Damping of differential rotation in neutron stars

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We derive the transport relaxation times for quasiparticle-vortex scattering processes via nuclear force, relevant for the damping of differential rotation of superfluids in the quantum liquid core of a neutron star. The proton scattering off the neutron vortices provides the dominant resistive force on the vortex lattice at all relevant temperatures in the phase where neutrons only are in the paired state. If protons are superconducting, a small fraction of hyperons and resonances in the normal state would be the dominant source of friction on neutron and proton vortex lattices at the core temperatures $T \geq 10^7$ K.

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A broad class of problems related to the nonequilibrium spin dynamics of pulsars require the knowledge of the fraction of neutron star fluid interiors which is coupled to the crusts on short (unobservable) time scales. In practice, one needs to identify the fraction of the moment of inertia of the superfluid in the core of the star which is responding to spin perturbations as a rigid body. A large amount of matter in neutron star models based on the modern, moderately soft, equations of state (EoS) resides at and above twice the nuclear matter density. At the densities of matter, relativistic electrons couple to the anomalous magnetic moment of the neutron quasiparticles localized in the cores of vortex lines. The knowledge of the state of the matter above these densities is limited, in particular with respect to nucleonic pairing, strangeness content, etc.; thus one is led to consider several admissible variants of the composition of matter when treating the problem of transport. Here we shall adopt this type of an approach and examine phases with purely nucleonic constituents and those containing hypernuclear matter at large densities.

Several electromagnetic channels of interaction between the electrons and the nucleonic superfluids in the quantum liquid cores of neutron stars have been invoked as mechanisms of coupling of the superfluid to the normal matter. Relativistic electrons couple to the anomalous magnetic moment of the neutron quasiparticles localized in the cores of vortex lines. At the densities of interest, the pairing in neutron matter is driven by the attractive $P$-wave interaction; a neutron vortex in a rotating $^3P_2$ superfluid acquires an intrinsic magnetization due to the pairing in the $l = 1$ state. The electron coupling to this magnetization dominates the coupling to the neutron magnetic moment at low temperatures. If protons are superconducting even larger magnetization comes about from the entrainment effect: In addition the proton superconductor in the mixed state might mediate the coupling between the superfluid neutron and electron fluids.

In this Rapid Communication we examine the nuclear interaction channels of coupling between the neutron vortex lines and the bulk quasiparticles: we show that (1) when protons are normal their scattering off the neutron vortex lines dominates electromagnetic coupling channels at all relevant densities and temperatures; and (2) if protons are superconducting, the scattering of normal (non-superfluid) heavy baryons off vortices dominates the electromagnetic coupling at the core temperatures $T \geq 10^7$ K.

To start with, consider the quasiclassical Lundag-Boltzmann kinetic equation for the proton quasiparticle distribution function

$$\left\{ \frac{\partial}{\partial t} + \frac{\partial \varepsilon_p}{\partial p} \frac{\partial}{\partial r} - \frac{\partial \varepsilon_p}{\partial r} \frac{\partial}{\partial p} \right\} f(p, r, t) = [1 - f(p, r, t)] \Sigma^< (p, r, t) - f(p, r, t) \Sigma^> (p, r, t), \quad (1)$$

where $\varepsilon_p$ is the proton quasiparticle energy and other variables have their usual meaning. The transport vertex in the quasiparticle limit is given by

$$\Sigma^< (p, r, t) = \frac{n_v^{(n)}}{V} \sum_{p', q} W(p'q', pq) \delta(\varepsilon_p + \varepsilon_q - \varepsilon_{p'} - \varepsilon_{q'}) \times f(p', r, t) f(q', r, t) [1 - f(q, r, t)], \quad (2)$$

where $W(p'q', pq)$ is the transition probability, $p$ and $q$ denote the proton and neutron momenta, respectively, $n_v^{(n)}$ is the neutron vortex density per unit area, and $V$ is the dimensional normalization. The expression for scattering-out rate, $\Sigma^> (p, r, t)$, follows from Eq. (2) by interchanging the particle and hole occupations.

