Challenges and improvements in applying a particle image velocimetry (PIV) approach to granular flows

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Abstract. The particle image velocimetry (PIV) is a well-established non-invasive optical technique for measuring the velocity field in fluids. Recently, the PIV approach has been extended to granular flows, where the medium under investigation is composed of a discrete number of grains that are typically non-transparent and of super-millimetric size. Granular PIV (g-PIV) still represents a non-standard application, as some accuracy concerns arise. In particular, since granular flows can be highly sheared, the choice of appropriate interrogation windows for the PIV analysis is not trivial. As well, owing to the spatially-averaged nature of the PIV approach, the estimation of second-order statistics remains a very challenging task. Here, we report a laboratory investigation on dry granular flows composed of glass spheres in a rotating drum. The velocity measurements at the sidewall are obtained by using a window deformation multi-pass PIV approach, where the open-source code PIVlab has been specifically used. Different combinations of the number of PIV passes and of interrogation windows are investigated. A slightly modified version of PIVlab allowed us to carry out g-PIV calculations with an arbitrary number of passes (i.e. greater than 4). Comparisons among different analyses helped us to identify reliable settings for g-PIV applications.

1. Introduction
The particle image velocimetry (PIV) is a reliable and well-established experimental technique for measuring the velocity field in classical fluids (e.g., [1-2]). With reference to an interrogation window (IW) belonging to the region of interest (ROI), the PIV approach determines the most-likely displacement of the fluid by maximizing the correlation function between two images, delayed by a short time interval. To make the fluid optically visible, in classical PIV small neutrally-buoyant tracers are typically employed together with a laser sheet illuminating the plane under study (e.g., [3]).

More recently, the PIV approach has been extended to granular flows [4-9], which can be considered fluids only in a broader sense, as they are composed of a discrete number of solid particles of super-millimetric size immersed in a liquid or gas (e.g., [10]). Granular media are ubiquitously involved in geophysical phenomena (e.g. debris flows and avalanches) and industrial applications (e.g.
conveyance of powders and pellets). Since the dynamics of such flows is still not completely understood, theoretical and experimental researches on granular media have recently attracted strong interest of the scientific community [11-18]. In particular, the laboratory investigation of physical quantities, like the flow velocity and the solid volume fraction (e.g., [8,19]), is of undeniable importance for getting insight into the granular dynamics.

PIV on granular media, often referred to as granular PIV or g-PIV [5], still represents a non-standard application and requires facing some specific challenges, which involve finding an appropriate experimental setup and also a reliable setting for the PIV analysis. In general, the overall performance of a g-PIV application depends on the optical properties of the granular medium and on its flow regime. With the notable exception of refractive index-matching applications (e.g., [20,21]), granular media are typically non-transparent, which prevents optical measurements from being obtained inside the flow domain. Consequently, the flow can only be investigated nearby its boundaries. Moreover, the ROI cannot be properly illuminated by a laser sheet and, thus, different illumination approaches, like flash lights or no-flicker high-brightness LED lamps, are necessary [6,8,12]. On the other hand, an advantage of g-PIV over classic PIV applications is that seeding is generally not required, because the grains themselves can be exploited as tracers for the calculation of the cross-correlation function. Moreover, if the grains’ surfaces exhibit some sub-grain sized optical textures, these patterns further help the PIV analysis. Hence, a spatial resolution of measurements finer than the grain diameter, \(d\), has been demonstrated to be achievable [9]. The main error sources in classical PIV, which are also common to g-PIV, are the loss-of-pair errors and the gradient bias errors [22]. The first kind of errors can be usually limited by choosing an IW, which is well larger than the maximum displacement. Yet, the larger is the IW the lower is the spatial resolution of the velocity measurements. Conversely, the gradient bias error is due to the deformation of the fluid within the IW. This type of error, which can be limited by employing a higher-order reconstruction for the displacement distribution in the IW, is particularly detrimental in g-PIV applications, since granular flows may be highly sheared, especially in the cases of a no-slip bottom boundary condition or erodible bed. Hence, though single-pass zero-order matching PIV may still be suitable for simple applications, more sophisticated PIV codes are required for g-PIV. Recently, Sarno et al. [9] investigated the reliability of the multi-pass window deformation approach [23]. Three granular materials (glass spheres, Ottawa sand and POM beads) with very different optical properties were investigated in two common flow geometries, i.e. rotating drum and chute flow geometries [10]. The open-source code PIVlab [24-26], developed in Matlab environment and capable of multi-pass PIV analyses up to 4 passes, was employed. Sarno et al. [9] focused on the problem of identifying a robust PIV setting for reliably measuring both the time-averaged velocities profiles and the magnitude of the fluctuation velocities (i.e. the granular temperature). They found that the multi-pass window deformation approach is particularly beneficial for g-PIV applications, especially to increase the spatial resolution of the measurements and to limit gradient bias errors. Hence, one is able to increase the accuracy of PIV measurements by using a IW in the last PIV pass, even smaller than the grain size.

