NON-NUCLEONIC EFFECTS STUDIED BY NUCLEAR MOMENTS

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Measured magnetic moments of single-particle states in the Pb region differ significantly from their Schmidt values. We discuss the reasons for this in terms of meson-exchange currents, isobar currents, core polarisation and other effects.

1 Meson-exchange currents

Nuclear physics calculations are based on the nonrelativistic Schrödinger equation in which wavefunctions for a many-body state are computed as a first step and expectation values of observables calculated as a second step. For processes such as $\gamma$- and $\beta$-decay the observable is represented by one-body operators, whose particular form can easily be deduced once the relativistic interaction between the currents and the fields is written down. For example, the interaction of a nucleon charge current with an electromagnetic field is given by a Hamiltonian $H = -J_\mu A_\mu$ where $A_\mu$ is the vector potential describing the field and $J_\mu$ is the charge current of a single nucleon:

$$J_\mu(k) = i\pi(p') \left[ F_1 \gamma_\mu - \frac{F_2}{2M} \sigma_\mu k_\mu \right] u(p),$$  

where $k = p' - p$. Here $\pi(p')$ and $u(p)$ are plane-wave Dirac spinors, and $F_1$ and $F_2$ are the Dirac and anomalous coupling constants. A nonrelativistic form of the charge current $J_\mu = (\rho, J)$ is obtained by multiplying out the Dirac spinors with the Dirac matrices and keeping only terms to leading order in $1/M$ — this produces a one-body operator sandwiched between Pauli spinors:

$$\rho(k) = F_1 + O(1/M^2)$$
$$J(k) = \frac{F_1}{2M} (p' + p) + \frac{F_1 + F_2}{2M} i\sigma \times k + O(1/M^3),$$

where for economy the Pauli spinors associated with $\pi(p')$ and $u(p)$ have been omitted. A Fourier transform to coordinate space and a multipole decomposition leads to the familiar one-body electromagnetic operators, whose matrix elements are then evaluated in the many-body system using, for example, shell-model wavefunctions. When the coupling constants for the nucleon current embedded in the nuclear medium are taken from the free-nucleon system then the procedure is called the impulse approximation. It has been widely successful, but at some point it is known to break down. This is because the nucleons in the nucleus are interacting through the exchange of mesons and the perturbing electromagnetic field can disturb this exchange.
and even interact with the exchanged meson itself. As has been dramatically shown by Kubodera, Delorme and Rho with soft-pion theorems, the most important meson-exchange corrections (MEC) occur in magnetic moments and transitions (operators originating in the space component of the vector current, $J_\mu$) and in axial-charge transitions in beta decay (operators originating in the time part of the axial current, $J_{5\mu}$).

There are different approaches to the construction of the two-body MEC operators that are equivalent up to certain unitary transformations. They are: (i) the quasipotential formalism in which one starts from the covariant four-dimensional Bethe-Salpeter equation and uses the Blankenbecler-Sugar reduction to get a three-dimensional equation; (ii) the equations of motion method in which unwanted degrees of freedom are eliminated by a unitary FST transformation and the non-relativistic reduction effected by a Foldy-Wouthuysen transformation; and (iii) the extended S-matrix formalism in which the S-matrix is expanded in a series of Feynman diagrams. Our calculations are based on this latter approach using a phenomenological chiral Lagrangian for the isovector mesons, $\pi$, $\rho$, and $A_1$, and augmented by the standard isoscalar meson Lagrangians for $\sigma$ and $\omega$ mesons.

