Research on Fault Rate Prediction Method of T/R Component

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Abstract. T/R component is an important part of the large phased array radar antenna array, because of its large numbers, high fault rate, it has important significance for fault prediction. Aiming at the problems of traditional grey model GM(1,1) in practical operation, the discrete grey model is established based on the original model in this paper, and the optimization factor is introduced to optimize the background value, and the linear form of the prediction model is added, the improved discrete grey model of linear regression is proposed, finally, an example is simulated and compared with other models. The results show that the method proposed in this paper has higher accuracy and the solution is simple and the application scope is more extensive.

1. Introduction
Large phased array radar is the main equipment of china’s strategic early warning system, which plays an important role in strategic air defense and antimissile warning. Compared with conventional radars, Large phased array radar has strong adaptability of the targets, more achieved functions, short reaction time and high system reliability, more advantage in the target search, tracking and measurement. Due to the use of phased array technology system, and the antenna array is very large, multicomponent and maintenance timeliness requirements, equipped with a small number, high fault rate, its fault diagnosis and isolation are very difficult, the spare parts cannot satisfy the actual demand of the troops. Therefore, the fault prediction for large phased array radar antenna array can predict the time and quantity of the fault in advantage, based on fault rate prediction, the troops can prepare required manpower and material resources in advantage, improve its operational readiness and combat effectiveness, fault prediction has important practical significance for equipment support[1-2].

At present, many scholars have done a lot of researches on fault prediction[3-5], the main methods of fault prediction are:(1) Neural network prediction, the drawbacks are that neural network training requires a lot of data, the convergence speed is very slow, network structure is difficult to determine. (2) Gray prediction, the disadvantages are the prediction accuracy and stability are not high. (3) Particle filter, the disadvantages are particle degradation and the traceability of mutation status is poor. (4) Regression prediction, the disadvantages are requiring a large number of samples and having a good regularities of distribution.

Due to the highly unstable fault rate of T/R component, this paper analyzes and researches the original monitoring data, establishes a discrete grey model, and improves GM(1,1) model by ameliorating the background value and introducing linear regression. The new model overcomes the defects of forecast sequence singleness in the grey model and improves the accuracy of the model, good prediction results
are obtained in practical application.

2. Traditional GM(1,1) model

The grey theory was put forward by Professor Deng Julong of Huazhong University of Science and Technology at the International Economics Conference in 1982, and was widely used and studied. GM(1,1) model is focuses on the study of establishing differential equations through sequential accumulation and differential fitting to predicting the future trend[6-7].

The main steps of traditional GM(1,1) model prediction are:

(1) Incremental operation of original data.
Set $X^{(0)}$ is non-negative original sequence,

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right\}$$

A cumulative generated sequence is :

$$X^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right\}$$

Which, $x^{(i)}(t) = \sum_{i=1}^{t} x^{(0)}(i) \quad (t = 1, 2, \cdots, n)$.

(2)The establishment of grey differential equation.
About $x^{(1)}(t)$,the differential equations are :

$$\frac{dx^{(1)}}{dt} + ax^{(1)}(t) = b$$

In the unit time :

$$\frac{dx^{(1)}}{dt} = x^{(1)}(t + 1) - x^{(1)}(t) = x^{(0)}(t)$$

Obtain:

$$x^{(0)}(t) + ax^{(1)}(t) = b$$

(3)Generate background values using closely averaging method.

$$Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \cdots, z^{(1)}(n))$$

In formula above,

$$z^{(1)}(t) = 0.5(x^{(1)}(t) + x^{(1)}(t - 1))$$

Obtain:

$$x^{(0)}(t) + az^{(1)}(t) = b$$

In the formula,$a$ is development coefficient,$b$ is ash content.$Z^{(n)}$ is the generating sequence of $X^{(n)}$ using closely averaging method..

(4)Calculate the model.
Estimated by least square method:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y$$

In the formula, 

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \quad ; \quad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

Obtain the value of $a$ and $b$,into the formula(3):

$$x^{(i)}(t + 1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-at} + \frac{b}{a} \quad (t = 1, 2, 3, \cdots, n-1)$$
(5) Obtain prediction model.

\[ \hat{x}^{(t+1)} = \hat{x}^{(t+1)}(t) - \hat{x}^{(t)}(t) \]  

Residual error is (absolute error) \( \varepsilon^{(0)} = x^{(0)} - \hat{x}^{(0)} \), relative error is \( E^{(0)} = \varepsilon^{(0)} / x^{(0)} \times 1000 \).

