Clustering effects for explaining an anomalous JLab result on the $^9$Be structure function

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Abstract. An anomalous nuclear modification was reported by JLab measurements on the beryllium-9 structure function $F_2$. It is unexpected in the sense that a nuclear modification slope is too large to be expected from its average nuclear density. We investigated whether it is explained by a nuclear clustering configuration in $^9$Be with two $\alpha$ nuclei and surrounding neutron clouds. Such clustering aspects are studied by using antisymmetrized molecular dynamics (AMD) and also by a simple shell model for comparison. We consider that nuclear structure functions $F_A^2$ consist of a mean conventional part and a remaining one depending on the maximum local density. The first mean part does not show a significant cluster effect on $F_2$. However, we propose that the remaining one could explain the anomalous JLab slope, and it is associated with high densities created by the cluster formation in $^9$Be. The JLab measurement is possibly the first signature of clustering effects in high-energy nuclear reactions. A responsible physics could be an internal nucleon modification, which is caused by the high densities due to the cluster configuration.

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INTRODUCTION

Nuclear modifications of structure functions $F_2$ were found by the European muon collaboration, and it is often called the EMC effect. The nuclear modifications are now experimentally measured from relatively small $x$ to large $x$. Theoretical mechanisms are different depending on the $x$ region [1]. At small $x$, the modifications are caused by nuclear shadowing due to multiple scattering of a $q\bar{q}$ pair coming from the virtual photon. At medium $x$, nuclear effects are due to binding and a possible internal nucleon modification. Nucleon Fermi motion and short-range nucleon-nucleon correlation cause large-$x$ nuclear effects. Using nuclear $F_2$ data together with the other ones, optimum parton distribution functions have been determined in nuclei [2, 3].

Nuclear modifications at $x > 0.2$ are generally described by a convolution model with the nucleon structure function $F_2^N$ convoluted with the nucleon momentum distribution in a nucleus. The nucleon four-momentum distribution, which is called the spectral function, has been calculated in a conventional shell model possibly with the short-range correlations and internal nucleon modifications. On the other hand, it is known in low-energy nuclear physics that some nuclei have clustering configurations which cannot be described by simple shell models. It is an an interesting topic to investigate such clustering aspects in deep-inelastic structure functions [4].
An anomalous nuclear modification was found for the beryllium-9 nucleus at the Thomas Jefferson National Accelerator Facility (JLab) by measuring ratios $F_A^2/F_D^2$, where $A$ and $D$ denote a nucleus and the deuteron. Usually, nuclear modifications are smooth functions of the average nuclear density. JLab measurements showed the nuclear modification slope $d(F_A^2/F_D^2)/dx$ by approximating their $F_A^2/F_D^2$ data by straight lines at $0.35 < x < 0.7$ [5]. They are shown in Fig. 1 as a function of the scaled nuclear density. It is obvious from the figure that the nuclear modification slope of $^{9}$Be is too large to be expected from the average density.

In Ref. [4], we proposed that the anomalous JLab result could be interpreted by a clustering phenomenon in the $^{9}$Be nucleus. Here, the clustering means that the $^{9}$Be consists of two $^4$He nuclei with extra-neutron clouds. Such a nuclear clustering phenomenon had never been investigated in the structure functions, although small quark clusters such as a six-quark bag model was considered in the early stage of EMC-effect studies. We explain that the JLab measurement could be interpreted if the modification slope is plotted as a function of the maximum local density at a cluster by using a theoretical method of antisymmetrized molecular dynamics (AMD). In this article, we explain our theoretical approach for explaining the anomalous JLab result.

**NUCLEAR CLUSTERING EFFECTS IN $F_2$**

Nuclear structure functions at $x > 0.2$ are described by a convolution description shown in Fig. 2. The nuclear structure function $F_A^2$ is theoretically evaluated in two steps: first by calculating nucleon ($N$) momentum distribution in a nucleus ($A$), second by calculating the quark ($q$) momentum distribution in the nucleon. Then, the virtual photon from the charged lepton (electron in the JLab case) interacts with the quark. Namely, $F_A^2$ is given by the nucleonic one $F_N^2$ convoluted with the nucleon momentum distribution in the nucleus $f(y)$ [1, 6]:

$$F_A^2(x,Q^2) = \int_x^1 dy f(y) F_N^2(x/y,Q^2), \quad f(y) = \frac{1}{A} \int d^3p_N y \delta \left( y - \frac{p_N \cdot q}{M_N} \right) |\phi(p_N)|^2,$$  \hspace{1cm} (1)

where $y$ is the momentum fraction $y = M_A p_N \cdot q / (M_N p_A \cdot q)$.

