$U(1)_{B-L}$: Neutrino Physics and Inflation\footnote{Based on talks given by Q. Shafi at the 6th European Meeting From The Planck Scale To The Electroweak Scale (Planck 03), Madrid, Spain, May 26-31, 2003; at the 11th International Workshop on Neutrino Telescopes, Venice, Italy, 22-25 Feb 2005; and at the 11th International Symposium on Particles, Strings and Cosmology (PASCOS 2005), Gyeongju, Korea, 30 May - 4 Jun 2005.\footnote{Electronic address: shafi@bxclu.bartol.udel.edu} and Q. Shafi\footnote{Electronic address: nefer@udel.edu}

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A gauged $U(1)_{B-L}$ symmetry predicts three right handed handed neutrinos and its spontaneous breaking automatically yields the seesaw mechanism. In a supersymmetric setting this breaking can be nicely linked with inflation to yield $\delta T/T$ proportional to $(M_{B-L}/M_P)^2$, where $M_{B-L}$ ($M_P$) denote the $B-L$ breaking (Planck) scale. Thus $M_{B-L}$ is estimated to be of order $10^{16}$ GeV, and the heaviest right handed neutrino mass is less than or of order $10^{14}$ GeV. A second right handed neutrino turns out to have a mass of order $10 - 10^2 T_r$, where $T_r$ ($\lesssim 10^9$ GeV) denotes the reheat temperature. A $U(1)$ R symmetry plays an essential role in implementing inflation and leptogenesis, resolving the MSSM $\mu$ problem and eliminating dimension five nucleon decay. An unbroken $\mathbb{Z}_2$ subgroup plays the role of matter parity. The scalar spectral index $n_s = 0.99 \pm 0.01$ for the simplest models, while in smooth hybrid inflation $n_s \geq 0.97$. The tensor to scalar ratio $r$ is negligible, and $\mathrm{d}n_s/\mathrm{d} \ln k \lesssim 10^{-3}$.

PACS numbers: 98.80.Cq, 12.60.Jv, 04.65.+e

I. INTRODUCTION

Physics beyond the Standard Model (SM) is required by the following experimental observations:

- Neutrino Oscillations: $\Delta m_{\nu}^2 \lesssim 10^{-10} \text{ eV}^2 \ll$ (mass difference)$^2$ needed to understand atmospheric and solar neutrino observations;
- CMB Anisotropy ($\delta T/T$): requires inflation which cannot be realized in the SM;
- Non-Baryonic Dark Matter ($\Omega_{\text{CDM}} = 0.25$): SM has no plausible candidate;
- Baryon Asymmetry ($n_b/s \sim 10^{-10}$): Not possible to achieve in the SM.

Recall that at the renormalizable level the SM possesses a global $U(1)_{B-L}$ symmetry. If the symmetry is gauged, the anomaly cancellation requires the existence of three right handed neutrinos. An important question therefore is the symmetry breaking scale of $U(1)_{B-L}$. Note that this scale is not fixed by the evolution of the three SM gauge couplings. Remarkably, we will be able to determine the $M_{B-L}$ by implementing inflation. With $M_{B-L}$ well below the Planck scale the seesaw mechanism enables us to realize light neutrino masses in the desired range. Furthermore, it will turn out that leptogenesis is a natural outcome after inflation is over.

The introduction of a gauge $U(1)_{B-L}$ symmetry broken at a scale well below the Planck scale exacerbates the well known gauge hierarchy problem. There are at least four potential hierarchy problems one could consider:

- $M_W \ll M_P$;
- $M_{B-L} \ll M_P$ (required by neutrino oscillations);
- $m_\chi \ll M_P$ (where $m_\chi$ denotes the inflaton mass);
- $f_a \sim 10^{10} - 10^{12}$ GeV ($\ll M_P$), where $f_a$ denotes the axion decay constant.

Supersymmetry (SUSY) can certainly help here, especially if the SUSY breaking scale in the observable sector is of order TeV. Thus, it seems that a good starting point, instead of SM $\times U(1)_{B-L}$, could be MSSM $\times U(1)_{B-L}$. The $\mathbb{Z}_2$ ‘matter’ parity associated with the MSSM has two important consequences. It eliminates rapid (dimension four) proton decay, and it delivers a respectable cold dark matter candidate in the form of LSP. However, Planck scale suppressed dimension five nucleon decay is still present and one simple solution is to embed $\mathbb{Z}_2$ in a $U(1)_R$ symmetry. It turns out that the R symmetry also plays an essential role in realizing a compelling inflationary scenario and in the resolution of the MSSM $\mu$ problem. Finally it seems natural to extend the above discussion to larger groups, especially to $SO(10)$ and its various subgroups.

