Dark Matter in the MSSM Golden Region

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Abstract

Dark matter is examined within the “golden region” of the Minimal Supersymmetric Standard Model. This region satisfies experimental constraints, including a lower bound on the Higgs mass of 114 GeV, and minimizes fine-tuning of the Z boson mass. Here we impose additional constraints (particularly due to experimental bounds on $b \to s\gamma$). Then we find the properties of the Dark Matter in this region. Neutralinos with a relic density that provides the amount of dark matter required by cosmological data are shown to consist of a predominant gaugino (rather than higgsino) fraction. In addition, the U(1)$_Y$ gaugino mass parameter must satisfy $M_1 \lesssim 300$ GeV.

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I. INTRODUCTION

The Minimal Standard Supersymmetric Model (MSSM) is the simplest supersymmetric extension beyond the standard model of particle physics, and stands to be tested in the upcoming Large Hadronic Collider experiments at CERN. The MSSM not only addresses fundamental questions in particle physics, but also naturally provides a compelling dark matter candidate, the lightest supersymmetric particle (LSP). In particular, the neutralino, which is a linear combination of the supersymmetric partners of the photon, the Z boson, and the neutral scalar Higgs particles, has the right cross section and mass to automatically provide the observed density of cold dark matter in the universe. According to the analysis in [1], the latter has the value

$$\Omega_{\chi} h^2 = 0.1143 \pm 0.0034.$$  \hspace{1cm} (1)

Here subscript $\chi$ refers to neutralinos, $h$ is the Hubble constant $H_0$ in units of 100 km/s/Mpc, and $\Omega_{\chi} = \rho_{\chi}/\rho_c$ is the fraction of the neutralino density $\rho_{\chi}$ in units of the critical density $\rho_c = 3H_0^2/(8\pi G) \sim 10^{-29} h^2$ g/cm$^3$ (alternatively, $\Omega_{\chi} h^2$ is the neutralino mass density in units of 18.79 yg/m$^3$).

Perelstein and Spethmann [2] examined a particularly interesting region of MSSM parameter space which they dubbed the “golden region.” They argued that data and naturalness (i.e. a low degree of fine-tuning) point to a region within the Higgs and top sectors where the experimental bounds from non-observation of superpartners and the Higgs boson are satisfied and fine-tuning is close to the minimum possible value. They found that, in this region, (i) the two stop eigenstates have masses below 1 TeV, (ii) there is a significant mass splitting between the two stop mass eigenstates, typically $\delta m \geq 200$GeV, and (iii) the stop mixing angle must be nonzero. They then suggested collider signatures of the golden region that may be found at the LHC.

In this paper, we further examine this golden region, with a particular focus on discovering the properties of the dark matter within it. We use the numerical package DarkSUSY [3] to find the same golden region as [2], and apply additional constraints due to experimental bounds on $b \rightarrow s\gamma$ as well. Then we look for the parameter regime inside the remaining golden region that also gives the right relic density of neutralino dark matter.

Below we begin by reviewing the boundaries of the golden region, and then turn to the properties of the dark matter within it.
II. THE MSSM GOLDEN REGION

We work in the framework of the Minimal Supersymmetric Standard Model (MSSM). For practical reasons, and to match the choices in [2], we impose the following restrictions on the MSSM parameters: (1) we assume that all soft parameters are flavor-diagonal, (2) we assume a common soft mass for the first and second generation squarks, \( m_{\tilde{q}} = m_{Q^{1,2}} = m_{U^{1,2}} = m_{D^{1,2}} \), and for all sleptons, \( m_{\tilde{\ell}} = m_{L^{1,2,3}} = m_{E^{1,2,3}} \), (3) we set all tridiagonal terms \( A \) to zero except for \( A_t \), (4) we further assume that the third generation soft mass \( m_{D^3} = m_{\tilde{q}} \), but let \( m_{Q^3} \) and \( m_{U^3} \) vary independently. We remain with 11 free parameters: the Higgs mass parameter \( \mu \), the mass \( m_A \) of the CP-odd Higgs boson, the ratio \( \tan \beta \) of the Higgs vacuum expectation values, the gaugino mass parameters \( M_1, M_2, M_3 \), the soft parameters \( m_{\tilde{q}} \) and \( m_{\tilde{\ell}} \), and the third-generation soft parameters \( m_{Q^3}, m_{U^3} \) and \( A_t \). Furthermore, following [2], we replace the last three parameters \( (m_{Q^3}, m_{U^3} \) and \( A_t \) with the mass of the lightest stop \( \tilde{m}_1 \), the mass difference between the stop masses \( \delta m = \tilde{m}_2 - \tilde{m}_1 \), and the stop mixing angle \( \theta_t \). Finally, we use \( m_t = 174.3 \) GeV for the top-quark mass, and define all parameters at the weak scale.

