Beam normal spin asymmetry in the quasi-RCS approximation

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The two-photon exchange contribution to the single spin asymmetries with the spin orientation normal to the reaction plane is discussed for elastic electron-proton scattering in the equivalent photon approximation. In this case, hadronic part of the two-photon exchange amplitude describes real Compton scattering (RCS). We show that in the case of the beam normal spin asymmetry, this approximation selects only the photon helicity flip amplitudes of RCS. At low energies, we make use of unitarity and estimate the contribution of the πN multipoles to the photon helicity flip amplitudes. In the Regge regime, QRCS approximation allows for a contribution from two pion exchange, and we provide an estimate of such contributions. We furthermore discuss the possibility of the quark and gluon GPD’s contributions in the QRCS kinematics.

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I. INTRODUCTION

Recently, the new polarization transfer data for the electromagnetic form factors ratio $G_E/G_M$ [1] raised a lot of interest for the two photon exchange (TPE) physics in elastic electron proton scattering. These new data appeared to be incompatible with the Rosenbluth data [2]. A possible way to reconcile the two data sets was proposed [3], which consists in a more precise account on the TPE amplitude, the real part of which enters the radiative corrections to the cross section (Rosenbluth) and the polarization cross section ratio in a different manner. At present, only the IR divergent part of the two photon exchange contribution, corresponding to one of the exchanged photon being soft, is taken into the experimental analysis [4]. Two model calculations exist for the real part of the TPE amplitude [5, 6], and they qualitatively confirm that the exchange of two hard photons may be responsible for this discrepancy. In order to extract the electric form factor in the model independent way, one has to study the general case of Compton scattering with two spacelike photons. These two photon contributions are important for the electroweak sector, as well.

In view of this interest, parity-conserving single spin asymmetries in elastic $ep$-scattering with the spin orientation normal to the reaction plane regain an attention. These observables are directly related to the imaginary part of the TPE amplitude and have been an object of theoretical studies in the 1960’s and 70’s [5]. By analyticity, the real part of the TPE amplitude is given by a dispersion integral over its imaginary part. Therefore, a good understanding of this class of observables is absolutely necessary. Recently, first measurements of the beam normal spin asymmetry $B_n$ [1] have been performed in different kinematics [5].

Though small (several to tens ppm), this asymmetry can be measured with a precision of fractions of ppm. Before implementing different models for the real part, where an additional uncertainty comes from the dispersion integral over the imaginary part, one should check the level of understanding of the imaginary part of TPE. These checks have been done for the existing data. Inclusion of the elastic (nucleon) intermediate state only [5] led to negative asymmetry of several ppm in the kinematics of SAMPLE experiment but was not enough to describe the data. The description of the beam normal spin asymmetry within a phenomenological model which uses the full set of the single pion electroproduction [10] did not give satisfactory description at any of the available kinematics. Especially intriguing appears the situation with the SAMPLE data with electron lab energy $E_{lab} = 200$ MeV, which is just above the pion production threshold where the theoretical input is well understood. On the other hand, an EFT calculation without dynamical pions [11] was somewhat surprisingly very successful in describing this kinematics for $B_n$. This success suggests that to the given order of chiral perturbation theory, the role of the dynamical pions for this observable might be not too large. Finally, a recent calculation within the leading logarithm approximation appeared, where only few dominant multipoles were used as input [12]. Surprisingly, the authors of [12] were able to describe all the experimental data at very different beam energies and scattering angles, while the full calculation of [10] failed to describe any of them.

Though even at low energies the situation with the imaginary part of TPE amplitude is by far not clear, an attention has to be paid to high energies, as well, since the dispersion integral which would give us its real part, should be performed over the full energy range. Due to relative ease of measuring $B_n$ within the framework of parity violating electron scattering, new data from running and upcoming experiments [13] will stimulate further theoretical investigations of this new observable. A calculation of $B_n$ in hard kinematics regime

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$^1$In the literature, also the $A_n$ notation for beam normal spin asymmetry or vector analyzing power was adopted.
at high energy and momentum transfer was performed recently in the framework of generalized parton distributions (GPD’s) and resulted in asymmetries of \( \sim 1.5 \) ppm.\footnote{In elastic \( ep \)-scattering, the usual notation for the momentum transfer is \( Q^2 \neq -q^2 \) but we prefer the more general notation \( t \) to avoid confusion with the incoming and outgoing photon virtualities in forward doubly virtual Compton scattering we will be concerned in the following.}

Since a ppm effect measurement at high momentum transfers is an extremely complicated task, the forward kinematics seems more favorable. For this kinematics, a calculation exists\footnote{\[15\]}, where an observation is made that the contribution of the situation where the exchanged photons are nearly real and overtake the external electron kinematics is enhanced as \( \log^2(-t/m^2) \sim 100 \), with \( m \) the electron mass and \( t < 0 \) the elastic momentum transfer. Making use of the optical theorem, the authors obtained estimates of \( B_n \) in this kinematics as large as 25-35 ppm. The calculation of \[16\] appeared afterwards showed that this result is not adequate, since the squared log term can contribute with the Compton scattering amplitudes originating from the QRCS kinematics. We provide the calculation of \[16\], where an observation is made that the contributions which might be enhanced by the large log-term but conclude that these are negligibly small. We therefore flip amplitudes. We provide the calculation of \[16\], where an observation is made that the contributions which might be enhanced by the large log-term but conclude that these are negligibly small. We therefore...