II. SUPERSYMMETRIC HYBRID INFLATION MODELS

In this section, we review a class of supersymmetric hybrid inflation models \cite{1,2,3} where inflation can be linked to the breaking of $U(1)_{B-L}$. We compute the allowed range of the dimensionless coupling in the superpotential and the dependence of the spectral index on...
this coupling, in the presence of canonical supergravity (SUGRA) corrections.

The simplest supersymmetric hybrid inflation model \[1\] is realized by the renormalizable superpotential \[2\]

\[
W_1 = \kappa S(\Phi\overline{\Phi} - M^2)
\]

(1)

where \(\Phi, \overline{\Phi}\) denote a conjugate pair of superfields transforming as nontrivial representations of some gauge group \(G\), \(S\) is a gauge singlet superfield, and \(\kappa > 0\) is a dimensionless coupling. A suitable \(U(1)\) R-symmetry, under which \(W_1\) and \(S\) transform the same way, ensures the uniqueness of this superpotential at the renormalizable level \[3\]. In the absence of supersymmetry breaking, the potential energy minimum corresponds to non-zero derivatives with respect to the normalized real scalar components \(|\langle \nu_H \rangle| = |\langle \overline{\nu}_H \rangle|\) for \(\Phi\) and \(\overline{\Phi}\), while the VEV of \(S\) is zero. (We use the same notation for superfields and their scalar components.) Thus, \(G\) is broken to some subgroup \(H\) which, in many interesting models, coincides with the MSSM gauge group.

In order to realize inflation, the scalar fields \(\Phi, \overline{\Phi}, S\) must be displaced from their present minima. For \(|S| > M\), the \(\Phi, \overline{\Phi}\) VEVs both vanish so that the gauge symmetry is restored, and the tree level potential energy density \(\kappa^2 M^4\) dominates the universe, as in the originally proposed hybrid inflation scenario \[4, 5\]. With supersymmetry thus broken, there are radiative corrections from the \(\Phi - \overline{\Phi}\) supermultiplets that provide logarithmic corrections to the potential which drives inflation.

In one loop approximation the inflationary effective potential is given by \[6\]

\[
V_{\text{LOOP}} = \kappa^2 M^4 \left[ 1 + \frac{\kappa^2 N}{32\pi^2} \left( \frac{2}{\Lambda^2} \right)^2 \right] + \left( z + 1 \right) \ln \left( 1 + z^{-1} \right) + \left( z - 1 \right)^2 \ln \left( 1 - z^{-1} \right),
\]

(2)

where \(z \equiv x^2 \equiv |S|^2/M^2\), \(N\) is the dimensionality of the \(\Phi, \overline{\Phi}\) representations, and \(\Lambda\) is a renormalization mass scale.

The scalar spectral index \(n_s\) is given by \[7, 8\]

\[
n_s \simeq 1 - 6\epsilon + 2\eta, \quad \epsilon \equiv \frac{m_p^2}{2} V', \quad \eta \equiv \frac{m_p^2 V''}{V},
\]

(3)

where \(m_p \simeq 2.4 \times 10^{18}\ \text{GeV}\) is the reduced Planck mass \((M_p/\sqrt{8\pi})\). We are going to use units \(m_p = 1\), although sometimes we will write \(m_p\) explicitly. The primes denote derivatives with respect to the normalized real scalar field \(\sigma \equiv \sqrt{2}|S|\). For relevant values of the parameters \((\kappa \ll 1)\), the slow roll conditions \((\epsilon, \eta \ll 1)\) are violated only ‘infinitesimally’ close to the critical point at \(x = 1\) \(|S| = M\) \[2\]. So inflation continues practically until this point is reached, where it abruptly ends.

The number of \(e\)-folds after the comoving scale \(l\) has crossed the horizon is given by

\[
N_l = \frac{1}{m_p^2} \int_{\sigma_f}^{\sigma_l} \frac{V d\sigma}{\sqrt{V}},
\]

(4)

where \(\sigma_l\) is the value of the field at the comoving scale \(l\) and \(\sigma_f\) is the value of the field at the end of inflation. Using Eqs. \([2, 4]\), we obtain

\[
\kappa \approx \frac{2\sqrt{2\pi}}{\sqrt{N} \ln 0} \frac{M}{m_p},
\]

(5)

Here, the subscript 0 implies that the values correspond to the comoving scale \(l_0 = 2\pi/k_0\), with \(k_0 \equiv 0.002\ \text{Mpc}^{-1}\). \(N_0 \approx 55\) is the number of \(e\)-folds and

\[
y_0^2 = \int_1^{x_0} dz \sqrt{2\kappa^2 V} > 0,
\]

