Accurate phase analysis of interferometric fringes by the spatiotemporal phase-shifting method

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Abstract
Phase-shifting interferometry (PSI) has been widely applied in the field of accurate optical methodology. However, the fluctuation of background and amplitude intensities due to the instability of laser source, and phase-shifting error or vibration are significant problems for the PSI. In this study, the spatiotemporal phase-shifting method (ST-PSM) (Ri S \textit{et al} 2019 \textit{J. Opt.} 21 095702), which is a highly accurate and robust phase analysis method using spatial and temporal intensities information simultaneously, is first applied to laser interferometry to achieve a stable measurement. Through several simulations, three effects of fluctuations in background and amplitude intensities, phase-shifting error were investigated. As a result, we clarified that the periodic phase error with fundamental or second harmonic frequencies occurs in the conventional PSM method, whereas no periodic error occurs in the ST-PSM. Besides, the ST-PSM is also robust to the noise either for uniform or distorted interference fringe images. In the Michelson laser interferometer experiment, the ST-PSM realizes a much more stable measurement of phase and phase gradient distributions than the PSM. We have revealed the excellent performance and the striking advantage that ST-PSM is entirely free of periodic errors in a similar manner to the simulations. Therefore, laser interferometry using the ST-PSM can be expected to apply to various applications, including the extremely accurate non-contact shape and deformation measurement, as well as thickness measurement of transparent materials in life and material sciences.

Keywords: fringe analysis, phase measurement, optical metrology, phase-shifting error, laser interferometry

(Some figures may appear in color only in the online journal)
1. Introduction

Interferometry is a fundamental technology in research on electromagnetic engineering ranging from gamma rays to visible light, radio waves, and sound waves. Phase-shifting interferometry (PSI), including Michelson, Fizeau [1], and Twyman-Green [2] interferometers, is a widely used technique in the past half a century because of its high measurement sensitivity, short time and the full field of view.

With this optical methodology, various physical quantities (such as the shape, out-of-plane displacement, refractive index distribution, etc) can be successfully and quantitatively obtained by calculating the phase distribution using three or more recorded interference fringes [3, 4]. The PSI allows measurements of wavefront deviations of the order of $\lambda/200$ with repeatability close to $\lambda/1000$. Thus, the PSI has been widely used not only the surface profile measurement of diffuse object [5] and highly reflective object [6], analysis of thickness or thickness variation of transparent plates [7–9], displacement measurement [10] in the industrial field, but also ranging to the areas in meteorology for monitoring of global warming [11], astronomy for gravitational-wave detection [12, 13], and biomedical imaging for optical coherence tomography and cancer diagnosis [14, 15].

While the PSI is widely used, it is susceptible to the instability of the laser source and the disturbing environment, which can lead to measurement errors that have not been completely solved. Specifically, phase-shift errors due to external vibration [16], the nonlinearity of piezo-electric transducer (PZT) and drift [17], the changes in the background and amplitude intensities due to fluctuations in light source output and ambient light, random noise and nonlinearity of the camera, and speckle noise are arisen. Generally, it is necessary to use a stable laser light source for high-precision measurements by performing experiments on a high vibration isolation table. Therefore, the background and amplitude intensities fluctuation and phase-shifting error are eternally a problem for the PSI. Owing to the fluctuation of background and amplitude intensities and phase-shifting error, the differences between the theoretical values of a sinusoidal wave and the actual recorded intensities could generate a significant phase error in the conventional analysis. In extreme cases, a more complex periodical phase error might occur if these error sources are mixed. For instance, Schwider et al studied that the phase-shift errors in the N-step phase-shifting method (PSM) generated systematic phase error with a double frequency of the interferometric fringes [18]. Hartharan et al devised a five-step PSM to reduce constant phase-shift errors [19]. Creath reported that increasing the number of phase shift could be robust to phase-shifting error [3]. Besides, there are many previous studies on phase errors due to variations in background [20] and amplitude intensities in recorded interference fringes, especially for interferometry with a tunable laser diode (LD) [21, 22]. In the LD PSI, measurement accuracy is degraded by intensity modulation of the interferogram that corresponds to a laser-power change associated with variations in the current [23]. A phase error caused by the laser-power change has the same period as the interferometric fringe [24]. Importantly, the measurement error for two error factors also have studied [25] and numerous canceling methods have been proposed [26, 27].

