Abstract

Non-time-orthogonal analysis of rotating frames is applied to objects in gravitational orbits and found to be internally consistent. The object’s surface speed about its axis of rotation, but not its orbital speed, is shown to be readily detectable by any “enclosed box” experimenter on the surface of such an object. Sagnac type effects manifest readily, but by somewhat subtle means. The analysis is extended to objects bound in non-gravitational orbit, where it is found to be fully in accord with the traditional analysis of Thomas precession.

I. Introduction

An analysis\(^1\)\(^,\)\(^2\)\(^,\)\(^3\) has been carried out of the non-time-orthogonal (NTO) metric obtained when one makes a straightforward (and also the most widely accepted) transformation from the lab to a relativistically rotating frame. Rather than assuming, as have other researchers, that it is then necessary to transform to locally time orthogonal frames, one can proceed by considering the NTO metric to be a physically valid representation of the rotating frame.

When this is done, one finds time dilation and mass-energy dependence\(^4\) on tangential speed \(\omega r\) that is identical to the predictions of special relativity and the test data from numerous cyclotron experiments. One also finds resolutions of paradoxes inherent in the traditional analytical treatment of rotating frames. Further, the analysis predicts two different experimental results\(^5\)\(^,\)\(^6\) that, in the context of the traditional analysis, have heretofore been considered inexplicable.

One of these results is a persistent non-null signal found by Brillet and Hall\(^7\) in the most accurate Michelson-Morley type experiment to date, which as Aspden\(^8\) pointed out, would correspond to an earth surface speed of approximately 363 m/sec. It is noteworthy that the earth surface speed at the test site is 355 m/sec, and that no other experiment has been sensitive enough to test for this effect.

Though the NTO frames analysis predicts such a signal, the question arises as to why the earth surface speed should differ from the solar and galactic orbital speeds, which yield null signals in the same (and many similar) test(s). This article answers this question, as well as a related question concerning Thomas precession.

\(^1\) Robert D. Klauber, “New perspectives on the relatively rotating disk and non-time-orthogonal reference frames”, Found. Phys. Lett. 11(5), 405-443 (1998).
\(^2\) Robert D. Klauber, “Comments regarding recent articles on relativistically rotating frames”, Am. J. Phys. 67(2), 158-159, (1999).
\(^3\) Robert D. Klauber, “Non-time-orthogonal frames in the theory of relativity”, xxx.lanl.gov paper gr-qc/0005121, submitted for publication May 2000.
\(^4\) Ref 1, pp. 425-429, and ref. 3, eq (5).
\(^5\) Ref 1, pp. 434-436, and ref 3, section V.B.
\(^6\) Ref. 3, Section V.
\(^7\) A. Brillet and J. L. Hall, “Improved laser test of the isotropy of space,” Phys. Rev. Lett., 42(9), 549-552 (1979).
\(^8\) H. Aspden, “Laser interferometry experiments on light speed anisotropy,” Phys. Lett., 85A(8,9), 411-414 (1981).
II. NTO Analysis

A. Predictions

NTO frame analysis makes many of the same predictions as the traditional analysis for rotating frames, and as emphasized in reference 3, is in accord with fundamental principles of relativity theory. Analyses of time-orthogonal (TO) frames, including those described by Lorentz, Schwarzchild, and Friedman metrics, remains unchanged. The line element remains invariant, and differential geometry maintains its reign as descriptor of non-inertial systems, whether TO or NTO.

However, NTO analysis does predict some behavior that may seem strange from a traditional relativistic standpoint, though it appears corroborated by both gedanken and physical experiments. In particular, it was found that velocities in the circumferential direction add in a nontraditional way, i.e.

\[ u_{\text{circum}} = \frac{-\omega r + U_{\text{circum}}}{\sqrt{1 - (\omega r)^2/c^2}} = \frac{-v + U_{\text{circum}}}{\sqrt{1 - v^2/c^2}}, \tag{1} \]

where \( u_{\text{circum}} \) is circumferential speed in the rotating frame of an object having circumferential speed \( U_{\text{circum}} \) in the non-rotating frame, \( \omega \) is the angular velocity measured from the non-rotating frame, \( r \) is the radial distance from the center of rotation, and \( v = \omega r \). Note that when \( U_{\text{circum}} = 0 \), the object appears in the rotating frame to be moving opposite the direction of \( \omega \) at speed \( \omega r \) (to first order), as is physically reasonable. Time dilation effects account for the familiar factor in the denominator.