(6)

with

\[
f(z) = (z + 1) \ln (1 + z^{-1}) + (z - 1) \ln (1 - z^{-1}).
\]

(7)

The amplitude of the curvature perturbation \(P_R^{1/2}\) is given by

\[
P_R^{1/2} = \frac{1}{2\sqrt{3} \pi m_p} \frac{V^{3/2}}{|V'|}.
\]

(8)

Using Eqs. \([2, 5, 9]\), \(P_R^{1/2}\) is found to be \[5, 6, 9\]

\[
P_R^{1/2} \approx 2 \left( \frac{N_0}{3N} \right)^{1/2} \left( \frac{M}{m_p} \right)^2 x_0^{-1} y_0^{-1} f(x_0^2)^{-1}.
\]

(9)

Up to now, we ignored supergravity (SUGRA) corrections to the potential. More often than not, SUGRA corrections tend to derail an otherwise successful inflationary scenario by giving rise to scalar (mass)\(^2\) terms of order \(H^2\), where \(H\) denotes the Hubble constant. Remarkably, it turns out that for a canonical SUGRA potential (with minimal Kähler potential \(|S|^2 + |\Phi|^2 + |\overline{\Phi}|^2\)), the problematic (mass)\(^2\) term cancels out for the superpotential \(W_1\) in Eq. \([1, 4]\). This property also persists when non-renormalizable terms that are permitted by the \(U(1)_R\) symmetry are included in the superpotential.\(^3\)

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\(^1\) \(N_0 \approx 54 + (1/3) \ln(T_\gamma/10^9\ \text{GeV})+(2/3) \ln(\nu/10^{14}\ \text{GeV})\), where \(T_\gamma\) is the reheat temperature and \(\nu\) is the false vacuum energy density.

\(^2\) Note that the quadrupole CMB anisotropy \(\delta T/T = P_R^{1/2}/2\sqrt{15}\).

