Sensorless Control of Permanent Magnet Synchronous Linear Motor Based on Sliding Mode Variable Structure MRAS Flux Observation

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Abstract—The object of this paper is a permanent magnet synchronous linear motor (PMLSM), whose control method is based on a model-referenced adaptive system (MRAS), and it analyses the speed identification of a permanent magnet synchronous linear motor without position sensors. The article proposes a new model-referenced adaptive method, which utilises a sliding-mode variable structure control method (SMC), to replace the PI control algorithm utilised in conventional model-referenced adaptive algorithm. The control system of the PMLSM is therefore designed and studied based on the change of the adaptive law in model-referenced adaption. The mathematical model of the PMLSM itself is chosen as the reference model, and the feedback magnetic chain model of the motor output is chosen as the adjustable model, replacing the conventional current model and simplifying the control algorithm. The sliding mode surface of the sliding mode variable structure control algorithm is constructed using the reference model and the output error of the adjustable model. Through theoretical analysis and simulation models built by MATLAB/Simulink simulation software, the simulation results show that the designed PMLSM speed induction-free control system MRAS speed observer based on the sliding mode variable structure has strong robustness and excellent dynamic static performance. The advantages verified by the new algorithm achieve the experimental purpose of the expected assumptions.

1. INTRODUCTION

Permanent magnet linear synchronous motor integrates the advantages of permanent magnet motors and linear motors, high efficiency, large thrust density, large acceleration, fast speed, and high precision. It is widely used in applications such as high-precision CNC equipment, logistics transportation, and weapon launch [1]. The precision control of PMLSM requires the speed of the motor (primary) and location information, which requires speed and position sensor as the acquisition information. The sensor is installed which adds the cost of the motor and the action inertia of the motor, which affects the dynamic properties of the system. The sensor is easily interfered by the external environment to reduce the reliability of the system. Therefore, the control strategy of a non-position sensor in the permanent magnet linear synchronous motor control has become the development trend of linear motor speed control systems.

In recent years, speed sensorless control methods have been studied and discussed by a large number of scholars. The control is mainly divided into two types. The first type of method is based on the high-frequency signal injection method of the motor salient effect. The second type of method is based on the mathematical model of the motor, and such methods include open loop algorithms and closed loop algorithms. The open-loop algorithm mainly includes direct calculation method and back EMF integration method. The closed loop algorithm mainly has an extended Kalman filter algorithm, sliding mode observer, MRAS, etc. [2]. The above methods have their own advantages and disadvantages in different places. Among them, the closed loop algorithm of the second method is the hotspot direction of theoretical
research. Refs. [3–5] propose the control mode of high frequency signal injection. Take high frequency current signal as an example. The negative sequence component of the high frequency current carries the position information of the motor rotor in the phase. Therefore, the speed information of the motor can also be determined, but this method is only applicable to a motor having a salient effect and a low speed operation. Extended Kalman filtering algorithm is proposed in [6–8]. This algorithm can effectively reduce the effects of external interference and noise, and the need for motor parameters and the initial location information is not high. However, because too many factors are involved, and the model is complex, it has increased the degree of difficulties in parameter analysis, and experimental data requires multiple verifications. Refs. [9, 10] propose to use slip mode variable structural control as a positionless observer. Sliding mode control overcomes the uncertainty of the system and has strong robustness to interference and unsteadying dynamics. However, its problem is that the output of the system controller has jitter. Refs. [11–13] describe the model reference adaptive control algorithm. This algorithm has simple, good anti-interference performance, with high control precision, but its accuracy is highly dependent on the accuracy of the parameters of the motor.

In this paper, the advantages and disadvantages of the above algorithms are considered and compared, and the model reference adaptive non-position sensor control based on slip mode variable structure control is proposed. The method based on the stator magnetic chain replaces the stator current, which is simple to calculate. The superior robust characteristics of sliding mode control replace the traditional PI control and make up for the dependence of MRAS on motor parameters with its characteristics that it does not depend on the control object. This article compares these two MARS methods based on theoretical analysis and MATLAB simulation. Detecting the model reference adaptive control based on slip mode variable structure control has a fast response and strong robustness.

