Quantum bus building-block for a scalable quantum computer architecture

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Abstract

In superconducting quantum computers, qubits are usually only coupled to their nearest-neighbors. To overcome this limitation, we propose a scalable architecture to simultaneously connect several pairs of distant logical qubits via a dispersively coupled quantum bus. The building-block of the bus is composed of orthogonal coplanar waveguide resonators connected through ancillary flux qubits working in the ultrastrong coupling regime. This regime activates virtual processes that boost the effective qubit-qubit interaction, which results in quantum gates on the nanosecond timescale. The architecture we are proposing has also the benefit of allowing the logical qubits to remain at their optimal bias point, preserving their coherence time.

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Superconducting circuits are a very promising hardware platform for quantum computers with capabilities beyond the ones of classical computers (see, e.g., [1–5] and references therein). To perform quantum logic gates a basic requirement is to have controllable interaction among qubits (e.g., [6–9]). Obviously, quantum computers benefit from higher and better connectivity among qubits, and this becomes more challenging to achieve as the system is scaled up. Unfortunately, superconducting qubits usually have nearest-neighbor couplings [10]. Although the distant interaction between two or three qubits, mediated by a cavity bus, has been demonstrated (e.g., [11, 12]), this scheme cannot be used to connect many pairs of distant qubits simultaneously [13]. Indeed, in this case, the qubit-qubit interaction is activated by tuning qubit frequencies, leading to possible unwanted couplings and to a reduction of the coherence time of the qubits. In addition to applications to quantum computing, superconducting circuits are a very versatile platform to investigate new quantum phenomena and to engineer quantum devices (e.g., [14–21]). Note that the coupling between a superconducting artificial atom (e.g., [22–25]) and a resonator can be a significant fraction of the atom and cavity bare energies (e.g., [26–30]). In this ultrastrong coupling regime, the usual Jaynes-Cummings approximation breaks down and the counter-rotating terms must be taken into account [31, 32].

Here, we theoretically propose a scalable architecture to simultaneously couple several pairs of distant superconducting qubits. The building block of this architecture is composed of three waveguides. Two of them (C\(_1\) and C\(_2\)), see Fig. 1(a), are directly connected to the logical qubits (q\(_a\) and q\(_b\)), while a third (C\(_3\)) is connected to the first two in a Π-shape form. At the intersection point, the interaction is mediated by ancillary flux qubits (f\(_1\) and f\(_2\)) in the ultrastrong coupling regime. All components of the Π-connector are on resonance with each other. However, the two logical qubits are detuned with respect to the eigenenergies of the bus. The last condition guarantees that the coupling between logical qubits is mediated by virtual excitations, thereby not affecting their coherence. Moreover, the bus takes advantage of the counter-rotating terms activated by the ultrastrong coupling, enhancing the coupling between the logical qubits. This allows to perform fast two-qubit gates on nanosecond timescales. To achieve scalability, these building blocks can be arranged in an array, see Fig. 1(b), so that every qubit is connected with each other. Couplings between qubits can be switched on and off by tuning the ancillary flux qubit frequencies on and off resonance with the waveguides. Importantly, this allows the logical qubits to remain in their
FIG. 1. (a) Sketch of the Π-connector. Dark grey lines represent the coplanar waveguide resonators, $C_1$, $C_2$ and $C_3$. Red lines represent the flux qubits, $f_1$ and $f_2$, connecting the waveguides. Blue lines represent the logical qubits (transmons), $q_a$ and $q_b$. The inset inside the green square represents the connection between the flux qubit (red square) and the constriction of the center conductor of the two orthogonal waveguides (light grey). (b) An array of logical qubits (yellow disks, denoting connectors which are “ON”) at the bottom part are connected through a net of waveguides. At each node, a flux qubit tuned with the waveguides (orange disk) mediates the interaction between logical qubits (transmon here, but could also be other types). The grey disks (connector is “OFF”) denote detuned flux qubits.

