Domain splitting algorithms for the Li-ion battery simulation

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Abstract. Numerical simulation of electrochemical processes in rechargeable batteries has important applications in an energy technology. In this paper we have developed and compared three domain splitting algorithms for the Li-ion battery simulation. Li-ion battery simulation is based on microscopic model, which contains nonlinear equations for Li-ion concentration and potential. On the interface of electrodes and electrolyte the Lithium ions intercalation are described by nonlinear equation. This nonlinear interface condition affects the Newton’s method iterations and computation time. To simplify numerical simulations we use domain splitting algorithms, which split the original problem into three independent subproblems in two electrodes and electrolyte. We investigate the numerical convergence and efficiency of the algorithms on a 2D model problem.

1. Introduction

Microstructures and electrochemical processes are important in microscopic battery simulations [1]. We consider the problem in three subdomains: anode, electrolyte and cathode \(\Omega = \Omega_a \cup \Omega_e \cup \Omega_c\). We denote the external boundary of anode, electrolyte, cathode by \(\Gamma_a, \Gamma_e, \Gamma_c\), respectively and the interface boundary of anode and cathode by \(\Gamma_{aI}\) and \(\Gamma_{cI}\).

Mathematical model of electrochemical processes in Li-ion battery are described by equations for Li-ion concentration and potential [2,3]. For each subdomain the equations of concentration \(c_a, c_e, c_c\) are defined. The indices \(i = a, e, c\) define belonging function of subdomain: anode, electrolyte, cathode respectively. For simplicity we use dimensionless concentrations

\[
\tilde{c}_a = \frac{c_a}{C_a}, \quad \tilde{c}_e = \frac{c_e}{C_e}, \quad \tilde{c}_c = \frac{c_c}{C_c},
\]

where \(\tilde{c}_i, i = a, e, c\) is the absolute value of concentrations, \(C_a, C_c\) are the maximum value of concentrations in electrodes, \(C_c\) is the mean value of concentration in electrolyte. For the dimensionless concentrations we chose different dividers because it is more convenient for comparison and solving of independent problems.

We define equations for electrical potential \(\varphi_a, \varphi_c\) in the electrodes and electrochemical potential \(\varphi_e\) in the electrolyte. The current densities in electrodes are

\[
j_a = -\kappa_a \nabla \varphi_a, \quad x \in \Omega_a,
\]
\[
j_c = -\kappa_c \nabla \varphi_c, \quad x \in \Omega_c,
\]

and the current density in electrolyte is
\[ J_e = -\kappa_e \nabla \varphi_e + \kappa_e (1 - t_\epsilon) \frac{RT}{F} \nabla \ln c_e, \quad x \in \Omega_e, \]

where \( \kappa_i, i = a, e, c \) is the conductivity, \( t_\epsilon \) is the transport number of Lithium ions, \( R \) is the universal gas constant, \( T \) is the temperature, \( F \) is the Faraday constant.

The current densities in all subdomains are solenoidal vector fields
\[ \nabla J_i = 0, \quad x \in \Omega_i, \quad i = a, e, c. \]

The concentration fluxes in the electrodes are
\[ N_a = -C_a D_a \nabla c_a, \quad x \in \Omega_a, \]
\[ N_c = -C_c D_c \nabla c_c, \quad x \in \Omega_c, \]

and in the electrolyte is
\[ N_e = -C_e D_e \nabla c_e + \frac{t_e}{F} J_e, \quad x \in \Omega_e, \]

where \( D_i, i = a, e, c \) is the diffusion coefficients. The concentration fluxes satisfy to the continuity equation without sources
\[ C_i \frac{d c_i}{dt} + \nabla N_i = 0, \quad x \in \Omega_i, \quad i = a, e, c. \]