1.1 ‘External’ corrections

There are two types of Feynman diagrams to consider: (i) those in which the electromagnetic field interacts with the nucleon charge current, $J_\mu(k)$, to be known generically as “pair” graphs; and (ii) those in which the electromagnetic field interacts with the exchanged meson's charge current, to be known as “current” graphs. There is one comment we wish to make concerning the construction of the “pair” graph. The general form of its expression involves the product

$$J_\mu(k)S(Q)\Gamma(-q),$$

where $J_\mu(k)$ is the nucleon charge current, Eq. (1), $S(Q)$ is the nucleon propagator of momentum $Q$, and $\Gamma(-q)$ is the meson-production vertex of momentum $q$. To avoid double counting, the contribution from the standard one-nucleon current must be carefully identified and separated from the “pair” graph leaving the genuine two-nucleon MEC operators. This separation is usually done by dividing the nucleon propagator into positive and negative frequency components

$$S(Q) = S^{(+)}(Q) + S^{(-)}(Q)$$

and retaining only the negative frequency component, $S^{(-)}(Q)$. This gives the correct result providing $J_\mu(k)$ and $\Gamma(-q)$ are energy independent, i.e. $J_\mu(k)$ does not depend on $k_0$ and $\Gamma(-q)$ does not depend on $q_0$. However, for the magnetic moment, which derives from the spatial component of $J_\mu(k)$, Eq. (1), there is a dependence on $k_0$ coming from the anomalous term $(F_2/2M)\sigma_{ij}k_0$. In this case Adam et al define
\[ J_\mu(k) = \overline{J}_\mu(k) + k_0 J'_\mu(k), \]  

(5)

where \( \overline{J}_\mu(k) \) has no dependence on \( k_0 \). Thus \( J'_\mu(k) = \partial / (\partial k_0) J_\mu(k) \). Similarly defining

\[ S^{(+)}(Q) = \frac{1}{k_0} S'^{(+))(Q)} \]

(6)

then an additional term from the “pair” graph that contributes a genuine MEC involves the combination

\[ J'_\mu(k) S'^{(+))(Q)} \Gamma(-q). \]

(7)

Such corrections are called ‘external’ by Adam et al.\(^{11}\) Similarly when \( \Gamma(-q) \) depends on \( q_0 \) there are ‘vertex’ corrections. The external corrections contribute to leading order, \( \mathcal{O}(1/M^2) \), for scalar and vector meson exchanges, and to next-to-leading order in \( \mathcal{O}(1/M^4) \) for pseudoscalar meson exchanges. Here \( M \) is the nucleon mass. Vertex corrections only contribute to next-to-leading order. We take this opportunity to add the ‘external’ corrections to our previously computed\(^{13}\) MEC corrections for magnetic moments in the Pb region. The correction does not have a large impact, being of order ~ ±0.08\( \mu_N \) with a plus sign for a single-particle proton state with \( j = \ell + \frac{1}{2} \) or a neutron with \( j = \ell - \frac{1}{2} \), and a minus sign in the other two cases.

1.2 MEC corrections for states in Pb region

The procedure for constructing the two-body MEC operators for a magnetic moment calculation is as follows\(^{13}\):

- Write down expressions for the “pair” and “current” Feynman diagrams in terms of Dirac spinors for the nucleons for the case when the Lorentz index on the charge current, \( J_\mu(k) \), is space-like;
- Expand the Dirac spinors in powers of \((1/M)\) and retain leading terms, to be denoted \( J(k) \);
- Construct the magnetic moment operator \( \mu = -\frac{1}{2}i \nabla_k \times J(k) \big|_{k \to 0} \);
- Fourier transform to coordinate space to obtain the two-body operator, \( \mu(\mathbf{r}, R) \), where \( r = r_1 - r_2 \) and \( R = \frac{1}{2}(r_1 + r_2) \) are relative and centre of mass coordinates;
- Compute matrix elements of this operator in a many-body system using, for example, nuclear wavefunctions from a shell-model calculation.

We can simplify the last stage by only considering matrix elements of the magnetic moment operator in closed-shell-plus (or minus)-one configurations. For the impulse-approximation one-body operator the calculation reduces simply to the single-particle
Table 1. Meson-exchange current corrections to magnetic moments in $^{209}$Bi and $^{209}$Pb.

