A SEE-SAW MECHANISM FOR LARGE NEUTRINO MIXING
FROM SMALL QUARK AND LEPTON MIXINGS

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Abstract

I introduce and sketch the main features of those see-saw models where a large atmospheric mixing can be achieved starting from nearly diagonal matrices for charged leptons, Dirac neutrinos and Majorana right-handed neutrinos. It turns out that these models can be realized in Grand Unified Theories and they are well compatible with the related phenomenology of fermion masses and mixings.

1 Introduction

Recent impressive results from Superkamiokande have confirmed that the atmospheric neutrino anomaly can be successfully interpreted in terms of neutrino oscillations. Also the solar neutrino deficit, observed by several experiments, is probably an indication of a different sort of neutrino oscillations. Since neutrino oscillations imply neutrino masses, one is forced to look for viable extensions of the Standard Model. In doing this, it is worth to stress that the extreme smallness of neutrino masses in comparison with quark and charged lepton masses seems to point in favour of a different nature of the former, maybe linked to lepton-number violation. Experimental facts on neutrino masses could then provide an indication on the very large energy scale where lepton-number is violated. Grand Unified Theories (GUTs) are certainly a very attractive framework where neutrino masses can be analyzed, because they predict - besides, of course, unification of the three gauge coupling constants - lepton and baryon-number violation. Since in GUTs all fermion masses are related, neutrino masses and mixings could also provide an insight on the mechanism for the generation of charged fermion masses. In particular the observation of a nearly maximal mixing angle for $\nu_\mu \rightarrow \nu_\tau$ is particularly impressive. At present solar neutrino mixings can be either large or very small, depending on which particular solution will eventually be established by the data. Large mixings in the neutrino sector are very interesting because a first guess was in favour of small mixings, in analogy to what is observed for left mixings in the quark sector. If confirmed, single or double maximal mixings can provide an important hint on the mechanisms that generate neutrino masses.

In the context of GUTs, many theoretical descriptions of large neutrino mixing(s) have been discussed (see Ref. 4 for a review). In most models large mixings are already present at the level of Dirac and/or Majorana matrices for neutrinos. Instead, here I discuss the interesting class of models where large, possibly maximal, neutrino mixings are generated by the see-saw mechanism starting from nearly diagonal Majorana and Dirac matrices for neutrinos, without fine-tuning or stretching small parameters into becoming large. A more complete discussion of the subject can be found in Ref. 1.
With neutrino masses settled, observation of proton decay will be the next decisive challenge remained to support or eventually put in crisis GUTs. In fact, SuperKamiokande is giving lower bounds on the proton life-time which, for certain decay modes, yet exclude part of the range generally predicted by GUTs. Then it seems interesting to see if it is possible to construct a simple but realistic GUT model which not only correctly reproduces the informations on neutrino masses and the actual bounds on proton decay, but also overcome the typical problem of these theories, that is the doublet-triplet splitting problem, without destroying gauge coupling unification.

2 Starting Assumptions

Since the experimental status of neutrino oscillations is still very preliminary, one has to make a number of assumptions on how the data will finally look like. Here I assume that only two distinct oscillation frequencies exist, the largest being associated with atmospheric neutrinos and the smallest with solar neutrinos. I assume that the hint of an additional frequency from the LSND experiment will disappear, thus avoiding the introduction of new sterile neutrino species. Dealing with only the three known species of light neutrinos, the atmospheric neutrino oscillations are interpreted as nearly maximal $\nu_\mu \to \nu_\tau$ oscillations while the solar neutrino oscillations correspond to the disappearance of $\nu_e$ into nearly equal fractions of $\nu_\mu$ and $\nu_\tau$. One has to be open minded to all the three most likely solutions for solar neutrino oscillations: the two MSW solutions with small (SA) or large (LA) mixing angle, or the vacuum oscillation solution (VO). Assuming only two frequencies, given by

$$\Delta_{\text{sun}} \propto m_2^2 - m_1^2 \quad , \quad \Delta_{\text{atm}} \propto m_3^2 - m_{1,2}^2 \quad ,$$

