Ion trajectory analysis for micromotion minimization and the measurement of small forces

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Abstract. For experiments with ions confined in a Paul trap, minimization of micromotion is often essential. This is the case, for example, for experiments in quantum information science using trapped ions, in combined traps for neutral atoms and ions, and for precision measurements using trapped ions. In order to diagnose and compensate micromotion we have implemented a method that allows for finding the position of the radio-frequency (RF) null reliably and efficiently, in principle, without any variation of direct current (DC) voltages. We apply a trap modulation technique and tomographic imaging to extract 3d ion positions for various RF drive powers and analyze the power dependence of the equilibrium position of the trapped ion. In contrast to commonly used methods, the search algorithm directly makes use of a physical effect as opposed to efficient numerical minimization in a high-dimensional parameter space. The precise position determination of an harmonically trapped ion employed here can also be utilized for the detection of small forces. This is demonstrated by determining light pressure forces with an accuracy of 135 yN. As the method is based on imaging only, it can be applied to several ions simultaneously and is independent of laser direction and thus well-suited to be used with surface-electrode traps.

Keywords: trapped ions, micromotion, small forces

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1. Introduction

Trapped laser cooled ions are a prolific starting point for many experiments related to quantum information science \[1\] \[2\] and precision spectroscopy yielding the most accurate clock standards to date \[3\] \[4\] \[5\]. Ions can be trapped for long times and laser-cooled, even to the motional ground state \[6\] \[7\] \[8\], and they are one of the most promising candidates for quantum computation \[2\] and for quantum simulations where the quantum mechanical properties of a system difficult to directly investigate is simulated using the well-understood and -controlled quantum system of trapped ions \[9\] \[10\] \[11\] \[12\] \[13\] \[14\]. The size of this quantum system can be scaled up by entangling few ion systems in complex segmented trapping architectures \[15\] \[16\] \[17\].

The conceptual starting point for all this research is one or several ions at rest, more accurately close to the motional ground state of an effective, approximately harmonic trapping potential \[18\] \[19\]. The Laplace equation forbids the existence of electrostatic potential minima in vacuum (\textit{Earnshaw theorem}). This limitation is bypassed in RF quadrupole (Paul-) traps, where the ions are confined by an inhomogeneous oscillating field \[18\] \[19\], and the oscillation energy of the forced oscillation becomes position dependent. Upon the presence of a cooling mechanism such as laser cooling, an ion comes to rest at the minimum of the oscillating field amplitude. Additional DC fields from biased electrodes or surface charges, e.g. created by the loading process, can add forces which push the ion away from the RF null; in addition, a phase mismatch between RF electrodes might even prevent the existence of a time-independent RF null. In both cases, the ion will carry out a forced oscillation at the drive frequency, the so called micromotion, with an amplitude depending on DC and phase mismatch. Micromotion creates unwanted Doppler shifts and plays a key role in excess heating of trapped ions \[20\] \[21\].

Renewed interest in micromotion minimization comes from recent research with combined traps of neutral and charged atoms \[22\] \[23\] \[24\] where micromotion is a detrimental source of neutral atom loss and has up to now prevented effective collisional cooling of ions by a cold neutral atom cloud \[25\]. Here again, as an initial stage of combined traps, micromotion is carefully compensated and the neutral atom loss can be used as a figure of merit for optimization \[26\]. Micromotion can be overcome altogether by using a dipole trap for the ion. As the light interacts with the dipole moment of the ion instead of the charge, dipole traps are much softer and shallower and themselves require carefully balanced DC offset fields as a prerequisite \[27\]. Another motivation for low micromotion comes from precision spectroscopy and frequency standards: micromotion sidebands spoil the accuracy of the determination of atomic resonance frequencies due to Doppler shifts, and thus micromotion is an essential prerequisite for accurate ion trap frequency standards. One disadvantage, though not a matter of principle, is that ion clocks suffer in short term stability, as just one atom is probed. Massively parallel interrogation of many ions in common or separate traps, all micromotion compensated, is an important step to improve the short term stability of ion based frequency standards.
Micromotion minimization can thus be regarded as a common crucial initial step and prerequisite for good trap performance. Micromotion causes diverse signatures, and several methods have been developed based on the minimization of either of these signatures:

(i) Micromotion creates a periodic Doppler shift, and thus a temporal correlation between the scattering rate and the trap drive [29, 30, 28].

(ii) When DC fields are not nulled at the RF node, the equilibrium position depends on the power of the RF trap drive. A modulation of the trap drive power, fast compared to the imaging exposure, creates a directional smearing of the ion image [30].

(iii) A periodic Doppler shift corresponds to sidebands in the absorption spectrum, spaced by the trap drive frequency [30, 31, 32, 33, 34].

(iv) The Doppler effect also modulates the emission spectrum of an ion subject to micromotion, and creates sidebands, again spaced by the trap drive frequency [35].

(v) Micromotion allows to increase the heating rate when exciting parametric resonances by modulating the trap drive at the secular frequency [36, 37, 38].

(vi) The effect on normal mode spectra of mixed ion species crystals [39].

(vii) When trapping simultaneously neutral atoms and ions, kinetic energy of the micromotion can be transferred to neutral atoms by collisions and lead to atom loss [26].

In this paper, we revisit the trap modulation technique (ii). For different RF levels, we determine the 3d ion position from tomographic imaging. This position can be determined with an uncertainty far below the wavelength of light scattered off the ions for observation. From the ion positions measured at different RF powers, we can extrapolate the ion trajectory to infinite RF power, which ends at the RF null. With the knowledge gathered from a single trajectory, the ion can be moved to the RF null by changing the DC electric fields, which requires accurate a priori characterization of the trapping fields. As an alternative, trajectories recorded at different DC settings readily yield this characterization, together with the optimized compensation voltages.

The time required for the minimization process can be crucial if the DC fields are fluctuating. This can happen as a consequence of stray charges and, usually to lesser extent, by drifting fields from unstable voltage supplies [37, 40]. For surface traps trapping times can be in the second or minute range owing to smaller trap depths and might require frequent reloading. Patch potentials may change on a short timescale. In such cases it is desirable to carry out the minimization fast and repeatedly to track or compensate time dependent effects.

