USE OF ABCD-MATRIX METHOD FOR THE SIMULATION OF LASER-ULTRASONIC PROBING PULSE PROPAGATION IN A LAYERED MEDIUM OF THE OPTICAL-ACOUSTIC TRANSDUCER

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Abstract. As a first step methods for modeling the propagation of acoustic waves in layered media was analyzed and the need to develop a new, faster method is substantiated. As a consequence, presented a method for modeling the propagation of elastic waves based on signal transformation using ABCD matrices, for the implementation of which a computer program in the Python language has been developed. For the verification of the method was analyzed the correlation between the simulated signal and the signal obtained experimentally from a medium with predetermined parameters. Further on the basis of the data obtained, the applicability of this method for modeling the propagation of acoustic signals in a plane-layered medium has been proved. As a next step modern methods of ultrasound generation and their influence on the characteristics of the probing signal was reviewed. Confirmed that laser-ultrasound techniques allow the best control over the characteristics of the probe signal. Therefore, the influence of the duration of the laser pulse, the properties of light absorption and the acoustic characteristics of the generator medium on the probing signal amplitude and bandwidth of the spectrum was estimated. To reduce the influence of the considered characteristics on the diagnostic process using the ABCD-matrix method the optimal characteristics of the optical-acoustic transducer for diagnostics at low frequencies have been determined. Finally, the main advantage of the proposed modeling method is the high speed of calculating the shape and spectrum of the signal at any point in the medium at a given time.

1. Introduction
Monitoring and detecting heterogeneities in rocks and other environments and products, determining the structural strength of the support - all these tasks are an important part of modern mining operations. In this case, the main requirement for control systems is the need to diagnose the structure and condition of large objects made of heterogeneous materials. The most frequently used methods for these purposes are ground-penetrating radar (GPR) studies [1-3], thermography [4-7] and low-frequency acoustic methods [8-10].
As noted in the review [1], reflecting the 30-year history of the use of georadars, in the tasks of non-destructive testing, this method is used to diagnose the condition of buildings, roads, bridges, tunnels, engineering-geological and underground communications. GPR makes it possible to qualitatively assess the structure of objects made of heterogeneous materials and its change. However, the information obtained from the tracks of electromagnetic pulses reflected on inhomogeneities is not sufficient for the identification and localization of defects [2, 3].

Thermal imaging control, based on the registration of changes in the heat flow, allows you to determine the places of discontinuity in objects [4] and only qualitatively evaluate their geometric parameters. Therefore, this method mainly studies heat-shielding structures [4] and various types of composites [5-7].

The most informative in the diagnosis of the structure and properties of such complex heterogeneous materials as rocks are acoustic methods [8-10]. These include acoustic emission [8], vibrometry [9, 10] and ultrasonic methods using piezoelectric transducers and antenna arrays of piezoelements [11], as well as laser-ultrasonic methods [12-16]. In any of the above methods, the problem of received scattered or reflected signals from inhomogeneities digital processing and identification of structure inhomogeneities from them arises. Thus, taking into account the complexity of this simulation, new high-performance software solutions are required to build the corresponding models [17-21].

Among the modeling methods, the most common is the finite element method (FEM), as well as its variations [22-25]. The main advantage of FEM is the ability to use it for modeling structures of any complexity and with high calculation accuracy, which leads to an increase in the time required for model processing. In this case, the calculation speed decreases sharply with a slight complication of the model [22, 23]. Due to the versatility of the method, many researchers are engaged in FEM modifications. For example, the authors of [24] proposed a method based on FEM with scalable boundaries, which accelerates the calculations in the case of high-frequency signals; and in [25] it was proposed to use an extended FEM, also called XFEM, to solve the modeling problem. But the revision and modification of the FEM cannot completely solve the problem of the calculation speed. Thus, the development of an alternative, more efficient modeling method remains an urgent task.

This paper presents a semi-analytical method for calculating the propagation of a laser-ultrasonic probe pulse with a Gaussian transverse beam profile in a plane-layered medium, based on the ABCD matrix method [26-27], which is mainly used in optics. This approach makes it possible to identify reflections and reverberations in a flat-layered medium, as well as to take into account the change in the temporal shape of the probe pulse due to diffraction and attenuation.

