Possible violations of spacetime symmetries

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Abstract. The identification of symmetries has played a fundamental role in our understanding of physical phenomena. Nevertheless, in most cases such symmetries constitute only a zeroth-order approximation and they need to be broken so that the predictions of the theory are consistent with experimental observation. In particular, the almost sacred CPT and Lorentz symmetries, which are certainly part of the fundamental ideas of modern physics, need to be probed experimentally. Recently, such efforts have been intensified because different theoretical approaches aiming to understand the microstructure of space-time suggest the possibility that such symmetries could present minute violations. Up to now, and with increasing experimental sensitivities, no signs of violation have been found. Nevertheless, we observe that even the persistence of such negative results will have a profound impact. On one hand, they will provide those symmetries with a firm experimental basis. On the other, they will set stringent experimental bounds to be compared with the possible emergence of such violations in quantum gravity models based upon a discrete structure of space. We present a very general perspective of the research on Lorentz symmetry breaking, together with a review of a few specific topics.

1. Introduction
The intuitive idea we have about some object exhibiting symmetry is that of some organized structure which shows some kind of regularity. From the point of view of physics we require a more precise statement of what a symmetry is. The basic idea is that some configuration is symmetrical if we perform operations (motions, changes or transformations) on it and we recover the original pattern afterward. In mathematics, the idea of a transformation (e.g. translations, reflections and rotations, for example) is encoded in what we call a group of motions or simply a group. The elements of the group are a set of transformations that satisfy some well known rules. We say that a system is symmetric under such group if its time evolution does not change under the application of any operation of the group. This requirement puts severe restrictions upon the application of any operation of the group. This requirement puts severe restrictions upon the form of the Hamiltonian of a symmetric system and usually one can use group-theoretical methods to find the spectrum.

A more general approach to describe a symmetry is in terms of the action $S$ of the system, which contains all the information about its time evolution, both at the classical and the quantum level. The action is a functional of the fields $\Phi^M(x^\mu)$ which describe the system, and it is given by

$$S[\Phi^M] = \int_R d^4 x \mathcal{L}(\Phi^M, \Phi^{M\mu}) , \quad \Phi^{M\mu} = \frac{\partial}{\partial\Phi^{M\mu}}, \quad \partial_\mu \mathcal{L} = 0, \quad \partial_\mu = \frac{\partial}{\partial x^\mu},$$ (1)
where \( L \) is the Lagrangian density (LD), \( \mu = 0, 1, 2, 3 \) are Minkoswki spacetime indices (with metric \( \eta_{\mu \nu} = \text{diag}(+,-,-,-) \)) and \( M \) are generalized labels that run over internal, as well as over spinor and tensor indices which specify the type of field we are dealing with. Let us emphasize that we are only considering LDs which do not depend explicitly upon the spacetime coordinates. The Euler-Lagrange equations of motion, obtained by requiring the action to be an extreme keeping the fields fixed at the initial and final time slices, together with appropriate boundary conditions at spatial infinite in the region \( R \), are

\[
0 = \frac{\partial L}{\partial \Phi^M} - \partial_\mu \left( \frac{\partial L}{\partial (\Phi^M)_\mu} \right) \equiv \mathcal{E}_M. \tag{2}
\]

1.1. Global continuous symmetries

Suppose now that we perform some global transformations \( \delta_{x,B} x^\mu, \delta_{x,B} \Phi^M, \delta_{x,B} \Phi^M = \partial_\mu \left( \delta_{x,B} \Phi^M \right) \), depending upon some independent infinitesimal constant parameters \( \epsilon_{x,B} \), with \( k = 1, \cdots, p \), for each type of field \( B \). We say that these transformations are a symmetry of the system if we can write

\[
\delta_{x,B} L = \partial_\mu \Omega^{\mu}_{\epsilon_{x,B}} (\Phi^M, \Phi^M_\mu), \tag{3}
\]

without using the equations of motion. Here the function \( \Omega^{\mu}_{\epsilon_{x,B}} (\Phi^M, \Phi^M_\mu) \) depends linearly on the parameters \( \epsilon_{x,B} \). The transformations of the fields \( \Phi^M \), together with those of their derivatives, are dictated by the spin content of the indices \( M \). Calculating the explicit variation in the left hand side of Eq. (3), and now using the equations of motion (2), we obtain

