To the theory of convective flows in a rotating stratified medium over a thermally inhomogeneous surface

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Abstract. In a liquid (gaseous) medium above a thermally inhomogeneous horizontal surface in a gravity field, convective flows arise, which can play an important role, in particular, in the dynamics of the atmosphere. An extensive literature is devoted to their theory, but, due to the complexity of the problem and the variety of possible combinations of parameters, not all situations of interest have been sufficiently studied even in the linear approximation. In this work, a theoretical model of such thermal circulations is considered, which is more general than in a number of previous works. An analytical solution is found within the framework of a linear stationary two-dimensional model of convective flows in a semi-infinite stably stratified medium rotating around a vertical axis. The obtained relations allow analyzing the dependences of the components of velocity and helicity on the parameters of the problem. A number of rather general statements about the ratio of different helicity “components” in the considered thermal circulations, in particular, in atmospheric currents with characteristic horizontal scales of the order of hundreds of kilometres, have been proved. Examples of numerical calculations of the vertical distribution of these components are given.

1. Introduction

In a liquid (gaseous) medium above a thermally inhomogeneous horizontal surface in a gravity field, convective flows arise, which can play an important role, in particular, in the dynamics of the atmosphere. An extensive literature is devoted to their theory (see, for example, [1–6] and bibliography in these articles). Much useful information can be obtained already from the analysis of linear problems that make sense for relatively small amplitudes of thermal inhomogeneities (see, for example, [1, 3–6]). In these articles and in a number of other publications, there are developed the linear models of mesoscale flows in a stably stratified rotating medium over an inhomogeneously heated surface – mesoscale atmospheric flows. However, due to the complexity of the problem and the variety of possible combinations of parameters, not all situations of interest have been studied sufficiently. For example, in our recent paper [6], the Ekman-type boundary layer arising at the lower boundary is assumed to be rather thin as compared to other characteristic vertical scales of the problem. In problems of atmospheric dynamics, this condition is not always satisfied with confidence. In this paper, we consider a more general model free from this limitation.

In addition, as far as we know, the helicity of this type of convective flows has been little studied so far. The importance of this characteristic, in particular, for atmospheric currents, has long been discussed in the literature (see, for example, [7–13]). The present paper contains some relevant results.
2. Statement of the problem of thermal circulations over an inhomogeneously heated surface

Here we consider a semi-bounded volume of a stratified medium \( z \geq 0 \) rotating around an axis \( z \) directed vertically upward. By using the Boussinesq approximation, we assume that the density of the medium depends linearly on temperature perturbations (in problems of atmospheric physics, it is convenient to use the deviations of the potential temperature as the corresponding variable [14, 15])

\[
T : \quad \rho = \rho_0(1 - \alpha T).
\]

Here \( \rho \) is the density, \( \rho_0 \) is the background density value, \( \alpha \) is the thermal coefficient if expansion.

The linearized system of equations of dynamics and heat transfer in a rotating coordinate system for a stationary two-dimensional problem has the form [3-6]

\[
0 = -\frac{\partial P}{\partial x} + \nu \Delta u + f v, \quad 0 = \nu \Delta v - f u, \quad 0 = -\frac{\partial P}{\partial z} + \nu \Delta w + \alpha g T, \quad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad \gamma w = \kappa \Delta T.
\]

Here \( u, v, w \) are velocity vector components of \( \mathbf{v} \) along horizontal axes \( x, y \) and vertical axis \( z \) respectively; \( P = p / \rho_0, \) \( p \) is the pressure disturbance; \( g \) is the acceleration of gravity; \( \nu, \kappa \) are the exchange coefficients; \( \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \) is the two-dimensional Laplace operator; \( f \) is the Coriolis parameter (the doubled angular velocity of the background rotation). Temperature stratification is assumed to be stable, so that the background potential temperature gradient \( \gamma > 0 \).

Note the coincidence (up to coefficients) of the equations for the vorticity \( \partial v / \partial x \) and the vertical temperature gradient \( \partial T / \partial z \), which can be seen easily if we differentiate the second equations (2) and (3) by \( x \) and \( z \), respectively, and take into account the continuity equation. Therefore, the vorticity of convective currents arising in the field of Coriolis accelerations is quite simply related to horizontal inhomogeneities of the vertical heat flux.

