Coherent Flux Tunneling Through NbN Nanowires

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We demonstrate evidence of coherent magnetic flux tunneling through superconducting nanowires patterned in a thin highly disordered NbN film. The phenomenon is revealed as a superposition of flux states in a superconducting loop with the nanowire acting as an effective tunnel barrier for the magnetic flux, and reproducibly observed in different wires. We perform microwave spectroscopy and study the tunneling amplitude as a function of the wire width, compare the experimental results with theories, and estimate the parameters for existing theoretical models.

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Introduction. Superconducting electrical circuits containing Josephson tunnel junctions have provided an ideal testing ground for investigating the quantum mechanics of macroscopic variables, starting with the observation of quantum coherence of the superconducting phase difference across a Josephson junction [1] and leading to the development of superconducting qubits [2]. Recently, it was realized that due to the fundamental charge–phase duality exhibited by Josephson devices, exactly dual physics can be observed in circuits containing narrow nanowires of highly disordered superconductors in which coherent quantum phase slips (CQPS) can have a significant probability amplitude [3]. Thermally activated phase slips (PS) of the order parameter, corresponding to passage of a quantum of magnetic flux over the energy barrier represented by the wire, are a well-known origin of resistance below the critical temperature in superconducting wires [4–6]. At the lowest temperatures, transport measurements indicate a transition to PS by incoherent quantum tunneling [7–9]. Very recently CQPS was observed directly for the first time in strongly disordered InOx nanowires embedded into superconducting loops [10], demonstrating the concept of a PS flux qubit [11], dual to the single Cooper pair box [12]. However, several basic questions remain open, e.g., universality and reproducibility in different materials. Moreover, strongly disordered superconductors such as InOx exhibit a number of properties different from conventional superconductors, in particular the role of dissipation [13], which make the study of QPS an interesting problem in itself.

In this Letter, we report the observation of coherent flux superpositions in NbN loops, each containing a nanowire section as the tunnel barrier for magnetic flux (cf. Fig. 1). We observe the behavior in several loops on the same chip, characterize the dependence of the flux tunneling on the wire width, and compare the measurement results with the expected exponential dependence on the barrier width. Each of the two main findings of this work, (i) demonstration of coherent flux tunneling in a material different from InOx and (ii) its wire-width dependence, is underscored by the experimental results. We also compare our findings with the theoretical expectations, and discuss the implications of our results for the broader context of quantum phase slips in nanowires and nanorings.
dependence are of significant importance. They are crucial for developing more involved CQPS devices \[14\]–\[17\], utilizing physics dual to conventional Josephson ones.

**The device.** The scanning electron micrograph of a typical loop in Fig. 1(a) illustrates the working principle of a PS flux qubit \[9\]–\[11\]. A loop of NbN with nominal area \(S\) and high kinetic inductance \(L_k\) is placed in a perpendicular magnetic field \(B_{ext}\). Due to flux quantization in superconducting loops \[5\], the total flux through the loop is an integer \((N)\) multiple of the magnetic flux quantum \(\Phi_0 = \hbar/2e \approx 2 \times 10^{-15}\) Wb, and the energy of the loop is \(E_N = E_0(N - N^2)\), expressed in terms of the external flux \(f_{ext} = \Phi_{ext}/\Phi_0\) with \(\Phi_{ext} = B_{ext}S\) and the inductive energy \(E_L = \Phi_0^2/2L_k\) \[18\]. The CQPS process in the nanowire, described by the amplitude \(E_S\), lifts the degeneracy of the fluxoid states \(|N\rangle\) and \(|N + 1\rangle\) at \(\Phi_{ext} = (N + 1/2)\Phi_0\). The resulting energy band diagram is shown in Fig. 1(b), characterized by an avoided crossing of magnitude \(E_S\) \[11\].

