Inverse proximity effect in superconductor-ferromagnet structures: From the ballistic to the diffusive limit

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The inverse proximity effect, i.e. the induction of a magnetic moment in the superconductor in superconductor-ferromagnet (S/F) junctions is studied theoretically. We present a microscopic approach which combines a model Hamiltonian with elements of the well established quasiclassical theory. With its help we study systems with arbitrary degree of disorder, interface transparency and thickness of the layers. In the diffusive limit we recover the result of previous works: the direction of the induced magnetization is opposite to the one of the F layer. However, we show that in the ballistic case the sign of the induced magnetic moment may be positive or negative depending on the quality of the interface and thickness of the layers. We show that, regardless of its sign, the penetration length of the magnetic moment into the superconductor is of the order of the superconductor coherence length, which demonstrates that the effect has a superconducting origin.

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I. INTRODUCTION

The proximity effect in multilayered structures consisting of superconductors and normal metals has been studied intensively in the last decades. Since the sixties it is well known that superconducting correlations can penetrate into a normal region changing the physical properties of the latter [1]. More recently special interest has been paid to the study of the proximity effect in superconductor-ferromagnet structures. The presence of an exchange field acting on the spin of the electrons leads to interesting new physics, as for example oscillations in the superconducting critical temperature and in the density of states (DoS) of the ferromagnet as a functions of its thickness [2, 3, 4].

The interplay between superconductivity and magnetism also leads to a change of the sign of the Josephson current in S/F/S structures [5, 6] (for a review see [7]). More recently special interest has been paid to the study of the proximity effect in superconductor-ferromagnet (S/F) junctions is studied theoretically. We present a microscopic approach which combines a model Hamiltonian with elements of the well established quasiclassical Green functions formalism. The results of Ref. [8] demonstrate that the direction of the induced magnetic moment is opposite to the one in the ferromagnet. The authors of Ref. [8] also shown that this effect is related to the induction of a triplet component (with the zero projection of total spin) in the pairing amplitude.

The interest in the inverse proximity effect has been increased with the development of experimental techniques like neutron reflectometry [12] and muon spin rotation [13] which allow to determine accurately the spatial distribution of magnetic moments. For example experiments on multilayered system consisting of the high $T_C$ superconductor YBa$_2$Cu$_3$O$_7$ and the ferromagnet L$_2$3C$_1$/3MnO$_3$ layers have shown an induced magnetic moment in the S layers with a sign opposite to the magnetization of the ferromagnet [12]. However, a conclusive explanation of this experiment has not been given yet.

More recently the inverse proximity effect was studied theoretically in atomic-scale F/S/F trilayers [10]. It was shown that if the magnetization of the F layers points in the same direction, the S layer acquires a spin polarization parallel to the one in the F layers. At first glance this result contradicts the one of Ref. [8], but until now no detailed comparison between these two works has been done. For completeness we mention that in Ref. [8] the induced magnetization in a clean S/F bilayer was computed for highly polarized ferromagnets and a leakage of the magnetic moment into the superconductor over atomic distances was obtained. In Ref. [14] the induced magnetic moment in a clean superconductor in contact with a magnetic wall was studied.

In this work we present a microscopic model which allows to study the inverse proximity effect for systems with an arbitrary degree of disorder, interface transparency and layer thickness. The model is based on the Hamiltonian approach of Refs. [15, 16, 17]. Within this model we show how disorder can be introduced by an appropriate averaging procedure in such a way as to recover the quasiclassical theory in the diffusive limit.

