Hydro-gravitational fragmentation, diffusion and condensation of the primordial plasma, dark-matter and gas

Carl H. Gibson

Departments of Mechanical and Aerospace Engineering and Scripps Institution of Oceanography, University of California, San Diego, CA 92093-0411
cgibson@ucsd.edu

ABSTRACT

The first structures were proto-voids formed in the primordial plasma. Viscous and weak turbulence forces balanced gravitational forces when the scale of causal connection $L_H \equiv ct \approx L_{SV} \equiv (\gamma \nu/\rho G)^{1/2} \approx L_{ST} \equiv \epsilon^{1/2}/(\rho G)^{3/4}$ at time $t \approx 30,000$ years ($10^{12}$ s), with $c$ the speed of light, $\gamma \approx 1/t$ the rate of strain, and $\nu$ the kinematic viscosity, where $L_{SV}$ and $L_{ST}$ are viscous and turbulent Schwarz scales of hydro-gravitational theory (Gibson 1996). The photon viscosity $\nu \approx 4 \times 10^{26}$ m$^2$ s$^{-1}$ allows only weak turbulence from the Reynolds number $Re_H \equiv c^2 t/\nu \approx 200$, with fragmentation at $\rho L_{SV}^3 \approx 10^{16} M_\odot$ to give proto-supercluster voids, buoyancy forces, fossil vorticity turbulence, and strong sonic damping. The expanding, cooling plasma continued fragmentation to proto-galaxy-mass $\approx 10^{12} M_\odot$, with $\rho \approx 10^{-17}$ kg m$^{-3}$ and $\gamma \approx 10^{-12}$ s$^{-1}$ preserved as fossils of the weak turbulence and first structure. Turbulence fossilization by self-gravitational buoyancy explains the $\delta T/T \approx 10^{-5}$ cosmic microwave background temperature fluctuations, not sonic oscillations in cold-dark-matter fragments. After plasma to gas transition at $t \approx 300,000$ years ($10^{13}$ s), gas fragmentation occurred within proto-galaxies at $L_J \approx 10^4 L_{SV}$ and $L_{SV} \approx L_{ST}$ scales to form proto-globular-star-cluster (PGCs) clouds of $10^{12}$ small-planetary-mass primordial-fog-particles (PFPs). Dark PGC clumps of frozen PFPs persist as inner-galaxy-halo dark matter, supporting Schild’s 1996 quasar-microlensing interpretation. Non-baryonic dark matter, with $D \gg 10^{28}$ m$^2$ s$^{-1}$, diffused into the plasma proto-cluster-voids and later fragmented as outer-galaxy-halos at diffusive Schwarz scales $L_{SD} \equiv (D^2/\rho G)^{1/4}$, indicating $m \approx 10^{-35}$ kg weakly-collisional fluid particles. Observations (Gibson & Schild 2003) support the theory.

1Center for Astrophysics and Space Sciences, UCSD
1. Introduction

We consider the hydrodynamic evolution of the hot big bang expanding universe after mass-energy equality to determine when gravitational forces were first able to form structure under the influence of viscous and turbulent forces. All flows of plasmas and gases with large $Re \equiv \delta v \times L/\nu \geq Re_{cr}$ are unstable to the formation of turbulence according to the 1883 Reynolds number criterion for transition, where $Re_{cr}$ is a finite critical value of $Re$ above which laminar flows are impossible, $\delta v$ is the velocity difference on scale $L$ and $\nu$ is the kinematic viscosity of the fluid. From the first universal similarity hypothesis for turbulence (Kolmogorov 1941), the universal critical Reynolds number value $Re_{cr} \approx 25 - 100$ applies to the Hubble flow as it does for all others. Gluon-neutrino-photon viscosity values $\nu \approx c^2t$ before mass-energy equality at 25,000 years give subcritical $Re \approx 1$. Cosmic microwave background anisotropy extended Self Similarity (ESS) coefficients (Bershadskii & Sreenivasan 2002) closely matching those for high Reynolds number turbulence are attributed to fossils of pre-inflationary turbulence (Gibson 2001) at temperatures $10^{28} - 10^{32}$ K too hot for large lepton viscosities to exist that might otherwise prevent big bang turbulence. Predictions of spectral forms and other first and second order turbulent flow parameters from Kolmogorovian universal similarity theories for turbulence and turbulent mixing have been widely validated in numerous atmospheric, oceanic and laboratory flows and numerous fluids (Gibson 1991). No experimental counterexamples exist, either for the Reynolds number turbulence transition criterion or Kolmogorovian universal similarity at low order. Linear stability theories suggesting the possibility of steady inviscid flows (Rayleigh 1880) have been recognized as unreliable for real fluids since the (Prandtl 1921) discovery of viscous instabilities and because much larger values of $Re_{cr}$ are predicted by such theories than observed in laboratory experiments and numerical simulations (White 1991).

Prior to the 1993 discovery that the anisotropies $\delta T/T$ of the cosmic microwave background temperature are very small ($\approx 10^{-5}$) it was consistently assumed by all authors that $Re$ values of the expanding universe would be supercritical ($\gg 100$), so that both the plasma and the subsequent gas would be strongly turbulent with primordial turbulence the crucial factor in all subsequent gravitational structure formation. Density fluctuations produced and mixed by the turbulence would trigger gravitational collapse to form structures such as stars and galaxies at mass scales determined by the primordial turbulence. From energy arguments vol Weizsacker 1951 showed the Jeans 1902 linear acoustical criterion for gravitational
instability in ideal fluids fails in strongly turbulent flows. He proposed that Kolmogorov’s incompressible turbulence expression \( \delta v \sim L^{1/3} \) for velocity differences \( \delta v \) between points separated by distances \( L \) should be used to compute the turbulent kinetic energy of a possibly unstable gas or plasma cloud, asserting that the turbulent kinetic energy of the cloud should be less than the gravitational potential energy as the criterion for gravitational instability in such clouds. Chandrasekhar 1951 also rejected the Jeans 1902 criterion for the gravitational instability of strongly turbulent flows but overlooked Kolmogorov’s theory in any form and simply added a turbulence pressure \( p_T \sim \rho (\delta v)^2 \) to the fluid pressure \( p \) in the expression for Jeans’s length scale \( L_J \)

\[
L_J \equiv V_S/(\rho G)^{1/2} \approx (p/\rho^2 G)^{1/2},
\]

where \( \rho \) is the density, \( G \) is Newton’s gravitational constant and \( V_S \) is the sound speed, to give a Chandrasekhar turbulent Jeans scale \( L_{JCT} \equiv [(p + p_T)/\rho^2 G]^{1/2} \). Star formation rates in the cold molecular clouds of the Galaxy disk are about 50 times less than expected from Eq. 1, presumably because \( L_{JCT} \geq L \geq L_J \), where \( L \) is the size of the cloud, Scheffler and Elsasser 1988, p438. Doppler broadened molecular absorption lines give strong evidence of Kolmogorovian turbulence in such clouds (Falgarone and Phillips 1990). A dissipation rate \( \varepsilon \approx 10^{-6} \) m\(^2\)s\(^{-3}\) is estimated from the third order velocity structure function measured in the Ursa Major cirrus cloud (Miville-Deschenes et al. 1999), giving \( L_{ST} \approx 8 \times 10^{18} \) m (\( \rho \approx 10^{-19} \) kg m\(^{-3}\)) much larger than the cloud size so that star formation is prevented (see Table 1).

