Cosmic ray transport in the Galaxy

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Abstract. The problem of cosmic ray propagation is briefly discussed.

1. Introduction
The overall energy spectrum of cosmic rays at $E > 10^{10}$ eV where it is not significantly influenced by the solar wind modulation is well described by a power law with the evident feature, the “knee”, at $4 \times 10^{15}$ eV, the possible second knee at $3 \times 10^{17}$ - $10^{18}$ eV, and the flat supposedly extragalactic component at energies larger than about $3 \times 10^{18}$ eV. The origin of relativistic particles is associated with the most energetic astronomical objects in the Galaxy primarily with supernovae and also with pulsars, compact accreting systems, powerful stellar winds, and Galactic progenitors of gamma-ray bursts. The charged energetic particles diffuse in random magnetic fields that accounts for their high isotropy and relatively long confinement time in the Galaxy. The galactic diffusion model explains the data on particle energy spectra, composition, and anisotropy. It also provides a basis for the interpretation of radio-astronomical, X-ray and gamma-ray measurements since the non-thermal continuous radiation in space is produced by the energetic charged particles – electrons, protons, and nuclei. The diffusion approximation works for particles with energies not much larger than $10^{17}$ Z eV ($Z$ is the particle charge). The trajectory calculations in galactic magnetic field are employed at higher energies.

2. Nature of cosmic ray diffusion
The relatively old age of cosmic rays suggest that the energetic particles are "mixed" efficiently and are retained rather well in the Galaxy. The galactic magnetic field plays a crucial role in this process. The theory of energetic particle transport in galactic magnetic fields is constructed in much the same way as in the well studied case of particle transport in the interplanetary magnetic fields, see [33, 7]. The presence of random magnetic field causes the pitch-angle scattering and gives the spatial diffusion of cosmic rays. The scattering is of resonant character so that a particle with Larmor radius $r_g$ mainly interacts with magnetic homogeneities which have a wave number $k = 1/r_g$. A charged particle with momentum $p$ and charge $Ze$ has a Larmor radius $r_g = pc/ZeB \approx 10^{12} R_{GV}$ cm in a typical interstellar magnetic field $B = 3 \mu$G (here $R_{GV}$ is the particle magnetic rigidity in GV; the definition of particle magnetic rigidity is $R = pc/Ze$). The Larmor radius is of the order of the principal scale of interstellar turbulence $L \sim 100$ pc at particle energy $E \sim 10^{17}$ Z eV. The random component of interstellar magnetic field with an extended, approximately Kolmogorov, spectrum of inhomogeneities from 100 pc down to $10^8$ cm can provide the efficient particle scattering and the spatial diffusion of cosmic rays with energies less than $\sim 10^{17}$ Z eV.
The diffusion coefficient of cosmic rays can be roughly estimated as
\[ D \approx \frac{v r_g B^2}{3} \frac{\partial B_{\text{res}}}{r_{\text{g}}}, \]
where \( B_{\text{res}} \) is the amplitude of random field at the resonant scale \( k^{-1} = r_g \). This gives approximately \( D_K = 3 \times 10^{28} \) \( \beta R_{\text{GV}}^{1/3} \) cm\(^2\)/s for statistically isotropic random field with Kolmogorov spectrum that is in agreement with a value found in the empirical diffusion model of cosmic ray propagation in the Galaxy, see below. The large scale convective motion of the interstellar gas with a frozen magnetic field leads to the convective transport of energetic particles that supplements their resonant diffusion.

The diffusion is anisotropic locally and directed predominantly along the magnetic field but the large scale wandering of magnetic field lines makes diffusion close to isotropic on scales larger than 100 pc. The analysis of this anomalous perpendicular diffusion in quasistatic interstellar magnetic fields is not trivial and requires the consideration beyond the scope of standard theory of weak turbulence [13, 15, 11]. The cosmic ray diffusion on the “anisotropic in k-space” interstellar turbulence is the focus of attention of the present-day theoretical investigations, see [35] and references therein.

3. Basic diffusion model

The procedure of the modeling of cosmic ray propagation in the Galaxy can be summarized in the following way. One must first specify the cosmic ray sources, define the shape of the cosmic ray halo and the conditions at its boundaries (it is generally assumed that the energetic particles are ejected freely into intergalactic space, in which the cosmic ray density is negligible). The basic diffusion-convection equations for different cosmic ray species should incorporate possible energy loss and gain processes in the interstellar medium, nuclear fragmentation, and radioactive decay of unstable nuclei. One can then calculate the distribution functions of protons and the different types of nuclei. The empirical transport coefficients of cosmic rays (diffusion coefficient and convection velocity), the properties of cosmic ray sources (total power, energy spectra of different components, elemental and isotopic composition), and the size of confinement region of cosmic rays in the Galaxy can be found from the fit to all available data on cosmic rays.