|        | Proton $0h_{9/2}$ | Neutron $0i_{13/2}$ |
|--------|-------------------|----------------------|
|        | $\delta g_L$     | $\delta g_S$ | $\delta g_P$ | $\Delta \mu$ | $\delta g_L$ | $\delta g_S$ | $\delta g_P$ | $\Delta \mu$ |
| $\pi$-pair | 0.303 | 0.725 | 0.302 | 1.16 | -0.168 | -0.336 | 0.004 | -1.18 |
| $\pi$-current | -0.247 | -0.384 | -1.048 | -0.94 | 0.136 | 0.215 | 1.164 | 1.02 |
| $\rho$-$\pi$ | -0.000 | -0.002 | 0.019 | -0.00 | -0.000 | -0.001 | 0.017 | 0.00 |
| $\omega$-$\pi$ | -0.000 | -0.031 | 0.151 | -0.01 | 0.000 | 0.026 | -0.190 | 0.00 |
| $A_1$-$\pi$ | 0.000 | 0.002 | 0.019 | -0.00 | -0.000 | -0.001 | -0.014 | -0.00 |
| $\sigma$-pair | 0.033 | 0.227 | 0.500 | 0.02 | 0.002 | -0.186 | -0.560 | -0.13 |
| $\omega$-pair | -0.004 | -0.080 | -0.244 | 0.04 | -0.004 | 0.071 | 0.261 | 0.03 |
| $\rho$-pair | 0.078 | 0.072 | -0.057 | 0.36 | -0.042 | -0.009 | 0.046 | -0.25 |
| $\rho$-current | 0.036 | 0.046 | 0.029 | 0.16 | -0.020 | -0.013 | -0.019 | -0.13 |
| $A_1$-current | -0.013 | -0.006 | 0.007 | -0.06 | 0.007 | 0.003 | -0.007 | 0.04 |
| Total | 0.186 | 0.560 | -0.346 | 0.72 | -0.089 | -0.226 | 0.713 | -0.59 |

Matrix element for the valence orbital outside the closed shell (the Schmidt value). Recall that the impulse-approximation magnetic-moment operator is

$$\mu = g_L L + g_S S$$ \hspace{1cm} (8)

with $g_L = 1$ and $g_S = 5.586$ for a proton, and $g_L = 0$ and $g_S = -3.826$ for a neutron. The magnetic-moment expectation value for a single particle of angular momentum $a$ is then

$$\mu = \left( \frac{a}{a+1} \right)^{1/2} \langle a | g_L L + g_S S | a \rangle$$ \hspace{1cm} (9)

The reduced matrix elements are defined in the conventions of Brink and Satchler[14]. For the two-body MEC operators, the calculation becomes one of computing two-body matrix elements between the valence nucleon and one of the nucleons in the closed-shell core and summing over all the nucleons in the core. It is useful to express the result of this computation in terms of an equivalent one-body operator

$$\mu_{\text{eff}} = g_{L,\text{eff}} L + g_{S,\text{eff}} S + g_{P,\text{eff}} [Y_2, S]$$ \hspace{1cm} (10)

where $g_{L,\text{eff}} = g_L + \delta g_L$, etc. Here $g_L$ is the free-nucleon coupling constant of Eq. (8) and $\delta g_L$ the calculated correction.

In Table 1 we give some sample results for magnetic moments for a valence $0h_{9/2}$ proton and $0i_{13/2}$ neutron outside a $^{208}$Pb closed-shell core. Harmonic oscillator radial wavefunctions, $\hbar \omega = 7.0$ MeV, are used. To accommodate the role of short-range.
correlations in the nuclear medium, we have multiplied the two-body MEC operator by a correlation function \( g(r) \) chosen to be \( g(r) = \theta(r - d) \) with \( d = 0.7 \text{ fm} \). We also include vertex form factors at all meson-nucleon vertices, taking care to ensure that the equation of continuity remains satisfied.

Let us look at the \( \delta g_L \) value. In the S-matrix approach, the pion-pair and pion-current diagrams give large contributions but there is a significant cancellation between them. For example, for the \( h_9/2 \) proton at \(^{209}\text{Bi}\) we compute \( \delta g_L = 0.303 - 0.247 = 0.056 \) with vertex form factors and \( \delta g_L = 0.325 - 0.230 = 0.095 \) without. This latter value is very close to the estimate first obtained by Miyazawa\(^{17}\) of \( \delta g_L \approx 0.1 \) from pion-exchange currents. We find, however, that heavy mesons make a substantial contribution to \( \delta g_L \). Our result in Table 1 is \( \delta g_L = 0.056 + 0.130 = 0.186 \) for a \( 0h_9/2 \) proton and \( \delta g_L = -0.032 - 0.057 = -0.089 \) for a \( 0i_{13}/2 \) neutron, where the first figure represents contributions from pions, the second from heavy mesons. As a consequence, the magnetic-moment correction from MEC is positive for protons and negative for neutrons.

