Numerical and Experimental Investigation on Axial Rub Impact Dynamic Characteristics of Flexible Rotor Supported by Hybrid Gas Bearings

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Abstract
Gas bearings are widely used in micro- and small turbomachinery. Because of the pursuit of high efficiency, turbomachinery adopts small clearance of rotor and stator. The gas bearing rotor system easily suffers from rub impact due to the inherently low damping and load capacity of gas film. Axial rub impact may lead to catastrophic failure of gas bearing rotor system. Previous work put emphasis on radial rub, and only a few papers researched on the axial rub impact by simulation method. In this paper, dynamic responses of full annular axial rub are investigated numerically and experimentally. A single span flexible rotor test rig is established to support this research. Dynamic characteristics of full annular axial rub of this gas bearing rotor system are obtained with finite element language-APDL. Dynamic characteristics within full speed range are experimentally researched based on the test rig. The dynamic behaviors are analyzed by means of waterfall diagrams, frequency spectrums, orbit trails, and vibration amplitude waveforms. During speed up, half speed whirl and gas film oscillation occur in radial direction. During speed down, the full annular axial rub between rotor thrust disk and gas bearing occurs. When lightly axial rub impact happens, the vibration patterns in the radial direction change barely, and 0 Hz component appears in the axial direction. When serious full annular axial rub impact happens, 0 Hz component occurs in both radial and axial directions and rotor orbit shows transverse motion in radial direction. These forms of dynamic characteristics can be effectively used to diagnose the full annular axial rub impact.

Keywords
Gas bearing rotor system, axial rub impact, rotor dynamics

Introduction
Micro- and small turbomachinery operating at high speed employ gas bearings in an oil-free environment due to the benefits of low friction power consumption, high rotational speed, and long working life.¹,² In order to reduce weight and improve equipment efficiency, turbomachinery adopts compact structure design and small clearance of rotor- stator.³ Consequently, rub impact between rotor and stator becomes one of the serious malfunctions that may lead to catastrophic failure. Gas bearing rotor system more easily suffers from instability at high speed due to the inherently low damping and load capacity of gas film.⁴,⁵
The residual unbalance, bearing pedestal stiffness, and localized defect of bearing greatly affect the radial vibrations of rotor system, which may lead to the radial rub impact when the radial clearance is smaller than the vibration amplitude. Improper installation, thermal expansion, and axial force imbalance can even occupy the normal axial clearance, which leads to the axial rub impact of rotor with stator. For gas bearing rotor system, axial rub impact may cause more serious consequences compared with radial rub impact because of the uncoated axial contact surfaces and the possible failure of thrust bearing. On the other hand, it is difficult to find axial rubs through experiments because they usually combine with radial rub and are hidden by the latter. Large quantity of work have been done to research radial rub, whereas published studies focused on axial rub have hardly been so far.

In terms of radial rub impact, according to the rubbing area, full rub, fixed point rub, and short arc rub between rotor and seal were researched experimentally and numerically. The results showed that the multiple harmonic components \((n\times)\), \(1/2\) and \(1/3\) fractional harmonic components \((n/2\times, n/3\times)\), were obvious when rub impact happened. The research object in above literature was single rotor system. The radial rub dynamics of dual rotor system were researched in the following works. Multiple combined frequency components of inner and outer rotor fundamental frequencies were particularly fault frequencies. In addition to the above rotor/stator rub, the rotor blade rub was also numerically researched. The blade passing frequency and its multiple frequency components were typical characteristics of blade rub impact.

The following literatures studied the axial rub impact. He and Huang listed the axial rub impact in the mechanical fault examples. Fault characteristics of axial rub were difficult to be found in the radial vibration spectrum. Yuan et al. researched the axial rub impact caused by rotor's pitching vibration based on a full six degrees of freedom Jeffcott rotor. The rotor dynamic model considering unbalance force and axial rub impact was established using Lagrangian equation. It was concluded that \(4\times\) component of displacement response in axial direction signals the occurrence of axial rub impact. Qian studied the axial rub between rotor's side face and two fixed positions on stator. His research shows that the axial rub results in complicated behaviors, such as quasi-periodical or chaotic responses, and the rotordynamics are similar to the radial rub impact. Kunpeng and Qian numerically investigated the lateral–torsional coupling vibration of a rotor system influenced by axial contact/rubs between rotor disks and stator. The calculation shows that the lateral vibration changes from periodic to chaotic, and the torsional natural frequency appears as the occurrence of the axial rub. In addition, more or less same frequency components appear both in the lateral and the torsional motions because of the coupling effect. Abdelrhman et al. numerically researched dynamic characteristics of axial rub between rotor blades and stator blades. Blade passing frequency and its multiple harmonic frequencies appear in \(x\), \(y\), and \(z\)-direction when the axial rub happens. Amplitude of blade passing frequency increases with increasing of rotational speed. Wang et al. calculated the dynamic responses of rotor system with radial and axial rubs. The results indicate that the characteristics of axial rub are similar with unbalance response at high speed. At low speed, the characteristics are more like that of radial rub. Żywic and Kaczmarczyk experimentally researched the dynamic characteristics of axial rub impact between gas thrust bearing and keep plate. The \(1\times\) component, super-harmonic components, and neighboring frequencies occur during the rub procedure. Xu and Qin numerically studied the bending/torsional vibration of rotor system with axial contact, and results indicated that the vibration was sensitive to mass eccentricity of rotor. Furthermore, lateral dynamic characteristics of coupled radial/axial rubs were researched numerically. Calculations show that rotor vibration is very complex, but influences of axial rub are not considered.

Gas bearing rotor system is prone to full annular axial rub due to the axial force imbalance resulted by the low load capacity of thrust film. Most of the papers that study axial rub have limited guidance for the axial rub of gas bearing rotor system. First, almost all above literatures numerically investigated on the axial rub impact without experimental verification. Second, except for Abdelrhman et al. and Żywic and Kaczmarczyk, all axial rubs are point axial rub caused by pitching motion of disks attached to the bending shaft. Pitch angle of rotor thrust plate is essential to the calculation of rub force and moment.

