Three-point correlation functions from pulsating strings in $\text{AdS}_5 \times S^5$

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Abstract

One of the most important problems in any conformal field theory is the calculation of three-point functions of primary operators. In this paper we provide explicit examples of correlators with two scalar operators in $\mathcal{N} = 4$ super-Yang–Mills theory at large $N$, corresponding to pulsating semiclassical strings in $\text{AdS}_5 \times S^5$, and an operator with small quantum numbers at strong coupling.

1 Introduction

An extremely active area of research in theoretical high-energy physics in recent years has been the correspondence between gauge and string theories. Following the impressive conjecture made by Maldacena $^{[1]}$ that type IIB string theory on $\text{AdS}_5 \times S^5$ is dual to $\mathcal{N} = 4$ super-Yang–Mills theory with a large number of colors, an explicit realization of the AdS/CFT correspondence was provided in $^{[2]}$. Many convincing results have been achieved thereafter, paving the way for the subject to become an indispensable tool in probing such diverse areas as the dynamics of quark-gluon plasma and high-temperature superconductivity.

A key feature of the duality is the connection between planar correlation functions of conformal primary operators in the gauge theory and correlators of corresponding vertex operators of closed strings with $S^2$ worldsheet topology. Recently, some progress was...
accomplished in the study of three- and four-point functions with two and three “heavy” vertex operators with large quantum numbers at strong coupling. The remaining operators were chosen to be various “light” states (with quantum numbers and dimensions of order one). It was shown that the large $\sqrt{\lambda}$ behavior of such correlators is fixed by a semiclassical string trajectory governed by the heavy operator insertions, and with sources provided by the vertex operators of light states.

Initially this approach was utilized in the calculation of two-point functions of heavy operators in [3]–[7]. More recently the above procedure was extended to certain three-point correlators in [8–10]. A method based on heavy vertex operators was proposed in [11]. Further developments in the computation of correlators with two string states are presented in [12]. The main goal of these investigations is elucidation of the structure of three-point functions of three semiclassical operators [13].

Recently the authors of [14] noticed that the precise formulation of such correlators should involve string energy eigenstates, which necessitates a slight modification of previous methods. Namely, one should average over all string solutions with a given energy. Although this alteration does not invalidate the results for the correlation functions obtained so far, it turns out that in the case of pulsating strings [15,16] we need to apply strictly the procedure, described in [14], in order to get the correct answer. In the present paper we consider the three-point correlation function of two heavy operators, corresponding to a pulsating string solution in $S^3 \subset S^5$ [17], and one BPS (dilaton or chiral primary) operator. We provide some limiting cases and recover known results.

The paper is organized in the following way. In Section 2 we present a brief review of the method for obtaining two-point correlation functions. Their computation in the leading semiclassical approximation is closely related to utilizing an adequate classical string solution [5–8]. If $V_{H1}(\xi_1)$ and $V_{H2}(\xi_2)$ are the two heavy vertex operators, which are inserted at the $\xi_1$ and $\xi_2$ points on the string worldsheet, the corresponding two-point correlator in the limit of large ‘t Hooft coupling is obtained from the stationary point of the action

$$\langle V_{H1}(\xi_1)V_{H2}(\xi_2) \rangle \sim e^{-I},$$  

(2.1)
where $I$ is the action of the AdS$_5 \times S^5$ string sigma model in the usual embedding coordinates

$$ I = \frac{\sqrt{\lambda}}{4\pi} \int d^2\xi \left( \partial Y_M \partial Y^M + \partial X_k \partial X_k + \text{fermions} \right), \quad (2.2) $$

$$ Y_M Y^M = Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 - Y_5^2 = -R^2, \quad X_k X_k = 1. $$

Throughout the paper we apply conformal gauge and use a worldsheet with Euclidean signature. Correspondingly, the two-dimensional derivatives are $\partial = \partial_1 + i\partial_2$, $\bar{\partial} = \partial_1 - i\partial_2$. We also work with the Euclidean continuation of AdS$_5$. The embedding, global and Poincaré coordinates of AdS$_5$ assume the following form

$$ Y_5 = R \cosh \rho e^{it}, \quad Y_1 + iY_2 = R \sinh \rho \cos \Theta e^{i\phi_1}, \quad Y_3 + iY_4 = R \sinh \rho \sin \Theta e^{i\phi_2}, \qquad (2.3) $$

$$ Y_m = \frac{R x_m}{z}, \quad Y_4 = \frac{1}{2z}(-R^2 + z^2 + x^m x_m), \quad Y_5 = \frac{1}{2z}(R^2 + z^2 + x^m x_m), $$

where $x^m x_m = x_0^2 + x_i x_i$ ($m = 0, 1, 2, 3; \ i = 1, 2, 3$).

