Current Switch by Coherent Trapping of Electrons in Quantum Dots

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We propose a new transport mechanism through tunnel-coupled quantum dots based on the coherent population trapping effect. Coupling to an excited level by the coherent radiation of two microwaves can lead to an extremely narrow current antiresonance. The effect can be used to determine interdot dephasing rates and is a mechanism for a very sensitive, optically controlled current switch.

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The analogy between real and artificial atoms (quantum dots) suggests the transfer of concepts from atomic physics to ultrasmall semiconductor structures. If methods like optical coherent control are combined with the tunability of quantum dots, basic quantum mechanical effects like preparation in a superposition of states and quantum interference can be realized and controlled in artificial microscopic devices. The interaction with light has been used to create coherent superpositions of states in single\textsuperscript{[1]} and double quantum dots\textsuperscript{[2]}. Furthermore, external radiation fields lead to non-linear electron transport effects like photo-assisted tunneling and photo-sidebands\textsuperscript{[3,4]}.

In this Letter, we propose a new transport mechanism through tunnel-coupled quantum dots based on the coherent population trapping effect, a well-known effect in atomic laser spectroscopy\textsuperscript{[5]}. We predict that the interaction with coherent light of two frequencies can be used to pump a current through a double dot. As a function of the relative detuning of the two frequencies the current shows an extremely narrow antiresonance, i.e. an optically controlled abrupt transition from a conducting to a non-conducting state. We furthermore show that the vanishing of the current antiresonance due to dephasing of the coupled groundstates coherence (which can be controlled by tuning the tunnel coupling) can be used to obtain quantitative estimates for inelastic dephasing rates in coupled dots.

The effect appears in double quantum dots where electron transport involves tunneling through two bonding and antibonding ground states $|1\rangle$ and $|2\rangle$ and one additional excited state $|0\rangle$, see Fig. 1. Leads coupled to both dots have chemical potentials such that electrons can tunnel into the groundstates but leave the dot only through the excited state. The system is driven by two light (microwave) sources with frequencies $\omega_1$ and $\omega_2$ that are detuned off the two excitation energies by $\hbar \delta_1 := \varepsilon_0 - \varepsilon_1 - \hbar \omega_1$ and $\hbar \delta_2 := \varepsilon_0 - \varepsilon_2 - \hbar \omega_2$. Relaxation from the excited level by acoustic phonon emission traps the dot in a coherent superposition of the bonding and the antibonding state, if $\delta_R := \delta_2 - \delta_1$ is tuned to zero.

In this case, the excited level becomes completely depopulated. In the case of real atoms, the resulting trapping of the electron in a radiatively decoupled coherent superposition leads to ‘dark resonances’ in the fluorescence emission. In the double dot case discussed here, the dark resonance effect appears as a suddenly vanishing electron current for $\delta_R = 0$. We suggest that for low enough microwave intensity, the effect can serve as a very sensitive, optically controlled current switch.

Atomic dark states have been found to be extraordinarily stable against a number of perturbations\textsuperscript{[6]}. In the quantum dot case, due to the Pauli blocking of the leads, a trapped electron can not tunnel out of the ground state coherent superposition. Furthermore, this superposition is protected from incoming electrons due to Coulomb blockade (no second electron can tunnel in). These two mechanisms guarantee the robustness of the effect, which is limited only by dephasing from inelastic processes. The latter are due to spontaneous emission of phonons in double dots\textsuperscript{[7,8]} and can be controlled by tuning system parameters with gate voltages.

\begin{align*}
\Gamma' & \quad \Gamma \\
\varepsilon_0 & \quad \varepsilon_1 \\
\varepsilon_2 & \quad \varepsilon_3 \\
\mu & \quad \mu' \\
\omega_1 & \quad \omega_2 \\
\Delta & \quad \epsilon
\end{align*}
FIG. 1. Level scheme for two coupled quantum dots in Coulomb blockade regime. Two tunnel coupled groundstates $|G\rangle$ and $|G'\rangle$ (small inset) form states $|1\rangle$ and $|2\rangle$ from which an electron is pumped to the excited state $|0\rangle$ by two light sources of frequency $\omega_1$ and $\omega_2$. Relaxation by acoustic phonon emission is indicated by dashed arrows.

