World-Sheet Defects, Strings, and Quark Confinement

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Abstract

In this talk I give a preliminary account of original results, obtained in collaboration with John Ellis. Details and further elaboration will be presented in a forthcoming publication [1]. We present a proposal for a non-critical (Liouville) string approach to confinement of four-dimensional (non-abelian) gauge theories, based on recent developments on the subject by Witten and Maldacena. We discuss the effects of vortices and monopoles on the open world-sheets whose boundaries are Wilson loops of the target-space (non Abelian) Gauge theory. By appropriately employing ‘D-particles’, associated with the target-space embedding of such defects, we argue that the appearance of five-dimensional Anti-De-Sitter (AdS) space times is quite natural, as a result of Liouville dressing, required for a consistent description of the D-particle quantum fluctuations (‘recoil’). The D-particle ‘recoil’ may be considered as a dual description of the ‘distortion of space-time’ caused by the propagation of the heavy test particle along the Wilson loop of the target-space gauge theory. We isolate the world-sheet defect contributions to the Wilson loop by constructing an appropriate observable; this observable is the same as the second observable in the supersymmetric U(1) theory of Awada and Mansouri, but in our construction supersymmetry appears not necessary. When vortex condensation occurs, we argue in favour of a (low-temperature) confining phase, in the sense of an area law, for a large-$N_c$ (conformal) gauge theory at finite temperatures. A connection of the Berezinski-Kosterlitz-Thouless (BKT) transitions on the world-sheet with the critical temperatures in the thermodynamics of Black Holes in the five-dimensional AdS space is made. The upper critical temperature in that case is identified with the monopole condensation BKT temperature, which is argued to be responsible for setting the scale for new Physics.

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1 Introduction

The recent exciting developments in the understanding of non-perturbative effects in the theory formerly known as strings \[2\] have led to tantalizing glimpses of a broader framework for the Theory of Everything (TOE), referred to variously as \(M\) or \(F\) theory.

Moreover, quite recently, some of these developments, have been argued to play an important rôle in a non-perturbative description of the long-distance Physics of certain large \(N\) Gauge theories, and in particular in their confining aspects \[3\]. In the center of these developments lies the conjecture by Maldacena \[4\], that certain large \(N\) conformal field theories may be understood as being related to Supergravity or String theory in the bulk of certain Anti-de-Sitter (AdS) space times, whose boundary is a conformal Minkowski space-time, on which the large \(N\) conformal field theory lives.

Formally, the conjecture may be formulated as follows \[3\]. Let \(Z_S(\phi_0)\) be ye supergravity of (super)string action in the bulk of a \((d+1)\)-dimensional \(AdS_{d+1}\), computed with the boundary condition that at the boundary of \(AdS_{d+1}\), \(M_d\) (a conformal Minkowski space time), the field \(\phi\) approaches \(\phi_0\). The ansatz for the Conformal-Field-theory CFT/AdS correspondence, proposed in \[4\], and elaborated in \[3\], can then be stated as:

\[
\langle\exp\int_{M_d} \phi_0 \mathcal{O}\rangle = Z_S(\phi_0) \tag{1}
\]

where \(\mathcal{O}\) are operators in the CFT. The important property of \(AdS\) is its ‘holo-graphic’ nature, in the sense of uniqueness theorems \[28, 3\], that specify the bulk behaviour of the classical field (or string) theory of \(\phi\) in terms of the boundary value \(\phi_0\).

The conjecture \(\text{(1)}\) allows for a (non-perturbative) computation of correlation functions of certain supersymmetric conformal field theories, notably some conformal supersymmetric gauge theories in their strong coupling (confining) regime \[3\]. In view of \(\text{(1)}\) and the associated holographic nature of \(AdS\), this means that quantum information about the confining physics of a non-Abelian Gauge theory is thus encoded in classical geometries. It should be noticed that in order for the supergravity solutions used in \[4, 3\] to be trusted one must work in a large \(N_c\) limit of a strongly coupled supersymmetric and conformal \(U(N_c)\) gauge theory. The appropriate limits are taken in such a way that \(g_{YM}^2 N_c\) is fixed but \emph{large}, as \(N_c \to \infty\), with \(g_{YM}^2 \sim g_s\), with \(g_s\) the string theory coupling. The supersymmetry in the above approach is needed because the entire approach is based on critical-dimension ‘super-string’ theory, in the sense of tensoring the four-dimensional space-time manifold with appropriate compact manifolds, so as to ensure a critical total central charge.

It is the purpose of this talk to report on an attempt to extend these developments to a non-critical-dimension string theory, which might open the way for an extension of the above ideas to non-supersymmetric theories. This talk is based on original
research done in collaboration with John Ellis, and a detailed account of it will appear in a forthcoming publication \[1\]. We make a proposal concerning a Liouville string description of (non-supersymmetric) four-dimensional Gauge theories, which are conformal at zero temperatures. One of the most important points to make is that in such attempts one faces the necessity to include in the world-sheet of the string, whose boundary is the Wilson loop of the pertinent Gauge theory, non-trivial defects (monopoles or vortices). The world-sheet thermodynamical properties of such defects will be argued to dictate the various phases of the finite-temperature target-space Gauge theory, through a non-trivial correspondence with the embedding AdS space times, arising as a result of appropriate Liouville dressing.

The layout of the talk is as follows: In section 2, we review aspects of our previous analysis of Liouville representation of world-sheet defects. In section 3 we discuss the connection of AdS space times and D-brane recoil Physics, as well as how (conformal) QCD, viewed as a non-critical string theory, could fit into this picture. In section 4 we present the Awada-Mansouri approach \[5\] to the connection between Gauge theories and scale invariant (super)strings in target space, which we shall make use of in our attempt to discuss a dynamical scenario for the appearance of the string tension in (conformal invariant) string theories. In section 5 we discuss the rôle of world-sheet defects on the (long-distance) physics of quark confinement. We argue, in particular, that the presence of such defects on the world sheet makes the role of supersymmetry in the approach of \[5\] redundant \[4\]. We also give arguments in support of the rôle of world-sheet vortices and monopoles on inducing the confining aspects of large-\( n \) \( U(n) \) gauge theory, in the sense of an area law, and a non-perturbative computation of the string tension. We thus relate the various Berezinski-Kosterlitz-Thouless (BKT) transitions on the world-sheet to the thermodynamics of AdS Black Holes, which has been argued \[3\] to be relevant for the various phases of the Gauge theory. We also present some remarks clarifying the rôle of the Abelian projection to confinement in pure glue Yang-Mills theories. Conclusions and outlook are presented in section 6.

2 World-Sheet Defects in (Liouville) Superstring Theories

2.1 General Remarks

In this section we shall review some basic features of world-sheet defects, which we shall use in our approach. A vortex defect on the world sheet \[4, 7\] is obtained as the solution \( X_v \) of the equation

\[
\partial_z \bar{\partial}_z X_v = \frac{i \pi q_v}{2} [\delta(z - z_1) - \delta(z - z_2)]
\]

\[2\]

\[4\] In fact finite temperature field theories which are discussed in the recent literature \[4, 3\] have their target space supersymmetry (and conformal symmetry) broken.
where $q_v$ is the vortex charge and $z_{1,2}$ are the locations of a vortex and antivortex, respectively, which we may map to the origin and the point at infinity. The corresponding solution to (2) is

$$X_v = q_v \text{Im} \ln z$$

and we see that the vortex charge $q_v$ must be integer.

There are related ‘spike’ configurations which are solutions of the equation

$$\partial_z \bar{\partial}_z X_m = -\frac{\pi q_m}{2} [\delta(z - z_1) - \delta(z - z_2)]$$

given by

$$X_m = q_m \text{Re} \ln z$$

It is easy to see \[8\] that, in the presence of both types of defects, single-valuedness of the partition function imposes the following quantization condition:

$$2\pi \beta q_v q_m = \text{integer}$$

at finite temperature $T \equiv \beta^{-1} \neq 0$.

There is a vortex-monopole duality, which is manifested as the invariance of the defect partition function under \[9\]:

$$\pi \beta \longleftrightarrow \frac{1}{4\pi \beta}; \quad q_v \longleftrightarrow q_m$$

Consider now an Abelian gauge theory description of the defect dynamics, such that the world-sheet vector field is the pullback of some background gauge field $A_M(X)$, taken to be Abelian for definiteness, over which the string propagates. The ‘pullback’ of the gauge field on the world-sheet can be defined as:

$$A_\alpha(z, \bar{z}) = \partial_\alpha X^M A_M(X)$$

where Greek indices are world-sheet variables, $\alpha = 1, 2$, and upper case Latin indices $M = 1, \ldots D$ are target-space indices. The world-sheet ‘magnetic field’ corresponding to (8) reads:

$$\mathbf{B} = \epsilon^{\alpha\beta} \partial_\alpha A_\beta = \epsilon^{\alpha\beta} \partial_\alpha \partial_\beta X^M A(X)_M + \epsilon^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N \partial_N A_M(X)$$

Consider now the case where the world-sheet surface has the topology of a disc, whose boundary $C$ will be identified in our work with a Wilson loop for the field, $A_M$, in target (embedding) space:

$$W(C) = e^{ie \int_C A \cdot dl}$$

In such a case the world-sheet flux of $\mathbf{B}$ through the world-sheet $\Sigma(C)$, can be identified with the flux of the target-space gauge field $A_M$, in the embedding space. By the Stokes theorem on the world-sheet, we then see that in the flux $\int_{\Sigma(C)} \mathbf{B} \cdot dS = \ldots$
\( \int_C A. d\ell \), the contribution from the first term on the r.h.s. of (9) can be attributed to vortices \( X^v \) on the world-sheet, whilst the second contribution to monopoles, \( X^m \). This is in accordance with the general Hodge decomposition of an arbitrary gauge field \( A_\alpha \) on \( \Sigma(C) \). It is the second term which corresponds to target-space monopoles for the field \( A \), since it is gauge invariant in target space, and enters the computation of space-time fluxes \(^1\).

