Effects of a neutrino-dark energy coupling on oscillations of high-energy neutrinos

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If dark energy (DE) is a dynamical field rather than a cosmological constant, an interaction between DE and the neutrino sector could exist, modifying the neutrino oscillation phenomenology and causing CP and apparent Lorentz violating effects. The terms in the Hamiltonian for flavor propagation induced by the DE-neutrino coupling do not depend on the neutrino energy, while the ordinary components decrease as \(\Delta m^2/E_\nu\). Therefore, the DE-induced effects are absent at lower neutrino energies, but become significant at higher energies, allowing to be searched for by neutrino observatories. We explore the impact of the DE-neutrino coupling on the oscillation probability and the flavor transition in the three-flavor framework, and investigate the CP-violating and apparent Lorentz violating effects. We find that DE-induced effects become observable for \(E_\nu m^{\text{eff}} \sim 10^{-20}\) GeV\(^2\), where \(m^{\text{eff}}\) is the effective mass parameter in the DE-induced oscillation probability, and CP is violated over a wide energy range. We also show that current and future experiments have the sensitivity to detect anomalous effects induced by a DE-neutrino coupling and probe the new mixing parameters. The DE-induced effects on neutrino oscillation can be distinguished from other new physics possibilities with similar effects, through the detection of the directional dependence of the interaction, which is specific to this interaction with DE. However, current experiments will not yet be able to measure the small changes of \(\sim 0.03\%\) in the flavor composition due to this directional effect.

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I. INTRODUCTION

Dark energy (DE) is a well established hypothesis in cosmology, being the driving force behind the accelerated expansion of the Universe. It makes up for \(\sim 68\%\) of the total energy density in the current Universe [1]. However, the nature of this presumed DE is still unknown, and several possible explanations are being considered. It could be a cosmological constant, which is a constant-valued energy density through time and space [2,3]. The other possibility is that it is composed of a scalar field, like quintessence [4,5]. In the latter case, DE might be able to undergo interactions with standard model particles, which we can search for in experiments.

For instance, there could exist a coupling between neutrinos and dynamical field DE. Such a coupling gives rise to an effective potential, which engenders an effect on neutrino oscillations that influences the evolution equation in a way that one could compare with the Mikheyev-Smirnov-Wolfenstein (MSW) effect that occurs when neutrinos propagate through matter [6–9]. This interaction will change the oscillation probability, and therefore has an impact on the flavor ratios of the neutrinos detected at Earth. The DE-induced part in the Hamiltonian for flavor propagation is independent of the neutrino energy, while the normal vacuum part falls off as \(1/E_\nu\). Therefore, the effect becomes more significant for higher neutrino energies, and might be detectable in experiments sensitive to high-energy extraterrestrial neutrinos such as IceCube [10] and KM3NeT [11] and ultrahigh energy neutrinos such as ANITA [12] and Auger [13]. Furthermore, since the expansion of the Universe is going outward in all directions, the preferred frame of this cosmic expansion is orthogonal to surfaces of constant DE density. Therefore, since we as observers are not in the cosmic-microwave-background (CMB) rest frame, the effect of the DE-neutrino interaction does depend on the propagation direction of the neutrinos. This CPT and Lorentz violating coupling has been studied before in Ref. [14], and in this
work, we extend this idea to the case of three-neutrino mixing.

With the IceCube detector fully operating and KM3NeT to follow in the near future, a new window has opened for searches of new physics. A general study of new physics through high energetic neutrinos and their effect on the flavor ratio at Earth is performed in Ref. [15], by introducing effective operators. (See also Refs. [16–24] for earlier theoretical work.) The DE-neutrino coupling that we study is a model that predicts specific types of terms in the interaction Lagrangian, which engenders such new physics. Other neutrino interactions are investigated in Refs. [25] and [26], which explore couplings between neutrinos and dark matter, and between neutrinos and the cosmic neutrino background respectively. In Refs. [27,28] the parameter space for the flavor ratio at Earth is explored, considering several beyond-the-standard-model theories that have an impact at the production, propagation, and detection of astrophysical neutrinos. Recently, the IceCube Collaboration performed a search for signals of Lorentz violation in their data of high-energy atmospheric neutrinos [29], and obtained stringent constraints particularly for higher dimensional operators than the ones that we specifically study for DE-neutrino couplings. (See Refs. [30–34] for earlier constraints.) In addition, since, as we show later, the oscillation length of the DE-induced mixing is much larger than the travel distance of atmospheric neutrinos, the constraints obtained in [29] are not applicable on the DE-neutrino coupling that we study.

In this paper we study the impact of the possible DE-neutrino coupling on the flavor composition of high-energy extraterrestrial neutrinos and the consequences of this interaction for current and future experiments. We explore the behavior of the probability and the CP violating effects, as well as the effects of the directional dependence. We also determine the sensitivity for experiments to be able to measure those effects.