(1)

there are two extreme possibilities for the mass eigenvalues:

$$\begin{align*}
\text{A} & : \quad m_3 \gg m_{2,1} \\
\text{B} & : \quad m_1 \sim m_2 \sim m_3 
\end{align*}$$

(2)

Configuration B imply a very precise near degeneracy of squared masses: it would need a relative splitting $|\Delta m/m| \sim \Delta m_{\text{atm}}^2/2m^2 \sim 10^{-3} - 10^{-4}$ and a much smaller one for solar neutrinos, especially if explained by vacuum oscillations: $|\Delta m/m| \sim 10^{-10} - 10^{-11}$. Foreseeing a GUT framework, it is reasonable to assume that the Dirac neutrino matrix has a strongly hierarchical structure, as is the case for charged fermions. So, it seems quite implausible that, starting from hierarchical Dirac matrices, one end up via the see-saw mechanism into a nearly perfect degeneracy of squared masses. As a consequence, here I will focus on models of type A with large effective light neutrino mass splittings and large mixings.

3 A 2×2 Example

Reconciling large splittings with large mixing would seem difficult. Indeed, one could guess that, in analogy to what is observed for quarks, large splittings correspond to small mixings because only close-by states are strongly mixed. At the contrary, via the see-saw mechanism there are two particularly simple ways in which this can be realized.

Without loss of generality, leaving apart for the moment the eventual presence of flavor symmetries, one can go to the basis where both the charged lepton Dirac mass matrix $m_D$ and the Majorana matrix $M$ for the right-handed neutrinos are diagonal. For simplicity, let’s start assuming that the role of the first generation is not crucial in the mechanism for the generation of neutrino masses, so than one can, with good approximation, work in the 2 by 2 case (in the next section I will however relax this condition). If one writes $m_D$ (defined by $Rm_DL$) and $M$
in the most general way:

\[ m_D = v \begin{bmatrix} a & b \\ c & 1 \end{bmatrix}, \quad M = \begin{bmatrix} M_2 & 0 \\ 0 & M_3 \end{bmatrix}, \]  

where \( v \) is a vacuum expectation value, \( a, b \) and \( c \) are Yukawa couplings, then, via the see-saw, one obtains:

\[ m_\nu = \frac{v^2}{M_3} \begin{bmatrix} a^2/M_2 + c^2/M_3 & a b/M_2 + c/M_3 \\ a b/M_2 + c/M_3 & b^2/M_2 + 1/M_3 \end{bmatrix}. \]  

The request of large splittings among the light neutrino’s eigenvalues is equivalent to demanding that the determinant of the previous matrix is much smaller than its trace. It is then possible to see at first sight that two very natural cases arise, respectively when the terms with \( M_1 \) or \( M_2 \) at the denominator are dominant.

One simple example of the first case is realized if \( M_2 \sim M_3 \) and \( a, b \ll 1 \). In order to have a large splitting, one must have \( c \sim 1 \), that is the right-handed neutrino of the third generation couples with the same strength \( \sim \) to left-handed \( \nu_\mu \) and \( \nu_\tau \). The heaviest mass for light neutrinos results \( m_3 \sim v^2/M_3 \). Since in the hierarchical case, the data from SuperKamiokande suggest \( m_3 \sim 0.05eV \), if one assumes that \( v \) is a typical weak scale, namely 250 GeV, then \( M_3 \sim 10^{15}GeV \), just the order of a GUT scale. It is worth to stress that this first mechanism is based on asymmetric Dirac matrices with, in the case of the example, a large left-handed mixing already present in the Dirac matrix. It has been observed that in SU(5) GUT left-handed mixings for leptons tend to correspond to right-handed mixings for \( d \) quarks (in a basis where \( u \) quarks are diagonal). Since large right-handed mixings for quarks are not in contrast with experiment, viable GUT models that correctly reproduce the data on fermion masses and mixings can be constructed following this mechanism.

