Complexity and neutron stars structure

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Abstract

We apply the statistical measure of complexity introduced by López-Ruiz, Mancini and Calbet [1] to neutron stars structure. Neutron stars is a classical example where the gravitational field and quantum behavior are combined and produce a macroscopic dense object. Actually, we continue the recent application of Sañudo and Pacheco [2] to white dwarfs structure. We concentrate our study on the connection between complexity and neutron star properties, like maximum mass and the corresponding radius, applying a specific set of realistic equation of states. Moreover, the effect of the strength of the gravitational field on the neutron star structure and consequently on the complexity measure is also investigated. It is seen that neutron stars, consistent with astronomical observations so far, are ordered systems (low complexity), which cannot grow in complexity as their mass increases. This is a result of the interplay of gravity, the short-range nuclear force and the very short-range weak interaction.

Keywords: Shannon Entropy; Complexity; Self-Organization; Equation of state; Neutron stars.

1 Introduction

Information theory, founded by Shannon to provide a theoretical framework in communications [3], has been further employed as a useful tool to characterize physical systems during the next decades [4, 5, 6, 7, 8]. Since then, a
series of studies concerning the application of information theory to various physical systems has appeared in the literature, while their number exhibits a remarkable rise over the past decade. The dependence of various information-theoretic measures on some parameters of the physical systems has been studied, the presence of correlations has been quantified, connections with experimental data have been detected and universal properties have been extracted. Very recently, these investigations have been extended to include statistical complexity measures, in order to examine self-organizing characteristics of physical systems, patterns and correlations. Although a complete and universal definition for complexity is missing, the current framework provides interesting and satisfactory results expected from intuition. So far, various complexity measures are used taking into account conditions and constraints imposed by the physical system under consideration [9, 10, 11, 12]. Such information and complexity studies (focusing so far on the two statistical complexity measures SDL and LMC [1, 13, 14]) have been applied to various quantum many-body systems i.e. nuclei, atoms, atomic clusters, bosons and molecules [15]-[33]. Recently Sanûdo and Pacheco [2] were the first to extend those studies to an astronomical object i.e. a white dwarf. Specifically, the Shannon information entropy \( S \) and the statistical complexity \( C \) have been calculated in the two kinematics extremes (non-relativistic and relativistic cases) of the electron-degenerate matter of a dwarf.

In the present work we study the information content of another astronomical object, a neutron star. With a mass of 1.4 up to 3 solar masses, a radius of \( \sim 10 \) Km and an average density of \( 10^{14} \) g/cm\(^3\), the neutron star is one of the possible endpoints of stellar evolution. Further gravitational collapse is counterbalanced by repulsive forces originating from Pauli’s exclusion principle, if the mass of the compressed stellar core is less than the Oppenheimer-Volkoff limit of about 3 solar masses.

Neutron stars are systems with several similarities with atomic systems, but there are also fundamental differences. In an atomic system self-organization is reached through the competition between the Coulomb interaction and the Pauli principle. In fact, the long-range electromagnetic interaction is the main interaction among the particles of the system. In addition, atoms are microscopic systems with a typical dimension of a few Angstroms (\( 10^{-10} \) m). In contrast, a neutron star is a macroscopic system with typical dimension of \( 10^4 \) m, much more complicated than the atoms, in the sense that it is organized under the competition of mainly the following three interactions. The long-range force of gravity, whose pressure tends to compress the mass of
the star. The short-range nuclear interaction, which through the degeneracy pressure of the nucleons tends to extend the outer mass of the star. Finally, the very short-range weak interaction, which is a kind of regulator of the particle fraction and thus affects indirectly the properties of the neutron star. The above forces coexist in harmony in the interior of a neutron star. The main features of the structure of a neutron star (mass, pressure and radius) are described by the Tolman-Oppenheimer-Volkoff equations [34], while they also depend strongly on the applied nuclear equation of state.

In this Letter we present a study of the information properties of a neutron star and explore how they are connected with the characteristic properties of the structure of the system, i.e. its mass \( M \) and radius \( R \). Furthermore, we investigate the dependence of information and complexity measures on the nuclear forces, through the asymmetry energy parameter \( c \), and the gravitational constant \( G \). Also we comment on the effect of information measures on the stability of a neutron star, based on the fact that stability regions are characterized by the inequality \( dM/dR < 0 \).

