Semantics of HTS AC Loss Modeling: Theories, Models, and Experiments

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Abstract—Computer-assisted modeling is an essential approach to design new devices. It speeds up the process from the initial idea to an actual device and saves resources by reducing the number of built prototypes. This is also a significant practical motivator behind scientific research in contemporary high-temperature superconductor (HTS) ac loss modeling. However, in the scientific literature in this field, consistent practices about modeling terminology have not been established. Then, it is up to the reader to decide, what is the true intent and meaning of the authors. Consequently, the interpretation of such literature might be very much reader dependent. An inseparable part of the whole modeling process is the development of modeling approaches and numerical methods and comparing the predictions obtained via modeling to experimentally achieved results. It is commonplace to discuss the accuracy of modeling results or the validation of a model. In this article, we discuss the terminology related to theories, models, and experiments in the context of HTS ac loss modeling. We discuss the recursive nature of theories and models in this context, discuss the compatibility of discrete formulations of physics utilized in our field with the corresponding continuum description, and interpret the perceived meaning of validation of a self-consistent model, shedding light on the relationships between theories, models, and measurements. We present our view on understanding these relations in the familiar context of ac losses in HTS, studying case examples through simulations and literature. As a result, we end this article with four conjectures describing our views.

Index Terms—AC losses, experiment, high-temperature superconductors (HTS), model, modeling, theory.

I. INTRODUCTION

Computer simulations to predict ac losses in high-temperature superconductors (HTS) are required in the design phase of devices made out of such materials. This need has given rise to a niche subfield of science dedicated to numerical modeling of ac losses [1]. However, perhaps due to the relatively young age of this field, consistent practices about related terminology have not been established, which has the consequence of disconcerting author dependence and reader dependence of the scientific literature. The need for consistency in the terminology as well as the need for the recognition of some of the more philosophical aspects of the science conducted in this field were recently addressed in the Sixth International Workshop on Numerical Modelling of HTS in Caparica, Portugal, in the summer of 2018 [2]. This article is a natural outgrowth of the discussions initiated there.

“To understand science is to know how scientific models are constructed and validated.” (Hestenes [3])

A. Motivation and Background

As the criteria of successfullness of researchers are these days largely of quantitative nature, a natural consequence is that the perceived value of an incremental contribution has come closer to that of an average or even seminal contribution. This is also the case in our field. The multitude of papers published makes it arguably more and more difficult to pick out the important contributions among the published articles—whether the article is application-oriented, presents new modeling methodology, or discusses a completely new approach to a problem. This may lead to inefficient use of available overall resources. To deal with this in the field of HTS ac loss modeling, we believe organization and synthesis are useful. With this article, we aim to start this work. This article seeks to provide synthesis by unifying the terminology and considering some of the fundamental issues related to such scientific research. In this sense, this is a sister paper for Stenvall and Lahtinen [4] presented in the Applied Superconductivity Conference in Seattle, WA, USA, in the fall of 2018 [5]. Also, this article complements a recent effort with somewhat similar goals [6].

In our field of study and more generally, in engineering and natural sciences, the distinction between a theory and a model is often blurred in the everyday practice of scientific discussion, and the terms are used somewhat interchangeably. The nature and difference of these two related notions are also a central research issue in the philosophy of science. Modern literature encases a multitude of different views in terms of, e.g., the related ontology and semantics. A comprehensive review on such viewpoints can be found in [7]. However, our view is motivated by pragmatic benefits and provides us with a mental framework for the process of forming mathematical models of physical theories, and thus, yields also a clear distinction between the two concepts, suitable for our field. However, we also suggest that the distinction between a theory and a model is not an absolute notion but relative with respect to the chosen
abstraction level. We argue that acknowledging this recursive chain structure of theories and models benefits the scientific research in our field.

The mathematical models we utilize when modeling HTS ac losses are typically formalized in continuum either utilizing pointwise quantities and (partial) differential equations describing the phenomena or macroscopic quantities and algebraic equations. This yields the need for a discretization, a way to finite-dimensionalize the space from where to search for the solution. Using different discretization methods and formulating the utilized models in different ways, one obtains discrete descriptions that preserve different aspects of the original continuum-based description. The compatibility of a discretization in this sense should be understood as a key issue also in HTS ac loss modeling.