The hydro-gravitational theory (HGT) of gravitational structure formation (Gibson 1996) abandons the Jeans 1902 theory in its entirety; not only for strongly turbulent flows but for flows that are weakly turbulent or nonturbulent. \( L_J \) in Eq. 1 should not be interpreted as either the minimum scale or maximum scale of gravitational instability as proposed by Jeans 1902. Such misinterpretations have resulted in the dark matter paradox. For a self gravitating ideal gas of nearly uniform density \( \rho \) and temperature \( T \), \( L_J \) represents the maximum scale of acoustical pressure and temperature equilibration \( L_{IC} \equiv (RT/\rho G)^{1/2} \), where \( R \) is the gas constant and \( p \) is the pressure (Gibson and Schild 1999ab). Such a field of nearly uniform plasma with known properties formed after the big bang and turned to gas at 300,000 years. From HGT, non-acoustic density perturbations in the primordial plasma and gas are absolutely unstable to structure formation, and viscous or weakly turbulent fluid forces at \( L_{ST} \approx L_{SV} \approx L_K \equiv (v^3/\varepsilon)^{1/4} \approx L_H \equiv ct \), or diffusion at \( L_{SD} \) determine the smallest scales of gravitational instability, not \( L_J \), where \( L_K \) is the Kolmogorov scale and \( L_H \) is the Hubble scale of causal connection. In the hot primordial plasma \( L_J \geq L_H \), so by the Jeans 1902 criterion no structure could form. Cold-dark-matter (CDM) non-baryonic fluid was invented with small \( L_J \) values to permit gravitational structure formation consistent with observations (Padmanabhan 1993). However, the necessarily strong diffusivity \( D_{CDM} \gg c^2t \) of the
weakly collisional non-baryonic dark matter in the plasma epoch prevents its condensation and rules out CDM models (Gibson 2000) because \((L_{SD})_{CDM} \gg L_H\) in the plasma epoch.

To correct the Chandrasekhar 1951 expression, the turbulent pressure \(\sim \rho(\delta v)^2\) should be substituted rather than added to \(p\) in Eq. 1 and the complete Kolmogorov 1941 expression \(\delta v \approx (\varepsilon L)^{1/3}\) should be substituted for \(\delta v\). Solving for the critical length scale at which inertial forces match gravitational forces gives

\[
L_{ST} \equiv \varepsilon^{1/2}/(\rho G)^{3/4},
\]

where \(L_{ST}\) is defined as the turbulent Schwarz scale (Gibson 1996) and \(\varepsilon\) is the viscous dissipation rate of the turbulence.

If the turbulence of the primordial plasma flow is weak, as indicated by the small CMB fluctuations, then viscous forces \(F_V \approx \rho\nu\gamma L^2\) determine the smallest scale of gravitational instability, balancing gravitational forces \(F_G \approx \rho^2 GL^4\) at the viscous Schwarz scale \(L_{SV}\), where

\[
L_{SV} \equiv (\nu\gamma/\rho G)^{1/2},
\]

\(\nu\) is the kinematic viscosity of the fluid, \(\gamma\) is the rate of strain, and \(\rho\) is the density. The turbulent Schwarz scale of Eq. 2 is closely related to the Ozmidov length scale \(L_R \equiv (\varepsilon/N^3)^{1/2}\) of stably stratified turbulent flows, where the stratification frequency \(N\) has a physical significance similar to the inverse free fall time \((\rho G)^{1/2}\) and \(L_R\) is derived by matching turbulence forces with buoyancy forces to find the critical length scale. The viscous Schwarz scale of Eq. 3 near \(Re_{cr}\) is analogous to the buoyancy-inertial-viscous scale \(L_{BIV} \equiv (\nu/N)^{1/2}\) that arises in fossil turbulence theory (Gibson 1999a). Turbulence is strongly inhibited and rapidly fossilized by buoyancy forces in the ocean and atmosphere at \(L_R\) scales, and astrophysical turbulence is strongly inhibited and fossilized at \(L_{ST}\) scales in self gravitating fluids. Because kinetic and gravitational forces of a flat universe are closely matched at the horizon scale \(L_H\), it follows that whatever turbulence levels existed at the time of first structure formation (when, for the first time, \(L_{SV} \approx L_{ST} \leq L_H\)) would be rapidly damped by buoyancy forces and the horizon length, density, mass, and the hydrodynamic parameter (\(\varepsilon\) or \(\gamma\)) preserved by hydrodynamic fossils.

Silk and Ames 1972 suggest that the large size of \(L_J \gg L_H \equiv ct\) in the plasma epoch with sound speed \(V_S \approx c/3^{1/2}\) prevents gravitational condensation of plasma by the Jeans 1902 criterion. By their galaxy formation theory, strong turbulence produced density fluctuations that served as nuclei for galaxy formation at the time of photon decoupling when the sound speed \(V_S\) dramatically decreased by a factor of \(3 \times 10^4\). Other studies claiming that strong primordial turbulence should set the scale of galaxies include Gamov 1952, Ozernoi and Chernin 1968, Ozernoi and Chernin 1969, Oort 1970, and Ozernoi and Chebyshev 1971.
All such strong turbulence theories of structure formation were rendered moot by the 1993 measurements of very small temperature fluctuations $\delta T/T \approx 10^{-5} \approx \delta v/v \approx \delta p/p \approx \delta a/a$ in the cosmic microwave radiation (CMB) data from the 1989 COsmic Background Explorer (COBE) satellite, rather than values of $\delta v/v \approx 10^{-1} - 10^{-2}$ that would result from fully developed turbulence, where $a$ is the cosmic scale factor and $\delta (T, v, \rho, p, a)$ represent fluctuation magnitudes. A subcritical horizon scale Reynolds number $Re_H \equiv c^2 t/\nu \leq 10$ at the time $10^{13}$ s of plasma-gas transition requires an enormous kinematic viscosity $\nu \geq 10^{29}$ m$^2$/s to be subcritical, much larger than $\nu \approx 10^{25}$ m$^2$/s estimated for the primordial plasma then (Gibson 2000). For a terrestrial comparison, the kinematic viscosity of the Earth's upper mantle is $\nu \approx 10^{21}$ m$^2$/s from glacial rebound rates (Professor Robert Parker of SIO, personal communication). Implicitly it has been assumed in the astrophysics literature after these COBE observations that the Hubble flow of the expanding universe must somehow be intrinsically stable to turbulence formation, independent of Reynolds number. Textbooks on structure formation in the universe such as Padmanabhan 1993 make no mention of viscosity, diffusivity, turbulence, or Reynolds number in their discussions of the process. No reference in the literature has been found that attempts to justify this implicit (and unwarranted) assumption. An example of strong turbulence generated by the Hubble flow is shown in Figure 1. Powerful Hubble flow drag forces separate protosuperclusters, protoclusters, and protogalaxies as they form by gravitational fragmentation in the primordial plasma and early gas epochs according to HGT. Hubble flow galaxy Reynolds numbers of order $10^{12}$ shown in Figure 1 have decreased to values $\approx 10^4$ or less at present.

