A Five-Parameter Wind Field Estimation Method Based on Spherical Upwind Lidar Measurements

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Abstract. Turbine mounted scanning lidar systems of focussed continuous-wave type are taken into consideration to sense approaching wind fields. The quality of wind information depends on the lidar technology itself but also substantially on the scanning technique and reconstruction algorithm. In this paper a five-parameter wind field model comprising mean wind speed, vertical and horizontal linear shear and homogeneous direction angles is introduced. A corresponding parameter estimation method is developed based on the assumption of upwind lidar measurements scanned over spherical segments. As a main advantage of this method all relevant parameters, in terms of wind turbine control, can be provided. Moreover, the ability to distinguish between shear and skew potentially increases the quality of the resulting feedforward pitch angles when compared to three-parameter methods. It is shown that minimal three measurements, each in turn from two independent directions are necessary for the application of the algorithm, whereas simpler measurements, each taken from only one direction, are not sufficient.

1. Introduction
Light Detection and Ranging (Lidar) has emerged as a feasible, reliable and accurate remote sensing technology for wind speed measurements in wind energy.

With increasing wind turbine sizes and decreasing weight and size of lidar devices, the usage of nacelle based horizontally aligned, upwind directing lidars becomes more attractive, hence studies about applications based on lidar inflow measurements as yaw control, feedforward pitch control or power curve measurement have been published increasingly in recent years [1, 2, 3, 4].

While full 3D wind vector measurements require three beams focused on the measurement volume, which is for example aimed in the ‘Windscanner’ research project [5] applying ground based continuous focusing lidars, only one nacelle based laser source limits the information obtainable from the system to wind speeds along the beam direction, often referred to as line of sight wind speeds. This limitation results in an ambiguity of interpretations of the underlying wind field, sometimes referred to as ‘Cyclops dilemma’.

Especially the simultaneous determination of shear and direction – both horizontally and vertically – is difficult and often avoided, e.g. by either assuming zero shear and calculating a mean wind direction which can be used for yaw control, or by assuming perfect directional alignment and...
the interpretation as shear, which is useful for feedforward pitch control. In reality both assumptions are not fully given and lead to errors in the calculated values and hence to incorrect control signals. In this paper a method is proposed determining both shear and direction parameters by applying one laser source only. Therefore in Section 2 a five-parameter 3D wind field model is introduced. A parameter reconstruction algorithm based on the measurement at two distances is deduced for a minimum number of perfect measurements and extended to an abundant number.

In Section 3 the algorithm’s performance to estimate the relevant parameters of a wind field is studied with simulations. Three analyses are performed: the influence of one or two measurement distances, the reconstruction of five and three parameters and the influence of spatial weighting and wind evolution. Section 4 concludes the paper and gives an outlook on possible wind energy applications making use of the method and further need for research.

2. Five-Parameter Wind Field Modelling and Reconstruction

2.1. Five-Parameter Wind Field Model

Five parameters are proposed to characterize the mean behavior of a three-dimensional wind vector field within a two-dimensional plane \( x = 0 \) in front of the turbine:

1. \( u_0 \): averaged streamwise wind speed [m/s]
2. \( s_h \): linear horizontal shear [s\(^{-1}\)]
3. \( s_v \): linear vertical shear [s\(^{-1}\)]
4. \( \delta_h \): averaged horizontal stream direction [rad]
5. \( \delta_v \): averaged vertical stream direction [rad]

The mean wind speed \( u_0 \) is aligned with the rotor plane averaged wind directions \( \delta_h \) and \( \delta_v \). The shear values \( s_h \) and \( s_v \) describe the slope of linearly ascending streamwise wind speeds in \( y \)- and \( z \)-direction respectively.

According to the model, wind vectors \( \vec{v} \) are then given depending on their position \( (y, z) \) in the wind field plane to

\[
\vec{v}(y, z) = (u_0 + s_h y + s_v z) \begin{bmatrix} \cos(\delta_v) \cos(\delta_h) \\ \cos(\delta_v) \sin(\delta_h) \\ \sin(\delta_v) \end{bmatrix}.
\] (1)

2.2. Two Distances Wind Field Parameter Reconstruction

2.2.1. Projected 2D Wind Speed Information

In the following, 2D wind speed information in measurement planes upwind the turbine as illustrated in Figure 1a are assumed to be given.