The paper is organised as follows. In Sec. II, we introduce the theory behind the DE-neutrino coupling and derive the DE induced oscillation probability in the framework of three-neutrino mixing. Extra details can be found in the Appendix. In Sec. III A, we explore the effects of the coupling on the behavior of the oscillations of high-energy neutrinos and discuss the impact on the flavor composition. We also investigate the CP-violating effects. In Sec. III B, we determine the sensitivity to those effects for current and future experiments, and explore the directional effects in Sec. III C, followed by conclusions in Sec. IV.

**II. THEORY**

**A. Dark energy-neutrino interaction**

We consider the DE-Neutrino coupling, following the discussion in Ref. [14]. Considering three neutrino flavors, the neutrino fields are described by the Dirac spinor set $\{ \nu_e, \nu_\mu, \nu_\tau \}$, and their charge conjugates by the set $\{ \nu_e^c, \nu_\mu^c, \nu_\tau^c \}$. The six neutrino fields are combined in the object $\nu_A$, where $A$ runs over the neutrino flavors and their conjugates. The most general Lorentz/CPT-violating form of the equations of motion is then given by [19]

$$ (i\gamma^\mu \delta_\mu - M_{AB})\nu_B = 0, $$

where

$$ M_{AB} \equiv m_{AB} + im_{SAB}\gamma_5 + a_{AB}^\mu \gamma_\mu + b_{AB}^\mu \gamma_\mu + \frac{1}{2} H_{AB}^\mu_\sigma \gamma_\sigma. $$

The four-vectors $a^\mu$, $b^\mu$, and the antisymmetric tensor $H_{AB}^\mu$ in Eq. (2) parametrize Lorentz violation. $H_{AB}^\mu$ is only Lorentz violating, while the parameters $a^\mu$ and $b^\mu$ are CPT violating as well. These parameters are highly restricted in our case where the coupling with DE is responsible for the Lorentz/CPT violation. The expansion of the Universe has an outward direction, thus the unit four-vector that parametrizes the preferred frame of this cosmic expansion, $l^\mu$, is orthogonal to the surfaces of constant DE density, which is closely aligned with the surfaces of constant CMB temperature [14,35–38]. Therefore, $a^\mu \propto l^\mu$ and $b^\mu \propto l^\mu$, where $l^\mu = (1, 0, 0, 0)$ in the rest frame of the CMB. Also, $H_{AB}^\mu$ should be proportional to $l^\mu$, but since it is not possible to create an antisymmetric tensor from just one four-vector, $H_{AB}^\mu$ has to be zero in the case of our DE-neutrino coupling. Finally, the DE-neutrino coupling can be parametrized solely by the combination of $a^\mu$ and $b^\mu$, namely $(a^\mu)_{ab} \equiv (a + b)^\mu_{ab}$, where we have $(a^\mu)_{ab} \propto l^\mu$ [14,19]. Because the velocity of our solar system with respect to the CMB rest frame is $\sim 10^{-3}$ times the speed of light, we have $(a^\mu)_{ab} p_\nu \propto E(1 - \nu \cdot \hat{p})$, where $\nu$ is our velocity with respect to the CMB rest frame and $\hat{p}$ is the neutrino propagation direction.

A simple form of Langrangian that describes an interaction by the DE-neutrino coupling is given by

$$ \mathcal{L}_{int} = -\lambda_{ab} \frac{\partial_\nu \phi}{M_\nu^2} \bar{\nu}_a \gamma^\mu (1 - \gamma_5) \nu_\beta, $$

where $\phi$ is a quintessence field, $\lambda_{ab}$ is a coupling constant matrix and $M_\nu$ is the energy scale of the interaction. In this example, we have $a^\mu \sim \lambda_\phi(\nu^\mu)/M_\nu$.\footnote{This interaction may also give rise to scattering between the neutrinos and DE-induced particles, for which we show that the mean-free path for the neutrinos is much larger than the Hubble length in Appendix B.}

The effective Hamiltonian that describes the propagation of the flavor eigenstates to leading order is given by
where the indices $a, b$ run over the flavor eigenstates $e, \mu, \tau$. The upper left block describes the neutrino interactions, and the lower right the antineutrinos. Since the effective Hamiltonian is block diagonal, no mixing will take place between neutrinos and antineutrinos, and therefore we consider the two blocks for neutrinos and antineutrinos separately.

The Hamiltonian that describes the propagation of neutrinos in vacuum in the mass base is given by

$$H_m = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{bmatrix},$$

where $E_i = \sqrt{p^2 + m_i^2}$. The Hamiltonian in the flavor basis is then obtained by rotating the basis as

$$H_f = U H_m U^\dagger,$$

in which $k_i$ is a constant and the positive (negative) sign is for neutrinos (antineutrinos), and the Hamiltonian in the flavor basis is obtained through

$$V_f = U_{\text{DE}} V_m U_{\text{DE}}^\dagger,$$

where $U_{\text{DE}}$ is an independent unitary matrix.

