Interferometric measurement of the helical mode of a single photon

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Abstract. We present measurements of the helical mode of single photons and do so by sending heralded photons through a Mach–Zehnder interferometer that prepares the light in a helical mode with topological charge one, and interferes it with itself in the fundamental non-helical mode. Masks placed after the interferometer were used to diagnose the amplitude and phase of the mode of the light. Auxiliary measurements verified that the light was in a non-classical state. The results are in good agreement with theory. The experiments demonstrate in a direct way that single photons carry the entire spatial helical-mode information.

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1. Introduction

Light in helical modes has received much attention since its discovery because of the orbital angular momentum (OAM) that it carries [1]. A single photon carries \( \ell \hbar \) of OAM, with \( \ell \) being its topological charge or the winding of the phase about the beam center in multiples of \( 2\pi \). Much work has been devoted to the classical properties of helical beams and their
ability to transfer angular momentum to matter [2]. More recently, the high-dimensional spaces allowed by spatial modes has led to interest in using helical modes of light for encoding quantum information. Early studies on quantum entanglement [3] have led to implementations of high-dimensional entanglement [4] and hyper-entanglement [5]. Spatial modes have also been identified for encoding information [6]–[9]. Spatial modes, the building blocks of images, are also expected to play an important role in a new form of imaging that exploits quantum mechanics [10]–[13].

Previous experimental studies of quantal properties of helical modes have involved detecting the single photons using diffractive mode filters. Although all these studies assume (and reaffirm) that single photons can be in spatial helical modes, no study has actually measured the helical mode of single photons. A previous study measured the amplitude of a helical mode in a mode interference experiment with a Hong–Ou–Mandel interferometer [14]. Related studies have examined mode correlations of down-converted photons: one study imaged post-selected mode superpositions that revealed the mode rotation due to a varying Gouy phase [15], and a more recent study measured orbital-mode characteristics in the joint mode of down-converted photons [16].

In a recent conference report, we presented the mathematical formalism and preliminary measurements of imaging single photons in a helical mode [17]. In this paper, we present a conclusive experimental demonstration of the helical modes of single photons by imaging them directly. We do this by preparing and interfering the light in a helical mode using a Mach–Zehnder interferometer, and diagnosing it via movable amplitude masks placed after the interferometer. Three sets of data taken in distinct experimental configurations contribute, with complementary information supporting the main hypothesis. We also present a measurement that demonstrates that the light in the experiments was in a non-classical state.

2. The experimental method

We generated energy-degenerate photon pairs by spontaneous parametric down-conversion (SPDC). As a pump source we used an 18 mW, continuous-wave, 442 nm light beam from a helium–cadmium laser in a pure $\ell=0$ Gaussian mode. A schematic diagram of the apparatus is shown in figure 1. This figure shows an experimental configuration where the down-converted photons are not collinear. We also used a configuration where the down-converted photons were collinear [17]. The light from the laser was incident on a BBO crystal aligned for type-I SPDC. In the imaging experiments the pump light was focused to a 0.2 mm waist. The heralding photon emerging from the BBO crystal was sent to a single-photon avalanche photo-diode (APD) detector $A$ via a multimode fiber. The corresponding fiber coupler was preceded by a 10 nm band-pass filter and an iris. The second photon was sent to a Mach–Zehnder interferometer.

It is well accepted that SPDC light produced by a pump photon in the fundamental spatial mode ($\ell=0$) is in a coherent superposition of modes of opposite $\ell$ [18, 19]:

$$|\Psi\rangle = \sum_{\ell} c_\ell |u_\ell\rangle |u_{-\ell}\rangle,$$

where $c_\ell$ is the probability amplitude that the SPDC photons of a pair be in spatial modes with topological charges $\ell$ and $-\ell$, described by spatial mode functions $|u_\ell\rangle$ and $|u_{-\ell}\rangle$, respectively. The distribution of values $c_\ell$ is in general maximum for $\ell=0$ and decreases with increasing value of $\ell$. We made the distribution more pronounced toward $c_0$ by focusing of the pump
Figure 1. Schematic representation of the apparatus. Optical elements shown are:
non-polarizing 50–50 beam splitters (BS), beta-barium borate nonlinear crystal (BBO), spiral phase plate (SP), glass blank (GB), mask (M), band-pass filters of bandwidth 10 nm (F₁) and 40 nm (F₂). Small images are modeled distributions of probability, phase (in color scale: 0° → 60° → 120° → 180° → 240° → 300° = red → magenta → blue → cyan → green → yellow) and wavefront of the spatial mode of the light. Larger images are the corresponding measured single-photon images.