Intercalation processes occur on the contact of electrode and electrolyte. The current density of the intercalation on interface is described by the Butler-Volmer equation:
\[ i_i = 2i_{i,0} \sinh \left( \frac{F}{2RT} \eta_i \right), \quad i = a, c, \]

where \( i_{i,0} \) is the exchange current density
\[ i_{i,0} = \sqrt{C_e C_i k_i} \sqrt{c_e c_i (1 - c_i)}, \quad i = a, c, \]

and \( k_i \) is the reaction coefficient, \( \eta_i \) is the overpotential
\[ \eta_i = \varphi_i - \varphi_e - U_{i,0}(c_i), \quad i = a, c, \]

where \( U_{i,0}(c_i) \) is the open circuit potential. The current density and the concentration flux on interface satisfy following conditions
\[ j_a \cdot n_a = -j_e \cdot n_e = i_a (c_a, \varphi_a, c_e, \varphi_e), \quad x \in \Gamma_a, \]
\[ N_a \cdot n_a = -N_e \cdot n_e = \frac{1}{F} i_a (c_a, \varphi_a, c_e, \varphi_e), \quad x \in \Gamma_a, \]
\[ j_c \cdot n_c = -j_e \cdot n_e = i_c (c_e, \varphi_e, c_c, \varphi_c), \quad x \in \Gamma_c, \]
\[ N_c \cdot n_c = -N_e \cdot n_e = \frac{1}{F} i_c (c_e, \varphi_e, c_c, \varphi_c), \quad x \in \Gamma_c, \]

where \( n_i \) is the outside normal of subdomains.

For the concentration we set condition of non-permeability through outer boundary of all subdomains
\[ N_i \cdot n_i = 0, \quad x \in \Gamma_i, \quad i = a, e, c. \]

We fix the potential on the anode outer boundary
\[ \varphi_a = \varphi_a(t), \quad x \in \Gamma_a, \]

and set the current density on the cathode outer boundary
\[ j_c \cdot n_c = g(t), \quad x \in \Gamma_c. \]

There is no current through the electrolyte outer boundary
\[ j_e \cdot n_e = 0, \quad x \in \Gamma_e. \]

The initial concentrations and potentials are given as constants
\[ c_i = c_{i,0}, \quad \varphi_i = \varphi_{i,0}, \quad t = 0, \quad x \in \Omega_i, \quad i = a, e, c. \]

The initial potentials are calculated from the interface conditions
\[ \varphi_a^0 = U_{a,0}(c_{a,0}), \]
\[ \varphi_e^0 = \frac{2RT}{F} \sinh \left( \frac{\Gamma_a g(0)}{2 [\Gamma_a]_{a,0} (c_{a,0} - c_{a,e})} \right), \]
\[ \varphi_c^0 = \varphi_e^0 + U_{c,0}(c_{c,0}) - \frac{2RT}{F} \sinh \left( \frac{\Gamma_c g(0)}{2 [\Gamma_c]_{c,0} (c_{c,0} - c_{c,e})} \right), \]
where \([Γ_c]\) is the area of outer boundary \(Γ_c\), \([Γ_a]\) \([Γ_e]\) is the areas of interface boundaries of anode and electrolyte. We chose these initial conditions for potentials because the initial values must be significantly close to solution for Newton's method.

2. Discretization

To discretize by time we use the implicit scheme with uniform step \(\tau\) and denote the time \(t^n = nt, n = 0, \ldots, N, N = T/\tau\). For brevity, the function on current time layer we denote \(c_i = c_i^{n+1}\), and on previous time layer \(c_i = c_i^n\).