Microscopically, the neutron vortex lattice is a collection of spatially localized normal quasiparticles, which are a superposition of plane waves along the vortex circulation and two-dimensional bound states in the perpendicular direction. The continuum of Bardeen-Cooper-Schrieffer (BCS) paired state (neutron condensate) feels the intervening configuration space. The transition probability for scattering of protons off these localized states (vortex cores) is given by

$$W(p'q', pq) = \frac{2\pi}{h} \left| G^R(p'q', pq; \varepsilon_p + \varepsilon_q) \right|^2 S(q, q') \times 2\pi h \delta(p_{||} - p'_{||} + q_{||} - q'_{||}), \quad (3)$$
where the structure factor of the vortex core quasiparticles is expressed through the coherence factors \((u, v)\) in the configuration space as

\[
S^{1/2}(q, q') = \left[ u_q(r_\perp) u_{q'}(r_\perp) - v_q(r_\perp) v_{q'}(r_\perp) \right] \times \exp[i(q_\perp - q'_\perp) r_\perp],
\]

where \(\|\) and \(\perp\) denote components along and perpendicular to the vector of vortex circulation. The expression \(S\) relies on the Brueckner G-matrix approximation for the transport vertices \((G^R\) is respectively the retarded and \(\psi\) is the Fermi momentum, \(D\psi\) and the vortex lattice) a good trial function is to arrive at the relaxation time

\[
\tau_{pm}^{-1} = \frac{n^{(n)}_p}{V} \sum_{p,p',q,q'} \frac{1}{2} [\psi(p) - \psi(p')]^2 W(p'q', pq) \times f(p') f(q') [1 - f(p)] [1 - f(q)] D^{-1}
\]

where \(D = \sum_p \psi(p)^2 f(p) [1 - f(p)]\),

\[
\psi(p) \text{ is the trial function, and all the distribution functions are the equilibrium ones. For our purposes (macroscopic momentum exchange in the quasiparticle current and the vortex lattice) a good trial function is } \psi(p) = p \cdot v_L, \text{ where } v_L \text{ is the vortex velocity. In this case } D = \nu(pF) \tilde{\epsilon}_{pF}^2 / \beta, \text{ where } \nu(pF) = m^*_p m_p / \pi^2 h^3 \text{ is the density of states of protons at the Fermi surface, } pF \text{ is the Fermi momentum, } m^*_p \text{ is the effective mass of protons, and } \beta \text{ is the inverse temperature. Using the relation } f(\omega_1)[1 - f(\omega_2)] = g(\omega_1 - \omega_2)[f(\omega_1) - f(\omega_2)], \text{ where } g(\omega) \text{ are the Fermi and Bose distribution functions, Eq. } 3 \text{ can be rearranged:}
\]

\[
\tau_{pm}^{-1} = \frac{n^{(n)}_p}{V} \sum_{k} \frac{1}{2} (k \cdot v_L)^2 \times D^{-1} \int_{-\infty}^{\infty} d\omega A^{(p)}(k, \omega) A^{(n)}(k, \omega),
\]

where the spectral functions of protons and neutrons are

\[
A^{(p)}(k, \omega) = \frac{2\pi}{h} \sum_{K} |G^R(K + k/2, K - k/2; \epsilon_{p+} + \epsilon_{p-})|^2 \times [f(\epsilon_{p+}) - f(\epsilon_{p-})] \delta(\epsilon_{p+} - \epsilon_{p-} + \omega),
\]

\[
A^{(n)}(k, \omega) = \sum_{q} S(k_\perp; q_\parallel + k_\parallel/2, q_\parallel - k_\parallel/2) \times [f(\epsilon_{q+}) - f(\epsilon_{q-})] \delta(\epsilon_{q+} - \epsilon_{q-} + \omega),
\]

where \(k = p - p'\) and \(K = (p + p')/2\) are the momentum transfer and the center-of-mass momentum, \(p_{+/=} = K \pm k/2\) and \(q_{+/=} = q \pm k/2\). The angle integration in Eq. \(8\) can be carried out using an angle averaged on-shell retarded G matrix, which then depends on the magnitude of the momentum transfer. To the leading order in \(\omega/kv_F\) one finds

\[
A^{(p)}(k, \omega) = \frac{\nu(pF)}{4 k \omega} |(G^R(k, pF))|^2 \theta(2pF - k),
\]

where angular brackets denote the angle averaging, \(v_F\) is the Fermi velocity of protons, and \(\theta\) is the Heaviside step function. The retarded G-matrix can be further related to the in-medium \(n-p\) differential cross section

\[
\frac{d\sigma_{pn}}{d\Omega}(k, pF) = \frac{(\nu^*)^2}{2\pi \hbar^3} |(G^R(k, pF))|^2,
\]

where \(\nu^* = m^*_n m_p / (m^*_n + m^*_p)\) is the reduced effective mass.

