Shot noise of spin current

Baigeng Wang\textsuperscript{1,2}, Jian Wang\textsuperscript{3,4,15}, and Hong Guo\textsuperscript{4}

1. Department of Physics, The University of Hong Kong, Pokfulam Road, Hong Kong, China
2. National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing, P.R. China
3. Institute of Solid State Physics, Chinese Academy of Sciences, Hefei, Anhui, China
4. Department of Physics, McGill University, Montreal, Quebec, Canada H3A 2T8

We report an exact solution for the noise spectrum of spin-current without charge-current in a spin field effect transistor (SFET). For the SFET with two leads, it is found that both auto- and cross-correlation functions are needed to characterize the presence of spin-current. A shot noise is also generated for the charge-current even though the net charge-current is identically zero. The noise spectra of spin-current and charge-current depends on the coupling strength between the SFET and the leads, and they may behave qualitatively differently. We also present results of noise spectra in the adiabatic regime.

Due to the particle nature of carriers, the fluctuation of charge-current gives rise to the notion of shot noise\textsuperscript{1}. For a normal system, the current correlation between different probes (cross correlation) is negative for Fermions and positive for Bosons\textsuperscript{2,3}. The correlation can be more complicated if superconducting leads are present\textsuperscript{6–12}. Importantly, the correlation properties provide further information, in addition to those contained in conductance or charge-current, for mesoscopic conductors\textsuperscript{2}. So far all theoretical and experimental attention have been devoted to correlations of charge-current.

In this paper, we theoretically investigate the correlation of spin-current in the absence of charge-current in a two-probe device. Indeed, to be able to generate and control spin-current is of great importance for spintronics\textsuperscript{13}, but the noise spectra of spin-current has never been studied before. A pure spin-current can behave qualitatively differently from charge-current because spins can flip so that spin-current may not be conserved. Therefore the cross correlation of currents may not be directly related to the auto-correlation. On the other hand, the conservation of charge forces the cross correlation and auto-correlation of charge-current noise spectra to differ by just a minus sign in a two-probe system. This difference necessarily indicates that the noise spectra of spin-current is more complicated. To be more specific, we will investigate the spin-current correlation in a two-probe quantum spin field effect transistor (SFET)\textsuperscript{14}. The detailed working principle of the SFET is summarized elsewhere\textsuperscript{14} and we refer interested readers there: this device delivers an instantaneous spin-current but without generating a net charge-current by the help of a rotating magnetic field. Roughly, if all the spin-up electrons in a conductor move to one direction while an equal number of spin-down electrons move to the opposite direction, the net charge-current vanishes because $I_e = e(I_{\uparrow} + I_{\downarrow}) = 0$; and a finite spin-current results because $I_s = \hbar/2(I_{\uparrow} - I_{\downarrow}) \neq 0$. Here $(I_{\uparrow}, I_{\downarrow})$ are the electron current for spin-up and spin-down electrons respectively. The SFET provides a mechanism for this effect, and there are a number of other possibilities to produce $I_s \neq 0$ with $I_e = 0$ which is the subject of several recently studies both theoretically\textsuperscript{15–17} and experimentally\textsuperscript{18}.

For a two-lead device, correlations can be formed by quantities measured at the same lead—the auto-correlation, or by quantities measured at the two different leads—the cross-correlation. As discussed above, due to a lack of spin-current conservation, we need both auto- and cross-correlation to characterize the noise spectra of spin-current. It has been well known\textsuperscript{19} that anti-bunching in a Fermionic system gives rise to negative definite cross-correlation for charge-current, and much of the recent research has been devoted to situations where the sign of this correlation\textsuperscript{1} can be reversed. For spin-current, the situation is very different. We found that when the coupling between SFET and lead is strong, the cross-correlation function of spin-current is positive definite. In the intermediate or weak coupling regime, the cross-correlation of spin-current is positive when the system is far off a quantum resonance but it becomes negative near the resonance. In the adiabatic regime, the shot noise of a SFET involving only a single lead vanishes at resonance which corresponds to the quantization of delivered spin. As the coupling strength between the single lead and the SFET is varied, the shot noise is found to displays different number of peaks depending on the coupling strength. Many of these features persist in the non-adiabatic regime. Finally, an oscillation between positive and negative shot noise is observed as a SFET parameter is tuned. These findings are qualitatively different and much richer than the behavior of charge-current correlations.

