HALO DARK CLUSTERS OF BROWN DWARFS AND MOLECULAR CLOUDS

F. DE PAOLIS,1,2 G. INGRASSO,3 PH. JETZER,2 AND M. RONCADELLI4,5

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ABSTRACT

The discovery of massive astrophysical compact halo objects (MACHOs) in microlensing experiments makes it compelling to understand their physical nature, as well as their formation mechanism. Within the present uncertainties, brown dwarfs are a viable candidate for MACHOs, and the present paper deals with this option. According to a recently proposed scenario, brown dwarfs are clumped with cold molecular clouds into dark clusters—in several respects similar to globular clusters—that form in the outer part of the Galactic halo. Here we analyze the dynamics of these dark clusters and address the possibility that a sizable fraction of MACHOs are binary brown dwarfs. We also point out that Lyα absorption systems fit naturally within the present picture.

Subject headings: dark matter — Galaxy: halo — Galaxy: kinematics and dynamics — gravitational lensing — stars: low-mass, brown dwarfs

1. INTRODUCTION

Observations of microlensing events (Alcock et al. 1993; Aubourg et al. 1993) toward the Large Magellanic Cloud (LMC) strongly suggest that a substantial fraction of the Galactic halo should be in the form of dark compact objects, called MACHOs (massive astrophysical compact halo objects) (De Rújula, Jetzer, & Massó 1992).

Actually, the MACHO collaboration has recently announced the discovery of several new events during their second year of observations (Alcock et al. 1997); eight microlensing events have been detected so far.6

Although the limited statistics presently available prevents us from drawing clear-cut conclusions from experimental data (Gates, Gyuk, & Turner 1996), the evidence for such a discovery is firm and its implications are striking. In fact, under the assumption that MACHOs are indeed located in the Galactic halo, the inferred halo mass in MACHOs within 50 kpc turns out to be 2.0^{+1.2}_{-0.7} \times 10^{11} M_\odot (Alcock et al. 1997), which is several times larger than the mass of all known stellar components of the Galaxy and represents a relevant portion of the Galactic dark matter. Remarkably enough, this result is almost independent of the assumed Galactic model. Unfortunately, this circumstance contrasts with the strong model dependence of the average MACHO mass. It has become customary to take the standard spherical halo model as a baseline for comparison. Regrettably, because of the low statistics, different data analysis procedures lead to results that are only marginally consistent.

Specifically, within the standard halo model, the average MACHO mass reported by the MACHO team is 0.46^{+0.3}_{-0.2} M_\odot (Alcock et al. 1997), whereas the mass moment method (De Rújula, Jetzer, & Massó 1991) yields 0.27 M_\odot (Jetzer 1996).

What can be reliably concluded from the existing data set is that MACHOs should lie in the mass range 0.05–1.0 M_\odot (see also Table 9 of Alcock et al. 1997), but stronger claims are unwarranted because of the high sensitivity of the average MACHO mass to the uncertain properties of the particular Galactic model under consideration (Evans 1996; De Paolis, Ingrosso, & Jetzer 1996).

Mass values of >0.1 M_\odot suggest that MACHOs should be either M dwarfs or white dwarfs. Observe that these mass values naturally arise within the standard halo model.

As a matter of fact, the M dwarf option can look problematic upon deeper consideration. The null results of several searches for low-mass stars both in the disk and in the halo of our Galaxy (Hu et al. 1994) suggest that the halo cannot be mostly in the form of hydrogen-burning main-sequence M dwarfs. Optical imaging of high-latitude fields taken with the Wide Field Camera of the Hubble Space Telescope indicates that less than ~6% of the halo mass can be in this form (Bahcall et al. 1994). A more detailed analysis that accounts for the fact that halo stars are likely to have a lower metallicity (with respect to solar), leads to an even more stringent upper limit of less than ~1% (Graff & Freese 1996). We emphasize that these results are derived under the assumption of a smooth spatial distribution of M dwarfs, and become less severe in the case of a clumpy distribution (Kerins 1997a). In the latter case, as pointed out by Kerins (1997b), the dynamical limits and HST observations require that the overwhelming fraction of M dwarfs, at least 95%, must still reside in clusters at present. His analysis shows that there exists a wide range of cluster masses and radii that are consistent with these requirements.

As we said, an alternative explanation for MACHOs can be provided within the standard spherical halo model by white dwarfs, and a scenario with white dwarfs as a major constituent of the Galactic halo dark matter has been explored (Tamanaha et al. 1990; Fields, Mathews, & Schramm 1997; Adams & Laughlin 1996; Chabrier, Segretain, & Méra 1996). However, even this proposal encoun-

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1 Bartol Research Institute, University of Delaware, Newark, DE 19716-4793.
2 Paul Scherrer Institut, Laboratory for Astrophysics, CH-5232 Villigen PSI, and Institute of Theoretical Physics, University of Zurich, Winterthurerstrasse 190, CH-8057 Zurich, Switzerland.
3 Dipartimento di Fisica, Università di Lecce, Via Arnesano, CP 193, 73100 Lecce, Italy, and Institute Nazionale di Fisica Nucleare, Sezione di Lecce, Via Arnesano, CP 193, 73100 Lecce, Italy.
4 Instituto Nazionale di Fisica Nucleare, Sezione di Pavia, Via Bassi 6, 1-27100, Pavia, Italy.
5 Work partially supported by Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, Pavia, Italy.
6 It should be mentioned that the MACHO team has found at least seven more events (which are reported on the Alert list), but a full analysis of them has not yet been published.
ters difficulties. Apart from requiring a rather ad hoc initial mass function (IMF) of the progenitor stars, sharply peaked somewhere in the range 1–8 $M_\odot$, and a halo age larger than ~16 Gyr, strong constraints on the number density of halo white dwarfs arise from present-day metal abundances in the interstellar medium (Ryu, Olive, & Silk 1990; Gibson & Mould 1997) and from deep Galaxy counts (Charlot & Silk 1995). In any case, future HST deep field exposures will either find the white dwarfs or put constraints on their fraction in the halo (Kawaler 1996).

Mass values $\lesssim 0.1 M_\odot$ make brown dwarfs an attractive candidate for MACHOs. In fact, these mass values are supported by several nonstandard halo models. One example is the maximal disk model (van Albada & Sancisi 1986; Persic & Salucci 1990; Sackett 1997), in which more matter is contained within the disk and the halo is less massive than in the standard halo model. The latter condition implies a falling rotation curve, and so a smaller transverse velocity of MACHOs. Hence, the microlensing timescale gets longer for a given MACHO mass, which means a smaller implied MACHO mass for a given observed timescale. We stress that a reduced transverse velocity of MACHOs also arises in other nonstandard halo models. Either a radially anisotropic velocity distribution or a halo rotation, for instance, would do the job (Alcock et al. 1997; Evans 1996; De Paolis et al. 1996). We also note that the EROS collaboration (Renault et al. 1997) has shown that MACHOs in the mass range $10^{-7}$–$2 \times 10^{-2} M_\odot$ do not contribute significantly (less than 20%) to the halo dark matter (this result is consistent with the MACHO experiment for objects of mass 0.1 $M_\odot$ to 1 $M_\odot$).

Although present uncertainties do not permit us to make any sharp statement about the nature of MACHOs, brown dwarfs still look like a viable possibility to date, and we shall stick to it throughout.

Even if MACHOs are indeed brown dwarfs, the problem nevertheless remains of explaining their formation, as well as the nature of the remaining dark matter in galactic halos.

We have previously proposed a scenario in which dark clusters of brown dwarfs and cold molecular clouds, mainly of H$_2$, naturally form in the halo at Galactocentric distances larger than 10–20 kpc (De Paolis et al. 1995a, 1995b, 1995c, 1995d). Similar ideas have also been put forward by Gerhard & Silk (1996). Dark clusters of brown dwarfs have been extensively investigated by Ashman & Carr (Ashman & Carr 1988; Ashman 1990). A slightly different picture, based on the presence of a strong cooling flow phase during the formation of our Galaxy, has been considered by Fabian & Nulsen (1994, 1997) and leads to a halo made of low-mass objects. In addition, Pfenniger, Combes, & Martinet (1994) suggested that H$_2$ clouds may constitute the dark matter in the disk of our Galaxy.

The model in question encompasses the model first proposed by Fall & Rees (1985) to explain the formation of globular clusters, and no substantial additional hypothesis is required. Various resulting observational implications have also been addressed. In particular, (1) the $\gamma$-ray flux arising from halo molecular clouds through the interaction with high-energy cosmic-ray protons has been estimated (De Paolis et al. 1995b, 1995c), (2) an anisotropy in the cosmic background radiation (CBR) is predicted to show up when looking at the halo of the M31 galaxy (De Paolis et al. 1995a), and (3) the infrared emission from MACHOs located in the halo of the M31 galaxy should be observable with the detector on the Infrared Space Observatory (ISO) or with the next generation of satellite-borne detectors (De Paolis et al. 1995a).

We would like to stress that a large proportion of MACHOs (up to 50% in mass) may well consist of binary brown dwarfs, formed either by the same fragmentation process that produces individual brown dwarfs or later, when the dark clusters start to undergo core collapse.

The aim of the present paper is to discuss in a systematic fashion further aspects of the above scenario. Basically, we try to figure out the dynamics of dark clusters. More specifically, we investigate the constraints that ensure their survival against various kinds of gravitational perturbations. We also demonstrate that because of dynamical friction on molecular clouds in the dark cluster cores (to be referred to as frictional hardening), the present orbital radius of binary brown dwarfs that are not too hard turns out to be typically of the order of the Einstein radius for microlensing toward the LMC. As a consequence, and also taking into account the adopted selection procedure in the data analysis, we understand why they have not been resolved so far; still, we argue that they can be resolved in future microlensing experiments with a more accurate photometric observation. Finally, we show that Ly$\alpha$ absorption systems naturally fit within our model.

The plan of the paper is as follows. In § 2 we recall the main points of the considered picture of the formation of dark clusters. Various dynamical constraints are thoroughly analyzed in § 3, paying particular attention to the phenomenon of core collapse. In § 4 we study the process whereby brown dwarfs form close binary systems (as a consequence of core collapse), and we investigate the mechanism of frictional hardening in great detail. In § 5 we turn our attention to the thermal balance in halo molecular clouds. Section 6 contains a short discussion of the relevance of Ly$\alpha$ absorption systems for the present scenario. Our conclusions are offered in § 7.

2. SCENARIO FOR DARK CLUSTER FORMATION

As shown elsewhere (De Paolis et al. 1995b, 1995c), the model in question encompasses the model first considered by Fall & Rees (1985) for the formation of globular clusters, and relies on the conclusion of Palla, Salpeter, & Stahler (1983; hereafter PSS) that the lower bound on the Jeans mass in a collapsing metal-poor cloud can be as low as $10^{-2} M_\odot$, provided that certain environmental conditions are met.

Let us begin by summarizing the ideas of Fall & Rees (1985) from the point of view that is most convenient for our considerations.

After the initial collapse, the protogalaxy (PG) is expected to reach a quasi–hydrostatic equilibrium state with a virial
temperature \( \sim 10^6 \) K. Fall & Rees (1985) have shown that in such a situation a thermal instability develops; density enhancements grow rapidly as the gas cools to lower temperatures. In fact, irregularities in the inflow during the gas collapse and fluctuations in the distribution of nonbaryonic dark matter (if present on the Galactic scale) would introduce perturbations with a wide range of sizes and amplitudes. As a result, randomly distributed overdense regions will form inside the PG. For reasons that will become clear later, these overdense regions will be referred to as proto-globular cluster (PGC) clouds.

