Heavy Baryon Transitions in a Relativistic Three-Quark Model

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Abstract

Exclusive semileptonic decays of bottom and charm baryons are considered within a relativistic three-quark model with a Gaussian shape for the baryon-three-quark vertex and standard quark propagators. We calculate the baryonic Isgur-Wise functions, decay rates and asymmetry parameters.

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1 Introduction

The investigation of semileptonic (s.l.) decays of heavy hadrons allows one to determine the unknown Cabibbo-Kabayashi-Maskawa (CKM) matrix elements, i.e. $V_{bc}$ and $V_{bu}$ in bottom meson and baryon decays. These play a fundamental role in the physics of weak interactions. The CKM matrix elements can be extracted from the inclusive s.l. width of heavy hadron [1] or decay spectra [2] and from the exclusive differential rates of $B \to D^{*}l\nu$, $\Lambda_b \to \Lambda_c l\nu$, ..., extrapolated to the point of zero recoil [1, 3]-[4]. Other characteristics of semileptonic decays (momentum dependence of transition form factors, exclusive decay rates, asymmetry parameters and etc.) are also important for our understanding of the heavy hadron structure.

From a modern point of view the appropriate theoretical framework for the analysis of hadrons containing a single heavy quark is the Heavy Quark Effective Theory (HQET) [5]-[11] based on a systematic $1/m_Q$-expansion of the QCD Lagrangian. The leading order of the HQET-expansion, when the heavy quark mass goes to infinity, corresponds to the case of Heavy Quark Symmetry (or Isgur-Wise symmetry) [6]. Due to the Isgur-Wise (IW) symmetry the structure of weak currents of low-lying baryons is simplified. The form factors of these transitions are expressed through a few universal functions. Unfortunately, HQET can give predictions only for the normalization of the form factors at zero recoil. Once one moves away from the zero recoil point one has to take recourse to full nonperturbative calculations.

This paper focuses on exclusive s.l. decays of the ground state bottom and charm baryons. Recently, the activity in this field has started to make contact with experiment due to the observation of the CLEO Collaboration [12] of the heavy-to-light s.l. decay mode $\Lambda_c^+ \to \Lambda e^+\nu_e$. Also the ALEPH [13] and OPAL [14] Collaborations expect to observe the exclusive mode $\Lambda_b \to \Lambda_c l\nu$ in the near future. Therefore, a theoretical study of the s.l. decays of heavy baryons seems to be very important.

In [15, 16] a model for QCD bound states composed of light and heavy quarks was proposed. The model is the Lagrangian formulation of the NJL model with separable interaction [17, 18] but its advantage consists in the possibility of studying baryons as relativistic systems of three quarks. The general framework was developed for light mesons [15, 16] and baryons [16, 19], and also for heavy-light hadrons [20]. Particularly, in ref. [15, 16] the pion weak decay constant, the two-photon decay width, as well as the form factor of the $\gamma^*\pi^0 \to \gamma$ transition, the pion charge form factor, and the strong $\pi NN$ form factor have been calculated and good agreement with the
data has been achieved with three parameters. Two of the parameters are range parameters characterizing the size of mesons and baryons. The remaining parameter is the constituent quark mass. In ref. [19] the approach developed in [13, 14] was applied to a calculation of the electromagnetic form factors of nucleons. Some preliminary results on s.l. decays of heavy-light baryons were already presented in [20].

The purpose of the present work is to give a description of the properties of baryons containing a single heavy quark within the framework proposed in [13, 14] and developed in [19, 20]. Namely, we report the calculation of observables in semileptonic decays of bottom and charm baryons: Isgur-Wise functions, asymmetry parameters, decay rates and distributions.

2 Model

We start with a brief review of our approach [13, 14] based on interaction Lagrangians coupling hadrons with constituent quarks and vice versa. It was found [13, 14, 19, 20] that this approach successfully describes low-energy hadronic properties like decay constants, form factors, etc. Here we are going to apply this approach to the calculation of baryonic observables when the baryons contain a heavy (b or c) quark.

Let \( y_i \) (i=1,2,3) be the spatial 4-coordinates of quarks with masses \( m_i \), respectively. They are expressed through the center of mass coordinate (\( x \)) and relative Jacobi coordinates (\( \xi_1, \ldots \)) as

\[
\begin{align*}
y_1 &= x - 3\xi_1 \frac{m_2 + m_3}{\sum_i m_i} \\
y_2 &= x + 3\xi_1 \frac{m_1}{\sum_i m_i} - 2\xi_2 \sqrt{3} \frac{m_3}{m_2 + m_3} \\
y_3 &= x + 3\xi_1 \frac{m_1}{\sum_i m_i} + 2\xi_2 \sqrt{3} \frac{m_2}{m_2 + m_3}
\end{align*}
\]

where 
\[
x = \frac{\sum_i m_i y_i}{\sum_i m_i}, \quad \xi_1 = \frac{1}{3} \left( \frac{m_2 y_2 + m_3 y_3}{m_2 + m_3} - y_1 \right), \quad \xi_2 = \frac{y_3 - y_2}{2\sqrt{3}}.
\]

We assume that the momentum distribution of the constituents inside a baryon is modeled by an effective relativistic vertex function \( F \left( \frac{1}{16} \sum_{i<j} (y_i - y_j)^2 \right) \) which depends on the sum of relative configuration space coordinates only. Its fall-off is sufficient to guarantee ultraviolet
convergence of matrix elements. At the same time the vertex function is a phenomenological
description of the long distance QCD interactions between quarks and gluons.

Then the general form of the interaction Lagrangian of baryons with quarks is written as

\[
L^\text{int}_B(x) = g_B \bar{B}(x) \int dy_1 \int dy_2 \int dy_3 \delta \left( x - \frac{\sum_i m_i y_i}{\sum_i m_i} \right) F \left( \frac{1}{18} \sum_{i<j} (y_i - y_j)^2 \right) \times J_B(y_1, y_2, y_3) + h.c.
\]  

with \( J_B(y_1, y_2, y_3) \) being the 3-quark current with quantum numbers of a baryon \( B \):

\[
J_B(y_1, y_2, y_3) = \Gamma_1 q^{a_1}(y_1)q^{a_2}(y_2)C \Gamma_2 q^{a_3}(y_3)\varepsilon^{a_1 a_2 a_3}.
\]  

Here \( \Gamma_1(2) \) are strings of Dirac matrices, \( C = \gamma^0\gamma^2 \) is the charge conjugation matrix, and \( a_i \) are the color indices.

The choice of baryonic currents depends on two different cases:
a) light baryons composed from \( u, d, s \) quarks,
b) heavy-light baryons with a single heavy quark \( b \) or \( c \).

In the case of light baryons we shall work in the limit of isospin invariance by assuming that the masses of \( u \) and \( d \) quarks are equal to each other, i.e. \( m_u = m_d = m \). The breaking of \( SU(3) \) symmetry is taken into account via a difference of strange and nonstrange quark masses \( m_s - m \neq 0 \). Thus, for baryons composed either of \( u \) or \( d \) quarks (nucleons, \( \Delta \)-isobar) or of \( s \) quarks (\( \Omega \)-hyperon) the coordinates of quarks may be written as

\[
y_1 = x - 2\xi_1 \quad y_2 = x + \xi_1 - \xi_2\sqrt{3} \quad y_3 = x + \xi_1 + \xi_2\sqrt{3}
\]

If a light baryon contains a single strange quark with mass \( m_s \) and two nonstrange quarks \( (u \) or \( d) \) with a mass \( m \) each as in \( \Lambda \) and \( \Sigma \)-hyperons one gets

\[
y_1 = x - 6\xi_1 \frac{m}{2m + m_s} \quad y_2 = x + 3\xi_1 \frac{m_s}{2m + m_s} - \xi_2\sqrt{3}, \quad y_3 = x + 3\xi_1 \frac{m_s}{2m + m_s} + \xi_2\sqrt{3}
\]

where \( y_1 \) is the coordinate of the strange quark and \( y_2 \) and \( y_3 \) are the coordinates of nonstrange quarks.
For a baryon with two strange quarks and a single nonstrange quark (as e.g. in the Ξ–hyperons) one obtains

\[
y_1 = x - 6\xi_1 \frac{m_s}{2m_s + m}
\]
\[
y_2 = x + 3\xi_1 \frac{m}{2m_s + m} - \xi_2 \sqrt{3},
\]
\[
y_3 = x + 3\xi_1 \frac{m}{2m_s + m} + \xi_2 \sqrt{3}
\]

where \(y_1\) now is the coordinate of the nonstrange quark and \(y_2\) and \(y_3\) are the coordinates of the strange quarks.

The spin-flavor structure of light baryonic currents with quantum numbers \(J^P = \frac{1}{2}^+\) and \(J^P = \frac{3}{2}^+\) has been studied in detail in the papers [21]-[28]. It was shown that there are two possibilities to choose the baryonic currents with \(J^P = \frac{1}{2}^+\):

vector variant \(J^V_B(y_1, y_2, y_3) = \gamma^\mu \gamma^5 q^{a_1}(y_1) q^{a_2}(y_2) C \gamma_\mu q^{a_3}(y_3) \varepsilon_{a_1 a_2 a_3}\) (4)

tensor variant \(J^T_B(y_1, y_2, y_3) = \sigma^{\mu \nu} \gamma^5 q^{a_1}(y_1) q^{a_2}(y_2) C \sigma_{\mu \nu} q^{a_3}(y_3) \varepsilon_{a_1 a_2 a_3}\) (5)

Both of these forms have been used in [20, 27] for studying the electromagnetic and strong properties of light baryons. It was shown that the tensor variant is more suitable for the description of the data. For this reason we will use the tensor current in the approach developed in this paper. For convenience the tensor current can be transformed into a sum of pseudoscalar \((\Gamma_1 = I, \Gamma_2 = \gamma_5\) in Eq. (3)) and scalar currents \((\Gamma_1 = \gamma_5, \Gamma_2 = I\) in Eq. (3)) using the Fierz transformations:

\[
(\sigma^{\mu \nu} \gamma^5)_{i_1 i_2} (C \sigma_{\mu \nu})_{i_3 i_4} = -2[I_{i_1 i_2} (C \gamma_5)_{i_3 i_4} + \gamma^5_{i_1 i_2} C_{i_3 i_4}] + 4[I_{i_1 i_4} (C \gamma_5)_{i_3 i_2} + \gamma^5_{i_1 i_4} C_{i_3 i_2}]
\]

