SU(3) glueballs on coarse, anisotropic lattices

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Results from a calculation of the low-lying glueball spectrum of pure-gauge SU(3) are used as a test of the effectiveness of improved discretisation schemes in reducing finite spacing errors. Glueball masses are extracted from simulations on anisotropic lattices, where the temporal lattice spacing is much shorter than the spatial ones. This allows clearer resolution of the decay of glueball correlators.

1. INTRODUCTION

Current interest in the use of Symanzik’s improvement scheme, supplemented by mean-field link renormalisation, to provide a better discretisation scheme for QCD has stemmed from recent successes of coarse lattice simulations. Coarse lattices offer the advantage of a significant reduction in computational overheads, since the approach to the continuum limit is accompanied by a rapid increase in CPU time requirements for Monte Carlo calculations. The improvement scheme allowed, for example, calculations of the static potential at lattice spacings up to $\approx 0.4$ fm with discretisation errors of only 5%. An attempt to use these actions on coarse lattices to examine the glueball spectrum of QCD proved difficult, however. The QCD glueballs are massive states and their evaluation on the lattice is notoriously difficult due to the large vacuum fluctuations of glueball operators. On a lattice with coarse temporal discretisation, the glueball correlator can barely be resolved before it is lost in the noisy vacuum. The use of a large operator basis set to optimise the ground state overlap was hampered by a technical difficulty caused by the $2 \times 1$ rectangle in the improved action. This term couples next-to-nearest neighbouring time-slices and thus modifies the transfer matrix, making it non-hermitian (physically, this term leads to the presence of unwanted extra modes in the improved gluon dispersion relation). A hermitian, positive transfer matrix is required for the validity of the variational calculation employed and to ensure that the effective mass approaches its asymptotic value from above. These problems naturally suggest breaking the explicit Euclidean symmetry of the action and using a different lattice spacing in the spatial and temporal directions. With a shorter temporal lattice spacing, clear evidence for plateaux in the effective masses can be seen before the signal is lost. Once discretisations of this form are used, a solution to the transfer matrix problem arises too.

In this article, a study of three of the lighter glueball states at six different lattice spacings is discussed. For all but the scalar glueball, the results show the expected scaling behaviour.

2. ANISOTROPIC LATTICE QCD

The improvement scheme can be extended readily to anisotropic lattices. In this study, the following action was used:

$$S_{II} = -\beta \sum_{x,s,s'>x} \frac{a_t}{a_s} \left\{ \frac{5}{3} \frac{P_{ss'}}{u_s^4} - \frac{1}{12} \frac{R_{ss'}}{u_s^6} - \frac{1}{12} \frac{R_{ss's}}{u_s^6} \right\}$$

$$-\beta \sum_{x,s} \frac{a_s}{a_t} \left\{ \frac{4}{3} \frac{P_{st}}{u_s^2 u_t^2} - \frac{1}{12} \frac{R_{st}}{u_s^2 u_t^2} \right\},$$

with $P_{\mu\nu}$ the plaquette in the $\mu-\nu$ plane, and $R_{\mu\nu}$ the $2 \times 1$ $\mu-\nu$ plane rectangle (with longest side parallel to the $\mu$ axis). This action, intended for use with $a_t \ll a_s$, has $O(a_s^4, a_t^2)$ discretisation errors. The coefficients were determined using tree-
level perturbation theory and tadpole improvement \cite{4}. The spatial mean link, \( u_s \), was tuned such that the input value in the action matched its measured value as defined by \( u_s = \langle P_{ss'} \rangle^{1/4} \). This tuning is rapidly convergent and requires a minimal amount of work. The temporal mean link, \( u_t \), was fixed to \( u_t = 1 \), since its value in Landau gauge differs from unity by \( O(a_t^2/a_s^2) \).

3. SIMULATION DETAILS

A set of six simulations were performed at fixed anisotropy, \( a_t/a_s = 1/3 \), with the intention of examining the scaling behaviour of the glueball masses using this action. The run parameters are given in Table 1. Configurations were generated using the Cabibbo-Marinari (CM) pseudo-heatbath and Creutz over-relaxation (OR) methods. In our previous coarse lattice calculation \cite{2}, large statistical samples were gathered in an attempt to resolve the glueball correlators. For this work, much smaller statistical samples were required and the largest data set collected (for the coarsest lattice) consisted of 100 bins, with each bin containing an average of 100 measurements, each measurement separated by 3 sweeps of a 1-3 hybrid CM/OR update. This is to be contrasted with the 334 bins of 1000 measurements from the old calculation, which yielded far less information.

3.1. Setting the scale

The lattice spacing was set by fitting the on-axis static potential to a standard ansatz,

\[
V(r) = V_0 - \frac{e}{r} + \sigma r, \tag{2}
\]

then using the fit to determine the hadronic scale, \( r_0 \), defined as \( r^2dV/dr = 1.65 \) at \( r = r_0 \), which corresponds roughly to \( r_0 \approx 0.5 \) fm. This definition gives a lattice scale that is less dependent on the choice of potential fit ansatz than the string tension scale. The results of this determination are given in Table 1. In all these calculations, the input ratio \( a_t/a_s \) has been used since its renormalisation is known to be small \cite{4}. In all runs, the lattice volumes were chosen to be large enough such that the masses of the scalar and tensor states should lie within 1% of their infinite volume limits as determined by Lüscher's finite volume analysis \cite{5}.

