Electric vehicle routing problem optimal solution

F Mesa¹, J R González Granada¹, and G Correa Velez¹
¹ Departamento de matemáticas, Universidad tecnológica de Pereira, Pereira, Colombia

E-mail: femesa@utp.edu.co

Abstract. The large distribution companies of goods have the challenge of solving the problem of offering an optimal service in which different interests are faced which determine the decisions of the company, the problem is reduced to providing the best customer assistance, with low restrictions. operating cost on a route. Due to this, we see the need to make the programming of the routes of each vehicle as equitable as possible in terms of the capacity of the vehicle and the available fuel, in addition, it seeks to reduce the distances in which the vehicle travels without load. In this article we will demonstrate in detail in a document the optimal solution to the electric vehicle routing problem, which consists of determining optimal routes for a transport company to make efficient use of available resources, saving electricity and time, among other variables. Initially, the basic concepts about the operation of electric vehicles are described, later the concepts and modalities of vehicle routing are discussed, among which are the problem of routing open vehicles and the problem of routing vehicles with backhauls, later the scenario to develop the respective routes. and show the results with the optimal routes and routes, finally the conclusions obtained from the development of the techniques are exposed.

1. Introduction
The companies of distribution of goods seek to offer an optimal service; This consists of offering the best assistance to the user, including low operating costs on the route. Therefore, we see the need to make the programming of the routes of each vehicle as equitable as possible in terms of the capacity of the vehicle and fuel available, also, it seeks to reduce the distances in which the vehicle travels without load, due to this, the vehicle routing problem with backhaul (VRPB) is being studied, which deals with a better approximation to the real behavior taking advantage of the fact that it analyzed the route of the delivery customers and has the available capacity to make a new route picking up merchandise on the way back to the warehouse.

Electric Vehicles (Ves) have often been suggested as a useful solution to reduce oil consumption and emissions of air pollutants. Due to energy efficiency and environmental advantages over conventional vehicles, the future of electric vehicles looks promising, however, the integration of Ves in electric power systems raises new economic techniques, policies, and regulatory challenges.

Due to the reduced scope of the Ves it is necessary to operate recharging or battery exchange points that affect the current power system to a minimum and make them competitive in the economic field compared to conventional internal combustion vehicles. This causes that this case of optimal charge of batteries combined with the optimal routing of Ves with delivery and return customers establish a problem of great importance and topicality for the cargo transport sector, this problem is named as electric vehicle routing problem with backhaul (EVRPB) [1].

2. Background

Next, we will describe the process on which the methodology will be applied to solve the routing problem.

2.1. Electric vehicles routing

The first reference of electric vehicle is the nineteenth century, at which time research related to electromagnetic theory progressed quickly and among its first applications was the motorization of vehicles. The electric locomotive of Robert Davidson was the first electric vehicle created in 1838, later Camille Jenatzy in 1889 designed a convertible car in which reached 105 kilometers per hour, the fame of electric vehicles was increasing until in 1911 Thomas Edison incorporates a nickel-iron battery to the fleet of its vehicles in production, both how Henry Ford made investments in electric vehicles predicting a good future [2].

The electric vehicle began to acquire great importance for its simplicity, its ability, its reasonable autonomy for the time and its price was acceptable, in addition the combustion vehicles were very polluting, dirty, noisy, and required gasoline to operate. However, the development of the combustion engine, the appearance of the assembly line with the Ford model T and the irruption of cheap oil gave predominance to the combustion vehicle leading the electric vehicle to disappear until the mid-60s.

Currently there are three types of electric vehicle: hybrid with internal combustion engine and electric motor, which is powered with interchangeable battery without external power, hybrid plug-in electric vehicle with the characteristics of the previous vehicle except that the power of the electric motor is with rechargeable battery and the third; the purely electric vehicle with external power. The panorama of electric vehicles is currently quite positive as the companies Tesla and Panasonic along with some allies have assumed the mission of accelerating the transition process towards a new era of vehicles, currently the biggest problem is the high cost of producing batteries functional and safe that give autonomy to the electric vehicles and position them as the best transport alternative [3].

