Detecting Black Hole Binaries by *Gaia*

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Abstract

We study the prospects of the *Gaia* satellite to identify black hole (BH) binary systems by detecting the orbital motion of the companion stars. Taking into account the initial mass function, mass transfer, common envelope phase, interstellar absorption, and identifiability of the systems, we estimate the number of BH binaries that can be detected by *Gaia* and their distributions with respect to the BH mass. Considering several models with different parameters, we find that ~200–1000 BH binaries could be detected by *Gaia* during its ~5 years operation. The shape of the BH mass distribution function is affected strongly by the zero-age main sequence (ZAMS) stellar mass–BH mass relation. We show that once this distribution is established observationally, we will be able to constrain the currently unknown ZAMS mass–BH mass relation.

Key words: astrometry – binaries: general – black hole physics

1. Introduction

More than $10^{8–9}$ stellar-mass black holes (BHs) are believed to reside in our Galaxy (Brown & Bethe 1994; Timmes et al. 1996). Today, 59 X-ray binaries are considered to harbor a BH (Corral-Santana et al. 2016). These are the only representatives of those numerous BHs that have been observationally identified so far. The masses of the BHs and the companion star in these binaries are estimated from optical and X-ray observations. Özel et al. (2010) presented the Galactic BH mass distribution, which is based on the dynamically measured masses of 16 BHs in transient, low-mass X-ray binaries. The observed BH mass distribution is quite narrowly centered around $7.8 \pm 1.2 \; M_\odot$. BHs in the mass range of 2–5 $M_\odot$ are absent in the Galaxy (at least as far as X-ray binaries are concerned). Farr et al. (2011) perform a Bayesian analysis using the observed BHs masses of 20 X-ray binaries and reach the similar results.

LIGO has detected Binary BH mergers in distant galaxies (Abbott et al. 2016a, 2016b, 2017). Somewhat surprisingly, these BHs are significantly more massive than the observed Galactic X-ray binary BHs, although there were indications that the masses of BH binaries in gravitational waves should be higher than those in X-rays (Belczynski et al. 2010; Bulik et al. 2011). The corresponding masses range from ~7.5 to ~36 $M_\odot$. At present, it is not clear if the difference in the observed mass distribution is due to a different origin of the progenitors, to differences in the initial mass function (IMF), or simply due to the LIGO’s sensitivity that is larger for more massive BHs.

In X-ray binaries, X-ray emission originates from a mass transfer (MT) from the companion star to the BH. Such a MT is expected when the radius of a companion star is larger than the Roche lobe radius of the system. However, if the orbital separation of a binary is too wide, there will not be a MT and no X-ray emission. The BH does not emit any electromagnetic radiation in such cases. However, we can still discover such a BH and estimate its mass if we are able to measure the orbital period and the semimajor axis with astrometry for its companion (Breivik et al. 2017; Kawanaka et al. 2017; Mashian & Loeb 2017). This idea is also mentioned in *Gaia’s* white paper (Barstow et al. 2014).

The astrometric satellite *Gaia*, which was launched at the end of 2013, is an ideal tool to perform the needed observations. *Gaia* can perform absolute astrometric measurement with a great precision on objects brighter than $G < 20$ mag, where $G$-band covers wavelength between 0.3 and 1.0 $\mu$m (de Bruijne 2012). In the case of a BH binary, if the companion is sufficiently bright, *Gaia* will detect its motion from which the existence of the BH can be inferred. Our goal here is to estimate, following Kawanaka et al. (2017), the expected number of BH binaries that can be detected by *Gaia* over its 5 year mission. In this work, we follow the binary formation and evolution, taking into account the IMF, common envelope (CE) phase, and MT, and we estimate the total number of Galactic BH-main sequence star binaries without mass accretion in our Galaxy, as well as their distribution with respect to their masses and orbital separations. We then estimate their detectability and identifiability, taking into account the interstellar absorption and obtain the number of such binaries detectable by *Gaia* during its operation (~5 years). We consider BH binaries that are bright enough to be detectable and their orbits shorter than *Gaia’s* mission lifetime, yet they are far enough not to involve MT so that they are not active X-ray sources. Recently, Mashian & Loeb (2017) estimated the number of BH binaries detectable by *Gaia* over its 5 year mission as nearly $2 \times 10^5$. However, they do not take into account the change of the orbital parameters due to MT from the primary star (i.e., the BH’s progenitor) to the secondary or the CE phase. In addition, they do not take into account the interstellar absorption. These effects could reduce significantly the number of detectable BH binaries. Breivik et al. (2017) also estimated the number of such BH binaries using the binary population synthesis code COSMIC, and predict that *Gaia* will be able to discover 3800–12,000 BH binaries. However, they also do not take into account the interstellar absorption, so this value may be overestimated. Moreover, it is not clear how their results depend on the relation between the initial stellar mass and the remnant BH mass, which is closely related to the mechanism of core collapse supernovae.
We also take another IMF given by Kroupa et al. (1993, 2001; Kobulnicky & Kudritzki 1995; Kroupa & Weidner 2001) into a BH close to unity separation. We consider two cases separately: a mass ratio of their distribution with respect to their masses and orbital parameters at birth. For the primary star, we use the Kroupa (2001) IMF as the fiducial one:

\[
\Psi_{\text{K01}}(M_1)dM_1 \propto \begin{cases} M_1^{-1.3}dM_1 & 0.08 M_\odot \leq M_1 < 0.5 M_\odot, \\ M_1^{-2.2}dM_1 & 0.5 M_\odot \leq M_1 < 100 M_\odot. \end{cases}
\]

