Role of system size on freezeout conditions extracted from transverse momentum spectra of hadrons

Ajay Kumar Dash,¹,² Ranbir Singh,¹,² Sandeep Chatterjee,² Chitrasen Jena,³ and Bedangadas Mohanty¹
¹School of Physical Sciences, National Institute of Science Education and Research, HBNI, Jatni, 752050, India
²AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, al. Mickiewicza 30, 30-059 Krakow, Poland
³Indian Institute of Science Education and Research, Tirupati, 517507, India

Abstract

The data on hadron transverse momentum spectra in different centrality classes of p+Pb collisions at √sNN = 5.02 TeV has been analysed to extract the freezeout hypersurface within a simultaneous chemical and kinetic freezeout scenario. The freezeout hypersurface has been extracted for three different freezeout schemes that differ in the way strangeness is treated: i. unified freezeout for all hadrons in complete thermal equilibrium (1FO), ii. unified freezeout for all hadrons with an additional parameter γs which accounts for possible out-of-equilibrium production of strangeness (1FO+γs), and iii. separate freezeout for hadrons with and without strangeness content (2FO). Unlike in heavy ion collisions where 2FO performs best in describing the mean hadron yields as well as the transverse momentum spectra, in p+Pb we find that 1FO+γs with one less parameter than 2FO performs better. This confirms expectations from previous analysis on the system size dependence in the freezeout scheme with mean hadron yields: while heavy ion collisions that are dominated by constituent interactions prefer 2FO, smaller collision systems like proton + nucleus and proton + proton collisions with lesser constituent interaction prefer a unified freezeout scheme with varying degree of strangeness equilibration.

I. INTRODUCTION

The knowledge of the surface of last scattering of the hadrons produced in a heavy ion collision (HIC) event is of utmost significance as it contributes to the calibration of the hadronic physics baseline to be contrasted with data to extract information of the quark gluon plasma (QGP) phase [1,2] as well as those of the QCD critical point [3,4]. The hadron resonance gas model has been the main phenomenological model to extract the freezeout hypersurface by comparing to the data of hadron yields [5,10] as well as spectra [11,14]. The surface where the hadrons cease to interact inelastically is known as the chemical freezeout surface. The hadrons freeze here. The surface where the hadrons cease to interact even elastically is known as the kinetic freezeout surface. The shape of the transverse momentum spectra of hadrons get fixed here. Depending on the model assumptions, the chemical and kinetic freezeout surfaces could be separate [13,17] or together [11,12,13,20]. In this study, we have worked with the THERMINATOR event generator where a combined freezeout of both yields as well as spectra at the same surface is implemented [21,22].

Traditionally, a single unified freezeout of all hadrons have been studied (1FO) [1,2]. However, the data from LHC have thrown open the interpretation of freezeout and several alternate schemes have been proposed [3,22]. In the standard picture, freezeout is interpreted as a competition between fireball expansion and interaction of the constituents. Thus it is natural to expect system size dependence in freezeout conditions, since constituent interactions decrease as one goes from nucleus-nucleus (A+A) to proton-nucleus (p+A) and proton-proton (p+p) collisions. On the contrary, it was found that 1FO provides equally good description of data on mean hadron yields of $e^+e^−, p+p$ and A+A [33]. This lack of sensitivity of the 1FO approach on the varying rate of interaction amongst the constituents and fireball expansion across system size raises doubt on the standard interpretation of freezeout as a competition between expansion and interaction.

