Study on the critical speeds of car’s movement with low and ultra-low profile tires in a turn

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Abstract. This work proposes an improved algorithm for determining the critical speeds of car’s movement with low and ultra-low profile tires in a turn, where sliding of its axles occurs and overturning. The critical speeds are determined by the characteristics of the lateral slipping calculated for their critical normal loading. On the basis of the algorithm an interactive programming system in the MATLAB environment is developed. Graphic dependencies for the radius of the turn are presented, taking into account the slip of the tires with different speeds of a car Honda Civic Aerodeck 1.5iLS with low profile tires of size 185/60R14. Results have been obtained on critical speeds against sliding in the front and rear axles and against the car’s overturning. Critical speed against overturning are determined in the absence of normal reactions for the inner steered wheels in a turn. The critical speeds of sliding are found to be lower than the overturning, and thus appear to be decisive.

1. Introduction

When the car is moving in a turn, stability may be disrupted by effects of different nature as a result of which lateral overturning or sliding can occur [1,2,3,4]. Critical speeds of movement against overturning or sliding are studied at settled turn, and where the vehicle’s theory [5,6,7,8] does not account the influence of tire lateral slipping. Some authors explore the critical speed of elastic tire sliding in the front axle only [9]. This study [10] suggests dependencies for the critical speeds against overturning and sliding of the front and rear axle of a car, but it does not account the changes in the resistance to lateral slipping coefficient \( k_{\delta 0} \) of the elastic front and rear wheels as well as the change in grip coefficient of lateral direction \( \varphi_y \).

The purpose of the present work is to determine through the improvement interactive programming system presented in [11], the critical speeds against sliding of the front and rear axle and overturning of cars with low and ultralow profile tires, taking into account the redistribution of wheel load and its impact on the lateral slipping coefficient \( k_{\delta 0} \) with the proposed [11,12,13] dependencies.

2. Algorithm for determining the critical speeds of a car’s movement in a turn

For critical speeds of movement in a turn on a horizontal road, without taking into account the elasticity of the tires [5,6], it is recommended to use the dependencies:

- against overturning

\[
V_c = \sqrt{gR \frac{B}{2h}}
\]  
(1)
against sliding

\[ V_{\psi} = \sqrt{gR\varphi_y} \]  

where \( g \) is acceleration of gravity; 
\( \varphi_y \) - the grip coefficient of lateral direction; 
\( R \) - radius of the turn; 
\( B \) - the track width of the vehicle; 
\( h \) - the height of the centre of gravity.

In the case of a steady motion of a car in a turn, with taking into account the lateral slip of tires, for the radius of the turn \( R_\delta \), on the basis of \([1,2,3,5,6,7,10]\) and figure 1, the following dependency is applied

\[ R_\delta = \frac{L}{\tan(\theta_\psi-\delta_1) + \tan \delta_2}. \]  

To determine the distance \( R_\psi \) from the centre of the turn to the centre of gravity of the vehicle according to \([11]\) the dependence is

\[ R_\psi = \sqrt{R_\delta^2 - (b - R_\delta \tan \delta_2)^2}. \]  

The values in formulas (3) and (4), similarly to \([5,6]\) are shown on figure 1.

![Figure 1. Scheme of a car in a turn.](image)

According to \([11]\) for the centrifugal force the following can be given by

\[ F_c = \frac{GV^2}{gR_\delta^2} R_\psi \cos^2 \delta_2. \]  

From figure 1 for the forces and reactions acting on the front and rear axles, vector triangles, shown in figure 2 can be constructed. Using a sinus theorem and transforming the trigonometric expressions for the summed lateral forces acting on the front and the rear axle, the following dependencies are obtained.
\[ F_{y1} = \frac{GbV_2^2 \cos^2 \delta_2}{gLR_d \cos (\theta_{ax} - \delta_1)} \]  
\[ F_{y2} = \frac{G(L - b)V_2^2 \cos^2 \delta_2}{gLR_d} , \]  

where \( L \) is the wheelbase of the vehicle; 
\( G \) - the weight of the vehicle; 
\( V_2 \) - the speed of the rear axle, determine the speed of motion of the vehicle in a turn; 
\( b \) - the longitudinal coordinate of the centre of gravity of the vehicle; 
\( \theta_{ax} \) - the average angle of the steered wheels.

