CP violation in Semi-Leptonic $\tau$ decays

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We study CP violation in semi-leptonic $\tau$ decays and we determine the conditions necessary to be able to define a observable CP asymmetry. We apply these conditions in both models, the standard model for the electroweak interactions and its supersymmetric extensions. In the first case, the leading order contribution to the direct CP asymmetry in $\tau^\pm \to K^{\pm} \pi^0 \nu_\tau$ decay rates is evaluated. In the second case, we compute the SUSY effective hamiltonian that describes the $\Delta S = 1$ semileptonic decays of tau leptons. We show that SUSY contributions may enhance the CP asymmetry of $\tau \to K \pi \nu_\tau$ decays by several orders of magnitude compared to the standard model expectations.

INTRODUCTION

Experimental searches for CP violating asymmetries in tau lepton semileptonic decays have been carried out in the $\tau \to \pi \nu_\tau$ and $\tau \to K\pi \nu_\tau$ modes. Motivation for these searches in the context of beyond the Standard Model approaches were provided in refs. [3, 4]. In ref. [2], the missing evidence for a non-zero CP asymmetry was interpreted as assuming its origin in the interference of scalar and vector form factors. In this talk, we shall first determine the general conditions to get observable CP asymmetry. We apply these conditions in both models, the Standard Model and its supersymmetric extensions. Then we should apply it to Standard Model and its supersymmetric extensions of scalar and vector form factors (at 90% c.l.) has been derived. The CP-odd observable studied in [2] depends upon two variables of a particular kinematical distribution of semileptonic tau decays as long as this effect is assumed to have its origin in the interference of scalar and vector form factors. In this talk, we shall first determine the general conditions to get observable CP asymmetry.

The general amplitude for $\tau^- (p) \to K^- (k) \pi^0 (k') \nu_\tau (p')$ is given by

$$M = \frac{G_F V_{us}}{\sqrt{2}} \left\{ \bar{u}(p')\gamma^\mu (1 - \gamma_5) u(p) F_V (t) \left[ (k - k')_\mu - \frac{\Delta^2}{t} q_\mu \right] ight.$$

$$+ \bar{u}(p')(1 + \gamma_5) u(p) m_\tau F_S (t) \frac{\Delta^2}{t}$$

$$+ F_T (K \pi |s\sigma_{\mu\nu} u(0)| \bar{u}(p')\sigma^{\mu\nu} (1 + \gamma_5) u(p)) \right\},$$

where $q = k + k'$ ($l = q^2$) is the momentum transfer to the hadronic system, $\Delta^2 \equiv m^2_K - m^2_\pi$ and $F_{V, S, T} (t)$ are the effective form factors describing the hadronic matrix elements (In Standard Model, $F_T = 0$, $\Delta = 1$)

$$\sum_{\text{pol}} |M|^2 \sim |F_V|^2 (2p.Q p'.Q - p.p'.Q^2) + |\Delta|^2 |F_S (t)|^2 M^2 p.p'$$

$$+ 2 Re \Delta \cdot Re (F S F^*_V) M m_{\tau p'.Q} - 2 Im \Delta \cdot Im (F S F^*_V) M m_{\tau p'.Q}$$

where $Q_\mu = (k - k')_\mu - \frac{\Delta^2}{t} q_\mu$ and $F_T$ contributions have been neglected. The last term is odd under a CP transformation but the last two terms disappear once we integrate on the kinematical variable $u$.

It is not possible to generate a CP asymmetry in total decay rates corresponding to this process unless:

- $F_{V, S} = f_{V, S} + a_{V, S}$ which $a_{V, S}$ containing a weak CP violating phase and a different strong phase than $f_{V, S}$
- Interference between tensorial and vector contribution $\Rightarrow$ we need to go beyond SM.
- to look for the double differential distribution ($d^2 \Gamma/dudt$) or a variant of it as CLEO collaboration did [2].

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FIG. 1: Higher order contributions to CP asymmetry in Standard Model

CP ASYMMETRY IN STANDARD MODEL

The form factor \( f_{\nu,S}(t) \) are dominated at tree level by a single vector or scalar strange resonance:

\[
 f_i(t) = \frac{f_i(0)m_i^2}{m_i^2 - t - im_i\Gamma_i}, \quad i = V, S,
\]

where \((m_i, \Gamma_i)\) denote the mass and width of the resonance in the vector or scalar configuration (respectively the \(K^*(892)\) or \(K^*_0(1430)\)).

