$S^3$: Learnable Sparse Signal Superdensity for Guided Depth Estimation

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Abstract

Dense depth estimation plays a key role in multiple applications such as robotics, 3D reconstruction, and augmented reality. While sparse signal, e.g., LiDAR and Radar, has been leveraged as guidance for enhancing dense depth estimation, the improvement is limited due to its low density and imbalanced distribution. To maximize the utility from the sparse source, we propose Sparse Signal Superdensity ($S^3$) technique, which expands the depth value from sparse cues while estimating the confidence of expanded region. The proposed $S^3$ can be applied to various guided depth estimation approaches and trained end-to-end at different stages, including input, cost volume and output. Extensive experiments demonstrate the effectiveness, robustness, and flexibility of the $S^3$ technique on LiDAR and Radar signal.

1. Introduction

Dense depth estimation is crucial in the field of 3D reconstruction [14], 3D object detection [44, 47], and robotic vision [25, 28]. Many works have proposed to estimate depth from RGB images or stereo pairs. Yet, the stereo estimation could be unreliable on homogeneous planes, large illumination changes, and repetitive textures [38, 43]; while monocular depth estimation is an ill-posed problem [11] and inherently ambiguous and unreliable [20, 24]. To attain a higher level of robustness and accuracy, modern solutions commonly leverage raw sparse signal, such as LiDAR [2, 34, 24] and Radar [5, 29], to improve depth estimation results or object detection for the challenging outdoor scenes, termed guidance in this paper.

Despite the success of those sparse-guidance methods, however, we still find two big problems with sparse signal. First, raw sparse signal can be ignored by networks when it is largely different from depth predicted with RGB (shown in Figure 1a). This situation stems from the low density property of the sparse signal, which is a common problem in many large-scale dataset. For example, KITTI dataset [12] wraps up an average density of 4.0% and nuScenes dataset [4] has an average of less than 50 Radar points over a $900 \times 1600$ image. Actually, the guidance module tends to ignore the accurate but sparse signals when they strongly disagree with the original prediction.

Furthermore, imbalance guidance is also the main problem. As shown in Figure 1b, the algorithms only focus on the small region with high signal density while barely correct the low-density region between scanning lines and

![Figure 1: Major problems of sparse depth signal.](image-url)
(a) Low Density Problem.
(b) Imbalanced Signal Distribution.

Best viewed in zoomed digital.
cause non-smoothing result. However, these low-density parts neither implicate less importance nor less confidence. In fact, there could be important objects like cars at these parts, and the imbalanced guidance stems from the uneven signal distribution of sensing devices in space. For example, LiDAR signals are mostly localized on the scanning lines with the same polar angles in the spherical coordinate, and the azimuth resolution of Radar signals is poor [10, 37]. As a result, for previous methods that conduct experiments under the assumption of uniformly distributed signal can be unreliable for real-world imbalanced cases.

To tackle the critical low density and imbalanced distribution problems, we propose a novel framework, Sparse Signal Superdensity ($S^3$), to enhance the density and mitigate imbalanced sparse signal for guided depth estimation. $S^3$ consists of two components: (1) sparse signal expansion (2) confidence weighting. For sparse signal expansion, $S^3$ first estimates the expanded area for each sparse signal based on the RGB image, and then assigns appropriate depth value to the expanded region. For confidence weighting, $S^3$ measures the confidence of the assigned depth to control the amount of influence to the sparse-guidance methods. Our method effectively utilizes confidence weighting to increase the density of the sparse signal.

$S^3$ framework, implemented with a light-weight network, can be applied to existing sparse-guidance depth estimation methods. For instance, embedding it in existing depth networks and trained in an end-to-end fashion. Losses are developed to allow $S^3$ network to learn sparse signal expansion and confidence weighting from data either for pre-training purposes or training jointly with depth networks. We conduct qualitative experiments to show the effectiveness of $S^3$ network on LiDAR and Radar guidance methods. The experimental results show that using our proposed $S^3$ can solve the low density and imbalanced distribution problems. Our method can highly increase the utility of the sparse signal and make substantial improvements on four typical sparse-guidance schemes on KITTI [13, 27, 41] and nuScenes [4] dataset.

To sum up, our contributions are highlighted as follows,

- The first work to point out the defective properties of the sparse signal and the subsequent influence to the depth estimation results.
- The novel and general framework Sparse Signal Superdensity ($S^3$) enhances the density of sparse signal, mitigates the imbalanced distribution problem, and provides extra confidence cues for depth estimation.
- $S^3$ largely increases the robustness and accuracy on depth estimation tasks using sparse signals, e.g., LiDAR and Radar.

2. Related Work

In this section, we will introduce guided depth estimation approaches and review related ideas about signal expansion.