The assumption made by CDM hierarchical clustering cosmology models (CDMHCCs) that the Hubble flow is stable to the formation of turbulence is inconsistent with the universal similarity theory of turbulence, which is the basis of HGT. Strong turbulence in the plasma epoch is ruled out from HGT by the small values $\delta T/T \approx 10^{-5}$ of the CMB observations, so buoyancy forces resulting from gravitational structure formation must have dominated the damping of turbulence because viscous forces are inadequate and no other fluid forces exist. Turbulent transition cannot fail by lack of triggering perturbations since $\delta T/T$ fluctuations are observed at scales $L > ct$ in the CMB that can nucleate growth of vorticity and structure once they enter the horizon. Neither can it be argued that a lack of time prevents self gravitational or fluid mechanical nonlinearity. Once $L_K \leq L_H$ at turbulence transition the eddy overturn time is $t$. The decreasing viscous stresses in the baryonic component permit fragmentation of supercluster to galaxy masses in the plasma epoch so that the hierarchical clustering of subgalactic scale CDM halos to form these structures in the gas epoch is unnecessary, even if such small CDM halos were physically possible (they are not). Because the nonbaryonic dark matter is necessarily strongly diffusive, such small CDM halos are excluded by HGT (Gibson 2000). Observations of galaxy-QSO correlations and
discordant cluster red shifts rule out CDMHCCs (Gibson & Schild 2003). CDMHCCs are also excluded by observed density distributions near galaxy cluster cores that fail to match universal forms computed by numerical simulation (Sand et al. 2002).

In the following §2 we consider whether an inviscid expanding universe is stable or unstable to the formation of turbulence. If it is unstable according to the conventional Reynolds number criterion, what constraints on viscosities and structure formation in the plasma epoch can be inferred from observed CMB anisotropies? We then examine the hydrodynamic parameters and structures to be expected from the (Gibson 1996) nonlinear gravitational structure formation theory during the plasma epoch, in §3, and in the early gas epoch, in §4. Conclusions are summarized in §5.

2. The absolute instability of inviscid flows

The instability of expanding flows is discussed in §23 of Landau and Lifshitz 1959. The equations of momentum conservation in a fluid may be written

$$\frac{\partial \vec{v}}{\partial t} = -\nabla B + \vec{v} \times \vec{\omega} + \nu \nabla^2 \vec{v} + \vec{F}_M + ...$$

(4)

where $B \equiv p/\rho + v^2/2 + \phi$ is the Bernoulli group of mechanical energy terms, $\vec{\omega} \equiv \nabla \times \vec{v}$ is the vorticity, $\vec{v} \times \vec{\omega}$ is the inertial vortex force that causes turbulence, $\nu \nabla^2 \vec{v}$ is the viscous force that damps it out, $\vec{F}_G = -\nabla \phi$ is the gravitational force and has been absorbed in $B$, $\phi$ is the gravitational potential energy per unit mass in the expression $\nabla^2 \phi = 4\pi \rho G$, $G$ is Newton’s constant, $\vec{F}_M$ is the magnetic force, and other forces have been neglected. Eq. 4 applies in a gas or plasma when a sufficient number of particles are assembled, so that the particle separation $L_P$ and the collision distance $L_C$ are much smaller than the size $L$ of the assemblage or the scale of causal connection $L_H \equiv ct$, where $c$ is the speed of light and $t$ is the age of the universe. Turbulence develops whenever the inertial-vortex force of the flow is larger than the other terms; that is, if the Reynolds number $Re \equiv (\vec{v} \times \vec{\omega})/(\nu \nabla^2 \vec{v})$, Froude number $Fr \equiv (\vec{v} \times \vec{\omega})/\vec{F}_G$, and all other such dimensionless groups exceed critical values.

In Landau and Lifshitz 1959 §23 an exact solution of Eq. 4 for an incompressible viscous fluid attributed to G. Hamel 1916 (usually termed the Jeffrey-Hamel flow, White 1991) gives multiple maxima and minima for expanding flows between plates. This solution is used to illustrate the relative instability of expanding flows compared to converging flows. The Jeffrey-Hamel converging flow solution approaches the solution for converging ideal fluid flow and thus might appear to be stable to the formation of turbulence at high Reynolds number because the turbulent intensity $\delta v/v$ decreases along a streamline as $v$ increases. Converging sections are used in wind and water tunnels before test sections to decrease
the turbulent intensity, but the turbulent viscous dissipation rate $\varepsilon$ and turbulent velocities actually increase in such flows (Batchelor 1953). Can steady inviscid flows of any kind be stable?

What about the stability of the expanding universe which is not incompressible but is a uniform expansion with rate-of-strain $\gamma \approx 1/t$, where $\gamma$ is generally termed the “Hubble constant” and the expansion is termed the “Hubble flow”? Instead of decreasing along a streamline with $1/x$ as for the incompressible diverging Jeffrey-Hamel flow, the speed $v \approx \gamma x$ increases with distance $x$. Does this mean the expanding Hubble flow is stable, similar to the converging Jeffrey-Hamel flow where the speed also increases with distance? Does this mean that the small CMB temperature anisotropies simply reflect the fundamental stability of a Hubble flow, and does not imply that large viscous or buoyancy forces must have been present in the plasma epoch? Are self gravitating fluids fundamentally different from stratified natural fluids in that the first turbulence of the Hubble flow is caused by gravitational forces rather than inhibited by them as in stratified flows?

According to the further analysis and discussion in Landau and Lifshitz 1959, in §27 titled “The onset of turbulence”, steady inviscid flows are absolutely unstable. Thus, all flows should develop turbulence at high enough Reynolds numbers, including the diverging Hubble flow of the expanding universe. In their derivation, maximum amplitudes of Fourier modes $|A|_{max} \sim (Re - Re_{crit})^{1/2}$ are expressed as functions of their departures from critical Reynolds numbers $Re_{crit}$ and it is shown that the individual modes grow to finite values with increasing Reynolds number, but with an ever increasing number of modes as $Re \to \infty$. Landau-Lifshitz admit that prediction of the mode amplitudes is mathematically difficult and that such stability analysis has had limited success in predicting the transition to turbulence except to confirm the 1883 Reynolds criterion, for which there is no experimental counterexample. As we have seen, in apparent counterexamples such as the Jeffrey-Hamel converging incompressible flow the increasing velocity along streamlines masks the developing turbulence, but does not prevent it.

