Logit Pairing Methods Can Fool Gradient-Based Attacks

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Abstract
Recently, several logit regularization methods have been proposed in [Kannan et al., 2018] to improve the adversarial robustness of classifiers. We show that the proposed computationally fast methods – Clean Logit Pairing (CLP) and Logit Squeezing (LSQ) – just make the gradient-based optimization problem of crafting adversarial examples harder, without providing actual robustness. For Adversarial Logit Pairing (ALP) we find that it can give indeed robustness against adversarial examples and we study it in different settings. Especially, we show that ALP may provide additional robustness when combined with adversarial training. However, the increase is much smaller than claimed by [Kannan et al., 2018]. Finally, our results suggest that evaluation against an iterative PGD attack relies heavily on the parameters used and may result in false conclusions regarding the robustness.

1 Introduction

It has been shown in [Szegedy et al., 2013] that many state-of-the-art classifiers are not robust against small perturbations of the inputs, known as adversarial examples. Since then, many new attacks have been proposed aiming at better ways of crafting adversarial examples, and also many new defenses to increase the robustness of classifiers. Notably, almost all defenses that were proposed were heuristical and could be broken by applying different attacks [Carlini and Wagner, 2017, Athalye and Sutskever, 2017, Athalye and Carlini, 2018, Athalye et al., 2018]. The only heuristical defense that could not be

(a) Logit Squeezing     (b) Clean Logit Pairing     (c) Adv. Logit Pairing     (d) Adv. Training

Figure 1: Input loss surfaces of MNIST models in a random subspace around an input image with $\epsilon = 38.25$. We can clearly see a distorted loss surface for the logit regularization methods, which can prevent gradient-based attacks from succeeding. Additional visualizations are found in Figures 3, 4, 5, and 6 in the appendix.

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broken so far is adversarial training [Goodfellow et al., 2014, Madry et al., 2017]. There is also a line of work on provable robustness of classifiers [Hein and Andriushchenko, 2017, Wong and Kolter, 2018, Raghunathan et al., 2018], that cannot be broken by definition, as they derive and report a lower bound on the adversarial accuracy, while any attack provides only an upper bound. The problem with many recently proposed defenses is that they evaluate only upper bounds, which might be arbitrary loose, i.e. there might exist an attack which is able to reduce the adversarial accuracy significantly compared to some baseline attack. However, one of the main issues with lower bounds is that usually they are too small, so a special way of maximizing them is applied during training, which may interfere with a proposed heuristical defense which one aims to evaluate. Thus, providing non-trivial lower bounds – with or without special training – is an important and active area of research [Wong et al., 2018, Zhang et al., 2018, Xiao et al., 2018, Croce et al., 2018]. However, these methods do not scale yet to large-scale datasets like ImageNet, and thus one still has to rely solely on upper bounds on adversarial accuracy to estimate the robustness for these datasets. Given the recent history of breaking most of the heuristical defenses accepted at ICLR 2018 in [Athalye et al., 2018], it is now natural to question any new heuristical defense. Many recent papers [Buckman et al., 2018, Kannan et al., 2018, Yao et al., 2018] that claim robustness of their models mainly rely on the PGD attack from [Madry et al., 2017] with the default settings. They assume that they evaluate their models against a “strong adversary”, so that adversarial accuracy that they obtain is close to the minimal possible. In this paper, we show that it is not the case for CLP, LSQ and some ALP models proposed in [Kannan et al., 2018].

Threat model and attack settings. We consider classification of images with pixel values in [0, 255]. We consider the white-box threat model, i.e. an attacker has complete knowledge of the model. We consider adversarial perturbations bounded with respect to $L_\infty$ norm of $\epsilon = 76.5$ for MNIST, and $\epsilon = 16$ for CIFAR-10 and Tiny ImageNet. We follow the settings of [Kannan et al., 2018] and evaluate MNIST and CIFAR-10 models with untargeted attacks. Tiny ImageNet models were evaluated with targeted attacks which is consistent with [Athalye et al., 2018]. We used the PGD attack [Madry et al., 2017] with maximum adversarial perturbation $\epsilon$ and experimented with different number of iterations $n$, step sizes $\epsilon_i$, and restarts $r$. When crafting untargeted adversarial examples, we maximize the loss using the true label.

