Quantum gravity predictions for black hole interior geometry

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In a previous work we derived an effective Hamiltonian constraint for the Schwarzschild geometry starting from the full loop quantum gravity Hamiltonian constraint and computing its expectation value on coherent states sharply peaked around a spherically symmetric geometry. We now use this effective Hamiltonian to study the interior region of a Schwarzschild black hole, where a homogeneous foliation is available. Descending from the full theory, our effective Hamiltonian preserves all relevant information about the graph structure of quantum space and encapsulates all dominant quantum gravity corrections to spatially homogeneous geometries at the effective level. It carries significant differences from the effective Hamiltonian postulated in the context of minisuperspace loop quantization models in the previous literature. We show how, for two geometrically and physically well motivated choices of coherent states, the classical black hole singularity is replaced by a homogeneous expanding Universe. The resultant geometries have no significant deviations from the classical Schwarzschild geometry in the pre-bounce sub-Planckian curvature regime, evidencing the fact that large quantum effects are avoided in these models. In both cases, we find no evidence of a while hole horizon formation However, various aspects of the post-bounce effective geometry depend on the choice of quantum states.

A primary aim for the quantization program of gravitational field is to determine the fate of classical spacetime singularities as predicted by general relativity. It has been speculated that quantum gravitational effects smooth out spacetime singularities, analogously to how an ultraviolet completion of quantum fields tames ultraviolet divergences. Fortunately, the canonical approach to loop quantum gravity (LQG) is now sufficiently developed to systematically resolve this key issue from the full theory perspective.

To provide some historical context, we point out that the first generation of quantum gravity predictions were made in cosmology and within the minisuperspace quantization scheme. There the loop quantization program was implemented by applying LQG inspired techniques to a (model dependent) symmetry reduced phase space of general relativity parametrized by the Ashtekar connection and densitized triad [1]. The resultant models, which later became known as loop quantum cosmology (LQC), unanimously predict that the classical spacetime singularity is replaced by a quantum bounce [2, 3]. They offered the first glimpse of how quantum gravity could resolve spacetime singularities.

However, attempts to generalize the loop quantization program to a black hole geometry were not equally successful at producing generally accepted predictions [4–13]. The starting point of all these previous attempts was the observation that the Schwarzschild black hole interior can be described in terms of a contracting anisotropic Kantowski–Sachs model, allowing one to apply LQC techniques to this highly symmetric geometry. This approach presented two major drawbacks.

Firstly, we were still confronted with the fundamental issue of whether the microscopic degrees of freedom removed by the symmetry reduction at the classical level could have been relevant, affecting the final physical predictions of the theory. In the case of LQC techniques applied to the Schwarzschild interior, this was clearly manifest in the use of point holonomies for the construction of the reduced kinematical Hilbert space associated with the 2-spheres that foliate the spacelike surfaces, which completely eliminated all information about the 3-D structure of the graph. Secondly, the reintroduction, by hand, of some ‘elementary plaquette’ at the end of the quantization procedure, in order to fix the quantum parameters entering the construction of the states, was an ambiguous prescription, with different choices yielding different physical scenarios, as well as undesirable features.

In order to construct a more robust and reliable model of quantum black hole geometry that addresses the aforementioned issues, a new framework, the so-called ‘quantum reduced loop gravity’ approach [14–19], has recently been developed in which one starts from the full LQG theory and then performs the symmetry reduction at the quantum level by means of coherent states encoding the information about a semi-classical spherically symmetric black hole geometry. More precisely, computing the action of the quantum Hamiltonian operator on a partially gauge fixed kinematical Hilbert space results in a semi-classical, effective Hamiltonian constraint $H_{\text{eff}}$ that now replaces the Hamiltonian constraint of general relativity. Unlike its predecessors, the newly obtained effective Hamiltonian constraint for the Schwarzschild geometry [19] descends from the LQG quantum Hamiltonian constraint and it encodes all the relevant graph structures inherited from the full theory.

