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One-loop self-energies in the electroweak model with a nonlinearly realized gauge group

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We evaluate at one loop the self-energies for the W, Z mesons in the electroweak model where the gauge group is nonlinearly realized. In this model the Higgs boson parameters are absent, while a second mass parameter appears together with a scale for the radiative corrections. We estimate these parameters in a simplified fit on leptons and gauge bosons data. We check physical unitarity and the absence of infrared divergences. Landau gauge is used. As a reference for future higher order computations, the regularized $D$-dimensional amplitudes are provided. Eventually the limit $D \rightarrow 4$ is taken on physical amplitudes.

I. INTRODUCTION

A consistent strategy for the all-orders subtraction of the divergences in nonlinearly realized gauge theories has recently been developed [1–5] by extending some tools originally devised for the nonlinear sigma model in the flat connection formalism [6]. The approach relies on the local functional equation for the one particle irreducible (1-PI) vertex functional [6] (encoding the invariance of the group Haar measure under local left transformations), the weak power-counting theorem [7], and the pure pole subtraction of properly normalized 1-PI amplitudes [8]. This scheme of subtraction fulfills all the relevant symmetries of the vertex functional. Physical unitarity is established as a consequence of the validity of the Slavnov-Taylor identity [9].

This strategy was first applied to the nonlinearly realized $SU(2)$ massive Yang-Mills theory [3]. The full set of one-loop counterterms and the self-energy have been obtained in [4].

The extension to the electroweak model based on the nonlinearly realized $SU_L(2) \otimes U_Y(1)$ gauge group introduces a number of additional nontrivial features [1,2]. The direction of the spontaneous symmetry breaking fixes the linear combination of the hypercharge and of the third generator of the weak isospin, giving rise to the electric charge. Despite the fact that both the hypercharge and the $SU_L(2)$ symmetry are nonlinearly realized, the Ward identity for the electric charge has a linear form on the vertex functional.

The anomalous couplings are forbidden by the $U_Y(1)$ invariance together with the weak power-counting. However, two independent mass parameters for the vector mesons are allowed. Thus the ratio of the vector meson masses is not given by the Weinberg angle anymore.

As a first step toward a detailed analysis of the radiative corrections of the nonlinearly realized electroweak theory, we provide in this paper the vector meson self-energies in the one-loop approximation.

The dependence of the self-energies on the second mass parameter is important in order to establish a comparison with the linear realization of the electroweak group based on the Higgs mechanism.

We provide a rough estimate of both the extra mass parameter and the scale of the radiative corrections. We fix some of the parameters on measures taken at (almost) zero momentum transfer, while the one-loop corrections are confronted with measures at the resonant value of the vector bosons energies. The resulting values are challenging: the departure from the Weinberg relation between the vector meson mass is very small, and the scale of the radiative corrections is of the order of a hundred GeV.

The aim of the present work is to provide the amplitudes in $D$ dimensions for future high order computations and to provide a preliminary assessment of the predictivity of the electroweak model based on the nonlinearly realized gauge group, including the one-loop self-energy corrections. Electroweak physics is described with very reasonable parameters (the second mass parameter and the scale of the radiative corrections).

The cancellations among unphysical states required by physical unitarity can be easily traced out. The physical amplitudes are shown to be free of infrared divergences. It is also remarkable that they do not depend on the spontaneous symmetry breaking parameter $v$. This fact has been discussed in Refs. [1,2,4].

The computation is done in the symmetric formalism on the $SU(2)_L$ flavor basis. This choice greatly simplifies both the Feynman rules and the actual computation; in fact, symmetry arguments turn out to be very useful in the...
calculation of the invariant functions. The symmetric formalism puts emphasis on the fact that the entering parameters are not renormalized (e.g. as in the on-shell renormalization procedure) and are fixed at the end, by means of the comparison with the experimental data. Moreover, the symmetric formalism makes the underlying symmetric structure encoded by the local functional equation more transparent.

II. FEYNMAN RULES

The classical action is written in order to establish the Feynman rules. We omit all the external sources which are needed in order to subtract the divergences at higher loops. We refer to the previous publications [1,2] where the procedure is described at length. The field content of the electroweak model based on the nonlinearly realized $SU(2)_L \otimes U(1)$ gauge group includes the $SU(2)_L$ connection $A_\mu = A_{\alpha \mu} \frac{i}{2} (\tau_{a}, a = 1, 2, 3$ are the Pauli matrices), the $U(1)$ connection $B_\mu$, the fermionic left doublets collectively denoted by $L$, and the right singlets, i.e.

$$
L \equiv \left\{ \left( \begin{array}{c} l^u_{ij} \\ V_{jk} q^d_{lk} \end{array} \right) \right\}, \quad j, k = 1, 2, 3.
$$

$$
R \equiv \left\{ \left( \begin{array}{c} l^u_{ij} \\ R_{kl} \end{array} \right) \right\}, \quad j, k = 1, 2, 3.
$$

In the above equation the quark fields $(q^U_{ij}, j = 1, 2, 3) = (u, c, t)$ and $(q^d_{ij}, j = 1, 2, 3) = (d, s, b)$ are taken to be the mass eigenstates in the tree-level Lagrangian; $V_{jk}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Similarly, we use for the leptons the notation $(l^c_{ij}, j = 1, 2, 3) = (\nu_e, \nu_\mu, \nu_\tau)$ and $(l^\ell_{ij}, j = 1, 2, 3) = (e, \mu, \tau)$. The single left doublets are denoted by $L^L_{ij}, j = 1, 2, 3$ for the leptons, and $L^R_{ij}, j = 1, 2, 3$ for the quarks. Color indices are not displayed.

One also introduces the $SU(2)$ matrix $\Omega$,

$$
\Omega = \frac{1}{v} (\phi_0 + i \phi_a \tau_a), \quad \Omega^\dagger \Omega = 1 \Rightarrow \phi_0^2 + \phi_a^2 = v^2.
$$

The mass scale $v$ gives $\phi$, the canonical dimension at $D = 4$. We fix the direction of spontaneous symmetry breaking by imposing the tree-level constraint

$$
\phi_0 = \sqrt{v^2 - \phi_a^2}.
$$

The $SU(2)$ flat connection is defined by

$$
F_\mu = i \Omega \partial_\mu \Omega^\dagger.
$$

A. Classical action

Discarding the neutrino mass terms, the classical action for the nonlinearly realized $SU(2) \otimes U(1)$ gauge group with two independent mass parameters for the vector mesons can be written as follows, where the dependence on $\Omega$ is explicitly shown:

$$
S = \Lambda^{D-4} \int d^D x \left\{ - \frac{1}{4} G_{\mu \nu} G^{\mu \nu} - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \right\} + M^2 \text{Tr} \left( \left( g A_\mu - \frac{g \theta}{2} \Omega^\dagger \tau_3 B_\mu \Omega + F_\mu \right)^2 \right) + M^2 \text{Tr} \left( \left( g A_\mu - \frac{g \theta}{2} \Omega^\dagger \tau_3 B_\mu \Omega + F_\mu \right) \right)^2 + \sum_L \left[ \text{Tr} \left( \left( \frac{g \theta}{2} B_\mu \Omega^\dagger \tau_3 L^L_j + m_{q_j} V^{J^L}_{k_j} \tau_3 \Omega^\dagger L^L_j \right) \right) \right] + \sum_{j} \left[ m_j R_j \frac{1 - \tau_3}{2} \Omega^\dagger L^L_j - m_{q_j} V^{J^L}_{k_j} \tau_3 \Omega^\dagger L^L_j \right] + \text{H.c.}
$$

In $D$ dimensions the doublets $L$ and $R$ obey

$$
\gamma_D L = -L, \quad \gamma_D R = R,
$$

with $\gamma_D$ a gamma matrix that anticommutes with every other $\gamma^\mu$.

