Realizing evaluation strategies by hierarchical graph rewriting

Petra Hofstedt
Brandenburg University of Technology Cottbus
hofstedt@informatik.tu-cottbus.de

Abstract. We discuss the realization of evaluation strategies for the concurrent constraint-based functional language ccfl within the translation schemata when compiling CCFL programs into the hierarchical graph rewriting language LMtal. The support of LMtal to express local computations and to describe the migration of processes and rules between local computation spaces allows a clear and simple encoding of typical evaluation strategies.

1 Introduction

The Concurrent Constraint Functional Language CCFL is a new multiparadigm constraint programming language combining the functional and the constraint-based paradigms. CCFL allows a pure functional programming style, but also the usage of constraints for the description and solution of problems with incomplete knowledge on the one hand and for the communication and synchronization of concurrent processes on the other hand.

CCFL compiles into another multiparadigm language, i.e. the language LMtal (pronounced "elemental") [UK05,UKHM06]. LMtal realizes a concurrent language model based on rewriting hierarchical graphs. One of its major aims is to unify various paradigms of computation and, thus, we chose LMtal as a base model and target language for the CCFL compilation.

In this paper we discuss the implementation of evaluation strategies for CCFL within the compilation schemata. The support of LMtal to express local computations and to describe the migration of processes and rules between local computation spaces allows the realization of typical evaluation strategies in a clear and simple way.

Sect. 2 introduces into programming with CCFL and presents the main language features by example. Sect. 3 is dedicated to the compiler target language LMtal and its evaluation principles. We discuss the encoding of evaluation strategies in Sect. 4.

In [HL09] we took another approach with an abstract machine on a parallel multicore architecture as compilation target and enabled even programming with typical parallelization patterns in CCFL.
Program 2.1 A simple (functional) ccfl program

\[
\begin{align*}
\text{def } & \text{add } x \ y = x + y \\
\text{def } & \text{addOne } = \text{add } 1 \\
\text{def } & \text{fac } x = \text{case } x \text{ of } 1 \rightarrow x \\ & \text{ } \quad \text{n } \rightarrow \text{n } \ast \text{fac } (\text{n } - 1)
\end{align*}
\]

Program 2.2 ccfl: list length

\[
\begin{align*}
data & \text{List } a = \text{Nil } | \text{Cons } a (\text{List } a) \\
def & \text{length } l = \\
& \text{case } l \text{ of } \text{Nil} \rightarrow 0 \\ & \text{Cons } x \text{ xs } \rightarrow 1 + \text{length } xs
\end{align*}
\]

2 Constraint-functional programming with CCFL

The Concurrent Constraint-based Functional Language ccfl combines concepts from the functional and the constraint-based paradigms. We briefly sketch on the main conceptual ideas. For a detailed presentation of ccfl’s full syntax and semantics and application examples we refer to [HL09,Hof08].

Functional programming. ccfl’s functional sub-language syntactically borrows from HASKELL. The language allows the typical constructs such as case- and let-expressions, function application, some predefined infix operations, constants, variables, and constructor terms, user-defined data types, higher-order functions and partial application and it has a polymorphic type system.

Example 1. Prog. 2.1 shows a simple functional ccfl program. We stress on this example later again. The following derivation sequence uses a call-by-value strategy. As usual, we denote the \(n\)-fold application of the reduction relation \(\rightarrow\) by \(\rightarrow^n\) and its reflexive, transitive closure by \(\rightarrow^*\). We underline innermost redexes. Note, that the given sequence is one of several (equivalent) derivations.

\[
\begin{align*}
\text{add} & \left( \text{addOne } (6 + 1) \right) \left( \text{addOne } 8 \right) \rightarrow \text{add} \left( \text{addOne } 7 \right) \left( \text{addOne } 8 \right) \\
& \rightarrow \text{add} \left( \text{addOne } 7 \right) \left( \text{add } 1 \ 8 \right) \rightarrow^2 \text{add} \left( \text{addOne } 7 \right) 9 \rightarrow^3 \text{add } 8 \ 9 \rightarrow^2 17
\end{align*}
\]

Free variables. In ccfl, expressions may contain free variables. Function applications with free variables are evaluated using the residuation principle [Smo93], that is, function calls are suspended until the variables are bound to expressions such that a deterministic reduction is possible. For example, a function call \(4 + x\) with free variable \(x\) will suspend. In contrast, consider Prog. 2.2 defining the data type List \(a\) and a length-function on lists. To proceed with the computation of the expression length \((\text{Cons } x (\text{Cons } 1 (\text{Cons } y \text{ Nil})))\) a concrete binding of the variables \(x\) and \(y\) is not necessary. The computation yields 3.

