Multi-body dynamics simulation and analysis of a nuclear power waste crane

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Abstract. The dynamic performance of nuclear waste crane is an important index to ensure its safety and reliability. In this paper, a multi-body dynamic model of nuclear power waste crane is established by using the method of Udwadia-Kalaba equation, and its simulation analysis is carried out to verify the feasibility of the application of this method in heavy equipment design.

1. Introduction
Due to the particularity of the workplace, nuclear waste crane must ensure strict safety, reliability and positioning accuracy. In order to achieve this goal, there is a high demand for the dynamic performance of nuclear power crane, but due to the limitation of cost, it is impossible to use physical prototype test to measure its dynamic index in the development stage. Therefore, the multi-body dynamic method has become an important means to analyze the nuclear waste crane.

In the 1990s, Udwadia F E and Kalaba R E of the University of Southern California in the United States put forward the Udwadia-Kalaba equation on the basis of the present perfect theory of multi-body dynamics. This equation is a theoretical method to describe the dynamics of multi-body systems with constraints. Without Lagrangian multipliers, it is a breakthrough in the field of analytical mechanics to obtain the equations of kinematics and dynamics and the analytical expressions of system constraints.

The main characteristics of this method are divided into three aspects [1]: (1) based on D'Alembert's principle and Gauss's theorem, the basic dynamic equation of multibody system under ideal constraint condition is proposed, and the analytical expression of total constraint force of the system is given; (2) aiming at the limitation of D'Alembert's principle in non-ideal constraint system, the dynamic equation of the system is improved, and the non-ideal constraint force is derived. (3) for the case of singular mass matrix, the system equation is extended to solve the problem of no solution when the mass matrix is singular.

Because of the above advantages, this method has been widely used in the field of dynamic modeling and trajectory control in recent 20 years. Chen YH, Professor of Georgia Institute of Technology in the United States, designed a servo control system for track tracking of mechanical system by using the second-order constraint of the method, and believed that good tracking control effect can be achieved through reasonable design [2-4]. Braun D and Goldfarb M use this equation to propose a constraint motion equation embedded with constraint modification, which effectively solves the defect of error accumulation caused by constraint drift [5,6]. Then Cho H and Udwadia F E use this theory to propose a method to solve the problem of attitude keeping of nonlinear configuration system. Without linearization approximation, the exact closed expression of control force can be obtained [7].
2. Establishment of nuclear waste crane model based on Udwadia-kalaba equation

2.1. Description method of position and orientation of spatial rigid body

In order to describe the position and attitude of a spatial rigid body, the common method in classical mechanics is to attach a dynamic coordinate system $O_B^{-x_B,y_B,z_B}$ to the rigid body (set as B) and establish a global coordinate system $O_g^{-x_g,y_g,z_g}$ fixed to the reference object. The general motion of a rigid body can be regarded as the combination of straight-line motion around a fixed point and rotation around a coordinate axis. Therefore, the position and orientation of the rigid body can be described according to the position of the origin of the rigid body coordinate system $O_B^{-x_B,y_B,z_B}$ in the global coordinate system $O_g^{-x_g,y_g,z_g}$ and the angle of rotation in a certain order. At the same time, the position coordinates of any point of the rigid body in space can be described according to the principle of homogeneous transformation, as shown in Figure 1.

![Figure 1. Schematic diagram of any point location of spatial rigid body](image)

In the picture, $^g r_{O_B}$ is the position vector of the original point of dynamic coordinate system $O_B^{-x_B,y_B,z_B}$ in the global coordinate system $O_g^{-x_g,y_g,z_g}$, $^B r_p$ is position vector of the arbitrary point P on rigid body B in the dynamic coordinate system $O_B^{-x_B,y_B,z_B}$, and $^g r_p$ position vector of point P in the global coordinate system $O_g^{-x_g,y_g,z_g}$.

The translation term of rigid body B is the displacement vector $^f r_{O_B}$, and the rotation term is the rotation attitude transformation matrix formed by rotating the moving coordinate system $O_B^{-x_B,y_B,z_B}$ to be parallel to the global coordinate system $O_g^{-x_g,y_g,z_g}$ according to a certain rotation order, which is set as $^g R$. According to the definition of vehicle coordinate system, the order of roll-pitch-yaw in the direction cosine method is selected to describe the transformation process of rigid body's rotation attitude.

Let the dynamic coordinate system $O_B^{-x_B,y_B,z_B}$ make three consecutive rotations around the x, y, and z axes of the global coordinate system. The first rotation angle $\alpha$ around the X axis is called the roll angle, and its direction cosine matrix is 9.

