Attitude Interpolation Algorithm for Industrial Robot Based on Quaternion Method

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Abstract. In order to achieve the smooth transition of the end-tool pose of the industrial robot, after determining the pose of the teaching points of the robot. Firstly, the path of the pose change between the adjacent teaching points is constructed by using the SLERP method, then, the position of the start and end points of the attitude transition curve is determined, and the expression of the attitude trajectory curve is obtained. According to the relation between the derivative of quaternion and Angular Velocity, the conversion between unit quaternion and angular velocity is realized by using Newton-Cortez formula. Finally, the velocity of the fitted industrial robot's tool's attitude trajectory is planned, and the whole attitude trajectory is interpolated.

1. Introduction
The industrial robot is composed of many joints in series, and the end of the manipulator can be equipped with different actuators to complete the work instead of human. Each joint of the robot is driven by a Servo Motor, and all the joints make up the joint motion space of the robot; the motion space of the robot's end effector is usually called Descartes space, Descartes Space and Joint Motion Space can be transformed through the forward and inverse kinematics of the robot[1]. Industrial robots usually perform task by means of teaching and reappearing [2]. Firstly, the position and attitude coordinates of the end effector are set artificially, and then the trajectory planning is carried out between teaching points, after path planning between teaching points, time variable is added to speed planning to complete the trajectory planning of robot in Descartes Space [3]. The trajectory planning of robot end effector can be divided into position trajectory planning and attitude trajectory planning, in which the research of position trajectory planning is based on the computer graphics [4][5]. In contrast, the pose trajectory of the robot is much more complicated.

Document studied the fairing method of the linear path in the workpiece coordinate system, and obtained the equal distance double NURBS path satisfying the continuous precision [6]. The dual quaternion method is used to re-express the position and attitude of the tool, and the quaternion b-spline function is used to fit the tool path, and the equidistant double NURBS tool path describing the position and attitude data is interpolated. Document [7] studies the method of generating the path of B-spline curve for extracting the key points of tool position, sets up the node vector according to the key points and adopts the least squares approximation, defines a new format of g code for NURBS interpolation, the method of generating dual NURBS interpolation G code is also studied. Yang aiming of breakpoints in the dual NURBS tool path which do not meet the fitting accuracy, proposed a fairing processing method based on the quaternion Bezier vector function, improved the curve fitting
accuracy of the tool path, the servo performance of the robot joint axis is improved to a certain extent [8]. In the paper [9] the related problem of using NURBS curve interpolation to restrain nonlinear error is studied, and the constant speed interpolation based on NURBS model is used.

Zhang [10] studies the quaternion representation of tool posture, establishes the mapping relationship between Cutter Position Vector points and quaternion points on the unit sphere, and transforms the interpolation of tool posture into a curve design problem in quaternion space, the attitude curves of the unit spherical tool in the form of parametric curves are obtained. In view of the problem of the distance fluctuation between the parametric curve path, the tool center curve and the tool direction vector curve from the discrete tool position data, the method of tool direction vector NURBS curve optimization on the unit sphere is studied, and the better effect is obtained [11]. In order to solve the problem of discontinuity of teaching point data, the NC code is used to restore the effective path and NURBS fitting under error constraint, and the rotation axis path is interpolated by spline curve, the mapping relationship between the rotating axis path and the machining path is established[12].

Based on the above literature, the tool path smoothing technology is helpful to improve the application effect of robot in machining technology, and the parametric curve is becoming the research hotspot in recent years. However, the variation of tool vector obtained by NURBS interpolation cannot accurately reflect the curvature variation of the processed object. It is necessary to continue to study the curve interpolation technique which can control the position and attitude of the tool.

According to the Chasles theory, the space motion of a rigid body can be expressed as a spiral motion composed of rotation and translation. There are three common ways to represent the rotation of space, namely, Matrix, Euler angles, and quaternion[13]. The rotation of a rigid body in a 3D can be effectively dealt with by the algebraic operation of quaternions. In order to describe the spiral motion of the tool, the quaternion method is used to interpolate the position and attitude of the tool.

2. Quaternion and tool attitude interpolation

2.1 Dual quaternion and space transformation

2.1.1 The concept of quaternions. The general form of a quaternion is[14]

\[ Q = (q_0, q_1, q_2, q_3) = q_0 + q_1i + q_2j + q_3k \]  

Among them, \( q_0 \), \( q_1 \), \( q_2 \) and \( q_3 \) are the 4 components of Quaternion, \( q_0 \) is the real part and \( q_1i + q_2j + q_3k \) is the imaginary part.

