Technical Note

Multi-Morphological Characteristics of a Crushed Granitic Rock of Varying Sizes

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Abstract: Crushed rock fragments are usually a mixture of particles with different sizes and morphologies comprising particle shape and surface texture scales. The significant role of particle morphology in the engineering behavior of granular materials has been increasingly appreciated. However, studies on morphology are mainly on particles of similar sizes, and the few studies that reported particles with varying sizes are limited to particle shape scale alone, especially when 3D morphological characteristics are considered. In this paper, we investigate the multi-scale morphological characteristics of crushed rock with a size ranging from sand to gravel by employing a 3D laser scanner and micro-Computed Tomography (µCT) using quantifiers of sphericity, aspect ratio, and roundness for particle shape, and fractal dimension for surface texture. Crushed granitic rock is used as the testing material to elucidate the morphological characteristics of crushed materials, which are not uncommon in geotechnical applications. For the tested crushed granitic rock, as particle size decreases, the overall shape becomes slightly angular, the corner of the particle becomes more rounded, and the surface becomes smoother. Differences in the morphological descriptors for small particles, mainly those with singular mineral composition, have also been observed and might be explained in terms of hardness and fracture features. The observed dependence of morphological descriptors on particle size and mineralogy bears significance for investigations using reconstructed particles of different sizes using 3D-printing techniques and numerical methods.

Keywords: 3D multi-scale morphology; crushed particles; sand and gravel

1. Introduction

The particle morphology of granular materials, such as soils or rock fragments which are usually a mixture of particles of different sizes, comprises different characteristic scales, including particle shape and surface texture. The particle morphology emerges to play an important role in the engineering properties of granular materials, e.g., [1–7]. However, most of the studies on three-dimensional particle morphology are on particles of similar sizes, with only a few exemptions on particles with varying sizes [8–11], where the characterizations are limited to the particle shape scale alone.

Surface texture, though, despite containing important information, e.g., transportation history and formation process [12–17], has not been taken into account, probably due to the complexities of surface measurements and a lack of a proper quantification parameter. Currently, successful measurements of the surface texture of sand have been made with an optical microscope [17–20]. Due to its high resolution, only a small measuring area could be made, which makes the testing apparatus most appropriate for sand-sized particles. For gravel, the larger surface area would make surface measurements using the optical microscope time-consuming, and an alternative has been proposed by using a 3D laser scanner [21]. For its quantification, a statistical parameter—the square root of the surface heights to a mean plane $S_q$, e.g., [18] has been frequently used. However, it has long been
recognized that $S_q$ is resolution-dependent, e.g., [22]. This has prompted the use of a scale-independent fractal dimension which arises from a hierarchical structure of the surface, e.g., [22]. However, using an advanced fractal method involving a spectrum revealing the periodical features of the surface, the authors showed that the fractal parameters could well characterize the surface texture of sand [20,23] and gravel [21]. A complete characterization of particle morphology at both particle shape and surface texture scales with varying particle sizes, though pivotal in revealing the formation and transportation history of granular materials and the generation of artificial particles. Using either numerical methods or 3D printing techniques, has yet not been reported.

In this note, 3D morphological descriptors for not only particle shape but also surface texture have been obtained for particles with sizes spanning from sand to gravel. Crushed granite, the result of mechanical degradation of parent rocks, not uncommon in geotechnical engineering, such as in mechanized tunneling in moderate to hard ground and mining engineering [24], is taken as the exemplar material. Specifically, crushed granitic rocks with particle sizes from 0.6 mm to 25 mm were tested by using a micro-CT and a 3D laser scanner. The adopted tested particles can be distinguished between different mineral types at small particle sizes in which particles mainly consist of a single mineral so that insights into the role of mineralogy on the particle shape can also be gained.

2. Materials and Methods

2.1. Sand and Gravel Particles

Tested particles are made of crushed granitic rock and are of low weathering grade as the fresh rock colors are generally retained but stained near joint surfaces [20]. Particle crushing was completed before they were obtained from a quarry in Hong Kong. These granular particles are common materials used for concrete aggregate. The particle size, obtained from sieving analysis, ranges from 0.6 mm up to 25 mm and were categorized into five size ranges 0.6~1.18 mm, 1.18~2 mm, 2~5 mm, 5~10 mm, and 10~25 mm. Particles at the first two small size ranges (0.6~1.18 and 1.18~2 mm) are mainly composed of a single mineral, as observed by naked eyes, and were further divided into four groups according to their main mineral compositions, i.e., K-feldspar, plagioclase, biotite, and quartz. In total, more than three hundred particles have been tested.

