A new linguistic decision making method—FLM-VIKOR

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Abstract. Fuzzy linguistic term set is a powerful and flexible tool to express evaluation of experts in decision making. In this paper, we propose a VIKOR method based on fuzzy linguistic multiple set to solve the fuzzy linguistic multiset multi-criteria group decision making. Firstly, we develop the conception of fuzzy linguistic multiset, its correlate properties and operations. Then, we aggregate all decision matrices of all experts by these operations, obtaining final decision making matrix. Moreover, we develop the distance of two fuzzy linguistic multisets. Lastly, an illustrative example is given to testify the effectiveness of the developed method.

1. Introduction
Decision making is a selection process. We select the most satisfactory decision alternative from a set of possible decision alternatives. In practical, decision making problems are usually uncertain. Especially, in the big data era, with the decision making alternatives and evaluation indexes become larger, the sources of evaluation information become more diverse. It makes the decision making process more difficult and complex. Recently, many decision making methods have been proposed to solve different decision making problems [1-3]. Generally, decision matrix is composed of alternative, criteria and corresponding evaluation information. In view of the characteristics and properties of evaluation indexes, there are various representations of evaluation information, such as quantitative and qualitative representation. Quantitative representation includes accurate number, interval number, probability, fuzzy number [4-6]. Qualitative representation includes order relation, preference relation, linguistic representation [7-8]. Different representations require different methods to solve corresponding decision making problems. Since fuzzy linguistic is closest to human cognitive process and users can understand its semantics through membership function intuitively. Fuzzy linguistic is an important way to represent uncertain information in decision making. At present, fuzzy linguistic decision making method is one of the hot topics in decision making [3].

The core of fuzzy linguistic decision making method is fuzzy linguistic processing [2]. Classical fuzzy linguistic processing method is based on the membership function and fuzzy reasoning. Calculating with words has some disadvantages, such as computational complexity, inaccuracy and information lost easily [2]. Therefore, Herrera et al. [2] proposed binary linguistic model for fuzzy linguistic information representation. Formally, the binary linguistic \((s_i, \alpha)\), where \(s_i \in S = \{s_0, \cdots, s_g\}\) be the initial fuzzy linguistic term set, and \(\alpha \in [-0.5,0.5]\) represents the difference of approximation between the actual linguistic value and the initial value \(s_i\). For example, the initial fuzzy linguistic term
set $S = \{s_1(\text{bad}), s_2(\text{little bad}), s_3(\text{general}), s_4(\text{little good}), s_5(\text{good})\}$ is used to evaluate the quality of several products, and the aggregation result is $(s_1, -0.2)$. In other words, the quality of products is close to "a little bad", and the difference is -0.2. The binary linguistic model has the properties which is simple to compute, no information lost and easy to understand. So far, the binary linguistic model has been heavily studied. Many generalization models have been proposed, such as proportional bivariate linguistic model, virtual linguistic model and hesitant fuzzy set, etc. Meanwhile, the linguistic decision making method based on the binary linguistic model has also been widely studied [2, 9].

Multiset is an effective tool which can handle the issue with repeated information and avoid information lost [6, 10]. The linguistic evaluation information of experts often repeatedly in the process of linguistic decision making. In this paper, we develop multiset to represent multi-criteria group decision making problems with linguistic assessment information, which is called Fuzzy Linguistic Multiset (FLM). Combining with VIKOR method, a method to solve fuzzy linguistic multiset multi-criteria group decision making problem (FLM-MCGDM) is proposed, which is called fuzzy linguistic multiset VIKOR method (FLM-VIKOR). Specifically, we propose the conception of FLM firstly. And then we analysis its related properties and operations. Inspired by [8], we develop the score function and variance of a FLM. So as to obtain the positive and negative ideal solutions, the distance of two FLMs is developed. Lastly, we illustrate that the proposed method is reasonable and effective, through comparing with the hesitant fuzzy linguistic TOPSIS (HFL-TOPSIS) method [11], the hesitant fuzzy linguistic VIKOR (HFL-VIKOR) method [8] and the fuzzy linguistic multiset TOPSIS (FLM-TOPSIS) method [6].

2. Preliminaries

2.1. Multiset

**Definition 1[10]** A multiset $M$ over a set $A$, called the base set of $M$, is an unordered collection of elements of $A$, where each element in $A$ can occur zero or more times in $M$. $M$ has associated with it a function $m_M : A \rightarrow \mathbb{N}$, called multiplicity function or characteristic function, where $m_M(x)$ is the multiplicity of $x \in A$ in $M$ (the number of times $x \in A$ occurs in $M$). The set of distinct elements of $M$, denoted by $\operatorname{set}(M)$, is called the support set of $M$.

