Damage identification in beams using discrete wavelet transforms
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ABSTRACT
Researchers, over the years, have done extensive research on damage identification. In the recent years, there has been an increased impetus to conduct research on damage identification using wavelet transforms. In the present work, discrete wavelet transform (DWT) has been used to detect, locate and quantify damage in beam elements. A generalised method has been proposed to solve damage identification in beams with a single dominant crack. The proposed quantification methodology uses only the measurements made in the current (damaged) state of the beam structure. The generalised method has been proposed using a Damage Intensity Factor (DIF) to quantify damage in all beams with similar edge conditions. Formulae have also been proposed to help in quantifying the damage in simply supported and fixed beams. The proposed method has been validated using problems taken from the literature. The applicability of the proposed method to solve multiple damage cases has been demonstrated using literature problems.

Keywords: Discrete Wavelet Transform, damage identification, simply supported beam, fixed beam, damage quantification, finite element analysis.

1. Introduction
Occurrence of damage is very common in all structures. The most common form of damage in beams occurs as dominant crack(s). Damage in any form degrades the performance of the structure, sometimes also increasing the vulnerability to failure. Hence, damage identification is performed to detect, locate and quantify damage in the structural elements. Damage identification helps plan the repair works required to restore the structure to normal use.

Damage identification has been an area of active research over the years. The most commonly used damage identification techniques are ultrasonic methods (Frietzen et al., 2002) and vibration-based methods (Rytter, 1993). The use of ultrasonic and vibration based methods have their own well-known short-comings in field conditions. Hence, there has been a continued search for identifying more effective and efficient damage identification techniques. In the recent years, researchers have been investigating the applicability of wavelet transform to perform damage identification for structural applications. Wavelet transform is a powerful signal processing tool. It is capable of identifying even small changes in the signal that is being processed. Hence, wavelet transform can be used for identifying even small changes in the structural response caused due to small damages.

Wavelet transform (WT) has been investigated for damage identification applications in civil engineering structures, since the 1990s. The first researcher known to have applied wavelets to vibration analysis problem is Newland (1994, “a”; 1994, “b”). Several works have been carried out to investigate the applicability of vibration characteristics of structures (mode shapes and frequencies) to identify the damage (Gentile and Messina, 2003; Kim et al., 2006;
Hong et al., 2002; Chang and Chen, 2003 and 2005). But, it is well known that the frequency-based methods have a few weaknesses. Small local damages do not greatly affect the frequency. Hence, the insensitivity problem is unavoidable. Similarly, there are a few drawbacks in employing the mode shape, as sufficient resolution is required for singularity problems near the inflection points of the mode shapes. This makes it difficult to detect the damage, especially in large structures. In the same way, the difficulty in employing mode shape curvature is that even the small errors in mode shape measurements get magnified in the central difference process. Similarly, identifying the mode shape of tall and large structures is very difficult.

Hence, there is continued research to identify observable signals which are sensitive to small damage(s) occurring in the structure and to arrive at the methodology that can help to effectively detect, locate and quantify damage. Haase and Widjajakusuma (2003) used different bars to analyse the transient vibration measurements due to impact loading to identify the ridges and maxima in the CWT coefficients. The applicability of wavelet packets for damage identification was investigated both analytically and experimentally by using the dynamic signals generated with impulse loading (Han et al., 2005). Ovanesova and Suárez (2004) used CWT of static and dynamic deflection responses to detect damage location in beams and plane frames. Pakrashi et al (2006) used the first mode shape and static deflection under a vertical load to solve the damage identification problem. A new wavelet-kurtosis based calibration scheme was also performed. Bridge beam structures have been investigated under moving loads by Zhu and Law (2006) using the deflection response to perform damage identification. Li et al (2006) used flexural wave obtained from FEM and experiments to locate the damage, its extent and orientation. Pai and Young (2001) used the operational deflected shapes to identify damage. Rucka and Wilde (2006, “a”) used the static deflection to prove the ability of wavelet transforms in crack identification. Both analytical and experimental investigation results were successfully used to solve the damage identification problem. Recently, Kim and Melhem (2004) made a detailed review of the damage detection works done using wavelet analysis. Although several structural responses are continuously being tried out, one of the structural responses that has been successfully tried and tested is the deflection profile/response of the structural system. Hence, we have used the deflection response in our work.

In the present work, discrete wavelet transform has been used to carry out the studies. It has been well established that wavelet transform can be used to detect and locate damage. The focus of the work is to propose a methodology for quantification of the damage using only the current damaged state of the beam. For this a parameter for representing the damage level has been proposed. The variations in the values of this parameter, namely the Damage Intensity factor (DIF), for various damage intensities have been studied and formulae have been arrived. Then, the methodology has been extended to solve the damage quantification problem in all types of beams with different geometrical and / or material properties, subjected to different load intensities. Both simply supported and fixed beams have been used for the studies. Later, the method has been extended to check for its applicability to solve multiple dominant cracks, by solving problems taken from the literature.

2. Damage identification using wavelet transforms

Damage identification has been performed in different levels in structures. They are: Level 1: detection of damage; Level 2: identifying location of damage; Level 3: quantification of the level of damage; and Level 4: estimation of the remaining service life. The present work focuses on performing damage identification using the first three levels.
2.1 Damage modelling – Single dominant crack

One of the key issues in solving a damage identification problem analytically is to model the damage correctly. Damage can be modelled as a reduction in stiffness, represented by reducing the flexural rigidity. Some researchers use rotational flexible springs to represent the flexibility in the damaged element. In this work, the first method of reducing the flexural rigidity of the damaged element has been employed. But the mass of the damaged element is assumed to remain constant.

In the present work, finite element modelling is used to represent the beam. The damage case considered is that of a beam with single dominant crack. The flexural rigidity of the undamaged finite element is \( EI = E \frac{bd^3}{12} \) where \( E \) is the Young’s modulus, \( I \) is the moment of inertia, \( b \) is the breadth and \( d \) is the depth of the beam. The damage is represented by a finite element with reduced flexural rigidity given by \( EI_{\text{damaged}} = E b \left( \frac{d - d_{\text{crack}}}{12} \right)^3 \), where \( d_{\text{crack}} \) is the crack depth in the finite element with crack, at \( x \) from the left end. This results in reduced stiffness. The schematic representation of the beam is shown in Figure 1.

