Detuning control of vortex oscillations in light matter coupling

Amir Rahmani

Department of Physics, Yazd university, Yazd, Iran

We study analytically the dynamics of vortices in strongly-coupled exciton–photon fields in the presence of energy detuning. We derive equations for the vortex core velocity and mass, where they mainly depend on Rabi coupling and the relative distance between the vortex cores in photon and exciton fields, and as the result core positions oscillate in each field. We use Magnus force balanced with a Rabi induced force to show that the core of the vortex behaves as an inertial-like particle. Our analysis reveals that the core is lighter at periphery of the beam and therefore it is faster at that region. While detuning induces oscillations in population imbalance of normal modes through relative phase between coupled fields, in the presence of topological charges detuning can control the orbital dynamics of the cores. Namely, it brings the vortex core to move on larger or smaller orbits with different velocities, and changes angular momentum and energy content of vortex field.

I. INTRODUCTION

Rotation of an object is extremely fascinating and challenging throughout the ages. Examples of angular motions are given in classical physics [1, 2] and engineering [3]. Particular example is a swirling fluid [4], where the term vortex is used to denominate a revolving flow around a central region. Basically, vortices occur in the wake of fluid motion where viscosity is the underlying physical mechanism. To quantify the rotation of a fluid one can introduce the concepts of vorticity, defined as the curl of velocity field, and circulation. The later is defined as the contour integral of the tangential velocity component, and is connected to the total vorticity by Stokes’s theorem.

In contrast to a conventional fluid that can take an arbitrary circulation, a Bose–Einstein condensate (BEC) is constrained to carry quantized circulation. Such an intrinsic property is typical for the complex-valued wavefunction, where its density profile can include a depleted region surrounded by quantized circulations. Flow in condensate has an intrinsic property, namely, it is incompressible, i.e., there is no viscosity to bring the fluid to stop. Mathematically, a condensate with vortex is not simply-connected and then Stokes’s theorem of classical regime does not apply. The idea of quantized vortices was introduced for the first time in studying the superfluid Helium, and was extended to other condensates. Examples of quantized vortices are reported in superconductor [5], superfluid [6], atomic BEC [7], and polariton [8, 9].

Quantized vortices are recognized by a null–density region where phase wrapping is quantized in units of $2\pi$, and then fields with phase defect carry quantized orbital angular momentum [10]. In this study we emphasize on the interplay between regime of strong coupling and topology of the fields when there is an energy detuning. To this end, we use microcavity polariton [11], a quasiparticle resulting from strong coupling between microcavity photon and exciton quantum well. Polaritons have some essential attributes. They have very light effective mass (due to photonic component), interact through excitonic part, and the photon and exciton components can also be easily detuned. Condensation phase of polaritons was reported in 2006 [12] which has casted light on many fundamental researches, including quantum hydrodynamics [13, 14], superfluidity [8, 15], Josephson effects [16], and vortices [8, 17–20], among others. The regime of strong coupling appears as the Rabi oscillations between photonic and excitonic components [21], and equivalently as the Rabi splitting between upper and lower polariton branches in dispersion diagrams.

Regarding fields that are free from phase defects, energy detuning between modes has marked effects on the dynamics of the fields: nonzero detuning triggers oscillations in relative phase resulting in an oscillation in population imbalance [22, 23]. Particular example is shown in Fig. 1. Here, population imbalance between normal modes $\rho \equiv (N_{ph} - N_{ex})/(N_{ph} + N_{ex})$, with $N_c$ and $N_x$ for population in photon and exciton fields respectively, is initially zero: $N_{ph}(0) = N_{ex}(0)$, and for zero energy detuning there is no oscillations in $\rho$, although there would be oscillation in populations due to Rabi coupling and imbalance in initial relative phases of normal modes. However, by increasing or decreasing the detuning there exist oscillations. With detuning population imbalance has a nonzero mean value, which increases with detuning; it also changes the frequency of oscillation.

In this work we study analytically the vortex dynamics.

FIG. 1. (a): Dispersion of polariton for positive detuning, when it shifts the branches upwardly. (b): Effect of detuning on population imbalance $\rho = (N_{ph} - N_{ex})/(N_{ph} + N_{ex})$, for three detuning energies: $\delta = 0.3\text{meV}$ in red, $\delta = 0\text{meV}$ in cyan, and $\delta = 1\text{meV}$ in blue color. $g$ is the Rabi frequency.
in regime of strong coupling in the presence of energy detuning. We restrict our analysis to an ideal polariton gas in Hamiltonian regime, where Rabi coupling is the predominant interaction in the system and it drives vortex core to move periodically. To describe the core motion, we exploit Magnus force [24, 25] balanced with a force originated from Rabi coupling. This provides an expression for the initial mass of the vortex core, which changes in time and is in order between 10^{-13} kg and 10^{-12} kg, depending on the position of the core. Such behavior is particular for polariton system, as its binary nature gives the core a time dependent attribute. Based on our analysis we demonstrate that the core behaves as an inertial particle, that is, it has position, velocity, and mass. We show that upon changing detuning, core moves faster or slower which is the result of particle transfer between the two coupled fields; then detuning addresses a controlling mechanics on the dynamics of the core. Also, it controls the angular momentum content of the field.

Our paper is organized as follows: Section II presents the relevant theoretical description of polariton BECs based on its photonic and excitonic components. In section III we show the oscillation of the core and its components. In section IV we focus on the effect of detuning on quantum averages. Finally we present conclusion remarks in section V.

