Ergodic Capacity Analysis of Remote Radio Head Associations in Cloud Radio Access Networks

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Abstract—Characterizing user to Remote Radio Head (RRH) association strategies in cloud radio access networks (C-RANs) is critical for performance optimization. In this letter, the single nearest and $N$-nearest RRH association strategies are presented, and the corresponding impact on the ergodic capacity of C-RANs is analyzed, where RRHs are distributed according to a stationary point process. Closed-form expressions for the ergodic capacity of the proposed RRH association strategies are derived. Simulation results demonstrate that the derived uplink closed-form capacity expressions are accurate. Furthermore, the analysis and simulation results show that the ergodic capacity gain is not linear with either the RRH density or the number of antenna per RRH. The ergodic capacity gain from the RRH density is larger than that from the number of antennas per RRH, which indicates that the association number of the RRH should not be bigger than 4 to balance the performance gain and the implementation cost.

Index Terms—Cloud radio access networks, cell association, performance analysis, large scale cooperation

I. INTRODUCTION

Cloud radio access networks (C-RANs) are by now recognized to curtail both capital and operating expenditures, as well as to provide high energy-efficiency transmission bit rates. The Remote Radio Heads (RRHs) in C-RANs operate as soft relays by compressing and forwarding the signals received from mobile users to a centralized Base Band Unit (BBU) through the backhaul links. The outage probability for distributed beamforming and best base station selection schemes in C-RANs are presented when the user and base stations are each configured with a single antenna and the path loss exponent is 2, and the minimal number of RRHs for the desired user to meet a predefined quality of service is analyzed as well. In particular, the outage probability and the closed-form ergodic capacity achieved by both the single nearest and $N$-nearest RRH association strategies for C-RANs are characterized, where multiple antennas are used in the RRH, and the path loss exponent is 4. Secondly, based on the proposed outage probability and ergodic capacity performance metrics, the impact of the number of antennas and the RRH density is characterized. Closed-form expressions for the ergodic capacity are derived for special cases in this letter though they are too complex to be analyzed mathematically. According to the analysis and simulation results, the association number of the RRH should not be larger than 4 to balance performance gains and implementation complexity.

II. SYSTEM MODEL

Consider C-RAN uplink systems, in which a group of RRHs, each having $L$ antennas, help the signals of a single-antenna user to be decoded in the BBU. The locations of the RRHs are assumed to be the atoms of a two-dimensional Poisson Point Process (PPP) $\Phi$ having intensity $\lambda$ in a disc $D^2$, whose radius is $R$. Without any loss of generality, we assume the desired user (denoted by $U$) is located at the origin of $D^2$. Let $\zeta(U) \in \Phi$ signify that $U$ is associated to an RRH. The number $N_R$ of RRHs in $D^2$ is random with probability distribution $P(N_R) = (\mu_D N_R/(N_R!)) e^{-\mu_D}$, where $\mu_D = \pi R^2 \lambda$. The large-scale fading is represented by $r_i^{-\alpha}$, where $\alpha$ is the path loss exponent, and $r_i$ is the distance between $U$ and the $i$-th RRH. When maximal ratio combining (MRC) is used for achieving full-diversity gains, the small-scale fading between $U$ and the $i$-th RRH is given by

$$H_i = \sum_{l=1}^{L} |h_{il}|^2,$$

where $h_{il}$ is the fading gain between $U$ and the $l$-th antenna of the $i$-th RRH, and can be modeled as a complex Gaussian random variable with zero mean and unit variance, i.e. $h_{il} \sim CN(0,1)$. Thus, for the case of a large number of antennas, $H_i$ follows the gamma distribution, i.e., $H_i \sim \Gamma(L,1)$. The probability density function (pdf) of $H_i$ can thus be written...
as
\[
f_{H_i}(x) = \frac{x^{L-1}e^{-x}}{(L-1)!}.
\] (2)

We let \( P_U \) denote the transmit power of \( U \). Two RRH association strategies are investigated in this letter.

1) Single nearest RRH association: The desired user \( U \) associates with the nearest RRH, which has the maximum receiving power when the shadow fading remains constant. The associated RRH \( i \) for user \( U \) is thus \( i = \arg \max_i P_U r_i^{-\alpha} H_i \).

