Triangle singularities in $\bar{B}^0 \to \chi_{c1} K^- \pi^+$ relevant to $Z_1(4050)$ and $Z_2(4250)$

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$Z_1(4050)$ and $Z_2(4250)$ observed in $\bar{B}^0 \to \chi_{c1} K^- \pi^+$ by the Belle Collaboration are candidates of charged charmonium-like states that minimally includes two quarks and two antiquarks. While $Z_1(4050)$ and $Z_2(4250)$ have been interpreted as tetraquark states previously, we propose a completely different scenario based on a kinematical effect called the triangle singularity. We demonstrate that the triangle singularities cause in the $\chi_{c1} \pi^+$ invariant mass distribution resonance-like bumps that fit very well the Belle data. If these bumps are simulated by the $Z_1(4050)$ and $Z_2(4250)$ resonance excitations, the spin-parity of them are predicted to be $1^-$ for $Z_1(4050)$ and $1^+$ or $1^-$ for $Z_2(4250)$. The bump corresponding to $Z_1(4050)$ has a highly asymmetric shape, which the Belle data exactly indicate. We show that the asymmetric shape originates from an interplay between the triangle singularity and the opening of the $X(3872){\pi}^+$ channel near the triangle-singularity energy. This characteristic lineshape could be used to discriminate different interpretations of $Z_1(4050)$.

INTRODUCTION

$Z_1(4050)$ and $Z_2(4250)$ ($X(4050)$ and $X(4250)$ in the Particle Data Group (PDG) notation [1]) were observed in the Belle experiment as resonance-like structures in the $\chi_{c1} \pi^+$ invariant mass distribution of $\bar{B}^0 \to \chi_{c1} K^- \pi^+$ [2]. It was not possible to determine the spin($J$)-parity($P$) of these states; $J = 0$ and 1 assignments were examined. The following analysis in the BaBar experiment [3] did not confirm them because the resonance-like signals were only barely discernible and insignificant. $Z_1(4050)$ and $Z_2(4250)$ are clearly candidates of charged charmonium-like states that minimally include four quarks and thus do not belong to the conventional quark model picture. In order to understand the QCD dynamics and its consequence in the non-perturbative regime, it is highly desirable to establish their existence with higher statistics data in the experimental side, and to clarify their identities such as tetraquark, hadron molecule, or kinematical effect in the theoretical side.

Previous theoretical interpretations of $Z_1(4050)$ and $Z_2(4250)$ are mainly categorized into two types: tetraquark and hadron-molecule. Within the tetraquark picture: (1) a diquark-antidiquark state is [not] assigned to $Z_2(4250)$ [$Z_1(4050)$] [4]; (2) $Z_1(4050)$ is described by a molecular-like tetraquark picture [5]; (3) $Z_1(4050)$ and $Z_2(4250)$ are described with tetraquarks based on a color flux-tube model [6]; (4) $J^{PC} = 0^{++}$ diquark-antidiquark state is not assigned to $Z_1(4050)$ and $Z_2(4250)$ using QCD sum rule (QCDSR) [7]; (5) a tetraquark state is assigned to $Z_2(4250)$ using QCDSR [8]. Meanwhile, within the hadron-molecule picture: (1) meson-exchange models disfavor hadron-molecule pictures for $Z_1(4050)$ and $Z_2(4250)$ [9,11]; (2) $D_s \bar{D}$ molecule state is assigned to $Z_2(4250)$ based on QCDSR [12]. For a more complete summary, see reviews [13,14].