II. THEORY

Polariton in exciton–photon basis is an example of a two–component Bose system. Equations of motions for fields in binary Bose system are described in the formalism of quantum field theory [26], where we have the following Hamiltonian for our study:

\[ \hat{H} = \frac{\hbar^2}{2 m_{ph}} |\nabla \psi|^2 + E_{ph} \psi^\dagger \psi + \frac{\hbar^2}{2 m_{ex}} |\nabla \varphi|^2 + E_{ex} \varphi^\dagger \varphi + \hbar g (\psi^\dagger \varphi + \varphi^\dagger \psi) + \hbar \Omega |\varphi|^4. \]

(1)

Here, the two first terms describe the free evolution of photon (\(\psi\)) and exciton (\(\varphi\)) fields, respectively; the first term in second line describes Rabi coupling between photon and exciton fields, which transfers excitations between the two fields with a Rabi frequency \(g\); the last term accounts for self–interaction energy in exciton field, with \(\hbar \Omega\) as a exciton–exciton interaction strength.

A. Main Equations

Based on Hamiltonian in Eq. (1), the dynamics of 2D interacting polaritons is described by a set of coupled Schrödinger equation and Gross Pitaevskii equation for the photon \(\psi\) and exciton \(\varphi\) fields, respectively:

\[ i \hbar \partial_t \begin{pmatrix} \psi(x, y, t) \\ \varphi(x, y, t) \end{pmatrix} = \mathcal{L} \begin{pmatrix} \psi(x, y, t) \\ \varphi(x, y, t) \end{pmatrix}, \]

(2)

where

\[ \mathcal{L} = \left( -\frac{\hbar^2 \nabla^2}{2 m_{ph}} + E_{ph} - \frac{\hbar^2 \nabla^2}{2 m_{ex}} + E_{ex} + \hbar \Omega |\varphi|^2 \right). \]

Here, \(m_{ex} = (m_e + m_h)m_0\) stands for the exciton mass and \(m_{ph} \approx n \pi \hbar / (\ell_c) = (n^2 / c^2)(E_x + \delta)\) is the effective photon mass , with \(E_{ph} \approx \hbar c n / (\ell_c)\) where \(\ell_c\) is the microcavity length. \(E_{ex}\) is the exciton energy that is given by \(E_{ex} = 13.6 eV \cdot m_{ex}\), where \(\mu\) is the reduced effective mass of the exciton and \(m_0\) is the bare electron mass.

Theoretical works [27] have found the self–interaction strength as \(\hbar \Omega \approx 6 E_x a_B\), where \(a_B\) is the exciton Bohr radius. Calculating \(\hbar \Omega / (\hbar^2 / 2m_p)\), with \(m_p \approx 10^{-4} m_0\) as the polaron mass, one finds polaritons in the regime of weakly interacting limit. However, Experimental evidence [28, 29] has recently informed us that the interaction constant \(\hbar \Omega\) could be in order of several meV \(\cdot \mu m\), which putting polaritons in the regime of strong self–interaction. In this work we aim at considering the effect of energy detuning on the dynamics of coupled fields with phase defect in each, while emphasis is on the Rabi coupling interaction; Such regime of vortex dynamics has been reported recently in Ref. [9], which could be justified in the realm of linear dynamics of coupled fields; then hereafter we ignore the effect of self–interaction energy in our calculations.

B. Initial states

Our objective is to study the dynamics of vortices in the regime of strong coupling. To this aim we introduce vortices in initial conditions as:

1. In the photon field: a superposition of a topological charge \(TC = 1\) Gaussian with a \(TC = 0\) Gaussian, the latter centered at \((x_c(0), y_c(0))\).

2. In the exciton field: a \(TC = 1\) Gaussian, with its core centered at \((x_e(0), y_e(0))\).

This reads

\[ \psi_0 \equiv \psi(x, y, t = 0) = \frac{e^{-(x^2+y^2)/2w^2}}{w \sqrt{\pi (w^2 + |z_c(0, 0, 0)|^2)}} \cdot \]

(4a)

\[ \varphi_0 \equiv \varphi(x, y, t = 0) = \frac{e^{-(x^2+y^2)/2w^2}}{w \sqrt{\pi (w^2 + |z_c(0, 0, 0)|^2)}} \cdot \]

(4b)

where \(z_j(x, y, t) = x - x_j(t) + i(y - y_j(t))\) with \(j = \{c, e\}\). Here, each field is normalized to one, and \(w\) control the spot size of Gaussian. It is worth to mention that one can
consider coupled fields with different topological charge in each, however, we found out initial states \( w \)'s are the simplest choice to show the physics. Also, we found out the initial condition with photon field only does not develop any dynamics in the vortex core.

**III. OSCILLATION OF THE CORE**

In this section we discuss on solutions for main equation \( \psi \), when \( \Omega = 0 \). Despite of the fact that equations are linear and simple we will see that the solutions are not trivial.