In the present work we extended the investigation of [9] by reporting further experiments, carried out in the rotating drum geometry at different rotation speeds. Specifically, a modification of the open-source code PIVlab allowed us to remove the limitation of the original code related to the maximum number of 4 PIV passes, and, thus, allowed us to perform PIV analyses with an arbitrary number of passes. Therefore, new PIV settings, involving 5 and 6 PIV passes, could be investigated. We can anticipate that the reported PIV measurements are generally found to well capture the typical shape of the velocity profiles, expected in a rotating drum geometry. Namely, such profiles, previously investigated in literature by using different measuring techniques (e.g. particle tracking method, nuclear magnetic resonance etc...), are characterized by a lower almost static region, where the velocity shows an exponential tail, and by an upper region, where the velocity has an almost linear behaviour (see e.g. [11] and [27]). Since in the present investigation the drum rotation speed was precisely estimated by an independent tracking method, the accuracy of various PIV analyses at the basal surface of the drum could be quantitatively assessed. Interestingly, a further improvement, over the previously investigated PIV settings [9], is found by using in the first PIV pass a IW much larger than \(1d\) and a high enough number of passes, so that the IW in the last pass is kept smaller than \(1d\).
Clearly, this improvement comes with a slightly increased computational cost, which will be discussed. These results underline the significance of choosing an appropriate number of PIV passes and suitable sizes of IWs at all PIV passes.

The paper is composed of the following parts. In Sec. 2 the experimental setup and the PIV method are briefly described. The results of the experimental campaign are reported and discussed in Sec. 3. The conclusions are, finally, summarized in Sec. 4.

2. The experimental setup and the methods

2.1 Granular avalanches in a rotating drum geometry

The experimental setup and the g-PIV method are here described.

The laboratory facility, located at the National Chi Nan University (Puli, Taiwan), is the same one used by Sarno et al. [9]. It consists of a steadily rotating drum with radius \( R = 60 \text{cm} \) and width \( W = 10 \text{cm} \). The sidewalls and the basal surface of the drum are made of transparent smooth Plexiglas, so as to allow the employment of optical measurement techniques. The drum was operated at constant clockwise rotation speeds, \( \Omega \), thanks to an electric motor regulated by an inverter. A sketch of the drum and some pictures of it are reported in figure 1. The measuring apparatus is composed of a high-speed digital camera, model IDT X-Stream XS-3, which is capable of recording 2000 frames per second (fps) with a spatial resolution of \( 1280 \times 1024 \) pixels. The camera, located in front of the drum at a distance of \( \approx 40 \text{cm} \), is equipped with a bright lens, model Sugitoh TSL-50095 (with focal length of 50mm). A no-flicker LED, model U&U 100W-LED, with a luminous output of \( \approx 9000 \) lumens, is employed to ensure a consistent photographic exposure. It is worth underlining that a constant illumination is crucial to get reliable g-PIV measurements, since the cross-correlation between two images relies on the steadiness of their brightness values. The lens aperture and the camera shutter were set at \( f/2 \) and at 100\( \mu \text{s} \), respectively, while the frame rate was 2 KHz.

