$D^0 - \bar{D}^0$ Oscillations as a Probe of Quark-Hadron Duality

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Abstract

It is usually argued that the Standard Model predicts slow $D^0 - \bar{D}^0$ oscillations with $\Delta M_D, \Delta \Gamma_D \leq 10^{-3} \cdot \Gamma_D$ and that New Physics can reveal itself through $\Delta M_D$ exceeding $10^{-3} \cdot \Gamma_D$. It is believed that the bulk of the effect is due to long distance dynamics that cannot be described at the quark level. We point out that in general the OPE yields soft GIM suppression scaling only like $\left(\frac{m_s}{\mu_{\text{hadr}}}\right)^2$ and even like $m_s/\mu_{\text{hadr}}$ rather than $m_s^4/m_c^4$ of the simple quark box diagram. Such contributions can actually yield $\Delta M_D, \Delta \Gamma_D \sim \mathcal{O}(10^{-3}) \cdot \Gamma_D$ without invoking additional long distance effects. They are reasonably suppressed as long as the OPE and local duality are qualitatively applicable in the $1/m_c$ expansion. We stress the importance of improving the sensitivity on $\Delta \Gamma_D$ as well as $\Delta M_D$ in a dedicated fashion as a laboratory for analyzing the onset of quark-hadron duality and comment on the recent preliminary study on $\Delta \Gamma_D$ by the FOCUS group.
$K^0-\bar{K}^0$ oscillations have played a crucially important role in the development of the Standard Model (SM). Likewise the observation of $B_d-\bar{B}_d$ oscillations had an essential impact. In both cases the oscillation rate is rather similar to the decay rate. On the other hand one expects $D^0-\bar{D}^0$ oscillations to be very slow even on the scale of a second-order weak amplitude. Searching for them thus represents primarily a high sensitivity probe for New Physics in the electroweak sector with almost zero background from SM dynamics. Active searches are being undertaken using high quality data already in hand or soon to become available.

In this note we look closely at the SM predictions. It is found that the operator product expansion includes contributions exhibiting only very soft GIM suppression. They are much larger than the usual quark box contribution and can dominate overall mixing. We address quark-hadron duality (or duality for short), the role it plays here and what a signal or lack thereof can teach us about it and QCD. Such a lesson is not merely of a conceptual nature, but can give us information on the scale above which duality holds at least approximately.

1 SM Estimates of $D^0-\bar{D}^0$ Oscillations

1.1 General Features

Oscillations in general are described by two dimensionless quantities:

$$x = \frac{\Delta M}{\Gamma}, \quad y = \frac{\Delta \Gamma}{2\Gamma}$$

where

$$\Delta M \equiv M_2 - M_1, \quad \Delta \Gamma \equiv \Gamma_1 - \Gamma_2, \quad \bar{\Gamma} = \frac{1}{2} (\Gamma_1 + \Gamma_2).$$

The leading contributions to $\Delta M_K$ as well as $\Delta M_B$ are obtained from the well-known quark box diagram. The fields in the internal loop – $c$ in the former and $t$ quarks in the latter case besides $W$ bosons – are much heavier than the external quark fields. Those heavy degrees of freedom can be integrated out leading to $\Delta M_{K,B}$ being described by the expectation value of a local operator.\footnote{It turns out that $\Delta M_K$ is not completely dominated by local contributions reflecting dynamics operating around the scale $m_c$: a significant fraction is due to long distance dynamics characterized by low scales $\sim \Lambda_{QCD}$. This would change dramatically if $m_c$ were larger.} This has been a highly successful ansatz: within the present theoretical uncertainties the SM can reproduce the observed values of $\Delta M_K$ and $\Delta M_{B_d}$ without forcing any parameter.

While the size of $\Delta \Gamma_K$ is naturally understood as due to $K \to 3\pi$ being severely phase space restricted, it cannot be inferred from such a local operator, as described above. For neutral $B$ mesons, on the other hand $\Delta \Gamma_B$ can be estimated through short distance dynamics \cite{1}.

On very general grounds one expects $D^0-\bar{D}^0$ oscillations to be quite slow within the SM, since two structural reasons combine to make $x_D$ and $y_D$ small:
• While the bulk of charm decays is Cabibbo allowed, the amplitude for $D^0 \leftrightarrow \bar{D}^0$ transitions is necessarily twice Cabibbo suppressed – as is therefore the ratio between oscillation and decay rate: $\Delta M_D/\Gamma_D, \Delta \Gamma_D/\Gamma_D \propto \mathcal{O}(\sin^2 \theta_C)$. The amplitudes for $K^0 \leftrightarrow \bar{K}^0$ and $B_d \leftrightarrow \bar{B}_d$ are Cabibbo and KM suppressed, respectively – yet so are their decay widths allowing the oscillation and decay rate to be quite comparable.

• Due to the GIM mechanism one has $\Delta M = \Delta \Gamma = 0$ in the limit of flavor symmetry. Yet flavor symmetry breaking driving $K^0 \rightarrow \bar{K}^0$ is characterized by $m_s^2 \neq m_u^2$ and therefore no real suppression arises. On the other hand $SU(3)$ breaking controlling $D^0 \rightarrow \bar{D}^0$ is typified by $m_s^2 \neq m_d^2$ (or, in terms of hadrons, $M_K^2 \neq M_D^2$) as compared to the scale $M_B^2$; it provides a significant reduction.

Having two Cabibbo suppressed classes of decays one concludes on these very general grounds:

$$\Delta M_D, \Delta \Gamma_D \sim SU(3) \text{ breaking } \times 2 \sin^2 \theta_C \times \Gamma_D \quad (3)$$

The proper description of $SU(3)$ breaking thus becomes the central issue.