The mixing matrices $U$ and $U_{\text{DE}}$ are parametrized as

$$U_{\text{DE}} = \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\ -s_{12} c_{23} - s_{13} s_{23} c_{12} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & -s_{23} c_{13} - s_{13} s_{23} c_{12} e^{i \delta} \\ s_{12} s_{23} - s_{13} c_{23} e^{i \delta} & -s_{23} c_{13} - s_{13} s_{23} c_{12} e^{i \delta} & c_{13} c_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i \beta_1} & 0 \\ 0 & 0 & e^{i \beta_2} \end{bmatrix}. $$

In the standard PMNS matrix $U$, $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, where $\theta_{ij}$ are the vacuum mixing angles and $\delta$ is the $CP$-violating phase. For the values of the vacuum parameters, we use the best fit values from the Particle Data Group [40]. The equivalent mixing matrix for the DE-induced interaction is given by $U_{\text{DE}}$, where $c_{ij,\text{DE}} = \cos \theta_{ij,\text{DE}}$ and $s_{ij,\text{DE}} = \sin \theta_{ij,\text{DE}}$, with $\theta_{ij,\text{DE}}$ and $\delta_{\text{DE}}$ the extra DE-induced mixing angles and $CP$-violating phase. The matrix on the right contains the two independent Majorana phases, relevant in the case of Majorana neutrinos. However, the oscillation probability does not depend on the Majorana phases, and therefore (DE induced) neutrino oscillations are not sensitive to these (DE induced) phases.

**B. Oscillation probabilities**

The Schrödinger equation in the flavor basis is given by

$$i \frac{d}{dt} \psi_f(t) = \mathcal{H}_f \psi_f(t),$$

where

$$\mathcal{H}_f = U H_m U^\dagger + U_{\text{DE}} V_m U_{\text{DE}}^\dagger.$$
\[ P_{\alpha \rightarrow \beta} = |A_{\alpha \beta}|^2. \] (15)

Since \( T \) is Hermitian (\( T^\dagger = T \)), the three eigenvalues \( \lambda_a \) are all real. We now define

\[ c_a = \cos(L\lambda_a), \] (16)

\[ s_a = \sin(L\lambda_a), \] (17)

\[ R_{\alpha \alpha'} = \text{Re}[M_{\alpha \alpha'}], \] (18)

\[ I_{\alpha \alpha'} = \text{Im}[M_{\alpha \alpha'}], \] (19)

and rewrite the oscillation probability as

\[ P_{\alpha \beta} = \delta_{\alpha \beta} - 4 \sum_a \sum_{b < a} [(R_{\alpha \alpha'} R_{\beta \beta'} + I_{\alpha \alpha'} I_{\beta \beta'}) \sin^2 x_{ab}] + 2 \sum_a \sum_{b < a} [(R_{\alpha \alpha'} I_{\beta \beta'} - R_{\beta \beta'} I_{\alpha \alpha'}) \sin 2x_{ab}]. \] (23)

Here in obtaining the first term, we used the fact that \( P_{\alpha \beta} = \delta_{\alpha \beta} \) at \( L = 0 \).

Rather than on the individual parameters \( k_i \) that show up in the Hamiltonian of Eq. (7), the probability will depend on the differences \( k_j - k_i \), which we call the effective mass parameter, \( m_{\text{eff}, \alpha \beta} \equiv k_j - k_i \). Since we consider the three-flavor case, two of them are independent: \( m_{\text{eff}, 1} \equiv k_2 - k_1 \) and \( m_{\text{eff}, 2} \equiv k_3 - k_1 \). When both independent effective mass parameters equal to zero, Eq. (23) returns the vacuum oscillation probability.

For distances much larger than the oscillation length, we may replace \( \sin^2 x_{ab} \to 1/2 \) and \( \sin 2x_{ab} \to 0 \), while for distances much shorter than the oscillation length, it is not possible to observe effects induced by the DE-neutrino coupling. For example, if the effective mass parameter has a value of \( m_{\text{eff}} = 10^{-23} \) GeV, the oscillation length is approximately \( L_{\text{osc}} \approx 10^{14} \) km. Since in our case, we are interested in astrophysical neutrinos, the probability that we use reduces to

\[ P_{\alpha \beta} = \delta_{\alpha \beta} - 2 \sum_a \sum_{b < a} [(R_{\alpha \alpha'} R_{\beta \beta'} + I_{\alpha \alpha'} I_{\beta \beta'}) \sin^2 x_{ab}] + 2 \sum_a \sum_{b < a} [(R_{\alpha \alpha'} I_{\beta \beta'} - R_{\beta \beta'} I_{\alpha \alpha'}) \sin 2x_{ab}]. \] (24)

This is justified especially for sources at cosmological distances, \( L \sim H_0^{-1} \), which is equivalent to assuming \( m_{\text{eff}} \gg H_0 \approx 10^{-42} \) GeV. In the next section, we shall see that this is indeed the case for the values of \( m_{\text{eff}} \) that we consider.

