New Insights into the Plateau-Insulator Transition in the Quantum Hall Regime

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Abstract

We have measured the quantum critical behavior of the plateau-insulator (PI) transition in a low-mobility InGaAs/GaAs quantum well. The longitudinal resistivity measured for two different values of the electron density follows an exponential law, from which we extract critical exponents \( \kappa = 0.54 \) and \( \kappa = 0.58 \), in good agreement with the value \( \kappa = 0.57 \) previously obtained for an InGaAs/InP heterostructure. This provides evidence for a non-Fermi liquid critical exponent. By reversing the direction of the magnetic field we find that the averaged Hall resistance remains quantized at the plateau value \( h/e^2 \) through the PI transition. From the deviations of the Hall resistance from the quantized value, we obtain the corrections to scaling.

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Despite its relatively long history of study, the exact nature of the transitions between adjacent quantum Hall states is still a subject of debate. In the framework of the scaling theory of the quantum Hall effect, the plateau transitions are interpreted as quantum phase transitions with an associated universal critical exponent \( \kappa = p/2 \chi \) [1]. Here \( p \) is the exponent of the phase breaking length \( l_\varphi \) at finite temperature \( T \) (i.e. \( l_\varphi \sim T^{-p/2} \)) and \( \chi \) the critical exponent for the zero \( T \) localization length \( \xi \). Pioneering magnetotransport experiments by Wei et al. [2] on the plateau-plateau (PP) transitions of a low mobility InGaAs/InP heterojunction resulted in a value of \( \kappa = 0.42 \). From subsequent current scaling transport measurements [3] a value \( p = 2 \) was extracted. Hence, an experimental estimate for the localization length exponent is \( \chi \sim 2.4 \), which is close to the free electron value \( 7/3 \) obtained from numerical simulations [4]. This Fermi liquid result has been quite puzzling, as a microscopic theory of the quantum Hall effect should have important ingredients such as localization effects and Coulomb interactions. It turns out however that the PP transitions suffer from systematic errors that are inherently due to macroscopic sample inhomogeneities [5,6]. More recently, we have conducted magnetotransport experiments [6,7,8] on the plateau-insulator (PI) transition of...
an InGaAs/InP heterostructure. By employing a new methodology, which recognizes the fundamental symmetries of the quantum transport problem, the magnetotransport data at the PI transition can be used to separate the universal critical behavior from the sample dependent aspects [6]. This resulted in a value of the exponent $\kappa = 0.57$. Since $p$ is bounded by $1 < p < 2$ [9], this value of $\kappa$ implies that the correlation length exponent is bounded by $0.9 < \chi < 1.8$, which is in conflict with Fermi liquid ideas.

This important new result asks for experimental verification by investigating different low-mobility semiconductor structures. The use of low-mobility structures is dictated by the scattering mechanism, which should predominantly be short-range random potential scattering, which results in a wide temperature regime for scaling. Here we report magnetotransport experiments at the PI transition of an In$_{0.2}$Ga$_{0.8}$As/GaAs quantum well for two different values of the electron density $n$.

The In$_{0.2}$Ga$_{0.8}$As/GaAs quantum well (QW) was grown by the MBE technique. The two-dimensional electron gas is located in a 12 nm thick In$_{0.2}$Ga$_{0.8}$As layer, sandwiched between GaAs. Carriers are provided by Si $\delta$-doping, separated from the quantum well by a spacer layer of 20 nm. Hall bars were made with Au-Ni-Ge contacts. The width of Hall bar equals 75 $\mu$m, and the distance between the voltage contacts along the Hall bar is 390 $\mu$m. The sample is insulating in the dark state. An infra-red LED was used to create carriers.

Magnetotransport data up to 9 T were taken in a vacuum loading Oxford dilution refrigerator. High-magnetic field data up to 20 T were taken at the Grenoble High Magnetic Field Laboratory using an Oxford top-loading plastic dilution refrigerator. The longitudinal $R_{xx}$ and Hall $R_{xy}$ resistances were measured using standard lock-in techniques at a frequency of 2.1 Hz. Since the longitudinal resistance exponentially increases at the PI transition, the phase change of the signal was carefully monitored. For the data presented here the out-of-phase signal is always less than 10%.

In order to characterize our In$_{0.2}$Ga$_{0.8}$As/GaAs QW we have first measured the low-temperature magnetotransport properties in fields up to 9 T (see Fig.1). The sample was illuminated such that the electron density, as determined from the initial slope of the Hall resistance at $T = 4.5$ K, amounts to $n = 1.9 \times 10^{11}$ cm$^{-2}$. The transport mobility $\mu = 160000$ cm$^2$/Vs. The spin resolved $\nu = 2 \rightarrow 1$ PP transition takes place around 5.4 T and is well defined below $\sim 1.5$ K. The presence of weak macroscopic sample inhomogeneities is illustrated by a small (1%) carrier density gradient along the Hall bar (i.e. between the two pairs of Hall contacts) [5]. The $R_{xy}(B)$ and $R_{xx}(B)$ curves of the $\nu = 2 \rightarrow 1$ PP transition, measured at low $T$, do not show a proper crossing point, while the peak height $R_{xx}(B)$ remains temperature dependent. The analysis of the PP transition requires higher-order correction terms in the transport problem, and will be reported elsewhere.