In our model, we consider a double quantum dot in the strong Coulomb blockade regime that is determined by transitions between states of fixed particle number $N$ and $N+1$. The two tunnel coupled $N+1$–particle groundstates $|G\rangle$ and $|G'\rangle$ (see Fig. 1 inset) have energy difference $\varepsilon := \varepsilon_G - \varepsilon_{G'}$ and hybridize into states $|1\rangle$ and $|2\rangle$ of energy difference $\Delta := \varepsilon_2 - \varepsilon_1 = (\varepsilon^2 + 4T_c^2)^{1/2}$. Here, $T_c$ denotes the tunnel coupling matrix element. The system is irradiated with two coherent microwave sources with frequencies $\omega_1$ and $\omega_2$, driving the transitions $|1\rangle \rightarrow |0\rangle$ and $|2\rangle \rightarrow |0\rangle$. Here, $|0\rangle$ is the first excited state of the same electron number $N+1$ in the right dot with energy $\varepsilon_0$. Furthermore, the energy of the first excited level $|0'\rangle$ of the other (left) dot is assumed to be in off–resonance for transitions to and from the two ground states. If the energy difference $\varepsilon_0 - \varepsilon_0$ is much larger than $T_c$, the hybridization of $|0'\rangle$ with $|0\rangle$ can be neglected.

The microwave radiation pumps electrons into the excited level $|0\rangle$ such that transport through $N+1$ particle states becomes possible if both dots are connected to reservoirs of free two–dimensional electrons. We assume the Coulomb charging energy $U$ to be so large that states with two additional electrons can be neglected. Typical values are $1\text{meV} \lesssim U \lesssim 4\text{meV}$ in lateral double dots [7]. The chemical potentials $\mu$ and $\mu'$ are tuned slightly above $\varepsilon_2$; this excludes the co–tunneling like reentrant resonant tunneling process that can exist in three–level dots [5].

The light coupling is described by an interaction Hamiltonian in the rotating wave approximation,

$$H_I(t) = -\frac{\hbar}{2} \Omega_1 e^{-i\omega_1 t}|0\rangle\langle 1| - \frac{\hbar}{2} \Omega_2 e^{-i\omega_2 t}|0\rangle\langle 2| + h.c.,$$

where non–resonant terms have been neglected and $\Omega_j = (E_j/\hbar)\langle 0|e z| j\rangle$, $j = 1, 2$, are the Rabi frequencies, where $E_j$ is the projection of the electric field vectors of the light onto the dipole moments for the transitions $1 \rightarrow 0$, $2 \rightarrow 0$. The coupling of the dot groundstates to the leads is described by the standard tunnel Hamiltonian

$$H_V = \sum_{k_i = G, G'} \left( V_{k i} c_{k i}^\dagger |E\rangle \langle i| + c.c. \right),$$

and correspondingly for the excited state $|0\rangle$. Here, $|E\rangle$ denotes the ‘empty’ double dot $N$–particle state before tunneling of an additional electron, $c_{k i}^\dagger$ creates an electron with quantum number $k$ in the reservoir connected to the dot groundstate $i = G$ or $i = G'$, and $V_{k i}$ denotes the corresponding tunnel matrix element. The rates $\Gamma$ (right dot) and $\Gamma'$ (left dot) for tunneling between the dots and the connected reservoirs can be calculated from $H_V$ by second order perturbation theory.

If the chemical potentials $\mu$ and $\mu'$ are as indicated in Fig. 1, electron tunneling occurs by in–tunneling that changes $|E\rangle$ into $|G\rangle$ at a rate $\Gamma$ and $|E\rangle$ into $|G'\rangle$ at the rate $\Gamma'$, whereas out–tunneling from $|G\rangle$ and $|G'\rangle$ is Pauli blocked. The corresponding rates $\gamma_1$ and $\gamma_2$ for tunneling into the hybridized states $|1\rangle$ and $|2\rangle$ are $\gamma_{1,2} = [(\Delta + \varepsilon)^2 + 4T_c^2]/[(\Delta + \varepsilon)^2 + 4T_c^2]$.

On the other hand, electrons can leave the dot only by tunneling out of the state $|0\rangle$ (but not in) at the rate $\Gamma$. This tunneling is only into the right lead because we assumed negligible hybridization of $|0\rangle$ with $|0'\rangle$. Here and in the following, we neglect the energy dependence of $\Gamma$ and $\Gamma'$ for simplicity.

