Thermodynamics with density and temperature dependent particle masses and properties of bulk strange quark matter and strangelets

X. J. Wen, 1 X. H. Zhong, 2 G. X. Peng, 1, 4 P. N. Shen, 3, 1 P. Z. Ning 3, 2

1 Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
2 Department of Physics, Nankai University, Tianjin 300071, China
3 China Center of Advanced Science & Technology (World Lab.), Beijing 100080, China
4 Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, 77 Mass. Ave., Cambridge, MA 02139-4107, USA

Thermodynamic formulas for investigating systems with density and/or temperature dependent particle masses are generally derived from the fundamental derivation equality of thermodynamics. Various problems in the previous treatments are discussed and modified. Properties of strange quark matter in bulk and strangelets at both zero and finite temperature are then calculated based on the new thermodynamic formulas with a new quark mass scaling, which indicates that low mass strangelets near \( \beta \) equilibrium are multi-quark states with an anti-strange quark, such as the pentaquark \((u^3d^2\bar{s})\) for baryon number 1 and the octaquark \((u^3d^3\bar{s})\) for dibaryon etc.

PACS numbers: 24.85.+p, 12.38.Mh, 12.39.Ba, 25.75.-q

I. INTRODUCTION

One of the most exciting possibility from QCD is that hadronic matter undergoes a rich and varied phase landscape with increasing densities. At extremely high densities, the mass of strange quarks becomes unimportant and all the three flavors of \( u, d, \) and \( s \) quarks can be treated on an equal footing. Consequently, quark matter is in the color-flavor-locked (CFL) phase \( 1 \) and/or a new gapless CFL phase \( (gCFL) 2 \). However, if densities are not that high, the strong interactions between quarks become important, and quark matter is in the unpaired phase. Presently, RHIC is teaching us about the properties of the hot but not asymptotically hot quark gluon plasma \( 3, 4 \). Actually the future FAIR project which will be built in the coming years at GSI in Germany is targeted towards the physics of ultradense matter like it is found in neutron stars \( 5 \).

The original idea of Witten is that strange quark matter (SQM), rather than the normal nuclear matter, might be the true ground state of the strong interaction \( 6 \). Immediately after Witten’s conjecture, Farhi and Jaffe showed that SQM is absolutely stable near the normal nuclear saturation density for a wide range of parameters \( 7 \). Now SQM has been investigated for more than two decades since the pioneer works of many authors \( 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 \). A common feature of these investigations is that quark masses are constant, so the normal thermodynamic formulas can be used without thermodynamic inconsistency problems. Actually, however, it is well known in nuclear physics that particle masses vary with environment, i.e., the density and temperature. Such masses are usually called effective masses \( 23, 24, 25, 26 \). Effective masses and effective bag constants for quark matter have been extensively discussed, e.g., within the Nambu–Jona-Lasinio model \( 27 \) and within a quasiparticle model \( 28 \). In principle, not only masses will change in the medium but also the coupling constant will run in the medium \( 29 \). The question now is how to treat the thermodynamic formulas which do not violate the fundamental principles of thermodynamics, when introducing density and/or temperature dependent masses. In fact, a lot of problems have been caused in this field.

There exist in literature several kinds of thermodynamic treatments. The first one uses all the thermodynamic formulas exactly the same as in the constant-mass case \( 30, 31, 32 \). The second treatment adds a new term, originated from density dependence of quark masses, to both pressure and energy \( 33 \). These two treatments were later proved to be inconsistent with the necessary thermodynamic requirement: their free energy minimum do not correspond to zero pressure \( 34 \). The third treatment adds the term from the density dependence of quark masses to the pressure but not to the energy, and the inconsistency disappear \( 34 \). Another treatment is the addition of a new term to the thermodynamic potential. This has been done when masses depend on either temperature \( 35 \) or chemical potential/density \( 28, 36 \). However, if particle masses depend on both chemical/density and temperature, this way meets difficulties. We will discuss it further in the next section.

With the third thermodynamic treatment and the cubic root scaling \( 37 \), Zheng et al. have studied the viscos-
ity of SQM and calculated the damping time scale due to the coupling of the viscosity and r mode. This model has also been applied to investigating the quark-diquark equation of state and compact star structure. Another important progress has been made recently by Zhang et al. They extended the quark mass density dependent model to finite temperature to let the model be able to describe phase transition. In this case, the quark masses depend on both density and temperature and the permanent confinement is removed. They use the third thermodynamic treatment mentioned above and parametrize the common interacting part of quark masses as

$$m_1 = \frac{B_0}{3n_B} \left[ 1 - a \frac{T}{T_c} + b \left( \frac{T}{T_c} \right)^2 \right].$$

(1)

This extension of the model has soon been applied to the study of strangelets, dibaryons, and proto strange stars. The purpose of the present paper is two-folded. First, we would like to point out that the thermodynamic formulas suitable for the present case. We here explicitly write out the arguments to make the meaning of partial derivatives clear in the following. If the masses $m_i$ are constant, other thermodynamic quantities can be obtained from normal formulas available in textbooks. Here the quark masses are density and/or temperature dependent, i.e.,

$$m_i = m_i(n_b, T),$$

(4)

where the baryon number density $n_b$ is connected to the particle numbers $\{N_i\}$ and volume $V$ by

$$n_b = \frac{\sum_i n_i}{3} \text{ with } n_i = \frac{\sum_i N_i}{V}.$$ 

(5)

To study this question, we start from the fundamental derivation equality of thermodynamics, i.e.,

$$d\bar{E} = Td\bar{S} - PdV + \sum_i \mu_id\bar{N}_i.$$ 

(6)

This is nothing but the combination of the first and second laws of thermodynamics. It means that the energy $\bar{E}$ is the characteristic function, i.e., all other quantities can be obtained from it, if one takes the entropy $\bar{S}$, the volume $V$, and the particle numbers $\{\bar{N}_i\}$ as the full independent state variables. But it is sometimes convenient to take $(T, V, \{\mu_i\})$ as the full independent state variables. In this case, the characteristic function is the thermodynamic potential $\Omega$ which is defined to be

$$\Omega = \bar{E} - T\bar{S} - \sum_i \mu_i\bar{N}_i.$$ 

(7)

II. THERMODYNAMICS WITH DENSITY AND TEMPERATURE DEPENDENT MASSES

Quasi particle models have been explored in great detail over the past 10-15 years. It is well understood how to construct thermodynamically consistent models when the masses depend on the chemical potential and temperature. In the present model, however, there are three differences. First, the particle masses depend on density and temperature, not directly on chemical potential and temperature. Secondly, the density and temperature dependence is independently determined from other arguments. And thirdly, the finite size effects have to be included. Therefore, we derive, in the following, the thermodynamic formulas suitable for the present case. Suppose the thermodynamic potential is known as a function of the temperature $T$, volume $V$, chemical potentials $\{\mu_i\}$, and particle masses $\{m_i\}$, i.e.,

$$\bar{\Omega} = \bar{\Omega}(T, \{\mu_i\}, \{m_i\}, V).$$

(3)

We here explicitly write out the arguments to make the meaning of partial derivatives clear in the following. If the masses $m_i$ are constant, other thermodynamic quantities can be obtained from normal formulas available in textbooks. Here the quark masses are density and/or temperature dependent, i.e.,

$$m_i = m_i(n_b, T),$$

(4)

where the baryon number density $n_b$ is connected to the particle numbers $\{\bar{N}_i\}$ and volume $V$ by

$$n_b = \sum_i \frac{n_i}{3} \text{ with } n_i = \sum_i \bar{N}_i/V.$$ 

(5)

