Prediction of Flow Stress Curve of Metallic Foam using Compressible Constitutive Equation

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Abstract. Deformation of metallic foams in forming processes is complicated due to large volume change so that the prediction is not easy. In this study, cylindrical billets of open-cell type nickel foam were compressed uniaxially. Multi-step compression tests were also conducted to reveal changes in apparent density and diameter during the compression. For numerical prediction, Oyane’s yield criterion and the associated flow rule were assumed. With considering volume change in the compression, incremental analysis was performed. Changes in apparent density, diameter and flow stress curves were successfully reproduced with constants $a=2.12$, $m=0.3$ and $n=2.0$.

1. Introduction

For industrial applications of metallic foams, it is demanded to develop forming processes and technologies [1]. However, deformation of metallic foams is complicated due to changes in volume or apparent density during the process so that numerical prediction is not easy. Deshpande et al. introduced plastic Poisson ratio to express the volume change [2], however it seems to be difficult to model the change of plastic Poisson ratio with deformation. Alternatively, yield criterion with hydrostatic-pressure dependence and the associated flow rule were used to predict the deformation. Yoshimura et al. [3] applied Oyane’s yield criterion [4], which was originally developed for sintered metals, to predict deformation of a closed-cell aluminum foam. Only the initial apparent density was considered in early studies, though change in apparent density during deformation is large. Therefore the prediction methods are not applicable to industrial bulk forming processes, e.g., forging, extrusion, rolling. In our previous study, multi-step uniaxial compression of metallic foams were conducted to investigate changes in apparent density and diameter experimentally [5, 6]. Changes in apparent density and diameter were successfully reproduced by numerical analysis with the determined constants, $a=2.12$ and $m=0.3$ [5, 7]. However, precise prediction of flow stress curve has not been reported. In this study, uniaxial compression tests of open-cell type nickel foam (Celmet) were conducted. Flow stress curve and change in apparent density are reproduced by Oyane’s equation with the same constants ($a=2.12$, $m=0.3$) and the newly determined constant $n=2.0$. It is found that the increase in flow stress is mainly due to the increase in apparent density.
2. Compression test

2.1. Experimental conditions

2.1.1. Material used. Open-cell nickel foam (Celmet ® by Sumitomo Electric Industries Ltd. [8]) with mean pore size of 0.8 mm was used. Cylindrical billets with the diameter of 7.7 mm and the height of 10.1 mm as shown in figure 1 were prepared by wire discharge cutting. The apparent density calculated with the weight and the apparent volume was \( \rho_{\text{apparent}} = 0.392 \text{ Mg/m}^3 \). The buoyant density calculated by the Archimedes method from the weight under atmosphere and that in distilled water was \( \rho_{\text{buoyant}} = 5.68 \text{ Mg/m}^3 \). The buoyant density \( \rho_{\text{buoyant}} \) was lower than the density of pure nickel \( \rho_{\text{Ni}} = 6.98 \text{ Mg/m}^3 \) due to micro pores existing in ligaments. As the buoyant density of Celmet does not change with compression [6], porosity \( p \) was defined by the following equation. The initial porosity of the used foam Celmet was 93-95%.

\[
p = 1 - \left( \frac{\rho_{\text{apparent}}}{\rho_{\text{buoyant}}} \right)
\]  

2.1.2. Compression test. The compression test was conducted on a universal testing machine at room temperature. The billet was compressed at crosshead speed of 1 mm/min between two flat steel dies without lubrication. The compression was interrupted by every 1.0 mm decrease of height to measure the height and the diameter of the billets under unloaded condition. The height was reduced by 90% in total by 9 steps. For the comparison, single-step compression was also conducted under the same conditions. In both cases, Celmet billets were compressed by 90% in height successfully without fracture.

2.2. Results

Nominal stress \( s \) and nominal strain \( e \) were defined with apparent dimensions as follows,

\[
s = \frac{F}{A} \quad \text{and} \quad e = \frac{\Delta h}{h_0}
\]

\( F \) is the load, \( A \) is the cross-sectional area, \( \Delta h \) is the height reduction, and \( h_0 \) is the initial height of the billet.

![Figure 1. Appearance of Celmet billet used.](image)

![Figure 2. Nominal stress-strain curves of the single-step and the multi-step compression tests.](image)
\[ s = \frac{F}{A_0} \quad (2) \]

\[ e = \frac{(h_0 - h)}{h_0} \quad (3) \]

where \( F \) is the compression load, \( A_0 \) is the initial apparent cross-sectional area of the billet, \( h_0 \) is the initial height and \( h \) is the current height of the billet.

