Periodic distributed delivery routes planning subject to operation uncertainty of vehicles travelling in a convoy

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ABSTRACT
This paper considers a Problem of Periodic Routing of Vehicles Platoons (PRVPP), which is a novel extension of the Periodic Vehicle Routing Problem (PVRP). The PRVPP boils down to scheduling of the fleet of vehicles travelling in a convoy while constituting the timetable for the passage of individual vehicles along the planned routes. The imprecise nature of transport/service operation times means there is a need to take into account the fact that the accumulating uncertainty of previously performed operations results in increased uncertainty of timely execution of subsequent operations. This raises the question as to the method required to avoid additional uncertainty introduced during aggregating uncertain operation execution deadlines. Due to the above fact, an algebraic model for calculating fuzzy schedules for individual vehicles, and for planning time buffers enabling the adjustment of the currently calculated fuzzy schedules, is developed. The model uses Ordered Fuzzy Numbers (OFNs) to reflect the uncertainty of times. The conducted research demonstrated that the proposed model allows us to develop conditions following the calculability of arithmetic operations of OFNs and guarantee the interpretability of the results obtained.

1. Introduction
In real-life settings of Out-Plant Operating Supply Networks (OPOSN), Kilic et al. (2012) and Meyer (2015), apart from randomly occurring disturbances (changes in the execution of already planned requests/orders and the arrival of new ones, traffic jams, accidents, etc.), an important role is played by the imprecise nature of the parameters that determine the timeliness of the services/deliveries performed (Polak-Sopinska, 2019). The imprecise nature of these parameters is caused by the operator’s psychophysical disposition, disturbances in the flow of traffic, etc. Therefore, the time values of the operations performed vary and are uncertain.
Also, to avoid the occurrence of blockages of concurrently executed delivery processes, it is necessary to introduce appropriate mechanisms, e.g. employing dispatching rules that synchronize the processes. In the context of cyclic flow being deadlock-free, the NP-hard problem of deadlock handling may be treated as equivalent to the problem of cyclically-executed local processes synchronization, simply because cyclical delivery excludes the appearance of vehicles congestion. Thus the periodicity of the overall distributed delivery network depends on the periodicities of processes carried out by individual platoons. Conversely, the delivery period in a network formed by a set of periodically routed platoons depends on the cycle of this network. Consequently, the throughput of a delivery network is maximized by minimization of its cycle time.

The non-stationary nature of the uncertainty of the parameters mentioned, and the usually small set of available historical samples in practice, limits the choice of a formal data model to a fuzzy numbers driven one. It means the uncertainty of OPOSN data connected with traffic disturbances as well as changes in service delivery dates requires the use of a model based on the formalism of fuzzy sets. However, it is worth noting that the specificity of the processes involved in the course of planning a services delivery schedule makes it necessary to determine the sequentially cumulative uncertainty of the performance of the operations involved in it. The question that arises concerns the method of avoiding additional uncertainty introduced in the combinations of aggregating uncertainties of cyclically executed operations in cyclic production (Bocewicz et al., 2020) or distribution (Bocewicz et al., 2016). In this context, in contrast to standard fuzzy numbers, the support of a fuzzy number obtained by algebraic operations performed on the Ordered Fuzzy Numbers (OFNs) domain does not expand. However, the possibility of carrying out algebraic operations is limited to selected domains of computability of these supports.

The current paper aims to fill this gap by introducing the OFN algebra framework. In other words, the objective is to develop an algebraic model aimed at rapid calculation of fuzzy schedules for individual vehicles as well as planning time buffers that permit the user to adjust currently calculated fuzzy schedules to baseline schedules assuming that operation times are deterministic values. The contribution especially, which is an extended version of our earlier work (Rudnik et al., 2021), especially focuses on the development of sufficient conditions to allow the calculability of algebraic operations that guarantee the interpretability of the results obtained.

The quality of the functioning evaluation of a given system depends decisively on which model is used to assess it. Available model classes are divided into those using an imperative method and those based on a declarative approach. The differences between both kinds of models rest on the fact that imperative approaches focus on how (to solve) while declarative ones focus on what (to describe/specify). Therefore, declaratively expressed models are independent of architectural and software details, and only depend on the conceptualization (paradigm) adopted, and are thus easier to expand and communicate as long as the basic conceptualization is agreed upon. Consequently, due to the nature of the research carried out underlying the semantics of the natural systems being modelled rather than the algorithms that calculate their changing states while covering both the analysis and synthesis of distributed delivery planning, a declarative modelling framework is implemented.

The main contributions to this study can be summarized as follows:
(1) Elaborate a declarative reference model for distributed delivery networks and conditions sufficient to guarantee operators of a OFNs algebra return values that are easy to interpret (so-called proper OFNs).

(2) Provide decision support for the rationale behind vehicle fleet cyclic scheduling in the OPOSNs while taking into account the uncertainty of the deliveries operation times.

(3) Analyse the impact of vehicle fleet size as well as time buffers size and allocation on the OPOSN design.

(4) Provide a trade-off between the sizes of delivery cycles and the size of time buffers (laytimes), taking into account the uncertainty of the deliveries operation times and the data characterizing the OPOSN.

The remainder of this paper is organized as follows. Section 2 elaborates related works. Section 3 provides a description of the methodology used to model and solve periodic distributed delivery in terms of the PRVPP. Section 4 presents the conditions guaranteeing operators of a OFNs algebra return values being proper OFNs. Section 5 formulates the declarative model of distributed deliveries planning in terms of the Ordered Fuzzy Constraint Satisfaction Problem. Section 6 presents experiments conducted and elaborates results obtained. Finally, Section 7 provides the conclusions followed by the description of future research.