To overcome the problem mentioned above, stabilizing the PSI by a ‘hardware’ approach is proposed by Yoshino and Yokota’s group [28–30], which is implemented with an acousto-optic frequency shifter, oscilloscope and several optical components including beam splitter, polarizer, mirror, and a quarter-wave plate. A closed-loop phase-shifting interferometer is controlled in the phase domain where optical phases are detected by a two-frequency optical heterodyne method [28]. With feedback control, the interferometer is insensitive to external disturbances and laser frequency fluctuation [29]. As a result, a surface profile measurement showed the reproducibility of $1/60$th of a wavelength even under an unstable environmental condition [30].

Besides, some ‘software’ approaches (i.e. algorithms) are also proposed to reduce the phase error in PSI. For instance, Okada et al [31] developed an iterative algorithm extract phase shifts from interferograms with background and contrast fluctuations. Wang et al [32] proposed an iterative algorithm that converges rapidly and accurately, but only interferograms with a constant background and contrast are compatible. Liu et al energetically developed several improved vibration-resistant phase retrieval methods to reduce the phase error due to background and contrast (amplitude) variation [33–35]. In these iteration methods, generally, the convergence iteration is needed more than four to ten times to obtain an accurate solution. Even so, there remains a small periodical error. To our best knowledge, no paper discussing the effects of variation of background and amplitude intensities and phase-shifting error at the same time has yet been reported. Furthermore, no analysis method has been proposed that can cancel these errors simultaneously. Thus, an analysis method that can perform extra accurate measurement of phase distribution for interferometric fringe is desirable from the viewpoint of the following point: (i) no calibration procedure, (ii) no iteration calculation, and (iii) no hardware changes.

We recently developed an accurate phase analysis technique called the spatiotemporal phase-shifting method (ST-PSM) [36]. The ST-PSM utilizes phase-shifted moiré fringe patterns both in the spatial and temporal domains to determine the phase distribution, based on a two-dimensional discrete Fourier transform (2D-DFT) algorithm. In our previous study, we confirmed that this method has a strong tolerance to random noise and a self-neutralizing function to eliminate the periodical phase error due to the nonlinearity of detector, intensity saturation. Therefore, this method is expected to be resistant to other disturbances such as variations of background and amplitude intensities, phase-shifting error, or their combination.

In this study, we first apply the ST-PSM to PSI and investigate the phase measurement accuracy. It mainly focuses on the variation of background and amplitude intensities and phase-shifting error. Through computer simulations and experiments, we demonstrate that the ST-PSM can overcome the problems of the fluctuation of a laser source, the change of environment light, and the phase-shifting error, to achieve a stable and accurate measurement in interferometry. Compared
with the ‘hardware’ approach [28–30], it should be emphasized that the ST-PSM can automatically remove the various disturbance noises (such as the phase-shifting error, background, and amplitude) without any additional optical components, which can be avoided the problems of large, cumbersome and costly systems.

2. Principle

2.1. Outline of the conventional phase-shifting method

In the well-known N-step phase-shifting algorithm based on Bruning’s method [37], the acquired intensity \( f(x,y;k) \) for the \( k \)-th image frame at the pixel \((x,y)\) in the sensor plane is given by

\[
\begin{align*}
  f(x,y;k) &= A(x,y) + B(x,y) \cos \left[ 2 \pi \frac{x}{P} + \varphi_0(x,y) + 2 \pi \frac{k}{N} \right] \\
  &= A(x,y) + B(x,y) \cos \left( \varphi(x,y) + 2 \pi \frac{k}{N} \right), \\
  \quad (k = 0, 1, \cdots, N - 1)
\end{align*}
\]

(1)

where \( A(x,y) \) and \( B(x,y) \) represents the background (the average intensity) and amplitude (the intensity modulation) intensities of the fringe pattern, respectively. \( P \) is the fringe pitch in the x-direction, and \( \varphi_0(x,y) \) is the position-dependent phase value according to the shape of the object; and \( \varphi(x,y) \) is the desired phase value to be determined. By solving equation (1) with a 1D-DFT algorithm, the desired phase information can be obtained.

\[
\varphi(x,y) = \arg \left\{ \sum_{n=0}^{N-1} f(x,y;n) W_N^k \right\}
\]

(2)

where \( \arg \{ \cdot \} \) is the operator of argument function for complex numbers calculation, and \( W \) is the twiddle factor in the DFT algorithm, and is defined as

\[
W_k = \exp \left( -\frac{2\pi i k}{K} \right), \quad (k = 0, 1, \cdots, K - 1).
\]

(3)

