Dynamical projection of atoms to Feshbach molecules at strong coupling

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The dynamical atom/molecule projection, recently used to probe fermion pairing, is fast compared to collective fermion times, but slow on the Feshbach resonance width scale. Theory of detuning-induced dynamics of molecules coupled to resonantly associating atom pairs, employing a time-dependent many-body Green’s function approach, is presented. An exact solution is found, predicting a 1/3 power law for molecule production efficiency at fast sweep. The results for s- and p-wave resonances are obtained and compared. The predicted production efficiency agrees with experimental observations for both condensed and incoherent molecules away from saturation.

Cold atomic Fermi gases, magnetically tuned to a Feshbach resonance region, host an intriguing strongly interacting many-body system. Recently, pairing of fermions near the resonance was probed with the help of dynamical projection of atomic state on the molecular state, achieved by a sweep through the resonance, followed by the detection of molecular Bose-Einstein condensate. The sweep could be made very fast compared to typical fermion time scales, such as the collision frequency or inverse Fermi bandwidth and pairing energy gap, making the process a “snapshot probe” with regard to the collective fermion processes.

On a single particle level, however, broad Feshbach resonances studied in Refs. 6, 7, exhibit strong atom-molecule coupling in a relatively wide detuning range. In this sense, the sweep speed corresponds to essentially adiabatic atom/molecule conversion, slow on the scale of the resonance width. For example, in the JILA experiment, the Feshbach resonance width $\Delta B \approx 10$ G translates into $\Delta \nu \approx 180$ MHz in detuning frequency, while the characteristic time of the magnetic field sweep, $\tau_n = (dt/\nu)^{1/2} \approx 1\mu$s, is about $10^2$ times longer than $\Delta \nu^{-1}$. A similar estimate applies to the MIT fast projection experiment. Somewhat paradoxically, the fermions involved in this “slow” molecule formation are the same whose many-body state is being analyzed by the dynamical “snapshot” projection. Thus a correct physical picture of the molecular state swept through the resonance must combine the adiabatic single-particle and the “snapshot” many-body aspects in a seamless way.

Continuing efforts to use atom/molecule projection as investigative tool call for better understanding of the driven molecular state. The Landau-Zener model, which fits the data well near saturation, focuses on the adiabatic aspects, ignoring molecule dissociation into continuous spectrum of atom pairs. The dynamical mean field approach, which can be justified for bosons in the atomic BEC regime, lacks firm foundation in the fermion case. Recently, the many-body state overlap models were put forward. While providing some guidance, these approaches do not account for the experimentally relevant situation of broad resonance when the “snapshot” many-body projection is slow on the scale of individual molecule formation.

Our objective is to describe molecules at a sweep fast compared to the elastic collisions, when only the quantum-mechanical processes are relevant. We develop a theoretical framework which accounts for resonance dissociation/association in the presence of time-dependent detuning as well as for fermion pairing correlations. We describe the molecules swept through the resonance using a time-dependent Green’s function which fully accounts for relative motion of the atoms associating to form molecules. While our method is quite general and applicable to Feshbach resonances with any angular momentum, here we focus, for the sake of concreteness, on the s-wave case. We consider the evolution from equilibrium at $\nu = \nu_0$, followed by an abrupt linear sweep:

$$\nu(t) = \begin{cases} \nu_0, & t < 0 \\ \nu_0 - \alpha t, & t > 0 \end{cases}$$

with $\alpha$ the sweep rate. The generalization to the p-wave resonances will be straightforward (see below).

Finding the time-dependent Green’s function is a non-trivial mathematical problem, here solved exactly using an idea similar to that of the Wiener-Hopf method. The important time scale, characterizing the adiabaticity of the sweep (see Fig. 1 inset), is found to be

$$\tau_0 = (\hbar / \lambda^3)^{1/3}, \quad \lambda = g^2 m^{3/2} / 4\pi \hbar^3,$$

with $g$ the atom-molecule coupling (see Eq. (3)), and $m$ the atom mass. The time scale $\tau_0$ can also be inferred, as noted by Altman and Vishwanath from the adiabaticity condition $\dot{\omega} \lesssim \omega^2$ for the time-dependent molecule energy $\hbar \omega$. Different regimes arise depending on the relation between $\tau_0$ and $\nu_0/\alpha$, the time it takes the sweep to reach the resonance (Fig. 1). The atom-to-molecule transformation takes place at times less than $\tau_0$ after crossing the resonance, where the evolution is non-adiabatic. At later times, the molecules, dressed by atom pairs, evolve adiabatically. For a fast sweep, $\alpha \tau_0 \gg \nu_0$, the number of produced molecules scales with the sweep rate as $\alpha^{-1/3}$, while for slower sweep, $\alpha \tau_0 \ll \nu_0$, the number of molecules scales as $\alpha^{-1}$. 
in the magnetic field units. The characteristic atom-
ν responds to 0.