beam [18]. In addition, we increased the amplitude that the detected SPDC photons be in ℓ = 0 by restricting the aperture of the collimator for the heralding photon. The latter reduced the numerical aperture of the fiber and hence the modes that propagated inside. From our data and discussion below, we are confident that this projected the state of the photons heading toward the interferometer to predominantly ℓ = 0 states (i.e. $c_0 \gg c_\ell'$ with $\ell' \neq 0$, in equation (1)). The standard way to do this is to detect the heralding photon by passing it through a single-mode fiber before reaching the detector, which projects the state of the pairs strictly to ℓ = 0 modes [3]. However, our pump beam was too weak to allow this operation and having high-enough count rates for doing the single-photon imaging described below.

The interferometer had 50–50 non-polarizing beam splitters, dielectric mirrors, a commercial ℓ = 1 spiral phase plate in one of the arms and a glass blank in the other arm (see figure 1). In some experiments we included neutral density filters in the interferometer arms to vary the total intensity of the interfering modes. The phase plate was made of a polymer deposited on a glass substrate [20]. Photons with a topological charge ℓ = 0 transmitted through the phase plate were put in a helical mode with a topological charge ℓ = 1. When a Gaussian beam with topological charge ℓ goes through such a phase plate the transmitted mode in the far field is mostly a Laguerre–Gauss mode $LG_{p}^{\ell}$ with radial index $p = 0$ and angular index $\ell + 1$ [21]. The glass blank compensated the path length of the substrate of the spiral phase plate and also served to advance the interferometer phase via the adjustment of its tilt. If we assume that the spatial mode of the photon entering the interferometer was ℓ = 0 then past the interferometer the photon was in a superposition of spatial modes with topological charges ℓ = 0...
and $\ell = 1$

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|u_1\rangle + |u_0\rangle e^{i\delta}),$$

(2)

where $\delta$ is the phase due to the path length difference of the interferometer. The photons leaving the interferometer were sent through a 40 nm band-pass filter and a large-aperture fiber collimator coupled to a multimode fiber. The latter channeled the light to an APD detector. The state of the photon with the spatial mode of equation (2) and with a bandwidth of 10 nm was obtained by coincident detection of the imaged photon and the heralding photon.

3. Results

The smallest images in the top-left part of figure 1 are simulations of the helical mode of the light: one is the computed spatial probability density of detecting photons at points in a transverse plane, a second frame is a color-coded mapping of the phase of the wave-function at each point in the transverse plane, and the third frame is a model of the helical wavefront. The transverse probability density has the characteristic ‘doughnut’ profile of a Laguerre–Gauss beam with a minimum at its center, which is where the optical phase vortex is located. The latter is easily identified in the phase map and helical-wavefront axis of the mode of figure 1.

The larger image adjacent to the probability and phase simulations is the measured discretized single-photon image of the probability density. The image was taken by recording the coincident-photon counts obtained by scanning a 1 mm diameter aperture in a two-dimensional (2D) plane. The aperture was moved by stepper-motor-driven translation stages in a $10 \times 10$ grid (0.5 mm step per grid point); in effect, it is a picture taken one pixel at a time. The aperture was located 162 cm (56 cm) from the BBO crystal (interferometer). The image was taken while blocking the arm of the interferometer that had the glass blank. The intensity at the center of the image is not zero as in the simulation because it represents the integral of the probability density about the 1 mm diameter aperture, and thus is limited by the image resolution. A model Laguerre–Gauss $\ell = 1$ mode [22] convoluted with the experimental resolution was fit to the data. Figure 2(a) shows three sections of the measured profile (symbols) and the corresponding sections of the fit (solid line). Fitting parameters were the center of the profile ($4.4 \pm 0.2$ mm, $4.2 \pm 0.3$ mm), the width ($2.1 \pm 0.2$ mm), the amplitude and a constant background. The fits are very good, especially the visibility of the dark central region, as seen in the section for $y = 4$ mm in the figure, which passes near the center of the profile.