\(^3\) In general, \(K\) is expanded as \(K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 - (\kappa/2)|S|^2/m_p^2 + \ldots\) and only the \(|S|^4\) term in \(K\) generates a (mass)\(^2\) for \(S\), which would spoil inflation for \(\alpha \sim 1\). From the requirement \(\sigma < m_p\), one obtains an upper bound on \(\alpha \sim 10^{-2}\) for \(\kappa \approx 10^{-2}\), \(\sim 10^{-5}\) for \(\kappa \sim 10^{-5}\). We should note that, since the superpotential is linear in the inflaton, the presence of other fields with Planck scale VEVs also induce an inflaton mass of order \(H\). Some ways to suppress the inflaton mass are discussed in \([12, 13]\).
The scalar potential is given by
\[
V = e^K \left( \frac{\partial^2 K}{\partial z_i \partial z_j^*} \right)^{-1} D_{z_i} W D_{z_j}^* W^* - 3|W|^2 \right) + V_D ,
\]
with
\[
D_{z_i} W = \frac{\partial W}{\partial z_i} + \frac{\partial K}{\partial z_i} W ,
\]
where the sum extends over all fields $z_i$, and $K = \sum_i |z_i|^2$ is the minimal Kähler potential. The D-term $V_D$ vanishes in the D-flat direction $|\Phi| = |\Phi|$. From Eq. (10), with a minimal Kähler potential one contribution to the inflationary potential is given by \[2, 13, 14, 15\]
\[
V_{\text{SUGRA}} = \kappa^2 M^4 \left[ \frac{|S|^4}{2} + \ldots \right] .
\]
There are additional contributions to the potential arising from the soft SUSY breaking terms. In $N = 1$ SUGRA these include the universal scalar masses equal to $m_{3/2} \sim (\text{TeV})$, the gravitino mass. However, their effect on the inflationary scenario is negligible, as discussed below. The more important term is the $A$ term $(2 - A)m_{3/2} \kappa M^2 S (-\text{h.c.})$. For convenience, we write this as $a m_{3/2} \kappa M^2 |S|$, where $a \equiv 2 - |A| \cos(\arg S + \arg(2 - A))$. The effective potential is approximately given by Eq. (2) plus the leading SUGRA correction $\kappa^2 M^4 |S|^4/2$ and the $A$ term:
\[
V_1 = \kappa^2 M^4 \left[ 1 + \frac{\kappa^2 N}{32\pi^2} \left( 2 \ln \frac{\kappa^2 |S|^2}{A^2} + (z + 1)^2 \ln(1 + z^{-1}) - (z - 1)^2 \ln(1 - z^{-1}) \right) + \frac{|S|^4}{2} + a m_{3/2} \kappa M^2 |S| \right] .
\]
We perform our numerical calculations using this potential, taking $|a m_{3/2}| = 1 \text{ TeV}$. It is, however, instructive to discuss small and large $\kappa$ limits of Eq. (13). For $\kappa \gg 10^{-3}$, $1 \gg \sigma \gg \sqrt{2} M$, and Eq. (13) becomes
\[
V_1 \approx \kappa^2 M^4 \left[ 1 + \frac{\kappa^2 N}{32\pi^2} 2 \ln \frac{\kappa^2 \sigma^2}{2 \lambda^2} + \frac{\sigma^4}{8} \right].
\]
To a good approximation. Comparing the derivatives of the radiative and SUGRA corrections one sees that the radiative term dominates for $\sigma^2 \lesssim \kappa \sqrt{N}/2\pi$. From $3H\dot{\sigma} = -V'$, $\sigma_0^2 \approx \kappa^2 N \sigma_0 / 4\pi^2$ for the one-loop effective potential, so that SUGRA effects are negligible only for $\kappa \ll 2\pi/\sqrt{N} \approx 0.1/\sqrt{N}$. (For $N = 1$, this essentially agrees with [13]).
\[
\mathcal{P}_{R}^{1/2} \text{ is found from Eq. (14) to be}
\]
\[
\mathcal{P}_{R}^{1/2} \approx \frac{1}{\sqrt{3}} \frac{\kappa M^2}{\sigma_0} .
\]
In the absence of the SUGRA correction, the gauge symmetry breaking scale $M$ is given by Eq. (9). For $\kappa \gg 10^{-3}$, $x_0 \gg 1$ and $x_0 y_0 f(x_0^2) \rightarrow 1$. $\mathcal{P}_{R}^{1/2}$ in this case turns out to be proportional to $(M/m_p)^2$. Using the WMAP best fit $\mathcal{P}_{R}^{1/2} \approx 4.7 \times 10^{-5}$, $M$ approaches the value $N^{1/4} \times 6 \times 10^{15}$ GeV. The presence of the SUGRA term leads to larger values of $\sigma_0$ and hence larger values of $M$ for $\kappa \gtrsim 0.06/\sqrt{N}$. For $\kappa \ll 10^{-3}$, $|S_0| \approx M$ where $S_0$ is the value of the field at $k_0$, i.e. $z \approx 1$. (Note that due to the flatness of the potential the last 55 or so e-folds occur with $|S|$ close to $M$.) From Eqs. (8, 13), as $z \rightarrow 1$
\[
\mathcal{P}_{R}^{1/2} = \frac{2 \sqrt{2\pi}}{3\sqrt{N}} \ln(2) \kappa^2 M^4 N^{1/2} \approx 2 \sqrt{2\pi} \kappa^2 M^4 \left[ \ln(2) \kappa^2 M^4 N^{1/2} \approx 2 \sqrt{2\pi} \kappa^2 M^4 \right].
\]
The denominator of Eq. (16) contains the radiative, SUGRA and the $A$ terms respectively. Comparing them, we see that the radiative term can be ignored for $\kappa \lesssim 10^{-4}$. There is also a soft mass term $m_{3/2}^2 |S|^2$ in the potential, corresponding to an additional term $8\pi^2 m_{3/2}^2 \kappa M^4$ in the denominator. We have omitted this term, since it is insignificant for $\kappa \gtrsim 10^{-5}$. For a positive $A$ term ($a > 0$), the maximum value of $\mathcal{P}_{R}^{1/2}$ as a function of $M$ is found to be
\[
\mathcal{P}_{R}^{1/2} \max = \frac{1}{27/10 \pi/3} \left( \frac{\kappa^6}{a m_{3/2}} \right)^{1/5} .
\]
Setting $\mathcal{P}_{R}^{1/2} \simeq 4.7 \times 10^{-5}$, we find a lower bound on $\kappa$ ($\simeq 10^{-5}$). For larger values of $\kappa$, there are two separate solutions of $M$ for a given $\kappa$. The solution with larger $M$ is not valid if the symmetry breaking pattern produces cosmic strings. For example, strings are produced when $\Phi, \overline{\Phi}$ break $U(1)_R \times U(1)_L$ to $U(1)_Y \times Z_2$ matter parity, but not when $\Phi, \overline{\Phi}$ are $SU(2)_R \times U(1)_L$ doublets. For $a < 0$, there are again two solutions, but for the solution with a lower value of $M$, the slope changes sign as the inflaton rolls for $\kappa \lesssim 10^{-4}$ and the inflaton gets trapped in a false vacuum.

