Improved 3D limit-cycle navigation method for path planning quad rotor

Fajril Akbar\textsuperscript{1,2,3} and Yasir Mohd Mustafah\textsuperscript{1}
\textsuperscript{1}Faculty of Engineering, International Islamic University, Jalan Gombak, 53100, Kuala Lumpur, Malaysia
\textsuperscript{2}Faculty of Information Technology, Andalas University, Limau Manis, 25163, Padang, Indonesia
E-mail: ijab@ft.unand.ac.id

Abstract. Quad rotors as a type of rotary wing aerial vehicle must planned their flight path effectively and avoid any disturbance to complete their mission. The extension of 2D limit-cycle navigation into 3D, made it possible to apply for aerial vehicle. The extended limit-cycle has been introduced, but it generate unsuitable path in certain condition. In this research, rendering the obstacle into a cylinder will generate the efficient path and provide the best way in avoiding stationary obstacle. By defining obstacle position, a simulation and performance study is done using limit-cycle characteristic method. The simulations results illustrated the path generation using improved 3D limit cycle with different obstacle condition.

1. Introduction
Quad rotor as one of the rotary wing aerial vehicle is becoming more preferable for various task in society. The capability to fly indoor as well as outdoor with unmanned capabilities make this autonomous system provide safety, security and convenience to performing s desired task in a any kind of environment. The minimal space required for takeoff and landing, the ability to maneuver sharply and hover precisely and capable of maintain static position is becoming important contribution of these aircraft.

Quad rotor must have good tracking characteristic and ability to reject any disturbance of the flight path. Thus, several researches has been done in planning flight path for UAV such as path planning based on visibility lines(VL) method \cite{1}, based on genetic algorithm\cite{2}, and multi-UAVs based on Rapidly-exploring Random Tree(RRT) algorithm \cite{3}. Providing accurate path with obstacle avoidance is also become a key challenge in planning the flight path in rural area. Several researches has been done in UAV navigation strategy to avoid the obstacle along the path. A method for path planning based on velocity vector field investigated in \cite{4}. In \cite{5}, the author presented a fuzzy logic approach for avoiding stationary and moving obstacle by sensing the distance and the angle between UAV and obstacles. A stereo vision for obstacle detection with probabilistic roadmaps for path planning is introduced in \cite{6}. The difficulties in detecting thin obstacles and inefficiency in memory for 3D maps representation is need improvement to make it more reliable.

The limit-cycle navigation method introduced in \cite{7} as one of the appropriate path planning methods for two dimension (2D) environment. By using the concept horizontal and vertical plane, the three dimension (3D) limit-cycle navigation also apply in planning the flight path. It still has several

\textsuperscript{3} To whom any correspondence should be addressed.
drawbacks because it only generated the limit-cycle based on the center of gravity position of the obstacle.

In this research, it is investigated and improved the extended limit cycle navigation method by rendering an obstacle into a cylinder in order to generate an effective path planning for aerial vehicle. By defining obstacle position, a simulation and performance study is done using limit-cycle characteristic method. Simulation studies are conducted and the results illustrated the effectiveness of path generation using 3D limit cycle with different obstacle condition.

2. The limit-cycle navigation

Limit cycle navigation based on characteristic of 2nd-order nonlinear system [7]:

\[
\begin{align*}
\dot{x} &= y + x(1 - x^2 - y^2) \\
\dot{y} &= -x + y(1 - x^2 - y^2)
\end{align*}
\]

by inspecting phase portrait of this equation and replacing \(1\) with \(r\), we will get a periodic orbit as shown in Figure 1.

\[
\begin{align*}
\dot{x} &= y + x(r^2 - x^2 - y^2) \\
\dot{y} &= -x + y(r^2 - x^2 - y^2)
\end{align*}
\]

