A Complex Circuit Tolerance Analysis Method Based on Affine Analysis

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Abstract: In this paper, the selection of tolerance analysis objects and the uncertainty in the process of tolerance analysis for complex circuit systems are studied. A relatively perfect principle for determining the key circuits of tolerance analysis is formed. And considering the linear and nonlinear problems of circuits, based on the affine analysis method, a tolerance analysis method considering the influence of random factors is studied. Then the oretical basis and implement steps of the method is given finally, and the application of the typical circuit in a product circuit system is verified.

1. Introduction

The Circuit Performance Parameter $f(x_i)$ is subject to the related parameter value of its component $x_i$, and the tolerance capability of the components determines the stability and reliability of the circuit to a certain extent.

In general, the tolerance of a circuit is characterized by the sensitivity to a parameter, the sensitivity of circuit performance parameter to one component parameter is the degree of the influence by its variation on circuit performance when the parameter value of other components is constant. Sensitivity has magnitude and direction. If it is positive, the circuit performance parameter value becomes increases (decreases) when the component parameter value becomes increases (decreases). Or if the sensitivity is negative, the circuit performance parameter value decreases (increases) when the component parameter value increases (decreases). The higher absolute value of parameter sensitivity is, the higher influence of component parameter value on circuit performance parameter value is.

When carrying out circuit tolerance analysis, the monotonicity of circuit performance function should be determined firstly, and then calculate the sensitivity.

If $x_1, x_2, \ldots, x_n$ are the parameters of a component, the expression of the function should be as follows:

$$y = f(x_1, x_2, \ldots, x_n)$$

In this formula, $y$ represents the circuit performance parameter.

Find the partial derivative of $f$ to $x_i$, and solve the equation shown below in the given parameter interval $[a, b]$ of $x_i$:

$$\frac{\partial f}{\partial x_i} = 0$$

If the equation has no solution, the performance parameter $y$ is the monotone function of the component parameter $x_i$ in the interval $[a, b]$. Otherwise, the performance parameter $y$ is the non-monotone function of the component parameter $x_i$ in the interval $[a, b]$. 
Define $s_i$ as the sensitivity of the performance parameter $y$ to the parameter $x_i$, $s_i$ is the partial derivative of the function $f$ to $x_i$, denote as

$$s_i = \frac{\partial f}{\partial x_i}$$

In the formula, the values of the parameters $x_1, x_2, ..., x_n$ are taken from their respective means.

If a circuit performance parameter $y$ is a non-monotonic function of the component parameter $x_i$, the extreme point and its extreme value of the performance function in $(x_{min}, x_{max})$ are solved. For nonmonotonic performance functions, there may be more than one extreme point. In the worst-case analysis, each case should be treated separately:

a) There is only one extreme point, and define the extreme point $x_{j1}$. If the sensitivity changes from $(+)$ to $(-)$ within $(x_{min}, x_{max})$, it will takes the worst-case minimum at $x_{min}$ and takes the worst-case maximum at $x_{j1}$ when $f(x_{min})<f(x_{max})$. When $f(x_{min})>f(x_{max})$, it will takes worst-case minimum at $x_{max}$ and takes the worst-case maximum at $x_{j1}$. If the sensitivity changes from $(-)$ to $(+)$ within $(x_{min}, x_{max})$, it will takes the worst-case maximum at $x_{max}$ and takes the worst-case minimum at $x_{j1}$ when $f(x_{min})<f(x_{max})$. When $f(x_{min})>f(x_{max})$, it will takes the worst-case maximum at $x_{min}$ and takes the worst-case minimum at $x_{j1}$.

b) There are multiple extreme points as $x_{j1}, x_{j2}, ..., x_{jn}$. The worst-case minimum value is taken at the parameter values $x$ corresponding to $\min(f(x_{min}), f(x_{max}), f(x_{j1}), f(x_{j2}), ...)$, and the worst-case maximum value is taken at the parameter values $x$ corresponding to $\max(f(x_{min}), f(x_{max}), f(x_{j1}), f(x_{j2}), ...)$. And the worst-case minimum value is taken at the parameter values $x$ corresponding to $\min(f(x_{min}), f(x_{max}), f(x_{j1}), f(x_{j2}), ...)$.

Monte Carlo method, worst-case analysis method and moment method are the main methods of circuit tolerance analysis. Among them, moment method is the poorest method in engineering practice which one has limitation in describing large circuit model. The Worst-Case Analysis method is used to study the system response under the assumption that the Worst-Case of all components is assumed to be the Worst-Case, it takes the maximum error of all the components, and its disadvantage is overemphasizing the influence of the components on the system. Monte Carlo method is the main method used in computer-aided circuit tolerance analysis, but there is no model which can fully describe the change of component performance. In the process of analysis, It is often necessary to make an artificial assumption, which will result in the inaccurate results.