In Ref. [34], the yield data was analysed within three different approaches: i. 1FO, ii. single unified freezeout of all hadrons with an additional parameter γs accounting for non-equilibrium production of strangeness (1FO+γs), and iii. separate freezeout surface for hadrons with and without strangeness content (2FO). The data on hadron yield was analysed across systems: p+p, p+Pb and Pb+Pb enabling one to study the freezeout condition for mid-rapidity charged particle multiplicity as well as the system volume varying over three orders of magnitude. It was found that while 1FO and 1FO+γs schemes are blind to system size, 2FO exhibits a strong system size dependence. While for central and mid-central collisions, 2FO provides the least chi-square per degree of freedom, for peripheral Pb+Pb to all centralities of p+Pb and min bias p+p, 1FO+γs provides a better description. This emphasizes a plausible freezeout scenario: in case of large system sizes, the freezeout dynamics is dominated...
by hadron interactions and hence flavor dependence in hadron-hadron cross sections play a role resulting in 2FO being the preferred freezeout scheme. On the other hand, in small systems the freezeout is mostly driven by rapid expansion and little interaction resulting in a sudden and rapid freezeout and hence disfavoring 2FO.

In this paper, we extend the above line of argument by studying the data on hadron spectra. The 2FO prescription has been already demonstrated to describe better the data on hadron spectra than 1FO in Pb+Pb at \( \sqrt{s_{\text{NN}}} = 2.76 \text{ TeV} \) \(^{[14]}\). Here we study the data on hadron spectra in p+Pb at \( \sqrt{s_{\text{NN}}} = 5.02 \text{ TeV} \) \(^{[35-37]}\) and finally connect to our previous findings with the spectra data in Pb+Pb \(^{[14]}\). The spectra of \( \pi^+ \pi^- \), \( K^+K^- \), \( p\bar{p} \), \( \phi \), \( \Lambda \bar{\Lambda} \), \( \Xi \bar{\Xi} \) and \( \Omega \bar{\Omega} \) are used for this study which are measured in the mid rapidity \( 0<\eta_{\text{cen}}<0.5 \) by the ALICE collaboration. We have performed the centrality dependence of this study by analyzing the data in seven centrality classes, 0-5%, 5-10%, 10-20%, 20-40%, 40-60%, 60-80% and 60-100%.

The paper is arranged in the following way. In Sec. II we discussed the model used for this study. The results from the model and data are compared in Sec. III. Finally we summarize our findings in Sec. IV.

II. MODEL

We have studied the data on hadron spectra in 3 schemes: 1FO, 1FO+\( \gamma_S \) and 2FO using the THERMINATOR event generator \(^{[21,22]}\). While 1FO is implemented in the standard version of THERMINATOR, in Ref. \(^{[14]}\) the standard version of THERMINATOR was extended to include the 2FO scheme. We now briefly describe the implementation of the freezeout hypersurface and the relevant parameters to be extracted in this approach.

The Cooper-Frye prescription provides the hadron spectra emanating from a freezeout hypersurface

\[
\frac{d^2N}{dy_{\text{pt}}dp_{\text{T}}} = \int d\Sigma \cdot pf(p \cdot u, T, \gamma_S, \mu)
\]  \hspace{1cm} (1)

where \( T \) is the temperature, \( \mu = \{\mu_B, \mu_Q, \mu_S\} \) refer to the three chemical potentials corresponding to the three conserved charges of QCD, \( u^\mu \) is the 4-velocity, \( d\Sigma \) is the differential element of the freezeout hypersurface over which the integration in Eq. (1) is supposed to be, \( p \) is the four momentum. There could be different choices for the parametrization of the freezeout hypersurface and \( u^\mu \). We have worked within the Krakow model \(^{[11]}\) whereby the freezeout is assumed to occur at a constant proper time \( \tau_f \)

\[
\tau_f^2 = t^2 - x^2 - y^2 - z^2
\]  \hspace{1cm} (2)

while \( u^\mu \) is chosen to be

\[
u^\mu = x^\mu/\tau_f
\]  \hspace{1cm} (3)

where \( (t, x, y, z) \) is the space-time coordinate.