Guided Mono Estimation. Previous works guide monocular depth estimation networks with external active sensors to address the technically ill-posed problem [11] and improve performance [50, 17, 24, 23, 39, 52, 41] known as Depth Completion. Cheng et al. [8] fuse the sparse depth as input and propagate the information to the surrounding pixels. Cadena et al. [3] concatenate the features of the cross-modality data to learn an auto-encoder for completing the partial or noisy depth. Ma and Karaiman [24] fuse different modalities in the first encoder-decoder to generate high-resolution depth. The methods aim at completing the depth from sparse depth signal and an image.

Guided Stereo Estimation. Previous works guide stereo matching results with external sparse signal for better predicted results [21, 2, 30, 38, 9]. Stereo matching leverages epipolar geometry to match pixels across image pairs and produce disparity [51], which can be transformed to depth by triangulation. PSMNet [6] and GANet [48] are renowned stereo backbones. Poggi et al. [32] propose guided techniques on cost volume to alleviate the domain shift. Yet, their method assumes sparse signal to be uniformly distributed, which does not consider imbalanced signal problem. You et al. [47] propose a graph-based depth correction algorithm to refine the stereo results in 3D domain with cheap LiDAR sensors. Nonetheless, their algorithm design does not take the imbalanced signal issue into account. Wang et al. [43] propose input fusion and regularize batch normalization conditioning on LiDAR signal. The above methods utilize the raw sparse signal for guidance or correction, which puts little emphasis on the inherent problems of the sparse signal mentioned.

Signal Expansion. The expansion idea has shown in tasks like superpixel segmentation [1, 42, 46, 36], depth completion, and depth sampling [15, 22, 45]. Superpixel aggregates pixels with similar semantics, but they do not imply similar depth values. Depth completion and depth sampling complete the sparse depth, but most of the previous works do not measure the confidence of the expanded depth and rely on heavily computational resources.

Shivakumar et al. [38] propose promotion of the depth signal to the neighboring pixels in the cost volume to improve depth estimation. The incentive to promote the sparse signal is close to our application on cost volume. However, their methods are only applicable to Semi Global Matching [16] algorithm. Furthermore, there are lots of hand-
3. Method

3.1. Intuition of Sparse Signal Superdensity

To solve the issues of low density and imbalanced distribution, we propose expanding the sparse cues to the neighborhood region. Our idea is that neighboring pixels with similar color intensities belong to the same image structure or object and thus have similar depth values.

Intuitively, the ad-hoc method is to expand points by color thresholds inspired by cross-based support window method [49]. To be specific, let \( I, G \) and \( G_{\text{exp}} \) be the color intensity map, sparse signal map and expanded map. Given a central pixel \((i, j)\) (the coordinate of the source point), we greedily expand from the central value \(G(i, j)\) to its neighbor pixels \((i', j')\) and fill in the expanded pixels \(G_{\text{exp}}(i', j')\) with \(G(i, j)\) as shown in Figure 3. The expansion stops until the maximum of color intensity differences is larger than a threshold or the expansion size reaches the limit.

![Sparse Signal Superdensity](image)

Figure 3: Intuition for sparse signal expansion by ad-hoc method. Sparse depth map (right) is expanded according to RGB image (left) presented in one channel here. Zero in the sparse map means no signal. The example expands the center signal according to difference of color intensity with threshold = 2.

Although the expanded map \(G_{\text{exp}}\) can substitute the sparse \(G\) to perform any guidance techniques in depth estimation, the expanded points may provide false guidance to the estimating process, especially for occlusions or pixels across object boundary. As a result, instead of applying the same level of guidance to all pixels, we provide a confidence map \(C\) to measure the reliability of the expanded value in \(G_{\text{exp}}\) and the level of guidance to apply for depth estimation.

3.2. Learnable Sparse Signal Superdensity

We propose leveraging a neural network to learn how to expand sparse signals and the corresponding confidence with the concept of sparse signal expansion and confidence weighting from Section 3.1. We expand each sparse signal to a patch by a \(S^3\) network and aggregate all the expanded patches to form the final output.

To be specific, we predict how confident the sparse depth \(G(i, j)\) can expand from the central pixel \((i, j)\) to the neighboring pixel \((i', j')\) with \(S^3\) network. We set the expansion space to be a square patch of size \(2L \times 1\) for each sparse signal, where \(|(i, j) - (i', j')| \leq L\). The input of the \(S^3\) network is a crop of the intensity map \(I(i - L : i + L, j - L : j + L)\). The output is a confidence patch of the same size and saved in \(C_{k}(i - L : i + L, j - L : j + L) \in [0, 1]\), where \(k\) is the index of \(k\)’th sparse depth signal and \(C_k = 0\) for other pixels out of the patch. Then, we aggregate the confidence patches to be the expanded depth map \(G_{\text{exp}}\) by the following interpolation equation:

\[
G_{\text{exp}}(i', j') = \frac{\sum_{k \in S_k} C_k(i', j') \cdot G(i_k, j_k)}{\sum_{k \in S_k} C_k(i', j')},
\]

where \((i_k, j_k)\) is the pixel coordinate of the \(k\)’th sparse signal and \(S_k\) is the set of indices of the sparse signal. The
operation means that a pixel with no signal from depth sensors is assigned with an interpolated depth value from its nearby sparse signal values. Consequently, the more confident $S^3$ network considers the source signal to be, the more likely the assigned depth value is to be. Finally, we aggregate the confidence maps by taking the maximum among the confidence patches.