The main results of the paper can be summarized as follows: For an atomic-size bilayer in the pure ballistic limit, in which the component of the momentum parallel to the plane of the layers is conserved, the induced magnetic moment has a positive sign in accordance with the results of Ref. [10]. However this is only valid for low values of the interface transmission coefficient $\tau_n$. When $\tau_n$ is of the order of unity the sign of the induced magnetization may be negative. These results can be understood in terms of the behavior of the bilayer Andreev states as discussed in Sect. III. When disorder is included and the interface becomes diffusive we recover the result obtained in Ref. [8], according...
to which the induced magnetic moment in S is negative regardless of the interface transparency. We also consider
the intermediate situation and show that one can go smoothly from the ballistic to the diffusive regime. In section
IV we consider the case of a S layer with thickness much larger than the superconducting coherence length $\xi_S$. We
show that the sign of the induced magnetic moment in the superconductor depends on the thickness of the F layers
as well as on the transmission of the interface. We also show that the penetration of the magnetic moment into
the superconductor is in all cases of the order of $\xi_S$. This demonstrates that the effect is governed by superconducting
correlations and is not just a “magnetization leakage” over atomic distances into the S region.

The rest of the paper is organized as follows: in Sect. II we introduce the model Hamiltonian. Details of the model
are presented in the appendix, where we also show the equivalence between our model and the quasiclassical approach
in the appropriate limit. In section III we study the inverse proximity effect in a S/F bilayer of atomic-size, while in
section IV we consider layers of arbitrary thickness. Finally section V is devoted to the concluding remarks.

II. THEORETICAL APPROACH

In order to describe a S/F layered structure we use a lattice model Hamiltonian as in Ref. [17]. This can be written
as $H = H_S + H_F + H_{SF}$ where

$$H_S = \sum_{i,\sigma=\pm} \epsilon_S a_{i,\sigma}^+ a_{i,\sigma} + \sum_i \Delta_i (a_{i,+}^+ a_{i,-}^+ + a_{i,-} a_{i,+}) + \sum_{<i,j>,\sigma=\pm} t_{ij} (a_{i,\sigma}^+ a_{j,\sigma} + a_{i,\sigma}^+ a_{j,\sigma})$$

$$H_F = \sum_{i,\sigma=\pm} \epsilon_F a_{i,\sigma}^+ a_{i,\sigma} - \sum_i h_i (a_{i,+}^+ a_{i,+} - a_{i,-} a_{i,-}) + \sum_{<i,j>,\sigma=\pm} t_{Fij} (a_{i,\sigma}^+ a_{j,\sigma} + a_{i,\sigma}^+ a_{j,\sigma}),$$

are the terms describing the uncoupled superconducting and ferromagnetic regions. Indexes $i, j$ label different sites
along the system. The coefficients $t_{ij}$ are the hopping amplitudes between neighboring sites. The site energies $\epsilon_i$ are
measured from the Fermi level. $\Delta_i$ is the local order parameter and $h_i$ is the local exchange field in the ferromagnet.
Throughout this paper we assume that $\Delta$ ($h$) is a given parameter which is constant and finite in the superconductor
(ferromagnet) and is equal to zero in the ferromagnet (superconductor).

Finally $H_{SF}$ is the term which describes the coupling between the superconducting and ferromagnetic regions, which is given by

$$H_{SF} = \sum_{i \in S, j \in F, \sigma=\pm} t_{SF,ij} (a_{i,\sigma}^+ a_{j,\sigma} + a_{i,\sigma}^+ a_{j,\sigma}).$$

Written in this way the Hamiltonian can be used to describe many different physical situations. In particular we
would like to consider not only ballistic but also diffusive systems. One can include disorder either in the diagonal
terms, i.e. introducing a random correction $\delta \epsilon_i$ for the site energies, or $\delta t_{ij}$ for the hopping terms. One can also model
a disordered interface by adding a random term $\delta t_{ij}$ to the hopping elements of Eq. (3).

