Photon–photon interaction under light localization in a system of conducting nanoparticles

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Received April 5, 2015; Revised August 3, 2015; Accepted August 3, 2015; Published September 26, 2015

It is shown that, under conditions of light localization in a system of scatterers, effective photon–photon interaction appears. This interaction is related to neither nonlinearity of medium nor nonlocal interaction of polarization-entangled photon pairs. It is related to the complex topology of photon trajectories. Taking into account this interaction, the scattering cross section of photon pairs is calculated. It is shown that this cross section contains only an extra degree of the small Rayleigh factor in comparison with the classic Rayleigh cross section. The proposed approach could potentially open a gate for controlling light by alternative light fluxes, eliminating the need for slow optoelectronic converters.

Subject Index D04, I81, I95

1. Introduction

Recent interest in effective photon–photon interaction was ignited by a paper published by Firstenberg and colleagues [1], in which attraction between a pair of polarization-entangled photons was observed. It is well known that photon–photon interaction in linear media is impossible in principle. On the other hand, it is not prohibited in nonlinear media if the field intensity is of sufficient magnitude. Under excitation of Rydberg states of atoms, the medium becomes nonlinear in weak fields, as shown in Ref. [1], where attraction of entangled photons was demonstrated. In the current paper, we show that some alternative process of effective photon interaction could exist. This interaction does not require any nonlinearity of a medium and is not related to quantum entanglement. This interaction could occur under conditions of light localization in a system of small non-absorbing resonance scatterers.

It was shown [2] that, even in a situation of photon scattering on a pair of resonance particles, the trajectory of a virtual photon becomes isomorphic to Antoine’s chain set (Antoine’s necklace), possessing zero topological dimension. For this reason, the trajectory acquires specific mechanical rigidity due to the singularity of the energy density along it. Analysis of Feynman’s diagrams shows that the photon mass operator interlaces rigid sections of Antoine’s necklace. As a result of such interlacing, the photon is able to localize even in this simplest system. The photon localization in a system of many resonance particles could be interpreted similarly [3]. This interpretation adds to the interpretation of localization as the creation of a bound state of a pair of virtual photons passing a closed loop on a trajectory in two alternative ways: clockwise and counterclockwise.

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Similar interlacing is also possible between Antoine’s sections of two real photons localized in a system of non-absorbing particles; one of them could pull out another one localized earlier in the system. Coincidence of the photon frequencies is not an indispensable condition. This interlacing can be considered as an alternative mechanism of effective photon–photon interaction.

2. Basic equations and results

Propagation of a photon in a densely packed system of small conducting particles at frequencies in the vicinity of the frequency of a dipole surface plasmon resonance $\omega_1$ is considered. The photon differential scattering cross section is related to an irreducible four-point vertex $\Gamma$ by the first line in Fig. 1 (see also Ref. [3]). The equation for $\Gamma$ in the most-crossed-diagram approximation [4] is represented by the second line in Fig. 1. This function describes the interference of amplitudes corresponding to two virtual photons moving in opposite directions along a closed loop on a trajectory. A mass operator $\Sigma$ (the third line in Fig. 1) is related to $\Gamma$ by the Ward identities. Thin horizontal lines represent vacuum photon propagators in a gauge with zero scalar potential, whilst the thick line is the averaged one-photon propagator defined by the Dyson equation (the sixth line in Fig. 1). Wavy lines represent wave functions of real photons, and a dotted line is an interaction potential. A circle stands for a packing factor $f$, and a coherent potential approximation [5] is used for the t-matrix of a photon scattering on a separate particle (the last line in Fig. 1).

Let us consider two photons (“black” and “red”) propagating inside the system. Two characteristic most-crossed diagrams describing the localization contribution to the differential cross section are presented in Fig. 2(a) for a case when both photons propagate independently. However, the other diagram (Fig. 2(b)) demonstrates that when both photons scatter on the same particles the processes cease to be independent.

The twice-differential cross section $d^2 \sigma/dn_1dn_2$ of cooperative photon scattering is presented in Fig. 3 ($\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors along the scattering directions of both quanta). The internal block $L$ in Fig. 3(a) represents the solution to an integral equation shown in Fig. 3(b). Flux $j_1$, representing “red” photons scattering along the direction $\mathbf{n}_1$, is related to $d^2 \sigma/dn_1dn_2$ and the intensities $I_1$ and $I_2$ of both photons by the relation $j_1 = I_1I_2d^2 \sigma/dn_1dn_2$. The classic scattering corresponding to each diagram in Fig. 2(a) yields $j = l d\sigma/dn_1$. Then, subsequent calculations can be reduced to the solution of the “four-story” equation shown in Fig. 3(b) for the irreducible eight-point vertex $L$.