2. Related work

Most Periodic Vehicle Routing Problems (PVRP) are aimed at searching for an optimal periodic distribution policy, i.e. a plan regarding whom to serve, how much to deliver, and what regularly repeated routes to travel by what fleet of vehicles (Braekers et al., 2016; Mor & Speranza, 2020). Examples of such problems (Holborn, 2013) include both simple ones, such as the Mix Fleet VRP, Multi-depot VRP, Split-up Delivery VRP, Pick-up and Delivery VRP, VRP with Time Windows, and VRP with Backhauls, and more complex ones, such as a combination of variants of the VRP with multiple trips, VRP with a time window, and VRP with pick-up delivery as well as the multi-period VRP with mixed pickup and delivery with time windows, heterogeneous fleet, duration time, and rest area (Kamsopa et al., 2021).

Against the background of these extensions, the problems of Periodic Routing of Vehicles Platoons have started to occupy a special place (Chen et al., 2015; Mushtaq et al., 2021). In such problems, collision-free routings of a fleet of vehicles forming platoons are sought that meet the expected deliveries at given points and dates subject to the relevant topological constraints of the distribution network. By vehicle platoons we mean groups of separate vehicles travelling in a convoy formation where each vehicle follows another. The attractiveness of such solutions results from military needs as well as expectations related to the development of autonomous vehicle traffic.

Regardless of the problems that accentuate the dynamic or static character of vehicle routing (Hanshar & Ombuki-Berman, 2009; Holborn, 2013; Khosiawan et al., 2018; Okulewicz & Mandziuk, 2019; Pavone et al., 2008; Sung & Nielsen, 2020), the goal is always to search for optimal solutions. In these studies, assumptions regarding congestion-free flow of concurrently executed transport processes and/or robustness of planned routings
and schedules to assumed disturbances and/or uncertainty of variables are tacitly accepted (Liu & Huang, 2010; Pavone et al., 2008; Saez et al., 2008). An exhaustive review of VRP taxonomy-inspired problems can be found in Braekers et al. (2016) and Pillac et al. (2013).

The PVRPs developed so far have limited use due to the data uncertainty observed in practice. The values describing parameters, such as transport time or loading/unloading times, depend on the human factor, which means they cannot be determined precisely. It is difficult to account for data uncertainty by using fuzzy variables due to the imperfections of the classical fuzzy numbers algebra (Bocewicz et al., 2016; Khairuddin et al., 2021). Equations that describe the relationships between fuzzy variables (variables with fuzzy values) using algebraic operations (in particular, addition and multiplication) do not meet the conditions of the Ring. In addition, algebraic operations based on standard fuzzy numbers follow Zadeh’s extension principle, which means that the uncertainty of variables increases with successive cycles of the system operation (i.e. caused by the need of intermediate approximations), until the information about their value is no longer useful. Despite this, the Fuzzy VRP assuming vagueness for fuzzy customer demands to be collected and fuzzy service or travel times are the subject of a growing body of research (Bansal et al., 2017; Bocewicz et al., 2020; Ghannadpour et al., 2015; Nucci, 2020; Saez et al., 2008).

Works introducing the OFNs formalism seems to be helpful in this respect. Taking advantage of this opportunity, this paper can be seen as an extension of our previous works limited to algebraic sum and difference operations on OFNs and a contribution providing a formal analytical approach enabling both qualitative and quantitative refinement of delivery routings. The current paper aims to fill this gap by introduction the OFN algebra framework. In other words, the objective is to develop an algebraic model aimed at rapid calculation of fuzzy schedules for individual vehicles as well as planning time buffers that permit to adjust currently calculated fuzzy schedules to baseline schedules assuming that operation times are deterministic values. Particularly, the paper focuses on the development of sufficient conditions that allow the calculability of arithmetic operations that guarantee the interpretability of the results obtained.

3. Delivery planning with uncertainty parameters

Let us consider graph $G = (N, E)$ modelling the OPOSN. The set of nodes $N = \{N_1, \ldots, N_M, \ldots, N_n\}$ includes one node representing distribution centre $N_1$ and $N_2, \ldots, N_n$ nodes representing customers. The set of edges $E = \{(N_i, N_j) \mid i, j \in N, i \neq j\}$ determines the possible connections between nodes. A fleet of vehicles $U = \{U_1, \ldots, U_k, \ldots, U_K\}$ is given. The customers are cyclically serviced (with period $T$) by vehicles $U_k$ travelling from node $N_1$. Variable $Q_k$ denotes the payload capacity of vehicles $U_k$. Execution of the ordered delivery $z_{\lambda}$ by the customer $N_\lambda$ takes place in the period $t_{\lambda}$. The moment of starting the mission of vehicles $U_k$ (on the distribution centre $N_1$) is indicated by variable $s_k$. In turn, the moment when the vehicle $U_k$ starts delivery to the customer $N_\lambda$ is indicated by variable $y_{\lambda}^k$. The deliveries ordered by the customer $N_\lambda$ are carried out in the period $\Delta_\lambda = [ld_{\lambda}; ud_{\lambda}]$ (delivery service deadline), i.e. $y_{\lambda}^k \geq ld_{\lambda}$ and $y_{\lambda}^k + t_{\lambda} \leq ud_{\lambda}$. It is assumed that the variable $d_{\beta,\lambda}$ determines travelling time between nodes $N_\beta, N_\lambda$, where: $(N_\beta, N_\lambda) \in E$. The routes of $U_k$ are represented by sequences: $\pi_k = (N_{k_1}, \ldots, N_{k_i}, N_{k_{i+1}}, \ldots, N_{k_i})$, where: $k_i \in \{1, \ldots, K\}$.
Moreover, the following assumptions are met: $Z$ denotes a set of required amounts of goods $z_{\lambda}, \lambda = 1 \ldots n$; $\Pi$ denotes a set of routes $\pi_k, k = 1 \ldots K$; node $N_1$ representing distribution centre occurs only once in each route of the set $\Pi$; node representing the customer $N_{\lambda} (\lambda > 1)$ occurs only once in the route belonging to the set $\Pi$; the amount of goods transported by $U_k$ cannot exceed payload capacity $Q_k$, deliveries are being made over a given periodically repeating time horizon $T$. The adopted assumptions allow us to formulate the question: Does there exist a set of routes $\Pi$ of fleet $U$ guaranteeing the timely delivery (according to given delivery service deadlines $\Delta_{\lambda}$ and in a time window $T$) of the required amount of goods $Z$, to the customers from the set $\mathcal{N}$?

