One-step implementation of entanglement generation on microwave photons in distant 1D superconducting resonators

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We present a scalable quantum-bus-based device for generating the entanglement on microwave photons (MPs) in distant superconducting resonators (SRs). Different from the processors in previous works with some resonators coupled to a superconducting qubit (SQ), our device is composed of some 1D SRs \( r_j \) which are coupled to the quantum bus (another common resonator \( R \)) in its different positions simply, assisted by superconducting quantum interferometer devices. By using the technique for catching and releasing a MP state in a 1D SR, it can work as an entanglement generator or a node in quantum communication. To demonstrate the performance of this device, we propose a one-step scheme to generate high-fidelity Bell states on MPs in two distant SRs. It works in the dispersive regime of \( r_j \) and \( R \), which enables us to extend it to generate high-fidelity multi-Bell states on different resonator pairs simultaneously.

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Quantum entanglement on photons plays a critical role in quantum communication [3]. There are some interesting ways for entanglement generation, including those based on linear optical elements and the input-output process of single photons assisted by cavity quantum electrodynamics (QED) in which a two-level atom is coupled to a single-mode field. By far, some artificial atoms have also been widely studied for quantum information processing, such as quantum dots [2], superconducting qubits [3, 4], and diamond nitrogen-vacancy centers [3]. In recent years, quantum information processing on microwave photons assisted by a superconducting qubit (SQ) [6–9] has attracted much attention because of its good performance on the extinction of the transmitted microwave photon (MP) [10, 11] and the long-distance traveling along a superconducting transmission line [12].

Recently, catching and releasing a MP state in a 1D superconducting resonator (SR) has been realized in experiments with high fidelity [13], which gives us another way to generate the entanglement on MPs for quantum communication with the following processes: (1), MPs can be caught from the transmission line (TL) to a device based on 1D SRs; (2), quantum entangling operations are performed on localized MPs assisted by the circuit QED; (3), MPs are released from the processor to the TL. The entanglement generation and the universal quantum gates on the localized MPs in different 1D SRs have been studied both in experiment [14, 15] and in theory [16, 17]. For example, in 2011, Wang et al. [15] realized the entangled NOON state on two 1D storage SRs in a device composed of a coupling resonator coupled to two phase qubits and each of the qubits is coupled to a storage SR, and Hu and Tian presented a scheme to deterministically generate the entangled photon pairs in a SR array. In 2012, Strauch [18] proposed an interesting all-resonant method to control the quantum state of 1D SRs in a processor composed of two 1D SRs coupled to a SQ and Yang et al. [19] presented a scheme to generate a Greenberger-Horne-Zeilinger state of \( n \) photons in \( n \) 1D SRs coupled to a SQ. In 2015, Hua et al. [20] constructed the fast universal quantum gates on 1D SRs with resonance operations.

To generate the particular entanglement on MPs, one always needs the SQ to apply the nonlinear interaction and acts as the coupler element in previous works. In the processor composed of some resonators coupled to a SQ [18, 20], to integrate more resonators, one should take small coupling strength between each resonator and the SQ and the large tunable range of the frequency of the SQ or take the tunable coupling technique between each resonator and the SQ. Small coupling strength leads to a slow quantum entangling operation on the SQ and the SR, which limits the performance of the processor. The large tunable range of the SQ or the tunable coupling between each resonator and the SQ will increase the difficulty for the implementation in experiment. On one hand, the long-coherence-time transmon qubit [21], which is always used to deal with the quantum computing, can just be tuned effectively with a frequency range of \( \sim 2.5 \) GHz [22]. On the other hand, the tunable coupling is just realized between a SQ and a SR [23]. Moreover, the length of the SQ (\( \sim \) tens of nm) can only deal with the entangling operation on SRs in a small range, which limits the ability for the large-scale integration of the processor.

