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Abstract
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Measurement of the Fermi constant by FAST

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A B S T R A C T
An initial measurement of the lifetime of the positive muon to a precision of 16 parts per million (ppm) has been performed with the FAST1 detector at the Paul Scherrer Institute. The result is \( \tau_{\mu} = 2.197083(32)(15) \) \( \mu s \), where the first error is statistical and the second is systematic. The muon lifetime determines the Fermi constant, \( G_F = 1.166352(9) \times 10^{-5} \) GeV\(^{-2} \) (8 ppm).

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1. Introduction

The Standard Model has three free parameters in the bosonic sector: the electromagnetic coupling constant, \( \alpha \), the mass of the Z boson, \( m_Z \), and the Fermi coupling constant, \( G_F \). The theory becomes predictive when these and several other fundamental parameters have been determined experimentally. By progressively improving the precision of these parameters, the theoretical predictions become increasingly precise and, in turn, the experimental measurements are increasingly sensitive to new physics beyond the Standard Model. Therefore, on quite general grounds, it is important for each of the fundamental parameters of the Standard Model to be measured with the highest possible experimental precision.

The Fermi coupling constant is determined from the measurement of the positive muon lifetime, \( \tau_{\mu} \), through the relationship

\[
\frac{1}{\tau_{\mu}} = \frac{G_F^2 m_{\mu}^5}{192\pi^2}(1 + \Delta q).
\]  

(1)

In order to avoid uncertainties in the capture rate of negative muons on the target nuclei, the more precise value of \( G_F \) is derived from the positive muon lifetime. In this equation \( \Delta q \) encapsulates the phase space factor for finite-mass leptons, and the higher-order QED and QCD corrections calculated in the Fermi theory, in which the weak charged current is described by a contact interaction. In contrast with the case for \( \alpha \), \( G_F \) is not afflicted with hadronic uncertainties at the 1-loop level since they are suppressed by a factor \( m_f^2/M_W^2 \), where \( m_f \) is a light fermion mass. This is discussed in detail by van Ritbergen and Stuart [1], who have computed the second-order QED corrections. The residual theoretical uncertainty in the determination of \( G_F \) due to hadronic uncertainties and higher-order QED corrections is less than 0.3 ppm [1].

Among the other parameters in Eq. (1), the error on the muon mass, \( m_{\mu} \), contributes an uncertainty of 0.21 ppm to \( G_F \). The contribution from the uncertainty in the muon neutrino mass is negligible, assuming the mass limits implied by neutrino oscillation experiments. Under the assumption of a pure V–A interaction in muon decay, the largest uncertainty in \( G_F \) comes from the error on \( \tau_{\mu} \), which is 18 ppm in the 2006 Review of Particle Properties [2], and recently reduced by a new measurement from the MuLan experiment at PSI to a precision of 11 ppm [3]. In this Letter, an initial measurement of \( \tau_{\mu} \) is presented from the FAST experi-

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1 FAST is an acronym of Fibre Active Scintillator Target.
dependent and simultaneous measurement of a detectors placed side-by-side, each one capable of making an in-
tom-designed updating discriminators with dual-threshold differ-
get of 32 beamline of the 590 MeV proton cyclotron at the Paul Scherrer
muons. Immediately in front of the target (BC2) and 4 m upstream (BC1).
the vertical position of the beam particle as it enters the target.
A wedge-shaped beam degrader ensures that the pion stopping po-
sitions are distributed uniformly through the target. A local copper
beam collimator defines a beam aperture of 160 × 100 mm² and
suppresses pion stops near the edges of the target. An array of 16
finger counters (Z counters), each 10 × 10 × 200 mm³, measures
the vertical position of the beam particle as it enters the target.
Two additional plastic scintillator beam counters are located im-
mediately in front of the target (BC2) and 4 m upstream (BC1).
The beam composition at the FAST detector has been determined
by measuring the time-of-flight of particles from the production
target. The beam comprises 80% pions, 14.5% positrons and 5.5%
uquons.