At \(\Phi_{ext} = (N + 1/2)\Phi_0\) the ground and first excited states correspond to symmetric and antisymmetric superpositions of \(|N\rangle\) and \(|N + 1\rangle\), respectively. The energy splitting of this effective two level system is \(E_{N+1} = \sqrt{\varepsilon^2 + E_S^2}\). Here, \(\varepsilon = 2L_p\delta\Phi\), with the persistent current \(I_p = \Phi_0/2L\) and \(\delta\Phi = \Phi_{ext} - (N + 1/2)\Phi_0\), gives the difference \(E_{N+1} - E_N\) away from the degeneracy. To probe \(f_0\) and hence \(E_S\), we couple the loop to a coplanar NbN resonator via a section of shared kinetic inductance [bottom loop edge in Fig. 1(a)], enabling readout of multiple qubits located close to each other on a single chip \[10\]. We perform dispersive readout of the coupled qubit–resonator system by monitoring the amplitude and phase of transmitted microwaves \[19\] while varying \(\Phi_{ext}\).

**Experimental methods.** Generally, the materials optimal for CQPS should be highly disordered and characterized by large normal state resistivity that translates into large impedance in superconducting state \[11\]. At the same time this high degree of disorder should not suppress the superconducting gap or introduce subgap states as this would introduce dissipation and decoherence \[10\]. Transport data \[20\]–\[21\] in combination with STM measurements \[22\]–\[25\] indicate that materials favorable for CQPS include InO\(_x\), TiN, and NbN films.

Our samples were fabricated by first depositing a NbN film of thickness \(d \approx 2 - 3\) nm on a Si substrate by DC reactive magnetron sputtering \[20\]. Details of the deposition process can be found in Refs. \[27\]–\[29\]. The overview in Fig. 1(d) displays coplanar lines connecting to the external microwave circuit as well as the CPW resonator Au groundplanes. In the last step, the resonator center line and the nanowire loops were patterned into a negative resist (MicroChem SU-8 2000) by electron beam lithography (EBL) and transformed to a NbN pattern by reactive ion etching in CF\(_4\) plasma. The chip was enclosed in a sample box and microwave characterization was performed at the base temperature of 40 mK.

We focus on two out of several measured devices, with identified qubits belonging to 7 (10) out of the 20 loops for sample A (B), respectively. Referring to the enlarged view in Fig. 1(d), they are numbered from 1 to 20, starting from the smallest, i.e., the leftmost loop. The nominal wire width increases from \(\gtrsim 20\) nm in loop 1 to \(\approx 75\) nm in loop 20. The samples reported here were fabricated simultaneously from the same film, and both feature a resonator with capacitive coupling. Step impedance resonators \[10\] performed comparably.

To characterize the qubits, we use a vector network analyzer and measure the complex microwave transmission coefficient \(t\) through the resonator as a function of the frequency \(f_p\) and the external field \(B_{ext}\) \[29\]. In addition, a second continuous microwave tone at \(f_s\) can be used to excite the qubits through the resonator. The resonant modes are given by \(f_n = n\nu/2L\), \(n = 1, 2, 3, \ldots\), where \(L\) is the resonator length (1.5 mm and 1.25 mm for sample A and B, respectively), \(\nu = 1/(L/C_1)\) the effective speed of light, and \(L_1 (C_1)\) the inductance (capacitance) per unit length. The qubit loops have stronger coupling to the odd modes with maximum current close to the center of the resonator. Transmission measurements yield an average mode spacing of 2.43 GHz and 2.92 GHz for sample A and B, respectively. Using the estimate \(C_1 \approx 7 \times 10^{-11}\) F/m, we find \(f \approx 7.5 \times 10^6\) s and \(L_1 \approx 2.7 \times 10^{-11}\) H/m, corresponding to the square inductance \(L_{Q} \approx 1.3\) nH and characteristic impedance \(Z_1 = (L_1/C_1)^{1/2} \approx 2\) k\(Ω\) \(\gg\) \(Z_0 = 50\) \(Ω\) of the line. Figure 1(c) shows the squared amplitude of \(t\) for sample A, at probing frequencies \(f_p\) in a narrow range around \(f_3 = 7.7306\) GHz, and normalized by the maximum transmission at \(f_p = f_3\). A Lorentzian fit to the peak of \(|t|^2\) gives the photon decay rate \(\kappa = 2\pi \times 6.6\) MHz, corresponding to a loaded quality factor \(Q_L \approx 1.1 \times 10^4\).