The Green functions $\hat{G}_i(k)$, which are $4 \times 4$ matrices in the particle-hole (Nambu) $\otimes$ spin space should be determined
from the Dyson equation (see for example Refs. [17,18])

$$\hat{G}_S(k) = \hat{g}_S(k) + \hat{g}_S(k) \hat{\Sigma}_S(k) \hat{G}_S(k),$$

where $\hat{g}_S(k)$ is the Green function at the interface on the S(F) side for the uncoupled regions. As discussed in the
appendix A the self-energy $\hat{\Sigma}_S(k)$ in the ballistic case is given by

$$\hat{\Sigma}_S^b(k) = t_{SF}^2 \tau_3 \hat{g}_S^b(k) \tau_3,$$

while in the case of a diffusive interface has the form

$$\hat{\Sigma}_S^d(k) = \delta t^2 \tau_3 <\hat{G}_S(k)\tau_3>,$$

where $< ... >$ denotes integration over parallel momentum $k$. The problem is then reduced to a closed set of self-
consistent equations which can be solved as discussed in the next sections.

With the help of this model we can also describe a diffusive metal by assuming that also the hopping terms between
the planes inside the S and F layers are randomly distributed. As we show in the appendix B after averaging over
disorder our model leads to a discretized version of the well known Usadel equation [19]. Thus, our model reproduces,
in the diffusive limit, the results of Refs. [8, 20] for the induced magnetization.
An intermediate situation between the purely ballistic and the diffusive limits can be analyzed by introducing a parameter $\alpha$ controlling the proportion of the hopping term which is random, i.e. $t_B = \alpha t_{SF}$ and $\delta t = (1 - \alpha) t_{SF}$ in such a way that $\alpha = 1$ ($\alpha = 0$) corresponds to the ballistic (diffusive) case. In a general intermediate case the self-energy can be written as a sum of a ballistic $\hat{\Sigma}_b$ and a diffusive $\hat{\Sigma}_d$ contribution, calculated according to Eqs. (5) and (6).

III. THIN S/F BILAYER

Let us consider two thin S/F layers, such that the unperturbed Green functions are given by Eq. (A1). Our aim is to calculate the magnetic moment $M$ induced in the superconductor, which is given by

$$M = \mu_B T \sum_\omega \text{Tr} \left[ (\hat{G}_{S+}(k, \omega) - \hat{G}_{S-}(k, \omega)) \right].$$

As it was mentioned above we assume a linear dispersion relation (flat bands). This means that

$$\langle G \rangle = \nu \int d\epsilon G,$$

where $\nu$ is the normal density of states at the Fermi level. Notice that in the wide-band approximation that we are using here the magnetization arises from changes in the density of states close to the Fermi level.

The exact normal Green functions $\hat{G}_{S,F,\pm}$ for spin up and spin down can be written in the more general case as (see Eq. (11))

$$\hat{G}_{S,\pm} = \left[ \hat{g}_{S,\pm}^{-1} - \hat{\Sigma}_b^{S,\pm} - \hat{\Sigma}_d^{S,\pm} \right]^{-1} \tag{7}$$

$$\hat{G}_{F,\pm} = \left[ \hat{g}_{F,\pm}^{-1} - \hat{\Sigma}_b^{F,\pm} - \hat{\Sigma}_d^{F,\pm} \right]^{-1}. \tag{8}$$

The self-energies $\hat{\Sigma}_b,\hat{\Sigma}_d$ correspond to the terms defined in Eqs. (5,6). We thus obtain the following set of coupled equations

$$< G_{S,\pm} > = -i \left( \frac{D_{\pm} \Omega_{\pm} - i \epsilon_{k} \langle \hat{G}_{S,\pm} \rangle}{t_B^4 + 2t_B^2(\omega_{\pm} \Omega_{\pm} - \epsilon_{k}^2) + D_{\pm}(\Omega_{\pm}^2 + \Delta_{\pm}^2 + \epsilon_{k}^2)} \right) \tag{9}$$

$$< F_{S,\pm} > = \left( \frac{D_{\pm} \Delta_{\pm}}{t_B^4 + 2t_B^2(\omega_{\pm} \Omega_{\pm} - \epsilon_{k}^2) + D_{\pm}(\Omega_{\pm}^2 + \Delta_{\pm}^2 + \epsilon_{k}^2)} \right) \tag{10}$$

