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PACS numbers: Valid PACS appear here

Introduction

Dark matter (DM) is one of the hot topics in fundamental physics and continues to question the scientific community concerning its origin and composition. DM leaves indirect evidence of its existence through its gravitational interaction with our Universe. However, no direct detection has yet been made [1], leading to the development of a multitude of experiments probing different models covering a large mass range [1, 2].

The search for ultra-light DM has recently seen a strong surge thanks to the excellent sensitivities provided by the latest advances in time/frequency metrology [3–15]. In this model, DM is a scalar-field (SF) non-universally coupled to the Standard Model (SM) fields. This SF can induce a violation of the equivalence principle through e.g. an oscillation of the fundamental constants, making it potentially detectable by a variety of experiments as reviewed in e.g. [10, 16].

Ultra-light scalar field oscillation

The theory of ultra-light SF has been developed in, e.g. refs. [4, 5, 17]. Within this framework, a SF $\varphi(t, x)$ is linearly coupled to the SM Lagrangian, which creates a variation of any SM fundamental constant $X(\varphi)$ from its value $X_0$ in the absence of the SF. The coupling constants $d_X$ parametrize the variation of the constants of Nature [17] such that $X(\varphi) = X_0 (1 + d_X \varphi)$ where $X$ can be the fine structure constant $\alpha$, the mass of fermions $m_f$, $m_d$, $m_u$, $m_s$, $m_{\nu}$, $m_\ell$ (electron, quarks) and the QCD mass scale $\Lambda_{QCD}, \Lambda_g$.

In this paper, we will focus on low masses $(m_\varphi \ll eV)$ for which $\varphi$ can be treated classically. Oscillating solutions $\varphi(t, x) = \varphi_0 \cos(\omega_m t)$ are natural solutions for the scalar field $[4, 5, 7]$. The amplitude of the oscillations is directly proportional to the local DM density $\rho_{DM}$ (0.4 GeV/cm$^3$ [18]) and $\omega_m$ is the Compton-De Broglie frequency of the SF.

Variations of the fine structure constant and/or electron mass could result in a variation of atomic transition frequencies and of the Bohr radius $a_0 = \hbar/(m_e c \alpha)$, which in turn leads to variation of the frequency of atomic clocks and the length of solids. Most experimental work in the search for “DM-oscillations” has explored the very low mass region $(\leq 10^{-14}\sim eV)$ [6–8, 11] owing to the fact that atomic clocks have typical measurement rates of no more than about 1 Hz. The development of new experiments is therefore necessary to cover higher masses. First steps in that direction were recently reported in [19, 20]. We present a new experimental approach that improves on those results by several orders of magnitude.

In addition, searches for a Yukawa-like violation of the universality of free fall also provide constraints on the couplings between matter and the SF, see e.g. [10, 21, 22]. Those constraints are independent of the identification of the SF as DM.

Stochastic fluctuations of bosonic DM

In standard models of galaxy formation, galactic DM must be virialized [23, 24] and has a velocity distribution $f_{DM}(v)$ with a characteristic width $\sigma_v \sim 10^{-3} c$ [25–27]. The Compton frequency of the SF, $\omega_m \sim m_\varphi c^2/\hbar (1 + v^2/(2c^2))$, is “Doppler broadened” because of the DM velocity distribution. This broadening introduces a coherence time $\tau_c = (\omega_m \sigma_v^2/c^2)^{-1} \sim 10^6 \omega_m^{-1}$ [28]. The DM distribution therefore implies that the scalar

Search for DArk Matter with a Non-Equal Delay interferometer: The DAMNED experiment

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(Dated: June 15, 2020)

The theory of oscillations

ϕ

the SM Lagrangian, which creates a variation of any SM

constant

X

fundamental constant

the SM

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field has a stochastic component from the sum of all the SF allowed by the velocity distribution. The effective field takes the following form [26, 29]:
\[
\varphi(t) = \sum_{j=1}^{N_j} \alpha_j \sqrt{f_{\text{DM}}(f_j) \Delta f \cos(2\pi f_j t + \phi_j)},
\]
where \(\alpha_j\) are stochastic amplitudes following a Rayleigh distribution [26, 29], \(\phi_j\) are random phases following a uniform distribution and \(f_{\text{DM}}(f)\) is the DM velocity distribution expressed in the frequency domain (see supplemental material and [25]). \(N_j\) defines the number of points used to discretize the DM frequency distribution curve \((N_j \Delta f \geq 1/\tau_c)\), where \(\Delta f\) is the frequency resolution of the data. When the experimental duration \(T_{\text{exp}}\) is longer than \(\tau_c\), this stochastic broadening needs to be taken into account in the data analysis [25, 26] (see supplemental material for details). The reference beatnote between the AOM arms provides the putative DM signal resulting from the DM effect.

**Experimental setup** Our experimental setup, dubbed the DAMNED (DArk Matter from Non Equal Delays) experiment is a three-arm Mach-Zender interferometer as shown in Fig. 1. A 1542nm laser source is locked to an ultra-stable cavity. The beam is then split between three arms and recombined to have access to the DM signal (long vs AOM arms) and the experimental reference (short vs AOM arms).

The fibre delay is given by \(T = L/f_\text{cavity}\) and can reach \(\approx 10^9\) s/c where \(f_\text{cavity}\approx 50\) MHz signal) at a sampling rate of 500 kHz.

**DM effect** The cavity frequency \(\omega(t)\) will oscillate following oscillations of the cavity length induced by oscillations of the Bohr radius due to the evolution of the SF. Similarly, the fibre delay \(T(t)\) will oscillate because of oscillations of its length and of its refraction index that are both induced by oscillations of the constants of Nature related to the evolution of \(\varphi\).

