Load Restoration in Islanded Microgrids: Formulation and Solution Strategies

Shourya Bose ©, Graduate Student Member, IEEE, and Yu Zhang ©, Member, IEEE

Abstract—Adverse circumstances, such as extreme weather events, can cause significant disruptions to normal operation of electric distribution systems (DSs), which include isolating parts of the DS due to damaged transmission equipment. In this article, we consider the problem of load restoration in a microgrid (MG) that is isolated from the upstream DS. The MG contains sources of distributed generation, such as microturbines and renewable energy sources as well as energy storage systems (ESSs). We formulate the load restoration task as a nonconvex optimization problem. This problem embodies the physics of the MG by leveraging a branch flow model while incorporating salient phenomena in islanded MGs, such as the need for internal frequency regulation and complementarity requirements arising in ESS operations. Since the formulated optimization problem is nonconvex, we introduce a convex relaxation that can be solved through model predictive control as a baseline method. However, in order to solve the problem considering its full nonconvexity, we leverage a policy-learning method called constrained policy optimization, a tailored version of which is used as our proposed algorithm. The aforementioned approaches, along with an additional deep learning method, are compared through extensive simulations.

Index Terms—Constrained policy optimization (CPO), convex relaxation, electric power networks, islanded microgrids, load restoration.

I. INTRODUCTION

Extreme weather events such as wildfires, hurricanes, and winter storms pose a big threat to the reliable operation of electric distribution systems (DSs) [1]. Those events can disrupt the operation of DS by damaging electric transmission equipment such as overhead power lines, thereby curbing the delivery of electric power. Traditionally, DSs have been designed to be reliable during nominal operations and in the face of predictable off-nominal operating conditions. Recently, a new paradigm of resilience has been explored by the power engineering community, which posits that a DS must be capable of rapidly recovering to a state of nominal operations post extreme weather events [2]. As localized DSs and microgrids (MGs) facilitate system resilience which are equipped with distributed power generation including microturbines (MTs) and renewable energy sources (RESs). RES may contain energy from photovoltaics, wind turbines, geothermal sources, etc. The intermittent nature of power production of RESs necessitates energy storage systems (ESSs). Thus, MGs have sources of power generation as well as energy storage. They are an ideal candidate for restoring power demand for loads such as residential homes, industries, and critical services such as hospitals; especially when they become disconnected (islanding mode) from the upstream DS. Coordination of distributed generation sources within islanded MGs occurs through a bihierarchical control scheme: The lower-level primary control allows for communication-free fast response to disturbances, whereas the higher-level secondary and tertiary controls are used to generate setpoints for primary control. The latter act over longer timescales while leveraging the communication infrastructure available to the MG [3]. Secondary and tertiary control in MGs is achieved with an MG controller (MGC) [4], which is a central computer capable of communicating with and controlling generation and storage elements in the MG. Since load restoration involves the coordination of sources and loads in the face of constraints arising due to network physics and finite generation capacities, the MGC algorithm is best posed as an optimization problem [5], which can be solved in real time by the MGC.

Load restoration, when posed as an optimization problem, features several unique characteristics, which inform strategies to solve it. The first challenge arises from the nonconvexity of ac power flow (ACPF) equations, which embody the physics of power transfer. Since the ACPF equations are a part of the constraints in load restoration, any solution thereof cannot be certified as globally optimal [6]. In this article, we use the DistFlow equations instead of ACPF equations since the former is equivalent to the latter in power networks with radial topologies [7], which is typical of MGs. The DistFlow equations, which lend themselves to intuitive convex relaxations, were first introduced in the literature by Baran and Wu [8]. The second challenge involves discrete decisions, which must be made as a part of the load restoration process. An important example is the principle of ESS complementarity, which states that an ESS may not simultaneously charge and discharge at any given time [9]. This principle has conventionally been encoded using nonconvex [10] or integer [11] constraints or enforced through penalty terms in the objective function [12]. The third challenge
is based on the observation that in islanded MGs, there is no external source of ac frequency regulation, and therefore, it must be done using internal sources, such as voltage source inverters (VSIs). Thus, the dependence of voltages and ac frequency on real and reactive power generation must be modeled as a (possibly convex) constraint [13]. The last challenge involves the uncertainty in forecasts of renewable sources, which renders the calculated solution suboptimal as the quality of forecasts degrades substantially.

As shown in the sequel, formulating load restoration as an optimization problem considering the aforementioned challenges results in a nonconvex nonlinear program defined over multiple time steps. The nonconvexity implies that there are no guarantees on whether a candidate’s feasible solution is globally optimal. Two approaches may be used to remedy this drawback: 1) the use of heuristic solution algorithms or 2) the relaxation of the problem into a more tractable form. For the latter approach, it is desirable to generate a linear or convex relaxation of the problem, which can then be solved through model predictive control (MPC). MPC is a popular technique for optimal control of dynamical systems wherein a control task, posed as an optimization problem defined over a long time horizon, is decomposed into smaller sequential subproblems and solved. MPC has been applied to several applications, such as voltage stability assurance [14], demand response in industrial loads [15], Volt-VAR control [16], and scheduling PV storage systems [17].

The other approach is to use heuristic solution strategies. In this article, we consider reinforcement learning (RL) for this purpose. RL and deep RL (in which deep neural networks are used to approximate various functions used in RL) are concerned with determining actions, which an agent interacting with an environment should take to maximize its accumulated reward. Over the last decade, RL has been considered for various power systems applications, such as Volt-VAR control [18], EV charge scheduling [19], power management in networked MGs [10], and optimal control of ESS in MGs [20]. A specific RL algorithm that can be useful in the current setting is constrained policy optimization (CPO) [21]. CPO aims to find a policy, which takes as input the current system state and outputs the optimal action with respect to the reward while satisfying multiple constraints on state and action. The policy is represented as a neural network, which can be trained [22] such that its outputs approach optimality. While CPO has recently been considered for power system applications [10], [23], significant challenges remain in tailoring the training procedure of CPO to a given application. Tailoring CPO for load restoration is considered in this article.

We conclude this section by mentioning other strategies used in load restoration literature. Many frameworks for load restoration consider reconfigurable MGs with inelastic loads, wherein the decision variable includes discrete switching actions and dispatch of generation. Such problems may be solved by posing it as a maximum coverage problem [24] or using a spanning tree search [25]. On the contrary, we consider a fixed-topology radial MG and elastic loads while focusing on the optimum dispatch of MTs, RESs, and ESSs. Such formulations have been solved in the literature using stochastic optimization [11], scenario generation and pruning [26], and RL aided by power flow simulators [27]. Many formulations also assume additional infrastructure such as mobile ESSs [28]. We restrict our attention to cases without specialized infrastructure and do not consider explicit scenario generation: The CPO agent implicitly generates scenarios as it steers the grid along different trajectories during simulation-based training.

A. Motivation

The principal motivation of this article is to compare the performance of an RL-based MGC for load restoration over conventional optimization-based techniques, such as MPC. There are two advantages that RL has over MPC. First, neural-network-based controllers (alternatively known as policy) trained with RL require significantly lower computational resources for implementation as compared to optimization solvers. Second, since RL involves learning from exploratory experience, it is possible for the policy to learn mispredictions in forecasts, thereby increasing the robustness of generated solutions. However, the downside to RL is that it produces black-box models, which may generate suboptimal or infeasible solutions. To that end, we seek to tailor CPO policy training such that the solutions generated are feasible with respect to DistFlow equations and other operational constraints. This would also alleviate the biggest drawback of interior-point methods (IPMs) conventionally used to solve MPC: nonconvexity and complementarity constraints may lead to ill-conditioning or exponential solve times of IPM-based solvers.

B. Contribution Statement

In this article, we consider load restoration for a fixed-topology islanded MG as an optimization problem, in which we incorporate constraints arising from different elements participating in the MG. As a baseline solution strategy, we consider MPC, for which we propose a convex relaxation of the problem. However, in order to solve the exact problem approximately, we consider CPO, whose training procedure is adapted specifically for load restoration. Finally, we compare MPC and CPO solution quality through simulations, and both these methods are compared to a brute-force learning approach from a dataset.

We make a few standing assumptions for our formulation of load restoration. First, we assume that the MG has a fixed radial topology, and line parameters such as resistance and reactance are known. Second, we assume that the loads are flexible, i.e., they can accept a fraction of the load power demand, and we also assume that the RES output can be fractionally curtailed. Third, we consider the load demands to be static over the entire duration of the restoration procedure while forecasts of RES outputs vary in time and are assumed to have errors over the same duration.

