Semi-leptonic $B \to S$ decays in the standard model and in the universal extra dimension model

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Abstract

In this article, we assume the two scalar nonet mesons below and above 1 GeV are all $\bar{q}q$ states, and study the semi-leptonic decays $B \to Sl^-\bar{\nu}_l$, $B \to Sl^+\ell^-$ and $B \to S\ell\nu$ both in the standard model and in the universal extra dimension model using the $B - S$ form-factors calculated by the light-cone QCD sum rules in our previous work. We obtain the partial decay widths and decay widths, which can be confronted with the experimental data in the future to examine the nature of the scalar mesons and constrain the basic parameter in the universal extra dimension model, the compactification scale $1/R$.

PACS number: 12.15.Ji, 13.20.He

Key words: $B$-meson, Semi-leptonic decays

1 Introduction

The natures of the scalar mesons are not well established theoretically, and their underlying structures are under hot debating [1]. Irrespective of the two-quark state, tetraquark state and glueball assignments, the underlying structures determine their production and decays. In previous work, we assume that the scalar mesons are all $\bar{q}q$ states, the underlying structures determine their production and decays. In previous work, we assume that the scalar mesons are all $\bar{q}q$ states, in case I, the scalar mesons $\{f_0(600), a_0(980), \kappa(800), f_0(980)\}$ below 1 GeV are the ground states, in case II, the scalar mesons $\{f_0(1370), a_0(1450), K_0^*(1430), f_0(1500)\}$ above 1 GeV are the ground states; and study the $B - S$ transition form-factors with the light-cone QCD sum rules [2]. The transition form-factors in the semi-leptonic decays are highly nonperturbative quantities. They not only depend on the dynamics of strong interactions among the quarks in the initial and final mesons, but also depend on the under structures of the involved mesons. In this article, we take the $B - S$ form-factors as basic input parameters, and study the semi-leptonic decays $B \to Sl^-\bar{\nu}_l$, $B \to Sl^+\ell^-$ and $B \to S\ell\nu$ both in the standard model and in the universal extra dimension model to examine the nature of the scalar mesons and search for new physics beyond the standard model.

The semi-leptonic $B$-decays are excellent subjects in studying the CKM matrix elements and CP violations in the standard model. They also serve as a powerful probe of new physics beyond the standard model in a complementary way to the direct searches, the indirect probe plays an important role in identifying the new physics and its properties [3]. At the quark level, the semi-leptonic $B \to S$ decays take place through the transitions $b \to u(c)\ell^-\bar{\nu}_\ell$, $b \to s(d)\ell^+\ell^-$ and $b \to s(d)\bar{\nu}_\ell\nu_\ell$. In the standard model, the decays $b \to u(c)\ell^-\bar{\nu}_\ell$ take place through the exchange of the intermediate $W$ boson at the tree-level, while the decays $b \to s(d)\ell^+\ell^-$ and $b \to s(d)\bar{\nu}_\ell\nu_\ell$ take place through the penguin diagrams and other diagrams at the one-loop level. Those processes induced by the flavor-changing neutral currents $b \to s(d)$ provide the most sensitive and stringent test of the standard model at the one-loop level. The branching fractions of the semi-leptonic decays $B^0(b\bar{d}) \to S(ud)\ell^-\bar{\nu}_\ell$, $B^-(b\bar{u}) \to S(u\bar{u})\ell^-\bar{\nu}_\ell$, $B^0_s(b\bar{s}) \to S(u\bar{s})\ell^-\bar{\nu}_\ell$ are expected to be large, which favors examining the theoretical predictions in the standard model. The branching fractions of the semi-leptonic decays $B^0(b\bar{d}) \to S(sd)\ell^+\ell^-$, $B^-(b\bar{u}) \to S(s\bar{u})\ell^+\ell^-$, $B^0_s(b\bar{s}) \to S(s\bar{s})\ell^+\ell^-$, $B^0_s(b\bar{d}) \to S(sd)\bar{\nu}_\ell\nu_\ell$, $B^-(b\bar{u}) \to S(s\bar{u})\bar{\nu}_\ell\nu_\ell$, $B^0_s(b\bar{s}) \to S(s\bar{s})\bar{\nu}_\ell\nu_\ell$ are expected to be small, which favors searching for new physics beyond the standard model. New physics effects manifest themselves in the rare $B$-decays in two different ways, either through new contributions to the Wilson coefficients or through the new operators in the effective Hamiltonians, which are absent in the standard model.

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The universal extra dimension (UED) models are promising models among various models of the new physics beyond the standard model [4], where all standard model fields are allowed to propagate in all available dimensions. The simplest model is the Appelquist, Cheng and Dobrescu (ACD) model, which has only one extra universal dimension [5]. The topology of the fifth dimension is the orbifold $S^1/Z_2$, the coordinate $y = x_5$ runs from 0 to $2\pi R$, where the $R$ is the compactification radius. The orbifold has two fixed points at $y = 0$ and $y = \pi R$, respectively, the boundary conditions in the two fixed points determine the Kaluza-Klein (KK) mode expansion of all the fields. In four dimensions after compactification, there are the standard model fields, the series of their KK partners and additional series of KK modes having no correspondence to the standard model fields. The only additional free parameter is the compactification scale $1/R$, the masses of the new KK particles and the interactions among KK particles and the standard model particles are described by the additional parameter $1/R$ and the parameters of the standard model. The presence of the boundaries of the $S^1/Z_2$ orbifold breaks translational invariance along the extra dimension and therefore leads to the violation of the KK-number at the loop level but still preserves a $Z_2$ symmetry (or KK-parity). The KK-parity warrants the stability of the lightest KK-excitation $2$.