$$< G_{F,\pm} > = -i \left( \frac{D_{\pm} \Lambda_{\pm} + \omega t_B^4}{t_B^4 + 2t_B^2(\omega_{\pm} \Lambda_{\pm} - \epsilon_{k}^2 + \Delta \delta t^2 < F_{S,\pm} > + D_{\pm}(\epsilon_{k}^2 + \delta t^4 < F_{S,\pm} >)} \right) \tag{11}$$

$$< F_{F,\pm} > = \left( \frac{\epsilon_{k}^2 + \delta t^2 < F_{F,\pm} > + D_{\pm}(\epsilon_{k}^2 + \delta t^4 < F_{F,\pm} >)}{t_B^4 + 2t_B^2(\omega_{\pm} \Lambda_{\pm} - \epsilon_{k}^2 + \Delta \delta t^2 < F_{S,\pm} > + D_{\pm}(\epsilon_{k}^2 + \delta t^4 < F_{S,\pm} >)} \right) \tag{12}$$

where we have defined the following quantities: $D_{\pm} = \omega_{\pm}^2 + \epsilon_{k}^2$; $D_S = \omega^2 + \epsilon_{k}^2 + \Delta^2$; $\Omega_{\pm} = \omega + i \delta t^2 < G_{F,\pm} >$; $\Delta_{\pm} = \pm \Delta + \delta t^2 < F_{F,\pm} >$ and $\Lambda_{\pm} = \omega_{\pm} + i \delta t^2 < F_{F,\pm} >$. This set of equations must be solved in a self-consistent

FIG. 1: Geometry of the S/F structure.
way. However for the pure ballistic case, when $\delta t = 0$, the equations in both layers are decoupled. For example it is easy to show that

$$G_{S+} = -\frac{i\omega (\omega^2 + \epsilon_1^2) + it^2\omega_+}{(\epsilon_k^2 - \epsilon_1^2)(\epsilon_k^2 - \epsilon_2^2)},$$

(13)

where

$$\epsilon_{1,2} = \sqrt{-\omega^2 - \omega_+^2 - \Delta^2 + 2t^2 \pm \sqrt{\omega^2 - \omega_+^2 + \Delta^2} - 4t^2 [\omega + \omega_+ + \Delta^2]}$$

(14)

For $G_{S-}$ we obtain a similar expression substituting $\omega_+$ by $\omega_-$. Now the integration over $\epsilon$ can be performed by closing the integration path in the upper half-plane and using the residue theorem.

In Fig. 2 we show the dependence of the induced magnetic moment on the interface transparency, for very low temperature ($T \ll \Delta$) and different values of the parameter $\alpha$ and of the exchange field $h$. $M$ is given in units of the Pauli paramagnetic moment $M_P$ defined as

$$M_P = 2\mu_B h.$$

The interface transparency is characterized by the transmission coefficient $\tau$, which within this model is given by $\tau = 4(t_{SF})^2/(1 + (t_{SF})^2)^2$. Throughout this paper all energies are given in units of $t_S = t_F$.

According to Fig. 2 in the pure ballistic case ($\alpha = 1$) the dependence $M(\tau)$ has a nonmonotonic behavior: in accordance with the results obtained in Ref. [10] for low transparency, the induced magnetic moment is positive. However, at some $\tau$ (when $t_{SF}$ is of the order of $h$) $M$ changes to negative values. If the exchange field is large enough this change of sign would never take place and the magnetization remains positive for all values of $\tau$.

Figure 2 also shows that by decreasing $\alpha$, i.e. by increasing the disorder at the interface, the curves become flatter. In the limiting case of a diffusive interface, i.e. when $\alpha = 0$, our model leads to the same result as the one obtained using the quasiclassical approach in Ref. [8]. This is not surprising since, as it is shown in the appendix, our model in the diffusive limit ($\alpha = 0$) leads to the Usadel equation and therefore all conclusions of Ref. [8] also hold here. In

![Figure 2](image-url)