The eigenfunctions of neutron core quasiparticles in the case of \(1S_0\) pairing (and in a spinor notation) are

\[
\left( \begin{array}{c} u_{q, \mu}(r_\perp) \\ v_{q, \mu}(r_\perp) \end{array} \right) = e^{i\theta} \left( \begin{array}{c} e^{i\theta - \frac{\pi}{4}} \\ e^{i\theta + \frac{\pi}{4}} \end{array} \right) \left( \begin{array}{c} u_{\mu}(r) \\ v_{\mu}(r) \end{array} \right),
\]

where \(r, \theta, z\) are cylindrical coordinates with the axis of symmetry along the vortex circulation, and \(\mu\) is the azimuthal quantum number, which assumes half-integer positive values. The radial functions are

\[
\left( \begin{array}{c} u_{\mu}(r) \\ v_{\mu}(r) \end{array} \right) = \left( \frac{2}{2\pi q_{\perp} r} \right)^{1/2} e^{-K(r)} \left( \begin{array}{c} \cos \left(q_{\perp} r - \frac{\pi\mu}{2} \right) \\ \sin \left(q_{\perp} r - \frac{\pi\mu}{2} \right) \end{array} \right),
\]

where \(q_{\perp} = \sqrt{q^2 - q_F^2}\), \(q_F\) being the neutron Fermi momentum, and

\[
K(r) = \frac{q_F}{q_{\perp} \Delta_{\infty}} \int_0^r d\rho' \Delta(\rho') d\rho' \approx \frac{q_F}{q_{\perp} \xi} \left( 1 + \frac{\xi e^{-\rho/r}}{r} \right).
\]

To arrive at the second equality in Eq. \(14\) we assumed a radial dependence of the gap function \(\Delta(\rho) = \Delta_{\infty} (1 - e^{-\rho/\xi})\) \(\Delta_{\infty}\) being the value at \(r/\xi \to \infty\), where \(\xi\) is the coherence length of neutron condensate. In the low temperature limit, the transitions with a change of the azimuthal quantum number can be neglected, and the summation can be restricted to the lowest order term \(\mu = \mu' = 1/2\). The same approximation should be used for the eigenvalues of quasiparticle energies.

Substituting Eqs. \(10\) and \(13\) in Eq. \(8\) and taking into account the normalization of states, we find

\[
S^{1/2}(k_\perp) = \int dy J_0(x, y) \cos (2xy - \pi\mu) e^{-2K(x, y)} \times \left\{ \int dy e^{-2K(x, y)} \right\}^{-1},
\]

where \(J_0(x, y)\) is the Bessel function of the first kind.
where \( J_0 \) is the Bessel function, \( x = k_1 \xi \) and \( y = r/\xi \). For a fixed value of \( q_F \xi \), the parametrical dependence of \( S \) on \( x \) can be well fitted by a Lorenzian after calculating the integrals numerically. For a typical value \( q_F \xi = 10 \), e.g., we find

\[
S(x) = \frac{a \gamma}{(x-x_0)^2 + \gamma^2},
\]

(16)

where the amplitude is \( a = 0.11 \), the maximum point \( x_0 = 1.1 \) and the width \( \gamma = 0.4 \). The neutron quasiparticle spectrum is given by

\[
\varepsilon_\mu(q) = \frac{\mu \hbar}{\sqrt{q^2_F - q_z^2}} \int_0^\infty dr \frac{\Delta(r)}{r} e^{-2K(r)} \times \left\{ \int_0^\infty d\epsilon e^{-2K(\epsilon)} \right\}^{-1} \varepsilon^0_\mu \left( 1 + \frac{q_z^2}{2q^2_F} \right),
\]

