Coupling of dust particles in a weakly collisional plasma with an ion flow

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Abstract. We investigate the coupling of dust particles into self-confined vertical pairs in a weakly collisional plasma with a directed ion flow. We neglect the horizontal external confinement and assume that the lateral confinement of the lower particle is provided by the horizontal wake field, while the vertical confinement of both particles is due to the sheath field. The conditions to predict the development of dissipative instability in a self-confined particle pair are obtained depending on the Mach number, the plasma screening parameter, the particle charge ratio, and the normalized particle mass.

1. Introduction
Vertical particle pairing was experimentally studied in layered structures of dust grains levitating above the lower electrode in a capacitively coupled radio-frequency discharge [1, 2]. The phenomenon, however, has been observed only for the extended structures of a crystalline type where each particle pair was confined horizontally by their neighbors and vertically by an electric field of the sheath. In some studies, particles of different masses were used to assemble their vertical configuration [3–6]. On the other hand, the alignment of particles was often created by applying a radial external electric confinement [5, 7–10].

In a recent laboratory experiment, a specially designed “pair state” of complex plasma was prepared by Nosenko et al where particles formed multiple pairs that were vertically oriented and levitated at the same height above the electrode but were weakly coupled with each other and did not form any regular structure [11].

For particles coupling in the dust subsystem there have to be attractive forces between dust particles. Vladimirov and Nambu [12] were the first to show that, when negatively charged particles are immersed in a collisionless plasma with an ion flow, attraction is possible between them due to electric field oscillations inside a circular cone downstream the particles, called wake. Collisional processes in plasmas, Landau damping and non-Maxwellian distribution of ions substantially suppress the oscillating structure of the wake potential behind the grain, whereby there is a single maximum of the electrostatic potential (so-called ion focus) [13–16]. Thus, the other microparticles interact with both the negatively charged grain and the positive ion focus, which leads to some effective non-reciprocal attraction between the negatively charged particles which in turn may lead to their pairing.
In this paper, we investigate the coupling of dust particles into self-confined vertical pairs in a weakly collisional plasma with an ion flow. According to [11] we neglect the horizontal external confinement and assume that the lateral confinement of the lower particle is provided by the horizontal wake field, while the vertical confinement of both particles is due to the sheath field. Because of the effect of positive ions focusing downstream of negatively charged dust particles, where a positively charged area (ion trace) arises, the interaction potential differs from the Debye one, here we use the following model of the wake potential for a point charged particle in a weakly collisional plasma in the presence of an electric field [14]:

$$\phi(r, \theta) = \frac{Q}{r} \exp(-\frac{r}{\lambda}) - 2\sqrt{\frac{2}{\pi}} \frac{Q M \lambda^2}{r^3} \cos \theta - \left(2 - \frac{\pi}{2}\right) \frac{Q M^2 \lambda^2}{r^3} (3 \cos^2 \theta - 1), \quad (1)$$

where $Q$ is the particle charge, $\lambda = \sqrt{\frac{T_i}{4\pi n_i e^2}}$ is the thermal ion Debye length in a plasma, $M = \frac{|\bar{u}_i|}{V_T}$ is the thermal Mach number (the ion drift velocity normalized to the thermal ion velocity $V_T = \sqrt{\frac{T_i}{m_i}}$), $\theta$ is the angle with respect to the electric field vector $E_0$. Equation (1) is valid for the subthermal ion flow regime ($M \lesssim 1$) when the ion mean free path due to ion–neutral collisions is much larger than the Debye length $\lambda$ and distances $r$ where we want to analyze the interparticle interaction.

2. Basic equations

To study the conditions for the development of dissipative instability in an open system with chain-like structure and with anisotropic interparticle interaction, the problem of the stability of a system consists of two interacting particles (as a basic unit of a chain), which is collinear to the electric field vector $E_0$ (the Z axis) is first step.

We consider the linearized equations of motion for two particle system with non-reciprocal interparticle interaction for small deviations in the radial and vertical directions from the equilibrium position, and also under the assumption that the Brownian (random) component of the particle motion is small.

The problem of transverse stability for the vertical configuration of like-charged dust particles is considered for the case of comparatively small horizontal component of the external applied electrostatic field, which is a usual quality of the sheath of a rf discharge. According to our previous study [17] we obtain the condition for the transverse stability of the vertical configuration of particles:

$$F^x_{21} + Q^* F^x_{12} > 0, \quad (2)$$

here index “1” corresponds to the lower particle and “2” corresponds to the upper particle of the two-particle system, $Q^* = Q_1/Q_2$ is the ratio of the particle charges and the directional derivative of the interaction force which acts from the particle $i$ to the particle $j$ is determined by the potential (1), and has the following form:

$$F^x_{ij} = \frac{Q_i Q_j}{\Delta^3} \left[ \exp(-\kappa)(1 + \kappa) + (-1)^j 8\sqrt{\frac{2}{\pi \kappa^2}} - 12 \left(2 - \frac{\pi}{2}\right) \frac{M^2}{\kappa^2} \right], \quad (3)$$

here $\Delta$ is the mean interparticle distance and $\kappa = \Delta/\lambda$ is the screening parameter. The violation of the condition (2) corresponds to the absence of restoring force in response to a radial displacement of charged particle which leads to a qualitative transition from the vertical orientation of the particles to their new stable state.