In order to reach the final goal of a 1 ppm measurement of \( G_F \),
FAST must handle a factor 100 higher data sample than earlier
muon lifetime experiments and, at the same time, reduce the sys-
tematic errors by at least an order of magnitude. The high rate
is achieved by means of a highly granular and fast detector that
can measure several muon decays in parallel. The imaging target
(Fig. 1) essentially comprises a large number of replicated mini-
detectors placed side-by-side, each one capable of making an in-
dependent and simultaneous measurement of a \( \mu^- \) decay. With
regards to systematic uncertainties, the detector is designed to
suppress several effects that limited previous experiments. Firstly,
pile-up effects are reduced by a fast, imaging detector. Secondly,
possible gain shifts and additional event losses at the start of the
muon decay period due to the higher counting rate are avoided
by operating in a DC beam (uniform counting rate). Thirdly, muon
spin polarisation systematics, arising from spin precession in the
geomagnetic field and non-isotropic positron detection efficiency,
are strongly suppressed by (a) using a \( \pi^- \) beam (spin 0) and tag-
ging the \( \pi \rightarrow \mu \) transition, (b) applying range cuts that further
suppress residual beam muons, (c) designing a uniform detector
with large angular acceptance, and (d) providing an 80 G magnetic
field across the target, which causes rapid precession of the muon
spins so that any residual spin effects can be observed and mea-
sured.

A two level trigger system is designed to identify the \( \pi \rightarrow \mu \)
chain in the detector. The first level trigger (LV1) selects beam pi-
on and provides the zero-time reference \( (t_e = 0) \) for the event.
The second level trigger (LV2) reconstructs the pion stop pixel and
identifies the subsequent \( \pi \rightarrow \mu \) decay.

The PSPM signals are passed through pre-amplifiers into cus-
tom-designed updating discriminators with dual-threshold differ-
ential ECL outputs. The low level (LL) threshold is satisfied by
minimum ionizing particles, and the LL pulses are sent to the time-
to-digital converters (TDCs). The high level (HL) threshold is set
to efficiently detect the large pulses from stopping pions or from
\( \pi \rightarrow \mu \) decays, but to suppress minimum ionizing particles. The HL pulses are sent to the LV2 trigger.

The time measurement is performed with 16 CAEN V767 128-
channel multihit TDCs [5]. They provide a fast time stamp for each
PSPM pulse and then load the pixel address and time into mem-
ory. Dual hit registers on each input channel of the TDCs ensure a
short minimum time separation of 10 ns for two hits to be regis-
tered on the same channel, although a longer deadtime of up to
several 100 ns occurs before a third hit can be registered on the
same channel since the contents of the hit registers must first be
written into the local event buffer. The TDCs are driven by an ex-
ternal 30 MHz Rb atomic clock. This defines a precise so-called
course TDC tick which is then divided by the delay locked loop of
the TDC chip into 32 fine TDC ticks, each of 1.041667 ns. The time
window of the TDCs is set to measure decays between \( \pm 8 \) µs and
\( \pm 22 \) µs relative to the pion stop time. Measurement of events with
a negative decay times allows the backgrounds to be studied.

The LV1 trigger defines an incident beam particle by a coinci-
dence of three beam counters (BC1, BC2 and Z) with the acceler-
ator RF signal. The leading edge of the LV1 trigger is determined
by the machine RF pulse (period \( \sim 19 \) ns). The LV1 deadtime is
50 ns, implying that no further triggers are accepted in the two
subsequent RF buckets.

The LV2 trigger uses the HL data from the target to reconstruc-
the \( \pi \rightarrow \mu \) decay. Having identified a prompt stopping beam pion,
in-time with LV1, the LV2 trigger is satisfied by a delayed second
pulse—within a 15–100 ns window—on any of the 3 × 3 pixels
centred on the pion stop pixel (the range of 4.1 MeV muons is
1.4 mm). The LV2 trigger then provides a selective trigger pulse
for the TDCs subtended by a 7 × 7 pixel array centred on the pion
stop pixel. This serves to reduce the data bandwidth requirements
of the data acquisition system (DAQ) since, on average, only 2.4 out
of the 16 TDC modules are triggered per event. The \( x \) and \( y \)
coordinates of the muon stop pixel are also sent to TDC data inputs
for use in the subsequent event analysis. The experiment reads
out only TDC data; each 128 ch TDC has 96 ch devoted to target
readout and 32 ch devoted to other data, including the pion stop
coordinates. The LV2 trigger operates at up to 1 MHz trigger rate
and can process multiple LV1 triggers in parallel, provided they are
separated by at least 50 ns.

