Cellular Automaton Approach to Pedestrian Dynamics - Theory

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Abstract. We present a 2-dimensional cellular automaton model for the simulation of pedestrian dynamics. The model is extremely efficient and allows simulations of large crowds faster than real time since it includes only nearest-neighbour interactions. Nevertheless it is able to reproduce collective effects and self-organization encountered in pedestrian dynamics. This is achieved by introducing a so-called floor field which mediates the long-range interactions between the pedestrians. This field modifies the transition rates to neighbouring cells. It has its own dynamics (diffusion and decay) and can be changed by the motion of the pedestrians. Therefore the model uses an idea similar to chemotaxis, but with pedestrians following a virtual rather than a chemical trace.

1 Introduction

Efficient computer simulations of large crowds consisting of hundreds or thousands of individuals require simple models which nevertheless provide an accurate description of reality. One such class of models, so-called cellular automata (CA), has been studied in statistical physics for a long time [1, 2].

In CA space, time and state variables are discrete which makes them ideally suited for high-performance computer simulations. However, CA modelling differs in several respects from continuum models. These are usually based on coupled differential equations which often can not be treated analytically. One has to solve them numerically and therefore the equations have to be discretized. In general, only space and time variables become discrete whereas the state variable is still continuous. One important point is now that CA are discrete from the beginning and that this discreteness is already taken into account in the definition of the model and its dynamics. This allows to obtain the desired behaviour in a much simpler way. On the other hand, the numerical solution of (discretized) differential equations is only accurate in the limit $\Delta x, \Delta t \to 0$. This is different in the CA where $\Delta x$ and $\Delta t$ are finite and accurate results can be obtained since the rules (dynamics) are designed such that the discreteness is an important part of the model.
In order to achieve complex behaviour in a simple fashion one often resorts to a stochastic description. A realistic situation seldomly can be described completely by a deterministic approach. Already minor events can lead to a very different behaviour due to the complexity of the interactions involved. For the problem of pedestrian motion this becomes evident e.g. in the case of a panic where the behaviour of people seems almost unpredictable. But also for “normal” situations a stochastic component in the dynamics can lead to a more accurate description of complex phenomena since it takes into account that we usually do not have full knowledge about the state of the system and its dynamics. Here one has to keep in mind that in general some sort of average over different realizations of the process (e.g. different sequences of random numbers) has to be taken. Even if there are single realizations which yield unrealistic behaviour the average process will be a good description of the real process. Furthermore a stochastic description allows to answer questions like “What is the probability that the evacuation of this building will take longer than 3 minutes?” in a natural way.

In the following we will present a detailed description of the model and the basic philosophy of our approach. Applications are presented in Part II [3].

2 Other Modelling Approaches

During the last decade considerable research has been done on the topic of highway traffic using methods from physics [4–10]. Cellular automata inspired by the pioneering works [11, 12] compose by now an important class of models. Most studies have been devoted to one-dimensional systems, where several analytic approaches exist to calculate or approximate the stationary state.

On the other hand, pedestrian dynamics has not been studied as extensively as vehicular traffic, especially using a cellular automata approach. One reason is probably its generically two-dimensional nature. In recent years, continuum models have been most successful in modelling pedestrian dynamics. An important example are the social force models (see e.g. [7, 10, 13] and references therein). Here pedestrians are treated as particles subject to long-ranged forces induced by the social behaviour of the individuals. This leads to (coupled) equations of motion similar to Newtonian mechanics. There are, however, important differences since, e.g., in general the third law (“actio = reactio”) is not fulfilled.

In contrast to the social force models our approach is closer in spirit to the general strategy of modelling (elementary) forces on a microscopic level by the exchange of mediating particles which are bosons. It is therefore similar to active walker models [14, 15] used so far mainly to describe trail formation, chemotaxis (see [16] for a review) etc. Here the walker leaves a trace by modifying the underground on his path. This modification is real in the sense that it could be measured in principle. For trail formation, vegetation is destroyed by the walker and in chemotaxis he leaves a chemical trace. In contrast, in our model the trace is virtual. Its main purpose is to transform effects of long-ranged interactions

\[1\] In the following we use “pedestrian” and “particle” interchangeably.
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(e.g. following people walking some distance ahead) into a local interaction (with
the “trace”). This allows for a much more efficient simulation on a computer.