The effective Hamiltonian in [19] was derived for a general spacetime foliation, well defined for both the exte-
rior and the interior Schwarzschild regions. Solving for the effective geometry in both regions cannot be done within the minisuperspace quantization program as a homogeneous slicing that covers the entire spacetime is not available. One is forced to use an inhomogeneous slicing and subsequently work with an infinite dimensional phase space. In particular, the effective scalar constraint equation $H_{\text{eff}} = 0$ is now a highly non-linear, non-local ODE. While work is in progress to develop numerical techniques to solve this equation, a simpler task is to restrict attention to the interior region where the standard Schwarzschild foliation is now homogeneous and the coherent states are peaked around this homogeneous geometry. In this way, the effective Hamiltonian derived in [19], as well as the Hamilton’s evolution equations that it generates, assume a more tractable form which can be solved through a combination of analytical and numerical tools. In this letter we report on the results of this analysis and show how the classical black hole singularity is replaced by an expanding Bianchi type I Universe.

Let us emphasize that our restricting to the interior minisuperspace in order to solve the interior effective dynamics is not equivalent to the minisuperspace quantization models of the previous literature. In fact, as it will be elucidated in detail, our $H_{\text{eff}}$ encapsulates all dominant quantum gravity corrections coming from the full theory to spatially homogeneous geometries at the effective level. It supersedes and generalizes the previously used Hamiltonian constraint of the minisuperspace loop quantization procedure, which so far has been postulated based on the example of homogeneous and isotropic cosmology [20]. At the same time, having the full theory (graph) structure to begin with, our analysis provides a consistent and well motivated geometrical procedure to identify the quantum parameters entering the effective dynamics. This allows us to investigate, in a controlled way, how different choices of quantum coherent states result in different physical predictions of the theory.

**Phase space and effective dynamics.** We are interested in the effective description for the interior geometry of a spherically symmetric black hole. Practically, this amounts to considering quantum gravitational effects for which the metric continues to be spatially homogeneous. Therefore, let us specialize to a coordinate system in which the metric is

$$ds^2 = -N(\tau)^2d\tau^2 + \Lambda(\tau)^2dx^2 + R(\tau)^2d\Omega^2. \quad (1)$$

Here $\Lambda(\tau)$ and $R(\tau)$ are the two dynamical metric functions and $d\Omega^2$ is the unit round 2-sphere line element. Due to spatial homogeneity, the only gauge freedom is to rescale proper time by choosing a lapse function $N$. As commonly done, in order to avoid having a divergent symplectic structure, we require $x \in [0, L_0]$ where $L_0$ is some infrared cut-off in the $x$ direction. The covariant phase space of solutions consists of the metric functions $\Lambda$ and $R$ and their conjugate momenta

$$P_R := -\frac{1}{GN} [\dot{\Lambda} + \dot{R}], \quad P_\Lambda := \frac{R \dot{R}}{GN}, \quad (2)$$

where dot denotes differentiation with respect to $\tau$. Note that the functional dependence of $P_R$ and $P_\Lambda$ on $R$ and $\Lambda$ coincides with their classical expressions. The symplectic 2-form for this phase space is $\omega = L_0(dR \wedge dP_R + d\Lambda \wedge dP_\Lambda)$, whence the Poisson brackets assume the familiar form $\{ R, P_R \} = \{ \Lambda, P_\Lambda \} = 1/L_0$. To avoid confusion with the previous LQG literature, we stress that the $L_0$ factor appearing in the symplectic structure is not absorbed in the phase space variables. Hence, in all subsequent equations, the phase space variables must be regarded as invariants under the rescaling of this cut-off. In particular, as it will become clear below, the effective dynamics is $L_0$-independent and the undesirable features related to this scale dependence that appear in some of the previous minisuperspace models are absent in our treatment.

