New coorbital dynamics in the solar system

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Abstract

Following the discovery that asteroid (3753) Cruithne was a coorbital companion of the Earth, a new theory of coorbital motion has been developed whereby planets or satellites can maintain companion objects in the same orbit as themselves. This has led to the prediction of hitherto unknown types of stable motion, all of which are seen in the evolution of specific near-Earth asteroids. The slow diffusion of such objects through the Earth’s coorbital region is shown to lead to temporary capture, suggesting the existence of undiscovered retrograde moons of the Earth.

1. Introduction

Lagrange\(^1\) demonstrated the existence of five equilibrium positions in the three-body problem. Those that form an equilateral triangle with the Sun and the planet, the leading \(L_4\) point and trailing \(L_5\) point, are stable to small displacements. However, it was not until the subsequent discovery of the first Trojan asteroid, (588) Achilles, near the \(L_4\) point in the Sun-Jupiter system that a real example of such motion was first observed. There are now known to be at least 400 Trojan asteroids in the orbit of Jupiter, and at least one, (5261) Eureka, near Mars’s \(L_5\) point;\(^2\) there are also three examples of Trojan moons in the Saturnian system.\(^3\) In fact, all the known Trojan objects are librating about \(L_4\) or \(L_5\) points and their orbits are often referred to as tadpoles (or T orbits) because of their shape with respect to the equilibrium position, as viewed in the frame rotating with the mean angular velocity (mean motion) of the planet (see Fig 1).

When the angular amplitudes of the leading and trailing tadpoles are sufficiently large, the orbits merge near the unstable \(L_3\) point located 180° from the planet; the resulting paths in the rotating frame are referred to as horseshoes (or H orbits). A variation on such a structure is seen in the orbits of Janus and Epimetheus, the coorbital satellites of Saturn.\(^4\) In this particular case, a good theoretical understanding of the peculiar orbital dynamics is possible because of the low eccentricities (\(e < 0.01\)) and inclinations (\(I < 0.4°\)) of both objects. In contrast, it has recently been discovered that asteroid (3753) Cruithne, previously designated 1986TO, performs a temporary horseshoe-like orbit with respect to the Earth.\(^5\) This is a more extreme example that has been difficult to incorporate in an analytical theory because of Cruithne’s large eccentricity (\(e = 0.515\)) and inclination (\(I = 19.8°\)).

In this article, we present aspects of the general theory of coorbital motion that accounts for all types of coorbital behavior.\(^6\) This shows that asteroid (3753) Cruithne is only a member of a more general class of objects that are captured in the coorbital regions of the planets, and that involve new types of coorbital motion at large eccentricity and inclination. We also report the identification of these new orbits in the motion of specific asteroids. For the cases we have examined, the additional planetary perturbations can cause near-Earth objects to be trapped into the coorbital regions of the terrestrial planets without any risk of collisions. The structures we found provide a new paradigm for coorbital motion in the three-body problem and we believe that they have played an important role in such diverse problems as the origin of retrograde outer satellites and the accretional processes in the early solar system.

2. Types of motion

In the Sun-planet-asteroid three-body problem, the motion of an asteroid on a small eccentricity and inclination orbit is constrained by the value of the Jacobi constant through the ‘excluded’ regions associated with the zero velocity curves\(^7\) (Fig 1). The T and H librations are found to be limited to the annulus between the Lagrangian points \(L_1\) and \(L_2\) defined as the coorbital region of the planet; its half-width equals the radius of the planet’s sphere of influence:

\[
\Delta_P = \left[ m_P / 3(m_P + m_S) \right]^{1/3} a_P
\]  

(1)
where $a_P$ is the semi-major axis of the planet’s orbit, and $m_P$ and $m_S$ are the masses of the planet and the Sun respectively. The outer boundary of the coorbital region is made up of irregular orbits and separates the H orbits from the passing orbits (or P orbits).