In this work, we consider elastic electron-proton scattering process \( e(k) + p(p) \rightarrow e(k') + p(p') \) for which we define:

\[
P = \frac{p + p'}{2}
\]
\[
K = \frac{k + k'}{2}
\]
\[
q = k - k' = p' - p,
\]
and choose the invariants \( t = q^2 \), \( u = (p + k)^2 \), \( u = (p - k)^2 \), and \( s = u = 4Mv \) through \( s + u + t = 2M^2 \). For convenience, we also introduce the usual polarization parameter \( \varepsilon \) of the virtual photon, which can be related to the invariants \( \nu \) and \( t \) (begrudging the electron mass \( m \)):

\[
\varepsilon = \frac{\nu^2 - M^2\tau(1 + \tau)}{\nu^2 + M^2\tau(1 + \tau)},
\]
with \( \tau = -t/(4M^2) \). Elastic scattering of two spin 1/2 particles is described by six independent amplitudes. Three of them do not flip the electron helicity \( p \):

\[
T_{\text{no flip}} = \frac{e^2}{-t} \bar{u}(k')\gamma_\mu u(k)
\]
\[
\cdot \bar{u}(p') \left( \hat{G}_M\gamma_\mu - \hat{F}_2 \frac{P_\mu}{M} + \hat{F}_3 \frac{K P_\mu}{M^2} \right) u(p),
\]
while the other three are electron helicity flipping and thus have in general the order of the electron mass \( M \):

\[
T_{\text{flip}} = \frac{m e^2}{M - t} \left[ \bar{u}(k') u(k) \cdot \bar{u}(p') \left( \hat{F}_4 + \hat{F}_5 \frac{K}{M^2} \right) u(p) \right.
\]
\[
+ \hat{F}_6 \bar{u}(k') \gamma_\gamma u(k) \cdot \bar{u}(p') \gamma_\gamma u(p) \right]
\]

In the one-photon exchange (Born) approximation, two of the six amplitudes match with the electromagnetic form factors,

\[
\hat{G}^{\text{Born}}_M(\nu, t) = G_M(t),
\]
\[
\hat{F}^{\text{Born}}_2(\nu, t) = F_2(t),
\]
\[
\hat{F}^{\text{Born}}_{3,4,5,6}(\nu, t) = 0
\]

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\]
\[
\hat{F}^{\text{Born}}_{3,4,5,6}(\nu, t) = 0
\]
where $G_M(t)$ and $F_2(t)$ are the magnetic and Pauli form factors, respectively. For further convenience we define also $\tilde{G}_E = G_M - (1 + \tau) F_2$ and $\tilde{F}_1 = G_M - F_2$ which in the Born approximation reduce to electric form factor $G_E$ and Dirac form factor $F_1$, respectively. For a beam polarized normal to the scattering plane, one can define a single spin asymmetry,

$$B_n = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}, \quad (6)$$

where $\sigma_+$ ($\sigma_-$) denotes the cross section for an unpolarized target and for an electron beam spin parallel (antiparallel) to the normal polarization vector defined as

$$S_{2\mu}^{\nu} = \frac{1}{2} \left( \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|} \right), \quad (7)$$

normalized to $(S \cdot S) = -1$. Similarly, one defines the target normal spin asymmetry $A_n$. It has been shown in the early 1970’s [10] that such asymmetries are directly related to the imaginary part of the $T$-matrix. Since the electromagnetic form factors and the one-photon exchange amplitude are purely real, $B_n$ obtains its finite contribution to leading order in the electromagnetic constant $\alpha_em$ from an interference between the Born amplitude and the imaginary part of the two-photon exchange amplitude. In terms of the amplitudes of Eqs.(3,4), the beam normal spin asymmetry is given by:

$$B_n = -\frac{m}{M}\sqrt{2\varepsilon(1 - \varepsilon)\sqrt{1 + \tau}} \left( \tau G_M^2 + \varepsilon G_E^2 \right)^{-1} \cdot \left\{ \tau G_M \text{Im} \tilde{F}_3 + G_E \text{Im} \tilde{F}_4 + F_1 \frac{\nu}{M} \text{Im} \tilde{F}_3 \right\}. \quad (8)$$

For completeness, we also give here the expression of target normal spin asymmetry $T_n$ in terms of invariant amplitudes:

$$T_n = \sqrt{2\varepsilon(1 + \varepsilon)\sqrt{\tau}} \left( \tau G_M^2 + \varepsilon G_E^2 \right)^{-1} \cdot \left\{ (1 + \tau) \left[ F_1 \text{Im} \tilde{F}_2 - F_2 \text{Im} \tilde{F}_1 \right] + \frac{2\varepsilon}{1 + \varepsilon} G_E - G_M \right\} \frac{\nu}{M} \text{Im} \tilde{F}_3 \right\}. \quad (9)$$

III. TWO PHOTON EXCHANGE

\[ \begin{array}{c}
  \text{k} \quad \text{k} \quad \text{k}' \\
  \text{q}_1 \quad \text{q}_2 \quad \text{p} \quad \text{p}' \\
  \text{A}_1
\end{array} \]

\[ \text{FIG. 1: Two-photon exchange diagram.} \]

The imaginary part of the two-photon exchange (TPE) graph in Fig.1 is given by

$$\text{Im}M_{2\gamma} = e^2 \int \frac{|\vec{k}_1|^2 d|\vec{k}_1| d|\vec{k}_2|}{2E_1(2\pi)^4} \bar{u}^{\nu}(\bar{k}_1 + m)\gamma_{\nu}(\bar{k}_1 + m)u \left[ 1 - \frac{1}{Q_1^2 Q_2^2} W^{\mu\nu}(u^2, Q_1^2, Q_2^2) \right], \quad (10)$$

where $W^{\mu\nu}(u^2, Q_1^2, Q_2^2)$ is the imaginary part of doubly virtual Compton scattering tensor. $Q_1^2$ and $Q_2^2$ denote the virtualities of the exchanged photons in the TPE diagram, and $w$ is the invariant mass of the intermediate hadronic system. We next study the kinematics of the exchanged photons. Neglecting the small electron mass and using the c.m. frame of the electron and proton, one has:

$$Q_{1,2}^2 = 2|\vec{k}|^2 (1 - \cos \Theta_{1,2}), \quad (11)$$

with $|\vec{k}| = \frac{\sqrt{s} - M^2}{2\sqrt{s}} = k$ the three momentum of the incoming (and outgoing) electron, $|\vec{k}_1| = \sqrt{(\frac{\sqrt{s} - u^2 + m^2}{2\sqrt{s}})^2 - m^2}$ that of the intermediate electron, and $\cos \Theta_2 = \cos \Theta \cos \Theta_1 + \sin \Theta \sin \Theta_1 \cos \phi$. The kinematically allowed values of the virtualities of the exchanged photons (the restriction is due to the fact that the intermediate electron is on-shell) are represented by the internal area of the ellipses shown in Fig. 2. The ellipses are drawn inside a square whose side is defined through the external kinematics ($k$) and the invariant mass of the intermediate hadronic state ($u^2$ or $k_1$), while the form solely by the scattering angle. If choosing higher values of the mass of the hadronic system $u^2 < s$, it leads to scaling the size of the ellipse by a factor of $\frac{\sqrt{s} - u^2}{\sqrt{s} - m^2}$. In the limit...
\( w^2 = (\sqrt{s} - m_e)^2 \), the ellipses shrink to a point at the origin and both photons are nearly real. This is not a soft photon (IR) singularity, however, since the real photons’ energy remains large enough in order to provide the transition from nucleon with the mass \( M \) to the intermediate state \( X \) with the mass \( w \). Instead, the intermediate electron is soft, \( k_l^2 \approx (n_e, 0) \), therefore this kind of kinematics does not lead to an IR divergency which can only occur if the intermediate hadronic state is the nucleon itself. In the following we are going to study this kinematical situation in more detail.

IV. QUASI-RCS APPROXIMATION

We consider the kinematical factors under the integral over the electron phase space of Eq. (10) at the upper limit of the integration of the invariant mass of the intermediate hadronic state, \( w \rightarrow \sqrt{s} - m_e \):

\[
\frac{k_l^2}{E_1 Q_1^2 Q_2^2} \sim \frac{1}{4k^2 E_1} \sim \frac{1}{m^2 k^2}.
\]  

(12)

In this range of the hadronic mass \( w \), the exchanged photons are real, and the contribution of real Compton scattering (RCS) to the \( 2\gamma \)-exchange graph is enhanced by a factor of \( 1/m \) (it is not a singularity since \( B_n \) has a factor of \( m \) in front).

We next rewrite the contraction of the hadronic and the leptonic tensors in Eq. (10) identically as

\[
l_{\mu\nu} W_{\mu\nu}(W^2, Q_1^2, Q_2^2) = l_{\mu\nu}^{(0)} W_{\mu\nu}(s, 0, 0) + l_{\mu\nu}^{(1)} W_{\mu\nu}(s, 0, 0) + l_{\mu\nu} [W_{\mu\nu}(W^2, Q_1^2, Q_2^2) - W_{\mu\nu}(s, 0, 0)],
\]  

(13)

where we expanded the leptonic tensor \( l_{\mu\nu} = l_{\mu\nu}^{(0)} + l_{\mu\nu}^{(1)} \) with

\[
l_{\mu\nu}^{(0)} = m\bar{u}\gamma_\mu\gamma_\nu u,
\]

\[
l_{\mu\nu}^{(1)} = \bar{u}\gamma_\mu k_1 \gamma_\nu u.
\]  

(14)

In Eq. (13), only the first term does not vanish in the RCS limit, while the second and third are at least linear in \( k_1 \). In the limit \( W^2 \rightarrow s \), one has \( k_1 \rightarrow m\gamma_0 \), which is to be compared to the term \( m \cdot 1 \) in \( l_{\mu\nu}^{(0)} \). Though on the first glance they are of the same order, the additional \( \gamma \)-matrix picks an extra electron mass \( m \) from one of the projectors \( \gamma^\pm m \) when performing the summation over electron spins. Quasi-real Compton scattering (Q RCS) approximation consists in assuming the first term to be dominant due to the kinematical enhancement under the integral and in neglecting the second one. In general, this kind of contributions coming from RCS kinematics will always be present in the full calculation, since the second term in Eq. (13) is constructed in such a way that the resulting integral is regular at the RCS point. In the following, the RCS approximation will be used. Hence, the hadronic tensor can be taken out of the integration over the electron phase space. The remaining integral is

\[
I_0 = \int \frac{d^3 E_1}{2E_1(2\pi)^3} \frac{1}{Q_1^2 Q_2^2},
\]  

(15)

The result of the integration reads:

\[
I_0 = \frac{1}{-32\pi^2 t} \left\{ \ln^2 \left( \frac{-t E^2}{m^2 E^2} \right) + 8Sp \left( \frac{E_{thr}}{E} \right) \right\},
\]  

(16)

where \( Sp(x) \) is the Spence or dilog function, \( Sp(x) = -\int_0^1 \frac{1}{t} \ln(1-xt) \text{d}t \). With \( Sp(1) = \pi^2/6 \). In the high energy limit, \( E_{thr} \rightarrow 1 \), we recover the result of Ref. [15]. For the details, we address the reader to the Appendix. This \( \ln^2 \left( \frac{-t}{m^2} \right) \) factor serves as the justification of the QRCS approximation. The full result may be represented as a “Taylor expansion” in powers of \( \ln \left( \frac{-t}{m^2} \right) \sim 10 \). By studying the Q RCS approximation we show in a model independent way that the leading term in this expansion is quadratic. Since this leading term is large, we can (under certain kinematical conditions) neglect further terms.

The RCS tensor may be taken for instance in the basis of Prange [18], or, equivalently of Berg and Lindner [19],

\[
W_{\mu\nu}^{\text{RCS}} = \tilde{N}_t \left\{ \frac{P_\mu P_\nu}{P^2 (B_1 + \mathbf{k}_B)} + \frac{n_\mu n_\nu}{n^2 (B_3 + \mathbf{k}_B)} \right\},
\]  

(17)

where the vectors defined as \( P_\mu = P - \frac{K}{K} K, n_\mu = \varepsilon^{\mu\alpha\beta} P_\alpha K_\beta \) such that \( P' \cdot K = P' \cdot n = n \cdot K = 0 \). The amplitudes \( B_i \) are functions of \( n \) and \( t \). This form of Compton tensor is convenient due to the simple form of the tensors appearing in Eq. (17).