The non-Abelian field strength $G_{\mu \nu}$ is defined by

$$
G_{\mu \nu} = G_{a \mu \nu} \frac{\tau_a}{2} = \left( \partial_\mu A_{\nu} - \partial_\nu A_{\mu} + g e_{abc} A_{b \mu} A_{c \nu} \right) \frac{\tau_a}{2}.
$$

while the Abelian field strength $F_{\mu \nu}$ is

$$
F_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.
$$

In the above equation the phenomenologically successful structure of the couplings has been imposed by hand.

However, the same structure is required by the weak power-counting requirement, as discussed in Ref. [2].

B. Gauge fixing

In order to set up the framework for the perturbative quantization of the model, the classical action in Eq. (5) needs to be gauge fixed. The ghosts associated with the $SU(2)_L$ symmetry are denoted by $c_a$. Their antighosts are denoted by $\bar{c}_a$, and the Nakanishi-Lautrup fields by $b_a$. It is also useful to adopt the matrix notation

$$
c = c_a \frac{\tau_a}{2}, \quad b = b_a \frac{\tau_a}{2}, \quad \bar{c} = \bar{c}_a \frac{\tau_a}{2}.
$$

The Abelian ghost is $c_0$, the Abelian antighost is $\bar{c}_0$, and the Abelian Nakanishi-Lautrup field is $b_0$.
In the Landau gauge the propagator matrix for the bilinear external sources is provided in Ref. [2]. Then the gauge-fixing part of the classical action is

\[ S_{GF} = \Lambda^{(D-4)} \int d^D x (b_0 \partial_\mu B^\mu - \bar{c}_0 \Box_0 c_0 + 2 Tr(b \partial_\mu A^\mu - \bar{c} \partial_\mu D[A]_\mu c). \]  

(10)

C. Boson symmetric formalism

The bilinear part of the boson sector is

\[ \frac{M^2}{2} \left[ \left(gA_{a\mu} - g' B_\mu \delta_{3a} - \frac{2}{v} \partial_\mu \phi_a \right)^2 + \kappa \left(GZ_\mu - \frac{2}{v} \partial_\mu \phi_3 \right)^2 \right] + b_0 \partial_\mu B^\mu + b_a \partial_\mu A^\mu_a 
= M^2 g^2 \left[ W^+_\mu - \frac{2}{v g} \partial_\mu \phi_3 \right]^2 + b^+ \partial_\mu W^- 
+ b^- \partial_\mu W^+ + \frac{M^2}{2} (G^2)(1 + \kappa) \left[ Z - \frac{2}{vG} \partial_\mu \phi_Z \right]^2 + b_\mu \partial_\mu Z^- + b_\mu \partial_\mu A^- \mu. \]  

(11)

We use the notations

\[ G = \sqrt{g^2 + g'^2}, \quad c = \frac{g}{G}, \quad s = \frac{g'}{G} \]  

(12)

\[ M_W = gM, \quad M_Z = \sqrt{(1 + \kappa)GM} \]

and

\[ W^+ = \frac{1}{\sqrt{2}} (A_1 - iA_2), \quad Z = \frac{1}{G} (gA_3 - g'B), \]

\[ A = \frac{1}{G} (g' A_3 + g B). \]  

(13)

In the Landau gauge the propagator matrix for the bilinear form

\[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} (V_\mu - \frac{2}{v} \partial_\mu \phi_3)^2 + b \partial_\mu V^\mu \]  

is given by

\[ \begin{pmatrix} V_\mu & -i & b & \frac{1}{2} \phi \\ b & \frac{1}{2} & 0 & -i \frac{v}{2p} \phi \\ \phi & 0 & -i \frac{v}{2p} & \frac{v^2}{4m^2} \frac{1}{p^2} \end{pmatrix}. \]  

(15)

In the symmetric notation we have

\[ \langle (A_1 A_1)_+ \rangle = \langle (A_2 A_2)_+ \rangle = \langle (W^+ W^-)_+ \rangle, \]

\[ \langle (A_1 A_2)_+ \rangle = \langle (A_1 A_3)_+ \rangle = \langle (A_2 A_3)_+ \rangle = 0, \]

\[ \langle (A_3 A_3)_+ \rangle = \frac{1}{G^2} \langle ((gZ + g'A)(gZ + g'A))_+ \rangle = \frac{1}{G^2} (g^2 \Delta_{M_Z} + g^2 D), \]

\[ \langle (A_3 B)_+ \rangle = \frac{1}{G^2} \langle ((gZ + g'A)(-g'Z + gA))_+ \rangle = \frac{g g'}{G^2} (-\Delta_{M_Z} + D), \]

\[ \langle (BB)_+ \rangle = \frac{1}{G^2} \langle ((-g'Z + gA)(-g'Z + gA))_+ \rangle = \frac{1}{G^2} (g^2 \Delta_{M_Z} + g^2 D), \]  

(16)

where we used a shorthand notation, e.g.

\[ \Delta_{M_Z} \rightarrow -i \frac{p^2 - M^2_Z}{p^2} \left( g_{\mu \nu} - \frac{p_{\mu} p_{\nu}}{p^2} \right), \]

\[ D \rightarrow -i \frac{p^2}{p^2} \left( g_{\mu \nu} - \frac{p_{\mu} p_{\nu}}{p^2} \right). \]  

(17)

D. Boson trilinear couplings

For the one-loop calculation of the vector boson self-energies, one needs the usual Feynman rules and the trilinear couplings generated by the two mass invariants in Eq. (5). The first mass invariant generates the trilinear couplings

\[ \frac{M^2}{2} \text{Tr} \left\{ 2 \left( gA_{\mu} - g' \frac{1}{2} \Omega B_{\mu} T_0 \Omega^\dagger - F_{\mu} \right)^2 \right\}_\text{trilinear} \]

\[ = \frac{M^2}{2} \left\{ -4 \frac{G}{v^2} Z_\mu \epsilon_{3bc} \partial_\mu \phi_b \phi_c \right. \]

\[ - 4 \frac{g}{v^2} \sum_{a=1,2} A_{a\mu} \epsilon_{abc} \partial_\mu \phi_b \phi_c \]

\[ + 4 \frac{g g'}{G v} (-g'Z + gA)_{\mu} \epsilon_{3ab} \phi_b \]

\[ - 8 \frac{g'}{G v} (-g'Z + gA)_{\mu} \epsilon_{3ab} \partial_\mu \phi_b \phi_c \}. \]  

(18)

The second mass invariant yields

\[ \frac{\kappa M^2}{2} \left( \text{Tr} \left[ g \Omega^1 A_{\mu} \Omega T_0 - \frac{g'}{2} B_{\mu} + i \Omega^1 \partial_\mu \Omega T_0 \right] \right)^2 \]

\[ = \frac{\kappa M^2}{2} \left( 4 \frac{1}{v^2} GZ_\mu \epsilon_{3bc} (\partial_\mu \phi_b) \phi_c \right. \]

\[ - 4g \frac{1}{v} GZ^\mu A_{a\mu} \epsilon_{3ab} \phi_b + 8 \frac{1}{v} g A_{a\mu} \partial_\mu \phi_3 \epsilon_{3ab} \phi_b \}. \]  

(19)

We put everything together as follows:
E. Fermion contribution

The evaluation of the fermion loops requires a rule on how to handle the $\gamma_5$ in dimensional regularization. Our mechanism for the removal of divergences is based on a regularization that respects the local gauge invariance; therefore $\gamma_5$ must anticommute with any $\gamma_\mu$. At one loop this is possible, as it is well known, since there are no chiral anomalies. For higher loop calculations any trace involving $\gamma_5$ must be considered as an independent amplitude up to the end of the subtraction procedure. Eventually we evaluate the limit at $D = 4$ for physical amplitudes.