The absolute instability of steady inviscid flows can be understood from the first two terms of Eq. 4, shown in Eq. 5. Such a flow must be irrotational to remain steady with $\partial \vec{v}/\partial t = 0$ and $B$ constant. Otherwise the vorticity $\vec{\omega}$ would produce inertial vortex forces $\vec{v} \times \vec{\omega}$ that would spread the rotational region indefinitely to larger and smaller scales by undamped turbulent diffusion. If a variation in speed occurs along one of the streamlines, then accelerations

$$\partial \vec{v}/\partial t = -\nabla B$$

develop that amplify any perturbations in $v$ with increasing time. Increasing $v$ requires increases in both $B$ and its gradient, and decreasing $v$ decreases both $B$ and its gradient.
From Eq. 5, positive speed perturbations increase \(-\nabla B\) and cause speed increases, and negative speed perturbations cause decreases in \(-\nabla B\) and cause speed decreases. Vorticity \(\vec{\omega} > 0\) develops and forms turbulence, which will grow in size and kinetic energy. This positive feedback is independent of the continuity equation or the equation of state for the fluid. Finite length scale perturbations of any of the hydrophysical parameters \((v, p, \rho)\) in a steady, inviscid, irrotational flow will cause local perturbations in the vorticity on the same finite scale, with resulting formation and growth of turbulent inertial vortex forces \(\vec{v} \times \vec{\omega}\) and thus turbulence at larger and smaller scales, drawing energy from the assumed variations of \(v\) along streamlines. Even the extreme case of steady flow is unstable to a vorticity perturbation, since the rotational region of the \(\vec{\omega}\) perturbation without viscous damping will spread its vorticity and kinetic energy, and thus turbulence, to indefinitely larger volumes by turbulent diffusion.

We conclude that steady inviscid flows are absolutely unstable, confirming the 1959 Landau-Lifshitz result and the conventional Reynolds criterion for turbulence formation. Viscosity is not necessary to the formation of turbulence, only its evolution. From the vorticity conservation equation following a fluid particle in a fluid with variable density

\[
D\vec{\omega}/Dt = \partial\vec{\omega}/\partial t + (\vec{v} \cdot \nabla)\vec{\omega} = \vec{\omega} \cdot \tilde{\vec{e}} + (\nabla \rho \times \nabla p)/\rho^2 + \nu \nabla^2 \vec{\omega}
\]  

(6)

we see variations in the density of the fluid can produce vorticity if pressure and density gradients are not aligned, at rate \((\nabla \rho \times \nabla p)/\rho^2\), leading to unconstrained inertial vortex forces \(\vec{v} \times \vec{\omega}\) and thus turbulence. Vorticity is produced by vortex stretching at a rate \(\vec{\omega} \cdot \tilde{\vec{e}}\), where \(\tilde{\vec{e}}\) is the rate of strain tensor. Turbulence is defined as an eddylike state of fluid motion where the inertial vortex forces of the eddies are larger than any other forces that tend to damp the eddies out (Gibson 1999a). Turbulence always starts at the smallest possible scale permitted by viscous forces, and cascades to larger scales by a process of eddy pairing and entrainment by the turbulence of irrotational fluid (Gibson 1991). Fourier modal analysis fails to properly describe the formation of turbulence, gravitational structure formation, or small scale turbulent mixing at small Prandtl numbers. These failures result from sacrificing realistic physical models for mathematical convenience by considering the linear behavior of sine waves rather than the nonlinear behavior of finite-scale local perturbations (Gibson 1996).

What about cosmic drag? It is sometimes argued that turbulence is prevented by the expansion of the universe because momentum decreases as \(V(t) = V_0/a(t)\) from general relativity, where \(V_0\) is an initial velocity perturbation and \(a(t)\) is the cosmic scale factor which monotonically increases with time \(t\) as the universe expands. Although the momentum and velocity of a perturbation may decrease, the proper length scale of the perturbation \(L(t) = L_0 a(t)\) will increase, so that \(a(t)\) in the Reynolds number \(Re(t) \equiv VL/\nu\) will cancel. To first order, the Reynolds number after inflation and before mass-energy equivalence is
Re \approx 1 because V \approx c, L \approx ct, and \nu \approx c^2t. Before inflation, much larger Reynolds numbers were possible (Gibson 2000).

It is not true that simply because the initial perturbation of a nonlinear process is small that the process can be accurately described by linear theories. In particular, just because remnant density perturbations \(\delta\rho/\rho \approx 10^{-5}\) from big bang quantum gravitational chaos are small does not mean that their evolution can be accurately described by linear methods once they reenter the horizon. Decreasing the size of \(\delta\rho/\rho\) by a factor of \(10^{-5}\) increases the gravitational condensation time by less than a factor of two. Cold dark matter theories that suggest an acoustic peak in the CMB temperature spectrum are therefore questionable. The gravitational response to density perturbations is always nonlinear and requires nonlinear fluid mechanics for its description independent of the size of the perturbation or the predictions of the linear, acoustic theory of Jeans 1902.

What about energy conservation? Won’t pressure support or thermal support prevent gravitational condensation at scales smaller than \(L_J\)? Won’t continued gravitational collapse require a loss of thermal energy to prevent pressure stabilization, and won’t this require a spontaneous and highly efficient flow of heat from a cold object into a hot environment? These misconceptions are all part of Jeans’s 1902 legacy. Consider a volume of initially stagnant, constant density gas, smaller than the horizon, with mass perturbation \(M'\) suddenly placed near its center. This system is absolutely unstable to gravitational condensation or void formation, depending on whether \(M'\) is positive or negative. Gravitational acceleration starts immediately with radial velocity \(v_r \approx -GM'(t)/r^2\), and mass flux \(4\pi r^2 \rho v_r(t) \approx 4\pi \rho G M'(t)t = dM'(t)/dt\) independent of radius. Thus \(M'(t) = M'(0) exp[2\pi \rho G t^2] = M'(0) exp[2\pi (t/\tau_G)^2]\). The density, temperature, and dynamical pressure \(p/\rho + v^2/2\) remain constant during the gravitational free fall process except in the small space-time region of the nonacoustic density nucleus at \(r \ll L_J\) and \(t \approx \tau_G \equiv (\rho G)^{-1/2}\) (Gibson 1999a, Gibson & Schild 1999a). Everything happens at once when \(t \to \tau_G\). Since it takes \(t \approx \tau_G\) for information to propagate a distance \(L_J\), no pressure support mechanism is possible to prevent the self gravitational collapse or void formation at nonacoustic density perturbations.

In any real fluid, the Hubble flow is unstable at all scales where the Reynolds number exceeds a universal value \(Re_{crit} \approx 100\). Thus, \(Re \approx \delta v \times x/\nu \approx \gamma x^2/\nu \approx 100\) at a critical length scale \(x_{crit} \approx 10(\nu/\gamma)^{1/2}\). The viscous dissipation rate \(\varepsilon \approx \nu \gamma^2\), so

\[
x_{crit} \approx 10(\nu^{3/2}/\varepsilon^{1/2})^{1/2} \approx 10L_K,
\]

where

\[
L_K \equiv (\nu^3/\varepsilon)^{1/4} = (\nu/\gamma)^{1/2}
\]
is the 1941 Kolmogorov length scale. Turbulence always begins at scales of \( \approx 10L_K \) and is inhibited at smaller scales by viscous forces. These small eddies pair, pairs of eddies pair with other eddy pairs, and so forth. Irrotational (and therefore nonturbulent) fluid is entrained into the interstices of the turbulent domain as ideal flows, is made turbulent at Kolmogorov scales by viscous forces, and supplies the kinetic energy of the turbulence. We now use these results to examine the formation of turbulence, and its inhibition, during the plasma epoch before \( 10^{13} \) s (300,000 years).