(17)

where \( \varepsilon^0_\mu = \mu \pi \Delta \xi_{/2} \epsilon_{F,\mu} \) with \( \epsilon_{F,\mu} \) being the Fermi energy of neutrons. The second equality is obtained by keeping the next-to-leading terms in small quantity \( q/q_F \) and approximating the integral as \( \Delta_r/\xi \). With the approximations above the spectral function for neutrons in the low temperature limit \( e^{-\beta \omega} \ll 1 \) is

\[
A^{n}(k,\omega) = \frac{q^2_F}{\pi \hbar \epsilon_{1/2}^0} \exp \left[ -\beta \varepsilon_{1/2}^0 \left( 1 + \frac{\omega^2 q^2_F}{2 \epsilon_{1/2}^0 k^2 F_n} + \frac{k_z^2}{8 q^2_F} \right) \right] \times \sinh \left( \frac{\beta \omega}{2} \right) S_{z,\frac{1}{2}}(k_z).
\]

(18)

We further substitute the spectral functions in Eq. (8) and perform the \( \omega \) integration. Further progress can be made if we assume that the scattering amplitude is independent of \( k_z \) and carry out the \( k_z \) integration. We find

\[
\tau_{pn}^{-1} = 6e^{1/2 \pi} K_0 \left( \frac{1}{2 \pi} \right) n^{(n)}_v \left( \frac{m^*_n}{m^*_p} \right)^2 \left( \frac{\epsilon_{F,n}}{\epsilon_{F,p}} \right) \times \frac{1}{\beta \varepsilon_{1/2}^0} \frac{e^{-\beta \varepsilon_{1/2}^0}}{m^*_p s_x^2} \frac{d\sigma_{pn}^*}{d\Omega},
\]

(19)

where

\[
\frac{d\sigma_{pn}^*}{d\Omega} = \int_{0}^{x_1} dx d^2 S(x) \frac{d\sigma_{pn}}{d\Omega}(x,p_F),
\]

(20)

with \( x_1 = 2p_F \xi/\hbar \); here \( K_0 \) is the modified Bessel function. The temperature dependence of the relaxation time is due to protons contributing the \( \beta^{-1} \) (thermal smearing of the Fermi surface) and vortex core quasiparticles contributing the \( \exp(-\varepsilon^0_{1/2}) \) (the probability of exciting a neutron quasiparticle inside the core of a neutron vortex). The \( n-p \) scattering cross section is modified in the medium, first, due to the suppression of the intermediate state two-particle propagation and, second, due to the modification of the density of states. We incorporate the medium modifications of the cross section using the code of Ref. [8] with the separable form of the Paris potential and a modified Pauli-blocking operator which keeps only the intermediate occupation numbers of neutrons and sets \( f(p) = 0 \) for a discussion of the related issues see also [1].

Table I shows the density and temperature dependence of the relaxation times \( \tau_{en} \) (Ref. [10]), \( \tau_{M} \) (Ref. [11]), and \( \tau_{pm} \) (present work). The microscopic parameters are from Ref. [12] (the proton \( 1S_0 \) gap, according to [12], vanishes at \( n = 0.4 \) fm\(^{-3} \), while \( 2P_2 \) neutron gap persists through the whole density region of interest; recent calculations of Ref. [14], which include the tensor coupling, indicate that superfluid neutron–normal proton mixture exists in the density region \( 0.43 \leq n \leq 0.45 \) fm\(^{-3} \) while matter is normal at larger densities).

The ratio of electron–neutron-vortex to proton–neutron-vortex relaxation times \( \tau_{en}/\tau_{pm} \) is of the order \( 10^6 \) and mainly reflects the difference in the spin–magnetic-moment and nuclear interaction matrix elements. As the temperature decreases, both time scales increase and for \( \beta \gg \varepsilon_{F,n}/\pi \Delta^2 \) tend to infinity due to the exponential cutoff of the scattering rates. Note that the relaxation time for electron scattering off the \( P \)-wave vortex magnetization, \( \tau_{en} \), is temperature independent; the crossover temperature corresponding to \( \tau_{en} \geq \tau_{pm} \) is of the order of \( 3 \times 10^6 \) K.