\[
0 = \partial_\mu \left( \frac{\partial L}{\partial (\Phi^M)_\mu} \delta_{x,B} \Phi^M - \Omega^{\mu}_{\epsilon_{x,B}} \right) \equiv \epsilon_{x,B} \partial_\mu J^\mu_{x,B}, \tag{4}
\]

from where we can read the conserved currents \( J^\mu_{x,B} \). This is the celebrated first Noether’s theorem, which states that each independent global transformation of coordinates and fields which is a symmetry of the LD as defined in Eq. (3), induces a conserved current given by Eq. (4) [1]. With appropriate boundary conditions, the corresponding charges

\[
Q_{x,B}(t) \equiv \int d^3 x J^0_{x,B}(t, x) \tag{5}
\]

satisfy \( dQ_{x,B}(t)/dt = 0 \) and generate the Lie algebra of the corresponding symmetry group, after rewriting them in terms of the canonical variables of the system. The most typical examples of the application of these ideas are: (1) energy-momentum conservation, arising from the translation symmetry \( \delta_{x} x^\mu = \epsilon^\mu \) of the LDs which do not depend explicitly upon the coordinates and (2) angular momentum (spin plus orbital) conservation arising from Lorentz invariance expressed in the infinitesimal form \( \delta_{\omega_{\alpha \beta}} x^\mu = \omega_{\mu \alpha} x_\alpha \) with \( \omega_{\alpha \beta} = -\omega_{\beta \alpha} \). As usual, indices in Minkowski space are raised and lowered by the metric \( \eta_{\mu \nu} \) and its inverse \( \eta^{\mu \nu} \).

1.2. Local continuous symmetries

They are also known as gauge symmetries and are characterized by spacetime-dependent infinitesimal parameters \( \theta_k(x) \) such that the field variations can be written as \( \delta \Phi^M = P_{M,k}(\Phi, \Phi_\mu) \theta_k(x) + Q^M_{M,k}(\Phi, \Phi_\mu) \partial_\mu \theta_k(x) \). Here we further restrict ourselves to internal transformations where \( \delta x^\mu = 0 \). Imposing that such local transformations leave the action invariant and calculating the variation of the Lagrangian density under them, for arbitrary fields \( \Phi \) and without using the equations of motion, leads to the relation

\[
\int d^4 x \left[ \mathcal{E}_M \delta \Phi^M + \partial_\mu \left( \frac{\partial L}{\partial (\Phi^M)_\mu} \delta \Phi^M \right) \right] = 0, \tag{6}
\]
where $\mathcal{E}_M$ was defined in the Eq. (2). After imposing the conditions that $\theta_k(x)$ and $\partial_\mu \theta_k(x)$ are zero on the boundary, but arbitrary in the bulk region, we obtain the second Noether’s theorem

$$E_M P_{M,k} - \partial_\mu \left( E_M Q^\mu_{M,k} \right) = 0,$$

which provides $p$ relations among the equations of motion. This is an alternative route to see that not all the equations in a gauge theory are independent, in such a way that arbitrary functions appear in their solutions, which is a consequence of the constraints that arise in the Hamiltonian formulation. In the well known case of electrodynamics coupled to an external current $J_\mu$, we have $\Phi^M \rightarrow A^\mu$ together with the gauge transformation $\delta A_\mu = \partial_\mu \theta$ (only one parameter), which leads to $P_{\mu,1} = 0$ and $Q^\mu_{\nu,1} = \delta^\mu_\nu$. Then Eq. (7) reduces to $0 = \partial^\mu E_\mu = \partial^\mu (\partial^\nu F^\nu_\mu - J_\mu) = \partial^\mu J_\mu$, thus yielding the conservation of the external current. Here $F_{\nu\mu} = \partial_\nu A_\mu - \partial_\mu A_\nu = -F_{\mu\nu}$.

The standard point of view is that gauge invariance forbids the term $m^2 A_\mu A^\mu$ in the LD of electrodynamics, which states that the photon has zero mass. The same argument follows for the additional gauge bosons: the gluons and the graviton. In subsection (4.3) we present an alternative way of looking at gauge bosons in terms of spontaneous Lorentz symmetry breaking.