![Figure 1: Modelling of the beam with a dominant crack](image)

2.2 Damage detection and location

The change in the stiffness of the damaged finite element causes a change in the structural response. This cannot be noticed through our naked eye, especially for small changes. On performing the wavelet transform on the structural response, one can get the exact locations where the changes in characteristics occur in the signal. In the present work, static deflection has been used as the signal for performing damage identification.

**Basics of wavelet transforms**

The mother wavelet forms the basis of the wavelet analysis. The mother wavelet is a waveform that has limited duration and an average value of zero. Based on this mother wavelet, the wavelet kernel can be expressed by

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi \left( \frac{t - b}{a} \right)
\]

where \( t \) is the time, \( a \) and \( b \) are real. Here, the scale and translation parameters \( (j \) and \( k) \) are chosen based on powers of two, and there exists \( \psi(t) \) with good time-frequency localisation property. The set of functions \( \psi(t) \) are denoted as
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\[ \psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \]  \hspace{1cm} (2)

which constitute an orthonormal basis for \( L^2(\mathbb{R}) \). Here \( j \) and \( k \) are integers and \( L^2(\mathbb{R}) \) denotes the class of measurable square-integrable function \( x(t) \) in \( \mathbb{R} \). Any signal \( x(t) \) in \( L^2(\mathbb{R}) \) can be expressed as a sum over \( j,k \) of the product of [the inner product of \( f \) and \( \psi_{j,k} \)] and \( \psi_{j,k}(t) \), expressed as

\[ x(t) = \sum_{j,k} \langle f, \psi_{j,k} \rangle \psi_{j,k}(t) \]  \hspace{1cm} (3)

This is called the discrete wavelet transform (DWT). One of the important characteristics of wavelet which help in selection of a particular wavelet for a particular problem is the number of vanishing moments of the wavelet. The \( n^{th} \) moment of a function \( \psi(t) \) is represented by

\[ \int_{-\infty}^{\infty} t^n \psi(t) dt \]  \hspace{1cm} (4)

When the wavelet’s \( m+1 \) moments are zero,

\[ \int_{-\infty}^{\infty} t^n \psi(t) dt = 0 \] for \( n=0, \ldots, m \)  \hspace{1cm} (5)

then it is said that the number of Vanishing Moment of the wavelet is ‘\( m \)’. When a wavelet has ‘\( m \)’ vanishing moments, suppression of signals that are polynomials of a degree lower than or equal to ‘\( m \)’ is ensured. In other terms, if a wavelet is ‘\( m \)’ times differentiable, the wavelet has at least ‘\( m \)’ vanishing moments. More theoretical details on wavelet transform can be obtained from Newland (1993).

**Spline extrapolation – Edge distortion**

During wavelet analysis, peaks in the wavelet coefficients also occur at the supports. These peaks (at the supports) are misleading in detecting damage and also influence the magnitude of the peak coefficients at the damage locations, if the damage is present near the support. For this, the signal to be analysed is extrapolated at the edges using spline for a specified length. The length for which the signal is to be extrapolated is determined by trial and error. Then, the wavelet analysis shall be carried out. After performing the wavelet analysis, the coefficients corresponding to the extrapolated region of the signal shall be removed for further analysis. Now, it can be observed that wavelet coefficient peaks do not occur at the supports. Thus, even damages near the support can be identified. This spline based extrapolation technique has been proposed by Rucka and Wilde (2006, “b”). In the present work, the spline based extrapolation technique has been coded and applied to all signals before applying wavelet transforms, to avoid the edge distortion problem.

**2.3 Damage quantification**

After performing the wavelet analysis, the best available information is the wavelet coefficients and the location of damage. It is with this knowledge that the quantification of the damage needs to be done. In the present work, a Damage Intensity Factor (DIF) has been proposed, to represent the level of damage. Damage Intensity Factor (DIF) is defined as the
ratio of the wavelet coefficient at the damage location when the load is placed on the damage location (WC\textsubscript{damage}) to the unbiased wavelet coefficient obtained at the load location when the damage is not at or near the load location (WC\textsubscript{Wloc}). This can be represented as

\[
\text{DIF} = \frac{\text{WC}}{\text{WC}}\text{\textsubscript{Wloc}}
\]

It shall be noted that the DIF is computed using the wavelet coefficients obtained during the analysis. The DIF is found to be dependant on the location of damage and the level of damage. Extensive studies have been carried out to study the variation of this factor and formulae have been proposed relating the DIF and level of damage, for all damage cases in beams with similar edge conditions. Hence, with the knowledge of the DIF and the location of damage, we can easily quantify the level of damage.

3 Proposed methodology

The present work focuses on damage identification using discrete wavelet transform (DWT). Based on the earlier studies (Sivasubramanian et al, 2007), symlets with four vanishing moments have been found to be effective and have been used for damage identification in the present work. The static deflection has been considered as the measured signal, to detect the location and to assess the damage level. Single span beams with simply supported and fixed edge conditions have been considered for the studies.

In all, only three measurements are required to solve the damage identification problem. As discussed earlier at least two measurements have to be made at first to detect and locate the damage. In the present work, it has been decided to place the load at 0.33L and 0.67L to obtain the two measurements, respectively. The magnitude of the load is kept constant for the complete analysis (to obtain the three measurements). Spline extrapolation is carried out in all the cases to avoid edge distortion in the signal (Rucka and Wilde, 2006 “b”). Now, consider a case where the damage is at 0.3L for some level of damage. It can be observed that on applying the wavelet transform on the first signal, the wavelet coefficient peak under the load (at 0.33L) is biased and influenced by the coefficient peaks near the damage location (at 0.3L). But, the measurement obtained by placing the load at 0.67L is used to determine the unbiased wavelet coefficient peak under the load (WC\textsubscript{Wloc}). From the two measurements, with the load locations known and the distortions in the edges avoided, the damage location(s) can be identified by noting the occurrence of the wavelet coefficient peak, at the damage point. It has been observed through the studies carried out by us that the value of WC\textsubscript{Wloc} remains constant irrespective of changes in load positions or boundary conditions.