The fermion contribution can be easily cast into a global formula,

$$\Gamma_{\mu \nu}[\text{ABST}] = -\langle 0| (\bar{\psi}(x) \gamma_\mu (A + B \gamma_5) \psi(x)) |0\rangle \times \gamma_\rho (S + T \gamma_5) \bar{\psi}(0) \psi(0) = \langle 0| \text{Tr} \left\{ \frac{dp}{(2\pi)^D} \int \frac{dq}{(2\pi)^D} \gamma_\mu (A + B \gamma_5) \right. \\
\times \left( \frac{\not{p} + \not{q} + m}{(q + p)^2 - m^2} e^{ipx} \gamma_\nu (S + T \gamma_5) \right) \left. \times \left( \frac{\not{q} + M}{q^2 - M^2} \right) \right\} \rangle,$$

(21)

where $A, B, S, T$ are matrix elements corresponding to the flavor and the color of the fermions with mass $m$ and $M$, and they can be obtained from the classical action (5). In particular, the neutral sector is

$$G \gamma^\mu \bar{\psi} \left[ \frac{\tau_3}{4} - s^Q \right] \gamma_\mu - \frac{\tau_3}{4} \gamma_\mu \gamma_5 \bar{\psi} + eA^\mu \bar{\psi} Q \gamma_\mu \psi,$$

$$e = \frac{g g'}{G}.$$

(22)

One then gets the transverse part of the contribution of the fermions (for the notations see the Appendix),

$$\Gamma_T[\text{ABST}] = \langle 0| \text{Tr} \left\{ (AS + BT) \frac{(2 - D)}{2} (\Delta_m + \Delta_M) + H(m, M) \times \left[ mM(AS - BT)D + \frac{(2 - D)}{2} (AS + BT)(-p^2 + M^2 + m^2) \right] \right. \left. - \frac{1}{p^2} (AS + BT)(m^2 - M^2) i(\Delta_M - \Delta_m) \right. \left. - \frac{1}{p^2} H(m, M) \left[ mM(AS - BT)p^2 + \frac{1}{2} (AS + BT) \right. \times \left. ((m^2 - M^2)^2 - p^2(m^2 + M^2)) \right] \right\} \rangle.$$

(23)

and the longitudinal part of the contribution of the fermions,

$$\Gamma_L[\text{ABST}] = \frac{4}{p^2} \text{Tr} \left\{ \frac{1}{2} (AS + BT)(i(-m^2 + M^2)\Delta_m + i(m^2 - M^2)\Delta_M) + H(m, M) \times \left[ mM(AS - BT)p^2 + \frac{1}{2} (AS + BT) \right. \times \left. (M^4 + m^4 - 2m^2M^2 - p^2(m^2 + M^2)) \right] \right\},$$

(24)

III. SELF-ENERGY AMPLITUDES IN D DIMENSIONS

The presentation of the results (Landau gauge in $D$ dimensions) is as follows. First we report the result of the calculation for the transverse and for the longitudinal parts,

$$\Sigma_{\mu \nu} = \left( g_{\mu \nu} - \frac{p_\mu p_\nu}{p^2} \right) \Sigma_T + \frac{p_\mu p_\nu}{p^2} \Sigma_L.$$

(25)

For each of them we report the contributions of the single graphs. Subsequently, we evaluate the diagonal amplitudes $\Sigma_T$ on shell, where we discuss the validity of physical unitarity and the absence of any infrared singularity. Finally, the on-shell amplitudes are taken at $D = 4$.

We omit, for the sake of conciseness, the self-energies at $D = 4$ for generic momentum (the procedure is straightforward). We do not use on-shell renormalization: $M_w$ and $M_z$ are dummy parameters, as are $c$ and $s$. The massless photon is a source of some infrared problems in the Landau gauge. In the Appendix the analytical tools needed to handle these difficulties are provided [see Eqs. (A2)–(A5)].

A. The $D = 4$ amplitudes

The $D = 4$ amplitudes are recovered as the finite parts in the Laurent expansion of the generic dimensional regularized amplitudes, normalized by the factor

$$\Lambda^{-(D-4)}.$$

(26)

This point has been discussed at length in Refs. [6,8]. In this procedure we essentially encounter the following cases.
\[ \Lambda^{-(D-4)} \Delta_{m|D-4} = \frac{m^2}{(4\pi)^2} \left( \frac{2}{D-4} - 1 + \gamma + \ln \left( \frac{m^2}{4\pi\Lambda^2} \right) \right) \]  
(27)

\[ \Lambda^{-(D-4)} H(m,M)(p^2)|_{D-4} = \frac{i}{(4\pi)^2} \left( \frac{2}{D-4} + \gamma - \ln(4\pi) \right) + \int_0^1 dx \ln \left( \frac{m^2}{\Lambda^2} (1-x) \right) + \frac{M^2}{\Lambda^2} x - \frac{p^2}{\Lambda^2} x (1-x) \right) \right\}. \]
(28)

In the Appendix we give the value of the last integral in Eq. (28).

\[ \Lambda^{-(D-4)} G(M)(p^2)|_{D-4} = \frac{i}{(4\pi)^2} \left[ \frac{1}{p^2 - M^2} \left( \frac{2}{D-4} + \gamma - \ln(4\pi) \right) \right] + \frac{1}{p^2 - M^2} \left[ \frac{M^2}{p^2} \ln \frac{M^2 - p^2}{M^2} + \ln \frac{M^2 - p^2}{\Lambda^2} \right] \frac{4}{D-4} + O((D-4)^2). \]

At \( D \approx 4 \) we use

\[ \frac{\Gamma(\frac{D}{2} - 2) \Gamma(\frac{D}{2} - 2)}{\Gamma(D-4)} \approx \frac{4}{D-4} + O((D-4)^2). \]

Then

\[ \Lambda^{4-D} \frac{\partial}{\partial M^2} G(M)|_{M=0} \sim -\frac{i}{(4\pi)^2} \left[ (\frac{D}{2} - 2) \right] \times \left[ 1 + \left( \frac{D}{2} - 2 \right) \log \left( -\frac{p^2}{(4\pi)^2} \right) \right] \times \left[ \frac{4}{D-4} + O((D-4)^2) \right] \]

\[ = -\frac{2i}{(4\pi)^2} \left[ -p^2 \right] \left[ \frac{2}{D-4} - 1 + \gamma + \log \left( -\frac{p^2}{(4\pi)^2} \right) \right]. \]

(31)

All other limit expressions can be reduced to Eqs. (27)–(29).

**B. The counterterms**

The counterterms are given by the pole parts of the same Laurent expansion taken with a minus sign and finally multiplied by the common factor \( \Lambda^{(D-4)} \). In Eq. (29) the pole in \( D - 4 \) dangerously multiplies a nonlocal term. However, we shall find that \( G(M) \) always enters with a factor \( p^2 - M^2 \).

**IV. WW SELF-ENERGY**

We first list the contributions to the transverse part.

**A. Transverse WW self-energy**

The Goldstone bosons contribution to the transverse part of \( \Sigma_{WW} \) is

\[ i \Sigma_{T{WW}_{\text{Goldstone}}} = -i \Delta_{M_Z} \frac{M^2_W}{4(D-1)} \left[ -p^2 \right] \left[ \frac{g^2 + \kappa G^2}{G} \right]^2 \left( \frac{1}{M^2_Z} + \frac{1}{p^2} \right) + \frac{G(0)}{D-1} M^2_W \left[ \frac{g^2 G^2}{4} \right]^2 p^2 + \frac{H(0,0)}{4(D-1)G^2 M^2_Z} \times \left[ -p^2 M^2_W (g^2 + \kappa G^2)^2 + g^2 M^2_Z [2(2-3) + 2D] g^2 M^2_W + G^2 p^2 (1 + \kappa) \right] + \frac{H(0,M_Z)}{4G^2} \times \left[ g^2 + \kappa G^2 \right] (4 + \left( \frac{M^2_Z - p^2}{(D-1)M^2_Z p^2} \right). \]

(32)

The Faddeev-Popov contribution is

\[ i \Sigma_{T{WW}_{\text{FP}}} = \frac{g^2}{2(D-1)} p^2 H(0,0). \]

(33)

The vector boson tadpole contribution is

\[ i \Sigma_{T{WW}_{\text{tadpole}}} = i \frac{(D-1)^2}{D} g^2 (\Delta_{M_W} + e^2 \Delta_{M_Z}). \]

(34)

