Self-gravitational Instability of Partially Ionized Plasma with Radiative Effects

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The gravitational instability of two-component plasma is discussed to investigate the effect of the radiative heat-loss function, thermal conductivity, collision frequency of neutrals, finite electrical resistivity and viscosity of the medium on the Jeans instability. The usual Magnetohydrodynamics (MHD) equations are used for the present configuration with radiative heat-loss function and thermal conductivity. A general dispersion relation is obtained from perturbation equations using the normal mode analysis method. We find that the Jeans condition of self-gravitational instability is modified due to the presence of radiative heat-loss function and thermal conductivity. Numerical calculations have been performed to show the effect of the various parameters on the growth rate of the self-gravitational instability.

1. Introduction

The fragmentation of the interstellar gas is an important phenomena and the self-gravitation plays a role in star formation. The self-gravitational instability of molecular clouds is connected to the cloud collapse and the formation of stars. The most common condition of matter in the universe is plasma, but there are some regions of low temperatures in the universe where a partially ionized plasma medium with neutral gas is present. HII regions of cool interstellar clouds, chromospheres and photospheres of stars are such regions. Thus the interstellar gas is not completely ionized and it is permeated with neutral atoms and due to this reason two-component theory is discussed. Thus the two-component i.e. plasma and neutrals interact with each other through mutual collisions. The magnetic field interacts only with the charged particles in the interstellar gas. The collisions of plasma with neutral gas in the clouds are responsible for coupling of magnetic field to interstellar plasma clouds.

In this connection the gravitational instability of infinite homogeneous self-gravitating magnetized plasma is discussed by Chandrasekhar [1]. Bhatia and Gupta [2] have studied the gravitational instability of composite plasma taking into account of frictional effects of neutrals with finite ion Larmor radius corrections, Hall current and finite conductivity. But they have neglected the contribution of pressure gradient of neutral, viscosity of neutral and contribution of self-gravitation force of neutrals. Herrnegger [3] has investigated the problem of gravitational instability of infinitely conducting, isotropic, inviscid two-component plasma with frictional effects of neutrals and FLR corrections only for the transverse mode of propagation. He has included the contribution of pressure gradient of neutrals and considered the gravitational potential to be dependent on the densities of both neutral and ionized component of plasmas. Chhonkar and Bhatia [4] have discussed the effects of neutral gas friction, magnetic resistivity, viscosity, FLR and Hall current on the growth rate of gravitational instability of two-component plasma with pressure, viscosity and self-gravitation of neutrals. Chhajlani and Vaghela [5] have investigated gravitational stability of finitely conducting two-component plasma considering the effects of neutral gas friction, viscosity, permeability, porosity and magnetic field with pressure, viscosity and self-gravitation of neutrals. Vaghela and Chhajlani [6] have carried out the investigation of gravitational instability of two-component viscous plasma with finite conductivity flowing through porous medium with FLR corrections. They have considered the
contribution of pressure, viscosity and self-gravitation of neutrals but in these studies of two-
component plasma, thermal and radiative effects were not considered.

It is well known that thermal and radiative effects play an important role in the investigation of
plasma instabilities. The thermal instability arising due to various heat loss mechanisms may be
responsible for astrophysical condensation and formation of astrophysical objects. In this direction
Bora and Talwar [7] have investigated the magnetothermal instability with finite electrical resistivity,
Hall current, electron inertia and radiative effects of self-gravitating plasma but they have not
considered the contribution of neutral component. The thermal instability in a cooling and expanding
medium including self-gravity and conduction in the neutral fluid dynamics has been investigated by
Gomez-Pelaez and Moreno-Insertis [8]. Radwan [9] has studied the gravitational instability of
radiating, rotating gas cloud streams with non uniform velocity. Recently Shaikh et. al [10] have
discussed the gravitational instability of thermally conducting partially-ionized plasma in variable
magnetic field taking the effect of Hall current, finite conductivity, ion viscosity and collision with
 neutrals. But they have not considered the contribution of neutral pressure, neutral viscosity and self-
gravitation of neutral with radiative effects.

From the above discussed problems [2-7, 10] we find that none of the authors have considered
the combined effects of neutral collisions, thermal conductivity and radiative heat-loss function on the
self-gravitational instability of partially ionized, finitely conducting, viscous, magnetized two-
component plasma with pressure, viscosity and self-gravitation of neutrals. Thus we investigate the
effects of radiative heat-loss function, thermal conductivity and neutral collisions on the self-
gravitational instability of partially ionized, finitely conducting, viscous, magnetized two-component
plasma with the neutral pressure gradient, neutral viscosity and self-gravitation of neutral particle in
the present problem. The above work is applicable to dense molecular interstellar clouds and
cometary plasma which contains a significant fraction of neutral atoms.