Under the assumption that the plasma in the PG is in collisional ionization equilibrium, it turns out that the cooling rate (as a function of density \( \rho \) and temperature \( T \)) has the form

\[
\Lambda(\rho, T) = \rho^2 L(T)
\]

the expression of \( L(T) \) can be found, e.g., in Einaudi & Ferrara (1991). The cooling time is

\[
t_{\text{cool}} = \frac{3p_k T}{2\mu(\Lambda - \Gamma)},
\]

where the heating rate \( \Gamma \) attributable to external heating sources has been taken into account (here \( \mu \approx 1.22m_p \) is the mean molecular mass of the primordial gas). Since at the high temperatures under consideration the heating rate can safely be neglected, it follows that \( t_{\text{cool}} \sim \rho^{-1} \). On the other hand, the free-fall time is

\[
t_{\text{ff}} \sim (G\rho)^{-1/2},
\]

so we see that \( t_{\text{cool}} \) decreases faster than \( t_{\text{ff}} \) as \( \rho \) increases. As the above quasi-equilibrium state of the PG is characterized by the condition \( t_{\text{cool}} \sim t_{\text{ff}} \), it is clear that inside the PGC clouds we have \( t_{\text{cool}} < t_{\text{ff}} \). That is to say, the PGC clouds cool more rapidly than the rest of the PG. This process continues until hydrogen recombination occurs, because as soon as this happens, at a temperature of \( \sim 10^4 \) K, the cooling rate decreases precipitously, under the ionization equilibrium assumption (Dalgarno & McCray 1972). Therefore, the regime \( t_{\text{cool}} > t_{\text{ff}} \) should now be established even in the PGC clouds, so that the PG can be regarded at this stage as a two-phase medium, with cold PGC clouds in pressure equilibrium with the external (inter-PGC clouds) diffuse hot gas.

However, Kang et al. (1990) realized that the fast radiative cooling of the PGC clouds (from \( 10^5 \) K to \( 10^4 \) K) implies that the plasma inside these clouds cools more rapidly than it recombines, so that the above ionization equilibrium assumption is violated. Actually, the out-of-equilibrium recombination results in an enhanced ionization fraction. This fact does not affect the previous conclusions for temperatures \( > 10^4 \) K, but entails drastic changes at lower temperatures. Indeed, the existence of a sizable number of protons and electrons at temperatures \( < 10^4 \) K gives rise to \( \text{H}_2 \) formation via the reactions

\[
H + p \rightarrow \text{H}_2^+ + \gamma, \quad H + e \rightarrow \text{H}^- + \gamma
\]

and

\[
\text{H}_2^+ + H \rightarrow \text{H}_2 + p, \quad H + \text{H}^- \rightarrow \text{H}_2 + e.
\]

As a consequence, the PGC clouds undergo a further cooling below \( 10^4 \) K. Specifically, there is a direct radiative cooling via reactions (eq. [4]) and a radiative cooling via excitation of rovibrational transitions of \( \text{H}_2 \) (observe that \( \text{H}_2 \) is further produced by the reactions in eqs. [5] and [6] below). We stress that the latter process is very effective (much more than the former) at temperatures \( < 10^4 \) K and plays a crucial role in our considerations. Because now \( t_{\text{cool}} \ll t_{\text{ff}} \) in the PGC clouds, the collapse goes on and the PGC cloud density rises steadily. When the number density in the PGC clouds exceeds \( 10^8 \) cm\(^{-3} \), the \( \text{H}_2 \) production increases dramatically thanks to the three-body reactions

\[
\text{H} + \text{H} + \text{H} \rightarrow \text{H}_2 + \text{H}, \quad \text{H} + \text{H} + \text{H}_2 \rightarrow \text{H}_2 + \text{H}_2,
\]

as pointed out by PSS. In effect, these reactions are so efficient that virtually all the atomic hydrogen is converted rapidly to \( \text{H}_2 \). Correspondingly, the cooling of the PGC clouds is strongly enhanced and their evolution can proceed as in the scenario proposed by PSS.

Still, it goes without saying that \( \text{H}_2 \) can be dissociated by various sources of ultraviolet (UV) radiation, such as an active galactic nucleus (AGN) or a population of massive young stars (Population III; see Carr, Bond, & Arnett 1984) at the center of the PG. So, the ultimate fate of the PGC clouds strongly depends on (apart from other environmental conditions, which will be discussed later) the survival of \( \text{H}_2 \).

In the early phase of the PG, an AGN is expected to form at its center, along with Population III stars, through the disruption of central PGC clouds. This indeed happens, since the cloud collision time is shorter than the cooling time in the central region of the PG.

Thus, \( \text{H}_2 \) will be dissociated at Galactocentric distances smaller than a certain critical value \( R_{\text{crit}} \). Following the analysis of Kang et al. (1990), it is straightforward to evaluate \( R_{\text{crit}} \). Consider first the case of a UV flux arising from a central AGN. Then we find

\[
R_{\text{AGN}} \approx \left( \frac{L_{\text{AGN}}}{2 \times 10^{42} \text{ ergs s}^{-1}} \right)^{1/2} \text{kpc} \quad \text{for typical luminosities up to } L_{\text{AGN}} \approx 10^{45} \text{ ergs s}^{-1}.
\]

Equation (7) yields \( R_{\text{AGN}} \approx 20 \text{ kpc} \). On the other hand, when the UV dissociating flux is produced by massive young stars mainly located at the center of the PG, the critical Galactocentric distance turns out to be

\[
R_{\text{crit}} \approx \frac{10^{-3} \text{ kpc}^{-3}}{n_0} \frac{L_{\text{tot}}}{L_\star} \text{kpc} \quad \text{for } \text{bolometric luminosity } L_{\text{tot}} \approx 2 \times 10^{38} \text{ ergs s}^{-1} \quad \text{and central density } n_0 \approx 10^3 \text{ kpc}^{-3}.
\]

Equation (8), \( L_\star \approx 2 \times 10^{38} \text{ ergs s}^{-1} \) is the bolometric luminosity of a single B0 V star. Assuming a total stellar luminosity \( L_{\text{tot}} \approx 2 \times 10^{43} \text{ ergs s}^{-1} \) and a central number density \( n_0 \approx 10^3 \text{ kpc}^{-3} \), we find \( R_{\text{crit}} \approx 10 \text{ kpc} \). In conclusion, \( \text{H}_2 \) should remain undissociated at Galactocentric distances larger than \( 10-20 \) kpc.

2.1. Globular Clusters

According to the preceding analysis, in the inner Galactic halo (that is, for Galactocentric distances smaller than
10–20 kpc), H₂ gets dissociated, thus preventing any further cooling of the PGC clouds below $T \sim 10^4$ K. Therefore, these clouds remain for a long time in quasihydrostatic equilibrium at $T \sim 10^4$ K (namely, we have $t_{\text{cool}} > t_{\text{ff}}$ during this period). In such a situation a characteristic mass scale is imprinted on the PGC clouds by the gravitational instability, thereby resulting in a strongly peaked mass spectrum of the PGC clouds. In fact, Fall & Rees (1985) have shown that the Jeans mass of the PGC clouds after the long permanence at $T \sim 10^4$ K is given (as a function of the Galactocentric distance $R$) by

$$M_{\text{PGC}}(R) \approx 5 \times 10^5 \frac{R}{\text{kpc}}^{1/2} M_\odot ,$$

while the PGC radius turns out to be

$$r_{\text{PGC}}(R) \approx 20 \frac{R}{\text{kpc}}^{1/2} \text{pc} .$$

Observe that in the present case, the propagation of sound waves erases all large-scale perturbations, leaving only those at small scales.

Surely this is not the end of the story, since the UV flux is expected to eventually decrease. So, after some time a nontrivial fraction of H₂ should form in any case, causing a sudden drop of the PGC cloud temperature to well below $\sim 10^4$ K (the clouds now enter the regime $t_{\text{cool}} < t_{\text{ff}}$). What happens next is a rapid growth of the small-scale perturbations, which lead directly (in one step), because of the thermal instability, to the formation of stars inside the PGC clouds (Murray & Lin 1989). It goes without saying that we(Murray 1989).

First, turbulence should be negligible. Indeed, turbulence effects would make fragments collide, which, as shown by PSS, would increase the Jeans mass. Second, no sizable gravitational perturbations (shocks, supernova explosions, etc.) should be present, since otherwise they would induce gravitational collapse before cooling has succeeded in lowering the fragment Jeans mass down to the aforementioned values. Because these environmental conditions are likely to have occurred in the outer halo, the mass of the produced compact objects is expected to be close to the corresponding Jeans mass.

In addition to individual brown dwarfs, it seems quite natural to suppose that in this case, in much the same way as happens for ordinary stars, the fragmentation process should also produce a substantial fraction of binary brown dwarfs. For reasons that will become clear, these will be referred to as primordial binaries (we shall come back to this issue in the next sections). It is important to keep in mind that the mass fraction of primordial binaries can be as large as 50% (Spitzer & Mathieu 1980). So, we are led to the conclusion that MACHOs consist of both individual and binary brown dwarfs in this scenario.

However, we do not expect the fragmentation process to be able to convert all the gas in a PGC cloud into brown dwarfs. For instance, standard stellar formation mechanisms lead to an upper limit of at most 40% for the conversion efficiency (Scalo 1985). Therefore, a fairly large amount of gas (mostly H₂) should have been left over. What is its fate? At variance with the case of globular clusters, strong stellar winds are now manifestly absent, so this gas should remain gravitationally bound in the dark clusters. This conclusion is further supported by the following arguments (provided that dark clusters comprise a consistent fraction

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12 This indeed occurs for a wide range of UV fluxes and PGC cloud densities (Kang et al. 1990).
13 We stress that the $R$ dependence in equations (9) and (10) holds for $R < 20$ kpc only (see the discussion in Vietri & Pesce 1995), while for $R > 20$ kpc the mass of globular clusters tends rather to decrease.
14 The reader should keep this point in mind, in spite of our focus on brown dwarfs. Of course, quantitative changes in some results will occur, because M dwarfs are more massive than brown dwarfs (this is especially true concerning discussions in §§ 3 and 4).
15 This circumstance is consistent with our assumption that the stellar IMF in the dark clusters is more sharply peaked than the IMF in the globular clusters and in the disk.
of the Galactic dark matter). First, the gas cannot have diffused into the whole halo, for otherwise it would have been heated by the gravitational field to a virial temperature $\sim 10^6$ K, thereby becoming observable in the X-ray band; this option is ruled out by the available upper limits (Dickey & Lockman 1990). Second, the alternative possibility that the gas wholly collapsed into the disk is also excluded, because the disk mass would then be of the order of the inferred dark halo mass. Now, the virial theorem entails that the temperature $T_{DC}$ of a diffuse gas component inside a dark cluster is

$$T_{DC} \simeq 1.1 \left( \frac{M_{DC}}{M_\odot} \right)^{2/3} K .$$

(11)

Accordingly, a large fraction of diffuse gas would presumably give rise to an unobserved radio emission. Thus, we conclude that the amount of virialized diffuse gas inside a dark cluster must be low (it will henceforth be neglected). This circumstance implies in turn that most of the leftover gas should be in the form of self-gravitating clouds clumped in the dark clusters (since in this case the virial theorem applies to individual clouds). As we shall see, there are good reasons to believe that the central temperature $T_m$ of the molecular clouds in question should be very low, in fact close to that of the CBR. Accordingly, the molecular cloud mass $M_m$ and median radius $r_m$ are related by the virial theorem as

$$r_m \simeq 4.8 \times 10^{-2} \frac{M_m}{M_\odot} \text{ pc} .$$

(12)

Presumably, the fraction of cluster dark matter in the form of molecular clouds should be a function of the Galactocentric distance $R$, depending on environmental conditions such as the UV flux and the collision rate for the PGC clouds.