In the following, we will use the HASKELL-typical notions for lists, i.e. \([]\) and e.g. \([1,6]\) for an empty and non-empty list, resp. , and "::" as the list constructor.
Program 2.3 CCFL: a game of dice

1 fun game :: Int -> Int -> Int -> C
2 def game x y n =
3 case n of
4   0 -> x := 0 & y := 0 ;
5   m -> with x1, y1, x2, y2 :: Int
6     in dice x1 & dice y1 &
7     x := x1 + x2 & y := y1 + y2 &
8     game x2 y2 (m-1)

9 fun dice :: Int -> C
10 def dice x =
11 member [1,2,3,4,5,6] x
12 fun member :: List a -> a -> C
13 def member l x =
14 l := y:ys -> x := y |
15 l := y:ys -> case ys of [] -> x := y ;
16                  z:zs -> member ys x

Constraint-based programming. CCFL features equality constraints on functional expressions, user-defined constraints, and conjunctions of these which enables the description of cooperating processes and non-deterministic behavior.

As an example consider Prog. 2.3. In lines 2–7 we define a constraint abstraction (or user-defined constraint, resp.) game. A constraint abstraction has a similar form like a functional definition. However, it is allowed to introduce free variables using the keyword with, the right-hand side of a constraint abstraction may consist of several body alternatives the choice of which is decided by guards, and each of these alternatives is a conjunction of constraint atoms. A constraint always has result type C.

The constraint abstraction game initiates a game between two players throwing the dice n times and reaching the overall values x and y, resp.

In lines 3–7 we see a conjunction of constraints which are either applications of user-defined constraints, like (dice x1) and (game x2 y2 (m-1)), or equalities e1 := e2 on functional expressions.

Constraints represent processes to be evaluated concurrently and they communicate and synchronize by shared variables. This is realized by suspending function calls (see above) and constraint applications in case of insufficiently instantiated variables.

 Guards in user-defined constraints enable to express non-determinism. For example the member-constraint in lines 12–15 non-deterministically chooses a value from a list. Since the match-constraints of the guards of both alternatives are the same (lines 13 and 14), i.e. l := y:ys, the alternatives are chosen non-deterministically which is used to simulate the dice.

\[2\] The integration of external constraint domains (and solvers) such as finite domain constraints or linear arithmetic constraints is discussed in \[\text{Hof08}\].
Note that alternatives by case-expressions as in lines 3 and 4 and alternatives by guarded expressions as in lines 13 and 14 are fundamentally different concepts. They do not only differ syntactically by using the keyword case-of and the separation mark ";" on the one hand and constraints as guards and the mark "|" on the other hand. Of course, the main difference is in the evaluation: While case-alternatives are tested sequentially, guards are checked non-deterministically for entailment.

The constraint evaluation in ccfl is based on the evaluation of the therein comprised functional expressions. Thus, we can restrict our presentation of evaluation strategies to the reduction of functional expressions in this paper.

Equality constraints are interpreted as strict. That is, the constraint \( e_1 =:= e_2 \) is satisfied, if both expressions can be reduced to the same ground data term \([HAB+06]\). While a satisfiable equality constraint \( x =:= fexpr \) produces a binding of the variable \( x \) to the functional expression \( fexpr \) and terminates with result value Success, an unsatisfiable equality is reduced to the value Fail representing an unsuccessful computation.

ccfl is a concurrent language. Thus, constraints within conjunctions are evaluated concurrently. Concerning the functional sub-language of ccfl, we allow the concurrent reduction of independent sub-expressions.

3 LMTal

The hierarchical graph rewriting language LMTal is the target language of the compilation of ccfl programs. One of its major aims is to "unify various paradigms of computation" \([UK05]\) and, thus, it lent itself as base model and target language for the compilation of ccfl programs. We briefly introduce the principles of LMTal by example, in particular the concepts necessary to explain our approach in the following. For a detailed discussion of LMTal’s syntax, semantics, and usage see e.g. \([UK05],[UKHM06],[LMN10]\).

An LMTal program describes a process consisting of atoms, cells, logical links, and rules.