The method discussed here relies on position determination, which in 2d can be carried out within milliseconds with high precision and does not require any narrow transition and ultra stable lasers for sideband spectroscopy. If the position determination
and the minimization are carried out in 3d, we need to use a tomographic method and the approach is slowed down by about an order of magnitude. On the other hand, even then, no constraints on the propagation direction of the laser limits the optimization procedure. As the tomographic method is based on imaging, the optimization might even be carried out for several ions in parallel, which is of particular interest for frequency standards.

The method discussed here is fast and has a sensitivity comparable to other approaches and is yet general, simple, and economic.

The paper is structured as follows: in Sec. 2, we model the displacement of the ion’s equilibrium position. The experimental setup is detailed in Sec. 3. The measurement procedure and results are presented in Sec. 4. As the position determination can be precise to a nanometer level, we use this for the measurement of small forces on the example of the light pressure in Sec. 5.

2. Theory

An ion (mass $m$, charge $Q$) trapped in a Paul trap (operated with an AC voltage $V \cos \Omega t$ and a DC voltage $U$) in the presence of an electric stray field $\vec{\varepsilon}$ obeys the inhomogeneous Mathieu equations [30]

$$\ddot{r}_k + \left[ a_k + 2q_k \cos(\Omega t) \right] \frac{\Omega^2}{4} r_k = \frac{Q \varepsilon_{\text{stray},k}}{m}, \quad k = x, y, z, \quad (1)$$

where $\vec{r} = \sum_k r_k \hat{k}$ is the ion position, $\hat{k} = \hat{x}, \hat{y}, \hat{z}$ are the cartesian unit vectors, $a_k$ and $q_k$ are the commonly called trapping parameters and the electric stray field $\varepsilon_{\text{stray}} = \sum \varepsilon_{\text{stray},k} \hat{k}$ is approximated to be static and uniform over the small volume occupied by the ion trajectory.

Such a stray field might be a real physical field (e.g. originating from charged isolators near the trap), but also effects lifting ideal symmetry and displace the DC origin with respect to the RF (e.g. misaligned trap electrodes or machining imperfections) as well as other homogeneous static forces (e.g. the light pressure force exerted by a cooling laser) can effectively be treated as contributions to the stray field.

Solving Eq. (1) using the adiabatic approximation ($|a_k|, q_k^2 \ll 1$) yields the approximate motion of the ion [30]

$$r_k(t) \approx \left[ r_{\varepsilon,k} + r_{0,k} \cos(\omega_k t + \varphi_k) \right] \left( 1 + \frac{q_k}{2} \cos(\Omega t) \right)$$

$$= \left[ r_{0,k} \cos(\omega_k t + \varphi_k) \right] \left( 1 + \frac{q_k}{2} \cos(\Omega t) \right) + r_{\varepsilon,k} + \frac{q_k r_{\varepsilon,k}}{2} \cos(\Omega t), \quad (3)$$

where $r_0$ and $\varphi_k$ depend on the initial conditions and $\omega_k = \frac{\Omega}{2} \sqrt{a_k + q_k^2/2}$. Eq. (3) describes a motion characterized by two frequency components: the secular motion component oscillating at frequency $\omega_k$ and the micromotion component oscillating at the much higher frequency $\Omega$ and much smaller amplitude $q_k r_{0,k}/2$. The secular motion can be reduced by cooling mechanisms as e.g. laser cooling. This reduces the micromotion amplitude as well as it is proportional to the secular motion. The last two terms of
Eq. (3) describe the effects of the additional electric field. The field $\vec{\varepsilon}_{\text{stray}}$ shifts the average position of the ion out of the RF potential node to the position

$$\vec{r}_\varepsilon \approx \frac{Q}{m} \sum_k \frac{\varepsilon_{\text{stray},k}}{\omega_k^2} \hat{k},$$

(4)

at which the RF electric field causes oscillations with amplitudes $q_k r_{\varepsilon,k}/2$ – the excess micromotion. This motion is a driven motion and thus cannot be significantly reduced by cooling techniques.

In the pseudo- or ponderomotive potential approach \[18\], the motion of the ion due to the RF electric field is averaged over one period of the micromotion and the so-called RF pseudopotential represents the kinetic energy due to micromotion,

$$\phi_{\text{eff}}^{\text{rf}} (\vec{r}) = \frac{m}{2} \sum_k \omega_{\text{rf},k}^2 r_k^2, \quad \omega_{\text{rf},k}^2 = \frac{\Omega^2 q_k^2}{4}.$$

(5)

The $\omega_{\text{rf},k}$ are called the RF trap frequencies and represent the RF contribution to the secular frequencies $\omega_k$.

The ion can then be treated as being confined in an effective potential that is the sum of the RF pseudopotential $\phi_{\text{eff}}^{\text{rf}}$ and all DC potential contributions (including DC electrodes ($\phi_{\text{dc},i}$) as well as stray potentials ($\phi_{\text{stray}}$)),

$$\phi_{\text{eff}}(\vec{r}) = \phi_{\text{eff}}^{\text{rf}} (\vec{r}) + Q \sum_i \phi_{\text{dc},i}(\vec{r}) + Q\phi_{\text{stray}}(\vec{r}),$$

(6)

where $-\nabla \phi_{\text{stray}}(\vec{r}) = \vec{\varepsilon}_{\text{stray}}$. Here the $\phi_{\text{dc},i}$ are taken to be ideal potentials in the aforementioned sense: contributions causing a shift of the DC origin relative to the RF node (that is contributions linear in the ion’s position) are absorbed into $\phi_{\text{stray}}$.

Near the center of the confining potential (e.g. along the axis of a linear trap) the sum of DC potentials can be approximated as a quadrupole potential

$$Q \sum_i \phi_{\text{dc},i}(\vec{r}) = \frac{m}{2} \sum_k \omega_{\text{dc},k}^2 r_k^2.$$  

(7)

Similar to the $\omega_{\text{rf},k}$ the $\omega_{\text{dc},k}$ are the DC trap frequencies at a specific setting of all DC trapping voltages ($\omega_{\text{dc},k}^2 = \Omega^2 a_k/4$ in the case of an ideal linear trap). For the sake of simplicity the principal axes of the RF and DC potentials have been chosen here to be parallel and equal to the cartesian axes.