In the experimental practice, due to the fluctuation of laser source or phase-shifting error of the phase shifter, the recorded interferometric fringes are depicted as follows:

\[
\begin{align*}
  f(x,y;k) &= A(x,y;k) \\
  &+ B(x,y;k) \cos \left[ \varphi(x,y) + 2 \pi \frac{k}{N} + \Delta \varphi(k) \right] \\
  &= \{ A(x,y) + \Delta A(k) \} + \{ B(x,y) + \Delta B(k) \} \\
  &\times \cos \left[ \varphi(x,y) + 2 \pi \frac{k}{N} + \Delta \varphi(k) \right], \\
  \quad (k = 0, 1, \cdots, N - 1).
\end{align*}
\]

(4)

There are three error factors due to the changes of background \( \Delta A(k) \), amplitude intensities \( \Delta B(k) \), and the phase-shifting error or vibration, \( \Delta \varphi(k) \). In most studies, the effects of \( \Delta A(k) \) and \( \Delta B(k) \) is ignored.

Figure 1 is a schematic diagram of phase errors in the images of the N-step phase-shifted in the time domain, arising from deviations from the ideal intensity (brightness). In this study regarding the PSM, we consider the fluctuations of the background and amplitude intensities and phase-shifting error, which have not been discussed in our previous study. In this study, therefore, we investigate the phase errors of these three error factors and the combination for our developed ST-PSM, compared with the conventional PSM.

2.2. Principle of the spatiotemporal phase-shifting method

The ST-PSM [36] is based on the N-step PSM—a temporal technique commonly used for phase extraction of fringe pattern—combined with the sampling moiré (SM) method [38, 39], as a spatial phase analysis technique similar to the windowed Fourier transform [40].

Figure 2 illustrates the fundamental principle of the ST-PSM, based on the PSM and SM principles. In the ST-PSM, the phase information at any pixel is determined from a 2D cosinusoidal wave by use of the spatial and temporal intensities data from a local space of \( N \times T \) pixels, as shown in figure 2(b). Figure 3 indicates the structure of the ST-PSM for phase calculation by utilizing the 2D-DFT algorithm after the normalization, down-sampling and intensity interpolation procedure.

After performing a normalization, down-sampling and intensity interpolation with a sampling pitch \( T \) near equal to \( P \) in the x-direction to all of the N-step phase-shifted fringe
adding the sampling phase correctly.

In the phase determination by the ST-PSM, the 2D-DFT calculation using the spatial and temporal phase-shifted intensity information simultaneously is a vital operation to determine the phase distribution of the fringe pattern. This feature can eliminate phase errors caused by many disturbances, such as random noise of the camera, saturation and nonlinearity of the recorded fringe pattern [36], and this algorithm could be also effect to background and amplitude variations and/or phase-shifting error.

3. Simulation

In PSMs, the random noise, the nonsinusoidal waveform error and the phase-shifting error are major error sources. In the previous study, we confirmed the excellent performance of the ST-PSM for the error sources of (i) random noise of the camera, (ii) nonlinearity (gamma distortion effect) of the detector or projector, and (iii) intensity saturation, respectively, and their combination (see figures 3 and 4 in [36]). For interferometric measurements using a laser source, the fluctuation of the bias, amplitude, and phase-shifting error (or vibration) are also need to be considered. Therefore, in this section, with the help of simulated fringe patterns, we compare the performances of the conventional N-step PSM and the ST-PSM. Although more than 80 improved temporal PSMs [41, 42] have been proposed nowadays, the four-step phase-shifting algorithm based on Bruning’s method [37] is more extensively used for broad applications. Therefore, we provide a detailed analysis of phase error for the Bruning’s four-step PSM and the four-step ST-PSM for all simulations.

3.1. Simulation results of single error source

To understand the nature of phase error that appears in the interferometric measurement by the conventional PSM and the effectiveness of the ST-PSM, we carried out a series of computer simulations to focus and investigate three factors of single error source and their combinations: (i) fluctuation of background intensity, (ii) fluctuation of amplitude intensity, (iii) phase-shifting error, respectively.