An atom \( p(X_1, \ldots, X_n) \) has a name \( p \) and logical links \( X_1, \ldots, X_n \) which may connect to other atoms and build graphs in this way. For example, the atoms \( f(A,B,E), A = 7, g(D,B) \) are interconnected by the links \( A \) and \( B \). Note that the links of an atom are ordered such that the above atoms can also be denoted by \( E = f(A,B), A = 7, B = g(D) \). As a shortened notation we may also write \( E = f(7,g(D)) \). \( A \) and \( B \) are inner links in this example; they cannot appear elsewhere because links are bilateral connections and can, thus, occur at most twice.

A cell \( \{a^*, r^*, c^*, l^*\} \) encloses a process, i.e. atoms \( a \), rules \( r \) (see below), and cells \( c \) within a membrane "{}" and it may encapsulate computations and express hierarchical graphs in this way. Links \( l \) may also appear in cells where they are used to interconnect with other cells and atoms.
Program 3.1 LMNtal: non-deterministic bubble sort

1 \( L = [X,Y] L2 \) : \( X > Y \) | \( L = [Y,X] L2 \).
2 \( aList = [2,1,5,0,4,6,3] \).

Program 3.2 LMNtal: membranes to encapsulate computations

1 \{ @r, \{ $p \}, \$s \} :- \{ \{ @r, \$p \}, \$s \}.
2 \{ \{ @r, \$p \}\, /, \$s \} :- \{ @r, \$p, \$s \}.
3 \{ addOne(A,B) :- B = 1 + A. \}
4 \{ addOne(2,D), \{ addOne(4,E)\} \}

Example 2. Consider the following two cells.

\{ addOne(A,B) :- B=1+A. E=5, \{ addOne(2,D)\}, \{ +E \} \}

The first cell encloses a rule \( addOne(A,B) :- B=1+A. \), an atom \( E=5 \), and an inner cell \( \{ addOne(2,D)\} \) enclosing an atom itself. The second (outer) cell just contains a link \(+E\) connecting into the first cell onto the value 5.

Rules have the form \( lhs :- rhs \) and they are used to describe the rewriting of graphs. Both, the left-hand side \( lhs \) and the right-hand side \( rhs \) of a rule are process templates which may contain atoms, cells and rules and further special constructs (see e.g. \[UK05\]) among them process contexts and rule contexts. Contexts may appear within a cell and they refer to the rest of the entities of this cell. Rule contexts \( @r \) are used to represent multisets of rules, process contexts \( \$p \) represent multisets of cells and atoms.

Consider the bubble sort rule and a list to sort in Prog. 3.1 as a first and simple example. In the bubble sort rule, link \( L \) connects to the graph \( [X,Y] L2 \) representing a list. Thus, \( [X,Y] L2 \) does not describe the beginning of a list but an arbitrary cutout such that the rule is in general applicable onto every list position where \( X > Y \) holds. Since LMNtal does not fix an evaluation strategy, the sorting process is non-deterministic.

As a second example consider the LMNtal Prog. 3.2. Lines 3–4 show a cell, i.e. a process encapsulated by a membrane \( \{\}\). It consists of an \( addOne\)-rewrite rule in a PROLOG-like syntax and two cells each enclosing an \( addOne\)-atom by membranes in line 4. \( A, B, D, E \) are links. The \( addOne\)-rewrite rule cannot be applied on the \( addOne\)-atoms in line 4 because they are enclosed by extra membranes which prevent them from premature evaluation. The rules in lines 1 and 2 however, operate on a higher level and they allow to describe the shifting of the \( addOne\)-rule into the inner cells and backwards. At this, \( @r \) is a rule-context denoting a (multi)set of rules, and \( \$p \) and \( \$s \) are process-contexts which stand for (multi)sets of cells and atoms. The template \( \{ @r, \$p \}/ \) in line 2 has a stable flag \( ”/” \) which denotes that it can only match with a stable cell, i.e. a cell containing no applicable rules.

In the current situation, the rule in line 1 is applicable to the cell of the lines 3–4 where \( @r \) matches the \( addOne\)-rule, \( \$p \) matches one of the inner
addOne-atoms and $s$ stands for the rest of the cell contents. A possible reduction of the this cell (in the context of the rules of lines 1–2) is, thus, the following; we underline the elements reduced in the respective steps:

\[
\{\text{addOne}(A,B) : -B = 1 + A. \{\text{addOne}(2,D)\}, \{\text{addOne}(4,E)\} \} \rightsquigarrow_{(1)} \{\\}
\]

\[
\{\\{\text{addOne}(A,B) : -B = 1 + A. \text{addOne}(4,E)\}, \{\text{addOne}(2,D)\} \} \rightsquigarrow_{\text{addOne}}
\]

\[
\{\\{\text{addOne}(A,B) : -B = 1 + A. \overline{E = 1+4}\}, \{\text{addOne}(2,D)\} \} \rightsquigarrow_{+}
\]

\[
\{\\{\text{addOne}(A,B) : -B = 1 + A. E = 5\}, \{\text{addOne}(2,D)\} \} \]