2. Linearized perturbation equations of the problem
We consider an infinite, homogeneous, self-gravitating, viscous, thermally conducting and radiating
composite fluid plasma consisting of a finitely conducting ionized component of density $\rho$ and
neutral component of density $\rho_n$. The uniform magnetic field $H(0,0,H)$ interacts with conducting
components and it gets coupled with the neutral gas through collisions of the two-components.
Assuming the initial velocities of both the components zero and the gravitational potential depends on
the density of both the components. Let the pressure gradient of both the components be comparable.
If $\delta \rho, \delta u (u_x, u_y, u_z)$ are the respective perturbations in pressure, density and velocity.
$h(h_x, h_y, h_z), \delta T, \delta U, L$ are perturbations in magnetic field, temperature, gravitational potential and
heat-loss function respectively. Let subscript $n$ refer to neutral component of the gas, respectively, the
linearized perturbation equations of the system are
\[
\rho \frac{\partial \mathbf{u}}{\partial t} = - \nabla \delta \rho + \rho \nabla \delta U + \frac{1}{4\pi} (\nabla \times \mathbf{h}) \times \mathbf{H} + \rho u \left[ \nabla^2 \mathbf{u} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{u}) \right] + \rho \left( u_n \cdot \mathbf{u} \right),
\]
(1)
\[
\frac{1}{\gamma - 1} \frac{\partial \delta \rho}{\partial t} - \rho \frac{\partial \delta \rho}{\partial t} + \rho (L_\rho \delta \rho + L_\tau \delta T) - \lambda \nabla^2 \delta T = 0,
\]
(2)
\[
\frac{\partial \delta \rho}{\partial t} = - \rho \nabla \cdot \mathbf{u},
\]
(3)
\[
\frac{\delta \rho}{\rho} = \frac{\delta T}{T} - \frac{\delta \rho}{\rho}.
\]
(4)
\[
\frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}) + \eta \nabla^2 \mathbf{h}, \quad (5)
\]
\[
\nabla \cdot \mathbf{h} = 0, \quad (6)
\]
\[
\rho_n \frac{\partial \mathbf{u}_n}{\partial t} = -\nabla \delta p_n + \rho_n \mathbf{V} + \rho_n \mathbf{u}_n \left[ \nabla^2 \mathbf{u}_n + \frac{1}{3} \nabla (\nabla \cdot \mathbf{u}_n) \right] - \rho_n \mathbf{e}_c (\mathbf{u}_n - \mathbf{U}), \quad (7)
\]
\[
\frac{\partial \delta \rho_n}{\partial t} = -\rho_n \nabla \cdot \mathbf{u}_n, \quad (8)
\]
\[
\delta \rho_n = \delta \rho_n C_n^2, \quad (9)
\]
\[
\nabla^2 \delta \mathbf{U} = -4\pi G (\delta \mathbf{p} + \delta \mathbf{p}_n), \quad (10)
\]

where \(\rho, \mathbf{V}, L_T, T, \eta, \mathbf{v}_c, C, G\) and \(\gamma\) are the density, kinematic viscosity, temperature dependent heat loss function, density dependent heat-loss function, thermal conductivity, temperature, electrical resistivity, ion-neutral collision frequency, sound velocity, universal gas constant and ratio of two specific heats. We assume all the perturbed quantities are varying as
\[
e^{i(k_x x + k_y y + \omega t)}, \quad (11)
\]
where \(\omega\) is the frequency of perturbation and \(k_x, k_z\) are the components of the wave vector \(k\) in \(x, z\) directions so that \(k^2 = k_x^2 + k_z^2\).

Introducing the displacement vectors \(\xi(\xi_x, \xi_y, \xi_z)\) and \(\xi_n(\xi_{nx}, \xi_{ny}, \xi_{nz})\) such that
\[
\mathbf{u} = \frac{\partial \xi}{\partial t}, \quad \mathbf{u}_n = \frac{\partial \xi_n}{\partial t}. \quad (12)
\]
The components of equation (5) can be written as
\[
d\xi_x = \imath \omega k_z \xi_n, \\
d\xi_y = \imath \omega k_x \xi_x, \\
d\xi_z = -\imath \omega k_y \xi_x, \quad (13)
\]
where \(d = (\omega + \eta k^2)\) and \(\omega = \imath \sigma\).

