Visual analysis of electrical signals

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Abstract. The sinusoidal spectrum of electrical signals is analyzed by using The Fourier series method and the Fast Fourier change method. Amplitude-frequency characteristic diagram and phase-frequency characteristic diagram are obtained respectively. The amplitude-frequency characteristic and the phase-frequency characteristic are superimposed in time domain and compared with the original signal. The analysis uses fast Fourier transform to analyze the minimum sampling frequency of electrical signals. The GUI module in MATLAB is used for programming, which makes the experimental parameters easy to modify. A more general conclusion can be drawn. Through experiments, we find that when the sampling frequency is more than twice the cut-off frequency of the original periodic signal, the fast Fourier transform method can better restore the original signal. When the sampling frequency is small, the signal will be seriously distorted. The amplitude-frequency and phase-frequency characteristics obtained by Fourier series can better restore the original signal when the harmonic frequency is large, but it is different from the original signal to some extent.

Keywords: Fourier series, Fast Fourier transform, Cutoff frequencies, Sampling frequency.

1. Introduction
We can't live without signal processing. In real life, we often encounter periodic or aperiodic time domain signals. With the rapid development of the computer, we usually need to change the analog signal into digital signal, and then use the computer for digital processing, including frequency domain transformation, digital filtering, recognition, synthesis and so on. However, when continuous time signal sampling becomes digital signal, if the sampling frequency is wrong, serious waveform distortion will occur. In this paper, the condition of waveform is verified by simulation experiment.

2. Fourier series analysis of periodic signals
Any periodic signal can be represented by its DC component, fundamental component and each harmonic component. The time domain analysis of a periodic signal can be transformed into the frequency domain analysis of each frequency component of the signal. These frequency components are called the frequency spectrum of periodic signals. The graph of each frequency component drawn with angular frequency as abscissa is called spectrum diagram. The spectrum diagram drawn with the amplitude of each frequency component is called amplitude spectrum diagram, also known as
amplitude-frequency characteristic. The spectrum diagram drawn with the initial phase of each frequency component is called phase spectrum, also known as phase frequency characteristic. For periodic time domain signals, we can analyze them by means of Fourier series method. In other words, the amplitude-frequency characteristic and phase-frequency characteristic of the signal are obtained by the analytic method of the calculation formula.

2.1. A subsection
When the time domain signal is periodic, the harmonic and DC components of the signal can be calculated by the following formula. The amplitude-frequency and phase-frequency characteristics of the signal can be obtained through analysis.

\[
a_0 = \frac{1}{T_0} \int_{T_0}^{T_0} f_p(t) dt
\]

\[
a_k = \frac{2}{T_0} \int_{T_0}^{T_0} f_p(t) \cos k\Omega_0 dt
\]

\[
b_k = \frac{2}{T_0} \int_{T_0}^{T_0} f_p(t) \sin k\Omega_0 dt
\]

And by the formula we can see that when the signal is symmetric in the time domain, the sine component of the signal is equal to zero. In order to simplify the solution, periodic square wave signals and periodic triangular wave signals are selected as modulation signals for analysis. When the signal is of these two kinds, the signal parameters are further simplified due to the symmetrical time axis in the time domain. When the signal is a periodic square wave signal, the formula for solving the Fourier series is reduced to the following formula.

\[
a_0 = \frac{\tau}{T_0} A, a_k = \frac{\tau\Omega_0}{\pi} A \left| \text{Sa} \frac{\tau k\Omega_0}{2} \right| \phi_k = \arg \left( \text{Sa} \frac{\tau k\Omega_0}{2} \right)
\]

When the signal is a periodic triangular wave, the formula for solving the Fourier series is reduced to the following formula.

\[
a_0 = \frac{\tau}{T_0} A, a_k = \frac{\tau\Omega_0}{\pi} A \left| \text{Sa} \frac{\tau k\Omega_0}{2} \right| \phi_k = \arg(a_k)
\]

We can see from the formula that when the signal is a triangular wave signal, the phase angle is always equal to zero because the amplitude of the signal is real. Means that the harmonic phase of the signal is equal, that is, there is no phase difference.

2.2. Use Matlab to analyze the signal
In MATLAB, sinusoidal spectrum analysis method is used to analyze periodic rectangular pulse signal, triangular wave signal and other periodic signals. The analytic formula is programmed, and the program can process periodic rectangular pulse signal, triangular wave signal and other periodic signals. Because this paper is a spectrum analysis of electrical signals, the spectrum selected is sinusoidal spectrum, which contains only positive frequency but not negative frequency. Due to the limited computing power of computer, it is impossible to calculate all harmonic components. This paper selects the first eight harmonics for analysis. By analyzing the two periodic time domain signals respectively, the following spectrum and phase frequency characteristic graphs are obtained.
The spectrum of periodic signal is discrete, and the interval of two adjacent spectral lines is the fundamental wave angular frequency of periodic signal. The larger the period of periodic signal is, the smaller the interval of adjacent spectral lines is, and the denser the spectral lines are. The fundamental frequency selected for this article is 1Hz. The spectrum of the periodic signal contains an infinite number of spectral lines, which indicates that the periodic signal contains an infinite number of frequency components. The high frequency component tends to decay. The frequency band width of the periodic signal is usually defined as the frequency range from zero frequency to a certain frequency. The signal with limited bandwidth is called spectrum limited signal and band limited signal. We usually consider the cut-off frequency of the periodic time domain signal as the first zero crossing. According to the amplitude-frequency and phase-frequency characteristics of periodic square wave signal, the cut-off frequency of periodic square wave signal is 4Hz, and that of periodic triangular wave signal is 2Hz.

