Diffractive Vector Photoproduction using Holographic QCD

Chang Hwan Lee* and Hui-Young Ryu†
Department of Physics, Pusan National University, Busan 609-735, South Korea

Ismail Zahed‡
Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA
(Dated: July 27, 2018)

We discuss diffractive photon-production of vector mesons in holographic QCD. At large \( \sqrt{s} \), the QCD scattering amplitudes are reduced to the scattering of pair of dipoles exchanging a closed string or a pomeron. We use the holographic construction in AdS\(_5\) to describe both the intrinsic dipole distribution in each hadron, and the pomeron exchange. Our results for the heavy meson photon-production are made explicit and compared to some existing experiments.

PACS numbers: 12.39.Jh, 12.39.Hg, 13.30.Eg

I. INTRODUCTION

Diffractive scattering at high energy is dominated by pomeron exchange, an effective object corresponding to the highest Regge trajectory. The slowly rising cross sections are described by the soft Pomeron with a small intercept \((0.08)\) and vacuum quantum numbers. Reggeon exchanges have even smaller intercepts and are therefore subleading. Reggeon theory for hadron-hadron scattering with large rapidity intervals provide an effective explanation for the transverse growth of the cross sections [1]. In QCD at weak coupling the pomeron is described through resummed BFKL ladders resulting in a large intercept and zero slope [2, 3].

The soft Pomeron kinematics suggests an altogether non-perturbative approach. Through duality arguments, Veneziano suggested long ago that the soft Pomeron is a closed string exchange [4]. In QCD the closed string world-sheet can be thought as the surface spanned by planar gluon diagrams. The quantum theory of planar diagrams in supersymmetric gauge theories is tractable in the double limit of a large number of colors \(N_c\) and \(\ 't\) Hooft coupling \(\lambda = g^2 N_c\) using the AdS/CFT holographic approach [5].

In the past decade there have been several attempts at describing the soft pomeron using holographic QCD [6–11]. In this paper we follow the work in [10] and describe diffractive \(\gamma + p \rightarrow V + p\) production through the exchange of a soft pomeron in curved AdS\(_5\) geometry with a soft or hard wall. This is inherently a bottom-up approach [12] with the holographic or 5th direction playing the role of the scale dimension for the closed string, interpolating between two fixed size dipoles. We follow the suggestion in [13, 14] and describe the intrinsic dipole size distribution of hadrons on the light cone through holographic wave functions in curved AdS\(_5\). Diffractive production of vector mesons was investigated in the non-holographic context by many in [15]. Recently a holographic description was explored in [16] in the context of the color glass condensate, and reggeized gravitons in [17].

The organization of the paper is as follows: In section 2 we briefly review the set up for diffractive scattering through a holographic pomeron as a closed surface exchange in curved AdS\(_5\) with a (hard) wall. In section 3, we detail the construction of the light cone wavefunctions including their intrinsic light cone dipole distributions. In section 4 and 5 we make explicit the AdS\(_5\) model with a (soft) wall to describe the intrinsic dipole distributions of massive vector mesons. As a check on the intrinsic wavefunctions, we calculate the pertinent vector electromagnetic decay constants. Our numerical results for the partial cross sections and their comparison to vector photoproduction data are given in section 6. Our conclusions are summarized in section 7.

II. DIPOLE-DIPOLE SCATTERING

In this section we briefly review the set-up for dipole-dipole scattering using an effective string theory. For that we follow [11] and consider the elastic scattering of
two dipoles

\[ D_1(p_1) + D_2(p_2) \rightarrow D_1(k_1) + D_2(k_2) \]  

(1)
as depicted in Fig. 1. \( b \) is the impact parameter and the relative angle \( \theta \) is the Euclidean analogue of the rapidity interval [18, 19]

\[ \cosh \chi = \frac{s}{2m^2} - 1 \rightarrow \cos \theta \]  

(2)
with \( s = (p_1 + p_2)^2 \).

A. Dipole-dipole correlator

Following standard arguments as in [11], the scattering amplitude \( \mathcal{T} \) in Euclidean space is given by

\[ \frac{1}{2is} \mathcal{T}(\theta, q) \approx \int d^2b \; e^{iq \cdot b} \mathcal{W} \]  

(3)
with \( \mathcal{W} \) the connected correlator of two Wilson loops, each represented by a rectangular loop sustained by a dipole and slated at a relative angle \( \theta \) in Euclidean space as shown in Fig. 1. The leading \( 1/N_c \) contribution from a closed string exchange is

\[ \mathcal{W} = g_s^2 \int \frac{dT}{2T} \mathcal{K}(T) \]  

(4)
where

\[ \mathcal{K}(T) = \int_{T} \mathcal{D}[x] \; e^{-S[x]+\text{ghost}} \]  

(5)
is the string partition function on the cylinder topology with modulus \( T \). The sum is over the string world-sheet with specific gauge fixing or ghost contribution. Here \( g_s \) is the string coupling.