The first experimental indication that the proton \( g_L \) value is enhanced in nuclei by about 10% over its free-nucleon value came in the measurement of Yamazaki et al.\(^{18}\) of the magnetic moment of the \( 11^- \) isomer in \(^{210}\text{Po}\). Nagamiya and Yamazaki\(^{19}\) later showed the enhancement to be a general phenomenon present throughout the whole mass region. A systematic analysis\(^{20}\) of all magnetic moment data in the Pb region produces as best-fit values: \( \delta g_L = 0.15 \pm 0.02 \) for a proton, and \( \delta g_L = -0.03 \pm 0.02 \) for a neutron. Our results in Table 1 are in reasonable accord with this expectation, but there are other corrections, core polarisation, isobar currents etc., to be considered before a serious comparison with experimental data can be made.

1.3 Isobar currents

The MEC currents constructed so far involve nucleons interacting with mesons. There is a further process to be considered, in which the meson prompts the nucleon to be raised to an excited state, the \( \Delta \)-isobar resonance, which is then deexcited by the electromagnetic field. Such processes as these are called ‘isobar currents’. In Table 2 it is seen that, the isobar current mainly quenches the spin \( g_s \)-factor by about 10% over its free-nucleon value producing a magnetic-moment correction that is positive for protons in \( j = \ell + \frac{1}{2} \) orbits and neutrons in \( j = \ell - \frac{1}{2} \) orbits, and negative otherwise.

2 Core polarisation

The nuclear wave function in a nucleus such as \(^{209}\text{Bi}\) is more complicated than a single particle coupled to a closed-shell core and will have components of \( 2p-1h \) and \( 3p-2h \) configuration whose impact on the magnetic-moment expectation value can be estimated in perturbation theory. To first order in the residual interaction, \( V \), the only
contributions arise from particle-hole states coupled to the same angular momentum as the multipolarity of the operator, \( \lambda \); namely, \( \lambda = 1 \) for magnetic moments. At the \(^{208}\text{Pb}\) closed shell, there are two such particle-hole states: proton \((h_{11/2}^{-1} h_{9/2})\) and neutron \((i_{13/2}^{-1} i_{11/2})\), and their contribution to the magnetic moment of \(^{209}\text{Bi}\) to first- and second-order is given by the expression:\[\tag{11}\]

\[
\langle a | \Delta \mu | a \rangle = 2 \sum_{\alpha} T_{\alpha} \frac{L_{\alpha}}{\epsilon_{\alpha}} + 2 \sum_{\alpha \beta} T_{\alpha} \frac{(A - B)_{\alpha\beta} L_{\beta}}{\epsilon_{\alpha} \epsilon_{\beta}}
\]

where Greek letters represent the particle-hole coupled states, viz. \(| \alpha \rangle = | (h_{-1}^{-1} p_{\alpha}) \lambda \rangle\) of multipolarity \( \lambda \) and the following notation is introduced:

\[
T_{\alpha} = \langle 0 | \mu | (h_{-1}^{-1} p_{\alpha}) \lambda \rangle
\]
\[
L_{\alpha} = -\hat{a}^{-1} \langle (h_{-1}^{-1} p_{\alpha}) \lambda | V | (a^{-1} a) \lambda \rangle
\]
\[
A_{\alpha\beta} = \langle (h_{-1}^{-1} p_{\alpha}) \lambda | V | (h_{-1}^{-1} p_{\beta}) \lambda \rangle
\]
\[
B_{\alpha\beta} = \langle (h_{-1}^{-1} p_{\alpha}) \lambda | V | (h_{-1}^{-1} p_{\beta}) \lambda \rangle
\]
\[
= (-)^{h_{-1}^{-1} - p_{\alpha} + \lambda} \langle (p_{-1}^{-1} h_{\alpha}) \lambda | V | (h_{-1}^{-1} p_{\beta}) \lambda \rangle
\]
\[
\epsilon_{\alpha} = \epsilon_{h_{-1}^{-1} - p_{\alpha} + \lambda}
\]

