Bose-Einstein condensation of polaritons in graphene in a high magnetic field

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The Bose-Einstein condensation (BEC) of magnetoexcitonic polaritons in a graphene layer embedded in a optical microcavity in a high magnetic field is predicted. The essential property of this system (in contrast, e.g., to a quantum well embedded in a cavity) is stronger influence of magnetic field and weaker influence of disorder. A two-dimensional (2D) magnetoexcitonic polaritons gas is considered in a planar harmonic electric field potential applied to excitons or a parabolic shape of the optical cavity causing the trapping of microcavity photons. It is shown that the effective polariton mass $M_{\text{eff}}$ increases with magnetic field as $B^{1/2}$. The BEC critical temperature $T_{\text{c}}^{(0)}$ decreases as $B^{-1/4}$ and increases with the spring constant of the parabolic trap. The Rabi splitting related to the creation of a magnetoexciton in a high magnetic field in graphene is obtained.

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In the past decade, Bose coherent effects of 2D excitonic polaritons in a quantum well embedded in a semiconductor microcavity have been the subject of theoretical and experimental studies\textsuperscript{1}. To obtain polaritons, two mirrors placed opposite each other form a microcavity, and quantum wells are embedded within the cavity at the antinodes of the confined optical mode. The resonant exciton-photon interaction results in the Rabi splitting of the excitation spectrum. Two polariton branches appear in the spectrum due to the resonant exciton-photon coupling. The lower polariton (LP) branch of the spectrum has a minimum at zero momentum. Recently, the polaritons in a harmonic potential trap have been studied experimentally in a GaAs/AlAs quantum well embedded in a GaAs/AlGaAs microcavity\textsuperscript{2}. In this trap, the exciton energy is shifted using a stress-induced band-gap. In this system, evidence for the BEC of polaritons in a quantum well has been observed\textsuperscript{2}. The theory of the BEC and superfluidity of exciton polaritons in a quantum well (QW) without magnetic field in a parabolic trap has been developed in Ref.\textsuperscript{3}.

A novel type of 2D electron system was experimentally obtained in graphene, which is a 2D honeycomb lattice of the carbon atoms that form the basic planar structure in graphite\textsuperscript{4,5}. Graphene is a gapless semiconductor with massless electrons and holes which have been described as Dirac-fermions. The energy spectrum and the wavefunctions of electron-hole pairs in a magnetic field, in graphene have been calculated in Refs.\textsuperscript{6,7}. In high magnetic field electron-hole pairs in graphene form two-dimensional magnetoexcitons. The BEC and superfluidity of spatially-indirect magnetoexcitons with spatially separated electrons and holes have been studied in graphene bilayer\textsuperscript{8} and graphene superlattice\textsuperscript{9}. The electron-hole pair condensation in the graphene-based bilayers have been studied in\textsuperscript{10,11,12,13}. The similar effects in the system of spatially-indirect excitons (or electron-hole pairs) in coupled quantum wells (CQWs), with and without a magnetic field were studied in Refs.\textsuperscript{14,15,16,17,18,19}. The experimental and theoretical interest to study these systems is partially due to the possibility of the BEC and superfluidity of indirect excitons, which can manifest in the CQW as persistent electrical currents in each well and also through coherent optical properties and Josephson phenomena\textsuperscript{14,15,16,17,18,19}. The great experimental success was achieved now in this field\textsuperscript{20,21,22,23}. Besides, the essential experimental progress was achieved in experimental studies of exciton polaritons in the system of a QW embedded in optical microcavity\textsuperscript{24,25,26}. However, while the exciton polaritons have been studied in a QW, the formation of the polaritons in graphene in high magnetic field have not yet been considered. Moreover, the polaritons formed as superposition of magnetoexcitons and cavity photons in magnetic field have not yet been studied. It is interesting to study a 2D system of polaritons in graphene embedded in a microcavity from the point of view of the existence of the BEC within it. The BEC of polaritons in high magnetic field in graphene embedded in a microcavity is perspective, since the random field in graphene is weaker than in a QW, particularly, because in a QW the random field is generated due to the fluctuations of the width of a QW. Let us mention that if the interaction of bosons with the random field is stronger, the BEC critical temperature is lower\textsuperscript{27}. Moreover, the BEC of polaritons in graphene embedded in a microcavity can exist at much lower magnetic field than in a QW, because the distance between electron Landau levels in graphene is much higher than in a QW at the same magnetic field, and, therefore, the lower magnetic field is required in graphene than in a QW to neglect the electron transitions between the Landau levels. The purpose of this Letter is to point out the existence of the BEC of the magnetoexciton polaritons in a graphene layer embedded in a semiconductor microcavity.
in a strong magnetic field and to discuss the conditions of its realization.

