Black-Hole Polarization and Cosmic Censorship

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(November 2, 2021)

Abstract

The destruction of the black-hole event horizon is ruled out by both cosmic censorship and the generalized second law of thermodynamics. We test the consistency of this prediction in a (more) ‘dangerous’ version of the gedanken experiment suggested by Bekenstein and Rosenzweig. A $U(1)$-charged particle is lowered slowly into a near extremal black hole which is not endowed with a $U(1)$ gauge field. The energy delivered to the black hole can be red-shifted by letting the assimilation point approach the black-hole horizon. At first sight, therefore, the particle is not hindered from entering the black hole and removing its horizon. However, we show that this dangerous situation is excluded by a combination of two factors not considered in former gedanken experiments: the effect of the spacetime curvature on the electrostatic self-interaction of the charged system (the black-hole polarization), and the finite size of the charged body.

The cosmic censorship hypothesis, introduced by Penrose [1] thirty years ago, is one of the corner stones of general relativity. Moreover, it is being envisaged as a basic principle of nature. The validity of this conjecture is, however, still an open question in classical general relativity.

The destruction of a black hole (i.e., its event horizon) is ruled out by this principle because that would expose naked singularities to distant observers. Moreover, the horizon area $A$ of a black hole is associated with entropy $S_{bh} = A/4$ (we use gravitational units in...
which \( G = c = 1 \). Therefore, without any obvious physical mechanism to compensate for the loss of (black-hole) entropy, the destruction of the black-hole event horizon is expected to violate the generalized second law of thermodynamics [2]. For these two reasons, any process which seems, at first sight, to have a chance of removing the black-hole horizon is expected to be unphysical. For the advocates of the cosmic censorship principle the task remains to find out how such candidate processes eventually fail to remove the horizon.

In this paper we inquire into the physical mechanism which protects the black-hole horizon from being eliminated by the capture of a charged particle which “supersaturate” the extremality condition for black holes. As is well known, the Kerr-Newman metric with \( M^2 < Q^2 + a^2 \) (where \( M, Q \) and \( a \) are the mass, charge, and angular momentum per unit mass of the configuration) does not contain an event horizon, and therefore describes a naked singularity.

We begin with the type of gedanken experiment suggested by Bekenstein and Rosenzweig [3] and independently by Hiscock [4] (our version of the gedanken experiment is, however, more ‘dangerous’ than the one considered in [3]): suppose there exist two different types of local charge, namely type-\( q \in U(1) \) and type \( k \in U'(1) \), e.g., electric and magnetic charge. A Reissner-Nordström spacetime with two different charges \( Q \in U(1) \) and \( K \in U'(1) \) displays an event horizon only if \( Q^2 + K^2 \leq M^2 \).

The black hole is assumed to be a near extremal Reissner-Nordström black hole, possessing a \( U'(1) \) charge \( K \), but no \( U(1) \) charge. Thus, the black hole is not endowed with a \( U(1) \) gauge field, and an infalling charge \( q \) [where \( q \in U(1) \)] seems to encounter no repulsive electrostatic potential barrier. Therefore, at first sight, the particle is not forbidden from crossing (and removing) the black-hole horizon. An assimilation of a charged body with proper energy (energy-at-infinity) \( E \), and charge \( q \), results with a change \( dM = E \) in the black-hole mass and a change \( dQ = q \) in its charge; Thus, a necessary condition for removal of the horizon after the assimilation of the body is \( E \leq (q^2 + K^2 - M^2)/2M \leq q^2/2M \).

Bekenstein and Rosenzweig [3] considered the infall of the charged particle from spatial infinity. For this case \( E \geq \mu \), where \( \mu \) is the particle’s rest mass, and a (necessary) condition
for removal of the horizon is therefore

\[ q^2/\mu \geq 2M . \]  

This condition simply says that the classical radius of the charged body (the analogous of the well-known classical radius of the electron) is larger than the black-hole size. Thus, if the body is capable of fitting in the black hole, it cannot satisfy condition (I), and thus cannot remove the horizon.

A question immediately arises: what physical mechanism insures the stability of the horizon if the charged particle is lowered slowly towards the black hole? In this case the energy delivered to the black hole can be arbitrarily red-shifted by letting the assimilation point approach the black-hole horizon. Therefore, this type of gedanken experiment is more 'dangerous' than the one considered in [3]. At first sight, therefore, the particle is not hindered from entering the black hole and removing its horizon. However, in this paper we shall show that the dangerous particle fails to remove the horizon; this conclusion is a direct consequence of two factors not considered in [3]: the black-hole polarization, and the finite size of the charged body.