Cellular automata for pedestrian dynamics have been proposed in [17–19].
These models can be considered as generalizations of the Biham-Middleton-
Levine model for city traffic [12]. Most works have focussed on the occurrence of
a jamming transition as the density of pedestrians is increased. All models have
only nearest-neighbour interactions, except for the generalization proposed in
[19] which is used for analyzing evacuation processes on-board passenger ships.
The other models use a kind of “sublattice-dynamics” which distinguishes be-
tween different types of pedestrians according to their preferred walking direc-
tion. Such an update is not easy to generalize to more complex situations where
the walking direction can change. To our knowledge so far most other collective
effects encountered empirically [7, 10, 20–23] have not been reproduced using
these models. Another discrete model has been proposed earlier by Gipps and
Marksjö [24]. This model is somewhat closer in spirit to our model than the cel-
lular automata approaches of [17–19] since the transitions are determined by the
occupancies of the neighbouring cells. However, this model can not reproduce
all collective effects either. In [25] a discretized version of the social force model
has been introduced. The repulsive potentials by the pedestrians are stored in a
global potential, with pedestrians reacting to the gradients of this global poten-
tial. Although this model is able to reproduce collective effects it is not flexible
enough to treat individual reactions to other pedestrians, and collision-avoidance
is not always guaranteed for velocities greater than 1.

3 Basic Principles of the Model

First we discuss some general principles we took into account in the develop-
ment of our model [26]. The implementation of the interactions between the
pedestrians uses an idea similar to chemotaxis. The pedestrians leave a virtual
trace which then influences the motion of other pedestrians. This allows for
a very efficient implementation on a computer since now all interactions are
local. The transition probabilities for all pedestrians just depend on the occu-
pation numbers and strength of the virtual trace in his neighbourhood, i.e. we
have translated the long-ranged spatial interaction into a local interaction with
“memory”. The number of interaction terms in other long-ranged models, e.g.
the social-force model, grows proportionally to the square of the number of par-
ticles whereas in our model it grows only linearly.

The idea of a virtual trace can be generalized to a so-called floor field. This
floor field includes the virtual trace created by the pedestrians as well as a static
component which does not change with time. The latter allows to model e.g.
pREFERRED areas, walls and other obstacles. The pedestrians then react to both
types of floor fields.

To keep the model simple, we want to provide the particles with as little
intelligence as possible and to achieve the formation of complex structures and
collective effects by means of self-organization. In contrast to older approaches
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we do not make detailed assumptions about the human behaviour. Nevertheless the model is able to reproduce many of the basic phenomena.

The key feature to substitute individual intelligence is the floor field. Apart from the occupation number each cell carries an additional quantity (field) which can be either discrete or continuous. This field can have its own dynamics given by diffusion and decay coefficients.

Interactions between pedestrians are repulsive for short distances. One likes to keep a minimal distance to others in order to avoid bumping into them. In the simplest version of our model this is taken into account through hard-core repulsion which prevents multiple occupation of the cells. For longer distances the interaction is often attractive. E.g. when walking in a crowded area it is usually advantageous to follow directly behind the predecessor. Large crowds may also be attractive due to curiosity.

With two particle species moving in opposite directions, each with its own floor field, effects can be observed which are so far only achieved by continuous models [13]: lane formation and oscillation of the direction of flow at doors. We consider this model to be another example for the ability of cellular automata to create complex behaviour out of simple rules and the great applicability of this approach to all kinds of traffic flow problems.

In contrast to vehicular traffic the time needed for acceleration and braking is negligible. The velocity distribution of pedestrians is sharply peaked [20]. These facts naturally lead to a model where the pedestrians have a maximal velocity \( v_{\text{max}} = 1 \), i.e. only transitions to neighbour cells are allowed. Furthermore, a greater \( v_{\text{max}} \), i.e. pedestrians are allowed to move more than just one cell per timestep, would be harder to implement in two dimensions, especially when combined with parallel dynamics, and reduce the computational efficiency. The number of possible target cells increases quadratically with the interaction range. Furthermore one has to check whether the path is blocked by other pedestrians. This might even be ambiguous for diagonal motion and crossing trajectories. Also higher velocity models lead to timescales which are much too small (see Sec. 4).