For any multiset $M_1$ and $M_2$ over the base set $A$, for any $x \in A$, we define:

- **Union:** $\operatorname{count}_{M_1 \cup M_2}(x) = \max\{\operatorname{count}_{M_1}(x), \operatorname{count}_{M_2}(x)\}$;
- **Intersection:** $\operatorname{count}_{M_1 \cap M_2}(x) = \min\{\operatorname{count}_{M_1}(x), \operatorname{count}_{M_2}(x)\}$;
- **Sum:** $\operatorname{count}_{M_1 + M_2}(x) = \operatorname{count}_{M_1}(x) + \operatorname{count}_{M_2}(x)$;
- **(Multiset) Difference:**
  
  $\operatorname{count}_{M_1 - M_2}(x) = \begin{cases} 
  \operatorname{count}_{M_1} - \operatorname{count}_{M_2}, & \text{if } \operatorname{count}_{M_1} > \operatorname{count}_{M_2} \\
  0, & \text{otherwise}
  \end{cases}$.

We can find that the intersection, union and sum are associative and commutative [16].

2.2. VIKOR Method

The VIKOR method was developed for multicriteria optimization of complex systems. It determines the compromise ranking list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with the initial (given) weights. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the multicriteria ranking index based on the particular measure of “closeness” to the “ideal” solution [8].

The multicriteria measure for compromise ranking is developed from the $L_p$-metric used as an aggregating function in a compromise programming method [8].
The various n alternatives are denoted as $A = \{a_1, a_2, \cdots, a_n\}$. For alternative $a_i$, the rating of the $I$ aspect is denoted by $f_{ij}$, i.e. $f_{ij}$ is the value of $j$th criterion function for $a_i$. Development of the VIKOR method started with the following form of $\mathcal{L}$ metric:

$$L_{ij} = \left\{ \sum_{j=1}^{r} \left[ w_j (f_{ij}^+ - f_{ij}^-) / (f_{ij}^+ - f_{ij}^-) \right]^p \right\}^{1/p} \quad 1 \leq p \leq \infty, \quad j = 1, 2, \cdots, r$$

Within the VIKOR method $L_{si} = S_i$ and $L_{ri} = R_i$ are used to formulate ranking measure. The solution obtained by $\min S_i$ with a maximum group utility (“majority” rule), and the solution obtained by $\min R_i$ with a minimum individual regret.

The compromise ranking algorithm VIKOR steps as below:

1. Determine the best $f_{ij}^+$ and the worst $f_{ij}^-$ values of all criterion functions

$$f_{ij}^+ = \max_{i, j} f_{ij}, C_j \in B$$

$$f_{ij}^- = \min_{i, j} f_{ij}, C_j \in C$$

(1)

$$f_{ij}^+ = \max_{i, j} f_{ij}, C_j \in B$$

$$f_{ij}^- = \min_{i, j} f_{ij}, C_j \in C$$

(2)

Where $B$ represents a benefit, $C$ represents a cost.

2. Compute the values $S_i$ and $R_i$ by the relations

$$S_i = L_{si} = \sum_{j=1}^{r} w_j \left( f_{ij}^+ - f_{ij}^- \right) / f_{ij}^+ - f_{ij}^-$$

$$R_i = L_{ri} = \max_{j} \left( w_j \left( f_{ij}^+ - f_{ij}^- \right) / f_{ij}^+ - f_{ij}^- \right)$$

(3)

(4)

Where $w_j$ are the weight of criteria.

3. Compute the values $Q$ by the relation

$$Q_i = v S_i - S^* + (1 - v) R_i - R^*$$

Where $S^* = \min_{i} S_i$, $S^* = \max_{i} S_i$, $R^* = \min_{i} R_i$, $R^* = \max_{i} R_i$ and $v$ is introduced as a weight for the strategy of “the majority of criteria” (or “the maximum group utility”), whereas $1 - v$ is the weight of the individual regret.

4. Rank the alternatives, sorting by the values $S$, $R$ and $Q$ in decreasing order.

5. Propose a compromise solution the alternative $a^{(2)}$ which is the best ranked by the measure $Q$ (minimum) if the following two conditions are satisfied:

1. “Acceptable Advantage”:

$$Q(a^{(2)}) - Q(a^{(0)}) \geq \frac{1}{n-1}$$

(6)

Where $a^{(2)}$ is the alternative with second position in the ranking list by $Q$.