### A. Approximate Solution

There is no close form solution for Eq. \( \psi \), even in the absence of nonlinear term. However, one can find approximate or series solutions, for example, by employing spectral method \[30\] or homotopy analysis method \[31\]. In this paper we use spectral method, where we use Hermite \( H_n(x) \) function as basis functions; then we expand wavefunctions \( \psi(x,y) \) and \( \varphi(x,y) \) as:

\[
\psi(x,y) = \sum_{n,m=0} u_{n,m}(x,y) a_{n,m}(t) , \quad (5a) \\
\varphi(x,y) = \sum_{n,m=0} u_{n,m}(x,y) b_{n,m}(t) , \quad (5b)
\]

where we introduce basis function \( u_{n,m}(x,y) \) as:

\[
u_{n,m}(x,y) = c_{n,m} H_n(x/w) H_m(y/w) e^{-(x^2+y^2)/(2w^2)} , \quad (6)
\]

with \( c_{n,m} = \frac{1}{w^{2n+m+1} \Gamma(n+1) \Gamma(m+1)} \). Here, \( \Gamma(n) \) stands for gamma function. Coefficients \( a_{n,m} \) and \( b_{n,m} \) satisfies the following initial value differential equations:

\[
\frac{da_{n,m}}{dt} = -\frac{\hbar^2}{2m_e w^2} (-1+n+m)a_{n,m} + \sqrt{(n+1)(n+2)} a_{n+2,m} + \sqrt{(m+1)(m+2)} a_{n,m+2} \\
\frac{db_{n,m}}{dt} = -\frac{\hbar^2}{2m_e w^2} (-1+n+m)b_{n,m} + \sqrt{(n+1)(n+2)} b_{n+2,m} + \sqrt{(m+1)(m+2)} b_{n,m+2}
\]

These provide an infinite hierarchy of equations, coupled by Rabi and kinetic energy terms. One way to truncate the hierarchy is to note that for \( w^2 \gg \xi^2 \), only those \( a_{n,m} \)'s and \( b_{n,m} \)'s that have nonzero initial values are several orders larger than other coefficients with zero initial values; it is the case when \( w \) is in order of several micrometers, which is a typical choice in experimental reports \[9\]. Then we approximate the wavefunctions as:

\[
\psi(x,y,t) \approx a_{00}(t) u_{00} + a_{1,0} u_{1,0} + a_{0,1} u_{0,1} , \quad (8a) \\
\psi(x,y,t) \approx b_{00}(t) u_{00} + b_{1,0} u_{1,0} + b_{0,1} u_{0,1} . \quad (8b)
\]

equations for \( a_{0,0}(t) \), \( a_{0,1}(t) \), \( a_{1,0} \), \( b_{0,0}(t) \), \( b_{0,1}(t) \), and \( b_{1,0}(t) \) are given in appendix \[A\].

### B. Dynamics of the core

Examples of solutions for zero detuning are shown in Fig. \[2\]. Cores are initially located at \( x_c(0) = -aw, y_c(0) = 0 \) in photon field and at \( x(0) = 0, y_x(0) = 0 \) in exciton field. While they move periodically on their orbits, the background densities oscillate similarly. Here fields are normalized initially and they remain normalized in time. This implies that there is no particle exchanging between coupled fields, however, background densities are oscillating while the core moves. Motion of a core is induced by the distanced presence of the other core, by ambient density of the other field, and by Rabi coupling. The combined effect is that the vortex core moves with non-constant speed.

We now explain the dynamics based on our main equations. One notes that the core in each field never disappears, which implies that the solutions can be written as the product of a background density and a complex time dependent function \( z_j(t) = x - x_j(t) + iy - y_j(t) \) for depleted point in the fields, as we have:

\[
\psi = \rho_{ph}(x,y,t) e^{i \phi_{ph}(x,y,t)} z_c(t) , \quad (9a) \\
\varphi = \rho_{ex}(x,y,t) e^{i \phi_{ex}(x,y,t)} z_x(t) , \quad (9b)
\]

By adding them in Eq. \( \psi \), one can find two central equa-
FIG. 2. Oscillation of the core in photon (upper panels) and in exciton (lower panels) fields, which is shown in one period of motion. Wavepacket is initially normalized to one and energy detuning is zero, which implies no particle transformation between the two fields, and they remain normalized in time. Each fields carries winding number of $l = 1$. Photon field is initially deformed by displacing core to a position far from origin, while core in exciton field is located at the origin. In dynamics, the ambient density oscillates similarly. Core dose not move with constant speed. We also show the stream lines of superfluid velocity of each field, as circles centered on vortex core position. Here, we used: $\hbar g = 2.75 \text{meV}$, $a = 1.2$, $w = 25 \mu m$.

Evaluations for core velocities:

$$
\mathbf{v}_e = \frac{\hbar}{m_{ph}}((\nabla \phi_{ph} - \hat{k} \times \nabla \ln \rho_{ph})|_{x=x_c, y=y_c}
+ g \rho_{ex} (j \Delta r \cdot \hat{S}_+ + i \hat{k} \times \Delta r \cdot \hat{S}_+) ,
$$

$$
\mathbf{v}_x = \frac{\hbar}{m_{ex}}((\nabla \phi_{ex} - \hat{k} \times \nabla \ln \rho_{ex})|_{x=x_c, y=y_c}
- g \rho_{ph} (j \Delta r \cdot \hat{S}_- + i \hat{k} \times \Delta r \cdot \hat{S}_-) ,
$$

where $\Delta r = r_e - r_x$ gives the relative position vector of cores, and $\hat{S}_+ = \cos(\phi_{ph} - \phi_{ex}) \hat{i} \pm \sin(\phi_{ph} - \phi_{ex}) \hat{j}$. These equations have the same mathematical form even in the presence of self-interactions and external potentials, although with potential the dynamics of the cores have different nature. Also, we note that there are three factors that determine the variation of velocities in time; one factor depends on the distance between the cores in each field which is followed by Rabi frequency $g$. The other two factors depend on the gradient of ambient phase and density of each field; these two factors come from kinetic energy through dependency on $\hbar/m_{ph,ex}$. Comparing $\hbar/m_c w$ and $gw$ to approximate the order of factors in cores velocity, we find out based on experimental evidences that $g \gg \hbar/m_c w^2$, which implies that Rabi-induced velocity is dominate contribution to the vortex core motion in polariton field. For $g = 0$, each field turns to a free field, and there is no motion for the core. In our case there is no external potential, and motion of a core is mediated by interplay between Rabi coupling and topology of the deformed field when a core is displaced to a point far from center of the field. As the core is moving on a curved path it has an angular velocity $\mathbf{\omega}$ which is given:

