constraints for this example are summarized in Table I and the net results based on the algorithm described in Fig. 2 are summarized in Table II. It is observed from Table II that the applications of dual-mode horns and trimode horns with optimum ratios improve the cross-pol performance by 4.674 and 19.021 dB, respectively, compared to the horns with single-mode excitations. These results are consistent with the findings published in [2], which presented results for a single-feed horn.

III. CONCLUSIONS

A systematic algorithm for suppressing the cross-polarized component of single-offset reflector antennas illuminated by a cluster of multimode horn feeds using a constrained minimization routine has been developed. The algorithm allows for design constraints to be imposed during the minimization process. The trimode-type horns provide for the greatest improvement in cross-polarization performance. While shown here for a scanned beam case, the algorithm would also be useful for minimizing cross polarization in shaped-beam applications as well.

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I. INTRODUCTION AND FORMULATION

The Kirchhoff-type aperture integration (AI) is the simplest way to calculate the radiation of an open-ended waveguide (OEW) excited by a mode. Reducing the AI to a line integration (LI) may provide a significant improvement in terms of computational efficiency. This is particularly useful in the framework of a method of moments (MoM) procedure, which is formulated in terms of mode-shaped basis functions. Furthermore, a line integral representation of the aperture field is particularly well suited for introducing fringe augmentation that may be provided by incremental techniques such as the physical theory of diffraction (PTD) [1] and the incremental theory of diffraction (ITD) [2].

In this letter, a method for asymptotically reducing the AI to a LI is presented. One formulation starts from the radiation in freespace of the corresponding semi-infinite distribution of the electric wall currents associated with the unperturbed mode. Recently, the equivalence between the field predicted by AI and the above wall current integration has been rigorously demonstrated for OEW’s with arbitrary cross-section [3]. Since this latter approach resembles the physical optics (PO) method for scattering problems, it will be referred to as PO as well.

Let us consider an OEW of arbitrary cross section, which is excited by either a TE or a TM mode of arbitrary order; the field of this mode is denoted by \( E_{\text{mod}} \), \( H_{\text{mod}} \) and its propagation constant by \( k_{\text{mod}} \). It is useful to define a reference system with its origin inside the aperture and its \( z \) axis parallel to the waveguide generatrix; its relevant spherical coordinate system is denoted by \( (r, \theta, \psi) \). The integration point on the wall is indicated by \( P' \equiv (l', z') \) where \( l' \) is a curvilinear parameter that describes the waveguide contour in the transverse plane [Fig. 1(a)]. A local coordinate system \( (x', y', z') \) is also introduced with its origin at the point \( P' \), its \( z' \) axis parallel to \( z \), and its \( y' \) axis tangent to the surface at \( P' \) [Fig. 1(a)]; a spherical coordinate system \( (\hat{R}, \hat{\Theta}, \hat{\Psi}) \) and its pertinent unit vectors \( (\hat{R}, \hat{\Theta}, \hat{\Psi}) \) is associated with it.

In [3], it has been demonstrated that the AI field is exactly equal to the PO field when observing outside the waveguide. Thus, it is useful to represent the AI field in terms of the spherical components \( E_{\text{mod}}^{\psi} \) of the PO field; i.e.,

\[
E_{\psi}^{\psi} = \int_{l'} F_{\psi}^{\psi}(l') \, dl'; \quad \psi = r, \theta, \psi
\]

Table I

| Constraints for Cross-Polarization Minimization |
|----------------------------------------------|
| Constraints | \( G_{d}(\text{dB}) \) | \( G_{c}(\text{dB}) \) | \( A_{c}(\text{dB}) \) |
| Dualmode Horns | 95.0 | 49.0 | 53.0 |
| Trimode Horns | 95.0 | 49.0 | 53.0 |

Table II

| Modes | \( G_{d}(\text{dB}) \) | \( G_{c}(\text{dB}) \) | \( A_{c}(\text{dB}) \) |
|-------|-----------------|-----------------|-----------------|
| Dualmode Horns | 95.0 | 49.0 | 53.0 |
| Trimode Horns | 95.0 | 49.0 | 53.0 |

Abstract—A line integral representation of the Kirchhoff-type aperture integration is derived for an open-ended waveguide with arbitrary cross section, excited by an arbitrary mode. The problem is formulated by taking advantage of the equivalence between the radiation of the aperture and the radiation of the modal currents along the semi-infinite waveguide walls.

