Large mass splittings between charged and neutral Higgs bosons in the MSSM

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Abstract

We show that large ($\gtrsim 100$ GeV) mass splittings between the charged Higgs boson ($H^\pm$) and the neutral Higgs bosons ($H^0$ and $A^0$) are possible in the Minimal Supersymmetric Standard Model (MSSM). Such splittings occur when the $\mu$ parameter is considerably larger than the common SUSY scale, $M_{SUSY}$, and have significant consequences for MSSM Higgs searches at future colliders.

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1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) is currently the leading candidate for physics beyond the Standard Model (SM). At the tree–level the Higgs sector of the MSSM takes the form of a two Higgs doublet model (2HDM), where the coefficients of the quartic scalar terms are functions of the electroweak gauge couplings. Five physical Higgs bosons are predicted: a charged pair ($H^+, H^-$), two CP–even scalars ($h^0, H^0$), and a CP–odd $A^0$. In the tree–level approximation the lightest Higgs $h^0$ satisfies $M_{h^0} \leq M_Z$, while $M_{H^\pm} \approx M_{H^0} \approx M_{A^0}$ for $M_{H^\pm} \gtrsim 200$ GeV. Only two parameters are needed to fully parametrize the tree–level potential, and these are usually taken to be $\tan \beta (= v_2/v_1$, where $v_i$ is the vacuum expectation value of a Higgs doublet) and one of $M_{H^\pm}, M_{A^0}$.

At the 1–loop level the coefficients in the Higgs potential receive contributions from virtual particle loops and thus become complicated functions of several (SUSY) parameters. In addition, new quartic scalar terms are generated. This changes the tree–level mass relationships, most notably weakening the above mass bound to $M_{h^0} \lesssim 130$ GeV. The near degeneracy relationship, $M_{H^\pm} \approx M_{A^0} \approx M_{H^0}$ for the large ($\geq 200$ GeV) mass region, is only slightly affected, resulting in mass splittings of $O(10)$ GeV. Recently there has been much interest in the 1–loop effective potential of the MSSM with unconstrained CP violating phases. It has been shown that the mass splitting between the two heavier neutral scalars (now mixed states of CP) can be increased up to 30 GeV for large phases.

In this paper we will show that large mass splittings between the charged and neutral Higgs bosons, $|M_{H^\pm} - M_{A^0, H^0}| \gtrsim 100$ GeV, are possible in the CP conserving MSSM in a previously ignored parameter space. These large splittings occur when $\mu$ is larger than the supersymmetric (SUSY) scale (defined as the arithmetic mean of the stop masses) by a factor of 4 or more. Although sizeable corrections to the tree–level mass sum rules can be implicitly found in previous works e.g. [8], we identify the parameter space for the largest splittings and discuss the phenomenological consequences. Importantly we show that this parameter space is consistent with current experiments.

Such splittings would break the commonly assumed degeneracy relation $M_{H^\pm} \approx M_{H^0} \approx M_{A^0}$ for $M_{H^\pm} \gtrsim 200$ GeV, which should no longer be taken as a prediction of the MSSM. The Higgs mass spectrum of the MSSM may resemble that of a general 2HDM or other extended (SUSY) Higgs sectors with scalar singlets etc. Knowledge of the maximum possible mass splittings among the Higgs bosons in the MSSM may be crucial in distinguishing different models at future colliders. In particular, measurements of the mass splittings provide important information on the structure of the underlying Higgs sector, especially since Higgs branching ratios (BR) and cross–sections are often very similar in many popular models. Moreover, new decays channels involving $H^\pm, H^0$ and $A^0$ would be open and may possess large branching ratios, thus affecting proposed MSSM Higgs search strategies at future colliders.