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix}$$  (1)

Among them:
The second rotation angle $\beta$ around the y-axis is called pitch angle, and its direction cosine matrix is:

$$R_2(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$ (3)

The third rotation angle $\gamma$ around the Z axis is called the yaw angle, and its direction cosine matrix is:

$$R_3(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (4)

After three rotations to be parallel to the global coordinate system $O_{g}x_{g}y_{g}z_{g}$, the transformation matrix of rotation attitude is as follows:

$$^{B}_{g}R = R_3(\gamma) R_2(\beta) R_1(\alpha)$$

$$= \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$ (5)

Therefore, according to the principle of homogeneous transformation, the position vector of any point $P$ on rigid body $B$ in the global coordinate system $O_{g}x_{g}y_{g}z_{g}$ is expressed as:

$$^g r_P = ^g r_{O_{a}} + ^B_{g}R^B r_P$$ (6)

By deriving the time $t$ from the above formula, the velocity vector of point $P$ in the global coordinate system can be obtained:

$$^g \dot{r}_P = ^g \dot{r}_{O_{a}} + ^g \phi_B \times ^B_{g}R^B r_P + ^B_{g}R^B \dot{r}_P$$ (7)

Where, $^g \phi_B$ is the angular velocity vector of rigid body $B$ in the global coordinate system $O_{g}x_{g}y_{g}z_{g}$, $\times$ representing the cross product of the vector. Since the rigid body does not have deformation, the last item $^B_{g}R^B \dot{r}_P$ is 0.

Therefore, for any rigid body in space, after determining the global coordinate system, the origin position of the dynamic coordinate system consolidated in the rigid body, the attitude angle of the dynamic coordinate system and the position vector of any point of the rigid body in the dynamic coordinate system, the spatial position and speed of any point can be described by homogeneous transformation. In this paper, this method is used to describe the position and pose of each rigid body, the position and speed of key hard points.
2.2. Establishment of dynamic model

For any mechanical system with constraints, according to the basic idea of Udwadia-Kalaba equation, the establishment of a complete system dynamics model can be divided into the following three steps:

(1) Firstly, the constraint relationship among the components of the system is released, and the components are regarded as free state. Under the condition of no constraint, the n-DOF dynamic equation of the system is established. If the system has i rigid bodies, then \( n = 6i \). When the generalized coordinate vector \( \mathbf{q} \in \mathbb{R}^n \) is used to represent the n-dimensional generalized coordinate, \( \dot{\mathbf{q}} \) is the generalized velocity and the \( \ddot{\mathbf{q}} \) is generalized acceleration of the system, the dynamic equation of the unconstrained system is as follows:

\[
M(\mathbf{q},t)\ddot{\mathbf{q}} = Q(\mathbf{\dot{q}},\mathbf{q},t)
\]

Where \( M(\mathbf{q},t) \in \mathbb{R}^{nxn} \) is the positive definite mass (inertia) matrix of the system, \( Q(\mathbf{\dot{q}},\mathbf{q},t) \in \mathbb{R}^n \) is the generalized active force of the unconstrained system, such as gravity, spring force and external force, and \( t \) is the time variable. This step can be realized by Lagrange method or Newton mechanics method.

(2) Secondly, considering the influence of constraints on the system, the constraint relationship among the degrees of freedom in the system is analyzed, and then the constraint equation of the system is established. Assuming that there are \( m \) constraints in the system, the constraint equations of the system are as follows:

\[
C_i(\mathbf{\dot{q}},\mathbf{q},t) = 0, i = 1, 2, \cdots m
\]

In order to facilitate the combination of equation (8), the second-order form of the system constraint is obtained by deriving the time \( t \) from the system constraint equation (9), and the coefficient matrix is extracted, which is expressed as:

\[
A(\mathbf{\dot{q}},\mathbf{q},t)\ddot{\mathbf{q}} = \mathbf{b}(\mathbf{\dot{q}},\mathbf{q},t)
\]

Where, \( A(\mathbf{\dot{q}},\mathbf{q},t) \in \mathbb{R}^{nxm} \) is the coefficient matrix of the second-order constraint of the system, and \( \mathbf{b}(\mathbf{\dot{q}},\mathbf{q},t) \in \mathbb{R}^m \) is the \( m \)-dimensional column vector.