with \( \hat{a} = (2a + 1)^{1/2} \). Here \( a \) represents the valence nucleon outside the closed-shell core. The energy denominators, \( \epsilon_{\alpha} \), are estimated from known spin-orbit splittings of \(-5.6\) MeV for the proton \( h \)-orbitals and \(-5.85\) MeV for the neutron \( i \)-orbitals. The residual interaction, \( V \), is taken as a one-boson-exchange potential involving \( \pi, \rho, \omega, \sigma \) and \( A_1 \) mesons and, for use in finite nuclei, multiplied by a short-range correlation function as described by Towner.\[\tag{12}\]

The first- and second-terms in \( V \) given in Eq. (11) can easily be extended to higher orders in the random phase approximation (RPA). It must be stressed that these are not the only second-order terms; there are others that will discuss further in Sec. 3. However, these terms are the simplest to evaluate. The selection rule that limited the number of particle-hole states, \( \alpha \), to just two still applies to all orders in RPA. The generalization of Eq. (11) is now

\[
\langle a | \Delta \mu | a \rangle = 2 \sum_{\alpha \beta} T_{\alpha} \frac{I - (A - B)/\epsilon_{\alpha \beta}^{-1} L_{\beta}}{\epsilon_{\alpha} \epsilon_{\beta}} \tag{13}
\]

where the matrix \([I - (A - B)/\epsilon]^{-1}\) is first constructed and then inverted. Here \( I \) is the unit matrix.

In Table 2 we give some results for the core-polarisation calculation, based on Eq. (13) and denoted CP(RPA). The calculation based on Eq. (11) leads to an alternating sign series: quenching in first order, enhancement in second order and so on. As a consequence, the RPA sum is between 30 and 40% less than the first-order
Table 2. Contributions to the equivalent effective one-body operator from all sources in the OBEP model for a $0\hbar$-proton and a $0i$-neutron in the Pb region.

|               | Proton $0\hbar_{9/2}$ | Neutron $0i_{13/2}$ |
|---------------|------------------------|----------------------|
|               | $\delta g_L$ | $\delta g_S$ | $\delta g_P$ | $\Delta \mu$ | $\delta g_L$ | $\delta g_S$ | $\delta g_P$ | $\Delta \mu$ |
| MEC           | 0.186     | 0.560     | -0.346     | 0.72        | -0.089     | -0.226     | 0.713       | -0.59        |
| Isobars       | -0.003    | -0.452    | 0.484      | 0.12        | 0.002      | 0.402      | -0.680      | 0.16         |
| CP(RPA)       | 0.005     | -1.167    | 0.481      | 0.45        | -0.005     | 1.034      | 0.102       | 0.50         |
| Vib           | -0.030    | -0.338    | -0.01      | -0.05       | 0.010      | 0.118      | 0.12        |              |
| Rel           | -0.024    | -0.152    | -0.041     | -0.05       | 0.010      | -0.075     | 0.000       | 0.50         |
| CP(2nd)$^a$   | -0.150    | -1.030    | -0.32      | -0.05       | 0.080      | 0.350      | 0.66        |              |
| MEC-CP$^b$    | 0.122     | 0.352     | 0.46       | 0.05        | 0.000      | 0.107      | -0.176      | -0.46        |
| Total         | 0.106     | -2.227    | 0.578      | 1.36        | -0.064     | 1.609      | 0.135       | 0.44         |

$^a$From ref\([23]\)  $^b$From ref\([24]\)

calculation alone. The calculation mainly impacts on the spin $g_s$-factor reducing it by about 25% compared to the free-nucleon value. The magnetic-moment correction is positive for protons in $j = \ell - \frac{1}{2}$ and neutrons in $j = \ell + \frac{1}{2}$, and negative otherwise.

