Entropy production from decay of GeV scale right-handed neutrinos and the primordial gravitational wave

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Abstract. Right-handed neutrinos are attractive particles beyond the standard model introduced in the seesaw mechanism [1] that can explain tiny neutrino masses. We revisit the entropy production from the decay of right-handed neutrinos with GeV scale masses. This additional entropy production dilutes the dark matter and baryon abundances. Furthermore, we point out that it gives the effect on the primordial gravitational wave spectrum. Primordial gravitational waves are closely related to the thermal history of the universe, and then the modified history by right-handed neutrino is imprinted in the gravitational wave spectrum12.

1. Introduction
We have observed the phenomena beyond the standard model. For example, they are neutrino oscillation that show neutrinos have masses, baryon asymmetry of the universe and dark matter.

The $\nu$MMSM (neutrino minimal standard model)[3, 4] is one of the minimal model extended by the three right-handed neutrinos. It can solve the problems of neutrino masses, baryon asymmetry of the universe and dark matter simultaneously. In this model, the lightest right-handed neutrino is a candidate of the dark matter with keV scale mass and the other two right-handed neutrinos have GeV scale masses that are responsible to the baryogenesis via neutrino oscillation.

In this talk we focus on the GeV scale right-handed neutrinos. They realize the seesaw mechanism that can explain a tiny neutrino mass and also baryogenesis. An attractive feature of the model is the testability of the right-handed neutrinos. Experimental searches of low scale right-handed neutrinos has been done (see the review, e.g., [5]), but no positive signals has been detected. We therefore, need to investigate further more to test these particles. Motivation of this talk is whether there is another method to explore the right-handed neutrinos. We discuss the possibility by the gravitational wave observation, i.e., the entropy production due to the decay of the right-handed neutrinos that modifies the gravitational wave spectrum. The entropy production by GeV scale right-handed neutrinos have been discussed [6, 7]. We revisit this dynamics with the new perspective of verification of right-handed neutrinos by gravitational waves.

1 This talk is based on the work with T.Asaka [2]
2 This work was supported by the Sasakawa Scientific Research Grant from The Japan Science Society.
2. Entropy production
We consider the Lagrangian extended by singlet right-handed neutrinos.

\[ \mathcal{L} = \mathcal{L}_{SM} + i \bar{\nu}_R \gamma^\mu \nu_R - \left( F \bar{\nu}_R \Phi - \frac{M_M}{2} \bar{\nu}_R \nu_R^c + h.c. \right), \]  

(1)

The mass term of neutrinos is given as follow and we assume type-I seesaw mechanism \( |M_D| \ll M_M \).

\[ -\mathcal{L}_m = \frac{1}{2} \left( \bar{\nu}_L, \bar{\nu}_R^c \right) \begin{pmatrix} 0 & M_D \\ M_D^T & M_M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} + h.c. \]  

(2)

where \( M_D \) is the Dirac mass matrix and \( M_M \) is the Majorana mass matrix. Mixing of light neutrinos \( \nu \) and heavy neutrinos \( N \) are given by

\[ \nu_L = U \nu + \Theta N^c \quad \text{with} \quad \Theta = \frac{M_D}{M_M} = \frac{F \langle \Phi \rangle}{M_M}, \]  

(3)

where \( U \) is the PMNS matrix. The right-handed neutrinos decay into the standard model particle through this mixing. We take into account the Cassas-Ibarra parameterization [8]. The Yukawa coupling is then given by

\[ F = \frac{i}{\langle \Phi \rangle} U D^\nu_1 D^1_\nu \Omega D^1_N, \]  

(4)

where \( D_\nu = \text{diag}(m_1, m_2, m_3) \) is left-handed neutrino mass matrix and \( D_N = \text{diag}(M_{N_1}, M_{N_2}) \) is the right-handed neutrino mass matrix. The matrix \( \Omega \) is complex \( 3 \times 2 \) matrix and given by

\[ \Omega_{NH} = \begin{pmatrix} 0 & 0 \\ \cos \omega & -\sin \omega \\ \xi \sin \omega & \xi \cos \omega \end{pmatrix}, \quad \Omega_{IH} = \begin{pmatrix} \cos \omega & -\sin \omega \\ \xi \sin \omega & \xi \cos \omega \\ 0 & 0 \end{pmatrix}. \]  

(5)

Here \( \Omega_{NH} [IH] \) is for the normal hierarchy (Inverted hierarchy) case and \( \xi = \pm 1 \) is sign parameter, \( \omega \) is complex parameter. We define a parameter \( X_\omega \equiv \exp(\text{Im} \omega) \).

We consider the entropy production by decay of the right-handed neutrinos. This dynamics occurs in the radiation dominated era after the inflation. Our scenario is following.

(i) Right-handed neutrinos are produced in thermal scattering.
(ii) Right-handed neutrinos decouple from thermal bath at the decoupling temperature \( T_{\text{dec}} \).
(iii) The number of right-handed neutrinos is frozen at this time.
(iv) If right-handed neutrinos have sufficiently long lifetimes, the energy densities of the right-handed neutrinos will dominate the energy of the universe.

