Modeling and Simulation of Excitation System for Third Harmonic Brushless Synchronous Generator

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Abstract. The excitation system of third harmonic excitation brushless synchronous generator (HESG) can improve the voltage stability and is widely used in small and medium generator set. In order to establish the model of harmonic brushless excitation system, the formula of third harmonic EMF is derived. Then, based on the traditional synchronous generator model, the influence of harmonic windings is counted in. Finally, the simulation model of closed-loop excitation system is analyzed with the automatic voltage regulator (AVR) as the core. On the basis of theoretical analysis, simulation and comparative analysis are carried out according to parameters from actual system. The results show that compared with normal synchronous generator, HESG has the advantage of excellent dynamic characteristics and automatic voltage adjustment.

1. Introduction
The excitation system of a synchronous generator is important for output voltage stabilizing. The HESG uses the harmonic energy of the air gap magnetic field as the input of the excitation system, and realizes brushless excitation with the rotating rectifier. The third harmonic induction electromotive force (EMF) can be adjusted with changes in load and power factor. So, it has the function of automatic voltage adjustment. The harmonic brushless excitation system has the advantages of simple and reliable and good dynamic characteristics.

The structure of HESG is different from that of normal synchronous generator. The present literature focuses on HESG magnetic field analysis and winding structure design[1-4]. There are few articles that analysing its characteristics and establishing its excitation system model. This paper introduces the structural characteristics of the third harmonic excitation (THE) system. Firstly, the generation and influencing factors of the third harmonic induction EMF are analysed. The characteristics of the automatic voltage adjustment are pointed out. Then, based on the traditional model of synchronous generator, the THE windings are considered. Finally, the model of the excitation control system is analysed. According to the theoretical analysis, the simulation model is built by MATLAB/Simulink, and the dynamic characteristics of the excitation system are observed. The accuracy of the established model is verified, which has important theoretical significance for further research on HESG.

2. Mathematical Model of Excitation System
The excitation system consists of HESG and AVR. It introduces a harmonic power from the air gap magnetic field by adding a THE winding in the stator slot. Under the adjustment of AVR, the excitation power is used as the input of the excitation system, to the rotating armature type AC exciter. The output of the exciter is converted into direct current by the rotating rectifier, and the excitation
winding of the generator is excited by it without brushes. The principle of the system is shown in figure 1.

![Figure 1](image-url)

**Figure 1.** The principle diagram of brushless harmonic excitation system.
1-generator main winding. 2-generator third harmonic winding. 3-generator excitation winding. 4-exciter excitation winding. 5-exciter armature winding.

2.1 The third harmonic induction electromotive force
In no-load condition, the air gap magnetic field of the synchronous generator only contains the main magnetic field $B_0$ generated by the main pole magnetomotive force (MMF) $F_f$. In load condition, the air gap synthesis magnetic field $B$ is caused by the armature MMF $F_a$ and the main pole MMF $F_f$. So, the EMF of third harmonic winding is related to the rotor magnetic field, and the d-q-axis armature reaction magnetic field.

For salient generator, the excitation windings are concentrated on the main pole of the rotor, the main pole MMF is a rectangular wave, and the distribution of the main magnetic field $B_0$ in the air gap is a flat-top wave, as shown in figure 2. It can be seen that except the fundamental wave $B_{01}$, it also contains a strong third harmonic wave $B_{03}$. According to formula $E = \sqrt{2} \pi f N K_w B \Phi$, $\Phi = \frac{2}{\pi} B \tau l$, the relationship between the EMF of THE winding and the main excitation winding is as follows[5]:

$$E_{03} = \frac{N_i K_{w3} B_{03}}{N_f K_{w1} B_{01}} E_{01} = k_1 E_{01} \quad (1)$$

Where: $N$ is the total number of turns in the winding; $K_w$ is the winding factor; $B$ is the amplitude of the magnetic field; $k_1$ can be approximated as constant, the amplitude $E_{01}$ can be expressed as $X_{ad} l_f$ when the saturation is ignored.
In load condition, according to the two-reaction theory, $F_a$ will be decomposed into two components, one is the q-axis armature MMF $F_{aq}$, and the other is the d-axis armature MMF $F_{ad}$:

$$F_a = F_{ad} + F_{aq}$$

$$F_{ad} = F_a \sin \psi = \frac{3Nk_{w1}}{\pi p} I \sin \psi = \frac{3Nk_{w1}}{\pi p} I_d$$

Where, $\psi$ is the inner power factor angle; $p$ is the number of pole pairs. It is known that $F_{ad}$ is proportional to $I_d$, and $F_{aq}$ is proportional to $I_q$.

Since the air gap permeance under the d-axis is much larger than that under the q-axis, the distributions of magnetic field $B_{ad}$ and $B_{aq}$, which generated by $F_{ad}$ and $F_{aq}$, are the peak top wave and the concave top wave, as shown in figure 3.