THERMINATOR accounts for both primary production as well as secondary contribution from resonance decays when evaluating the distribution function \( f \). The integration in Eq. (2) occurs over the freezeout hypersurface coordinates, namely the spacetime rapidity \( \eta \) whose integration range is from minus infinity to plus infinity, the azimuthal angle \( \phi \) which is integrated from 0 to \( 2\pi \) and \( \rho = \sqrt{x^2 + y^2} \), the perpendicular distance between the \( Z \)-axis and the freezeout hypersurface. \( \rho \) is integrated from 0 to \( \rho_{\text{max}} \). Thus, we have 3 parameters within the 1FO scheme: \( T, \tau_f \) and \( \rho_{\text{max}} \) to be extracted by comparison with data.

The choice of the thermodynamic ensemble is a relevant topic whenever one discusses system size dependence. In p+p collisions at the highest SPS and RHIC energies, the use of canonical ensemble or strangeness canonical ensemble has been suggested \(^{[35-37]}\). At the LHC energies, grand canonical ensemble was found to work best in describing the hadron yields \(^{[40]}\). Similar recent studies on the role of thermodynamic ensemble in small systems can be found in Refs. \(^{[41,43]}\). Here, we work with the grand canonical ensemble as well. Since we work with the LHC data that shows a very good particle-antiparticle symmetry, we have set all the chemical potentials to zero. In 1FO+\( \gamma_S \), there is also the additional parameter \( \gamma_S \) in \( f \) that accounts for out-of-equilibrium production of strangeness. In 2FO, we have different parameter sets for parametrising the non-strange \( (T_{\text{ns}}, \tau_{f_{\text{ns}}} \) and \( \rho_{\text{max ns}} \) and strange \( (T_s, \tau_{f_s} \) and \( \rho_{\text{max s}} \) freezeout hypersurfaces.

III. RESULT

We have varied the \( T \) in the range from 145 to 162 MeV in the steps of 1–2 MeV whereas \( \rho_{\text{max}} \) and \( \tau_f \) are varied in the range 1.5 to 4.1 fm and 1.5 to 3.1 fm, respectively in steps of 0.1 fm. The goodness of the parameter set in describing the data is ascertained from the \( \chi^2/ndf \), where

\[
\chi^2 = \sum_i \left( \frac{\text{Data}(p_T_i) - \text{Model}(p_T_i)}{\text{Error}(p_T_i)} \right)^2
\]  \hspace{1cm} (4)

and \( ndf = \text{Number of data points} - \text{Number of free parameters} \).

The sum goes over all available p+Pb data points up to \( p_T = 2.5 \text{ GeV/c} \) \(^{[35,37]}\). For 1FO, we have varied all the three parameters \( T, \rho_{\text{max}} \) and \( \tau_f \) to arrive at the best
FIG. 1. (Color online) The comparison of $p_T$ spectra in p+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV obtained from the THERMINA-TOR 21, 22 and data 35, 37 is shown for three centralities: 0 – 5%, 10 – 20% and 60 – 80% in three different FO schemes at $0 < t_{cm} < 0.5$. The gross features of the spectra comparison seems to be independent of the freezeout scheme. The bottom panels show the ratio of data to model calculation.
We have plotted the extracted freeze-out parameters in the 1FO, 1FO+γs and 2FO scheme in p+Pb collisions at √s_{NN} = 5.02 TeV. The average error on \(T\) and \(γ_s\) is 2 MeV and 0.02, respectively, whereas the error on \(ρ_{max}\) and \(τ_f\) is around 15\% for all centrality classes.