In the case of a Schwarzschild spacetime with a gravitational mass $m$, the above phase space variables assume the following trajectories (in the rest of the letter we denote classical solutions with a subscript $c$) \footnote{We work in $c = \hbar = 1$ units unless otherwise stated.}

$$R_c(\tau) = 2Gm \, e^{\tau/2Gm},$$

$$\Lambda_c(\tau) = \sqrt{e^{-\tau/2Gm} - 1},$$

$$P_{R_c}(\tau) = \frac{1}{2G} \left[ 2 - e^{-\tau/2Gm} \right],$$

$$P_{\Lambda_c}(\tau) = -2m \, e^{\tau/4Gm} \sqrt{1 - e^{\tau/2Gm}}, \quad (3)$$

for

$$N_c = -\frac{R^2}{2G^2mP_\Lambda} \quad (4)$$

as the choice for the lapse function. Here the range $-\infty < \tau < 0$ covers the entire interior region of the Schwarzschild black hole, with $\tau = 0$ corresponding to the horizon and $\tau = -\infty$ to the classical singularity. As we will see shortly, a convenient choice for the lapse function that we utilize in the effective theory reduces to (4) in the limit $\hbar \to 0$.

In the effective theory, the phase space variables are evolved in time $\tau$ via the following Poisson brackets:

$$\dot{R} = \{ R, H_{\text{eff}}[N] \} = \frac{1}{L_0} \frac{\partial H_{\text{eff}}[N]}{\partial P_R},$$

$$\dot{P}_R = \{ P_R, H_{\text{eff}}[N] \} = -\frac{1}{L_0} \frac{\partial H_{\text{eff}}[N]}{\partial R},$$

$$\dot{\Lambda} = \{ \Lambda, H_{\text{eff}}[N] \} = \frac{1}{L_0} \frac{\partial H_{\text{eff}}[N]}{\partial P_\Lambda},$$

$$\dot{P}_\Lambda = \{ P_\Lambda, H_{\text{eff}}[N] \} = -\frac{1}{L_0} \frac{\partial H_{\text{eff}}[N]}{\partial \Lambda}. \quad (5)$$
where $H_{\text{eff}}[N]$ is the smearing of the effective Hamiltonian constraint by the effective metric. As gravity is described by a constrained Hamiltonian system, we also have

$$H_{\text{eff}} = 0$$ (6)

at all times. This constraint equation is implied by Eq. (5), provided that it is satisfied at some initial time.

In order to find the solutions to Eq. (5), let us introduce $H_{\text{eff}}$ and discuss in some detail the graph structure that we have adapted to constant $\tau$ slices. By specializing the general effective Hamiltonian derived in [19] to the spatially homogeneous metric (1) and integrating over the angular coordinates, we obtain

$$H_{\text{eff}} = -\frac{L_0}{4\gamma^2G\epsilon_\pi\epsilon^2} \left[ eR \sin \left( \frac{\gamma Ge_\pi P_R - P_\Lambda \Lambda}{R^2} \right) \right.$$\vspace{-10pt}

$$\times \left\{ 2 \sin \left( \frac{\gamma Ge_\pi P_R}{R} \right) + \pi H_0 \left( \frac{Ge_\pi P_R}{R} \right) \right\}$$\vspace{-10pt}

$$+ \epsilon_\pi \Lambda \left\{ 8\gamma^2 \cos (\epsilon) \sin \left( \frac{\epsilon}{2} \right)^2 + \pi \sin \left( \frac{\gamma Ge_\pi P_R}{R} \right) \right\} \times H_0 \left( \frac{Ge_\pi P_R}{R} \right) \right\}. \quad (7)$$

Here $H_0(x)$ is the Struve function of zeroth order and $\gamma$ is the Barbero–Immirzi parameter that, for the sake of numerical calculation presented below, we fix it to be approximately $0.274$ in consistency with (some) black entropy calculations in LQG [21, 22]. The two quantum parameters in (7), $\epsilon$ and $\epsilon_\pi$, are the angular and longitudinal coordinate lengths of the so-called plaquettes, cubic cells that are sewn together to make up a graph or a discrete geometric structure on constant $\tau$ surfaces. Due to how the partially gauged fixed LQG kinematical Hilbert space was constructed in [19] and the peakedness properties of the coherent states defined on the associated graph structure, these parameters are related to the semi-classical data as

$$\epsilon = \sqrt{\frac{8\pi\gamma \ell_p \sqrt{J_0}}{R}}, \quad \epsilon_\pi = \sqrt{\frac{8\pi\gamma \ell_p j}{\Lambda \sqrt{J_0}}}, \quad (8)$$

where $\ell_p = \sqrt{\hbar G/c^4}$ is the Plank length and $J_0$ and $j$ are some quantum numbers associated respectively with the angular and the longitudinal links of the coherent states. It is important to emphasize that the quantum parameters (8) are local quantities that are invariant under a rescaling of global quantities, such as the infrared cut-off $L_0$ \(^3\). Therefore, the dynamics derived from the effective Hamiltonian (7) with the expressions (8) for the quantum parameters does not contain any information about $L_0$, rendering our key physical predictions independent of this scale as well.