For large eccentricities, the previous picture of coorbital motion is modified substantially because the Jacobi constant can no longer exclude spatial regions for the motion of the asteroid. In addition, the large eccentricity of the asteroid’s orbit can lead to physical collisions with the planet. There are two facts that allows us to develop a theory that is valid for all types of coorbital motion: the first is the realisation that the asteroid’s motion can be described as the superposition of two types of motion: (i) a fast three dimensional gyration with amplitudes of $O(aAe)$ and $O(aA \sin I)$ and a frequency equal to the asteroid’s mean motion and (ii) the slow evolution of the mean position of the asteroid with respect to the planet. The latter is referred to as the motion of the guiding center and is represented by the conjugate variables: relative semi-major axis, $a_r = (a_A - a_P)/a_P$, and relative mean longitude, $\lambda_r = \lambda_A - \lambda_P$.

The second fact is the regularity of $e$, $I$ and the argument of perihelion $\omega$ away from the boundary of the coorbital region; this allows us to derive the motion of the guiding center by perturbation analysis. The Jacobi constant can then be used to constrain the motion of the guiding center instead of the full motion, by averaging with respect to the fast gyration represented by $\lambda_A$. The guiding center $a_r(\lambda_r)$ can be written as:

$$a_r^2 = C - \frac{8m_P}{3(m_P + m_S)} S(\lambda_r, e, I, \omega)$$  \hspace{1cm} (2)

where $C$ is a constant that defines the nature of the orbit, and $S$ is related to the averaged interaction potential by:

$$S = \frac{1}{2\pi} \int_{-\pi}^{\pi} (|r_A - r_P|^{-1} - r_A \cdot r_P) \, d\lambda_A \bigg|_{a_r=0}.$$  \hspace{1cm} (3)

In equation (3), $r_A$ and $r_P$ are the position vectors, and the second term is due to the motion of the Sun; the distances are scaled to $a_P$. The motion along the guiding center can now be viewed as that of a particle in the unidimensional potential well $S$ (Fig 2).

The fact that the shape of the guiding center depends on $e$, $I$ and $\omega$, has three major consequences for the coorbital motion: firstly, the effective Lagrangian points $L_4$ and $L_5$ can be displaced appreciably from the equilateral configuration (Fig 2 b,c,d). In addition, they no longer correspond necessarily to the same Jacobi constant, which fact implies an asymmetry in the distribution of T orbits.

Secondly, retrograde satellite orbits (or RS orbits) appear outside the planet’s sphere of influence as they are the only family of bounded orbits around the planet that remain stable with increasing eccentricity (Compare a and b in Fig 2). For the planar motion, it can be seen that this family is separated from the H and T orbits by the collision singularity, $r_A = r_P$, at $\lambda_r \approx 2e$ for $e \leq 0.7$. This implies that RS orbits and H orbits merge inside the inner boundary of the P orbit domain and therefore, that collisional orbits are unstable in two dimensions.

In three dimensions, however, physical collisions occur only when the nodes of the asteroid’s orbit cross the orbit of the planet; this corresponds to a specific argument of perihelion given by $\cos \omega \approx e$. Consequently, the collision singularity of $S$ is generally removed and replaced by maxima whose existence and relative magnitudes depend on $e$, $I$ and $\omega$.

This leads to the third major consequence of large $e$ and $I$: the appearance of compound and transition orbits (Fig 2 c,d). The first correspond to the merger of H or T orbits with RS orbits; asteroid (3753) Cruithne is the first example of a compound H-RS orbit in the solar system. The second orbits correspond to the maxima of $S$; however, they are not long-lived because of the secular variations of $e$, $I$ and $\omega$, in time which modify the magnitudes of the corresponding maxima. Transition orbits therefore appear in the orbital evolution only to permit the passage between different orbit families.

The secular stability that was found in the three-body problem, together with the presence of asteroid Cruithne on an H-RS orbit, suggested the robustness of this new dynamics when exposed to planetary perturbations. This motivated our search for further examples in the solar system.

### 3. New coorbital asteroids

We have conducted a search in the catalogue of near-Earth objects (NEOs) which is maintained in the Minor Planet Center web-site for objects with semi-major axes closer than approximately $\Delta_P$ from the semi-major axis of a terrestrial planet. We carefully avoided objects with poorly determined orbital elements, choosing to
include in our sample only multi-apparition asteroids. We stress the need here for further observations that refine the orbits of the known near-Earth asteroids. Our selection criteria yielded a total of five objects, ~1% of the known NEO population, as favorite candidates for transient coorbital behavior. These asteroids are (3753) Cruithne, (3362) Khufu, 1989 VA, 1993 WD and 1994 TF2. Here, we present results for three of those objects (see Table 1) as representative of what is observed for the full sample.