The first inner cell is now stable such that no rule is applicable inside. Thus, we can apply the rule from line 2.

\[
\{\\{\text{addOne}(A,B) : -B = 1 + A. E = 5\}, \{\text{addOne}(2,D)\} \} \rightsquigarrow_{(2)}
\]

\[
\{\text{addOne}(A,B) : -B = 1 + A. E = 5, \{\text{addOne}(2,D)\} \}
\]

In this state, again the first outer rule (line 1) is applicable which yields the following rewriting sequence and final state:

\[
\{\text{addOne}(A,B) : -B = 1 + A. E = 5, \{\text{addOne}(2,D)\} \} \rightsquigarrow_{(1)}
\]

\[
\{\{\text{addOne}(A,B) : -B = 1 + A. \overline{\text{addOne}(2,D)}\}, E = 5\} \rightsquigarrow_{\text{addOne}}
\]

\[
\vdots
\]

\[
\{\text{addOne}(A,B) : -B = 1 + A. E = 5, D = 3\}
\]

As one can see by the above example, LMNTal supports a PROLOG-like syntax. However, there are fundamental differences. Our example already demonstrated the use of process-contexts, rule-contexts, membrane enclosed cells, and the stable flag. Different from other languages, the head of a rule may contain several atoms, even cells, rules, and contexts. A further important difference to other declarative languages are the logical links of LMNTal. What one may hold for variables in our program, i.e. $A$, $B$, $D$, $E$, are actually links. Their intended meaning strongly differs from that of variables. Declarative variables stand for particular expressions or values and, once bound, they stay bound throughout the computation and are indistinguishable from their value. Links in LMNTal also connect to a structure or value. However, link connections may change. While this is similar to imperative variables, links are used to interconnect exactly two atoms, two cells, or an atom and a cell to build graphs and they have, thus, at most two occurrences. The links $D$ and $E$ in the above example occur only once and, thus, link to the outside environment. In rules, logical links must occur exactly twice.

Semantically, LMNTal is a concurrent language realizing graph rewriting. It inherits properties from concurrent logic languages. LMNTal does not support evaluation strategies. The rule choice is non-deterministic, but can be controlled by guards (used e.g. in Prog. 4.1 for the fac-rules, see below).

As shown in the example, the encapsulation of processes by membranes allows to express local computations, and it is possible to describe the migration of processes and rules between local computation spaces. We will use these techniques to implement evaluation strategies for CCFL.
Program 4.1 Intermediate LMNtal compilation result

1  add(X, Y, V0) :- V0 = X+Y.
2  addOne(X, V0) :- app(add, 1, V1), app(V1, X, V0).
3  fac(X, V0) :- X == 1 | V0 = 1.
4  fac(X, V0) :- X =\= 1 | V0 = X*V1, V2 = X-1, app(fac, V2, V1).
5  app(fac, V1) :- fac(V1).
6  app(fac, V1, V2) :- fac(V1, V2).
7  ... 
8  app(add, V1, V2, V3) :- add(V1, V2, V3).
9  app(V2, V3, V4), add(V1, V2) :- add(V1, V3, V4).
10 ... 

4 Encoding evaluation strategies

Now, we discuss the compilation of CCFL programs into LMNtal code. We start
with a presentation of the general translation schemata and show the realization
of a call-by-value strategy and an outermost evaluation strategy subsequently.

Our compilation schemata are partially based on translation techniques [Nai91, Nai96, War82] for functional into logic languages.

A CCFL function definition is translated into a set of LMNtal rules. CCFL data
elements and variables are represented and processed by means of certain heap
data structures during run-time. However, to clarify the presentation in this
section, we represent CCFL variables directly by LMNtal links \(^3\) instead and data
structures by LMNtal atoms. For a detailed discussion of the heap data structures
see [Hof08]. CCFL infix operations are mapped onto their LMNtal counterparts.
Function applications are realized by an atom \textit{app}(...) and an according \textit{app}-rule
which is also used for higher-order function application and partial application
as discussed in [Hof08]. Case-expressions generate extra LMNtal rules for pattern
matching, let-constructs are straightforwardly realized by equalities.