This paper is motivated to research dynamic characteristics of full annular axial rub impact between gas bearing and rotor. First, a test bench of single span flexible rotor supported by hybrid gas bearings is introduced in “Gas bearing rotor system test rig” section. Second, force model of the full annular axial rub impact model is proposed in “Force model of full annular axial rub impact” section. Based on the actual structure of the test rig, the finite element model of the rotor system is introduced in “Finite element model of gas bearing rotor system” section. Third, the vibration characteristics of axial rub impact are researched in “Results and discussion” section. Finally, the conclusions are drawn in the final section. Unlike the force model of point axial rub impact listed in reported papers, the pitch angle of rotor thrust disk is ignored in the force model of full annular axial rub impact. Furthermore, transient dynamic characteristics at different rub states are researched. It is expected that some
valuable vibration features of the axial rub impact can be observed, and the research can provide some useful information for the fault diagnostics of gas bearing rotor system.

**Gas bearing rotor system test rig**

A gas bearing-flexible rotor test rig is established to fulfill this research. Figure 1 gives a schematic view of this test rig. It is composed of gas bearing rotor system, vibration data acquisition and analysis system, high pressure air feeding system, and monitoring system.

As shown in Figure 1, the rotor is made of 2Cr13. Density, elastic module, and Poisson’s ratio are 7750 kg/m³, 228 GPa, and 0.27, respectively. The rotor is supported by hybrid bearings and driven by a single-stage coaxial radial turbine. The bearing material is graphite alloy. O-rings are used in bearings to work as high pressure seals and provide damping. Bearing B1 is radial-thrust hybrid gas bearing and provides gas film in both radial and axial directions. Bearings B2 and B3 are pure thrust hybrid gas bearing and pure radial hybrid gas bearing, respectively. Figure 2 presents the schematics of gas bearings. Dynamic pressure grooves and orifices are located on the working surfaces of bearings. Herringbone grooves are placed in the internal surface of radial bearings. Spiral grooves are placed in the thrust surface of thrust bearings. The radial bearings have two lines of feeds holes in the axial direction and 30 holes equably located in the circumferential direction for each line. The distance between two lines is 27.5 mm. Thirty holes are placed in the thrust surface of thrust bearing. The main parameters of gas bearings and rotor are listed in Table 1.

As shown in Figure 3, axial distances between rotor thrust disk and bearings B1 and B2 are named as \( h_1 \) and \( h_2 \), respectively. “Axial clearance” represents the sum of \( h_1 \) and \( h_2 \) in this paper.

The vibration data acquisition and analysis system monitors rotor vibration real time and conducts data offline processing. Displacement transducers measure rotor vibration responses during experiment. Vibration frequency domain signals are obtained through fast Fourier transform algorithm. The sampling frequency is 4000 Hz in the experiment. A resolution of 2 Hz is adopted during data offline analysis. As depicted in Figure 1, eddy current displacement transducers are used to measure the radial, axial, and key phase signals of the rotor. Transducers placed in the radial direction measure relative displacement of rotor to sensors, and transducers placed in the axial direction measure absolute displacement of rotor. The increasing of axial vibration amplitude means that the thrust disk moves toward bearing B1, and decreasing means the contrary tendency.

High pressure air feeding system consists of pipelines, valves, and sensors. The maximum pressure of air is 1.20 MPa, and the flow rate is 1000 Nm³/h. The monitoring system includes cameras, control circuits, and computers.

**Force model of full annular axial rub impact**

Dynamic characteristics of full annular axial rub impact between rotor thrust disk and bearing B2 are numerically researched. As depicted in Figure 4, axial rub impact can be divided into point axial rub impact and full annular
axial rub impact according to different rub positions. $\delta_i$ and $\delta$ are static and transient axial clearances between rotor disk and stator. The point axial rub impact between rotor and stator occurs when $\delta$ is negative, and the full annular axial rub impact occurs when the surface of rotor thrust disk is in full contact with stator.

Forces caused by point axial rub impact include axial rub impact force $F_{p}$, tangential rub impact force based on the coulomb friction law $F_{pf}$, and the friction torque $M_{pf}$. At the same time, the rotor is also subjected to thrust gas film forces $F_{B1}$, $F_{B2}$, and $F_{T}$, originating from bearings B1, B2, and turbine. Point axial rub impact is not the focus of this study and will not be discussed in depth here.

Forces caused by full annular axial rub impact include axial rub impact force $F_{A}$ and tangential rub impact force based on the Coulomb friction law $F_{Af}$. At the same time, the rotor is also subjected to thrust gas film forces $F_{B1}$ and $F_{T}$. In the range of rotational speed of full annular axial rub impact, the thrust gas film of bearing B2 ruptures and $F_{B2}$ is ignored. $F_{Af}$ acts on the whole rub surface and forms moment $M_{Af}$, which is opposite to the rotation direction.

The contact form of thrust disk and bearing thrust surface is wear form. During full annular rub impact, the force $F_A$ and moment $M_{Af}$ are stated as equation (1), where $K_{axi}$ is the axial stiffness coefficient of the stator, $A$ is the area of rotor thrust disk, and $\mu$ is the axial friction coefficient between thrust disk and bearing B2. $F_{B1}$ is calculated based on Tun et al. $^{31}$ and Yunfei. $^{32}$ Static pressure carrying capacity of bearing B1 is calculated based on the assumption that airflow at outlet of the orifice uniformly flows in the radial direction, and dynamic pressure carrying capacity is calculated based on the incompressible narrow groove theory. During the full annular axial rub impact process, $F_{B1}$ is 550 N and remains unchanged.

$$F_A = K_{axi}(R \sin \theta + z_a)R \in \{(-R_2, -R_1)(R_1, R_2)\}$$
$$dM_{Af} = \frac{2\pi R^2 F_A}{A} dR$$
$$M_{Af} = \begin{cases} 
\frac{2\pi R^2 F_A}{A} \left[ \sin \theta / 4 (R_2^4 - R_1^4) + z_a / 3 (R_2^3 - R_1^3) \right] & R \in (R_1, R_2) \\
\frac{2\pi R^2 F_A}{A} \left[ \sin \theta / 4 (R_1^4 - R_2^4) + z_a / 3 (R_1^3 - R_2^3) \right] & R \in (-R_2, -R_1) 
\end{cases}$$

Figure 2. Sketch of hybrid gas bearing.