2.2. Testing Apparatus

X-ray micro-CT (Toscaner-3000, Toshiba, Tokyo, Japan) has been used for particles of sizes from 0.6~5 mm (Figure 1). During scanning, a cylindrical container of 20 mm-diameter filled with silica oil was used to accommodate particles of size 0.6~2 mm. A voxel size of around 15 µm has been achieved, whereas for particles of 2~5 mm, a glass slide was used, and the voxel size is around 29 µm. Processing of the obtained images follows the method of Yang et al. [20].

A 3D laser scanner (LPX-60, Roland DG Corporation, Hamamatsu, Japan) was used for gravel particles. The chamber of the scanner can accommodate samples with dimensions up to 203 mm in diameter and 304.8 mm in height. The precision was set to be the highest, which is up to 0.2 mm/scanning pitch in the vertical direction and 0.2 mm/degree in the circumferential direction. The processing of data involves principal component analysis, power spectral analysis, and fractal analysis of a square area retrieved from digitized surfaces. For details of the data processing technique, refer to the work of Yang et al. [21].

2.3. Morphological Descriptors

Morphological features at the form and roundness scale (as in Figure 2) are referred to as particle shape, which is described by sphericity, aspect ratio (including Elongation and Flatness indexes), and roundness, while the surface texture is described by fractal dimensions. They are briefly described below, whereas for detailed descriptions, one would refer to respective references.
Figure 1. Volume rendering of micro-CT images for a layer of particles (mica of size 0.6–1.18 mm).

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Sphericity describes how close a particle to a sphere is and is defined as [25]:

\[ S = \sqrt[3]{\frac{36\pi V^2}{SA}} \]  

(1)
where \( V \) and \( SA \) are the volume and surface area of the particle, respectively. The Aspect ratio comprises of the Elongation and Flatness indexes [26] and are defined as: 
\[
EI = \frac{b}{a} \quad \text{and} \quad FI = \frac{c}{b}
\]
where \( a \geq b \geq c \) represents the major, intermediate, and minor axes, respectively. EI and FI have been determined by performing principal component analysis on the coordination of the obtained point cloud so that the major axis of the particle could be aligned with the axis of the Cartesian coordinate e.g., [27].

Roundness describes how angular the corner of the particle is. Here it was calculated using the method of Zhao and Wang [28], in which simplification of the triangular mesh (representing the particle surface) and mean curvature at a point is involved. Overall, 1500 triangular elements were used. The 3D roundness is calculated by:
\[
R = \frac{\sum g(i)|k_m|^{-1}}{NR_{\text{ins}}}
\]
where \( R_{\text{ins}} \) is the largest inscribed sphere inside the particle, \( k_m \) is the mean curvature determined based on one ring surrounding triangles around a vertex using the method of Dong and Wang [29], and \( g(k) \) is 1 when \(|k_m| < R_{\text{ins}}\) and is 0 when \(|k_m| \geq R_{\text{ins}}\).

Fractal Dimension (FD) has been used to describe the surface texture. Fractal parameters have been shown to capture the surface texture of sand [23] and gravel well [21] and also show a potential to link the two [20]. The same approach is adopted here. The power spectral density (PSD) function, which reveals the periodical features of the surface, is applied to a square area that is cut from the particle surface. Fractal parameters are approximated from PSD given as:
\[
PSD(q \geq q_c) = C_0 \left( \frac{q}{q_c} \right)^{2D_{PSD} - 8}
\]
where the PSD is given by [22]:
\[
PSD(q_x, q_y) = \frac{1}{(2\pi)^2} \int \int A(x, y)e^{-i(q_x x + q_y y)} dx dy
\]
where \( A(x, y) \) is the auto-correlation function of surface heights \( h(x, y) \) and \( q \) is the spatial frequency or wavevector (in mm\(^{-1}\)).

3. Results
3.1. Morphological Descriptors at Different Scales

The sphericity, indicated by the relationship between surface area and volume for all tested particles (Figure 3), is bounded by values of 1 and 0.55, with the best-fit value being around 0.73.

The aspect ratio (EI and FI) is scattered, from 0.4 to 1 for EI and from 0.3 to 1 for FI (Figure 4a), which indicates the particles are less spheroid and are more prolate and oblate. The average aspect ratio defined as the average of EI and FI appears to be positively correlated with sphericity (Figure 4b), i.e., a more spherical particle has a larger value in the average aspect ratio, which is consistent with the findings of Bagheri [11].

The roundness is mostly in the range of 0.55 to 0.8 and appears to be in an inverse relationship with sphericity, i.e., larger sphericity is accompanied by a smaller roundness (Figure 5a). The FD spans a wide range from around 2.1 to 2.7 (Figure 5b). The FD for particles larger than 5 mm falls into a narrow range of 2.5–2.6 and has less variance with roundness than particles smaller than 5 mm, for which the mineral composition is mostly singular.
where the PSD is given by [22]:

\[
\frac{1}{V} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x,y) \cos(2\pi q x + 2\pi q y) \, dx \, dy
\]

(4)

where \(A(x,y)\) is the autocorrelation function of surface heights \(h(x,y)\) and \(q\) is the spatial frequency or wavevector (in \text{mm}^{-1}).