FIG. 1: Molecule number 
\( N_m \) vs. the sweep rate \( (\lambda^4/\alpha)^{1/3} \)
at different initial detuning \( \nu_0 \). The asymptotic regimes, \( N_m \propto \alpha^{-1}, \alpha^{-1/3}, \) correspond to slow and fast sweep. Inset: Molecular energy time dependence \( \mathbf{1} \) with the time interval corresponding to nonadiabatic evolution marked.

These results agree with the molecular number and condensate production efficiency reported by JILA group \( \mathbf{4} \). The sweep speeds \( |dt/dB| \approx 10 - 80 \mu s/G \) \( \mathbf{4} \) correspond to \( \nu_0/\alpha \approx 1 - 100 \mu s \) with \( \nu_0 = 0.1 - 1 \) G in the magnetic field units. The characteristic atom-
molecule coupling \( \lambda^2 \approx 1 \) GHz gives the adiabaticity time \( \tau_0 \approx 10 - 20 \) μs depending on the sweep speed. Thus with \( 0.2 < \alpha \tau_0/\nu_0 < 10 \) both the fast and the slow regimes are realized. Indeed, the molecule number obtained for different sweep speeds below saturation (see Fig.5 in Ref. \( \mathbf{4} \) displaying the data for \( \nu_0 = 0.12 \) G) can be fitted quite accurately with the 1/3 power law dependence, \( N_m \propto |dt/dB|^{1/3} \), in agreement with our results. Also reasonable, by order of magnitude, is the predicted total number of produced molecules. Our conclusions regarding the incoherent molecule production channel are consistent with the observed independence of the condensate fraction \( \mathbf{4} \) of the sweep speed. We obtain the same production efficiency for condensed and incoherent molecules (Eq.(28)), except near saturation.

Let us recall the form of the two-channel Hamiltonian \( \mathbf{4} \), describing pairs of fermions binding to form molecules at the resonance:

\[
H = H_a + H_m + \sum_{p,p'} \left( g_{p+p'} \sigma_{p^+} \sigma_{p'^-} + \text{h.c.} \right)
\]  

with \( H_a = \sum_{p,p'} \frac{\hbar^2}{2m} a_{p^+}^\dagger a_{p'}^\dagger a_p a_{p'} \), \( H_m = \sum_k (\nu + \Delta^2) b_k^\dagger b_k \), and \( a_{p^+}, \sigma_k \) the atom and molecule operators, \( \sigma \) the spin (\( k = 1 \)). The detuning \( \nu \) is determined by the molecule and two-atom Zeeman energy difference, \( \nu = \Delta \mu (B - B_0) \).

The single-atom Green’s function, obtained from Dyson equation \( \mathbf{28} \), has the form

\[
G(\omega, k) = \frac{1}{\omega - \nu - \Sigma(\omega)} , \quad \omega = \frac{k^2}{4m} + i0 , \quad (4)
\]

where \( \Sigma(\omega) = \lambda (-\omega)^{1/2} \) is the self-energy describing molecule dissociation \( (s\text{-wave}) \), which arises after integrating over the 3d density of atom pair states \( N(\epsilon) \propto \epsilon^{1/2} \) along with a suitable ultraviolet regularization \( \mathbf{8} \).

