New Multiplicative Arithmetic-Geometric Indices

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http://dx.doi.org/10.22147/jusps-A/290601

Acceptance Date 10th May, 2017, Online Publication Date 2nd June, 2017

Abstract

In this Paper, we introduce the second, third, fourth and fifth multiplicative arithmetic-geometric indices of a molecular graph. We compute the fifth multiplicative arithmetic-geometric index of line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of \( TUC_4 \), \( C_8 \)[p, q].

Key words: molecular graph, fifth multiplicative arithmetic-geometric index, nanostructures.

Mathematics Subject Classification: 05C05, 05C12, 05C35.

1. Introduction

Let \( G \) be a finite, simple connected graph with a vertex set \( V(G) \) and an edge set \( E(G) \). The degree \( d_G(v) \) of a vertex \( v \) is the number of vertices adjacent to \( v \). The line graph \( L(G) \) of a graph \( G \) is the graph whose vertex set corresponds to the edges of \( G \) such that two vertices of \( L(G) \) are adjacent if the corresponding edges of \( G \) are adjacent. The subdivision graph \( S(G) \) of \( G \) is the graph obtained from \( G \) by replacing each of its edges by a path of length two. We refer to [1, 2] for undefined term and notation.

A molecular graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numeric quantity from structural graph of a molecule. These indices are useful for establishing correlation between the structures of a molecular compound and its physico-chemical properties.

Very recently Kulli\(^3\) introduced the first multiplicative arithmetic-geometric index of a graph \( G \) and it is defined as

\[
AG_{I\!I}(G) = \prod_{u,v \in E(G)} \frac{d_G(u) + d_G(v)}{2\sqrt{d_G(u)d_G(v)}}
\]

Many other multiplicative indices were studied, for example, in\(^4,5,6,7,8,9,10,11\).

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Motivated by the definition of the first multiplicative arithmetic-geometric index and by previous research on topological indices, we propose the second, third, fourth and fifth multiplicative arithmetic-geometric indices of a graph as follows:

The second multiplicative arithmetic-geometric index of a graph $G$ is defined as

$$AG_{II}^2(G) = \prod_{uv \in E(G)} \frac{n_u + n_v}{2\sqrt{n_u n_v}}$$

where the number $n_u$ of vertices of $G$ lying closer to the vertex $u$ than to the vertex $v$ for the edge $uv$ of a graph $G$.

The third multiplicative arithmetic-geometric index of a graph $G$ is defined as

$$AG_{II}^3(G) = \prod_{uv \in E(G)} \frac{m_u + m_v}{2\sqrt{m_u m_v}}$$

where the number $m_u$ of edges of $G$ lying closer to the vertex $u$ than to the vertex $v$ for the edge $uv$ of a graph $G$.

The fourth multiplicative arithmetic-geometric index of a graph $G$ is defined as

$$AG_{II}^4(G) = \prod_{uv \in E(G)} \frac{\varepsilon(u) + \varepsilon(v)}{2\sqrt{\varepsilon(u) \varepsilon(v)}}$$

where the number $\varepsilon(u)$ is the eccentricity of vertex $u$.

The fifth multiplicative arithmetic-geometric index of a graph $G$ is defined as

$$AG_{II}^5(G) = \prod_{uv \in E(G)} \frac{S_G(u) + S_G(v)}{2\sqrt{S_G(u) S_G(v)}}, \text{ where } S_G(u) = \sum_{uv \in E(G)} d_G(v).$$

We need the following results.

*Lemma 1* 1. Let $G$ be a $(p, q)$ graph. Then $L(G)$ has $q$ vertices and $\frac{1}{2} \sum_{i=1}^{p} d_G(u_i)^2 - q$ edges.

*Lemma 2* 1. Let $G$ be a $(p, q)$ graph. Then $S(G)$ has $p+q$ vertices and $2q$ edges.

In this paper, we compute the fifth multiplicative arithmetic-geometric index of line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$.