Expanding on (1), NTO analysis finds the specific result for the speed of light in the circumferential direction for rotating (NTO) frames to be non-invariant, non-isotropic, and equal to

\[ u_{\text{light,circum}} = \frac{-\omega r \pm c}{\sqrt{1 - (\omega r)^2/c^2}} = \frac{-v \pm c}{\sqrt{1 - v^2/c^2}}, \tag{2} \]

where the sign before \( c \) depends on the circumferential direction of the light ray at \( r \). Note the circumferential light speed varies to first order with \( \omega r \).

Relationship (2) leads readily to the prediction of i) the Sagnac effect, and ii) a signal due to the earth surface speed \( v \) precisely like that found by Brillet and Hall. For bodies in gravitational orbit (2) does not hold, and \( |u_{\text{light,circum}}| = c \), as in traditional relativity. This is because such bodies are in free fall, and are essentially inertial, Lorentzian, time orthogonal (TO) frames. They are not subject to the idiosyncrasies of non-time-orthogonality, so their orbital speed would result in a null Michelson-Morley signal.

B. Orbital Speeds Reconsidered

While the statements at the end of the preceding subsection may at first seem reasonable, under scrutiny they are seen, not only as somewhat superficial, but also in apparent conflict with logic used to form the basis of the NTO analysis.

In particular, the gedanken experiment of reference 3 addresses an observer fixed to the rim of a rotating disk who sends out two very short pulses of light (of length 1/360 of the circumference) that travel in opposite directions around the rim of the disk. From the lab frame both light pulses have speed \( c \). As the two light pulses are traveling the disk is rotating ccw, so from the lab frame it is readily apparent that the cw pulse strikes the original observer before the ccw pulse. The conclusion reached by the disk observer, who knows that both pulses traveled the same distance around the rim in his frame, is that in his frame the cw speed of light must be greater than the ccw speed of light. This seeming contradiction of the relativistic tenet that light speed is invariant, isotropic, and equal to \( c \) is resolved by the NTO analysis leading to (2). That is, in TO frames (local, physical) light speed is invariant and isotropic, but in NTO frames, such as the rotating frame, it is not.

Applying the same logic to a sun centered rotating frame in which the earth is fixed, one would expect the same result, i.e., different ccw and cw light speeds as seen from the earth leading to a non-null Michelson-Morley result.

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9 Ref 3, section II.
10 Ref. 1, pg. 425, eq. (19) modified by the time dilation factor discussed in the subsequent paragraphs to yield physical velocity., and pg. 430, eq. (33).
11 E.J. Post, "Sagnac effect," Mod. Phys. 39, 475-493 (1967).
Yet, as claimed above, the earth in that frame is in gravitational orbit, in free fall, and in a Lorentz frame. In such a frame the speed of light must always be equal to $c$, and the analysis appears to be internally inconsistent.

### III. Resolution of Apparent Inconsistency

#### A. Non-spinning Body in Orbit

As a first step in answering the above conundrum, consider a planet in orbit about a star where the planet is not rotating on its own axis relative to distant stars, i.e., one solar day equals one year. (See Figure 1.) K is an inertial frame with its origin fixed at the center of the sun. The $K_0$ frame is fixed to the planet and has $\omega = 0$, though it has orbital angular velocity about the sun of $\Omega$ relative to K. $R$ is the distance in K from the sun center to the planet center, so $\Omega R = V$ is the planet’s orbital speed in K. For simplicity, all velocities are co-planar, the orbit is circular, and unless otherwise noted, analysis is confined to first order effects in velocity, time, and distance (higher order effects are not measurable.)

![Fig 1. Non-spinning Planet](image)

![Fig 2. Paths of Light Rays Seen from Non-spinning Planet Frame](image)

Note that an observer on the planet doing experiments (Foucault pendulum, Coriolis effects, etc.) inside a closed laboratory (similar to Einstein’s gedanken intergalactic elevator) would measure zero angular velocity, and be unable to determine $\Omega$.

Now reconsider our gedanken experiment of section II.B with $V = c/3$. At time $T_A=0$, two light rays are emitted from the origin of $K_0$, one in the "forward" or positive $Y_0$ direction and one in the "backward" or negative $Y_0$ direction. These light rays are reflected off of mirrors placed suitably in orbit such that they travel around a circumference in K at the orbital radius. In K the speed of light is invariant and equal to $c$. Therefore from K we would expect the two rays to arrive back at the $K_0$ origin at different times, $T_B$ and $T_C$ ($= 2T_B$), since $K_0$ moves along the orbit while the light rays are in transit.