Consider now neutron star matter which, in addition to the neutron-proton-electron (npe) mixture, contains also heavy baryons in the normal state. Among these the lowest threshold densities have the \( \Sigma^- \) and \( \Lambda \) hyperons and the \( \Delta^- \) resonance. The threshold densities in the case of noninteracting Fermi gases and relativistic mean-field models are low, a few times nuclear saturation density \( (n_{sat} = 0.17 \) fm\(^{-3} \)) [1][11]. Correlations tend to shift the threshold densities to higher values in general.

| \( n \) (fm\(^{-3} \)) | \( x_p \) | \( \Delta \) (MeV) | \( \sigma_{pn}^* \) (\( \text{MeV} \cdot \text{fm}^2 \)) | \( \tau_{en} \) (yr) | \( \tau_{M} \) (yr) |
|---|---|---|---|---|---|
| 0.4 | 0.060 | 0.7 | 87.2 | 0.004 | 0.105 | 87.175 |
| 0.6 | 0.070 | 1.3 | 51.7 | 0.015 | 12.426 | 51.699 |
| 0.8 | 0.084 | 1.3 | 57.7 | 0.022 | 19.175 | 57.865 |
| 1.0 | 0.090 | 1.2 | 65.1 | 0.024 | 10.811 | 65.098 |

TABLE I. The relaxation times for the spin period of the Vela pulsar \( P = 0.089 \) s (\( \tau \) scale \( \propto P \)) and for temperatures \( \beta^{-1} \approx 10^8, 10^7, \) and \( 3 \times 10^6 \) K (in descending order). \( x_p \) is the proton fraction.
While the uncertainties in the hypernuclear matter calculations are still considerable, we shall, for the purpose of order of magnitude estimate, make several simplifying assumptions below. In the high density \( n^{(s)} p eY \) phase, where superscript \( s \) stands for the superfluid state and \( Y = \Sigma^-, \Lambda, \Delta^- \), the \( Y-n \) channel due to the heavy baryons would contribute additively to the scattering rate in the \( p-n \) channel; in this case no new qualitative effects are expected.

The situation is different in the \( n^{(s)} p eY \) phase, where in addition to the neutron condensate, the proton superconductor is in the mixed state. The relaxation time for the dominant electromagnetic coupling via scattering of electrons by the magnetic flux of a vortex is

\[
\tau_{e\Phi}^{-1} = \frac{3\pi^3}{64} \frac{n^{(n/p)}_e}{(k_F\lambda)^2}, \tag{21}
\]

where \( k_F \) is the electron Fermi wave vector and \( \lambda \) is the penetration depth of the proton superconductor, \( n^{(n/p)}_e \) is the density of either neutron or proton vortex lines.

To compare this time with the case of neutral scattering of heavy baryons by neutron and proton vortex core quasiparticles we assume in Eq. (3) (with obvious modifications) \( \sigma_{Yn} = 40 \text{ mb} \) and replace the effective masses of heavy baryons by their bare masses \( m_{\Sigma^-} = 1116 \), \( m_{\Lambda} = 1197 \), and \( m_{\Delta^-} = 1232 \text{ MeV} \). Once the threshold density is achieved, the hyperon population grows steeply to about 10% and stays further almost constant [10,11].

Table II shows the net relaxation time [Eq. (19)] due to \( \Lambda, \Sigma^-, \Delta^- \) scattering off neutron or proton quasiparticles localized in the vortex cores at the threshold density \( n = 0.4 \text{ fm}^{-3} \), and for a well developed hyperon core at \( n = 0.5 \text{ fm}^{-3} \). The relative abundances for heavy baryons are assumed equal and fixed at 1% and 10% for each case respectively. The values of \( \gamma_{Yn} \) are larger than that of \( \gamma_{Yp} \), at \( n = 0.4 \text{ fm}^{-3} \) mainly due to the large ratio of neutron to proton Fermi energies, i.e., more abundant neutrons are more effective in scattering of hyperons.