Here, we consider that the temperature of a star is \( T = 0 \), in the context that the Fermi energy is much greater than \( kT \). However, it should be of interest to extend our study and try to connect the thermodynamic properties of a hot neutron star with the information content of the system. Furthermore, it is important to examine the information properties of other astronomical objects e.g. stars consisting of fermions or bosons, with arbitrary masses and interaction strengths. Such a work is in progress.

The outline of this Letter is the following: In Section 2, we define the information and complexity measures employed here, together with a model of neutron stars. In Section 3, we present our results and a discussion, while Section 4 contains a summary.

## 2 The model

### 2.1 Theoretical information measures

The Shannon information entropy \( S \) [3] for a continuous probability distribution \( \rho(r) \), denoting a measure of the amount of uncertainty associated with a probability distribution, is defined as

\[
S = - \int \rho(r) \ln \rho(r) \, dr,
\]  

(1)
while the disequilibrium $D$, being a quadratic distance from equiprobability, is

$$D = \int \rho^2(r) \, dr,$$  \hspace{1cm} (2)

with dimension of inverse volume.

For a continuous probability distribution the disequilibrium is indeed the same measure as the information energy defined by Onicescu [35].

For discrete probability distributions $\{p_i\} = \{p_1, p_2, \ldots, p_N\}$, the information entropy $S = -\sum_{i=1}^{N} p_i \ln p_i$ is minimum ($S_{\text{min}} = 0$) for the distribution of a completely regular system (absolutely localized), where one of the $p_i$’s equals unity, while all the others vanish. The maximum value ($S_{\text{max}} = \ln N$) is attained for the equiprobable distribution (completely delocalized), where $p_i = 1/N$, $i = 1, \ldots, N$. On the other hand, the disequilibrium $D = \sum_{i=1}^{N} (1/N - p_i)^2$, is maximum, $D_{\text{max}} = 1 - 1/N \to 1$ (for large $N$) for a completely regular system, while it is minimum, $D_{\text{min}} = 0$ for an equiprobable distribution.

In the continuous case, an equiprobable probability distribution can be defined as a rectangular function, while a completely regular system corresponds to a $\delta$-like probability distribution function, where the width of the distribution becomes very narrow and its peak extremely high.

In order to study the statistical complexity defined by López-Ruiz, Mancini and Calbet (LMC) [1], we use a slightly modified definition introduced in [36]

$$C = H \cdot D,$$  \hspace{1cm} (3)

where

$$H = e^S,$$  \hspace{1cm} (4)

is the information content of the system, while the exponential functional preserves the positivity of $C$.

The aforementioned definitions of information entropy and disequilibrium in the case of neutron stars are modified as follows:

$$S = -b_0 \int \bar{\epsilon}(r) \ln \bar{\epsilon}(r) \, dr,$$  \hspace{1cm} (5)

and

$$D = b_0 \int \bar{\epsilon}(r)^2 \, dr,$$  \hspace{1cm} (6)
where $b_0 = 8.9 \times 10^{-7} \text{Km}^{-3}$ is a proper constant satisfying the condition that both information entropy $S$ and disequilibrium should be dimensionless quantities, while $\bar{\epsilon}(r)$ is the dimensionless energy density of the system. It is equivalent to the density mass $\rho(r)$, obtained by solving the structure equations characterizing the system.

### 2.2 Neutron star structure equations

In order to calculate the gross properties of a neutron star, we assume that the star has a spherically symmetric distribution of mass in hydrostatic equilibrium and is extremely cold ($T = 0$). Effects of rotations and magnetic fields are neglected and the equilibrium configurations are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations \[34\]

\[
\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2} \left(1 + \frac{P(r)}{c^2 \rho(r)}\right) \left(1 + \frac{4\pi r^3 P(r)}{c^2 M(r)}\right) \left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1},
\]

\[
\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) = \frac{4\pi r^2 \epsilon(r)}{c^2},
\]

where $P(r)$ and $M(r)$ are the pressure and the mass functions of the star respectively.