Example 3. Consider the CCFL Prog. 2.1. It compiles into the (simplified) LMNtal
code given in Prog. 4.1 (not yet taking an evaluation strategy into consideration).

The additional link arguments \(V0\) of the \textit{add}, \textit{addOne}-, and \textit{fac}-rewrite rules
are used to access the result of the rule application which is necessary because
LMNtal explicitly deals with graphs while a computation with a (constraint-) functional language like CCFL yields an expression as a result. The two \textit{fac} rules
result from the case distinction in the according CCFL function.

Note that the right-hand side of the \textit{addOne} rule in line 2 represents the
LMNtal expression \((\texttt{app(app add 1) X})\) resulting from the \textit{ccfl addOne} definition
in Prog. 2.1 by \(\eta\)-enrichment.

\(^3\) Moreover, we tolerate \(n\)-fold occurrences of links in rules, where \(n \neq 2\). This is also
not conform with LMNtal, where links must occur exactly twice in a rule, but the problem
disappears with the introduction of heap data structures as well.
The rules of lines 5-10 are a cutout of the rule set generated to handle function application including higher-order functions and partial application (according to a schema adapted from [Na96,War82]). Thus, in these rules the root symbols appear with different arities.

LMNtal evaluates non-deterministically, and it does a priori not support certain evaluation strategies. Thus, the control of the order of the sub-expression evaluation for CCFL is integrated into the generated LMNtal code. We realized code generation schemata for different evaluation strategies for CCFL by encapsulating computations by membranes using similar ideas as demonstrated in Prog. 3.2. We discuss the realization of a call-by-value and an outermost reduction strategies in the following.

A call-by-value strategy
To realize evaluation strategies expressions are destructured into sub-expressions which are encapsulated by membranes and interconnected by links. These links allow to express dependencies between sub-expressions on the one hand, but to hold the computations apart from each other, on the other hand. Consider the CCFL application

\[ add \left( \text{addOne}(6+1) \right) \left( \text{addOne} \right) 8 \]  

from Example 1. It is destructured and yields the following LMNtal atoms:

\[ Z = add \left( X,Y,Z \right), X = addOne \left( W \right), W = 6+1, Y = addOne \left( 8 \right) \]

or in an equivalent notation, resp.:

\[ add \left( X,Y,Z \right), addOne \left( W,X \right), W = 6+1, addOne \left( 8,Y \right) \]

The idea to realize a call-by-value evaluation strategy is now provide the expressions to be reduced first (i.e. the inner calls \( W = 6+1 \) and \( addOne \left( 8,Y \right) \)) with the ruleset and to delay the outer calls by holding them apart from the rules until the computation of the inner redexes they depend on is finished. To enable a concurrent computation of independent inner sub-expressions as discussed in Example 1 we must assign each independent inner expression or atom, resp., a separate membrane (including a copy of the rules). This yields the following structure (where every atom is held within separate membranes for organizational reasons).

\[ \{ \{ add(X,Y,Z) \} \{ \{ addOne(W,X) \} \{ @rules, \{ W=6+1 \} \{ @rules, \{ addOne(8,Y) \} \} \} \} \]  

Fig. 1 visualizes an evaluation of 3 using a call-by-value strategy with concurrent evaluation of independent sub-expressions. Reduction step numbers are given in brackets at the right margin. Membranes are represented as enclosing ellipses. Interdependencies of atoms by links control the order of the evaluation and they are represented by arrows.

In the initial state, the atoms \( W=6+1 \) and \( addOne(8,Y) \) are inner redexes to be reduced first. We mark these by a gray color. They are provided with the LMNtal rules @rules generated from the CCFL program. The atoms \( add(X,Y,Z) \)
and $addOne(W,X)$ represent outer calls to be delayed until the computation of their inner sub-expressions have been finished. Thus, we put them into extra protecting membranes.

To control the order of sub-expression evaluation we need three things:

1. the destructured expression as given in (3),
2. LMNTal rules (denoted by \texttt{@rules} in Fig. 1) generated from the CCFL program: These rules take the destructuring of expressions into consideration and realize local call-by-value evaluations.
3. a general LMNTal ruleset reorganizing the computation spaces in case that local computations are finished.

(i) and (ii) The destructuring of expressions in LMNTal rules (ii) generated from the CCFL program is handled similarly to (i) as discussed above. Prog. 4.2 shows the LMNTal code generated from Prog. 2.1 taking the intended call-by-value evaluation into consideration.

