Profit Allocations for Restricted Coalition With Hesitation Degrees in Cooperative Game Theory

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ABSTRACT Profit allocation plays an important role in the decision-making field. In this paper, we study an allocation method on restricted coalition cooperation with intuitionistic fuzzy coalitions, in which the partners have some hesitation degree and different risk preferences when they participate in a limited communication structure game. In order to sufficiently analyze the profit allocation strategy, an average tree solution (A-T solution) with Choquet integrals and hesitation degrees is studied. In particular, a simple solving method for the A-T solution is proposed by proving that the characteristic functions of the cooperative game satisfy the monotonicity condition. Using this method, the upper and lower bounds of the A-T solution can be calculated directly from the upper and lower bounds of the interval characteristic functions. This method avoids the subtraction of interval numbers. Furthermore, the properties of the A-T solution according to an axiomatic system are proved in this paper. Finally, the applicability and superiority of the proposed approach are demonstrated through comparison with other methods.

INDEX TERMS Profit allocation, cooperative game, average tree solution, restricted coalition, intuitionistic fuzzy coalition.

I. INTRODUCTION

Currently, research into cooperative games is based mainly on the hypothesis of arbitrary coalitions being formed. However, due to the limitation of resources, status, and culture, cooperation is always limited in reality [1]. In this situation, direct or indirect connection among players is a necessary condition for forming an alliance, and it is a cooperative game with a limited communication structure, [2]. This game can be categorized as a limited graph cooperative game, in which the alliances are restricted. As the restricted coalition cooperation can effectively reflect complicated cooperation, the corresponding solutions have been proposed [3]–[6]. An average tree solution (A-T solution) is defined for an acyclic restricted coalition cooperative game [7], which satisfies the properties of component efficiency and component fairness. The A-T solution is analyzed by presenting a weaker condition that the super-additivity belongs to the core [8]. Later, the A-T solution for an acyclic restricted coalition cooperative game with communication structure was generalized [9], [10]. And a version of the A-T solution appropriate for a game based on an acyclic graph was studied [11]. The A-T solution of a restricted coalition cooperative game is concerned more and more because its good properties [12]: it must lie in the core if the game is super-additive but the Myerson value must not, and it is much simpler to find the marginal eigenvector of the allocation than the Shapley value.

In addition, fuzzy cooperative games have attracted great attention from researchers due to players often being unable to evaluate situations accurately in a real game. Fuzzy cooperative games with fuzzy coalitions were introduced to extend crisp coalitions [13], in which the degree of participation is a real number in the interval [0,1] instead of 1 or 0. This means that players may not fully participate in a coalition, but only partially participate in it. For a restricted coalition cooperative game, a fuzzy coalition Myerson value was proposed by...
using the proportional and the Choquet model [14]. Then the Myerson value of cooperative games with communication structure and fuzzy coalitions was studied [15]. In the above, all fuzzy coalitions are described by the set of real numbers, and the non-membership degree is simply the complement of the membership degree to 1. In practice, however, players do not often express the non-membership degree of a given element as the complement of the membership degree. In other words, players may have some hesitation degree. For example, if a player knows his/her participation degree in a coalition is at least 0.6, and the non-participation degree is 0.3, then their hesitation degree is 0.1. In order to incorporate the hesitation degree into cooperative game theory and to make it more applicable, we analyzed it using intuitionistic fuzzy information. An intuitionistic fuzzy set [16], [17] can express more abundant information by using membership degree, non-membership degree and hesitation degree, and this plays an important role in decision-making. However, as the calculation and complete ranking problems of intuitionistic fuzzy numbers, research into cooperative games is limited. Currently, only set solutions are generally studied, such as least square prenucleolus [18] and core [19]. Our other paper [20] studied A-T solution with intuitionistic fuzzy coalition based on players’ risk preferences weighted form.

In this paper, we study the restricted coalition cooperative game with intuitionistic fuzzy coalitions which satisfies the monotonicity condition and proposes a corresponding A-T solution based on Choquet integrals. There are lower and upper degrees of participation of players when we introduce confidence levels to hesitation degrees, and then the characteristic functions can be integrated into interval numbers. Nowadays, interval numbers are widely used in fuzzy decision, and cooperative games are no exception. The core of interval-valued cooperative games based on an interval-valued square dominance core and interval-valued dominance core is extended [21]. And an axiomatic characterization of the interval-valued Shapley-like value of a subclass of interval-valued cooperative games is given [22], in which the interval-valued cooperative game is called size monotonicity. In addition, [23] studied the interval Shapley function based on the extended Hukuhara difference [24], which is an interval population monotonic allocation function when the games are convex. Most of the aforementioned works used the partial subtraction operator [25] and Moore’s [26] and Hukuhara [24] interval subtractions, which are irreversible. What’s worse, Moore’s subtraction usually increases the uncertainty of the resulted interval, and Hukuhara subtraction may not be used when the interval value does not satisfy the defined condition [27]. In view of the facts mentioned above, this paper proposes a simple method to the profit allocations for restricted coalition with hesitation degrees in cooperative game theory. There are two important contributions: (1) we combining the idea of intuitionistic fuzzy coalitions and the restricted coalition cooperative games to study the fuzzy profit allocation problem. (2) due to the interval subtraction is complex, and is not invertible in the compute process, we propose a simple method for the A-T solution by proving that the characteristic functions of the cooperative game satisfy the monotonicity condition. Then the upper and lower bounds of the A-T solution can be calculated directly from the upper and lower bounds of their interval characteristic functions, and this can effectively avoid interval subtraction or interval order relation. The method of this paper is completely different from author’s another paper [20], which aggregated the characteristic functions of intuitionistic fuzzy coalitions into crisp numbers by the attitude factors of risk preferences. It means that the essence of the cooperative game in [20] is a crisp cooperative game.

The remaining content is organized as follows: Section 2 is preliminaries; Section 3 defines a restricted coalition cooperative game with intuitionistic fuzzy coalitions, and gives the characteristic function using Choquet integral form; Section 4 proposes a formula for the intuitionistic fuzzy coalition A-T solution and proves the existence and rationality of the solution according to an axiomatic system in view of the crisp cooperative game; Section 5 illustrates the proposed method with numerical examples and comparisons with other methods to show its applicability and superiority; and Section 6 shows conclusions.

II. PRELIMINARIES

A. INTUITIONISTIC FUZZY SET

Definition 1 [16]: Let $X$ be a nonempty set of the universe. If two mappings on the set $X$ are $\mu_X : X \rightarrow [0, 1]$ and $\nu_X : X \rightarrow [0, 1]$, so that $0 \leq \mu_X(x) \leq 1$, $0 \leq \nu_X(x) \leq 1$ and $0 \leq \mu_X(x) + \nu_X(x) \leq 1$ for $x \in X$, then $\mu_X$ and $\nu_X$ are called an intuitionistic fuzzy set $X$, denoted by $A = \{< x; \mu_X(x), \nu_X(x) > | x \in X \}$, where $\mu_X$ and $\nu_X$ are the membership degree and non-membership degree of $A$ respectively. The set of the intuitionistic fuzzy on the universal set $X$ is denoted by $IF(X)$.