In $K_0$ the speed of light must also be $c$. The conundrum dissolves when we note the paths of the two light rays, as depicted in Figure 2, are not the same as seen from $K_0$. This is because $K_0$ does not rotate and hence the two light pulses do not travel the same circular path as seen from $K_0$. In this particular case, the path of the ccw pulse is twice that of the cw pulse as seen from $K_0$. Hence for invariant light speed in $K_0$, $T_C = 2T_B$ as was found in $K_0$.

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12 This result, as well as Figure 2, can be found by inputting the components of the displacement vector from the origin of $K_0$ to the light rays as a function of time into a spreadsheet or other computer program.

13 To simplify the discussion we are considering weak gravitational fields, first order effects on speed, time, and distance, and hence $K_0$ as an effectively Lorentzian frame even at distances removed from the $K_0$ origin. Essentially, we are restricting observable effects
Note that a Michelson-Morley (MM) interferometer on the surface of the planet (i.e., in a Lorentz frame) would detect no variance or anisotropy in the speed of light. Yet the Sagnac effect around the orbit would manifest completely.

B. Body in Orbit Spinning with $\omega = \Omega$

Consider next Figure 3 where the planet spins on its own axis (perpendicular to the orbital plane) at $\omega = \Omega$. This is similar to the rotating disk case in that the same side of the planet faces its sun at all times, just as every element on the disk maintains the same alignment relative to the disk center. The frame of the rotating planet is designated $k_{\omega}=\Omega$.

For the frame K, fixed relative to the distant stars, the analysis of two light paths in opposite directions around the planet’s orbit is identical to that for two light rays as seen from the lab frame in the rotating disk case of section II.B. For orbital speed of the planet $V$ in K equal to one-third the speed of light, we would again find $T_C=2T_B$.

From the point of view of the planet frame $k_{\omega}=\Omega$, rotation is taking place about the planet’s center. Hence the angular velocity our experimentalist on the planet would measure would be $\omega = \Omega$. She would then determine the tangent velocity of the planet’s surface to be

$$v_{\omega=\Omega} = \omega r_p = \Omega r_p \tag{3}$$

where $r_p$ is the radius of the planet, not the orbital radius about the star.

Based on NTO analysis, a Michelson-Morley experiment on the surface of the planet would find a non-null signal corresponding to the same velocity. Hence, a number of independent experiments could detect the planet surface tangent velocity, though not the orbital tangent velocity.

As seen from the rotating planet frame $k_{\omega}=\Omega$, the two light pulses would travel in opposite directions along the same circular path. (See Figure 4.) Each pulse has a different speed, according to the $+/-$ sign in (2), but with the notable difference that these speeds are now variable since $r$, the distance from the planet center to a given light pulse is no longer constant. That is, circumferential light speed in the planet frame is a function of distance $r$ from the origin of the frame to the light pulse, i.e.,

$$u_{\text{light,orbit},k_\Omega}(r) = \frac{-\omega r \pm c}{\sqrt{1 - (\omega r)^2/c^2}} = \frac{-v(r) \pm c}{\sqrt{1 - v^2(r)/c^2}} \tag{4}$$

to those arising from the numerator of (2) and ignoring effects such as that from the denominator, which are too subtle to measure in experiments.
Note that the velocity corresponding to this speed is \textit{perpendicular} to the \( r \) vector corresponding to distance \( r \). That is, (4) is only one component of the light velocity along the path traveled as seen in \( \omega=\Omega \).

One can then use the two speeds of (4) to determine each pulse speed along the paths traveled. When this speed is integrated with respect to time between zero and arbitrary time \( t \), and the result set equal to the distance traveled \( 2\pi R \), one finds \( t \) equals (to first order) \( T_B \) for the ccw light pulse and \( T_C = 2T_B \) for the cw pulse, as before.\[14\]

In the \( \omega=\Omega \) frame, the paths traveled are equal, but the speeds differ. In the \( K \) frame the speeds are equal, but the paths differed. In both frames the times of arrival of each pulse are the same.