Arriving at a discrete description of the phenomenon we are modeling, suitable for a computer and perhaps compatible with the corresponding continuum description in some sense, we can make predictions. The question arises: How do we know if the predictions we make are any good? We must compare the predictions of our models to predictions of other models, and perhaps more preferably, to experimental data. This brings us to the notion of model validation. The central question here is: What is the relation between simulation results and experiments, and in what sense, if any, does such model validation validate a model? This is also one of the central issues of this article.

B. Structure of This Article

In Section II, we discuss the recursive nature of theories and models. Then, in Section III, we consider how we formulate our models and discuss also the compatibility of our discrete descriptions with the corresponding continuum descriptions. In Section IV, we take on the issue of relating modeling to experiments. Finally, Section V concludes this article.

II. THEORIES VERSUS MODELS: A RECURSIVE SYSTEM

In this section, we discuss the nature of and relationship between theories and models. In mathematics, a theory is a collection of axioms, taken as a starting point for deduction, that concern a set of undefined elementary entities. Theories of physics build upon the frameworks provided by mathematical theories by postulating some defining properties for the mathematical objects of a mathematical theory. Naturally, such a theory is typically based on physical intuition and mathematics is the way to formalize this intuition. As our field of interest is ac loss modeling of HTS, we shall restrict our discussion to mathematical theories and models of physics.

A. Maxwell’s Theory

Eventually, modeling physical phenomena utilizing models about nature is about solving equations. However, we know that models and theories utilized are something more than the formulas used to represent them. In the end, we want to represent something from the real world utilizing mathematics and often computers, to predict natural phenomena. Still, models seem to arise from axioms independently of how nature behaves. And even more clearly, things from the real world do not need any models—models are tools for predicting reality. As an example, real superconductors do not obey the Bean’s model [8], or the power-law (PL) model [9], which describe their transition between superconducting and normal conducting states, but indeed, both models are very useful in predicting the behavior of superconductors in certain situations. Of course, this does not mean that a model fully characterizing the electromagnetic behavior of a superconductor in a wider range of situations could not exist—most likely such a model can be formulated. However, we might later find examples of circumstances where this model would again fail. In this sense, real-world objects do not exhibit obedience to models.

But how do we form models from theories? And what are (mathematical) theories of physics in the first place? To take an example familiar to an ac loss modeler, consider Maxwell’s theory of electromagnetism, which, in our field, is typically stated as

\[
\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \tag{1}
\]

\[
\nabla \times \mathbf{H} = \mathbf{J} + \partial_t \mathbf{D} \tag{2}
\]

\[
\nabla \cdot \mathbf{B} = 0 \tag{3}
\]

\[
\nabla \cdot \mathbf{D} = q \tag{4}
\]

with the constitutive laws

\[
\mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}, \quad \mathbf{E} = \rho \mathbf{J}. \tag{5}
\]

In these equations, \( \mathbf{E}, \mathbf{D}, \mathbf{H}, \) and \( \mathbf{B} \) are the electric field intensity, electric flux density, magnetic field intensity, and magnetic flux density, respectively. These are mathematical beings called vector fields. The electric charge density \( q \) is a scalar field. Moreover, these fields are connected through the material parameters \( \varepsilon, \mu, \) and \( \rho. \) Typically, in our field, we work within the magnetoquasistatic regime, where we assume \( \partial_t \mathbf{D} = 0. \)

The standard approach for 3-D modeling is to assume that the modeling domain in which we are solving (1)–(5) is embedded in \( \mathbb{R}^3. \) Furthermore, we typically assume that the metric tensor of our modeling domain is Euclidean, which in \( \mathbb{R}^3 \) induces the standard norm that can be represented as \( ||\mathbf{x}|| = \sqrt{\sum_i x_i^2}, \) where \( x_i \) are the components of the vector \( \mathbf{x}. \) In fact, if we read the equations (1)–(5) literally, we have even fixed the coordinate system to be Cartesian. If, e.g., \( \nabla \times \mathbf{H} \) means literally the cross product of the formal vector of partial derivatives and the vector field \( \mathbf{H}, \) Maxwell’s equations as presented earlier do not hold for, e.g., spherical coordinates. However, if \( \nabla \times \mathbf{H} \) is merely a notational tool to express the curl of \( \mathbf{H}, \) then we are not necessarily bound to Cartesian frames. Nonetheless, Maxwell’s equations in the form presented earlier require the structures of metric tensor and volume form, as the definitions of curl and divergence require them, respectively [10].