$$C(i', j') = \max_{k \in S_k} C_k(i', j').$$  

(2)

Note that $C(i', j') = 0$ if $(i', j')$ has no expanded signal.  

$G_{exp}(i', j') = G(i', j')$ and $C(i', j') = 1$ if $(i', j') = (i_k, j_k)$ for a $k \in S_k$.

We formulate a general method to learn $S^3$ network along with any depth backbone. Here, the confidence value can act as the weights between the guided depth $D$. That is,

$$D_{out} = G_{exp} \cdot C + D \cdot (1 - C).$$  

(3)

With the depth ground truth $D^*$, the supervised loss on the output depth $D_{out}$ can be formed as $L_{sup} = \|D^* - D_{out}\|$. We also supervise $G_{exp}$ with $D^*$ and add regularization

$$L_{S^3} = \lambda_1 \cdot C \cdot \|D^* - G_{exp}\| + \lambda_2 \cdot \|C\|.$$  

(4)

The first term in Equation 4 means the more confident the expanded depth is, the more accurate it should be. The second term prevents excessive confidence for pretrained. In practice, the gradient of $C$ of the first term is detached, otherwise, $C = 0$ can be a bad local minimum. The model is trained end-to-end so that the expansion process is learned from data. The main difference between having and not having $S^3$ is that $S^3$ increases the density of the sparse signal by providing an additional confidence map to tell the subsequent depth estimation algorithms how reliable the expanded depth is.

4. Application of $S^3$

$S^3$ network can learn to expand different modality data, including the most widely used LiDAR and Radar. Furthermore, $S^3$ works on both depth and disparity representation, allowing users to use our module in various applications. For instance, disparity is preferred for robotic tasks due to the need to provide higher accuracy in the nearby region.

Many works have proposed signal-guidance schemes to enhance depth estimated from RGB as addressed in Section 1 and 2. These methods can be divided into three categories: (1) Guidance on Input and Output (2) Guidance on Cost Volume (3) Guidance on 3D Space. We will introduce how to apply our module for each type of methods (overview in Figure 2) in the following.

4.1. Guidance on Input and Output

For guidance on input, the most intuitive way is to concatenating these external sparse signal as one of the input to the neural network. This strategy is widely used in dense depth estimation domain for either monocular [50, 23, 24] or stereo [43] depth estimation. For these approaches, we can simply replace the original raw sparse signal as our expanded signal along with the confidence map.

For guidance after the output of the depth prediction network, a naive way is to add the accurate but sparse signal to the predicted depth. Similar schemes are used by Chen et al. [7], called shortcut connection in the paper, and You et al. [47], who ignores the sparse signals largely different from stereo results to avoid numerical error and add those signals back to the corrected depth. We modify the naive method by interpolation with Equation 3 so that more pixels are guided with the expanded $G_{exp}$ and confidence $C$.

4.2. Guidance on Cost Volume

Many practices have tried to modify the cost volume, an intermediate representation of matching relationships between pixels, either guidance with external cues [32, 40, 38] or confidence measure [33] in the field of stereo matching. The cost volume in the stereo network consists of 3D features with geometric and contextual information that allows the subsequent convolution to regress the disparity probability [18, 6, 48]. Here, we take Guided Stereo Matching (GSM) [32] as an example to explain how $S^3$ framework is applied to cost volume. Another example, CCVNorm [43], is presented in the supplementary materials.

GSM [32] peaks the correlated features of the cost volume suggested from the sparse signal with Gaussian function to provide guidance to the network. Specifically, let $G \in \mathbb{R}^{H \times W}$ be external sparse but accurate data, $V$ specifies a binary mask whether $G$ has signal on pixel coordinate $(i, j)$, and the cost volume is $CV \in \mathbb{R}^{H \times W \times D_{max} \times F}$, where $D_{max}$ is the max disparity and $F$ is the feature number. Given the pixel coordinate $(i, j)$ and disparity value $G(i, j)$ from external cue $G$, they apply Gaussian function

$$f^{GSM}(i, j, d) = h \cdot e^{-\frac{(d - G(i, j))^2}{2w^2}}$$  

(5)

on the features $CV(i, j, d) \leftarrow ((1 - V(i, j)) \cdot 1 + V(i, j) \cdot f^{GSM}(i, j, d)) \cdot CV(i, j, d)$ of the cost volume, where $h$ and $w$ are hyper-parameters to control the height and width of the Gaussian, $\forall d \in \{0, 1, \ldots, D_{max} - 1\}$. The function $f^{GSM}$ enlarges the feature values having positive relation to sparse cues, while suppressing others.

