Abstract. We propose a scheme for the reconstruction of the quantum state without a priori knowledge about the measurement setup. Using the data pattern approach, we develop an iterative procedure for obtaining information about the measurement which is sufficient for an estimation of a particular signal state. The method is illustrated with the examples of reconstruction with on/off detection and quantum homodyne tomography.
1. Introduction

For the successful reconstruction of the quantum state, several technical as well as mathematical pre-requisites are needed. A well-calibrated detection device making a sufficiently accurate measurement of the signal is an indispensable part of any reconstruction protocol. However, calibration is a formidable task in itself, especially if it is to be done with few-photon states. This problem has received increased attention in recent years; in particular, the so-called ‘absolute’ calibration was developed, see [1–6]. Mathematically, the problem of characterizing an unknown measurement device is closely related to the dimensionality of the parameter space [11]. For a search subspace of dimension \( N \), there are about \( N^2 \) parameters characterizing a density matrix, and therefore \( N^4 \) elements of a rank-4 tensor are needed for the description of the link between an arbitrary input signal and the measured outcome. A detailed discussion of detector tomography and related techniques can be found in [8]. High dimensions of the reconstruction spaces are often required for a faithful characterization of complex quantum systems [7]. Despite the recent progress in the field of quantum process estimation [9, 10], the problem is far from being solved.

This work was motivated by a simple question: is it really necessary to know all about the measurement device before the reconstruction starts? Intuitively, it is clear that since the supposedly signal state is localized in some region of the search subspace, one can be satisfied with the device features associated with the probing of only this region. Then it is sufficient to calibrate the device only in this region. The question is: how to distinguish these features without a priori information about the signal state and by relying only on the results of measurements made on the signal state?

Surprisingly, there is a simple way to do this. It will be shown that the calibration can be incorporated into the reconstruction process. Information about the device can be updated on the basis of the reconstruction results obtained in each step of the estimation procedure. In this sense this proposal reminds us of adaptive algorithms known as ‘neural networks’, where self-learning is an important part of the reconstruction protocol. Here we will discuss such an adaptive reconstruction scheme based on the recently formulated pattern tomography [12]. This scheme makes use of fitting measurement data obtained for an unknown signal state on the
set of data obtained for known probe states. It requires neither calibration of the measurement
nor any assumptions about the search subspace, which is naturally defined by the choice of
probe states. Such a reconstruction procedure was shown to be analogous to the classical image
processing [13]. Here we further elaborate this idea into a versatile and economical tool that
provides a means of making quantum-state estimation tomography without knowledge about
the measurement apparatus, using limited quasi-classical resources. First, we demonstrate how
a general quantum state can be represented as a mixture of quasi-classical coherent and thermal
states with positive and negative weights. We describe the procedure for constructing such a
representation with a minimal number of appropriately chosen known quasi-classical states.
In particular, it will be shown that the single-photon state can be represented with arbitrary
accuracy with just two thermal states. Then, we develop an estimation procedure implementing
an adaptive choice of probe states for improved fit and more precise estimation. New probe
states are selected using the available data patterns in an adaptive fashion. We also demonstrate
that the proposed data pattern reconstruction is experimentally feasible and particularly well
suited for the inference of photon-number distributions.

The outline of the paper is as follows. In section 2 we introduce the representation of non-
classical states with phase-averaged coherent states, thermal states and coherent state projectors.
In section 3 the data pattern reconstruction is discussed in detail for two measurement schemes,
namely on/off detection and quantum homodyne tomography.

2. Representation by mixtures of classical states

The key point of our discussion is a possibility of representing an arbitrary quantum state as a
mixture of appropriately chosen states with positive and negative weights. The density matrix,
\( \rho \), of the signal state is supposed to be represented as a mixture

\[
\rho = \sum_{j=1}^{N} x_j \sigma_j,
\]

where the coefficients \( x_j \) are real scalars (not necessarily positive) and \( \sigma_j \) are density
matrices describing a set of linearly independent (generally, non-orthogonal) probe states. For
concreteness, let us assume Gaussian probe states, namely coherent or thermal states.

Discrete representation (1) represents a generalization of the concept of P-representation [14, 15] and can be sufficiently accurate depending on the number of
terms in the sum. Note that for calculating averages with arbitrary precision, one can approx-
imate even a highly singular P-function by an infinitely differentiable function [16]. Then, a
sufficiently dense discretization can be adopted for the required approximation of the state
analogously to [17]. Coherent state projectors used for P-representation can be replaced by
continuous representation of arbitrary Gaussian operators [18].