The frequency variation of the light exiting the cavity \(\delta \omega(t)\) is inversely proportional to its length variation
\[
\frac{\delta \omega(t)}{\omega_0} = \epsilon_L (\mathcal{E}_c(1 + \alpha) \cos(\omega_n t) + \mathcal{E}_s \beta \sin(\omega_m t)),
\]
where \(\omega_0\) is the unperturbed frequency, \(\epsilon_L\) is the fractional length change due to variation of the Bohr radius and \(\alpha, \beta, \mathcal{E}_c, \mathcal{E}_s\) are coefficients related to the mechanical resonances of the cavity and its finesse \(F\) reflecting the multiple passes of the light in the cavity. For our high finesse cavity \((F \approx 800000)\) and frequencies of interest \((f \in [10, 200])\) kHz we have \(\mathcal{E}_c, \mathcal{E}_s \approx 1\). For our \(\approx 0.1\) m ULE cavity the resonant frequencies are \(\omega_n \approx 2\pi n 27.6\) kHz where \(n\) is an integer \((n \geq 1)\), and are therefore well within our frequency region of interest \((10, 200)\) kHz. Only odd resonances are excited due to the symmetry of the length change. At resonance \((\omega_m = \omega_1)\), \(\alpha \approx 0\) and \(\beta = 8Q_1/\pi^2\), with the quality factor of our ULE cavity \(Q_1 \approx 6.1 \times 10^4\) and we therefore expect a significant enhancement of the signal. Below resonance \((\omega_m < \omega_1)\) both \(\beta, \alpha \approx 0\). A detailed derivation of the coefficients \(\alpha, \beta, \mathcal{E}_c, \mathcal{E}_s\) is provided in the supplemental material. Similar analysis was also carried out in [35, 36] giving similar results.

The fibre delay is given by \(T(t) = L_f(t)n(t)/c\), where \(L_f(t)\) and \(n(t)\) are the fibre length and refractive index respectively, which may both vary with the SF. Using the
approach described in [37, 38] we find,
\[
\frac{\delta T(t)}{T_0} = \frac{\omega_0}{n_0} \frac{\partial n}{\partial \omega} \left( \frac{\delta \omega(t)}{\omega_0} - \epsilon_n \cos(\omega_m t) - \epsilon_L \cos(\omega_m T_0) \right),
\]
where \( \delta \omega(t)/\omega_0 \) is the relative frequency variation at the entrance of the fibre, which in our case is given by Eq. (2) and \( \epsilon_n \) is the fractional refractive index change. For the telecom fibres that we use the pre-factor of (3) is typically \( \approx 10^{-2} \).

Both the cavity frequency and fibre delay oscillations can be integrated to obtain the phase difference \( \Delta \Phi(t) \) between the delayed and non-delayed signals:
\[
\Delta \Phi(t) = 2 \frac{\omega_0}{\omega_m} \sin \left( \frac{\omega_m T_0}{2} \right) \left[ C_{\Delta \Phi} \cos \left( \frac{\omega_m t - \omega_m T_0}{2} \right) \right.
\]
\[
+ \left. S_{\Delta \Phi} \sin \left( \omega_m t - \omega_m T_0 \right) \right].
\]
(4)
where \( C_{\Delta \Phi} \) and \( S_{\Delta \Phi} \) are derived from (2) and (3), to leading order:
\[
C_{\Delta \Phi} \simeq \epsilon_L \alpha - \epsilon_n \frac{\omega_0}{n_0} \frac{\partial n}{\partial \omega}; \quad S_{\Delta \Phi} \simeq \epsilon_L \beta.
\]
(5)
The DM coupling constants \( \epsilon_L \) and \( \epsilon_n \) are defined as
\[
\epsilon_n \equiv \varphi_0 (2d_c + d_m_s + (d_m_s - d_g)/2 - 0.024(d_m_s - d_g)),
\]
\[
\epsilon_L \equiv \varphi_0 (d_c + d_m_s),
\]
(6)
where \( \varphi_0 \) is the amplitude of the SF oscillation. The dependence on \( d_g \) and \( d_m_s \) arises from the phonon frequencies in the fibre that determine its refractive index [37, 38].

Although (4) has extinctions for \( \omega_m T_0 = n2\pi \), the use of two different fibre lengths (thus different delays \( T_0 \)) allows to recover sensitivity over the whole desired frequency range. For the reference arm \( T_0 \simeq 0 \), and the signal of (4) vanishes, which allows its use to characterise systematic effects and identify false DM signals.

**Experimental results** The parallel acquisitions of the signal and reference phase data lasted 12 days each for the two different fibre lengths (52.64 and 56.09 km) at a 500 kHz sampling rate. The total raw phase data (~ 4 x 2.1 TB) requires digital pre-processing to compute Fourier transforms with a spectral resolution limited to ~ 3 mHz 1. Figure 2 shows the power spectral density (PSD) computed over the full 12 days duration of the experiment. Only the “signal” branch for the 52 km acquisition is shown here, but all results are similar for the 56 km fibre, as well as for the “reference” branch.