The orbits were integrated numerically\textsuperscript{11} with the eight major planets (Mercury to Neptune) using a Runge-Kutta-Nystrom 12th order scheme with adaptive step-size control.\textsuperscript{12} This presents an advantage over the scheme used in ref 5 due to its ability to model close encounters, a vital part of the mechanism which gives rise to stable orbit transitions. The integration time-span was set to be 200,000 years centered on the present as a reasonable trade-off between integrator accuracy, which may deteriorate for longer time-spans, and the richness of the coorbital dynamics that we expect to observe.

All the objects examined exhibited transient coorbital behavior. Figures 3 and 4 show the orbits’ evolution during the coorbital capture of (3753) Cruithne and (3362) Khufu by the Earth, and 1989 VA by Venus. We confirm the existence of the new structures discussed in the previous section, such as compound orbits (Cruithne, 1989 VA), displaced tadpole librations (Cruithne), retrograde satellites of the Earth (Khufu, Cruithne) and the transitions between different coorbital modes. Owing to the intervening periods of stochastic drift, mainly in $a_r$, we cannot claim that we observe the actual evolution of the asteroids in the integration time-span. What we can say, however, is that each one of those asteroids is likely to have been or will be captured in such orbits. This claim is supported by our numerical experiments with slightly modified orbits for the asteroids (i.e. clone asteroids).

4. Discussion

Unlike (3753) Cruithne which is currently on an H-RS orbit, well inside the coorbital region of the Earth, the remaining objects that we examined are not currently in coorbital libration with any planet. This fact, however, did not induce distinct types of global evolution in the sample. For instance, during the evolution, Cruithne happened to exit the coorbital region; in contrast, the other objects could be captured by the Earth or Venus. This shows that Cruithne’s status is less special than previously thought: it is simply a member of the population of NEOs that evolve inside the inner solar system with a slowly varying semi-major axis.\textsuperscript{13}

This seemingly disappointing conclusion about the true nature of Cruithne’s evolution can be viewed positively in the context of the protection of the Earth from collisions with nearby asteroids. The evolution of the mentioned asteroids and their clones indeed suggests that the Earth and Venus are well protected from the class of NEOs that drift (in semi-major axis) towards and get close to the coorbital regions of the terrestrial planets. A key factor in this protection is the quasi-absence of the chaotic layer at the edges of the coorbital region, found at small $e$ and $I$, and responsible for the instability during the crossing of coorbital region.\textsuperscript{6,14} This suggests the presence of undiscovered coorbital asteroids of the terrestrial planets, that display one of the new coorbital modes. In particular, the connection between retrograde satellite orbits and the classic types of motion is remarkable because it allows a fraction of the drifting NEOs to become, temporarily, retrograde satellites of the terrestrial planets. Clear examples of this behavior are the past trapping of Khufu that lasted 35,000 years and the future 3,000 year RS phase of Cruithne’s evolution. This same connection also offers a new route for the capture of the known retrograde satellites of the Jovian planets: the dissipative processes of early planetary formation such as the planet’s growth\textsuperscript{15} or the drag due to the circumplanetary nebula,\textsuperscript{16} can produce permanent capture by reducing the Jacobi constant of a Trojan asteroid that undergoes a transient RS motion. This results in a distant retrograde satellite with large planetocentric eccentricity and inclination, which is reminiscent of the orbits of the known retrograde satellites of the Jovian planets.\textsuperscript{17}

The evolution of our sample shows that the coorbital asteroids of Earth and Venus with large $e$ and $I$ are generally expelled from the coorbital regions because of the close encounters with the other planets. In that respect, Mercury is a better candidate for finding permanent coorbitals because of its distant location from Venus. The presence of a specific population of minor bodies close to Mercury has already been proposed in the context of the vulcanoid hypothesis for the chronology of the geological evolution the planet.\textsuperscript{18} However, the dynamical constraints of the model assumed that the vulcanoids had small eccentricities and inclinations and the recent observational searches\textsuperscript{18} produced negative results. In view of (1) the large eccentricity of Mercury ($e = 0.2$), (2) the action of nearby secular resonances and (3) the stability of the structures we found, the putative coorbital vulcanoids are likely to have stable large eccentricity and inclination orbits. The evidence of
such objects would argue for the early presence and slow depletion of a larger reservoir of bodies that contributed significantly to the cratering history of Mercury.