In this way, the presence of a magnetic monopole in the embedding space may be pull-backed as a monopole (or vortex) defect of the (world-sheet) surface \( \Sigma(C) \). This will always by understood in our discussion in this work. We also note that in the context of two-dimensional target-space embeddings, world-sheet monopole defects can be mapped \(^4\) to topologically non-trivial two-dimensional target space times with Schwarzschild black holes \(^14\). In the higher-dimensional case, discussed in this work, we shall find an analogous phenomenon: the physics of world-sheet monopoles will be associated with the physics of Black Holes in (five-dimensional) Anti-de-Sitter (AdS) space times.

### 2.2 Vortices and Monopoles in Liouville Strings

The analysis of \(^8\), which we have adopted and extended in our approach \(^12\), indicated that it is possible to interpret the Liouville theory \(^13\) in the dangerous region of the central charge \( 1 < D < 9 \) (or 25) in terms of defect configurations in a two-dimensional world-sheet theory with finite (inverse) temperature, given by the appropriate ‘matter’ central charge deficit \(^1\). For our purposes, of representing QCD (in some limits) as a non-critical string theory, we shall review only the supersymmetric case.

In our non-critical string interpretation, the ‘matter’ central charge, \( D \), in general differs from the number of space-time dimensions \(^13\), \( D \), by the matter screening charge, or, in general, by the ‘dilaton’ fields:

\[
D = D + 12Q^2
\]

In our case we assume \( D = 4 \) (the target-space dimensionality of ordinary gauge theories). In such a case, one restores criticality by allowing a Liouville central charge deficit, \( c_L \), such that \( c_{\text{total}} = D + c_L = 10 \). The Liouville mode is ‘time-like’ \(^1\), if \( D > 9 \) and space-like if \( D < 9 \). The Liouville mode decouples at \( D = 10 \).

The supersymmetrization of the world-sheet defects may be represented using a sine-Gordon theory with local \( n = 1 \) supersymmetry, which has the following

\[^2\text{As a side remark, it is interesting to note that the quantity } \sigma^{MN} = \epsilon^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N \text{ enters the Schild action } \[10\], recently claimed \[11\] to be associated with a matrix-model representation of D-brane dynamics.}

\[^3\text{As we shall discuss, in our case, the relevant quantity for the confining aspects of gauge theory is } \sigma^M = \epsilon^{\alpha\beta}\partial_\alpha \partial_\beta X^M \text{, which is also non-vanishing for a vortex (angular field), or at the monopole core.}

\[^4\text{However, one can also work in the ‘stringy’ region for the central charge } D > 9 \text{, since the analysis of } \[8\] \text{ applies in that case as well.} \]
monopole deformation operator:[8]

\[ V_m = \bar{\psi}\psi : \cos\left(\frac{e}{\beta_{n=1}}(\phi(z) - \phi(\bar{z}))\right) : \]

(12)

where the \( \psi, \bar{\psi} \) are world-sheet fermions with conformal dimensions \((1/2, 0), (0, 1/2)\) respectively, and \( \phi \) is a Liouville field. The effective temperature \( 1/\beta_{n=1} \) is related to the matter central charge by

\[ \beta_{n=1} = \frac{2}{\pi|D - 9|} \]

(13)

The corresponding vortex deformation operator is

\[ V_v = \bar{\psi}\psi : \cos[2\pi q^{1/2}(\phi(z) + \phi(\bar{z}))] : \]

(14)

where \( q \) is the vortex charge. The duality (7) is valid also in this approach.

Including the conformal dimensions of the fermion fields, we find that the conformal dimensions of the vortex and monopole operators are

\[ \Delta_v = \frac{1}{2} + \frac{1}{2}\pi\beta_{n=1}q^2 = \frac{1}{2} + \frac{q^2}{|D - 9|}, \quad \Delta_m = \frac{1}{2} + \frac{1}{8\pi\beta_{n=1}}e^2 = \frac{1}{2} + \frac{e^2|D - 9|}{16} \]

(15)

respectively.

Let us first consider the case \( D > 9 \). We see that the supersymmetric vortex deformation with minimal charge \( |q| = 1 \) is marginal when \( D = 11 \). Below this dimension, the vortex deformation is irrelevant, and this is the case which shall interest us in the context of the present work. The simplest case is to consider minimum charge defects \( |q| = 1 \), and study the various phases of such a world-sheet deformed theory, which will be characterized by different values of the matter central charge \( D \). The quantization condition (6) tells us that the allowed charge for a monopole defect, in the presence of a \( |q| = 1 \) vortex, is \( |e| = \frac{m}{4}(D - 9) \), \( m \in \mathbb{Z} \).

In this case, for \( m = 1 \) we find three regions \([8, 7, 16]\):

\( n = 1 \) world-sheet supersymmetry:

\[ 9 < D < 11 \quad \text{monopole vacuum unstable, vortices bound} \]

\[ 11 < D < 14.04 \quad \text{both monopole and vortex vacua unstable} \]

\[ 14.04 < D \quad \text{monopoles bound, vortex vacuum unstable} \]

(16)

The \( D > 9 \) case includes critical string theories, as well as their generalizations to M and F theory [16].

We next consider the case \( D < 9 \), which is most relevant for our low-energy description of gauge theories, as we shall discuss later on. A similar analysis reveals

\[ 4 \text{ It is understood that, in the general case, the critical values depend on the charges } q, e. \]
the following phase diagram:

\begin{align*}
n &= 1 \text{ world} - \text{sheet supersymmetry} : \\
\mathcal{D} &< 3.96 \quad \text{monopoles bound, vortex vacuum unstable} \\
3.96 &< \mathcal{D} < 7 \quad \text{both monopole and vortex vacua unstable} \\
7 &< \mathcal{D} < 9 \quad \text{monopole vacuum unstable, vortices bound} \\
\end{align*}

Thus, in the \( \mathcal{D} < 9 \) case the monopole BKT transition occurs at lower values of \( \mathcal{D} \) than the corresponding transition for vortices, which is the opposite of the situation encountered in the \( \mathcal{D} > 9 \) case.

However, from the point of view of BKT temperatures, in both cases the temperatures \((13)\) for the vortex condensation is lower than that of monopoles:

\begin{align*}
n &= 1 \text{ world} - \text{sheet supersymmetry/BKT Temperatures} : \\
T &< T_{\text{vortex}} \quad \text{vortices bound, monopole vacuum unstable} \\
T_{\text{vortex}} &< T < T_{\text{monop}} \quad \text{both monopole and vortex vacua unstable} \\
T_{\text{monop}} &< T < \infty \quad \text{monopoles bound, vortex vacuum unstable} \\
\end{align*}

where the temperatures are understood to be given in units of the corresponding string tension. Notice, that as a result of monopole-vortex duality there is no region where both of these defects are stable.

In standard string theory \([6]\), compactified on a circle \( S^1 \), the inverse temperature is the square of the compactification radius \( \beta = R^2 \) (in units of \( \alpha' = 1 \)). Usually in such a case \([6]\) one considers only one type of defects, since the other can always be derived by a \( T \) duality transformation. In that case the vortex condensation BKT temperature is identified with the Hagedorn temperature, where the winding modes of the string become tachyonic. However, in our approach we shall be more general and consider both types of world-sheet defects. As we have just seen, this leads to richer phase diagrams. In that case, for compactified (Bosonic) strings we have for the vortex condensation \([8]\):

\[
\frac{1}{2\pi} R_H^2 q^2 = 1 \quad \text{if } |q| = 1 \rightarrow R_H = \sqrt{2/\pi} \\
\]

whilst the monopole condensation occurs at:

\[
\frac{e^2}{8\pi R_M^2} = 1 \quad \text{if } |e| = 1 \rightarrow R_M = 1/(2\sqrt{2\pi}) \\
\]

From the above point of view, then, there appears to be an \textit{intermediate temperature}, corresponding to the self-dual radius of compactification

\[
R_1 = 1/\sqrt{2\pi} \\
\]

Notice the following relation between \( R_H, R_M, \) and \( R_1 \):

\[
R_1^4 = (R_M R_H)^2 \\
\]
We shall come back to a comparison of this phase diagram with the Thermodynamics of AdS in section 5.

The $n = 2$ world-sheet supersymmetric case, responsible for $\mathcal{N} = 1$ target-space supersymmetry, is less interesting, in the sense that the vortex or monopole vacua are stable, in the region $D > 1$, $D < 1$, while the point $D = 1$ appears as degenerate. So, this theory has no such phase transitions. In [14] we have discussed ways of breaking the $n = 2$ supersymmetry down to $n = 1$; from the target space-time point of view this implies breaking of the target $\mathcal{N} = 1$ supersymmetry. In this work we shall be interested mostly in phase transitions at finite temperatures, where supersymmetry is broken. From this point of view, the $n = 1$ supersymmetric world-sheet case (or the corresponding bosonic, which, however, we do not discuss here) becomes of interest.

We have argued in [17] that, in the context of string theory, the world-sheet defects correspond to $D$ (Dirichlet) branes [4], since correlators involving defects and closed-string operators have cuts for generic values of $\Delta_{v,m}$. These cause the theory to become effectively that of an open string. One may then impose Dirichlet boundary conditions on the boundaries of the effective world sheet, i.e., along the cuts, obtaining solitonic $D$-brane configurations. In this work we shall deal with QCD, and in particular we shall offer a scenario for a non-critical string representation. In our approach Dirichlet particles, and in particular their ‘recoil’ [18], will play a crucial rôle. An association of QCD with non-critical strings has been advocated by Polyakov [19], but our approach will be different, since we shall use world-sheet defects, condensation of which will be argued to be responsible for the confining aspects of the (long-distance) physics of the non-Abelian gauge theory. In this respect, we shall associate the regions (16) to the confinement-deconfinement phase transition of QCD, following and extending the work of [3], to connect the B-K-T world-sheet transitions to the critical temperatures of a gas of Black Holes in AdS space times [20].

To understand the connection with anti-de-Sitter space times let us first discuss how such space times can arise in our recoil approach to $D$ branes. The use of $D$-branes, in the dual string picture, may be seen as an alternative way of dealing with a gauge theory. Indeed, it is widely accepted today [2, 21] that $m$ parallel branes have a gauge $U(m)$ symmetry, and their dynamics is equivalent (in the sense of string dualities) to the dynamics of certain supersymmetric Yang-Mills gauge theories with group $U(m)$. Such issues are best understood in the regions where the number $m \to \infty$, so such an approach may be considered as a stringy picture of $U(N_c)$ gauge theories, with $N_c$ large.