2.1 Coupling to core-vibrational states

For single-particle states in the Pb region, especially the high-spin states, their wavefunctions are not those of pure single-particle configurations, but have some admixtures with low-lying vibrational states in the neighbouring even-even nucleus. For example, the lowest $13/2^+$ state in $^{209}$Bi is not simply a proton $i_{13/2}$ state but has a sizeable admixture of $h_{9/2} \times 3^-$. Hamamoto\([21]\) has estimated the impact of core excitations on the magnetic-moment expectation values in a particle-vibration model. The correction, denoted ‘Vib’ in Table 2, is small, $\leq 0.06\mu_N$, in all cases except the proton $i_{13/2}$ orbital and, to a lesser extent, the neutron $i_{13/2}$ orbital.

2.2 Relativistic correction

In deriving the MEC operators, a nonrelativistic reduction is made of the meson-nucleon interaction. This reduction is in essence an expansion in terms of $p/M = v/c$, where $p$ is the typical nucleon momentum and $M$ its mass. Terms to order $(p/M)^2$ were kept. The standard one-body operator, Eq. (8), has only been determined to order $(p/M)$ and we will consider its extension to the next-to-leading order, $O(p/M)^3$. The magnetic moment operator then becomes\([24]\)
\[ \mu = g_L \left\{ L \left( 1 - \frac{p^2}{2M^2} \right) - \frac{p^2}{2M^2} (S - (\mathbf{S} \cdot \hat{p}) \hat{p}) \right\} + g_S S \left( 1 - \frac{p^2}{2M^2} \right) \]  

(14)

It remains to estimate the expectation value of \( \langle \frac{p^2}{2M^2} \rangle \) for which we use harmonic oscillator wavefunctions. Since a second derivative is involved, this expectation value is very sensitive to the choice of radial wavefunction. The main result, denoted by ‘Rel’ in Table 2, is a small reduction in the calculated magnetic moment of less than 5%.

### 3 Higher orders

There are other second-order core-polarization corrections not contained in the RPA series discussed in Sec. 4. Typically these terms have an operator structure, \( V \mu V \), where the residual interaction first excites a single-particle state to a \( 2p-1h \) or \( 3p-2h \) intermediate state where the one-body magnetic-moment operator acts, then a second interaction reduces the intermediate state back to a single-particle state. There are no selection rules to limit the number of intermediate states, so the calculations are computationally time consuming and have only been completed without approximation in light nuclei. In heavy nuclei, Shimizu 22 has explored a closure approximation to estimate these terms.

There is another set of terms of the same order, namely first order in \( V \) but fourth order in meson-nucleon couplings, that have an operator structure \( VM + MV \), where \( M \) is the two-body meson-exchange current operator. Again there are no restrictions on the intermediate-state summations. The importance of these terms was first pointed out by Arima et al. 23, 24, where they are known as the ‘crossing terms’. Their contribution to the magnetic moment is of opposite sign to that from second-order core-polarization terms involving one-body operators and cancels a large part of it.

In Table 2, we quote the results of Arima et al. 23, 24, where the cancellation between the second-order core polarisation, CP(2nd), and the ‘crossing’ term, MEC-CP, is clearly evident, but in detail it varies from case to case. The quoted numbers are model dependent and parameter dependent.

Finally, we bring together all the computed corrections from core polarisation, CP(RPA), meson-exchange currents, isobar excitations and together under ‘Other’, the coupling to core-vibrational states, the relativistic correction and the higher-order processes, CP(2nd) and MEC-CP. In Table 2 the results are expressed in terms of an equivalent effective one-body operator. Consider first the orbital \( \delta g_L \) value. Our results of \( \delta g_L = 0.106 \) for protons and \( \delta g_L = -0.064 \) for neutrons are not too far from the empirical values determined in a best-fit analysis of magnetic-moment data in the Pb region of Yamazaki et al. 20 of 0.15±0.02 and −0.03±0.02 respectively. The spin \( \delta g_s \) values are dominated by core polarisation. Our results of \( \delta g_s/g_s = -40\% \) for protons
Table 3. Sum of all corrections to magnetic moments and $B(M1)$ in the Pb region (last line gives $\pi g_{7/2}$ case at $^{132}$Sn) in the OBEP model compared with experiment.

