Nuclear charge radii as signature for structural changes

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Abstract. The correlation of nuclear charge radii with other ground and excited state nuclear observables is considered. An empirical approach is used to deal with a large amount of experimental information, which is properly handled to obtain interesting correlations among different observables as one moves away from the line of stability. Especially the appearance of new magic numbers and/or disappearance of traditional ones as well as the onset of deformation in the region of light nuclei ($A < 30$) are discussed.

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1. Introduction

Recently, a large table of experimental charge radii $R$ that contains more than 900 isotopes has been published [1]. It is obtained by applying appropriately chosen statistical methods for analysis of two types of experimental data: i) radii changes determined from optical and Kα X-ray isotope shifts and ii) absolute radii $R = \langle r^2 \rangle^{1/2}$ measured by muonic spectra and electronic scattering experiments. The optical isotope shifts are the main source of information on mean square radii changes $\delta \langle r^2 \rangle$. Due to modern laser spectroscopic methods with their high precision and sensitivity, isotope shifts away from the line of stability are accessible. This determines the extension of the radii tabulation towards the neutron numbers far from stability.

The table of experimental charge radii $R = R(N,Z)$ can be considered from different aspects. Those include the influence of the magic nucleon numbers, of deformation and of the odd-even staggering [2]. Also a comparison of the radii trend with other nuclear observables becomes accessible. This is possible not only in isotopic but also in isotonic dependences of $R$ over the whole nuclide chart. Remarkable correlations between nuclear charge radii and other nuclear observables have been recognized in their isotopic evolution [3] for $A > 100$ where sufficient data exist to make meaningful conclusion. The authors proposed this as a simple approach that can be exploited to predict trends of behaviour in more exotic systems. In [4], our knowledge is extended over the broad region of nuclear chart by investigating the correlations of $R$ and other nuclear observables that elucidate nuclear structure and its evolution with $Z$. 
When necessary also the evolution with $N$ was additionally considered. Special attention is paid to the region of light nuclei where structural variations are better seen. The abrupt changes in the behaviour of charge radii have been compared to similar abrupt behaviour of other nuclear observables.

This paper presents brief highlights of new single $Z$ or double $(Z, N)$ magic numbers in the region of light nuclei determined in [4]. The same empirical approach is used as well to show other nuclei in this region for which the structural changes – subshell closure and onset of deformation - are reflected in a correlated behaviour of the discussed nuclear observables.

2. Observables discussed

We will search for the correlations between the nuclear charge radii and the following nuclear observables (see [3, 4]): the energies, $E_i(2^+)$, of the first excited $2^+$ state; the ratios of the energies of the first $4^+$ and $2^+$ states - $R_{42} = E_i(4^+)/E_i(2^+)$; the reduced transition probabilities $B(E2)^+ = B(E2; 2^+ \rightarrow 0^+)$ and—in some cases—the effective two nucleon shell gap $\Delta_p(\Delta_n)$ expressed as the difference of the two nucleon separation energy (see [4] and [5]). All these exhibit familiar behaviour at closed shells and at the onset of deformation. The excitation energy $E_i(2^+)$ sensitively reflects the structure of even-even nuclei, peaks at magic numbers, drops at the onset of deformation and thus leads to clear patterns over the whole nuclide chart. The ratio $R_{42}$ is an indicator of nuclear structure (although not its unambiguous determinant) that takes values < 2 for nuclei in the closed vicinity of magic numbers, $R_{42} \sim 2.0$–2.4 in vibrational nuclei and $R_{42} \sim 3.33$ in well-deformed rotors. The $B(E2; 2^+ \rightarrow 0^+)$ values have a similar behaviour as $R_{42}$. The effective two-neutron and two-proton shell gap goes toward maximum at shell closures and vanishes in deformed nuclei. In the case of light nuclei, a peak of $\Delta_p(\Delta_n)$ for $Z = N$ nuclei is dominant due to the Wigner effect that makes the $Z = N$ nuclei more tightly bound than their neighbours.

The evolution of radii with neutron or proton numbers is well known and discussed in a large amount of papers. At magic nucleon numbers, $R(N)$ and $R(Z)$-curves show characteristic kinks: the slope of radii curve goes to saturation before and increases steep after a magic number. The shape transition regions are characterized with a sharp rise of $R$. In each case, sources of empirical information are the most recent databases (see references [1, 6–7]).

