Wi-Fi Coexistence with Duty Cycled LTE-U

Yimin Pang
University of Colorado Boulder
ECEE Dept, 425 UCB
Boulder, Colorado 80309
Email: yipa5803@colorado.edu

Alireza Babaei, Jennifer Andreoli-Fang, Belal Hamzeh
Cable Television Laboratories
858 Coal Creek Circle
Louisville, CO 80027
Email: ar.babaei@gmail.com, {j.fang, b.hamzeh}@cablelabs.com

Abstract—Coexistence of Wi-Fi and LTE in unlicensed spectrum (LTE-U) is recently drawing significant concern in industry. In this paper, we investigate the Wi-Fi performance in the presence of duty cycle based LTE-U transmission on the same channel. More specifically, one LTE-U cell and one Wi-Fi basic service set (BSS) coexist by allowing LTE-U devices transmit their signals only in predetermined duty cycles. Wi-Fi stations, on the other hand, simply contend the shared channel using the distributed coordination function (DCF) protocol without cooperation with the LTE-U system or prior knowledge about the duty cycle period or duty cycle of LTE-U transmission. We define the fairness of the above scheme as the difference between Wi-Fi performance loss ratio (considering a defined reference performance) and the LTE-U duty cycle (or function of LTE-U duty cycle). Depending on the interference to noise ratio (INR) being above or below -62 dbm, we classify the LTE-U interference as strong or weak and establish mathematical models accordingly. The average throughput and average service time of Wi-Fi are both formulated as functions of Wi-Fi and LTE-U system parameters using a probabilistic framework. Then, we use the Monte Carlo analysis to demonstrate the fairness of Wi-Fi and LTE-U air time sharing.

Index Terms—coexistence, LAA-LTE, LTE-U, LTE unlicensed, medium access delay, service time, throughput, Wi-Fi, WLAN

I. INTRODUCTION

The rapidly growing demand of wireless network services has led the mobile network operators (MNOs) to look into the possibility of exploring unlicensed spectrum to offload the data traffic from the licensed bands. Most recently, 3GPP is considering extending LTE into unlicensed spectrum as a promising technique, known henceforth as LTE unlicensed (LTE-U)[1], to offload part of the LTE data traffic onto unlicensed spectrum. Compared to data offload using Wi-Fi, this approach has the advantage of seamless integration into the existing LTE evolved packet core (EPC) architecture. In a proposal outlined in [1], three LTE-U modes are introduced as supplement downlink, TD-LTE-U carrier aggregation and standalone, the first two of which have been proposed to 3GPP.

Coexistence of heterogeneous networks such as Wi-Fi and LTE-U on the same band and their uncoordinated operations can potentially cause significant interference, degrading the performance of both systems. Solutions to manage the interference between such systems are therefore necessary for their successful coexistence. One straightforward method is to split the common radio channel through air time sharing between the Wi-Fi and LTE-U sub-systems. With this approach, LTE-U operates over the shared channel periodically and during each period (the so-called duty cycle period, denoted as $T$), only a portion (defined by the LTE-U duty cycle $\alpha$) of the time is utilized for the LTE-U transmission, as shown in Fig. 1. In this scheme, Wi-Fi has no cooperation with LTE-U. Wi-Fi stations have neither knowledge about the time length of duty cycle period nor the duty cycle, they simply access the shared channel by standard channel sensing and random back-off mechanisms.

In an LTE system, each of the UEs communicates with eNB in a deterministic way through a centralized channel access control mechanism and synchronization, i.e. the access time and OFDM sub-carriers of an LTE frame are predetermined at eNB, where the MAC scheduler incorporates the radio measurement and quality of service needed for each UE. Given the above time sharing scheme and the LTE’s centralized access structure, computation of the LTE-U performance in terms of throughput and service time is relatively simple and straightforward. However, the medium access at the Wi-Fi side, controlled by distributed coordination function (DCF) protocol defined in IEEE 802.11 standard, is random and distributed. Therefore, we focus on the impact of LTE-U interference on Wi-Fi performance in this paper.

A. Previous works

LTE-U and Wi-Fi coexistence is a relatively new area of research. Previous works in this area are summarized as follows: In [2], Wi-Fi and LTE-U coexistence in single floor and multiple floors environment at various densities are simulated. The results show that without any interference management scheme, LTE-U system performance is slightly affected from

---

1Since July 2014, 3GPP has been using License Assisted Access (LAA) as the official 3GPP term, and ETSI amended “Listen before talk” into LAA. By the time this paper was completed, both FDD and TDD modes were introduced in TR36.889. Meanwhile, the LTE-U forum, formed by Verizon and a few other vendors in 2014, released an LTE supplemental downlink (SDL) coexistence specification, where an adaptive duty cycle based coexistence scheme was introduced, and the term “LTE-U” was used instead of “LAA” in the specification. Hence, there are now technical and business related differences between LTE-U and LAA. Various vendors and operators have developed positions in support of one or the other. However, since this paper focuses on duty cycle based LTE on unlicensed spectrum, we still use the short and conventional term LTE-U for convenience.