B. Theory-Model Recursion—Is Maxwell’s Theory a Theory or a Model?

It is commonly agreed that Maxwell’s theory of electromagnetism as presented earlier is a theory. To obtain models from this theory, we can instantiate the free pieces in it. For example, we
are left to decide the material properties, the modeling domain \( R \subset \mathbb{R}^3 \), and source fields of this theory as we wish to instantiate a model and use it for modeling the electromagnetic phenomena in a given real-world situation. This is how we make models from theories: we instantiate the free pieces, the \textit{structures} left uninstantiated in the theory. However, we could have represented the same equations in a more general setting on a Riemannian manifold \( \Omega \) utilizing differential geometry [11]. From this perspective, the theory presented earlier is merely an instance of this more general theory—its model. We would have not had to instantiate the modeling domain to be Euclidean; such a domain is just an instance of a Riemannian manifold.\(^1\)

The aforementioned example suggests that the difference between a theory and a model is not an absolute notion. Instead, whether to call such a set of defining properties, a theory or a model is a decision left for the modeler. Hence, we can think of Maxwell’s theory of electromagnetism, as presented earlier, either as a theory or a model of a more general theory. However, given this freedom, it is crucial for the modeler to distinguish between a theory and its models in order to make meaningful predictions. Note also how, when understood in this sense, e.g., the PL model and the critical state model (CSM) are indeed different models of the same theory, (magnetoquasistatic) Maxwell’s theory. They differ in their instantiation of the constitutive equation connecting \( E \) and \( J \).

The key difference between a theory and its model is that the model instantiates something in the theory. In the science of modeling, it can sometimes be useful to go toward more abstract to look for something new or to a “silo” where no one has ever been yet. It is of crucial importance to acknowledge the abstraction level one is working at: what there is to instantiate and what can be found by raising the abstraction level. As an example, consider again Maxwell’s theory of electromagnetism. The whole theory is implied by the theory of quantum dynamics but not vice versa [13]. The whole classical theory of electromagnetism is thus reducible to a mere consequence, or a special case, of a more fundamental theory. In a related manner, by raising the abstraction level, we can \textit{unify} several seemingly different theories or models under one, more abstract entity containing them.\(^2\)

\( \text{C. Concluding Remarks} \)

We propose that theories and models form a recursive system, as shown in Fig. 1. Instantiating the free structures in a theory, one travels toward a more concrete description and forms models of the theory. On the other hand, free pieces may still exist at this abstraction level and in this sense, the obtained model is equivalent to a theory, from which more concrete models may again be obtained. New opportunities in research can arise from the recognition of the recursive theory-model chain. Via identification of structures, whether free or instantiated, in one’s model, opportunities for science arise. An example of this in the context of ac loss modeling can be found in [18].

\( \text{III. Formulating Numerical Models} \)

Acknowledging the recursive nature of theories and models does not carry a modeler very far by itself. To utilize models of continuum physics for computer simulations, a \textit{discretization} is required. Naturally, there are many ways to achieve such a discretization and, in addition, different starting points for modeling altogether. In this section, we discuss formulating physics with differential, pointwise quantities and algebraic, macroscopic quantities. Then, we go on to discuss discretizing such a formulation and the theoretical soundness of such a discrete description, i.e., the soundness of a \textit{numerical model} via the \textit{compatibility} of a discretization.

\( \text{A. Algebraic Versus Differential Formulation} \)

The typical way to formulate physics is to use pointwise quantities, such as vector fields or differential forms, governed by differential equations. This is exactly how we presented Maxwell’s theory of electromagnetism in Section II. Another option is to use macroscopic quantities: integrals of vector fields, or macroscopic counterparts of differential forms called cochains [19]. See Fig. 2 for clarification. For example, Maxwell’s theory can be represented either using pointwise quantities and differential equations or macroscopic quantities and algebraic equations.
∀ et al. makes the cochain equations give rise to infinite-dimensional function spaces in a similar manner as differential equations holding pointwise do [24].