An alternative possibility is to have the dominance of the terms with \( M_2 \) at the denominator. This is achieved for any \( c < 1 \) if \( a^2, b^2 > M_2/M_3 \). The request for large splitting is then equivalent to require also \( a \sim b \). Now it is the second generation right-handed neutrino which is particularly light and which couples with the same strength \( \sim \) to left-handed \( \nu_\mu \) and \( \nu_\tau \). In order to be more specific, consider one particular example with symmetric matrices. These matrices are interesting because, for instance, one could want to preserve left-right symmetry at the GUT scale. Then, the observed smallness of left-handed mixings for quarks would also demand small right-handed mixings. Starting from

\[ m_D = v \begin{bmatrix} \epsilon & xe \\ xe & 1 \end{bmatrix}, \quad M^{-1} = \frac{1}{M_3} \begin{bmatrix} r_2 & 0 \\ 0 & 1 \end{bmatrix}, \]  

where \( \epsilon \) is a small number, \( x \) is of O(1) and \( r_2 \equiv M_3/M_2 \), then, via the see-saw, it is sufficient that \( \epsilon^2 r_2 \gg 1 \) in order to have approximately:

\[ m_\nu = \frac{v^2}{M_3} \epsilon^2 r_2 \begin{bmatrix} 1 & x \\ x & x^2 \end{bmatrix}. \]  

The determinant is naturally vanishing so that the mass eigenvalues are widely split and for \( x \sim 1 \) the mixing is nearly maximal. It is exactly maximal if \( x = 1 \). The see-saw mechanism has created large mixing from almost nothing: all relevant matrices entering the see-saw mechanism are nearly diagonal, that is they are diagonalized by transformations that go into the identity in the limit of vanishing \( \epsilon \). Clearly, the crucial factorization of the small parameter \( \epsilon^2 \) only arises if the light Majorana eigenvalue is coupled to \( \nu_\mu \) and \( \nu_\tau \) with comparable strength, that is \( x \sim 1 \). An interesting feature of this second case, in connection with a possible realization within a GUT scheme, is that it requires \( M_3 > v^2/m_3 \), so that one can push \( M_3 \), the scale of lepton-number violation, beyond the GUT scale. This is desirable because, for instance, this is expected in SU(5) if right-handed neutrinos are present and also in the breaking of SO(10) to SU(5).
Summarizing, the second case require a peculiar hierarchy in the Majorana eigenvalues in order to work, but it has however the good characteristic that it can be realized even with nearly diagonal matrices.

4 Generatization to the $3 \times 3$ Case

It is straightforward to extend the previous model to the 3 by 3 case. One simple class of examples with symmetric mass matrices is the following one. Starting from

$$m_D = v \begin{bmatrix} e'' & e' & y e' \\ e' & e & x e \\ y e' & x e & 1 \end{bmatrix}, \quad M^{-1} = \frac{1}{\Lambda} \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix},$$

(7)

where, unless otherwise stated, $x$ and $y$ are $O(1)$; $e$, $e'$ and $e''$ are independent small numbers and $r_i \equiv M_3/M_i$. One expects $e'' \ll e' \ll e \ll 1$ and, perhaps, also $r_1 \gg r_2 \gg r_3 = 1$, if the hierarchy for right-handed neutrinos follows the same pattern as for known fermions. Depending on the relative size of the ratios $r_i/r_j$, $e/e'$ and $e'/e''$, it is possible to have models with dominance of any of the $r_{1,2}$. For example, setting $x = 1$ (keeping $y$ of $O(1)$) and assuming $r_2 e^2 \gg r_1 e^2; r_3$, together with $r_2 e^2 \gg r_1 e^{''2}$ and $r_2 e \gg r_1 e''$, with good accuracy we obtain:

$$m_\nu = \frac{v^2}{\Lambda} r_2 e^2 \begin{bmatrix} \frac{e'^2}{\epsilon^2} & \frac{e'}{\epsilon} & \frac{e'}{\epsilon} \\ \frac{e'}{\epsilon} & 1 + \frac{r_4 e^2}{r_2 e^2} & 1 \\ \frac{e'}{\epsilon} & 1 + \frac{r_4 e^2}{r_2 e^2} \end{bmatrix}.$$  

(8)

Since the subdeterminant of the 23 block is vanishing, the eigenvalues are widely split. Having set $x = 1$ the atmospheric neutrino mixing is nearly maximal. The solar neutrino mixing is instead generically small in these models, being proportional to $e'/\epsilon$. Thus the SA-MSW solution is obtained. It is easy to find set of parameter values that lead to an acceptable phenomenology within these solutions. As an illustrative example take:

$$e \sim \lambda^4, \quad e' \sim \lambda^6, \quad e'' \sim \lambda^{12}, \quad r_1 \sim \lambda^{-12}, \quad r_2 \sim \lambda^{-9},$$