3. The plasma epoch

What about the formation of turbulence in the plasma epoch? Since the Hubble flow is unstable to the formation of turbulence, either viscous forces or buoyancy forces, or both, must have been present to prevent strong turbulence. When did the first turbulence form? What was the viscosity of the plasma required to prevent turbulence?

From COBE to WMAP, numerous experiments have been undertaken to resolve the small scale fluctuations of the CMB. Super-horizon contributions to the \( \delta T \) variance are approximately constant with a Sachs-Wolfe plateau of about \( 2 \times 10^{-5} \) K for angular separations \( \theta \) greater than about 1-2 degrees corresponding to the horizon scale \( L_H \approx 3 \times 10^{21} \) m existing at this plasma-gas transition time \( 10^{13} \) s (Lineweaver 1999). From measurements at smaller sub-horizon scales a sonic, or doppler, peak of about \( 8 \times 10^{-5} \) K at \( \theta \approx 0.5 \) degrees and smaller-amplitude, smaller-scale, harmonics are attributed to undamped sound waves in the plasma sloshing in CDM clump potential wells in the gravitational potential.

This sonic peak explanation of the CMB is questionable for at least three reasons: 1. the postulated CDM fluid with \( L_{SD} > L_H \) (see Eq. 11 below) is too diffusive to condense; 2. no sound source of any kind exists, and certainly not the non-turbulent super-powerful sound source that would be required to match the observations; 3. even if a super-powerful source of sound could be identified, the sound would be rapidly damped by viscous forces because the sonic attenuation coefficient \( \alpha \approx \nu/V_S \lambda^2 \) is \( \gg \lambda^{-1} \) since \( \nu \approx V_S L_C \) and \( L_C \gg \lambda \) for all the relevant sonic wavelengths \( \lambda \).

Reason 3. is why sonic fluctuations of temperature in the relatively noisy atmosphere of the earth rarely exceed the 1 db reference level \( \delta T/T \approx 10^{-10} \), Pierce and Berthelot 1990, and why whales near Japan can be heard from California but eagles cannot. Time \( t_{FS} \approx 10^{12} \) s (30,000 years) is indicated as the time of first structure formation since this is the time when the increasing horizon mass \( \rho(ct)^3 \) just matches the observed mass of superclusters \( \approx 10^{46} \) kg (Gibson 1997b). This supercluster mass is \( 10^{-6} \) times the present
horizon mass \((ct)^3 \rho_{\text{crit}} = 10^{52}\) kg since the observed supervoid size is \(10^{-2} \times L_H\). The observed globular star cluster density \(\approx 10^{-17}\) kg m\(^{-3}\) just matches the baryonic density existing at \(t \approx 10^{12}\) s, also indicating \(t_{FS} \approx 10^{12}\) s as the time when the plasma first began fragmentation. Voids formed at that time should expand for a brief period as rarefaction waves with velocities limited by the sound speed \(V_S = c/3^{1/2}\), giving a structural rather than sonic peak in the range \(0.6 > \theta_{SP} > 0.1\) degrees, as observed, with a monotonic decrease of the \(\delta T\) power spectrum reflecting fragmentation to galactic scales, possibly with acoustical harmonics from the rarefaction waves. Further fragmentation at smaller and smaller scales limits the amplitude of \(\delta \rho/\rho\) to small values as \(M_{SV}\) decreases toward proto-galaxy masses in the cooling, expanding plasma.

To prevent turbulence at the horizon scale at decoupling requires a viscosity \(\nu_{\text{crit}} \approx c^2 t/100 \approx 10^{28}\) m\(^2\) s\(^{-1}\), which is too large for the baryonic component by any known mechanism. Setting \(x_{\text{crit}} = L_H = 10L_K\) in Eqs. 7 and 8 with \(\gamma = 1/t\) gives a value of \(\nu = (ct/100)^2 \gamma = 9 \times 10^{26}\) m\(^2\) s\(^{-1}\) for our estimated \(t_{FS} \approx 10^{12}\) s. This large value of \(\nu\) is only slightly larger than that required to prevent turbulence at the time of first structure. Once gravitational structure formation begins, buoyancy forces will inhibit turbulence.

Densities were larger at this earlier time (30,000 yr) so mean free paths for collisions \(L_C \approx (\sigma n)^{-1}\) were shorter, where \(\sigma\) is the collision cross section and \(n\) is the particle density. The physical mechanism of viscous stress in the plasma epoch is photon collisions with the free electrons of the plasma (Silk & Ames 1972, Thomas 1930). The electrons then drag along the protons and alpha particles of the primordial plasma to maintain electrical neutrality. The kinematic viscosity is then

\[
\nu \approx L_C \times v = c/\sigma_T n_e
\]

where \(\sigma_T = 6.65 \times 10^{-29}\) m\(^2\) is the Thomson cross section for scattering and \(n_e\) is the number density of the free electrons. Substituting \(n_e \approx 10^{10}\) m\(^{-3}\) for the electron number density at \(t = 10^{12}\) s (Weinberg 1972) gives \(\nu \approx 4 \times 10^{26}\) m\(^2\) s\(^{-1}\), which is close to our estimated minimum \(\nu\) value required to inhibit turbulence. The collision distance \(L_C \approx 1.5 \times 10^{18}\) m is less than the horizon scale \(L_H = 3 \times 10^{20}\) m, so the assumption of collisional fluid dynamics in Eq. 9 is justified. The viscous dissipation rate \(\varepsilon \approx \nu \gamma^2 \approx 4 \times 10^{-2}\) m\(^2\) s\(^{-3}\) gives a Kolmogorov scale \(L_K \approx 2 \times 10^{20}\) m from Eq. 8. Since \(10L_K \geq L_H\), the Hubble flow of plasma should be viscous and laminar or weakly turbulent.

The baryonic density at \(t \approx 10^{12}\) s was \(\rho \approx 2 \times 10^{-17}\) kg m\(^{-3}\) (Weinberg 1972). The strain rate at turbulence fossilization was \(10^{-12}\) s\(^{-1}\). Thus, from Eq. 3

\[
L_{SV} \approx (10^{-12} \times 4 \times 10^{26}/2 \times 10^{-17} \times 6.672 \times 10^{-11})^{1/2} \approx 5 \times 10^{20} \text{ m},
\]

(10)
approximately matching the horizon scale \( L_H = 3 \times 10^{20} \) m. The horizon scale baryonic mass \( M_H \equiv L_H^3 \times \rho = (ct)^3 \rho = 5 \times 10^{44} \) kg is close to the baryonic mass of superclusters \( (M_{SC} \approx 10^{46} \) kg includes the non-baryonic component), so these are suggested as the first structures of the universe, formed by fragmentation when the viscous Schwarz scale first matches the horizon scale, Gibson 1996, 1997ab, 1999ab.

