Numerical Results for Transport Coefficients of Quark Gluon Plasma with Iwasaki’s Improved Action

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Numerical results for the transport coefficients of quark gluon plasma are calculated by lattice simulation of SU(3) pure gauge model. The bulk viscosity is consistent with zero. The shear viscosity is finite and increases with temperature $T$ roughly as $T^3$, and around the finite temperature transition points, it is slightly smaller than the typical hadron masses.

1. Introduction

In the phenomenological study of quark gluon plasma (QGP), when its bulk properties are concerned, the system of quarks and gluons is usually treated as gas or liquid. Then the fundamental parameters of QGP such as transport coefficients, are very important information. The aim of this work is to calculate them from the fundamental theory of QCD.

The calculation of transport coefficients is formulated in the framework of linear response theory of Kubo[1–4]. They are expressed by the space time integral of retarded Green’s function of energy momentum tensors at finite temperature. The shear viscosity $\eta$ is expressed as follows.

\[
\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{i(t_1-t)} \times \int_{-\infty}^{t_1} dt' < T_{12}(x, t)T_{12}(x', t') >_{ret}
\] (1)

Similarly the bulk viscosity $\zeta$ and heat conductivity $\chi$ are expressed in terms of the retarded Green’s function of $T_{11}$ and $T_{41}$ components of energy momentum tensor. In the pure gauge model the energy momentum tensors are written by the field strength tensor.

The direct calculation of the retarded Green’s function at finite temperature is very difficult. Then the shortcut is to calculate Matsubara Green’s function($G_\beta$) and then by the analytic continuation, we obtain the retarded Green’s function at finite temperature. The analytic continuation is carried out by the use of the fact that the spectral function of Fourier transform of both Green’s functions is the same. For the spectral function we use the following simplest ansatz[3].

\[
\rho(p=0, \omega) = \frac{A}{\pi \left( (m-\omega) - \gamma \right)^2 + \gamma^2} - \frac{\gamma}{(m+\omega)^2 + \gamma^2}
\] (2)

where $\gamma$ partially represents the effects of the interactions and is related to the imaginary part of self energy. Under this ansatz, the transport coefficients are calculated as,

\[
\alpha \times a^3 = 2A \frac{2\gamma m}{(\gamma^2 + m^2)^2}
\] (3)

where $\alpha$ means $\eta$, $\frac{4}{3} \eta + \zeta$ and $\chi \cdot T$.

We notice that if $m = 0$ or $\gamma = 0$, transport coefficient becomes zero. In order to determine these parameters, at least three independent data points in $G_\beta$ are required in the temperature direction. In the pioneering work of Karsch and Wyld[5], they performed lattice QCD calculations on $8^3 \times 4$ lattice and the resolution was not enough to determine these three parameters independently.
2. Numerical Results

We carry out our simulation on $16^3 \times 8$ lattice. We have started the the simulation from U(1) gauge theory. It is found that the fluctuation of $G_\beta$ is very large that it need about $10^6$ data for the determination of $G_\beta$ in the deconfined phase. Further in the confined phase, the fluctuations become still larger and we could not obtain the Green’s functions even with the $\sim 1.5 \times 10^6$ data. Similar results are obtained in the case of SU(2) gauge theory. Then in the calculation of SU(3) gauge theory, we carry out our simulation only in the deconfined phase.

It is also found that the fluctuation of $G_\beta$ becomes larger as we proceed to $U(1) \to SU(2) \to SU(3)$. Then it is a very important problem to reduce the fluctuation of $G_\beta$. We find that by using the Iwasaki’s improved action, the fluctuation is much reduced as shown in Fig.1. Then in the

![Figure 1. Time histories of $G_\beta$ of $T_{11}$ at $T = 2$ with improved and standard action](image)

following we apply Iwasaki’s Improved action for the simulation of SU(3) gauge theory.

From roughly $0.5 \times 10^6$ data, we obtain $G_\beta$ for $T_{11}$ and $T_{12}$ . But they have still rather large errors. The fit of $G_\beta$ with parameters in the spectral function given by Eq.(2) is done with SALS. The shear and bulk viscosities are calculated by these parameters by Eqs.(3) and the error are estimated by the Jackknife method. The results for them are shown in Fig.2. It is found that the bulk viscosity is zero within errors. This result

![Figure 2. Shear and bulk viscosity $\times a^3$](image)

is consistent with the arguments of S. Gavin that in the pure gauge theory, the bulk viscosity and heat conductivity should be zero. In our calculation, $G_\beta$ of $T_{14}$ from which the heat conductivity is calculated, has large background and it has been impossible to get signal from it.

In order to know the shear viscosity in the physical unit, we should determine the lattice spacing $a$ in these $\beta$ values. For this purpose we have started to determine the finite temperature transition points $\beta_c$ at $N_T = 8$. We made a histogram analysis on $16^3 \times 8$ lattice. The results for the Polyakov susceptibility are shown in Fig.3.

The spacial volume is too small to make a precise determination of $\beta_c$ on our lattice, but we find that the transition region is $2.71 \leq \beta_c \leq 2.73$. By using the finite size scaling formula and the transition temperature $T_c \sim 276(3)(2)\text{MeV}$ determined by Tsukuba group and assuming asymptotic scaling for $\beta \geq 2.73$ region, the lattice spacing is estimated.

The preliminary results for the shear viscosity in the physical unit are shown in Fig.4. It is found that it increases with temperature. The $T$ dependence is consistent with $T^3$, because $\eta \times a^3$ is almost independent of $\beta$ as shown in Fig.2 and
Figure 3. The polyakov susceptibility on $16^3 \times 8$.

Figure 4. Shear viscosity in the physical unit $a \propto 1/T$ when $N_T$ is fixed. But clearly more accurate data are necessary. It is also seen that it seems finite around $T_c$, which is slightly smaller than the typical hadron massess. What is the physical effects on the phenomenology of quark gluon plasma when it has shear viscosity with this magnitudes, is a very interesting problem.

3. Conclusions and Further Problem

We have obtaind the transport coefficients of quark gluon plasma from the lattice QCD calculation by using Iwasaki’s improved action, as shown in Figs.2 and 4. The shear viscosity increases with temperarure roughly as $T^3$.

The bulk viscosity is consistent with zero. And the heat conductivity is very difficult to calculate because of large background for $G_\beta$ of $T_{14}$.

Our results depend strongly on the ansatz of spectral function in Eq.(2). In order to study the functional form for the spectral function, we have started the simulation with anisotropic lattices.

We could not calculate the $G_\beta$ in the confined phase. We think that this is because the energy momentum tensor operators should be written by the hadron fields, rather than the gluon fields. In order to certify this hypothesis, we have started the calculation of gluon propagator at finite temperature with improved action. The preliminary results on the small lattice $8^3 \times 4$ shows that in the confined phase, gauge copies are found in lattice version of Lorentz gauge and the gluon propagators show the peculiar behavior, which Nakamura group has found on the large lattice with standard action.

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Su(3), T=2

Stand, $\beta=6.25$: $<G(2)>_{all}=1.7 \times 10^{-6}$

Imp, $\beta=3.3$: $<G(2)>_{all}=6.4 \times 10^{-7}$

Fig. 1
Fig. 2
Fig. 3
Shear Viscosity

Fig. 4