CP violation of the two-body charmless hadronic $B$ decays in the minimal supergravity model

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By choosing two typical input parameter points in the minimal supergravity (mSUGRA) model and using the QCD factorization (QCDF) approach, we studied the supersymmetric effects to the CP violation of the two-body charmless hadronic $B$ meson decays. We found that though the SUSY contributions can give large corrections to the CP asymmetries for some decay channels, they could not be distinguished experimentally from the SM values because of the large theoretical errors dominated by calculating the annihilation contributions in the QCDF approach.

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As one of the main goals of $B$ experiments, measurements of CP asymmetries in various $B$ decay processes are at the center of attention. Since in the standard model (SM) all the measured CP asymmetries have to be consistently explained by the complex phase $\delta$ in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, precise measurements of CP asymmetries at the current $B$ factories can be used to test the CKM mechanism and sequentially the SM[1, 2].

Experimentally, Belle collaboration has given the first experimental evidence for the direct CP violation in the decay mode $B \rightarrow \pi^+\pi^-[3]$. And presently large direct CP violation in neutral $B$ meson decay $B^0 \rightarrow K^+\pi^-$ has been measured by both BaBar[4] and Belle[5] collaborations, but no CP violation has been found for the charged $B$ meson decay $B^+ \rightarrow K^+\pi^0$. The world averages given by the Heavy Flavor Averaging Group (HFAG) [6] are

$$A_{CP}(B^0 \rightarrow K^+\pi^-) = -0.115 \pm 0.018,$$
$$A_{CP}(B^+ \rightarrow K^+\pi^0) = -0.04 \pm 0.04.$$  (1)

In the SM and with the QCD factorization (QCDF) approach, the CP-violating asymmetry for these two channels should be naturally very similar in size and positive in sign for a set of favored hadronic parameters [2]. In the perturbative QCD (PQCD) factorization approach[7], however, the SM predictions are $A_{CP}(B^0 \rightarrow K^+\pi^-) \approx -0.18$ and $A_{CP}(B^0 \rightarrow K^+\pi^-) \approx -0.15$: the first prediction agrees well with the measured value as given in Eq. (1), but the second one shows a large deviation with the data.

For the measurement of the time-dependent CP asymmetries, great efforts have been made by BaBar, Belle and other collaborations. The recently reported measurements of

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time-dependent CP asymmetries in $B \to \phi K_s$ decays and other s-penguin modes lead to

$$\sin(2\beta)_{\text{ave}} = 0.50 \pm 0.06,$$

which is a 2.6σ deviation from that for $B \to J/\psi K_s$ and other charmonium modes,

$$\sin(2\beta)_{\text{ave}} = 0.687 \pm 0.032.$$

Therefore, something must be there to account for the large deviations. Since the $B \to \phi K_s$ and other s-penguin modes are induced at the one loop level, while $B \to J/\psi K_s$ and other charmonium modes are governed by tree level decays, it is tempting to expect that the new physics (NP) contributions are more significant to the former and do not show up clearly to the latter.

Recently, many works about various NP models have been done in the $B$ meson system [9]. In our previous papers [10], we have studied the minimal supergravity (mSUGRA) model by calculating the branching ratios of $B \to M_1 M_2$ ($M_i$ stands for the light pseudoscalar (P) or vector (V) mesons) decays, and have found some interesting results. Based on the previous works [10], here we will try to calculate how much can the SUSY contributions in the mSUGRA model affect the CP asymmetries of the $B \to M_1 M_2$ decays by using the same input parameters as listed in the Appendix of Ref. [10].

In the mSUGRA model [11], only four continuous free parameters and an unknown sign are left, say $\tan \beta$, $m_{1/2}$, $m_0$, $A_0$ and $\text{sign}(\mu)$. The new effects on the rare $B$ meson decays from this model will manifest themselves through the corrections to the Wilson coefficients of the same operators involved in the SM calculation. In Ref. [10], we have found that the SUSY contributions to the Wilson coefficients $C_k (k = 3 \sim 6)$ are always small and can be neglected safely. But for the Wilson coefficients $C_7 (m_b)$ and $C_8 (m_b)$, the SUSY contributions can be rather large and even make them have an opposite sign with the SM ones.