To demonstrate that single photons also carry all the phase characteristics of the helical mode, we needed it to interfere with itself in the fundamental ($\ell = 0$) mode. Simulations of the theoretical probability density, phase map and wavefront for this mode and the measured single-photon discretized probability are shown in the images in the bottom-left side of figure 1. The measured image, taken the same way as for the helical mode but with the other interferometer arm blocked, is consistent with the expectation. Fits of a Laguerre–Gauss $\ell = 0$ mode convoluted with the experimental resolution are very good; figure 2(b) shows a few sections of the profile with the corresponding sections of the fit. The fitted parameters were consistent with those of figure 2(a): center ($4.6 \pm 0.3$ mm, $4.5 \pm 0.3$ mm) and width ($2.7 \pm 0.4$ mm). The fitted width was slightly larger than that of the $\ell = 1$ mode. However, the excellent agreement between the fits and the data gives convincing evidence that the apparatus detected a mode input to the interferometer that was predominantly $\ell = 0$. 

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Figure 2. Sections of the measured profiles (symbols) of photons leaving the interferometer in the $\ell = 1$ (a) and $\ell = 0$ (b) modes along with the corresponding fits with Laguerre–Gauss mode-functions convoluted with the experimental resolution.

When both arms were unblocked the expected probability density of finding a photon in a transverse plane has a well-known asymmetric amplitude pattern carrying an off-center phase singularity, as shown in the simulations in the right side of figure 1. The larger frame on the bottom-right side of the figure shows an experimental single-photon image of the interference that indeed confirms the approximate intensity pattern that is expected. This image was taken at twice the resolution of the component images shown.

The phase encoded in the mode can be observed by changing the interferometer phase $\delta$, which preserves the shape of the probability distribution but makes the pattern rotate about its center, completing one full turn for a phase change of $360^\circ$. Figure 3 shows the measured patterns when $\delta$ was increased sequentially by $90^\circ$. We changed $\delta$ by tilting the glass blank in one of the arms of the interferometer. We attenuated the light from the zero-order arm by about 40% to better appreciate the pattern. Fits to the patterns of figure 3 with a superposition of $\ell = 1$ and $\ell = 0$ Laguerre–Gauss mode-functions gave excellent agreement, as expected from a visual inspection of the figure.

An additional quantitative comparison between the theory and experiment can be obtained by passing the light through a rotatable angular mask. In a second series of experiments the light leaving the interferometer went through a mask that consisted of a wedge-shaped angular aperture with an angular opening $\Delta \phi = 19.5 \pm 0.5^\circ$, and with its apex located at the axis of the light beam. This is shown schematically in figure 1. The probability of a photon passing through the wedge oriented at an angle $\phi$ can be obtained by integrating the spatial probability. For pure $p = 0$ Laguerre–Gauss modes [22] it is given by

$$P = \frac{\Delta \phi}{2\pi} \left[ 1 - \sqrt{\frac{\pi}{\Delta \phi}} \sin \left( \frac{\Delta \phi}{2} \right) \cos \left( \phi + \delta + \frac{\Delta \phi}{2} \right) \right].$$

(3)

It is interesting to note that it is a harmonic function of $(\phi + \delta)$. Its visibility is not zero and approaches $\sqrt{\pi}/2 = 0.886$ in the limit of zero-wedge angle. This is because the probability is zero only at the singularity. Past the mask the light was sent to a non-polarizing beam splitter.
Figure 3. Contour maps of single-photon images of the interference of $\ell = 0$ and $\ell = 1$ spatial modes. They were taken by scanning a 1 mm-diameter aperture in a 2D plane in increments of 0.5 mm (spacing of tic marks in images is 1 mm). The color scale is in units of coincidences taken in 10 s intervals. The sequence of frames correspond to phase increments of $90^\circ$.