![Figure 1](image)

**Figure 1.** The rotating drum facility. (a) \( x-z \) sketch showing the investigated ROI, (b) \( y-z \) sketch showing the position of the high-speed camera, (c) front picture, (d) transverse picture (\( y-z \) view).

The granular medium is composed of black and white mono-disperse glass spheres of diameter \( d = 2 \text{mm} \). With the aim of improving the SNR ratio of the PIV cross-correlation, we employed black and white glass spheres in equal proportions (50\%-50\%). Before the measurements, the granular medium was loaded into the drum (figure 1(c)), and the drum was set in motion at a constant rotation speed, so as to obtain a steady granular avalanche. In order to for video-record the intermediate region of the avalanche, a rectangular ROI of size \( 3.2 \text{cm} \times 8.4 \text{cm} \) (i.e. \( 16d \times 42d \)), located where the tangent...
to the drum forms 41° with the horizontal (cf. figure 1(a)), is considered. The resulting imaging scale is 1px=0.1026mm (i.e. 1d≈19.5px).

Two nominal rotation speeds, \( \Omega_{\text{nom}} \), are investigated: 0.6 round per minute (rpm) and 3rpm. The basal surface is smooth compared to the internal friction of the spheres. Nonetheless, owing to the non-negligible sidewall resistances [8,27-30], not only a no-slip condition is observed at the basal surface, but also a lower zone (with a depth of several \( d \)) is found to rigidly move together with the basal surface without deformation. Therefore, the effective rotation speed of the drum could be precisely assessed by manual tracking the first layer of spheres in touch with the basal surface. Owing to the imaging scale and the resolution of digital images, this tracking method allowed us to estimate the linear velocity of the lowest layer of spheres, i.e. \( \dot{u}_{\text{z}} \big|_{z=d/2} \), with an accuracy of \( \approx 10^{-4} \text{m/s} \), and, consequently, allowed us to precisely estimate \( \Omega_{\text{eff}} \) (for details, please see [9]). This independent velocity measurement is also employed for validating the subsequent PIV analyses. For each experiment, we investigated an intermediate steady-state time interval of duration 1s. Preliminary investigations, where time-averages of the measured quantities were calculated on larger time intervals (up to 3s), showed no significant differences.

Example pictures of the ROI, showing the flow at the two investigated rotation speeds, are reported in figure 2. It can be noted that the investigated granular material has a glossy surface with barely visible sub-grain sized textures. Therefore, as it will be clearer hereafter, the investigated granular medium represents a particularly challenging test bench for g-PIV applications. The main details of the two experiments are listed in table 1, where \( \dot{u}_{\text{z}} \big|_{z=d/2} \) is reported in both [m/s] and [px/frame] pixel units. As shown by table 1, the flow depth, \( h \), is approximately the same in the two experiments, since the spreading of the avalanche, increasing with \( \Omega_{\text{eff}} \), is counterbalanced by the decrease of the volume fraction due to dilatancy. Conversely, the inclination of the free surface, \( \vartheta \), with respect to the horizontal is found to increase with the rotation speed, due to increasing resistances (e.g., [28]).

![Example pictures of the ROI, showing the flow at the two investigated rotation speeds.](image)

**Figure 2.** Digital photographs of the ROI, showing the granular flow of glass spheres (diameter \( d=2\text{mm} \)). (a) E-0.6rpm, (b) E-3rpm.

**Table 1.** List of experiments (the anticlockwise convention is used: hence clockwise rotation speeds are negative).

| Experiment ID code | \( \Omega_{\text{nom}} \) [rpm] | \( h \) [mm] | \( \vartheta \) [°] | \( \dot{u}_{\text{z}} \big|_{z=d/2} \) [px/frame] | \( \dot{u}_{\text{z}} \big|_{z=d/2} \) [m/s] | \( \Omega_{\text{eff}} \) [rpm] |
|-------------------|-----------------|--------------|-------------|-----------------|-----------------|-----------------|
| E-0.6rpm          | -0.6            | 74.1         | 23.5        | 0.172           | 0.0355          | -0.565          |
| E-3rpm            | -3              | 75.3         | 28.3        | 0.901           | 0.1857          | -2.960          |
2.2 Gramalar PIV

In the present application we employed the open-source code PIVlab (version 1.42), which incorporates state-of-the-art multi-pass window deformation algorithms [25]. The main features of PIVlab are here briefly recalled with specific attention to g-PIV applications. We refer the reader to [25] for further details.

