Dust-acoustic modes in plasmas with dust distributions and charge fluctuations

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New Journal of Physics 5 (2003) 21.1–21.8 (http://www.njp.org/)
Received 28 November 2002
Published 14 March 2003

Abstract. Charge fluctuations in dusty plasmas are discussed starting from charge, mass, momentum and current equations for the combined dusty plasma. This allows a generic description of low-frequency waves in plasmas with dust distributions, and leads to the damping and other modifications of the dust-acoustic and related modes, like generalized dust-Coulomb modes.

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1. Introduction

Dusty plasmas are encountered in space and in the laboratory, as mixtures of ordinary plasma particles and charged dust grains. Astrophysical dust is present in many solar system and interstellar environments, where dust grains come with a whole range of characteristics, the description of which ideally requires polydisperse distributions, contrary to experiments that rely on reasonably calibrated and essentially monodisperse grains.

A fundamental difference from ordinary plasmas is that dust grains are charged due to interactions with the plasma and radiation environments in which they are submersed. Fluctuations disturb these charging processes, and any change in the grain charge means that...
unit charges are captured from or released into the ambient plasma. Such source/sink effects are difficult to model, taking into account the relative importance of primary electron and ion collection, photoelectron emission, secondary electron emission, charge exchange with neutral grains, photo ablation, particle creation and loss mechanisms, ionization and recombination and other processes [1]–[3].

Describing wave processes in transparent and macroscopic terms like overall charge and mass conservation and injection leads to global characteristics, and the use of charging/attachment frequencies allows a generic discussion of low-frequency wave phenomena while avoiding or postponing the intricacies of the real charging processes [4]. In the latter paper, however, we implicitly assumed that a unique dust fluid velocity could be defined. When dust grains with different charge-to-mass ratios are followed in their charge or mass motion, however, different velocities result, unless all grains have the same charge-to-mass ratio [5, 6]. This assumption is equivalent to having a monodisperse distribution for the dust [6], and there is thus a need to carefully rework the analysis leading to the dispersion characteristics of, for example, the dust-acoustic mode that has been theoretically [7] and experimentally [8] studied. Depending on how the dust plasma frequencies and dust-acoustic velocities are defined in truly polydisperse dust, differences occur compared with a monodisperse treatment, as we now show.

2. Basic formalism

We consider a model that contains, besides the electrons and (plasma) ions, a collection of charged dust grains with differing characteristics, in contrast to the classic picture of dusty plasmas where all dust grains are assumed to have the same average mass and charge [1]–[3]. To describe dust charge (and possibly mass) fluctuations, source/sink terms are included in the relevant equations, because the dust grains capture/release plasma particles depending on the plasma potentials and other details of the charging mechanisms, these being very model dependent. For the electron and (plasma) ion continuity equations we thus have

\[
\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha u_\alpha) = S_\alpha. \tag{1}
\]

The electrons (\(\alpha = e\)) and ions (\(\alpha = i\)) have densities \(n_\alpha\), fluid velocities \(u_\alpha\) and, to be introduced later, charges \(q_\alpha (q_i = e\) and \(q_e = -e\)) and masses \(m_\alpha\). Source/sink terms \(S_\alpha\) model the capture/release of plasma particles.

Because of their multiple and changing identities, all charged dust grains will be described together, with charge density \(\sigma_d\) and mass density \(\rho_d\). The conservation of global charge is expressed as

\[
\frac{\partial}{\partial t} (\sigma_d + n_i e - n_e e) + \nabla \cdot (\sigma_d v_d + n_i e u_i - n_e e u_e) = Q. \tag{2}
\]