For example, a tensor current for the proton

\[
J^T_p(y_1, y_2, y_3) = \sigma^{\mu \nu} \gamma^5 u^{a_1}(y_1) u^{a_2}(y_2) C \sigma_{\mu \nu} u^{a_3}(y_3) \varepsilon_{a_1 a_2 a_3}
\]

written in \(S + P\) form becomes

\[
J^T_p(y_1, y_2, y_3) = 4[u^{a_1}(y_3) u^{a_2}(y_2) C \gamma_5 d^{a_3}(y_1) + \gamma_5 u^{a_1}(y_3) u^{a_2}(y_2) C \gamma_5 d^{a_3}(y_1)] \varepsilon_{a_1 a_2 a_3}
\]

in the Fierz transformed form. After exchanging the variables \(y_1 \leftrightarrow y_3\) in the interaction Lagrangian of the proton with quarks we have

\[
L^{\text{int,T}}_p(x) = 4g^T_p \bar{p}(x) \int dy_1 \int dy_2 \int dy_3 \delta \left(x - \frac{\sum m_i y_i}{\sum m_i}\right) F \left(\frac{1}{18} \sum_{i<j} (y_i - y_j)^2\right)
\]
\[
\times [u^{a_1}(y_1) u^{a_2}(y_2) C \gamma_5 d^{a_3}(y_3) + \gamma_5 u^{a_1}(y_1) u^{a_2}(y_2) C \gamma_5 d^{a_3}(y_3)] \varepsilon_{a_1 a_2 a_3} + h.c.
\]
Table 1. Three-Quark Currents of Light Baryons

| Baryon     | Three-Quark Current                                                                 |
|------------|-----------------------------------------------------------------------------------|
| Proton     | $J_T^p(y_1, y_2, y_3) = [u^a(y_1)u^b(y_2)C\gamma^5d^c(y_3) + \gamma^5u^a(y_1)u^b(y_2)Cd^c(y_3)]\varepsilon^{abc}$ |
| Neutron    | $J_T^n(y_1, y_2, y_3) = [d^a(y_1)d^b(y_2)C\gamma^5u^c(y_3) + \gamma^5d^a(y_1)d^b(y_2)Cu^c(y_3)]\varepsilon^{abc}$ |
| $\Xi^-$-hyperon | $J_T^{\Xi^-}(y_1, y_2, y_3) = [s^a(y_1)s^b(y_2)C\gamma^5d^c(y_3) + \gamma^5s^a(y_1)s^b(y_2)Cd^c(y_3)]\varepsilon^{abc}$ |
| $\Lambda^0$-hyperon | $J_T^{\Lambda^0}(y_1, y_2, y_3) = [s^a(y_1)u^b(y_2)C\gamma^5d^c(y_3) + \gamma^5s^a(y_1)u^b(y_2)Cd^c(y_3)]\varepsilon^{abc}$ |

Table 1 contains a set of tensor currents for nucleons, $\Lambda^0$ and $\Xi^-$-hyperons in the $S + P$ form which will be used in our calculations.

Next we turn to the discussion of heavy-light baryonic currents. Suppose that the heavy-quark mass is much larger than the light-quark masses ($m_Q \gg m_{q_1}, m_{q_2}$). From Eq. (1) one then obtains:

$y_1 = y_Q = x$

$y_2 = y_{q_1} = x + 3\xi_1 - 2\xi_2\sqrt{3} \frac{m_{q_2}}{m_{q_1} + m_{q_2}}$ and $y_3 = y_{q_2} = x + 3\xi_1 + 2\xi_2\sqrt{3} \frac{m_{q_1}}{m_{q_1} + m_{q_2}}$

where $y_1$ is the coordinate of heavy quark, and $y_2$ and $y_3$ are the coordinates of light quarks $q_1$ and $q_2$.

It is convenient to transform the Jacobi coordinates of Eq.(1) to remove the light-quark mass dependence

$\xi_1 \rightarrow \xi_1 - \frac{\xi_2}{\sqrt{3}} \frac{m_{q_1} - m_{q_2}}{m_{q_1} + m_{q_2}}$

$\xi_2 \rightarrow \xi_2$

Then we have

$y_1 = y_Q = x, \quad y_2 = y_{q_1} = x + 3\xi_1 - \xi_2\sqrt{3}, \quad y_3 = y_{q_2} = x + 3\xi_1 + \xi_2\sqrt{3}$
The problem of the spin-flavor structure of heavy-light baryonic currents was analyzed in ref. [23]-[25]. It was shown that, in the static limit $\vec{p}_Q \to 0$ (this is equivalent to the heavy quark limit $m_Q \to \infty$), Λ-type baryons ($\Lambda_Q$, $\Xi_Q$) containing a light diquark system with zero spin may be described by either of the following nonderivative three-quark currents

$$J^P_{\Lambda hQ} = \varepsilon^{abc} h^a_Q u^b C \gamma^5 d^c, \quad J^A_{\Lambda hQ} = \varepsilon^{abc} h^a_Q u^b C \gamma^\alpha \gamma^5 \sigma^{\alpha\beta} d^c$$

where $h_Q$ denotes the effective static field of the heavy quark.

In the same vein there are two currents for Ω-type baryons ($\Omega_Q$, $\Sigma_Q$ and $\Omega^* Q$, $\Sigma^* Q$) containing a light diquark system with spin 1

$$J^V_{\Omega hQ} = \varepsilon^{abc} \gamma_5 h_Q^a s^b C \gamma s^c, \quad J^{V;\mu}_{\Omega hQ} = \varepsilon^{abc} \gamma_5 h_Q^a s^b C \gamma^\mu s^c$$

$$J^T_{\Omega hQ} = \varepsilon^{abc} \gamma_5 h_Q^a s^b C \gamma_5 s^c, \quad J^{T;\mu}_{\Omega hQ} = \varepsilon^{abc} \gamma_5 h_Q^a s^b C \gamma_5 \gamma^\mu s^c$$

where $k = 1, 2, 3$, $(\gamma^k)^2 = -3$. The currents $J^{I;\mu}_{\Omega hQ}$ ($I = V, T$) satisfy the spin-3/2 Rarita-Schwinger condition $\gamma^k J^{I;\mu}_{\Omega hQ} = 0$. In this paper we work with Lorentz-covariant representations of the HQET heavy-light currents mentioned above [23]-[25].

Our currents are listed below

| variant         | $J^I_{\Lambda hQ}$ | $J^I_{\Omega hQ}$ |
|-----------------|-------------------|-------------------|
| **pseudoscalar variant** | $J^P_{\Lambda hQ}$ | $J^P_{\Omega hQ}$ |
| **axial variant**              | $J^A_{\Lambda hQ}$ | $J^A_{\Omega hQ}$ |
| **vector variant**             | $J^V_{\Omega hQ}$ | $J^V_{\Omega hQ}$ |
| **tensor variant**             | $J^T_{\Omega hQ}$ | $J^T_{\Omega hQ}$ |
The currents \( J_{\Omega Q}^{(\perp)I;\mu} \) (\( I = V, T \)) are orthogonal to the corresponding baryon field with spin 3/2: \( \bar{\Omega}_Q \cdot J_{\Omega Q}^{(\perp)I;\mu} = 0 \) and can, therefore, be omitted in the interaction Lagrangian (2). Thus, for heavy-light baryons with spin 3/2 we use the currents \( J_{\Omega Q}^V \) (9) and \( J_{\Omega Q}^T \) (11).

In Table 2 we give the quark content, the quantum numbers (spin-parity \( J^P \), spin \( S_{qq} \) and isospin \( I_{qq} \) of light diquark) and the experimental (when available) and theoretical mass spectrum of heavy baryons \([28, 29]\) which will be analyzed in this paper. Square brackets \([\ldots]\) and round brackets \(\{\ldots\}\) denote antisymmetric and symmetric flavor and spin combinations of the light degrees of freedom.

The Lagrangian that describes the interaction of \( \Lambda_b^0 \) with \( b, u, d \)-quarks is then written as

\[
\mathcal{L}_{\Lambda_b^0}^{\text{int}}(x) = g_B \Lambda_b^0(x) \Gamma_1 b^a(x) \int d\xi_1 \int d\xi_2 F \left( \xi_1^2 + \xi_2^2 \right) \times u^b(x + 3\xi_1 - \xi_2\sqrt{3}) \Gamma_2 d^c(x + 3\xi_1 + \xi_2\sqrt{3}) \varepsilon^{abc} + h.c. \tag{12}
\]

where

\[
\Gamma_1 \otimes \Gamma_2 = \begin{cases} 
I \otimes C \gamma^5 & \text{pseudoscalar current} \\
\gamma_{\mu} \otimes C \gamma^5 \gamma^\mu & \text{axial current}
\end{cases}
\]

The vertex form factor \( F(\xi_1^2 + \xi_2^2) \) characterizes the distribution of \( u \) and \( d \) quarks inside the \( \Lambda_b^0 \) baryon. The Fourier-transform of the vertex form factor is defined as

\[
F(\xi_1^2 + \xi_2^2) = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \exp(-ik_1\xi_1 - ik_2\xi_2) F(k_1^2 + k_2^2) \tag{13}
\]