$$
\mathbf{\omega}_j = \frac{\mathbf{r}_j \times \mathbf{v}_j}{x_j^2 + y_j^2} ,
$$

with $j = \{c, x\}$, and $\mathbf{r}_j$ denotes the position vector of the core in photon ($j = c$) or in exciton ($j = x$).

Till now we show that a core has both linear and angular velocities and positions, and its cinematic is then transparent. To address the dynamics of a core we also need to know about its cinetic, namely, force associated to the core. In classical fluid, circulation of one vortex can derive any other vortex at a given distance, which results an effective force perpendicular to the velocity of
the vortex core\cite{32}. Expanding this picture to a quantum fluid, it was shown that such force is connected to the gradient of total energy of the system for BEC in harmonic trap\cite{33,34}, and ambiguously for two unbounded coupled BECs\cite{32} in nonlinear regime. For polariton in linear regime, one can show that the predominated energy scale in the system is Rabi energy which is defined as:

$$E_R(x_c, y_c, x, y) = 2\hbar g\text{Re}(\psi|\varphi),$$  \hspace{1cm} (12)$$

and depends on the positions of the vortex cores. Taking gradient with respect to the core positions one can introduce a force through $F_R = -\nabla E_R$. For core in photon field, $x$ and $y$ components of this force has a linear dependency on $x_c, y_c$ through an equation like $g(\alpha_1(t)x_c + \alpha_2(t)y_c)$, while for core in exciton field it behaves like $g(\beta_1(t)x_c + \beta_2(t)y_c)$, where $\alpha_i$ and $\beta_i$ are some real coefficients depended on parameters of the system. On the other hand, there would be a Magnus force that depends on the velocity of the vortex core. To find this, we multiply velocities in Eqs. (10) from the left respectively to $-2\pi\hbar\rho_{ph}\hat{k}$ and $-2\pi\hbar\rho_{ex}\hat{k}$, and we have:

$$F_{M_{ph}}^{Mag} + F_{M_{ph}}^{Rabi} = F_{M_{ph}}^{Kin},$$  \hspace{1cm} (13a)$$

$$F_{M_{ex}}^{Mag} + F_{M_{ex}}^{Rabi} = F_{M_{ex}}^{Kin},$$  \hspace{1cm} (13b)$$

where we introduce the following expressions: for the core in photon field:

$$F_{Mag}^{ph} = \rho_{ph}K_{ph} \times (v_c - \frac{\hbar}{m_{ph}}\nabla\varphi_{ph}),$$  \hspace{1cm} (14a)$$

$$F_{Kin}^{ph} = -\frac{\hbar\rho_{ph}}{m_{ph}}K_{ph} \times (\hat{k} \times \nabla\ln\rho_{ph}),$$  \hspace{1cm} (14b)$$

$$F_{Rabi}^{ph} = g\rho_{ex}K_{ph} \times (j\Delta r \cdot \hat{S}_+ + \hat{i} \hat{k} \times \Delta r \cdot \hat{S}_-),$$  \hspace{1cm} (14c)$$

and for the core in exciton field:

$$F_{Mag}^{ex} = -\rho_{ex}K_{ex} \times (v_x - \frac{\hbar}{m_{ex}}\nabla\varphi_{ex}),$$  \hspace{1cm} (15a)$$

$$F_{Kin}^{ex} = \frac{\hbar\rho_{ex}}{m_{ex}}K_{ex} \times (\hat{k} \times \nabla\ln\rho_{ex}),$$  \hspace{1cm} (15b)$$

$$F_{Rabi}^{ex} = -g\rho_{ph}K_{ex} \times (j\Delta r \cdot \hat{S}_+ + \hat{k} \times \Delta r \cdot \hat{S}_+),$$  \hspace{1cm} (15c)$$

where $K_j = 2\pi\hbar\hat{k}$. Here, we have a balance between Magnus force $F_{Mag}^{ph}$ added to a Rabi mediated force $F_{Rabi}^{ph}$ and a gradient force $F_{Kin}^{ph}$. Later force has an origin in quantum kinetic energy or zero point energy, and is related to the quantum pressure. Examining the orbital motion of the core, as it is shown in Fig. (2), One can conclude that a possible force that derives the core should depend on the relative distance between the cores, as when $x_c(0) = x_c(0)$ and $y_c(0) = x_c(0)$ there is no motion for the core. This implies that any effective force $F_R$ related to the gradient of Rabi energy dose not reproduce expected dynamics of the vortex core in our binary system.