Therefore, a series of new tolerance analysis techniques, such as incremental analysis method[2] and analysis method based on chaos theory[3], have been created in engineering practice. These methods have solved the practical problems in some specific fields, but they also have some limitations. For example, the incremental analysis method mainly uses the knowledge of mathematical analysis to judge whether the characteristic function of the circuit is a monotonic function in the analysis interval. If the function is monotonic, the maximum and minimum of the characteristic function are at the vertices of the analysis interval. If it is not a monotone function, then the interval is divided into two sub-intervals to determine whether the monotonicity is valid. If not, continue dividing, until all intervals are monotonic. This method is not suitable for complex circuit or characteristic function monotony judgment. The analysis method based on chaos theory is mainly based on chaos theory to carry out the worst-case analysis. It optimizes the initial population by using the chaotic technique in the particle swarm optimization algorithm, introduces the chaotic disturbance term when the position is updated and carries on the boundary constraint after the position is updated, and carries on the simulation, the results are significant in terms of accuracy, stability and global search capability. However, the probability distribution of the error is not considered, and the circuit with the probability distribution is easy to lose the information.

Based on the theory and technology of circuit tolerance analysis at home and abroad, combined with the circuit design situation and characteristics of a typical aviation product, in this paper, the tolerance analysis technology of complex circuit system is studied, and a relatively perfect principle for determining the key circuit of tolerance analysis is formed.

2. Determination principle of key circuit in tolerance analysis

GJB/Z89-97 Guide to the Circuit Tolerance Analysis provides the general criteria for determining key
circuits that require tolerance analysis. It is mainly based on the importance of the task, the limitation of funds, schedule and the result of FMEA, or other analysis to determine the critical circuit which needs tolerance analysis in each development phase[4]. Mainly include:

- The circuit that seriously affects the safety of the products;
- The circuit that seriously compromises the mission;
- Expensive circuit;
- The circuit that are difficult to purchase or product;
- Circuits that require special protection.

In practical engineering application, the circuit which requires high precision is usually paid attention by designers, and its tolerance to parameter fluctuation determines the tolerance level of the whole circuit system in a certain sense. At the same time, the circuit which is sensitive to external environment, such as electromagnetic environment, should be considered in the design process. In practice, therefore, three additional circuits are required:

- The circuit that is more sensitive;
- The circuit that requires a lot of precision;
- The circuit that is often out of tolerance during testing.

3. The basic principle of affine analysis method

The interval \([x]\) is expressed as \(\bar{x} = x_0 + \sum_{i=1}^{n} x_i \xi_i\) in affine analysis method [5]. The interval symbol \([x]\) is replaced by \(\bar{x}\) in order to distinguish the representation. \(x_0\) is the center value of the interval, and \(x_i\) is the deviation component, and \(\xi_i\) is called noise coefficient (\(\xi_i \in [-1,1]\)). In the operation of the affine analysis method, the independence of each interval should be guaranteed as far as possible. The operations of the affine analysis method can be divided into two categories, linear operations or nonlinear operations.

3.1 Linear operations

In affine analysis, the result after operation can still be expressed by the same noise coefficient. We assume that:

\[
\bar{x} = x_0 + \sum_{i=1}^{n} x_i \xi_i
\]

Define their linear operation as follows:

\[
\bar{x} = x_0 + \sum_{i=1}^{n} y_i \xi_i
\]

\[
\bar{x} = x_0 + \alpha + \sum_{i=1}^{n} x_i \xi_i
\]

In the formula, \(\alpha \in R\).

3.2 Nonlinear operations.

In affine analysis, the result after operation can not be expressed by the original noise coefficient. Other noise coefficients must be added. We assume that:

\[
\bar{x} = x_0 + \sum_{i=1}^{n} x_i \xi_i
\]

\[
\bar{x} = y_0 + \sum_{i=1}^{n} y_i \xi_i
\]

the result of the nonlinear operation can be expressed as

\[
\bar{x} = f^m(\bar{x}, \bar{y}) = f^m(\xi_1, \xi_2, \ldots, \xi_n) = f^m(\xi_1, \xi_2, \ldots, \xi_n) + z_{N+1}\xi_{N+1}
\]

In the formula: \(|z_{N+1}| \geq \max \{k^m(\xi_1, \xi_2, \ldots, \xi_n)\}, \quad \xi_i \in [-1,1], i = (1,2,\ldots,n)\)
\[ e^{aw}(\xi_1, \xi_2, \cdots \xi_n) = f^{aw}(\xi_1, \xi_2, \cdots \xi_n) - f^w(\xi_1, \xi_2, \cdots \xi_n) \]
\[ f^w(\xi_1, \xi_2, \cdots \xi_n) = z_0 + \sum_{i=1}^{N} z_i \xi_i \]

In affine analysis, the value of \( z_i \) depends on \( x_i \) and \( y_i \) corresponding to the same noise.
Coefficient \( \xi_i \) in \( \sum_{i=1}^{N} \xi_i \) represents the newly generated noise coefficient expression, \( \xi_{N+1} \) and \( \xi_i \) are independent of each other. Using \( \xi_{N+1} \) instead of \( e^{aw}(\xi_1, \xi_2, \cdots \xi_n) \) will result a certain error, but the error can be reduced as long as the selected method is appropriate.