In this work, we propose a completely different interpretation of $Z_1(4050)$ and $Z_2(4250)$. This is to associate $Z_1(4050)$ and $Z_2(4250)$ with triangle singularities (TS) [15,17], which is a kinematical effect, arising from triangle diagrams depicted in Figs. 1(a) and 1(b) (we refer to them as the triangle diagrams A and B hereafter), respectively. The diagrams consist of experimentally well-established hadrons including $X(3872)$ ($\chi_{c1}(3872)$ in the PDG). The TS can occur only when three particles in the loop are kinematically allowed to be on-shell at the same time, and can generate a resonance-like spectrum bump; see an illustrative discussion in Ref. [18] for a mathematical detail. Applications of TS to phenomenology have become popular these days [18–30] because recent experimental developments have made relevant data available. Some examples are an explanation of the large isospin violation in $\eta(1405)/1475 \to 3\pi$ [19,20], and interpretations of recently discovered resonance-(like) states such as the hidden charm pentaquark $P_c(4450)^+$ [18,21,22] and $a_1(1420)^{23,24}$.

Recently we also applied TS [31] to interpreting $Z_c(4430)$ [32,33] and $Z_c(4200)$ [33,35] which are candidates of charged charmonium-like states and were observed in $\bar{B}^0 \to \psi(2S)K^- \pi^+$ and $J/\psi K^- \pi^+$. We successfully explained their properties ($J^P$, mass, width, Argand plot) extracted in the experiments. The presence [absence] of $Z_c(4200)$/$Z_c(4430)$-like contribution in $\Lambda_b^0 \to J/\psi K^- \pi^+$ [33] was also explained in terms of the TS.

This work shows that $Z_1(4050)$ and $Z_2(4250)$ can also be consistently interpreted as TS, provided the TS have experimentally detectable strengths. We demonstrate that the triangle diagram A [B] creates a $Z_1(4050)$ [$Z_2(4250)$]-like bump in the $\chi_{c1}\pi^+$ invariant mass ($m_{\chi_{c1}\pi^+}$) distribution of $\bar{B}^0 \to \chi_{c1} K^- \pi^+$. Simulating the bumps with the $Z_1(4050)$ and $Z_2(4250)$ resonance excitations, $J^P = 1^-$ and $1^+$ are predicted, respectively. The Breit-Wigner masses and widths fitted to the bumps agree very well with those of $Z_1(4050)$ and $Z_1(4250)$ from...
the Belle analysis [2]. The $Z_1(4050)$-like bump has a highly asymmetric shape as the Belle data exactly indicates. We clarify that the opening of the $X(3872)\pi^+$ channel near the TS energy of $m_{\chi_{c1}\pi} \sim 4.05$ GeV is responsible for it. This characteristic bump shape could discriminate different interpretations of $Z_1(4050)$.

**MODEL**

We calculate the $\bar{B}^0 \rightarrow \chi_{c1}K^-\pi^+$ decay amplitudes due to the triangle diagrams A and B of Fig. 1 using a model similar to those of Ref. [3]. A general formula for the decay amplitude is given by

$$T_{abc,H} = \int dp_1 v_{ab;23}(p_a,p_b;p_2,p_3) \frac{\Gamma_{3c,1}(p_3,p_c;p_1)}{E - E_2(p_2) - E_3(p_3) - E_c(p_c) + i\epsilon} \times \left( \frac{1}{E - E_1(p_1) - E_2(p_2)} \Gamma_{12, H}(p_1,p_2;p_{H}) \right),$$

(1)

where we have used the particle labels and their momenta in Fig. 1(c). The total energy in the center-of-mass (CM) frame is denoted by $E$, while the energy of a particle $x$ is $E_x(p_x) = \sqrt{p_x^2 + m_x^2}$ with the mass $m_x$ and momentum $p_x$. For unstable intermediate particles 1 and 2, we use $E_j(p_j) = m_j + p_j^2/2m_j - i\Gamma_j/2$ ($j = 1, 2$) where $\Gamma_j$ is the width. We use the mass and width values of the PDG average [1]. Because $X(3872)$ has a very small width ($\Gamma_{\chi_{c1}\pi} < 1.2$ MeV), we set it to zero in calculations.