The information may be obtained either

- by focusing the beams of two lidars on the same measurement volume, the beams originating from two positions on the \( x \)-axis (Figure 1a), or
- by focusing the beam of one lidar subsequently on two upwind positions, the focus points being separated in \( x \)-direction (Figure 1b).

While the first method requires the use of two lidar beams and the ability to place their radiation outlets significantly separated in \( x \)-direction, the second method needs one beam outlet only but requires refocusing and re-adjusting the direction of the scan. Also, the second method needs the Taylor assumption [6] that the first measurement \( \vec{v}_{1,i} \) can be propagated with mean wind speed to the second measurement distance ignoring loss of information. In Section 2.3.4 this loss will explicitly be regarded. The index \( i \) refers to the \( i \)th measurement location in the \( y-z \)-plane.
Assuming now that two measurements $\vec{v}_{1,i}$ and $\vec{v}_{2,i}$ in different beam direction span a measurement plane $P_i$, a corresponding vector $\vec{m}_i$ lying in plane $P_i$ can be calculated, so that $\vec{v}_{1,i}$ and $\vec{v}_{2,i}$ are orthogonal projections of $\vec{m}_i$, see Figure 1c. If $\vec{v}_{1,i}$ and $\vec{v}_{2,i}$ are not parallel, $\vec{m}_i$ is uniquely given.

The vectors $\vec{m}_i$ can be interpreted as the orthogonal projections of the three dimensional wind speed vectors $\vec{v}(y_i, z_i)$ to planes $P_i$ and will hence be referred to as projection measurements $\vec{m}_i$ in the following.

In the next subsection three of the projection measurements $\vec{m}_i$ are used to reconstruct a wind field as modeled in Section 2.1 comprising five plane-averaged parameters.

**Figure 1a.** Two lidar beams (dashed), both focused on one point, the outlets being separated in $x$-direction.

**Figure 1b.** Refocusing lidar setup, subsequently measuring $\vec{v}_{1,i}$ and $\vec{v}_{2,i}$. $\vec{v}_{1,i}$ is shifted in space and time and assumes perfect approach to the second measurement distance.

**Figure 1c.** Determination of the wind vector projection $\vec{m}_i$ (bold) as the intersection of the orthogonal lines (gray) to the measurements $\vec{v}_{1,i}$ and $\vec{v}_{2,i}$ in beam direction.
2.2.2. Wind Field Reconstruction using Three Projection Measurements

Now the wind speed at position \((y, z)\) is given by the five-parameter model (1). Its projections \(\vec{m}_i\) are assumed to be determined with known components \([m_{u,i}, m_{v,i}, m_{w,i}], i = \{1,2,3\}\).

The three measurement planes are spanned by \(\hat{e}_u\) and \(\hat{e}_l\), respectively, which can be constructed pairwise orthonormally, i.e. \((\hat{e}_u, \hat{e}_l) = 0\) and \(\|\hat{e}_l\| = 1\) \(\forall i\).

The projection of \(\vec{v}(y, z)\) on \(P_i\) can be formulated using (1) to

\[
\vec{m}_i = \begin{bmatrix} m_{u,i} \\ m_{v,i} \\ m_{w,i} \end{bmatrix} = \langle \vec{v}, \hat{e}_u \rangle \hat{e}_u + \langle \vec{v}, \hat{e}_l \rangle \hat{e}_l
\]

\[
= \begin{bmatrix} (u_0 + s_h y_l + s_v z_l) \cos(\delta_v) \cos(\delta_h) \\ (u_0 + s_h y_l + s_v z_l) \left( e_{v,i}^2 \cos(\delta_v) \sin(\delta_h) + e_{v,i} e_{w,i} \sin(\delta_v) \right) \\ (u_0 + s_h y_l + s_v z_l) \left( e_{v,i} e_{w,i} \cos(\delta_v) \sin(\delta_h) + e_{w,i}^2 \sin(\delta_v) \right) \end{bmatrix}
\]

Equation (2) evaluated at \(i = \{1,2,3\}\) produce a system of nine nonlinear equations for the five unknown parameters.

Without loss of generality it is assumed that \(e_{v,1} = 1\) and \(e_{w,1} = 0\), i.e. the first measurement plane is oriented horizontally, see Figure 2.

