Modelling of field emitter array characteristics in diode system

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Abstract. In terms of a mathematical model, this paper studies features of the current characteristics of a thin-film matrix field emission array cathode with vertical blade-shaped emitters. Microscopical distribution of electric field strength was calculated and its dependence on geometric parameters of the structures was defined.

1. Introduction
This study considers a thin-film matrix field emission cathode with vertical blade-shaped emitters [1, 2]. Such vertical blade-like field emission matrices are suitable for many applications: an emitter with circular blade-like margin was regarded as a prospective element of a vacuum nanotriode [3], a part of electron guns [4], with consideration of different production technologies [5], [6]. As such, modeling of such structures if of current interest [7–17].

Efficiency of a matrix field emission cathode is defined by the value of emission current density. The current density grows with increase of the electric field strength and area of emitting surface. However, increase of the latter via curvature radius of the emitter top does not necessarily lead to increase in emission current, as the field strength decreases in more blunt emitter margin. The goal of this study is thus defining the dependence between the emission current and the curvature radius of each emitter in the array in order to increase the efficiency of a cathode.

2. Mathematical model
In order to achieve the given goal, it is necessary to solve the problem of finding out the distribution of electrostatic potential in a bi-electrode system comprising of a cathode matrix and an anode. The surface of the matrix and the emitters were modeled as an ideal conductor with no adsorbed insulators. Metal emitters shaped as hollow round cylinders with several nanometers thick walls and sharp top margins are situated in the nodes of square matrix with a step of 5 μm on a flat substrate of highly doped silicon (Figure 1). Distance between the emitters is 2.3 μm, emitter height is 0.2 μm, curvature radius of the cylinders is varied between 1 and 20 nm (the technology of their production allows to set those values independently).

The influence of the spatial charge was neglected. We used 3d Laplace equation for electrostatic potential φ with boundary conditions corresponding to the Dirichlet conditions of the equipotential
surfaces of the cathode $\Gamma_1$, anode $\Gamma_2$, and the spherical surface $\Gamma_3$ around the computation area that models the behavior of the potential at the point at infinity.

$$\Delta \varphi = 0,$$

$$\varphi|_{\Gamma_1} = 0, \varphi|_{\Gamma_2} = V, \varphi|_{\Gamma_3} = 0. \quad (1)$$

The boundary problem (1), (2) can only be solved numerically. But for a singular emitter an analytical method for calculation of an electrostatic potential of diode system also exists [18]. This analytical method is not applicable to a matrix cathode, because it has mutual influence between different emitters that is absent at the open boundary of the cathode but influences homogeneity of emission throughout the array.

In this study an approximate solution for the boundary problem (1), (2) is found by the method of finite elements in form of a linear combination of quadratic basis functions using Comsol Multiphysics. Discretization parameters of adaptive tetrahedral finite-element lattice are chosen in a way that for a singular emitter there is a correspondence between numerical solution and the result of analytical computation by the method of pairing equations with reducing the boundary problem to a second-order Fredholm integral equation with quadrature right-hand side and the core represented explicitly [18].

![Computation area and depiction of a fragment of a matrix cathode discretized via finite element method.](image)

**Figure 1.** Computation area and depiction of a fragment of a matrix cathode discretized via finite element method.

### 3. Results

Software implementation of the model in Comsol Multiphysics with terminal-based input of geometrical parameters (that allowed to conduct computational experiments) takes in account the specifics of emission systems in which the computational area includes the boundaries of an emitter with highly curved surface and small size. This leads to a significant variance of characteristic measurements in the same geometric configuration, while the exponential dependence of current density on field strength requires particularly strict precision of defining the boundary conditions on the emitter. The solution has a rapidly changing gradient in the emission area (i.e. the blade margin), and thus the finite-element lattice grew more dense in the neighborhood of emitter blade so that the rate of convergence of the solution towards the exact solution would not significantly decrease and the number of unknown variables (i.e. dimensions of the finite-element linear algebraic system) would not grow.
Figure 2. Field strength at the margin of one emitter in four symmetrically defined points (marked by dots colors of which correspond to colors of the plots) relative the size of the largest mesh element.

Figure 2 shows the calculated values for field strength in certain points on emitter margin with different levels of discretization. One can see that there is a definite upper bound for the lattice element size below which the symmetrical properties of the solution are conserved and there's reasonable convergence of the finite-element solutions in embedded lattices.

The current density was calculated by the Fowler-Nordheim equation (which does not influence the qualitative conclusions) and integrating of the current density for obtaining total current was performed numerically. As a result we've constructed a dependence between current characteristics and curvature radius of the emitter margin shown in figure 3.

Figure 3. Influence of the curvature radius of the emitter blade on the emission current (on normalized values).
From figure 3 it is easy to see that in given range of the values of curvature radius of emitter margin its influence on the emission current is monotonous: the sharper the margin is, the higher the field strength and the emission current density, even though the emission area might decrease.

4. Conclusion
This paper studies the microscopic distribution of an electric field along the surface of a cathode matrix with cylindrical blade emitters in a flat diode configuration. The current characteristics and their influence on the geometric parameters were also assessed.

Acknowledgments
Financial support has been obtained from RFBR (13-01-00150) and partially from Saint-Petersburg State University (9.38.673.2013). Research was carried out using computational resources provided by Resource Center "Computer Center of SPbU" (http://cc.spbu.ru/en) and using experimental equipment of the Interdisciplinary Resource Center for Nanotechnology of St. Petersburg State University.

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