Next we discuss the model parameters. First, there are the baryon-quark coupling constants and the vertex function in the Lagrangian (2). The coupling constants are calculated from the compositeness condition (see, ref. [30]), i.e. the renormalization constant of the baryon wave function is set equal to zero, \( Z_B = 1 - g_B^2 \Sigma_B(M_B) = 0 \), with \( \Sigma_B \) being the baryon mass operator (see, Fig.1a for light baryons and Fig.1b for heavy baryons). Actually, the compositeness condition is equivalent to the normalization of the elastic form factors to one at zero momentum transfer. This may be readily seen from the Ward identity which relates the vertex function with the mass operator on mass shell. We have

\[
J_{\Omega Q}^{T;\mu} = -i\varepsilon^{abc} \gamma_\mu Q^a s^b C \sigma^{\mu\nu} s^c \tag{11}
\]

\[
J_{\Omega Q}^{(\perp)T;\mu} = \frac{i}{4} \varepsilon^{abc} \gamma_\mu \gamma_\alpha Q^a s^b C \sigma^{\alpha\nu} s^c \tag{11}
\]
Table 2. Quantum Numbers of Heavy-Light Baryons

| Baryon | Quark Content | $J^P$ | $(S_{qq}, I_{qq})$ | Mass (GeV) |
|--------|---------------|-------|--------------------|------------|
| $\Lambda_c^+$ | c[ud] | $\frac{1}{2}^+$ | (0,0) | 2.285 |
| $\Xi_c^+$ | c[us] | $\frac{1}{2}^+$ | (0,1/2) | 2.466 |
| $\Sigma_{c}^{++}$ | c{uu} | $\frac{1}{2}^+$ | (1,1) | 2.453 |
| $\Omega_c^0$ | c{ss} | $\frac{1}{2}^+$ | (1,0) | 2.719 |
| $\Sigma_{c}^{*++}$ | c{uu} | $\frac{3}{2}^+$ | (1,1) | 2.510 |
| $\Omega_{c}^{*0}$ | c{ss} | $\frac{3}{2}^+$ | (1,0) | 2.740 |
| $\Lambda_b^0$ | b[ud] | $\frac{1}{2}^+$ | (0,0) | 5.640 |
| $\Xi_b^+$ | b[us] | $\frac{1}{2}^+$ | (0,1/2) | 5.800 |
| $\Sigma_b^+$ | b{uu} | $\frac{1}{2}^+$ | (1,1) | 5.820 |
| $\Omega_b^-$ | b{ss} | $\frac{1}{2}^+$ | (1,0) | 6.040 |

Fig.1a. Light baryon mass operator.  
Fig.1b. Heavy-light baryon mass operator.
\[ \Lambda_{B \to B \gamma}(p, p) \bigg|_{y=M_B} = g_B^2 \left. \frac{\partial \Sigma_B(p)}{\partial p^\mu} \right|_{y=M_B} = \gamma^\mu g_B^2 \Sigma'_B(p) \bigg|_{y=M_B} \]  

(14)

where the vertex function is related to the baryon elastic form factor by

\[ \Lambda_{B \to B \gamma}(p, p) = \gamma^\mu F_B(0). \]  

(15)

From this the normalization of the form factor mentioned above immediately follows.

The vertex function is an arbitrary function except that it should make the Feynman diagrams ultraviolet finite, as we have mentioned above. In the papers [15, 16] we have found that the basic physical observables of pion and nucleon low-energy physics depend only weakly on the choice of the vertex functions. In this paper we choose a Gaussian vertex function for simplicity. In Minkowski space we write

\[ F(k_1^2 + k_2^2) = \exp \left( \frac{k_1^2 + k_2^2}{\Lambda_B^2} \right) \]

where \( \Lambda_B \) is the Gaussian range parameter which may be related to the size of a baryon.

Note that all calculations are done in the Euclidean region \((k_i^2 = -k_{iE}^2)\) where the above vertex function decreases very rapidly. It was found in [19] that for nucleons \((B = N)\) the value \( \Lambda_N = 1.25 \text{ GeV} \) gives a good description of the nucleon's static characteristics (magnetic moments, charge radii) and its form factors in the space-like region for \( Q^2 \) up to 1 GeV². In this work we will use the value \( \Lambda_{B_q} \equiv \Lambda_N = 1.25 \text{ GeV} \) for light baryons and take the value \( \Lambda_{B_Q} \) for the heavy-light baryons as an adjustable parameter.

For light quark propagator with a mass \( m_q \) we shall use the standard form of the free fermion propagator

\[ <0|T(q(x)\bar{q}(y))|0> = \int \frac{d^4k}{(2\pi)^4i} e^{-ik(x-y)}S_q(k), \quad S_q(k) = \frac{1}{m_q - k} \]  

(16)

For the heavy quark propagator we will use the leading term in the inverse mass expansion. Suppose \( p = M_{BQ}v \) is the heavy baryon momentum. We introduce the parameter \( \bar{\Lambda}_{(Qq)} = M_{(Qq)} - m_Q \) which is the difference between the heavy baryon mass \( M_{(Qq)} \equiv M_{BQ} \) and the heavy quark mass. Keeping in mind that the vertex function falls off sufficiently fast such that
the condition $|k| \ll m_Q$ holds where $k$ is the virtual momentum of light quarks, one has

$$S_Q(p+k) = \frac{1}{m_Q - (\not p + \not k)} = \frac{m_Q + M_{BQ} \not p + \not k}{m_Q^2 - M_{BQ}^2 - 2M_{BQ}v_k - k^2} = S_v(k, \bar{\Lambda}_{\{q_1q_2\}}) + O\left(\frac{1}{m_Q}\right)$$

$$S_v(k, \bar{\Lambda}_{\{q_1q_2\}}) = -\frac{(1 + \not p)}{2(v \cdot k + \bar{\Lambda}_{\{q_1q_2\}})}$$

In what follows we will assume that $\bar{\Lambda} \equiv \bar{\Lambda}_{uu} = \bar{\Lambda}_{dd} = \bar{\Lambda}_{su} = \bar{\Lambda}_{ds}$. Thus there are three independent parameters: $\bar{\Lambda}$, $\bar{\Lambda}_s$, and $\bar{\Lambda}_{ss}$.

A drawback of our approach is the lack of confinement. This can in principle be corrected by changing the analytic properties of the light-quark propagator. We leave the investigation of this possibility for future studies. For the time being we shall avoid the appearance of unphysical imaginary parts in the Feynman diagrams by postulating the following condition: the baryon mass must be less than the sum of constituent quark masses $M_B < \sum_i m_q$.

In the case of heavy-light baryons the restriction $M_B < \sum_i m_q$ implies that the parameter $\bar{\Lambda}_{\{q_1q_2\}}$ must be less than the sum of light quark masses $\bar{\Lambda}_{\{q_1q_2\}} < m_{q_1} + m_{q_2}$. The last constraint serves as the upper limit for our choices of the parameter $\bar{\Lambda}_{\{q_1q_2\}}$.

Thus, there are three sets of adjustable parameters in our model: the constituent light quark masses $m_q$ ($m = m_u = m_d$ and $m_s$), the range cutoff parameters $\Lambda_B$ ($\Lambda_B$, and $\Lambda_{BQ}$) and a set of $\bar{\Lambda}_{\{q_1q_2\}}$ subsidiary parameters: $\bar{\Lambda}$, $\bar{\Lambda}_s$ and $\bar{\Lambda}_{ss}$. The parameters $m=420$ MeV and $\Lambda_B=1.25$ GeV were fixed in ref. [19] from a best fit to the data on electromagnetic properties of nucleons. The parameters $\Lambda_{BQ}$, $m_s$, $\bar{\Lambda}$ are determined in this paper from the analysis of the $\Lambda_c^+ \to \Lambda^0 + e^+ + \nu_e$ decay data. The following values are obtained: $\Lambda_Q=2.5$ GeV, $m_s=570$ MeV and $\bar{\Lambda}=710$ MeV.

The parameters $\bar{\Lambda}_s$ and $\bar{\Lambda}_{ss}$ cannot be adjusted at present since at present there are no experimental data on the decays of heavy-light baryons containing one or two strange quarks.
3 Matrix Elements of Semileptonic Decays of Bottom and Charm Baryons

In our model the semileptonic decays of bottom and charm baryons are described by the standard triangle quark diagram (Fig.2). The matrix elements describing heavy-to-heavy ($b \rightarrow c$) and heavy-to-light ($c \rightarrow s$) transitions can be written as

- $b \rightarrow c$ transition

\[
\bar{u}(v') M_{\Gamma}(v, v') u(v) = g_{B_b} g_{B_c} \int \frac{d^4 k}{\pi^2} \int \frac{d^4 k'}{\pi^2} \, \text{Tr} \left[ \Gamma_1' S_q \left( \frac{k' - k}{2} \right) \Gamma_2' S_q \left( \frac{k' + k}{2} \right) \right] \exp \left( \frac{18k^2 + 6k'^2}{\Lambda_{B_Q}^2} \right) \bar{u}(v') \Gamma_1 S_{v'}(k, \bar{\Lambda}) \Gamma_2 S_v(k, \bar{\Lambda}) \Gamma_2 u(v)
\]

- $c \rightarrow s$ transition

\[
\bar{u}(p') M_{\Gamma}(p, v') u(v) = g_{B_s} g_{B_c} \int \frac{d^4 k}{\pi^2} \int \frac{d^4 k'}{\pi^2} \, \text{Tr} \left[ \Gamma_1' S_q \left( \frac{k' - k}{2} \right) \Gamma_2' S_q \left( \frac{k' + k}{2} \right) \right] \exp \left( \frac{9k^2 + 3k'^2}{\Lambda_{B_Q}^2} \right) \exp \left( \frac{9(k + \alpha p')^2 + 3k'^2}{\Lambda_{B_q}^2} \right) \times \bar{u}(p') \Gamma_1 S_s(k + p') \Gamma_2 S_v(k, \bar{\Lambda}) \Gamma_2 u(v)
\]

\[
\alpha = \frac{2m}{2m + m_s}
\]

Here $\text{Tr}[...]$ corresponds to the light quark loop obtained after a standard transformations which involves the charge conjugation matrix $C$.

\[
(\Gamma_1')^{\alpha \mu} \xi^{\mu \nu} \left( \frac{k' - k}{2} \right) (\Gamma_2')^{\nu \beta} \xi^{\beta \gamma} \left( \frac{k' + k}{2} \right) = \text{Tr} \left[ \Gamma_1' S_q \left( \frac{k' - k}{2} \right) \Gamma_2' S_q \left( \frac{k' + k}{2} \right) \right]
\]

Calcutational details of the matrix elements (18) and (19) are given in Appendix A.