2. Spacetime symmetries

They include the discrete symmetries C (charge conjugation), P (parity) and T (time reversal) together with the continuous ones associated to translational invariance and Lorentz transformations. Even in the broken symmetry cases, we will always respect the translational invariance of the LD in order to have energy-momentum conservation. It is also interesting to observe that P and CP are not exact symmetries in Nature, since they are violated in the weak interactions. In fact, in 1956 Lee and Yang proposed tests to probe parity symmetry [2] and in the next year C.S. Wu et al. found P violation in the beta decay of Co\textsuperscript{60} [3]. In the same year, this violation was independently confirmed by Garwin et al. [4]. Later in 1964, Cronin and Fitch found CP violation in the decay of neutral kaons [5].

This contribution will focus on the discussion of some specific topics in Lorentz symmetry breaking (LSB) and does not do any justice to the huge literature in this field. Before going into the details it is worth recalling two basic theorems relating spacetime symmetries. The first one is the CPT theorem [6], which states that any Lorentz invariant, local, quantum field theory with an hermitian Hamiltonian must exhibit CPT invariance. The second one is called the anti-CPT theorem [7], which proves that CPT violation implies Lorentz violation, but not the converse. That is to say, in LSB theories we can have both CPT even and CPT odd contributions in the Lagrangian.

3. Why testing Lorentz symmetry

3.1. Motivation

There are at least two important reasons for probing this symmetry. The most obvious one is that, since physics is an experimental science, it is of fundamental importance to test, with the utmost precision, the validity of such a fundamental symmetry as Lorentz invariance. In order to do this one must have models that include and parameterize a violation of Lorentz symmetry, in order to design experiments that are able to set up bounds upon these parameters, determining the range of its validity. The second reason is related to the description of space at very short distances, where one assumes that quantum gravity becomes relevant. Most of the contending theories in this field start from the idea that space is no longer a continuum in that limit, but rather that is has a discrete or granular structure. Naively, one can entertain the possibility that Lorentz symmetry, basically rooted on a continuum space, is no longer valid under such circumstances and that very small violations will permeate to standard model energies and distances. A definite answer to this question will come only after the semiclassical limit of
such quantum gravities theories is achieved, that is to say after the reconstruction of standard Minkowski space is attained. Then it will be possible to decide upon the impact of such discrete structure in the continuum limit associated with the Lorentz symmetry. The construction of such semiclassical approximation is still an open problem in quantum gravity theories [8]. In this process, the strict bounds for LSB already determined will serve as a set of experimental benchmarks for contending quantum gravity theories, providing an experimental basis to decide upon the correct one. For a review of the present day status in testing Lorentz invariance see for example Ref. [9].

3.2. The Standard model extension
To test Lorentz symmetry we require models that break this invariance, in order to introduce parameters that quantify the violations, which subsequently are bounded by experiments. The most general of such models, which has been extensively used by experimentalists to present their results, is the Standard Model Extension (SME) [10]. This is an effective field theory which extends the LD of the Standard Model (SM) of particle physics by adding all possible dimension three and four Lorentz violating operators that can be constructed with the fields appearing in the SM, thus being consistent with the particle content and interactions of the SM. The SME has been also generalized to incorporate the gravitational interaction [11] and also to include higher order Lorentz violating operators [12]. The full range of applications of the SME, both at the theoretical and experimental levels, can be appreciated in Refs. [13]. The basic idea is that Lorentz symmetry breaking (LSB) arises from non-zero vacuum expectation values (VEVs) of tensorial fields in a spontaneous symmetry breaking of a more fundamental theory. Such VEVs fill all spacetime, define privileged directions and couple to the fields of the SM thus yielding the LSB operators. Since LSB is expected to be suppressed by some huge quantum gravity scale, in a first approximation one considers only linear modifications arising from such VEVs. When the gravitational interaction is not included, the dynamics of such VEVs is not relevant and they behave as non-dynamical fields entering in the action. In this case LSB is equivalent to an explicit breaking, in contrast with a spontaneous breaking which is mandatory in the presence of gravity [11, 14]. It is important to emphasize that we are not violating what is called observer (passive) Lorentz invariance, which is the freedom we have to select an arbitrary coordinate frame to describe the system. When passing from one reference frame to another, both the dynamical fields as well as the tensorial VEVs transform accordingly. Here one must be careful with the effects of boosts, which may kinematically amplify the values of the VEVs, giving rise to enormous effects in the LSB which are not consistent with observation. The natural reference frames in which we describe observations in the laboratory are such that LSB effects are much suppressed and we want to restrict ourselves only to these set of frames, which are called concordant frames. What we violate is particle (active) Lorentz transformations which, in a given reference frame, are applied to the dynamical fields but not to the VEVs. In a pictorial representation we can imagine that the earth is immersed in the flux of constant tensor fields (VEVs) defining fixed directions in space. As the earth moves through this aether, the relation among the relevant fields of our experiment in the laboratory changes with respect to such directions, thus providing sidereal or daily variations in the signals produced by the coupling of matter and gauge fields to the VEVs. This type of variations is what most of the experiments searching for LSB look for. A more vivid analogy is that of a physicist confined to a laboratory immersed in a constant magnetic field and not having access to the exterior. For him, a privileged direction appears and rotational symmetry is violated.