2For more details, see [3].
Wi-Fi, whereas Wi-Fi is significantly impacted by the LTE-U transmissions. This result is reinforced in [3] by computing the Wi-Fi successful channel accessing probability in the presence of LTE-U transmission. However, in [1], LTE-U is described as a better neighbor to Wi-Fi than Wi-Fi to itself as long as a proper coexistence mechanism (called CSAT) is applied. Authors in [4] present simulation results on spectrum efficiency comparison between Wi-Fi and LTE-U in a sparse deployment scenario. The paper, however, lacks sufficient details on the coexistence features and their effectiveness. Cano and Leith [5] proposed a duty-cycle mechanism for LTE-U, which, by selecting an appropriate probability to access the channel and transmission duration, ensures proportional fairness among LTE-U and Wi-Fi nodes. Specifications regarding duty-cycled based LTE-U are released and maintained by LTE-U forum [6], where CSAT is officially introduced as the access mechanism.

On the other hand, the 3GPP study item technical report document [7] has listed Listen-Before-Talk (LBT) as the required function for clear channel assessment for LTE LAA. The application of LBT may potentially enhance the coexistence behavior of Wi-Fi and LTE. Some analysis and performance test have been reported in [8]–[10].

B. Main results

In this paper, we define Wi-Fi average saturation throughput $\mathcal{R}(T, \alpha, \mathcal{H})$ (in terms of bits/Wi-Fi slot time) and average service time $D(T, \alpha, \mathcal{H})$ (in terms of Wi-Fi time slots) to be functions of $T$, $\alpha$ and $\mathcal{H} = \{q, L, n\}$, where the set $\mathcal{H}$ represents an $n$-clients Wi-Fi sub-system with LTE-U to Wi-Fi collision probability (Wi-Fi transmission failure probability due to LTE-U transmission) $q$ and Wi-Fi data payload length $L$ (the length of MAC data payload, in terms of bytes). Given $\mathcal{H}$, the throughput fairness (cf. Def.2) of a $(T, \alpha)$ air time sharing scheme is measured by the difference between average Wi-Fi saturation throughput loss ratio (with respect to the corresponding non-LTE-U duty cycle scenario $(\infty, 0, \mathcal{H})$ performance) and LTE-U duty cycle $\alpha$, i.e., $\frac{\mathcal{R}(\infty, 0, \mathcal{H}) - \mathcal{R}(T, \alpha, \mathcal{H})}{\mathcal{R}(\infty, 0, \mathcal{H})} - \alpha$. In a similar way, the average service time fairness is defined as $\frac{D(T, \alpha, \mathcal{H}) - D(\infty, 0, \mathcal{H})}{D(\infty, 0, \mathcal{H})} - \frac{1}{T-\alpha}$. These two fairness measures indicate whether Wi-Fi will lose less or more than $\alpha$ portion of its performance (in the absence of LTE-U transmission) if $\alpha$ portion of the channel resource is shared with LTE-U.

Our first step is to analytically formulate $\mathcal{R}(T, \alpha, \mathcal{H})$ and $D(T, \alpha, \mathcal{H})$ using a probabilistic framework. The following two key techniques are employed:

a) Only one labeled client station among the $n$ Wi-Fi stations being affected by LTE-U interference: As first introduced by [11], Wi-Fi DCF can be formulated into a Markov chain model, which was generalized later in [12] and [13]. But when LTE-U is considered, the Markov property no longer holds, because the chain state transition probability becomes time variant. We make an assumption that only one client station among the $n$ stations, labeled as Sta-A, is affected by the LTE-U interference. The other $n-1$ stations render the Wi-Fi background traffic for the labeled station. When $n$ is chosen to be large enough, the background traffic could still be approximately modeled using the existing framework in [11]–[13]. Under this assumption, functions $\mathcal{R}(T, \alpha, \mathcal{H})$ and $D(T, \alpha, \mathcal{H})$ are with respect to Sta-A.

b) Different interference levels lead to different formulations: The Wi-Fi DCF employs CSMA/CA with binary exponential back-off algorithm. Depending on the energy level being detected, the back-off timer may or may not be frozen. In short, an LTE-U transmission with interference to noise ratio (INR) greater than -62dbm or a neighbor Wi-Fi station transmission with INR greater than -82dbm will cause the interfered Wi-Fi station freeze its back-off timer. We refer to weak interference as the LTE-U interference with its INR being less than -62dbm, and strong interference as interference with INR greater than -62dbm. The mathematical formulations as well as the performance results are quite different between the cases of weak and strong LTE-U interference.