As can be seen from the DE-induced Hamiltonian in Eq. (8), the DE-induced part of the probability has different sign for neutrinos and antineutrinos; i.e., \( CP \) is violated. It also does not depend on the neutrino energy, while the vacuum probability falls off over \( E_\nu \). Therefore, the impact of DE on neutrino oscillations will become more significant for higher neutrino energies, and thus the effect could be explored through experiments such as IceCube and KM3NeT.

Finally, the DE-induced part is frame dependent. It depends on our velocity with respect to the CMB rest frame, and the propagation direction of the incoming neutrino.

To summarize, the probability will depend on three new mixing angles, one extra \( CP \)-violating phase, and two independent effective mass parameters. We will investigate the impact of the DE-neutrino coupling on neutrino oscillations and explore how the probability behaves for different values of the new mixing parameters in the next section. Throughout this work, we assume normal mass hierarchy.

### III. RESULTS

#### A. Behavior of the probability

To explore the effect of the DE-neutrino coupling on what we detect here at Earth, we determined the possible final flavor compositions at the time of detection in the presence of this coupling. The result can be seen in Fig. 1. We varied all the values of the new mixing parameters, and determined the final flavor composition for several starting flavor ratios at the source. The expected composition for vacuum oscillation is also included, for which the mixing parameters are fixed at the best-fit values of the Particle Data Group [40].

As can be seen, the part of the composition-triangle that could be reached at Earth, depends on the flavor composition at the source. The cyan colored area corresponds to the source composition \( 1:2:0 \) for the flavors \( e', \mu', \tau \), which is the characteristic flavor composition from pion decays. This is the main channel in which astrophysical neutrinos...
are expected to be produced. In the case that there is no new physics, the expected flavor composition measured at detection is approximately 1:1:1 as shown as the “cross” symbol.

Starting from a purely single flavor state, the possible area after these DE-neutrino interactions can occupy only one-third of the entire triangle. No astrophysical process is known to produce r neutrinos. In Fig. 2, the possible flavor compositions are shown for all flavor compositions at the source consisting of a combination of $\nu_e$ and $\nu_\mu$. The cyan colored region corresponds to the case that there is no new physics. If the observed flavor composition lies outside the cyan region, then it is not compatible with normal oscillation, and regarded as an indication of new physics. If the ratio lies in the magenta region, this could be due to the DE-neutrino coupling. The lower left part of the triangle cannot be reached by conventional astrophysical neutrinos even with an effect of the DE-neutrino coupling we study. Therefore, it requires both $\nu_e$ production at the source and non-standard neutrino oscillation such as the DE-neutrino interaction (Fig. 1).

In Fig. 3, we explore the behavior of the probability as a function of energy. In these plots, the flavor composition at the source is set to 1:2:0, and the values of the two independent effective mass parameters are set to $m_{\text{eff}12} = m_{\text{eff}13}/2 = 10^{-20}$ GeV. We set the new CP-violating phase equal to zero, and consider the cases that $\theta_{\text{DE}12} = 0.25\pi$, $\theta_{\text{DE}13}, \theta_{\text{DE}23} = 0$ [Fig. 3(a)]...
at earth changes from approximately $1:1:1$, to exactly $1:1:1$. This is also the case in Fig. 3(c).

### B. Sensitivity

Figure 1 shows that all the possible final flavor compositions from the initial flavor ratio of $1:2:0$ are still allowed in light of the IceCube constraints [42]. To this end, we also investigated the sensitivity for experiments to be able to detect the effects from the DE-neutrino coupling. For this, we compare the total amount of muon neutrinos with the null hypothesis that no new physics is detected. We calculate this for the case that $\theta_{13}^{\text{DE}}, \theta_{23}^{\text{DE}} = 0$, corresponding to the case explored in Fig. 3(a). We set limits on the parameter space for the effective mass parameter $m_{\text{eff}}$ and the mixing angle $\theta_{12}^{\text{DE}}$. If a number of $N_{\nu}^{\text{tot}}$ neutrino events is detected at the experiment, assuming a flavor ratio of $1:2:0$ at the source, the number of $\nu_{\mu}$ we expect to measure is given by

\begin{align*}
N_{\nu_{\mu}} = N_{\nu_{\mu}}^{\text{tot}} & \int_{E_{\text{min}}}^{E_{\text{max}}} E^{-2} P_{\nu_{\mu}} \text{d}E \\
& + \frac{2N_{\nu_{\mu}}^{\text{tot}}}{3} \int_{E_{\text{min}}}^{E_{\text{max}}} E^{-2} P_{\nu_{\mu}} \text{d}E. \quad (25)
\end{align*}

