Topological properties of the bond-modulated honeycomb lattice

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We study the combined effects of lattice deformation, e-e interaction and spin-orbit coupling in a 2D honeycomb lattice. We adopt different kinds of hopping modulation—generalized dimerization and a Kekulé distortion—and calculate topological invariants for the non-interacting system and for the interacting system. We identify the parameter range (Hubbard U, hopping modulation, spin-orbit coupling) where the 2D system behaves as a trivial insulator or Quantum Spin Hall Insulator.

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The study of novel topological phases of matter is an extremely active research field. Quantum Spin Hall Insulators (QSHI) are a remarkable example of topology at work; in these two-dimensional insulating systems the non-dissipative spin current carried by gapless edge states owes its robustness to the topology of bulk bands described by the non-zero value of the $\mathbb{Z}_2$ topological invariant. The two-dimensional graphene-like lattice with intrinsic spin-orbit coupling has been identified as paradigmatic example of QSHI. The spin-orbit chiral interaction, described by a nearest neighbor spin-dependent complex hopping, opens a gap in the otherwise linear spectrum of honeycomb lattice and at the same time induces a metallic behavior on the edges.

A band gap opening, the conditio sine qua non for the emergence of topological features, can be achieved—at least conceptually—in different ways, not all of them with the same topological consequences. A gapped phase on the honeycomb lattice may be induced by modulating the tight-binding hopping amplitudes to describe different kinds of bond dimerization or by including many-body e-e interactions. The interplay between these three types of “gapping” interactions—spin-orbit couplings, hopping modulation and on-site e-e interaction—has been recently studied assuming the bond dimerization that can be associated to uniaxial strain. Another interesting hopping modulation is the one leading to a Kekulé distortion where stronger and weaker nearest-neighbor links alternate on the honeycomb lattice in a $\sqrt{3} \times \sqrt{3}$ arrangement. This structure turns out to be the stable in the presence of nearest-neighbor and next-to-nearest-neighbor e-e interactions, resulting, at the mean field level, in an effective bond dimerization of Kekulé type.

In this work, we explore the combined effects of spin-orbit couplings, hopping modulation and on-site e-e interaction. We superimpose different kinds of hopping modulation on a Kane-Mele-Hubbard model describing a 2-dimensional honeycomb lattice in the presence of both spin-orbit coupling and local e-e interaction and we identify the topological properties of this interacting system in terms of topological invariants. This will be done by solving the many-body problem within Cluster Perturbation Theory (CPT) and extracting topological invariants from the many-body Green’s function. The goal is to clarify how the topological phases that stem from the intrinsic spin-orbit coupling are modified by different kinds of hopping texturing and by e-e interaction.

The paper is organized as follows: in section I we consider non-interacting electrons in the presence of intrinsic spin-orbit coupling and different kinds of hopping modulation; section II extends the results to the interacting case and last section is devoted to discussion and conclusions.

I. SINGLE-PARTICLE DESCRIPTION

In the present section we consider a honeycomb model of non-interacting electrons represented by a single particle hamiltonian with a spin-dependent hopping term

$$\hat{H} = \sum_{i,l,i',l'} t_{il,i'l'}(s) \hat{c}_{i'l'}^\dagger \hat{c}_{il} s .$$

Here $i,i'$ run over the atomic positions within the unit cell and $l,l'$ refer to lattice vectors identifying the unit cells of the lattice, $s=1,2$ is for spin up and down. The hopping term $t_{il,i'l'}(s)$ includes both the first-neighbor spin-independent hopping and the Haldane-Kane-Mele second-neighbor spin-orbit coupling given by $t_{KM} s_z (d_1 \times d_2)_z$, where $d_1$ and $d_2$ are unit vectors along the two bonds that connect site $il$ with site $i'l'$. This hamiltonian preserves time-reversal symmetry and parity symmetry. In turn, this implies that—by Kramer’s theorem—states come in time-reversal pairs and the Chern number computed from the bulk occupied bands identically vanishes, and so does the charge conductivity. However the spin-conductivity may be non-vanishing as it depends upon the difference between the two spin-filtered Chern numbers.