The basic model for the investigation of cosmic-ray propagation in the Galaxy is the flat halo diffusion model [17, 7]. The model has simple geometry which reflects, however, the most essential

![Figure 1](image.png)

**Figure 1.** A schematic representation of the region in which cosmic rays propagate in the Galaxy. The Sun location is indicated
features of the real system. It is assumed that the system has the shape of a cylinder with a radius \( R \) (~20 kpc) and a total height \( 2H \) \( (H > 3 \text{ kpc}) \), see Figure 1. The cosmic-ray sources are distributed within an inner disk having characteristic thickness \( 2h \) (~300 pc).

Hundreds of stable and radioactive isotopes should be included in the calculations of nuclear fragmentation and transformation of energetic nuclei in the course of their interaction with interstellar gas. The electron-positron component and the energetic antiprotons, which are produced in a course of cosmic-ray propagation, also need to be incorporated in the calculation procedure. The approximate analytical solutions, e.g. [16, 21], and the full-scale numerical solutions, e.g. [31], of the diffusion transport equations for the entire Galaxy and for all cosmic-ray species are used. The simulation of cosmic-ray transport and comparison with the data on cosmic-ray energy spectrum, composition, and anisotropy allow the determination of parameters of the model: the source power (about \( 3 \times 10^{40} \text{ erg/s} \) in the entire Galaxy), the energy dependent diffusion coefficient (with a characteristic value \( 3 \times 10^{28} \text{ cm}^2/\text{s} \) at energy \( \sim 1 \text{ GeV/n} \)), the size of cosmic-ray halo (about 4 Kpc in the static model without Galactic wind), the leakage time of cosmic rays from the Galaxy (about \( 10^8 \text{ yr} \) at energy \( \sim 1 \text{ GeV/n} \)), the velocity of possible convective transport of cosmic rays (about 20 km/s at distances up to a few kiloparsecs above the Galactic plane), etc.

The necessary degree of complexity and detalization of the galactic model employed in cosmic ray research depends on the specific problem under the investigation. Below we illustrate this statement by a few examples.

The crude "leaky box" approximation may work in some cases. Under this approximation, the diffusive-convective transport of cosmic rays from the Galaxy is described by some escape time \( T_{lb} \) and no spatial dependence of cosmic ray distribution, source density, escape time, and any other parameters are taken into account. It can be shown that for an observer in the galactic disk and for the calculations of abundances of not very heavy stable nuclei, the leaky box model gives the same results as a flat halo diffusion model. The escape length, \( X_{lb} = \rho v T_{lb} \) (\( \rho \) is the mean density of interstellar gas), the parameter which determines the nuclear spallation of cosmic rays in the leaky box model, can be expressed in the terms of diffusion model as \( X_{lb} = \mu v H/(2D) \), where \( \mu \) is the surface gas density of Galactic disk. It is obvious however that the homogeneous leaky box approximation can not be used for studies of very inhomogeneous radioastronomical and gamma-ray diffuse radiation of our Galaxy.

The caution should be also used in the studies of rapidly decaying cosmic-ray secondary isotopes (\( ^{10}\text{Be}, \text{Al}^{27}, \text{Cl}^{36} \) and other) which have very inhomogeneous distribution in the Galaxy. The observed abundance of these isotopes depends on the value of the diffusion coefficient and on the details of complex spatial distribution of interstellar gas in the local Galactic vicinity of a few hundred parsecs around the Solar system.

The usual approximation of a continuous steady state distribution of cosmic ray sources can not be always used in the calculations. This is the case for the transport of very high energy electrons. The energy loss time of electrons with energy about \( 1 \text{ TeV} \) is only \( \sim 10^5 \text{ yr} \) in the interstellar medium. These particles may diffuse to the Solar system only from a dozen of nearby sources (supernova remnants) so that the knowledge about the exact location and temporary evolution of these local sources is absolutely essential.

The different approach is required for the analysis of cosmic ray anisotropy. In principle, the tensor character of cosmic ray diffusion and the structure of magnetic field in the vicinity of an observer should be taken into account in the calculation of cosmic ray angular distribution.

