Departure time and route choices with accurate information under binary stochastic bottleneck capacity in the morning commute

YUN YU1, XIAO HAN1,2, RUI JIANG1, JUSTIN DARR2, BIN JIA1

1Institute of Transportation System Science and Engineering, Beijing Jiaotong University, Beijing, 100044, China
2Department of Civil and Environmental Engineering, University of California, Davis, CA 95616, United States

Corresponding author: Xiao Han (han.xiao@bjtu.edu.cn), Bin Jia (bjia@bjtu.edu.cn).

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ABSTRACT Uncertainty, a critical factor of causing congestion and extra travel costs in the commute, can be mitigated by providing information. This paper studies the welfare effects of accurate pre-trip information on departure time and route choices in the morning commute under binary stochastic bottleneck capacity. We consider a classical two-route network. Each route has a single bottleneck where congestion occurs during the rush hours. The two routes’ bottleneck capacities vary from day-to-day due to events such as bad weather, accidents, and temporary road closures. We derive all equilibrium solutions in consideration of the differences between routes in free-flow travel time, the shadow value of travel time, the severity of bottleneck capacity reductions, and the degree of correlation between two routes in travel conditions. Furthermore, we investigate the benefit changes from zero-information to full-information and prove that accurate pre-trip information about the bottleneck conditions is strictly welfare-improving. Finally, these theoretical results are supplemented by case studies that show examples of benefit gains from pre-trip information.

INDEX TERMS Departure time choice, route choice, bottleneck, congestion, pre-trip information, uncertainty.

I. INTRODUCTION

Traffic congestion occurs when traffic demand exceeds road capacity and produces many negative impacts such as travel delay, excess greenhouse gas (GHG) emissions, and road rage [1]–[3]. In 2018, an American driver lost an average of 97 hours, with an estimated cost of $1,348 per driver totaling $87 billion, due to traffic congestion [4]. A major cause of congestion is unpredictable events such as adverse weather, road accidents, unannounced road work, malfunctioning traffic control devices, and special events [5], [6]. Fortunately, thanks to the recent development of advanced traveler information systems (ATIS) [7], it is easy to deliver information to drivers via television, radio, smartphones, and traffic websites (e.g., waze.com). Commuters can utilize this information to assist them in choosing departure times, routes, modes, parking locations, and so on. While it is expected that providing traffic information to travelers will reduce congestion and improve social welfare, it is valuable to understand how much benefit traveler information systems may provide under different circumstances.

Pre-trip information is messages that will deliver travel conditions to travelers before they depart. For decades, many analytical, experimental, and simulation studies have been conducted to examine the effects of pre-trip information on travel-related decision-making and its impact on traffic congestion [8]–[12]. A majority of them concluded that pre-trip information was beneficial to reduce congestion and improve social welfare [13]–[17]; however, some studies revealed that pre-trip information had a negative effect on social welfare in some cases, which is known as the ‘information paradox.’ The main reasons for information adversely affecting route choice, departure time, and other...
decisions include concentration, overreaction, and oversaturation [9]. For example, the overreaction to the provided information might increase congestion and reduce social welfare [18], [19]. Verhoef et al. [20] and Emmerink et al. [21] found that too much new information might increase travel costs and decrease social welfare. De Palma et al. [22] studied information acquisition and route-choice decisions of risk-averse drivers on a simple road network with four information regimes: no information, free information, costly information and private information. They found that free information might be welfare-reducing when a large proportion of the population is highly risk averse. Lindsey et al. [23] found that accurate pre-trip information was more likely to be welfare-reducing when free-flow travel costs were appreciably different, travel cost functions were convex, capacity degradations were similar on the two routes, and route conditions were positively correlated. In particular, the free-flow travel costs and correlation in conditions between routes were two critical factors that affect commuters’ choice behaviors. A companion experiment conducted by Rapoport et al. [24] revealed a rather complex and vibrant pattern of collective and individual behaviors, and supported the conclusions in Lindsey et al. [23]. Acemoglu et al. [25] found that informational Braess’ Paradox occurred in a network congestion game, characterized by commuters having information about additional routes being worse off than those who do not.

In addition to route choice, many studies have investigated the effects of pre-trip information on departure time choice. Arnott et al. [26]–[28] considered the effects of information in the bottleneck model with stochastic demand and travel conditions. They found that improving the accuracy of information might exacerbate congestion and reduce welfare compared to no information provided. Khan and Amin [29] examined the effects of heterogeneous information and information penetration on traffic congestion with an unreliable bottleneck link by using a Bayesian game. They found that departure time decisions of high accuracy information population made low accuracy information population worse off relative to under zero information, and increasing the fraction of informed commuters might exacerbate congestion and drive up costs. Liu and Liu [30] studied optimal information provision at bottleneck equilibrium with risk-averse travelers in which the free-flow travel time followed a uniform distribution. They found that improving information quality might not always reduce the total system costs when the cost of providing the information was considered. Zhu et al. [31] investigated the dynamical evolution of departure time choice in stochastic capacity bottleneck models with bounded rationality and various information perceptions. They found that travelers’ sensitivity towards real-time information could negatively impact the convergence to the boundedly rational user equilibrium.

Furthermore, some studies also considered the value of information in cases with both departure time and route choices. Arnott et al. [32] studied the effects of information on departure time and route choices under stochastic bottleneck capacity. They found if there are a random number of commuters, both perfect and noisy information might increase travel costs in contrast to no information. Soriguera [33] studied the value of highway travel time information by modeling the departure time and route choice grounded on the expected utility theory, where drivers tried to maximize the expected value of their perceived utility. Numerical examples and empirical data (measured on a highway corridor accessing the city of Barcelona, Spain) revealed that information systems with very high precision did not produce better results than the information with an acceptable level of precision. Furthermore, that study also revealed that information systems with very poor precision might even be detrimental. Most of these studies, however, use a traffic simulation rather than an equilibrium analysis, consider one choice dimension (e.g. route choice or departure time choice), or assume constant bottleneck capacities. It is not clear whether their findings still hold in cases with both departure time and route choices under stochastic bottleneck capacity. It is also worth noting that the previous studies about departure time choice usually neglected free-flow travel time, which has proven to be an essential factor in accurate pre-trip information causing welfare reduction in route choice [23].

In addition, previous studies usually assumed that the feelings of commuters about different routes in the morning commute are the same [32]–[36]. However, commuters exhibit a wide range of knowledge of the network topology and route selection criteria, such as minimizing time or stress or maximizing the aesthetic experience of a trip. Commuters may have different feelings when traveling on different routes because of differentiated route attributes such as road facility layouts, traffic control plans, the number of intersections and traffic lights, the complexity of the paths, the aesthetics of the route, and travel time reliability [37]–[42]. Therefore, the effects of different routes with different shadow values of travel time on departure time and route choices should be investigated.

In this paper, we study the effects of accurate pre-trip information on commuters’ departure time and route choices under binary stochastic bottleneck capacity. To this end, we consider a classical one origin-destination (OD) pair with two routes and two bottlenecks, a network commonly used in these studies. Two regimes are devised for comparison: the zero information regime and the full information regime. In the zero-information regime, commuters only know the unconditional probability distribution of bottleneck conditions on the two routes, while in the full-information regime they are provided the actual bottleneck conditions before choosing travel routes and departure times. In the full information regime, two scenarios are considered: the two routes’ bottleneck capacities are perfectly correlated or they are uncorrelated. Natural hazards and human activities are the two primary causes of uncertainty in transportation. The bottleneck conditions are usually correlated when the
two routes are not far apart and are affected by a natural hazard such as adverse weather. However, with regard to some incidents caused by human activities, such as car accidents and temporary lane closures, the two routes’ bottleneck conditions are usually uncorrelated. Furthermore, the departure flow patterns between the correlated and uncorrelated scenarios in the full-information regime are different. Therefore, we consider these two scenarios in the study. The value of pre-trip information is defined as the difference of the expected travel costs between the full-information and zero-information regimes [23].