Contribution. First, we give empirical evidence that CLP, LSQ, and some ALP models distort the loss surface in the input space, thus fooling gradient-based attacks, but not providing actual robustness. This can be seen as a particular case of masked or obfuscated gradients [Papernot et al., 2017, Athalye et al., 2018]. We illustrate this by analyzing the input space loss surface in two random directions (see Figure 1). We provide an extensive experimental evaluation of the robustness of CLP, LSQ, and ALP models on MNIST, CIFAR-10, and Tiny ImageNet datasets against the PGD attack with a large number of iterations and random restarts reducing the adversarial accuracy of e.g. MNIST-LSQ model from 70.6% to 5.0% (see Table 1). Our results suggest, that while CLP and LSQ do not provide actual robustness, ALP may give additional robustness on top of adversarial training. However, the increase is much smaller than claimed by [Kannan et al., 2018] (Table 3). Finally, we elaborate on the importance of performing many random restarts of PGD, especially when the loss surface is distorted. We illustrate this by plotting the distribution of the loss values over different restarts of PGD (Figure 2).

Related work. Recently, [Engstrom et al., 2018] have evaluated the robustness of ALP on a single ImageNet model. However, there are important differences compared to our work. First, they do not explore the robustness of computationally cheap methods CLP and LSQ, which is a significant contribution of [Kannan et al., 2018]. Second, they only test an ALP model that was trained on clean examples, while [Kannan et al., 2018] mainly advocate for the usage of ALP combined with mixed-minibatch PGD (i.e. training on 50% clean and 50% adversarial examples), which we explore in detail. Finally, we show that the conclusions depend on the dataset, and thus the evaluation of only a single model using a single dataset may not allow for general statements.

2 Experiments

Following [Kannan et al., 2018] we augment the inputs with Gaussian noise (denoted by $N(\mu, \sigma)$ in all tables) when applying CLP and LSQ. On MNIST we used the same LeNet architecture as [Kannan et al., 2018].
We visualize the cross-entropy loss in a two-dimensional subspace of the input space in the vicinity of with ALP below CLP from with different number of iterations $n$, step sizes $\epsilon_i$, and restarts $r$. AT denotes adversarial training.

We use the Cleverhans library [Papernot et al., 2018]. In order to deal with this and in contrast to the results given in [Kannan et al., 2018] and [Engstrom et al., 2018], we run our attacks with multiple random restarts and report the adversarial accuracy over the most successful restarts.

### 2.1 Results on MNIST

The results of our evaluation on MNIST are given in Table 1. We find that when performing only a single restart of PGD, the model trained with LSQ provides an adversarial accuracy of 70.6%. By increasing the step size as well as the number of iterations and restarts, we can significantly reduce it to 5.0%. Following the same approach, we can reduce the adversarial accuracy for the model trained with CLP from 62.4% to 4.1%. For the adversarially trained models, the situation is different. Even our strongest attack could not reduce the accuracies of the models combining adversarial training with ALP below 89.9% and 85.7%. Further, the ALP model which was trained on clean samples only, achieves a comparable accuracy of 88.9% against our strongest attack, giving an improvement of 1.7% over the adversarial accuracy of the models trained using adversarial training only.

| Model | Accuracy | Adversarial accuracy ($L_\infty$ PGD, $\epsilon = 16.0$) |
|-------|----------|------------------------------------------------------|
| Plain | 99.2% | 0.0% |
| CLP ($\lambda = 0.5$) + $\lambda(0, 0.5)$ | 98.8% | 0.0% |
| LSQ ($\lambda = 0.5$) + $\lambda(0, 0.5)$ | 98.8% | 0.0% |
| Plain + ALP ($\lambda = 1$) | 99.5% | 96.0% |
| 50% AT + ALP ($\lambda = 1$) | 98.3% | 97.2% |
| 50% AT | 99.1% | 95.6% |
| 100% AT + ALP ($\lambda = 1$) | 98.4% | 96.6% |
| 100% AT | 98.9% | 95.2% |
| 50% AT + ALP ($\lambda = 0$) | 99.0% | 96.1% |
| 50% AT | 99.1% | 95.6% |
| 100% AT + ALP ($\lambda = 0$) | 98.5% | 96.0% |
| 100% AT | 98.9% | 95.2% |

Table 2: Clean and adversarial accuracy against different attacks on CIFAR-10. Adversarial accuracy is evaluated against PGD attack using $\epsilon = 16$ with different number of iterations $n$, step sizes $\epsilon_i$, and restarts $r$. AT denotes adversarial training.
Table 3: Clean and adversarial top-1 accuracy against different PGD attacks using random target label on Tiny ImageNet. Adversarial accuracy is evaluated against PGD attack using $\epsilon = 16$ with different number of iterations $n$, step sizes $\epsilon_i$, and restarts $r$. The suffix LL denotes that adversarial examples used for training were crafted with least-likely target.