The LMNTal rules’ right-hand sides consist of cells containing the destructured expressions. Outermost expressions are encapsulated by extra membranes to protect them against premature evaluation as necessary for an innermost strategy. This effect is observable for the $addOne$-rule in line 2 and the second $fac$-rule in line 5 while for the other rules the flat term structure of the right-hand sides of the CCFL functions is just carried over to the generated LMNTal rules. To simplify the presentation in Fig. 1 however, we inlined the \texttt{app}-calls for function

Fig. 1. A call-by-value computation sequence
Program 4.2 Generated LMNTal code for a call-by-value strategy

1 \{add(X,Y,V0), \$p\} := \{V0 = X+Y, \$p\}.
2 \{addOne(X,V0), \$p\} := \{app(add,1,V1), \{app(V1,X,V0), \$p\}\}.
3 \{fac(X,V0), \$p\} := X ::= 1 \mid \{V0 = 1, \$p\}.
4 \{fac(X,V0), \$p\} := X := 1 \mid \{V0 = X*V1, \$p\}, \{V2 = X-1\}, \{app(fac,V2,V1)\}.
5 \{app(fac,V1), \$p\} := \{fac(V1), \$p\}.
6 \{app(fac,V1,V2), \$p\} := \{fac(V1,V2), \$p\}.
7 \{app(fac,V1), \$p\} := \{fac(V1), \$p\}.
8 \{app(fac,V1,V2), \$p\} := \{fac(V1,V2), \$p\}.

Program 4.3 LMNTal code for a call-by-value strategy, simplified version

1 \{add(X,Y,V0), \$p\} := \{V0 = X+Y, \$p\}.
2 \{addOne(X,V0), \$p\} := \{add(1,X,V0), \$p\}.
3 \{fac(X,V0), \$p\} := X ::= 1 \mid \{V0 = 1, \$p\}.
4 \{fac(X,V0), \$p\} := X := 1 \mid \{V0 = X*V1, \$p\}, \{V2 = X-1\}, \{app(fac,V2,V1)\}.

Applications (e.g. we applied the \texttt{app}-rule of line 7 on the call \texttt{app (fac, V2,V1)} of line 5) and used instead of Prog. 4.2 the accordingly simplified Prog. 4.3.

(iii) The reorganization of computation spaces for the call-by-value strategy is mainly realized by two LMNTal rules given in Prog. 4.4. They are visualized in Fig. 2 for better understanding. These rules organize the evaluation of outer calls when the inner redexes they depend on have been completely reduced. In the following, we discuss the rules semantics by means of Fig. 2.

![Fig. 2. LMNTal: rules emulating a call-by-value strategy](image)

Both rules, (A) and (B), are only applicable when the cell \{@rules, \{p(\ldots,L)\}\} is stable, i.e. the rules cannot be applied on the atom \texttt{p(\ldots,L)} (or \texttt{L=p(\ldots)}, resp.) further. For rule (A), the atom \texttt{q(\ldots,L,\ldots)} does not contain any further link connected to a process representing a sub-expression evaluation. Thus, both atoms are ready for their combined evaluation, and they are put into one computation cell or space, resp., together with the rules. An example for the application of this rule is the computation...
Program 4.4 LMNTal rules to control the reorganization of computation spaces for a call-by-value reduction of the CCFL compilation result

```
ruleA @@
{ @rules, {$procs_p, +L, inLinks_(0)$} } /,
{ { $procs_q, -L, inLinks_(N)$ } } :-
N ::= 1 | { @rules, {$procs_p, $procs_q, inLinks_(0)$} } .
```

```
ruleB @@
{ @rules, {$procs_p, +L, inLinks_(0)$} } /,
{ { $procs_q, -L, inLinks_(N)$ } } :-
N > 1 | { { $procs_p, $procs_q, inLinks_(M)$, M = N-1} } .
```

Program 4.5 LMNTal compilation result for an outermost strategy

```
{ fac(X, V0), $p$ } :- X ::= 1 | { V0 = 1, $p$ } .
{ fac(X, V0), $p$ } :- X =\= 1 |
{ V0 = X*V1, $p$ } , { { V2 = X-1} , { fac(V2, V1) } } .
```

step (2) in Fig. 1 where the atoms W=7 and addOne(W,X) are brought together into one membrane and join into the atom addOne(7,X).