An averaged one-photon propagator $D$, shown in Fig. 1, could be presented in a form similar to the vacuum propagator in the selected gauge:

$$D_{\alpha\beta}(\mathbf{r}, \mathbf{r}') = \left( \delta_{\alpha\beta} - \frac{e^2}{\varepsilon_0 \omega^2} \nabla_{\alpha} \nabla'_{\beta} \right) \exp \left( \frac{-i \omega |\mathbf{r} - \mathbf{r}'|}{c} \right),$$

where $\varepsilon$ is the effective dielectric permeability, $\omega$ is the frequency, $c$ is the light velocity in vacuum, and $\delta_{\alpha\beta}$ is the Kronecker symbol. The interaction potential of a photon with a separated particle could be presented in the form [3,6,7]

$$\pi^a_{\alpha\beta}(\mathbf{r}, \mathbf{r}') = \frac{\varepsilon - \varepsilon_0}{4\pi} \omega^2 \delta_{\alpha\beta} \delta \left( \mathbf{r} - \mathbf{r}' \right) \theta \left( R - |\mathbf{a} - \mathbf{r}| \right),$$

where $\varepsilon = 1 - \omega_0^2/\omega^2$ is the dielectric permeability of the electron gas of a particle, $\omega_0$ is the classic plasma frequency of the unbounded electron gas, $\delta(\mathbf{r})$ is the Dirac delta-function, $\theta(x)$ is the unit step function, $\mathbf{a}$ is the coordinate of a particle center, and $R$ is the particle radius.
Solving the system shown in Fig. 1 yields the following equation for $\bar{\varepsilon}$ [3]:

$$\frac{\bar{\varepsilon} - 1}{3\bar{\varepsilon}} = \frac{f}{\varepsilon + 2\bar{\varepsilon}},$$

where one of many common effective media approximations [8] could be easily recognized. Then, the solution to the equation for the $\Gamma$-block in a form of the four-time gradient of some function was
found in a similar way to the procedure described in detail in Refs. [2,3]:

\[ \Gamma_{\alpha\beta\sigma\nu}(r, r', \bar{r}, \bar{r}') = \delta_{\alpha\beta}\delta_{\sigma\nu} \delta(r-r') \varphi(r-\bar{r}) + \frac{\partial^4}{\partial \bar{r}_\alpha \partial \bar{r}_\beta \partial \bar{r}'_\sigma \partial \bar{r}'_\nu} \Phi(r, r', \bar{r}, \bar{r}'), \]

where the first term on the right-hand side represents the contribution of the first diagram in Fig. 1 for \( \Gamma' \):

\[ \varphi(r-\bar{r}) = \frac{n_0}{1-f} \omega^4 |A|^2 \int \theta(R - |a - r|) \theta(R - |a - \bar{r}|) da \]

and

\[ A = \frac{3}{4\pi} \frac{\epsilon - \bar{\epsilon}}{\epsilon + 2\bar{\epsilon}}. \]

As a result, calculation of \( \Phi \) can be reduced to the solution of a simple algebraic equation. Taking into account only the unretarded part of the propagators and neglecting components of the order of \( \omega R/c \ll 1 \), the cross section per particle could be presented as:

\[ \frac{1}{N} \frac{d^2\sigma}{dn_f} = \left( \frac{\omega R}{c} \right)^4 \frac{R^2}{1-f} \left| \frac{4\pi}{3} A \right|^4 (e_i e_f)^2 - \left( 1 + \frac{16}{\alpha^3} + \frac{192}{\alpha^6} \int_0^\alpha \frac{x^2 dx}{x^3 - 3\alpha^2 x + 2\alpha^3 - 4} \right) F(\theta), \]

where

\[ F(\theta) = \left[ \frac{(e_i k_f)^2 (e_f k_i)^2}{(k_i + k_f)^4} (1 - \delta_{k_i,-k_f}) + \frac{1 + 2 (e_i e_f)^2}{15} \delta_{k_i,-k_f} \right], \]

where \( k_i \) and \( k_f \) are the wave vectors of incident and scattered photons, \( e_i \) and \( e_f \) are the corresponding unit polarization vectors, \( \theta \) is the scattering angle, and

\[ \alpha^3 = \frac{18}{1-f} \left| \frac{\epsilon - \bar{\epsilon}}{\epsilon + 2\bar{\epsilon}} \right|^2. \]

The first component in brackets corresponds to routine Rayleigh scattering. The scattering indicatrix described by Eq. (1) is presented in Fig. 4. The angular widths of the slit and cog in the indicatrix of p- and s-polarized light, respectively, are equal to zero (in the first case, the polarization vector belongs to the scattering plane; in the second case, this vector is orthogonal to it). Enhanced scattering to the back hemisphere could be considered as one more reason for localization.
Fig. 4. Angular dependence of scattered light intensity for both polarizations. Curves 1 and 2 are the Rayleigh scattering of p- and s-polarization, respectively. Curves 3 and 4 are the same, taking into account the localization. Packing factor $f = 0.6$, $\omega / \omega_1 = 1.8$.