Before contracting the leptonic and hadronic tensors, we notice that the amplitude \( B_T \) can only contribute to the invariant amplitude \( \bar{F}_6 \), since it contains the \( \tilde{N}_t \gamma_5 N \) structure. \( \bar{F}_6 \) does not contribute at leading order in \( m \) to neither observable of interest, therefore \( B_T \) will be neglected in the following. The remaining tensors in Eq. (17) are symmetric in indices \( \mu \nu \).

\[
\text{Im} M^{Q\text{RCS}}_{\gamma_2} = e^2 m I_0 \bar{u} u W^{\mu\nu}_{\text{RCS}} g_{\mu\nu}.
\]  

(18)

Finally, we identify different terms in Eq. (18) with the structures, together with amplitudes \( \bar{F}_{1,3,6} \) parametrizing elastic ep-scattering amplitude in Eqs. (18) and find for the invariant amplitudes for the elastic electron-proton scattering in the Q RCS approximation:

\[
\text{Im} G^{Q\text{RCS}}_M = \text{Im} \bar{F}_2^{Q\text{RCS}} = \text{Im} \bar{F}_3^{Q\text{RCS}} = 0
\]  

(19)

\[
\text{Im} F_{4}^{Q\text{RCS}} = -M I_0 \text{Im}(B_1 + B_3)
\]  

(20)

\[
\text{Im} F_{5}^{Q\text{RCS}} = -M^2 I_0 \text{Im}(B_2 + B_4)
\]  

(21)
We obtain for $B_n$ in the QRCS approximation:

$$B_n = m t I_0 \sqrt{2 \pi (1 - \varepsilon) \sqrt{1 + \tau}} \left( \tau G^2_{3T} + \varepsilon G^2_{E} \right)^{-1} \cdot \{G E \text{Im}(B_1 + B_3) + \nu F_1 \text{Im}(B_2 + B_4) \}.$$  

(22)

The result of Eq. (22) is obtained by only using the assumption that the QRCS kinematics dominate the integral in Eq. (16). The combinations of the RCS amplitudes appearing in the final result can be further expressed in terms of the helicity amplitudes of real Compton scattering. With these latter defined as $T_{ij} = \varepsilon_{ij}^{\mu \nu} \varepsilon_{ij}^{\rho \sigma} W_{\mu \nu}^{RCS}$, one has [21,22]:

$$B_1 + B_3 = -\frac{1}{\sqrt{-t}} \left( T_{-1 - \frac{1}{2}, 1 \frac{1}{2}} + T_{+1 - \frac{1}{2}, -1 \frac{1}{2}} \right)$$

$$B_2 + B_4 = \frac{2M}{s-M^2} \sqrt{-t} \left( T_{-1 - \frac{1}{2}, 1 \frac{1}{2}} + T_{+1 - \frac{1}{2}, -1 \frac{1}{2}} \right) + \frac{2}{s-su-s-M^2} T_{1 \frac{1}{2}, -1 \frac{1}{2}} \tag{23}$$

As can be seen, only photon helicity-flipping amplitudes enter the final result for $B_n$ in the QRCS approximation. This is the main result of this work.

$$\text{V. RESULTS}$$

In this section, we consider the impact of the QRCS contributions onto the beam normal spin asymmetry in different kinematics: low energies ($\pi N$ intermediate states), high energies and forward angles, i.e. Regge regime ($2\pi$ exchange in the $t$-channel) and hard regime (handbag diagrams and two gluon exchange).

$$\text{A. Low energies: }\pi N\text{ multipoles}$$

Above the pion production threshold, the imaginary part of the RCS helicity amplitudes can be related to the pion photo- and electro-production multipoles. These relations read [20,22] for the three amplitudes entering

$$B_n:$$

$$\text{Im}T_{-1,1 \frac{1}{2}} = \sin \frac{\Theta}{2} 16\pi q_\pi \sqrt{s} \sum_{k \geq 0} (k + 1)^2$$

$$\times \left[ |A_{k+}|- |A_{(k+)-}| \right]$$

$$\times F(-k, k + 2, 2, \sin^2 \Theta / 2)$$

$$\text{Im}T_{-1 - \frac{1}{2}, 1 \frac{1}{2}} = -\sin^3 \frac{\Theta}{2} 8 \pi q_\pi \sqrt{s} \sum_{k \geq 1} k^2 (k + 2)^2$$

$$\times \left[ |B_{k+}|- |B_{(k+)-}| \right]$$

$$\times F(-k + 1, k + 3, 4, \sin^2 \Theta / 2)$$

$$\text{Im}T_{+1,1 \frac{1}{2}} = \sin^2 \Theta \cos \frac{\Theta}{2} 8 \pi q_\pi \sqrt{s} \sum_{k \geq 1} k^2 (k + 2)^2$$

$$\times \Re \left[ B_{k+} A^*_k - B_{(k+)-} A^*_{(k+)-} \right]$$

$$\times F(-k + 1, k + 3, 3, \sin^2 \Theta / 2), \tag{27}$$

where $q_\pi$ is the c.m. pion three-momentum and the hypergeometric function is defined as

$$F(a, b, c, x) = 1 + \frac{a x}{c 1!} + \frac{a(a+1) b(b+1) x^2}{c(c+1) 2!} + \ldots \tag{28}$$

We keep only few first multipoles in this infinite series, namely $A_{0+}, A_{1+}, A_{1-}, A_{2-}, B_{2-}$ which obtain their leading contributions from threshold pion production, $\Delta(1232)$ and $D_{13}(1520)$ resonances. The result for $B_n$ reads:

$$B_n = -8\pi m q_\pi \frac{st^2}{(s-M^2)^2} I_0 \sqrt{2\pi (1-\varepsilon) \sqrt{1+\tau}} \left( \tau G^2_{3T} + \varepsilon G^2_{E} \right)^{-1} \cdot \left\{ \left( \frac{E_{lab}}{M} F_2 - F_1 \right) \right.$$

$$\times \left[ |A_{0+}|^2 - |A_{1-}|^2 + 4(|A_{1+}|^2 - |A_{2-}|^2) \right]$$

$$\frac{6\sqrt{s}}{M} F_1 \text{Re} \left( B_{1+} A^*_1 - B_{2-} A^*_2 \right)$$

$$-\frac{3}{2} \sin^3 \frac{\Theta}{2} \left[ \left( \frac{E_{lab}}{M} F_2 - F_1 \right) \right.$$

$$\times \left[ 4(|A_{1+}|^2 - |A_{2-}|^2) + |B_{1+}|^2 - |B_{2-}|^2 \right]$$

$$+2 \left( \frac{k}{M} F_2 - \frac{E}{M} F_1 \right)$$

$$\times \text{Re} \left( B_{1+} A^*_1 - B_{2-} A^*_2 \right) \right\}. \tag{29}$$