\( \Rightarrow \) the tree-level strong phase fixed by the decay width of these resonances, while the weak phase is absent at the tree-level. Higher order contributions can induce weak phase contributions \( (a_{\nu,S}) \) (see figure 1).

\[
 A_{CP} = \frac{\Gamma(\tau^+ \rightarrow K^+\pi^0\bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow K^-\pi^0\bar{\nu}_\tau)}{\Gamma(\tau^+ \rightarrow K^+\pi^0\bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow K^-\pi^0\bar{\nu}_\tau)} \approx -\frac{\sqrt{2}G_F^3m_\tau^5\text{Im}(V_{us}V_{td}V_{ts}^*)f_Kf_\pi}{768\pi^3\Gamma(\tau^+ \rightarrow K^+\pi^0\bar{\nu}_\tau)} \times I_{CP},
\]

where

\[
 I_{CP} = \frac{1}{m_\tau^6} \int_{(m_K+m_\pi)^2}^{m_\tau^2} \frac{dt}{t^3} (m_\tau^2 - t)^2 h(t) \left(1 + \frac{2t}{m_\tau^2}\right) \lambda^{3/2}(t, m_K^2, m_\pi^2)
\]

where

\[
 \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz
\]

\[
 A_{CP} \approx -\frac{\sqrt{2}G_F^3m_\tau^5\text{Im}(V_{us}V_{td}V_{ts}^*)f_Kf_\pi}{20B(\tau^+ \rightarrow K^+\pi^0\bar{\nu}_\tau)} \times I_{CP} \approx 2.3 \times 10^{-12},
\]

with \(B(\tau^+ \rightarrow K^+\pi^0\bar{\nu}_\tau) = (4.5 \pm 0.3) \times 10^{-3} \) and \(m_c = 1.35 \text{ GeV}\).

CP ASYMMETRY IN SUPERSYMMETRIC MODELS

Strangeness-changing \( |\Delta S| = 1 \) decays of tau leptons are driven by the \( \tau^- \rightarrow \bar{u}s\nu_\tau \) elementary process. The effective Hamiltonian \( H_{eff} \) derived from SUSY can be expressed as

\[
 H_{eff} = \frac{G_F}{\sqrt{2}}V_{us} \sum_i C_i(\mu)Q_i(\mu),
\]

1 for details of the computation, see reference [2].
where $C_i$ are the Wilson coefficients and $Q_i$ are the relevant local operators at low energy scale $\mu \simeq m_\tau$. The operators are given by

$$Q_1 = (\bar{\nu} \gamma^\mu L \tau)(\bar{s} \gamma_\mu L u),$$

$$Q_2 = (\bar{\nu} \gamma_\mu L \tau)(\bar{s} \gamma_\mu R u),$$

$$Q_3 = (\bar{\nu} R \tau)(\bar{s} L u),$$

$$Q_4 = (\bar{\nu} R \tau)(\bar{s} R u),$$

$$Q_5 = (\bar{\nu} \gamma_{\mu \nu} R \tau)(\bar{s} \gamma^{\mu \nu} R u).$$

SUSY contributions to the Hamiltonian of $\tau^- \rightarrow \bar{u}s\nu_\tau$ transitions can be generated through two topological box diagrams as shown in Figs. 2 and 3. Other SUSY contributions (vertex corrections) are suppressed either due to small Yukawa couplings of light quarks or because they have the same structure as the SM in the hadronic vertex.

