Modeling the Multiband Light Curves of the Afterglows of Three Gamma-Ray Bursts and their Associated Supernovae

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Abstract

Some dozen supernovae (SNe) associated with long gamma-ray bursts (GRBs) have been confirmed. Most of the previous studies derive the physical properties of the GRB-SNe by fitting the constructed (pseudo-)bolometric light curves. However, many GRB-SNe only have a few filter data, for which the (pseudo-)bolometric light curves are very difficult to construct. Additionally, constructing (pseudo-)bolometric light curves rely on some assumptions. In this paper, we use the multiband broken power-law plus $^{56}$Ni model to fit the multiband light curves of the afterglows and the SNe (SN 2001ke, SN 2013dx, and SN 2016ja) associated with three GRBs (GRB 011121, GRB 130702A, and GRB 161219B). We find our model can account for the multiband light curves of the three GRB-SNe (except for the late-time z-band light curve of two events), indicating that the model is a reliable model. The $^{56}$Ni masses we derive are higher than those in the literature. This might be due to the fact that the $^{56}$Ni masses in the literature are usually obtained by fitting the pseudo-bolometric light curves whose luminosities are usually (significantly) underestimated. We suggest that the multiband model can not only be used to fit the multiband light curves of GRB-SNe that have many filter observations, but also fit those having sparse data.

Unified Astronomy Thesaurus concepts: Type Ic supernovae (1730); Gamma-ray bursts (629)

1. Introduction

Gamma-ray bursts (GRBs) are the most powerful explosions in the universe. It is widely believed that GRBs come from the relativistic jet launched by the central engine (Woosley 2011). The interactions between the jets with the surrounding medium would produce X-ray, UV-optical–near-infrared (NIR), and radio afterglows (see Zhang 2018 and references therein). According to the observation of prompt emission duration, GRBs is divided into long-duration bursts (LGRBs) and short-duration bursts (SGRBs) with a dividing line of $\sim 2$ s (Kouveliotou et al. 1993). The observations and analysis for some dozen supernovae (SNe) associated with LGRBs (Hjorth et al. 2003; Matheson et al. 2003; Stanek et al. 2003; Malesani et al. 2004; Deng et al. 2005; Campana et al. 2006; Mirabal et al. 2006; Modjaz et al. 2006; Sollerman et al. 2006; Maeda et al. 2007; Chornock et al. 2010; Starling et al. 2011; Bufano et al. 2012; Melandri et al. 2012, 2014, 2019; Olivares et al. 2012; Singer et al. 2013; Schulze et al. 2014; D’Elia et al. 2015; Toy et al. 2016; Cano et al. 2017a; Volnova et al. 2017; Ashall et al. 2019; Hu et al. 2021) indicate that most LGRBs are produced by the explosions of massive stars. On the other hand, the confirmation of SSS17a/AT2017gfo, which is a kilonova associated with GW170817 that is a gravitational wave emitted by a merger of a neutron star binary and GRB 170817A that is an SGRB (Abbott et al. 2017; Arcavi et al. 2017; Coulter et al. 2017; Shappee et al. 2017), supports the conjecture that at least a fraction of SGRBs are produced by the mergers of compact binary stars.

The SNe associated with LGRBs are called GRB-SNe (Woosley & Bloom 2006; Hjorth & Bloom 2012; Cano et al. 2017b).

On average, one or two GRB-SNe can be found every year. To date, there are about 60 LGRBs that have been confirmed to be associated with SNe. Almost all GRB-SNe are broad-lined Ic (Ic-BL) SNe whose optical spectra are hydrogen deficient and show broad absorption line features. The spectral features indicate that the progenitors of GRB-SNe are highly stripped, and might be Wolf-Rayet stars (Price et al. 2002; Sonbas et al. 2008). The broad absorption lines are indicative of huge ejecta velocities $\gtrsim 2 \times 10^9$ cm s$^{-1}$. Therefore, a major fraction of GRB-SNe (and the SNe Ic-BL without accompanying GRBs) become so-called hypernovae (HNes) whose kinetic energy is $\gtrsim 10^{52}$ erg, which is about 10 times that of normal SNe. The explosion mechanisms of GRB-SNe are still elusive.