Then from (2) the three equations for \(\vec{m}_1\) become

\[
\begin{bmatrix} m_{u,1} \\ m_{v,1} \\ m_{w,1} \end{bmatrix} = \begin{bmatrix} (u_0 + s_h y_l + s_v z_l) \cos(\delta_v) \cos(\delta_h) \\ (u_0 + s_h y_l + s_v z_l) \cos(\delta_v) \sin(\delta_h) \\ 0 \end{bmatrix}
\]

and can be condensed to

\[
\tan(\delta_h) = \frac{m_{v,1}}{m_{u,1}}
\]
Analogously an equation for the vertical averaged wind direction can be derived from another set of three equations of the second projection measurement \( \vec{m}_2 \), including result (3):

\[
\tan(\delta_\nu) = \frac{m_{v,2}}{m_{u,2}} - e_{v,2} \frac{m_{v,1}}{m_{u,1}} \left( 1 + \left( \frac{m_{v,1}}{m_{u,1}} \right)^2 \right)^{-1/2} \left( e_{v,2} e_{w,2} \right)^{-1}.
\] (4)

Equation (4) gives the vertical averaged wind direction if \( \vec{P}_2 \) is not parallel or perpendicular to \( \vec{P}_1 \), for example 120° turned. A perpendicular second plane is also possible and gives a simpler equation for \( \delta_\nu \).

As an intermediate result it can be observed that two projection measurements \( \vec{m}_1, \vec{m}_2 \) on two arbitrary turned measurement planes which are not parallel are sufficient to get both the horizontal and vertical averaged wind directions \( \delta_h \) and \( \delta_\nu \).

To calculate the parameters \( u_0, s_h \) and \( s_\nu \), there remain three independent equations from the \( u \)-component of equation (2) for \( i = \{1,2,3\} \), giving the equation:

\[
\begin{bmatrix}
1 & y_1 & z_1 \\
1 & y_2 & z_2 \\
1 & y_3 & z_3 \\
\end{bmatrix}
\begin{bmatrix}
u_0 \\
s_h \\
s_\nu \\
\end{bmatrix}
= \left( \cos(\delta_h) \cos(\delta_\nu) \right)^{-1}
\begin{bmatrix}
m_{u,1} \\
m_{u,2} \\
m_{u,3} \\
\end{bmatrix}.
\] (5)

The linear equation (5) has exactly one solution if the determinant of the coefficient matrix is not zero. It can be shown that this is always the case if the measurement points \( (y_1, z_1), (y_2, z_2) \) and \( (y_3, z_3) \) are located at differing positions on a circle.

Using (3), (4) and (5) all five wind field parameters can be calculated assuming three given projection measurements \( \vec{m}_i \). Note that the average wind direction requires two projection measurements only. Hence mean values of \( \delta_h \) and \( \delta_\nu \) could be calculated from more than two projection measurements.

2.2.3. Wind Field Reconstruction using more than Three Projection Measurements

If more than three projection measurements are available and can be assigned to one common wind field, reconstruction of the five wind field parameters can be improved and made more robust against in-wind-field turbulence, i.e. local deviations of wind speeds from the model.

The parameter estimation can then be formulated as the optimization problem:

\[
\| \vec{m} - \vec{f}(\vec{x}) \|_2^2 \rightarrow \min
\] (6)

where \( \vec{m} \) represents the array of projection measurements \( \vec{m}_i, i \geq 3 \), and \( \vec{f} \) represents an array of modeled projection measurements \( \vec{f}(\vec{x}) \), \( \vec{x} \) being the vector of the five wind field parameters.

The problem (6) can be solved by varying the parameters \( \vec{x} \) and minimizing the sum of squares of the differences between model and measurement. The Levenberg-Marquardt algorithm (\( i \geq 3 \)) or Trust-region-reflective algorithm (\( i \geq 5 \)) give good numerical results.

Beneficially, (6) represents a convex optimization problem, the linear equations (5) become an over-determined system when extended with additional measurements. Consequently a local minimum if exists is always a global minimum, making the algorithm independent from initial search values.