Using equations (2)-(4) and (13) for plasma components and equations (7) and (8) for the neutral components along with equations (9)-(12), we may write the following algebraic equations for the components of equations (1) and (6) as
\[
\xi_x \left\{ \omega^2 + \omega \left[ (1 + \beta) \frac{V^2 k^2}{d} + \mathbf{v} \left( k_x^2 + k_z^2 \right) + \mathbf{v}_c \frac{k_x^2 + \Omega_T^2}{3} \right] - \xi_{nx} \left( \omega \mathbf{v}_c + \frac{k_x^2}{k^2} 4\pi G \rho_n \right) \right\} - \xi_{nx} \left( \omega \mathbf{v}_c + \frac{k_x^2}{k^2} 4\pi G \rho_n \right) = 0, \quad (14)
\]
\[
\xi_y \left\{ \omega^2 + \omega \left[ (1 + \beta) \frac{V^2 k^2}{d} + \mathbf{v}_c \right] - \xi_{ny} \left( \omega \mathbf{v}_c + \frac{k_y^2}{k^2} 4\pi G \rho_n \right) \right\} = 0, \quad (15)
\]
\[ \xi_n \left\{ \omega^2 + \omega \left[ v \left( k^2 + \frac{k_n^2}{3} \right) + \frac{k_n^2}{k^2} \Omega_T^2 \right] + \frac{k_n^2}{k^2} \Omega_T^2 \right\} - \xi_{nT} \left( \omega \frac{v}{\beta} + \frac{k_n^2}{k^2} 4\pi G \rho_n \right) \]
\[ + \frac{k_n^2}{k^2} \left\{ \xi_x \left( \omega \frac{v_n}{\beta} + \frac{k_n^2}{k^2} \right) - 4\pi G \rho_n \xi_{nT} \right\} = 0, \]  
(16)

\[ \xi_{nT} \left\{ \omega^2 + \omega \left[ v_n \left( k^2 + \frac{k_n^2}{3} \right) + \frac{v}{\beta} \right] + \frac{k_n^2}{k^2} J_n^2 \right\} - \xi_x \left( \omega \frac{v}{\beta} + \frac{k_n^2}{k^2} 4\pi G \rho \right) \]
\[ + \frac{k_n^2}{k^2} \left\{ \xi_n \left( \omega \frac{v_n}{\beta} + \frac{k_n^2}{k^2} \right) - 4\pi G \rho \xi_x \right\} = 0, \]  
(17)

\[ \xi_{nT} \left\{ \omega^2 + \omega \left[ v_n \left( k^2 + \frac{k_n^2}{3} \right) + \frac{v}{\beta} \right] - \frac{\omega}{\beta} \xi_y \right\} = 0, \]  
(18)

\[ \xi_{nT} \left\{ \omega^2 + \omega \left[ v_n \left( k^2 + \frac{k_n^2}{3} \right) + \frac{v}{\beta} \right] + \frac{k_n^2}{k^2} J_n^2 \right\} - \xi_x \left( \omega \frac{v}{\beta} + \frac{k_n^2}{k^2} 4\pi G \rho \right) \]
\[ + \frac{k_n^2}{k^2} \left\{ \xi_n \left( \omega \frac{v_n}{\beta} + \frac{k_n^2}{k^2} \right) - 4\pi G \rho \xi_x \right\} = 0, \]  
(19)

we have made following substitutions in above equations

\[ \beta = \frac{\rho_n}{\rho}, \quad V^2 = \frac{H^2}{4\pi \rho_0}, \quad J^2 = C^2 k^2 - 4\pi G \rho, \quad J_n^2 = C_n^2 k^2 - 4\pi G \rho_n, \]
\[ \rho_0 = \rho + \rho_n, \quad \Omega_T^2 = \frac{\Omega_T^2 + \omega J^2}{B + \omega}, \quad \Omega_T^2 = k^2 A - 4\pi G B, \]
\[ A = (\gamma - 1) \left( TL_T - \rho L_T + \frac{\lambda k^2 T}{\rho} \right), \quad B = (\gamma - 1) \left( \frac{T L_T}{\rho} + \frac{\lambda k^2 T}{\rho} \right) \]

3. Dispersion relation

From the above six equations (14)-(19) the dispersion relation is derived both for longitudinal and transverse directions to the magnetic field and discussed separately.