Considering a generic IW belonging to the ROI under investigation, a 2D PIV algorithm works by maximizing the cross-correlation function, \( C \), between the brightness fields of two digital images, delayed by a short time interval \( \Delta t \). The cross-correlation optimization in its discrete form, which is valid for applications on digital images, is written as follows

\[
\max_{m,n} \left( \sum_{i,j=1}^{M} f_i(i,j) f_{i+\Delta t}(i+m,j+n) \right)
\]

where \( f_i \) and \( f_{i+\Delta t} \) are the grey intensities of the two digital images at time points \( t \) and \( t+\Delta t \), \( M \) is the total number of pixels in the IW, \( m \) and \( n \) are the components of the displacement, \( s = (m,n) \), in the two perpendicular directions of the imaging plane. The maximum value of the cross-correlation function corresponds to the most likely displacement and, since the time interval is known \textit{a priori}, it allows for the estimation of the spatially-averaged velocity in the IW. As it is usual for a better computational efficiency, also in the PIVlab code the cross-correlation optimization (1) is solved in the frequency domain through a Fast Fourier Transform (FFT), instead of in the spatial domain. Some overlap between contiguous IW is typically adopted to enhance the spatial resolution of PIV measurements. Nevertheless, it is well-known that overlaps higher than 50% are not beneficial, since the spatial wavelength response undergoes a cut-off for lengths smaller than half of the IW size owing to the Nyquist-Shannon sampling theorem [1]. In classical PIV applications the brightness pattern is made visible thanks to neutrally-buoyant tracers (with size of \( \sim \mu m \)) previously injected into the flow. The quality of the PIV analysis can be evaluated by the signal-to-noise ratio (SNR), defined as the ratio between the first and the second highest peaks of the cross-correlation function. To get a satisfactory SNR and to limit loss-of-pair errors, the maximum displacement has to be less than 1/4 of the IW size [31], which is often referred to as the 1/4-displacement rule, and the optimal tracers' density is typically \( \approx 5-10 \) per IW [3]. These requirements clearly represent significant limitations to the maximum spatial resolution achievable by PIV, especially if the range of variation of velocities is large in the ROI (e.g. in highly-sheared flows). Moreover, as highlighted by Sarno et al. [9], a large IW is also detrimental for the correct estimation of second-order statistics (e.g. the standard deviation of the fluctuating velocities or the granular temperature) due to spatial-averaging errors intrinsic to the PIV method. A more sophisticated approach, convenient in g-PIV applications, is represented by multi-pass iterative PIV algorithms [22,23]. Indeed, these algorithms overcome the aforementioned limitations by allowing a progressive spatial refinement of the IW. Specifically, the 1/4-displacement rule has to be fulfilled only by the first-pass IW in a multi-pass analysis. Moreover, higher-order reconstructions of the displacement field within each IW help to reduce the gradient-bias errors, which are caused to flow deformations [22,32]. PIVlab, in its original version, allows multi-pass PIV analyses up to 4 passes and incorporates a computationally cost-effective window deformation method, where a 9-point bilinear reconstruction of the IW is performed at the end of each PIV pass based on the trial displacements calculated in the previous pass [25].