Other assumptions are just inherited from the existing framework by [11]–[13] on Wi-Fi DCF: 1) A transmission from one Wi-Fi station can be heard by all the other $n-1$ Wi-Fi stations, and Wi-Fi to Wi-Fi INR is always greater than -82dbm; 2) Collisions between Wi-Fis or LTE-U to Wi-Fi are the only causes to Wi-Fi failure transmission.

Then, we analyze the performance as well as the fairness numerically. Both the analytical functions built for the weak and strong LTE-U interference are computationally inefficient and characterizing the two performance functions in closed form is hard. On the other hand, implementing Monte Carlo analysis based on these two functions is simple. It is also difficult and meaningless to show the fairness over all possible combinations of $T$, $\alpha$ and $\mathcal{H}$. We focus our attention on
the cases when LTE-U to Wi-Fi collision probability $q = 1$, which has wide measure over real systems where LTE-U INR and Wi-Fi SNR are comparable. Other parameters are also selected in a reasonable range according to practical system setting. The results demonstrated in Section IV, support the conclusions below:

1) Fix $q = 1$: Under strong interference, air time sharing scheme could approximately achieve the fairness for some $(T, \alpha)$; Under weak interference, air time sharing scheme is generally unfair;

2) The fairness measure degrades almost linearly when LTE-U to Wi-Fi collision probability $q$ increases;

3) The fairness measure degrades almost linearly when Wi-Fi payload length $L$ increases.

The rest of the paper is organized as follows: Section II presents the Wi-Fi and LTE-U coexistence model and formulates the problem; Section III characterizes the average Wi-Fi saturation throughput and average service time in the presence of LTE-U duty cycle; The impact of duty cycled LTE-U interference to Wi-Fi is discussed in Section IV; Section V concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we first introduce the generalized Markov chain model for Wi-Fi DCF, then we formulate the Wi-Fi and duty cycled LTE-U coexistence, lastly we define the fairness measure.

A. Generalized Markov chain model for Wi-Fi DCF

In [11], the Wi-Fi DCF is formulated into a two dimensional Markov chain, the $i$-th floor in the Markov chain (refer to Fig. 2) stands for the random back-off process before the $i$-th transmission attempt, where $0 \leq i \leq M$, with contention window size $CW_i = 2^i CW_0$, where $CW_0$ is the contention window size of the 0-th back-off. This Markov chain has transition probability $p(i_{n+1}, j_{n+1} \mid i_n, j_n)$ (With a slight abuse of notation, we temporarily use $n$ to denote the state at $n$-th discrete moment).

\begin{equation}
\begin{cases}
1 & i_{n+1} = i_n; j_{n+1} = j_n - 1 \\
1 - p_{in} & i_{n+1} = 0, j_n \neq M - 1; \\
& j_{n+1} \in \{0, \cdots, CW_0\}, j_n = 0 \\
\frac{p_{in}}{2^{n+1} CW_0} & i_{n+1} = i_n + 1, i_{n+1} \neq M - 1; \\
& j_{n+1} \in \{0, \cdots, 2^n + CW_0\}, j_n = 0 \\
1 & i_{n+1} = 0, i_n = M - 1; \\
& j_{n+1} \in \{0, \cdots, CW_0\}, j_n = 0
\end{cases}
\end{equation}

Let $\sigma$ be the duration of the Wi-Fi system slot time as defined in IEEE 802.11 standard. Throughout the paper, we normalize all the time variables to $\sigma$, which means 1s is normalized to $1/\sigma$. During each $(i, 0)$ state, a Wi-Fi station senses the channel, with probability $p_i$ it detects clear channel and transmits (or re-transmits) a packet. If successful, the station

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Markov model for Wi-Fi DCF (refer to [11])}
\end{figure}
according to the transition probability defined in (1), where \( p(i,j) \) is the stationary distribution of the Markov chain. There is no close form expression of the solution to \( p_c \) and \( \tau \), but given the system parameters and number of stations, they can be numerically computed. When the Wi-Fi system is saturated, i.e. the buffer in each station is never empty, i.e. \( \lambda = 0 \) and \( p_c = 1 - (1 - \tau)^{n-1} \).