B. From Continuum to Discrete: Compatibility of a Discrete Formulation With the Continuum Theory

In the finite-element method (FEM), for example, we approach a discrete formulation of a physical problem from the traditional direction by discretizing partial differential equations. That is, we have our model of physics described by such pointwise quantities as vector fields, which are governed by partial differential equations. Solving such problems, infinite-dimensional function spaces are encountered, and hence, a finite-dimensionlization, i.e., a discretization is needed. Utilizing an algebraic formulation, e.g., in CM, one discretizes the infinite-dimensional cochain complex by finding a finite-dimensional subcomplex. As described in [24], this is in fact the same thing one does in FEM. Here, our emphasis is on FEM formulations as they are mainly utilized in our field.

Whether we use a differential or an algebraic formulation to begin with, to get reliable predictions, the discretization that we make should be theoretically as sound as possible. That is, the discrete description should be compatible with the continuum one in the sense that as much of the structure as possible should be preserved in the discretization. This is our main focus here. We call this compatibility of the discrete description with the corresponding continuum description. Why care about compatibility? To ensure the reliability of the simulations. A continuum description of reality may not readily transfer all its properties to the discrete setting if attention to this is not paid. Several works explain this idea from different perspectives; see, e.g., [25] for a collection of such papers. Also, we discussed our views recently on the subject broadly in [24]. Be that as it may, the main idea boils down to preserving structures. For example, the properties of the differential operators, invariants of the theory, properties of the modeled quantities, or properties of the space.

1) Compatibility in FEM: The method we most often utilize to achieve a discrete description of the nonlinear magnetoquasistatics problem we are solving to predict the ac losses is FEM. Several formulations have been utilized for this purpose [26]–[33]. Lahtinen et al. [12] presented a comparison of some of the formulations and Grilli et al. [34] presented a comprehensive review of ac loss computation altogether. However, perhaps the most popular formulation is the so-called $H$-formulation, made popular in this context by Brambilla et al. [26].

In (Galerkin) FEM, one first forms the weighted residual formulation from the strong form of the partial differential equations by utilizing an inner product in a Hilbert space. Then, the weak formulation is obtained through weakening the differentiability requirements via integration by parts. Finally, this weak formulation is discretized by finding a suitable finite-dimensional subspace of the function space from where one is searching the solution for the problem at hand. The choice of the subspace and its basis is of crucial importance for the reliability of the numerical model, as we will discuss below.
We will not delve deeper into the details of FEM within the scope of this article, as the average reader is likely to be familiar with the method to some extent. For more, see, e.g., [35].

What kind of tools ensure the preservation of structures in a FEM discretization? As an example, the $H$-formulation is such a powerful and reliable tool for HTS ac loss modeling, because of its use of edge elements or Whitney 1-forms [36]. The Whitney form discretization ensures that the continuity properties of the electromagnetic field quantities are preserved in the process. Furthermore, Whitney forms inherit the exterior derivative, which is a generalization of the vector differential operators $\text{grad}$, $\text{curl}$, and $\text{div}$, from the more general setting of differential forms, preserving its properties. Moreover, the so-called Whitney map and the de Rham map, crucial in the Whitney form discretization, are ways to travel between simplicial cochains (integrals of differential forms on a simplicial mesh) and differential forms (Whitney forms). For formal definitions within a modern treatment, see, e.g., [19]. The properties of these mappings guarantee that the cohomology groups related to differential forms (i.e., in the continuum description) and those related to simplicial cochains (i.e., in the discrete description) are the same for modeling intents and purposes [19], [24], [37], [38]. Thus, important topological properties of the space are preserved in the Whitney form discretization too.

The spurious solutions plaguing electromagnetics in the 1980s were disposed of via the introduction of Whitney forms [39]. A good example from recent HTS ac loss modeling literature can be given in the context of FEM discretization of the $A-\psi-J$-formulation of magnetoequistatics. In [12] and [40], the current density $J$ was discretized using nodal elements, thus rendering $J$ tangentially continuous.