The equilibrium position of the ion is then given by zero net force

$$-\nabla \phi_{\text{eff}}(\vec{r}) = -\nabla \left( \frac{m}{2} \sum_k \left( \omega_{\text{rf},k}^2 + \omega_{\text{dc},k}^2 \right) r_k^2 + Q\phi_{\text{stray}}(\vec{r}) \right) \equiv \vec{0}$$

(8)

$$\Rightarrow \vec{r}_0 = \frac{Q}{m} \sum_k \frac{\varepsilon_{\text{stray},k}}{\omega_{\text{rf},k}^2 + \omega_{\text{dc},k}^2} \hat{k} = \frac{Q}{m} \sum_k \frac{\varepsilon_{\text{stray},k}}{P_{\text{rf}} \omega_{\text{rf},k}^2 + \omega_{\text{dc},k}},$$

(9)

which is equivalent to the average displacement $\vec{r}_\varepsilon$ from Eq. (4), as $\omega_k^2 = \omega_{\text{rf},k}^2 + \omega_{\text{dc},k}^2$. Here $P_{\text{rf}}/P_0$ has been introduced as a convenient scaling factor. $P_{\text{rf}}$ is the applied RF power and $\omega_{\text{rf},k}$ the RF trap frequencies at an applied RF power of $P_0$ ($P \propto V^2 \propto q_k^2$).

The shift of the equilibrium position out of the RF potential minimum depends on the stray field and the applied RF power (see Fig. (1)).
As an intuitive point of view, it is also useful to combine the effects of all stray fields (real as well as effective fields) as a translation $\hat{\delta}(\varepsilon_{\text{stray}})$ of the DC potentials origin explicitly and rewrite the effective potential as

$$
\phi_{\text{eff}}(\vec{r}) = \frac{m}{2} \sum_k \frac{P_{\text{rf}}}{P_0} \omega_{\text{rf},k}^2 r_k^2 + \frac{m}{2} \sum_k \omega_{\text{dc},k}^2 (r_k - \delta_k(\varepsilon_{\text{stray}}))^2.
$$

(10)

In a one-dimensional trap, the electric stray field would shift the DC potential by

$$
\delta = -\frac{Q\varepsilon_{\text{stray}}}{m\omega_{\text{dc}}^2}
$$

and the ion’s position in the trap by

$$
r_0 = \frac{Q}{m} \frac{\varepsilon_{\text{stray}}}{P_{\text{rf}} P_0 \omega_{\text{rf}}^2 + \omega_{\text{dc}}^2}.
$$

(12)

As the applied RF power is increased the ion’s position would then approach the minimum of the RF pseudopotential ($r_0 = 0$). By monitoring $r_0$ as a function of the RF power an extrapolation of the RF node in the limit of $P_{\text{rf}} \to \infty$ is possible.

Instructive cases of two dimensional potential configurations displaying characteristic features of the ion’s movement are shown in Fig. (2). The electric stray potential has been included as a shift of the DC potential’s origin (see Eq. (10)). RF potential isolines are drawn in red, DC potential isolines are drawn in blue. The colored dots indicate the respective potential’s origin. The thick black line marks the equilibrium positions of the ion while tuning the applied RF power from infinity to zero.

Fig. (2a) displays a configuration of confining RF and DC potentials with degenerate trap frequencies ($\omega_{\text{rf},x}^2 = \omega_{\text{rf},y}^2 > 0$, $\omega_{\text{dc},x}^2 = \omega_{\text{dc},y}^2 > 0$), as it can be realized in the central horizontal plane of a ring trap with a positive DC bias on the ring electrode. Due to radial symmetry, the ion’s equilibrium positions will move on a straight line between the RF and DC node as the RF power is varied.
Figure 2. Two dimensional potential configurations of the DC (blue isolines) and RF (pseudo-)potential (red isolines). The potentials’ origins are indicated by respectively colored dots. The thick black line marks the ion’s trace as the strength of the RF potential is tuned from infinity to zero. The dashed line shows the full hyperbola and its asymptotes describing the ion’s movement. (a) A configuration of degenerate trap frequencies ($\omega_{rf,x}^2 = \omega_{rf,y}^2 > 0$, $\omega_{dc,x}^2 = \omega_{dc,y}^2 > 0$). Due to radial symmetry, the ion’s equilibrium positions will move on a straight line. (b) For broken DC radial symmetry ($\omega_{dc,x}^2 < \omega_{dc,y}^2$) the ion’s trace changes to a hyperbola. (c) A potential configuration with an unstable DC trapping in $\hat{x}$-direction ($\omega_{dc,x}^2 < 0$) as realized in the transverse plane of a linear Paul trap. (d) Additionally broken RF radial symmetry ($\omega_{rf,x}^2 > \omega_{rf,y}^2$) and a rotation of the DC potential’s principal axis causes a shearing and rotation of the hyperbolic trace, featuring non-perpendicular asymptotes.

When radial symmetry is broken (e.g. $\omega_{dc,x}^2 < \omega_{dc,y}^2$) the ion’s trace changes to a hyperbola (Fig. 2b)). The dashed line indicates the full hyperbola and its asymptotes. As the RF potential still features rotational symmetry, the hyperbola’s asymptotes are parallels to the DC potential’s principal axis.

For now we have assumed the RF and DC potential to be confining in all directions. In a typical linear Paul trap, the DC potential will be used to create DC confinement in axial direction. Since the DC potential has to obey Laplace’s equation ($\Delta \phi_{dc}(\vec{r}) \equiv 0$) at least in one of the radial directions the DC potential must be repulsive. Fig. 2c) shows the same configuration as Fig. 2b) but with an unstable DC trapping in $\hat{x}$-direction ($\omega_{dc,x}^2 < 0$), resembling the potentials in the transverse plane of a linear Paul trap. The extreme values of the two potentials do not reside on the same branch of
the hyperbola but are separated by it’s pole. When decreasing the RF power and such lowering the RF confinement, the ion’s equilibrium position will move away from both, the RF minimum and the DC saddle point, finally escaping from the trap when the RF confinement becomes weaker than the DC repulsion.