Low-temperature high-field magnetotransport data near the PI transition were taken for two different carrier densities, $n = 1.5 \times 10^{11}$ cm$^{-2}$ and $2.0 \times 10^{11}$ cm$^{-2}$. For these densities the PI transition takes place at $B_c = 10.7$ and 15.7 T, respectively. Typical results are shown in Fig. 2, where we have plotted $\rho_{xx}$ in units $h/\kappa^2$ on a logarithmic scale versus the filling factor $\nu$. The data, taken in the temperature range 0.08-1.07 K,
The longitudinal resistivity $\rho_{xx}$ as a function of filling factor $\nu$ at temperatures 1.07, 0.80, 0.60, 0.45, 0.34, 0.26, 0.19, 0.142, 0.107 and 0.08 K for the InGaAs/GaAs QW. The crossing point indicates the PI transition. Near the critical point the resistivity $\rho_{xx}$ obeys Eq. (1). The carrier density $n = 2.0 \times 10^{11}$ cm$^{-2}$.

show a sharp crossing point at the critical filling factor $\nu_c = 0.58$ and 0.53 for lower and higher densities, respectively. At the critical point $\rho_{xx,c} = h/e^2$ (to within 1 %), as it should. In the vicinity of $B_c$, the longitudinal resistance $\rho_{xx}$ follows the empirical law

$$\ln(\rho_{xx}/\rho_0) = -\Delta\nu/\nu_0(T)$$

where $\rho_0 = \rho_{xx,c}$ and $\Delta\nu = \nu - \nu_c$. The slope $1/\nu_0$ of the curves in Fig. 2 has been determined by a fitting procedure and obeys power law behavior ($\nu_0 \propto T^\kappa$) over more than one decade in $T$. From the log-log plot of $\nu_0$ vs $T$ we extract a critical exponent $\kappa = 0.54 \pm 0.02$ and $0.58 \pm 0.02$, respectively.

In order to corroborate this result further, we present in this paragraph an alternative way of investigating scaling at the PI transition. Notice that we have chosen the temperatures $T_i$ of the field sweeps such that these are equally spaced on a logarithmic temperature scale, i.e. $\ln T_i = \ln T_0 - \alpha i$, where $T_0$ and $\alpha$ are constants and $i$ is an integer which labels the different curves. In the case of scaling $\ln\nu_0(T) = \kappa \ln T$. Hence, the parameter $\nu_0$ changes with index $i$ as

$$\ln\nu_0(T) = -\kappa \ln(T)$$

where $\rho_{xx}$ is expressed in units of $h/e^2$. The absolute value here is used to take into account both positive and negative values of $\Delta\nu$ and $\ln\rho_{xx}$, depending whether $B < B_c$ or $B > B_c$. Under the condition of scaling, a plot of $\ln|\ln\rho_{xx}|$ vs $\ln|\Delta\nu|$ transforms the experimental data into two sets of parallel lines, equally spaced by the amount $\alpha\kappa$ along the abscissa. This is illustrated in Fig. 3 for the data set shown in Fig. 2 ($B_c = 15.7$ T). The derived values of $\kappa$ are identical to the values quoted above. The advantage of this method, which relies on the presence of a sharp well-defined crossing point, over the “traditional” method is that no fitting procedure is needed to visualize scaling behavior. Hence, errors in the determination of the critical exponent due to arbitrary fitting constraints are minimized. Besides, scaling behavior can be verified away from the critical field $B_c$. Furthermore, this

![Fig. 2](image.png)

Fig. 2. The longitudinal resistivity $\rho_{xx}$ as a function of filling factor $\nu$ at temperatures 1.07, 0.80, 0.60, 0.45, 0.34, 0.26, 0.19, 0.142, 0.107 and 0.08 K for the InGaAs/GaAs QW. The crossing point indicates the PI transition. Near the critical point the resistivity $\rho_{xx}$ obeys Eq. (1). The carrier density $n = 2.0 \times 10^{11}$ cm$^{-2}$. Inset: temperature dependence of the fitting parameter $1/\nu_0$ (open symbols) for two different values of carrier densities as indicated. Solid lines represent linear fits.