Having interfaces between materials with differing resistivities, this leads to unphysical current density profiles, as depicted in Fig. 3. Morandi’s approach to this problem of incompatibility between the discrete and continuous descriptions was to introduce doubled nodes on the material interface, thus relaxing the condition of tangential continuity [29]. Another option could be to build the numerical model using Whitney 2-forms for $J$, as $J$ is naturally expressed as a differential 2-form in continuum.

Another example of incompatibility due to modeling decisions is how the standard formulation in our field is often utilized: the $H$-formulation. In this formulation, one makes the (physically justified) modeling decision that nonconducting regions are in fact regions with very high resistivity. However, when one sets the Dirichlet boundary conditions for the magnetic field, fixing the magnetic field intensity at the boundary to be the applied external field, one also imposes a net current to flow through the cross sections of the whole domain. Then, imposing a net current in the conductors separately, a significant current

\[ J \]

will have to flow in the highly resistive regions of the modeling domain, as discussed, e.g., in [12] and [30]. Note that this is not as much a matter of incompatibility between the discrete and the continuous as it is a matter of incompatibility between the model and our physical intuition. This can be fixed by taking the self-field into account in the boundary conditions using the Biot–Savart law or with a different discretization altogether, using a truly nonconducting domain and cohomology basis functions to set the net currents [30], [31], [41], [42]. In such a formulation utilizing cohomology, also the time derivative of Gauss’s law (3) is typically imposed in an explicit sense, unlike in the usual form of the $H$-formulation. However, also in the $H$-formulation, (3) is satisfied as long as the initial condition satisfies it. Still, accumulating numerical errors can cause deviation from the zero-divergence condition in the $H$-formulation in long simulations.

2) Case Example: Utilizing Cohomology in a Compatible FEM Formulation: As discussed earlier, using an $H$-oriented formulation with cohomology basis functions, a compatible FEM formulation is attained. We have discussed the benefits of such a formulation in terms of compatibility via avoiding leak currents in the air regions in [30] and presented the formulation in detail and discussed its benefits in terms of faster simulations in [31]. Here, we would like to emphasize the “naturality” of applying current constraints in an ac loss modeling problem using such a formulation as well as the ramifications on the practicality of full 3-D simulations.

Consider setting current constraints for each of the tapes individually for the Roebel cable mesh utilized in [43], which represents 1/14th of the transposition length of a Roebel cable consisting of 14 YBCO tapes.1 In the $H$-formulation, this is done by introducing an algebraic constraint for each of the currents, thus rendering the equation system to be solved differential algebraic. As this mesh consists of 708 389 edges, posing an ac loss modeling problem in the domain with the $H$-formulation results in a similar number of degrees of freedom, with possible

\[ x 10^3 \]

\[ 0 \]

\[ -1 \]

\[ -2 \]

\[ -3 \]

Fig. 3. Current density profile obtained for a superconductor in applied magnetic field utilizing the $A-\psi-J$-formulation in which the current density $J$ is discretized using nodal basis functions, i.e., Whitney 0-forms. The profile is nonsmooth near the material interface. [12].

\[ 3 \]

\[ 2 \]

\[ 1 \]

\[ 0 \]

\[ -1 \]

\[ -2 \]

\[ -3 \]

\[ \text{Note that when we are solving for the magnetomotive forces over the edges of the mesh in the } H \text{-formulation, we are actually solving for the coefficients of the cochains that we obtain as integrals of the Whitney 1-forms, which are utilized as the basis functions in this FEM formulation.} \]

\[ J \]

\[ \text{has to be spanned with basis functions because of the nonlinearity of the } \text{PL visible in the constitutive equation between } E \text{ and } J \text{. This way, } \psi \text{ can be expressed without using the time derivative of the (unknown) magnetic vector potential } A \text{ and, thus, generic time-stepping algorithms may be utilized [40].} \]

\[ \text{The mesh is available in COMSOL format at the HTS Modelling Workgroup webpage. [Online]. Available: http://www.htsmodelling.com} \]
Fig. 4. Edges making up the cohomology basis functions that can be used for setting net currents through the tapes of the Roebel cable mesh. Each of the tapes has a corresponding cohomology basis function reaching from the boundary of the modeling domain up to the boundary of the tape. To minimize the total support of the cohomology basis, the supports of the basis functions overlap. Fixing these cohomology basis functions fixes the integrals of $H$ around the tapes in the $H$-$\varphi$-$\Psi$-formulation.