Here we suggest to use an appropriately chosen non-orthogonal basis unconnected with
the quasi-probability discretization. Moreover, that basis of Gaussian density matrices might be
very different from the set obtained as a result of the discretization (here, it is useful to mention
a curious example of representing a thermal squeezed state with a few coherent projectors
localized in two circles on the phase plane [19]).
Figure 1. Examples of representing quantum states by thermal and phase-averaged coherent basis states (3). Panels (a) and (b) present coefficients $x_j$ and matrix elements of the two-photon Fock state approximated by $N = 5$ thermal states (distance (2) is about 0.0015); panels (c) and (d) present the single-photon state with $N = 8$ phase-averaged coherent states, $d = 2.96 \times 10^{-5}$. Panels (e) and (f) present coefficients $x_j$ and matrix elements of the approximated three-photon Fock state for $N = 8$ phase-averaged coherent basis states, $d = 0.0157$. Note that panels (b), (d) and (f) zoom in on the very small deviations from the state being represented.

2.1. Representing phase-averaged states

Let us start with a simple case of states which are diagonal in the Fock-state representation. Despite its simplicity, this case is highly relevant for quantum-state diagnostics with conventional single-photon detectors. In particular, such a representation can be applied for inferring the photon-number distributions of a signal state using on/off detection schemes [20–22]. This will be considered in the next section.

First of all, we note that a weak phase-averaged quantum states can be efficiently represented as a mixture of just a few thermal states. Indeed, a single-photon Fock state can be represented as the difference of two weak thermal states. For example, subtracting a vacuum state from a thermal state with $\langle n \rangle \ll 1$, up to the normalization factor one obtains

$$\rho \approx \sum_{n=0}^{\infty} \langle n \rangle^n |n+1\rangle\langle n+1|,$$

where the vector $|n\rangle$ denotes the Fock state with $n$ photons. Similarly, one can represent Fock states with a larger number of photons. In figures 1(a) and (b), an illustration is shown of the
two-photon Fock state representation with just five weak thermal states. Here we use the distance

\[ d = \sqrt{\sum_{k,l=0}^{\infty} |\rho_{kl}^{\text{est}} - \rho_{kl}^{\text{true}}|^2} \]  

(2)

to characterize the quality of the representation, where \( \rho^{\text{est}} \) is the approximated density matrix and \( \rho^{\text{true}} \) is the true one; elements \( \rho_{kl} \) are taken in the Fock-state basis.

Thermal states are not the only choice for an experimentally feasible representation of diagonal states. Phase-averaged coherent states

\[ \sigma_j = \sum_{k=0}^{\infty} \frac{|\alpha_j|^k}{k!} e^{-|\alpha_j|^2} |k\rangle \langle k| \]  

(3)

can be used as well, as has already been demonstrated with figure 1. Again, a small number of phase-averaged coherent states are sufficient for obtaining a reasonably accurate representation of weak diagonal states. In contrast with the thermal-state representation, a state with larger average numbers of photons can be used.

Simulations in this subsection were performed using the standard MATLAB procedures of least-square fitting. Note that even for a comparatively small number of terms in the representation (\( \approx 10 \)) problems of bad conditioning might arise, making the straightforward least-square procedure problematic (the same kind of problems are plaguing attempts to implement a direct matrix inversion for the reconstruction, see, e.g., [20]). However, imposing linear constraints, such as the requirement of positivity of the diagonal elements of the density matrix in the Fock state basis and unit trace of them, stabilizes the least-square fit.

2.2. Designing an iterative approximation

The results presented in figure 1 hint at designing an iterative approximation: in the first step a predefined set of points is used and then probe states are added/modified depending on the residual errors. This procedure is expected to work better, provided the representation is well localized in the phase space.