Finally in Fig. 3 we have plotted the extracted freeze-out parameters corresponding to the least \(χ^2/ndf\) with event multiplicity across different centralities in p+Pb and Pb+Pb that vary over three orders of magnitude. While the \(T\) remains mostly flat between 145 – 160 MeV, \(ρ_{max}\) and \(τ_f\) show a growth of 5–7 times. The growth rate is smooth across system size. We also note that the difference between the non-strange and strange freeze-out parameters systematically increase as we go to events with higher multiplicity, signifying the role of interaction. However, currently the uncertainties over the extracted parameters in the non-strange and strange sectors are large and does not allow us to further quantify the magnitude of the hierarchy in freezeout of the strange and non-strange flavors. \(γ_s\) in p+Pb steadily grows from 0.74 to about 0.94 across peripheral to central collisions. The approach to strangeness equilibration with more central p+Pb events could be related to the larger entropy deposition in the initial state in central p+Pb collisions as opposed to peripheral events [14]. We use similar errors on \(T\), \(ρ_{max}\) and \(τ_f\) as for Pb+Pb results [14] since the errors are mostly system size independent.

| Centrality (%) | 1FO | 1FO+γs | 2FO |
|---------------|-----|--------|-----|
|               | Non-Strange | Strange | Total | Non-Strange | Strange | Total | Non-Strange | Strange | Total |
| 0-5           | 248 (62) | 374 (66) | 622 (131) | 256 (61) | 208 (65) | 464 (130) | 225 (59) | 206 (63) | 432 (128) |
| 5-10          | 361 (62) | 325 (66) | 686 (131) | 297 (61) | 207 (65) | 504 (130) | 290 (59) | 257 (63) | 547 (128) |
| 10-20         | 377 (62) | 445 (66) | 822 (131) | 324 (61) | 204 (65) | 528 (130) | 338 (59) | 246 (63) | 584 (128) |
| 20-40         | 487 (62) | 560 (66) | 1046 (131) | 392 (61) | 219 (65) | 612 (130) | 441 (59) | 320 (63) | 761 (128) |
| 40-60         | 549 (62) | 822 (66) | 1371 (131) | 494 (61) | 315 (65) | 808 (130) | 495 (59) | 401 (63) | 896 (128) |
| 60-80         | 716 (62) | 1463 (66) | 2179 (131) | 615 (61) | 251 (65) | 866 (130) | 671 (59) | 873 (63) | 1544 (128) |
| 80-100        | 824 (62) | 2428 (64) | 3253 (129) | 721 (61) | 311 (63) | 1032 (128) | 746 (59) | 931 (61) | 1677 (126) |

| Centrality (%) | 1FO | 1FO+γs | 2FO |
|---------------|-----|--------|-----|
|               | Non-Strange | Strange | Total | Non-Strange | Strange | Total | Non-Strange | Strange | Total |
| 0-5           | 155 (158) | 3.9(3.8) | 2.7(2.7) | 0.94 |
| 5-10          | 157 (158) | 3.5(3.5) | 2.6(2.6) | 0.92 |
| 10-20         | 157 (158) | 3.3(3.4) | 2.5(2.5) | 0.90 |
| 20-40         | 156 (158) | 3.1(3.0) | 2.4(2.4) | 0.88 |
| 40-60         | 156 (156) | 2.7(2.7) | 2.2(2.2) | 0.84 |
| 60-80         | 155 (155) | 2.2(2.3) | 2.0(2.1) | 0.80 |
| 80-100        | 154 (153) | 1.6(1.7) | 1.9(1.9) | 0.74 |

TABLE II. Thermal freezeout parameters in the 1FO, 1FO+γs and 2FO scheme in p+Pb collisions at √s_{NN} = 5.02 TeV. The average error on \(T\) and \(γ_s\) is 2 MeV and 0.02, respectively, whereas the error on \(ρ_{max}\) and \(τ_f\) is around 15\% for all centrality classes.
IV. SUMMARY AND OUTLOOK