The most prevailing model adopted to account for the light curves of GRB-SNe is the $^{56}$Ni cascade decay ($^{56}$Ni $\rightarrow ^{56}$Co $\rightarrow ^{56}$Fe) model (the $^{56}$Ni model, Arnett 1979, 1980, 1982, 1996). Some very luminous GRB-SNe cannot be explained by the $^{56}$Ni model, and alternative or additional energy sources (e.g., the magnetar spinning down, the fallback accretion, etc.) are employed to account for the light curves.

Previous studies focusing on GRB-SNe usually construct the pseudo-bolometric light curves of the SNe and derive the physical properties of GRB-SNe by fitting the constructed pseudo-bolometric light curves. However, it should be noted that the process of constructing the pseudo-bolometric light curves might underestimate the luminosities of the SNe and therefore underestimate the $^{56}$Ni masses.

Recently, the model directly fitting the multiband light curves (Nicholl et al. 2017) has been adopted to fit the light curves of superluminous SNe (Nicholl et al. 2017; Moriga et al. 2018), the tidal disruption events (Mockler et al. 2019), the luminous rapidly evolving optical transients (Wang et al. 2019), and ordinary SNe Ib and Ic (S. Q. Wang et al. 2022, in preparation).

In this paper, we collect published data of the (UV-optical–NIR counterparts (GRB 011121/SN 2001ke, GRB 130702A/
Notes.

1 Schalicky & Finkbeiner (2011).

2 (1) Bloom et al. (2002); (2) Price et al. (2002); (3) Garavich et al. (2003); (4) Greiner et al. (2003); (5) Toy et al. (2016); (6) Volnova et al. (2017); (7) D’Elia et al. (2015); (8) Buckley et al. (2016); (9) Mazaeva et al. (2016); (10) Martin-Carrillo et al. (2016); (11) Fujiwara et al. (2016); (12) Cano et al. (2017a); (13) Ashall et al. (2019); (14) Laskar et al. (2018). (The UVOT white (U/V) band data are not included, since U/V is not narrow band data; however, the clear band data are included and labeled as R band, since the two are closely approximated.)

3 Küpcü Yoldaş et al. (2007).

4 Toy et al. (2016); assuming that the value of the total to selective extinction ratio (Rv) is 3.1, which is the typical value of the Rv of the Milky Way, Schultz & Wiemler (1975).

5 Cano et al. (2017a).

Table 1
Properties of GRB 011121/SN 2001ke, GRB 130702A/SN 2013dx, and GRB 161219B/SN 2016jca

| GRB 011121/SN 2001ke | GRB 130702A/SN 2013dx | GRB 161219B/SN 2016jca |
|----------------------|------------------------|------------------------|
| RA (°)               | Decl. (°)              | z                      |
| 11°34′29″.67         | -76°01′41″.6            | 0.362                  |
| E(B − V)_MW(μ)       | E(B − V)_host(μ)       | Data Sources           |
| 0.419                | ≤0.08                  | 1, 2, 3, 4             |
| GRB 130702A/SN 2013dx|                        |                        |
| 14°29′14″.78         | +15°46′26″.4            | 0.145                  |
| 0.024                | 0.032                  | 5, 6, 7                |
| GRB 161219B/SN 2016jca|                      |                        |
| 06°06′51″.37         | -26°47′29″.7            | 0.1475                 |
| 0.028                | 0.017 ± 0.012          | 8, 9, 10, 11, 12, 13, 14 |