Just to give an example, the LD corresponding to the photon sector of the SME is

\[
L = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (kF)^{\mu\alpha\beta} F_{\mu\nu} F^{\alpha\beta} - J^\mu A_\mu, \tag{8}
\]
where $F_{\mu\nu}$ is the Faraday tensor. We have the standard Maxwell term plus corrections that incorporate LSB via the constant dimensionless VEV $(k_F)^{\mu\alpha\beta}$. This tensor has zero epsilon-trace plus Riemann-like symmetries, yielding 19 independent components which have been bounded by different experiments or observations. The system described in Eq. (8) can be cast in the standard form of electrodynamics in a medium by introducing the corresponding constitutive relations in terms of $3 \times 3$ permittivity, permeability and mixing matrices. Further linear combinations of these matrix elements $(\tilde{\kappa}_e)_{ij}$, $(\tilde{\kappa}_o)_{ij}$, $(\tilde{\kappa}_t)_{ij}$, are subjected to bounds from different sources. For example, astrophysical tests produce the bounds $(\tilde{\kappa}_e)_{ij} \leq 10^{-32}$, $(\tilde{\kappa}_o)_{ij} \leq 10^{-32}$ and $(\tilde{\kappa}_t)_{ij} \leq 10^{-14}$, while resonator tests lead to $(\tilde{\kappa}_e)_{ij} \leq 10^{-17}$ together with $(\tilde{\kappa}_o)_{ij} \leq 10^{-14}$. The situation for the remaining sectors of the SME is completely similar.

The amount of experimental work dedicated to search for LSB has been enormous, ranging from tabletop experiments in atomic physics and data analysis in high energy physics to astrophysical observations. No signal of LSB has been detected so far, but a huge list of bounds obtained for the VEVs codifying LSB in the SME has been obtained. These bounds, together with their sources are reported in the Data Tables for Lorentz and CPT Violation [15], which is revised every year in order to include the improved results obtained from recent experiments.

The results obtained in the study of LSB can be considered an important contribution to the subject of Quantum Gravity Phenomenology, which attempts to provide experimental guidance to the problem of constructing a viable quantum gravity theory [16]. Other approaches to study the impact of space granularity upon the propagation of particles at standard model energies have been considered. Some of them are: Deformed (Double) Special Relativity [17], Minimal Length Scenarios [18], Finsler Geometries [19], Horava-Lifshitz Gravity [20] and Very Special Relativity [21], for example.

4. Some particular topics in LSB

4.1. Modified dispersion relations and photon propagation

One of the simplest kinematical consequences of LSB is the modification of the energy-momentum relations for propagating particles, as well as the changes introduced in the threshold conditions that allow some specific reaction. Of course, a detailed description of a particular process requires the knowledge of the dynamics, via the cross section for example, which can be calculated in a given LSB field theory, as it is the case of the SME, for example. Much attention has been devoted to the information that extragalactic photons could provide by observing gamma ray bursts (GRB), pulsars and active galactic nuclei (AGN), among others.