2. “Acceptable stability in decision making”: The alternative $a^{(0)}$ must also be the best ranked by $S$ or/and $R$. This compromise solution is stable within a decision making process, which could be the strategy of maximum group utility. If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives $a^{(0)}$ and $a^{(2)}$ if only the condition 2 is not satisfied.
- Alternatives $a^{(0)}$, $a^{(2)}$, ..., $a^{(i)}$ if the condition 1 is not satisfied. $a^{(0)}$ is determined by the relation (7) to seek for the maximum $I$. 


\[ Q(a^{(i)}) - Q(a^{(0)}) < \frac{1}{n-1} \]  

3. Fuzzy Linguistic Multiset VIKOR Method

3.1. Fuzzy linguistic multi-criteria group decision making matrix

In general, a fuzzy linguistic decision making problem can be described as: a group of decision makers \( E = \{d_i, \ldots, d_m\} \) evaluate the decision alternatives \( A = \{a_i, \ldots, a_n\} \) according to the criteria \( C = \{c_1, \ldots, c_r\} \), using the fuzzy linguistic term set \( S = \{s_1, \ldots, s_g\} \). Depending on the evaluation linguistic information provided by the decision makers, the fuzzy linguistic decision method provides the comprehensive linguistic evaluation results of each decision object and is used to select satisfactory decision alternative. In existing linguistic decision making methods, the decision matrix is as follows:

\[
D = \begin{bmatrix}
  e_{i1} & \cdots & e_{ir} \\
  \vdots & \ddots & \vdots \\
  e_{im} & \cdots & e_{mr}
\end{bmatrix}
\]

(8)

Where, \( e_{ij} \in S \) represents the decision maker \( d_i \) use the linguistic terms \( e_{ij} \) to evaluate alternative \( a_j \) for criteria \( c_j \).

Definition 2 Let \( S = \{s_1, \ldots, s_g\} \) be a linguistic term set, \( M = \{e_1, \ldots, e_m\} \) is said to be a fuzzy linguistic multiset (FLM) over \( S \). Where \( e_k \in S \), \( k = 1, 2, \ldots, m \).

Note that if the elements of \( M \) are all different, then it is a classical set.

For the fuzzy linguistic decision problem described as previously, any two decision makers \( d_i \), \( d_j \) give the decision matrices are \( D_i \), \( D_j \) respectively

\[
D_i = \begin{bmatrix}
  e_{i1} & \cdots & e_{ir} \\
  \vdots & \ddots & \vdots \\
  e_{im} & \cdots & e_{mr}
\end{bmatrix}
\]

\[
D_j = \begin{bmatrix}
  e_{j1} & \cdots & e_{jr} \\
  \vdots & \ddots & \vdots \\
  e_{jm} & \cdots & e_{mr}
\end{bmatrix}
\]

The proposed method combines the decision matrix \( D_i \), \( D_j \) into the final decision matrix \( D \). The element of \( D \) is a FLM which aggregate all elements at corresponding position in the \( D_i \), \( D_j \). In this paper, we denote this operation as \( \mathcal{M} \). Specifically,

\[
D = D_i \mathcal{M} D_j = \begin{bmatrix}
  e_{i1} & \cdots & e_{ir} \\
  \vdots & \ddots & \vdots \\
  e_{im} & \cdots & e_{mr}
\end{bmatrix}
\begin{bmatrix}
  e_{j1} & \cdots & e_{jr} \\
  \vdots & \ddots & \vdots \\
  e_{jm} & \cdots & e_{mr}
\end{bmatrix}
= \begin{bmatrix}
  \{e_{ij}, e_{ij}\} & \cdots & \{e_{ir}, e_{ir}\} \\
  \{e_{ij}, e_{ij}\} & \cdots & \{e_{ir}, e_{ir}\} \\
  \{e_{ij}, e_{ij}\} & \cdots & \{e_{ir}, e_{ir}\}
\end{bmatrix}
\]

(9)

Definition 3 In general, the operation of \( m \) matrixes given by \( m \) decision makers is

\[
D = M_i \mathcal{M} D_j \mathcal{M} \cdots \mathcal{M} D_n, \text{ denoted as}
\]

\[
D = \mathcal{M}^m D_n
\]

3.2. The operations and distance

Definition 4 For any FLMs \( M_1, M_2, \ldots, M_n \), for all \( e \in S \)

Element: \( e \in M \Leftrightarrow \text{count}_{M_i}(e) \geq 1; e \in M \Leftrightarrow \text{count}_{M_i}(e) = 0 \);

Inclusion: \( M_1 \subseteq M_2 \Leftrightarrow \text{count}_{M_2}(e) \leq \text{count}_{M_1}(e) \);