2. PMLSM MATHEMATICAL MODEL

Ideally ignoring the magnetic saturation effect, the mathematical model of the PMLSM in the rotating coordinate system d-axis and q-axis orientations of the rotor magnetic field is:

The voltage equation can be expressed as:

\[
\begin{align*}
    u_d &= R_s i_d + \frac{d}{dt} \varphi_d - \omega_e \varphi_q \\
    u_q &= R_s i_q + \frac{d}{dt} \varphi_q + \omega_e \varphi_d
\end{align*}
\]

(1)

Flux linkage equation is expressed as follows:

\[
\begin{align*}
    \varphi_d &= L_d i_d + \varphi_f \\
    \varphi_q &= L_q i_q
\end{align*}
\]

(2)

The thrust equation is expressed as:

\[
F_e = \frac{3\pi}{2\tau} P_n [\varphi_f i_q + (L_d - L_q) i_d i_q]
\]

(3)

The mechanical motion equation is expressed as:

\[
M \frac{dv}{dt} = F_e - F_L - Bv
\]

(4)

\(u_d\) and \(u_q\) are d-axis and q-axis voltages, respectively; \(i_d\) and \(i_q\) are d-axis and q-axis currents; \(L_d\) and \(L_q\) are d-axis and q-axis inductances; \(\varphi_d\) and \(\varphi_q\) are d-axis and q-axis magnetic chains; \(\varphi_f\) is a permanent magnet magnetic chain; \(R_s\) is the armature resistance; \(\omega_e\) is the electrical angle speed; \(F_e\) is the export thrust; \(F_L\) is the load resistance; \(\tau\) is the power of the motor; \(P_n\) is the polar number of the motor; \(M\) is the quality of motor; \(v\) is a straight line speed of motor; \(B\) is a viscous friction coefficient. Fig. 1 shows the MATLAB simulation of PMLSM. Fig. 2 is a linear motor cross-sectional schematic.
3. SPEED OBSERVER BASED ON MODEL REFERENCE ADAPTIVE

3.1. MRAS Observer Based on PI Adaptive Law

The basic idea of the model reference adaptive observation method is to use an equation containing the estimated parameters as an adjustable model, and the motor body model is used as a reference model. The difference between them is through the action of adaptive law. The adjustable model can track the reference model quickly and accurately. The selection of adaptive law has a great influence on the stability and convergence of the system. At present, the design methods of adaptive law mainly include Popov hyperstability theory and Lyapunov stability theory [14]. MRAS’s structural form can be divided into parallel according to the location of the reference model and the adjustable model, parallel, series, string parallel [15]. This paper uses the parallel as shown in Fig. 3. The reference model is a secondary magnetic chain vector $\varphi_f$ based on a linear motor. Adjustable model is an estimated secondary magnetic chain vector $\hat{\varphi}_f$.

In order to simplify the modeling, formula (2) is substituted into formula (1). The voltage equation is rewritten into the form of flux $\varphi_d, \varphi_q$ as the state variable.
The reference model can be expressed as:

\[
\frac{d}{dt} \begin{bmatrix} \varphi_d \\ \varphi_q \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_d} & \omega_e \\ -\omega_e & -\frac{R_s}{L_q} \end{bmatrix} \begin{bmatrix} \varphi_d \\ \varphi_q \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_d + \frac{R_s}{L_d} \varphi_f \\ u_q \end{bmatrix}
\] (5)

The parameter adjustable estimation flux linkage model can be expressed as:

\[
\frac{d}{dt} \begin{bmatrix} \hat{\varphi}_d \\ \hat{\varphi}_q \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_d} & \hat{\omega}_e \\ -\hat{\omega}_e & -\frac{R_s}{L_q} \end{bmatrix} \begin{bmatrix} \hat{\varphi}_d \\ \hat{\varphi}_q \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_d + \frac{R_s}{L_d} \hat{\varphi}_f \\ u_q \end{bmatrix}
\] (6)

Define formula (5) minus (6) to obtain the state equation of flux linkage error as:

\[
\frac{d}{dt} \begin{bmatrix} \varphi_d - \hat{\varphi}_d \\ \varphi_q - \hat{\varphi}_q \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_d} & \omega_e \\ -\omega_e & -\frac{R_s}{L_q} \end{bmatrix} \begin{bmatrix} \varphi_d - \hat{\varphi}_d \\ \varphi_q - \hat{\varphi}_q \end{bmatrix} + (\hat{\omega}_e - \omega_e) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{\varphi}_d \\ \hat{\varphi}_q \end{bmatrix}
\] (7)

Define that a generalized error is \( e = \varphi - \hat{\varphi} \), \( \lim_{t \to \infty} e(t) = 0 \). The output of the estimation model is consistent with the output of the reference model. Simplify the above formula as follows:

\( \dot{e} = Ae - Q \) (8)

In the above formula:

\[
A = \begin{bmatrix} -\frac{R_s}{L_d} & \omega_e \\ -\omega_e & -\frac{R_s}{L_q} \end{bmatrix}, \quad Q = (\hat{\omega}_e - \omega_e) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.
\]

The error system established by Equation (8) is represented as a state equation:

\[
\begin{cases}
\dot{e} = Ae - W \\
V = Ce
\end{cases}
\] (9)

The structural diagram of an error system is analyzed by Equation (9), as shown in Fig. 4. The matrix \( C \) in the figure is a linear compensator, and its function is to keep the stability of the system. To simplify calculations, usually \( C \) is a unit array. The dashed box represents a linear forward path.