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1 RAM limitation reduces the maximum amount of data duration that can be loaded to ~268 s. The maximum spectral resolution is therefore \( 1/T_{RAM} > 1/T_{exp} \).
profile with a typical width of $\sim$ Hz. This is much too large for a putative DM signal whose width should be $\sim 10^{-6} f_{m} \leq 0.02$ Hz. Furthermore, the mean frequency of the peaks drifts coherently over the 12 days which allows us to assume that their physical origin is the same. The other cavities available in the laboratory [32] do not show peaks at the same locations even when the acquisition is performed in parallel with our main source for the experiment. The absence of common peaks between the cavities, expected if the signal is induced by DM, allows us to deduce that these peaks are due to systematics. Although the exact technical origin of the peaks are unknown, we are convinced of their common physical origin and they are likely coming from the laser/cavity. The profile of the $\sim 101$ and $\sim 103$ kHz peaks is also Gaussian with a width of $\sim 10$ Hz and the evolution of the peak position is correlated with the room temperature. In this frequency range, the temperature dependence and the presence of “double” peaks strongly suggest that the signal is due to resonances in the piezoelectric block used to control the laser frequency. Therefore, the set of peaks visible on the PSD are all systematic effects that either have a spectral shape and width incompatible with the DM signal, and are present only in the cavity used for the experiment and not in the other cavities available in the laboratory, or are also present in the “reference” branch which is insensitive to coupling with DM. Therefore, we report no detection of ultra-light DM in the frequency range $[10, 200]$ kHz. However, that conclusion does not apply at the frequencies of the systematic peaks, which might mask a putative DM signal, and we thus exclude those frequency regions from our results, as summarised in tab. I. All the peak positions are drifting relatively to their mean value $\langle f \rangle$ by a factor $r_f$. With the peak widths $\sigma_f$, we define a conservative exclusion interval $\{\langle f \rangle \times (1 - r_f) - 3\sigma_f, \langle f \rangle \times (1 + r_f) + 3\sigma_f\}$.

| Origin       | $(f)_{/Hz} r_f \sigma_f_{/Hz}$ | $(f)_{/Hz}$ | $(f)_{/Hz}$ |
|--------------|---------------------------------|-------------|-------------|
| Unknown      | $26178 \ 10^{-4}$              | 1           | 26172.382, 26183.618 |
|              | $50069 \ 10^{-4}$              | 1           | 50060.993, 50077.007 |
|              | $59364 \ 10^{-4}$              | 1           | 59355.064, 59372.936 |
| Piezo        | $101684 \ 10^{-3}$             | 3           | 101573.316, 101794.684 |
|              | $103525 \ 10^{-3}$             | 3           | 103412.975, 103638.025 |
| Eittus       | Multiples of 7629.395 Hz, $\sigma_f = 3$ mHz |

TABLE I. Excluded frequency regions due to systematic effects.

**Constraints** In order to constrain the DM model, the coupling constants must be extracted from the coefficients $C_{\Delta \Phi}$ and $S_{\Delta \Phi}$ available in the Fourier transform of our data. The stochastic nature of the signal (1) requires the adjustment of the following parameters: the linear combination of the coupling constants defined in equations (6), $N_j$ amplitudes $\alpha_j$ and $N_j$ phases $\phi_j$, where $N_j$ is chosen to sufficiently sample the DM frequency distribution $f_{DM}(f)$. The a priori knowledge of the probability distribution of amplitudes (Rayleigh distribution) and phases (uniform distribution) favours the use of a Bayesian approach. Working in the frequency domain the corresponding posterior distributions can be analytically marginalised over the $N_j$ amplitudes $\alpha_j$ and phases $\phi_j$, which makes the problem numerically solvable (see [25, 26, 29] and the supplemental material). The result is a posterior probability distribution for the coupling constants that appear in Eqs. (6) for each DM mass, providing the corresponding 95% upper limit. To simplify, we concentrate on $d_e$ and $d_{m_e}$ and assume that only one of them is non-zero in turn, a so called “maximum reach approach”. We use our acquisitions with two different fibre lengths (52.96 and 56.09 km) and combine both likelihoods to infer a unique upper limit at 95% confidence. These upper limits for the galactic DM model as well as the Earth relaxation halo one are presented in Fig. 3. The latter is a model proposed in [30], in which DM may form gravitationally bound objects with the possibility that such objects are formed in gravitational fields of the Sun or the Earth. The corresponding so called “relaxation halo” of DM around the Earth then has a peculiar DM density and velocity distribution very different from that of the galactic DM model.

The constraints show large “peaks” at the resonant frequencies ($n = 1, 3, 5, 7$) of our cavity, and at frequencies where the combination of two different fibre lengths does not fully solve the loss of sensitivity due to the $\sin(\omega_m T_0/2)$ term in (4). In between the peaks the constraints come from a combination of the length and index changes of the cavity and the fibres.

In the case of the galactic DM model (top graphs), our experiment exceeds best existing constraints on $d_{m_e}$ from torsion balance experiments [10, 21, 40] by about an order of magnitude, but only over a narrow-frequency band around the cavity resonance.

In the relaxation halo model from [30] (bottom graphs) our experiment improves on best existing constraints for almost all of the probed DM masses, by up to 5 orders of magnitude for $d_e$ and 6 orders of magnitude for $d_{m_e}$.

The underlying reason for the difference in sensitivity in the two models comes from the fact that experiments like ours or [20] depend on the local DM density while torsion balance experiments search for a Yukawa interaction between the Earth and the test masses, which is independent of the identification of the SF as DM [10] and are thus independent of the local DM density.

In all cases our experiment improves on the recent experiment reported in [20], which directly probes the same DM models as ours, by typically two orders of magnitude over the DM mass region where the two overlap.

\[^2\text{Note that in [20] the authors do not take the factor } \sim 10 \text{ sensi-}\]
**Conclusion**  The DAMNED experiment has not revealed any sign of scalar DM for masses in the $[4.1 \times 10^{-11}, 8.3 \times 10^{-10}]$ eV region, but we have improved existing bounds on the DM-SM coupling constants by up to 6 orders of magnitude, depending on the considered mass and DM distribution model.

Our main limitation is the cavity noise, and we plan to improve on the results presented here over the next years, and also test other models (e.g. axion couplings), using similar set-ups but with an improved optical cavity currently under construction.