We consider a charged body of rest mass \( \mu \), charge \( q \), and proper radius \( b \), which is lowered towards a (near extremal) black hole. The total energy \( E \) of a body moving in a black-hole spacetime is made up of the energy \( E_0 \) of the body's mass (red-shifted by the gravitational field), and the electrostatic self-energy \( E_{self} \) of the charged body (there is no repulsive electrostatic force between the charges \( K \) of the black hole and \( q \) of the body, since these charges belong to different \( U(1) \) gauge fields).

The first contribution, \( E_0 \), is given by Carter's [3] integrals of the Lorentz equations of motion of a charged particle moving in a black-hole background [3]:

\[ E_0 = \frac{\mu(r_+ - r_-)^{1/2}}{r_+} \delta^{1/2} \{ 1 + O[\delta/(r_+ - r_-)] \} , \]  

where \( \delta = r - r_+ \), and \( r_\pm = M \pm (M^2 - K^2)^{1/2} \) are the locations of the black-hole (event and inner) horizons.
The gradual approach to the black hole must stop when the proper distance from the body’s center of mass to the black-hole horizon equals \( b \), the body’s radius. Thus, one should evaluate \( E_0 \) at \( r = r_+ + \delta(b) \), where \( \delta(b) \) is determined by

\[
\int_{r_+}^{r_+ + \delta(b)} (g_{rr})^{1/2} dr = b ,
\]

with \( g_{rr} = r^2/\left(r - r_+\right)\left(r - r_-\right) \). Integrating Eq. (3) one finds (for \( b \ll r_+ \))

\[
\delta(b) = (r_+ - r_-)b^2/4r_+^2 .
\]

Thus, we obtain \( E_0 = \mu b(r_+ - r_-)/2r_+^2 \).

The second contribution, \( E_{\text{self}} \), reflects the effect of the spacetime curvature on the particle’s electrostatic self-interaction. The physical origin of this force is the distortion of the charge’s long-range Coulomb field by the spacetime curvature. This can also be interpreted as being due to the image charge induced inside the (polarized) black hole \[7,8\]. The self-interaction of a charged particle in the black-hole background results with a repulsive (i.e., directed away from the black hole) self-force. A variety of techniques have been used to demonstrate this effect in black-hole spacetimes \[9-17\]. In particular, the contribution of this effect to the particle’s (self) energy in the Reissner-Nordström background is \( E_{\text{self}} = Mq^2/2r^2 \[14,15\] \), which implies \( E_{\text{self}} = Mq^2/2r_+^2 \) to leading order in \((b/r_+)^2\).

An assimilation of the charged body results with a change \( dM = E \) in the black-hole mass and a change \( dQ = q \) in its \( Q \)-type charge. The condition for the black hole to preserve its integrity after the assimilation of the body is

\[
q^2 + K^2 \leq (M + E)^2 .
\]

Substituting \( E = E_0 + E_{\text{self}} \) we find that a necessary condition for removal of the black-hole horizon is

\[
\mu < \frac{q^2}{b} - \frac{(r_+ - r_-)(r_+^2 - q^2)}{4Mb} < \frac{q^2}{b} .
\]

The total mass of the charged body is given by \( \mu = \mu_0 + fq^2/b \), where \( \mu_0 \) is the mechanical (nonelectromagnetic) mass, and \( f \) is a numerical factor of order unity which depends on how
the charge is distributed inside the body. The Coulomb energy attains its minimum, \( q^2/2b \), when the charge is uniformly spread on a thin shell of radius \( b \), which implies \( f \geq 1/2 \) (an homogeneous charged sphere, for instance, has \( f = 3/5 \)). Therefore, any charged body which respects the weak (positive) energy condition must be larger than \( r_c \equiv q^2/2\mu \).