| Model | Accuracy | Adversarial accuracy ($L_\infty$ PGD, $\epsilon = 16.0$, random target) |
|-------|----------|--------------------------------------------------------------------------|
| Plain | 53.0%    | 3.9% 0.4% 0.4%                                                             |
| CLP ($\lambda = 0.25$) + $N(0, 0.06)$ | 48.5% 12.2% 1.7% | 49.4% 12.8% 1.3% |
| LSQ ($\lambda = 0.05$) + $N(0, 0.06)$ | 53.5% 17.2% 2.0% | 72.0% 31.8% 10.0% |
| Plain + ALP LL ($\lambda = 0.5$) | Finetuned Plain + ALP LL ($\lambda = 0.5$) | 53.5% 17.2% 2.0% | 72.0% 31.8% 10.0% |
| 50\% AT LL | 46.3% 25.1% 13.5% | 50\% AT LL + ALP LL ($\lambda = 0.5$) | 45.2% 26.3% 18.7% |
| 100\% AT LL | 41.2% 25.4% 19.6% | 100\% AT LL + ALP LL ($\lambda = 0.5$) | 37.0% 25.4% 19.6% |

Figure 2: The histogram of the loss values for a single point for 10000 random restarts of the PGD attack for CLP model trained on MNIST. We show 4 typical cases, which illustrate that there exist points for which the loss can be successfully maximized only with a good starting point. The vertical red line denotes the loss value of $-\ln(0.1)$, which guarantees that for this and higher values of the loss an adversarial example is found. More histograms can be found in Figure 7 in the appendix.

2.2 Results on CIFAR-10

Results on CIFAR-10 can be found in Table 2. Again, we find that both CLP and LSQ do not give actual robustness and the accuracy of the models trained using either CLP or LSQ can be reduced to 0.0\% and 1.7\%, respectively. This clearly shows that the robustness of 27.0\% of the LSQ model against the baseline PGD attack is misleading. On the other hand, we find that ALP can lead to some robustness even against our strongest PGD attack and outperforms adversarial training by 3.4\%, which is in agreement with our findings on MNIST.

2.3 Results on Tiny ImageNet

The results of our experiments on Tiny ImageNet are given in Table 3. We again find that both CLP and LSQ models do not provide actual robustness. Next, we analyze the model provided by [Kannan et al., 2018] “Finetuned Plain + ALP LL ($\lambda = 0.5$)”, which was fine tuned from a model trained on full ImageNet. Our results show that we can reduce the adversarial accuracy from 31.8\% to 3.6\%. This suggests that this model does not provide state-of-the-art robustness against white-box PGD attacks. However, when combined with training only on targeted adversarial examples, ALP marginally improves the adversarial accuracy over plain targeted adversarial training by 0.2\% while sacrificing 4.2\% of clean accuracy. In additional experiments (see Tables 4 and 5 in the appendix) we confirm the hypothesis of [Engstrom et al., 2018] that adversarial training using untargeted PGD attack leads to improved adversarial accuracy.
3 Conclusions

We perform an empirical evaluation investigating the robustness of logit pairing methods introduced by Kannan et al. [2018]. We find that both CLP and LSQ deteriorate the input space loss surface and make crafting adversarial examples difficult, without providing actual robustness. This suggests that the current practice of evaluating against PGD attack with default settings can be misleading and one should consider performing many random restarts, which helps to reliably find adversarial examples in such cases (Figure 2). Finally, we show that the ALP models of Kannan et al. [2018] do not improve drastically over adversarial training alone.

Acknowledgments. We would like to thank Michael Hedderich and Francesco Croce for their valuable feedback on drafts of this paper. Marius Mosbach acknowledges partial support by the German Research Foundation (DFG) as part of SFB 1102.

