Simultaneous readout of two charge qubits

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We consider a system of two solid state charge qubits, coupled to a single read-out device, consisting of a single-electron transistor (SET). The conductance of each tunnel junction is influenced by its neighboring qubit, and thus the current through the transistor is determined by the qubits’ state. The full counting statistics of the electrons passing the transistor is calculated, and we discuss qubit dephasing, as well as the quantum efficiency of the readout. The current measurement is then compared to readout using real-time detection of the SET island’s charge state. For the latter method we show that the quantum efficiency is always unity. Comparing the two methods a simple geometrical interpretation of the quantum efficiency of the current measurement appears. Finally, we note that full quantum efficiency in some cases can be achieved measuring the average charge of the SET island, in addition to the average current.

I. INTRODUCTION

A solid state charge qubit, formed by a single electron trapped in a double quantum dot (DQD), is an interesting system for studying quantum coherence in the solid state.$^{1,2,3,4}$

The state of charge qubits can be read out by sensitive electrometers, such as the radio-frequency single-electron transistor (RF-SET)$^5$. The straightforward scheme is to couple the charge qubit capacitatively to the small SET island.$^6$ The SET acts as a variable resistor, dependent on the qubit state. The SET is included in an LC-tank circuit, and the qubit state can be detected by measuring the tank circuits dissipation. Theoretical estimates for the back-action on the qubit indicate that single-shot qubit readout is possible.$^7,8$

In principle the RF-SET can also be used to detect the charge of a QD.$^9,10$ But a conceptually even simpler method is to place the QD close to a quantum point contact (QPC)$^{11}$. The charge of the QD will influence the transparency of the QPC, and by applying a driving voltage across the QPC, the charge can be determined through a current measurement. This method has been successfully used to read out the charge of a DQD.$^{12,13}$

Starting with Ref.$^{14}$ there are numerous theoretical works on the measurement induced dephasing rate $\Gamma_\varphi$ of a charge qubit read out by a QPC, see e.g. Ref.$^{15}$ and references therein. The dephasing rate can be compared with the measurement time $t_{ms}$, and from fundamental principles of quantum measurement$^{16}$, the quantum efficiency $\eta = (t_{ms} \Gamma_\varphi)^{-1}$ has an upper bound of unity, implying that one cannot distinguish two states without destroying the quantum coherence between them.

An interesting combination of the SET and QPC measurement techniques was proposed by Tanamoto and Hu.$^{17}$, and is also the focus of our present paper. The setup allows for reading out two charge qubits using a single SET. The two qubits are positioned close to the SET source and drain junction respectively (see Fig.$^1$). Thus the state of the left/right qubit will influence the conductance of the left/right tunnel junction. By applying a source-drain voltage and measuring the current, both qubits can be read out. In Ref.$^{17}$ the focus was made on the transient dynamics of qubit-detector system, as well as the ensemble averaged current.

In this paper we concentrate on the single-shot readout properties, by calculating the measurement times from the noise and full counting statistics.$^{18}$ of the charge transport. The quantum efficiency is obtained by comparing with the measurement induced dephasing rates. From the quantum inefficiency, in detecting certain qubit states, we conclude that the qubits get entangled with degrees of freedom not measured in current measurements. This leads to an analysis of a measurement setup, where instead of the current, the charge of the SET island is detected, in real-time. We find that this measurement always give full quantum efficiency.

The outline of the paper is as follows. In Secs.$^{II}$ and $^{III}$we derive a master equation for the dynamics of the density matrix of the detector-qubit system, using an open quantum systems approach.$^{19}$ In Sec.$^{IV}$ we use this formulation to obtain the single-shot measurement time, as well as the dephasing rates, and consequently also the quantum efficiency. We then apply the general results to a few analytically tractable cases. In Sec.$^{V}$ the analysis is extended to the case when the charge of the SET island is monitored. The properties of that setup is compared with the current measurements in Sec.$^{VI}$ leaving the conclusion and discussion for Sec.$^{VII}$.

II. THE MODEL

We consider two double-dot charge qubits, read out by a quantum dot single-electron transistor. The SET island contains one spin-degenerate level with energy $E_d$, and the Coulomb interaction gives rise to an additional energy $U$ when it is doubly occupied. Each qubit is electrostatically coupled to one of the SET’s quantum point contacts, as shown in Figure $1$. There is also an electrostatic interaction between the qubits, causing a direct qubit-qubit coupling with strength $J$. The two qubits...
have a Hamiltonian of the following form

\[ H_{qb} = \sum_{\alpha=L,R} \left( \Omega_\alpha \sigma_{\alpha x} + \Delta_\alpha \sigma_{\alpha z} \right) + J \sigma_{Lz} \sigma_{Rz}, \]

(1)

where \( \sigma_{\alpha x} \) and \( \sigma_{\alpha z} \) are Pauli matrices (\( \alpha \in \{L, R\} \)). \( \Omega_\alpha \) denotes the tunnel coupling strength between the two dots of each qubit, and \( \Delta_\alpha \) the charging energy difference within each qubit. We can also express the Pauli matrices in terms of the creation and destruction operators of the extra electron in the qubit quantum dots,

\[ \sigma_{\alpha x} = a_\alpha^\dagger b_\alpha + b_\alpha^\dagger a_\alpha, \quad \sigma_{\alpha z} = a_\alpha^\dagger a_\alpha - b_\alpha^\dagger b_\alpha, \]

(2)

where \( a_\alpha^\dagger \) and \( a_\alpha \) denote the creation and destruction operators in the upper dot of the \( \alpha \in \{L, R\} \) qubit, and \( b_\alpha^\dagger \) and \( b_\alpha \) operates on the lower dot.