Table 2
AB Magnitudes of the Host Galaxies of GRB 130702A/SN 2013dx and GRB 161219B/SN 2016jca

| Band | GRB 130702A/SN 2013dx | GRB 161219B/SN 2016jca |
|------|-----------------------|------------------------|
| K    | 20.73 ± 0.11          |                        |
| H    | 20.73 ± 0.07          |                        |
| J    | 23.15 ± 0.48          | 20.80 ± 0.07           |
| z    | 22.97 ± 0.18          | 20.67 ± 0.04           |
| i    | 22.96 ± 0.09          | 20.86 ± 0.03           |
| R    | 23.17 ± 0.06          | 21.33 ± 0.11           |
| r    | 23.05 ± 0.06          | 21.13 ± 0.05           |
| V    | 23.52 ± 0.06          | 21.48 ± 0.03           |
| g    | 23.39 ± 0.20          | 21.63 ± 0.03           |
| B    | 22.79 ± 0.31          | 20.65 ± 0.03           |
| u    | 22.62 ± 0.12          | 20.95 ± 0.12           |
| U/VW | 22.62 ± 0.12          | 20.95 ± 0.12           |
| UVM2 | 23.77 ± 0.49          | 23.77 ± 0.49           |
| U/VW2| 23.41 ± 0.23          |                        |

SN 2013dx, GRB 161219B/SN 2016jca) of three GRBs (GRB 011121, GRB 130702A, GRB 161219B; here, the GRBs represent their afterglows) and use the broken power-law plus 56Ni model to fit their multiband light curves. In Section 2, we model the multiband light curves of the three GRB-SNe using the 56Ni model. In Section 3, we compare the parameters and bolometric properties of the SNe to those in the literature. We draw some conclusions in Section 4. The values of the foreground reddening of the Milky Way (E(B − V)_MW) are from Schalicky & Finkbeiner (2011). The standard cosmological parameters (Ω_m = 0.315, Ω_λ = 0.685, and H_0 = 67.3 km s^{-1} Mpc^{-1}, Planck Collaboration et al. 2014) are adopted throughout this paper.

2. Modeling the Multiband Light Curves of Three GRB-SNe Using the 56Ni Model

The properties of GRB 011121/SN 2001ke, GRB 130702A/SN 2013dx, and GRB 161219B/SN 2016jca are listed in Table 1. The flux of the host galaxy of GRB 011121/SN 2001ke is negligible, and the flux of the host galaxies of GRB 130702A/SN 2013dx and GRB 161219B/SN 2016jca have been subtracted (the magnitudes of the two host galaxies are from Volnova et al. 2017 and Laskar et al. 2018, respectively, and listed in Table 2). Then the flux of the (UV)-optical–NIR counterpart of a GRB-SN (F_{UV,AG}(t)) can be divided into that of the GRB afterglow (F_{GRB}(t)) and that of the SN (F_{SN}(t)) associated with the GRB, i.e., F_{UV,AG}(t) = F_{GRB}(t) + F_{SN}(t).

The flux density of an afterglow is proportional to a broken power-law decay function (F_{AG}(t) ∝ (t/t_0)^{-α + β}) (Beuermann et al. 1999), and can be expressed as F_{AG}(t) = A_{AG} · (t_{AG}/t_0)^{-α + β}, where the definitions of α, α, β, t_0, and β are presented in Table 3.

We assume that the SN associated with a GRB was powered by 56Ni cascade decay. The bolometric luminosity of 56Ni-powered SNe powered is (see e.g., Arnett 1982; Chatzopoulos & Wang 2012; Wang et al. 2015; Wang & Gan 2022)

\[ L_{SN}(t) = \frac{2}{\tau_m} e^{-t/\tau_m} \int_0^t e^{-t'/\tau_m} \left( e^{\kappa_m M_\text{Ni} e^{-t/\tau_m}} + e^{\kappa_\text{Co} M_\text{Ni} e^{-t/\tau_\text{Co}}} \right) \left( e^{-t'/\tau_\text{Co}} - e^{-t'/\tau_m} \right) \frac{1}{1 - \tau_m/\tau_\text{Co}} \, dt', \]