2) \( N \)-nearest RRH association: The desired user \( U \) associates with the \( N \) nearest RRHs amongst the total \( N_R \) RRHs \((N \leq N_R)\). To avoid the calculation of distances from \( U \) to the total \( N_R \) RRHs, the \( N \) RRHs with the maximum average received power during the observed interval will be selected when the transmit powers of all RRHs are the same.

Obviously, the higher diversity gains can be achieved by selecting the \( N \) best RRHs (i.e., the \( N \) RRHs with maximum instantaneous received power taking all kinds of fading into account) than \( N \) nearest RRHs. However, to access the \( N \) best RRHs, the instantaneous channel state information (CSI) is necessary and the backhaul signalling overhead increases with \( N_R \), which challenges the implementation complexity. Consequently, this letter focuses on the practicable \( N \)-nearest RRH association strategy that selects the \( N \) RRHs with the largest received power at the BBU.

III. PERFORMANCE ANALYSIS

The received signal-to-noise-ratio (SNR) for \( U \) at a distance \( r_i \) from the \( i \)-th RRH is
\[
\gamma_i = \frac{P_U r_i^{-\alpha} H_i}{\sigma^2},
\] (3)
where \( \sigma^2 \) is the additive thermal noise power.

A. Single nearest RRH association

For this scheme, the \( U \) associated with the nearest RRH and the subscript \( i \) can dropped in (3). An outage occurs when the received SNR at the associated RRH is smaller than a predefined threshold \( T \).

**Lemma 1:** The outage probability achieved by the single nearest RRH association strategy in C-RAN uplink systems is
\[
P_{\text{out-IR}} = \int_0^\infty \frac{\varepsilon(L, \frac{r^2T}{\rho})}{(L-1)!} e^{-\lambda \pi r^2} 2\pi \lambda dr,
\] (4)

where \( \rho = \frac{\lambda}{\varepsilon(a, b)} \) and \( \varepsilon(a, b) = \int_0^b a^{-1} e^{-u} du \).

**Proof:** The lemma is proved in two steps: first obtain the pdf of the distance \( r \) between \( U \) and the serving RRH, and then find the outage probability.

Following [5], the pdf of \( r \) is given by
\[
f_r(r) = e^{-\lambda \pi r^2} 2\pi \lambda r, r > 0
\] (5)

Based on (5), the outage probability that the received SNR \( \gamma \) to access the nearest RRH is smaller than threshold \( T \) can be written as
\[
P_{\text{out-IR}} = \Pr[\gamma < T] = E \left[ \Pr \left[ \rho H r^{-\alpha} < T \right] \right]
\] (6)

The ergodic capacity for the proposed single nearest RRH association strategy is specified in the following proposition:

**Proposition 1:** For high SNR, the uplink ergodic capacity (bps/Hz) for the single nearest RRH association strategy in C-RAN system approximates
\[
C_{1R} = \frac{\sum_{i=1}^{L-1} \frac{1}{\lambda} + \frac{\alpha}{2} [\ln(\pi \lambda) + C] - C + \ln(P/\sigma^2)}{\ln(2)},
\] (7)
where \( C \) is Euler’s constant.

**Proof:** The ergodic capacity can be calculated as
\[
C_{1R} = \int_0^\infty \frac{\varepsilon(L, \frac{r^2T}{\rho})}{(L-1)!} e^{-\lambda \pi r^2} 2\pi \lambda dr,
\] (8)

where \( \varepsilon(L, \frac{r^2T}{\rho}) \) is the pdf of the SNR \( \gamma \). Using the definition of pdf and the outage probability in (4), we have
\[
f_{\gamma_1R}(\gamma) = \frac{\partial}{\partial T} \left( \int_0^\infty \frac{\varepsilon(L, \frac{r^2T}{\rho})}{(L-1)!} e^{-\lambda \pi r^2} 2\pi \lambda dr \right)
\] (9)

Since \( H \sim \Gamma(L, 1) \) described in (2), (9) can be written as
\[
f_{\gamma_1R}(\gamma) = \int_0^\infty a^{L-1} e^{-ax} e^{-\lambda \pi r^2} 2\pi \lambda dr,
\] (10)

where \( a = r^\alpha / \rho \).