One of the major channels of information about cosmic-ray propagation is the abundance of secondary energetic nuclei produced as the result of spallation of more heavy primary nuclei interacting with the interstellar gas. The observed ratio of fluxes of secondary to primary nuclei, for example the Boron to Carbon ratio, is decreasing with energy at \( E > 1 \text{ GeV/nucleon} \) that is naturally explained by the increase of the diffusion coefficient with magnetic rigidity \( D \sim v B^a \). The index \( a = 0.55 \) was found in the empirical plain diffusion model, and \( a = 0.3 \) was determined in the diffusion model with cosmic ray distributed reacceleration by the interstellar turbulence. The leakage time of
cosmic rays from the Galaxy decreases correspondingly. It does not refer to the energies less than 1 GeV/n where the rising with energy ratio of fluxes of secondary to primary nuclei is observed. Several versions of the basic diffusion model were considered in an attempt to determine which may explain the peak in secondary to primary ratio at 1 GeV/n and gives a better fit to the data at high energies, see e.g. [21]. The most popular explanations employ the effect of cosmic ray convection transport in the presence of galactic wind flow, and the influence of distributed stochastic reacceleration of cosmic rays in the interstellar medium on the spectra of primary and secondary species.

4. Anisotropy of cosmic rays
The reliable data on secondary nuclei in cosmic rays are available for energies up to about 100 GeV/n. The absence of considerable changes in cosmic ray spectrum at higher energies implies that the source spectrum and the energy dependence of cosmic ray escape from the Galaxy remain approximately the same as they are at low energies at least up to the knee at $4\times10^{15}$ eV. The important information on cosmic ray propagation at very high energy is provided by the measurements of cosmic ray anisotropy at energies above a few hundred GeV/n where the observations are not affected by the solar wind flow which modulates the intensity of galactic cosmic rays.

Figure 2 shows the results of calculations of cosmic ray anisotropy in the frameworks of basic diffusion model for two types of scaling: $D \sim v R^{0.55}$ (the plain diffusion model; dash lines) and $D \sim v R^{0.3}$ (the reacceleration model; solid lines).

![Figure 2. Cosmic ray anisotropy for an observer at the solar system: dash lines – plain diffusion, solid lines – reacceleration model. The data are taken from [2, 3].](image)

It is assumed that diffusion is isotropic and the cosmic rays are produced during the point instant bursts of supernovae distributed in the Galaxy according to [10]. The supernova rate is $1/(30 \text{ yr})$ and
the same amount of cosmic rays is released after every SN burst. It is assumed that the power-law source spectrum of an ion with charge \( Z \) experiences a break at \( 4 \times 10^{15} \) eV that reproduces the knee in the observed all particle spectrum.

The average anisotropy produced by the global leakage of cosmic rays from the Galaxy is shown by the thick lines in Figure 2. The statistical nature of SN bursts results in the fluctuation anisotropy shown separately by the thin lines. This “typical” fluctuation anisotropy was calculated under the assumption of random distribution of SN events in space and time. The theory of cosmic ray fluctuations [23, 7] shows that the main contribution to the fluctuation anisotropy is expected from the nearby (the distances up to 1 kpc) and recent (the ages up to \( 10^5 \) yr) SN events. The effect of such supernovae including Vela, RX1713.7-3946, S147, S185 and some others is also shown in Figure 2.

It is clear from Figure 2 that the data on cosmic ray anisotropy agree satisfactorily with the model characterized by the weak dependence of diffusion on energy \( D \sim v R^{0.3} \). The plain diffusion model with its strong dependence of diffusion coefficient on energy \( D \sim v R^{0.55} \) is difficult to reconcile with the data on anisotropy.

It should be noted that a nearby SNR might be responsible for some small features (peaks) in the observed cosmic ray spectrum between \( 10^{16} \) and \( 10^{17} \) eV [14].

5. Collective effects: galactic wind driven by cosmic rays
The galactic cosmic rays are not always treated as test particles moving in given magnetic fields. The energy density of cosmic rays estimated as \( w_{cr} \sim 1 \) eV/cm\(^3\) is approximately equal to the energy density of magnetic field and to the energy density of turbulent motions of the interstellar gas. The presence of non-thermal component in the interstellar medium leads to the collective (plasma) effects and in particular to the cosmic ray streaming instability. The cosmic ray streaming instability of magnetohydrodynamic waves, which develops when the bulk velocity of cosmic rays exceeds Alfven velocity, is the most important. As a result, the cosmic rays produce turbulence which selfconsistently determines the cosmic ray diffusion coefficient, see e.g. [12]. Also, the pressure of energetic particles is an essential factor in the large-scale dynamics of interstellar gas [24]. In particular, the cosmic ray pressure can drive the Galactic wind [20].