We also take another IMF given by Kroupa et al. (1993) and Kroupa & Weidner (2003) to investigate dependence of results on IMF:

\[
\Psi_{\text{K03}}(M_1)dM_1 \propto \begin{cases} M_1^{-1.3}dM_1 & 0.08 M_\odot \leq M_1 < 0.5 M_\odot, \\ M_1^{-2.7}dM_1 & 0.5 M_\odot \leq M_1 < 1.0 M_\odot, \\ M_1^{-2.2}dM_1 & 1.0 M_\odot \leq M_1 < 100 M_\odot. \end{cases}
\]

For the secondary mass, we assume a flat mass ratio distribution as the fiducial case (Kuiper 1935; Kobulnicky & Fryer 2007):

\[
\Phi(q) = \frac{1}{(1 - M_{\text{min}}/M_1)} q^{0.5}.
\]

We also try the cases of the index of $q$, $-1$ and $+1$. Here, we set the lower limit of $q$ as $M_{\text{min}}/M_1$, where $M_{\text{min}} = 0.08 M_\odot$ is the minimal initial mass of a star.

We assume that the recent specific star formation rate in our Galaxy to be a constant (Belczynski et al. 2007),

\[
\int d\dot{M} \dot{M} \Psi(\dot{M}) = 3.5 M_\odot \text{ yr}^{-1},
\]

where $\Psi = \Psi_{\text{K01}}$ or $\Psi_{\text{K03}}$. This assumption is justified as the life times of stars that we examine are much shorter than the evolution times of our Galaxy.

The distribution of initial binary separations is assumed to be logarithmically flat (Abt 1983):

\[
\Gamma(\bar{A}) = \frac{\Gamma_0}{\bar{A}}.
\]

The normalization factor $\Gamma_0$ is determined by the range of $\bar{A}$:

\[
\int_{\bar{A}_{\text{min}}}^{\bar{A}_{\text{max}}} d\bar{A} \Gamma(\bar{A}) = 1.
\]

We set the lower limit of this integral, $\bar{A}_{\text{min}}$, as the distance such that the primary fills its Roche lobe at the periastron (Eggleton 1983):

\[
\bar{A}_{\text{min}} = \frac{0.6q^{-2/3} + \ln(1 + q^{-1/3})}{0.49q^{-2/3}} R_1.
\]

With this definition of $A_{\text{min}}$ and the normalization condition, $\Gamma_0$, $A$ is a function of $M_1$ and $q$. We set the upper limit as $\bar{A}_{\text{max}} = 10^4$ au. Recent studies show that the double stars with the separation up to $\sim 10^4$ au can stay gravitationally bound for long timescales (Andrews et al. 2017; Oelkers et al. 2017).

As we are interested in binaries that contain a BH (as a remnant of the primary star) and a secondary star that has not collapsed, the age of the system should be between $t = t_{1,1}$ and $t = t_{1,2}$, where $t_{1,1}$ is the lifetime of a star with an initial mass $M_1$. We adopt here the lifetime suggested by Eggleton (1983). For the primary star, $t_{1,1}$ is the time when it collapses into a BH.

Turning now to the current conditions of the system, we note that with no current MT from the secondary star to the BH the radius of the secondary should be smaller than its Roche lobe:

\[
R_L(M_2/M_{\text{BH}}, A) > R_2.
\]

Using the mass–radius relation for terminal age main sequence (Demircan & Kahraman 1991) and the formula by Eggleton (1983), we can rewrite the condition (8) as

\[
A > A_{\text{min}} = \frac{0.6 + \left(\frac{M_1}{M_{\text{min}}}\right)^{2/3} \ln \left[ 1 + \left(\frac{M_1}{M_{\text{min}}}\right)^{-1/3}\right]}{0.49} \left(\frac{M_2}{M_\odot}\right)^{0.83} R_\odot.
\]

This condition set the lower limit of the initial binary separation, $\bar{A} > \bar{A}_{\text{RL}}$.

An upper limit, $\bar{A}_{\text{period}}$, is determined by the condition that the orbital period of the binary should be shorter than $P_{\text{max}}$. 

\[
\bar{A}_{\text{period}} = \frac{0.6 + \left(\frac{M_1}{M_{\text{min}}}\right)^{2/3} \ln \left[ 1 + \left(\frac{M_1}{M_{\text{min}}}\right)^{-1/3}\right]}{0.49} \left(\frac{M_2}{M_\odot}\right)^{0.83} R_\odot.
\]
which is given in Section 2.7:

\[ A < A_{\text{max}} = \left[ \frac{G(M_1 + M_2)}{(\rho_{\text{max}}/2\pi)^3} \right]^{1/3}. \]  

(10)

In the following sections, we related the current \( A_{\text{max}} \) to an upper limit \( \bar{A}_{\text{period}} \) on the initial separation.