Table 1. Main parameters of gas bearing rotor system.

| Parameter                        | Unit | Value |
|----------------------------------|------|-------|
| Outer radius of bearing          | mm   | 55    |
| Inner radius of bearing          | mm   | 25    |
| Inner radius of grooves on thrust surface | mm | 27.5  |
| Outer radius of grooves on thrust surface | mm | 45    |
| Orifice diameter                 | mm   | 0.4   |
| Diameter of orifice position circle on thrust surface | mm | 80.4  |
| Bearing length                   | mm   | 60    |
| Rotor length                     | mm   | 698   |
| Thrust disk radius               | mm   | 45    |
| Rotor mass                       | kg   | 10.36 |
| Bearing span                     | mm   | 525   |
| Mean radial clearance            | mm   | 0.06  |
Figure 3. Sketch of rotor thrust disk and bearings.

Figure 4. Models of axial rub impact. (a) sketch of point axial rub and (b) sketch of full annular axial rub.

Finite element model of gas bearing rotor system

Figure 5 shows the finite element model of the rotor system depicted in Figure 1. The rotor is simulated by 78,112 eight-node solid elements. The volume of the largest solid element is $2.019 \times 10^{-7}$ m$^3$. Parameters of rotor material are the same with real rotor. The radial bearings are simulated by eight spring elements. Each bearing is simulated by four spring elements. The radial turbine is simulated by lumped mass. The mobile ends of spring elements are located at nodes on the surface of rotor. The reference ends of spring elements are located at nodes outside the rotor. The coordinates of spring elements in the positive y-direction are marked in Figure 5. A and B listed in picture are nodes analyzed in the following, and units of coordinates are given in millimeters.

In this study, the finite element design language-APDL is used for finite element modeling and carrying out the transient force vibration analysis. The load conditions of this analysis are set as transient tabular forces, which consist of rotor unbalance force, inertial force owing to speed changes, gas film force, and axial rub force and moment. As shown in Figure 6, the unbalance force and inertia force are applied to the central node of thrust disk. The axial rub force is applied to the surface of thrust disk, and moment is also applied to this surface by
generating a rigid region. Thrust gas film force of bearing B1 is applied to rotor thrust surface on the drive-end side. All freedom degrees of the reference ends of spring elements are constrained.

The calculation time is 0.2 s, and the step length is \(5 \times 10^{-5}\) s. The beginning speed of this transient analysis is 57,000 r/min, and the rotational speed is updated based on the moment caused by full annular axial rub impact. In this study, the finite difference method is used to calculate the main stiffness of gas bearings. Gas film radial stiffness is set as \(4 \times 10^6\) N/m, and damping is neglected.

Distance between thrust disk and bearing B2, namely \(h_2\) indicated in Figure 3, is inputted as transient tabular value. \(h_2\) is larger than 0 \(\mu\)m before 0.06 s. From 0.06 to 0.2 s, \(h_2\) decreases from 0 to –0.11 \(\mu\)m linearly. \(h_2\) changing from positive to negative means the happening of full annular axial rub impact. Axial rub force and moment are calculated based on equation (1). Parameters used in the calculation are listed in Table 2. The transient response calculation contains two load steps:

**Step 1:** Only the unbalance force is considered during this step. \(h_2\) is larger than 0 \(\mu\)m, and the axial forces \(F_{B1}\), \(F_{B2}\), and \(F_T\) are assumed balanced (resultant force is 0 N). The friction moments of gas film forces with thrust disk are ignored owing to the low friction coefficient.

**Step 2:** \(h_2\) decreases from 0 to –0.11 \(\mu\)m linearly, and the full annular axial rub impact between thrust disk and bearing B2 occurs. The unbalance force, inertia force, \(F_A\), \(F_{B2}\), and \(M_{Af}\) are applied to the rotor. \(F_T\) acts in the same direction as force \(F_{B2}\), and it is much less than \(F_{B2}\). So that \(F_T\) is ignored in the calculation.

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**Figure 5.** Finite element model of flexible rotor system.

**Figure 6.** Schematic diagram of forces applied to rotor.
Results and discussion

Results of finite element calculation

The modes and frequencies of the first three natural frequencies are listed in Figure 7. The rotor filled with blue indicates the deformed rotor, and the rotor filled without color indicates the undeformed rotor. The first two modes are rigid body modes, and the third the first-order bending mode. The frequency errors between the calculated and experimental values are, respectively, 7.43%, –1.19%, and 2.19%. The calculation results indicate that the element number is enough to obtain the accurate results.

Figure 8 gives the rotor deformation characteristics at 57,000 r/min with only unbalance force consideration. Pitching angles of rotor thrust disk around x and y axes are calculated based on displacements of nodes P1 and P2. Figure 9 depicts the characteristics of pitch angles in several rotation cycles. The pitch angles are so small that they are ignored in the calculation of axial rub force and moment.

Figures 10 to 13 are waveforms, frequency spectrums, and orbit trails of displacement and angle responses of node B in different time intervals.