2.3. Single Distance Wind Field Parameter Reconstruction

As mentioned in Section 1, the simultaneous determination of wind shear and direction from measurements taken at a single distance is affected by ambiguities and numerical issues. Nevertheless, here a method is developed to give the best possible reconstructed parameters using one single measurement distance. Its performance is compared to the proposed reconstruction method using two measurement distances in Section 3.1.
Wind vectors given by the five-parameter model as in Equation (1) are assumed to be projected on conical lidar beam directions. The measured velocities $v_L$ in beam direction depend on the azimuthal position $\theta$ of the scan and the five wind field parameters

$$v_L(\theta) = \frac{1}{R} \left( u_0 + s_h R_x \cos(\theta) + s_v R_z \sin(\theta) \right)$$

$$\left[ \cos(\delta_v) \left( R_x \cos(\delta_h) - R_z \cos(\theta) \sin(\delta_v) \right) - R_z \sin(\theta) \sin(\delta_v) \right]$$

with $R$, the radius of the measurement circle in the y-z-plane, $R_x$ measurement distance in $x$-direction, $R$ the focus length.

Term (7) can be rewritten employing trigonometric identities to get

$$v_L(\theta) = \frac{1}{R} \left( a_1 + a_2 \cos(\theta) + a_3 \sin(\theta) + a_4 \sin(2\theta) + a_5 \cos(2\theta) \right)$$

where

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} u_0 R_x \cos(\delta_h) \cos(\delta_v) - \frac{1}{2} s_h R_x^2 \sin(\delta_h) \cos(\delta_v) - \frac{1}{2} s_v R_z^2 \sin(\delta_v) \\ s_h R_x R_z \cos(\delta_h) \cos(\delta_v) - u_0 R_z \sin(\delta_h) \cos(\delta_v) \\ s_v R_x R_z \cos(\delta_h) \cos(\delta_v) - u_0 R_x \sin(\delta_h) \cos(\delta_v) \\ \frac{1}{2} R_x^2 (-s_v \sin(\delta_h) \cos(\delta_v) - s_h \sin(\delta_v)) \\ \frac{1}{2} R_z^2 (-s_h \sin(\delta_h) \cos(\delta_v) + s_v \sin(\delta_v)) \end{bmatrix}$$

are non-physical parameters. It can be proven that the parameterization of $v_L$ by $a$ is unique, so that in a first step $a$ can be obtained using a least squares fit of the measurements $v_L(\theta)$. Secondly the physical wind field parameters have to be determined by solving the nonlinear equation (9) which emerges to be numerically unstable due to multiple neighboring solutions and large gradients. The best solutions in terms of root mean square errors can be determined using optimal initial values in algorithms solving (9).

3. Performance Evaluation

3.1. Comparison: Five-Parameter Estimation with Single- vs. Two-Distances Approach

The presented methods to reconstruct five wind field parameters, using one and two measurement distances respectively, are compared in the following. The method relying on one measurement distance only, and that on one measurement direction per focused wind volume element, is expected to result in larger errors in cases of turbulence within the rotor plane, due to observed numerical issues of the nonlinear Equation (9). In Subsection 3.1.1 the turbulence intensity as seen by a lidar is experimentally studied, so that a simple turbulence model for simulations is generated. In Subsection 3.1.2 this model is used to run Monte Carlo Simulations of turbulent wind measurements to evaluate the reconstruction algorithms.

3.1.1. Consideration of Turbulent Deviations of the Five-Parameter Model

By using experimental data of a conical scanning spinner mounted lidar, a simple empirical model of turbulence as seen by the lidar is derived. With the simple model, later, different turbulent conditions seen by the lidar are generated for numerical simulations.

The chosen experimental data were gathered during daytime at a near shore site at 59 m height at turbulence intensities of 7.6 to 10.1% and low vertical shear $<0.1 \text{ s}^{-1}$. In other real cases higher turbulent conditions might occur.

In a timeseries of approximately 40 minutes with 115,175 conical measurements at approximately 50 Hz the deviation $\Delta v_L$ to an a posteriori fit to equation (8) was calculated, see Figure 3a. The deviations were found to fulfill the Kolmogorov-Smirnov test for normality in 75% of the samples and are therefore modeled Gaussian distributed in the following.
Further turbulent fluctuations of the measured wind speed in beam direction are quantified and referred to as ‘conical turbulence intensity’, defined by

\[ T_{I,c} = \frac{\sigma(\Delta v_L)}{\overline{v}_L} \]  

(10)

with standard deviation \( \sigma(\Delta v_L) \) of the difference of measured velocities to modeled velocities in beam direction assuming the five-parameter model, and \( \overline{v}_L \) indicating the mean velocity in beam direction, both with respect to all measurements during one conical scan.