The stationary two-dimensional horizontal periodic thermal inhomogeneities, as well as non-slip and impermeability conditions are specified at the lower boundary:

\[
c \rho_0 \kappa \frac{\partial T}{\partial z} = -Q \cos k x, \quad w = u = v = 0 \quad \text{at} \quad z = 0.
\]

Here \( c \) is heat capacity of medium, and the sense of parameters \( Q \) and \( k \) is obvious. It is assumed that far from the surface (at \( z \rightarrow \infty \)) all disturbances damp.

3. The system of equations for the amplitudes and its general solution

The problem posed above in a number of respects is close to those considered earlier in the papers [3, 4, 6]. Below, a solution will be found that is free from some of the previously adopted restrictions. We are looking for horizontally periodic solutions in the form

\[
\begin{align*}
u(x, z) & = U(z) \sin k x, \quad v(x, z) = V(z) \sin k x, \quad w(x, z) = W(z) \cos k x, \\
P(x, z) & = \Phi(z) \cos k x, \quad T(x, z) = \theta(z) \cos k x.
\end{align*}
\]

The system of equations for the amplitudes has the form

\[
-k \Phi = \nu \left( \frac{d^2 U}{dz^2} - k^2 U \right) + f V, \quad 0 = \nu \left( \frac{d^2 V}{dz^2} - k^2 V \right) - f U, \quad \frac{d \Phi}{dz} = \nu \left( \frac{d^2 W}{dz^2} - k^2 W \right) + \alpha g \theta, \quad k U + \frac{d W}{dz} = 0, \quad \gamma W = \kappa \left( \frac{d^2 \theta}{dz^2} - k^2 \theta \right).
\]

\[
\begin{align*}
u(x, z) & = U(z) \sin k x, \quad v(x, z) = V(z) \sin k x, \quad w(x, z) = W(z) \cos k x, \\
P(x, z) & = \Phi(z) \cos k x, \quad T(x, z) = \theta(z) \cos k x.
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P(x, z) & = \Phi(z) \cos k x, \quad T(x, z) = \theta(z) \cos k x.
\end{align*}
\]
By eliminating all unknowns from the last system, except \( W \), we obtain the equation

\[
\left( \frac{d^2}{dZ^2} - 1 \right)^3 W = -Ta \frac{d^2 W}{dZ^2} + RW, \tag{9}
\]

\[
R = \frac{N^2}{k \nu k^4}, \quad Ta = \frac{f^2}{\nu^2 k^4} = \left( \frac{4}{\kappa h_E} \right)^4, \quad N = (\alpha g \gamma_T)^{1/2}, \quad h_E = \left( \frac{2v}{f} \right)^{1/2}.
\]

Here, the buoyancy frequency \( N \) and the dimensionless variable \( Z = k z \) are introduced; the dimensionless parameters \( R > 0 \), \( Ta > 0 \) are some analogs of the Rayleigh and Taylor numbers.

We seek a solution to equation (9) in the standard way in the form of a linear combination of exponentials of the type \( \exp(\sigma Z) \). The characteristic equation has the form

\[
\left( \sigma^2 - 1 \right)^3 = -Ta \sigma^2 + R. \tag{10}
\]

Taking into account the damping condition of disturbances at \( z \to \infty \), the solution for the vertical velocity is a linear combination of three exponentials

\[
w(x, z) = \sum_{j=1}^{3} C_j \exp(k \sigma_j z) \cos kx,
\]

where the roots \( \sigma_j \) with negative real parts are selected (here it is assumed that these roots are different); the integration constants \( C_j \) are determined from the boundary conditions. From the equation of continuity

\[
u(x, z) = -\sum_{j=1}^{3} C_j \sigma_j \exp(k \sigma_j z) \sin kx. \tag{12}
\]

The expressions for the perturbations of temperature, pressure, and the vortex component of the horizontal velocity, as it is easy to see, can also contain terms with the fourth exponent \( \exp(-k z) \):

\[
T(x, z) = \left[ C_4 \exp(-k z) + \frac{\gamma}{kk} \sum_{j=1}^{3} \frac{C_j}{\sigma_j^2 - 1} \exp(k \sigma_j z) \right] \cos kx, \tag{13}
\]

\[
v(x, z) = \left[ C_5 \exp(-k z) - \frac{f}{\nu k^2} \sum_{j=1}^{3} \frac{C_j \sigma_j}{\sigma_j^2 - 1} \exp(k \sigma_j z) \right] \sin kx. \tag{14}
\]