For wave propagation parallel to the magnetic field \( k (0,0,k) \), equations (14)-(19) have non-trivial solutions if the determinant of the equations vanishes.
For wave propagation perpendicular to the magnetic field \( \mathbf{k} = (k, 0, 0) \), equations (14)-(19) have non-trivial solutions if the determinant of the equations vanishes.

\[
\begin{vmatrix}
P & 0 & 0 & -\omega_0 c & 0 & 0 \\
0 & P & 0 & 0 & -\omega_0 c & 0 \\
0 & 0 & R & 0 & 0 & -D \\
-\omega_0 c / \beta & 0 & 0 & Q & 0 & 0 \\
0 & -\omega_0 c / \beta & 0 & 0 & Q & 0 \\
0 & 0 & -E & 0 & 0 & S \\
\end{vmatrix}
= 0. \quad (20)
\]

where we have made following assumptions:

\[
P = M + \omega(1 + \beta) \frac{V^2}{d}, \quad M = \omega^2 + \omega(\nu k^2 + \nu_v), \quad Q = \omega^2 + \omega(\nu_n k^2 + \frac{\nu_u}{\beta}),
\]

\[
R = \omega^2 + \omega \left( \frac{4}{3} \nu k^2 + \nu_v \right) + \Omega^2, \quad S = \omega^2 + \omega \left( \frac{4}{3} \nu_n k^2 + \frac{\nu_u}{\beta} \right) + J_n^2, \quad D = (\omega_0 c + 4\pi G \rho_n).
\]

\[
E = \left( \omega \frac{\nu_u}{\beta} + 4\pi G \rho \right), \quad F = R + \omega(1 + \beta) \frac{V^2}{d}.
\]

The dispersion relation for longitudinal wave propagation from the determinant of matrix of equation (20) is

\[
(RS - DE) \left( PQ - \frac{\omega^2 \nu_u^2}{\beta} \right)^2 = 0. \quad (22)
\]

The dispersion relation for transverse wave propagation from the determinant of matrix of equation (21) is
The both dispersion relations (22) and (23) have two independent factors which may be discussed separately.

4. Discussion

4.1. Longitudinal Propagation

Equating first part of equation (22) to zero, i.e. \( RS - DE = 0 \), on solving we get

\[
\omega^2 + \omega^4 \left[ v_x \left( 1 + \frac{1}{\beta} \right) + \frac{4}{3} v_0 k^2 + \frac{4}{3} v_0 k^2 + B \right] + \omega^2 \left[ B \left( 1 + \frac{1}{\beta} \right) + \frac{4}{3} v_0 k^2 \left( B + \frac{v_0}{\beta} + \frac{4}{3} v_0 k^2 \right) \right] + \frac{4}{3} v_0 k^2 \left( B + v_0 \right) + J^2 + J^2 \right] + \omega^2 \left[ \frac{4}{3} v_0 k^2 \left( Bv_0 + J^2 + \frac{4}{3} v_0 k^2 B \right) + \frac{4}{3} v_0 k^2 \left( B \frac{v_0}{\beta} + J^2 \right) + \Omega^2 \right] + k^2 \left[ -16 \pi^2 G^2 \rho_n \rho \right] \]

\[
+ k^2 \left[ 2AC_n^2 - 4\pi G (A\rho_n + B\rho C_n^2) \right] = 0. \tag{24}
\]

The above equation (24) represents the dispersion relation of self-gravitating, viscous two-component plasma having collisions with thermal and radiative effects. There is no effect of magnetic field and finite electrical conductivity of plasma in this case.

The condition of instability from the constant term of equation (24) is given as

\[
k^2 \left[ 2AC_n^2 - 4\pi G (A\rho_n + B\rho C_n^2) \right] < 0, \tag{25}
\]

or \( k < k_{J1} \), where

\[
k_{J1}^2 = \left[ 4\pi G \left( \frac{\rho_n}{C_n^2} + \frac{\rho}{C^2} \right) + \frac{\rho^2 L_p}{\lambda T} - \frac{\rho L_T}{\lambda} \right] \pm \left[ 4\pi G \left( \frac{\rho_n}{C_n^2} + \frac{\rho}{C^2} \right) + \frac{\rho^2 L_p}{\lambda T} - \frac{\rho L_T}{\lambda} \right]^2 \]

\[
+ \frac{16 \pi G \rho}{\lambda} \left( \frac{\rho_n L_T}{C_n^2} - \frac{\rho L_p}{C^2} \right) \right]^{1/2} \tag{26}
\]

In the absence of thermal and radiative effects the condition of instability is obtained from equation (24) which is given as

\[
k^2 \left[ C^2 - 4\pi G (\rho n C^2 + \rho C_n^2) \right] < 0, \tag{27}
\]

or \( k^2 < k_{J2}^2 = 4\pi G \left( \frac{\rho_n}{C_n^2} + \frac{\rho}{C^2} \right) \tag{28}\)

The above equation (28) is identical to equation (50) of Chhonkar and Bhatia [4] and equation (25) of Vaghela and Chhajlani [6]. On comparing equations (27), (28) with equations (25), (26) we find that the condition of instability and expression of critical Jeans wave number both are modified due to the presence of thermal conductivity and radiative heat-loss function. Thus the present results
are the improvement of Chhonkar and Bhatia [4] and Vaghela and Chhajlani [6] having radiative effects.