From our point of view, the presence of a (dual) D-particle, or a collection thereof, may also represent ‘back reaction’ of the space-time due to the sudden movement of a heavy test particle (quark-antiquark pair) along the Wilson loop in pure glue gauge theory; such a movement causes distortion of the surrounding space time, and the corresponding fluctuations can be described by the excitation of a D-particle ‘recoil’ operator [18, 17, 22], in a $\sigma$-model approach. Notice that the mass of such a particle is inversely proportional to the (dual) string coupling $g_s$, which therefore is weak.
As we shall discuss below, the presence of the D-particle ‘recoil’ is crucial in yielding AdS space-times in a *dynamical way*.

3 Anti-de-Sitter Space Times from D-particle Recoil

3.1 Recoil and Liouville Dressing: General Formalism

In this section we would like to discuss how the anti-de-Sitter space time arises from our Liouville approach to D-brane recoil \[17\]. The analysis was motivated originally by attempts, in collaboration with J. Ellis and D.V. Nanopoulos \[23\], to understand the structure of the elusive \(M\)-theory description of ‘grand-unified strings’. Below I will present a version suitable for the purposes of the present work. The basic point of view we shall adopt here is that the encounter of a closed string state with the world-sheet defect (monopole or vortex) produces recoil, described by a pair of logarithmic deformations \[24\] corresponding to the collective coordinate and velocity of the recoiling D-particle \[18, 22\], which the world-sheet defect is mapped to.

These two operators are slightly relevant \[18\], in a world-sheet renormalization group sense, with anomalous dimension \(\Delta = -\frac{\epsilon^2}{2}\). Thus, the recoiling D-particle ceases to be described by a conformal theory on the world sheet, despite the fact that before the recoil the theory was conformal invariant. To restore conformal invariance one has to invoke Liouville dressing \[13\], thereby increasing the target-space time dimensionality to \(D + 1\). The Liouville field in this approach plays the rôlé of time, due to the supercriticality \[15\] of the central charge \(D\) of the stringy \(\sigma\)-model, which was critical (conformal) before recoil. Such a procedure leads to an effective curved space-time \(F\)-manifold in \(D + 1\) dimensions, with signature at least as \(F = (1, D)\), which according to the analysis of \[25\] is described by a metric of the form:

\[
G_{00} = -1, \quad G_{ij} = \delta_{ij}, \quad G_{0i} = G_{i0} = f_i(y_i, t) = \epsilon(\epsilon y_i + u_i t), \quad i, j = 1, ..., D
\]  

where \(\epsilon \to 0^+\), is a regulating parameter, related \[18\] to the world-sheet size \(L\) via

\[
\epsilon^{-2} \sim \eta \ln(L/a)^2,
\]

where \(\eta = -1\) for a Minkowski signature Liouville mode, \(t\), and \(a\) a world-sheet short-distance cut-off. The quantities \(y_i\) and \(u_i\) represent the collective coordinates and velocity of a \(D\)-dimensional D(irichlet)-particle. In the \(\sigma\)-model approach of \[22, 17\] \(y_i, u_i\) are viewed as *exactly marginal* \(\sigma\)-model couplings. Notice that, in view of the identification \(24\), for Minkowskian signature Liouville mode \(t\), \(\epsilon^2 < 0\). This will be important in yielding negative curvature anti-de-Sitter space times.

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\(^5\)In the context of a \(\sigma\)-model path integral it is convenient to work with Euclideanized ‘times’ \(X^0\) in the \(D\)-dimensional space time. The time \(X^0\) is distinct from the Minkowskian signature Liouville time \(t\). In the case of gauge theories, the Euclidean \(X^0\) may be thought of as temperature.
We recall from the analysis of [25] that the components of the Ricci tensor for the above (D+1)-dimensional $F$-manifold are:

\begin{align}
R_{00} &= -\frac{1}{(1 + \sum_{i=1}^{D} f_i^2)^2} \left( \sum_{i=1}^{D} f_i \frac{\partial f_i}{\partial t} \right) \left[ \sum_{j=1}^{D} \frac{\partial f_j}{\partial y_j} \left( 1 + \sum_{k=1, k \neq j}^{D} f_k^2 \right) \right] \tag{25} \\
R_{ii} &= \frac{1}{(1 + \sum_{k=1}^{D} f_k^2)^2} \left\{ \frac{\partial f_i}{\partial y_i} \left( \sum_{j=1}^{D} f_j \frac{\partial f_j}{\partial y_j} \right) - \left( 1 + \sum_{k=1}^{D} f_k^2 \right) \frac{\partial f_i}{\partial y_i} \frac{\partial f_i}{\partial t} \right\} + \frac{\partial f_i}{\partial y_i} \left[ \sum_{j=1, j \neq i}^{D} \frac{\partial f_j}{\partial y_j} \left( 1 + \sum_{k=1, k \neq j}^{D} f_k^2 \right) \right] \tag{26} \\
R_{0i} &= \frac{f_i}{(1 + \sum_{k=1}^{D} f_k^2)^2} \left\{ \frac{\partial f_i}{\partial y_i} \left( \sum_{j=1}^{D} f_j \frac{\partial f_j}{\partial y_j} \right) - \left( 1 + \sum_{k=1}^{D} f_k^2 \right) \frac{\partial f_i}{\partial y_i} \frac{\partial f_i}{\partial t} \right\} \tag{27} \\
R_{ij} &= \frac{1}{(1 + \sum_{k=1}^{D} f_k^2)^2} f_i f_j \frac{\partial f_i}{\partial y_i} \frac{\partial f_j}{\partial y_j} \tag{28}
\end{align}

In [25] it was argued that due to the abrupt change in the surrounding space-time at the moment of impact of the closed string state $t = 0$ with the defect, decoherence will be induced for a spectator particle. Below we shall consider the asymptotic case where $t \gg 0$.

We shall also restrict ourselves to the case where the recoil velocity $u_i \rightarrow 0$. This case is encountered if the D-particle is very heavy, with mass $M \propto 1/g_s$, where $g_s \rightarrow 0$ is the (dual) string coupling. In such a case the closed string state splits into two open ones ‘trapped’ on the D-particle defect. The collective coordinates of the latter, though, are still allowed to quantum fluctuate with fluctuations of order \[ \Delta y_i \sim |e^2 y_i| \]. From the analysis of [25], then, we can infer that, in the $u_i \rightarrow 0$ case, the particle creation, which was found to be proportional to $u_i^2$, vanishes, and thus the asymptotic limit of $t \gg 0$ may be treated as a coherent quantum string theory, i.e. as a closed system. From the world-sheet point of view [8], the very heavy D-particle case corresponds to a strongly coupled defect. Indeed, the coupling $e$ of the world-sheet defect is related to the (dual) string coupling $g_s$ via

\[ e \sqrt{\pi/3} \propto \frac{1}{\sqrt{g_s}} \tag{30} \]

which implies a world-sheet/target-space strong/weak coupling duality. In the case where one views the open world-sheet as the area enclosed by a Wilson loop of the
Figure 1: Schematic representation of the $D$-dimensional (Euclideanized) anti de Sitter space time, arising in our Liouville approach to a recoil $D$-particle, as the interior of a $D$-dimensional ball $B_D$; the boundary of the anti-de-Sitter is a $D-1$ dimensional conformal sphere $S^{D-1}$ (compactified Minkowski space time). Any field theory defined on $S^{D-1}$ may be viewed as corresponding uniquely to a $D$-brane supergravity theory in the bulk of the $AdS_D$.

gauge theory in target space, then, the world-sheet defect charge $e$ becomes proportional to the target-space gauge theory coupling, $g_{YM}$. Thus, the above approach allows one to study a strongly coupled gauge theory by using a weakly coupled (dual) $D$-string $g_s$.

We now observe from (29) that, in the case of $u_i \to 0$, the only non-vanishing components of the Ricci tensor are:

$$ R_{ii} \simeq \frac{\partial f_i}{\partial y_i} \left( \sum_{j=1, j \neq i}^{D} \frac{\partial f_j}{\partial y_j} [1 + O(\epsilon^4)] \right) \frac{1}{(1 + \sum_{k=1}^{D} f_k^2)^2} \simeq \frac{-(D-1)/|\epsilon|^4}{\left(\frac{1}{|\epsilon|^2} - \sum_{k=1}^{D} |y_k|^2\right)^2} + O(\epsilon^8) $$

(31)

where we have taken into account (24) for Minkowskian signature of the Liouville mode $t$. Thus, in this limiting case, and for large $t >> 0$, the Liouville mode decouples, and one is effectively left with a $D$-dimensional manifold. We may now write (31) as

$$ R_{ij} = G_{ij} R $$

(32)

with $G_{ij}$ a diagonal (dimensionless) metric, corresponding to the line element:

$$ ds^2 = \frac{|\epsilon|^{-4} \sum_{i=1}^{D} dy_i^2}{\left(\frac{1}{|\epsilon|^2} - \sum_{i=1}^{D} |y_i|^2\right)^2} $$

(33)

which is the metric describing the interior of a $D$-dimensional ball, depicted in figure 1. This is the Euclideanized version of an anti-de-Sitter space time. In its Minkowski version, one can easily check that the curvature corresponding to (33) is constant and negative, $R = -4D(D-1)/|\epsilon|^4$.

The interesting property of the space time (33) lies on the fact that there exist a coordinate singularity in the metric (33) at $\sum_{i=1}^{D} |y_i|^2 = |\epsilon|^{-4}$, which prevents a
naive extension of the open ball $B_D$ to the closed ball $\overline{B}_D$, with boundary the sphere $S^{D-1}$. The metric that extends to $\overline{B}_D$ is provided by a conformal transformation of (13) [3]:

$$d\tilde{s}^2 = f^2 ds^2$$ (34)

Choosing, say, $f = \frac{1}{|\epsilon|^2 - \sum_{i=1}^{D} |y_i|^2}$, results in $d\tilde{s}^2$ being associated with the metric on a sphere $S^{D-1}$ of radius $\frac{1}{|\epsilon|^2}$. In general $f$ may be changed by any conformal transformation, thereby leading to the appearance of a conformal invariant (Euclidean) $S^{D-1}$ space as the boundary of an anti-De-Sitter space time, whose metric is invariant under the Lorentz group $SO(1,D)$.