### 3. The problems of traditional GM(1,1) model in prediction

The time distribution curve of the T/R component fault rate of a phased array radar is obtained by state monitoring as shown in Fig. 1.

![Fault rate curve in the second half of 2015](image)

**Figure 1.** Fault rate curve in the second half of 2015

Through the analysis of Fig. 1, the data basically obey exponential distribution, but there is obvious volatility, the fault rate increases with time. Problems using traditional GM(1,1) model to predict are:

1. Traditional GM(1,1) model in prediction of functions obey exponential distribution has certain advantages, while in practical application, through analyzing the function curve of fault rate, the most of data collected are undulatory, do not accord with the strict exponential function, there is a large error in prediction of exponential growth sequence using GM(1,1) model.

2. In the traditional GM(1,1) model, the method of constructing of background value is closely mean value, such as formula (7), and the prediction accuracy of the model lies in the construction of \( z^{(t)}(t) \), the different data selected in the same sequence makes the values of \( z^{(t)}(t) \) different. For the accuracy of the model, the method of selecting closely mean value in the structure of \( z^{(t)}(t) \) is inappropriate in some cases.

3. In different stages, the fault rate curve of equipment has different forms, which basically obey exponential distribution and linear distribution[8], according to less fault rate of the original data, use GM(1,1) model to predict, but GM(1,1) model is used to simulate the data of exponential function, generally applies only to the exponential change, it is difficult to describe the change of other forms of sequence data. For the data with linear variation, the GM(1,1) model not only cannot obtain better prediction results, but also has a large prediction error.

### 4. Improved GM(1,1) model

Through the above analysis combined with the actual situation of equipment fault prediction, in order to obtain better prediction results, the traditional GM(1,1) model is improved from the following aspects[9-13].

#### 4.1 Dispersed GM(1,1) model

The dispersed GM(1,1) model is:

\[ x^{(t+1)} = \hat{\beta}_1 x^{(t)}(t) + \beta_2 \]  

In the formula, the estimation formula of \( \hat{\beta} = [\beta_1, \beta_2]^T \) and formula(9) are similar, that is \( \hat{\beta} = [\beta_1, \beta_2]^T = (B^T B)^{-1} B^T y \). Which,
According to the formula of \( x^{(i)}(1) = x^{(0)}(1) \), the recursive function of the discrete GM(1,1) model is:

\[
\dot{x}^{(i)}(t+1) = [x^{(0)}(1) - \beta_2 \beta_1^{-1}] \beta_1 + \beta_2 \beta_1^{-1}, \quad t = 1, 2, \ldots, n-1
\]  

(13)

Obtain the estimated value of \( \dot{x}^{(i)}(t+1) \) through putting the value of \( \beta_1 \) and \( \beta_2 \) into the formula (13).

Obtain predicted value of the improved GM(1,1) model by subtracting consecutively and deoxidizing:

\[
\ddot{x}^{(0)}(t+1) = \dot{x}^{(i)}(t+1) - \dot{x}^{(i)}(t), \quad t = 1, 2, \ldots, n-1
\]  

(14)

4.2 Background value optimization

Restructure the formula (7) by introducing optimization factor \( w \), that is

\[
z^{(i)}(t) = wx^{(i)}(t) + (1-w)x^{(i)}(1-t) , \quad t = 2, 3, \ldots, n
\]  

(15)

The \( w \) is coefficient of background value optimization \( 0 < w < 1 \), formula (7) is the special case when \( w = 0.5 \). The value of \( w \) is when the average relative error of the original value subtracts the predicted value reaches the minimum, here the average relative error is:

\[
\bar{\Delta} = \frac{1}{n} \sum_{i=1}^{n} \frac{|x^{(0)}(t) - \ddot{x}^{(0)}(t)|}{x^{(0)}(t)} = \text{min}
\]  

(16)

4.3 Grey linear regression forecasting model

Fit formula (10) with the linear regression equation, obtain:

\[
\dot{x}^{(1)}(t) = C_1 e^{\nu t} + C_2 t + C_3
\]  

(17)

Here, \( \nu \) and \( C_1, C_2, C_3 \) are parameters to be determined.

