The explicit formula of the distributions of the nonoverlapping words and its applications to statistical tests for random numbers

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Abstract—Bassino et al. 2010 and Regnier et al. 1998 showed the generating functions of the distributions of the number of the occurrences of words (distributions of words for short) in finite string in the form of rational functions. However the coefficients of the expansion of the rational functions are complicated and we do not have a simple formula of the exact distributions of words from rational functions. In this paper we study the finite dimensional generating functions of the distribution of nonoverlapping words for each fixed sample size and show the explicit formula of the distributions of words for Bernoulli model. We demonstrate that 1) the tests based on the distributions of words reject the random number generator in BSD Library with p-value almost zero and 2) computation of the distributions of words in the human DNA size strings.

Key words: suffix tree, partial word, inclusion-exclusion principle and, statistical tests, pseudo random numbers

I. INTRODUCTION

We study the distributions of the number of the occurrences of the words (the distribution of words for short) of finite alphabet, which plays an important role in information theory, ergodic theory, computer science, and DNA analysis, see [3], [9], [12], [17], [26]. The data compression scheme LZ are based on the statistics of the occurrences of words [19], [27], [28] and are applied to nonparametric statistics [10].

Régnier and Szpankowski [15] derived generating functions of the distributions of words in a finite sample under finite Markov processes. Goulden and Jackson [7] and Bassino et al. [1] obtained the generating functions of the distributions of words via cluster method and inclusion-exclusion principle. In [5], [11], [14] average return time of words are analyzed. We can detect the partial word where the length of the word are large. Our theorems can be applied to nonoverlapping partial words. We can detect the difference of the distributions of large size sparse pattern of letters (nonoverlapping partial words) with practical sample size.

In Section III we introduce partial word that allows the special case of the theorem in Guibas and Odlyzko [8] (pp.61). Since

\[ F(z) = \frac{1}{1 - 2z + z^2}, \]

\[ = \frac{1}{(1 - z)^2} = \sum_n z^n = \sum_n (n + 1)z^n, \]

we have \( f(n) = n + 1 \). For example, \( f(3) = 4 \) and the four words, 000, 100, 110, and 111 do not contain the word 01 among the strings of length 3.

In general, the multivariate generating functions of the distributions of words with the variables of the number of words and sample size are rational functions and the asymptotic formula of the distributions of words are obtained from the generating functions [6], [9]. However, except for simple cases, the exact value of the coefficients of the expansion of the rational functions are complicated and we do not have a simple formula of the exact distributions of words from the rational functions [5].

In this paper, for each fixed sample size, we show the finite dimensional generating functions of the probabilities of the number of nonoverlapping words by inclusion-exclusion principle and present the explicit formula of the exact distributions of words (Theorem 1).

The number of the appearance of 01 is almost same to the half of the number of runs. For example the number of runs and the word 01 in 00110011 are 4 and 2, respectively. Statistical tests based on the number of the appearance of words are considered to be a generalization of the run tests.

Let \( w_0w_1 \cdots \) be a pseudo random numbers where \( w_n = f(w_{n-1}) \) for all \( n \) and \( f \) is a pseudo random number generator. Then \( w_{n-1} \neq w_n \) for all \( n \) and the empirical distribution of the word \( ww \) in the sequence \( w_0w_1 \cdots \) will be different from the fair-coin flipping if \( |w| \) is large. It is unpractical to test the number of the occurrence of the word \( ww \) when \( |w| \) is large since we need large sample to detect the small difference between the empirical distribution and null hypothesis with Kolmogorov-Smirnov test. To overcome this difficulty, in Section IV we consider statistical tests based on the distributions of words (sliding sampling test) and the distribu-
tions of the number of blockwise occurrence of words (blockwise sampling test). We show that the power function of the sliding sampling test is much larger than that of the blockwise sampling test.

In Section 7, we apply the distribution of words to statistical test for random numbers.

Finally in Section 7, we report the experiment on the computation of the distribution of words in human DNA size string.

The preliminary versions of the paper have been presented at [20]–[25]. For the proof of Theorem 2 see [20].