In coupled quantum dots, decay of excited levels is due to spontaneous emission of phonons rather than photons [9]. We denote the corresponding decay rates for the state $|0\rangle$ and $|2\rangle$ by $\Gamma_0$ and $\Gamma_2$, respectively. The lowest state $|1\rangle$ is stable against decay. For the moment, we take these rates as given and discuss quantitative estimates below. We are then in the position to set up equations of motion for the time–dependent occupation probabilities $p_j(t)$, $j = E, 1, 2, 0$, of the four double dot states. The spontaneous photon emission and the single electron tunneling gives rise to an incoherent dynamics, while the electron–light interaction in treated fully coherently. One has

$$\dot{p}_E = -(\gamma_1 + \gamma_2)p_E + \Gamma_{00},$$
$$\dot{p}_0 = -(\Gamma_0 + \Gamma)p_0 + 3\mu (\Omega_1 p_{10} + \Omega_2 p_{20}),$$
$$\dot{p}_1 = \alpha_1 \Gamma_0 p_0 + \gamma_1 p_E + \Gamma_2 p_2 - 3\mu (\Omega_1 p_{10}),$$
$$\dot{p}_2 = \alpha_2 \Gamma_0 p_0 + \gamma_2 p_E - \Gamma_2 p_2 - 3\mu (\Omega_2 p_{20}).$$

Here, $\alpha_1 = 1 - \alpha_2 = (\Delta + \varepsilon)^2/[(\Delta + \varepsilon)^2 + 4T_c^2]$ and $\rho_{00} = \rho_{00} e^{i\omega_0 t}$ are slowly–varying off–diagonal matrix elements of the reduced density operator of the double dot, whose equations of motion close the set (3). One has

$$\dot{\rho}_{10} = -D_1 \rho_{10} + \frac{\Omega_1^2}{2}(p_1 - p_0) + i\frac{\Omega_2}{2} \rho_{12},$$
$$\dot{\rho}_{02} = -D_2 \rho_{02} - i\frac{\Omega_2}{2}(p_2 - p_0) - i\frac{\Omega_1}{2} \rho_{12},$$
$$\dot{\rho}_{12} = -[i\delta_H + \Gamma_2/2]\rho_{12} - i\frac{\Omega_1}{2} \rho_{02} + i\frac{\Omega_2}{2} \rho_{10},$$

where we defined resonance denominators $D_j := (-1)^j i\delta_j + \alpha_j \Gamma_0 + \Gamma_2/2$ that appear in the solution for the coherences in the stationary case for large times which we consider from now on. Together with the normalization condition $p_E + p_1 + p_2 + p_0 = 1$, the stationary solution is then easily obtained.

Before discussing the stationary tunnel current, we estimate the inelastic rates $\Gamma_0$ and $\Gamma_2$ which determine if or not the effect can be observed in quantum dots at all. In the following, we restrict ourselves to lateral dots. Relaxation from the excited dot level $|0\rangle$ is due to acoustic phonon emission at a rate
\[ \Gamma^0 = (2\pi/h) \sum_\mathbf{Q} |\lambda_\mathbf{Q}|^2 \delta(\hbar\omega_\mathbf{Q} - \varepsilon_0) F_z(q_z)G(q_y), \]  

where \( \lambda_\mathbf{Q} \) is the deformation potential matrix element, \( \mathbf{Q} = (q_y, q_z) \) the phonon vector, \( \omega_\mathbf{Q} = c|\mathbf{Q}| \), and \( F_z \) and \( G \) are the quantum well and lateral dot form factor which cut off phonons with \( |q_z| \gtrsim l^{-1} \) and \( |q_y| \gtrsim l^{-1} \), where \( l_z \) is the quantum well width and \( l \) an estimate for the dot diameter. For \( \varepsilon_0 \lesssim 0.5\text{meV} \) and a typical well width of \( l_z \sim 50\text{Å} \), only the lateral cutoff \( G \) is effective here at energies above \( \hbar\omega = \hbar c/l \), where \( c \) is the longitudinal speed of sound [11]. The explicit form of \( G \) depends on the shape of the many–electron wave functions \( \langle x|0 \rangle \) and \( \langle x|i \rangle \), \( i = 1, 2 \) and is never known exactly for realistic dots with \( N \gtrsim 10 \) electrons. Assuming a form \( G(q) = (q^2)\delta^2/[1 + (q^2)^2] \) that smoothly interpolates between \( G(0) = G(\infty) = 0 \) and using material parameters for GaAs and \( l = 200 \text{nm} \), we find rates \( \Gamma^0(\varepsilon_0 = 0.5\text{meV}) = 6 \cdot 10^8 \text{s}^{-1} \) and \( \Gamma^0(\varepsilon_0 = 10\text{meV}) = 2 \cdot 10^{10} \text{s}^{-1} \).

