Field redefinition's help in constructing non-abelian gauge theories

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\section*{ABSTRACT}

We study, using the example of general covariance, to what extent a would-be non-abelian extension of free field abelian gauge theory can be helped by a field redefinition; answer – not much! However, models resulting from dimensional reduction also include non-gauge fields needing to be integrated out, thereby offering a wider choice of redefinitions whose effects may indeed change the situation.

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A pervasive feature in attempts to construct nonabelian gauge theories that are ultimately seen to be inconsistent is that the first – abelian invariant quadratic – action term exists, as does the next, cubic one, taken as the product 

\[ J^\alpha A^\alpha = J^\mu A^\mu + J^\nu A^\nu \]

...
derivative, order, useful field redefinitions must be algebraic, here
$h_{\mu\nu} \rightarrow h_{\mu\nu} + (h^3)_{\mu\nu}$. [There can obviously be no \((h^2)_{\mu\nu}\) redefi-
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tions because they would disturb the (assumed trivially correct) cubic terms.] The resulting quartic modification is
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$$
\Delta A(4) \sim \int d^4 x h_{\mu\nu} \Omega^{\mu\nu\rho\sigma} (h^3)_{\rho\sigma} = \int d^4 x G^{\rho\sigma} (\ln(h^3))_{\rho\sigma} .
$$
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not a very general form, even allowing for integrations by parts in
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the \(h^2\) structure \(\Omega^{\mu\nu\rho\sigma}\) operator. Thus IF and only IF the culprit
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part of the \(h^4\) term in \(A\) can be put in this manifestly “field-
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redefinable” form is there a hope of success, though even that is
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rather unlikely given the quintic effects of this redefinition; at best
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there would be an infinite series of higher power redefinitions re-
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quired. For our purposes, focusing on the first dangerous – quartic
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– deviation, the condition (3) already suffices to rule out most can-
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didates. At that, GR is the most favorable case because all terms
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are of the same derivative order, while models such as YM are of
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finite number and decreasing derivative order, so obviously even
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less amenable to field redefinitions, that we have seen start at sec-
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ond derivative order due to the quadratic kinematical term.¹

Our main take-home point is that the key test of nonabelian
structure and local symmetry occurs at the fourth order in fluctua-
tion fields, be this in a gauge theory such as Yang-Mills or in a
gravitational theory. In the full analysis of a complicated system
such as dimensional reduction on a manifold without killing sym-
metries, preservation of lower-dimensional local symmetry, and
consistency with the anticipated realization of such symmetry is to
be expected only after carefully integrating out heavy (non-zero-
mode) fields. Alternatively, field redefinitions including massive
non-gravitational fields could be made prior to integrating them
out, but these would have to prepare the eventual massive mode
integrations for a structure in which they made no changes in the
pure gravitational part of the theory. In either case, the crucial task
becomes how to obtain the correct anticipated pure gravitational
structure at fourth order in fluctuation fields after all heavy fields
are integrated out.

An example of a system which initially appears to generate
just such problems is reduction of type IIA supergravity on a non-
compact \(\mathcal{H}^{(2,2)}\) space which nonetheless yields an effective lower
dimensional theory as a result of a mass gap in the spectrum of the
(corresponding transverse wave function \(\xi\) [3]. Expansion of the
appropriate effective action initially reveals just such difficulties
at fourth order in the lower dimensional gravitational \(h_{\mu\nu}\). More-
over, similar difficulties at fourth order can be encountered in a
toy model variant of ordinary dimensional reduction of \(D = 5\) GR
where instead of an extra dimensional circle one reduces on a line
interval with mixed boundary conditions for the transverse wave
function: Dirichlet \(\xi(0) = 0\) on one side and Robin \(\xi'(1) = 0\) on the other.²

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References

[1] G. ’t Hooft, M.J.G. Veltman, One loop divergencies in the theory of gravitation, Ann. Inst. Henri Poincaré A, Phys. Théor. 20 (1974) 69.
[2] J.C. Criado, M. Pérez-Victoria, Field redefinitions in effective theories at higher orders, J. High Energy Phys. 1903 (2019) 038, https://doi.org/10.1007/JHEP03(2019)038, arXiv:1811.09413 [hep-ph].
[3] B. Crampton, C.N. Pope, K.S. Stelle, Braneworld localisation in hyperbolic spacetime: J. High Energy Phys. 1412 (2014) 035, https://doi.org/10.1007/JHEP12(2014)035, arXiv:1408.7072 [hep-th].
[4] M.J. Duff, B.E.W. Nilsson, C.N. Pope, N.P. Warner, On the consistency of the Kaluza-Klein ansatz, Phys. Lett. B 149 (1984) 90, https://doi.org/10.1016/0370-2693(84)91558-2.
[5] M.J. Duff, C.N. Pope, Consistent truncations in Kaluza-Klein theories, Nucl. Phys. B 253 (1985) 355, https://doi.org/10.1016/0550-3213(85)90140-3.
[6] M.J. Duff, S. Ferrara, C.N. Pope, K.S. Stelle, Massive Kaluza-Klein modes and effective theories of superstring modulus, Nucl. Phys. B 333 (1990) 783, https://doi.org/10.1016/0550-3213(90)90139-S.

¹ Gravitational field redefinitions were introduced, in a different context, by G. ’t Hooft and M. Veltman [1]. A recent list of some of the literature on field redefi-
2 nitions in effective theories may be found in [2].\[2\]

² The various problems involving technically inconsistent dimensional reduction are a large topic. Older literature on problems of consistent and technically incon-
sistent Kaluza-Klein reductions can be found in Refs. [4]. Details of the effective theory resulting from the \(\mathcal{H}^{(2,2)}\) reduction of type IIA supergravity and of the mixed
Dirichlet-Robin reduction of \(D = 5\) GR will be given in [5].