Let us point out that, not surprisingly, in the classical limit where $\hbar \to 0$, $\epsilon$ and $\epsilon_\pi$ vanish, the discrete graph structure disappears, and the effective Hamiltonian in Eq. (7) reduces to its classical value

$$\frac{H_\epsilon}{L_0} = -\frac{G P_R P_\Lambda}{R} + \frac{G \Lambda P_\Lambda^2}{2R^2} - \frac{\Lambda}{2G}. \quad (9)$$

The appearance of the Struve function in (7) represents the main departure from the minisuperspace quantization models in the previous literature. It is a direct reflection of including ab initio the graph structure of the 2-spheres in the quantum reduced kinematical Hilbert space. The other significant difference, also originating from the SU(2) holonomies along angular links, is encoded in the term proportional to $\gamma^2$ inside the second curly brackets. This term derives from the Lorentzian part of the LQG Hamiltonian constraint and it contains corrections to all orders in $\epsilon$, while in its minisuperspace counterpart only the leading term is present.

In order to distinguish the classical regime, in which general relativity is expected to be a valid description of gravitational field, from the region where quantum effects are expected to become relevant, we rely on the Kretschmann scalar, which for the classical Schwarzschild metric functions given in Eq. (3) becomes

$$K_c := R_{abcd}R^{abcd} = \frac{3}{4Gm^4}. \quad (10)$$

Quantum gravity heuristics suggest that any effective metric encoding quantum gravity corrections should begin to deviate significantly from the classical one in the region where curvature becomes (super-) Planckian, namely as $K_c \gtrsim 1/\ell_p^4$, this happens for times $\tau \lesssim \tau_\star = (Gm/3)\log [3\ell_p^4/(4Gm^4)]$. We can thus define the parameter

$$\rho := \frac{R_c(\tau)}{R_c(\tau_\star)} \quad (11)$$

and expect quantum effects to become dominant at about $\rho \sim 1$.

The relations (8) do not uniquely fix the graph structure. Different choices of $\epsilon$ and $\epsilon_\pi$ can appear depending on what one assumes for the quantum numbers $J_0$ and $j$. Below we present the effective metrics corresponding to two possible choices of graph regularization scheme.

**Quantum parameters as phase space functions.** The first natural option for the quantum parameters is to

\[\text{A rescaling of } L_0 \text{ is compensated by a rescaling of the number of plaquettes in the construction of the quantum coherent state used to compute the expectation value of the quantum Hamiltonian constraint, leaving the expression for } \epsilon_\pi \text{ independent of this rescaling.}\]
make them dynamical, namely to keep their dependence on the effective phase space variables as given in (8) (in the old literature, this corresponds to the so-called $\bar{\mu}$-scheme [6]). In this case, the quantum parameters have non-trivial contributions to the Poisson brackets (5) and lead to a very complicated system of ODEs that is difficult to analytically integrate. We thus numerically solve the effective Hamilton’s evolution equations starting with classical initial data very near the black hole event horizon, namely when $\rho \sim (Gm/\ell_p)^{2/3}$. We specialize to the following choice of lapse function

$$N = -\frac{\gamma \epsilon R}{Gm} \left[ \sin \left( \frac{2G\epsilon P_0}{R} \right) + \frac{\pi}{2} H_0 \left( \frac{2G\epsilon P_0}{R} \right) \right],$$

which will prove especially useful for the subsequent case of graph structure. This lapse reduces to $N_c$ given in Eq. (4) in the limit $\hbar \to 0$. In presence of a non-zero $\epsilon$ and for $m$ much larger than the Planck mass $m_p = \sqrt{\hbar c/G}$, the black hole event horizon is still nearly at $\tau = 0$. $\tau$ can be extended all the way to $-\infty$ unless the coordinate system breaks down at a finite value of $\tau$ due to, e.g., reaching a Killing horizon.