Index Terms—Aperture antennas.
where

\[
F_{\xi}^{\alpha}(l') = \int_{-\infty}^{0} f_{\xi}(z', l') e^{-jklz/(z', l')} e^{-jz'z_{l} \mod d z'}
\]

(2)

and

\[
f_{\xi}(z', l') = -j k o_{\xi} \cdot \left( \hat{\theta} + \hat{\psi} \right) \cdot (\hat{n}_{\omega} \times \hat{R}^{\text{mod}}).
\]

(3)

In (2), the functional dependence of \( R \) on the integration variables has been explicitly indicated. In (3), \( \hat{n}_{\omega} \) is the normal to the wall (pointing inside the OEW, and the gradient operator \( \nabla \) has been replaced by its relevant asymptotic approximation \(-j k \hat{R}\). The term \( F_{\xi}^{\alpha}(l') \, dl' \) in (1) represents the radiation of an elementary, semi-infinite strip of traveling-wave current, parallel to the \( z \) axis [Fig 1(b)].

For each \( \ell' \), the integrand in (1) is asymptotically evaluated by its stationary phase point (SPP) contribution \( F_{\xi}^{\alpha}(l') \) and its end-point (EP) contribution \( F_{\xi}^{\alpha}(l') \), i.e.,

\[
F_{\xi}^{\alpha}(l') \sim F_{\xi}^{\alpha}(l') + F_{\xi}^{\alpha}(l')
\]

(4)

where

\[
F_{\xi}^{\alpha}(l') = U(\theta_{0} - \theta_{t}) f_{\xi}(z_{t}, l') \frac{1}{4j}
\]

(5)

and

\[
F_{\xi}^{\alpha}(l') = f_{\xi}(0, l') \frac{e^{-j kl_{0}}}{4\pi R_{0}} \frac{2j}{\pi k R_{0} \sin \theta_{0}} e^{-j z_{l} \cos(\theta_{0} - \theta_{t})}
\]

(6)

In (5) and (6), \( (R_{0}, \Theta_{0}) \equiv (R, \Theta)_{l', \theta_{t}} \) [Fig. 1(a)]; \( \theta_{t} \) is the incident-mode ray angle that is defined by \( k \cos \theta_{t} = \mathcal{F}(x) \) is the transition function of the uniform theory of diffraction (UTD) [4]; \( z_{t} = z - R_{0} \sin \theta_{0} \cot \theta_{t} \) is the SPP, and \( U(x) \) is a Heaviside unit-step function, which is unity when the SPP lies on the strip and zero elsewhere. The above expressions have been calculated by applying to the integral in (2) a simplified version of the method presented in [5].

In the following, \( F_{\xi}^{\alpha} \) and \( F_{\xi}^{\alpha} \) will be referred to as incremental geometrical optics contribution (IGOC) and incremental end-point contribution (IEC), respectively. As expected from the radiation of a traveling-wave current, each IGOC is a conical wave propagating in the direction \( \theta_{t} \). According to \( U(\theta_{0} - \theta_{t}) \) the conical wave exists within the region \( \theta_{0} > \theta_{t} \) and has its shadow boundary cone (SBC) at \( \theta_{0} = \theta_{t} \) [Fig. 1(b)]. Each IEC is a spherical wave arising from a point of the edge; close to SBC of the corresponding IGOC, the IEC exhibits a transition into a conical wave behavior that allows compensation for the discontinuity of the IGOC. The sum of the IGOC plus the IEC is continuous everywhere and easily integrable.

