Effect of sound-absorbing coatings on the disturbance evolution in a flow of a mixture of vibrationally excited gases

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Abstract. The flow around a solid plate and a plate with a sound-absorbing coating at a non-zero angle of attack in a hypersonic flow ($M_e=8.44$) of a mixture of vibrationally excited carbon dioxide and nitrogen is considered. Numerical simulations are performed by solving two-dimensional unsteady Navier–Stokes equations with a two-temperature model of relaxing flows. The vibrational energy as a function of time is defined by the Landau–Teller equation. A skeleton model, which is a set of square elements arranged in a staggered order, is used for simulating the porous coating made of foamed nickel with a porosity coefficient of 95%. The distance between the elements is equal to the pore diameter of the real sound-absorbing material. Data on the evolution of disturbances on the solid plate and on the plate with the sound-absorbing coating are presented for various angles of attack and CO$_2$ concentrations in the mixture. The experimental and calculated data on pressure fluctuations on the plate surfaces are found to be in good agreement. The effects of various parameters of the sound-absorbing coating (depth, length, and location at the flat plate) are considered. It is shown that the sound-absorbing coating significantly reduces the intensity of pressure fluctuations on the plate surface as compared to the solid surface (up to 50% depending on the length and location of the sound-absorbing coating).

1. Introduction

The possibility of controlling the intensity of disturbances in boundary layers and the transition to turbulence is one of the important aspects of the development of perspective hypersonic aircraft. Real gas effects associated with excitation of vibrational degrees of freedom of molecules and flow nonequilibrium are manifested under real flight conditions, where high speeds and temperatures are observed. Real gas properties can significantly affect the generation and evolution of disturbances in the boundary layer and, as a consequence, the transition to turbulence. Nowadays large scientific experience has been accumulated in studying the effect of physical and chemical processes in gases on the mean flow around bodies at hypersonic velocities [1]. However, there are only a few papers on stability of such flows (e.g., [2-4]).

Disturbances formed in the shock layer are carried downstream and affect the evolution of disturbances and the laminar-turbulent transition in the hypersonic boundary layer on a model. One of the known methods for controlling the laminar-turbulent transition in moderately hypersonic flows is the method of applying a sound-absorbing coating onto the surface [5]. The method of sound-absorbing coatings has proved its effectiveness for low-temperature flows at $M_e=21$ [6]. It is shown [7] that the method of sound-absorbing coatings is also effective in hypersonic flows of vibrationally excited carbon dioxide. However, the intensity of relaxation processes and their influence on the evolution of disturbances in a mixture with other gases can differ from the processes in pure CO$_2$.

The present research is focused on experiments conducted in the IT-302M hot-shot wind tunnel based at ITAM SB RAS [4] where the characteristics of the mean flow and pressure fluctuations on the surface of a solid plate and a plate with a porous insert made of foamed nickel (a porosity coefficient of 95%) were measured in mixtures of air and CO$_2$. The rectangular sound-absorbing insert was
located on a plate of length \( L = 200 \text{ mm} \), width of 50 mm, and thickness of 25 mm at a distance of 90 mm from the leading edge. The depth of the porous insert did not exceed the plate thickness.

The flow of a mixture of vibrationally excited carbon dioxide and nitrogen past a flat plate and a plate with a sound-absorbing coating at a nonzero angle of attack under the action of external acoustic disturbances was numerically simulated.

2. Computational domain and skeleton model of a porous material

The computational domain in numerical simulations of flow around the solid plate is a rectangle, and its lower boundary coincides with the plate surface, where the no-slip condition and the condition of a constant surface temperature \( T_w = 300 \text{ K} \) are imposed. The left and upper boundaries are subjected to the free-stream conditions (pressure \( P_\infty \), Mach number \( M_\infty \), temperature \( T_\infty \), and vibrational energy of carbon dioxide molecules \( e_{\text{vib}} \)). The exhaustion condition is imposed on the right boundary and on the lower boundary that is not occupied by the plate surface. The computational domain height is chosen from the condition that the bow shock wave does not interact with the upper boundary. The computational grid consists of rectangular cells 0.18 mm \( \times \) 0.1 mm (the number of cells for the plate length of 200 mm is 322000). The computational domain and regular grid were constructed in the WORKBENCH program.

In the problem of the flow past the plate with a porous sound-absorbing coating, the computational domain is supplemented with a subdomain where the skeleton of a porous material made of foamed nickel with a porosity coefficient of 0.95 is directly simulated. The skeleton model is a set of square elements arranged in a staggered order. The distance between the elements is determined by the pore size of the real high-porosity cellular material (the pore diameter is d = 2 mm). The size of the square elements (0.4 mm \( \times \) 0.4 mm) was calculated in accordance with the porosity coefficient of 95% (the skeleton was only 5% of the area of the porous insert). The computational domain is covered by regular rectangular mesh inside the porous zone. The upper boundary of the porous zone is permeable. The no-slip condition and the condition of a constant surface temperature \( T_w = 300 \text{ K} \) are set on other boundaries, including the skeleton walls. The total number of cells in the porous zone is 26419 for the insert with a length of 80 mm and depth of 14 mm.