In our computations of Wilson coefficients, we will work in the mass insertion approximation (MIA), where gluino and neutralino are flavor diagonal. Denoting by $(\Delta_{ij})_{ab}$ the off-diagonal terms in the sfermion mass matrices where
\[ \langle \tilde{f}_A \tilde{f}_B \rangle = i(k^2 I - \tilde{m}^2 I - \Delta^f_{AB})^{-1} \approx \frac{i\delta_{ab}}{k^2 - \tilde{m}^2} + \frac{i(\Delta^f_{AB})_{ab}}{k^2}
abla^2 + O(\Delta^2), \] (8)

where \( \tilde{f} \) denotes any scalar fermion, \( a, b = (1, 2, 3) \) are flavor indices, \( I \) is the unit matrix, and \( \tilde{m} \) is the average sfermion mass. It is convenient to define a dimensionless quantity \( (\delta^f_{AB})_{ab} = (\Delta^f_{AB})_{ab}/\tilde{m}^2 \). As long as \( (\Delta^f_{AB})_{ab} \) is smaller than \( \tilde{m}^2 \) we can consider only the first order term in \( (\delta^f_{AB})_{ab} \) of the sfermion propagator expansion. In our analysis we will keep only terms proportional to the third generation Yukawa couplings and terms of order \( \lambda = V_{us} \).

The complete expressions for the Wilson coefficients \( C_i \) at \( m_W \) scale induced by SUSY computed from Figs. can be found in reference [4]. As can be seen in ref. [3], the \( C_i \) are given in terms of several mass insertions that represent the flavor transitions between different generations of quarks or leptons. In general, these mass insertions are complex and of order one. However, the experimental limits of several flavor changing neutral currents impose severe constraints on most of these mass insertions. In the following, we summarize all the important constraints on the relevant mass insertions for our process.

1. From the experimental measurements of \( BR(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11} \), the following bounds on \( |(\delta^f_{12})_{AB}| \) and \( |(\delta^v_{12})_{AB}| \) are obtained [2]: For \( M_1 \sim M_2 = 100 \text{ GeV} \) and \( \mu = \bar{m}_l = 200 \text{ GeV} \),

\[
|\delta^f_{12}\rangle_{LL} \lesssim 10^{-3}, \quad |(\delta^f_{12})_{LR}| \lesssim 10^{-6}, \quad |(\delta^v_{12})_{LL}| \lesssim 6 \times 10^{-4}, \quad |(\delta^v_{23})_{LL}| \lesssim 4 \times 10^{-4}, \quad |(\delta^v_{23})_{LR}| \lesssim 7 \times 10^{-4}.
\] (9)

2. From \( BR(\tau \rightarrow \mu\gamma) < 1.1 \times 10^{-6} \), one gets the following constraint on \( |(\delta^f_{23})_{LR}| \) [3]:

\[
|\delta^f_{23}\rangle_{LR} \lesssim 2 \times 10^{-2},
\] (11)

and from \( BR(\tau \rightarrow e\gamma) < 2.7 \times 10^{-6} \), one finds [3]:

\[
|\delta^f_{13}\rangle_{LR} \lesssim 1 \times 10^{-1}.
\] (12)

3. The mass insertions \( (\delta^f_{12})_{AB} \) are constrained by the \( \Delta M_K \) and \( \epsilon_K \) as follows [3]:

\[
\sqrt{|\text{Im}[(\delta^f_{12})_{LL}]^2|} \lesssim 3 \times 10^{-3}, \quad \sqrt{|\text{Im}[(\delta^f_{12})_{LR}]^2|} \lesssim 3 \times 10^{-4}.
\] (13)

4. The mass insertion \( (\delta^v_{12})_{AB} \) are constrained by the \( \Delta M_B \) as follows [9]:

\[
|\delta^v_{12}\rangle_{LL} \lesssim 1.7 \times 10^{-2}, \quad |\delta^v_{12}\rangle_{LR} \lesssim 2.4 \times 10^{-2}.
\] (15)

Here, three comments are in order. i) Due to the Hermiticity of the LL sector in the sfermion mass matrix, \( (\delta^f_{AB})_{LL} = (\delta^f_{AB})_{LL}^\dagger = (\delta^v_{AB})_{LL} \), where \( A, B = 1, 2, 3 \). ii) The above constraints imposed on the mass insertions \( (\delta^f_{AB})_{LL,LR} \) are derived from supersymmetric contributions through exchange of gluino or neutralino which preserves chirality, therefore same constraints are also imposed on the mass insertions \( (\delta^f_{AB})_{RR,RL} \). iii) The mass insertions \( (\delta^f_{AB})_{LR,RL} \) are not, in general, related to the mass insertions \( (\delta^f_{BA})_{LR,RL} \). Taking the above constraints into account, one finds that the dominant contribution to the \( \tau^\rightarrow u\bar{s}\nu^\tau_\tau \) is given in terms of \( (\delta^f_{12})_{LR}, (\delta^v_{23})_{RL}, (\delta^v_{21})_{RL}, \) and \( (\delta^v_{21})_{LR} \). Notice that the effective Hamiltonian (eq. [1]) derived in this section can induce supersymmetric effects in all the \( |\Delta S| = 1 \) exclusive \( \tau \) lepton decay.