4 Definition of the Model and its Dynamics

The area available for pedestrians is divided into cells of approximately \( 40 \times 40 \text{ cm}^2 \). This is the typical space occupied by a pedestrian in a dense crowd [23]. Each cell can either be empty or occupied by exactly one particle (pedestrian). For special situations it might be desirable to use a finer discretization, e.g. such that each pedestrian occupies four cells instead of one.

The update is done in parallel for all particles. This introduces a timescale into the dynamics which can roughly be identified with the reaction time \( t_{\text{reac}} \). In the deterministic limit, corresponding to the maximal possible walking velocity in our model, a single pedestrian (not interacting with others) moves with a velocity of one cell per timestep, i.e. \( 40 \text{ cm} \) per timestep. Empirically the average velocity of a pedestrian is about \( 1.3 \text{ m/s} \) [23]. This gives an estimate for the real time corresponding to one timestep in our model of approximately 0.3 sec which is...
of the order of the reaction time $t_{\text{reac}}$, and thus consistent with our microscopic rules. It also agrees nicely with the time needed to reach the normal walking speed which is about $0.5 \text{ sec}$. This corresponds to at least $v_{\text{max}}$ timesteps if the pedestrian can only accelerate by one unit per timestep. Therefore in models with large $v_{\text{max}}$ a timestep would correspond to a real time shorter than the smallest relevant timescale. This makes the model more complicated than necessary and reduces the efficiency of simulations.

4.1 Basic Rules

Each particle is given a preferred walking direction. From this direction, a $3 \times 3$ matrix of preferences is constructed which contains the probabilities for a move of the particle. The central element describes the probability for the particle not to move at all, the remaining 8 correspond to a move to the neighbouring cells (see Fig. 1). The probabilities can be related to the velocity and the longitudinal and transversal standard deviations (see [24,27] for details). So the matrix of preferences contains information about the preferred walking direction and speed. In principle, it can differ from cell to cell depending on the geometry and aim of the pedestrians. In the simplest case the pedestrian is allowed to move in one direction only without fluctuations and in the corresponding matrix of preference only one element is one and all others are zero. In the following it is assumed that a matrix of preferences is given at every timestep for each pedestrian. These can e.g. be obtained from some model for route selection which assigns certain routes to each pedestrian.

![Figure 1: A particle, its possible transitions and the associated matrix of preference $M = (M_{ij})$.](image)

This ansatz can easily be extended by fixing the direction of preference for each cell separately, e.g. to handle structures inside buildings. Then the particles would use the matrix belonging to the cell they occupy at a given step. However, a similar effect can be obtained much simpler by introducing a second floor field (see Sec. 4.2).
In each update step for each particle a desired move is chosen according to these probabilities. This is done in parallel for all particles. If the target cell is occupied, the particle does not move. If it is not occupied, and no other particle targets the same cell, the move is executed. If more than one particle share the same target cell, one is chosen according to the relative probabilities with which each particle chose their target. This particle moves while its rivals for the same target keep their position (see Fig. 2).

\[
\begin{align*}
p_1 &= \frac{M^{(1)}}{M^{(1)} + M^{(2)}} \\
p_2 &= \frac{M^{(2)}}{M^{(1)} + M^{(2)}}
\end{align*}
\]

Figure 2: Solving conflicts according to the relative probabilities for the case of two particles with matrices of preference \(M^{(1)}\) and \(M^{(2)}\).

The rules presented up to here are a straightforward generalization of the CA rules used so far for the description of traffic flow [17, 18]. The main difference is that in principle transitions in all directions are possible and each pedestrian \(j\) might have her own preferred direction of motion characterized by a matrix of preferences \(M^{(j)}\). The only interaction between particles taken into account so far is hard-core exclusion.

4.2 Floor Field

In order to reproduce certain collective phenomena it is necessary to introduce further longer-ranged interactions. In some continuous models this is done using the idea of a social force [7, 10, 13]. Here we present a different approach. Since we want to keep the model as simple as possible we try to avoid using a long-range interaction explicitly. Instead we introduce the concept of a floor field which is modified by the pedestrians and which in turn modifies the transition probabilities. This allows to take into account interactions between pedestrians and the geometry of the system (building) in a unified and simple way without
loosing the advantages of local transition rules. The floor field modifies the transition probabilities in such a way that a motion into the direction of larger fields is preferred.