3. Thermodynamic model of the flow

A system of two-dimensional unsteady Navier–Stokes equations (1)-(3), which is supplemented with the Mendeleev-Clapeyron equation (4) and with the conservation equation of vibrational energy (5) for each vibrational degree of freedom of \( \text{CO}_2 \) molecules (see below) is solved. The viscosity is calculated by Sutherland’s formula; the thermal conductivity is found from the kinetic theory of gases; the heat capacity is determined as a function of temperature within the framework of the thermally perfect gas model:

\[
\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{u}) = 0
\]  
\[
\frac{\partial}{\partial t} (\rho \vec{u}) + \nabla (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \tau \mu
\]  
\[
\frac{\partial}{\partial t} (\rho e_t) + \nabla [\rho (\rho e_t + p)] = (\lambda \nabla T_{tr} + \tau_{\mu} \vec{u}) + \nabla \tau_{\mu} - \sum_{i=1}^{4} q_{tr}^{n} (\text{CO}_2 - \text{CO}_2) - \sum_{i=1}^{4} q_{tr}^{n} (\text{CO}_2 - \text{N}_2)
\]  
\[
p = \rho R T_{tr} \sum_{i=1,2} Y_i
\]  
\[
\frac{\partial}{\partial t} (\rho e_t^n) + \nabla (\rho u e_t^n) = q_{tr}^{n} (\text{CO}_2 - \text{CO}_2) + q_{tr}^{n} (\text{CO}_2 - \text{N}_2), \quad n = 1, ..., 4
\]

Here \( \rho, p, \) and \( \lambda \) are the density, pressure, and thermal conductivity, \( \vec{u} \) is the velocity vector, \( T_w \) is the temperature of translational-rotational degrees of freedom, \( n \) is the mode number of vibrational degrees of freedom, \( \tau_{\mu} \) is the viscous stress tensor, \( e_t \) is the gas energy, \( R \) is the universal gas constant, and \( Y_i \) is the mole fraction of the \( i \)-th component of the mixture.
The model of a thermally perfect gas is considered in this paper. The heat capacity of the gas depends on the temperature \( c_v = f(T) \) due to excitation of degrees of freedom of carbon dioxide molecules. If the vibrational relaxation time \( \tau_v \) is much shorter than the time of convective changes in flow parameters \( \tau_t \), the gas is considered to be equilibrium. If these times are comparable \( (\tau_v \sim \tau_t) \), then the gas is nonequilibrium, and it is necessary to take into account the time of energy exchange between the translational-rotational and vibrational degrees of freedom of molecules.

Four equations (5), which describe the conservation of vibrational energy for each vibrational degree of freedom of \( \text{CO}_2 \), are added to the Navier–Stokes equations (1)-(3) in the present paper to take into account the effect of vibrational relaxation of \( \text{CO}_2 \) molecules. The energy exchange between the vibrational and translational-rotational degrees of freedom of carbon dioxide molecules with a finite relaxation time is taken into account by supplementing Eqs. (5) with source terms \( q_v \) (7), (8) calculated by using the Landau–Teller equation, which includes the vibrational relaxation times of \( \text{CO}_2 \) molecules in collisions with \( \text{CO}_2 \) molecules [8] and in collisions with \( \text{N}_2 \) molecules [9]:

\[
q_v^{n(\text{CO}_2-\text{CO}_2)} = \rho(e_v^{eqn} - e_v^n)\left(\frac{\tau_v^{(\text{CO}_2-\text{CO}_2)}}{\tau_v^{n(\text{CO}_2-\text{CO}_2)}}\right)^{-1}
\]

\[
q_v^{n(\text{CO}_2-\text{N}_2)} = \rho(e_v^{eqn} - e_v^n)\left(\frac{\tau_v^{(\text{CO}_2-\text{N}_2)}}{\tau_v^{n(\text{CO}_2-\text{N}_2)}}\right)^{-1}
\]

Here \( q_v \) is the energy flux between the translational and vibrational degrees of freedom, \( \tau_v \) is the vibrational relaxation time, \( e_v^{eq} \) is the equilibrium vibrational energy of the gas, and \( e_v \) is the nonequilibrium vibrational energy of gas. The source terms (7) and (8) with the opposite sign are added to the equation of translational-rotational energy (3) to avoid disturbing of the energy balance of the system of the Navier–Stokes equations.