In the high SNR regime, \( \log_2(1 + \gamma) \sim \log_2(\gamma) \). Substituting (10) into (8), the ergodic capacity expression can be approximated as
\[
C_{1R} \approx \frac{1}{\ln(2)} \int_0^\infty \left( \sum_{i=1}^{L-1} i - C - \ln \left( \frac{\alpha}{\rho} \right) \right) e^{-\lambda \pi r^2} 2\pi \lambda dr
\] (11)

The derived closed-form capacity expression in (11) indicates that the ergodic capacity from the single nearest RRH association strategy is non-linearly increasing with the number of antennas per RRH \( L \), the spatially intensity of RRHs \( \lambda \) and the user’s transmit power \( P_U \). Furthermore, the impact on the ergodic capacity of \( \lambda \) is larger than that of \( L \).

B. \( N \)-nearest RRH association

When associating with the \( N \) nearest RRHs amongst \( N_R \) RRHs, the received SNR with the MRC can be written as
\[
\gamma_N = \sum_{i=1}^{N} \frac{P H_i r_i^{-\alpha}}{\sigma^2}.
\] (12)
For simplicity of description, the case \( N=2 \) will be presented first, followed by the \( N > 2 \) case.

Case 1: Associated With 2 RRHs \((N = 2)\)

When associating with 2 RRHs in terms of the distances of \( r_1 \) and \( r_2 \) (assuming \( 0 \leq r_1 \leq r_2 \)) and the fading gains of \( H_1 \) and \( H_2 \), we have

**Lemma 2:** The outage probability for the 2-nearest RRHs association strategy is

\[
P_{\text{out}2\text{R}} = \Pr \left[ \rho_1^{-\alpha} H_1 + \rho_2^{-\alpha} H_2 < T \right]
\]

\[
\approx \int_{(\rho_1,\rho_2,\rho_1+\rho_2)} \left( \frac{4\pi^2\lambda^2 r_1 r_2 e^{-\pi\lambda^2/2 dr_1 dr_2}}{\left( r_2 - (\theta_1,\theta_2,\rho_2) \right) \left( r_2 - (\theta_1,\theta_2,\rho_1) \right)} \right) e^{-\pi\lambda^2/2 dr_2}.
\]

\( (a) \) follows from the fact that the two shortest distances from the desired user \( U \) are governed by the joint pdf \( f(r_1, r_2) = 4\pi^2\lambda^2 r_1 r_2 e^{-\pi\lambda^2/2} \) (Proof: See Appendix A).

Based on the expectation of \( H_i \) (i.e., \( L_i \)) in \( (b) \), the double integral in \( (a) \) can be changed to be a single integral.

According to the derived single integral in \( (15) \), the SNR pdf for the 2-nearest RRH association strategy can be approximated as

\[
f_{r_{2\text{R}}} (\gamma) = \int_{(2\lambda^2)} e^{-\pi\lambda^2/2 dr_2} d(T - (L\rho)t^{-\frac{\alpha-1}{\alpha}}\rho L e^{-\pi\lambda^2/2 dt}).
\]

(14)

Thus, the uplink ergodic capacity can be characterized by \( (15) \) located on the top of the next page. Note that this formula applies for arbitrary \( \alpha > 2 \), which is an extension of \( (2) \).

Furthermore, for the case of \( \alpha = 4 \), a simple closed-form ergodic capacity expression can be derived as \( (16) \), which shows that the number of antennas \( L \) has the same impact on the capacity as the RRH density \( \lambda^2 \).

Case 2: Associated with \( N \) RRHs \((N > 2)\)

To extend to the arbitrary \( N \) case, the main challenge is that an exact pdf expression for \( \sum_{i=1}^{N} r_i^{-\alpha} \) is difficult to derive. Consider the stochastic geometry property that the points of the two-dimensional PPP of intensity \( \lambda \) can be mapped to a one-dimensional PPP. The pdf of the random variable \( \pi\lambda r_i^2 \) can be expressed as \( f(x) = \frac{1}{\pi} x^{-\frac{1}{\alpha}} e^{-\frac{\alpha}{\alpha} \frac{x}{\alpha}} \). Hence, the expectation of \( r_i^{-\alpha} \) can be written in