The SET Hamiltonian can be split into three pieces: \( H_{SET} = H_{res} + H_{is} + H_T. \) \( H_{res} \) describes the left and right reservoirs

\[ H_{res} = \sum_{\alpha=L,R} \sum_{s=\uparrow,\downarrow} \sum_k E_{k\alpha} c_{k\alpha}^\dagger c_{k\alpha}, \]

(3)

where \( E_{k\alpha} \) is the electron energy in the \( \alpha \) reservoir at wave vector \( k \), and \( c_{k\alpha}^\dagger \) and \( c_{k\alpha} \) are corresponding electron creation and destruction operators. For the SET island, we have,

\[ H_{is} = \sum_{s=\uparrow,\downarrow} E_d d_s^\dagger d_s + U d_s^\dagger d_s, \]

(4)

where \( d_s^\dagger \) and \( d_s \) are creation and destruction operators of the electrons on the island. We consider the transport through the QPCs to be in the low transparency regime, described by a tunnel Hamiltonian,

\[ H_T = \sum_{\alpha=L,R; s=\uparrow,\downarrow} \sum_k V_{k\alpha} \left( c_{k\alpha s}^\dagger d_s e^{i\chi_\alpha} + d_s^\dagger c_{k\alpha s} e^{-i\chi_\alpha} \right), \]

(5)

where we assume that the tunnelling strengths \( V_{k\alpha} \) of electrons between the reservoirs and the Coulomb island are independent of the spin degree of freedom, and the spin \( s \) is conserved during the tunnelling processes. The counting fields \( \chi_\alpha \) for \( \alpha \in \{L, R\} \) are introduced to keep track of the number of electrons \( m_{L/R} \) which passed through the \( L/R \) junction from the SET island to the \( L/R \) lead. The operators \( e^{\pm i\chi_\alpha} \) changes this quantum number by one, \( e^{\pm i\chi_\alpha} |m_\alpha \rangle = |m_\alpha \pm 1 \rangle \).

The qubits electrostatically interact with the QPCs, and influence the tunnelling rates according to the following Hamiltonian,

\[ H_T^{qb} = \sum_{\alpha,s,k} \delta V_{k\alpha} \left( c_{k\alpha s}^\dagger d_s e^{i\chi_\alpha} + d_s^\dagger c_{k\alpha s} e^{-i\chi_\alpha} \right) \sigma_{\alpha z}, \]

(6)

where \( \delta V_{k\alpha} \) are coupling strengths between the QPCs and the qubits. Depending on the qubits’ states, the tunnelling strengths of the left and right QPCs change from \( V_{k\alpha} \) to \( V_{k\alpha} \pm \delta V_{k\alpha} \). For simplicity, we neglect the relative phase between these tunnelling strengths, but we still allow for a weak energy dependence of their magnitudes.

\[ \text{III. A KINETIC EQUATION} \]

One conventional approach to continuous quantum measurement problems is called the open system approach[2]. In general terms, an open system is a limited quantum system coupled to another quantum system with a large number of degrees of freedom, called the environment. In our case, the reservoirs (leads) connected to the left and right QPCs constitute an environment for the limited quantum system consisting of the two qubits and the SET island. The system hamiltonian is \( H_S = H_{qb} + H_{is} \), while the hamiltonian of the environment is \( H_{res} \) and the interaction hamiltonian is given by the tunnelling terms \( H_{int} = H_T + H_T^{qb} \).
The starting point is the Liouville equation on integro-differential form
\[
\partial_t \tilde{\rho}_{\text{tot}}(t) = -i[\tilde{H}_{\text{int}}(t), \rho_{\text{tot}}(0)] - \int_0^t dt' \left[ [\tilde{H}_{\text{int}}(t'), \rho_{\text{tot}}(t')] \right],
\]
where \( \tilde{H}_{\text{int}}(t) \) is the interaction hamiltonian and \( \tilde{\rho}_{\text{tot}}(t) \) the density matrix in the interaction picture. Then we proceed to a markoffian kinetic equation for the reduced density matrix of the system \( \tilde{\rho}(t) \), making the Born and Markov approximations,
\[
\dot{\tilde{\rho}}(t) = -\int_0^t dt' \text{Tr}_{\text{res}} \left\{ [\tilde{H}_{\text{int}}(t'), \tilde{\rho}(t')] \right\},
\]
where \( \text{Tr}_{\text{res}} \) indicates taking the trace over the reservoir degrees of freedom, which are taken to be in thermal equilibrium, at their respective chemical potential. The integrals on the right-hand side of Eq. (8) can be approximately evaluated, taking the lower limit of the integral to minus infinity. They have in general both real and imaginary parts, where the imaginary parts effectively can be absorbed into renormalized system energies. The real parts give the dissipative dynamics, i.e. the electron tunnelling rates. A typical example of a term contributing to the tunnelling rate from lead \( \alpha \) to the island is
\[
\int_{0}^{\infty} d\tau \sum_{k,s} e^{\pm i (E_{ks} - E_d) \tau} V_{k\alpha}^2 f_{\alpha}(E, T) \rightarrow \pi g_{\alpha}(E_d) V_{\alpha}^2(E_d) f_{\alpha}(E_d, T),
\]
where we have taken the sum to an integral by introducing the density of states in the leads \( g_{\alpha}(E) \), and the energy dependent tunnelling amplitude \( V_{\alpha}(E) \), and \( f_{\alpha}(E, T) \) is the Fermi function of lead \( \alpha \). The bias voltage between the left and right lead enters through the shifted Fermi distributions. For details, please see Ref. 20.

A. The non-zero tunnelling rates

Since our quantum system has 16 degrees of freedom the reduced density matrix has 256 elements, and thus the kinetic equation in general 256 \times 256 terms, we have to make some simplifying assumptions to arrive at a tractable set of equations.