The pion-charmonium interaction is denoted by $v_{ab;23}$ in Eq. (1). The particles 2 is either $X(3872)|J^P = 1^-\rangle$ or $\psi(3770)|1^-\rangle$, the particle $a$ is $\chi_{c1}|1^-\rangle$, and the particles 3 and b are pions$|0^-\rangle$. In calculating the triangle diagram A, we use an s-wave interaction:

$$v_{ab;23}^{(A)}(p_a,p_b;p_2,p_3) = f_{ab}^{01}(p_{ab}) f_{23}^{01}(p_{23}) \epsilon_a^* \cdot \epsilon_2,$$

(2)

where polarization vectors for the particles $a$ and $2$ are denoted by $\epsilon_a$ and $\epsilon_2$, respectively. The quantities $f_{ab}^{01}(p_{ab})$ and $f_{23}^{01}(p_{23})$ are form factors that will be defined in Eq. (3): the momentum of the particle $i$ in the $ij$-CM frame is denoted by $p_i$ and $p_{ij} = |p_{ij}|$. This interaction leaves an s-wave $\chi_{c1}\pi^+$ pair in the final state.

Therefore, if a spectrum bump is created by the triangle diagram A in the $\chi_{c1}\pi^+$ invariant mass distribution and it is simulated by a resonance-excitation, the resonance has $J^P = 1^-\rangle$.

Regarding $v_{ab;23}$ for the triangle diagram B, where the intrinsic parity is different between the incoming and outgoing states, we use

$$v_{ab;23}^{(B)}(p_a,p_b;p_2,p_3) = f_{ab}^{11}(p_{ab}) f_{23}^{01}(p_{23}) \epsilon_a^* \cdot \epsilon_2 \times p_{ab},$$

(3)

which converts s-wave $\psi(3770)\pi^+$ into p-wave $\chi_{c1}\pi^+$. Therefore, a resonance that simulates a $\chi_{c1}\pi^+$ spectrum bump from the triangle diagram B has $J^P = 1^+\rangle$. In Eqs. (2) and (3), the incoming 23-pair is in s-wave and can create a sharp TS bump, avoiding a suppression due to the centrifugal barrier. For the triangle diagram B, however, we also examine an interaction of $p$-wave $\psi(3770)\pi^+$ going to s-wave $\chi_{c1}\pi^+$ because the $\psi(3770)\pi^+$ threshold is rather below the TS energy ($\sim 4.25$ GeV) and the centrifugal barrier would not be so effective. Such an interaction is obtained from Eq. (4) by interchanging $p_{ab}$ and $p_{23}$ on the right-hand-side, and the $\chi_{c1}\pi^+$ pair seems to be from a $J^P = 1^-\rangle$ resonance.

The vertex function for a $R \rightarrow ij$ decay, $\Gamma_{ij,R}$ in Eq. (1), is given by

$$\Gamma_{ij,R}(p_i,p_j;p_R) = \sum_{LS} f_{ij}^{LS}(p_{ij}) \langle s_i s_j^* \epsilon_i \epsilon_j | SS^* \rangle \times (LMSS^*|SR S_R^* ) Y_{LM}(\hat{p}_R),$$

(4)

with $Y_{LM}$ being spherical harmonics. Clebsch-Gordan coefficients are written by $(abcd;ef)$ in which the spin of a particle $x$ is denoted by $s_x$ and its z-component $s_x^z$. We use the form factor $f_{ij}^{LS}(p_{ij})$ in the form of

$$f_{ij}^{LS}(p) = g_{ij}^{LS} \frac{p_L}{E_1(p) E_2(p)} \left( \frac{A^2}{A^2 + p^2} \right)^{1+(L/2)},$$

(5)

which is parametrized with a coupling $g_{ij}^{LS}$ and a cutoff $\Lambda$. For the 1 $\rightarrow 3c$ and 23 $\rightarrow ab$ interactions, a non-zero value of $g_{ij}^{LS}$ is allowed for only one set of $\{L,S\}$ due to the selection rule. While the actual values of $g_{ij}^{LS}$
for the $1 \rightarrow 3c$ processes can be determined using the $K^+(892)$ and $K_2^*(1430)$ decay widths, experimental and Lattice QCD inputs are currently missing to determine the couplings for the $23 \rightarrow ab$ interactions. Experimentally, $X(3872) \rightarrow \chi_{c1}\pi^+\pi^-$ has not yet been seen [37], perhaps because $X(3872)$ has a very small width and the phase-space for this final state is small; $\psi(3770) \rightarrow \chi_{c1}\pi\pi$ is not kinematically allowed. Here we assume that these couplings are strong enough.