It is easy to see from Definition 1 that an intuitionistic fuzzy set is defined by a pair of membership and non-membership degrees (functions), which are more or less independent of each other, and the sum of the membership degree and non-membership degree is not greater than 1.

B. INTERVALS AND THEIR ARITHMETIC OPERATIONS

Denote an interval $\tilde{a} = [a_L, a_R] = \{a | a \in \mathbb{R}, a_L \leq a \leq a_R \}$, where $\mathbb{R}$ is the set of real numbers. $a_L \in \mathbb{R}$ and $a_R \in \mathbb{R}$ are called the lower bound and the upper bound of the interval $\tilde{a}$ respectively. Clearly, interval numbers are a generalization of real numbers. Conversely, real numbers are a special case of intervals.

In the following, we give some interval arithmetic operations such as the equality, the addition, and the scalar multiplication.

Definition 2 [26]: Let $\tilde{a} = [a_L, a_R]$ and $\tilde{b} = [b_L, b_R]$ be two intervals on the set $\mathbb{R}$, and $\gamma$ is any real number. The interval arithmetic operations are defined as follows:
(1) Interval equality: \( \tilde{a} = \tilde{b} \) if and only if \( a_D = b_D \) and \( a_R = b_R \).

(2) Interval addition: \( \tilde{a} + \tilde{b} = [a_D + b_D, a_R + b_R] \).

(3) Interval’s scalar multiplication:

\[
\gamma \tilde{a} = \begin{cases} 
[\gamma a_D, \gamma a_R] & \text{if } \gamma \geq 0 \\
[\gamma a_R, \gamma a_D] & \text{if } \gamma < 0.
\end{cases}
\]

The above interval arithmetic operations are an extension of arithmetic operations on real numbers. However, interval subtraction is complex, and is not invertible. The common subtraction is as follows:

Moore’s interval subtraction [26]:

\[
\tilde{a} - \tilde{b} = [a_D - b_D, a_R - b_R];
\]

Partial subtraction operator [25]:

\[
\tilde{a} - \tilde{b} = [a_D - b_D, a_R - b_R] \text{ if } a_R - a_D \geq b_R - b_D;
\]

Hukuhara subtraction [24]: if \( \tilde{a} = \tilde{b} + \tilde{c} \) and \( \tilde{c} = \tilde{a} - \tilde{b} \), then the subtraction of \( \tilde{a} \) and \( \tilde{b} \) is denoted by \( \tilde{c} = \tilde{a} - \tilde{b} = [a_D - b_D, a_R - b_R] \) if \( a_R - b_R \geq a_D - b_D \).

### III. A RESTRICTED COALITION COOPERATIVE GAME WITH INTUITIONISTIC FUZZY COALITIONS AND ITS CHARACTERISTIC FUNCTION

#### A. FUZZY COALITION AND INTUITIONISTIC FUZZY COALITION

In a cooperative game with fuzzy coalitions, the set \( N = \{1, 2, \cdots, n\} \) composed by all the fuzzy coalitions is denoted as \( F(N) \), and arbitrary element \( \tilde{S} \) represents a fuzzy coalition and can be demonstrated by a fuzzy vector as:

\[
\tilde{S} = (\tilde{S}(1), \tilde{S}(2), \cdots, \tilde{S}(n)) : F(N) \rightarrow \mu_{\tilde{S}}(i)
\]

where \( \mu_{\tilde{S}}(i) \) is the degree of participation of the player \( i \) \( (i = 1, 2, \cdots, n) \) in the coalition \( \tilde{S} \), namely the ratio of the resources invested by player \( i \) to the required resources. Currently, \( \mu_{\tilde{S}}(i) \) describes a real number in the interval \( [0, 1] \), and the cooperative game with fuzzy coalitions \([0, 1]^n\) is still a crisp number. It shows that there is no uncertain information, let alone players’ hesitation degrees.

From the definition of intuitionistic fuzzy sets, for intuitionistic fuzzy coalitions, the membership degrees express players’ participation levels, the non-membership degrees are the level of not participation, and the rest are players’ hesitation degrees. Therefore, we can generalize the cooperative game with fuzzy coalition \( F(N) \) into a cooperative game with intuitionistic fuzzy coalition \( IF(N) \). For any intuitionistic fuzzy coalition element \( \tilde{S}, \mu_{\tilde{S}}(i) \subseteq [0, 1] \) is a participation degree of player \( i \) \( (i = 1, 2, \cdots, n) \) in the coalition \( \tilde{S}, \psi_{\tilde{S}}(i) \subseteq [0, 1] \) is non-participation degree, and \( 0 \leq \mu_{\tilde{S}}(x) + \psi_{\tilde{S}}(x) \leq 1 \).

Then the hesitation degree of player \( i \) is \( \pi_{\tilde{S}}(i) = 1 - \mu_{\tilde{S}}(x) - \psi_{\tilde{S}}(x) \). We can express intuitionistic fuzzy coalition as

\[
\tilde{S} = \{ \langle 1; \mu_{\tilde{S}}(1), \psi_{\tilde{S}}(1) \rangle, \langle 2; \mu_{\tilde{S}}(2), \psi_{\tilde{S}}(2) \rangle, \\
\cdots, \langle n; \mu_{\tilde{S}}(n), \psi_{\tilde{S}}(n) \rangle \}
\]

Especially, if \( \tilde{S} = \{ \langle 1; 0, 0 \rangle, \langle 2; 0, 0 \rangle, \cdots, \langle n; 0, 0 \rangle \} \), then the alliance is empty; and \( \tilde{S} = \{ \langle 1; 1, 0 \rangle, \langle 2; 1, 0 \rangle, \cdots, \langle n; 1, 0 \rangle \} \) is a crisp grand coalition. Obviously, if \( \mu_{\tilde{S}}(x) + \psi_{\tilde{S}}(x) = 1 \), then the intuitionistic fuzzy coalition becomes a fuzzy coalition.

#### B. RESTRICTED COALITION COOPERATIVE GAMES WITH INTUITIONISTIC FUZZY COALITIONS

In a crisp cooperative game with transferable utility (a TU-game), the triad \((N, v, L)\) constitutes a restricted coalition cooperative game, where \( N = \{1, 2, \cdots, n\} \) is the set of players, \( v : 2^N \rightarrow \mathbb{N} \) is a mapping defined on subsets of \( N \), and \( L \subseteq \{(i, j) | i \neq j, i, j \in N\} \) is a set of links in the communication graph. The function \( v \) is the payoff function of a game, where \( v(S) \) is the worth of each coalition \( S \subseteq 2^N \), with the property \( v(\emptyset) = 0 \).

If \( L = \{(i, j) | i \neq j, i, j \in N\} \), then players are free to choose their own cooperative partners in the game, and the corresponding graph is a complete graph \((N, L)\). In this case, \((N, v, L)\) is a cooperative game with complete communication structure or a complete graph cooperative game, and usually the well-known cooperative game refers to this one, which can be abbreviated as \((N, v)\). In this game, any player in the game can form coalitions with others freely and without restriction. If \( L \neq \{(i, j) | i \neq j, i, j \in N\} \) and \( L \) is non-null, then \((N, v, L)\) is a cooperative game with a limited communication structure. In this game, players may have a cooperation if and only if they are interconnected, and coalitions forming is restricted. This paper discusses a cooperative game with acyclic and restricted communication structure.