First we consider a source-drain voltage across the SET large enough compared to temperature so that we may neglect backwards tunnelling. Thus, without the coupling to the qubits, and with spin-independent tunnelling, there are four different rates to consider; tunnelling onto the unoccupied SET island from the left lead (\( \Gamma_L \)), tunnelling onto the singly occupied SET island from the left lead (\( \Gamma'_L \)), tunnelling off the singly occupied SET island to the right lead (\( \Gamma_R \)), and finally tunnelling off the doubly occupied SET island to the right lead (\( \Gamma'_R \)), see Fig. 2.

FIG. 2: The four non-zero tunneling rates for electrons, on and off the SET island. The applied source-drain voltage \( eV \) creates a difference in chemical potentials between the left and right lead \( eV = \mu_L - \mu_R \).

Including the coupling to the qubits each rate will acquire a state-dependent shift \( \Gamma_{\alpha}^{\pm} = \Gamma_{\alpha} \pm \delta \Gamma_{\alpha} \) and \( \Gamma'_{\alpha}^{\pm} = \Gamma'_{\alpha} \pm \delta \Gamma'_{\alpha} \), see Fig. 3. Not to complicate the system further we assume that qubit energies \( \Delta_{\alpha}, \Omega_{\alpha}, \) and \( J \) are small enough so that the tunnelling rates including a flip of the qubits’s state are identical to the ones without flipping the qubit state.

We are now ready to present the kinetic equation for the elements of the reduced density matrix. As basis we chose the four qubit product states \( |A\rangle = |\downarrow\downarrow\rangle, |B\rangle = |\downarrow\uparrow\rangle, |C\rangle = |\uparrow\downarrow\rangle, |D\rangle = |\uparrow\uparrow\rangle \), the SET island states corresponding to
no electron \( |a\rangle \), one spin-up/down electron \( |b\rangle \) and \( |c\rangle \) and doubly occupied \( |\rangle \), and the number of electrons which has tunnelled across the right junction \( |m\rangle \):

\[
\rho^{u}_{z_{1}z_{2}}(m) = \langle m | \otimes \langle u | \otimes \langle z_{1} | \rho | z_{2} \rangle \otimes | u \rangle \otimes | m \rangle ,
\]

where \( z_{1}, z_{2} \in \{ A, B, C, D \} \) and \( u \in \{ a, b_{1}, b_{j}, c \} \). Here we only keep terms off-diagonal in the qubit states. It is easy to verify that terms off-diagonal in the island state do not couple to the diagonal terms, and they decay on a time-scale given by \( \Gamma_{L/R}^{-1} \). Because the kinetic equation is translationally invariant in \( m \)-space, we Fourier transform it with respect to this variable \( \rho^{u}_{z_{1}z_{2}}(k) = \sum_{m} e^{ikm} \rho^{u}_{z_{1}z_{2}}(m) \), and arrive at the following kinetic equation for the elements of the reduced density matrix

\[
\dot{\rho}_{z_{1}z_{2}}^{a} = [i(J_{z_{2}} - J_{z_{1}}) - (\Gamma_{L}^{z_{1}} + \Gamma_{L}^{z_{2}})]\rho_{z_{1}z_{2}}^{a} - i\Omega R \left[ \rho_{g_{r}(z_{1}),z_{2}}^{a} - \rho_{z_{1}g_{r}(z_{2})}^{a} \right] - i\Omega L \left[ \rho_{g_{l}(z_{1}),z_{2}}^{a} - \rho_{z_{1}g_{l}(z_{2})}^{a} \right] + \sqrt{\Gamma_{R}^{z_{1}} \Gamma_{R}^{z_{2}}} e^{ik} \left( \rho_{z_{1}z_{2}}^{b_{1}} + \rho_{z_{1}z_{2}}^{b_{2}} \right),
\]

\[
\dot{\rho}_{z_{1}z_{2}}^{b_{1}} = \left[ i(J_{z_{2}} - J_{z_{1}}) - \frac{\Gamma_{L}^{z_{1}} + \Gamma_{L}^{z_{2}} + \Gamma_{R}^{z_{1}} + \Gamma_{R}^{z_{2}}}{2} \right] \rho_{z_{1}z_{2}}^{b_{1}} - i\Omega R \left[ \rho_{g_{r}(z_{1}),z_{2}}^{b_{1}} - \rho_{z_{1}g_{r}(z_{2})}^{b_{1}} \right] - i\Omega L \left[ \rho_{g_{l}(z_{1}),z_{2}}^{b_{1}} - \rho_{z_{1}g_{l}(z_{2})}^{b_{1}} \right] + \sqrt{\Gamma_{R}^{z_{1}} \Gamma_{R}^{z_{2}}} e^{ik} \rho_{z_{1}z_{2}}^{c},
\]

\[
\dot{\rho}_{z_{1}z_{2}}^{b_{2}} = \left[ i(J_{z_{2}} - J_{z_{1}}) - \frac{\Gamma_{L}^{z_{1}} + \Gamma_{L}^{z_{2}} + \Gamma_{R}^{z_{1}} + \Gamma_{R}^{z_{2}}}{2} \right] \rho_{z_{1}z_{2}}^{b_{2}} - i\Omega R \left[ \rho_{g_{r}(z_{1}),z_{2}}^{b_{2}} - \rho_{z_{1}g_{r}(z_{2})}^{b_{2}} \right] - i\Omega L \left[ \rho_{g_{l}(z_{1}),z_{2}}^{b_{2}} - \rho_{z_{1}g_{l}(z_{2})}^{b_{2}} \right] + \sqrt{\Gamma_{R}^{z_{1}} \Gamma_{R}^{z_{2}}} e^{ik} \rho_{z_{1}z_{2}}^{c},
\]