For the purpose of illustration, let us consider network $G$ shown in Figure 1, where 10 nodes (1 distribution centre and 9 customers) are serviced by fleet $U = \{U_1, U_2, U_3\}$. The following routes: $\pi_1 = (N_1, N_3, N_9, N_2)$, $\pi_2 = (N_1, N_5, N_{10}, N_7)$, and $\pi_3 = (N_1, N_6, N_8, N_4)$ guarantee the delivery of the required amount of goods to all customers cyclically (within the period $T=1800$).

The solution was determined assuming that the vehicle payload capacity $Q_k$ is equal to 120. The corresponding cyclic schedule is shown in Figure 2(a). This solution assumes that both travelling times $d_{\beta,\lambda}$ (see CN rows in Figure 3) as well as times of node occupation $t_{\lambda}$ (time of the service provided) are crisp ($t_{\lambda} = 120, \lambda = 1 \ldots 10$). The vehicles of the $U$ fleet deliver services on the dates specified by delivery service deadline $\Delta_{\lambda}$. The delivery cycle period $T$ is equal to 1800. The vehicles complete their missions (return to node $N_1$) on times: $1710 + c \times T$, $1485 + c \times T$, $1790 + c \times T$, ($c \in \mathbb{N}$) respectively, and are waiting at the distribution centre for further deliveries to begin. Laytimes, which are assumed to be represented by crisp values $w_1^1, w_1^2, w_1^3$ (where subscripts and superscripts of $w_k^\lambda$ correspond to $N_\lambda$ and $U_k$), spent in the so called distribution centre are: 270, 435, and 400, respectively (see CN rows in so-called Figure 3).

In this context, the service point located in distribution centre $N_1$ can be treated as a time buffer that allows for adjusting the schedule of implemented missions in situations of accumulating uncertainty, caused by delays following disruptions occurring in the course of the transport operations. It is worth noting that the issue concerning laytimes

![Figure 1](image-url) Figure 1. Graph $G$ modelling the considered OPOSN.
sizing and allocation guaranteeing the existence of schedules that meet the ordered delivery times in the OPOSN is still an open problem.

An example illustrating the imperfections of the classic fuzzy numbers algebra (Bocewicz et al., 2020, 2016) is shown in Figure 2(b), where the level of uncertainty increases with successive cycles. Fuzzy values of variables (i.e. fuzzy travel times $\hat{d}_{b}$, $l$ and fuzzy laytimes $\hat{w}_{1}$, $\hat{w}_{2}$, $\hat{w}_{3}$) used are collected in the Figure 3 (see FN rows). Figure 2 distinguishes the fuzzy values of the start/end moments of service operations carried out on nodes $N_7$ and $N_1$ located along the route selected for $U_2$.

As opposed to a schedule assuming crisp data (Figure 2(a)) the moments of commencement of operations in subsequent cycles take on increasingly uncertain fuzzy values (with a larger support), e.g. ‘about 1035’, ‘about 2835’, ‘about’ for $N_7$ and ‘about 1365’, ‘about 3165’, ‘about 4965’ for $N_1$. As a result, in place of the ‘exact’ routes from Figure 2(a) (distinguished by red, blue and green lines) we get a ribbon-like lines, which determines the area of the real routes execution (each ‘ribbon’ defines the boundaries within which the real route can be performed). It should be noted that due to Zadeh’s principle the accumulation of uncertainty in subsequent cycles, the ‘ribbons’ expand to such a size until they begin to cover almost the time windows (see the 3rd cycle). In such a situation, it is practically impossible to assess the implementation of the actual routes.

Figure 2. Base line (a) and fuzzy (b) schedules corresponding to the routes from Figure 1.
In particular, this means that the moments of starting the mission $s^t(q)$ of $U_k$ in subsequent $q$ cycles assume fuzzy values with increasingly larger supports. Therefore, the degree of the cumulative uncertainty $\Sigma(q)$ of the fleet $U = \{U_1, \ldots, U_k, \ldots, U_k\}$ carried over to subsequent cycles of $q$ can be expressed by the largest support of variables $\hat{s}^t(q)$:

$$\Sigma(q) = \max_{k=1..K} \{\text{supp}_{\hat{s}^t(q)}\},$$

where $\text{supp}_X$ is the support of $X$. For example, Figure 2(b) shows how the support of $\hat{s}^2(q)$ for the vehicle $U_2$ increases in the following cycles, i.e. incrementing by a constant value of 270: 0, 270, 540, …, respectively. Consequently, in the considered example (Figure 2(b)) the degree of cumulative uncertainty $\Sigma(q)$ is respectively:

1. $\Sigma(1) = 0$ when $\text{supp}_{s^1(1)} = 0$; $\text{supp}_{s^2(1)} = 0$ (see Figure 2(b)); $\text{supp}_{s^3(1)} = 0$;
2. $\Sigma(2) = 640$ when $\text{supp}_{s^1(1)} = 640$; $\text{supp}_{s^2(1)} = 410$ (see Figure 2(b)); $\text{supp}_{s^3(1)} = 270$;
3. $\Sigma(3) = 1280$ when $\text{supp}_{s^1(1)} = 1280$; $\text{supp}_{s^2(1)} = 820$ (see Figure 2(b)); $\text{supp}_{s^3(1)} = 540$.

It means that, in the considered case, the fuzzy variable carriers of the successive cycles $q$ are increased by 640.

In this framework, the nascent problem boils down to setting, for the given ranges of uncertainty (e.g. concerning travel and service delivery operation times) the time buffers...
of size (laytimes) that allow us to reduce the cumulative uncertainty (\( \Sigma(q) = 0 \) for \( q = 1 \ldots Q \)) to such an extent that it does not transfer to the next cycles.