Here we assume that the neutrino energy spectrum multiplied by the effective area roughly scales as $E^{-2}$. In case that no new physics is detected, the expected number of muon neutrinos is $N_{\nu_{\mu}} = N_{\nu_{\mu}}^{\text{tot}} / 3$. To obtain the limits on $m_{\text{eff}}$ and $\theta_{12}^{\text{DE}}$ with 95% confidence level, we solve

\begin{align*}
N_{\nu_{\mu}} < \frac{N_{\nu_{\mu}}^{\text{tot}}}{3} + 2 \sqrt{\frac{N_{\nu_{\mu}}^{\text{tot}}}{3}} \quad (26)
\end{align*}

for $m_{\text{eff}}$ and $\theta_{12}^{\text{DE}}$, where we choose $m_{\text{eff}1} = 2m_{\text{eff}3}$. In Fig. 5(a), we show the sensitivity to probe for the value of $m_{\text{eff}}$ for experiments measuring neutrino events in the
energy range from 100 TeV to 10 PeV—which holds for, for example, IceCube and KM3NeT—in case that they measure 100, 1000, and 10 000 neutrino events, as a function of the mixing angle $\theta_{12}^{\text{DE}}$. Given that IceCube already has found tens of neutrino events above $\sim$10 TeV [44,45], proper analysis will enable to exclude $m_{\text{eff}} \gtrsim 10^{-27}$ GeV in the near future. In Fig. 5(b), we show the same for (future) experiments sensitive to ultrahigh-energy (UHE) neutrinos, capable of detecting neutrinos in the energy range between 100 PeV and 10 EeV. Values of $m_{\text{eff}}$ and the corresponding values of $\theta_{12}^{\text{DE}}$ that lie above the coloured curves would result in an atypical increase of the amount of $\nu_\mu$ at the detector. In a similar way, this could be calculated for $\nu_e$ and $\nu_\tau$.

Looking back at the toy model for a possible Lagrangian of the DE-neutrino coupling could look like in Eq. (3), we follow Ref. [14] to explore the mass scale of the interaction corresponding to a certain value of the effective mass

![Diagram](a)

![Diagram](b)

**FIG. 4.** The flavor ratios as a function of neutrino energy for different values of $\delta_{\text{DE}}$, for both neutrinos and antineutrinos. The values of the new mixing parameters are set to $\theta_{12}$, $\theta_{13}$, $\theta_{23} = 0.25\pi$, $m_{\text{eff},12} = \frac{1}{2}m_{\text{eff},11} = 10^{-26}$ GeV and the starting flavor ratio is set to 1:2:0. The usual CP-violating phase $\delta_{\text{BE}}$ is set to zero. The new DE-induced CP-violating phase is set to (a) $0.25\pi$, (b) $0.5\pi$, and (c) 0. The effect in (c) comes solely from the sign difference of $m_{\text{eff}}$ in the oscillation probability for neutrinos and antineutrinos.

![Diagram](a)

![Diagram](b)

**FIG. 5.** The sensitivity for neutrino experiments capable of distinguishing muon neutrinos to probe for the value of $m_{\text{eff}}$, in case of a detection of 100, 1000 and 10 000 events, for different neutrino energy ranges: (a) 0.1–100 PeV and (b) 0.1–100 EeV. When the genuine values of the parameters $m_{\text{eff}}$ and $\theta_{12}$ lie above the colored lines, more $\nu_\mu$ than compatible with standard physics will be detected. In this example, the new mixing angles are set to $\theta_{13}$, $\theta_{23} = 0$. The flavor composition at the source is set to 1:2:0.
parameter $m_{\text{eff}}$. We have $a_L'' \sim \lambda \dot{\phi}(t)/M_*$ and $m_{\text{eff}} \sim \Delta \dot{\phi}(t)/M_*$, with $\Delta \lambda$ the difference between the eigenvalues of $\lambda_{\text{eff}}$. For quintessence we assume $\dot{\phi} \sim M_{\text{Pl}} H_0(1 + w)^{1/2}$ [5, 14], where $M_{\text{Pl}}$ is the Planck mass. The energy scale of the interaction is therefore given by

$$M_* \approx 10^6 (\Delta \lambda) \left( \frac{1 + w}{0.01} \right)^{1/2} \left( \frac{10^{-30} \text{ GeV}}{m_{\text{eff}}} \right) \text{GeV}. \quad (27)$$

Therefore, the experiments corresponding to Figs. 5(a) and 5(b) probe up to mass scales of $M_* \sim 10^5$ GeV and $M_* \sim 10^8$ GeV respectively.