We now turn to the discussion of matrix elements of heavy-to-heavy baryonic decays. In this paper we consider decays of bottom baryons ($\Lambda_b^0$, $\Xi_b^0$, $\Sigma_b^+$ and $\Omega_b^-$) into pseudoscalar charmed baryons ($\Lambda_c^+, \Sigma_c^{++}$ and $\Omega_c^0$) and pseudovector states ($\Sigma_c^{*++}$ and $\Omega_c^{*+}$). The matrix elements describing weak transitions between heavy baryons can be decomposed into a set of relativistic form factors. In the HQL these form factors are proportional to three universal functions.
Fig. 2. Semileptonic decay of heavy-light baryon.

\[ \zeta, \xi_1, \xi_2 \] of the variable \( \omega = v \cdot v' \), the so-called Isgur-Wise functions [31, 32]. The function \( \zeta(\omega) \) describes the \( b - c \) transitions of \( \Lambda \)-type baryons. The functions \( \xi_1(\omega) \) and \( \xi_2(\omega) \) describe transitions of \( \Omega \)-type baryons.

Weak hadronic currents describing the transition of a heavy baryon \( B_b(v) \) with four-velocity \( v \) to a heavy baryon \( B_{\epsilon}^{(s)}(v) \) with \( v' \) are written as [31]-[34]

\[ \Lambda_b \rightarrow \Lambda_c \text{ Transition} \]

\[ \langle \Lambda_c(v') | \bar{b} \Gamma c | \Lambda_b(v) \rangle = \zeta(\omega) \bar{u}(v') \Gamma u(v), \]

\[ \Omega_b \rightarrow \Omega_c(\Omega^*_c) \text{ Transition} \]

\[ \langle \Omega_c(v') \text{ or } \Omega^*_c(v') | \bar{b} \Gamma c | \Lambda_b(v) \rangle = \bar{B}^{\mu}(v') \Gamma B_{b}^{\mu}(v)[ -\xi_1(\omega) g_{\mu\nu} + \xi_2(\omega) v_\mu v'_\nu ], \]

where the spinor tensor \( B_{b}^{\mu}(v) \) satisfies the Rarita-Schwinger conditions \( v_\nu B_{b}^{\mu}(v) = 0 \) and \( \gamma_\nu B_{b}^{\mu}(v) = 0 \). The spin wavefunctions are written as

\[ B_{\Omega}^{\mu}(v) = \frac{\gamma^\mu + v^{\mu}}{\sqrt{3}} \gamma^5 u_{\Omega}(v) \text{ for } \Omega \text{ states} \quad \text{and} \quad B_{\Omega^*}^{\mu}(v) = u_{\Omega^*}(v) \text{ for } \Omega^* \text{ states} \]
where \( u_{\Omega Q}(v) \) is the usual spin 1/2 spinor and the spinor \( u_{\Omega Q}^\mu(v) \) is the usual Rarita-Schwinger spinor. Note that the Ward identity between the derivative of the mass operator of heavy-light baryons and the vertex function (18) with \( \Gamma = \gamma \mu \) and \( v = v' \) ensures the correct normalization of the functions \( \zeta(\omega) \) and \( \xi_1(\omega) \) at \( \omega = 1. \)

In the heavy quark limit the matrix element of the transition of heavy baryon containing a scalar light diquark into light baryons is described by two relativistic form factors \( f_1 \) and \( f_2. \) For example, the typical hadronic current for \( \Lambda_c \rightarrow \Lambda^0 \) transition is written as

\[
< \Lambda(p') | \bar{s} O_{\mu} c | \Lambda_c(v) > = \bar{u}_\Lambda(p') [f_1(p' \cdot v) + \not{v} f_2(p' \cdot v)] O_{\mu} u_{\Lambda_c}(v)
\]

4 Results

In this section we give numerical results on the observables of semileptonic decays of bottom and charm baryons: the baryonic Isgur-Wise functions, decay rates and asymmetry parameters in the two-cascade decays \( \Lambda_b \rightarrow \Lambda_c[\rightarrow \Lambda_s \pi] + W[\rightarrow \ell \nu_\ell] \) and \( \Lambda_c \rightarrow \Lambda_s[\rightarrow p \pi] + W[\rightarrow \ell \nu_\ell]. \) Our model contains a number of parameters. The cutoff parameter \( \Lambda_B q \) and the light quark mass \( m_q \) are taken from a fit to proton and neutron data [19]. The cutoff parameter \( \Lambda_B Q \) relevant for heavy-light baryons, the binding energy \( \bar{\Lambda} = M_B Q - m_Q \) and the strange quark mass \( m_s \) are fixed by comparison with the experimentally measured decay \( \Lambda_c^+ \rightarrow \Lambda^0 + e^+ + \nu_e. \) We have checked that the Isgur-Wise functions \( \xi_1 \) and \( \xi_2 \) satisfy the model-independent Bjorken-Xu inequalities [35]. We give a detailed description of the \( \Lambda_c^+ \rightarrow \Lambda^0 + e^+ + \nu_e \) decay, which was recently measured by CLEO Collaboration [34]. In what follows we will use the following values for the CKM matrix elements: \(|V_{bc}| = 0.04, |V_{cs}| = 0.975. \)

4.1 Baryonic Isgur-Wise Functions

In sec. 2 we have introduced heavy-light baryonic currents. We present a full list of possible currents (without derivatives) with the quantum numbers of baryons \( J^P = \frac{1}{2}^+ \) and \( J^P = \frac{3}{2}^+. \) For simplicity we restrict ourselves to only one variant of the three-quark currents for each kind of heavy-light baryon: pseudoscalar current [8] for \( \Lambda_Q \)-type baryons and vector currents (8,9) for \( \Omega_Q \)-type baryons. A justification of this procedure may be taken from the QCD sum
rule analysis of [25] where it was found that, using the axial current for \( \Lambda_Q \) baryons and tensor currents for \( \Omega_Q^{(*)} \) baryons, one obtains results which are not very different from the ones with the pseudoscalar current and the vector currents. The direct calculation of the IW-functions with currents (6) and (8,9) gives the following results

\[
\zeta(\omega) = \frac{F_0(\omega)}{F_0(1)}, \quad \xi_1(\omega) = \frac{F_1(\omega)}{F_1(1)}, \quad \xi_2(\omega) = \frac{F_2(\omega)}{F_1(1)}
\]  

(20)

\[
F_1(\omega) = \int_0^\infty dx \int_0^\infty \frac{dyy}{(y+1)^2} \int_0^1 d\phi \int_0^1 d\theta R_1(\omega) \exp[-6S(\beta)(4\mu^2_q - \bar{\lambda}^2)] \times \exp[-12x^2S(\beta)(1-\phi)(\omega - 1) - 6S(\beta)(x - \bar{\lambda})^2 - 24\mu^2_q(1-2\theta)^2 \frac{y^2}{1+y}]
\]

where

\[
R_0(\omega) = \mu^2_q + \frac{1}{6S(\beta)(1+y)} + \frac{x^2\beta}{4(1+y)^2}(1 + 2\phi(1-\phi)(\omega - 1))
\]

\[
R_1(\omega) = \mu^2_q + \frac{1}{12S(\beta)(1+y)} + \frac{x^2\beta}{4(1+y)^2}(1 + 2\phi(1-\phi)(\omega - 1))
\]

\[
R_2(\omega) = \frac{x^2\beta}{2(1+y)^2}\phi(1-\phi) = \frac{R_1(\omega) - R_1(1)}{\omega - 1}
\]

\[
\beta = 1 + 2y + 4y^2\theta(1-\theta), \quad S(\beta) = \frac{2}{3} + \frac{\beta}{3(1+y)}, \quad \mu_q = \frac{m_q}{\Lambda_Q}, \quad \bar{\lambda} = \frac{\bar{\Lambda}}{\Lambda_Q}
\]

All mass-dimension variables are scaled by the parameter \( \Lambda_Q \). Hence the IW-functions depend only on two parameters \( \mu_q \) and \( \bar{\lambda} \). We reiterate that the functions \( \zeta \) and \( \xi_1 \) are normalized to one at zero recoil due to the existence of a Ward identity relating the vertex function with the derivative of the heavy-light baryon mass operator as discussed after Eq.(13). Contrary to this the normalization of the \( \xi_2 \)-function is model-dependent. In our model the value \( \xi_2(1) \) satisfies the inequality \( 0 < \xi_2(1) < 1/2 \) and depends on the choice of the parameters \( \mu_q \) and \( \bar{\lambda} \).

It is easy to show that the baryonic IW-functions can be rewritten in the form

\[
\zeta(\omega) = \sum_{N=0}^\infty C_N^\zeta \bar{\lambda}^N \Phi_N(\omega) \leq \Phi_0(\omega) = \frac{\ln(\omega + \sqrt{\omega^2 - 1})}{\sqrt{\omega^2 - 1}},
\]
\[\xi_1(\omega) = \frac{\sum_{N=0}^{\infty} C_{N}^{\xi_1} \lambda^N \Phi_N(\omega)}{\sum_{N=0}^{\infty} C_{N}^{\xi_1} \lambda^N} \leq \Phi_0(\omega) = \frac{\ln(\omega + \sqrt{\omega^2 - 1})}{\sqrt{\omega^2 - 1}},\]

\[\xi_2(\omega) = \frac{\sum_{N=0}^{\infty} C_{N}^{\xi_2} \lambda^N (\Phi_N(\omega) - \Phi_{N+1}(\omega))}{(\omega - 1) \sum_{N=0}^{\infty} C_{N}^{\xi_2} \lambda^N} < \frac{\Phi_0(\omega) - \Phi_1(\omega)}{\omega - 1} = \frac{1}{\omega^2 - 1} \left(\frac{\omega \ln(\omega + \sqrt{\omega^2 - 1})}{\sqrt{\omega^2 - 1}} - 1\right)\]

Here

\[\Phi_N(\omega) = \int_0^1 \frac{d\phi}{[1 + 2(w - 1)\phi(1 - \phi)]^{N/2+1}} \leq \Phi_0(\omega) \text{ for } \forall N \geq 0,\]

\[C_N^F = \frac{(2\sqrt{6})^N \Gamma(N/2 + 1)}{12 \Gamma(N)} \int_0^1 d\theta \int_0^\infty dy y^{S^{N/2-1}(\beta)} \exp(-24\mu_y^2 y) \Delta_F > 0, \ F = \zeta, \xi_1, \xi_2\]

\[\Delta_\zeta = \mu_q^2 + \frac{1}{6S(\beta)(1+y)} + \left(\frac{N}{2} + 1\right) \frac{\beta}{24S(\beta)(1+y)^2}\]

\[\Delta_{\xi_1} = \mu_q^2 + \frac{1}{12S(\beta)(1+y)} + \left(\frac{N}{2} + 1\right) \frac{\beta}{24S(\beta)(1+y)^2}\]

\[\Delta_{\xi_2} = \left(\frac{N}{2} + 1\right) \frac{\beta}{24S(\beta)(1+y)^2}\]

\[\Gamma(N) = \int_0^\infty dt t^{N-1} \exp(-t) \text{ is the } \gamma \text{- function}\]