The main contributions of our work are summarized as follows.

1) Alongside existing relaxations for DistFlow equations, we present new convex relaxations for ESS complementarity and the dependence of inverter voltages on reactive power. These relaxations serve to render MPC...
subproblems convex, thereby allowing the use of convex optimization for solving the same.

2) Alternatively, in order to solve the unrelaxed load restoration problem, we consider the use of CPO. Training a policy with CPO involves updating learnable weights of the policy using a quadratically constrained optimization problem multiple times for every episode. In Proposition 1 and Lemma 3, we present a tailored method for efficiently calculating coefficients of the said optimization problem.

3) In Section V, we compare the relaxed MPC controller with the CPO policy. The comparisons entail verifying the performance of both under erroneous forecasts of RES output. Furthermore, we compare the feasibility gaps accrued due to the MPC relaxation with those resulting from the CPO policy.

The rest of the article is organized as follows. Section II presents the problem formulation of load restoration. Section III shows how the problem can be relaxed and solved by MPC. Section IV details our proposed CPO approach along with a procedure on how to train the CPO policy. Section V compares the solutions obtained by the two approaches, with a third approach learning the solutions generated by MPC with a deep neural network. Finally, Section VI concludes this article. All detailed proofs of lemmas and propositions are deferred to the Appendix.

C. Notation

The notations $\mathbb{R}$, $\mathbb{C}$, $\mathbb{N}$, and $\mathbb{R}_+$ denote the sets of real numbers, complex numbers, natural numbers, and nonnegative reals, respectively. For a complex number $c \in \mathbb{C}$, $\Re(c)$ and $\Im(c)$ denote its real and imaginary parts, respectively. $\converge(A)$ is the convex hull of set $A$. $D_{KL}(p_1 || p_2)$ denotes the Kullback–Leibler (KL)-divergence between probability distributions $p_1$ and $p_2$. For a real number $a \in \mathbb{R}$, $[a]^+$ and $[a]^-$ denote $\max\{a, 0\}$ and $\max\{-a, 0\}$, respectively. Boldface variables represent vectors and matrices. $a^T$ and $a^H$ represent the transpose and complex conjugate transpose of vector $a$, respectively. $I_n$ is the identity matrix of size $n \times n$. $a \preceq b$ denotes the elementwise inequality between two vectors. $A \otimes B$ denotes the Kronecker product of $A$ and $B$. $\|a\|_2$ denotes the 2-norm of vector $a$. $N(\mu, \Sigma)$ denotes a multivariate Gaussian distribution with mean vector $\mu$ and covariance matrix $\Sigma$. For a random variable $X$, $\mathbb{E}[X]$ denotes its expectation.

II. LOAD RESTORATION PROBLEM FORMULATION

Consider an MG that is islanded from the upstream DS due to an extreme weather event. Let the MG be represented by a directed graph $G = (N, E)$, where $N$ represents the set of buses and $E$ is the set of power lines. We assume that $G$ is a radial network, i.e., $G$ is a tree. $N$ is the union of disjoint sets $N^L$, $N^{MT}$, $N^{RES}$, and $N^{ESS}$, where $N^L$ represents the load buses, $N^{MT}$ represents the buses connected to an MT, $N^{RES}$ represents the buses with RES, and $N^{ESS}$ represents the buses connected to an ESS. We consider the load restoration problem over a time horizon of $T \triangleq \{1, 2, \ldots, T\}$, and time steps are indexed by $t$. We let $s_{i,t} \in \mathbb{C}$ and $v_{i,t} \in \mathbb{R}_+$ denote the complex power injection and squared magnitude of voltage phasor at bus $i \in N$, respectively, at time $t$. For every line $(i, j) \in E$, we let $S_{ij,t} \in \mathbb{C}$, and $l_{ij,t} \in \mathbb{R}_+$ denote the sending-end complex power flow and squared magnitude of the current phasor, respectively. For buses $i$ and $j$, notation $i \rightarrow j$ indicates the presence of a power line in between, i.e., $(i, j) \in E$. Note that any equation involving the index $t$, unless stated otherwise, is assumed to hold for all $t \in T$.

A. Objective Function

The objective is to maximize load restoration while minimizing MT fuel consumption. To that end, an appropriate objection function can be represented as

$$J_T \triangleq \sum_{t \in T} \left( \sum_{i \in N} C^L_{i,t}(|\Re(S_{i,t})|) + \sum_{i \in N^{MT}} C^MT_{i,t}(|\Re(S_{i,t})|) \right)$$

(1)

where the (possibly time varying) functions $C^L$ and $C^MT$ are concave functions considering the sign convention that generated power is positive while consumed power is negative. The objective function is meant to incentivize the amount of load restored while disincentivizing power produced from MT in favor of utilizing ESSs or RESs. In practice, $C^L$ is often linear with coefficients representing the priority order of load restoration while $C^MT$ can be linear or concave quadratic.

B. Power Flow Constraints

Let $z_{ij} \in \mathbb{C}$ represent the impedance of line $i \rightarrow j$. We use the DistFlow equations for quantifying the power flows that are given as follows:

$$s_{i,t} = \sum_{j \rightarrow k} S_{jk,t} - \sum_{i \rightarrow j} (S_{ij,t} - z_{ij}l_{ij,t}) \quad \forall j \in N \quad (2a)$$

$$v_{i,t} = v_{i,t-1} - 2\Re(z_{ij}S_{ij,t}) + |z_{ij}|^2l_{ij,t} \quad \forall i \rightarrow j \quad (2b)$$

$$v_{i,t}l_{ij,t} = |S_{ij,t}|^2 \quad \forall i \rightarrow j \quad (2c)$$

It is known that in radial networks, voltage angles can be recovered from any solution of (2a)–(2c) [7, Th. 2]. The voltage and current constraints at each bus and line, respectively, needed for the nominal operation of the MG, are codified as follows:

$$v \leq v_{i,t} \leq \bar{v} \quad \forall i \in N \quad (3)$$

$$l_{ij,t} \leq \bar{l}_{ij} \quad \forall i \rightarrow j. \quad (4)$$

C. MT Constraints

The power generation of each MT is subject to constraints on per-time step generation, as well as those on rates of ramping up/down. They are given as

$$\sum_{i \in N^{MT}} \bar{l}_{i,t} \leq \Re(S_{i,t}) \leq \bar{l}_{i,t} \quad \forall i \in N^{MT} \quad (5a)$$

$$\sum_{i \in N^{MT}} \bar{l}_{i,t} \leq \Re(S_{i,t}) - \Re(S_{i,t-1}) \leq \bar{l}_{i,t} \quad \forall i \in N^{MT} \quad (5b)$$

$$\Re(S_{i,t}) \leq \bar{l}_{i,t} \quad \forall i \in N^{MT}, \quad (5c)$$

wherein (5b) holds for all $t \in T \setminus \{1\}$. Each MT is assumed to have a fixed amount of fuel at the beginning of $T$. This
constrains the total amount of real power produced over \( \mathcal{T} \) as 
\[
(\sum_{t \in \mathcal{T}} \mathbb{R}(s_{i,t})) \tau_i \leq E_i \quad \forall i \in \mathcal{N}_{MT},
\]
where \( E_i \) is the total fuel initially available at MT \( i \), and \( \tau_i \) is the power-to-fuel conversion factor. The total fuel constraint can be reformulated as a recursive relation by denoting with \( \zeta_{i,t} \) the amount of fuel remaining in MT \( i \) at time \( t \) and observing that
\[
\zeta_{i,t} = \zeta_{i,t-1} - \tau_i \mathbb{R}(s_{i,t}) \quad \forall i \in \mathcal{N}_{MT} \tag{6a}
\]
\[
\zeta_{i,t} \geq 0 \quad \forall i \in \mathcal{N}_{MT} \tag{6b}
\]
with the initial condition \( \zeta_{i,0} = E_i \).