2.2. Electric batteries

Electric batteries are devices that convert the chemical energy directly contained in their active elements into electrical energy by means of an oxidoreduction process. A battery is composed of a series/parallel array of cells, which contain the active materials that convert chemical energy into electrical energy (anode and cathode). The Table 1 shows the main types of batteries to drive electric vehicles, note that the lower the specific energy, the cheaper the battery, but this causes that these types of batteries are inappropriate for the electric vehicles.

| Battery type                        | Acid of lead | Ni-Cd  | Ni-MH  | Li-ion |
|-------------------------------------|--------------|--------|--------|--------|
| Cost                                | Low          | Medium | High   | Very high |
| Voltage per cell (V)                | 2            | 1.25   | 1.25   | 3.6    |
| Charge current                      | Low          | Very Low | Moderate | High |
| Number of cycles (charge/discharge) | 200-500      | 1000   | 1000   | 1200   |
| Activity requirement (days)         | 180          | 30     | 90     | None   |
| Self-discharge per month (% of total) | Low (5%)  | High moderate (20%) | High (30%) | Low (10%) |
| Environmental precaution            | High         | High   | Low    | High   |
| Specific energy (Wh/kg)             | 30-50        | 50-80  | 40-100 | 160    |

The electric batteries, which are the elements in charge of providing autonomy to the electric vehicles, have been responsible for the electric vehicles have not dominated the automotive market throughout the history of the automotive industry. The previous statement arises because these elements are and always have been the arduous aspect in the development of the electric vehicles, due to their low durability, reduced storage capacity and in addition to their great weight and volume [4].
3. The vehicle routing problem

Different companies dedicated to the provision and distribution of goods aim to offer the best service with the lowest possible cost of operation, these costs are associated with labor, vehicle use and others. For this reason, some companies invest in software or research that leads to plan the best strategies for operating your system. A large number of real-world applications, both in North America and in Europe, have amply demonstrated that the use of computer procedures for the planning of the distribution process produces substantial savings (estimated between 5% and 20%) in the global transportation costs. A decrease in operating costs in real problems, translates into an increase in the company’s profit or a decrease in the additional costs of the product. Operations research seeks to optimize the benefit of the distribution companies, using mathematical models that describe the process to be improved, based on a previously stopped objective. These models are later solved by exact, metaheuristic techniques and hybrid techniques known as metaheuristics. The vehicle routing problem (VRP) is a formulation of the mixed whole linear type [5].

The use of exact techniques in this type of problems is restricted to the complexity of the mathematical model that describes it, such as, for example, the number of discrete variables or the number of restrictions. These problems generate a combinatorial explosion that is reflected in the high computation times to obtain the global optimal solution, additionally these times grow exponentially with the size of the problem, which is why, in terms of the computational complexity, it is classified within the category NP-complex. The variables involved in the VRP problem can be time, number of vehicles, nature of the vehicles means homogeneous or heterogeneous, time, number of deposits, geographical location, customer demand, transportation cost. Here the study of 2 particular problems is interesting, the main one is the vehicle routing problem with backhauls, but the model is made based on the studies done for the open vehicle routing problem (OVRP).

3.1. Open vehicle routing problem

The open vehicle routing problem is that case where the company delivering goods did not have its own fleet of vehicles or its private fleet was inadequate to meet the demand of customers. Therefore, contractors who were not employees of the distribution company used their own vehicles to make the delivery, in this case, the vehicles do not need to return to the warehouse after their delivery routes because the company was only hired to attend until the Last client of the route. Consequently, the objective of the OVRP is to design a set of roads that satisfy the demand of the clients. In practice, the OVRP represents situations such as: delivery of parcels and newspapers, bus route of a school, routes of coal mines, transport of hazardous waste [6].