We assume that the spatial distribution of BH binaries in the Galactic plane follows the stellar distribution. According to Bahcall & Soneira (1980), we can write the star formation number density per unit mass bin in the Galactic disk as a function of the distance from the Galactic center \( r \) in the Galactic plane, and the distance perpendicular to the Galactic plane \( z \) as:

\[ \rho_b(r, z, \bar{M}) = \Psi(\bar{M}) \cdot \rho_{d,0} \exp \left[ -\frac{z}{h_z} - \frac{r - r_0}{h_r} \right], \]

(11)

where \( r_0 = 8.5 \) kpc is the distance from the Galactic center to the Sun, and \((h_r, h_z) = (250 \text{ pc}, 3.5 \text{ kpc})\) are the scale lengths for the exponential stellar distributions perpendicular and parallel to the Galactic plane, respectively. Hereafter, we consider only the disk component because the binaries in the Galactic bulge would not be observed due to the interstellar absorption. Then, we can determine the normalization factor, \( \rho_{d,0} \) by

\[ 4\pi \int_0^{r_{\text{max}}} dr \int_0^{z_{\text{max}}} dz \rho_{d,0} \exp \left[ -\frac{z}{h_z} - \frac{r - r_0}{h_r} \right] = 1. \]

(12)

We describe this distribution with respect to the spherical coordinate centered at the Earth, \((D, b, l)\), where

\[ r = [r_0^2 + D^2 \cos^2 b - 2 Dr_0 \cos b \cos l]^{1/2}, \]

(13)

\[ z = D \sin b, \]

(14)

and \(D, b, \) and \(l\) are the distance from the Earth, Galactic latitude, and Galactic longitude.

The total number of BH binaries without mass accretion detectable by Gaia can be obtained as a multidimensional integral over the initial primary mass, the mass ratio, the initial separation, and the position:

\[ N = \frac{f_{\text{bin}}}{1 + f_{\text{bin}}} \int_{M_{\text{min,BH}}}^{100 M_\odot} dM_1 \int_{q_{\text{min}}}^{1} dq \int_{0}^{1} d\Gamma_0 \Phi(q)
\cdot \left[ \frac{1}{A} \right] \left[ (t_{l,2} - t_{l,1}) \right] \int_{0}^{\bar{M}_{\text{max}}} \int_{(\bar{M}_{\text{min},A_{\text{det}}}-\bar{M}_{\text{max}})}^{\bar{M}_{\text{min},A_{\text{det}}}} d\bar{M} \int_{0}^{\pi/2} d\ell \int_{0}^{\pi/2} \cos b db \int_{0}^{D_{\text{max}}} D^2 dD_\rho_b(D, b, l, \bar{M}_1), \]

(15)

where we set the lower limit of the integration with respect to \( \bar{M}_1 \) as \( M_{\text{min,BH}} = 20 M_\odot \), above which a primary star would form a BH after its collapse, and \( f_{\text{bin}} \) is the binary fraction. Hereafter, we assume \( f_{\text{bin}} = 0.5 \), which means that we have 50 binaries and 50 single stars out of 150 stars. In addition, \( q_{\text{min}} \) represents the minimum mass ratio defined in Section 2.6, and \( A_{\text{det}} \) is defined by considering the condition for the BH identification with Gaia (Section 2.7). \( D_{\text{max}} \) is set as 10 kpc in Section 3. Note that while the integration looks simple, it involves numerous implicit dependences, e.g., \( t_{l,i} \) (\( i = 1, 2 \)) depend on \( \bar{M}_1 \).

As shown below, the final binary separation, \( A \), can be described as

\[ A = \bar{A} \cdot a(q), \]

(16)

where \( a(q) \) is a function of the initial mass ratio \( q \); then, we can describe the differential number distribution of BH binaries of interest as

\[ \frac{dN}{dM_{\text{BH}} dM_2 dAdR} = \frac{dN}{dM_{\text{BH}} dM_2 d\bar{A}dR} \cdot \frac{\partial(M_1, M_2, \bar{A})}{\partial(M_{\text{BH}}, M_2, A)} \]

\[ = \frac{2f_{\text{bin}} \Gamma_0}{1 + f_{\text{bin}}} \cdot 2 \int_{0}^{2\pi} \cos b db \int_{0}^{\pi/2} d\ell \int_{0}^{D_{\text{max}}} dD \rho_b(R, b, l, \bar{M}_1)
\cdot \frac{1}{M_1} (t_{l,2} - t_{l,1}) \frac{1}{A} \frac{\partial(M_1, M_2, \bar{A})}{\partial(M_{\text{BH}}, M_2, A)}, \]

(17)

where \( \partial(M_1, M_2, \bar{A})/\partial(M_{\text{BH}}, M_2, A) \) is the Jacobian of the variable transformation from the initial to final parameters.

