Implementing Liquid State Machines on Adiabatic Quantum Computers

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February 1, 2008

Abstract
This paper outlines the theory of Liquid State Machines, and describes how they may be used to filter the Hamiltonian space of a qubit network. The underlying principle of this kind of quantum computer is that when enough of the nodes in a filter activate, a desired output will be generated. A liquid state machine implemented on an adiabatic quantum computer is capable of solving NP and Sharp-P problems in BQP time.

1 Introduction
The paper by Wolfgang Maass and Henry Markram (2002) showed that a wide variety of computations can be accomplished using a Liquid State Machine (LSM). This paper explores the problem of implementing a LSM on an adiabatic quantum computer. In this context, the purpose of the LSM is to associate an input signal with a particular Hamiltonian space. This paper attempts to model waves in the neuronal network by modifying the propagation of signals through the network. The LSM network can be used to solve NP and #P problems in BQP time.

2 Description of the Quantum Liquid State Machine

2.1 Overview and Background
A Hamiltonian space is represented in the network by the state of the system. The system itself is an adiabatic quantum computer, and the rules of the liquid state machine depend on the network. The network nodes are represented by qubits in this space. Adjacent nodes influence each other directly, but even distant pairs can influence each other by creating a wave that propagates through the medium of the space (i.e., the network). The purpose of the network is to find a set of filters that associates a desired Hamiltonian space (the output) with the input signal. This is accomplished by training the network on known or model data and updating the parameters controlling signal propagation between adjacent nodes.
2.2 Formal Description of a Quantum Liquid State Machine

From now on, the term ‘network’ will be used to mean a specific set of rules existing in the adiabatic computer. In a quantum system, these rules define a one-parameter family of Hamiltonians.

**Definition 1**

\[ H(t) = \tilde{H}(t/T) \]  \quad (2.1)

\[ H(s)|\ell; s\rangle = E_\ell(s)|\ell; s\rangle \]  \quad (2.2)

\[ E_0(s) \leq E_1(s) \leq \cdots \leq E_{N-1}(s) \]  \quad (2.3)

\[ |\psi(0)\rangle = |\ell = 0; s = 0\rangle . \]  \quad (2.4)

\[ \lim_{T \to \infty} |\langle \ell = 0; s = 1 | \psi(T) \rangle| = 1 . \]  \quad (2.5)

The adiabatic computer is set up as a quantum Liquid State Machine with fading memory and input separability. The LSM can be defined by initially connecting the qubits to each other in a random manner. Each *qubit node* receives an independent input signal, which may vary over time. (A qubit node is defined as a circuit whose state is stored in a qubit.) This type of LSM is a universal machine, so it can approximate any function or operation.

The liquid state machine evolves in accordance with the adiabatic theorem [3]:

**Theorem 1**

\[ H(t) = (1 - t/T)H_B + (t/T)H_P \]  \quad (2.6)

Thus, from (2.7) we have

\[ \tilde{H}(s) = (1 - s)H_B + sH_P \]  \quad (2.7)

\[ H_C(t) = (1 - t/T)H_{B,C} + (t/T)H_{P,C} \]  \quad (2.8)

and accordingly

\[ \tilde{H}_C(s) = (1 - s)H_{B,C} + sH_{P,C} . \]  \quad (2.9)

Then we have

\[ H(t) = \sum_C H_C(t) \]  \quad (2.10)

and

\[ \tilde{H}(s) = \sum_C \tilde{H}_C(s) . \]  \quad (2.11)

The Hamiltonian space is approximated according to the usual principles of liquid state machines.

For any \( m \in N \), any compact set \( X \subseteq \mathbb{R}^m \), any continuous function \( h : \mathbb{R} \to \mathbb{R} \), and any constant \( \rho > 0 \), there exists some \( f \) in \( \mathcal{F} \) such that \( |h(x) - f(x)| \leq \rho \) for all \( x \in X \) [4].

It has been proven in the paper just cited that under certain conditions on their basis filters \( \mathcal{B} \) and the class of possible readout functions \( \mathcal{F} \), liquid state machines have *universal computational power*. These conditions are described below.
The basis filters $\mathcal{B}$ must have the property of pointwise separation: for any two input functions $u(\cdot), v(\cdot) \in U^n$, where $u(s) \neq v(s)$ for some $s \leq 0$, there exists some filter $B \in \mathcal{B}$ that separates $u(\cdot)$ and $v(\cdot)$. Furthermore, for any $u : R \rightarrow \mathbb{R}$ one can find an operator $\psi = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$ in $\mathcal{H}$ which gives the desired output.

The domain of input functions $U$ has the following restrictions:

$$U = \{ u : \mathbb{R} \rightarrow [-K, K]; |u(s) - u(t)| \leq K' \cdot |s - t| \forall s, t \in \mathbb{R} \}$$

For some arbitrary constants $K$ and $K'$.

The qubit circuits must also have the property of fading memory. Thus, we create suitable filters defined as follows:

**Definition 2** Fix some space $U$ of input functions $u : \mathbb{R} \rightarrow \mathbb{R}$, and let $Q$ be a qubit circuit that assigns some output $Q(x_I, u[t, t])$ to any initial state $x_I \in I$, any $t \in \mathbb{R}$, and any finite segment $u[t, t]$ of a Hamiltonian space $u(\cdot) \in U^n$. We say that $Q$ has fading memory (relative to $U$ and $I$) if there exists for every $t \in \mathbb{R}$ and every $\varepsilon > 0$ some $\delta > 0$ and $t_I < t$ such that $|C(x_I, u[t_I, t]) - C(x_{I'}, u[t_I, t'])| < \varepsilon$ for all $u, v \in U^n$ and for all $x_I, x_{I'} \in U^n$, provided that $\|u(t') - v(t')\| \leq \delta$ for all $t' \in [t_I, t]$.