Equality: \( M_1 = M_2 \Leftrightarrow \text{count}_{M_1}(e) = \text{count}_{M_2}(e) \);

Sum: \( M_1 \oplus M_2 \Leftrightarrow \text{count}_{M_1 \oplus M_2}(e) = \text{count}_{M_1}(e) + \text{count}_{M_2}(e) \);
Union: \( M_1 \cup M_2 \leftrightarrow \text{count}_{M_1 \cup M_2}(e) = \text{count}_{M_1}(e) \lor \text{count}_{M_2}(e) \); 
Intersection: \( M_1 \cap M_2 \leftrightarrow \text{count}_{M_1 \cap M_2}(e) = \text{count}_{M_1}(e) \land \text{count}_{M_2}(e) \).

Definition 5 For any FLM \( M = \bigcup_{s \in \mathcal{M}} \{s^l \mid l = 1, \ldots, M \} \) its score function and variance are below:

\[
\rho(M) = \frac{1}{M} \sum_{s \in \mathcal{M}} s_j = s_j \sum_{l \in \mathcal{L}}
\]

\[
\sigma(M) = \frac{1}{M} \sqrt{\sum_{s \in \mathcal{M}} (s_j - s) \sum_{l \in \mathcal{L}}}
\]

Where \( \rho(M) \) and \( \sigma(M) \) are called score function and variance, respectively. \( s, s_j \in S \), \( l, k \in \{0, 1, \ldots, M\} \), \( \mathcal{M} \) is the number of linguistic terms in \( M \). And the compare of two FLMs are defined bellow:

Definition 6 For any two FLMs \( M_p, M_q \)

1. if \( \rho(M_p) > \rho(M_q) \), then \( M_p > M_q \); 
2. if \( \rho(M_p) = \rho(M_q) \), and \( \sigma(M_p) < \sigma(M_q) \), then \( M_p > M_q \); 
3. if \( \rho(M_p) = \rho(M_q) \), and \( \sigma(M_p) = \sigma(M_q) \), then \( M_p = M_q \).

We define the positive and negative ideal solutions of each criteria as:

\[
M^+ = \max M^j
\]

\[
M^- = \min M^j
\]

Then the positive and negative solution of FLM-MCGDM are:

\[
M^+ = \bigcup_{j=1}^r M^j+
\]

\[
M^- = \bigcup_{j=1}^r M^j-
\]

Definition 5 For any FLMs \( M_p, M_q \), the distance between them is defined as:

\[
d(M_p, M_q) = \frac{1}{M} \sum_{l \in \mathcal{L}} |p^l - q^l|
\]

Where \( M_p = \bigcup_{s \in \mathcal{M}_p} \{s^l \mid l = 1, \ldots, M \} \), and \( M_q = \bigcup_{s \in \mathcal{M}_q} \{s^l \mid l = 1, \ldots, M \} \).

It satisfy the following properties:

Normality: \( d(M_1, M_2) \geq 0, \ d(M_1, M_2) = 0 \iff M_1 = M_2 \); 
Symmetry: \( d(M_1, M_2) = d(M_2, M_1) \); 
Inequality: \( d(M_1, M_2) \leq d(M_1, M_3) + d(M_3, M_2) \);

Proof: 1), 2) obviously. We only prove the last one.

\[
d(M_1, M_2) = \frac{1}{M} \sum_{l=1}^M |\delta^l - \delta^l| = \frac{1}{M} \sum_{l=1}^M |\delta^l - \delta^l + \delta^l - \delta^l| \leq \frac{1}{M} \sum_{l=1}^M (|\delta^l - \delta^l| + |\delta^l - \delta^l|)
\]

\[
= \frac{1}{M} \sum_{l=1}^M |\delta^l - \delta^l| + \frac{1}{M} \sum_{l=1}^M |\delta^l - \delta^l| = d(M_1, M_3) + d(M_3, M_2).
\]

3.3. FLM-VIKOR method

The specific decision making steps of FLM-VIKOR method are bellow:

1. Obtain the fuzzy linguistic value multiset multi-criteria decision making matrix by equation (8) and (9).
(2) Find the positive and negative ideal solutions by equation (15) and (16).

(3) Compute the maximum group utility $S_i$ and the minimum individual regret $R_i$ of each object, where

$$S_i = \sum_{j=1}^{n} w_j \frac{d\left(M_j^{+}, M_i^{+}\right)}{d\left(M_j^{+}, M_j^{-}\right)}$$

(18)

$$R_i = \max_{j} w_j \frac{d\left(M_j^{+}, M_i^{-}\right)}{d\left(M_j^{+}, M_j^{-}\right)}$$

(19)

Note that the weight $w_j$ can be omitted.