With typical nonleptonic $D$ decay channels exhibiting sizeable $SU(3)$ breaking – see our discussion in Sect. 2 – a priori one cannot count on this suppression to amount to more than a factor of about two or three in the $D$ width difference. There are reasons to believe that a larger reduction may occur for the mass difference $\Delta M_D$ driven by virtual intermediate states. Yet an order of magnitude reduction, in particular in $\Delta \Gamma_D$ would seem unjustifiably pessimistic. Thus

$$\frac{\Delta M_D}{\Gamma_D} \lesssim \frac{\Delta \Gamma_D}{\Gamma_D} \lesssim \frac{1}{3} \times 2 \sin^2 \theta_C \sim \text{ few } \times 0.01 \quad (4)$$

represents a conservative bound for overall mixing based on very general features of the SM; for the mass difference this estimate can actually be seen on the cautious side.

The following line of arguments is usually employed: (i) Quark-level contributions are estimated by the usual quark box diagrams and yield only insignificant contributions to $\Delta M_D$ and $\Delta \Gamma_D$ (see below). (ii) Various schemes employing contributions of selected hadronic states are invoked to estimate the impact of long distance dynamics; the numbers typically resulting are $x_D, y_D \sim 10^{-4} - 10^{-3}$ \footnote{One can argue that because of the $\Delta C = -\Delta Q$ rule in semileptonic charm decays one should write the nonleptonic rather than the total width in Eq. (5); yet this difference is small for $D^0$ and certainly is in the theoretical ‘noise’.} \cite{2,3,4}. (iii) These findings lead to the following widely embraced conclusions: An observation of $x_D > 10^{-3}$ would reveal the intervention of New Physics beyond the SM, while $y_D \simeq y_D|_{SM} \leq 10^{-3}$ has to hold since New Physics has hardly a chance to contribute to it.
Beyond the general property that both $\Delta M_D$ and $\Delta \Gamma_D$ have to vanish in the $SU(3)$ limit, the dynamics underlying them have different features: $\Delta M_D$ receives contributions from virtual intermediate states whereas $\Delta \Gamma$ is generated by on-shell transitions. Therefore the former is usually considered to represent a more robust quantity than the latter; actually it has often been argued that quark diagrams cannot be relied upon to even estimate $\Delta \Gamma$. A folklore has arisen that theoretical evaluations of the two quantities rest on radically different grounds.

Yet we note that despite these differences there is no fundamental distinction in the theoretical treatment of $\Delta M_D$ and $\Delta \Gamma_D$: both can be described through an operator product expansion, and its application relies on local quark-hadron duality for both $\Delta M_D$ and $\Delta \Gamma_D$. Only the numerical aspects differ, as does the sensitivity to New Physics.

### 1.2 Operator Product Expansion

Following the general treatment of inclusive weak transitions, see Refs. [5, 6, 7], we can describe $D^0 - \bar{D}^0$ oscillations by considering a correlator

$$ \hat{T}_{D\bar{D}}(\omega) = \frac{1}{2} \int d^4x e^{-i\omega t} iT\{\mathcal{H}_W(x)\mathcal{H}_W(0)\}, \quad T_{D\bar{D}}(\omega) = \frac{1}{2M_D} \langle \bar{D} | \hat{T}_{D\bar{D}}(\omega) | D \rangle \quad (5) $$

as a function of a complex variable $\omega$. Here $\mathcal{H}_W$ is the $\Delta C = -1$ Hamiltonian density. With the mixing amplitude of interest

$$ A(\omega) = 2 \sum_n \frac{1}{2M_D} \frac{\langle \bar{D} | \mathcal{H}_W | n(\vec{0}) \rangle \langle n(\vec{0}) | \mathcal{H}_W | D \rangle}{E_n - M_D + \omega + i\epsilon} \equiv -\Delta \tilde{M}_D(\omega) + \frac{i}{2} \Delta \tilde{\Gamma}_D(\omega) \quad (6) $$

one has $4 T_{D\bar{D}}(\omega) = A(\omega) + A(-\omega)$, whereas $\Delta M_D = \Delta \tilde{M}_D(0)$, $\Delta \Gamma_D = \Delta \tilde{\Gamma}_D(0)$. The summation above runs over all intermediate states $|n\rangle$ with energies $E_n$ and vanishing spacelike momentum. $\Delta M_D$ can be expressed by a dispersive integral over $\Delta \tilde{\Gamma}_D(\omega)$ evaluated through the principal value prescription:

$$ \Delta M_D = \frac{1}{2\pi} \text{VP} \int d\omega \frac{\Delta \tilde{\Gamma}_D(\omega)}{\omega}. \quad (7) $$

Applying the operator product expansion (OPE) to Eq. (4) provides us with a consistent evaluation of the transition rates through an expansion in powers of $1/m_c$.

With the charm quark mass exceeding the typical scale of strong interactions $\mu_{\text{hadr}}$ by a modest amount only, one cannot count on obtaining a reliable quantitative description in this way; yet it still yields a useful classification of various effects. This is briefly reviewed below.

The leading term for $\Delta C = -2$ transitions comes from dimension-6 four-fermion operators of the generic form $(\bar{u}c)(\bar{u}c)$ with the corresponding Wilson coefficient receiving contributions from different sources.
(a) Effects due to intermediate $b$ quarks are most simply calculated since they are highly virtual and Euclidean:

$$
\Delta M_D^{(bb)} \simeq \frac{G_F^2 m_b^2}{8\pi^2} |V_{cb}^* V_{ub}|^2 \frac{1}{2M_D} \langle D^0 | (\bar{u} \gamma_\mu (1-\gamma_5)c)(\bar{u} \gamma_\mu (1-\gamma_5)c) | D^0 \rangle ; \tag{8}
$$

however they are highly suppressed by the tiny KM mixing with the third generation. Using factorization to estimate the matrix element one finds:

$$
x_D^{(bb)} \sim \text{few} \times 10^{-7} . \tag{9}
$$

Loops with one $b$ and one light quark likewise are suppressed.