This approach to AdS space time and their connection to conformal invariant Minkowskian space times is due to Witten’s elaboration [3] on the conjecture by Maldacena [4] that certain large N-limit conformal field theories in $d$ dimensions can be described in terms of supergravity (or string theory) on the product of a $(d + 1)$ dimensional AdS space with a compact manifold. In our approach above we have shown how AdS string backgrounds can arise naturally in the Liouville $\sigma$-model approach to D-brane recoil advocated in [17]. The discussion then of refs. [3], [4] applies to the resulting anti de Sitter space times [1]. The important property of Anti-de-Sitter (AdS) space times is the existence of powerful theorems which imply that if a classical field theory is specified in the boundary of the space, then there are unique extensions to the bulk [3, 26]. This is the main point of the holographic nature of field/string theories in anti-de-Sitter space times, in the sense that all the information about the bulk theory of an anti-de-Sitter space is ‘stored’ in its boundary. By the Gauge Theory/Conformal-field-theory correspondence of [4, 3], then, information about quantum aspects of gauge theories on the boundary of AdS, are encoded in classical geometry in the bulk of the $AdS$ space.

We may now interpret the Ricci tensor (31) as corresponding to the low-energy $O(\alpha')$, $\alpha' \ll 1 - \beta$ function of a one-string-loop corrected $\sigma$-model, propagating in this background. Notice that the one-string-loop correction is necessary so as to yield the constant-curvature nature of the background in a way compatible with the conformal invariance [28]. Indeed, in a perturbative expansion in terms of $\alpha'$, the $O(\alpha')$ $\sigma$-model $\beta$-function is proportional to the Ricci tensor $R_{ij}$ alone [27]. The graviton background conformal invariance conditions, imply the relation

$$\beta^G_{ij} = R_{ij} = \nabla_i \partial_j \Phi$$ (35)

with $\Phi$ a dilaton field. At first sight, it seems that the Ricci tensor (32), describing a constant curvature space time, cannot be a consistent string background, compatible with conformal invariance (35), to order $\alpha'$. However, as shown in [28], this conclusion is false. Indeed, if one includes higher-string loop corrections, then one obtains

---

6 It should be stressed, however, that our approach, as discussed in [25], is much more general. The AdS space time was obtained in the limit of zero recoil velocity $u_i$, whilst the inclusion of such velocity corrections complicates the asymptotic definition of a local quantum field theory on the boundary of our space time, due to induced decoherence as a result of quantum recoil degrees of freedom.
an induced (target-space) cosmological constant, corresponding to a dilaton tadpole, which renders the constant-curvature backgrounds consistent with the conformal-invariance conditions, corrected by string loops. The mechanism of [28] concerns one loop corrections, which in our case are sufficient, in view of the smallness of the (dual) string coupling $g_s << 1$. According to [28], there are zero momentum target space dilatons, non-propagating delocalized modes, which fail to decouple from the one loop partition function in pinched world-sheet loops (see figure 2). Such modes yield non-zero constant dilaton tadpoles, $J$.

Such pinched loops lead to extra world-sheet divergences, whose regularization in lower genus (tree level) world-sheet surfaces induces corrections to the graviton $\beta$ functions of the form

$$\beta_{G, \text{1-ring-loop}} = \beta_{G, \text{tree-level}} + g_s G_{ij} J, \quad J = \text{zero momentum dilaton tadpole}$$

thereby leading to a constant curvature space time, with cosmological constant $g_s J$, as a consistent solution of the string-loop corrected conformal invariance conditions. In our case above, $g_s J \propto -|\epsilon|^4$.

Notice that the one-string-loop correction is consistent with the fact that in the context of our original \(\sigma\)-model, the recoil operators are themselves viewed as arising from one string loop corrections in the propagating string matter states (see fig. 3a), due to the cancellation of extra world-sheet divergences arising from pinched world-sheet loops (see figure 3b), in the presence of Dirichlet boundary conditions [28, 17]. Also notice that the the fact that the solution is of anti-De-Sitter type, allows extension of these ideas to target-space supergravity theories, given that that local supersymmetry in a space of constant curvature for \(D > 2\) requires that the space time is of Anti-de-Sitter type [30].

The above discussion has indicated the appearance of an anti-de-Sitter (target) space time, due to the recoil of a very heavy D-particle. As well understood by now [2], a single D-particle corresponds to a (compact) \(U(1)\) gauge theory [2], whilst
Figure 3: (a) World-sheet annulus diagram for the leading quantum correction to the propagation of a string state in a $D$-brane background, and (b) the pinched annulus configuration which is the dominant divergent contribution to the quantum recoil.

$m$ D-particles, or $m$ parallel non-interacting branes, correspond to a $U(m)$ gauge symmetry [2, 21]. In the latter case, the formal extension of the above approach is computationally non-trivial. The formalism is complicated due to technicalities involved in the path ordering of the non-Abelian Wilson loops [21]. However, there appear to be no conceptual problems with such an extension. What we conjecture as happening, which we shall justify in this article, is that, in the non Abelian case, the topology of the anti-de-Sitter space time may be more involved. In particular, in the five-dimensional case, of relevance to QCD, the above-described Liouville approach to $D$-particle recoil may lead, in certain cases, to AdS space-times involving Black Holes [20], thereby generalizing non-trivially the $B_{d+1}$ (33), encountered in the single-D-particle case above.

### 3.2 Liouville-String Approach to Four-Dimensional Gauge Theories

Let us discuss now the connection of D-brane recoil (viewed as quantum fluctuations of a heavy D-particle in a (dual) space time) to a Gauge Theory at finite temperatures, formulated in real time formalism. In that case, the gauge theory lives in a five-dimensional space time manifold, four space-time Minkowskian, with a time $t$,
and a compactified time direction, playing the rôle of temperature.

Consider first the Matsubara formalism, where the finite-temperature field theory is obtained by compactifying the time direction to a circle $S^1$ of period the inverse temperature $\beta$. We consider the case where the space is flat. In this case, the string theory, living on the two-dimensional open string, whose boundary is the Wilson loop of the Gauge theory, may be taken to be a non-critical Liouville string theory with $\sigma$-model action:

$$S_\sigma = \int_{\Sigma(C)} \left\{ (\partial \phi)^2 + Q \phi R^{(2)} + \Phi(X) R^{(2)} + G_{ij}(X) \partial X^i \partial X^j + V_v(\phi) + V_m(\phi) + \ldots \right\}$$  \hspace{1cm} (37)

where $X^i, i = 1, \ldots 4$, span a manifold of topology $R^3 \otimes S^1$, $\phi$ denotes the Liouville mode, and the dots denote appropriate supersymmetrizations or other deformations, relevant in the description of the Wilson loop, such as antisymmetric tensors etc. $Q$ is related to the matter central charge deficit in a standard way. In our case, we initially take the matter central charge to be $c = 4$, coinciding with the dimensionality of the space-time manifold $X$. There are world-sheet vortices and monopoles in such a picture, since the Liouville theory is in the dangerous region, $1 < c < 9$, so the deformations $V_v, V_m$ should be thought of as the analogue of the Liouville ‘cosmological constant’ operators in the $c < 1$ case. These deformations are given by $L_{\phi} = L_{\phi}^{(1)} + L_{\phi}^{(2)}$, respectively. The Liouville mode $\phi$ is ‘space-like’ in this case. Moreover, if one views the Liouville field $\phi$ as a local scale on the world-sheet, then the space-time metric should be renormalized by $\phi$, which leads to curved five-dimensional manifolds, $G_{AB}(X, \phi)$, $A, B = 1, \ldots 5$. Thus, a four-dimensional (three spatial dimensions and a temperature) string is thereby dressed with an extra space-like target dimension, in order to become critical and thus quantized self-consistently.

We next come to examine the rôle of a Minkowskian time in such a formalism, which turns out to be a second (time-like) Liouville field, that determines the consistent string ($\sigma$-model) backgrounds for the five-dimensional metric $G_{AB}$. Including the time $t$ as a dynamical field in the $\sigma$-model (37), corresponds, from the point of view of the original gauge theory, to a ‘real time’ approach to the finite temperature field theory. Let us see now how we can incorporate the field $t$ consistently.

To this end, we first recall that a Wilson loop may be thought of as describing a heavy test particle propagating along the loop. At the beginning of time, $t = 0$, the particle starts moving, which causes sufficient distortion of the surrounding space-time, described in the dual string picture by the recoiling $D$-particle. In terms of the ‘Wilson-loop $\leftrightarrow$ string correspondence’, discussed above, the situation may be thought of as corresponding to placing Dirichlet boundary conditions on the dual virtual disc describing the two-dimensional extension of the Loop curve $C$ (Stokes’ theorem). Consider the case where the $D$-particles are quite heavy, so any velocities $u_i \to 0$. Then, in a weakly dual string $\sigma$-model picture the deformation is equivalent to the position quantum fluctuations of the $D$-particles,

\footnote{Such boundary conditions may arise by viewing $T$ duality as a canonical transformation of the string world sheet.}
appearing at time \( t = 0 \), and it is given by the recoil operator [18]

\[
y_i \int_{\partial \Sigma = C} \Theta_\epsilon(t) \partial_n X^i = y_i \int_{\Sigma(C)} \partial_\alpha(\Theta_\epsilon(t) \partial^\alpha X^i)
\]

(38)

where we have used Stokes theorem to write the boundary recoil operator as a bulk operator [17]. Notice that, the size \((L/a)^2\) (where \(a\) a world-sheet cut-off), of the world-sheet disc is to be identified with the size of the quark-anti quark loop, so the \( \epsilon^2 \sim 1/\ln(L/a)^2 \to 0 \) is consistent with considering large loops, relevant for the confining aspects of the Gauge theory (!). The collective coordinates for the position of the D-particles \( y_i \) describe the space-time coordinates of the gauge theory, or equivalently the target space of the non-critical string theory living on the world-surface surrounding the Wilson loop.