The modified dispersion relations (MDR) for photons propagating with momentum $k$ and energy $E$ can be presented as

$$c^2 k^2 = E^2 \left( 1 \pm \xi \left( \frac{E}{E_{QG}} \right) + O(E^2) + \cdots \right), \quad \rightarrow \quad |\mathbf{v}| = |\mathbf{\nabla}_k E| = c \left( 1 \pm \xi \left( \frac{E}{E_{QG}} \right) + \cdots \right),$$

where $E_{QG}$ is the quantum gravity scale and $\xi$ is a parameter of order one. One must keep an open mind regarding $E_{QG}$, which is usually taken as the Planck mass $M_P = 10^{19}$GeV. Such MDR predicts an energy dependent velocity, which produces a time delay $\Delta t$ in the arrival of photons having an energy difference $\Delta E$, which were simultaneously emitted at a distance $L$ from the detector. A first approximation gives

$$\Delta t = \frac{L \Delta E}{c E_{QG}}. \quad (10)$$

As emphasized in the seminal proposal [22], the large suppression arising from $1/E_{QG}$ requires a huge amplifying factor in such a way that the effect becomes observable. This is provided by
taking photon sources located at cosmological distances. Some typical numbers like $\Delta E = 20$ MeV, $E_{QG}/\xi = 10^{19}$ GeV, together with the distance $L = 10^{10}$ l.y. produce $\Delta t = 10^{-3}$ s, which is within the precision of cosmological gamma ray detection. Numerous observations of this kind have been carried on and they have set lower bounds for $E_{QG}/\xi$, which range from $1.8 \times 10^{15}$ GeV [23] to $(1.4 - 122) \times 10^{19}$ GeV [24]. The latter bound either indicates that modified linear dispersion relations are not consistent with observation, or that the quantum gravity scale needs to be revised. Soon after the proposal of Ref. [22], some specific models of quantum gravity corrections to standard particle dynamics incorporating LSB appeared [25, 26, 27, 28].

The first heuristic derivation of a quantum gravity induced modified Hamiltonian for Maxwell equations in a Loop Quantum Gravity inspired description of Einstein-Maxwell theory was reported in [25]. This sparked a great deal of interest and it was subsequently followed by generalizations to the fermionic case [27], further elaborations in the Maxwell case [28] and the first steps towards the Yang-Mills case [29]. The idea behind all these heuristic derivations starts from the corresponding well defined operators in the quantum theory [30] and relies in the construction of quantum gravity semiclassical states which approximate a flat geometry at macroscopic distances while retaining their quantum granular structure at scales of the order of the Planck length. Since those states were not known, their expected properties were postulated and estimations of the required expectation values were performed. Accordingly, this procedure left overall dimensionless coefficients undetermined. Furthermore, extracting the dynamics is a process less clear to develop due to the known difficulties with time evolution. [31]. The construction of such semiclassical states is still an open problem in canonical quantum gravity [8] and the main difficulty resides in finding the appropriate quantum gravity ground state which yields the Minkowski space.

4.2. Radiative corrections

Most of the quantum gravity inspired corrections to the dynamics at standard model energies arise through powers of the factor $(E/E_{QG})$, which are directly relevant at inaccessible energies $E \approx E_{QG}$. An alternative possibility of probing such high energies is through the inclusion of radiative corrections, because the internal momenta are integrated up to the maximum allowed in a given reference frame. The standard folklore with respect to any new physics entering at high energy scales (here Planck scale) via an effective low energy field theory is that such scales should produce negligible effects upon the leading-order low-energy physics (here standard model energies). Contrary to this belief, the first indications that this is not necessarily true in the LIV case appeared in Ref. [32], where a specific form of the coupling was proposed in order to avoid such contributions. Subsequently it was shown that the proposed cure did not survive higher orders in perturbation theory [33]. The fact that LIV induced by modeling space granularity via the introduction of a physical cutoff, defined in a preferred reference frame, lead to unsuppressed dimension four LIV contributions, was finally recognized in Ref. [34]. To this end, the calculation of one-loop radiative corrections in the Yukawa model