2.2. Relation between the ZAMS Mass and BH Mass

We assume for simplicity that the BH mass satisfies the equation

\[ M_{\text{BH}} = k\bar{M}_1, \]

(18)

where we adopt \( k = 0.2 \) as the fiducial case. This is a conservative assumption. A good approximation is given by Equation (2.42) in Eggleton (2011). In short, \( k \) increases with mass from about 0.1 at \( M_l = 0.8 M_\odot \) to about 0.4 at \( M_l = 40 M_\odot \), and then it flattens toward 0.5 at very large masses. In reality, the WR stars will lose a lot of matter in the wind later, and the BH will have a smaller mass than the mass of \( kM \). Our choice of \( k = 0.2 \) takes into account these two processes. We also try \( k = 0.1 \) and 0.5.

However, the real relation between the ZAMS mass and BH mass should be more complicated. Thus, we assume the mass relation as follows:

\[ M_{\text{BH}} = \frac{2}{\ln 3} \ln(\bar{M}_1 - 19) + 2, \]

(19)

where we use the mass relation in Belczynski et al. (2008) as a reference, and this function satisfies \( M_{\text{BH}} = 2 \) for \( \bar{M}_1 = 20 \) and \( M_{\text{BH}} = 10 \) for \( \bar{M}_1 = 100 \). The model with this mass relation is named “curved.”

2.3. Formation Paths of Binaries with a BH

A general scenario for formation of binaries with a BH involves a binary that initially contains a massive star and a companion. We assume that the initial orbit is circular. The more massive star—the BH progenitor—will evolve faster and will initiate the first MT when it sufficiently expands and fills the Roche lobe. Here, we adopt \( 3 \times 10^4 R_\odot \) as the maximum radius of the stars with \( \bar{M}_1 > 20 M_\odot \). This value is given by the formula of the radius of the asymptotic giant branch star with mass of \( 20 M_\odot \) and the luminosity of \( 10^{53} L_\odot \) (Hurley et al. 2000). Although the radius depends on the mass and luminosity of a star, the uncertainty is no more than a factor of a few, which has little influence on the result. We discuss this issue in detail in Section 4. We discuss the outcome of the MT in detail for two cases of the mass ratio in Sections 2.4 and 2.5.
In our calculation, we make several simplifying assumptions. We neglect the wind mass loss from the stars. The wind mass loss in the pre-MT phase will lead to tightening of the orbit, and therefore it is degenerate with the initial orbital separation. The mass loss from the primary will also decrease the mass that can be transferred to the companion in the large mass ratio case. However, the contribution to the observed number of binaries in this case is small, which is discussed in detail in Section 4. We assume that a BH forms through direct collapse in the case. However, the contribution to the observed number of binaries in this case is small, which is discussed in detail in Section 4. We assume that a BH forms through direct collapse. Additionally, we assume that the BH receive no natal kicks of the compact core with no mass loss during the process. We neglect the wind mass loss from the stars. The wind mass loss from the stars. We assume that the MT would continue until the MT=0.1 if the stellar radius is larger than 1 kpc in the V-band. As the lower limit of the luminosity of the observable star gets higher, the lower limit of the corresponding stellar mass gets higher. In this paper, we equate the Gaia band with the V-band, which is valid when the star is bluer than G-type stars whose color V−I ≲ 1 (Jordi et al. 2010).

The interstellar extinction in the V-band A_V affects the relation between the absolute magnitude M_V, the apparent magnitude m_V, and the distance D:

\[ M_V = m_V - 5(2 + \log_{10} D_{\text{kpc}}) - A_V(D_{\text{kpc}}). \]  

where the distance is normalized by 1 kpc and we adopt A_V(D_{kpc}) = D_{kpc}. In addition, the absolute magnitude is related to the companion mass, M_2, as

\[ M_2 = \begin{cases} 10^{-0.1(M_V-4.8)} & M_V < 8.5 \\ 1.9 \times 10^{-0.17(M_V-4.8)} & M_V > 8.5 \end{cases} \]

where we adopt the empirical mass–luminosity relation in Smith (1983). The absolute magnitude M_V = 8.5 corresponds to a companion mass M_2 = 0.4 M_\odot. By combining Equations (26) and (27), we find the companion mass whose apparent magnitude and distance are m_V and D_{kpc} respectively. Thus, given the limiting magnitude of Gaia and distance of the system, we obtain M_{2,\text{min}} which is defined to be the minimum mass of the
companion observable by Gaia. Therefore, $q_{\min}$ in Equation (15) is now defined as $\max(M_2(M_{2,\min}/M_1, M_{\min}/M_1).

2.7. Constraints Required for the BH Identification

We need impose constraints on various parameters of the binaries to identify the primary of the binary system as a BH. The robust way to do so is to measure its mass. The astrometric observations of the companion star enables us to estimate the mass of the other unseen object through the measurements of the semimajor axis and the orbital period. The mass $M_{BH}$ can be expressed by $M_2$, the orbital period $P_{\text{orb}}$, and the angular semimajor axis, $a_\ast$:

$$\frac{(M_{BH} + M_2)^2}{M_{BH}^3} = \frac{G}{4\pi^2} \frac{P_{\text{orb}}^2}{(a_\ast D)^3}, \quad (28)$$

where $G$ is the gravitational constant. This equation means that the identification of BH requires measurements of $M_2$, $P_{\text{orb}}$, $a_\ast$, and $D$ with a sufficient accuracy. In what follows, we estimate the inferred standard errors of these parameters for the BH identification and constraints on these parameters.