We propose fusings the expanded disparity map $G_{exp}$ and the correspondent confidence map $C$ in a novel approach:

$$f^{Ours}(i, j, d) = C \cdot \left(h \cdot e^{-\frac{(d - G_{exp}(i, j))^2}{2w^2}}\right) + s.$$  

(6)
Then, it forms a neighborhood-relation graph considering as an example, the algorithm first projects the dense depth space is an intuitive alternative. Take Graph-based Depth Correction (GDC) algorithm proposed by You or cost volume, performing sparse signal guidance on 3D based approaches like CCVNorm [43] on cost volume.

Lastly, our module is flexible to apply to other guidance-visualized in Figure 4. Additionally, GSM is a subset of ours.

are learnable and confidence-based expansion, which is viable. The shift range \( s \) preserves the minimum feature value when \((d - G_{exp}(i,j))^2\) is large or \( C = 0 \). When \( s \) is positive, value in cost volume \( CV(i,j,d) \) will not be suppressed to zero so that the gradient of network would not be blocked during back-propagating. \( s \) can be a learnable parameter for training. The confidence value \( C \) acts as a switch to control how much guidance should be applied according to the expanded guidance \( G_{exp} \).

The largest difference between our approach and others are learnable and confidence-based expansion, which is visualized in Figure 4. Additionally, GSM is a subset of ours. Lastly, our module is flexible to apply to other guidance-based approaches like CCVNorm [43] on cost volume.

### 4.3. Guidance on 3D Space

In addition to using sparse signal information on input or cost volume, performing sparse signal guidance on 3D space is an intuitive alternative. Take Graph-based Depth Correction (GDC) algorithm proposed by You et al. [47] as an example, the algorithm first projects the dense depth estimated from monocular or stereo network to 3D space. Then, it forms a neighborhood-relation graph considering depth value via \( k \)-nearest neighbor.

\[
W = \arg \min_W \| Z - WZ \|^2,
\]

(7)

where \( Z \) denotes the depth vector, and \( W \) denotes the edge weight between two points. Given the sparse 3D point cloud data, it then corrects the projected points with the relation graph in an optimization manner.

\[
Z' = \arg \min_{Z'} \| Z' - WZ' \|^2,
\]

(8)

where \( Z'_{1:n} = G \). The first \( n \) points are set to their correct depth value from the hint of the sparse signals, and the algorithm corrects the rest of points \( Z'_{n+1} \) by minimizing the reconstruction loss. The algorithm corrects the neighbors of the sparse signal points via the relation built from \( W \), and the neighbors of the neighbors would also be corrected. The algorithm would propagate the correct depth value via the graph relation for the sparse signals in the long run.

We improve the algorithm with the expanded depth \( G_{exp} \) and confidence \( C \) in the following approach. Suppose there are \( n_e \) expanded points and \( m \) points to be corrected, we first build the graph in Equation 7, and then minimize the reconstruction error considering the confidence.

\[
Z' = \arg \min_{Z'} \| (C'G_{exp} + (I - C')Z') - W(C'G_{exp} + (I - C')Z') \|^2.
\]

(9)

Here \( C' \in \mathbb{R}^{m\times n_e \times m} \) is a diagonal matrix, where \( C'_{kk} = 1 \) for \( k \in \{1, \cdots, n\} \), \( C'_{kk} = C \) for \( k \in \{n+1, \cdots, n_n + n_e\} \), and \( C'_{kk} = 0 \), otherwise. The modification differs from Equation 8 in that \( Z'_{n+1:n_n} \) is interpolated to the suggested value \( G_{exp} \) with confidence \( C \). For \( C \) close to 0, the influence of the guidance value is negligible. For \( C \) close to 1, the guidance value is as confident as the one from sparse signal. Such modification not only allows more points to be corrected by the algorithm, but also takes the magnitude of guidance into consideration.

### 5. Experiment

#### 5.1. Experimental Setting

**Dataset.** We use SceneFlow [26], KITTI Stereo 2012 [13], and 2015 [27] to conduct experiments for LiDAR sparse signal, and NuScenes v1.0 dataset [4] for Radar sparse signal. SceneFlow [26] dataset is a large-scale synthetic stereo dataset mainly for pretraining purpose. KITTI Stereo 2012 [13] and KITTI Stereo 2015 [27] datasets contain stereo and LiDAR data with an application to autonomous driving. Due to no dense depth ground truth provided on NuScenes, we accumulate consecutive frames of LiDAR signals (5 before and 5 after the frame of interest) for evaluation as KITTI dataset did [13].