From the point of view of an observer in a third frame \( k_S \), centered on the sun and rotating at \( \Omega \), the speeds (again to first order) of the two pulses are -\( \Omega R + c \) and -\( \Omega R - c \). (These are not, as mentioned, the speeds a free fall observer in orbit at radius \( R \) would measure.) In \( k_S \) the path lengths of the two light pulses are equal, but the difference in speeds results once again in the same values for \( T_B \) and \( T_C \).

\[C. \text{ Comparison with Sagnac Experiment}\]

The above analysis agrees with the Sagnac experimental results reported by Post\[11\]. In the Sagnac experiment the light beams are not short as in our gedanken experiment so arrival times are supplanted by interference fringing of the two beams. The number of fringes difference between the two beams varies with wave maxima arrival times, and so increases directly with the rotational speed. Post\[15\] gives the empirically determined relative fringe shift \( \Delta Z = \Delta \lambda/\lambda_0 \) as

\[
\Delta Z = 4\Omega \cdot \mathbf{A}/\lambda_0 c,
\]

where \( \Omega \) is the angular velocity, and \( \mathbf{A} \) is the area enclosed by the paths of the two oppositely directly light beams with the vector direction orthogonal to the area surface. Although this is an empirical result, as noted in reference 1, it can be readily derived from (2).

To find arrival time difference between light beams multiply (5) by \( \lambda_0/c \) to yield

\[
T_C - T_B = 4\Omega \cdot \mathbf{A}/c^2.
\]

Post acknowledges that (5) (and therefore also (6)) is only accurate to first order. He also clearly points out that the area \( \mathbf{A} \) does not have to be circular, nor does it have to have the axis of rotation at its center. Further, (5) and (6) can be used by observers in either a rotating or non-rotating frame, since all quantities used therein are readily measured by experimenters in either type of frame.

Hence, all three frames considered in section III.B would yield the same experimental results, since to first order all have the same \( \mathbf{A} \) and \( \Omega \) values. Thus the experimental results would be in full accord with the NTO analysis of section III.B.

\[D. \text{ Body in Orbit with Arbitrary } \omega\]

For a situation like the earth where \( \omega >> \Omega \), or more generally for any \( \omega \), similar logic would hold. In the former case, light following the path of the earth’s orbit would seem from the earth frame to follow a corkscrew-like path with varying velocity, yet all Sagnac type results (interference fringing, arrival times) would be the same regardless of the frame from which they are analyzed. The light speed anisotropy would in all cases, however, be related to the earth surface velocity relative to the inertial frame in which the earth axis is stationary. In particular, the earth surface speed at its equator would be determined by Michelson-Morley, Foucault pendulum, Coriolis, etc. experiments to be

\[
v_{eq} = \omega r_{eq},
\]

where \( r_{eq} \) is the equatorial radius of the earth.

\[14\] One does not have to actually make this tedious calculation. The times \( T_B \) and \( T_C \) have been calculated in the frame \( K \), and those values can be transformed to the frame \( k_S \) (introduced in a subsequent paragraph), where they will be found to be unchanged to first order. From \( k_S \) they can be again transformed to frame \( \omega=\Omega \), and found unchanged once again. The first transformation is shown by both references 1 and 3 to have \( t_s = T_K \). The second transformation is purely a spatial coordinate value shift and has no effect on time.

\[15\] Ref. 11, equation (1).
IV. Thomas Precession

A. Traditional Analysis of Thomas Precession

Figure 1 can be used as an aid in discussing Thomas precession. However, instead of a gravitationally bound orbit we now consider Figure 1 to depict a charged object such as a classical electron held in orbit by a central charge of opposite sign such as an atomic nucleus. Although the orbiting K₀ frame is not spinning, the orbiting (Bohr) electron is spinning (unlike the planet in Figure 1) and thereby possesses both intrinsic angular momentum and a magnetic moment. Consider that at time \( T=0 \) a projection of the angular momentum vector (which is not necessarily orthogonal to the orbital plane) onto the plane of the figure would be aligned with the \( X₀ \) axis (and hence the \( X \) axis as well).

In Newtonian theory, as the origin of the \( X₀-Y₀ \) axes orbits the central charge at radius \( R \), the angular momentum vector orientation remains fixed (relative to K, the frame of the distant stars), and so its projection would remain aligned in the direction of the \( X \) axis. In traditional relativity theory, however, the spin (angular momentum) axis precesses as seen from K, due to Lorentz contraction effects which vary in time due to the oscillating (from the point of view of K) centrifugal acceleration. This precession is called Thomas precession after its discoverer.

Note that the \( X₀-Y₀ \) axes frame is not an NTO frame, but an accelerating TO frame. It is no different in this regard from any frame that undergoes (variable or constant) acceleration without rotation. Hence NTO analysis is not in conflict with the traditional treatment of Thomas precession for orbital electrons, which is based on TO frames only.