II. MAIN RESULTS

Let $X^n := X_1 \ldots X_n$ be random variables that take value in finite alphabet $A$ and $N(w_1, \ldots, w_l; X^n)$ be the vector of the number of the appearance of the word $w_1, \ldots, w_l$ in an arbitrary position of $X^n$. For example $N(10, 11; 1011101) = (2, 2)$. Let $|x|$ be the length of $x$. A word $x$ is called overlapping if there is a word $z$ such that $x$ appears at least 2 times in $z$ and $|z| < |x| + |y|$ otherwise $x$ is called nonoverlapping. A pair of words $x, y$ is called overlapping if there is a word $z$ such that $x$ and $y$ appear in $z$ and $|z| < |x| + |y|$ otherwise the pair is called nonoverlapping. A word $x$ is overlapping if and only if $(x, x)$ is overlapping. A finite set of words $S$ is called nonoverlapping if every pair $(x, y), x, y \in S$ are nonoverlapping. If a finite set of words $S$ is nonoverlapping, each element of $S$ is nonoverlapping since $(x, x), x \in S$ is nonoverlapping. For example $11, \{10, 01\}$, and $\{00, 11\}$ are overlapping, and 10 and $\{00111, 00101\}$ are nonoverlapping.

**Theorem 1** Let $X_1 X_2 \cdots X_n$ be an i.i.d. process of fixed sample size $n$ of finite alphabet. Let $w_1, \ldots, w_l$ be the set of nonoverlapping words of finite alphabet. Let $m_i := |w_i|$ and $P(w_i)$ be the probability of $w_i$ for $i = 1, \ldots, l$. Let

\[
A(k_1, \ldots, k_l) = \left( n - \sum_{k_1 \ldots k_l} m_i k_i + \sum_{k_1 \ldots k_l} k_i \right) \prod_{i=1}^l P_{k_i}(w_i),
\]

\[
B(k_1, \ldots, k_l) = P\left( \sum_{i=1}^n I_{X_i = w_j} = k_j, j = 1, \ldots, l \right),
\]

\[
F_A(z_1, \ldots, z_l) = \sum_{k_1 \ldots k_l} A(k_1, \ldots, k_l) z_1^{k_1} \cdots z_l^{k_l}, \text{ and}
\]

\[
F_B(z_1, \ldots, z_l) = \sum_{k_1 \ldots k_l} B(k_1, \ldots, k_l) z_1^{k_1} \cdots z_l^{k_l}.
\]

Then

\[
F_A(z_1, z_2, \ldots, z_l) = F_B(z_1 + 1, z_2 + 1, \ldots, z_l + 1),
\]

and

\[
P(N(w_1, \ldots, w_l; X^n) = (s_1, \ldots, s_l))
= \sum_{k_1 \ldots k_l : s_1 \leq k_1, \ldots, s_l \leq k_l, 0 \leq n - \sum_{i=1}^l m_i k_i}
\left( -1 \right)^{\sum_{i=1}^l k_i - s_i} \left( \sum_{s_1, \ldots, s_l} k_i \right) \prod_{i=1}^l P_{k_i}(w_i).
\]

(3)

**Theorem 2** Let $X_1 X_2 \cdots X_n$ be an i.i.d. process of fixed sample size $n$ of finite alphabet. Let $w_1 \sqsubset w_2 \sqsubset \cdots \sqsubset w_l$ be strictly increasing nonoverlapping words of finite alphabet, i.e., $w_i$ is a prefix of $w_j$ and $m_i < m_j$, where $m_i$ is the length of $w_i$, for all $i < j$. Let $P(w_i)$ be the probability of $w_i$ for $i = 1, \ldots, l$. Let $A, B, F_A, F_B$ be the functions defined with the same manner in (2). Then

\[
F_A(z_1, z_2, \ldots, z_l) = F_B(z_1 + 1, z_1 + z_2 + 1, \ldots, z_1 + \cdots + z_l + 1).
\]

With slight modification of Theorem 2, we can compute the number of the occurrence of the overlapping increasing words. For example, let us consider increasing overlapping words 11, 111, 1111 and the number of their occurrences. Let 011, 0111, 01111 then these words are increasing overlapping words. The number of occurrences 11, 111, 1111 in sample of length $n$ is equivalent to the number of occurrences 011, 0111, 01111 in sample of length $n + 1$ that starts with 0.

In [15], expectation, variance, and CLTs (central limit theorems) for the occurrences of words are shown. We show the higher moments for nonoverlapping words.