The hadron yields and $p_T$ spectra are the standard observables to throw light on freezeout dynamics. Contrary to expectations, the 1FO scheme is known to be blind to system size dependence in freezeout \[33\]. However, simultaneous analysis of the hadron yields in Pb+Pb, p+Pb and p+p revealed an interesting system size dependence of the preferred freezeout scheme: 2FO is preferred over 1FO and 1FO+γS in Pb+Pb while in small systems like p+Pb and p+p, 1FO+γS is preferred \[34\]. In order to put this hypothesis on a more strong footing, here we extend the previous analysis to hadron spectra. While 2FO is known to describe the hadron spectra better in Pb+Pb, here we analyse the data for different centralities in p+Pb. We find that allowing for a different hypersurface for the freezeout of the strange hadrons do not improve the quality of the fits. This is in accordance to our previous study with the hadron yields \[34\]. Thus, our current analysis with the data on hadron spectra reaffirms the hypothesis on the system size dependence of freezeout scheme: flavor dependent freezeout scheme is preferred in large systems while unified freezeout is preferred in small systems. Thus, the role of interaction in larger system is mostly to delay the freezeout of the non-strange hadrons.

V. ACKNOWLEDGEMENT

AKD and RS acknowledge the support of XIIth plan project no. 12-R&D-NIS-5.11-0300 of Govt. of India. BM acknowledges financial support from J C Bose National Fellowship of DST Govt. of India. SC acknowledges the support of the AGH UST statutory tasks No. 11.11.220.01/1 within subsidy of the Ministry of Science and Higher Educations and by the National Science Centre Grant No. 2015/17/B/ST2/00101.

[1] M. Gyulassy and L. McLerran, Nucl. Phys. A750, 30 (2005), arXiv:nucl-th/0405013 [nucl-th]
[2] J. Adams et al. (STAR), Nucl. Phys. A757, 102 (2005), arXiv:nucl-ex/0501009 [nucl-ex]
[3] K. Rajagopal and F. Wilczek (2000) pp. 2061–2151, arXiv:hep-ph/0011333 [hep-ph]
[4] M. A. Stephanov, Prog. Theor. Phys. Suppl. 153, 139 (2004), [Int. J. Mod. Phys.A20,4387(2005)], arXiv:hep-ph/0402115 [hep-ph]
[5] P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, Phys. Lett. B365, 1 (1996), arXiv:nucl-th/9508020 [nucl-th]
[6] G. D. Yen and M. I. Gorenstein, Phys. Rev. C59, 2788 (1999), arXiv:nucl-th/9808012 [nucl-th]
[7] J. Cleymans and K. Redlich, Phys. Rev. C60, 054908 (1999)
FIG. 3. (Color online) The extracted freezeout parameters with best goodness of fit for different centralities in p+Pb and Pb+Pb collisions. There is a gradual preference for sequential freezeout of the strange and non-strange flavors as we go to higher multiplicity events. The $\gamma_S$ value is fixed at one for Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

[14] S. Chatterjee, B. Mohanty, and R. Singh, Phys. Rev. C92, 024917 (2015) arXiv:1411.1718 [nucl-th]
[15] E. Schnedermann, J. Sollfrank, and U. W. Heinz, Phys. Rev. C48, 2462 (1993) arXiv:nucl-th/9307020 [nucl-th]
[16] S. V. Afanasiev et al., Nucl. Phys. A715, 161 (2003) arXiv:nucl-ex/0208014 [nucl-ex]
[17] J. M. Burward-Hoy (PHENIX), Nucl. Phys. A715, 498 (2003) arXiv:nucl-ex/0210001 [nucl-ex]
[18] J. Letessier, J. Rafelski, and A. Tounsi, Phys. Lett. B328, 499 (1994) arXiv:hep-ph/9711321 [hep-ph]
[19] T. Csorgo and L. P. Csernai, Phys. Lett. B333, 494 (1994) arXiv:1312.1487 [nucl-th]
[20] L. P. Csernai and I. N. Mishustin, Phys. Rev. Lett. 74, 5005 (1995)
[21] A. Kisiel, T. Taluc, W. Broniowski, and W. Florkowski, Comput. Phys. Commun. 174, 669 (2006) arXiv:nucl-th/0504047 [nucl-th]
[22] M. Chojnacki, A. Kisiel, W. Florkowski, and W. Broniowski, Comput. Phys. Commun. 183, 746 (2012) arXiv:1102.0273 [nucl-th]
[23] J. Steinheimer, J. Aichelin, and M. Bleicher, Phys. Rev. Lett. 110, 042501 (2013) arXiv:1203.5302 [nucl-th]
[24] F. Becattini, M. Bleicher, T. Kollegger, J. Schuster, J. Steinheimer, and R. Stock, Phys. Rev. Lett. 111, 082302 (2013) arXiv:1212.2431 [nucl-th]
[25] M. Petrn, J. Letessier, V. Petrek, and J. Rafelski, Phys. Rev. C88, 034907 (2013), arXiv:1303.2098 [hep-ph].