This periodic orbit is called a limit-cycle. Figure 1a shows the trajectories from all points move towards the unit circle clockwise. On the other hand the unit circle counter-clockwise as shown in figure 1b, can be derived as follows:

\[
\begin{align*}
\dot{x} &= -y + x(r^2 - x^2 - y^2) \\
\dot{y} &= x + y(r^2 - x^2 - y^2)
\end{align*}
\]

to applied in navigation plan. Figure 2 shows how the limit cycle method can drive an aircraft into destination point and avoid an obstacle along its path.
Based on (7), there are some definitions using in limit-cycle navigation that:

- Variable obstacle \( O_v \) : By assuming a robot as a point mass, the variable obstacle is defined as circle with the radius is the total of its relative position to the aircraft, the total size of the obstacle and the aircraft. The aircraft will follow the boundary of the circle \( O_v \) in order to avoid any collision with obstacle. The radius of the variable obstacle \( r_v \) is defined as

\[
\tilde{r}_v = \tilde{r}_v + \tilde{r}_o + \delta
\]

- Disturbing obstacle \( O_d \) is variable obstacle that is in the way between the aircraft and target point. The nearest obstacle to the aircraft is defines as \( O_{d1} \), and so on. Non-disturbing obstacle \( O_{n1} \) is vice versa. \( O_{n1} \) is defined as the nearest obstacle to the aircraft and \( O_{n2} \) is the next one, etc.

Based on that definition, the steps to develop a path planning using the limit-cycle method are:

1. draw a line \( l \) from the robot to the target in a global coordinate \( \Sigma_{OXY} \) as follows:

\[
ax + by + c = 0
\]  
(5)

2. Define a variable obstacles as disturbing obstacle as \( O_d \) if the line \( l \) crosses them, or \( O_n \) if it is non-disturbing ones.

3. move towards the target if there is no \( O_d \)

4. Calculate the distance \( d \) from the center of nearest disturbing obstacle \( O_d \) to the line \( l \), using

\[
d = \frac{aQ_x + bQ_y + c}{\sqrt{a^2 + b^2}}
\]  
(6)

where \((Q_x, Q_y)\), \((G_x, G_y)\) and \((R_x, R_y)\) are the xy-values of the center positions of the obstacle, the target and the aircraft respectively. Calculate the desired direction of the aircraft at each position using

\[
\begin{align*}
\dot{x} &= \frac{d}{|d|} y + x(r_v^2 - x^2 - y^2) \\
\dot{y} &= -\frac{d}{|d|} x + y(r_v^2 - x^2 - y^2)
\end{align*}
\]  
(7)

where \( x \) and \( y \) are relative values to the obstacle. The aircraft avoids the obstacle \( O_d \) clockwise if \( d \) is positive and vice versa if \( d \) is negative. Calculate \( r_v \) as the total size of the aircraft and the relative position to the obstacle as follows:

\[
r_v = r_v + r_o + \delta
\]  
(8)

where \( r_v \) and \( r_o \) are the radius of the aircraft and the obstacle respectively. If obstacle is disturbing one, \( r_o = r \), where \( r \) is the radius of the real obstacle and \( r_o = 0 \) if it is non-disturbing obstacles. As the aircraft moves, the line \( l \) varies, so repeat the steps 2 to 4 until the destination reached.
3. 3D limit-cycle method
A several research have been introduced to adopt the limit cycle characteristic in planning the flight path for aircraft navigation. Because of the obstacle information is known in advance and it has to be carefully pre-planning the safe flight path for the vehicle.

The limit-cycle navigation should be extent into three dimensional environments for aerial vehicle application. In [8], the extension of 2D into 3D is considered as two plane that is horizontal and vertical which perpendicular each other. The existing limit-cycle method is applied in horizontal plane that affected only in xy-plane. The angle $\varphi$ is defined as the angle between the current position and the desired position as shown in figure 3.

