Noncommutative QED and $\gamma\gamma$ scattering

Namit Mahajan*

Department of Physics and Astrophysics,
University of Delhi, Delhi-110 007, India.

Abstract

We study $\gamma\gamma$ scattering in noncommutative QED (NCQED) where the gauge field has Yang-Mills type coupling, giving new contributions to the scattering process and making it possible for it to occur at tree level. The process takes place at one loop level in the Standard Model (SM) and could be an important signal for physics beyond SM. But it is found that the Standard Model contribution far exceeds the tree level contribution of the noncommutative case.

Keywords: Noncommutative, gamma-gamma scattering

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Introduction

Noncommutativity of a pair of conjugate variables forms the central theme of quantum mechanics in terms of the Uncertainty Principle. We are quite familiar with the noncommutativity of rotations in ordinary Euclidean space. The idea of noncommutative (NC) space-time can be traced back to the work of Snyder [1]. But more recently, string theory arguments have motivated an extensive study of Quantum Field Theory (QFT) on NC spaces [2]. The noncommutativity of space-time is realised by the coordinate operators, $x_\mu$, satisfying

$$[x_\mu, x_\nu] = i\Theta_{\mu\nu}$$

with $\Theta_{\mu\nu} = \theta\epsilon_{\mu\nu}$. $\theta$ is the noncommutativity parameter with dimensions $(mass)^{-2}$ and $\epsilon_{\mu\nu}$ is a dimensionless antisymmetric matrix with elements $O(1)$. The field theories formulated on such spaces are non-local and violate Lorentz symmetry. The deviation from the standard theory manifests as violation of Lorentz invariance. We can still expect manifest Lorentz invariance for energies satisfying $E^2\theta << 1$. In the limit $\theta \to 0$, one expects to recover the standard theory. This is true for the theory at classical level. But at the quantum level, the limit $\theta \to 0$ does not lead to the commutative theory [3]. The theory of electrons in a strong magnetic field, projected to the lowest Landau level, is a classic example of NC field theory.

Various attempts, both theoretical and phenomenological, have been made to study QFT on NC spaces. The study of perturbative behaviour and divergence structure [4],

*E-mail: nm@ducos.ernet.in, nmahajan@physics.du.ac.in
C, P and T properties and renormalisability \([5]\) of such theories has been undertaken. It has been shown that quantum theories with time-like noncommutativities are not unitary \([6]\). We shall therefore restrict our discussion to the theories with space-like noncommutativities, although it has been shown that light-like noncommutative theories are also free of pathologies \([7]\). To this end, the coordinate commutator simply reads
\[
[x_i, x_j] = i\theta\epsilon_{ij} \tag{2}
\]

There have been attempts to write down particle physics models, in particular SM, on such NC spaces \([8]\). From a phenomenological point of view, various scattering processes have been analysed \([9, 10]\) along with the attempts to calculate additional contributions to the precisely measured quantities like anomalous magnetic moment \([11]\) and Lamb shift \([12]\) in the noncommutative version of QED.

\(\gamma\gamma\) scattering in NCQED

Consider NCQED i.e. a U(1) noncommutative theory coupled to fermions. The noncommutative version of a theory can be written by replacing the field products by what is called the 'star product'. The star (*) product for any two functions is given by
\[
f(x) * g(x) = f(x) e^{i\frac{\theta}{2} \Theta^{\alpha\beta} \partial_\alpha \partial_\beta} g(x) \tag{3}
\]

The NCQED action, using the above line of reasoning, is
\[
S_{NCQED} = \int d^Dx \left(-\frac{1}{4g^2} F^{\mu\nu}(x) * F_{\mu\nu}(x) + i \bar{\psi}(x) \gamma^\mu * D_\mu \psi(x) - m \bar{\psi}(x) * \psi(x) \right) \tag{4}
\]
where \(g\) is the coupling and
\[
F_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + i\theta [A_\mu(x), A_\nu(x)]_* \tag{5}
\]
The covariant derivative is given by
\[
D_\mu \psi(x) = \partial_\mu \psi(x) + i\theta A_\mu(x) * \psi(x) \tag{6}
\]
The action is invariant under the noncommutative U(1) transformations obtained by replacing all the products in the standard transformations by the corresponding star products.