(b) For the light intermediate quarks the momentum scale is set by the external mass $m_c$, and the corresponding factor is given by $G_F^2 m_c^2 / 8\pi^2 \sin^2 \theta_C \cos^2 \theta_C$ (from now on we will often omit the KM factors when they are obvious). However, it is highly suppressed by the GIM factor $\left(\frac{m_s^2-m_d^2}{m_c^2}\right)^2$ leading to

$$
\hat{T}_{DD}(\omega) = \frac{G_F^2}{16\pi^2} |V_{cs}^* V_{us}|^2 \left(\frac{(m_s^2-m_d^2)^2}{(m_c-\omega)^2} + \frac{(m_s^2-m_d^2)^2}{(m_c+\omega)^2}\right) \times

\left[(\bar{u} \gamma_\mu (1-\gamma_5)c)(\bar{u} \gamma_\mu (1-\gamma_5)c) + 2(\bar{u}(1+\gamma_5)c)(\bar{u}(1+\gamma_5)c)\right] . \tag{10}
$$

Hence we read off for its contribution to the mass difference

$$
\Delta M_D^{(s,d)} \simeq -\frac{G_F^2 m_c^2}{4\pi^2} |V_{cs}^* V_{us}|^2 \frac{(m_s^2-m_d^2)^2}{m_c^4}

\times \frac{1}{2M_D} \langle D^0 | (\bar{u} \gamma_\mu (1-\gamma_5)c)(\bar{u} \gamma_\mu (1-\gamma_5)c) + 2(\bar{u}(1+\gamma_5)c)(\bar{u}(1+\gamma_5)c) | D^0 \rangle . \tag{11}
$$

(This expression differs from what is usually quoted in the literature, see, e.g. Ref. [2].) As follows from Eq. (10) bare quark loops do not contribute to $\Delta \Gamma_D$ at this order. The latter is suppressed by additional powers of $m_s/m_c$, or by $\alpha_s/\pi$ when gluon corrections are accounted for (e.g., through the anomalous dimension of the light quark mass).

The GIM suppression by two powers of $m_s/m_c$ for each quark line is inevitable for left-handed weak vertices. This feature persists for Penguin operators, albeit in a slightly different way. Numerically one finds:

$$
\Delta \Gamma_D^{\text{box}} < \Delta M_D^{\text{box}} \sim \text{few} \times 10^{-17} \text{ GeV} \equiv \hat{x}_D^{\text{box}} \sim \text{few} \times 10^{-5} \tag{12}
$$

However, since the leading Wilson coefficient is highly suppressed, one has to consider also the contributions from higher dimensional operators. It turns out that

This contribution is obviously saturated at the momentum scale $\sim m_c$, and thus refers to the Wilson coefficient of the $D=6$ operator. We disagree with statements that these are long-distance contribution simply because they are proportional to $m_s^2$.\footnote{This contribution is obviously saturated at the momentum scale $\sim m_c$, and thus refers to the Wilson coefficient of the $D=6$ operator. We disagree with statements that these are long-distance contribution simply because they are proportional to $m_s^2$.}
the $SU(3)$ GIM suppression is in general not as severe as $(m_s^2 - m_d^2)/m_c^2$ per fermion line: it can be merely $m_s/\mu_{\text{hadr}}$ if the fermion line is soft. In the so-called practical version of the OPE this is described by condensates contributing to the next terms in the $1/m_c$ expansion.

There is a simple rule of thumb: cutting a quark line, we pay the price of a power suppression $\sim \mu^3_{\text{hadr}}/m_c^3$; yet the GIM suppression now becomes only $m_s/\mu_{\text{hadr}}$. Altogether this yields a factor $\sim 4\pi^2 \mu^2_{\text{hadr}}/(m_s m_c)$ which can result in an enhancement. In particular, we keep in mind that $SU(3)$ breaking effects in condensates are not significantly suppressed, and the ratio between $\mu_{\text{hadr}}$ and $m_c$ is not much smaller than unity.

An example is given by the diagram in Fig. 1a. It yields a six-fermion operator of the generic form $(\bar{u}c)(\bar{u}c)(\bar{d}d - \bar{s}s)$ with the Wilson coefficient $\sim m_s^2 G_F^2 m_c^{-1}$. This contribution thus scales like $4\pi^2 m_s^2 \mu^3_{\text{hadr}}/m_c^5$ compared to the “standard” factor $G_F^2 f_D^2 m_c^2 M_D$. Explicitly, this contribution (neglecting gluon corrections) is given by

$$\Delta M_{D}^{(D=9)} = 2 \sin^2 \theta_C \cos^2 \theta_C \frac{G_F^2 m_s^2}{m_c^2} \times$$

$$\frac{1}{2M_D} \langle \bar{D} | \bar{u}^i \gamma_\mu (1-\gamma_5) c^j \bar{u}^k \gamma_\nu (1-\gamma_5) \gamma_0 c^l \left( \bar{d}^j \gamma_\mu \gamma_0 \gamma_\nu (1-\gamma_5) s^k - \bar{d}^j \gamma_\mu \gamma_0 \gamma_\nu (1-\gamma_5) d^k \right) | D \rangle.$$ (13)

Here $i, j, k$ are color indices. Note, that the $c$ quark operators here are normalized at the low momentum scale. Therefore, $\frac{\mu_{\text{hadr}}}{2} c(x)$ describes only annihilation of $c$ quark, and $\frac{\mu_{\text{hadr}}}{2} c(x)$ only creation of charmed antiquark. This implies, for example, that the relation $\gamma_0 c(x) \times c(y) = - c(x) \times \gamma_0 c(y)$ always holds for the considered matrix elements, since only two combinations $\frac{\mu_{\text{hadr}}}{2} c(x) \times \frac{\mu_{\text{hadr}}}{2} c(y)$ survive. It is interesting that for the “neutral current” type color flow in the both weak vertices, the contributions of Figs. 1a (proportional to $a_s$) vanish to the leading order in $1/m_c$.