We scan the 11-dimensional parameter space by generating random values of the parameters. In most of our results, we use the parameters within the following ranges (all dimensionful parameters are in GeV):

\[
\begin{align*}
80 < \mu < 500, & \quad 100 < m_A < 2000, \quad \tan \beta = 10, \\
100 < M_1 < 400, & \quad 100 < M_2 < 2000, \quad 100 < M_3 < 2000, \\
100 < m_{\tilde{q}} < 2000, & \quad 100 < m_{\tilde{\ell}} < 2000, \\
100 < \tilde{m}_1 < 1000, & \quad 100 < \delta m < 600, \quad \theta_t = \pi/4.
\end{align*}
\]

Notice that we fixed the value of \( \theta_t \) and \( \tan \beta \) to reproduce one of the panels in Fig. 2 of [2]. We define our “default scan” to be the case where these values are fixed at \( \tan \beta = 10 \) and \( \theta_t = \pi/4 \). We also produced other special scans: we randomized \( \tan \beta \) in the range 0.5 to 30; we implemented the GUT relation between \( M_1, M_2, \) and \( M_3 \); we separately extended \( M_1 \) up to 2000 GeV.

The points in Fig. illustrate the golden region we obtain in our default scan. We have plotted the mass difference between the stop masses \( \delta m = \tilde{m}_2 - \tilde{m}_1 \) as a function of the
FIG. 1: The Improved Golden Region: Mass difference between the stop masses $\delta m = \tilde{m}_2 - \tilde{m}_1$ as a function of the mass of the lightest stop $\tilde{m}_1$. We have scanned SUSY parameter space with constraints imposed from the bound on the Higgs mass, fine tuning, and other experimental constraints including $b \rightarrow s\gamma$ to find an “improved” golden region; here we have fixed $\theta_t = \frac{\pi}{4}$ and $\tan\beta = 10$ as our “default scan” in parameter space. For comparison, the area enclosed by the solid line shows the corresponding golden region previously found in Fig. 2 of [2]. Note that our lowest value of $\delta m = 150\text{GeV}$ is lower than that in the previous work as discussed in the text.

mass of the lightest stop $\tilde{m}_1$. This plot can be directly compared with Fig. 2 of [2], from which the solid triangular region is drawn. Using DarkSUSY, we have improved upon the previous work by applying more accurate calculations of the Higgs boson mass and of the $b \rightarrow s\gamma$ branching ratio, as described below. A variety of other experimental constraints are applied as well, using their implementation in DarkSUSY. In particular, LEP2 searches for direct production of charginos and stops constrain the chargino and stop masses, resulting in $\mu \gtrsim 80\text{ GeV}$ and $\tilde{m}_1 \gtrsim 90\text{ GeV}$. The $b \rightarrow s\gamma$ decay rate [7] also constrains the golden region. Whereas [2] noted that including this constraint was mostly beyond the scope of their paper, we have implemented this constraint throughout. Hence our results in Fig. [1]
illustrate the “improved” golden region in the presence of this additional constraint (and of a more accurate calculation of the Higgs boson masses).