The presented issue of planning of the concurrently realized cyclical deliveries belongs to the Discrete Event Systems domain and concerns a well-known class of the Systems of Concurrent Cyclic Processes (SCCP) (Bocewicz et al., 2015). The SCCP is understood as a set of processes executing cyclic operations on the set of shared resources (processors, machines, means of transport, etc.). The periodicity of the behaviour of these systems determines the cyclical nature of the processes carried out in them. The analysis of the SCCP behaviour and synthesis of structures that meet their expected behaviour make up the main directions of such systems research. In that context, the study of various SCCP extensions, i.e. assuming uncertainty of operation times and/or vehicles travelling in a convoy, comes down to the estimation of their cycle time. Therefore, in a given transport structure of the network, the different delivery processes with different cycle times can be implemented. This, in turn, means that any delivery network can be associated with a set of cyclic steady states (CSSs) encompassing the flows of delivery processes following particular cycle time. For the space of states understood in this way, a new problem arises related to the existence of transient states connecting arbitrarily assumed CSSs. The existence of transients states determines the possibility of the system behaviour changing without having to stop it, i.e. allowing its ‘smooth’ transition between the assumed CSSs. Thus, the corresponding problem is a search for the sufficient conditions, guaranteeing the existence of transient states between the selected elements of thr CSSs space. Among the many approaches to CSS reachability modelling, the following ones seem to be most promising: algebraic methods (e.g. based on algebra (max, +)) and declarative modelling (e.g. employing constraints programming techniques).

4. Ordered fuzzy numbers algebra

In the case of classic fuzzy numbers \( \hat{a} \), \( \hat{b} \), \( \hat{c} \) (fuzzy numbers are marked with the symbol ‘\( \hat{\ldots} \)’), the following implication \( (\hat{a} + \hat{b} = \hat{c}) \Rightarrow [(\hat{c} - \hat{b} = \hat{a}) \land (\hat{c} - \hat{a} = \hat{b})] \) does not hold. This makes it impossible to solve a simple equation \( \hat{A} + \hat{X} = \hat{C} \). This fact significantly hinders the application of approaches based on declarative models. Therefore, we propose a declarative model based on OFN algebra in which it is possible to solve algebraic equations. OFNs can be defined (Kosinski et al., 2003) as a pair of continuous real functions \( (f_A, g_A) \), i.e.:

\[
\hat{A} = (f_A, g_A), \quad \text{where : } f_A, g_A : [0, 1] \rightarrow \mathbb{R}. \tag{1}
\]

Assuming that \( f_A \) is increasing and \( g_A \) is decreasing as well as that \( f_A \leq g_A \), the membership function \( \mu_A \) of the OFN \( \hat{A} \) is as follows (see OFN in Figure 3):

\[
\mu_A(x) = \begin{cases} 
  f_A^{-1}(x) & \text{when } x \in UPA \\
  g_A^{-1}(x) & \text{when } x \in DOWNA \\
  1 & \text{when } x \in CONSTA \\
  0 & \text{in the remaining cases}
\end{cases} \tag{2}
\]

where, \( UPA = (l_A0, l_A1) \), \( CONSTA = (l_A1, p_A1) \), and \( DOWNA = (p_A1, p_A0) \).
OFNs have two types of orientations (Kosinski et al., 2003; Prokopowicz & Slezak, 2017): positive, when \( \hat{A} = (f_A, g_A) \); negative, when \( \hat{A} = (g_A, f_A) \).

In the further formulated theorems, the following sequences describing OFN \( \hat{A} \) are used:

\[
\vec{S}_A = (\text{SUP}_A, \text{SCOA}_A, \text{SDOA}_A)
\]

where: \( \text{SUP}_A = \frac{f_A(0)}{f_A(1)}, \text{SCOA}_A = \frac{g_A(1)}{g_A(1)}, \text{SDOA}_A = \frac{g_A(0)}{g_A(0)} \) and \( f_A(1) > 0, g_A(1) > 0, g_A(0) > 0, \)

\[
\vec{S}_A = (\text{SUP}_A, \text{SCOA}_A, \text{SDOA}_A)
\]

where: \( \text{SUP}_A = \frac{f_A(1)}{f_A(0)}, \text{SCOA}_A = \frac{g_A(1)}{g_A(1)}, \text{SDOA}_A = \frac{g_A(0)}{g_A(0)} \) and \( f_A(0) > 0, f_A(1) > 0, g_A(0) > 0. \)

The definitions of algebraic operations used in the proposed model are as follows:

**Definition 4.1:** Let \( \hat{A} = (f_A, g_A), \hat{B} = (f_B, g_B) \), and \( \hat{C} = (f_C, g_C) \) be OFNs. \( \hat{A} \) is a number equal to \( \hat{B} \) (\( \hat{A} = \hat{B} \)), or (\( \hat{A} > \hat{B}; \hat{A} < \hat{B} \)), (\( \hat{A} \leq \hat{B} \)) if:

- \( x \in [0, 1] \) \( f_A(x) \times f_B(x) \land g_A(x) \times g_B(x) \), where: ‘\( \times \)’ stands for: \( =, >, \geq, \leq, \), or \( \leq \). The operations of:
  - \( \hat{C} = \hat{A} + \hat{B} \);
  - \( \hat{C} = \hat{A} - \hat{B} \);
  - \( \hat{C} = \hat{A} \times \hat{B} \);
  - \( \hat{C} = \hat{A} \div \hat{B} \) are defined as follows:

An example of ordered fuzzy numbers \( \hat{A}, \hat{B} \) following the relation \( \hat{A} < \hat{B} \) is presented in Figure 4. In turn, examples of operations of addition \( \hat{C} = \hat{A} + \hat{B} \) and subtraction \( \hat{A} = \hat{C} - \hat{B} \) showing the fulfillment of the implication: (\( \hat{A} + \hat{B} = \hat{C} \)) \( \Rightarrow [([\hat{C} - \hat{B} = \hat{A}) \land (\hat{C} - \hat{A} = \hat{B})] \) are given in Figure 5. It is worth noting that the operation of addition of ordered fuzzy numbers with different orientations leads to the ‘reduction of fuzziness’ (the support of the OFN \( \hat{C} \) is smaller than the supports of OFNs \( \hat{A} \) and \( \hat{B} \)); however, its result is not always a proper OFN. An example of such a situation is illustrated in Figure 6, where the sum \( \hat{A} + \hat{B} \) is an ordered fuzzy number \( \hat{C} \) characterized by \( \mu_C(x) \), which is not a function.