To study this question, we start from the fundamental derivation equality of thermodynamics, i.e.,

$$d\bar{E} = Td\bar{S} - PdV + \sum_i \mu_i d\bar{N}_i.$$ 

(6)

This is nothing but the combination of the first and second laws of thermodynamics. It means that the energy $\bar{E}$ is the characteristic function, i.e., all other quantities can be obtained from it, if one takes the entropy $\bar{S}$, the volume $V$, and the particle numbers $\{\bar{N}_i\}$ as the full independent state variables. But it is sometimes convenient to take $(T, V, \{\mu_i\})$ as the full independent state variables. In this case, the characteristic function is the thermodynamic potential $\Omega$ which is defined to be

$$\Omega = \bar{E} - T\bar{S} - \sum_i \mu_i\bar{N}_i.$$ 

(7)
because adding \(-d(T\bar{S} + \sum_i \mu_i N_i)\) to both side of Eq. \(10\) will give
\[
d\bar{\Omega} = -\bar{S}dT - PdV - \sum_i \bar{N}_i d\mu_i.
\] (8)

Another important characteristic function is the free energy \(\bar{F}\). It is defined to be
\[
\bar{F} \equiv \bar{E} - T\bar{S}.
\] (9)

Then the corresponding basic derivation equation is
\[
d\bar{F} = -\bar{S}dT - PdV + \sum_i \mu_i d\bar{N}_i
\] (10)

which can be obtained by adding \(-d(T\bar{S})\) to Eq. \(10\). Therefore, the independent state variables are \((T, V, \{\bar{N}_i\})\) in this case, i.e.,
\[
\bar{F} = \bar{F}(T, V, \{N_i\}).
\] (11)

According to the second term on the right hand side of Eq. \(10\), one has a general expression for the pressure
\[
P = -\frac{d\bar{F}}{dV}\bigg|_{T,\{N_i\}}.
\] (12)

Here \(\bar{F}\) should be expressed as a function of \((T, V, \{\bar{N}_i\})\), and the derivative is taken with respect to the volume at fixed \(T\) and \(\{\bar{N}_i\}\). Comparing Eqs. \(7\) and \(9\) leads to the basic relation between thermodynamics and statistics, i.e.,
\[
\bar{F} = \bar{\Omega} + \sum_i \mu_i \bar{N}_i
\] (13)

Substituting this into Eq. \(12\) gives
\[
P = -\frac{d\bar{\Omega}}{dV}\bigg|_{T,\{N_i\}} - \sum_i \bar{N}_i \frac{\partial \mu_i}{\partial V}.
\] (14)

Because the independent state variables here are \((T, V, \{N_i\})\), the chemicals \(\{\mu_i\}\) in \(\bar{\Omega}\) [see Eq. \(4\)] should be expressed as a function of \((T, V, \{N_i\})\), i.e.,
\[
\mu_i = \mu_i(T, V, \{N_i\}).
\] (15)

So the total derivative of \(\bar{\Omega}\) with respect to \(V\) at fixed \(T\) and \(\{N_i\}\) is
\[
\frac{d\bar{\Omega}}{dV}\bigg|_{T,\{N_i\}} = \frac{\partial \bar{\Omega}}{\partial V} + \sum_i \frac{\partial \bar{\Omega}}{\partial \mu_i} \frac{\partial \mu_i}{\partial V} + \sum_i \frac{\partial \bar{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial V},
\] (16)

where
\[
\frac{\partial m_i}{\partial V} = \frac{\partial m_i}{\partial V}\bigg|_{T,\{N_i\}} = \frac{\partial m_i}{\partial n_b} \frac{\partial n_b}{\partial V} = -\frac{n_b}{V} \frac{\partial m_i}{\partial n_b}.
\] (17)

Consequently, substitution of Eq. \(10\) into Eq. \(12\) gives
\[
P = -\bar{\Omega} - V\frac{\partial \Omega}{\partial V} + n_b \sum_i \frac{\partial \Omega}{\partial m_i} \frac{\partial m_i}{\partial V} - T \sum_i \frac{\partial \Omega}{\partial \mu_i} \frac{\partial \mu_i}{\partial V} - \sum_i \bar{N}_i \frac{\partial \mu_i}{\partial \mu_i}.
\] (18)

Similarly, for the entropy, we have
\[
\bar{S} = -\frac{d\bar{F}}{dT}\bigg|_{V,\{N_i\}} = -\frac{d\bar{\Omega}}{dT}\bigg|_{V,\{N_i\}} - \sum_i \bar{N}_i \frac{\partial \mu_i}{\partial T}.
\] (19)

Substitution of
\[
\frac{d\bar{\Omega}}{dT}\bigg|_{V,\{N_i\}} = \sum_i \frac{\partial \bar{\Omega}}{\partial \mu_i} \frac{\partial \mu_i}{\partial T} + \sum_i \frac{\partial \bar{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial T}
\] (20)

leads to
\[
\bar{S} = -\sum_i \frac{\partial \bar{\Omega}}{\partial T} \frac{\partial \mu_i}{\partial T} - \sum_i \left( \frac{\partial \bar{\Omega}}{\partial \mu_i} \frac{\partial \mu_i}{\partial T} \right).
\] (21)

The energy can be obtained from Eq. \(7\),
\[
\bar{E} = \bar{\Omega} + \sum_i \mu_i \bar{N}_i + T\bar{S}.
\] (22)

Replacing \(\bar{S}\) here with the expression in Eq. \(21\), one has
\[
\bar{E} = \bar{\Omega} + \sum_i \mu_i \bar{N}_i - T \frac{\partial \bar{\Omega}}{\partial T} - T \sum_i \frac{\partial \bar{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial T} - T \sum_i \left( \frac{\partial \bar{\Omega}}{\partial \mu_i} \frac{\partial \mu_i}{\partial T} \right)
\] (23)

Now we need \(N_i\) (number of flavors) equations to connect \(T, V, \{\mu_i\}\), and \(\bar{N}_i\), so that the functions \(\mu_i(T, V, \{N_k\})\) in Eq. \(15\) can be obtained. Presently, nearly all relevant models adopt \(25, 30, 33, 34, 35, 40, 41, 42, 43, 44, 45\), and Eq. \(24\) become, respectively,
\[
P = -\bar{\Omega} - V\frac{\partial \Omega}{\partial V} + n_b \sum_i \frac{\partial \Omega}{\partial m_i} \frac{\partial m_i}{\partial V} - n_b \sum_i \frac{\partial \Omega}{\partial m_i} \frac{\partial m_i}{\partial V},
\] (25)

\[
E = \bar{\Omega} - \sum_i \mu_i \frac{\partial \bar{\Omega}}{\partial \mu_i} - T \frac{\partial \bar{\Omega}}{\partial T} - T \sum_i \frac{\partial \bar{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial T},
\] (26)

\[
S = -\frac{\partial \bar{\Omega}}{\partial T} - \sum_i \frac{\partial \bar{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial T}.
\] (27)
$$F = \Omega - \sum_i \mu_i \frac{\partial \Omega}{\partial \mu_i},$$  
(28)

$$n_i = -\frac{\partial \Omega}{\partial \mu_i}$$  
(29)

Compared with thermodynamic formulas in the normal case, both the entropy and energy have a new term from the temperature dependence of the masses, while the pressure has a new term due to the density dependence of the masses. The second term in Eq. (25) exists when the finite size effect cannot be ignored, no matter the masses are constant or not. These new terms are understandable with a view to the equalities

$$S = - \frac{d \Omega}{dT}|_{T, \mu}, \quad P = - \frac{d \Omega}{dV}|_{T, \mu},$$  
(30)

and the total derivative rules in mathematics. Here one should pay special attention to the difference between the total derivatives and the partial derivatives in Eq. (18).