where \( \tau_m = (2\kappa M_\text{ej}/\beta SN \phi c)^{1/2} \) is the diffusion timescale, \( \tau_r = 3\kappa M_\text{ej}/4\pi \rho c^2 \) is the optical depth to γ-rays (Chatzopoulos et al. 2009, 2012), \( \kappa \) is the optical opacity of the ejecta which is set to be 0.07 cm^2 g^{-1}, c is the speed of light, \( \beta \approx 13.8 \) is a constant (Arnett 1982), \( c_{\text{Ni}} = 3.9 \times 10^{10} \text{erg s}^{-1} \text{g}^{-1} \) (Sutherland & Wheeler 1984; Cappellaro et al. 1997), \( \tau_{\text{Co}} = 8.8 \text{days} \), \( c_{\text{Co}} = 6.8 \times 10^9 \text{erg s}^{-1} \text{g}^{-1} \) (Maeda et al. 2003), and \( \tau_{\text{Co}} = 111.3 \text{days} \). Assuming the early-time photosphere radii of the SNe is proportional to the time, and the ejecta cool to constant temperatures (\( T_f \)), the temperatures and radii can be given by (Nicholl et al. 2017):

\[ T_{\text{ph}}(t) = \begin{cases} \frac{L_{SN}(t)}{4\pi \sigma v_\text{ph}^2 t^2} & \frac{L_{SN}(t)}{4\pi \sigma v_\text{ph}^2 t^2} > T_f \\
T_f & \frac{L_{SN}(t)}{4\pi \sigma v_\text{ph}^2 t^2} \leq T_f \end{cases} \]
suppose that the spectral energy distributions (SEDs) of the SNe can be described by the UV-absorbed blackbody model whose range is 0.248 mag. Additionally, we assume that the host galaxy extinction, while Küpcü Yoldaş et al. 2017; Prajs et al. 2017 can be set to be constants. Hence, the multiband $^{56}$Ni model fitting GRB 130702A/SN 2013dx and GRB 161219B/SN 2016jca, which cannot be well fitted by the multiband model (see the top and bottom-left panels of Figure 1). The UV band light curves cannot be well fitted by the model. As shown in Figure 2 of Laskar et al. (2018), the values of $\alpha_2$ of different UV bands of GRB 161219B/SN 2016jca are different. Hence, assuming a same value of $\alpha_2$ for all bands can result in a bad fit. To improve the fit, we assume that the values of $\alpha_2$ in different UV bands are different from each other, and different from the value in the optical and NIR bands. The new fit for the light curves of GRB 161219B/ SN 2016jca and the corresponding corner plot are presented in the bottom-right panel of Figures 1 and A4, respectively. The parameters of the new fit are listed in the last column of Table 4. We find that the new fit is better than the first fit since the UV bands are also well matched by the model. There are two (possible) reasons that might explain the bad quality of the fits for the $z$-band light curves of the two GRB-SNe. (1). Their late-time $z$-band light curves show fluctuation features that cannot be fully fitted by the theoretical light curves, which are smooth. (2). Their late-time SEDs deviate from the blackbody function the in the $z$ band. The derived masses of $^{56}$Ni of SN 2001ke, SN 2013dx, and SN 2016jca are 0.46 ± 0.01, 0.74 ± 0.01, and 0.33 ± 0.00 $M_\odot$, respectively. The ejecta masses of the three GRB-SNe are 4.02 ± 0.53, 3.71 ± 0.03, and 1.64 ± 0.02 $M_\odot$, respectively. The respective velocity of the ejecta of the three GRB-SNe are 4.22 ± 0.54 × 10$^9$, 2.61 ± 0.02 × 10$^9$, and 2.17 ± 0.03 × 10$^9$ cm s$^{-1}$. The parameters are roughly consistent with the parameter ranges in the literature.

3. Discussion

Here, we compare the values of the $^{56}$Ni masses, the ejecta masses, the ejecta velocity, and the kinetic energy of the ejecta of the three GRB-SNe to that in the literature and discuss the reasons causing the discrepancies. Moreover, we discuss the theoretical bolometric light curves of the three GRB-SNe.
3.1. $^{56}\text{Ni}$ Masses of the Three GRB-SNe