The main contributions of the work are summarized as follows:

- A morning commute model is studied by combining Vickrey’s bottleneck model in a two-route network in consideration of binary stochastic bottleneck capacity, different shadow values of travel time, and different free-flow travel time on each route.
- We derive the departure flow patterns at user equilibrium under binary stochastic bottleneck capacity with and without pre-trip information.
- We investigate the effects of accurate pre-trip information on departure time and route choices under binary stochastic bottleneck capacity and find that accurate pre-trip information is strictly welfare-improving.
- Key factors that can impact the benefit gains from accurate pre-trip information are investigated.

The rest of this paper is organized as follows: Section II gives a brief introduction of Vickrey’s bottleneck model under deterministic bottleneck capacity and binary stochastic bottleneck capacity. Section III describes the morning commute model with departure time and route choices under binary stochastic bottleneck capacity and provides the equilibrium solutions. Section IV analyzes benefit changes from the zero-information regime to the full-information regime. Section V presents the results of the sensitivity analysis. Section VI discusses the results and presents our final conclusions.

II. DEPARTURE TIME CHOICE ON ONE ROUTE

The notations used throughout the paper are listed in Table 1.

A. DETERMINISTIC BOTTLENECK CAPACITY

The classical single bottleneck model proposed by Vickrey [43] concerns a highway with a bottleneck connecting a residential district (RD) with a central business district (CBD). A fixed number of \( N \) commuters travel from RD to CBD to work every morning during the rush hours. Congestion occurs at the bottleneck through which at most \( s \) vehicles can pass per unit of time. When the arrival rate at the bottleneck exceeds \( s \), a queue develops. Let \( Q(t) \) be the queue length at departure time \( t \). The travel time from RD to CBD can be described as,

\[
T(t) = \frac{Q(t)}{s} + T^f.
\]

in which \( T^f \) is the free-flow travel time if commuters do not experience congestion. We consider the highway is congested during the rush hours. Let \( t_0 \) and \( t_e \) denote the times at which queue begins and ends during the rush hours, respectively. Then queue length at time \( t \) can be obtained from,

\[
Q(t) = \max\{R(t) - s(t - t_0), 0\},
\]

where \( R(t) \) is the cumulative departures, which can be formulated as,

\[
R(t) = \int_{t_0}^{t} r(x) dx,
\]

where \( r(x) \) is the departure rate at time instant \( x \). We assume that \( t^* \) is the preferred arrival time (working time). Therefore, besides travel time costs, commuters might have schedule delay costs. If commuters arrive CBD before \( t^* \), they incur schedule delay early costs; if commuters arrive CBD after \( t^* \), schedule delay late costs occur. Therefore, the travel cost of a commuter traveling from RD to CBD includes two components, the travel time cost and the schedule delay cost. The travel cost of a commuter who departs at time \( t \) can be denoted as,

\[
C(t) = \alpha T(t) + \begin{cases} 
\beta(t^* - t - T(t)) & \text{if } t^* \geq t + T(t) \\
\gamma(t + T(t) - t^*) & \text{if } t^* < t + T(t)
\end{cases}
\]

where \( \alpha, \beta \) and \( \gamma \) indicate the shadow values of travel time, schedule delay early (SDE), and schedule delay late (SDL), respectively. Moreover, in correspondence to the empirical findings by Small [44], the relationship between the three parameters is,

\[
\gamma > \alpha > \beta > 0.
\]

User equilibrium (UE) is reached when no commuter can reduce his/her travel costs by unilaterally altering his/her departure time. All commuters at UE will experience congestion except the first and last departure time, and the departure rate is piecewise constant and given by

\[
r(t) = \begin{cases} 
\frac{\alpha}{\alpha + \gamma} & \text{if } t_0 \leq t < t^* \\
\frac{\alpha}{\alpha + \gamma} & \text{if } t^* < t \leq t_e
\end{cases}
\]

The travel cost of a commuter under UE is,

\[
C^* = \frac{\beta\gamma N}{\beta + \gamma s} + \alpha T^f.
\]

B. BINARY STOCHASTIC BOTTLENECK CAPACITY

The basic setting under binary stochastic bottleneck capacity is the same as deterministic bottleneck capacity except for the bottleneck conditions. Because of bad weather, accidents, roadwork, or special events, the bottleneck capacity may vary from day to day, where \( \omega \in \Omega = \{G, B\} \) is the set of possible bottleneck conditions. The capacity of the bottleneck in bad conditions is lower than that in good conditions, which can be formulated as,

\[
s_B = ks_G.
\]
where $k$ ($0 < k < 1$) is the capacity degradation rate, indicating the severity of bottleneck capacity reduction. The probabilities of bad conditions or good conditions occurring are $\pi$ and $1 - \pi$, respectively.

According to many existing studies, the following assumptions in the morning commute model with binary stochastic bottleneck capacity are adopted throughout the paper.

**Assumption 1.** Commuters are risk-neutral concerning costs [23].

**Assumption 2.** The free-flow travel time is free from the influence of capacity degradation [45], [46].

**Assumption 3.** The bottleneck capacity is constant within a day but fluctuates from day to day [45]– [47].

**Assumption 4.** Commuters are homogeneous with the same shadow values of travel time and schedule delay [43], [48].

**Assumption 5.** The shadow values of travel time of different routes might be different [37]– [42].

The expected travel cost of a commuter at departure time $t$ under binary stochastic bottleneck capacity can be described as,

$$E[C(t)] = E[\alpha T(t) + \beta SDE(t) + \gamma SDL(t)],$$

where $E[]$ is the expectations operator, and $SDE(t)$ and $SDL(t)$ are the schedule delay early and late cost for the commuter who chooses departure time $t$, respectively, and can be expressed as,

$$\begin{align*}
SDE(t) &= \max\{0, t^* - T(t) - t\} \\
SDL(t) &= \max\{0, T(t) + t - t^*\}
\end{align*}$$