The noncommutativity is encoded in the star product and from the above expressions it is quite evident that the field strength, even in the case of U(1), theory is nonlinear in gauge field and it is precisely this nonlinearity that gives rise to additional vertices for the gauge field. It is now a straight forward task to derive the Feynman rules from the above action \([11]\), Arfaei et.al \([12]\). It is found that apart from generating the three and four point vertices for the gauge field self interaction, each interaction vertex picks up a momentum dependent phase factor, whose argument typically has the structure \(\frac{1}{2} p \land k\).

The \(\land\) product, in general, is defined as
\[
p \land k = p_\mu \Theta^{\mu\nu} k_\nu \tag{7}
\]
In the case of theories with only space-like noncommutativities, only the space-space elements contribute and using Eq.(2) it simply reduces to the usual vector cross-product of the two three momenta i.e.

\[ p \wedge k = \vec{p} \times \vec{k} \]  

(8)

The process, $\gamma \gamma \rightarrow \gamma \gamma$ takes place at the one loop level in standard QED as well as SM and thus is quite suppressed. But the presence of Yang-Mills type coupling for the photon field in NCQED enables the process to take place at the tree level. This makes the above process a plausible candidate to look for physics beyond SM at the tree level.

The diagrams contributing to the scattering process are

Due to the noncommutative nature of the coordinates, the theory is not Lorentz invariant and the results are frame dependent. In writing down the amplitudes corresponding to each of the above diagrams, we assume that \( \theta \ll 1 \) and make the substitution \( \sin(a\theta) \rightarrow a\theta \), where \( a \) is used to generically denote the quantity appearing in the argument of the sine function multiplied to \( \theta \).

Choosing to work in the center of mass frame, we find that the s-channel diagram vanishes. The square of the matrix amplitude reads

\[ |M_{NC}|^2 = \left( \frac{e\theta}{16} \right)^4 \left[ 100s^4 + 96t^4 + 204st^3 + 360s^2t^2 + 250s^3t \right] \]  

(9)

and the total (unpolarised) cross section is

\[ \sigma_{NC} = (1.5 \times 10^{-3})\alpha_{em}^2 s^3 \theta^4 \]  

(10)
which for $\sqrt{s} \sim \text{TeV}$ and $\theta \leq (10^4 \text{ GeV})^{-2}$ as in [12] gives $\sigma_{NC} \sim 10^{-10} \text{ fb}$, to be compared with the SM contribution, $\sigma_{SM} \sim \text{fb}$ at the same center of mass energy [14]. It is found that at low energies the fermion contribution dominates the SM cross-section while at higher $\sqrt{s} (> 100 \text{ GeV})$, it is the W contribution that becomes important. The SM contribution gradually decreases as $\sqrt{s}$ crosses the 500 GeV range. Although, in contrast to SM, the NC cross-section increases monotonously with $\sqrt{s}$, it can never catch up with the SM cross-section for the same energy.

Conclusions

In this article we have computed the NCQED contribution to the $\gamma\gamma$ scattering and found that even though in this case the process occurs at the tree level as opposed to SM, where it takes place at the one loop level, the SM contribution far exceeds the NC contribution. It is clear that the NCQED contribution will start showing up only when $\theta$ is much larger than the value used here.

The process has been studied in context of NCQED by Hewett et.al [10] but the authors argue that inspired by recent theories of extra dimensions [13], where the effective scale of gravity is $\mathcal{O} \sim \text{TeV}$ as opposed to the Planck scale, the scale of noncommutativity can, too, be chosen to be around TeV. But a more physical approach would be to use the value of $\theta$, as obtained from studies like Lamb shift [12], to calculate the new contributions. Also the authors have taken into account time-like noncommutativity that may lead to possible non-unitary S-matrix elements. Even for $\theta \sim (\text{TeV})^{-2}$ as taken by the authors, the SM contribution is still overwhelmingly large. Thus with the present day and near future experiments, it doesn’t seem possible to get a signal of NCQED from $\gamma\gamma$ scattering.

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