B. Holographic Pomeron

In flat \( 2 + D_\perp \) dimensions, the effective string description for long strings is the Polyakov-Luscher action with \( D_\perp = 2 \). However, the dipole sources for the incoming Wilson loops vary in size within a hadron. To account for this change and enforces conformality at short distances, we follow [9] and identify the dipole size \( z \) with the holographic direction. The stringy exchange in (4) is now in curved AdS in \( 2 + D_\perp \) with \( D_\perp = 3 \). At large relative rapidity \( \chi \) this exchange is dominated by the string tachyon mode with the result [9]

\[ \mathcal{W}(z, z', b_\perp) \simeq -\frac{g_s^2}{4} \frac{(2\pi^2)^2}{\lambda^4} \frac{(zz')^2}{z_0^4} N(\chi, z, z', b_\perp) \]  

(6)
and

\[ N(\chi, b_\perp, z, z') = \frac{z_0^2}{zz'} \Delta(\chi, \xi) + z^2 \Delta(\chi, \xi_\star) \]  

(7)
\( \Delta(\chi, \xi) \) refers to the tachyon propagator in walled AdS. It solves a curved diffusion equation in the metric defined by

\[ ds^2 = \frac{z_0^2}{z^2} (db_\perp^2 + dz^2) \]  

(8)
within \( 0 \leq z \leq z_0 \) with a zero current at the wall,

\[ \Delta(\chi, \xi) = \frac{\exp[(\alpha \mathbf{p} - 1)\chi] \xi \exp[-\xi^2/(4D\chi)]}{(4\pi D\chi)^{3/2} \sinh(\xi)} \]  

(9)
with the chordal distances given by

\[ \cosh \xi = \cosh(u' - u) + \frac{b^2}{2\pi^2} e^{u'+u}, \]
\[ \cosh \xi_\star = \cosh(u' + u) + \frac{b^2}{2\pi^2} e^{u'-u}. \]  

(10)
and \( u = \ln \frac{z_0}{2} \) and \( u' = \ln \frac{z_0}{2} \). The holographic Pomeron intercept and diffusion constant are respectively given by

\[ \alpha_p = 1 + \frac{D_\perp}{12} - \frac{1}{2\sqrt{\lambda}}, \]
\[ D = \frac{1}{2\sqrt{\lambda}}. \]  

(11)
The string coupling in walled AdS is identified as \( g_s = \kappa g/4\pi N_c \) and \( \alpha'/z_0^2 = 1/\sqrt{\lambda} \). Here \( \kappa \) is an overall dimensionless parameter that takes into account the arbitrariness in the normalization of the integration measure in (4). This analysis of the holographic Pomeron is different from the (distorted) spin-2 graviton exchange in [8] as the graviton is massive in walled AdS. Our approach is similar to the one followed in [11] with the difference that \( 2 + D_\perp = 5 \) and not 10 [9]. It is an effective approach along the bottom-up scenario of AdS which yields a dipole-dipole total cross section that is similar to the one following from BFKL exchange [20, 21], and a wee-dipole density that is consistent with saturation at HERA [22].

III. PHOTON-HADRON SCATTERING

In a valence quark picture an incoming meson is considered as a dipole made of a quark-diquark pair, while a baryon is considered as a dipole made of a pair of a quark-diquark. The quantum scattering amplitude follows by assigning
to the scattering pairs dipole sizes $r_{1,2}$ and distributing them within the quantum mechanical amplitude of the pertinent hadron. At large $\sqrt{s}$ the scattering particles propagate along the light cone and are conveniently described by light cone wave functions. Typically, the latters are given in terms of an intrinsic wavefunction \( \Psi(x, r) \) for a dipole of size \( r \) with a fraction of parton longitudinal momentum \( x \). With this in mind, the scattering amplitude for the diffraction process for vector meson photo-production \( \gamma + p \rightarrow V + p, \) reads

\[
\mathcal{A} = -2is \int d^2b_\perp e^{-i q \cdot b_\perp} \\
\times \int \frac{d^2r_1 dx_1}{4\pi} \Psi_+^\dagger \Psi_0(x_1, r_1) \\
\times \int \frac{d^2r_2 dx_2}{4\pi} \Psi_0^\dagger \Psi_-(x_2, r_2) (-\text{WW}(r_1, r_2, b_\perp)) \\
= -2is \int \frac{d\theta_2}{2\pi} J_0 \left( \sqrt{\theta_2} \right) \\
\times \int \frac{d^2r_1 dx_1}{4\pi} \Psi_+^\dagger \Psi_0(x_1, r_1) \\
\times \int \frac{d^2r_2 dx_2}{4\pi} \Psi_0^\dagger \Psi_-(x_2, r_2) (-\text{WW}(r_1, r_2, |b_\perp|))
\] (13)