**D. ESS Constraints**

Each ESS is an energy reservoir, which may discharge power into the MG when required, and otherwise charge in order to replenish its energy reserves. We denote by \( S_{i,t} \) the state of charge of ESS \( i \) at time step \( t \), which takes values in \([S_{i}, \bar{S}_i]\). Letting \( P_{i,t}^{ch} \) and \( P_{i,t}^{dis} \) denote the charge and discharge powers of the same, the power input/output constraints of the ESS are
\[
0 \leq P_{i,t}^{dis} \leq \bar{P}_i \quad 0 \leq P_{i,t}^{ch} \leq \bar{P}_i \quad \forall i \in \mathcal{N}_{ESS} \tag{7a}
\]
\[
P_{i,t}^{ch} P_{i,t}^{dis} = 0 \quad \forall i \in \mathcal{N}_{ESS} \tag{7b}
\]
\[
\mathbb{R}(s_{i,t}) = P_{i,t}^{dis} - P_{i,t}^{ch} \quad \forall i \in \mathcal{N}_{ESS}. \tag{7c}
\]
Note that (7b) denotes the complementarity constraint, which may alternatively be denoted as an integer constraint using a charge/discharge indicator variable. The evolution of \( S_{i,t} \) is given as
\[
S_{i,t} = S_{i,t-1} + \eta_{i,t}^{ch} P_{i,t}^{ch} \Delta t - \frac{1}{\eta_{i,t}^{dis}} P_{i,t}^{dis} \Delta t \quad \forall i \in \mathcal{N}_{ESS} \tag{8a}
\]
\[
S_{i,0} = S_{i,init}, \quad S_{i,t} \in [S_{i}, \bar{S}_i] \quad \forall i \in \mathcal{N}_{ESS} \tag{8b}
\]
where \( (8a) \) holds for all \( t \in \mathcal{T}, \eta_{i,t}^{ch} \in (0, 1] \) and \( \eta_{i,t}^{dis} \in (0, 1] \) denote the charge and discharge efficiency for ESS \( i \), \( \Delta t \) denotes the time duration corresponding to each time step, and \( S_{i,init} \) denotes the initial SoC of ESS \( i \). Considering other factors such as battery temperature, the efficiencies \( \eta_{i,t}^{ch} \) and \( \eta_{i,t}^{dis} \) can be modeled as time-varying [29].

**E. RES Constraints**

Since the power output of RES is stochastic in nature and cannot be predicted with certainty ahead of time, forecast \( \hat{P}_{i,t}^{RES} \) is used as a stand-in for the actual output. We assume each RES has the capability to curtail its power output and denote by \( \kappa_{i,t} \in [0, 1] \) the curtailment ratio of the real power. The RES real power constraint is given as
\[
\mathbb{R}(s_{i,t}) = (1 - \kappa_{i,t}) \hat{P}_{i,t}^{RES} \quad \forall i \in \mathcal{N}_{RES}. \tag{9}
\]

**F. Reactive Power and Droop Bus Constraints**

The MT, RES, and ESS buses may be interfaced to the MG through inverters, which convert dc to ac power. Such inverters are capable of supplying and absorbing reactive power to and from the MG, constrained as
\[
|\mathbb{S}(s_{i,t})| \leq \bar{Q}_i \quad \forall i \in \mathcal{N}_{MT} \cup \mathcal{N}_{ESS} \cup \mathcal{N}_{RES} \tag{10}
\]
where \( \bar{Q}_i \) is the nameplate capacity of the inverter at bus \( i \). Other representations of inverter capacity that limits the total apparent power of the inverter (by way of an upper bound on \(|s_{i,t}|\)) may also be used in lieu of (10).

Many inverters in MGs may operate on the principle of droop control, also referred to as VSI control, wherein the inverter acts as an ac voltage source, with voltage and frequency depending on the real and reactive power output. Letting \( \mathcal{N}_{droop} \subset \{\mathcal{N} \setminus \mathcal{N}_{L}\} \) denote the set of generation buses operating on droop control (“droop buses”), and \( \omega_i \) denote the system frequency at time \( t \), the droop bus constraints are given as [30]
\[
\omega_i = \omega_i^* - k_P (\mathbb{R}(s_{i,t}) - P_i^*) \quad \forall i \in \mathcal{N}_{droop} \tag{11a}
\]
\[
\sqrt{\mathbb{V}_{i,t}^2 - k_Q (\mathbb{S}(s_{i,t}) - Q_i^*)} \quad \forall i \in \mathcal{N}_{droop} \tag{11b}
\]
\[
\omega \leq \omega_i \leq \bar{\omega} \tag{11c}
\]
wherein \( \omega^*, \mathbb{V}_i^*, P_i^*, \) and \( Q_i^* \) are operational setpoints of the inverter while \( k_P \) and \( k_Q \) are the droop constants. Since the MG receives no frequency signals from the upstream DS during islanding, droop buses are essential for internal frequency regulation of the system, which is achieved through constraint (11c).

**G. Load Constraints**

We let time-varying forecasts of active and reactive power demands of the loads be denoted by \( P_{i,t}^{L} \) and \( Q_{i,t}^{L} \) respectively. We let \( \rho_{i,t} \in [0, 1] \) denote the pickup ratio (i.e., the ratio of load served with respect to demand) of load \( i \in \mathcal{N}_L \) at time \( t \). Assuming a constant power factor, we have the constraints
\[
\mathbb{R}(s_{i,t}) = -\rho_{i,t} P_{i,t}^{L}, \quad \mathbb{S}(s_{i,t}) = -\rho_{i,t} Q_{i,t}^{L} \quad \forall i \in \mathcal{N}_L. \tag{12}
\]
A monotonically increasing pickup ratio is preferable over the frequent dropping of loads already picked up. This is ensured by the almost-monotonic load restoration constraint
\[
\rho_{i,t} - \rho_{i,t-1} \geq -\epsilon \quad \forall i \in \mathcal{N}_L \forall t \in \mathcal{T} \setminus \{1\} \tag{13}
\]
where parameter \( \epsilon \geq 0 \) allows for a small leeway in monotonicity of load pickup and is chosen by the system operator.

**H. Optimization Problem**

We collect all decision variables at time \( t \) as
\[
X_t = \{ s_{i,t}, v_{i,t}, i_{j,t,ij}, \zeta_{i,t}, \rho_{i,t} \}_{i \in \mathcal{N}_{MT} \cup \mathcal{N}_{ESS} \cup \mathcal{N}_{RES}} \tag{14a}
\]
\[
\text{s.t. } (1) - (13). \tag{14b}
\]
Obtaining an optimal solution to (14) allows the MGC to implement said solution for load restoration. Note that the constraints (2c), (7b), and (11b) are nonconvex, thereby making (14) overall nonconvex and devoid of global optimality guarantees.
III. CONVEX RELAXATION AND SOLUTION USING MPC

In this section, we consider convex relaxations to problem (14), which makes it amenable to MPC. The MPC approach involves solving (14) over subhorizons of $T$, followed by using the subhorizon-optimal solutions to construct a near-optimal solution over $T$ [31]. The nonconvexity arising from DistFlow (2c) can be addressed using a well-studied second-order cone relaxation. We use the aforementioned relaxation for the MPC formulation. Next, we consider the droop bus (11b), which is nonconvex due to the presence of the term $\sqrt{v_i^T}$. It can be relaxed to a second-order cone as follows.

**Lemma 1 [Convex relaxation of (11b)]:** The constraint $\sqrt{v_i^T} = v_i^T - k_Q(\bar{S}(s_i,t) - \bar{Q}_i)$ can be relaxed to a second-order cone given as

$$\left\| \left[\sqrt{v_i^T} - k_Q(\bar{S}(s_i,t) - \bar{Q}_i)\right] v_i + \frac{1}{2} \right\|_2 \leq v_i + \frac{1}{2}.$$ 

**Proof:** See Appendix A.

Finally, we consider the nonconvex ESS complementarity constraints (CC) constraints. We address the nonconvexity by relaxing the joint feasible region of $P_{i,t}^{ch}$ and $P_{i,t}^{dis}$ to its convex hull, whose closed form is given as follows.