The total amplitude (SM and SUSY) of the \( \tau \rightarrow K\pi(p)\pi(p')\nu_\tau(p') \) decay as

\[
A_T(\tau \rightarrow K\pi\nu) = \frac{G_F V_{us}}{\sqrt{2}} \left[ (1 + C_1) \langle K\pi | \bar{s}\gamma_\mu u(0)\bar{\nu}(p')\gamma^\mu L \tau(p) \right.
\]

\[
+ \left. (C_3 + C_4) \langle K\pi | \bar{s}u(0)\bar{\nu}(p')R \tau(p) + C_5 \langle K\pi | \bar{s}g_{\mu\nu} u(0)\bar{\nu}(p')\sigma^{\mu\nu} R \tau(p) \right],
\]
where $C_i$ stand for $C_i^{SUSY}$. We consider the following two interesting scenarios:

1. The case of $C_3$ or $C_4$ gives relevant contributions while $C_5$ is negligible. In this case, SUSY induces a relative weak phase between the vector and scalar form factors describing this process.

2. The case of $C_5$ gives relevant contributions while $C_{3,4}$ are negligible. In this case, SUSY induces a relative weak phase between the vector and tensor form factors. A CP asymmetry in decay rate could be measured.

First scenario

\[
\langle K\pi|\bar{s}\gamma_\mu u|0\rangle = f_V(t)Q_\mu + f_S(t)(q + q')\mu , \Rightarrow \langle K\pi|\bar{s}u|0\rangle = \frac{t}{m_s - m_u}f_S(t) ,
\]

\[
A_T(\tau \to K\pi\nu) = \frac{G_FV_{us}}{\sqrt{2}}(1 + C_1) \times \left\{ f_VQ^\mu\bar{u}(p')\gamma_\mu Lu(p) + \left[ m_\tau + \left( \frac{C_3 + C_4}{1 + C_1} \right) \frac{t}{m_s - m_u} \right] f_S\bar{u}(p')Ru(p) \right\} .
\]

Using CLEO limit, we can translate this bound into:

\[-0.010 \leq \text{Im} \left( \frac{C_3 + C_4}{1 + C_1} \right) \leq 0.004 ,
\]

where we have used $m_s - m_u = 100$ MeV, and the average value $\langle t \rangle \approx (1332.8$ MeV$)^2$. Using $M_1 = 100$ and $M_2 = 200$ GeV and $\mu = M_{\tilde{q}} = 400$ GeV and $\tan \beta = 20$, one gets

\[
\text{Im} \left( \frac{C_3 + C_4}{1 + C_1} \right) \approx 1.3 \times 10^{-5} \text{Im}(\delta_{21}^d)_{RL}
\]

Second scenario

\[
A_T(\tau \to K\pi\nu) = \frac{G_FV_{us}}{\sqrt{2}}(1 + C_1) \left\{ f_V(t)Q_\mu\bar{u}(p')\gamma_\mu Lu(p) \right\} + \frac{C_5}{1 + C_1} \langle K\pi|\bar{s}\sigma_{\mu\nu}u|0\rangle\bar{u}(p')\sigma^{\mu\nu}Ru(p) \right\}
\]

\[
\langle K\pi|\bar{s}\sigma_{\mu\nu}u|0\rangle = \frac{ia}{m_K} \left[ (p_\tau)\mu (p_K)\nu - (p_\tau)\nu (p_K)\mu \right]
\]

where $a$ is a dimensionless quantity which fixes the scale of the hadronic matrix element.