**Transmission measurements.** Figure 2(a) displays the result of the main qubit characterization measurement of sample B: \(|t|\) in a range of \(f_p\) around \(f_3 \equiv f_3\), and over a range of \(B_{ext}\). Avoided crossings typical for coherently coupled qubit–resonator systems are observed, with corresponding features present also in \(\arg(t)\) (not shown). Measuring over a wider range of \(B_{ext}\) and extracting the periodicity in field of each feature in Fig. 2(a) allows us to identify the loop from which they originate. For 4 qubits, the lines in Fig. 2(a) show the two lowest transitions at frequencies \(f_{\pm} = (f_0 + f_i)/2 \pm (2g_e/h)^2 + (f_0 - f_i)^2/2\), calculated according to the Jaynes–Cummings model \[19\] by considering at a time only a single qubit coupled to the resonator. Here, \(g_e = gE_S/(h f_0)\) is the renormalized qubit–resonator coupling energy with \(g = M L_0 f_0\), where \(M\) denotes the coupling inductance and \(L_0\) the zero-point current fluctuation in the resonator.

To determine \(E_S\) and \(f_0\) of the qubits (from the minimum value and slope of \(f_n\), respectively), we perform two-tone spectroscopy by continuously monitoring \(t\) at the fixed frequency \(f_p = f_3\), while simultaneously sweep-
ton processes with curves with the same line type correspond to multiphoton noise typical for two-level fluctuations is observed. We calculated short range of over a wide range [30]. The result for sample A over a ing the frequency  of the additional spectroscopy tone over a wide range [30]. The result for sample A over a short range of is shown Fig. 2 (b), including calculated  for selected qubits. The vertically offset curves with the same line type correspond to multiphoton processes with . In some cases, telegraph noise typical for two-level fluctuations is observed. We attribute this to background charge fluctuators affecting .

We proceed to find from the type of vs. measurements in Fig. 2 (a), under the approximation it has the same value for all qubits. To achieve this, for each qubit detected in spectroscopy, we use the total qubit decoherence rate as an adjustable parameter, and compare the measured with simulations and analytical expressions valid in the weak driving limit [31]. Finally, we find of the qubits not distinguishable from the low frequency noise floor in two-tone measurements (usually with , cf. Table 1). In this case we treat as an adjustable parameter along with while keeping at the above-determined value. We obtain satisfactory agreement between the calculation and the measurements with . Moreover, agrees with our expectations.

We observe to decrease as expected with increasing total number of squares of NbN film in a loop. Corresponding to from spectroscopic and transmission measurements (probing local film properties), we find kinetic inductances in the range agreeing with our expectations. For both samples, we focus on the qubits from (sample A) and (sample B), featuring smallest relative roughness in a loop and B: B1–B6), featuring smallest relative roughness in a loop and B: B1–B6), featuring smallest relative roughness in a loop.

![FIG. 2. (color online) (a) Amplitude of the normalized transmission coefficient around the resonator mode  (sample B). For four qubits, the lines show transition frequencies between the ground state and the two lowest dressed energy levels of the coupled qubit–resonator system. (b) Typical two-tone spectroscopy (sample A). The lines correspond to calculated qubit frequencies  vs. for four qubits. The horizontal features originate from the resonator modes.](image)

TABLE I. Qubit energies and wire widths.