The vertical configuration of particle system with anisotropic non-reciprocal interparticle interaction trapped in an external electrostatic field can become unstable and change the configuration due to the longitudinal instability. The stability condition for the pair of particles which initially have the vertical arrangement can be written as:

$$-F^z_{21} - Q^* F^z_{12} - \beta Q_1 > 0, \quad (4)$$
Figure 1. The region of the stable existence of vertical configuration of a particle pair with wake interaction (1) is depicted by red filled area depending on the Mach number $M$ and the coupling parameter $\kappa$ (equal charges case, $Q^* = 1$): solid curve corresponds to condition (2); dashed curve—(4); dot curve—(6).

Figure 2. The evolution of the region of the stable existence of a vertical particle system with a change in the particle charges ratio in the chain: the red area—$Q^* = 1$; the green area—$Q^* = 0.95$; the blue area—$Q^* = 0.8$. Solid curve corresponds to the condition (2); dashed curve—(4); dot curve—(6).

Here $\beta = \partial E_0 / \partial z > 0$ is the external electric field gradient, and the directional derivative of the force is described as follows:

$$ F_{ij}^z = -\frac{Q_i Q_j}{\kappa} \left[ 2 \exp(-\kappa)(1 + \kappa + 0.5\kappa^2) + (-1)^j 24 \sqrt{\frac{2}{\pi}} \frac{M}{\kappa^2} + 24 \left( 2 - \frac{\pi}{2} \right) \frac{M^2}{\kappa^2} \right]. \quad (5) $$

When the particle system is in equilibrium state, the force balance condition must be satisfied, which allows us to obtain one more criterion to restrict the range of interparticle interaction parameters:

$$ \beta \Delta = \frac{F_{21}}{Q_1} - \frac{F_{12}}{Q_2} + \frac{M g}{Q_2} - \frac{M g}{Q_1} > 0, \quad (6) $$

Here

$$ F_{ij} = \frac{Q_i Q_j}{\Delta^2} \left[ \exp \left( -\frac{r}{\lambda} \right) \left( 1 + \frac{r}{\lambda} \right) + (-1)^j 6 \sqrt{\frac{2}{\pi}} M \kappa^{-2} - 6 \left( 2 - \frac{\pi}{2} \right) M^2 \kappa^{-2} \right]. \quad (7) $$

3. Results

The conditions of equations (2), (4), (6) limit the range of the structural stability of the system under consideration within $M(\kappa)$ space. In the general form, the solution of the system of inequalities (2), (4), (6) for potential (1) is determined by the quadratic equations (relative to the Mach number) and has a rather complicated form. However, for the case of equal charges...
Figure 3. Evolution of the region of stable existence of a vertical particle system with a change in the ratio $C = Mg\Delta^2/(Q_2Q_1)$ for different values of the particle charge ratio: $Q^* = 0.99$ (a) and 0.85 (b); the curve numbers correspond to the different values of the parameter $C$: 1 — $C = 1$; 2 — $C = 10$; 3 — $C = 100$. The solid curve corresponds to the condition (2), the dashed curve corresponds to the condition (4), the dot curve corresponds to the condition (6).

For $Q^* = 1$ we can simplify the conditions (2), (4), (6) to the following system:

\[
\begin{align*}
\kappa \sqrt{\frac{\exp(-\kappa)(1 + \kappa)}{12(2 - \frac{\k}{2})}} &< M < \kappa \sqrt{\frac{\exp(-\kappa)(1 + \kappa)}{6(2 - \frac{\k}{2})}}, \\
M &< \kappa \sqrt{\frac{\exp(-\kappa)(3 + 3\k + \k^2)}{30(2 - \frac{\k}{2})}}.
\end{align*}
\]

Stability diagrams in the $M(\kappa)$ space for a system of two interacting particles arranged along the ion flow were obtained. For the case of equal charges, $Q^* = 1$, the stability region is shown in figure 1, it is bounded from below by a curve corresponding to the condition for transverse instability, and from above by curves that correspond to the condition for longitudinal instability and the condition for maintaining the balance of forces.

Figure 2 shows the evolution of the ranges of $M$ and $\kappa$ parameters, which provide the system stable state, in response to a change in the charge ratio of dust particles in the system. In figure 2 we see that the condition of the balance of forces limits the stability region for a vertical configuration of particles with comparatively small differences in particle charges $Q^* > 0.95$.

Figure 3 depicts the evolution of the stability region of the system in response to a change in the ratio $Mg\Delta^2/(Q_2Q_1)$ for different values of the charge ratio of dust particles.

4. Conclusions

Within the study of the conditions for the development of dissipative instability in an open particle system with chain-like structure, the problem of the stability of a system consists of two interacting particles (as a basic form of a chain), which is collinear to the electric field vector was solved.
For the model of wake potential of a point charged particle suspended in a weakly collisional plasma with a directed ion flux the structural stability conditions of the vertical configuration of two interacting particles were obtained depending on the Mach number, the plasma screening parameter, the particle charge ratio, and the ratio of the gravity force to the Coulomb interaction force.

The directed ion flow and the wake interaction between dust particles have a dual effect on the stability of vertical chain structure of the system of interacting particles. On the one hand, it is precisely the presence of directed ion flux in the area where dust particles are suspended that causes vertical ordering of dust particles. On the other hand, nonreciprocal interaction with a positively charged ion cloud provides a very efficient mechanism for converting the energy of the flowing ions into the kinetic energy of dust particles, which can lead to the development of another kind of instability, the amplitude instability. The problem of the development of amplitude instability in an open system of dust particles with chain-like structure immersed in asymmetric nonequilibrium medium is an interesting matter for future research.

The results of this work can be useful for experimental analysis particularly for estimating the parameters of the sheath of an rf discharge.

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