Statistical Analysis and Study on Joint Distribution of the Extreme Value of Wind Speed and Wind Direction

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Abstract. The joint distribution of wind speed and direction is one of basic characters of wind load. the distribution function at the location of building have to be acquired before structural wind resistance reliability analysis. In this paper, the joint probability distribution function was further studied based on previous references. An analytical method of joint probability density function of wind speed and wind direction from extreme value sample is proposed in this paper. By using this method, the probability distribution feature of any wind direction can be easily calculated which is of great benefits for safety and reliability analysis of wind-resistant structures. Then the joint probability of wind speed and direction near a civil structure was derived on the basis of datum sampled from weather station on the assumption that the wind direction and gradient wind speed of the weather station is same as that at location of the civil building. The data of wind speed and corresponding direction from Shanghai Baoshan weather stations were analysed as a case according to the method discussed in this paper. The conclusions in this paper is useful for analysis of reliability of structural wind resistance.

1. Introduction

The measurement and analysis of near ground wind characteristics is a very important basic and long-term topic in the wind resistance research of civil engineering structures. The uncertainty of wind parameters is one of the most important factors affecting the accuracy of wind resistance design of structures [1]. In accordance with the current wind resistance design methods of structures, the analysis of extreme value samples (such as the annual maximum wind speed) is usually emphasized in the data processing of measured wind speed, and the action direction of extreme wind in the samples is seldom considered. However, the distribution of wind speed is random not only in time but also in space. In each direction, the intensity and frequency of wind field are different. The measured records show that the wind direction is concentrated when the wind speed is high, but the distribution of wind direction is very discrete when the wind speed is low. It is obviously not enough to analyse the wind resistance reliability of maintenance structure, so the distribution of wind direction needs to be paid special attention. Specifically, the joint effect of wind speed and wind direction at the structure location should be considered. Once the joint probability distribution function of the wind speed and wind direction (hereinafter referred to as JPDF) characteristic information is mastered, designers can...
accurately grasp the wind loads acting on different directions of the structure, so as to design a more reasonable and economic structure. For large-scale structures, the influence of wind resistance design is more significant.

There are three main ideas in the study of JPDF: 1) to establish the theoretical model of JPDF by means of theoretical methods such as directional extremum [2], multivariable extreme value [3] ARMA time series model [4], multidimensional stochastic process and other theoretical methods are mainly used for the analysis of extreme wind speed; 2) the study of JPDF is transformed into the study of probability characteristics of structural response under wind influence[5]. This idea can be used when the research work focuses on structural response characteristics rather than load characteristics; 3) the research results of all wind direction are extended to the given wind direction, and the correlation between different wind directions are further assumed. The third method is extension of the all wind direction method, and in this method both extreme value samples and population samples can be taken as study object. For example, research work in reference [6] is focus on the joint probability distribution of wind speed and direction in Shanghai. The practical expression of joint probability density function surface of wind speed and direction is obtained by using the method of statistics and surface fitting, but the correlation between different wind directions is not directly given. In reference [7] the joint distribution of average wind speed and direction based on the measured extreme wind speed sample data is taken as research object. The specific statistical analysis methods of JPDF mainly include stationary random process method, maximum wind direction coefficient method and joint distribution probability method [8]. The stationary random process method regards the average wind speed as a two-dimensional stationary random vector process, which reflects the probability distribution characteristics of the average wind speed and wind direction with time; however, due to the current technical level, this method has not been applied in practice. The maximum wind direction coefficient method is only applicable to the static wind load analysis of the rigid structure which is less affected by aeroelasticity. The joint distribution probability model is used to study the joint effect of wind speed and wind direction. It is suitable for all types of structures and is the most effective method to resolve problem of the joint effect of wind speed and direction.