Following the definition of UE, we have,

$$dE[C(t)]/dt = 0, \text{ if } r(t) > 0$$

### TABLE 1. Notational glossary.

| Variable | Description |
|----------|-------------|
| $F$ | Full information regime |
| $Z$ | Zero information regime |
| $B$ | Bad condition |
| $G$ | Good condition |
| $\omega$ | Condition, $\omega \in \Omega$ |
| $\Omega$ | Set of conditions, $\Omega = \{G, B\}$ |
| $\lambda$ | State on the two routes simultaneously, $\lambda \in \Lambda$ |
| $\Lambda$ | Set of states, $\Lambda = \{GG, GB, BG, BB\}$ |
| $s$ | Bottleneck capacity |
| $s_\omega$ | Bottleneck capacity in condition $\omega$ |
| $s_{i,\omega}$ | Bottleneck capacity on route $i$ in condition $\omega$ |
| $k$ | Bottleneck capacity degradation rate ($0 < k < 1$) |
| $k_i$ | Bottleneck capacity degradation rate on route $i$ ($0 < k_i < 1$) |
| $N$ | Total number of commuters |
| $N_i, \omega$ | Number of commuters on route $i$ in condition $\omega$ |
| $N_i$ | Number of commuters on route $i$ |
| $\bar{N}_i$ | Number of commuters on route $i$ in the static route-choice model |
| $\pi$ | Degradation probability of capacity ($0 < \pi < 1$) |
| $T(t)$ | Travel time at departure time $t$ |
| $R(t)$ | Cumulative departures at departure time $t$ |
| $Q(t)$ | Queue behind the bottleneck at departure time $t$ |
| $Q_{i,\omega}(t)$ | Queue behind the bottleneck on route $i$ in condition $\omega$ at departure time $t$ |
| $T_f^i$ | Free-flow travel time |
| $T_{f^i}$ | Free-flow travel time on route $i$ |
| $r(t)$ | Departure rate at departure time $t$ |
| $r_h$ | Departure rate of the $h$-th situation, $h = 1, 2, \ldots, 6$ |
| $C$ | Travel cost |
| $C_i$ | Travel cost on route $i$ |
| $C^*$ | Travel cost under deterministic bottleneck capacity |
| $C_{i,\omega}$ | Travel cost on route $i$ in condition $\omega$ |
| $C_{i,\lambda}$ | Travel cost under UE when the two routes are in state $\lambda$ |
| $C_i^*$ | Travel cost on route $i$ in the static route-choice model |
| $C_{i,\omega}^*$ | Travel cost on route $i$ in condition $\omega$ in the static route-choice model |
| $H$ | Difference of free-flow travel cost between two routes |
| $E[]$ | Expectation operator |
| $m_\omega$ | Congestion coefficient |
| $m_i$ | Congestion coefficient on route $i$ |
| $\tilde{m}_i$ | Congestion coefficient on route $i$ in the static route-choice model without pre-trip information |
| $\alpha$ | Shadow value of travel time |
| $\alpha_i$ | Shadow value of travel time on route $i$ |
| $\beta$ | Shadow value of schedule delay early |
| $\gamma$ | Shadow value of schedule delay late |
| $t^*$ | Preferred arrival time |
| $t_0$ | Start time for the queue |
| $t_{i,0}$ | Start time for the queue on route $i$ |
| $t_e$ | End time for the queue |
| $t_{i,e}$ | End time for the queue on route $i$ |
| $t_{j,j^*}$ | Critical time point between $j$-th situation and $j^*$-th situation on route $i$ |
| $SDE(t)$ | Schedule delay early cost at departure time $t$ |
| $SDL(t)$ | Schedule delay late cost at departure time $t$ |
| $G_{ZF}$ | Absolute benefit gain from shifting from zero information to pre-trip information |
| $\varphi$ | Relative benefit gain from shifting from zero information to pre-trip information |
| $B$ | Travel cost reduced from UE to system optimal (SO) with congestion tolling |
| $B_{i,\omega}$ | Travel cost reduced from UE to SO with congestion tolling under state $\omega$ |
| $TC$ | Total travel cost |
| $TTC$ | Total travel time cost |
| $SDC$ | Total schedule delay cost |
| $TTC_i$ | Total travel time cost on route $i$ |
| $SDC_i$ | Total schedule delay cost on route $i$ |
| $TC_{i,\omega}$ | Total travel cost on route $i$ in condition $\omega$ |
| $TTC_{i,\omega}$ | Total travel time cost on route $i$ in condition $\omega$ |
| $SDC_{i,\omega}$ | Total schedule delay cost on route $i$ in condition $\omega$ |
| $TTCC_i$ | Total time travel cost under state $\lambda$ |
| $SDC_{i,\lambda}$ | Total schedule delay cost under state $\lambda$ |
| $a_{i,\omega}$ | Congestion coefficient on route $i$ in condition $\omega$ in the static route-choice model |
| $b_{i,\omega}$ | Free-flow travel cost on route $i$ in condition $\omega$ in the static route-choice model |
Unlike deterministic bottleneck, the capacity of the stochastic bottleneck fluctuates from day to day. Therefore, commuters departing at the same time of one day may pass through the bottleneck in good conditions and arrive at CBD early, or cross the bottleneck in bad conditions and arrive late. Similarly, commuters may experience different queue times due to the variation in the bottleneck capacity.

The three schedule delay types and two queuing experience types shown in Table 2 lead to six combination-situations. According to Equations (9–11), we obtain the departure rates at UE in the six situations, summarized in Table 3.

Table 4 shows the six possible traffic flow patterns under different $k$ and $\pi$. The details of deriving the critical time points for the six possible patterns under UE can be found in the Appendix.

Figure 1 shows the illustration of the six possible traffic flow patterns under UE. Let $t_{j,j'}$ denotes the watershed lines that separate the $j$-th situation and $j'$-th situation. The details of the six possible departure patterns under UE are presented as follows:

**P1.** This pattern includes three situations, S1, S2, and S3. As shown in Figure 1 (a), Commuters departing before $t_{12}$ experience schedule delay early and always experience queuing; commuters departing during $[t_{12}, t_{23}]$ possibly experience schedule delay either early or late and always experience queuing; commuters departing after $t_{23}$ experience schedule delay late and always experience queuing.

**P2.** Three situations (i.e., S1, S2, and S5) occur in this pattern. As shown in Figure 1 (b), commuters departing during $[t_0, t_{12}]$ experience schedule delay early and always experience queuing; commuters departing during $[t_{12}, t_{25}]$ possibly experience schedule delay either early or late and always experience queuing; commuters departing during $[t_{25}, t_e]$ possibly experience schedule delay either early or late and possibly experience queuing.

**P3.** There are two situations (i.e., S4 and S5) in this pattern. As shown in Figure 1 (c), commuters departing during $[t_0, t_{45}]$ experience schedule delay early and possibly experience queuing; commuters departing during $[t_{45}, t_e]$ possibly experience schedule delay either early or late and possibly experience queuing.

**P4.** When $\pi > \frac{\alpha - \gamma}{\alpha + \gamma}$, commuters departing after $t_{36}$ experience queuing in bad conditions and always experience schedule delay, otherwise, there is no commuter departing after $t_{36}$.

**P5** and **P6.** When $\pi > \frac{\alpha - \gamma}{\alpha + \gamma}$, commuters departing after $t_{56}$ always experience schedule delay and experience queuing in bad conditions, otherwise, there is no commuter departing after $t_{56}$.

According to the six possible patterns, we can unify the expected travel cost of a commuter at UE as

$$E[C] = Nm + \frac{36}{\pi^3},$$

in which $m$ has different values under different $\pi$ and $k$ (see Table 5).
The following propositions are summarized to present the properties under binary stochastic bottleneck capacity on one route.

**Proposition 1**: The expected travel cost for every commuter under binary stochastic bottleneck capacity at user equilibrium is a strictly monotonically increasing function of the number of commuters, i.e., $\partial E[C]/\partial N > 0$.

**Proof**: Because $\partial E[C]/\partial N = m$ and $m > 0$, then $\partial E[C]/\partial N > 0$.

**Proposition 2**: The length of the peak period is decreasing as $s_B$ approaches $s_G$.

**Proof**: All of the departure rates at UE in Table 3 decrease as the capacity degradation rate $k$ decreases. Therefore, $t_e - t_0$ is monotonically increasing as $k$ decreases. This proposition coincides with Proposition 3 in Xiao et al. [45].

**Proposition 3**: When bad conditions dominate, i.e., $\pi > \frac{\gamma}{\alpha + \gamma}$, the departure flow patterns in the zero information scenario lead to the same expected travel costs as those of the deterministic model with bottleneck capacity $s_B$.

**Proof**: When $\pi > \frac{\gamma}{\alpha + \gamma}$, there are three possible departure flow patterns under UE. The expected TC under the three possible departure flow patterns is $\frac{N^2 \beta^2}{(\beta + \gamma)s_B}$, which is the same as the expected TC of the deterministic model with bottleneck capacity $s_B$.