**Lemma 2 (Convex hull of nonconvex ESS CC feasible set):** Define the set $\mathcal{P}_i^t \triangleq \{(P_{i,t}^{ch}, P_{i,t}^{dis}) | (7a) - (7b)\}$. Then, the convex hull of $\mathcal{P}_i^t$ is given as

$$\mathcal{P}_i^{t,\text{conv}} = \left\{(P_{i,t}^{ch}, P_{i,t}^{dis}) | P_{i,t}^{ch} \geq 0, P_{i,t}^{dis} \geq 0, \frac{P_{i,t}^{ch}}{P_{i,t}^{dis}} + \frac{P_{i,t}^{dis}}{P_{i,t}^{ch}} \leq 1 \right\}.$$ 

**Proof:** See Appendix B.

The relaxation of CC presented in Lemma 2 replaces the constraints (7). A concern is that the relaxed solution may be physically unimplementable due to noncomplementarity. However, based on results presented in [32], it is possible to recover a charge/discharge schedule, which results in the same power injections (7c) while satisfying CC. For an ESS $i \in \mathcal{N}^{\text{ESS}}$, consider a charge/discharge schedule satisfying constraints in Lemma 2, given as $\mathbf{P}_{i,t}^{ch} \triangleq \{P_{i,t}^{ch} | t \in T\}$ and $\mathbf{P}_{i,t}^{dis} \triangleq \{P_{i,t}^{dis} | t \in T\}$. Consider an alternate schedule given as $\tilde{P}_{i,t}^{ch} \triangleq \{P_{i,t}^{ch} | t \in T\}$ and $\tilde{P}_{i,t}^{dis} \triangleq \{P_{i,t}^{dis} | t \in T\}$, such that $\tilde{P}_{i,t}^{ch} \geq |P_{i,t}^{ch} - P_{i,t}^{dis}|$. The alternate schedule results in the same net injection since

$$\tilde{P}_{i,t}^{dis} - \tilde{P}_{i,t}^{ch} = \left[|P_{i,t}^{dis} - P_{i,t}^{ch}| + |P_{i,t}^{ch} - P_{i,t}^{dis}|\right].$$ 

Furthermore, if $(P_{i,t}^{ch}, P_{i,t}^{dis}) \in \mathcal{P}_i^{t,\text{conv}}$, then it can be verified that $\tilde{P}_{i,t}^{ch}$ and $\tilde{P}_{i,t}^{dis}$ satisfy (7a) - (7b). However, the alternate charging/discharging schedule leads to the original SoC being an underestimate, which we show through induction. Let $\hat{S}_{i,t}$ be the SoC at time step $t$ under the schedule $\mathbf{P}_{i,t}^{ch}$ and $\hat{P}_{i,t}^{dis}$ while $S_{i,t}$ is the SoC under $\mathbf{P}_{i,t}^{ch}$ and $\mathbf{P}_{i,t}^{dis}$. Furthermore, assume that $\hat{S}_{i,t} \geq S_{i,t}$ holds for some time step $t$. Since it holds that $P_{i,t}^{dis} \geq [P_{i,t}^{ch} - P_{i,t}^{dis}]$ and $\eta_i^{ch} \leq 1$, the difference between SoC on subsequent time steps can be expressed as

$$S_{i,t+1} - S_{i,t} = \Delta t \left[\hat{v}_{i,t} - \left(P_{i,t}^{ch} - P_{i,t}^{dis}\right) - \left(\eta_i^{ch} \frac{1}{\eta_i^{dis}} P_{i,t}^{dis}\right)\right] \leq \Delta t \left[\hat{v}_{i,t} - \left(P_{i,t}^{ch} - P_{i,t}^{dis}\right)\right].$$

and therefore, $\hat{S}_{i,t+1} - S_{i,t+1} \geq \hat{S}_{i,t} - S_{i,t}$. Combined with the induction assumption, and starting off from the same initial SoC (i.e., $\hat{S}_{i,0} = S_{i,0}$), it follows that $\hat{S}_{i,t} \geq S_{i,t}$ for all $t \in T$. Therefore, we showed that the convex relaxation presented in Lemma 2 can be converted post hoc into a schedule, which respects ESS CC, at the cost of underestimating the ESS SoC.

A. MPC Implementation

In this section, we briefly sketch out the implementation details of MPC. Even the convexified problem may be difficult to solve over long time horizons, i.e., when $T$ is large since the number of decision variables scales as $O(T)$. MPC posits that (14) may be approximately solved by dividing it into $T$ subproblems, with the $t$th subproblem defined over the variables $\{X_{t}, \ldots, X_{t+H-1}\}$ and considering the relaxed constraints restricted to those involving the variables $\{X_{t-1}, \ldots, X_{t+H-1}\}$. Since $X_{t-1}$ is not a decision variable in the $t$th subproblem, its value is fixed as the optimal value of $X_{t-1}$ derived from the $(t-1)$th subproblem. Thus, MPC allows for a solution of (14) by solving $T$ subproblems over time horizons of size $H$, instead of one problem over a time horizon of size $T$. The scheme is represented visually in Fig. 1. Note that choosing $H = T$ and considering only the first subproblem reduces to solving the relaxed instance of (14) in a one-shot fashion.

IV. SOLUTION USING CPO

We motivate the use of CPO to solve (14) by considering whether the decision variables and available information for the $t$th MPC subproblem (which starts at time $t$), denoted by $X_t \triangleq \{X_t | t \leq t+H-1 \}$ can be split into three groups, viz. 1) state, 2) action, and 3) observation variables, such that they have the following properties.
1. The *state* at time $t$, denoted by $x_t$, represents the smallest set of variables in $X_t$, plus exogenous data such as forecasts $\hat{p}_{\text{RES}}$, $\hat{p}_{L}$, and $Q_{L}^{i}$, which can adequately describe the system at time $t$.

2. The *action* at time $t$, denoted by $u_t$, represents the set of variables in $X_t$, which the MGC can control.

3. The *observation* at time $t$, denoted by $o_t$, denotes all variables in $X_t \setminus \{u_t\}$. Intuitively, the observation variables should be fully specified once $x_t$ and $u_t$ are known.

If such a split is possible, it allows for the introduction of a *policy*, which is a randomized map from a given state to a distribution over possible actions. Should we also define an objective function, which, in the RL framework, is called the *reward*, it becomes a well-posed problem to seek an optimal policy that maps state to actions in a way that maximizes the total accumulation of reward. Similar to MPC, each action is restricted to the subhorizon \( t, \ldots, t + H - 1 \), and load restoration is done in a rolling-horizon fashion.

Armed with the motivation, we pose problem (14) as a constrained Markov decision process (CMDP) under the RL policy-learning framework of CPO. A CMDP is defined as the 6-tuple \( \{X, U, p, R, C, \gamma\} \), where $X$ is the state space, $U$ is the action space, $p: X \times U \times X \mapsto [0, 1]$ is the state transition probability, $R: X \times U \mapsto \mathbb{R}$ is the reward function, $C: X \times U \mapsto \mathbb{R}^M$ is the constraint function, and $\gamma \in (0, 1]$ is the *discount factor* used to de-emphasize the contribution of uncertain future quantities to the reward. For many systems including the one under consideration, $p$ or the state-transition probabilities are deterministic, based on known rules. The constraint function $C$ is used to encode $M \geq 0$ constraints by representing them as $C(x_t, u_t) \leq 0$ on every time step $t$.

We now describe load restoration in the CPO framework by defining salient variables and functions. Then, we describe a tailored training process to find an optimal policy. Finally, we discuss the issue of evaluating various mathematical expressions, which arise during the training process. We define the state $x_t$ and the action $u_t$ at time $t$ as

$$
\begin{align*}
x_t &\triangleq \left\{ S_{i,t-1} \in \text{INRESS}, \ z_{i,t-1} \in \text{INMT}, \ \rho_{i,t-1} \in \text{INL}, \right. \\
&\left. \begin{cases} 
P_{i,t}^{\text{RES}}, \ P_{i,t}^{\text{L}} \end{cases} \in \text{INRESS}, \ V_{i,t} \in \text{INMT}, \ E_{i,t} \in \text{IN}, \right. \\
&\left. \begin{cases} 
\phi_{i,t}, \ Q_{i,t} \end{cases} \in \text{INL}, \right. \\
&\bigg\{ \begin{cases} 
\gamma(s_{i,t}, e_{i,t}) \in \text{INRESS}, \ s_{i,t} \in \text{INRESS}, \ \rho_{i,t} \in \text{INL}, \right. \\
&\left. \begin{cases} 
\phi_{i,t}^{\text{RES}}, \ P_{i,t}^{\text{L}} \end{cases} \in \text{INRESS}, \ \gamma(s_{i,t}, e_{i,t}) \in \text{INRESS}, \\
&\bigg\} \right\}_{t=1}^{t+H-1}.
\end{align*}
$$

The constraints on the variables in action $u_t$ are denoted by the vector-valued function $C(x_t, u_t) \leq 0$. Note that we consider only inequality constraints (“$\leq$”) from load restoration in $\gamma^C$ while equality constraints are embedded implicitly in the observation generation procedure, which solves (14) for all the unspecified variables. This allows for a simulator-free implementation of CPO; indeed, (14) can itself be considered a “simulator” in the current application. We define the reward function and constraint function at time $t$ as

$$
\begin{align*}
\gamma^R(\theta^*, x_t) &\triangleq \mathbb{E}_{u_t \sim \pi^*_\theta} \left[ R(x_t, u_t) | x_t \right] \\
\gamma^C(\theta^*, x_t) &\triangleq \mathbb{E}_{u_t \sim \pi^*_\theta} \left[ C(x_t, u_t) | x_t \right].
\end{align*}
$$

The load restoration problem can be solved by determining a policy $\theta^*$ such that for any state $x$, $\pi^*_\theta$ maximizes $\gamma^R(\pi^*_\theta, x)$ while respecting the constraints $\gamma^C(\pi^*_\theta, x) \leq 0$. Since finding a $\theta^*$, which produces the optimal action for all possible states, is an intractable problem, we instead adopt the framework of training $\theta$ episodically. Thus, a near-optimal policy $\theta^*$ can be found in an
Algorithm 1: Training Load Restoration Policy With CPO.