2.3. Compare the acquired spectrum characteristics with the original signal
The amplitude-frequency and phase Angle of each harmonic of the periodic signal have been solved in the previous step. The frequency and phase Angle of the harmonics are substituted into the cosine function to generate the time-domain signal of each harmonics, and the function of the original signal and each harmonic component is put into a THREE-DIMENSIONAL image to obtain the amplitude-frequency and phase-frequency composite graph. When the duty ratio is 25% and the frequency is 1Hz, the dc component, one to eight harmonics and the original function image output are obtained as shown in the figure.

![Figure 1. Spectral characteristics of square waves](image1)

![Figure 2. Triangular wave spectrum characteristics](image2)
In the previous step, the amplitude-frequency and phase Angle of the harmonic of periodic triangular waves have been solved. The frequency and phase Angle of the harmonics are substituted into the cosine function to generate the time-domain signals of each harmonics, and the functions of the original signal and each harmonic component are put into a THREE-DIMENSIONAL image to obtain the amplitude-frequency and phase-frequency composite graphs. When the period is 0.02s, take the DC component, one to eight harmonics and the output of the original function image as shown in the figure.

And by the nature of the Fourier series we know that if you add up all the harmonics of the signal you can theoretically restore the original signal perfectly. This is very important in practical applications. The DC components and harmonics solved in the previous section are superposed according to the form of Fourier series, and the formula is

$$f_p(t) = c_0 + \sum_{k=1}^{8} c_k \cos(k\Omega_0 t + \phi_k)$$

The expanded function can be obtained, and the fitted expanded image can be compared with the original function image, as shown in the figure.
Figure 5. Square wave expansion fitting diagram

In the same way, the expansion fitting diagram of periodic triangular wave signal and the comparison diagram of original function can be obtained.

Figure 6. Triangular wave expansion fitting diagram

By comparing the original signal with the expanded fitting diagram, it can be found that the expanded fitting diagram basically conforms to the original signal time domain diagram, but due to the limited number of harmonics to be expanded, there is a small fluctuation in the expanded fitting diagram. When the number of unrolled harmonics increases further, the fluctuation will decrease. When the number of unrolled harmonics is infinite, the theoretical unrolled fitting diagram is completely consistent with the time-domain diagram. It is proved that the analysis of periodic signal by Fourier series method is effective.

3. Fast Fourier transform of the signal

For time domain signals, we often do not know the period of the signal, so it is difficult to analyze the signal by Fourier series. For a general signal, we usually extend the signal and take the Fourier transform. Since computers cannot process continuous signals, they can only process discrete signals. So we sample the original signal into a discrete sequence of signals. Then the spectrum characteristics of the signal sequence are calculated.

There are FFT integration functions in MATLAB, no need to program yourself. Just call the function correctly. However, if the sampling frequency is too low, there will be a large error with the original
signal. According to the sampling theorem, the sampling frequency is required to be greater than twice the cut-off frequency of the original signal. By programming, the spectrum characteristics of the sinusoidal spectrum are shown in the figure.

![Spectral characteristics of square waves](image1)

**Figure 7.** Spectral characteristics of square waves

By analyzing the image, it can be concluded that the amplitude-frequency characteristics of the periodic square wave are symmetric with respect to \((n-1)/2\) pair, and the phase-frequency characteristics are symmetric with respect to \((n-1)/2\) odd symmetry. The correctness of FFT is preliminarily verified. According to theoretical analysis, if the first half of the points obtained are synthesized by cosine series, the original signal should be approximately reflected. However, if the sampling frequency of the signal is less than two times of the original signal cutoff frequency, serious distortion will occur. The developed fitting diagram is far from the original signal.

![Square wave expansion fitting diagram](image2)

**Figure 8.** Square wave expansion fitting diagram

The cut-off frequency of this square wave is 4 times of the fundamental wave frequency, and the sampling frequency is 3 times of the fundamental wave frequency for testing. Get the image as shown in the figure.
By comparing the original signal with the fitted signal, it can be found that the fitted signal is quite different from the original signal. The signal can be considered distorted. After a few more experiments, we find that when the sampling frequency is more than twice the cut-off frequency, the fitted signal can better restore the original signal. It is concluded that the sampling signal should be more than twice the cut-off frequency of the original signal if the original signal is to be better recovered.

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