Regarding the weak vertices for the $H \rightarrow 12$ decays, $g_{ij}^{L,S} \neq 0$ is allowed for several sets of $\{L,S\}$ but their values are currently difficult to estimate due to the lack of data. However, the details of these vertices would not be crucial in this work because main conclusions are essentially determined by the kinematical effects once the structure of $v_{ab;23}$ is fixed as Eqs. (2) and (3). Thus we assume simple structures and detectable strengths. We set $g_{ij}^{L,S} \neq 0$ only for $S = |s_1 - s_2|$ (exception: $S = 2$ when using Eq. (3) with $p_{ab} \leftrightarrow p_{23}$) and the lowest allowed $L$: $g_{ij}^{L,S} = 0$ for the other $\{L,S\}$. We use the cutoff $\Lambda = 1$ GeV in Eq. (3) throughout because main conclusions are robust in a reasonable cutoff range: $\Lambda = 0.7 - 1.3$ GeV.

The interactions of Eqs. (2), (3) and (4), evaluated in the CM frame of the two-body subsystem, are further multiplied by kinematical factors to account for the Lorentz transformation to the total three-body CM frame; see Appendix C of Ref. [38]. The Dalitz plot distribution for $H \rightarrow abc$ is calculated with $T_{abc;H}$ of Eq. (4) following the procedure detailed in Appendix B of Ref. [38].

**RESULTS**

In Fig. 2 we present the $\chi_{c1}\pi^+$ invariant mass distributions for $B^0 \rightarrow \chi_{c1}K^-\pi^+$. The triangle diagram A [B] gives the red [blue and magenta] solid curve in Fig. 2(a) [2(b) and 2(c)]. We also show the phase-space distributions (black dotted curves). The triangle singularity creates clear resonance-like peaks at $m_{\chi_{c1}\pi} \sim 4.05$ GeV in panel (a) and $m_{\chi_{c1}\pi} \sim 4.22$ GeV in panels (b) and (c). It is interesting to observe in Fig. 2(a) that the bump has a significantly asymmetric shape.

Now we extract the resonance mass and width from the spectra within the conventional resonance-excitation mechanism. The Dalitz plot distributions from the triangle diagrams A and B are fitted with the mechanism of $B^0 \rightarrow ZK^-$ followed by $Z \rightarrow \chi_{c1}\pi^+$. The Breit-Wigner form of Ref. [33] is used for the $Z$ propagation. The kinematical region included in the fit covers the Dalitz plot distribution larger than 20% of the peak height. The obtained fits are shown by the green dash-dotted curves in Fig. 2. As expected, the highly asymmetric bump, the red solid curve in Fig. 2(a), is not well fitted with the Breit-Wigner form, while the bumps in Figs. 2(b) and 2(c) are reasonably fitted. We present the resulting Breit-Wigner parameters in Table I along with those of $Z_1(4050)$ and $Z_2(4250)$ from the Belle analysis [2]. The agreement is quite good for $Z_1(4050)$. Meanwhile, the $Z_2(4250)$ mass and width from the Belle analysis have rather large errors, and thus our result easily agrees with them. It is however noted that the masses from our analysis is fairly close to the central value from the experiment. From this analysis, we still cannot eliminate the $J^P = 1^-$ assignment to $Z_2(4250)$.