**Proposition 4**: Providing accurate pre-trip information about the bottleneck conditions always reduces the expected travel cost for every commuter at user equilibrium on one route.

**Proof**: When the commuters are provided the bottleneck conditions each day before departure, the expected travel cost for every commuter is $E[C^*] = \frac{N^2 \beta^2}{(\beta + \gamma)s_B} \left( \frac{\pi}{s_B} + \frac{1 - \pi}{s_G} \right)$, where $E[C] > E[C^*]$.

### III. THE MODEL WITH DEPARTURE-TIME AND ROUTE CHOICES IN A TWO-ROUTE NETWORK

In this model, we suppose there are two routes (i.e., route 1 and route 2) connecting RD and CBD. A fixed number of $N$ commuters who are treated as continuum travel from RD to

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**TABLE 5.** The formulas of $m$ with different values of $\pi$ and $k$. The details of derivations can be found in Appendix.

| $\frac{\pi}{\alpha + \gamma} < k < 1$ | $0 < \frac{\pi}{\alpha + \gamma} < \frac{\beta}{\alpha + \gamma}$ | $\frac{\beta}{\alpha + \gamma} < \pi < 1$ |
|-------------------------------------|-------------------------------------|-------------------------------------|
| $\frac{\pi}{\alpha + \gamma} < k < 1$ | $0 < \frac{\pi}{\alpha + \gamma} < \frac{\beta}{\alpha + \gamma}$ | $\frac{\beta}{\alpha + \gamma} < \pi < 1$ |

The formulas of $m$ with different values of $\pi$ and $k$.

**FIGURE 1.** Six possible patterns under binary stochastic bottleneck capacity. The free-flow travel time is eliminated, i.e., $T^f = 0$. 
CBD. The bottleneck capacity of each route \((s_{\omega i}, i = 1, 2)\) may vary from day to day, where \(\omega \in \Omega = \{G, B\}\) is a condition of route \(i\). The bottleneck capacity in bad conditions is lower than that in good conditions, i.e., \(s_{1B} = k_1s_{1G}\), where \(k_1 (0 < k_1 < 1)\) is the capacity degradation rate of route \(i\). The shadow values of travel time of the two routes are denoted as \(\alpha_1\) and \(\alpha_2\), respectively. Two pre-trip information regimes, zero information (Z) and full information (F), are considered. In the zero-information regime, commuters only know the probabilities in bad conditions (\(\pi\)) and in good conditions (\(1 - \pi\)). In the full-information regime, all commuters have been informed of the bottleneck conditions of the two routes before departure. To investigate the effects of pre-trip information on the system efficiency, we first derive all equilibrium solutions in the zero-information and full-information regimes as follows.

### A. USER EQUILIBRIUM IN THE FULL-INFORMATION REGIME

In the full-information regime, commuters are informed of the bottleneck conditions of the two routes each day before departure. Let \(C^F_{\omega i}\) denote the travel cost of a commuter and \(N^F_{\omega i}\) denote the number of commuters choosing route \(i\) in the full-information regime with bottleneck condition \(\omega\). The equilibrium conditions in the full-information regime can be formulated as,

\[
\begin{align*}
&dC^F_{\omega i}(t)/dt = 0 \\
&dC^F_{\omega j}(t)/dt = 0 \\
&C^F_{\omega i}(t) = C^F_{\omega j}(t).
\end{align*}
\]

By applying the above conditions, we obtain the travel cost of a commuter under UE when the two routes are in certain conditions in the full-information regime as follows,

\[
C^F_\lambda = \frac{\beta \gamma N}{(\beta + \gamma)(s_{1\omega} + s_{2\omega})} + \frac{H s_{2\omega}}{s_{1\omega} + s_{2\omega}} + \alpha_1 T^f_1, \tag{14}
\]

where \(H\) is the free-flow travel cost difference between two routes, i.e., \(H = \alpha_2 T^f_2 - \alpha_1 T^f_1\) and \(\lambda \in \Lambda = \{GG, GB, BG, BB\}\). Without loss of generality, we let the free-flow travel cost on route 1 less than the cost on route 2, i.e., \(H \geq 0\). When the parameter \(H\) is not equal to zero, the travel cost under UE in the full information regime not only depends on aggregate capacity but also on how it is allocated between routes. The constraint that ensures \(N^F_{\omega 1} > 0\) is \(0 \leq H < H_1\), where \(H_1 = \beta \gamma N/s_{1G}\).

Furthermore, we consider whether the conditions of the two routes in the full-information regime are correlated. For the correlated scenario, the expected travel cost of a commuter can be denoted as,

\[
E[C^F] = (1 - \pi)C^F_{GG} + \pi C^F_{BB}. \tag{15}
\]

By comparing the expected travel costs between the uncorrelated and correlated scenarios, we have the following proposition.

**Proposition 5:** In the full-information regime, the expected travel cost at user equilibrium in the uncorrelated scenario is strictly less than that in the correlated scenario.

**Proof:** The proof of the proposition can be found in the Appendix.

### B. USER EQUILIBRIUM IN THE ZERO-INFORMATION REGIME

In the zero-information regime, commuters only know the probabilities in bad conditions (\(\pi\)) and in good conditions (\(1 - \pi\)). The equilibrium conditions in the zero-information regime can be formulated as,

\[
\begin{align*}
&dE[C^Z_1(t)]/dt = 0, \text{ if } r_1(t) > 0 \\
&dE[C^Z_2(t)]/dt = 0, \text{ if } r_2(t) > 0 \\
&E[C^Z_1(t)] = E[C^Z_2(t)],
\end{align*}
\]

Solving the above equations, we have

\[
\begin{align*}
&E[C^Z_1] = N^Z_1 m_1 + \alpha_1 T^f_1 \\
&E[C^Z_2] = N^Z_2 m_2 + \alpha_2 T^f_2 \\
&E[C^Z] = E[C^Z_1] = E[C^Z_2],
\end{align*}
\]

in which the expressions of \(m_1\) and \(m_2\) are the same as \(m\) for one route shown in Table 5 except for the specialized parameters for different routes (to be concrete, the parameters \(\alpha_i, s_{1\omega}, s_{2\omega}\) and \(k\) in \(m\) are specialized as \(\alpha_i, s_{1G}, s_{1B}\) and \(k_i\) in \(m_i\)), and \(N^Z_1\) and \(N^Z_2\) are the number of commuters choosing route 1 and route 2, respectively. The general formula of the expected travel cost at UE in the zero-information regime can be denoted as,

\[
E[C^Z] = \frac{m_1}{m_1 + m_2} (Nm_2 + H) + \alpha_1 T^f_1. \tag{19}
\]

From the above equation, we can see that the influence of the free-flow travel time on the expected travel costs cannot be negligible in the zero-information regime.

We have now obtained all expected travel costs at UE in the zero-information and full-information regimes. We find that the travel costs caused by the free-flow travel cost difference between the two routes not only depend on the shadow value of travel time (\(\alpha_i\)), but they also relate to the shadow values of schedule early and delay time (\(\beta\) and \(\gamma\)) in the two regimes. In the following section, we investigate whether pre-trip information can reduce travel costs based on the above equilibrium solutions under different parameters.

### IV. BENEFIT GAINS FROM PRE-TRIP INFORMATION

To discover the benefit gains from pre-trip information in our morning commute model with departure time and route choices under binary stochastic bottleneck capacities, we compare the expected travel cost of a commuter at UE between the zero-information regime and the full-information regime.
The absolute benefit gains in our proposed model at UE are defined as,

\[ G^{ZF} = E[C^Z] - E[C^F]. \]  \hspace{1cm} (20)

The constraint \( 0 < H < H_1 \) ensures the number of commuters choosing between the two routes are both greater than zero for the zero information and the full information regimes.