Our work is organized as follows. In Section 2 we outline our approach for evaluating the mass splittings. Section 3 presents our numerical results, while section 4 contains our conclusions.
2 Mass splittings in the MSSM

The most general CP violating Higgs potential of the MSSM may be described by the following effective Lagrangian:

$$\mathcal{L}_V = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + m_{12}^2 \Phi_1^\dagger \Phi_2 + m_{12}^* \Phi_2^\dagger \Phi_1 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2$$

$$+ \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1)^2$$

$$+ \lambda_7 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \lambda_8 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_2) + \lambda_9 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1)(1)$$

At tree–level $\lambda_1 \to \lambda_4$ are functions of the $SU(2)$ and $U(1)$ gauge couplings while $\lambda_5 = \lambda_6 = \lambda_7 = 0$. In the 1-loop effective potential all $\lambda_i$ receive sizeable corrections from the enhanced Yukawa couplings of the third generation squarks. In particular $\lambda_5 \neq \lambda_6 \neq \lambda_7 \neq 0$ and are complex in the presence of SUSY phases in $A_t, A_b$ and $\mu$. Explicit formulae may be found in [4].

The restricted form of the tree–level potential limits the possible mass splittings between $M_{H^\pm}, M_{H^0}$ and $M_A$. It can be shown that $M_{H^\pm}$ and $M_A$ are related by:

$$M_{H^\pm}^2 - M_A^2 = (\lambda_4/2 - Re(\lambda_5))v^2 \quad (2)$$

At tree–level $\lambda_4 = g_W^2/2$ and $\lambda_5 = 0$, and the above relationship reduces to the familiar sum rule

$$M_{H^\pm}^2 - M_A^2 = M_W^2 \quad (3)$$

With the current LEP bound of $M_A \geq 90$ GeV one finds $M_{H^\pm} - M_A \leq 30$ GeV, with approximate degeneracy for $M_A \geq 200$ GeV.

At the 1–loop level $\lambda_4$ and $\lambda_5$ may be written as (keeping the dominant terms):

$$\lambda_4 \approx \frac{g_W^2}{2} - \frac{3}{96\pi^2} \left[ h_t^4 \left( \frac{3|\mu|^2}{M_{SUSY}^2} - \frac{3|\mu|^2|A_t|^2}{M_{SUSY}^4} \right) + h_b^4 \left( \frac{3|\mu|^2}{M_{SUSY}^2} - \frac{3|\mu|^2|A_b|^2}{M_{SUSY}^4} \right) \right]$$

$$+ \frac{3}{8\pi^2} h_t^2 h_b^2 \left[ \frac{1}{2} X_{tb} \right] \quad (4)$$

$$\lambda_5 \approx \frac{3}{192\pi^2} \left[ h_t^4 \left( \frac{\mu^2 A_t^2}{M_{SUSY}^4} \right) + h_b^4 \left( \frac{\mu^2 A_b^2}{M_{SUSY}^4} \right) \right]$$

where $X_{tb}$ is a function of $A_t, A_b, M_{SUSY}$ and $\mu$. From now on we will focus on the case of $\mu, A_t, b$ being real, with mention given to the case of complex phases where appropriate.

It can be seen from eq. (4) that $\mu, A_t$ or $\mu, A_b \geq 4M_{SUSY}$ would overcome much of the suppression from the small coefficients, permitting $\lambda_4, \lambda_5 = \mathcal{O}(1)$. From eq. (2) this would give rise to large mass differences $M_{H^\pm} - M_A$.

One can see from eq. (2) that $M_{H^\pm} > M_A$ requires $\lambda_4 > \lambda_5$ while $M_A > M_{H^\pm}$ requires $\lambda_5 > \lambda_4$. We note that both terms in $\lambda_5$ are positive definite and the term proportional to $h_t^4$ will dominate unless $\tan \beta$ is large (which enhances $h_b$). In the expression for $\lambda_4$ there may be both constructive and destructive interference among the various terms and so the SUSY correction to the tree–level value may take either sign.
For the CP even Higgs bosons one has a mass matrix $\mathcal{M}_S$ which when diagonalized gives the mass eigenstates $h^0$ and $H^0$. Note that the 1–loop corrected $\mathcal{M}_S$ involves all $\lambda_i$, $i = 1 \rightarrow 7$, and its explicit form may be found in [4]. Of interest to us is the mass splitting between $M_{H^\pm}$ and $M_{H^0}$ which may be given approximately by:

$$M^2_{H^\pm} - M^2_{H^0} \sim v^2 \left(\frac{\lambda_4}{2} + \text{Re}\lambda_5 + \frac{2\text{Re}\lambda_6}{\tan\beta}\right). \quad (5)$$