Note that the $A$ term depends on $\arg S$, so it should be checked whether $\arg S$ changes significantly during inflation. Numerically, we find that it does not, except for a range of $\kappa$ around $10^{-4}$ [2]. For this range, if the initial value of the $S$ field is greater than $M$ by at least a factor of two or so, the $A$ term and the slope become negative even if they were initially positive, before inflation can suitably end. However, larger values of the $A$ term, or the mass term coming from a non-minimal Kähler potential (or from a hidden sector VEV) would drive the value of $M$ in that region up, allowing the slope to stay positive (see [17] for the effect of varying the $A$ term and the mass).

The dependence of $M$ on $\kappa$ is shown in Fig. 4. Note that with inflation linked to the breaking of MSSM$\times U(1)_{B-L}$, $M$ corresponds to the $U(1)_{B-L}$ breaking scale, which is not fixed by the evolution of the three SM gauge couplings. The amplitude of the curvature perturbation (or, equivalently, $\delta T/T$) determines this scale
to be close to the SUSY GUT scale, suggesting that $U(1)_{B-L}$ could be embedded in $SO(10)$ or its subgroups. For example, $M$ can be determined in flipped $SU(5)$ from the renormalization group evolution of the $SU(3)$ and $SU(2)$ gauge couplings. The values are remarkably consistent with the ones fixed from $\delta T/T$ considerations \cite{13}. Here, some remarks concerning the allowed range of $\kappa$ is in order. As discussed above, a lower bound on $\kappa$ is obtained from the inflationary dynamics and the amplitude of the curvature perturbation. An upper bound on $\kappa$ is obtained from the value of the spectral index, which we discuss next. The gravitino constraint provides a more stringent upper bound ($\kappa \lesssim 10^{-2}$), as discussed in the next section. If cosmic strings form (as would be the case for $N = 1$), the range of $\kappa$ is also restricted by the limits on the cosmic string contribution to $\delta T/T$, however most of the range may still be allowed \cite{17}.

In the absence of SUGRA corrections, the scalar spectral index $n_s$ for $\kappa \gg 10^{-3}$ is given by \cite{4}

$$n_s \approx 1 + 2\eta \approx 1 - \frac{1}{N_0} \approx 0.98,$$

while it approaches unity for small $\kappa$. When the SUGRA correction is taken into account, from Eq. \cite{13},

$$n_s \approx 1 + 2\eta \approx 1 + 2 \left( 3\sigma^2 - \frac{\kappa^2 N}{8\pi^2 \sigma^2} \right),$$

and it exceeds unity for $\sigma^2 \gtrsim \kappa \sqrt{N}/2\sqrt{3}\pi$. For $x_0 \gg 1,

$$N_0 = \int_{\sigma_{\text{end}}}^{\sigma_0} \frac{V}{V'} d\sigma \approx \frac{\pi}{2\sigma_0^2 \kappa_c} \left( \frac{\pi}{2\kappa_c} \right),$$

where $\kappa_c = \pi^2/\sqrt{N}N_0 \approx 0.16/\sqrt{N}$. Using Eq. \cite{20}, one finds that the spectral index $n_s$ exceeds unity for $\kappa \approx 2\pi/\sqrt{3\sqrt{N}N_0} \approx 0.06/\sqrt{N}$. The dependence of $n_s$ on $\kappa$ is displayed in Fig. \cite{2} $dn_s/d\ln k$ is small and the tensor to scalar ratio $r$ is negligible, as shown in Fig. \cite{3}.

The experimental data disfavor $n_s$ values in excess of unity on smaller scales (say $k \lesssim 0.05 \text{ Mpc}^{-1}$), which leads to $\kappa \lesssim 0.1/\sqrt{N}$ for $n_s \lesssim 1.04$. Thus, even for the largest allowed $\kappa$ the vacuum energy density during inflation is considerably smaller than the symmetry breaking scale, and the tensor to scalar ratio $r \lesssim 10^{-4}$.

Note that the initial WMAP analysis suggests a running spectral index, with $|dn_s/d\ln k| \lesssim 10^{-3}$ disfavored at the $2\sigma$ level \cite{18}. On the other hand, an analysis including the constraints from the Sloan Digital Sky Survey (SDSS) finds no evidence for running, with $n_s = 0.98 \pm 0.02$ and $dn_s/d\ln k = -0.003 \pm 0.010$ \cite{21}. Clearly, more data is necessary to resolve this important issue. Modifications of the models discussed here, generally involving two stages of inflation, have been proposed in \cite{13,21} and elsewhere to generate a much more significant variation of $n_s$ with $k$.