The highly asymmetric shape of the bump from the
triangle diagram A seems to play an important role in explaining the Belle data in the $Z_1(4050)$-region. To see this, let us superimpose the spectra from the triangle diagrams A and B on the Belle data (Fig. 14 of Ref. [2]), as shown in Fig. 3. Although this is a qualitative comparison where any interferences among different mechanisms are not taken into account, the spectrum bumps from the triangle diagrams fit the data very well. In particular, the asymmetric shape from the triangle diagram A, which has a very sharp rise and a moderate fall-off, is exactly what the data show. In the Belle analysis [2]

[Table I. Spin ($J$), parity ($P$), and Breit-Wigner mass ($M_{BW}$ in MeV) and width ($\Gamma_{BW}$ in MeV) for $Z_1(4050)$ and $Z_2(4250)$. The Breit-Wigner parameters for $Z_1(4050)$ [$Z_2(4250)$] are extracted by fitting the Dalitz plot distributions for $B^0 \rightarrow \chi_{c1}K^-\pi^+$ generated by triangle diagram of Fig. 1(a) [1(b)]]. The parameters from the Belle analysis [2] are also shown; the first (second) errors are statistical (systematic).]

|       | $Z_1(4050)$ | $Z_2(4250)$ |
|-------|-------------|-------------|
|       | Fig. 1(a)   | Belle [2]   |
|       | Fig. 1(b)   | Belle [2]   |
| $J^P$ | $1^-$       | $?^+$       |
| $1^-$ | $1^-$       | $?^+$       |
| $M_{BW}$ | $4060$         | $4051 \pm 4^{+20}_{-41}$ |
| 4221  | 4288        | 4248$^{+14}_{-9}$          |
| $\Gamma_{BW}$ | $81$        | $82^{+21}_{-17}$          |
| 332   | 539         | 177$^{+54}_{-39}$          |

[FIG. 3. $\chi_{c1}\pi^+$ invariant mass distributions for $B^0 \rightarrow \chi_{c1}K^-\pi^+$ calculated with the triangle diagram A. The red, blue, and magenta solid curves in Figs. 2(a-c) are superimposed on the Belle data (Fig. 14 of Ref. [2]); each of the curves is multiplied by a constant factor and an incoherent constant background is added to fit the data.]

[FIG. 4. $\chi_{c1}\pi^+$ invariant mass distributions for $B^0 \rightarrow \chi_{c1}K^-\pi^+$ calculated with the triangle diagram A. The red solid, blue dashed, green dotted, and magenta dash-dotted curves are calculated using the $X(3872)\pi^+$ threshold energy smaller than the PDG value by 0, 50, 100, and 150 MeV, respectively; see the text for details. All these curves, being scaled, have the same peak height. The black dash-two-dotted curve is obtained from the red solid one by turning off the on-shell $X(3872)\pi^+$ contribution.

where the Breit-Wigner form was used to simulate this bump, their model does not seem to fit this sharp peak of the data very well, as seen in Fig. 14 of the reference, perhaps because the Breit-Wigner shape is not what the data call for. As seen in Fig. 2(a), the spectrum shape from the triangle diagram A is significantly different from the Breit-Wigner.

Considering the importance of the asymmetric shape in explaining the Belle data, it would be worthwhile addressing how this peculiar shape come about from the triangle diagram A. By closely observing the spectrum shown in Fig. 3(a) or an enlarged one shown by the red solid curve in Fig. 4 the sharp rise of the spectrum starts from an abrupt bend at $m_{\chi_{c1}\pi} \sim 4.01$ GeV where the $X(3872)\pi^+$ channel opens. We indeed confirm this idea, as shown by the black dash-two-dotted curve in Fig. 4 by turning off the on-shell $X(3872)\pi^+$ contribution arising from $+i\epsilon$ in the denominator of Eq. (1).