We start by considering the SFET which generates spin-current without charge-current, described by the following Hamiltonian\textsuperscript{14}

\begin{equation}
H = \sum_{k, \sigma, \alpha = L,R} \varepsilon_k C_{k,\alpha\sigma}^\dagger C_{k,\alpha\sigma} + \sum_{\sigma} \left[ \epsilon + \sigma B_0 \cos \theta \right] d_{\sigma}^\dagger d_{\sigma} + \gamma \left[ \exp(-i\omega t) d_{\downarrow}^\dagger d_{\uparrow} + h.c. \right] + \sum_{k, \sigma, \alpha = L,R} \left[ T_{k,\alpha} C_{k,\alpha\sigma}^\dagger d_{\sigma} + h.c. \right]
\end{equation}

where the first term stands for the non-interacting electrons in lead $\alpha = L, R$ and $C_{k,\alpha\sigma}^\dagger$ is the creation oper-
ator. Note that we have set the same chemical potential for the leads $L$ and $R$ because we are only interested in the quantum pumping effect$^{20}$ of the SFET. The second term describes the scattering region of the SFET which is a quantum dot characterized by an energy level $\epsilon$ and spin $\sigma$. A gate voltage $v_g$ is applied to control the energy level. The SFET generates spin-dependent particle current operator $\bar{C}_{\alpha\sigma}$ via a time-dependent magnetic field $B(t) = B_0[\sin \theta \cos \omega t + i \sin \theta \sin \omega t \hat{J} + \cos \theta \hat{k}]$. The last term of the Hamiltonian denotes tunneling between the leads and the dot with tunneling matrix elements $T_{\alpha\sigma}$. Importantly, we apply a rotating magnetic field (the term proportional to $\gamma$) rather than an oscillating field: a counter-clockwise rotating field allows a spin-down electron to absorb a photon and flip to spin-up, and it does not allow a spin-up electron to emit a photon and flip to spin-down. As schematically shown in the upper inset of Fig. (2), it is this symmetry breaking field that provides the driving force to deliver a pure DC spin-current to the leads$^{21}$.

To analyze the noise spectrum of spin-current, we define a spin-dependent particle current operator ($\hbar = 1$)

$$J_{\alpha\sigma} = \sum_k \frac{d[C_{k\alpha\sigma}^\dagger C_{k\alpha\sigma}]}{dt} = -i \sum_k [\mathcal{T}_{k\alpha} C_{k\alpha\sigma} d_{\sigma} - h.c.]$$

(2)

Then the charge-current operator is $I_{aq} = \sum_\sigma \tilde{J}_{a\sigma}$. An important quantity for our study is the correlation between spin-dependent particle currents in lead $\alpha$ and $\beta$,

$$S_{\alpha\beta}^{\sigma\sigma'} = \langle [\tilde{J}_{\alpha\sigma}(t_1) - \tilde{J}_{\alpha\sigma}(t_2)] [\tilde{J}_{\beta\sigma'}(t_1) - \tilde{J}_{\beta\sigma'}(t_2)] \rangle$$

(3)

with $\tilde{J}_{a\sigma} = \langle \tilde{J}_{a\sigma} \rangle$ and $\sigma, \sigma'$ denoting spin indices. Here $\langle \cdots \rangle$ denotes both statistical average and quantum average on the nonequilibrium state. The noise spectra of both charge-current and spin-current can be obtained from the correlation $S_{\alpha\beta}^{\sigma\sigma'}$.

We calculate $S_{\alpha\beta}^{\sigma\sigma'}$ using standard Keldysh nonequilibrium Green’s function (NEGF) formalism$^{14}$. Briefly, we substitute Eq.(2) into Eq.(3), define NEGF $G^{<}$ and $G^{>}$ with properly contour ordered operators, and apply the theorem of analytic continuation$^{11}$ so that the contour Green’s functions are extended to the real time axis. This standard and widely used NEGF technique allows us to obtain the exact expression for the zero-frequency spin-dependent correlation$^{22}$.

We now examine noise spectrum in the low temperature limit $kT << \hbar \omega$, i.e. the shot noise. First, setting $\alpha = L, \beta = R$, we investigate shot noise of the charge-current (cross correlation). It has been shown in Ref. 14 that the net charge-current of the SFET is identically zero$^{21}$, however here we find that shot noise of it is nonzero. The shot noise of the charge-current is

$$S_{\text{ele}} = \langle \Delta I_L \Delta I_R \rangle = \sum_{\sigma\sigma'} S_{LL}^{\sigma\sigma'}$$

$$= -q^2 \Gamma_L \Gamma_R \int \frac{dE}{2\pi} \left[ |G_{\uparrow\downarrow}^{\sigma\sigma'}|^2 + |G_{\downarrow\uparrow}^{\sigma\sigma'}|^2 \right] f_\uparrow(1 - f_\uparrow)$$