If the mass of the hidden companion is larger than $3 M_\odot$, with a $n - \sigma$ confidence level, the object can be identified as a BH. Note that $3 M_\odot$ is the fiducial minimum mass of BH expected from the maximum mass of neutron stars (Kalogera & Baym 1996). This condition is expressed as

$$M_{BH} - n\sigma_{MBH} > 3 M_\odot, \quad (29)$$

where $\sigma_{MBH}$ is a standard error of the mass estimate of the unseen primary, and we adopt $n = 1$. Using Equation (28) $\sigma_{MB}/M_{BH}$ is related to $\sigma_{M2}$, $\sigma_{P}$, $\sigma_{a}$, and $\sigma_{D}$ (the standard errors of the companion mass, orbital period, semimajor axis, and distance, respectively) as

$$\left( \frac{\sigma_{MB}}{M_{BH}} \right)^2 = \left( \frac{3}{2} - \frac{M_{BH}^2}{M_{BH} + M_2} \right)^{-2} \left[ \left( \frac{M_2}{M_{BH} + M_2} \right)^2 \sigma_{M2}^2 + \sigma_{P}^2 + \left( \frac{9}{4} \frac{\sigma_{a}^2}{a_\ast^2} + \frac{\sigma_{D}^2}{D^2} \right) \right], \quad (30)$$

where we assume that for all parameters the ratios of the standard errors to the parameters themselves are smaller than 1, and the correlation between errors of these parameters can be neglected. From Equations (29) and (30), we can constrain the ratios of the standard errors to the parameters, $\sigma_{M2}/M_2$, $\sigma_{P}/P_{\text{orb}}$, $\sigma_{a}/a_\ast$, and $\sigma_{D}/D$. If the following constraints are satisfied, Equation (29) is also satisfied, when $M_{BH} > 5 M_\odot$:

$$M_2 < 0.1 \sigma_{M2}, \quad P_{\text{orb}} < 0.1 \sigma_{P}, \quad a_\ast < 0.1 \sigma_{a}, \quad \text{and} \quad D < 0.1 \sigma_{D}. \quad (31)$$

When $M_{BH} < 5 M_\odot$, conditions more stringent than in Equation (31) are required for satisfying Equation (29). However, empirically few BHs weigh less than $5 M_\odot$ (Özel et al. 2010), so that we expect that the conditions in Equation (31) is useful for identifying most BHs. We note that these condition equations (Equation (31)) are given to simplify the following reduction, and therefore just necessary conditions.

The constraints on the standard errors of orbital period, semimajor axis, and distance in Equation (31) are reduced to conditions on BH binaries detectable by Gaia. The constraint on the standard error of the distance imposes a condition on the magnitude of the companion at a distance $D$. Because the distance is inversely proportional to the parallax $\pi$, $\sigma_{D}/D$ can be expressed as $\sigma_{\pi}/\pi$, where $\sigma_{\pi}$ is the standard error of the parallax for the Gaia astrometry, which is related to the apparent magnitude of the Gaia band (de Bruijne 2012). Assuming that the apparent magnitude of the Gaia band is equal to $V$-band magnitude as done in the previous section, the condition $\sigma_{\pi}/\pi$ is represented as

$$(-1.631 + 680.8 \cdot z(mv) + 32.73 \cdot z^2(mv))^{1/2} < \frac{10^2}{D_{\text{kpc}}}. \quad (32)$$

where the expression of $\sigma_{\pi}$ appears in Gaia Collaboration et al. (2016), and the function $z(mv)$ is

$$z(mv) = 10^{0.4(mv - 15)}. \quad (33)$$

In addition, we neglect the factor including $V - I$ of the original expression of $\sigma_{\pi}(G)$ because this factor changes $\sigma_{\pi}$ by only a few percent. We note that Equation (32) gives the maximum detectable apparent $V$ magnitude $m_{v,\text{max}}$ for a fixed distance as

$$m_{v,\text{max}} \sim 17.5 + 2.5 \log_{10} \left[ \sqrt{1 + \left(0.6 D_{\text{kpc}} \right)^{-2}} - 1 \right], \quad (34)$$

where we assume that the maximum function in Equation (33) is simply equal to $m_v$, which is valid for $D_{\text{kpc}} \ll 10$. This maximum magnitude enables us to obtain the minimum companion mass $M_{2,\min}$ by using Equations (26) and (27), that is, $M_{2,\min} = M_3(m_{v,\text{max}})$.