When an undoped electron system in graphene in a magnetic field without an external electric field is in the ground state, half of the zeroth Landau level is filled with electrons, all Landau levels above the zeroth one are empty, and all levels below the zeroth one are filled with electrons. We suggest using the gate voltage to control the chemical potential in graphene by two ways: to shift it above the zeroth level so that it is between the zeroth and first Landau levels (the first case) or to shift the chemical potential below the zeroth level so that it is between the first negative and zeroth Landau levels (the second case). In both cases, all Landau levels below the chemical potential are completely filled and all Landau levels above the chemical potential are completely empty. Based on the selection rules for optical transitions between the Landau levels in single-layer graphene \([28]\), in the first case, there are allowed transitions between the zeroth and the first Landau levels, while in the second case there are allowed transitions between the first negative and zeroth Landau levels. Correspondingly, we consider magnetoexcitons formed in graphene by the electron on the first Landau level and the hole on the zeroth Landau level (the first case) or the electron on the zeroth Landau level and the hole on the Landau level \(-1\) (the second case). Note that by appropriate gate potential we can also use any other neighboring Landau levels \(n\) and \(n+1\).

For the relatively high dielectric constant of the microcavity, \(\epsilon \gg e^2/(\hbar \nu_F) \approx 2\) \((\nu_F = \sqrt{3} at/(2\hbar))\) is the Fermi velocity of electrons in graphene, where \(a = 2.566\ \text{Å} \) is a lattice constant and \(t \approx 2.71\ \text{eV} \) is the overlap integral between the nearest carbon atoms \([29]\) the magnetoexciton energy in graphene can be calculated by applying perturbation theory with respect to the strength of the Coulomb electron-hole attraction analogously as it was done in \([30]\) for 2D quantum wells in a high magnetic field with non-zero electron \(m_e\) and hole \(m_h\) masses. This approach allows us to obtain the spectrum of an isolated magnetoexciton with the electron on the Landau level 1 and the hole on the Landau level 0 in a single graphene layer, and it will be exactly the same as for the magnetoexciton with the electron on the Landau level 0 and the hole on the Landau level \(-1\). The characteristic Coulomb electron-hole attraction for the single graphene layer is \(e^2/(\epsilon r_B)\), where \(\epsilon\) is the dielectric constant of the environment around graphene, \(r_B = \sqrt{\hbar / (e B)}\) denotes the magnetic length of the magnetoexciton in the magnetic field \(B\), and \(c\) is the speed of light. The energy difference between the first and zeroth Landau levels in graphene is \(\hbar \nu_F/r_B\). For graphene, the perturbative approach with respect to the strength of the Coulomb electron-hole attraction is valid when \(e^2/(\epsilon r_B) \ll \hbar \nu_F/r_B\) \([31]\). This condition can be fulfilled at all magnetic fields \(B\) if the dielectric constant of the surrounding media satisfies the condition \(e^2/(\epsilon \hbar \nu_F) \ll 1\). Therefore, we claim that the energy difference between the first and zeroth Landau levels is always greater than the characteristic Coulomb attraction between the electron and the hole in the single graphene layer at any \(B\) if \(\epsilon \gg e^2/(\hbar \nu_F) \approx 2\). Thus, applying perturbation theory with respect to weak Coulomb electron-hole attraction in graphene embedded in the GaAs microcavity \((\epsilon = 12.9)\) is more accurate than for graphene embedded in the SiO\(_2\) microcavity \((\epsilon = 4.5)\). However, the magnetoexcitons in graphene exist in high magnetic field. Therefore, we restrict ourselves by consideration of high magnetic fields.