\[
\dot{\rho}_{z_{1}z_{2}}^{c} = [i(J_{z_{2}} - J_{z_{1}}) - (\Gamma_{R}^{z_{1}} + \Gamma_{R}^{z_{2}})]\rho_{z_{1}z_{2}}^{c} - i\Omega R \left[ \rho_{g_{r}(z_{1}),z_{2}}^{c} - \rho_{z_{1}g_{r}(z_{2})}^{c} \right] - i\Omega L \left[ \rho_{g_{l}(z_{1}),z_{2}}^{c} - \rho_{z_{1}g_{l}(z_{2})}^{c} \right] + \sqrt{\Gamma_{L}^{z_{1}} \Gamma_{L}^{z_{2}}} e^{ik} \left( \rho_{z_{1}z_{2}}^{b_{1}} + \rho_{z_{1}z_{2}}^{b_{2}} \right),
\]

where

\[
\begin{align*}
\Gamma^{A}_{L} = \Gamma^{B}_{L} = \Gamma^{c}_{L}, & \quad \Gamma^{A}_{R} = \Gamma^{D}_{R} = \Gamma^{c}_{R}, & \quad \Gamma^{D}_{R} = \Gamma^{D}_{R} = \Gamma^{c}_{R}, \\
\Gamma^{A}_{L} = \Gamma^{B}_{L} = \Gamma^{c}_{L}, & \quad \Gamma^{C}_{L} = \Gamma^{D}_{L} = \Gamma^{c}_{L}, & \quad \Gamma^{C}_{R} = \Gamma^{D}_{R} = \Gamma^{c}_{R}, & \quad \Gamma^{B}_{R} = \Gamma^{D}_{R} = \Gamma^{c}_{R}, \\
J_{A} = -\Delta_{L} - \Delta_{R} + J, & \quad J_{B} = -\Delta_{L} + \Delta_{R} - J, & \quad J_{C} = \Delta_{L} - \Delta_{R} - J, & \quad J_{D} = \Delta_{L} + \Delta_{R} + J,
\end{align*}
\]

and

\[
g_{1}(A) = C, \quad g_{1}(B) = D, \quad g_{3}(C) = A, \quad g_{3}(D) = B, \quad g_{r}(A) = B, \quad g_{r}(B) = A, \quad g_{r}(C) = D, \quad g_{r}(D) = C.
\]

We note that, neglecting the counting field \( (k \rightarrow 0) \), the above equations coincide with the rate equations derived by Tanamoto and Hu (see Appendix in Ref. [17], using a different approach). Eqs. (11) to (14) form one of the main
results of this paper, and they constitute the basis of our further analysis. One may note that although the rates $\Gamma_{L/R}'$ and $\Delta \Gamma_{L/R}'$ are intimately connected to the microscopic tunneling Hamiltonians in Eqs. (5) and (6), we may now consider different operating regimes in terms of the value of these rates themselves, rather than specifying the coefficients of the tunnel Hamiltonians.

B. Numerical methods

To solve for the time-dependence of the $k$-dependent reduced density matrix, we rewrite the kinetic equations on matrix form $\dot{\tilde{\rho}} = M \cdot \tilde{\rho}$ as follows

$$\begin{pmatrix}
\dot{\rho}_{AA}^k \\
\dot{\rho}_{AB}^k \\
\vdots \\
\dot{\rho}_{DD}^k
\end{pmatrix}
= \begin{pmatrix}
-\frac{2\Gamma^L}{i\Omega} & i\Omega & \cdots & 0 \\
\frac{2\Gamma^R}{i\Omega} & -2i(\Delta R - J) - 2\Gamma^R & \cdots & 0 \\
0 & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & -2\Gamma^R
\end{pmatrix}_{64 \times 64}
\begin{pmatrix}
\rho_{AA}^k \\
\rho_{AB}^k \\
\vdots \\
\rho_{DD}^k
\end{pmatrix}_{64}.
$$

(17)

Thus the reduced density matrix elements at arbitrary time $t$ are given by the evolution matrix $M$

$$\tilde{\rho}(t) = e^{Mt} \tilde{\rho}(0).$$

(18)

By considering the time-derivative of the average number of electrons which have tunnelled across the right junction $\langle \dot{m} \rangle = Tr \{ m | \tilde{\rho} \} = Tr \{ -i \partial_t \tilde{\rho} (k=0) \}$ we get the expression for the time-dependent average current

$$\langle I_R(t) \rangle = e \sum_z \left\{ \Gamma^R_z \left[ \rho_{zz}^b(t) + \rho_{zz}^c(t) \right] + 2\Gamma^R_z \rho_{zz}^c(t) \right\},$$

(19)

which also agrees with the one used in Ref. [17]. To discuss the single-shot measurement time, needed to separate different qubit states, we will also need the time-dependent probability distribution of the number of electrons transferred through the detector $P(m, t)$. This we obtain by tracing out the qubits’ and island’s degrees of freedom,

$$P(m, t) = \sum_{z, u} \rho(z, z; u, u; m, m; t),$$

(20)

which is straightforward using the Fourier transformed $k$-dependent density matrix,

$$P(m, t) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{-ikm} P(k, t) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{-ikm} \sum_{z, u} \rho_{zz}^u(k, t).$$

(21)

We are now ready to discuss the properties of reading out two charge qubits using a single-electron transistor, in a few different parameter regimes.