In g-PIV applications the non-transparent grains and also their sub-grain sized textures (whenever visible) can be successfully exploited as tracers. Hence, seeding is typically not required [5]. Moreover, Sarno et al. [9] showed that the aforementioned general guideline about the optimal tracers density can be somehow relaxed in g-PIV, thanks to the exploitation of the sub-grain sized optical patterns. Also in the present investigation all PIV analyses have been performed without additional tracers. It should be noted that the cross-correlation task in its discrete form (1) yields a pixel accuracy, as the displacements are investigated in the domain of integers. In classical PIV, sub-pixel accuracy can be straightforwardly achieved by interpolating the cross-correlation peak through suitable continuous functions (typically of Gaussian type). To do so, \( 2 \times 3 \) and 9-point Gaussian interpolations (e.g., [33]) are incorporated in the PIVlab code. Under the ideal conditions of synthetic
image pairs with disk-shaped tracers and an optimal tracers’ density, PIVlab is reported to yield a reasonably low random error of ≈0.02 px/frames, which is also comparable with several commercial codes [25]. By investigating granular media with different optical properties, Sarno et al. [9] experimentally found that the correlation peak can be correctly interpolated by a Gaussian function also in g-PIV applications, although it is typically more spread out than in classic applications, probably due to the higher recurrence of optical patterns in granular media. Nonetheless, investigating semi-synthetic image pairs of granular media obtained by shifting the first image by a predetermined displacement, they found small random PIV errors, only slightly higher than 0.02 px/frames. For illustration, the auto-correlation function of a squared \(3d \times 3d\) image of glass spheres, i.e. the correlation function of the image with itself (corresponding to \(s = (0,0)\)), is shown in figure 3.

![Auto-correlation function of the investigated granular medium (black and white glass spheres, \(d = 2\)mm), calculated on a squared IW of size \(\approx 3d\) (elaborated from Sarno et al. [9]).](image)

Analogous to [9], in all multi-pass PIV analyses, hereafter reported, the size of the IW is halved from one pass to the next one. Indeed, a higher degree of refinement is found to be detrimental to the overall accuracy of measurements. A modification of the PIVlab open-source code allowed us to remove the upper limit of 4 PIV passes, so that we could carry out multi-pass analyses with an arbitrary number of passes. Specifically, PIV settings with 5 and 6 passes have been investigated in this work. While the number of passes and the size of IWs were systematically varied, in all analyses a 50\% overlap of the IW is employed at each pass and the sub-pixel accuracy is always achieved by using the computationally efficient \(2 \times 3\)-point Gaussian interpolation. Finally, it is worth mentioning that PIVlab also incorporates some traditional pre-processing algorithms (e.g. CLAHE local contrast equalization, intensity capping, Wiener filter etc.), which could be useful to enhance the image quality before cross-correlation analysis. However, in the present investigation no image pre-processing was employed, since we preliminary verified the already good quality of the pictures.

### 3. Results and discussion

In this study we investigated several PIV settings by varying the number of passes (from 1 to 6) and the size of the IWs. For all analyses we preliminarily verified that the IW size in the first-pass fulfilled the \(1/4\)-dispalcement rule. All analyses were performed over 1000 non-overlapping image pairs taken from a steady-state time interval of 1s, so that 1000 instantaneous measurements were obtained for each spatial node. The complete list of investigated PIV settings, employed on both runs E-0.6rpm and E-3rpm, is reported in table 2. Moreover, table 3 lists the computation times required by different PIV settings.

To get rid of the drum rotation speed in the plots hereafter reported, a moving frame of reference is considered, so that the \(x'\)-component velocity can be calculated as follows

\[
u_{x'}(z) = u_x(z) - \Omega_{\text{eff}} (R - z)
\]

(2)
where $u_x(z)$ is the $x$-component of the PIV velocity in the fixed frame of reference $Oxyz$ (cf. figure 1). With reference to the middle cross-section $x=0$ located at the centre of the ROI, the time-averaged velocity profiles ($u_x$ and $u_z$) of E-0.6rpm and the standard deviations of the instantaneous velocities ($\sigma_{u_x}$ and $\sigma_{u_z}$), related to the same time interval, are reported in figures 4-5. These latter quantities are defined as

$$
\sigma_{u_x}(z) = \sqrt{\frac{\sum_{i=1}^{N} u_x(z,t) - \bar{u}_x^2}{N-1}} , \quad \sigma_{u_z}(z) = \sqrt{\frac{\sum_{i=1}^{N} u_z(z,t) - \bar{u}_z^2}{N-1}}
$$

where $N=1000$ represents the total number of instantaneous velocity measurements. As well, the time-averaged and standard deviations profiles of E-3rpm are shown in figures 6-7.