Recently it has been shown that the SUSY radiative corrections to the effective Yukawa couplings [9] can be important (for a review see [10]). Such corrections comprise loops involving gluino–sbottom and chargino–stop. The modified $h_b$ has already been shown to sizeably affect $b \rightarrow s\gamma$ [11],[12], the effective $H^\pm tb$ coupling [13], and the Higgs decay $h^0 \rightarrow b\bar{b}$ [4],[13]. In our analysis we shall restrict ourselves to $\tan\beta \leq 20$, which is the region where the above corrections to $h_b$ have minor impact.

Note that such large mass splittings occur more naturally in extended versions of the MSSM which include a singlet Higgs field in the superpotential e.g. [16]. In such models eq. (2) is modified to include a term $\sim \lambda v^2$, where $\lambda = \mu/v_s$ ($v_s$ is the vacuum expectation value of the singlet Higgs field). Thus $\lambda = \mathcal{O}(g_{\text{ew}})$ (i.e. gauge coupling strength) may be attained with $\mu = \mathcal{O}(100)$ GeV, giving rise to large mass splittings.

3 Numerical results

We now present our numerical results for the mass splittings $M_{H^\pm} - M_{H^0}$ and $M_A - M_{H^\pm}$. We take two representative sets of parameters: set (A) gives large $M_{H^\pm} - M_{H^0}$, and set (B) gives large $M_A - M_{H^\pm}$.

| $\tan\beta$ | $M_{H^\pm}$ | $M_{Q_3}$ | $M_1$ | $M_2$ | $\mu$ | $A_t$ | $A_b$ |
|------------|-------------|-----------|-------|-------|-------|-------|-------|
| (A)        | 11          | 250       | 500   | 550   | 4000  | 1900  | 0     |
| (B)        | 10          | 150       | 250   | 200   | 500   | 2800  | 0     |

(6) Here all masses are in GeV; $M_{Q_3}, M_1, M_2$ refer to third generation squark soft SUSY breaking masses; $A_t$ and $A_b$ are the analogous trilinear couplings. We take $\mu$ and $A_{t,b}$ to be real in the numerical analysis. For these values of $\tan\beta$ the terms $\sim h^4_t$ dominate the terms $\sim h^4_b$ in the expressions for the $\lambda_i$. We define $M_{SUSY}$ as

$$M^2_{SUSY} = (m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})/2 \quad (7)$$

where $\tilde{t}_1$ and $\tilde{t}_2$ refer to the lighter and heavier stop eigenstates respectively. For set(A) $M_{SUSY} \approx 450$ GeV and for set(B) $M_{SUSY} \approx 280$ GeV. For both parameter sets the mass splitting $m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}$ comfortably satisfies the RG analysis requirement [4]

$$\frac{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2} + m^2_{\tilde{t}_1}} \lesssim 0.5 \quad (8)$$
Note that the large value of $\mu$ taken in set (A) and (B) does not imply an unacceptably light $\tilde{t}_1$, since the corresponding entry in the stop mass matrix is $\mu \cot \beta$, which is comfortably suppressed for the assumed values of $\tan \beta$. Larger values of $\tan \beta$ together with large $\mu$ would invariably generate values of $m_{b_1}$ which are lighter than the current experimental bounds. We also require that all $\lambda_i$ remain in the perturbative region.