A quaternion is a complex number in 3D, and has three orthogonal imaginary units \( i, j, \) and \( k \). The units vector can be treated as a quaternion with a real part equal to 0, while a real number can be treated as a quaternion with an imaginary part equal to 0.

Quaternions can be expressed in the form of the following axis-angle Pairs

\[ Q = \left[ q_0 \quad (q_1 \  q_2 \  q_3) \right] = [\cos(\theta/2) \ \sin(\theta/2) \ n_x \ \sin(\theta/2) \ n_y \ \sin(\theta/2) \ n_z] \]  

Where \( n \) is the axis of rotation and \( \theta \) is the angle of rotation. The unit quaternion is denoted by \( Q = [1 \quad 0 \quad 0 \quad 0] \), which means that there is no angular displacement around the vector.

The conjugate of quaternion \( Q \) is \( Q^* = [q_0 \quad -(q_1 \  q_2 \  q_3)] \) and the inverse is \( Q^{-1} = Q^* / \|Q\| \). In Formula (2), \( n \) is the unit vector, i. e. \( \|Q\| = 1 \), so

\[ Q^{-1} = [q_0 \quad -(q_1 \  q_2 \  q_3)] \]  

The space vector point \( N = xi + yj + zk \) can be expressed as \( N = [0 \quad (x \  y \  z)] \) in the form of quaternions. Let \( Q_1 = [\cos(\theta_1/2) \ \sin(\theta_1/2) \ n_x] \) and \( Q_2 = [\cos(\theta_2/2) \ \sin(\theta_2/2) \ n_z] \) be the other two quaternions. If \( N \) is rotated at an angle of \( n_1 \) around the axis \( \theta_1 \), the new vector point is
\[ N' = Q_1NQ_1^{-1} \]  \hspace{1cm} (4)

If it is rotated at an angle of \( \theta \) around the axis \( n \), then
\[ N'' = Q_2N'Q_2^{-1} = Q_2(Q_1NQ_1^{-1})Q_2^{-1} = (Q_2Q_1)N(Q_2Q_1)^{-1} \]  \hspace{1cm} (5)

2.1.2. Dual quaternion. Dual number is an extension of the concept of complex number, it is an ordered combination of real units 1 and dual units \( \varepsilon \) and two real elements \( b \) and \( b' \). Its general form is[15]
\[ \tilde{b} = b + \varepsilon b' \]  \hspace{1cm} (6)

Where \( \varepsilon \) is the Clifford operator, followed by \( \varepsilon^2 = 0 \), the two pairs of even numbers \( \tilde{b}_1 \) and \( \tilde{b}_2 \), for example, the Algorithm is
\[
\begin{aligned}
\tilde{b}_1 + \tilde{b}_2 &= (b_1 + b_2) + \varepsilon (b'_1 + b'_2) \\
\tilde{b}_1\tilde{b}_2 &= b_1b_2 + \varepsilon (b_1b'_2 + b'_1b_2)
\end{aligned}
\]  \hspace{1cm} (7)

Replacing the scalar in formula (6) with a vector constitutes a dual vector. If \( \tilde{b}_1 \) and \( \tilde{b}_2 \) are two quaternions, it is called a dual quaternion. Let \( \tilde{Q} = Q + \varepsilon Q' \) be a dual quaternion composed of quaternion \( Q = q_0 + q_1i + q_2j + q_3k \) and \( Q' = q'_0 + q'_1i + q'_2j + q'_3k \), then the conjugacy of \( \tilde{Q} \) is
\[ \tilde{Q}^* = Q^* + \varepsilon Q'^* = (q_0 + q'_0) - (q_1 + q'_1)i - (q_2 + q'_2)j - (q_3 + q'_3)k \]  \hspace{1cm} (8)

The measure value of a dual quaternion is
\[ \|\tilde{Q}\| = \sqrt{\tilde{Q}Q^*} = \sqrt{\tilde{Q}^*\tilde{Q}} = \|Q\| + \langle \tilde{Q}, Q^* \rangle \]  \hspace{1cm} (9)

The unit dual quaternion is defined as
\[ \tilde{Q}Q^* = (Q + \varepsilon Q')(Q^* + \varepsilon Q'^*) = QQ^* + \varepsilon (QQ'^* + Q'O') = 1 \]  \hspace{1cm} (10)