optimal working point, preserving their coherence times.
I. RESULTS

The Hamiltonian describing the Π-connector in Fig. I(a) is \( \hat{H} = \hat{H}_{qb} + \hat{H}_\Pi + \hat{H}_{int} \), where \( \hat{H}_{qb} = \frac{1}{2} \sum_{i=a,b} \omega_{qi} \hat{\sigma}^{(i)}_z \) represents the logical qubits \( (\hbar = 1) \),

\[
\hat{H}_\Pi = \frac{1}{2} \sum_{i=1,2} \omega_i \hat{\sigma}^{(i)}_z + \sum_{i=1}^3 \omega_c \hat{a}^\dagger_{(i)} \hat{a}_{(i)} + \sum_{i=1,2} \lambda_s \hat{\sigma}^{(i)}_x \left( \hat{X}_i + \hat{X}_3 \right)
\]

(1)

is the Hamiltonian of the ultrastrongly coupled quantum bus, and \( \hat{H}_{int} = \lambda \left( \hat{\sigma}_x^{(a)} \hat{X}_1 + \hat{\sigma}_x^{(b)} \hat{X}_2 \right) \) represents the interaction between the logical qubits and the quantum bus. Here, \( \hat{\sigma}^{(i)}_z \) and \( \hat{\sigma}^{(i)}_x \) are Pauli operators for the logical qubits and for the flux qubits, with transition energies \( \omega_{qi} = \omega_q \) and \( \omega_i = 3 \omega_q \), respectively. We set the fundamental frequency of all resonators \( C_k \) to be \( \omega_c = 3 \omega_q \), and we denote the annihilation, creation, and quadrature operators by \( \hat{a}_{(k)}, \hat{a}^\dagger_{(k)} \), and \( \hat{X}_k = \hat{a}_{(k)} + \hat{a}^\dagger_{(k)} \), respectively. The resonators \( C_1 \) and \( C_2 \) are, respectively, connected to the resonator \( C_3 \) via the ultrastrongly coupled flux qubits \( f_1 \) and \( f_2 \), with coupling strength \( \lambda_s = 0.26 \omega_c \). The coupling strength between the logical qubits \( q_a \) (\( q_b \)) and the resonators \( C_1 \) (\( C_2 \)) is set to \( \lambda = 0.05 \omega_q \). All elements of the II-connector are detuned with respect to the logical qubits, as guaranteed by the condition \( \omega_c - \omega_q = 2 \omega_q \). Therefore, only virtual excitations mediate the qubit-qubit interaction, whose fingerprint is the avoided-level crossing shown in Fig. 2(a). At \( \omega_{q_b} = \omega_q \), the splitting \( \omega_R \) is twice the effective qubit-qubit coupling, \( \lambda_{eff} = 5.64 \times 10^{-4} \omega_q \), and the system states are the symmetric and antisymmetric superposition of \( |e,g\rangle \) and \( |g,e\rangle \), where \( |g\rangle \) and \( |e\rangle \) represent the ground and exited states of the qubits. By preparing the qubits in the state \( |e,g\rangle \) with a \( \pi \)-pulse applied to \( q_a \), we show in Fig. 2(b) that when \( t = \pi/\omega_R \) the excitation is coherently transferred from qubit \( q_a \) to qubit \( q_b \), \( |e,g\rangle \rightarrow |g,e\rangle \). Setting an interaction time \( t_{\text{swap}} = \pi/2\omega_R \), we obtain the universal \( \sqrt{i}\text{SWAP} \) gate. For \( \omega_q/2\pi = 5 \text{GHz} \), the gate time is 44.3 ns, three orders of magnitude lower than the typical transmon coherence-times \( 33 \). We note that higher-energy modes in the resonators can activate other virtual paths, possibly increasing the effective qubit-qubit coupling. Note that, since the ground state of the II-connector is a dressed state and the system is in the dispersive regime, there is a very low shift of the logical qubit transition frequency that does not affect the transfer of the excitation between qubits.