Dirichlet and periodic boundary conditions reducing the number slightly compared to the total number of edges. However, the problem can be set up utilizing the $H$-$\varphi$-$\Psi$-formulation, which makes use of nodal basis functions for scalar potential and cohomology basis in the air regions and uses edge elements only in the conducting regions. Not considering Dirichlet or periodic boundary conditions for simplicity, this results in 175 792 total degrees of freedom. There is no need for algebraic constraints as currents are fixed simply by fixing the cohomology basis. Hence, one can solve a pure ordinary differential equation system. A close-up of the cohomology basis for the Roebel cable, which can be used to set the net current constraints for the tapes, is depicted in Fig. 4. The cohomology basis was computed using the cohomology solver implemented within the Gmsh mesh generator software [44].

For demonstrative purposes, we solved two cases using this mesh with the $H$-$\varphi$-$\Psi$-formulation utilizing our in-house magnetoquasistatics solver LoST [45]: an applied field case with a sinusoidal magnetic field with frequency $f = 50$ Hz, amplitude $B_{\text{app}} = 100$ mT, and a transport current case with each tape carrying a sinusoidal current of amplitude $I = 0.5 I_c$, the $I_c$ for each tape being 100 A. With cross-sectional dimensions of $10 \, \mu m \times 2 \, mm$, this yields $J_c = 0.5 \times 10^{10}$ A/m$^2$. In the PL, we chose $E_c = 10^{-4}$ V/m and $n = 18$. The current penetration profiles for these example simulations are depicted in Fig. 5(a) and (b).

Hence, utilizing cohomology basis functions have instant ramifications on the practicality of such large 3-D simulations via reduced number of degrees of freedom. This is especially the case with such high aspect ratio structures as Roebel cables, as the conducting subdomains remain small compared to the air region required in the modeling. Moreover, setting the current constraints is done naturally by fixing the cohomology basis functions to the desired current values. As there is no need to constrain currents with separate algebraic equations, an ordinary differential equation system instead of a differential algebraic one may be solved. This makes the spatial discrete formulation compatible with a larger class of time-discretization schemes.

IV. MODEL VALIDATION: RELATING SIMULATIONS AND EXPERIMENTS

The compatibility of a discrete description with the continuum model is crucial for the internal consistency of the numerical model. Moreover, algebraic formulations of continuum physics can provide an intuitive and measurementlike interpretation for us. However, none of this says anything about compatibility with measurements. This is when we begin to discuss model validation.

Modeling need not even take a stand if a model is right or wrong. Instead, it is crucial to study the usefulness of models. If we, as modelers, predict something that deviates from what we

\footnote{Note that the cohomology basis was not utilized in the applied field case, as we applied no current constraints.}
observe, it does not mean that the physical theory we are using is wrong; indeed, another model of the same theory may yield predictions more compatible with the observations. Moreover, we have built the physical theory inside a mathematical one, and obviously, a mathematical theory is consistent without any reference to physical reality. It is not significant to ask if a model or a theory is right or wrong, as long as it is self-consistent.

Hence, to validate a model cannot mean to test its rightfulness. But if it is not that, then what is it exactly? In [46], the authors differentiate between verification and validation. They define verification as the assessment of the accuracy of the solution (this is related to our term compatibility in Section III) and validation as the assessment of the accuracy of a simulation by comparison with experiments. Hence, their term verification is related to benchmarking with, e.g., analytical solutions and validation to benchmarking with measurement data.

Let us approach model validation in the context of HTS ac loss modeling via a literature example.

A. Literature Example: Ripple Field Losses in Direct Current (DC) Biased Superconductors

Ripple field losses in HTS biased with dc have been investigated in some recent works [47]–[49]. In terms of ac loss modeling, such situations exhibit very long transients compared to typical ac situations, such as losses due to ac, ac field, or combinations of those. Additionally, it seems that different types of behavior can be predicted depending on the chosen E-J relation in the model [47]. The fact that ac ripple fields in applications, such as motors and generators, tend to be small compared to the dc bias accentuates the difference between, e.g., the PL and the CSM. PL predicts a nonzero loss for any value of current density, whereas CSM only predicts losses above \( J_c \). This leads to spatial homogenization of the current density profiles over time in PL-based simulations, further complicating the long transient behavior, as discussed in [47].