Dual conjugation is also used to describe the transformation of spaces, which is defined as
\[ \tilde{Q} = Q - \varepsilon Q' = (q_0 - q'_0) - (q_1 - q'_1)i - (q_2 - q'_2)j - (q_3 - q'_3)k \]  \hspace{1cm} (11)

2.2. Quaternion interpolation of tool attitude
Quaternions are suitable for interpolation because they are normalizable 4D vectors. The LINEAR interpolation formulas for two adjacent discrete unit quaternions \( Q_1 \) and \( Q_2 \) of tool attitude are
\[ Q(t) = (1-t)Q_1 + tQ_2, \quad t \in [0,1] \]  \hspace{1cm} (12)

As can be seen from Fig. 1, the vector length can not be kept constant by General Linear interpolation. This can be improved by
\[ Q(t) = ((1-t)Q_1 + tQ_2)\| (1-t)Q_1 + tQ_2 \|, \quad t \in [0,1] \]  \hspace{1cm} (13)

The endpoint of the interpolation vector \( V(t) \) will always fall on an arc on the surface of the
sphere from which the vectors $V_S$ and $V_E$ are located, thus keeping the same length. Because the method is carried out in the direction of chord length, the angular velocity cannot be kept constant and the smoothness of vector rotation is poor. The following is an interpolation model for the smooth rotation of quaternions.

$$\text{SLERP}(V_S, V_E, t) = \frac{\sin((1-t)\theta)}{\sin \theta} V_S + \frac{\sin(t\theta)}{\sin \theta} V_E, \quad t \in [0,1]$$

(14)

In the formula, $\theta$ is the angle between the above two unit quaternions on the sphere, i.e. $\theta = \arccos(V_S \cdot V_E)$. It's not hard to see that there's a relationship between the two endpoints. $\text{SLERP}(V_S, V_E, 0) = V_S$ holds.

When the interpolation parameters change uniformly in the defined domain, the rotation between the first and last directions can be realized smoothly. Taking the NURBS curve of tool position point in Fig. 3(a) as an example, the tool attitude trajectory is located on the unit sphere, and the spherical sector obtained by SLERP method tool attitude interpolation is shown in Fig. 3(b).

Taking a NURBS curve in figure 3 as an example, linear interpolation and quaternion smooth interpolation are used for tool attitude. Fig. 4 shows the curve of change of tool attitude rotation angle. The standard deviation $\sigma$ of the two is 0.0468 and 0.0023 respectively. Thus, the curve of quaternion method is much smoother.
The ideal angular velocity planning can be obtained by Quaternion interpolation of tool attitude. In order to obtain a smooth tool motion path, the translational motion in the workpiece coordinate system should be considered together with the rotational motion of the tool.

3. Tool path generation based on dual quaternion

3.1. Dual quaternion representation of rigid body motion in space

A coordinate system \( C_m \) is fixed to the moving rigid body, and its origin coincides with the fixed coordinate system \( C_f \). The rotation displacement of a rigid body relative to the coordinate system \( C_f \) can be described by a 3-order Matrix \( A = [X_{mf}, Y_{mf}, Z_{mf}] \), where \( X_{mf} \), \( Y_{mf} \) and \( Z_{mf} \) are the coordinates of the unit vector in coordinate system \( C_m \) in \( C_f \). Any point \( P \) in a rigid body is assumed to have a coordinate vector of \( p = (p_x, p_y, p_z)^T \) in the fixed coordinate system \( C_f \) and a coordinate value of \( p' = (p'_x, p'_y, p'_z)^T \) in the fixed coordinate system \( C_m \).

\[
p = Ap'
\]  (15)

The following euler parameters can be obtained from the rotation Matrix \( A \):

\[
\begin{align*}
c_0 &= \cos(\theta/2) \\
c_1 &= \sin(\theta/2)s_x \\
c_2 &= \sin(\theta/2)s_y \\
c_3 &= \sin(\theta/2)s_z
\end{align*}
\]  (16)

Where \( s = (s_x, s_y, s_z) \) is the unit vector in the direction of the rotation axis and \( \theta \) is the rotation angle.