Figure 10 displays the dynamic behaviors at 57,000 r/min before axial rub impact. It is shown that displacement responses in x and y directions are synchronous vibrations. The displacement response in z direction maintains about 0 m, and the frequency spectrum shows the 0 Hz frequency, which can be neglected because of the low amplitude.

Figure 11 displays the dynamic behaviors from 0.055 to 0.1 s. At 0.06 s, the axial rub impact between rotor thrust disk and bearing B2 happens. After the axial rub impact happens, the waveform in x direction shifts to the positive direction and in y direction to the negative direction. Correspondingly, 0 Hz component appears in the frequency spectrums of both x and y directions because of the overall shift of waveform. In z direction, the waveform increases almost linearly because of the effects of $F_A$ and $F_{B2}$, which results in 0 Hz frequency. With the increasing of axial rub impact, the shift distance of waveforms and amplitude of 0 Hz frequency increases.

Remarkably, 0 Hz component is observed in frequency spectrums of displacement responses of all the three directions. This is a new phenomenon that means the occurrence of full annular axial rub impact.

Figure 12 displays the dynamic behaviors of angle responses from 0.055 to 0.1 s. At 0.06 s, waveforms of $\theta_x$ and $\theta_y$ both shift. Correspondingly, 0 Hz component occurs in frequency spectrums. The waveform of $\theta_z$ decreases

Table 2. Parameter values used in calculation.

| Parameter                                | Unit       | Value          |
|------------------------------------------|------------|----------------|
| Stiffness of axial rub impact            | N/m        | $5 \times 10^8$ |
| Polar moment of inertia of rotor         | kg m$^2$   | $3.56 \times 10^{-3}$ |
| Unbalance magnitude                      | g mm       | 4.46           |
| Mass of radial turbine                   | kg         | 0.033          |
| Polar moment of inertia of radial turbine| kg m$^2$   | $4.65 \times 10^{-5}$ |
| Diameter moment of inertia of radial turbine | kg m$^2$ | $2.33 \times 10^{-5}$ |
| Friction coefficient                     |            | 0.3            |

Figure 7. Natural frequency modes of rotor.
linearly because of the effect of friction torque, which causes the 0 Hz component to play a dominant role in the frequency spectrum. Characteristics of $h_x$ and $h_y$ represent the internal bending-torsion coupling.\textsuperscript{21} With the increasing of axial rub impact, internal bending-torsion coupling deepens, and amplitude of 0 Hz component increases.

Figure 8. Deformation characteristics of rotor at 57,000 r/min.

Figure 9. Pitch angle characteristics of rotor thrust disk.

Figure 10. Waveforms and frequency spectrums of displacement responses from 0.04 to 0.05 s.
Rotor orbits at different time are depicted in Figure 13. During the full annular axial rub impact process, the rotor orbit has a displacement in the radial direction. With the increasing of axial rub, the displacement becomes larger and larger. This phenomenon can be concluded from the waveforms of displacement responses and is the typical behavior of full annular axial rub impact.

Figure 11. Waveforms and frequency spectrums of displacement responses from 0.055 to 0.1 s.

Figure 12. Waveforms and frequency spectrums of angle responses from 0.055 to 0.1 s.
Calculation results show that 0 Hz component appears in the responses of all the six degrees of freedom. This frequency originates from the axial force and reverse torque caused by full annular axial rub impact. With the deepening of full annular axial rub impact, amplitude of 0 Hz component and displacement of rotor orbit increase.

Experimental results and discussion

Steady state and rotor dynamic behaviors are experimentally researched in the following. In steady state, gas supply pressures of bearings are all 0.8 MPa, and the axial clearance is measured. $h_1$ and $h_2$ are, respectively, 87.61 $\mu$m and 33.48 $\mu$m, which indicates that the axial rub impact would happen when amplitude of axial vibration decreases to 54.13 $\mu$m. The dynamic experiment is carried out on the basis of static experiment, in which bearing gas supply pressures and axial clearance remain unchanged.

Radial vibration 3D waterfall plot during run-up process is shown in Figure 14. The waterfall plot is made with frequency as horizontal, rotational speed as longitude and amplitude as vertical. Rotor critical speeds are 7547, 11,638, and 33,000 r/min. Due to small damping of gas bearing rotor system, vibration amplitude of synchronous motion decreases abruptly after the rotor goes through the first bending critical speed.

Low frequency eddy occurs at 32,344 r/min and disappears at 33,867 r/min. Whirl ratio of it stays at 0.50, which is typical half-frequency whirl characteristic. At 49,805 r/min, sub-harmonic vibration occurs, which is represented by curve AZ in the picture. The emergence speed of AZ is greater than twice the cone dynamic critical speed. Frequency value of AZ stays almost constant with the increasing of rotational speed, and it locks near the cone dynamic critical speed. AZ shows the typical characteristics of gas film oscillation.

3D waterfall diagram during run-down process shown in Figure 15 demonstrates the characteristics of synchronous unbalance response and sub-harmonic vibrations. The run-down process is divided into two different speed ranges based on the sub-harmonic vibration characteristics. The two different speed ranges are named as stage one and stage two in this study.

Speed range of stage one is from 56,718 to 48,750 r/min. Amplitude of synchronous response stays almost constant, and the sub-harmonic vibrations play dominant roles in stage one. The main low frequencies contain $1/3\times$ and $2/3\times$ components, represented by curves FU and IR, respectively. In addition, the main sub-harmonic frequencies are accompanied by fractional harmonic components.

Speed range of stage two is from 47,461 to 30,586 r/min, and it goes through the first bending critical speed. Sub-harmonic vibrations appear at 47,343 r/min, and the main frequency components are represented by curves JQ and KP. Frequency ratios of above two curves are $1/3$ and $2/3$. They are continuation of curves FU and IR. In addition, the fractional harmonic components around JQ and KP are more abundant than those in stage one. When rotational speed decreases to 32,109 r/min, the vibration amplitude of working frequency increases, and the sub-harmonic vibrations display contrary tendency.