The \( \gamma W \) loop is

\[ i \Sigma_{T{WW}_{\gamma W}} = -G(0) \frac{g^2 g^2}{4(D-1)G^2 M^2_W} - H(0,0) \frac{(D-2) g^2 G^2 p^4}{(D-1)G^2 M^2_W} + G(M_W) \frac{g^2 g^2}{4(D-1)G^2 M^2_W} (M^2_W - p^2)^2 \times \left[ M^4_W + 2(2D-3) p^2 M^2_W + p^4 \right] + H(0,M_W) \frac{g^2 G^2}{(D-1)G^2 M^2_W} (M^2_W + p^2)(M^2_W + (D-2)M^2_W + 2(3-2D) p^2 M^2_W + (D-2) p^4 - i \Delta_{M_W} \frac{g^2 G^2}{4(D-1)G^2 M^2_W} (D(4D-7)M^2_W + 2[D(2D-1) - 2] p^2 M^2_W + D(4D-7) p^4). \]

(35)

The \( Z W \) loop is
The quark contribution to the transverse $\Sigma_{\ell \ell}$ is obtained from Eq. (23) by using

$$A_S - B_T = 0, \quad A_S + B_T = \frac{g^2}{4}. \quad (37)$$

Therefore

$$i\Sigma_{\ell \ell}^{\text{quarks}} = \frac{g^2}{2} \left( \frac{1}{D-1} \sum_{l=e,\mu,\tau} \left( 2 - D \right) i \Delta_{\ell l} + \frac{M_{\ell}^2}{p^2} i \Delta_{\ell l} \right)$$

$$+ H(0, M_{\ell})(-p^2 + M_{\ell}^2) \left[ (2 - D) - \frac{M_{\ell}^2}{p^2} \right]. \quad (38)$$

The quark contribution to the transverse part of $\Sigma_{\ell \ell}$ is given by Eq. (23), where

$$A_S - B_T = 0, \quad (A_S + B_T)_{ab} = \frac{g^2}{4} 3V_{ab}V_{ab}^*, \quad (39)$$

with $V_{ab}$ the CKM matrix. Thus

$$i\Sigma_{\ell \ell}^{\text{quarks}} = \frac{3g^2}{2(D-1)} \sum_{ab} V_{ab}V_{ab}^* \left[ i \Delta_{m_a} \left( 2 - D \right) + \frac{1}{p^2} m_a^2 \right]$$

$$\times \left( m_a^2 - m_b^2 \right) + i \Delta_{m_b} \left( 2 - D \right) - \frac{1}{p^2} \left( m_a^2 - m_b^2 \right)$$

$$+ H(m_a, M_b) \left[ (2 - D)(-p^2 + M_b^2 + m_a^2) \right]$$

$$- \frac{1}{p^2} \left( (m_a^2 - M_b^2)^2 - p^2(m_a^2 + M_b^2) \right). \quad (40)$$

**B. Longitudinal WW self-energy**

In a similar way we list the contributions for the longitudinal parts of $\Sigma_{\ell \ell}$. The Goldstone boson contribution to the longitudinal part of $\Sigma_{\ell \ell}$ is

$$i\Sigma_{\ell \ell}^{\text{Goldstone}} = i \Delta_{m_{L}} M_{W}^{2} \left[ \frac{\kappa g^2 + g'^2}{4g^2} \left( \frac{1}{p^2} + \frac{1}{M_{Z}^2} \right) \right]$$

$$- \left[ G(0) M_{W}^{2} - \frac{g^2 g'^2}{4g^2} \right] H(0, M_{Z})$$

$$\times \left( \frac{\kappa g^2 + g'^2}{4g^2 M_{W}^{2} p^2} \left( M_{Z}^2 - p^2 \right)^2 + \frac{1}{4} H(0, 0) \right)$$

$$\times \left[ M_{W}^{2} \left( 2g^2 g'^2 + (\kappa g^2 + g'^2)^2 \right) \right]$$

$$\times \left[ \frac{M_{W}^{2}}{G^2 M_{Z}^{2}} - \frac{g^2 g'^2}{\kappa + 1} \right]. \quad (41)$$

The Faddeev-Popov contribution to the longitudinal part of $\Sigma_{\ell \ell}$ is

$$i\Sigma_{\ell \ell}^{\text{FP}} = - \frac{g^2}{2} p^2 H(0, 0). \quad (42)$$

The vector boson tadpole contribution to the longitudinal part is

$$i\Sigma_{\ell \ell}^{\text{tadpole}} = i \frac{(D-1)^2}{D} \left( g^2 (\Delta_{m_{L}} + c^2 \Delta_{M_{L}}) \right). \quad (43)$$

The $\gamma W$ loop contribution to the longitudinal part is

$$i\Sigma_{\ell \ell}^{\text{\gamma W}} = - G(W) \frac{g^2 g'^2}{4g^2 p^2} (M_{W}^2 - p^2)^2 - i \Delta_{m_{L}} \frac{g^2 g'^2}{2G^2 p^2}$$

$$\times \left[ \left( \frac{7}{2} - 2D \right) M_{W}^2 + \left( 2D - 3 + \frac{2}{D} \right) p^2 \right]$$

$$+ H(0, M_{W}) \frac{g^2 g'^2}{2G^2 p^2} \left[ -2(D - 2) M_{W}^4 \right]$$

$$- p^2 M_{W}^2 + p^4 \right]. \quad (44)$$

The $ZW$ loop contribution to the longitudinal part is
The charged lepton contribution to the longitudinal $\Sigma_{LW}$ is obtained from Eqs. (24) and (37),

$$i \Sigma_{\text{leptons}}^{LW} = \frac{g^2}{2p^2} \sum_{i=e, \mu, \tau} \{-i M_i^2 \Delta M_i + H(0, M_i) \times (-p^2 + M_i^2) M_i^2\}.$$  

(46)

The quark contribution to the longitudinal $\Sigma_{LW}$ is obtained from Eqs. (24) and (39),

$$i \Sigma_{\text{quarks}}^{LW} = -\frac{g^2}{2p^2} \sum_{ab} V_{ab} V_{ab}^* \{i \Delta M_{ab}(m_a^2 - M_b^2) - i \Delta M_{ab}(m_a^2 - M_b^2) - H(m_a, M_b) \times [(m_a^2 - M_b^2)^2 - p^2(m_a^2 + M_b^2)]\}.$$  

(47)

V. ZZ SELF-ENERGY

We first list the contributions to the transverse part.

A. The transverse ZZ self-energy

The Goldstone contribution to the transverse part of the ZZ self-energy is

$$i \Sigma_{\text{Goldstone}}^{TZZ} = -i \Delta M_w \frac{(M_w^2 + p^2)(\kappa G^2 + g^2)^2}{2(D-1)G^2 p^2} + H(0, M_w) \frac{(\kappa G^2 + g^2)^2}{2(D-1)G^2 p^2} [M_w^2 + 2(2D - 3) \times p^2 M_w + p^4] + H(0, p) \frac{p^2}{4(D-1)G^2} \times [g^4 - 2(\kappa G^2 + g^2)^2 - (\kappa G^2 + g^2)^2].$$  

(48)

The Faddeev-Popov contribution to the transverse part of the ZZ self-energy is

$$i \Sigma_{\text{FP}}^{TZZ} = -\frac{g^4}{G^2} \frac{1}{2(D-1)} p^2 H(0, 0).$$  

(49)

The vector boson tadpole contribution to the transverse part is

$$i \Sigma_{\text{tadpole}}^{TZZ} = \frac{g^4}{G^2} \frac{(D-1)^2}{D} \Delta M_w.$$  

(50)

The WW loop contribution to the transverse part of the ZZ self-energy is

$$i \Sigma_{\text{WW}}^{TZZ} = H(0, M_w) \frac{g^4 p^6}{4(D-1)G^2 M_w^4} - H(M_w, M_w) \times \frac{g^4(4M_w^2 - p^2)}{4(D-1)G^2 M_w^4} [4(D-1)M_w^4 + 4(2D - 3)p^2 \times M_w^4 + p^4] - H(0, M_w) \frac{g^4(M_w^2 - p^2)^2}{2(D-1)G^2 M_w^4 p^2} \times [M_w^2 + 2(2D - 3)p^2 M_w + p^4] - i \Delta M_w 2(D-1)DG^2 M_w^4 p^2 \times [-DM_w^4 + (5D - 4)p^2 M_w^4 + D(4D - 7)p^4].$$  