Proper ordered fuzzy numbers are specified by a convex membership function \( \mu_A(x) \). Consequently – limiting the following considerations to the linear, monotonic forms of the functions \( f \) and \( g \) – a proper OFN can be defined as follows.

![Figure 4. An example illustrating the relation \( \hat{A} < \hat{B} \).](image-url)
Definition 4.2: The ordered fuzzy number \( \hat{A} \) is a proper OFN (Prokopowicz & Slezak, 2017) when one of the following conditions is met: \( f_A(0) \leq f_A(1) \leq g_A(1) \leq g_A(0) \) (for positive orientation) or \( g_A(0) \leq g_A(1) \leq f_A(1) \leq f_A(0) \) (for negative orientation).

Figure 5. An example of addition \( \hat{C} = \hat{A} + \hat{B} \) and subtraction \( \hat{A} = \hat{C} - \hat{B} \) for OFNs with the same (positive) orientations (a) and for OFNs with different orientations (b).

Figure 6. An example of operation \( \hat{A} + \hat{B} \) returning an improper OFN \( \hat{C} \).
According to Definition 4.2, the membership function \( \mu_A(x) \) for the number \( \hat{A} \) corresponds to the membership function of a classic fuzzy set, and assuming that the functions \( f \) and \( g \) are linear – it corresponds to the convex membership function \( \mu_A(x) \). All numbers from Figures 4 and 5 are proper OFNs according to Definition 4.2. Figure 6 shows an example of proper OFNs \( \hat{A} \) and \( \hat{B} \), the algebraic sum of which is an improper OFN \( \hat{C} \). Similar results of improper OFNs can be obtained for other algebraic operations: difference, multiplication, and division. Definition 4.2 allows us to specify the conditions that guarantee that the result of algebraic operations is a proper OFN:

**Theorem 4.3:** Let \( \hat{A} \) and \( \hat{B} \) be proper OFNs with different orientations: \( \hat{A} \) (positive orientation), \( \hat{B} \) (negative orientation). If one of the following conditions holds:

\[
(|UP_A| - |UP_B| \geq 0) \land (|CONST_A| - |CONST_B| \geq 0) \land (|DOWN_A| - |DOWN_B| \geq 0),
\]

\[
(|UP_B| - |UP_A| \geq 0) \land (|CONST_B| - |CONST_A| \geq 0) \land (|DOWN_B| - |DOWN_A| \geq 0),
\]

then the result of the operation \( \hat{A} + \hat{B} \) is a proper OFN \( \hat{C} \).

**Proof.** The proof follows directly from Bocewicz et al. (2021).

**Theorem 4.4:** Let \( \hat{A} \) and \( \hat{B} \) be proper OFNs with different orientations: \( \hat{A} \) (positive orientation), \( \hat{B} \) (negative orientation). If one of the following conditions holds:

1. \( (SUP_A - SUP_B \leq 0) \land (SCO_A - SCO_B \leq 0) \land (SDO_A - SDO_B \leq 0), \)
2. \( (SUP_B - SUP_A \leq 0) \land (SCO_B - SCO_A \leq 0) \land (SDO_B - SDO_A \leq 0), \)

then the result of the multiplication operation \( \hat{A} \times \hat{B} \) is a proper OFN \( \hat{C} \).

**Proof.** First we will prove condition (1) that the simultaneous fulfillment of the inequalities: \( SUP_A - SUP_B \leq 0 \); \( SCO_A - SCO_B \leq 0 \); \( SDO_A - SDO_B \leq 0 \) guarantees a result of \( \hat{A} \times \hat{B} \) that is a proper OFN \( \hat{C} = (f_C, g_C) \) with a positive orientation:

1. If \( SUP_A - SUP_B \leq 0 \) then: \( \frac{f_A(0)}{f_A(1)} - \frac{f_B(0)}{f_B(1)} \leq 0 \). Thus, \( f_A(0) \times f_B(0) \leq f_A(1) \times f_B(1) \). According to Definition 1, for function \( f_C \) of OFN \( \hat{C} \): \( f_C(0) = f_A(0) \times f_B(0) \); \( f_C(1) = f_A(1) \times f_B(1) \), we get: \( f_C(0) \leq f_C(1) \).
2. If \( SCO_A - SCO_B \leq 0 \) then: \( \frac{g_A(1)}{g_A(0)} - \frac{g_B(1)}{g_B(0)} \leq 0 \). Thus, \( f_A(1) \times f_B(1) \leq g_A(1) \times g_B(1) \). According to Definition 1, for function \( g_C \) of OFN \( \hat{C} \): \( g_C(1) = f_A(1) \times f_B(1) \); \( g_C(0) = g_A(1) \times g_B(1) \), we get: \( g_C(1) \leq g_C(0) \).
3. If \( SDO_A - SDO_B \leq 0 \) then: \( \frac{g_A(1)}{g_A(0)} - \frac{g_B(1)}{g_B(0)} \leq 0 \). Thus, \( g_A(1) \times g_B(1) \leq g_A(0) \times g_B(0) \). According to Definition 1, for function \( g_C \) of OFN \( \hat{C} \): \( g_C(1) = g_A(1) \times g_B(1) \); \( g_C(0) = g_A(0) \times g_B(0) \), we get: \( g_C(0) \leq g_C(1) \).