In the most general case (Fig. (2d)) also the RF potential’s radial symmetry is broken ($\omega_{rf,x}^2 > \omega_{rf,y}^2$) and the directions of the DC potential’s principal axis are rotated with respect to the RF potential. This causes a shearing and rotation of the hyperbolic trace, featuring non-perpendicular asymptotes.

A particularly interesting consequence of non-degenerate radial RF trap frequencies is the existence of a non-vanishing axial RF component. In the quadrupole approximation the RF potential can be written as

$$\phi_{\text{real}}^{\text{rf}}(\vec{r},t) = \sum_k \alpha_k(V) \cos(\Omega t) r_k^2,$$

where the $\alpha_k$ are depending on the actual geometry of the trap and the RF voltage $V$. Using Eq. (5) and $q_k = \frac{2Q}{m \Omega^2} \alpha_k$ the RF trap frequencies are given by

$$\omega_{rf,k}^2 = \frac{Q^2}{2m^2 \Omega^2} \alpha_k^2.$$

Since the RF potential fulfills Laplace’s equation $\Delta \phi_{\text{real}}^{\text{rf}}(\vec{r},t) \equiv 0 \forall t$, it follows that

$$\Delta \phi_{\text{real}}^{\text{rf}}(\vec{r},t) = 2 \cos(\Omega t) \sum_k \alpha_k \equiv 0 \Rightarrow \sum_k \alpha_k \equiv 0.$$

A vanishing axial RF component $\omega_{rf,z}$ requires $\alpha_z = 0$, which immediately yields the degeneracy of the radial RF trap frequencies,

$$\alpha_x = -\alpha_y \Rightarrow \omega_{rf,x}^2 = \omega_{rf,y}^2.$$

Above considerations allow to detect excess micromotion by monitoring the ion’s position while varying the applied RF power. Using Eq. (9) (adjusted for a rotation of the DC potential principal axis), in principle the measurement of a single trace of the ion’s movement during an RF variation without any variation of DC voltages is sufficient to extrapolate the location of the RF node and therefore the location of zero excess micromotion.

In practice insufficient knowledge of the trapping parameters (e.g. relative rotation of the RF and DC potential principal axis, electric field generated by compensation electrodes) might require the combination of variations of the RF power and several DC voltage settings. An example of such a combination is given in Fig. (3). The RF and DC potential configuration is the same as in Fig. (2d), with three additional settings for compensation voltages, shifting the origin of the original DC potential (blue) to $(1.0, -0.5)$ (green), $(-0.5, -0.5)$ (orange) and $(-0.5, 1.0)$ (red). The DC potential origins are marked by the respectively colored dots. The ion’s traces (thick lines) for all of these settings converge to the RF node in the limit of infinite RF power. A parallel analysis of the measurements for different DC configuration yields then the position of the RF null as well as the required DC compensation voltages.
Figure 3. A combination of different DC potential shifts for a potential configuration as shown in Fig. (2d). The RF (pseudo-)potentials isolines are drawn in gray. The ion’s traces (thick lines) are shown for the original DC potential (blue) and shifts of it’s origin to (1.0, −0.5) (green), (−0.5, −0.5) (orange) and (−0.5, 1.0) (red). Dashed lines showing the respective full hyperbolas, colored dots mark the DC potentials’ origins. All traces converge to the RF node (black dot) in the limit of infinite RF power. Near the RF node, the set of all trajectories resembles a star, as expected by intuition. The directions of the hyperbola’s asymptotes are unchanged by the shifts.

3. Experimental setup

A schematic view of the region of the microstructured segmented linear Paul trap used in the experiments presented here is shown in Fig. (4). The trap electrodes have a separation of 500 µm in \( \hat{y} \) direction and 350 µm in \( \hat{x} \) direction. The DC electrodes are divided into segments of 250 µm width (labeled DC\(_{Up,i}\) and DC\(_{Low,i}\) in Fig. (4)). The full trap features 33 pairs of DC electrodes composing a wide section (described above) and a narrow section (electrode width 100 µm, \( \hat{y} \) separation 250 µm) section connected by a tapered transfer section. The trap and the supporting experimental setup has been described in detail in [31, 41, 42].

For this work we trapped single \(^{172}\text{Yb}^+\) ions, which are produced by a two-photon ionization process from neutral Ytterbium vapor emitted from the atom oven [43]. The ion is Doppler cooled on the \( S_{1/2} - P_{1/2} \) transition. The ionization laser (399 nm), the cooling laser (369 nm) and two additional repumpers (638 nm and 935 nm) are overlapped outside the trap [41] and propagate through the center of the trapping region, enclosing angles of approximately 35° with the \( \hat{x} \) direction and 89° with the \( \hat{y} \) direction (see Fig. (4)).

For basic trapping the central electrode pair (DC\(_{Up/0}\)) is set to −1 V, providing axial confinement, whereas all other DC electrodes are set to ground. The RF electrodes are driven through a helical resonator with voltage of 210 V\(_{pp}\) at a frequency of 13.2 MHz. The secular frequencies of an ion trapped with this configuration are \( \omega_{x'} = 2\pi \times 1.054 \text{ MHz} \), \( \omega_{y'} = 2\pi \times 1.112 \text{ MHz} \) and \( \omega_z = 2\pi \times 199 \text{ kHz} \). The principal radial axes of the trapping potential (\( \hat{x}' \) and \( \hat{y}' \)) are rotated by about 64° with respect to the \( \hat{x} \) and \( \hat{y} \) axis.
Static and approximately uniform fields for micromotion minimization are created using three compensation voltages $U_C$ (see Fig. (4b)). $U_{C1}$ is added as an antisymmetrical bias voltage to all upper ($+U_{C1}/2$) and lower ($-U_{C1}/2$) DC segments, causing a field roughly along the $\hat{x} + \hat{y}$ direction near the trap axis. A second compensation voltage $U_{C2}$ is applied to the aperture of the nearby atom oven. The aperture extends mainly in the $y$-$z$ plane and its size largely exceeds the dimensions of the central trap slit, creating a field mainly along the $\hat{x}$ direction. A bend of the aperture along the $\hat{y}$ direction near the trapping site causes a non-negligible $z$ component of the compensation field created by $U_{C2}$ (see Fig. (4c)). Therefore a counteracting field is created by adding a calibrated bias voltage $U_B(U_{C2})$ antisymmetrically to all outer left (DC$_{Up/Low,i}$, $i \leq -2$) and outer right ($i \geq 2$) DC electrodes (colored orange), that cancels the unwanted $z$ component of the aperture’s field. The third compensation voltage $U_{C3}$
is applied antisymmetrically to the same outer electrodes if control of the ion’s axial position is desired.