In this letter, we present a scalable device for generating entanglement on microwave photons in distant SRs by using another common SR \( R \) as the quantum bus. In this device, multiple SRs \( r_j \) (act as quantum information carriers) can be coupled to the bus in its different positions simply, assisted by superconducting quantum interferometer devices (SQUIDs). It does not require SQs, far different from previous quantum processors on MPs. As a critical task for quantum communication, we give

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a one-step implementation scheme to generate the Bell state on two distant SRs $r_1$ and $r_2$ with a high fidelity. This scheme works in the dispersive regime of $r_j$ and $R$ ($g_j/\Delta_j \ll 1$) and it can be extended to generate multi-Bell states on different pairs of resonators simultaneously. Here and after, $g_j$ is the coupling strength between $r_j$ and $R$. $\Delta_j = \omega_j - \omega_R$ in which $\omega_j$ and $\omega_R$ are the frequencies of $r_j$ and $R$, respectively. By using the technique for catching and releasing a MP in a SR, our device can work as a Bell-state generator or a node for long-distance quantum communication.

FIG. 1: (Color online) The scalable quantum-bus-based device for generating the entanglement on microwave photons (MPs) in different superconducting resonators. Here the resonator $r_j$ can catch and release a microwave photon from or to the transmission line (TL) by turning on the coupling $C$. The resonator $R$ acts as a quantum bus. The subsystem composed of three resonators (TRQ) shown in the dashed-line box is the setup for producing Bell states on microwave photons in the two distant resonators $r_1$ and $r_2$.

Our device for generating Bell states on microwave photons in distant SRs is composed of some 1D SRs $r_j$ ($j = 1, 2, \cdots$) coupled to a quantum bus $R$ assisted by the SQUIDs, shown in Fig. 1. $r_j$ are placed far enough to each other to avoid the mutual capacitances and mutual inductive interactions among them. Quantum information on $|0\rangle_j$ and $|1\rangle_j$ in resonators $r_j$ can be released and caught by turning on the coupler $C$. To show the principle of our device, one just needs to consider the three-resonator-qubit (TRQ) subsystem composed of $r_1$, $r_2$, and $R$, shown in the dashed-line box in Fig. 1. Here, the coupling $C$ should be turned off. In the interaction picture, the Hamiltonian of the TRQ subsystem can be expressed as ($\hbar = 1$, under the rotating-wave approximation) $^{24}$

$$H = \sum_j g_j (a^+ b_j e^{-i\Delta_j t} + a b_j^+ e^{i\Delta_j t}).$$  

Here, $g_j$ is decided by the coupling position on $R$ and can be tuned by an external flux field through the SQUID $^{24}$. $a^+$ and $b_j^+$ are the creation operators of the resonators $R$ and $r_j$, respectively.

In order to generate the Bell state on $r_1$ and $r_2$ in the TRQ subsystem, let us consider the dispersive regime of $r_j$ and $R$. In the Schrödinger picture, Hamiltonian $H$ can be rewritten as

$$H' = \omega_{a} a^+ a + \omega_{b} b^+ b + \omega_{d} d^+ d + g_1 (a^+ b_1 + a b_1^+ ) + g_2 (a^+ b_2 + a b_2^+ ).$$  

(2)

By taking the unitary transformation $^{3}$

$$U = \exp \left[ \frac{g_1}{\Delta_1} (a b_1^+ - a^+ b_1) + \frac{g_2}{\Delta_2} (a b_2^+ - a^+ b_2) \right]$$  

(3)

to the second order in the small parameters $g_j/\Delta_j$, one can get

$$H'' = U H' U^\dagger \approx \omega'_{a} a^+ a + \omega'_{b} b^+ b + \omega'_{d} d^+ d + g' (b_1 b_2^+ + b_1^+ b_2).$$  

(4)

