Characterisation of Surface Roughness for Ultra-precision Freeform Surfaces

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Abstract. Ultra-precision freeform surfaces are widely used in many advanced optics applications which demand for having surface roughness down to nanometer range. Although a lot of research work has been reported on the study of surface generation, reconstruction and surface characterization such as MOTIF and fractal analysis, most of them are focused on axial symmetric surfaces such as aspheric surfaces. Relative little research work has been found in the characterization of surface roughness in ultra-precision freeform surfaces. In this paper, a novel Robust Gaussian Filtering (RGF) method is proposed for the characterisation of surface roughness for ultra-precision freeform surfaces with known mathematic model or a cloud of discrete points. A series of computer simulation and measurement experiments were conducted to verify the capability of the proposed method. The experimental results were found to agree well with the theoretical results.

1. Introduction

Due to the geometrical feature and functional speciality, ultra-precision freeform surfaces are widely used in advanced optics applications for various industries such as aviation, space flight, manufacturing, telecommunication and other technology [1]. Ultra-precision freeform surfaces are complex surfaces which possesses non-rotational symmetry. To meet the optical functional performance of the advanced optical applications, it imposes stringent surface roughness requirement in the fabrication of ultra-precision freeform surfaces. This demands for having surface roughness down to nanometer range.

However, there are currently lack of definitive international standards and techniques for the characterization of surface roughness of ultra-precision freeform surfaces. Although some surface parameters have been proposed by the ISO project, their applicability of practical inspection is limited to optical surfaces possessing rotational symmetry. Moreover, most of the previous research work has been focused on the study of surface generation and reconstruction [2]. Relative little research work has been reported on the development robust methods for the characterization of surface roughness for ultra-precision freeform surfaces. As a result, this paper presents a novel Robust Gaussian Filtering (RGF) method for the characterization of surface roughness of ultra-precision freeform surfaces. The RGF method is verified through a series of simulation and measurement experiments.
2. Novel RGF algorithm for the characterization of surface roughness

Surface is integrally determined by form and micro-geometrical structures with different dimensions, such as roughness, waviness and random features, which directly affect the functional properties and surface quality of ultra-precision freeform surfaces [3]. Before separating these micro-geometrical surface components with filtering, it is important to firstly remove local form and form error.

2.1 Form removal

In general, ultra-precision freeform surfaces are generated based on such two requirements as the known surface model and the given discrete data because of the geometric complexity. Therefore, surface characterization of ultra-precision freeform surface is realized in the different ways according to the different generated surface patterns.

Assume a 3D ultra-precision freeform surface is \( z(x, y) \), the residual surface \( z'(x, y) \) is

\[
z'(x, y) = \rho \left( z(x, y) - f(x, y) \right)
\]

(1)

after the form component \( f(x, y) \) is removed based on the minimum function \( \rho \).

For the surface with known model, \( f(x, y) \) is the known mathematic expression. For the surface with unknown surface model, \( f(x, y) \) is the cubic B-spline fitted surface based on a cloud of known discrete points as described in Equation (2).

\[
f(x, y) = \sum_{i=0}^{m} \sum_{j=0}^{n} d_{i,j} \phi_{i,j}(x) \phi_{i,j}(y)
\]

(2)

where \( d_{i,j} \) are the control points, and

\[
\phi_{i,j}(x) = \frac{1}{6} \sum_{k=0}^{m+1} (-1)^k \binom{m+1}{k} \left( \frac{x - x_0}{\Delta x} + 2 - k \right)_+^m
\]

(3)

where \( x_0 \) and \( \Delta x \) are the original point and the sampled interval, respectively.

2.2 Robust Gaussian Filtering

After form removal, the residual surface \( z'(x, y) \) is separated robustly in the whole measured length into two parts which include the low frequency reference waviness \( w(x, y) \) and the high frequency roughness \( r(x, y) \).

\[
w(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_b(x, y) z'(x, y) q(x, y) dxdy
\]

(4)

in which the modified dynamic weighting function is used to remove the boundary effect

\[
h_b(\xi, \eta) = h(\xi, \eta) / \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\xi, \eta) d\xi d\eta
\]

(5)

and the Gaussian weighting function

\[
h(x, y) = \frac{1}{\alpha \lambda_{xc} \lambda_{yc}} \exp\left(-\frac{\pi \alpha^2}{\beta \lambda_{xc} \lambda_{yc}} \left( \frac{x^2}{\lambda_{xc}^2} + \frac{y^2}{\lambda_{yc}^2} \right) \right)
\]