At present, the joint probability distribution model proposed by Yang Yongxin et al. [3] reflects the joint distribution function of wind speed and wind direction. Its extreme value distribution function is composed of wind direction frequency function \( f(\theta) \) and wind speed distribution function \( P_v(U < u, \theta) \) in each wind direction

\[
F(u, \theta) = f(\theta)P_v(U < u, \theta)
\]

On this basis, further research work is carried out in this paper. Firstly, four hypotheses of joint probability distribution characteristics of wind speed and direction at the same location are introduced: 1) the wind speed of any given wind direction follows the same probability distribution; 2) It is assumed that the probability parameters of average wind speed distribution in different directions of the same location are independent, and the distribution parameters are estimated by the extreme wind speed samples in the same direction. 3) the correlation between different wind directions is reflected by the relative frequency of wind speed data; 4) the variation of probability parameters between different wind directions meets the harmonic function. Thus, the joint distribution function of wind speed and direction in any wind direction range can be obtained [9,10,11]. It is assumed that the wind speed in a given wind direction conforms to the generalized extreme value distribution (GeV) with two parameters. The statistical analysis of the joint distribution of wind speed and direction can be divided into the following four steps: 1) sampling of extreme wind speed samples; 2) testing and parameter estimation of the probability model of wind speed distribution within a given wind direction interval; 3) fitting the frequency function and parameters in the distribution function of wind direction interval with harmonic function; 4) The joint distribution function of wind speed and wind direction at the location of weather station is obtained, and the maximum wind speed corresponding to different return periods is further obtained. According to the requirements of wind reliability analysis of maintenance
structure, this paper analyses the joint distribution of wind speed and direction based on the data measured by Shanghai Baoshan meteorological station through the above steps.

2. Statistical analysis of joint distribution of extreme wind speed and direction

2.1. Fitting and parameter estimation of joint probability model of wind speed and direction

After obtaining the frequency statistics table of extreme wind speed samples in all wind directions with wind speed range, it is necessary to fit the probability model of extreme wind speed distribution according to the statistical results. Dargahi-Noubari summarized the probability distribution model of extreme wind speed into three types: extreme value type I (Gumbel) distribution, extreme value type II (Frechet) distribution and extreme value type III (Weibull) distribution [7]. The research results showed that the extreme value type I was more suitable for the annual maximum wind speed [12]. In reference [13], Weibull distribution is considered as the universal optimal probability model of annual maximum wind speed. According to three probability models of extreme wind speed distribution, assuming that the probability parameters of average wind speed distribution in different directions at the same place are independent, three joint distribution models of joint distribution of wind speed and direction can be obtained by estimating distribution parameters from extreme wind speed samples in the same direction.

I: Gumbel distribution

\[ F_{\text{Gumbel}}(x) = f(\theta) \exp[-\exp(-\frac{x-\mu(\theta)}{\sigma(\theta)})] \]  

(2)

II: Frechet distribution

\[ F_{\text{Frechet}}(x) = f(\theta) \exp\left[-\left(-\frac{x}{\sigma(\theta)}\right)^{\kappa(\theta)}\right] \]  

(3)

III: Weibull distribution

\[ F_{\text{Weibull}}(x) = f(\theta) \left[1 - \exp\left(-\left(-\frac{x}{\sigma(\theta)}\right)^{\kappa(\theta)}\right)\right] \]  

(4)

In above formula, \( f(\theta) \) is the frequency function of wind direction describing the distribution of extreme wind speed samples in every wind direction interval, which can be obtained from the last column of the frequency statistics table of extreme wind speed samples in every wind direction within wind speed range. The other parameters in the joint distribution function are all functions of wind direction. After the selected extreme wind speed distribution probability model is determined, it is necessary to fit the corresponding extreme wind speed distribution probability parameters by means of parameter estimation. A variety of parameter estimation methods have been developed for the three extreme probability distributions, including frequently used moment method and maximum likelihood method and specifically used digital feature method, sequential statistics method, least square method, probability weight method, reliability moment method and percentage point method [13]. In this paper, the least square method is used for its simplicity in principle and easy to implement. The least square method is based on the principle of minimizing the sum of squares of deviations to determine the undetermined parameters in the equation. Since the parameters in the joint distribution of wind speed and direction are all functions of wind direction, the estimation of parameters needs to be carried out separately in each wind direction. Since there are two distribution parameters of each joint distribution probability in formula (2), (3) and (4), linear function fitting can be applied to fit parameters in the parameter estimation method.