In [47], two models of the magnetoquasistatic Maxwell’s theory, the PL model and CSM were utilized to predict losses in a dc-biased ReBCO tape in ac ripple fields. Furthermore, the predictions of the models were compared against experimental data. Or, as one often says, the models were validated against measurements. What could be inferred? The authors report that the qualitative and quantitative agreement between the predictions of PL and CSM was good in a wide range. Furthermore, qualitative and quantitative agreement with measurements was also good in a wide range of situations. However, discrepancies at low ac fields with significant dc bias were observed, prominently when CSM was utilized. On the other hand, the PL-based model exhibited discrepancies especially in terms of magnetization loss.

A lot of uncertainty is of course related to such simulations and measurements. The authors note that there was a possible degradation of the critical current density \( J_c \) near the tape edges that was not taken into account in the simulations. Also \( J_c \) was possibly underestimated at low fields, and the potential current sharing with the stabilization layer was not taken into account. Furthermore, the noise-to-signal ratio at low fields is not good, adding further uncertainty to the results. Finally, the discretization methods were in fact of totally different nature, and the mutual compatibility of these discrete descriptions was not detailedly analyzed.

B. Concluding Remarks

As reported in [47], one model can reflect measurements better in a range of situations while one in another. So were the models validated in [47]? At least not in the sense that one model or another would have been proven right or wrong. In ac loss modeling, we tend to look at the double integral of the loss density \( P = E \cdot J \)

\[
\int_T \int_{\Omega_{sc}} P dV dt
\]  

over the superconducting regions \( \Omega_{sc} \) and cycle \( T \) of the ac quantity and compare models and measurements solely based on this real number. Such lumping can easily hide some essential properties of the utilized models. A theory-model system is or is not internally consistent regardless of model validation. Moreover, model validation like this does not validate a model in terms of compatibility of the discretization, which is rarely discussed in the context of model validation. To conclude, model validation demonstrates the applicability of the modeling methodology in some particular cases via comparison with measured data.

V. Conclusion

Methodological and terminological issues of HTS ac loss modeling have been addressed in recent workshops and conferences. Discussions have indicated a need for addressing such questions in more formal format. In this article, we have started this discussion and considered some of these issues with (subjectively) of high importance. We discussed the general recursive nature of models and theories and the formulation of numerical models, emphasizing the compatibility of such models with the continuum theory. Finally, we discussed the relation of modeling and experiments, scrutinizing the term model validation via an example from the literature.

Validation of a model via comparison with experiments is of crucial importance in the endeavor to produce predictions of ac losses in HTS. However, we feel that the verification of the reliability of the results in terms of theoretical soundness is currently underrepresented in our field and should be addressed more carefully. Approaching numerical modeling also from an algebraic point of view, in addition to the traditional differential equations approach, could broaden the horizons of researchers working on ac loss modeling via increased intuition and the necessarily different mathematical machinery, which is after all inherently present in the discretized versions of differential equation based formulations too.

To conclude and summarize the ideas presented in this work, we would like to end this article with four conjectures regarding modeling in general, but learned in the context of and especially valid for HTS ac loss modeling.
Conjecture 1: Theories and models should be considered as a recursive system. Whether a mathematical description of nature should be considered a model or a theory depends on the abstraction level from where one is looking.

Conjecture 2: While modeling, it is important to recognize the free structures left for the modeler to instantiate. Furthermore, the modeler can benefit from seeing the model as an instance of a more abstract theory.

Conjecture 3: The compatibility of a discrete formulation with the continuum formulation is of great importance in achieving reliable predictions: the properties of the continuum description should be preserved in a discretization as well as possible.

Conjecture 4: “Model validation” as it is understood, never fully validates a model. Demonstrating the applicability of a modeling methodology via comparison with measurements is necessary, but there are further aspects to validation and verification of a model that should be considered.

We hope these conjectures serve as a springboard for further discussions within the numerical superconductor modeling community in general and among HTS ac loss modelers in particular.

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