IV. READ-OUT IN THE QUBITS’ EIGENBASIS

To approach a quantum non-demolition (QND) measurement for a weakly coupled read-out device, it is necessary to measure in the qubits’ eigenbasis. For the setup considered here, the measurement is performed in the $z$-direction of each qubit, i.e. the measurement determines in which dot the electron of each qubit is located. Thus we consider the case when the inter-dot coupling is switched off, i.e. $\Delta L_z = \Delta R_z = 0$. We then start from Eqs. (14,15) and derive simplified kinetic equations. Due to the spin degeneracy of the tunneling, we may treat the two singly occupied island states together by defining $\rho_{z_1z_2}^k = \rho_{z_1z_2}^b + \rho_{z_1z_2}^c$. Also, we note that the terms proportional to $i(J_{z_2} - J_{z_1})$ vanish by going to the rotating frame, with respect to these coherent $z$-rotations, i.e. $\tilde{\rho}_{z_1z_2}(t) = e^{-i(J_{z_2} - J_{z_1})t} \rho_{z_1z_2}(t)$. We arrive at the following equations for the density matrix in the rotating frame $\tilde{\rho}(t)$,

$$\begin{align*}
\dot{\rho}_{z_1z_2}^a &= -(\Gamma^L_{z_1} + \Gamma^R_{z_1}) \rho_{z_1z_2}^a + \sqrt{\Gamma^L_{z_1} \Gamma^R_{z_1}} \rho_{z_1z_2}^b, \\
\dot{\rho}_{z_1z_2}^b &= -\frac{\Gamma^L_{z_1} + \Gamma^R_{z_1} + \Gamma^L_{z_2} + \Gamma^R_{z_2}}{2} \rho_{z_1z_2}^b + 2\sqrt{\Gamma^L_{z_1} \Gamma^R_{z_1} \Gamma^L_{z_2} \Gamma^R_{z_2}} \rho_{z_1z_2}^c + 2 \sqrt{\Gamma^L_{z_1} \Gamma^R_{z_1}} \rho_{z_1z_2}^c, \\
\dot{\rho}_{z_1z_2}^c &= -(\Gamma^L_{z_1} + \Gamma^R_{z_1}) \rho_{z_1z_2}^c + \sqrt{\Gamma^L_{z_1} \Gamma^R_{z_1}} \rho_{z_1z_2}^b,
\end{align*}$$

(22)

(23)

(24)

where we have suppressed the ‘tilde’ for notational brevity.
A. Dephasing rates and the quasistationary distribution

Since the interdot coupling is switched off ($\Delta_L = \Delta_R = 0$), the equations for density matrix elements with different qubit state indices ($z_1, z_2$) separate. To analyze these equations further we now consider the elements of the reduced qubit density matrix $\rho_{z_1, z_2} = \rho_{a z_1, z_2} + \rho_{b z_1, z_2} + \rho_{c z_1, z_2}$, obeying the equation

$$\dot{\rho}_{z_1, z_2} = -\Gamma_{\varphi L}^{a z_1, z_2} \rho_{z_1, z_2}^{a} - \frac{1}{2} \left( \Gamma_{\varphi R}^{a z_1, z_2} + \Gamma_{\varphi L}^{a z_1, z_2} \right) \rho_{z_1, z_2}^{b} - \Gamma_{\varphi R}^{a z_1, z_2} \rho_{z_1, z_2}^{c};$$

(25)

where we defined the basic dephasing rates

$$\Gamma_{\varphi L}^{a z_1, z_2} = \sqrt{\Gamma_{L}^{z_1, z_2} - \Gamma_{L/R}^{z_2}} \text{ and } \Gamma_{\varphi R}^{a z_1, z_2} = \sqrt{\Gamma_{L/R}^{z_1, z_2} - \Gamma_{R}^{z_2}}.$$  

(26)

It is clear that these rates are only non-zero for off-diagonal elements ($z_1 \neq z_2$). Thus the diagonal elements $\rho_{zz}$ are conserved, i.e. $\rho_{zz} = 0$, as required by probability conservation. However, all off-diagonal elements decay, which is the definition of dephasing. From this we may conclude that the measurement will eventually destroy all coherence between the different qubit product states $|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$. The expressions in Eq. (26) are similar to the ones presented in Ref. [17], and for each junction and tunnel event separately the rates agrees with the result for a single qubit read-out by a single QPC.

In the regime of weak measurement $\delta \Gamma_{L/R}/\Gamma_{L/R} \ll 1$, the dephasing rates are small

$$\Gamma_{\varphi} = \frac{\delta \Gamma^2}{\Gamma} \left[ 1 + O \left( \frac{\delta \Gamma^2}{\Gamma^2} \right) \right],$$

(27)

so the dynamics will have two different timescales. On the short timescale given by $\Gamma_{L/R}^{-1}$, all reduced density matrix elements will be approximately conserved, and we obtain the quasistationary distribution

$$\rho_{z_1, z_2}^{a} = \frac{\Gamma_{L}^{z_1, z_2} \Gamma_{R}^{z_1, z_2}}{\Gamma_{L}^{z_1, z_2} \Gamma_{R}^{z_1, z_2} + 2 \Gamma_{L/R}^{z_1, z_2} \Gamma_{R}^{z_1, z_2} + \Gamma_{R}^{z_1, z_2} \Gamma_{L/R}^{z_1, z_2}} \equiv P^{a}_{z_1, z_2},$$

$$\rho_{z_1, z_2}^{b} = \frac{\Gamma_{L/R}^{z_1, z_2}}{\Gamma_{L/R}^{z_1, z_2} \Gamma_{R}^{z_1, z_2} + 2 \Gamma_{L/R}^{z_1, z_2} \Gamma_{R}^{z_1, z_2} + \Gamma_{R}^{z_1, z_2} \Gamma_{L/R}^{z_1, z_2}} \equiv P^{b}_{z_1, z_2},$$

$$\rho_{z_1, z_2}^{c} = \frac{\Gamma_{L/R}^{z_1, z_2}}{\Gamma_{L/R}^{z_1, z_2} \Gamma_{R}^{z_1, z_2} + 2 \Gamma_{L/R}^{z_1, z_2} \Gamma_{R}^{z_1, z_2} + \Gamma_{R}^{z_1, z_2} \Gamma_{L/R}^{z_1, z_2}} \equiv P^{c}_{z_1, z_2},$$