The ion is detected by imaging fluorescence of the cooling transition along the $-\hat{x}$ direction on an EMCCD (Andor iXon Blue), see Fig. (5): The light gathering objective (numerical aperture of 0.4) is mounted on a motorized travelling stage and allows a positioning of the focus of the imaging system with a resolution of 8 nm along the $x$-axis. The objective collects the ion’s fluorescence with a magnification of approximately 12.5 onto the chip of the EMCCD consisting of $512 \times 512$ pixels with an edge length of $16 \times 16 \mu m^2$.

4. Method and Results

4.1. Tomographic position determination

Since our experiment only features imaging of the ion from a single direction, we determine the full three dimensional position of the ion in our trap during the RF and DC variation by employing tomographic imaging (see Fig. (5)).

For every position determination, we take several images of the ion with the focus swept over $\pm 15 \mu m$. For every single image a two-dimensional gaussian is fitted to the intensity distribution yielding $(y, z)$ center coordinates of the distribution as well as the gaussian spot size. The $x$ coordinate of the ion is then deduced by fitting the spot size of a gaussian beam to the spot sizes obtained from the single images (see Fig. (6)). Then the center coordinates of the intensity distribution imaged with the focus setting closest to the ion’s $x$ coordinate are taken as the $y$ and $z$ coordinates of the ion. Using

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5}
\caption{A schematic view of the tomographic three dimensional determination of the ion’s position. The light gathering objective is mounted on a motorized travelling stage, that is used to sweep the focal plane (black dashed lines) while imaging a series of intensity distributions (blue lines). The ion’s $x$ coordinate is determined from a fit of a gaussian beam waist to the spot sizes at the different focus settings (dashed red line). The $y$ and $z$ coordinate are taken as the center of a two-dimensional gaussian fitted to the intensity distribution imaged closest to the $x$ coordinate.}
\end{figure}
Figure 6. Tomographic three dimensional determination of the ion’s position. Fitting two-dimensional gaussians to the intensity distributions taken at different focus displacements yields the center coordinates \((y, z)\) of the distribution as well as the spot size. Exemplary fits for focus displacements of \(\pm 13.5 \mu m\) and \(0 \mu m\) are given in the right graph. The white lines indicate the distributions isolines at one and two FHWM. The ion’s \(x\) coordinate is deduced by fitting the spot size of a gaussian beam waist to the spot sizes obtained from the single images (left graph). The errorbars for large focus displacements are overestimated as the assumption of a gaussian beam only holds near to the focus. The center coordinates of intensity distribution imaged with the focus setting closest to the ion’s \(x\) coordinate are taken as the ion’s \(y\) and \(z\) coordinates.

This procedure, one is able to determine the average ion position with an accuracy that is significantly below the wavelength of the fluorescence light. In principle the accuracy scales as the inverse square root of the fluorescence and is only limited by their signal to noise ratio and the mechanical stability of the imaging system. For an average number of about 7000 photons per image (observed scattering rates of about 200 kHz to 400 kHz) and images taken at 25 focal positions we routinely achieve an accuracy of \((\sigma_x, \sigma_y, \sigma_z) = (98 \text{ nm}, 39 \text{ nm}, 35 \text{ nm})\).

4.2. Micromotion minimization in 3d

By construction an ideal linear trap features no axial RF electric field component and therefore no axial micromotion. For all realizations with finite size electrodes, particularly for segmented traps, regions of non-negligible axial micromotion cannot be avoided, although recent results have demonstrated trap designs with small residual axial RF field components [37, 45, 46, 28].

The focus of segmented traps, as the one used here, lies often in the storage and manipulation of ion crystals. Such operations, e.g. shuttling, splitting and merging make use of an extended region along the almost micromotion free symmetry axis of the linear trap. In a similar sense, when experimenting with extended linear ion crystals oriented along this axis, finding a single point of vanishing axial micromotion compensation is not meaningful. Any axial offset from this ideal position, as long as it is much smaller than the extent of the crystal will have a small effect. So the level of precision aimed
Figure 7. Results of a 3d optimization procedure performed in a trap region with non-negligible axial RF field strength. The minimization was performed using 4 \((U_{C1}, U_{C2})\) \times 3 \((U_{C3})\) settings of compensation voltages. Dots represent the average ion position as obtained from tomographic imaging, with their color indicating the respective RF power. Example errorbars indicating \(\pm \sigma_x\) and \(\pm 2\sigma_{y,z}\) are given for four points. A fit of the trajectories is indicated by dashed lines. The black dot marks the position of the RF node.

for in the determination of the axial position will be on the order of a micrometer or slightly below.

If the axial positions of trapped ions are not constraint by other experimental requirements, minimization of axial as well as radial micromotion can be performed.

A 3d micromotion minimization is demonstrated qualitatively by extending the procedure for 2d minimizations (as detailed below in Sec. 4.3) by a variation of the axial compensation voltage \(U_{C3}\). The model of the effective trapping potential given in Eq. \((17)\) is complemented by adding the respective terms for axial components of the RF pseudopotential, DC potential and the linear potential created by \(U_{C3}\). Fig. \((7)\) demonstrates a 3d optimization procedure performed in a region of our trap suffering from significant axial RF electric field strength. The minimization was performed with 4 different settings for the radial compensation voltages \((U_{C1}, U_{C2})\) and 3 different settings for the axial compensation voltage \(U_{C3}\), yielding 12 ion trajectories of varying RF powers. Dots represent the average ion position with their color indicating the respective RF power. A fit of the trajectories from the 3d effective trapping potential are given by dashed lines. As described in Sec. \((2)\) all trajectories approach the RF node (black dot) for increasing RF field strength.
4.3. Micromotion minimization in 2d

In most usecases of segmented linear traps the axial positions of ions are not free parameters and a spacecurve of positions of minimized radial micromotion along the axial direction may be used to optimize the operation of the trap.