For representations discussed in the previous subsection, a predefined small set of phase-averaged or thermal states was used. However, choosing this set is not a trivial task. For the given number of probe states, the quality of the representation depends rather strongly on the choice of amplitudes or temperatures (see figure 2). For choosing them we have developed a simple and rather general iterative approach. First of all, we specify a set of available probe states. For the phase-averaged signal states considered in this section, this set is represented by a sequence of coherent state amplitudes or the average numbers of thermal photons. For example, it can be an equidistant grid with the distance between neighboring points corresponding to the average photon-number difference between the corresponding states much less than unity (since we are interested here in the reconstruction of signal states with a few photons on average). In figure 2 one can see an example of how the quality of the two-photon Fock signal state representation with phase-averaged coherent states depends on the choice of the grid, described by the distance between neighboring points, \( \delta \). As should be expected, when \( \delta \) remains much less than unity, the representation is good (however, the quality can be changed by orders of magnitude by adding just one state). When the step approaches unity, the representation deteriorates.
2. Representing generic quantum states

A generic quantum state can be represented as a mixture of coherent-state projectors or thermal-state density matrices as before. However, choosing probe states for the representation is a more complicated task. In [12] it was demonstrated how to represent different non-classical states.
with the same set of coherent-state projectors with amplitudes forming a finite square lattice on the phase plane. Taking into account the discussion in the previous subsection, one can guess that such a representation is far from being the most efficient one. In this subsection we further elaborate on the representation with a set of coherent probes outlining ways of reducing their number without accuracy loss.

First of all, one can try reducing the number of required probe states using a set with amplitudes forming a lattice on the phase plane by taking the lattice with the maximum possible period in a given region. However, one can hardly lower significantly the number of required probes in this way. Accuracy drops rather sharply. As an example, in figure 3, a representation of the following non-classical state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |2\rangle)$$

Figure 3. Different representations of the non-classical state equation (4) with coherent projectors and thermal states. Panel (a) shows the real parts of the density matrix elements of the represented state in the Fock-state basis. The set of the coherent probe state amplitudes used is shown in panel (b). Stars denote coherent states with coefficients $x_j$ exceeding the threshold of 1% of the maximal $x_j$ values. Panel (c) shows the coefficients, $x_j$, of the representation. Panel (d) shows how the quality of the representation improves with increasing the density of the coherent-state lattice spanning the same region of the phase plane; the size of the lattice is $N \times N$. Black bars show the quality of the representation for the same $N \times N$ latices plus one extra thermal state with the average number of photons $\langle n \rangle = 0.22$.
is shown using the set of $11 \times 11$ coherent-state projectors with amplitudes forming a square lattice. In that case, the representation is highly accurate ($d < 0.003$). But, as can be seen in figure 3(d), an attempt at 10\% reduction of the number of states using the square lattice of amplitudes leads to a significant drop in accuracy; an attempt to halve the number of probe states leads to drastic loss of accuracy. Thus, using a set of probe states on a lattice it would be necessary using a few hundreds of probe states for the representation of few-photons states.

However, this number can be more substantially reduced just by adding a single thermal state to the set of probe states. An example is depicted in figure 3(d): by adding just one probe thermal state with the average number of photons of $\langle n \rangle = 0.22$, it is possible to improve accuracy by an order of magnitude. The reason for such an improvement is simple and rather general: a thermal state has significant (and rather close in value) overlap with any few-photon state. Actually, it plays a similar role in the representation as when the unit operator is added to the set of probe states. Also, as should be expected, adding another thermal state does not lead to any noticeable improvement of the fit.

It is worth noting that for a given regular lattice, only a small part of basis states contributes to the representation, see figures 3(b) and (c). Such ‘major contributing’ probe states can be found using the adaptive procedure along the lines indicated in the previous subsection. An example of such a search procedure is shown in supplementary video 1 (see the supporting material, available from stacks.iop.org/NJP/15/025038/mmedia). Here, the iterative procedure representing the single-photon state with just eight coherent-state projectors is demonstrated. The procedure goes as follows: eight initial points (coherent amplitudes) on the small-step square lattice are chosen. For the representation, the region $[-1, 1]$ for both the real and imaginary parts of amplitudes was chosen. This region was covered by a $11 \times 11$ square grid. Initial points were taken on the border of the region (actually, the choice of points for the given grid is not really relevant). Then, each amplitude was consequently replaced by the amplitude corresponding to the neighboring grid point, and the change in the quality of the fit was checked. The replacement leading to maximal improvement of the fit is adopted. This optimization is done sequentially for each point and is repeated until no further improvement becomes possible. The total number of points is kept constant. In the example in Supplement I, the distance $d < 0.01$ was achieved using only 25 iterations of the procedure. Thus, one can achieve an accurate representation of few-photon states using only a few tenths of probe states instead of hundreds for a pre-defined set of probe states.