In Fig. 1 (a) and (b), we plot $M_{H^\pm} - M_{H^0}$ and $M_A - M_{H^\pm}$ as a function of $\tan \beta$ for $\mu=4500, 4000, 3500$ GeV (from above) and $\mu=3300, 2800, 2300$ GeV (from above), respectively. For the other parameters we take the choices in (6).

In both Figs. 1 and 2, the line in thin typeset signifies that the corresponding mass for one of $\tilde{t}_1$, $\tilde{b}_1$ and $M_{H^0}$ becomes smaller than the present experimental bound.

In Fig. 1(a), we can understand the behaviour of the splittings $M_{H^\pm} - M_{H^0}$ from eqns.(4) and (5). In eq. (4), increasing $\tan \beta$ enhances the term $h_b^2$ which is negative in this parameter space, and thus reduces the positive $\lambda_4/2$. $\lambda_5$ is also positive and is not so sensitive to $\tan \beta$ since the top quark Yukawa term dominates. However, $2\lambda_6/\tan \beta$ is negative and its modulus becomes smaller as $\tan \beta$ increases. The combined effect is as follows: for small $\tan \beta$ the negative term in eq. (5) cancels the positive terms and the splitting is small. As $\tan \beta$ increases the positive terms dominate, giving rise to large splittings. Further increases in $\tan \beta$ reduce the still positive $\lambda_4$, while $\lambda_5$ remains approximately constant, which explains the descent of the curve for larger $\tan \beta$. We have also checked that the splittings are relatively insensitive to variations in $A_t$ since the relevant terms are suppressed by $h_b^4$. One might expect that the splittings could be further increased by changing the sign of $A_t$, which makes $\lambda_6$ positive. However it is not easy to satisfy the scalar quark mass bound with this choice in the CP conserving scenario. Larger values of $\mu$ give rise to larger splittings, which is explained in Fig. 2(a) below.

In Fig. 1(b) one can see that $M_A - M_{H^\pm}$ increases with $\tan \beta$, which is explained by the fact that $\lambda_4$ is decreasing (see above) and thus enhances the splitting from eq. (2). Large $\mu$ provides the largest splittings (see below).

In Fig. 2 (a) we plot $M_{H^\pm} - M_{H^0}$ as a function of $\mu/M_{SUSY}$ for $A_t = 2000, 1900, 1800$ GeV (from above). For the other parameters we take the choices in (6). In Fig. 2 (a), we can see that $\mu/M_{SUSY} \gtrsim 6$ allows splittings $M_{H^\pm} - M_{H^0} \gtrsim 80$ GeV. Increasing $A_t$ enhances the positive contributions to $\lambda_4$ and $\lambda_5$, which in turn enhances $M_{H^\pm} - M_{H^0}$ (from eq. (5)).

In Fig. 2 (b), we plot $M_A - M_{H^\pm}$ as a function of $\mu/M_{SUSY}$ for $M_{H^\pm} = 150, 250, 350$ GeV (from above), since it turns out that the splittings are not very sensitive to $A_t$. As expected, we can see the decoupling behaviour, with the splitting decreasing for increasing $M_{H^\pm}$. For low values of $\mu/M_{SUSY}$ the 1-loop corrections are not so large and so one has approximately the tree–level result $M_{H^\pm} \gtrsim M_A$. As $\mu/M_{SUSY}$ increases, $\lambda_5$ is enhanced to large positive values. In addition, $\lambda_4$ is decreased from its positive tree-level value. Hence the mass splitting $M_A - M_{H^\pm}$ increases and exceeds 80 GeV for $\mu/M_{SUSY} \gtrsim 10$.