It remains to specify the distribution of unit decrement time \( T_d \). Let \( T_s \) and \( T_c \) be the time duration, normalized to the system slot time, of one successful and failed (collided) transmission, respectively. If CTS/RTS mechanism is used, \( T_s \) and \( T_c \) can be calculated as follows

\[
T_s = \text{RTS} + \text{CTS} + \text{HDR} + L + \text{ACK} + 3 \times \text{SIFS} + \text{DIFS} \quad (4)
\]

\[
T_c = \text{RTS} + \text{DIFS} \quad (5)
\]

otherwise

\[
T_s = \text{HDR} + L + \text{ACK} + \text{SIFS} + \text{DIFS} \quad (6)
\]

\[
T_c = \text{HDR} + L + \text{DIFS} \quad (7)
\]

In both cases, \( L \) denotes the length of the data payload of a Wi-Fi frame. Let \( p_s \) be the probability that one of the other \( n - 1 \) Wi-Fi station transmit successfully\(^3\), i.e.

\[
p_s = \frac{(n-1)}{n} - \frac{1}{n} \quad (8)
\]

The unit decrement time \( T_d \) has following pmf,

\[
p_{T_d}(t_d) = \begin{cases} 
1 - p_c & t_d = 1 \\
p_c - p_s & t_d = T_c \\
p_s & t_d = T_s \\
0 & \text{o.w.}
\end{cases}
\]

B. Formulation of Wi-Fi and duty cycled LTE-U coexistence

Consider an infrastructure-based Wi-Fi network coexisting with an LTE-U network on the same unlicensed band, where interference is coming from LTE-U sub-system to the Wi-Fi station labeled Sta-A. Considering a duty cycle period which extends \( T \) Wi-Fi system slots (refer to Fig. 1) the eNB or UEs in LTE-U sub-system transmit during the LTE-U ON stage of duration \( \alpha T \), where \( \alpha \in [0, 1] \), and keep silence during the LTE-U OFF stage\(^4\). The variables \( T \) and \( \alpha \) are defined to be LTE-U duty cycle period and duty cycle, respectively. The Wi-Fi sub-system does not cooperate with LTE-U nor has any prior knowledge about LTE-U interference, it simply transmits data frame based on the DCF mechanism.

As has been introduced before, assuming only one out of the \( n \) Wi-Fi stations receive LTE-U interference is for the purpose of maintaining the Markov properties to model the rest of \( n - 1 \) stations, so when \( n \) is large enough, the collision probability \( p_c \) and unit decrement time \( T_d \) can still be approximately computed using the generalized Markov chain model. The \( n - 1 \) non-interference stations actually provide a stationary background Wi-Fi traffic for Sta-A.

We denote a Wi-Fi sub-system as \( \mathcal{H}(q, L, n) \), where \( q \in [0, 1] \) is the Wi-Fi collision probability subject to LTE-U interference (the Wi-Fi transmission failure probability when a Wi-Fi frame and an LTE-U frame transmits simultaneously, it only applies to Sta-A), \( L \) is the data payload length in each transmission, and \( n \) the number of clients in the Wi-Fi sub-system which determines the Wi-Fi to Wi-Fi collision probability (i.e., Wi-Fi collision probability in the absence of LTE-U transmission). Furthermore let \( (T, \alpha) \) denote an air time sharing scheme with duty cycle period \( T \) and LTE-U duty cycle \( \alpha \).

Instead of adopting the uniformed Wi-Fi throughput as in\(^1\), we evaluate the Wi-Fi throughput (saturation throughput of Sta-A, the same premise keeps for future discussion) \( \mathcal{R}(T, \alpha, \mathcal{H}) \) as the number of bits can be successfully transmitted per Wi-Fi slot time. The Wi-Fi service time \( D(T, \alpha, \mathcal{H}) \), or the medium access delay, is defined to be the time interval (also normalized to system slot time) from the time instant that a packet becomes the head of the queue and starts to contend for transmission, to the time instant that either the packet is acknowledged for a successful transmission or the packet is dropped. Note both \( \mathcal{R}(T, \alpha, \mathcal{H}) \) and \( D(T, \alpha, \mathcal{H}) \) are random variables according to above definition. Finally, we define the average Wi-Fi throughput and average service time \( \mathcal{R}(\infty, \alpha, \mathcal{H}) \) (in terms of bits/Wi-Fi slot time) and \( D(\infty, \alpha, \mathcal{H}) \) (in terms of Wi-Fi time slots) respectively for a coexistence system with Wi-Fi sub-system \( \mathcal{H}(q, L, n) \) and air time sharing scheme \( (T, \alpha) \) as

\[
\mathcal{R}(T, \alpha, \mathcal{H}) = \mathbb{E}[\mathcal{R}(T, \alpha, \mathcal{H})] \quad (10)
\]

\[
D(T, \alpha, \mathcal{H}) = \mathbb{E}[D(T, \alpha, \mathcal{H})] \quad (11)
\]

For convenience, we sometimes omit the underlying variables \( (T, \alpha, \mathcal{H}) \), and just use a simple notation as letter \( \mathcal{R} \) or \( \mathcal{R} = \mathbb{E}[\mathcal{R}] \) for short.