**Definition 3:** According to the crisp cooperative game with communication structure \((N, v, L)\), if the fuzzy payoffs \( \tilde{v} \) of \((N, \tilde{v}, L)\) are mapping functions from intuitionistic fuzzy coalitions \( F(N) \) to a fuzzy number set \( \tilde{\mathbb{R}} \), namely, \( \tilde{v} : IF(N) \rightarrow \tilde{\mathbb{R}} \) with \( \tilde{v}(\emptyset) = 0 \), then \((N, \tilde{v}, L)\) is a restricted coalition cooperative game with intuitionistic fuzzy coalitions. For conciseness, the restricted coalition cooperative games with intuitionistic fuzzy coalitions is denoted \( \tilde{G} \), and the entirety of \( \tilde{G} \) is denoted \( \tilde{G}^n \).

This paper discusses the most common cooperative game, which satisfies general properties of convexity and super additivity.

**Definition 4:** Let \((N, \tilde{v}, L)\) be a restricted coalition cooperative game with intuitionistic fuzzy coalitions. An intuitionistic fuzzy coalition \( \tilde{T} \in IF(N) \) is a carrier of \((N, \tilde{v}, L)\) if it satisfies

\[
\tilde{v}(\tilde{S} \cap \tilde{T}) = \tilde{v}(\tilde{S})
\]

for any coalition \( \tilde{S} \in IF(N) \) with \( \tilde{S} \cap \tilde{T} = \emptyset \).

**Definition 5:** Let \((N, \tilde{v}, L)\) be a restricted coalition cooperative game with intuitionistic fuzzy coalitions, it is said to be convex if it satisfies

\[
\tilde{v}(\tilde{S} \cup \tilde{T}) + \tilde{v}(\tilde{S} \cap \tilde{T}) \geq \tilde{v}(\tilde{S}) + \tilde{v}(\tilde{T})
\]

for any coalitions \( \tilde{S}, \tilde{T} \in IF(N) \) with \( \tilde{S} \cap \tilde{T} = \emptyset \).
Definition 6: Let \((N, \tilde{v}, L)\) be a restricted coalition cooperative game with intuitionistic fuzzy coalitions, it is called super-additive if it satisfies
\[ \tilde{v}(\tilde{S} \cup \tilde{T}) \geq \tilde{v}(\tilde{S}) + \tilde{v}(\tilde{T}) \]
for any coalitions \(\tilde{S}, \tilde{T} \in IF(N)\) with \(\tilde{S} \cap \tilde{T} = \emptyset\).

In a \((N, \tilde{v}, L)\), the minimum participation degrees is \(\mu_{\tilde{S}}(i)\), and the maximum participation degrees depend on the hesitation degree \(\pi_{\tilde{S}}(i) = 1 - \mu_{\tilde{S}}(i) - \nu_{\tilde{S}}(i)\). If the whole hesitation degree is meant to participate in the cooperation, then the participation degree \(\eta_{\tilde{S}}(i)\) and the upper is different for different attitudes of hesitation degree. Thus the participation degree of player \(i\) \((i \in N)\) can be expressed as an interval value, the lower of which is \(\mu_{\tilde{S}}(i)\), and the upper is different for different attitudes of hesitation degree. Therefore, the participation degree \(\eta_{\tilde{S}}(i)\) of player \(i\) can be defined as a closed interval number with a confidence level \(\alpha \in [0, 1]\):

\[
\eta_{\tilde{S}}(i) = [\mu_{\tilde{S}}(i), \mu_{\tilde{S}}(i) + \alpha \pi_{\tilde{S}}(i)] \quad (1)
\]

It is clear that the participation degree of player \(i\) is \(\eta_{\tilde{S}}(i) \subseteq [0, 1]\).

C. CHARACTERISTIC FUNCTIONS OF RESTRICTED COALITION COOPERATIVE GAME WITH INTUITIONISTIC FUZZY COALITIONS

In a restricted coalition cooperative game with intuitionistic fuzzy coalitions \((N, \tilde{v}, L)\), for any coalition \(\tilde{S} \in IF(N)\), let \([\tilde{S}]_{h_1} = \{i \in N | \mu(i) \geq h_1\}\) be crisp coalitions with all the players’ participation degree \(\mu(i) > h_1\), where \(h_1 \in [0, 1]\), \(h_0 = 0, l = 1, 2, \ldots, d(\tilde{S}); [\tilde{S}]_{q_0} = \{i \in N | \eta(i) \geq q_0\}\) be crisp coalitions with all the players’ hesitation degree \(\eta(i) > q_0\), where \(q_0 \in [0, 1]\), \(m = 1, 2, \ldots, d'(\tilde{S})\). Let arbitrary \(i \in Supp(\tilde{S}), D(\tilde{S}) = \{ \mu(i) | \mu(i) > 0, i \in N \}, D'(\tilde{S}) = \{ \eta(i) | \eta(i) > 0, i \in N \}\), and \(d(\tilde{S}), d'(\tilde{S})\) be the number of elements in \(D(\tilde{S})\). If the elements in \(D'(\tilde{S})\) are arranged in an ascending order \(0 < h_1 \leq h_2 \leq \ldots \leq h_{d(\tilde{S})} \leq 1\) and \(0 < q_1^0 \leq q_2^0 \leq \ldots \leq q_{d'(\tilde{S})}^0 \leq 1\), then the characteristic function by Choquet integral form [28] of \((N, \tilde{v}, L)\) can be expressed as:

\[
\begin{align*}
\tilde{v}_D(\tilde{S}) &= \sum_{l=1}^{d(\tilde{S})} [v([\tilde{S}]_{h_l}) (h_l - h_{l-1})]; \\
\tilde{v}_R(\tilde{S}) &= \sum_{m=1}^{d'(\tilde{S})} [v([\tilde{S}]_{q_m}) (q_m - q_{m-1})]
\end{align*}
\]

(2)

where \(v([\tilde{S}]_{h_l})\) and \(v([\tilde{S}]_{q_m})\) are the payoffs of a restricted coalition cooperative game with crisp coalitions \([\tilde{S}]_{h_l}\) and \([\tilde{S}]_{q_m}\) respectively.

According to Eq. (2), the characteristic functions of \((N, \tilde{v}, L)\) are interval numbers due to the fact that intuitionistic fuzzy coalitions of the restricted coalition cooperative game are interval values when the confidence level of hesitation degree are considered. The payoff functions \(\tilde{v}(\tilde{S}) = [\tilde{v}_D(\tilde{S}), \tilde{v}_R(\tilde{S})]\) are the expected payoffs of the intuitionistic coalition \(\tilde{S}\), which are mappings of the coalition \(\tilde{S}\) to the interval value set \(\tilde{\Pi}\), namely \(\tilde{v} : \tilde{F}(N) \rightarrow \tilde{\Pi}\), and \(\tilde{v}(\emptyset) = 0\).