Here \(v_d\) is an average dust fluid velocity following its charge, and we refer to recent papers on how this can be computed from a dust charge and mass distribution in phase space [5, 6]. Charge sources/sinks external to the combined plasma are represented by \(Q\), if the dusty plasma is viewed as an open or partially open system in which the electron and ion losses to the charged dust are replenished, as in an experiment if the discharge is maintained. With the help of the continuity equation (1) for the electrons and ions we thus get

\[
\frac{\partial \sigma_d}{\partial t} + \nabla \cdot (\sigma_d v_d) + S_i e - S_e e = Q. \tag{3}
\]
and this serves as the dust charge continuity equation. From this equation it is obvious that if \( Q = 0 \), the electrons, ions and dust grains can only exchange charges between themselves, whereas for \( Q \neq 0 \), charges can be injected from or lost to the outside. Now we perform a similar computation for the mass and momentum densities; we express conservation of mass as

\[
\frac{\partial}{\partial t} (\rho_d + n_i m_i + n_e m_e) + \nabla \cdot (\rho_d \mathbf{u}_d + n_i m_i \mathbf{u}_i + n_e m_e \mathbf{u}_e) = N, \tag{4}
\]

and obtain the dust mass continuity equation

\[
\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d \mathbf{u}_d) + S_i m_i + S_e m_e = N. \tag{5}
\]

Following the dust in its mass motion defines a velocity \( \mathbf{u}_d \) [5], that usually differs from \( \mathbf{v}_d \) owing to different charge and mass weightings, unless all dust grains have the same charge-to-mass ratios at all times [6]. Also, \( N \) refers to the possible external addition or removal of mass, although changes in mass will be an extremely small effect compared with external injection or loss of charge.

As we will focus on electrostatic modes, we need the electron (\( \alpha = e \)) and ion (\( \alpha = i \)) equations of motion,

\[
\left( \frac{\partial}{\partial t} + \mathbf{u}_\alpha \cdot \nabla \right) \mathbf{u}_\alpha + \frac{q_\alpha}{m_\alpha} \mathbf{u}_\alpha + \frac{v_{T\alpha}^2}{n_\alpha} \nabla n_\alpha = M^\alpha - \frac{S^\alpha}{n_\alpha} \mathbf{u}_\alpha, \tag{6}
\]

where \( \phi \) represents the electrostatic potential. The pressure terms have been written in the simplest possible way, involving up to the linear level only scalar and barotropic processes, through suitably defined thermal velocities \( v_{T\alpha} \). The momentum source/sink terms in these equations include the direct losses/gains in momentum \( M^\alpha \), through, for example, elastic-like collisions between plasma particles and/or dust. The second term, on the other hand, expresses momentum gain or loss when electron and ion number densities are not conserved.

Because of the different charge-to-mass ratios involved, we had to introduce two average dust velocities, \( \mathbf{u}_d \) from following its mass motion and \( \mathbf{v}_d \) from following its charge motion. This leads to two evolution equations, namely the classic momentum equation for \( \rho_d \mathbf{u}_d \) and an equation for the evolution of the dust current \( \mathbf{e}_d \mathbf{v}_d \) [5, 6]. To keep matters tractable, we will neglect effects of dust pressure and write the evolution equations with the help of (3) and (5)

\[
\left( \frac{\partial}{\partial t} + \mathbf{u}_d \cdot \nabla \right) \mathbf{u}_d + \frac{\sigma_d}{\rho_d} \nabla \phi = M^{(m)}_d + \frac{\mathbf{u}_d}{\rho_d} (S_i m_i + S_e m_e - N),
\]

\[
\left( \frac{\partial}{\partial t} + \mathbf{v}_d \cdot \nabla \right) \mathbf{v}_d + \frac{\xi_d}{\sigma_d} \nabla \phi = M^{(q)}_d + \frac{\mathbf{v}_d}{\sigma_d} (S_e e - S_e e - Q). \tag{7}
\]

