Neutron-hole strength in the $N = 81$ isotones

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Abstract. The distribution of neutron-hole strength has been studied in the $N = 81$ isotones $^{137}$Ba, $^{139}$Ce, $^{141}$Nd and $^{143}$Sm through the single-neutron removing reactions ($p,d$) and ($^3$He,$\alpha$), at energies of 23 and 34 MeV, respectively. Systematic cross section measurements were made at angles sensitive to the transferred angular momentum, and spectroscopic factors extracted through a distorted-wave Born approximation analysis. Application of the MacFarlane-French sum rules indicate an anomalously low summed $g_7/2$ spectroscopic factor, most likely due to extensive fragmentation of the single-particle strength. Single-particle energies, based upon the centroids of observed strength, are presented.

1. Introduction

Interest in the evolution of single-particle energies (spe's) has been stimulated in recent years by theoretical studies of effective nucleon-nucleon interactions and their effect on nuclear shell structure [1]. It is suggested that the tensor component of the interaction may be responsible for driving relative shifts in spe's between high-$j$, spin flip orbitals. In brief, a repulsive interaction is predicted between $j_>$ ($j = \ell + s$) or $j_<$ ($j = \ell - s$) pairs, and an attractive interaction between $j_>$ and $j_<$ pairs. This interaction is expected to be greatest between high-$\ell$ orbitals, and further enhanced where the radial overlap is large.

A prime example of this effect is seen in the Sb isotopes, where a splitting of $\pi g_{7/2}$ ($j_<$) and $\pi h_{11/2}$ ($j_>$) energies has been observed with increasing neutron excess [2]; this is attributed to the filling of the $j_>$ orbital, repelling the $\pi h_{11/2}$ while attracting the $\pi g_{7/2}$. A similar energy splitting has also been observed in the $N = 83$ system between $i_{13/2}$ and $h_{9/2}$ orbitals, with increasing numbers of $g_{7/2}$ protons [3]. The magnitude of the splitting in both cases is in good agreement with calculations that include a tensor interaction.

If the inclusion of the tensor force is required to produce a robust effective interaction, its effect should be apparent also in the single-hole structure beneath the $N = 82$ shell closure. In particular the high-$\ell$ $g_{7/2}$ and $h_{11/2}$ orbitals should be modified by the increasing $\pi g_{7/2}$ occupancy. The existing transfer data for this system are from multiple disparate studies, introducing significant systematic uncertainties which are reflected in inconsistencies between the reported spectroscopic data (see for example Refs. [4–8]). The aim of this work is to obtain a consistent set of spectroscopic data across the odd-mass isotones $^{137}$Ba, $^{139}$Ce, $^{141}$Nd and $^{143}$Sm, Published under licence by IOP Publishing Ltd
with particular emphasis on minimising systematic uncertainties. The interest in high-$j$ strength necessitates the use of a reaction with large $Q$-value, well matched for $\ell$ transfer of 4-5 $\hbar$. For this purpose the $(^3\text{He},\alpha)$ reaction was used. In the interest of also identifying low-$j$ strength, and providing higher-resolution data, the $(p,d)$ reaction has been included in this study as well.

2. Experimental details
Experimental data were collected at the Wright Nuclear Structure Laboratory located at Yale University. The ESTU tandem accelerator was used to deliver beams of protons and $^3\text{He}$ nuclei, at energies of 23 and 34 MeV, respectively, onto carbon-backed $N = 82$ oxide targets of thickness 40-140 $\mu$g/cm$^2$. Outgoing light reaction products were momentum analysed using an Enge split-pole spectrograph and transported to a position-sensitive ionisation chamber (ic). Ions were stopped after leaving the ic in a 6.4-mm thick plastic scintillator. Particle identification was achieved through comparison of the ion energy losses in the ic and scintillator.

Representative focal-plane position spectra are shown in Fig. 1 for both $(p,d)$ and $(^3\text{He},\alpha)$ data, calibrated to give excitation energy in the residual nucleus. Excitation energy resolutions of approximately 25 and 75 keV were obtained for the $(p,d)$ and $(^3\text{He},\alpha)$ data, respectively. With reference to Fig. 1, the influence of reaction $Q$-value on angular momentum transfer is readily apparent.

![Figure 1.](image_url) 

Figure 1. (Colour online) Focal plane spectra from the $(p,d)$ and $(^3\text{He},\alpha)$ reactions at 42$^\circ$ and 15$^\circ$, respectively, on a $^{142}$Nd target. The $(^3\text{He},\alpha)$ data are scaled by a factor of 0.5 in the inset spectrum to more clearly show the structure above 2 MeV excitation. Low-lying peaks have been labelled with the transferred angular momentum to highlight the dependence on reaction $Q$-value.

The total beam incident on each target was recorded using a current integrator connected to a tantalum beam stop immediately behind the target position. Both beam stop and target were biased to $+300$ V to suppress electron sputtering. A Si monitor detector mounted inside the target chamber, at 30$^\circ$ relative to the beam axis, provided a constant monitor of the target integrity through elastically scattered beam; no changes in the ratio of scattered to on-target beam current above the few percent level were observed throughout the experiment,
indicating negligible change in target thickness. To enable absolute cross section measurements a calibration of target thickness and spectrometer entrance aperture was performed with 15-MeV $\alpha$-particles at a laboratory angle of 20°; under these conditions the total elastic cross section is Rutherford to within 1%.

For each target cross section data were measured at angles of 5°, 20°, 35° and 42° using ($p, d$), corresponding to the peaks of the angular distributions for $\ell$ transfer of 0, 2, 4 and 5, respectively ($\ell = 0$ transfer actually peaks at 0°, however 5° is the furthest forward angle experimentally accessible). Cross sections for ($^3$He,$\alpha$) were measured at 5° and 15°, corresponding to where the ratio of $\ell = 4$ to $\ell = 5$ yield is expected to be greatest.