If CP violating phases are allowed in $\mu$ and $A_{t,b}$ then $\lambda_{5,6}$ become complex numbers in general. The mass eigenstates $H_0^0$ and $H_0^0$ are now mixed states of CP, and the large splittings $M_{H^\pm} - M_{H_2^0}$ or $M_{H_1^0} - M_{H_{H^\pm}}$ are possible in a wider range of parameter space which satisfies the mass bounds on $H_1^0$ and $\tilde{t}_1$ ($\tilde{b}_1$).
The magnitude of the mass splittings has important consequences for the phenomenology of the MSSM. Our results show that \( M_{H^\pm} \) need not be close in mass to \( H^0 \) and \( A^0 \), and the mass differences \( |M_{H^\pm} - M_{H^0,A^0}| \) may be as large as the analogous values in a general 2HDM or in other extended Higgs sectors. The relation \( M_{H^\pm} \approx M_{H^0,A^0} \) for \( M_{H^\pm} \geq 200 \) GeV, which is assumed in existing phenomenological analyses, may be broken in the region of large \( \mu/M_{\text{SUSY}} \). We stress that the large values of \( \mu \) considered here do not arise in popular SUSY models, such as minimal supergravity or gauge mediated models. As discussed in [6], in order to be compatible with electroweak symmetry breaking, such large \( \mu/M_{\text{SUSY}} \) would require the soft SUSY breaking mass parameters \( m_1^2 \) and \( m_2^2 \) to be of the order of \( |\mu| \) and negative, and considerable fine-tuning is necessary.

If \( M_{H^\pm} - M_{H^0} \geq 80 \) GeV then the 2 body decays \( H^\pm \rightarrow H^0 W^\pm \) would be open. This would offer a new discovery channel for \( H^\pm \) at the LHC, and one which is expected to offer a very promising signature. Ref. [17] presented a signal–background analysis showing that the background is small, and any model which allows a large \( \text{BR}(H^\pm \rightarrow H^0 W^\pm) \) would provide a very clear signal in this channel. We shall show below that such large BRs are possible in the MSSM, and consequently would aid the search for \( H^\pm \) at the LHC.

If \( M_A - M_{H^\pm} \geq 80 \) GeV then the 2 body decays \( A^0 \rightarrow H^\pm W^\mp \) would be open. This would have important consequences for the process \( \mu^+\mu^- \rightarrow H^\pm W^\mp \) at a muon collider, which receives contributions from \( A^0, H^0 \) mediated s–channel diagrams, and was shown to be a promising production mechanism for \( H^\pm \) in the MSSM with \( M_{H^\pm} \approx M_{H^0,A^0} \) [18]. Any splittings with \( M_A \geq M_{H^\pm} \) would enhance the rate for \( \sigma(\mu^+\mu^- \rightarrow H^\pm W^\mp) \) compared to that in [18]. For \( M_A - M_{H^\pm} \geq 80 \) GeV one could have resonant \( H^\pm \) production via \( \mu^+\mu^- \rightarrow A^0 \rightarrow H^\pm W^\mp \), which was shown to allow very large cross–sections (> 1 pb) in the case of the general 2HDM [19]. These possibilities will be pursued in a future article [20].

In Fig. 3 we plot branching ratios (BRs) for the decay \( H^\pm \rightarrow H^0 W^\pm \) as a function of \( \tan \beta \). This two body decay is open if \( M_{H^\pm} - M_{H^0} \geq 80 \) GeV, and is proportional to \( \sin^2(\beta - \alpha) \). Also plotted is \( \text{BR}(H^\pm \rightarrow h^0 W^\pm) \) which \( \sim \cos^2(\beta - \alpha) \). We take the same parameters as Fig. 1 (a). The thick (thin) lines represent \( \text{BR}(H^\pm \rightarrow H^0(h^0) W^\pm) \) for \( \mu = 4500, 4000, 3500 \) (from above), respectively. Note that we do not consider decays to SUSY particles \( H^\pm \rightarrow t\bar{b}, \chi^{\pm}\chi^0 \) etc. The decays \( H^\pm \rightarrow \tilde{t}_1\tilde{b}_1 \) are not open for this choice of \( M_{H^\pm} \) and \( H^\pm \rightarrow \chi^{\pm}\chi^0 \) can be closed by choosing suitable values for \( M_1,M_2 \) etc. It can be seen from Fig.3 that \( \text{BR} \gtrsim 20\% \) is attainable for intermediate values of \( \tan \beta \), clearly showing the importance of this new decay channel.