(9)

where $\lambda \sim \sin \theta_C$, $\theta_C$ being the Cabibbo angle. The neutrino mass matrices than become

$$m_D = v \begin{bmatrix} \lambda^{12} & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{bmatrix}, \quad M = \frac{1}{\Lambda} \begin{bmatrix} \lambda^{12} & 0 & 0 \\ 0 & \lambda^9 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$  

(10)

and, in units of $v^2/\Lambda$, we obtain: $m_3 \sim 1/\lambda$, $m_2 \sim 1$, $m_1 \sim \lambda^4$. The solar mixing angle $\theta_{12}$ is of order $\lambda^2$, suitable to the SA-MSW solution. Also $\theta_{13} \sim \lambda^2$.

Models based on symmetric matrices are directly compatible with left-right symmetry and therefore are naturally linked with SO(10). This is to be confronted with models that have large right-handed mixings for quarks, which, in SU(5), can be naturally translated into large left-handed mixings for leptons. In this connection it is interesting to observe that the proposed textures for the neutrino Dirac matrix can also work for up and down quarks. For example, the matrices

$$m_D^\nu \propto \begin{bmatrix} 0 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{bmatrix}, \quad m_D^d \propto \begin{bmatrix} 0 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix},$$

(11)

where for each entry the order of magnitude is specified in terms of $\lambda \sim \sin \theta_C$, lead to acceptable mass matrices and mixings. In fact $m_u : m_c : m_t = \lambda^8 : \lambda^4 : 1$ and $m_d : m_s : m_b = \lambda^4 : \lambda^2 : 1$. The $V_{CKM}$ matrix receives a dominant contribution from the down sector in that the up sector angles are much smaller than the down sector ones. The same kind of texture can also be adopted in the charged lepton sector.
5 Bimixing

The solar mixing angle is generically small in the class of models explicitly discussed above. However, small mixing angles in the Dirac and Majorana neutrino mass matrices do not exclude a large solar mixing angle. For instance, this is generated from the asymmetric, but nearly diagonal mass matrices:

\[
m_D = v \begin{pmatrix} \lambda^6 & \lambda^6 & 0 \\ 0 & \lambda^4 & \lambda^4 \\ 0 & 0 & 1 \end{pmatrix}, \quad M = \Lambda \begin{pmatrix} \lambda^{12} & 0 & 0 \\ 0 & \lambda^{10} & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\] (12)

They give rise to a light neutrino mass matrix of the kind:

\[
m_\nu = \begin{pmatrix} \lambda^2 & \lambda^2 & 0 \\ \lambda^2 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{v^2}{\lambda^2 \Lambda},
\] (13)

which is diagonalized by large \(\theta_{12}\) and \(\theta_{23}\) and small \(\theta_{13}\). The mass hierarchy is suitable to the large angle MSW solution.

6 Outlook and Conclusions

Other comments about this mechanism are contained in Ref. 1. For instance here we show that:

i) the results obtained are stable under renormalization from the high energy scale where the mass matrices are produced down to the electroweak scale; in fact, if light neutrino masses are hierarchical, one always expects renormalization effects to be negligible\(^4\);

ii) it is possible to construct specific realizations of the mechanism sketched here, e.g. in the context of SU(5) \(\times\) broken horizontal flavour symmetries.

Summarizing, in most models\(^11\),\(^12\),\(^15\) that describe neutrino oscillations with nearly maximal mixings, there appear large mixings in at least one of the matrices \(m_D, m_D^T, M\) (i.e. the neutrino and charged-lepton Dirac matrices and the right-handed Majorana matrix). In this contribution I have discussed the peculiar possibility that large neutrino mixing is only produced by the seesaw mechanism starting from all nearly diagonal matrices. Although this possibility is certainly rather special, models of this sort can be constructed without an unrealistic amount of fine-tuning and are well compatible with grand unification ideas and the related phenomenology for quark and lepton masses.

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