### C. Directional dependence

There are multiple new physics hypotheses that could result in a flavor composition that is not compatible with normal physics (see, e.g., Refs. [15, 27, 28]), and the DE-neutrino coupling is just one of the possibilities. However, the directional dependence of the DE-neutrino coupling is very specific to this model. The DE-induced part of the probability is proportional to $(1 - v \cdot \hat{p})$, and therefore results in a different mixing probability for identical neutrinos with different propagation directions. Since our velocity with respect to the CMB rest frame is $\sim 10^{-3} c$, the effects of the directional component will be small compared to the general effects of the DE-neutrino coupling. To calculate the directional effect, we follow Ref. [14] with some modifications to set our coordinate system. The origin is set at the south pole of the Earth, with the $z$-axis aligned along the rotational axis of the Earth, such that the north pole lies on the positive axis. The $x$-axis is set along the direction to the Sun at spring equinox, while the $y$-axis is set along this direction at summer solstice. The seasonal rotation can be expressed by the azimuthal angle $\phi_s$, where $\phi_s = 0$ and $\phi_s = \pi$ for spring and autumn equinox respectively. Since the velocity of the Sun with respect to the CMB rest frame is $v_\odot = 369 \text{ km s}^{-1}$ in the direction $\alpha = 168^\circ$, $\delta = -7.22^\circ$, where $\alpha$ and $\delta$ are right ascension and declination respectively, in our coordinate system this velocity is $v_\odot = v_\odot (\cos \delta \cos \alpha, \cos \delta \sin \alpha, \sin \delta) = (-385, 76.1, -46.4) \text{ km s}^{-1}$. Because the Earth moves around the Sun with an average orbital speed of...
\(v_\oplus = 29.8 \text{ km s}^{-1}\), the velocity of the Earth with respect to the CMB rest frame is
\[
v_\oplus = v_\odot + v_\oplus = \begin{pmatrix}
\sin \phi_v \\
-\cos \phi_v \cos \theta_{\text{inc}} \\
-\cos \phi_v \sin \theta_{\text{inc}}
\end{pmatrix} \text{ km s}^{-1},
\]
\[
= \begin{pmatrix}
-358 + 29.8 \sin \phi_v \\
76.1 - 27.3 \cos \phi_v \\
-46.4 - 11.9 \cos \phi_v
\end{pmatrix} \text{ km s}^{-1}, \tag{28}
\]
where \(\theta_{\text{inc}}\) is the inclination between the \(x - y\) plane and the orbital plane around the Sun. In our coordinate frame, the south pole is set to \((0, 0, 0)\). The propagation direction of the incoming neutrino is therefore described by the unit vector
\[
\hat{p} = \begin{pmatrix}
\cos \theta_v \cos \phi_v \\
\sin \theta_v \cos \phi_v \\
\sin \phi_v
\end{pmatrix}, \tag{29}
\]
where \(\phi_v\) and \(\theta_v\) are the polar and azimuthal angle of the incoming neutrino at the south pole respectively. Since the source lies outside Earth, the propagation direction of the neutrino path with respect to the CMB background does not depend on the rotation of the Earth, although it would in the case of an Earth-based neutrino beam. The mixing probability has terms that are proportional to \((1 - v \cdot \hat{p})^2\), \((1 - v \cdot \hat{p})\) and terms that are not dependent on \((1 - v \cdot \hat{p})\) at all. We expect the effect to be larger when the terms \(\propto (1 - v \cdot \hat{p})\) dominate, which is the case in the transition phase.

In Fig. 6, the effect on the final flavor composition as a function of the different variable angles is shown. In Fig. 6(a), it is seen that the effect is much smaller than the effects of the other new parameters explored earlier in this paper. We have to zoom in on one particular flavor to be able to visualize the effect. In Fig. 6(b), the fraction of \(\nu_\mu\) is plotted as a function of the seasonal shift \(\phi_v\). The effect is extremely small, with a maximal change of \(\sim 0.002\%\) depending on the season in which the neutrinos are detected. Clearly, at this moment, it is far beyond our current abilities to measure such small differences. The advantage of the seasonal shift is that, in the case that a source produces neutrinos on a regular basis such as blazars, the flavor ratio of neutrinos originating from that source could be evaluated in different seasons to search for a seasonal effect. The effect of the propagation direction of the neutrinos is slightly larger than the seasonal effect, as can be seen from Figs. 6(c) and 6(d), in which the fraction of \(\mu\) is plotted as a function of the incoming directions \(\theta_v\) and \(\phi_v\) respectively. The maximal change between the flavor fractions depending on the incoming direction is \(\sim 0.03\%\). Although this effect is an order of magnitude larger than the effect of the seasonal shift, it is still outside our observational reach to detect such small effects. However, eventually future experiments might become more sensitive, and meanwhile in the years or decades to come, data are being collected, contributing to better statistics. In the case that new physics with the effects described in Sec. III A is found, the directional effect would be the evidence for a DE-neutrino coupling rather than some other solution. It would be also evidence for a noncosmological constant type of DE.

