Wormhole Dominance Proposal And Wave Function Discord

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Using the wormhole dominance proposal, it is shown that quantum corrections to the usual WKB ansatz for the wave function of the universe ably circumvent many of the drawbacks present in the current proposals. We also find that the recent criticism by Hawking and Turok does not apply to the tunneling proposal.

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I. INTRODUCTION

There are currently three proposals, namely the no boundary proposal [1], the tunneling proposal [2] and Linde’s proposal [3] on the wave function of the universe. Recently a fourth proposal [4] is given and we call it wormhole dominance proposal. Apart from the debate that the above three proposals do or not lead to sufficient inflation, recently the dispute arises whether the pair production of blackholes during inflation would lead to catastrophic instability to the de Sitter space or not. It is argued by Bousso and Hawking [5] that the tunneling wave function would lead to catastrophic instability to the de Sitter space. This claim was analyzed in ref. [6] and was shown to be unfounded. On the other hand Linde [7] demands an unacceptably low values for the density parameter if one uses Hartle- Hawking wavefunction. Hawking and Turok [8] misunderstood the Linde’s wavefunction as the tunneling wavefunction and assert that the tunneling wavefunction would lead to unstable perturbations about a homogeneous cosmological background and is meaningful for homogeneous minisuperspace models.

Vilenkin [6] objects to the pair production of blackholes as a process of independent nucleation and considers the pair production of black holes as pair production of massive particles in de Sitter space. He starts with the tunneling wavefunction and obtains the solution of the Wheeler-DeWitt equation in the region \(m >> H\), where the quantum state of the scalar field (which are produced) are obtained using the tunneling boundary conditions. He shows the emergence of de Sitter invariant Bunch-Davies vacuum. Using Mellor and Moss’ result [9] for the nucleation rate of blackholes, it is shown that the tunneling boundary condition also reproduces this result. This makes him conclude that the tunneling wavefunction is not an opposite prescription to the Hartle-Hawking wavefunction and since the blackholes productions are suppressed, the tunneling wavefunction would not lead to catastrophic instability. Incidentally, the Hartle-Hawking proposal also leads to the same mode functions for the scalar field as is obtained by Vilenkin. This implies the emergence of Bunch-Davies vacuum with the suppression of blackhole production rate substantiating the validity of Hartle-Hawking proposal and this is why Bousso and Hawking [5] obtains the same conclusion from the wavefunction calculation. In Bousso and Hawking, the blackhole production rate is given by

\[
\Gamma = \frac{P_{S DS}}{P_{DS}} = \exp\left(-\frac{1}{3H^2}\right),
\]

where \(P_{S DS}\) is the nucleation probability of a pair of blackholes in \(S^2 \times S^1\) Schwarzschild-de Sitter universe and \(P_{DS}\) is the same in de Sitter universe without the pair of blackholes. The nucleation probability in Hartle-Hawking proposal is given by

\[
\Gamma \propto \exp\left(-\frac{1}{6H^2}\right),
\]
at the nucleation point and (1) is obtained using the expression in (2) for $SDS$ and $DS$ background separately, $S_{h}$ being the euclidean action. It is argued that since the tunneling wavefunction $\psi_{t}$ grows like $e^{\pm |S_{h}|}$, hence (1) and (2) impart catastrophic instability for a tunneling wavefunction for small $H$.

In the present paper, we show that the wormhole dominance in the wavefunction and the concept of WKB Complex trajectory (CWKB) results in a most general wavefunction of the universe. The boundary conditions corresponding to the no boundary proposal and the tunneling proposal when introduced in $\psi_{WD}$ gives the respective wavefunctions. We also show that $\psi_{t}$ is also given by $\exp (-S_{h})$, when quantum corrections are taken into account. The quantum corrections is interpreted in our previous work [4] as due to wormhole contribution. Hence the name is the wormhole dominance proposal. Deviating from our previous work, we now interpret the quantum corrections in terms of Lorentzian sector, as well as in terms of wormhole contribution. If quantum corrections are not taken into the Hartle-Hawking wavefunction, it would have the drawback of not allowing sufficient inflation. Moreover we show that both the proposals give the same nucleation probability and hence we get the same Bunch-Davies vacuum in both the cases.