For spatial approximation we use a finite element methods. Due to concentration and potential discontinues on the interface we consider problem on electrodes and electrolyte domains and connect by interface condition. We formulate the variational problem by presenting the equation in the weak formulation. Taking into account the interface (4) and the boundary conditions (5), (7), (8), the equations (1), (2) are written in the weak formulation

\[
\begin{align*}
F_{a,c} &= \int_{Ω_a} (C_a \frac{c_a-c_a^0}{\tau} v_a - N_a \cdot \nabla v_a) \, dx + \int_{Γ_a} i_n^a v_a \, ds, \\
F_{a,φ} &= - \int_{Ω_a} j_a \cdot \nabla q_a \, dx + \int_{Γ_a} i_a q_a ds, \\
F_{e,c} &= \int_{Ω_e} (C_e \frac{c_e-c_e^0}{\tau} v_e - N_e \cdot \nabla v_e) \, dx - \int_{Γ_e} i_n^e v_e \, ds - \int_{Γ_e} i_c q_e \, ds, \\
F_{e,φ} &= - \int_{Ω_e} j_e \cdot \nabla q_e \, dx - \int_{Γ_e} i_a q_e \, ds - \int_{Γ_e} i_c q_e \, ds, \\
F_{c,c} &= \int_{Ω_c} (C_c \frac{c_c-c_c^0}{\tau} v_c - N_c \cdot \nabla v_c) \, dx + \int_{Γ_c} i_n^c v_c \, ds, \\
F_{c,φ} &= - \int_{Ω_e} j_c \cdot \nabla q_e \, dx + \int_{Γ_e} i_c q_e ds + \int_{Γ_c} gq_c ds, \\
\end{align*}
\]

where \(v_i, q_i, i = a, e, c\) are the test functions.

Nonlinear variational problem is as follows:

Find functions \((c_a, φ_a, c_e, φ_e, c_c, φ_c) \in C_a × Φ_a × C_e × Φ_e × C_c × Φ_c\) such that

\[
\begin{align*}
F_{a,c} + F_{a,φ} + F_{e,c} + F_{e,φ} + F_{c,c} + F_{c,φ} &= 0, \\
∀(v_a, q_a, v_e, q_e, v_c, q_c) \in V_a × Q_a × V_e × Q_e × V_c × Q_c, \\
\end{align*}
\]

where

\[
\begin{align*}
C_a &= H^1(Ω_a), Φ_a = \{φ_a \in H^1(Ω_a) : φ_a = φ_a, x \in Γ_a\}, \\
C_e &= Φ_e = H^1(Ω_e), \\
C_c &= Φ_c = H^1(Ω_c), \\
V_a &= H^1(Ω_a), Q_a = \{q_a \in H^1(Ω_a) : q_a = 0, x \in Γ_a\}, \\
V_e &= Q_e = H^1(Ω_e), \\
V_c &= Q_c = H^1(Ω_c),
\end{align*}
\]

where \(H^1(Ω_i)\) are Sobolev spaces for subdomains \(Ω_i\). Since the problem is considered in several subdomains and has two unknown functions, we define the general norm as follows:

\[
\| (c_a, φ_a, c_e, φ_e, c_c, φ_c) \| = \| c_a \|^2 + \| φ_a \|^2 + \| c_e \|^2 + \| φ_e \|^2 + \| c_c \|^2 + \| φ_c \|^2 \|^\frac{1}{2},
\]

where function norm in subdomains

\[
\| u_i \|^2 = \int_{Ω_i} u_i^2 \, dx, \quad i = a, e, c.
\]

To get a linear variational problem from the nonlinear problem we use Newton's method [4]. We introduce the following notation for the unknown of iteration \(k + 1\)

\[
c_i^{k+1} = c_i^k + δc_i^k, \quad φ_i^{k+1} = φ_i^k + δφ_i^k, \quad i = a, e, c,
\]