In deriving the lower bound on particle’s size \( r_c \) one neglects the mechanical mass of the body. In fact, large stresses may be placed inside the charged body and the charge distribution must have forces of nonelectromagnetic character holding it stable. A purely classical electromagnetic model therefore has little relevance to the real world. Nonelectromagnetic forces imply a large contribution \( \mu_0 \) to the mass of the body from such forces. The large nonelectromagnetic contribution will prevent us from coming close to the minimum size limit \( r_c \): Atomic nuclei, for instance, are bounded by strong forces, which are often much stronger than the force exerted by the surface electric field. In fact, even atomic nuclei, which are the densest charged objects (with negligible self-gravity) in nature, satisfy the relation \( b/r_c \sim 10^2 \sim 10^3 \) and are therefore far larger than \( r_c \) ! Black holes with their extreme gravitational binding character are in fact the only objects in nature whose size can come close to the limit \( r_c \): An extremal Reissner-Nordström black hole, in particular, satisfies the relation \( b/r_c = 2 \) (other black holes satisfy \( b/r_c > 2 \)). Therefore, even the gravitational interaction in its extremal form as displayed in black holes cannot allow a charged object to be as small as \( r_c \).

Thus, one may safely conjecture that a charged body which respects the weak (positive) energy condition must satisfy the restriction \( b/r_c \geq 2 \) (where the equality is only saturated by the extremal Reissner-Nordström black hole), and we therefore conclude that the black-hole horizon cannot be removed by an assimilation of such a charged body – cosmic censorship is upheld!

For an elementary charge which is subjected to Heisenberg’s quantum uncertainty principle with \( b \sim \hbar/\mu \), \( r_c \) is not the measure of particle size. In fact, for \( U(1) \) charges found free in nature (weak coupling constant \( q^2 \ll \hbar \), e.g., an electron), the classical radius \( r_c \) is far smaller than the Compton length. This is incompatible with the necessary condition Eq.
(4), and we therefore recover our previous conclusion that the black-hole horizon cannot be removed.

The gedanken experiment can be generalized to astrophysically realistic black holes, i.e., to rotating Kerr black holes. We consider a charged body which is lowered towards the black hole along its symmetry axis. For this case one finds \( \delta(b) = (r_+ - r_-)b^2/4(r_+^2 + a^2) \), where \( r_\pm = M \pm (M^2 - a^2)^{1/2} \). This yields (see e.g., [18]) \( E_0 = \mu b(r_+ - r_-)/2(r_+^2 + a^2) \). The self-interaction \( E_{\text{self}} \) of a charged particle on the symmetry axis of a Kerr black hole has been determined in [15,16], yielding \( E_{\text{self}} = Mq^2/(r^2 + a^2) \). The assimilation of the charged particle by the black hole produces the following changes in the black-hole parameters:

\[
M \to M + E \quad ; \quad a \to a[1 - 2E/M + O(E^2/M^2)] \quad ; \quad Q = 0 \to q .
\] (7)

Hence, the condition for the black hole to preserve its integrity after the assimilation of the body is (to leading order in \( E/M \)) \( q^2 + a^2(1 - E/M) \leq M^2(1 + 2E/M) \). Substituting \( E = E_0 + E_{\text{self}} \) we find that a necessary condition for removal of the black-hole horizon is

\[
b < \frac{q^2}{\mu} \frac{M^2}{M^2 + a^2} - \frac{M(r_+ - r_-)(r_+^2 + a^2 - q^2)}{4(M^2 + a^2)\mu} < \frac{q^2}{\mu} .
\] (8)

This is the same as condition (4), and we therefore recover our previous conclusion that classical charged bodies which satisfy the weak energy condition, and elementary particles which are subjected to Heisenberg’s quantum uncertainty principle cannot remove the black-hole horizon – the cosmic censorship conjecture is respected!

The synthesis of these two gedanken experiments [namely, the capture of a particle with \( q \in U(1) \) charge by a Kerr-Newman black hole of charge \( K \in U'(1) \)] is trivial. We obtain the same condition (8) for the destruction of the black-hole horizon, where now \( r_\pm = M \pm (M^2 - K^2 - a^2)^{1/2} \). This implies, again, that the horizon cannot be removed in our ‘dangerous’ gedanken experiment.

Evidently, Eqs. (6) and (8) implies that not only processes which transcend the extremality condition are forbidden, but also processes which saturate this condition are forbidden as well. In other words, a (near extremal) black hole cannot be transformed into an extremal
one by assimilating a charged particle. This conclusion generalizes the results of [19] to the gedanken experiments considered in this paper.