The stationary solution satisfies the string equations of motion with singular sources given by $V_{H1}(\xi_1)$ and $V_{H2}(\xi_2)$. Utilizing the conformal symmetry of the theory, we are able to map the $\xi$-plane worldsheet to a Euclidean cylinder with $(\tau, \sigma)$ coordinates

$$ e^{\tau + i\sigma} = \frac{\xi - \xi_2}{\xi - \xi_1}. \quad (2.4) $$

Under this Schwarz–Christoffel mapping the singular solution on the $\xi$-plane goes to a smooth solution on the cylinder $[5, 6, 7]$, with $t = \kappa \tau$, where $\kappa$ is a constant parameter proportional to the string energy. The quantum numbers of the latter solution coincide with the quantum numbers of the heavy vertex operators, guaranteeing that there is no loss of information.

The considerations above can also be applied to a physical integrated vertex operator dependent on a point $x$ on the boundary of the Poincaré patch of AdS$_5$ $[3, 5]$

$$ V_H(x) = \int d^2\xi \, V_H(\xi; x), \quad V_H(\xi; x) \equiv V_H(z(\xi), x(\xi) - x, X_k(\xi)) \quad (2.5) $$

Again the semiclassical two-point correlation function $\langle V_{H1}(x_1) V_{H2}(x_2) \rangle$ is determined by the classical action evaluated on the stationary point solution. Applying the conformal mapping $[2, 3]$, we obtain the corresponding smooth spinning string solution in terms of Poincaré coordinates, with the boundary condition $[5]$

$$ \tau \rightarrow -\infty \Rightarrow z \rightarrow 0, \quad x \rightarrow x_1, \quad \tau \rightarrow +\infty \Rightarrow z \rightarrow 0, \quad x \rightarrow x_2. \quad (2.6) $$

In a similar fashion we can calculate three-point correlation functions with two heavy and one light operators $[9, 11]$

$$ G_3(x_1, x_2, x_3) = \langle V_{H1}(x_1) V_{H2}(x_2) V_L(x_3) \rangle \quad (2.7) $$

$$ = \int \mathcal{D}x^M e^{-I} \int d^2\xi_1 d^2\xi_2 d^2\xi_3 \, V_{H1}(\xi_1; x_1) V_{H2}(\xi_2; x_2) V_L(\xi_3; x_3), $$

$^1$We refer to [7] for details.
where \( \int \mathcal{D}X^{M} \) is the integral over \((Y_{M},X_{k})\). We note that the contribution of the light operator in the stationary point equations can be neglected, so that one can use the same classical string solution as in the case of the two-point function of two heavy operators. In this way we obtain 11

\[
\frac{G_{3}(x_{1},x_{2},x_{3})}{G_{2}(x_{1},x_{2})} = \int d^{2}\xi \, V_{L}(z(\xi),x(\xi) - x_{3},X_{k}(\xi)),
\]

(2.8)

where \((z(\xi),x(\xi),X_{k}(\xi))\) denote the respective string solution with the same quantum numbers as the heavy vertex operators, and with the boundary conditions in (2.6) mapped to the \(\xi\)-plane by the Schwarz–Christoffel mapping (2.4). Using the two-dimensional conformal invariance, we can also provide (2.8) in terms of the cylinder (\(\int d^{2}\sigma = \int_{-\infty}^{\infty} d\tau \int_{0}^{2\pi} d\sigma\))

\[
\frac{G_{3}(x_{1},x_{2},x_{3})}{G_{2}(x_{1},x_{2})} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} d\tau_{0} \int d^{2}\sigma \, V_{L}(z(\tau - \tau_{0},\sigma),x(\tau - \tau_{0},\sigma) - x_{3},X_{k}(\tau,\sigma)) e^{-(\Delta_{2} - \Delta_{1})\kappa\tau_{0}},
\]

(2.9)

where, as was detailed in [14], we have averaged over all solutions in AdS (parameterized by different values of \(\tau_{0}\)) with the same energy in order to obtain the needed energy eigenstate. We have denoted the conformal dimension of \(V_{H_{1}}\) with \(\Delta_{1}\) and that of \(V_{H_{2}}\) with \(\Delta_{2}\).