Similar to Ref. [10], considering the constraints coming from the well measured $B \to X_s \gamma$ decays and so on, we take two typical parameter points, say Case A and B as listed in the following tabular. For Case C in Ref. [10], the Wilson coefficient $C_7 (m_b)$ in the mSUGRA model is also SM-like (negative) as that in Case A. Hence in our calculations we will only consider Case A and B. The numerical values of the ratio $R_7 = C_7 (m_b)/C_7^{\text{SM}} (m_b)$ for the two cases have also been given in the tabular. It is easy to see that the Wilson coefficient $C_7 (m_b)$ in the mSUGRA model is SM-like (negative) for Case-A, but nonstandard (positive) for Case-B.

| Case | $m_0$ | $m_{1/2}$ | $A_0$ | $\tan \beta$ | $\text{Sign}[\mu]$ | $R_7$ |
|------|-------|-------------|-------|--------------|------------------|------|
| A    | 300   | 300         | 0     | 2            | −                | 1.10 |
| B    | 369   | 150         | −400  | 40           | +                | −0.93|

Just take a look at the two typical sets of SUSY parameters, both of them are chosen to be real for the constraint condition coming from the measured electric dipole moment (EDM) of neutron and electron. Therefore no new weak phase has been introduced and the mechanism of CP violation in the mSUGRA model is the same as that in the SM. But when we calculate the amplitudes of $B$ decays by using the QCDF approach [2],

$$\sin(2\beta)_{\text{ave}} = 0.50 \pm 0.06,$$
the real and imaginary part of the factorized coefficient \( a_i(M_1M_2) \) or \( \alpha_i(M_1M_2) \) (See Ref. 2 for detailed definitions) will be modified by the SUSY contributions and hence the CP-violating parameter will be affected in this model.

We begin with a discuss of the CP asymmetries in \( B \rightarrow PP \) decays. Firstly, for those channels which have only direct CP violations, one can define

\[
A_{CP} = \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)}. \tag{4}
\]

As we have discussed in Ref. [10], the potential SUSY contributions are mainly embodied in \( \alpha_{p}^{q}(M_1M_2) \) and \( \alpha_{p,ew}^{q}(M_1M_2) \). Therefore, we naturally expect a large new physics corrections to those penguin dominated \( B \) meson decays. According to our calculations, things are absolutely so. For the tree-dominated decay channels, say \( B^{\pm} \rightarrow \pi^{\pm}(\pi^{0},\eta^{(')}) \), the SUSY corrections are less than 10% for both Case A and B. However for other channels which are penguin-dominated, we can see from Table I: (a) in Case A, all the corrections are still small and less than 10% but in Case B the corrections are large and the largest one is about \(-35\%\) for \( B \rightarrow \pi^{\pm}K^{\mp} \). (b)Though the corrections in Case B are large for those channels, experimentally they can not be distinguished from the SM values for they are all of the same order of magnitude as SM values. (c) For the very interesting decay \( B \rightarrow \pi^{\pm}K^{\mp} \), \( A_{CP} \) in both the SM and mSUGRA model have a different sign with the data in Eq. (1). If we consider the large uncertainties from the annihilation contributions, \( A_{CP} \) is

\[
A_{CP}(\pi^{\pm}K^{\mp}) = \begin{cases} 
4.5^{+8.9}_{-9.4}, & \text{SM} \\
4.9^{+9.5}_{-10.2}, & \text{Case } A \\
3.0^{+5.8}_{-6.1}, & \text{Case } B 
\end{cases} \tag{5}
\]

and can only be consistent with the data within about three standard deviations. However, in Refs. [12], by using the PQCD approach or considering the final state interaction (FSI) modifications to short-distance (SD) predictions in QCDF approach, the authors found the consistency between the theory and the experiments for \( B \rightarrow \pi^{\pm}K^{\mp} \) decay. For more details, one can see these references.

### TABLE I: Theoretical results in percent for those penguin dominated and having only direct CP violations \( A_{CP} \) channels of \( B \rightarrow PP \) in the SM and mSUGRA model, where superscripts “f+a” and “f” denote the predictions with or without weak annihilation contributions considered.

| Decay Modes | SM | Case A | Case B |
|-------------|----|--------|--------|
| \( B^d \rightarrow \pi^{\pm}K^{\mp} \) | 5.36 | 4.54 | 5.75 | 3.47 |
| \( B^{\pm} \rightarrow \pi^{0}K^{\pm} \) | 8.28 | 7.45 | 8.70 | 6.09 |
| \( B^{\pm} \rightarrow \pi^{\pm}K^{0} \) | 1.00 | 0.91 | 1.03 | 0.81 |
| \( B^{\pm} \rightarrow K^{\mp}\eta \) | -13.7 | -12.8 | -14.1 | -11.5 |
| \( B^{\pm} \rightarrow K^{\pm}\eta^{(')} \) | 2.38 | 2.04 | 2.46 | 1.94 |
| \( B^{\pm} \rightarrow K^{\pm}K^{0} \) | -26.8 | -24.0 | -27.6 | -22.0 |
Secondly, for the CP asymmetries of the left $B \to PP$ channels, they are time-dependent and can be described by