References

[1] J. L. Lagrange, *Oeuvres*, M. J.-A. Serret, Ed. (Gauthier-Villars, Paris, 1867–1892).
[2] S. Mikkola, K. A. Innanen, K. Muinonen and E. Bowell, *Celest. Mech. Dyn. Astron*. **58**, 53 (1994).
[3] P. Laques and J. Lecacheux, *IAU Circular 3457* (1980). P. K. Seidelmann et al., *Icarus* **47**, 282 (1981). B. A. Smith, *IAU Circular 3602* (1981).
[4] S. F. Dermott and C. D. Murray, *Icarus* **48**, 12 (1981). C. F. Yoder, G. Colombo, S. P. Synnott and K. A. Yoder, *Icarus* **53**, 431 (1983).
[5] P. A. Wiegert, K. A. Innanen and S. Mikkola, *Nature* **387**, 685 (1997), *Astron. J.* **115**, 2604 (1998).
[6] F. Namouni, *Icarus* **137**, 293 (1999). A general classification of the types of coorbital motion is done in F. Namouni, in preparation.
[7] V. Szebehely, *The theory of orbits* (Academic Press, New York, 1967). S. F. Dermott and C. D. Murray, *Icarus* **48**, 1 (1981).
[8] F. Namouni and C. D. Murray, in preparation.
[9] M. Hénon, *Astron. Astroph*. **1**, 223 (1969).
[10] G. V. Williams, Minor Planet Center: [http://cfa-www.harvard.edu/iau/mpc.html](http://cfa-www.harvard.edu/iau/mpc.html) (1999).
[11] A. A. Christou, *Icarus* (submitted).
[12] J. R. Dormand, M. E. A. El-Mikkawy and P. J. Prince, *IMA J. Num. Analysis* **7**, 423 (1987).
[13] Ch. Froeschlé, G. Hahn, R. Gonczi, A. Morbidelli and P. Farinella, *Icarus* **117**, 45 (1995).
[14] J.-M. Petit and M. Hénon, *Icarus* **66**, 536 (1986).
[15] T. A. Heppenheimer and C. Porco, *Icarus* **30**, 385 (1977).
[16] J. B. Pollack, J. A. Burns and M. E. Tauber, *Icarus* **37**, 587 (1979).
[17] *Satellites*, J. A. Burns and M. S. Matthews, Eds. (University of Arizona Press, Tucson, 1986). B. J. Gladman et al., *Nature* **392**, 897 (1998).
[18] M. A. Leake, C. R. Chapman, S. J. Weindenschilling, D. R. Davis, and R. Greenberg, *Icarus* **71**, 350 (1987). H. Campins, D. R. Davis, S. J. Weindenschilling and M. Magee, *ASP Conf. Series* **107**, 85 (1996).
[19] A. Chamberlin et al. HORIZONS ephemeris system: [telnet ssd.jpl.nasa.gov 6775](telnet ssd.jpl.nasa.gov 6775) (1999).
[20] We thank H. Morais, P. Nicholson and I. Williams for discussions, S. Collander-Brown for the use of his integration code and D. Hamilton for his comments on an earlier version of the paper. AAC was supported by the Sofoclis Achiliiopoulosp Foundation, Greece. The authors acknowledge financial support from the UK Particle Physics and Astronomy Research Council.
Figure captions

1. Representative zero velocity curves for three values of the Jacobi constant and the mass ratio \( m_P/(m_P + m_S) = 0.01 \). The locations of the Lagrangian equilibrium points \( L_1-L_5 \) are indicated by small open circles. The dashed line denotes a circle of radius equal to the planet’s semi-major axis. The letters T (tadpole), H (horseshoe) and P (passing) denote the type of orbit associated with the curves. The regions enclosed by each curve (shaded) are excluded from the spatial motion of the asteroid that has the corresponding value of the Jacobi constant. Note that the curve between the \( L_1 \) and \( L_2 \) points permits satellite orbits around the planet inside the Hill sphere of radius \( \Delta_P \) (Eq 1).