where \(c_i^k, φ_i^k\) are the known concentrations and potentials of the previous iteration, \(δc_i^k, δφ_i^k\) are the correction functions, which we want to find. Also we introduce the following auxiliary notations
\[ \delta N^k_i = \frac{\partial N^k_i}{\partial c_i} \delta c^k_i + \frac{\partial N^k_i}{\partial \varphi_i} \delta \varphi^k_i, \quad i = a, e, c, \]
\[ \delta F^k_i = \frac{\partial F^k_i}{\partial c_i} \delta c^k_i + \frac{\partial F^k_i}{\partial \varphi_i} \delta \varphi^k_i, \quad i = a, e, c, \]
\[ \delta \ell_a = \frac{\partial \ell_a}{\partial c_a} \delta c^k_a + \frac{\partial \ell_a}{\partial \varphi_a} \delta \varphi^k_a, \]
\[ \delta \ell_e = \frac{\partial \ell_e}{\partial c_e} \delta c^k_e + \frac{\partial \ell_e}{\partial \varphi_e} \delta \varphi^k_e, \]
\[ \delta \ell_c = \frac{\partial \ell_c}{\partial c_c} \delta c^k_c + \frac{\partial \ell_c}{\partial \varphi_c} \delta \varphi^k_c. \]

Using the introduced notations we write equations for the corrections
\[ F_{a,c}^{k+1} = \int_{\Omega_a} (c_a \delta c_a^k / \tau v_a - \delta N_a^k \cdot \nabla v_a) \, dx + \int_{\Gamma_a} \delta \ell_a^k / \tau v_a ds + F_{a,c}^k, \]
\[ F_{e,c}^{k+1} = \int_{\Omega_e} (c_e \delta c_e^k / \tau v_e - \delta N_e^k \cdot \nabla v_e) \, dx - \int_{\Gamma_e} \delta \ell_a^k / \tau v_a ds + F_{e,c}^k, \]
\[ F_{e,\varphi}^{k+1} = -\int_{\Omega_e} \delta \varphi_e^k / \tau v_e \, dx - \int_{\Gamma_e} \delta \ell_a^k / \tau v_a ds - \int_{\Gamma_e} \delta \ell_e^k / \tau v_e ds + F_{e,\varphi}^k, \]
\[ F_{c,\varphi}^{k+1} = \int_{\Omega_c} (c_c \delta c_c^k / \tau v_c - \delta N_c^k \cdot \nabla v_c) \, dx + \int_{\Gamma_c} \delta \ell_a^k / \tau v_a ds + F_{c,\varphi}^k, \]
\[ F_{c,\varphi}^{k+1} = -\int_{\Omega_c} \delta \varphi_c^k / \tau v_c \, dx + \int_{\Gamma_c} \delta \ell_a^k / \tau v_a ds + F_{c,\varphi}^k, \]

where
\[ F_{i,c}^k = F_{i,c}, \quad F_{i,\varphi}^k = F_{i,\varphi}, \quad i = a, e, c. \]

Then the linear variational problem for the corrections will be as follows: Find functions correction \((\delta c_a, \delta \varphi_a, \delta c_e, \delta \varphi_e, \delta c_c, \delta \varphi_c)\) \(\in V_a \times Q_a \times V_e \times Q_e \times V_c \times Q_c\) such that
\[ F_{a,c}^{k+1} + F_{a,\varphi}^k + F_{e,c}^{k+1} + F_{e,\varphi}^k + F_{c,\varphi}^{k+1} + F_{c,\varphi}^k = 0, \]
\[ \forall (v_a, q_a, v_e, q_e, v_c, q_c) \in V_a \times Q_a \times V_e \times Q_e \times V_c \times Q_c. \]

As initial approximations for Newton's method we use the values from the previous time layer. The Newton iterations are performed while the relative norm of corrections \(\| \delta (c, \varphi) \| / \| (c, \varphi) \| \) is less than relative tolerance \(t_a = 10^{-6}\), or absolute norm of corrections \(\| \delta (c, \varphi) \| \) is less than absolute tolerance \(t_a = 10^{-20}\). In the transition from continuous subdomains \(\Omega_i, i = a, e, c\) to discrete subdomains \(\Omega_i^k, i = a, e, c\) we use Lagrange elements of the 1st order.

3. Algorithms

To reduce the calculation time and to simplify the problem we use domain splitting algorithms. Due to the relatively small time derivative of functions in the electrolyte, we assume that: we can do not change the functions in electrolyte over few time layers. Using this assumption we consider several algorithms of splitting into independent subproblems.