As will be shown in the next section, a similar iterative procedure can be adopted for the quantum-state reconstruction.

Also, note that in finding suitable coefficients $x_j$, the question of the positive definiteness of the approximation arises. For the predefined set of probe states on a lattice, the linear semidefinite programming approach was adopted [23]. The linear programming procedure goes as follows: to guarantee that the chosen region on the phase plane is indeed sufficient for the representation of the signal state, we require the representation to be localized, i.e. we minimize the overlap of our state with the states lying on the lattice border

$$\text{Tr} \left\{ \rho \sum_{\sigma_j \in \text{border}} \sigma_j \right\}.$$
An accurate representation implies that the fitting equations
\[ \text{Tr}\{\rho \sigma_j\} = \sum_l x_l \text{Tr}\{\sigma_l \sigma_j\} \]
be satisfied, and these are used as linear constraints for the procedure. In general, this problem is unfeasible. But a sufficiently precise solution, where the constraints are approximated in the sense of the least-square fit, can be calculated with help of convex optimization routines, such as the MATLAB package CVX (http://cvxr.com/cvx/). The positivity constraint on \( \rho \) can be easily incorporated as well.

For the adaptive procedure shown in supplementary video 1 (available from stacks.iop.org/NJP/15/025038/mmedia), we have used the least-square fit with a positive semidefinite constraint on matrix variables as given by the MATLAB package CVX.

3. Data pattern reconstruction

The idea underlying the quantum-state reconstruction using data patterns [12] bears an obvious similarity to image processing [13]. A measurement described by a positive-valued operator measure (POVM), \( \Pi_i \), is carried out on a set of probe states, \( \sigma_j \). The obtained data represent data patterns used for fitting the data measured for an the unknown state to be reconstructed. Consequently, due to the linearity, the unknown signal state can be expanded (represented) as a mixture of probe states with the same set of coefficients. Apart from the fact that this expansion is done on the basis of registered data, this is fully analogous to the fitting procedure discussed in the previous section. Responses of the measurement setup to the probe states are described by the matrix of probabilities
\[ \tilde{P}_{lj} = \text{Tr}\{\Pi_l \sigma_j\}. \]  
(5)
The result of a practical estimation with a finite number of copies is a matrix of frequencies—the data pattern matrix \( P_{lj} \). Now, the response \( f_l \) to an unknown signal can be expanded in the responses to probe states
\[ f_l = \sum_j x_j P_{lj} \]  
(6)
and the coefficients \( x_j \) can be estimated. Those coefficients are then used in the superposition equation (1) to yield the estimated state \( \rho \). The quantumness of such a process is manifested by the positivity constraint imposed on the fitting procedure: the fitting of equation (6) must yield a set of \( x_j \) making the reconstruction \( \rho \) obtained from equation (1) positive semi-definite. The practical feasibility of this scheme was demonstrated in [12]. An essential feature of the fitting procedure is that one does not need any knowledge about the POVM elements. The knowledge required for the reconstruction is obtained in the process of building the data pattern matrix. It is not necessary to perform a complete tomography of the measurement setup, since only the probe states contributing significantly to the representation of a particular signal state are needed. The data pattern procedure can be reformulated as the process of updating information about the measurement setup until a sufficiently accurate fit is obtained.

Note that the measurement setup defines the limit on the precision of the probe state generation. Obviously, if the difference between the patterns corresponding to supposedly different probes is within the range of the statistical/systematic error, then, effectively, these two probes are identical, see also [12]. Hence, in the case of coherent probe states with amplitudes
forming a regular lattice in the phase space, the errors in coherent amplitudes generation can be neglected provided that they are much less than the lattice period.

The suggested operational procedure is the following: at first the responses to the signal state are recorded. Then, patterns generated by a set of probe states are recorded, and reconstruction of the signal is done. The quality of the fit is characterized by introducing a cost function characterizing the difference between measurement outcomes and probabilities predicted by the estimation. In the next iteration, more probe states are used and the new fit is analyzed. The choice of new probe states is as described in the previous section: one either adds more states from a pre-defined set until found sufficient for the representation of the signal state (e.g. a new row or column of the lattice of coherent amplitudes) or uses the results of the previous step to find the best probe states to be added (similarly to what was done in the adaptive procedure described in the previous section and depicted in supplementary video 1); an important difference is that here we want to incorporate all the previously measured probe states into the fit in order to maximize the information gain. The procedure stops when the fit does not change significantly upon adding more probe states.