Nominal stress-strain curves by the single-step (dashed curve) and the multi-step (solid curve) compression tests are compared for two specimens in figure 2. The initial porosities \( p_0 \) of the specimen A and B were slightly different 94.6% and 93.7%, respectively. Although the stress-strain curves of the two billets are not identical due to the non-uniformity in microstructure, two stress-strain curves of the multi-step compression are close to those of the single-step compression \( (p_0=93.5\%) \) so the change of apparent density in compression can be evaluated by that of multi-step compression. Increase in nominal stress \( s \) with an increase in nominal strain \( e \) is small when \( e<0.4 \). Nominal stress \( s \) increases steeply with increasing strain \( e \) when \( e>0.6 \). These trends well agree with the stress-strain curves in the previous study [6].

Change in diameter \( d \) with increasing nominal strain \( e \) is shown in figure 3(a). Ideal change of diameter under volume constancy is shown by the dotted curve on the figure. Diameter change of Celmet is much smaller than that of the nonporous metals and negligible when \( e<0.4 \). Change in apparent density \( \rho \) with nominal strain \( e \) is shown in Figure 3(b). Apparent density increases gradually until \( e=0.4 \). Then apparent density increases steeply with increasing nominal strain \( e \) when \( e>0.6 \). By 90% reduction in thickness, the density increases to 3.84 Mg/m\(^3\), while the porosity was reduced to 32.4%.

3. Plasticity theory of porous material

3.1. Yield criterion with hydrostatic-pressure dependence

In order to explain the above-mentioned experimental results including apparent volume change, a yield criterion with hydrostatic-pressure dependency was used. The most popular yield criterion of compressive metals, Oyane’s criterion which was developed for sintered metals [4], was selected. The criterion is described by equation (4),

\[ \text{Figure 3. Changes in diameter (a) and apparent density (b) with nominal axial strain.} \]
\[
\bar{\sigma} = \frac{1}{\varphi^{n-1}} \left\{ \left( \sigma_1 - \sigma_2 \right)^2 + \left( \sigma_2 - \sigma_3 \right)^2 + \left( \sigma_3 - \sigma_1 \right)^2 \right\}^{1/2} + \left( \frac{\sigma_m}{f} \right)^2 \quad (4)
\]
\[
f = \frac{1}{a(1-\varphi)^m} = \frac{1}{ap^m} \quad (5)
\]

where \( \bar{\sigma} \) is the equivalent stress, \( \varphi \) is the relative density \( (\varphi = 1 - p) \), \( n, a \) and \( m \) are constants which will be discussed later, \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the principal stresses, \( \sigma_m \) is the mean stress. \( f \) is the parameter defined by equation (5) showing the dependence on the mean stress. For instance, when \( \varphi = 1 \) (solid metal), \( f = \infty \), so equation (4) attributes to the von Mises’s yield criterion. However, according to equation (4), yielding may take place also by mean stress or hydrostatic pressure.

3.2. Constitutive equation

If the associated flow rule (equation (6)) is assumed, constitutive equations can be derived from the yield criterion (equation (4)),
\[
d\varepsilon_{ij} = d\lambda \frac{\partial \bar{\sigma}}{\partial \sigma_{ij}} \quad (6)
\]
The principal strain increments \( d\varepsilon_k \) is derived from equations (4)-(6) by taking partial derivatives of \( \bar{\sigma} \) with respect to \( \sigma_k \) as,
\[
d\varepsilon_k = \frac{3}{2} \frac{1}{\varphi^{n-1}} \frac{\partial \varepsilon}{\partial \sigma} \left\{ \sigma_k - \frac{(1-\varphi^2)}{9f^2} \sigma_m \right\} \quad (7)
\]
where \( \frac{\partial \varepsilon}{\partial \sigma} \) is the increment of equivalent strain defined by,
\[
\frac{\partial \varepsilon}{\partial \sigma} = \frac{1}{2} \left\{ \left( \varepsilon_1 - \varepsilon_2 \right)^2 + \left( \varepsilon_2 - \varepsilon_3 \right)^2 + \left( \varepsilon_3 - \varepsilon_1 \right)^2 \right\}^{1/2} + \left( \frac{f_d\varepsilon}{3} \right)^2 \quad (8)
\]

