigate this problem, here we explore the possibility that the MSs implement a successive interference cancellation receiver in which the intra-cell uplink signal is first decoded and then cancelled before decoding the intended signal. Now, using standard results on the capacity region of multiple access channels (see, e.g., [8] Sec. 4.6), the rate achievable with single-cell processing is given as

\[
R_d = \min \{q(t_1, t_2 - R_u, t_3), C_d\}
\]

with \( t_1, t_2, \) and \( t_3 \) equal to \( C \left(\frac{(p_d(1 + 2\alpha^2 P_d + 2\beta_d^2 P_u))}{(1 + 2\alpha^2 P_d + 2\beta_d^2 P_u)}\right) \) and \( C \left(\frac{(p_d(1 + 2\alpha^2 P_d + 2\beta_d^2 P_u))}{(1 + 2\alpha^2 P_d + 2\beta_d^2 P_u)}\right) \), respectively. Finally, the equal per-cell rate can be calculated as

\[
R_{eq} = \max_{p_u \leq P_u, p_d \leq P_d} \min \{R_u, R_d\}.
\]

B. C-RAN

1) Uplink: As discussed above, in the uplink of a C-RAN, the signals received by the RUs, after D-U self-interference cancellation, are compressed and forwarded to the CU, which then performs joint decoding. Each RU produces a compressed version (5) of the received signal. Similar to (6), the quantization noise power \( \sigma_d^2 \) is calculated as

\[
1 + (1 + 2\alpha^2) P_u + 2\beta_d^2 P_u (1 + R_d(2)) P_d,
\]

with \( R_d(\tau) = \sum_{k} g_k g_k^{\tau-} \) is the correlation function of the downlink filter. Note that the third term in the numerator of (11) quantifies the contribution of the D-U interference, which is given as \( E[|\beta_d x_{k-1} + \beta_d x_{k+1}|^2] \). Based the received signals (5), the CU first cancels the D-U interference. Note that this is possible since the downlink signals are known to the CU. Then, the CU performs joint decoding. Similar to (7), the corresponding achievable rate per cell can be written as

\[
R_u = \int_0^1 C \left(\frac{P_u H(f)^2}{(1 + \sigma_u^2)}\right) df.
\]

2) Downlink: We adopt linear precoding as discussed in Sec. [H-B2]. Accordingly, if U-D intra-cell interference is treated as noise, the achievable per-cell rate is given as (13). If instead successive interference cancellation is performed at the MSs, the rate achievable in C-RANs can be written as
\[ R_d = C \left( \frac{p_d(1 - 2^{-C_d})\tilde{h}_d^2}{1 + 2p_d(1 - 2^{-C_d})\sum_{k>0} \tilde{h}_k^2 + (2\beta_{ud}^2 + \gamma_{ud}^2)p_u + p_d2^{-C_d}(1 + 2(\alpha^2))} \right) . \]  (13)

\( R_d = q(t_1, t_2 - R_u, t_3), \) where \( t_1, t_2 \) and \( t_3 \) can be calculated similar to Sec. IV-A from (13). For instance, \( t_1 \) equals (13) but with the term \( \gamma_{ud}^2p_u \) removed from the denominator. In summary, for any given precoding filter \( g \), the per-cell equal rate is equal to (4) with \( R_u \) in (12) and \( R_d \) in (13).

**Figure 2.** Equal per-cell rate \( R_{eq} \) versus the fronthaul capacities \( C_d = C_u \) with \( P_u = P_d = 20dB \) \( \alpha = 0.4, \beta_{du} = 0.4, \beta_{ud} = 0.04, \) and \( \gamma_{ud} = 4. \)