**Theorem 3** Let $w$ be a nonoverlapping word.

\[
\forall t \; E(N_w^n) = \sum_{s=1}^{\min\{T, t\}} A_{t,s} \left( n - s|w| + s \right) P_s(w).
\]

\[
A_{t,s} = \sum_r \binom{s}{r} r^t (-1)^{s-r}, \quad T = \max\{t \in \mathbb{N} : n - t|w| \geq 0\}.
\]

In the above theorem, $A_{t,s}$ is the number of surjective functions from $\{1, 2, \ldots, t\} \rightarrow \{1, 2, \ldots, s\}$ for $t, s \in \mathbb{N}$, see [16].

III. NONOVERLAPPING PARTIAL WORDS

We introduce the symbol $?$ to represent arbitrary symbols. Let $A$ be a finite alphabet. A word consists of extended alphabet $A \cup \{?\}$ is called partial word [2]. The word $w'$ consists of $A$ is called a realization of the partial word $w$ if $w'$ consists of $A$ and coincides with $w$ except for the symbol $?$. A partial word is called nonoverlapping if the set of the realization is nonoverlapping. For example 001?1 is a nonoverlapping partial word with alphabet $A = \{0, 1\}$ and its realizations are 00101 and 00111. We write $w_1 \sqsubset w_2$ for two partial words if $w_1$ is a prefix of $w_2$ with the alphabet $A \cup \{?\}$. For example 01?1 $\sqsubset$ 01?11 but 01?1 $\nsubseteq$ 01111.
The probability of partial word $w$ is defined by
\[ P(w) := \sum_{w': \text{realization of } w} P(w'). \]

Let $\tilde{w}$ be the word obtained by removing the letter ? from the partial word $w$. For example $\tilde{w} = 011$ if $w = 01?1$.

**Proposition 1** Let $P$ be i.i.d. and $w$ be a partial word. Then
\[ P(w) = P(\tilde{w}). \]

In particular if $P$ is the fair-coin flipping,
\[ P(w) = 2^{-|\tilde{w}|}. \]

The theorems above still hold for nonoverlapping partial words.

**Corollary 1** Theorem 2 and Theorem 3 holds for nonoverlapping partial words. Theorem 2 holds for nonoverlapping increasing partial words.

We can find many nonoverlapping partial words. For example, $0^m(1?^m)\pi^m$ are nonoverlapping for all $m$, $m$. Here $w^m$ is the $m$ times concatenation of the word $w$. For example, $0^3(1?^2)^2 = 0001??1?1$. We can construct large size partial words that have large probabilities

**Proposition 2** Let $P$ be the fair-coin flipping and
\[ w(m) := 0^m(1?^m)\pi^m1 \quad (4) \]
then $w(m)$ is nonoverlapping for any $m$, $|w(m)| = m^2 + 1$, and $P(w(m)) = 2^{-2m}$.

For example let $m = 10$ then we have 101 length partial word and its probability is $2^{-20}$ for fair-coin flipping, which is significantly large compared to its length. The author expect that $w(m)$ achieves almost maximum probability among the nonoverlapping partial words with length $|w(m)|$.

**Corollary 2** Let $P$ be the fair-coin flipping. There is a sequence of nonoverlapping partial words $w_1, w_2, \ldots$ with increasing length $|w_n| < |w_{n+1}|$ and $\lim_n |w_n| = \infty$ such that
\[ \lim_{n \to \infty} \frac{- \log P(w_n)}{|w_n|} = 0. \]

**IV. Power function of nonoverlapping words**

**Tests**

In [4], [15], CLT for the occurrences of words is shown.
\[ P\left(\frac{N_w - E(N_w)}{\sqrt{V(N_w)}} < x\right) \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}x^2} dx, \]

where $w$ is a nonoverlapping word,
\[ E(N_w) = (n - |w| + 1)P(w) \]
and
\[ V(N_w) = E(N_w) + (n - 2|w| + 2)(n - 2|w| + 1)P^2(w) - E^2(N_w). \]

Let
\[ N'_w := \sum_{i=1}^{\lfloor n/|w|\rfloor} I_N^{(i+1)-|w|-1} = w. \]

$N'_w$ obeys binomial law if the process is i.i.d. We call $N_w$ sliding block sampling and $N'_w$ block-wise sampling.

As an application of CLT approximation, we compare power functions of sliding block sampling $N_w$ and block-wise sampling $N'_w$.