The helicity multipoles are related to the electromagnetic multipoles $E_{i+,i+1-} - M_{i+,i+1-}$ as

$$E_{0+} = A_{0+}, \quad M_{1-} = A_{1-}, \quad \tag{30}$$
and for \( l \geq 1 \)

\[
E_{l+} = \frac{1}{l+1} \left[ A_{l+} + \frac{l}{2} B_{l+} \right]
\]

\[
M_{l+} = \frac{1}{l+1} \left[ A_{l+} - \frac{l+2}{2} B_{l+} \right]
\]

\[
E_{(l+1)-} = -\frac{1}{l+1} \left[ A_{(l+1)-} - \frac{l+2}{2} B_{(l+1)-} \right]
\]

\[
M_{(l+1)-} = -\frac{1}{l+1} \left[ A_{(l+1)-} + \frac{l}{2} B_{(l+1)-} \right]
\]

(31)

It is informative to consider the contributions of \( E_{0+} \) and \( M_{1+} \) which are dominant at low energies. Neglecting other multipoles, we get:

\[
B_n = -8\pi m_q \frac{\Theta}{s-M^2} I_0 \sqrt{2(1-\varepsilon)} \sqrt{1+\tau} \left\{ \frac{E_{lab}}{M} (F_2 - F_1) \left[ |E_{0+}|^2 + |M_{1+}|^2 \right] + \frac{3\sqrt{\tau}}{M} F_1 + 3\sin^2 \frac{\Theta}{2} \frac{\sqrt{\tau} + M}{2\sqrt{sM}} \frac{\sqrt{\tau} + M}{M} (F_2 - F_1) \right\}
\]

(32)

The first term in the brackets is negative for \( E_{lab} \lesssim M \approx 0.45 \text{ GeV} \) for the proton target, and always negative for the neutron target. Due to the overall minus sign, the beam normal spin asymmetry necessarily obtains a large negative contribution from the threshold pion production. Moving towards the \( \Delta(1232) \) resonance position, we see that the dominant contribution now comes from the second term. The factor multiplying the \( |M_{1+}|^2 \) term is always positive for the proton and negative for the neutron. Therefore one expects that the \( \Delta \) resonance give a large negative contribution to \( B_n \) on the proton target, and a positive one on the neutron target. We use the MAID2003 multipoles as input to Eq. (32) and present in Fig. 4 the energy dependence of the beam normal spin asymmetry for the proton target over the resonance region. This result can be compared to the results of Refs. [10, 12]. If confronted to the full calculation [10], we find agreement with their findings in the QRCs approximation. In Ref. [12], the authors do nominally the same approximation as we do, keeping the \( \ln^2(Q^2/m^2) \) and the RCS part of the hadronic tensor only but arrive to the same sign of the contributions of \( E_{0+} \) and \( M_{1+} \) multipoles for the proton. They are furthermore able to describe both low energy backward angle data from SAMPLE and intermediate energy and angle data from MAMI within the same approximation. It has been shown in [10] that the QRCs approximation does work well at backward angles (in the sense that it represents the dominant part of the full integration range) but drops short at forward angles, where the exchange of at least one hard virtual photon is important (for very forward angles, see [13]). Therefore, even if the QRCs approximation did work in the forward regime, one should not take this success too seriously, since the neglected double or single virtual Compton scattering effects are important. We show the angular distributions of \( B_n \) on the proton target for different values of the electron \( \text{lab} \) energy in Fig. 5. In Fig. 6 we display the energy distributions of \( B_n \) on the neutron over the resonance region.

It is necessary to note that the resonance region does not seem to be favorable for the QRCs approximation. In this approximation, the value of the hadronic amplitude is taken at the largest energy available and is not integrated over the full spectrum. Therefore, if one goes above a resonance position, it is only tail of the resonance that contribute, which causes the asymmetry to drop quite fast (see the sharp resonance behaviour at \( \Delta(1232), D_{13}(1520) \) in Fig. 5). In a full calculation, also intermediate energies of the hadronic system do contribute and the asymmetry does not have such a sharp energy dependence. So, it is preferable to use the QRCs approximation in the region where the energy dependence is rather smooth. Furthermore, the quality of the QRCs approximation is function of the scattering angle, as well. It works better at backward angles and worse at forward angles. The reason for this is clear: taking the limit of the soft intermediate electron, \( k_1 \approx 0 \) and neglecting the dependence of the hadronic and leptonic tensors on \( k_1 \) corresponds to performing Taylor expansion of the integration result in powers of \( \ln \frac{Q^2}{m^2} \) and neglecting all the terms beyond the first leading log term \( \sim \ln^2 \frac{Q^2}{m^2} \).
larger $Q^2$, the better the approximation works. This behaviour was first observed in Ref. [10]. Instead, results of Ref. [12] suggest that the loop integral in $B_n$ is completely dominated by the QRCS region in all the kinematics (backward regime for SAMPLE data and rather forward regime for MAMI data).