If the sonic speeds in the plasma and neutral gas are taken equal i.e. \( C = C_n = c \), then condition (28) reduces to
\[
k^2 < k_{J1}^2 = \frac{4 \pi G \rho_0}{c^2}.
\] (29)
This is the original Jeans condition of instability for a self-gravitating gas cloud.

The condition of radiative instability is obtained from equation (24) for purely plasma component and given as
\[
k^2 A - 4 \pi G \rho B < 0.
\] (30)
The above inequality is similar to that obtained by Bora and Talwar [7] and can be solved to get the following expression of critical Jeans wave number
\[
k_{J1}^2 = \frac{1}{2} \left\{ \frac{4 \pi G \rho}{C^2} + \frac{\rho L_p}{\lambda T} - \frac{\rho L_T}{\lambda} \right\} \pm \left[ \left( \frac{4 \pi G \rho}{C^2} + \frac{\rho L_p}{\lambda T} - \frac{\rho L_T}{\lambda} \right)^2 + \frac{16 \pi G \rho^2}{\lambda C^2} \frac{L_T}{\lambda} \right]^{1/2}.
\] (31)

It may be noted here that the modified critical Jeans wave number involves, derivatives of temperature dependent and density dependent heat-loss function and thermal conductivity of the medium. On comparing equations (30), (31) with equations (25), (26) we conclude that the condition of instability and expression of critical Jeans wave number both are modified due to the presence of neutral particle. Thus the present results are the improvement of Bora and Talwar [7].

For purely neutral component equation (24) reduces to
\[
0 = \frac{4}{3} n \left( C_n^2 k^2 - 4 \pi G \rho_n \right).
\] (32)
The condition of instability from above equation is
\[
C_n^2 k^2 - 4 \pi G \rho_n < 0,
\] (33)
or
\[
k^2 < k_{J2}^2 = \frac{4 \pi G \rho_n}{C_n^2}.
\] (34)

Now on comparing equations (28), (29), (31) and (34) with (26) it is obvious that the Jeans condition of instability is modified because of thermal conductivity and heat-loss function and also for a two-component plasma only because of the different sonic speeds of the two-components.

In absence of viscosity and collisions (i.e. \( \nu = \nu_n = \nu_n' = 0 \)) the dispersion relation (24) gives
\[
\left[ k^2 + \omega^2 \right] \left[ C_n^2 + C_n^2 \right] = 4 \pi G \rho_n B
\]
\[
+ \omega^2 \left[ k^2 \left( C_n^2 + C_n^2 \right) - 4 \pi G \rho_n \right] + \omega^2 \left[ k^2 \left( A + B \right) + 4 \pi G \rho_n B \right]
\]
\[
+ \omega \left[ k^2 \left( C_n^2 + C_n^2 \right) - 4 \pi G \rho_n \right] = 0.
\] (35)
The condition of instability obtained from constant term of above equation (35) is same as equation (25) hence viscosity and collisions of the two-components do not affect the condition of instability. However they cause damping effect.

According to Herrnegger [3] the necessary condition for instability can be seen only from the constant term of the dispersion relation. So when constant term is less than zero, one of the roots must be real and hence shows unstable mode.

For a better insight, the graphical presentation of the condition of instability from constant term of equation (24) after making it non-dimensional is given as
where the various non-dimensional parameters are defined as

\[ k^* = \frac{kC}{(4\pi G\rho)^{1/2}}, \quad C_0^2 = \frac{C_0^2}{C^2}, \quad A^* = \frac{1}{\gamma} (L_T^* + \lambda^* k^{*2}) - L_p^*, \quad B^* = (L_T^* + \lambda^* k^{*2}), \]

\[ L_T^* = \frac{(\gamma - 1)pTL_T}{p(4\pi G\rho)^{1/2}}, \quad L_p^* = \frac{(\gamma - 1)pL_p}{C^2(4\pi G\rho)^{1/2}}, \quad \lambda^* = \frac{(\gamma - 1)\gamma\lambda}{pC^2}. \]

Figure 1. The normalized function \( F(k^*) \) is plotted against wave number \( k^* \) for three values of parameter \( L_T^* = 0, 0.5, 1.5 \) keeping the other parameters fixed \( L_p^* = 1, \lambda^* = 1, C_0^2 = 1 \), \( \beta = 1 \) and \( \gamma = 5/3 \).