The conditions for the semimajor axis and orbital period in Equation (31) constrain the range of the semimajor axis. The standard error of the semimajor axis of the stellar orbit $\sigma_a$ is expected to be similar to $\sigma_{a}$, because the semimajor axis of the stellar orbit $a_\ast$ is roughly the size of the orbit on the celestial sphere. Thus, we can assume $\sigma_{a} \sim \sigma_{a}$, and therefore, the constraint on the semimajor axis of the binary system A can be written as

$$A > 10 \frac{M_{BH} + M_2}{M_{BH}} \frac{\sigma_{\pi}(m_v) D}{M_{BH}} \equiv A_{\text{ad}}. \quad (35)$$

According to orbital solutions in the Hipparcos and Tycho catalog (ESA 1997), all binaries with the orbital period less than $2/3$ of the mission period of Hipparcos show the standard error of the orbital period less than $1/10$ of the orbital period. Thus, we expect that for BH binaries with the orbital period less than $\sim 3$ years, the orbital period will be measured with Gaia at the standard error less than $10\%$. Therefore, we adopt $3$ years as the maximum orbital period. The minimum orbital period might be determined by Gaia’s cadence for each object, which is roughly 50 days, so that we adopt 50 days as the minimum orbital period. These conditions give a range of the semimajor axis:

$$A(P_{\text{orb}} = 50 \text{ days}) < A < A(P_{\text{orb}} = 3 \text{ years}). \quad (36)$$

The typical standard error of the stellar mass is $\sim 10\%$ (Tetzlaff et al. 2011), where stellar masses are measured by using their luminosities and temperature, so that we
expect that the standard error of the stellar mass measured by Gaia is also \( \sim 10\% \). Therefore, \( \tilde{A}_{\text{det}} \) is defined to be \( \max(\tilde{A}(A_{\text{ast}}), \tilde{A}(A(P_{\text{orb}} = 50 \text{ days})) \).

In the next section, we obtain the numbers of BH binaries detectable with Gaia by integrating Equation (17) for various models in which the parameters are different from each other. The parameters in models are shown in Table 1. We show just distributions of the BH mass for models other than the fiducial one. We have four quantities as integration variables. The integral range of \( M_{\text{BH}} \) are assumed to be [4, 30 \( M_\odot \)] in the case of the fiducial model. The minimal companion mass \( M_2 \), \( M_{2, \text{min}} \), is given in Section 2.7. The maximal companion mass is \( \tilde{M}_1[1 + \beta(1 - k)] \) for a mass ratio larger than 0.5 or \( \tilde{M}_1 \) for a mass ratio smaller than 0.5. The integral interval of the semimajor axis is the range such that Equations (35) and (36) are satisfied, where we note that the semimajor axis of all binaries should be between \( \tilde{A}_{\text{min}} a(q) \) and \( \tilde{A}_{\text{max}} a(q) \).

### 3. Results

The calculated number of BH binaries detectable by Gaia for the fiducial model is \( \sim 500 \). These include BH binaries whose companions’ brightness is \( m_V \sim 20 \) magnitude. This number is smaller by 400 than the one found by Mashian & Loeb (2017) and by about one order of magnitude than the one found by Breivik et al. (2017). In our calculation, \( \sim 1 \) million BH-main

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**Figure 1.** Calculated distributions of BH binaries in the case of the fiducial model. The upper-left panel shows the distance distribution between 0.1 and 10 kpc. The upper-right panel and lower panel show the BH mass and the companion mass distributions, respectively. The red line in the distribution of BH mass shows the power-law function whose index is \( -2.3 \).

**Table 1**

| Parameters/Model Names | Fiducial | lin01 | lin05 | Curved | K03 | al01 | al20 | \( q - 1 \) | \( q + 1 \) |
|------------------------|---------|------|------|--------|-----|------|------|----------|----------|
| Coefficient in Equation (18) | 0.2 | 0.1 | 0.5 | ... | ... | ... | ... | ... | ... |
| Index of Initial mass function | 2.3 | ... | ... | ... | 2.7 | ... | ... | ... | ... |
| \( \alpha \lambda \) | 1.0 | ... | ... | ... | 0.1 | 2.0 | ... | ... | ... |
| Power-law index of the \( q \) distribution | 0 | ... | ... | ... | ... | ... | -1 | 1 | ... |

Note. “…” indicate the same value as the fiducial model.
sequence binaries exists in whole Milky Way. This is easily estimated from the star formation rate and IMF. The companion stars in a few percent of those are observable with Gaia, i.e., the companions are brighter than 20 magnitude for G-band. Furthermore, Gaia can detect the orbital motion of a few percent of those and identify them as BH binaries.

Figure 1 depicts the distributions of BH binaries that are detectable by Gaia. Almost all detectable BH binaries are within 1–10 kpc. The peak is at ~7 kpc. The distribution within ~5 kpc increases monotonically, corresponding to the increasing volume. Above ~7 kpc the number of BH binaries drastically decreases, because Gaia cannot measure the distance and semimajor axis of most binaries accurately enough to identify those objects as BH binaries.