Rule (B) describes the case that the atom q (...) does contain at least one further link connected to a process itself under evaluation. These represent sub-expressions of q (... L ...) or inner redexes, resp., and are denoted by incoming links resp. arrows in Fig. 2. Thus, while p (... L) and q (... L ...) can be combined in one membrane, they are not ready for evaluation yet such that we omit the rules @rules on the right-hand side here. An example is step (6) in Fig. 1: the atoms add(X,Y,Z) and Y=9 are combined in one membrane but not provided with the rules.

An outermost strategy For call-by-name and lazy evaluations the computation proceeds on the outermost level. As we will see in the following, thus, copying of the rule-set, like for the innermost strategy, (which may become expensive) is not necessary. Besides, the mechanisms are are quite similar.

Consider the CCFL faculty function from Prog. 2.1. The (simplified) LMNTal compilation result taking an outermost strategy into consideration is given in Prog. 4.5. In contrast to Prog. 4.3, now innermost expressions on the right-hand sides are encapsulated by extra membranes to protect them against premature evaluation. (The flat term structure of the right-hand sides of the CCFL functions add and addOne is again just carried over to the generated LMNTal rules and yields the same results as in Prog. 4.3).

Fig. 3 shows an outermost evaluation sequence of an LMNTal process corresponding to the CCFL expression add (addOne (6+1)) (addOne 8). We did denote the ruleset @rules only in the first computation state; there is in general only one copy and it resides on the top level where the computation executes. An outgoing link from q (... L ...) would accordingly represent its parent expression. Such links are allowed, of course, but omitted in Fig. 2 for easier understanding.
An outermost computation sequence for an LMNTal process corresponding to the \textit{ccfl} expression \texttt{fac (addOne (addOne 3))} is shown in Fig. 5.

We show two examples of evaluation sequences to allow a direct comparison between the two evaluation sequences in Fig. 1 and Fig. 3 for the expression \texttt{add (addOne (6+1)) (addOne 8)} on the one hand, but to illustrate a particular aspect of outermost strategies (i.e. the unprotect-protect mechanism as described in particular for the sequence in Fig. 5 below), too.

The two main rules realizing the outermost evaluation are given in Fig. 4. We mark cells on the outermost level, i.e. cells to be reduced first, by the dark gray color as before. Again, the rules are only applicable if the cells with dark gray color are stable, i.e. they cannot be reduced further.

Rule (C) lifts the (“first”) inner expression $p\ (\ldots, L)$ on the evaluation level in case that the outermost term $q\ (\ldots, L, \ldots)$ is not (yet) reducible. However, for such (light-gray marked) sub-expressions (and except for arithmetic expressions) only one reduction step is allowed before protecting its result again by an extra membrane. This is realized by a variant of each rule of Prog. 4.5 (not shown there). Rule (C) is applied e.g. in the first computation step of Fig. 5. Afterwards the described unprotect-protect-mechanism applies. In steps (3) and (4) of Fig. 5 rule (C) is applied even twice which allows to make a reduction step onto the inner redex \texttt{addOne}(3,Y) in step (6). For the applications of rule (C) in e.g. step (4) of Fig. 5 and (2,3) in Fig. 3 the unprotect-protect mechanism does not apply because the outermost expressions are arithmetic ones, here, such that we use rule (D) afterwards.
Rule (D) can only be applied on expressions with arithmetic built-in operators as root. Because, in this case, the outermost term can only be evaluated if the inner expressions are completely evaluated, we lift them onto the outermost level, in general. The rule is applied in the step 4 of Fig. 3 and step 5 of Fig. 5.

We realized a call-by-name strategy using the presented approach. The encoding of a call-by-need strategy is possible by additionally introducing heap data structures as anyway needed to deal with free variables and constraints in ccfl (see above and [Hof08]). These allow sharing as needed for lazy evaluation.

Residuation Function applications in ccfl may contain free variables. In such a case, we apply the residuation principle [Smo93] (see above). This is realized in the resulting LmNtal program by according atoms in the rules left-hand sides as in the following example (or guards resp. as e.g. in Prog. 4.1 for the fac-rules) checking for concerning variable bindings.

Example 4. Consider again the ccfl Prog. 2.2 defining a length-function. Prog. 4.6 shows a cut-out of the generated LmNtal program. From a not sufficiently instantiated ccfl expression (length y) the compiler generates an according LmNtal atom length (Y, V). The LmNtal evaluation of this process together with Prog. 4.6 suspends as long as there is no connection of the link Y to an appropriate atom or graph, i.e. we need an atom nil (Y) or cons (H, T, Y).