3.1.1. Fluctuation of background intensity. Figures 4(a) and (b) show the phase errors by the PSM due to the fluctuations of bias (background intensity). Even if one of the four phase-shifted images have a fluctuation of bias (figure 4(a), black line), a periodical error with the same spatial frequency of the fringe pattern (10-pixel period) appears. An additional (negative) fluctuation in the third image leads to slight increase of error amplitude as well as the initial phase to be shifted (figure 4(a), grey line). The increase of negative fluctuation in the third image gives rise to further enhancement of error amplitude together with the initial phase slight shift (figure 4(a), blue line). Figure 4(b) illustrates the phase errors when bias...
fluctuates at three images. If positive fluctuations in intensity increases linearly from the second to the fourth images, errors with about 10-pixel period appears (figure 4(b), black line). In the case of larger tendency of linear increase of fluctuations in intensity, amplitude of error increases without any initial phase change (figure 4(b), gray line). Reversed sigh of fluctuation leads to reversed phase in error with the same amplitude (figure 4(b), blue line). In striking contrast to the PSM, the ST-PSM method is perfectly free of error in all cases and found to be extremely robust to fluctuations in any background intensity, as shown in figure 4(c).

3.1.2. Fluctuation of amplitude intensity. Figure 5 shows the simulation result of the phase error due to the fluctuations of amplitude intensity. If amplitude in intensity decreases linearly from the first to the fourth images, a periodical phase error with a double frequency of fringe pitch (5-pixel period) appears (figure 5(a), black line). If the tendency of amplitude in intensity increases linearly, the phase of the periodical phase error is reversed (figure 5(a), gray line). In the case where the intensity at the second and the fourth images are suddenly changed (80% of the original intensity), the same periodical phase error with a double frequency of fringe pitch occurs, and the amplitude of periodic phase error increases (figure 5(a), blue line). In contrast, the ST-PSM is perfectly free of error in all cases and found to be extremely robust to fluctuations in any amplitude intensity, as shown in figure 5(b).

3.1.3. Phase-shifting error. Figure 6 shows the simulation result of the phase error due to the phase-shifting error (or vibration). Even a single vibration in one of the four phase-shifted images gives rise to a periodic phase error with a double frequency of the fringe pitch (figure 6(a), black line). Additional phase-shifting error with opposite sign in other phase-shifted image increases the amplitude of the periodic phase error (figure 6(a), gray line). Moreover, phase-shifting errors in three images further enhance the error amplitude (figure 6(a), blue line). These results suggest that vibration errors in each phase-shifted image are accumulated to the total phase error. In case of the ST-PSM for a single vibration error, small constant initial phase error appears (figure 6(b), black line). Two vibration errors with the same magnitude but opposite in sign lead to the cancellation of two periodic phase errors with opposite sign, resulting in nearly zero errors (figure 6(b), gray line). Phase-shifting errors in three images give largest error with a double frequency of fringe pitch (5-pixel period) appears (figure 5(a), black line). If the tendency of amplitude in intensity increases linearly, the phase of the periodical phase error is reversed (figure 5(a), gray line). In the case where the intensity at the second and the fourth images are suddenly changed (80% of the original intensity), the same periodical phase error with a double frequency of fringe pitch occurs, and the amplitude of periodic phase error increases (figure 5(a), blue line). In contrast, the ST-PSM is perfectly free of error in all cases and found to be extremely robust to fluctuations in any amplitude intensity, as shown in figure 5(b).
phase error due to the accumulation of each error originating in individual vibration errors (figure 6(b), blue line). The average phase-shifting error appears as the initial phase error for the ST-PSM.

To summarize briefly the simulation results of single error source, fluctuations in the background intensity give rise to periodic errors with the same period of the grating pitch; fluctuations in amplitude intensity and phase-shifting error generate periodic errors with a double frequency (the half period) of the fringe pitch for the conventional PSM. These results match other studies that the phase errors in the fluctuation of background and amplitude intensities are due to the first-order spectral zeroes in four-step PSM \[42\]. On the other hand, the ST-PSM is completely free of periodical error for all cases. Next, we consider phase error owing to the combination of more than two error sources.