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Appendix A

Additional results and visualizations

| Model | Accuracy | Adversarial accuracy ($L_\infty$ PGD, $\epsilon = 16.0$, random target) |
|-------|----------|--------------------------------------------------------------------------------------------------|
| Plain | 53.0%    | 3.9% 0.4% 0.4%                                    |
| CLP ($\lambda = 0.25$) + $N(0, 0.06)$ | 48.5% | 12.2% 1.7% 0.7%                                    |
| LSQ ($\lambda = 0.05$) + $N(0, 0.06)$ | 49.4% | 12.8% 1.3% 0.8%                                    |
| Plain + ALP ($\lambda = 0.5$) | 53.3% | 17.4% 3.1% 1.4%                                    |
| Plain + ALP LL ($\lambda = 0.5$) | 51.5% | 12.2% 0.9% 0.3%                                    |
| Finetuned Plain + ALP LL ($\lambda = 0.5$) | **72.0%** | **31.8%** 10.0% 3.6%                              |
| $\epsilon_1 = 2.0, n = 10, r = 1$ | $\epsilon_3 = 4.0, n = 100, r = 1$ | $\epsilon_3 = 4.0, n = 100, r = 100$ |
| $\epsilon_1 = 2.0, n = 10, r = 1$ | $\epsilon_3 = 4.0, n = 100, r = 1$ | $\epsilon_3 = 4.0, n = 100, r = 100$ |

Table 4: Clean and adversarial top-1 accuracy against different PGD attacks using random target label on Tiny ImageNet. The suffix LL denotes that adversarial examples used for training were crafted with least-likely target. If omitted, adversarial examples were crafted using untargeted attack.

| Model | Accuracy | Adversarial accuracy ($L_\infty$ PGD, $\epsilon = 16.0$, least-likely target) |
|-------|----------|--------------------------------------------------------------------------------------------------|
| Plain | 53.0%    | 5.6% 0.1% 0.0%                                    |
| CLP ($\lambda = 0.25$) + $N(0, 0.06)$ | 48.5% | 11.6% 0.9% 0.0%                                    |
| LSQ ($\lambda = 0.05$) + $N(0, 0.06)$ | 49.4% | 11.5% 0.3% 0.0%                                    |
| Plain + ALP ($\lambda = 0.5$) | 53.3% | 23.7% 6.2% 1.3%                                    |
| Plain + ALP LL ($\lambda = 0.5$) | 51.5% | 17.0% 1.9% 0.0%                                    |
| Finetuned Plain + ALP LL ($\lambda = 0.5$) | **72.0%** | **38.3%** 14.8% 4.2%                              |
| $\epsilon_1 = 2.0, n = 10, r = 1$ | $\epsilon_3 = 4.0, n = 100, r = 1$ | $\epsilon_3 = 4.0, n = 100, r = 100$ |
| $\epsilon_1 = 2.0, n = 10, r = 1$ | $\epsilon_3 = 4.0, n = 100, r = 1$ | $\epsilon_3 = 4.0, n = 100, r = 100$ |

Table 5: Clean and adversarial top-1 accuracy against different PGD attacks using least-likely target label on Tiny ImageNet. The suffix LL denotes that adversarial examples used for training were crafted with least-likely target. If omitted, adversarial examples were crafted using untargeted attack.
with many random restarts of PGD attack. Our quantitative results in Table 1 show that this model is resistant even to our strongest attack for the first 8 test examples. The loss surface has a local maximum at the input point. At the same time, our quantitative results in Table 1 show that this model does not provide actual robustness and a gradient-based PGD attack must use many random restarts to successfully craft adversarial examples.

![Figure 3: Input loss surfaces of CLP model on MNIST in a random subspace with $\epsilon = 38.25$ for the first 8 test examples. The loss surface contains many local maxima and hence, makes gradient-based attacks much more difficult. This is in line with our quantitative results in Table 1, showing that this model does not provide actual robustness and a gradient-based PGD attack must use many random restarts to successfully craft adversarial examples.](image)

![Figure 4: Input loss surfaces of Plain + ALP model on MNIST in a random subspace with $\epsilon = 38.25$ for the first 8 test examples. The loss surface has a local maximum at the input point. At the same time, our quantitative results in Table 1 show that this model is resistant even to our strongest attack with many random restarts of PGD attack.](image)
Figure 5: Input loss surfaces of LSQ model on CIFAR-10 in a random subspace with $\epsilon = 16.0$ for the first 8 test examples. For some points, the loss surface contains local maxima and thus may pose a problem for gradient-based attacks. This is in line with our quantitative results in Table 2, showing that this model does not provide actual robustness, and a successful gradient-based attack must use many random restarts of PGD attack.

Figure 6: Input loss surfaces of Plain + ALP model on CIFAR-10 in a random subspace with $\epsilon = 16.0$ for the first 8 test examples. The loss surface may be suitable for gradient descent. This is in line with our quantitative results in Table 2 showing that there is only a small benefit in using random restarts of PGD attack.
Figure 7: The histogram of the loss values across 10000 random restarts of the PGD attack on CLP model trained on MNIST for the first 16 test examples. The vertical red line denotes the loss value of $-\ln(0.1)$, which guarantees that for this and higher values of the loss an adversarial example is found.