Model and Optimization of Low-Frequency Sound Absorption Performance of Multilayer Microperforated Panel Absorber

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Abstract. Optimization of the low-frequency sound absorption performance of the multilayer microperforated panel (MPP) absorber is favourable to promote its practical application in the noise reduction. Theoretical model of the multilayer MPP absorber was constructed based on the Maa’s theory for the further optimization. For target frequency range of 100-500Hz, parameters of the multilayer MPP absorbers with layer number of 1 to 4 were optimized through the cuckoo search algorithm according to the certain optimization objectives and the constraint conditions. Judging from the achieved optimal structural parameters for the multilayer MPP absorber with the target frequency ranges and the corresponding average sound absorption coefficients, it could be concluded that the parameter optimization was not only effective, but also be quite necessary. Meanwhile, it could be seen that average sound absorption coefficients of the optimal multilayer MPP absorbers decreased gradually along with increase of the layer number from 1 to 4. Through the optimization of structural parameters of the multilayer MPP absorbers, not only the sound absorption performance could be improved, but also the fabrication cost could be reduced. The research results exhibited outstanding low-frequency absorption performance, and it would be favourable to promote their practical applications to reduce the industrial noise in urban area.

1. Introduction

The ever-growing noise pollution not only impairs the human being’s health, but also destroys balance of the earth’s ecosystem, especially those in the low-frequency ranges [1-4]. Judging from these research achievements on the noise pollution [1-4], it can be found that low-frequency noise reduction is not only necessary and essential, but also urgent and pressing. In order to reduce the industrial noise with the variable frequency ranges in the selected urban area, the multilayer MPP absorbers with layer number of 1 to 4 were investigated in this study, objectives of which were to achieve maximum average sound absorption coefficients for target frequency range of 100-500 Hz.

Theoretical sound absorption model was built firstly based on the classical Maa’s theory [5-7], which supplied theoretical basement for the further optimization of the multilayer MPP absorbers. Secondly, with the summarized constraint conditions and certain optimization objectives, structural parameters of the investigated multilayer MPP absorbers were optimized through the cuckoo search algorithm [8-11]. Meanwhile, the comparisons of sound absorption performance of the multilayer MPP absorbers with the different layer numbers were conducted, which aimed to find the optimal acoustic absorbers for the target frequency range and provide meaningful guidance to develop the appropriate acoustic absorbers for other conditions. Through the optimization of structural parameters of the multilayer MPP absorbers, not only the sound absorption performance could be improved, but also the fabrication cost could be reduced. The research results would be favorable to promote their practical applications.
2. Materials and Methods

2.1. Multilayer MPP absorber
Schematic diagram of the studied multilayer MPP absorber is shown in the Figure 1, which consists of \( n \) monolayer MPP absorbers. In this research, the microperforated holes were circular, and they were arranged in linear and equidistant distribution. For each monolayer MPP absorber, structural parameters included the thickness of the panel \( t_i (i = 1, 2, \ldots, n) \), diameter of the microholes \( d_i (i = 1, 2, \ldots, n) \), distance between the neighboring microholes \( b_i (i = 1, 2, \ldots, n) \), and the length of the rear cavity \( D_i (i = 1, 2, \ldots, n) \), which determined its sound absorption performance. Thus, for the studied \( n \)-layers MPP absorber, total number of its structural parameters was \( 4n \).

![Figure 1. Schematic diagram of the investigated multilayer MPP absorber in this study.](image)

2.2. Theoretical sound absorption model
Theoretical sound absorption model of the multilayer MPP absorbers could be constructed through the transfer matrix method [12-14], which required to derive transfer matrix of each MPP and that of each cavity in the multilayer MPP absorbers based on the Maa’s theory [5-7].