(6)

where \( \lambda_{xc0} \) and \( \lambda_{yc0} \) are the cut-off wavelength in \( x \) and \( y \) directions, respectively; \( \alpha = \sqrt{\ln 2 / \pi} \), \( \beta = \alpha^2 \). The robust weighting function \( q(x, y) \) is defined as [4]:

\[
q(x, y) = \begin{cases} 
1 & |d_x| \leq a \\
\frac{a}{|d_x|} & |d_x| \in (a, b] \\
\frac{b}{|d_x|^2} & |d_x| \in (b, c] \\
0 & |d_x| > c
\end{cases}
\]

(7)
where the following requirements are met: \( dv = d/s \) and \( d = z(x, y) - w(x, y) \), the scale parameter \( s = \text{median}[d - \text{median}(d)] \), the trimmed coefficients \( a, b \) and \( c \). So the 3D robust roughness is

\[
    r(x, y) = zf(x, y) - w(x, y)
\] (8)

The 3D discrete surface is obtained with the sampling interval \( \Delta x \) and \( \Delta y \) in \( x \) and \( y \) directions, respectively. The discrete Robust Gaussian Filtering can be expressed as follows:

\[
    w(x, y,j) = \sum_{g \in G} \sum_{k \in K} h_k [(i-k)\Delta x, (j-g)\Delta y] \cdot z(k \cdot \Delta x, g \cdot \Delta y) \cdot q(k \cdot \Delta x, g \cdot \Delta y) \cdot \Delta x \cdot \Delta y
\] (9)

To analyze the entire measured surface, the subscript \( k_1 \) and \( k_2 \) should be chosen according to the following rules:

\[
    \begin{align*}
    k_1 &= -i; k_2 = m & (i = 1, 2, \cdots, m - 1) \\
    k_1 &= -m; k_2 = m & (i = m, m + 1, \cdots, M - m) \\
    k_1 &= -m; k_2 = M - i & (i = M - m + 1, M - m + 2, \cdots, M)
    \end{align*}
\] (10)

in which \( m \) is half of the Gaussian window width and \( M \) is the number of sampled points in \( x \)-direction, respectively. The other subscripts \( g_1 \) and \( g_2 \) are obtained by obeying the same rule.

3. Computer Simulation and Experimental Analysis

3.1 Computer simulation

Before the practical case is studied, it is useful to examine the proposed method with the ideal freeform surface as shown in Figure 1 that consists of the form surface \( f = \sin(0.2x) + \cos(0.3y) \) and micro-structural synthetic sinusoidal waveforms.

![Figure 1. Original simulated surface](image)
![Figure 2. Form surface](image)

Figure 1. Original simulated surface   Figure 2. Form surface

![Figure 3. Waviness surface](image)
![Figure 4. Roughness surface](image)

Figure 3. Waviness surface   Figure 4. Roughness surface

A MATLAB program is purposely built for implementing the computer simulation and the novel RGF algorithm. Figure 2 shows the removed form surface with the known mathematical model. The waviness and roughness surface extracted with RGF are shown in Figures 3 and 4, respectively. As compared with the original simulated surface, several surface components are separated effectively according to their differences in frequency ranges.
3.2 Case Study
An ultra-precision freeform surface is machined by a five-axis ultra-precision freeform machine (Freeform 705G). The ideal design surface is described in Equation (11). The Form Talysurf PGI 1240 Freeform Measurement System is used to measure the original 3D freeform surface as shown in Figure 5. The sampled area is \((2 \times 2) \text{ mm}^2\).

With the use of the RGF characterization technique, the frequency-based separation of surface components of the ultra-precision surface has been carried out. As shown in Figure 6, the form surface is successfully extracted by the form removal operation with known mathematical model. Then the RGF method is used to separate the waviness and roughness components. The robust processing results are illustrated in Figures 7 and 8. It is found that the reference waviness surface is rather smooth and the high-frequency component is extracted and kept in the roughness surface.

4. Conclusions
A novel RGF method is proposed to characterize surface roughness of ultra-precision freeform surfaces. The freeform surface can be built either from known mathematic model or a cloud of discrete points. Computer simulation and measurement experiments are performed to verify the RGF method. The experimental results indicate that the RGF method can efficiently separate the surface components from ultra-precision freeform surfaces with different frequencies and dimensions on the whole measured area without any information loss. The present study is of prime importance in the advancement of the freeform surface metrology.

![Figure 5. Original measured surface](image1)

![Figure 6. Form surface](image2)

![Figure 7. Waviness surface](image3)

![Figure 8. Roughness surface](image4)

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