$SU(3)$ suppression can be further softened by cutting both fermion lines. To transfer a large momentum one has to add a gluon, like in Fig. 1b (for this reason another loop factor of $4\pi^2$ is replaced by $4\pi\alpha_s$). These yield eight-fermion opera-
tors with the flavor structure 

\[ (\bar{u}c)(\bar{u}c) \left[ (\bar{d}d)(\bar{d}d) + (\bar{s}s)(\bar{s}s) - (\bar{d}d)(\bar{s}s) - (\bar{s}s)(\bar{d}d) \right] \].

With the SU(3) suppression in the matrix element due to double antisymmetrization between \( s \) and \( d \) only \( m_s^2/\mu^2_{\text{had}} \), this contribution scales like \( 4\pi^2 m_s^4/\mu^4_{\text{had}} \cdot G_F^2 f_D^2 m_c^2 M_D \).

It is interesting to note that, in principle, the SU(3) suppression can be as mild as only the first power of \( m_s \). Namely, if we schematically define

\[ \langle \bar{D} | (\bar{u}c)(\bar{u}c) \left[ (\bar{d}d)(\bar{d}d) + (\bar{s}s)(\bar{s}s) - (\bar{d}d)(\bar{s}s) - (\bar{s}s)(\bar{d}d) \right] | D \rangle \equiv \zeta \langle \bar{D} | (\bar{u}c)(\bar{u}c)(\bar{d}d)(\bar{d}d) | D \rangle \]

then, at small \( m_s \), the SU(3) suppression factor \( \zeta \) can scale as \( m_s/\mu_{\text{had}} \). Indeed, if the matrix element, as a function of the two quark masses is given by

\[ \langle \bar{D} | (\bar{u}c)(\bar{u}c)(\bar{q}_1q_1)(\bar{q}_2q_2) | D \rangle = A + B \cdot (m_1 + m_2) \ln \frac{\mu_{\text{had}}}{m_1 + m_2} \]

then \( \zeta \simeq -2Bm_s\ln 2 \). More accurately, for the actual operator the leading, linear in \( m_s \) term in the “soft GIM” factor \( \zeta \) is determined by the matrix element

\[ \frac{M_{K^0}^2}{16\pi^2 f_\pi^2} < \bar{D}^0 | (\bar{u}L\Gamma c_L)^2 \bar{d}_L \bar{s}_L \Gamma d_L | D^0 > \] (14)

in the limit of massless \( s \) and \( d \) and is due to the chiral \( \bar{K}^0\eta(\pi^0)K^0 \) loop. The explicit structure of the Lorentz and color matrices \( \Gamma \) above follows from the operator given below in Eq. (15).

![Diagram of chiral loop leading to the effect linear in \( m_s \).](image)

**Figure 2:** Chiral loop leading to the effect linear in \( m_s \).

Let us sketch this step. At small \( m_s \) (for simplicity we put \( m_d = 0 \), but \( m_u \) can be arbitrary) it is convenient to use the current quark fields \( s', d' \) instead of mass eigenstates, and perform an expansion around the symmetry point \( m_s = m_d \). Then the transition operator has the simple flavor structure \( (\bar{u}c)^2(s'd')^2 \). Its matrix element between \( \bar{D}^0 \) and \( D^0 \) vanishes to zeroth order since the operator has \( \Delta S' = - \Delta D' = -2 \). The mass perturbation, however, has a \( \Delta S' = - \Delta D' = 1 \) piece

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\[ ^4 \text{There are actually additional contributions not obtained merely by cutting quark lines; they are reminiscent of the soft part of Penguin contributions, see below.} \]
\[
\sin \theta_C \cos \theta_C \, m_s \, \bar{d}' s'.
\]
The nonvanishing second-order correction to the matrix element \(\langle \bar{D}^0 | \bar{(u)c}^2 (s'd')^2 | D^0 \rangle\) is then generally proportional to \(m_s^2\). The only exception comes from the pseudo-goldstone loop of Fig. 2 shaped by
\[
\int \frac{d^4k}{(2\pi)^4 i} \frac{1}{(M^2 - k^2)^3} = \frac{1}{32\pi^2 M^2}
\]
with \(M^2 \propto m_s\). This contribution is proportional to the zero-momentum amplitude \(\langle \bar{D}^0 \bar{K}^0 K^0 | (\bar{u}c)^2 (\bar{s}d)^2 | D^0 \rangle\) which, by PCAC is related to the chiral limit matrix element of the double commutator of the operator with the \(\bar{d}s\) axial charge. This yields the stated equation.

The above estimate serves only as an existence proof. Most probably, such infrared effects involving pion loops are not the dominant source. Even without such nonanalytic terms the matrix elements typically depend strongly on the quark masses, and in the actual world the double subtraction present in the above operators, can result in only a mild suppression factor in spite of being formally of the order of \(m_s^2\).