The proximity of the $X(3872)\pi^+$ threshold to the TS energy ($\sim 4.05$ GeV) is also an important factor to create the large asymmetry. To see this, we change the $X(3872)$ and $K^*(892)$ masses to lower the $X(3872)\pi^+$ threshold while keeping the peak position of the spectrum almost the same. We use, in unit of MeV, $(m_X(3872), m_{K^*}(892)) = (3822, 1084), (3772, 1218), (3722, 1330)$ to lower the threshold by 50, 100, and
150 MeV, respectively. The spectra calculated with these altered masses, presented in Fig. 4, show that the rise of the bump becomes significantly more moderate as the threshold is lowered. In this way, the asymmetric shape of the $Z_1(4050)$ bump observed in the Belle data is explained with well-founded physics, TS and the channel opening near the TS energy, included in the triangle diagram A.

The asymmetric shape of the $Z_1(4050)$ bump could sensitively discriminate different interpretations of $Z_1(4050)$. A compelling model should address not only the mass, width, and $J^P$ of $Z_1(4050)$, but also its characteristic spectrum shape. So far, only our model has successfully addressed this question. It is also highly desirable to establish the spectrum shape with higher statistics data, considering that the Belle data still have large error bars.

Finally, we present Argand plots from the triangle diagrams A and B because this information may be measured in future experiments; here we use Eq. (3) for the diagram B. Because $Z_1$ ($Z_2$) and $K^-$ are relatively in $p$-wave, we obtain the angle-independent part of the amplitude by

$$A(m_{ab}^2) = \int d\Omega p_c d\Omega p_a Y_{\ell,-s_{\chi_{c1}}}^z (\hat{p}_c) Y_{\ell 0} (\hat{p}_{ab}) M_{abc,H}$$

with $\ell = 0$ and 1 for the diagrams A and B, respectively; $s_{\chi_{c1}}^z$ is the $z$-component of the $\chi_{c1}$ spin and $m_{ab}$ the $ab$ invariant mass. See Eq. (B3) of Ref. [38] for the relation between the invariant amplitude $M_{abc,H}$ and $T_{abc,H}$ of Eq. (1). The angle-independent amplitudes $A$ are shown in Fig. 5 as Argand plots. Both the triangle diagrams A and B create counterclockwise behaviors, seemingly similar to resonances.

**CONCLUSION**

We demonstrated that TS from the triangle diagrams of Figs. 1(a) and 1(b) cause the bumps in the $\chi_{c1}\pi^+$ invariant mass distribution of $B^0 \to \chi_{c1}K^-\pi^+$, and that their positions and shapes, and thus Breit-Wigner parameters fitted to the bumps, agree very well with those found and named as $Z_1(4050)$ and $Z_2(4250)$ in the Belle experiment [2]. Within the resonance-based simulation of these bumps, $J^P = 1^-$ is predicted for $Z_1(4050)$ and $J^P = 1^+$ or $1^-$ for $Z_2(4250)$. The highly asymmetric shape of $Z_1(4050)$-like bump found by the Belle is well reproduced by our model: the opening of the $X(3872)\pi^+$ channel near the TS energy causes the abrupt increase of the spectrum. This characteristic lineshape, which could discriminate different interpretations of $Z_1(4050)$, is yet to be accounted for by any other hadron structure models. The kinematic effects, TS and the channel opening, essentially determine the shape and position of the spectrum bumps, once the spin-parity of the $\chi_{c1}\pi^+$ system is specified by Eqs. (2) and (3); the uncertainty of the remaining dynamical details would not largely change the presented results.

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![Argand plots from the triangle diagrams of Figs. 1(a) and 1(b), corresponding to the spectra of Fig. 2(a) and 2(b), respectively. Six curved segments belong to six bins equally-separating the range of $\Gamma_{BW}^\ell = m_{\chi_{c1}\pi^\ell} \leq M_{BW}^\ell + \Gamma_{BW}; M_{BW}$ and $\Gamma_{BW}$ are taken from Table 1. $m_{\chi_{c1}\pi^\ell}$ increases counterclockwise.](image-url)
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