(4)

where $G_{\sigma\sigma'}^{\alpha} = \gamma/[(E - \epsilon + i\Gamma/2)(E - \epsilon + \sigma \omega + i\Gamma/2) - \gamma^2]$ is obtained from Ref. 14. Here $\Gamma_\alpha$ is the linewidth function, $f_\uparrow = f_\uparrow(E)$, and $f_\uparrow = f_\uparrow(E - \omega)$. We conclude that $S_{\text{ele}} \neq 0$ although the instantaneous charge-current is zero$^{14}$. The excess noise has the following lineshape in the adiabatic limit (i.e. $\omega \rightarrow 0$),

$$S_{\text{ele}} = -\frac{q^2 \omega}{2\pi} \frac{2\Gamma_L \Gamma_R \gamma^2}{(\epsilon^2 + \Gamma^2/4 - \gamma^2)^2 + \Gamma^2 \gamma^2}$$

(5)

here $\epsilon$ is the resonant level of the quantum dot and we have set $\theta = \pi/2$ and $E_F = 0$. As expected, the excess noise (cross correlation) in charge-current is always negative. A single peak is found at $\epsilon = 0$ for $\gamma \leq \Gamma/2$ while a double peak structure occurs when $\gamma > \Gamma/2$. This can be understood as follows. When the spin flip magnetic field $\gamma$ is turned on, the spin degeneracy of resonant state is broken. Two resonant states are found at $\epsilon \pm \gamma - i\Gamma/2$ with level spacing $2\gamma$ and half width $\Gamma/2$. When $\gamma > \Gamma/2$, the two resonant states do not overlap and we obtain two peaks. However, if $\gamma \leq \Gamma/2$, only one peak is showed up due to the overlapping of resonant states. Finally, the auto-correlation is obtained by setting $\alpha = \beta = L$, $S_{\text{ele}}^{\alpha\beta} = S_{\alpha\beta}^{\sigma\sigma'}$, and it is straightforward to show $\langle \Delta I_L \Delta I_L \rangle = \langle \Delta I_L \Delta I_R \rangle$. This is the expected result for charge-current which is a conserved quantity, i.e. $I_L + I_R = 0$.

Next, we calculate the shot noise for spin-current. The cross correlation spin-current shot noise is found to be:

$$S_{\text{spin,1}} = \langle \Delta J_{\text{LL}} \Delta J_{\text{LL}} \rangle [\Delta J_{\text{RR}} - \Delta J_{\text{LR}}]$$

$$= (S_{11}^{\text{LL}} + S_{22}^{\text{LL}} - S_{12}^{\text{LL}} - S_{21}^{\text{LL}})/4 = \int \frac{dE}{8\pi} f_\uparrow(1 - f_\uparrow)$$

$$\Gamma_L \Gamma_R [\langle |G_{\uparrow\downarrow}^{\sigma\sigma'}|^2 + |G_{\downarrow\uparrow}^{\sigma\sigma'}|^2 - 2\Gamma^2 (|G_{\uparrow\downarrow}^{\sigma\sigma'}|^4 + |G_{\downarrow\uparrow}^{\sigma\sigma'}|^4) \rangle]$$

(6)

and the auto-correlation is found to be:

$$S_{\text{spin,2}} = \langle \Delta J_{\text{LL}} \Delta J_{\text{LL}} \rangle [\Delta J_{\text{LL}} - \Delta J_{\text{RR}}]$$

$$= (S_{11}^{\text{LL}} + S_{22}^{\text{LL}} - S_{12}^{\text{LL}} - S_{21}^{\text{LL}})/A = \int \frac{dE}{8\pi} f_\uparrow(1 - f_\uparrow)$$

$$\{2[-\Gamma_L^2 \Gamma_R^2 (|G_{\uparrow\downarrow}^{\sigma\sigma'}|^4 + |G_{\downarrow\uparrow}^{\sigma\sigma'}|^4) + \Gamma_L \Gamma_R (|G_{\uparrow\downarrow}^{\sigma\sigma'}|^2 + |G_{\downarrow\uparrow}^{\sigma\sigma'}|^2)] \}$$

(7)

In contrast to the shot noise of charge-current, we need both the cross- and the auto-correlation to characterize the shot noise for spin-current for the two-lead SFET. The reason is because a spin-current is not conserved due to the spin flip mechanism introduced by the rotating magnetic field. In the adiabatic limit, the cross correlation reduces to

$$S_{\text{spin,1}} = \frac{\omega}{4\pi} \Gamma_L \Gamma_R \gamma^2 (\epsilon^2 + \Gamma^2/4 - \gamma^2 + \Gamma \gamma)$$

$$\times (\epsilon^2 + \Gamma^2/4 - \gamma^2 - \Gamma \gamma)$$

(8)