Now, the third measurement is obtained by placing the load at the damage location and measuring the response once again. Apply spline extrapolation and then perform wavelet transform on the signal. Obtain the wavelet coefficient at the damage point (WC\textsubscript{damage}), where the load is also placed. Based on the studies conducted, this wavelet coefficient (WC\textsubscript{damage}) is observed to vary with change in damage location and damage level. Also, it varies with the change in geometrical and material properties of the beam. On analysing the results of the studies conducted in the present work, it has been proposed to use a Damage Intensity Factor (DIF) for representing the quantity of damage. The main advantage is that the DIF for a particular level of damage occurring at a particular location remains constant for all beams with similar edge conditions. This is also highlighted with the help of numerical studies later.
4. Damage identification as an inverse problem

Damage detection can be solved as an inverse problem. Inverse problems are concerned with the determination of inputs or causes (loads, stresses) from the observed output or responses (i.e. strains, displacements, natural frequencies, mode shapes, modal curvatures, etc.), in contrast to direct problems in which outputs or responses are determined using knowledge of the inputs or sources.

In order to perform the studies the following assumptions are made:

1. The finite element model developed to represent the damaged beam satisfies the Euler’s beam theory.
2. Field measurements are error free and perfect. Here, error refers to any noise that occurs in the measurement. Noisy data can lead to misleading results.
3. The structural response (deflection) is measurable with sufficient accuracy, although there are issues related to the number of sensors, their placement, and the level of accuracy in measurement.
4. The beam has only one dominant crack in the beam. Although smeared crack is possible in concrete members; it is assumed that the final failure occurs because of propagation of a single dominant crack due to strain localization.
5. The damage is considered as non-progressive, at least while taking the measurements.

Since damage is represented by stiffness and since stiffness of an element depends on system characteristics like Young’s modulus ($E$), length ($L$), cross-sectional dimensions like breadth ($b$) and depth ($d$), load ($W$), load location ($W_{loc}$) and edge conditions, the influence of these characteristics have been considered for further studies. Density of the structural member which is a parameter representing material property, does not directly affect the stiffness of the beam and hence the deflection of the beam, its influence has not been considered for studies in the present work.

Let us now define, the damage location ratio ($\lambda$) as a ratio of the location of damage from the left end ($x$) to the total length of the beam ($L$).

$$\lambda = \frac{x}{L} \quad (7)$$

Similarly, the crack damage level ($\phi$) shall be represented as a ratio of the effective depth with cracking ($d - d_{crack}$) to the total depth of the beam ($d$).

i.e. $$\phi = \frac{d - d_{crack}}{d} \quad (8)$$

We need to study the relation between the DIFs for different damage locations and damage levels. The DIF will depend on the wavelet transform coefficients and therefore on the signal characteristics. In the context of damage detection for a particular problem, the geometric and material properties remain constant. Also in the present work, the load intensity has been kept constant to obtain the three measurements. Now the relation between DIF, damage location ratio and the crack damage level can be represented as

$$\therefore \text{DIF} = F(\lambda, \phi) \quad (9)$$
From Equation (9), it can be observed that for any damage of intensity $\phi$ occurring at $\lambda$, a unique intensity factor (DIF) exists. For any problem, the damage location can be obtained by performing a DWT on the deflection of the beam. Knowing $\lambda$ and the DIF, the damage level $\phi$ need to be determined. Now, the crack identification inverse problem can be described by

$$(\phi) = G^{-1}(\text{DIF}, \lambda)$$  \hspace{1cm} (10)$$

It can be observed that if this relation (given in Equation (10)) can be established, then given the DIF and the damage location, the damage level can immediately be obtained. On considering the effect of material and geometric properties and load intensity on the DIF, the generalised relation can be arrived to solve all problems, irrespective of changes in material or geometric properties or load intensity.

5 Numerical studies and discussion

5.1 Simply supported beam

The effectiveness of the proposed damage quantification methodology has been demonstrated through numerical simulations conducted on a literature problem (Problem 1) given by Chang and Chen (2005). The beam is of length 1m and is considered to be simply supported at both the ends. The depth of the beam ($d$) is 2.5cm and the breadth ($b$) is 1.5cm, the Young's modulus ($E$) is 210 GPa and the density ($\rho$) is 7870kg/m$^3$.

A point load ($W$) of 2 kN is applied on the beam at ‘Wloc’ from the left of the beam to obtain the deflection profile. ‘Wloc’ will be 0.33L or 0.67L or ‘x’ for the first, second and third measurements, respectively. The beam has been discretised into hundred finite elements to perform the analysis. The deflected shape of the damaged beam is used as the signal for damage identification.

Using the wavelet transform of the signal, the location of damage can be obtained. It shall be noted that the spline extrapolation technique is used on all signals before applying the discrete wavelet transform. Then, using the procedure discussed earlier, the DIF at the damage location can be obtained for the damaged beam.

**Damage Quantification**

In order to arrive at a quantification procedure for the problem, a detailed study is conducted to obtain the DIF values (based on Equation (6)) for various damage levels ($\phi$) occurring at different locations ($\lambda$) along the length of the beam. The values are plotted in Figure 2.
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Figure 2: DIF values for different damage locations and damage levels for a simply supported beam

The values given in Figure 2, help in understanding the relation between the (unknown) level of damage and the two known values, namely the location of damage and the DIF. From the figure, we observe nine curves are plotted for damages of 5% to 45% occurring at 0.1L to 0.9L, at every 0.1L. From the figure it is observed that the DIF obtained at x and (L-x) are nearly equal, for the same damage levels. Thus, we can approximate the data by considering only five inverse relation curves corresponding to crack locations 0.1L to 0.5L, that is by assuming the data to be symmetrical about the mid-point. By fitting polynomial curves to these curves we obtain

\[
\phi_{(0.1L)} = -5.89e + 0.05782\times DIF^2 + 0.2206\times DIF^3 + 4.598\times DIF - 5.644
\]