### 3.2. Simulation results of two error sources

Figure 7 shows the simulation results of the phase errors obtained by the PSM and the ST-PSM in the case of two error factors are mixed. As for the phase errors in background and amplitude in the PSM (figure 7(a), blue line), the phase errors in the background intensity bring phase errors with 10-pixel period which (frequency \(f\) error) is similar to the gray line in figure 4(b), although the magnitude of phase errors in figure 7(a) is larger than that in figure 4(b) because of larger background errors assumed in figure 7(a). Owing to the amplitude errors, phase errors with 5-pixel period (frequency \(2f\) error) is expected from the results of figure 5(a). Since the magnitude of amplitude errors assumed in figure 7(a) is smaller than that in figure 5(a), \(2f\) errors are not explicitly recognized in figure 7(a). However, dominating frequency \(f\) errors in figure 7(a) is distorted due to the small \(2f\) errors, resulting in larger sharpness (kurtosis) in the positive side of phase error as compared to the negative side. Also, the wavy pattern of phase error is slightly skewed from the sinusoidal wave. In the case of background intensity and phase-shifting errors in the PSM (figure 7(b), blue line), a periodical phase error with two different frequencies occurs. Interestingly, the dominating \(f\) errors due to background error are distorted by \(2f\) errors due to vibration errors (figure 6(a)). As for the errors in amplitude and vibration in the PSM (figure 7(c), blue line), only \(2f\) errors appear because both of amplitude errors (figure 5(a)) and vibration errors (figure 6(a)) give rise \(2f\) errors. When the \(f\) periodic error superposes with the \(2f\) periodic error in the PSM (figures 7(a) and (b); blue line), these periodic errors cannot be eliminated perfectly by conventional smoothing filter, low-pass filter, sine/cosine filter, or other spatial filters.
In contrast, the ST-PSM is free of any periodic errors for any two combination of the background, amplitude and vibration errors (figure 7, red line), because the ST-PSM does not produce any periodic error due to individual error factors, as indicated in figures 4(c), 5(b), and 6(b).

3.3. Simulation results of three error sources

Figure 8 shows the simulation results of the phase and phase error distributions obtained by the PSM and the ST-PSM in case of background and amplitude intensities, the phase-shifting error are mixed for four-step phase-shifted fringe patterns, as one example. We assumed that the bias and the amplitude intensities gradually increase, corresponding to the gradual intensification and stabilization of the power of laser source after power supply. To put the assumption concretely in terms of equation (4), the incremental errors in background, amplitude are given as $\Delta A(k) = [0, +10, +20, +30]$ and $\Delta B(k) = [0, +5, +10, +15]$ for $k = [0, 1, 2, 3]$, respectively. Besides, the phase-shifting errors are given as $\Delta \varphi(k) = [0, \pi/30, -\pi/10, \pi/15]$ for each $k$. For simplicity, we did not consider any random noise here. The simulated four-step phase-shifted fringe patterns are a cross-sectional intensity data in the middle of the horizontal line, under a noise-free situation when both the background and amplitude intensities and phase-shifting error exist, are shown in figure 3(a). The grating pitch is 10 pixels, and the sampling pitch of 10 pixels was used to analyze the phase distribution by the ST-PSM. Figures 3(b) and (c) indicate the phase and phase error distributions obtained by the PSM. The PSM method shows a twisted phase distribution and resulting a periodical phase error with two different frequencies. On the other hand, a theoretical linear phase distribution and no phase error occurs in the ST-PSM algorithm, as shown in figures 3(d) and (e).

From these facts, it confirmed firstly that the ST method is not affected by the three effects on the fluctuation of the background and amplitude intensities, and the phase-shifting error, which are practical problems in conventional laser interferometry.

3.4. Simulation results of four error sources

The photon shot noise (random noise) of a complementary metal-oxide-semiconductor (CMOS) or charge-coupled
device (CCD) camera also affects the measurement accuracy for any optical methods. Figure 9 illustrates the simulation results of the phase errors of the obtained by the PSM and the ST-PSM in the case of three error factors without and with random noises. In the case without random noise (figure 9(a)), the PSM leads to the combination of $f$ and $2f$ periodic phase errors, while the ST-PSM shows no phase error (as the same result of figures 8(c) and (e)). Compared with the case of noiseless, 10% random noise distorts the phase error randomly if we use the PSM. On the other hand, the ST-PSM reduces phase errors to nearly zero. The phase errors due to the combination of three error sources (background, amplitude, vibration) and the random noise by the ST-PSM is much smaller than those by the PSM.

3.5. Simulation results of distort fringe patterns

Figure 10 shows the simulation results for the distorted fringe patterns without any background, amplitude, and phase-shifting errors for noiseless and 10% random noise cases. The distorted fringe patterns, as shown in figure 10(a) was generated according to [43]; equation (19) with peaks = 0.5. The lower parts of each circle in the upper panels of figure 10(b1) are disturbed with random noise as compared to the upper parts of circles without noise. In contrast to the homogeneous distribution of the theoretical amplitude (figure 10(b2)), the theoretical phase is not homogeneous due to the distortion included in fringe patterns (figure 10(b3)). This leads to the inhomogeneous distribution of phase gradient (first derivative, figure 10(b4)) and curvature (second derivative, figure 10(b5)).