2.2.1. Transfer matrix of the MPP
For the \( i^{th} \) layer MPP in the Figure 1, its transfer matrix \( T_{mopi} \) could be calculated by the Eq. (1) based on the Maa’s theory [5-7]. Here \( Z_s \) was corresponding acoustic impedance rate of the \( i^{th} \) layer MPP, which included the real part \( R_i \) and imaginary part \( X_i \), as shown in the Eq. (2).

\[
T_{mopi} = \begin{bmatrix} 1 & Z_s \\ 0 & 1 \end{bmatrix} 
\]

(1)

\[
Z_s = R_i + jX_i 
\]

(2)

In the Eq. (2), \( j \) was symbol of the imaginary number, \( j = \sqrt{-1} \); The real part \( R_i \) and imaginary part \( X_i \) could be calculated by the Eqs. (3) and (4) respectively.

\[
R_i = \frac{32(\mu + \nu) \rho}{\varepsilon_i} \cdot \frac{t_i}{d_i^2} \cdot k_{ni} 
\]

(3)

\[
X_i = \frac{t_i \omega \rho}{\varepsilon_i} \cdot k_{ni} 
\]

(4)

In the Eqs. (3) and (4), \( \mu \) was viscosity coefficient of the air, \( 1.506 \times 10^{-5} \text{ m}^2/\text{s} \); \( \nu \) was temperature conduction coefficient of metal panel, \( 2.0 \times 10^{-5} \text{ m}^2/\text{s} \); \( \rho \) was density of the air with room temperature.
and atmospheric pressure, 1.21 kg/m$^3$; $\varepsilon_{i}$ was the microperforation rate, which could be calculated by the Eq. (5); $t_{i}$ and $d_{i}$ were thickness of the panel and diameter of the microholes respectively, which were consistent with those symbols in the Figure 1; $k_{ri}$ and $k_{mi}$ were acoustic resistance constant and acoustic mass constant, which could be calculated through the Eqs. (6) and (7) respectively; $\omega$ was the angular frequency, which was obtained by the Eq. (8), and $f$ was the sound frequency.

$$\varepsilon_{i} = \frac{\pi}{4} \left( \frac{d_{i}}{b_{i}} \right)^{2}$$  \hspace{1cm} (5)

$$k_{ri} = \sqrt{\frac{1 + k_{i}^2}{32} + \frac{\sqrt{2}}{8} k_{i} \cdot \frac{d_{i}}{t_{i}}}$$  \hspace{1cm} (6)

$$k_{mi} = 1 + \frac{1}{\sqrt{9 + k_{i}^2}} + 0.85 \cdot \frac{d_{i}}{t_{i}}$$  \hspace{1cm} (7)

$$\omega = 2\pi f, f \in [f_{\text{min}}, f_{\text{max}}]$$  \hspace{1cm} (8)

In the Eq. (5), $d_{i}$ and $b_{i}$ were the diameter of the microholes and distance between the neighboring microholes respectively, which were consistent with those symbols in the Figure 1. In the Eqs. (6) and (7), $i$ was the perforated panel constant, which could be calculated through Eq. (9), and meanings of the symbols $\omega$, $\mu$, $\nu$, and $d_{i}$ were consistent with these definitions above. By the Eqs. (1) to (9), transfer matrix $T_{\text{appi}}$ of the $i_{th}$ layer MPP was obtained.

$$k_{i} = \frac{\omega}{\mu + \nu} \cdot \frac{d_{i}}{2}$$  \hspace{1cm} (9)

2.2.2. Transfer matrix of the cavity

For the rear cavity behind the $i_{th}$ layer MPP, its transfer matrix $T_{\text{cavi}}$ could be calculated by the Eq. (10) based on the Maa’s theory [5-7]. Here $D_{i}$ was length of the rear cavity, which was consistent with the symbol in the Figure 1, and meanings of these symbols $\omega$, $c$, and $j$ were consistent with the corresponding definitions in the section 2.2.1.

$$T_{\text{cavi}} = \begin{bmatrix} \cos \left( \frac{\omega}{c} D_{i} \right) & j \rho c \sin \left( \frac{\omega}{c} D_{i} \right) \\ j \rho c \sin \left( \frac{\omega}{c} D_{i} \right) & \cos \left( \frac{\omega}{c} D_{i} \right) \end{bmatrix}$$  \hspace{1cm} (10)