$$\mathcal{L} = \frac{1}{2} (\partial \Phi)^2 - \frac{1}{2} \mu^2 \Phi^2 + \bar{\Psi} (i\gamma^\mu \partial_\mu - m) \Psi + g \Phi \bar{\Psi} \Psi + \text{(LVT)}$$

was considered. Here LVT refers to the highly suppressed zeroth order Lorentz violating terms that take into account the previously discussed free particle dynamical modifications. The space granularity was modeled by introducing a physical cut-off $\Lambda$, in such a way that the magnitude of the three-momentum in any loop is bounded by this quantity. The parameter $\Lambda$ defines the onset of the scale at which the granularity of space becomes manifest. A convenient way of incorporating this requirement is to introduce the physical cutoff function $f(|k|/\Lambda)$ with $f(0) = 1$, $f(\infty) = 0$, which suppresses the internal momenta having $|k| \geq \Lambda$. The one-loop
calculation of the boson self-energy $\Pi(E, p, \Lambda)$ produces a finite contribution

$$\delta \Pi(E, p, \Lambda) = \eta \langle p_\mu p_\nu W^\mu W^\nu, \quad \eta = \frac{g^2}{12\pi^2} \left[ 1 + 2 \int_0^\infty dx x (f'(x))^2 \right],$$

(12)

where $W^\mu$ is the four-velocity of the frame in which the momentum cut-off $\Lambda$ is defined. Using standard model couplings, the coefficient is estimated in the range $\eta \geq 10^{-3}$. Similar results are found in the calculation of the fermion self-energy. These huge corrections would make LSB easily observable, which contradicts the tight bounds found so far. This result also poses serious problems for the preferred frame interpretation of the explicit dynamical modifications possibly induced by quantum gravity effects [34]. Nevertheless, spontaneous Lorentz symmetry breaking still remains an open possibility.

4.3. Gauge particles as Goldstone bosons

Since a definite signal for LSB has not being found so far, it is interesting to consider a somewhat opposite point of view regarding Lorentz violation which, having a long history in physics, has been recently revisited in the literature. The main goal in this approach is to determine the conditions under which spontaneous Lorentz symmetry breaking (SLSB) in a more fundamental theory becomes unobservable, thus providing a dynamical origin to the massless gauge bosons. The motivation comes from the property that spontaneous symmetry breaking of a global symmetry produces zero-mass excitations (Nambu-Goldstone bosons (NGB)). In particular, the global breaking of continuous spacetime symmetries including tensorial VEVs, can be adjusted to produce massless spin one and spin two fields describing the corresponding NGB. Then, the question whether or not it is possible to identify such NGB with the corresponding gauge particles (photons and gravitons, for example) arises. An affirmative answer would provide a dynamical basis for the gauge principle, which has played a fundamental role in the construction of modern physics. This idea goes back to Dirac [35], Bjorken [36] and Nambu [37], among others, and has been recently revisited in the context of Electrodynamics [38], Yang-Mills theories [39] and gravity [40]. In analogy with the nonlinear sigma model describing pion physics, a very efficient way to deal with the low energy sector of a theory exhibiting SLSB, which includes only the corresponding NGB, can be adjusted to produce massless spin one and spin two fields describing the corresponding NGB. Then, the question whether or not it is possible to identify such NGB with the corresponding gauge particles (photons and gravitons, for example) arises. An affirmative answer would provide a dynamical basis for the gauge principle, which has played a fundamental role in the construction of modern physics. This idea goes back to Dirac [35], Bjorken [36] and Nambu [37], among others, and has been recently revisited in the context of Electrodynamics [38], Yang-Mills theories [39] and gravity [40]. In analogy with the nonlinear sigma model describing pion physics, a very efficient way to deal with the low energy sector of a theory exhibiting SLSB, which includes only the corresponding NGB, is by considering the so called Nambu models. Their LD is given by that of the gauge theory which gauge bosons one tries to interpret as NGB, plus a nonlinear constraint (NLC) that encodes the SLSB. The simplest case, from which electrodynamics arises, is the abelian Nambu model coupled to a conserved current [37]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J^\mu A_\mu, \quad A^\mu A_\mu = (n_\beta n^\beta) M^2, \quad \partial_\mu J^\mu = 0.$$  