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tivity loss pointed out in [29] into account, contrary to our work (see supplemental material).
In this appendix we model the resonant cavity in the presence of a temporal oscillation of the fundamental constants. As shown in [35, 36] using a simple "mass-spring" model, the effect of the Bohr radius change is a "driving" force of the harmonic oscillator whose equation of motion is then

\[ \ddot{D}(t) + \frac{\omega_r}{Q_0} \dot{D}(t) + \omega_r^2 D(t) = -\epsilon_L L_0 \omega_m^2 \cos(\omega_m t), \]  

(7)

where \( \omega_r \) is the resonant frequency and \( Q_0 \) its quality factor. We define the displacement \( D(t) \equiv L(t) - L_{eq}(t) \) where \( L(t) \) is the cavity length and \( L_{eq}(t) \equiv L_0(1 - \epsilon_L \cos(\omega_m t)) \) the equilibrium length. It is deviations with respect to \( L_{eq}(t) \) that give rise to restoring and damping forces.

The simple mass-spring model can be generalised to an elastic solid cavity using the standard methods described in e.g. [42, 43]. The harmonic oscillator (7) becomes a wave equation for the function \( D(t, x) \) representing the displacement with respect to the (time varying) equilibrium position of any segment at position \( x \) (we choose \( x = 0 \) at the cavity centre):

\[ \ddot{D}(t, x) - \frac{\partial^2}{\partial x^2} \left( \kappa D(t, x) + \gamma \dot{D}(t, x) \right) = -x \epsilon_L \omega_m^2 \cos(\omega_m t), \]  

(8)

where \( \kappa, \gamma \) are material dependent constants.

Boundary conditions (free ends) impose the spatial modes \( u^{(n)}(x) \) of form

\[ u^{(n)}(x) = \sqrt{\frac{2}{L_0}} \cos \left( \frac{n \pi}{L_0} \left( x + \frac{L_0}{2} \right) \right), \]  

(9)

where \( n \) is an integer. The steady state solution is then given by a superposition of those modes and can be written as \( D(t, x) = \sum_{n=1}^{\infty} D^{(n)}(t) u^{(n)}(x) \), where the \( D^{(n)}(t) \) must oscillate at \( \omega_m \) and satisfy

\[ \ddot{D}^{(n)}(t) + \frac{\omega_n}{Q_n} \dot{D}^{(n)}(t) + \omega_n^2 D^{(n)}(t) = -\epsilon_L \omega_m^2 \cos(\omega_m t) \int_{-L_0/2}^{L_0/2} x u^{(n)}(x) \, dx \]  

\[ = \tilde{b}^{(n)} \epsilon_L L_0 \omega_m^2 \cos(\omega_m t), \]  

(10)

where \( \omega_n = nv_{s}\pi/L_0 \approx n \times 173 \text{ kHz} \) (\( v_s \) is the phase velocity of longitudinal elastic waves in ULE), \( Q_n = \frac{K}{\omega_n} \) with \( K \) a constant and \( Q_1 \approx 6.1 \times 10^4 \). The factor \( \tilde{b}^{(n)} = \frac{2^{3/2} \sqrt{n}}{n \pi^{3/2}} \) for odd \( n \), and zero for even \( n \). So only modes with odd \( n \) are excited as one would expect from the symmetry of the driving force. Equations (10) have analytical solutions giving the final result

\[ L(t) = L_0 \left( 1 - \epsilon_L \left( (1 + \alpha) \cos(\omega_m t) + \beta \sin(\omega_m t) \right) \right), \]  

(11)

with

\[ \alpha = \sum_{i=1}^{\infty} \frac{8}{n^2 \pi^2} Q_n^2 \omega_m^2 \left( \omega_n^2 - \omega_m^2 \right) \]  

\[ \beta = \sum_{i=1}^{\infty} \frac{8}{n^2 \pi^2} Q_n^2 \omega_m^3 \]  

(12)

where \( n = 2i - 1 \). The sums in (12) can be evaluated with a limited number of terms, as for DAMNED we are interested in the frequency region up to about \( n = 7 \) and the contribution of higher resonances quickly decreases.

Below resonance (\( \omega_m < \omega_1 \)) both \( \beta, \alpha \approx 0 \) and the cavity length follows \( L_{eq}(t) \) and the Bohr radius change. Above resonance (\( \omega_m > \omega_1 \)) the coefficients converge to \( \beta = 0 \) and \( \alpha = -1 \), meaning the cavity can no longer follow the oscillations of the equilibrium length.

**SUPPLEMENTARY MATERIAL A : CAVITY RESONANCE**

The description of the resonant light field inside a Fabry-Perot cavity of oscillating length \( L(t) = L_0 \cos(\omega_m t) \) has been treated extensively in the context of gravitational wave detectors like LIGO, Virgo, and more recently MIGA and described in detail in e.g. [42, 43]. Those analyses apply directly to our cavity and we only recall the main results, for details the reader is referred to the original papers.
We follow in particular the analysis in annex A of [43], starting from equ. (35) of [43], which gives the phase variation of the resonant light field exiting a cavity whose length is varying as $L(t) = \zeta_c L_0 \cos(\omega_m t)$ (with $\zeta_c \ll 1$),

$$\phi(t) \simeq \frac{2\zeta_c L_0 \omega_0 r^2}{c (r^4 - 2r^2 \cos(2\nu) + 1)} \left( (r^2 - 1) \cos(\nu) \cos(\omega_m t) - (r^2 + 1) \sin(\nu) \sin(\omega_m t) \right),$$

(13)

where $r$ is the reflection coefficient of the cavity mirrors and $\nu \equiv \omega_m L_0 / c$. For our cavity with finesse $F \approx 800000$ [31] we have $1 - r^2 \approx 4 \times 10^{-6}$ ($\frac{r^2}{1 - r^2} \approx F / \pi$) and $\nu \approx [2, 40] \times 10^{-5}$ for our frequency range of $[10, 200]$ kHz, so we will neglect the first term in (13).