C. Definition of fairness

It is a critical task to define what fairness means in this context, since there could be many ways to describe the fairness in such a coexistence scenario. One straightforward way is to compare the Wi-Fi performance with and without the presence of LTE-U. More specifically, we want to find answer to the question: Will the performance loss (throughput degradation and service time increase) due to time sharing be proportional to the duty cycle \( \alpha \)? Also, what reference values should be used when we characterize the performance loss? The definition below gives an intuitive way of measuring fairness.

**Definition 1.** For a given \( \mathcal{H}(q, L, n) \), assume the reference Wi-Fi performance to be \( \mathcal{R}(\infty, 0, \mathcal{H}) \) and \( D(\infty, 0, \mathcal{H}) \). The throughput fairness \( \phi_R(T, \alpha, \mathcal{H}) \) is the difference between the average throughput loss ratio and the LTE-U duty cycle \( \alpha \), i.e.

\[
\phi_R(T, \alpha, \mathcal{H}) = \frac{\mathcal{R}(\infty, 0, \mathcal{H}) - \mathcal{R}(T, \alpha, \mathcal{H})}{\mathcal{R}(\infty, 0, \mathcal{H})} - \alpha \quad (12)
\]
and service time fairness $\phi_D(T, \alpha, H)$ is the difference of average service time increase ratio to $\frac{1}{1-\alpha}$, i.e.

$$
\phi_D(T, \alpha, H) = \frac{D(T, \alpha, H) - D(\infty, 0, H)}{D(\infty, 0, H)} - \frac{\alpha}{1-\alpha} \quad (13)
$$

Depending on $H$ and $(T, \alpha)$, the fairness measures $\phi_R$ and $\phi_D$ can be negative, positive or zero. If both these two parameters ($\phi_R$ and $\phi_D$) are zero, Wi-Fi performs at exact $(1-\alpha)$ “portion” of the non-LTE duty cycle performance. We consider such a time sharing scheme to be acceptable when $\phi_R$ and $\phi_D$ are trivial i.e. interference causes almost no impact to Wi-Fi system which is not in the scope of this paper. It is obvious that very low INR/SNR interference causes almost no impact to Wi-Fi system which is trivial i.e. $q \approx 0$ and $\phi_R, \phi_D \leq 0$. This paper focus on the situations when INR and SNR are comparable, and in most subsequent discussions we assume $q = 1$, which means a Wi-Fi transmission will definitely fail if an LTE-U transmission occurs at the same time. Additionally, we will show in Section IV how $\phi_D$ and $\phi_R$ decay when $q$ increases from 1 to 0 in a numerical example. Readers are reminded that $q = 1$ can happen to either strong or weak interference cases.

### III. IMPACT OF DUTY CYCLED LTE-U INTERFERENCE

During the LTE-U ON period, the $i$-th attempt of Wi-Fi transmission fails with probability

$$
p_i = 1 - (1 - p_c)(1 - q) \quad (14)
$$

Whether the failure probability should be chosen as $p_i$ or $p_i'$ depends on whether the LTE-U is ON or OFF, therefore the Sta-A DCF could no longer be modeled by Markov chain. Instead, we characterize the Sta-A throughput and service time in three steps:

1) Suppose a Wi-Fi packet leaves Sta-A buffer at time $T_0 = t_0$, where $t_0 \in \{0, 1, \cdots, T - 1\}$, and at time $T_e$, where $T_e \in \{t_0, \cdots, \infty\}$, the packet will either be sent out successfully or dropped. Conditioning on $t_0$, we compute the conditional distribution $p_{T_e | T_0}(t_e | t_0)$ of the finish time $T_e$, the conditional mean service time $E[D | t_0]$ and conditional mean throughput $E[R | t_0]$;

2) Let $T'_e$ be a function of $T_e$ that

$$
T'_e = T_e \mod (T - 1)
$$

the conditional distribution of $T'_e$ can be derived from $p_{T_e | T_0}(t_e | t_0)$ as

$$
p_{T'_e | T_0}(t'_e | t_0) = \sum_{T_e: T_e \mod (T-1)=t_e} p_{T_e | T_0}(t_e | t_0)
$$