\[
\begin{array}{ccccccc}
 n & x_p & x_Y & \Delta_p & \Delta_n & \tau_{Yp} & \tau_{Yn} & \tau_{e\Phi} \\
 (\text{fm}^{-3}) & & & (\text{MeV}) & (\text{MeV}) & (\text{MeV}) & (\text{MeV}) \\
 0.4 & 0.05 & 0.01 & 0.4 & 0.7 & 0.036^a & 0.0002^a & 292.4^4 \quad 21.8^a \\
 0.5 & 0.1 & 0.1 & 0.1 & 1.0 & 0.027^a & 0.0002^a & 0.195^a \\
\end{array}
\]

At \( n = 0.5 \text{ fm}^{-3} \) the trend is opposite mainly due to an order of magnitude difference in the pairing gaps. One should note, however, that for typical pulsar magnetic fields, \( B \sim 10^{12} - 10^{13} \text{ G} \), the number of proton vortices per neutron vortex is very large, \( n_v^{(p)} / n_v^{(n)} \sim 10^{12} - 10^{13} \) and \( \gamma_Y \) is the main quantity of interest. For core temperatures \( T \sim 10^8 \text{ K} \), typical for not extremely old neutron stars, the hyperon scattering appears to be the dominant source of friction. It could be also important in the well developed hyperon cores at \( T \sim 10^7 \text{ K} \), below the densities at which the proton superconductivity is quenched because of the high proton fraction.

In summary, our calculations above show that scattering of baryons via nuclear force off vortex core quasiparticles in neutron star’s superfluid interiors may be dominant in a number of cases. In the so-called \( n^{(s)} p e \) (or \( n^{(s)} p eY \)) phase, where only the neutrons form a condensate, the baryon (including hyperons and resonances) scattering off neutron vortex core quasiparticles dominates other channels driven by the electromagnetic interactions [10,11]. In the \( n^{(s)} p eY \) phase, where the type-II proton superconductor is in the mixed state, a trace of heavy baryons (such as \( \Sigma^-, \Lambda \), and \( \Delta^- \)) in the normal state would produce a resistive force on vortices which will dominate at temperatures \( \sim 10^8 \text{ K} \) and will give the way to electron flux scattering at lower temperatures \( \leq 10^7 \text{ K} \) [4,3]. Which of the channels dominates at the intermediate temperatures \( \sim 10^7 \text{ K} \), depends on the details of the microphysical input, such as the pairing gaps and baryon abundances. For practical applications, our main result, Eq. (19), can be cast in a compact form (\( h = c = 1 \))

\[
\tau_{pn} = 1.5 \left( \frac{k_F}{m_p^*} \right)^4 \left( \frac{\beta}{\sigma_{pn}^*} \right)^4 \exp \left( \frac{0.038}{m_n^* \beta \Delta_n^2 \left( k_F/m_p^* \right)^2} \right) P, \tag{22}
\]

where the relations \( n_Y^{(n)} = (4m_n^* / h)P^{-1}, \) where \( P \) is the spin period, and \( \sigma_{pn}^*/4\pi = d\sigma_{pn}^*/d\Omega \) have been used.

As an illustrative example consider first a neutron star model of canonical mass 1.4 \( M_\odot \) based on the EoS of Ref. [13]. The \( n^{(s)} p e \)-phase in this model occupies the radial range \( 0 \leq R \leq 8 \text{ km} \) for the gap profiles of Ref. [12] and \( 7 \leq R \leq 7.7 \text{ km} \) for the profiles of Ref. [14] (star radius is 10.13 km). For the maximum mass, \( M = 1.65 M_\odot \), star model of Ref. [13] (star radius is 11.4 km), the \( n^{(s)} p eY \)-phase occupies the shell \( 7 \leq R \leq 9.5 \) while at smaller radii the \( 1S_0 \) proton pairing vanishes (\( n^{(s)} p eY \)-phase). These numbers by no means provide a consistent overall picture of the distribution of different superfluid phases in a neutron star, for the results on the pairing gaps and EoS stem from different (BCS-BHF [12,14] vs respectively variational [13] and mean-field [14]) approaches to the nuclear many-body problem. Nevertheless, our observation of the dominance of nuclear interaction channels when any kind of baryons is in the normal state is robust, i.e., independent of the details of the microphysical input. Its implementation, however, as given by the example above, is connected with the uncertainties due to...
limited knowledge of the interactions and pairing in the high density nuclear matter as well as the heavy baryon concentrations and their density thresholds.

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