2. 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$

We consider the graph of 2D-lattice, manotube and nanotorus of $TUC_4C_8[p, q]$ where $p$ and $q$ denote the number of squares in a row and the number of rows of squares respectively. These graphs are shown in Figure 1.
By algebraic method, we get $|V(G_i)| = 4pq$, $|E(G_i)| = 6pq – p – q$; $|V(H_i)| = 4pq$, $|E(H_i)| = 6pq – p$; $|V(K_i)| = 4pq$, $|E(K_i)| = 6pq$.

3. Results for 2D-lattice of $TUC_4C_8[p, q]$

The line graph of the subdivision graph of 2D-lattice of $TUC_4C_8[p, q]$ is shown in Figure 2(b).

**Theorem 1.** Let $G$ be the line graph of the subdivision graph of 2D-lattice of $TUC_4C_8[p, q]$. Then

$$AG_5II(G) = \begin{cases} 
\left(\frac{9}{4\sqrt{5}}\right)^4 \times \left(\frac{13}{4\sqrt{10}}\right)^{4(p+q-2)} \times \left(\frac{17}{12\sqrt{2}}\right)^{4(p+q-2)}, & \text{if } p > 1, q > 1, \\
\left(\frac{9}{4\sqrt{5}}\right)^4 \times \left(\frac{13}{4\sqrt{10}}\right)^{4(p-1)} \times \left(\frac{17}{12\sqrt{2}}\right)^{4(p-1)}, & \text{if } p > 1, q = 1.
\end{cases}$$

Proof: The 2D-lattice of $TUC_4C_8[p, q]$ is a graph with $4pq$ vertices and $6pq – p – q$ edges. By Lemma 2, the subdivision graph of 2D-lattice of $TUC_4C_8[p, q]$ is a graph with $10pq – p – q$ vertices and $2(6pq – p – q)$ edges. Thus by Lemma 1, $G$ has $2(6pq – p – q)$ vertices and $18pq – 5p – 5q$ edges. It is easy to see that the vertices of $G$ are either of degree 2 or 3, see Figure 2(b). Therefore we have partition of the edge set of $G$ as follows.

| $S_G(u), S_G(v)uv \in E(G)$ | (4, 4) | (4, 5) | (5, 5) | (5, 8) | (8, 9) | (9, 9) |
|---|---|---|---|---|---|---|
| Number of edges | 4 | 8 | 2($p+q–4$) | 4($p+q–2$) | 8($p+q–2$) | 2($9pq+10$) – $19(p+q)$ |

| $S_G(u), S_G(v)uv \in E(G)$ | (4, 4) | (4, 5) | (5, 5) | (5, 8) | (8, 8) | (8, 9) | (9, 9) |
|---|---|---|---|---|---|---|---|
| Number of edges | 6 | 4 | 2($p–2$) | 4($p–1$) | 2($p–1$) | 4($p–1$) | $p–1$ |
Case 1. Suppose \( p > 1 \) and \( q > 1 \).

By algebraic method, we obtain \(|V_4|=8, |V_5|=4(p+q-2), |V_8|=4(p+q-2)\) and \(|V_9|=2(6pq-5p-5q+4)\). Thus the edge partition based on the degree sum of neighbor vertices of each vertex is obtained, as given in Table 1.

To compute \( AG_2(G) \), we see that

\[
\begin{align*}
AG_2(G) &= \prod_{uv \in E(G)} \frac{S_G(u) + S_G(v)}{2\sqrt{S_G(u)S_G(v)}} \\
&= \left(\frac{4+4}{2\sqrt{4 \times 4}}\right)^4 \left(\frac{4+5}{2\sqrt{4 \times 5}}\right)^8 \left(\frac{5+5}{2\sqrt{5 \times 5}}\right)^{2(p+q-4)} \left(\frac{5+8}{2\sqrt{5 \times 8}}\right)^{4(p+q-2)} \\
&\quad \times \left(\frac{8+9}{2\sqrt{8 \times 9}}\right)^{8(p+q-2)} \left(\frac{9+9}{2\sqrt{9 \times 9}}\right)^{(2pq+10)-19(p+q)} \\
&= (1)^4 \left(\frac{9}{4\sqrt{5}}\right)^8 \left(\frac{13}{4\sqrt{10}}\right)^{4(p+q-2)} \left(\frac{17}{12\sqrt{2}}\right)^{8(p+q-2)}.
\end{align*}
\]

Case 2. Suppose \( p>1 \) and \( q=1 \).