If we regard the $T$ time slots (labeled as $0, \cdots, T - 1$) in a duty cycle period as $T$ states, those $T$ states form the state space of a one dimension Markov chain, with transition probability from state $t_0$ to state $t_e$ of

$$
p_{T_e | T_0}(t_e | t_0)
$$

compare $t_e$ with the values of $\alpha T, (1+\alpha)T, 2T, \cdots$, it could be figured out if the $i$-th attempt falls within the LTE-U ON period, record the result by a bool function

$$
g(\zeta_i) = \begin{cases} 0 & \alpha T \leq \zeta_i \mod T < (\zeta_i + T) \mod T < T \\ 1 & \text{otherwise} \end{cases}
$$

combining (15) and (16), we have

$$
p(\zeta(w^{(m)}) | t_0) = \left( \prod_{i=0}^{\lfloor \frac{m-2}{2}\rfloor} \frac{1 - (1 - p_c)(1 - qg[\zeta_i(w^{(m)})])}{\frac{CW_i}{CW_{m-1}}(1 - p_c)(1 - qg[\zeta_{m-1}(w^{(m)})])} \right)
$$

$$
\times \left( \frac{1 - p_c}{\frac{CW_{m-1}}{CW_{m-1}}(1 - p_c)(1 - qg[\zeta_{m-1}(w^{(m)})])} \right)
$$

The end time $T_e$ is a function of random vector $W^{(m)}$, $T_e(W^{(m)})$ is the sum of the total waiting time and the time each transmission of back-off pattern $W^{(m)}$. Let $b(W^{(m)})$.
be a function of random variable $W^{(m)}$ of the successful retransmission probability of the $m$-th retrial,

$$h(W^{(m)}) = (1 - p_c)(1 - qg(\zeta_{m-1}(W^{(m)})))$$ (18)

According to the DCF, the mapping $f : (W^{(m)}, t_0) \rightarrow T_e$ is $f(W^{(m)}, t_0) =$

$$\begin{cases} 
  t_0 + E[T_d] \zeta_{m-1} + T_s & 1 \leq m < M + 1 \\
  t_0 + h(W^{(M)}) E[T_d] \\
  \times \left( \sum_{i=0}^{M} W_i + M T_c + T_s \right) & m = M + 1 \\
  +(1 - h(W^{(M)})) E[T_d] \\
  \times \left( \sum_{i=0}^{M} W_i + (M + 1) T_c \right)
\end{cases}$$ (19)

Note in (19), to make $f(W^{(m)}, t_0)$ a map, we have to deal with the map between $(W^{(M+1)}, t_0) \rightarrow T_e$ so it has unique image, we take the last re-transmission duration to be its expectation. The finish time $T_e$ has pmf

$$p_{T_e|T_0}(t_e|t_0) = \sum_{w: f(w) = t_e} p_{W_l|T_0}(w|t_0)$$ (20)

Knowing the distribution of $T_e$, the conditional mean $E(D|t_0)$ can be written as

$$E[D|t_0] = E[T_e - t_0]$$ (21)

For $E[R|t_0]$, we need to find out the probability that a packet transmission started at $t_0$ being dropped, denoted as $p_{dv}(t_0)$,

$$p_{dv}(t_0) = \sum_{w^{(M+1)}} p_{W_l|T_0}(w^{(M+1)}|t_0) [1 - h(w^{(M+1)})]$$ (22)

the conditional mean throughput can be written as

$$E[R|t_0] = L(1 - p_{dv}(t_0)) E \left\{ \frac{1}{D} | t_0 \right\}$$ (23)

B. The conditional probability $p_{T_e|T_0}(t_e|t_0)$ under strong LTE-U interference

Under strong interference, the Wi-Fi back-off timer will be blocked when LTE-U is ON. As will be demonstrated later, it effectively helps Wi-Fi to eliminate the LTE-U interference. The LTE-U interference not only cause the collision to Wi-Fi, but also the unit decrement time $T_d$ at Wi-Fi part will have a time variant pmf. When LTE-U is ON, the Wi-Fi mean unit decrement time $E[T_d]$ become $\alpha T$\footnote{When $\alpha T$ is chosen at a reasonable value, for instance $T \geq 100$.}. As a result, equation (15) and (19) which both reply on $E[T_d]$ no longer hold true. Computation of $\zeta_w$ and $T_e = f(W^{(m)}, t_0)$ needs iterative algorithm, which is given below, this algorithm has complexity $O(n^2)$. Shortly speaking, the counting process keeps checking if $T_e \mod T < \alpha T$ in every iteration, if it is true then a constant $(\alpha T - S \mod T)$ is added to the partial sum of $T_e$.