Figure 2. The normalized function \( F(k^*) \) is plotted against wave number \( k^* \) for three values of parameter \( \lambda^* = 2, 4, 6 \) keeping the other parameters fixed \( L_p^* = 0, L_T^* = -0.5, C_0^2 = 1 \), \( \beta = 1 \) and \( \gamma = 5/3 \).

From figure 1 we conclude that on increasing temperature dependent heat loss function region of instability increases but critical Jeans wave number decreases; hence increasing temperature dependent heat loss function tends system towards stabilization. From figure 2 we conclude that on increasing thermal conductivity region of instability increases. Hence thermal conductivity tends to destabilize the system. Basically the effect of thermal conductivity depends on the sign of \( L_T^* \) and \( L_p^* \).

It may be mentioned here that the above result from figure 1 and figure 2 in the present configuration are obtained only from the consideration of the sign of the last term (i.e. constant term) in the dispersion relation (24). But an actual computation of roots of dispersion relation (24) will provide us the exact growth rate of unstable modes in the present configuration.

To solve equation (24) numerically we introduce the dimensionless quantities

\[ k^* = \frac{kC}{(4\pi G\rho)^{1/2}}, \quad C_0^* = \frac{C_0}{C}, \quad A^* = \frac{1}{\gamma} (L_T^* + \lambda^* k^{*2}) - L_p^*, \quad B^* = (L_T^* + \lambda^* k^{*2}) \]
Numerical calculations were performed to determine the roots of $\omega$ from dispersion relation (24), as a function of wave number $k$ for several values of the different parameters involved, taking $\gamma = 5/3$. Out of the five modes only one mode is unstable for which the calculations are presented in figures 3-7, where the growth rate is plotted against the wave number to show the dependence of the growth rate on the different physical parameters.

$$\omega^* = \frac{\omega}{(4\pi G \rho)^{1/2}}, \quad L_T^* = \frac{(\gamma-1)\rho T}{p(4\pi G \rho)^{1/2}}, \quad L_p^* = \frac{(\gamma-1)\rho L_p}{C^2(4\pi G \rho)^{1/2}}, \quad \lambda^* = \frac{(\gamma-1)\rho \lambda(4\pi G \rho)^{1/2}}{pC^2}$$

$$\beta = \frac{\rho_n}{\rho}, \quad \nu^* = \frac{\nu (4\pi G \rho)^{1/2}}{C^2}, \quad \nu_n^* = \frac{\nu_n (4\pi G \rho)^{1/2}}{C^2}, \quad \nu_p^* = \frac{\nu_p}{(4\pi G \rho)^{1/2}}.$$

Figure 3. The growth rate (positive values of $\omega^*$) against wave number $k^*$ for three values of parameter $L_T^*$ keeping the other parameters fixed. $C_0^* = 1, \beta = 1, L_T^* = 1, \lambda^* = 1, \nu^* = 1, \nu_n^* = 1, \nu_p^* = 1$.

1. $L_T^* = 0.0$, 2. $L_T^* = 2$ and 3. $L_T^* = 7$.

Figure 4. The growth rate (positive values of $\omega^*$) against wave number $k^*$ for three values of parameter $L_p^*$ keeping the other parameters fixed. $C_0^* = 1, \beta = 1, L_T^* = 1, \lambda^* = 1, \nu^* = 1, \nu_n^* = 1, \nu_p^* = 1$.

1. $L_p^* = 0.0$, 2. $L_p^* = 2$ and 3. $L_p^* = 7$.

Figure 5. The growth rate (positive values of $\omega^*$) against wave number $k^*$ for three values of

Figure 6. The growth rate (positive values of $\omega^*$) against wave number $k^*$ for three values of
parameter $v_0^*$, keeping the other parameters fixed.

$$C_0^2 = 1, \beta = 1, L_0^* = 1, \lambda^* = 1, v_0^* = 1, L_T^* = 1.$$

(1) $v_0^* = 0.0$, (2) $v_0^* = 2$ and (3) $v_0^* = 7$.

Parameter $\lambda^*$, keeping the other parameters fixed.

$$C_0^2 = 1, \beta = 1, L_0^* = 1, v_0^* = 1, v_n^* = 1, L_T^* = 1.$$

(1) $\lambda^* = 0.0$, (2) $\lambda^* = 2$ and (3) $\lambda^* = 7$.