The effective metric functions $R(\tau)$ and $\Lambda(\tau)$ are plotted in FIG. 1 for the choice of $m = 10^{12}m_p$ and $j = j_0 = 100$. The qualitative behavior of the two metric functions remains the same for different values of the mass and quantum numbers, as long as $m$ is reasonably above the Planck mass (a stellar mass black hole has $m \approx 10^{28}m_p$).

The plots in FIG. 1 show how the effective metric agrees with general relativity in the low curvature region all the way till $\tau \sim \tau_*$, where the Planck regime is reached and quantum geometry corrections become dominant, as expected. No large quantum effects near the horizon appear. In particular, $R$ decreases with $\tau$ till it reaches a minimum value $R_{\min} \sim R(\tau_*)$. It then bounces and grows exponentially as $\tau \to -\infty$. Therefore, the area of 2-spheres foliating constant $\tau$ surfaces never shrinks to zero, i.e., the singularity is effectively resolved. The surface $\tau \sim \tau_*$ marks the transition for the 2-spheres from being trapped in the past ($\tau_* < \tau < 0$) to being anti-trapped in the future ($-\infty < \tau < \tau_*$). In order to determine if a white hole horizon forms in the future of $\tau_*$, we need to analyze the behavior of the second metric function $\Lambda$.

As can be seen in FIG. 1, numerical integration suggests that $\Lambda$ approaches a constant non-zero value for $\tau \ll 0$. This observation is confirmed by an asymptotic analysis of the Hamilton’s dynamical equations, which provides us with the following estimates that are valid in the $\tau \to -\infty$ limit so long as the spin quantum numbers coincide:

$$N(\tau) \to \text{constant} \sim j^{1/2} \frac{\ell_p}{Gm},$$

$$\Lambda(\tau) \to \text{constant} \sim j^{-1/6} \left( \frac{Gm}{\ell_p} \right)^{1/3},$$

$$R(\tau) \sim j^{2/3} \ell_p^{4/3} (Gm)^{-1/3} e^{-\tau/2Gm}. \quad (13)$$

In this limit, the interior geometry becomes a product of a 2-D pseudo-Euclidean space and a round 2-sphere whose proper area is blowing up exponentially. Note that as $\hbar \to 0$ the classical singularity reappears. As it is clear from the asymptotic forms (13), the interior metric is not asymptotically flat. In fact, the Ricci scalar approaches the non-vanishing asymptotic value $3/(2G^2m^2N^2) \sim 1/(j^2\ell_p^2)$. Some of the other curvature invariants, such as the Ricci squared $R_{ab}R^{ab}$ and the Kretschmann scalar remain non-vanishing but bounded everywhere, while the Weyl squared $C^{abcd}C_{abcd}$ vanishes asymptotically. Interestingly enough, the upper bounds as well as the asymptotic values of the aforementioned curvature scalars are all independent of the black hole mass $m$ and only carry information about the quantum structure. It follows without much difficulty from the asymptotic relations (13) that the interior metric is in fact geodesically complete. The null energy condition is violated for $\tau \lesssim \tau_*$.  

Quantum parameters as Dirac observables. A second possibility is to choose the coherent states such that $\epsilon$ and $\epsilon_\tau$ are constants of motion, as would be the case if Eq. (8) is evaluated at a given (judiciously chosen) instant of time. In this case, the dynamics simplifies further. Before making a specific choice, let us mention a few results that follow in this analysis. Firstly, it can be verified that
\[
\mathcal{M} = \frac{R^2 \sin \left( \frac{\gamma G \xi}{h^2} [P_R R - P_A \Lambda] \right)}{\gamma G \xi}
\]  

(14)

is a Dirac observable, namely \( \{ \mathcal{M}, H_{\text{eff}} \} \approx 0 \) (where \( \approx \) denotes evaluation on the constraint surface), that reduces to the classical gravitational mass \( m \) in the limit \( h \to 0 \).