In summary, we have tested the consistency of the cosmic censorship conjecture and the generalized second law of thermodynamics in the simple context of a charged particle lowered into a (near extremal) black hole. In particular, we have considered a (more) ‘dangerous’ version of the gedanken experiment suggested by Bekenstein and Rosenzweig [3]: The particle is lowered slowly into the black hole and the energy delivered to the black hole is, therefore, red-shifted. Hence, at first sight, the particle is not hindered from entering the black hole and removing its horizon. We have shown, however, that the black-hole horizon is protected by a combination of two factors not considered in former gedanken experiments: the influence of the spacetime curvature on the electrostatic self-interaction of the charged body, and the finite size of the charged system.

ACKNOWLEDGMENTS

I thank Jacob D. Bekenstein and Avraham E. Mayo for helpful discussions. This research was supported by a grant from the Israel Science Foundation.

APPENDIX A: LOWERING AN ELEMENTARY CHARGE WITH $q^2 > \hbar$

Throughout this paper we have assumed that the charged body obeys the weak (positive) energy condition. However, an elementary charge with $b \sim \hbar/\mu$ in a strongly coupled QED-type theory (with $q^2 > \hbar$), has $b < q^2/\mu$, and we cannot rule out condition (8). Nevertheless, Bekenstein and Rosenzweig [3] have shown that for a quantum charge the horizon is saved because in order to avoid the Landau ghost, the effective coupling constant cannot be large enough to accomplish the removal of the horizon (see [3] for a detailed analysis). Actually, the analysis of [3] concerning quantum particles is also applicable to our case, in which the particle is lowered slowly into the black hole. In particular, Eq. (19) of [3] still holds in our case:
\[ [q(\zeta M)]^2 < \frac{3\pi \hbar}{2} \ln(\zeta \xi M \mu / \hbar) \]  

(A1)

with \( \zeta > 1 \) and \( \xi > 1 \) (see again [3] for details). The necessary condition for removal of the black-hole horizon \( b < q^2 / \mu \) implies \( \mu b \ln(\zeta \xi M \mu / \hbar) < 3\pi \hbar / 2 \). However, since \( \zeta \xi \) must be few times unity, this condition can be satisfied only if \( b = O(M) \). The motion of a particle of this size in the black-hole background [which has a characteristic length scale of \( O(M) \)] cannot be treated classically; its evolution in the black-hole background is quantum mechanical in its nature. Therefore, we cannot conclude that the horizon can be removed by these particles [3].
REFERENCES

[1] R. Penrose, Riv. Nuovo Cimento 1, 252 (1969).

[2] J. D. Bekenstein, Phys. Rev. D 9, 3292 (1974).

[3] J. D. Bekenstein and C. Rosenzweig, Phys. Rev. D 50, 7239 (1994).

[4] W. A. Hiscock, Ann. Phys. (N. Y.) 131, 245 (1981).

[5] B. Carter, Phys. Rev. 174, 1559 (1968).

[6] Our task is to challenge the validity of the cosmic censorship conjecture in the most ‘dangerous’ situation, i.e., when the charge-to-energy ratio of the particle is as large as possible. Therefore, we consider a body which is captured from a radial turning point of its motion. This minimize the energy delivered to the black hole (for a given charge of the body).

[7] B. Linet, J. Phys. A: Math. Gen. 9, 1081 (1976).

[8] J. D. Bekenstein and A. E. Mayo, e-print gr-qc/9903002.

[9] C. M. DeWitt and B. S. DeWitt, Physics (N. Y.) 1, 3 (1964).

[10] F. A. Berends and R. Gastmans, Ann. Phys. (N. Y.) 98, 225 (1976).

[11] C. H. MacGruder III, Nature (London) 272, 806 (1978).

[12] A. Vilenkin, Phys. Rev. D 20, 373 (1979).

[13] A. G. Smith and C. M. Will, Phys. Rev. D 22, 1276 (1980).

[14] A. I. Zel’nikov and V. P. Frolov, Sov. Phys. -JETP 55, 191 (1982).

[15] D. Lohiya, J. Phys. A: Math. Gen. 15, 1815 (1982).

[16] B. Léauté and B. Linet, J. Phys. A: Math. Gen. 15, 1821 (1982).

[17] B. Léauté and B. Linet, Int. J. Theor. Phys. 22, 67 (1983).
[18] J. D. Bekenstein, Phys. Rev. D 8, 2333 (1973).

[19] B. Wang, R. K. Su, P. K. N. Yu and E. C. M. Young, Phys. Rev. D 57, 5284 (1998).