Estimates of the actual size of these contributions at present suffer from considerable uncertainties, primarily in the matrix elements. Direct computation of the bare diagram yields the following cumbersome result:
\[
-2 \hat{T}(0) = -\frac{8\pi \alpha_s G_F^2}{m_c^4} \sin^2 \theta_C \cos^2 \theta_C \, G \, O^{(12)} ,
\]
\[
O^{(12)} = \left\{ \bar{u} \Gamma_\alpha c \bar{d} \Gamma_0 t^a c - \bar{u} \Gamma_0 d \bar{s} \Gamma^\alpha t^a c + i \epsilon^{0\mu\alpha\beta} \bar{u} \Gamma_\beta t^a d \bar{s} \Gamma_\mu c \right\} \times \left\{ \bar{u} \Gamma_\alpha s \bar{d} \Gamma_\alpha t^a c - \bar{u} \Gamma_0 s \bar{d} \Gamma_0 t^a c + i \epsilon_{\mu\nu\alpha\gamma} \bar{u} \Gamma_\gamma t^a s \bar{d} \Gamma_\nu c \right\}
\]
where \(\Gamma_\mu = \gamma_\mu (1 - \gamma_5)\) and \(t^a = \frac{\lambda^a}{2}\) are color matrices. The symbol \(G\) denotes GIM-type subtraction; its action on a generic operator with four \(s\) and \(d\) quarks like in \(O^{(12)}\) is defined as
\[
G (\bar{s}U d)(\bar{d}V s) =
\]
\[
(\bar{s}U d)(\bar{d}V s) - (\bar{s}U s)(\bar{d}V d) - (\bar{d}U d)(\bar{s}V d) + (\bar{d}U s)(\bar{d}V d) + (\bar{d}U d)(\bar{s}V d)
\]
(the last two terms do not have counterparts in the box diagram). The exact coefficients and the color structure in \(O^{(12)}\) are modified by straightforward renormalization of the weak decay operators, yet the major uncertainty lies in the matrix elements.

In the spirit of the “Educated dimensional analysis” of Ref. 6 we estimate the magnitude of the matrix element of \(G \, O^{(12)}\) as
\[
\zeta f_D^2 M_D \cdot (0.3 \text{ GeV})^6 \approx \zeta \cdot 7 \cdot 10^{-5} \text{ GeV}^9
\]
where \(\zeta\) accounts for the \(SU(3)\) GIM suppression,
\[
\langle \bar{D}^0 | G \, O^{(12)} | D^0 \rangle = \zeta \langle \bar{D}^0 | O^{(12)} | D^0 \rangle .
\]
Note that $\alpha_s$ enters at the charm mass scale, and for consistency must be evaluated in the $V$- rather than the $\overline{\text{MS}}$-scheme, which is obvious in the BLM approximation. Numerically we end up with

$$\delta^{(12)}_{x_D} \sim \mathcal{O}(10^{-3}).$$

(18)

It is actually conceivable that medium-size instantons yield enhanced contributions to the corresponding matrix element of the eight-fermion operators in question, due to the induced t’ Hooft vertex of the form $(\bar{u}_L u_R)(\bar{d}_R d_L)(\bar{s}_L s_R)$. Note that it can yield the effect $\propto m_s^2$ which is not formally related to the spontaneous symmetry breaking since would violate only the anomalous singlet $U_A(1)$.

Diagrams in Figs. (1) and (2) literally do not produce an absorptive part and, therefore contribute to $\Delta M_D$, but not $\Delta \Gamma_D$. Yet the latter can be generated, for example, through a cut across the gluon propagator in Fig. 2 if it is dressed; in $\Delta M_D$ this would contribute to the anomalous dimension. This is the leading contribution in the BLM approximation. It amounts to replacing $\alpha_s$ by $\frac{3}{2} i\alpha_s^2/(1 + 81/16\alpha_s^2)$ in Eq. (15). Such approximation is justified if other contributions to the anomalous dimension can be neglected. At the charm scale these modifications do not seem to lead to a particular numerical suppression of $\Delta \Gamma_D$ compared to $\Delta M_D$. Therefore, we arrive at

$$x_D, \ y_D \sim \mathcal{O}(10^{-3}).$$

(19)

**In summary:** We have shown that the high degree of $SU(3)$ invariance and related GIM suppression $(m_s^2 - m_d^2)^2/m_c^2$ exhibited by quark box diagrams for $D^0 \rightarrow \bar{D}^0$ is not typical for the process. Terms in the $D^0 \rightarrow \bar{D}^0$ amplitude can be proportional to $m_s^2$ or even $m_s^1$. Such contributions arise naturally in the OPE through condensate contributions containing higher dimensional operators. While those are formally suppressed by powers in the heavy quark mass, this does not constitute a very significant factor for the case of charm. It had been noted long ago (see, for example, Ref. [10]) that estimates of the absorptive part of the $D^0 \rightarrow \bar{D}^0$ amplitude are very sensitive to low energy parameters: evaluating it on the quark level one encountered much more effective $SU(3)$ cancellations than in its potential hadronic counterparts $\propto M_R^1/m_c^1$. Application of the OPE treatment allows to clarify and justify those, rather tentative suggestions identifying the possible source of such enhanced contributions in the framework of the $1/m_c$ expansion. Moreover, it becomes clear that the same applies to the mass difference $\Delta M_D$ as well.

Our numerical estimates are rather uncertain. However we note that the OPE naturally accommodates the size of mixing previously discussed in the literature as a possible effect of general long-distance dynamics. At the same time, we think that $x_D, \ y_D$ exceeding $5 \cdot 10^{-3}$ cannot be attributed to the OPE contributions in the framework of standard assumptions.