After the above parameters are obtained, the unit quaternion which represents the rotation displacement can be expressed as

\[
Q = c_0 + c_1i + c_2j + c_3k
\]

The General Space Displacement is described by \( [A, d] \), where \( A \) is a three-order unit orthogonal Matrix, representing the direction of the coordinate system \( C_m \) relative to the coordinate system \( C_f \), and the three-dimensional vector \( d = (d_x, d_y, d_z)^T \) represents the position vector of the origin of the coordinate system \( C_m \) in the coordinate system \( C_f \). Its corresponding quaternion is written as \( D = 0 + d_xi + d_yj + d_zk \). Any point \( P \) on a rigid body can be expressed as

\[
P = Ap' + d
\]  (17)

The above rigid body space motion can be described by dual quaternions as
Where \( Q = q_0 + q_1 i + q_2 j + q_3 k \) is a quaternion derived from Matrix \( A \) and \( R = r_0 + r_1 i + r_2 j + r_3 k \) is the dual part, determined by both rotational and translational motion, which means
\[
R = \frac{1}{2} DQ
\] (19)

The dual quaternion \( \hat{Q} = (Q, R) \), which describes the spiral motion of a rigid body in space, satisfies the following two conditions:

(i) The relationship between \( Q \) and \( R \) is: \( Q \cdot R = q_0 r_0 + q_1 r_1 + q_2 r_2 + q_3 r_3 = 0 \);

(ii) \( \hat{Q} = (Q, R) \) is homogeneous, that is, if \( k > 0 \), then \( (Q, R) \) and \( (kQ, kR) \) express the same spatial displacement.

3.2. Dual quaternion description of robot tool pose

Before the industrial robot starts the task, the linear discrete data obtained from the teaching operation are the positions and postures of the tools at the end of the robot. In the workpiece coordinate system \( O_WX_WY_WZ_W \), the tool position and pose can be expressed as a six-dimensional vector
\[
P_{\text{tool}} = (p_x, p_y, p_z, a_x, a_y, a_z), \quad \text{of which } P_{\text{tip}} = (p_x, p_y, p_z) \text{ is the tool position vector and } a = (a_x, a_y, a_z) \text{ is the tool unit vector.}
\]

Let us calculate the dual quaternion \( \hat{Q} \) of \( P_{\text{tool}} \).

The quaternion corresponding to the tool position point is \( D = 0 + p_x i + p_y j + p_z k \), and the quaternion corresponding to the tool vector is \( Q = [\cos(\theta/2) \quad \sin(\theta/2) n] \). The rotation axis \( n = (n_x, n_y, n_z) \) and rotation angle \( \theta \) of \( Q \) are calculated respectively. As shown in Fig. 5, a secondary coordinate system \( O_W'X_W'Y_W'Z_W' \) is set up at tool position \( P_{\text{tip}} \) in the same direction as coordinate system \( O_WX_WY_WZ_W \).

![Fig. 5 Dual quaternion representation of tool pose](image)

As can be seen in auxiliary coordinate system \( O_W'X_W'Y_W'Z_W' \), \( n \) is actually the axis of rotation that rotates the tool vector \( a = (a_x, a_y, a_z) \) in the direction of \( Z_W' \) axis \( e_z = (0,0,1) \) of the workpiece coordinate system
\[
n = \frac{e_z \times a}{|e_z \times a|} = \frac{1}{\sqrt{a_x^2 + a_y^2}} (a_x i - a_y j)
\] (20)

The quaternion \( Q \) has a rotation angle \( \theta \)
\[
\theta = \arccos\left(\frac{e_z \cdot a}{|e_z| \cdot |a|}\right) = \arccos(a_z)
\] (21)
If we plug formulas 20 and 21 into \( Q = [ \cos(\theta / 2) \ \sin(\theta / 2) \ n] \), we get
\[
Q = \cos \left( \frac{\arccos(a_x)}{2} \right) + \sin \left( \frac{\arccos(a_x)}{2} \right) \left[ \frac{1}{\sqrt{a_x^2 + a_y^2}} (a_y - a_x j + 0k) \right]
\] (22)

The dual quaternions of the tool pose is \( \hat{Q} = Q + \varepsilon R \), of which \( R \) are defined by \( Q \) for rotation and \( D \) for translation, that is
\[
R = \frac{1}{2} DQ = \frac{1}{2} (p_x i + p_x j + p_x k) \left[ \cos \left( \frac{\arccos(a_x)}{2} \right) + \frac{a_x i - a_y j}{\sqrt{a_x^2 + a_y^2}} \sin \left( \frac{\arccos(a_x)}{2} \right) \right]
\] (23)

By substituting formulas 22 and 23 into \( \hat{Q} = Q + \varepsilon R \), the dual quaternion representation of robot tool position and pose can be obtained. Thus, the dual quaternion \( \hat{Q} \) contains all the information needed to describe the pose of the tool.