\[
E \{ r_i^{-\alpha} \} = (\pi\lambda)^{-\frac{\alpha}{\alpha}} \int_{0}^{\infty} x^{-\frac{1}{\alpha}} f(x) dx = (\pi\lambda)^{-\frac{\alpha}{\alpha}} \Gamma \left( i - \frac{\alpha}{\alpha} \right)
\]

where \( \Gamma \left( i - \frac{\alpha}{\alpha} \right) \) is finite only for the case \( i < \alpha/2 \), and therefore, we should derive the additional part when \( i \geq \lceil \alpha/2 \rceil + 1 \), where \( \lceil \cdot \rceil \) is the floor function. The outage probability can be expressed as

\[
P_{\text{out}N\text{R}} = \Pr \left\{ \sum_{i=1}^{N} H_i r_i^{-\alpha} + \sum_{i=\lceil \alpha/2 \rceil + 1}^{N} \rho L_i r_i^{-\alpha} < T \right\}
\]

\[
\approx \Pr \left\{ \sum_{i=1}^{\lceil \alpha/2 \rceil} \rho H_i r_i^{-\alpha} + \rho L \sum_{i=\lceil \alpha/2 \rceil + 1}^{N} (\pi\lambda)^{2 \Gamma \left( i - \frac{\alpha}{\alpha} \right)} \right\} < T.
\]

(18)

In the special case of \( \alpha = 4 \), \( (18) \) can be simplified to

\[
P_{\text{out}N\text{R}}^\alpha = \Pr \left\{ \sum_{i=1}^{2} \rho H_i r_i^{-4} + \rho L (\pi\lambda)^{2 N - 2} < T \right\}
\]

\[
= \int_{(\pi\lambda r_2)^{\frac{1}{\alpha}}}^{\infty} \frac{2\pi^2\lambda^2 r_2}{\left( r_2 - \frac{\rho L^2}{T} \right)^2} e^{-\pi\lambda^2/2 dr_2}.
\]

(19)

Based on \( (19) \), the uplink ergodic capacity for the \( N > 2 \) RRH association strategy can be derived as \( (20) \).

Since a closed-form pdf expression for the SNR exists in the special case of \( N \to \infty, \alpha = 4 \) as follows

\[
f_{\infty} (\gamma) = \frac{\pi\lambda\sqrt{T^2\pi\lambda}}{2T^{3/2}} \exp \left( -\frac{L\rho^3\lambda^4}{4T} \right),
\]

we can write an upper bound on the ergodic capacity as

\[
C_{\text{upper}} = \int_{0}^{\infty} \frac{\pi\lambda^{2\Gamma \left( i - \frac{\alpha}{\alpha} \right)}}{2T^{3/2}} \exp \left( -\frac{L\rho^3\lambda^4}{4T} \right) \log_2 (1 + T) dT
\]

\[
C \approx \sum_{j=0}^{\infty} \frac{\lambda^{2\Gamma \left( i - \frac{\alpha}{\alpha} \right)}}{(j+1)!} + \ln \frac{1}{4T^4}.
\]

(21)

The derived expression in \( (22) \) shows that the upper capacity bound is related to the parameters \( L, P_U, \) and \( \lambda \), with \( \lambda \) entering quadratically. Hence, compared with \( L \) and \( P_U \), \( \lambda \) is the primary factor impacting on the ergodic capacity.

IV. NUMERICAL RESULTS

In this section, the accuracy of the above closed-form expressions and the impact of \( \lambda, L, \) and \( P_U \) on capacity performance are evaluated. The number of antennas per RRH \( L \) is set 4, and the expected value of \( H_i \) is utilized. The path loss exponent \( \alpha \) is set 4, the radius \( R \) of the disc is set at 600m, and the intensity of RRHs, \( \lambda \), is assumed to be \( 10^{-4}, \) i.e., \( \mu R \approx \pi R^2 \lambda = 11.304. \) The power spectral density \( \sigma^2 \) is\( -174\)dBm/Hz, and the spectral bandwidth is 100MHz.