| Loop |  |  |  |  |  |  |
|------|---|---|---|---|---|---|
|      |  |  |  |  |  |  |
| A1   | 27.4 | 21.6 | 2.3 | 12.6 |  |
| A2   | 26.8 | 20.2 | 2.6 | – |  |
| A3   | 29.2 | 25.1 | 2.0 | 2.3 |  |
| A4   | 30.0 | 24.9 | 2.2 | 1.0 |  |
| A5   | 34.0 | 29.6 | 2.0 | – |  |
| A6   | 31.5 | 27.2 | 1.9 | 0.9 |  |
| B1   | 28.0 | 22.2 | 2.4 | 7.0 | 7.0 |  |
| B2   | 29.6 | 23.2 | 3.0 | 7.3 | 5.5 |  |
| B3   | 29.0 | 24.1 | 1.7 | 1.4 | 0.9 |  |
| B4   | 29.1 | 24.8 | 2.2 | 0.8 | 1.0 |  |
| B5   | 30.7 | 26.8 | 1.9 | 1.6 | 2.5 |  |
| B6   | 30.8 | 26.2 | 1.5 | – | 1.3 |  |

a Re-measurement of sample B after thermal cycling to 300 K  
b Wire length 750 nm by design (500 nm for wires 1–5); is normalized by 500/500  
c determined from measurement to approximately ±50% accuracy (vs. ≤ 100 MHz with two-tone spectroscopy)

Analysis of the phase slip amplitude. Table I and Fig. 3 summarize the results. In Table I we collect the average wire widths , the minimum widths , and the width standard deviations together with the experimentally derived and , the latter obtained after thermal cycling of sample B to 300 K. Figure 3 shows versus for both samples, we focus on the qubits from loops 1–6 with wires of better quality (sample A: A1–A6 and B: B1–B6), featuring smallest relative roughness in width. During EBL, the nominally narrowest wires in these loops were written as single pixel lines, resulting in . In contrast, of the other detected qubits (from loops 7–12, patterned in area mode with sub-optimal dose, yielding ) do not follow any apparent dependence on , indicating that these wires behave as multiply constricted rather than uniform barriers for the flux tunneling. We take the SEM resolution into account in the wire width derivation, while additional unknown systematic error can remain in the absolute values of . Effective can also be reduced by a few nanometers due to oxidation at the edges. Nevertheless, it should not affect the overall dependence. Note that almost all wires 1–6 work as good tunnel barriers for the magnetic flux. However, signatures from loops A2
and A5 with minimal and maximal $\tilde{w}$ are not found. We suppose that this is due to too high (more than 15 GHz) and too low (less than 0.5 GHz) $E_S/\hbar$ to be detected by our methods, consistent with our expectations.

We now compare the data with the theoretical expectations. As any quantum tunneling, the phase slip process is expected to be exponential in the tunnel barrier width:

$$E_S = E_0 \exp(-\kappa \tilde{w})$$

(1)

where $E_0/\hbar$ is related to an attempt frequency and $\kappa^{-1}$ gives the width at which the wire becomes essentially a one dimensional channel characterized by large quantum fluctuations. Qualitatively, the trend in Fig. 3 agrees with this exponential dependence. However, the $E_S$ values exhibit large scatter. It can originate from small non-uniformities in material parameters or film thickness, or the remaining wire width roughness. In addition, because of the exponential dependence of the tunneling rate on the number of conduction channels $N_{ch}$, mesoscopic fluctuations of the conductance $\delta G \sim e^2/\hbar$ are expected to result in large fluctuations $\delta \ln E_S \sim \delta N_{ch} \sim 1$.