In the most advanced model of magnetohydrodynamic wind driven by cosmic rays in a rotating galaxy [36, 26], the relativistic particles operate "at full power". The cosmic ray pressure accelerates the wind flow, and the current of energetic particles moving along the regular spiral magnetic field out of the galaxy generate Alfvénic turbulence. The galactic wind velocity increases with distance from the galactic midplane from about 30 km/s at a few kpc to the asymptotic value \( \sim 450 \) km/s at distances more than a few hundred kpc. The Galactic wind termination shock is supposedly located at approximately 300 kpc. The selfconsistently determined diffusion coefficient is of the order \( D_s \sim 10^{27} \beta R_{GV} \gamma_s^{-1} \) cm\(^2\)/s, where \( \gamma_s \) is the exponent of the power law source spectrum on rigidity.

The last value is one-two orders of magnitude smaller than \( D_K \) at 1 GV but more rapidly increases with rigidity. The cosmic ray streaming instability can not work in the region adjusted to the galactic midplane (the full thickness of this region is 1-3 kpc) where it is suppressed by the dissipation on the ion-neutral collisions in thermal plasma. The turbulence in this region can be created by external sources (the supernovae and stellar winds) and the diffusion coefficient is probably close to \( D_K \).

It should be pointed out that the mere existence of Galactic wind and in particular the wind driven by cosmic rays remains unproved. However, in general, the diffusive nature of cosmic ray motion in the Galaxy with or without a wind is strongly supported by the studies of cosmic ray transport in different empirical models.

6. Nature of double knee structure in cosmic ray spectrum
The knee might just indicate the limit of proton acceleration by supernova shocks. The efficient acceleration of cosmic rays with energies up to \( 10^{14} \) eV in young supernova remnants has been established by the observations of non-thermal X-ray and TeV gamma-ray radiation from SNRs and is consistently described by the theory of diffusive shock acceleration. In contrast, the explanation of
cosmic rays at higher energies remains a challenge. The standard theoretical estimate of the maximum energy of accelerated particles in SNRs is a few $10^{14}$ Z eV (Z being the nuclear charge), e.g. [6]. It is based on the Bohm limit for cosmic ray diffusion in interstellar magnetic field and is close to the observed position of the knee at $3\times10^{15}$ eV.

Further acceleration could be achieved in a two-stage model where individual supernova remnants accelerate particles up to the knee, followed by a subsequent, collective reacceleration on shocks produced by other supernovae [9]. Also, plerions (the supernova remnants with a pulsar in their center) might re-accelerate particles via mechanisms related to the pulsar activity [4].

Another mechanism of cosmic ray spectrum extension to higher energies may be related to plasma instabilities in supernova shocks. Recent analyses [5, 27] demonstrated the possibility to gain much higher particle energies than allowed by the standard Bohm limit which is based on undisturbed interstellar magnetic fields. The maximum energy can be as high as $10^{17}$ Z eV, i.e. close to the expected galactic limit. It was shown in [28] that the average spectrum of cosmic rays injected in the interstellar medium over the course of adiabatic SNR evolution (the Sedov stage) is close to $E^{-2}$ at energies larger than about 30 Gev/n and with the maximum particle energy that is close to the knee position in the observed cosmic ray spectrum. At earlier stage of SNR evolution - the ejecta dominated stage described by the Chevalier-Nadoyzhin solution - the particles are accelerated to higher energies but have rather steep power-law distribution on energy. These results suggest that the knee may mark the transition from the ejecta-dominated to the adiabatic evolution of SNR shocks which accelerate cosmic rays.

Whereas the galactic origin of energies beyond a few $10^{15}$ eV requires the mentioned extensions of the original SNR model, the steepening of the spectrum above $10^{15}$ eV may in principle be explained by an effect not related to acceleration itself. The escape probability from the Galaxy increases with energy and may explain the knee, even if the source spectrum follows a single power law. In particular, the knee might occur as a result of an interplay between the diffusion of cosmic rays along magnetic field lines and the drift (Hall diffusion) perpendicular to the average, predominantly azimuthal, galactic magnetic field [25, 29]. This explanation implies that the source spectrum has no significant breaks up to about $10^{18}$ eV.

An essential common feature of the processes of particle reacceleration and diffusion is the dependence on magnetic rigidity, i.e. on $p/Z$ (the momentum per unit charge). Passing the knee region, one expects an increase of the contribution of heavy nuclei with increasing energy. This prediction is convincingly confirmed by recent data from EAS-TOP [1], EAS-TOP & MACRO [8], KASCADE [22], MSU [34], TUNKA [18] and SPASE/AMANDA [30] experiments. The detailed behavior of the spectra of individual mass components in the total cosmic ray flux, in particular the expected steepening of the iron spectrum at about $10^{17}$ eV, is still to be determined.