Input: Initial weights $\theta$, number of episodes $E$, batch size $B$, look-ahead window $H$, stale update parameter $m$, multiple RES and load forecasts $\{\hat{p}_{\text{RES}}, \hat{p}_I, \hat{Q}_L\}$, and not for all $\epsilon > 0$.

Output: Trained policy weights $\theta$ .

1: for $t \in \mathcal{T}$ do
2: Generate state $x_{t-1}$ from $o_{t-1}$, forecasts
3: Sample $N(0, I_d^f)$ $B$ times as $\{e^{(b)}\}_{b \in [B]}$
4: Generate coefficients $a_t, B_t, c_t, F_t$ using Prop. 1 and Lemma 3 and exploration by policy
5: if mod($t, m$) = 0 then ▷FIM stale update
6: Generate $\Sigma^{-1}_\theta$ using Prop. 1 and Lemma 3
7: end if
8: Update $\theta$ by solving (20)
9: end for
10: end for
11: return $\theta$

episodic fashion by sequentially solving the following problem:

$$\theta_{t+1} = \arg \max_\theta \mathcal{R}(\pi_\theta, x_t)$$

where $\delta > 0$ is the trust region parameter to ensure that successive policies do not have large variations. The final policy $\theta'$ is then determined as $\theta' = \lim_{t \to -\infty} \theta_t$. Since the original problem (14) is only defined for $t \in \mathcal{T}$ and not for all $t \in \mathbb{N}$, we “loop back” to a different initial state $x_0$ at time $T + 1$. This idea of training multiple times over $\mathcal{T}$ using different initial conditions is referred to as episodic training.

Directly solving (19) is challenging due to the highly nonlinear and nonconvex optimization landscape of (19), which arises due to the nonlinearity of FNNs. In order to alleviate this challenge, we replace (19) with a quadratically constrained linear program (QCLP) approximation, which is well known in the literature [21]

$$\theta_{t+1} = \arg \max_\theta a_t^\top (\theta - \theta_t)$$

where $\delta > 0$ is the trust region parameter to ensure that successive policies do not have large variations. The final policy $\theta'$ is then determined as $\theta' = \lim_{t \to -\infty} \theta_t$. Since the original problem (14) is only defined for $t \in \mathcal{T}$ and not for all $t \in \mathbb{N}$, we “loop back” to a different initial state $x_0$ at time $T + 1$. This idea of training multiple times over $\mathcal{T}$ using different initial conditions is referred to as episodic training.

The aforementioned QCLP approximation is obtained by replacing the objective function (19a) and constraint function (19b) with their first-order Taylor approximations (20a) and (20b), respectively. The KL divergence constraint (19c) is replaced with its second-order Taylor approximation (20c) since its first-order approximation vanishes [33]. $F_t$ is positive definite by construction [33], and therefore, constraint (20c) is convex.

The challenge in adapting CPO to any given application is to find an accurate and efficient algorithm for computing $a_t, B_t, c_t,$ and $F_t$ on every time step $t$. In the following proposition, we present a procedure to generate said variables on every time step, which simply requires an expectation over a standard normal distribution, along with the partial derivative of the reward and constraint functions with respect to action. This is a significant improvement over the training procedure presented in [10], which requires multiple matrix inversions and matrix exponential evaluations per time step.

Proposition 1 [Parameters in problem (20)]: Define the variables $\mu_\theta \triangleq L_{\mu}(f_\theta(x_t))$ and $\Sigma_\theta \triangleq M(L_{\Sigma}(f_\theta(x_t)))$. Then, we have

$$a_t = \left( E_{x \sim N(0, I_d^f)} \left[ \frac{\partial R_t}{\partial u_t} \left( (\epsilon \otimes I_d^d) \frac{\partial \nu_\theta}{\partial \theta} + \frac{\partial \mu_\theta}{\partial \theta} \right) x_t \right] \right)_{\theta = \theta_t},$$

$$B_t = \left( E_{x \sim N(0, I_d^f)} \left[ \frac{\partial C_t}{\partial u_t} \left( (\epsilon \otimes I_d^d) \frac{\partial \nu_\theta}{\partial \theta} + \frac{\partial \mu_\theta}{\partial \theta} \right) x_t \right] \right)_{\theta = \theta_t},$$

$$c_t = \left( \Sigma_\theta^{-1} \frac{\partial \nu_\theta}{\partial \theta} + \frac{\partial \mu_\theta}{\partial \theta} \right)_{\theta = \theta_t},$$

$$F_t(i, j) = \left[ \frac{\partial \mu_\theta}{\partial \theta(i)} \Sigma_\theta^{-1} \frac{\partial \mu_\theta}{\partial \theta(j)} + \frac{1}{2} \Tr \left( \Sigma_\theta^{-1} \frac{\partial \Sigma_\theta}{\partial \theta(i)} \Sigma_\theta^{-1} \frac{\partial \Sigma_\theta}{\partial \theta(j)} \right) \right]_{\theta = \theta_t},$$

where $R_t \triangleq R(x_t, u_t)$, $C_t \triangleq C(x_t, u_t)$, $\nu_\theta \triangleq \text{vec}(\Sigma_\theta)$, and $F_t(i, j)$ and $\theta(i)$ are the $(i, j)$th and $i$th element of $F_t$ and $\theta$, respectively.

Proof: See Appendix C.

In the following remark, we highlight some implementation details of the formulae presented in Proposition 1.

Remark 1 (Implementation of Proposition 1): First, we observe that in their definitions, $a_t$ and $B_t$ are defined in the form of an expectation over $\epsilon \sim N(0, I_d^f)$, which would result in the linear term $\epsilon^\top$ inside the expression being nullified. In practice, we implement the expectation through a sample average over samples of $\epsilon$, which are produced by a random number generator, and since we do not use infinite samples, the term $\epsilon^\top$ is not nullified. Second, we note that evaluating $F_t$ requires inverting the matrix $\Sigma^{-1}_\theta \in \mathbb{R}^{d \times d}$ on every time step. As a workaround, we implement stale updates, which

...
means that $\Sigma_\theta^{-1}$ is updated only once every $m$ time steps. Third, note that once $x_t$ and $u_t$ are known, the variables in $o_t$ can be derived by solving a constraint satisfaction problem using the constraints of (14). This is equivalent to using (14) as an environment for the CPO agent in lieu of external simulators.