### Table 2. List of the investigated multi-pass PIV analyses.

| PIV analysis | No. of passes | Sizes of the interrogation windows (IWs) | Overlap (50%) |
|--------------|---------------|----------------------------------------|---------------|
| PIV-1        | 1             | 40px=2d                                 | 20px          |
| PIV-2        | 2             | 40px=2d, 20px=1d                        | 10px          |
| PIV-3        | 3             | 40px=2d, 20px=1d, 10px=0.5d             | 5px           |
| PIV-4        | 4             | 40px=2d, 20px=1d, 10px=0.5d, 5px=0.25d  | 3px           |
| PIV-5        | 4             | 80px=4d, 40px=2d, 20px=1d, 10px=0.5d    | 5px           |
| PIV-6        | 5             | 160px=8d, 80px=4d, 40px=2d, 20px=1d, 10px=0.5d | 5px |
| PIV-7        | 6             | 160px=8d, 80px=4d, 40px=2d, 20px=1d, 10px=0.5d, 5px=0.25d | 3px |

### Table 3. Run-times of the PIV analyses, obtained on an Intel Core I7-2600@3.40 GHz machine.

| PIV analysis | Run-time for E-0.6rpm [s] | Overhead versus PIV-1 (E-0.6rpm) | Run-time for E-3rpm [s] | Overhead versus PIV-1 (E-3rpm) |
|--------------|---------------------------|-----------------------------------|-------------------------|--------------------------------|
| PIV-1        | 342                       | 1.00x                             | 350                     | 1.00x                          |
| PIV-2        | 565                       | 1.65x                             | 573                     | 1.68x                          |
| PIV-3        | 805                       | 2.35x                             | 816                     | 2.39x                          |
| PIV-4        | 1095                      | 3.20x                             | 1107                    | 3.24x                          |
| PIV-5        | 982                       | 2.87x                             | 1000                    | 2.92x                          |
| PIV-6        | 1126                      | 3.29x                             | 1130                    | 3.30x                          |
| PIV-7        | 1438                      | 4.20x                             | 1440                    | 4.21x                          |

From figures 4(a) and 6(a), it emerges that, independently from the specific choice of PIV setting, the behaviour of the stream-wise velocity, $u_x$, is in qualitative agreement with other experimental investigations on similar geometries [8,27,28,34]. A stratified flow, composed of a lower almost static region (corresponding to a frictional regime) and of an upper roughly linear zone (corresponding to a more collisional regime), can be well identified [8,35]. Considering the no-slip kinematic boundary condition (KBC) at the bed and that the bed surface is rigid, the following requirements should be fulfilled by a reliable and accurate PIV analysis: $u_x = 0$ and $u_z = 0$. Moreover, the occurrence of an almost static lower layer implies that the fluctuating velocities there should be approximately null: $\sigma_{u_x} \approx \sigma_{u_z} \approx 0$. The aforementioned conditions are used as criteria to evaluate the performance of the PIV settings.
Figure 4. (a) $x'$- and (b) $z$-component of the flow velocity profiles, related to experiment E-0.6rpm and obtained by employing various PIV settings. Inset figures: (c) $u_x'$-profiles near the free surface, (d) $u_x'$-profiles near the basal surface.