The model can be used to various forms of generate sequences, when \( C_1 = 0 \), the model can be used for linear growth sequences. When \( C_2 = 0 \), the model is the ordinary GM(1,1) model. In following steps, the parameters in formula (17) are calculated:

Order \( z(t) = \dot{x}^{(1)}(t+1) - \dot{x}^{(1)}(t) \), obtain by formula (17):

\[
z(t) = C_1 (e^\nu - 1) e^{\nu t} + C_2 , \quad t = 1, 2, \ldots, n-1
\]  

(18)

Order \( y_{m}(t) = z(t+m) - z(t) \), obtain by formula (18):

\[
y_{m}(t) = C_1 e^{\nu t} (e^{\nu} - 1)(e^{-m} - 1), \quad m = 1, 2, \ldots, n-3; n = 1, 2, \ldots, n-m - 2
\]  

(19)

Obtain by formula (19): \( y_{m}(t+1)/y_{m}(t) = e^\nu \), so the value of \( \nu \) is:

\[
\nu = \ln \left[ y_{m}(t+1)/y_{m}(t) \right]
\]  

(20)

In formula (17), change \( x^{(0)}(t) \) to \( \ddot{x}^{(0)}(t) \), the approximate value \( v_m \) of \( \nu \) can be obtained, different \( m \) values will obtain different \( v_m \), in order to minimize the error, thus using their average \( \bar{v}_m \) as the value of the last \( \nu \).

\[
\bar{v}_m = \ln \left[ y_{m}(t+1)/y_{m}(t) \right]
\]  

(21)

Among it, \( m = \frac{(n-2)(n-3)}{2} \), obtain
\[ \bar{v}_m = \frac{\sum_{n=1}^{m-1} \sum_{t=1}^{m-2} v_{m}(t)}{(n-2)(n-3)/2} \]  

(22)

Order \( l(t) = e^t \), then formula (17) is expressed as:

\[ \hat{x}^{(1)}(t) = C_1 l(t) + C_2 t + C_3 \]  

(23)

The estimated value of parameter \( C_1, C_2, C_3 \) can be obtained by least square method. Order:

\[ X^{(0)} = \begin{bmatrix} x^{(0)}(1) \\ x^{(0)}(2) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, A = \begin{bmatrix} l(1) & 1 \\ l(2) & 1 \\ \vdots & \vdots \\ l(n) & 1 \end{bmatrix}, C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \]

Obtain:

\[ C = (A^T A)^{-1} A^T X^{(1)} \]  

(24)

Calculate the value of \( v, C_1, C_2, C_3 \), into formula (17), obtain the predicted value of the primary data order by subtracting consecutively and reduction:

\[ \hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1) \]  

(25)

The relative average error \( \Delta \) of the model is:

\[ \Delta = \frac{1}{n} \sum_{t=1}^{n} \Delta(t) = \frac{1}{n} \sum_{t=1}^{n} (\hat{x}^{(0)}(t) - x^{(0)}(t)) \]  

(26)

4.4 Computational procedures

On the basis of previous analysis, the application steps of improved GM(1,1) model are:

**Step 1** Select \( n \) data of the state monitoring data to establish initial data \( X^{(0)} \), obtain order \( X^{(0)} \) by accumulating procession.

**Step 2** Construct background value function \( \hat{x}^{(1)}(t) \) of discrete grey model by formula (15), obtain the optimal factor \( w \) to satisfying the minimum value of \( \bar{\Delta} \).

**Step 3** By using the obtained value of \( B \), obtain the value of \( \hat{\beta} = [\beta_1, \beta_2]^T \) using least square method, put into formula (13), obtain the predicted value of \( \hat{x}^{(1)}(t+1) \).

**Step 4** Structure grey linear regression model, by formula (22)(23)(24), calculate the parameters \( v, C_1, C_2, C_3 \), obtain \( \hat{x}^{(1)}(t) \).

**Step 5** Obtain the predicted value \( \hat{x}^{(0)}(t) \) of the model through subtract consecutively reduction finally.