Regardless of the size of the matrix elements involved, we still can state that $D^0 - \bar{D}^0$ mixing must be suppressed in the SM whenever it makes sense to speak of
it in the $1/m_c$ expansion. The natural yardstick for the unsuppressed level is the overall nonleptonic decay width (up to inherent CKM mixing factors). As illustrated above, smallness of the higher-order terms compared to the formally leading in $1/m_c$ effect for mixing cannot serve as a valid universal criterion. Yet a certain suppression of the higher-order terms compared to the (parton) decay width \textit{per se} is a necessary condition for applying sensibly the $1/m_c$ expansion. Since the identified effects with mild GIM cancellation emerge in relatively high order in $1/m_c$, the minimal suppression must amount to a noticeable factor, as asserted above.

### 1.3 Quark-Hadron Duality

Beyond the question about the size of the higher order corrections in the $1/m_c$ expansion there is the more fundamental one about local quark-hadron duality; i.e., to which degree of accuracy does a quark level result derived from the OPE describe an inclusive quantity involving hadrons? In other words, how well knowledge of $\Delta M(\omega)$, $\Delta \Gamma(\omega)$ in Eq. (6) at large (compared to $\Lambda_{\text{QCD}}$) complex $\omega$ based on the short-distance expansion, determines their values at $\omega=0$ measured in experiment.

It was pointed out in Ref. [11] that with heavy quarks one often has to deal with a novel aspect, referred to as \textit{global} duality: a Euclidean dispersive integral will reproduce the contributions coming from \textit{all} cuts in the Minkowski domain, while some of them are unphysical for the considered decay process. One then has to filter out the contributions to the integral that correspond to the individual process of interest. Since this can be done only to a certain accuracy, it introduces a further source of theoretical uncertainty.

Even though the cuts in $T_{D\bar{D}}(\omega)$ describe the same physical channel, this complication still persists in a certain form for flavor oscillation processes: with $T_{D\bar{D}}(\omega) = A(\omega) + A(-\omega)$, it has two cuts which overlap. However, since one of them starts at $\omega \simeq -m_c$ and another at $\omega \simeq m_c$, they can be disentangled at $|\omega| \ll m_c$ in the $1/m_c$ expansion of ‘practical’ OPE in the same way as in the usual heavy quark decay widths [11]. The central issue is again the validity of local duality, both for $\Delta \Gamma$ and $\Delta M$. Its more dedicated explanation and its implementations can be found in various reviews (see, for example, Ref. [5, 6, 7]).

That also $\Delta M_D$ is sensitive to duality violations is readily seen by imagining the presence of a narrow resonance of appropriate quantum numbers close to the $D$ meson mass: it would significantly affect the size of $\Delta M_D$. Yet, practically, it is natural to expect local duality violations to be smaller in $\Delta M_D$ than in $\Delta \Gamma_D$; i.e., the onset of duality should occur at a lower scale for the former than for the latter. For $\Delta \Gamma_D$ is directly given by the discontinuity in the corresponding $D \to D$ transition amplitude, whereas $\Delta M_D$ can be represented by the principal value of the dispersion integral, Eq. (7). The latter provides a measure of averaging that reduces the sensitivity to the resonances or thresholds and thus local duality violations. This has been illustrated by a simple model with a single resonance in Ref. [3].

While no general proof has been given for the validity of local duality, consider-
able evidence has been accumulated over the years that it does apply for sufficiently heavy flavors. Detailed studies of OPE and duality have been performed recently within model field theories, in particular the t' Hooft model \[12\] which is QCD in 1+1 dimensions with $N_c \to \infty$. Analytical analyses showed that the OPE in the inverse powers of heavy quark mass holds for the heavy quark decay widths \[7\] in spite of certain doubts which had been voiced. The above papers did not consider the width difference between the two neutral heavy meson eigenstates. Yet using the technique developed there, it is not difficult to establish the similar correspondence between the hadronic saturation and the quark box diagrams at least through the next-to-leading order in $1/m_Q$.

Concluding duality to be valid asymptotically – for $m_Q \to \infty$ – one turns to the question at how low a scale duality emerges to apply with some accuracy. Most authors would expect it to be valid for $m_Q \sim m_b$; yet assuming duality to hold already at the charm scale even in a semiquantitative fashion would appear to be a rather iffy proposition for semileptonic transitions, let alone for nonleptonic ones. As argued above, experimental observation of a stronger suppression of $D^0 - \bar{D}^0$ oscillations compared to the phenomenological estimate Eq.\((\text{4})\), would be an indication of a relatively low onset of duality for the inclusive decay widths. The width difference $\Delta\Gamma_D$ is an even more sensitive, undiluted probe for duality violations than $\Delta M_D$. A scenario with a sizeable $\Delta\Gamma_D$ and a somewhat smaller value of $\Delta M_D$ could still imply the nearby onset of local duality.

One reservation has to be made though, due to a notorious complication peculiar to local duality violation. Since the duality violating component ‘oscillates’ (as a function of $m_c$), it can actually vanish for certain mass values. Determining the size of such effects at a single scale cannot yield a definite conclusion since an accidental vanishing at that scale cannot be ruled out. Yet we have two measures for mixing, namely $\Delta M_D$ and $\Delta\Gamma_D$, and their oscillatory dependence on $m_c$ in general will be out of phase.