The \( \frac{1}{4\pi} \) normalization conforms with the light cone rules. Note that in flat \( D_\perp \)-space (also for \( \xi \ll 1 \)), the propagator (9) simplifies

\[
\Delta_F(\chi, \xi) = \Delta_F(\chi, \xi_e) \\
= \frac{\exp[(\alpha - 1)\chi]}{(4\pi D(\chi))^3/2} \\
\times \exp[-(b_\perp^2 + (z - z')^2)/(2\alpha')]
\] (14)

after the substitution \( z_0 D \rightarrow z_0' \). For an estimate of (12) we may insert (14) into (12), ignore the wall and assume \( z \sim z' \) to carry out the integration in (12) exactly

\[
\mathcal{A}_F = 2 \times \frac{g_2^2 (2\pi)^2}{4} \left( \frac{2is z_0^2}{(4\pi D(\chi)^3/2)} \right) \frac{s}{s_0}^{\alpha_F(t) - 1} \\
\times \int \frac{d^2r_1 dx_1}{4\pi} \Psi_+^\dagger \Psi_0(x_1, r_1) \\
\times \int \frac{d^2r_2 dx_2}{4\pi} \Psi_0^\dagger \Psi_-(x_2, r_2)
\] (15)

with the Pomeron trajectory

\[
\alpha_F(t) = \alpha_P + \frac{\alpha'}{2}t
\] (16)

### A. Photon wave function

The description of the light cone photon wave function in terms of a \( qq \) pair follows from light cone perturbation theory as described in [29]. Let \( Q^2 \) be the virtuality of the photon of polarization \( h \). The amplitude for finding a \( qq \) pair in the virtual photon with light cone momentum fractions \( (x, \bar{x}) \) is given by [15, 23]

\[
\psi_{h, h}^\gamma(r, x : Q^2, m_f) = \sqrt{N_c ee_f} \delta_h \delta_{h'} x \bar{x} 2Q \frac{K_0(\epsilon r)}{2\pi} \\
\psi_{h, h}^{\gamma T}(r, x : Q^2, m_f) = \sqrt{2N_c ee_f} \left[ i e^\pm i\theta_r (x \delta_{h\pm} \delta_{h'\mp} + \bar{x} \delta_{h\mp} \delta_{h'\pm}) \right] K_0(\epsilon r) \\
\] (17)

with \( \Psi_{h, h}^\gamma \) the matrix entries in helicity of \( \Psi_h \) in (12). Here \( ee_f \) is the charge of a quark of flavor \( f \), \( e^2 = x\bar{x}Q^2 + m_f^2 \), and \( K_{0,1} \) are modified Bessel functions. Also \( (r, \theta_r) \) are the 2-dimensional dipole polar coordinates. While the photo-production analysis to be detailed below corresponds to \( Q^2 = 0 \), we will carry the analysis for general \( Q^2 \) for future reference.

### B. Hadron wave functions

We start by defining the proton (squared) wave function for a pair of quark-diquark as

\[
|\psi_p(x, r)|^2 = \frac{2}{r_p} \delta \left( x - \frac{1}{2} \right) \delta(r - r_p)
\] (18)

by simply assuming equal sharing of the longitudinal momentum among the pair, and a fixed dipole size \( r_p \), with the normalization

\[
\int d^2r dx |\psi_p(x, r)|^2 = 1
\] (19)

The vector meson wave function on the light cone will be sought by analogy with the photon wave function given above. Specifically we write

\[
\psi_{h, h}^{V, L}(r, x : M_V, m_f) = \delta_h \delta_{h'} x \bar{x} f_L(x, r) \\
\psi_{h, h}^{V, T}(r, x : M_V, m_f) = \left[ i e^\pm i\theta_r (x \delta_{h\pm} \delta_{h'\mp} + \bar{x} \delta_{h\mp} \delta_{h'\pm}) \right] f_T(x, r)
\] (20)

where \( \Psi_{h, h}^V \) are the matrix entries in helicity of \( \Psi_V \) in (12). The intrinsic \( f_{L,T}(x, r) \) dipole distributions for the vector mesons will be sought below in the holographic construction by identifying the holographic direction in the description of massive vector mesons with the dipole size [13, 14].
C. Partial cross sections