3.2. Vehicle routing problem with backhauls

The Vehicle routing problem with backhauls is established as the problem of determining a group of vehicle routes that visits all customers that are divided into 2 subsets. The first subset contains the vertices of the delivery clients, where there is a quantity of product requested by each client. The second subset contains the return clients, where a quantity of product must be collected at each vertex and taken to the warehouse. The objective is to take into account the routes to be made from the deposit to the client by a fleet of homogeneous vehicles in order to satisfy the demand of the clients (products to be collected or products to be delivered). In this case, vehicles must first follow customers with delivery requirements before customers with return requirements, said subsets of customers must be linked by a path, which is known as the link. For some transport companies it is essential to avoid the reorganization of the products inside the vehicles at each delivery point. The withdrawals and deliveries of merchandise in a mixed manner or simultaneously cause difficulties, due to the rearrangements of the goods on board, when it refers to reordering is to collect and deliver merchandise at the same time. In the VRPB, the precedence constraint, which stipulates that in each circuit the vertices of delivery precede the vertices of return, leads to the approach of an exact model of the traditional literature seen as the capacitated vehicle routing problem (CVRP), this a complex task [7,8].
Figure 1 and Figure 2 shows the differences in the routes when studying the VRPB and the EVRPB, respectively, the superscript in each node is the demand in goods for delivery clients (circles) and return clients (boxes), likewise, recharging stations are described by diamonds. The nomenclature for the sets, variables and parameters of the model proposed for the EVRPB is described below [8]. Likewise, the dotted lines are the roads that close the path on each route, but they are dotted so that they do not violate the cyclical restriction, that is, it is a phantom line.

Figure 1. VRPB Vehicle routing problem with backhauls.

Figure 2. EVRPB Electric Vehicle routing problem with backhauls.

3.3. Definition of the variables, parameters and sets of the problem

The nomenclature for the sets, variables, and parameters of the proposed model for the EVRPB is described below.

3.3.1. Sets of the problem. \( L \) is the set of delivery customers \( L = \{1, ..., n\} \); \( B \) is the set of return clients \( B = \{n + 1, ..., n + m\} \); \( K \) is the set of charging stations \( K = \{n + m + 1, ..., n + m + k\} \); \( L_o \) is the set of delivery customers including deposit \( L_o = \{0\} \cup L \); \( B_o \) is the set of return customers including the deposit and charging stations \( B_o = \{0\} \cup B \cup K \); \( C_u \) is the set of delivery, return and loading stations \( C_u = L \cup B \cup K \); and \( V \) Node is the set \( V = C_u \cup \{0\} \).
### 3.3.2. Parameters of the problem

D_{ij} is the distance between nodes i and j, C_{ij} is the cost associated with the path between the nodes i and j, D_{ij} is the quantity of product delivered or collected to customers j ∈ C_u, KL is the minimum number of vehicles needed to serve delivery customers, KB is the minimum number of vehicles needed to serve return customers, Q is the capacity in merchandise of vehicles (identical vehicles), and E^{max} is the maximum electrical capacity of the battery in units of length.

### 3.3.3. Variables of the problem

s_{ij} is the binary variable that indicates whether the path between the nodes i, j ∈ V it is active, ξ_{ij} is the binary variable that indicates whether the arc between the nodes i ∈ L; j ∈ B_o it is active, l_{ij} is the amount of merchandise transported between the nodes i and j, and p_{ij} is the state of charge of the vehicle in units of distance, when it goes from the node i to the node j.

### 3.4. Mathematical formulation of the problem

The objective function (Equation (1)) minimizes operating costs, consists of 2 terms. The first considers the cost related to visiting delivery customers, return customers and charging stations. The second seeks to minimize the cost associated with the link between the last return client with a station, with the deposit, or with the delivery client [9].

$$
\min \sum_{i \in V} C_{ij} s_{ij} + \sum_{j \in B_o} C_{ij} \xi_{ij},
$$

(Equation 1)