**FIG. 2**: Induced magnetization $M/M_P$ as a function of the transmission $\tau$ for (a) $h = 0.2$ and (b) $h = 0.4$ and different values of $\alpha$. We have chosen $\Delta = 0.1$ and $T = 10^{-4}$. All energies are given in units of $t_S = t_F$. 
particular the physical picture given in Ref. 8 of Cooper pairs sharing their electrons between both the S and the F layer, may explain the negative induced magnetic moment. From Eqs. (9-12) one can easily check that in the Usadel limit (α = 0) and for low transparencies the first correction to the density of states, in both layers, is proportional to δt^4, i.e. to the square of the interface transparency. This indicates that in this limit the inverse proximity effect arises from higher order processes requiring the induction of a non-vanishing paring amplitude in the ferromagnet. In fact, the Green functions in the F region have a BCS-like form with an induced minigap in the DoS 8 21. We are thus dealing with two coupled condensates with Cooper pairs sharing the electrons between the S and the F layer. This coupling tends to align the spins of both regions in opposite direction.

It would be interesting to find a physical argument for explaining the observation of a positive magnetization in the ballistic regime. In this case one can check from Eqs. (9-12) that if α = 1 and for low transparency, the first correction to the induced magnetic moment is of order t_{SF}^2 and is positive (cf. Ref. 10). However, when τ becomes larger the induced magnetization reaches a maximum and then decreases down to negative values. One can understand this behavior in terms of Andreev states induced in the DoS. In Fig.3 we sketch the position of these states in the spectral density. For values of t_{SF} smaller than h the state corresponding to the majority spin is located below the Fermi energy and thus, the induced magnetization has a positive sign. The corresponding density of states is shown in the inset of Fig. 3 where a peak in the spin-up channel (solid line) can be identified. By increasing the interface transparency and when t_{SF} ~ h the Andreev states for both spin orientations eventually cross. A further increase of τ leads to a negative magnetization, since now the Andreev state below the Fermi level corresponds to the spin-down orientation (the dashed curve in the right inset of Fig. 3 shows the DoS for the spin-down electrons).

The S/F bilayer of atomic thickness analyzed so far should not be considered as a mere idealized example but it could also describe the situation in real high T_C materials consisting of alternating magnetic and superconducting layers.

IV. S/F LAYERS OF ARBITRARY THICKNESS

Let us consider a superconductor of thickness d_S ≫ ξ_S (where ξ_S = 2αt_S/πΔ is the coherence length) in contact with a ferromagnet of arbitrary thickness which is the situation in most experiments 12 22. This case can be analyzed theoretically considering the superconductor as a semi-infinite system. We shall first consider the ballistic regime in which electrons are specularly reflected (k is conserved) at the outer surface of the ferromagnet. In Fig. 4 we show the magnetic moment induced in the superconductor at the S/F interface as a function of the transmission.
FIG. 4: The induced magnetization $M/M_P$ at the S side of the interface as a function of the transmission coefficient $\tau$ for different values of the F-layer thickness $d_F$. The latter is given in units of $\pi\xi_F$. The S layer is assumed to be semi-infinite. We have chosen $h=0.2$, $\Delta=0.1$ and $T=0$. 

Coefficient $\tau$ and for different thickness $d_F$ of the F layer. In the tunnelling limit the effect is very weak as the induced magnetization is proportional to $t_{SF}^4$. However, in the region of high transparency an appreciable magnetic moment can be induced in the superconductor. As it can be observed in Fig. 4, its sign depends on the ratio between the thickness of the F layer and the ferromagnetic length defined as $\xi_F = \alpha t_F / \pi h$: while for thin layers ($d_F \leq \pi\xi_F$) is negative, by increasing the F layer thickness it becomes positive reaching a maximum for $d_F \sim 2\pi\xi_F$. For $d_F \gg \xi_F$ the induced magnetization tends to zero. Since the structure is purely ballistic we expect that interference effects take place leading to a nonmonotonic behavior. The change of sign can be understood within the description of Andreev bound states similar to the one depicted in the previous section. According to the calculations of Ref. [17] if $\tau$ is of the order of unity, the crossing of the Andreev peaks associated to the majority and minority spins, and thus the change of sign of the induced magnetization, occurs at $d_{F0} \sim 4\xi_F$. This is in agreement with the present results.