Since the recoil operator (38) has small but negative anomalous dimension [18] \(-\epsilon^2/2\), one may dress this theory with a time-like Liouville field, which introduces an extra time-like dimension \( \varphi \), since, as mentioned previously, the five-dimensional \( \sigma \)-model is assumed critical, before the recoil, as a result of the (space-like) Liouville dressing. By identifying \( \varphi \) with \( t \), as in [17], one, then, obtains a \( \sigma \)-model in six dimensions, with signature (1,5). This six-target-dimensional Liouville dressed theory is described by a target-space metric around the D-particle of the form (23), leading to a target-space time Ricci curvature of the form (29). In such a case, the \( t \)-Liouville-dressed couplings \( y_{i,L} \) are given to lowest order by:

\[
y_{i,L} = \epsilon y_i e^{\epsilon \varphi}; \quad \varphi \equiv t
\]

(39)

and the approach of [17] applies straightforwardly to this case, since the only source of non-criticality is the operator (38).

To find the consistent geometry of the five-dimensional Euclidean manifold, one should then follow the steps outlined in the previous section, leading to (23), which is achieved straightforwardly by partial differentiating the bulk vertex operators (38), as in [17]. Then, in the limit of a heavy D-particle (weakly coupled string \( g_s << 1 \)) the time (extra time - like Liouville) coordinate \( t \) decouples, and the distortion of space time may be described by anti-de-Sitter geometries for the Euclidean 5-dimensional space.

Recalling that the latter theory has a ‘matter central charge’ 4, and hence a space-like Liouville mode, then, according to our discussion in the previous section, we observe that the \( \sigma \)-model associated with this AdS background has a similar world-sheet phase diagram for the vortices and monopoles with the one described in [17],[18]. The various phases are described by non-critical strings with varying matter central charges, \( D \), related to the temperature via (13). In such a non-critical string picture, the deficit of the target-space dimensionality \((D=5)\) is compensated by condensates of the background dilaton (and possibly other) fields. The rôle of the dilaton condensate may be associated with condensates of glueball fields, related to the scale anomaly in effective Lagrangian descriptions of ordinary QCD [33].

We hope to come back to a detailed discussion of these issues in a forthcoming publication [1].
As we shall discuss below, there is an elegant interpretation of this non-critical string approach to the real-time formalism of finite-temperature gauge theories, in terms of thermodynamical properties of a gas of (classical) AdS black holes [20], as a result of the AdS/CFT correspondence [4,3]. The above-described world-sheet defect description, allows an identification of the pertinent ‘Liouville temperature’ (13) with the temperature of the (macroscopic) $AdS_5$ [20], whose boundary is the conformal space time, on which the (conformal) gauge theory in question lives. In this way one may obtain non-perturbative information on the phases of the gauge theory, which can be identified with the phases (17) of the corresponding Liouville theory. However, in the present work, we shall also point out an alternative possibility, according to which the pertinent $AdS_5$ Black Hole structures, are not associated with classical macroscopic geometries as in [3], but with microscopic fluctuations, whose size is set by the UV cutoff of the low-energy theory. The latter induce an area law for large Wilson loops, but with string tension considerably weaker than the previous macroscopic $AdS_5$ geometries. This latter case is probably relevant for the appearance of the fundamental string tension in grand-unified models of strings.

4 Coupling of Scale Invariant (Super)String theories to Gauge Theories

As a prelude to our results, we consider it as instructive to review first briefly the work of [5], according to which certain flat-target-space scale invariant Green-Schwarz superstring theories can be shown to be equivalent to Abelian gauge theories. This equivalence should be understood in the sense of relations of the form:

$$<W(C)> \sim e^{iS_{\sigma}}$$

with $S_{\sigma}$ a world-sheet action, expressing area of the Wilson loop in some sense, and $W(C)$ some combination of observables, expressed in terms of chiral currents $e^{\int J_A}$, which goes beyond the standard Wilson loops. The action $S$ becomes a standard world-sheet action upon appropriate dynamical generation of a string scale due to some sort of condensation. In [5], Supersymmetry was argued to be essential for such an equivalence. It is the point of our work to argue that if monopoles or vortices are present on the world sheet, then the supersymmetry becomes unnecessary, and in fact one may get a natural mechanism for exhibiting the confining aspects of the finite-temperature gauge theory, even in non-supersymmetric (but still conformal) cases.

The analysis of ref. [5] was performed for four-dimensional target spaces, but it can be straightforwardly generalized to higher dimensionalities as well. For reasons of concreteness and calculational simplicity, we shall review below the analysis in the four dimensional case, where the gauge field theory coupling is dimensionless. We shall first review the (supersymmetric) case where world-sheet defects have been ignored [5]. We shall comment on the non-trivial rôle of our defects in the next section.
Let us consider an Abelian Supersymmetric Gauge Theory, described by a standard Maxwell Lagrangian in a superfield form. The connection with string theory follows from standard arguments, by considering the Stokes theorem on a two-dimensional surface, Σ, whose boundary is the loop \( C \) in question. If one parametrizes the curve \( C \) by \( \lambda \), then he may write the exponent of the Wilson loop as

\[
S_{\text{int}} = ie \int_C d\tau A(X(\tau)) \frac{\partial}{\partial \tau} X(\tau)
\]

and by applying Stokes’ theorem

\[
S_{\text{int}} = ie \frac{2}{2} \int_{\Sigma(C)} d^2 \sigma \epsilon^{ab} F_{\alpha \beta}, \quad a, b = 1, 2,
\]

where \( X^M, M = 1, \ldots D, \) is a \( D \)-dimensional space-time coordinate for the gauge theory; we denote by lower-case Latin indices the two-dimensional coordinates of the surface \( \Sigma \), which plays the rôle of the world-sheet of the string, equivalent to the gauge theory in question [19]. The quantity \( F_{ab} = \partial_a X^M \partial_b X^N F_{NM} = \partial_a A_b - \partial_b A_a \) is then the pull back of the Maxwell tensor on the ‘world-sheet’ \( \Sigma \), with \( A_a \) the corresponding projection of the gauge field on \( \Sigma \) (c.f. (9):

\[
A_a = v^M_a A_M; \quad v^M_a \equiv \partial_a X^M, a = 1, 2; M + 1, \ldots D(= 4).
\]

From a two-dimensional view-point, the term looks like a Chern-Simons term for a two-dimensional gauge theory on \( \Sigma \), bounded by the loop \( C \).

The novel observation of [5] is the possibility, in a supersymmetric gauge theory, of constructing, a second superstring-like observable, in addition to the Wilson loop, which again is defined on the two surface \( \Sigma \), and is consistent with all the symmetries of the theory. The second observable is easily understood in the two-dimensional superfield formalism

\[
Z^A \equiv (X^M, \theta^m, \theta^\hat{m})
\]

The pull-back basis \( v^M_a \) in (33) is now extended to \( v^A_a = E^A_B \partial_a z^B \), with the following components [5]:

\[
\begin{align*}
v^\alpha a &= \partial_a x^{\alpha \hat{a}} - \frac{i}{2} \left( \theta^\alpha(\sigma) \partial_a \theta^\hat{a}(\sigma) + \theta^\hat{a}(\sigma) \partial_a \theta^\alpha(\sigma) \right) \\
v^a A &= \partial_a \theta^\alpha(\sigma) \\
v^{\hat{a}} a &= \partial_a \theta^\hat{a}(\sigma)
\end{align*}
\]

in the standard notation [34], where Greek dotted and undotted indices denote superspace components, with \( x^{\alpha \hat{a}} \equiv X^M \), etc.

Following [5] we now define:

\[
\begin{align*}
C_{ab}^{\alpha \beta} &= \frac{i}{2} v_{a \beta}^\alpha v_{b \alpha}^\beta \\
C_{ab}^{\alpha \hat{a}} &= v_a^{\alpha \hat{a}} v_{b \hat{a}}^\alpha
\end{align*}
\]

with the properties \( C^{\alpha \beta} = \epsilon^{ab} C_{ab}^{\alpha \beta}, \quad C^{\alpha \hat{a}} = \epsilon^{ab} C_{ab}^{\alpha \hat{a}} \) (46)
with similar relations holding for appropriately defined dotted components of \( C \), which can be found in [5]. Notice that \( C^\alpha \) vanishes in the absence of supersymmetry, whilst \( C^\alpha{}^\beta \) exists even in non supersymmetric gauge theories as well. In the presence of defects, as we shall later, this is no longer the case of course, and \( C^\alpha \) can have non-supersymmetric remnants.

The supersymmetric Wilson loop, expressing the interaction between the super-particle and a supersymmetric gauge theory in the approach of [5], is now expressed in terms of \( C' \)’s as:

\[
W(C) = e^{S^{(1)}_{\text{int}}}, \quad S^{(1)}_{\text{int}} \equiv \frac{i}{2} \int_{\Sigma(C)} d^2 \sigma \epsilon^{ab} F_{ab} \\
F_{ab} \equiv \epsilon_{ab}\left\{ \frac{1}{2} C^\alpha{}^\beta (\sigma) D_\alpha W_\beta (x(\sigma), \theta(\sigma)) + C^\alpha (\sigma) W_\alpha (x(\sigma), \theta(\sigma)) + \text{h.c.} \right\} \tag{47}
\]
where \( W_\alpha (x(\sigma), \theta(\sigma)) \) is the chiral superfield of the supersymmetric Abelian gauge theory. The exponent \( S^{(1)}_{\text{int}} \) clearly reduces to the standard expression (41) in non-supersymmetric cases.