To solve the set of equations (7) for $P(r)$ and $M(r)$, one can integrate outwards from the origin ($r = 0$) to the point $r = R$, where the pressure becomes zero. This point defines $R$ as the radius of the star. To do this, one needs an initial value of the pressure at $r = 0$, called $P_c = P(r = 0)$. The radius $R$ and the total mass of the star, $M \equiv M(R)$, depend on the value of $P_c$. To be able to perform the integration, one also needs to know the energy density $\epsilon(r)$ (or the density mass $\rho(r)$) in terms of the pressure $P(r)$. This relationship is the equation of state (EOS) for neutron star matter and here, has been calculated applying a phenomenological nuclear model.

We can modify equations (7), so that they become suitable for program-
ming, in the following form:

\[
\frac{d\bar{P}(r)}{dr} = -1.474 \frac{\bar{\epsilon}(r) \bar{M}(r)}{r^2} \left(1 + \frac{\bar{P}(r)}{\bar{\epsilon}(r)}\right) \left(1 + 11.2 \times 10^{-6} r^3 \frac{\bar{P}(r)}{\bar{M}(r)}\right)
\times \left(1 - 2.948 \frac{\bar{M}(r)}{r}\right)^{-1},
\]

\[
\frac{d\bar{M}(r)}{dr} = 11.2 \times 10^{-6} r^2 \bar{\epsilon}(r).
\]

In Eqs. (8), the quantities \(\bar{P}(r), \bar{\epsilon}(r)\) and \(\bar{M}(r)\) are dimensionless. The radius \(r\) is measured in Km. More specifically:

\[
M(r) = \bar{M}(r) \, M_\odot, \quad \epsilon(r) = \bar{\epsilon}(r) \, \epsilon_0, \quad P(r) = \bar{P}(r) \, \epsilon_0, \quad \epsilon_0 = 1 \text{ MeV fm}^{-3}.
\]

(9)

It is obvious from Eqs. (8) that

\[
\bar{M}(R) = 11.2 \times 10^{-6} \int_0^R r^2 \bar{\epsilon}(r) dr = b_0 \int \bar{\epsilon}(r) dr.
\]

(10)

2.3 Nuclear equation of state

In general, the energy per baryon of neutron-rich matter may be written to a very good approximation as

\[
\frac{E(n, x)}{A} = \frac{E(n, 1/2)}{A} + (1 - 2x)^2 E_{\text{sym}}(n),
\]

(11)

where \(n\) is the baryon density \(n = n_n + n_p\) and \(x\) is the proton fraction \((x = n_p/n)\). The symmetry energy \(E_{\text{sym}}(n)\) can be expressed in terms of the difference of the energy per baryon between neutron \((x = 0)\) and symmetrical \((x = 1/2)\) matter. Here, we consider a schematic equation for symmetric nuclear matter energy (energy per baryon \(E/A\) or equivalently the energy density per nuclear density \(\epsilon/n\)), given by the expression [37]

\[
\frac{E(n, 1/2)}{A} = \frac{\epsilon_{\text{sym}}}{n} = m_N c^2 + \frac{3}{5} E_F^0 u^{2/3} + V(u), \quad u = n/n_0,
\]

(12)

where \(E_F^0 = (3/5)(\hbar k_F^0)^2/(2m_N)\) is the mean kinetic energy per baryon in equilibrium state and \(n_0\) is the saturation density.
The density dependent potential \( V(u) \) of the symmetric nuclear matter is parameterized, based on the previous work of Prakash et al. \([37, 38]\), as follows

\[
V(u) = \frac{1}{2} Au + \frac{Bu^\sigma}{1 + B' u^{\sigma-1}} + 3 \sum_{i=1,2} C_i \left( \frac{\Lambda_i}{p_F^0} \right)^3 \left( \frac{p_F}{\Lambda_i} - \arctan \frac{p_F}{\Lambda_i} \right),
\]

where \( p_F \) is the Fermi momentum, related to \( p_F^0 \) by \( p_F = p_F^0 u^{1/3} \). The parameters \( \Lambda_1 \) and \( \Lambda_2 \) parameterize the finite-range forces between nucleons. The values employed here are \( \Lambda_1 = 1.5 p_F^0 \) and \( \Lambda_2 = 3 p_F^0 \). The parameters \( A, B, B', \sigma, C_1 \) and \( C_2 \) are determined using the constraints provided by the empirical properties of symmetric nuclear matter at the saturation density \( n_0 \). Then, the values of the above parameters are determined in order that

\[
E(n = n_0)/A - m_N c^2 = -16 \text{ MeV}, \quad n_0 = 0.16 \text{ fm}^{-3}, \quad K_0 = 240 \text{ MeV}.
\]