From above equation first factor gives $\omega^2 = 0$, we get marginal stable modes.

Equating second factor of equation (22) to zero, i.e.

$$\left( PQ - \frac{\omega^2 v_0^2}{\beta} \right)^2 = 0,$$

on solving we get

$$\omega^2 \left\{ \omega^3 + \omega^2 \left[ v_c \left( 1 + \frac{1}{\beta} \right) + \frac{\kappa^2}{\eta} \right] + \omega \left[ (1 + \frac{1}{\beta}) \eta k^2 + v_n^* k^2 + \frac{\eta k^2}{\beta} \right] + (1 + \frac{\beta}{v_c^2} ) \nu^2 k^2 \right\} = 0,$$

$$+ v_c \left[ \frac{\kappa^2}{\beta} + \frac{\eta k^2}{\beta} \right] + \frac{\kappa^2}{\eta} \left[ \frac{v_c^2}{\beta} + \frac{v_n k^2}{\beta} \right] = 0.$$  (37)

From above equation first factor gives $\omega^2 = 0$, we get marginal stable modes.
The second factor of equation (37) is a cubic equation in powers of \( \omega \) whose all the coefficients are positive and the principal diagonal minors of Hurwitz's matrix are also positive hence this mode shows stability. The effect of self-gravitation, thermal conductivity and radiative heat-loss function is not exhibited in this case.

4.2. Transverse Propagation
Equating the first part of equation (23) to zero, i.e. \((FS - DE = 0)\), on solving we get

\[
\omega^6 + \omega^3 \left[ v_c \left( 1 + \frac{1}{\beta} \right) + \frac{4}{3} \nu k^2 + \frac{4}{3} \nu n k^2 + \eta k^2 + B \right] + \omega^4 \left[ \eta k^2 \left[ v_c \left( 1 + \frac{1}{\beta} \right) + \frac{4}{3} \nu k^2 + \frac{4}{3} \nu n k^2 + B \right] \right]
\]

\[
+ B \left[ v_c \left( 1 + \frac{1}{\beta} \right) + \frac{4}{3} \nu k^2 + \frac{4}{3} \nu n k^2 \right] + \frac{4}{3} \nu n k^2 v_c + \frac{4}{3} \nu k^2 + \frac{4}{3} v_c \right] + \frac{4}{3} \nu n k^2 v_c + (1 + \beta) \nu^2 k^2 + J^2 + J_n^2 \]

\[
+ \omega^2 \left[ \eta k^2 \left[ B \left( v_c \left( 1 + \frac{1}{\beta} \right) + \frac{4}{3} \nu k^2 + \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right) + \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right] + J_n^2 \left( \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right) \right]
\]

\[
+ \Omega_n^2 + J_n^2 \left( \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right) - v_c \left( \frac{4\pi G \rho_n}{\beta} + 4\pi G \rho \right) \right] + \omega^2 \left[ \eta k^2 \left[ B \left( \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right) + \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right] \right]
\]

\[
+ B \left( \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right) + J_n^2 \left( \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right) - v_c \left( \frac{4\pi G \rho_n}{\beta} + 4\pi G \rho \right) \right]
\]

\[
+ B \left[ \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right] + J_n^2 \left( \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right) - v_c \left( \frac{4\pi G \rho_n}{\beta} + 4\pi G \rho \right) \right] + \Omega_n^2 \left( \frac{4}{3} \nu n k^2 + \frac{4}{3} v_c \right)
\]

\[
+ \eta k^2 \left[ 4\pi G (A \rho_n + B \rho C_n^2) \right] = 0.
\]

The above dispersion relation represents the combined influence of viscosity, magnetic field, thermal and finite electrical conductivity, collision frequency and heat-loss function on self-gravitational instability of two component plasma. The condition of instability from constant term of the equation (38) is given as

\[
\left[ k^2 AC_n^2 - 4\pi G (A \rho_n + B \rho C_n^2) \right] < 0.
\]

This is already discussed in equation (25).

In absence of thermal and radiative effects we get the same results as obtained earlier in equation (28).