IV. INTUITIONISTIC FUZZY COALITION A-T SOLUTION AND ITS PROPERTIES

A. CRISP RESTRICTED COALITION COOPERATIVE GAME AND ITS A-T SOLUTION

For an undirected graph \((N, L)\), a coalition of players \(K \subseteq N\) is a network of \((N, L)\) if \(K\) is connected, i.e. between any two members of \(K\), there is a path with in \(L\). A network is called a component if no larger network contains it. We denote by \(\tilde{C}^L(N)\) the set of all components of \((N, L)\). An n-tuple \(B = (B_1, \cdots, B_n)\) of subsets of \(N\) is admissible if \(|1\) for all \(i \in N, i \in B_i;\) \(2\) for some \(j \in N, B_j = N;\) \(3\) for all \(i \in N\) and \(K \in \tilde{C}^L(B_i)\), there exists \(j \in N\) such that \(K = B_j\) and \([i, j] \in L\). In a crisp restricted coalition cooperative game \((N, v, L)\), a A-T solution is defined as follows [7]:

\[
AT_i(N, v, L) = \frac{1}{|B|^2} \left[ \sum_{B \in B^L} [v(B_i) - \sum_{K \in \tilde{C}^L(B_i) \backslash \{i\}} v(K)] \right] (3)
\]

where \(i = 1, 2, \ldots, n\), \(B^L\) is the collection of all admissible n-tuple of coalitions \(B\), \(|B|^2\) represents the number of components of \(B^L\).

It has been proved that A-T solution is characterized by efficiency, dummy, linearity, independent, and it satisfies properties of component efficiency, component fairness, and additivity [12].

B. A-T SOLUTION OF A RESTRICTED COALITION COOPERATIVE GAME WITH INTUITIONISTIC FUZZY COALITIONS

Given the characteristic functions of a restricted coalition cooperative game with intuitionistic fuzzy coalitions can be transformed into interval values from Section 3.2, the arithmetic operations of interval numbers are necessary for solving an intuitionistic fuzzy coalition A-T solution. However, the interval subtraction may result in irrational conclusions as it is not an invertible operator [24]. In this paper, we focus on developing an effective and simplified method for computing an intuitionistic fuzzy coalition A-T solution by using monotonicity, rather than the special interval subtraction operator or ranking method.

For any interval-valued characteristic functions of \((N, \tilde{v}, L)\), we can convert it by introducing weighting factor as follows:

\[
\tilde{v}(\lambda)(K) = (1 - \lambda)v_D(K) + \lambda v_R(K) \quad (K \subseteq N) (4)
\]

where \(\lambda \in [0, 1]\) is any real number, which can be interpreted as an attitude factor of players (or decision makers), i.e., it reflects the attitude of players towards uncertainty.
According to Eq. (3), we can obtain the intuitionistic fuzzy coalition A-T solution \(\psi_i(\check{\nu}(\lambda)) = (\psi_1(\check{\nu}(\lambda)), \psi_2(\check{\nu}(\lambda)), \ldots, \psi_n(\check{\nu}(\lambda)))^T\) of \((N, \check{\nu}, L) \in \check{G}^n\), where

\[
\psi_i(\check{\nu}(\lambda)) = \frac{1}{B^L} \sum_{B \in B^L} \left( \check{\nu}(\lambda)(B_i) - \sum_{K \in \check{C}^L(B_i \setminus \{i\})} \check{\nu}(\lambda)(K) \right)
\]

which can further be rewritten as follows:

\[
\psi_i(\check{\nu}(\lambda)) = \frac{1}{B^L} \sum_{B \in B^L} ((1 - \lambda)v_D(B_i) + \lambda v_R(B_i))
\]

- \(\frac{1}{B^L} \sum_{B \in B^L} \sum_{K \in \check{C}^L(B_i \setminus \{i\})} ((1 - \lambda)v_D(K) + \lambda v_R(K))
\]

Clearly, \(\check{\nu}(\lambda)\) is a continuous function of \(\lambda \in [0, 1]\) due to Eq. (4). Accordingly, the intuitionistic fuzzy coalition A-T solution \(\psi_i(\check{\nu}(\lambda)) (i = 1, 2, \ldots, n)\) is a continuous and non-decreasing function of the parameter \(\lambda \in [0, 1]\).

**Theorem 1:** For any \((N, \check{\nu}, L) \in \check{G}^n\), if the following system of inequalities

\[
\sum_{B \in B^L} \left( (v_R(B_i) - v_D(B_i)) - \sum_{K \in \check{C}^L(B_i \setminus \{i\})} (v_R(K) - v_D(K)) \right) \geq 0
\]

is satisfied, then the A-T solution \(\psi_i(\check{\nu}(\lambda))\) of \((N, \check{\nu}, L) \in \check{G}^n\) is monotonic and non-decreasing function of the parameter \(\lambda \in [0, 1]\).

**Proof:** For any \(\lambda \in [0, 1]\) and \(\lambda' \in [0, 1]\), according to Eq. (4), we have

\[
\psi_i(\check{\nu}(\lambda)) - \psi_i(\check{\nu}(\lambda'))
\]

\[
= \frac{1}{B^L} \sum_{B \in B^L} \left( \check{\nu}(\lambda)(B_i) - \sum_{K \in \check{C}^L(B_i \setminus \{i\})} \check{\nu}(\lambda)(K) \right)
\]

\[
- \frac{1}{B^L} \sum_{B \in B^L} \left( \check{\nu}(\lambda')(B_i) - \sum_{K \in \check{C}^L(B_i \setminus \{i\})} \check{\nu}(\lambda')(K) \right)
\]

\[
= (\lambda - \lambda') \frac{1}{B^L} \sum_{B \in B^L} (v_R(B_i) - v_D(B_i))
\]

\[
- (\lambda - \lambda') \frac{1}{B^L} \sum_{B \in B^L} \sum_{K \in \check{C}^L(B_i \setminus \{i\})} (v_R(K) - v_D(K))
\]

where \(i = 1, 2, \ldots, n\).

If \(\lambda \geq \lambda'\), then combined with the assumption, i.e., Eq. (6), we have

\[
\psi_i(\check{\nu}(\lambda)) - \psi_i(\check{\nu}(\lambda')) \geq 0 \quad (i = 1, 2, \ldots, n),
\]

i.e., \(\psi_i(\check{\nu}(\lambda)) \geq \psi_i(\check{\nu}(\lambda'))\), which means that \(\psi_i(\check{\nu}(\lambda))\) \((i = 1, 2, \ldots, n)\) are monotonic and non-decreasing functions of the parameter \(\lambda \in [0, 1]\). This completes the proof for Theorem 1.