Most important for the observation of the population trapping effect in dots is the relaxation rate \( \Gamma_{21} \). In GaAs/AlGaAs lateral double dots, \( \Gamma_{31} \) is mainly due to the spontaneous emission of phonons [9]. In experiments, gate voltages can be applied to tune the ground state level splitting to small values. Here, we assume \( \Delta \lesssim 20\text{meV} \) where form factor cutoffs are no longer effective. One obtains

\[ \Gamma_{21}(\Delta) \approx 2\pi \left( \frac{T_e}{\Delta} \right)^2 g \frac{\Delta}{\hbar} \left[ 1 - \frac{\sin(\Delta/\hbar\omega_d)}{\Delta/\hbar\omega_d} \right], \]

where \( \omega_d := c/d, \ g \lesssim 0.05 \) the dimensionless coupling constant, \( d \) is the distance between the dot centers, and we assumed identical shapes of both dots for simplicity and neglected the small overlap between the states \( |G \rangle \) and \( |G' \rangle \). Furthermore, a simplified model with bulk piezoelectric phonons has been adopted. Important here is that in contrast to real atoms the spontaneous rate \( \Gamma_{21} \) can be tuned in gated double dots by varying \( T_e \) and or \( \varepsilon \). This allows one to study how the coherent superposition of states is destroyed due to the interaction with the phonon bath as discussed now.

![FIG. 2. Tunnel current antiresonance through double dot system from Fig. 1 with groundstate energy difference \( \varepsilon = 10\text{µeV} \). The Rabi frequencies \( \Omega_1 \) and \( \Omega_2 \) are taken to be equal, parameters are \( \Omega_R = 0.21\Gamma^0, \Gamma = \Gamma' = \Gamma^0 = 10^9\text{s}^{-1} \), and \( \Gamma^0 \) is the relaxation rate due to acoustic phonon emission from \( |0 \rangle \). Inset: Inelastic rate \( \Gamma_{21} \) in \( \mu \text{eV}/\text{h} \), Eq. (6), with \( \hbar\omega_d = 20\text{meV} \). Dashed line indicates the crossover at \( \Gamma_{21}/2 = |\Omega_R|^2/2[\Gamma^0 + \Gamma] \), cf. Eq. (6).](image)

![FIG. 3. Current for fixed coupling \( T_e \) and different tunnel rates \( \Gamma = \Gamma' \). Parameters are \( \varepsilon = 10\text{µeV}, \ \Gamma^0 = 10^9\text{s}^{-1}, \ \Omega_R = 1.0\Gamma^0 \), \( T_e = 1\text{µeV} \). The stationary electric current \( I \) is obtained from the net flow of electrons with charge \( -e < 0 \) through either of the tunnel barriers connecting the dot to the reservoirs, \( I = -e\Gamma[p_0 - p_E]_{\text{stat}} = -e\Gamma' [p_E]_{\text{stat}} \). Fig. 3 shows the result for \( I \) as a function of the Raman detuning \( \delta_R \) for \( \Omega_1 = \Omega_2 \) and ground state energy difference \( \varepsilon = 10\text{µeV} \). Our calculations have been done for zero temperature \( T = 0 \). For finite \( T \), reabsorption of phonons which would smear the ground state levels can be suppressed by choosing a sufficiently large \( \varepsilon \gtrsim k_B T \). Close to \( \delta_R = 0 \), the overall Lorentzian profile breaks in and shows a sharp current antiresonance. For fixed microwave intensity (fixed Rabi frequency \( \Omega_R := (\Omega_1^2 + \Omega_2^2)^{1/2} \)) and increasing tunnel coupling \( T_e \), the inelastic rate \( \Gamma_{21} \), Eq. (6), increases (inset). As a result, the antiresonance becomes broader and finally disappears for larger tunnel coupling \( T_e \). The half–width \( \delta_{1/2} \) of the current antiresonance can be found via the stationary solution of Eq. (3) and Eq. (6) from the pole of a two–photon denominator as a function of \( \delta_R \). We find for the symmetric case \( \varepsilon = 0 \)