The fractional frequency variation ($\delta \omega(t)/\omega_0 = \dot{\phi}(t)/\omega_0$) is given by

$$\frac{\delta \omega(t)}{\omega_0} = -2\zeta_c \nu r^2 (1 + r^2) \sin(\nu) \cos(\omega_m t).$$

(14)

The result for $L(t) = \zeta_s L_0 \sin(\omega_m t)$ is simply obtained from (14) by shifting $\omega_m t \rightarrow \omega_m t - \pi/2$ i.e. replacing $\cos(\omega_m t) \rightarrow \sin(\omega_m t)$ and $\zeta_c \rightarrow \zeta_s$.

We identify $\zeta_c = -\epsilon_L (1 + \alpha)$ and $\zeta_s = -\epsilon_L \beta$, and comparing to (2) we finally obtain

$$\mathcal{E}_c = \mathcal{E}_s = \frac{2 \nu r^2 (1 + r^2) \sin(\nu)}{r^4 - 2r^2 \cos(2\nu) + 1}.$$  

(15)

Evaluating (15) for our cavity and frequency range we have $\mathcal{E}_c, \mathcal{E}_s \in [0.991, 0.99998]$ i.e. $\approx 1$.

SUPPLEMENTARY MATERIAL C : CAVITY NOISE FLOOR

Although the cavity was characterized in detail in [32], the noise floor changed due to an air conditioning failure in the lab. Modelling the cavity noise floor is necessary to estimate the constraints on the parameters since the signal-to-noise ratio has to be constructed for Bayesian analysis.

The unequal-arm interferometer allows us to compare the signal from the cavity to itself after a delay corresponding to the propagation time $T$ of a photon in the fibre spools. The cavity noise $\Phi_{\text{cav}}$ creates an interferometer phase noise $\Delta \Phi_{\text{cav}}(t) = \Phi_{\text{cav}}(t) - \Phi_{\text{cav}}(t - T)$. The cavity noise PSD $S_{\Delta \Phi_{\text{cav}}}$ can be linked to the interferometric cavity-noise floor $S_{\Delta \Phi_{\text{cav}}}$:

$$S_{\Delta \Phi_{\text{cav}}}(f) = 4 \sin^2 \left( \pi f T \right) S_{\Phi_{\text{cav}}}(f).$$

(16)

The $4 \sin^2(\pi f T)$ transfer function is responsible for the extinctions of the PSD when $f = n/T$ as seen in figure 2. In order to obtain the full PSD, we had to split the dataset in $\sim 268$ s-long subsets due to RAM-limitation. In doing so, we calculate the PSD for each subset of data and ensure that the cavity noise level has not changed over the duration of the acquisition. Thanks to the stationarity of the noise, we can average these PSDs to model the characteristic noise floor of the cavity. This averaged PSD is shown in orange in figure 4. The peaks are the systematic effects discussed in the main section. The model fitted to the averaged PSD is shown in black and incorporates only broad trends so as not to adjust for potential DM traces or systematic effects.

SUPPLEMENTARY MATERIAL D : DATA ANALYSIS

We describe the data analysis used, based on [25, 26, 29].
FIG. 4. PSD of phase fluctuations $S_{\Delta \Phi}(f)$ over the 12 days acquisition computed using small segment of 268s in blue. The orange line represents the average over all the 268s-segment PSD which is used to fit the cavity noise-floor model shown as a black line. The bottom plot shows full and mean PSDs normalized by the model.

**Dark Matter velocity and frequency distribution**

We assume that the DM velocity distribution has a Gaussian profile characterized by a central velocity $v_{\text{obs}}$, the galactic velocity of the Solar System, and by a virial velocity $\sigma_v$.

$$f_{\text{DM}}(v) = \frac{1}{(2\pi \sigma_v^2)^{3/2}} e^{-\frac{(v-v_{\text{obs}})^2}{2\sigma_v^2}}. \quad (17)$$

This leads to a distribution in term of velocity amplitude $v$ given by

$$f_{\text{DM}}(v) = \sqrt{\frac{2}{\pi}} \frac{v}{v_{\text{obs}} \sigma_v} e^{-\frac{v^2 + v_{\text{obs}}^2}{2\sigma_v^2}} \sinh \left( \frac{v v_{\text{obs}}}{\sigma_v^2} \right). \quad (18)$$

Typical values for the two velocity parameters are: $v_{\text{obs}} \sim 230 \text{ km/s}$ and $\sigma_v \sim 150 \text{ km/s}$ [26]. The top panel of Fig. 5 shows this velocity distribution.

Using the fact that the Compton frequency for the scalar field is related to the DM velocity through

$$f = \frac{m \phi c^2}{h} \left( 1 + \frac{v^2}{2c^2} \right), \quad (19)$$

the DM velocity distribution can be transformed into a frequency distribution [25, 26]

$$f_{\text{DM}}(f) = \frac{h}{m \phi c^2} \sqrt{\frac{2}{\pi}} \frac{c^2}{v_{\text{obs}} \sigma_v} e^{-\frac{c^2(f-1) + 2 \pi v_{\text{obs}}^2}{2\pi}} \sinh \left( \frac{c v_{\text{obs}}}{\sigma_v^2} \sqrt{2(f-1)} \right), \quad (20)$$

where $\bar{f}$ is a dimensionless frequency defined as

$$\bar{f} = \frac{f h}{m \phi c^2}. \quad (21)$$

Fig. 5 presents the shape of both velocity and frequency DM distribution. In particular, it is interesting that the shape of the frequency distribution is highly asymmetric because of the dispersion relation from Eq. (20) and has a lower cut-off at the frequency $m \phi c^2/h$. This feature is particulary interesting to identify DM in the power spectrum of an experiment.

---

3 Most of this section is presented within the standard galactic DM model, but its application to the Earth relaxation halo model is straightforward as discussed in the last subsection.
FIG. 5. Top: DM velocity distribution from Eq. (18). Bottom: DM frequency distribution from Eq. (20). The green filled area has a width of 3 FWHM and its range is given by Eq. (25) with $a = 3$. This is the frequency domain over which the scalar field is modeled in Eq. (22).

