Intrinsic Josephson Effect in the Layered Two-Dimensional $t$-$J$ Model

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The intrinsic Josephson effect in the high-$T_c$ superconductors is studied using the layered two-dimensional $t$-$J$ model. The d.c. Josephson current which flows perpendicular to the $t$-$J$ planes is obtained within the mean-field approximation and the Gutzwiller approximation. We find that the Josephson current has its maximum near the optimum doping region as a function of the doping rate.

KEYWORDS: Two-dimensional $t$-$J$ model, Gutzwiller approximation, layered system, intrinsic Josephson effect

Since the discovery of high-$T_c$ superconductors, there have been revealed many novel superconducting properties which can not be expected for conventional superconductors. One of these is the $d$-wave symmetry of the pair potential giving rise to novel properties of phase coherence which has been confirmed by both experimental and theoretical studies in tunneling and Josephson effects. The strong two dimensionality and the anomalous transport along $c$-axis direction have brought about another important feature of the existence of the intrinsic Josephson effects in Bi$_2$Sr$_2$CaCu$_2$O$_8$ (BSCCO). It was reported that a small BSCCO single crystal behaved like a series of Josephson junctions. It is natural to consider a BSCCO superconductor as a stack of superconducting sheets consisting of CuO bilayers separated by BiO and SrO layers acting as insulators. After the discovery of intrinsic Josephson effect by Kleiner, there are several evidences which support the existence of Josephson coupling between adjacent CuO bilayers. Up to now there are several theoretical works about this problem from the view point of the layered structure. However, existing theories do not treat the two important facts, i.e., short coherence length and the strong correlation on the equal footing. Taking account of the short coherence length effect, it is suitable to express BSCCO by the lattice model. One of the authors calculated the intrinsic Josephson current in layered attractive Hubbard model within mean field approximations where the $s$-wave conventional pairing is considered. Later, Schmitt et al. calculated the Josephson current in the same model based on the Quantum Monte Carlo Calculation and established the existence of intrinsic Josephson current from the numerical calculations with much more accuracy. However, in these two papers, strong correlation effect and the resulting $d$-wave symmetry were not considered. To overcome this problem, in this paper, we study the intrinsic Josephson current using the layered two-dimensional (2D) $t$-$J$ model, which includes the important features above mentioned.

The $t$-$J$ model is one of the promising models which explain the low-energy excitations in high-$T_c$ superconductors. Although the analytic solution of this model have not yet been obtained, phase diagrams as a function of doping rate $\delta$ and a superexchange interaction $J$ are numerically studied at $T = 0$ for one-dimensional and two-dimensional. Especially, in the 2D $t$-$J$ model, the obtained phase diagram as a function of doping is consistent with actual high-$T_c$ superconductors.

In this letter, we study the intrinsic Josephson effect in the layered two-dimensional $t$-$J$ model. The intrinsic Josephson current which flows perpendicular to the $t$-$J$ planes is calculated and the current-phase relations are obtained for several doping rates. The Josephson current obtained as a function of the doping concentration $\delta$ has a maximum near the optimum doping region.

The Hamiltonian of the $m$-th $t$-$J$ plane is written as

$$\mathcal{H}_{t-J}^m = -t \sum_{\langle i,j \rangle \sigma} c^m_{i\sigma} c^m_{j\sigma} + \text{h.c.} + \sum_{\langle i,j \rangle} (JS_i^m \cdot S_j^m - \frac{J_N}{4} n_i^m n_j^m)$$

where the Hilbert space is defined on the subspace without double occupancy, $i, j$ refer to planar sites on a square lattice, and $c^m_{i\sigma} \mid c^m_{i\sigma}$ represent fermion operators within the $m$-th layer. The spin and the number operators in this layer are defined as

$$S_i^m = \sum_{\alpha \beta} c^m_{i\alpha} (\frac{1}{2})_{\alpha \beta c_{i\beta}^m} \quad n_i^m = \sum_{\sigma} c^m_{i\sigma} c^m_{i\sigma},$$

respectively. There seems to be an agreement in the conclusion that the $c$-axis conductivity in the normal state generally cannot be explained as a coherent interlayer transport. We assume, however, that the coherence among the layers is restored in the superconducting state. Thus we allow for the electron hopping between the $m$-th and the $m+1$-th layers,

$$\mathcal{H}_z^m = -t_z (c^m_{i\sigma} c^m_{i\sigma} + \text{h.c.}).$$

Now the total Hamiltonian of the layered $t$-$J$ model is
written as
\[ H = \sum_n (H_{l,n}^m i + H_{z,n}^m) - \mu \sum_{m,i,\sigma} c_{i\sigma}^m \bar{c}_{i\sigma}^m \] (3)
where \( \mu \) is the chemical potential of the whole system.