![Figure 4. Error system structure diagram.](image-url)
Since the relationship between the output $V$ and feedback amount $W$ is not clear, use a feedback path to represent their contact. In the solid line box, it is a nonlinear feedback pathway.

According to Popov super stability theory, if the error system is kept stable, the nonlinear time-changing feedback link in the above figure needs to meet the following two conditions:

1) Transfer function matrix $G(s) = C(sI - A)^{-1}$ of linear forward passage is a strictly positive matrix.
2) Nonlinear feedback path to meet Popov integral inequality:

$$\eta(0, t_1) = \int_0^{t_1} V^T W dt \geq -\gamma_0^2 (\forall t_1 \geq 0)$$  \hspace{1cm} (10)

where $\gamma_0^2$ is any limited positive number, with $\lim_{t \to \infty} e(t) = 0$, and the model reference adaptive system is asymptotically stable. When $C = I$, $V = e$. Solve Equation (10) in reverse. Select PI adaptive law to ensure that $\eta(0, t_1) \geq -\gamma_0^2$ is established. The identification algorithm of the primary estimated speed of PMLSM can be obtained as follows:

$$\dot{\hat{\omega}}_e = \int_0^{t_1} K_I (\hat{\varphi}_q \hat{\varphi}_d - \varphi_q \varphi_d) \, dt + K_P (\hat{\varphi}_q \varphi_d - \varphi_q \hat{\varphi}_d) + \hat{\omega}_0$$  \hspace{1cm} (11)

The primary position estimation can be obtained by integrating formula (11). As shown below:

$$\dot{\hat{\theta}} = \int \hat{\omega}_e \, dt$$  \hspace{1cm} (12)

### 3.2. MRAS Observer Based on Sliding Mode Variable Structure

Sliding mode control is a variable structure control. Its difference from traditional control is the discontinuity of control, that is, the control system structure has a time-variable switching characteristic. According to the current state of the system, the destination is continuously changed, so that the system moves with a predetermined "sliding modal" state trajectory, that is, sliding mode variable structure control [16]. The sliding mode can be designed according to the system requirements, and the parameters of the system are independent of the external disturbance, so the sliding mode variable structure has strong robust characteristics. Its mathematical model is expressed as:

$$u = \begin{cases} u^+(x), & s(x) > 0 \\ u^-(x), & s(x) < 0 \end{cases}$$  \hspace{1cm} (13)

In the above formula, $u^+(x) \neq u^-(x)$ makes the sports trajectory to arrive sliding surface within a limited time, equivalent to $\dot{s}s < 0$.

The speed observer based on sliding mode variable structure adaptation constructs an adaptive law by using the characteristics of sliding mode variable structure. According to the design principle $s(c) = 0$ of sliding mode surface, the design switching function is $s(x) = \hat{\varphi}_d \hat{\varphi}_d - \varphi_q \hat{\varphi}_q$. Derive switching function:

$$\dot{s}(x) = \left( \frac{R_s}{L_q} + \frac{R_d}{L_d} \right) \left( \varphi_q \hat{\varphi}_d - \varphi_d \hat{\varphi}_q \right) + u_q \left( \varphi_d - \hat{\varphi}_d \right) + \left( \omega_e - \hat{\omega}_e \right) \left( \varphi_d \hat{\varphi}_d + \varphi_q \hat{\varphi}_q \right)$$  \hspace{1cm} (14)

Simplify the above formula:

$$\dot{s}(x) = f \left( R_s, L_d, L_q, \varphi_f, u_q, u_d, \varphi_d, \varphi_q \right) - k \left( \varphi_d \hat{\varphi}_d + \varphi_q \hat{\varphi}_q \right) \text{sgn}(s)$$  \hspace{1cm} (15)