We start by considering the modulation in the hopping amplitudes among nearest neighboring sites that...
may arise as a consequence of a non-uniform shear strain. As shown in Ref. 15 this corresponds to different values for the three nearest neighbor hopping parameters (Fig. 1(a)). Since the system has time-reversal and inversion symmetry we may identify the topological character of the system through the Z_2 parity invariant defined as the exponent \( \Delta \) in the expression

\[
(-1)^\Delta = \prod_{\text{TRIM}} \prod_{n=1}^{N} \eta_n(\Gamma_i).
\]

where \( \eta_n(\Gamma_i) = \pm 1 \) are the parity eigenvalues of the occupied bands for any of the two spin sectors, calculated at time-reversal invariant momenta (TRIM) \( \Gamma_i \) defined by the condition that \( -\Gamma_i = \Gamma_i + \mathcal{G} \) with \( \mathcal{G} \) a reciprocal lattice vector. The value of \( Z_2 \) topological invariant distinguishes trivial insulators (\( \Delta = 0, \mod 2 \)) from topological QSH insulators (\( \Delta = 1, \mod 2 \)).

The eigenvalues and eigenvectors for the honeycomb lattice as well as the parity eigenvalues at TRIM points from QSHI and TTI phase the single particle parity invariant defined as

\[
\eta(\Gamma) = \prod_{n=1}^{N} \eta_n(\Gamma_i)
\]

In the lower panel of Fig. 2 we show the evolution of the band structure for \( t_{KM}/t_1 = 0.1 \) along the line in parameter space where \( t_1 = t_3 \). By increasing the value of \( t_2 \) the positions in k-space of the band gap move along a line parallel to \( k_y = \sqrt{3}k_x \) and merge at M points where the gap closes down for \( t_2/t_1 = 2 \), signalling the topological transition from QSHI to TTI. We stress again that being the spin-orbit term isotropic, it does not alter the \( C_3 \) symmetry and as such it does not affect the gap position in k-space. After the transition the gap remains open.

We turn now to the Kekulé distortion. In this case the unit cell contains six atoms, with alternating values of the nearest neighbor hopping parameters \( t \) and \( t' \) as shown in Fig. 1(b).

Interestingly, these quantities depend just on first nearest neighbor hopping parameters and not on the spin-orbit interaction \( t_{KM} \). Still, the spin-orbit interaction is essential in order to obtain a non-trivial topological behavior: for \( t_{KM} = 0 \) a modulation in the hopping parameters would only transform a semi-metal into a trivial insulator while for \( t_{KM} \neq 0 \) the system is always an insulator (except just at the phase boundary—see below). On the contrary for \( t_{KM} \neq 0 \), by tuning the hopping parameters \( t_i \) we may go from a trivial insulating phase to a topological Quantum Spin Hall insulating regime. The values where a phase transition occurs—either between a QSHI and a topologically trivial insulator (TTI) for \( t_{KM} \neq 0 \) or between a semimetal and a TTI for \( t_{KM} = 0 \)—are identical and, as far as the phase separations are concerned, the phase diagram that we obtain (Fig. 2(a)) coincides with the one reported in refs. 15 and 16 for \( t_{KM} = 0 \). We observe that right at the transition points from QSHI and TTI phase the single particle gap \( \Delta_{sp} \) closes down and the system recovers a semi-metallic behavior. We analyze in particular the behavior of the system varying just one hopping parameter \( t_2 \), moving in the parameter space along the line shown in (Fig. 2(a)) where \( t_3 = t_1 \). For this choice of parameters the gap between filled and empty states evolves as shown in Fig. 2(b). Before the transition, the absolute value of the energy gap depends on \( t_{KM} \) but after the transition it becomes independent on \( t_{KM} \) and increases linearly as \( E_g = 2(t_3 - 2t_1) \).