Therefore, for any coalition monotonicity-like \((N, \check{\nu}, L) \in \check{G}^n\), i.e., it satisfies Eq. (6), then it can be derived directly from Theorem 1 and Eq. (4), and the lower and upper bounds of the components (intervals) \(\psi_i(\check{\nu}) (i = 1, 2, \ldots, n)\) of the intuitionistic fuzzy coalition A-T solution \(\psi(\check{\nu}) = (\psi_1(\check{\nu}), \psi_2(\check{\nu}), \ldots, \psi_n(\check{\nu}))^T\) are given as follows:

\[
\psi_D(\check{\nu}) = \psi_i(\check{\nu}(0))
\]

\[
= \frac{1}{B^L} \sum_{B \in B^L} \left( v_D(B_i) - \sum_{K \in \check{C}^L(B_i \setminus \{i\})} v_D(K) \right)
\]

and

\[
\psi_R(\check{\nu}) = \psi_i(\check{\nu}(1))
\]

\[
= \frac{1}{B^L} \sum_{B \in B^L} \left( v_R(B_i) - \sum_{K \in \check{C}^L(B_i \setminus \{i\})} v_R(K) \right)
\]

So \(\psi_i(\check{\nu})\) for the players \(i (i = 1, 2, \ldots, n)\) in the coalition monotonicity-like \((N, \check{\nu}, L) \in \check{G}^n\) are directly and explicitly expressed as follows:

\[
\psi_i(\check{\nu}) = \left[ \frac{1}{B^L} \sum_{B \in B^L} \left( v_D(B_i) - \sum_{K \in \check{C}^L(B_i \setminus \{i\})} v_D(K) \right),
\]

\[
\frac{1}{B^L} \sum_{B \in B^L} \left( v_R(B_i) - \sum_{K \in \check{C}^L(B_i \setminus \{i\})} v_R(K) \right) \right]
\]

Therefore, the lower bounds of the intervals \(\psi_i(\check{\nu})\) \((i = 1, 2, \ldots, n)\) of the intuitionistic fuzzy coalition A-T solution can be obtained by distributing the lower bounds of the interval-valued coalitions’ payoffs to player \(i\) who are in the coalition. Similarly, we can obtain the upper bounds of \(\psi_i(\check{\nu})\) for player \(i\).

It is obvious that Eq. (6) is an important condition which ensures that the intuitionistic fuzzy coalition A-T solution \(\psi_i(\check{\nu})\) possesses monotonicity. It requires that the sum of all interval-valued characteristic lengths of bigger coalitions with player \(i\) to that of the coalitions without player \(i\) is monotonic

\[
\sum_{B \in B^L} (v_R(B_i) - v_D(B_i)) \geq \sum_{B \in B^L} \sum_{K \in \check{C}^L(B_i \setminus \{i\})} (v_R(K) - v_D(K))
\]

It is clear that the monotonicity of Eq. (6) is a good property, and it can be satisfied more easily than traditional monotonicity of a cooperative game \((N, \check{\nu}, L) \in \check{G}^n\):

\[
len(\check{\nu}(B)) \geq len(\check{\nu}(B \setminus \{i\})) \quad (i = 1, 2, \ldots, n)
\]

where \(B\) is an admissible coalition as above in section IV, and \(len(\check{\nu}(B))\) is the interval length of characteristic function of \(B\).

The interpretation is that the interval length of the payoffs of the bigger coalition is not necessarily bigger than that of a coalition without player \(i \in B\). That is, if an \((N, \check{\nu}, L) \in \check{G}^n\) satisfies Eq. (6), even if \(len(\check{\nu}(B)) \leq len(\check{\nu}(B \setminus \{i\}))\), we can use the method to easily compute intuitionistic fuzzy coalition A-T solution.
C. PROPERTIES OF INTUITIONISTIC FUZZY COALITION A-T SOLUTION

In this section, we will discuss some useful and important properties of the intuitionistic fuzzy A-T solution of monotonicity-like restricted coalition cooperative games according to the relevant properties of crisp A-T solution.

Theorem 2: The intuitionistic fuzzy coalition A-T solution \( \psi(\tilde{v}) \) determined by Eq. (9) of any \((N, \tilde{v}, L) \in \tilde{\mathcal{G}}^n\) satisfies the following properties:

P1 (Component Efficiency): For any \((N, \tilde{v}, L) \in \tilde{\mathcal{G}}^n\), if the intuitionistic fuzzy coalition A-T solution \( \psi(\tilde{v}) : \tilde{G}^n \rightarrow \tilde{\mathcal{H}} \) satisfies component efficiency, then \( \sum_{i \in K} \psi_i(\tilde{v}) = \tilde{v}(K) \) for any \( K \in \hat{\mathcal{C}}^l(N) \).

P2 (Component Fairness): For any \((N, \tilde{v}, L) \in \tilde{\mathcal{G}}^n\), if the intuitionistic fuzzy coalition A-T solution \( \psi(\tilde{v}) : \tilde{G}^n \rightarrow \tilde{\mathcal{H}} \) satisfies component fairness, then

\[
\frac{1}{|K^h|} \sum_{i \in K^h} (\psi_i(\tilde{v}) - \psi_j(\tilde{v}\backslash L[h, l])) = \frac{1}{|K^l|} \sum_{j \in K^l} (\psi_j(\tilde{v}) - \psi_j(\tilde{v}\backslash L[h, l])),
\]

where \( K^h \) and \( K^l \) express the link networks containing nodes \( i \) and \( j \) when the edge \( L[h, l] \) is deleted in component.

P3 (Additivity): For any \((N, \tilde{v}_1, L) \in \tilde{\mathcal{G}}^n\) and \((N, \tilde{v}_2, L) \in \tilde{\mathcal{G}}^n\), if the intuitionistic fuzzy coalition A-T solution \( \psi(\tilde{v}) : \tilde{G}^n \rightarrow \tilde{\mathcal{H}} \) satisfies additivity, then has \( \psi_i(\tilde{v}_1 + \tilde{v}_2) = \psi_i(\tilde{v}_1) + \psi_i(\tilde{v}_2) \).

P4 (Efficiency): For any \((N, \tilde{v}, L) \in \tilde{\mathcal{G}}^n\), if the intuitionistic fuzzy coalition A-T solution \( \psi(\tilde{v}) : \tilde{G}^n \rightarrow \tilde{\mathcal{H}} \) satisfies efficiency, then exists \( \sum_{i=1}^{n} \psi_i(\tilde{v}) = \tilde{v}(N) \).

P5 (Dummy): For any \((N, \tilde{v}, L) \in \tilde{\mathcal{G}}^n\), if the intuitionistic fuzzy coalition A-T solution \( \psi(\tilde{v}) : \tilde{G}^n \rightarrow \tilde{\mathcal{H}} \) satisfies dummy, then \( \psi_i(\tilde{v}) = 0 \) whenever \( \tilde{v}(i) = 0 \) for all intuitionistic coalition \( S \subseteq N \).

P6 (Linearity): For any two games \((N, \tilde{v}_1, L) \in \tilde{\mathcal{G}}^n\) and \((N, \tilde{v}_2, L) \in \tilde{\mathcal{G}}^n\), \( a, b \in \mathcal{B} \), if the intuitionistic fuzzy coalition A-T solution \( \psi(\tilde{v}) : \tilde{G}^n \rightarrow \tilde{\mathcal{H}} \) satisfies linearity, then \( \psi_i(a\tilde{v}_1 + b\tilde{v}_2) = a\psi_i(\tilde{v}_1) + b\psi_i(\tilde{v}_2) \).