In general, the parameter values for three possible values of the compression modulus \( K_0 \left( K_0 = 9 n_0^2 \frac{d^2(E/A)}{dn^2} \right|_{n_0} \) are displayed in Table I, in \([37]\).

To a very good approximation, the nuclear symmetry energy \( E_{\text{sym}} \) can be parameterized as follows \([39]\)

\[
E_{\text{sym}}(u) \simeq 13 \frac{u^2}{3} + 17 F(u),
\]

where the first term of the right-hand side part of Eq. (14) is the contribution of the kinetic energy and the second term comes from the interaction energy. For the function \( F(u) \), that parametrizes the interaction part of the symmetry energy, we apply the following form

\[
F(u) = u^c,
\]

where the parameter \( c \) (hereafter called potential parameter) varies between \( 0.4 < c < 1.5 \) leading to reasonable values for the symmetry energy. In order to construct the nuclear equation of state, the expression of the pressure is needed. In general, the pressure, at temperature \( T = 0 \), is given by the relation

\[
P = n^2 \frac{d(\epsilon/n)}{dn} = n \frac{de}{dn} - \epsilon.
\]

Employing equations (11), (12) and (16), we find the contribution of the baryon to the total pressure:

\[
P_b = \left[ \frac{2}{5} E_F^0 n_0 u^{5/3} + u^2 n_0 \frac{dV(u)}{du} \right] + n_0 (1 - 2x)^2 u^2 \frac{dE_{\text{sym}}(u)}{du}.
\]
The leptons (electrons and muons), originating from the condition of the beta stable matter, contribute also to the total energy and total pressure \[39\]. To be more precise, the electrons and the muons, which are the ingredients of the neutron star, are considered as non-interacting Fermi gases. In that case their contribution to the total energy and pressure is

\[
\epsilon_{e^-\mu^-} = \frac{m_e^4 c^5}{8\pi^2 \hbar^3} \left[ (2z^3 + z)(1 + z^2)^{1/2} - \sinh^{-1}(z) \right], \quad (18)
\]

\[
P_{e^-\mu^-} = \frac{m_e^4 c^5}{24\pi^2 \hbar^3} \left[ (2z^3 - 3z)(1 + z^2)^{1/2} + 3 \sinh^{-1}(z) \right], \quad (19)
\]

where \( z = k_F/m_l c \). Now the total energy and total pressure of charge neutral and chemically equilibrium nuclear matter are

\[
\epsilon_{\text{tot}} = \epsilon_b + \sum_{l=e^-\mu^-} \epsilon_l, \quad (20)
\]

\[
P_{\text{tot}} = P_b + \sum_{l=e^-\mu^-} P_l. \quad (21)
\]

From equations (20) and (21) we can construct the equation of state in the form \( \epsilon = \epsilon(P) \). In order to calculate the global properties of the neutron star, i.e. the radius and mass, we solved numerically the TOV equations (7) with the given equations of state constructed employing the present model. For very low densities \( n < 0.08 \text{ fm}^{-3} \) we use the equation of state according to Feynman, Metropolis and Teller \[40\] and also from Baym, Bethe and Sutherland \[41\].

### 3 Results and Discussion

The starting point of our study is the solution of Eq. (8) for three different equations of \( \beta \)-stable nuclear matter. More precisely, we employ three values of the parameter \( c \), which characterizes the density dependence of the nuclear symmetry energy, i.e. \( c = 0.7 \) (soft equation of state), \( c = 1.0 \), and \( c = 1.5 \) (stiff equation of state). In Fig. 1(a), we plot the nuclear symmetry energy \( E_{\text{sym}} \), in Fig. 1(b) the corresponding equations of state and in Fig. 1(c) the mass-radius diagrams for each of the three cases.