The $^{56}\text{Ni}$ mass of GRB 011121/SN 2001ke is $0.46 \pm 0.01 M_\odot$, we do not find the literature’s value. The $^{56}\text{Ni}$ mass of GRB 130702A/SN 2013dx is $0.74 \pm 0.01 M_\odot$, which is $\sim 2.0$ and $\sim 3.7$ times those of the values derived by Toy et al. (2016) ($0.37 \pm 0.01 M_\odot$) and D’Elia et al. (2015) ($0.2 M_\odot$). The $^{56}\text{Ni}$ mass of GRB 161219B/SN 2016jca is $0.33 \pm 0.00 M_\odot$, which is $\sim 1.50^{+0.86}_{-0.40}$ and $\sim 1.22^{+0.28}_{-0.19}$ times those of the values...
derived by Cano et al. (2017a) \((0.22 \pm 0.08 \, M_\odot)\) and Ashall et al. (2019) \((0.27 \pm 0.05 \, M_\odot)\), respectively. The discrepancy might be due to the facts that Toy et al. (2016), Cano et al. (2017a), and Ashall et al. (2019) derived the \(^{56}\text{Ni}\) masses by fitting the pseudo-bolometric light curves\(^1\), which are dimmer than the bolometric light curves and that our blackbody multiband fits correspond to the bolometric light curves.

The \(^{56}\text{Ni}\) mass of GRB 130702A/SN 2013dx is rather large, but comparable to the \(^{56}\text{Ni}\) mass of SN 1998bw, which is \(0.4-0.7 \, M_\odot\) (Iwamoto et al. 1998; Nakamura et al. 2001) or \(0.54^{+0.08}_{-0.07} \, M_\odot\) (Lyman et al. 2016). Therefore, we suggest that the \(^{56}\text{Ni}\) mass is reasonable.

### 3.2. Properties of the Ejecta

The ejecta masses of SN 2013dx and SN 2016jca are \(3.71 \pm 0.03\) and \(1.64 \pm 0.02 \, M_\odot\), respectively, which are lower than the values derived in the literature \((3.1 \pm 0.1 \, M_\odot\) (Toy et al. 2016) or \(\sim 7 \, M_\odot\) (D’Elia et al. 2015) for SN 2013dx, \(5.8 \pm 0.3 \, M_\odot\) (Cano et al. 2017a), or \(6.5 \pm 1.5 \, M_\odot\) (Ashall et al. 2019) for SN 2016jca).

For example, Toy et al. (2016) and D’Elia et al. (2015) assume that the velocities of SN 2013dx are \(2.13 \times 10^9\) and \(\sim 2.9 \times 10^9 \, \text{cm s}^{-1}\), respectively; Cano et al. (2017a) and Ashall et al. (2019) assume that the velocities of SN 2016jca are \(2.97 \pm 0.15 \times 10^9\) and \(3.5 \pm 0.7 \times 10^9 \, \text{cm s}^{-1}\), respectively.

Our derived early-time photospheric velocities of SN 2013dx and SN 2016jca are \(2.61 \pm 0.02 \times 10^9\) and \(2.17 \pm 0.03 \times 10^9 \, \text{cm s}^{-1}\), respectively. The former is between the two values adopted by Toy et al. (2016) \((2.13 \times 10^9 \, \text{cm s}^{-1})\) and D’Elia et al. (2015) \((2.7 \times 10^9 \, \text{cm s}^{-1})\); the latter is lower than those derived by Cano et al. (2017a) \((2.97 \pm 0.15 \times 10^9 \, \text{cm s}^{-1})\) and Ashall et al. (2019) \((3.5 \pm 0.7 \times 10^9 \, \text{cm s}^{-1})\).

The kinetic energy \((E_k = \frac{3}{10} M_\odot v_k^2)\) of the ejecta of SN 2001ke is \(4.27^{+1.88}_{-1.60} \times 10^{52} \, \text{erg}\). The kinetic energy of the ejecta of SN 2013dx is \(1.51 \pm 0.04 \times 10^{52} \, \text{erg}\), which is comparable to the value derived by Toy et al. (2016) \((8.2 \times 10^{51} \, \text{erg})\) and significantly lower than the value inferred by D’Elia et al. (2015) \((3.5 \times 10^{52} \, \text{erg})\). The kinetic energy of the ejecta of SN 2016jca is \(4.6 \pm 0.2 \times 10^{51} \, \text{erg}\), which is significantly lower than that derived by Cano et al. (2017a) \((5.1 \pm 0.8 \times 10^{52} \, \text{erg})\).