The sparse signal for KITTI Stereo datasets is obtained according to the original paper. For Guided Stereo Matching (GSM) [32] experiments, we sub-sample 15% of pixels from the semi-dense disparity maps. For Graph-based Depth Correction (GDC) [47] experiments, we obtain the 4-beam LiDAR signal by slicing point clouds into separate lines by an elevation step of 0.4°.

**Training Protocol.** For GSM [32], we pretrain on SceneFlow, fine-tune on the training set of KITTI Stereo 2012, and test on the training set of KITTI Stereo 2015, following the protocols in the original paper. We also fine-tune on KITTI Stereo 2015, and test on KITTI Stereo 2012. For
Implementation Detail. We implement the proposed methods with PyTorch [31] framework. The architecture of $S^3$ network is a light-weight version of U-Net [35] structure with patch size 32 with the last Sigmoid layer to normalize the confidence map. The number of parameters for $S^3$ network is 0.7M and only takes 11% of the depth network like PSMNet [6]. The inference time of the module is 0.14ms per patch for a single thread on one NVIDIA TESLA V100 GPU with batch size 512, which can be sped up by parallelism of patch operations. $S^3$ network is pretrained on SceneFlow for 8000 iterations end-to-end with PSMNet [6] optimized with Adam [19] and 0.001 learning rate. Following previous works [6, 48], we randomly crop 256 by 512 for training and pad to full resolution for testing on SceneFlow and KITTI datasets. For nuScenes, we rescale input images to 288 by 512 and train sparse-to-dense [23] monocular backbone from scratch for 35k iterations. Then, the depth is guided by $S^3$ network pretrained from SceneFlow.

Evaluation Metric. We follow standard metrics to evaluate the results. For disparity maps, we use average pixel error (Avg) and n-pixel error rate (> n). The “Avg” is defined as $\frac{1}{N} \sum |D_{pred} - D_{gt}|$, where $N$ denotes the number of pixels included in valid ground truth disparity map. The “> n” represents the percentage of disparity error that is greater than n. We evaluate depth maps with root mean squared (RMS) error, mean absolute relative error (REL), and $\delta_i$. The $\delta_i$ means the percentage of the relative error within a threshold of 1.25. Except for $\delta_i$, the other metrics are the smaller the better.

| Model          | Avg Disp Error ↓ | > n Disp Error Rate (%) ↓ | 1 | 2 | 3 | 4 | 5 |
|----------------|------------------|--------------------------|---|---|---|---|---|
| In             | 0.891            | 22.72                    | 6.12 | 3.02 | 2.09 | 1.63 |    |
| In + Ours      | 0.851            | 21.93                    | 5.98 | 2.77 | 1.78 | 1.34 |    |
| Out            | 0.935            | 26.37                    | 8.29 | 3.98 | 2.59 | 1.94 |    |
| Out + Ours     | 0.418            | 8.90                     | 1.97 | 1.05 | 0.73 | 0.55 |    |

Table 1: Guidance on Input (In) and Output (Out) Experiments on KITTI Stereo 2015. (Section 5.2.1)

GDC [47], we use the officially released SceneFlow pre-training from PSMNet [6] and fine-tune on the training sets of KITTI Stereo 2012 and 2015, and test on 2015 and 2012, respectively. For monocular depth estimation on nuScenes dataset, the network is trained supervisedly with L1 loss on LiDAR signal and guided with two algorithms: (1) Guidance on Output in Section 4.1 (2) GDC in Section 4.3.

5.2. Guidance Experiment

5.2.1 Guidance on Input and Output.

In Table 1, even though our input guidance simply concatenating the super-density as input, our approach can still improve upon the guided results with PSMNet. On the other hand, we contribute the huge gain of our output guidance to the density of the sparse signal, since the only difference is that more pixels are guided by expanded signal. Also, the improvement strengthens our idea that neighboring pixels of the sparse signal have similar depth and are able to be modeled with confidence by the center depth value.

5.2.2 Guidance on Cost Volume

In Table 2, applying our method in Section 4.2 on GSM can boost a large gap of performance. In the visualization results of Figure 5, GSM does not correct much depth pixel from the stereo output, but it does when applying $S^3$. This tells that the network tends to ignore sparse signal when the density is not high enough, which consents to the low density problem and our motivation of solution. Note that we use GANet [48] as the backbone for no fine-tuning cases because we fail to reproduce GSM results on PSMNet [6].