The ACD model has potentially many phenomenological interest, such as the semi-leptonic and radiative $B$-decays [5], semi-leptonic $\Lambda_b$ decays [6], etc. The electro-weak precision tests yield a bound of $1/R > 500\text{GeV}$ in case of a UED Higgs boson with the mass about 125 GeV [10, 11]. Other analysis of the electro-weak precise measurements and the inclusive radiative $b \to s \gamma$ decay imply $1/R > 600\text{GeV}$ [12, 13]. While the LHC searches for the dilepton resonances lead to $1/R \geq 715\text{GeV}$ [14]. In this article, we neglect the long-distance contributions to the low energy processes occur at the energy scales $\mu \ll 1/R$. The local operators in the low energy effective Hamiltonians are the same both in the standard model and in the ACD model, and the effects of the KK modes amount to modifying the Wilson coefficients [6, 7].

In the following, we write down the effective Hamiltonian $\mathcal{H}_{eff}$ responsible for the transitions $b \to u\ell^-\bar{\nu}_\ell$, $b \to s\bar{\ell}^+\ell^-$ and $b \to s\bar{\nu}\nu$ in the standard model and in the UED model [6, 7, 15, 16, 17],

\[ \mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ub} \bar{u} \gamma_\alpha (1 - \gamma_5) b \bar{\ell} \gamma^\alpha (1 - \gamma_5) \nu_\ell - \frac{G_F V_{tb} V_{ts}^*}{\sqrt{2}} \frac{e^2}{8\pi^2} \left\{ c_{\gamma 1} \bar{s} \gamma_\alpha (1 - \gamma_5) b \bar{\ell} \gamma^\alpha \ell + c_{\gamma 2} \bar{s} \gamma_\alpha (1 - \gamma_5) b \bar{\ell} \gamma^\alpha \gamma_5 \ell - \frac{2i m_b c_{\gamma 3}^{eff}}{q^2} \bar{s} \gamma_\alpha (1 + \gamma_5) q^2 b \bar{\ell} \gamma^\alpha \ell + \frac{G_F V_{tb} V_{ts}^*}{\sqrt{2}} \frac{e^2}{8\pi^2 \sin^2 \theta_W} \eta_X X(x_1) \bar{s} \gamma_\alpha (1 - \gamma_5) b \bar{\ell} \gamma^\alpha (1 - \gamma_3) \nu_\ell , \right\} \]

where we have neglected the terms proportional to $V_{ub} V_{us}^*$ according to the value $|V_{ub} V_{us}^*/V_{tb} V_{ts}^*| \sim 10^{-2}$. No new operators are induced in the ACD model, the effects of the KK contributions are implemented by modifying the Wilson coefficients which also depend on the additional parameter, the compactification radius $R$. In the present case, we only need to specify the relevant Wilson coefficients $c_{\gamma 1}^{eff}$, $c_{\gamma 2}^{eff}$, $c_{\gamma 3}^{eff}$ and $X(x_1)$ [6, 7]. In this article, we neglect the long-distance contributions come from the four-quark operators near the $c\bar{c}$ resonances, such as the $J/\psi, \psi', \cdots$, which can be experimentally removed by applying appropriate kinematical cuts in the neighborhood of the resonances [18].

2 The decay widths in the standard model and in the universal extra dimension model

In the following, we write down the effective Hamiltonian $\mathcal{H}_{eff}$ responsible for the transitions $b \to u\ell^-\bar{\nu}_\ell$, $b \to s\bar{\ell}^+\ell^-$ and $b \to s\bar{\nu}\nu$ in the standard model and in the UED model [6, 7, 15, 16, 17],
Now, we write down the Wilson coefficients $C_{eff}^{c_7}, C_{eff}^{c_9}$ and $C_{eff}^{c_{10}}$, explicitly,

$$
C_{eff}^{c_7} \left( \mu, \frac{1}{R} \right) = \eta^6 C_7 \left( \mu_W, \frac{1}{R} \right) + \frac{8}{3} \left( \eta^6 - \eta^2 \right) C_8 \left( \mu_W, \frac{1}{R} \right) + C_2 \left( \mu_W, \frac{1}{R} \right) \sum_{i=1}^{8} h_i \eta^{a_i},
$$

$$
C_{eff}^{c_9} \left( \mu, \frac{1}{R} \right) = C_{eff}^{c_{NDR}} \left( \mu, \frac{1}{R} \right) \eta(s) + h(z, \tilde{s}) (3C_3 + C_2 + 3C_5 + C_4 + 3C_5 + C_6)
$$

$$
- \frac{1}{2} h(1, \tilde{s}) (4C_3 + 4C_4 + 3C_5 + C_6)
$$

$$
- \frac{1}{2} h(0, \tilde{s}) (3C_3 + 3C_4) + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6),
$$

$$
C_{eff}^{c_{10}} \left( \mu, \frac{1}{R} \right) = - \frac{Y (x_t, \frac{1}{\pi})}{\sin^2 \theta_W},
$$

where $\eta = \frac{\alpha_s(\mu_W)}{\alpha_s(\mu)}$, 

$$
C_2 \left( \mu_W, \frac{1}{R} \right) = 1,
$$

$$
C_7 \left( \mu_W, \frac{1}{R} \right) = - \frac{1}{2} D' \left( x_t, \frac{1}{R} \right),
$$

$$
C_8 \left( \mu_W, \frac{1}{R} \right) = - \frac{1}{2} E' \left( x_t, \frac{1}{R} \right),
$$

$$
C_{eff}^{c_{NDR}} \left( \mu, \frac{1}{R} \right) = P_{0 NDR} + \frac{Y (x_t, \frac{1}{\pi})}{\sin^2 \theta_W} - 4Z \left( x_t, \frac{1}{R} \right) + P_E E (x_t),
$$

and

$$
a_i = (\frac{14}{23}, \frac{16}{23}, \frac{6}{23}, -\frac{12}{23}, 0.4086, -0.4230, -0.8994, 0.1456),
$$

$$
h_i = (2.2996, -1.0880, -\frac{3}{2}, -\frac{1}{4}, -0.6494, -0.0380, -0.0186, -0.0057).
$$