The absolute benefit changes from the zero-information regime to the correlated full-information regime can be formulated as,

\[ G^{ZF} = g_1 + g_2 H, \]  \hspace{1cm} (21)

where \( g_1 = \frac{N m_1 m_2}{m_1 + m_2} - \frac{N \beta \gamma}{\pi^{1+\gamma}} \left( \frac{s_{1G} + s_{2G}}{s_{1G} + s_{2G}} \right) \) and \( g_2 = \frac{m_1}{m_1 + m_2} - \frac{1}{\pi^{1+\gamma}} \left( \frac{s_{1B} + s_{2B}}{s_{1G} + s_{2G}} \right). \) We observe that the free-flow travel cost difference between the two routes may have positive or negative influences on the benefit gains, which should be taken into consideration when providing pre-trip information to commuters.

After comparing the expected travel costs between the zero information regime and the full information regime, we have the following proposition.

**Proposition 6:** Providing pre-trip information can always reduce travel costs in the morning commute model with departure time and route choices under binary stochastic bottleneck capacity.

**Proof:** The proof of the proposition can be found in the Appendix.

**V. SENSITIVITY ANALYSIS**

In this section, we conduct a series of sensitivity analysis to illustrate how pre-trip information affects benefit gains from the zero-information regime to the full-information regime in the proposed model. According to the empirical findings in Small [44], we set the shadow values of travel time, early arrival time, and late arrival time as \( \alpha_1 = 6.4(\$/h) \), \( \alpha_2 = 5(\$/h) \), \( \beta = 3.9(\$/h) \) and \( \gamma = 15.21(\$/h) \), respectively. The remaining parameters are: \( N = 8000(veh/h) \), \( s_{1G} = 4000(veh/h) \), \( s_{2G} = 2000(veh/h) \), \( T_1^f = 0.2(h) \), \( T_2^f = 0.3(h) \).

**A. BENEFIT GAINS UNDER DIFFERENT CAPACITY DEGRADATIONS.**

Figure 2 plots the results of benefit gains by shifting from the zero-information regime to the full-information regime with fixed probability \( \pi = 0.25 \). The benefit gains rise steadily with the decreases of \( k_1 \) and \( k_2 \), in both the correlated and uncorrelated scenarios. As expected, the value of information increases with the capacity degradation. Therefore, an accurate forecast is of vital importance to avoid loss and reduce travel costs before a serious bottleneck capacity degradation.

To further assess the benefit gains from pre-trip information, it is necessary to find a metric that is independent of scale and units of measurement. Therefore, we use the following index \( \varphi \) to measure the relative benefit gains,

\[ \varphi = \frac{N \cdot G^{ZF}}{N \cdot G^{ZF} + B^F}, \]  \hspace{1cm} (22)

where \( B^F \) denotes the travel cost reduced from UE to system optimal (SO) with congestion tolling in the full-information regime [49]. The values of \( B^F \) in the correlated and uncorrelated scenarios are described as,

\[ B^F = \left\{ \begin{array}{l}
(1 - \pi) B_{GG}^F + \pi B_{BG}^F \\
(1 - \pi)^2 B_{GG}^F + \pi^2 B_{BB}^F + \pi(1 - \pi)(B_{GB}^F + B_{BG}^F)
\end{array} \right. \]  \hspace{1cm} (23)

where \( B_{\alpha}^F = B_{\alpha 1}^F + B_{\alpha 2}^F = \frac{\bar{\alpha}_\gamma}{2(\beta + \gamma)} \left( \frac{N_{\alpha 1}^F}{s_{\alpha 1}} + \frac{N_{\alpha 2}^F}{s_{\alpha 2}} \right). \)

Figure 3 shows the relative benefit gains \( \varphi \) as a function of the influence of capacity degradation on each route in the correlated and uncorrelated scenarios. Similar to the results of absolute benefit gains, the relative benefit gains increase steadily in each dimension. For example, the \( \varphi \) is close to 0.95 when the values of \( k_1 \) and \( k_2 \) converge to 0 in the uncorrelated scenario. The results illustrate that providing pre-trip information does more to optimize network performance than congestion tolling when the bottleneck capacity experiences a large reduction.

**B. THE IMPACTS OF CAPACITY DEGRADATIONS ON TTC AND SDC.**

The total travel cost (TC) is combined by two kinds of costs, i.e., the total travel time cost (TTC) and the total schedule delay cost (SDC). In the full-information regime, the TC for the commuters who choose route \( i \) at UE can be described as,

\[ TC_{i\omega}^F = TTC_{i\omega}^F + SDC_{i\omega}^F, \]  \hspace{1cm} (24)

in which \( TTC_{i\omega}^F \) and \( SDC_{i\omega}^F \) can be denoted as,

\[ \begin{align*}
TTC_{i\omega}^F &= \frac{\bar{\beta}_\gamma}{2(\beta + \gamma)} \frac{N_{i\omega}^F}{s_{i\omega}} + N_{i\omega}^F \alpha_i T_1^f \\
SDC_{i\omega}^F &= \frac{\bar{\beta}_\gamma}{2(\beta + \gamma)} \frac{N_{i\omega}^F}{s_{i\omega}}.
\end{align*} \]  \hspace{1cm} (25)

Then, we can obtain the values of \( TTC^F \) and \( SDC^F \) for all commuters in the correlated scenario,

\[ \begin{align*}
TTC^F &= (1 - \pi) TTC_{GG}^F + \pi TTC_{BB}^F \\
SDC^F &= (1 - \pi) SDC_{GG}^F + \pi SDC_{BB}^F.
\end{align*} \]  \hspace{1cm} (26)

Similarly, we can obtain the values of \( TTC^F \) and \( SDC^F \) in the uncorrelated scenario,

\[ \begin{align*}
TTC^F &= (1 - \pi)^2 TTC_{GG}^F + (1 - \pi)\pi TTC_{GB}^F + (1 - \pi)\pi TTC_{BG}^F + \pi(1 - \pi)(TTC_{BB}^F + SDC_{BB}^F) \\
SDC^F &= (1 - \pi)^2 SDC_{GG}^F + (1 - \pi)\pi SDC_{GB}^F + (1 - \pi)\pi SDC_{BG}^F + \pi(1 - \pi)(SDC_{BB}^F + SDC_{BB}^F).
\end{align*} \]  \hspace{1cm} (27)
FIGURE 2. Absolute benefit gains from pre-trip information. (a) the correlated scenario, (b) the uncorrelated scenario.

FIGURE 3. Relative benefit gains from pre-trip information. (a) the correlated scenario, (b) the uncorrelated scenario.

FIGURE 4. Absolute benefit gains for TTC and SDC from pre-trip information. (a) The correlated scenario, (b) the uncorrelated scenario. The green region labeled ‘TTC, SDC’ denotes $\text{TTC}^Z > \text{TTC}^F$ and $\text{SDC}^Z > \text{SDC}^F$, the blue region labeled ‘SDC’ denotes $\text{SDC}^Z > \text{SDC}^F$ and $\text{TTC}^Z < \text{TTC}^F$. 
In the zero-information regime, total travel time is the area between the cumulative departures under UE and the arrival curves. This is true for both bad and good conditions. Therefore, the values of TTC of choosing route \( i \) in the zero-information regime can be described as follows.