P7 (Independence): For any \((N, \tilde{v}, L) \in \tilde{\mathcal{G}}^n\) with \( \tilde{v}_T(S) \neq 0 \), if the intuitionistic fuzzy coalition A-T solution \( \psi(\tilde{v}) : \tilde{G}^n \rightarrow \tilde{\mathcal{H}} \) satisfies independence, then \( \psi_i(\tilde{v}_T) = \psi_i(\tilde{v}_{T\backslash i}) \) when coalition \( T \subseteq S \) and otherwise \( \tilde{v}_T(S) = 0 \).

Proof P1 (Component Efficiency): According to Eqs. (7)-(8) and case (2) of Definition 2, and combine with the component efficiency of the A-T solution for a crisp restricted coalition cooperative game, we have

\[
\sum_{i \in K \in \hat{\mathcal{C}}^l(N)} \psi_i(\tilde{v}_D) = \sum_{i \in K \in \hat{\mathcal{C}}^l(N)} \frac{1}{|B^l|} \sum_{B \in B^l} \left( \bar{v}_D(B_i) - \sum_{K \in \hat{\mathcal{C}}^l(B_i \backslash [l])} \bar{v}_D(K) \right)
\]
According to Eq. (8), we can easily prove that
\[ A\bar{T}(i, v) = A\bar{T}(i, w) \]
and otherwise according to case (1) of Definition 2, we obtain additivity
\[ A\bar{T}(i, v) = A\bar{T}(i, \tilde{w}) \quad (i = 1, 2, \ldots , n). \]

**P4 (Efficiency):** According to the component efficiency of the A-T solution for a crisp restricted coalition cooperative game, if the \((N, \tilde{v}, L) \in \tilde{G}\) is a complete game, the intuitionistic fuzzy coalition A-T solution \(A\bar{T}(\tilde{v}) = \tilde{v}(N)\) satisfies the efficiency. It is easy to see that the A-T solution is the Shapley value when the cooperative game has an unrestricted communication structure.

**P5 (Dummy):** In \((N, \tilde{v}, L) \in \tilde{G}\), the coalitions with player \(i\) have \(\tilde{v}(S) = \tilde{v}(S\setminus \{i\})\) when \(\tilde{v}(i) = 0\) for all intuitionistic coalition \(S \in N\) and according to case (2) of Definition 2, we have \(A\bar{T}(B) = A\bar{T}(B\setminus \{i\})\). Combined with Eq. (9),
\[
\psi(\tilde{v}) = \frac{1}{|B|^L} \sum_{\tilde{B} \in B^L} \left( \psi(\tilde{v}) - \sum_{K \in \tilde{C}^L(B\setminus \{i\})} \psi(K) \right) = 0,
\]
\[
\psi(\tilde{v}) = \frac{1}{|B|^L} \sum_{\tilde{B} \in B^L} \left( \psi(\tilde{v}) - \sum_{K \in \tilde{C}^L(B\setminus \{i\})} \psi(K) \right) = 0
\]

Therefore, there exists \(\psi(\tilde{v}) = 0\) \((i = 1, 2, \ldots , n)\) whenever \(\tilde{v}(i) = 0\) for all intuitionistic coalition \(S \in N\).

**P6 (Linearity):** For any \((N, \tilde{v}, L) \in \tilde{G}\), \((N, \tilde{v}, L) \in \tilde{G}\), from the additivity of intuitionistic fuzzy coalition A-T solution \(\tilde{v}(S) : \tilde{G}^{n} \rightarrow \tilde{N}\), there have
\[
\psi(\tilde{v}) = \frac{1}{|B|^L} \sum_{\tilde{B} \in B^L} \left( \psi(\tilde{v}) - \sum_{K \in \tilde{C}^L(B\setminus \{i\})} \psi(K) \right) = 0,
\]
\[
\psi(\tilde{v}) = \frac{1}{|B|^L} \sum_{\tilde{B} \in B^L} \left( \psi(\tilde{v}) - \sum_{K \in \tilde{C}^L(B\setminus \{i\})} \psi(K) \right) = 0
\]

Combined with the linearity of characteristic function of Eq. (9), we can easily get \(\psi(a\tilde{v} + b\tilde{w}) = a\psi(\tilde{v}) + b\psi(\tilde{w})\).

**P7 (Independence):** In a restricted coalition game \((N, \tilde{v}, L) \in \tilde{G}\), there exists \(\tilde{v}(T) \neq 0\) when coalition \(T \subseteq \tilde{S}\) and otherwise \(\tilde{v}(T) = 0\). If a player joins a coalition in the game, it holds the independence property [12]: \(A\bar{T}(i, \tilde{v}) = A\bar{T}(i, \tilde{v}T_{\cup \{i\}})\) when \(i \in T, i \in S, j \notin T\) and \(j \notin S\). We therefore have
\[ A\bar{T}(i, \tilde{v}) = A\bar{T}(i, \tilde{v}T) = A\bar{T}(i, \tilde{v}T_{\cup \{i\}}), \]
i.e., \(A\bar{T}(i, \tilde{v}) = A\bar{T}(i, \tilde{v}T_{\cup \{i\}})\).

Independence means that if a player joins a coalition, the payoff of any player in the coalition not linked to that player remains the same, because players can form coalitions if and only if they are interconnected. And the A-T solution treats each player to the number of agents they are connected to outside of the coalition.

**Theorem 3:** If \((N, \tilde{v}, L) \in \tilde{G}\) is a monotonicity-like restricted coalition cooperative game, then the intuitionistic fuzzy coalition A-T solution is an reasonable existence allocation of \((N, \tilde{v}, L)\).

**Proof:** If \(i \in N\), according to component efficiency of intuitionistic fuzzy coalition A-T solution, we have \(\sum_{i \in K} \psi(i) = \tilde{v}(K)\) for any \(K \in \tilde{C}^L(N)\). That is to say, \(\psi(i)\) satisfies group rational.

In addition, as
\[
\psi(\tilde{v}) = \frac{1}{|B|^L} \sum_{\tilde{B} \in B^L} \left( \tilde{v}(B) - \sum_{K \in \tilde{C}^L(B\setminus \{i\})} \tilde{v}(K) \right) \geq 0
\]

Therefore, \(\psi(\tilde{v})\) satisfies the group rationality and individual rationality conditions from allocations. Hence, the intuitionistic fuzzy coalition A-T solution is an efficient allocation.

**Theorem 4:** If \(\tilde{v}\) is an interval-valued complete communication graph game, then the intuitionistic fuzzy coalition A-T solution \(A\bar{T}(\tilde{v})\) is equivalent to the interval-valued Shapley value \(\phi(\tilde{v})\).