We denote the total squared momentum of the lepton pairs as $q^2$ and the introduce the variables $\tilde{s}$ and $z$ with $\tilde{s} = \frac{s}{m^2}$ and $z = \frac{m^2}{m^2}$.

$$
\eta(\tilde{s}) = 1 + \frac{\alpha_s(\mu)}{\pi} \omega(\tilde{s}),
$$

$$
\omega(\tilde{s}) = -\frac{2}{9} \pi^2 - \frac{4}{3} \text{Li}_2(\tilde{s}) - \frac{2}{3} \log \tilde{s} \log(1 - \tilde{s}) - \frac{5 + 4\tilde{s}}{3(1 + 2\tilde{s})} \log(1 - \tilde{s}) - \frac{2\tilde{s}(1 + \tilde{s})(1 - 2\tilde{s})}{3(1 - \tilde{s})^2(1 + 2\tilde{s})} \log \tilde{s}
$$

$$
+ \frac{5 + 9\tilde{s} - 6\tilde{s}^2}{6(1 - \tilde{s})(1 + 2\tilde{s})},
$$

$$
h(z, \tilde{s}) = -\frac{8}{9} \ln \frac{m_b}{\mu} - \frac{8}{9} \log z + \frac{8}{27} + \frac{4}{9} x
$$

$$
- \frac{2}{9} (2 + x)(1 - x)^{1/2} \left\{ \log \left( \frac{\sqrt{x+1}}{\sqrt{x-1}} \right) - i\pi, \text{ for } x \equiv \frac{4x^2 - 1}{2},
$$

$$
2 \arctan \frac{1}{\sqrt{x-1}}, \text{ for } x \equiv \frac{4x^2}{s} > 1,
$$

$$
h(0, \tilde{s}) = \frac{8}{27} \frac{8}{9} \log \frac{m_b}{\mu} - \frac{4}{9} \ln \tilde{s} + \frac{4}{9} i\pi.
$$

We take the leading logarithmic approximation for the Wilson coefficients $C_i$ with $i = 1 - 10$ in
the standard model \[10\], where the NDR is the abbreviation for naive dimensional regularization,

\[
P_0^{NDR} = \frac{\pi}{\alpha_s(m_W)} \left( -0.1875 + \sum_{i=1}^{8} p_i \eta_i^{a_i+1} \right) + 1.2468 + \sum_{i=1}^{8} \eta_i^{a_i} \left( r_i^{NDR} + s_i \eta \right),
\]

\[
P_E = 0.1405 + \sum_{i=1}^{8} q_i \eta_i^{a_i+1},
\]

\[
C_j = \sum_{i=1}^{8} k_{ji} \eta_i, \quad i = 1 - 6,
\]

with the parameters

\[
k_{1i} = (0, 0, \frac{1}{3}, -\frac{1}{2}, 0, 0, 0, 0),
\]

\[
k_{2i} = (0, 0, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0),
\]

\[
k_{3i} = (0, 0, -\frac{1}{14}, \frac{1}{14}, 0.0510, -0.1403, -0.0113, 0.0054),
\]

\[
k_{4i} = (0, 0, -\frac{1}{14}, -\frac{1}{6}, 0.0984, 0.1214, 0.0156, 0.0206),
\]

\[
k_{5i} = (0, 0, 0, 0, -0.0397, 0.0117, -0.0025, 0.0304),
\]

\[
k_{6i} = (0, 0, 0, 0, 0.0335, 0.0239, -0.0462, -0.0112),
\]

\[
p_i = (0, 0, -\frac{80}{200}, \frac{8}{35}, 0.0433, 0.1384, 0.1648, -0.0073),
\]

\[
r_i^{NDR} = (0, 0, 0.8966, -0.1960, -0.2011, 0.1328, -0.0292, -0.1858),
\]

\[
s_i = (0, 0, -0.2009, -0.3579, 0.0490, -0.3616, -0.3554, 0.0072),
\]

\[
q_i = (0, 0, 0, 0, 0.0318, 0.0918, -0.2700, 0.0059).
\]

The Wilson coefficients \( F \left( x_t, \frac{1}{R} \right) \) generalize the corresponding standard model Wilson coefficients \( F_0(x_t) \) according to the formula,

\[
F \left( x_t, \frac{1}{R} \right) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n),
\]

where \( x_t = \frac{m_t^2}{m_W^2}, x_n = \frac{m_n^2}{m_W^2} \) and \( m_n = \frac{m_n}{R} \). Now we write down on the Wilson coefficients \( X \left( x_t, \frac{1}{R} \right) \), \( Y \left( x_t, \frac{1}{R} \right) \), \( Z \left( x_t, \frac{1}{R} \right) \), \( D' \left( x_t, \frac{1}{R} \right) \) and \( E' \left( x_t, \frac{1}{R} \right) \), explicitly \([6, 7]\),

\[
X \left( x_t, \frac{1}{R} \right) = X_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n),
\]

\[
Y \left( x_t, \frac{1}{R} \right) = Y_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n),
\]

\[
Z \left( x_t, \frac{1}{R} \right) = Z_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n),
\]

\[
D' \left( x_t, \frac{1}{R} \right) = D'_0(x_t) + \sum_{n=1}^{\infty} D'_n(x_t, x_n),
\]

\[
E' \left( x_t, \frac{1}{R} \right) = E'_0(x_t) + \sum_{n=1}^{\infty} E'_n(x_t, x_n).
\]
where

\[
X_0(x_t) = \frac{x_t}{8} \left[ \frac{x_t + 2}{x_t - 1} + \frac{3x_t - 6}{(x_t - 1)^2} \log x_t \right],
\]

\[
Y_0(x_t) = \frac{x_t}{8} \left[ \frac{x_t - 4}{x_t - 1} + \frac{3x_t}{(x_t - 1)^2} \log x_t \right],
\]

\[
Z_0(x_t) = \frac{18x_t^3 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t - 1)^3} + \left[ \frac{32x_t^4 - 38x_t^3 - 15x_t^2 + 18x_t}{72(x_t - 1)^4} - \frac{1}{9} \right] \log x_t,
\]

\[
D_0(x_t) = \frac{x_t(8x_t^2 + 5x_t - 7)}{12(x_t - 1)^3} + \frac{x_t^2(2 - 3x_t)}{2(x_t - 1)^4} \log x_t,
\]

\[
E_0(x_t) = \frac{x_t(x_t^2 - 5x_t - 2)}{4(x_t - 1)^3} + \frac{3x_t^2}{2(x_t - 1)^2} \log x_t.
\]