II. WORMHOLE DOMINANCE PROPOSAL

We start with the Wheeler-DeWitt equation

\[ \left[ \frac{d^{2}}{da^{2}} - a^{2}(1 - H^{2}a^{2}) \right] \psi(a) = 0, \tag{3} \]

for a homogeneous, isotropic and closed universe with constant vacuum energy $\rho_{v}$, where $H = \frac{1}{3}G \rho_{v}^{1/2}$. The classical solution of the model is the de Sitter space

\[ a(t) = H^{-1} \cosh (Ht). \tag{4} \]

The WKB solutions are :

\[ \psi^{\pm} (a > H^{-1}) = [p(a)]^{-1/2} \exp \left[ \pm i \int_{H^{-1}}^{a} p(a')da' \mp \frac{i \pi}{4} \right], \tag{5} \]

\[ \psi^{\pm} (a < H^{-1}) = [p(a)]^{-1/2} \exp \left[ \pm i \int_{H^{-1}}^{a} p(a')da' \right], \tag{6} \]

where $p(a) = \left[ -a^{2}(1 - H^{2}a^{2}) \right]^{1/2}$. The wavefunctions corresponding to the Hartle-Hawking ($\equiv \psi_{h}$) and the tunneling ($\equiv \psi_{t}$) proposals are :

\[ \psi_{h}(a < H^{-1}) = \psi^{-}(a), \tag{7} \]

\[ \psi_{t}(a < H^{-1}) = \psi^{+}(a) - \frac{i}{2} \psi^{-}(a). \tag{8} \]

The CWKB solution of (3) is obtained as follows. Identifying

\[ S(a_{f}, a_{i}) = \int_{a_{i}}^{a_{f}} p(a')da', \tag{9} \]

where $a$ may be complex, the solution of (3) at a point $a$, real or complex is obtained as
\[
\psi(a) = \sum_{\text{CWKB paths}} \exp \left[ \pm iS(a, a_0) \right],
\]

(10)

where \(a_0\) is an arbitrary point, where the boundary conditions are known or fixed. We now consider the classically unallowed region with \(a < H^{-1}\). For a wave moving from left to right, we take the negative sign in (10) and call it the direct trajectory. The wave moving right to left is called reflected trajectory and corresponds to the positive sign. The classical trajectory corresponding to (3) is

\[
a(t) = H^{-1} \sin (Ht),
\]

(11)

where \(t\) may be complex. Here \(a = 0\) and \(a = H^{-1}\) are the turning points and act as reflection point for trajectories that move towards it. We consider a point \(a < H^{-1}\) and start from \(a = 0\).

Thus in CWKB [4], neglecting the WKB preexponential factor

\[
\psi(a) \sim (\psi_{\text{DT}}(a) - i\psi_{\text{RT}}) \left[ \text{Repeated reflections between the turning points } a = 0, H^{-1} \right],
\]

(12)

where

\[
\psi_{\text{DT}}(a) \sim \exp [-iS(a, 0)],
\]

(13)

\[
\psi_{\text{RT}}(a) \sim \exp [-iS(H^{-1}, 0) + iS(a, H^{-1})],
\]

(14)

Repeated reflections \equiv \frac{1}{1 - \exp [-2iS(H^{-1}, 0)]}.

(15)

Using (12) to (15), we get for (3), using (9)

\[
\psi^\pm \sim \frac{\exp (\pm \frac{i}{3H}a^2)}{1 - \exp (\pm \frac{i}{3H}a^2)} \left[ C_\pm \exp (\mp \frac{1}{3H^2}(1 - a^2H^2)^{3/2}) - d_\pm i \exp (\mp \frac{1}{3H^2}(1 - a^2H^2)^{3/2}) \right].
\]

(16)

Here \(C_\pm\) and \(d_\pm\) are two constants, \(\pm\) come from the negative sign under the square root in \(p(a)\). Using (5,6), we write as

\[
\psi^\pm(a < H^{-1}) = N_\pm \left[ C_\pm \psi^\mp(a) - id_\pm \psi^\pm(a) \right].
\]

(17)

Eq.(17) is the most general wavefunction in CWKB. Here \(N_\pm\) is given by the prefactor outside the square bracket in (16).