Note that \(\zeta(\omega)\) and \(\xi_1(\omega)\) become largest when \(\lambda = 0\).

\[\zeta(\omega) \equiv \xi_1(\omega) \equiv \Phi_0(\omega) \quad (21)\]

An increase of \(\lambda\) leads to a suppression of the IW-functions in the physical kinematical region of the variable \(\omega\), i.e. in the region

\[1 \leq \omega \leq \omega_{\text{max}} = \frac{M_B^2 + M_{B'}^2}{2M_B M_{B'}}\]

(22)

The radii of the form factors \(\zeta\) and \(\xi_1\) are defined as

\[F(\omega) = 1 - \rho_F^2(\omega - 1) + ..., \quad F = \zeta, \xi_1\]

(23)

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It is easy to show that $\rho_\xi^2$ and $\rho_{\xi_1}^2$ have the lower bound

$$
\rho_\xi^2 = \frac{1}{3} + 2 \bar{\lambda}\frac{I(2, 2)}{I(1, 2)} \geq \frac{1}{3}, \quad \rho_{\xi_1}^2 = \frac{1}{3} + 2 \bar{\lambda}\frac{I(2, 1)}{I(1, 1)} \geq \frac{1}{3}
$$ (24)

since the integral

$$
I(M, N) = \int_0^1 d\theta \int_0^\infty dx \int_0^\infty dy \frac{y}{(1+y)^2} S(\beta) x^M \left[ \mu_q^2 + \frac{N}{12 S(\beta)(1+y)} + \frac{x^2 \beta}{4(1+y)^2} \right] \times \exp \left[ -6 S(\beta) x(x - 2 \bar{\lambda}) - 24 \mu_q^2 y \right]
$$

is always positive.

As was shown in [35], the IW-functions $\xi_1$ and $\xi_2$ must respect the two model-independent Bjorken-Xu inequalities. The first inequality

$$
1 \geq B(\omega) = \frac{2 + \omega^2}{3}\xi_1^2(\omega) + \frac{(\omega^2 - 1)^2}{3}\xi_2^2(\omega) + \frac{2}{3}(\omega - \omega^3)\xi_1(\omega)\xi_2(\omega)
$$ (25)

is derived from the Bjorken sum rule for semileptonic $\Omega_b$ decays to the ground state and to low-lying negative-parity excited charmed baryon states in the HQL. The inequality (25) implies a second inequality, namely a model-independent restriction of the slope (radius) of the form factor $\xi_1(\omega)$

$$
\rho_{\xi_1}^2 \geq \frac{1}{3} - \frac{2}{3}\xi_2(1)
$$ (26)

Let us check whether our IW-functions $\xi_1$ and $\xi_2$ respect these inequalities. First, the inequality (26) for the slope of the $\xi_1$-function can be seen to be satisfied because from Eq. (24) one has $\rho_{\xi_1}^2 \geq 1/3$ and further $\xi_2(1) > 0$ from Eq. (24).

To check the inequality (26) we rewrite it in the form

$$
1 \geq B(\omega) = \frac{2}{3}\xi_1^2(\omega) + \frac{1}{3}(\omega \xi_1(\omega) - \xi_2(\omega)(\omega^2 - 1))^2
$$ (27)

One can show that the combination $\omega \xi_1(\omega) - \xi_2(\omega)(\omega^2 - 1)$ satisfies the following condition $\xi_1(\omega) \leq \omega \xi_1(\omega) - \xi_2(\omega)(\omega^2 - 1) \leq \omega \xi_1(\omega)$. Hence,

$$
\xi_1^2(\omega) \leq B(\omega) \leq \frac{2 + \omega^2}{3}\xi_1^2(\omega)
$$ (28)
From the inequalities (27) and (28) one finds an upper limit for the function \( \xi_1(\omega) \):

\[
\xi_1(\omega) \leq \sqrt{\frac{3}{2 + \omega^2}}
\]

The results for the IW-functions \( \zeta(\omega) \) and \( \xi_1(\omega) \) are plotted in Fig.3-7 in the physical region \( 1 \leq \omega \leq \omega_{\text{max}} \). The function \( \xi_1(\omega) \) is shown for the two cases: a) decay of \( \Sigma_b \)-baryon and b) decay of \( \Omega_b \)-baryon. In Figs.3 and 4 we demonstrate the sensitivity of the \( \zeta \)-function on the choice of the parameters \( \bar{\Lambda} \) and \( \Lambda_Q \) when one is varied and the other one is fixed. Below (in sec.4.2) we will show that the best description of the experimental data for \( \Lambda_c^+ \to \Lambda^0 + e^+ + \nu_e \) decay is obtained with the choice of parameters \( \Lambda_Q=2.5 \text{ GeV}, \bar{\Lambda}=710 \text{ MeV} \) and \( m_s=570 \text{ MeV} \). In Fig.3 the \( \zeta(\omega) \) function is shown for \( \bar{\Lambda} \) values between 600 MeV to 800 MeV, where the parameter \( \Lambda_Q \) is assumed to be 2.5 GeV. It is seen that an increase of \( \bar{\Lambda} \) leads to a suppression of the baryonic IW-function \( \zeta \). In Fig.4 the dependence of \( \zeta \) on the value \( \Lambda_Q \) is plotted for \( \bar{\Lambda}=710 \text{ MeV} \). One can see that a decrease of \( \Lambda_Q \) leads to a suppression of \( \zeta(\omega) \). In Fig.5 we give the best fit for the IW-function \( \zeta \) (\( \Lambda_Q=2.5 \text{ GeV}, \bar{\Lambda}=710 \text{ MeV} \)). For comparison the results of other phenomenological approaches are shown too where we compare with results obtained from QCD sum rules [24], IMF models [38, 39], MIT bag model [12], a simple quark model (SQM) [44] and the dipole formula [39]. Our result is close to the QCD sum rule result [24]. For quick reference we want to remark that in the physical region our function \( \zeta \) can be well approximated by the formula

\[
\zeta(\omega) \approx \left[ \frac{2}{1 + \omega} \right]^{1.7+1/\omega}
\]

In Fig.6 and 7 we analyse the \( \omega \)-dependence of the \( \xi_1 \) form factor. We exhibit the dependence of \( \xi_1(\omega) \) on the choice of \( \bar{\Lambda} \) for \( \Sigma_b \) baryon decays (Fig.6) and for \( \Omega_b \) baryon decays (Fig.7). For both cases \( \Lambda_Q \) is put equal to 2.5 GeV. In the analysis of the \( \Omega_b \) form factor we use \( m_s=570 \text{ MeV} \). We also present results on the upper limit (22) for the function \( \xi_1(\omega) \). In Fig.7 we also compare to a simple quark model calculation of [45]. We want to emphasize that for both cases, \( \Sigma_b \) and \( \Omega_b \) baryon decays, our \( \zeta_1 \) does not exceed the upper limit (22) except in a narrow region of very small (unphysical) values of \( \bar{\Lambda} \): \( \bar{\Lambda} \leq 60 \text{ MeV} \). Thus we conclude that the Bjorken-Xu inequality is respected by our model.

The results for the charge radii are listed in Tables 3-6 for various sets of the adjustable parameters. For comparison we quote the results for the charge radii predicted by other
phenomenological approaches: $\rho_\zeta^2 = 3.04$ (IMF model) [39], $\rho_\zeta^2 = 1.78$ (dipole formula) [38], $\rho_\zeta^2 = 2.28$ (MIT bag model) [42], $\rho_\zeta^2 = 1$ and $\rho_{\xi_1}^2 = 1.02 \div 1.18$ (simple quark model) [44, 45], $\rho_\zeta^2 = 0.55 \pm 0.15$ (QCD Sum Rules) [46].

**Table 3.** The Charge Radius $\rho_\zeta^2$ of $\Lambda_b$ baryon at $\Lambda_Q=2.5$ GeV.

| $\bar{\Lambda}$ (MeV) | 600 | 625 | 650 | 675 | 700 | 710 | 725 | 750 | 675 | 690 | 800 |
|------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\rho_\zeta^2$         | 1.04| 1.09| 1.12| 1.22| 1.30| 1.33| 1.38| 1.47| 1.59| 1.68| 1.76|

**Table 4.** The Charge Radius $\rho_\zeta^2$ of $\Lambda_b$ baryon at $\bar{\Lambda}=710$ MeV.

| $\Lambda_Q$ (GeV) | 1.25 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 | 2.1 | 2.3 | 2.5 |
|-------------------|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\rho_\zeta^2$    | 2.93 | 2.82| 2.63| 2.47| 2.31| 2.15| 2.0 | 1.87| 1.75| 1.64| 1.46| 1.33|

**Table 5.** The Charge Radius $\rho_{\xi_1}^2$ of $\Sigma_b$ baryon at $\Lambda_Q=2.5$ GeV.

| $\bar{\Lambda}$ (MeV) | 600 | 625 | 650 | 675 | 700 | 710 | 725 | 750 | 675 | 690 | 800 |
|------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\rho_{\xi_1}^2$       | 1.05| 1.09| 1.12| 1.22| 1.32| 1.35| 1.38| 1.50| 1.59| 1.68| 1.80|

**Table 6.** The Charge Radius $\rho_{\xi_1}^2$ of $\Omega_b$ baryon at $\Lambda_Q=2.5$ GeV.

| $\bar{\Lambda}_{ss}$ (MeV) | 800  | 850  | 875  | 900  | 925  | 950  | 975  | 1000 | 1025 | 1050 | 1075 | 1100 |
|-----------------------------|------|------|------|------|------|------|------|------|------|------|------|------|
| $\rho_{\xi_1}^2$            | 1.44 | 1.58 | 1.66 | 1.74 | 1.82 | 1.92 | 2.02 | 2.12 | 2.25 | 2.39 | 2.56 | 2.79 |
4.2 Rates, Distributions and Asymmetry Parameters in $b \to c$ Baryonic Decays