Before proceeding further, an important issue should be addressed. Given the supposed existence of a large amount of gas in the dark clusters, one would expect a star formation process to be presently operative. However, things are not so simple. Under the above environmental conditions, only stars of mass smaller than the cloud mass can be formed. Evidently, these stars are either brown dwarfs or M dwarfs. So, we see that undetected bright stars should not form in the dark clusters to the extent that our assumptions hold true. One might also wonder whether a sizable quantity of gas is eventually left over. As already pointed out, we argue that this should be the case, since otherwise it would mean that the brown or M dwarf formation mechanism would be much more efficient than any known star formation mechanism. Moreover, Gerhard & Silk (1996) have shown that the cluster gravitational field can stabilize the clouds against collapse.

Unfortunately, the lack of any observational information about dark clusters would make any effort to understand their structure and dynamics hopeless, were it not for some remarkable insights that our unified treatment of globular and dark clusters provides us.

In the first place, it seems quite natural to assume that dark clusters have a denser core surrounded by an extended spherical halo. For simplicity, we suppose throughout that the core density profile can be taken as constant. Moreover, it seems reasonable to imagine (at least tentatively) that dark clusters have the same average mass density as globular clusters. Hence, we obtain

$$r_{DC} \simeq 0.12 \left( \frac{M_{DC}}{M_\odot} \right)^{1/3} \text{ pc} ,$$

(13)

where $M_{DC}$ and $r_{DC}$ denote the mass and the median radius of a dark cluster, respectively. In addition, dark clusters (just like globular clusters) presumably stay for a long time in a quasi-stationary phase, with an average central density $\rho_g(0)$ slightly lower than $10^4 M_\odot \text{ pc}^{-3}$ (which is the observed average central density for globular clusters).

As a further implication of the present model, we stress that, at variance with the case of globular clusters, the mass spectrum of the dark clusters should be smooth, since the monotonic decrease of the PGC cloud temperature fails to single out any particular mass scale. As will be shown in § 3, dark clusters in the mass range $3 \times 10^2 M_\odot \lesssim M_{DC} \lesssim 10^6 M_\odot$ should survive all disruptive effects, so we restrict our attention to such a mass range throughout.

As far as dark clusters are concerned, we have seen that the brown-dwarf mass is expected to lie in the range $10^{-2} \cdots 10^{-1} M_\odot$. For definiteness (and with an eye to microlensing results), we imagine that all individual brown dwarfs have the same mass, $m \simeq 0.1 M_\odot$. So, binary brown dwarfs are twice as heavy. As a consequence, the mass stratification instability (Spitzer 1969) will drive them into the dark cluster cores, which then tend to be composed chiefly of binaries. Furthermore, an average MACHO mass somewhat larger than $\simeq 0.1 M_\odot$ can naturally be accounted for.

Finally, let us consider molecular clouds. Since they also originate from the above-mentioned fragmentation process, we suppose (for definiteness) that they lie in the mass range $10^{-3} M_\odot \lesssim M_m \lesssim 10^{-1} M_\odot$. Correspondingly, equation (12) entails $4.8 \times 10^{-5} \text{ pc} \lesssim r_m \lesssim 4.8 \times 10^{-3} \text{ pc}$ and $2.7 \times 10^{10} \text{ cm}^{-3} \gtrsim n_m \gtrsim 2.7 \times 10^9 \text{ cm}^{-3}$, respectively, where $n_m$ denotes the number density in the clouds.

Before closing this section, some comments are in order. There is little doubt that the foregoing considerations are qualitative in nature. Nevertheless, they provide nontrivial insights into several questions that arise in connection with the discovery of MACHOs. Specifically, a sharply peaked IMF in the range $10^{-2} \cdots 10^{-1} M_\odot$ comes out naturally, without having to invoke any new physical process. This explains the observed absence of a substantial quantity of main-sequence stars inside the dark clusters. Therefore, the observed absence of a large number of planetlike objects in the halo (Renault et al. 1997) is automatically explained. Furthermore, we can understand why brown dwarfs clumped into dark clusters form copiously in the outer halo, but not in the inner halo or in the disk. Indeed, the different stellar content of these regions is here traced back to the different environments in which the same star formation mechanism operates. It goes without saying that various issues addressed above require further investigations.

3. DYNAMICAL CONSTRAINTS ON DARK CLUSTERS

As we have seen, MACHOs are clumped into dark clusters when they form in the outer Galactic halo. Still, the further fate of these clusters is quite unclear. They might either evaporate or drift toward the Galactic center. In the latter case, encounters with globular clusters might have

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16 Incidentally, the same argument also rules out a halo primarily made of unclustered brown dwarfs (as well as white and M dwarfs).
dramatic observational consequences, and dynamical friction could drive too many MACHOs into the Galactic bulge. So, even if dark clusters are unseen, nontrivial constraints on their characteristic parameters arise from the observed properties of our Galaxy. Moreover, in order to play any role as a candidate for dark matter, MACHOs must have survived until the present in the outer part of the Galactic halo. Finally, it is important to know whether dark clusters are still clumped into clusters today, especially because an improvement in the statistics of microlensing observations permits us to test this possibility (Maoz 1994; Metcalf & Silk 1996).

We remark that previous work on dynamical constraints on clusters of brown dwarfs (Carr & Lacey 1987; Carr 1994; Wasserman & Salpeter 1994; Kerins & Carr 1994; Moore & Silk 1995; Gerhard & Silk 1996; Kerins 1997a) rests on the hypothesis of an initial dark cluster distribution that follows rather closely the analysis of Tremaine & Binney (1987). Accordingly, becomesequation (14). equation (20) Keeping in mind that in our model \( R > 10^{-20} \) kpc and \( M_{\text{DC}} \leq 10^6 M_\odot \) (see later discussion), we see that \( \Delta R \leq 5.8 \times 10^{-2} \) kpc. Therefore, dark clusters are still confined to the outer Galactic halo. As a consequence, encounters between dark and globular clusters as well as disk and bulge shocking of dark clusters are dynamically irrelevant, as long as they move on not too highly elongated orbits (in this way, an effective circularization of the orbits is achieved).

3.2. Encounters between Dark Clusters

Encounters between dark clusters may, under the circumstances to be analyzed below, lead to their disruption. For orientation, we note that an estimate of the one-dimensional velocity dispersion \( \sigma_\star \) of MACHOs and molecular clouds within a dark cluster is supplied by the virial theorem and reads

\[
\sigma_\star \approx 6.9 \times 10^{-2} \left( \frac{M_{\text{DC}}}{M_\odot} \right)^{1/3} \text{ km s}^{-1},
\]

where equation (13) has been used. Because in the present scenario \( M_{\text{DC}} \leq 10^6 M_\odot \) (see later discussion), we get \( \sigma_\star \leq 6.9 \) km s\(^{-1}\). Therefore, the one-dimensional velocity dispersion of dark clusters, which we naturally suppose to be just \( \sigma_\star \), is much larger than \( \sigma_\star \). Hence, it makes sense to work within the impulse approximation, whose range of validity is established more precisely by the condition \( M_{\text{DC}} \leq 10^{10} M_\odot \), which is evidently always met in our model.

In order to proceed further, we introduce \( \Delta E \) as the change of the internal energy of a dark cluster in a single encounter. Then (following Binney & Tremaine 1987) we find that encounters with impact parameter \( b \) in the range \( b_{\text{min}} \leq b \leq b_{\text{max}} \) increase the cluster’s energy at the rate

\[
\dot{E}(R) \approx \sqrt{\frac{\pi}{\sigma}} \frac{n_{\text{DC}}(R)}{\sigma^3} \int_0^\infty dv v^3 e^{-v^2/4\sigma^2} \int_{b_{\text{min}}}^{b_{\text{max}}} db b \, \Delta E,
\]

where \( v \) and \( n_{\text{DC}}(R) \) are the cluster velocity and number density (in the halo), respectively. We let \( \gamma \) stand for the fraction of halo dark matter in the form of dark clusters, so that we have \( n_{\text{DC}}(R) = \gamma \rho(R)/M_{\text{DC}} \), with \( \rho(R) \) given by equation (14). Accordingly, equation (20) becomes

\[
\dot{E}(R) \approx \frac{1}{2} \gamma \frac{\sigma G M_{\text{DC}}}{R^2 + a^2} \int_0^\infty dv v^3 e^{-v^2/4\sigma^2} \int_{b_{\text{min}}}^{b_{\text{max}}} db b \, \Delta E.
\]

Now, a natural definition of the time required by encounters to dissolve a cluster is provided by \( t_\text{d}(R) = E_{\text{bind}}/\dot{E}(R) \), where the binding energy \( E_{\text{bind}} \) is expressed in terms of the cluster properties as \( E_{\text{bind}} \approx 0.2 GM_{\text{DC}}^2/P_{\text{DC}} \).

3.2.1. Distant Encounters

Let us first consider distant encounters. Correspondingly, \( \Delta E \) is to be evaluated in the tidal approximation (Spitzer...
clusters in question, exceeds the age of the universe. We insert this quantity into equation (21), assuming now and in this way, we find
\[ t_d(R) \approx 4.7 \times 10^{-11} \left( \frac{M_{\odot}}{M_{\text{DC}}} \right)^{1/3} (R_2 + 1) \text{ yr} , \]  
where use has been made of equation (13). Assuming \( R > 10^{-20} \text{ kpc} \) and keeping in mind that \( \gamma \leq 1 \) and \( M_{\text{DC}} \lesssim 10^6 M_{\odot} \) (see later discussion), we get that for all the dark clusters in question, \( t_d(R) \) exceeds the age of the universe.

### 3.2.2. Close Encounters

In order to deal with close encounters, dark clusters must be regarded as extended objects. As in the case of globular clusters, this task is most simply accomplished by modeling the dark clusters by means of a Plummer potential with core radius \( \alpha \). Correspondingly, \( \Delta E \) is found to be
\[ \Delta E \approx \frac{G^2 M_{\text{DC}}^3}{3 \pi^2 v^2} . \]  
As before, we insert this quantity into equation (21), assuming now \( b_{\text{min}} \approx 0 \) and \( b_{\text{max}} \approx r_{\text{DC}} \). In this way, we obtain
\[ t_d(R) \approx 2.4 \times 10^{-11} \left( \frac{\alpha}{r_{\text{DC}}} \right)^2 \frac{\text{pc}}{r_{\text{DC}}} (R_2^3 + 1) \text{ yr} . \]  
We proceed by recalling that
\[ \alpha = \left[ \frac{3M_{\text{DC}}}{4\pi \rho_*(0)} \right]^{1/3} , \]  
where \( \rho_*(r) \) denotes the dark cluster mass density. Taking \( \rho_*(0) \approx 10^6 M_{\odot} \text{ pc}^{-3} \) (as for globular clusters today) and using equation (13), we can rewrite equation (25) as
\[ t_d(R) \approx 10^{-11} \left( \frac{M_{\odot}}{M_{\text{DC}}} \right)^{1/3} (R_2^3 + 1) \text{ yr} . \]  
Assuming \( R > 10^{-20} \text{ kpc} \) and remembering that \( \gamma \leq 1 \), we are led to the conclusion that dark clusters are not disrupted by close encounters, provided that \( M_{\text{DC}} \lesssim 10^6 M_{\odot} \).\(^{18}\)

### 3.3. Evaporation

Various dynamical effects conspire to make dark clusters evaporate within a finite time. Relaxation via gravitational two-body encounters leads to the escape of MACHOs approaching the unbounded tail of the cluster velocity distribution. Tidal truncation by the Galactic gravitational field enhances this process. A more substantial effect is caused by the gravothermal instability, when the inner part of the dark clusters contracts (core collapse) and the envelope expands.