3.3 Application to uniaxial compression

In the uniaxial compression of a cylindrical billet \( (\sigma_1 = \sigma_r = 0, \sigma_2 = \sigma_\theta = 0, \sigma_3 = d\varepsilon_3 = d\varepsilon_0 = d\varepsilon_r = d\varepsilon_\theta) \), the mean stress \( \sigma_m \) and volumetric strain \( d\varepsilon_v \) can be calculated by,
\[
\sigma_m = \frac{(\sigma_r + \sigma_\theta + \sigma_z)}{3} = \sigma_z / 3 \quad (9)
\]
\[
d\varepsilon_v = d\varepsilon_r + d\varepsilon_\theta + d\varepsilon_z = 2d\varepsilon_r + d\varepsilon_z \quad (10)
\]
Considering the conditions of the uniaxial compression equations (9) and (10) into equation (7), the equations were summarized as,
\[
d\varepsilon_r = d\varepsilon_\theta = -\frac{3}{2} \frac{1}{\varphi^{n-1}} \frac{d\varepsilon}{\sigma} \left\{ (1-\frac{2}{9f^2}) \frac{\sigma_z}{3} \right\} \quad (11)
\]
\[
d\varepsilon_z = \frac{3}{2} \frac{1}{\varphi^{n-1}} \frac{d\varepsilon}{\sigma} \left\{ (2+\frac{2}{9f^2}) \frac{\sigma_z}{3} \right\} \quad (12)
\]
Eliminating \( \frac{d\varepsilon}{\bar{\sigma}} \) from equations (11) - (14), the equations can be further simplified as,
\[
\frac{d\varepsilon_r}{d\varepsilon_z} = \frac{2 - 9f^2}{2 + 18f^2}, \quad \frac{d\varepsilon_v}{d\varepsilon_z} = \frac{3}{9f^2 + 1}
\] (13)

Using equation (13), the radial strain increment \(d\varepsilon_r\) and the volumetric strain increment \(d\varepsilon_v\) can be calculated incrementally with the axial strain increment \(d\varepsilon_z\).

4. **Prediction of change in diameter and apparent density**

Change in apparent density during the compression should be updated properly for prediction of deformation characteristics. For the purpose, the deformation characteristics is predicted by an incremental approach with the above-mentioned theory of plasticity.

Deformation was predicted by an incremental numerical analysis on a spread sheet. For every 0.1mm decrease in height, axial strain increment \(\Delta\varepsilon_z\), radial strain increment \(\Delta\varepsilon_r\) and volumetric strain increment \(\Delta\varepsilon_v\) were predicted by equations (13) and (14). Then, the diameter of the billets \(d\) and the porosity \(p\) were updated using the following equations,

\[
\varepsilon_r^j = \sum_{i=0}^{j} \Delta\varepsilon_r^i = \ln\left(\frac{d}{d_0}\right)
\]

\[
\varepsilon_v^j = \sum_{i=0}^{j} \Delta\varepsilon_v^i = \ln\left(\frac{V}{V_0}\right) = \ln\left(\frac{\phi_l}{\phi_0}\right) = \ln\left(\frac{1-p_0}{1-p^j}\right)
\] (14)

Where \(V\): volume of the billet. Superscript \(j\) means \(j\)-th increment of the numerical analysis, while subscript \(0\) means the initial state before the compression.

After porosity and diameter were updated, next 0.1mm decrease in height was analysed. Accordingly, changes in diameter and in apparent density were predicted and are shown in figure 4 (a) and (b). In this prediction \(a=2.12\) was assumed because no diameter change is expected when the porosity is close to 100\% [9]. The predictions are shown as a function of \(m\) value. It is found that \(m=0.3\) gives good agreement with the experiments.

We could assume no work hardening of nickel ligaments in Celmet because Vickers hardness change by the compression is little. Therefore, flow stress curve of the Celmet can be predicted as,

\[
\sigma_{zz} = Y\phi_l^m / \sqrt{1 + \frac{1}{9f_j^2}}
\] (16)

![Figure 4](image-url) **Figure 4.** Changes in diameter (a) and apparent density (b) with increasing nominal strain.
Where $\sigma_{zj}$ is the axial stress at $j$-th increment, $Y$ is the yield stress of ligaments and assumed to 138.2 MPa [10]. The prediction was shown in figure 5 as a function of $n$. It is found that $n=2.0$ gives good agreement with the experiment. It implies that the increase in flow stress is mainly due to the decrease in porosity. Because no work hardening of ligaments during the compression was assumed in the prediction.

In other words, changes in apparent density, diameter as well as flow stress curves were successfully reproduced by Oyane’s yield criterion with the associated flow rule using the material constants, $a = 2.12$, $m = 0.3$ and $n = 2.0$.

![Figure 5](image)

**Figure 5.** Nominal stress-strain curve as a function of constant $n$.

5. Conclusion

Cylindrical billets of open-cell type nickel foam (Celmet) were compressed uniaxially. Flow stress shows gradual increase at the early stage, then steep increase after the densification. Changes in apparent density and diameter were revealed by multi-step compression test. It is found that diameter change is small, however increases slightly with reduction in height due to decrease in porosity. Not only changes in apparent density and diameter but also flow stress curves were successfully reproduced by Oyane’s yield criterion with the associated flow rule using the determined constants, $a = 2.12$, $m = 0.3$ and $n = 2.0$. It is found that the increase in flow stress is mainly due to the decrease in porosity.

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