For time-independent \( \nu \), the molecular state described by atom pairs, is described by the Green’s function pole:

\[
G_0(\omega) = \frac{Z(\omega)}{\omega - \omega(k) + i0} \quad (5)
\]

with \( \omega(k) \) given by \( \omega - \Sigma(\omega) = \nu \). Near the resonance, at \( |\nu| \ll \Delta E_s \approx \lambda^2 \), neglecting \( \omega \) compared to \( \Sigma(\omega) \), one obtains molecular energy quadratic in detuning:

\[
\omega(k) = -\left( \frac{\nu}{\lambda} \right)^2 + \frac{k^2}{4m} . \quad (6)
\]

At \( \nu < 0 \), Eq.(6) gives the energy of molecules, while at \( \nu > 0 \) it describes a resonance in the two-fermion scattering mediated by virtual molecules \( \mathbf{24} \). The residue \( Z \) defines the bare molecule weight in the physical molecule state, \( Z^{-1}(\omega) = dG^{-1}/d\omega = 1 + i\frac{\omega}{k^2}(\omega - \tilde{\omega})^{-1/2} \), which varies from zero to one across the resonance, at \( -\Delta E_s \ll \nu < 0 \).

At relatively small detuning, \( |\nu|/\Delta E_s \ll 1 \), \( Z \) increases linearly: \( Z(\omega) \approx 2|\nu|/\lambda^2 \).

To investigate molecule formation at the resonance, we consider the Green’s function for the problem with time-dependent detuning \( \nu(t) \). In this case, due to nonlocal character of \( \Sigma \) in the time domain, the molecule evolution is described by an integral-differential equation \( \mathbf{17, 18} \)

\[
\left( i\partial_t - \nu(t) - \frac{k^2}{4m} \right) b_k(t) - \int \Sigma_k(t, t') b_k(t') dt' = \eta_k(t) \quad (7)
\]

with \( \eta_k(t) = \int \frac{d^3x}{\lambda^2} \psi_\sigma(x, t) \tilde{\psi}_\sigma(x, t) \) the pairing field, and \( \psi_\sigma(x, t) = \sum_p a_{p, \sigma} e^{i p x - i \nu(t)} \). Here the self-energy is

\[
\Sigma_k(t > t') = \frac{\int \Sigma_k(\tilde{\omega}) e^{-i\omega(t-t')} d\omega}{2\pi} = \frac{ae^{-i\frac{k^2}{2m}(t-t')}}{(t-t')^{3/2}} , \quad (8)
\]

with \( a = \frac{1}{\sqrt{2\pi}} \) and \( \Sigma(t < t') \) vanishes due to the causality.

The pairing field \( \eta \), which acts as a source in Eq.(7), should be taken as a c-number for the condensed molecules (with \( k = 0 \)), and as an operator for the incoherent molecules. Generally, its correlation function includes both the coherent and incoherent parts:

\[
\langle \eta_{k, \omega} \eta_{k', \omega'} \rangle = (2\pi)^4 |\eta_0|^2 \delta(\omega - \mu) \delta(k) + K(\omega, k) , \quad (9)
\]

where \( \eta_0 \) describes a finite amplitude for two fermions to have opposite momenta in the paired state, with \( \mu < 2E_F \) the chemical potential of a pair, and \( K(\omega, k) = \langle \eta_{k, \omega} \eta_{k, \omega} \rangle \) the dynamical pair correlator.
which is nonzero even for ideal Fermi gas. We first consider the coherent molecule production, treating both $\eta = \eta_0 e^{-i\omega t}$ and $b(t)$ as c-numbers. The incoherent pair source $K(\omega, k)$ will be discussed subsequently below.

The evolution problem (7) is non-elementary due to nonlocality of $\Sigma(t, t')$. Our approach employs an idea similar to that used in the Wiener-Hopf method. We first handle an auxiliary problem in which the linear sweep $\nu(t) = v_0 - at$ extends from $-\infty$ to $\infty$, and then modify the solution to describe the situation of interest (11).

The auxiliary problem in question is to find $b(t)$, $-\infty < t < \infty$, which obeys a linear integral-differential equation

$$
(\dot{\omega} - \nu_0 - \Sigma(\omega) + \alpha t) b(t) = \eta(t), \quad \dot{\omega} = i\partial_t
$$

with a source term $\eta(t)$ of a generic form. It is convenient to go to Fourier representation, in which the problem is reduced to an ordinary differential equation $(\omega - \Sigma(\omega) - \nu_0 - i\alpha \partial_\omega) b(\omega) = \eta(\omega)$ for $b(\omega) = \int e^{i\omega t} b(t) dt$. Solution of this equation, first order in $\partial_\omega$, is found using the gauge transformation $b(\omega) \to e^{i\varphi(\omega)} b(\omega)$ with the phase $\varphi$ satisfying

$$
-\alpha \varphi'(\omega) = D_0(\omega) \equiv \omega - \Sigma(\omega) - \nu_0.
$$