(51)

The neutrino contributions to the transverse $\Sigma_{TZZ}$ are obtained from Eq. (23) by using

$$\Delta - BT = 0, \quad \Delta + BT = \frac{G^2}{8}. $$  

(52)

Therefore the neutrinos yield

$$i \Sigma_{\text{TZZ}}^{\nu} = -3p^2 \frac{G^2 (2-D)}{4(D-1)} H(0, 0).$$  

(53)

For the charged leptons, as well for the up and down quarks, the contribution to the self-energy $\Sigma_{TZZ}$ has the same form (23),

$$i \Sigma_{TZZ}^{\text{charged fermions}} = \sum_{j} \frac{1}{D-1} \left\{ i(\Delta + BT)(2-D) \Delta_m, \right.$$  

$$+ H(m_j, m_j) \left[ 2m_j^2((2-D)BT + \Delta) \right.$$  

$$- p^2 \frac{(2-D)}{2} (\Delta + BT) \right\}.$$  

(54)

where the sum is over the flavors. For the leptons

$$\Delta - BT = G^2 s^2[-\frac{1}{2} + s^2], \quad \Delta + BT = G^2 t^2[\frac{1}{8} - \frac{1}{2} s^2 + s^2].$$  

(55)

For up quarks

$$\Delta - BT = 3G^2 s^2[-\frac{1}{2}Q_u + s^2Q_u^2], \quad \Delta + BT = 3G^2 t^2[\frac{1}{8} - \frac{1}{2} s^2 Q_u + s^2 Q_u^2], \quad Q_u = \frac{2}{3}.$$  

(56)
For the down quarks
\[
AS - BT = 3G^2s^2[\frac{1}{2}Q_d + s^2Q_d^2],
\]
\[
AS + BT = 3G^2[\frac{1}{8} + \frac{1}{2}s^2Q_d + s^4Q_d^2],
\]
\[
Q_d = -\frac{1}{3}.
\]

(57)

B. The longitudinal ZZ self-energy

Now we discuss the longitudinal part of ZZ self-energy. The Goldstone contribution to the longitudinal part of the ZZ self-energy is
\[
i\Sigma_{LZZ}^{\text{Goldstone}} = i\Delta_{M_w} \frac{(\kappa G^2 + g^2)^2}{2G^2p^2}(M_{W}^2 + p^2)
+ H(0,0) \frac{(\kappa G^2 + g^2)^2}{2G^2p^2}p^2
- H(0,M_{W}) (\kappa G^2 + g^2)^2 (M_{W}^2 - p^2)^2.
\]

(58)
The Faddeev-Popov contribution to the longitudinal part of the ZZ self-energy is
\[
i\Sigma_{LZZ}^{\text{FP}} = -\frac{g^2c^2}{2}p^2H(0,0).
\]

(59)
The vector boson tadpole contribution to the longitudinal part of the ZZ self-energy is
\[
i\Sigma_{LZZ}^{\text{tadpole}} = i\Delta_{M_w} \frac{g^4}{2G^2p^2}(D - 1)^2 \Delta_{M_w}.
\]

(60)
The WW loop contribution to the longitudinal part of the ZZ self-energy is
\[
i\Sigma_{LZZ}^{\text{WW}} = H(0,M_{W}) \frac{g^4}{2G^2p^2}(M_{W}^2 - p^2)^2 - i\Delta_{M_w} \frac{g^4}{2G^2p^2}
\times \left\{M_{W}^2 + \frac{1}{D}(D^2(4D - 7) + 4)p^2\right\}.
\]

(61)

VI. \(\gamma\gamma\) SELF-ENERGY

We first list the contributions to the transverse part.

A. The transverse \(\gamma\gamma\) self-energy

The Goldstone contribution to the transverse part of the \(\gamma\gamma\) self-energy is
\[
i\Sigma_{T\gamma\gamma}^{\text{Goldstone}} = -i\Delta_{M_w} \frac{g^2g^2}{2(D - 1)G^2p^2}(M_{W}^2 + p^2)
+ H(0,0) \frac{g^2p^2g^2}{2(D - 1)G^2p^2}
+ H(0,M_{W}) \frac{g^2g^2}{2(D - 1)G^2p^2}p^2
\times [M_{W}^4 + 2(2D - 3)p^2M_{W}^2 + p^4].
\]

(63)
The Faddeev-Popov contribution to the transverse part of the \(\gamma\gamma\) self-energy is
\[
i\Sigma_{T\gamma\gamma}^{\text{FP}} = -\frac{e^2}{2(D - 1)}p^2H(0,0).
\]

(64)
The vector boson tadpole contribution to the transverse part of the \(\gamma\gamma\) self-energy is
\[
i\Sigma_{T\gamma\gamma}^{\text{tadpole}} = i\Delta_{M_w} \frac{g^2g^2}{2G^2}2^2(\frac{D - 1)^2}{D} \Delta_{M_w}.
\]

(65)
The WW loop contribution to the transverse part of the \(\gamma\gamma\) self-energy is
\[
i\Sigma_{T\gamma\gamma}^{\text{WW}} = H(0,0) \frac{g^2g^2}{4(D - 1)G^2M_{W}^2}M_{W}^2H(M_{W},M_{W}) \frac{g^2g^2}{4(D - 1)G^2M_{W}^4}
\times \left\{4M_{W}^2 - p^2\right\}[4(D - 1)M_{W}^2 + 4(2D - 3)p^2M_{W}^2 + p^4]
- H(0,M_{W}) \frac{g^2g^2}{2(D - 1)G^2M_{W}^4}p^2(M_{W}^2 - p^2)^2[M_{W}^2 + 2(2D - 3)p^2M_{W}^2 + p^4]
- i\Delta_{M_w} \frac{g^2g^2}{2(D - 1)DG^2M_{W}^2p^2}\left\{-DM_{W}^4 + (5D - 4)p^2M_{W}^2 + D(4D - 7)p^4\right\}.
\]

(66)

The electromagnetic interaction gives
\[
AS - BT = eQ, \quad AS + BT = eQ.
\]

(67)

and then
\[
i\Sigma_{T\gamma\gamma}^{\text{fermion}} = -8BT \sum_{j=\text{leptons,quarks}} m_j^2H(m_j, m_j).
\]

(62)

where B, T are taken from Eqs. (55)–(57).

For small \(p^2\) one gets
\[
i\Sigma_{T\gamma\gamma}^{\text{fermion}} = 8BT \sum_{j=\text{leptons,quarks}} m_j^2H(m_j, m_j).
\]

(62)

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\[ i\Sigma_{L\gamma\gamma}^{\text{fermion}} = 0. \]  

(69)

**B. The longitudinal \( \gamma\gamma \) self-energy**

The Goldstone contribution to the longitudinal part of the \( \gamma\gamma \) self-energy is

\[
i\Sigma_{L\gamma\gamma}^{\text{Goldstone}} = -\frac{1}{2p^2} M_W^2 \left[ \frac{g^4}{G} \int M_W^2 \left( \frac{p^2}{M_W^2} - 1 \right)^2 H(0, M_W) \right.
- \left. \frac{p^4}{M_W^2} H(0, 0) - i\Delta_{M_W} \left( \frac{p^2}{M_W^2} + 1 \right) \right].
\]  

(70)

The Faddeev-Popov contribution to the longitudinal part of the \( \gamma\gamma \) self-energy is

\[
i\Sigma_{L\gamma\gamma}^{\text{FP}} = -\frac{e^2}{2} p^2 H(0, 0).
\]  

(71)

The vector boson tadpole contribution to the longitudinal part of the \( \gamma\gamma \) self-energy is

\[
i\Sigma_{L\gamma\gamma}^{\text{tadpole}} = i\frac{g^2}{G^2} \frac{2(D - 1)^2}{D} \Delta_{M_W}.
\]  

(72)

The WW loop contribution to the longitudinal part of the \( \gamma\gamma \) self-energy is

\[
i\Sigma_{L\gamma\gamma}^{WW} = i\frac{g^2}{G^2} \frac{2(D - 1)^2}{D} \Delta_{M_W} \left[ -\frac{p^2}{2} + \frac{M_W^2}{2} + \frac{2p^2}{D} + 2Dp^2 - 3p^2 \right] + \frac{1}{2} H(0, M_W)(p^2 - M_W^2)^2.
\]  

(73)

For the longitudinal part we get

\[
i\Sigma_{L\gamma\gamma}^{\text{fermion}} = 0.
\]  

(74)

It is remarkable that the sum of all the contributions (70)–(73) amounts to zero photon longitudinal self-energy. This is in agreement with the Ward identity for QED derived in Ref. [1].