![Figure 3. The scheme of extended limit cycle navigation method in 3D](image)

It can be obtained as :

$$d_h = (dx^2 + dy^2)^{1/2}$$  \hspace{1cm} (10)

$$\varphi = \tan^{-1}\left(\frac{dz}{dh}\right)$$  \hspace{1cm} (11)

where $dh$ is a tangent line between the current position and the tangent of an obstacle circle. By getting the angle $\varphi$, the $z$-value can be obtained by

$$\Delta h = (\Delta x^2 + \Delta y^2)^{1/2} \tan \varphi$$  \hspace{1cm} (12)

$$z = \Delta h + \bar{z}$$  \hspace{1cm} (13)

where $\bar{z}$ is the current $z$ position. The result in extending the 2D limit-cycle into 3D is shown in figure 3 as solid lines.

By applying this method, it shows a flight path for three dimensional environment. But it also has several drawbacks. First of all, it provide unsuitable path if the aircraft position below the sphere of variable obstacle. It clearly seen in figure 4 where it generate the path that impossible to follow if we consider $z = 0$ as ground level of the earth.
The second problem of extended limit-cycle navigation is not providing the best way to reach the destination point. As example, it provide inefficient flight path if the actual obstacle height is not defined as the limit of safe region. Figure 5 is the clear example of inefficient path that provided by this method. This two problem occur because the method is generates the path by only considering the center of gravity position of the obstacle.

Rendering the obstacle into a cylinder is the solution to solve this entire problem. The obstacle is considered as cylinder with radius is $r_o$ and height is $h$ and the xy-plane as a base of the cylinder. It defined as

$$(x - x_c)^2 + (y - y_c)^2 = r^2, \quad 0 \leq (z - z_c) \leq h$$

with $x_c$ and $y_c$ is x and y obstacle position respectively.

By applying this rule, the straight path will apply if the current position of the aircraft above the obstacle height although they intersect in their xy-plane. The determinant indicates whether an intersection occurs. The intersection with the z-coordinates must be in the range $0 \leq (z - z_c) \leq h$ if it identify as intersection points.
4. Result and discussion
In this section, the proposed path planning algorithm is verified using MATLAB. In this simulation work, we render the obstacle into a cylinder and apply the limit-cycle navigation in avoiding the obstacle that intersects with the cylinder.

Figure 6 shows comparison between the previous extended limit-cycle and the proposed method with obstacle render as cylinder.

![Simulation result of extended limit-cycle method](image)

(a)

![Simulation result of proposed method](image)

(b)

**Figure 6.** Simulation result of extended limit-cycle method: (a) previous method, (b) proposed method

The result of the simulation in figure 6 shows that the aircraft moved from initial point (0, 0, 0) to destination position (30,55,35) while avoiding obstacle at (25,40,55). Figure 6.a shows the result of previous extended limit-cycle simulation. It generates some path in certain position which have a value of \( z < 0 \). It will make an aircraft fall down to earth and crash. Figure 6.b shows how the obstacle that render as a cylinder is generated a safe flight path using the proposed limit-cycle navigation method.
Figure 7. Result when aircraft flight above the obstacle; (a) previous method, (b) proposed method

Figure 7 also shows an inefficient path generate using previous extended limit-cycle. We set up obstacle position at (25,40,15), it clearly seen that figure 7b is generate efficient path compare to the previous method as shown in figure 7a. The aircraft will straight forward if their current position is above the obstacle height. The previous method take 95.4519 point as the distance while the proposed method take 44.4934 point. It is 46.6134% efficiency in flight distance and it is the shortest path to reach destination point.

Consequently, the paths generated in these simulations were produce suitable and efficient path for aerial vehicle application.

5. Conclusion

The previous extended limit-cycle navigation method is always developed the path based on center of gravity position of the obstacle. Accordingly, the previous extended limit-cycle navigation method is generated inefficient path in certain condition. In this paper, we verified that rendering an obstacle as cylinder will improve the drawback of the previous method. The simulation results show that the proposed extended limit cycle method is provide an effective path compare to the previous method.
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