(13)

Here $n_\mu$ denotes the direction of the vacuum which breaks the symmetry, $M$ is the energy scale at which such breaking occurs and the nonlinear constraint (NLC) indicates that we are taking the VEV $\langle A_\mu \rangle = M n_\mu$. This constraint is to be solved and substituted into the LD. An adequate parametrization to solve the constraint in Eq. (13) is by introducing the field $a_\mu$ as

$$A_\mu = a_\mu + n_\mu \left( M^2 - n_\beta a^\beta \right)^{1/2} / n^2, \quad n_\beta a^\beta \neq 0, \quad n^2 a_\mu = 0.$$  

(14)

Since $a_\mu$ is orthogonal to the vacuum it includes only the NGB of the model. The simplest realization of the ANM arises from taking standard QED plus the nonlinear constraint in Eq. (13), which is subsequently solved in terms of $a_\mu$ according to Eq. (14), in such a way that

$$\mathcal{L}_{ANM}(\bar{\psi}, \psi, a_\mu) = \mathcal{L}_{QED}(\bar{\psi}, \psi, A_\mu(a_\alpha)).$$  

(15)
The substitution $A_\mu(a_\alpha)$ has to be rewritten as an expansion in powers of $a^\mu/M$ and yields a highly nonlinear LD, which provides additional LSB contributions to standard QED processes, where photons are now described by the NGB field $a_\mu$. Perturbative calculations in the tree and one loop approximation have been reported for some specific processes, with the surprising result that all LSB contributions cancel, yielding the standard QED amplitudes [37, 41]. Motivated by these unexpected conclusions and as a first step to understand this question, we have examined in a non-perturbative way, the conditions under which the equivalence between a non-abelian Nambu model (NANM) and its corresponding Yang-Mills theory is achieved. The NANM is the direct generalization of the ANM in Eq. (13) where $A_\mu \to A_\mu^a$, $F_{\mu\nu} \to F_{\mu\nu}^a$, $J_\mu \to J_\mu^a$, $n_\mu \to n_\mu^a$, still with only one nonlinear constraint. Here $a$ labels the corresponding group index. For the sake of simplicity we next illustrate the main steps of the procedure in terms of the ANM, given by Eq. (13), and refer the reader to Refs. [42] for the generalization to the NANM. The strategy is to start from the canonical formulation of the ANM and arrive to the standard canonical formulation of ED in terms of the canonically conjugated fields $A_i, E^j$, $i, j = 1, 2, 3$, with Hamiltonian density

$$\mathcal{H}_{ED} = \frac{1}{2} \left( \mathbf{E}^2 + \mathbf{B}^2 \right) + J^i A_i - \left( \partial_i E_i - J^0 \right) A_0, \quad E_i = -\partial_0 A_i - \partial_i A_0, \quad \partial_i E_i - J^0 \approx 0. \quad (16)$$

To begin we introduce the fields $\Phi_k$, where we split the indices $k = 1, 2, 3$ into $\vec{k} = 1, 2$ and 3, together with the alternative parametrization

$$A_0 = \Phi_3 \left[ 1 + \frac{N}{4 \Phi_3 \Phi_3} \right], \quad A_3 = \Phi_3 \left[ 1 - \frac{N}{4 \Phi_3 \Phi_3} \right], \quad A_\vec{k} = \Phi_\vec{k}, \quad (17)$$

where $N = \Phi_\vec{k} \Phi_\vec{k} + n^2 M^2$. In terms of the new coordinates $\Phi_k$ the NLC in Eq. (13) is identically satisfied. Let us now focus on the invertible change of variables $A_i = A_i(\Phi_j)$, which yields $\dot{A}_i = \dot{A}_i(\Phi, \dot{\Phi})$ and $\dot{A}_0 = \dot{A}_0(\Phi, \dot{\Phi})$, recalling that we also know $\dot{A}_0 = A_0(\Phi)$. The first relation gives

$$\dot{A}_i = \dot{\Phi}_k \frac{\partial A_i}{\partial \Phi_k}, \quad \dot{\Phi}_k \frac{\partial A_i}{\partial \Phi_k} = \dot{\Phi}_k \frac{\partial A_i}{\partial \Phi_k}. \quad (18)$$