For a new quark mass scaling for the strong interactions of the second term in Eq. (31) is indeed fulfilled by Eq. (24), hence the thermodynamic for- 

model assumption Eq. (24), Eq. (31) is a model dependent requirement. Consequently, the thermodynamic for- 

dence of the masses. The second term in Eq. (25) exists when the finite size effect cannot be ignored, no matter the masses are constant or not. These new terms are understandable with a view to the equalities

$$S = - \frac{d \Omega}{dT}|_{V, \mu}, \quad P = - \frac{d \Omega}{dV}|_{T, \mu},$$  
(30)

and the total derivative rules in mathematics. Here one should pay special attention to the difference between the total derivatives and the partial derivatives in Eq. (18) and Eq. (24).

From the viewpoint of quasiparticle models, the first and second terms in the pressure Eq. (25) are the normal quasiparticle contributions, while the last extra term is the contribution of mean-field interactions.

Normally, the first term on the right hand side of Eq. (27) goes to zero when the temperature approaches to zero, i.e., \(\lim_{T \to 0} \partial \Omega/\partial T = 0\). However, the first factor of the second term does not, i.e., \(\lim_{T \to 0} \partial \Omega/\partial n_i \neq 0\). Therefore, one must require

$$\lim_{T \to 0} \frac{\partial \Omega}{\partial T} = 0$$  
(31)

to be consistent with the third law of thermodynamics, i.e., \(\lim_{T \to 0} S = 0\). Because Eq. (27) depends on the model assumption Eq. (24), Eq. (31) is a model dependent requirement. Consequently, the thermodynamic formulas Eqs. (25)-(29) are merely valid for systems whose interactions meet this requirement. In the subsequent section, we will see that Eq. (31) is indeed fulfilled by a new quark mass scaling for the strong interactions of quarks.

In Ref. [34], we mentioned a necessary condition any consistent thermodynamic treatment must satisfy. At finite temperature, we also have a similar criterion:

$$P = - \frac{d \hat{F}}{dn_b}|_{T, \{N_b\}} \frac{\partial n_b}{\partial n_b} = n_b^2 \frac{d}{dn_b} \left( \frac{F}{n_b} \right)|_{T, \{N_b\}}.$$  
(32)

In obtaining the second equality of Eq. (32), the equalities \(V = \sum_i N_i/(3n_b), \partial n_b/\partial V = -n_b/V\), and \(\hat{F} = V F\) have been used. Because \(F/n_b = \hat{F}/(\sum_i N_i/3)\) is the free energy per baryon, Eq. (32) shows explicitly that the free energy extreme occurs exactly at zero pressure. This is why people are interested in the free energy minimum, rather than the energy minimum, at finite temperature, to look for mechanically stable states.

The extra term or the last term in Eq. (25) is important even in the MIT bag model when one introduces a density dependent bag constant \(B(n_b)\). In this context, the extra term is \(n_b dB/dn_b\). If it is not included, the zero pressure will not be located at the free energy minimum. In Ref. [47], this term has been considered in the calculation of hadron-quark phase transition in dense matter and neutron stars within the bag model.

The derivation process of the pressure is also instructive to the case when one wants to include the Coulomb contribution. In this case, the energy density, accordingly the thermodynamic potential density, gets a term \(E_{\text{Coul}}(\{n_b\}, R)\). Correspondingly the pressure gets

$$P_{\text{Coul}} = -E_{\text{Coul}} + \sum_i n_i \frac{\partial E_{\text{Coul}}}{\partial n_i} - \frac{R}{3} \frac{\partial E_{\text{Coul}}}{\partial R}. \quad (33)$$

In literature, there is another approach to have thermodynamic consistency by adding a term \(B^*\) to the original thermodynamic potential density. This makes the total thermodynamic potential density becomes \(\Omega + B^*\). We comment that this can be unconditionally done merely in two special cases, i.e., at finite temperature with zero chemical potential and at finite chemical potential with zero temperature. The expression of the added term for the former case is [35]:

$$B^*(T) = - \int_{T_0}^T \frac{d \Omega}{d m_i} \frac{d m_i}{dT} dT,$$  
(34)

while that for the latter case is [28]:

$$B^*(\mu) = - \int_{\mu_0}^{\mu} \frac{d \Omega}{d m_i} \frac{d m_i}{d \mu} d\mu.$$  
(35)

However, if one wants to extend this to both chemical potential/density and temperature dependent masses, difficulties arise. To perform the integration in Eqs. (34) and (35), one should use \(m = m(T)\) to replace the \(m\) on the right hand side of Eq. (34) or apply \(m = m(\mu)\) to the right hand side of Eq. (35). If \(m_i = m_i(T, \{\mu_k\})\), the directly extended integration is

$$B^*(T, \mu) = - \int \left[ \sum_i \frac{\partial \Omega}{\partial m_i} \frac{d m_i}{dT} + \sum_{i,j} \frac{\partial \Omega}{\partial m_i} \frac{d m_i}{d \mu_j} d\mu_j \right]. \quad (36)$$

In the above, \((T_0, \mu_0)\) is some reference point. Eq. (36) is a multi-dimensional integration. To let it be path-independent, one must mathematically require Cauchy conditions. If all chemical potentials are equal, for example, the Cauchy condition is

$$\sum_i \left[ \frac{\partial^2 \Omega}{\partial m_i \partial \mu} \frac{d m_i}{d \mu} - \frac{\partial^2 \Omega}{\partial m_i \partial T} \frac{d m_i}{dT} \right] = 0. \quad (37)$$

Such an example has recently been given in Ref. [48], where the Cauchy condition Eq. (37), or the equivalent Maxwell relation Eq. (7) in Ref. [48], is fulfilled by solving the equation for the coupling in the masses, As the
Eq. (8) of Ref. 48 indicated. However, if the masses are completely determined from other arguments, and so there are no adjustable parameters, this thermodynamic treatment may fail. When masses depend on density and temperature, rather than directly on chemical potential and temperature, the case becomes much more involved. Also, if finite size effects can not be ignored, or in other words, $$\Omega$$ depends explicitly on the volume or radius, not merely the integration in Eq. (43) becomes much more difficult or impossible, the integration in Eq. (44) or in Eq. (65) has an unknown function of the volume or radius as well.

These difficulties are not surprising. In fact, we have proved, from Eq. (6) to Eq. (24), that Eqs. (25)-(28) are inevitable consequences of Eq. (3) with Eq. (24). In this section, we will apply these formulas to the calculation of properties of both bulk SQM and strangelets.

### III. DERIVATION OF QUARK MASS SCALING

In the preceding section, we have derived the thermodynamic formulas suitable for systems with density and/or temperature dependent masses. In this section, we derive quark mass scaling by a similar method as in Ref. 37.

Let’s schematically write the QCD hamiltonian density for the three flavor case as

$$H_{\text{QCD}} = H_k + \sum_{q=u,d,s} m_q \bar{q}q + H_I,$$

(38)

where $$m_q$$ ($$q=u,d,s$$) are the quark’s current mass, $$H_k$$ is the kinetic term, $$H_I$$ is the interacting part.