To minimize micromotion in the radial plane at a specific axial position we choose a set of voltage settings \( \{ (U_{C1}, U_{C2})_m \} \) and apply a set of RF powers \( \{ P_{rf, n} \} \). For all combinations \((m, n)\) a tomographic position determination is carried out yielding the ion’s equilibrium positions \( \{ \vec{r}_{0,m,n} \} \). We then model our effective trapping potential as a superposition of the RF pseudopotential, the unshifted DC potential and two additional linear potentials as created by our compensation electrodes,

\[
\Phi_{\text{eff}}^{(m,n)}(x, y) = \frac{m P_{rf, n}}{2} \left( \vec{r} - \vec{r}_{rf} \right)^\top R_{\alpha_{rf}} M_{\alpha_{rf}}^{-1} \left( \vec{r} - \vec{r}_{rf} \right) \\
+ \frac{m}{2} \left( \vec{r} - \vec{r}_{rf} \right)^\top R_{\alpha_{dc}} M_{\alpha_{dc}}^{-1} \left( \vec{r} - \vec{r}_{rf} \right) \\
+ s_{C1} (U_{C1,m} - \tilde{U}_{C1}) y^\top R_{\alpha_{C1}} \left( \vec{r} - \vec{r}_{rf} \right) \\
+ s_{C2} (U_{C2,m} - \tilde{U}_{C2}) x^\top R_{\alpha_{C2}} \left( \vec{r} - \vec{r}_{rf} \right),
\]

where \( R_\alpha \) is a rotation by \( \alpha \), \( M_{\omega_{rf}^2} \) and \( M_{\omega_{dc}^2} \) are the diagonal matrices of squared RF and DC trap frequencies \((\omega_{rf, 0, k}^2, \omega_{dc, k}^2)\) and \( s_{C1} \) and \( s_{C2} \) are geometrical factors depending on the compensation electrodes shape and placement. \( \tilde{U}_{C1} \) and \( \tilde{U}_{C2} \) are the compensation voltages that cancel the stray potentials and shift the origin of the DC potential to the position of the RF null \( \vec{r}_{rf} \). The trap frequencies are determined from independent measurements by exitation of the ion’s motion with a voltage modulation of one DC segment for various settings of the applied RF power \( P_{rf} \). All remaining parameters are determined by fitting the equilibrium positions obtained from Eq. (17) to the \( \{ \vec{r}_{0,m,n} \} \).

Fig. (8) shows the results of an optimization procedure with 12 different setting for the compensation voltages \( (U_{C1}, U_{C2}) \) and a variation of the applied RF power over 12.5 dB in 25 steps (varying the RF voltage from 420 V_{pp} to 99 V_{pp}). The dots represent the ion’s positions \( \vec{r}_{0,m,n} \) with color-coded RF power, the dashed line represents the fit and the position of the RF node \( \vec{r}_{rf} \) is marked by the black dot. After applying the optimized compensation voltages as yielded by the fit, we perform a similar measurement but without any variation of the compensation voltages. While lowering the RF confinement, we detect no change of the ion’s average position before the ion escapes the trap (at an RF voltage of about 62 V_{pp}), where the escape happens faster than time resolution of our position determination. Using Eq. (9) and the weighted standard deviation of the ion positions along the principal radial axis of our trapping potential, we derive a residual electric stray field uncertainty of \((\Delta \varepsilon_{\text{stray,x}}, \Delta \varepsilon_{\text{stray,y}}) = (0.09 \text{ V/m}, 1.09 \text{ V/m})\) at respective trap frequencies of \( \omega_x' = 2\pi \times 132.5 \text{ kHz} \) and \( \omega_y' = 2\pi \times 387.0 \text{ kHz} \).
5. Position determination application: light pressure measurement

Another application of precise position measurements is the determination of small forces. We demonstrate this on the example of the light pressure force \cite{47} acting on a laser cooled ion in a Paul trap.

When micromotion is minimized using any scheme that utilizes a signal from an incoherent scattering process, the light pressure force due to the fluorescence inducing laser is compensated automatically. As a consequence, the settings obtained from the minimization are only valid at the scattering rate used during the process and will be different if the interaction is changed (e.g. change of laser intensity or usage of a different transition). This is especially true for coherent manipulation of the ion as there is no light pressure force present.

If minimized micromotion is required for situations without photon scattering, it is necessary to perform the minimization for several scattering rates and extrapolate the minimization settings to the zero scattering level. Given sufficient knowledge about the DC potential, one can monitor the ion’s position as a function of the scattering rates.
and counteract solely the light pressure induced shift.

The force corresponding to the time averaged momentum transfer of absorbed laser photons (or light pressure force) \( F_{lp} \) is given by

\[
\vec{F}_{lp} = \langle p_{ph} \rangle_t = \Gamma \hbar \vec{\kappa} \tag{18}
\]

with rate of absorption \( \Gamma \), the Planck constant \( \hbar \) and the laser’s wave vector \( \vec{\kappa} \).

This force can effectively be treated as originating from a stray potential and adds as \( F_{lp,k}/m \) to the ride-hand-side of Eq. (1). In analogy to Eq. (4), the light pressure force shifts the ion’s average position by

\[
\vec{r}_{lp} = \frac{1}{m} \sum_k F_{lp,k} \omega_k \hat{k}' \tag{19}
\]

with \( M_{\omega^2}^{-1} \) being the inverse matrix of the squared secular frequencies. In the frame of the potential’s principal axis \( M_{\omega^2}^{-1} \) is diagonal and Eq. (19) reduces to \( \vec{r}_{lp} = \frac{1}{m} \sum_k F_{lp,k} \omega_k \hat{k}' \) (see Eq. (4)).

When the light field interacting on the cooling transition is detuned by \( \delta \) from resonance, the scattering rate \( \Gamma \) and hence the rate of absorption of the ion is given by

\[
\Gamma = \frac{s_0}{1 + s_0} \frac{\gamma/2}{1 + (2\delta/\gamma')^2} \tag{20}
\]

with the on-resonance saturation parameter \( s_0 = 2\Omega^2/\gamma^2 \), the Rabi frequency \( \Omega \) of the interaction, the natural line width of the transition \( \gamma \) and the saturation broadened line width \( \gamma' = \gamma \sqrt{1 + s_0} \).