In this section we present numerical results for rates, distributions and asymmetry parameters in the $b \to c$ flavor changing baryon decays. The standard expressions for observables of semileptonic decays of bottom baryons (decay rates, differential distributions, leptonic spectra and asymmetry parameters) have simple forms when expressed in terms of helicity amplitudes $H_{\lambda_f \lambda_W}$ \cite{36, 28}, where $\lambda_f$ is helicity of the final state baryon and $\lambda_W$ is the helicity of the off-mass-shell W-boson. The HQL helicity amplitudes describing transitions of bottom baryon into charm ones are expressed through IW-functions in the following way:

\[ H_{\pm \frac{3}{2} \pm 1} = -2\sqrt{M_i M_f (\sqrt{\omega - 1} \mp \sqrt{\omega + 1})} \times \begin{cases} 
\zeta(\omega) & \Lambda_b \to \Lambda_c \text{decay} \\
\frac{1}{3} \xi_T(\omega) & \Omega_b \to \Omega_c \text{decay} \\
\pm \frac{\sqrt{2}}{3} \xi_T(\omega) & \Omega_b \to \Omega^*_c \text{decay}
\end{cases} \]

\[ H_{\pm \frac{3}{2} 0} = \frac{1}{\sqrt{\omega_{\text{max}} - \omega}} \times \begin{cases} 
\zeta(\omega)[M_+ \sqrt{\omega - 1} \mp M_- \sqrt{\omega + 1}] & \Lambda_b \to \Lambda_c \text{decay} \\
\frac{1}{3}[M_+ \sqrt{\omega - 1} \xi_{L+}(\omega) \mp M_- \xi_{L-}(\omega) \sqrt{\omega + 1}] & \Omega_b \to \Omega_c \text{decay} \\
\frac{\sqrt{2}}{3}[M_+ \sqrt{\omega - 1} \xi_{L+}(\omega) \mp M_- \xi_{L-}(\omega) \sqrt{\omega + 1}] & \Omega_b \to \Omega^*_c \text{decay}
\end{cases} \]

\[ H_{\pm \frac{3}{2} \pm 1} = \mp 2\xi_1(\omega) \sqrt{\frac{2}{3} M_i M_f [\sqrt{\omega - 1} \mp \sqrt{\omega + 1}]} & \Omega_b \to \Omega^*_c \text{decay} \]

where

\[ M_{\pm} = M_i \pm M_f, \quad \xi_T = \xi_1 \omega - \xi_2 (\omega^2 - 1), \]

\[ \xi_{L_{\pm}} = \xi_1 (\omega \pm 2) - \xi_2 (\omega^2 - 1), \quad \xi_{L^*_{\pm}} = \xi_1 (\omega \mp 1) - \xi_2 (\omega^2 - 1), \]

\[ \omega_{\text{max}} = \frac{M_i^2 + M_f^2}{2M_i M_f}. \]

The decay rates of semileptonic decays are then given by
\[ \Gamma = \int_{\omega_{\text{max}}}^{\omega} \left( \frac{d\Gamma}{d\omega} - \frac{d\Gamma_T}{d\omega} + \frac{d\Gamma_T}{d\omega} + \frac{d\Gamma_L}{d\omega} - \frac{d\Gamma_L}{d\omega} \right) \]  

where the indices \( T \) and \( L \) denote partial contributions of transverse (\( \lambda_W = \pm 1 \)) and longitudinal (\( \lambda_W = 0 \)) components of the current transitions. Partial differential distributions are given by

\[ \frac{d\Gamma_T^{\pm}}{d\omega} = \kappa_\omega |H_{\pm^{1/2} \pm 1}|^2 \quad \text{for } \frac{1}{2}^+ \to \frac{1}{2}^+ \text{ transition} \]

\[ \frac{d\Gamma_L^{\pm}}{d\omega} = \kappa_\omega |H_{\pm^{1/2} \pm 0}|^2, \quad \kappa_\omega = \frac{G_F^2}{(2\pi)^3} |V_{bc}|^2 \frac{M_3^3}{6} (\omega_{\text{max}} - \omega) \sqrt{\omega^2 - 1} \]

Tables 7-11 list our predictions for the semileptonic rates of beauty baryons. In Table 7 we present the results for total and partial rates for various \( b \to c \) decay modes. The adjustable parameters are chosen as \( m_s = 570 \text{ MeV}, \Lambda_Q = 2.5 \text{ GeV}, \bar{\Lambda} = 710 \text{ MeV}, \bar{\Lambda}_s = 850 \text{ MeV} \) and \( \bar{\Lambda}_{ss} = 1000 \text{ MeV} \). In Table 8 we compare our results for total rates with the predictions of other phenomenological approaches: constituent quark model [28], spectator quark model [37], nonrelativistic quark model [43]. The dependence of the total rates on the parameters \( \bar{\Lambda}, \bar{\Lambda}_s \) and \( \bar{\Lambda}_{ss} \) are shown in Tables 9-11.

**Table 7.** Decay Rates of Bottom Baryons (in \( 10^{10} \text{ sec}^{-1} \)) for \( |V_{bc}| = 0.04 \)

| Process       | \( \Gamma_{\text{total}} \) | \( \Gamma_T \) | \( \Gamma_T^{+} \) | \( \Gamma_T^{-} \) | \( \Gamma_L \) | \( \Gamma_L^{+} \) | \( \Gamma_L^{-} \) |
|---------------|-----------------|-------------|-----------------|-----------------|-------------|-----------------|-----------------|
| \( \Lambda_b^0 \to \Lambda_c^+ e^- \bar{\nu}_e \) | 5.39 | 2.07 | 0.53 | 1.54 | 3.32 | 0.11 | 3.21 |
| \( \Xi_b^0 \to \Xi_c^+ e^- \bar{\nu}_e \) | 5.27 | 2.02 | 0.54 | 1.48 | 3.25 | 0.11 | 3.14 |
| \( \Sigma_b^+ \to \Sigma_c^{++} e^- \bar{\nu}_e \) | 2.23 | 0.33 | 0.08 | 0.25 | 1.90 | 1.49 | 0.41 |
| \( \Omega_b^- \to \Omega_c^0 e^- \bar{\nu}_e \) | 1.87 | 0.29 | 0.08 | 0.21 | 1.58 | 1.26 | 0.32 |
| \( \Sigma_b^+ \to \Sigma_c^{++} e^- \bar{\nu}_e \) | 4.56 | 2.07 | 0.54 | 1.53 | 2.49 | 1.09 | 1.40 |
| \( \Omega_b^- \to \Omega_c^0 e^- \bar{\nu}_e \) | 4.01 | 1.89 | 0.53 | 1.36 | 2.12 | 0.95 | 1.17 |
Table 8. Model Results for Rates of Bottom Baryons (in $10^{10}$ sec$^{-1}$) for $|V_{bc}|=0.04$

| Process                                      | Ref. [37] | Ref. [43] | Ref. [28] | Our results |
|----------------------------------------------|-----------|-----------|-----------|-------------|
| $\Lambda_b^0 \rightarrow \Lambda_c^+e^-\bar{\nu}_e$ | 5.9       | 5.1       | 5.14      | 5.39        |
| $\Xi_b^0 \rightarrow \Xi_c^+e^-\bar{\nu}_e$     | 7.2       | 5.3       | 5.21      | 5.27        |
| $\Sigma_b^+ \rightarrow \Sigma_c^{++}e^-\bar{\nu}_e$ | 4.3       |           |           | 2.23        |
| $\Sigma_b^+ \rightarrow \Sigma_c^{*+}e^-\bar{\nu}_e$ |           |           |           | 4.56        |
| $\Omega_b^- \rightarrow \Omega_c^0e^-\bar{\nu}_e$ | 5.4       | 2.3       | 1.52      | 1.87        |
| $\Omega_b^- \rightarrow \Omega_c^0e^-\bar{\nu}_e$ |           | 3.41      |           | 4.01        |

Table 9. Dependence of Rates on $\bar{\Lambda}$ for $|V_{bc}|=0.04$

| Process                                      | $\bar{\Lambda}$ (MeV) |
|----------------------------------------------|------------------------|
| $\Lambda_b^0 \rightarrow \Lambda_c^+e^-\bar{\nu}_e$ | 600  650  710  750  800 |
| $\Sigma_b^+ \rightarrow \Sigma_c^{*+}e^-\bar{\nu}_e$ | 4.99  4.81  4.56  4.35  4.03 |
Table 10. Dependence of Rates on $\bar{\Lambda}_s$ for $|V_{bc}|=0.04$

| Process         | $\bar{\Lambda}_s$ (MeV) |
|-----------------|--------------------------|
|                 | 760  | 800  | 850  | 900  |
| $\Xi_b^0 \to \Xi_c^+ e^- \bar{\nu}_e$ | 5.81 | 5.58 | 5.27 | 4.93 |

Table 11. Dependence of Rates on $\bar{\Lambda}_{ss}$ for $|V_{bc}|=0.04$

| Process         | $\bar{\Lambda}_{ss}$ (MeV) |
|-----------------|-----------------------------|
|                 | 900  | 950  | 1000 | 1050 | 1100 |
| $\Omega_b^− \to \Omega_c^0 e^- \bar{\nu}_e$ | 2.09 | 1.98 | 1.87 | 1.72 | 1.54 |
| $\Omega_b^− \to \Omega_c^* e^- \bar{\nu}_e$ | 4.44 | 4.23 | 4.01 | 3.75 | 3.43 |

The differential distributions for $\Lambda_b^0 \to \Lambda_c^+ e^- \bar{\nu}_e$ decay are plotted in Fig.8.