The upper-right panel shows the power-law distribution of the BH masses. The index of this power-law distribution is ~2.3, which is just the same as that of IMF. The distribution of companions masses is shown in the lower panel in Figure 1. The contribution of companions less massive than 20 $M_\odot$ is much smaller than that of those larger than 20 $M_\odot$. Binaries with $q < 0.3$ undergo a CE phase in which its separation decreases typically down to 1% of the original one (see Equation (24)). Therefore, these binaries will not be detected by Gaia because of their short orbital period (or small separation). This result is different from one obtained by Breivik et al. (2017). Although the minimal companions mass in the binaries that undergo a MT phase is 7 $M_\odot$, this phase, the companions receives a fraction of the matter of the primary. This leads to a minimal companion mass of 15 $M_\odot$ used in our fiducial model. We also see that the maximal mass of the companions reaches ~200 $M_\odot$, although the maximal mass of primaries is 150 $M_\odot$. This is because when the mass ratio is larger than 0.3, we assume that the companion mass increases due to the MT (see Section 2.4).

### 3.1. Dependence of the Distribution of BH Mass on the Models

The total numbers of detectable BH binaries for the models, lin01, lin05, and curved, are shown in Table 2. We see that the total number correlates with the coefficient $k$. This can be interpreted to be an effect of astrometric observation. If BH mass is larger, the orbit of companion is also larger, which increases the detectability of BH by Gaia. This can be also understood by Equation (35). Thus, as the coefficient $k$ increases, the total number increases.

Figure 2 shows distributions of BH mass when the relation between the ZAMS mass and BH mass is changed. In the model of the linear mass relation (fiducial, lin01, and lin05), the BH masses show the single power-law distributions, and their power-law indexes are similar to each other. On the other hand, the distribution of the model “curved” shows a peak at ~7 $M_\odot$. This feature arises clearly due to the projection of IMF onto the BH mass with the relation of Equation (19). The small number below 7 $M_\odot$ reflects the steep slope in the function of Equation (19), where we note that the minimal BH mass is 2 $M_\odot$, which is within the mass range of a neutron star. The drastic decrease over 7 $M_\odot$ is caused by the projection of IMF, whose mass range is [40 $M_\odot$; 150 $M_\odot$] onto the narrow mass range, i.e., [7 $M_\odot$; 10 $M_\odot$].

We also examine the IMF shown in Kroupa & Weidner (2003). The total number of detectable BH binaries is estimated to be ~200 (Table 2). The power-law index of this IMF for stars more massive than 1.0 $M_\odot$ is smaller than that of the fiducial one, so that the total number of primary star whose main sequence mass is >20 $M_\odot$ is less than that of the fiducial case. This leads to a smaller number of the detectable BH binaries than in the fiducial case. This smaller power-law index also affects the distribution of the BH mass, which is shown in the left panel of Figure 3. The slope of the distribution in the case of K03 is steeper than that of the fiducial case. Thus, the power-law index of IMF directly affects the mass distribution of BH detectable with Gaia.

The parameter $\alpha\lambda$ does not affect the total number of detectable BH binaries. This is because the detectable binaries do not undergo the CE phase. Therefore, the distributions of BH mass (Figure 3) show no difference between fiducial, al01, and al20 cases.

Table 2 also shows the total numbers of the cases in which the distribution of mass ratio $q$ is different. This indicates that the smaller the power-law index of the $q$ distribution, the smaller the total number. As the index becomes smaller, the fraction of massive stars becomes smaller. The massive stars contribute the total number because they undergo the MT phase, so that the total number decreases as a result. Figure 4 shows the corresponding distributions of BH mass, whose slopes are not so different from that of the fiducial case. This means that the $q$ distribution does not affect the shape of the distribution of BH masses.
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Breivik et al. (2017) find different results from ours. Their histogram of the companion mass indicates that the binaries whose companion mass is less than 10 $M_\odot$ are included, which is because in their calculation the binaries undergoing the CE phase are included in the detectable BH binaries. In addition, the distribution of BH mass shows a complicated shape, but it seems to be consistent with the BH mass distribution of our fiducial model.

4.1. Validity of Our Model

The number estimate of detectable BHs depends on various parameters other than those changed in this paper. Among those are the star formation rate (Equation (4)), the distribution of semimajor axis (Equation (5)), the binaries spatial distribution (Equation (11)), the range of the orbital period required for the detection of BHs (Equation (36)), and Gaia’s astrometric precision $\sigma_\pi$. Thus, the total number shown in Table 2 can easily vary if those value are different from the one that we used. However, we do not expect that reasonable variation of these parameters would change significantly the number of detected BHs.

Our calculations rely on several simplifying assumptions: we ignored the effects of the stellar wind mass loss, natal kicks, and the initial eccentricities. Before concluding, we review these assumptions here. First, we note that the orbital expansion due to the stellar wind mass loss does not affect the number count of detectable BH binaries significantly. The factor by which the orbital separation expands depends only on the ratio of the binary mass before mass loss to that after the mass loss, and it does not depend on the separation. Because we assume a logarithmically flat distribution of initial separation, such a modification of the orbital separation does not change the final count of detectable BH binaries. Second, the stellar wind mass loss may affect whether the binaries would experience a CE phase or a MT phase because the mass ratio would be modified from that in the ZAMS phase. However, when the mass is maximally lost from the primary star by the time of the CE or MT phase, the mass would be $kM_1$, and then the initial mass ratio that divides two cases (CE or MT) would be no less than $\geq 0.3k$. In the fiducial model, the distribution of the initial mass ratio is flat, and then such a modification would affect the final result only by a factor. Third, as for the effect of natal kicks,
Figure 4 of Breivik et al. (2017) shows that kicks do not affect the final results such as the number of detectable BH binaries or its mass distribution. Finally, as for the initial eccentricity, Figure 4 of Breivik et al. (2017) shows that most of the detectable BH binaries have eccentricities that are nearly equal to zero. This means that our estimates obtained by assuming circular orbits for all binaries do not deviate so much from the estimates obtained taking into account the non-zero eccentricity.