We recall that the charge density \( \sigma_d \) corresponds to averaging the dust distribution by charge, while the mass density \( \rho_d \) does so by mass. A third dust density \( \xi_d \) occurs in a natural way and corresponds to the average over all dust grains of the ratio of the charge squared to the mass. It is this density that gives the correct plasma frequency for polydisperse dust through \( \omega_{pd}^2 = \xi_d / \varepsilon_0 \) [5, 6]. Finally, Poisson’s equation for the electrostatic potential,

\[
\nabla^2 \phi = \frac{1}{\varepsilon_0} (en_e - en_i - \sigma_d), \tag{8}
\]

completes the set of basic equations.
3. Dust-acoustic modes

We linearize the basic equations around a stationary state, denote the perturbation amplitudes by a tilde and get for wave propagation along the $z$ axis from (1) and (6) for the electron ($\alpha = e$) and ion ($\alpha = i$) variables that

\[
\begin{align*}
\frac{\partial \tilde{n}_\alpha}{\partial t} + n_{\alpha 0} \frac{\partial \tilde{u}_\alpha}{\partial z} &= \tilde{S}_\alpha, \\
\frac{\partial \tilde{u}_\alpha}{\partial t} + \frac{q_\alpha}{m_\alpha} \frac{\partial \tilde{\phi}}{\partial z} + \frac{v_{T_\alpha}^2}{n_{\alpha 0}} \frac{\partial \tilde{n}_\alpha}{\partial z} &= \tilde{M}_\alpha.
\end{align*}
\]

(9)

The dust equations (3), (5) and (7), on the other hand, give

\[
\begin{align*}
\frac{\partial \tilde{\rho}_d}{\partial t} + \rho_{d0} \frac{\partial \tilde{u}_d}{\partial z} + m_e \tilde{S}_e + m_i \tilde{S}_i - \tilde{N} &= 0, \\
\frac{\partial \tilde{\sigma}_d}{\partial t} + \sigma_{d0} \frac{\partial \tilde{v}_d}{\partial z} + e \tilde{S}_e - e \tilde{S}_i - \tilde{Q} &= 0, \\
\frac{\partial \tilde{u}_d}{\partial t} + \frac{\sigma_{d0}}{\rho_{d0}} \frac{\partial \tilde{\phi}}{\partial z} &= \tilde{M}^{(m)}_d, \\
\frac{\partial \tilde{v}_d}{\partial t} + \frac{\xi_{d0}}{\sigma_{d0}} \frac{\partial \tilde{\phi}}{\partial z} &= \tilde{M}^{(q)}_d,
\end{align*}
\]

(10)

to be considered together with (8).

Since we study the dust-acoustic mode in the presence of generic charge fluctuations, we will use the common assumption that, compared with the heavy dust grains, the electrons and ions can be treated as effectively massless. Formally, the limit $m_e,i \rightarrow 0$ is taken, but we keep $m_\alpha v_{T_\alpha}^2 = \kappa T_\alpha$ finite, where $T_\alpha$ is the electron or ion temperature. Only the Boltzmann result

\[
\tilde{n}_\alpha = -\frac{n_{\alpha 0} q_\alpha}{\kappa T_\alpha} \tilde{\phi}
\]

(11)
remains of the second equation of (9). The electrons and ions no longer carry any mass, so that there are no momentum exchanges with the dust, but the influence of their changing number densities remains in the dust equations. We will also assume that, even though there may be charge injection from or loss to the outside, there is no such injection or loss of current.

To make further progress, the source/sink terms need to be made explicit. For simplicity, we assume that $S_\alpha$ depends on the species’ own density and on the dust charge and mass densities, so that we can perform a Taylor expansion and obtain for the variations that

\[
\tilde{S}_\alpha = \frac{\partial S_\alpha}{\partial n_\alpha} \big|_0 \tilde{n}_\alpha + \frac{\partial S_\alpha}{\partial \sigma_d} \big|_0 \tilde{\sigma}_d + \frac{\partial S_\alpha}{\partial \rho_d} \big|_0 \tilde{\rho}_d
\]

\[
\equiv -\eta_\alpha \tilde{n}_\alpha - \beta_{ad} \tilde{\sigma}_d - \gamma_{ad} \tilde{\rho}_d.
\]

(12)