**Proof** It has been proven that the A-T solution is equivalent to the Shapley value when the crisp cooperative game is a complete communication graph game [9], [12]. For interval-valued payoffs, we have
\[
\phi(\tilde{v}) = \phi(\tilde{w}), \quad \phi(\tilde{w}) = \phi(\tilde{v})
\]

Then there is \(\psi(\tilde{v}) = \psi(\tilde{w})(i = 1, 2, \ldots , n)\). Therefore, the intuitionistic fuzzy coalition A-T solution, \(A\bar{T}(\tilde{v})\) is equivalent to the fuzzy Shapley value \(\phi(\tilde{v})\).

**V. PROFIT ALLOCATION STRATEGY DECISION OF RESTRICTED COOPERATION AND COMPARISON**

A. PROFIT ALLOCATION STRATEGY DECISION OF RESTRICTED COOPERATION

Software cooperation is one of the important development models in the new era. Forming an alliance with appropriate partners and having a fair distribution mechanism are key issues. In a project of software cooperation, there is a knowledge value chain based on knowledge, skills and
capital investors, and the coalition formation is limited to the upstream and downstream of knowledge value chain. For conciseness, upstream, middle-stream and downstream are called player 1, player 2, and player 3. Current research into software cooperation is based on the assumption that players have equal power and can form coalitions freely [29]. However, this is untrue. Restricted cooperation in software cooperation is obvious, as different players have different marginal contributions. Due to the weak marginal revenue of players 1 and 3, this means that they cannot cooperate directly, but that player 1 must deal with player 2 first, then join the coalition with player 3 via the intermediary role of player 2. In another words, player 2 is the crucial player in this cooperation. The essence of this is a cooperative game with limited communication structure, and all the potential coalitions are \{1,2\}, \{2,3\}, \{1,2,3\}. This phenomenon is particularly common in social and economic cooperation, such as watershed governance and supply chain management. In this cooperation, there are upper, middle and lower reaches, the upstream and downstream cannot communicate directly due to restricted communication, and they must turn to the key player, middle reach, in order to create an alliance. With restricted cooperation, the alliance structure of cooperation and players’ location within the structure are incredibly different from traditional cooperation.

In other words, if three players participate fully in a restricted cooperation as above, everyone can invest 100% of the required resources into the alliance, i.e. crisp cooperation, then the profits of crisp coalitions are: \( v(\{1\}) = 20, v(\{2\}) = 10, v(\{3\}) = 20, v(\{1,2\}) = 60, v(\{2,3\}) = 50, v(1,2,3) = 70. \) But due to limited capacity or resources, or in order to reduce investment risk, the three players do not fully participate in the coalition.

The intuitionistic fuzzy sets of fuzzy coalitions are \( \tilde{A}_1 = \langle 1; 0.1, 0.5 >, \tilde{A}_2 = \langle 2; 0.3, 0.4 >, \tilde{A}_3 = \langle 3; 0.5, 0.3 >, \) where player 1 invests at least 10% into the coalitions, 50% must not be invested, while \( \pi_2(1) = 1 - \mu_2(1) - \nu_2(1) = 40\% \) is his/her hesitation degree. Therefore, the minimum participation degree of player 1 is 0.1, and the maximum participation degree is 0.5. Similarly, the intuitionistic fuzzy coalition of players 2 and 3 can be explained.

According to Eq. (4), the characteristic functions of an intuitionistic fuzzy coalition \( \{1, 2, 3\} \) with \( \alpha = 1 \) are
\[
tv_{\alpha}(\{1, 2, 3\})
= \frac{1}{3} \sum_{h_{1} \in \tilde{A}_1} v([1, 2, 3]; h_{1})(h_{1} - h_{1-1})
= v([1, 2, 3]; h_{1})(h_{1} - h_{1-1}) + v([2, 3]; (h_{2} - h_{1})) + v([3]; h_{3} - h_{2})
= 70 \times (0.1 - 0) + 50 \times (0.3 - 0.1) + 20 \times (0.5 - 0.3) = 21
\]

for \( \alpha = 1 \) are
\[
tv_{\alpha}(\{1, 2, 3\}) = \frac{1}{3} \sum_{m=1}^{q_{m}} v([1, 2, 3]; q_{m})(q_{m} - q_{m-1})
= v([1, 2, 3]; q_{m})(q_{m} - q_{m-1}) + v([2, 3]; (q_{2} - q_{1})) + v([3]; q_{3} - q_{2})
= 70 \times (0.5 - 0) + 50 \times (0.6 - 0.5) + 20 \times (0.7 - 0.6) = 42
\]

In a similar way, we can obtain all the characteristic functions of intuitionistic fuzzy coalitions, as shown in Table 1.

| \( IF(N) \) | \( tv(\tilde{S}) \) |
|-----------------|-----------------|
| \( \tilde{S}_{(1)} = \langle 1; 0, 1, 0.5 >, 0, 0 \rangle \) | [2,10] |
| \( \tilde{S}_{(2)} = \langle 0, 0, < 3; 0.5, 0.3 > \rangle \) | [10,14] |
| \( \tilde{S}_{(3)} = \langle 2; 0.5, 0.4 >, < 2; 0.3, 0.4 > \rangle \) | [8,31] |
| \( \tilde{S}_{(2,3)} = \langle 0, < 2; 0.3, 0.4 >, < 3; 0.5, 0.3 > \rangle \) | [19,32] |
| \( \tilde{S}_{(2,3)} = \langle 1; 0, 1, 0.5 >, < 2; 0.3, 0.4 >, < 3; 0.5, 0.3 > \rangle \) | [21,42] |

In other words, the characteristic functions of intuitionistic fuzzy coalitions, as shown in Table 1. Then, by using the above characteristic functions of \( (N, \tilde{v}, L) \in \tilde{G}^3 \), we have
\[
\sum_{B \in B^L} \left( tv(B_1) - tv(B_1) \right) - \sum_{K \in \hat{C}^L(B_1 \setminus \{1\})} \left( tv(K) - tv(K) \right)
= 8 + 8 + 21 - 13 > 0
\]

In the same way, we can obtain all the characteristic functions of intuitionistic fuzzy coalitions, as shown in Table 1. Therefore, according to Eq. (7), we can obtain the lower bounds of \( \psi_{i}(\tilde{v}) \) \( (i = 1, 2, \cdots, n) \)
\[
\psi_{1}(tv) = \frac{1}{3} \sum_{B \in B^L} \left( tv(B_1) - tv(B_1) \right)
= \frac{1}{3} \left( tv([1]) + tv([1]) + tv([1, 2, 3]) - tv([2, 3]) \right) = 2
\]
\[
\psi_{2}(tv) = \frac{1}{3} \sum_{B \in B^L} \left( tv(B_2) - tv(B_2) \right)
= \frac{1}{3} \left( tv([1, 2]) - tv([1]) + tv([2, 3]) - tv([3]) \right)
+ \frac{1}{3} \left( tv([1, 2, 3]) - tv([1]) - tv([3]) \right) = 8
\]
\[
\psi_{3}(tv) = \frac{1}{3} \sum_{B \in B^L} \left( tv(B_3) - tv(B_3) \right)
= \frac{1}{3} \left( tv([2]) - tv([2]) + tv([2, 3]) - tv([3]) \right)
+ \frac{1}{3} \left( tv([2, 3]) - tv([2]) - tv([3]) \right) = 8
\]
\[ \frac{1}{3} \sum_{b \in \mathcal{B}} \left( v_D(B_3) - \sum_{K \in \mathcal{K}} v_D(K) \right) \]
\[ = \frac{1}{3} (v_D(\{3\}) + v_D(\{3\}) + v_D(\{1, 2, 3\}) - v_D(\{1, 2\})) = 11 \]

In the same way, we can obtain the upper bounds according to Eq. (8). Therefore, the intuitionistic fuzzy coalition A-T solution of \( \tilde{\nu} \in \mathcal{G} \) are AT(\( \tilde{\nu} \)) = ([2, 10], [8, 19], [11, 13]).