Now we want to include interaction effects within an equivalent mass $$m_q$$. For this purpose we define an hamiltonian density of the form

$$H_{\text{eqv}} = H_k + \sum_{q=u,d,s} m_q \bar{q}q,$$

(39)

where $$m_q$$ is our equivalent mass to be determined. We firstly divide it into two parts, i.e.

$$m_q = m_{eq} + m_1.$$  

(40)

The first part $$m_{eq}$$ ($$q = u, d, s$$) are the quark current masses while $$m_1$$ is a common part for all the three flavors to mimic the strong interaction. Obviously we must require that the two hamiltonian densities $$H_{\text{eqv}}$$ and $$H_{\text{QCD}}$$ have the same eigenenergy for any eigenstate $$|\Psi\rangle$$, i.e.

$$\langle \Psi | H_{\text{eqv}} | \Psi \rangle = \langle \Psi | H_{\text{QCD}} | \Psi \rangle.$$  

(41)

Applying this equality, respectively, to the state $$|n_b, T\rangle$$ and the vacuum $$|0\rangle$$, and then taking the difference, we have

$$\langle H_{\text{eqv}} \rangle_{n_b, T} - \langle H_{\text{eqv}} \rangle_0 = \langle H_{\text{QCD}} \rangle_{n_b, T} - \langle H_{\text{QCD}} \rangle_0.$$  

(42)

where the symbol definitions $$\langle O \rangle_{n_b, T} \equiv \langle n_b, T | O | n_b, T \rangle$$ and $$\langle O \rangle_0 \equiv \langle 0 | O | 0 \rangle$$ have been used for an arbitrary operator $$O$$. Then solving for $$m_1$$ from this equation gives

$$m_1 = \frac{\sum_{q=u,d,s} \langle H_I \rangle}{\langle \bar{q}q \rangle_{n_b, T} - \langle \bar{q}q \rangle_0},$$  

(43)

where $$\langle H_I \rangle \equiv \langle H_{\text{eqv}} \rangle_{n_b, T} - \langle H_{\text{eqv}} \rangle_0$$ is the interacting part of the energy density from strong interactions between quarks. It can be linked to density $$n_b$$ and temperature $$T$$ by

$$\langle H_I \rangle = 3n_b v(\bar{r}, T).$$  

(44)

Here

$$\bar{r} = \left(\frac{2}{\pi n_b}\right)^{1/3}$$  

(45)

is the average distance of quarks at the density $$n_b$$, $$v(\bar{r}, T)$$ is the interaction between quarks at density $$n_b$$ and temperature $$T$$. Because we are interested in the confinement effect, while the lattice simulation 49 and string model investigation 50 show that the confinement is linear, we write

$$v(n_b, T) = \sigma(T)\bar{r}. $$  

(46)

The temperature dependence of the string tension $$\sigma(T)$$ can be obtained by combining the Eqs. (94) and (91) in Ref. 51:

$$\sigma(T) = \sigma_0 - \frac{4T}{a} \exp\left(-\frac{2\sigma_0 a}{T}\right),$$  

(47)

where $$a$$ is the lattice spacing while $$\sigma_0$$ is the string tension at zero temperature. The value of $$\sigma_0$$ from potential models varies in the range of (0.18, 0.22) GeV$^2$ 52.

For convenience, let’s define a dimensionless constant $$\lambda = 2\sigma_0 a/T_c$$ with $$T_c$$ being the critical temperature. Then substituting $$a = \lambda T_c/(2\sigma_0)$$ into Eq. (47) gives

$$\sigma(T) = \sigma_0 \left[1 - \frac{8T}{\lambda T_c} \exp\left(-\frac{\lambda T_c}{T}\right)\right].$$  

(48)

Because the string tension should become zero at the deconfinement temperature, the value of $$\lambda$$ is determined by the equation $$\sigma(T_c) = 0$$ whose solution is

$$\lambda = \text{LambertW}(8) \approx 1.60581199632.$$  

(49)

Accordingly, Eq. (46) becomes

$$v(n_b, T) = \frac{(2/\pi)^{1/3} \sigma_0}{n_b^{1/3}} \left(1 - \frac{8T}{\lambda T_c} \exp(-\lambda T_c/T)\right).$$  

(50)

The inter-quark potential has been also studied by comparison to lattice data in Ref. 52. Replacing the factor $$\exp(-\mu r)$$ in the Eq. (2.8) there with $$1 - \mu r$$, one
will find the linear confining part. The temperature factor used there is $1 - (T/T_c)^2$, and this temperature factor has also been used in Ref. [10]. However, because of a problem with the radius-temperature relation mentioned in the introduction part, the factor has been modified to $1 - 0.65 T/T_c - 0.35 (T/T_c)^2$. The temperature factor derived in the present paper is $1 - \frac{8T}{\lambda T_c} e^{-\lambda T_c/T}$. These temperature factors are compared in Fig. 1.

Chiral condensates have been extensively studied in literature [54, 55]. In principle, they depend on both density and temperature. Presently for simplicity, we only consider their density dependence and use the model-independent result

$$\langle \bar{q}q \rangle_{n_b} = 1 - \frac{n_b}{\rho^*}, \quad (51)$$

with

$$\rho^* = \frac{m_N^2 f^2}{\sigma_N}. \quad (52)$$

When taking 140 MeV for the pion mass $m_\pi$, 93.3 MeV for the pion decay constant $f_\pi$, and 45 MeV for pion-nucleon sigma term $\sigma_N$, one has $\rho^* \approx 0.49$ fm$^{-3}$.

Substituting Eqs. (51), (50), and (44) to Eq. (43), we obtain

$$m_1(n_b, T) = \frac{D}{n_b^0} \left[ 1 - \frac{8T}{\lambda T_c} \exp \left( -\frac{\lambda T_c}{T} \right) \right]. \quad (53)$$

Here $z = 1/3$. Please note, many density and temperature independent constants such as the vacuum condensates, the string tension at zero temperature, and $\rho^*$ et al. are grouped into a constant $D$, i.e.,

$$D = \frac{3(2/\pi)^{1/3} \sigma_0 \rho^*}{\lambda T_c^4}. \quad (54)$$

Taking $\sigma_0 = 0.18$ GeV$^2$ and $\rho^* = 0.49$ fm$^{-3}$, $\sqrt{D}$ value is in the range of (147, 270) MeV when $-\sum_q \langle \bar{q}q \rangle_0$ varies from $3 \times (300$ MeV$)^3$ to $3 \times (200$ MeV$)^3$. Because of many uncertainties within the relevant quantities and the fact that Eq. (51) is not exactly valid, at least it ignores temperature dependence and higher orders in density, we do not try to use the relevant quantities to calculate the value for $D$ from Eq. (54). Instead, we treat $D$ as a free parameter to be determined by stability arguments, i.e., it makes the energy per baryon $E/n_b$ at zero temperature is greater than 930 MeV for two flavor quark matter in order not to contradict standard nuclear physics, but less than 930 MeV for three flavor quark matter so that SQM can have a chance to be absolutely stable. Obviously, the range determined by this method depends on the thermodynamic formulas and the values of quark current masses. In the present calculation, we use $m_{u,0} = 5$ MeV, $m_{d,0} = 10$ MeV, and $m_{s,0} = 120$ MeV for the current masses involved. These conditions constrain $\sqrt{D}$ to a very narrow range of (154.8278, 156.1655) MeV, and we take $D = (156$ MeV$)^2$ in this paper.

The model described in the above is a combination of a chiral model and a string model. There should be nothing really wrong with it. In fact, its zero-temperature form has been successfully applied to studying the properties of SQM [34, 37], calculating the damping time scale of strange stars due to the coupling of the viscosity and r mode [35], and investigating the quark di-quark equation of state and compact star structure [36]. Furthermore, various applications of the conventional quasi particle models with chemical and temperature dependent masses have turned out to be very successful [46, 48]. Therefore, we will apply the specific model presented here to the investigation of SQM in bulk and strangelets in the following.