**VII. Z\( \gamma \) SELF-ENERGY**

We first list the contributions to the transverse part.

**A. The transverse Z\( \gamma \) self-energy**

The Goldstone contribution to the transverse part of the Z\( \gamma \) self-energy is

\[
i\Sigma_{T\gamma\gamma}^{\text{Goldstone}} = H(0, 0) \frac{g^4 g'}{2(D - 1)G^2} \frac{2(D - 1)G^2}{2(D - 1)G^2} \frac{2}{p^2} \Delta_{M_W} + \frac{2}{p^2} \Delta_{M_W} (M_W^2 - p^2)^2 \frac{2(D - 1)M_W^2 + 4(2D - 3)p^2 M_W^2 + p^4}{2(D - 1)G^2 M_W^2}.
\]

(77)

The Faddeev-Popov contribution to the transverse part of the Z\( \gamma \) self-energy is

\[
i\Sigma_{T\gamma\gamma}^{\text{FP}} = -\frac{g^4 g'}{2(D - 1)} \frac{1}{p^2} H(0, 0).
\]  

(76)

The vector boson tadpole contribution to the transverse part of the Z\( \gamma \) self-energy is

\[
i\Sigma_{T\gamma\gamma}^{\text{tadpole}} = i\frac{g^2}{G^2} \frac{2(D - 1)^2}{D} \Delta_{M_W}.
\]  

(77)

The WW loop contribution to the transverse part of the Z\( \gamma \) self-energy is

\[
i\Sigma_{T\gamma\gamma}^{WW} = \frac{g^4 g'}{2(D - 1)G^2} \frac{2(D - 1)G^2}{2(D - 1)G^2} \frac{2}{p^2} \Delta_{M_W} + \frac{2}{p^2} \Delta_{M_W} (M_W^2 - p^2)^2 \frac{2(D - 1)M_W^2 + 4(2D - 3)p^2 M_W^2 + p^4}{2(D - 1)G^2 M_W^2}.
\]

(78)

The fermion contribution to the transverse part of \( \Sigma_{Z\gamma} \) is

\[
i\Sigma_{T\gamma\gamma}^{\text{fermion}} = \sum_{j=\text{leptons,quarks, color}} \frac{4(A S)_j}{D - 1} \left[ i(2 - D)\Delta_{m_j} + H(m_j, m_j) \left( \frac{p^2}{2} + 2m_j^2 \right) \right].
\]  

(79)

where the constants \( A, B, S, T \) are as follows.

Neutrinos:

\[
AS - BT = 0, \quad AS + BT = 0.
\]  

(80)

Charged leptons:

\[
AS - BT = AS + BT = eG[\frac{1}{3} - s^2].
\]  

(81)

Up quarks:

\[
AS - BT = AS + BT = eQ_a G[\frac{1}{3} - s^2 Q_a]. \quad Q_a = \frac{2}{3}
\]  

(82)

Down quarks:

\[
AS - BT = AS + BT = -eQ_d G[\frac{1}{3} + s^2 Q_d]. \quad Q_d = -\frac{1}{3}
\]  

(83)
B. The longitudinal $Z\gamma$ self-energy

The Goldstone contribution to the longitudinal part of the $Z\gamma$ self-energy is

$$i\Sigma_{LZ\gamma}^{\text{Goldstone}} = \frac{1}{2} \frac{1}{p^2} \left[ M_W^2 \left( \frac{g'^2}{G} + \kappa G \right) \frac{g g'}{G} \right]$$

$$\times \left\{ M_W^2 \left( \frac{p^2}{M_W^2} - 1 \right)^2 H(0, M_W) \right\}$$

$$- \frac{p^4}{M_W^2} H(0, 0) - i\Delta_{M_W} \left( \frac{p^2}{M_W^2} + 1 \right).$$

(84)

The Faddeev-Popov contribution to the longitudinal part of the $Z\gamma$ self-energy is

$$i\Sigma_{LZ\gamma}^{\text{FP}} = - \frac{g^3 g' g}{G^2} \frac{1}{2} p^2 H(0, 0).$$

(85)

The vector boson tadpole contribution to the longitudinal part of the $Z\gamma$ self-energy is

$$i\Sigma_{LZ\gamma}^{\text{tadpole}} = \frac{g^3 g' g}{2 G^2} \left( \frac{D - 1}{D} \right) \Delta_{M_W}.$$

(86)

The $WW$ loop contribution to the longitudinal part of the $\gamma Z$ self-energy is

$$i\Sigma_{LZ\gamma}^{WW} = \frac{g^3 g'}{G^2 p^2} \left\{ - i\Delta_{M_W} \left[ \frac{- p^2}{2} + M_W^2 + \frac{2 p^2}{D} + 2 D p^2 

- 3 p^2 \right] \right\} \left( \frac{1}{2} H(0, M_W) (p^2 - M_W^2)^2 \right).$$

(87)

For the longitudinal part, one has

$$i\Sigma_{LZ\gamma}^{\text{fermion}} = 0.$$

(88)

VIII. PHYSICAL UNITARITY FOR DIAGONAL ELEMENTS

It is simple to trace, at the one-loop level, the contributions due to unphysical modes (Faddeev-Popov ghosts, Goldstone bosons, and scalar parts of the vector mesons). These contributions have to cancel when we evaluate the transverse part of the self-energies on shell. Here we show that this cancellation works in generic $D$ dimensions when the transverse part is taken on shell.

A. $\Sigma_{TWW}$: The unphysical $H(0, 0)$, $H(0, M_Z)$ and $G(M_W), G(0)$

We collect the terms in the self-energy in Eqs. (32)–(36) proportional to $H(0, 0)$ and $H(0, M_Z)$, in order to check physical unitarity. They must vanish at $p^2 = M_W^2$ since they are due to the presence of unphysical modes (Goldstone and longitudinal parts of the vector bosons). We get

$$H(0, 0) \left( \frac{g^2 (p^2 - M_W^2)}{4(D - 1)G^2 (k + 1)M_W^4} \right) (g^4 p^2 (p^2 + M_W^2)$$

$$- 2 g^2 g^2 (1 + k) M_W^2 \left[ (-3 + 2D) M_W^2 + 2 (-2 + D) p^2 \right]$$

$$- 2 g^4 (1 + k) M_W^2 \left[ (-3 + 2D) M_W^2 + 2 (-2 + D) p^2 \right])$$

(89)

and

$$- H(0, M_Z) \left( \frac{M_W^2 - p^2}{4(D - 1)G^2 (k + 1)M_W^4 p^2} \right) \left[ g^4 (k + 1)^2 M_W^4$$

$$+ 2 (2D - 3) g^2 g^2 (k + 1) p^2 M_W^2 + g^4 p^4 \right]$$

$$\times \left\{ \frac{(2k + 1) M_W^2 - p^2}{M_W^2} g^2 + 2 g^2 (k + 1) M_W^2 \right\}.$$ 

(90)

Thus they are zero on shell. Similarly, one can prove that on-shell $p^2 = M_W^2$ the coefficients of $G(0)$ and of $G(M_W)$ are zero in generic $D$ dimensions.