Based on Figure 15, the abundant low frequency components in the radial direction indicate the occurrence of radial rub impact. The sub-harmonic frequencies are mainly $1/3\times$ and $2/3\times$ motions, accompanied by fractional harmonic components.
Curves of axial vibration amplitude and rotational speed during run-down process are depicted in Figure 16. The asymmetric sinusoidal waveform of axial vibration and the high sampling frequency result in the black area in Figure 16. The upper boundary of the black area represents the maximum rotor axial vibration and the lower boundary the minimum. To analyze the axial rub impact, the lower boundary is the main concern in this paper.

As shown in Figure 16, axial vibration amplitude decreases to 50 μm at 54,298 r/min, which indicates the occurrence of lightly axial rub between rotor thrust disk and thrust surface of bearing B2. Axial vibration amplitude abruptly decreases at 56,718 r/min, which leads to full annular axial rub impact of rotor thrust disk with bearing B2. Then the rotor enters into stage one, and the axial vibration amplitude decreases from 15.08 to –54.50 μm within this stage. Due to severe axial rub impact, rotational speed decreases rapidly, and the drop rate reaches 6804 r/min/s. In stage two, the black area is larger than stage one, which indicates the increasing of peak-to-peak value of axial vibration. This is due to the first-order bending deformation of rotor in stage two. In the following, the frequency spectrums and rotor orbit trails will be used to analyze the characteristics of full annular axial rub impact in different speed ranges.

Figure 17 shows the radial and axial vibration frequency spectrums during lightly axial rub. Working frequency, 2× component, and oscillation frequency, namely curve AZ marked in Figure 14, are coexisting motion types in both directions. Moreover, 0 Hz component occurs in this process because of the decreasing of axial vibration amplitude.

Radial and axial vibration frequency spectrums in stage one are depicted in Figure 18. Sub-harmonic vibration patterns existed in the radial direction can also be observed in axial direction, and vibration amplitudes of these low frequency components are larger than the synchronous response. In both radial and axial directions, the 0 Hz frequency appears, and amplitude of it is larger than all the other frequencies. Experimental results and numerical calculation results reveal that 0 Hz frequency originates from the axial rub force and rub torque. 1/3× and 2/3× frequency components are marked in Figure 18. 2/3× and fractional harmonic components around it are close to the first bending natural frequency, which leads to amplitude of those components that are larger than 1/3× and 1× components.
Figure 18(c) gives the rotor orbit trails within stage one, the black-dashed line is orbit trail before axial rub, the blue line is during axial rub, and the red-dashed line is exiting axial rub. After the axial rub occurs, the rotor orbit is disordered, and the vibration amplitude increases. The rotor orbit changes back to circle at the end of axial rub. During the full annular axial rub impact, the rotor orbit has a displacement along the $x$ direction, and the displacement is about 20 $\mu$m.

As depicted in Figure 16, the axial vibration amplitude continuously decreases in stage two, which indicates the existence of full annular axial rub impact. Frequency spectrums in radial and axial directions are depicted in Figure 19(a) and (b). In both two directions, the synchronous and sub-harmonic vibrations dominate, and the super-harmonic vibration is negligible. The low frequency components occur in radial direction can also be observed in axial direction. Characteristics of 0 Hz frequency in stage one and stage two are the same. Amplitude of working frequency is larger than sub-harmonic frequencies because the rotor is passing through the first bending critical speed. In radial direction, amplitudes of sub-harmonic vibrations are almost equal to those in stage one. In axial direction, amplitudes of sub-harmonic vibrations are larger than those in stage one owing to the coupling of radial and axial directions at the first bending critical speed.

Within stage two, the displacement of rotor orbit is larger than it in stage one, which indicates that the full annular axial rub impact in stage two is more serious than it in stage one.

Above analysis indicates that only axial rub impact exists during the lightly full annular axial rub impact process, and radial rub is accompanied by serious full annular axial rub impact in stages one and two. During the lightly full annular axial rub impact process, characteristics of rotor lateral response remain unchanged, and 0 Hz component appears in the axial direction. During serious full annular axial rub impact process, the synchronous and sub-harmonic frequencies are both obvious. The sub-harmonic vibrations in the radial direction are caused by radial
Figure 18. Dynamic characteristics of stage one. (a) radial vibration frequency spectrum of stage one, (b) axial vibration frequency spectrum of stage one and (c) rotor orbits within stage one.

Figure 19. Dynamic characteristics of stage two. (a) radial vibration frequency spectrum of stage two, (b) axial vibration frequency spectrum of stage two and (c) rotor orbits within stage two.
rub impact, and axial component of the radial rubbing contact force causes the same frequency components in the axial direction. As the axial vibration amplitude decreases, the axial rub impact force and friction torque acting on the rotor increase, which leads to the amplitude of 0 Hz component and displacement of rotor orbit become larger and larger.

Because of the occurrence of rub impact, it is decided to disassemble the test rig and check the technical condition of all its components. Figure 20 gives the status of bearing and rotor. Visual inspection of the components reveals clear signs of wear of bearing B2 and rotor. As shown in Figure 20(a), rotor thrust surface was painted red before experiment. After experiment, the red color was worn totally, which signified the occurrence of full annular axial rub. Features of full annular axial rub impact could also be observed on the thrust surface of bearing B2. Grooves on bearing thrust surface were almost flattened. Besides, features of wear could also be observed on rotor journal, which resulted from the radial rub between rotor and bearing.

**Figure 20.** Technical conditions of gas bearing and rotor. (a) status of rotor thrust surface before experiment, (b) status of rotor thrust surface after experiment, (c) status of thrust surface of bearing B2 after experiment, and (d) sign of radial rub impact between rotor with bearing B3.