Since \( \theta \) in the above formula is the discrete value of given wind direction, sometimes the distribution characteristics on the whole circumference are needed. Therefore, Chen Jun [11] and others further assume that the frequency of wind direction and the variation of distribution function parameters on the circumference can be fitted by harmonic function, so as to obtain the joint distribution function of wind speed and direction in any wind direction interval. In formula (2), (3), (4), \( \sigma(\theta) , \mu(\theta) , \kappa(\theta) \) are scale parameters, position parameters and shape parameters of the given
wind direction \( \theta \). \( f(\theta) \) and parameters \( \sigma(\theta) \), \( \kappa(\theta) \) can be obtained from the measured data. According to the assumption ④ mentioned in the previous chapters of this paper, the variation of wind direction frequency function, scale parameter, position parameter and shape parameter on the circumference can be represented by the harmonic function:

\[
f(\theta) = a' + \sum_{m=1}^{n_\sigma} b_m' \cos(m\theta - c_m')
\]

(5a)

\[
\sigma(\theta) = a'' + \sum_{m=1}^{n_\mu} b_m'' \cos(m\theta - c_m'')
\]

(5b)

\[
\mu(\theta) = a''' + \sum_{m=1}^{n_\kappa} b_m''' \cos(m\theta - c_m''')
\]

(5c)

\[
\kappa(\theta) = a^{\prime\prime\prime} + \sum_{m=1}^{n_\kappa} b_m^{\prime\prime\prime} \cos(m\theta - c_m^{\prime\prime\prime})
\]

(5d)

Here \( f(\theta) \) is the distribution function representing frequency of wind direction by means of harmonic function fitting; \( \sigma(\theta) \), \( \mu(\theta) \), \( \kappa(\theta) \) are the parameters in the distribution function; \( a', b_m', c_m' \) are the undetermined coefficients in the harmonic function; \( n_\sigma \), \( n_\mu \), \( n_\kappa \) are the order of the harmonic function. Equations (2), (3), (4) and (5) constitute a complete joint probability distribution function of wind speed and direction for extreme wind speed samples. The advantage of the above calculation method is that it can be used to calculate the probability distribution characteristics of any given wind direction, which is convenient for the reliability analysis of wind resistant structures. According to the harmonic function expressed in formula (5), the probability of occurrence of 16 wind direction intervals and the parameters of wind speed distribution function corresponding to each wind direction interval are fitted, thus the joint distribution function of wind direction and wind speed can be obtained.

2.2. Joint distribution function of wind speed and direction and design wind speed corresponding to different return periods

In this section, the conversion method from single measurement points to calculation points is described, which is used to calculate the joint distribution function of wind speed and direction at the building location and the design wind speed corresponding to different return periods. In the reliability analysis of actual maintenance structure, it is necessary to consider the law of wind speed distribution at the building structure location. After obtaining the joint distribution probability function of wind speed and direction at the meteorological station, and assuming that the wind direction at the weather station is the same as that at the building location, the joint distribution function at the building location can be obtained according to the joint distribution probability function of wind speed and direction at the building location. The transformation process of the two is as follows. The joint probability distribution function of wind speed and direction is supposed to be

\[
P(U_o < u_o, \theta) = f(\theta)g(u_o)
\]

(6)

Here \( f(\theta) \) is the probability in each wind direction interval; \( g(u_o) \) is the probability distribution function of wind speed; \( U_o \) is the wind speed variable at a certain height of the weather station. Because gradient wind speed at the weather station is the same as that at the building, the relationship between the wind speed variable \( U_o \) at weather station and the wind speed variable \( U_b \) at building location can be established:

\[
U_b/U_o = (H_o/H_s)^\alpha (b_o/H_s)^\kappa
\]

(7)

Here \( H_o \) and \( H_s \) are the gradient wind height at the location of the weather station and the building.
respectively; \( h_o \) and \( h_b \) are the wind speed height at the location of the weather station and the building; \( \alpha_o \) and \( \alpha_b \) are the ground roughness index at the location of the weather station and the building respectively.

According to formula (6) and formula (7), the joint probability distribution function of wind speed and direction at the building location can be obtained as follows:

\[
P(U_s < u_s, \theta) = g\left((h_b/H_o)^\nu (H_o/h_b)^\nu U_s < u_s, \theta\right)
\]

If the location of a building in Shanghai is of class C landform, the wind speed distribution of Shanghai Baoshan meteorological station adopts Gumbel distribution, Frechet distribution and Weibull distribution respectively. The wind speed and direction joint distribution functions for building at height \( h \) are as follows:

\[
P(U_s > u_s, \theta) = f(\theta) \left\{ 1 - \exp\left[-\left(\frac{u_s - \mu(\theta)}{\sigma(\theta)}\right)^\eta\right]\right\},
\]

\[
P(U_s > u_s, \theta) = f(\theta) \left\{ 1 - \exp\left[-\left(\frac{u_s}{\sigma(\theta)}\right)^\kappa\right]\right\},
\]

\[
P(U_s > u_s, \theta) = f(\theta) \left\{ \exp\left(-\left(\frac{u_s}{\sigma(\theta)}\right)^\kappa\right)\right\}
\]

here, \( \eta = (H_o/h_b)^\nu (h_b/H_o)^\nu \).