We can see that, both $B_{ad}$ and $B_{aq}$ contain a larger third harmonic. Since $B_{ad3}$ and $B_{aq3}$ differ by $90^\circ$ (the third harmonic electrical angle), the EMF phasors $\dot{E}_{ad3}$ and $\dot{E}_{aq3}$ produced by $B_{ad3}$ and $B_{aq3}$ in the THE windings are orthogonal.
When considering the d-axis armature reaction, the relationship between the harmonic winding EMF $E_{ad3}$ and the main winding EMF $E_{ad1}$ is given by:

$$E_{ad3} = \frac{N_3 K_{w3} B_{ad3}}{N_1 K_{w1} B_{ad1}} E_{ad1} = k_2 E_{ad1} \quad (2)$$

In the formula, $k_2$ can be approximated as a constant; since $E_{ad1}$ is proportional to $B_{ad1}$, when magnetic saturation is not counted, $B_{ad1}$ is proportional to $F_{ad}$ and $F_{ad}$ is proportional to $I_d$. That is, $E_{ad1}$ can be expressed as $I_d X_{ad}$.

Similarly, when considering the q-axis armature reaction, the formula is as follows:

$$E_{aq3} = \frac{N_3 K_{w3} B_{aq3}}{N_1 K_{w1} B_{aq1}} E_{ad1} = k_3 E_{aq1} \quad (3)$$

In the formula, $k_3$ can be approximated as a constant; $E_{aq1}$ can be expressed as $I_q X_{aq}$.

In general condition, when the synchronous generator has a resistance-inductance load, the d-axis armature usually exhibits demagnetization, that is, $F_{ad}$ opposite to $F_f$. At this time, $B_{ad1}$ and $B_{01}$ are in the opposite direction, then $B_{ad3}$ and $B_{03}$ are in the same direction. It means that the phases of $\hat{E}_{ad3}$ and $\hat{E}_{03}$ are identical, and the total EMF of third harmonic winding $\hat{E}_3$ can be obtained as:

$$\hat{E}_3 = \hat{E}_{q3} + \hat{E}_{ad3} + \hat{E}_{aq3}$$

$$E_3 = \sqrt{(E_{q3} + E_{ad3})^2 + E_{aq3}^2} = \sqrt{(k_1 X_{ad} i_d + k_2 X_{ad} i_q)^2 + (k_3 X_{aq} i_q)^2} \quad (4)$$

It can be further seen that when the load current increases, both $\hat{E}_{ad3}$ and $\hat{E}_{aq3}$ increase, and $\hat{E}_{03}$ is in the same phase with $\hat{E}_{ad3}$, and the quadrature phase with $\hat{E}_{aq3}$. Thus, the total EMF of third harmonic winding $\hat{E}_3$ also increases. That is why the THE system has the characteristics of automatically compensating for load variation and better dynamic performances.

### 2.2 Synchronous generator mathematical model

For the Park equation of synchronous generator, when considering the electromagnetic transient process of five windings and the mechanical transient process of rotor, the equation is a seventh-order model. The most important simplification assumption in various practical models is to ignore the stator winding transient. This simplification assumes that only the positive sequence fundamental wave current is passed through the stator windings of synchronous machine. However, in the dynamic load test, the sudden change of the load will cause a large negative-sequence current component, and the stator transient needs to be considered. Therefore, the Park equation with no simplification should be used in this simulation[6].

Based on the ideal generator assumption, the per-unit equation for dq0 coordinates is:

1. **Voltage equation**
\[
\begin{align*}
\dot{u}_d^* &= p\psi_d^* - \omega_e^*\psi_q^* - r_d^*i_d^* \\
\dot{u}_q^* &= p\psi_q^* + \omega_e^*\psi_d^* - r_q^*i_q^* \\
\dot{u}_0^* &= p\psi_0^* - r_0^*i_0^* \\
\dot{u}_f^* &= p\psi_f^* + r_f^*i_f^* \\
\dot{u}_D^* &= p\psi_D^* + r_D^*i_D^* = 0 \\
\dot{u}_Q^* &= p\psi_Q^* + r_Q^*i_Q^* = 0
\end{align*}
\]

The variables in the above formulas are per-unit values, and \( p \) is the derivative operator of per-unit value of time; \( \omega_e \) is the electrical angular velocity, it is related to the mechanical angular velocity, \( \omega_e = p\omega_m \).