A natural question that arises at this point concerns the penetration length of the induced magnetization. In Fig. 5 we show the spatial dependence of $M$ in the superconductor for two different thicknesses of the F layer and the same values of $h$ and $\Delta$ as in the previous figure. We have chosen $d_F = \pi\xi_F$ and $2\pi\xi_F$ in order to show that whatever the sign of the induced magnetization is, the penetration length is of the order $\xi_S$. This demonstrates that the effect described here is due to the superconducting proximity effect and it is not just a magnetization leakage over atomic distances as in Ref. [17].

Finally we analyze the case in which the two layers are connected by a disordered interface. In Fig. 6 we show the dependence of $M/M_P$ on the transmission for different values of the parameter $\alpha$ and $d_F = 2\pi\xi_F$. We observe that by increasing $\alpha$, the induced magnetization is reduced. However, even in the case of a fully disordered interface, a finite magnetization of the order of $10^{-2}M_P$ for large transparency is obtained, demonstrating that the effect is robust against disorder.

V. CONCLUSIONS

We have studied the inverse proximity effect in S/F structures with arbitrary thickness, type of interfaces and degree of disorder. For this purpose we have presented a microscopic model which combines a model Hamiltonian with elements of the quasiclassical theory. We have shown that the sign and magnitude of the magnetic moment induced in the superconductor is highly dependent on the system parameters. In the diffusive limit, where our model leads to a discretized version of the well known Usadel equation, the induced magnetization in S and the one in F
FIG. 5: The spatial dependence of the magnetization induced in a semi-infinite S layer in contact with a F layer of thickness $\pi \xi_F$ and $2\pi \xi_F$. We have chosen $h = 0.2$ and $\Delta = 0.1$.

have opposite directions, in agreement with Ref. [8]. However, we have shown that the induced magnetization can exhibit either sign when the system deviates from the diffusive regime.

We have first considered thin layers and shown that in the purely ballistic case and for low transparency of the interface the induced magnetization is positive. By increasing the transmission the magnetization reaches a maximum and then decreases to negative values. The change of sign takes place when the coupling parameter between the layers is of the order of the exchange field. This behavior can be understood in terms of localized Andreev states and their evolution when varying the coupling strength. The crossover between the ballistic and diffusive limits was also analyzed.

Finally we have studied a bulk superconductor attached to a ferromagnetic layer of arbitrary thickness $d_F$. We have shown that for high interface transparency and small $d_F$ the induced magnetization $M$ is negative. For increasing thickness and when $d_F$ is of the order of $\xi_F$ a change of sign in the induced magnetization takes place. In both cases (positive or negative sign of the magnetization) $M$ penetrates into the superconductor over a distance comparable to the superconducting coherence length demonstrating that the effect is long-ranged.

It is worth mentioning that band structure effects as well as a $d$-wave symmetry for the superconducting order parameter can be straightforwardly included in our Hamiltonian approach. This will be the object of future works.

We expect that experimental techniques such as neutron reflectometry or muon spin rotation used recently in Refs. [12, 13] may help to confirm our predictions.

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APPENDIX A: THE SELF-ENERGY TERM FOR DIFFERENT TYPES OF INTERFACES

Let us consider the purely ballistic case which occurs when the mean free path of electrons is much larger than the characteristic dimension of the system. As we are considering layered structures, with translational invariance parallel to the interface ($y-z$ plane in Fig. 1), it is convenient to perform a change of basis from the local one used in writing the model Hamiltonian into a mixed basis of Bloch waves in the direction parallel to the interface and localized states in the perpendicular direction. Thus, we subdivide the layers into planes parallel to the interface, separated
FIG. 6: Magnetization in the superconductor as a function of \( \tau \) for different type of interfaces: \( \alpha = 1 \) corresponds to the ballistic while \( \alpha = 0 \) to the diffusive one. The other values chosen for \( \alpha \) are 0.5 and 0.75. Here \( d_F = 2\pi \xi_F, h = 0.2, \Delta = 0.1 \) and \( T = 0 \).