After obtaining the joint distribution function of wind speed and direction for buildings at height of \( h_b \). The maximum wind speed used to calculate the reliability of the maintenance structure in each wind direction interval is the maximum design wind speed for a given recurrence period. Therefore, it is also necessary to determine the maximum wind speed for a fixed recurrence period \( T \) (year) in each wind direction angle at this height. \( N \) is defined as the time interval of sampling in a year. For example, when sampling interval is one day, \( N \) is equal to 365. In other words, if the return period \( T \) is in years, when sampling is in days, the return period should be in days, and the actual return period is \( 365T \).

\( u_{\text{max}}(\theta) \) is defined as the maximum wind speed for return period \( T \) in different wind directions. The following is the relationship between the maximum wind \( u_{\text{max}}(\theta) \) and the return period at different wind directions based on the joint distribution function. For Gumbel distribution, there is the following relationship expression [7]:

\[
\frac{1}{NT} = P(U_s > u_s, \theta) = f(\theta) \left\{ 1 - \exp\left[-\left(\frac{u_s - \mu(\theta)}{\sigma(\theta)}\right)^\eta\right]\right\}.
\]

The physical meaning of the formula (10) is very clear: if the return period is \( NT \) days, within \( NT \) days, the probability of wind speed exceeding the maximum design wind speed \( u_{\text{max}}(\theta) \) on a certain day is \( 1/NT \), which is the failure probability.

According to formula (10), there is:

\[
u_{\text{max}} = \eta \left\{ \mu(\theta) - \sigma(\theta) \ln\left[\ln\left(\frac{NTf(\theta)}{NTf(\theta) - 1}\right)\right]\right\}
\]

The maximum design wind speed for Frechet distribution and Weibull distribution can be obtained as follows:
In formula (11):

\[ N \text{ sampling interval in 1 year} \]
\[ T \text{ return period (in years)} \]

The corresponding assurance rates for return period T of 10 years, 30 years, 50 years, 100 years, and 1000 years are 90\%, 96.66\%, 98\%, 99\%, and 99.9\% for respectively. The program in this paper can calculate the maximum design wind speed (basic wind speed) corresponding to any guarantee rate under any wind condition. From the relationship between the basic wind speed and the return period, it can be seen that the essence of the influence of wind direction distribution frequency function \( f(\theta) \) on the basic wind speed is the reduction of the return period, through which the difference of the number of samples in different wind directions can be considered, and the parameter N is the multiplication of the return period.

3. Example and analysis

3.1. Description of original wind speed and direction sample

The wind speed samples used in this paper are from the data provided by Baoshan meteorological station in Shanghai (refer to table 1). Shanghai is a coastal city. With the change of seasons, the statistical characteristics of wind speed and direction will change constantly [14]. The meteorological station records the maximum hourly wind speed at 10m height (10 min average time interval) for 33 years from January 1, 1975 to December 31, 2007, with a total of 33 * 365 * 24 = 289080 data and corresponding wind direction. The sampling method used in this paper is consistent with the sampling method in reference [7] (stage extreme value method). In this paper, the whole data processing and analysis process is implemented by MATLAB programming. Generally, daily maximum wind speed samples are extracted from the whole wind speed data for analysis. The difference of sample number among different wind directions is considered by introducing wind direction frequency function. Table 1 is the statistical data table of frequency (%) of wind speed samples in each wind direction and within wind speed range provided by Baoshan wind speed station. The data in the table reflect the distribution of extreme value samples in 17 wind speed intervals by column, and the distribution of extreme value samples in 16 wind directions by row. The total number of samples is set as 100\%.