The very high data rate of the FAST detector makes it impracti-
cal to store all events for later analysis offline. Not only would the
disk space requirements be huge—about 7 TB/day—but the read-
ning and reprocessing time for these events would far exceed the
original time taken for their collection. The adopted solution is to
carry out a full analysis of all data online, and to record approxi-
ately each hour a set of about 1200 separate muon lifetime his-
tograms that contain all the information required for the lifetime
measurement and for the study of systematic errors. In addition,
and, from the pixel hit pattern recorded in the TDCs, identifies any
hardware trigger.

The analysis procedure uses the muon coordinates given by LV2
and \( t_e \) from the pixel hit pattern recorded in the TDCs, identifies any
position candidate emerging from the muon pixel in the \( \sim 8 \) to
22 µs window. Positron tracks are identified using a set of 512
mask topologies in a so-called \( 5 \times 5 \) superpixel matrix centred on
the muon pixel. The mask topologies are based purely on geo-
metrical considerations and are chosen to be highly efficient for
accepting true positrons from muon decays (including the effect of pixel inefficiencies), while suppressing false positrons due to tracks from other beam particles or decays elsewhere in the target that overlap the superpixel. For example, a hit pattern corresponding to a track that appears to traverse the full superpixel is rejected since it is highly unlikely be caused by a true positron but has the signature of an overlapping background track. Once a candidate positron is accepted, the absolute decay time is taken as the average of all the positron pixels within the superpixel.

The readout of the TDCs is done via PVIC boards [6] which provide an interface between the DAQ PCs and the 4 VME crates containing the TDC modules. The data are transferred at the full VME bandwidth limit of 20 MB/s per crate into 4 DAQ PCs. A Gigabit ethernet switch and collector PC then build the events from the DAQ PCs into time slices and pass these slices in turn to a farm of event analyser PCs. The event analyser PCs process the events and write the histograms and other information to disk.

3. Data sample

The data were recorded in December 2006 at an average LV2 trigger rate of 30 kHz, which corresponds to 80 kHz LV1 and an average of about 3 beam particles per 30 µs trigger window. The muon lifetime histogram for the full data sample of $1.073 \times 10^{10}$ events is shown in Fig. 2(a), measured with the resolution of a fine TDC tick (1.041667 ns). The positron time is measured relative to the time of the beam particle as defined by the LV1 pulse (which is synchronised with the accelerator RF time). The analysis of the $5 \times 5$ superpixel surrounding a muon stop pixel allows one and only one positron candidate to be found in the $-8 \text{ to } 22 \mu s$ window; events failing this criterion are rejected. Events are also rejected if the pion stop position falls outside an x range of pixels defined by the associated Z counter (this suppresses a small contamination of beam muons since their range is 21 pixels further than pions). After some additional geometrical and quality criteria are applied, 42% of the LV2-triggered events are accepted for inclusion in the lifetime histograms (Fig. 2).

The appearance of muon decays at negative times involves the loss of the true positron and the detection of a fake positron. As seen in the inset of Fig. 2(a), the fake positron background is structured with the accelerator RF period and dominated by incoming beam particles. The events with negative decay times allow a precise description to be obtained for the accidental background. The events with positive lifetime are used to perform the muon lifetime measurement.