The set of restrictions (Equation (2) to Equation (5)) models the OVRP for the delivery route, where Equation (2) and Equation (3) impose the requirements for radiality in the connection. Here it is imposed that the cardinality of L (number of delivery clients) must be equal to the number of arcs used in the optimal solution and is guaranteed by Equation (2). However, this one condition is not enough for the tree solution, it is necessary to consider the connectivity properties through (Equation (3)) allowing the flow of demand for goods in each delivery client. The degree restrictions impose them (Equation (4) and Equation (5)), forcing an exact arc are between and from each vertex associated with the delivery clients respectively. The restrictions for the VEs in the delivery routes are assigned by (Equation (6) to Equation (9)), where in Equation (6) the auxiliary variable that takes the distance between the nodes i and j, this restriction with (Equation (7) to Equation (9)) allows battery autonomy to go from one delivery client to another [10]; subject to:

$$
\sum_{j \in L} l_{ij} = |L|,
$$

(Equation 2)

$$
\sum_{j \in L} l_{ij} - \sum_{k \in L} l_{ik} = D_{ij},
$$

(Equation 3)

$$
\sum_{k \in L} s_{ijk} + \sum_{k \in B_o} s_{ijk} = \sum_{j \in L} s_{ij},
$$

(Equation 4)

$$
\sum_{k \in L} s_{ik} + \sum_{k \in B_o} s_{ik} = -D_{ij},
$$

(Equation 5)

$$
\sum_{j \in L} s_{ij} + \sum_{k \in L} s_{ij} = \sum_{j \in L} s_{ij},
$$

(Equation 6)

$$
\sum_{k \in L} s_{oi} + \sum_{j \in L} s_{oi} = \sum_{j \in L} s_{ij},
$$

(Equation 7)

$$
\sum_{i \in B} D_{ij} s_{ij} + \sum_{j \in L} D_{ij} s_{ij} + \sum_{k \in L} D_{ij} s_{ij},
$$

(Equation 8)

$$
\sum_{i \in C_u} p_{ij} - \sum_{k \in B} p_{lk} = p_{ij}.
$$

(Equation 9)

### 4. Results and analysis

Figure 3 and Figure 4, the computational results about VRPB and EVRPB are presented, VEs with different defined electrical capacity (E^{max}) were used. It was used as a reference to build a new database denoted as GJ dataset, was proposed by Goetschalconck and Jacobs-Blecha and contains 62 instances with a number of clients between 20 and 150.

Here new vertices are proposed as load points already built, some instances were modified in capacity (Q), minimum number of vehicles (K_b, K_L) in order to facilitate the solution, since a large interval between these 2 values makes the model takes much longer to find an optimum because some vehicles
must return to the warehouse after finishing delivery clients, these instances are shown with the sign (+). Next, tests performed to verify the compliance of the model are shown using 3 instances of the EVRPB case of small, medium and large size respectively. The routes optimal found, and the total distance that the vehicle of limited electric capacity (VE) should travel, in some of these routes it is necessary that the vehicle look for recharging points to be able to continue with its route, this happens when the distance to travel exceeds the capacity of the vehicle [11,12].

**Figure 3.** Optimal routes for EVRP of medium size.

**Figure 4.** Optimal routes for EVRP of large size.

5. **Conclusions**

The development of the proposed technique allowed solving the problem of electric vehicles with delivery and return customers electric vehicle routing problem with backhaul. Here it is proved that the implemented implementation is very useful for companies that distribute goods. It is possible to observe the complexity of the model in the G instances due to the reduced number of return customers compared to the delivery clients, one of the causes is due to the electrical limitations of the model, since it reduces the space of feasible solutions and find an optimal cost to have delivery routes of such magnitude that they must subsequently go to a loading station, also subject to the minimum number.
This work is contributed to discussions of different authors about different models with various proposed restrictions in recent years, it also shows that the proposed connectivity was possible thanks to the general-purpose model, which finds results efficiently. The study is a pillar for further research on the impact of the use of VEs in the energy distribution system as it is of great importance for the electricity companies in the sector and for governments that are beginning to implement the use of new technologies.

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