According to [5], there is a second superstring-like observable, as noted in [5], defined on the world-sheet surface \( \Sigma \), \( \Psi(\Sigma) \), which is constructed out of the \( C^\alpha{}^\beta \) components, and hence - in the absence of world-sheet defects - exists only in supersymmetric gauge theories, as mentioned previously:

\[
\Psi(\Sigma) \equiv e^{i S^{(2)}_{\text{int}}}, \quad S^{(2)}_{\text{int}} \equiv \kappa \int_{\Sigma(C)} d^2 \sigma \sqrt{-\gamma} \gamma^{ab} C^\alpha{}^\beta (\sigma) W_\alpha (x(\sigma), \theta(\sigma)) + \text{h.c.} \tag{48}
\]
where \( \gamma^{ab} \) is the metric on \( \Sigma \). This term, unlike the standard Wilson loop, is not a total world-sheet derivative, and, therefore, lives in the bulk of the world-sheet \( \Sigma \), and thus depends on the metric \( \gamma \). The coupling constant \( \kappa \) is defined as an independent coupling classically. However, one expects that quantum effects will relate it to the gauge coupling constant \( e \). We shall come to this point later. In [5] this second observable has been expressed in terms of ‘chiral’ currents on \( \Sigma \), located at the string source:

\[
S^{(2)}_{\text{int}} = \int d^6 Z \left( J^\alpha W_\alpha + \text{h.c.} \right), \quad J^\alpha \equiv \kappa \int_{\Sigma(C)} d^2 \sigma \sqrt{-\gamma} \gamma^{ab} C^\alpha{}^\beta (\sigma) \delta^{(6)} (Z - Z(\sigma)), \quad \delta^{(6)} (Z - Z(\sigma)) = \delta^{(4)} (Z - Z(\sigma) (\theta - \theta(\sigma))^2 \tag{49}
\]

Upon integrating out the gauge field components in (48), i.e. considering the vacuum expectation value (vev): \( < \Psi(\Sigma) > \), with \( < \ldots > \) denoting averaging with respect to the Maxwell action for the gauge field, the authors of [5] have obtained a superstring-like action, which is scale invariant in target space, as well as on the world-sheet:

\[
< \Psi(\sigma) >_{\text{Maxwell}} = e^{S_0 + S_1}
\]
\[ S_0 \equiv \frac{\kappa_0^2}{16\pi} \int_{\Sigma} d^2\sigma \sqrt{-\gamma} \gamma^{ab} v^M_a v^N_b \sigma_M \sigma_N, \]
\[ S_1 \equiv \frac{\kappa_1^2}{4\pi} \int_{\Sigma(C)} \sqrt{-\gamma} \gamma^{ab} v^M_a v^N_b \eta_{MN} \sigma^K \sigma_K \]

(50)

where upper case latin indices denote target space indices \((M, N, K = 1, \ldots D(= 4))\), and

\[
v^M_a \equiv \partial_a X^M(\sigma) - i \theta^m(\sigma) \Gamma^M \partial_a \theta_m(\sigma)
\]
\[
\sigma^M = \sqrt{-\gamma^{ab}} \partial_a v^M_b, \quad G_{ab} \equiv v^M_a v^N_b \eta_{MN}
\]

(51)

in standard four-component notation in target space \((m\) are spinor indices, and \(\Gamma^M\) are Dirac four-dimensional matrices). The (dimensionless) coupling constants \(\kappa_{0,1}\) appear arbitrary at a classical level, but one expects them to be related (proportional) to the (dimensionless) gauge coupling \(e\), in the quantum theory.

The important observation in [5] was the fact that the world-sheet action \(S_2\) resembles the classical Green-Schwarz superstring action in flat four-dimensional target space, provided that condensation occurs for the ‘composite field’

\[
\Phi \equiv \sigma^M \sigma_M, \quad M = 1, \ldots D(= 4),
\]

(52)

in such a way that

\[
\frac{\kappa_1^2}{4\pi} < \Phi > = \mu_{\text{string tension}}
\]

(53)

In this way a dimensionful scale (string tension) could be obtained from a gauge theory without dimensionful parameters. The physically interesting question is what causes this condensation, which presumably is responsible for a ‘spontaneous breaking’ of the scale invariance of four dimensional string theory.

In the next section we shall present a scenario on the formation of such a condensate, which is an extension of the ideas of [3]. We shall associate the second observable \([18]\) of [3] to the condensation of non-trivial world-sheet defects. The latter, in turn, will be associated with topology changes in the five-dimensional Liouville AdS space time, argued previously to arise in the non-critical string approach to the four-dimensional gauge theory, described in section 3. In such a picture, the standard Wilson loop \(W(C)\) will exhibit the area law in the confining (low-temperature) phase, with string tension \([53]\), which can be related to the radius of the AdS5 space time. Notice that discussions in the recent literature on non-perturbative prescriptions for the computation of Wilson loops \([33]\) apply to our case as well, provided one takes proper account of the Liouville dynamics, and the non-trivial world-sheet topology due to the presence of the defects. We hope to give detailed reports on such delicate matters in the future. At the moment we note that the condensate of \(\Phi\) \([52]\) may be viewed as being related to a vacuum expectation value of a target-space dilaton field, which in this approach may be related to scalar glueballs, appearing due to scale anomalies in standard treatments of effective Lagrangians for QCD \([33]\). We plan to discuss these issues elsewhere.
5 World-Sheet Monopoles and Quark Confinement

5.1 A Small Digression on Anti-De-Sitter Black Holes

One possible mechanism for the appearance of the condensate \[53\] would be the one associated with four-dimensional quantum gravity effects at Planckian scales. In such a case the resulting string tension would probably describe fundamental strings, responsible for the unification in higher dimensional string models \[8\].

A more elegant scenario, which is probably more relevant for our long-distance confining gauge theory physics, may be based on the recent interpretation \[9\] of confinement in pure glue non-Abelian gauge theories, by means of a holographic principle encoding confinement quantum physics to classical geometries of (uncompactified) five-dimensional anti-De-Sitter space times. In such a picture, the four-dimensional conformally invariant Minkowski space time is viewed as the boundary of a five-dimensional Anti-de-Sitter space time, in the spirit of \[10, 11\]. Macroscopic Black Holes defined in anti-de-Sitter space times disappear at temperatures \(T_0\), as we go down in temperature from a high-temperature phase. Notice that Macroscopic Black holes in anti-de-Sitter space times have been found to possess well-defined thermodynamical properties \[20\]. Indeed, the analysis of \[20\] has shown that there exists a critical temperature, \(T_0\), above which black holes can be thermodynamically stable. Below \(T_0\) only radiation-dominated universes exist. Witten \[12\], has associated this temperature to a confining-deconfining phase transition for quarks, and stressed the fact that for spatial Wilson loops the are law and the associated string tension is obtained only for finite temperature field theory; this is due to the conformal invariance of the zero-temperature four-dimensional gauge theory, whose Renormalization Group \(\beta\) function vanishes identically.

It is instructive for our purposes to review briefly the relevant properties of Anti-De-Sitter space times \[21, 13\]. For concreteness we shall explicitly describe the case of \[21\], which is \(AdS_4\). The generalization to \(AdS_d\), for general \(d\) is straightforward \[13\].

The (Minkowskian signature ) AdS Schwarzschild Black hole solution of \[21\] corresponds to a line metric element of the form:

\[
ds^2 = -V(dt)^2 + V^{-1}(dr)^2 + r^2d\Omega^2
\]  

(54)

with \(d\Omega^2\) the line element on a round two-sphere, and \(r\) the radial coordinate of \(AdS\), \(t\) is the time coordinate.

The Anti-de-Sitter Black Hole space time, which is obtained as a consistent classical solution of Einstein’s equations, in a space with cosmological constant \(\Lambda <\)

Notice that if one defines the composite field \(\Phi\) as above, then one may extend the above analysis to higher dimensional cases by simply contracting the integrand of \[13\] with appropriate powers of \(\Phi\) so as to make the coupling constant \(\kappa\) dimensionless. This can be understood as the definition of the superstring observable \(\Psi\) in higher than four target space dimensions. It is important to note that the dimensionality of the coupling constant \(\kappa\) is the same as that of the gauge theory coupling \(e\) only for space-time dimensionality four, six and ten, where supersymmetric theories exist as well.
0, becomes a smooth geometry if one compactifies the time direction, to a special period $\beta$, defining a temperature for the AdS Black Holes

$$\beta = \frac{4\pi b^2 r_+}{b^2 + 3r_+^2}, \quad b = \sqrt{-\frac{3}{\Lambda}}$$

(55)

where $r_+$ is the maximum of the roots of the equation

$$V \equiv 1 - \frac{2M}{m_p^2 r} + \frac{r^2}{b^2},$$

(56)

where $m_p$ is the AdS space Planck mass. The Euclidean version of the space time has the topology of

$$X_2 = B^2 \times S^{n-1} \quad (n = 3 \text{ for ref. [20]})$$

(57)

According to the analysis of [20], there are three critical temperatures in the AdS black hole system: (i) at the first one $T_0$ the specific heat of a gas of Black Holes changes sign at a critical temperature, $T_0$. The critical temperature is the one corresponding to the maximum of $\beta$ (55):

$$T_0 = (2\pi)^{-1}\sqrt{3}b^{-1}$$

(58)

For $T < T_0$ there is only radiation, and the topology of the finite-temperature space time is described by:

$$X_1 = B^n \times S^1 \quad (n = 3 \text{ for ref. [20]})$$

(59)

The line element for this metric, without Black Holes, is given by:

$$ds^2_{x_1} = (1 + \frac{r^2}{b^2})dt^2 + \frac{1}{1 + \frac{r^2}{b^2}}dr^2 + r^2d\Omega^2$$

(60)

(ii) For temperatures above $T_0$ the topology of the space time changes to (57) to include black holes, but in the region $T_0 < T < T_1$, where:

$$T_1 = \frac{1}{\pi}b^{-1}$$

(61)

the free energy of the black hole is positive, so the black hole would have the tendency to evaporate.

(iii) For $T > T_1$, the free energy of the configuration with the black hole and thermal radiation is lower than the corresponding configuration with just thermal radiation, so the radiation would just tunnel to black holes, whilst

(iv) for temperatures greater than a third value $T > T_2$:

$$T_2 = (m_p^2)^{1/4}(3)^{1/4}b^{-1/2}$$

(62)
there is no equilibrium configuration without a black hole.