While we assume the uniform binary fraction (50%) for entire stellar mass for simplicity, recent studies show that massive stars are expected to be found in binaries at higher rates (e.g., Sana et al. 2012; Moe & Di Stefano 2017). A larger binary fraction increases linearly the number of BH binaries. If we assume the binary fraction ~0.7 (Sana et al. 2012), we can easily obtain the total numbers of BH binaries by multiplying those in Section 3 by ~1.2.

We adopt a uniform value of $3 \times 10^3 R_\odot$ as the maximal radius of massive stars. Although this value depends on the stellar mass (and the luminosity) and can be larger for more massive stars, a variation of this value does not affect the results. Our results show that the detectable binaries should have passed through a MT phase, which implies that the separation does not change so much after that phase for a binary with a massive companion. In addition, the orbital period of the binary whose Roche radius is $\sim 3 \times 10^3 R_\odot$ is $\sim 10$ yrs. This is larger than the period of the detectable binaries, which is given in Section 2.7. Thus, even if the maximal radius of massive stars is larger than $3 \times 10^3 R_\odot$, Gaia cannot identify them as BH binaries, therefore the results are not influenced by this uncertainty.

We assume that the $G$-band magnitude is equal to the $V$-band magnitude. This is basically valid for nearby blue stars. Our results show that the companions of all detectable binaries are blue stars, but most of them are located at $\sim 7$ kpc, which means that these stars suffer from significant interstellar extinction. The extinction can be estimated to be $A_V \sim 7$ using “1 magnitude per kpc.” The ratio of extinction $A_G/A_V$ given in Jordi et al. (2010) shows that $A_G/A_V \sim 0.9$ for the bluest stars ($V-I \sim 0.4$; $T_{\text{eff}} = 50,000$ K in Jordi et al. (2010)) if $A_V = 5$. Correspondingly, we expect that, due to the linearity of $A_G/A_V$ with respect to $A_V$, $A_G/A_V \sim 0.8$ for $A_V \sim 7$. Thus, we expect $A_G \sim 6$. This difference in the extinction does not affect the results because the apparent $V$ magnitude of these stars is $\sim 13$ ($L_{\text{star}} \sim 10^5$ and distance $\sim 7$ kpc), which is much brighter than Gaia’s limiting magnitude.

4.2. Note on Astrometric Measurements

The semimajor axis and the parallax are not generally degenerate in the Gaia astrometric measurements. There are two reasons. One is that the periods of projected motion are different. The period of parallax is just 1 year and the orbital period is generally different from that. Thus, we can separate these two motions by the Fourier analysis. The other is that the phase and shape of these two motions are different. The phase and shape (ellipticity and position angle) of motion due to parallax is determined by the ecliptic longitude and latitude, respectively. On the other hand, those of the orbital motion are generally independent of the ecliptic coordinate. In this paper, we assume that the astrometric signature due to the orbital motion is large enough to be detected, so that these two motions can be separated. Of course, we note that if the orbital period is just the same as one year, and if the shape of orbital motion is the same as that of parallax, which occurs when the orbit is circular (the same as Earth orbit) and the inclination angle is equal to the ecliptic latitude, we expect that these two motions are degenerate. However, such case must be rare.

5. Conclusion

We have investigated the prospect of detecting of Galactic BH binaries by Gaia by observing the orbital motion of the BH’s companion. We have taken into account the orbital change due to a MT and a CE phase. In addition, we have taken into account interstellar absorption that was not considered before. We have calculated the distribution of BH binaries detected by Gaia adopting a signal-to-noise ratio of 10 for both the semimajor axis and the parallax. We show that, assuming our fiducial model, Gaia will be able to identify $\sim 500$ stars as companions of BHs. varying various parameters in this model we find the uncertainty of this estimate is in the range 200–1000. This values are much smaller than those obtained by Mashian & Loeb (2017) and by Breivik et al. (2017).

The shape of the mass distribution function of the detectable BHs depends strongly on the relation between the ZAMS mass of a star $M_\text{init}$ and its remnant mass $M_\text{BH}$. This mass relation is difficult to estimate from observations because the number of identified Galactic BH binaries is still small and their ZAMS is not known. Our results shows that the BH mass distribution obtained by Gaia will enable us to estimate this ZAMS mass–BH mass relation. This relation is important for the understanding of the stellar evolution process as well as understanding the core collapse process in massive stars.

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