\[
a_{CP} = \frac{\Gamma(\tau^- \to K^-\pi^0\nu_\tau) - \Gamma(\tau^+ \to K^+\pi^0\nu_\tau)}{\Gamma(\tau^- \to K^-\pi^0\nu_\tau) + \Gamma(\tau^+ \to K^+\pi^0\nu_\tau)} \approx \frac{a}{2} \text{Im} C_5
\]

\[
\approx 1.4 \times 10^{-7} a \text{Im}(\delta_{21}^d)_{RL}
\]
The goal of this work is to illustrate the difficulties to define an observable CP asymmetry in semi-leptonic $\tau$ decays. We show that in standard model, a CP asymmetry in the total decay rate can be induced through higher order contributions but the absolute value of this CP asymmetry is too small to be once accessible to experiments. In the supersymmetric case, we have computed the effective hamiltonian derived from SUSY for $|\Delta S| = 1$ tau lepton decays using the mass insertion approximation. Supersymmetric extensions of the SM could induce CP violating asymmetry in the double differential distribution as CLEO collaboration did and could also induce CP asymmetry in total decay rate due to interference between $O_3$ and $O_1$ operators. A direct consequence of our computation is that any CP asymmetry in the channel under consideration bigger than $10^{-6}$ will be a clear evidence of not only Physics beyond Standard Model but also an evidence of Physics beyond SUSY extensions of the SM.

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APPENDIX

SUSY CONTRIBUTIONS TO WILSON COEFFICIENTS

Here we provide the complete expressions for the supersymmetric contributions, at leading order in MIA, for the Wilson coefficients of $\tau^+ \to s \nu\tau$ transition, $C_i(M_W)$, $i = 1, \ldots, 5$. The dominant SUSY contributions are given by chargino-neutralino box diagram exchanges, as illustrated in Fig. 2.

The effective Hamiltonian $H_{\text{eff}}$ derived from SUSY can be expressed as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{us} \sum_i C_i(\mu) Q_i(\mu),$$

$$= \sum_i \tilde{C}_i(\mu) Q_i(\mu),$$

where $C_i$ are the dimensionless Wilson coefficients and $Q_i$ are the relevant local operators at low energy scale $\mu \simeq m_\tau$. In terms of the vertex, one can write the complete vertex as a product of the vertex coming from leptonic sector and of the vertex coming from hadronic sector. In this respect we can also write the Wilson coefficients as

$$\tilde{C}_i = C_i^{l(\tau-\chi^-)} \left( C_{i(\tau-\chi^-)-s}^0 + C_{i(\tau-\chi^-)-u}^q \right) + C_i^{q(\tau-\chi^0)} \left( C_{i(\tau-\chi^0)-s}^q + C_{i(\tau-\chi^0)-u}^q \right)$$

where the $C_i^l$ is due to the leptonic vertex and $C_i^q$ is from the quark sectors. If we expand $C_i^{l,q}$ in terms of the mass insertions, one finds that the leading contributions are given by

$$\tilde{C}_3 = C_3^{l(\tau-\chi^-)} C_3^{q(1)} I_n(x_i, x_j) + C_3^{l(0)} C_3^{q(1)} I_n(x_i, x_j) + O(\delta^2),$$

where $I_n(x_i, x_j)$ is defined below and $x_i = m_{\tilde{\chi}^0_i}/\tilde{m}^2$ and $x_j = m_{\tilde{\chi}^0_j}/\tilde{m}^2$.

$$C_3^{l(\tau-\chi^-)} = g(h_e)_{33} N_{i3} U^*_{j1}(U_{MNS}^*)_{33}$$

$$-g\sqrt{2} \tan \theta_w N_{i1} U_{j2}(h_e)_{33} (U_{MNS}^*)_{33},$$

$$C_3^{l(0)} = -(h_e)_{33} \frac{g}{\sqrt{2}} (N_{i2} - \tan \theta_w N_{i1}) U_{j2}(U_{MNS}^*)_{33},$$

$$\text{(16)}$$

$$\text{(17)}$$

$$\text{(18)}$$

$$\text{(19)}$$

$$\text{(20)}$$

$$\text{(21)}$$
\[ C_{3(\tau - \chi^o - s)}^{(1)} = \left( -\frac{1}{8} \right) \left( \frac{g^2}{\sqrt{2}} - \frac{2}{3} \tan \theta_w N_{i1}^* U_{j1}^* (V_{C_{KM}}^*)_{1a} (\delta_{RL}^d)_{2a} \right. \\
- \left. \frac{g}{\sqrt{2}} \tan \theta_w N_{i1}^* U_{j2}^* (h_d)_{33} (V_{C_{KM}}^*)_{13} (\delta_{RR}^d)_{23} \right), \]  
(22)