The topology change in the five-dimensional AdS space time from $X_1$ to $X_2$, at $T = T_0$, prompted Witten to conjecture, based on the CFT/AdS correspondence, that there exists a connection of this phase transition, which in the case of a gas of AdS Black Holes is a first order transition, with the confinement-deconfinement phase transition of the Boundary large $N_c U(N_c)$ gauge theory at finite temperatures. The temporal Wilson lines, $P$, acquire non-zero vacuum expectation values above $T_0$, thereby inducing a spontaneous breaking of the center of the gauge group, while the spatial Wilson loops acquire an area law below $T_0$, with (constant, temperature independent) string tension $T_{str}$, given by the square of the (large) AdS Radius of curvature:

$$T_{str} \propto b^2 \propto (-\Lambda)^{-1}$$

in units of an elementary string tension (e.g. type IIB string in the approach of [3]).

### 5.2 World-sheet defects, AdS space-times, and application to Quark Confinement in non-supersymmetric theories

Let us now examine the above phase diagram from a world-sheet defect point of view. We first mention that the inclusion of world-sheet defects then, the above picture of obtaining a finite string tension, is valid also in non-supersymmetric theories. Indeed, at the position $\sigma_0$ of the defect (center of the loop $C$), where the target-space field $X(\sigma)$ diverges the quantity $\sigma^M$, entering the definition of the field $\Phi$, acquires contributions also from the bosonic parts $\partial_a X^M$, since in that case there is a non-zero vorticity:

$$\epsilon^{ab} \partial_a \partial_b X^M \neq 0$$

It is important to notice that condensation of composite operators containing $\epsilon^{ab} \partial_a \partial_b X^M$, e.g. appropriate powers as in (52), can occur for both vortex and monopole configurations. This allows the passage to the non-critical-dimension string picture of QCD, sketched in section 3.2.

We also note that in a dilute-gas approximation for the defects, where only a single defect is considered on the world-sheet, the two-dimensional surface $\Sigma$ with boundary $C$ which encircles the defect does not contain any other defects. In that case, the quantities $\sigma^M$ have a non-supersymmetric counterpart, due to the non-trivial vorticity. However, in a situation of many defects, as we expect to be the case beyond dilute gas approximation, one encounters defect singularities on $\Sigma(C)$. For such cases, a non-supersymmetric remnant of the observable exists, and the pertinent world-sheet action equals the standard bosonic string Nambu-Goto action, upon (53).

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9The quantity $\epsilon^{ab} \partial_a \partial_b X^M$ is non trivial for vortices, since the vortex angular variable is not differentiable due to its non-trivial winding number around a closed loop. Also (64) is not well defined at the monopole core, and requires regularization, e.g. by cutting small loops around the singularity.
Let us first examine the case studied in [3], which is based on critical-dimension (super)strings. In that case, one may consider the role of vortices on the world-sheet of the string, wrapped around the compact dimension $S^1$ in the manifold with topology $X_1$, where such configurations exist [6]. Then, condensation of such vortices, bound into pairs with antivortices, results in the quantity $\Phi$ (52) acquiring a non-trivial v.e.v. $< \Phi > \neq 0$. In the classical AdS picture, described above, it is the non-trivial Planckian dynamics of a five-dimensional space time which is responsible for the above phenomenon. In such a case the standard BKT phase transitions of vortex condensation on the world sheet of critical strings [5], may be identified with $T_0$ of the AdS Black Hole space time. If then this transition is associated with the traditional ‘Hagedorn’ temperature, where vortex fields become tachyonic [6], then one has a nice (non-perturbative) picture where the confining phase transition coincides with the Hagedorn temperature.

In the picture advocated in section 2, where both types of defects are present, one has a nice correspondence of the entire phase diagram of AdS Black Holes presented above, with the the higher temperature $T_2$ -associated with Planckian scales in the $AdS_5$ space time- corresponding to monopole condensation. This would define an effective Regge slope for the string

$$\alpha' = \frac{8\pi^3}{m_p'^{1/2}(-\Lambda)^{3/4}}$$

which may be thought of as the inverse of a string tension. Then, the definition of the string tension (63) is consistent with (64), provided one identifies $m_p \sim (-\Lambda)^{-7/2}$ in some fundamental string units. Notice that, in this way, $m_p^2 >> |\Lambda|$, (65) is small, and the entire approach is consistent. In this picture, then, one has a higher-dimensional analogue of the correspondence between world-sheet monopole/anti monopole pairs and Schwarzschild Black Holes in two-target-space dimensional strings [7].

The above phase diagram applies intact also to the non-critical string approach to Gauge Theory, advocated in section 3.2. However, in this more general case one may think of the scales $-\Lambda$ and $m_p$ as being independent. As we shall discuss later on, in our Liouville approach, the quantity $m_p$ may also receive contributions from microscopic fluctuations of the AdS space. Hence, in this picture, one obtains the following phase diagram (c.f. (18) for the non-Abelian gauge theory, or a compact Abelian Gauge theory:

$$(i) \ T < T_{BKT-vortex} : \ \text{Confining Phase}$$

$$(ii) \ T_{BKT-vortex} < T < T_{BKT-monop} : \ \text{Deconfining phase}$$

where $T_{BKT-monop} >> T_{BKT-vortex}$

$$(iii) \ T > T_{BKT-monop} : \ \text{Planckian Physics}$$

Notice that in this approach, the string tension appearing in the Planckian Physics regime, due to world-sheet monopole condensation (corresponding to stable Black Holes in $AdS_5$ [20]), is not the same as the confining string tension in the low-energy
phase of the theory; the Planckian tension may be viewed as a fundamental string tension, due to quantum-gravitational effects. The existence of a higher temperature, such as $T_2$ or the monopole condensation temperature in our world-sheet picture, which sets the scale for new Physics (Planckian), and hence may lead to a unified picture involving effective and fundamental (grand-unified) strings, is not in disagreement with generic conjectures made in some simple models of QCD, based on the effective Lagrangian approach [33, 36]. Also notice that in such models, the gluon transition had been identified with the deconfinement transition, at least for large number of colours, and argued to be of first order.

There still remains the issue on the nature of the $T_1$ intermediate temperature, appearing in the phase diagram of [24]. In view of the vortex-monopole duality [7], as well as the discussion preceding (24), one would be tempted to identify $T_1$ with the temperature corresponding to the self-dual radius. This, however, does not work. Indeed, if such were the case then the following relation should be valid (c.f. (24)):

$$T_2 = T_0^2 / T_1$$

which, in view of (58), (61), (62) would imply $m_p = \frac{4}{\sqrt{\Lambda}} (-\Lambda)^{1/2}$. This contradicts the assumption $m_p \gg -\Lambda$ on which the above analysis is based. Notice, in this respect, that the effective Regge slope (55), combined with (52), defines a self-dual temperature $T_{sd} \propto (-\Lambda)^{-1/2}$ such that $T_2 >> T_{sd} >> T_1 >> T_0$ for $|\Lambda|$ small.

However, there may not be a phase transition at $T_1$. Indeed, in the region $T_0 < T < T_1$ the black hole has a tendency to evaporate (but in a unitary way), whilst above $T_1$ the radiation tunnels to the Black Hole space time. A possible scenario describing the pertinent tunneling phenomena would be that there exist some other relevant operators in the world-sheet theory, e.g. instantons, which cause this instability; indeed that picture would match the picture in the two-dimensional black hole case [12, 37], with a cigar-type target-space metric to which a world-sheet monopole-antimonopole pair is equivalent [7]. For such metrics, there are instanton solutions on the world-sheet, which constitute relevant operators. This was argued in ref. [12] to describe transition among black hole states. Notice that the spherically symmetric part of the metric for the four-dimensional AdS pf [24], (14), (16) approaches a cigar type metric for $b >> 1$, so one may conjecture that at least in the limit of large $b$ there are world-sheet instantons in the $n$-dimensional case too, in the corresponding $\sigma$-model. A formal construction of an exact conformal field theory for such Black Hole space times, which would be the higher-dimensional analogue of [14], is still eluding us, but we think that the present work offers a sufficient number of arguments, and non-trivial consistency checks, in favour of its existence.

We now note that if we accept the above picture, and associate the string tension with the $AdS_5$ radius, $b^2$, then in our ‘recoil’-induced anti-de-Sitter space $b^2 \propto |\epsilon|^{-4}$. Notice that $|\epsilon|^{-2}$ is proportional (24) to the size of the world-sheet disc [18]. If we interpret such world-sheet discs as the minimal area enclosed by Wilson loops, we may then consider the limit $\epsilon \to 0^+$, where the analysis of [18] can be trusted, as representing macroscopic loops, appropriate for quark confinement. If one, then,
applies the approach of [3] to this case, by identifying the string tension with the squared radius of the $ADS_5$, one obtains a logarithmic scaling violation of the area law, manifested as a dependence of the string tension on the logarithm of the area of the large quark loop:

$$T_{str} \sim \ln^2 A, \quad A = \text{Loop minimal area} \quad (68)$$

Then, following (53), one would be tempted to interpret such a result as reflecting an asymptotic freedom on the coupling $\kappa_1$. As we mentioned previously, this coupling is expected to be proportional to the gauge coupling of the original theory. The fact that the confinement occurs at the region of strong gauge couplings constitutes a nice consistency check of the approach.

We now mention an alternative possibility, which may also be responsible for the appearance of a non-vanishing string tension in the sense of (53). Instead of large AdS radii, one may consider microscopic five-dimensional $AdS_5$ black holes, whose intersection with the surface $\Sigma(C)$ bounded by the macroscopic Wilson loop, would correspond to small 2-d regions bounded by microscopic loops (of the size of the Planckian length, which is the effective UV cut-off in the target-space low-energy theory). In our D-string picture, such microscopic loops correspond formally to $|\epsilon|^2 \to \infty$, viewed as a world-sheet renormalization group scale. Although in such a case the approximations of [18] break down, however, one may consider a formal extension of the above arguments; in that case, such small loops on $\Sigma(C)$, could be viewed as ‘regularized’ world-sheet defects. Our Liouville theory approach, then, applies, at least formally, to this case as well. However, the thermodynamical analysis of [20] may not be valid. If one accepts though, the existence of a temperature $T'_0$, which induces topology changes for the microscopic fluctuations of space time, then such a temperature could be identified with the corresponding monopole condensation BKT transitions in our Liouville approach [19]. In this regime, the string tension could arise via (53), through, say, condensation of these microscopic fluctuations (monopoles in a world-sheet picture). However, the string tension in such a case is expected to depend on the microscopic AdS radii $b^2 \lesssim L_{Planck}^2$, and thus it will be much weaker than the one obtained in the macroscopic $AdS_5$ case; this is why we expect this latter approach, if correct, to be relevant for the appearance of a string tension in grand-unified strings. Certainly, such non-trivial issues require further investigations before any serious claims can be made.