If so desired, other dependences besides $\eta_\alpha$, $\beta_{ad}$ and $\gamma_{ad}$ can be included through additional attachment coefficients, the detailed computation of which necessitates a model for the charging processes. A similar procedure for the external charge inflow or outflow $\tilde{Q}$ gives

\[
\tilde{Q} = \frac{\partial Q}{\partial n_e} \bigg|_0 \tilde{n}_e + \frac{\partial Q}{\partial n_i} \bigg|_0 \tilde{n}_i + \frac{\partial Q}{\partial \sigma_d} \bigg|_0 \tilde{\sigma}_d + \frac{\partial Q}{\partial \rho_d} \bigg|_0 \tilde{\rho}_d
\]

\[
\equiv e(\eta^E \tilde{n}_i - \eta^E \tilde{n}_e + \beta_{E} \tilde{\sigma}_d + \gamma_{E} \tilde{\rho}_d).
\]

(13)
and factors \( e \) have been introduced to streamline subsequent notation. As charge fluctuations are not supposed to depend on changes in dust number density \([2, 9]\), we have a generic relation of the form

\[
(\beta_{id} - \beta_{ed} + \beta_E)\sigma_{d0} + (\gamma_{id} - \gamma_{ed} + \gamma_E)\rho_{d0} = 0. 
\]

(14)

The validity of this relation can be verified immediately for the standard orbital-motion-limited (OML) spherical probe model to describe dust charging \([2]\). To see how this comes about in general, we introduce formally a kind of average dust number density \( \tilde{n}_d \), so that \( \sigma_d = n_d m_d \) and \( \rho_d = n_d m_d \). The fluctuations about equilibrium are then \( \tilde{\sigma}_d = \tilde{n}_d q_{d0} + n_{d0}\tilde{q}_d \) and \( \tilde{\rho}_d = \tilde{n}_d m_{d0} + n_{d0}\tilde{m}_d \), and saying in (10) that

\[
\tilde{S}_i - \tilde{S}_e - \frac{\tilde{Q}}{e} = (\eta_e + \eta^E_e)\tilde{n}_e - (\eta_i + \eta^E_i)\tilde{n}_i + (\beta_{ed} - \beta_{id} + \beta_E)(\tilde{n}_d q_{d0} + n_{d0}\tilde{q}_d)
+ (\gamma_{id} - \gamma_{ed} + \gamma_E)(\tilde{n}_d m_{d0} + n_{d0}\tilde{m}_d)
\]

(15)

cannot depend on \( \tilde{n}_d \) yields the condition (14), when we go back to equilibrium charge and mass rather than number densities for the dust. With the help of (12) and (13) we now Fourier transform (10), use (14) and get

\[
\omega\tilde{\rho}_d = \rho_{d0}k\tilde{u}_d,
\]

\[
\omega\tilde{\sigma}_d = \sigma_{d0}k\tilde{v}_d + i e(\eta_i + \eta^E_i)\tilde{n}_i - i e(\eta_e + \eta^E_e)\tilde{n}_e + i e(\beta_{ed} - \beta_{id} + \beta_E)(\tilde{\sigma}_d - \frac{\sigma_{d0}\tilde{\rho}_d}{\rho_{d0}}),
\]

(16)

Elimination of all quantities in favour of \( \tilde{\sigma}_d \) yields

\[
\tilde{\sigma}_d = \frac{\omega k^2 c_{da}^2 + i B k^2 e^2_{da} - i A\omega^2}{\omega^2(\omega + i B)\lambda_{De}^2}\tilde{\phi},
\]

(17)

where the coefficients

\[
A = \frac{(\eta_i + \eta^E_i)\lambda_{De}^2 + (\eta_e + \eta^E_e)\lambda_D^2}{\lambda_{De}^2},
\]

\[
B = e\beta_{ed} - e\beta_{id} - e\beta_E,
\]

(18)

have the dimensions of a frequency. We repeat that the different attachment coefficients \( \eta_e, \eta^E_e, \eta_i, \eta^E_i, \beta_{ed}, \beta_{id} \) and \( \beta_E \) can be computed when the charging mechanisms are made explicit, so that then \( A \) and \( B \) become known frequencies.