Effective coupling. As explained in the Methods section, to calculate the effective qubit-qubit coupling we perform a projection of the full Hamiltonian \( \hat{H} \) into the ground
FIG. 2. (a) Energy levels of the system as a function of the transition energy $\omega_{q_2}$, calculated in the point of the avoided level crossing resulting from the coupling between the $|e,g\rangle$ and $|g,e\rangle$ states. (b) Coherent state transfer between qubits $q_a$ and $q_b$. The plot shows the time evolution of $\langle \hat{\sigma}_z^{(a)} \rangle$ (blue) and $\langle \hat{\sigma}_z^{(b)} \rangle$ (dashed green) when a $\pi$-pulse is applied to qubit $q_a$ at $t = 0$.

state of the bus Hamiltonian $\hat{H}_\Pi$. Considering the dispersive regime between logical qubits and the bus, the effective coupling becomes

$$\lambda_{\text{eff}} = \sum_k \frac{g_k^{(1)} g_k^{(2)}}{\omega_q - \Delta E_k},$$

where $E_k$ and $|\tilde{k}\rangle$ are the eigenenergies and eigenstates of $H_\Pi$, and where $g_k^{(1)} = \lambda \langle \tilde{k} | \hat{X}_1 | \tilde{0} \rangle$, $g_k^{(2)} = \lambda \langle \tilde{k} | \hat{X}_2 | \tilde{0} \rangle$, and $\Delta E_k = E_k - E_0$ [34]. In Fig. 3, we numerically computed the effective coupling as a function of $\lambda_s$ using the full Hamiltonian $\hat{H}$, and compared it with Eq. (2). The agreement is very good, with deviations more evident for $\lambda_s > 0.26 \omega_c$ due to the approximations in the model. According to perturbation theory at sixth-order [35, 36], the virtual process that provides the main contribution to the qubit-qubit effective interaction is the one that connects the state $|e,g\rangle|0\rangle$ to $|g,e\rangle|0\rangle$ (where $|0\rangle = |g,g,0,0,0\rangle$) through states with the lowest energy differences with the initial state, $|e,g\rangle|0\rangle$. It appears clear now that the main process, Fig. 4 (red solid arrows), is the one that transfers one excitation
FIG. 3. Effective coupling calculated numerically using the full Hamiltonian $\hat{H}$ (solid blue curve), dropping the counter-rotating terms (dashed black curve) and calculated using the semi-analytical expression in Eq. (2) (red dots).

through all the elements that compose the bus. In the same diagram, it is also shown a virtual process (red dashed arrows) involving the simultaneous excitation of the flux qubit $f_1$ and the resonator $C_3$, which is activated by the counter-rotating terms in the interacting part of the bus Hamiltonian $\hat{H}_{\Pi}$. In the ultrastrong coupling regime, the counter-rotating terms become relevant and activate virtual processes that strongly boost the effective coupling. To prove this, we have numerically calculated the effective coupling after dropping the counter-rotating terms in $\hat{H}$ (see Fig. 3, dashed curve). Comparing this with the results from the full Hamiltonian (blue solid curve), we notice that $\lambda_{\text{eff}}(\lambda_s)$, calculated with the counter-rotating terms, increases much faster compared to the one calculated without it, as a function of the coupling $\lambda_s$.

Switch-off of the effective interaction. To realize a properly scalable system, it is important to be able to switch-off the interaction between arbitrary logical qubits. We achieve this by controlling the transition frequency of the ancillary flux qubit by varying the external flux $\Phi_{\text{ext}} = f \Phi_0$ threading it [21]. We set the switch-on condition at the optimal bias point, $f \to f_{\text{on}} = 0.5$, where the flux qubit has a symmetric potential energy and maximum dipole moment $M_{\text{on}}$ [37]. To switch-off the interaction we move the flux qubit away from its optimal point, by changing the external flux, $f \to f_{\text{off}}$. If we detune $f_2$ from the $\Pi$-connector in Fig. 1(a), using $f_{\text{off}} = 0.53$, the flux qubit transition-frequency becomes $\approx 55 \omega_q$, the dipole
FIG. 4. The main path (solid arrows) connecting the states \(|ge\rangle\) and \(|eg\rangle\) (blue states) through the virtual bus states (black states). The order of the labels in the black kets is \(|f_1, f_2, C_1, C_2, C_3\rangle\). The dashed arrows indicate a path due to the counter-rotating terms.

moment becomes \(M_{\text{off}} = 7.4 \times 10^{-3}M_{\text{on}}\), and the effective coupling between logical qubits becomes \(\approx 1.6 \times 10^{-6}\lambda_{\text{eff}}\) (other parameters are provided in Methods). In this regime, the logical qubits \(q_a\) and \(q_b\) can be considered decoupled.