The inflationary scenario based on the superpotential $W_1$ in Eq. \cite{1} has the characteristic feature that

\footnote{Larger values of $\kappa$ are allowed in models where dissipative effects are significant. Such effects become important for large values of $\kappa$, provided the inflaton also has strong couplings to matter fields \cite{18}.}
end of inflation essentially coincides with the gauge symmetry breaking. Thus, modifications should be made to \(W_1\) if the breaking of \(G\) to \(H\) leads to the appearance of topological defects such as monopoles or domain walls. For instance, the breaking of \(G_{PS} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R\) to the MSSM by fields belonging to \(\Phi(1,1,2), \bar{\Phi}(1,1,2)\) produces magnetic monopoles that carry two quanta of Dirac magnetic charge \([23]\). As shown in \([24]\), one simple resolution of the topological defects problem is achieved by supplementing \(W_1\) with a non-renormalizable term:

\[
W_2 = \kappa S(\bar{\Phi} \Phi - v^2) - \frac{S(\bar{\Phi} \Phi)^2}{M_S^2},
\]

where \(v\) is comparable to the SUSY GUT scale \(M_{GUT} \approx 2 \times 10^{16}\) GeV and \(M_S\) is an effective cutoff scale. The dimensionless coefficient of the non-renormalizable term is absorbed in \(M_S\). The presence of the non-renormalizable term enables an inflationary trajectory along which the gauge symmetry is broken. Thus, in this ‘shifted’ hybrid inflation model the topological defects are inflated away.

The inflationary potential is similar to Eq. \(13\) \([24]\); the common VEV at the SUSY minimum \(M = \sqrt[3]{\langle \sigma \rangle}\) is comparable to the SUSY GUT scale \(M_{GUT} \approx 2 \times 10^{16}\) GeV. This term, however, does not have a significant effect for the range of parameters where \(v \lesssim 10^{17}\) GeV.

\[
V_3 = S\left(-v^2 + \frac{(\Phi \bar{\Phi})^2}{M_S^2}\right),
\]

where the dimensionless parameter \(\kappa\) is absorbed in \(v\). The resulting scalar potential possesses two (symmetric) valleys of local minima which are suitable for inflation and along which the GUT symmetry is broken. The inclination of these valleys is already non-zero at the classical level and the end of inflation is smooth, in contrast to inflation based on the superpotential \(W_1\) (Eq. \(1\)). An important consequence is that, as in the case of shifted hybrid inflation, potential problems associated with topological defects are avoided.

The value of the field \(\sigma\) at the SUSY minimum \(M = \sqrt[3]{\langle \sigma \rangle}\) is \(2.8 \times 10^{17}\) GeV. Note that, if we express Eq. \(24\) in terms of the coupling parameter \(\kappa\), this value corresponds to \(\kappa \sim O(v^2/M_{GUT}^2) \sim 10^{-2}\). The value of the field \(\sigma\) is \(1.1 \times 10^{17}\) GeV at the end of inflation (corresponding to \(\eta = -1\)) and is \(\sigma_0 \approx 2.4 \times 10^{17}\) GeV at \(k_0\). In the absence of the SUSY GUT scale \((\delta T/T)_{\nu} \propto M^{10/3}/(M_S^2 M_{\nu}^{4/3})\) and the spectral index is given by \([24]\)

\[
n_s \approx 1 - \frac{5}{3N_0} \approx 0.97,
\]

a value which coincides with the prediction of some D-brane inflation models \([24]\). This may not be surprising since, in the absence of SUSY GUT scale, the potential \(V_3\) (Eq. \(24\)) has a form familiar from D-brane inflation. The SUSY GUT scale raises \(n_s\) from 0.97 to 1.0 for \(M \approx 10^{16}\) GeV, and above unity for \(M \gtrsim 1.5 \times 10^{16}\) GeV (Fig. \(4\)).

### III. LEPTOGENESIS IN SUPERSYMMETRIC HYBRID INFLATION MODELS

An important constraint on SUSY hybrid inflation models arises from considering the reheating temperature \(T\) after inflation, taking into account the gravitino problem which requires that \(T_r \lesssim 10^9\) GeV. \([27]\). This constraint on \(T_r\) depends on the SUSY breaking mechanism and the gravitino mass \(m_3/2\). For gravity mediated SUSY breaking models with unstable gravitinos of
The low-energy neutrino data indicates that the right handed neutrinos in this case will also be hierarchical. As discussed in Ref. 55, setting the Dirac masses strictly equal to the up-type quark masses and fitting to the neutrino oscillation parameters generally yields strongly hierarchical right handed neutrino masses ($M_1 \ll M_2 \ll M_3$), with $M_1 \sim 10^8$ GeV. The lepton asymmetry in this case is too small by several orders of magnitude. However, it is plausible that there are large radiative corrections to the first two family Dirac masses, so that $M_1$ remains heavy compared to $T_r$.