The integral in (1) could be calculated, however, at $\alpha > \sqrt{2}$, which corresponds to a frequency close to the frequency $\omega_1$ of the dipole surface plasmon in a separate particle; it diverges and exists only in the sense of a principal value [3]. The peculiarities of similar integrals were discussed in Refs. [2,3]. Here, the imaginary part related to bypassing the pole lying on the real axis appears. The total scattering cross section in this case is presented in the form $\sigma = \sigma_0 + (\sigma_1 + i \sigma_2)$, where $\sigma_0$ is the Rayleigh cross section, and the components inside the brackets are related to localization. Approaching the localization region causes the appearance of the component $\sigma_1$. This component describes additional scattering relating to the radiation transition to the localization state. Then, another component $\sigma_2$ appears. Localized light is considered as photons moving along closed-loop trajectories. Their number inside an arbitrary closed volume varies in time. The imaginary part of the cross section $\sigma_2$ solely describes this oscillating flux.

Regarding $d^2 \sigma / d n_1 d n_2$, a solution to the equation provided in Fig. 3(b) could be obtained in the form of an octuple gradient of some function. Then, the block $L$ in Fig. 3(b) becomes a tensor of fourth rank, depending on eight vector variables. Such a substitution reduces the determination of $L$ to the solution of some simple algebraic equation. Then, in order to obtain the cross section, one ought to join eight wave functions of real photons to $L$ and calculate the corresponding integrals. Consideration of a special case when both photons shown in Fig. 3 move along the same trajectory results in:

$$
\frac{1}{N} \frac{d^2 \sigma}{d n_1 d n_2} = \frac{1}{1 - f} \left( \frac{\omega R}{c} \right)^8 R^4 \left| \frac{\epsilon - \bar{\epsilon}}{\epsilon + 2 \bar{\epsilon}} \right|^4 \left( (e_1, e_{1f})^2 (e_2, e_{2f})^2 \right)
$$

$$
- \frac{3}{2} \left( 1 + \frac{16}{\alpha^3} + \frac{192}{\alpha^6} \int_0^\alpha \frac{x^2 dx}{x^3 - 3 \alpha^2 x + 2 \alpha^3 - 4} \right) F (\theta_1) F (\theta_2) \right)_{n_2 = n_1}.
$$

As is seen, the calculated cross section contains an extra degree of the small Rayleigh factor $(\omega R / c)^4$ as compared to the classic expression. However, the small magnitude of this factor is compensated by an extra square of the resonant factor $s = |\bar{\epsilon} (\epsilon - \bar{\epsilon}) / (\epsilon + 2 \bar{\epsilon})|$. The dependence of $s^4$ on the dimensionless frequency $\omega / \omega_1$ is presented by curve 3 in Fig. 5.
3. Conclusions

It has been shown that, under light localization conditions in a system of small non-absorbing closely packed resonance particles, the effective photon–photon interaction appears due to peculiarities of the photon trajectories. Some analysis of Feynman’s diagrams shows that the trajectory of the localized photon becomes isomorphic to Antoine’s chain set with zero topological dimension and singular energy density along it. Realization of this interaction does not require any nonlinearity of medium. This interaction is not related to the interaction of polarization-entangled photons and the condition of coincidence of frequencies of interacting photons is not required. The elastic scattering cross section taking into account this interaction contains an extra degree of the small Rayleigh factor in comparison with the classic expression. This interaction manifests itself at sufficiently large values of the packing factor and has to be close to the frequency of the dipole surface plasmon in the particle. Only in this range does the imaginary part of the effective dielectric permeability appear (see Fig. 5). This phenomenon appears as an emission of localized photons by means of other photons with different frequencies due to interlacing of the rigid Antoine’s trajectories.

The possibility of effective photon–photon interaction at small-intensity electromagnetic fields introduces the possibility of controlling light with alternative light fluxes, eliminating the need for complex and slow optoelectronic converters.

Acknowledgements

This work was supported by RSCF (No 14-03-00507).

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