**IV. CONCLUSION**

We are only at the beginning stage of collecting data from high energy neutrinos, and exciting times lie ahead. It will not take long before IceCube and KM3NeT will determine if the measured flavor ratio at Earth is compatible with normal physics. We explore a possible origin for new physics results in neutrino telescopes and how this would establish in measurements here on Earth.

The physics we investigate is a possible coupling between dark energy (DE) and neutrinos, which engenders an additional source for neutrino mixing. Such a coupling might exist in the case that DE is a dynamical field rather than a cosmological constant. We study the impact on neutrino oscillations in the three-neutrino framework and find that this could result in significant observable effects on Earth. The part of the oscillation probability that is induced by DE is independent of energy in the propagation Hamiltonian, has different sign for neutrinos and anti-neutrinos, and contains a directional component. Furthermore, the probability depends on three extra mixing angles, one new \(CP\)-violating phase, and two independent mass parameters \(m_{\text{eff}}\). Because of the energy independency of the DE induced part of the Hamiltonian, while the vacuum oscillation term is proportional to \(\sim \frac{\Delta m_{\odot}}{2\pi}\), the effect of the DE-neutrino coupling becomes larger for higher neutrino energies. Below are our main findings.

1. The transition from the energy scale in which vacuum oscillation dominates, to the energy scale where the DE-induced mixing dominates, happens around \(E_\nu m_{\text{eff}} \sim 10^{-20} \text{ GeV}^2\).
2. We explored the effect of the coupling on the flavor composition of astrophysical neutrinos that we would measure on Earth. Depending on the flavor composition at the source and the values of the new mixing parameters, the possible final flavor ratios cover the entire flavor composition triangle, while vacuum oscillation covers only a limited area. If no tau-neutrinos are produced in astrophysical sources, however, part of the flavor composition triangle cannot be reached even through mixing induced by DE.
3. We also explored the effect on the flavor composition due to the sign difference in the probability between the neutrinos and antineutrinos and the new \(CP\)-violating phase. Neutrinos and antineutrinos behave differently over a wide energy range, which might be possible to detect if the effective mass parameter \(m_{\text{eff}}\) happens to have a value between
The matrix $M$ Organisation for Scientific Research (NWO) through a Vidi Research on Matter (FOM), which is part of the
ments at an early phase of this project. This work is part of
We thank Marc Kamionkowski for his insightful com-
Because the cosmic expansion has a preferred frame,
We also determined the sensitivity for current and
future experiments to probe the value of the effective mass parameters $m_{\text{eff}}$. We find that current experi-
ments are able to measure anomalous effects due to
the DE-neutrino coupling, and can probe the values of
the new mixing parameters, for a genuine value of
the effective mass parameter down to $m_{\text{eff}} \sim 10^{-27}$,
depending on the number of detected neutrino events. Experiments capable of detecting ultra-
high-energy neutrons could probe further down
to $m_{\text{eff}} \sim 10^{-30}$.

(5) Because the cosmic expansion has a preferred frame,
namely the rest frame of the CMB, the value of $m_{\text{eff}}$
does depend on our velocity with respect to the
CMB rest frame, and gets slightly altered for differ-
ent propagation directions of the incoming neutrinos
(directional dependence), as well as the position of
the Earth with respect to the Sun (the seasonal
dependence). These effects are small, resulting only
in differences between the flavor composition on
a subpercentage level, but of big importance in the
case new physics is found, since this effect is an
unique feature of the DE-neutrino coupling.