Polaritons are linear superpositions of excitons and photons. In high magnetic fields, when magnetoexcitons may exist, the polaritons become linear superpositions of magnetoexcitons and photons. Let us define the superpositions of magnetoexcitons and photons as magnetopolaritons. It is obvious that magnetopolaritons in graphene are two-dimensional, since graphene is a two-dimensional structure. The Hamiltonian of magnetopolaritons in the strong magnetic field is given by

\[
\hat{H}_{tot} = \hat{H}_{mex} + \hat{H}_{ph} + \hat{H}_{mex-ph} ,
\]

where \(\hat{H}_{mex}\) is a magnetoexcitonic Hamiltonian, \(\hat{H}_{ph}\) is a photonic Hamiltonian, and \(\hat{H}_{mex-ph}\) is the Hamiltonian of magnetoexciton-photon interaction.

Let us analyze each term of the Hamiltonian for magnetopolaritons \([11]\). The effective Hamiltonian and the energy dispersion for magnetoexcitons in graphene layers in a high magnetic field derived in Ref. \([9]\) are given by

\[
\hat{H}_{mex} = \sum_{\mathbf{P}} \varepsilon_{mex}(P) \hat{b}^\dagger_{\mathbf{P}} \hat{b}_{\mathbf{P}} , \quad \varepsilon_{mex}(P) = E_{\text{band}} - E^{(b)}_B + \varepsilon_0(P) ,
\]

where \(\hat{b}^\dagger_{\mathbf{P}}\) and \(\hat{b}_{\mathbf{P}}\) are magnetoexcitonic creation and annihilation operators obeying the Bose commutation relations. In Eq. \([2]\), \(E_{\text{band}} = E^{(0)}_{1,0} = \sqrt{2} \hbar \nu_F/r_B\) is the band gap energy, which is the difference between the Landau levels 1 and 0 in graphene. \(E^{(b)}_B\) is the binding energy of a 2D magnetoexciton with the electron in the Landau level 1 and the hole on the Landau level 0 in a single graphene layer, and \(\varepsilon_0(P) = P^2/(2m_B)\), where \(m_B\) is the effective magnetic mass of a 2D magnetoexciton with the electron on the Landau level 1 and hole on the Landau level 0 in a single graphene layer. The binding energy \(E^{(b)}_B\) and effective magnetic mass \(m_B\) of a magnetoexciton in graphene obtained using the first order perturbation respect to the electron-hole Coulomb attraction similarly to the case of a single quantum well.
The magnetoexcitons are located at a distance $R \sim E$ of their dipole-dipole interaction is described in Ref. [30] for 2D magnetoexcitons in a quantum well. The dipole moment of each exciton in a magnetic field is $d_{1,2} = e \rho_0 = r_B^2 (B \times P_1,2) / B$ [30], where $P_1$ and $P_2$ are the magnetic momenta of each exciton and $P_1, P_2 \ll 1 / r_B$. The magnetoexcitons are located at a distance $R \gg r_B$ from each other. The corresponding contribution to the energy of their dipole-dipole interaction is $\sim \mathcal{E}_B^{(b)} (r_B / R)^3 P_1 P_2 r_B^2 / \epsilon \sim (r_B / R)^3 P_1 P_2 / (\epsilon M_0) \ll e^2 r_B^2 / (\epsilon R^3)$. Inputting the radius of the magnetoexciton in graphene $r_{0.1} \sim r_B$ [8], we obtain that the van der Waals attraction of the exciton at zero momenta is proportional to $\sim (r_0 / R)^6 \sim (r_B / R)^6$. Therefore, in the limit of a strong magnetic field for a dilute system $r_B \ll R$, both the dipole-dipole interaction and the van der Waals attraction vanish, and the 2D magnetoexcitons in graphene form an ideal Bose gas analogously to the 2D magnetoexcitons in a quantum well given in Ref. [30]. Thus, the Hamiltonian (1) does not include the term corresponding to the interaction between two direct magnetoexcitons in graphene on a single graphene layer. So in high magnetic field there is the BEC of the ideal magnetoexcitonic gas in graphene.