Fig. 2(b) shows the same data after rebinning by exactly the accelerator RF period ("RF rebinning"), as described below, which eliminates the beam structure. A slight exponential behaviour can be seen in the background near the boundaries of the measurement window, as anticipated in the original study of systematic errors for the FAST proposal [4]. The increase in background around $-8 \mu s$ is due to positrons from another pion that stops in the...
signal muon pixel before the early edge of the window, whereas the increase in background around +22 μs is due to another pion that stops in the signal muon pixel (thereby faking a positron) but whose subsequent positron emerges after the late edge of the window. Either of these background event types would be eliminated if it produced two candidate positrons within the window. In both cases, therefore, the background grows exponentially as the edge of the window is approached, with the rise time given by the muon lifetime.

4. Muon lifetime fit

The measurement of the muon lifetime is performed in three steps. First, the beam period is obtained with high precision from the negative decay region in Fig. 2(a). Then, the lifetime histogram is rebinned using the measured beam period as the bin width (Fig. 2(b)). Finally a fit to the rebinned histogram is performed with the muon lifetime as one of the free parameters. The rebinning minimises the influence of the periodic background structure on the measurement of the lifetime. A systematic error due to this fit method is assessed by comparing the lifetime value obtained from the data without rebinning (Section 5).

Two different methods were used to obtain the beam period. In both cases, the decay interval from −7000 to −1000 TDC ticks was used, since this maximised the statistics while minimising the possible systematic uncertainties related to the exponential tail. The first method consists of folding the full interval into a single period. The dispersion of the points around the mean shape of the background is a minimum for the real beam period (Section 5). The second method uses the Fourier transform of the data to determine the frequency with the highest power, which corresponds to the beam frequency. Both methods agree and determine the beam period to be 18.960051 TDC ticks, with a precision of better than 1 ppm. Since this does not correspond to an integer number of TDC ticks, the data are “RF rebinned” by dividing fractional TDC ticks into the two adjacent bins according to a linear sharing algorithm.

A further periodicity is introduced in the data by a non-linearity of the TDCs. Although the coarse ticks are perfectly paced by the 30 MHz Rb atomic clock, the 32 fine ticks are determined by the TDC chip interpolation. This was found to have a non-linearity with an amplitude of 0.1%, introducing periodicities of 16 and 32 ticks in the fit residuals. An identical TDC non-linearity was measured in both the beam data (by folding the residuals with a period of 1 Rb coarse clock tick) and in laboratory measurements (using random flat data).

The positive region of the lifetime distribution in Fig. 2(b) is fitted with the function

\[ N(t_\varepsilon) = f_\text{TDC}(t_\varepsilon)(Ae^{-t_\varepsilon/\tau_\mu} + Be^{t_\varepsilon/\tau_\mu} + C) \]

where \( t_\varepsilon \) is the positron time relative to the RF bucket of the beam pion, \( f_\text{TDC}(t_\varepsilon) \) accounts for the TDC non-linearity, and the parameters \( A, B \) and \( C \) describe the amplitude of each component. The first term represents the muon decay signal, the second term accounts for the small exponential rise of the background at the positive edge of the window due to the beam pion component, and the final term accounts for the flat, uncorrelated background. The free parameters of the fit are \( A, B, C \) and \( \tau_\mu \).

A binned maximum likelihood fit to the data is performed over the interval from 600 to 20000 TDC ticks.\(^2\) The lower limit is chosen to avoid the “third hit” TDC inefficiency at earlier times, and the upper limit avoids undue sensitivity to the rising background near the edge of the time window. The result for the lifetime is \( \tau_\mu = [2095.200 \pm 0.031] \) TDC ticks [2.197083(32) μs], with \( \chi^2/\text{dof} = 1.01 \) for 1020 degrees of freedom, corresponding to 40.1% probability. The residuals of the fit show no systematic trends versus decay time (Fig. 3(a)) and are Gaussian distributed, with the expected mean and width (Fig. 3(b)). The fitted signal fraction is 96.82%, while the positive exponential contribution is 0.02% and the flat background accounts for 3.15%.