4. Case Study
Based on the key circuit criterion of tolerance analysis, the second-order infinite gain multi-channel feedback filter circuit is selected as the object of application verification. The circuit is shown in figure 1.

\[ U_o(S) = \frac{GB}{S^2 + (\frac{B}{2} + \alpha_0)}U_i(S) \]

In the formula, the Gain \( G = \frac{R_2}{2R_1} \), the Central Angular Frequency \( \omega_c = \frac{1}{R_1C}(\frac{1}{R} + \frac{1}{R_2}) \), the Central Frequency \( f_c = \frac{\sqrt{R_1 + R_2}}{2\pi C \sqrt{R_1R_2}}(\alpha_0 = 2\pi f_c) \), the Bandwidth \( B = \frac{1}{\pi R C} \), and \( R_1 = R_2 = 1k\Omega \), \( C = 1\mu F \).

It is assumed that only the resistance value varies with the environmental conditions, and the Tolerance Range is [0.95kΩ, 1.05kΩ]. It is also assumed that other parameters of the circuit are less affected by the environmental conditions and are regarded as constants.

Establish tolerance analysis simulation circuit model with Saber software.

\[ \text{Fig. 1 Infinite gain multiplex feedback filter circuit} \]

\[ \text{The transfer function of the circuit is as follow:} \]

\[ \text{Fig. 2 Tolerance analysis simulation circuit model} \]

After simulation calculation:
The experimental results of the circuit are as follows:

\[ [f_0] = [217.2\, Hz, 234\, Hz] \]
\[ [B] = [309\, Hz, 331.5\, Hz] \]

According to the basic principle of affine analysis method, the central frequency \( f_0 \) and the bandwidth \( B \) tolerance of the second order infinite gain multiplex feedback band-pass filter circuit can be obtained:

\[
\begin{align*}
\widetilde{R}_1 &= \widetilde{R}_2 = \widetilde{R}_3 = 1000 + 50\xi_1, \\
\widetilde{R}_1 + \widetilde{R}_2 &= 2000 + 100\xi_1, \\
\widetilde{R}_1 \times \widetilde{R}_2 \times \widetilde{R}_3 &= (1000 + 50\xi_1)^3
\end{align*}
\]

\( \xi_1 \in [-1, 1] \), 
\[ f_0 = \frac{\sqrt{\widetilde{R}_1 + \widetilde{R}_2}}{2\pi C \sqrt{\widetilde{R}_1 \widetilde{R}_2 \widetilde{R}_3}} = [214.3\, Hz, 237.1\, Hz] \]
\[ B_0 = \frac{1}{\pi \widetilde{R}_1 C} = [303.2\, Hz, 335.1\, Hz] \]

5. Conclusion

In view of the uncertainty in the process of tolerance analysis and the selection of tolerance analysis objects in complex circuit systems, the principles and methods for determining the key circuits of tolerance analysis are studied. Finally, the theoretical basis and implementation steps of the method are given, and the application of the method is verified with a typical circuit in a product circuit system. Compared with the measured results, the result obtained by affine analysis method is effective, not only is easier to realize, but also the analysis error is smaller. So it has a better applicability.

References

[1] Jones J, Hayes J. (1999) A Comparison of Electronic Reliability Prediction Models. IEEE Transactions on Reliability, 48: 127-134.
[2] Michael W, Tian, Richard Shi C J. (2005) Worst Case Tolerance Analysis of Linear Analog Circuits Using Sensitivity Bands. IEEE Trans. On Circuits and Systems I, 47: 1138-1145.
[3] Zhou Qiangfeng, Zhao Shumin, AnNing, Chi Chengzh. (2011) Tolerance Analysis of Array Antennas by Chaos-particle Swarm Optimization. Computer Simulation, 28: 211-214.
[4] (1997) Guide to circuit tolerance analyses (GJB/Z89-97). Approved by the National Defense Science, Technology and Industry Commission.
[5] Nicols Femia, Giovanni Spagnuolo. (2000) True Worst case Circuit Tolerance Analysis Using Genetic Algorithms and Affine Arithmetic. IEEE Transon Circuits and Systems I, 9: 1285-1296.