In the following subsections we describe particular realizations of the scheme described above for both the diagonal state inference and the complete state reconstruction.

3.1. Reconstruction of a photon-number distribution

Consider now the reconstruction of a photon-number distribution using the data-pattern scheme. As will be shown, the proposed scheme is feasible, efficient and fast and allows to avoid a meticulous and difficult procedure of calibrating the detection device at the single-photon level. The photon-number inference is performed using a measurement sensitive to the diagonal elements of the density matrix expressed in the Fock-state basis:

\[ p_j = \sum_n \Pi_{jn} \rho_{nn}. \]

A well-known example of such measurements (7) is the already well-established on/off detection and its modifications [20–22]. Simulated data pattern reconstructions with the simplest version of the on/off detection device are presented in figure 4. Absorbers are placed before the single-photon detector to change the quantum efficiency of the detection. The data comprise the number of clicks detected for each efficiency setting for a sequence of identical copies of the signal state.

An example of the data pattern reconstruction from the simulated data using the iterative procedure discussed in the previous section is shown in figure 4. Phase-averaged coherent states were considered as probes. The values of the probe-state amplitudes are taken from the predefined equidistant lattice with a period much smaller than unity (which can be considered rather as the general rule for the case, since we are interested in the reconstruction of few-photon signals); \( \delta = 0.02 \) for the example. The vacuum state was chosen as a starting point; on each subsequent step of the procedure the next probe state on the grid was added. The Kullback–Leibler divergence

\[ K_d = \sum_j f_j \log \left( \frac{f_j}{p_j} \right) \]

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Figure 4. Reconstruction of the single-photon Fock state by an iterative procedure described in the text. Phase-averaged coherent probe states are used. (a) Reconstructed photon-number distributions for $N = 8$ probe states (gray bars) and $N = 3$ probe states (black bars). Panels (b) and (c) show coefficients $x_j$ and amplitudes of the probe states for $N = 8$ (b) and $N = 3$ (c). Panel (d) shows the Kullback–Leibler divergence equation (8) for different numbers of probe states $N$. In total, 30 detector efficiencies were used for the reconstruction equidistantly distributed in the interval [0.1, 0.9], and $10^5$ copies were measured for each of those settings. The estimation was performed using standard MATLAB packages for the linearly constrained least-squares estimation.

is used as a cost function quantifying the quality of the fitting procedure. Here $f_j$ are the measured frequencies for the signal state, and $p_j$ are the probabilities predicted from the reconstructed state. Note that the photon-number distribution of a single photon state can be reconstructed with just a few semi-classical probe states. Also note that the information about the actual detector efficiencies was not used in the estimation process. It can be seen that adding more responses from additional probe states would typically improve the fit (figure 4(d)), although some oscillations in the quality with adding new terms might also occur.

The possible non-monotonic character of the quality of the fit can be well illustrated with reconstruction based on thermal probe states. The results for the single-photon state estimation with thermal probe states and a realistic fiber-loop detector are shown in figure 5 [24, 25]. In the loop detector, the signal travels several times around the fiber loop. Each time, a portion of the signal is split off at a fiber beam splitter and gets detected at the single-photon detector with on/off result. For $M$ passes, $2^M$ different outcomes are thus registered. One can see in figure 5 that for a single-photon state, quite a precise fit is achieved with just two thermal probe states. With adding more probe states the quality first drops down and then improves gradually with adding more probe states. Still, the procedure is efficient in comparison with full calibration of the measurement setup, since just a few probe states are needed here.
3.2. The complete quantum-state reconstruction

Finally, we will apply the proposed data pattern approach to the complete quantum-state reconstruction. To be specific, we will consider quantum homodyne tomography [26] as the measurement scheme and coherent probe states of amplitudes $\alpha_j$ as the probe states. In this case the response of the measurement device is characterized by the probabilities of measuring quadrature values $x_l(\theta)$, where $\theta$ is the phase of the local oscillator. This probability has a simple Gaussian form

$$\tilde{P}_{lj} = \sqrt{\frac{2}{\pi}} \exp\{-2(x_l(\theta) - y(\alpha_j, \theta))^2\},$$

(9)