(11)

\[
\phi_{(0.2L)} = -5.72e - 0.06\times DIF^2 + 0.0009783\times DIF^3 - 0.06517\times DIF^3 + 2.407\times DIF - 2.68
\]

(12)

\[
\phi_{(0.3L)} = -1.91e - 0.06\times DIF^2 + 0.004258\times DIF^3 - 0.03703\times DIF^3 + 1.794\times DIF - 1.818
\]

(13)

\[
\phi_{(0.4L)} = -1.11e - 0.06\times DIF^2 + 0.0002832\times DIF^3 - 0.02809\times DIF^3 + 1.555\times DIF - 1.479
\]

(14)

\[
\phi_{(0.5L)} = -9.42e - 0.07\times DIF^2 + 0.0002497\times DIF^3 - 0.0258\times DIF^3 + 1.489\times DIF - 1.384
\]

(15)

Thus, the inverse damage quantification problem for this beam problem can be solved using Equation (11) to (15) and by interpolating the values for the intermediate points. The method discussed so far can be used to solve any damage case, but only in the Problem 1.

Generalising the methodology

Now, let us generalise the methodology for all similar beam problems. For this the variations in DIF values caused by varying the geometrical or material properties have been studied. To carry out these studies, the properties of the beam in Problem 1 (Chang and Chen, 2005) has been modified one at a time to arrive at five new problems, the details of which are given in Table 1. Problems 2 to 4 are arrived by varying the geometrical properties and the Problems 5 and 6 are obtained by changing the load intensity and material property. All these problems are arrived by arbitrarily varying one of the properties by a small magnitude.
Table 1: Various problems considered for numerical simulations

| Problem   | L      | b      | d      | W    | E     |
|-----------|--------|--------|--------|------|-------|
| Problem 1 | 1.00 m | 0.015m | 0.025m | 2000N| 206Gpa|
| Problem 2 | 1.25 m | 0.015m | 0.025m | 2000N| 206Gpa|
| Problem 3 | 1.00 m | 0.025m | 0.025m | 2000N| 206Gpa|
| Problem 4 | 1.00 m | 0.015m | 0.035m | 2000N| 250Gpa|
| Problem 5 | 1.00 m | 0.015m | 0.025m | 3000N| 206Gpa|
| Problem 6 | 1.00 m | 0.015m | 0.025m | 2000N| 250Gpa|

DIF values have been obtained for some arbitrarily presumed damage instances of Problem 1. The details of the damage cases considered and the corresponding DIF values obtained are given in Table 2. Similarly, the DIF values are obtained for Problems 2 to 6. The error in percentage between the DIF obtained for Problem 1 and that of the other problems are computed and plotted in Figure 3. From the figure, it can be observed that the error between the DIF obtained for the six different problems is negligible (less than 0.01%). Hence, it can concluded that the DIF values obtained using Problem 1 can be used to solve the damage quantification in any simply supported beam problem, irrespective of the changes in the geometrical properties, material property and magnitude of the point load. Since, the DIF values remain constant the inverse relations represented using Equation (11) to (15), obtained for Problem 1, also remain the same for all simply supported beams irrespective of the changes in geometrical or material properties or load intensity.

From the results shown in Table 2 and Figure 3, an important observation also made from Figure 1, has been noted. From Figure 1, it has been observed that the DIF values obtained for different levels of damage at x and (L-x) are the nearly same. The same observation has been observed with the help of DIF values obtained for Problems 1 to 6. Table 3 shows the error in percentage between the DIF values obtained at 0.25L and 0.75L, for different damage levels. It can be observed that the variations, represented using the percentage error, is very small (less than 2 percentage). Hence, it can be assumed that the DIF values vary symmetrically about the mid-point of the beam, when symmetry exists in the beam, in the structural properties and boundary conditions.

Table 2: DIF obtained for different damage cases of Problem 1 – simply supported

| Damage case | Damage location in L | Damage percentage | WC_{loc} | WC_{damage} | DIF |
|-------------|----------------------|-------------------|----------|-------------|-----|
| Problem 1   | 0.25                 | 10                | -4.91E-07| -5.53E-08  | 8.875577|
|             | 0.25                 | 20                | -1.17E-06| -5.53E-08  | 21.192505|
|             | 0.25                 | 30                | -2.30E-06| -5.53E-08  | 41.579937|
|             | 0.5                  | 10                | -4.85E-07| -5.53E-08  | 8.780382|
|             | 0.5                  | 20                | -1.11E-06| -5.53E-08  | 20.156818|
|             | 0.5                  | 30                | -2.16E-06| -5.53E-08  | 38.987646|
|             | 0.75                 | 10                | -4.99E-07| -5.53E-08  | 9.022627|
|             | 0.75                 | 20                | -1.19E-06| -5.53E-08  | 21.569628|
|             | 0.75                 | 30                | -2.34E-06| -5.53E-08  | 42.337620|
Table 3: Test for symmetry in the DIF values of Problem 1 to 6

| Error in % between 0.25L and 0.75L | Problem 1 | Problem 2 | Problem 3 | Problem 4 | Problem 5 | Problem 6 |
|-----------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| 10                                | 1.630     | 1.630     | 1.630     | 1.630     | 1.630     | 1.630     |
| 20                                | 1.748     | 1.748     | 1.748     | 1.748     | 1.748     | 1.748     |
| 30                                | 1.790     | 1.790     | 1.789     | 1.790     | 1.790     | 1.790     |

Figure 3: Comparison of error in DIF for problems 2 to 6 w.r.t. that of problem 1

Validation Studies

The proposed method has been validated using three problems taken from the literature. The details related to the validation problems (VP) are given in Table 4. Several damage cases have been simulated and the proposed method has been used to solve the damage identification problems. The location of damage has been precisely determined by applying the wavelet transform. The level of damage has been computed by interpolating the data represented in Figure 1 or by using the Equation (11) to (15). The errors in estimating the level of damage are given in Table 5. From the Table, it is clear that the damage estimated using the equations are comparable to those obtained using the Figure 2. Also, the errors in estimating the damage levels are smaller in both the cases. Hence, the equations can be used for further studies.