In the lower panel of Fig. 1 we show the evolution of the band structure for \( t_{KM}/t_1 = 0.1 \) along the line in parameter space where \( t_1 = t_3 \). By increasing the value of \( t_2 \) the positions in k-space of the band gap move along a line parallel to \( k_y = \sqrt{3}k_x \) and merge at M points where the gap closes down for \( t_2/t_1 = 2 \), signalling the topological transition from QSHI to TTI. We stress again that being the spin-orbit term isotropic, it does not alter the \( C_3 \) symmetry and as such it does not affect the gap position in k-space. After the transition the gap remains open.
\[-t - 2t', t + 2t', t - t' - 3\sqrt{3}t_{KM}, -t + t' - 3\sqrt{3}t_{KM}, t - t' + 3\sqrt{3}t_{KM}, -t + t' + 3\sqrt{3}t_{KM}\]

and the gap closure occurs when one of these two conditions is verified

\[
t - t' - 3\sqrt{3}t_{KM} = 0
\]
\[
t - t' + 3\sqrt{3}t_{KM} = 0
\]

This leads to the following analytic expression for the $Z_2$ invariant as a function of the hopping parameters

\[
(-1)^\Delta = \text{sign} \left[ (t - t')^2 - \left(3\sqrt{3}t_{KM}\right)^2 \right]
\]

This relation has been checked numerically in terms of parity eigenvalues according to eq. 2. We obtain in this way the phase diagram of Fig. 3(a): any $t_{KM} \neq 0$ defines a range of $t'/t$ where the system behaves as a QSHI, and this range depends on the strength of the spin-orbit coupling. The hopping modulation has different effects if $t_{KM} = 0$ or $t_{KM} \neq 0$ as shown in Fig. 3(b) where the evolution of the gap value is reported: for the undistorted system in particular we have zero gap and maximum gap for $t_{KM} = 0$ and $t_{KM} \neq 0$ respectively.

\[
\hat{H} = \sum_{i,\Delta l, \sigma} t_{i\Delta l, \sigma} \hat{c}_{i\Delta l, \sigma}^\dagger \hat{c}_{i\Delta l, \sigma} + U \sum_{i} \hat{c}_{i\uparrow}^\dagger \hat{c}_{i\uparrow} \hat{c}_{i\downarrow}^\dagger \hat{c}_{i\downarrow} .
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\]
acting topological insulator. In this case, in order to topologically characterize the system, we face two distinct problems: on one side we need to substitute the single particle band structure with the quasi-particle excitation energies that can be obtained from the many-body Green’s function; on the other side we must extend the \(Z_2\) parity invariant, originally associated to single particle state, to the interacting case. In Refs. 19 and 20 it has been demonstrated that the \(Z_2\) invariant is determined by the behavior of the one-particle propagator at zero frequency only: the inverse of the Green’s function at zero frequency defines a fictitious noninteracting topological hamiltonian:

\[
h_{\text{topo}}(k) \equiv -G^{-1}(k, 0)
\]

and its eigenvectors

\[
h_{\text{topo}}(k)|k, n\rangle = \epsilon_n(k)|k, n\rangle
\]

are the quantities that in eq. replace the non-interacting band eigenvectors to obtain the topological invariant for the interacting system.

These concepts have been recently applied to identify the topological character of heavy fermion mixed valence compounds and of the half-filled honeycomb lattice also in the presence of uniaxial bond dimerization.