IV. PROPERTIES OF BULK STRANGE QUARK MATTER AT FINITE TEMPERATURE

As usually done in this model, the quasiparticle contribution to the total thermodynamic potential density of SQM is written as

$$\Omega = \sum_i \Omega_i(T, \mu_i, m_i). \quad (55)$$

where the summation index $i$ goes over $u, d, s$ quarks and electrons. Anti-particles are treated as a whole with particles, i.e., the contribution of the particle type $i$ to the thermodynamic potential density is

$$\Omega_i = -\frac{g_i}{2\pi^2} \int_0^\infty \left\{ \ln \left[ 1 + e^{-(\epsilon_i - \mu_i)/T} \right] \right\}$$
\[ n_i = \frac{g_i}{2\pi^2} \int_0^\infty \left[ \frac{1}{1 + e^{(\epsilon_{i,p} - \mu_i)/T}} - \frac{1}{1 + e^{(\epsilon_{i,p} + \mu_i)/T}} \right] p^2 dp, \]  

(56)

where \( m_i, \mu_i, \), and \( T \) are, respectively, the particle masses, chemical potentials, temperature, and \( \epsilon_{i,p} = (p^2 + m_i^2)^{1/2} \) is the dispersion relation. The particle number density corresponding to the particle type \( P \) is

\[ F \]

\[ = \partial \Omega / \partial \mu_i, \]

(57)

The free energy density \( F \), the entropy density \( S \), and the pressure \( P \) are, respectively,

\[ F = \sum_i F_i = \sum_i (\Omega_i + \mu_i n_i), \]

(59)

\[ S = \sum_i S_i = \sum_i \left( \frac{\partial \Omega_i}{\partial T} - \frac{\partial \Omega_i}{\partial m_i} \frac{\partial m_i}{\partial T} \right), \]

(60)

\[ P = \sum_i P_i = \sum_i \left( -\Omega_i + n_b \frac{\partial m_i}{\partial n_b} \frac{\partial \Omega_i}{\partial m_i} \right). \]

(61)

In the above, the partial derivatives relevant to \( \Omega_i \) are

\[ \frac{\partial \Omega_i}{\partial m_i} = \frac{g_i m_i}{2\pi^2} \int_0^\infty \left[ \frac{1}{1 + e^{(\epsilon_{i,p} - \mu_i)/T}} + \frac{1}{1 + e^{(\epsilon_{i,p} + \mu_i)/T}} \right] p^2 dp \]

(62)

\[ \epsilon_{i,p} \]

and

\[ \frac{\partial \Omega_i}{\partial T} = \frac{g_i}{2\pi^2} \int_0^\infty \left\{ \ln[1 + e^{-(\epsilon_{i,p} - \mu_i)/T}] + \frac{(\epsilon_{i,p} - \mu_i)/T}{1 + e^{(\epsilon_{i,p} - \mu_i)/T}} \right. \]

\[ + \ln[1 + e^{-(\epsilon_{i,p} + \mu_i)/T}] + \frac{(\epsilon_{i,p} + \mu_i)/T}{1 + e^{(\epsilon_{i,p} + \mu_i)/T}} \right\} p^2 dp. \]

(63)

The masses of electrons/positrons (0.511 MeV) and neutrinos/anti-neutrinos (if any) are extremely small. So they can be treated to be zero. For massless particles, the relevant integrations in the above can be carried out to give

\[ \Omega_i = -\frac{g_i}{24} \left( \frac{\mu_i^4}{\pi^2} + 2\mu_i^2 T^2 + \frac{7}{15} \pi^2 T^4 \right), \]

(64)

\[ n_i = \frac{g_i}{6} \mu_i \left( T^2 + \frac{\mu_i^2}{\pi^2} \right), \]

(65)

\[ S_i = \frac{g_i}{6} T \left( \frac{\mu_i^2}{\pi^2} + \frac{7}{15} \pi^2 T^2 \right), \]

(66)

\[ E_i = \frac{g_i}{8} \left( \frac{\mu_i^4}{\pi^2} + 2\mu_i^2 T^2 + \frac{7}{15} \pi^2 T^4 \right), \]

(67)

\[ F_i = \frac{g_i}{8} \left( \frac{\mu_i^4}{\pi^2} + \frac{2}{3} \mu_i^2 T^2 - \frac{7}{45} \pi^2 T^4 \right). \]

(68)

At zero temperature, we have the familiar results

\[ \Omega_i = -\frac{g_i}{48\pi^2} \left[ \frac{1}{m_i^4} \sqrt{\frac{\mu_i^2}{m_i^2}} - m_i^2 (2\mu_i^2 - 5m_i^2) \right] + 3m_i^4 \ln \left[ \frac{\mu_i^2}{m_i^2} + \mu_i \right], \]

(69)

\[ n_i = \frac{g_i m_i}{6\pi^2 |\mu_i|} \left( \frac{\mu_i^2}{m_i^2} \right)^{3/2}, \]

(70)

\[ E_i = \frac{g_i}{16\pi^2} \left[ \frac{1}{m_i^4} \sqrt{\frac{\mu_i^2}{m_i^2}} - m_i^2 (2\mu_i^2 - m_i^2) \right] - m_i^4 \ln \left[ \frac{\mu_i}{m_i} + \sqrt{\mu_i^2 - m_i^2} \right], \]

(71)

\[ \frac{\partial \Omega_i}{\partial m_i} = \frac{g_i m_i}{4\pi^2} \left[ \frac{1}{m_i^4} \sqrt{\frac{\mu_i^2}{m_i^2}} - m_i^2 \right] - m_i^4 \ln \left[ \frac{\mu_i}{m_i} + \sqrt{\mu_i^2 - m_i^2} \right]. \]

(72)

The above formulas show that the number density is an odd function of the corresponding chemical potential, i.e., particle and anti-particle numbers are opposite in sign. But other quantities, such as the thermodynamic potential, entropy, energy, and free energy, are all even functions.

Suppose weak equilibrium is always reached within SQM by the weak reactions such as

\[ d, s \leftrightarrow u + e + \nu_e, \quad s + u \leftrightarrow u + d. \]

(73)

Correspondingly, relevant chemical potentials satisfy

\[ \mu_d = \mu_s, \]

(74)

\[ \mu_d + \mu_u = \mu_u + \mu_e. \]

(75)

We also have the baryon number density equality

\[ \frac{1}{3} (n_u + n_d + n_s) = n_b \]

(76)

and the charge neutrality condition

\[ \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0. \]

(77)

Neutrinos are assumed to enter and leave the system freely. So their chemical potential \( \mu_\nu \) is zero. From Eqs. (62)-(68), they have no contribution at zero temperature, but contribute at finite temperature.
For a given $T$ and $n_b$, the quark masses are obtained from Eqs. (40) and (53):

$$m_q = m_{q0} + \frac{D}{n_b^2} \left[ 1 - \frac{8T}{\lambda T_c} \exp \left( -\frac{T_c}{T} \right) \right].$$  \(78\)

The corresponding partial derivatives are easy to get

$$\frac{\partial m_q}{\partial n_b} = -\frac{zD}{n_b^{2+1}} \left[ 1 - \frac{8T}{\lambda T_c} \exp \left( -\frac{T_c}{T} \right) \right],$$  \(79\)

$$\frac{\partial m_q}{\partial T} = -\frac{8D}{n_b^2} \left[ \frac{1}{\lambda T_c} + \frac{1}{T} \right] \exp \left( -\frac{T_c}{T} \right).$$  \(80\)

The chemical potentials $\mu_i$ ($i = u, d, s, e$) are obtained by solving Eqs. 47-48, all other thermodynamic quantities can then be obtained.