\[ C_{3(\tau - \chi^o - s)}^{(1)} = \left( -\frac{1}{8} \right) \left( \frac{2 g^2}{3 \sqrt{2}} \tan \theta_w N_{i1}^* U_{j1}^* (V_{C_{KM}}^*)_{1a} (\delta_{RL}^d)_{2a} \right. \\
+ \left. \frac{2 g}{3 \sqrt{2}} \tan \theta_w N_{i1}^* U_{j2}^* (h_u)_{33} (V_{C_{KM}}^*)_{13} (\delta_{RR}^u)_{32} \right), \]  
(23)

\[ \tilde{C}_4 = C_{4(\tau - \chi^o - u)}^{(0)} C_{4(\tau - \chi^o - u)}^{(1)} \tilde{I}_n(x_i, x_j) + C_{4(\tau - \chi^o - u)}^{(0)} C_{4(\tau - \chi^o - u)}^{(1)} \tilde{I}_n(x_i, x_j) + O(\delta^2), \]  
(24)

where \( \tilde{I}_n(x_i, x_j) \) is given below.

\[ C_{4(\tau - \chi^o - u)}^{(0)} = -\left( h_c \right)_{33} \frac{g}{\sqrt{2}} (N_{i2} - \tan \theta_w N_{i1}) U_{j2}^* (U_{MNS})_{33} \]  
\[ = C_{3(\tau - \chi^o - u)}, \]  
(25)

\[ C_{4(\tau - \chi^o - u)}^{(0)} = g(h_c)_{33} N_{i1} U_{j1}^* (U_{MNS})_{33} \]  
\[ - g \sqrt{2} \tan \theta_w N_{i1} U_{j2}^* (h_u)_{33} (U_{MNS})_{33}, \]  
(26)

\[ C_{4(\tau - \chi^o - u)}^{(1)} = \left( -\frac{1}{8} \right) \left( -\frac{4 g^2}{3 \sqrt{2}} \tan \theta_w N_{i1} U_{j1}^* (V_{C_{KM}}^*)_{a2} (\delta_{LR}^d)_{a1} \right. \\
+ \left. \frac{4 g}{3 \sqrt{2}} \tan \theta_w N_{i1} U_{j2}^* (h_u)_{33} (V_{C_{KM}}^*)_{32} (\delta_{RR}^u)_{31} \right), \]  
(27)

\[ C_{4(\tau - \chi^o - u)}^{(1)} = \left( -\frac{1}{8} \right) \left( -\frac{4 g^2}{3 \sqrt{2}} \tan \theta_w U_{j1}^* N_{i1} (V_{C_{KM}}^*)_{a2} (\delta_{LR}^u)_{a1} \right. \\
+ \left. \frac{4 g}{3 \sqrt{2}} \tan \theta_w U_{j2}^* N_{i1} (h_u)_{33} (V_{C_{KM}}^*)_{32} (\delta_{RR}^u)_{31} \right). \]  
(28)

\[ \hat{C}_5(\tau - \chi^o - u) = \frac{1}{4} \hat{C}_4(\tau - \chi^o - u), \]  
(29)

\[ \hat{C}_5(\tau - \chi - - u) = \frac{1}{4} \hat{C}_4(\tau - \chi - - u). \]  
(30)

The loop integrals \( I_n(x_i, x_j) \) and \( \tilde{I}_n(x_i, x_j) \) are defined as follows:

\[ I(x_i, x_j) = \frac{1}{16\pi^2 m^2} \left( \frac{1}{x_i - x_j} \right) \left( \frac{1}{x_i - x_j} \right) \left( \frac{x_i^2 - x_j^2 + x_i x_j \log x_i}{(1 - x_i)^2} \right) - (x_i \leftrightarrow x_j), \]  
(31)
The expression for the other Wilson coefficients can be found in ref. [6].

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