**P1-TTC:**

\[
\text{TTC}_i^Z = \alpha_i N_i^Z T_i^f + (1 - \pi) \alpha_i \int_{t_{i,0}}^{t_{i,2Q}} Q_{i,G}(t) \, dt + \pi \alpha_i \int_{t_{i,0}}^{t_{i,0} + \frac{N_i^Z}{s_{i,B}}} Q_{i,B}(t) \, dt,
\]

**P2-TTC:**

\[
\text{TTC}_i^Z = \alpha_i N_i^Z T_i^f + (1 - \pi) \alpha_i \int_{t_{i,0}}^{t_{i,2Q}} Q_{i,G}(t) \, dt + \pi \alpha_i \int_{t_{i,0} + \frac{N_i^Z}{s_{i,B}}}^{t_{i,0} + \frac{N_i^Z}{s_{i,B}}} Q_{i,B}(t) \, dt,
\]

**P3-TTC:**

\[
\text{TTC}_i^Z = \alpha_i N_i^Z T_i^f + \pi \alpha_i \int_{t_{i,0} + \frac{N_i^Z}{s_{i,B}}}^{t_{i,0} + \frac{N_i^Z}{s_{i,B}}} Q_{i,B}(t) \, dt,
\]

**P4-TTC:**

\[
\text{TTC}_i^Z = \alpha_i N_i^Z T_i^f + (1 - \pi) \alpha_i \int_{t_{i,0}}^{t_{i,2Q}} Q_{i,G}(t) \, dt + \pi \alpha_i \int_{t_{i,0}}^{t_{i,0} + \frac{N_i^Z}{s_{i,B}}} Q_{i,B}(t) \, dt,
\]

**P5-TTC:**

\[
\text{TTC}_i^Z = \alpha_i N_i^Z T_i^f + (1 - \pi) \alpha_i \int_{t_{i,0}}^{t_{i,2Q}} Q_{i,G}(t) \, dt + \pi \alpha_i \int_{t_{i,0} + \frac{N_i^Z}{s_{i,B}}}^{t_{i,0} + \frac{N_i^Z}{s_{i,B}}} Q_{i,B}(t) \, dt,
\]

**P6-TTC:**

\[
\text{TTC}_i^Z = \pi \alpha_i \int_{t_{i,0} + \frac{N_i^Z}{s_{i,B}}}^{t_{i,0} + \frac{N_i^Z}{s_{i,B}}} Q_{i,B}(t) \, dt + \alpha_i N_i^Z T_i^f,
\]

where \( Q_{i,G}(t) \) and \( Q_{i,B}(t) \) are the cumulative queue lengths at departure time \( t \) on route \( i \) in good and bad conditions, respectively. All the values of critical time points and the number of cumulative departures at critical time points can be found in Figure 1 and in the Appendix. The values of TTC and SDC in the zero-information regime can be calculated from TTC\(_Z\) = TTC\(_1\) + TTC\(_2\) and SDC\(_Z\) = \( N \times E[\text{C}_Z] \) - TTC\(_Z\), respectively.

Figure 4 presents the absolute benefit gains for TTC and SDC by shifting from the zero-information regime to the full-information regime, in which the parameters are as same as Figure 2. When facing relatively small capacity degradations, the values of TTC and SDC both decrease when pre-trip information is provided. When capacity degradation increases, providing pre-trip information lowers the benefits for SDC, but it has negative impacts on TTC. The increase of TTC may bring more burdens to transportation systems. Therefore, from the perspective of alleviating traffic congestion, providing pre-trip information to commuters is not ideal. In contrast to the uncorrelated scenario, the areas that only reduce SDC are larger in the correlated scenario. When the capacity degradations on the two routes are large enough, provided information can reduce the values of TTC and SDC at the same time again.

**C. A COMPARISON BETWEEN OUR PROPOSED MODEL AND THE STATIC ROUTE-CHOICE MODEL**

Until now, we have proved that accurate pre-trip information is strictly welfare-improving in our proposed model considering both departure time and route choices. However, previous studies such as Lindsey et al. [23] found that providing accurate information might reduce social welfare in a model in which commuters are only required to choose a route. We refer to the model with only route choice as the static route-choice model. In that model, negative externalities caused by congestion are considered, and travel costs are an increasing function in the number of commuters choosing the same route. The relationship between route choice and congestion can typically be described by a linear cost function [23]. To compare the welfare effects of accurate pre-trip information between our proposed model and the static route-choice model, we adopt the linear cost functions in the static route-choice model,

\[
\tilde{C}_{i,\omega}(N_i) = a_{i,\omega} N_i + b_{i,\omega}
\]

in which \( a_{i,\omega} \) is the congestion coefficient, and \( b_{i,\omega} \) is the free-flow travel cost on route \( i \). Furthermore, we can see that Eq. 34 has the same form as Eq. 7. Let \( a_{i,\omega} = \frac{\beta \gamma}{(\beta + \gamma) s_{i,\omega}} \) and \( b_{i,\omega} = \alpha_i T_i^f \), then we have

\[
\tilde{C}_{i,\omega}(N_i) = \frac{\beta \gamma}{(\beta + \gamma) s_{i,\omega}} N_i + \alpha_i T_i^f.
\]

When commuters are informed of the two routes’ bottleneck conditions, the expected travel cost of a commuter in the static route-choice model is the same as that in our proposed model with pre-trip information. However, when there is no pre-trip information, commuters only know the unconditional probability distribution of travel conditions in the static route-choice model. Let \( \bar{N}_i^Z \) denote the number of commuters who choose route \( i \) in the static model without pre-trip information. Therefore, the expected travel cost of a commuter is,

\[
E[\tilde{C}_{i,\omega}(\bar{N}_i^Z)] = \tilde{N}_i^Z m_i + \alpha T_i^f.
\]

where \( m_i = \frac{\beta \gamma}{\beta + \gamma} \left( \frac{\pi^i}{\pi^i} + \frac{1 - \pi}{\pi^G} \right) \). Then, according to the UE condition \( E[\tilde{C}_{i,\omega}(\bar{N}_i^Z)] = E[\tilde{C}_{2,\omega}(\bar{N}_i^Z)] \), we obtain the expected travel cost at UE in the static route-choice model without pre-trip information as,

\[
E[\tilde{C}_Z] = \frac{\bar{m}_1}{\bar{m}_1 + \bar{m}_2} (N \bar{m}_2 + \alpha H) + \alpha T_i^f
\]

where \( 0 < H < N \bar{m}_1 / \alpha \).

Figure 5 shows a travel cost comparison between our proposed model and the static route-choice model. We can see that information paradox occurs in the range of \( 0.035 < k_2 < 0.5 \), i.e., \( E[C_{FP}] > E[C_Z] \), in the static route-choice model. However, in our proposed model, \( E[C_{FP}] \) is always less than \( E[C_Z] \), indicating that the information paradox disappears when considering both departure time and route choices. The main reason why the information
paradox in Lindsey et al. [23] disappears in our proposed model is that commuters can adjust their decisions from not only the spatial dimension but also the time dimension. The choices in the time dimension will generate extra schedule delay costs or congestion costs, further increasing the expected travel costs under UE when no pre-trip information is provided.

VI. CONCLUSIONS AND DISCUSSION

In this paper, we study a morning commute model with departure time and route choices under binary stochastic bottleneck capacities. The model is based on the classical two-route network and Vickrey’s single bottleneck model. However, unlike the classical Vickrey’s single bottleneck model, the bottleneck capacity on each route changes from good conditions to bad conditions with a certain probability because of the influences of bad weather, accidents, and other disturbances. We analyze the welfare effects of accurate pre-trip information by studying the benefit changes from the zero-information regime to the full-information regime. Additionally, different shadow values of travel time and free flow travel time on each route are considered. In the zero-information regime, commuters only know the unconditional probability distribution of bottleneck conditions on the two routes, while in the full-information regime, they are entirely and accurately informed of the condition of the bottleneck of each route. According to user equilibrium analysis, we derive all theoretical solutions of the proposed model in the zero-information and full-information regimes.