A histogram of the conical turbulence intensity was determined from the experimental spinner lidar data, cf. Figure 3b. In 95% of all samples the conical turbulence intensity does not exceed the range 0.01 to 0.08.

3.1.2. Monte Carlo Simulations of Turbulent Lider Measurements

Simulated lidar measurements of synthetic wind fields built using the five-parameter model are assumed to be affected by statistical errors \( \Delta v_L \) in the following. A setup with beam width \( R = 96 \) m, opening angle \( \beta = 30^\circ \) and 62 equi-azimuthal measurement focus locations on a circle are assumed to result in 62 measured wind velocities in lidar beam direction. Inspired by the experimental results of Section 2.4, the error \( \Delta v_L \) is modeled \((0, \sigma)\)-normal distributed with standard deviation \( \sigma(\Delta v_L) = T_{I,c} \cdot \overline{v}_L \). As usual for Monte-Carlo-Simulations, a large number of random samples is produced, in this case 1000 series of 62 conical measurement errors each. For each experiment, both five-parameter reconstruction algorithms, single distance and two distances, are employed to calculate parameter estimates. The two distance method additionally uses a second independent series of measurement errors. Root mean square errors of all experiments are then calculated for each parameter estimate and both estimation methods.

All simulations and calculations are repeated with stepwise varied conical turbulence intensity in the range of \( 10^{-3} \) to \( 10^{-1} \) for a fixed parameter set \([u_0, s_h, s_v, \delta_h, \delta_v] = [10 \text{ m/s}, 0.003 \text{ s}^{-1}, 0.02 \text{ s}^{-1}, 6^\circ, 10^\circ] \), being in a typical order during inflow conditions at rated wind speed of a wind turbine.

The results are shown in Figure 4. It can be seen that at vanishing conical turbulence intensities both methods give perfect estimates of all five parameters. At increasing turbulence the single distance method produces large estimation errors. In the region of experimentally measured conical turbulence intensity of around 0.04 the direction error for instance exceeds 5° horizontal and 10° vertical.

The estimation method based on two measurement planes shows much better performance with very low estimation errors, for instance with direction errors below 1° over the whole region of the considered turbulence intensities. The remaining dependence on the conical turbulence intensity can
be explained by the subset of conical measurements taken from the whole windfield, and by the simplification of independent fluctuations in both of the two distances.

**Figure 4.** Root mean square errors of parameter estimations calculated applying a Monte Carlo method, for the parameters mean wind speed, horizontal shear, vertical shear (left) and horizontal and vertical wind direction (right).

Gray: Histogram indicating typically occurring conical turbulence intensity measured by a horizontal circular scanning lidar.

### 3.2. Comparison: Five- vs. Three-Parameter Estimation

In [7] and [8], the extraction of three wind field parameters for the mean wind speed, vertical shear and horizontal flow angle is described for a commercial wind lidar and studied using experimental data.

Here, the proposed five-parameter estimation (5pe) method using two measurement distances is compared to a three-parameter estimation (3pe) method which reconstructs the following parameters:

1. $u_0$: mean streamwise wind speed [m/s]
2. $s_v$: linear vertical shear [s$^{-1}$]
3. $\delta_h$: mean horizontal flow angle [rad]

For numerical experiments, full turbulent wind fields with 169 grid points are generated with TurbSim [9]. 50 azimuthal measurements taken by a hub mounted lidar at $R_Z = 45$ m are simulated in Matlab. The 3pe method uses 50 measurements in an axial distance $d_{x1}$, the 5pe method additionally 50 measurements in a second distance $d_{x2}$. As a simplification the measurements at one focus distance...
are assumed to be gathered fast enough, i.e. simultaneously. First, the spatial averaging effect of the lidar and turbulent changes within the wind field during approaching are neglected.

In Figure 5 exemplary timeseries of both 5pe and 3pe methods are illustrated and compared to the true parameters (dashed), which are calculated based on the full wind information averaged over the rotor plane.