The IGOC’s can also be interpreted as incremental fields that arise from the SPP’s \( P_{s} = \left( l', z_{s} \right) \) defined by \( \Theta = \theta_{t} \). These points belong to a line \( L \), which is the intersection between the waveguide and a ray cone with vertex in \( P \) and aperture angle \( \theta_{t} \) [Fig. 1(c)]. When \( L \) does not intersect the waveguide rim, the integration of IGOC’s asymptotically reconstructs the field of an infinite waveguide, namely the modal field inside the infinite waveguide and zero outside. To speed-up practical calculations this latter condition (null field outside the infinite waveguide) can be enforced \textit{a priori}. If the line \( L \) intersects the rim [Fig. 1(d)], only a part of \( L \) has to be integrated.
the integration end-points are determined by the condition \( z_s = 0 \)
and described by the unit step function in (5).

It is worth noting that for large apertures in terms of a wavelength,
\( F_\xi^\ast \left( t' \right) \) is a rapidly oscillating function of \( t' \).
Consequently, the integration in (3) can be asymptotically evaluated by its stationary
phase point contributions, thus, leading to a UTD-type ray-field
representation. However, this latter fails in describing the field close
to and at the axial caustic and it has also been found less accurate
with respect to the present numerical line integration for moderate
sized apertures.

II. NUMERICAL RESULTS

Numerical results from AI (continuous line) have been compared
with those from LI (dashed line) for the case of a circular OEW with
radius \( a \). In particular, Fig. 2(a) shows results for a waveguide with
radius \( a = 0.5\lambda \), excited by the TE\(_{11} \) mode. The \( \psi \) component
of the electric field in the H plane is plotted at a distance \( r = 1.5\lambda \)
and \( \theta = 0.7\lambda \), respectively. Both curves are normalized with respect
to the maximum value obtained in the case \( r = 0.7\lambda \); furthermore,
the field is calculated in the region external to the waveguide; i.e.,
\( \theta < 130^\circ \) for \( r = 0.7\lambda \) and \( \theta < 160^\circ \) for \( r = 1.5\lambda \).
Normalized near field patterns for TM\(_{11} \)-mode excitation are presented in Fig. 2(b).
The \( \theta \) component of the electric field in the H plane is plotted for the
two cases \( a = 0.65\lambda \), \( r = 1.5\lambda \), and \( a = \lambda \), \( r = 2\lambda \), respectively.
The curves corresponding to this latter case are shifted 10 dB down
to render the figure more readable.

In spite of the moderate size of the apertures, the agreement
between the AI and its corresponding LI has been found quite
satisfactory over the total 40-dB dynamic range. The small glitches
arise from the fact that the IGCO integration has been turned off when
\( L \) does not intersect the edge [see Fig. 1(c)]. The result presented here
also suggests an effective method to speed-up practical calculations
of the interaction between modes [6].

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An Efficient Formulation for Calculating the Modal
Coupling for Open-Ended Waveguide Problems

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Abstract—A double line integral representation of the mutual coupling
between open-ended waveguides of arbitrary cross section is presented,
which is useful to speed up calculations inside the framework of a
Galerkin method of moments.

Index Terms—Aperture antennas, electromagnetic coupling.

I. INTRODUCTION AND FORMULATION

The theorem demonstrated in [1] establishes a rigorous equivalence
between the field predicted by the Kirchhoff-type aperture integration
(AI) and that radiated by the physical optics (PO) wall current.
By using this equivalence, a formulation has been presented [2] to
asymptotically reduce the AI into a line integration (LI) along the
waveguide edge.

In this paper, the equivalence between PO and AI [1] is applied
to modal coupling between OEW’s of arbitrary cross-sections. This
allows for the derivation of a convenient double integral expression
for the modal coupling to be applied in the framework of a method
of moments (MoM) for arrays of OEW’s and horns. Note that a
MoM Galerkin mutual impedence is generally given in terms of a
quadruple integral in the space domain and the reduction to a
double line integral is possible only for rectangular coplanar apertures
[3]. When closed-form Fourier transform representations of modal
coplanar distributions are available, one can resort to a spectral-
domain approach; however, the resulting double spectral integrals
are improper and slowly convergent. The method presented here is
independent from the waveguide cross sections and, although it is
developed here only for coplanar apertures, it can be easily extended
to the noncoplanar case.

Let us consider two open-ended waveguides OEW1 and OEW2
of arbitrary cross sections. For the sake of simplicity, but without
loss of generality, we will assume the two axes of the OEW’s to be
parallel. Two reference systems are introduced in which
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