**Interaction between a flux qubit and two orthogonal coplanar waveguides.** The ultrastrong coupling between a flux qubit and two superconducting coplanar stripline resonators has been experimentally realized [38]. However, our scheme further requires the waveguides to cross and the resonator modes not to be significantly modified by the coupling with the flux qubit. The inset in Fig. 1(a) represents a sketch of the connection between the orthogonal waveguides mediated by the ancillary flux qubit. The latter is directly connected to both the center conductor of the coplanar waveguide transmission-line resonators, see also Fig. 5 in Methods. At the insertion point, the width of the center conductor is narrower and the local inductance is larger, to enhance the coupling between the flux qubit and the resonator [26]. The three Josephson junctions forming the flux qubits must be inserted in the two tiny flux qubit arms that connect the center conductors of both waveguides. In this way, the current in the resonator flows predominantly through the center conductor constrictions of the waveguides and the resonator modes are not significantly modified. There is a small overlap between the center conductors, where a dielectric or insulating buffer material must be inserted in between to avoid contacts.

**Scalable architecture.** Figure 1(b) shows a possible scalable architecture for quantum computation using the Π-connector. In the bottom part of Fig. 1(b), we represent an array of logical qubits. In the upper part (colored background) we present the quantum bus. At each node, ancillary flux qubits can either couple (orange disks) or decouple (grey disks) to the waveguides, depending on their frequency. In this way, it is possible to control the
connectivity among arbitrary couples of logical qubits. For example, in Fig. 1(b) qubit 1 is connected to qubit 3, and qubit 2 is connected to qubit \(N\). It is also possible to connect more than two qubits simultaneously. Since the distribution of the electromagnetic field is not uniform in the resonator (the fundamental mode is a half-wave), we suggest to fabricate waveguides with progressively narrower constrictions, in order to maintain a uniform coupling for all qubits. Alternatively, one can increase the coupling strength by inserting a variety of Josephson junctions in the constrictions with a progressively increasing inductance along the waveguide \[26\].

II. DISCUSSION

By taking advantage of the large coupling between flux qubits and the modes of waveguides or LC resonators, we proposed an architecture which allows to control the coupling between distant qubits. We numerically showed that the effective coupling is boosted by the counter-rotating terms of the Rabi Hamiltonian, whose contribution become more relevant in the ultrastrong coupling regime. The switch-on and -off of the interaction between logical qubits is controlled by the magnetic fluxes threading the flux qubits, which tune their transition frequencies to the bus. We also showed that the system is scalable, as it allows to simultaneously couple several logical qubits. This architecture might lead to a new generation of quantum computer architectures controlled by elements largely detuned from the logical one, allowing to increase the complexity of the system without affecting the coherence times. A natural evolution could be the connection of a matrix of logical qubits through waveguides in a 3D circuit \[19\].

III. METHODS

**Effective coupling.** In this section, we derive an effective model to describe the dynamics of two logical qubits in contact with a quantum bus. We do this by projecting the full dynamics (which takes place in the total Hilbert space \(\mathcal{H}\) of both logical qubits and bus) into the subspace \(\mathcal{H}_{\text{eff}} = PHP\), where the bus is in the ground state. Here, \(P = \mathbb{I}_\text{qb} \otimes |\tilde{0}\rangle \langle \tilde{0}|\) denotes the projector into the ground state \(|\tilde{0}\rangle\) of the bus (\(\mathbb{I}_\text{qb}\) being the identity operator on the logical qubits).
As a first step, we decompose the total Hamiltonian \( \hat{H} \) into a “diagonal” contribution \( \hat{H}_0 \) (which preserves \( \mathcal{H}_{\text{eff}} \), i.e., \( \{ \hat{H}_0, P \} = 0 \)) and an “off-diagonal” contribution \( \hat{V} \) (for which \( \{ \hat{V}, P \} \neq 0 \)). By defining a complementary projector \( Q \) such that \( P + Q = I \), we can write

\[
\hat{H} = (P + Q)\hat{H}(P + Q) = \hat{H}_0 + \hat{V}
\]

where \( \hat{H}_0 = \hat{H}P + Q\hat{H}Q \) and \( \hat{V} = \hat{H}_{\text{int}}Q + Q\hat{H}_{\text{int}}P \). The potential \( \hat{V} \) can be explicitly written as