by a lattice spacing \( a \) \[27\]. We label these planes by integer numbers \( n \). The new basis is given by the state vectors \( |\vec{k}, n > = \sum_{j \in n} e^{i\vec{k}\vec{R}_j} |j > \), where \( \vec{k} \) is the momentum parallel to the plane of the junction, while \( \vec{R}_j \) is the position of the site \( j \) within the plane. A similar model was considered in Ref. \[23\].

After changing the basis, the field operators in Eqs.(1-3) are now labelled by \( \vec{k} \) and \( n \). We assume that the superconducting order parameter is homogeneous in the superconductor and therefore \( \Delta_k = \Delta \). In principle the dispersion relation \( \epsilon_k \) may be non-linear. However in the present work we assume that all energies involved in the problem (\( \Delta, h, \) etc.) are much smaller than the Fermi energy and thus we can assume a linear and isotropic dispersion relation, i.e. \( \epsilon_k \sim v_F(k - k_F) \), where \( v_F \) is the Fermi velocity. A generalization of this model beyond this quasiclassical approximation and including a more realistic band structure could be implemented straightforwardly.

Since the system is purely ballistic, the parallel momentum \( \vec{k} \) is conserved across the interface. This means that the hopping matrices \( \vec{T}_S, \vec{T}_F, \vec{T}_{SF} \) in the new basis are diagonal, i.e.

\[
< \vec{k}' n | \vec{T} | \vec{k} n + 1 > = \delta_{kk'} t_k.
\]

For simplicity we shall assume that the lattice model is such that \( t_k = \sum_{<i,j>} e^{i\vec{k}\vec{R}_{ij}} t_{ij} \) do not depend on \( \vec{k} \). This is consistent with the approximation of a linear dispersion relation introduced above. Thus, the self-energy term in the Dyson equation Eq.(4) is given by

\[
\Sigma^b_{S(F)}(k) = t_{SF}^2 \tau_3 \gamma_{F(S)}(k) \tau_3,
\]

In order to calculate the interface Green functions \( \gamma_{S(F)} \) of the uncoupled regions one needs to know the functions \( \gamma_{S(F)}^{(0)} \) of the uncoupled planes. These are given by \[18\]

\[
\gamma_{S(F)}^{(0)} = \begin{pmatrix}
\hat{\gamma}_{S(F)}^{(0)+} + \hat{f}_{S(F)}^{(0)+} & 0 \\
0 & \hat{\gamma}_{S(F)}^{(0)-} + \hat{f}_{S(F)}^{(0)-}
\end{pmatrix},
\]

where the normal and anomalous components, \( \hat{\gamma}_{S(F)}^{(0)} \) and \( \hat{f}_{S(F)}^{(0)} \), for spin up (“+”) and down (“−”) are known and
Given by

\[
\begin{align*}
g_{S+}^{(0)} &= \frac{-i\omega - \epsilon_k \tau_3}{\omega^2 + \Delta^2 + \epsilon_k^2}, \quad (A2) \\
g_{S-}^{(0)} &= \frac{+\Delta \tau_1}{\omega^2 + \Delta^2 + \epsilon_k^2}, \quad (A3) \\
g_{F+}^{(0)} &= \frac{i\omega + \epsilon_k \tau_3}{\omega^2 + \epsilon_k^2}, \quad (A4)
\end{align*}
\]

where \(\tau_i\) are the Pauli matrices in particle-hole space, \(\omega_\pm = \omega \mp i\hbar\), and \(\omega = \pi T (2n+1)\) are the Matsubara frequencies. It is clear that the anomalous component \(\tilde{G}\) given by Eq. (A5) does not satisfy the Dyson equation like in the ballistic case (Eq. (4)) but with a different self-energy. In order to obtain an approximate self-energy for the disordered case \(\tilde{G}\) and \(\tilde{\Sigma}\) we use the self-consistent Born approximation, whose diagrammatic representation is sketched in Fig. 7 [24, 25].