The global conformal \(\text{SO}(2,4)\) symmetry fixes the spacetime dependence of two- and three-point functions 2

\[
G_{2}(x_{1},x_{2}) = \frac{C_{12}}{x_{12}^{\Delta_{1} + \Delta_{2}}}, \quad x_{ij} \equiv |x_{i} - x_{j}|,
\]

(2.10)

\[
G_{3}(x_{1},x_{2},x_{3}) = \frac{C_{123}}{x_{12}^{\Delta_{1} + \Delta_{2} - \Delta_{1}}x_{13}^{\Delta_{1} + \Delta_{3} - \Delta_{2}}x_{23}^{\Delta_{2} + \Delta_{3} - \Delta_{1}}},
\]

(2.11)

where \(\Delta_{i}\) are the dimensions of corresponding operators. Choosing properly \(x_{i}\), we can suppress the dependence on \(x_{ij}\) in (2.9), and apply the prescription given in (2.9) to compute the structure constant \(C_{123}\) 9, 11. Having in mind that \(\Delta_{1} \approx \Delta_{2}\) and setting \(C_{12} = 1\) in (2.10), we determine that

\[
\frac{G_{3}(x_{1},x_{2},x_{3} = 0)}{G_{2}(x_{1},x_{2})} = C_{123} \left( \frac{x_{12}}{|x_{1}||x_{2}|} \right)^{\Delta_{3}}.
\]

(2.12)

For further details we refer the interested reader to [6, 7, 11, 14].

### 3 Three-point correlators from pulsating strings in \(\mathbb{R} \times S^{3}\)

In the present Section we use the approach outlined above for the calculation of specific three-point correlators. Without loss of generality we can fix \(x_{1} = (-1,0,0,0)\) and \(x_{2} = \)

2We assume that \(V_{H_{2}} = V_{H_{1}}^{*}\), which is valid for the correlation functions we are interested in.
(1,0,0,0), from which follows that \( R = 1 \). We consider a particular pulsating string in \( \mathbb{R} \times S^3 \subset \text{AdS}_5 \times S^5 \) [17] as the string solution that describes the semiclassical trajectory. Using that the string energy is \( E = \sqrt{\lambda \kappa} \) and the spin is \( J = \sqrt{\lambda J} \), the solution is defined as

\[
t = \kappa \tau, \quad \rho = 0, \quad \cos \theta(\tau) = a - \text{sn}\left(im a_+ \tau, \frac{a_+}{a_-}\right),
\]

(3.1)

where we have assumed the notation of [18] for Jacobi elliptic functions, and \((\theta, \varphi_1, \varphi_2)\) parameterize \( S^3 \subset S^5 \) with metric

\[
ds^2_{S^3} = d\theta^2 + \sin^2 \theta d\varphi_1^2 + \cos^2 \theta d\varphi_2^2.
\]

(3.2)

In Poincaré coordinates (2.3) the AdS part of the solution is

\[
z = \frac{1}{\cosh[\kappa(\tau - \tau_0)]}, \quad x_0 = \tanh[\kappa(\tau - \tau_0)], \quad x_i = 0,
\]

(3.3)

where we have left the integration constant \( \tau_0 \) unfixed, because we will need to average our expressions over it. It can be shown that the above solution possesses the right asymptotic behavior, namely, \( \lim_{\tau \to \pm \infty} z = 0 \) and \( \lim_{\tau \to \pm \infty} x_0 = \pm 1 \). Note that by taking \( J = 0 \) we would get the original solution for pulsating strings in \( \mathbb{R} \times S^2 \) [15].

We will proceed with the study of the corresponding three-point correlation functions with two heavy and one light operators. We will examine two choices for the light operator – dilaton or superconformal primary scalar (chiral primary operator).

### 3.1 Dilaton as light operator

It is known that the ten-dimensional dilaton field is decoupled from the metric in the Einstein frame [19]. Consequently, it is described by a free massless ten-dimensional Laplace equation in \( \text{AdS}_5 \times S^5 \). The respective string vertex operator is proportional to the worldsheet Lagrangian \((j \geq 0 \text{ is the } S^5 \text{ momentum of the dilaton})

\[
V_L(x = 0) = V_j^{(\text{dil})}(0) = \hat{c}_\Delta K_\Delta X^j \left[ (\partial x_m \tilde{\partial} x^m + \partial z \tilde{\partial} z)/z^2 + \partial X_k \tilde{\partial} X_k + \text{fermions} \right],
\]