Proto-superclusters formed by fragmentation rather than condensation because void formation is augmented by the expansion of the universe but condensation is inhibited. Thus proto-supercluster-voids expand in the plasma epoch while the proto-superclusters between these voids also grow, but more slowly, by internal fragmentation, preserving the density of the fragments. Further fragmentation at \( L_{SV} \) scales down to protogalaxy masses with little change in the baryonic density due to fossil density turbulence formation is proposed by HGT (Gibson 1996). Turbulence formation is inhibited at every stage of the plasma epoch by a combination of viscous and buoyancy forces, and there is no energy source for sound other than the gravitational void formation. Temperature fluctuations observed in the CMB are proposed as fossils of big bang turbulence and fossils of the first structure formation, Gibson 2000.

In contrast, Silk 1989 Fig. 10.1 traces the evolution of an adiabatic galaxy mass pressure fluctuation as it drops below the Jeans mass at a redshift of \( z \approx 10^8 \) and oscillates as an undamped sound wave in the necessarily inviscid plasma epoch with \( 10^4 \) density contrast until decoupling at \( z \approx 10^3 \). It seems unlikely that any such loud sounds (\( > 100 \) db) could start at that time, less than a week after the big bang. If somehow they were started they would be rapidly damped, within another week, by the large photon viscosity, not to mention damping by the expansion of the universe (cosmic drag). Sonic pressure fluctuations \( p \approx p_0 \exp[-\alpha x] \), where \( p_0 \) is the initial pressure, \( x \) is the direction of propagation. The sonic attenuation coefficient \( \alpha \approx \nu \omega^2/V_S^3 = \nu/V_S \lambda^2 \), where the frequency \( \omega = V_S/\lambda \), \( \lambda \) is the wavelength, and \( V_S \) is the sound speed (Pierce and Berthelot 1990). Thus, \( p/p_0 \approx \exp[-(\nu/V_S \lambda^2)x] \ll 1 \) for distance \( x \geq \lambda \) if \( \nu \geq V_S \lambda \), and this will be true since \( \nu \) increases with time as the universe density decreases and \( \lambda \leq ct \) is limited in size by the time \( t \) when the sound wave was created.

What about the non-baryonic dark matter (NB) required to make up the critical density of a flat universe? Its cross section \( \sigma_C \) for collisions with ordinary matter must be very small or it would have been detected based on the expression \( \sigma_C = m_p(GM/r)^{1/2}/\rho D_{NB} \), where \( m_p \) is the particle mass and \( D_{NB} \) is the diffusivity inferred from outer-halo dimensions \( r \) of galaxies or clusters of mass \( M \) (Gibson 2000). Thus such material must have large mean free paths for collisions and large diffusivities \( D_{NB} \equiv L_{NB} \times v_{NB} \) compared to \( D_B \) for baryonic matter since \( L_{NB} \gg L_B \) and \( v_{NB} \approx v_B \). From measurements of the mass profile of Abell
1689 by Tyson & Fischer 1995, Gibson 1999b estimates the non-baryonic dark matter of the dense galaxy cluster is \( D_{NB} \approx 10^{28} \text{ m}^2 \text{ s}^{-1} \) by setting the radius of curvature of the profile to \( L_{SD} \). Since neutrinos are now known to have mass, an obvious non-baryonic candidate is neutrinos, which have densities comparable to the density of photons and very small cross sections for collisions since they interact with baryonic matter mostly through the weak force. Large numbers of neutrinos were formed in nucleosynthesis and their unknown number of flavors and abilities to convert between flavors leaves their total mass a mystery. Assuming a neutrino collision cross section of \( \sigma_n \approx 10^{-40} \text{ m}^2 \) and number density \( n_n \approx 10^{20} \text{ m}^{-3} \) gives a mean free path \( L_n \) of 10\( ^{20} \) m, so collisional dynamics apply. Cross sections for light (\( \approx 10^{-35} \text{ kg} \)) particles like neutrinos give such small \( \sigma \) values, but \( 10^{-25} \text{ kg} \) particles like neutralinos give \( \sigma \approx 10^{-22} \text{ m}^2 \), much larger than \( \approx 10^{-46} \text{ m}^2 \) theoretical values or the \( \leq 10^{-42} \text{ m}^2 \) values excluded by experiments (Gibson 2000).

In the case of strongly diffusive matter in weakly turbulent flows, gravitational condensation is limited by a match between the diffusion velocity of an isodensity surface \( V_D \approx D/L \) and the gravitational free fall velocity \( V_G \approx L/\tau_G \), giving the diffusive Schwarz scale

\[ L_{SD} \equiv (D^2/\rho G)^{1/4} \tag{11} \]

where the diffusivity \( D_n \approx L_n \times c \approx 3 \times 10^{28} \text{ m}^2 \text{ s}^{-1} \). This gives \( L_{SD} \approx 10^{21} \text{ m} \) during the plasma epoch, much larger than any of the structures formed and larger than the horizon for part of the epoch. Any such nonbaryonic material would diffuse away from the protogalaxies and proto-superclusters as they fragment, to fill the voids between. Non-baryonic materials fragment as the last stage of gravitational structure formation to form protosuperhalos when the baryonic protosuperclusters separate by scales larger than \( L_{SD} \). This is contrary to cold dark matter models that require CDM condensation as the first rather than last stage of structure formation, producing, rather than being produced by, the baryonic structure.

The necessary condition for the diffusive Schwarz scale \( L_{SD} \) of Eq. 11 to determine the minimum scale of gravitational condensation is

\[ D \geq \nu \gamma \tau_G \tag{12} \]

for viscous flows. Since \( \gamma \tau_G \geq 1 \) and \( D \approx \nu \) for baryonic matter, the scale \( L_{SD} \) only applies to nonbaryonic matter. Substituting \( D_n \approx 3 \times 10^{28} \text{ m}^2 \text{ s}^{-1} \) and \( \rho \approx 10^{-23} \text{ kg} \text{ m}^{-3} \) for the density of a galaxy cluster gives \( L_{SD} \approx 3 \times 10^{22} \text{ m} \text{ (Mpc)} \) as the scale for gravitational fragmentation of the non-baryonic dark-matter halo of a small galaxy cluster with total mass \( \approx 4 \times 10^{14} \text{ kg} \).