We note that [2] defined a benchmark point in their Table 1, with particular choices of the other parameters, and did include $b \rightarrow s\gamma$ ratio when they checked this point using SuSpect. SuSpect gave them an acceptable $b \rightarrow s\gamma$ ratio for their benchmark model, but DarkSUSY (which includes better expressions for the branching ratio with NLO corrections) gives an unacceptable value.

The golden region in Fig. 1 contains 7000 points satisfying all relevant bounds. The region is a triangle, with the lower boundary due to the constraint on the Higgs mass, the leftmost boundary due to the bounds on the $\rho$ parameter, and the upper boundary due to fine-tuning. We now discuss each of these bounds in turn.

A. Lower boundary of golden triangle: Higgs mass

The lower boundary of the golden region triangle is set by bounds on the Higgs mass. The LEP2 lower bound on the standard model Higgs mass is

$$m(h^0) \geq 114\text{GeV}.$$  \hspace{1cm} (6)

For generic MSSM parameter choices, the limit on the lightest Higgs is very close to this value as well and we may use this bound. At tree level, the MSSM predicts $m(h^0) \leq m_Z|\cos2\beta|$, so that large loop corrections are required to satisfy this bound. The dominant one-loop corrections are from top and stop loops. The numerical package FeynHiggs [6] is incorporated into DarkSUSY to properly compute the Higgs mass and to apply the experimental bound. We note that the resulting boundary to the golden region that we find is slightly different from that of [2]: e.g. at $\tilde{m}_1 = 440\text{GeV}$, our lowest value of $\delta m = 150\text{GeV}$ is quite a bit lower than their lowest value of $\delta m \sim 280$. The reason for this discrepancy is that their analytic approximations for the Higgs masses are less accurate than the values we find using the numerical package.
B. Upper boundary of golden triangle: Fine-Tuning Constraint

Fine-tuning of the Z mass also constrains the Higgs sector. At tree level, the Z mass in the MSSM is given by

$$m_Z^2 = -m_u^2 \left( 1 - \frac{1}{\cos 2\beta} \right) - m_d^2 \left( 1 + \frac{1}{\cos 2\beta} \right) - 2|\mu|^2,$$

(7)

where

$$\sin 2\beta = \frac{2b}{m_u^2 + m_d^2 + 2|\mu|^2}.$$  

(8)

Since one of the two CP-even Higgs masses must satisfy $m_{u,d}^2 < 0$ for electroweak symmetry breaking, and since experimentally it is found that at least one of $m_{u,d}, |\mu| \gg m_Z$, cancellation of the terms on the right hand side is required in order to get the right value of $m_Z$. Following Barbieri and Giudice [4], one may quantify this fine-tuning by computing

$$A(\xi) = \left| \frac{\partial \log m_Z^2}{\partial \log \xi} \right|$$

(9)

where $\xi = m_u^2, m_d^2, b, \mu$ are the relevant Lagrangian parameters. Then

$$A(\mu) = \frac{4\mu^2}{m_Z^2} \left( 1 + \frac{m_A^2 + m_Z^2}{m_A^2} \tan^2 2\beta \right),$$

$$A(b) = \left( 1 + \frac{m_A^2}{m_Z^2} \right) \tan^2 2\beta,$$

$$A(m_u^2) = \frac{1}{2} \cos 2\beta + \frac{m_A^2}{m_Z^2} \cos^2 \beta - \frac{\mu^2}{m_Z^2} \times \left( 1 - \frac{1}{\cos 2\beta} + \frac{m_A^2 + m_Z^2}{m_A^2} \tan^2 2\beta \right),$$

$$A(m_d^2) = -\frac{1}{2} \cos 2\beta + \frac{m_A^2}{m_Z^2} \sin^2 \beta - \frac{\mu^2}{m_Z^2} \times \left( 1 + \frac{1}{\cos 2\beta} + \frac{m_A^2 + m_Z^2}{m_A^2} \tan^2 2\beta \right),$$

(10)

where it is assumed that $\tan \beta > 1$. The overall fine-tuning $\Delta$ is defined by adding the four $A$’s in quadrature; values of $\Delta$ far above one indicate fine-tuning. Following [2], we require $\Delta \leq 100$, corresponding to fine tuning of 1% or better; this bound is implemented in our work and produces an upper bound $\mu \lesssim 440$ GeV for $\tan \beta \geq 0.5$. This matches the upper bound on $\mu$ found in [2]. We impose the constraints on the chargino mass from LEP2 chargino searches, which select $\mu \gtrsim 80$ GeV.