B. $\Sigma_{TZZ}$ terms proportional to $H(0, 0)$ and $H(0, M_W)$

We collect the terms in the self-energy in Eqs. (48)–(51) proportional to $H(0, 0)$ and $H(0, M_W)$:

$$H(0, 0) \frac{p^2}{D - 1} \frac{g^4}{4 M_W^4} \left( \frac{g^4 p^2}{D - 1} \right),$$

(91)

and similarly

$$\frac{g^4 H(0, M_W)}{2(D - 1)G^2 M_W^2 p^2} \frac{p^2 - M_Z^2}{(2 M_W^2 - M_Z^2 - p^2)}$$

$$\times \left( [M_W^4 + 2 (2D - 3) p^2 M_W^2 + p^4].$$

(92)

Thus, physical unitarity is again working for the self-mass of $Z$.

C. $\Sigma_{T\gamma\gamma}$: The limit $p^2 = 0$

We collect the terms in the self-energy in Eqs. (63)–(66) proportional to $H(0, 0)$ and then we put $p^2 = 0$ in order to check that the photon remains with zero mass. One verifies that

$$H(0, 0) \frac{p^2}{D - 1} \frac{g^2 g^2}{G^2} \left\{ \frac{1 - \frac{1}{2} + \frac{p^4}{4 M_W^4} - \frac{1}{2} \right\} \left| p^2 = 0 \right| = 0.$$

(93)

For the terms involving $H(0, M)$, one needs the identity

$$H(0, M) = \frac{i\Delta_{M}}{M^2} \left\{ 1 - \frac{p^2}{M^2 (D - 4)} \right\} + O(p^4).$$

(94)

It is then straightforward to verify that

$$\lim_{p^2 \to 0} i\Sigma_{T\gamma\gamma} = 0.$$

(95)

Thus the mass of the photon remains null.
D. Unitarity for the $\Sigma_{T Z Y}$

The unitarity properties of $\Sigma_{T Z Y}$ are strictly connected to the process where this graph contributes (e.g. $Z \rightarrow t + \bar{t}$). Thus, more graphs are necessary in order to verify physical unitarity. This subject is outside the scope of the present work.

IX. W AND Z SELF-MASSES

By using the procedure of extracting the finite parts from the $D$-dimensional amplitudes described in Sec. III, we evaluate the self-masses for $W$ and $Z$ bosons. Since we have already thoroughly examined the properties of the amplitudes in $D$ dimensions at the on-shell momenta, the self-masses can be evaluated by any computer algorithm. We do not reproduce the results in the present paper.

X. PARAMETER FIT

In this section we provide an estimate of the parameters introduced in the model. The parameters $g$, $g'$, $M$ can be fixed by experiments that are essentially at low momentum transfer: for instance, $\alpha$, $G_\mu$, and the $\nu - e$ scattering that provides a precise value of $\sin^2 \theta_W$. Our calculation of the self-energies can be checked on the physics of the vector bosons $W$, $Z$. The physical masses are the input for the determination of the extra parameters of the model: $\kappa$ and $\Lambda$.

For the processes at nearly zero momentum transfer, we can use the Particle Data Group [10] values,

$$\alpha = 1/137.059 \pm 0.006 \; \text{1}(46),$$

$$G_\mu = 1.16637(1) \times 10^{-5} \; \text{GeV}^{-2},$$

and from $\nu - e$ scattering [11],

$$\sin^2 \theta_W = 0.2324 \pm 0.011.$$  \hspace{1cm} (97)

We get

$$g = \frac{e}{s} = \frac{\sqrt{4\pi\alpha}}{s} = 0.6281, \quad g' = \frac{e}{c} = 0.3456,$$

$$M = \frac{1}{\sqrt{4\sqrt{2}G_F}} = 123.11 \; \text{GeV}.$$  \hspace{1cm} (98)

With these inputs we can evaluate the values for the other two parameters by imposing the conditions on the mass corrections,

$$(gM)^2 + \Delta m_W^2 = (80.428 \pm 0.039)^2 \; \text{GeV}^2,$$

$$M^2 G^2 (1 + \kappa) + \Delta m_Z^2 = (91.1876 \pm 0.0021)^2 \; \text{GeV}^2.$$  \hspace{1cm} (99)

One gets

$$\kappa = 0.0085, \quad \Lambda = 283 \; \text{GeV}.$$  \hspace{1cm} (100)

The widths of the vector mesons obtained from the imaginary parts of the self-energies (all fermions are taken to be massless except the top with $M_{top} = 174.2 \; \text{GeV}$) are

$$\Gamma_Z = 2.203 \; \text{GeV} \quad \exp. (2.4952 \pm 0.0023 \; \text{GeV}),$$

$$\Gamma_W = 1.818 \; \text{GeV} \quad \exp. (2.141 \pm 0.041 \; \text{GeV}).$$  \hspace{1cm} (101)

These values are quite encouraging for the calculation of further radiative corrections. However, one should consider only the order of magnitude of these numbers. In fact, they depend strongly on the value of $\sin^2 \theta_W$. Only a fit including other sensitive quantities will be able to reduce their variability.

XI. COMPARISON WITH THE STANDARD MODEL

The comparison with the linear theory is performed in the limit $\kappa = 0$; i.e. the tree-level Weinberg relation between the masses of the intermediate vector mesons $W$ and $Z$ holds. The standard model self-energy corrections are given by the same diagrams of the nonlinear model evaluated at $\kappa = 0$, plus the amplitudes involving one internal Higgs line. The latter have been collected in [12].

We list below the amplitudes contributing to the transverse part of the $W$ self-energy with an internal Higgs line. The results are valid in the Landau gauge and in the limit $D = 4$. The Higgs tadpole is

$$i \Sigma_{TWW}^{\text{Higgs tad}} = \frac{g^2}{4} \Delta m_H.$$  \hspace{1cm} (102)

The Higgs-gauge bubble is

$$i \Sigma_{TWW}^{\text{Higgs gauge}} = -\frac{g^2}{12} \left[ i \left(1 + \frac{M_W^2 - m_H^2}{p^2} \right) \Delta m_W - i \frac{M_W^2}{p^2} \Delta m_H \right. - \frac{2i}{(4\pi^2)^2} M_W^2 \left. + \left( \frac{M_W^2 - m_H^2}{p^2} \right) \frac{1}{p^2} + \frac{p^2}{2} \right] \nonumber$$

$$+ \left. 10M_W^2 - 2m_H^2 \right) H(M_W, m_H) \nonumber$$

$$- \left( 1 - \frac{m_H^2}{p^2} \right)^2 p^2 H(0, m_H).$$  \hspace{1cm} (103)

The Higgs-Goldstone bubble is

$$i \Sigma_{TWW}^{\text{Higgs Goldstone}} = -\frac{g^2}{12} \left[ i \left(1 + \frac{m_H^2}{p^2} \right) \Delta m_H \right. \nonumber$$

$$\left. - \frac{2i}{(4\pi^2)^2} \left( \frac{m_H^2 - p^2}{3} \right) \nonumber$$

$$+ \left( 1 - \frac{m_H^2}{p^2} \right)^2 p^2 H(0, m_H).$$  \hspace{1cm} (104)

We list here the various contributions to the transverse part of the $Z$ self-energy with an internal Higgs line. The Higgs tadpole is

$$i \Sigma_{TTZ}^{\text{Higgs tad}} = \frac{g^2}{4} \Delta m_H.$$  \hspace{1cm} (105)
The Higgs-gauge bubble is

\[
\begin{align*}
\Sigma_{TZZ}^{\text{Higgs gauge}} &= -\frac{G^2}{12} \left[ i \left( 1 + \frac{m_H^2}{p^2} \right) \Delta m_H - i \frac{M_Z^2}{p^2} \Delta m_H \right. \\
&\quad - \left. \frac{2i}{(4\pi)^2} \frac{M_Z^2}{9} \left( \frac{m_H^2 - p^2}{3} \right) \right. \\
&\quad \left. + \left( 1 - \frac{m_H^2}{p^2} \right)^2 p^2 H(0, m_H) \right]. 
\end{align*}
\]