Modeling of the scalar field

In general, for a given scalar field mass $m_{\phi}$, the scalar field can be written as [26]

$$\varphi(t) = \sqrt{\frac{4\pi G_{\text{DM}}}{m_{\phi} c^2}} \sum_{j=1}^{N_i} \alpha_j \sqrt{f_{\text{DM}}(f_j) \Delta f} \cos \left[ \omega_j t + \phi_j \right], \quad (22)$$

where

$$\omega_j = 2\pi f_j = \frac{m_{\phi} c^2}{\hbar} \left( 1 + \frac{v_j^2}{2c^2} \right). \quad (23)$$

The amplitudes $\alpha_j$ follow a Rayleigh distribution [26] while the phases $\phi_j$ are uniformly distributed, i.e.

$$P[\phi_j] = \frac{1}{2\pi} \quad \text{for} \quad 0 \leq \phi_j \leq 2\pi, \quad (24a)$$

$$P[\alpha_j] = \alpha_j e^{-\alpha_j^2/2}. \quad (24b)$$

The number of terms involved in the sum depends on the frequency resolution of the experiment $\Delta f = \frac{1}{T_{\text{exp}}}$ and of the typical width of the frequency distribution. As can be noticed from Fig. 5, the full width half max (FWHM) of the frequency distribution, a good estimator of its width, is given by $\sim 10^{-6} m_{\phi} c^2/\hbar$. In practice, we use a sampling of the DM frequency distribution that covers a FWHM starting at the cut-off frequency. In other words, the frequencies $f_j$ included in the sum from Eq. (22) are the Fourier frequencies (i.e. $f_j = j \Delta f = j f_s/N$ with $f_s$ the sampling frequency and $N$ the number of measurements) contained in the range

$$\left[ \frac{m_{\phi} c^2}{\hbar}, \frac{m_{\phi} c^2}{\hbar} (1 + a \times 10^{-6}) \right], \quad (25)$$

where in practice we use $a = 3$. The frequency region covered by this sampling is indicated by the green shaded area in Fig. 5.
The energy density for a scalar field is given by
\[ \rho_{\varphi} = \frac{c^2}{8\pi G} \left[ \dot{\varphi}^2 + \frac{c^4 m^2}{\hbar^2} \varphi^2 \right]. \]  
(26)

For the scalar field from Eq. (22), this quantity is a stochastic quantity. We can perform an ensemble average of the energy density for the scalar field using the distribution from Eqs. (24) to demonstrate that average energy density for the scalar field is the local DM energy density, i.e. \( \langle \rho_{\varphi} \rangle = \rho_{\text{DM}} \).

**Modeling of the phase measurements**

Eq. (4) from the main part of the paper gives the relationship between the phase measurement and the scalar field. If we take into account the fact that the scalar field has several frequencies (see Eq. (22)) and taking into account only the contribution from \( d_e \) and \( d_{me} \), the phase measurements are modeled as
\[ \Delta \Phi(t) = \omega_0 T_0 + \sum_j \alpha_j \left( d_e \tilde{A}_j + d_{me} \bar{A}_j \right) \cos \left( \omega_j t + \phi_j + \tilde{\phi}_j \right), \]  
(27)

with
\[ \tilde{A}_j = \sqrt{\frac{4\pi G \rho_{\text{DM}}}{c^2}} \sqrt{f_{\text{DM}}(f_j) \Delta f} \sqrt{K_e(\omega_j) + L^2(\omega_j)}, \]  
(28a)
\[ \bar{A}_j = \sqrt{\frac{4\pi G \rho_{\text{DM}}}{c^2}} \sqrt{f_{\text{DM}}(f_j) \Delta f} \sqrt{K_{me}(\omega_j) + L^2(\omega_j)}, \]  
(28b)

where the functions \( K(\omega) \) and \( L(\omega) \) are obtained from (5) and (6):
\[ K_e(\omega) = 2\frac{\omega_0}{\omega} \sin \left( \frac{T_0}{2} \right) \left[ \alpha(\omega) - 2\frac{\omega_0}{n_0} \frac{\partial n}{\partial \omega} \right], \]  
(29a)
\[ K_{me}(\omega) = 2\frac{\omega_0}{\omega} \sin \left( \frac{T_0}{2} \right) \left[ \alpha(\omega) - 3\frac{\omega_0}{2} \frac{\partial n}{\partial \omega} \right], \]  
(29b)
\[ L(\omega) = 2\frac{\omega_0}{\omega} \sin \left( \frac{T_0}{2} \right) \beta(\omega), \]  
(29c)

and \( \tilde{\phi}_j \) are constants, such that \( \phi_j + \tilde{\phi}_j \) are also uniformly distributed.

In this analysis, we use a “maximum reach approach” which means that we are considering \( d_e \) and \( d_{me} \) independently in two independent analysis where we fix one of these parameters to 0. We can then write the signal that is used in our data analysis as
\[ s(t, \gamma, \{ \alpha_j \}, \{ \varphi_j \}) = \omega_0 T_0 + \gamma \sum_{j=1}^{N_j} \alpha_j A_j \cos \left[ \omega_j t + \tilde{\phi}_j \right], \]  
(30)

where we use \( \gamma = d_e \) and \( A_j = \tilde{A}_j \) when we consider the coupling parameter to electromagnetism and \( \gamma = d_{me} \) with \( A_j = \bar{A}_j \) when we consider the coupling parameter to the electron mass.

**Fourier transform**

In order to infer the value of \( \gamma \), the linear combination of DM coupling constants, we choose to analyse the data using a Bayesian inference method on a discrete Fourier transform (DFT) of our full dataset. In this section, we briefly remind our notation for DFT and useful relations. We follow closely the Appendix of [25].