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APPENDIX A: THE AMPLITUDE OF THE FLAVOR TRANSITION

The exponential of an $N \times N$ matrix $M$ can be expressed as
$a_0 I + a_1 M + \ldots + a_{N-1} M^{N-1}$.  
\begin{equation}
(1)
\end{equation}
The matrix $M$ can also be expressed as
$M = M_0 + \frac{1}{N} (\text{tr}M) I$,
\begin{equation}
(2)
\end{equation}
where $M_0$ is an $N \times N$ traceless matrix. By combining
Eqs. (1) and (2), and defining the complex phase $\phi \equiv e^{-i \lambda T} / 2$ and the traceless matrix $T \equiv \lambda I - (\text{tr}H_I) / 3$, we can write
\begin{equation}
e^{-iH_I L} = \phi e^{-i L T} = \phi(a_0 I - i L T a_1 - L^2 T^2 a_2).
\end{equation}
The coefficients $a_0$, $a_1$, and $a_2$ can be computed from the following system of linear equations:
\begin{equation}
e^{-i L a_0} = a_0 - i L a_1 - L^2 a_2,
\end{equation}
\begin{equation}
e^{-i L a_1} = a_0 - i L a_2 - L^2 a_3,
\end{equation}
\begin{equation}
e^{-i L a_2} = a_0 - i L a_3 - L^2 a_3,
\end{equation}
where $\lambda_1$, $\lambda_2$, and $\lambda_3$ are the eigenvalues of $T$, by solving
\begin{equation}
a = \Lambda^{-1} e,
\end{equation}
where
\begin{equation}
e = \left( e^{-i L a_1}, e^{-i L a_2}, e^{-i L a_3} \right), \quad \Lambda = \left( \begin{array}{lll}
1 - i L \lambda_1 - L^2 \lambda_1^2 \\
1 - i L \lambda_2 - L^2 \lambda_2^2 \\
1 - i L \lambda_3 - L^2 \lambda_3^2
\end{array} \right), \quad a = \left( \begin{array}{c}
a_0 \\
a_1 \\
a_2
\end{array} \right).
\end{equation}
such that eventually we have
\begin{equation}
U_f(L) \equiv e^{-i H_I L} = \frac{1}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \phi e^{-i L a_0} \lambda_2 \lambda_3 I - (\lambda_2 + \lambda_3) T + T^2 \\
+ \frac{1}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \phi e^{-i L a_1} \lambda_1 \lambda_3 I - (\lambda_1 + \lambda_3) T + T^2 \\
+ \frac{1}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \phi e^{-i L a_2} \lambda_1 \lambda_2 I - (\lambda_1 + \lambda_2) T + T^2.
\end{equation}
The eigenvalues $\lambda_i$ of $T$ are solutions of the equation
\begin{equation}
\lambda^3 + c_2 \lambda^2 + c_1 \lambda + c_0 = 0,
\end{equation}
where
\begin{equation}
c_0 = - \text{det} T,
\end{equation}
\begin{equation}
c_1 = T_{11} T_{22} - T_{12} T_{21} + T_{13} T_{33} - T_{13} T_{31} \\
+ T_{22} T_{33} - T_{23} T_{32},
\end{equation}
\begin{equation}
c_2 = - \text{tr} T.
\end{equation}

APPENDIX B: ESTIMATION OF THE MEAN FREE PATH

In the case that DE is a scalar field, in theory there could be a nonzero cross section for neutrino scatterings with

\begin{equation}
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\end{equation}
DE-induced particles. In this Appendix, we show that this would have no effect on the neutrino propagation that we discuss in this paper. To show this, we estimate the mean free path due to this DE-neutrino scattering interaction, following [46]. In order to simplify the estimate, here we assume the weak interaction for the coupling strength. We neglect the neutrino mass, the cross section in the rest frame of the DE-induced particle is approximately

\[ \sigma_{\nu \phi} \sim G_F E m_\phi \sim 10^{-38} \text{cm}^2 \left( \frac{E m_\phi}{\text{GeV}} \right) , \]  

(B1)

where \( G_F \) is the Fermi coupling constant, \( E \) is the neutrino energy, and \( m_\phi \) is the mass of the DE-induced particle. The mean free path can then be calculated by \( l' \sim (n_\phi \sigma_{\nu \phi})^{-1} \), where \( n_\phi \) is the number density of the DE-induced particles. Considering a neutrino of 100 PeV and assuming that the energy density of the DE-induced particle is of similar order as the local dark matter density, \( \rho \sim 0.4 \text{GeV/cm}^3 \) [47–49], we estimate \( l' \sim 10^4 \text{ Mpc} \), which is much larger than the Hubble length. For lower neutrino energies, this value will be even larger. Thus, we can safely ignore any effect of a possible non-zero cross section for interactions between neutrinos and DE-induced particles.

\[ \cdots \]

\[ \begin{align*}
\text{PHYS. REV. D} & \quad 97, 063006 (2018) \\
\end{align*} \]
[40] K. A. Olive et al. (Particle Data Group), Chin. Phys. C 38, 090001 (2014).
[41] T. Ohlsson and H. Snellman, J. Math. Phys. (N.Y.) 41, 2768 (2000); 42, 2345(E) (2001).
[42] M. G. Aartsen et al. (IceCube Collaboration), Astrophys. J. 809, 98 (2015).
[43] S. L. Glashow, Phys. Rev. 118, 316 (1960).
[44] M. G. Aartsen et al. (IceCube Collaboration), Astrophys. J. 833, 3 (2016).
[45] M. G. Aartsen et al. (IceCube Collaboration), Proc. Sci., ICRC2017 (2017) 981.
[46] C. Giunti and C. W. Kim, Fundamentals of Neutrino Physics and Astrophysics (Oxford University Press, New York, 2007), p. 710.
[47] R. Catena and P. Ullio, J. Cosmol. Astropart. Phys. 08 (2010) 004.
[48] M. Pato, F. Iocco, and G. Bertone, J. Cosmol. Astropart. Phys. 12 (2015) 001.
[49] Q. Xia, C. Liu, S. Mao, Y. Song, L. Zhang, R. J. Long, Y. Zhang, Y. Hou, Y. Wang, and Y. Wu, Mon. Not. R. Astron. Soc. 458, 3839 (2016).