![Fig. 3](image.png)

Fig. 3. The $\rho_{xx}$ data for the InGaAs/GaAs QW ($B_c = 15.7$ T) from Fig. 2, plotted versus $\Delta\nu$ in the insulating (a) and quantum Hall phase (b). The axis are rescaled to illustrate the validity of Eq. 3. Equally spaced parallel lines signify scaling.

$$\ln\nu_0(T) = -\alpha\kappa + \text{const.}$$

Taking the logarithm of Eq. (1) and taking into account Eq. (2), we obtain

$$\ln|\ln\rho_{xx}| = \ln|\Delta\nu| + \alpha\kappa - \text{const},$$

where $\rho_{xx}$ is expressed in units of $h/e^2$. The absolute value here is used to take into account both positive and negative values of $\Delta\nu$ and $\ln\rho_{xx}$, depending whether $B < B_c$ or $B > B_c$. Under the condition of scaling, a plot of $\ln|\ln\rho_{xx}|$ vs $\ln|\Delta\nu|$ transforms the experimental data into two sets of parallel lines, equally spaced by the amount $\alpha\kappa$ along the abscissa. This is illustrated in Fig. 3 for the data set shown in Fig. 2 ($B_c = 15.7$ T). The derived values of $\kappa$ are identical to the values quoted above. The advantage of this method, which relies on the presence of a sharp well-defined crossing point, over the “traditional” method is that no fitting procedure is needed to visualize scaling behavior. Hence, errors in the determination of the critical exponent due to arbitrary fitting constraints are minimized. Besides, scaling behavior can be verified away from the critical field $B_c$. Furthermore, this

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method allows for a direct check of particle-hole symmetry, \( \rho_{xx}(\Delta \nu) = 1/\rho_{xx}(-\Delta \nu) \). In this case, the curves for \( B > B_c \) (Fig. 3a) and \( B < B_c \) (Fig. 3b) should be identical as can be verified by plotting the data from both panels in the same graph.

By reversing the magnetic field polarity, the \( \rho_{xy} \) data for the PI transition remain identical. This is an important experimental result, as it indicates that the admixture of \( \rho_{xy} \) into \( \rho_{xx} \) is negligible, which simplifies the analysis [6]. Thus a proper critical exponent can be derived directly from the symmetric (with respect to the magnetic field) \( \rho_{xx} \) data. Notice that this is not the case for the PP transition, where the \( \rho_{xx} \) data for fields up and down are slightly different.

On the other hand, the Hall resistance data are strongly affected by reversal of the magnetic field polarity. After averaging over both field polarities we find that the antisymmetric part of the Hall resistance, \( \rho_H \), remains quantized at the value \( h/e^2 \) in the insulating state for \( T \leq 0.2 \) K. With increasing temperature, however, \( \rho_H(B) \) starts to deviate from the quantized value. In Fig. 4 we show the averaged Hall resistance for a density \( 2.0 \times 10^{11} \text{ cm}^{-2} \) (\( B_c = 15.7 \) T) in the field range 13-18 T at three different temperatures. At \( B_c \), the deviation from exact quantization can be written as \( \rho_H = 1 + \eta(T) \), where \( \eta(T) = (T/T_1)^{y_\nu} \) contains the corrections to scaling. In the inset of Fig. 4 we have plotted \( \eta(T) \) (dashed line) from which we extract parameters \( y_\nu = 2.6 \) and \( T_1 = 4.5 \) K, which are similar to the values reported earlier for an InGaAs/InP heterostructure [6].

In summary, we have studied the quantum critical behavior of the PI transition in a low-mobility InGaAs/GaAs quantum well for two different values of the electron density. From the \( \rho_{xx} \) data we extract critical exponents \( \kappa = 0.54 \) and 0.58, in good agreement with the value (\( \kappa = 0.57 \)) previously obtained for an InGaAs/InP heterostructure. This provides further evidence for a non-Fermi liquid quantum critical point. By reversing the direction of the magnetic field, we find that the averaged Hall resistance remains quantized at the plateau value \( h/e^2 \) through the PI transition. From the deviations of the Hall resistance from the quantized value, we obtain the corrections to scaling.

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References
[1] A. M. M. Pruisken, Phys. Rev. Lett. 61, 1297 (1988); H. Aoki and T. Ando, Phys. Rev. Lett. 54, 831 (1985).
[2] H. P. Wei, D. C. Tsui, M. Paalanen and A. M. M. Pruisken, Phys. Rev. Lett. 61, 1294 (1988).
[3] H. P. Wei, S. Y. Lin, D. C. Tsui and A. M. M. Pruisken, Phys. Rev. B 45, 3926 (1992).
[4] B. Huckestein and B. Kramer, Phys. Rev. Lett. 64, 1437 (1990).
[5] L. A. Ponomarenko et al., cond-mat/0306063
[6] A. M. M. Pruisken et al., cond-mat/0109043
[7] R. T. F. van Schaijk, A. de Visser, S. M. Olsthoorn, H. P. Wei and A. M. M. Pruisken, Phys. Rev. Lett. 84, 1567 (2000).
[8] D. T. N. de Lang, L. A. Ponomarenko, A. de Visser, C. Possanzini, S. M. Olsthoorn, A. M. M. Pruisken, Physica E 12, 666 (2002); D. T. N. de Lang et al., to be published.
[9] M. A. Baranov et al., cond-mat/0106446 and references therein.