3. Results

The cross section data have been analysed within the framework of the distorted wave Born approximation (DWBA) in order to extract spectroscopic information. DWBA calculations were performed using the code PTOLEMY [9]. Optical model parameters for protons and deuterons were taken from Refs. [10, 11] and parameters for $^3$He and $\alpha$-particles from Refs. [12, 13]. A wider range of potentials from the literature were selected to assess the systematic uncertainty introduced by the choice of potentials. It was found that the absolute spectroscopic factor varied by $\sim$20% between optical parameters, however the relative change between different $\ell$ or target nuclei was only $\sim$5%. The choice of bound state parameters, particularly the radii, also had a dramatic impact on absolute spectroscopic factors, but only affected relative values at the $\sim$10% level.

![Figure 2](Image)

Figure 2. (Colour online) Example angular distributions for (a) $\ell = 4$ and 5 transfer from ($^3$He,$\alpha$) on $^{138}$Ba and (b) $\ell = 0$ and 2 transfer from ($p, d$) on $^{142}$Nd. The solid curves represent DWBA calculations for the assigned $\ell$ fitted to the data, and are labelled according to the excitation energies of states in the residual nucleus. The dotted curves in (a) show whichever $\ell$ was not assigned for comparison. Note that the data for $\ell = 0$ in (b) are scaled by a factor of 1/100.
States populated through $\ell = 0, 2, 4$ and 5 transfer are assumed to carry $3s_{1/2}$, $2d_{3/2,5/2}$, $1g_{7/2}$ and $1h_{11/2}$ strength, respectively. While the majority of states observed have $\ell$ established through previous work (for example Refs. [4–8]), these assignments were able to be confirmed, and unknown states identified, through comparison of angular distributions to DWBA predictions. Sample distributions are shown for reference in Fig. 2.

A normalisation, $N$, is applied to the ‘absolute’ spectroscopic factors, taken as the ratio of experimental to DWBA cross section, to yield relative values such that the summed spectroscopic strength for a given orbital reflects its total occupancy. Since the $N = 82$ nuclei have a magic number of neutrons, all orbitals beneath the shell closure are assumed to be fully occupied; therefore

$$N \sum S_i = 2j + 1,$$

(1)

where $S$ is the spectroscopic factor and the sum runs over states of one particular $n, l, j$.

As per Eqn. 1, the spectroscopic strength associated with each single-particle orbital was summed for each $N = 81$ isotope. For the $(p, d)$ data a normalisation of $N_{(p,d)} = 1.72 \pm 0.06$ was found based upon the total $s_{1/2}$ strength observed and for $(^3\text{He}, \alpha)$ a value of $N_{(^3\text{He},\alpha)} = 1.08 \pm 0.06$ was determined from the $h_{11/2}$ strength. Fig. 3 shows the variation in summed strength for each $\ell$ across the isotones, expressed as the fractional occupancy. Apart from $g_{7/2}$, the summed occupancies are consistent both with each other and between isotones to within the $\sim 15\%$ level expected.

**Figure 3.** (Colour online) Summed spectroscopic strength for each $\ell$ using data from (a) $(p, d)$ and (b) $(^3\text{He}, \alpha)$, expressed as the fraction of expected strength for full occupancies. A common normalisation is applied to the data as discussed in the text. The solid lines indicate confidently assigned strength while the dotted lines include also tentatively assigned states. Since $d_{3/2}$ and $d_{5/2}$ strength are indistinguishable in this work a combined result for $\ell = 2$ is provided, assuming a total occupancy of 10.

It should be noted that the population of $d_{3/2}$ and $d_{5/2}$ strength both occur through $\ell = 2$ transfer and so are indistinguishable by angular distributions alone. For this reason the two
strengths are combined in Fig. 3 and taken to have a combined occupancy of 10. It is further noted that \( \sim 40\% \) of the total \( \ell = 2 \) strength is consistently found in the ground state peaks which are known to have spin \( 3/2^+ \). It may therefore be inferred that all excited \( \ell = 2 \) strength is characterised by \( d_{5/2} \).

The ‘missing’ \( g_{7/2} \) strength could be interpreted as a non-full occupancy; this is extremely unlikely, however, given that it is well beneath the Fermi surface and that no significant \( \ell = 4 \) strength has been observed in neutron adding reactions on \( N = 82 \) (see for example Ref. [3]). A more plausible explanation is that the \( g_{7/2} \) strength is severely fragmented and that the missing strength is accounted for in either weak states masked by the continuum background, or above the \( \sim 3.5 \) MeV excitation window studied in this work.

Single-particle energies are reconstructed from the spectroscopic data using a weighted sum over the observed states

\[
E_{n,l,j} = \frac{\sum_i E_i S_i}{\sum_i S_i},
\]

where \( E_i \) and \( S_i \) are the energies and spectroscopic factors for all states of a given \( n, l, j \). These centroids are shown in Fig. 4 as a function of the binding energy of the orbital. The \( d_{3/2} \) and \( d_{5/2} \) values are subject to the assumptions discussed above. The values for \( g_{7/2} \) should be taken as upper limits on the binding energies, due to the apparent non-observation of higher excitation fragments mentioned earlier.

![Figure 4. (Colour online) Single-particle binding energies reconstructed from the fragmented hole-state strength. The errors shown are only statistical and the dotted curves are to help guide the eye. The result for \( g_{7/2} \) can be interpreted as an upper limit on the binding energy, see the text for details.](image)

A more detailed description and interpretation of this work is forthcoming.

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