The Hamiltonian and the energy spectrum of non-interacting photons in a semiconductor microcavity are given by [31]:

$$
\hat{H}_{ph} = \sum_P \varepsilon_{ph}(P) \hat{a}_P^\dagger \hat{a}_P , \\
\varepsilon_{ph}(P) = (c / n) \sqrt{P^2 + \hbar^2 \pi^2 L_C^2} ,
$$

where $\hat{a}_P^\dagger$ and $\hat{a}_P$ are photonic creation and annihilation Bose operators. In Eq. (4), $L_C$ is the length of the cavity, $n = \sqrt{\epsilon_C / \epsilon}$ is the effective refractive index and $\epsilon_C$ is the dielectric constant of the cavity. We assume that the length of the microcavity has the following form:

$$
L_C(B) = \frac{\hbar \pi c}{n (\epsilon_{band} - \mathcal{E}_B^{(b)})} ,
$$

(5)

corresponding to the resonance of the photonic and magnetoexcitonic branches at $P = 0$, i.e. $\varepsilon_{mex}(0) = \varepsilon_{ph}(0)$. As it follows from the energy spectra in (2) and (4), and Eqs. (3) and (5), the length of the microcavity, corresponding to a magnetoexciton-photon resonance, decreases with the increment of the magnetic field as $B^{-1 / 2}$. The resonance between magnetoexcitons and cavity photonic modes can be achieved either by controlling the spectrum of magnetoexcitons $\varepsilon_{mex}(P)$ by changing magnetic field $B$ or by choosing the appropriate length of the microcavity $L_C$.

The Hamiltonian of the magnetoexciton-photonic coupling has the form (see Refs. [32, 33, 34]):

$$
\hat{H}_{mex-ph} = \hbar \Omega_R \sum_P \hat{a}_P^\dagger \hat{b}_P + h.c. ,
$$

(6)

$$
\hat{b}_P = X_P \hat{p}_P - C_P \hat{u}_P , \\
\hat{a}_P = C_P \hat{p}_P + X_P \hat{u}_P ,
$$

(7)

where the magnetoexciton-photonic coupling energy represented by the Rabi constant $\hbar \Omega_R$ is obtained below, $\hat{p}_P$ and $\hat{u}_P$ are lower and upper magnetoexciton Bose operators, respectively, $j X_P$ and $C_P$ are coefficients of the unitary Bogoliubov transformation [32, 34], and the energy spectra of the lower/upper magnetoexcitons are

$$
\varepsilon_{LP / UP}(P) = \frac{\varepsilon_{ph}(P) + \varepsilon_{mex}(P)}{2} + \frac{1}{2} \sqrt{(\varepsilon_{ph}(P) - \varepsilon_{mex}(P))^2 + 4 |\hbar \Omega_R|^2} ,
$$

(8)

Eq. (8) implies a splitting of $2 \hbar \Omega_R$ between the upper and lower states of polaritons at $P = 0$, which is known as the Rabi splitting.

Substituting (7) into Hamiltonians (2), (4) and (6) and diagonalizing the Hamiltonian (1), finally we obtain

$$
\hat{H}_{tot} = \sum_P \varepsilon_{LP}(P) \hat{p}_P^\dagger \hat{p}_P + \sum_P \varepsilon_{UP}(P) \hat{u}_P^\dagger \hat{u}_P ,
$$

(9)
where $\hat{p}_p^\dagger$, $\hat{p}_p$, $\hat{\sigma}_p^\dagger$, $\hat{\sigma}_p$ are the Bose creation and annihilation operators for the lower and upper magnetopolaritons, respectively. The Hamiltonian Eq. (3) describes magnetopolaritons in a single graphene layer in a high magnetic field. Our particular interest is the lower energy magnetopolaritons which produce the BEC. The lower polaritons have the lowest energy within a single graphene layer.