Table 4: Details of the validation problems

| Validation problem Number | L in mm | b in mm | d in mm | E in N/m² | ρ in kg/m³ | Reference        |
|---------------------------|--------|--------|---------|-----------|------------|-----------------|
| VP1                       | 300    | 20     | 20      | 3.10E+09  | 1000       | Douka et al (2003) |
| VP2                       | 1200   | 20     | 20      | 7.00E+10  | 2700       | Hong et al (2002)  |
| VP3                       | 515    | 12     | 20      | 2.06E+11  | 7917       | Xiang et al (2006) |
Table 5: Validation studies results for simply supported beam

| Val. Prob. VP−i | Simulated crack damage | W in N | WC\textsubscript{damage} | WC\textsubscript{loc} | DIF | Estimated crack damage using Figure 2 |
|-----------------|------------------------|-------|--------------------------|----------------------|-----|--------------------------------------|
|                 |                        |       |                         |                      |     | Loc. in L | Level in % (x) |
| VP1             | 0.2 7 3000 -1.01E−06  -2.2E-07 | 4.538278 | 0.2 | 6.9345 |
| VP1             | 0.5 7 3000 -1.39E−06  -2.2E-07 | 6.265674 | 0.5 | 6.9254 |
| VP1             | 0.7 7 3000 -1.22E−06  -2.2E-07 | 5.507629 | 0.7 | 6.8973 |
| VP1             | 0.2 12 3000 -1.63E−06  -2.2E-07 | 7.33003 | 0.2 | 12.0884 |
| VP1             | 0.5 12 3000 -2.37E−06  -2.2E-07 | 10.6525 | 0.5 | 12.0247 |
| VP1             | 0.7 12 3000 -2.04E−06  -2.2E-07 | 9.195778 | 0.7 | 12.0383 |
| VP2             | 0.2 14 1500 -2.72E−06  -3.1E-07 | 8.639037 | 0.2 | 14.0623 |
| VP2             | 0.5 14 1500 -4.00E−06  -3.1E-07 | 12.7025 | 0.5 | 13.9953 |
| VP2             | 0.7 14 1500 -3.44E−06  -3.1E-07 | 10.9193 | 0.7 | 14.0453 |
| VP2             | 0.5 18 1500 -3.67E−06  -3.1E-07 | 11.6493 | 0.2 | 17.8478 |
| VP2             | 0.7 18 1500 -5.49E−06  -3.1E-07 | 17.4278 | 0.5 | 18.0009 |
| VP2             | 0.7 18 1500 -4.69E−06  -3.1E-07 | 18.4920 | 0.7 | 17.914 |
| VP3             | 0.2 23 2500 -3.84E−07  -2.3E-08 | 16.3464 | 0.2 | 23.1382 |
| VP3             | 0.5 23 2500 -5.83E−07  -2.3E-08 | 24.8007 | 0.5 | 22.9902 |
| VP3             | 0.7 23 2500 -4.95E−07  -2.3E-08 | 21.0907 | 0.7 | 23.0366 |
| VP3             | 0.2 25 2500 -6.74E−07  -2.3E-08 | 28.6871 | 0.2 | 32.0659 |
| VP3             | 0.5 25 2500 -1.04E−06  -2.3E-08 | 44.1718 | 0.5 | 32.0648 |
| VP3             | 0.7 25 2500 -8.78E−07  -2.3E-08 | 37.3765 | 0.7 | 31.9566 |

| Val. Prob. VP−i | Simulated crack damage | W in N | (x−y1) | Error using Eq. (11) to (15) | Error using Eq. (11) to (15) |
|-----------------|------------------------|-------|---------|-----------------------------|-----------------------------|
|                 |                        |       |         | Error                       | Error                       |
|                 |                        |       |         | Loc. in L | Level in % (y2)            | Loc. in L | Level in % (y2)            |
| VP1             | 0.2 7 3000 0.935714 | 0.2 | 7.0115 | -0.16429 |
| VP1             | 0.5 7 3000 1.065714 | 0.5 | 6.9874 | 0.18 |
| VP1             | 0.7 7 3000 1.467143 | 0.7 | 6.9915 | 0.12143 |
| VP1             | 0.2 12 3000 -0.73667 | 0.2 | 11.8503 | 1.2475 |
| VP1             | 0.5 12 3000 -0.20583 | 0.5 | 11.8632 | 1.14 |
| VP1             | 0.7 12 3000 -0.31917 | 0.7 | 11.8573 | 1.189167 |
| VP2             | 0.2 14 1500 -0.445 | 0.2 | 13.848 | 1.085714 |
| VP2             | 0.5 14 1500 0.033571 | 0.5 | 13.8551 | 1.035 |
| VP2             | 0.7 14 1500 -0.32357 | 0.7 | 13.8895 | 0.789286 |
| VP2             | 0.2 18 1500 0.845556 | 0.2 | 17.8954 | 0.197222 |
| VP2             | 0.5 18 1500 -0.005 | 0.5 | 17.9706 | 0.163333 |
| VP2             | 0.7 18 1500 0.477778 | 0.7 | 18.0008 | -0.00444 |
| VP3             | 0.2 23 2500 -0.60087 | 0.2 | 23.1156 | -0.5026 |
| VP3             | 0.5 23 2500 0.042609 | 0.5 | 23.1372 | -0.59652 |
| VP3             | 0.7 23 2500 -0.15913 | 0.7 | 23.1488 | -0.64696 |
| VP3             | 0.2 32 2500 -0.20594 | 0.2 | 31.9614 | 0.120625 |
| VP3             | 0.5 32 2500 -0.2025 | 0.5 | 32.0161 | -0.05031 |
| VP3             | 0.7 32 2500 0.135625 | 0.7 | 32.0085 | -0.02656 |

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Multiple damage case

The proposed methodology is used to detect and quantify damage in a beam with a single dominant crack. However, it has been observed through numerical simulations that the same method can be used to detect and quantify more than one dominant crack in beams. This aspect has been demonstrated with the help of numerical simulations of certain damage cases conducted on the validation problems (VP1, VP2 and VP3).