The edge partition based on the degree sum of neighbor vertices of each vertex is obtained, as given in Table 2.

\[
\begin{align*}
AG_2(G) &= \prod_{uv \in E(G)} \frac{S_G(u) + S_G(v)}{2\sqrt{S_G(u)S_G(v)}} \\
&= \left(\frac{4+4}{2\sqrt{4 \times 4}}\right)^6 \left(\frac{4+5}{2\sqrt{4 \times 5}}\right)^4 \left(\frac{5+5}{2\sqrt{5 \times 5}}\right)^{2(p-2)} \left(\frac{5+8}{2\sqrt{5 \times 8}}\right)^{4(p-1)} \\
&\quad \times \left(\frac{8+8}{2\sqrt{8 \times 8}}\right)^{2(p-1)} \left(\frac{8+9}{2\sqrt{8 \times 9}}\right)^{4(p-1)} \left(\frac{9+9}{2\sqrt{9 \times 9}}\right)^{(p-1)} \\
&= (1)^6 \left(\frac{9}{4\sqrt{5}}\right)^4 \left(\frac{13}{4\sqrt{10}}\right)^{4(p-1)} \left(\frac{17}{12\sqrt{2}}\right)^{4(p-1)} \times (1)^{(p-1)} \\
&= \left(\frac{9}{4\sqrt{5}}\right)^4 \left(\frac{13}{4\sqrt{10}}\right)^{4(p-1)} \left(\frac{17}{12\sqrt{2}}\right)^{4(p-1)}.
\end{align*}
\]

4. Results for \( TUC_5C_4[p, q] \) nanotube
The line graph of the subdivision graph of $TUC_4C_8[p, q]$ nanotube is shown in Figure 3(b).

(a) Subdivision graph of $TUC_4C_8[4, 2]$ nanotube
(b) line graph of subdivision graph of $TUC_4C_8[4, 2]$ nanotube

Theorem 2. Let $H$ be the line graph of the subdivision graph of $TUC_4C_8[p, q]$ nanotube. Then

$$AG_{SI}(H) = \left(\frac{13}{4\sqrt{10}}\right)^{4p} \times \left(\frac{17}{12\sqrt{2}}\right)^{8p}, \quad \text{if } p > 1 \text{ and } q > 1,$$

$$= \left(\frac{13}{4\sqrt{10}}\right)^{4p} \times \left(\frac{17}{12\sqrt{2}}\right)^{4p}, \quad \text{if } p > 1 \text{ and } q = 1.$$

Proof: The $TUC_4C_8[p, q]$ nanotube is a graph with $4pq$ vertices and $6pq - p$ edges. By Lemma 2, the subdivision graph of $TUC_4C_8[p, q]$ nanotube is a graph with $10pq - p$ vertices and $12pq - 2p$ edges. Thus by Lemma 1, $H$ has $12pq - 2p$ vertices and $18pq - 5p$ edges. We see that in $H$, there are $4p$ vertices, are of degree 2 and remaining all vertices are of degree 3. Therefore we have partition of the edge set of $H$ as follows:

| $S_H(u), S_H(v)uv \in E(H)$ | (5, 5) | (5, 8) | (8, 9) | (9, 9) |
|-----------------------------|-------|-------|-------|-------|
| Number of edges             | 2$p$  | 4$p$  | 8$p$  | 18pq – 19p |

| $S_H(u), S_H(v)uv \in E(H)$ | (5, 5) | (5, 8) | (8, 8) | (8, 9) | (9, 9) |
|-----------------------------|-------|-------|-------|-------|
| Number of edges             | 2$p$  | 4$p$  | 2$p$  | 4$p$  | $p$ |

Case 1. Suppose $p > 1$ and $q > 1$.