**V. NUMERICAL RESULTS AND CONCLUDING REMARKS**

In this section, we provide some numerical results to bring insights into the performance of the discussed approaches. Fig. 2 we plot the equal per-cell rate versus the fronthaul capacities \( C_d = C_u \) with \( P_u = P_d = 20dB \) \( \alpha = 0.4, \beta_{du} = 0.4, \beta_{ud} = 0.04, \) \( \gamma_{du} = 0 \) and \( \gamma_{ud} = 4. \) Note that the parameter \( \gamma_{ud} \) does not play a role in the analysis. The inter-cell D-U interference gain \( \beta_{du} \) is chosen to be comparable to the inter-cell gain \( \alpha \), while the U-D intra-cell interference gain \( \gamma_{ud} \) is significantly larger and the corresponding inter-cell gain \( \beta_{ud} \) is instead significantly smaller than \( \alpha \). This setting appears to be in line with what is expected in a dense small-cell scenario in which the RUs are placed in a more advantageous position than the MSs. A ZF precoder is assumed for the downlink, and, unless stated otherwise, successive interference cancellation (SIC) is employed in the downlink.

The figure shows that C-RAN solutions have a significant advantage over the corresponding single-cell processing (SCP) approaches for both half-duplex (HD) and full-duplex (FD) operations as long as the fronthaul capacities are large enough. Note that the spectral efficiency of the fronthaul links is expected to at least one order of magnitude larger than the downlink or uplink spectral efficiencies, which is well within the range shown in the figure. Moreover, when the fronthaul capacities are sufficiently large, FD-C-RAN provides a gain of around 1.7 as compared to HD-C-RAN, which falls short of the maximum gain of 2 due to the interference between uplink and downlink.

We finally study the impact of U-D intra-cell interference in Fig. 3. The parameters are the same as for the previous figure. For the full-duplex approaches, we consider the rate \( R_{eq} \) achievable with and without SIC as a function of \( \gamma_{ud} \). It is seen that, FD-C-RAN is advantageous only if we have small intra-cell interference \( \gamma_{ud} \) or if the MSs implement SIC and the gain \( \gamma_{ud} \) is large enough. This suggests that, in practice, FD-C-RAN should only be used in conjunction with an appropriate scheduling algorithm that ensures one of these two conditions to be satisfied.

Overall, the results herein confirm the significant potential advantages of the C-RAN architecture in the presence of full-duplex base stations, as long as sufficient fronthaul capacity is available and appropriate MS scheduling or successive interference cancellation at the MSs is implemented.

**REFERENCES**

[1] A. Sabharwal, P. Schniter, D. Guo, D. W. Bliss, S. Rangarajan and R. Wichman, “In-band full-duplex wireless: Challenges and opportunities,” arXiv:1311.0456.

[2] Y.-S. Choi and H. Shirani-Mehr, “Simultaneous transmission and reception: Algorithm, design and system level performance,” IEEE Trans. Wireless Commun., vol. 12, no. 12, pp. 5992–6010, Oct. 2013.

[3] S. Goyal, P. Liu, S. Panwar, R. DiFazio, R. Yang, J. Li, and E. Bala, “Improving small cell capacity with common-carrier full duplex radios,” in Proc. IEEE Int. Conf. Commun. (ICC 2014), to appear.

[4] S. Barghi, A. Khajastepour, K. Sundaresan and S. Rangarajan, “Characterizing the throughput gain of single cell MIMO wireless systems with full duplex radios,” in Proc. Int. Symp. Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt 2012), pp.68-74, May 2012.

[5] S. Goyal, P. Liu, S. Hua, and S. Panwar, “Analyzing a full-duplex cellular system,” in Proc. Conference on Information Sciences and Systems (CISS 2013), Mar. 2013.

[6] China Mobile, “C-RAN: the road towards green RAN,” White Paper, ver. 2.5, China Mobile Research Institute, Oct. 2011.

[7] O. Simeone, N. Levy, A. Sanderovich, O. Somekh, B. M. Zaidel, H. V. Poor and S. Shamai (Shitz), “Cooperative wireless cellular systems: an information-theoretic view,” Foundations and Trends in Communications and Information Theory, vol. 8, nos. 1-2, pp. 1-177, 2012.

[8] A. E. Gamal and Y.-H. Kim, Network information theory, Cambridge University Press, 2011.