This problem is solved by the function

$$
b(\omega) = -i\alpha^{-1} e^{i\varphi(\omega)} \int_0^{+\infty} e^{-i\varphi'(\omega')} \eta(\omega') d\omega'.
$$

To verify (12), one can compare its behavior to that of $b_0(\omega) = C e^{i\varphi(\omega)}$, the solution to the homogeneous problem (10). For $\omega$ large and positive, since $(-\omega - 0)^{1/2} = -i\sqrt{\omega}$, we obtain the asymptotic behavior $D_0 \approx i\lambda \omega^{1/2}, \varphi \approx -i(2\lambda/3\alpha)\omega^{3/2}, e^{i\varphi} \propto e^{i\alpha \omega^{3/2}} (a = 2\lambda/3\alpha)$. Thus $C = 0$ is required to eliminate exponential growth. Indeed, the asymptotic behavior of the integral in (12) at large positive $\omega$ is non-exponential: $\int_\omega^{+\infty} e^{-i\varphi'(\omega')} \eta(\omega') d\omega' \approx e^{-i\varphi(\omega)} \eta(\omega)/\varphi'(\omega)$. (For any physical source, $\eta(\omega \to \infty)$ is algebraic.) At the same time, the behavior at large negative $\omega$ does not require special attention: $\varphi$ is real at $\omega < 0$, and so the exponentials $e^{i\varphi}$ oscillate as $e^{i\alpha \omega^{3/2}}$ without giving rise to “dangerous” asymptotic behavior.

Now, having found the solution for the sweep spanning the entire range $-\infty < t < +\infty$, let us consider the sweep trajectory (11). In this case, it is convenient to represent the function $b(t)$ as a sum $b_\prec(t) + b_\succ(t)$, with $b_\succ(t)$ nonzero only at $t \geq 0$ ($t \leq 0$), respectively, obtained by restricting $b(t)$ on the half-line $t \geq 0$ ($t \leq 0$). Then the evolution equation, in operator form written as $(\dot{\omega} - \nu(t) - \Sigma(\omega)) b(t) = \eta(t)$, can be represented as

$$
[\hat{D}_0 b_\succ + (\hat{D}_0 + \alpha t) b_\prec] = \eta_\prec + \eta_\succ
$$

with $\eta_\succ, \prec = \theta(\pm t \eta(t)$ having the same meaning as $b_\succ, \prec(t)$, and $\hat{D}_0 \equiv D_0(\omega)$ defined in Eq.(11).

Let us project the terms on the left hand side on the regions $t \geq 0$, $t \leq 0$, taking into account the constraints due to causality. The integral operator $\Sigma$ acts only forward, not backward in time, the property explicit in Eq.(8). Thus the function $(\hat{D}_0(\omega) + \alpha t) b_\succ$ is nonzero only at $t > 0$, while the function $\hat{D}_0(\omega) b_\prec$ has both the $t > 0$ and the $t < 0$ parts. This observation allows to write the problem as two separate problems for $b_\succ, \prec(t)$ as follows:

$$
[\hat{D}_0 b_\prec]_\prec = \eta_\prec, \quad (\hat{D}_0 + \alpha t) b_\succ + [\hat{D}_0 b_\prec]_\succ = \eta_\succ,
$$

(14)

where $[\ldots]_\prec$ denotes the part of the function at $t > 0$ ($t < 0$), with zero value on the opposite half-line. Now, we can solve the first equation for $b_\prec$ and substitute the result in the second equation, which (after some algebra) can be brought to the form

$$
(\hat{D}_0 + \alpha t) b_\succ = \hat{D}_0 \left[ \hat{D}_0^{-1} \eta \right]_\succ.
$$

(15)

We note that $b_\succ$ and the function on the right-hand side are both nonzero only at $t > 0$. This allows to treat this equation as Eq.(10), formally extending the linear time dependence $\alpha t$ to negative $t$. Using the above result, we obtain the answer in Fourier representation of the form (12) with $\eta$ replaced by

$$
\tilde{\eta}(\omega) = \hat{D}_0 \left[ \hat{D}_0^{-1} \eta \right]_\succ = D_0(\omega) \int_\delta^{-\infty} \frac{D_0^{-1}(\omega') \eta(\omega')}{\delta - i(\omega - \omega')} \frac{d\omega'}{2\pi}.
$$