Consider a beam to have more than one dominant crack. The damage or crack locations can be determined by analysing the structural response, obtained through different measurements made by placing a point load each time at different locations. The $W_{W_{\text{loc}}}$ for the problem is obtained by placing the load such that the wavelet coefficient at the load location is not influenced by the wavelet coefficients at/near any of the cracks in the beam. It has been observed through studies that the $W_{W_{\text{loc}}}$ for a particular beam subjected to a constant magnitude of the load always remains constant.

Consider a beam to have more than one dominant crack. The damage or crack locations can be determined by analysing the structural response, obtained through different measurements made by placing a point load each time at different locations. The $W_{W_{\text{loc}}}$ for the problem is obtained by placing the load such that the wavelet coefficient at the load location is not influenced by the wavelet coefficients at/near any of the cracks in the beam. It has been observed through studies that the $W_{W_{\text{loc}}}$ for a particular beam subjected to a constant magnitude of the load always remains constant.

Now the load is placed on the crack 1. The wavelet coefficient at the damage location (corresponding to crack 1) is noted as $W_{\text{damage}}$ and the DIF is computed for the crack 1. This value of DIF is used to quantify the level of damage at that location. The quantification has been done using the Equation (11) to (15). Similarly, place the load at the location of the next crack to compute the DIF at that location and thus quantify the damage. Then, follow the same procedure for each of the other cracks. Numerical simulations have been conducted to highlight this damage identification capability of the proposed methodology using three cracks (C1, C2, and C3) in each case. The results of the studies are shown in Table 6.

From Table 6, it can be observed that the applicability of the proposed method for the damage identification of a single crack can be successfully extended to solve the multiple damage cases in beams. The results show that the level of damage has been determined with very less error (less than 3%). However, based on the numerical simulations conducted, it has been observed that the method can only be used to detect the damages provided they are not close to each other. With the present level of discretisation of the structural beam model, it has been observed that the distance between any two cracks have to be atleast $0.1L$, in order to apply this method. Otherwise, the wavelet coefficients at a crack location get influenced by the wavelet coefficients of the nearby crack, which will lead to misinterpretation of the results.
Table 6: Multiple damage cases solved using the validation problems – simply supported beams

| Val. Prob. VP-i | Simulated crack damage (Location in L, Damage Level in %) | W in kN | DIF for C1 | DIF for C2 | DIF for C3 |
|----------------|--------------------------------------------------------|--------|------------|------------|------------|
|                | C1 (0.2, 5)     |        |            |            |            |
| VP 1           | (0.4, 10)       | 3.0    | 3.580      | 8.484      | 10.291     |
|                | (0.6, 12)       |        |            |            |            |
| VP 1           | (0.3, 10)       | 3.0    | 7.604      | 5.340      | 4.556      |
|                | (0.6, 6)        |        |            |            |            |
| VP 1           | (0.42, 15)      | 3.0    | 13.480     | 11.511     | 14.322     |
|                | (0.56, 13)      |        |            |            |            |
| VP 2           | (0.2, 5)        | 1.5    | 3.580      | 8.484      | 10.291     |
|                | (0.4, 10)       |        |            |            |            |
| VP 2           | (0.3, 10)       | 1.5    | 7.604      | 5.340      | 4.556      |
|                | (0.6, 6)        |        |            |            |            |
| VP 2           | (0.42, 15)      | 1.5    | 13.479     | 11.511     | 14.322     |
|                | (0.56, 13)      |        |            |            |            |
| VP 3           | (0.2, 5)        | 2.5    | 3.580      | 8.484      | 10.291     |
|                | (0.4, 10)       |        |            |            |            |
| VP 3           | (0.3, 10)       | 2.5    | 7.604      | 5.340      | 4.556      |
|                | (0.6, 6)        |        |            |            |            |
| VP 3           | (0.42, 15)      | 2.5    | 13.480     | 11.512     | 14.322     |
|                | (0.56, 13)      |        |            |            |            |

5.2 Fixed beam

Damage quantification

The damage quantification method is now extended to solve fixed beams. The Problem 1 (Chang and Chen, 2005) has been considered again but with both ends fixed. The proposed methodology of measuring three set of structural response and analysing the first two to locate damage and compute WC_{Wloc} and then using the third to compute the WC_{damage}, are the same for this problem also. As has been done for the simply supported beam, the DIF values for the fixed beams are computed by assuming different levels of damage to occur at various locations along the length of the beam. The values are plotted in Figure 4.
From the Figure 4, we observe nine curves are plotted for damages of different levels (from 5% to 45% damage in depth) occurring at 0.1L to 0.9L by considering damages at every 0.1L. The values in the figure help in understanding the relation between the level of damage and the two known, namely the location of damage and the DIF. From the figure it is observed that the DIFs obtained for different levels of damage at x and (L-x) are nearly same. Thus, we can reduce the number of curves to five. By fitting polynomial curves to these curves (corresponding to crack locations 0.1L to 0.5L) we obtain the five inverse relations, as represented by Equation (16) to (20).

\[
\phi_{(0.1L)} = -9.32e -02 \times \text{DIF}^4 + 1.885 \times \text{DIF}^3 - 14.69 \times \text{DIF}^2 + 57.27 \times \text{DIF} - 58.22 \\
\phi_{(0.2L)} = -6.97e -04 \times \text{DIF}^4 + 0.03793 \times \text{DIF}^3 - 0.8083 \times \text{DIF}^2 + 9.421 \times \text{DIF} - 11.8 \\
\phi_{(0.3L)} = -7.19e -05 \times \text{DIF}^4 + 0.006617 \times \text{DIF}^3 - 0.2386 \times \text{DIF}^2 + 4.794 \times \text{DIF} - 5.989 \\
\phi_{(0.4L)} = -2.35e -05 \times \text{DIF}^4 + 0.002829 \times \text{DIF}^3 - 0.1332 \times \text{DIF}^2 + 3.508 \times \text{DIF} - 4.258 \\
\phi_{(0.5L)} = -1.66e -05 \times \text{DIF}^4 + 0.002176 \times \text{DIF}^3 - 0.1114 \times \text{DIF}^2 + 3.19 \times \text{DIF} - 3.82
\]

Using the equations the damage level for a particular location of damage can be determined, provided the DIF and the location of damage are known. The damage levels for damages that fall in between those considered locations can be obtained by interpolation. Numerical studies have been conducted later where the determination of damage level has been done using these equations and then using interpolation. However, it shall be noted that these inverse relations are derived for the fixed-fixed case of the Problem 1.