$$T_e = t_0$$

for $i = 0 : m - 1$

for $j = 0 : w_i - 1$

if $(T_e \mod T < \alpha T)$

$T_e = T_e + \alpha T - T_e \mod T$

else

$T_e = T_e + E[T_d]$

end

end

$\zeta_i = T_e$;

if $(i < m - 1)$

$T_e = T_e + T_c$

else

if $(m \neq M + 1)$

$T_e = T_e + T_s$

else

$T_e = T_e + (1 - h(w)) T_c + h(w) T_s$

end

end

IV. FAIRNESS EVALUATION – MONTE CARLO ANALYSIS

It is computationally unpractical to characterize the distribution of $p_D(\cdot|t_0)$ as a function of $p_{W_l|t_0}$ in closed form, since the sample space of random vector $W_l^{(M)}$ has cardinality of $\Pi_{i=0}^{M-1} CW_i$, which is at the scale of $10^{11}$. However, based on the analytical discussion in the previous section, it is easy to implement a Monte Carlo analysis for both the weak and strong interference cases. In this section, we refer average throughput and average service time simply as throughput and service time.

We numerically discuss the impact of LTE-U duty cycle in terms of the following parameters:

- duty cycle period $T$;
- the duty cycle $\alpha$;
- LTE-U to Wi-Fi collision probability $q$;
- Wi-Fi payload length $L$.

On the other hand, the parameters below are fixed:

- Wi-Fi system is saturated, i.e. $\lambda = 0$;
- RTS/CTS is applied, slot time $\sigma = 9$ms, Wi-Fi physical layer bit rate is $1$Mb/s, for the random back-off, $M=6$, $CW_0 = 16$, other parameters respect to IEEE 802.11n standard in the 5GHz band;
- The scenario contains 17 Wi-Fi client stations, according to (12), we know the Wi-Fi to Wi-Fi collision probability $p_c = 0.3739$.

A. The role of duty cycle period $T$

Fixing the duty cycle $\alpha = 0.3$, $q = 1$ and $L = 1$KB, fig.\footnote{fig.3} shows the impact of duty cycle period $T$ on throughput and service time, for both weak and strong interference scenarios.
It can be observed that $T \leq 600ms$ (note one LTE-U frame duration is 10ms) will cause significant Wi-Fi performance degradation which is unfair to Wi-Fi. When $T$ is large enough, the air time sharing tends to cause less unfairness to Wi-Fi under strong interference, more numerical results shows (omitted due the page limit) large $T$ cause less unfairness under weak interference as well.

### B. The role of duty cycle $\alpha$

For $T = 500ms$, $q = 1$, and $L = 1KB$, the result is demonstrated in Fig. 4. When the interference is strong, throughput loss ratio is almost $\alpha$ and is linear. However, if interference is weak but significant ($q= 1$), there can be additional reduction on throughput. When $\alpha \approx 0.4$, the air time sharing cause greatest throughput unfairness. When $\alpha \leq 0.3$, the delay seems to be linear, however, when $\alpha \rightarrow 1$, the service time increase exponentially. Recall the definition of fairness and it can be inferred for in the given network setting, a fair air time sharing scheme should have at least $\alpha \leq 0.3$. Considering the throughput only, strong interference approaches the throughput fairness over almost any $\alpha \in [0, 1]$.

### C. LTE-U to Wi-Fi collision probability $q$

Fixing the duty cycle $T = 500ms$ and $\alpha = 0.3$ and $L = 1KB$, Fig. 5 shows how the fairness varies with $q$, it degrades almost linearly with $q$. Particularly, $q$ has less effect to fairness under strong interference because interference over -62dbm will freeze the Wi-Fi back-up timer and the only possible LTE-U to Wi-Fi collision occurrence is when an LTE-U transmission starts after a Wi-Fi transmission.

### D. The role of Wi-Fi payload length $L$

Fixing $T = 500ms$, $q = 1$, and $\alpha = 0.3$, Fig 6 illustrates the impact of data length $L$, for both weak and strong interference, fairness degrades almost linearly with $L$.

### E. Why weak interference is worse at $q = 1$?

In the strong interference case, Sta-A eliminates interference by freezing the back-off timer, after LTE-U being off, Sta-A could be immediately released from LTE-U interference, so the total loss ratio of the performance is very close to the the duty cycle $\alpha$, as can be seen in Fig. 4. However, when interference is weak, Sta-A continues to contend the channel, and it eliminates the interference by enlarging the contention window size and increasing the number of attempts during the LTE-U-ON stage. Once LTE-U switches OFF, Sta-A will not start transmitting until the current back-off timer being reset, in the worst case, the recovery time could take as long as $2^{q}C_{W}\frac{E[T_d]}{T_d}$

$E[T_d]$ in the above experiments is about 2.6ms. In case $T$ is about 100ms, Sta-A would wait for 10 duty cycle periods long before sensing the channel again. This effect is demonstrated clearly in Fig. 3 and 4.