The fulfillment of the above three conditions implies the following inequality: \( f_C(0) \leq f_C(1) \leq g_C(1) \leq g_C(0) \), which, according to Definition 4.2, means that the number \( \hat{C} \) is a proper OFN with a positive orientation.

The proof of condition (2) is analogous.

An example of multiplying two numbers \( \hat{A} \times \hat{B} \) is presented in Figure 7. In the first case (Figure 7(a)), the multiplied numbers satisfy (1) condition of the Theorem 4.4, which
means that the result $\hat{C}$ is a proper OFN. In the second case (Figure 7(b)) the multiplied numbers do not meet the condition (1) ($SCOA \rightarrow SCOB \leftarrow C_2 0$) and result $\hat{C}$ is an improper OFN.

**Theorem 4.5:** Let $\hat{A}$ and $\hat{B}$ be proper OFNs with the same (e.g. positive) orientations: $\hat{A}, \hat{B}$. The result of the multiplication operation $\hat{A} \times \hat{B}$ is always a proper OFN $\hat{C}$ (with the same orientation).

**Proof.** For equally oriented numbers, the conditions (1) and (2) of Theorem 4.4 always hold. For numbers with a positive orientation: $SUP_A \rightarrow \rightarrow SCOA \rightarrow SCOB \leftarrow SDOA \rightarrow SDOB \leftarrow \left[0, 1\right]$ and $SUP_B \leftarrow \leftarrow SCOB \rightarrow SCOA \leftarrow SDOB \rightarrow SDOA \leftarrow \left[1, \infty\right]$ hold, which means that their difference is always $\leq 0$ (condition (1) always holds). An analogous property occurs for the condition (2).

**Theorem 4.6:** Let $\hat{A}$ and $\hat{B}$ be proper OFNs with the same (e.g. positive) orientations: $\hat{A}, \hat{B}$. If one of the following conditions holds:

1. $(SUP_A - SUP_B \geq 0) \land (SCOA - SCOB \geq 0) \land (SDOA - SDOB \geq 0)$,
2. $(SUP_B - SUP_A \geq 0) \land (SCOB - SCOA \geq 0) \land (SDOB - SDOA \geq 0)$,

then the result of the division operation $\hat{A}/\hat{B}$ is a proper OFN $\hat{C}$ (with the opposite orientation).

**Proof.** First, we will prove condition (1) that the simultaneous fulfillment of the inequalities: $SUP_A - SUP_B \geq 0; SCOA - SCOB \geq 0; SDOA - SDOB \geq 0$ guarantees a result of $\hat{A}/\hat{B}$ (with positive orientations) that is a proper OFN $\hat{C} = (f_C, g_C)$ with a negative orientation:

1. If $SUP_A - SUP_B \geq 0$ then: $\frac{f_A(0)}{f_B(0)} - \frac{f_A(1)}{f_B(1)} \geq 0$. Thus, $\frac{f_A(0)}{f_B(0)} \geq \frac{f_A(1)}{f_B(1)}$. According to Definition 4.1, for function $f_C$ of OFN $\hat{C}$: $f_C(0) = f_A(0)/f_B(0); f_C(1) = f_A(1)/f_B(1)$, we get: $f_C(0) \geq f_C(1)$. 

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**Figure 7.** Results of proper (a) and improper (b) OFNs multiplication
(2) If \( \overrightarrow{SCOA} - \overrightarrow{SCOB} \geq 0 \) then: \( \frac{f_A(1)}{g_A(1)} - \frac{f_B(1)}{g_B(1)} \geq 0 \). Thus, \( \frac{f_A(1)}{g_A(1)} \geq \frac{g_A(1)}{g_B(1)} \). According to Definition 4.1, for function \( f_C \) of OFN \( \hat{C} \): \( f_C(1) = f_A(1)/f_B(1) \); \( g_C(1) = g_A(1)/g_B(1) \), we get: \( f_C(1) \geq g_C(1) \).

(3) If \( \overrightarrow{SDOA} - \overrightarrow{SDOB} \geq 0 \) then: \( \frac{g_A(1)}{g_B(1)} - \frac{g_A(0)}{g_B(0)} \geq 0 \). Thus, \( \frac{g_A(1)}{g_B(1)} \leq \frac{g_A(0)}{g_B(0)} \). According to Definition 4.1, for function \( f_C \) of OFN \( \hat{C} \): \( g_C(1) = g_A(1)/g_B(1) \); \( g_C(0) = g_A(0)/g_B(0) \), we get: \( g_C(1) \geq g_C(0) \).

The fulfillment of the above three conditions implies the following inequality: \( f_C(0) \geq f_C(1) \geq g_C(1) \geq g_C(0) \), which, according to Definition 4.2, means that the number \( \hat{C} \) is a proper OFN with a negative orientation.

The proof of condition (2) is analogous.

An example of dividing two numbers \( \hat{A}/\hat{B} \) is presented in Figure 8. In the first case (Figure 8(a)), the divided numbers satisfy (i) condition of Theorem 4, which means that the result \( \hat{C} \) is a proper OFN. In the second case (Figure 8(b)), the divided numbers do not meet the condition (i) \( \overrightarrow{SDOA} - \overrightarrow{SDOB} \neq 0 \) and result \( \hat{C} \) is improper OFN.

**Theorem 4.7** Let \( \hat{A} \) and \( \hat{B} \) be proper OFNs with different orientations: \( \hat{A} \) (negative orientation), \( \hat{B} \) (positive orientation). The result of the division operation \( \hat{A}/\hat{B} \) is always a proper OFN \( \hat{C} \) (with the negative orientation).

**Proof.** For opposite oriented numbers, conditions (1) and (2) of Theorem 4.5 always hold. For numbers with a negative orientation: \( \overrightarrow{SUPA}, \overrightarrow{SCOA}, \overrightarrow{SDOA} \in [1, \infty] \) and for numbers with a positive orientation \( \overrightarrow{SUPA}, \overrightarrow{SCOA}, \overrightarrow{SDOA} \in [0, 1] \) hold, which means that their difference is always \( \geq 0 \) (condition (1) always holds). An analogous property occurs for the condition (2).