The expected light pressure force of a laser saturating \( s_0 = 1 \) the \( S_{1/2} - P_{1/2} \) transition of \(^{172}\text{Yb}^+\) red detuned by \( \gamma/2 \) is \( |\vec{F}_{lp}| \approx 35 \text{zN} \). For the parameters given as setting (a) in Fig. [9] this force would cause a shift of \( \vec{r}_{lp} \approx (4 \text{nm}, 1 \text{nm}, -50.0 \text{nm}) \), which is on the order of the level of accuracy of the position determination reported above for the \( z \)-component and significantly below for the \( x \)- and \( y \)-component. To be able to observe the light pressure shift averaging over several position determinations is a necesssity in our present setup. Furthermore the mechanical stability of our imaging system requires differential measurements to cancel slow movements of the objective mount causing a virtual shift of the ion’s position.

We measure the shift of the ion’s center coordinates in the \( z-y \) plane as a function of the change in observed scattering rate for different sets of trap frequencies. The ion’s scattering rate is varied by tuning the intensity of the 369 nm light field using an AOM in 15 steps. Exposure times are adjusted such that the amount of collected photons for each intensity setting is approximately constant to avoid systematic effects in the position determination. We obtain the shifts of the ion’s central positions \( (\Delta y, \Delta z) \) with respect to the lowest intensity setting and average these shifts over 500 repetitions of the intensity variation for each trapping configuration, which improves the accuracy to the nanometer range. The change in observed scattering rate \( \Delta \Gamma_{obs} \) is determined by extracting the number of collected fluorescence photons from the ion images and referencing to the lowest intensity setting as well. Using a part of the image with
Figure 9. Shifts of the ion’s $y$ (blue markers) and $z$ (red markers) position as a function of the change in observed scattering rate for three different trap configurations. Corresponding symbols and trap frequencies (in kHz) are specified in the legend. Set (a) gives strong confinement in all three directions, set (b) features weakened axial confinement resulting in an increased shift along the $\hat{z}$ direction. Set (c) is weakened in terms of radial confinement and causes a substantial shift along the $\hat{y}$ direction. The inset shows a zoom in to the region of small shifts. The straight lines are fits of the linear dependence of the position shift on the change of the light pressure force (see text).

negligible contributions from the ion’s intensity distribution we monitor and correct for the background.

Fig. (9) shows the shifts of the ion’s average position in $\hat{y}$ (blue) and $\hat{z}$ direction (red) versus the change in observed scattering rate for three sets of trap frequencies: (a) strong confinement in all three directions, (b) weakened axial confinement and (c) weakened radial confinement. Detailed trap frequencies and corresponding symbols are given in the legend of Fig. (9). The inset shows a zoom in where data from the weak axial confinement has been left out for clarity. The linear dependence of the ion’s position shift on the observed scattering rate and thus the light pressure force given by Eq. (19) and Eq. (18) is clearly visible.

We fit a linear model $\Delta r_{lp,k} = \alpha_k \cdot \Delta \Gamma_{obs} + \beta_k (k = y, z)$ to the data, where $\alpha_k = \gamma \frac{\hbar}{m} \left( M_{\omega_k}^{-1} \right)_k$ and $\gamma$ being a scaling factor accounting for the overall efficiency of our imaging system ($\Gamma = \gamma \cdot \Gamma_{obs}$). When we weaken the axial confinement of the ion but keep a comparable radial confinement (going from case (a) to (b)), the movement in $\hat{z}$ direction increases from $\alpha^{(a)}_z = -0.19(3) \text{ nm/kHz}$ to $\alpha^{(b)}_z = -4.66(4) \text{ nm/kHz}$ whereas the shift in $\hat{y}$ direction ($\alpha^{(a)}_y = -0.01(1) \text{ nm/kHz}$) changes only negligibly. Keeping the axial confinement but lowering the radial confinement (going from case (a) to (c)), we observe no change in the shift along the $\hat{z}$ direction but an increased shift along $\hat{y}$ of
Table 1. Comparison of the ratios of $\alpha_z$ as obtained from our fits and the squared inverse ratios of the trap frequencies $\omega_z$ ($\alpha_z \propto \frac{1}{\omega_z^2}$) for three different trapping configurations (a), (b) and (c) (see text and Fig. 9).

| (i),(j) | (a),(b) | (c),(b) | (c),(a) |
|--------|--------|--------|--------|
| $\left(\frac{\omega_z^{(i)}}{\omega_z^{(j)}}\right)^2$ | 0.046(3) | 0.046(3) | 0.9947(15) |
| $\alpha_z^{(i)}/\alpha_z^{(j)}$ | 0.041(3) | 0.044(5) | 0.92(16) |

$\alpha_y^{(c)} = 0.144(13)$ nm/kHz that agrees with the approximate orientation of the trapping potential’s principal axes.

Since $M_{\omega_z^{-1}}$ is not diagonal in our frame of observation, the ion’s displacements cannot in general be associated to only one of the trap frequencies and as the principal axes of the DC and RF effective trapping potentials differ, changing the radial confinement also changes the orientation of the radial trapping axes. For the shifts of the ion position along the $\hat{z}$ direction however $\alpha_z = \gamma \frac{\hbar}{m} \frac{\kappa_z^2}{\omega_z^2}$, as the axis of our linear trap is aligned in parallel to $\hat{z}$. So the ratio of $\alpha_z$ for two different axial confinements should be equal to the squared inverse ratio of the trap frequencies $\omega_z$. Ratios obtained from the measurements displayed in Fig. 9 and the respective trap frequency ratios for the axial position shifts are given in table (1), yielding good agreement.

Best sensitivity and uncertainty is achieved for light weighted particles and low trap frequencies along the direction of the scattering force (see Eq. (19)). As the sensitivity depends on position accuracy, the measurement can again be either optimized for accuracy or speed. For our setup we report a sensitivity for the measurement of the light pressure force along the $\hat{z}$ direction of $s_{F_{lp}} = 633 \text{ yN}/\sqrt{\text{Hz}}$ (total exposure of a differential measurement: 38 ms), with our minimal absolute uncertainty being $\sigma_{F_{lp}} = 135 \text{ yN}$.