Now, let us consider the source $\eta_\prec(\omega) = \eta_0 e^{-i\omega t}$, describing coherent fermion pairs with the chemical potential $\mu/2$ per particle. In this case, $\eta_\prec(\omega) = 2\pi \eta_0 \delta(\omega - \mu)$ and $\eta_\succ(\omega) = D_0^{-1}(\mu) \eta_0 D_0(\omega)/(\delta - i(\omega - \mu))$. Inserting $\eta_\prec$ in Eq.(12), and using the identity $(\delta - i(\omega - \mu))^{-1} = \int_\delta^{-\infty} e^{i(\omega - \mu)\tau} d\tau$, we find a closed form representation

$$
b(\omega) = \frac{A\eta_0}{\delta - i(\omega - \mu)} + \Delta b(\omega),
$$

(16)

$$
\Delta b(\omega) = iA\eta_0 e^{i\varphi(\omega)} \int_0^{+\infty} e^{-i\omega \tau} \tau \int_\omega^{+\infty} e^{i\omega' \tau - i\varphi'(\omega')} d\omega' d\tau
$$

with $A = D_0^{-1}(\mu)$. (To obtain (16), we transformed the integral over $\omega'$ by writing $D_0(\omega') = \alpha d(\omega' - \varphi(\omega'))/d\omega' - \alpha t$ and integrating by parts.) Since the first term of (16) gives the would be $b(\omega)$ in the absence of the sweep, $\Delta b(\omega)$ describes the effect of the sweep.

Now, let us analyze the asymptotic behavior of $b(t) = \int e^{-i\omega t} b(\omega) d\omega/2\pi$ at large positive $t \gg \eta_0, \nu_0/\alpha$. In this case, the integral over $\omega$ is controlled by large negative $\omega$, which can be seen with the help of the stationary phase approximation. Indeed, the saddle point $\omega_\ast$ of $-\omega t + \varphi(\omega)$, obtained from $\omega' = t$, at $t \to +\infty$ implies $\omega \to -\infty$. With that in mind, we obtain the asymptotic for $b(t)$ by setting the lower integration limit in Eq.(16) at $\omega = -\infty$, leading to the central result of this work:

$$
\Delta b(t) = -\frac{A\eta_0}{\pi t} F^*(t) \int_0^{+\infty} e^{-i\omega \tau} F(\tau) d\tau,
$$

(17)
\[ F(t) = \int_{-\infty}^{\infty} e^{i\omega't - i\varphi'(\omega')} \, d\omega'. \]  

The qualitative behavior of \( F(t) \) can be analyzed using the stationary phase approximation. We obtain \( F(t) = \sqrt{-2\pi i/\varphi''(\omega_0)} e^{i\varphi'(\omega_0) - i\varphi''(\omega_0)} \), where the stationary phase equation for \( \omega_0 \), given by \( D_0(\omega_0) + \alpha t = 0 \), has a real solution \( \omega_0 = -\alpha^2(t - t_0)^2/\lambda^2 \) only for \( t > t_0 = \nu_0/\alpha \). Relating the curvature \( \varphi'' \) and the Green’s function residue, \( -\alpha \varphi'' = D_0 = Z^{-1} \), yields the asymptotic form

\[ F(t > t_0) = (2\pi i \alpha Z(\omega_0))^{1/2} e^{-i\alpha^2(t - t_0)^3/4}. \]  

with \( Z(\omega_0) = 2\alpha(t - t_0)/\lambda^2 \). The self-energy-dominated \( D_0(\omega) = -\nu_0 - \Sigma(\omega) \), appropriate for broad s-wave Feshbach resonance, was used in the above estimates.) Thus \( F(t) \) grows as \( (t - t_0)^{1/2} \) and oscillates at \((t - t_0)/\tau_0 \gg 1 \), decreasing exponentially at \( t - t_0 < 0 \).