Below, we shall address these issues separately. As is well known, a key element in such analyses is the relaxation time,
\[ t_{\text{relax}}(r) = 0.34 \frac{\sigma_*^3}{G^3 \rho_*(r) \ln(0.4N)} , \]  
where \( N \) is the number of MACHOs per cluster. As in the case of globular clusters, \( \rho_*(r) \) is expected to vary by various orders of magnitude in different regions of a single dark cluster; this dependence obviously shows up in \( t_{\text{relax}}(r) \). Therefore, for reference purposes, it is often more convenient to characterize a dark cluster by a single value of the relaxation time. This goal is achieved by introducing the median relaxation time (Spitzer & Hart 1971),
\[ t_{\text{rh}} = \frac{6.5 \times 10^{10}}{\ln(0.4N)} \left( \frac{M_{\text{DC}}}{10^5 M_{\odot}} \right)^{1/2} \frac{M_{\odot}}{m} \left( \frac{r_{\text{DC}}}{\text{pc}} \right)^{3/2} \text{ yr} . \]  
Explicitly, using equation (13) together with equation (19) and \( m \approx 0.1 M_{\odot} \), equation (28) takes the form\(^{19}\)
\[ t_{\text{relax}}(r) \approx 5 \times 10^{-7} \frac{M_{\text{DC}}}{M_{\odot}} \left( \frac{M_{\odot} \text{ pc}^{-3}}{\rho_*(r)} \right) \frac{1}{1.4 + \ln(M_{\text{DC}}/M_{\odot})} \text{ yr} . \]  
In the same fashion, equation (29) becomes
\[ t_{\text{rh}} \approx 8 \times 10^{-8} \frac{M_{\text{DC}}}{M_{\odot}} \frac{1}{1.4 + \ln(M_{\text{DC}}/M_{\odot})} \text{ yr} . \]  

#### 3.3.1. Spontaneous Evaporation

As is well known, any stellar association evaporates spontaneously\(^{20}\) within a finite time as a result of relaxation via gravitational two-body encounters. Specifically, a single close encounter between two MACHOs can leave one of them with a speed larger than the local escape velocity. So, the MACHO under consideration gets ejected from the dark cluster. We find for the ejection time (Hénon 1969)
\[ t_{\text{ej}} \approx 1.1 \times 10^5 \ln(0.4N) t_{\text{rh}} \approx 9 \times 10^8 \frac{M_{\text{DC}}}{M_{\odot}} \text{ yr} . \]  
Alternatively, several more distant, weaker encounters can gradually increase the energy of a given MACHO until a further weak encounter is sufficient to make it escape from the cluster. In this case, the evaporation time turns out to be (Spitzer & Thuan 1972)
\[ t_{\text{evap}} \approx 300t_{\text{rh}} \approx 2.4 \times 10^8 \frac{M_{\text{DC}}}{M_{\odot}} \frac{1}{1.4 + \ln(M_{\text{DC}}/M_{\odot})} \text{ yr} . \]  
Since \( t_{\text{ej}} \) is in any case longer than \( t_{\text{evap}} \), we focus our attention on the latter quantity. By demanding that \( t_{\text{evap}} \) should exceed the age of the universe, we conclude that dark clusters with \( M_{\text{DC}} > 3 \times 10^2 M_{\odot} \) are not yet evaporated.

#### 3.3.2. Tidal Perturbations

Dark clusters, just like globular clusters, are tidally disrupted by the Galactic gravitational field unless \( r_{\text{DC}} \) is

\(^{18}\) It should hardly come as a surprise that close encounters yield a more stringent bound on \( M_{\text{DC}} \) than distant encounters.

\(^{19}\) For simplicity, we here neglect the fact that binaries have masses larger than individual brown dwarfs. Furthermore, given the logarithmic \( N \)-dependence, we can safely take \( N \approx M_{\text{DC}}/m \approx 10(M_{\text{DC}}/M_{\odot}) \).

\(^{20}\) We use this terminology for the evaporation process that is neither induced by external perturbations nor specific to the gravitational interactions.
smaller than their tidal radius. So, the survival condition reads
\[ r_{DC} < \left( \frac{M_{DC}}{3M_\odot(R)} \right)^{1/3} R, \]
where \( R \) should be understood here as the perigalactic distance of the dark cluster and \( M_\odot(R) \) denotes the Galaxy mass inside \( R \). From equation (14), we obtain
\[ M_\odot(R) \approx 5.5 \times 10^{10} R_d(1 - R_d^{-1} \text{arc} R) M_\odot, \]
and from equation (13), equation (34) becomes
\[ R_d(1 - R_d^{-1} \text{arc} R)^{-1/2} > 0.047, \]
which is always satisfied for \( R > 10\)–20 kpc. Thus, the dark clusters under consideration are not tidally disrupted by the Galactic gravitational field.

### 3.3.3. Core Collapse

Core collapse plays an important role in the considerations that follow. It is now well established that the initial stage of this process is triggered by evaporation, which leads to a shrinking of the core as a consequence of energy conservation. Nuclei of dark clusters have shown that the dynamics of the core is correctly described by a sequence of King models (Cohn 1980). However, once the cluster density reaches a certain critical value, core collapse is dramatically accelerated by the gravothermal instability (Antonov 1962; Lynden-Bell & Wood 1968). Indeed, the negative specific heat of the core implies that the internal velocity dispersion \( \sigma_* \) increases, thereby enhancing evaporation, as the average kinetic energy decreases through evaporation itself. Moreover, the unbalanced gravitational energy makes the core contract, so its density rises by several orders of magnitude in a runaway manner. Numerical simulations show that the central velocity dispersion and the number of stars in the core, \( N_* \), scale as
\[ \sigma_* \sim \rho_*^{0.05}, \]
\[ N_* \sim \rho_*^{-0.36}, \]
respectively (Binney & Tremaine 1987). Incidentally, the somewhat surprising slow rise of \( \sigma_* \) in equation (37) is due to the larger mass loss from the core, as follows from equation (38).

When does the gravothermal instability show up? Unfortunately, a clear-cut answer does not exist, since the corresponding time \( t_{GI} \) depends on how clusters are modeled as well as on their concentration (Quinlan 1996). Manifestly, the lack of observational data for dark clusters makes a precise determination of \( t_{GI} \) impossible. The best we can do is to suppose that dark clusters behave like globular clusters as far as core collapse is concerned. In this way, we are led to the order-of-magnitude estimate (Binney & Tremaine 1987)
\[ t_{GI} \approx 3 t_{fr} \approx 2.4 \times 10^6 \frac{M_{DC}}{M_\odot} \frac{1}{1.4 + \ln(M_{DC}/M_\odot)} \text{yr}. \]

Comparing \( t_{GI} \) with the age of the universe, we conclude that dark clusters with \( M_{DC} \leq 5 \times 10^4 M_\odot \) are expected to have started core collapse.

As we have said, the central density grows dramatically during the second stage of core collapse, so the central relaxation time gets shorter and shorter. Detailed studies of the gravothermal instability have shown that if nothing opposes the collapse, the time needed to complete core collapse \( t_{coll} \) starting from an arbitrary time \( t \) proceeds as \( t_{\text{relax}}(0) \), with the latter quantity computed for the particular value taken by \( \rho_* \) at \( t \). Computer simulations of globular cluster dynamics entail \( t_{\text{coll}} \approx 330 t_{\text{relax}}(0) \) (Cohn 1980).

Because of the huge increase in the central density, close two-body encounters lead to the formation of bound binary systems by converting enough kinetic energy into internal energy (tidal capture). As we will see in § 4, binary brown dwarfs are produced in this way in the dark cluster cores during the early phase of core collapse. These binaries, which will be referred to as tidally captured binaries, and happen to be very hard, play a crucial role in this context, since they ultimately stop and reverse the collapse. Schematically, the argument goes as follows. Because hard binaries necessarily get harder in collisions with individual stars (Heggie 1975), the internal binding energy released by a binary is transformed into kinetic energy of both the star and the binary. Actually, the exchanged energy is so large that they both leave the cluster. However, as is not the case with evaporation, the kinetic energy (per unit mass) of the cluster is unaffected, while mass ejection obviously increases the potential energy. That is, the binding energy given up by the binaries ultimately becomes gravitational energy of the core. As a result of the unbalanced kinetic energy, the core starts expanding. Moreover, because of the negative specific heat, the increased potential energy makes \( \sigma_* \) decrease, thereby slowing down mass ejection. In this manner, core collapse is halted and reversed (Spitzer 1987).

As a matter of fact, the presence of binaries in appreciable quantities can also modify to some extent the standard scenario of core collapse as outlined above. Indeed, numerical Fokker-Planck simulations have shown that in this case the collapse is also driven by the mass stratification instability. As a consequence, the collapse proceeds faster than and starts before the few-binary case (the latter point makes eq. [39] more plausible than it might appear at first sight). This phenomenon is found to occur for both tidally captured binaries (Statler, Ostriker, & Cohn 1987) and primordial binaries (Spitzer & Mathieu 1980).

### 3.4. Discussion

What the above analysis shows is that dark clusters within the mass range \( 3 \times 10^2 M_\odot \leq M_{DC} \leq 10^6 M_\odot \) should have survived all disruptive effects arising from gravitational perturbations and are at present expected to populate the outer part of the Galactic halo. In addition, clusters with \( 3 \times 10^2 M_\odot \leq M_{DC} \leq 5 \times 10^4 M_\odot \) should undergo core collapse. Unfortunately, it is practically impossible to predict the future fate of those dark clusters that are in the postcollapse phase today. A priori it seems natural to imagine that bounce and subsequent reexpansion should follow core collapse (Cohn & Hut 1984; Heggie & Ramamani 1989). Perhaps also a whole series of core contractions and expansions may take place, giving rise to so-called gravothermal oscillations (Bettwieser & Sugimoto 1984). However, this conclusion crucially depends on the unknown model that correctly describes dark clusters. For instance, in tidally truncated models the cluster is completely destroyed within a finite time (Stodolkiewicz 1985; Ostriker, Statler, & Lee 1985). Moreover, what certainly happens in either case is that the number of MACHOs in...
the core monotonically decreases with time. So, an unclustered MACHO population is expected to coexist with dark clusters in the outer Galactic halo (unless all dark clusters have $M_{DC} \gtrsim 5 \times 10^4 M_\odot$); detection of unclustered MACHOs would therefore not rule out the present scenario.

4. MACHOS AS BINARY BROWN DWARFS

As has already been pointed out, it seems natural to suppose that a fraction of primordial binary brown dwarfs, possibly as large as 50% in mass, should form along with individual brown dwarfs as a result of the fragmentation process of the PGC clouds. Subsequently, because of the mass stratification instability, primordial binaries will concentrate inside the dark cluster cores, which are therefore expected to be chiefly composed of binaries and molecular clouds. In addition, as far as dark clusters with $M_{DC} \lesssim 5 \times 10^4 M_\odot$ are concerned, a population of tidally captured binary brown dwarfs ought to form in the dark cluster cores because of the increased central density caused by core collapse. Thus, a large fraction of binaries should be present inside the dark cluster cores at a late stage of their evolution. Below, we will try to make the discussion of this issue as quantitative as possible.