A reasonable mass pattern is therefore $M_1 < M_2 \ll M_3$, which can result from either the dimensionless couplings $\gamma_{ij}$ or additional symmetries (see e.g. 21). The dominant contribution to the lepton asymmetry is still from the decays with $N_3$ in the loop, as long as the first two family right handed neutrinos are not quasi degenerate. Under these assumptions, the lepton asymmetry is given by 36

$$n_L/s \lesssim 3 \times 10^{-10} \frac{T_r}{m_\chi} \left( \frac{M_i}{10^9 \text{ GeV}} \right) \left( \frac{m_\nu}{0.05 \text{ eV}} \right),$$

where $M_i$ denotes the mass of the heaviest right handed neutrino the inflaton can decay into. The decay rate $\Gamma_\chi = (1/8\pi)(M_3^2/M^2)m_\chi/3$, and the reheat temperature $T_r$ is given by

$$T_r = \left( \frac{45}{2\pi^2g_*} \right)^{1/4} \left( \frac{\Gamma_\chi m_\rho}{1/2} \right) \simeq 0.063 \left( \frac{m_\rho m_\chi}{M} \right)^{1/2} M_i. \quad (28)$$

From the experimental value of the baryon to photon ratio $n_B = 6.1 \times 10^{-10}$ 16, the required lepton asymmetry is found to be $n_L/s \simeq 2.5 \times 10^{-10}$ 37. Using this value, along with Eqs. (27)–(28), we can express $T_r$ in terms of the symmetry breaking scale $M$ and the inflaton mass $m_\chi$:

$$T_r \gtrsim \left( \frac{10^{16} \text{ GeV}}{M} \right)^{1/2} \left( \frac{m_\chi}{10^{11} \text{ GeV}} \right)^{3/4} \times \left( \frac{0.05 \text{ eV}}{m_\nu} \right)^{1/2} 1.6 \times 10^7 \text{ GeV}. \quad (29)$$

Here $m_\chi$ is given by $\sqrt{2}\kappa M$, $\sqrt{2}\kappa M\sqrt{1-4\kappa}$ and $2\sqrt{2}\nu^2/M$ respectively for hybrid, shifted hybrid and smooth hybrid inflation. The value of $m_\chi$ is shown in Figs. 5 and 6. We show the lower bound on $T_r$ calculated using this equation (taking $m_\nu = 0.05 \text{ eV}$) in Figs. 6–8.

Eq. (28) also yields the result that the heaviest right handed neutrino the inflaton can decay into is about 400 (6) times heavier than $T_r$, for hybrid inflation with $\kappa = 10^{-5} (10^{-2})$. For shifted hybrid inflation, this ratio does not depend on $\kappa$ as strongly and is $\sim 10^2$ 38. This is consistent with ignoring washout effects as long as the lightest right handed neutrino mass $M_1$ is also $\gg T_r$.

Both the gravitino constraint and the constraint $M_1 \gg T_r$ favor smaller values of $\kappa$ for hybrid inflation, with
FIG. 5: The inflaton mass $m_{\chi}$ vs. $\kappa$, for SUSY hybrid inflation with $N = 1$ (solid), and for shifted hybrid inflation (dot-dashed for $M_S = m_P$, dotted for $M_S = 5 \times 10^{17}$ GeV). The grey segments denote the range of $\kappa$ for which the change in $\arg S$ is significant.

![Graph](image1)

FIG. 6: The inflaton mass $m_{\chi}$ vs. the symmetry breaking scale $M$ for smooth hybrid inflation.

![Graph](image2)

$T_r \gtrsim 2 \times 10^7$ GeV for $\kappa \sim 10^{-5}$. Similarly, the gravitino constraint favors $\kappa$ values as small as the inflationary trajectory allows for shifted hybrid inflation, and $T_r \gtrsim 5 \times 10^7$ GeV for $M_S = m_P$. Smooth hybrid inflation is relatively disfavored since $T_r \gtrsim 4 \times 10^9$ and $M_2/T_r \simeq 4 (12)$ for $M = 5 \times 10^{15}$ GeV ($2 \times 10^{16}$ GeV).$^6$