Leptonic spectra $d\Gamma/dE_\ell$ are calculated according to the sum

$$
\frac{d\Gamma}{dE_\ell} = \frac{d\Gamma_{T\pm}}{dE_\ell} + \frac{d\Gamma_{T\mp}}{dE_\ell} + \frac{d\Gamma_{L\pm}}{dE_\ell} + \frac{d\Gamma_{L\mp}}{dE_\ell}
$$

(32)

Expressions for partial leptonic spectra are given by

$$
\frac{d\Gamma_{T\pm}}{dE_\ell} = \int d\omega \kappa_E (1 \pm \cos \Theta)^2 |H_{\pm\pm1}|^2 \frac{d\Gamma_{T\pm}}{dE_\ell} = \int d\omega \kappa_E (1 - \cos^2 \Theta)^2 |H_{\pm\pm0}|^2
$$

$$
\kappa_E = \frac{G_F^2}{2(2\pi)^3} |V_{bc}|^2 \frac{M_{\Lambda_c}}{8} (\omega_{\max} - \omega), \quad \cos \Theta = \frac{E_{\max} - 2E_\ell + M_{\Lambda_c} (\omega_{\max} - \omega)}{M_{\Lambda_c} \sqrt{\omega^2 - 1}}
$$

$$
E_{\max} = \frac{M_{\Lambda_b} - M_{\Lambda_c}^2}{2M_{\Lambda_b}}, \quad \omega_{\min}(E_\ell) = \omega_{\max} - 2 \frac{E_\ell E_{\max} - E_\ell}{M_{\Lambda_c} M_{\Lambda_b} - 2E_\ell}
$$
Our results on leptonic spectra in semileptonic $\Lambda_b \to \Lambda_c$ transitions are shown in Fig. 9.

Finally, we consider the cascade decay $\Lambda_b \to \Lambda_c \to \Lambda_s \pi + W \to \ell \nu$ which is characterized by a set of asymmetry parameters. The formalism and a detailed analysis of the asymmetry parameters is presented in [36, 28]. In terms of helicity amplitudes the asymmetry parameters of nonpolarized $\Lambda_b$ decays ($\alpha, \alpha', \alpha'', \gamma$) and polarized $\Lambda_b$ decays ($\alpha_P, \gamma_P$) are given by the following expressions

$$
\alpha = \frac{H_T^+ + H_L^-}{H_T^+ + H_L^+}, \quad \alpha' = \frac{H_T^-}{H_T^+ + 2H_L^+}, \quad \alpha'' = \frac{H_T^+ - 2H_L^+}{H_T^+ + 2H_L^+}, \quad \gamma = \frac{2H_\gamma}{H_T^+ + H_L^+},
$$

$$
\alpha_P = \frac{H_T^- - H_L^-}{H_T^+ + H_L^+}, \quad \gamma_P = \frac{2H_{\gamma P}}{H_T^+ + H_L^+},
$$

$$
H_T^\pm = |H_{1/2 1}|^2 \pm |H_{-1/2 -1}|^2, \quad H_L^\pm = |H_{1/2 0}|^2 \pm |H_{-1/2 0}|^2
$$

$$
H_\gamma = Re(H_{-1/2 0}H_{1/2 1}^* + H_{1/2 0}H_{-1/2 -1}^*) \quad H_{\gamma P} = Re(H_{1/2 0}H_{-1/2 -1}^*)
$$

We evaluate the average magnitudes of the asymmetry parameters ($<\alpha>, <\alpha'>, \text{ etc.}$) as results of separate $\omega$ integrations of numerators and denominators. Results for average magnitudes are given in Table 12. Also the results of paper [39] are quoted for comparison.

| Model  | $\alpha$ | $\alpha'$ | $\alpha''$ | $\gamma$ | $\alpha_P$ | $\gamma_P$ |
|--------|----------|-----------|------------|----------|------------|------------|
| Our    | -0.76    | -0.12     | -0.53      | 0.56     | 0.39       | -0.16      |
| IMF    | -0.71    | -0.12     | -0.46      | 0.61     | 0.33       | -0.19      |

4.3 Heavy-to-Light Baryon Decays

In this subsection we consider the heavy-to-light semileptonic modes. In particular the process $\Lambda_c^+ \to \Lambda^0 + e^+ + \nu_e$ which was recently investigated by the CLEO Collaboration [12] is studied
in detail. In the heavy mass limit \((m_C \rightarrow \infty)\) its transition matrix element is defined by two form factors \(f_1\) and \(f_2\) (see, section 3). Assuming identical dipole forms for the form factors (as in the model of Körner and Krämer [36]), CLEO found that \(R = f_2/f_1 = -0.25 \pm 0.14 \pm 0.08\).

Our form factors have different \(q^2\) dependences. In other words, the quantity \(R = f_2/f_1\) has a \(q^2\) dependence in our approach. In Fig.10 we plot the results for \(R\) in the kinematical region \(1 \leq \omega \leq \omega_{\text{max}}\) for different magnitudes of the \(\bar{\Lambda}\) parameter.

It is seen that larger values of \(\bar{\Lambda}\) lead to an increase of the ratio \(R\). The best fit to the experimental data is achieved for the following set of parameters: \(m_s = 570\) MeV, \(\Lambda_Q = 2.5\) GeV and \(\bar{\Lambda} = 710\) MeV. In this case the \(\omega\)-dependence of the form factors \(f_1, f_2\) and their ratio \(R\) are shown in Fig.11. Particularly, we get \(f_1(q_{\text{max}}^2) = 0.75, f_2(q_{\text{max}}^2) = -0.17, R = -0.23\) at zero recoil (\(\omega = 1\) or \(q^2 = q_{\text{max}}^2\)) and \(f_1(0) = 0.38, f_2(0) = -0.06, R = -0.16\) at maximum recoil (\(\omega = \omega_{\text{max}}\) or \(q^2 = 0\)). Note that our results for \(q_{\text{max}}^2\) are close to those of the nonrelativistic quark model [43]: \(f_1(q_{\text{max}}^2) = 0.75, f_2(q_{\text{max}}^2) = -0.17, R = -0.23\).

Our result for \(R\) agree well with the experimental data [12] \(R = -0.25 \pm 0.14 \pm 0.08\). The predictions for the decay rate \(\Gamma(\Lambda_c^+ \rightarrow \Lambda^0 e^+\nu_e) = 7.22 \times 10^{10}\) sec\(^{-1}\) and for the asymmetry parameter \(\alpha_{\Lambda_c} = -0.812\) also coincide with the experiment: \(\Gamma_{\exp} = 7.0 \pm 2.5 \times 10^{10}\) sec\(^{-1}\) and \(\alpha_{\exp} = -0.82^{+0.09}_{-0.06} + 0.06\) respectively as well as with the result of [13] \(\Gamma = 7.1 \times 10^{10}\) sec\(^{-1}\). Note that the agreement with the experimental rate measurement crucially depends on the use of the \(\Lambda^0\) three-quark current in its \(SU(3)\)-flavor symmetric form (see, Table 1.) which leads to the presence of the flavor-suppression factor \(N_{\Lambda_c\Lambda} = 1/\sqrt{3}\) for \(\Lambda_c^+ \rightarrow \Lambda^0 e^+\nu_e\). If the \(SU(3)\) symmetric structure of \(\Lambda^0\) hyperon is not taken into account the predicted rate for \(\Lambda_c^+ \rightarrow \Lambda^0 e^+\nu_e\) becomes too large (see, discussion in ref. [28, 43]).

In Table 13 we present our predictions for some modes of semileptonic heavy-to-light transitions (for \(\bar{\Lambda}_s = 850\) MeV, \(\bar{\Lambda}_{ss} = 1000\) MeV). Also the results obtained in other approaches are tabulated. Note that the flavor-suppression factor for the modes \(\Xi_c^0 \rightarrow \Xi^- e^+\nu_e, \Lambda_b^0 \rightarrow p e^- \bar{\nu}_e\) and \(\Lambda_c^+ \rightarrow n e^+\nu_e\) is equal to \(1/\sqrt{2}\).

Finally, in Table 14 we give the predictions for the average magnitudes of the asymmetry parameters for the cascade decay \(\Lambda_c \rightarrow \Lambda_s [\rightarrow p\pi] + W [\rightarrow \ell \nu_{\ell}]\) which are expected to be measured in near future by the COMPASS Collaboration [19]. For comparison, the results of paper [36] for \(R = f_2/f_1 = -0.25\) are also given.
Table 13. Heavy-to-Light Decay Rates (in $10^{10} \text{s}^{-1}$) for $|V_{bc}|=0.04$, $|V_{cs}|=0.975$.

| Process | Quantity | Ref. [37] | Ref. [43] | Ref. [47] | Ref. [48] | Our | Exp. [29] |
|---------|----------|-----------|-----------|-----------|-----------|-----|----------|
| $\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$ | $\Gamma$ | 9.8 | 7.1 | 5.36 | 7 | 7.22 | 7.0± 2.5 |
| $\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$ | $\Gamma$ | 8.5 | 7.4 | 9.7 | 8.16 |
| $\Lambda_b^0 \rightarrow p e^- \bar{\nu}_e$ | $\Gamma/|V_{bu}|^2$ | 6.48×$10^2$ | | | 7.47×$10^2$ |
| $\Lambda_c^+ \rightarrow n e^+ \nu_e$ | $\Gamma/|V_{cd}|^2$ | | | 0.17×$10^2$ | 0.26×$10^2$ |

Table 14. Asymmetry parameters of $\Lambda_c$ decay

| Model | $\alpha$ | $\alpha'$ | $\alpha''$ | $\gamma$ | $\alpha_P$ | $\gamma_P$ |
|-------|---------|----------|-----------|--------|--------|--------|
| Our   | -0.81   | -0.13    | -0.56     | 0.50   | 0.40   | -0.15 |
| Körner & Krämer [33] | -0.82 | -0.13 | -0.56 | 0.47 | 0.39 | -0.14 |

5 Conclusion

We have developed a relativistic model [15, 16], [19, 20] for QCD bound states composed of light quarks and a heavy quark. In fact, this model is the Lagrangian formulation of the NJL model with separable interaction [17, 18] and its advantage consists in the possibility of studying baryons as three-quark states as multiquark and exotic objects. We have used our approach to study the properties of baryons containing a single heavy quark. We have calculated the observables of semileptonic decays of bottom and charm baryons: Isgur-Wise functions, asymmetry parameters, decay rates and distributions. We obtained analytical expressions for the baryon IW-functions: $\zeta$ ($\Lambda_b \rightarrow \Lambda_c$ transition), $\xi_1$ and $\xi_2$ ($\Omega_b \rightarrow \Omega_c^{(*)}$ transition). We checked the model-independent Bjorken-Xu inequalities for the $\xi_1$ and $\xi_2$ functions and their derivatives.
at the zero recoil point. It is shown that inequality for the charge radius of $\xi_1$ (see, Eq. (26)) is automatically respected in our model. The inequality (23) for the $\omega$-dependence of $\xi_1$ and $\xi_2$ leads to an upper limit for $\xi_1$ (see, 24) which is respected in our model for reasonable values of the parameter $\bar{\Lambda}$. We have also applied our model to the calculation of heavy-to-light semileptonic decay processes motivated by the recent experimental observation of the $\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$ decay by the CLEO Collaboration [12]. Our predictions for the form factor ratio, the decay rate and the asymmetry parameter $\alpha$ are in good agreement with measured values [12, 29]. The success in reproducing the correct experimental rate requires the use of the $\Lambda^0$ three-quark current in the $SU(3)$-flavor symmetric form (see, Table 1.). Predictions for other semileptonic heavy-to-light rates are also given. Finally, we have given predictions for the asymmetry parameters of the cascade decay $\Lambda_c \rightarrow \Lambda_s [\rightarrow p\pi] + W [\rightarrow \ell\nu_\ell]$.