(10)

The last term in \(C^{NDR}_{\mu} (\mu, \frac{1}{\mu})\) is numerically negligible, we take the approximation \(E (\mu, \frac{1}{\mu}) = E_0(x_t),\)

\[
E_0(x_t) = \frac{x_t(x_t^2 + 11x_t - 18)}{12(x_t - 1)^3} + \frac{x_t^2(15 - 16x_t + 4x_t^3)}{6(x_t - 1)^4} \log x_t - \frac{2}{3} \log x_t,
\]

(11)

and

\[
C_n(x_t, x_n) = \frac{x_t}{8(x_t - 1)^2} \left[ \frac{x_t^2 - 8x_t + 7 + (3 + 3x_t + 7x_n - x_t x_n) \log \frac{x_t + x_n}{1 + x_n}}{1 + x_n} \right],
\]

\[
D'_n(x_t, x_n) = \frac{x_t(44x_t + 17x_t^2 - 37 + 6x_n^2(10 - 9x_t + 3x_t^2) - 3x_n(21 - 54x_t + 17x_t^2))}{36(x_t - 1)^3}
\]

\[
- \frac{(x_n + 3x_t - 2)(x_t + 3x_t^2 + x_n(3 + x_t) - x_n[1 + x_t(x_t - 10)])}{6(x_t - 1)^4} \log \frac{x_n + x_t}{1 + x_n},
\]

\[
E'_n(x_t, x_n) = \frac{x_t(x_t^2 - 8x_t - 17 - 3x_n(21 - 6x_t + 3x_t^2) - 6x_n^2(10 - 9x_t + 3x_n^2))}{12(x_t - 1)^3}
\]

\[
+ \frac{(1 + x_n)(x_t + 3x_t^2 + x_n(3 + x_t) - x_n[1 + x_t(x_t - 10)])}{2(x_t - 1)^4} \log \frac{x_n + x_t}{1 + x_n},
\]

\[
- \frac{1}{2} x_n(1 + x_n)(3x_n - 1) \log \frac{x_n}{1 + x_n}.
\]

(12)

The summation of the coefficients \(C_n(x_t, x_n), D'_n(x_t, x_n)\) and \(E'_n(x_t, x_n)\) over \(n\) leads to the
then we take into account the definitions for the transition form-factors, formula [6, 7],

\[
\sum_{n=1}^{\infty} C_n(x_t, x_n) = \frac{x_t(7 - x_t)}{16(x_t - 1)} - \frac{\pi m_W R x_t}{16(x_t - 1)^2} \left[ 3(1 + x_t)J(R, -\frac{1}{2}) + (x_t - 7)J(R, \frac{1}{2}) \right],
\]

\[
\sum_{n=1}^{\infty} D_n(x_t, x_n) = \frac{x_t [37 - x_t(44 + 17x_t)]}{72(x_t - 1)^3} + \frac{\pi m_W R}{2} \left[ \int_0^1 dy \frac{2y^2 + 7y^2 + 3y^2}{6} \coth(pm_W R \sqrt{y}) \right]
\]

\[
\sum_{n=1}^{\infty} E_n(x_t, x_n) = \frac{x_t [17 + x_t(8 - x_t)]}{24(x_t - 1)^3} + \frac{\pi m_W R}{4} \left[ \int_0^1 dy \frac{y^2 + 7y^2 + 3y^2}{6} \coth(pm_W R \sqrt{y}) \right]
\]

where

\[
J(R, \alpha) = \int_0^1 dy \alpha \left[ \coth(pm_W R \sqrt{y}) - x_t^{1+\alpha} \coth(pm_t R \sqrt{y}) \right].
\]

The masses of the KK states increase monotonously with increase of the value of $1/R$, in the limit $1/R \to \infty$. the KK states decouple from the low-energy processes and the standard model phenomenology are recovered.

Now we study the semi-leptonic decays $B \to S\ell^- \nu_\ell$, $B \to S\ell^+ \ell^-$, $B \to S\bar{\nu}_\ell \nu_\ell$ with the effective Hamiltonian $\mathcal{H}_{\text{eff}}$ and write down the transition amplitudes,

\[
\langle \bar{\ell}(k_1)\ell^-(k_2)S(p)|\mathcal{H}_{\text{eff}}|B(p') \rangle = -\frac{G_F V_{tb}^*}{\sqrt{2}} \langle S(p)|\bar{u}(0)\gamma^\alpha \gamma_5 b(0)B(p') \rangle \bar{u}(k_2)\gamma_\alpha (1 - \gamma_5)v(k_1),
\]

\[
\langle \ell^+(k_1)\ell^-(k_2)S(p)|\mathcal{H}_{\text{eff}}|B(p') \rangle = \frac{G_F V_{tb}^*}{\sqrt{2}} \alpha \frac{2}{2\pi} \left\{ C_7^{\gamma f f} \frac{2|m_u|}{q^2} \langle S(p)|\bar{s}(0)\gamma^\alpha \gamma_5 q_0 b(0)B(p') \rangle \bar{u}(k_2)\gamma_\alpha v(k_1) + C_9^{\gamma f f} \langle S(p)|\bar{s}(0)\gamma^\alpha \gamma_5 b(0)B(p') \rangle \bar{u}(k_2)\gamma_\alpha \gamma_5 v(k_1) \right\},
\]