5.2.3 Guidance on 3D Space

In Table 3, the results show consistent improvement when applying our method in Section 4.3. The performance gain of GDC is smaller than GSM because the number of points of 4-beam LiDAR is less than the sub-sampled one from GSM. The visualization in the fourth row of Figure 5 illustrates the imbalanced signal distribution problem is reduced with our method. The results are presented in the disparity domain, since the Pseudo-LiDAR point cloud [44] originates from stereo matching. Also, we evaluate on the task
Figure 5: Visualization on KITTI Stereo Datasets with Methods GSM [32] and GDC [47]. We show the original depth color map and the zoomed one (visually enhanced) to compare results with (5th column) and without (4th column) our method, which is best viewed in zoomed digital and color. The first row shows that our $S^3$ can fix the unreliable matches on the distant cars which is the low density region. The second row demonstrates that the noise from domain shift cannot completely be removed without our method. The third row illustrates that $S^3$ reduces the imbalanced signal distribution problem, which the scanning lines of LiDAR are obvious in the results of GDC [47]. The last example shows that the edge of cars are better preserved with our method.

Table 3: Experiments on GDC Algorithm Proposed in Pseudo-LiDAR++ [47]. (Section 5.2.3)

of depth estimation instead of object detection because the focus of this paper is to improve depth estimation results.

5.3. Radar Guidance

We test the effectiveness of our module for Radar signal on nuScenes [4] dataset, which is one of the first datasets containing Camera, Radar, and LiDAR in diverse scenes and weather conditions. We choose guidance on output and guidance on 3D (GDC [47]) to improve the prediction of monocular depth estimation shown in Table 4. The improvement of “GDC + Ours” on LiDAR modality is significant compared to Table 3 because the LiDAR source here is 32-beam instead of 4-beam. The improvement from Radar modality is minor compared to LiDAR because the number of Radar point cloud is extremely sparse due to small elevation degree. However, with the help of $S^3$, the performance gain can be amplified. The experiment demonstrates the success of our proposed $S^3$ framework on both Radar and LiDAR sparse signals.

5.4. Ablation Study

Effectiveness of Each Component. We decompose our module with the expansion part and the confidence part.

Table 4: Experiments of Radar Signal on NuScenes [4] Dataset. “Out” means guidance on output and GDC is graph-based depth correction [47]. We demonstrate the ability of our method to gain improvement even on extremely sparse Radar signal. (Section 5.3)

In Table 5, the main improvement comes from the expansion design, which realizes our arguments that expanding the sparse signal before guidance can improve. When considering the confidence of the expanded signal, $S^3$ network is allowed to learn the fine-grained magnitude of influence.
### Table 5: Ablation Study of GSM [32] on KITTI 2012.
The best combination is to add both Expansion and Confidence on Sparse Signal. “No Correction” refers to the raw stereo output. (Section 5.4)

| Model                  | Avg Error | Error Avg (Fine-tune) |
|------------------------|-----------|-----------------------|
| No Correction          | 1.010     | 16.87                 |
| + Sparse Signal        | 0.526     | 6.45                  |
| + Expansion            | 0.383     | 4.90                  |
| + Confidence           | 0.342     | 3.83                  |

Table 6: Ablation Study of Different Expansion Methods on KITTI 2012 Applied with GSM [32]. (Section 5.4)

| Expansion Model       | Avg Error | Avg Error (Fine-tune) |
|-----------------------|-----------|-----------------------|
| No Expansion          | 1.370     | 0.526                 |
| Ad-hoc Method         | 1.155     | 0.582                 |
| SLIC [1]              | 1.027     | 0.489                 |
| Ours                  | 0.836     | 0.342                 |

Sparsity Expansion. We discuss on how to expand the sparse signal in Table 6. Two baseline models closely related to the idea of expansion are chosen for the experiment: (1) The ad-hoc method mentioned in Section 3.1. (2) A superpixel algorithm, SLIC [1], which iteratively clusters the neighbor pixels based on color and distance. Confidence weighting is applied to the baselines by considering the inverse distance of the expanded point to the source point, i.e., expanded depth closer to the source has higher confidence.

In Table 6, performing expansion on the sparse signal is better than no expansion for no fine-tuning case. This tells that increasing the density of the external signal can help reduce the domain shift problem, where a network is initially trained on a synthetic dataset and tested on real imagery when real data is insufficient. This also meets the goal of improving the overall accuracy without retraining mentioned in GSM [32].

For fine-tuning case, simple expansion by color thresholds, like ad-hoc expansion, is worse than no expansion. This implies the stereo network can learn to leverage the sparse signal better than simple expansion techniques. Nevertheless, our proposed $S^3$ can jointly learn with the depth network to achieve better results.

The assumption of the confidence weighting for baseline methods may not hold all the time. The expansion of baselines can enlarge the guided field, but it would also provide false guidance to disparity discontinuous areas, where disparity changes sharply. The ablation study results demonstrate the learnable confidence weighting can avoid the ill assumption and improve performance.