The partial diffractive cross sections for the production of longitudinal and transverse vector mesons are given by

\[
\frac{d\sigma_{L,T}}{dt} = \frac{1}{16\pi s^2} |A_{L,T}|^2
\]  

with the virtual-photon-vector-meson transition amplitudes following from the contraction of the helicity matrix elements (17-20). The results are

\[
L: \quad \Psi_\psi \psi = \frac{2\sqrt{N_c}}{\pi} e e_V (x\bar{x})^2 QK_0 f_L(x, r)
\]

\[
T: \quad \Psi_\psi \psi = \frac{\sqrt{N_c}}{\pi \sqrt{2}} e e_V \times \left( \eta_M (x^2 + \bar{x}^2) K_1 (-\partial_r) + \frac{m^2_V}{M_V} K_0 \right) f_T(x, r)
\]

The vector charge \( e_V \) is computed as the average charge

\[ e_V = \left| \sum f_a f_f \right| \]

in a state with flavor content \( V = \sum f_a f_f \). The elastic differential cross section follows as

\[
\frac{d\sigma_{el}}{dt} = \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt}
\]

IV. \( f_{L,T} \) FROM HOLOGRAPHY

The intrinsic light cone distributions in the vector mesons is inherently non-perturbative. Our holographic set-up for the description of the \( \gamma + p \rightarrow V + p \) process as a dipole-dipole scattering through a holographic pomeron in AdS_5 suggests that we identify the intrinsic light cone distributions \( f_{L,T} \) with the holographic wave function of massive Spin-1 mesons in AdS_5. The mass will be set through a tachyon field in bulk.

A. AdS model for Spin-1

With this in mind, consider an AdS_5 geometry with a vector gauge field \( A \) and a dimensionless tachyon field \( X \) described by the non-anomalous action

\[
S = \int d^4xdz \left( \frac{1}{2} \frac{1}{z} F^{MN} F_{MN} - \frac{1}{z^3} |DX|^2 + \frac{2}{z^5} |X^2|^2 \right)
\]

with \( DX = dX + AX \) and \( F = dA, M, N = 0, 1, 2, 3, z \) and signature \((-++, ++, +, +)\) The coupling \( g_5^2 = 12\pi^2/N_c \) is fixed by standard arguments [12]. The background tachyon field satisfies

\[
\frac{d}{dz} \left( \frac{1}{z^3} \frac{dX}{dz} \right) + \frac{3}{z^5} X = 0
\]

which is solved by

\[ X(z) \approx c_1 z + c_2 z^3 \]

The constants in (27) are fixed by the holographic dictionary [5, 12] near the UV boundary (\( z \approx 0 \))

\[ X(z) \approx Mz + \langle \bar{Q}Q \rangle z^3 \]

In the heavy quark limit \( \langle \bar{Q}Q \rangle \rightarrow 0 \), so \( X(z) \approx Mz \).

In the presence of \( X(z) \), the vector gauge field satisfies

\[
D^M F_{MN} + \frac{4g_5^2}{z^2} X^2 A_N = 0
\]

We now seek a plane-wave vector meson with 4-dimensional spatial polarization \( \epsilon_\mu \) in the form

\[
A_M(x, z) = e^{ixx\mu} (\sqrt{z} \varphi(z)) \delta_{M\mu} \epsilon_\mu(p)
\]

which yields

\[
-\varphi'' + \frac{3}{z^2} \varphi + \frac{4g_5^2}{z^2} X^2 \varphi = -p^2 \varphi
\]

We now use the solution for \( X(z) \approx c_1 z + c_2 z^3 \) with \( c_2 = 0 \) (no heavy chiral condensate), and identify \( 4g_5^2 c_1 = (2m_f)^2 \) with \( m_f \) the (constituent) quark mass. Thus near the boundary

\[
-\varphi'' + \frac{3}{z^2} \varphi \approx (-p^2 - (2m_f)^2) \varphi
\]

We can now either solve (32) using a hard-wall by restricting (32) to the slab geometry \( 0 \leq z \leq z_0 \), or introducing a soft wall [24]. The former is a Bessel function with a spectrum that does not Reggeizes, while the latter is usually the one favored by the light-cone with a spectrum that Reggeizes. The minimal soft wall amounts

\[
-\varphi'' + \frac{3}{z^2} \varphi + \kappa^2 z^2 \varphi = (-p^2 - (2m_f)^2) \varphi
\]