By algebraic method, we obtain $|V_5| = 4$p$, $|V_8| = 4$p$ and $|V_9| = 2(6pq - 5p)$ in $H$. Thus the edge partition based on the degree sum of neighbor vertices of each vertex is obtained, as given in Table 3.

$$AG_{SI}(H) = \prod_{uv \in E(H)} \frac{S_H(u) + S_H(v)}{2S_H(u)S_H(v)}$$

$$= \left(\frac{5 + 5}{2\sqrt{5 \times 5}}\right)^{2p} \times \left(\frac{5 + 8}{2\sqrt{5 \times 8}}\right)^{4p} \times \left(\frac{8 + 9}{2\sqrt{8 \times 9}}\right)^{8p} \times \left(\frac{9 + 9}{2\sqrt{9 \times 9}}\right)^{18pq - 19p}$$
\[
\text{Case 2. Suppose } p > 1 \text{ and } q = 1.
\]

The edge partition based on the degree sum of neighbor vertices of each vertex is obtained, as given in Table 4.

\[
AG_{SII}(H) = \prod_{uv \in E(H)} \frac{S_H(u) + S_H(v)}{2S_H(u)S_H(v)}
\]

\[
= \left(\frac{5 + 5}{2/5 \times 5}\right)^{2p} \times \left(\frac{5 + 8}{2/5 \times 8}\right)^{4p} \times \left(\frac{8 + 8}{2/8 \times 8}\right)^{2p} \times \left(\frac{8 + 9}{2/8 \times 9}\right)^{4p} \times \left(\frac{9 + 9}{2/9 \times 9}\right)^{p}
\]

\[
= (1)^{2p} \times \left(\frac{13}{4\sqrt{10}}\right)^{4p} \times (1)^{2p} \times \left(\frac{17}{12\sqrt{2}}\right)^{4p} \times (1)^{p} = \left(\frac{13}{4\sqrt{10}}\right)^{4p} \times \left(\frac{17}{12\sqrt{2}}\right)^{4p}.
\]

5 Results for \(TUC_4C_8[p, q]\) nanotorus

The line graph of the subdivision graph of \(TUC_4C_8[p, q]\) nanotorus is shown in Figure 4(b).

![Figure 4](image)

(a) subdivision graph of \(TUC_4C_8[4,2]\) nanotorus

(b) line graph of subdivision graph of \(TUC_4C_8[4,2]\) nanotorus.

Theorem 3. Let \(K\) be the line graph of the subdivision graph of \(TUC_4C_8[p, q]\) nanotorus. Then

\[\text{AG}_{SII}(K) = 1.\]

Proof: The graph of \(TUC_4C_8[p, q]\) nanotorus has \(4pq\) vertices and \(6pq\) edges. Then by Lemma 2, the subdivision graph of \(TUC_4C_8[p, q]\) nanotorus is a graph with \(10pq\) vertices and \(12pq\) edges. Thus by Lemma 1, \(K\) has \(12pq\) vertices and \(18pq\) edges. We see easily that in \(K\), \(|V_K| = 12pq\) and we have edge partition based on the degree sum of neighbor vertices of each vertex, as given in Table 5.

| Table 5. Edge partition of \(K\). |
|----------------------------------|
| \(S_K(u), S_K(v) \)uv \in E(K) | \(9, 9\) |
| Number of edges                  | \(18pq\) |

\[
AG_{SII}(K) = \prod_{uv \in E(K)} \frac{S_K(u) + S_K(v)}{2S_K(u)S_K(v)} = \left(\frac{9 + 9}{2/9 \times 9}\right)^{18pq} = 1.
\]
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