Moreover, in the case of constant \( \epsilon \)'s, the convenient choice (12) for the lapse function (where we can now replace \( m \) with \( \mathcal{M} \)) allows for the explicit solution

\[
R(\tau) = 2GM e^{\frac{3}{2} \pi \epsilon \tau} \sqrt{1 + \frac{\tau^2 \pi^2}{64G^2M^2} e^{-\frac{3}{2} \pi \tau} + 1}.
\]

(15)

In the limit \( h \to 0 \), we recover the classical metric function given in Eq. (3). It is clear from Eq. (15) that the aerial coordinate reaches a minimum value

\[
R_{\text{min}}(\tau_b) = \sqrt{\gamma \epsilon \pi GM}
\]

(16)

at the moment \( \tau_b = GM \log \left( \gamma \epsilon / 8GM \right) \). This is to say that the area of the concentric 2-spheres that foliate constant \( \tau \) surfaces never shrinks to zero, evidencing how the quantum corrections eliminate the classical singularity and replace it with a cosmological bounce at \( \tau = \tau_b \).

Let us now provide a physical argument to fix the quantum parameters to be constants along the dynamical trajectories. A natural choice would be to equate \( R_{\text{min}}(\tau_b) \) with the classical aerial radius when the curvature is Planckian, namely

\[
R_{\text{min}}(\tau_b) = R_{\epsilon}(\tau_b).
\]

(17)

This immediately yields

\[
\epsilon = \frac{3^{1/3} 2^{4/3} \ell_p^{4/3}}{\gamma (GM)^{1/3}},
\]

(18)

where we have implicitly assumed that the effective geometry approaches the classical one in proximity of the black hole event horizon, \( i.e. \mathcal{M} \) is the Schwarzschild gravitational mass. This is correct so long as the space-time curvature is sub-Planckian near the event horizon, or equivalently as long as \( \mathcal{M} \gg m_p \). We can then fix \( \epsilon \) by demanding that at the bouncing time \( j_0 = 1/2 \), namely

\[
\epsilon = \frac{4\pi \gamma \ell_p^2}{\ell_p \gamma \epsilon \pi G M} = \frac{2^{1/3} \sqrt{\pi \gamma \ell_p^{1/3}}}{3^{1/6} (GM)^{1/3}},
\]

(19)

where we used Eq. (8).

Numerical investigations of Eq. (5) show that all commonly used spacetime curvature invariants, such as the Kretschmann scalar, remain bounded for all times \( -\infty < \tau < 0 \), solidifying the resolution of black hole singularity in this model. Moreover, we numerically confirm a feature found in previous literature that suggests that the upper bounds for these curvature invariants are mass independent as long as \( \mathcal{M} \gg m_p \) (see, e.g., [13]).

It would be important to gain more insight on the quantum-corrected interior geometry. It turn out to be difficult to find analytical expressions for all of the metric functions. Nevertheless, the following asymptotic estimates that are valid in the limit \( \tau \to -\infty \) can be obtained:

\[
N(\tau) \sim \frac{\epsilon R}{GM} e^{-\tau/2GM},
\]

\[
\Lambda(\tau) \sim \frac{\epsilon G}{\\xi} e^{-\tau/2GM},
\]

\[
R(\tau) \sim \epsilon x e^{-\tau/2GM}.
\]

(20)

Here

\[
\mathcal{L}(\epsilon) = \frac{2\gamma^2 \ell_p^2 [2 + \pi H_{\text{eff}} \rho (\pi)]}{\pi^2 H_{\text{eff}}^2 (\pi)} + \mathcal{O}(\epsilon^3),
\]

(21)

which is numerically found to be greater than zero for \( \epsilon \ll 1 \), but significantly suppressed compared to unity. This asymptotic behavior of \( \Lambda \) is confirmed by the numerical solution, as shown in FIG. 2.

\begin{itemize}
  \item[(a)] A plot of the effective metric functions \( \Lambda \) (blue line) in comparison with the classical metric function \( \Lambda_c \) (red line).
  \item[(b)] A closer view of the effective metric function \( \Lambda \) after the bounce occurred at \( \tau_b \sim -3.763 \times 10^{13} \).
\end{itemize}

FIG. 2. Metric function \( \Lambda \) for \( m = 10^{12} m_p \) in the second regularization scheme.