Defining \( E = M^2 - (2m_f)^2 \), it follows that

\[
M^2 = 4\kappa^2(n+1) + (2m_f)^2
\]

\[ \varphi_n(z) \sim (\kappa z)^{\frac{1}{2}} e^{-\frac{1}{2}\kappa^2 z^2} L_n(\kappa^2 z^2) \]

The meson spectrum Reggeizes. The value for \( \kappa = \sqrt{\sigma_T/2} \approx \frac{1}{2} \text{GeV} \) is fixed by the string tension.
B. Intrinsic wave functions

We now suggest that the holographic wavefunction

$$\varphi_{n=0}(z) \sim (\kappa z)^{\frac{3}{2}} e^{-\frac{1}{2}\kappa^2 z^2}$$  \hspace{1cm} (35)$$

can be related to the intrinsic amplitudes $f_{L,T}$ for the dipole distribution in the light cone wavefunctions for the vector mesons in (20). For that we note that the main part of the transverse vector in (20) satisfies $\bar{\Psi}^T \sim \sqrt{v} f_T$. With this in mind, we identify the holographic coordinate $z$ with the relative dipole size $r$ through $z = \sqrt{x^2 + y^2}$. Solving for $f_T$ we obtain

$$f_T(x,r) = 2\kappa(x\bar{x})^{\frac{3}{2}} e^{-\frac{1}{2}\kappa^2 x\bar{x}^2}$$ \hspace{1cm} (37)$$

which normalizes to 1

$$\int \frac{d^2 r dx}{4\pi} |f_T(x,r)|^2 = 1$$ \hspace{1cm} (38)$$

For a massive spin-1 meson with the helicity content and quark mass analogous to the $\gamma^*$ ~ $\bar{q}q$ content as ansatz in (37), we will assume the holographic dipole content derived in (37), with instead general overall constants

$$f_{T,L}(x,r) \rightarrow N_{T,L}(x\bar{x})^{\frac{3}{2}} e^{-\frac{1}{2}\kappa^2 x\bar{x}^2}$$ \hspace{1cm} (39)$$

(39) is in agreement with the intrinsic dipole wave function developed in [14] using the light cone holographic procedure for $m_f = 0$. We note that (35) describes a massive spin-1 gauge field in AdS$_5$.

V. LEPTONIC DECAY CONSTANTS

The size of the light cone wavefunction is empirically constrained by the electromagnetic decay width $V \rightarrow e^+e^-$ as captured by the measured vector decay constant $f_V$ for each of the vector mesons,

$$\langle 0 | J^\mu_{em} (0) | \psi_{V,L,T} (q) \rangle = e f_V M_V e^\mu (T,L)(q)$$ \hspace{1cm} (43)$$

This puts an empirical constraint on the longitudinal and transverse light cone wavefunctions (37) using the holographic intrinsic wavefunctions (39) as suggested earlier.

A. Longitudinal

More specifically, the longitudinal wavefunction gives for the right-hand-side in (43)

$$e f_V M_V e^\mu_L(q) \rightarrow e f_V q^+$$ \hspace{1cm} (44)$$

as $q^+ \rightarrow \infty$, with the conventions $q^2 = q^+ q^- = -Q^2 = -M^2_V$. The left-hand-side in (43) can be reduced using the light cone rules in the Appendix of [23] together with the longitudinal wavefunction (20) to have

$$\langle 0 | J^\mu_{em} (0) | \psi_{V,L,T} (q) \rangle = \int_0^1 dx \left( e e f_V \sqrt{N_L} \delta_{h,-h} \frac{2\bar{x}x}{\sqrt{x^2 + y^2}} q^+ \right) \times \left( \int \frac{d^2 r d^2 k}{(2\pi)^4} e^{ikr} \delta_{h,-h} x\bar{x} f_L(x,r) \right)$$ \hspace{1cm} (45)$$

The first bracket refers to the reduction of the current, and the second bracket to the reduction of the longitudinal wavefunction. The result for the vector decay constant from the longitudinal current $J^\mu_{em}$ is

$$\frac{f_L}{\kappa} = e f_V \sqrt{N_L} \frac{M^2_V}{32}$$ \hspace{1cm} (46)$$

after the use of the normalization $N_L$ as given in (42). For example, for the rho meson $f_\rho/\kappa = 9\sqrt{5}/(32\sqrt{2})$, while for the phi meson $f_\phi/\kappa = 3\sqrt{5}/32$.