\[
\langle \bar{\nu}(k_1)\nu(k_2)S(p)|\mathcal{H}_{\text{eff}}|B(p') \rangle = -\frac{G_F V_{tb}^*}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \eta X(x_1) \langle S(p)|\bar{s}(0)\gamma^\alpha \gamma_5 b(0)B(p') \rangle \bar{u}(k_2)\gamma_\alpha (1 - \gamma_5)v(k_1),
\]

then we take into account the definitions for the transition form-factors,

\[
\langle S(p)|\bar{\sigma}_\mu \gamma_5 \gamma_\mu b(0)B(p') \rangle = -2i F_+(q^2) p_\mu - i \left[ F_+(q^2) + F_-(q^2) \right] q_\mu,
\]

\[
=-i \left[ F_1(q^2) P_\mu - \frac{m_B^2 - m_S^2}{q^2} q_\mu \right] + F_0(q^2) \frac{m_B^2 - m_S^2}{q^2} q_\mu,
\]

\[
\langle S(p)|\bar{\sigma}_\mu \gamma_5 q^\mu b(0)B(p') \rangle = -\frac{2F_T(q^2)}{m_B + m_S} (q^2 p_\mu - q \cdot p q_\mu),
\]
where $P = p' + p$, $p' = p + q$, $q = k_1 + k_2$ and 

$$F_1(q^2) = F_+(q^2),$$

$$F_0(q^2) = F_+(q^2) + \frac{q^2}{m_B^2 - m_S^2}F_-(q^2).$$

(17)

Finally we obtain the partial decay widths,

$$\frac{d\Gamma(B \rightarrow S\ell^- \nu_\ell)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{384\pi^3m_B^4} \frac{(q^2 - m_\ell^2)^2}{q^6} \sqrt{\lambda(m_B^2, m_S^2, q^2)}$$

$$\left\{ \lambda(m_B^2, m_S^2, q^2) \right\},$$

(18)

$$\frac{d\Gamma(B \rightarrow S\ell^+ \ell^-)}{dq^2} = \frac{G_F^2\alpha^2|V_{ub}V_{ts}^*|^2}{512\pi^3m_B^4} \frac{q^2}{3q^2} \left\{ \left[ C_{7}^{\text{eff}} \frac{2m_BF_7^2(q^2)}{m_B + m_S} \right] + C_{9}^{\text{eff}} F_9(q^2) \right\} \lambda(m_B^2, m_S^2, q^2)$$

$$\left( q^2 + 2m_\ell^2 + C_{10}^{\text{eff}} C_{10}^{\text{eff}} [F_2^2(q^2)\lambda(m_B^2, m_S^2, q^2) (q^2 - 4m_\ell^2) + F_0^2(q^2)6m_\ell^2 (m_B^2 - m_S^2)^2] \right),$$

(19)

$$\frac{d\Gamma(B \rightarrow S\ell\nu)}{dx} = \frac{3G_F^2\alpha^2|V_{ub}V_{ts}^*|^2}{384\pi^5m_B^5 \sin^4\theta_W} \eta_X^2 \lambda(m_B^2, m_S^2, q^2) \sqrt{\lambda(m_B^2, m_S^2, q^2)} F_1^2(q^2),$$

(20)

where $x = \frac{E_{\text{miss}}}{m_B}$, $m_B = m_\ell^2 + m_q^2$, $m_\ell^2$, $\alpha$ denotes the missing energy in the decays $B \rightarrow S\ell\nu$ and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$. In calculating the decay widths, it is more convenient to use the form-factors $F_1(q^2)$ and $F_0(q^2)$.

### 3 Numerical results and discussions

The input parameters are taken as $m_B = 5279.55$, $\tau_B = 1.519 \times 10^{-12}$, $m_{B_s} = 5366.7$ MeV, $\tau_{B_s} = 1.512 \times 10^{-12}$, $m_{B_s(980)} = 980$ MeV, $m_{B_s(980)} = 682$ MeV, $m_{B_s(980)} = 990$ MeV, $m_{B_s(1450)} = 1474$ MeV, $m_{B_s(1430)} = 1425$ MeV, $m_{B_0(1500)} = 1505$ MeV, $m_{c} = 0.511$ MeV, $m_{\mu} = 105.68$ MeV, $m_{\tau} = 1776.82$ MeV, $\alpha = 1/137$, $\sin^2\theta_W = 0.23$, $G_F = 1.16637 \times 10^{-5}$ GeV$^{-2}$, $V_{ts} = 0.0405$, $V_{tb} = 0.9911$ [19], $m_b = 4.8$ GeV, $m_c = 1.4$ GeV, $m_W = 80$ GeV, $m_t = 170$ GeV, $\Lambda_{QCD} = 0.225$, $\alpha_s(\mu) = 4\pi / \beta_0 \log(\mu^2/\Lambda^2)$, $\beta_0 = 43/3$, $\beta_1 = 116/3$, $\mu = 5.0$ GeV and $\eta_X = 1$ [16] [17].

In previous work [2], we calculate the $B - S$ form-factors by taking into account the perturbative $\mathcal{O}(\alpha_s)$ corrections to the twist-2 terms using the light-cone QCD sum rules, and fit the numerical values of the form-factors into the single-pole forms,

$$F_i(q^2) = \frac{F_i(0)}{1 - a_i \frac{q^2}{m_B^2}},$$

(21)

where $m_B = 5.28$ GeV, $i = +, -, T$, the values of the $F_i(0)$ and $a_i$ are shown explicitly in Tables 1-3. In calculations, we observe that the uncertainties induced by the uncertainties $\delta a_i$ are greatly
amplified in the regions $(m_b - m_s)^2 - 2(m_b - m_s)\chi \leq q^2 \leq (m_B - m_s)^2$ with $\chi \approx 500$ MeV, and even larger than the central values, while the uncertainties originate from the uncertainties $\delta F_i(0)$ are moderate. In the light-cone QCD sum rules, the operator product expansion is valid at small and intermediate momentum transfer squared $q^2$, $0 \leq q^2 \leq (m_b - m_s)^2 - 2(m_b - m_s)\chi$, the extrapolations to large values of the $q^2$ maybe out of control. So we only retain the uncertainties $\delta F_i(0)$ and neglect the uncertainties $\delta a_i$.