Comparison of numerical results with experiment

Rotor dynamics of full annular axial rub impact are researched with finite element method. The calculation results show that 0 Hz frequency and offset of rotor orbits are typical dynamic characteristics of full annular axial rub impact.

In the experiment, during slightly axial rub impact, dynamic characteristics in the radial direction change barely, and 0 Hz frequency occurs in the axial direction. In stages one and two, 0 Hz frequency appears in all
the six degrees of freedom, and orbits show transverse motion in the radial direction. Amplitude of 0 Hz frequency and transverse displacement of orbits increase with deepening of axial rub. The numerical results are consistent with the experimental results.

In the experiment, gas film whirl and oscillation occur, and dynamic characteristics of radial rub impact are observed in radial and axial directions. Because only main stiffness of gas bearings and full annular axial rub impact are considered in numerical calculation, the numerical results do not show gas film whirl and oscillation and radial rub impact dynamics.

Conclusions

This paper numerically and experimentally investigates on the full annular axial rub impact in gas bearing rotor system. The radial and axial vibration characteristics within rub speed range are analyzed in the time and frequency domains, and conclusions are summarized as given below.

The finite element design language-APDL is used for rotor dynamics analysis of axial rub impact. During the calculation, the unbalance force, the inertia force, and the axial rub force and torque on rotor thrust disk are considered. After the full annular axial rub impact occurs, 0 Hz component appears in all the six degrees of freedom, and amplitude of it increases with deepening of full annular axial rub impact. Correspondingly, the rotor orbit has a displacement in the radial direction, and the displacement becomes larger with deepening of axial rub impact.

In dynamic experiment, the lightly and serious full annular axial rub impact are investigated. Within lightly axial rub impact speed range, the unbalance response and gas film oscillation are two dominant motion types in radial direction, and frequency characteristics of axial vibration are almost the same with those in radial direction except for the 0 Hz frequency.

In dynamic experiment, amplitude of axial vibration decreases rapidly resulting in serious full annular axial rub impact. Meanwhile, the radial rub impact occurs during this process. Non-synchronous motions especially the sub-harmonic vibrations can be observed in both axial and radial directions. Sub-harmonic frequency components are caused by radial rub impact. The axial components of forces caused by radial rub lead to the same vibration frequencies in axial direction. The axial rub impact force and torque cause the 0 Hz frequency, and it can also be observed in the radial direction. As a result, the rotor orbit has a displacement in x direction.

It can be concluded from the numerical calculation and experiment that the 0 Hz frequency and the displacement of rotor orbit are signature features caused by the full annular axial rub impact.

The force model of full annular axial rub impact is simplified reasonably. Pitch angle of thrust plate and cross stiffness and damping of gas bearings are ignored in this paper. This causes the numerical results to have a difference with the experimental phenomenon. In the following research, the support model of gas bearings and coupling effect of radial and axial rub impacts of gas bearing rotor system will be researched deeply. It is beneficial to the recognition of rub impact of gas bearing rotor system.

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References

1. Dessornes O, Landais S, Valle R, et al. Advances in the development of a microturbine engine. J Eng Gas Turbine Power 2014; 136: 071201.
2. San Andrés L, Kim TH, Ryu K, et al. Gas bearing technology for oil-free microturbomachinery: research experience for undergraduate (re) program at Texas A&M University. In: ASME turbo expo 2009: power for land, sea, and air. New York, NY: American Society of Mechanical Engineers, 2009, pp.845–857.

3. San Andrés L and Kim TH. Forced nonlinear response of gas foil bearing supported rotors. Tribol Int 2008; 41: 704–715.

4. Waumans T, Vleugels P, Peirs J, et al. Rotordynamic behaviour of a micro-turbine rotor on air bearings: modelling techniques and experimental verification. In: International conference on noise and vibration engineering (ISMA2006), Leuven, Belgium, September 2006, pp.18–20.

5. San Andrés L and Ryu K. Hybrid gas bearings with controlled supply pressure to eliminate rotor vibrations while crossing system critical speeds. J Eng Gas Turbine Power 2008; 130: 062505.

6. Yang Y, Yang Y, Cao D, et al. Response evaluation of imbalance-rub-pedestal looseness coupling fault on a geometrically nonlinear rotor system. Mech Syst Signal Process 2018; 118: 423–442.

7. Liu J. A dynamic modelling method of a rotor-roller bearing-housing system with a localized fault including the additional excitation zone. J Sound Vib 2020; 469: 115144.

8. Liu J and Shao Y. Dynamic modeling for rigid rotor bearing systems with a localized defect considering additional deformations at the sharp edges. J Sound Vib 2017; 398: 84–102.

9. Bently DE and Hatch C T. Fundamentals of rotating machinery diagnostics. Mech Eng-CIME 2003; 125: 450.

10. Yuan Z, Chu F, Hao R, et al. Simulation of rotor’s axial rub-impact in full degrees of freedom. Mech Mach Theory 2007; 42: 763–775.

11. Chu F and Lu W. Experimental observation of nonlinear vibrations in a rub-impact rotor system. J Sound Vib 2005; 283: 621–643.

12. Ma H, Shi C, Han Q, et al. Fixed-point rubbing fault characteristic analysis of a rotor system based on contact theory. Mech Syst Signal Process 2013; 38: 137–153.

13. Silva A, Zarzo A, González JMM, et al. Early fault detection of single-point rub in gas turbines with accelerometers on the casing based on continuous wavelet transform. J Sound Vib 2020; 487: 115628.

14. Pennacchi P, Bachschmid N, Tanzi E, et al. Light and short arc rubs in rotating machines: Experimental tests and modelling. Mech Syst Signal Process 2020; 225–2227.

15. Yang Y, Cao D, Yu T, et al. Prediction of dynamic characteristics of a dual-rotor system with fixed point rubbing – theoretical analysis and experimental study. Int J Mech Sci 2016; 253–261.