B. Transverse

For a consistency check, the same rules apply to the transverse component of the current $J^\mu_{em}$. The transverse wavefunction gives for the right-hand side of (43)
\[ e f^T \gamma_5 M_V e^{\gamma_5}(q) \rightarrow e f^T \gamma_5 M_V \left( -\frac{1}{\sqrt{2}} \right) \] (47)

The left-hand-side can be reduced using also the light cone rules

\[ \langle 0 | J^1_{\text{em}}(0) | \psi^T(q) \rangle = \int_0^1 dx \int \frac{d^2r d^2k}{(2\pi)^3} e^{ik \cdot r} \times \frac{e e_V \sqrt{N_c}}{\sqrt{2 x \bar{x}}} \left( \frac{xk^- - \bar{x}k^+}{\sqrt{x \bar{x}}} \delta_{h,-h} - \frac{m_f}{\sqrt{x \bar{x}}} \delta_{h,h} \right) \times \left( \frac{i f_{T}}{M_V} \right) \frac{1}{r} \left( -\frac{1}{r} \right) f_T(x, r) \times f_T(x, 0) \] (49)

Inserting (49) in (48) gives for the left-hand-side

\[ \langle 0 | J^1_{\text{em}}(0) | \psi^T(q) \rangle = \frac{e e_V \sqrt{N_c}}{2\pi} \int_0^1 dx \left( \frac{m_f^2 + \kappa^2 x \bar{x}(x^2 + \bar{x}^2)}{M_V x \bar{x}} \right) f_T(x, 0) \] (50)

which reduces to

\[ f^T f^T \kappa = \frac{e e_V \sqrt{N_c}}{\pi \sqrt{2}} \left( \frac{40 M^2_V \kappa^2}{\kappa^2 + 20 m_f^2} \right)^{\frac{1}{2}} \times \int_0^1 \frac{dx}{\sqrt{x \bar{x}}} \left( \frac{m_f^2}{M^2_V} + x \bar{x} (x^2 + \bar{x}^2) \frac{\kappa^2}{M^2_V} \right) \] (51)

Substituting the value of \( \kappa \) from the Regge spectrum (34) yields the transverse to longitudinal ratio for the decay constants

\[ \frac{f^T f^T}{f^T f^T} = \frac{1}{6 \sqrt{3}} \frac{59 \zeta^2 + 5}{(1 + 19 \zeta^2) \zeta^2} \] (52)

with \( \zeta = 2m_f/M_V \). In Fig. 2 we show the behavior of (52) in the range \( \zeta = 0, 1 \) from the massless to the heavy quark limit where it reaches 1.

### Table I: Holographic parameters along with the model prediction for \( M_V, f_V \). See text.

| Parameter | \( M_V \) [GeV] | \( f_V \) [MeV] | \( f^T/f^T \) | \( m_f \) [MeV] | \( \kappa \) | \( g^* \) |
|----------|-----------------|-----------------|--------------|-----------------|-------------|-------------|
| \( \rho \) | 1000            | 186.9           | 1.087        | 0.380           | 0.325       | 0.63        |
| \( \omega \) | 971             | 44.8            | 1.114        | 0.302           | 1.46        |
| \( \phi \) | 1172            | 78.67           | 1.096        | 0.450           | 0.375       | 0.44        |
| \( J/\Psi \) | 3185            | 153.3           | 1.344        | 1.550           | 0.366       | 1.50        |
| \( \Upsilon \) | 9472            | 49.0            | 1.367        | 4.730           | 0.234       | 5.00        |
| \( s_0 \) | 0.1 GeV^2       |                 |              |                 |             |             |
| \( z_0 \) | 1.8 GeV^{-1}    |                 |              |                 |             |             |

### VI. Numerical Analysis

To carry out the numerical analysis, we can partially eliminate the model dependence in the transition amplitudes (22), by trading \( \kappa \) in the normalizations \( N_{L,T} \) in (42) with (46) to obtain

\[ L: \quad \Psi^+_V \Psi_\omega = \frac{128}{3\pi} e f^L_V (x \bar{x})^2 Q K_0 F(x, \kappa r) \]

\[ T: \quad \Psi^+_V \Psi_\omega = \frac{128}{3\pi} e f^L_V \left( \frac{1}{3} + 57 \zeta^2 \right) \frac{1}{2} \times \left( \frac{e}{M_V} (x^2 + \bar{x}^2) K_1 (\kappa r) + \frac{m_f^2}{M_V} K_0 \right) F(x, \kappa r) \] (53)

Here we have set
\( F(x, \kappa r) = (x\bar{x})^{1/2} e^{-\frac{1}{2} \kappa^2 x\bar{x}r^2} \) \hspace{1cm} (54)

with \( \kappa \) fixed by the ground state meson mass in (34)

\[ \kappa = \frac{M_V}{2} \sqrt{1 - \zeta^2} \] \hspace{1cm} (55)