On the other hand, the results of studies using any ultrasonic methods strongly depend on the characteristics of the generator and receiver of ultrasonic signals. It should be noted that, according to this criterion, the most accurate and controllable method of generating ultrasound today is lasing, since it is impossible to effectively excite and receive broadband ultrasonic signals in traditional piezoelectric transducers. However, even in the case of lasing, the effect of optical-acoustic conversion on the shape and spectrum of the probe signal has not been fully investigated. To register scattered and reflected signals with this method of excitation, broadband piezoelectric receivers are used, in which various methods are used to improve the quality of the conversion of mechanical signals into electrical ones [28-30].

Thus, the determination of the characteristics of the optical-acoustic transducer, which have the main effect on the probe signal, and finding their optimal combination for diagnostics of geomaterials remains an urgent task.

2. Modeling ABCD-method

The following problem was considered. Let a plane light wave with an intensity $I = I_0 f(t) H(\vec{r}_1)$ fall from a transparent medium along the normal to the boundary ($z = 0$) with an absorbing medium, where $I_0$ is the amplitude value of the intensity, and $f(t)$ and $H(\vec{r}_1)$ ($\vec{r}_1 = \{x, y\}$), the time envelope and the cross-sectional distribution of the beam intensity, respectively. Without loss of generality, we assume that the time envelope $f(t)$ and its Fourier transform $\tilde{f}(\omega)$ have a Gaussian shape:
\[
\begin{aligned}
    f(t) &= \pi^{-1/2} \exp[-(t/\tau_L)^2] \\
    \tilde{f}(\omega) &= \tau_L \exp[-\omega^2\tau_L^2/4]
\end{aligned}
\]  

(1)

where \(\tau_L\) is the characteristic duration of the laser pulse.

For effective excitation of an ultrasonic wave, it is necessary that strong absorption of optical radiation be realized in the optical-acoustic generators (OAG) medium, that is, the light absorption coefficient \(\alpha\) should be of the order of \((10^3- 10^4)\) cm\(^{-1}\). In this case, the width of the optical beam \(\alpha\) is much greater than the depth \(\alpha^{-1}\) of light penetration into the generator medium, and a step-by-step approach can be used.

At the first stage, it is assumed that the spatial distribution of the laser radiation intensity does not affect the generation process, the transverse gradients of the thermal field can be neglected, and only a longitudinal wave is excited. Then the process of generation of the vibrational velocity \(\vec{v}\) of particles in a longitudinal wave is described by the system of equations [32]:

\[
\begin{aligned}
    c_L^{-2} \frac{\partial \vec{v}}{\partial t} - \Delta \vec{v} &= -\beta \left( 1 - 4 \frac{\zeta^2}{\omega_p^2} \right) \frac{\partial^2 T}{\partial t \partial z} \\
    \frac{\partial T}{\partial t} &= \chi \Delta T + \frac{a_1}{\rho_0 c_p} f(t) H(\vec{r}) e^{-\alpha z}
\end{aligned}
\]  

(2)

where \(z > 0\), \(\chi\) is the thermal diffusivity, \(\beta = V^{-1}(\partial V/\partial T)_p\) is the temperature coefficient of volumetric expansion, \(c_p\) is the specific heat capacity of the generator medium at constant pressure \(p\), \(\rho\) and \(\rho_0\) are the density and its equilibrium value, \(c_L, c_T\) - velocities of longitudinal and transverse waves. This system of equations must be supplemented with boundary conditions at the interface \(z = 0\), which ensure the continuity of displacements and normal velocity components. Then outside the heating region \(|z| > \max (\alpha^{-1}, \sqrt{(\chi_1 / \omega)})\) (subscript 1 at the thermal diffusivity refers to the material of the transparent prism, 2 - for the OAG medium) there are two purely traveling acoustic waves. One of these waves will propagate in the prism and is the reference signal, the second - in the generator and is the probe signal. Each harmonic \(\omega\) of these waves is described by the expression:

\[
\vec{v}_{1/2} = K_{12}(\omega)|l_0| \hat{f}(\omega) \exp(i\omega(t \pm z/c_L))
\]  

(3)

where index 1 and sign “-” correspond to the reference wave, and index 2 and sign “+” - to the probe wave. Transfer functions \(K_{12}(\omega)\) for each medium are discussed in article [32]. Note that formula (3) is a special case of the general formula for a Gaussian beam in a medium, which can be described by an ABCD matrix in the paraxial approximation for a Gaussian intensity distribution in the cross section [26, 27]:

\[
u(r_L, z) = \frac{1}{A + B/q_1} \exp \left[-tk \frac{r_L^2}{2q_0} \right] \]  