The observable quantities are calculated by averaging over disorder configurations. This averaging procedure restores the translational case (Eq. (B1)) but we use the self-consistent Born approximation within the self-consistent Born approximation. The thick lines in this figure correspond to the fully dressed Green function, the crosses represent the hopping processes and the dashed semicircle indicates averaging over the parallel momentum \(k\).

**APPENDIX B: COMPARISON WITH QUASICLASSICS**

According to our model a diffusive metal can be also described by a random coupling \(\delta t_{ij}\) between the layers with a distribution given by Eq. (A3). The Green function at the plane \(n\) is determined by the Dyson equation

\[
G_n = g_n + g_n \delta t_2 < \tau_3 G_{n+1} \tau_3 > G_n + g_n \delta t_2 < \tau_3 G_{n-1} \tau_3 > G_n , \quad (B1)
\]

where \(g_n\) denotes the Green function for the uncoupled plane (cf. Eqs. (A4) and (A3)) and \(< ... >\) the average over momentum. Multiplying this equation by \(g_n^{-1}\) from the left and by \(\tau_3\) from the right and subtracting the conjugated equation multiplied by \(g_n^{-1}\) from the right and by \(\tau_3\) from the left one obtains

\[
\begin{align*}
\tau_3 g_n^{-1}, \tilde{G} &= \left[ \Sigma_{n+1}, \tilde{G}_n \right] + \left[ \Sigma_{n-1}, \tilde{G}_n \right] \\
\end{align*}
\]

where \(\tilde{G}_n = G_n \tau_3, \Sigma_n = t^2 < \tilde{G}_n >\) and \([ , ]\) denotes a commutator.

As the dependence of \(g_n^{-1}\) on the momentum only appears in the term proportional to \(\tau_3\) which vanishes when subtracting the two Dyson equations, the integration over momentum can be done easily and is equivalent to substitute \(\tilde{G}\) by \(< \tilde{G} >\). One obtains

\[
\begin{align*}
\tau_3 g_n^{-1}, \tilde{G}_n &= t^2 \left( < \tilde{G}_{n+1} > + < \tilde{G}_{n-1} > \right) < \tilde{G}_n > - t^2 < \tilde{G}_n > \left( < \tilde{G}_{n+1} > + < \tilde{G}_{n-1} > \right) \\
\end{align*}
\]
Note that by subtracting Eq. (B1) from its conjugate the inhomogeneous term also cancels out. Thus, we need a second equation in order to determine the Green functions. As in the derivation of the quasiclassical equations, in the wide-band approximation considered in this work, the Green functions satisfies the normalization condition $\langle \tilde{G}^2 \rangle = c$, where $c$ is a constant.

One can associate the terms in the r.h.s of Eq. (B3) with the second spatial derivative of a continuous model by means of the identification

$$\frac{<\tilde{G}_{n+1}> + <\tilde{G}_{n-1}> - 2 <\tilde{G}_n>}{a^2} \equiv \partial^2_{xx} \tilde{G}$$

(B4)

From the normalization condition follows that the second derivative of $\tilde{G}$ vanishes and therefore

$$2\partial_x (\tilde{G} \partial_x \tilde{G}) = \tilde{G} \partial^2_{xx} \tilde{G} - \partial^2_{xx} \tilde{G} \tilde{G}$$

(B5)

Thus, using Eqs. (B3, B5) we obtain following equation for $\tilde{G}$

$$[\tau g^{-1}, \tilde{G}] = -2t^2 a^2 \nu \partial_x (\tilde{G} \partial_x \tilde{G})$$

Identifying the factor $t^2 a^2 \nu$ with the diffusion coefficient one sees that this expression is equivalent to the well known Usadel equation [19].

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