(3.4)

where \( \hat{c}_\Delta \) is a constant determined by the normalization of the dilaton. The conformal dimension of the dilaton is \( \Delta = 4 + j \) to the leading order in the large ’t Hooft coupling expansion. The corresponding operator in the dual gauge theory is proportional to \( \text{tr}(F^2_{mn}Z^j + \ldots) \). For \( j = 0 \) it is given by the SYM Lagrangian.
From (2.9), (2.12) and (3.4) we obtain that

\[ C_{123} = 4c_\Delta \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} d\tau_0 \int_{-\infty}^{\infty} d\tau \int_{0}^{\pi/2} d\sigma K_\Delta U e^{-(\Delta_2 - \Delta_1)\kappa\tau_0} \]  

(3.5)

\[ U = X^j [\partial x_m \partial x^m + \partial z \partial z] z^2 + \partial X_k \partial X_k], \quad c_\Delta = 2^{-\Delta} \hat{c}_\Delta. \]  

(3.6)

The authors of [11] calculated the normalization constant of the dilaton \( \hat{c}_\Delta \) as

\[ \hat{c}_\Delta = \hat{c}_{4+j} = \frac{\sqrt{\lambda}}{8\pi N} \sqrt{(j+1)(j+2)(j+3)}. \]  

(3.7)

Evaluating \( U \) on the pulsating string solution (3.1), we get

\[ U = (\kappa^2 + \dot{\theta}^2 - \frac{J^2}{\sin^2 \theta} + m^2 \cos^2 \theta) \dot{e}^{ij\varphi_1} = 2m^2 \cos^2 \theta \dot{e}^{ij\varphi_1}, \]  

so that the expression in (3.5) takes the following form

\[ C_{123} = 8m^2 c_\Delta \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} d\tau_0 \int_{-\infty}^{\infty} d\tau \int_{0}^{\pi/2} d\sigma \frac{\cos^2 \theta \dot{e}^{ij\varphi_1} - j\dot{J} \tau_0}{\cosh^{4+j}[\kappa(\tau - \tau_0)]} \]  

(3.8)

where in the integral over \( \tau_0 \) we have changed the integration variable to \( \tau' = \tau_0 - \tau \). The first integral in the second line could be computed in terms of hypergeometric functions. The second integral, however, is difficult to calculate analytically due to the presence of an elliptic integral of the third kind in the exponent. Therefore, we will study the structure constant for particular values of the parameters. First, we note that when \( m = 0 \) we get the three-point function with light operator corresponding to a point-like string. In this case, as has been explained in [17], the equation of motion for \( \theta \) leads to \( \theta = \pi/2 \). Thus, it follows that \( \kappa = \dot{J} \), which means that \( E = J \) as expected for a BPS solution. It can be easily seen that if we set \( m = 0 \) in (3.9), we will indeed get a vanishing structure constant.

Next, let us concentrate on the most significant case of \( j = 0 \). It can be obtained that

\[ C_{123} = \frac{16\pi m^2}{3} \frac{c_\Delta}{\kappa} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} d\tau \cos^2 \theta \]  

\[ = \frac{16\pi m^2 a_2^2}{3} \frac{c_\Delta}{\kappa} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} d\tau \sin^2 \left(ima_+\tau, \frac{a_-}{a_+} \right). \]  

(3.9)

The resulting integral is divergent. In order to obtain a finite result, we analytically continue \( m \rightarrow -im \). We will reverse this operation in the end. We get for the integral

\[ C_{123} = \frac{16\pi m^2 a_2^2}{3} \frac{c_\Delta}{\kappa} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} d\tau \sin^2 \left(ima_+\tau, \frac{a_-}{a_+} \right) \]  

\[ = \frac{16\pi m^2 a_2^2}{3} \frac{c_\Delta}{\kappa} \lim_{T \to \infty} \frac{1}{ma_+ T} \int_{-ma_+T/2}^{ma_+ T/2} dx \sin^2 \left( x, \frac{a_-}{a_+} \right). \]  

(3.10)
The integrand is a periodic function over the real numbers, so we need to integrate over only one period in order to obtain the average

\[ C_{123} = -\frac{16\pi}{3} \frac{m^2 a_2}{\kappa} c^2 \int_{-\frac{a_2}{a_+}}^{\frac{a_2}{a_+}} \frac{dx}{x} \frac{\mathrm{sn}^2(x, \frac{a_2}{a_+})}{K(\frac{a_2}{a_+})}. \] (3.12)

We go back to real \( m \), and finally get

\[ C_{123} = \frac{16\pi}{3} \frac{m^2 a_2^2}{\kappa} c\Delta \left(1 - \frac{E(a_{2\pm})}{K(a_{2\pm})}\right). \] (3.13)

As pointed out in [10, 14], the structure constant should be proportional to the derivative of the string energy with respect to the square root of the ’t Hooft coupling

\[ C_{123} = \frac{16\pi}{3} \frac{c^2}{\kappa} \frac{\partial E(J, I_\theta, m, \sqrt{\lambda})/\partial \sqrt{\lambda}}{\partial \sqrt{\lambda}}, \] (3.14)

where \( I_\theta \) is the action variable corresponding to \( \theta \). Differentiating the expression for \( I_\theta \), obtained in [17], we are able to confirm the validity of (3.14).