4.1. Survival and Hardness of Binary Brown Dwarfs

The first question to be addressed is whether a binary brown dwarf, produced by whatever mechanism long ago, survives until the present. To this end, we recall that a binary system is “hard” when its binding energy exceeds the kinetic energy of field stars (otherwise it is “soft”). In the present case, binary brown dwarfs happen to be hard when their orbital radius $a$ obeys the constraint

$$a \lesssim 1.4 \times 10^{12} \left( \frac{M_\odot}{M_{DC}} \right)^{2/3} \text{ km} . \quad (40)$$

As is well known, soft binaries always get softer, while hard binaries always get harder because of encounters with individual stars (Heggie 1975). So, if individual-binary encounters were the only relevant process, we would conclude that hard binary brown dwarfs should indeed survive. However, binary-binary encounters also play an important role in the dark cluster cores, where binaries are expected to be far more abundant than individual brown dwarfs. Now, in the latter process, one of the two binaries is often disrupted (which cannot happen for both binaries, given that they are hard, while fly-bys are rather infrequent), thereby leading to the depletion of the binary population. We will address this effect in § 4.3, where we find that for realistic values of the dark cluster parameters, the binary breakup does not take place.

4.2. Tidally Captured Binary Brown Dwarfs

As far as globular clusters are concerned, it is now well known that the most efficient mechanism for late binary formation is dissipative tidal capture in the core (Fabian, Pringle, & Rees 1975; Press & Teukolsky 1977; Lee & Ostriker 1986). Hence, we expect a similar situation to occur in dark clusters.

Let us now analyze this phenomenon in a quantitative fashion. As a first step, we observe that the radius $r_\bullet$ of a brown dwarf of mass $\approx 0.1 M_\odot$ is $r_\bullet \approx 0.7 \times 10^3$ km (Saumon et al. 1996). Furthermore, the Safronov number (Binney & Tremaine 1987) is

$$\Theta = \frac{G m}{2 \sigma_a r_\bullet} \approx 2 \times 10^7 \left( \frac{M_\odot}{M_{DC}} \right)^{2/3} , \quad (41)$$

which turns out to be much larger than 1. Within this approximation, the time for brown dwarf tidal capture can be written as (Lee & Ostriker 1986)

$$t_{tid} \approx 10^{12} \frac{10^5 \text{ pc}^{-3}}{n_{IBD}(0)} \left( \frac{M_\odot}{100 \text{ km s}^{-1}} \right)^{1/2} \left( \frac{r_\bullet}{M_\odot} \right)^{0.9} \left( \frac{M_{DC}}{M_\odot} \right)^{1.1} \text{ yr} , \quad (42)$$

where $n_{IBD}(0)$ is the number density of individual brown dwarfs in the core. Obviously, we have $n_{IBD}(0) \approx f_{IBD} \rho_*(0)/m$, where $f_{IBD}$ denotes the mass fraction of individual brown dwarfs in the core. From equation (37), we see that $\sigma_*$ increases very slightly, and so core-collapse effects on $\sigma_*$ can safely be neglected. Accordingly, we can rewrite equation (42) as

$$t_{tid} \approx 1.6 \times 10^{14} \frac{M_\odot}{f_{IBD} M_{DC}} \text{ pc}^{-3} \left( \frac{M_{DC}}{M_\odot} \right)^{0.4} \text{ yr} , \quad (43)$$

using equation (19). Comparing $t_{tid}$ with the age of the universe, we see that practically all individual brown dwarfs in the core are tidally captured into binaries provided that 21

$$\rho_*(0) \gtrsim 3.2 \times 10^4 \left( \frac{M_{DC}}{M_\odot} \right)^{0.4} \text{ M}_\odot \text{ pc}^{-3} . \quad (44)$$

According to the above assumptions, we expect $\rho_*(0) \approx 10^4 M_\odot$ pc$^{-3}$ just before core collapse. Therefore, we see that tidal capture requires an increase of $\rho_*(0)$ by a factor in the range $31/f_{IBD} - 242/f_{IBD}$, corresponding to $M_{DC}$ in the range $3 \times 10^2 - 5 \times 10^4 M_\odot$. Thus, the formation of tidally captured binaries would occur during the early phase of core collapse (the same conclusion was reached in a different way by Statler, Ostriker, & Cohn [1987] for globular clusters). However, this conclusion depends on the fractional (mass) abundance, $f_{PB}$, of primordial binaries in the core, since $f_{IBD}$ necessarily becomes small for large $f_{PB}$.

Next, we compute the (average) orbital radius of tidally captured binary brown dwarfs, following the procedure outlined by Statler et al. (1987). Correspondingly, we find $a \approx 2.5 \times 10^3$ km (this value is practically independent of $M_{DC}$). As a consequence, we see that they are so hard that the condition of equation (40) is always abundantly met.

Let us finally try to estimate the fractional abundance of tidally captured binary brown dwarfs in the dark cluster cores soon after their formation, namely, when the inequality of equation (44) just starts to be satisfied (their total number at this stage will be denoted by $N_{TCB}$). Thanks to equation (38), we easily get

$$N_{TCB} \approx 3.3 f_{IBD}^{1.36} \left( \frac{M_{DC}}{M_\odot} \right)^{0.86} \frac{M_c}{M_\odot} , \quad (45)$$

where $M_c$ denotes the core mass just before core collapse.

---

21 Although equation (42) has been derived for main-sequence stars, it seems plausible that it may also apply to brown dwarfs, since it is not very sensitive to the particular stellar model (Lee & Ostriker 1986).

22 More precisely, we are demanding that the rate for tidal capture times the age of the universe should exceed 1. Note that the former quantity is one-half of $t_{tid}$, so we require that $2t_{tid}$ should be smaller than $10^{14}$ yr (we are actually following the same procedure used by Press & Teukolsky 1977).
[corresponding to \( \rho_*(0) \approx 10^4 \, M_\odot \, \text{pc}^{-3} \)]. Further defining \( N_{\text{IBD}}^{\text{tot}} \) as the total number of individual brown dwarfs in a dark cluster before core collapse, we find

\[
\frac{N_{\text{TCB}}}{N_{\text{IBD}}} \approx 0.33 f_{\text{IBD}}^{3/10} \left( \frac{M_\odot}{M_{\text{DC}}} \right)^{0.14} \frac{M_*}{M_{\text{DC}}} \, \text{yr}^{-1}.
\] (46)

Realistically, even in the extreme case of a fully baryonic halo we expect \( f_{\text{IBD}} \approx 3 \) (of course, a large \( f_{\text{PB}} \) would imply a \( f_{\text{IBD}} \) considerably smaller than that). Moreover, \( M_*/M_{\text{DC}} \) sensitively depends on how the core is defined; the analogy with globular clusters entails, however, that it should in any case be less than 20%. Accordingly, we conclude that the fractional abundance of tidally captured binary brown dwarfs should not exceed 1% (again in remarkable agreement with the result of Statler et al. [1987] for globular clusters), and so they are irrelevant from the observational point of view.

### 4.3. Primordial Binary Brown Dwarfs

Primordial binaries are a very different story. Not only are they expected to be much more abundant than tidally captured binaries, but they are also presumably much less hard, since all values for their orbital radius consistent with the conditions of equation (40) should in principle be contemplated. So, hardening effects ought to play a crucial role for primordial binaries (as may be guessed intuitively, effects of this kind turn out to be totally negligible for tidally captured binaries; this will be demonstrated later on).

#### 4.3.1. Collisional Hardening

Let us begin by considering collisional hardening, namely, the process whereby hard binaries get harder in encounters with individual brown dwarfs. We recall that the associated average hardening rate (Spitzer & Mathieu 1980) presently reads

\[
\dot{E} \approx -2.8 \frac{G^2 m^3 n_{\text{IBD}}(0)}{\sigma_*} \, \text{ergs yr}^{-1}.
\] (47)

From the definition of \( n_{\text{IBD}}(0) \) and equation (19), equation (47) becomes

\[
\dot{E} \approx -1.7 \times 10^{-32} f_{\text{IBD}} \left( \frac{M_\odot}{M_{\text{DC}}} \right)^{1/3} \frac{\rho_*(0)}{M_\odot \, \text{pc}^{-3}} \, \text{ergs yr}^{-1}.
\] (48)

Observe that a characteristic feature of collisional hardening is that \( \dot{E} \) is independent of hardness, and so is time independent.

As is well known, the internal energy of a binary is \( E = -\frac{G m^2}{2a} \), which yields

\[
\dot{E} = \frac{G m^2}{2a^2} \dot{a}.
\] (49)

Hence, by combining equations (48) and (49) and integrating the ensuing expression, we get

\[
\frac{\text{km}}{a_2} \approx \frac{\text{km}}{a_1} + 1.3 \times 10^{-20} f_{\text{IBD}} \left( \frac{M_\odot}{M_{\text{DC}}} \right)^{1/3} \frac{\rho_*(0)}{M_\odot \, \text{pc}^{-3}} \, t_{21} \, \text{yr},
\] (50)

where \( a_1 \) is the initial orbital radius and \( a_2 \) is the orbital radius after a time \( t_{21} \).

Assuming momentarily that no other hardening mechanism is operative and taking \( t_{21} \) equal to the age of the universe, we find that the present orbital radius of a binary brown dwarf is given by

\[
\frac{\text{km}}{a_2} \approx \frac{\text{km}}{a_1} + 1.3 \times 10^{-10} f_{\text{IBD}} \left( \frac{M_\odot}{M_{\text{DC}}} \right)^{1/3} \frac{\rho_*(0)}{M_\odot \, \text{pc}^{-3}}.
\] (51)

Of course, collisional hardening works to the extent that \( a_2 \) becomes considerably smaller than \( a_1 \). Correspondingly, we see from equation (51) that this is indeed the case, provided that

\[
a_1 \approx 8 \times 10^9 f_{\text{IBD}}^{-1} \left( \frac{M_{\text{DC}}}{M_\odot} \right)^{1/3} \frac{M_\odot}{\rho_*(0)} \, \text{km}.
\] (52)

Using equation (51), equation (52) in turn yields

\[
a_2 \approx 8 \times 10^9 f_{\text{IBD}}^{-1} \left( \frac{M_{\text{DC}}}{M_\odot} \right)^{1/3} \frac{M_\odot}{\rho_*(0)} \, \text{km}.
\] (53)

Physically, the emerging picture is as follows. Only those binaries whose initial orbital radius obeys the condition given in equation (52) undergo collisional hardening, and their present orbital radius turns out to be almost independent of the initial value. We can make the present discussion somewhat more specific by noticing that our assumptions strongly suggest \( \rho_*(0) \approx 10^4 M_\odot \, \text{pc}^{-3} \), in which case both equation (52) and equation (53) acquire the form

\[
a_{1,2} \approx 8 \times 10^9 f_{\text{IBD}}^{-1} \left( \frac{M_{\text{DC}}}{M_\odot} \right)^{1/3} \, \text{km}.
\] (54)

Evidently, very hard primordial binaries, which fail to meet the condition given in equation (54), do not suffer collisional hardening, and the same is true for tidally captured binaries.