The floor field can be thought of as a second grid of cells underlying the grid of cells occupied by the pedestrians. It can be discrete or continuous. As already explained in Sec. 3 we distinguish two types of fields which will be called static and dynamic floor fields, respectively.

The dynamic floor field \( D \) is just the virtual trace left by the pedestrians (see Sec. 3). It is modified by the presence of pedestrians and has its own dynamics, i.e. diffusion and decay. Usually the dynamic floor field is used to model a (“long-ranged”) attractive interaction between the particles. Each pedestrian leaves a “trace”, i.e. the floor field of occupied cells is increased. Explicit examples where such an interaction is relevant are given in Part II [3]. The dynamic floor field is also subject to diffusion and decay which leads to a dilution and finally the vanishing of the trace after some time.

The static floor field \( S \) does not evolve with time and is not changed by the presence of pedestrians. Such a field can be used to specify regions of space which are more attractive, e.g. an emergency exit (see the example in 3) or shop windows. This has an effect similar to a position-dependent matrix of preference, but is much easier to realize.

The transition probability \( p_{ij} \) in direction \((i, j)\) (see Fig. 1) now depends on four contributions:

(i) the matrix of preference \( M_{ij} \) which contains the information about the aim and average velocity of the pedestrian.

(ii) the value \( D_{ij} \) of the dynamic floor field at the target cell. This contribution takes into account the effects of the motion of the other pedestrians. In many applications (see 3) it is attractive to “follow the crowd”, i.e. transitions in directions \((i, j)\) with a large value of the dynamic floor field are preferred.

(iii) the value \( S_{ij} \) of the static floor field. It allows to model effects of the geometry. E.g. in a corridor it is usually less attractive to walk close to the walls. Such an effect can be incorporated in a static floor field which decreases near the walls.

(iv) the occupation number \( n_{ij} \) of the target cell. A motion in direction \((i, j)\) is only allowed if the target cell is empty \((n_{ij} = 0)\) and forbidden if it is already occupied \((n_{ij} = 1)\).

One simple possibility to take into account all contributions (i)–(iv) is to define the transition probability in direction \((i, j)\) by

\[
p_{ij} = N M_{ij} D_{ij} S_{ij} (1 - n_{ij}).
\]

\( N \) is a normalization factor to ensure \( \sum_{(i,j)} p_{ij} = 1 \) where the sum is over the nine possible target cells. There are also slightly more general forms of the transition probabilities which have been studied in [26, 27].

Since the total transition probability is proportional to the dynamic floor field it becomes more attractive to follow in the footsteps of other pedestrians. This
effect competes with the preferred walking direction specified by $M_{ij}$ and the
effects of the geometry encoded in $S_{ij}$. The relative influence of the contributions
(i)–(iii) is controlled by coupling parameters. These depend on the situation to
be studied. Consider for example a situation where people want to leave a large
room (see [3]). Normal circumstances, where everybody is able to see the exit,
can be modelled by solely using a static floor field which decreases radially with
the distance from the door. Since transitions in the direction of larger fields are
more likely this will automatically guarantee that everybody is walking in the
direction of the door. If, however, the exit can not be seen by everybody, e.g.
in a smoke-filled room or in the case of failing lights, people will try to follow
others hoping that they know the location of the exit. In this case the coupling
to the dynamic floor field is much stronger and the static field has a considerable
influence only in the vicinity of the door. This example will be studied in more
detail in [3].

4.3 Dynamics of the Floor Field

In contrast to the static floor field $S$ the dynamic floor field $D$ is changed by
the motion of pedestrians. Furthermore it is subject to diffusion and decay. Its
dynamics consists of three steps:

(a) If a pedestrian leaves a cell $(x, y)$ the dynamic floor field $D_{xy}$ corresponding
to this cell is increased by $\Delta D_{xy}$. The increment $\Delta D_{xy}$ is a parameter of the
model and can either be discrete or continuous.

(b) To model the diffusion, a certain amount of the field is distributed among
the neighbouring cells.