We have one set of \( N \) measurements sampled at the frequency \( f_s = \frac{1}{\Delta T} \) over a period \( T_{\text{exp}} = N/f_s \) with colored noise characterized by its PSD from Eq. (16), i.e. the dataset can be described as a set of \( \{(t_i, d_i)\} \) measurements characterized by a covariance matrix \( C_{ij} \).
For any time depend function $x(t)$, we will write the value of $x(t)$ at the $l$th sampling time by $x_l = x(l\Delta t)$ where $l \in \{0, 1, \ldots, N - 1\}$. A tilde quantity will denote the discrete Fourier Transform (DFT) of a quantity

$$
\tilde{x}_k = \sum_{l=0}^{N-1} e^{-2\pi i \frac{kl}{N}} x_l,
$$

where $k \in \{0, 1, \ldots, N - 1\}$. Note that $\tilde{x}_k$ corresponds to the frequency $f_k = kf_s/N$, it is periodic $\tilde{x}_{k+N} = \tilde{x}_k$ and if $x$ is a real signal, it is symmetric $\tilde{x}_{N-k} = \tilde{x}_k$ where a bar denotes the complex conjugate. In vectorial notation, on can write the last equation as

$$
\tilde{x} = \sqrt{N} U x,
$$

where we introduced the rotation matrix $U$ whose components are $U_{kl} = \exp(-2\pi i \frac{kl}{N})/\sqrt{N}$. This matrix $U$ is symmetric and unitary ($U \cdot U^\dagger = U^\dagger \cdot U = \delta$ with $\delta$ the identity).

Let us introduce the noise time series $\mathbf{n}$ which has a vanishing expectation $E[n_i] = 0$. The noise covariance matrix then is given by $C_{ij} = E[n_i n_j]$. A simple calculation shows that the covariance matrix of the DFT $\tilde{\mathbf{n}}$ is given by $\tilde{C} = NU \cdot C \cdot U^\dagger$ such that

$$
C^{-1} = NU^\dagger \cdot \tilde{C}^{-1} \cdot U.
$$

$\tilde{C}$ is known as the two-sided PSD matrix, which for a stationnary process is diagonal [25]. We can introduce the two-sided PSD by $\tilde{C}_{ij} = N f_s \delta_{ij} S_j$ or in other words

$$
S_j = S(f_j) = \frac{E[|\tilde{n}_j|^2]}{N f_s}.
$$

**Bayesian inference**

Working in the context of Bayesian inference, we will use a Gaussian likelihood (i.e. the probability to get the dataset given the model and some model parameters) with a colored noise, which writes

$$
L = P[\mathbf{d}|\gamma, \alpha, \phi] = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} \exp\left( -\frac{1}{2} (\mathbf{d} - \bar{s})^T \cdot C^{-1} \cdot (\mathbf{d} - \bar{s}) \right),
$$

where $\mathbf{d} = (d_0, d_1, \ldots, d_{N-1})$ is the vector of data and $\mathbf{s}(\mathbf{p}) = (s(t_0, \mathbf{p}), s(t_1, \mathbf{p}), \ldots, s(t_{N-1}, \mathbf{p}))$ is the model given by Eq. (30) while $\mathbf{C}$ is the noise covariance matrix. Note that the model depends on one coupling parameter $\gamma$ and on a set of amplitudes $\alpha$ and of phases $\phi_j$.

Using the DFT of the data and of the signal such as introduced in Eq. (32) as well as Eq. (33), the likelihood becomes

$$
L = \prod_j (2\pi S_j f_s)^{1/2} \exp\left( -\frac{1}{2} (\tilde{\mathbf{d}} - \tilde{\bar{s}})^T \cdot C^{-1} \cdot (\tilde{\mathbf{d}} - \tilde{\bar{s}}) \right) = \prod_j (2\pi S_j f_s)^{1/2} \exp\left( -\frac{1}{2} \sum_{k=0}^{N-1} \frac{\tilde{d}_k - \tilde{s}_k}{\tilde{C}_{kk}} \right)
$$

where $\beta_k = 1/2$ for $k = 0$, $\beta_k = 1/2$ for $k = N/2$ when $N$ is even and $\beta_k = 1$ otherwise. In our case, we will not consider the 0 and higher frequency in our analysis so $\beta_k = 1$ (but see the Appendix of [25] for a general case). In the end, the log-likelihood writes

$$
-\ln L(\mathbf{d}|\gamma, \alpha, \phi) = \sum_{k=0}^{\lfloor N/2 \rfloor} -\ln L_k = \sum_{k=0}^{\lfloor N/2 \rfloor} \tilde{\chi}_k^2 + \ln (S_k f_s),
$$

(37)
with

\[ \chi_k^2 = \frac{\left| \tilde{d}_k - \tilde{s}_k \right|^2}{C_{kk}} = \frac{\left| \tilde{d}_k - \tilde{s}_k \right|^2}{Nf_sS_k}. \]  

(38)

Since the signal is modeled as a sum of harmonic components at Fourier frequencies (see Eq. (30) where \( f_j \) have been chosen as \( f_j = \frac{k}{N} f_s \)), the DFT of the signal can easily be computed

\[ \tilde{s}_k = \frac{\gamma N A_k \alpha_k e^{i \phi_k}}{2} \quad \text{for} \quad k > 0. \]  

(39)

The \( \chi_k^2 \) that appears in the expression of the likelihood from Eq. (38) becomes

\[ \chi_k^2 = \frac{\left| \tilde{d}_k - \tilde{s}_k \right|^2}{Nf_sS_k} = \frac{\left| \tilde{d}_k \right|^2}{Nf_sS_k} + \frac{\left| \tilde{s}_k \right|^2}{Nf_sS_k} - \frac{2}{Nf_sS_k} \Re[e(\tilde{d}_k \tilde{s}_k)], \]  