We consider the following test for sliding block sampling: Let $E_0 = E(N_w)$ and $V_0 = V(N_w)$ if $P(w) = \theta$. Null hypothesis: $P(w) = \theta^*$ vs alternative hypothesis $P(w) < \theta^*$. Reject null hypothesis if and only if $N_w < E_0 - 5\sqrt{V_0}$. The likelihood of the critical region is called power function, i.e.,
\[ P\text{ow}(\theta) := P_0(N_w < E_0 - 5\sqrt{V_0}) \quad \text{for } \theta \leq \theta^*. \]

From (6) power function will be large if the variance is small.

We construct a test for block-wise sampling: Null hypothesis: $P(w) = \theta^*$ vs alternative hypothesis $P(w) < \theta^*$. Reject null hypothesis if and only if $N'_w < E'_0 - 5\sqrt{V'_0}$, where $E'_0 = [n/|w|]\theta$ and $V'_0 = [n/|w|]\theta(1 - \theta)$.

The following table shows powers of sliding block sampling and block wise sampling at $\theta = 0.2, 0.18, 0.16$ under the condition that alphabet size is 2 (binary data), $\theta^* = 0.25, |w| = 2$, and $n = 500$.

| $\theta$  | 0.2   | 0.18  | 0.16  |
|-----------|-------|-------|-------|
| Power of Sliding block | 0.316007 | 0.860057 | 0.995681 |
| Power of Block wise | 0.000295 | 0.002939 | 0.021481 |

Figure 1 shows the graph of power functions for sliding block words test and block-wise sampling.

**V. Experiments on statistical tests for pseudo random numbers**

In [13], a battery of statistical tests for pseudo random number generators are proposed, and chi-square test is recommended to test the pseudo random numbers with $N_w$ for nonoverlapping word $w$. 
In this section, we apply Kolmogorov Smirnov test to the empirical distribution of pseudo random numbers with the true distribution of nonoverlapping words \( w \). Let sample size \( n = 1600 \) and \( l = 1 \) in \( 3 \) and null hypothesis \( P \) be fair-coin flipping. For each nonoverlapping words \( w = 10 \) and \( n = 11110 \), we consider the three distributions, 1) true distributions of \( N_w \) (eq. \( 3 \) in Theorem 1), 2) binomial distributions \( \left( \begin{array}{c} n \\ k \end{array} \right) p^k (1-p)^{n-k}, \ p = 2^{-|w|}, \ k = 1, \ldots, n, \) and 3) empirical distributions of \( \sum_{i=1}^{n} I_{X_i+w−1=i=w} \) generated by Monte Carlo method with BSD RNG random, 20000 times iteration of random sampling. Figure 2 shows the graph of the three distributions for \( w = 11110 \).

The expectation of binomial distribution is \( pn \), which is almost same to the expectation of \( N_w \). However the variance of binomial distribution and \( N_w \) are different. Sliding block sampling \( N_w \) have strong correlations even if the process is i.i.d. For example, if a nonoverlapping word has occurred at some position, then the same nonoverlapping word do not occur in the next position. From Figure 2 we see that binomial distributions have large variance compared to the true distributions. This is because, in the binomial model, the correlations of words are not considered. We see that the binomial model approximations of the distributions of the words are not appropriate.

Figure 2 shows that the empirical distributions (Monte Carlo method) is different from the true distribution. We see that BSD RNG random do not simulate the sliding block sampling correctly.

\[
F(t) = \sum_{i=1}^{n} I_{X_i+w−1=i=w} \approx \frac{1}{2} \lambda \approx \frac{1}{2} \frac{32 \cdot 10^8}{200000} = 0.001376
\]

\[
P_{0 \leq x < \infty} |F_t(x) - F(x)| \leq 0.001376
\]

\[
P_{0 \leq x < \infty} |F_t(x) - F(x)| \leq 0.001409
\]

\[
P_{0 \leq x < \infty} |F_t(x) - F(x)| \leq 0.843306
\]

\[
P_{0 \leq x < \infty} |F_t(x) - F(x)| \leq 0.822066
\]

VI. Computation of the distribution of words in human DNA

Let \( A = \{a, b, c, d\} \), \( |X| = 32 \cdot 10^8 \), and the probability of each letter is \( 1/4 \). \( X \) is the human DNA size and \( P(w) = 4^{-|w|} \). If \( |w| \geq 14 \), our algorithm can compute the exact distribution of \( w \) within few seconds with desktop computer and we identified that the exact distribution \( 3 \) is numerically almost same to Poisson distribution, \( \text{Po}(N_w = k) = e^{-\lambda} \frac{\lambda^k}{k!} \) and \( \lambda = E(N_w) \). In case that \( |w| \) is small, the exact distributions will be well approximated with CLT.