**Generalising the methodology**

Now, the methodology needs to be generalised for all beam problems with similar edge conditions. For this, the variations in DIF values caused due to changes in the geometrical or material properties or load intensity have been studied. To perform these studies, the geometrical and material properties of the beam and load intensity in Problem 1 (Chang and Chen, 2005) have been modified arbitrarily, one at a time to arrive at five new problems, the details of which are given in Table 1.
Some arbitrarily presumed damage instances of Problem 1 have been considered to conduct the studies. The details related to the damage cases and the corresponding DIF values obtained are given in Table 7. Similarly, the DIF values are computed and the error in percentage between the DIF obtained for Problem 1 and that of the Problems 2 to 6 are plotted in Figure 5. From the figure, it can be observed that the errors in the DIF obtained for the six different problems are negligible (less than 0.01%). Hence, it can concluded that the DIF values obtained using Problem 1 (fixed case) can be used to solve the damage quantification in any fixed beam problem, irrespective of the changes in the geometrical properties, material property and magnitude of the point load. Since, the DIF values remain constant the inverse relation represented using Equation (16) to (20), obtained for Problem 1, also remain the same for all fixed-fixed beams irrespective of the changes in geometrical or material properties or load intensity.

**Table 7: DIF obtained for Problem 1 – fixed beam**

| Damage case | Damage location in L | Damage percentage | WC_{Wloc}   | WC_{damage} | DIF     |
|-------------|----------------------|------------------|-------------|-------------|---------|
| 1           | 0.1                  | 10               | -1.07E-07   | -5.53E-08   | 1.93219 |
| 2           | 0.1                  | 20               | -1.43E-07   | -5.53E-08   | 2.584532|
| 3           | 0.1                  | 30               | -2.01E-07   | -5.53E-08   | 3.62881 |
| 4           | 0.3                  | 10               | -2.23E-07   | -5.53E-08   | 4.031906|
| 5           | 0.3                  | 20               | -4.38E-07   | -5.53E-08   | 7.928301|
| 6           | 0.3                  | 30               | -7.87E-07   | -5.53E-08   | 14.23769|
| 7           | 0.5                  | 10               | -2.83E-07   | -5.53E-08   | 5.118907|
| 8           | 0.5                  | 20               | -5.92E-07   | -5.53E-08   | 10.7165 |
| 9           | 0.5                  | 30               | -1.10E-06   | -5.53E-08   | 19.84336|

**Figure 5:** Comparison of error in DIF for problems 2 to 6 w.r.t. that of problem 1 – fixed beam
**Validation Studies**

The proposed method for fixed beams has been validated using three problems taken from the literature, as given in Table 7. Arbitrarily chosen damage cases have been simulated and the proposed method has been used to solve the damage identification problems. The location of damage has been precisely determined by using the wavelet transform. The level of damage has been computed by interpolating the data represented in Figure 4 and also by using the Equation (16) to (20). The errors in estimating the level of damage have been computed and are given in Table 8. From the Table, it is clear that the equations can be used as a replacement for the data to determine the level of damage with lesser error.

**Multiple damage case**

As has been demonstrated using the simply supported beams, the proposed methodology can be used to detect multiple cracks also, in fixed beams. This aspect has been demonstrated with the help of numerical simulations conducted on the validation problems, given in Table 4.

Follow the same procedure as in the simply supported case to obtain the \( WC_{WL} \), \( WC_{damage} \) and DIF for each of the cracks. The quantification has been done using the Equation (16) to (20). The results of the studies are shown in Table 9.

It can be observed from the table that the proposed method can be used to detect and quantify damage with very small errors. Based on the studies carried out in the present work and the observations made, we can solve damage identification problems in all simply supported and fixed beams, with very small errors.

**Table 8: Validation results for the fixed beam problems**

| Val. Prob. VP-i | Simulated crack damage | W in N | \( WC_{damage} \) | \( WC_{WL} \) | DIF | Estimated crack damage using Figure 4 |
|-----------------|------------------------|-------|-------------------|-------------|-----|---------------------------------------|
|                 | Loc in L | Level in % (x) |       |       |       | Loc in L | Level in % (y) |
| VP 1            | 0.1     | 7              | 3000  | -3.97E-07 | -2.22E-07 | 1.79     | 0.1  | 6.87713 |
| VP 1            | 0.3     | 7              | 3000  | -7.03E-07 | -2.22E-07 | 3.16     | 0.3  | 7.1125 |
| VP 1            | 0.5     | 7              | 3000  | -8.61E-07 | -2.22E-07 | 3.87     | 0.5  | 6.9484 |
| VP 1            | 0.1     | 12             | 3000  | -4.53E-07 | -2.22E-07 | 2.04     | 0.1  | 11.8711 |
| VP 1            | 0.3     | 12             | 3000  | -1.04E-06 | -2.22E-07 | 4.68     | 0.3  | 11.8977 |
| VP 1            | 0.5     | 12             | 3000  | -1.34E-06 | -2.22E-07 | 6.04     | 0.5  | 11.9858 |
| VP 2            | 0.1     | 14             | 1500  | -6.80E-07 | -3.15E-07 | 2.16     | 0.1  | 14.4414 |
| VP 2            | 0.3     | 14             | 1500  | -1.69E-06 | -3.15E-07 | 5.38     | 0.3  | 13.8691 |
| VP 2            | 0.5     | 14             | 1500  | -2.22E-06 | -3.15E-07 | 7.06     | 0.5  | 13.9757 |
| VP 2            | 0.1     | 18             | 1500  | -7.65E-07 | -3.15E-07 | 2.43     | 0.1  | 17.2813 |
| VP 2            | 0.3     | 18             | 1500  | -2.20E-06 | -3.15E-07 | 7.00     | 0.3  | 17.9897 |
| VP 2            | 0.5     | 18             | 1500  | -2.95E-06 | -3.15E-07 | 9.38     | 0.5  | 17.9711 |
| VP 3            | 0.1     | 23             | 2500  | -6.69E-08 | -2.35E-08 | 2.85     | 0.1  | 23.869 |
| VP 3            | 0.3     | 23             | 2500  | -2.23E-07 | -2.35E-08 | 9.50     | 0.3  | 22.9984 |
| VP 3            | 0.5     | 23             | 2500  | -3.05E-07 | -2.35E-08 | 12.98    | 0.5  | 22.9803 |
| VP 3            | 0.1     | 32             | 2500  | -9.18E-08 | -2.35E-08 | 3.91     | 0.1  | 32.428 |
| VP 3            | 0.3     | 32             | 2500  | -3.75E-07 | -2.35E-08 | 15.9     | 0.3  | 32.0472 |
| VP 3            | 0.5     | 32             | 2500  | -5.25E-07 | -2.35E-08 | 22.3     | 0.5  | 32.0467 |
Table 9: Validation of wavelet quantification in multiple crack cases – fixed beams