Let us describe two particular cases of (3.13). If we consider the case of large energy, namely large \( \kappa = E/\sqrt{\lambda} \), we will get for the structure constant

\[ C_{123} = \frac{8\pi}{3} \frac{c\Delta}{\kappa} \left(1 - \frac{8J^2 - m^2}{8\kappa^2} - \frac{4m^2 J^2 - m^4}{16\kappa^4} + \ldots\right). \] (3.15)

Another interesting case is when \( J \ll \kappa \). Then we get

\[ C_{123} \approx \frac{16\pi}{3} \frac{c\Delta}{\kappa^2} \left(1 - \frac{E(m)/\kappa}{K(\frac{m}{\kappa})}\right). \] (3.16)

### 3.2 Superconformal primary scalar as light operator

The string state that corresponds to the chiral primary operator results from the trace of the graviton in the \( S^5 \) section of the geometry [19, 20]. As detailed in [9, 21], the bosonic part of the respective operator takes the form\(^3\)

\[ V_L(x = 0) = V_j^{(\text{CPO})}(0) = \hat{c}_\Delta K_{\Delta} X^j \left[ (\partial x_m \partial x^m - \partial z \partial z)/z^2 - \partial X_k \partial X_k \right], \] (3.17)

\[ K_{\Delta} \equiv \left(\frac{z}{z^2 + x^m x_m}\right)^\Delta, \quad X \equiv X_1 + iX_2 = e^{i\phi}, \]

\(^3\)We neglect derivative terms that will not influence our calculations since we have made the restriction \( x_1 = -x_2 \).
where $\hat{c}_\Delta$ is again given by the normalization. The corresponding operator in the dual gauge theory is the BMN operator $\text{tr}Z^j$ with dimension $\Delta = j$.

We can infer from (2.9), (2.12) and (3.17) that

$$C_{123} = 4c_\Delta \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} d\tau_0 \int_{-\infty}^{\infty} d\tau \int_0^{\pi/2} d\sigma K_\Delta U e^{-(\Delta_2-\Delta_1)\kappa\tau_0}$$

$$U = X^j [\left( \partial x_m \partial x^m - \partial z \partial \bar{z} \right)/z^2 - \partial X_k \partial \bar{X}_k], \quad c_\Delta = 2^{-\Delta} \hat{c}_\Delta,$$

where the constant $\hat{c}_\Delta$ of the superconformal scalar is

$$\hat{c}_\Delta = \hat{c}_j = \sqrt{\lambda/8\pi N} (j + 1) \sqrt{j}.$$  

The expression for $U$, evaluated on the solution (3.1), leads to

$$U = 2 \left( \frac{\kappa^2}{\cosh^2[\kappa(\tau - \tau_0)]} - \frac{J^2}{\sin^2 \theta} - m^2 \cos^2 \theta \right) e^{ij\varphi_1},$$

so that (3.18) gives

$$C_{123} = 4\pi c_\Delta \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} d\tau' \int_{-\infty}^{\infty} d\tau'' \int_{-\infty}^{\infty} \frac{d\tau_0}{\cosh^2[\kappa(\tau - \tau_0)]} \left( \frac{\kappa^2}{\cosh^2[\kappa(\tau - \tau_0)]} - \frac{J^2}{\sin^2 \theta} - m^2 \cos^2 \theta \right).$$

Analogously to the dilaton case the integrals cannot be calculated analytically, so we take $J$ to be small and consider only the first term in the resulting series. We also change the variable $\tau_0$ to $\tau' = \tau_0 - \tau$ and get

$$C_{123} = 4\pi c_\Delta \int_{-\infty}^{\infty} \frac{d\tau'}{\cosh^2(\kappa \tau')} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{d\tau''}{\cosh^2(\kappa \tau'')} \left( \frac{\kappa^2}{\cosh^2(\kappa \tau')} - m^2 \cos^2 \theta \right)$$

$$= 4\pi \kappa^2 c_\Delta \int_{-\infty}^{\infty} \frac{d\tau'}{\cosh^2(\kappa \tau')} - 4\pi m^2 c_\Delta \int_{-\infty}^{\infty} \frac{d\tau'}{\cosh^2(\kappa \tau')} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} d\tau' \cos^2 \theta$$

$$= 4\pi^3 / 2 \kappa c_\Delta \frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2} + j)} \left( \frac{E(\frac{m}{\kappa})}{K(\frac{m}{\kappa})} - \frac{1}{1 + j} \right).$$