Let us calculate the norm of the wavefunction \(\psi(0^+)\) and nucleation probability according to (16):

\[
\psi^+(0^+) = \frac{-i(e^{\frac{2i}{3H^2}} + i)}{1 - e^{\frac{2i}{3H^2}}},
\]

(18)

\[
\psi^-(0^+) = \frac{e^{\frac{2i}{3H^2}} - i}{1 - e^{\frac{2i}{3H^2}}},
\]

(19)

so that we find \(|\psi^+(0)| = |\psi^-(0)|\). This coincides with the norm given by Klebanov et al. [10] and which is identified due to wormholes contributions at \(a = 0\). The nucleation amplitude is given by

\[
|\psi^\pm(H^{-1})| = \frac{e^{\pm \frac{2a}{3H^2}} \left[ |C_\pm|^2 + |d_\pm|^2 \right]^{1/2}}{1 - e^{\pm \frac{2a}{3H^2}}}.\]

(20)
Let us see how do the standard wavefunctions emerge from (17). In the present work we propose a more transparent interpretation of \( N_\pm \) in terms of Lorentzian sector instead of wormhole dominance. In CWKB, \( a(t) \) may be complex and we write

\[
a(t) = H^{-1} \sinh \left( t_n + it_i \right). \tag{21}
\]

Consider \( t_n = H^{-1} \frac{\pi}{2} \), Eq. (21) gives

\[
a(t) \rightarrow H^{-1} \frac{\pi}{2} H^{-1} \cosh \left( H t_i \right). \tag{22}
\]

This corresponds to a Lorentzian de Sitter universe and corresponds to outgoing and ingoing trajectories for \( t_i > 0 \) and \( t_i < 0 \). Thus a trajectory from \( a = 0 \) to \( a = H^{-1} \) and then parallel to the imaginary axis gives rise to both the outgoing and ingoing modes where \( t_i \) serves as Lorentzian time. In the region \( a < H^{-1} \), the euclidean trajectory is \( \exp \left[ -i S(H^{-1}, 0) \right] = \exp \left( \frac{-1}{3H^2} \right) \). As \( H^{-1} \) is a turning point, the ingoing and outgoing modes must have equal amplitudes at the point \( a = H^{-1} \). This corresponds to the Hartle-Hawking proposal. The wavefunction \( \psi_i \) is then given by \( \psi_+ \) with \( C_+ = 1, d_+ = 0 \) and hence no repeated reflections. Thus

\[
\psi_+ (a < H^{-1}) = \exp \left( \frac{1}{3H^2} \right) \psi^-(a) \equiv \psi_i (a < H^{-1}), \tag{23}
\]

\[
\psi_+ (a > H^{-1}) = \exp \left( \frac{1}{3H^2} \right) (\tilde{\psi}^+(a) - \psi^-(a)) \equiv \psi_i (a > H^{-1}). \tag{24}
\]

The nucleation probability is

\[
|\psi_i (a = H^{-1})|^2 = \exp \left( \frac{2}{3H^2} \right). \tag{25}
\]

When we consider \( \psi_- \), we should start from Lorentzian sector, where we have only outgoing modes from \( H^{-1} \). This implies both growing and decaying exponentials and hence also repeated reflections between \( a = 0 \) and \( a = H^{-1} \). This corresponds to the tunneling proposal. Thus taking \( \psi_-(a < H^{-1}) \) and \( C_- = +1, d_- = \frac{1}{2} \), we get

\[
\psi^- (a < H^{-1}) = \frac{\exp \left( \frac{-1}{3H^2} \right)}{1 - \exp \left( \frac{-1}{3H^2} \right)} \left[ \psi^+ - i/2\psi^- \right] \equiv \psi_i (a < H^{-1}). \tag{26}
\]