### 3.3. Theoretical Bolometric Light Curves

We use the derived best-fitting parameters to yield the bolometric light curves of the three GRB-SNe we study, see Figure 2. We find that the peak bolometric luminosities of SN 2001ke, SN 2013dx, and SN 2016jca are \(1.37 \times 10^{43}\), \(1.92 \times 10^{43}\), and \(1.04 \times 10^{43} \, \text{erg s}^{-1}\), respectively.

For comparison, the peak (pseudo-)bolometric luminosities of the three GRB-SNe derived by the literature are \(6 \times 10^{42} \, \text{erg s}^{-1}\) (Cano et al. 2017b), \(1 \times 10^{43} \, \text{erg s}^{-1}\) (Toy et al. 2016), and \(6.3 \times 10^{42} \, \text{erg s}^{-1}\) (Ashall et al. 2019) or \(4.6 \times 10^{42} \, \text{erg s}^{-1}\) (Cano et al. 2017a), respectively.

By comparing our derived peak bolometric luminosities of SN 2001ke, SN 2013dx, and SN 2016jca to their peak (pseudo-)bolometric luminosities in the literature, we find that the former are respectively 2.28, 1.92, and 1.65 (or 2.26) times that of the latter.

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1 D’Elia et al. (2015) construct the pseudo-bolometric light curve of SN 2013dx and derive the \(^{56}\text{Ni}\) mass by scaling the pseudo-bolometric light curve of SN 2003dh.

2 This value is derived from the medians of the ejecta mass (7 \(M_\odot\)) and kinetic energy \((35 \times 10^{51} \, \text{erg})\) provided by D’Elia et al. (2015), assuming \(E_k = \frac{3}{10} M_\odot v_k^2\).

3 The SN velocities inferred from the spectra evolve (usually decrease) with time. Toy et al. (2016) find that the spectral velocity of SN 2013dx inferred from the Si II lines at days 9.3, 11.3, 14.2, 31.3, and 33.3 are 2.81, 2.52, 2.13, 1.17, and 1.08 \(\times 10^9 \, \text{cm s}^{-1}\), respectively. D’Elia et al. (2015) find that the velocity of SN 2013dx declines from \(\sim 2.7 \times 10^9 \, \text{cm s}^{-1}\) at day 8 to \(\sim 3.5 \times 10^9 \, \text{cm s}^{-1}\) at day 40. Previous studies fitting the (pseudo-)bolometric light curves usually adopt the velocity derived from the spectra obtained around maximum light or earlier epochs.
The discrepancies of the peak luminosities of bolometric light curves we derive and those of the pseudo-bolometric light curves might be due to the fact that the latter omit the flux in UV and/or IR bands. Toy et al. (2016) construct the pseudo-bolometric light curve of SN 2013dx by integrating the flux in the $g'r'i'z'yJ$ bands, and more flux is neglected. Cano et al. (2017a) use the $griz$ band data to construct the pseudo-bolometric light curve of SN 2016jca, and the flux might also be underestimated.

Our derived rise time of SN 2001ke and SN 2013dx are respectively 11.8 and 13.7 days, which are respectively smaller than and comparable to the rise time of the two SNe in the literature, which are $\sim$17.5 days (Cano et al. 2017b) and $\sim$14 days (Toy et al. 2016). Our derived rise time of SN 2016jca is 10.7 days, which is slightly larger than in the literature, which is $\sim$10 days (Ashall et al. 2019).

4. Conclusions

In the past two decades, a few dozen LGRBs have been confirmed to be associated with SNe Ic, most of which are SNe Ic-BL and HNe. While the kinetic energy of most GRB-SNe is $\geq$10 times that of normal SNe Ic, most of which are SNe Ic, their average peak luminosities are not significantly higher than those of SNe Ic. Therefore, the $^{56}$Ni model adopted to account for the light curves of normal SNe Ic have also been used to explain the light curves of GRB-SNe. However, many studies exploring the energy sources of GRB-SNe construct the pseudo-bolometric light curves and fit them. This method might underestimate the $^{56}$Ni masses needed to power the light curves of SNe.