2 Contributions to $D^0 \leftrightarrow \bar{D}^0$ from Exclusive Channels

We have stated above that the OPE expectation of in particular $y_D \sim \mathcal{O}(10^{-3})$ is highly remarkable since \textit{a priori} one would estimate it to be an order of magnitude larger, see Eq.\((\text{4})\). We will illustrate this point by considering transitions to two pseudoscalar mesons, which are common to $D^0$ and $\bar{D}^0$ decays and can thus communicate between them:

$$D^0 \xrightarrow{CS} K^+ K^-, \pi^+ \pi^- \xrightarrow{CS} \bar{D}^0,$$

$$D^0 \xrightarrow{CA} K^- \pi^+ \xrightarrow{CS^2} \bar{D}^0,$$

$$D^0 \xrightarrow{CS^2} K^+ \pi^- \xrightarrow{CA} D^0.$$  

(20)
where $CA$, $CS$ and $CS^2$ denotes the channel as Cabibbo allowed, Cabibbo suppressed and doubly Cabibbo suppressed, respectively.

In the $SU(3)$ limit one obviously has $\Delta \Gamma(D^0 \to K\bar{K}, \pi\pi, K\pi, \pi\bar{K}) = 0$ since the amplitudes for Eqs. (21) would then be equal in size and opposite in sign to those of Eq. (20). Yet the measured branching ratios [13]

$$BR(D^0 \to K^+K^-) = (4.27 \pm 0.16) \cdot 10^{-3} \quad (22)$$
$$BR(D^0 \to \pi^+\pi^-) = (1.53 \pm 0.09) \cdot 10^{-3} \quad (23)$$
$$BR(D^0 \to K^-\pi^+) = (3.85 \pm 0.09) \cdot 10^{-2} \quad (24)$$
$$BR(D^0 \to K^+\pi^-) = (2.8 \pm 0.9) \cdot 10^{-4} \quad (25)$$

show very considerable $SU(3)$ breakings:

$$\frac{BR(D^0 \to K^+K^-)}{BR(D^0 \to \pi^+\pi^-)} \simeq 2.8 \pm 0.2 \quad (26)$$
$$\frac{BR(D^0 \to K^+\pi^-)}{BR(D^0 \to K^-\pi^+)} \simeq (3 \pm 1) \cdot \tan^4 \theta_C \quad (27)$$

compared to ratios of unity and $\tan^4 \theta_C$, respectively, in the symmetry limit.

One would then conclude that the $K\bar{K}, \pi\pi, K\pi, \pi\bar{K}$ contributions to $\Delta \Gamma$ should be merely Cabibbo suppressed with flavor $SU(3)$ providing only moderate further reduction – similar to the general expectation of Eq. (4):

$$\Delta \Gamma \frac{\mid_{D \to K\bar{K}, \pi\pi, K\pi, \pi\bar{K}}}{\Gamma} \sim \mathcal{O}(0.01) \quad (28)$$

Yet despite these large $SU(3)$ breakings an almost complete cancellation takes place between their contributions to $D^0 - \bar{D}^0$ oscillations:

$$BR(D^0 \to K^+K^-) + BR(D^0 \to \pi^+\pi^-) - 2\sqrt{BR(D^0 \to K^-\pi^+)BR(D^0 \to K^+\pi^-)} \simeq$$
$$\left(-\delta_{12}^{+12}_{-10}\right) \cdot 10^{-4} \quad (29)$$

to be compared to

$$BR(D^0 \to K^-\pi^+) + BR(D^0 \to K^+K^-) + BR(D^0 \to \pi^+\pi^-) + BR(D^0 \to K^+\pi^-) \simeq$$
$$\left(4.46 \pm 0.01\right) \cdot 10^{-2} \quad (30)$$

In principle, a note of caution should be sounded here: In writing down Eq. (29) we have ignored the possibility that $SU(3)$ breaking final state interactions can generate a strong phase shift $\delta_{K^+}$ between $D^0 \to K^-\pi^+$ and $D^0 \to K^+\pi^-$ amplitudes. If this happens, the last interference term then gets multiplied by a factor $\cos \delta_{K\pi}$. Here and in what follows we neglect this phase shift as motivated by the naive quark level
diagrams. Whether it is small as suggested by some is not clear \[14\]; in any case it could have a significant impact on the cancellations among the different terms. Yet we meant this discussion only as a qualitative illustration of our argument on the relation between $SU(3)$ symmetry and duality.

There is evidence that Eq. (26) overstates the amount of $SU(3)$ breaking in inclusive transitions: the data on Cabibbo suppressed four body modes read \[13\]

\[
\text{BR}(D^0 \to K^+K^-\pi^+\pi^-) = (2.52 \pm 0.24) \cdot 10^{-3} \\
\text{BR}(D^0 \to \pi^+\pi^-\pi^+\pi^-) = (7.4 \pm 0.6) \cdot 10^{-3};
\]

(31)
i.e., again these exclusive channels exhibit very sizeable $SU(3)$ breaking

\[
\frac{\text{BR}(D^0 \to K^+K^-\pi^+\pi^-)}{\text{BR}(D^0 \to \pi^+\pi^-\pi^+\pi^-)} \simeq 0.34 \pm 0.04;
\]

(32)
Yet adding these two- and four-body modes then leads to a result which is quite compatible with equality of the combined rates:

\[
\frac{\text{BR}(D^0 \to K^+K^-, K^+K^-\pi^+\pi^-)}{\text{BR}(D^0 \to \pi^+\pi^-, \pi^+\pi^-\pi^+\pi^-)} \simeq 0.8 \pm 0.1.
\]

(33)
While this sum cannot be unambiguously related to the violation of $SU(3)$ in the fully inclusive rates $\Gamma(c \to s\bar{s}u)$ vs. $\Gamma(c \to d\bar{d}u)$, the observed trend is at least suggestive.