We assume that the original equation in the electrolyte can be replaced by the following equations
\[ \frac{c_e - \tilde{c}_e}{\tau} = 0, \quad \frac{\varphi_e - \tilde{\varphi}_e}{\tau} = 0, \]
then the interface conditions (4) for electrodes can be replaced with the following conditions
\[ f_a \cdot n_a = i_a(c_a, \varphi_a, \tilde{c}_e, \tilde{\varphi}_e), \quad x \in \Gamma_a^i, \]
\[ N_a \cdot n_a = \frac{1}{F} i_a(c_a, \varphi_a, \tilde{c}_e, \tilde{\varphi}_e), \quad x \in \Gamma_a^i, \]
\[ f_c \cdot n_c = i_c(c_c, \tilde{\varphi}_e, c_e, \varphi_c), \quad x \in \Gamma_c^i, \]
\[ N_c \cdot n_c = \frac{1}{F} i_c(c_c, \tilde{\varphi}_e, c_e, \varphi_c), \quad x \in \Gamma_c^i. \]

If we use interface conditions (14) then the original nonlinear variational problem is split into two independent subproblems in electrodes: Find the functions \((c_i, \varphi_i) \in C_i \times \Phi_i\) such that
for each electrodes subdomain $i = a, c$. In Newton's method for each subproblem expressions (12) are replaced by the following expressions

$$
\delta i_a = \frac{\partial i_a}{\partial c_a} \delta c_a + \frac{\partial i_a}{\partial \phi_a} \delta \phi_a, \\
\delta i_c = \frac{\partial i_c}{\partial c_c} \delta c_c + \frac{\partial i_c}{\partial \phi_c} \delta \phi_c.
$$

Then the linear variation subproblems for corrections in electrodes are as follows: Find correction functions $\delta c_i, \delta \phi_i \in V_i \times Q_i$ such that

$$
F_{i,c}^{k+1} + F_{i,\phi}^{k+1} = 0, \quad \forall (v_i, q_i) \in V_i \times Q_i,
$$

for each electrodes subdomain $i = a, c$.

Below we describe the first domain splitting algorithm of the problem simulation

**Algorithm 1.** The functions are fixed in electrolyte.

```plaintext
while $t < t_f$ do
    repeat
        $t = t + \tau, \ Solve(\Omega^h)$
    until $\|c_e - c^h_e\| < \delta$
    for $i = 1$ to $N$ do
        $t = t + \tau, \ Solve(\Omega^h_a), \ Solve(\Omega^h_c), \ c_e = \bar{c}_e, \ \phi_e = \bar{\phi}_e$
    end for
end while
```

In the algorithm 1 firstly for each time layer we solve the complete system while the concentration change is sufficiently small in electrolyte. When the time derivative of the concentration in electrolyte is less than $\delta$ we solve the separated subproblems. In next time layers we solve the subproblems in electrodes and fix solution in electrolyte. We repeat these steps until the simulation ends.

We propose the second domain splitting algorithm, which based on reformulating the original boundary condition on anode. We use Neumann BC instead of Dirichlet BC on anode boundary. Also we don't fix functions in electrolyte and solve independent problem in electrolyte.

The absolute value of potential do not affect the mathematical model, since the only potentials difference and gradient are used. Thus if not considering the Dirichlet boundary condition, then for all potentials we can add any constant that does not affect the equations. We introduce the new unknown depending on the potential of the original problem

$$
\phi_i = \phi_i + \phi_D, \quad i = a, e, c, \quad x \in \Omega_i, \quad \phi_D \in \mathbb{R}.
$$

Instead of the Dirichlet boundary conditions (6), we introduce the Neumann boundary condition

$$
\int_a n_a \cdot g(t), \quad x \in \Gamma_a,
$$

and choose the constant

$$
\phi_D = \phi_a - \phi_D, \quad x \in \Gamma.
$$

Then we can remove the explicit definition of the Dirichlet condition, but we select the appropriate constant $C$ which satisfies this condition. Note we assume that potential is the same on whole anode outer boundary due to high conductivity of contacting battery shell.