By differentiating the first equation (2) with respect to \( z \), and the last with respect to \( x \), it is easy to verify the relation

\[
C_5 = \frac{\kappa \alpha g}{f} C_4. \tag{15}
\]

4. The simplifications based on a scale analysis

In the general case, the solution is rather cumbersome. But it is useful to bear in mind that the values of parameters \( R \), \( Ta \) in the atmosphere are usually quite large, so it makes sense to analyze some relatively simple limiting cases. For example, at \( N = 10^{-2} \text{ s}^1 \), \( f = 10^{+4} \text{ s}^{-1} \), \( \kappa = \nu = 1 \text{ m}^2/\text{s} \) (effective coefficients of turbulent exchange), \( k = 10^{-5} \text{ m}^1 \) (that corresponds to a horizontal half-wave length of about 300 km) \( R \sim 10^{16} \), \( Ta \sim 10^{12} \). If everything else being equal, \( \kappa = \nu = 10 \text{ m}^2/\text{s} \), then \( R \sim 10^{14} \), \( Ta \sim 10^{10} \). The roots of the characteristic equation \( \sigma_j \) in such situations are large in absolute value compared to unity. In [3, 6], there were considered the values of the parameters, for which
\[ 1 < R^{2/3} < \frac{\alpha}{R} \leq \frac{\alpha}{R}. \] (16)

All of these inequalities hold well for the above estimate at \( \kappa = \nu = 1 \) \text{ m/s}. But the intensity of turbulent exchange in the atmosphere can be very different, and often a more adequate estimate is \( \kappa = \nu = 10 \) \text{ m/s}. In this case, the second of inequalities (16) becomes softer, and some of the assumptions made in [3, 6] are not fulfilled strictly. In this paper, we consider the case of the softer inequality \( \frac{\alpha}{R} < \frac{\alpha}{R} \leq \frac{\alpha}{R} \). In this case, the assumptions about the smallness of some dimensionless parameters cease to be fulfilled, so that the model becomes more complicated. The approximate expressions for three roots of the characteristic equation appearing in (11) — (14) in this case coincide with [3, 6]:

\[ \sigma_1 \approx -b, \quad \sigma_2,3 \approx -(1 \pm i)a, \quad a = \left( \frac{\alpha}{4} \right)^{1/4}, \quad b = \left( \frac{R}{\alpha} \right)^{1/2} = \left( \frac{\nu}{\kappa} \right)^{1/2} N / f. \] (17)

at that

\[ a > b; \quad |\sigma_2,3| = \alpha^{1/4} > |\sigma_1| \gg 1 \] (18)

(in [3, 6], the first inequality was stronger).

5. The approximate solution

Taking into account the lower boundary conditions, for the coefficients at exponentials, the system of equations is obtained:

\[ \sum_{j=1}^{3} C_j = 0, \quad \sum_{j=1}^{3} \sigma_j C_j = 0, \] (19)

\[ C_4 \approx \frac{f^2}{\alpha \kappa \nu k^2} \sum_{j=1}^{3} \frac{C_j}{\sigma_j}, \quad C_4 - \frac{\gamma}{kk^2} \sum_{j=1}^{3} \frac{C_j}{\sigma_j} \approx \frac{Q}{c \rho_0 \kappa k}. \] (20)

By eliminating \( C_4 \) from (20), we obtain

\[ \left( 1 - \frac{\kappa f^2}{\nu N^2} \right) \sum_{j=1}^{3} \frac{C_j}{\sigma_j} \approx -\frac{kQ}{c \rho_0 \gamma}. \]

For the considered characteristic values of the parameters, the second term in parentheses is much less than unity; therefore, together with two equations (19), we consider

\[ \sum_{j=1}^{3} \frac{C_j}{\sigma_j} \approx -\nu, \quad \nu = \frac{kQ}{c \rho_0 \gamma}. \] (21)

The approximate solution of the system (19), (21) has the form

\[ C_1 \approx \frac{b \nu}{B}, \quad C_{2,3} \approx -\frac{b \nu}{2B} [1 \pm i(1 - \delta)], \quad C_4 \approx -\frac{f^2}{N^2} \frac{Q}{c \rho_0 \nu k}. \] (22)

Here the dimensionless parameters are introduced

\[ \delta = \frac{b}{a} \left( \frac{4R^2}{\alpha^3} \right)^{1/4}, \quad B = 1 - \delta + \frac{\delta^2}{2}. \]