| Val. Prob. VP-i | Simulated crack details (Location in L, Level in %) | W in kN | DIF for C1 | DIF for C2 | DIF for C3 | Estimated damage level in % | Error in Estimating damage level in % |
|-----------------|-----------------------------------------------|--------|------------|------------|------------|-----------------------------|-------------------------------------|
|                 |                                               |        | C1         | C2         | C3         |                             |                                     |
| VP 1            | (0.2, 5) (0.4, 10) (0.6, 12)                  | 3.0    | 2.1524     | 4.8215     | 5.6884     | 0.2                         | 0.4                                 |
| VP 1            | (0.3, 10) (0.6, 6) (0.8, 7)                   | 3.0    | 4.0312     | 3.3385     | 2.4745     | 0.3                         | 0.6                                 |
| VP 1            | (0.42, 15) (0.56, 13) (0.78, 20)              | 3.0    | 7.2659     | 6.3869     | 5.8027     | 0.42                        | 0.56                                |
| VP 2            | (0.2, 5) (0.4, 10) (0.6, 12)                  | 1.5    | 2.1524     | 4.8215     | 5.6885     | 0.2                         | 0.4                                 |
| VP 2            | (0.3, 10) (0.6, 6) (0.8, 7)                   | 1.5    | 4.0311     | 3.3384     | 2.4745     | 0.3                         | 0.6                                 |
| VP 2            | (0.42, 15) (0.56, 13) (0.78, 20)              | 1.5    | 7.2656     | 6.3869     | 5.8028     | 0.42                        | 0.56                                |
| VP 3            | (0.2, 5) (0.4, 10) (0.6, 12)                  | 2.5    | 2.1525     | 4.8219     | 5.6885     | 0.2                         | 0.4                                 |
| VP 3            | (0.3, 10) (0.6, 6) (0.8, 7)                   | 2.5    | 4.0312     | 3.3386     | 2.4746     | 0.3                         | 0.6                                 |
| VP 3            | (0.42, 15) (0.56, 13) (0.78, 20)              | 2.5    | 7.2660     | 6.3870     | 5.8030     | 0.42                        | 0.56                                |

Error in Estimating damage level for:

| Val. Prob. VP-i | Simulated crack details (Location in L, Level in %) | W in kN | Estimated damage level in % | Error in Estimating damage level for |
|-----------------|-----------------------------------------------|--------|-----------------------------|-------------------------------------|
|                 |                                               |        | C1         | C2         | C3         | C1 | C2 | C3 |
| VP 1            | (0.2, 5) (0.4, 10) (0.6, 12)                  | 3.0    | 5.10         | 9.86        | 11.88      | -2.02 | 1.40 | 0.98 |
| VP 1            | (0.3, 10) (0.6, 6) (0.8, 7)                   | 3.0    | 9.89         | 6.08        | 7.11       | 1.11 | -1.36 | -1.59 |
| VP 1            | (0.42, 15) (0.56, 13) (0.78, 20)              | 3.0    | 15.19        | 13.00       | 20.82      | -1.28 | 0.00 | -4.09 |
| VP 2            | (0.2, 5) (0.4, 10) (0.6, 12)                  | 1.5    | 5.10         | 9.87        | 11.87      | -1.92 | 1.27 | 1.09 |
| VP 2            | (0.3, 10) (0.6, 6) (0.8, 7)                   | 1.5    | 9.88         | 6.06        | 7.10       | 1.25 | -0.95 | -1.46 |
| VP 2            | (0.42, 15) (0.56, 13) (0.78, 20)              | 1.5    | 15.04        | 13.05       | 20.84      | -0.24 | -0.42 | -4.20 |
| VP 3            | (0.2, 5) (0.4, 10) (0.6, 12)                  | 2.5    | 5.10         | 9.83        | 11.91      | -2.03 | 1.68 | 0.72 |
| VP 3            | (0.3, 10) (0.6, 6) (0.8, 7)                   | 2.5    | 9.87         | 6.06        | 7.10       | 1.31 | -1.08 | -1.41 |
| VP 3            | (0.42, 15) (0.56, 13) (0.78, 20)              | 2.5    | 15.05        | 13.06       | 20.77      | -0.33 | -0.45 | -3.84 |
5.3 Conclusion

The present work focuses on arriving at a generalised methodology to detect, locate and quantify damage in beams using discrete wavelet transforms. Both simply supported beams and fixed beams have been used to demonstrate the capability of the proposed method. Only single dominant cracks have been considered to propose the method. Equations have been arrived to obtain the level of damage for different damage locations. The level of damage for the other locations can be obtained by interpolating between the values obtained using the equations. With this method the level of damage can be determined with errors of very less order, as has been demonstrated through numerical studies. Validation of the proposed methodology has been done using problems taken from the literature. It has been observed that the proposed methodology can be used to solve any damage case in any simply supported or fixed beam problem. Though the method is proposed for single dominant crack case, the applicability of the method to solve multiple cracks in a beam has been highlighted using numerical simulations.

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