|                  | CP(RPA) | MEC | Isobars | Other | Sum   | ASBH$^a$ | Expt$^b$ |
|------------------|---------|-----|---------|-------|-------|---------|---------|
| $\pi h_{9/2}$    | 0.45    | 0.72| 0.12    | 0.08  | 1.36  | 1.49    | 1.49(0) |
| $\pi i_{13/2}$  | -0.44   | 1.07| -0.17   | -1.21 | -0.75 | -0.89   | -0.78(10)$^c$ |
| $\pi s_{1/2}^{-1}$| -0.45   | 0.32| -0.29   | -0.44 | -0.86 | -0.99   | -0.92(0) |
| $\pi d_{3/2}^{-1}$| 0.21    | 0.18| 0.10    | 0.18  | 0.67  | 0.65    | 0.64(19)$^d$ |
| $\pi f_{5/2} \rightarrow f_{7/2}$ | -0.37   | 0.12| -0.21   | -0.18 | -0.63 |         | -0.82(9)$^e$ |
| $\nu g_{9/2}$    | 0.38    | -0.37| 0.19    | 0.17  | 0.38  | 0.47    | 0.44(0) |
| $\nu p_{1/2}^{-1}$| -0.09   | -0.04| -0.01   | -0.05 | -0.18 | -0.18   | -0.05(0) |
| $\nu f_{5/2}^{-1}$| -0.30   | -0.23| -0.11   | -0.03 | -0.66 | -0.71   | -0.58(3) |
| $\nu i_{13/2}^{-1}$| 0.50    | -0.59| 0.16    | 0.37  | 0.44  | 0.75    | 0.90(3)$^f$ |
| $\nu p_{3/2}^{-1} \rightarrow p_{1/2}^{-1}$ | -0.23   | 0.04| -0.17   | -0.06 | -0.42 |         | -0.44(5)$^e$ |
| $\nu f_{7/2}^{-1} \rightarrow f_{5/2}^{-1}$ | -0.30   | 0.06| -0.18   | -0.02 | -0.44 |         | -0.52(11)$^e$ |
| $\pi g_{7/2}$    | 0.48    | 0.52| 0.08    | 0.13  | 1.21  |         | 1.28(1)$^g$ |

$^a$From ref. 25  
$^b$From ref. 26  
$^c$Deduced from the magnetic moment of the $11^-$ state in $^{210}$Po, ref. 18  
$^d$Deduced from the magnetic moment of the $3^+$ state in $^{205}$Tl, ref. 27  
$^e$Deduced from the magnetic moment of the $12^+$ state in $^{206}$Pb, ref. 29  
$^f$Deduced from the magnetic moment of the $11^-$ state in $^{210}$Po, ref. 18  
$^g$Deduced from the magnetic moment of the $13^+$ state in $^{207}$Pb, ref. 29

and $-42\%$ for neutrons show a large quenching consistent with an empirical relation often used: $g_{s,\text{eff}} = 0.6g_s$. In Table 3 we compare these corrections with all known magnetic moment and M1-transition data in closed-shell-plus (or minus)-one nuclei in the Pb region. We also list the calculated results obtained by Arima, Shimizu, Bentz and Hyuga (ASBH)$^{25}$ from core-polarisation and meson-exchange current processes. The agreement between theory and experiment is very good and is within 0.15$\mu_N$ in all cases except one, the neutron $i_{13/2}$ state in $^{207}$Pb, where the strong cancellation between core polarisation and MEC seems to be too severe. The ASBH calculation has less of a cancellation here.

Lastly, in Table 3 we have included one result for the $^{132}$Sn doubly-closed shell core. The magnetic moment of $^{133}$Sb has recently been measured at the OSIRIS mass separator of the Uppsala University Neutron Physics Laboratory by Stone et al. In terms of the calculated effective $g$-factors the results are very similar to $^{209}$Bi. The
computed magnetic moment is in good agreement with experiment.

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