It is obvious that the parameters $s_h$ and $\delta_v$ which are not estimated by the 3pe method are always affected by mean errors and fluctuating errors. But also the estimated parameters $s_v$ and $\delta_h$ show larger deviations, if their counterparts $\delta_v$ and $s_h$ respectively deviate from zero. The reason is the misinterpretation of shear as direction or vice versa.

In Table 1 the accurateness of the methods for the sample wind field is expressed in terms of root mean squares (rms) of the estimation errors. While the mean wind speed accuracy is both for the 3pe and the 5pe in the order of 0.5 m/s, shear is significantly better estimated (0.015 s\(^{-1}\) rms error) and direction can be obtained with 1°-2° accuracy applying the 5pe.

Table 1. Root mean squares of parameter estimation errors.

(IEC wind fields, turbulence intensity $l_T = 18\%$, mean parameters: $\overline{u}_0 = 16$ m/s, $\overline{s}_h = \overline{s}_v = \overline{\delta}_h = 0$, $\overline{\delta}_v = 6^\circ$, sampling rate $f_s = 1$ Hz)

| method ($d_{c1}, d_{c2}$) | rms $\epsilon(u_0)$ [m/s] | rms $\epsilon(s_h)$ [s\(^{-1}\)] | rms $\epsilon(s_v)$ [s\(^{-1}\)] | rms $\epsilon(\delta_h)$ [°] | rms $\epsilon(\delta_v)$ [°] |
|--------------------------|--------------------------|-----------------------------------|-----------------------------------|--------------------------|--------------------------|
| 5pe (80, 40)             | 0.50                     | 0.013                             | 0.013                             | 1.9                      | 1.4                      |
| 5pe (30, 10)             | 0.54                     | 0.015                             | 0.013                             | 1.5                      | 1.0                      |
| 3pe (80, – )             | 0.51                     | 0.023                             |                                   | 7.1                      |                          |
| 3pe (30, – )             | 0.68                     | 0.056                             |                                   | 2.9                      |                          |
Figure 5. Five-parameter and three-parameter estimation compared to the full information. Circular scan at 40 m (3pe) and 80 m, 40 m (5pe) is assumed.

(IEC wind field, turbulence intensity $I_T = 18\%$, mean parameters: $\overline{u}_0 = 16 \frac{m}{s}$, $\overline{s_h} = \overline{s_v} = \overline{\delta_h} = 0$, $\overline{\delta_v} = 6^\circ$, sampling rate $f_s = 1$ Hz)

3.3. Consideration of Lidar Spatial Weighting and Linearity of Wind Evolution
Two simplifications of Section 3.2 are regarded in the following two subsections, the lidar spatial averaging effect and the linearity of the wind evolution. After that, their influence on the parameter estimation is demonstrated exemplarily.

3.3.1. Lidar Spatial Weighting
Wind speed measurements taken by a lidar anemometer result from backscatter signals within a spatial range along the beam direction, because of finite signal lengths and focusing effects. According to
for continuous wave lasers the probability of backscattering can be expressed to be proportional to the Cauchy distribution

\[ C(\Delta l) = \frac{1}{\pi z_R^2 + \Delta l^2} \]

with distance \( \Delta l \) from the focus point and \( z_R \) the Rayleigh length which can be modeled as a device specific quadratic function of the focus distance. Thus, a lidar measurement can be interpreted as a weighted combination of measurements along the beam direction with weighting function \( C(\Delta l) \). In simulations for this paper the domain of the weighting function was adaptively limited so that the probability of backscatter from a point within the limited domain amounts to 80\%, corresponding to a weighting level of 1/10 of the maximum level in the focus distance, compare Figure 6.

\[ C(\Delta l) \] [-]

Figure 6. Weighting function of a continuous wave lidar for measurements along the beam direction, exemplarily for a focus distance of 80 m (solid) and 40 m (dotted).

3.3.2. Linearity of Wind Evolution

As the wind field approaches the rotor plane, high frequency turbulent wind speed fluctuations cannot be considered ‘frozen’ according to of Taylor’s hypothesis [6]. In [11] corner frequencies are experimentally determined below which the relation of two separated measurements can be considered linearly related. For this purpose, the linearity of the phase of the cross spectra of two separated lidar measurements was evaluated and corner frequencies as illustrated in Figure 7 were found.

Figure 7. Corner frequencies of linear wind evolution over separation distance. Mean wind speed in m/s is varied from 2 to 24.