Under the new boundary condition in variational problems we change only the function spaces

$$
\Phi_a = \{\phi_a \in H^1(\Omega_a)\}, \ Q_a = \{q_a \in H^1(\Omega_a)\}.
$$

The solution in electrolyte is computed using the concentrations and potentials of electrodes from the current time layer, which are known after solution in electrodes.

Nonlinear variational problem for subproblem in electrolyte is as follows: Find functions $(c_e, \phi_e) \in C_e \times \Phi_e$ such that

$$
F_{e,c} + F_{e,\phi} = 0, \quad \forall (v_e, q_e) \in V_e \times Q_e.
$$
In Newton's method for subproblem in electrolyte we need to replace the expressions (12) by the following expressions

\[
\delta i_a = \frac{\partial i_a^k}{\partial c_e} \delta c_e + \frac{\partial i_a^k}{\partial \varphi_e} \delta \varphi_e,
\]

\[
\delta i_c = \frac{\partial i_c^k}{\partial c_e} \delta c_e + \frac{\partial i_c^k}{\partial \varphi_e} \delta \varphi_e.
\]

Then linear variation subproblem for the corrections in electrolyte is as follows: Find correction functions \((\delta c^e, \delta \varphi^e) \in V_e \times Q_e\) such that

\[
F_{e,c}^{k+1} + F_{e,\varphi}^{k+1} = 0, \quad \forall (v_e, q_e) \in V_e \times Q_e.
\]

The second algorithm is the same as the algorithm 1, the only difference is that we solve subproblem in electrolyte.

**Algorithm 2.** The solution is separated into subproblems.

```plaintext
while t < t_f do
    repeat
        t = t + \tau, Solve(\Omega^h)
    until \|c_e - \bar{c}_e\| < \delta
    for i = 1 to N do
        t = t + \tau, Solve(\Omega_a^h), Solve(\Omega_c^h), Solve(\Omega_e^h)
    end for
end while
```

With the third algorithm we do not need to solve the complete system, i.e. we solve only split subproblems. Each time layer we solve subproblems in electrodes, and solve the subproblem in electrolyte using results of electrodes subproblem.

**Algorithm 3.** Domain splitting scheme.

```plaintext
while t < t_f do
    t = t + \tau, Solve(\Omega_a^h), Solve(\Omega_c^h), Solve(\Omega_e^h)
end while
```

Nonlinear and linear variational problems are the same as in the algorithm 2. We modify and substantially simplify the solution algorithm. In algorithm 3 we do not have parameters \(\delta\) and \(N\). This algorithm represent the domain splitting scheme.

4. Numerical experiments

We consider 2D model Li-ion battery problem with simple domain.

| Table 1. Problem parameters. |
|-----------------------------|
| Symbol | Parameter | Value | Unit |
|-------|-----------|-------|------|
| \(R\) | Universal gas constant | 8.3144621 | J/(Kmol) |
| \(T\) | Temperature | 300 | K |
| \(F\) | Faraday constant | 96485.33 | As/mol |
| \(C_e\) | Average concentration in electrolyte | 0.001 | mol/cm³ |
| \(t_a\) | Transport number of lithium ions, | 0.2 | | |
| \(k_e\) | Electrolyte conductivity | 0.001 | S/cm |
| \(D_e\) | Electrolyte diffusion coefficient | \(10^{-7}\) | cm²/s |
| \(C_a\) | Maximum concentration in anode | 0.02 | mol/cm³ |
| \(k_a\) | Anode reaction rate | 0.002 | \(Ac m^{2.5}/mol^{1.5}\) |
| Symbol | Description | Unit |
|--------|-------------|------|
| $U_{A,0}(c_a)$ | Anode open circuit voltage | $e^{-10\cdot c_a}$ V |
| $\kappa_a$ | Anode conductivity | 10 S/cm |
| $D_a$ | Anode diffusion coefficient | $5 \cdot 10^{-10}$ cm$^2$/s |
| $C_c$ | Maximum concentration in cathode | 0.02 mol/cm$^3$ |
| $k_c$ | Cathode reaction rate | 0.2 A cm$^2$/mol$^{1.5}$ |
| $U_{C,0}(c_c)$ | Cathode open circuit voltage | $4 - 100(c_c - 0.5)^5$ V |
| $\kappa_c$ | Cathode conductivity | 1 S/cm |
| $D_c$ | Cathode diffusion coefficient | $10^{-9}$ cm$^2$/s |