**B. Regge regime: 2π exchange**

The calculation of Ref. [17] estimates $B_n$ as

$$B_n \sim \sigma_{tot} \ln(-t/m^2),$$

which is based on taking the RCS amplitude in the exact forward limit. If one allows the momentum transfer to be non-zero, one however obtains the contribution from photon helicity flip amplitudes, which is the result of this work. Though it is logical to expect that these amplitudes be suppressed at low values of $t$, there is a competing factor of $\sim \ln^2 Q^2$, which can be of the order of several hundreds. In this subsection, we provide an estimate of such a contribution at forward kinematics. The combinations of the RCS amplitudes $B_1 + B_3$, $B_2 + B_4$ in Regge regime are known to have the quantum numbers of a scalar isoscalar exchange in the $t$-channel which was successfully described by $\sigma$-meson [20] or equivalently, by two pion exchange in the $t$-channel [21]. Since the effective $\sigma$-meson exchange does not result in a non-zero imaginary part in the $s$-channel, one should use the two pion exchange mechanism accompanied by multiparticle intermediate state in the $s$-channel. In this section we estimate $B_1 + B_3$ and $B_2 + B_4$ through $\pi N$ and $\rho N$ intermediate states contributions within the “minimal” Regge model for $\pi$ and $\rho$ photoproduction [25] where reggeized description of high energy pion production is obtained by adding the $t$-channel meson exchange amplitude and (in the case of $\gamma p$ reaction) $s$-channel Born amplitude which is necessary to ensure gauge invariance. The reggeization procedure naturally leads to the substitution of each $t$-channel Feynman propagator by its Regge counterpart,

$$\frac{1}{t-m^2_\pi} \rightarrow \mathcal{P}_\pi(\alpha_\pi(t)),$$

with

$$\mathcal{P}_\pi = \left( \frac{s}{s_0} \right)^{\alpha_\pi(t)} \frac{\pi \alpha'_\pi}{\sin \pi \alpha_\pi(t)} \frac{1}{\Gamma(1+\alpha_\pi(t))},$$

with $\alpha_\pi(t) = \alpha'_\pi(t-m^2_\pi)$ and $\alpha'_\pi = 0.7 \text{ GeV}^{-2}$. Gauge invariance requires the $s$-channel piece to be reggeized in the same way, i.e., to be multiplied by $(t-m^2_\rho)\mathcal{P}_\rho(\alpha_\rho(t))$. For compactness, we will use the shorthand [20],

$$A_1 \equiv \frac{1}{t} \left[ B_1 + B_2 + \nu(B_2 + B_4) \right]$$

for the combination of the $B_i$’s which enters the result for the asymmetry. Here, we list the results of the calculation.
of $A_1$:

$$\text{Im} A_1^\pi N = \frac{2m_e^2 C_\pi^2}{M(s-M^2)} (B_0^\pi - M^2 A_0^\pi),$$  

(36)

$$\text{Im} A_1^\rho N = \frac{m_\rho^2 C_\rho^2}{2M(s-M^2)} \left\{ \frac{M^2}{s-M^2} C_0^\rho 
+ \frac{s-3M^2+5m_\rho^2}{2} B_0^\rho + \frac{m_\rho^2 s + M^2}{2} A_0^\rho \right\},$$

where $C_\pi = 2\sqrt{2} Me^{\frac{N}{m_\pi}}$ with $f_{2N}/4\pi = 0.08,$ and $C_\rho = 2\sqrt{2} Me^{\frac{N}{m_\rho}}$ with $f_{\rho \pi} = 0.103$ \[28\]. Where possible, $m_\pi$ and $Q^2$ were neglected as compared to $s,$ $M,$ $m_\rho$ in order to simplify the final expression. The scalar integrals $A_0,$ $B_0,$ $C_0$ for both $\pi N$ and $\rho N$ contributions are given in the Appendix.

The result of Eq.\[36\] leads to negligibly small contribution to the $B_n$ of the order of $10^{-2}$ ppm for energies between 6 and 45 GeV, which is to be compared to $\approx 5$ ppm from Ref.\[17\]. There are three suppression factors which are responsible for this small result. Firstly, the Reggeization procedure which leads to suppression in energy $\rho_s(t).$ Secondly, the amplitude $A_1$ is defined as the combination $\frac{1}{2}[D_1 + B_3 + \nu(B_2 + B_1)],$ and the singularity $\frac{1}{2}$ in the definition only cancels if taking the gauge invariant combination as described in \[27\]. Therefore, we obtain an additional suppression factor of $t.$ The other feature of the result of Eqs.\[36,37\] is that both contributions are suppressed by factors $\frac{m_\pi^2}{s-M^2}$ and $\frac{m_\rho^2}{s-M^2},$ respectively. In the case of the pion, it is interesting to observe this fact in view of somewhat surprising success of the effective field description of the SAMPLE data point on $B_n$ without pion contribution. This might give a hint that the pion contribution to $B_n$ at low energies is suppressed by the pion mass, if calculated to the same order.

C. Hard regime: GPD’s

Recently, beam normal spin asymmetry was studied in the hard scattering regime \[14\] in the framework of the handbag mechanism. In this picture, the process factorizes into the hard part (electron scattering off an on-shell quark with the exchange of two spacelike photons) and the soft part which is parametrized by the generalized parton distributions (GPD’s). However, one cannot access the QRCS kinematics here, since it is impossible for an on-shell quark to absorb (emit) a hard real photon and remain on-shell. As it has been noticed before, the QRCS kinematics selects the RCS amplitudes with the photon helicity flip, i.e. the helicity changes by two units. In the context of the GPD’s, helicity amplitudes with the helicity changed by 2 units were considered in several works. In the collinear kinematics, this requires that scattering occurs on a parton of spin 1, that is gluon. The corresponding gluon helicity flip GPD’s were introduced in

$$\frac{1}{x} \int \frac{d\lambda}{2\pi} e^{i\lambda x} < p'S | F^{(\mu\nu} (-\frac{\lambda}{2} n) F^{\nu\beta}) (-\frac{\lambda}{2} n) | pS >$$

$$= H_{T_1}(x, \xi) \tilde{N}'(p', S') \frac{P(\mu\nu)^{(i} | q | q^{(j})}{M^2} N(p, S)$$

$$+ E_{T_1}(x, \xi) \tilde{N}'(p', S') \frac{P(\mu| q^{(i} | q | q^{(j})}{M^2} N(p, S)$$

(37)

where $H_{T_1}(x, \xi),$ $E_{T_1}(x, \xi)$ are the double helicity flip gluon GPD’s, $q = p' - p,$ and the matrix element is taken between the initial and final nucleon states with the momenta $p,$ $p',$ and spins $S,$ $S',$ respectively. The integration is performed along the light-cone vector $n^\mu = (1, 0, 0, 1).$ $x$ and $\xi$ are the longitudinal parton momentum fraction and skewness parameter, respectively. Furthermore, $[\mu\nu]$ and $[\nu\beta]$ are antisymmetric pairs of indices, while $(\ldots)$ means symmetrization and removal of the trace. These operations are essential since the product of operators must transform as irreducible representations of the Lorentz group. This requirement rules out the possibility to see these GPD’s in the QRCS kinematics, since it is exactly the trace of the RCS tensor that contributes to $B_n$ in the QRCS regime, c.f. Eq.\[15\]. So, our conclusion is that in the QRCS kinematics, contributions from generalized parton distributions are ruled out.