Figure 4. Scans of the transmission through the wedge mask taken by varying the voltage on the PZT that pushed one of the interferometer mirrors. Midway through the scans the mask was rotated by $90^\circ$ (graph (a)) and $180^\circ$ (graph (b)). Inserts show the orientation of the mask before and after the change. Solid curves are fits to the data. The horizontal scale is the fitted phase without the phase jumps.

The outputs of the beam splitter were sent to APD detectors via multimode optical fibers ($B$ and $C$). Large-aperture fiber collimators were preceded by 40 nm band-pass filters.

For this set of experiments we changed the interferometer phase by pushing one of the mirrors of the interferometer with a piezo-electric transducer (PZT). The PZT was placed as a spacer in a translation stage that mounted one of the interferometer mirrors. Figure 4 shows the data for transmission of heralded photons through the wedged mask and detected by detector $B$ by varying the voltage on the PZT, $V_p$. Because the phase varied slightly nonlinearly with $V_p$ we graph the data as a function of the calibrated interferometer phase instead of $V_p$. At three points through the scan, the data acquisition was paused and the mask was rotated by $90^\circ$, as shown schematically in the inserts to figure 4. This rotation of the mask (i.e. a change in $\phi$) effectively inflicted a phase shift in the transmitted pattern, as evident by the relation of...
equation (3). The solid lines in the graphs of figure 4 are separate least-square fits of the function $a[1 + v \cos(\alpha + \beta V_p + \gamma V_p^2)]$, where the fitting parameters were: half the overall maximum count $a$; the visibility of the pattern $v$; the offset phase $\alpha$; the linear variation of phase with voltage $\beta$; and a parameter, $\gamma$, that accounts for the nonlinearity of the PZT. We determined the phase jump by calculating the difference in phase of adjacent fits at the value of $V_p$ where the mask was rotated. In the case of figure 4 the fitted phase jumps were $133^\circ \pm 24^\circ$, $104^\circ \pm 23^\circ$ and $57^\circ \pm 28^\circ$. In general, these measurements are consistent with expectation: $90^\circ$. We took numerous other scans like that of figure 4. The variations between the expected and measured phases are typical, with the differences never exceeding $45^\circ$ in either direction. These variations may be due to drifts in the phase due to PZT creep while the scan was paused, or departures of the actual modal pattern from the modeled one. The fitted visibilities of the scans in the four sections were $0.78 \pm 0.13$, $0.89 \pm 0.12$, $0.87 \pm 0.12$ and $0.75 \pm 0.16$, consistent with expectation.

The graphs in figure 5 show the data taken in a mode where the mask was rotated about its center while keeping the interferometer phase fixed. These data were taken with the down-converted photons leaving the BBO crystal collinearly. A beam splitter placed after the crystal but before the interferometer split the photons half the time, and thereafter taking roles similar to the photons in the non-collinear case.

The interferometer phase was set by adjustment of the tilt of the blank in the interferometer. The data represented by the squares (blue), diamonds (red) and triangles (green) in figure 5 were taken for phases $\delta_i - \delta_0$, where $i = 0, 90^\circ, 180^\circ$, respectively. We calibrated the phase change by allowing the 442 nm pump beam to go through the interferometer, imaging the resulting interference pattern (a superposition of $\ell = 0$ and $\ell = 2$ modes) with a digital camera, and then adjusting the phase by appropriate rotations the pattern. We estimate the error in this determination to be about $10^\circ$. The scans were taken consecutively via a stepper-motor-driven rotation of the mask. For each case we took scans for three full turns of the mask in

![Figure 5. Data corresponding to the transmission of the light through the wedge mask as a function of the wedge orientation angle for different interferometer phases. The data described by diamonds (red) and triangles (green) were taken after advancing the interferometer phase from that of squares (blue) by 90° and 180°, respectively. Solid curves were fits to the data.](http://www.njp.org/)
Figure 6. Simulations of the intensity (top row) and color-coded phase of the interference of modes $\ell$ and $\ell + 1$ for different values of $\ell$.

increments of 15°. The data shown represent the average of the data modulo one turn. Fits to the graphs gave $\delta_{90} - \delta_0 = 112 \pm 16^\circ$ and $\delta_{180} - \delta_0 = 171 \pm 13^\circ$, respectively, consistent with the expected values.