Here, $g' = \frac{g_1 g_2}{\Delta_1 \Delta_2} (\Delta_1 + \Delta_2)$, $\omega'_{a} = \omega_a - \chi_1 - \chi_2$, $\omega'_{b} = \omega_b + \chi_1$, and $\omega'_{d} = \omega_d + \chi_2$ ($\chi_j = \frac{g_j}{\Delta_j}$). In the interaction picture, the Hamiltonian $H''$ is rewritten as

$$H_{eff} = g' (b_1 b_2^+ e^{-i\delta t} + b_1^+ b_2 e^{i\delta t}),$$  

(5)

in which the detuning value $\delta = \omega'_{a} - \omega'_{b}$. In the subspace composed of $|0\rangle_1 |1\rangle_2$ and $|1\rangle_1 |0\rangle_2$, the evolution of the state $|\psi(t)\rangle$ of the TRQ subsystem can be solved with the equation of motion

$$\frac{i}{\hbar} \frac{\partial |\psi(t)\rangle}{\partial t} = H_{eff} |\psi(t)\rangle.$$  

(6)

Here, $|\psi(t)\rangle$ is the linear combination of the states $|0\rangle_1 |1\rangle_2$ and $|1\rangle_1 |0\rangle_2$, that is,

$$|\psi(t)\rangle = c_{0,1}(t) |0\rangle_1 |1\rangle_2 + c_{1,0}(t) |1\rangle_1 |0\rangle_2$$  

(7)

in which $c_{0,1}(t)$ and $c_{1,0}(t)$ can be expressed as

$$\frac{\partial c_{0,1}}{\partial t} = -i g' c_{1,0} e^{i\delta t},$$  

$$\frac{\partial c_{1,0}}{\partial t} = -i g' c_{0,1} e^{-i\delta t}. $$  

(8)

The general solution for $c_{0,1}(t)$ and $c_{1,0}(t)$ is $^{24}$

$$c_{0,1}(t) = \{c_{0,1}(0) \left[ \cos \left( \frac{\Omega t}{2} \right) + \frac{i \delta}{\Omega} \sin \left( \frac{\Omega t}{2} \right) \right]$$

$$- \frac{2ig'}{\Omega} c_{1,0}(0) (0) \left[ \cos \left( \frac{\Omega t}{2} \right) \right] e^{-i\delta t/2},$$

$$c_{1,0}(t) = \{c_{1,0}(0) \left[ \cos \left( \frac{\Omega t}{2} \right) - \frac{i \delta}{\Omega} \sin \left( \frac{\Omega t}{2} \right) \right]$$

$$- \frac{2ig'}{\Omega} c_{0,1}(0) (0) \left[ \cos \left( \frac{\Omega t}{2} \right) \right] e^{i\delta t/2}.$$  

(9)

Here $\Omega^2 = 4g'^2 + \delta^2$.

Suppose the initial state of the TRQ subsystem is $|\Psi_1\rangle = |1\rangle_1 |0\rangle_2 |0\rangle_R$ which indicates that there is an MP in the resonator $r_1$. In the resonance regime of $r_1$ and $r_2$ (i.e., $\delta = 0$), the state of the TRQ subsystem evolves to

$$\Psi(t) = C_1(t) |\Psi_1\rangle + C_2(t) |\Psi_2\rangle$$  

(10)
at the time \( t \) with the coefficients

\[
C_1(t) = \cos g't, \quad C_2(t) = -i \sin g't. \tag{11}
\]

Here \( |\Psi_2\rangle = |0\rangle_1|1\rangle_2|0\rangle_R \). Under the condition \( \omega_j < \omega_R \) and after an operation time of \( g't = \frac{\pi}{4} \), one will get the final state

\[
|\Psi_{1,2}^+\rangle = \frac{1}{\sqrt{2}}((0)_1|1\rangle_2 + i|1\rangle_1|0\rangle_2). \tag{13}
\]

By taking \( g't = \frac{\pi}{4} \) and \( \omega_j > \omega_R \), one can get the state

\[
|\Psi_{1,2}^-\rangle = \frac{1}{\sqrt{2}}((0)_1|1\rangle_2 - i|1\rangle_1|0\rangle_2). \tag{14}
\]

Both \( |\Psi_{1,2}^+\rangle \) and \( |\Psi_{1,2}^-\rangle \) are the Bell states on microwave photons in the two distant resonators \( r_1 \) and \( r_2 \).