### 6. Conclusion

In the paper, we propose $S^3$ framework to improve depth estimation results by considering the defective property of sparse signals. Our idea is deployable to existing sparse-guidance methods. Extensive experiments show consistent improvement among guidance approaches, and strengthen the idea that expansion on sparse signal can solve low density and imbalanced distribution problem. Our $S^3$ framework could become an important reference for future exploration on sparse-guidance methods.

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Supplementary Material:

S\textsuperscript{3}: Learnable Sparse Signal Superdensity for Guided Depth Estimation

1. S\textsuperscript{3} Framework Tradeoff

We discuss the tradeoff between the performance and overhead by dividing the key factors into (1) patch size (2) sample rate, and (3) model size.

For (1) the patch size cropped by the center of sparse signals, doubling the size would quadruple the tensor memory and inference time, and the performance would improve but converge till the sparse cues are effective enough for a local structure.

For (2), the sample rate is the % of the sparse signals chosen for expansion by S\textsuperscript{3}, other sparse signals remain the same. Higher sample rate would cover and overlap more expanded region without extra memory but increase the computational cost linearly. We find that 25% sample rate can effectively reduce the inference time without hurting much performance.

For (3) the model size (altered by number of channels and convs), reducing the model size effectively reduces the memory usage and inference time, but suffers performance drop larger than (2) the sample rate.

Here we highlight the flexibility to apply our S\textsuperscript{3} framework. If a user prefers real-time usage, then he or she should use a small sample rate. And if a user prefers to reduce the model size, then he or she should use a small patch size. If a user wants to reach state-of-the-art performance, then he or she should use a small patch size. And if a user prefers real-time usage, then he or she should use a small sample rate. If a user prefers real-time usage, then he or she should use a small sample rate. If a user prefers to reduce the memory usage, then he or she should use a small sample rate.

Here we highlight the flexibility to apply our S\textsuperscript{3} framework. If a user prefers real-time usage, then he or she should use a small sample rate. And if a user wants to reach state-of-the-art performance, then he or she should maximize the sample rate and model size.

2. More Guidance on Cost Volume

2.1. Guidance on Batch Normalization

Wang et al. [43] propose to add guidance to the batch normalization in the cost volume. They leverage Conditional Batch Normalization (CBN) operation to predict the feature-wise affine transformation in dependence on the condition of sparse LiDAR signal \( L^s \). In particular, the CBN can be written in the following given a mini-batch of data indexed \( i \) and cost volume features \( F \in \mathbb{R}^{C \times H \times W \times D} \),

\[
F^{CCVN}_{\text{norm}} = \gamma_{i,c,h,w,d} \frac{F_{i,c,h,w,d} - \mathbb{E}_\beta[F_{i,c,h,w,d}]}{\sqrt{\text{Var}_\beta[F_{i,c,h,w,d}]} + \epsilon} + \beta_{i,c,h,w,d}
\] (10)

\[
\gamma_{i,c,h,w,d} = \begin{cases} 
\phi^\gamma(d)g_c(L^s_{i,h,w}) + \psi^\gamma(d), & \text{if } L^s_{i,h,w} \text{ is valid} \\
\bar{g}_{c,d}, & \text{otherwise}
\end{cases}
\] (11)

\[
\beta_{i,c,h,w,d} = \begin{cases} 
\phi^\beta(d)h_c(L^s_{i,h,w}) + \psi^\beta(d), & \text{if } L^s_{i,h,w} \text{ is valid} \\
\bar{h}_{c,d}, & \text{otherwise}
\end{cases}
\] (12)

The \( \gamma \) and \( \beta \) parameters are conditioned on the sparse source \( L^s_{i,h,w} \), if it is valid, otherwise the parameters are reduced to the unconditional ones. \( g_c \) and \( h_c \) compute the intermediate representations of the sparse signal. \( \phi \) and \( \psi \) modulate the final normalization parameters based on the intermediate representations. More details are presented in the original paper.

The following we demonstrate how our proposed S\textsuperscript{3} module is applied to the conditional batch normalization. With the expanded disparity \( L^{exp} \) and confidence \( L^{cnf} \) from S\textsuperscript{3}, we improve the batch normalization process as

\[
\begin{aligned}
\gamma_{i,c,h,w,d}^{Ours} &= \begin{cases} 
L^{cnf}_{i,c,h} \cdot \left( \phi^\gamma(d)g_c(L^{exp}_{i,h,w}) + \psi^\gamma(d) \right) + \\
(1 - L^{cnf}_{i,c,h}) \cdot \bar{g}_{c,d}, & \text{if } L^{exp}_{i,h,w} \text{ is valid} \\
\bar{g}_{c,d}, & \text{otherwise}
\end{cases}
\end{aligned}
\] (13)

\[
\begin{aligned}
\beta_{i,c,h,w,d}^{Ours} &= \begin{cases} 
L^{cnf}_{i,c,h} \cdot \left( \phi^\beta(d)h_c(L^{exp}_{i,h,w}) + \psi^\beta(d) \right) + \\
(1 - L^{cnf}_{i,c,h}) \cdot \bar{h}_{c,d}, & \text{if } L^{exp}_{i,h,w} \text{ is valid} \\
\bar{h}_{c,d}, & \text{otherwise}
\end{cases}
\end{aligned}
\] (14)