16. Xu H, Wang N, Jiang D, et al. Dynamic characteristics and experimental research of dual-rotor system with rub-impact fault. Shock Vib. Epub ahead of print 29 August 2016. DOI: 10.1155/2016/6239281.

17. Wang N, Jiang D and Xu H. Effects of rub-impact on vibration response of a dual-rotor system-theoretical and experimental analysis. Exp Tech 2020; 44: 299–311.

18. Han Q, Luo HT, Wen B, et al. Simulations of a dual-rotor system with local rub-impacts based on rigid-flexible multi-body model. Key Eng Mater 2009; 413–414: 677–682.

19. Ma H, Yin F, Wu Z, et al. Nonlinear vibration response analysis of a rotor-blade system with blade-tip rubbing. Nonlinear Dyn 2016; 84: 1225–1258.

20. Wang N, Liu C, Jiang D, et al. Casing vibration response prediction of dual-rotor-blade-casing system with blade-casing rubbing. Mech Syst Signal Process 2019; 118: 61–77.

21. He ZJ and Huang ZY. Selected examples of mechanical faults. Xian: Xian Jiaotong University Press, 1991. (in Chinese)

22. Qian D. Lateral vibration of a rotor/bearing system with axial rubs. J Mech Strength 2004; 26: 132–137.

23. Zhang K and Ding Q. Lateral and torsional vibrations of a two disk rotor-stator system with axial contact/rubs. Int J Appl Mech 2009; 1: 305–326.

24. Abdelrhman AM, S, Tang ES, Leong MS, et al. Numerical investigations on axial and radial blade rubs in turbo-machinery. In: IOP conference series, Curtin University, Malaysia, 20–21 April 2017.

25. Wang W, Gao J, Zhang Y, et al. Numerical and experimental investigation on the controlling for rotor-to-stationary part rubbing in rotating machinery. In: ASME 2011 turbo expo: turbine technical conference and exposition. New York, NY: American Society of Mechanical Engineers, 2011, pp.425–434.

26. Zywica G and Kaczmarczyk TZ. Experimental evaluation of the dynamic properties of an energy microturbine with deformations at the sharp edges. J Sound Vib 2013; 366: 115–135.

27. Xu KJ and Qin HQ. Nonlinear characteristics of axial contact rotor-stator system with coupled bending and torsion. Chin J Appl Mech 2006; 23: 577–582. (in Chinese)

28. Xu KJ and Qin HQ. Dynamic model of two disk rotor-stator rubbing action. J Vib Shock 2007; 26: 17–21. (in Chinese)

29. Xu KJ and Qin HQ. Numerical simulation analysis for rubbing model of a two-disk rotor-stator system. J Vib Shock 2007; 26: 74–79 + 84. (in Chinese)

30. Luan Y. GB1220-2007, stainless steel bars. Beijing: Standards Press of China, 2007.

31. Liu T, Liu Y and Jie CS. Static pressure gas lubrication. HarBin: Harbin Institute of Technology Press, 1990. (in Chinese)

32. Yunfei W. Gas lubricated theory and design manual of gas bearings. Beijing: China Machine Press, 1999. (in Chinese)

33. Qin ZY, Han QK and Chu FL. Analytical model of bolted disk-drum joints and its application to dynamic analysis of jointed rotor. Arch Proc IMechE, Part C: J Mechanical Engineering Science 2014; 228: 646–663.
Appendix

Notation

\( A \) area of rotor thrust disk
\( B_1 \) radial-thrust hybrid gas bearing
\( B_2 \) pure thrust hybrid gas bearing
\( F_A \) axial rub impact force of full annular axial rub impact
\( F_{Af} \) tangential force of full annular axial rub impact based on the coulomb friction law
\( F_{B1} \) thrust gas film force of bearing \( B_1 \)
\( F_{B2} \) thrust gas film force of bearing \( B_2 \)
\( F_p \) axial rub impact force of point axial rub impact
\( F_{pf} \) tangential force of point axial rub impact based on the coulomb friction law
\( F_T \) axial load of turbine
\( K_{axi} \) axial stiffness coefficient of stator
\( M_{Af} \) torque of full annular axial rub impact
\( M_{pf} \) friction torque of point axial rub impact
\( O_r \) center of rotor disk
\( R \) radius of thrust disk
\( R_1 \) rotor radius at thrust disk
\( R_2 \) outer radius of thrust disk
\( z_a \) axial vibration amplitude
\( \delta \) transient axial clearance between rotor disk and stator
\( \delta_i \) static axial clearance between rotor disk and stator
\( \mu \) axial friction coefficient between rotor thrust disk and bearing \( B_2 \)
\( \theta_x \) angle response around axis \( x \)
\( \theta_y \) angle response around axis \( y \)
\( \theta_z \) angle response around axis \( z \)

Appendix 1

The air film force of thrust bearing is composite of static pressure carrying capacity and dynamic pressure carrying capacity.

Equations of the outlet pressure of orifices \( p_d \) are as follows

\[
\frac{\beta^2 - \sigma^2}{\sigma \psi} = f_1 f_2 f_3 \tag{2}
\]

\[
\beta = \frac{p_d}{p_o} \tag{3}
\]

\[
\sigma = \frac{p_a}{p_o} \tag{4}
\]
\[
\psi = \begin{cases} 
\left[ \frac{k}{k+1} \left( \frac{2}{k+1} \right)^{(k+1)/k} \right]^{0.5} \frac{p_d}{p_o} \leq \left( \frac{2}{k+1} \right)^{k/(k+1)} \\
\left\{ \frac{k}{k+1} \left[ \left( \frac{p_o}{p_d} \right)^{(k+1)/k} - \left( \frac{p_o}{p_d} \right) \right] \right\}^{0.5} \frac{p_d}{p_o} > \left( \frac{2}{k+1} \right)^{k/(k+1)} 
\end{cases}
\]

(5)

\[
f_1 = \frac{\phi A n}{h^3}
\]