In order to solve the eigenvalue problem (8), in strict analogy with what is done in any standard Tight-Binding scheme for non-interacting hamiltonians, a Bloch basis expression of the topological hamiltonian, namely of the dressed Green’s function and of its inverse, is required:

\[
G_{ij}(k, \omega) = \langle \Psi_0| \hat{c}_{ki} \hat{G} \hat{c}_{kj} |\Psi_0\rangle + \langle \Psi_0| \hat{c}_{ki} \hat{G}^\dagger \hat{c}_{kj}^\dagger |\Psi_0\rangle
\]
Here \( \hat{G} = \frac{1}{\omega - \mu} \) and

\[
\hat{c}_{ki} = \frac{1}{\sqrt{L}} \sum_l e^{-i k (\mathbf{R}_l + \mathbf{r}_i)} \hat{c}_{li} ; \quad \hat{c}^\dagger_{ki} = \frac{1}{\sqrt{L}} \sum_l e^{i k (\mathbf{R}_l + \mathbf{r}_i)} \hat{c}^\dagger_{li}
\]

with \( \mathbf{R}_l \) the lattice vectors (\( L \to \infty \)) and \( \mathbf{r}_i \) the atomic positions inside the unit cell.

Here we calculate the dressed Green’s function by Cluster Perturbation Theory (CPT). CPT belongs to the class of Quantum Cluster theories that solve the problem of many interacting electrons in an extended lattice by a divide-and-conquer strategy, namely by solving first the many-body problem in a subsystem of finite size and then embedding it within the infinite medium. Different Quantum Cluster approaches (Dynamical Cluster Approach, Cellular Dynamical Mean Field Theory, Variational Cluster Approach) differ for the embedding procedure and/or for the way the lattice Green’s function—or the corresponding self-energy—is expressed in terms of the cluster one. The common starting point is the choice of the \( M \)-site cluster used to tile the extended lattice. By construction CPT is exact in the two limits \( U/t = 0 \) (non-interacting band limit), \( U/t = \infty \) (atomic limit); for intermediate values of \( U/t \) it opens a gap in metallic systems at half occupation.

In CPT the Green’s function (10) for the extended lattice is calculated by solving the equation

\[
G_{ij}(k, \omega) = G^C_{ij}(\omega) + \sum_{i''} B_{ii''}(k, \omega) G_{i''j}(k, \omega).
\]

Here \( G^C_{ij} \) is the cluster Green’s function in the local basis obtained by exact diagonalization of the interacting hamiltonian for the finite cluster; we separately solve the problem for \( N, N-1 \) and \( N+1 \) electrons and express the cluster Green’s function in the Lehmann representation at real frequencies.

The matrix \( B_{ii''}(k, \omega) \) is given by

\[
B_{ii''}(k, \omega) = \sum_l e^{i k \cdot \mathbf{R}_l} \sum_{i'''} G_{i'''}(\omega) t_{i'''}0,i'(s)
\]

where \( t_{i'''}0,i' \) is the hopping between site \( i' \) and \( i'' \) belonging to different clusters.

The key approximation in this derivation is the expression of the complete Green function in terms of Green functions of decoupled clusters and it is important to verify the accuracy of the results by using larger and larger cluster sizes. This procedure is limited by the dimensions of Hilbert space used in the exact diagonalization, dimensions that grow exponentially with the number of sites. A further limitation in the cluster choice arises by a symmetry requirement since only clusters that preserve the point group symmetries of the lattice must be used. In the case of the generalized dimerization we have verified that no significant change occur in the spectral functions assuming 2- and 8-site clusters and for this reason we report results obtained only for the smallest cluster size, namely 2- and 6-site clusters for the generalized bond dimerization and Kekulé distortion respectively. Notice that in this case the clusters used to “tile” the infinite lattice are those shown in Fig. 1 and that \( t_1 \) (\( t' \)) describe intra-cluster hoppings for the two distortions (generalized dimerization and Kekulé respectively).

Eq. (10) is solved by a \( M \times M \) matrix inversion at each \( k \) and \( \omega \). A second \( M \times M \) matrix inversion is needed to obtain the topological hamiltonian according to eq. (7). The topological hamiltonian is then diagonalized and its eigenvectors are used for the calculation of \( Z_2 \) according to (2).