The electron and ion Debye lengths are given through \( \lambda_{Da}^2 = \varepsilon_0 k T_a/n_e e^2 \), a global plasma Debye length \( \lambda_D \) through \( \lambda_{Da}^2 = \lambda_{De}^2 + \lambda_D^2 \) and there are two definitions of the dust-acoustic velocity, \( c_{da} = \lambda_D \omega_{pd} \) and \( \tilde{c}_{da} = \lambda_D \tilde{\omega}_{pd} \) \([5, 6]\). The latter, in turn, stem from the two possible dust plasma frequencies, one defined through \( \omega_{pd}^2 = \varepsilon_0/\varepsilon_0 \), based on the correct average over all dust charge squared to mass ratios and one given through \( \tilde{\omega}_{pd}^2 = \sigma_{d0}/\varepsilon_0 \rho_{d0} \), using essentially the average charges and masses computed separately. There is a general relation \( \tilde{\omega}_{pd} \leq \omega_{pd} \) \([5, 6]\). Inserting the expression (17) for the perturbed dust density in Poisson’s equation (8) gives a cubic dispersion law

\[
(1 + k^2 \lambda_D^2)\omega^3 + i[A + B(1 + k^2 \lambda_D^2)]\omega^2 - k^2 c_{da}^2 \omega - i Bk^2 e^2_{da} = 0.
\]

(19)

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This corresponds essentially to the dispersion law obtained by, among others, Varma [10], up to specific kinetic effects outside our approach, and if no distinction is made between both dust-acoustic velocities. At the same time it puts our previous result [4] in perspective for truly polydisperse dust, since in treatments with a unique dust-acoustic velocity all dust grains have the same charge-to-mass ratio, and this is formally equivalent to a monodisperse description [6].

In the absence of charge fluctuations \((A = B = 0)\), there are two roots of (19) corresponding to the dust-acoustic mode, \(\omega^2 = k^2 c^2_{da} / (1 + k^2 \lambda^2_D)\), whereas the zero-frequency root represents the remainder of the charging mode.

When the different attachment frequencies are small compared with the wave frequency \(|A|, |B| \ll \omega\), the dust-acoustic roots of (19) can be approximated by

\[
\omega \simeq \pm \frac{k c_{da}}{\sqrt{1 + k^2 \lambda^2_D}} - \frac{i A}{2(1 + k^2 \lambda^2_D)} - i B \left(1 - \frac{c^2_{da}}{c^2_{da}}\right).
\]

(20)

In the OML spherical probe theory, one can show [2] that \(A > 0\) and \(B > 0\), so that the dust-acoustic modes are indeed damped [11]–[13]. For space dust distributions observed in planetary rings, there occur power-law density decreases with grain size [14]–[16], for which then \(\bar{\omega}^2_{pd}\) can be up to 10–20% smaller than \(\omega^2_{pd}\), with similar conclusions for \(c^2_{da} < c_{da}^2\) [17]. Since this is not a huge difference, an average monodisperse description can usually be deemed adequate. In addition, the main damping in (20) comes from \(A\), because the coefficient of \(B\) tends to be fairly small. Hence it is chiefly the electron and ion density perturbations that influence the dust charge fluctuations. The third root of (19) is a charging mode with dispersion

\[
\omega \simeq -i B \frac{c^2_{da}}{c_{da}^2},
\]

(21)

and in this zero real frequency mode \(B\) functions as a kind of charging frequency, multiplied by a coefficient which is smaller for a polydisperse dust distribution than for the corresponding monodisperse approximation.