Thus a proper description of structure formation in the primordial self-gravitational fluids of the early universe requires more than the linearized Euler equation with gravity.
and the density equation without diffusion or gravity, as assumed by (Jeans 1902). All the forces in the momentum Eq. 4 are needed except ($\vec{F}_M + ...$). The appropriate non-acoustic density conservation equation near density maxima and minima is

$$\frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho = D_{\text{eff}} \nabla^2 \rho,$$

(13)

where $D_{\text{eff}} \equiv D - L^2/\tau_G$ and $D$ is the molecular diffusivity of the density; that is, on scales $L \leq L_J$ so that the pressure adjusts rapidly, and on scales $L \geq L_{S_X \text{max}}$ (the maximum Schwarz scale) where gravity dominates fluid forces and molecular diffusion (Gibson 1999b). The density $\rho$ depends on temperature and species concentration variations and their diffusivities, and not simply the pressure as assumed by Jeans. The problem is similar to the turbulent mixing problem (Gibson 1968) except for the remarkable fact that for clouds of fluid with sizes $L_J \geq L \geq L_{S_X \text{max}}$, gravitational diffusivity takes over and the effective diffusivity $D_{\text{eff}}$ becomes negative. Thus, rather than reaching a local equilibrium between local straining and diffusion at the Batchelor length scale $L_B \equiv (D/\gamma)^{1/2}$ near density extrema as in turbulent mixing theory with a monotonic decrease toward ambient values, densities in the self-gravitational fluids of astrophysics increase to large values or decrease toward zero at these points due to gravitational instability (Gibson and Schild 1999a).

4. The gas epoch

From standard cosmology and the CMB observations, the initial conditions of the gas epoch are precisely defined. Little or no turbulence was present, as discussed previously, so the rate of strain of the fluid was larger than $\gamma \approx 1/t \approx 10^{-13} \text{ s}^{-1}$ existing at that time and smaller than the fossil vorticity turbulence value in the structures $\gamma_{FS} \approx 10^{-12} \text{ s}^{-1}$. The density of the protogalaxies cannot have been much different from the fossilized initial fragmentation density $\rho_{FS} \approx 10^{-17} \text{ kg m}^{-3}$ since there was insufficient time for collapse. The temperature at decoupling was $T_o \approx 3000 \text{ K}$. The composition was 75% H and 25% He by mass. Therefore the kinematic viscosity of the primordial gas was about $3 \times 10^{12} \text{ m}^2 \text{ s}^{-1}$, from $\mu \equiv \rho \times \nu$ in standard gas tables with a weighted average $\mu(T_o)$, with gas constant $R$ about $3612 \text{ m}^2 \text{ s}^{-2} \text{ K}^{-1}$, (Gibson 1999b). Viscous dissipation rates were only $\varepsilon \approx \nu \gamma^2 \approx 3 \times 10^{-14} \text{ m}^2 \text{ s}^{-3}$ so the Kolmogorov scale $L_K \approx 5 \times 10^{12} \text{ m}$ from Eq. 10 was a factor of $5 \times 10^8$ smaller than the horizon scale.

Fragmentation of the neutral gas protogalaxies occurred simultaneously at both the Jeans scale $L_J$ of Eq. 1 and the viscous Schwarz scale $L_{SV}$ of Eq. 3, where for the primordial gas conditions $L_J \approx 10^4 L_{SV} \gg L_{SV}$. The physical mechanism of this Jeans scale fragmentation is not the mechanism proposed by Jeans 1902. Temperatures in growing voids at scales
smaller than $L_J \approx (RT/\rho G)^{1/2}$ adjust by particle diffusion to remain constant at $T = p/\rho R$ as the gravity driven rarefaction waves of void formation propagate, where the term “void” indicates a density deficiency rather than $\rho = 0$. As the density decreases the pressure decreases. Particle speeds and temperatures are constant as long as the particle diffusion time $\tau_P \equiv L/(RT)^{1/2}$ is less than the gravitational free-fall time $\tau_G$; that is, for scales $L \leq L_J$. For scales $L$ larger than $L_J$ the diffusion time $\tau_P$ is larger than $\tau_G$, causing temperatures in these large voids to decrease as the voids grow because particle diffusion cannot maintain constant temperature and acoustical equilibrium. When this happens, radiation heat transfer from the warmer surroundings increases the temperature, and thus also the pressure, within the voids, and the increased pressure accelerates the void formation, isolating blobs of gas at some multiple of the Jeans scale to form PGCs.

Substituting the $T$ and $\rho$ values of the primordial gas gives $L_J \approx 5 \times 10^{17}$ m, and $M_J \equiv L_J^3 \rho \approx 10^{35-36}$ kg. Substituting $\rho$, $\gamma$ and $\nu$ values in $L_{SV} \equiv (\nu \gamma / \rho G)^{1/2}$ gives $L_{SV} \approx 10^{14}$ m and $M_{SV} \equiv L_{SV}^3 \rho \approx 10^{24-25}$ kg, a factor of $\approx 10^{12}$ smaller than $M_J$. The Jeans scale objects are called “Proto-Globular-Clusters” (PGCs) and the $L_{SV}$ scale objects are called “Primordial Fog Particles” (PFPs). From the observational evidence it appears that many if not most PGCs have not yet dispersed and most of their PFPs have not yet accreted to form stars, so that both persist as the dominant component of galactic baryonic dark matter (Gibson 1996). The calculated masses of PGCs and PFPs depend on universal proportionality constants of order one that will emerge from observations. Observations of globular star clusters indicate a mass $10^{5-6} M_\odot$ matching our calculated PGC value of $10^{35-36}$ kg and densities close to the fossilized initial fragmentation density $\rho_{FS} \approx 10^{-17}$ kg m$^{-3}$. The calculated PFP mass $10^{24-25}$ kg matches observations of $\approx 10^{-6} M_\odot$ “rogue planets” by Schild 1996 as the dominant component of the lensing galaxy in a lensed quasar system. Evidence supporting HGT has recently been summarized (Gibson & Schild 2003), and includes the appearance of PFP candidates brought out of cold storage by evaporation near hot objects such as white dwarfs in planetary nebula. Figure 2 shows PFP candidates in the Helix planetary nebula which is the one closest to earth, photographed by the Hubble Space telescope. Thousands of cometary globules appear with mass values, densities, and separation distances as predicted by HGT.

5. Conclusions

We conclude that the small amplitude $\delta T/T \approx 10^{-5}$ of measured temperature fluctuations in the cosmic background radiation is evidence of strong turbulence damping by both a photon viscosity $\nu \approx 4 \times 10^{26}$ m$^2$ s$^{-1}$ and buoyancy forces of viscous-gravitational structure
formation beginning approximately 30,000 years after the Big Bang.

The hypothesis is rejected that the small CMB fluctuations reflect hydrodynamic stability of the Hubble flow. This would require a critical Hubble flow Reynolds number of $Re_{crH} \approx 10^5$, contrary to universal similarity hypotheses of Kolmogorov 1941 for turbulence and strong experimental evidence that the universal critical Reynolds number of transition is $Re_{cr} \lesssim 25$. Steady inviscid flows are absolutely unstable to the formation of turbulence, as shown in §2 and as derived by Landau & Lifshitz 1959. Buoyancy forces from gravitational structure formation in the plasma epoch are therefore required to explain the lack of turbulence at the time of plasma to gas transition.