Table 1 frequency statistics of overall samples in different wind directions and wind speed ranges of Baoshan meteorological station in Shanghai

| Wind Speed Direction | 0° | 1°~2° | 3°~4° | 5°~6° | 7°~8° | 9°~10° | 11°~12° | 13°~14° | 15°~16° | 17°~18° | Sum |
|----------------------|----|-------|-------|-------|-------|--------|---------|---------|---------|---------|-----|
| N                    | 4  | 0.0001| 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| NNE                  | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| NE                   | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| ENE                  | 0  | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005| 0.0006  | 0.0007  | 0.0008  | 0.0009  | 0.0010| 0.0011|
| E                    | 0  | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005| 0.0006  | 0.0007  | 0.0008  | 0.0009  | 0.0010| 0.0011|
| ESE                  | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| SE                   | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| SSE                  | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| S                    | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| SSW                  | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| SW                   | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| W                    | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| WSW                  | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
| NW                   | 0  | 0     | 0.0001| 0.0002| 0.0003| 0.0004| 0.0005  | 0.0006  | 0.0007  | 0.0008  | 0.0009| 0.0010|
3.2. Numerical calculation

Based on the theoretical analysis in Section 2.1, the parameter estimation values of three extreme value distributions are calculated based on the wind speed samples in all directions and 16 wind directions in Table 1. The numerical calculation of parameter estimation is calculated by MATLAB programming, and the calculation results are shown in Table 2.

Table 2 Distribution parameters of three distributions under all wind directions at Baoshan meteorological station in Shanghai

| Wind direction | Gumbel distribution | Frechet distribution | Weibull distribution |
|----------------|---------------------|----------------------|----------------------|
|                | Parameter σ | Parameter μ | Correlation coefficient r | Parameter σ | Parameter k | Correlation coefficient r | Parameter σ | Parameter μ | Correlation coefficient r |
| NNW            | 0.7421    | 5.7907    | 0.7434 | 7.0472    | 0.6136 | 9.0565    | 0.1585 | 0.8564 |

It can be seen from Table 2 that the correlation coefficient of Weibull distribution is \( r = 0.8564 \), which is greater than that of the other two distributions, so the wind speed extreme value distribution is considered as Weibull distribution.

According to the wind speed extreme value samples of 16 wind directions at Baoshan meteorological station in Table 1, the parameter estimation values of Gumbel distribution, Frechet distribution and Weibull distribution are estimated respectively. The numerical calculation of parameter estimation is calculated by MATLAB programming, and the result is shown in Table 3.

Table 3 Parameter estimation results of joint distribution of three wind speeds and directions at Baoshan meteorological station, Shanghai

| Wind direction | f(θ) | Gumbel distribution | Frechet distribution | Weibull distribution |
|----------------|------|---------------------|----------------------|----------------------|
|                | σ    | μ       | Correlation coefficient r | σ    | k       | Correlation coefficient r | σ    | μ       | Correlation coefficient r |
| N              | 0.0717 | 0.3809 | 5.4277 | 0.8462 | 3.5913 | 13.470i | 0.6880 | 9.9560 | 10.6285 | 0.6791 |
| NNE            | 0.1041 | 0.5473 | 5.7871 | 0.7637 | 3.8267 | 9.3348 | 0.6148 | 9.5687 | 13.8466 | 0.8742 |
| NE             | 0.0764 | 0.4453 | 5.5788 | 0.8153 | 3.6726 | 11.466 | 0.6560 | 9.4332 | 13.9470 | 0.8760 |
| ENE            | 0.0949 | 0.5664 | 5.7591 | 0.7389 | 3.7740 | 8.8704 | 0.5870 | 8.9604 | 9.9020 | 0.8413 |
| E              | 0.0770 | 0.4525 | 5.6467 | 0.7845 | 3.6766 | 11.000 | 0.6197 | 8.9250 | 10.0842 | 0.8514 |
| ESE            | 0.0851 | 0.4449 | 5.6275 | 0.8086 | 3.6975 | 11.375i | 0.6479 | 9.4567 | 13.8654 | 0.8731 |
| SE             | 0.0463 | 0.3011 | 4.4951 | 0.9035 | 3.1742 | 18.220 | 0.7862 | 9.7035 | 16.9683 | 0.8856 |
| SSE            | 0.0716 | 0.3194 | 4.7720 | 0.8995 | 3.2708 | 16.847 | 0.7626 | 9.1311 | 14.2679 | 0.8835 |
| S              | 0.0357 | 0.4432 | 5.5407 | 0.8057 | 3.6459 | 11.523 | 0.6485 | 9.3845 | 13.8161 | 0.8683 |
| SSW            | 0.0349 | 0.3819 | 5.6232 | 0.8106 | 3.7111 | 11.687 | 0.6540 | 10.034 | 16.5114 | 0.8761 |
| SW             | 0.0247 | 0.3155 | 4.7742 | 0.8920 | 3.2649 | 17.002 | 0.7541 | 9.0784 | 14.1484 | 0.8762 |
| W              | 0.0358 | 0.3791 | 5.5233 | 0.8446 | 3.6430 | 13.452i | 0.6824 | 10.024 | 16.6250 | 0.8816 |
| WNW            | 0.0760 | 0.3773 | 5.4546 | 0.8425 | 3.5888 | 13.558 | 0.6791 | 9.4075 | 14.0388 | 0.8805 |
| NW             | 0.0471 | 0.3416 | 5.0978 | 0.8799 | 3.4116 | 15.271 | 0.7275 | 9.2291 | 14.1594 | 0.8812 |
| NNW            | 0.0978 | 0.7528 | 5.9918 | 0.7309 | 4.0809 | 7.0784 | 0.6136 | 10.282 | 16.2982 | 0.8741 |
| All directions | 1     | 0.7421 | 5.7907 | 0.7454 | 3.8337 | 7.0709 | 0.6163 | 9.0564 | 10.1584 | 0.8564 |