3.2. Optimising the ground state overlap

The low signal-to-noise ratio of glueball calculations makes it crucial to find lattice operators with good ground state overlaps. Large bases of at least 24 operators were used and optimal correlators were determined using the variational method. The glueball operators were built from spatial Wilson loops constructed from either APE smeared or Teper fuzzed links \cite{7}. As in previous investigations, the smearing and fuzzing techniques were found to be far more efficient when they included a gauge-invariant projection of the new links back onto SU(3). In most cases, the APE smeared operators gave larger contributions to the ground state; the glueballs extend over a small number of sites of the lattice, and thus operators built from fuzzed “superlinks” are too large. At most, one iteration of Teper fuzzing proved useful. In all cases, these procedures gave excellent ground state overlaps and in many cases, the overlaps were statistically consistent with unity.

4. RESULTS

4.1. The scalar glueball

The effective mass for the scalar glueball for one of the simulations performed is shown in Figure 1. For this example, a correlator signal can be seen out to time-slice 7, and a good single exponential fit in the range 1 – 7 can be performed. A signal for the first excited state can be obtained.

| \( \beta \) | Lattice | \( u_s \) | \( a_s \) (fm) | \( L \) (fm) |
|---|---|---|---|---|
| 1.7 | 6\(^3\) \times 18 | 0.745 | 0.457(4) | 2.7 |
| 1.9 | 6\(^3\) \times 18 | 0.764 | 0.414(3) | 2.5 |
| 2.0 | 8\(^3\) \times 24 | 0.772 | 0.387(3) | 3.1 |
| 2.2 | 8\(^3\) \times 24 | 0.789 | 0.329(2) | 2.6 |
| 2.4 | 8\(^3\) \times 24 | 0.806 | 0.272(4) | 2.2 |
| 2.6 | 10\(^3\) \times 30 | 0.819 | 0.217(3) | 2.2 |
from the first excited variational eigenvector. The mass of this state is slightly less than twice that of the ground state, consistent with an excited state observed in Wilson action calculations [8]. The dependence on the lattice spacing is shown in Figure 3. The scalar mass shows a significant "dip" as the lattice spacing is increased, at the bottom of which the glueball is about 70% of its continuum value (from Wilson data). This dip is caused possibly by the presence of a critical point in the fundamental/adjoint plane, similar to that which affects the scalar glueball mass calculated using the Wilson action [9]. No continuum extrapolations to the scalar data have been included.

4.2. Other glueball states

Simulation data for the tensor (2++) and pseudovector (1+−) states were also examined. The β = 2.4 effective masses for the three lattice irreps (the T_2^{++} and E^{++} for the tensor and the T_1^{-+} for the pseudovector) with these continuum quantum numbers are shown in Figure 2. Again, reliable single exponential fits to these correlators can be performed over a range of time-slices. The scaling behaviours of these states are included in Figure 3. For the E^{++} and T_1^{-+} channels, no scaling violations are seen while the T_2^{++} mass does show some cut-off dependence. At the largest lattice spacing, this leads to a 15% split in the masses, caused by rotational invariance breaking. The cut-off dependence is consistent with the anticipated leading discretisation error, proportional to a^4 (however other functional forms can not be ruled out). Extrapolations to the continuum limit are summarised in Table 2. Here, the simplest functional form that gives a reasonable $\chi^2$ is used in the extrapolations. Note that in the continuum limit, the masses of the two tensor irreps are in excellent agreement, suggesting restoration of rotational symmetry.

5. CONCLUSIONS

The simulations discussed here clearly demonstrate the advantages of using anisotropic actions to study the glueball spectrum of QCD. Mean
The lattice irreps $A_{1}^{++}$, $E^{++}$, $T_{2}^{++}$ and $T_{1}^{+-}$ are labelled $\circ$, $\Box$, $\Diamond$ and $\ast$, respectively. Crosses indicate Wilson action data from [8]. The solid line is a fit to the scalar glueball Wilson data, the dashed lines are fits to the leading scaling behaviour from the improved action (see Table 2).

Except for the scalar glueball, all the states demonstrate the expected scaling behaviour for this action. Using $1/r_0 = 372$ MeV to set the scale gives a tensor mass of $2140 \pm 45$ MeV (from the $E^{++}$ irrep) and a pseudovector mass of $2620 \pm 60$ MeV. The errors quoted are purely statistical. In the case of the scalar, the improvement has reduced the cutoff contamination significantly.

Table 2

| Fit type | $\chi^2$/dof. | $m_g r_0$(continuum) |
|----------|---------------|-----------------------|
| $E^{++}$ | A 0.52        | 5.74 $\pm$ 0.12       |
| $T_{2}^{++}$ | B 0.38     | 5.77 $\pm$ 0.08       |
| $T_{1}^{+-}$ | A 0.29      | 7.04 $\pm$ 0.17       |

Alternative improved actions which could have less contamination from the adjoint/fundamental fixed point are under investigation. With the success in extracting the continuum masses of the three states considered, an extended calculation to compute the masses of the lightest states in all 20 lattice irreps seems feasible, perhaps using a higher anisotropy.

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