At present, the main problem of the data interpretation is shifting to the second knee in the cosmic ray spectrum. The natural assumption that all individual ions has only one knee at $4\times10^{15}$ Z eV and that the knee in the spectrum of iron expected at about $10^{17}$ eV explains the second knee in the all-particle spectrum does not agree with the observed position of the second knee at more than $3\times10^{17}$ eV, see also discussion [19]. Of considerable promise is the approach [32] where the dispersion of parameters of SN explosions in the calculations of the knee position and the maximum particle energy were taken into account. It leads to the widening of the energy interval between the two knees in the overall all-particle spectrum.

The second knee in the cosmic-ray spectrum at $3\times10^{17} - 10^{18}$ eV may be due to the severe decrease of efficiency of Galactic sources and hence indicates the approaching to the ultimate limit of acceleration in the Galaxy. But as in the case with the first knee, there is an alternative interpretation that the second knee occurs because of the rapid decrease of cosmic-ray confinement time in the Galaxy. It might be due to the decline of efficient cosmic-ray scattering in random galactic magnetic fields whose main scale is of the order of $L = 100$ pc which is the Larmor radius for $10^{17}$ Z eV particles. The scattering becomes inefficient at $r_g \gg L$. As noted above, the flux of galactic cosmic rays eventually dissolves in the extragalactic background at $3\times10^{18} - 10^{19}$ eV.
References

[1] Alessandro B for the EAS-TOP Collaboration 2001 *Proc. 27th ICRC, Hamburg* 1 124
[2] Ambrosio M et al. 2003 *Phys. Rev.* D67 042002
[3] Antoni T et al. 2004 *ApJ* 604 687
[4] Bell A R and Lucek S G 1996 *MNRAS* 283 1083
[5] Bell A R and Lucek S G 2001 *MNRAS* 321 433
[6] Berezhko E G et al. 1996 *J. Exp. Theor. Phys.* 82 1
[7] Berezinskii V S et al. 1990 *Astrophysics of Cosmic Rays* (Amsterdam: North-Holland).
[8] Bertain M et al. 2003 *Proc. 28th ICRC, Tsukuba* 1 115
[9] Bykov A M and Toptygin I N 2001 *Astron. Let.* 27 625
[10] Case G and Bhattacharya D 1996 *A&A* 120 437
[11] Casse F, Lemoine M. and Pelletier G. 2001 *Phys. Rev.* D65 023002
[12] Cesarsky C J 1980 *Ann. Rev. Astron. Astrophys* 18 289
[13] Chuvilgin L G and Ptuskin V S 1993 *A&A* 279 278
[14] Erlykin A A and Wolfendale A W 1997 *J Phys.* G23 979
[15] Giacalone J and Jokipii J R 1999 *ApJ* 520 204
[16] Ginzburg V L, Khazan Ya M and Ptuskin V S 1980 *Astrophys. Space Sci.* 68 295
[17] Ginzburg V L and Ptuskin V S 1976 *Rev. Mod. Phys.* 48 161
[18] Gress O A et al. 1999 *Nucl. Phys. B (Proc. Suppl.)* 75A 299
[19] Hillas A M 2005 *J. Phys.* G31 95
[20] Ipavich F M 1975 *ApJ* 196 107
[21] Jones F C et al. *ApJ* 547 264
[22] Kampert K-H et al. 2001 *Proc. 27th ICRC Hamburg* 2 792
[23] Lee M A 1979 *ApJ* 229 424
[24] Parker E N 1969 *Space Sci. Rev.* 9 651
[25] Ptuskin V S et al. 1993 *A&A* 268 726
[26] Ptuskin V S et al. 1997 *A&A* 321 434
[27] Ptuskin V S and Zirakashvili V N 2003 *A&A* 403 1
[28] Ptuskin V S and Zirakashvili V N 2005 *A&A* 429 755
[29] Roulet E 2004 *Intern. J. Modern Phys. A* 19 1133
[30] Rawlins K et al. 2003 *Proc. 28th ICRC, Tsukuba* 1 173
[31] Strong A W and Moskalenko I V 1998 *ApJ* 509 212
[32] Sveshnikova L G 2003 *A&A* 409 99
[33] Toptygin I N 1984 *Cosmic Rays in Interplanetary Magnetic fields* (Dordrecht: D. Reidel Publ. Co.)
[34] Vishnevskaya E A et al. 2002 *Izv. RAN* 66 1566
[35] Yan H and Lazarian A 2004 *ApJ* 614 757
[36] Zirakashvili V N et al. 1996 *A&A* 311 113