Hence, cross correlation $S_{\text{spin,1}}$ can be either positive or negative depending on a number of parameters: the
gate voltage which controls the energy level position, the linewidth function \( \Gamma \), and the external magnetic field strength \( \gamma \) (see Fig.1). Far away from resonance, the cross correlation is always positive when \( x > \sqrt{2} \) (with \( x \equiv |e|/\gamma \)) regardless of the coupling strength between quantum dot and the leads. On the other hand, for \( x < \sqrt{2} \), the cross correlation is positive definite when the coupling of the lead to the quantum dot is strong such that \( \Gamma > 2(1+\sqrt{2-x}) \gamma \). When the coupling to the lead is in the intermediate range \( 2\sqrt{2-x}-1<\gamma<2(1+\sqrt{2-x}) \gamma \), the cross correlation turns to negative. In the weakly coupled regime \( \Gamma < 2\sqrt{2-x}-1 \gamma \), the cross correlation becomes positive again. The cross correlation is zero when \( \Gamma = 2(\sqrt{2-x}+1) \gamma \) or \( \Gamma = 2\sqrt{2-x}-1 \gamma \) with \( x \leq 2 \). Interestingly, as one varies the ratio \( \Gamma/(2 \gamma) \), different lineshapes are found for the cross correlation: (1) in the strong coupling regime (compared with \( 2 \gamma \)): \( \Gamma/2 > (2 + \sqrt{3}) \gamma \), the cross correlation is positive definite with a broad peak at \( \epsilon = 0 \) (see inset of Fig.1); (2) as one decreases the coupling strength \( \Gamma \) such that \( \gamma < \Gamma/2 < (2 + \sqrt{3}) \gamma \), the positive peak at \( \epsilon = 0 \) becomes a local minimum and a double peak structure appears. This can also be seen in the shot noise for the charge-current (auto correlation)\(^{23} \). Due to the overlap of the two resonant states, two peaks are found (solid line in Fig.1); (3) as one further decreases \( \Gamma \), a third peak emerges at \( \epsilon = 0 \) when \( \gamma > \Gamma/2 > (2 - \sqrt{3}) \gamma \) (so that two resonant states do not overlap) (see dotted line in Fig.1); (4) finally, in the weak coupling regime \( (\Gamma/2 < (2 - \sqrt{3}) \gamma ) \), the third peak splits and a four-peak structure appears in the shot noise (dashed line in Fig.1). Because of the spin flip mechanism, both spin-up and spin-down electrons are contributing to the spin current. The cross correlation between spin up electrons (or spin down) is negative definite while the cross correlation between spin-up electron and spin-down electron is positive definite. The competition between these two contributions gives rise to either a positive or a negative cross correlation. Such a complicated behavior is qualitatively different from the correlation in charge-current.

In the adiabatic regime, the auto-correlation is obtained from Eq.(7),

\[
S_{\text{spin}}^{2} = \frac{\omega \Gamma L \gamma^{2}[(\Gamma_{L} + \Gamma)(\epsilon^{2} + \Gamma^{2}/4 - \gamma^{2})^{2} + \Gamma R \Gamma^{2} \gamma^{2}]}{4\pi[(\epsilon^{2} + \Gamma^{2}/4 - \gamma^{2})^{2} + \Gamma^{2} \gamma^{2}]^{2}}
\]

which is positive definite. When the cross correlation is zero, the auto correlation is \( \omega L \gamma^{2}/(8\pi \Gamma) \).

The SFET can operate with a single lead\(^{14} \). In this case, the shot noise is found to be

\[
S_{\text{spin}} = \frac{\omega \Gamma^{2} \gamma^{2}(\epsilon^{2} + \Gamma^{2}/4 - \gamma^{2})^{2}}{2\pi[(\epsilon^{2} + \Gamma^{2}/4 - \gamma^{2})^{2} + \Gamma^{2} \gamma^{2}]^{2}}
\]

We observe that this shot noise is positive definite and it displays a single to four-peak structure in the lineshape as \( \Gamma \) is changed. This is similar to the behavior of \( S_{\text{spin},1} \). When \( \gamma > \Gamma/2 \), the shot noise is zero at \( \epsilon = \pm \sqrt{\gamma^{2} - \Gamma^{2}/4} \). For a single lead SFET, the spin current was found before\(^{14} \),

\[
I_{s} = \frac{-\omega}{2\pi} \frac{\Gamma^{2} \gamma^{2}}{(\epsilon^{2} + \Gamma^{2}/4 - \gamma^{2})^{2} + \Gamma^{2} \gamma^{2}}
\]

When the shot noise is identically zero, \( I_{s} = \omega/(2\pi) \). This means that the SFET pumps out two spin quanta \( (h = 1) \) in a period \( \tau = 2\pi/\omega \). This gives an example of “optimal” spin pump. The analogous optimal charge pump\(^{24} \) was discussed in literature which is actually rather difficult to achieve\(^{25,26} \).