It is worth recalling that the eigenvalues of \( h_{\text{topo}} \) used to calculate the value of \( Z_2 \) invariant in principle have nothing to do with the quasi-particle excitation energies: they only contain topological information and the full Green’s function is needed to calculate quasi-particle spectral functions

\[
A(k, \omega) = \frac{1}{\pi} \sum_n \text{Im} G(k, n, \omega)
\]

with \( n \) the band index and \( \alpha^n_i(k) \) the eigenstate coefficients obtained by the single-particle band calculation.

Fig. 3 shows the results that we obtain for interacting electrons in the honeycomb lattice with the two kinds of hopping modulation (generalized dimerization and Kekulé distortion). By comparing the interacting case, where the \( Z_2 \) invariant is obtained from the eigenvectors of \( h_{\text{topo}} \), and the non-interacting one we notice that the QSHI/TTI phases are modified by the local e-e interaction for both lattice distortions. In the case of generalized dimerization (Fig. 4 (a)) the overall region in parameter space where the system is in the QSHI phase increases with \( U \) but at the cost of larger distortions; conversely, when the system is almost undistorted (lower corner on the left of Fig. 4 (a), where \( t_2 \) and \( t_3 \) are closer to \( t_1 \)) the effect of e-e is to extend the region of the TTI phase. For \( U \geq 3.5 \) the undistorted system is always topologically trivial.

The phase separation lines remain linear and independent on the strength of the spin-orbit coupling, as in the non interacting case. Indeed, the effect of e-e interaction is to induce a renormalization of the intra-cluster hopping parameters and therefore the topological hamiltonian of eq. (4) coincides with an effective single-particle hamiltonian with modified hopping terms. This is particularly evident when a 2-site cluster is used as a basic unit in CPT, but remains true with larger clusters. We have checked this by considering an 8-site cluster and we do not find significant differences.

In the Kekulé distortion the effect of e-e interaction favors even more clearly the TTI phase since to total area
where the system behaves as a QSHI is reduced with re-
spect to the non-interacting case and, for a given distor-
tion, larger values of the spin-orbit coupling are required
to have a non trivial topological character.

The lower panels of Fig. 4 show the spectral functions
that we obtain for the two kinds of hopping modulation
as a function of the intra-cluster hopping parameters ($t_1$
and $t'$ respectively) at fixed values of Hubbard $U$ and
of inter-cluster hopping parameters ($t_2 = t_3 = 1$ and
$t = 1$ in the two cases respectively). We notice that the
hopping modulation induces in both cases a closure of the
energy separation between filled and empty quasi-particle
states, signalling the topological phase transition.

**FIG. 4.** Phase diagram and spectral functions for the Kane-Mele model of the honeycomb lattice in the presence of on-site
*e-e* interaction for the generalized bond dimerization (right panel) and for the Kekulé distortion (left panel). Panel (a): phase
diagrams for the generalized bond dimerization obtained with different values of $U$ as a function of $t_1$ and $t_3$ at $t_{KM} = 0.1$. Panel
(b): phase diagram for the Kekulé distortion at a fixed value $U = 3$ as a function of $t'$ and $t_{KM}$. The filled area corresponds
in both cases to the non interacting result reported in Figs. 2 and 3. Spectral functions for the dimerized honeycomb lattice:
$t_{KM} = 0.1, U = 3, t_2 = t_3 = 1$ and (a) $t_1 = 1$, (b) $t_1 = 1.5$, (c) $t_1 = 2$. Spectral functions for the honeycomb lattice in the
Kekulé distortion: with $U = 3, t = 1$ and (d) $t' = 1$, (e) $t' = 1.3$, (f) $t' = 2$.