(2) flux linkage equation

\[
\begin{bmatrix}
\psi_d^* \\
\psi_q^* \\
\psi_0^* \\
\psi_f^* \\
\psi_D^* \\
\psi_Q^*
\end{bmatrix} = 
\begin{bmatrix}
X_d & 0 & 0 & X_{ad} & X_{ad} & 0 \\
0 & X_q & 0 & 0 & 0 & X_{aq} \\
0 & 0 & X_0 & 0 & 0 & 0 \\
X_{ad} & 0 & 0 & X_f & M_R & 0 \\
X_{ad} & 0 & 0 & M_R & X_D & 0 \\
0 & X_{aq} & 0 & 0 & X_Q & 0
\end{bmatrix}
\begin{bmatrix}
-\dot{i}_d^* \\
-\dot{i}_q^* \\
-\dot{i}_0^* \\
\dot{i}_f^* \\
\dot{i}_D^* \\
\dot{i}_Q^*
\end{bmatrix}
\]

(3) rotor motion equation

The electromagnetic torque equation:

\[
T_e^* = \psi_d^*i_q^* - \psi_q^*i_d^*
\]

The structure of the HESG is different from that of the normal synchronous generator. Considering the influence of the third harmonic winding on the rotor motion equation, if ignoring the resistance loss and the power supplied to the magnetic field, it can be considered that the output electric power of the winding is equal to the converted mechanical power. At this time, the equation of motion of the rotor is:

\[
\begin{align*}
2H^*\frac{d\omega}{dt} &= T_e^* - T_D^* - \frac{P^*}{\omega} \\
\frac{d\delta}{dt} &= \omega^* - 1
\end{align*}
\]

Where, \( \delta \) is the angle of q-axis leading synchronous coordinate real-axis x, the unit is rad; \( T_D \) is the output torque of the diesel engine; \( P^* \) is the value of the output power of the third harmonic winding; \( H \) is the inertia time constant, \( H = (J\omega_m^2)/(2S) \).

2.3 The model of excitation control system

The automatic voltage regulator (AVR) is the core of the excitation control system. The AVR of the harmonic brushless excitation system includes a voltage measurement circuit, a PID adjustment circuit,
and a thyristor trigger circuit. The generator terminal voltage $U_t$ is compared with the reference voltage $U_r$ after the measurement link. After the voltage error passes through the PID link, the control amount $U_A$ is outputted, which can change the conduction angle of the thyristor in the excitation main circuit to adjust the harmonic winding output power $U_R$. $U_R$ is the input of the main exciter, which can regulate the generator excitation voltage $E_f$. This system eventually forms a closed-loop control of the generator terminal voltage. In order to stabilize the excitation system and improve its dynamic quality, the excitation system stabilizer is introduced as the negative feedback link. The control block diagram of the excitation system is shown in figure 4.

3. Simulation and analysis

Based on the MATLAB/Simulink platform and the excitation system model shown in figure 5, a simulation model of the generator set is established, and the speed control system is also included in the model.

The test loads are linear resistive load (different power factor) and induction motor (direct start). The main parameters of the system are set as shown in the table 1.

| Unit of Model             | Parameters | Detail                      |
|--------------------------|------------|-----------------------------|
| Synchronize Generator    | 25kVA      | 400V/50Hz cosθ=0.8          |
| RL Load                  | 10kVA      | cosθ=0.8                    |
|                          | 20kVA      | cosθ=0.8                    |
| Induction Motor          | 7.5kW      | 400V/50Hz 1440r/min Mechanical Torque: 0 N·m |

In the simulation experiment of the suddenly loading and unloading the RL loads, a load of 10kVA is suddenly loaded at 0.3s, and is completely unloaded at 0.6s. Then a load of 20kVA is loaded at 0.9s, and is unloaded at 1.2s. The power factor of all loads is 0.8, and the generator has entered a steady
state, before the load changes. The voltage and current waveform of A-phase and the effective value of terminal voltage are shown in figure 6.

In the simulation experiment of directly starting the induction motor, the induction motor is connected at 0.3s, which is set to no mechanical load. The simulation result is shown in the figure 7.

**Figure 6. The experiment of RL loads.**

**Figure 7. The experiment of induction motor.**

Figure 8 compares the RMS of voltage and excitation current of the third harmonic excitation system (THES) and the normal excitation system (NES) when the resistance load is suddenly added. By comparison, it can be found that the THES has advantages of rapid excitation, fast terminal voltage recovery and excellent electrical performance.

**Figure 8. The comparison between THES and NES.**

4. **Conclusion**

In this paper, the relationship between the third harmonic EMF, the rotor main magnetic field, and the armature reaction are analysed. The EMF expression indicates that the harmonic excitation has an automatic voltage adjustment. The influence of the THE windings is considered on the basis of the general synchronous generator model, and the model of the excitation control system is analysed. Then the simulation modelling is carried out, and the simulation experiments of RL load and the induction motor are carried out. By comparing with the normal generator, the superiority of its dynamic performance is verified.

The simulation results show that the components of this system are modelled correctly and the control strategy is good. This digital simulation platform can be used to analyse the excitation control system for further research in the future.
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