The condition of instability obtained from equation (38) for an infinitely conducting plasma in absence of viscosity, neutral collisions, thermal conductivity and radiative effects \((v = v_n = v_c = \eta = \lambda = \lambda_T = \lambda_P = 0)\) is

\[
\left\{ J_n^2 \left[ (1 + \beta) \nu^2 k^2 + J^2 \right] - \nu^2 + 16\pi^2 G^2 \rho_n \rho \right\} < 0,
\]
or \[ k^2 < k_{j6}^2 = 4\pi G \left( \frac{\rho_p}{n_p^2} + \frac{\rho}{(1 + \beta) V^2 + C^2} \right) \] (41)

Now the condition of instability obtained from equation (38) for infinitely conducting plasma in absence of viscosity, neutral collisions, thermal conductivity, radiative effects and neutral particle is

\[ k^2 (C^2 + V^2) - 4\pi G \rho b < 0, \] (42)

or \[ k^2 < k_{j7}^2 = \frac{4\pi G \rho b}{C^2 + V^2}. \] (43)

The above equation (43) shows that magnetic field modifies the Jeans criterion. Since on increasing magnetic field the value of critical wave number decreases, hence magnetic field tends to stabilize the system.

For purely plasma component only i.e. by neglecting neutral component in equation (38) the condition of instability is identical to equation (30).

The condition of instability from equation (38) for infinitely conducting medium with purely plasma component only is

\[ k^2 (A + V^2) - 4\pi G \rho b < 0, \] (44)

or \[ k < k_{j8}, \]

\[ k_{j8}^2 = \frac{1}{2} \left[\left( \frac{\rho_p^2 L_p}{\lambda T} + 4\pi G \frac{\rho b}{C^2} \right) \left(1 + \frac{V^2}{C^2}\right)^{-1} - \frac{\rho L_T}{\lambda} \right] \pm \left[\left( \frac{\rho_p^2 L_p}{\lambda T} + 4\pi G \frac{\rho b}{C^2} \right) \left(1 + \frac{V^2}{C^2}\right)^{-1} - \frac{\rho L_T}{\lambda} \right]^2 \] (45)

This relation is same as given by Aggarwal and Talwar [11].

On comparing equation (44) with equation (30) we find that inclusion of finite conductivity removes the effect of magnetic field and hence the effect of finite conductivity is to destabilize the system.

Equating the second part of equation (23) to zero, i.e. \[ M_Q - \frac{\omega^2 v_c^2}{\beta} = 0, \] on solving we get

\[ \omega^2 \left( \omega^2 + \omega \left( 1 + \frac{1}{\beta} \right) + \frac{u k^2 + u_n k^2}{\beta} \right) + u_c \left( u_n k^2 + \frac{u k^2}{\beta} \right) + u_k^2 u_n k^2 = 0. \] (46)

For the first factor of equation (46) \( \omega^2 = 0, \) we get the marginal stable modes.

The second factor is a second order equation in powers of \( \omega \) whose all the coefficients are positive and the principal diagonal minors of Hurwitz’s matrix are also positive hence this mode shows stability. In this mode collision frequency and viscosity of the medium exhibits the damping effect. This dispersion relation is independent of radiative heat-loss function, thermal conductivity, magnetic field strength and self-gravitation.

In absence of viscosity equation (46) becomes

\[ \omega + u_c \left( 1 + \frac{1}{\beta} \right) = 0. \] (47)

This result is identical to Herrnegger [3].

This mode exists because of ion-neutral collision and in absence of any one of the two components the above mode vanishes. Here damping rate depends on the collision frequency and the density ratio of the two components.
5. Conclusion
Thus in the present paper, the effects of radiative heat-loss function, thermal conductivity, finite
electrical resistivity, viscosity and collision frequency on the self-gravitational instability of two-
component partially ionized plasma is investigated. For both the longitudinal and transverse wave
propagation, stable modes are obtained when velocity perturbations are taken perpendicular to wave
vector. The instability conditions are obtained on the basis of Jeans criterion. The value of the critical
Jeans wave number is depending on radiative heat-loss function and thermal conductivity and also
depending on the ratio of sonic speeds, density of the two components in some particular cases. The
stable modes are obtained due to viscosity and collision frequency of the two-components. The
magnetic field stabilizes the system and finite electrical conductivity destabilizes the system in
transverse mode of propagation. Numerical calculation shows stabilizing effect of temperature
dependent heat-loss function, collision frequency, thermal conductivity and viscosity, and
destabilizing effect of density dependent heat-loss function on the self-gravitational instability of two-
component plasma.

Acknowledgment
The authors are indebted to Dr. S.B. Shrivastava, Prof. and Head, S.S. in Physics Vikram University,
Ujjain; for his constant encouragements. One author (SK) is grateful to Er. Praveen Vashishtha,
Chairman of Mahakal Institute of Technology and Management under whose continuous support and
guidance this work was done. The authors thank to DST for providing research project to Dr. R.K.
Chhajlani.

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