#### 4.3.2. Frictional Hardening

As we will show, the presence of molecular clouds in the dark cluster cores, which is indeed the most characteristic feature of the model in question, provides a novel hardening mechanism for binary brown dwarfs. Basically, this is brought about by dynamical friction on molecular clouds.

It is not difficult to extend the standard treatment of dynamical friction (Binney & Tremaine 1987) to the relative motion of the brown dwarfs in a binary system that moves inside a molecular cloud. For simplicity, we assume that molecular clouds have a constant density profile \( \rho_m \). In the case of a circular orbit, the equations of motion imply that the time \( t_{21}^{(0)} \) needed to reduce the orbital radius \( a \) of a binary that moves all the time inside molecular clouds from \( a_1 \) down to \( a_2 \) is

\[
t_{21}^{(0)} \approx 0.17 \left( \frac{m}{M_\odot} \right)^{1/2} \frac{1}{\rho_m \ln \Lambda} \left( a_2^{-3/2} - a_1^{-3/2} \right),
\] (55)

where the Coulomb logarithm reads

\[
\ln \Lambda \approx \ln \left( \frac{r_m v_s^2}{G m} \right) \approx \ln \left( \frac{r_m}{a_1} \right),
\] (56)

where \( v_s \) is the circular velocity (approximately given by Kepler’s third law). Manifestly, the diffusion approximation upon which the present treatment is based requires that the orbital radius of a binary should always be smaller than the median radius of a cloud. As we are concerned henceforth with hard binaries, \( a_1 \) must obey the condition given in equation (40). On the other hand, \( a_2 \) is the larger value for the orbital radius in equation (55). So, we shall take for

\footnote{Indeed, the circularization of the orbit is achieved by tidal effects after a few periastron passages (Zahn 1977).}
definiteness (in the Coulomb logarithm only) $a_1 \approx 1.4 \times 10^{-2}(M_\odot/M_{\text{DC}})^{2/3}$ km. In addition, from equation (12) we have
\begin{equation}
\rho_m \approx 2.5 \left(\frac{r_m}{pc}\right)^2 M_\odot \text{ pc}^{-3}.
\end{equation}

Hence, putting everything together we obtain
\begin{equation}
\left(\frac{\text{km}}{a_1}\right)^{3/2} \approx \left(\frac{\text{km}}{a_1}\right)^{3/2} + 2 \times 10^{-26} \pi^{-1} \left(\frac{\text{pc}}{r_m}\right)^2 t_{21}^{(0)} \text{ yr},
\end{equation}
having set
\begin{equation}
\Xi \equiv \left[3 + \ln \left(\frac{r_m}{pc}\right) + 0.7 \ln \left(\frac{M_{\text{DC}}}{M_\odot}\right)\right]^{-1}.
\end{equation}

Specifically, the diffusion approximation demands $\Xi > 0$, which yields in turn
\begin{equation}
r_m > 5 \times 10^{-2} \left(\frac{M_\odot}{M_{\text{DC}}}\right)^{0.7} \text{ pc}.
\end{equation}

Observe that for $M_{\text{DC}} \ll 2.1 \times 10^4 M_\odot$, this constraint restricts the range of allowed values of $r_m$, as stated in §2.2.

Were the dark clusters completely filled by clouds, equation (58) would be the final result. However, the distribution of the clouds is lumpy. So, if we want to know the orbital radius $a_2$ after a time $t_{21}$, we must proceed as follows. First, we should compute the fraction $t_{21}^{(0)}$ of the time interval in question, $t_{21}$, spent by a binary inside the clouds. Next, we need to reexpress $t_{21}^{(0)}$ in equation (58) in terms of $t_{21}$. This will be achieved by the procedure outlined below.

Keeping in mind that both the clouds and the binaries have average velocities $v \approx (3)^{1/2} \sigma_\star$ (for simplicity, we neglect the equipartition of kinetic energy of the binaries), it follows that the time needed by a binary to cross a single cloud is
\begin{equation}
t_m \approx \frac{r_m}{\sqrt{2}v} \approx 5.6 \times 10^6 \frac{r_m}{\text{pc}} M_\odot \left(\frac{M_{\text{DC}}}{M_\odot}\right)^{1/3} \text{ yr}.
\end{equation}

As an indication, we notice that for $r_m \approx 10^{-3}$ pc ($M_\odot \approx 2 \times 10^{-2} M_\odot$) and $M_{\text{DC}} \approx 10^5 M_\odot$, we find $t_m \approx 1.2 \times 10^2$ yr. Therefore, frictional hardening involves many clouds. Specifically, during the time $t_{21}$, the number of clouds crossed by a binary is evidently
\begin{equation}
N_m \approx \frac{t_{21}}{t_m} \approx 1.8 \times 10^{-7} \frac{\text{pc}}{r_m} M_\odot \left(\frac{M_{\text{DC}}}{M_\odot}\right)^{1/3} t_{21}^{(0)} \text{ yr}.
\end{equation}

Let us now ask how many crossings of the core are necessary for a binary to traverse $N_m$ clouds. To this end, we estimate the number of clouds $N_c$ encountered during one crossing of the core. Describing the dark clusters by a King model, we can identify the core radius with the King radius. The cross section for binary-cloud encounters is $\pi r_m^2$, so we have
\begin{equation}
N_c \approx \left[\frac{9 \sigma_\star^2}{4\pi G \rho_m(0)}\right]^{1/2} n_{\text{CL}}(0) \pi r_m^2,
\end{equation}
where $n_{\text{CL}}(0)$ is the cloud number density in the core. From equation (12), we can write
\begin{equation}
n_{\text{CL}}(0) = f_{\text{CL}} \rho_m(0) M_m \approx 4.8 \times 10^{-2} f_{\text{CL}} \frac{\text{pc}}{r_m} M_\odot \rho_m(0) \text{ pc}^{-3}.
\end{equation}
where $f_{\text{CL}}$ denotes the fraction of core dark matter in the form of molecular clouds. Correspondingly, equation (63) becomes
\begin{equation}
N_c \approx 0.13 \times f_{\text{CL}} \left[\frac{\rho_m(0)}{M_\odot \text{ pc}^{-3}}\right]^{1/2} \left(\frac{M_{\text{DC}}}{M_\odot}\right)^{1/3} \frac{r_m}{\text{pc}},
\end{equation}
through equation (19). So, the total number of core crossings $N_{cc}$ that a binary has to make in order to traverse $N_m$ clouds is
\begin{equation}
N_{cc} \approx \frac{N_m}{N_c} \approx 1.4 \times 10^{-6} f_{\text{CL}} \left[\frac{M_\odot \text{ pc}^{-3}}{\rho_m(0)}\right]^{1/2} \left(\frac{r_m}{\text{pc}}\right)^2 t_{21}^{(0)} \text{ yr}.
\end{equation}

Because the core crossing time is
\begin{equation}
t_{cc} \approx \left[\frac{9 \sigma_\star^2}{4\pi G \rho_m(0)}\right]^{1/2} \frac{1}{v} \approx 7 \times 10^6 \left[\frac{M_\odot \text{ pc}^{-3}}{\rho_m(0)}\right]^{1/2} \text{ yr},
\end{equation}

which is the desired relationship between $t_{21}$ and $t_{21}^{(0)}$. We are now in position to reexpress equation (58) in terms of $t_{21}$. Accordingly, we get
\begin{equation}
\left(\frac{\text{km}}{a_1}\right)^{3/2} \approx \left(\frac{\text{km}}{a_1}\right)^{3/2} + 2 \times 10^{-27} f_{\text{CL}} \Xi^{-1} \frac{\rho_m(0)}{M_\odot \text{ pc}^{-3}} t_{21} \text{ yr}.
\end{equation}

In order to quantify the effect of frictional hardening, we may proceed in much the same way as in the case of collisional hardening, neglecting, however, the latter effect for the moment. Specifically, taking $t_{21}$ in equation (69) to be equal to the age of the universe, we find that the present orbital radius of a binary brown dwarf is given by
\begin{equation}
\left(\frac{\text{km}}{a_1}\right)^{3/2} \approx \left(\frac{\text{km}}{a_1}\right)^{3/2} + 2 \times 10^{-17} f_{\text{CL}} \Xi^{-1} \frac{\rho_m(0)}{M_\odot \text{ pc}^{-3}} \text{ yr}.
\end{equation}

Manifestly, frictional hardening is operative to the extent that $a_2$ becomes considerably smaller than $a_1$. Accordingly, from equation (70) we see that this is indeed the case, provided that
\begin{equation}
a_1 \gtrsim 1.3 \times 10^{11} f_{\text{CL}}^{-2/3} 2^{2/3} \left[\frac{M_\odot \text{ pc}^{-3}}{\rho_m(0)}\right]^{2/3} \text{ km}.
\end{equation}

From equation (71), equation (70) entails in turn
\begin{equation}
a_2 \approx 1.3 \times 10^{11} f_{\text{CL}}^{-2/3} 2^{2/3} \left[\frac{M_\odot \text{ pc}^{-3}}{\rho_m(0)}\right]^{2/3} \text{ km}.
\end{equation}

Physically, only those binaries whose initial orbital radius satisfies the condition of equation (71) are affected by frictional hardening, and their present orbital radius turns out to be almost independent of the initial value. We can make the present discussion somewhat more specific by noticing that our assumptions strongly suggest $\rho_m(0) \approx 10^4 M_\odot \text{ pc}^{-3}$, in which case both equations (71) and (72) acquire the
Finally, the fairly slow dependence of the conditions of collisional hardening decreases for smaller values of $f$. This makes a decrease, so $t_{\text{react}}$ increases with time. This effect can be taken into account by considering the average value $\langle t_{\text{react}} \rangle$ of the reaction time over the time interval in question (to be denoted by $T$), namely,

$$\langle t_{\text{react}} \rangle \equiv \frac{1}{T} \int_0^T dt \ t_{\text{react}}.$$

In order to compute $\langle t_{\text{react}} \rangle$, the temporal dependence of the binary orbital radius is needed. Because frictional hardening plays the major role, the latter quantity is evidently supplied by equation (69). Setting for notational convenience $t \equiv t_{21}$ and $a(t) \equiv a_2$, equation (69) yields

$$a(t) \approx \left( \frac{\text{km}}{a_1} \right)^{3/2} + 2.1 \times 10^{-27} \Xi^{-1} f_{\text{CL}} \left( \frac{\rho_\ast(0)}{M_\odot \text{ pc}^{-3}} \right) t^{3/2} \text{ km}.$$

Combining equations (76) and (78) and inserting the ensuing expression into equation (77), we get

$$\langle t_{\text{react}} \rangle \approx 1.9 \times 10^{46} f_{\text{PB}} f_{\text{CL}}^{1/2} \left( \frac{M_{\text{DC}}}{M_\odot} \right)^{1/3} \frac{\text{yr} (\text{km})^{5/2}}{T (\text{yr}) a_1} \ \times \left[ \frac{\rho_\ast(0)}{M_\odot \text{ pc}^{-3}} \right]^{2/3} \left[ 1 + 2.1 \times 10^{-27} \Xi^{-1} \right] \ \times f_{\text{CL}} \left( \frac{\rho_\ast(0)}{M_\odot \text{ pc}^{-3}} \right) \left( \frac{\text{yr}}{\text{km}} \right)^{3/2} \left[ \frac{a_1}{M_{\text{DC}}} \right]^{5/3} \left[ \frac{T}{\text{yr}} \right]^{1/3} - 1 \right \} \text{ yr}.$$

Let us now require $\langle t_{\text{react}} \rangle$ to exceed the age of the universe (taking evidently $T \approx 10^{10}$ yr). As is apparent from equation (76), $t_{\text{react}}$ is shorter for softer binaries. Hence, in order to contemplate hard binaries with an arbitrary orbital radius, we need to set $a_1 \approx 1.4 \times 10^{13} (M_\odot / M_{\text{DC}})^{2/3}$ km in equation (79). Correspondingly, the binary survival condition reads

$$f_{\text{PB}} \approx 8.2 \times 10^{-5} f_{\text{CL}}^{1/2} \left( \frac{M_{\text{DC}}}{M_\odot} \right)^{2/3} \left( \frac{M_{\odot} \text{ pc}^{-3}}{\rho_\ast(0)} \right) \times \left[ 1 + 35 f_{\text{CL}} \Xi^{-1} \left( \frac{\rho_\ast(0)}{M_\odot \text{ pc}^{-3}} \right) M_{\text{DC}}^{-5/3} \right] - 1 \}. \quad (80)$$

Although the presence of various dark cluster parameters prevents a clear-cut conclusion to be drawn from equation (80), in the illustrative case where $M_{\text{DC}} \approx 10^5 M_\odot$ and $f_{\text{CL}} \approx 0.5$, equation (80) entails, e.g., $f_{\text{PB}} \lesssim 0.3$ for $\rho_\ast(0) \approx 3 \times 10^3 M_\odot \text{ pc}^{-3}$. Thus, we infer that for realistic values of the parameters in question, a sizable fraction of primordial binary brown dwarfs survive binary-binary encounters in the dark cluster cores.