Several further tests of the quality of the fit have been performed. The Fourier transform of the residuals reveals no periodicities. Furthermore, the fit has been separated into four regions to check the stability and quality in samples that are statistically independent. The four regions are 600–3000, 3000–6000, 6000–10000, and 10000–20000 TDC ticks. Within statistical errors, the fitted lifetime is the same in all four regions, as is the fitted constant background term. As expected, the late decay region above 10000 ticks is most sensitive to the correct description of the background, while the early decays are essentially insensitive to background uncertainties. Overall, no evidence has been found to suggest any problems with the quality of the fit.

\(^2\) Since the histogram is rebinned, the actual fitting region goes from 593.8 to 20008.9 TDC ticks.
5. Muon lifetime fit with fine binning

An alternative analysis has been carried out using the original lifetime histogram with a fine bin size of 1 TDC tick (Fig. 2(a)). The data are fitted in two steps. First, a description of the shape of the background is obtained from the negative region. Second, the data in the positive lifetime region are fitted with this description plus the decay exponential plus two other small corrections (TDC non-linearity and exponential rise of the background near the end of the window). The systematic uncertainties of this analysis are somewhat larger than for the nominal fit with the RF rebinned data since a more precise description is required for the DC background structure and for the pulsed, positive exponential background at the end of the TDC window. Nevertheless, this second analysis is important in order to check the consistency of the nominal fit and to assign a systematic error to the fit method.

A precise description of the negative region is obtained by folding all the data from −7000 TDC ticks to −1000 TDC ticks into one single RF period. The folded data (Fig. 4(a)) clearly show the three components of the beam background: muons (−15 TDC ticks), positrons (−9 ticks) and pions (bump near −6 ticks). The different timings of the particles are due to their individual times-of-flight from the production target. Due to the different probabilities to fake a decay positron, their relative normalisation does not correspond to the actual beam composition. The structure is well-described using a spline with 150 knots. The negative decay region is then fitted by the sum of an exponential, a flat background and the spline, repeated with the measured RF frequency. The fit—a region of which is shown in Fig. 4(b)—has a $\chi^2/N_{\text{dof}} = 1.12$ for 5995 degrees of freedom. Once the shape of the background is fixed using the negative region, the positive lifetime region is fitted using the function:

$$N(t_\tau) = f_{\text{TDC}}(t_\tau)(Ae^{-t_\tau/\tau_{\mu}} + B\tau_{\pi}f_{\text{p}}(t_\tau) + C + \text{Spline})$$

where $t_\tau$ is the positron time relative to the beam pion, the parameters $A$, $B$, and $C$ describe the amplitude of each component, $f_{\text{TDC}}$ is the correction due to the TDC non-linearity and $f_{\text{p}}$ is a periodic function describing the shape of the beam pion contribution. The free parameters of the fit are $A$, $B$, $C$ and $\tau_{\mu}$.

A binned maximum likelihood fit is performed to the data between 600 and 16000 TDC ticks. The fit region excludes longer-lived muons since they are most sensitive to uncertainties in the background description. The muon lifetime obtained with these fine binned data is 5.2 ppm below the nominal value obtained with the RF rebinned data.

6. Systematic errors

Possible sources of systematic error have been identified and their influence on the lifetime has been evaluated, in turn, with the experimental data. As discussed below, there is no evidence for any systematic error that significantly biases the nominal fitted value, taking its statistical precision into account. The chosen procedure is therefore to apply no correction to the central value obtained from the fit, but rather to take the maximum deviation resulting from each source to provide the estimate of the associated systematic uncertainty. This is a conservative estimate of the systematic error, resulting in a larger quoted error than would be obtained by correcting the central value. The procedure is described in detail in Ref. [7]. The possible sources of systematic error are evaluated in turn below.

1. Fit method: The stability of the measured lifetime is checked by varying the starting point and the end point of the fit over a large region. The results are stable and no associated systematic error can be discerned.

   The influence of the sharing algorithm for RF rebinning has been checked using three different methods: simple sharing, linear interpolation and quadratic interpolation. The observed difference between the measured lifetimes is below 0.1 ppm.

   The systematic error on the TDC non-linearity correction has been estimated using two different corrections, with different amplitudes and phases. No effect is observed on the fitted lifetime.