Especially, when the intuitionistic fuzzy sets of input resources are \( A_1 = < 1; 0.1, 0.9 >, A_2 = < 2; 0.3, 0.7 >, A_3 = < 3; 0.5, 0.5 >, A_1 = < 1; 0.1, 0.5 > \), the intuitionistic fuzzy coalitions degenerate to normal fuzzy coalitions due to \( \mu \tilde{\nu}(i) + v_2(i) = 1 \). Then the allocation by the proposed intuitionistic fuzzy coalition A-T solution is AT(\( \tilde{\nu} \)) = ([2, 2], [8, 8], [11, 11]). It is obvious that the proposed A-T solution of the restricted coalition cooperative game is a more general method.

**B. COMPARISON WITH ALLOCATION BASED ON INTERVAL SUBTRACTION**

In order to show the applicability and superiority of the proposed method, we compare the allocations based on Moore’s interval subtraction [26], the partial subtraction operator [25] and Hukuhara subtraction [24].

By using the Moore's interval subtraction, i.e., \( \tilde{a} - \tilde{b} = [a_D - b_R, a_R - b_D] \), we directly have

\[ \psi_1(\tilde{\nu}) = \frac{1}{3} (\{2, 10\} + \{2, 10\} + \{(21, 42) - [19, 32]\}) \]
\[ = [-7/3, 43/3] \]

If using the partial subtraction operator, \( \psi_3(\tilde{\nu}) \) cannot be calculated because \([21, 42] - [8, 31] = [13, 11] \) does not satisfy \( a_R - a_D \geq b_R - b_D \). And it is irrational that the lower bound is larger than the upper bound, which conflicts with the notation of intervals.

According to the rule of interval addition, the Hukuhara subtraction cannot be used to calculate the above \( \psi_3(\tilde{\nu}) \) due to the fact that the interval numbers \([21, 42] \) and \([3, 31] \) cannot satisfy the condition \( a_D - b_R \geq a_D - b_D \).

In this example, for clarity, the allocation values of players based on different subtraction are shown in Table 2.

| \( \tilde{\nu}_{(1,2,3)} \) | Proposed method | Moore subtraction | Partial subtraction | Hukuhara subtraction |
|-----------------|-----------------|-------------------|--------------------|---------------------|
| \( \psi_1(\tilde{\nu}) \) | [2,10] | [210] | [2,10] | [2,10] |
| \( \psi_2(\tilde{\nu}) \) | [8,19] | [0,13] | [8,19] | [8,19] |
| \( \psi_3(\tilde{\nu}) \) | [11,13] | [10,3,6/3] | / | / |

It can be observed that the conditions of the partial subtraction and the Hukuhara subtraction are similar to Eq. (10), and it is always useless when the conditions cannot be satisfied. But the condition given by Eq. (6) is weaker than Eq. (10). That is to say, if Eq. (10) is satisfied, then Eq. (6) is always true. Therefore, if the intuitionistic fuzzy coalition A-T solution can be calculated by using the partial subtraction operator or Hukuhara subtraction, it can then be calculated using this proposed method. In other words, both subtractions are special cases of the proposed method in this paper.

**C. COMPARISON WITH SHAPLEY VALUE**

To highlight the superior applicability and effectiveness of the profit allocation for restricted coalitions, it is compared with the fuzzy Shapley value. In this case, there is no coalition between player 1 and player 3, if we hypothesize that a coalition exists, and its payoff equals the sum of independent payoff of each player. Based on this hypothesis, the coalition payoff with player 1 and player 3 can be given \( \tilde{\nu}(1, 3) = \tilde{\nu}(1) + \tilde{\nu}(3) = [12, 24] \). The allocation strategy for each player can then be obtained from the Shapley value in Table 3.

| \( \tilde{\nu} \) | Player 1 | Player 2 | Player 3 |
|-----------------|-----------|-----------|-----------|
| \( \psi_1(\tilde{\nu}) \) | [2,10] | [8,19] | [11,13] |
| \( \psi_2(\tilde{\nu}) \) | [5,2, 25/2] | [13/2, 87/6] | [12, 15] |

Comparing the interval-valued A-T solution with the interval-valued Shapley value, we can obtain \( \psi_1(\tilde{\nu}) < \psi_1(\tilde{\nu}) \), \( \psi_2(\tilde{\nu}) > \psi_2(\tilde{\nu}) \), \( \psi_3(\tilde{\nu}) < \psi_3(\tilde{\nu}) \). That is to say, the allocation of player 2 increases while the allocations of player 1 and player 3 both decrease according to the A-T solution, relative to the Shapley value. This is a result of highlighting the special status of player 2, which is indispensable in restricted coalition cooperative games with restricted communication structure. In the Shapley value, it is not objective or scientific enough that we suppose the unrealistic coalition (e.g. the coalition \{1, 3\}) is existing and use its payoff value in the cooperative game. In addition, we have demonstrated that players' profitability depends not only on their marginal degree of contribution to the coalition, but also on the communication structure of the coalition and players' positions in the cooperation. Therefore, the interval-valued A-T solution is more reasonable and appropriate than the interval-valued Shapley value in cooperative games with limited communication structure.

**VI. CONCLUSIONS**

In reality, cooperative games with restricted coalition and fuzzy information are very common. As A-T solution is an important single-valued solution in limited communication structure cooperative games, we propose the fuzzy A-T solution of restricted coalition cooperative games with intuitionistic fuzzy coalitions. The proposed method can take into account players' judgments and hesitations in cooperation. It is worth pointing out that the intuitionistic fuzzy coalition A-T solution is a generalized form of that of a classical cooperative game, and it extends the fuzzy cooperative game theory. To solve the intuitionistic fuzzy coalition A-T...
solution, the upper and lower limits of characteristic functions for the cooperative game are first obtained based on Choquet integrals and confidence levels. Then, a simplified method for the A-T solution is proposed, which can effectively avoid the irreversible interval subtraction. With this method, the upper and lower bounds of the A-T solution can be directly calculated from the upper and lower bounds by proving that the restricted coalition cooperative game is monotonic. Furthermore, we prove that the intuitionistic fuzzy coalition A-T solution satisfies several important and useful properties. In addition, the applicability and superiority of the proposed approach is demonstrated by comparing it with partial subtraction operator, Moore subtraction and Hukuhara subtraction. The method and model proposed in this paper can describe the restrictiveness and fuzziness of cooperation, and they are more practical and universal for allocation decision-making.

CONFLICTS OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this article.

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