$$A_{CP}(t) = \frac{\Gamma(\bar{B}^0(t) \to \bar{f}) - \Gamma(B^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(B^0(t) \to f)} = S_f \sin(\Delta M t) - C_f \cos(\Delta M t)$$ (6)

where $C_f$ and $S_f$ represent the direct and the mixing CP asymmetry, respectively, and they are given by

$$C_f = \frac{1 - |\lambda_{CP}|^2}{1 + |\lambda_{CP}|^2}, \quad S_f = \frac{2\text{Im}(\lambda_{CP})}{1 + |\lambda_{CP}|^2}. \quad (7)$$

The parameter $\lambda_{CP}$ is defined by

$$\lambda_{CP} = \frac{V_{td}^* V_{td} A(\bar{B}^0(0) \to \bar{f})}{V_{td}^* V_{td} A(B^0(0) \to f)}.$$ (8)

Through the numerical calculations, we found that for those neutral $B_d \to PP$ decays, the SUSY contributions to the two CP asymmetric parameters $C_f$ and $S_f$ are generally small or moderate in both Case A and B. The largest correction is only about 26% to $C_f$ of $B_d \to K_S^0 \pi^0$ and make it vary from 3.8% to 2.8%. Obviously, so small correction can not be distinguished experimentally and can be masked easily by the uncertainty coming from the annihilation contributions.

TABLE II: The same as Table II but for some $B \to PV$ channels.

| Decays         | SM | Case A       | Case B       |
|----------------|----|--------------|--------------|
| $B^\pm \to \pi^\pm K^{*0}$ | 2.01 | 1.71 | 1.29 | 1.27 |
| $B^\pm \to \pi^0 K^{*\pm}$ | 9.26 | 8.56 | 6.81 | 6.32 |
| $B^\pm \to K^{*\pm} \eta$ | 3.19 | 2.98 | 3.21 | 3.08 |
| $B^\pm \to K^{*\pm} \eta'$ | -27.5 | -19.9 | -25.1 | -64.5 |
| $B^\pm \to K^{0} \rho^\pm$ | -13.5 | -11.6 | -13.1 | -17.2 |
| $B^\pm \to K^{0} \phi$ | 0.12 | 0.16 | 0.12 | 0.19 |
| $B^\pm \to \pi^\pm K^{*0}$ | -6.61 | -5.90 | -6.43 | -8.21 |
| $B^\pm \to K^{\pm} \phi$ | 2.12 | 1.75 | 2.27 | 1.44 |
| $B^\pm \to K^{\pm} K^{*0}$ | -52.7 | -46.0 | -56.7 | -34.4 |
| $B^\pm \to K^{0} K^{\pm}$ | -2.55 | -2.92 | -2.43 | -4.03 |
| $B_d \to \pi^\pm K^{*\pm}$ | -0.32 | -0.37 | -0.28 | -0.26 |
| $B_d \to \pi^0 K^{*0}$ | -15.4 | -12.9 | -18.1 | -7.34 |
| $B_d \to K^{\pm} \rho^\pm$ | -4.51 | -3.25 | -4.20 | -8.18 |
| $B_d \to K^{*0} \eta$ | 4.93 | 4.45 | 4.97 | 4.69 |
| $B_d \to K^{*0} \eta'$ | -11.2 | -8.15 | -10.4 | -23.8 |
TABLE III: The same as Table II but for CP asymmetries parameter $C_f$ and $S_f$ of some $B_d \to PV$ channels.

| Decay Modes | SM     | Case A | Case B |
|-------------|--------|--------|--------|
| $B_d \to \pi^0\rho^0$ | $C_f$  | -4.47  | -3.03  | -2.94  | -15.8  |
|             | $S_f$  | -32.1  | -35.8  | -35.2  | -8.92  |
| $B_d \to \pi^0\omega$ | $C_f$  | 74.4   | 94.6   | 73.0   | 61.6   |
|             | $S_f$  | -62.5  | 32.3   | -56.8  | -64.9  |
| $B_d \to \eta\rho^0$ | $C_f$  | 6.28   | 1.94   | -4.51  | 47.6   |
|             | $S_f$  | -29.3  | -32.7  | -52.4  | 87.9   |
| $B_d \to \eta^\prime\rho^0$ | $C_f$  | 46.5   | 37.3   | 44.8   | 58.1   |
|             | $S_f$  | -69.5  | -63.3  | -73.6  | -24.6  |
| $B_d \to \eta\omega$ | $C_f$  | 8.36   | 5.48   | 7.14   | 16.9   |
|             | $S_f$  | -41.5  | -38.0  | -39.4  | -55.6  |
| $B_d \to \eta^\prime\omega$ | $C_f$  | -18.4  | -17.9  | -19.3  | -11.5  |
|             | $S_f$  | -28.9  | -25.5  | -27.0  | -42.9  |
| $B_d \to K^0_s\rho^0$ | $C_f$  | -9.15  | -7.91  | -8.85  | -12.3  |
|             | $S_f$  | -62.1  | -63.8  | -62.6  | -57.1  |
| $B_d \to K^0_s\omega$ | $C_f$  | 9.71   | 8.26   | 9.36   | 13.5   |
|             | $S_f$  | -89.8  | -87.9  | -89.5  | -93.9  |
| $B_d \to K^0_s\phi$ | $C_f$  | -2.12  | -1.85  | -2.27  | -1.44  |
|             | $S_f$  | 76.9   | 76.6   | 76.9   | 76.9   |