3. Results

3.1. Morphological Descriptors at Different Scales

The sphericity, indicated by the relationship between surface area and volume for all tested particles (Figure 3), is bounded by values of 1 and 0.55, with the best-fit value being around 0.73.

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Figure 3. Sphericity as inferred from the relationship between surface area and volume for all particles.

Figure 4. Correlation (a) between Flatness Index and Elongation Index where \(a \geq b \geq c\) represents the major, intermediate, and minor axes, respectively and (b) between average aspect ratio and sphericity.

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Figure 5. Cont.
3.2. Average Morphological Descriptors with Particle Size

In general, as particle size decreases, there is a slight decrease in aspect ratio (EI and FI) and sphericity (Figure 6), though the difference in the average values between each particle size is roughly in the range of standard deviation (Table 1). In contrast, for roundness and fractal dimension, the trend is more obvious. As particle size decreases, the fractal dimension decreases, whereas the roundness increases.

![Figure 5](image_url)  
**Figure 5.** Correlation between shape descriptors: (a) roundness and sphericity and (b) fractal dimension and roundness.

![Figure 6](image_url)  
**Figure 6.** Variation of morphological descriptors with particle sizes.
| Particle Size, mm | Aspect Ratio | Sphericity | Roundness | Fractal Dimension |
|------------------|-------------|------------|-----------|------------------|
|                  | EL          | FL         |           |                  |
|                  | Mean        | Standard Deviation | Mean        | Standard Deviation | Mean        | Standard Deviation | Mean        | Standard Deviation |
| 0.6~1.18         |             |             |           |                  |
| Quartz           | 0.71        | 0.10        | 0.72      | 0.12             | 0.77        | 0.04              | 0.70        | 0.03              | 2.34        | 0.12              |
| K-feldspar       | 0.63        | 0.14        | 0.69      | 0.16             | 0.75        | 0.05              | 0.70        | 0.03              | 2.36        | 0.13              |
| Plagioclase      | 0.70        | 0.15        | 0.61      | 0.17             | 0.72        | 0.05              | 0.71        | 0.04              | 2.35        | 0.15              |
| Mica             | 0.76        | 0.13        | 0.50      | 0.18             | 0.72        | 0.07              | 0.71        | 0.04              | 2.46        | 0.17              |
| Average          | 0.71        | 0.14        | 0.63      | 0.18             | 0.74        | 0.06              | 0.71        | 0.04              | 2.38        | 0.15              |
| 1.18~2           |             |             |           |                  |
| Quartz           | 0.74        | 0.09        | 0.69      | 0.16             | 0.77        | 0.05              | 0.71        | 0.03              | 2.33        | 0.12              |
| K-feldspar       | 0.73        | 0.15        | 0.60      | 0.15             | 0.73        | 0.05              | 0.73        | 0.04              | 2.37        | 0.12              |
| Plagioclase      | 0.70        | 0.13        | 0.59      | 0.17             | 0.71        | 0.07              | 0.73        | 0.05              | 2.38        | 0.16              |
| Mica             | 0.77        | 0.14        | 0.54      | 0.15             | 0.71        | 0.07              | 0.74        | 0.06              | 2.43        | 0.10              |
| Average          | 0.73        | 0.17        | 0.60      | 0.17             | 0.73        | 0.07              | 0.73        | 0.05              | 2.38        | 0.13              |
| 2~5              |             |             |           |                  |
| Quartz           | 0.70        | 0.13        | 0.66      | 0.17             | 0.72        | 0.05              | 0.70        | 0.04              | 2.39        | 0.15              |
| K-feldspar       | 0.73        | 0.12        | 0.61      | 0.17             | 0.77        | 0.08              | 0.61        | 0.04              | 2.56        | 0.04              |
| 5~10             |             |             |           |                  |
| Quartz           | 0.79        | 0.12        | 0.64      | 0.16             | 0.78        | 0.06              | 0.62        | 0.05              | 2.58        | 0.04              |

4. Discussion and Implications

4.1. Comparison of Morphological Descriptors of Crushed Particles with Particles of Weathered Origin

Grain formation due to mechanical and chemical degradation (weathering) from the parent material/rocks and transportation and depositional environments have a strong influence on particle morphology. Most of the studies using 3D parameters focused on weathered particles considering only the particle shape scale. Here particle morphology across the scales, i.e., from surface texture to shape, is examined. The effect of the particle formation process being either crushing or weathering, referred to as intact particles, is highlighted in the following. The results between crushed particles and intact natural soil particles in the literature (which used similar morphological descriptors as in this paper) are compared. Some noticeable differences are revealed:

Firstly, at the particle shape scale, the sphericity for crushed granitic particles (being in the range of around 0.55 to 0.87) is smaller than that of a highly decomposed granite of similar size (being in the range of around 0.75 to 0.94) that was reported in [30]. The sphericity is consistent with the results of Zhao and Wang [28] for the crushing product of Leighton Buzzard sand (LBS, sand formed in a fluvial environment).