Actually every pair \((R, M)\) in a mass-radius diagram is the outcome of the structure equations (Eqs. 8) for an arbitrary chosen initial value of the pressure \( P_c \) in the center of the star. Thus, varying the value of \( P_c \) in a reasonable
range, we can have a picture of the behavior of those substantial structure characteristics. We have to note here that the region where \( dM/dR < 0 \) corresponds to a stable neutron star, while \( dM/dR > 0 \) to an unstable one. The presence of the unstable region \( (M < M_{\text{max}} \text{ and } dM/dR > 0) \) seems to lead to double valued functions \( S(M) \) and \( C(M) \), for values of \( M \) close to \( M_{\text{max}} \) (insets in Fig. 2(a) and Fig. 2(d)). However, the study of that region is beyond the scope of the present work. Another important feature of a neutron star is the value of the maximum mass \( M_{\text{max}} \) for which the star can exist for the specific equation of state. As displayed in Fig. 1(c), \( M_{\text{max}} \) is strongly dependent on the equation of state, while a stiffer equation leads to larger \( M_{\text{max}} \).

In Fig. 2(a), we present the information entropy \( S \), given by Eq. (5), as a function of the mass \( M \). We find that \( S \) is a decreasing function of \( M \) in the region denoting a stable neutron star. The above result is a direct consequence of the fact that when the mass of the star increases, its radius decreases, so does its volume, while its energy density (or its mass density) becomes more localized. Thus, the star is less extended, more compact and \( S \) is smaller. The effect of the parameter \( c \) is just to shift the curve \( S \) versus \( M \).

In Fig. 2(b) we plot the information content \( H = e^S \) employed in the LMC definition \( C = H \cdot D \). It is seen that both \( S(M) \) and \( H(M) \) exhibit the same monotonic trend as functions of \( M \).

In Fig. 2(c) we display the disequilibrium \( D(M) \). Increasing \( M \) corresponds to a more concentrated density distribution, its energy density becomes more localized, resulting to a monotonically increasing \( D \). The rate of this increase is clearly greater in the region close to the value of \( M_{\text{max}} \). This is due to the fact that as \( M \) approaches to its maximum value, it becomes almost independent of \( R \). We also observe a reciprocal behavior of the trends of \( S(M) \) and \( D(M) \), as expected from their definitions.

Complexity \( C \), is plotted in Fig. 2(d). In the region denoting a stable neutron star, \( C \) is a monotonically decreasing function of the star mass \( M \). The most interesting result in this figure is that a neutron star can not grow in complexity as its mass increases towards the limit of \( M_{\text{max}} \). Considering the fact that in nature the most probable values for the mass of a neutron star vary between 1.4 \( M_\odot \) and 3 \( M_\odot \), we note that a neutron star is eventually a physical system of minimum complexity. It is an ordered system, since in the corresponding region the rate of decrease of \( C \) becomes very small and can be considered as a plateau of minimum (zero) complexity.
This result becomes more striking in the following set of figures, Fig. 3, where we plot in three-dimensions (3D) information and complexity measures, as functions of both $M$ and $R$, taking advantage of the fact that each choice of initial values in the equation of state provides a different pair $(R, M)$, reflecting the competition between the gravitational and degenerate gas pressures. The facts that the most probable radii of a neutron star are close to 10 Km, together with the aforementioned comment on the most likely masses, lead us to conclude that a neutron star is in general, a system of minimum complexity. Furthermore, it can not grow in complexity as the mass or radius increase inside the regions imposed and commented above.

The neutron star is an ordered system. From the 3D plots of Fig. 3 we can visualize the variation of $S$ and $D$ as functions of $R$, by keeping $M$ constant. Information entropy $S$ is an increasing function of $R$, i.e. a larger radius corresponds to a larger volume, the energy density (or its mass density) becomes more delocalized, and hence the system is more extended and the information describing it increases. On the other hand, the disequilibrium $D$ is a monotonically decreasing function of $R$ corroborating the fact that the system tends to equiprobability as $R$ increases.