5 Conclusion

We discussed the realization of typical evaluation strategies for functional programs based on hierarchical graph rewriting. The control of sub-expression evaluation was built into the translation schemata when compiling ccfl programs into the graph rewriting language LmNtal. LmNtal is a concurrent language not
Fig. 5. An outermost computation sequence, II

supporting strategies a priori. However, the abilities of LMtal to express hierarchical computation spaces and to migrate processes and rules between them enables a clear and simple strategy control.

Ueda [Ued08b, Ued08a] presents encodings of the pure lambda calculus and the ambient calculus, resp., using LMtal. Like in our approach, the membrane construct of LMtal plays an essential role for the encoding and allows a significantly simpler than previous encodings, like e.g. that of the lambda calculus by Sinot [Sin05] based on token passing in interaction nets. In [BFR05] Banâtre, Fradet, and Randenac identify a basic calculus $\gamma_0$ containing the very essence of the chemical calculus and which – similar to LMtal – is based on multiset rewriting. They show an encoding of the strict $\lambda$-calculus which is straightforward because of the strict nature of $\gamma_0$ and state that an encoding of a call-by-name $\lambda$-calculus is possible too, but more involved. In contrast, LMtal a priori does not support certain evaluation strategies. Instead, LMtal offers extended
features like the rewriting of rules and graph hierarchies which allows to en-
capsulate and migrate computations which was highly used in our modelling of
evaluation strategies as presented in the paper.

Acknowledgment This work has been supported by a postdoctoral fellowship No.
PE 07542 from the Japan Society for the Promotion of Science (JSPS).

References

BFR05. J.-P. Banatre, P. Fradet, and Y. Radenac. Principles of Chemical Program-
ing. Electr. Notes in Theor. Computer Science, 124(1):133–147, 2005.

HAB+06. M. Hanus, S. Antoy, B. Braβel, H. Kuchen, F.J. Lopez-Fraguas, W. Lux,
J.J. Moreno Navarro, and F. Steiner. Curry: An Integrated Functional
Logic Language. Technical report, 2006. Version 0.8.2 of March 28, 2006.

HL09. P. Hofstedt and F. Lorenzen. Constraint Functional Multicore Program-
ing. In Informatik 2009. Proceedings, volume 154 of LNI – Lecture Notes
in Informatics, pages 367, 2001–2015, 2009.

Hof08. P. Hofstedt. CCFL – A Concurrent Constraint Functional Lan-
guage. Technical Report 2008-08, Technische Universität Berlin, 2008.

LMN10. LMNtal PukiWiki. http://www.ueda.info.waseda.ac.jp/lmntal/ 2010.
last visited 12 May 2010.

Nai91. L. Naish. Adding equations to NU-Prolog. In Programming Language
Implementation and Logic Programming – PLILP, volume 528 of LNCS,
pages 15–26. Springer, 1991.

Nai96. L. Naish. Higher-order logic programming in Prolog. Technical Report
96/2, Department of Computer Science, University of Melbourne, 1996.

Sin05. F.-R. Sinot. Call-by-Name and Call-by-Value as Token-Passing Interaction
Nets. In P. Urzyczyn, editor, Typed Lambda Calculi and Applications –
TLCA, volume 3461 of LNCS, pages 386–400. Springer, 2005.

Smo93. G. Smolka. Residuation and Guarded Rules for Constraint Logic Program-
ing. In F. Benhamou and A. Colmerauer, editors, Constraint Logic Pro-
gramming. Selected Research, pages 405–419. The MIT Press, 1993.

Ued08a. K. Ueda. Encoding Distributed Process Calculi into LMNtal. ENTCS,
209:187–200, 2008.

Ued08b. K. Ueda. Encoding the Pure Lambda Calculus into Hierarchical Graph
Rewriting. In A. Voronkov, editor, Rewriting Techniques and Applications –
RTA, volume 5117 of LNCS, pages 392–408. Springer, 2008.

UK05. K. Ueda and N. Kato. LMNtal: a Language Model with Links and Mem-
branes. In Fifth International Workshop on Membrane Computing (WMC
2004), volume 3365 of LNCS, pages 110–125. Springer, 2005.

UKHM06. K. Ueda, N. Kato, K. Hara, and K. Mizuno. LMNtal as a Unifying Declar-
ative Language. In T. Schrijvers and T. Frühwirth, editors, Third Work-
shop on Constraint Handling Rules, Technical Report CW 452, pages 1–15.
Katholieke Universiteit Leuven, 2006.

War82. D.H.D. Warren. Higher-order extensions to PROLOG: Are they needed?
Machine Intelligence, 10:441–454, 1982.