For this Hamiltonian, we consider Gutzwiller-type variational wave functions \( P_G|\Phi\rangle \) with \( P_G = \Pi_m, l(1 - n_l^m n_l^m) \) being the Gutzwiller projection operator which excludes the double occupancy, and \( |\Phi\rangle \) being a one-body mean-field wave function. Due to the Gutzwiller projection, it is usually difficult to calculate the variational energy. Thus, we use a Gutzwiller approximation in which the weight factors. For the two-dimensional model, it is known that this approximation and the variational Monte Carlo simulation give very similar variational energies.\

In the Gutzwiller approximation, the expectation values of the terms in \( H_{l,n}^m i \) are estimated as
\[ \langle \bar{c}_{i\sigma}^m \bar{c}_{j\sigma}^m \rangle = g_t \langle \bar{c}_{i\sigma}^m \bar{c}_{j\sigma}^m \rangle_0, \]
\[ \langle S_i^m \cdot S_j^m \rangle = g_s \langle S_i^m \cdot S_j^m \rangle_0, \]
\[ \langle n_i^m n_j^m \rangle = g_n \langle n_i^m n_j^m \rangle_0, \] (4)
where \( \langle \cdot \cdot \cdot \rangle \) and \( \langle \cdot \cdot \cdot \rangle_0 \) represent the expectation values in terms of \( P_G |\Phi\rangle \) and \( |\Phi\rangle \), respectively. The renormalization factors \( g_t, g_s, \) and \( g_n \) are determined by the ratios of the probabilities of the corresponding physical processes in the states \( P_G |\Phi\rangle \) and \( |\Phi\rangle \):
\[ g_t = \frac{2 \delta}{1 + \delta}, \quad g_s = \frac{4}{(1 + \delta)^2}, \quad g_n = 1, \] (5)
where \( \delta = 1 - n \) is the doping rate. Since the double occupancy is excluded in each layer, we can naturally assume that the interlayer hopping term is also estimated as
\[ \langle \bar{c}_{i\sigma}^m \bar{c}_{j\sigma}^m \rangle = g_t \langle \bar{c}_{i\sigma}^m \bar{c}_{j\sigma}^m \rangle_0. \] (6)

Next, we consider the relation among the superconducting phases of the layers. We introduce order parameters in the \( m \)-th layer as
\[ \Delta^m = \langle \bar{c}_{i+x+y}^m \bar{c}_{i}^m \rangle_0, \] (7)
with \( \tau = x \) and \( y, i + \tau \) denotes the nearest neighbor of \( i \) in the \( \tau \) direction. Since we consider the \( d_{x^2-y^2} \)-wave symmetry, \( \Delta_x = -\Delta_y \). When the Josephson current flows along the \( z \)-direction, the order parameters satisfy the following phase relation:
\[ \Delta^m = \Delta^0 \exp(\im \phi). \] (8)
Thus, the fermion operator is subject to the following condition:
\[ \bar{c}_{i\sigma}^{m+1} = \exp(\pm \frac{\im}{2} \phi) \bar{c}_{i\sigma}^m. \] (9)

Using the Gutzwiller approximation and eq.(9), we can obtain the effective mean-field Hamiltonian of (3) in momentum space as
\[ H_{\text{eff}} = -\sum_{k,\sigma} T_k c_{k\sigma}^\dagger c_{k\sigma} + N_s \epsilon_0 \]
\[ -\sum_k \left[ \Delta_{k \tau} c_{-k\tau}^\dagger c_{k\tau} + \Delta^*_{k \tau} c_{k\tau}^\dagger c_{-k\tau} \right], \] (10)
where \( k = (k_x, k_y, k_z) \) and \( k_{\tau} = (k_x, k_y) \) are the three- and two-dimensional wavevectors, respectively, \( k_{2D} = (k_x, k_y) \) is the two-dimensional wavevectors, \( N_s \) is the total number of sites,
\[ T_k = (g_t + J_1 \xi) \gamma_{k_{2D}} - 2g_t \cos(k_z a_z - \frac{\phi}{2}) + J_N n + \mu, \]
\[ \Delta_{k_{2D}} = J_{2}|\Delta_0|^2 \gamma_{k_{2D}}, \]
\[ \epsilon_0 = 4J_2|\Delta_0|^2 + 4J_1 \xi^2 + \frac{J_N}{N} n^2, \]
\[ \gamma_{k_{2D}} = 2(\cos(k_x a_x) + \cos(k_y a_y)), \]
\[ \eta_{k_{2D}} = 2(\cos(k_x a_x) - \cos(k_y a_y)), \]
\[ J_1 = \frac{3}{4} g_s J - \frac{3}{4} J_N, \quad J_2 = \frac{3}{4} g_s J + \frac{1}{4} J_N, \]
and \( a_x, a_y, a_z \) are the lattice constants.