6. Performance

In [26] a comparison of several micromotion minimization approaches is given in terms of various figures of merit, as the DC field uncertainty, the micromotion amplitude and the average kinetic energy of micromotion which relates to the second order Doppler shift. For a specific experiment, one of those numbers might be more suitable or relevant than the others. The micromotion amplitude and the average kinetic energy depend to different extents on the ion mass, and on the drive and secular frequency. As these numbers vary substantially throughout the ion trapping community, we restrict the following discussion to the DC field uncertainty. In general, by increasing the RF drive power, one can, within the constraints given by the experiment, reduce the effect of micromotion.

In table (2) the outcomes of several minimization methods are shown. The final level of precision of micromotion minimization can be scaled in each method by changing the
Table 2. Comparison of different micromotion minimization methods in terms of the residual electric stray field $\Delta \epsilon$. The first five rows are an extension of the values given in [26], the last row states the results obtained in this work. If different residual electric stray fields for different trapping directions are reported, only the lowest value is given.

| Ref. | Method                                      | $\Delta \epsilon$ (V/m) |
|------|---------------------------------------------|--------------------------|
| [28] | photon-correlation spectroscopy              | 0.9                      |
| [31, 33, 32] | micromotional sideband spectroscopy       | 7, 1, 0.4                |
| [35] | ion-cavity emission spectroscopy            | 1.8                      |
| [38, 37] | parametric excitation of secular motion   | 6, 0.4                   |
| [26] | neutral atom loss                           | 0.02                     |
| this work | trajectory analysis                        | 0.09                     |

averaging time for data taking, as long as the stray fields to be compensated are stable. Ultimately, as our minimization approach relies on imaging, the position determination, and in turn the determination of the RF null location is shot noise limited. Thus another question to ask is on which timescale which level of accuracy is obtained. Here, almost no information was found in literature.

The observed fluorescence rate is limited by the $S_{1/2} - P_{1/2}$ transition of $^{172}$Yb$^+$ and its linewidth $\Gamma = 2\pi \times 19.2$ MHz, the solid angle covered by our light gathering optics (4% of $4\pi$) and its transmission, and the quantum efficiency of our camera (55%). With these constraints and the time required for the translation of our optics, recording a data set as shown in Fig. (8) takes about 620 s and directly yields all compensation voltages. We achieve a sensitivity of the residual electric field in the radial directions of $(s_{\text{stray},x'}, s_{\text{stray},y'}) = (2.3 \text{ V/m}\sqrt{\text{Hz}}, 27.1 \text{ V/m}\sqrt{\text{Hz}})$. Subtracting the time needed for objective translations (about 400 s) the achieved sensitivity increases to $(s_{\text{stray},x'}, s_{\text{stray},y'}) = (1.4 \text{ V/m}\sqrt{\text{Hz}}, 16.3 \text{ V/m}\sqrt{\text{Hz}})$.

7. Conclusion and outlook

In our present setup, fluorescence is detected normal to the plane of the trap chip, and thus vertical to the soft trapping axis along which confinement is almost purely realized by DC fields, labeled as the axial direction of our linear segmented trap (compare Fig. [4]). In an ideal linear trap, the RF effective potential and thus micromotion depend exclusively on the radial coordinates, vertical to the axial direction. In this case position changes upon RF variation will occur only in radial directions and thus partially in the direction of our line of sight. In real traps, there might be a residual axial RF electric field, which is minimized by careful design and typically much smaller than the radial RF field strength. In segmented traps, the intention is usually to exploit the entire axial span of the linear trap by shuttling etc. and thus axial micromotion might be present, but potentially cannot be minimized. Micromotion minimization always requires a determination of the radial ion displacement or motion, in our case also along our line of sight. We realize this by tomographic imaging as detailed above. In our
present setup this is one of the main limitations, in both accuracy and speed. As our detection of excess micromotion is based on a shift of the ion’s average position, the method as presented is insensitive to micromotion induced by a phase difference between the RF trap electrodes or an RF pick up of the DC electrodes that is phase-shifted by nearby filter circuits. These effects can be avoided by design to great extent, especially in surface traps, and are often of negligible size.

Detection along the axial direction of the trap would allow to obtain the relevant displacements without the time consuming physical translation of massive light gathering optics and reduces the number of required images considerably: right now, we take 25 focal steps and need to include additional times to wait for the completion of a translation action. Additionally, omitting a translation stage further improves the mechanical stability and thus the accuracy of the position determination. Taking into account translation and settling times as well as the reduction in the number of images, the speed of acquisition would increase by approximately 70, meaning that the entire minimization as shown in Fig. (8) would be finished in two dimensions in roughly 9s and the sensitivity of this method would increase to about \((s_{\text{stray},x'}, s_{\text{stray},y'}) = (0.3 \, \text{V/m} \sqrt{\text{Hz}}, 3.3 \, \text{V/m} \sqrt{\text{Hz}})\) for a 2d minimization. Speed is not only relevant to keep the time for calibration etc. low and leave most to the experiment of interest. It also allows to compensate more often and characterize and counteract time dependent drifts of the power supplies and patch potentials. In principle, when the effect of DC field changes of each electrode are well characterized, a single trajectory will suffice, and this might further improve the performance of the method in terms of speed.

Utilizing the same technique, precise position determination of a harmonically trapped ion may also be used for the simple detection of small forces. We demonstrated the detection of forces on the yoctonewton \((10^{-24} \, \text{N})\) scale on the example of the light pressure force exerted on the ion by the cooling laser. Depending on the experiment a precise knowledge of the light pressure force might be desirable to counter its effects and maintain minimal micromotion also in experimental conditions without incoherent scattering.

In conclusion, we presented a method, which allows to compensate micromotion due to DC stray fields in 3d using tomographic imaging as well as the detection of rather small forces. The method is general and does not require spectroscopy on a narrow transition or a specific orientation of laser beams, which is of particular interest for surface-electrode traps. If imaging is carried out along the symmetry axis of a linear trap and the optimization is carried out in the transverse direction only, no additional components and no tomographic imaging are required and the optimization can be carried out within seconds.

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Ion trajectory analysis

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