**Theorem 2** Any qubit circuit $C$ that has fading memory (relative to $U$ and $I$) defines a fading memory filter $F_c : U^n \rightarrow \mathbb{R}^\mathbb{R}$ through the definition

$$(F_c u)(t) := \lim_{t_I \rightarrow -\infty} C(x_I, u[t_I, t]),$$

where any initial state $x_I$ may be used on the r.h.s.

2.3 Unsupervised learning

The first unsupervised learning technique to use on a quantum wave network is that of applying the Hebbian postulate.

Let us assume that the persistence or repetition of a cyclical activity (or "trace") tends to induce lasting cellular changes that add to its stability. When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A’s efficiency, as one of the cells firing B, is increased.

This learning rule can be implemented as follows: if $a$ and $b$ commonly co-occur in the context $x$, then the presence of either $a$ or $b$ in the context $x$ will tend to increase the expectation that $b$ or $a$, respectively, will be present as well. This
is implemented using an Adaptive Resonance Theory network (ART), which can identify patterns/contexts in sub-populations of the input stream. These autonomously discovered patterns promote Hebbian reinforcement of the input layer (see Fig 1). All positive connections are forged bidirectionally, while negative connections are taken to emanate from the context nodes only. [?]  

2.4 Solving NP and Sharp-P Problems

According to the algorithm outlined in [1], one can implement NP and Sharp-P problems by using filters to achieve the desired results. Since the recurrent network is a nonlinear network, it can perform the operations described below.

Step 1. Give each node a $\pi/2$ rotation.

$$\psi = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i, 0\rangle$$  \hspace{1cm} (2.12)

Step 2. Store the input signal as data in the qubit nodes. The data must have the pairwise disjoint property:

$$\psi = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i, f(i)\rangle$$  \hspace{1cm} (2.13)

Step 3. The first qubit node is considered separately. All other qubit states are grouped into superposition pairs based on the values represented by their respective qubit nodes.

If all qubits are considered simultaneously there are $2^n$ possible configurations of the space. Now consider a subspace consisting of a two-qubit superposition state, where one qubit is a node in the LSM and the other is a flag qubit (used to define the output state of the network). There are $2^{n_1}$ possible configurations of the remaining qubits in this case. Each such subspace will be in one of the states listed below. We write the value of the first qubit followed by the value of the flag qubit, ignoring normalization constants. The result of applying a filter to the liquid state machine for this pair is as follows:

(a) $|00\rangle + |11\rangle$
(b) $|01\rangle + |10\rangle$
(c) $|00\rangle + |10\rangle$

[1]

(a) $|00\rangle + |11\rangle \rightarrow |01\rangle + |11\rangle$  \hspace{1cm} (b) $|01\rangle + |10\rangle \rightarrow |01\rangle + |11\rangle$
(c) $|00\rangle + |10\rangle \rightarrow |00\rangle + |10\rangle$

This transformation is stored as a filter in the liquid state machine.
Step 4. Repeat step 3 for all $n$ qubit nodes, defining a new filter for each node. After each iteration, the number of components in the superposition state with $|1\rangle$ for the flag qubit doubles. After iteration $n$ is complete, however, the flag qubit is no longer entangled with the first $n$ qubits. Either it is in the state $|1\rangle$ for every component of the superposition, or it is in the state $|0\rangle$ for every component of the superposition.

Step 5. Measure the flag qubit to determine the solution.

The algorithm can be altered slightly to solve $\#P$ problems, by redefining the flag qubit as a set of $\log_2 n$ qubit nodes and summing their solution values.

2.5 Implementation

A liquid state machine can be constructed using a classical computer as the control node, as described in Kaminsky et al. (year). The network of qubits would be slaved to the control node. All the qubit gates would be connected to each other in the manner of a Liquid State Machine, with fading memory and input separability. The Liquid State Machine is then used to approximate the desired Hamiltonian. The qubits are organized randomly by design, because the liquid state network will define a higher-level architecture gradually during the training process. A liquid state network has the ability to create and sustain multiple filters simultaneously [4]. We can monitor the adiabatic evolution of the network using the method outlined in [2].

3 Conclusion

This form of adiabatic computer has no size limitations, because more nodes can always be added to the network. Thus, it is perfectly scaleable. Since the computer uses nonlinear operations, it is also capable of solving Sharp-P problems in BQP time. [1].

References

[1] Daniel S. Abrams and Seth Lloyd. Nonlinear quantum mechanics implies polynomial-time solution for np-complete and $\#p$ problems. Physical Review Letters, 81:3992, 1998.

[2] M. Grajcar, A. Izmalkov, and E. Il’ichev. Possible implementation of adiabatic quantum algorithm with superconducting flux qubits. Physical Review B, 71:144501, 2005.

[3] William M. Kaminsky, Seth Lloyd, and Terry P. Orlando. Scalable superconducting architecture for adiabatic quantum computation, 2004.

[4] Wolfgang Maass and Henry Markram. On the computational power of recurrent circuits of spiking neurons. Electronic Colloquium on Computational Complexity Report No. 22, 2002.