(4) Compute the comprehensive evaluation value $Q_i$, where

$$Q_i = vS_i + (1-v)R_i$$

(20)

(5) Obtain the compromise solution.

Firstly, according to $S_i$ , $R_i$ , $Q_i$ , we can rank for each $a_i$ ; then according to the condition① and② whether satisfy or not; finally select the compromise solution.

4. Instance and Analysis

4.1. Instance

The board of directors of a company has five members who plan to make a strategic plan for the company in the next five years. There are three projects $a_i (i=1,2,3)$ to be evaluated. Considering the following four aspects, i.e. $c_1$: financial aspects; $c_2$: consumer satisfaction; $c_3$: international career development; $c_4$: growth of the company. The five directors used linguistic terms to express their linguistic evaluation value. Where, the linguistic term set is $\{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$. Their decision matrixes are as follows:

$$D = \begin{bmatrix} s_1 & s_4 & s_3 & s_5 \; s_3 & s_4 & s_3 & s_2 \; s_4 & s_3 & s_2 & s_4 \; s_5 & s_4 & s_3 & s_5 \; s_2 & s_4 & s_3 & s_2 \end{bmatrix}$$

(1)

By means of the mean complement method, the decision matrix becomes

$$D = \begin{bmatrix} s_1 & s_4 & s_3 & s_5 \; s_3 & s_4 & s_3 & s_2 \; s_4 & s_3 & s_2 & s_4 \; s_5 & s_4 & s_3 & s_5 \; s_2 & s_4 & s_3 & s_2 \end{bmatrix}$$

(2)

Find the $M^{+}$, $M^{-}$:

By equation (15) and (16), we can obtain the positive solution is $M^{+} = \{a_1, a_2, a_3, a_4\}$, and the negative solution is $M^{-} = \{a_1', a_2', a_3', a_4'\}$.

(3) Calculate the $S_i$, $R_i$, $Q_i$

By equation (17), we can obtain $S_1 = 1.377$, $S_2 = 4$, $S_3 = 1.78$; By equation (18), we can obtain $R_1 = 1.17$, $R_2 = 1$, $R_3 = 1$, By equation (19), we can obtain $Q_1 = 1.27$, $Q_2 = 2.5$, $Q_3 = 1.39$.

Rank $a_i$ according to the $S_i$, $R_i$, $Q_i$, the result as shown in table 1.
Table 1

|       | Rank       |
|-------|------------|
| $S_i < S_j < S_k$ | $a_i > a_j > a_k$ |
| $R_i = R_j < R_k$ | $a_j = a_i > a_k$ |
| $Q_i < Q_j < Q_k$ | $a_i > a_j > a_k$ |

Table 2

| Method       | Rank       | Optimal/compromise solution |
|--------------|------------|-----------------------------|
| FLM-TOPSIS [6] | $a_i > a_j > a_k$ | $a_i$ |
| HFL-TOPSIS [11] | $a_j > a_i > a_k$ | $a_j$ |
| SAB [3]       | $a_j > a_i = a_k$ | $a_j$ |
| FLM-VIKOR     | $a_i > a_j > a_k$ | $a_i, a_j$ |

4.2. Compare and Analysis

Compared with other methods, the calculation in this paper is compared with the example results in literature [3,6,11], the result as shown in table 2.

As shown in the table above, the method proposed in this paper is consistent with the sorting result obtained by FLM-TOPSIS [6], but the final solution is inconsistent with the sorting result obtained by HFL-TOPSIS [11] and SAB [3]. Which method is more reasonable? As shown in the comparison in [6], for this part of unknown linguistic information, it is more reasonable to use the FLM-TOPSIS method, and just as the advantages of the VIKOR method discussed above over the TOPSIS method, it can be seen that the ideal solution obtained by the FLM-VIKOR method is closer to the optimal solution than that by the FLM-TOPSIS method, so the result obtained by the method in this paper is more reasonable.

5. Conclusion

In this paper, we propose a new linguistic decision making method FLM-VIKOR. At first, we develop the FLM, and study its properties and operations. After that, we discuss the distance between FLMs. Then we combine it with VIKOR method to solve the fuzzy linguistic decision making problem, at the same time, we give the proposed method specific steps, and apply it on an instance. Finally, by comparing with existing methods, it is shown that the method proposed can obtain the ideal solution which closest to the optimal solution, meanwhile retaining the original information completely.

Acknowledgments

This work was financially supported by the Innovation Fund of Xihua University (ycjj2018187, ycjj2018062).

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