Cosmic Crystallography in Compact Hyperbolic Universes

H. V. Fagundes and E. Gausmann
Instituto de Física Teórica
Universidade Estadual Paulista
São Paulo, SP 01405-900, Brazil
e-mail: helio@ift.unesp.br, gausmann@ift.unesp.br

23 November 1998

Abstract

We try to apply the cosmic crystallography method of Lehoucq, Lachieze-Rey, and Luminet to a universe model with closed spatial section of negative curvature. But the sharp peaks predicted for Einstein-de Sitter closed models do not appear in our hyperbolic example. So we turn to a variant of that method, by subtracting from the distribution of distances between images in the closed model, the similar distribution in Friedmann’s open model. The result is a plot with much oscillation in small scales, modulated by a long wavelength quasi-sinusoidal pattern.

1 Introduction

The idea of cosmic crystallography was introduced by Lehoucq et al., and applied by them to several Einstein-de Sitter (EdS) models with closed (i.e., compact and boundless) spatial sections of Euclidean geometry and nontrivial topology. The simplest of these cases has the three-torus $T^3$ as spatial section - model E1 in Lachièze-Rey and Luminet review article on cosmic topology. This model provides the sharpest example of the crystallographic effect of the topology: if $T^3$ is based on a cube of side $L$ as
fundamental polyhedron (FP) then the distances between images of a single source are in the form $d = (l^2 + m^2 + n^2)^{1/2}L$, with $l, m, n$ integers, so that a plot of the distribution of distances $n(d)$ vs. $(d/L)^2$ provides neat peaks in the integral values $l^2 + m^2 + n^2$. In the other five EdS models with closed spatial sections the peaks tend to be in smaller numbers or less sharp, because the distances between some of the images of a single source depend on the latter’s position inside the FP. This is characteristic of locally but not globally homogeneous models [3], and can be seen in the study of model E4 by Fagundes and Gausmann [4]: when $L = 7200/h$ Mpc is of the order of magnitude of the observable universe’s radius, no peaks showed up in the distribution $n(d)$. These studies are based on computer simulated catalogs, and cannot yet be compared with real catalogs, given the present limitations of the latter.

The same absence of obvious peaks is true for a model with a closed hyperbolic manifold $M^3$ as spatial section, and hence the metric of Friedmann’s open model. This is illustrated in Figs. 1 and 2, with $M^3$ the second of Best’s [3] three manifolds with an icosahedron as FP, and listed as number $v2293(+3, 2)$ in the census of closed, orientable hyperbolic manifolds in Weeks’s computer program SnapPea 2.5.3 [6]. The figure shows plots of $n(d)$ against the distances in megaparsecs, assuming the density parameter $\Omega_0 = 0.3$ and Hubble’s constant $H_0 = 65$ km sec$^{-1}$Mpc$^{-1}$, and a radius of 12873 Mpc, equivalent to maximum redshift $Z = 1300$, which we take to be the position of the surface of last scattering (SLS) of the cosmic microwave background. Figures 1 and 2 show the plots for the distributions for the compact model and the infinite model, respectively. Although there is more wiggling in the former, the general shape is the same for both.

So we tried a further step in the crystallography method, which is to map the differences between the distributions for the compact and infinite cases.

In Sec. 2 we explain our method and present the results, and in Sec. 3 the latter are discussed.

2 Calculations

To fix our notation let the metric of the hyperbolic Friedmann model be written as

$$ds^2 = dt^2 - a^2(t)[d\chi^2 + \sinh^2 \chi(d\theta^2 + \sin^2 \theta d\varphi^2)].$$
To simulate a random distribution of sources in the FP, we first looked for such a distribution in the ball enclosed by the icosahedron’s circumscribing sphere, with radius $\chi_{out} = 1.38257$. This was done by dividing the ball into 13 shells of thickness $\Delta \chi = \chi_{out}/13$, with volume $\Delta V \approx 2\pi \sinh^2 \chi_m \Delta \chi$, where $\chi_m$ is the medium radius of the shell; and also dividing the solid angle space into ten zones, with $\Delta \Omega \approx 2\pi \sin \theta_m \Delta \theta$ in each zone, where $\Delta \theta = \pi/10$ and $\theta_m$ the zone’s medium value of $\theta$. Then a number (150) of pseudorandom positions in the ball were chosen by computer, with the condition that the occupations in each shell and zone were proportional to $\Delta V$ and $\Delta \Omega$ respectively. Finally a program routine excluded from the obtained points those that were outside the icosahedron. The 51 remaining ones were considered the positions of the sources in the FP. The radius of the SLS in normalized units is $\chi = \chi_{max} = 2.33520$ for the assumed values of $\Omega_0$ and $H_0$.