In [3, 6] the first of them was assumed to be a small parameter, that made it possible to significantly simplify the calculations. As it is easy to see, the smallness of this parameter means that \( B \approx 1 \), and the Ekman scale of the height

\[ h_E = \frac{1}{k} \left( \frac{4}{\alpha^3} \right)^{1/4} = \left( \frac{2 \nu}{f} \right)^{1/2} \]

is much less than another significant vertical scale.
\[ h_b = (k\sigma_1)^{-1} = \frac{1}{k} \left( \frac{\text{Ta}}{R} \right)^{1/2} = \left( \frac{\kappa}{v} \right)^{1/2} \frac{f}{kN}. \]

The present model is free from this simplification – the scales indicated can be comparable. For the sought variables, we get the expressions:

\[
\begin{align*}
    u & \approx \frac{b^2 \nu}{B} \left\{ \exp(-bkz) - \exp(-akz) \left[ \cos(akz) + 2 - \delta \sin(akz) \right] \right\} \sin kx, \\
v & \approx -\frac{f\nu}{vk^2} \left\{ \exp(-kz) - \frac{1}{B} \exp(-bkz) + \frac{\delta}{2B} \exp(-akz) \left[ (2 - \delta) \cos(akz) - \delta \cos(akz) \right] \right\} \sin kx, \\
w & \approx \frac{b\nu}{B} \left\{ \exp(-bkz) - \exp(-akz) \left[ \cos(akz) + (1 - \delta) \sin(akz) \right] \right\} \cos kx,
\end{align*}
\]

(23)

In the limiting case of small values of the dimensionless parameter \( \delta \), these expressions turn into the solution [6]. (Note that a misprint in the sign has crept into the expression for temperature [6] that did not affect further results).

Bearing in mind the calculation of the flow helicity, we also present the expressions for the vertical derivatives of the horizontal velocity components:

\[
\begin{align*}
    \frac{\partial u}{\partial z} & = -\frac{a^2 b k \nu}{B} \left[ \delta^2 \exp(-bkz) + 2 \exp(-akz) \left[ (1 - \delta) \cos(akz) - \sin(akz) \right] \right] \sin kx, \\
    \frac{\partial v}{\partial z} & = \frac{fb \nu}{vkB} \left[ b \exp(-kz) - \exp(-bkz) + \exp(-akz) \left[ \cos(akz) + (1 - \delta) \sin(akz) \right] \right] \sin kx.
\end{align*}
\]

(24)

Note the relative smallness of the second of these derivatives at the lower boundary, since at \( z = 0 \) all the terms in curly brackets, except for the small first term, are mutually compensated.

Let us estimate the amplitudes of the horizontal velocity perturbations for the typical conditions in the atmosphere. If the heat flux deviation is \( Q = 10 \text{ W/m}^2 \), \( k = 10^{-5} \text{ m}^4 \), \( \gamma = 3 \cdot 10^{-3} \text{ K}^{-1} \), \( \kappa = \nu = 10 \text{ m}^2/\text{s} \), then \( \nu \approx 3 \cdot 10^{-5} \text{ m/s} \), \( b = 100 \), \( \delta \approx 0.45 \), \( B \approx 0.65 \); the amplitudes \( u \) and \( v \) reach some meters in second. Strictly speaking, this is beyond the scope of the linear approximation used here, so that the use of the results obtained for smaller disturbance amplitudes is more justified.

The spatial scales included in the obtained solutions, generally speaking, are different significantly. One of the three exponentials included in the solution \( \exp(-kz) \) slowly decays with height, the other two exponentials decay much faster – on vertical scales of the order of \( h_E \) and \( h_b \). The slowly decaying exponent is not included in the expressions for \( u \) and \( w \) and makes a relatively small amplitude contribution to the temperature perturbation up to heights of the order of \( h_E \) and \( h_b \). But it largely determines the vortex component of the horizontal velocity \( \nu \). It is interesting to note that anticyclonic circulation occurs in areas of positive heat inflow (values \( \nu \) are negative). A detailed physical analysis of the solution is contained, for example, in the article [3], which is limited to a particular case \( \nu = \kappa \) and small values of the parameter \( \delta \).