The hypothesis is rejected that the CMB spectral peak with $L \approx 0.4L_H$ is a doppler or sound horizon with $L \approx V_st$ because no source of sound exists to produce the observed $\delta T/T \approx 8 \times 10^{-5}$ peak value, and because strong viscous damping in the plasma epoch would rapidly flatten any such sonic peaks. Persistent sonic oscillations of baryonic matter sloshing in CDM potential wells as a sound source is rejected because no CDM potential wells are possible in the plasma epoch, because viscous damping would occur, and because recent strong observational evidence excludes CDMHCC scenarios, Gibson & Schild 2003. Instead, the observed spectral peak at scales $L \approx ct$ is interpreted as evidence of the first hydro-gravitational structure formation. Secondary acoustic peaks observed may reflect rarefaction wave oscillations of hydro-gravitationally driven proto-supercluster void formation near sonic velocities. Hydro-gravitational theory suggests the first structures to form were proto-supercluster-voids at the viscous Schwarz scale $L_{SV}$, when $L_{SV} > L_H$ first matched the increasing horizon scale $L_H$. Rapid expansion of the universe during the plasma epoch prevented gravitational condensation but enhanced void formation, §3.

As shown in §4, fragmentation of the primordial gas occurred simultaneously at $L_J$ and $L_{SV}$ scales to form proto-globular-clusters (PGCs) and primordial-fog-particles (PFPs). Estimated PGC masses match the observed globular star cluster masses of $10^6 M_\odot$ and estimated PFP masses match the observed “rogue planet” dark matter masses of $10^{-6} M_\odot$ in lensed quasars (Schild 1996). Most PGCs and their PFPs are observed to persist as dark clumps of frozen planetoids, forming the dominant component of $\approx 100 kpc = 3 \times 10^{21}$ m galactic dark-matter inner-halos (Gibson & Schild 2003), with non-baryonic dark matter fragmenting to form outer galactic dark-matter halos at $L_{SD} \approx Mpc = 3 \times 10^{22}$ m scales.

All evidence suggests the early universe was an extremely gentle place, with practically no turbulence or sound anywhere after the big bang and prior to the formation of stars. Buoyancy and large photon viscosities damped sound and turbulence in the plasma epoch. Gravitational condensation formed PFPs and prevented turbulence in the early stages of the gas epoch. As the universe continued to expand and cool, some of these small-planetary-mass
objects experienced an accretional cascade to larger mass-scales to form the first very small stars. This cascade was a gentle process to produce the remarkably spherical distributions of these long-lived, tightly-packed stars in globular star clusters and the more numerous dark or dim PGC-PFP baryonic-dark-matter structures with the same $\rho \approx 10^{-17} \text{ kg m}^{-3}$ density and the PFP mass $\approx 10^{24} \text{ kg}$ preserved as fossils of the weak turbulence and large density at the time of first structure (Gibson 2000).

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Fig. 1.— Turbulent Hubble flow wakes of proto-galaxies (labeled PG) bound by gravity (dark double arrows) in a proto-cluster, soon after the plasma to gas transition at time $t \approx 10^{13}$ s (300,000 years) after the big bang. From HGT (Gibson 1996) the PG size is $\approx 10^{20}$ meters, with primordial H-He mass $\approx 10^{42}$ kg ($\approx 10^{12} M_\odot$) and density $\rho \approx 10^{-18}$ kg m$^{-3}$. The Hubble flow velocity $v_H = r \times \gamma_H \approx 10^7$ m s$^{-1}$ at the radius $r$ of the PGs. Hubble flow drag forces will separate the protogalaxies, as it separated the protoclusters and protosuperclusters also formed in the plasma epoch by gravitational fragmentation. Nonbaryonic dark matter moves freely through the galaxies by diffusion and the Hubble flow to fill the voids. The horizon scale $L_H$ where $v_H = c$ is $3 \times 10^{21}$ m (100 kpc), 10 times the size of the outer sphere shown.
Fig. 2.— PFP-like objects (Gibson & Schild 2003) observed by HST in the Helix Planetary Nebula, 4.5 × 10^{18} m from Earth (O’Dell & Handron 1996). Evaporation of the frozen H-He objects produces 10^{25} kg photo-ionized cocoons, with H-wakes pointing away from the hot (≈ 50,000 K) White Dwarf. The indicated density of the PN halo is \( \rho_{\text{Halo}} \approx M_{\text{PFP}} L_{\text{Sep.}}^{-3} \approx 10^{-17} \text{ kg m}^{-3} \), where the PFP separation distance \( L_{\text{Sep.}} \approx 10^{14} \text{ m} \). This matches the baryonic density at the time of first structure 10^{12} s (30,000 years) as a fossil of this time of first structure formation as predicted by HGT.
Table 1. Length scales of self-gravitational structure formation

| Length scale name           | Symbol  | Definition\(^a\)                                      | Physical significance\(^b\)                  |
|-----------------------------|---------|-----------------------------------------------------|---------------------------------------------|
| Jeans Acoustic              | \(L_J\) | \(V_S/\sqrt[1/2]{\rho G}\)                         | ideal gas pressure equilibration           |
| Chandrasekhar Turbulent     | \(L_{JCT}\) | \(\left[\left(p + p_T\right)/\rho^2 G\right]^{1/2}\) | turbulence balances gravitation             |
| Schwarz Diffusive           | \(L_{SD}\) | \([D^2/\rho G]^{1/4}\)                             | \(V_D\) balances \(V_G\)                   |
| Schwarz Viscous             | \(L_{SV}\) | \([\nu/\rho G]^{1/2}\)                             | viscous force balances gravitational force  |
| Schwarz Turbulent           | \(L_{ST}\) | \(\varepsilon^{1/2}/[\rho G]^{3/4}\)              | turbulence balances gravitation             |
| Kolmogorov Viscous          | \(L_K\) | \([\nu^3/\varepsilon]^{1/4}\)                     | turbulence force balances viscous force     |
| Batchelor Diffusive         | \(L_B\) | \([D/\gamma]^{1/2}\)                               | diffusion balances strain rate              |
| Collision                   | \(L_C\) | \(m\sigma^{-1}\rho^{-1}\)                          | distance between particle collisions        |
| Horizon, Hubble             | \(L_H\) | \(ct\)                                              | maximum scale of causal connection         |

\(^a\)\(V_S\) is sound speed, \(\rho\) is density, \(G\) is Newton’s constant, \(p\) is pressure, \(p_T \equiv \rho(\delta v)^2\), \(v\) is velocity, \(D\) is the diffusivity, \(V_D \equiv D/L\) is the diffusive velocity at scale \(L\), \(V_G \equiv L[\rho G]^{1/2}\) is the gravitational velocity, \(\gamma\) is the strain rate, \(\nu\) is the kinematic viscosity, \(\varepsilon\) is the viscous dissipation rate, \(m\) is the particle mass, \(\sigma\) is the collision cross section, light speed \(c\), age of universe \(t\).

\(^b\)Magnetic and other forces (besides viscous and turbulence) are negligible for the epoch of primordial self-gravitational structure formation considered here (Gibson 1996).