We observe that we will not require the explicit form of the transformations $A_i = A_i(\Phi_j)$ in the following. In this way, the LD of the ANM can be written as $\mathcal{L}_{ANM}(\Phi, \dot{\Phi})$. A convenient way to make later contact with ED is to rewrite the LD in Eq. (13) as

$$\mathcal{L}_{ANM}(\Phi, \dot{\Phi}) = \frac{1}{2} \left( \mathbf{E}^2 - \mathbf{B}^2 \right) - J^\mu A_\mu, \quad (19)$$

where $E_i, B_i = \epsilon_{ijk} \partial_j A_k$ and $A_i$ are just labels for the corresponding functions of $\Phi$ and $\dot{\Phi}$, according to previous definitions. The coordinates $\Phi_i$ together with the ANM canonically conjugated momenta $\Pi^j$ satisfy the standard canonical algebra and define a regular system. The usual definition of the electric field in Eq.(16), yields the relations

$$\Pi_i = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_i} = \frac{\partial A_j}{\partial \Phi_i} E_j, \quad E_i = \frac{\partial \Phi_j}{\partial A_i} \Pi_j, \quad \Pi_i \Phi_i = E_j \dot{A}_j, \quad (20)$$

which follow because the dependence on $\dot{\Phi}$ arises only in $E_i$, together with the relations in Eq. (18). Next we calculate the ANM Hamiltonian $\mathcal{H}_{ANM}(\Pi, \dot{\Phi}) = \Pi \dot{\Phi} - \mathcal{L}_{ANM}(\Phi, \dot{\Phi})$ and explicitly verify that it can be written as

$$\mathcal{H}_{ANM} = \frac{1}{2} \left( E_i^a E_i^a + B_i^a B_i^a \right) + J^{ai} A_i^a - \left( D_i E_i^b - J^{0i} \right) A_0^b, \quad (21)$$
in terms of the corresponding labels. Also, the relations (20) would allow the explicit calculation of the algebra among $A_i^a, E^b_j$, in terms of the ANM canonical algebra. Let us observe that in the Hamiltonian form of the action for the ANM, the last relation in Eq. (20) identifies $E_j^b$ as the canonically conjugated momenta to the coordinates $A_i$ of ED. Moreover, since the transformation among the variables $(E, A)$ and $(\Pi, \Phi)$ is generated only by the coordinate transformations $A_i = A_i(\Phi)$, we can assert that this is indeed a canonical transformation [42]. In this way we recover the canonical algebra for the ED phase-space variables $A_i, E_j$ without any further calculation. Summarizing, up to now we have proved that the ANM canonical algebra yields the canonical algebra of ED. Nevertheless, both theories are still not equivalent because Eq. (21) is not the Hamiltonian of ED, in spite of its almost identical form. In fact, $A_0$ is not an arbitrary function, but only a label for some combinations of the coordinates in the ANM. Also, the Gauss function $G \equiv (\partial_i E_i - J^0)$ is not a constraint, as it should be in a gauge theory. To proceed, we calculate the time evolution of the Gauss function under the ANM dynamics, obtaining

$$\dot{G} = -\partial_\mu J^\mu + MG + \partial_i (N^i G),$$

where $M, N^i$ are known functions of ANM phase space. Since we have coupled the ANM to a conserved current, it is enough to impose the Gauss constraint $G = 0$ at some initial time slice $t_0$ in order to recover gauge invariance. In fact, under this requirement Eq. (22) guarantees that $G = 0$ for all time. Then, such condition can be added to the Hamiltonian Eq. (21) as an additional constraint via a Lagrange multiplier $R$ which now replace $A_0$ by $S = A_0 + R$. The canonical algebra of ED, together with current conservation, guarantees that no additional constraints appear. Summarizing, the equivalence between ED and the ANM coupled to a conserved current is established once the Gauss law $G = 0$ is imposed as an initial condition upon the ANM dynamics.

Acknowledgments

LFU acknowledges support from the projects UNAM (Dirección General de Asuntos del Personal Académico) # IN104815 and CONACyT # 237503.

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