The partial derivatives of $v_\theta$, $\mu_\theta$, and $\Sigma_\theta$ in Proposition 1 with respect to $\theta$ may be calculated via backpropagation operation [22], Algorithms 6.3 and 6.4] on the FNN, which is a standard operation for any software capable of handling FNNs. Now, it only remains to develop a procedure to evaluate $\frac{\partial h_t}{\partial u_t}$ and $\frac{\partial h_t}{\partial \mu_t}$. The procedure we develop is unique to the DistFlow equations and is similar for both the terms, and therefore, we only demonstrate the evaluation of $\frac{\partial h_t}{\partial u_t}$. For generic variables $a$ and $b$, let the expressions $a \in R_t$ and $b \in u_t$ imply that $a$ makes an appearance in the closed form of $R_t$, and $b$ is a variable in $u_t$. The evaluation of $\frac{\partial h_t}{\partial u_t}$, therefore, boils down to the efficient evaluation of $\frac{\partial h_t}{\partial \theta}$. To this end, we collect the equations, which couple variables in $x_t$, $u_t$, and $o_t$ over the time horizon $\{t, \ldots, t+H-1\}$, and evaluate their total differentials as follows:

\begin{align}
    ds_{i,t} &= \sum_{j \in k} dS_{j,k,t} - \sum_{i \in j} (dS_{i,j,t} - z_{ij} dl_{ij,t}) \quad (21a) \\
    dv_{i,t} &= dv_{i,t} - 2\mathbb{R}(\bar{z}_{ij} ds_{i,j,t}) + |z_{ij}|^2 dl_{ij,t} \quad (21b) \\
    dl_{ij,t}v_{i,t} + l_{ij,t} dv_{i,t} = 2\mathbb{R}(\bar{z}_{ij} ds_{i,j,t}) \quad (21c) \\
    P_{i,t}^{\text{dis}}P_{i,t}^{\text{ch}} + P_{i,t}^{\text{ch}}P_{i,t}^{\text{dis}} = 0 \quad (21d) \\
    \mathbb{R}(ds_{i,t}) = dP_{i,t}^{\text{dis}} - dP_{i,t}^{\text{ch}} \quad (21e) \\
    dS_{i,t+1} = dS_{i,t} + (P_{i,t}^{\text{ch}}\Delta_t)P_{i,t}^{\text{dis}} - \left(\frac{\Delta_t}{P_{i,t}^{\text{ch}}}\right) dP_{i,t}^{\text{dis}} \quad (21f) \\
    d\xi_{i,t+1} = d\xi_{i,t} - \tau_i \mathbb{R}(ds_{i,t}) \quad (21g) \\
    \mathbb{R}(ds_{i,t}) = -d\xi_{i,t}P_{i,t}^{\text{RES}} \quad (21h) \\
    d\omega_t = -k_P \mathbb{R}(ds_{i,t}) \quad (21i) \\
    \left(\sqrt{\omega_t}\right)^{-1} dv_{i,t} = -2k_Q \mathbb{R}(ds_{i,t}) \quad (21k)
\end{align}

where (21a) holds for all $j \in N$, (21b)–(21c) hold for all $i \to j$, (21d)–(21f) hold for all $i \in N^{\text{ESS}}$ (21g) holds for all $i \in N^{\text{MT}}$ (21h) holds for all $i \in N^{\text{L}}$, (21i) holds for all $i \in N^{\text{RES}}$, and (21j)–(21k) hold for all $i \in N^{\text{loop}}$.

Note that (21) is a homogeneous system of linear equations in the differentials, which can be used to numerically calculate the required partial derivatives via the following result.

**Lemma 3:** The partial derivative $\frac{\partial h_t}{\partial \theta}$ for $a \in R_t$ and $b \in u_t$ may be evaluated by setting $db' = 0$ for all $b' \in u_t$ such that $b \neq b'$, and solving (21) for $da$ and $db$. A solution exists if the number of loads exceeds nonloads, in which case $\frac{\partial h_t}{\partial \theta} = \frac{da}{db}$.

**Proof:** See Appendix D.

All the steps involved in CPO training are summarized in Algorithm 1.

### A. MG Implementation

After we train the FNN for a large number of time steps $t$ in an episodic fashion, the trained FNN may be used as the MGC. If deterministic policies are desired, then the mean $\mu_x$ may be used while the covariance matrix $\Sigma_x$ may be nullified. On the other hand, if stochastic policies are desired, both $\mu_x$ and $\Sigma_x$ may be retained.

### V. Simulation Results

In this section, we use simulations to validate the performance of the proposed MPC and CPO approaches. In order to model the MG, we use a modified IEEE 37-bus test feeder [34], which prescribes network topology and power injection data. The 37-bus system is modified to a 36-bus system by deleting a bus interfaced with the network through a transformer, thereby maintaining the same voltage levels across the MG. We also consider an MG based on a larger 141-bus radial feeder derived from case141 in MATPOWER [35].

All simulations were carried out in Python on a PC with an Intel Core i7 CPU, NVIDIA 1060Ti GPU, and 32 GB of RAM. The Gurobi solver [36], interfaced with Python through CVXPY [37], was used to find solutions for the MPC problem as well as solving (20). All FNN operations, as well as gradient calculations for Lemma 3, are calculated with PyTorch and the automatic differentiation engine PyTorch Autograd.

We tested three algorithms, which can solve (14) are as follows.

1) **M1:** This method uses a stochastic CPO policy trained according to (20a) for 1000 episodes for the 36-bus MG and 3000 episodes for the 141-bus MG. We use $H = 5$ and $\gamma = 0.8, 0.9, 1.0$, and retain the value of $\gamma$ with the best performance. The parameters for the FNN used as the CPO policy, as well as other parameters involved in CPO, are displayed in Table II. In order to relieve the

| System Parameter | Value |
|------------------|-------|
| $T$ (hours)      | 24    |
| $(\rho(p,u), \rho(p,u))$ | $(0.95, 1.05)$ |
| $(\rho^{\text{MT}}(\text{kW}), \rho^{\text{MN}}(\text{kW}))$ | $(0, 250, 0.9)$ |
| $(\rho^{\text{MT}}(\text{kW}), \rho^{\text{MN}}(\text{kW}-1))$ | $(930, -30, 25)$ |
| $(\rho^{\text{MT}}(\text{kW}), \rho^{\text{MN}}(\text{kW}))$ | $(25, 25, 0.9, 0.9)$ |

| CPO Parameter | 36-bus MG | 141-bus MG |
|---------------|-----------|------------|
| FNN layer sizes | $(544, 100, 75, 440)$ | $(1403, 500, 250, 1630)$ |
| Activation function | tanh | tanh |
| Test region $\tau$ | 0.001 | 0.008 |
| $\epsilon$ samples per time step | 32 | 64 |
| State update param. $m$ | 5 | 5 |

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The 36-bus MG that is adapted from the IEEE 37-bus distribution feeder.

Fig. 3. The 36-bus MG that is adapted from the IEEE 37-bus distribution feeder.

computational burden, we choose $\Sigma_\theta$ to be a diagonal matrix.

2) $M2$: This method uses MPC to solve the proposed convex relaxation. We use $H = 5$ and the convex relaxations proposed in Section III.

3) $M3$: We refer to this method as brute-force learning (BFL). It involves generating state–action pairs $(x_k, u_k)$, wherein the optimal actions are generated via M2. The data are generated for various renewable forecasts, similar to Algorithm 1. Then, we train an FNN on the input–output pairs. This method evaluates the performance of deep architectures, which are trained in a supervised fashion, and are not privy to constraint-respecting training as in CPO.

A. 36-Bus MG

We first consider simulation results for the 36-bus MG. The parameters for various elements in the MG are listed in Table I, and a one-line diagram of the MG is presented in Fig. 3. We model the droop bus as an additional MT with similar parameters. The cost functions in (1) are chosen as $C_{\text{L}_i}^M(\Re(s_{i,t})) = -\Re(s_{i,t})$ (since load demands are negative) and $C_{\text{MT}_i}^M(\Re(s_{i,t})) = -0.75\Re(s_{i,t})$. The RES outputs are modeled as having a particular shape, as shown in Fig. 4(a) and (c), and during training, they are perturbed uniformly by a factor of $[0.75, 1.25]$ on each time step. The loads are chosen to be constant for the training in order to better demonstrate load restoration performance, and their values are derived from the case data.

The training curves for different values of $\gamma$ are presented in Fig. 4(f). In the figure, the solid line shows the mean reward collected while the shaded region indicates the variance of rewards over multiple training runs. In total, 500 training runs of Algorithm 1 were carried out and the best policy was used to generate final results and plots. From the figure, it can be seen that setting $\gamma = 1$ leads to the highest reward collection and also the lowest variance during training. Thus, $\gamma = 1$ is used for the remaining experiments with the 36-bus MG. Next, we compare the load restoration process with uncertain values of $P_{\text{RES}}$. In order to compare the load restoration performance of the three competing methods, we consider three scenarios, as shown in Fig. 4(a) and (c). In the first scenario with a perfect forecast, the performance of CPO is slightly worse than MPC and BFL, which demonstrates that under the availability of perfect information, MPC solutions can be of higher quality than CPO. However, in scenarios where the actual RES output is lower than forecasts, as shown in Fig. 4(b) and (c), CPO shows better performance in terms of initial load pickup, as well as achieving full load restoration. This is due to the capability of CPO to learn from experience, and therefore, during training, it learns the best schedule for load restoration even under imperfect information. As opposed to this, MPC is restricted to only using forecast inputs for the current time step to $H$ time steps in the future, which poses a disadvantage in uncertain systems.