5. THERMAL BALANCE IN HALO MOLECULAR CLOUDS

As far as the energetics of halo molecular clouds is concerned, we can identify two main heat sources. One is energy deposition from background photons, and the other is of gravitational origin (coming from frictional hardening of primordial binary brown dwarfs). Although we discuss

$$a_{1,2} \gtrsim 2.8 \times 10^8 f_{\text{CL}}^{-2/3} \Xi^{2/3} \text{ km}. \quad (73)$$

Evidently, very hard primordial binaries, which violate the condition of equation (73), do not suffer frictional hardening, and the same is true for tidally captured binaries.

4.3.3. Present Orbital Radius of Primordial Binaries

Which of the two hardening mechanisms under consideration is more effective? A straightforward implication of equations (53) and (72) is that frictional hardening is more efficient than collisional hardening whenever it happens that

$$f_{\text{IBD}} \approx 6.3 \times 10^{-2} f_{\text{CL}}^{1/3} \Xi^{-2/3} \left( \frac{M_{\text{DC}}}{M_\odot} \right)^{1/3} \left[ \frac{M_\odot \text{ pc}^{-3}}{\rho_\ast(0)} \right]^{1/3}. \quad (74)$$

Unfortunately, the occurrence of various dark cluster parameters in the condition given by equation (74) does not permit a sharp conclusion to be drawn, but in the illustrative case where $M_{\text{DC}} \approx 10^5 M_\odot$, $r_m \approx 10^{-3}$ pc, $f_{\text{IBD}} \approx 5$, and $\rho_\ast(0) \approx 10^3 M_\odot \text{ pc}^{-3}$, we get $f_{\text{IBD}} < 0.5$. As we expect in the cores $f_{\text{IBD}} \ll f_{\text{PB}}$ and $f_{\text{IBD}} \approx f_{\text{IBD}}$, we see that frictional hardening plays the dominant role. Moreover, from equations (53) and (72) it also follows that the effectiveness of collisional hardening decreases for smaller values of $f_{\text{IBD}}$. Finally, the fairly slow dependence of the conditions of equation (74) on $M_{\text{DC}}$ and $\rho_\ast(0)$ makes our conclusion rather robust.

Having shown that collisional hardening can effectively be disregarded, we see that the present orbital radius of primordial binary brown dwarfs is actually given by equation (72), as long as the condition of equation (71) is met. Taking again the above particular case as an illustration, we find $a_2 \approx 7 \times 10^8$ km, which is of the same order as the Einstein radius for microlensing toward the LMC (Gaudi & Gould 1997). Therefore, we argue that primordial binaries that are not too hard can be resolved in future microlensing experiments with a more accurate photometric observation, the signature being small deviations from standard microlensing light curves (Dominik 1996).

4.3.4. Gravitational Encounters

We are now in position to take up the question of the survival of binary brown dwarfs against gravitational encounters. As already pointed out, individual-binary encounters are harmless in this respect, since hard binaries are considered throughout, so we shall restrict our attention to binary-binary encounters.

As a first step, we recall that their average rate $\Gamma$ (Spitzer & Mathieu 1980) can be written as

$$\Gamma \approx \alpha \frac{G m a}{\sigma_\ast}, \quad (75)$$

with $\alpha \approx 13$. Correspondingly, from equation (19) the reaction time in the dark cluster cores turns out to be

$$t_{\text{react}} \approx 10^{19} \beta \frac{M_\odot \text{ pc}^{-3}}{\rho_\ast(0)} \left( \frac{M_{\text{DC}}}{M_\odot} \right)^{1/3} \text{ km yr}, \quad (76)$$

with $\beta \approx 6.7 f_{\text{PB}}$. Now, if no hardening were to occur, which means that the orbital radius $a$ would stay constant, the binary survival condition would simply follow by demanding that $t_{\text{react}}$ should exceed the age of the universe. However, hardening makes a decrease, so $t_{\text{react}}$ increases with time. This effect can be taken into account by considering the average value $\langle t_{\text{react}} \rangle$ of the reaction time over the time interval in question (to be denoted by $T$), namely,

$$\langle t_{\text{react}} \rangle \equiv \frac{1}{T} \int_0^T dt \ t_{\text{react}}.$$

Applying the same argument to tidally captured binaries shows that no depletion occurs in this way.

If the initial value of $f_{\text{IBD}}$ fails to satisfy the condition of equation (80), primordial binaries start to be destroyed in binary-binary encounters until their fractional abundance is reduced down to a value consistent with the condition given in equation (80).
them separately, it should be kept in mind that they both act at the same time.

5.1. Energy from Background Photons

We proceed to estimate $T_m$ by momentarily neglecting gravitational effects. To this end, we need to know the heating rate (due to external sources) and the cooling rate (due to the molecules). In the Galactic halo, the dominant heat source for molecular clouds is expected to be ionization from photons of the X-ray background, whose spectrum in the relevant range $1 \text{ keV} < E < 25 \text{ keV}$ (see below) can be parametrized in terms of the energy $E$ (expressed in keV) as $I(E) = 8.5 E^{-0.4} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ (O'Dea et al. 1994). The ionization rate per H$_2$ molecule (taking secondary ionization into account) is

$$\xi_X = 26 \int_{E_{\text{min}}}^{E_{\text{max}}} 4\pi I(E) \sigma_X(E) dE \text{ s}^{-1},$$

(81)

where $\sigma_X(E) = 2.6 \times 10^{-22} E^{-8/3} \text{ cm}^2$ is the absorption cross section in the above energy range for incoming X-rays on gas with interstellar composition (Morrison & McCammon 1983). In fact, $\sigma_X(E)$ decreases faster as $E$ increases when the gas metallicity is lower (see Fig. 1 of Morrison & McCammon 1983). So, the expression quoted above for $\sigma_X(E)$ is expected to yield an upper bound to the cross section for X-rays on halo molecular clouds. The integration limits $E_{\text{min}}$ (below which X-rays are absorbed) and $E_{\text{max}}$ (above which X-rays go through the whole cloud without being absorbed) depend on the cloud column density. Thus, taking as an orientation in the range $10^4 - 10^6 \text{ cm}^{-2}$, we get $1.25 \text{ keV} < E_{\text{min}} < 7 \text{ keV}$ and $10 \text{ keV} < E_{\text{max}} < 20 \text{ keV}$. It turns out that $\xi_X$ is rather insensitive to the upper limit, and we obtain $\xi_X = 2.2 \times 10^{-19} \text{ s}^{-1}$ for $E_{\text{min}} = 1.25 \text{ keV}$ and $\xi_X = 5.6 \times 10^{-21} \text{ s}^{-1}$ for $E_{\text{min}} = 7 \text{ keV}$.

Since each ionization process releases an energy of $\approx 8 \text{ eV}$, the heating rate per H$_2$ molecule $\Gamma$ turns out to be

$$\frac{3.5 \times 10^{-32} \text{ ergs s}^{-1}}{\text{H}_2} < \Gamma < \frac{1.5 \times 10^{-30} \text{ ergs s}^{-1}}{\text{H}_2}.$$  

(82)

Unfortunately, the cooling rate for the halo clouds in question is not well known, owing to the lack of detailed information about their chemical composition. Nevertheless, by merely considering the cooling rate due to H$_2$ as given by Goldsmith & Langer (1978), the equality between cooling and heating rate per molecule leads to $T_m \approx 10 \text{ K}$. More complete models for the cooling rate, which include the contribution from HD and heavy molecules, imply that the cooling efficiency is substantially enhanced, and thus make it very plausible that halo molecular clouds clumped into dark clusters should have a temperature close to that of the CBR, namely, $T_m \approx 3 \text{ K}$ (see Gerhard & Silk 1996 for similar conclusions).

5.2. Energy from Primordial Binary Brown Dwarfs

As the analysis in § 4.3 shows, dynamical friction transfers a huge amount of energy from primordial binary brown dwarfs to molecular clouds, so it looks compelling to investigate (at least) the gross features of the ensuing energy balance.

Let us start by evaluating the energy acquired by molecular clouds in the process of frictional hardening. Recalling that the traversal time for a single cloud is given by equation (61), equation (58) entails that after a binary with initial orbital radius $a_1$ has crossed $N$ clouds, its orbital radius is reduced down to

$$a_{N+1} \approx \left[ \frac{1}{3} \frac{1.1 \times 10^{-9} N \Xi^{-1} \text{ pc}}{r_m} \left( \frac{M_{\odot}}{M_{\text{DC}}} \right)^{1/3} \right] + \frac{(\text{km}/a_1)^{3/2}}{2} \text{ km}.$$  

(83)

Accordingly, we see that the orbital radius remains almost constant until $N$ reaches the critical value

$$N_0 \approx 9 \times 10^{18} \text{ pc} \left( \frac{M_{\text{DC}}}{M_{\odot}} \right)^{1/3} \left( \frac{\text{km}}{a_1} \right)^{3/2},$$  

(84)

whereas it decreases afterward. Because the energy acquired by the clouds is just the binding energy given up by primordial binaries, this information can be directly used to compute the energy $\Delta E_c(N)$ gained by the $N$th cloud traversed by a binary whose initial orbital radius was $a_1$. Manifestly, we have

$$\Delta E_c(N) = \frac{1}{2} G m^2 \left( \frac{1}{a_{N+1}} - \frac{1}{a_N} \right).$$  

(85)

From equation (83), a straightforward calculation shows that $\Delta E_c(N)$ stays practically constant,

$$\Delta E_c \approx 9.8 \times 10^{32} \Xi^{-1} \text{ pc} \left( \frac{M_{\odot}}{M_{\text{DC}}} \right)^{1/3} \left( \frac{a_1}{\text{km}} \right)^{1/2} \text{ ergs},$$  

(86)

as long as $N \approx N_0$, while it decreases subsequently. So, the amount of energy transferred to a cloud is maximal during the early stages of hardening. Now, since the binding energy of a cloud is

$$E_c \approx 7.7 \times 10^{42} \frac{r_m}{\text{pc}} \text{ ergs},$$  

(87)

it can well happen that $\Delta E_c > E_c$ (depending on $r_m$, $M_{\text{DC}}$, and $a_1$), which means that the cloud would evaporate unless it manages to efficiently dispose of the excess energy.