Now we study the semi-leptonic decays in the standard model firstly. In Figs.1-3, we plot the partial decay widths of the $B \to S\ell^-\bar{\nu}_\ell$, $B \to S\ell^+\ell^-$ and $B \to S\ell\nu$ with variations of the squared momentum $q^2$ of the leptonic pairs and the fractions $x$ of the missing energies, respectively, which can be confronted with the experimental data in the future. In Fig.2, there exist small

| $B - a_0(980)$ | $F_+(0)$ | $a_+$ |
|-----------------|----------|-------|
| $0.576 \pm 0.042$ | $0.987 \pm 0.251$ |
| $B - \kappa(800)$ | $0.504 \pm 0.039$ | $0.988 \pm 0.266$ |
| $B_a - \kappa(800)$ | $0.442 \pm 0.033$ | $0.904 \pm 0.274$ |
| $B_a - f_0(980)$ | $0.448 \pm 0.032$ | $0.952 \pm 0.257$ |
| $B - a_0(1450)$ | $0.549 \pm 0.071$ | $0.743 \pm 0.656$ |
| $B - K_0^*(1430)$ | $0.523 \pm 0.070$ | $0.795 \pm 0.669$ |
| $B_s - K_0^*(1430)$ | $0.458 \pm 0.062$ | $0.885 \pm 0.644$ |
| $B_s - f_0(1500)$ | $0.470 \pm 0.059$ | $0.941 \pm 0.595$ |

Table 1: The parameters of the transition form-factors $F_+(q^2)$.

| $B - a_0(980)$ | $-F_-(0)$ | $a_-$ |
|-----------------|----------|-------|
| $0.414 \pm 0.036$ | $0.904 \pm 0.319$ |
| $B - \kappa(800)$ | $0.390 \pm 0.034$ | $0.934 \pm 0.314$ |
| $B_a - \kappa(800)$ | $0.340 \pm 0.030$ | $0.829 \pm 0.342$ |
| $B_a - f_0(980)$ | $0.305 \pm 0.029$ | $0.830 \pm 0.377$ |
| $B - a_0(1450)$ | $0.287 \pm 0.067$ | $0.190 \pm 1.445$ |
| $B - K_0^*(1430)$ | $0.275 \pm 0.064$ | $0.330 \pm 1.402$ |
| $B_s - K_0^*(1430)$ | $0.240 \pm 0.058$ | $0.518 \pm 1.353$ |
| $B_s - f_0(1500)$ | $0.222 \pm 0.057$ | $0.565 \pm 1.418$ |

Table 2: The parameters of the transition form-factors $F_-(q^2)$.

| $B - a_0(980)$ | $F_T(0)$ | $a_T$ |
|-----------------|----------|-------|
| $0.778 \pm 0.062$ | $0.961 \pm 0.278$ |
| $B - \kappa(800)$ | $0.673 \pm 0.056$ | $0.970 \pm 0.288$ |
| $B_a - \kappa(800)$ | $0.596 \pm 0.049$ | $0.877 \pm 0.304$ |
| $B_a - f_0(980)$ | $0.596 \pm 0.048$ | $0.900 \pm 0.299$ |
| $B - a_0(1450)$ | $0.693 \pm 0.112$ | $0.511 \pm 0.893$ |
| $B - K_0^*(1430)$ | $0.657 \pm 0.109$ | $0.598 \pm 0.900$ |
| $B_s - K_0^*(1430)$ | $0.575 \pm 0.098$ | $0.718 \pm 0.874$ |
| $B_s - f_0(1500)$ | $0.570 \pm 0.095$ | $0.778 \pm 0.835$ |

Table 3: The parameters of the transition form-factors $F_T(q^2)$. 

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discontinuities in the decays to the final states $S\ell^+\ell^-$ and $S\mu^+\mu^-$, which originate from the discontinuities in the $h(z,\tilde{s})$ and $h(1,\tilde{s})$ functions, the discontinuities disappear in the decays to the final states $S\tau^+\tau^-$, as the value $q^2 \geq 4m^2$ is large enough to warrant that the variations of the $q^2$ do not pass the discontinuities in the $h(z,\tilde{s})$ and $h(1,\tilde{s})$ functions. From the Figs.1-2, we can see that the branching fractions of the decays to the final states $S\ell^-\bar{\nu}_\ell$ and $S\ell^+\ell^-$ with $\ell = e, \mu$ are much larger than the ones of the corresponding final states $S\tau^-\bar{\nu}_\tau$ and $S\tau^+\tau^-$ due to the much larger available phase-space.

The numerical values of the total branching fractions are shown in Table 4. From the table, we can see that the branching fractions of the decays induced by the transitions $b \rightarrow u\ell^-\bar{\nu}_\ell$, $b \rightarrow s\bar{\ell}^+\ell^-$ and $b \rightarrow s\bar{\nu}\nu$ are of the orders $10^{-4}$, $10^{-7}$ and $10^{-6}$, respectively. The magnitudes are compatible with the ones from other works based on the light-cone QCD sum rules [20, 21] and perturbative QCD [22] and light-front quark model [23]. The transitions $b \rightarrow u\ell^-\bar{\nu}_\ell$ take place at the tree-level through the intermediate $W$-boson, while the transitions $b \rightarrow s\bar{\ell}^+\ell^-$ and $b \rightarrow s\bar{\nu}\nu$ take place at the loop-level, so the decays $B \rightarrow S\ell^-\bar{\nu}_\ell$ have the largest branching fractions. Compared to the decays $B \rightarrow S\nu\nu$, the decays $B \rightarrow S\ell^+\ell^-$ have even smaller branching fractions due to the smaller phase-space. The semi-leptonic decays $B \rightarrow S\ell^-\bar{\nu}_\ell$ are optimal in testing the standard model predictions, we can examine the nature of the scalar mesons by confronting the predictions to the experimental data in the future, while the semi-leptonic $B \rightarrow S\ell^+\ell^-$ are optimal in searching for new physics beyond standard model.