Next, we consider two relaxations proposed in Section III. Fig. 4(d) shows the charge and discharge performance for the ESS. Recall that for MPC, we use the relaxation proposed in Lemma 2, and the same is also inherited BFL ensuring its training. The gray shaded areas represent the time steps when MPC does not respect CC while the blue shaded areas do the same for CPO. From here, it can be seen that CPO creates solutions that adhere better to CC than MPC. Furthermore, even in the time steps when CPO violated CC, the magnitude of its violation is much lower than the solution produced by MPC and BFL. Therefore, we conclude that it is possible to implement the charge/discharge schedule produced by CPO directly to the MG while MPC and BFL schedules require the post hoc modification as discussed in Section III.

Finally, we consider the relaxation for the voltage droop constraint presented in Lemma 1. It is possible to calculate the gap between the voltages derived as a result of the relaxed constraint and the actual voltage of the unrelaxed constraint by evaluating (11c) using reactive power injections. From here, it can be seen that MPC and BFL incur large gaps between the relaxation and the actual voltages, especially in the initial time steps. This shows that the convex relaxation proposed in Lemma 1 may not be tight in practice. However, the performance of CPO is much better in terms of the gap, thereby demonstrating that CPO is a better approach to satisfy the nonlinear droop constraints than relaxing them to a convex inequality constraint. However, further studies are required to establish conditions under which the proposed relaxation is tight and holds exactly for all time steps.

B. 141-Bus MG

We now consider the simulation results for the 141-bus MG. The load demands are chosen from the case data, and multiple generation sources are incorporated into the MG. We place 7 MTs at buses $\{3, 51, 56, 77, 91, 123, 138\}$, 3 RESs at buses $\{12, 43, 88\}$, 6 ESS at buses $\{16, 41, 77, 83, 124, 139\}$, and buses 56 and 91 operate under the principle of droop control.
Fig. 4. Simulation results for the 36-bus MG. (a) Loads restored by various algorithms under perfect RES output forecast. (b) Loads restored by various algorithms under imperfect RES output forecast. (c) Loads restored by various algorithms under highly imperfect RES output forecast. (d) Charge and discharge power trajectories at ESS on bus 5 for various algorithms. Complementarity violations are highlighted. (e) Violations of droop equation (11b) at bus 11 resulting from various algorithms. (f) Rewards accumulated during training as a function of episodes (epochs). The standard deviations of different training runs are plotted.

Fig. 5. Simulation results for the 141-bus MG. (a) Loads restored by various algorithms under imperfect RES output forecast. (b) Rewards accumulated during training as a function of episodes (epochs). The standard deviations of different training runs are plotted. (c) Time-averaged absolute value of gap in equation (2c) under different algorithms.

As in the 36-bus MG, the total load demands to be restored are constant and shown in Fig. 5(a) while the RES forecasts (with mispredictions) are also displayed in the same figure.

The training process of the CPO agent, as shown in Fig. 5(b), shows that the best choice for parameter $\gamma$ is $\gamma = 0.9$. Unlike the 36-bus MG, choosing $\gamma = 1$ leads to a very unstable training trajectory, wherein the rewards remain low throughout training. Thus, we choose $\gamma = 0.9$ for our experiments. The load restoration process under uncertain RES forecasts, as shown in Fig. 5(a), shows that CPO performs slightly better than MPC and BFL, both in terms of load restored on any given time step, as well as reaching full load restoration. Furthermore, we compare the performance of CPO in satisfying the nonconvex DistFlow (2c). Recall that MPC and BFL use the relaxed second-order cone version of this constraint discussed in Section III. In Fig. 5(c), we calculate the gap resulting from this constraint not binding for all three methods. It can be seen that the gap for CPO is an order of magnitude lower in terms of maximum value than MPC or BFL. On the other hand, MPC and BFL enjoy zero gap on most buses, but an extremely high value of the gap on certain buses. Thus, the solutions produced by these two methods enjoy lower exactness than the ones produced by CPO.

Finally, we discuss the time taken for both training and online runtime for all three methods, which are presented in Table III. The training time of CPO involves all steps presented in Algorithm 1 while the training time of BFL includes time taken for the generation of data samples as well as training the FNN on these samples. While the training time for CPO is significantly longer than BFL, it also produces small runtimes. On the other

| Method | 36-bus MG time | 141-bus MG time |
|--------|----------------|-----------------|
|        | Training       | Runtime         | Training       | Runtime         |
| CPO    | 4310s          | 0.51s           | 33780s         | 0.76s           |
| MPC    | -              | 2.76s           | -              | 7.59s           |
| BFL    | 345s           | 0.38s           | 1215s          | 0.62s           |
hand, since MPC has to solve multiple optimization problems per time horizon, it results in the longest runtime. BFL has a smaller runtime than CPO due to a simpler FNN structure, which does not take variance into account. From here, we see that CPO is competitive with respect to MPC in terms of runtime but this comes with a tradeoff of a far longer training time, which is absent in the case of MPC.

VI. CONCLUSION

In this article, we considered the load restoration problem for an islanded MG, which contains sources of distributed power generation such as RES and MTs as well as sources of energy storage such as ESS. Two approaches to find a solution to the problem, which can be implemented in the MGC, were studied. We considered MPC and proposed a convex relaxation of the load restoration problem, which can be efficiently solved. We also developed a CPO that finds an optimal policy through episodic training. Then, we compared the performance of MPC and CPO on 36-bus and 141-bus MGs. An important direction of future extension is to consider topology-switching MGs with discrete decision variables and implement policies for the same using CPO. Other directions of investigation involve reducing CPO training time through more efficient training strategies and modeling of power-sharing strategies in highly unbalanced multiphase MGs.

APPENDIX

A. Proof of Lemma 2

We will demonstrate that the provided second-order cone can be derived from the equation \( \sqrt{v_{i,t}} = \sqrt{v_{i}^2} - kQ(3s_{i,t}) - Q_i^1 \) through a sequence of relaxations. We denote relaxations through the \( \text{relax} \) symbol

\[
\begin{align*}
\sqrt{v_{i,t}} &= \sqrt{v_{i}^2} - kQ(3s_{i,t}) - Q_i^1 \\
\implies v_{i,t}^2 &= \left( \sqrt{v_{i}}^2 - kQ(3s_{i,t}) - Q_i^1 \right)^2 \\
\implies (v_{i,t})^2 + v_{i,t} + \frac{1}{4} &= \left( \sqrt{v_{i}}^2 - kQ(3s_{i,t}) - Q_i^1 \right)^2 \\
&+ (v_{i,t})^2 + \frac{1}{4} \\
\implies \frac{(v_{i,t} + \frac{1}{2})^2}{2} &= \left\| \left[ \sqrt{v_{i}}^2 - kQ(3s_{i,t}) - Q_i^1 \right] \left[ v_{i,t} \ 1 \right] \right\|_2^2 \\
\Rightarrow \text{relax} \ v_{i,t} + \frac{1}{2} &= \left\| \left[ \sqrt{v_{i}}^2 - kQ(3s_{i,t}) - Q_i^1 \right] \left[ v_{i,t} \ 1 \right] \right\|_2^2.
\end{align*}
\]

B. Proof of Lemma 1

Note that the points \((0,0), (0, P_{ch}), \) and \((P_{ch}, 0)\) are contained in \(P_i\), and therefore, any convex set that contains \(P_i\) should contain \( \text{conv}\{(0,0), (0, P_{ch}), (P_{ch}, 0)\} \), which is exactly \(P_{conv}^i\). Thus, \(P_{conv}^i \subseteq \text{conv}(P_i)\). On the other hand, \(P_{conv}^i\) is convex and contains the set \(P_i\), and therefore, \(\text{conv}(P_i) \subseteq P_{conv}^i\). It follows that \(\text{conv}(P_i) = P_{conv}^i\).