We collected photometric data of three well-observed GRB-SNe (GRB 011121/SN 2001ke, GRB 130702A/SN 2013dx, GRB 161219B/SN 2016jca) and use the multiband broken power-law plus $^{56}$Ni model to fit the multiband light curves of the total flux, which is the sum of those of the afterglows of the GRBs and the SNe. The multiband model we use fits the observed multiband data, rather than the pseudo-bolometric light curves constructed by making some assumptions. A larger data set could pose more stringent constraints on the physical parameters.

We find that the multiband light curves of GRB 011121/SN 2001ke can be fitted by the model we use; the multiband light curves of GRB 130702A/SN 2013dx and GRB 161219B/SN 2016jca can be fitted by the model (except their late-time $z$-band light curves). This indicates that the UV-optical–NIR SEDs of SNe associated with GRBs can be well described by the UV-absorbed blackbody model, and that our model can account for the multiband light curves of the three GRB-SNe.

Our derived $^{56}$Ni masses of SN 2013dx and SN 2016jca are $0.74 \pm 0.01$ and $0.33 \pm 0.00 M_\odot$, respectively. The former is about $\sim 2.0$ and $\sim 3.7$ times those of the values derived by Toy et al. (2016) and D’Elia et al. (2015), while the latter is $\sim 1.50_{-0.40}^{+0.36}$ and $\sim 1.22_{-0.19}^{+0.28}$ times those of the values derived by Cano et al. (2017a) and Ashall et al. (2019). This might be due to the fact that the constructed pseudo-bolometric light curves of SN 2013dx and SN 2016jca omit a fraction of the total flux. Therefore, we suggest that the $^{56}$Ni masses of at least a fraction of GRB-SNe have been underestimated, and the multiband $^{56}$Ni model can make it possible to avoid underestimating the luminosities of SNe and therefore the $^{56}$Ni masses.

The derived early-time photospheric velocities of SN 2013dx and SN 2016jca are $2.61 \pm 0.02 \times 10^9$ and $2.17 \pm 0.03 \times 10^9 \text{ cm s}^{-1}$, respectively, the former is between those adopted in the literature ($2.13$ or $2.7 \times 10^9 \text{ cm s}^{-1}$), while the latter is lower than that in the literature ($2.97$ or $3.5 \pm 0.15 \times 10^9 \text{ cm s}^{-1}$). The derived kinetic energies of SN 2013dx and SN 2016jca are $1.51 \pm 0.04 \times 10^{52}$ and $4.6 \pm 0.2 \times 10^{53}$ erg. While the former is (significantly) lower in the literature ($8.2 \times 10^{51}$ or $3.5 \times 10^{52}$ erg), the latter is significantly lower than that in the literature ($5.1 \pm 0.8 \times 10^{52}$ erg for SN 2016jca).

Our study demonstrates the validity of the multiband afterglow plus $^{56}$Ni model for the the fits of the multiband light curves of GRB-SNe. The model can be regarded as an independent model that do not rely on the (pseudo-)bolometric light curves constructed. Although the GRB-SNe we fit have ample data at many bands, we expect that the model can also be used for the multiband light curves of GRB-SNe observed in only one, two, or three bands at some or all epochs. For the GRB-SNe with sparse data, the multiband model can play a key role in determining their physical properties by fitting their multiband light curves, since constructing the (pseudo-) bolometric light curves is very difficult.

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Appendix

Figures A1–A4 show the corner plots of the model for GRB 011121/SN 2001ke, GRB 130702A/SN 2013dx, and GRB 161219B/SN 2016jca (two cases) in the main text.
Figure A1. Corner plot of the $^{56}$Ni model for the multiband light curves of GRB 011121/SN 2001ke.
Figure A2. Corner plot of the $^{56}$Ni model for the multiband light curves of GRB 130702A/SN 2013dx.
Figure A3. Corner plot of the $^{56}$Ni model for the multiband light curves of GRB 161219B/SN 2016jca.
Figure A4. Corner plot of the $^{56}$Ni model for the multiband light curves of GRB 161219B/SN 2016jca (assuming that the $\alpha_2$ values in UV bands are different from each other, and different from that in optical and NIR bands).

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