We now take a look at $B \to PV$ mode. Different from $B \to PP$ mode, the SUSY correction to this mode can interfere with the SM counterparts constructively or destructively [10]. Through the numerical results, we found the new physics corrections to the CP violations in Case B are large for most channels. For those channels having only direct CP violation, the penguin dominated ones are affected a lot. As shown in Table III in Case B the largest SUSY corrections is about 135% for $B^\pm \to K^{*\pm}\eta^\prime$ channel and make the size of its CP violation increased from 28% to 65%. Such large corrections may be measured experimentally.

For other neutral $B_d \to PV$ decays, they have both direct CP violation $C_f$ and mixing CP asymmetry $S_f$. For the CP-violating parameters of $B \to (\pi, \eta^\prime)\phi$ decays, since these channels receive no contributions from electromagnetic or chromo-magnetic penguin operators where the SUSY contributions are entered, they remain almost unchanged in the mSUGRA model. For other channels as shown in Table III the direct CP violations of most channels in Case B are greatly affected by the SUSY corrections. The largest corrections even reach a factor of 7 for the decay $B^0 \to \eta\rho^0$, about 253% increase for $B^0 \to \pi^0\rho^0$ and 100% enhancement for $B^0 \to \eta\pi$. As to the indirect CP violation $S_f$ in Case B, the SUSY corrections are also large. For $B_d \to \eta\rho^0$ decay, for example, the sign of its indirect CP violation has been changed and the size also increased by a factor of 3. But for the very interesting channels $B \to \phi K_s$, the SUSY contributions make little
TABLE IV: The same as Table II but for some $B \to VV$ channels.

| $A^{dir}(B \to VV)$ | SM | Case A | Case B |
|---------------------|----|--------|--------|
| $A_{CP}(B^0 \to K^{*-}\rho^+)$ | 17.8 | 19.3 | 10.5 |
| $A_{CP}(B^0 \to \bar{K}^{0}\rho^0)$ | −21.1 | −23.4 | −11.9 |
| $A_{CP}(B^- \to K^{*-}\rho^0)$ | 18.7 | 19.6 | 13.8 |
| $A_{CP}(B^- \to \bar{K}^{0}\rho^-)$ | 1.57 | 1.63 | 1.25 |
| $C_f(B^0 \to \bar{K}^{0}K^{*0})$ | −28.2 | −29.4 | −21.6 |
| $A_{CP}(B^- \to K^{*-}K^{*0})$ | 28.2 | 29.4 | 21.6 |
| $A_{CP}(B^0 \to \bar{K}^{0}\phi)$ | 12.8 | 13.4 | 9.68 |
| $A_{CP}(B^- \to K^{*-}\phi)$ | 32.5 | 34.3 | 22.4 |
| $A_{CP}(B^0 \to \bar{K}^{0}\omega)$ | 1.75 | 1.81 | 1.38 |
| $A_{CP}(B^- \to K^{*-}\omega)$ | 1.75 | 1.81 | 1.38 |

effects and hence the “$\phi K_s$” puzzle as mentioned above still cannot be solved here.

At last, let us talk about $B \to VV$ mode in brief. For $B \to VV$, we first give two remarks: (a) From Eq. (8) one can see that the parameter $\lambda_{CP}$ for $B \to VV$ mode is helicity-dependent since three decay amplitudes with $\lambda = (0, \pm1)$ are different. It follows that theoretically the indirect CP violation $S_f$ can be given only when the two vector mesons in the final state have a certain helicity. Hence in our calculation we only give the direct CP violation ($A_{CP}$ or $C_f$) of $B \to VV$ in Table IV. (b) The annihilation amplitude in the $VV$ case does not gain a chiral enhancement of order $M_P^2/(m_q m_b)$ as that in $B \to PP$ and $PV$ modes. Therefore, it is truly power suppressed in heavy quark limit and we have ignored such contributions in our calculation.