Based on the above equations for forces, we can have growing understanding about an effective mass associated to a core: To end this, we note that the application of a force normal to the velocity direction dose change the linear momentum of an object, and the rate of the change in momentum is equal to the applied force. This implies that there would be an rotational acceleration related to the cross product of linear and angular velocities, which in the case of core motion we have

$$F_c \equiv m_c\Delta\vec{v}_c \times \vec{v}_c = -\rho_{ph}K_{ph} \times \vec{v}_c,$$  \hspace{1cm} (16a)$$

$$F_x \equiv m_x\Delta\vec{v}_x \times \vec{v}_x = -\rho_{ex}K_{ex} \times \vec{v}_x,$$  \hspace{1cm} (16b)$$

where $m_c$ and $m_x$ are the core masses in photon and exciton fields, respectively, and we have:

$$m_c = \frac{-2\pi\hbar\rho_{ph}(x_c, y_c)}{\omega_c},$$  \hspace{1cm} (17a)$$

$$m_x = \frac{-2\pi\hbar\rho_{ex}(x_c, y_c)}{\omega_x},$$  \hspace{1cm} (17b)$$

which depend explicitly on angular velocity of the cores. As there is no dependency on Rabi frequency, one can be hopeful that even in the presence of self-interaction or external potentials, equations for masses $m_c$ and $m_x$ have the same mathematical form, and any effects associated with interactions and potential will be collected in angular frequencies of the cores and ambient densities.
Now we have a complete description of the core dynamics with its position, velocity, mass and force associated to the core. In Fig. 4(a) we show examples of the dynamical variables for zero detuning and for photon field. Core initially is located at (−aw, 0), indicated with a yellow point in Fig. 3(a). While core moves counterclockwisely, its speed |v_r| decreases to a minimum, when core is being positioned at origin, and then core speeds up as it approaches to its initial position. With such variation in speed, core decelerates in first half of its motion and then accelerates in the rest of its motion. Acceleration vector is pointed toward a point C inside the orbit, while Magnus force is pointed to opposite direction. This implies a negative mass, which is shown in Fig. 3(b) for one period of motion. Core is lighter when it takes its position on periphery of the ambient density. In our system there is no external potential, and such variation in inertial mass of the core originates exclusively from the binary nature of the fields. Certainly, with the presence of an external potential and/or self-interaction, the core mass behaves differently, depending on the interplay between Rabi energy and other energy scale of the system.

Now we turn to consider the effect of energy detuning. We first study orbital motions of the cores for different detunings, which are shown in Fig. 4(a) and (b) for photon and exciton fields, respectively. With nonzero detunings, which are shown in Fig. 4(a) and (b) for photon and exciton fields, respectively, with nonzero detunings in photon and exciton fields, respectively. With nonzero detunings in photon and exciton fields, respectively, with nonzero detunings, which are shown in Fig. 4(a) and (b) for photon and exciton fields, respectively. With nonzero detunings, which are shown in Fig. 4(a) and (b) for photon and exciton fields, respectively. With nonzero detunings, which are shown in Fig. 4(a) and (b) for photon and exciton fields, respectively.

IV. ANGULAR MOMENTUM AND ENERGY CONTENTS

Wavepacket with phase defect that rotates around the vortex core has nonzero value of angular momentum. The average angular momentum of the wavepacket Ψ is given by

$$\langle \hat{L}_z \rangle = -i\hbar \int r dr d\phi \langle \Psi | \frac{\partial}{\partial \phi} \Psi \rangle,$$

(18)

with \( r = \sqrt{x^2 + y^2} \) and \( \phi = \arg(x + iy) \) in polar coordinate. For initial wavepackets in Eqs. (4), one can find \( \langle \hat{L}_z \rangle_{ph}(t = 0) = \hbar w^2/(w^2 + |z_c|^2) \), and \( \langle \hat{L}_z \rangle_{ex}(t = 0) = \hbar w^2/(w^2 + |z_x|^2) \), respectively, for photon and exciton fields; then, with \( z_c \neq z_x \), corresponds to placing cores in different points in space, there is a nonzero initial imbalance in angular momentum \( \Delta l \equiv \langle \hat{L}_z \rangle_{ph} - \langle \hat{L}_z \rangle_{ex} \neq 0 \). This also changes the energy content of the coupled fields. To study it we first note that the kinetic energy is in order of \( \hbar g/2w^2 \) for photon filed and of \( (m_{ph}/m_{ex})\hbar g/2w^2 \) for exciton field, and for \( w \) equal to several ξ, these are at least two orders less than the Rabi energy that is proportional to \( \hbar g \); this means that one can neglect kinetic energy for \( w \) in the range of several ξ. For zero detuning, Rabi energy is constant of the motion and is given in \( t = 0 \) with \( E_R(t = 0) = 2\hbar g(w^2 + r_c \cdot r_c)/\sqrt{(w^2 + |z_c|^2)(w^2 + |z_x|^2)} \). Configuration with \( r_c = r_x \), is unstable to the displacement of the cores, as by increasing the distance between the two cores the energy of the system decreases correspondingly. One can deduce that states with cores at distances are more favorable than \( r_c = r_x \), corresponding to fields being at rest, and this decreasing in energy as well as inducing the nonzero \( \Delta l \), will be realized through the rotation of the vortex cores.

We describe the time evolution of energy and angular momentum in Fig. 5 for some negative and positive detuning. One can see in Fig. 5(a) the time evolution of...
FIG. 5. (a) shows the imbalance in angular momentum: \( \Delta l = (L_x)_v - (L_x)_e \). For positive detuning angular momentum content of photon field is larger. We see variation in period of oscillations with detuning. It is shown for \( \delta = 3\text{meV} \) in purple, \( \delta = 1\text{meV} \) in blue, \( \delta = 0\text{meV} \) in green, \( \delta = -1\text{meV} \) in orange, and \( \delta = -3\text{meV} \) in red. (b) Mean value of angular momentum for photon (blue—solid line) and exciton (red—dashed line) fields. For negative detuning, angular momentum is carried mostly by photon field. Mean value of population imbalance is also shown (in dark—thick solid line). In (c) we present the variations of Rabi energy in time for different detuning. In (d) we show population imbalance.