![FIG. 3. (color online) Dependence of $E_S$ on the average nanowire width $\tilde{w}$ extracted from SEM images by an automated procedure. In the main panel, the symbols denote experimental data, and the lines are exponential fits (see text for details). Inset: $E_S$ vs. $w_{\text{min}}$, the minimum wire width, together with a fit to the phenomenological model.](image)

The BCS-based theory of QPS in moderately disordered superconductors \cite{30, 43, 44} gives the parameters in Eq. (1) for $\tilde{w} \lesssim \xi$: $E_0 = \Delta (R_Q/R_\Box) l \bar{\omega} \xi^{-2}$ and $\kappa = a (R_Q/R_\Box) \xi^{-1}$. Here, $\Delta$ is the superconducting energy gap, $R_Q = \hbar/(4e^2)$ $\approx$ 6.4 kΩ the quantum resistance, $R_\Box$ the normal state sheet resistance of the film, $l = 500$ nm the wire length, $\xi$ the superconducting coherence length, and $a$ denotes a dimensionless parameter of order unity. We use $\Delta \approx 1.6 \pm 0.1$ meV inferred from direct measurements of the gap in NbN films similar to those used here, $\xi = 4$ nm known for thicker films \cite{45}, and the approximate low temperature resistance $R_Q \approx 2$ kΩ. A linear fit to $\ln(E_S)$ yields the reasonable value $a \approx 0.6$ (solid black line in Fig. 3), whereas the corresponding kinetic inductance $L_\Box = h R_\Box / \pi \Delta \approx 0.25$ nH expected from BCS theory deviates from the measured $L_\Box \approx 1.3$ nH. Poor applicability of the BCS theory, however, is not surprising for the strongly disordered material, and not strictly one-dimensional wires. Here also random charge distribution along the wire is not accounted, which results in $E_0 \propto l$.

Now, we compute $E_S$ according to the phenomenological model \cite{36, 37} of the strongly disordered superconductors, where the measured $L_\Box$ enters directly as an input parameter. In this model $E_0 = \rho \sqrt{l/\tilde{w}}$ and $\kappa = \eta \sqrt{v_F \bar{p}}$, where $\rho = (\hbar/2e^2) L_\Box^{-1}$ is the superfluid stiffness ($\rho/\hbar \approx 130$ GHz), the numerical parameter $\eta \approx 1$, and $v_F = 1/(2e^2 R_\Box D)$ is the Cooper pair density of states \cite{10, 35}. Based on the diffusion coefficient of the films $D \approx 0.45 \text{ cm}^2/\text{s}$ \cite{21} we fix $v_F \approx 35$ eV$^{-1}$ nm$^{-2}$. A fit then yields the reasonable value $\eta \approx 1.4$ (dashed red line in Fig. 3). Next, in the inset of Fig. 3 we show $E_S$ as a function of $w_{\text{min}}$. Assuming that $E_S$ is dominated by the tunneling amplitude via a single constriction, as suggested in Ref. \cite{39} we fit the data within the phenomenological model without the prefactor $\sqrt{l/\tilde{w}_{\text{min}}}$ and obtain $\eta \approx 1.2$ (dashed line). Note that estimates using $\eta = 1$ give the correct order of the $E_S$ without any fitting parameters.

Sample B was cooled down twice to study the effects of thermal cycling. As evident from Table 1, $E_S$ changes a little compared to the first measurement. This may be interpreted in terms of the Aharonov–Casher effect, i.e., interference of PS from different regions of the wire, and its dependence on the surrounding offset charges \cite{40, 41}. As argued in Ref. \cite{39} the PS nature of the wires is retained even if they contain weak constriction-type inhomogeneities: The requirement is that the constriction resistance is much smaller than the total wire resistance, a condition likely satisfied by our wires.

Besides the initial demonstration of CQPS in InO$_x$ wires and the NbN wires discussed in this Letter, we have recently observed qubit behavior in nanowires from ALD–grown TiN as well as purposely-made short constrictions in NbN and TiN. Similar to InO$_x$, the cause of strong decoherence in the nanowire qubits requires further study. For the fabrication of practical devices utilizing CQPS, the ideal would be a disordered material with highly reproducible fabrication process, together with minimized wire roughness. In conclusion, we find phase-slip flux qubit behavior with systematic wire-width dependence, in agreement with the theory of CQPS up to exponential accuracy.

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