Figure 2 shows the entropy per baryon as a function of temperature for different densities. It is an increasing function of the temperature and goes to zero at zero temperature. This is ensured by the fact that we have $\lim_{T \to 0} \frac{\partial m_q}{\partial T} = 0$ from Eq. (80). It is interesting to note that we did not require this in deriving the scaling Eq. (78) in Sec. III. We got this automatically and naturally.

Figure 3 gives the temperature dependence of the energy and free energy per baryon of SQM. The energy is an increasing function of the temperature while the free energy decreases with increasing temperature.

Figure 4 gives the density dependence of the energy and free energy per baryon of SQM at different temperature. On every energy lines, the zero pressure point is exactly located at the minimum. However, they are generally not at the same points for the energy per baryon, except zero temperature where the energy and free energy is identical.
the zero pressure, satisfying Eq. (32). However, these two points (zero pressure and the minimum) are generally not the same for the energy per baryon at finite temperature. But at zero temperature they coincide because the energy and free energy are equal to each other at zero temperature.

As pointed out in Ref. [40], a density and temperature dependent mass model should have the ability of investigating phase transition. This is demonstrated in Fig. 4. The figure is plotted by adjusting, for a given density, the temperature to such a value that it gives a definite pressure induced in the legend. It is obviously shown that all lines go to the deconfinement temperature $T_c$ as the density approaches to zero. But at zero temperature, the density is different for different pressure. Higher densities correspond to higher pressure.

V. PROPERTIES OF STRANGELETS

To study strangelets, the special problem is to include the finite size effect. We do this by applying the multi-reflection method, originally comprised by Balian and Bloch [57], later developed by Madsen [13], Farhi, Berger, and Jaffe [3, 11] etc., and applied to the mass density and temperature dependent model by Zhang and Su [42, 43, 44]. We express the quasiparticle contribution to the thermodynamic potential density of strangelets as $\Omega = \sum_i \Omega_i(T, \mu_i, m_i, R)$ with

$$\Omega_i = -T \int_0^{\infty} \left\{ \ln \left[ 1 + e^{-\left(\sqrt{p^2 + m_i^2} - \mu_i\right)/T} \right] + \ln \left[ 1 + e^{-\left(\sqrt{p^2 + m_i^2} + \mu_i\right)/T} \right] \right\} n'_i dp \quad (81)$$

where the density of state $n'_i(p, \mu_i, R)$ is given in the multi-expansion approach [57] by

$$n'_i(p, \mu_i, R) = g_i \left[ \frac{p^2}{2\pi^2} + \frac{3p}{R} f_S(x_i) + \frac{6}{R^2} f_C(x_i) \right]. \quad (82)$$

Here $x_i \equiv m_i/p$, the functions $f_S(x_i)$ and $f_C(x_i)$ are

$$f_S(x_i) = -\frac{1}{4\pi^2} \arctan(x_i) \quad (83)$$

and

$$f_C(x_i) = \frac{1}{12\pi^2} \left[ 1 - \frac{3}{2x_i} \arctan(x_i) \right]. \quad (84)$$

Then all other thermodynamic quantities are straightforward from Eqs. (25)-(29). Here is the number density for flavor $i$:

$$n_i = \int_0^{\infty} \left[ \eta^+_i - \eta^-_i \right] n'_i(p, m_i, R) dp, \quad (85)$$

where $\eta^\pm_i$ is the fermion distribution function, i.e.,

$$\eta^\pm_i \equiv \frac{1}{1 + e^{\left(\sqrt{p^2 + m_i^2} \mp \mu_i\right)/T}}. \quad (86)$$

Because we treat the particles and anti-particle as a whole, the number densities $n_i$ can be both positive and negative theoretically. A negative particle number means negative theoretically. A negative particle number means
chemical potential and negative number density in some special cases. We will see such special examples a little later. One may concern that non-monotone of the number density leads to $\partial n/\partial \mu < 0$ which violates the stability condition. However, our results are always located in the regime where the derivative is positive. The regime is marked with full lines in Fig. 4.

The free energy density is

$$F = \sum_i (\Omega_i + \mu_i n_i),$$

(87)

the energy density $E$ is

$$E = \sum_i \left[ \int_0^\infty (\eta_i^+ + \eta_i^-) \sqrt{p^2 + m_i^2} n_i'(p, m_i, R) dp - T \frac{\partial \Omega_i}{\partial m_i} \frac{\partial m_i}{\partial T} \right],$$

(88)

the entropy density is

$$S = \sum_i \left( -\frac{\partial \Omega_i}{\partial T} - T \frac{\partial \Omega_i}{\partial m_i} \frac{\partial m_i}{\partial T} \right),$$

(89)

and the pressure $P$ is

$$P = \sum_i \left( -\Omega_i + n_b \frac{\partial m_i}{\partial n_b} \frac{\partial \Omega_i}{\partial m_i} - \frac{R}{3} \frac{\partial \Omega_i}{\partial R} \right).$$

(90)

In the above, the relevant partial derivatives are

$$\frac{\partial \Omega_i}{\partial T} = \int_0^\infty \ln \left[ \frac{(1 - \eta_i^+)(1 - \eta_i^-)}{(1/\eta_i^+ - 1)^{n_i^+}} \right] n_i' dp,$$

(91)

FIG. 6: Chemical potential dependence of the particle number density at different temperature and mass. The radius is fixed to be 1 fm.

FIG. 7: Chemical potentials and energy per baryon from bulk strange quark matter to finite baryon number at zero temperature and zero pressure.

$$\frac{R}{3} \frac{\partial \Omega_i}{\partial R} = -g_i \int_0^\infty \left[ 2\sqrt{p^2 + m_i^2} + T \ln(\eta_i^+ \eta_i^-) \right] \times \left[ \frac{p}{R} f_S(x_i) + \frac{4}{R^2} f_C(x_i) \right] dp,$$

(92)

and

$$\frac{\partial \Omega_i}{\partial m_i} = \int_0^\infty \left\{ \frac{(\eta_i^+ + \eta_i^-) n_i'}{\sqrt{1 + p^2/m_i^2}} + 2\sqrt{p^2 + m_i^2} + T \ln(\eta_i^+ \eta_i^-) \right\} \frac{\partial n_i'}{\partial m_i} dp,$$

(93)

with

$$\frac{\partial n_i'}{\partial m_i} = \frac{3g_i}{4\pi^2 m_i R^2} \left[ \frac{p}{m_i} \arctan \left( \frac{m_i}{p} \right) - \frac{1 + R m_i}{1 + m_i^2/p^2} \right].$$

(94)

Quark masses and relevant derivatives are still given by Eqs. (88)-(91).

In Ref. [41], charge neutrality is also imposed for strangelets. This is convenient for checking whether the formulas are continuous from bulk SQM to finite baryon number. Fig. 7 gives the energy per baryon and chemical potentials versus baryon number at zero temperature and zero pressure. The horizontal lines are the corresponding bulk values at those temperatures and zero pressure. The radius is fixed to be 1 fm.