Secondly, at the surface texture scale, the fractal dimension for crushed particles at sand size in this study has a value of around 2.38. The results are compared with that in [21,31], in which a fractal method similar to this study was applied. The values are found to be smaller than that of LBS reported by [21], being approximately 2.7. (Note that a different correlation has been used, which necessitates the addition of 0.5 in the results). [31] found the fractal dimension is well correlated with sedimentary environments and, specifically, the average value of $D_{PSD}$ decreases from Aeolian (2.94), high energy subaqueous (2.85) to low energy subaqueous (2.69) environments, all of which are higher than that of crushed particles. The smaller fractal dimension for crushed particle surfaces is consistent with former qualitative observations: the relatively freshly created surfaces, as the case for crushed particles, are smoother than naturally weathered surfaces [14], as surfaces of minerals such as feldspar could develop micro-pores, etched pits, or depressions due to dissolution (e.g., [13,15]) and quartz particles could develop specific topographic features (e.g., v-shaped depressions and straight or slightly curved scratches) depending on the environment in which they evolved [12].

Thirdly, the negative relationship between sphericity and roundness has also been observed for LBS fragments [28] but not for intact LBS sand, highly decomposed granite particles [30], and Pingtan sand [32]. Therefore, it appears that the relationship between sphericity and roundness depends on how particles are created. However, further experimental substantiation is needed. No clear correlation between fractal dimension and...
roundness is observed for the tested particles, which is opposed to intact soil particles as in [30].

4.2. Role of Mineral Type

Figure 7 shows the morphological descriptors for particles with different mineralogy. In the order from quartz, K-feldspar, plagioclase to mica, the sphericity and FI decreases, whereas roundness increases, which indicates that the particle becomes more flat, angular, and rounded. The variation of morphological descriptors with mineralogy might be due to different mineral hardness and fracture features (fracture along cleavages or conchoidal fracture). For example, crushed particles are, in general, created by fracturing; further comminution by wear and attrition are induced by the further abrasion of fragments [33]. Therefore, fractures along cleavage for mica, K-feldspar, and plagioclase can contribute to the resulting flat and angular shape, whereas conchoidal quartz fractures form a less flat and less angular shape. Erdogan et al. [34] showed that the hardness of different minerals affected the grinding process and thus the particle shape. The more rounded particles and a rougher surface for mica could be explained by its softest hardness among the mineralogy investigated here so that it is more prone to deformation under external forces.

![Figure 7. Variation of shape descriptors with particle sizes for different minerals. Blue symbols refer to fractal dimension. Grey solid and dotted lines are for shape descriptors for particle size 1.18 to 2 mm.](image)

4.3. Implications

Investigations into the morphological features of crushed particles would provide valuable insights into engineering practices. For example, crushed rocks are usually found in the mediate to hard ground excavation and result in more angular particles. A higher angularity was found to be more abrasive [35]. For the interaction between excavating tool and rocks, higher wear on excavating tool would thus be expected [36], especially for quartz-rich rocks. On the other hand, the resulting more angular particles from crushing could indicate a strong inter-locking effect between particles. A higher possibility of blockage might be expected in ore passes through which ore materials flow down underground [24].
especially for soft particles whose surfaces are less resistant to external forces, and a rough surface ensues, leading to a higher inter-particle friction coefficient. Furthermore, with the increasing interest in 3D-printable geomaterials, the new data could serve as validation of the artificially generated particles, using such as 3D-printing techniques [37,38] and numerical methods with controlled morphology at either shape [39] or surface texture scale [40].

5. Conclusions

This paper presents an experimental testing program on a crushed granitic rock with the particle size ranging from 0.6 mm to 25 mm, based on which the multi-scale morphology is characterized. As particles become smaller as a result of crushing processes, the overall shape becomes slightly angular, the corner of the particle becomes more rounded, and the surface becomes smoother. Surface texture characterized by the fractal dimension has a smaller value for the crushed particles than that of weathered particles in the literature, indicating a less rough surface. When interpreting the morphological descriptors, the mineral type may also play a role. Compared to quartz, the K-feldspar, plagioclase, and mica particles are flatter and less spherical, which could be attributed to the different fracture features and mineralogical structures. The rounded corner and rougher surface of mica could attribute to its decreased hardness. The observed dependence of morphological descriptors on particle size and mineralogy bears significance for reconstructed particles using 3D-printing techniques and numerical methods with controlled morphology at either shape or surface texture scale.

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