Similar to $B \to PP$, only in the QCD penguin-dominated channels can their CP violation be affected a lot. From Table IV in Case B the mSUGRA predictions of CP violations for all the listed channels become smaller than corresponding SM predictions. As to $B^0 \to K^{*0}\rho^0$, the SUSY contributions in Case B can even make its CP-violating parameter decreased by about 44%. However, these SUSY contributions are of the same order of magnitude as SM values and therefore almost impossible to be distinguished experimentally.

To conclude, we have computed the CP asymmetries for two-body charmless $B \to M_1M_2$ decays in the mSUGRA model based on our previous works. Since the SUSY phases in the mSUGRA model are so small as to be ignored safely and the Yukawa couplings are the main source of the flavor structure, we found that the SUSY contributions to the CP asymmetries in charmless $B$ decays are generally small except in most $B \to PV$ channels. Currently, people have tried to measure the CP violation in $B$ decays, but the data are always error-weighted and consistent with zero except for $B \to \pi^\pm K^{\mp}$. Moreover, large theoretical uncertainties also exist for example in the QCDF approach where the weak annihilations and other potential power corrections are not well calculated. Therefore, low experimental statistics and large theoretical uncertainties together prevent us from testing the mSUGRA model through studies of the CP-violating asymmetries at
present. We are waiting for great progress in both the theory and experiment.

[1] I.I. Bigi and A.I. Sanda, *CP violation*, Cambridge University Press, 2000; G.C. Branco, L. Lavoura and J.P. Silva, *CP Violation*, Clarendon Press, Oxford, 1999; A. Ali, G. Kramer, and C.D. Lü, Phys. Rev. D 59, 014005 (1999). M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 43, 242 (1979);

[2] M. Beneke, et al., Phys. Rev. Lett. 83, 1914 (1999); Nucl. Phys. B 591, 313 (2000); *ibid*, 606, 245 (2001); M. Beneke and M. Neubert, Nucl. Phys. B 651, 225 (2003); *ibid*, 675, 333 (2003).

[3] Belle Collaboration, K. Abe, et al., Phys. Rev. Lett. 93, 021601 (2004).

[4] BaBar Collaboration, B. Aubert, et al., Phys. Rev. Lett. 93, 131801(2004); hep-ex/0408080.

[5] Belle Collaboration, K. Abe, et al., hep-ex/0507045.

[6] Heavy Flavor Averaging Group, hep-ex/0505100.

[7] H.-n. Li, Phys. Lett. B 348, 597 (1995); H.-n. Li and H.L. Yu, Phys. Rev. Lett. 74, 4388 (1995); Phys. Lett. B 353, 301 (1995), Phys. Rev. D 53, 2480 (1996).

[8] Y.-Y. Keum, H.-n Li and A.I. Sanda, Phys. Lett. B 504, 6 (2001); Phys. Rev. D 63, 054008 (2001).

[9] Z.J. Xiao, et al., Eur. Phys. J. C 18, 681 (2001); S. Khalil and E. Kou, Phys. Rev. D 67, 055009 (2003); Z.J. Xiao and L.B. Guo, Commun. Theor. Phys. 40, 77 (2003); J.F. Cheng, C.S. Huang, and X.H. Wu, Phys. Lett. B 585, 287 (2004); X.G. He, C.S. Li and L.L. Yang, Phys. Rev. D 71, 054006 (2005); S. Khalil, Phys. Rev. D 72, 035007 (2005); H.J. Lipkin, Phys. Lett. B 621, 126 (2005).

[10] Z.J. Xiao and W.J. Zou, Phys. Rev. D 70, 094008 (2004); W.J. Zou and Z.J. Xiao, Phys. Rev. D 72, 094026 (2005).

[11] R. Barbieri, S. Ferrara, and C.A. Savoy, Phys. Lett. B 119, 343 (1982); A.H. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. 49, 970 (1982); L. Hall, J. Lykken, and S. Weinberg, Phys. Rev. D 27, 2359 (1983); Nobuyoshi Ohta, Prog. Theor. Phys. 70, 542 (1983); A. Djouadi, et al., hep-ph/9901246.

[12] H.Y. Cheng, C.K. Chua, and A. Soni, Phys. Rev. D 71, 014030 (2005); Bi-Hai Hong and C.D. Lü, hep-ph/0505020; Hai-Yang Cheng, hep-ph/0508063.