The problem of the evolution of disturbances was solved in two stages. The first stage is the numerical simulation of a hypersonic flow past a plate without disturbances (steady problem). At the second stage, acoustic disturbances of the incoming flow are introduced into the computational domain, and an unsteady problem is solved until the unsteady solution reaches a periodic regime.

4. Numerical simulation of the flow of a mixture of vibrationally excited gases past a solid plate

A parametric study of the flow of a gas mixture of \( \text{CO}_2 \) (0.44) and \( \text{N}_2 \) (0.56) \( (P_c=374.5 \text{ Pa}, M_c=8.44, \text{Re}_c=13.6\times10^6 \text{ m}^{-1}) \) past a plate with a length \( L=0.4 \text{ m} \) at various angles of attack \( \alpha=5°-20° \) and various frequencies of external acoustic disturbances \( f=40-160 \text{ kHz} \) is performed.

Figure 1 shows the root-mean-square (RMS) pressure fluctuations on the plate \( (L=400 \text{ mm}) \) aligned at the angles of attack \( \alpha=5° \) and \( 20° \). One can see not only the disturbance growth regions, but also the regions of local maximums and minimums of the disturbance intensity.

![Figure 1](image)

Figure 1. RMS pressure fluctuations on the surface of the solid plate under the action of a fast acoustic wave with the frequency \( f=120 \) (a), and 160 kHz (b); \( \alpha=5° \) (1), 10.2° (2), 15° (3), and 20° (4); \( M_c=8.44 \) and \( \text{Re}_c=13.6\times10^6 \text{ m}^{-1} \).
The good agreement between the experimental and numerical data of the growth rates of the disturbances on the plate surface (the ratio of the amplitudes of pressure fluctuations in two positions, near the model nose \( x_1 = 80 \) mm and at the plate end \( x_2 = 180 \) mm, i.e. \( p'_2/p'_1 \)) is demonstrated in Figure 2a. Figure 2b shows that the growth rate of disturbances as a function of the external acoustic disturbance frequency decreases with increasing angle of attack.

![Figure 2. Growth rates of pressure fluctuations on the surface of the solid plate under the action of fast acoustic waves (\( M_\infty = 8.44 \) and \( Re_1 = 13.6 \times 10^6 \) m\(^{-1}\)). (a) – comparison with the experiment for \( \alpha = 10.2^\circ \): experimental data for the mixture of CO\(_2\) and air (1); calculated data for the mixture of CO\(_2\) and N\(_2\) (2); (b) calculated data: for \( \alpha = 5^\circ \) (3), 10.2° (2), \( \alpha = 15^\circ \) (4), and \( \alpha = 20^\circ \) (5).](image)

The intensity of vibrational relaxation of carbon dioxide molecules depends on the CO\(_2\) concentration in the mixture, because the equilibrium between the translational-rotational and vibrational degrees of freedom is established through collisions between gas molecules. In this paper, the effect of the CO\(_2\) concentration in the mixture with N\(_2\) (both gases are vibrationally excited) on the mean flow characteristics and the evolution of disturbances in the shock layer on the solid plate at the angle of attack \( \alpha = 10.2^\circ \) is investigated.

![Figure 3. RMS pressure fluctuations on the surface of the solid plate under the action of a fast acoustic wave with the frequency \( f = 120 \) (a) and 160 kHz (b) for different concentrations of CO\(_2\) in the mixture: 0.22 (1), 0.44 (2), and 0.88 mol (3) (\( M_\infty = 8.44 \) and \( Re_1 = 13.6 \times 10^6 \) m\(^{-1}\)).](image)

Figure 3 shows the distributions of the RMS pressure fluctuations on the plate surface for various concentrations of CO\(_2\) in the mixture with N\(_2\) under the action of a fast acoustic wave with the frequency \( f = 120 \) and 160 kHz. The excitation and nonequilibrium of the vibrational degrees of freedom of CO\(_2\) molecules are more pronounced in the flow past a plate with a higher content of carbon dioxide in the mixture. Figure 3 shows that this leads to rapid enhancement of the disturbance amplitude along
the plate; however, the maximum of the pressure fluctuation amplitude decreases in magnitude and is shifted toward the leading edge of the plate. This behavior of pressure fluctuations testifies to a damping effect of vibrational relaxation of CO$_2$ molecules, similar to that observed in [2].

5. Effect of the sound-absorbing coating on the mean flow characteristics and on the evolution of disturbances

Investigations of the evolution of disturbances on a solid plate are necessary for solving the problem of the effect of the sound-absorbing coatings aimed at its optimization. Sound-absorbing inserts 80 mm long and 14 mm deep are included in the computational domain at distances of 90 mm (insert I), 200 mm (insert II), and 270 mm (insert III) from the leading edge of the plate. These sound-absorbing zones are simulated by using a skeleton model (see Section 2). The calculations are performed for the mixture of CO$_2$ (0.44) and N$_2$ (0.56) ($P_\infty$=374.5 Pa, $M_\infty$=8.44, $Re_1$=13.6×10$^6$ m$^{-1}$, and $T_\infty$=215 K) for the angle of attack $\alpha$=10.2°.