## 4 Conclusion

The AdS/CFT correspondence has been through significant development in recent years. One of the active areas of research has been the holographic calculation of three-point functions at strong coupling. The correlation functions of three massive string states escape full comprehension so far [13], but we have uncovered almost all features of correlators containing two heavy and one light states in the semiclassical approximation [9]–[12].

In the present paper we calculated three-point correlation functions of two string and one supergravity states from string theory in $\text{AdS}_5 \times S^5$ at strong coupling, applying
the approach of [11] for computing correlators using the respective vertex operators. We examined the method, which had been correctly modified by the authors of [14], for the occasion of a particular pulsating string solution, providing some limiting cases.

One of the possible future directions for exploration is the connection of our work to recent developments in the calculation of correlation functions with heavy states based on integrability methods in $\mathcal{N} = 4$ SYM [22].

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References

[1] J. M. Maldacena, “The large $N$ limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [hep-th/9711200].

[2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. B 428, 105 (1998) [hep-th/9802109]. E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2, 253 (1998) [hep-th/9802150].

[3] A. M. Polyakov, “Gauge fields and space-time,” Int. J. Mod. Phys. A 17, 119 (2002) [hep-th/0110196].

[4] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “A semi-classical limit of the gauge/string correspondence,” Nucl. Phys. B 636, 99 (2002) [hep-th/0204051].

[5] A. A. Tseytlin, “On semiclassical approximation and spinning string vertex operators in $\text{AdS}_5 \times S^5$,” Nucl. Phys. B 664, 247 (2003) [hep-th/0304139].

[6] E. I. Buchbinder, “Energy-Spin Trajectories in $\text{AdS}_5 \times S^5$ from Semiclassical Vertex Operators,” JHEP 1004, 107 (2010) [arXiv:1002.1716].

[7] E. I. Buchbinder and A. A. Tseytlin, “On semiclassical approximation for correlators of closed string vertex operators in $\text{AdS/CFT}$,” JHEP 1008, 057 (2010) [arXiv:1005.4516].

[8] R. A. Janik, P. Surowka and A. Wereszczyński, “On correlation functions of operators dual to classical spinning string states,” JHEP 1005, 030 (2010) [arXiv:1002.4613].

[9] K. Zarembo, “Holographic three-point functions of semiclassical states,” JHEP 1009, 030 (2010) [arXiv:1008.1059].

[10] M. S. Costa, R. Monteiro, J. E. Santos and D. Zoakos, “On three-point correlation functions in the gauge/gravity duality,” JHEP 1011, 141 (2010) [arXiv:1008.1070].
[11] R. Roiban and A. A. Tseytlin, “On semiclassical computation of 3-point functions of closed string vertex operators in $\text{AdS}_5 \times S^5$,” Phys. Rev. D 82, 106011 (2010) [arXiv:1008.4921].