Now we treat strangelets in another different way. Instead of imposing the charge neutrality Eq. (77), we require that the electron and positron number densities are...
zero, i.e.,

\[ n_e = 0 \]  \hspace{1cm} (95)

because the radius of strangelets is much smaller than the Compton wavelength of electrons and positrons. Electrons and positrons are not involved in the strong interaction, or in other words, they are not confined, so the finite size terms in Eq. (82) vanish for them. From Eq. (81), we have

\[ n_e = \frac{\mu_e}{3} \left( T^2 + \frac{\mu_e^2}{\pi^2} \right). \]  \hspace{1cm} (96)

Therefore, zero \( n_e \) means zero \( \mu_e \). With a view to the chemical equilibrium Eqs. (74) and (75), we naturally get

\[ \mu_u = \mu_d = \mu_s. \]  \hspace{1cm} (97)

In fact, Eq. (97) is the condition to find out the configuration, which has the lowest energy per baryon, from all possible strangelets with a fixed baryon number \( A \). Due to Eq. (97), only one chemical potential is left independent. And it can then be determined by solving

\[ \frac{1}{3}(n_u + n_d + n_s) = \frac{A}{(4/3)\pi R^3} \]  \hspace{1cm} (98)

for a given baryon number \( A \), temperature \( T \), and radius \( R \).

Figure 8 shows the energy and free energy per baryon as a function of the radius for \( A = 20 \) at different temperature. The points marked with an open circle are the mechanically stable radius where the pressure is zero. The minimum of each line is marked with a triangle. Again we see that these two points are always the same on the free energy line. But they are different on the energy line at finite temperature. However, they coincide at zero temperature because the energy and free energy are equal at zero temperature.

For a given \( A \) and \( T \), the mechanically stable radius is obtained by adjusting it so that the free energy is minimized, or, simply by solving the equation

\[ P = 0. \]  \hspace{1cm} (99)

The temperature dependence of the stable radius for \( A = 20 \) is plotted in Fig. 9 with a solid line. It is obviously an increasing function of temperature. The corresponding energy per baryon is also plotted in the same figure, labeled on the right axis with a dotted line. It is also an increasing function of temperature. However, the free energy per baryon (dashed line) decreases with increasing temperature.

Because charge neutrality is not imposed, strangelets here are charged. The electric charge can be calculated by

\[ Z = \frac{4}{3}\pi R^3 \left( \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s \right). \]  \hspace{1cm} (100)

In Fig. 10 we plot the charge to baryon number ratio
as a function of the baryon number at different temperature. This figure shows that the charge to baryon number ratio is a decreasing function of the baryon number and temperature.

A noticeable feature is that the charge to baryon number ratio at lower baryon numbers ($A \leq 5$ at zero temperature) becomes greater than $1/2$. This is very different from that in the bag model where the charge to baryon number ratio is very small. For normal nuclei, this ratio reaches its biggest value $1/2$. So it seems difficult to understand a heavy positive charge at first sight. In fact, it is caused by the fact that $f_s$ is negative for $1 \leq A \leq 5$ and $T = 0$, i.e., anti-strange quarks, rather than strange quarks, appear. One can see this phenomenon clearly in Fig. 11 where the free energy per baryon and the chemical potential have also been shown. Because of finite size effects, strangelets with very low baryon numbers are metastable for the parameters chosen. If one choose a bigger value for the current quark mass, the value for $D$ should be smaller, and consequently, the mass for strangelets would be smaller.

Fig. 12 shows the quark configuration for low baryon numbers. To see the results clearly, we give the corresponding data in Tab. II. The first column is the baryon number, the second to fourth column are, respectively, the quark numbers $N_u$, $N_d$, and $N_s$. Because shell effects are not taken into account and beta-equilibrium is imposed, fractional quark numbers appear in Tab. 1. Actual quark numbers should naturally be integers. So we approximate these real numbers to integers by $\text{int}(N_i) = \lfloor N_i \rfloor$ ($i = u, d, s$; int means the number before the decimal point). The results are shown in the fifth column. For $A = 1$, we have the pentaquark state ($u^2d^2s^{-1}$). For $A = 2$, we have the dibaryon ($u^4d^3s^{-1}$) or octaquark state. For $A = 3, 4,$ and $5$, we respectively have the multi-quark states ($u^3d^2s^{-1}$), ($u^2d^3s^{-1}$), and ($u^5d^5s^{-1}$). A common feature of these states is that they all include an anti-strange quark. So we use ‘slet’ as the title of the fifth column. The charge number of these slets are given in the sixth column, while the seventh column gives the energies calculated by the present parameters ($D^{1/2} = 156$ MeV and $m_{\pi} = 120$ MeV) with perfect $\beta$ equilibrium. If one would like to produce 1540 MeV (the actual $\Theta^+$ resonant mass) for $A = 1$, then one has to take $D^{1/2} = 186$ MeV and get 2856 MeV for $A = 2$. So we expect that the mass of the octaquark ($u^4d^5s^{-1}$), if truly exists, is near 2856 MeV. For $D^{1/2} = 186$ MeV and $A \geq 3$, the strange quark number becomes positive. So in this case we have only the pentaquark and the octaquark. Because of uncertainties in parameters, and also many other factors e.g. the perturbative interaction has not been included (the quark mass scaling is derived by assuming that the linear confinement interaction dominates), the concrete values should not be taken seriously, and further studies are needed.

Recently, the pentaquark state $\Theta^+(1540)$ has aroused a lot of interest. The width of $\Theta^+(1540)$ is very narrow with upper limit as small as 7 MeV. Cahn and trilling have extracted $\Gamma(\Theta^+) = 0.9 \pm 0.3$ MeV from an analysis of Xenon bubble chamber. Although other hadrons like...
strangelets look like and might decay for different lifetime. With the similar ideas, Ref. [12] studied strangelets within density-and-temperature dependent quark masses and extending the findings in Ref. [13] to finite temperature. Present investigation concentrate on deriving thermodynamical formulas and quark mass scaling, and finding the lowest-energy configuration from the strangelets with a fixed baryon number. Naturally, the observation of heavily positively charged strangelets, or multi-quark states with an anti-strange quark, depends on the value of the parameter $D$. If we took a much larger $D$ value, the charge to baryon number ratio would also be small, or the anti-strange quark would not appear. However, bulk SQM has no chance to be absolutely stable in that case.

VI. SUMMARY

When masses are density and/or temperature dependent, the thermodynamical formulas are different from that for constant masses. We have derived a new set of thermodynamical formulas which can be used to calculate the properties of quark matter within a density and/or temperature dependent quark mass model. The new formulas are also instructive when one introduces a density and/or temperature dependent bag constant in the bag model etc.

We have also argued for a new quark mass scaling at finite temperature. The basic feature is that quark masses and their partial derivative with respect to the temperature go to zero when the temperature approaches to zero. This ensures that all quantities restore to the density dependent model at zero temperature. It is especially important that the entropy goes naturally to zero when the temperature approaches to zero, satisfying the third law of thermodynamics.

With the new thermodynamical formulas and new quark mass scaling, we have studied the properties of bulk SQM and strangelets. It is shown that the free energy minimum corresponds exactly to the zero pressure, both at zero and finite temperature. The mechanically stable strangelet radius increases with temperature. An interesting new observation is that low mass strangelets are heavily positively charged, or appear as multi-quark states with an anti-strange quark, such as the pentaquark ($u^2d^2s$) and the octaquark ($u^2d^3s$) etc., if bulk SQM is absolutely stable.