The wind angle frequency and distribution parameters of the three extreme value distributions in Table 3 are fitted. In the calculation process, the first seven terms of the fitting polynomial are used for fitting, with a total of \( 2 \times 6 + 1 = 13 \) coefficients to be solved. The numerical calculation of parameter estimation is calculated by MATLAB programming, and the calculation results are shown in Table 4.

Table 4 The fitting results of four meteorological extreme values of Baoshan meteorological station

| Wind direction | \( f(θ) \) | Gumbel distribution | Frechet distribution | Weibull distribution |
|----------------|--------|---------------------|----------------------|----------------------|
|                | σ      | μ       | Correlation coefficient r | σ      | k       | Correlation coefficient r | σ      | μ       | Correlation coefficient r |
| N              | 0.062  | 0.031  | -0.741 | 0.006  | 1.217  | -0.003 | 0.092  | 0.003  | -0.549 | 0.003  | -0.512 | 0.008  | -0.514 |
| NNE            | 0.43   | 0.082  | -0.393 | 0.03   | -0.67  | 0.051  | 1.795  | 0.04   | 0.697  | -0.059 | -0.957 | -0.028 | -0.421 |
| NE             | 5.415  | 0.273  | -0.099 | -0.177 | 1.319  | -0.157 | -1.062 | 0.279  | -0.19  | 0.245  | 2.522  | 0.019  | 1.888 |
| E              | 3.605  | 0.158  | -0.078 | -0.073 | 1.644  | 0.082  | 1.827  | 0.129  | -0.043 | -0.147 | -0.762 | -0.029 | -0.533 |
| ESE            | 12.739 | -2.246 | -0.444 | 1.114  | 1.716  | 1.432  | -1.043 | -1.552 | -0.031 | 1.691  | -0.722 | 0.27   | -0.822 |
The harmonic function curves of several fitted parameters are listed below:

According to formula (11) and the parameter estimation results of wind speed extreme value samples of 16 wind directions in Table 3, the basic wind speed of each wind direction in Shanghai (Baoshan meteorological station) is calculated, and the results are listed in Table 5. The data in the last row of the table represents different basic wind speeds that correspond to the wind speed distribution obtained by the traditional calculation method without considering the joint distribution of wind speed and direction.

Table 5 Basic wind speed of each wind direction calculated by joint distribution probability model (m/s)