The behaviour of the BRs can be qualitatively understood as follows: for low \( \tan \beta \) the splittings are not large enough to open the channel and for large \( \tan \beta \) the newly opened channel \( H^\pm \rightarrow H^0 W^\pm \) cannot compete with the fermionic decay modes. Thus the maximum impact of \( H^\pm \rightarrow H^0 W^\pm \) decays is for intermediate values of \( \tan \beta \), which is the most problematic region for the \( H^\pm \) search at the LHC. The current search strategies utilize \( H^\pm \rightarrow \tau\nu_\tau \) decays [21], which is most effective for \( \tan \beta \geq 15 \), or \( H^\pm \rightarrow t\bar{b} \) decays which cover the regions \( \tan \beta \leq 3 \) and \( \tan \beta \geq 25 \) [22]. We can also see that \( \text{BR}(H^\pm \rightarrow h^0 W^\pm) \), although suppressed in the decoupling limit, can be significant. The sizeable BRs shown in Fig. 3 would offer very good detection prospects in these channels
In general, the mass splittings presented here may give rise to a large contribution to the precisely measured electroweak parameter $\rho$. For mass splittings $\geq 20$ GeV we find that the Higgs contribution is always positive and violates the $\delta\rho$ constraint at the $2(3)\sigma$ level for $|M_{H^\pm} - M_{H^0,A}| \geq 150(200)$ GeV. The dominant SUSY particle contribution ($t\bar{b}$ loops) to $\delta\rho$ has the same sign as the Higgs contribution, thus further reducing the maximum allowed value of $|M_{H^\pm} - M_{H^0,A}|$. Therefore the mass splittings $|M_{H^\pm} - M_{H^0,A}|$ presented here are consistent with the $\delta\rho$ constraints.

4 Conclusions

We have shown that very large mass splittings $M_{H^\pm} - M_{H^0,A}$ are possible in the MSSM. Such splittings occur in a previously ignored region of the MSSM parameter space and may exceed 100 GeV, thus strongly violating the commonly assumed degeneracy relation $M_{H^\pm} \approx M_{H^0} \approx M_A$. The largest splittings arise for relatively large $\mu \geq 6M_{\text{SUSY}}$. The previously neglected 2 body decays $H^\pm \rightarrow H^0W^\pm$ and $A^0 \rightarrow H^+W^-$, $H^-W^+$ may become the dominant channels, which would have important consequences for Higgs searches at future colliders. If $M_A - M_{H^\pm} \gtrsim 80$ GeV, a muon collider could copiously produce $H^\pm$ at resonance.

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Figure 1: $M_{H^\pm} - M_{H^0}$ (left panel) and $M_A - M_{H^\pm}$ (right panel) as a function of $\tan \beta$ for $
mu = 4500, 4000, 3500 \text{ GeV}$ (from above, left panel) and $\nmu = 3300, 2800, 2300 \text{ GeV}$ (from above, right panel), respectively. The thin lines violate the experimental mass bounds on $h^0, \tilde{t}_1$ or $\tilde{b}_1$. For other parameters, see the text.
Figure 2: $M_{H^\pm} - M_{H^0}$ (left panel) and $M_A - M_{H^\pm}$ (right panel) as a function of $\mu/M_{SUSY}$ for $A_t = 2000, 1900, 1800$ GeV (from above, left panel) and $M_{H^\pm} = 150, 250, 350$ GeV (from above, right panel), respectively. The thin lines violate the experimental mass bounds on $h^0$, $\tilde{t}_1$ or $\tilde{b}_1$. For other parameters, see the text.
Figure 3: BR($H^\pm \rightarrow H^0(h^0)W^\pm$) (thick (thin) lines) as a function of tan $\beta$ for the same parameter choice as Fig. 1 (a).