5. Example analysis

T/R component is an important part of large phased array radar antenna array, which mainly completes the functions of transmitting power amplification, transmitting receiving isolation and receiving low noise amplification. Account for more than 80% of the total equipment, due to the huge numbers and high fault rate, its BITE coverage reached 100%, the state monitoring of the phased array antenna array T/R fault rate [14-15] in the second half of 2015, as show in Tab.1.

| Table 1. T/R component fault rate statistics |
|---------------------------------------------|
| Month(t) | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Fault rate \( \lambda(t) \) | 3.5 | 3.6 | 3.9 | 4.0 | 4.1 | 4.3 | 4.7 |
As can be seen from the data in Tab.1, the fault rate of T/R components shows an increasing trend with the increase of time, but it exists a greater volatility, according to the calculation steps of the improved GM(1,1) model provided in this paper, combined with MATLAB simulation, the fault rate of T/R components is predicted, and the prediction results are obtained. In order to show the superiority of the improved GM(1,1) model, respectively, compared with linear regression model and GM(1,1) model, the predicted and relative error results are shown in Tab.2.

Table 2. Comparisons of prediction results of different models

| Month | Actual value | Linear regression model | GM(1,1) model | Proposed method |
|-------|--------------|-------------------------|--------------|----------------|
|       | Predicted value | Average error | Predicted value | Average error | Predicted value | Average error |
| 7     | 3.6           | 3.51                   | 2.5          | 3.41           | 5.28           | 3.59          | 0.28         |
| 8     | 3.9           | 3.80                   | 2.56         | 3.69           | 5.38           | 3.85          | 1.28         |
| 9     | 4.0           | 4.08                   | 2.0          | 3.99           | 0.25           | 3.97          | 0.75         |
| 10    | 4.1           | 4.37                   | 6.6          | 4.31           | 5.1            | 4.14          | 1.0          |
| 11    | 4.3           | 4.65                   | 8.1          | 4.67           | 8.6            | 4.43          | 3.0          |
| 12    | 4.7           | 4.94                   | 5.1          | 5.04           | 7.2            | 4.74          | 0.9          |

Average relative error % 4.58 4.11 1.03

The analysis of Tab.2 can be concluded that although three methods can predict the fault rate of the T/R component, but through comparisons and analysis of the results, the average relative error from the point of view, the improved GM(1,1) model of the minimum error, the prediction accuracy compared with linear regression model, increased by 77.5%, and GM(1,1) model, the prediction accuracy is improved by 74.9%. in order to show the accuracy of three methods more clearly, the comparison curves of predicted and actual values are given respectively, and the relative error curves of the three methods are shown in Fig.2 and Fig.3.

![Figure 2. Comparison curve of predicted and actual values](image)

![Figure 3. Relative error curves of the three methods](image)

From Fig.2, compared with the actual data curve, the predicted data of proposed method and the actual data are more closer, more in line with the actual situation, showing strong volatility, and linear regression and GM(1,1) model and the actual data difference is large, cannot well reflect the fluctuations of the data.

From Fig.3, the error of proposed method in this paper is minimal in the errors of three methods, the errors of linear regression model and GM(1,1) model are larger, the prediction error of GM(1,1) model is maximum, it is said that the exponential growth of the sequence, proposed method in this paper is better than linear regression model, linear regression model is better than GM(1,1) model.

Through the analyses above, the improved GM(1,1) model is used to fit the fault rate curve by linear...
regression equation and exponential equation, gives full play to the advantage of less data modeling of grey system and regression model factors correlation, complex utilizes many information such as linearity and exponential, and does not require data which must accord with a typical regularities of distribution, it is more coincident with the factual situation. Therefore, the improved GM(1,1) model is better than the linear regression model and the traditional GM(1,1) model in the accuracy prediction of T/R component fault rate, which has some practicability in the prediction of radar equipment fault rate.

6. Summary
In the prediction of T/R component fault rate, according to the problems arisen in the prediction of traditional GM(1,1) model, in this paper, the model is improved, discrete GM(1,1) model is established to solve the problems of large error in predicting exponential growth sequence, introduce optimization factor to reconstruct model background value, which can adjust the value of the background according to the change of the original data and improve the accuracy of the model, based on that, linear regression model is added to solve the situation of different distribution of fault rate in different prediction stages, greatly expand the prediction range and accuracy, the model has universal applicability in practical fault prediction. At last, the prediction results of linear regression model, GM(1,1) and this method are compared and analyzed with examples. Results show that the improved GM(1,1) model proposed in this paper not only improves the accuracy of the model, but also broadens the application scope of the model and it has wider application value.

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