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7 Appendix

A. The Calculation Technique

To elucidate the calculation of the matrix elements (18) and (19) we consider the two generic integrals in a Euclidean space

\[ I_1(p'_E, w_E) = \int \frac{d^4k_E}{\pi^2} \int \frac{d^4k'_E}{\pi^2} \exp \left( -\frac{9k_E^2}{\Lambda_{Bq}^2} \right) \exp \left( -\frac{9(k_E + \alpha p'_E)^2}{\Lambda_{Bq}^2} + \frac{9k'_E^2}{\Lambda_{Bq}^2} \right) \]

\[ \times \frac{1}{m^2 + (k_E + k'_E)^2/4} \frac{1}{m^2 + (k_E - k'_E)^2/4} \]

\[ \times \frac{1}{m_s^2 + (k_E + p'_E)^2} \frac{1}{k_E v_E - \Lambda} \]

\[ \alpha = \frac{2m}{2m + m_s} \] (A.1)

\[ I_2(w_E) = \int \frac{d^4k_E}{\pi^2} \int \frac{d^4k'_E}{\pi^2} \exp \left( -\frac{18k_E^2}{\Lambda_{Q}^2} + \frac{6k'_E^2}{\Lambda_{Q}^2} \right) \]

\[ \times \frac{1}{m^2 + (k_E + k'_E)^2/4} \frac{1}{m^2 + (k_E - k'_E)^2/4} \]

\[ \times \frac{1}{k_E v_E - \Lambda} \frac{1}{k_E v'_E - \Lambda} \] (A.2)

where \( \alpha \) is defined in (19). The final light baryon state carrying the Euclidean momenta \( p'_E \), is on mass-shell: \( p'_E^2 = -M'^2 \). The dimensionless variable \( w_E \) is defined as \( w_E = v_E \cdot p'_E / M' = -w \).

The first integral appears in the calculation of heavy-to-light form factors, the second one in the calculation of the heavy-to-heavy case.

Scaling all momentum variables in (A.1) by \( \Lambda_{Bq} \) and (A.2) by \( \Lambda_{BQ} \) and using the Feynman parametrization

\[ \frac{1}{A} = \int_0^\infty d\alpha \exp(-\alpha A) \]

we have

\[ I_1(-M'^2, w_E) = 2\Lambda_{Bq} [6(1 + R)]^4 \exp[-36m^2(1 + R) - 9M'^2\alpha^2] \] (A.3)
\[ \times \int_0^\infty \ldots \int_0^\infty d\beta_1 \ldots d\beta_4 t^2(\beta) \int \frac{d^4k_F}{\pi^2} \int \frac{d^4k'_F}{\pi^2} \]

\[ \times \exp \left[ -3(1 + R)(1 + \beta_3 + \beta_4) \left( k'_F + k_F \frac{\beta_3 - \beta_4}{1 + \beta_3 + \beta_4} \right)^2 \right] \]

\[ \times \exp \left[ -3(1 + R)t(\beta)(1 + \beta_2) \left( k_F + \frac{v_E\beta_1 + v'_E r_1}{1 + \beta_2} \right)^2 \right] \]

\[ \times \exp \left[ \frac{-3(1 + R)t(\beta)}{1 + \beta_2} \left( \beta_1 + M'\beta_2 - \bar{\Lambda}(1 + \beta_2)^2 + 6M'(1 + R)t(\beta)(w_E + 1) \frac{\beta_1 \beta_2}{1 + \beta_2} \right) \right] \]

\[ \times \exp \left[ 18M'^\alpha (w_E\beta_1 - M'r_2) - 12m^2(1 + R) \frac{(\beta_3 - \beta_4)^2}{1 + \beta_3 + \beta_4} - 3(1 + R)t(\beta)(4m^2 - \bar{\Lambda}^2) \right] \]

\[ \times \exp \left[ -3(1 + R)t(\beta)\beta_2(m_2^2 - (M' - \bar{\Lambda})^2) \right] \]

\[ I_2(w_E) = 4\Lambda^2_{Bq} 12^4 \exp[-72m^2] \int_0^\infty \ldots \int_0^\infty d\beta_1 \ldots d\beta_4 t^2(\beta) \int \frac{d^4k_F}{\pi^2} \int \frac{d^4k'_F}{\pi^2} \quad (A.4) \]

\[ \times \exp \left[ -6(1 + \beta_3 + \beta_4) \left( k'_F + k_F \frac{\beta_3 - \beta_4}{1 + \beta_3 + \beta_4} \right)^2 \right] \]

\[ \times \exp \left[ -6t(\beta)(1 + \beta_2)(k_F + v_E\beta_1 + v'_E \beta_2)^2 \right] \]

\[ \times \exp \left[ -6t(\beta)(\beta_1 + \beta_2 - \bar{\Lambda})^2 + 12t(\beta)(w_E + 1)\beta_1 \beta_2 \right] \]

\[ \times \exp \left[ -24m^2 \frac{(\beta_3 - \beta_4)^2}{1 + \beta_3 + \beta_4} - 6t(\beta)(4m^2 - \bar{\Lambda}^2) \right] \]

The notation is as follows:

\[ R = \frac{\Lambda^2_{Bq}}{\Lambda^2_{Bq}}, \quad t(\beta) = \frac{3 + 4(\beta_3 + \beta_4) + 4\beta_3 \beta_4}{1 + \beta_3 + \beta_4}, \]

\[ r_1 = \beta_2 + \frac{3\alpha}{(1 + R)t(\beta)}, \quad r_2 = \beta_2 + \frac{3\alpha}{2(1 + R)t(\beta)} \]

After a change of variables for \( k'_E, k_E \) and integrations we arrive at

\[ I_1(-M'^2, -w) = 32\Lambda_{Bq} \exp[-36m^2_q(1 + R) - 9m^2\alpha^2] \quad (A.5) \]
\[ I_2(-w) = 64 \Lambda Q B \exp[-72m^2] \int_0^\infty \int_0^\infty \frac{d\beta_1 \ldots d\beta_4}{(1 + \beta_3 + \beta_4)^2} \left[ -3(1 + R)t(\beta)\beta_2(m_s^2 - (M' - \bar{\Lambda})^2) \right] \]

\[ \times \exp\left[ -\frac{3(1 + R)t(\beta)}{1 + \beta_2}(\beta_1 + M'\beta_2 - \bar{\Lambda}(1 + \beta_2))^2 - 6M'(1 + R)t(\beta)(w - 1)\frac{\beta_1\beta_2}{1 + \beta_2} \right] \]

\[ \times \exp\left[ -\frac{18M'\alpha}{1 + \beta_2}(w\beta_1 + M'r_2) - 12m^2(1 + R)\frac{(\beta_3 - \beta_4)^2}{1 + \beta_3 + \beta_4} - 3(1 + R)t(\beta)(4m^2 - \bar{\Lambda}^2) \right] \]

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Fig. 3 $\zeta(\omega)$ function
1. $\bar{\Lambda}=600$ MeV
2. $\bar{\Lambda}=650$ MeV
3. $\bar{\Lambda}=710$ MeV
4. $\bar{\Lambda}=750$ MeV
5. $\bar{\Lambda}=800$ MeV

Fig. 4 $\zeta(\omega)$ function
1. $\Lambda_Q=2.5$ GeV
2. $\Lambda_Q=2.0$ GeV
3. $\Lambda_Q=1.7$ GeV
4. $\Lambda_Q=1.5$ GeV
5. $\Lambda_Q \equiv \Lambda_q=1.25$ GeV

Fig. 5 $\zeta(\omega)$ function
1. SQM (Ref. [45]), 2. QCD SR (Ref. [24]),
3. Our result ($\bar{\Lambda}=710$ MeV, $\Lambda_Q=2.5$ GeV),
4. Dipole (Ref. [28]), 5. MIT Bag (Ref. [42]),
6. IMF (Ref. [28]), 7. IMF (Ref. [38])
Fig. 6 $\xi_1(\omega)$ function ($\Sigma_b$-decay)

1. $\bar{\Lambda}=600$ MeV
2. $\bar{\Lambda}=650$ MeV
3. $\bar{\Lambda}=710$ MeV
4. $\bar{\Lambda}=750$ MeV
5. $\bar{\Lambda}=800$ MeV

Fig. 7 $\xi_1(\omega)$ function ($\Omega_b$-decay)

1. $\bar{\Lambda}_{ss}=800$ MeV
2. $\bar{\Lambda}_{(ss)}=900$ MeV
3. $\bar{\Lambda}_{(ss)}=1000$ MeV
4. $\bar{\Lambda}_{(ss)}=1050$ MeV
5. $\bar{\Lambda}_{(ss)}=1100$ MeV
Fig. 10 Form factor ratios $R = \frac{f_2}{f_1}$ for $\Lambda_+^c \rightarrow \Lambda^0 + e^+ \nu$-decay
1. $\bar{\Lambda} = 650$ MeV
2. $\bar{\Lambda} = 710$ MeV
3. $\bar{\Lambda} = 725$ MeV
4. $\bar{\Lambda} = 750$ MeV
5. $\bar{\Lambda} = 775$ MeV
6. $\bar{\Lambda} = 800$ MeV

Fig. 11 Form factors and their ratio for $\Lambda_+^c \rightarrow \Lambda^0 + e^+ \nu$-decay
Fig. 8 Differential Distribution

Fig. 9 Leptonic Spectrum