### III. CONCLUSIONS

We have studied the joint effects of intrinsic spin-orbit
coupling, hopping modulation and on-site *e-e* interaction
on the topological properties of the 2D honeycomb lat-
tice. The goal was to understand how the topological
phases induced by intrinsic spin-orbit coupling are mod-
ified by different kinds of lattice distortions and by *e-e*
interaction. The main results may be summarized as fol-
lows. In the non-interacting case the shape of the phase
diagram obtained assuming a generalized dimerization
does not depend on the value of the intrinsic spin-orbit coupling: the phase separation lines are identical for any value of $t_{KM}$, with the noteworthy difference that in the parameter range where the system is for $t_{KM} = 0$ a semimetal, for any other $t_{KM} \neq 0$ the system is a QSHI. In the absence of spin-orbit coupling the Kekulé distortion makes the system insulating and in this case the parameter space where the system behaves as a QSHI increases with the value of $t_{KM}$.

We have extended the analysis in terms of topological invariants to the case of interacting electrons by calculating the dressed Green’s function within CPT and the topological hamiltonian. For both lattice distortions, in the regime of relatively small deformations, the effect of e-e interaction is to reduce the region where the system behaves as a QSHI. In this sense we may conclude that lattice distortions and e-e interaction do not cooperate in inducing a non trivial topological phase but rather reduce the possibility of finding the honeycomb lattice in a non-trivial topological state.