(3) Finally, the system constraints are added to the unconstrained system, and the effect of constraints in the mechanical system is to produce certain constraints on the system, and the complete dynamic equation of the system including constraints is obtained.

\[
M(\mathbf{q},t)\ddot{\mathbf{q}} = Q(\mathbf{\dot{q}},\mathbf{q},t) + Q_c(\mathbf{\dot{q}},\mathbf{q},t)
\]

Where, \( Q_c(\mathbf{\dot{q}},\mathbf{q},t) \) is the system additional force, i.e. constraint force, including ideal constraint force and non-ideal constraint force. The Udwadia-kalaba equation extends the Lagrangian method, assuming that the system constraint force is \( \mathbf{c}(\mathbf{\dot{q}},\mathbf{q},t) \in \mathbb{R}^n \) and the constraint work is:

\[
W = \mathbf{v}^T \mathbf{c}(\mathbf{\dot{q}},\mathbf{q},t) \in \mathbb{R}^n
\]

Where, \( \mathbf{v}^T \in \mathbb{R}^n \) is the generalized virtual displacement. The ideal constraint force is:

\[
\mathbf{v}^T \mathbf{Q}_c = 0
\]

The work of non-ideal constraint force is:

\[
\mathbf{v}^T \mathbf{Q}_{nic} \neq 0
\]
According to the derivation process of constraint force in Udwadia-kalaba equation, the ideal constraint force $Q_{ic}$ in the system is expressed as:

$$Q_{ic}(q,q,t) = M^{1/2}D^+(b - AM^{-1}Q)$$  \hspace{1cm} (15)$$

Among them $D = AM^{1/2}$, non-ideal constraint force $Q_{nic}$ is expressed as:

$$Q_{nic}(q,q,t) = M^{1/2}(I - D^+D)M^{-1/2}c$$ \hspace{1cm} (16)$$

+ represents the generalized inverse, $I$ is the unit matrix, and vector $c$ is the compensation factor. This equation can be derived from Gauss theorem and Darlambert principle. Therefore, the detailed expression of formula (11) is:

$$M(q,t)\ddot{q} = Q(q,q,t) + M^{1/2}D^+(b - AM^{-1}Q) + M^{1/2}(I - D^+D)M^{-1/2}c$$ \hspace{1cm} (17)$$

When $c = 0$, the above formula is the complete dynamic equation of the ideal constrained system.

According to the above mathematical derivation process, the dynamic model calculation program of nuclear waste crane is compiled in MATLAB, and the model is solved.

Figure 2. Udwadia-kalaba dynamics model of crane based on MATLAB

3. Simulation results
According to the motion requirements of the crane, and considering the influence of the three motions of the installation platform on the crane, the driving equation of each moving part is obtained, as shown in table 1. In this case, the dynamic simulation of the crane is carried out, and the binding force between each moving pair is analyzed.

| Motion equation of each motion pair          | Equation                                      |
|---------------------------------------------|-----------------------------------------------|
| Translational equation in X direction       | (-0.15/2.75*cos(2.75*t-5.5)+0.15*t)*1000      |
| Translational equation in Y direction       | (-0.15/4.82*cos(4.82*t-9.64)+0.15*t)*1000    |
| Translational equation in Z direction       | (-0.15/6.2*cos(6.2*t-12.4)+0.15*t)*1000      |
| Equation of roll motion (around X-axis)     | 5/180*pi*sin(pi/4*t)                           |
| Equation of pitch motion (around Y-axis)    | 4/180*pi*sin(pi/4*t)                           |
| Equation of global vertical motion          | -2500*sin(pi/4*t)                              |

Some of the results are as follows:

Input the mass center of each component and the motion equation of the whole machine, the simulation time is 3S, and the simulation results are shown in the figure 3.
a) X-direction force of frame point 2

b) Y-direction force of frame point 2

c) Z force of frame point 2

d) X-direction force of point 2 of X-direction transfer device
e) Y-direction force of point 2 of X-direction transfer device

f) Z-direction force of point 2 of X-direction transfer device

g) X-direction force of point 3 of X-direction transfer device

h) Y-direction force of point 3 of X-direction transfer device
i) Z-direction force of point 3 of X-direction transfer device

j) X-direction force of point 2 of Y-direction transfer device

k) Y-direction force of point 2 of Y-direction transfer device

l) Z-direction force of point 2 of Y-direction transfer device
m) X-direction force of point 3 of Y-direction transfer device

n) Y-direction force of point 3 of Y-direction transfer device

o) Z-force of point 3 of Y-direction transfer device

p) X-direction force of point 2 of Z-direction transfer device
4. Conclusion
In this paper, the multi-body dynamic model of nuclear power waste crane is established by using the Udwadia-kalaba equation, and its simulation analysis is carried out. The dynamic load between the key moving pairs in the crane working process is obtained, which verifies the feasibility of this method in the simulation analysis of heavy machinery products.

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