\( \Delta l \). There will be an exchange of angular momentum between components of coupled fields, which is periodic in time, as we expect form Rabi nature of the coupling. By placing the photon vortex core at a distance to the exciton vortex core, there exists a nonzero (here, negative) initial imbalance in angular momentum, as photon part has initially less rotational content than exciton part, and since the total angular momentum should be constant, this imbalance triggers oscillations in angular momentum. Depending on energy detuning there will be oscillations with different periods and behaviors. First we see that by increasing the detuning there is a decrease in period of oscillations; then, we see for positive detuning there will be longer time interval with positive imbalance in angular momentum, i.e., photon filed has more angular content in average. On the other hand, by negative detuning there would be an state with always \( \Delta l < 0 \) in time, which implies that rotational content of photon field are always less than the exciton field. In this case, the core will be faster is photon field, as it being pushed to periphery of the beam, while the core in exciton field is slower (see Fig. 4). For an object with positive mass, one expects that by increasing the angular momentum content, it goes faster, however, in the case of the core something reverse happens. Actually, a core is an absence of the mater, and it has a negative mass, which results in motion of the core toward regions of low density. It is shown in part (d) in Fig. 5 that for negative detuning the photon field has more population, and indeed the field has higher density in central region, which finally pushes the core with negative mass, to lower density region, i.e., periphery of the wavepacket. In part (b) we show the mean value of angular momentum (right y axis) and of population imbalance (left y axis). For positive detuning the population is concentrated in exciton side and at the same time photon field has more angular content. The reverse happens for negative detuning. One notes that, while the mean value of population imbalance is symmetric with respect to detuning variations, it is not the case for angular momentum content. It makes the negative detuning a special in dynamics of the vortex core. Indeed, if we imagine \( r_c = (-aw, 0) \) and \( r_c = (0, 0) \), one can show that for \( \delta \approx -\hbar g(a^2w^2 + \sqrt{1 + a^2})/w^2\sqrt{1 + a^2} \), there is no evolution for angular momentum in time, i.e., \( \Delta l = \text{cons.} \). Total energy content of the system \( E_T \approx E_R(t) + \delta N_{ph}(t) \) also changes with detuning, although it is the constant of the motion, i.e., it dose not change with time. Increasing \( |\Delta r| \) and applying detuning, one can set the energy of the system to a desire value. We present the time evolution of Rabi energy in part (c). Here, as we mentioned before, there is no oscillation in time for zero detuning, due to zero imbalance in population and normalization in wavepackets. Nonzero detuning will induce oscillations in Rabi energy. To understand this we first note that \( \partial_t E_R \propto -\delta \text{Im}(\langle \psi | \varphi \rangle) \) and \( \partial_t \text{Im}(\langle \psi | \varphi \rangle) \propto \delta E_R \); then as \( E_R(t = 0) \) is nonzero, any nonzero detuning will derive \( \langle \psi | \varphi \rangle \), and therefore there is an oscillation in relative phase \( \arg(\langle \psi | \varphi \rangle) \), which derives population imbalance; hence, nonzero detuning induces self trapping for both population and angular content of the coupled fields.

V. CONCLUSION

In conclusion, we studied the dynamics of the vortices in coupled exciton–photon field, where the binary nature of the coupling fields set the vortex cores into motion. We found out that the vortex core behaves like a particle as we described its position, velocity, and mass. We found the later based on Magnus force. Then, we showed the effect of energy detuning to control the dynamics of the vortex core. It provides controlling over both cinematic (positions & velocities) and cinetic (energy & angular momentum) of the field plus the core positioned in the field. This controlling over the vortex dynamics, that is special for polaritonic system, can have applications in many fields related to angular momentum in mater and light; in particular, memory reading and writing, and information processing with angular momentum.

It also could be used for manipulating and maintaining vortices in superconductor phases. [35] [37].
Appendix A: Details for Equations 8

We mentioned in the main text that equations of motions for $a_{n,m}$ and $b_{n,m}$ provide an hierarchy in general. To solve such systems of ordinary equations, one needs to truncate them based on some criterion. We found out that when $w$ is several order larger than $\xi$, we can approximate the solutions by keeping those terms that are initially nonzero, while removing other terms due to their small values. Our initial conditions in subsection II B give the following values for nonzero coefficients in $t = 0$:

\begin{align*}
a_{0,0}(0) &= \frac{-(x_0 + iy_0)}{\sqrt{w^2 + |z_0(0,0)|^2}}, \\
a_{0,1}(0) &= \frac{wv}{2}\sqrt{\frac{1}{w^2 + |z_0(0,0)|^2}}, \\
a_{1,0}(0) &= \frac{wv}{2}\sqrt{\frac{1}{w^2 + |z_0(0,0)|^2}}, \\
b_{0,0}(0) &= \frac{-(x_0 + iy_0)}{\sqrt{w^2 + |z_0(0,0)|^2}}, \\
b_{0,1}(0) &= \frac{wv}{2}\sqrt{\frac{1}{w^2 + |z_0(0,0)|^2}}, \\
b_{1,0}(0) &= \frac{iwv}{2}\sqrt{\frac{1}{w^2 + |z_0(0,0)|^2}},
\end{align*}

which yield the following relations:

\begin{align*}
a_{0,0}(t) &= \frac{1}{g_c} e^{-\frac{\mu}{4w}(g(1+m_0)+2(E_{ph}+E_{ex})w^2)} \left( -i(a_{0,0}(0)g + 2w^2(a_{0,0}(0) \frac{\delta}{\hbar} + b_{0,0}(0)(0)g)) \sin(\frac{gc}{4w^2}) + a_{0,0}(0)g_c \cos(\frac{gc}{4w^2}) \right), \\
a_{0,1}(t) &= \frac{1}{g_d} e^{-\frac{\mu}{4w}(g(1+m_0)+2(E_{ph}+E_{ex})w^2)} \left( -i \sin(\frac{gd}{2w^2})(a_{0,1}(0)(g + \frac{\delta}{\hbar} w^2) + 2gw^2b_{1,0}(0)) + a_{0,1}(0)g_d \cos(\frac{gd}{2w^2}) \right), \\
a_{1,0}(t) &= \frac{1}{g_d} e^{-\frac{\mu}{4w}(g(1+m_0)+2(E_{ph}+E_{ex})w^2)} \left( -i \sin(\frac{gd}{2w^2})(a_{1,0}(0)(g + \frac{\delta}{\hbar} w^2) + 2gw^2b_{1,0}(0)) + a_{1,0}(0)g_d \cos(\frac{gd}{2w^2}) \right), \\
b_{0,0}(t) &= \frac{1}{g_c} e^{-\frac{\mu}{4w}(g(1+m_0)+2(E_{ph}+E_{ex})w^2)} \left( i(b_{0,0}(0)g + 2w^2(b_{0,0}(0) \frac{\delta}{\hbar} - 2a_{0,0}(0)g)) \sin(\frac{gc}{4w^2}) + b_{0,0}(0)g_c \cos(\frac{gc}{4w^2}) \right), \\
b_{0,1}(t) &= \frac{1}{g_d} e^{-\frac{\mu}{4w}(g(1+m_0)+2(E_{ph}+E_{ex})w^2)} \left( i \sin(\frac{gd}{2w^2})(b_{0,1}(0)(g + \frac{\delta}{\hbar} w^2) - 2gw^2a_{0,1}(0)) + b_{0,1}(0)g_d \cos(\frac{gd}{2w^2}) \right), \\
b_{1,0}(t) &= \frac{1}{g_d} e^{-\frac{\mu}{4w}(g(1+m_0)+2(E_{ph}+E_{ex})w^2)} \left( i \sin(\frac{gd}{2w^2})(b_{1,0}(0)(g + \frac{\delta}{\hbar} w^2) - 2gw^2a_{0,1}(0)) + b_{0,1}(0)g_d \cos(\frac{gd}{2w^2}) \right),
\end{align*}

where $\delta = E_{ph} - E_{ex}$ and we introduce:

\begin{align*}
g_c &= \sqrt{g^2 + 4gw^2 \frac{\delta}{\hbar} + 4w^4(4g^2 + \frac{\delta}{\hbar})^2}, \\
g_d &= \sqrt{g^2 + 2gw^2 \frac{\delta}{\hbar} + w^4(4g^2 + \frac{\delta}{\hbar})^2}.
\end{align*}

We present in Fig. 6 some examples for variations of $a_{n,m}$ and $b_{n,m}$ coefficients for two values of $w = 30\xi$ and $w = \xi$. One notes that for $w = 30\xi$, we can safely keep $a_{0,0}$, $a_{0,1}$, and $a_{1,0}$, as they are four orders larger than...
other coefficients. However, with $w = \xi$, we need to take into account more terms for approximation of the solution.