(28)

where $\Gamma_{L/R}^{z_1, z_2} = \Gamma_{L/R}^{z_1} + \Gamma_{L/R}^{z_2}$, $\Gamma_{L/R}^{z_1 z_2} = \Gamma_{L/R}^{z_1} + \Gamma_{L/R}^{z_2}$, and $P^{\alpha}_{z_1, z_2}$ is the probability to find the island in the state $\alpha \in \{a, b, c\}$ given that the qubits are in the product state $|z\rangle \in \{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$. In order to get a qualitative understanding it is useful to note that According to Eq. (25), the dephasing will take place on the much longer timescale $\Gamma_{L/R}^{-1}/\delta \Gamma_{L/R}^2$, and as we will see, this is also the time-scale for the measurement. Inserting this quasi-stationary distribution into Eq. (25) we find the total dephasing rate between the product states $|z_1\rangle$ and $|z_2\rangle$.

$$\Gamma_{\varphi} = \frac{1}{2} \left[ P^{a}_{z_1, z_2} 2 \Gamma_{L}^{z_1, z_2} + P^{b}_{z_1, z_2} \left( \Gamma_{L/R}^{z_1, z_2} + \Gamma_{R}^{z_2} \right) + P^{c}_{z_1, z_2} 2 \Gamma_{L/R}^{z_1, z_2} \right].$$

(29)

One simple interpretation of this formula is that the coherence between state $z_1$ and $z_2$ will decrease only due to tunnel events across a junction where the two states differ.

B. Average current, noise and the measurement time

The average current will approach a steady-state value on the short time-scale given by $\Gamma_{L/R}^{-1}$ (see Fig. [4]). In the case when the qubits initially are in the product state $|z\rangle$ the steady-state average current

$$I_z = e \left[ \rho^{b}_{zz} \Gamma_{R}^{z} + P^{c}_{zz} 2 \Gamma_{L/R}^{z} \right],$$

(30)

is obtained by setting $\rho_{zz}(t) = 1$ in Eq. (25) and then inserting $\rho^{a}_{zz} = \rho^{b}_{zz} + \rho^{c}_{zz}, \rho^{b}_{zz}$ and $\rho^{c}_{zz}$ in Eq. (19).
For times much longer than $\Gamma_{L/R}^{-1}$, when the average number of electrons which has tunneled through the SET is large $\langle m \rangle \gg 1$, the distribution $P(m, t)$ will approach a Gaussian with average $\langle m \rangle \approx I_z t/e$, and a width given by the zero-frequency current noise $S_I$. The current noise is readily obtained from the full counting statistics as:

$$S_I = 2e^2 \left( - \frac{\partial^2}{\partial t^2} \lambda_0(k) \right), \quad (31)$$

where $\lambda_0(k)$ is the eigenvalue of the evolution matrix $M$ which approaches zero in the limit $k \to 0$. The noise is sometimes expressed in terms of the Fano factor $f_z$, through the relation $S_I = 2eI f_z$. After some algebra we obtain an analytic expression for the Fano factor corresponding to the qubit product state $|z\rangle$,

$$f_z = \frac{S_I^z}{2eI_z^z} = \frac{\left( \Gamma_{L}^z + \Gamma_{R}^z \right)^2 (\Gamma_{L}^z)^2 + (\Gamma_{R}^z)^2 - \left( \Gamma_{L}^z + \Gamma_{R}^z \right)^2 + 4(\Gamma_{L}^z)^2(\Gamma_{R}^z)^2}{(\Gamma_{L}^z + 2\Gamma_{R}^z)^2}. \quad (32)$$

We now define the time needed to distinguish between two different product states $|z_1\rangle$ and $|z_2\rangle$ as:

$$t_{ms}^{z_1 z_2} = \left( \frac{\sqrt{S_I^{z_1}} + \sqrt{S_I^{z_2}}}{I_{z_1} - I_{z_2}} \right)^2, \quad (33)$$

after which the two corresponding counting distributions are distinguishable. Distinguishable implies that the overlap of the counting distributions fulfills:

$$\sum_m \sqrt{P_{z_1}(m, t_{ms}^{z_1 z_2})} P_{z_2}(m, t_{ms}^{z_1 z_2}) \leq \frac{1}{e}, \quad (34)$$

where $P_z(m, t)$ corresponds to the qubits’ state $|z\rangle$.

According to fundamental principles of quantum mechanics, two different states cannot be distinguished classically before the quantum coherence between these states has decayed. With our definitions this inequality reads $\Gamma_{z_1 z_2} t_{ms}^{z_1 z_2} \geq 1$, leading to the definition of the quantum efficiency

$$\eta_{z_1 z_2} = \frac{1}{\Gamma_{z_1 z_2} t_{ms}^{z_1 z_2}} \leq 1, \quad (35)$$

for distinguishing the two states $|z_1\rangle$ and $|z_2\rangle$, using our suggested measurement scheme. One may note that other definitions of distinguishability, and thus measurement time, gives other numerical values for the limit of the quantum efficiency. This is the reason why the quantum limit sometimes in literature is given as $1/2$. The foundation is now established for discussing the properties of the suggested readout scheme in some analytically tractable, and relevant cases.

C. A single junction detecting a single qubit

For comparison we may check the limit of a single QPC measuring the state of a single qubit. This situation is...
(δΓ_R = 0), and if the escape rate through the right junction is much larger than the in-rates Γ_L ≫ Γ_R. To leading (zeroth) order in Γ_R/Γ_L the island is always empty, i.e. P_{zz}^0 = 1, the average current is I_z = 2eΓ_L^z, and the Fano factor is one. The time needed to separate the product state which differ in the state of the left qubit is obtained from Eq. (33),
\[
\frac{I_{ms}}{\Delta I_{ms}} = \frac{t_{AD}}{t_{BC}} = \frac{t_{BD}}{t_{CD}} = 1 + \frac{1}{2} \frac{\delta \Gamma}{\Gamma_L} \left[ 1 + O \left( \frac{\delta \Gamma^2}{\Gamma_L^2} \right) \right],
\]
where the last equality is valid in the weak coupling regime (δΓ_L ≪ Γ_L). The corresponding dephasing rates are given by Eq. (39).
\[
\Gamma_{\varphi}^{AC} = \Gamma_{\varphi}^{BC} = \Gamma_{\varphi}^{BD} = 2 \left( \Gamma_L - \sqrt{\Gamma_L^2 - \delta \Gamma^2} \right) \approx \delta \Gamma \left[ 1 + O \left( \frac{\delta \Gamma^2}{\Gamma_L^2} \right) \right].
\]
Thus we find that this readout is indeed quantum limited η^{AD} = 1, η^{BC} = η^{CD} = 1, which was first discussed in Ref. 14.