\[ V_k = \frac{1}{2 \cos \delta_2} \sqrt{gR_d \frac{B_1 + B_2}{h}} \] 
\[ V_{4\theta} = \sqrt{gR_d \varphi_2} . \]

In this work, the critical speeds against sliding of the front \( V_{1k\theta} \) and the rear \( V_{2k\theta} \) axle and against overturning are defined by improvement interactive system based on the proposed in [11] for movement of a car with low and ultra-low profile tires in a turn. Initially, the minimum angles of turning of the steered wheels are set \( \left( \theta_{1\theta}, \theta_{2\theta} \right) \) and the average angle of the steered wheels \( \theta_{ax} \) is calculated. For each value of \( \theta_{ax} \), the speed of the car \( V_2 \) changes a small step until the critical speed is reached of the front axle \( V_{1k\theta} \) or the rear \( V_{2k\theta} \) axle. The calculation starts with the initial values of the slipping \( \delta_1 = \delta_2 = \delta_{1v} = \delta_{1m} = \delta_{2v} = \delta_{2m} = 0 \) and by [11,12,13] the wheel loads are determined \( F_{ze1v}, F_{ze1m}, F_{ze2v}, F_{ze2m} \).

By the proposed [11,12,13] dependencies and the improved graphical method of K. Enke separately for the front and the rear axle, with the obtained values of the normal loads, the characteristics of the lateral slipping are constructed, taking into account the changes in the coefficient of resistance against lateral slipping \( k_{s0} \), according to [12,13]. The characteristics of the outer wheels are displaced, respectively, at a distance \( F_{y1} \) for the front and \( F_{y2} \) for the rear axle by a mirror image [11]. The intersection point of the front axle determines the slipping \( \delta_1 \), while the intersection point of the rear axle - \( \delta_2 \).
With the resulting values of $\delta_1$ and $\delta_2$ by the formula (3) and (4) $R_{\delta}$ and $R_c$ are calculated. With a small value of deviation of the new $R_c^{\text{new}}$ from the old $R_c^{\text{old}}$ value, the iteration process stops. This has the following form:

$$100 \left( \frac{R_c^{\text{new}}}{R_c^{\text{old}}} - 1 \right) < \Delta$$

where $\Delta$ is the set accuracy of approximation, $\Delta = 0.1\%$.

If the inequality (10) is not met with the obtained values of $\delta_1$, $\delta_2$ and $R_{\delta}$ the new wheel loads are determined $F_{1v}$, $F_{1m}$, $F_{2v}$, $F_{2m}$ and the lateral forces $F_{y1}$ and $F_{y2}$. The characteristics of slipping are constructed $\delta_{1v} = f\left(F_{y1}\right)$, $\delta_{1m} = f\left(F_{y1m}\right)$, $\delta_{2v} = f\left(F_{y2}\right)$ and $\delta_{2m} = f\left(F_{y2m}\right)$ by the method of K. Enke [11]. The new values of $R_{\delta}$, $\delta_1$, $\delta_2$ and $R_c^{\text{new}}$ are determined, until the inequality is fulfilled (10). The radius of the turn is specified $R_c$, the slipping of the front and rear axle at the set value of $\theta_{\mu}$. Then the speed is increased and the calculations are repeated until the inequality is met (10). The speed increase is performed until, as shown in figure 3 there is no intersection point on the characteristics of lateral slip for inner and outer wheels for the front or rear axle. If there is no intersection between the slips $\delta_{1v}$ and $\delta_{1m}$ the front axle is sliding and the critical speed is determined $V_{1k\mu}$, while in the absence of the intersection point between the slips $\delta_{2v}$ and $\delta_{2m}$ the rear axle is sliding - the critical speed is $V_{2k\mu}$.

![Figure 3](image.png)

**Figure 3.** Graphical determination of critical speeds against sliding of the front and rear axle.

### 3. Interactive system for determining the critical speeds of car’s movement in a turn

Figure 4 presents a block-scheme of the improved program, operating on the basis of the presented in [11] and presently algorithm. The user’s communication with the created program is by the dialogue mode, i.e. question-answer. Initially, the geometric and mass parameters of the test vehicle are entered by the user: the initial vehicle speed value $V$, the initial values for the angles of rotation of the inner $\theta_{i}$ and outer $\theta_{m}$ wheels around the steering axis and the step $\Delta \theta_{\mu}$, with which the average angle of the steered wheels changes. This allows at different speeds of movement to specify the accuracy in determining the critical speeds against sliding and overturning. The interactive system performs a quick and accurate calculation of the parameters of the turn. The program can be used to determine which of the two axles is first sliding and what the value of the critical speed against sliding is. It is checked at what speed the vehicle will overturn, by monitoring the value of $\theta_{\mu}$, in which the vertical reactions $(F_{2v}, F_{2v})$ of the inner wheels become zero or negative.
Figure 4. Block-scheme of the interactive system.
The resistance is determined at the smallest of the three speeds \( V_k \cdot V_{1k} \cdot V_{2k} \).