The examples considered in [3, 4] and below (section 3.2) probe a variety of structural features that present a unified view of the structural evolution and, that can be applied to the study of exotic nuclei.

3. Nuclei with $Z \leq 16$

3.1. Summary of earlier empirical results

The detailed analysis performed in [3, 4] shows an excellent agreement among the evolution of nuclear charge radii, $E_i(2^+)$, $R_{42}$, $B(E2)^+$ and $\Delta_p(\Delta_n)$ for the major proton and neutron magic numbers 50 and 82. This gives an unambiguous experimental confirmation that charge radii can be considered as an additional signature for shell closures in the case of nuclei with $A > 100$. The approach has been exploited also in the regions of lower proton magic numbers 28 and 20 [4]. For some light nuclei around $(Z, N) = 20, 28$ and below an abnormal behaviour was detected. It is shown that for low $Z$ the similarity with the corresponding pictures around the traditional proton magic numbers 50 and 82 is too poor. It turns out that the abrupt changes of the magic number signatures which occurs in heavy nuclei, are either absent in light nuclei or somewhat displaced from traditional magic numbers. This is due to two reasons: i) the lack of sufficient information in the region of light nuclei and ii) the appearance of exotic properties.

The comparison of the $(Z, N)$-radii evolution with the other shell closure signatures gives an evidence that also for light nuclei $R$ can be considered as a reliable indicator for structural changes. Unambiguous experimental confirmation is obtained [4] that $Z = 6$ (8) and 14 (16) are magic-like for several $N$ values: $Z = 6$ is magic for $N = 8$; $Z = 8$ is magic not only for $N = 8$ but also for $N = 6$; $Z = 14$ is magic for $N = 16$, $Z = 16$...
\( Z = 14 \) and may be \( Z = 16 \) are magic for \( N = 20 \) [4]. Strong evidences are presented that \( ^{34}\text{Si} \) and the mirror nuclei \( ^{14}\text{C} - ^{14}\text{O} \) are new double magic-like nuclei. The predicted double-magicity of \( ^{34}\text{Si} \) is also discussed but no unambiguous evidence is found.

Special attention is paid to the mirror nuclei \( ^{30}\text{Si} \) and \( ^{30}\text{S} \). Those are very interesting due to the conflict between the theoretical predictions of [9, 10] and empirical rule of the so called “j-j” coupling [11]. The latter claims a double magicity of both nuclei. According to our previous work [4] the double magicity of \( ^{30}\text{Si} \) and \( ^{30}\text{S} \) appears to be as yet questionable contrary to the statement of [11]. This may be a consequence of the insertion of additional observable to the usually considered \( E_1(2^+) \) and \( B(E2) \) and/or the use of more recent experimental data.

3.2. Light nuclei around \( Z = 10 \)

![Figure 1](Varna2015 IOP Publishing Journal of Physics: Conference Series 724 (2016) 012032 doi:10.1088/1742-6596/724/1/012032)

**Figure 1.** Isotopic sequences of Ne and Mg: comparison of charge radii development with \( E_1(2^+) \), \( R_{4/2} \) and \( B(E2) \). Except charge radii development all curves refer to even-even nuclei. Error bars are shown only if larger than the symbols. The dashed vertical lines indicate the traditional magic number \( N = 20 \); the short dashed lines mark the anomalies at \( N = 14 \). These presentation illustrates the breakdown of \( N = 20 \) shell gap for Ne and Mg and the appearance of structural changes around \( N = 14 \) or \( N = 16 \).

Usually, rapid nuclear structure changes appear in localized regions of the nuclear chart. Most of the theoretical predictions and experimental data refer to light nuclei from neutron drip line [15]. Concerning
the nuclei around \( Z = 10 \) different theoretical predictions exist: e.g. according to [16] \( N = 14 \) is magic for \( Z = 7 \)-10, while [17] predicted magicity of \( N = 16 \) for \( Z = 10 \)-18. We test the situation from an experimental point of view using the correlation between nuclear charge radii and the observables pointed above. The charge radii evolution in the \( \text{Ne} \) [12], \( \text{Na} \) [13], and \( \text{Mg} \) [14] isotopic chains, spanning the neutron vsd shell and beyond, provide excellent examples for considering different types of abrupt changes in nuclear structure.