(c) To model the decay of the field, the field strength is reduced by a decay
constant $\delta$.

In (a) the virtual trace left by the motion of the pedestrians is created. (b) is
necessary because pedestrians do not necessarily follow exactly in the footsteps
of others. Diffusion leads to broadening and dilution of the trace. (c) implies
that the lifetime of the trace is finite and that it will vanish after some time.
Diffusion and decay of the dynamic field lead to an effective interaction strength
between the pedestrians which decays exponentially with the distance [27].

In [26] we have introduced two variants of the floor field, a discrete and a
continuous one. In the discrete case the field strength $D_{xy}$ can be interpreted as
the number of bosonic particles (“bosons”) at the cell $(x, y)$. In (a) the number
of bosons is increased by one. In (b) bosons can move with probability $\gamma$ to
neighbouring cells and in (c) bosons are removed with probability $\alpha$. In the
continuous case the dynamics in (b) and (c) is described by a diffusion-decay
equation

$$\frac{\partial D}{\partial t} = d \cdot \Delta D - \delta \cdot D$$  \hspace{1cm} (2)

where $d$ is the diffusion constant and $\delta$ the decay constant. Details can be found
in [26] [27].
4.4 Summary of the Update Rules

The update rules of the full model including the interaction with the floor fields then have the following structure:

1. The dynamic floor field $D$ is modified according to its diffusion and decay rules (see Sec. 4.3).
2. For each pedestrian, the transition probabilities $p_{ij}$ for a move to an unoccupied neighbour cell $(i, j)$ are determined by the matrix of preferences and the local dynamic and static floor fields, e.g. $p_{ij} \propto M_{ij} D_{ij} S_{ij}$ (see Sec. 4.2).
3. Each pedestrian chooses a target cell based on the probabilities of the transition matrix $P = (p_{ij})$.
4. The conflicts arising by any two or more pedestrians attempting to move to the same target cell are resolved, e.g. using the procedure described in Sec. 4.1.
5. The pedestrians which are allowed to move execute their step.
6. The pedestrians alter the dynamic floor field $D_{xy}$ of the cell $(x, y)$ they occupied before the move (see Sec. 4.3).

These rules have to be applied to all pedestrians at the same time (parallel dynamics). This introduces a timescale into the dynamics which corresponds to approximately 0.3 sec of real time. This allows e.g. to translate evacuation times measured in computer simulations into real times.

5 Conclusions

We have introduced a stochastic cellular automaton to simulate pedestrian behaviour. We focused here on the general concept. The effects which can be observed with the basic approach will be presented in [3] together with simple applications.

The key mechanism is the introduction of a floor field which acts as a substitute for pedestrian intelligence and leads to collective phenomena. This floor field makes it possible to translate spatial long-ranged interactions into non-local interactions in time. The latter can be implemented much more efficiently on a computer. Another advantage is an easier treatment of complex geometries. In models with long-range interactions, e.g. the social-force models, one always has to check explicitly whether pedestrians are separated by walls in which case there should be no interaction between them.

The general idea in our model is similar to chemotaxis. However, the pedestrians leave a virtual trace rather than a chemical one. This virtual trace has its own dynamics (diffusion and decay) which e.g. restricts the interaction range (in time). It is realized through a dynamical floor field which allows to give the pedestrians only minimal intelligence and to use local interactions. Together with the static floor field it offers the possibility to take different effects into account in a unified way, e.g. the social forces between the pedestrians or the geometry of the building.
In Part II we will demonstrate that the approach indeed is able to reproduce the known collective effects and self-organization phenomena. Therefore the model is a good starting point for realistic applications.

The model can also be applied to more complex geometries and various characteristics of a crowd can be simulated without major changes. So it should be possible to study the effects of panic (see and references therein). In we show results for simple evacuation simulations.

The description of pedestrians using a cellular automaton approach allows for very high simulation speeds. Therefore, we have the possibility to extract the complete statistical properties of our model using Monte Carlo simulations.

Finally it should be emphasized that we have presented only the basic ideas of the approach. For realistic applications modifications might be appropriate, e.g. smaller cell sizes etc. One can also introduce more than just one species of pedestrians (e.g. two groups moving in opposite directions). In this case each species interacts with its own floor field. In the simplest case these fields are independent from each other.

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