The above theorem allows the construction of models in which decision variables resulting from algebraic operations (in particular sums) take values that are proper OFNs (numbers that are easy to interpret). Moreover, the fulfillment of the conditions underlying the above theorem may lead to a reduction in the fuzziness of the sum of OFNs with different orientations.

![Figure 8](image-url)
5. Ordered fuzzy constraint satisfaction problem

The problem being considered can be perceived as an Ordered Fuzzy Constraint Satisfaction (OFCS) Problem (3):

\[ \widehat{FCS} = \left( \left( \widehat{V}, \widehat{D} \right), \widehat{C} \right), \]  

where:

\( \widehat{V} \) – a set of decision variables, including variables representing routes \( x_{\beta,\lambda}^k \) and schedule \( (c^k, s^k, y_{\lambda}^k, w_{\lambda}^k, \Sigma) \):

\( x_{\beta,\lambda}^k \): binary variable indicating the travel of \( U_k \) between nodes \( N_{\beta}, N_{\lambda} \):

\[ x_{\beta,\lambda}^k = 1 \text{ if } U_k \text{ travels from node } N_{\beta} \text{ to node } N_{\lambda}; x_{\beta,\lambda}^k = 0 \text{ otherwise}, \]

\( c^k \): weight of goods delivered to \( N_{\lambda} \) by vehicle \( U_k \),

\( y_{\lambda}^k \): fuzzy time at which vehicle \( U_k \) arrives at node \( N_{\lambda} \), (OFN representation),

\( w_{\lambda}^k \): laytime at node \( N_{\lambda} \) for \( U_k \) (OFN representation),

\( s^k \): take-off time of vehicle \( U_k \) (OFN representation),

\( \Sigma \): cumulative uncertainty \( (\Sigma = \Sigma(1), \text{i.e. for } q=1) \),

\( \widehat{D} \) – a finite set of decision variable domains: \( \widehat{s}^k, \widehat{y}_\lambda^k, \widehat{w}_\lambda^k \in F \) \( (F \text{ is a set of OFNs } (1)), \)

\( x_{\beta,\lambda}^k \in \{0, 1\}, \)

\( \widehat{C} \) – a set of constraints specifying the relationships between decision variables, such as:

\[ \widehat{s}^k \geq 0; \quad k = 1 \ldots K, \]  

\[ \sum_{j=1}^{n} x_{ij}^k = 1; \quad k = 1 \ldots K, \]  

\[ \left( x_{ij}^k = 1 \right) \Rightarrow \left( y_{ij}^k = \widehat{s}^k + \widehat{d}_{ij} \right); \quad j = 1 \ldots n, \]  

\[ \left( x_{ij}^k = 1 \right) \Rightarrow \left( y_{ij}^k = y_{ij}^k + \widehat{d}_{ij} + \widehat{t}_i + \widehat{w}_\lambda^k \right), \]  

\[ \widehat{s}^k + T = \widehat{y}_1^k + \widehat{t}_1 + \widehat{w}_\lambda^k; \quad k = 1 \ldots K, \]  

\[ \widehat{y}_{ij}^k \geq 0; \quad i = 1 \ldots n; \quad k = 1 \ldots K, \]  

\[ \sum_{j=1}^{n} x_{ij}^k = \sum_{j=1}^{n} x_{ji}^k; \quad i = 1 \ldots n; \quad k = 1 \ldots K, \]
\[ c_i^k \leq Q_k \times \sum_{j=1}^{n} x_{ij}^k; \quad i = 1 \ldots n; \quad k = 1 \ldots K, \quad (11) \]

\[ \left( x_{ij}^k = 1 \right) \Rightarrow c_i^k \geq 1; \quad k = 1 \ldots K; \quad i = 1 \ldots n, \quad (12) \]

\[ \sum_{k=1}^{K} c_i^k = z_i; \quad i = 2 \ldots n, \quad (13) \]

\[ \hat{y}_i^k \leq T; \quad i = 1 \ldots n; \quad k = 1 \ldots K, \quad (14) \]

\[ x_{ij}^k = 0; \quad i = 1 \ldots n; \quad k = 1 \ldots K, \quad (15) \]

\[ \text{ld}_i + \hat{t}_i \leq \hat{y}_i^k + \hat{t}_i + c \times T \leq \text{ud}_i, \quad i = 1 \ldots n \quad (16) \]

\[ \Sigma = \max_{k=1\ldots K} \{ \text{supp}_k \} = 0. \quad (17) \]

Constraints (4)–(17) describe the relationship between routes (represented by the variables \( x_{ij}^k \)) and the delivery schedule (variables \( \hat{y}_i^k \) and \( \hat{s}_i^k \)). Moreover, it is assumed that deliveries are made in accordance with the adopted routes (4-7, 9), and routes form closed loops (8, 10). Constraints (11)–(17), link routes \( x_{ij}^k \) to the amounts of delivered goods \( c_i^k \).

It is assumed that the arithmetic operations contained in the above constraints meet the conditions of Definition 4.1 and Theorems 4.3–4.7.

In practice, this means that the use of OFN algebra operators allows to take into account the reduction of uncertainty resulting from the periodic operation of the system. The developed operators can also in models that have relatively complex constraints, e.g. describing nonlinear relations between decision variables (see battery consumption models elaborated by Thibbotuwawa et al., 2020).

6. Computational experiments

Let’s consider network G shown in Figure 1, which parameters are described by fuzzy variables (i.e. fuzzy travel times \( \hat{d}_{b_i} \) and fuzzy laytimes \( \hat{w}_i^1, \hat{w}_i^2, \hat{w}_i^3 \)) collected in Figure 3. The answer to the following question is sought: Does there exist a set of time buffers (laytimes) allowing for the reduction of the cumulative uncertainty \( \Sigma(q) = 0 \) for \( q = 1 \ldots Q \)?