In order to study in more detail the connection between the information and complexity measures with the nuclear interaction and gravity, we plot $S$, $D$ and $C$ (which correspond to a maximum mass of a neutron star), as functions of $R$ and $M$, varying $c$ and $G$ respectively. In Fig. 4 and Fig. 5 we present the results for the effect of the nuclear interaction in two cases. This is done by modifying the equation of state by varying $c$ from 0.7 to 1.5 and then we plot information and complexity measures, first as functions of $M$ in Fig. 4 (for a fixed value of $R = 11.5$ Km) and second, of $R$ (for a fixed value of $M = 1.5 M_\odot$) in Fig. 5.

Therefore, by keeping $R$ fixed and studying the dependence of $S$, $D$ and $C$ on the nuclear interaction indirectly employing $M$, we note that $S$ is almost a linear decreasing function of $M$ (Fig. 4(a)), $D$ is increasing exponentially (Fig. 4(c)), resulting to a fast exponential decrease of $C$ versus $M$ (Fig. 4(d)). Since $R$ is fixed and so does the volume, increasing $M$ corresponds to a more localized energy density, hence $S$ decreases with decreasing $D$.

The approximate linear and exponential expressions for $S(M)$, $D(M)$ and
Approximate expressions have been obtained by the application of the least squares fitting (LSF) method. We use linear relations for fitting of the form \( y = c_1 x + c_2 \), while exponential relations are of the form \( y = c_1 e^{-x/c_2} + c_3 \).

On the other hand, we keep \( M = 1.5 \, M_\odot \) and we see that all measures \( S, D, \) and \( C \) are linearly depended on \( R \), as \( c \) varies. Information entropy \( S \) increases (Fig. 5(a)), while disequilibrium \( D \) decreases (Fig. 5(c)). For fixed \( M \) an increasing \( R \) corresponds to a larger volume. Then, the star becomes more extended and accordingly, its energy density becomes less localized. Complexity \( C \) increases as a result of that delocalization, since the system becomes less ordered (Fig. 5(d)).

The approximate (fitted) linear expressions for \( S(R), D(R) \) and \( C(R) \) are (Fig. 5, \( M = 1.5 \, M_\odot \)):

\[
\begin{align*}
S &= -7.371 M + 2.581, \quad (22) \\
D &= 7.027 e^{2.307 M} + 313.550, \quad (23) \\
C &= 454.949 e^{-5.624 M} - 0.001. \quad (24)
\end{align*}
\]

We repeat the same series of calculations examining this time the gravitational dependence of information and complexity measures, by varying the gravitational parameter \( G \) in the range from 0.9 \( G \) to 1.1 \( G \), while the equation of state is fixed. Our aim is to see how the variation of \( G \) affects quantitatively \( S \) in a neutron star and compare with the results of the previous case. These results are presented in Fig. 6 and Fig. 7. A general comment is that the two cases are almost equivalent. The trends and the behavior of \( S, D \) and \( C \) obtained by varying the gravitational parameter \( G \), keeping the equation of state fixed, are almost the same with the corresponding trends obtained by varying the equation of state for fixed \( G \).

Specifically, first, for fixed \( R = 11.5 \, \text{Km} \), \( S \) decreases linearly with \( M \) (Fig. 6(a)), \( D \) increases exponentially (Fig. 6(c)), resulting to a fast exponential decrease of \( C \) with \( M \) (Fig. 6(d)). The energy density of the system becomes more localized as \( G \) increases for a fixed \( R \) (fixed volume).
The (fitted) approximate linear and exponential expressions for $S(M)$, $D(M)$, and $C(M)$ are (Fig. 6, $R = 11.5$ Km):

$$S = -7.081M + 2.027,$$
$$D = 362.941 e^{0.712M} - 531.607,$$
$$C = 336.821 e^{-5.446M}.\quad (28)$$

Second, for fixed $M = 1.5$ M$_\odot$, $S$ increases linearly with $R$ (Fig. 7(a)), $D$ decreases exponentially (Fig. 7(c)), resulting to a linear increase of $C$ with $R$ (Fig. 7(d)). The energy density of the system becomes less localized as $G$ increases for a fixed $M$, as a result of the radius increase.