VI. SUMMARY

In summary, we presented a calculation of the beam normal spin asymmetry. This observable obtains its leading contribution from the imaginary part of the two photon exchange graph times the Born amplitude and is directly related to the imaginary part of doubly virtual Compton scattering. The resulting loop integral obtains a large contribution from the kinematics when both exchanged photons are nearly real and collinear to the external electrons. We adopt the QRCS or equivalent photons approximation which allows to take the hadronic part out of the integral and to perform the integration over the electron phase space analytically. For the hadronic part, we use the full real Compton scattering amplitude and show, that only photon helicity flipping amplitudes contribute in this observable in the QRCS approximation. At low energies, we relate this helicity flipping Compton amplitude to the $\pi N$ multipoles and discuss the contributions of different multipoles.

At high energies and forward scattering angles (Regge regime), we provide an explicit calculation which is due to two pion exchange in the $t$-channel. The resulting values of the asymmetry are of the order $10^{-2}$ ppm for the energies in the range 6 – 45 GeV. We conclude that the double logarithmic enhancement does not dominate $B_n$ in forward regime since it comes with helicity-flip Compton amplitude which highly suppresses this behaviour.

Finally, we consider possible contributions to $B_n$ in the hard kinematics, among which contributions from quark
and gluon GPD’s and show that such contributions are ruled out in the QRCS kinematics.

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VII. APPENDIX A: INTEGRALS OVER ELECTRON PHASE SPACE

In this section we present calculation of the integrals over electron phase space appearing in the QRCS approximation. First we calculate the scalar integral $I_0$,

$$I_0 = \int_0^{k_{thr}} \frac{k_{thr}^2 dk_1}{2E_1(2\pi)^3} \int \frac{d\Omega_{k_1}}{(k-k_1)^2(k'-k_1)^2}, \quad (38)$$

where the upper integration limit $k_{thr}$ corresponds to the inelastic threshold (i.e. pion production), $k_{thr} = \sqrt{(s-(M+m))2} - m^2$. We next introduce integration over the Feynman parameter $\frac{1}{x} = \int_0^1 \frac{dx}{[a(x)+(1-a)x]}$. We chose the polar axis such as $k_1(\vec{k}-x\vec{q}) = k_1[\vec{k}-x\vec{q}]\cos\Theta_1$ with $|\vec{k}-x\vec{q}|^2 = k^2 + x(1-x)t$ and perform angular integration. We furthermore change integration over $dk_1$ to integration over dimensionless $z = E_1/E$.

$$I_0 = \frac{1}{-8\pi^2 t} \int_0^{E_{thr}} \frac{dz}{\sqrt{z^2 - m^2}} \left[ \ln \left( \frac{z^2 - m^2}{m^2 E^2} \right) - \ln \left( \frac{z^2 - m^2}{m^2 E^2} \right) \right] \cdot \ln \left( \frac{z^2 - m^2}{m^2 E^2} \right)$$

To perform the integration over the electron energy, we follow here the main details of the calculation in Appendix A of Ref. [17]. The result reads

$$I_0 = \frac{1}{-32\pi^2 t} \left\{ \ln^2 \left( \frac{-t E_{thr}^2}{m^2 E^2} \right) + 8Sp \left( E_{thr} \right) \right\}, \quad (40)$$

where $Sp(x)$ is the Spence or dilog function, $Sp(x) = -\int_0^1 \frac{1}{1-xt} \ln(1-xt)$ with $Sp(1) = \pi^2/6$. In the high energy limit, $k_{thr} \rightarrow 1$, we recover the result of Ref. [17].

VIII. APPENDIX B: SCALAR INTEGRALS FOR HELIQUARK EXCHANGE AMPLITUDE

The vector and tensor integrals can be reduced to the scalar ones by means of standard methods [25]. The remaining integrals to be calculated are the two, three, and four-point scalar integrals. Here we are only interested in the imaginary part of these, therefore there are only three integrals with non-zero imaginary part: the two-point integral

$$C_0 = \frac{1}{8\pi} \int \frac{d^4p}{4\pi^3} \frac{1}{p^2 - m^2} \left( P + K - p^2 - M^2 \right)$$

and finally, the four-point integral:

$$B_0 = \frac{1}{8\pi s^2} \int \frac{d^4p}{4\pi^3} \frac{1}{p^2 - m^2} \left( P + K - p^2 - M^2 \right)$$

These integrals should however be reggeised as described in Section [14] by substituting the Regge propagator instead of the Feynman one. Denoting $t_1 = (k - p)2$ and $t_2 = (k' - p)2$ the momentum transferred by the pions in the t-channel, we have for the reggeized version of scalar integrals:

$$(C_0^R) = \frac{1}{32\pi^2} \frac{p^2}{\sqrt{s}} \int d\Omega_{\pi} (t_1 - m_{\pi}^2)P_{\pi}^R(\alpha_{\pi}(t_1))$$

$$(B_0^R) = \frac{1}{32\pi^2} \frac{p^2}{\sqrt{s}} \int d\Omega_{\pi} (t_2 - m_{\pi}^2)P_{\pi}^R(\alpha_{\pi}(t_2))$$