A deeper insight into this issue can be gained as follows (we focus on the early stages of hardening, when the effect under consideration is more dramatic). Imagine that a spherical cloud is crossed by a primordial binary that moves along a straight line, and consider the cylinder $\Delta$ traced by the binary inside the cloud (its volume being approximately $\pi a^2 r_m$). Hence, by $n_m \approx 62.2 (\text{pc}/r_m)^2 \text{ cm}^{-2}$ (which follows from eq. [12]), the average number of molecules inside $\Delta$ will be

$$N_\Delta \approx 5.8 \times 10^{19} (\frac{a_1^2}{\text{km}})^2 \frac{\text{ pc}}{r_m}.$$  

(88)

Physically, the energy $\Delta E_c$ is first deposited within $\Delta$ in the form of heat. Neglecting thermal conductivity (which will be discussed later), the temperature inside $\Delta$ accordingly becomes

$$T_\Delta \approx \frac{2}{3} \frac{\Delta E_c}{N_\Delta k_B} \approx 8.1 \times 10^{17} \Xi^{-1} \left( \frac{M_{\odot}}{M_{\text{DC}}} \right)^{1/3} \left( \frac{\text{km}}{a_1} \right)^{3/2} \text{ K},$$  

(89)

26 It is straightforward to verify that the ionization rate $\xi_X$ due to the halo cosmic rays is less important in this context. In fact, assuming the ionization rate per H$_2$ molecule typical for disk cosmic rays $\xi_0 = 5 \times 10^{-23} \text{ s}^{-1}$ (see, e.g., van Dishoeck & Black 1986) and rescaling for the diffusion of the cosmic rays in the Galactic halo (De Paolis et al. 1995c), we infer $\xi_X \approx 10^{-23} \text{ s}^{-1}$. 
\( k_B \) being the Boltzmann constant. Taking into account equations (40) and (72), equation (89) yields

\[
0.5 \Xi^{-1} \left( \frac{M_{DC}}{\dot{M}_\odot} \right)^{2/3} \ K \lesssim T_\Lambda \lesssim 16.2 f_{cl} \Xi^{-2} \\
\times \rho_s(0) \frac{M_\odot}{M_{DC}} \left( \frac{\dot{M}_\odot}{\dot{M}_{DC}} \right)^{1/3} \ K , \tag{90}
\]

which in the illustrative case where \( f_{cl} \approx 0.5, \rho_s(0) \approx 3 \times 10^3 \frac{M_\odot}{pc^{-3}}, \) and \( M_{DC} \approx 10^5 \frac{M_\odot}{pc}, \) entails in turn \( 5.3 \times 10^3 K \lesssim T_\Lambda \lesssim 1.3 \times 10^4 K. \) As a consequence of the increased temperature, the molecules within \( \Delta \) will radiate, thereby reducing the excess energy in the cloud. In order to see whether this mechanism actually prevents the cloud from evaporating, we notice that the characteristic time needed to accumulate the energy \( \Delta E_c \) inside \( \Delta \) is just the traversal time \( t_m. \) Therefore, this energy will be totally radiated away provided that the cooling rate per molecule \( \Lambda \) exceeds the critical value \( \Lambda_0 \) given by the equilibrium condition

\[
N_\Lambda \Lambda_0 t_m \approx \Delta E_c . \tag{91}
\]

Specifically, equation (91) yields

\[
\Lambda_0 \approx 10^{-12} \Xi^{-1} \left( \frac{pc}{r_m} \right)^{3/2} \left( \frac{km}{a_1} \right)^{2.9} \ ergs \ s^{-1} \ molecule^{-1} , \tag{92}
\]

taking into account equations (61), (86), and (88). Moreover, in the present case, in which most of the molecules are \( H_2, \) the explicit form of \( \Lambda \) is (see, e.g., O'Dea et al. 1994; Neufeld, Lepley, & Melnick 1995)

\[
\Lambda \approx 3.8 \times 10^{-31} \left( \frac{T_\Lambda}{K} \right)^{2.9} \ ergs \ s^{-1} \ H_2^{-1} , \tag{93}
\]

which, from equation (89), becomes

\[
\Lambda \approx 3.4 \times 10^{21} \Xi^{-2.9} \left( \frac{km}{a_1} \right)^{4.35} \ ergs \ s^{-1} \ H_2^{-1} \tag{94}
\]

(note that \( \Lambda \) is almost independent of \( M_{DC} \)). Now, from equations (92) and (94) it follows that the condition \( \Lambda \gtrsim \Lambda_0 \) implies

\[
a_1 \approx 6 \times 10^{11} \Xi^{-0.7} \left( \frac{r_m}{pc} \right)^{0.35} \ km \ . \tag{95}
\]

For a wide range of dark cluster and molecular cloud parameters, it turns out that equation (95) is fulfilled for hard primordial binaries.

Thus, we conclude that the energy given up by primordial binary brown dwarfs and temporarily acquired by molecular clouds is efficiently radiated away, so that the clouds are not dissolved by frictional hardening.

As a final comment, we stress that our estimate for \( T_\Lambda \) should be understood as an upper bound, since thermal conductivity has been neglected. In addition, the above analysis implicitly relies upon the assumption that \( T_\Lambda < 10^4 K, \) which ensures the survival of \( H_2. \) Actually, in spite of the fact that the condition given by equation (90) suggests that this may well not be the case, our conclusion nevertheless remains true. This is because higher temperatures would lead to the depletion of \( H_2, \) which correspondingly is replaced by atomic and possibly ionized hydrogen. As is well known, in either case the resulting cooling rate would exceed that for \( H_2, \) and so cooling would be even more efficient than estimated above (Böhringer & Hensler 1989).

6. Ly\( \alpha \) ABSORPTION SYSTEMS

It is well known that quasar Ly\( \alpha \) absorption lines provide detailed information on the evolution of the gaseous component of galaxies (see, e.g., Fukugita, Hogan, & Peebles 1996 and references therein). These lines are seen for a neutral hydrogen column density \( N_{HI} \) ranging from \( \sim 3 \times 10^{12} \ cm^{-2} \) (the detection threshold) to \( \sim 10^{22} \ cm^{-2}. \)

At the upper limit of this range \( (N_{HI} \gtrsim 2 \times 10^{20} \ cm^{-2}), \) the lines are classified as damped Ly\( \alpha \) systems (mostly associated with metal-rich objects), and it is generally believed that they are the thick progenitors of galactic disks. The H I distribution in damped Ly\( \alpha \) systems is usually flatter than the corresponding surface brightness of the optical disks and extends to much larger radii (up to \( \sim 40 \ kpc \) in giant galaxies). In the outer galactic regions, Ly\( \alpha \) systems show sharp H I edges at a level of \( N_{HI} \approx 10^{17} \ cm^{-2}, \) the so-called Lyman limit. Damped Ly\( \alpha \) systems are observed up to redshift \( z \sim 3.5, \) and the survey results suggest that the average mass of neutral hydrogen per absorption system decreases with time, in agreement with the hypothesis of gas consumption into stars (Lanzetta, Wolfe, & Turnshek 1995).

Within our picture, it is tempting to identify damped Ly\( \alpha \) systems with the PGC clouds in the inner Galactic halo, where they undergo disruption.

Below the Lyman limit (i.e., for \( 3 \times 10^{12} \ cm^{-2} < N_{HI} < 3 \times 10^{15} \ cm^{-2}; \) see, e.g., Fig. 2 in Cristiani 1996), the absorption lines are classified as Ly\( \alpha \) forest and are generally ascribed to a large number of intervening clouds (in some cases extending up to \( \sim 200 \ kpc \) from a central galaxy) along the line of sight to distant quasars. Studies of Ly\( \alpha \) forest lines have made rapid progress recently, and some observational trends are firmly established. In particular: (1) The evolution of the comoving number density of systems per unit interval in redshift shows that the number of Ly\( \alpha \) forest clouds decreases rapidly with time for \( z > 2, \) while it is approximately constant for \( z < 2. \) (2) The correlation between the thermal Doppler parameter \( b \) and the number of clouds can be fitted with a Gaussian distribution of median \( \langle b \rangle \approx 30 \ km \ s^{-1}, \) and dispersion \( \approx 10 \ km \ s^{-1}, \) corresponding to a temperature of a few \( 10^4 K. \)

Within our model, it seems natural to identify Ly\( \alpha \) forest clouds with the molecular clouds clumped into dark clusters located in the outer halo. Indeed, we expect that the clouds contain in their external layers an increasing fraction of H I gas and that the outer regions are even ionized, because of the incoming UV radiation. Observe that so far we have been assuming that halo molecular clouds have a constant temperature. This was a good approximation as far as the previous analysis was concerned, but would be too poor in the present discussion. An H I column density of \( \sim 10^{14} \ cm^{-2}, \) corresponding to a layer of about \( 10^{-6} \ pc, \) is sufficient to shield the incoming radiation. Remarkably enough, \( N_{HI} \approx 10^{14} \ cm^{-2} \) corresponds to the average value of the observed Ly\( \alpha \) forest distribution (Cristiani 1996).

Since we expect the UV flux to decrease as the Galactocentric distance increases, clouds lying at a larger distance may thus have a smaller H I column density, while clouds closer to the Galactic disk may have a higher column density. This fact explains the observed distribution of the column density.

Moreover, the number of clouds is expected initially to decrease with time because of both MACHO and H\( _2 \) for-
mation, in this way explaining the observed evolution of Lyα forest clouds according to the above-mentioned point (1). As a final comment, we mention that very recently, Röttgering, Miley, & Van Ojik (1996), by considering the filling factors and the physical parameters derived from their Lyα forest observations, pointed out that galactic halos may contain \( \sim 10^9 M_\odot \) of neutral hydrogen gas and are typically composed of \( \sim 10^{12} \) clouds, each of size \( \sim 40 \) light days. It seems remarkable that these are the typical parameters of molecular clouds dealt with in this paper.

A thorough quantitative analysis requires, however, further investigations that are beyond the scope of the present paper.

7. CONCLUSIONS

Looking back at what we have done, it seems to us fair to say that present-day results of microlensing experiments toward the LMC are amenable to a very simple explanation in terms of what has repeatedly been proposed as a natural candidate for baryonic dark matter—namely, brown dwarfs. Once this idea is accepted, a few almost obvious steps follow. First, given the fact that ordinary halo stars form in (globular) clusters, it seems likely that brown dwarfs also form in clusters rather than in isolation (this circumstance has been repeatedly recognized in the last few years). Of course, whether brown dwarfs are still clumped into dark clusters today is a different and nontrivial question; we have seen that core collapse can liberate a considerable fraction of brown dwarfs from the less massive dark clusters. Moreover, the lack of any observational information makes any attempt to figure out the physics of dark clusters almost impossible, were it not for some remarkable structural analogies with globular clusters suggested by the model upon which our discussion is based. What should in any case be clear is that a host of different phenomena can occur. Some of them have intentionally been neglected here, in order not to make the present paper either excessively long or too speculative. Yet many others are likely to occur, that we have not even been able to imagine.

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27 See, e.g., the ongoing experiments by the GMAN and PLANET collaborations (Proc. 2nd International Workshop on Gravitational Microlensing Surveys, Orsay, 1996).

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