So far we have not addressed the $\mu$ problem and the relationship to $T_r$ in the present context. The MSSM $\mu$ problem can naturally be resolved in SUSY hybrid inflation models in the presence of the term $\lambda Sh^2$ in the superpotential, where $h$ contains the two Higgs doublets $\bar{S}$. (The ‘bare’ term $h^2$ is not allowed by the $U(1)$ R-symmetry.) After inflation the VEV of $S$ generates a $\mu$ term with $\mu = \lambda \langle S \rangle = -m_{3/2}\lambda/\kappa$, where $\lambda > \kappa$ is required for the proper vacuum. The inflaton in this case predominantly decays into higgses (and higgsinos) with $\Gamma_h = (1/16\pi)\lambda^2 m_\chi$. As a consequence the presence of this term significantly increases the reheat temperature $T_r$. Following $^{10}$, we calculate $T_r$ for the best case scenario $\lambda = \kappa$. We find a lower bound on $T_r$ of $4 \times 10^8$ GeV in hybrid inflation, see Fig. 7 $T_r \gtrsim 4 \times 10^9$ GeV for shifted hybrid inflation with $M_S = m_P$, and $T_r \gtrsim 10^{12}$ GeV for smooth hybrid inflation. An alternative resolution of the $\mu$ problem in SUSY hybrid inflation involves a Peccei-Quinn (PQ) symmetry $U(1)_{\text{PQ}}$ $^{21}$. $^{11}$

The lower bounds on $T_r$ are summarized in Table 1. Even for hybrid and shifted models, there is some tension between the gravitino constraint and the reheat temperature required to generate sufficient lepton asymmetry, particularly for gravity mediated SUSY breaking models, and if hadronic decays of gravitinos are not suppressed. However, we should note that having quasi degenerate neutrinos would increase the lepton asymmetry per neutrino decay $\epsilon$ $^{42}$ and thus allow lower values of $T_r$ corresponding to lighter right handed neutrinos. Provided that the neutrino mass splittings are comparable to their decay widths, $\epsilon$ can be as large as $1/2$ $^{43}$. The lepton asymmetry in this case is of order $T_r/m_\chi$ where

$^6$ A new inflation model related to smooth hybrid inflation is discussed in $^{11} 21$, where the energy scale of inflation is lower and consequently lower reheat temperatures are allowed.

$^7$ We take $\lambda/\kappa = 2/(14\xi - 1)$ for shifted hybrid inflation. Some scalars belonging to the inflaton sector acquire negative (mass)$^2$ if $\lambda$ is smaller. $\kappa \sim 10^{-4}$ corresponds to $\lambda/\kappa \simeq 3$. 

FIG. 7: The lower bound on the reheat temperature $T_r$ vs. $\kappa$, for SUSY hybrid inflation with $N = 1$ (solid), and for shifted hybrid inflation (dot-dashed for $M_S = m_P$, dotted for $M_S = 5 \times 10^{17}$ GeV). The segments in the top left part of the figure correspond to the bounds in the presence of a $\lambda Sh^2$ coupling. The grey segments denote the range of $\kappa$ for which the change in $\arg S$ is significant.

![Graph](image3)

FIG. 8: The lower bound on the reheat temperature $T_r$ vs. the symmetry breaking scale $M$ for smooth hybrid inflation.

![Graph](image4)
TABLE I: Lower bounds on the reheat temperature (GeV)

| System                        | without $\lambda S h^2$ | with $\lambda S h^2$ |
|-------------------------------|--------------------------|-----------------------|
| SUSY hybrid inflation         | $2 \times 10^7$          | $4 \times 10^8$       |
| Shifted hybrid inflation      | $5 \times 10^7$          | $4 \times 10^8$       |
| Smooth hybrid inflation       | $4 \times 10^9$          | $\gtrsim 10^{12}$     |

$m_s \sim 10^{11}$ GeV for $r \sim 10^{-5}$, and sufficient lepton asymmetry can be generated with $T_r$ close to the electroweak scale.

IV. SUMMARY

- MSSM $\times Z_2$ (matter parity), when extended to MSSM $\times U(1)_{B-L} \times U(1)_R$ helps us to implement inflation and leptogenesis; natural extensions include $SO(10)$ and its various subgroups;

- The $U(1)_{B-L}$ breaking scale is constrained from $\delta T/T$ to be of order the SUSY GUT scale;

- The spectral index is predicted to be $0.99 \pm 0.01$, with a negligible tensor to scalar ratio and $\Delta n_s/\Delta n_k \lesssim 10^{-3}$. For smooth hybrid inflation $n_s \gtrsim 0.97$;

- $U(1)_R$ contains $Z_2$ matter parity, eliminates Planck suppressed dimension five proton decay, and may play an essential role in resolving the MSSM $\mu$ problem.

V. ACKNOWLEDGEMENTS

This work is partially supported by the US Department of Energy under contract number DE-FG02-91ER40626.
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