We have taken \( d_- = 1/2 \) to have equal amplitude at the turning point. If we keep quantum corrections to both the proposals, we have

\[
\psi^+ (a < H^{-1}) = \frac{\exp \left( \frac{-1}{3H^2} \right)}{1 - \exp \left( \frac{-1}{3H^2} \right)} \psi^- (a), \tag{27}
\]

\[
\psi^- (a < H^{-1}) = \frac{\exp \left( \frac{-1}{3H^2} \right)}{1 - \exp \left( \frac{-1}{3H^2} \right)} \left[ \psi^+ - i/2\psi^- \right], \tag{28}
\]

and the probability of nucleation

\[
P(a = H^{-1}) \equiv |\psi^\pm (H^{-1})|^2 = \exp \left( \frac{2}{3H^2} \right), \tag{29}
\]

the same in the two proposals. This result is a bit surprising. Let us consider the most general expression of \( \psi_\pm \). For \( a < H^{-1} \), the four sphere having the boundary as 3 sphere of radius \( a \), the action is
\[ S_{\pm} = -\frac{1}{3H^2} \left[ 1 \pm (1 - H^2a^2)^{3/2} \right], \]  

(30)

where the plus (minus) sign denotes the action that corresponds to filling in the 3 sphere with more (less) than half the 4 sphere. In terms of CWKB this corresponds to the reflected trajectory, and the direct trajectory. We now write the \( \psi_{WD}^{\pm} \) in terms of \( S_{\pm} \). We get from (26)

\[
\psi^+(a < H^{-1}) = \frac{1}{1 - \exp \left( \frac{2}{3H^2} \right)} \left[ C_+ \exp (-S_-) - id_+ \exp (-S_+) \right],
\]

(31)

\[
\psi^-(a < H^{-1}) = -\frac{1}{1 - \exp \left( \frac{2}{3H^2} \right)} \left[ C_- \exp (-S_+) - id_+ \exp (-S_-) \right].
\]

(32)

Thus we see that the quantum corrections arising out of repeated reflections do all the necessary job to cast the tunneling wavefunction also in the form \( \psi \sim \exp (-S_E) \). Not only that, the repeated reflections also save the Hartle-Hawking wavefunction from the drawback for not having sufficient inflation. If \( H \) is small, then (27) gives

\[
|\psi^+(a = H^{-1})|^2 \sim e^{-\frac{2}{3H^2}},
\]

with small nucleation probability for large universes.

III. CONCLUSION

Thus our conclusion is that if we do not take quantum corrections either through wormhole dominance or repeated reflections at the turning points, the discord among the proposals would sustain. Our proposal in terms of CWKB paths gives a plausible answer to the current discord on the wavefunction of the universe. The normalization and other aspects of \( \psi_{WD} \) have already been discussed in our previous work \[4\]. We have shown that \( \psi_T \) now grows as \( \exp (-S_E) \) and hence Bousso and Hawking’s criticism does not apply to it.

Allowance of repeated reflections in the tunneling proposal is quite natural since we have both \( \psi^+ \) and \( \psi^- \) like terms in the region \( a < H^{-1} \). But in the Hartle-Hawking proposal, it cannot be obtained since it has only \( \tilde{\psi}^- \) like term in the regions \( a < H^{-1} \). The wormholes require a contribution \( \sim \exp \left( \frac{2}{3H^2} - \frac{1}{2}a_{\min}^2 \right) \) i.e., a \( \psi^+ \)-like term, where \( a_{\min} \) is the radius of the wormhole throat [see ref. \[4\] and ref. \[10\]]. The absence of \( (1 - \exp \left( \frac{2}{3H^2} \right)) \) like term in \( N_+ \) would then imply for having not sufficient inflation in the Hartle-Hawking proposal.

With respect to pair production of blackholes we mention that since we have the same nucleation probability in both the proposal, using Mottola’s arguments \[11\] we can show that energy density of the produced pairs is given by

\[
T_{ab} \propto |\beta|g_{ab} \sim e^{-\frac{2}{3H^2}} g_{ab}.
\]

Hence the suppression of blackholes for small \( H \) and \( m >> H \) is not forbidden by any of the proposals so long as \( a = H^{-1} \) acts as reflection point. More details in this regard would be explored shortly.

[1] J.B.Hartle and S.W.Hawking, Phys. Rev. D28 2960 (1983).
[2] A.Vilenkin, Phys. Rev. D33 3560 (1986).
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