Acknowledgments

The authors would like to thank support from the DOE (DF-FC02-94ER40818) and NSF (10375074, 90203004, 10475089, 10435080, 10275037). G.X.P also acknowledges hospitality at MIT-CTP. In particular, he is grateful to Prof. E. Farhi, R. Jackiw, R. L. Jaffe, J. W. Negele, K. Rajagopal, and F. Wilczek, for helpful conversations.
[1] K. Rajagopal and F. Wilczek, Phys. Rev. Lett. 86, 3492 (2001).
[2] M. Alford, C. Kouvaris, and K. Rajagopal, Phys. Rev. Lett. 92, 222001 (2004).
[3] M. Gyulassy and L. McLerran, Nucl. Phys. A 750, 30 (2005).
[4] E. V. Shuryak, hep-ph/0405066.
[5] For the FAIR project, one can visit http://www.gsi.de/zukunftsprojekt/kernmateriephysik.html
[6] E. Witten, Phys. Rev. D 30, 272 (1984).
[7] E. Farhi and R. L. Jaffe, Phys. Rev. D 30, 2379 (1984).
[8] A. R. Bodmer, Phys. Rev. D 4, 1601 (1971).
[9] S. A. Chin and A. K. Kerman, Phys. Rev. Lett. 43, 1292 (1979).
[10] M. Creutz, Nucl. Phys. B (Proc. Suppl.) 94, 219 (2001).
[11] M. S. Berger and R. L. Jaffe, Phys. Rev. C 35, 213 (1987).
[12] E. P. Gilson and R. L. Jaffe, Phys. Lett. 71, 332 (1993).
[13] J. Madsen, Phys. Rev. Lett. 70, 391 (1993); Phys. Rev. D 50, 3328 (1994); Phys. Rev. D 47, 5156 (1993).
[14] J. Madsen, Phys. Rev. Lett. 85, 4687 (2000).
[15] J. Schaffner-Bielich, C. Greiner, A. Diener, and H. Stöcker, Phys. Rev. C 55, 3038 (1997).
[16] B. C. Parija, Phys. Rev. C 48, 2483 (1993); 51, 1473 (1995).
[17] Y. B. He, C. S. Gao, X. Q. Li, and W. Q. Chao, Phys. Rev. C 53, 1903 (1996); Y. B. He, W. Q. Chao, C. S. Gao, and X. Q. Li, Phys. Rev. C 54, 857 (1996).
[18] J. Madsen, Phys. Rev. 87, 172003 (2001).
[19] T. Schafer, Nucl. Phys. A 702, 167 (2002).
[20] C. Ratti, Europhys. Lett. 61, 314 (2003).
[21] P. Wang, V. E. Lyubovitskij, T. Gutsche, A. Faessler, Phys. Rev. C 67, 015210 (2003).
[22] J. W. Harris, J. Phys. G 30, S613 (2004).
[23] J. D. Walecka, Oxford Stud. Nucl. Phys. 16, 1.
[24] E. M. Henley and H. Müther, Nucl. Phys. A 513, 667 (1990).
[25] G. E. Brown and M. Rho, Phys. Rev. Lett. 66, 2720 (1991).
[26] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. Lett. 67, 961 (1991); Phys. Rev. C 45, 1881 (1992).
[27] M. Buballa and M. Oertel, Phys. Lett. B 457, 261 (1999).
[28] K. Schertler, C. Greiner, and M. H. Thoma, Nucl. Phys. A 616, 659 (1997).
[29] E. S. Fraga and P. Romatschke, hep-ph/0412298, and quoted therein for a general treatment using perturbative QCD.
[30] S. Chakrabarty, S. Raha, and B. Sinha, Phys. Lett. B 229, 112 (1989); S. Chakrabarty, Phys. Rev. D 43, 627 (1991); Phys. Rev. D 48, 1409 (1993).
[31] J. Cleymans, S. V. Ilyin, S. A. Smolyanski, and G. M. Zinovjev, Z. Physik C 62, 75 (1994).
[32] J. Letessier, J. Rafelski, and A. Tounsi, Phys. Lett. B 323, 393 (1994).
[33] O. G. Benvenuto and G. Lugones, Phys. Rev. D 51, 1989 (1995); G. Lugones and O. G. Benvenuto, ibid. 52, 1276 (1995).
[34] G. X. Peng, H. C. Chiang, B. S. Zou, P. Z. Ning, and S. J. Luo, Phys. Rev. C 62, 025801 (2000).
[35] M. I. Gorenstein and S. N. Yang, Phys. Rev. D 52, 5206 (1995).
[36] P. Wang, Phys. Rev. C 62, 015204 (2000).
[37] G. X. Peng, H. C. Chiang, J. J. Yang, L. Li, and B. Liu, Phys. Rev. C 61, 015201 (2000).
[38] X. P. Zheng, X. W. Liu, M. Kang, and S. H. Yang, Phys. Rev. C 70, 015803 (2004).
[39] G. Lugones and J. E. Horvath, Int. J. Mod. Phys. D 12, 495-508 (2003).
[40] Y. Zhang and R. K. Su, Phys. Rev. C 65, 035202 (2002).
[41] Y. Zhang, R. K. Su, S. Q. Ying, and P. Wang, Europhys. Lett. 56, 361 (2001).
[42] Y. Zhang and R. K. Su, Phys. Rev. C 67, 015202 (2003).
[43] Y. Zhang and R. K. Su, Mod. Phys. Lett. A 18, 143 (2003).
[44] Y. Zhang and R. K. Su, J. Phys. G 30, 811 (2004).
[45] V. K. Gupta, A. Gupta, S. Singh, D. Anand, Int. J. Mod. Phys. D 12, 583 (2003).
[46] A. Peshier, B. Kämpfer, and G. Soff, Phys. Rev. D 66, 094003 (2002); P. Léveaux and U. Heinz, Phys. Rev. C 57, 1879 (1998).
[47] G. F. Burgio, M. Baldo, P. K. Sabu, and H. J. Schulze, Phys. Rev. C 66, 025802 (2002).
[48] A. Peshier, B. Kämpfer, and G. Soff, Phys. Rev. C 61, 045203 (2000).
[49] V. M. Belyaev and Ya. I. Kogan, Phys. Lett. B 136, 273 (1984); K. D. Born, E. Laermann, N. Pirch, T. F. Walsh, and P. M. Zerwas, Phys. Lett. D 40, 1653 (1989).
[50] N. Isgur and J. Paton, Phys. Lett. B 247, 247 (1983); Phys. Lett. D 31, 2910 (1985).
[51] A. Ukafor, in the proceedings of the 1993 Uehling Summer School: Phenomenology and lattice QCD, eds. G. Kicup and S. Sharpe (World Scientific, 1993), p. 231.
[52] S. Veseli and M. G. Olsson, Phys. Lett. B 338, 109 (1996).
[53] C. Y. Wong, Phys. Rev. C 65, 034902 (2001).
[54] For revies see, e.g., G. E. Brown and M. Rho, Phys. Rep. 363, 85 (2002); M. C. Birse, J. Phys. G 20, 1537 (1994); S.P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).
[55] G. X. Peng, Nucl. Phys. A 747, 75 (2005); G. X. Peng, M. Loewe, U. Lombardo, and X. J. Wen, Nucl. Phys. B (Proc. Suppl.) 133, 259 (2004); G. X. Peng, H. C. Chiang, P. Z. Ning, U. Lombardo, and M. Loewe, Int. J. Mod. Phys. A 18, 3151 (2003); G. X. Peng, U. Lombardo, M. Loewe, and H. C. Chiang, Phys. Lett. B 548, 189 (2002).
[56] E. G. Drukarev, M. G. Ryskin and V. A. Sadovnikova, Prog. Part. Nucl. Phys. 47, 73 (2001); Eur. Phys. J. A4, 171 (1999); Z. Phys. A353, 455 (1996).
[57] R. Balian and C. Bloch, Annu. Phys. 60, 401 (1970).
[58] For experimental review, see S. Efimel and et al. (Particle Data Group), Phys. Lett. B 592, 1 (2004).
[59] T. Nakano and K. Hicks, Mod. Phys. Lett. A 19, 645 (2004).
[60] R. N. Cahn and G. H. Trilling, Phys. Rev. D 69, 011501(R) (2004).