The Higgs-Goldstone bubble is

\[
\Sigma_{TZZ}^{\text{Higgs Goldstone}} = -\frac{G^2}{12} \left[ i \left( 1 + \frac{m_H^2}{p^2} \right) \Delta m_H \\
- \frac{2i}{(4\pi)^2} \left( \frac{m_H^2 - p^2}{3} \right) \\
+ \left( 1 - \frac{m_H^2}{p^2} \right)^2 p^2 H(0, m_H) \right].
\]

These results can be used in order to estimate the numerical impact of the Higgs corrections to the self-masses. We choose as a reference value \( m_H = 165 \) GeV and evaluate the corrections with the same input parameters in Eq. (98) and \( \Lambda = 283 \) GeV. The shifts in the self-masses are

\[
\Delta m_W^{\text{Higgs}} = 0.629 \text{ GeV}, \quad \Delta m_Z^{\text{Higgs}} = 0.531 \text{ GeV}. \tag{108}
\]

These estimates are rather intriguing. \( \Delta m_W^{\text{Higgs}} \) and \( \Delta m_Z^{\text{Higgs}} \) strongly depend on the value of \( \Lambda \) (they vary by more than 20% in the range from \( \Lambda = 200 \) GeV to \( \Lambda = 350 \) GeV). Compensations of the Higgs contributions to electroweak observables may be triggered by a change in the scale \( \Lambda \) of the radiative corrections. A more refined fit to the electroweak precision observables is required in order to discriminate between the linear and the nonlinear theory.

**XII. CONCLUSIONS**

The one-loop evaluation of self-energies for the vector mesons in the electroweak model based on a nonlinearly realized gauge group has been explicitly performed in \( D \) dimensions. The finite amplitudes in \( D = 4 \) have been achieved according to the procedure suggested by the local functional equation associated with the local invariance of the path integral measure. In practice, this implies the minimal subtraction of poles in \( D - 4 \) on properly normalized amplitudes. Thus, in this model the Higgs sector is absent and the parameters are fixed by the classical Lagrangian (no free parameters for the counterterms and therefore no on-shell renormalization). Two new parameters appear: a second mass term parameter and a scale of radiative corrections. The spontaneous symmetry breaking parameter \( v \) is not a physical constant. The scheme is very rigid and it should be checked by comparison with the experimental measures.

The calculation has been performed in the Landau gauge and by using the symmetric formalism whenever it was possible. We checked the physical unitarity and the absence of \( v \) in the measurable quantities.

A very simple evaluation has been performed for the parameters of the classical action, by using leptonic processes. The parameter that describes the departure from the Weinberg relation between \( M_W \) and \( M_Z \) is very small, and the scale of the radiative corrections is of the order of a hundred GeV. This means that the model is on solid ground, and it is reasonable to make further efforts for the evaluation of the radiative corrections in other processes.

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**APPENDIX: LIMIT D = 4 FOR THE LOGARITHMIC INTEGRAL**

We collect, in this appendix, some relevant formulas.

\[
\begin{align*}
\Delta_m &= \frac{1}{(2\pi)^{D/2}} \int d^D q \frac{i}{q^2 - m^2}, \\
H(m, M) &= -\frac{1}{(2\pi)^{D/2}} \int d^D q \frac{1}{q^2 - m^2} \frac{1}{(p + q)^2 - M^2}. 
\end{align*}
\tag{A1}
\]

The following identities allow one to prove the cancellation of infrared divergences due to the massless photon.

\[
\begin{align*}
G(M) &= \frac{\partial}{\partial m^2} H(m, M)|_{m^2=0} \\
&= \frac{i}{(4\pi)^{D/2}} \frac{\Gamma(3 - \frac{D}{2})}{\Gamma(2)} \int_0^1 dx (1 - x) \\
&\quad \times [M^2 x - p^2 x (1 - x)]^{(D/2) - 3}. 
\end{align*}
\tag{A2}
\]

\[
\begin{align*}
G(0) &= \lim_{M \to 0} G(M) \\
&= \frac{i}{(4\pi)^{D/2}} \left[ - p^2 \right]^{(D/2) - 3} \frac{\Gamma(3 - \frac{D}{2})}{\Gamma(2)} \\
&\quad \times \frac{\Gamma(\frac{D}{2} - 2) \Gamma(\frac{D}{2} - 1)}{\Gamma(D - 3)}.
\end{align*}
\tag{A3}
\]
\begin{equation}
\lim_{m \to 0} \frac{1}{m^2} \Delta_m = \lim_{m \to 0} \frac{1}{(4\pi)^{D/2}} \frac{\Gamma(1 - \frac{D}{2})}{\Gamma(1)} (m^2)^{(D/2)-2} = 0.
\end{equation}
(A4)

\begin{equation}
\frac{\partial}{\partial M^2} G(M) \bigg|_{M-0} = - \frac{i}{(4\pi)^{D/2}} \frac{\Gamma(4 - \frac{D}{2})}{\Gamma(2)} \left[ -p^2 \right]^{D/2-4} \Gamma(\frac{D}{2} - 2) \Gamma(D - 4).
\end{equation}
(A5)

For the integral in Eq. (28), use the notation
\[ a = p^2, \quad b = -p^2 + M^2 - m^2, \quad c = m^2 \] (A6)
i.e.
\begin{equation}
\int_0^1 dx \ln(p^2 x^2 + [M^2 - m^2 - p^2] x + m^2) = \int_0^1 dx \ln(ax^2 + bx + c).
\end{equation}
(A7)

If one or two masses are zero, one gets
\begin{equation}
\int_0^1 dx \ln(a[p^2 x + M^2 - p^2]) = -2 + \ln[p^2 - M^2] + \frac{M^2}{p^2} \ln \left| \frac{M^2}{p^2 - M^2} \right| - i \pi \frac{p^2 - M^2}{p^2} \theta(p^2 - M^2).
\end{equation}
(A8)

Let
\begin{equation}
\Delta = [m^2 + M^2 - p^2]^2 - 4m^2M^2.
\end{equation}
(A9)

By following the Feynman prescription, one obtains the following:
For \( 0 < p^2 < (M - m)^2 \), \( \Delta > 0 \) and then the integral is
\begin{equation}
-2 + \ln(a + b + c) + \frac{b}{2a} \ln \left( \frac{a + b + c}{c} \right) + \frac{\sqrt{\Delta}}{2a} \times \ln \frac{2c + b - \sqrt{\Delta}}{2c + b + \sqrt{\Delta}}.
\end{equation}
(A10)

for \( p^2 > (M + m)^2 \), \( \Delta > 0 \) and then the integral is
\begin{equation}
-2 + \ln(a + b + c) + \frac{b}{2a} \ln \left( \frac{a + b + c}{c} \right) + \frac{\sqrt{\Delta}}{2a} \times \ln \frac{2c + b - \sqrt{\Delta}}{2c + b + \sqrt{\Delta}} - i \frac{\sqrt{\Delta}}{a};
\end{equation}
(A11)

for \( p^2 = (M - m)^2 \) or \( p^2 = (M + m)^2 \), \( \Delta = 0 \) and then the integral is
\begin{equation}
-2 + \ln(a + b + c) + \frac{b}{2a} \ln \left( \frac{a + b + c}{c} \right);
\end{equation}
(A12)

for \( (M - m)^2 < p^2 < (M + m)^2 \), \( \Delta < 0 \) and then the integral is
\begin{equation}
-2 \frac{b}{2a} \ln[c + \left( 1 + \frac{b}{2a} \right) \ln(a + b + c)] + \frac{\sqrt{\Delta}}{a} \left\{ \tan^{-1} \left( \frac{2a + b}{\sqrt{-\Delta}} \right) - \tan^{-1} \left( \frac{b}{\sqrt{-\Delta}} \right) \right\}.
\end{equation}
(A13)

where \( -\frac{x}{2} < \tan^{-1}(x) < \frac{x}{2}, x \in \mathbb{R} \).

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