We applied the ST-PSM with the fringe pitch $P = 20$ pixels and the sampling pitch $T = 20$ pixels to the distorted fringe patterns. For the distorted fringes, the phase is compensated by phase curvature $C_{xx}$, assuming $C_{xx} = ((P - 1)/2)^2$. In the case of noiseless, the upper part of circle for background (figure 10(c1)), phase (figure 10(c3)), phase gradient (figure 10(c4)), phase curvature (figure 10(c5)) are almost similar to the theoretical distributions (lower panels of figure 10(b1)), although amplitude (figure 10(c2)) contains some inhomogeneity owing to the irregularity of fringe pitch in space. The high accuracy of phase (figure 10(c3)) is also confirmed small error.
Figure 11. Comparison results obtained by the PSM and the ST-PSM for three error factors: (a) simulated four-step distorted phase-shifted fringe patterns, (b) background, (c) amplitude, (d) phase, (e) phase gradient, (f) phase curvature, (g) phase error. The upper, middle, and lower rows show the PSM, the theory, and the ST-PSM results.

The quality of the lower parts of circles for 10% random noise is almost identical to the lower of circles for noiseless (figures 10(c1)–(c5)) except for slightly large phase error (figure 10(c6)). On the other hand, the results obtained by the PSM are largely disturbed by random noise, as is indicated by disturbances in the lower parts of circles in figure 10(d). These results suggest that the ST-PSM is robust to random noise even for distorted fringe patterns as compared to the PSM.

Figure 11 indicates the simulation results for the distorted fringe patterns with errors in background, amplitude and phase-shifting simultaneously. In this simulation, background and amplitude of brightness are increased from the first phase-shifted image $(k = 0)$ to the fourth phase-shifted image $(k = 3)$ at a step of 10 and 5 intensity, respectively. The intervals of phase-shift deviate from the quarter of the fringe period.

Concretely, the second, the third and the fourth shifted images contain $-\pi/20$, $\pi/10$, $-\pi/15$ errors, respectively. The results by the ST-PSM are almost identical to theoretical distributions, implying the ST-PSM is not affected by the errors in the background, amplitude and phase-shifting. On the other hand, the background obtained by the PSM is contaminated by frequency $f$ error (figure 11(d1)) even without any noise (the upper part of the circle). Similarly, amplitude and phase are contaminated by frequency $2f$ error (figures 11(d2) and (d3)). As a result, large frequency errors appear in phase gradient (figure 11(d4)) and phase curvature (figure 11(d5)) even without any noise (the upper part of the circle). In the case of 10% random noise, the quality of phase gradient and phase curvature is severely deteriorated, as is indicated by the lower part of figures 11(d4) and (d5). The conventional PSM is found to be extremely vulnerable to errors in background,
amplitude and phase-shifting, as well as the random noise.

3.6. Summary of simulation results
Judging from a series of simulation analyses (figures 4–11), we confirmed that the ST-PSM has the great advantage of reducing phase error owing to errors in the background and amplitude intensities, the phase-shifting error, which has been problematic obstacles to conventional laser interferometer measurements. Our simulations have revealed the following findings:

(1) In the case of the conventional PSM, fluctuations in the background (\(\Delta A\)) and amplitude (\(\Delta R\)) intensities give rise to a periodic error with a spatial frequency of \(f\) (corresponding to the frequency of fringe images) and \(2f\), respectively. In contrast, the ST-PSM enables high accuracy measurement without any influence of these errors.

(2) In the case of phase-shifting error (\(\Delta \phi\)) in at least one image of \(N\)-step phase-shifting images, the conventional PSM gives rise to a periodic error with a spatial frequency of \(2f\). On the other hand, we confirmed that the ST-PSM does not produce any periodic errors. In the ST-PSM, the average of errors in \(N\)-step phase-shifting images is found to emerge as a deviation in an initial phase, which can be eliminated by correction procedure.

(3) The ST-PSM demonstrated that any two or all combinations of three error factors (\(\Delta A, \Delta R, \Delta \phi\)) do not give rise to any periodic errors. Furthermore, even in the presence of random noise superposed with the three error factors, the ST-PSM accuracy is greatly higher than that of the PSM. The ST-PSM is also robust to random noise, which is problematic for laser interferometry.

(4) The ST-PSM is also useful for distorted images. Since the ST-PSM does not produce any periodic errors, the ST-PSM is extremely efficient for the accurate measurement of the phase gradient, which is the first derivative of phase distribution. All these advantages of the ST-PSM suggest that the ST-PSM is suitable for nonhomogeneous measurements such as rough surface and defect of the target object.