Figure 1 shows an example of a solution for speed components at \( \kappa = \nu = 1 \text{ m/s} \) (the values of other parameters are given above). In this case \( \sigma_1 = -100 \), \( \sigma_{2,3} \approx -(1 \pm i) \cdot 700 \), \( a \approx 700 \), \( b = 100 \), \( h_b = 1000 \text{ m} \), \( h_E \approx 140 \text{ m} \), \( \delta \approx 0.14 \), \( B \approx 0.9 \); \( k^{-1} \gg h_b \gg h_E \). Velocity components
$u, w$ practically do not go beyond the thickness of the layer $h_b$ (a pronounced maximum is contained in a much thinner Ekman layer – “Ekman inflow” into the heat release region).

**Figure 1.** Vertical dependencies of speed components at $\kappa = v = 1 \text{ m}^2/\text{s}$: $u$ (thick line, normalized to $4b^2\nu / B$), $v$ (dashed line, normalized to $f\nu / \nu k^2$) on the vertical $kx = \pi / 2$; $w$ on the vertical $x = 0$ (thin line, normalized to $b\nu / B$).

Figure 2 shows the example of profiles for the case $\kappa = v = 10 \text{ m}^2/\text{s}$. In this case $\sigma_1 = -100$, $\sigma_{2,3} \approx -(1 \pm i) \cdot 220$, $a \approx 220$, $b = 100$, $h_b = 1000 \text{ m}$, $h_E \approx 450 \text{ m}$, $\delta \approx 0.45$, $B \approx 0.65$; $k^{-1} \gg h_b > h_E$. The thickness of the Ekman layer is significantly greater than in the first example; although $h_E$ remains less than $h_b$, it has the same order of magnitude. The inflow of mass into the region of heat release occurs in a much thicker layer.

**Figure 2.** The same as in figure 1, but at $\kappa = v = 10 \text{ m}^2/\text{s}$ and normalization $u$ to $b^2\nu / B$.

The thickness of the perturbation region at the lower boundary, on the one hand, depends on rotation and stratification (scale $h_b$), and on the other hand, on viscosity (scale $h_E$). In the first of the considered examples, the mentioned scales are quite different, so that boundary layers of different nature can be distinguished. In the second example, these scales are of the same order of magnitude, therefore, effects of different natures "mix".

6. **The analysis of thermal circulations helicity**

Recall the definition of helicity:

$$H = \mathbf{v} \cdot \text{rot} \mathbf{v}.$$  \hspace{1cm} (25)
For two-dimensional flows in the rotating coordinate system considered here

\[ H = -u \frac{\partial v}{\partial z} + v \frac{\partial u}{\partial z} - v \frac{\partial w}{\partial x} + w \frac{\partial v}{\partial x}, \]

(26)

where "azimuthal" components are underlined (see below).

We analyze the solution derived above in this context. Since we are talking about solving a linear problem, and helicity is quadratic in velocity, in this case it makes sense to consider not the absolute values of helicity (in the linear approximation they are obviously very small), but its spatial distribution. In addition, it is of interest to analyze the relative contribution of different terms in (26). For example, in [11-13], where the axisymmetric problem was considered, the contributions of the "radial" component (its analogue is the first term in the right-hand side of (26)), "azimuthal" (underlined second and third terms), and "vertical" helicity (last term) were compared.

Let us first dwell on the dependence of (26) on the horizontal coordinate \( x \). As one can see easily, all the terms in the right-hand side (26) are proportional \((\sin kx)^2\), or (the last) \((\cos kx)^2\). When averaged over the wavelength of the considered horizontal harmonic, this gives the same factor \( \frac{1}{2} \) for all terms. The wavelength-averaged values of the last two terms obviously coincide, and the sum of these terms is uniform horizontally, since

\[ -v \frac{\partial w}{\partial x} + w \frac{\partial v}{\partial x} = V(z)W(z)2kx + V(z)W(z)\cos^2 kx = V(z)W(z) \]

(27)

The third term in (26) (and, consequently, the fourth) in amplitude should be much less than the second, since near the maxima of the vertical profiles the ratio \((\partial w/\partial x)/(\partial u/\partial z) \sim (w/u) \cdot (kh_E)\); under the sign of the last modulus, each of the two expressions in parentheses is much less than unit. Thus, from the scale analysis it follows that the main contribution to the helicity integrated over space should be made by the first two terms on the right-hand side of (26).