The upper boundary of the golden region triangle is determined by further restrictions on the fine-tuning. Quantum corrections to naturalness also constrain the size of the quantum corrections to the parameters in Eq. (7). Following [2], here we consider the largest correction...
in the MSSM, namely the one-loop contribution to the \( m_u^2 \) parameter from top and stop loops:

\[
\delta m_u^2 \approx \frac{3}{16\pi^2} \left( y_t^2 \left( \tilde{m}_1^2 + \tilde{m}_2^2 - 2m_t^2 \right) + \frac{(\tilde{m}_2^2 - \tilde{m}_1^2)^2}{4v^2} \sin^2 2\theta_t \right) \log \frac{2\Lambda^2}{\tilde{m}_1^2 + \tilde{m}_2^2}.
\]  

(11)

Here \( m_t \) is the top mass, \( \Lambda \) is the scale at which the logarithmic divergence is cut off, and finite (matching) corrections have been ignored. The correction to the Z mass induced by this effect is

\[
\delta t m_Z^2 \approx -\delta m_{H_u}^2 \left( 1 - \frac{1}{\cos 2\beta} \right).
\]  

(12)

To measure the fine-tuning between the bare (tree-level) and one-loop contributions, \cite{2} introduced

\[
\Delta_t = \left| \frac{\delta_t m_Z^2}{m_Z^2} \right|.
\]  

(13)

Choosing the maximum allowed value of \( \Delta_t \) selects a region in the stop sector parameter space, \( (\tilde{m}_1, \tilde{m}_2, \theta_t) \), whose shape is approximately independent of the other parameters. This constraint is outlined by the upper edge in Fig. \ref{fig:golden_triangle} which corresponds to \( \Delta_t \leq 33.3 \) (3% fine tuning). Note that the particular value of \( \Delta_t \) depends on the scale \( \Lambda \); we choose \( \Lambda = 100 \) TeV in this figure. As pointed out in \cite{2}, the shape of the \( \Delta_t \) contours and the obvious trend for fine tuning to increase with the two stop masses is independent of \( \Lambda \).

C. Left Boundary of Golden Triangle: \( \rho \) parameter

The left boundary of the golden region triangle is set by measurements of the \( \rho \) parameter, which obtains corrections from stop and sbottom loops. We compute the \( \rho \) parameter using DarkSUSY and require

\[
(2 - 8) \times 10^{-4} \leq \rho - 1 \leq (2 + 8) \times 10^{-4},
\]  

(14)

which represents the 2\( \sigma \) range from \cite{8}.

III. DARK MATTER

Now that we have found the improved golden region with the \( b \to s\gamma \) bound implemented, we can investigate the properties of the dark matter in this region. Using DarkSUSY, we
find the neutralino relic density for each set of MSSM parameter values in the improved golden region.

Fig. 2 shows the relic density as a function of neutralino mass for all our default points in the golden region. As discussed previously, our default scan is defined by fixing $\tan \beta = 10$ and $\theta_t = \pi/4$ and scanning over other parameters. The lines illustrate the band that satisfies the cosmological requirement of $\Omega_\chi h^2$ in Eq.(1). Notice that there are points with the correct density for practically all neutralino masses in the golden region (with perhaps a tiny exception at the largest masses). Although most of the points in the figure have small $\Omega_\chi h^2$, we remind the reader that the density of points, in this and all the other plots is arbitrary. The simulation uses a random number generator in the parameter domain. The dots are used for a plot as long as they satisfy the criteria for the golden region. Thus, the dot density doesn’t necessarily have a physical meaning. It simply represents how the random number generator creates the dots.