2. Sensitivity to the periodic beam background has been estimated by varying this parameter over one full beam period. The associated systematic error is $\Delta \tau_{\mu} < 0.1$ ppm. Furthermore the central value of the period has been varied by ±100 ppm, which is far outside the range of experimental uncertainty. The influence on the measured lifetime is $\Delta \tau_{\mu} < 0.2$ ppm.

   Four different methods have been used to perform the fit to test if there is any sensitivity to the analytical method: minimise the $\chi^2$, maximise the likelihood, minimise the Kullback-Leibler discrepancy and minimise the absolute value of the residuals. The various methods are sensitive to different features of the distribution. Since no differences are found, we conclude that the fitted lifetime is insensitive to the analytical method.

   Finally, in order to evaluate possible biases of the rebinned method, the results have been compared with an alternative evaluation using the fine binned data. The method used to obtain the lifetime from the fine binned histogram is described above. The results for both methods are shown in Fig. 5(a). In addition, the fine binned method has been applied to the data rebinned into integer
The binning of these histograms ranges from 1 to 20 TDC ticks per bin, depending on the effect under study.
Fig. 5. Examples of the evaluation of systematic errors, showing the muon lifetimes obtained as a function of various parameters: (a) bin size, (b) PSPM containing the $\pi$ stop pixel, (c) $\mu$ pixel location within the PSPM, and (d) position of the $\mu$ pixel relative to the $\pi$ stop pixel (number 5). In the first panel, the point at 19 TDC ticks corresponds to the nominal lifetime fit, using RF-rebinned data. The other points are derived from fits to the fine-binned data (Section 5). The statistical error bars are only shown for the results obtained on the RF-rebinned and the non-rebinned. There is a systematic difference between large bins and small bins due to the required precision in the description of the background structure. In the final three panels, the lifetimes are expressed in number of standard deviations away from the nominal value. The inserts explain the numbering schemes for the horizontal axes.

6. TDC performance: Occasionally during data taking, a TDC chip could lose synchronisation of the fine tick phase lock. This was automatically identified when it happened by a characteristic broadening of the time distribution of the beam pion pixels. The DAQ then ended the run, tagged that run as bad, re-initialised the TDCs, and started a new run. The tagged data samples were removed from the analysis. To eliminate the possibility of any residual effect from untagged events with broadened timing, the bad runs have been collected together and fitted for the muon lifetime. The value obtained is compatible with the nominal lifetime, within the relatively large statistical errors, $\Delta \tau_\mu = 15 \pm 60$ ppm. Further evidence against a residual error from this source is that there was a significant difference in susceptibility to this problem among the 16 TDCs, yet there is no sign of any different lifetime measurement from the susceptible TDCs. In conclusion, there is no evidence for any systematic effect from this source.

7. Muon spin rotation ($\mu$SR): Muon spin rotation effects are highly suppressed in FAST as a result of the basic experimental design. Nevertheless, spin effects can be extracted from the data by selection of special event topologies. These make use of the fact that the muon spin is 100% polarised in the direction pointing back to the parent pion. A $\mu$SR signal can be obtained, for example, by selecting events where the $\pi$, $\mu$ and positron are in the same pixel, or where the $\pi$ and $\mu$ are in the same pixel but not the positron. From these data, the $\mu$SR period is measured to be $T_{\mu SR} = 931$ TDC ticks (970 ns), in agreement with that expected for $\sim 80$ G magnetic field.

In order to understand $\mu$SR effects in the lifetime measurement, it is helpful to consider an idealised case where the pion is exactly centred on a pixel. In this case the daughter muon will lie somewhere on a sphere centred on the pion and of radius about 1.4 mm (corresponding to the range of a 4.1 MeV muon), with its spin pointing radially inwards at the time of the $\pi \to \mu$ decay. When projected onto the readout plane, this effectively produces a circular distribution of muon stop positions located (in this idealised case) near the edge of the 4 mm pixel, with spins pointing radially inwards. The net muon spin is zero and so, assuming uniform detection efficiency, no asymmetry should be observed in the positron emission directions. The muons then begin to precess about the magnetic field, with the largest precession angle for the forward–backward muons (i.e. along the beam direction and with spin direction perpendicular to the magnetic field) and the smallest for the left–right muons (transverse to the beam direction and with spin direction along the magnetic field).