4. Results

The subject of the study is a passenger car Honda Civic Aerodeck 1.5iLS with the following parameters: mass of the car \( m = 1665 \, kg \), track width of the front \( B_1 = 1,475 \, m \) and rear \( B_2 = 1,470 \, m \) axle, wheelbase \( L = 2,62 \, m \), maximal angel of rotation of the inner \( \theta_i = 36^\circ \) and outer \( \theta_o = 30^\circ \) wheels around the steered axes, the coordinates of the mass centre of the vehicle, respectively \( b = 1,286 \, m, \) \( h = 0,50 \, m \), low profile tires of size 185/60R14 and maximum speed \( V = 191 \, km/h \).

The graphical determination of the slip of the front \( \delta_1 \) and rear \( \delta_2 \) axles at average angle of the steered wheels \( \theta_{sr} = 1,9^\circ \) and at driving speed \( V = 85 \, km/h \) are presented on figure 5 and 6.

![Figure 5](image1.png)

**Figure 5.** Determination of the slip \( \delta_1 \).

![Figure 6](image2.png)

**Figure 6.** Determination of the slip \( \delta_2 \).

Determination of the critical speeds against sliding of the front and rear axles at average angle of turning of the steered wheels \( \theta_{sr} = 1,9^\circ \) are presented in figure 7 and 8. The obtained critical speed is \( V = 87 \, km/h \).

![Figure 7](image3.png)

**Figure 7.** Determination of the critical speed against sliding of the front axle.

![Figure 8](image4.png)

**Figure 8.** Determination of critical speeds against sliding of the rear axle.

Figure 9 shows the dependence of the radius of the turn \( R_\delta \) of a car on the average angle of turning of the steered wheels in the plane of the road at driving velocity of 20, 50, 80, 110, 140 and 165 \( km/h \).
The analysis in figure 9 shows that by increasing vehicle speed in a turn, the angle of the steered wheels decreases and the radius increases significantly. By increasing speed respectively from 20 to 50, 80, 110, 140 to 165 km/h, the program limits the maximum angle of turning of the steered wheels. From $\theta_s = 32$°, they respectively are reduced to 5,95°, 2,15°, 1,15°, 0,55°, 0,35°, which indicates that the interactive programming system quickly and accurately determines the minimum value of the radius of the turn $R_s$, taking into account the slipping.

Figure 10 and 11 present the results for changes of the vertical reactions of the inner wheels due to the driving speed. Critical speeds against overturning of the front and rear axles at average angle of the steered wheels $\theta_s = 3,2$ are defined.

Figure 12 shows the changes of the critical speeds of the car, taking into account the lateral slip of low profile tires.
Figure 12. Changing critical speed sliding and overturning from the average angle of the steered wheels of the car.

The analysis in figure 12 shows that by increasing the average angle of the steered wheels \( \theta_{sr} \) (respectively the inner and outer wheel in respect to the turn), sliding occurs at lower speeds. The critical speeds against sliding \( V_{k} \) for the two axles of the car are practically the same, due to the virtually symmetrical position of the centre of gravity, and they are presented with one line. The critical speed against overturning is above against sliding, which means that the sliding occurs before the car is overturned.

5. Conclusion
From the expose and the results for the radius of the turn at different speeds and angles of the steered wheels of a car, as well as for the critical speeds against sliding and overturning, the following conclusions are made:

- The proposed improved interactive programming system to study the critical speed of a motion of a car with low and ultra-low profile tires in a turn allows their determination to be made in the design stage of the car.
- The critical speeds against sliding of the front and rear axle for studied car are practically equal.
- When a car with low-profile tires turns, the critical speeds against overturning are higher than critical speeds against sliding, and depending on the condition of the road sliding of the front or rear axle may occur.

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