This Hamiltonian can be diagonalized using a standard Bogoliubov transformation:
\[ H_{\text{eff}} = \sum_k \left( E_k - 2g_t \sin k_z a_z \sin \frac{\phi}{2} \right) \left( A_{k}^\dagger A_{k} + B_{-k}^\dagger B_{-k} \right) \]
\[ + \sum_k (\xi_k - E_k) + N_s \epsilon_0, \] (11)
where
\[ E_k = \sqrt{\xi_k^2 + |\Delta_{k_{2D}}|^2}, \quad \xi_k = -\frac{1}{2}(T_k + T_{-k}) \] (12)

The parameters \( \Delta_0, \xi, \mu \) are determined by the following self-consistent equations:
\[ 1 = \frac{J_2}{4N_s} \sum_k \frac{\eta_{k_{2D}}}{2E_k}, \] (13)
\[ \xi = -\frac{1}{4N_s} \sum_k \frac{\xi_k \gamma_{k_{2D}}}{2E_k}, \] (14)
\[ \delta = \frac{1}{N_s} \sum_k \frac{\xi_k}{E_k}. \] (15)

Then we obtain the ground state energy per site as the function of \( \phi \):
\[ E_0(\phi)/N_s = \frac{1}{N_s} \sum_k (\xi_k - E_k) \]
\[ + 4J_2|\Delta_0|^2 + 4J_1 \xi^2 + \frac{J_N}{2} n^2. \] (16)
In the following calculation, we take \( J/t = J_N/t = 0.2 \) and \( t_z/t = 0.1 \).

First, we look at the \( \phi \)-dependence of \( E_0(\phi)/N_s \). In order to see the variation in the energy clearly, we define \( \Delta E(\phi) = (E_0(\phi) - E_0(0))/N_s \). In Fig.2, \( \Delta E(\phi) \) is plotted against the phase \( \phi \) for various values of the doping rate \( \delta \). We can see that \( \Delta E(\phi) \) is the monotonic in-
creasing function of $\phi$, and that $\Delta E(\phi)$ increases with $\delta$ for arbitrary values of $\phi$. When $\delta \geq 0.25$ (the line (c)), however, $E(\phi)$ exceeds the energy of the normal state, i.e., the superconducting state becomes unstable in the region $\phi \geq 0.3\pi$. Let us discuss this situation a little more. From eq.(6), we can interpret $g_{t_z}$ as the effective amplitude of the interlayer hopping. Thus, as $\delta$ increases, the coupling strength between layers increases. On the other hand, the gap amplitude $\Delta_0$ linearly decreases with increasing of $\delta$. If the gap amplitude $\Delta_0$ is smaller than the effective interlayer hopping amplitude $g_{t_z}$, the quasiparticle energy
\[
\sqrt{\xi_0^2 + |\Delta_{kz}|^2} - 2g_{t_z} \sin k_z a_z \sin \frac{\phi}{2} \quad (17)
\]
becomes negative for $\phi \geq \phi_c$, where $\phi_c$ is a critical phase. Consequently, the superconducting state is no more stable as compared to the normal state.

Next, we calculate the Josephson current defined as
\[
\frac{e}{\hbar} \frac{\partial}{\partial \phi} \frac{\partial}{\partial \phi} \frac{\partial}{\partial E_0(\phi)}.
\]
(18)

Figure 2 shows the Josephson current-phase relations obtained for various values of $\delta$. The currents show the sinusoidal $\phi$-dependence as expected and have their maximum values $j_{\text{max}}(\delta)$ at $\phi = \frac{\pi}{2}$. When $\delta \leq 0.2$, $j_{\text{max}}$ increases with $\delta$. This is because the effective mass of a Cooper pair, that is, its localization tendency in a layer decreases in rough proportion to $1/|\Delta_0|$ with $|\Delta_0|$ being the decreasing function of $\delta$. On the contrary, when $\delta \geq 0.25$, $j_{\text{max}}$ decreases with increasing of $\delta$ because the Josephson current vanishes for $\phi \geq \phi_c$. Thus, $j_{\text{max}}$ reaches its maximum in the region $0.2 \leq \delta \leq 0.25$. In order to see this situation in more detail, the doping dependence of $j_{\text{max}}$ is plotted explicitly in Fig.3. We can see that $j_{\text{max}}$ has its maximum value at $\delta \approx 0.23$ in the present case.

In the recent experiment\cite{4}, it has been reported that the intrinsic Josephson current increases with hole-doping (in our notation, $\delta$) when the system is in the under-doped region. We note here that, if we do not assume the doping dependence of interlayer hopping as in Eq. (10), i.e., the interlayer hopping amplitude has no doping dependence, $j(\phi)$ becomes a decreasing function of $\delta$. Since the renormalization of interlayer hopping in eq.(10) is a natural assumption to treat the layered $t$-$J$ model based on the Gutzwiller approximation, the doping dependence of the intrinsic Josephson current observed in actual experiments can be used to check the validity of the Gutzwiller approximations of the layered $t$-$J$ model. Of course there is only a few experiments concerning a doping dependence of the Josephson current so far, so that more extensive comparison between the theory and experiments is necessary.
In this letter, we have calculated the d.c. Josephson current which flows perpendicular to the CuO$_2$ planes in high $T_C$ superconductors using the layered $t$-$J$ model within the mean field theory and Gutzwiller approximation. The maximum Josephson current has its maximum near the optimum doping rate. This is due to the competition between the enhancement of the effective transfer energy along the $c$-axis and the decrease of the magnitude of the order parameter with the increase of the doping rate. We hope such a behavior will be observed in experiments near future.

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