Because of $(\varphi_d \hat{\varphi}_d + \varphi_q \hat{\varphi}_q) > 0$, there is a large enough $k$ to make $\dot{s}(x) = 0$. So this means that $s(x)$ satisfies the condition of $\lim_{s \to 0^+} \dot{s} < 0$, $\lim_{s \to 0^-} \dot{s} > 0$. At the same time this means that it satisfies the condition of $\dot{ss} < 0$. When $\dot{ss} < 0$ is satisfied, the system reaches the ideal sliding mode. The control law generally selects the constant switching control law $u = C \text{sgn} \left( S(x) \right)$ and $C$ as constants. When the switching function and control law are determined, the sliding mode variable structure control system can be established. Further calculate the speed of the motor which is equivalent to:

$$\dot{\hat{\omega}}_{e1} = k \cdot \text{sgn} \left( \varphi_d \hat{\varphi}_q - \varphi_q \hat{\varphi}_d \right)$$  \hspace{1cm} (16)

Sliding mode variable structure adaptive speed observer structure is shown in Fig. 5.
4. SLIDING MODE VARIABLE STRUCTURE ADAPTIVE OBSERVER

This paper selects PMLSM as the simulation motor to verify the feasibility and correctness of the sliding mode variable structure MRAS speed observer. The rated parameters of the motors are shown in Table 1.

Table 1. Motor parameters.

| Parameter                      | Value         |
|--------------------------------|---------------|
| Simulation speed               | $n = 0.2 \text{ m/s}$ |
| $D$-axis inductors             | $L_d = 0.0019 \text{ H}$ |
| $Q$-axis inductors             | $L_q = 0.0019 \text{ H}$ |
| Permanent magnet chains        | $\varphi_f = 0.0385 \text{ Wb}$ |
| Polar logarithm                | $p = 1$ |
| Resistors                      | $R_s = 4.4 \Omega$ |
| Friction coefficient           | $J = 0.004$ |
| Polar distance                 | $\tau = 0.001 \text{ m}$ |
| Primary kinetic mass           | $M = 0.5 \text{ kg}$ |

The motor module uses MATLAB simulation of linear motor shown in Fig. 1. The simulation uses MATLAB/Simulink to model the control system. It adopts $i_d = 0$ vector control strategy, and the speed loop and current loop are controlled by PI. The speed observer is modeled by MRAS observer based on PI adaptive law and MRAS observer based on sliding mode variable structure adaptive law. The simulation results are compared and analyzed. Fig. 6 shows the model reference adaptive control system.

Figure 7 shows the simulation diagram of MRAS sensorless motor control using traditional PI adaptive law, including the following relationship among estimated speed, motor speed, and given speed. Fig. 8 is a simulation diagram of SMC as adaptive law control, and Fig. 9 compares the speed error of PI control with that of SMC control.

Comparing the speed curves in Fig. 7 and Fig. 8, it can be seen that the following conditions of the two control methods for a given speed are different. From the speed diagram of PI control, it can be seen that in the process of PI control, the overshoot between motor speed and estimated speed is very large, and there is chattering phenomenon. It can be clearly seen from Fig. 9 that the error of PI control is greater than the speed error of SMC control. It further shows that the model reference adaptive system of sliding mode variable structure control has strong stability when the speed changes suddenly.

Figures 10 to 12 are simulation diagrams of the setting contents of the second group of experiments. Fig. 10 is the simulation result diagram of PI control, which is compared with the simulation result...
Figure 6. Model reference adaptive control system structure diagram.

Figure 7. PI controlled variable speed.

Figure 8. SMC control variable speed.

Figure 9. Speed error comparison.

Figure 10. PI control variable load.
diagram of SMC control in Fig. 11. It can be observed that the initial overshoot is still large, and the jitter is obvious. Moreover, the estimated speed and motor speed do not follow well after sudden load at 0.5 seconds. In Fig. 11, the load has little effect on the estimated speed following the motor speed.

As seen from Fig. 12, the difference between the estimated speed of the two control modes and the motor speed during loading is more obvious. The error of estimated speed is mapped. The sudden load experiment shows that the model reference adaptive control strategy based on sliding mode variable structure has strong robustness to external disturbances.

5. CONCLUSION

In this paper, the speed sensorless PMLSM is analyzed and studied. MRAS speed observer based on PI adaptive law and MRAS speed observer based on sliding mode variable structure adaptive law are built, respectively. The variable speed no-load simulation experiment and constant speed variable load simulation experiment are carried out for linear motor. Experimental results show that the MRAS speed observer control based on sliding mode variable structure adaptive law has high tracking accuracy, small jitter, and strong ability to suppress external interference. The new method of speed observer is verified by simulation experiments.

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