Physical constants and properties of the battery components are given in Table 1. The boundary value for the anode potential is assigned as constant $\varphi_D = U_{i,0}(c_i^0)$. We set the current density at the cathode outer boundary as fixed number $g = \alpha C_{rate}$, where $\alpha \in \mathbb{R}$, $C_{rate}$ is the current density at which the battery is theoretically fully charged or discharged in 1 hour, depending on the sign

$$C_{rate} = \frac{C_c F[\Omega_c]}{3600[\Gamma_c]}$$

where $[\Omega_c]$ is the size of domain $\Omega_c$. In all calculations we used $\alpha = -1$, which means we simulate the battery charging. We take the initial conditions for the concentrations as follows

$$c_a^0 = 0.1, \quad c_e^0 = 1, \quad c_c^0 = 0.9,$$

and the potentials we calculated using the formula (10). The simulation end time is $T = 2900$ s.

![Figure 1](image1.png) *Figure 1.* Computational mesh with 610 nodes. Cell size is 1$\mu$m. Domain size is 50$\mu$m $\times$ 7.5$\mu$m.

![Figure 2](image2.png) *Figure 2.* Concentrations in $t = 2900$ s with separate legends, mol/cm$^3$.

![Figure 3](image3.png) *Figure 3.* Potentials in $t = 2900$ s with separate legends, mol/cm$^3$. 
In Figure 1 we show the computational mesh which has double unknowns on interface boundary. Numerical problem is solved in $\mu m$ scale and domains size is $[\Omega] = 375 \mu m^2$. Example of solution is showed in Figures 2 and 3. Numerical experiments with varying mesh size $h$, time step $\tau$, parameters $N$ and $\delta$ showed that the algorithms are numerically converge. In the algorithms comparison we use error norm of solution in following form
\[ \varepsilon = \| (c_a - c_{a*}, \varphi_a - \varphi_{a*}, c_e - c_{e*}, \varphi_e - \varphi_{e*}, c_c - c_{c*}, \varphi_c - \varphi_{c*}) \|, \]
where $\| (c_a, \varphi_a, c_e, \varphi_e, c_c, \varphi_c) \|$ is numerical solution with $\tau = 10 s$ and $\| (c_{a*}, \varphi_{a*}, c_{e*}, \varphi_{e*}, c_{c*}, \varphi_{c*}) \|$ is the reference solution with $\tau = 2.5 s$. Note that the general norm of constant function with value one $\| I \| = \| (I_a, I_a, I_e, I_e, I_c) \| = (2[\Omega_a]^2 + 2[\Omega_e]^2 + 2[\Omega_c]^2)^{1/2}$, where $l_i = 1, x \in \Omega_i, i = a, e, c$. Then for our mesh $\| I \| \approx 332.6 \mu m^2$

5. Conclusion
The main aim of this article is to develop and compare domain splitting algorithms for the problem describing electro-chemical processes in Li-ion batteries. We have developed and compared three domain splitting algorithms based on splitting by interface boundary. Our results showed that the algorithm 3 (splitting scheme) error is comparable with implicit scheme error (see Figure 4). The algorithms 1 and 2 computationally more complex than the algorithm 3 due to the solution of full system. The algorithm 3 is fully decouple the general problem into 3 independent subproblems and preserve precise accuracy, which can be comparable with the implicit scheme accuracy.

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