Our exact solutions Eqs.(6) and (7) also allow the investigation of shot noise of spin-current in the non-adiabatic regime \( (\omega \neq 0) \). We fix field strength \( \gamma = 0.02 \) in the following calculation. When frequency \( \omega \neq 0 \), the noise becomes asymmetric with respect to the gate voltage. For \( \Gamma = 0.017 \), the influence of frequency is shown in the left inset of Fig.2. We observe that when \( \omega = 0.01 \) (solid line), the depths of the two minima decrease. As frequency changes to \( \omega = 0.02 \) (dotted line), the noise is still negative for a wide range of gate voltages and is positive for some negative gate voltages. Finally, when \( \omega = 0.03 \), the noise becomes positive definite. The noise as a function of frequency is depicted in Fig.2. The oscillatory behavior between positive and negative noise of the spin-current is observed, which is due to photon assisted process.

In summary, we have, for the first time, presented an exact solution of the shot noise for spin-current generated by the SFET which is essentially a quantum spin pump at arbitrary pumping frequency. For the device discussed here, the instantaneous charge current is zero but its shot noise is finite. Due to the spin flip mechanism in the SFET, the spin-current is not conserved. As a result, both auto-correlation and cross correlation must be used to characterize the shot noise of spin-current for our two-lead system. Away from a quantum resonance, we found that the cross correlation becomes positive due to the occurrence of carriers with different spin. In the adiabatic regime, the auto-correlation of a quantum dot connected to a single lead is calculated and found to be zero when the pumped spin is quantized. As the coupling strength between the single lead and the quantum dot is varied, we found that the shot noise displays different lineshapes in the different coupling regimes.

ACKNOWLEDGMENTS

We gratefully acknowledge support by a RGC grant from the SAR Government of Hong Kong under grant number HKU 7113/02P and from NSERC of Canada and FCAR of Quebec (H.G).

\(^{a)} \) Electronic mail: jianwang@hkusub.hku.hk
Briefly, the SFET works as follows. The z-component of field $B(t)$ splits the level $\epsilon$ into $\epsilon_\downarrow < \epsilon_\uparrow$. The chemical potential $\mu$ of the leads is adjusted so that $\epsilon_\downarrow < \mu < \epsilon_\uparrow$ as shown in the upper inset of Fig.(2). A spin-down electron can tunnel into $\epsilon_\downarrow$ from the left lead, it absorbs a photon due to the rotating field and flip its spin to occupy $\epsilon_\uparrow$. Because $\epsilon_\uparrow > \mu$, the now spin-up electron tunnels out to the leads easily. The same process occurs to spin-down electrons coming from the right lead. Therefore, spin-down electrons flow toward the quantum dot while spin-up electrons flow away from the dot, giving rise to a zero net charge-current and a finite spin-current. See Ref. 14 for more details.

The detailed derivation will be published elsewhere.

FIG. 1. The shot noise for spin current $S_{\text{spin},1}$ versus the gate voltage $v_g$ for different coupling strength: (1). $\Gamma = 0.05$ (solid line); (2). $\Gamma = 0.017$ (dotted line); (3). $\Gamma = 0.004$ (dashed line). Inset: $S_{\text{spin},1}$ vs $v_g$ when $\Gamma = 0.16$. Here $\gamma = 0.02$, $\Gamma_l = \Gamma_R$, and the shot noise is plotted in the unit of $\omega/(16\pi)$.

FIG. 2. $S_{\text{spin},1}/\omega$ versus frequency for $v_g = 0.007$ and $\Gamma = 0.016$. The unit is $1/(4\pi)$. Inset: $S_{\text{spin},1}$ versus gate voltage at different frequency when $\Gamma = 0.017$: $\omega = 0.01$ (solid line); $\omega = 0.02$ (dotted line); $\omega = 0.03$ (dashed line). The unit is $\omega/(4\pi)$ with $\omega = 0.005$. 

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