In order to illustrate clustering effects on $F_A^2$, we calculate the momentum density $|\phi(p_N)|^2$ in two methods, a simple shell model with the harmonic-oscillator potential $M_N \omega^2 r^2 / 2$ and the AMD. It is known that the wave functions of the harmonic-oscillator potential are expressed by Laguerre polynomials and spherical harmonics. The AMD (or FMD (fermionic molecular dynamics)) is a typical method in investigating clustering aspects of nuclear structure. The advantage of the AMD is that it does not assume
any specific structure, cluster- or shell-like configuration, on nuclei. The AMD is a variational method, in which a nuclear wave function is given by the Slater determinant of single-particle wave packets: \[ \Phi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \det[\phi_1(\vec{r}_1), \phi_2(\vec{r}_2), \ldots, \phi_A(\vec{r}_A)]/\sqrt{A!}. \]

Here, \(\phi_i(\vec{r}_j)\) is the single-particle wave function given by \(\phi_i(\vec{r}_j) = \phi_i(\vec{r}_j) \chi_i \tau_i\) with spin and isospin states \(\chi_i\) and \(\tau_i\). The function \(\phi_i(\vec{r}_j)\) is the space part expressed as \[\phi_i(\vec{r}_j) = (2\nu/\pi)^{3/4} \exp[-\nu(\vec{r}_j - \vec{Z}_i/\sqrt{\nu})^2].\] Supplying simple NN interactions, we determine the variational parameters \(\nu\) and \(\vec{Z}_i\) by minimizing the system energy. Calculated AMD space densities are shown in Figs. 3 and 4 for \(^4\)He and \(^9\)Be, respectively. The \(^4\)He AMD density is almost the same as the shell-model one. However, as shown in Fig. 4, the \(^9\)Be density is totally different from a monotonic shell-model density because the \(^9\)Be has the configuration of two \(\alpha\) clusters with surrounding neutron clouds. Using both densities of the shell and AMD models, we calculate \(F_A^2\) in Eq. (1).

**FIGURE 3.** Density distribution of \(^4\)He [4].  

**FIGURE 4.** Density distribution of \(^9\)Be by AMD [4].

Although the two-dimensional \((x, y)\) density for \(^9\)Be in Fig. 4 is very different from a shell-model one, inhomogeneity of the nuclear density is washed out if the angular average is taken [4]. Then, transforming the densities to the momentum space, we obtain the results in Fig. 5 for \(^4\)He and \(^9\)Be. It is interesting to find that the AMD wave function of \(^9\)Be has larger high-momentum components than the shell-model one, whereas both densities are the same in \(^4\)He. The high-momentum components are created by the cluster development in \(^9\)Be because nucleons are confined mainly in the two small-space clusters.

Using the momentum densities, we calculate the nuclear structure functions \(F_A^2\) at \(Q^2=5\text{ GeV}^2\) by using Eq. (1). The results are shown in Fig. 6 for \(F_A^{\text{Be}}/F_A^{\text{D}}\) together with experimental data. Since the short-range correlations and internal nucleon modifications are not taken into account in our formalism, the curves do not fully agree with the data. How-

**FIGURE 5.** Nucleon-momentum distributions in \(^4\)He and \(^9\)Be by shell and AMD models [4].

**FIGURE 6.** Nuclear modifications in \(^9\)Be by shell and AMD models [4].
ever, it is interesting to find some clustering effects in $^9$Be as shown in Fig. 6 because there are differences between the two curves of the shell and AMD models, whereas both are the same in $^4$He. If the current experimental errors and other effects such as the correlations are considered, it may not be easy to find a clear clustering signature in $F_2^A/F_2^D$ from the conventional mean part.

We found that the conventional binding and Fermi-motion contributions do not show a significant clustering effect. Next, we consider the possibility that the high densities due to the cluster formation in $^9$Be could be the origin for the anomalous JLab data. We plotted the JLab data by the maximum local densities calculated by both models in Fig. 7. The curve interpolates the shell-model data points except for $^9$Be. If the $^9$Be slope data is shown by the maximum density of the shell model, it is again too large to be expected from other nuclei. However, if it is shown by the maximum density of the AMD with the cluster structure, it is on the curve. In this way, the “anomalous” JLab result could be explained by the cluster structure in the nucleus. There is also a small difference between the AMD and shell model in $^{12}$C of Fig. 7, which is caused by the mixing of a cluster-like configuration in the AMD model of $^{12}$C.

Since it may be confusing for the reader that the $^9$Be slope is understood by the cluster structure although the effect is rather small in Fig. 6, we would like to explain our viewpoint. The nuclear structure functions consist of the mean conventional part and the remaining one depending on the maximum local density:

$$F_2^A = (\text{mean part}) + (\text{part created by large densities due to cluster formation}). \quad (2)$$

The first part is described by the usual convolution calculation with the spectral function given by the averaged nuclear density distribution. The remaining part is associated with the inhomogeneity of the nuclear density, before taking the average of nuclear wave function, given by the nuclear cluster structure. The physics behind the latter part could be a nuclear-medium modification of internal nucleon structure. Our studies suggest that the physics mechanism, associated with the high densities created by the clusters in $^9$Be, could be the origin for the anomalous slope $d(F_2^{^9\text{Be}}/F_2^D)/dx$. Such studies of possible cluster structure in deep inelastic scattering will be continued at JLab [7], and a more elaborated theoretical model needs to be developed for comparison with future data.

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