To summarize the discussion in this Section:

- The $SU(3)$ breaking in exclusive nonleptonic channels is naturally expected to be sizeable, and this is indeed what is observed, see Eq. (26). The deviations from the symmetric case are actually substantially larger than what had been anticipated by most authors.

- Quark based calculations lead to the prediction that inclusive $D$ decays exhibit $SU(3)$ invariance to a high degree, since the symmetry breaking in described by $m_s^2/m_c^2 \sim \mathcal{O}(0.01)$.

- Emerging data provide the first indication that $SU(3)$ breaking is quite reduced when one sums up over various nonleptonic channels, see Eq. (33).

- Likewise the overall contributions to $\Delta \Gamma$ from channels with two pseudoscalar mesons in the final state appear to be considerably reduced, see Eq. (29).

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5This fully conforms to the spirit of the OPE description whose prediction we are trying to examine at the level of hadrons.
3 Experimental Bounds and Lessons on Duality

The present experimental landscape can be portrayed by the following numbers inferred from various analyses of \( D^0 \rightarrow K^+K^- \) vs. \( D^0 \rightarrow K^-\pi^+ \) and \( D^0 \rightarrow K^+\pi^- \) vs. \( D^0 \rightarrow K^-\pi^+ \). From general bounds on mixing one can infer:

\[
|x_D|, |y_D| \leq 0.028, \quad 95\% \text{ C.L.} \quad \text{CLEO [15]} \tag{34}
\]

Targeting more specifically width differences one finds

\[
-0.04 \leq y_D \leq 0.06, \quad 90\% \text{ C.L.} \quad \text{E791 [16]} \tag{35}
\]

\[
-0.058 \leq y_D' \leq 0.01, \quad 95\% \text{ C.L.} \quad \text{CLEO [15]} \tag{36}
\]

The CLEO study analyzes the time evolution of \( D^0(t) \rightarrow K^+\pi^- \) and is thus sensitive to

\[
y_D' = y_D \cos \delta_{K\pi} - x_D \sin \delta_{K\pi} \tag{37}
\]

where \( \delta_{K\pi} \) denotes the strong phase between \( D^0 \rightarrow K^+\pi^- \) and \( \bar{D}^0 \rightarrow K^+\pi^- \). A very recent and still preliminary FOCUS study compares the lifetimes for \( D \rightarrow K^+K^- \) and \( D \rightarrow K\pi \):

\[
y_D = 0.0342 \pm 0.0139 \pm 0.0074 \quad \text{FOCUS [17]} \tag{38}
\]

At this point we want to summarize and draw the following conclusions:

- Based on general grounds one expects

\[
\Delta M_D, \Delta \Gamma_D \sim SU(3) \text{ breaking } \times 2\sin^2 \theta_C \times \Gamma_D \tag{39}
\]

The observation of large deviations from \( SU(3) \) invariance in nonleptonic \( D \) decays suggests a conservative estimate

\[
\frac{\Delta \Gamma_D}{\Gamma_D} \leq \text{few } \times 0.01 \tag{40}
\]

with \( \Delta M_D / \Gamma_D \) being somewhat smaller.

- Specific dynamical features have to intervene to suppress \( D^0 - \bar{D}^0 \) below these levels. Such features arise naturally in a quark level treatment of \( SU(3) \) symmetry breaking as it arises in an OPE. Assuming local duality one obtains from the OPE the prediction

\[
x_D, y_D \sim \mathcal{O}(10^{-3}) \tag{41}
\]

without invoking additional long distance contributions. The main uncertainty in this prediction rests in the size of the relevant hadronic matrix elements. The OPE allows for rather mild \( SU(3) \) GIM cancellations. Consequently, for
sufficiently small values of $m_c$ there could be unsuppressed contributions to the oscillation rate. Yet for the actual charm mass such contributions should be reasonably suppressed compared to Eq. (40) since they emerge in higher orders in $1/m_c$. Therefore, we would consider the degree of suppression of $x_D$, $y_D$ below the 1% level as a measure of applicability of local duality. From this perspective, a stronger suppression of $x_D$ compared to $y_D$ seems the natural situation.

- The data have reached the general bound of Eq. (4). Any further reduction in the experimental bound on $y_D$ means that $D^0 - \bar{D}^0$ oscillations proceed more slowly than can be understood on the basis of general selection rules (a “symmetry level”).

- There is some tentative evidence that inclusive decays might exhibit the effective $SU(3)$ invariance expected to arise on the quark level.

- If the suggestion coming from the FOCUS data is confirmed that actually $y_D \sim \mathcal{O}(0.01)$ holds then one of two conclusions can be drawn: Either $\Delta M_D$ is just “around the corner”, i.e. a moderate improvement in experimental sensitivity should reveal a nonvanishing value for it without establishing the intervention of New Physics. This would mean we had seriously underestimated the size of the relevant matrix elements. Or it would represent a clear-cut violation of local quark-hadron duality at the charm scale.

Note added: After submitting this paper we were informed about the publication [18] (N.U. is grateful to A. Petrov for bringing our attention to it) where it was first proposed that contributions due to chiral symmetry breaking that are subleading in $1/m_c$ can generate a moderately larger value for $\Delta M_D$ than the SM box estimate. That paper focussed on an analogue of the OPE for the $\Delta C = 1$ transitions rather than directly for $\Delta C = 2$. Our analysis differs in a number of conceptual as well as technical aspects. We analyze the OPE for $\Delta C = 2$ transitions and obtain the formally leading term linear in $m_s$ that had been missed in [18]. We also address the difference between $\Delta M_D$ and $\Delta \Gamma_D$ explicitly.

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