We find that accurate pre-trip information is strictly welfare-improving in our proposed model, and this information can be more beneficial in the uncorrelated scenario than in the correlated scenario. A welfare effect comparison between our proposed model and the static route-choice model further sheds light on the importance of taking departure time choice into account when investigating commute behaviors. When taking both departure time and route choices into account, providing accurate pre-trip information is always beneficial to commuters. We also analyze the changes in the total travel time costs and the total schedule delay costs after providing pre-trip information. We find that although accurate pre-trip information can reduce the total travel costs, it might generate more congestion on the road, leading to an increase in total travel time costs. Our findings in the study shed light on the importance of developing ATIS in reducing travel costs in the morning commute.

Our study has some limitations that provides opportunities for future research. We have identified six of these limitations that should be addressed. First, we only consider the welfare effect of accurate pre-trip information. However, the provided information might not be one hundred percent accurate. Therefore, the effects of inaccurate pre-trip information on commuters’ departure time and route choices should be investigated. Second, we only adopt the classical two-route network to highlight the welfare effects of pre-trip information. Future work should explore the effects of pre-trip information on commuters’ choices based on more complex networks with multiple origin-destination pairs and links. Third, we only consider departure time and route choices. In reality, commuters usually adjust their decisions from multiple aspects, e.g., travel routes, departure times, travel modes, and parking locations. Additional work on this topic should consider the theoretical analysis of multiple choices for the commuter. Fourth, Although we have proved that accurate pre-trip information is strictly welfare-improving when commuters are required to choose both departure time 

![Figure 5](image-url)
and routes under binary stochastic bottleneck capacity, it may have a negative influence on the traffic congestion when the commuters consider additional aspects of the trip. Therefore, reducing traffic congestion while maintaining social welfare needs to be explored. Fifth, we only consider the equilibrium solutions under binary stochastic bottleneck capacity. It should be investigated whether our conclusions still hold when the stochastic bottleneck capacity is assumed to follow a general distribution. Sixth, although our study is based on a rigorous theoretical analysis by the principle of user equilibrium, people’s choices might not strictly follow this principle in reality. Therefore, whether accurate pre-trip information is always welfare-improving should be verified further, especially when relative benefit gains are not large. This can be accomplished with laboratory experiments designed to investigate travel choice problems with lower costs and improved control of both context and game settings [50]–[52]. After collecting subjects’ choice data, analysis methods such as time series analysis and post-hoc tests can be applied to verify the effects of accurate pre-trip information on choice behaviors.

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APPENDIX

A. THE CRITICAL TIME POINTS IN THE SIX POSSIBLE PATTERNS.

P1: The four critical points in P1 are,

\[
\begin{align*}
& t_0 = t^* - \frac{aN}{N_{\alpha\gamma}(\rho_s - \frac{1}{sG})} - T^f \\
& t_{12} = t^* - \frac{aN}{N_{\alpha\gamma}sG}(\frac{1}{r_1} - \frac{1}{sG}) - T^f \\
& t_{25} = t^* - \frac{aN}{N_{\alpha\gamma}sG}(\frac{1}{r_3} - \frac{1}{sG}) - T^f \\
& t_e = t^* - T^f.
\end{align*}
\]

The expected travel cost of a commuter under binary stochastic bottleneck capacity at UE is,

\[
E[C] = \frac{N\beta(\alpha + \gamma)(\pi/k + 1 - \pi) - a}{(\beta + \gamma)sG} + \alpha T^f.
\]

P2: The four critical points in P2 are,

\[
\begin{align*}
& t_0 = t^* - \frac{aN}{N_{\alpha\gamma}(\rho_s - \frac{1}{sG})} - T^f \\
& t_{12} = t^* - \frac{aN}{N_{\alpha\gamma}sG}(\frac{1}{r_1} - \frac{1}{sG}) - T^f \\
& t_{25} = t^* - \frac{aN}{N_{\alpha\gamma}sG}(\frac{1}{r_3} - \frac{1}{sG}) - T^f \\
& t_e = t^* - T^f.
\end{align*}
\]

The expected travel cost of a commuter under binary stochastic bottleneck capacity at UE is,

\[
E[C(t)] = \frac{N\beta(\alpha + \gamma)}{(\beta + \gamma)sG} + \alpha T^f.
\]

P3: The three critical points in P3 are,

\[
\begin{align*}
& t_0 = t^* - \frac{N_{\alpha\gamma}}{\beta + \pi(\alpha + \gamma)\frac{sG}{sG}} - T^f \\
& t_{45} = t^* - \frac{N_{\alpha\gamma}}{\beta + \pi(\alpha + \gamma)\frac{sG}{sG}}(1 - \frac{sG}{r_4}) - T^f \\
& t_e = t^* - T^f
\end{align*}
\]

The expected travel cost of a commuter under binary stochastic bottleneck capacity at UE is,

\[
E[C(t)] = \frac{N\beta(\alpha + \gamma)}{(\beta + \pi(\alpha + \gamma))ksG} + \alpha T^f.
\]

P4: The five critical points in P4 are,

\[
\begin{align*}
& t_0 = t^* - \frac{N}{\beta + \gamma s_B} - T^f \\
& t_{12} = t^* - \frac{N}{\beta + \gamma s_B}(1 - s_B) - T^f \\
& t_{23} = t^* - \frac{N}{\beta + \gamma s_B}(1 - \frac{s_B}{r_1} - \frac{s_B}{r_2}) - T^f \\
& t_{36} = t^* + \frac{N}{\beta + \gamma s_B}(\frac{r_1 - s_B}{r_2}) - T^f \\
& t_e = t^* + \frac{N}{\beta + \gamma s_B} - T^f
\end{align*}
\]

The expected travel cost of a commuter under binary stochastic bottleneck capacity at UE is,

\[
E[C(t)] = \frac{N\beta(\alpha + \gamma)}{(\beta + \gamma)sB} + \alpha T^f.
\]

P5: The five critical points in P5 are,

\[
\begin{align*}
& t_0 = t^* - \frac{N}{\beta + \gamma s_B} - T^f \\
& t_{12} = t^* - \frac{N}{\beta + \gamma s_B}(1 - s_B) - T^f \\
& t_{25} = t^* + \frac{N}{\beta + \gamma s_B}(\frac{r_1 + r_2s_B}{r_2} - s_B) - T^f \\
& t_{56} = t^* - T^f \\
& t_e = t^* + \frac{N}{\beta + \gamma s_B} - T^f
\end{align*}
\]

The expected travel cost of a commuter under binary stochastic bottleneck capacity at UE is,

\[
E[C(t)] = \frac{N\beta(\alpha + \gamma)}{(\beta + \gamma)sB} + \alpha T^f.
\]

P6: The four critical points in P6 are,

\[
\begin{align*}
& t_0 = t^* - \frac{N}{\beta + \gamma s_B} - T^f \\
& t_{45} = t^* - \frac{N}{\beta + \gamma s_B}(1 - s_B) - T^f \\
& t_{56} = t^* - T^f \\
& t_e = t^* + \frac{N}{\beta + \gamma s_B} - T^f
\end{align*}
\]

The expected travel cost of a commuter under binary stochastic bottleneck capacity at UE is,

\[
E[C(t)] = \frac{N\beta(\alpha + \gamma)}{(\beta + \gamma)sB} + \alpha T^f.
\]

We find that even though there are six possible departure patterns, there are only three different formula for the expected travel cost of a commuter. Moreover, according to the critical points in the six possible departure patterns, we can obtain the boundary constraints. For example, there are four critical points in P1 which \( t_e > t_{25} > t_{12} > t_0 \). According to Eq. 38, we have the boundary constraint for P1, \( N_{\alpha\gamma}(\rho_s - \frac{1}{sG}) < k < 1 \).
B. THE PROOF OF PROPOSITION 5.