Similarly to the Bose atoms in a trap in the case of a slowly varying external potential [33], we can make the quasiclassical approximation, assuming that the effective magnetoexciton mass does not depend on a characteristic size $l$ of the trap and it is a constant within the trap. This quasiclassical approximation is valid if $P \gg \hbar/l$. The harmonic trap is formed by the two-dimensional planar potential in the plane of graphene. The potential trap can be produced in two different ways. In case 1, the potential trap can be produced by applying an external inhomogeneous electric field. The spatial dependence of the external field potential $V(r)$ is caused by shifting of magnetoexciton energy by applying an external inhomogeneous electric field. The photonic states in the cavity are assumed to be unaffected by this electric field. In this case the band energy $E_{\text{band}}$ is replaced by $E_{\text{band}}(r) = E_{\text{band}}(0) + V(r)$. Near the minimum of the magnetoexciton energy, $V(r)$ can be approximated by the planar harmonic potential $\gamma r^2/2$, where $\gamma$ is the spring constant. Note that a high magnetic field does not change the trapping potential in the effective Hamiltonian [30]. In case 2, the trapping of magnetopolaritons is caused by the inhomogeneous shape of the cavity when the length of the cavity is determined by Eq. (4) with the term $\gamma r^2/2$ added to $E_{\text{band}} - E_{\text{B}}^{(0)}$, where $r$ is the distance between the photon and the center of the trap. In case 2, the $\gamma$ is the curvature characterizing the shape of the cavity. In case 1, for the slowly changing confining potential $V(r) = \gamma r^2/2$, the magnetoexciton spectrum is given in the effective mass approximation by Eq. (2) provided we added the term $\gamma r^2/2$ added to $E_{\text{band}} - E_{\text{B}}^{(0)}$ in the r.h.s., where $r$ is now the distance between the center of mass of the magnetoexciton and the center of the trap. The Hamiltonian and the energy spectrum of the photons in this case are shown by Eq. (5), and the length of the microcavity is given by Eq. (6).

In case 2, for the photonic spectrum in the effective mass approximation is given by substituting the slowly changing shape of the cavity length depending on the term $\gamma r^2/2$ into Eq. (4) representing the spectrum of the cavity photons. This quasiclassical approximation is valid if $P \gg \hbar/l$, where $l = (\hbar/(m_B\omega_0))^{1/2}$ is the size of the magnetoexciton cloud in an ideal magnetoexciton gas and $\omega_0 = \sqrt{\gamma/m_B}$. The Hamiltonian and energy spectrum of magnetopolaritons in this case are given by [2].

At small momenta $\alpha = 1/2(m_B^{-1} + (c/n)L_C/\hbar\pi)P^2/|h\Omega_R| \ll 1$ ( $L_C = \hbar\pi c/n \left(E_{\text{band}} - E_{\text{B}}^{(0)}\right)^{-1}$ ) and weak confinement $\beta = \gamma r^2/|h\Omega_R| \ll 1$, the single-particle lower magnetopolariton spectrum obtained through the substitution of Eq. (2) into Eq. (5), in linear order with respect to the small parameters $\alpha$ and $\beta$, is

$$\epsilon_0(P) \approx \frac{c}{n}\hbar\pi L_C^{-1} - |h\Omega_R| + \frac{\gamma}{4} r^2 + \frac{1}{4} \left(m_B^{-1} + \frac{c L_C(B)}{n\hbar\pi}\right) P^2.$$  

(10)

Let us emphasize that the spectrum of non-interacting magnetopolaritons $\epsilon_0(P)$ at small momenta and weak confinement is given by Eq. (10) for both physical realizations of confinement: case 1 and case 2.