In order to fill this space without gaps we needed 92 copies of the icosahedron, besides the FP itself. They were produced by the 20 face-pairing generators $\gamma_k$ (one-letter words) of the covering group $\Gamma$, 60 two-letter, and 12 three-letter words in the $\gamma_k$’s. This set of icosahedra cover hyperbolic space up to a radius $\chi = 2.33947$, slightly larger than $\chi_{max}$. The 92 operators were then applied to each of the 51 sources, the result being accepted as an image if it lay inside the SLS ($\chi < \chi_{max}$). The total of potential images (they are potential in the sense that, because of evolution and other factors, many of them may not be observable) thus obtained, including the sources, was 1570.

Then we looked for the present (or comoving) distances between each pair of these images, by multiplying the normalized distances by the curvature radius of the universe (see, for example, [7]), $R_0 = cH_0^{-1}(1 - \Omega_0)^{-1/2} = 5512.62$ Mpc. A list was prepared by grouping the numbers of occurrence of distances in bins of 100 Mpc width. The corresponding plot is shown in Fig. 1.

We proceeded to obtain a similar distribution in Friedmann’s open model, with the same parameters as above, but with the same number of pseudorandom sources as there are images, i. e., 1570 sources. A list similar to the one above mentioned was plotted in Fig. 2. As indicated in the Introduction, we could not see a significant difference between these plots.

Finally we subtracted the values of the second list from those of the first. Fig. 3 is a plot of these differences. One notices wild oscillations in the scale of the bin width, modulated by a broad pattern, on the scale of $R_0$. We fitted this plot with a five-term Fourier series of the form
\[ f(x) = \sum_{n=1}^{5} a_n \sin \frac{n\pi x}{\lambda}, \]

where \( x \) = nearest integer to \( d/(100 \text{ Mpc}) \), \( \lambda = 258 \), and \( \{a_n\} = \{96.6, 134.4, -211.7, 127.0, -89.4\}. \)

### 3 Discussion

A *Clifford translation* of a metric space \( S \) is an isometry \( g \) of \( S \) such that \( \text{distance}(p, gp) \) is independent of point \( p \in S \). Cf. Wolf \[8\]. In a recent paper, Gomero et al. \[9\] link the existence of sharp peaks in cosmic crystallography to that of elements of \( \Gamma \) which are Clifford translations of the universal covering space of \( M^3 \). (See also Lehoucq et al. \[10\].) But in Fig. 1 of \[4\] we see a significant peak that may not be produced by a Clifford translation. The matter deserves further investigation.

In the case of the hyperbolic space \( \nu 2293 (+3, 2) \) treated here, there seems indeed to be no sharp peaks, as predicted by \[8\] and \[10\]. (It should be interesting to look for them in the case of a highly symmetrical manifold, like the hyperbolic dodecahedron space \[11\].) This is why we proceeded to look for the differences in the distribution of distances between this model and Friedmann’s open model, whose plot does show a significant pattern.

Of course we would not expect to be able to infer the correct cosmic topology from a single plot, even one with sharp peaks. The topology of the universe will probably be revealed slowly and progressively, as more and better data are obtained, and several topology searching methods are applied to them.

E. G. thanks Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq - Brazil) for a doctorate scholarship and financial help for participation in this Symposium. H. V. F. thanks CNPq for partial financial support, and Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) for a grant to attend this event.

### References
[1] R. Lehoucq, M. Lachièze-Rey, and J.-P. Luminet, Astron. Astrophys. 313, 339 (1996)

[2] M. Lachièze-Rey and J.-P. Luminet, Phys. Rep. 254, 135 (1995)

[3] H. V. Fagundes, Gen. Relat. Gravit. 24, 199 (1992); Addendum in Gen. Relat. Gravit. 30 (1998) - to appear

[4] H. V. Fagundes and E. Gausmann, Phys. Lett. A 238, 235 (1998)

[5] L. A. Best, Canadian J. Math. 23, 451 (1971)

[6] J. R. Weeks, SnapPea: A Computer Program for Creating and Studying Hyperbolic Manifolds, obtained by anonymous ftp from geom.umn.edu; latest version 2.5.3 (1998)

[7] E. W. Kolb and M. S. Turner, The Early Universe (Addison-Wesley, Reading, 1994), p. 51

[8] J. A. Wolf, Spaces of Constant Curvature (McGraw-Hill, New York, 1967), Sec. 2.7

[9] G. I. Gomero, A. F. F. Teixeira, M. J. Rebouças, and A. Bernui, preprint gr-qc/9811038

[10] R. Lehoucq, J.-P. Luminet, and J.-P. Uzan, preprint astro-ph/9811107

[11] H. Seifert and W. Threlfall, A Textbook of Topology, ed. J. S. Birman and J. Eisner (Academic, New York, 1980)
FIGURE CAPTIONS:

FIGURE 1. A plot of the number of occurrences of the distances in the abscissa, in bins of 100 Mpc, for model with the finite, multiply connected manifold $v^{2293}(+3, 2)$ as spatial section, with a simulated catalog of 51 sources and 1570 potential images.

FIGURE 2. A plot similar to that of Fig. 1, now for the spatially infinite Friedmann’s open model and a simulated catalog of 1570 sources or potential images.

FIGURE 3. The abscissa is the same as in Figs. 1 and 2, the ordinate is the difference of their ordinates.