The travel cost difference under UE between the correlated and the uncorrelated scenarios is,

\[ U = \pi(1-\pi)(C_{FG}^B + C_{FB}^B - C_{F}^B) \]

\[ \pi(1-\pi)(k_1-1)s_{1G}s_{2G}(u_1+u_2H) \]

where \( u_1 = \frac{N\beta\pi}{\beta+\pi}[(k_1 \pm 1)s_{1G} + (k_2 \pm 1)s_{2G}] \), and \( u_2 = k_2^2s_{2G}^2 - k_1^2s_{1G}^2 \). Let \( u(H) = u_1 + u_2H \), where \( u(H) \) is a linear function with respect to \( H \). If \( u_2 \geq 0 \), then we have \( u(H) \geq u_1 \geq 0 \); otherwise, we obtain the minimum value of \( u(H) \) with \( H = \min(1) \) as,

\[ u(H_1) = \frac{N\beta\gamma s_{1G}^2 + (k_2 \pm 1)s_{1G}s_{2G} + k_2^2s_{2G}^2}{\beta+\gamma} > 0 \]  

Thus, we have proven that the correlation in conditions between the two routes can increase travel costs in the full information regime. In other words, pre-trip information is more efficient in forecasting some uncorrelated events, such as road accidents.

C. THE PROOF OF PROPOSITION 6.

It is straightforward to prove that \( g_1 > 0 \) with the transitive property of inequality states. Moreover, we find the increase or the decrease of \( G^ZF \) with respect to \( H \) depends on the value of \( g_2 \).

(1) If \( g_2 > 0 \), then \( G^ZF \) is a monotone increasing function of \( H \). Thus, we have \( G^ZF \geq g_1 > 0 \).

(2) If \( g_2 = 0 \), then \( G^ZF = g_1 > 0 \).

(3) If \( g_2 < 0 \), then \( G^ZF \) is a monotone decreasing function of \( H \). We can obtain the minimum value \( G^ZF \) in this situation as,

\[ \hat{G}^ZF = \frac{N}{m_1 \pm m_2(\frac{\beta\gamma}{\beta+\gamma})s_{1G}} - \frac{(1-\pi)\beta\gamma}{(\beta+\gamma)s_{1G}} \]  

The task is now becoming to prove \( \hat{G}^ZF > 0 \). Assume \( \alpha_1 \leq \alpha_2 \) (similar logic applies if \( \alpha_1 > \alpha_2 \)).

(1) When \( 0 < \pi < \frac{\gamma}{\alpha_1+\gamma} \), the partial derivative of \( \hat{G}^ZF \) with respect to \( s_{2G} \) is,

\[ \frac{\partial \hat{G}^ZF}{\partial s_{2G}} = \frac{N}{m_1(1+m_2)} \frac{\partial (m_2)}{\partial s_{2G}} - \frac{\pi\beta\gamma k_2(k_1-1)}{(\beta+\gamma)(k_1s_{1G} + k_2s_{2G})} \]

where \( b = \frac{\beta\gamma}{(\beta+\gamma)s_{1G}} \). Substituting \( m_1 = \frac{a_1\beta}{s_{1G}^2} \), \( m_2 = \frac{a_2\beta}{s_{2G}^2} \) and \( \frac{\partial m_2}{\partial s_{2G}} = -\frac{a_2\beta}{s_{2G}^3} \) into the above equation, we have,

\[ \frac{\partial \hat{G}^ZF}{\partial s_{2G}} = \frac{\beta N}{\beta+\gamma} \frac{a_1s_{1G} + a_2s_{1G}}{2(k_1s_{1G} + k_2s_{2G})^2} \]  

where \( \xi = a_1a_2(\gamma-\frac{(\beta+\gamma)a_1}{k_1s_{1G} + k_2s_{2G}} - \frac{\pi\gamma k_2(k_1-1)}{(a_1s_{2G} + a_2s_{1G})^2} - 1) \). We can obtain the second order partial derivative of \( \xi \) with respect to \( s_{1G} \) and \( s_{2G} \) as,

\[ \frac{\partial^2 \xi}{\partial s_{1G} \partial s_{2G}} = 2a_1a_2k_2\eta, \]

where \( \eta = \pi(1+\frac{\gamma}{\beta+\gamma}a_1k_1 - \pi\gamma(k_1-1)) \). If \( \frac{\pi(\alpha_1+\gamma)}{\beta+\gamma(\alpha_1+\gamma)} < \eta \), then \( \alpha_1 + \gamma > \eta \). Now we obtain,

\[ \eta = \alpha_1 \xi(k_1 - 1) < 0 \]

Then, we have,

\[ \xi < \xi s_{1G} + 0, s_{2G} \rightarrow 0 = 0 \]  

Therefore, we have proven that the formula of \( \hat{G}^ZF \) is a decreasing function of \( s_{2G} \). Finally, we have,

\[ \hat{G}^ZF > \hat{G}^ZF s_{2G} + 0, s_{2G} \rightarrow +\infty = 0 \]

If \( 0 < k_1 \leq \frac{\pi(\alpha_1+\gamma)}{\beta+\pi(\alpha_1+\gamma)} \), then \( a_1 = \frac{\pi(\alpha_1+\gamma)}{k_1(\beta+\pi(\alpha_1+\gamma))} \), we can obtain,

\[ \eta = (1-\pi)\gamma k_1 + \pi\gamma - \frac{\pi(\alpha_1+\gamma)(\beta+\gamma)}{\beta+\pi(\alpha_1+\gamma)} \]

where \( \eta \) is an increasing function of \( k_1 \). Because \( k_1 \leq \frac{\pi(\alpha_1+\gamma)}{\beta+\pi(\alpha_1+\gamma)} \), we can obtain,

\[ \eta \leq \frac{-\pi\alpha_1\beta}{\beta+\pi(\alpha_1+\gamma)} < 0 \]

Then, we have,

\[ \xi < \xi s_{1G} + 0, s_{2G} \rightarrow 0 = 0 \]

Therefore, we obtain,

\[ \hat{G}^ZF > \hat{G}^ZF s_{2G} + 0, s_{2G} \rightarrow +\infty = 0 \]

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XIAO HAN received his bachelor’s degree from Shandong University in 2012, and Ph.D. degree in systems theory from Beijing Normal University in 2017. He was a post-doctoral researcher with Beijing Jiaotong University from 2017 to 2018. He is currently a post-doctoral researcher with Department of Civil and Environmental Engineering, University of California, Davis. His research interests include connected automated vehicle, travel behavior modeling, and game theory.

RUI JIANG received the B.S. degree from University of Science and Technology of China, in 1998, where he received the Ph.D. degree in engineering thermophysics, in 2003. He is currently a Professor at Beijing Jiaotong University. His research interests include travel behavior modeling, traffic control, and traffic flow theory.

JUSTIN DARRE received his B.S. degree in Civil and Environmental Engineering from the University of California, Davis, in 2017. He is currently pursuing an M.S. and a Ph.D. in Civil and Environmental Engineering at the University of California, Davis. His research interests include studying accessibility, micromobility, and public transit in relation to equity and health in transportation.

BIN JIA received the B.S. and M.S. degrees from Inner Mongolia University of Science & Technology, China, in 1997 and 2000, respectively, and the Ph.D. degree in fluid mechanics from University of Science and Technology of China, in 2003. He is currently a Professor at Beijing Jiaotong University. His research interests include connected automated vehicle, traffic big data mining and application, and traffic flow theory.