To apply Eq. (17) to the experimental situation we take into account that \( \mu \ll \hbar/\tau_0, \hbar/\tau_0 \). (Indeed, \( \mu \lesssim 2E_F \), with \( E_F = 0.35 \mu K = 50 \text{ KHz} \) in Ref. (6).) We evaluate the integral in (17) using the stationary phase form (19):

\[ \Delta b(t) = (2\alpha Z(t))^{1/2} A_0 e^{i\alpha(t - t_0)^3/8} C_1(t_0 + C_2 \tau_0), \]  

\( C_1 = (i/3)^{1/2} \Gamma(1/2), \quad C_2 = (i/3)^{1/6} \Gamma(5/6). \) The asymptotic number of molecules at time \( t \) is evaluated as \( N_m = |\Delta b(t)|^2 \) (see Fig. 1). The fast and slow sweep regimes can be identified: at \( t_0 \ll \tau_0 \) we obtain \( N_m \propto \alpha^{-1/3} \), while at \( t_0 \gg \tau_0 \) we have \( N_m \propto \alpha^{-1} \).

The incoherent molecule production can be studied in a similar manner. Using the operator \( \eta_k \) as a source in \( \mathbf{S} \), and averaging over its dynamical correlations \( \mathbf{B} \) we obtain the molecule momentum distribution

\[ N_m(k) = \sum_{\omega} \frac{A}{2\pi} F^*(t) \int_{-\infty}^{\infty} e^{-i\omega \tau} F(\tau) d\tau \left| K(\omega, k) \right|^2. \]  

with \( A = D_0^{-1}(\omega) \). As a function of frequency, \( K \) is nonzero at \( \omega \lesssim 2E_F \). At \( E_F \ll \hbar/\tau_0, \hbar/\tau_0 \), the expression \(|.|^2 \) is \( \omega \)-independent, as above. Factoring it out, we conclude that the condensed and incoherent molecule production efficiencies are identical. The molecule condensate fraction is then expressed through the fermion pair fraction:

\[ \frac{N_m}{N_m^{(0)}} + \sum_k N_m(k) \]  

\[ \frac{N_m^{(0)}}{N_m^{(0)} + \sum_k N_m(k)} = \frac{|(\psi_\uparrow(x)\psi_\downarrow(x))|^2}{(\tilde{n}_\uparrow(x)\tilde{n}_\downarrow(x))} \]  

\( \tilde{n}_\sigma(x) = \tilde{\psi}_\sigma(x)\tilde{\psi}_\sigma^\dagger(x) \). We note that the incoherent contribution exists even in the absence of pairing. For ideal fermions at density \( n \), we have \( \sum K(\omega, k) = \frac{\hbar^2}{2m}(1 - u)^2 + \frac{u}{2}(1 - u) \), \( u = k^2/2p_F \), which corresponds to a broad molecule momentum distribution with \( k \ll 2p_F \).

The approach presented above yields accurate results for the atom/molecule projection in a wide range of sweep rates, fast and slow, as long as the times \( \tau_0, t_0 \) are short on the scale of \( E_F \). The only limitation stems from the assumption of a steady source, which describes the situation when the fraction of atoms converted into molecules is small. The depletion effects, which are different for the condensed and incoherent molecules, can be incorporated in the framework of a quantum kinetic equation.

The above method is applicable to the \( p \)-wave Feshbach resonance case, with the essential modification in the self-energy form \( \Sigma(\omega) \sim (\omega - \omega_k)^{3/2} \). This self-energy is an irrelevant perturbation near the resonance, so we have the atom/molecule conversion time \( \tau_0 \propto \alpha^{-1/2} \) and \( \alpha \), as for weak coupling. Thus one can use \( D_0(\omega) = \omega - \nu_0 \) in (14), yielding the result identical to (17) with

\[ F(t) = \int e^{i\omega(t - t_0) + \omega^2/2\alpha} d\omega = (2\pi i \alpha)^{1/2} e^{-i\alpha(t - t_0)^3/2}. \]  

The number of produced molecules, both condensed and incoherent, scales as inverse sweep rate \( \alpha^{-1} \). The production is less efficient than in the \( s \)-channel case due to weaker coupling at resonance.

In summary, molecule production at Feshbach resonance is considered as a many-body problem for which the exact Green’s function is obtained using Wiener-Hopf method. The theory is applied to the \( s \)-wave and \( p \)-wave resonances. The slow and fast sweep regimes are identified in the \( s \)-wave case, controled by the adiabaticity time scale \( \alpha \). The predicted power law \( 1/3 \) for the molecule production, as well as the total molecule number, are found to be in agreement with observations away from saturation \( \mathbf{G} \). The independence of the produced condensate fraction on the sweep rate observed at fast sweep \( \mathbf{R} \) is also explained by this theory.

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