The results of the mean flow simulation show that the porous coatings practically do not affect the mean flow characteristics. The translational temperature profiles ($x$=0.25 m) are plotted in Figure 4 for the solid plate and the plate with a sound-absorbing coating. The section $x$=0.25 m is located downstream of the coating for the plate with insert I, in the porous region for the plate with insert II, and upstream of the porous region on the plate with insert III. It can be seen that the temperature profiles on the sound absorbing plate coincide with the profiles on the solid plate in all cases. This fact suggests that the porous coating practically does not disturb the structure of the mean flow.

![Figure 4](image-url). Computed translational temperature profiles on the solid plate (1) and on the plate with a sound-absorbing coating (2) in the section $x$=0.25m: (a) plate with insert I; (b) plate with insert II; (c) plate with insert III ($M_\infty$=8.44 and $Re_1$=13.6×10$^6$ m$^{-1}$).

The sound-absorbing inserts I and II are located in the region of the growth of disturbances for the plate at the angle of attack 10.2°, whereas insert III is located in the region of the maximum of pressure fluctuations. In all three cases, the sound-absorbing coatings effectively reduce the intensity of pressure fluctuations at frequencies higher than 80 kHz (Figure 5). However, the sound-absorbing coatings located closer to the maximum of the disturbance amplitude are more effective in reducing the intensity of pressure fluctuations.
Figure 5. RMS pressure fluctuations on the plate surface under the action of a fast acoustic wave with the frequency $f=120$ (a) and $160$ kHz (b); solid plate (1), plate with insert I (2), plate with insert II (3), and plate with insert III (4) ($M_\infty=8.44$ and $Re_1=13.6\times10^6$ m$^{-1}$).

Two cases are considered to investigate the influence of the porous coating length: the flow past a plate with insert II 80 mm long located in the region of the maximum disturbances and with insert IV with a doubled length (160 mm). Figure 6a shows the field of instantaneous pressure fluctuations on the plate with the sound-absorbing porous coating IV. It can be seen that the intensity of pressure fluctuations is significantly reduced due to interaction with the porous zone. A quantitative comparison of the effect of the porous coating length on the evolution of disturbances is shown in Figure 6b. It is seen that the porous coating placed in the region of the local maximum of disturbances effectively reduces pressure fluctuations (in particular, up to 95% on the surface of the plate with the porous coating IV).

Figure 6. (a) Fields of instantaneous pressure fluctuations with the frequency $f=160$ kHz on the plate with the porous coating IV; (b) RMS pressure fluctuations on the solid plate surface (1), on the plate with the porous coating II (2), and on the plate with the porous coating IV (3) ($M_\infty=8.44$ and $Re_1=13.6\times10^6$ m$^{-1}$).

6. Conclusions

The flow of a mixture of carbon dioxide and nitrogen past a solid plate and a plate with a sound-absorbing coating at a nonzero angle of attack was numerically simulated for experimental conditions in the IT-302M hot shot wind tunnel based at ITAM SB RAS. Two channels of vibrational relaxation of CO$_2$ molecules (in collisions with CO$_2$ molecules and N$_2$ molecules) were taken into account within the framework of the two-temperature model of relaxing flows.

Data on the evolution of disturbances on the solid plate in a hypersonic flow of a vibrationally excited mixture of carbon dioxide and nitrogen were obtained for different angles of attack, CO$_2$ concentrations in the mixture, and frequencies of the external acoustic disturbance. The growth rate and intensity of pressure fluctuations on the solid plate surface were demonstrated to increase with increas-
ing angle of attack. An increase in the CO\textsubscript{2} concentration in the mixture was found to enhance the thermal nonequilibrium, and the intensity of pressure fluctuations on the solid plate surface decreased.

The calculated and experimental data on pressure fluctuations on the solid plate surface were found to agree well.

A skeleton model of the porous medium, which is a set of square elements arranged in a staggered order, was implemented for numerical simulations of the flow past a plate with a sound absorbing coating made of a high-porosity cellular material. The distance between the elements was equal to the diameter of the real porous sound-absorbing coating (d=2 mm).

It was shown that the sound-absorbing coating appreciably reduces the pressure fluctuations on the surface. Inserts located in regions of the local maximum or of intensely growing disturbances were found to be the most effective ones. In addition, suppression of disturbances is greatly enhanced (up to 95\%) as the porous region length increases. Thus, the method of porous sound-absorbing coatings ensures effective suppression of pressure fluctuations in the flow of vibrationally excited gases.

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