The first check of $\mu$SR effects is therefore to see if there is any variation of the lifetime with respect to the relative position of the $\mu$ and the $\pi$. This is shown in Fig. 5(d); no systematic effects are observed.

The second check of $\mu$SR effects is to look at the time dependence of the residuals of the lifetime fit with respect to the direction of positron emission. This is shown in Fig. 6(a) and reveals...
a clear $\mu$SR signal in the forward–backward ($\phi=0^\circ$ and $180^\circ$) and left–right ($90^\circ$ and $270^\circ$) directions. The $\mu$SR amplitude is about 0.1% and the phase at $t_e=n\cdot T_{\mu SR}$, with $n=0,1,\ldots$, gives a peak deficiency for $0^\circ$ and $180^\circ$, and a peak excess for $90^\circ$ and $270^\circ$. This is due to a small difference in the solid angle for positrons to be classified as $0^\circ$ between muons located near the upstream compared with downstream edges of the pion pixel, which arises from the relatively coarse pixel granularity. For example, at $t_e=0$, the downstream muons have a higher solid angle for classification as $\phi=0^\circ$ decays than do the upstream muons, which leads to a net deficiency. The mirror effect (with the same phase) occurs for positrons at $180^\circ$. The small $t_e=0$ deficiency at $0^\circ$ and $180^\circ$ appears as a symmetric excess at $90^\circ$ and $270^\circ$, and cancels almost completely along the diagonal directions. This pattern then oscillates with the $\mu$SR frequency and gradually decays as the muons depolarise.

In short, the small $\mu$SR signal in the $0^\circ$ and $180^\circ$ directions due to muons emitted in the forward–backward direction is causing—and is therefore exactly compensated by—an anti-phase $\mu$SR signal in the $90^\circ$ and $270^\circ$ directions. This is confirmed by the central distribution in Fig. 6(a), which shows no $\mu$SR signal when all positron directions are combined. Fig. 6(b) shows the corresponding Fourier transform for each of the positron directions. The $\mu$SR signal is seen to be completely absent in the summed data. We conclude that no systematic errors are present due to $\mu$SR effects. Furthermore, the equality of the $\mu$SR signal in the $0^\circ$ and $180^\circ$ directions confirms that there is a negligible residual component of beam muons, which would produce a forward–backward $\mu$SR asymmetry.

7. Final result and conclusion

A summary of the estimated contributions to the systematic uncertainty is shown in Table 1. The total systematic error is obtained by first summing the same-sign errors in quadrature and then taking the average of the absolute values of the positive and negative errors [7]. This leads to an estimated total systematic error of $\pm 6.5$ ppm. The measured value of the $\mu^+$ lifetime is therefore $2109.200 \pm 0.031 \pm 0.014$ TDC ticks, i.e.

$$\tau_{\mu} = 2.197083(32)(15) \mu s,$$

where the first error is statistical and the second is systematic. This measurement, with 16 ppm overall precision, is consistent with both the previous world average (18 ppm precision) [2] and the new result from MuLan (11 ppm precision) [3]. From Eq. (1), our measurement of the muon lifetime determines the Fermi constant to be

$$G_F = 1.166352(9) \times 10^{-5} \text{ GeV}^{-2},$$

which may be compared to the world average value of $1.166371(1) \times 10^{-5} \text{ GeV}^{-2}$ in the 2006 Review of Particle Properties [2].

The uncertainty of our present measurement of $G_F$ to 8 ppm precision is dominated by the statistical error. The size of the data sample also limits the determination of the systematic errors. It is expected that both will decrease with larger data samples, allowing the FAST experiment to reach its final goal of determining the Fermi constant $G_F$ to 1 ppm precision.
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