[12] R. Hernández, “Three-point correlation functions from semiclassical circular strings,” J. Phys. A 44, 085403 (2011) [arXiv:1011.0408]. • S. Ryang, “Correlators of Vertex Operators for Circular Strings with Winding Numbers in $\text{AdS}_5 \times S^5$,” JHEP 1101, 092 (2011) [arXiv:1011.3573]. • D. Arnaudov and R. C. Rashkov, “Semiclassical calculation of three-point functions in $\text{AdS}_4 \times CP^3$,” Phys. Rev. D 83, 066011 (2011) [arXiv:1011.4669]. • G. Georgiou, “Two and three-point correlators of operators dual to folded string solutions at strong coupling,” JHEP 1102, 046 (2011) [arXiv:1101.5181]. • J. G. Russo and A. A. Tseytlin, “Large spin expansion of semiclassical 3-point correlators in $\text{AdS}_5 \times S^5$,” JHEP 1102, 029 (2011) [arXiv:1012.2760]. • C. Park and B. Lee, “Correlation functions of magnon and spike,” Phys. Rev. D 83, 126004 (2011) [arXiv:1012.3293]. • E. I. Buchbinder and A. A. Tseytlin, “Semiclassical four-point functions in $\text{AdS}_5 \times S^5$,” JHEP 1102, 072 (2011) [arXiv:1012.3740]. • D. Bak, B. Chen and J. Wu, “Holographic Correlation Functions for Open Strings and Branes," JHEP 1106, 014 (2011) [arXiv:1103.2024]. • A. Bissi, C. Kristjansen, D. Young and K. Zoubos, “Holographic three-point functions of giant gravitons,” JHEP 1106, 085 (2011) [arXiv:1103.4079]. • D. Arnaudov, R. C. Rashkov and T. Vetsov, “Three- and four-point correlators of operators dual to folded string solutions in $\text{AdS}_5 \times S^5$,” Int. J. Mod. Phys. A 26, 3403 (2011) [arXiv:1103.6145]. • R. Hernández, “Three-point correlators for giant magnons,” JHEP 1105, 123 (2011) [arXiv:1104.1160]. • X. Bai, B. Lee and C. Park, “Correlation function of dyonic strings,” Phys. Rev. D 84, 026009 (2011) [arXiv:1104.1896]. • C. Ahn and P. Bozhilov, “Three-point Correlation functions of Giant magnons with finite size,” Phys. Lett. B 702, 286 (2011) [arXiv:1105.3084]. • B. Lee and C. Park, “Finite size effect on the magnon’s correlation functions,” Phys. Rev D 84, 086005 (2011) [arXiv:1105.3279]. • D. Arnaudov and R. C. Rashkov, “Quadratic corrections to three-point functions,” Fortschr. Phys. 60, 217 (2012) [arXiv:1106.0859]. • G. Georgiou, “SL(2) sector: weak/strong coupling agreement of three-point correlators,” JHEP 1109, 132 (2011) [arXiv:1107.1850]. • P. Bozhilov, “More three-point correlators of giant magnons with finite size,” JHEP 1108, 121 (2011) [arXiv:1107.2645]. • M. Michalcik, R. C. Rashkov and M. Schimpf, “On semiclassical calculation of three-point functions in $\text{AdS}_5 \times T^{1,1}$,” Mod. Phys. Lett. A 27, 1250091 (2012) [arXiv:1107.5795]. • P. Bozhilov, “Three-point correlators: finite-size giant magnons and singlet scalar operators on higher string levels,” Nucl. Phys. B 855, 268 (2012) [arXiv:1108.3812]. • A. Bissi, T. Harmark and M. Orselli, “Holographic 3-point function at one loop,” JHEP 1202, 133 (2012) [arXiv:1112.5075]. • P. Caputa, R. Koch and K. Zoubos, “Extremal vs. Non-Extremal Correlators with Giant Gravitons,” JHEP 1208, 143 (2012) [arXiv:1204.1724]. • H. Lin, “Giant gravitons and correlators,” JHEP 1212, 011 (2012) [arXiv:1209.6624]. • B. Lee, B. Gwak and C. Park, “Correlation functions of the ABJM model,” Phys. Rev. D 87, 086002 (2013) [arXiv:1211.5838]. • P. Bozhilov, “Leading finite-size effects on some three-point correlators in $\text{AdS}_5 \times S^5$,” Phys. Rev. D 87, 066003 (2013) [arXiv:1212.3485]. • G. Georgiou, B. Lee and C. Park, “Cor-
relators of massive string states with conserved currents,” JHEP 1303, 167 (2013) [arXiv:1301.5092].

C. Kristjansen, S. Mori and D. Young, “On the Regularization of Extremal Three-point Functions Involving Giant Gravitons” [arXiv:1507.03965].

• T. Klose and T. McLoughlin, “A light-cone approach to three-point functions in AdS$_5 \times S^5$,” JHEP 1204, 080 (2012) [arXiv:1106.0495].

S. Ryang, “Extremal Correlator of Three Vertex Operators for Circular Winding Strings in AdS$_5 \times S^5$,” JHEP 1111, 026 (2011) [arXiv:1109.3242].

• R. A. Janik and A. Wereszczynski, “Correlation functions of three heavy operators: The AdS contribution,” JHEP 1112, 095 (2011) [arXiv:1109.6262].

• Y. Kazama and S. Komatsu, “On holographic three point functions for GKP strings from integrability,” JHEP 1201, 110 (2012) [arXiv:1110.3949].

E. I. Buchbinder and A. A. Tseytlin, “Semiclassical correlators of three states with large $S^5$ charges in string theory in AdS$_5 \times S^5$,” Phys. Rev. D 85, 026001 (2012) [arXiv:1110.5621].

• J. Minahan, “Holographic three-point functions for short operators,” JHEP 1207, 187 (2012) [arXiv:1206.3129].