[26] R. Bellwied, S. Borsanyi, Z. Fodor, S. D. Katz, and C. Ratti, Phys. Rev. Lett. 111, 202302 (2013), arXiv:1305.6297 [hep-lat].

[27] S. Chatterjee, R. M. Godbole, and S. Gupta, Phys. Lett. B727, 554 (2013), arXiv:1306.2006 [nucl-th].

[28] K. A. Bugaev, D. R. Oliinychenko, J. Cleymans, A. I. Ivanytskyi, I. N. Mishustin, E. G. Nikonov, and V. V. Sagun, Europ. Phys. Lett. 104, 22002 (2013), arXiv:1308.3594 [hep-ph].

[29] M. Floris, Nucl. Phys. A931, 103 (2014), arXiv:1408.6403 [nucl-ex].

[30] J. Noronha-Hostler and C. Greiner, (2014), arXiv:1405.7298 [nucl-th].

[31] A. Bazavov et al., Phys. Rev. Lett. 113, 072001 (2014), arXiv:1404.6511 [hep-lat].

[32] P. Alba, V. Vovchenko, M. I. Gorenstein, and H. Stoecker, Nucl. Phys. A974, 22 (2018), arXiv:1606.06542 [hep-ph].

[33] F. Becattini, P. Castorina, A. Milov, and H. Satz, Eur. Phys. J. C66, 377 (2010), arXiv:0911.3026 [hep-ph].

[34] S. Chatterjee, A. K. Dash, and B. Mohanty, J. Phys. G44, 105106 (2017), arXiv:1608.00643 [nucl-th].

[35] B. B. Abelev et al. (ALICE), Phys. Lett. B728, 25 (2014), arXiv:1307.6796 [nucl-ex].

[36] J. Adam et al. (ALICE), Phys. Lett. B758, 389 (2016), arXiv:1512.07227 [nucl-ex].

[37] J. Adam et al. (ALICE), Eur. Phys. J. C76, 245 (2016), arXiv:1601.07868 [nucl-ex].

[38] I. Kraus, J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys. Rev. C76, 064903 (2007), arXiv:0707.3879 [hep-ph].

[39] I. Kraus, J. Cleymans, H. Oeschler, and K. Redlich, Phys. Rev. C79, 014901 (2009), arXiv:0808.0611 [hep-ph].

[40] S. Das, D. Mishra, S. Chatterjee, and B. Mohanty, Phys. Rev. C95, 014912 (2017), arXiv:1605.07748 [nucl-th].

[41] V. V. Begun, V. Vovchenko, M. I. Gorenstein, and H. Stoecker, (2018), arXiv:1805.01901 [nucl-th].

[42] N. Sharma, J. Cleymans, and L. Kumar, Eur. Phys. J. C78, 288 (2018), arXiv:1802.07972 [hep-ph].

[43] N. Sharma, J. Cleymans, and B. Hippolyte, (2018), arXiv:1803.05409 [hep-ph].

[44] P. Castorina, S. Plumari, and H. Satz, Int. J. Mod. Phys. E26, 1750081 (2017), arXiv:1709.02706 [nucl-th].