(4)

Further transformations are considered in the article [32]. Thus, the field on the axis \((r_L = 0)\) is given by the expression:

\[
\vec{v}(\omega, z) = \frac{\beta}{\rho_1 c_p} \frac{-t\omega/\omega_0}{1 + (\omega/\omega_0)^2} \frac{AE_p}{\pi a^2} \exp \left[ -\frac{\omega^2 \tau_L^2}{4} \right] \frac{4}{1 + Z_2^2} (1 + Z_2) \]  

(5)

\[
1 + \left( \frac{L_3 + c_3 c_2 L_2}{c_1^3} \right) \frac{2tc_1}{\omega a z}
\]
1.1. Results
Figures 1 (a, b) show: dashed lines - temporal waveforms of signals obtained experimentally, solid lines - temporal forms of signals modeled by the method described above. Figures 1 (c, d) - spectral forms, respectively.

Figure 1. a) Time waveform of the signal from the sensor from the free surface (air), b) Time waveform of the signal from the sensor from the steel plate, c) Spectral waveform of the signal from the sensor from the free surface (air), d) Spectral waveform of the signal from the sensor from steel plate.

As can be seen from Figure 1 (a-d), the simulated signals are close to the experimental ones, and the deviations of these signals are due to the difference in the real physical and mechanical properties of the media and the properties of the media specified in the simulation program.

3. Determination of optics-acoustic converter of a low-frequency optimal characteristics for diagnostics of geomaterials
In order to assess the effect of the duration of laser radiation on the probe signal, simulation was carried out for three different values: 10 ns, 100 ns, and 1 μs. Figures 2-4 show the spectra of the probing signal at these values. Figures 2-4 (a) - the spectrum of the signal immediately after generation, Figures 2-4 (b) - after passing through the converter, a centimeter steel plate and back to the receiver.
Figure 2. The spectrum of the probing signal at $\tau_i = 10$ ns: a) Immediately after generation, b) After passing the model.

Figure 3. The spectrum of the probing signal at $\tau_i = 100$ ns: a) Immediately after generation, b) After passing the model.

Figure 4. Spectrum of the probing signal at $\tau_i = 1 \mu$s: a) Immediately after generation, b) After passing the model.
It can be seen from Figures 2-4 that a decrease in the duration of a laser pulse leads to a noticeable expansion of the initial spectrum of the signal, while an increase in it leads to a narrowing. Thus, changing the duration of the laser pulse allows one to control the width of the probe signal spectrum.

![Graphs](image)

**Figure 5.** The spectrum of the probing signal at different absorption rates: (a) 10,000 (1 / m), (b) 30,000 (1 / m), (c) 50,000 (1 / m).

To assess the effect of the light absorption coefficient of the generator medium, simulations were also carried out for three different values. Figure 5 shows the signal spectra at different light absorption rates. As can be seen from the figure, with a decrease in the value of the absorption coefficient, the signal amplitude noticeably increases, which in turn simplifies diagnostics, allowing one to observe the response at a lower laser radiation power. It is also worth noting the possibility of selecting the material of the generator plate for a specific diagnostic task.

**4. Conclusions**

In the course of studying methods for modeling the propagation of ultrasonic beams in layered media, the need was revealed to develop a method for describing the process of propagation of acoustic signals in a layered medium using the ABCD matrix method, which is characterized by a faster process of model calculation. To implement this method, a computer program was created in the Python language. The result of the program work is a graphical representation of the time and spectral waveforms at any point in the analyzed environment. The tools presented in the program allow you to adjust the physical parameters of the environment and signal generation, which gives results that are close to those obtained experimentally. The program can be used to simulate laser generation and propagation of acoustic signals in any plane-parallel layered media, and the propagation simulation is possible for any predetermined signal.
The program was used to simulate the sensor and the medium under study to assess the effect of the characteristics of the optical-acoustic transducer on the probing signal. Based on the data obtained, it was revealed that the properties of the piezoelectric transducer and the conditions of lasing can be used to control the characteristics of the probe signal. So changing the duration of the laser pulse allows you to control the width of the probe signal spectrum. At the same time, the light absorption index of the generator medium makes it possible to change the amplitude of the probing signal without changing the laser radiation power, which makes it possible to select the sensor material for specific studies. But to simplify the selection of the optimal material of the generator medium, further research is required, in particular, experiments on various materials in order to determine their light absorption and acoustic attenuation rates.

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