From Table 4, we can also see that the branching fractions of the decays to the light scalar mesons $a_0(980)$, $\kappa(800)$, $f_0(980)$ below 1 GeV are much larger than that of the corresponding decays to the heavy scalar mesons $a_0(1450), K_0^{*}(1430), f_0(1500)$ above 1 GeV due to the much larger energy released in the decays.

In Figs.4-5, we plot the branching fractions of the semi-leptonic decays $B \rightarrow S\ell^+\ell^-$ and $B \rightarrow S\nu\nu$ with variations of the compactification scale $1/R$, respectively. From the figures, we can see that the branching fractions decrease monotonously with increase of the values $1/R$ at the region $1/R \geq 800$ GeV, the branching fractions almost reach constants, i.e. the KK states almost decouple from the low energy observables, while at the region $1/R \leq 600$ GeV, the impact of the KK states on the decays $B \rightarrow S\ell^+\ell^-$ are significant, at the region $1/R \leq 400$ GeV, the impact of the KK states on the decays $B \rightarrow S\nu\nu$ are significant. If the constraint $1/R \geq 715$ GeV obtained from the LHC searches for dilepton resonances is robust [14], the semi-leptonic decays $B \rightarrow S\ell^+\ell^-$ are not the optimal processes in studying the UED model. In the limit $1/R \rightarrow \infty$ or $R \rightarrow 0$, the summation of the coefficients $C_n(x_t, x_n)$, $D_n^e(x_t, x_n)$ and $E_n(x_t, x_n)$ over $n$ does not vanish, but approach some constants which are independent on the $R$. The constants modify the Wilson coefficients slightly, and lead to slightly larger branching fractions, it is difficult to distinguish the new physics effects from the standard model contributions.

4 Conclusion

In previous work, we assume the two scalar nonet mesons below and above 1 GeV are all $\bar{q}q$ states, in case I, the scalar mesons below 1 GeV are the ground states, in case II, the scalar mesons above 1 GeV are the ground states, and calculate the $B - S$ form-factors by taking into account the perturbative $O(\alpha_s)$ corrections to the twist-2 terms using the light-cone QCD sum rules. In this article, we take those form-factor as basic input parameters, and study the semi-leptonic decays the $B \rightarrow S\ell^-\bar{\nu}_\ell$, $B \rightarrow S\ell^+\ell^-$ and $B \rightarrow S\nu\nu$ both in the standard model and in the UED model. We obtain the partial decay widths and decay widths, which can be confronted with the experimental data in the future to examine the nature of the scalar mesons and constrain the basic parameters in the UED model, the compactification scale $1/R$. 
Figure 1: The partial decay widths with variations of the $q^2$, where the 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 denote the semi-leptonic decays $\bar{B}^0 \rightarrow a_1^+(980)e^−\bar{\nu}_e$, $\bar{B}^0 \rightarrow a_1^+(980)\mu^−\bar{\nu}_\mu$, $\bar{B}^0 \rightarrow a_1^+(980)\tau^−\bar{\nu}_\tau$, $\bar{B}_s \rightarrow \kappa^+(800)e^−\bar{\nu}_e$, $\bar{B}_s \rightarrow \kappa^+(800)\mu^−\bar{\nu}_\mu$, $\bar{B}_s \rightarrow \kappa^+(800)\tau^−\bar{\nu}_\tau$, $\bar{B}^0 \rightarrow a_0^+(1450)\mu^−\bar{\nu}_\mu$, $\bar{B}^0 \rightarrow a_0^+(1450)\tau^−\bar{\nu}_\tau$, $\bar{B}_s \rightarrow K_0^{*+}(1430)e^−\bar{\nu}_e$, $\bar{B}_s \rightarrow K_0^{*+}(1430)\mu^−\bar{\nu}_\mu$ and $\bar{B}_s \rightarrow K_0^{*+}(1430)\tau^−\bar{\nu}_\tau$, respectively.
Figure 2: The partial decay widths with variations of the $q^2$, where the 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 denote the semi-leptonic decays $\bar{B}^0 \to \kappa^0(800)e^+e^-$, $\bar{B}^0 \to \kappa^0(800)\mu^+\mu^-$, $\bar{B}^0 \to \kappa^0(800)\tau^+\tau^-$, $\bar{B}_s \to f_0(980)e^+e^-$, $\bar{B}_s \to f_0(980)\mu^+\mu^-$, $\bar{B}_s \to f_0(980)\tau^+\tau^-$, $\bar{B}^0 \to K^{*0}(1430)e^+e^-$, $\bar{B}^0 \to K^{*0}(1430)\mu^+\mu^-$, $\bar{B}^0 \to K^{*0}(1430)\tau^+\tau^-$, $\bar{B}_s \to f_0(1500)e^+e^-$, $\bar{B}_s \to f_0(1500)\mu^+\mu^-$ and $\bar{B}_s \to f_0(1500)\tau^+\tau^-$, respectively.