The intuitive explanation for \( \gamma^{Ours} \) is that we interpolate the valid value and invalid one of \( \gamma \) in Equation 11 by the expanded confidence \( L^{cnf} \) if \( L^{exp} \) is valid.
2.2. Experiments on Batch Normalization

We follow the training protocols and implementation details in the original paper to conduct our experiments. We apply both the input and cost volume guidance with $S^3$ following their proposed model. In Table 7, we present the results on KITTI Depth Completion dataset [41]. We find that the performance gain is smaller than the one in Table 2 of the main paper. We contribute it to the amount of training data, where KITTI Stereo contains about 200 pairs of data while KITTI Depth Completion is hundred times larger. Ideally, it is more likely to have large performance gains for small datasets, which highlights our framework is useful when small amount of data is available in hand.

3. Details about the Confidence of $S^3$

3.1. Confidence Aggregation

The main paper mentions that we use maximum operation to aggregate confidence patches into the final confidence map in Equation 2 ($C'(i', j') = \max_{k \in S_k} C_k(i', j')$). The following we discuss why choosing the maximum operation. Suppose a pixel coordinate $(i', j')$ without sparse signals $(i', j') \neq (i_k, j_k), \forall k \in S_k$ and $(i', j')$ is expanded by three nearby sparse signal sources with depth $(d_1, d_2, d_3)$ and confidence $(c_1, c_2, c_3)$, we consider two alternative aggregation operations (1) averaging the confidence and (2) interpolation with the confidence itself.

For (1) average the confidence $C(i', j') = \frac{c_1 + c_2 + c_3}{3}$, suppose the ground truth $D(i', j') = 50, (d_1, d_2, d_3) = (50, 100, 100), (c_1, c_2, c_3) = (1, 0.001, 0.001)$. The nearby depth 100 is not apparent not similar to the depth 50, so the estimated values for $c_2$ and $c_3$ are reasonable to be close to zero. Nonetheless, the values of $c_2$ and $c_3$ lower the averaged confidence to about 0.33, which does not make sense. This case particularly happens to the occlusions or object edges.

For (2) interpolation with confidence itself $C(i', j') = \frac{c_1 + c_2 + c_3}{3}$, suppose two cases: (a) $(d_1, d_2, d_3) = (50, 50, 50)$, $(c_1, c_2, c_3) = (1, 0.01, 0.01)$ and (b) $(d_1, d_2, d_3) = (50, 50, 50)$, $(c_1, c_2, c_3) = (1, 0.9, 0.9)$. The expectation of the final confidence for case (b) should be greater than or at least equal to the final confidence for case (a), since the expanded signals in case (b) vote for higher confidence values. However, the interpolated confidence for case (b) is apparently not similar to the depth 50, which does not make sense.

4. Impact of Sparse Signal Superdensity

An insightful comment from one of the reviewers is that the confidence maps along the stacked axis may relate to the slanted surfaces. Suppose there are three sparse depth pixels lying on the same slanted surface (e.g., road), and a neighboring pixel on the surface is interpolated by the three pixels with confidence predicted from $S^3$ network, the four pixels should form a slanted surface by projecting them to the 3D space with the intrinsic matrix. To this end, one could develop geometric constraints on the confidence from $S^3$ network via the projection matrix and the assumption that neighboring points fall on the same surface. In addition, one could leverage normal visualizations to help distinguish a good confidence prediction if the slanted assumption holds. We appreciate the idea and are open to have future discussions.
Figure 7: Impact of expansion. Applying our expanded signal of $S^3$ on GSM [32] can improve more depth points with the same source of sparse signal. Red points represent the pixels improved for more than 5, 2, and 0.5 disparity value from left column to right, respectively. More depth points are guided with our method by comparing the two top and bottom sub-figures. Best viewed in color.

Here we visualize an example in Figure 7 to show the guided pixels (red) with (b) and without (a) our method. The region improved with the sparse signal is also imbalanced pixels (red) with (b) and without (a) our method.

5. More Visualization

We visualize the LiDAR and Radar signals with low density and imbalanced distribution problems in Figure 8. The elevation degree of the Radar sensor is poor so the points are mostly located at the horizontal vision line. Also, filtering operation is applied to the Radar point cloud to reduce the noise. As a result, the Radar signal is extremely sparse and imbalanced. The LiDAR signal is sparse and mostly located on the scanning lines.

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