$\eta$ being the efficiency of the measurement setup and

$$y(\alpha_j, \theta) = \sqrt{\eta} \left(\text{Re}(\alpha_j) \cos\theta + \text{Im}(\alpha_j) \sin\theta\right).$$

To illustrate how the knowledge is accumulated and how the accuracy of the estimate improves in the course of measurements, let us again consider the reconstruction of a single-photon signal state, whose response is described by a simple function

$$p_l = \sqrt{\frac{2}{\pi}} \left(1 - \eta(1 - 4x_l(\theta)^2)\right) \exp\{-2x_l(\theta)^2\}.$$  

(10)

In figure 6 examples of iterative data pattern reconstruction are given for the standard quantum homodyne tomography setup (six phases of the local oscillator and about $2 \times 10^5$ measurements for each phase). A realistic value of quantum efficiency of $\eta = 0.8$ was used.
Figure 6. Data pattern reconstruction of the single-photon Fock state by the homodyne tomography setup using coherent probe states. The origin-centered square lattice of coherent probe states with the period of 0.0154 was used. Panel (a) shows the Kullback–Leibler divergence (8) for different sizes of $N \times N$ probe lattices. Panels (b)–(f) show the real and imaginary parts of the reconstructed matrices for lattices with $N = 7$ (b, e); $N = 10$ (c, f); and $N = 14$ (d). In all simulations, the homodyne measurement was simulated with six phases of the local oscillator equidistantly distributed in the interval $[0, \pi]$ with $2 \times 10^5$ detections per phase, the quadrature values $x(\theta)$ being discretized into 64 bins, and the quantum detection efficiency being $\eta = 0.8$.

in the simulations. The amplitudes of the coherent probe states were chosen from the square lattice in the phase space, similar to those depicted in figure 3(b). The search scheme is the following. In each step a subset of $N \times N$ probe states are taken centered at the origin of the phase space. The fitting procedure is performed with the positive-semidefinite least-square estimation algorithm as described in section 2.3. This is repeated with $N$ incremented by one. The Kullback–Leibler divergence (8) is calculated as the measure of the fit quality. One can see how the quality improves with each step, see figure 6(a). When the probe states are too few and their amplitudes are too close to the origin on the phase plane, the reconstructed state is close to the vacuum state, see figures 6(b) and (e). Increasing the probe state number the single-photon component gradually emerges in the reconstructed state, see figures 6(c) and (f). For a sufficiently large set of probe states, a faithful reconstruction of the signal states is obtained as shown in figure 6(d).

To conclude this example, let us point out that not all the probe states are contributing equally to the final state reconstruction. As seen here, only a part of the basis states contributes significantly to the reconstruction. Respective weights of each pattern may change strongly with
each iteration. So, curiously, if one has a set of probe states which is found to be sufficient for the reconstruction, even random sampling of probe states from this set might be more efficient for the reconstruction than the procedure described above. This is illustrated in supplementary video 2 (available from stacks.iop.org/NJP/15/025038/mmedia), where the square lattice of \(15 \times 15\) probes spanning the region \(\text{Re}(\alpha) \in [-2, 2], \text{Im}(\alpha) \in [-2, 2]\) was chosen as the set to choose the probe state from. Then, the probe states were chosen randomly and the quality of the fit was checked after each sampling. As the result for the single-photon state reconstruction, the subset needed for reconstruction is about 30\% less than that in the case of the deterministic iterative procedure, where the lattice sites were fixed.

4. Conclusion

We have presented the data pattern approach to quantum tomography. It appears that this technique may become an efficient and feasible tool in experimental quantum-state reconstruction, the most prominent feature being the ability to perform reconstruction without ever knowing the exact properties of the measurement apparatus. The knowledge required for the precise estimation of a particular signal state can be obtained \textit{a posteriori}, after the measurement on the signal state is done. Characterizing the quality of the estimation by means of the Kullback–Leibler divergence, one can decide which additional probe states might be helpful in further improving of the reconstruction. This strategy was demonstrated with simulated estimations of the diagonal elements of the density matrix using on/off detection and also with a complete quantum-state reconstruction using the homodyne tomography setup. We have explicitly demonstrated that coherent and thermal probe states provide adequate quantum resources even for the reconstruction of highly non-classical signal states with a few photons. This is a considerable advantage for the experimenters, since calibrating the measurement setups for such weak signals can be rather a challenging task.

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