| Wind direction | Gumbel distribution | Frechet distribution | Weibull distribution |
|----------------|---------------------|----------------------|----------------------|
|                | Return period 10 years | Return period 30 years | Return period 50 years | Return period 100 years | Return period 10 years | Return period 30 years | Return period 50 years | Return period 100 years | Return period 10 years | Return period 30 years | Return period 50 years | Return period 100 years |
| N              | 19.4928             | 20.739               | 21.3156              | 22.0968              | 12.2946             | 13.3548               | 13.8756              | 14.6382              | 25.6866             | 25.9656               | 26.0772              | 26.2226              |
| NNE            | 19.437              | 20.6646              | 21.2412              | 22.0224              | 12.2574             | 13.3176               | 13.8384              | 14.5824              | 25.668              | 25.947                | 26.0586              | 26.2074              |
| NE             | 19.3812             | 20.6088              | 21.1668              | 21.948               | 12.2338             | 13.2618               | 13.7826              | 14.5452              | 25.6494             | 25.9284               | 26.042               | 26.1888              |
| ENE            | 19.3254             | 20.5344              | 21.111               | 21.8736              | 12.2016             | 13.2286               | 13.7454              | 14.4894              | 25.6308             | 25.9098               | 26.0214              | 26.1702              |
| E              | 19.2696             | 20.4786              | 21.0366              | 21.7929              | 12.1644             | 13.1874               | 13.7082              | 14.4522              | 25.6122             | 25.8912               | 26.0028              | 26.1516              |
| ESE            | 19.2138             | 20.4228              | 20.9808              | 21.7248              | 12.1458             | 13.1502               | 13.6711              | 14.3964              | 25.5936             | 25.8726               | 25.9842              | 26.1332              |
| SE             | 19.158              | 20.367               | 20.9064              | 21.669               | 12.1086             | 13.1131               | 13.6338              | 14.3592              | 25.575              | 25.854                | 25.9656              | 26.1144              |
| SSE            | 19.1208             | 20.2926              | 20.8506              | 21.5946              | 12.0714             | 13.0758               | 13.5966              | 14.3212              | 25.5564             | 25.8334               | 25.9472              | 26.0958              |
| S              | 19.065              | 20.2368              | 20.7948              | 21.5388              | 12.0528             | 13.0572               | 13.5594              | 14.2662              | 25.5378             | 25.8168               | 25.9284              | 26.0772              |
| SSW            | 19.0092             | 20.181               | 20.739               | 21.4644              | 12.0342             | 13.02                 | 13.5222              | 14.2229              | 25.5192             | 25.7982               | 25.9098              | 26.0586              |
In this paper, the locations of weather stations and buildings are B and C respectively. The probability density function of the total wind speed samples, and the extreme wind speed samples of every day, every 8 days, every half month, every quarter, every half year and every year are statistically fitted. Finally, the maximum design wind speed is calculated for three extreme value distributions corresponding return periods of 10 years, 30 years, 50 years and 100 years are calculated respectively under 16 wind directions or all directions.

4. Conclusion and discussion

1. The numerical results in this paper show that the joint distribution function of wind speed and direction obtained by monthly maximum wind speed sampling (or more intensive sampling) can be used to analyse the wind resistance reliability of maintenance structures. The annual (or even half yearly) maximum wind speed is not suitable for analysis of wind resistance reliability of maintenance structures. Because the number of samples is too small to take the annual maximum wind speed as the sample, the probability of low wind speed appearing in some wind directions is zero according to the joint distribution function of wind speed and direction, which will lead to loss of small wind speed conditions in many wind directions.

2. In addition, it should be noted that the larger the sampling interval is, the larger the extreme value samples are, and the fewer samples are in the low wind speed range, which leads to poor fitting result.

3. The maximum value 31.14m/s in omnidirectional wind speed results is 40.9% higher than that of 22.10m/s determined by considering joint distribution of wind speed and direction. It is suggested that Gumbel distribution be used for the joint distribution function obtained from the wind speed and direction data of Baoshan meteorological station in Shanghai. The maximum wind speed corresponding to the 100 years return period obtained from the meteorological data with the influence of wind direction is smaller than that obtained without the influence of wind direction.

4. Compared with the results fitting by three extreme value distributions, the goodness of fit of low wind speed samples (with dense sampling) and the three extreme value distributions are extremum III < extremum I < extremum II; and the goodness of fit of high wind speed samples (sparse sampling) with the three extreme value distributions is extremum I < extremum III < extremum II. Under normal wind conditions, the North (high wind speed area) is more suitable for extreme value type I, and the South (low wind speed area) is more suitable for extreme value type III.

In this paper, the analysis and processing of extreme wind speed data are completed by MATLAB programming. The variation law of the probability parameters along the circumference is defined by harmonic fitting method. Therefore, the joint distribution function of wind speed and direction in any direction can be given, so as to bring convenience for the wind resistance reliability analysis of structures. The method is simple and practical, and has very useful value of engineering.

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