(6)

\[
f_2 = \frac{12\eta}{\pi} \sqrt{\frac{2}{p_o p_a}}
\]

(7)

\[
f_3 = \frac{\ln \left( \frac{p_o}{p_c} \right) \ln \left( \frac{R_o}{R_i} \right)}{\ln \left( \frac{p_o}{p_c} \right) + \ln \left( \frac{R_o}{R_i} \right)}
\]

(8)

Equations of static pressure carrying capacity \( W \) are as follows

\[
W = \pi R_i^2 p_d \left[ 1 - \frac{\sigma}{\beta} \left( \frac{R_i}{R_o} \right)^2 - \exp \left( -\frac{2}{G_2} \sqrt{\frac{2}{2}} \int \sqrt{\frac{\sigma}{\beta}} \exp(\gamma^2) \, d\gamma \right) \right] + \frac{\pi R_i^2 p_d}{2} \left[ \frac{\sigma}{\beta} \left( \frac{R_o}{R_i} \right)^2 - 1 - \sqrt{\frac{G_1}{2}} \exp \left( \sqrt{\frac{\sigma}{\beta}} \right) \int \sqrt{\frac{1}{\gamma}} \exp(-\gamma^2) \, d\gamma \right]
\]

(9)

\[
G_1 = \left[ 1 - \left( \frac{\sigma}{\beta} \right)^2 \right] / \ln \frac{R_o}{R_i}
\]

(10)

\[
G_2 = \left[ 1 - \left( \frac{\sigma}{\beta} \right)^2 \right] / \ln \frac{R_i}{R_i}
\]

(11)

where \( p_d \) is environmental pressure, \( p_o \) is bearing supply pressure, \( \phi \) is flow coefficient, generally 0.8, \( A \) is area of feed holes, \( h \) is thickness of gas film, \( n \) is number of orifices, \( \eta \) is viscosity coefficient of gas at 20°C, \( \rho \) is gas density at 20°C, \( k \) is gas constant, 1.4, \( R_i \) is inner radius of thrust bearing, \( R_o \) is outer radius of thrust bearing, and \( R_i \) is radius of orifice distribution circle. \( \beta \) can be obtained by numerical calculation or chart after \( f_1, f_2, \) and \( f_3 \) are calculated.

Equations of dynamic pressure carrying capacity \( W_p \) are as follows

\[
W_p = \frac{3\pi \mu \omega R_o^4}{2h^2} g_1(xH_1b_1) \left( \frac{r_{2d}}{R_o} \right)^4 \frac{R_{inv}}{(1 - R_g^2 - 2 \left( 1 - R_g^2 \right) \ln \left( \frac{R_i}{R_i} \right) g_5(xH_1b_1) - \left( \ln R_g \right) g_5(xH_1b_1) \right)}
\]

(12)

\[
H_1 = \frac{h}{h_1}
\]

(13)

\[
b_1 = \frac{b_1}{b_g}
\]

(14)
\[
\begin{align*}
\frac{r_{2ef}}{R_o} &= R_o e^{-\frac{L}{2\pi R_o (1 - \frac{b}{2R_o})}} 
\frac{2}{(2\pi R_o)} 
\left( 1 - H_1^2 \right) 
\left( 1 - H_1^2 \right) 
\left[ 1 - \frac{b_1 H_1^2 \tan \left( 1 + H_1 \right) + b_2 \left( 1 - H_1^2 \right) - H_1^2 \csc^2 \left( 1 + H_1 \right)}{b_2 \tan \left( 1 - H_1 \right) - H_1 \cot \left( 1 - H_1 \right) (1 + H_1^2)} \right]^{-1}
\end{align*}
\]  
\tag{15}

\[
\begin{align*}
\frac{r_g}{R_o} &= \sqrt{0.5 \left( R_i^2 + R_o^2 \right)} 
\end{align*}
\tag{16}

\[
\begin{align*}
R_g &= \frac{r_g}{r_{2ef}}
\end{align*}
\tag{17}

\[
\begin{align*}
g_1(zH_1 b_1) &= \frac{b_1 H_1^2 \cot \alpha (1 - H_1) \left( 1 - H_1^2 \right)}{(1 + b_1 H_1^2) (b_1 + H_1^2) + H_1^2 (\cot \alpha)^2 (1 + b_1)^2}
\end{align*}
\tag{18}

\[
\begin{align*}
g_3(zH_1 b_1) &= H_1^3 (1 + b_1) (1 + \cot^2 \alpha) \left( b_1 + H_1^2 \right)
\end{align*}
\tag{19}

\[
\begin{align*}
g_5(zH_1 b_1) &= (1 + b_1 H_1^2) (b_1 + H_1^2) + H_1^2 \cot^2 \alpha (1 + b_1)^2
\end{align*}
\tag{20}

\[
\begin{align*}
R_{\text{ew}} &= \left( R_o^3 \ln R_o + 0.5 - 0.5 R_o^2 \right) g_3 - \left[ R_g^2 \ln \left( \frac{r_g}{R_o} \right) - 0.5 R_g^2 + 0.5 \frac{R_g^2}{r_{2ef}^2} \right] g_5
\end{align*}
\tag{21}

\[
\begin{align*}
m_{r_2} &= 0.5 \left( 1 - \left( \frac{r_g}{R_o} \right)^2 \right) \frac{b_1 H_1^2 \cot \alpha (1 + b_1) (1 - H_1) (1 - H_1^2)}{\ln \left( \frac{r_g}{R_o} \right) g_5 - \ln \left( \frac{r_g}{R_o} \right) g_3}
\end{align*}
\tag{22}
\]

where \( h_1 \) is gap between bottom of bearing grooves and rotor thrust plate, \( \alpha \) is angle of grooves, \( b_r \) and \( b_g \) represent width of table and groove, and \( N_g \) is number of grooves.