2.2.3. Sound absorption coefficient of the multilayer MPP absorber

Total transfer matrix of the n-layers MPP absorber $T_{\text{total}}$ could be achieved by the Eqs. (1) to (10), as shown in the Eq. (11), and the theoretical sound absorption coefficient $a(f)$ could be calculated by the Eq. (12) according to the transfer matrix method [12-14]. The total transfer matrix $T_{\text{total}}$ was a 2$^n$ x 2$^n$ matrix, and the $T_{11}$ and $T_{21}$ in the Eqs. (11) and (12) were two components in it. Meanwhile, in the Eq. (12), meanings of the symbols $\rho$ and $c$ were consistent with the definitions in the section above. By this method, for
the given frequency \( f \), sound absorption coefficient of the multilayer MPP absorber could be calculated, and its sound absorption performance was decided by its structural parameters [5-7].

\[
T_{\text{total}} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \prod_{i=1}^{n} (T_{\text{mpp}_i} \cdot T_{\text{cav}_i})
\]

\[
\alpha = 4 \Re \left( \frac{T_{11}}{T_{21}} \cdot \frac{1}{\rho e} \right) \left[ 1 + \Re \left( \frac{T_{11}}{T_{21}} \cdot \frac{1}{\rho e} \right) \right]^{2} + \left| \Im \left( \frac{T_{11}}{T_{21}} \cdot \frac{1}{\rho e} \right) \right|^{2}
\]

3. Cuckoo search optimization algorithm

Based on the constructed theoretical sound absorption model in the section 2.2, the optimal structural parameters of the multilayer MPP absorbers could be obtained through the cuckoo search optimization algorithm [8-11], which required to confirm the optimization objectives and constraint conditions. Since average sound absorption coefficient \( \text{mean}(\alpha(f)) \) in a certain frequency range \([f_{\text{min}}, f_{\text{max}}]\) was a typical index to assess sound absorption performance of the MPP absorbers, its maximization was treated as the optimization objective in the cuckoo search algorithm, as shown in the Eq. (13).

\[
\text{max} \left( \text{mean}(\alpha(f)), f \in [f_{\text{min}}, f_{\text{max}}] \right)
\]

According to definition of the MPP [5-7], thickness of the panel \( t_i \) should be in the range of 0.1-1 mm. Since the proposed sound absorber was expected to reduce the industrial noise, strength and stiffness of the panel should be taken into consideration. Thus, aluminum (Al) plate was selected to prepare each monolayer MPP for the n-layers MPP absorbers in this research, and its thickness \( t_i \) was designated as 0.5 mm, as shown in the Eq. (14).

\[
t_i = 0.5 \text{ mm} \quad i = 1, 2, \ldots, n
\]

Meanwhile, fabrication cost of the MPP would rise obviously along with decrease of diameter of the microholes \( d_i \) and decrease of distance between the neighboring holes \( b_i \), which indicated more tiny holes. In order to promote practical application of the proposed multilayer MPP absorbers, fabrication cost should be taken into consideration. According to common processing capabilities of the normal machines to fabricate the microholes, constraint conditions for diameter of the microholes \( d_i \) and distance between the neighboring holes \( b_i \) for the n-layers MPP absorber were confirmed, as shown in the Eqs. (15) and (16) respectively. Moreover, length of each cavity \( D_i \) for the n-layers MPP absorber should be positive number, and the constraint condition for each cavity was shown in the Eq. (17).

\[
0.2 \text{ mm} \# d_i \quad 2 \text{ mm} \quad i = 1, 2, \ldots, n
\]

\[
1 \text{ mm} \# b_i \quad 20 \text{ mm} \quad i = 1, 2, \ldots, n
\]

\[
0 \leq D_i \quad i = 1, 2, \ldots, n
\]

Furthermore, total thickness of the sound absorber was also an important factors to evaluate its sound absorption efficiency, which determined the required space to install it. The acoustic absorber with large total thickness would limit its practical application. For the n-layers MPP absorber in this study, its total thickness \( L \) was determined by the thickness of each monolayer panel and length of each rear cavity, as shown in the Eq. (18). Taking the expected application of the multilayer MPP absorber into account,
limit of total thickness of the sound absorber was designated as 20 mm, which indicated that total thickness of the n-layers MPP absorber $L$ should be no more than 20 mm, as shown in the Eq. (19).