\[ \delta_{1/2} \approx \frac{\Gamma_{21}}{2} + \frac{|\Omega_R|^2}{2[\Gamma^0 + \Gamma]}. \]
Thus, $\delta_{1/2}$ increases with the inelastic rate $\Gamma_{21}$. For fixed microwave intensity, the vanishing of the antiresonance sets in for $\Gamma_{21} > |\Omega_R|^2/|\Gamma^0 + \Gamma|$, cf. the inset of Fig. 2. On the other hand, with increasing elastic tunneling $\Gamma$ out of the dot we recognize the striking fact that $\delta_{1/2}$ decreases down to its lower limit $\Gamma_{21}/2$. This behavior is shown in Fig. 3. For increasing tunnel rate $\Gamma$, the current increases until an overall maximal value is reached at $\Gamma \approx \Gamma^0$. The curve $I(\delta R)$ decreases again and becomes very broad if the elastic tunneling becomes much faster than the inelastic relaxation $\Gamma^0$. Simultaneously, the center antiresonance then becomes sharper and sharper with increasing $\Gamma$, its halfwidth $\delta_{1/2}$ approaching the limit $\Gamma_{21}/2$, Eq. (1).

The appearance of the sharp current antiresonance is due to a trapping of the additional electron in a coherent superposition of the two ground states $|1\rangle$ and $|2\rangle$. One can define linear combinations $|NC(t)\rangle := (\Omega_2/\Omega_R) |1\rangle - (\Omega_1/\Omega_R) e^{i(\omega_2 - \omega_1)t} |2\rangle$ and the orthogonal state $|C(t)\rangle$. At Raman resonance, only $|C(t)\rangle$ couples to the light, and excitation of the electron from $|C(t)\rangle$ to the excited state $|0\rangle$ with a subsequent decay into $|C(t)\rangle$ and $|NC(t)\rangle$ gradually pumps all the population into $|NC(t)\rangle$. This is because in the latter state the electron is decoupled from the light and can not be excited again.

We point out that the resonance effect described here differs physically from other transport effects in AC-driven systems, such as coherent destruction of tunneling [1], tunneling through photo-sidebands [2], or coherent pumping of electrons [13,14]. These phase-coherent effects are due to an additional time-dependent phase that electrons pick up while tunneling. Then, the time evolution within the system is ideally completely coherent with dissipation being a disturbance rather than necessary for the effect to occur. In contrast, the trapping effect discussed here requires incoherent relaxation (phonon emission) within the system in order to create the trapped coherent superposition of the ground states.

To conclude, our results suggest that the population trapping effect can be observed in the tunnel current through double dots irradiated with two microwaves. It offers the possibility to switch a current optically and to determine the interdot inelastic rate $\Gamma_{21}$ from the antiresonance linewidth $\delta_{1/2}$, Eq. (1). The microwave frequencies $\nu$ should be such that the first excited level in one of the dots is coupled by one-photon processes to the groundstates. An estimate with a single particle excitation energy of $\delta \varepsilon \approx 0.5\text{meV}$ yields $\nu = \delta \varepsilon / h = 120\text{GHz}$ which should be attainable with present day technology. The Raman shift $\delta R \equiv \delta_2 - \delta_1$ can be scanned through by fixing one of the frequencies (e.g., $\omega_1$) at resonance such that $\delta_1 \equiv 0$, and changing $\omega_2$ and therewith $\delta R = \omega_1 - \omega_2 - \Delta / h$. Both the relaxation rate $\Gamma^0$ and the dephasing rate $\Gamma_{21}$ then can be obtained from $I(\delta R)$-curves for different values of, e.g., the tunnel coupling $T_c$ or the energy difference $\varepsilon$.

Finally, we comment on the dephasing channel due to tunneling of electrons from the ground state coherent superposition into holes created by absorption of photons in the leads. The rate $\Gamma_r$ for such processes is proportional to $(\Omega_R/2\nu)^2$ [13] and turns out to be at least one order of magnitude less than the intrinsic dephasing rate $\Gamma_{21}$ unless one tunes to very small tunnel couplings $T_c \lesssim 0.5\mu\text{eV}$. In this regime, $\Gamma_r$ starts to dominate over $\Gamma_{21}$, and the halfwidth $\delta_{1/2}$ then becomes independent of $T_c$.

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