The approximate linear and exponential expressions for $S(R)$, $D(R)$, and $C(R)$ are (Fig. 7, $M = 1.5$ M$_\odot$):

$$S = 0.409R - 13.348,$$
$$D = 41451.805 e^{-0.407R} - 143.591,$$
$$C = 0.014R - 0.067.\quad (31)$$

Finally in Fig. 8, we present the direct dependence of complexity $C$ on the parameters $c$ and $G$. It is seen from Fig. 8(a) that complexity for a given $M_{\text{max}}$ is a decreasing function of the equation of state parameter $c$ (the trend is equivalent with the one in Fig. 4(d)), while it increases exponentially with the parameter of the gravitational field Fig. 8(b).

The corresponding (fitted) exponential expressions for $C(c)$, and $C(G)$ are:

$$C = 0.006 e^{-1.208c} + 0.003,$$
$$C = 5.45 \times 10^{-9} e^{13.82c} - 0.001.\quad (34)$$

4 Summary

We present a study of neutron stars from the point of view of information and complexity theories. It is shown that the measures of information entropy $S$ and disequilibrium $D$ can serve as indices of structure of a neutron star. More specifically, $S$ is a decreasing function of the mass of the star, while it is an increasing one of its radius. This result is consistent with the fact that as a neutron star’s mass increases, its radius decreases resulting to more
localized energy and mass densities. The disequilibrium $D$ shows an inverse behavior. It is an increasing function of the mass and a decreasing one of its radius. More localized energy and mass densities correspond to a distribution far from equiprobability and as a result the disequilibrium of the system is higher i.e. it is far from equilibrium.

The complexity $C$ of a neutron star is a decreasing function of its mass. It almost vanishes for a vast set of pairs of values $(R, M)$, while it increases rapidly for masses less than $1.5\ M_\odot$ and radii greater than 12 Km. But this is a not such a favorable case for a neutron star compared with astronomical observations done so far. The favorable one, for masses larger than $1.5\ M_\odot$ and radii less than 12 Km corresponds to almost vanishing complexity, supporting the conclusion that a neutron star is an ordered system, which cannot grow in complexity as its mass increases.

Furthermore, we investigate the impact of the equation of state parameter $c$ and the gravitational parameter $G$ on $S$ and $C$. The behaviors of information and complexity measures are equivalent in both cases. Complexity decreases exponentially with the mass, while it increases linearly with the radius. In direct calculations, complexity decreases exponentially with the equation of state parameter $c$, while it increases exponentially with the gravitational parameter $G$.

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5 Figures

Figure 1: (a) Symmetry Energy vs baryon density \( n \), (b) Energy vs Pressure, and (c) Mass vs Radius.
Figure 2: (a) Entropy $S(M)$, (b) Information Content $H(M)$, (c) Disequilibrium $D(M)$, and (d) Complexity $C(M)$. The insets are commented in the text.
Figure 3: 3D display of (a) Entropy $S(R, M)$, (b) Disequilibrium $D(R, M)$, and (c) Complexity $C(R, M)$, projected for each case on two planes: (a) $R - M$ and $S - R$, (b) $R - M$ and $D - R$, (c) $R - M$ and $C - R$. 
Figure 4: (a) $S(M)$, (b) $H(M)$, (c) $D(M)$, and (d) $C(M)$, by varying $c$ for a fixed radius $R = 11.5$ Km (see text, Eqs. (22)-(24)).
Figure 5: (a) $S(R)$, (b) $H(R)$, (c) $D(R)$, and (d) $C(R)$, by varying $c$ for a fixed mass $M = 1.5\ M_\odot$ (see text, Eqs. (25)-(27)).
Figure 6: (a) $S(M)$, (b) $H(M)$, (c) $D(M)$, and (d) $C(M)$, by varying $G$ for a fixed radius $R = 11.5$ Km (see text, Eqs. (28)-(30)).
Figure 7: (a) $S(R)$, (b) $H(R)$, (c) $D(R)$, and (d) $C(R)$, by varying $G$ for a fixed mass $M = 1.5 \, M_\odot$ (see text, Eqs. (31)-(33)).
Figure 8: (a) Complexity vs the equation of state parameter $c$, and (b) Complexity vs the gravitational parameter $G$, for a given $M = M_{\text{max}} = 1.5 M_\odot$ (see text, Eqs. (34)-(35)).

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