$$L = \sum_{i=1}^{n} (t_i + D_i) \text{ for } i = 1, 2, ..., n$$

(18)

$$L \leq 20 \text{ mm}$$

(19)

3.1. The optimal structural parameters

Based on the constructed theoretical sound absorption model in the section 2.2, the objective function $f(X)$ was defined, and the $n$ groups of the solutions $X = (X_1, X_2, ..., X_n)$ were randomly generated. Through setting the maximum discovery probability and the maximum iteration number, cuckoo search algorithm was initialized. Afterwards, through the iterations of calculation of the objective value, update of the next generation solution, achievement of the superior solution, and the judgement of whether the superior solution was satisfied, the final optimal solution and optimal objective value were obtained by the cuckoo search algorithm [8-11].

The achieved optimal structural parameters for frequency ranges of 100-500 Hz was summarized in Table 1, and the corresponding theoretical average sound absorption coefficients were also exhibited. It was interesting to note that for each target frequency range, the average sound absorption coefficient of the optimal multilayer MPP absorber did not keep growing with increase of the layer number from 1 to 4. It could be seen from Table 1 that average sound absorption coefficients of the optimal multilayer MPP absorbers decreased gradually along with increase of the layer number from 1 to 4. It was known that sound absorption band would be enlarged gradually with increase of layer number of the multilayer MPP absorber, which indicated that promotion of sound absorption performance of the sound absorber with large target frequency range would be better than that with small target frequency range. Sound absorption property of monolayer MPP was weakened along with increase of the layer number, because the available space for each MPP was reduced and superimposition of absorption peaks was destroyed.

Table 1. The achieved optimal structural parameters for the target frequency range of 100-500 Hz and the corresponding theoretical average sound absorption coefficients

| Multilayer MPP absorber | Layer series | Thickness of the panel (mm) | Diameter of the microholes (mm) | Distance between the neighboring holes (mm) | Length of the cavity (mm) | Average sound absorption coefficient |
|-------------------------|--------------|-----------------------------|---------------------------------|-------------------------------------------|--------------------------|-----------------------------------|
| 1-layer                 | 1-layer      | 0.5                         | 0.96                            | 20.00                                     | 19.50                    | 0.3528                            |
|                         | 2-layer      | 0.5                         | 0.99                            | 17.40                                     | 20.00                    | 0.3523                            |
| 2-layers                | 1-layer      | 0.5                         | 0.99                            | 17.40                                     | 19.50                    | 0.3523                            |
|                         | 2-layer      | 0.5                         | 2.00                            | 17.40                                     | 19.00                    | 0.3523                            |
| 3-layers                | 1-layer      | 0.5                         | 1.39                            | 19.75                                     | 19.75                    | 0.3488                            |
|                         | 2-layer      | 0.5                         | 1.43                            | 20.00                                     | 16.60                    | 0.3488                            |
|                         | 3-layer      | 0.5                         | 0.36                            | 19.72                                     | 1.80                     | 0.3419                            |
| 4-layers                | 1-layer      | 0.5                         | 1.04                            | 18.70                                     | 0.00                     | 0.3419                            |
|                         | 2-layer      | 0.5                         | 1.95                            | 19.51                                     | 5.84                     | 0.3419                            |
|                         | 3-layer      | 0.5                         | 1.67                            | 3.58                                      | 9.92                     | 0.3419                            |
|                         | 4-layer      | 0.5                         | 1.62                            | 5.51                                      | 2.24                     | 0.3419                            |

Judging from the achieved optimal structural parameters for the multilayer MPP absorber with the target frequency ranges and the corresponding average sound absorption coefficients in the Table 1, it could be concluded that the parameter optimization was not only effective, but also be quite necessary. Through the optimization of structural parameters of the multilayer MPP absorbers, not only the sound absorption performance could be improved, but also the fabrication cost could be reduced.
4. Conclusions
According to the selected optimization objectives and the confirmed constraint conditions, the structural parameters of the multilayer MPP absorbers were optimized by the cuckoo search algorithm based on the constructed theoretical sound absorption model, which provided the effective guidance for following preparation of the samples and evaluation of the sound absorption performance. The outstanding low-frequency sound absorption performance was achieved through the cuckoo search optimization of the multilayer MPP absorbers with the variable frequency range, which would be favorable to promote their practical applications to reduce the industrial noise in the urban area.