In figure 1, we show evolution of the energy densities of the radiation and the right-handed neutrinos and the entropy. The energy density of right-handed neutrinos dominates the energy of the universe at a certain time because the energy density of radiation \( \rho_R \propto a^{-4} \) and right-handed neutrino \( \rho_{\nu_R} \propto a^{-3} \). When the decay rate \( \Gamma \) of the right-handed neutrino becomes larger than the Hubble parameter, i.e., \( \Gamma/H(T) > 1 \), right-handed neutrinos decay into the standard model particles. The energy density of right-handed neutrinos decrease exponentially, then the energy density of radiations will increase and the additional entropy will be produced.

In figure 2, we show a result of the entropy production ratio and reheating the temperature in terms of \( X_\omega \). The late-time entropy production is constrained by the BBN which restricts the
3. Primordial gravitational wave

Primordial gravitational wave is the gravitational wave produced by quantum fluctuation in the inflation. Existence of the primordial gravitational wave is a strong evidence for the inflation. One of the characteristics of the primordial gravitational wave is that the thermal history of the universe is imprinted in its spectrum.

The gravitational wave spectrum is defined by dimensionless parameter.

$$\Omega_{GW}(f, a) = \frac{\rho_{GW}(f, a)}{\rho_{cr}(a)}, \quad \rho_{GW} = \frac{h^2(f, a)}{16\pi G} \left( \frac{2\pi f a_0}{a} \right)^2$$

Figure 1. Evolution of the energy density of radiation (red dotted line) and right-handed neutrino (blue dashed line) and the entropy production ratio (black solid line). Here $\Delta S = s_f a_0^3/s_i a_i^3$ is entropy production ratio which mean final entropy divided by initial entropy where $s_i(f)$ is initial (final) entropy density and $a_i(f)$ is initial (final) scale factor. We use common mass of right-handed neutrinos $M = (M_{N_2} + M_{N_1})/2 = 1GeV$, mass difference of right-handed neutrinos $\Delta M = (M_{N_2} - M_{N_1})/2 = 10^{-5}M$, majorana phase in PMNS matrix $\eta_1 = \eta_2 = 0$, $Re\omega = 0$, $X_\omega = 1$, $(Im\omega = 0)$.

Figure 2. The result of entropy production ratio and the reheating temperature in the case of $M = 1GeV$ and the mass difference of right-handed neutrinos $\Delta M = 10^{-5}M$. We calculate the evolution from the decoupling temperature that is set to be $T_{dec} = 1 GeV$. We also use $\eta_1 = \eta_2 = Re\omega = 0$.

reheating temperature to be larger than 1.8 MeV (e.g., see the recent paper [9]). The reheating temperature, we write $T_{reh}$, is defined by a relation $\Gamma = 3H(T_{reh})$. In this calculation we find $O(1)$ entropy production ratio. The entropy production ratio is large around $X_\omega = 1$. In the case of $X_\omega = 1$, the decay width of right-handed neutrino is small, and then the lifetime is long. In this case, the entropy is produced more effectively because right-handed neutrinos have long enough lifetime and the dominate the energy of the universe sufficiently long time.
where \( \rho_c(a) \) is the critical density of universe, \( \rho_{GW}(f, a) \) is the energy of gravitational waves per unit present logarithmic frequency \( f \), \( a \) is the scale factor for the cosmic time \( t \), \( a_0 \) is the current value of the scale factor and \( h \) is amplitude of the gravitational wave.

The modification of the gravitational wave spectrum by the entropy production is discussed in [10]. The analytic formula of the modified gravitational wave spectrum by the entropy production is given by,

\[
\Omega'_{GW}(f, a_0) = \Delta S^{\frac{4}{3}} \Omega_{GW}(f, a_{in}),
\]

where \( \Omega'_{GW}(f, a_0) \) is the spectrum with modified thermal history by entropy production, \( \Omega_{GW}(f, a_{in}) \) is the spectrum with standard cosmology and \( \Delta S \) is entropy production ratio at horizon reentering. This analytic equation shows that if the entropy production occurs, the spectrum is suppressed. In the case of right-handed neutrinos, the damping factor \( \Delta S^{\frac{4}{3}} \) will be \( O(10^{-1}) \) and the critical frequency can be around \( 10^{-11} - 10^{-8} \) Hz in this analysis. This frequency region is corresponding to the region of the pulsar timing array, like future SKA (Square Kilometre Array) experiment [11].

4. Summary

We discuss the entropy production by the decay of the GeV scale right-handed neutrinos and the modification of the primordial gravitational wave spectrum. If the lifetime of the right-handed neutrinos are long enough, their decay can produce the additional entropy production in the early universe. The entropy production changes the thermal history of the universe and affects the primordial gravitational spectrum. The suppression of the primordial gravitational wave spectrum by the entropy production appears in about \( 10^{-11} - 10^{-8} \) Hz (SKA range).

Unfortunately, the sensitivity of SKA can not reach the primordial gravitational wave, but the evidence of the modified history of right-handed neutrino may be found in the future experiments if the sensitivity will be improved. Even if we would not observe the right-handed neutrinos in terrestrial experiments, we may be able to find their signals by the cosmological observations such as gravitational waves.

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