C. Proof of Theorem 1

The main idea of the proof is to replace the reward function, constraint function, and KL-divergence terms in (19) with their respective Taylor-series approximations around \(\theta_i\). The first-order approximation of the objective function is given as

\[
\mathcal{J}^R(\pi_\theta, x_i) = \mathcal{J}^R(\pi_{\theta_i}, x_i) + \nabla_\theta \mathcal{J}^R(\pi_{\theta_i}, x_i) \big|_{\theta = \theta_i} (\theta - \theta_i)
\]

and by comparing the previous approximation with (20), we see that \(a_k = \nabla_\theta \mathcal{J}^R(\pi_{\theta_i}, x_i) \big|_{\theta = \theta_i} \). In order to compute \(a_k\), we need a closed form of the gradient

\[
\nabla_\theta \mathcal{J}^R(\pi_{\theta_i}, x_i) = \nabla_\theta \mathbb{E}_{x_i \sim \pi_{\theta_i}} \left[ R(x_i, u_i) \mid x_i \right]
\]

which is difficult to evaluate since the gradient is with respect to \(\theta\) that parameterizes the distribution over which the expectation is being taken. To alleviate this difficulty, we use the reparametrization trick. For a standard normal vector \(\epsilon \sim \mathcal{N}(0, I^p_{\theta})\), it holds that

\[
\Sigma_\theta \epsilon + \mu_\theta \sim \mathcal{N}(\mu_\theta, \Sigma_\theta \Sigma_\theta^\top).
\]

Therefore, letting \(u_i = \Sigma_\theta \epsilon + \mu_\theta\) is equivalent to defining \(u_i\) as a Gaussian random vector with mean and variance given in (18). Thus, we have

\[
\begin{align*}
\nabla_\theta \mathbb{E}_{x_i \sim \pi_{\theta_i}} \left[ R(x_i, u_i) \mid x_i \right] &= \nabla_\theta \mathbb{E}_{\epsilon \sim \mathcal{N}(0, I^p_{\theta})} \left[ R(x_i, \Sigma_\theta \epsilon + \mu_\theta) \mid x_i \right] \\
&= \mathbb{E}_{\epsilon \sim \mathcal{N}(0, I^p_{\theta})} \left[ \partial R_i \partial u_i \partial \Sigma_\theta \partial \theta + \partial R_i \partial u_i \partial \mu_\theta \partial \theta \mid x_i \right] \\
&= \mathbb{E}_{\epsilon \sim \mathcal{N}(0, I^p_{\theta})} \left[ \partial R_i \partial u_i \partial \Sigma_\theta \partial \theta + \partial R_i \partial u_i \partial \mu_\theta \partial \theta \mid x_i \right] \mathcal{N}(0, I^p_{\theta}).
\end{align*}
\]

Computing \(\frac{\partial u_i}{\partial \Sigma_\theta} \frac{\partial \Sigma_\theta}{\partial \theta}\) involves a multiplication of two tensors, which can be bypassed by vectorizing \(\Sigma_\theta\). It follows that:

\[
\frac{\partial u_i}{\partial \Sigma_\theta} \frac{\partial \Sigma_\theta}{\partial \theta} = \frac{\partial u_i}{\partial \Sigma_\theta} \frac{\partial \Sigma_\theta}{\partial \theta} = (\epsilon \otimes I^p_{\theta}) \frac{\partial \Sigma_\theta}{\partial \theta}.
\]

On the other hand,

\[
\frac{\partial u_i}{\partial \mu_\theta} \frac{\partial \mu_\theta}{\partial \theta} = \frac{\partial \mu_\theta}{\partial \theta} \times \frac{\partial \Sigma_\theta \epsilon + \mu_\theta}{\partial \theta} = \frac{\partial \epsilon}{\partial \theta}.
\]

This verifies the closed form of \(a_k\) given in the theorem. The closed form of \(B_i\) can be similarly derived by computing the Jacobian of \(\mathcal{J}^C(\pi_{\theta_i}, x_i)\) at \(\theta = \theta_i\) (with \(c_i\) simply being the zeroth-order term in the Taylor expansion), carrying out the reparametrization trick and then deriving the closed form of the partial derivatives.

The first-order term in the Taylor approximation of constraint (19c) vanishes, and the second-order term is used in the relaxed constraint (20c). Matrix \(F_i\) is the Fisher information matrix (FIM) that is positive semidefinite by construction. As provided in the theorem statement, the closed form of FIM for a Gaussian vector is well known, see e.g., [33].

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D. Proof of Lemma 3

The numerical result proposed in the Lemma arises from the following observation.

**Remark 2 (Evaluating partial from total derivatives):** For a generic problem, let the independent variable \( p \) and dependent variable \( q \) be related through the implicit equation \( F(p, q) = 0 \), where \( F \) is a smooth function. The total differential of \( F(p, q) = 0 \) is given as

\[
\left[ \frac{\partial F(p, q)}{\partial p} \right]^T dp + \left[ \frac{\partial F(p, q)}{\partial q} \right]^T dq = 0.
\]  
(23)

To calculate \( \frac{\partial F}{\partial q} \), we set \( dp_j = 0 \) for all \( q \neq j \) (a partial derivative with respect to \( p_j \) means that any variables \( p_g \) with \( g \neq j \) are assumed to be constant), and solve (23) for \( dp_j \) and \( dq \). From here, we have \( \frac{\partial q}{\partial p_j} \).

Now, we show that the system of (21) always admits a solution when all action variables in \( \mathbf{u}_t \) except one are nullified. We consider the real and imaginary parts of any of the complex differential equivalent to two real independent differentials. First, we calculate the number of linearly independent equations in (21). In order to detect linear dependencies, we note that equations among (21d)–(21k), which only contain a single term of the form \( \Re(ds_{i,j}) \) or \( \Im(ds_{i,j}) \), may be combined into (21a). Using this observation, we note that (21e) and (21g)–(21k) are linearly dependent on (21a). The total number of linearly independent equations, denoted by \( \lambda_{\text{eqn}} \), is, therefore, given as

\[
\lambda_{\text{eqn}} = H \left( 2|\mathcal{N}| + 2|\mathcal{E}| + 2|\mathcal{N}_{\text{ESS}}| + 2|\mathcal{N}_{\text{dEEP}}| \right).
\]

On the other hand, the number of variables in \( \mathbf{u}_t \) is given as

\[
\lambda_{\text{action}} = H \left( 2|\mathcal{N}_{\text{MT}}| + 2|\mathcal{N}_{\text{RES}}| + 3|\mathcal{N}_{\text{ESS}}| + |\mathcal{N}_d| \right).
\]

while the total number of variables in (21) is given as

\[
\lambda_{\text{vars}} = H \left( 3|\mathcal{N}| + 3|\mathcal{E}| + 3|\mathcal{N}_{\text{ESS}}| + |\mathcal{N}_{\text{MT}}| \right)
+ |\mathcal{N}_{\text{RES}}| + |\mathcal{N}_d| + 1.
\]

From the above, it can be verified that \( \lambda_{\text{vars}} - \lambda_{\text{action}} + 1 - \lambda_{\text{eqn}} > 0 \) when \( |\mathcal{N}_d| \geq |\mathcal{N}_{\text{MT}}| + |\mathcal{N}_{\text{RES}}| + |\mathcal{N}_{\text{ESS}}| \). Thus, the system of (21) when all but one action variable differentials are nullified is a strictly underdetermined system of homogeneous equations. Thus, it has a nonempty nullspace, and therefore, a solution always exists.

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Shourya Bose (Graduate Student Member, IEEE) received the B.E. and M.Sc. degrees in electrical and electronics engineering and mathematics from BITS Pilani, KK Birla Goa Campus, Sancoale, India. He is currently working toward the Ph.D. degree in electrical and electronics engineering with the University of California, Santa Cruz, Santa Cruz, CA, USA.

His research interests involve addressing problems in power systems engineering using tools from optimization theory, machine learning, and control theory.

Mr. Bose was a co-receipient of the Early Career Best Paper Award given by the Energy, Natural Resources, and the Environment (ENRE) section of the Institute of Operations Research and the Management Sciences (INFORMS) in 2021.

Yu Zhang (Member, IEEE) received the Ph.D. degree in electrical and computer engineering from the University of Minnesota, Minneapolis, MN, USA, in 2015. He is currently an Assistant Professor with ECE Department, University of California, Santa Cruz (UCSC), Santa Cruz, CA, USA. Prior to joining UCSC, he was a Postdoctoral Researcher with the University of California, Berkeley, USA, and Lawrence Berkeley National Laboratory, Berkeley. His research interests include cyberphysical systems, smart power grids, optimization theory, machine learning, and Big Data analytics.

Dr. Zhang was the recipient of the Hellman Fellowship in 2019. He was also the co-recipient of the Early Career Best Paper Award given by the Energy, Natural Resources, and the Environment (ENRE) section of the Institute of Operations Research and the Management Sciences (INFORMS) in 2021.