If we measure the energy relative to the $P = 0$ lower magnetopolariton energy $(c/n)\hbar\pi L_C^{-1} - |h\Omega_R|$, we obtain the resulting effective Hamiltonian for trapped magnetopolaritons in graphene in a magnetic field. At small momenta $\alpha \ll 1$ ($L_C = \hbar\pi c/n \left(E_{\text{band}} - E_{\text{B}}^{(0)}\right)^{-1}$) and weak confinement $\beta \ll 1$, this effective Hamiltonian is

$$\hat{H}_{\text{eff}} = \sum_{\mathbf{P}} \left(\frac{P^2}{2 M_{\text{eff}}(B)} + \frac{1}{2} V(r)\right) \hat{p}_p^\dagger \hat{p}_p ,$$  

(11)

where the sum over $\mathbf{P}$ is carried out only over $P \gg \hbar/l$ (only in this case the quasiclassical approach used in Eq. (2) is valid), and the effective magnetic mass of a magnetopolariton is given by

$$M_{\text{eff}}(B) = 2 \left(m_B^{-1} + \frac{c L_C(B)}{n\hbar\pi}\right)^{-1}.$$  

(12)

According to Eqs. (12) and (3), the effective magnetopolariton mass $M_{\text{eff}}$ increases with the increment of the magnetic field as $B^{1/2}$. Let us emphasize that the resulting effective Hamiltonian for magnetopolaritons in graphene in a magnetic field for the parabolic trap is given by Eq. (11) for both physical realizations of confinement represented by case 1 and case 2.

Neglecting anharmonic terms for the magnetoexciton-photon coupling, the Rabi splitting constant $\Omega_R$ can be estimated quasiclassically as

$$|h\Omega_R|^2 = \langle f | \hat{H}_{\text{int}} | i \rangle^2, \quad E_{\text{ph}0} = \left(\frac{2\pi\hbar\omega}{cW}\right)^{1/2}, \quad \hat{H}_{\text{int}} = -\frac{v_F e}{c} \hat{\sigma} \cdot \vec{A}, \quad \vec{A} = \frac{v_F e}{i\omega} \hat{\sigma} \cdot \vec{E}_{\text{ph}0}.$$  

(13)
where \( \vec{\sigma} = (\sigma_x, \sigma_y) \), \( \sigma_x \) and \( \sigma_y \) are Pauli matrices, \( \hat{H}_{\text{int}} \) is the Hamiltonian of the electron-photon interaction corresponding to the electron in graphene described by Dirac dispersion, \( E_{\text{ph}0} \) is the electric field corresponding to a single cavity photon, \( W \) is the volume of microcavity, \( \omega \) is the photon frequency. The initial \(|i\rangle \) electron state corresponds to the completely filled Landau level 0 and completely empty Landau level 1. The final \(|f\rangle \) electron state corresponds to creation of one magnetoexciton with the electron on the Landau level 1 and the hole on the Landau level 0. The transition dipole moment corresponding to the process of creation of this magnetoexciton is given by \( d_{12} = e r_B / 4 \). Let us note that in Eq. (13) the energy of photon absorbed at the creation of the magnetoexciton is given by \( \hbar\omega = \varepsilon_1 - \varepsilon_0 = \sqrt{2} \hbar v_F r_B \) (we assume that \( E_{\text{ph}}^{(b)} \ll \varepsilon_1 - \varepsilon_0 \)). Substituting the photon energy and the transition dipole moment from into Eq. (13), we obtain the Rabi splitting corresponding to the creation of a magnetoexciton with the electron on the Landau level 1 and the hole on the Landau level 0 in graphene: \( \hbar \Omega_R = e (\pi \hbar v_F r_B / (\sqrt{2} \varepsilon W))^{1/2} \).

Thus, the Rabi splitting in graphene is related to the creation of the magnetoexciton, which decreases when the magnetic field increases and is proportional to \( B^{-1/4} \). Therefore, the Rabi splitting in graphene can be controlled by the external magnetic field. It is easy to show that the Rabi splitting related to the creation of the magnetoexciton, the electron on the Landau level 0 and the hole on the Landau level 1 will be exactly the same as for the magnetoexciton with the electron on the Landau level 1 and the hole on the Landau level 0. Let us mention that dipole optical transitions from the Landau level 0 to the Landau level 0, as well as from the Landau level 0 to the Landau level 1, are allowed by the selection rules for optical transitions in single-layer graphene.