From the information theory perspective, when LTE-U interference is strong, Wi-Fi has accurate and updated channel state information (CSI) on whether LTE-U is on or off, hence Wi-Fi could use this CSI to skip the interference. When LTE-U interference is weak, Wi-Fi has very delayed CSI since the AP knows the interference only after detecting the failure of a previous transmission, which causes significant performance degradation.

### V. Conclusion

In this paper, we study the performance of an infrastructure-based Wi-Fi network when its operating channel in the unlicensed spectrum is air time shared with an LTE-U network. We define and characterize the Wi-Fi average performance and fairness in the presence of duty cycled LTE-U as functions of Wi-Fi sub-system parameters and the air time sharing scheme being used. Through Monte Carlo analysis, we numerically demonstrate the fairness under typical coexistence settings. It can be observed from the results that Wi-Fi and LTE-U coexistence using simple air time sharing is generally unfair to Wi-Fi. We conclude that some other schemes (e.g., similar to the listen-before-talk mechanism used in 802.11 networks) need to be developed for LTE-U networks in order to overcome the unfairness rooted in the distributed medium access mechanism of Wi-Fi.

### References

[1] “Extending the benefits of LTE-A to unlicensed spectrum,” Qualcomm Whitepaper, April 2014.

[2] A. M. Cavalcante and et al., “Performance evaluation of LTE and Wi-Fi coexistence in unlicensed bands,” in Vehicular Technology Conference (VTC Spring), 2013 IEEE 77th. IEEE, 2013, pp. 1–6.

[3] A. Babaei, J. Andreoli-Fang, and B. Hamzeh, “On the impact of LTE-U on Wi-Fi performance,” in Proceedings of IEEE PIMRC, 2014.

[4] “U-LTE: Unlicensed spectrum utilization of LTE,” Huawei Whitepaper.

[5] C. Cano and D. J. Leith, “Coexistence of wifi and lte in unlicensed bands: A proportional fair allocation scheme,” in 2015 IEEE International Conference on Communication Workshop (ICCW). IEEE, 2015, pp. 2288–2293.

[6] “LTE-U SDL Coexistence Specifications,” LTE-U Forum.

[7] “Technical Specification Group Radio Access Network; Study on Licensed-Assisted Access to Unlicensed Spectrum; (Release 13), 3GPP TR 36.899.0.4.0.

[8] R. Ratasuk, N. Mangalvedhe, and A. Ghosh, “Lte in unlicensed spectrum using licensed-assisted access,” in 2014 IEEE Globecom Workshops (GC Wkshps). IEEE, 2014, pp. 746–751.

[9] A. M. Voicu, L. Simić, and M. Petrova, “Coexistence of pico-and femtocellular lte-unlicensed with legacy indoor wi-fi deployments,” in 2015 IEEE International Conference on Communication Workshop (ICCW). IEEE, 2015, pp. 2294–2300.

[10] R. Kwan, R. Pazhyannur, J. Seymour, V. Chandrasekhar, S. Saunders, D. Bevan, H. Osman, J. Bradford, J. Robson, and K. Konstantinou, “Fair co-existence of licensed assisted access lte (laa-lte) and wi-fi in unlicensed spectrum,” in Computer Science and Electronic Engineering Conference (CEEC), 2015 7th. IEEE, 2015, pp. 13–18.

[11] G. Bianchi, “Performance analysis of the IEEE 802.11 distributed coordination function,” Selected Areas in Communications, IEEE Journal on, vol. 18, no. 3, pp. 535–547, 2000.

[12] H. Zhai, Y. Kwon, and Y. Fang, “Performance analysis of IEEE 802.11 MAC protocols in wireless LANs,” Wireless communications and mobile computing, vol. 4, no. 8, pp. 917–931, 2004.

[13] A. Banchs, P. Serrano, and A. Azzouz, “End-to-end delay analysis and admission control in 802.11 DCF WLANs,” Computer Communications, vol. 29, no. 7, pp. 842–854, 2006.
(a) throughput fairness vs $T$

(b) service time fairness vs $T$

Fig. 3: Impact of duty cycle period $T$

(a) throughput fairness vs $\alpha$

(b) service time fairness vs $\alpha$

Fig. 4: Impact of duty cycle $\alpha$
Fig. 5: The role of LTE-U to Wi-Fi collision probability $q$

Fig. 6: Impact of Wi-Fi payload length $L$