We also tested the quantum nature of the light source by performing a measurement of the second-order correlation coefficient $g_2(0)$ [23]. We did this using the non-collinear setup shown in figure 1 by detecting the heralded photons reaching $B$ and $C$, and recording the double coincidences ($N_{AB}$ and $N_{AC}$) and triple coincidences ($N_{ABC}$). We then calculated $g_2(0) = N_{ABC}N_{A}/N_{AB}N_{AC}$ at a fixed interferometer phase and mask position that yielded a maximum in $N_{AB}$ and $N_{AC}$. The result was $g_2(0) = 0.35 \pm 0.08$. A value $g_2(0) < 1$ means that the light source is anti-bunched, and as a consequence, non-classical. A source with classical, wave-like properties would give $g_2(0) \geq 1$.

4. Discussion and conclusions

Returning to discussion of the spatial mode of the photon entering the interferometer, if we assume that the mode has a topological charge $\ell$ then the interferometer produces a superposition of $\ell$ and $\ell + 1$ modes. Figure 6 shows simulations of the mode of the light leaving the interferometer for different input modes $\ell$. It is interesting to observe the similarities and differences. Among the similarities is the fact that in all cases the amplitude is a single angular structure centered about the same angle. Simulations of the patterns for other phase differences between the $\ell$ and $\ell + 1$ modes show that they all rotate at the same rate and in the same direction. Furthermore, the integration of the amplitude over an angular mask for the different cases gives a harmonic dependence with the angular coordinate, similar in form to equation (3). Thus, it is not possible to unambiguously infer the value of $\ell$ from the interference pattern, a result that is not intuitively obvious. The differences between these cases can be appreciated in the phase maps of figure 6, which show distinct patterns of vortices of differing charge and helicity: cases $\ell = 0$ and $\ell = -1$ have both one off-axis vortex with a topological charge of one at the same position but with opposite helicity; cases $\ell = +1$ and $\ell = -2$ have two charge-one vortices, one central and another off-axis (peripheral [24]) also with opposite helicities; and case $\ell = 2$ has a central vortex with a topological charge of two and another peripheral vortex with a topological charge of one. Single-photon superpositions of spatial modes such as these ones may warrant a study in their own right.

In summary, the presented measurements constitute the first direct measurements of the helical mode of single photons. In these measurements, the light was put in a superposition
of two fundamental modes: with topological charges $\ell = 1$ and $\ell = 0$. Since images are superposition of modes we made a basic demonstration that a single photon carries image information. Verifying a helical mode implicitly confirms the OAM of single photons. The analysis of the last section shows that even in the case where the input mode is a superposition of modes, the interference is of modes with at least one mode being helical, and with the resulting imaged mode having at least one optical vortex and a complex helical wavefront. This demonstration implies that a single photon should exhibit geometric phases due to mode transformations [25], as is the case with polarization transformations [26]. These results also support visualizing the photon as extending in the transverse direction via the spatial mode and longitudinally via the coherence length.

It is interesting to inquire if there are optical modes that are strictly non-classical (i.e. not mimicked by an experiment with classical light). These would undoubtedly be contained in entangled states of modes. Recent imaging studies [15, 16] have revealed modes containing singularities in the post-selected states of SPDC-generated many-state entanglement. Engineering of quantum-modal states may allow the generation of entangled states of desired mode superpositions [27], which may carry an image in their correlation, visible only when both photons are detected, but invisible by detection of the individual photons. Thus, the quantum states of spatial modes provide an interesting future for encoding non-classical image information.

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