To show the feasibility of our one-step scheme for generating the Bell state \( |\Psi_{1,2}^+\rangle \), we numerically simulate its fidelity with the feasible parameters. The evolution of the TRQ subsystem can be described by the master equation

\[
\frac{d\rho}{dt} = -i[H, \rho] + \kappa_q D[a] \rho + \kappa_s D[b_1] \rho + \kappa_s D[b_2] \rho. \tag{15}
\]

Here, \( \kappa_q (n = R, 1, 2) \) is the decay rate of the resonator \( r_n \), \( D[L] \rho = (2L \rho L^+ - L^+ L \rho - \rho L^+ L) / 2 \). \( \rho \) is the realistic density operator after our Bell-state generation scheme with the initial state \( |\Psi_1\rangle \) and the realistic Hamiltonian \( H \). The influence from the SQUIDs has been neglected as their frequencies are designed to detune with those of the resonators largely \([24]\). By taking the parameters as: \( \omega_1/(2\pi) = 5.75 \) GHz, \( \omega_2/(2\pi) = 6.25 \) GHz, \( g_1/(2\pi) = g_2/(2\pi) = 50 \) MHz, and \( \kappa_q^{-1} = \kappa_s^{-1} = \kappa_s^{-1} = \kappa_R^{-1} = 10 \) \( \mu \)s, we numerically simulate the populations of a microwave photon in resonators \( r_1 \) and \( r_2 \) varying with the operation time, shown in Fig.2(a). The definition of the population is

\[
P_m = \langle \Psi_m | \rho(t) | \Psi_m \rangle. \tag{16}
\]

Here \( m = 1, 2 \). The fidelity of the state \( |\Psi_{1,2}^+\rangle \) generated with the initial state \( |\Psi_1\rangle = |1\rangle_1|0\rangle_R|0\rangle_2 \) in the TRQ subsystem varying with the operation time is simulated by using the definition

\[
F = \langle \Psi_{1,2}^+ | \rho(t) | \Psi_{1,2}^+ \rangle \tag{17}
\]

and it is shown in Fig.2(b), in which the fidelity varies with the operation time and different decay rates of resonators (those with \( \kappa^{-1} = 10 \) \( \mu \)s and \( \kappa^{-1} = 3 \) \( \mu \)s are plotted with the black solid line and the dark green dashed line, respectively). Here the maximal fidelity with the decay rate of \( \kappa^{-1} = 10 \) \( \mu \)s can reach 99.73% within about 25 ns and the one with the decay rate of \( \kappa^{-1} = 3 \) \( \mu \)s can reach 99.1%. It is worth noticing that the best quality factor of a 1D superconducting resonator has reached \( Q \sim 2 \times 10^6 \) \([20]\), which indicates the life time of a microwave photon in the resonator is about 50 \( \mu \)s with considering the relation \( \kappa = \omega_R / Q \) \([3]\) (\( \omega_R \) is the frequency of the resonator). The coupling strengths \( g_1 \) and \( g_2 \) taken here are the theoretic result discussed in Ref.\([21]\) which fulfill the rotating-wave approximation in Eq.\([1]\).

More resonators \( r_j (j = 1, 2, 3, 4 ... ) \) can be coupled to the quantum bus \( B \) simply in our device for generating multi-Bell states on different resonator pairs simultaneously in the dispersive regime of \( r_j \) and \( R \). Here, we take the generation of two Bell states on two resonator pairs as an example to describe its principle. In detail, the five-resonator-qubit (FRQ) subsystem composed of \( r_1, r_2, r_3, r_4, \) and \( R \) is used to generate two pairs of Bell states on MPs. Similar to \( r_1 \) and \( r_2 \) shown in Fig.2, \( r_3 \) and \( r_4 \) are coupled to the bus \( R \) and they are not drawn. In the dispersive regime of \( r_j \) and \( R \), \( r_j \) are detuned with \( R \) largely. Meanwhile, if one takes the frequencies of \( r_1 \) and \( r_2 \) to detune with those of \( r_3 \) and \( r_4 \) largely, our one-step scheme for the generation of the Bell state on \( r_1 \) and \( r_2 \) will works independently with the one on \( r_3 \) and \( r_4 \). Moreover, \( g' \) of \( r_1 \) and \( r_2 \) should be equivalent to the one of \( r_3 \) and \( r_4 \) for generating the Bell states simultaneously in the FRQ subsystem. We numerically simulate the fidelity \( F_{1,2,3,4} \) for generating the state \( \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 + i|1\rangle_1|0\rangle_2) \) on \( r_1 \) and \( r_2 \) and the state \( \frac{1}{\sqrt{2}}(|0\rangle_3|1\rangle_4 - i|1\rangle_3|0\rangle_4) \) on \( r_3 \) and \( r_4 \) simultaneously in this FRQ subsystem. The parameters of \( r_3 \) and \( r_4 \) are taken as \( \omega_3/(2\pi) = 6.75 \) GHz,
The fidelity of the multi-Bell state $|\Psi\rangle$ vs the operation time $t$.