Fig. 1 shows the ergodic capacity performance under different numbers of association RRHs \( N \) with the varying transmit power \( P_U \), where the capacity grows monotonically as the transmit power increases because the interference can be avoided due to the cooperative processing inherited from the C-RAN architecture. The Monte Carlo simulation results match well with those indicated by the presented closed-form ergodic capacity expressions. When \( L = 4 \) and \( \lambda = 10^{-4}, \) the capacity gain from the single nearest RRH association to the 2-nearest RRH association is significant. However, the capacity gaps among the 4 and 8 and even infinite RRH associations are not large, which indicates that no more than 4 RRHs should be associated for each user when considering the balance between the performance gains and implementation cost.

The impact of the number of antennas per RRH \( L \) on the uplink C-RAN ergodic capacity performance is shown in Fig. 2 where \( P_U = 10mW \) and \( \lambda = 10^{-4}. \) Similarly to the influence of the transmission power \( P_U \) shown in Fig. 1, the uplink ergodic capacity increases with an increasing number of antennas per RRH \( L \), and the performance gain is significant.
when no more than 4 RRHs are associated. Specially, when
fixing $L = 4$ and increasing the RRH association number $N$
from 1 to 2 and from 4 to infinite, the capacity performance
improves about 0.58bps/Hz and 0.28bps/Hz, respectively.
The ergodic capacity for the case of $L = 8$, $N = 2$ is about
9.21bps/Hz, while it is about 8.06 bps/Hz for the case
of $L = 2$, $N = 8$. This result demonstrates that more antennas are
preferred to increase capacity when the RRH density remains static.

\[ C_{2R} = \int_0^\infty \int_0^\infty \frac{2\pi^2 \lambda^2 (L\rho)^{\frac{3}{2}}}{\alpha} (Z - (L\rho) t^{-\frac{3}{2}}) \frac{-\frac{3}{2} e^{-\pi\lambda t} \log (1 + Z)}{dt\,dz}. \]  \hspace{1cm} (15)

\[ C_{\alpha=4}^{2R} = \int_0^\infty \frac{2\pi^2 \lambda^2 (L\rho)^{\frac{1}{2}} e^{-\pi\lambda t}}{4\ln 2} \left\{ \frac{\ln Z}{\sqrt{Z - (L\rho) t^{-2}}} + \frac{2}{\sqrt{(L\rho)t^{-2}}} \arctan \left[ \sqrt{Z - (L\rho) t^{-2}} - \sqrt{(L\rho)t^{-2}} \right] \right\} dt. \]

\[ C_{\alpha=4}^{N=4} = \int_0^\infty \pi^2 \lambda^2 e^{-\pi\lambda t} \sqrt{L\rho t} \left[ \ln (2L\rho + St^2) - \ln (t^2) + \frac{2}{L\rho + St^2} \arctan \left( \sqrt{L\rho + St^2} \right) \right] dt. \]  \hspace{1cm} (20)

V. CONCLUSION

In this paper, closed-form ergodic capacity expressions for
both single nearest and $N$-nearest RRH association strategies
when the pathloss exponent is 4 have been presented. Both
analytical and simulation results have shown that the RRH
association number should not be larger than 4 in order to
balance the performance gain and implementation cost, and the
RRH density $\lambda$ has a greater impact on performance gain than
the number of antennas per RRH $L$ does. However, when $\lambda$
is fixed, more antennas are preferred because this can provide
higher gains than increasing RRH association can.

VI. APPENDIX A

We need the joint distribution probability there is not more
than one RRH within a ring from the radius $r_1$ to $r_2$, that is
\[ \Pr (r_1, r_2) = \Pr (null \in \circ r_1, only one \in \phi_{r_1 r_2}) \]
\[ \cup \Pr (null \in \circ r_1, null \in \phi_{r_1 r_2}), \]  \hspace{1cm} (23)
where $\circ r_1$ denotes the circle centered at the origin of radius
$r_1$, and $\phi_{r_1 r_2}$ denotes the ring centered at the origin of radius
from $r_1$ to $r_2$. Since RRHs are distributed according to the
two-dimensional Poisson process distribution, thus the joint
probability can be written as
\[ \Pr (r_1, r_2) = \left( e^{-\lambda \pi (r_2^2 - r_1^2)} + (\lambda \pi (r_2^2 - r_1^2)) e^{-\lambda \pi (r_2^2 - r_1^2)} \right) e^{-\lambda \pi (r_1^2)}. \]  \hspace{1cm} (24)

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