• T. Bargheer, J. Minahan and R. Pereira, “Computing Three-Point Functions for Short Operators,” JHEP 1403, 096 (2014) [arXiv:1311.7461].

• Y. Kazama and S. Komatsu, “Three-point functions in the SU(2) sector at strong coupling,” JHEP 1403, 052 (2014) [arXiv:1312.3727].

• J. Minahan and R. Pereira, “Three-point correlators from string amplitudes: Mixing and Regge spins,” JHEP 1504, 134 (2015) [arXiv:1410.4746].
Type IIB Supergravity on AdS$_5 \times S^5$ and Three-point Functions in SYM$_4$ at Large $N$,” Phys. Rev. D 61, 064009 (2000) [hep-th/9907085]. • S. Lee, “AdS$_5$/CFT$_4$ Four-point Functions of Chiral Primary Operators: Cubic Vertices,” Nucl. Phys. B 563, 349 (1999) [hep-th/9907108].

[21] D. E. Berenstein, R. Corrado, W. Fischler and J. M. Maldacena, “The operator product expansion for Wilson loops and surfaces in the large $N$ limit,” Phys. Rev. D 59, 105023 (1999) [hep-th/9809188].

[22] J. Escobedo, N. Gromov, A. Sever and P. Vieira, “Tailoring Three-Point Functions and Integrability,” JHEP 1109, 028 (2011) [arXiv:1012.2475]. • J. Escobedo, N. Gromov, A. Sever and P. Vieira, “Tailoring Three-Point Functions and Integrability II. Weak/strong coupling match,” JHEP 1109, 029 (2011) [arXiv:1104.5501]. • J. Caetano and J. Escobedo, “On four-point functions and integrability in $\mathcal{N} = 4$ SYM: from weak to strong coupling,” JHEP 1109, 080 (2011) [arXiv:1107.5580]. • O. Foda, “$\mathcal{N} = 4$ SYM structure constants as determinants,” JHEP 1203, 096 (2012) [arXiv:1111.4663]. • G. Georgiou, V. Gili, A. Grossardt and J. Plefka, “Three-point functions in planar $\mathcal{N} = 4$ super Yang Mills Theory for scalar operators up to length five at the one-loop order,” JHEP 1204, 038 (2012) [arXiv:1201.0992]. • N. Gromov and P. Vieira, “Quantum Integrability for Three-Point Functions,” [arXiv:1202.4103]. • G. Grignani and A. Zayakin, “Matching Three-point Functions of BMN Operators at Weak and Strong coupling,” JHEP 1206, 142 (2012) [arXiv:1204.3096]. • I. Kostov, “Three-point function of semiclassical states at weak coupling,” J. Phys. A 45, 494018 (2012) [arXiv:1205.4412] • G. Grignani and A. Zayakin, “Three-point functions of BMN operators at weak and strong coupling II. One loop matching,” JHEP 1209, 087 (2012) [arXiv:1205.5279]. • A. Bissi, G. Grignani and A. Zayakin, “The SO(6) Scalar Product and Three-Point Functions from Integrability,” [arXiv:1208.0100]. • J. Caetano and J. Toledo, “$\chi$–Systems for Correlation Functions,” [arXiv:1208.4548]. • O. Foda, Y. Jiang, I. Kostov and D. Serban, “A tree-level 3-point function in the su(3)-sector of planar $\mathcal{N} = 4$ SYM,” JHEP 1310, 138 (2013) [arXiv:1302.3539]. • W. Schulgin and A. Zayakin, “Three-BMN Correlation Functions: Integrability vs. String Field Theory. One-Loop Mismatch,” JHEP 1310, 053 (2013) [arXiv:1305.3198]. • Y. Jiang, I. Kostov, F. Loebbert and D. Serban, “Fixing the Quantum Three-Point Function,” JHEP 1404, 019 (2014) [arXiv:1401.0384]. • J. Caetano and T. Fleury, “Three-point functions and su(1|1) spin chains,” JHEP 1409, 173 (2014) [arXiv:1404.4128]. • Y. Kazama, S. Komatsu and T. Nishimura, “Novel construction and the monodromy relation for three-point functions at weak coupling,” JHEP 1501, 095 (2015) [arXiv:1410.8533]. • Z. Bajnok and R. A. Janik, “String field theory vertex from integrability,” JHEP 1504, 042 (2015) [arXiv:1501.04533]. • B. Basso, S. Komatsu and P. Vieira, “Structure Constants and Integrable Bootstrap in Planar $\mathcal{N} = 4$ SYM Theory” [arXiv:1505.06745].