VII. Proofs

Proof of Theorem 1. For simplicity we prove the theorem for \( l = 1 \). The proof of the general case is similar. Let \( m = |w| \). Since \( w \) is nonoverlapping, the number of possible allocations such that \( k \) times appearance of \( w \) in the string of length \( n \) is

\[
\binom{n - mk + k}{k}
\]

This is because if we replace each \( w \) with additional extra symbol \( \alpha \) in the string of length \( n \) then the problem reduces to choosing \( k \) \( \alpha \)'s among the string of length \( n - mk + k \). Let

\[
A(k) = \binom{n - mk + k}{k} P^k(w). \tag{7}
\]

\( A \) is not the probability of \( k \) \( w \)'s occurrence in the string, since we allow any letters in the remaining place except for the appearance of \( w \). For example \( A \) may count the event that \( w \) appear more than \( k \) times. Let \( B(t) \) be the probability
that nonoverlapping words $w$ appear $k$ times. We have the following identity,

$$A(k) = \sum_{k \leq t} B(t) \binom{t}{k}. $$

Let $F_A(z) := \sum_k A(k) z^k$ and $F_B(z) := \sum_k B(k) z^k$ be generating functions for $A$ and $B$ respectively. Then

$$F_A(z) = \sum_k z^k \sum_{t \leq k} B(t) \binom{t}{k}$$

$$= \sum_t B(t) \sum_{k \leq t} \binom{t}{k} z^k$$

$$= \sum_t B(t) (z + 1)^t$$

$$= F_B(z + 1).$$

In the above second equality, we changed the order of the sum. We have

$$F_B(z) = F_A(z - 1)$$

$$= \sum_{k : n - mk \leq 0} \left( n - mk + k \right)^k (-1)^k P^k(w)$$

$$= \sum_{k,t : n - mk \leq 0, t \leq k} \left( n - mk + k \right)^k (-1)^{k-t} P^k(w)$$

$$= \sum_t \sum_{k : t \leq k, n - mk \geq 0} (-1)^{k-t} \left( n - mk + k \right)^k (-1)^{k-t} P^k(w),$$

and (3).

Proof of Theorem 3. For simplicity let $Y_i = I_{X_{i+|w|-1}} = 1$. Since $w$ is nonoverlapping, $Y_i Y_j = Y_i$ if $i = j$. $Y_i Y_j = 0$ if $\{i, i + 1, \ldots, i + |w| - 1\}$ \(\cap\) $\{j, j + 1, \ldots, j + |w| - 1\} \neq \emptyset$. We say that $\{i, i + 1, \ldots, i + |w| - 1\}$ is the coordinate of $Y_i$. We say that a subset of $\{Y_i\}_{1 \leq i \leq n - |w| + 1}$ is disjoint if their coordinate are disjoint.

Let $Y_{i,j} = Y_i$ for all $1 \leq j \leq t$. Then $(\sum_i Y_i)^t = \prod_{j=1}^t \sum_i Y_{i,j}$. Note that $E(\sum_{j=1}^t Y_{i,j}) = P^t(w)$ if and only if there is a disjoint set $Y_{n(j),j}, 1 \leq j \leq s$ such that $\prod_{j=1}^t Y_{i,j} = \prod_{j=1}^s Y_{n(j),j}$. Observe that the number of possible combination of disjoint sets of $Y_{n(j),j}, 1 \leq j \leq s$ such that $\prod_{j=1}^t Y_{i,j} = \prod_{j=1}^s Y_{n(j),j}$ is $A_t(s) \binom{n-s|w|+s}{s}$. Note that there is no disjoint sets of $Y_{n(j),j}, 1 \leq j \leq s$ if $s > \max\{t \in \mathbb{N} | n - t|w| \geq 0\}$. From the linearity of the expectation, we have the theorem.

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