Acknowledgments
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References
[1] Yang X C, Shen X M, Duan H Q, Zhang X N, Yin Q 2020 Identification of Acoustic Characteristic Parameters and Improvement of Sound Absorption Performance for Porous Metal *Metals* 10(3) 340.
[2] Bai P F, Yang X C, Shen X M, Zhang X N, Li Z Z, Yin Q, Jiang G L, Yang F 2019 Sound absorption performance of the acoustic absorber fabricated by compression and microperforation of the porous metal *Materials & Design* 167 107637.
[3] Yang X C, Bai P F, Shen X M, Zhang X N, Zhu J W, Yin Q, Peng K 2019 Theoretical Modeling and Experimental Validation of Sound-Absorbing Coefficient of Porous Iron *Journal of Porous Media* 22(2) 225-241.
[4] Bai P F, Shen X M, Zhang X N, Yang X C, Yin Q, Liu A X 2018 Influences of compression ratio on sound absorption performance of porous nickel-iron alloy *Metals* 8(7) 539.
[5] Yang X C, Bai P F, Shen X M, To S, Chen L, Zhang X N, Yin Q 2019 Optimal design and experimental validation of sound absorbing multilayer microperforated panel with constraint conditions *Applied Acoustics* 146 334-344.
[6] Duan H Q, Shen X M, Yang F, Bai P F, Lou X F, Li Z Z 2019 Parameter Optimization for Composite Structures of Microperforated Panel and Porous Metal for Optimal Sound Absorption Performance *Applied Sciences* 9(22) 4798.
[7] Yang X C, Chen L, Shen X M, Bai P F, To S, Zhang X N, Li Z Z 2019 Optimization of geometric parameters of the standardized multilayer microperforated panel with finite dimension *Noise Control Engineering Journal* 67(3) 197-209.
[8] Shen X M, Bai P F, Yang X C, Zhang X N, To S 2019 Low-frequency sound absorption by optimal combination structure of porous metal and microperforated panel *Applied Sciences* 9(7) 1507.
[9] Yang X C, Shen X M, Duan H Q, Yang F, Zhang X N, Pan M, Yin Q 2020 Improving and Optimizing Sound Absorption Performance of Polyurethane Foam by Prepositive Microperforated Poly(methyl methacrylate) Panel *Applied Sciences* 10(6) 2103.
[10] Yang F, Shen X M, Bai P F, Zhang X N, Li Z Z, Yin Q 2019 Optimization and Validation of Sound Absorption Performance of 10-layer Gradient Compressed Porous Metal *Metals* 9(5) 588.
[11] Yang X C, Shen X M, Bai P F, He X H, Zhang X N, Li Z Z, Chen L, Yin Q 2019 Preparation and characterization of gradient compressed porous metal for high-efficiency and thin-thickness acoustic absorber *Materials* 12(9) 1413.
[12] Shen X M, Bai P F, Chen L, To S, Yang F, Zhang X N, Yin Q 2020 Development of thin sound absorber by parameter optimization of multilayer compressed porous metal with rear cavity *Applied Acoustics* 159 107071.
[13] Yang X C, Peng K, Shen X M, Zhang X N, Bai P F, Xu P J 2017 Geometrical and dimensional optimization of sound absorbing porous copper with cavity *Materials & Design* 131 297-306.
[14] Duan H Q, Shen X M, Yin Q, Yang F, Bai P F, Zhang X N, Pan M 2020 Modeling and optimization of sound absorption coefficient of microperforated compressed porous metal panel absorber *Applied Acoustics* 166 107322.