The converse frequency regime means fast (dis)charging times, expressed here as \(\omega \ll |A|, |B|\), so that to lowest order

\[
\omega^2 = \frac{k^2 c^2_{da}}{1 + k^2 \lambda^2_D + A/B}.
\]

(22)

With the OML interpretation of \(A\) and \(B\) this corresponds to the result given by Rao [18, 19] when discussing the influence of fugacity. If \(|A/B| \ll 1 + k^2 \lambda^2_D\), we recover the dust-acoustic modes [7]. In the converse case, when \(1 + k^2 \lambda^2_D \ll |A/B|\), we obtain a mode with phase velocity given through

\[
\left(\frac{\omega}{k}\right)^2 \simeq B \frac{c^2_{da}}{A},
\]

(23)

much smaller than \(c^2_{da}\). This generalizes to more complicated charging mechanisms the notion of dust-Coulomb modes, explicitly introduced for dense dusty plasmas on the basis of the OML charging model [18]–[20] but already discussed in an earlier paper on charge fluctuations [11]. Such dense dusty plasmas are presumed to exist in certain astrophysical plasmas like the F ring of Saturn [11, 18]. Moreover, the phase velocity of the dust-Coulomb modes given in (23) has also been derived within the OML framework from a kinetic treatment, so that it is not an artefact of the fluid description. In the limit of stronger coupling between the dust grains, the phase velocity in (23) is close to the phase velocity of dust-lattice waves [21].
Given this, our general treatment is a strong indication that a concept like that of dust-Coulomb modes in dense dusty plasmas is more general than could be inferred from the OML charging model alone. Indeed, in the dense regime as defined here in a more abstract setting, it follows from (17) that
\[ \tilde{\sigma}_d = \tilde{n}_d q_d + n_{d0} \tilde{\sigma}_d \simeq 0, \] (24)
and the total dust space charge density remains approximately constant. There is thus at the level of Poisson’s equation (8) not only quasi-neutrality, but perturbations in the electron and ion charge densities do not contribute appreciably [22]. The response of the Coulomb shielding inside the Debye cloud around the dust is the main mechanism to sustain the waves [11, 18, 19, 22]. While the reasoning in papers dealing with dust-Coulomb modes is based on the OML charging mechanism, our treatment shows that the concept is a generic one in dense dusty plasmas with charge fluctuations.

In addition, there is a small damping (or maybe growth), determined from (19) by taking \( \omega = \omega_r + i \gamma \), where \( \omega_r^2 \) is given by the right-hand side of (22) and by supposing that \( \gamma \ll \omega_r \). This yields
\[ \gamma = \frac{A k^2 c_d^2 + B k^2 (c_d^2 - \bar{c}_d^2)(1 + k^2 \lambda_D^2)}{2 B^2 (1 + k^2 \lambda_D^2 + A/B)^2}, \] (25)
This generalizes results where OML [19] and secondary electron currents [23] have been used.

As pointed out already in our earlier paper [4], the open or closed nature of the dusty plasma does not alter these qualitative conclusions, since we include both possibilities, or even a mixture of these, in our treatment. Nevertheless, the determination of all the attachment frequencies crucially depends on consistent and complete dust charging models, a debate that is nowhere near finished yet.

4. Conclusions

Charge (and mass) fluctuations in dusty plasmas with dust distributions have been modelled by incorporating general sink/source terms in all basic equations. Expressing the overall evolution of charge, mass, momentum and current for electrons, ions and dust together leads to pairs of dust equations for continuity and motion.

When studying generic characteristics of dust-acoustic modes, the source/sink terms are represented through attachment coefficients. The precise computation of these demands a detailed charging model that is for the time being not generally available, except for partial results derived from the standard OML probe model and extensions thereof. Generic characteristics of dust-acoustic and dust-Coulomb modes with charge fluctuations encompass and enlarge earlier results from OML theory. In particular, polydisperse dust distributions necessitate different definitions of the dust plasma frequency and the dust-acoustic velocity, depending on how the averaging is done. We have also shown that the notion of dust-Coulomb modes in dense dusty plasmas with charge fluctuations is generic, transcending the OML model used in the first papers on this subject.

Our conclusions are qualitatively the same, regardless of whether the dusty plasma is considered as an open or a closed system, or a mixture of both. The elaboration of a truly consistent model for the interactions between and charging of dust particles in a plasma in a radiative environment remains an open problem, recent progress notwithstanding.
Acknowledgment

We thank the Fonds voor Wetenschappelijk Onderzoek (Vlaanderen) for support.

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