As known, the exact solution for the ground state of 2D magnetoexcitons in high magnetic fields is the Bose condensate of non-interacting magnetoexcitons. This is valid for both semiconductor quantum well and graphene. Although Bose-Einstein condensation cannot take place in a 2D homogeneous ideal gas at non-zero temperature, as discussed in Ref. [37], in a harmonic trap the BEC can occur in two dimensions below a critical temperature \( T^c \). In a harmonic trap at a temperature \( T \) below a critical temperature \( T^c \) (\( T < T^c \)), the number \( N_0(T, B) \) of non-interacting magnetoexcitons in the condensate is given in Ref. [37]. Applying the condition \( N_0 = 0 \), and assuming that the magnetoexciton effective mass is given by Eq. (12), we obtain the BEC critical temperature \( T_e^{(0)} \) for the ideal gas of magnetoexcitons in a single graphene layer in a magnetic field:

\[
T_e^{(0)}(B) = \frac{1}{k_B} \left( \frac{3 \hbar^2 \gamma N}{\pi \left( g_s^{(e)} g_v^{(e)} + g_s^{(h)} g_v^{(h)} \right) M_{\text{eff}}(B)} \right)^{1/2},
\]

where \( N \) is the total number of magnetoexcitons, \( g_s^{(e), (h)} \) and \( g_v^{(e), (h)} \) are the spin and graphene valley degeneracies for an electron and a hole, respectively, \( k_B \) is the Boltzmann constant. At temperatures above \( T_e^{(0)} \), the BEC of magnetoexcitons in a single graphene layer does not exist. \( T_e^{(0)} / \sqrt{N} \) as a function of magnetic field \( B \) and spring constant \( \gamma \) is presented in Fig. 1. In our calculations, we used \( g_s^{(e)} = g_v^{(e)} = g_s^{(h)} = g_v^{(h)} = 2 \). According to Eq. (14), the BEC critical temperature \( T_e^{(0)} \) decreases with the magnetic field as \( B^{-1/4} \) and increases with the spring constant as \( \gamma^{1/2} \). Note that we assume that the quality of the cavity is sufficiently high, so that the time of the relaxation to the Bose condensate quasi-equilibrium state is smaller than the life time of the photons in the cavity.

To conclude, we have studied the ideal gas of trapped cavity magnetoexcitons in a single graphene layer in a high magnetic field. The resonance between magnetoexcitons and cavity photonic modes can be achieved either by controlling the spectrum of magnetoexcitons \( \varepsilon_{\text{ex}}(P) \), by changing magnetic field \( B \) or by controlling the length of the microcavity \( L_C \). We analyzed two possible physical realizations of the trapping potential: a harmonic electric field potential coupled to magnetoexcitons and a parabolic shape of the semiconductor cavity causing the trapping of microcavity photons. We conclude that both realizations of confinement result in the same effective Hamiltonian. It is shown that the effective magnetoexciton mass \( M_{\text{eff}} \) increases with the magnetic field as \( B^{1/2} \). Meanwhile, the BEC critical temperature \( T_e^{(0)} \) decreases as \( B^{-1/4} \) and increases with the spring constant as \( \gamma^{1/2} \). The gas of magnetoexcitons in graphene in a high magnetic field can be treated as an ideal Bose gas since magnetoexciton-magnetoexciton interaction vanishes in the limit of a high magnetic field and a relatively high dielectric constant of the cavity \( \epsilon \gg 2 \). Let us mention that this condition for the high dielectric constant of the microcavity is valid only for graphene, and it is not valid for the quantum well. Besides, we have obtained the Rabi splitting related to the creation of a magnetoexciton in a high magnetic field in graphene which can be controlled by the external magnetic field \( B \).
Magnetic field, $B$ (T)

$T_c(0)/\sqrt{N}$ vs. $B$ at different spring constants $\gamma$. We assume the environment around graphene is GaAs with $\epsilon = 12.9$.

**FIG. 1**: The ratio of the BEC critical temperature to the square root of the total number of magnetopolaritons $T_c(0)/\sqrt{N}$ as a function of magnetic field $B$ at different spring constants $\gamma$. We assume the environment around graphene is GaAs with $\epsilon = 12.9$.

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