\[
\hat{V} = \sum_k \left[ g_k^{(1)} \hat{\sigma}_x^{(a)} (|k\rangle\langle 0| + |0\rangle\langle k|) + g_k^{(2)} \hat{\sigma}_x^{(b)} (|k\rangle\langle 0| + |0\rangle\langle k|) \right] \\
\approx \sum_k \left[ g_k^{(1)} \hat{\sigma}_-^{(a)}|k\rangle\langle 0| + g_k^{(2)} \hat{\sigma}_-^{(b)}|k\rangle\langle 0| \right] + \text{H.c.},
\]

where we made a rotating-wave approximation under the assumption that \( |g_k^{(1)}|, |g_k^{(2)}| \ll \omega_q, \Delta E_k \). We further assume to be in a dispersive regime where the detuning between the splitting of the logical qubits \( \omega_q \) and the transition energies of the bus \( \Delta E_k \) are much bigger than the couplings \( g_k^{(1)} \) and \( g_k^{(2)} \) (i.e., \( |\omega_q - \Delta E_k| \gg |g_k^{(1)}|, |g_k^{(2)}| \)). In this limit, it is possible to perturbatively define a rotating frame where the dynamics is effectively constrained in \( \mathcal{H}_{\text{eff}} \) (Schrieffer-Wolff transformation). Specifically, a change of frame \( \exp[\hat{S}] \) (for an antihermitian operator \( \hat{S} \) such that \( [\hat{H}_0, \hat{S}] = \hat{V} \)) allows to define the effective Hamiltonian

\[
\hat{H}_{\text{eff}} = Pe^{\hat{S}}\hat{H}e^{-\hat{S}}P \approx P\hat{H}_0P + \frac{1}{2}P[S, \hat{V}]P,
\]

at the lowest non-trivial order in \( S \). Specifically, by choosing

\[
S = \sum_{k>0} \left( \frac{g_k^{(1)}}{\omega_q - \Delta E_k} \sigma_+^{(a)} + \frac{g_k^{(2)}}{\omega_q - \Delta E_k} \sigma_+^{(b)} \right) |0\rangle\langle k| - \text{H.c.},
\]

and computing the commutator \( [S, \hat{V}] \) in Eq. \((4)\), we obtain the effective coupling between the logical qubits described in the main text.

**Flux qubit-resonator.** The energies and electric dipole moments were calculated considering a flux qubit composed of three Josephson junctions with energies \( E_{J1} = E_{J2} = E_J \), and \( E_{J3} = \alpha E_J \). The Hamiltonian of the flux qubit is \([37]\)

\[
H_F = E_C P_+ + \frac{E_C}{1 + 2\alpha} P_- + U(\varphi_+, \varphi_-),
\]

where \( U(\varphi_+, \varphi_-) \) represents the interaction between the qubits.
with \( U(\varphi_+, \varphi_-) = -E_J[2 \cos \varphi_+ \cos \varphi_- + \alpha \cos(2\pi f + 2\varphi_+)] \), having defined \( \varphi_+ = (\varphi_1 + \varphi_2)/2 \) and \( \varphi_- = (\varphi_1 - \varphi_2)/2 \), where \( \varphi_1 \) and \( \varphi_2 \) are the phase drops across the larger junctions. \( P_+ \) and \( P_- \) are the conjugate momenta of \( \varphi_+ \) and \( \varphi_- \). Choosing, e.g., \( E_J = 35 \ E_C, \ E_C = 33.7 \) GHz, and \( \alpha = 0.8 \), the dipole moment was determined by the matrix element \( \langle g | \sin(2\pi f + 2\varphi_+)|e \rangle \). The derivation of the flux qubit-resonator Hamiltonian \( \hat{H}_\Pi \) is standard [26], but here the voltage condition for the flux qubit (red loop in Fig. 5) is \( \sum_{i=1}^{3} \varphi_i + \Delta \psi_{C1} + \Delta \psi_{C2} = \Phi_{\text{ext}} \), where \( \Delta \psi_{C1} = \psi_{C1}(x_2) - \psi_{C1}(x_1) \) and \( \Delta \psi_{C2} = \psi_{C2}(y_1) - \psi_{C2}(y_2) \).

**FIG. 5.** Equivalent circuit diagram of the coplanar waveguides (black lines) connected to the flux qubit (red lines).

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