First, let us discuss the time-averaged velocity measurements. Analogous to that observed by Sarno et. al [9] by investigating higher rotation speeds, we found that the multi-pass PIV analyses with a low number of passes (PIV-1, PIV-2, PIV-3) systematically underestimate the absolute displacements with respect to the fixed frame of reference $Oxyz$ (i.e. the displacements in the fixed imaging plane). This error, which is due to spatial averaging errors, is evident at the basal surface of the drum where an overestimation of $u_x'$ is observed (inset figures 4(d) and 6(d)). For the same reason, a systematic underestimation of $u_x'$ is observed in the upper region of the flow near the free surface (inset figures 4(c) and 6(c)). Conversely, all PIV settings are found to capture quite correctly the other KBC $0z_u = 0$ at the basal surface (figures 4(b) and 6(b)). The PIV settings characterized by a finer spatial resolution (PIV-4, PIV-5, PIV-6 and PIV-7), made possible by the employment of a higher number of passes, are noticeably better in capturing the basal KBC $u_x' = 0$ (cf. figures 4(d) and 6(d)). Moreover, these analyses perform generally better along the entire profile (cf. figures 4(a)-(b) and 6(a)-(b)). By focusing in more detail on the estimations of the stream-wise velocities, $u_x'$, at the basal surface (figures 4(d) and 6(d)), it is interesting to note that the accuracy of the PIV analysis increases not only with refining spatial resolution (i.e. with decreasing IW size in the last pass) but also with increasing IW size in the first pass. This finding confirms the results by Sarno et al. [9] and it is corroborated by the slight further improvements provided by the PIV settings with 5 and 6 passes (PIV-6 and PIV-7), here investigated for the first time. In fact, these new settings yield a very good estimation of $u_x'$ at the bed. This improvement is made possible by the larger 1st-pass IW ($8d \times 8d$). In fact, it clearly helps to improve the initial cross-correlation, since it contains a larger amount of spheres and, thus, more information is available for the cross-correlation task. Finally, it is worth underlining that, despite the very large size of the 1st-pass IW, the modified PIVlab code allowed to refine the spatial resolution down to values smaller than $d$, so that the spatial averaging errors could be kept low.
Figure 5. (a) $x'$- and (b) $z$-component of the standard deviations profiles, related to E-0.6rpm and obtained by employing various PIV settings.

Figure 6. (a) $x'$- and (b) $z$-component of the flow velocity profiles, related to experiment E-3rpm and obtained by employing various PIV settings. Inset figures: (c) $u_x$-profiles near the free surface, (d) $u_x$-profiles near the basal surface.

Now, let us look at the standard deviations profiles (figures 5 and 7). First, one can observe that the typical magnitude of the stream-wise fluctuating velocities, $\sigma_{u_x}$, is larger than $\sigma_{u_z}$. As expected, such differences increase with increasing rotation speed, $\Omega_{eff}$ (cf. figures 5(a)-(b) and 7(a)-7(b)). Moreover, $\sigma_{u_x}$ and $\sigma_{u_z}$ are found to increase with the velocity and, thus, with $z$, owing to the higher grains’ collisionality in its upper zone. Two error sources typically occur in the estimation of second-order statistics. The first kind is represented by PIV random errors, which are not damped out but amplified by the variance quadratic operator in the formulas (3). The second error source, also common to the time-averaged estimations, is represented by the spatial average in the last-pass IW. Random errors cause a systematic overestimation of $\sigma_{u_x}$ and $\sigma_{u_z}$, while the second ones cause an underestimation of them. Due to PIV random errors, it should be noted that all PIV analyses exhibit a non-null noise floor in the lower region (cf. figures 5 and 7), whereas the true value of $\sigma_{u_x}$ and $\sigma_{u_z}$ should be approximately zero there. Sarno et al. [9] experimentally observed that the spatial averaging errors are
typically dominant in g-PIV applications, although the magnitude of the related underestimations are always found weaker than those expected under the hypothesis of no spatial correlation among grains [36]. Figures 5 and 7 confirm these observations. Indeed, if one looks at the active flow region in the upper part of the ROI (i.e. \(z/d>27\) for E-0.6rpm and \(z/d>25\) for E-3rpm), \(\sigma_{x'}\) and \(\sigma_{z'}\) are found to steadily increase with increasing spatial resolution, i.e. with decreasing size of IW in the last pass. To this regard, the employment of a IW smaller than 1\(d\) in the last pass is beneficial for reducing the spatial averaging errors also in the estimation of the second-order statistics. However, it should be noted that, in the case of the most spatially refined settings (PIV-4 and PIV-7), unnatural spikes of the fluctuating velocities are observed in the lower static layer. This indicates that the random errors become dominant over the spatial averaging errors. As signalled by Sarno et al. [9], this behaviour is due to the fact that a the last-pass IW is too small in terms of pixels (5px×5px) and, consequently, the quality of the cross-correlation significantly worsens. Considering the occurrence of these errors, it is clear that the lowest reliable size of IW is limited by the pixel resolution of the camera sensor.