10From the point of view of the gauge theory [3, 2], where the AdS-space radius of curvature is proportional to some positive power of $g_M^2 N_c = \alpha$, kept fixed as $N_c \to \infty$, the region of large $\epsilon$ corresponds to weak coupling, $\alpha \ll 1$, and, thus, to the high-energy region, where the gauge theory is asymptotically free. In this regime, the standard BKT world-sheet phase diagram, with three temperatures, for monopole and vortex condensation, and self-dual situation, satisfying (67), may be valid. However, the string background computations cannot be trusted.
5.3 Abelian Projection, Abelian Dominance Hypothesis and Quark Confinement

As already mentioned, one expects the above connection of Wilson loops with ‘black hole (classical) physics’ to be valid only for Non-Abelian gauge theories or compact Abelian Gauge theories with (magnetic) monopoles. To show this formally in the non-Abelian case would necessitate the extension of the formalism of \[5\] to take into account Path ordering in the definition of the Wilson Loop, or the second superstring observable, which is technically a non-trivial task. However we expect no conceptual problems with such an extension.

At the present stage, we have a formal proof of the equivalence between gauge fields and strings, in the sense of (40), only for Abelian gauge fields \[5\]. The confinement mechanism, described above, is valid only for non-Abelian gauge fields, or compact Abelian gauge fields, with magnetic monopoles. For other cases, although the formal equivalence (48) may be valid, however, the resulting string tension (53) should be zero. In our approach above this can be understood by the fact that the resulting scale invariant string appearing in (48) lives in a flat target space, in contrast to the non-Abelian case, where the bulk \textit{AdS} space, associated with it in the sense of \[4, 3\], has been conjectured to have non-trivial topologies.

However, even at the present stage, one is able of making some remarks on the long-distance physics concerning quark confinement, as we shall argue now. This is due to ‘t Hooft’s observation on the rôle of Abelian gauge theories on the low-energy confining physics.

For definiteness we shall consider the case of the Non-Abelian gauge group \(U(N_c)\), \(N_c \to \infty\), gauge theories. One may invoke the ‘Abelian Projection’ conjecture of ‘t Hooft \[38\], according to which one can fix the gauge in a non-Abelian gauge theory, in such a way that one can remove as many non-Abelian degrees of freedom as possible, in the sense that only a maximal torus group \(H\) of the gauge group \(G\) remains unbroken. For the group \(G=SU(N_c)\), for instance, \(H = U(1)^{N_c-1}\). According to ‘t Hooft’ the Abelian projected theory reduces to a \(U(1)^{N_c-1}\) abelian gauge theory \textit{supplemented} by a magnetic monopole. In such a case, according to Mandelstam, confinement manifests itself as \textit{superconductivity} (type II) in the dual theory, in the phase where monopole condense. In addition, to the Abelian projection hypothesis, a stronger statement had also been made in the literature, the so-called Abelian dominance hypothesis, according to which only the Abelian parts of a non-Abelian gauge group play an important rôle for confinement of quarks \[39\]. For a recent discussion on Abelian -projected QCD and confinement see \[40\]. An important ingredient in all these works is the decoupling of the ‘non-Abelian’ degrees of freedom at the expense of having magnetic monopoles in the dual theory.

We now come to our stringy case, for the gauge group \(G=SU(N_c)\), for large \(N_c\). The theory is conformal at zero temperatures, and leaves in the boundary of a dynamically appearing \textit{AdS} space, as explained above. According to the previous discussion, one may fix the gauge to the Abelian projected one, in which only the subgroup \(U(1)^{N_c-1}\) matters, but there is a dual magnetic monopole. Then, the
relevant Wilson loop or the superstring-like observable (18) factorize

\[ < \Psi_1(\Sigma) > N_c^{-1} = e^{-(N_c - 1)S_{1,\text{int}}^{(2)}} \tag{69} \]

where the subscript 1 indicates quantities in a $U(1)$ gauge theory, and we have passed into a Euclidean formalism. The large $N_c >> 1$ limit in such a case may be interpreted as implying an effective string tension $T_{st} \equiv N_c a_0^2 < \Phi >_1 \to \infty$, or a Regge slope $\alpha' \sim T_{st}^{-1} \to 0$, therefore the low-energy field theory (perturbation expansion in powers of $\alpha'$) may be trusted. This is analogous, but not identical, to the arguments why large $N$ is needed for the analysis of (48, 3). The relation (69) implies, then, Wilson’s criterion for confinement, if one interprets the world-sheet superstring action $S_{1,\text{int}}$ as an area of the pertinent loop. Notice here that, as a result of the spontaneous breaking of scale invariance, in the sense of the condensation (53), a confining area law makes perfect sense. This is the result of world-sheet vortex condensation. The relevant confinement-deconfinement phase transition has been argued, following (3), to occur due to topology change (appearance of singular space-time structures (black holes)) in the Liouville AdS. Notice that the presence of world-sheet vortex defects, associated with non-zero vorticity (64), can be understood, through the pull back (9), as implying a corresponding non-trivial dual magnetic monopole gauge field in target space, necessary for confinement in Abelian-projected theories (35). The dual nature of the target-space defect is associated with the fact that the magnetic flux through the surface surrounding the Wilson loop is associated with the second term in the rhs of (9), which is related to the world-sheet monopole contributions, as mentioned already. The world-sheet duality (1), which maps a monopole onto a vortex, is thereby associated with a standard electric-magnetic duality in target space (11).

At present we consider the above discussion as conjectural, given that a mathematically rigorous correspondence of the condensate (53) with AdS black holes, in the non-Abelian case, although plausible, has not yet become available, due to the aforementioned technical path-ordering difficulties associated with non-Abelian Wilson loops. However, the non-trivial consistency of the above plausibility arguments on the Abelian Projection points to the validity of this assumption.

\[ \text{From this point of view, our approach in this work may also be considered relevant to related studies on the role of magnetic monopoles on the confining aspects of compact Abelian gauge field theories, and, in particular, it may offer a way to describe the interaction between electric and magnetic (monopole) loops in such theories (11). In this latter respect, we would like to mention the work of (24), according to which the presence of massless point-like sources in compact U(1) 4d theories, leads naturally to two-dimensional shock wave configurations. Their magnetic counterparts (monopoles) have been argued to be responsible for a Coulomb-gas behaviour of the vacuum, exhibiting BKT-like transitions, as we find here. However, at present, it is not clear to us how to make a formal correspondence of our results with such analyses. This issue deserves further investigation.} \]
6 Conclusions

In this work [1] we have made a proposal for a non-critical Liouville string approach to the long-distance physics of four-dimensional (conformal) non-Abelian Gauge theories. We have given many plausibility arguments in its favour, and performed some non-trivial consistency checks. A crucial ingredient in the approach was the incorporation of world-sheet defects, monopoles and vortices. Their presence was necessitated by the fact that, the ‘matter’ central charge of the pertinent Liouville theory, was found to be in the ‘dangerous’ region $1 < D < 9$. We have identified the relevant contributions of the world-sheet defects to the Wilson loop of the 4-d Gauge Theory, and argued that they are described by the second superstring observable of [5], which in this case has non-supersymmetric remnants.

The association of the world-sheet defects with D-particle boundary deformations on the (open) world-sheet, bounded by the Wilson loop curve, lead to non-trivial recoil effects, argued to describe the ‘distortion’ of space time due to the motion of the heavy test particle that circulates along the Wilson loop. Such recoil effects have resulted in a dynamical appearance of five-dimensional Anti-de-Sitter space times, as a result of appropriate Liouville dressing.

We have presented plausibility arguments, supported by non-trivial consistency checks, that the incorporation of non-Abelian Gauge fields leads, by the above-described Liouville dressing, to Black Hole $AdS_5$ space times.

The various Berezinski-Kosterlitz-Thouless phase transitions of a gas of world-sheet defects have been associated with the thermodynamics of the $AdS_5$ black holes, following the argumentation of Witten [3], based on the AdS/CFT correspondence proposed in [4]. The resulting phase diagrams for the gauge theory have shown the existence of a deconfining phase, which occurs at the Hagedorn temperature, the latter being defined as the temperature at which world-sheet vortices condense [6]. Our analysis has also revealed the existence of an interesting phase structure above this transition, which is in correspondence with the Thermodynamics of the $AdS$ Black Holes discussed in [20]. There exists a temperature, much higher than the vortex condensation temperature, which sets the scale for Planckian Physics, at which world-sheet monopoles condense, and thus stable black holes occur in $AdS_5$. Such a result is not in disagreement with generic expectations from corresponding studies in the context of the effective Lagrangian approach to confining $QCD$ [33, 36], predicting a much higher temperature where new physics comes in. From the point of view of this work, this temperature has been associated with the Hagedorn temperature of grand-unified fundamental strings.

There are many things in our approach that should be checked, before the above claims can be considered rigorous. The first is the extension of [5] to the non-Abelian case, which is plagued with technical difficulties. The second is the fact that a proper treatment of non-critical (Liouville) boundary dynamics is still lacking. Due to world-sheet reparametrization invariance, the Liouville field is not simply a scalar field, as we have tacitly assumed in the presentation of the above arguments. This leads to complications in imposing boundary conditions [13], and may alter some
of the above results. What we hope, however, is that at least the major qualitative features of the above analysis, which are relevant for long-distance confining physics, will survive the proper treatment of boundary Liouville dynamics, especially in the dangerous regions for the matter central charges, which is still eluding us.

Before closing, we would also like to point out that our Liouville approach to four-dimensional gauge theories, which made use of non-trivial (logarithmic) boundary ‘D-particle recoil’ deformations, appears to offer a unified description of how the string tension may arise in various regimes of string theory, including high-energy (Planckian) string tensions, as a result of microscopic fluctuations of $AdS$ space. The latter might be relevant to searches for the elusive $M$-theory description of strings [2].

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