Figure 3: The partial decay widths with variations of the $x$, where the 1, 2, 3 and 4 denote the semi-leptonic decays $\bar{B}^0 \to \kappa^0(800)\bar{\nu}\nu$, $\bar{B}_s \to f_0(980)\bar{\nu}\nu$, $\bar{B}^0 \to K^{*0}(1430)\bar{\nu}\nu$ and $\bar{B}_s \to f_0(1500)\bar{\nu}\nu$, respectively.
| Decay channels                      | Branching fractions               |
|------------------------------------|-----------------------------------|
| $B^0 \to a_0^+(980)e^-\bar{\nu}_e$ | $(2.74 \pm 0.40) \times 10^{-4}$  |
| $B^0 \to a_0^+(980)\mu^-\bar{\nu}_\mu$ | $(2.74 \pm 0.40) \times 10^{-4}$  |
| $B^0 \to a_0^+(980)\tau^-\bar{\nu}_\tau$ | $(1.31 \pm 0.23) \times 10^{-4}$  |
| $B_s \to \kappa^+(800)e^-\bar{\nu}_e$ | $(2.06 \pm 0.31) \times 10^{-4}$  |
| $B_s \to \kappa^+(800)\mu^-\bar{\nu}_\mu$ | $(2.06 \pm 0.31) \times 10^{-4}$  |
| $B_s \to \kappa^+(800)\tau^-\bar{\nu}_\tau$ | $(1.07 \pm 0.19) \times 10^{-4}$  |
| $B^0 \to a_0^+(1450)e^-\bar{\nu}_e$ | $(1.48 \pm 0.38) \times 10^{-4}$  |
| $B^0 \to a_0^+(1450)\mu^-\bar{\nu}_\mu$ | $(1.47 \pm 0.38) \times 10^{-4}$  |
| $B^0 \to a_0^+(1450)\tau^-\bar{\nu}_\tau$ | $(0.54 \pm 0.15) \times 10^{-4}$  |
| $B_s \to K_0^{*+}(1430)e^-\bar{\nu}_e$ | $(1.27 \pm 0.35) \times 10^{-4}$  |
| $B_s \to K_0^{*+}(1430)\mu^-\bar{\nu}_\mu$ | $(1.27 \pm 0.35) \times 10^{-4}$  |
| $B_s \to K_0^{*+}(1430)\tau^-\bar{\nu}_\tau$ | $(0.54 \pm 0.16) \times 10^{-4}$  |
| $B^0 \to \kappa^{0}(800)e^+e^-$ | $(7.34 \pm 1.22) \times 10^{-7}$  |
| $B^0 \to \kappa^{0}(800)\mu^+\mu^-$ | $(7.31 \pm 1.21) \times 10^{-7}$  |
| $B^0 \to \kappa^{0}(800)\tau^+\tau^-$ | $(1.33 \pm 0.36) \times 10^{-7}$  |
| $B_s \to f_0(980)e^+e^-$ | $(5.16 \pm 0.79) \times 10^{-7}$  |
| $B_s \to f_0(980)\mu^+\mu^-$ | $(5.14 \pm 0.78) \times 10^{-7}$  |
| $B_s \to f_0(980)\tau^+\tau^-$ | $(0.74 \pm 0.17) \times 10^{-7}$  |
| $B^0 \to K_0^{*+}(1430)e^+e^-$ | $(4.14 \pm 1.17) \times 10^{-7}$  |
| $B^0 \to K_0^{*+}(1430)\mu^+\mu^-$ | $(4.12 \pm 1.17) \times 10^{-7}$  |
| $B^0 \to K_0^{*+}(1430)\tau^+\tau^-$ | $(0.11 \pm 0.03) \times 10^{-7}$  |
| $B_s \to f_0(1500)e^+e^-$ | $(3.74 \pm 0.99) \times 10^{-7}$  |
| $B_s \to f_0(1500)\mu^+\mu^-$ | $(3.72 \pm 0.99) \times 10^{-7}$  |
| $B_s \to f_0(1500)\tau^+\tau^-$ | $(0.13 \pm 0.04) \times 10^{-7}$  |
| $B^0 \to \kappa^{0}(800)\nu\bar{\nu}$ | $(6.30 \pm 0.97) \times 10^{-6}$  |
| $B_s \to f_0(980)\nu\bar{\nu}$ | $(4.39 \pm 0.63) \times 10^{-6}$  |
| $B^0 \to K_0^{*+}(1430)\bar{\nu}\nu$ | $(3.49 \pm 0.93) \times 10^{-6}$  |
| $B_s \to f_0(1500)\bar{\nu}\nu$ | $(3.12 \pm 0.78) \times 10^{-6}$  |

Table 4: The branching fractions in the standard model.
Figure 4: The branching fractions with variations of the 1/\( R \), where the 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 denote the semi-leptonic decays \( \bar{B}^0 \rightarrow \kappa^0(800)e^+e^- \), \( \bar{B}^0 \rightarrow \kappa^0(800)\mu^+\mu^- \), \( \bar{B}^0 \rightarrow \kappa^0(800)\tau^+\tau^- \), \( \bar{B}_s \rightarrow f_0(980)e^+e^- \), \( \bar{B}_s \rightarrow f_0(980)\mu^+\mu^- \), \( \bar{B}_s \rightarrow f_0(980)\tau^+\tau^- \), \( \bar{B}^0 \rightarrow K_0^{*0}(1430)e^+e^- \), \( \bar{B}^0 \rightarrow K_0^{*0}(1430)\mu^+\mu^- \), \( \bar{B}^0 \rightarrow K_0^{*0}(1430)\tau^+\tau^- \), \( \bar{B}_s \rightarrow f_0(1500)e^+e^- \), \( \bar{B}_s \rightarrow f_0(1500)\mu^+\mu^- \) and \( \bar{B}_s \rightarrow f_0(1500)\tau^+\tau^- \), respectively.
Figure 5: The branching fractions with variations of the $1/R$, where the 1, 2, 3 and 4 denote the semi-leptonic decays $B^0 \to \kappa^0(800)\bar{\nu}\nu$, $B_s \to f_0(980)\bar{\nu}\nu$, $B^0 \to K^{*0}(1430)\bar{\nu}\nu$ and $B_s \to f_0(1500)\bar{\nu}\nu$, respectively.

Acknowledgements

This work is supported by National Natural Science Foundation, Grant Numbers 11375063, and Natural Science Foundation of Hebei province, Grant Number A2014502017.

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