Similarly, in the case of the $\rho$-exchange in the s-channel, the integrals with non-zero imaginary part are
\begin{align*}
B_0^\rho &= \text{Im} \int \frac{d^4p_\rho}{(2\pi)^4} \frac{1}{p_\rho^2 - m_\rho^2} \frac{1}{(k - p_\rho)^2 - m_\pi^2} \\
&= - \frac{1}{8\pi(s - M^2)} \ln \frac{2E_\rho(s - M^2)}{Mm_\rho^2},
\end{align*}

\begin{align*}
A_0^\rho &= \text{Im} \int \frac{d^4p_\rho}{(2\pi)^4} \frac{1}{(k' - p_\rho)^2 - m_\rho^2} \frac{1}{(P + K - p_\rho)^2 - M^2} \\
&= \frac{1}{8\pi Q^2(s - M^2 + m_\rho^2)} \sqrt{1 + \frac{4\sigma^2}{Q^2m_\rho^2}} \\
&\cdot \ln \frac{\sqrt{1 + \frac{4\sigma^2}{Q^2m_\rho^2}} + 1}{\sqrt{1 + \frac{4\sigma^2}{Q^2m_\rho^2}} - 1}.
\end{align*}

with $|\vec{p}_\rho| = \sqrt{(s - M^2 + m_\rho^2)^2 - m_\rho^2}$, and $\sigma^2 = E^2m_\rho^2 - 2E\rho(m_\rho^2 - m_\pi^2) + \frac{1}{4}(m_\rho^2 - m_\pi^2)^2$. If neglecting the pion mass, $\sigma = \frac{Mm_\pi^2}{2s}$. The reggeization procedure is the same as for $\pi N$- intermediate state.

[1] M.K. Jones et al., Phys. Rev. Lett. 84 (2000) 1398; O. Gayou et al., Phys. Rev. Lett. 88 (2002) 092301; V. Punjabi et al., Phys. Rev. C 71 (2005) 055202, Erratum-ibid. C 71 (2005) 069902.
[2] L. Andivahis et al., Phys. Rev. D 50 (1994) 5491; M.E. Christy et al., Phys. Rev. C 70 (2004) 015206; I.A. Qattan et al., Phys. Rev. Lett. 94 (2005) 142301, arXiv:nucl-ex/0410010.
[3] P.A.M. Guichon, M. Vanderhaeghen, Phys. Rev. Lett. 91 (2003) 142303.
[4] L.W. Mo and Y.S. Tsai, Rev. Mod. Phys. 41 (1969) 205; L.C. Maximon and J.A. Tjon, Phys. Rev. C 62 (2000) 054320.
[5] P.G. Blunden, W. McNitchouk and J.A. Tjon, Phys. Rev. Lett. 91 (2003) 142304.
[6] Y.C. Chen, A. Afanasev, S.J. Brodsky, C.E. Carlson and M. Vanderhaeghen, Phys. Rev. Lett. 93 (2004) 122301.
[7] N.F. Mott, Proc. R. Soc. London, Ser. A135 (1935) 429; A.O. Barut and C. Fronsdal, Phys. Rev. 120 (1960) 1871; A. De Rujula, J.M. Kaplan and D. de Rafael, Nucl. Phys. B 35 (1971) 365.
[8] S.P. Wells et al. [SAMPLE Collaboration], Phys. Rev. C 63 (2001) 064001; F. Maas et al. [MAMI A1 Collaboration], Phys. Rev. Lett. 94 (2005) 082001.
[9] A. Afanasev, I. Akushevich and N.P. Merenkov, arXiv:hep-ph/0208260.
[10] B. Pasquini and M. Vanderhaeghen, Phys. Rev. C 70 (2004) 045206.
[11] L. Diaconescu and M.J. Ramsey-Musolf, Phys. Rev. C 70 (2004) 054003.
[12] D. Borisyuk and A. Kobushkin, nucl-th/0508053.
[13] SLAC E158 Experiment, contact person K. Kumar; G. Cates, K. Kumar and D. Lhuillier, spokespersons HAPPEX-2 Experiment, JLab E-99-115; D. Beck, spokesperson JLab/G0 Experiment, JLabE-00—6, E-01-116.
[14] M. Gorchtein, P.A.M. Guichon and M. Vanderhaeghen, Nucl Phys. A 741 (2004) 234.
[15] A.V. Afanasev, N.P. Merenkov, Phys. Lett. B 599 (2004) 48; Phys. Rev. D 70 (2004) 073002.
[16] M. Gorchtein, hep-ph/0505022.
[17] A.V. Afanasev, N.P. Merenkov, hep-ph/0407167 v2.
[18] R.E. Prange, Phys. Rev. 110 (1958) 240.
[19] R.A. Berg and C.N. Lindner, Nucl. Phys. 26 (1961) 259.
[20] A.I. L’vov, V.A. Petrun’kin, M. Schumacher, Phys. Rev. C 55 (1997) 359.
[21] D. Drechsel, M. Gorchtein, B. Pasquini, M. Vanderhaeghen, Phys. Rev. C 61 (1999) 015204.
[22] B. Pasquini, M. Gorchtein, D. Drechsel, A. Metz, M. Vanderhaeghen, Eur. Phys. A 11 (2001) 185.
[23] M. Gorchtein, PhD thesis, Universität Mainz.
[24] T.H. Bauer, R.D. Spital, D.R. Yennie and F.M. Pipkin, Phys. Rev. D 70 (1978) 261; [Erratum-ibid. 51 (1979) 407].
[25] M. Guidal, J.-M. Laget, M. Vanderhaeghen, Nucl. Phys. A 627 (1997) 645.
[26] A. Metz and D. Drechsel, Z. Phys. A 356 (1996) 351; A. Metz, PhD Thesis, Universität Mainz.
[27] G. Passarino and M. Veltman, Nucl. Phys. B160 (1979) 151.
[28] P. Hoodbhoy and X. Ji, Phys.Rev. D 58 (1998) 054006; N. Kivel, L. Mankiewicz and M.V. Polyakov, Phys.Lett. B 467 (1999) 263-270; A.V. Belitsky and D. Müller, Phys.Lett. B 486 (2000) 369-377.