With the exception of \( g_s, \kappa, m_f \), all holographic parameters \( D_\perp, \lambda, \sigma_0, \zeta_p \) are fixed by the DIS analysis in [9] as listed in Table I. For the light vector mesons, we have set \( m_{u,d,s} \) at their constituent values, and \( m_{c,b} \) at their PDG values. The value of \( \kappa \) is adjusted to reproduce the best value for the vector meson decay constants. The vector masses \( M_V \) are then fixed by (55) as listed in Table I. In our holographic set up, the lower decay constants for the heavier mesons imply smaller values of \( \kappa \) (string tension) for \( J/\Psi, \Upsilon \) in comparison to the \( \rho \) for instance. Since \( f_\rho^2 \) is a measure of the compactness of the wavefunction at the origin this is reasonable, although the spread in the transverse direction appears to be larger in the absence of the Coulombic interactions which are important for \( J/\Psi, \Upsilon \). Finally, the string coupling \( g_s \) is adjusted to reproduce the overall normalization of the cross section for each vector meson channel.

A. Radiative widths

In terms of (46), the radiative decay width \( \Gamma(V \to e^+e^-) \) is

\[ \frac{\Gamma}{e^V} = \frac{4\pi\alpha^2 f_L^2}{3M_V e^V} \] \hspace{1cm} (56)

We note that (46) is finite in the heavy quark limit as expected from the Isgur-Wise symmetry. Using (34), (56) gives

\[ (\rho : 9.32; \omega : 8.30; \phi : 10.6; J : 3.71; \Upsilon : 0.51) \text{ KeV} \] \hspace{1cm} (57)

The empirical ratios of the width to the squared charge are

\[ (\rho : 13.2; \omega : 12.8; \phi : 11.8; J : 10.5; \Upsilon : 10.6) \text{ KeV} \] \hspace{1cm} (58)

with \( e_V \) fixed by (23). The holographic decay widths are in agreement with the empirical ones for the light vector mesons \( \rho, \omega, \phi \), but substantially smaller for the heavy vector mesons \( J/\Psi, \Upsilon \). This maybe an indication of the strong Coulomb corrections in the heavy quarkonia missing in our current holographic construction. One way to remedy this is through the use of improved holographic QCD [35].

B. \( \gamma p \to \rho p, \omega p \)

In Fig. 3 we show the differential \( \rho \)-photoproduction versus \( |t| \) for \( E_\gamma = 2.8 \text{ GeV} \). At this energy the photon size is of the order of the hadronic sizes and sensitive to non-perturbative physics. In Fig. 4 we show the total cross section for \( \omega \)-photoproduction in the range of low mass photons. The discrepancy close to threshold maybe due to \( t \)-channel sigma-exchange and the \( s \)-channel photo-excitation of the \( \Delta(1230), N(1520), N(1720) \) in the intermediate nucleon state, not retained in our analysis. Note that both the \( \rho \) and \( \omega \) have comparable transverse sizes with \( 1/\kappa \approx 3 \text{ fm} \) but very different decay constants. We expect their differential and total cross sections to be in the ratio of their decay constant, say \( f_\omega^2/f_\rho^2 \approx \frac{1}{10} \).

FIG. 3: Differential cross section for \( \gamma p \to \rho p \) versus \( |t| \) for \( E_\gamma = 2.8 \text{ GeV} \): the solid line is this work, the filled circle are the data. Data are taken from [25].

FIG. 4: Total cross section for \( \gamma p \to \omega p \) versus \( E_\gamma \). The solid line is this work, the filled circles are the data. Data are taken from [26].
C. $\gamma p \rightarrow \phi p$

In Figs. 5-8, we present the total and differential cross section for the $\phi$-photoproduction $\gamma p \rightarrow \phi p$ process. In Fig. 6 we compare our results to the available CLAS and LEPS data. Our results agree with the backward angle data well, but overshoot the forward angle data. In Fig. 7-8, the differential cross sections are shown. The agreement at large $\sqrt{s}$ probes mostly the Pomeron exchange. Note that our overall fit to the $\phi$-decay constant implies a transverse size for the $\phi$ that is comparable to the $\rho, \omega$ sizes, which is reasonable. The differential and total cross sections are expected to be in the ratio of the squared decay constants $f_{\phi}^2/f_{\rho}^2 \approx \frac{1}{5}$ or $f_{\phi}^2/f_{\omega}^2 \approx \frac{1}{2}$.