As in the previous case, the interior metric is geodesically complete but not asymptotically flat. However, now the components of the Riemann tensor computed in an orthonormal frame vanish asymptotically. Moreover, the weak, strong and dominant energy conditions are violated only around the bounce \( \tau \sim \tau_b \sim \tau_\ast \).
Discussion of results and comparison with existing literature. We have shown how the black hole effective dynamics derived for the first time from the full LQG framework resolves the classical singularity. In fact, in both choices of coherent states considered here, the effective metric function \( R \) has a similar behavior; it follows the classical trajectory till the Planckian curvature regime is reached, where quantum geometry effects become dominant and generate a bounce after which an exponential expansion follows. All curvature scalars are finite at any time, with mass-independent upper bounds.

The two different choices of quantum parameters result in different predictions for several aspects of the asymptotic post-bounce effective geometry, as described in the main body of the letter. In the first case where the quantum parameters are chosen to be phase space functions, \( \Lambda \) ceases to change appreciably around the bouncing time and reaches a constant asymptotic value. In the second case where we fix the quantum parameters to be Dirac observables, \( \Lambda \) reaches a maximum around the bouncing time, then decreases until a minimum value greater than zero is reached, and then grows very slowly as \( \tau \to -\infty \). However, in both cases \( \Lambda \) never vanishes at any time \( \tau < 0 \), implying that no white hole horizon forms after the bounce.

Since similar versions of our regularization schemes have been previously applied in the LQC minisuperspace quantization literature, a comparison is merited to highlight the key role played by the graph structure corrections that are accounted for in our effective Hamiltonian (7). We focus on the two references which are closer to our choices of quantum parameters. The first investigation where the \( \bar{\mu} \)-scheme is applied to the Schwarzschild interior was carried out in [6]. There the effective metric was shown to be a Nariai type Universe in the asymptotic limit, with the effective \( R \) undergoing damped oscillations until a fixed finite value was reached as \( \tau \to -\infty \), while \( \Lambda \) blew up exponentially in the same limit. Aside from obtaining distinct qualitative behavior for the metric functions in the asymptotic limit, our effective description circumvents the physically undesirable large quantum effects near the classical event horizon that appeared in the \( \bar{\mu} \)-scheme of [6], and more generally in the polymer quantization approach. Let us elaborate further on this important point.

In order to quantify the quantum geometry corrections, we introduce the ratio

\[
\delta := (K/K_c)^{1/4}
\]  

(22)

of the effective over the classical Kretschmann scalar and analyze where significant deviations from 1 appear. In FIG. 3 we plot this ratio as a function of the parameter \( \rho \) defined previously. For both prescriptions we see no large quantum effects, with the effective geometry deviating significantly from the classical one only in the regime where curvature becomes Planckian, namely as \( \rho \sim 1 \).

The other previous investigation relevant for a comparison with our second regularization prescription in which the quantum parameters are fixed to be constants along the effective dynamical trajectories is [13]. In both cases, the two quantum parameters acquire a dependence on the Dirac observable related to the gravitational mass as a power of \(-1/3\) (see Eqs. (18), (19)). By omitting the quantum corrections coming from the 2-sphere graph structure, due to using point holonomies for the angular directions, the effective dynamics of [13] predicts the appearance of a white hole horizon at a finite instant of time \( \tau \). As emphasized in the beginning of this letter, the effective dynamics derived in [19] accounts for precisely these ignored degrees of freedom that are present in the full theory. In the expression of the effective Hamiltonian (7), the quantum effects associated with the discrete graph structure on the 2-spheres are encoded in the appearance of the Struve function of zeroth order, instead of a Sine function. This modification has drastic implications for predictions of the theory. In particular, it prevents the formation of a white hole horizon at any time.

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