[1] D. Kleppner and R. Kolenkow, *An Introduction to Classical Mechanics* (Cambridge University Press).
[2] P. G. Saffman, *Vortex Dynamics* (Cambridge University Press, 1992).
[3] J. L. Meriam and L. G. Kraige, *Engineering Mechanics-Dynamics* (Wiley, 2012).
[4] P. R. Childs, *Rotating Flow* (Elsevier, 2011).
[5] G. Blatter, M. V. Feigel’man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, *Rev. Mod. Phys.* **66**, 1125 (1994).
[6] A. J. Leggett, *Rev. Mod. Phys.* **71**, S318 (1999).
[7] M. R. Matthews, B. P. Anderson, P. C. Haljan, D. S. Hall, C. E. Wieman, and E. A. Cornell, *Phys. Rev. Lett.* **83**, 2498 (1999).
[8] K. G. Lagoudakis, M. Wouters, M. Richard, a. Baas, I. Carusotto, R. André, L. S. Dang, and B. Deveaux-Plédran, *Nat. Phys.* **4**, 706 (2008).
[9] L. Dominici, D. Colas, A. Gianfrate, A. Rahmani, C. S. Muoz, D. Ballarini, M. D. Giorgi, G. Gigli, F. P. Laussy, and D. Sanvitto, preprint at [https://arxiv.org/abs/1801.02580](https://arxiv.org/abs/1801.02580) (2018).
[10] A. M. Yao and M. J. Padgett, *Adv. Opt. Photonics* **3**, 161 (2011).
[11] A. V. Kavokin, J. J. Baumberg, G. Malpuech, and F. P. Laussy, *Microcavities* (Oxford University Press, 2016).
[12] J. Kasprzak, M. Richard, S. Kundermann, a. Baas, P. Jeambrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szymańska, R. André, J. L. Staehli, V. Savona, P. B. Littlewood, B. Deveaud, and L. S. Dang, *Nature* **443**, 409 (2006).
[13] A. Amo, S. Pigeon, D. Sanvitto, V. G. Sala, R. Hivet, I. Carusotto, F. Pisanello, G. Leménager, R. Houdré, E. Giacobino, C. Ciuti, and A. Bramati, *Science* **332**, 1167 (2011).
[14] S. Pigeon, I. Carusotto, and C. Ciuti, *Phys. Rev. B* **83**, 144513 (2011).
[15] A. Amo, J. Lefrère, S. Pigeon, C. Adrados, C. Ciuti, I. Carusotto, R. André, E. Giacobino, and A. Bramati, *Nat. Phys.* **5**, 805 (2009).
[16] M. Abbarchi, A. Amo, V. G. Sala, D. D. Solnyshkov, H. Flayac, L. Ferrier, I. Sagnes, E. Galopin, A. Lemaître, G. Malpuech and J. Bloch, *Nature* **9**, 275 (2013).
[17] K. G. Lagoudakis, T. Ostatnický, A. V. Kavokin, Y. G. Rubo, R. André, and B. Deveaux-Plédran, *Science* **326**, 974 (2009).
[18] D. Sanvitto, F. M. Marchetti, M. H. Szymańska, G. Tosi, M. Baudisch, F. P. Laussy, D. N. Krizhanovskii, M. S. Skolnick, L. Marrucci, A. Lemaître, J. Bloch, C. Tejedor, and L. Viña, *Nat. Phys.* **6**, 527 (2010).
[19] L. Dominici, G. Dagvadorj, J. M. Fellows, D. Ballarini, M. D. Giorgi, F. M. Marchetti, B. Piccirillo, L. Marrucci, A. Bramati, G. Gigli, M. H. Szymańska, and D. Sanvitto, *Sci. Adv.* **1**, e1500807 (2015).
[20] L. Dominici, R. Carretero-Gonzalez, A. Gianfrate, J. Cuevas-Maraver, A. S. Rodrigues, D. J. Frantzeskakis, G. Lerario, D. Ballarini, M. De Giorgi, G. Gigli, P. G. Kevrekidis, and D. Sanvitto, *Nature Communications* **9**, 1467 (2018).
[21] L. Dominici, D. Colas, S. Donati, J. P. Restrepo Cuartas, M. De Giorgi, D. Ballarini, G. Guirales, J. C. López Carreño, A. Bramati, G. Gigli, E. del Valle, F. P. Laussy, and D. Sanvitto, *Phys. Rev. Lett.* **113**, 226401 (2014).
[22] N. S. Voronova, A. A. Elistratov, and Y. E. Lozovik, *Phys. Rev. Lett.* **115**, 186402 (2015).
[23] Rahmani, Amir and Laussy, Fabrice P., *Scientific Reports* **6**, 28930 (2016).
[24] D. J. Thouless, P. Ao, and Q. Niu, *Phys. Rev. Lett.* **76**, 3758 (1996).
[25] P. Ao and D. J. Thouless, *Phys. Rev. Lett.* **70**, 2158 (1993).
[26] G. D. Mahan, *Many Particle Physics* (Plenum, 2000).
[27] F. Tassone and Y. Yamamoto, *Phys. Rev. B* **59**, 10830 (1999).
[28] Sun, Yongbao and Yoon, Yoseob and Steger, Mark and Liu, Gangqiang and Pfeiffer, Loren N. and West, Ken and Snobe, DavidW. and Nelson, Keith A., *Nature Physics* **13**, 870 (2017).
[29] I. Rosenberg, D. Liran, Y. Mazuz-Harpaz, K. West, L. Pfeiffer, and R. Rapaport, preprint at [https://arxiv.org/abs/1802.01123](https://arxiv.org/abs/1802.01123) (2018).
[30] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes* (Cambridge University Press, 2007).
[31] S. Liu, *Homotopy/Analysis Method in Nonlinear Differential Equations* (Springer, 2012).
[32] L. Calderaro, A. L. Fetter, P. Massignan, and P. Wittek, *Phys. Rev. A* **95**, 023605 (2017).
[33] B. Jackson, J. F. McCann, and C. S. Adams, *Phys. Rev. A* **61**, 013604 (1999).
[34] S. Donadello, S. Serafini, M. Tylutki, L. P. Pitaevskii, F. Dalfovo, G. Lamporesi, and G. Ferrari, *Phys. Rev. Lett.* **113**, 065302 (2014).
[35] T. Zhong, J. M. Kindem, J. G. Bartholomew, J. Rochman, I. Craiciu, E. Miyazono, M. Bettinelli, E. Cavalli, V. Verma, S. W. Nam, F. Marsili, M. D. Shaw, A. D. Beyer, and A. Faraon, *Science* **357**, 1392 (2017).
[36] J. Wang, J.-Y. Yang, I. M. Fazal, N. Ahmed, Y. Yan, T. C. H. Liew, *Phys. Rev. B* **90**, 014504 (2014).
[37] H. Sigurdsson, O. A. Egorov, X. Ma, I. A. Shelykh, and T. C. H. Liew, *Phys. Rev. B* **90**, 014504 (2014).
[38] J. Wang, J.-Y. Yang, I. M. Fazal, N. Ahmed, Y. Yan, T. C. H. Liew, *Phys. Rev. B* **90**, 014504 (2014).
[39] I. S. Veshchunov, W. Magrini, A. G. A. Godin, J.-B. Trebbia, A. I. Buzdin, P. Tamarit, and B. Lounis, *Nature Communications* **7**, 12801 (2016).