D. $\gamma p \rightarrow J/\Psi p, \Upsilon p$

In Figure 9 we show the differential cross section for $\gamma p \rightarrow J/\Psi p$ process, and in Figure 10 we show the differential cross section for $\gamma p \rightarrow \Upsilon p$ process. We note that $2m_f = 2.58, 8.83$ GeV respectively, so $\sqrt{s} > 10$ GeV are necessary to eikonalized the heavy quarks. These results are only exploratory, since the transverse sizes of the $J/\Psi, \Upsilon$ are large in our current construction as we noted earlier. To remedy this shortcoming requires including the effects of the colored Coulomb interaction which is important in these quarkonia states. In holography this can be achieved through the use of improved holographic QCD [35] which is beyond the scope of our current analysis.

VII. CONCLUSIONS

In QCD the diffractive photo-production of vector mesons on protons at large $\sqrt{s}$ is described as the scattering of two fixed size dipoles running on the light cone.
and exchanging a soft pomeron. In a given hadron the distribution of fixed size dipoles is given by the intrinsic dipole distribution in the light cone wavefunction. The soft pomeron exchange and the intrinsic dipole distribution are non-perturbative in nature. We use the holographic construct in AdS$_5$ to describe both.

The soft Pomeron parameters used in this work were previously constrained by the DIS data [9], so the extension to the photoproduction mechanism is a further test of the holographic construction. The new parameter characterizing the transverse size of the vector mesons was adjusted to reproduce the meson radiative decays and found to be consistent with the expected string tension characteristic of the vector Regge trajectory. Comparison of our results to the data for photoproduction of vectors show fair agreement with data for the $\rho, \omega, \phi$, although the inclusion of Reggeon exchanges may improve our description at low photon masses near threshold. At high photon masses, perturbative QCD scaling laws are expected. Our analysis of the photoproduction of $J/\Psi, \Upsilon$ is limited since the present construction does not account for the substantial Coulomb effects for these quarkonia. We hope to address this issue and others next.

VIII. ACKNOWLEDGEMENTS

This work was supported by the U.S. Department of Energy under Contract No. DE-FG-88ER40388. HYRyu and CHL were partially supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (No. 2015R1A2A2A01004238 and No. 2016R1A5A1013277).

[1] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. 100, 1 (1983).
[2] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP 45, 199 (1977) [Zh. Eksp. Teor. Fiz. 72, 377 (1977)].
[3] I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978) [Yad. Fiz. 28, 1597 (1978)].
[4] G. Veneziano, Nuovo Cim. A 57, 190 (1968).
[5] J. M. Maldacena, Int. J. Theor. Phys. 38, 1113 (1999) [Adv. Theor. Math. Phys. 2, 231 (1998)] [hep-th/9711200]; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B 428, 105 (1998) [hep-th/9802109]; E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998) [hep-th/9803131]; I. R. Klebanov and E. Witten,
[6] M. Rho, S. J. Sin and I. Zahed, Phys. Lett. B 466, 199 (1999) [hep-th/9907132].
[7] R. A. Janik and R. B. Peschanski, Nucl. Phys. B 586, 163 (2000) [hep-th/0003059].
[8] R. C. Brower, J. Polchinski, M. J. Strassler and C. I. Tan, JHEP 0712, 005 (2007) [hep-th/0603115]; R. C. Brower, M. J. Strassler and C. I. Tan, JHEP 0903, 092 (2009) [arXiv:0710.2730 [hep-th]].
[9] A. Stoffers and I. Zahed, Phys. Rev. D 87, 075023 (2013) [arXiv:1205.3223 [hep-ph]]; A. Stoffers and I. Zahed, Phys. Rev. D 88, no. 2, 025038 (2013) [arXiv:1211.3077 [nucl-th]].
[10] A. Stoffers and I. Zahed, arXiv:1210.3724 [nucl-th].
[11] G. Basar, D. E. Kharzeev, H. U. Yee and I. Zahed, Phys. Rev. D 85, 105005 (2012) [arXiv:1202.0831 [hep-th]].
[12] J. Ballam et al., Phys. Rev. D 74, 015005 (2006) [hep-ph/0602229].
[13] J. Nemchik, N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B 341, 228 (1994) [hep-ph/9405355]; J. Nemchik, N. N. Nikolaev, E. Predazzi and B. G. Zakharov, Phys. Lett. B 374, 199 (1996) [hep-ph/9604419]; J. Nemchik, N. N. Nikolaev, E. Predazzi and B. G. Zakharov, Z. Phys. C 75, 71 (1997) [hep-ph/9605231]; H. G. Dosch, T. Gousset, G. Kulzinger and H. J. Pirner, Phys. Rev. D 55, 2602 (1997) [hep-ph/9608203]; G. Kulzinger, H. G. Dosch and H. J. Pirner, Eur. Phys. J. C 7, 73 (1999) [hep-ph/9806352].