In the vicinity of the traditional neutron shell closure \( N = 20 \) the radii curves do not show the expected saturation but increase steeply signaling the weakening of the \( N = 20 \) magic number. This is confirmed by the evolutions of the other observables that are as well typical for an onset of deformation: an abnormal low energy of the \( 2^+ \) state, much larger \( B(E2) \) and \( R_{4/2} \) values than that expected from a shell closure. The decreased value of the effective two neutron shell gap toward \( N = 20 \) \( (\Delta_n = 2.2 \text{ MeV in Ne and } \Delta_n = 1.4 \text{ MeV in Mg}) \) is also in favour of the disappearance of the traditional neutron shell closure. As explained in [18, 19], this is caused by a deformed intruder configuration formed by promoting two neutrons from the sd shell across the \( N = 20 \) shell gap into the fp shell.

The development of the Ne and Mg radii, as well as of Na (see [13]), around a minimum at \( N = 14 \) (figure 1) holds the characteristic features of a subshell closure. For Ne the trends of radii and \( B(E2) \) are in agreement, showing a minimum at \( N = 14 \); \( R_{4/2} \) is not informative due to the lack of data at \( N = 16 \) [6]. The local “plateau” between \( N = 14 \) and \( N = 16 \) of a strongly enhanced \( E(2^+) \) correlates with the “plateau” of increased \( \Delta_n \) values \( (\Delta_n \sim 4.4 \text{ MeV}) \) suggesting a competition between \( N = 14 \) (closed \( 1\text{vd}_{5/2} \) subshell) and \( N = 16 \) (closed \( 2\nu s_{1/2} \) subshell) as candidates for magic-like neutron number for \( Z = 10 \).

In the case of Mg, the agreement between the behaviour of all observables presented in figure 1 proposes more definitely the persistence of shell closure at \( N = 14 \): the minimum of charge radii at \( N = 14 \) (if one neglect the odd-even staggering) agrees with the local maximum of \( E(2^+) \) and the local minima of \( R_{4/2} \) and \( B(E2) \). However, effective two neutron shell gap exhibits a well-defined maximum of 4.9 MeV at \( N = 16 \) instead at \( N = 14 \).

4. Outlook

The examples presented in [4] and above give an experimental evidence for new magic-like numbers, for disappearance of some traditional magic numbers and for unexpected onset of deformation. The interpretation concerning \( E(2^+) \) and \( B(E2) \) is not in itself new as namely these observables are considered in most papers. What we try to show is that sharp changes in nuclear charge radii can be an additional signature for sub-shell effects. The main focus is on light nuclei, where effects related to the shell or subshell changes are studied. Signatures of shell closure, as well as abnormal behaviour or deviations of expected one are explored and, as a result 1) new candidates for magic-like numbers are presented and 2) disappearance of other traditional magic numbers is predicted. The results confirm that a proper arrangement of raw empirical information can lead to the finding of hidden properties that show up once correlations among various observables are established. Such investigations provide important insight into the shell evolution and pose a stringent benchmark for theory.

Although the correlations point out to the existence (disappearance) of new (traditional) magic-like numbers, in some cases adding new observables to the usually discussed \( E(2^+) \) and \( B(E2) \) leads to discrepancies between different empirical data (see [4] and section 3.2). This happens even though some of the observables are related, e.g. \( E(2^+) \) and \( B(E2) \) (see e.g. figure 4b an figure 5a of Ref. [4]) as well as \( R \) and \( B(E2) \) through the quadrupole deformation. The reason is simply that different observables may differ widely in sensitivity to the specific physics, thus remaining in many cases open questions. For example, it is important to be cleared whether only the sudden change of \( E(2^+) \) toward a maximum can be considered as a signature for shell closure or it serves only as a benchmark for structural changes.
What would happen when the number of observables considered increases, e.g. may their growth switch the aspect considered over other equally important features of the nucleus?

The problem is widely discussed in the literature (see e.g. [20, 21], but no unambiguous answer is found. Though, it seems that the sudden change of a single observable does not give in itself a sufficient reason to draw convincing conclusion without further experimental evidence or proper theoretical background. This emphasizes the importance of the empirical approach applied. It lies primarily in new regions of exotic nuclei far from stability where the experimental information is very poor, usually limited to mass and energy level data. Examining the correlation among those signatures, on which information is available, it would be possible to draw implications on other observables. But this also presents a challenge because of the minimal data that will often be available, especially at the extremes of accessibility. “Therefore, just as it is essential to construct the facilities and instruments to access exotic nuclei, it is also critical to develop new approaches to extract the most physics from minimal information” [20].

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