$$g_3/(2\pi) = g_4/(2\pi) = 50 \text{ MHz}, \text{ and } \kappa_{3}^{-1} = \kappa_{4}^{-1} = 10 \mu s.$$  

The parameters of $r_1$, $r_2$, and $R$ are the same as the ones for generating the state $\frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 + i|1\rangle_1|0\rangle_2)$ on $r_1$ and $r_2$ in our TRQ subsystem. The definition of the fidelity is

$$F^{1,2,3,4} = \langle \Psi | \rho'(t) | \Psi \rangle$$

in which $\rho'(t)$ is the realistic density operator of the FRQ subsystem with the initial state $|1\rangle_1|0\rangle_2|1\rangle_3|0\rangle_4|0\rangle_R$. $|\Psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|0\rangle_2 + i|0\rangle_1|1\rangle_2) \otimes (|1\rangle_3|0\rangle_4 - i|0\rangle_3|1\rangle_4) \otimes |0\rangle_R$. The result of our simulation for the fidelity of the final state is shown in Fig. 3. It can reach 99.5% within about 25.5 ns. $g_j/\Delta_j$ taken here is not small enough to ignore the influence from the interactions between $r_j$ and $R$. That is, under the same parameters of $r_1$, $r_2$, and $R$ in the TRQ subsystem, the maximal fidelity of the state $\frac{1}{\sqrt{2}}(|1\rangle_1|0\rangle_2 + i|0\rangle_1|1\rangle_2)$ on $r_1$ and $r_2$ achieved in the FRQ subsystem is smaller than the one in the TRQ subsystem, and the fidelity is also reduced faster than the one in the TRQ subsystem.

Although our schemes work in the dispersive regime of $r_j$ and $R$, one can insert a SQUID in each resonator $r_j$ to tune their frequencies [27] to make $r_j$ resonate with $R$ and make the frequencies of MPs equivalent to each other before they are released to the TL. In our scheme, the MPs in the initial states $|\Psi_1\rangle$ and $|1\rangle_1|0\rangle_2|1\rangle_3|0\rangle_4|0\rangle_R$ can be caught from the TLs. The oscillations with small amplitudes of the populations and the fidelities in Fig. 2 and Fig. 3 come from the interactions between $r_j$ and $R$, which can be removed by increasing the detuning value $\Delta_j$ or reduce the coupling strength $g_j$. To generate more Bell states on different resonator pairs, one should also remove the influence from the bus $R$ effectively and take a smaller $g_i/\Delta_j$ to reduce the indirect interaction among more resonator pairs with difference frequencies.

In summary, we have proposed a scalable quantum-bus-based device for generating entangled states on microwave photons. It is composed of some resonators $r_j$ coupled to a quantum bus $R$ assisted by SQUIDS. With this device, we present a high-fidelity one-step scheme for the generation of the Bell state on the two distant resonators $r_1$ and $r_2$. The scheme works in the dispersive regime of $r_j$ and $R$. Besides, in the dispersive regime, the scheme can also be extended to generate the multi-pair Bell states on different pairs of SRs $r_j$ simultaneously and we take the case for generating two Bell states on two pairs of resonators simultaneously as an example to describe its principle, which can be achieved with a high fidelity as well.

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