### 3.3. Dual quaternion based dual NURBS tool path

After the tool pose is expressed by the dual quaternion \( \hat{Q} \), NURBS curves with \( \hat{Q} \) as the data unit can be interpolated to obtain the b-spline curve \( \hat{Q}(u) \) of tool pose \( \hat{Q} \) for \( p \) times. Then, the NURBS tool path, i.e. the tool position curve \( C_{up}(u) \) and the attitude curve \( C_{an}(u) \), are obtained.

With \( n + 1 \) discrete cutter positions \( L_k = (p_{k,1}, p_{k,2}, p_{k,3}, a_{k,1}, a_{k,2}, a_{k,3}) \), \((k = 0,1,\cdots,n)\), the corresponding dual quaternion is \( \hat{Q}_k \) \((k = 0,1,\cdots,n)\). Similar to the curve interpolation steps described in section 2.1, \( L_k \) is first parameterized by assigning a parameter value of \( \hat{u}_k \) to each cutter location data unit \( \hat{Q}_k \). The modulus \( |\hat{Q}_i - \hat{Q}_{i-1}| \), which is the difference between two adjacent dual quaternions, is still referred to as the ‘chord length’.

The total chord length \( d_i \) of the dual quaternion is
\[
d_i = \sum_{i=1}^{n} |Q_i - Q_{i-1}|
\] (24)

Among them
\[
|Q_i - Q_{i-1}| = |Q_i - Q_{i-1}| + \frac{(Q_i - Q_{i-1}) (R_i - R_{i-1})}{|Q_i - Q_{i-1}|}
\]

The value of the parameter \( \hat{u}_k \) corresponding to \( \hat{Q}_k \) is: \( \hat{u}_0 = 0, \hat{u}_n = 1 \), while the remaining term \( \hat{u}_k \) is
\[
\hat{u}_k = \hat{u}_{k-1} + \frac{|Q_i - Q_{i-1}|}{d_i}, \quad k = 1, \cdots, n - 1
\] (25)

The average value method is used to determine the node vector of B-spline curve \( \hat{Q}(u) \), that is
\[
u_0 = \cdots = u_p = 0;
\]
\[
u_{p+1} = \cdots = u_{n-p+1} = 1;
\]
\[
u_{j+p} = \frac{1}{p} \sum_{i=1}^{p} \hat{u}_i, \quad j = 1, \cdots, n - p
\] (26)

Then, the control vertices of \( \hat{Q}(u) \) are obtained by the method mentioned above, and finally the dual quaternion B-spline curves \( \hat{Q}(u) \) describing the tool trajectory are obtained. According to the motion trajectory of the discrete rigid-body tool, all points on the tool can be described by the rational
function, therefore, the tool motion path expressed by equidistant NURBS curves can be obtained according to the tool position points and the tool attitude motion path. Because dual quaternion $\hat{Q}_i$ integrates tool position and tool attitude information, the chord length parameterization method can not only reflect the distance between adjacent tool position points, but also take into account the angle between adjacent tool attitude. Therefore, the node vector obtained here can be shared by two NURBS curves.

The control vertices of record curves $C_{tip}(u)$ and $C_{axis}(u)$ are $\{P^p_i\}$ and $\{P^a_i\}$ respectively. According to the given tool position data point $L_k = (p_{k,1}, p_{k,2}, p_{k,3})$, tool attitude $a_k = (a_{k,1}, a_{k,2}, a_{k,3})$, set the distance between the tool position point and the tool position point tool $L_k$, then the attitude point $p^a_k$ is

$$p^a_k = (p_{k,1}, p_{k,2}, p_{k,3}) + L_{tool}(a_{k,1}, a_{k,2}, a_{k,3})$$ (27)

To sum up, the following system of linear equations can be obtained

$$p_k^p = C_{tip}(\hat{u}_k) = \sum_{i=0}^{n} N_{i,p}(\hat{u}_k)P^p_i,$$

$$p_k^a = C_{axis}(\hat{u}_k) = \sum_{i=0}^{n} N_{i,p}(\hat{u}_k)P^a_i$$ (28)

The control vertices $\{P^p_i\}$ and $\{P^a_i\}$ can be obtained by solving the equations, and the final NURBS trajectory curves representing the position and attitude of the tool at the end of the industrial robot can be obtained.

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