2. Types of coorbital motion. \( S \) is viewed as an effective potential well and the possible guiding centers are given by the levels \( 3(m_P + m_S)/8m_P \) (light lines). The dashed levels correspond to transition orbits. T, H, RS and P denote respectively tadpole, horseshoe and retrograde satellite and passing orbits. The hyphenated designations denote compound orbits. \( L_1 \) and \( L_2 \) are located at \( \lambda = 0^\circ \) and \( L_3 \) at \( \pm 180^\circ \). The bold ticks denote the locations of \( L_4 \) and \( L_5 \). The comparison of \( a \) (circular 2D orbits) and \( b \) (eccentric 2D orbits, \( e = 0.3 \)) shows the appearance of retrograde satellite orbits outside the Hill sphere and the displacement of \( L_4 \) and \( L_5 \) form the equilateral configuration \( \lambda = \pm 60^\circ \). In 3D, \( c: e = 0.3, I = 20^\circ, \omega = 60^\circ \); \( d: e = 0.5, I = 30^\circ, \omega = 0^\circ \), orbit transition and compound orbits become possible. Note the asymmetry in the location and the maxima of \( \lambda = 0^\circ \). Because \( S(\lambda, e, I, -\omega) = S(-\lambda, e, I, \omega) \), the coorbital motion also includes orbits that are symmetric to those in \( c \) with respect to \( \lambda = 0^\circ \).

3. Coorbital capture of near-Earth asteroids. The evolution of the relative semi-major axis, \( a_r \), and the difference in mean longitudes between the object and the relevant planet, \( \lambda_r \), are shown as functions of time. The plots correspond to (3753) Cruithne (\( a \), \( b \)) and (3362) Khufu (\( c \), \( d \)) coorbiting with the Earth, and 1989 VA (\( e \), \( f \)) coorbiting with Venus. The asteroids are seen to transit between many types of coorbital motion (Fig 2) for time-scales ranging from thousands to tens of thousands of years. Note that in all three cases, the transitions occur without the large jumps in semi-major axis which are indicative of unstable close approaches.

4. Coorbital librations of the guiding center. The trajectories observed in Fig 4 are shown in the \( (\lambda_r, a_r) \) plane. \( a: (3753) \) Cruithne, \( b: (3362) \) Khufu and \( c: 1989 \) VA. Note that the proximity of Khufu’s guiding center (\( \sim 1^\circ \)) to the Earth (of radius \( \sim 9^\circ \)) does not result in a near collision because the full motion of the asteroid includes the fast gyration that was averaged out and whose smallest amplitude is \( \sim a_\lambda e \). This means that the asteroid is \( \sim 0.5 \) AU away from the Earth. The temporary proximity to \( \lambda_r = 0^\circ \) implies that the trajectory associated with the fast gyration is centered on the Earth and does not precess secularly.

Table 1: Osculating elements for three coorbital candidates at JD 2451000.5 as given by the JPL HORIZONS ephemeris generation system.\(^{19}\) \( a_A, e, I, \omega \) and \( \Omega \) denote respectively the semi-major axis, the eccentricity, the inclination, the argument of perihelion and the longitude of the ascending node. \( a_{rP} \) is the difference between the semi-major axes of the coorbital planet and the asteroid; it is given in terms of the width of the planet’s coorbital region \( \Delta_P \). OCC \(^{19}\) (Orbit Condition Code) provides a rating of the quality of the computed orbit based on the available observations. In this scale 0 is the best and 7 the worst with 1 the typical value for numbered asteroids.

| Object     | \( a_A \) (AU) | \( e \) | \( I \) (°) | \( \omega \) (°) | \( \Omega \) (°) | \( a_{rP} \) | OCC |
|------------|---------------|-------|-----------|---------------|---------------|---------|-----|
| (3362) Khufu | 0.9895       | 0.469 | 9.9       | 54.8          | 152.6         | 1.05 \( \Delta_{\text{Earth}} \) | 1    |
| (3753) Cruithne | 0.9978       | 0.515 | 19.8      | 43.7          | 126.4         | 0.22 \( \Delta_{\text{Earth}} \) | 1    |
| 1989 VA     | 0.7287       | 0.595 | 28.8      | 2.8           | 225.7         | 0.57 \( \Delta_{\text{Venus}} \) | 3    |
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