**Figure 7.** (a) \(x'\)- and (b) \(z\)-component of the standard deviations profiles, related to E-3rpm and obtained by employing various PIV settings.

Finally, by considering again the fixed frame of reference \(Oxyz\) of figure 1, we could also quantitatively compare the PIV estimations of the time-averaged \(u_x\) nearest the bed of the drum with the linear velocities, calculated at the same distance from the bed by means of the effective rotation speed, \(\Omega_{eff}\) (cf. table 1). The absolute errors, here expressed in [px/frames] in order to be general, are reported in figure 8. It should be noted that the location of the first PIV measurement, closest to the basal surface, slightly varies with the PIV setting. Indeed, owing to the 50% overlap, the first PIV point is located at a distance from the bed equal to the 50% of the IW size in the last-pass. As clearly shown by figure 8 and in agreement with that already observed from figures 4(d) and 6(d), for both runs (E-0.6rpm and E-3rpm) the errors steadily decrease not only with increasing spatial resolution of the PIV calculation (i.e. from PIV-1 to PIV-4) but also with increasing size of IW in the 1st-pass (PIV-5, PIV-6, PIV-6). Indeed, the minimum error is obtained with settings PIV-6 and PIV-7: \(\approx0.005\) px/frame and \(\approx0.04\) px/frame for E-0.6rpm and E-3rpm, respectively.
Figure 8. Absolute value of the error on the time-averaged stream-wise velocity, $u_x$, at the basal surface of the drum, expressed in [px/frame]. (a) E-0.6rpm, (b) E-3rpm.

In the light of the aforementioned comparisons on time-averaged and second-order velocity estimations, we can conclude that the multi-pass window deformation approach is suitable for g-PIV applications, as long as a reasonably high number of passes is chosen and a careful choice about the sizes of the IWs is made. The best PIV settings, among the investigated ones, are PIV-5 and PIV-6. In particular, PIV-6 is found to deliver slightly better results than PIV-5, especially for the estimation of the time-averaged quantities. Obviously, this improvement comes at the cost of a higher computation effort: i.e. a $\approx 3.30x$ overhead (with respect to the single-pass analysis PIV-1) is observed for PIV-6, while a $\approx 2.90x$ overhead is observed for PIV-5 (cf. table 3). Conversely, the most time-consuming setting, PIV-7, is found to deliver worse results, due to the high noise in the estimation of the second-order statistics.

4. Conclusions
This experimental work reports the application of a multi-pass window deformation PIV to estimate the time-averaged velocities and the magnitude of the velocity fluctuations in granular avalanches in a rotating drum geometry. Two different rotation speeds of the drum were investigated, while the employed granular material consisted in black and white glass spheres. The open-source code, PIVlab [24-26], is employed in a slightly modified form, so that an arbitrary number of PIV passes is allowed. Different settings are systematically investigated, by varying the number of PIV passes (from 1 to 6) and the sizes of the interrogation windows at different passes (from $\approx 8d$ to $\approx 1/4d$). The comparisons of different PIV analyses revealed the crucial importance of employing a multi-pass approach in order to achieve a reasonably high spatial resolution of the measurements and also to limit as much as possible the spatial averaging errors, intrinsic to the PIV method. As well, the importance of a large interrogation window in the first pass of the PIV analysis emerged. In summary, from this investigation on granular flows it can be concluded that a first-pass IW larger than $\approx 4d$ together with a last PIV pass of size $\approx 1/2d$, yields reasonably accurate PIV estimations. Conversely, a too high spatial refinement in the last pass is found to be detrimental as regards the estimation of the magnitude of the fluctuating velocities, due to the limited resolution of the camera’s sensor. These results could be used as general guidelines for further g-PIV applications.

Acknowledgments
We would like to thank Prof. I. C. Liu for allowing us to use the rotating drum facility, located at the National Chi Nan University (Puli, Taiwan).
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