D. Negligible Coulomb energy

Now, going back to the case of two qubits detected by two QPCs, a limit which is easy to analyze is when the rates do not depend on whether the doubly occupied state is involved or not, i.e. Γ_{L/R}^z = Γ_{L/R}. This regime is e.g. obtained if the Coulomb energy can be neglected U zz = 0. The average current is then
\[
I_z = 2e \frac{\Gamma L^z}{\Gamma L + \Gamma R},
\]
and the noise is the same as for a non-interacting quantum dot
\[
f = \frac{(\Gamma L^z)^2 + (\Gamma R^z)^2}{(\Gamma L + \Gamma R)^2}.
\]
The occupation probabilities of the island are
\[
P^{zz}_{zz} = \frac{(\Gamma L^z)^2}{(\Gamma L + \Gamma R)^2}, \quad P^{zb}_{zz} = \frac{2\Gamma L^z \Gamma R^z}{(\Gamma L + \Gamma R)^2}, \quad P^{bc}_{zz} = \frac{(\Gamma R^z)^2}{(\Gamma L + \Gamma R)^2},
\]
resulting in the average steady-state current
\[
I_z = 2e \frac{\Gamma L^z}{2\Gamma L + \Gamma R},
\]
and Fano factor
\[
f = \frac{4(\Gamma L^z)^2 + (\Gamma R^z)^2}{(2\Gamma L + \Gamma R)^2}.
\]
We may note that the expressions for the current and Fano factor can formally be obtained from Eqs. (35) and
by letting $\Gamma_L \rightarrow \Gamma_R \rightarrow \Gamma_R^z/2$. Again looking at the symmetric situation where the left and right tunnelling rates are equal ($\Gamma_L = \Gamma_R = \Gamma$), and the qubits are equally strongly coupled to the SET ($\delta r_L = \delta L_L = \delta L$), we have the following expressions for the time needed to distinguish the different qubit product states,

$$t^{AD}_{ms} = \frac{5 \Gamma}{3 \delta L}, \quad t^{AB}_{ms} = t^{CD}_{ms} = \frac{15 \Gamma}{4 \delta L},$$

$$t^{AC}_{ms} = t^{BC}_{ms} = t^{BD}_{ms} = \frac{15 \Gamma}{\delta L},$$

where we again assumed weak coupling to the qubits $\delta L \ll \Gamma$, and presented the results to leading order in $\delta L / \Gamma$. The states $|A\rangle = |\downarrow\downarrow\rangle$ and $|D\rangle = |\uparrow\uparrow\rangle$ again have the largest difference in average current, and are the fastest to distinguish. The states $|B\rangle = |\downarrow\uparrow\rangle$ and $|C\rangle = |\uparrow\downarrow\rangle$ no longer give the same average current, and thus we can tell them apart. To distinguish all product states one has to measure for at least as long as the longest measurement time $t^{AB}_{ms}$. The corresponding quantum efficiencies are

$$\eta^{AD} = \frac{9}{10}, \quad \eta^{AB} = \eta^{CD} = \frac{4}{5},$$

$$\eta^{AC} = \eta^{BD} = \frac{1}{5}, \quad \eta^{BC} = \frac{1}{10},$$

leading order in $\delta L / \Gamma$. Here all quantum efficiencies are finite, since we can distinguish all product states. Also, the quantum efficiency for separating the $|A\rangle$ and $|D\rangle$ state is lowered to 9/10 compared to unity in the noninteracting case, which is due to the differences in island state probabilities between the $|A\rangle$ and $|D\rangle$ state. In fact, by choosing $\Gamma_L = \Gamma, \Gamma_R = 2\Gamma, \delta L_L = \delta L$ and $\delta L_R = \delta L$ the quantum efficiency $\eta^{AD}$ is again unity, and the measurement times and quantum efficiencies are given by the expressions for the noninteracting case in Eqs. (44) and (45).

V. MEASURING THE ISLAND STATE

When the tunneling rates are slow enough, one may perform a time-resolved measurement of the island charge state, e.g. using another RF-SET. Plotting the island charge as a function of time, each tunnel event appears as an almost vertical line, and from this information it is straightforward to determine the full counting statistics of the electrons passing the detector, as shown by Gustavsson et al.

A. Measuring the full time-trace

The time the system waits in island state $a$ ($t^{a}_{ms}$) and $c$ ($t^{c}_{ms}$) are random variables, exponentially distributed with parameters $\Gamma_L$ and $\Gamma_R$, respectively (neglecting the coupling to the qubits). From state $b$, the system escapes both to $a$ and $c$, so this waiting time is exponentially distributed with parameter $\Gamma_L + \Gamma_R$. By labeling the $b$ state waiting times also with the destination state ($a$ or $c$), one gets two separate waiting times ($t^{a}_{ms}$ and $t^{c}_{ms}$), each exponentially distributed, with parameter $\Gamma_R$ and $\Gamma_L$ respectively. The average waiting time equals the inverse rate, e.g. $\langle t^{a}_{ms} \rangle = \Gamma_L^{-1}$, and the variance is decreasing as $\Gamma_L^{-2} n^{-1}$, where $n$ is the number of tunnel events ($a \rightarrow b$) where $t^{a}_{ms}$ has been sampled. The average number of tunnel events increases linearly with time, $n_a = \langle t^{a}_{ms} \rangle = \Gamma L^{-1}$, $n_{ba} = \Gamma R^{-1}$, $n_{bc} = \Gamma R^{-1}$ and $n_c = \Gamma R^{-1}$.