Some rather general statements can be proved about the mean (vertically integrated) terms in the right-hand side of (26). Let us integrate the second of these terms over the entire vertical:

\[ \int_0^\infty v \frac{\partial u}{\partial z} dz = \sin^2 kx \int_0^\infty V \frac{\partial U}{\partial z} dz = \sin^2 kx \int_0^\infty \left( UV \right)_0^\infty \frac{\partial U}{\partial z} dz - \int_0^\infty \int_0^\infty \frac{\partial v}{\partial z} dz \]

(28)

(integration "by parts" was performed and it was taken into account that \( u \) vanishes at the lower boundary and decays with height). Thus, the integral contribution of the first two terms into the helicity in (26) is the same. Therefore, the integral contribution of the "radial" and "azimuthal" components is approximately the same.

From the second equation (2) it follows:

\[ u = \frac{v}{f} \Delta v \approx \frac{v}{f} \frac{\partial^2 v}{\partial z^2} \]

(29)

(in the approximate equality, the smallness of the horizontal derivatives is taken into account in comparison with the vertical derivatives). Hence it follows that the first term in (26) can be approximately represented in the form

\[ -u \frac{\partial v}{\partial z} \approx -v \frac{\partial^2 v}{f \partial z^2} \frac{\partial v}{\partial z} = - \frac{v}{2f} \frac{\partial}{\partial z} \left( \frac{\partial v}{\partial z} \right)^2. \]

Consequently,

\[ \left[ u \frac{\partial v}{\partial z} \right]_0^\infty \approx - \frac{v}{2f} \left( \frac{\partial v}{\partial z} \right)_0^\infty \left( \frac{\partial v}{\partial z} \right)_0^\infty = \frac{f \nu^2}{2\nu^2} \sin^2 kx \]

(30)

(the last equality takes into account the second formula (24)).

Figures 3, 4 show the vertical dependencies of the first two terms in the right-hand side (26) and their sums for the first and second numerical examples, respectively. The helicity is generated
effectively in the lower layers, where influence of the bottom boundary is significant. In the first example, first of all, this is rather sharply expressed Ekman boundary layer. The essential role of the Ekman layer in this context has long been noted in the literature for flows of various natures (for example, [7, 8]). But, in addition to the Ekman scale $h_E$, for flows of the nature under consideration, the influence of the lower boundary and the region of helicity generation are also determined by the vertical scale $h_b$, which does not depend on the viscosity. This is especially clearly seen in the second numerical example.

![Figure 3](image1.png)

**Figure 3.** Vertical dependencies of the horizontally averaged helicity components: $v(\partial u/\partial z)$ (thin line), $-u(\partial v/\partial z)$ (dashed line) and their algebraic sum (thick line) for the first numerical example. All profiles are normalized to $f b^3 b^2 / 2v k$.

![Figure 4](image2.png)

**Figure 4.** The same as in Fig. 3, but for the second numerical example.

Although the integral contribution of the “radial” and “azimuthal” helicity components turned out to be practically the same, but, as can be seen from the figures, their vertical dependencies differ markedly. The layers with positive and negative helicity in the considered examples compensate each other to a large extent, so that integral (30) over the entire region turned out to be much less in absolute value than the integrals over each of the mentioned relatively thin layers.

**7. Conclusion**

The considered theoretical model of convective flows over a thermally inhomogeneous surface generalizes some previous studies. In particular, this is free from the assumption that the Ekman boundary layer is relatively thin as compared to other characteristic vertical scales of the problem. This assumption (not always justified) greatly simplified the calculations, since it was associated with
the presence of a small parameter in the problem. Rejection of this assumption required a noticeable
complication of the calculations.

The obtained relations allow analyzing the dependencies of the components of velocity and helicity
on the parameters of the problem. For example, at first glance, one could assume that with the
intensification of rotation (an increase in the Coriolis parameter), the generation of helicity should
increase. But, as can be seen from the solution, the radial velocity rapidly decreases in absolute value c
– on sufficiently large horizontal scales, rotation prevents radial movements. Thus, helicity
decreases in absolute value (with the exception of the Ekman boundary layer, where the radial velocity
decreases relatively slowly with f ).

A number of rather general statements regarding the ratio of different helicity "components" in the
considered thermal circulations, in particular, in atmospheric currents with characteristic horizontal
scales of the order of hundreds of kilometers, have been proved. There are given examples of
numerical calculations of the vertical distribution of these components. It is shown that the integral
contribution of the "radial" and "azimuthal" helicity components in the situations under consideration
is practically the same, but their vertical dependences can differ significantly. The horizontal velocity
vertical shears, on which the generation of helicity depends, are determined not only by the Ekman
scale of the height, but also by the scale h_b that does not depend on the viscosity.

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