(40)

where \( \Re[x] \) denotes the real part of \( x \). If we introduce \( \theta_k \) such that \( \tilde{d}_k = |\tilde{d}_k| e^{i \theta_k} \) then

\[ \chi_k^2 = \frac{\left| \tilde{d}_k \right|^2}{Nf_sS_k} + \frac{\gamma^2 N A_k^2 \alpha_k^2}{4f_sS_k} - \frac{\gamma A_k \alpha_k |\tilde{d}_k|}{f_sS_k} \cos(\tilde{\phi}_k + \theta_k). \]  

(41)

The likelihood from Eq. (37) depends on a large number of parameters (\( \gamma, \tilde{\phi}_j \) and \( \alpha_j \)) making it hard to sample efficiently. Since we are not interested in the estimates of \( \alpha_j \) and \( \tilde{\phi}_j \), we can marginalize the likelihood over these parameters. The first step consists in integrating on the random phases \( \tilde{\phi}_j \) such that the likelihood marginalized over the phases \( \mathcal{L}(d|\gamma, \alpha) = \int d\tilde{\phi} \mathcal{L}(d|\gamma, \alpha, \tilde{\phi}) P[\tilde{\phi}] \), where the last term is the prior from Eq. (24a). One can treat the frequencies independently and

\[ \mathcal{L}_k(d|\gamma, \alpha_k) = \frac{1}{2\pi} \int_0^{2\pi} d\tilde{\phi}_k \frac{1}{2\pi f_sS_k} e^{-\chi_k^2}, \]  

with \( I_0(x) \) the Bessel function.

The second step is a marginalization over \( \alpha_j \). Once again, we can treat each frequency independently such that

\[ \mathcal{L}_k(d|\gamma) = \int_0^\infty d\alpha_k \mathcal{L}_k(d|\gamma, \alpha_k) P[\alpha_k], \]  

where the last term is the Rayleigh prior from Eq. (24b), i.e.

\[ \mathcal{L}_k(d|\gamma) = \frac{1}{2\pi f_sS_k} e^{-\frac{|\tilde{d}_k|^2}{Nf_sS_k}} \int_0^\infty d\alpha_k \alpha_k e^{-\alpha_k^2 \left( \frac{N\gamma^2 A_k^2}{4f_sS_k} + \frac{1}{2} \right)} I_0 \left( \frac{\gamma A_k \alpha_k |\tilde{d}_k|}{f_sS_k} \right). \]  

(43)

Fortunately, this expression is analytical since

\[ \int_0^\infty dx x e^{-ax^2} I_0(bx) = \frac{e^{b^2/4a}}{2a}, \]

and leads to an expression of the marginalized likelihood given by

\[ \mathcal{L}_k(d|\gamma) = \frac{1}{2\pi f_sS_k} \frac{1}{1 + \frac{N\gamma^2 A_k^2}{2f_sS_k}} \exp \left( -\frac{|\tilde{d}_k|^2}{Nf_sS_k} \frac{1}{1 + \frac{N\gamma^2 A_k^2}{2f_sS_k}} \right). \]  

(44)

Finally the posterior distribution on \( \gamma \) \( \mathcal{P}(\gamma|d) \) marginalized over all other parameters is given by the Bayes theorem and is \( \mathcal{P}(\gamma|d) \propto \mathcal{L}(d|\gamma) P[\gamma] \) where the last term is the prior on \( \gamma \) that is chosen as flat. At the end, the posterior is given by [25, 26, 29]

\[ -\ln \mathcal{P}(\gamma|d) = \text{cst} + \sum_{k=1}^{N/2} \frac{|\tilde{d}_k|^2}{Nf_sS_k} + \ln \left( 1 + \frac{N\gamma^2 A_k^2}{2f_sS_k} \right). \]  

(45)
The 95% upper value for $\gamma$ is determined from this posterior distribution by solving for

$$\int_{\gamma_{95\%}}^{\gamma_{95\%}} P(\gamma|d)d\gamma = 2 \int_{0}^{\gamma_{95\%}} P(\gamma|d)d\gamma = 0.95. \quad (46)$$

Summary of the data analysis in practice

From the raw measurements $d$, we compute the DFT and compute the $|\tilde{d}_k|^2$ values. Then, for a given mass of the scalar field $m_{\phi}$, we compute the range over which the DM frequency distribution is non-negligible, i.e. we use Eq. (25) with $a = 3$. We determine the Fourier frequencies $f_j = \frac{j}{N}f_s$ that are contained in this frequency range. For all these frequencies, we evaluate the values of $A_j$ that are given by Eq. (28) and the two-sided PSD $S_k$ is provided by Eq. (16). We can then evaluate the posterior $P(\gamma|d)$ using Eq. (45) and compute the 95% upper limit by solving numerically for Eq. (46). We iterate this procedure for all masses corresponding to the [10, 200] kHz frequency range.

Data analysis in the Earth relaxation halo model

In this model the density of DM on the Earth’s surface is much larger than in the galactic DM model (see supplementary figure 2 of [30]) and we simply include this as an additional frequency dependent pre-factor in eqs. (28). The velocity distribution of DM is also different but the corresponding coherence times are much larger than for the galactic DM distribution. As a consequence the width of the corresponding frequency distribution is smaller than the RAM limited frequency resolution of our DFT ($\sim 3$ mHz) and we use a single frequency $\omega_j$ in the sum of (22). The rest of the procedure is identical to the galactic DM case described above. Note that we still take the probability distribution of $\alpha_j$ and $\phi_j$ into account and marginalise over them. As a consequence the factor $\sim 10$ sensitivity loss pointed out in [29] is accounted for, contrary to e.g. [19, 20].