The answer to this kind of question requires us to solve \( FCS (3) \). That means, the values of the decision variables from the adopted set of domains for which the given constraints are satisfied must be determined. In other words, routings \( (x_{ij}^k) \) and the corresponding schedule \( (s^k, \hat{y}_i^k, \hat{w}_i^k, \Sigma) \) are sought that guarantee timely deliveries (according to given delivery service deadlines \( \Delta \)) despite travel times uncertainty \( (\hat{d}_{b_i}) \). Moreover, it is assumed that the feasible solution sought does not result in the accumulation of uncertainty in subsequent cycles (laytimes allow for uncertainty reduction \( \Sigma = 0 \)). For fuzzy travel times \( \hat{d}_{b_i} \) given in the form of an OFN (see Figure 3) solution (IBM ILOG CPLEX environment where OFN values are represented by discrete forms Bocewicz et al., 2016) to the problem of the vehicle fleet scheduling in the OPOSN from the Figure 1 is
shown in Figure 9. It should be noted that the cumulative uncertainty value is reduced at the end of each cycle ($\Sigma(q) = 0$). Fuzzy variables describing the waiting time of vehicles at node $N_i$ have a negative orientation (Figure 3 – laytimes $\hat{w}_{1i}, \hat{w}_{2i}$ and $\hat{w}_{3i}$), which means that the results of the algebraic operations (e.g. $\hat{y}_{1i} + \hat{t}_i + \hat{w}_{1i}$) using these variables lead to a decrease in uncertainty (support of $\hat{s}_{1i} = 1$). This means that, in contrast to the solution of Figure 2(b), the supports of the all OFN decision variables ($\hat{y}_{li}$) do not increase in subsequent schedule cycles.

For the case under consideration, a series of experiments are performed aimed to assess of cumulative uncertainty $\Sigma(q)$ reduction depending on changes in the cycle period ($T = 1300 –– 1850$) and laytimes ($\hat{w}_{ki} = \hat{0} \ldots \hat{550}$). Figure 11 shows the space of solutions, among which only solutions with cycle period $T \geq 1685$ enable the reduction of uncertainty ($\Sigma(q) = 0$). A proper example of such a solution is: $T = 1800; \hat{w}_{ki} = 270$ determining the schedule from Figure 9. On the other hand, assuming the value of $T = 1450$, makes it impossible to reduce the uncertainty, which is illustrated in Figure 10.

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**Figure 9.** Cyclic fuzzy schedules following Theorems 4.3–4.7 and corresponding to the routes from the Figure 1 – a cycle period $T = 1800$.

**Figure 10.** Cyclic fuzzy schedules following Theorems 4.3–4.7 and corresponding to the routes from the Figure 1 – a cycle period $T = 1450$. 

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Therefore, the presented trade-off (see Figure 11) allows us to determine the part of the solution space (above the limit $T_C = 1685$) in which the cycle period $T$ and the laytimes $\hat{w}_k^l$ guarantee timely deliveries without the accumulation of uncertainty in subsequent cycles.

In addition to the above experiments, the effectiveness of the proposed approach was evaluated for distribution networks of different sizes (different numbers of customers and

**Figure 11.** Trade-off between the cycle period $T$ and the laytime support $\hat{w}_k^l$.

| Number of nodes $n$ | Number of vehicles (1 vehicle / convoy) | Calculation time [s] | Number of convoys (2 vehicles / convoy) | Calculation time [s] |
|---------------------|----------------------------------------|----------------------|----------------------------------------|----------------------|
| 8                   | 1                                      | <1                   | 1                                      | 4                     |
| 8                   | 2                                      | 3                    | 2                                      | 12                    |
| 10                  | 1                                      | 11                   | 1                                      | 46                    |
| 10                  | 2                                      | 24                   | 2                                      | 154                   |
| 10*                 | 3                                      | 31                   | 3                                      | 421                   |
| 12                  | 1                                      | 151                  | 1                                      | 351                   |
| 12                  | 2                                      | 331                  | 2                                      | 785                   |
| 12                  | 3                                      | 574                  | 3                                      | >900                  |
| 14                  | 1                                      | 258                  | 1                                      | 567                   |
| 14                  | 2                                      | 464                  | 2                                      | >900                  |
| 14                  | 3                                      | 854                  | 3                                      | >900                  |
| 16                  | 1                                      | 521                  | 1                                      | >900                  |
| 16                  | 2                                      | 723                  | 2                                      | >900                  |
| 16                  | 3                                      | >900                 | 3                                      | >900                  |
| 18                  | 1                                      | >900                 | 1                                      | >900                  |
| 18                  | 2                                      | >900                 | 2                                      | >900                  |
| 18                  | 3                                      | >900                 | 3                                      | >900                  |

*Solution from Figure 9
vehicles). The results are collected in Table 1. The experiments were carried out for networks containing 6–18 nodes in which services were made by sets of 1–3 convoys including 1 or 2 vehicles in each one. In the case when convoys include 1 vehicle the corresponding delivery problem can be solved in online mode (<900s) for distribution network not exceeding 16 nodes. In turn for the case when convoys include 2 vehicles the delivery problem can be solved in online mode for distribution network not exceeding 12 nodes. In the case of larger networks, the effect of combinatorial explosion limits the practical use of this method.

7. Conclusions

Conditions of the theorem proposed allow us to construct models in which decision variables resulting from algebraic operations take values that are proper OFNs. Consequently, the arithmetic operations defined on OFNs enable us to avoid some drawbacks of the classical approach while the method can be characterized by easy to interpret convex membership functions.

In addition to its capability to handle the fuzzy nature of variables through an algebraic approach, the proposed method can be used to rapidly prototype the sizing and allocation of time buffers. The related issue of vehicle mission planning subject to changing order uncertainty constraints will be the subject of works preceding the achievement of the main goal, which is the integration of algebraic (e.g. based on (max, +) algebra) and declarative modelling (e.g. employing constraints programming techniques) methods in order to develop conditions ensuring the transition between the assumed CSSs.

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