In our default scan, we allowed the gaugino mass parameters $M_{1,2,3}$ to vary independently. If we instead impose the GUT relations

$$M_1 = \frac{5}{3} \tan^2 \theta_W M_2, \quad (15)$$

$$M_3 = \frac{\alpha_s(m_Z)}{\alpha} \sin^2 \theta_W M_2, \quad (16)$$

we still find points satisfying the cosmological constraint on $\Omega_\chi h^2$. This is shown in Fig. 3 obtained by using the parameter ranges in Eqs. 2-5 but with the additional GUT conditions on $M_1$, $M_2$, and $M_3$ imposed.

To understand the properties of the points with the cosmologically interesting values of $\Omega_\chi h^2$, we have analyzed the dependence of $\Omega_\chi h^2$ in our default sample on all of the 11 independent parameters in the Lagrangian. Most of the parameters showed no interesting connection with $\Omega_\chi h^2$, except for $M_1$, and the gaugino fraction $Z_g$.

Fig. 4 shows the relic density as a function of $Z_g/(1 - Z_g)$ where $Z_g$ is the gaugino fraction of the neutralino. The denominator $1 - Z_g$ is the higgsino fraction. Points in the cosmologically interesting band typically have $Z_g/(1 - Z_g) > 1$. Thus, the dark matter in the golden region that satisfies Eq.(1) is predominantly gaugino rather than higgsino.

Figs. 5(a) and (b) plot the relic density as a function of $M_1$, the mass of the $U(1)_Y$ gaugino. Fig. 5(a) is for our default scan, while Fig. 5(b) is for an extended scan of 1,700 points in which $\tan \beta$ varies in the range 0.5 to 30 and $M_1$ is allowed to be as large as 2000.
FIG. 2: Neutralino relic density $\Omega \chi h^2$ as a function of neutralino mass $m_\chi$ for our default scan in parameter space ($\tan \beta = 10$, $\theta_t = \pi/4$). Each dot represents a point in supersymmetric parameter space that lies within the golden region. The horizontal band shows the 2σ range in the measured value of the cosmological density of cold dark matter [1]. Notice that there are points falling into the cosmological band for $m_\chi \leq 300$ GeV.

One can see that points with the right relic density to explain WMAP have $M_1 < 300$ GeV.

IV. CONCLUSION

In conclusion we have imposed further experimental bounds ($b \rightarrow s\gamma$) on the golden region found by [2] and have searched for subsets of this region which provide today’s dark matter density in the form of neutralinos. We found that $M_1 < 300\text{GeV}$ is required, and that the neutralino that is the dark matter is predominantly gaugino. In the future, we plan to examine direct and indirect detection rates in concert with LHC tests of the golden region.
FIG. 3: Same as Fig. 2 except that $M_1$, $M_2$, and $M_3$ are related through the GUT relations, Eqs. (15-16). Notice that there are points falling into the cosmologically-interesting band.

V. ACKNOWLEDGEMENT

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FIG. 4: Neutralino relic density $\Omega_\chi h^2$ as a function of $\frac{Z_g}{1 - Z_g}$ for our default scan in parameter space. Each dot represents a point in supersymmetric parameter space that lies within the golden region. The horizontal band shows the $2\sigma$ range in the measured value of the density of cold dark matter [1]. Notice that there are points only above $\log \frac{Z_g}{1 - Z_g}$=0. One can see that the neutralino is typically predominantly gaugino rather than higgsino.

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FIG. 5: Neutralino relic density $\Omega_{\chi} h^2$ as a function of $M_1$ for (a) our default scan in parameter space, and (b) an extended scan with $0.5 < \tan \beta < 30$ and $100 \text{ GeV} < M_1 < 2000 \text{ GeV}$. Each dot represents a point in supersymmetric parameter space that lies within the golden region. The horizontal band shows the $2\sigma$ range in the measured value of the density of cold dark matter [1]. Notice that there are points in the golden region with the right cosmological neutralino density provided $M_1 < 300 \text{ GeV}$. 