For reference, we note that (to second order), where \( v=\Omega R \), Thomas precession equals

\[
\omega_T \cong -\frac{1}{2} \frac{v^2}{c^2} \Omega,
\]

and is the precession of the spin vector relative to the \( X \) axis direction of K, as seen from K.

B. Alternative Analysis of Thomas-like Effect

As is well known, Thomas precession of orbiting electrons alters the spin-orbit interaction (which is a function of the precession rate of the spin vector) and results in fine structure splitting of atomic spectra. There is, however, another way to find the same effect using NTO analysis.

Consider the same spinning electron in the same orbit, but analyze it using the orbiting frame \( k_S \) (which is sun centered and rotating at \( \Omega \) relative to K). Use the \( k_ω=\Omega \) frame (see Figure 3) as a convenient local representation of \( k_S \), and note that the projection of the spin angular momentum vector rotates relative to \( k_ω=\Omega \) (and hence also relative to \( k_S \)) at -\( \Omega \).

According to NTO analysis, there is no Lorentz contraction effect between the \( k_S \) and K frames, and hence no Thomas-like rotation can arise. However, there is time dilation in \( k_S \), and so the rate of rotation of the spin vector projection relative to the \( X_ω=\Omega \) axis will not be the same as seen from K as it is seen from \( k_S \). Specifically, if \( T_S \) is the time on a standard clock in \( k_S \) travelling with the electron for one full rotation of the spin vector relative to the \( X_ω=\Omega \) axis, then the time \( T_K \) for the same rotation as seen in K is

\[
T_K = \frac{T_S}{\sqrt{1 - v^2/c^2}}.
\]

Hence the rotation rates seen in the two frames are related by

\[
\Omega_K = \Omega_S \sqrt{1 - v^2/c^2} \cong \Omega_S - \frac{v^2}{2c^2} \Omega_S.
\]

So

16 Edwin F. Taylor and John Archibald Wheeler, *Spacetime Physics*, (W.H. Freeman and Co., San Francisco, 1966) pp. 169-174. Taylor and Wheeler have as lucid and readily assimilable a treatment of Thomas precession as any in the literature.
17 George P. Fisher, “The Thomas Precession”, *Am. J. Phys.*, 40, 1772-1781 (1972).
18 A. E. Ruark, and H.C. Urey, *Atoms, Molecules, and Quanta* (McGraw-Hill, New York, 1930) pp. 162-163.
19 Ref. 16, equation (134) on pg. 173.
Thus, from (11) one sees the difference in spin vector precession rate between that seen in K and that seen in k_S found from NTO analysis to be the same as that of (8) found from Thomas precession analysis using only TO frames. Hence the difference between energy levels measured in the lab and the corresponding energy levels that would be measured on a frame traveling with the electron is the same for both methods.

\[ \Delta \Omega \cong -\frac{v^2}{2c^2} \Omega_S \cong -\frac{v^2}{2c^2} \Omega. \]  

(11)

C. Rotating Disks and Thomas Precession

NTO analysis does not, however, predict any type of Thomas precession phenomena for objects such as a macroscopic rotating disk in which every element of the object rotates with the same angular velocity. According to such analyses, no Lorentz contraction effects manifest as internal disk stresses. This is in conflict with some analyses based on a more traditional approach\textsuperscript{20}, but in accord with the Phipps\textsuperscript{21} experiment, which found no evidence of the predicted Thomas precession-like effects for a spinning disk.

V. Summary

We have shown that NTO analysis of bodies in gravitational orbit is internally consistent and in agreement with expected results based on the Sagnac experiment. The speed of light on bodies in orbit that do not rotate relative to distant stars is invariant, isotropic and equal to c. For a rotating body in orbit, light speed on the surface is anisotropic and a function of both the angular velocity of the body and the radial distance from the axis of rotation.

NTO analysis does not contravene the traditional analysis of Thomas precession for a spinning object held in orbit by a non-gravitational force. It does not, however, predict any Thomas precession type effects for an object such as a rotating disk wherein every local element in the object rotates at the global rotation rate and maintains fixed distance to the axis of rotation.

\textsuperscript{20} Daniel P. Whitmire, “Relativistic Precessions of Macroscopic Objects”, \textit{Nature}, \textbf{239}, 207-207 (1972).

\textsuperscript{21} Thomas E. Phipps, Jr., “Kinematics of a Rigid Rotor”, \textit{Nuovo Cimento Lett.}, \textbf{9}, 467-470 (1974).