Now, including the coupling to the qubits, each of the four rates may have one of two possible values $\Gamma_L^{(+)} / \Gamma_R$ or $\Gamma_R^{(+)} / \Gamma_L^{(+)}$. The combined probability distribution for the four different waiting times will now depend on the qubits’ state. We again use Eq. (31) as the definition of the measurement time, with the difference that the probability distribution now is a function of four continuous random variables.

In the weak coupling regime $\delta L \ll \Gamma$ the individual distributions approach a Gaussian form, in the relevant regime of all $n_a, n_{ba}, n_{bc}, n_c \gg 1$. The overlap for the $t^{a}_{ms}$ distributions is

$$\int_0^\infty dt_w \sqrt{P_z(t_w,t)P_z(t_w,t)} = e^{-n_a \delta L^2 / 2 \Gamma_L^2},$$

where $n_a$ is the average number of $a \rightarrow b$ tunnel events, neglecting the coupling to the qubits. This expression is valid to lowest non-vanishing order in all $\delta L / \Gamma$. Since the waiting times are independently distributed random variables we get the total overlap as a product of the four individual overlaps, giving the measurement time

$$[\tau^{z1,2}_{ms}]^{-1} = \Theta_L(z_1, z_2) \left[ P_a \frac{\delta L^2}{2 \Gamma_L} + \frac{P_b (\delta L)^2}{2 \Gamma_L^2} \right] + \Theta_R(z_1, z_2) \left[ P_b \frac{\delta L^2}{2 \Gamma_R^2} + \frac{P_c (\delta L)^2}{2 \Gamma_R^2} \right],$$

where $\Theta_L^{(z_1, z_2)} = 1$ if the states $|z_1\rangle$ and $|z_2\rangle$ differ for the left/right qubit, and $\Theta_L^{(z_1, z_2)} = 0$ otherwise. Comparing with the weak coupling expression of the total dephasing rate in Eq. (28), we find the quantum efficiency of recording all tunnel events to be unity, to lowest order in $\delta L / \Gamma$. Thus, during the measurement, all the information about the qubits’ state which is extracted is transferred into information about the four different tunneling rates.

VI. QUANTUM EFFICIENCY OF CURRENT MEASUREMENTS REVISITED

Knowing that the full information is in the distribution of the rates, we can now reexamine the current measurement in terms of how much information it gives about the rates. In general the current is a function of all four rates, and the analysis gets complicated. For simplicity
we here only reanalyze the two cases we analyzed analytically above: negligible and large Coulomb energy. In both these cases there is only one independent non-zero rate per junction. Thus the current is a function of only two independent random variables, and we may straightforwardly visualize the situation. In Fig. 5 the pair of rates corresponding to the four different qubit product states A-D are shown as dots in a two-dimensional rate diagram. In the weak coupling regime the distributions decreased to $1/e$. Furthermore, the line connecting the B and C states is orthogonal to the current axis, giving a zero $\cos^2 (\pi/2) = 0$ quantum efficiency, i.e. the projection of the two distributions always overlap completely. This simple geometrical interpretation is in full agreement with Eq. (12).

In the other analytically tractable case of large Coulomb interaction, the current axis $I_b$ forms an angle $\gamma = \arctan (1/2)$ with the horizontal $\Gamma_R$-axis. Noting that $\cos^2 \gamma = 4/5$, and that $\cos^2 (\gamma - \pi/4) = 9/10$ we see that the geometrical interpretation also reproduces Eq. (47).

\section*{A. Measuring average island charge}

Measuring the full time-trace of all tunnel events has so far only been demonstrated for tunnel rates on the order of kHz\textsuperscript{30}. For faster readout schemes a more realistic setup would be to measure the average island charge $\langle n \rangle$, in addition to the average current through the device. The time-dependent distribution of the average charge $\langle n \rangle$ can also be analyzed in terms of the rate distributions. One may note that the vector corresponding to the average charge measurement is perpendicular to the current measurement vector in the symmetric setup, i.e. $\Gamma_L = \Gamma_R$ for negligible Coulomb energy, and $\Gamma_R = 2\Gamma_L$ for large Coulomb energy. In this case the distribution of average charge is independent of the current distribution, and we recover full quantum efficiency in separating all product states, even if the full time-trace cannot be recorded.

\section*{VII. CONCLUSION AND DISCUSSION}

We have described how to read out two charge qubits using a single SET. After deriving general kinetic equations for the whole system, we focussed on the properties of single-shot readout. Any interdot coupling during the measurement will reduce the fidelity, so we considered the case when the inter-dot qubit tunneling is turned off during measurement ($\Delta_L = \Delta_R = 0$). Detecting the charge of the SET island in real-time corresponds to measuring the individual tunnel rates, and is always quantum efficient. The current is a function of the tunneling rates, and the quantum efficiency in this case depends on which qubit states should be separated, as well as the biasing conditions of the SET. In some symmetric cases, a combined measurement of average current and average island charge gives full quantum efficiency for separating all four product states. The setup can not be used for entangling...
the qubits, since the measurement always projects onto a product state.

The possibility to have one readout device for two qubits can be used to optimize the design of many-qubit circuits. One drawback is that the qubits always have to be read out simultaneously. This may not be a severe problem, since in many quantum algorithms the readout is performed at the end. Also, there is an obvious advantage in reducing the number of readout lines attached to your circuit. This has to be weighed against the reduced signal-to-noise ratio in the readout, coming from the fact that four different states should be separated, instead of two.

VIII. ACKNOWLEDGEMENTS

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