4. Experiment and discussions
Based on the simulation results, we further evaluated the performance of our proposed method experimentally for three different conditions, including the interferometric fringe with both a large pitch and a small pitch, and an extremely high fluctuation condition.

4.1. Experimental setup
The diagram of the experimental setup of a Michelson interferometry is illustrated in figure 12(a). In the experiment, a semiconductor laser operating at 473 nm with 50 mW power was used as the coherent light source. The laser light source, after tuned to an appropriate light power by the neutral density filter, is collimated through a beam expander. Then, the collimated laser beam is divided into two beams by a beam splitter. One beam illuminates the measurable object (a flat mirror). The reflected beam from the object becomes an object beam and comes into the camera. The other, called the reference beam, is reflected from another mirror mounted on a PZT stage (PAZ0005, Thorlabs Inc.) and comes into camera and interferes with the object beam. As a result, the camera records a repetitive fringe pattern—the interferogram—generated by two plane waves, and it changes when the PZT shifts the optical paths of the reference beams.

Figures 12(b) show photograph of the experimental setup.

A CMOS camera (VCXU-50, Baumer Inc.) with a resolution of 2448 \(\times\) 2048 pixels and a PZT with a resolution of 5 nm in the closed-loop mode were utilized to record four phase-shifted interference fringe images. The pixel pitch of the monochrome CMOS camera is 3.45 \(\mu\)m. The four-step phase-shifting is realized using the PZT stage and the phase-shifting amount of the PZT was set to \(\pi/2\). Theoretical displacement of the mirror attached to the PZT stage should be 473 nm/8 = 59 nm. However, the phase-shifting amount of
the PZT is not perfectly equal to the theoretical value (i.e. $\pi/2$), since the surface of the mirror mounted on the PZT is not strictly perpendicular to the incoming beam generally. Because the shifting resolution of the PZT is 5 nm and a phase-shifting error of about 10% may occur. It should be noted that a PZT with sub-nanometer accuracy is extremely expensive. In
Figure 15. Experimental results for a small pitch interferometric fringe patterns under an extreme case. The average pitch of fringe patterns is 33.2 pixels.

Figure 14 depicts the results of a smaller pitch (34.2 pixels) interval of fringe patterns as compared with figure 12 (269.8 pixels) to investigate the dependence of measurement accuracy on pitch interval. Since the average pitch of fringe patterns is 34.2 pixels, we set the sampling pitch to 34 pixels for the ST-PSM. Note that the sampling pitch for the ST-PSM in figure 14 is much smaller than that in figure 13, which leads to the higher horizontal resolution of figure 14 than that of figure 13. Similar to figures 13(b) and (c), the background and amplitude intensities obtained by the PSM are contaminated by frequency errors and random noise, whereas the background and amplitude distributions by the ST-PSM is free of those errors (figures 14(b) and (c)). In the case of phase (figure 14(d)), the patterns obtained by the PSM and the ST-PSM look similar at a glance. However, in terms of the phase gradient (figure 14(e)), the phase gradient by the PSM is contaminated by random noise and periodical phase error due to phase-shifting error, whereas the phase gradient by ST-PSM is free of random noise.
and periodical phase error. Again, we confirmed the effectiveness and advantage of the ST-PSM over the PSM even for smaller pitch interval of fringe patterns.

### 4.3. Experimental results of extreme condition

Figure 15 indicates the results of fringe patterns with a small pitch, similar to figure 14 but with extreme image recording conditions. The average interval of fringe patterns is 33.2 pixels which is nearly the same as figure 12 (34.2 pixels). We set the sampling interval to 33 pixels for the ST-PSM. In this experiment, we assumed over-exposure condition for the first and third phase-shifted images severely under-exposure condition for the second and fourth phase-shifted images (figure 15(a)). This means that the background and amplitude intensity differ widely among four phase-shifted images. The background (figure 15(b)) and amplitude (figure 15(c)) distributions obtained by the PSM give rise to conspicuous frequency $f$ errors. The phase obtained by the PSM is contaminated by random noises and is far from linear change (figure 15(d)). As a result, the phase gradient by the PSM, as shown in figure 15(e), is extremely disturbed with large random noises.

On the contrary, the background (figure 15(b)) and amplitude (figure 15(c)) obtained by the ST-PSM are free of frequency $f$ errors. The phase obtained by the ST-PSM well reproduces linearity in spatial domain (figure 15(d)), leading to a constant phase gradient (figure 15(e)). In spite of a deplorable measurement condition of the recorded fringe patterns, we further confirmed the robustness of the ST-PSM.