To maintain only correlated information in the parameter estimates, the lidar measurements can then be low pass filtered before input in the estimation algorithm.
This allows the explicit consideration of the invalidity of Taylor’s hypothesis by filtering results in lower frequent parameter estimates, depending on the separation distance and the mean wind speed. For these studies a second order Butterworth filter is adaptively designed for each of the two scan distances and depending on the mean wind speed. The filter is applied with zero phase lag.

3.3.3. Spectral Analysis of the Lidar Weighting and Wind Evolution Effect

In Figure 8 the amplitude spectrum of one of the estimated parameters, $u_0$, of the simulation presented in Section 3.2 is shown applying successively

1. lidar scan and 5pe,
2. lidar scan and 5pe and lidar weighting,
3. lidar scan and 5pe and lidar weighting and wind evolution filtering.

The wind evolution low pass filter which avoids uncorrelated fluctuations limits the dynamics approximately to its corner frequency of around 0.15 Hz, confer Figure 7. Note that measurements from two distances with different corner frequencies are combined in the 5pe method.

The lidar’s spatial weighting has a minor effect compared to the wind evolution filtering.

The bandwidth of the estimation signal is primary limited by the evolution filter also at larger focus distances, as can be seen in further simulations.

From this analysis a minimal sampling frequency can be deduced, so that all possible information in the filtered signal is maintained. Half of the sampling frequency (Nyquist frequency) has to exceed the corner frequency of the low pass wind evolution filter, so that a sampling frequency more than 0.3 Hz (circular scans per second) can be sufficient.

\[ Y(f) \]

\[
\begin{align*}
10^1 & \quad 10^2 \\
10^3 & \quad 10^4 \\
10^0 & \quad 10^1 \\
\end{align*}
\]

\[
\begin{align*}
\text{5pe} & \\
\text{5pe + Lidar weighting} & \\
\text{5pe + Lidar weighting + Evolution filtering} & \\
\end{align*}
\]

\[
\begin{align*}
\text{Frequency (Hz)} & \\
10^{-2} & \quad 10^{-1} \\
10^0 & \quad 10^1 \\
\end{align*}
\]

Figure 8: Amplitude Spectrum of $u_0$ using scan distances (30 m, 10 m), including lidar weighting and evolution filtering.
4. Conclusion and Outlook

In this paper an algorithm was presented to robustly produce accurate wind field parameter estimates using a single scanning hub based continuous wave focused lidar device. The basic idea is to scan each wind field twice while approaching to get measurements from differing directions. By that, five important wind field parameters can be reconstructed in front of a wind turbine.

Based on wind field simulations with modeled conical turbulence intensities as seen by real lidars, it was demonstrated that for typical turbulent conditions the second measurement distance – and by that a second measurement angle – is necessary and reconstructions based on one measurement distance result in significant estimation errors.

The five-parameter estimation method with two distances was also compared to a three-parameter estimation approach applying turbulent IEC wind fields. The accuracy of shear and direction estimates is much higher in terms of root mean square errors.

Limitations in the usable dynamics of the output signals were examined. They are caused by the lidar’s spatial averaging and, predominantly, by the physics of the turbulent wind evolution and can be influenced by the lidar technology and by scan geometries.

In yaw control the use of an appropriate lidar and the five-parameter estimation method may be regarded as an opportunity to improve the wind direction signal, especially if faster yaw adjustment algorithms are in consideration. A possible practical implementation is proposed in Figure 9, where measurements on a sphere in two distances from two respective angles can be used to obtain the required data.

Another advantageous application of the five-parameter estimation method could be given in feedforward based individual pitch control (IPC). IPC can reduce asymmetric loads caused by shear or direction. Due to the fact that shear and direction properties require different phases in cyclic pitch control signals, the ability to distinguish between both is essential.

Further research is needed to verify whether changes of the wind speed or the directional drift of the wind field is problematic for the two distances reconstruction, and field tests under different atmospheric conditions are necessary.

Figure 9. Possible realization using a double spherical scan: The focus distance is switched between two distances. By that, measurements at the same radial distance $R_2$ are obtained from two different angles. The measurements are then combined as illustrated in Figure 1b.
While effects of linear shear on wind turbine rotors are well known and can be modeled to produce feedforward control signals [12], changing stream direction and its impact on the rotor aerodynamic is still an issue and needs further investigation.

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