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1. M. Z. Hasan and C. L. Kane. *Colloquium: Topological insulators*. Rev. Mod. Phys., 82:3045–3067, Nov 2010.
2. Y. Ando. Topological Insulator Materials. J. Phys. Soc. Japan, 82:100201, 2013.
3. Xiao-Liang Qi and Shou-Cheng Zhang. Topological insulators and superconductors. Rev. Mod. Phys., 83:1057–1110, Oct 2011.
4. C. L. Kane and E. J. Mele. Z$_2$ Topological Order and the Quantum Spin Hall Effect. Phys. Rev. Lett., 95:146802, Sep 2005.
5. Liang Fu and C. L. Kane. Time reversal polarization and a Z$_2$ adiabatic spin pump. Phys. Rev. B, 74:195312, Nov 2006.
6. Liang Fu and C. L. Kane. Topological insulators with inversion symmetry. Phys. Rev. B, 76:045302, Jul 2007.
7. Thomas C. Lang, Andrew M. Essin, Victor Gurarie, and Stefan Wessel. Z$_2$ topological invariants in two dimensions from quantum Monte Carlo. Phys. Rev. B, 87:205101, May 2013.
8. C. Weeks and M. Franz. Interaction-driven instabilities of a Dirac semimetal. Phys. Rev. B, 81:085105, Feb 2010.
9. Eduardo V. Castro, Adolfo G. Grushin, Belén Valenzuela, María A. H. Vozmediano, Alberto Cortijo, and Fernando de Juan. Topological Fermi Liquids from Coulomb Interactions in the Doped Honeycomb Lattice. Phys. Rev. Lett., 107:106402, Sep 2011.
10. Stephan Rachel and Karyn Le Hur. Topological insulators and Mott physics from the Hubbard interaction. Phys. Rev. B, 82:075106, Aug 2010.
11. Dong Zheng, Guang-Ming Zhang, and Congjun Wu. Particle-hole symmetry and interaction effects in the Kane-Mele-Hubbard model. Phys. Rev. B, 84:205121, Nov 2011.
12. M. Hohenadler, T. C. Lang, and F. F. Assaad. Correlation Effects in Quantum Spin-Hall Insulators: A Quantum Monte Carlo Study. Phys. Rev. Lett., 106:100403, Mar 2011.
13. M. Hohenadler, Z. Y. Meng, T. C. Lang, S. Wessel, A. Muramatsu, and F. F. Assaad. Quantum phase transitions in the Kane-Mele-Hubbard model. Phys. Rev. B, 85:15132, Mar 2012.
14. F. D. M. Haldane. Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly". Phys. Rev. Lett., 61:2015–2018, Oct 1988.
15. Vitor M. Pereira, A. H. Castro Neto, and N. M. R. Peres. Tight-binding approach to uniaxial strain in graphene. Phys. Rev. B, 80:045401, Jul 2009.
16. Yasumasa Hasegawa, Rikio Konno, Hiroki Nakano, and Mahito Kôhmoto. Zero modes of tight-binding electrons on the honeycomb lattice. Phys. Rev. B, 74:033413, Jul 2006.
17. M. Hohenadler and F. F. Assaad. Correlation effects in two-dimensional topological insulators. Journal of Physics: Condensed Matter, 25(14):143201, 2013.
18. Zi Yang Meng, Iisii-Hsuan Hung, and Thomas C. Lang. The characterization of topological properties in quantum Monte Carlo simulations of the Kane-Mele-Hubbard model. Modern Physics Letters B, 28(01):1430001, 2014.
19. Zhong Wang, Xiao-Liang Qi, and Shou-Cheng Zhang. Topological invariants for interacting topological insulators with inversion symmetry. Phys. Rev. B, 85:165126, Apr 2012.
20. Zhong Wang and Shou-Cheng Zhang. Simplified Topological Invariants for Interacting Insulators. Phys. Rev. X, 2:031008, Aug 2012.
21. Zhong Wang and Binghai Yan. Topological Hamiltonian as an exact tool for topological invariants. Journal of Physics: Condensed Matter, 25(15):155601, 2013.
22. Jan Werner and Fakher F. Assaad. Interaction-driven transition between topological states in a Kondo insulator. Phys. Rev. B, 88:035113, Jul 2013.
23. Xiaoyu Deng, Kristjan Haule, and Gabriel Kotliar. Plutonium Hexaboride is a Correlated Topological Insulator. Phys. Rev. Lett., 111:176404, Oct 2013.
24. Feng Lu, JianZhou Zhao, Hongming Weng, Zhong Fang, and Xi Dai. Correlated Topological Insulators with Mixed Valence. Phys. Rev. Lett., 110:096401, Feb 2013.
25. Tsuneya Yoshida, Robert Peters, Satoshi Fujimoto, and Norio Kawakami. Topological antiferromagnetic phase in a correlated Bernevig-Hughes-Zhang model. Phys. Rev. B, 87:085134, Feb 2013.
26. F. Grandi, F. Manghi, O. Corradini, C. M. Bertoni, and A. Bonini. Topological invariants in interacting Quantum Spin Hall: a Cluster Perturbation Theory approach. ArXiv e-prints, April 2014.
27. D. Sénéchal, D. Perez, and M. Pioro-Ladrière. Spectral Weight of the Hubbard Model through Cluster Perturbation Theory. Phys. Rev. Lett., 84:522–525, Jan 2000.
28. Thomas Maier, Mark Jarrell, Thomas Pruschke, and Matthias H. Hettler. Quantum cluster theories. Rev. Mod. Phys., 77:1027–1080, Oct 2005.
29. M. H. Hettler, A. N. Tahvildar-Zadeh, M. Jarrell, T. Pruschke, and H. R. Krishnamurthy. Nonlocal dynamical correlations of strongly interacting electron systems. Phys. Rev. B, 58:R7475–R7479, Sep 1998.
30. S. S. Kanchala, B. Kyung, D. Sénéchal, M. Civelli, M. Capone, G. Kotliar, and A.-M. S. Tremblay. Anomalous superconductivity and its competition with antiferromag-
netism in doped Mott insulators. *Phys. Rev. B*, 77:184516, May 2008.

31 M. Potthoff, M. Aichhorn, and C. Dahnken. Variational Cluster Approach to Correlated Electron Systems in Low Dimensions. *Phys. Rev. Lett.*, 91:206402, Nov 2003.

32 F Manghi. Multi-orbital cluster perturbation theory for transition metal oxides. *Journal of Physics: Condensed Matter*, 26(1):015602, 2014.

33 Notice that in Fig. 4(b) the asymmetry in the topological behaviour for $t' > t$ and $t' < t$ (slightly different absolute slope of the upper and lower blue lines) is due to CPT being more accurate for intra-cluster hopping larger than the inter-cluster one.