### 4.4. Experimental results of reproducibility

We also investigated the effect of the phase-shifting error experimentally by setting the PZT displacement for three different amounts, including 60 nm, 70 nm, and 80 nm. Besides, the reproducibility of the ST-PSM was confirmed by recording four steps of phase-shifted fringes for each PZT displacement measurement and repeating the measurement for a total of ten times under the same conditions.

Figures 16(a) and (b) indicated the analysis results of ten measurements for three different phase-shifting amounts of 60 nm, 70 nm and 80 nm obtained by the PSM and the ST-PSM algorithms, respectively. The down-sampling pitch of 30-pixel was used for ST-PSM, and the central area of 1000 by 1000 pixels was evaluated. The comparison of the mean of S. D. of phase gradient obtained by the PSM and the ST-PSM for three different phase-shifting error is shown in table 1. The results of these analyses reveal the following finding:

![Figure 16](image_url)
Table 1. Comparison of the mean of S. D. of phase gradient obtained by the PSM and the ST-PSM in case of the PZT step are 60 nm, 70 nm, and 80 nm, respectively.

| PZT step | Phase-shifting error (%) | Mean of S. D. |
|----------|--------------------------|---------------|
|          | PSM                      | ST-PSM        |
| 60 nm    | 1.2                      | 0.0433        |
| 70 nm    | 18.9                     | 0.0614        |
| 80 nm    | 36.5                     | 0.1049        |

(i) Even in the absence of phase-shifting error (i.e. PZT = 60 nm), the ST-PSM is extremely stable and reproducible, while the conventional PSM has significant variations.

(ii) As the phase-shifting error increases (i.e. PZT = 70 nm and 80 nm), the variation of the phase gradient in PSM further increases, but it was confirmed that ST-PSM has little change and can measure the stable phase gradient as well.

(iii) The results summarized in table 1 show that the PSM is very sensitive to phase-shifting errors. On the other hand, the variation of the phase gradient obtained by the ST-PSM is negligible (0.0015) for all conditions. This phase-shifting error immunity has been very valuable in measuring large optics for which mounting everything on a stable table may not be practical.

5. Discussion

By combining the finding of the present and our previous studies, it is revealed that the ST-PSM algorithm is robust to not only the change of the background and amplitude intensities, and phase-shifting error or a combination of these factors, but also random noise of the camera, saturation and nonlinearity of the recorded fringe patterns. Therefore, the ST-PSM is a robust phase analysis of interferometric fringe in the PSI to measure the phase and phase gradient distributions. Currently, we think the operation of 2D-DFT computation (step 3 in figure 3; equation (6)) is very important for the ST-PSM algorithm. It should be mentioned that the definition of 2D-DFT in this study refers to 1D of space and 1D of time, not 2D space. In principle, the main reason why the ST-PSM is robust to external disturbances originates in the exact cancellation of periodic phase errors by the 2D-DFT calculation using the spatial and temporal intensity information simultaneously. The derivation of a complete theoretical equations will be reported in detail in the next manuscript.

Compared with the PSM, one disadvantage of the ST-PSM is that complex fringe patterns with high-modulation or circular fringe patterns are not easy to analyze due to the down-sampling and intensity interpolation procedure in the ST-PSM algorithm currently. The further improvement of the ST-PSM apply to a more complex interferometric fringe patterns are our future work.

6. Conclusion

This study is the first attempt to apply the ST-PSM to PSI. We investigated how the measurement accuracy of the ST-PSM is affected by the instability of the light source, phase-shifting error, which have been problematic obstacles to conventional laser interferometer measurements.

We confirmed that the ST-PSM has the great advantage of canceling of phase error owing to errors in the background and amplitude intensities, the phase-shifting error, by computer simulations. In the experiment, the ST-PSM was applied firstly to the Michelson laser interference experiment targeted at a plane mirror. The distributions of phase and phase gradient based on four-step phase-shifting interference fringes are analyzed by the ST-PSM as well as by the conventional PSM. The ST-PSM realizes a much more stable measurement than the PSM, regardless of three error factors and random noise. Notably, we have revealed the excellent performance and the striking advantage that ST-PSM is perfectly free of periodic errors. Therefore, laser interferometry using the ST-PSM can be expected to apply to the measurement of non-contact shape and deformation measurement as well as thickness measurement of transparent materials in life and material sciences.

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