Industrial equipment sizes optimization

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Abstract. The method of optimizing the sizes of vessels from the conditions of minimum mass and minimum surface area of the housing has been considered. The dependences of the optimal diameter, height, mass and minimum surface area of the hull on the volume of the vessel are obtained. Optimization of dimensions can lead to significant savings of structural materials and, accordingly, to reduction of the cost of vessels.

1. Introduction
In various industries, vessels and tanks of different technological appointment are widely applied. Cylindrical vessels with elliptical, conical, flat and other bottoms (lids) are most common due to relatively simple and inexpensive manufacturing technology.

It is important to note that the optimization of the process equipment dimensions should take into account many factors [1, 2]: the required volume of the vessel, determined by the capacity of the process unit, the design pressure and temperature, the properties of the structural material used, the shape and the production technology of the vessel, etc.

There are two main approaches to optimizing the size of vessels. In most cases, optimization is carried out in order to reduce the consumption of structural material necessary for the manufacture of the vessel, and accordingly reduce the cost of equipment [3].

In some cases, it is advantageous to optimize the dimensions according to the minimum side surface area of the vessel. This reduces the quantity and cost of coating materials, such as thermal insulation materials [4].

The greatest savings in materials and a significant reduction in material costs can be obtained by optimizing the size of large-capacity vessels, for example, vertical cylindrical tanks that are widely used in the petrochemical industry, at the stage of their design.

In order to determine the optimum dimensions of the vessel, it is necessary to express its mass or side surface area through the desired dimension, for example the inner diameter. Further, the first derivative of the objective function is calculated, equated to zero and, solving the obtained equation, the desired optimal size value is determined. A prerequisite for the minimum value of the objective function is the negative value of its second derivative [2-4].

2. Optimization of dimensions of vessels operating under excessive pressure based on minimum material capacity of the housing
Consider a cylindrical vessel with an elliptical bottom and a lid. Expression the mass of a vessel:

\[ m = \rho_m \left( A_c S_c + A_b S_b + A_l S_l \right), \]  

(1)
where $\rho_m$ – density of the material from which the vessel is made, kg/m$^3$; $A_c$, $s_c$ – surface area (m$^2$) and wall thickness (m) of the vessel body; $A_b$, $s_b$ – surface area (m$^2$) and wall thickness (m) of the vessel bottom; $A_l$, $s_l$ – surface area (m$^2$) and wall thickness (m) of the vessel lid.

As a rule, at the design stage, the nominal volume $V$ of the vessel and the structural material are known. Therefore, after expressing the values included in equation (1) through the diameter $D$ of the vessel, a functional dependence of the mass of the vessel on the inner diameter is obtained:

$$m = f(D).$$  \hspace{1cm} (2)

In order to determine the optimal diameter corresponding to the minimum material intensity it is necessary to solve the equation:

$$\frac{dm}{dD} = 0.$$  \hspace{1cm} (3)

In addition, verify that the condition is met:

$$\frac{d^2m}{dD^2} > 0.$$  \hspace{1cm} (4)

When this condition is satisfied the optimum height of the vessel body $H_c$ can be determined using the found value $D$ and the given $V$.

The volume of the cylindrical part with $H_c$ height is equal to:

$$V_c = \pi D^2 H_c / 4.$$  \hspace{1cm} (5)

Volume of elliptical bottom (lid):

$$V_b = V_i = \pi D^3 / 24.$$  \hspace{1cm} (6)

Find the full volume of the vessel:

$$V = V_c + V_b + V_i = \pi D^3 H_c / 4 + \pi D^3 / 12.$$  \hspace{1cm} (7)

Let us express the height of the vessel body:

$$H_c = 4V / \pi D^3 - D / 3.$$  \hspace{1cm} (8)

Side surface area of cylindrical part:

$$A_c = 4V/D - 1.05D^2.$$  \hspace{1cm} (9)

Surface area of elliptical bottom (lid):

$$A_b = A_i = 1.24D^2.$$  \hspace{1cm} (10)

Then the total area of the side surface of the cylindrical vessel with the elliptical bottom and lid:

$$A = A_c + A_b + A_i = 4V/D + 1.43D^2.$$  \hspace{1cm} (11)

Thickness of cylindrical and elliptical wall of devices operating under internal pressure is determined by expression [5]:

$$s = pD/(2[\sigma]\phi - p) + c,$$  \hspace{1cm} (12)

where $p$ is the pressure in the vessel, Pa; $[\sigma]$ – the allowable stress of the structural material, Pa; $\phi$ – the weld strength factor; $c$ – the increase to the calculated wall thickness, m.

To simplify the expression, use a design complex that takes into account the pressure and allowable stress of the structural material [6]:

$$s = pD/((2[\sigma]\phi - p) + c).$$
\[ k_s = p \bigg( \frac{2[\sigma]\phi}{\sigma} - p \bigg), \]

then

\[ s = k_s D + c. \]  \hspace{1cm} (13)

Expression the mass of the vessel through its inner diameter:

\[ m = \rho_m \left[ 4V \left( k_s + c / D \right) + 1,43D^2 \left( k_s D + c \right) \right]. \]  \hspace{1cm} (14)

To determine the optimal diameter it is necessary to find the derivative of the obtained function by diameter:

\[ \frac{dm}{dD} = -4Vc / D^2 + 2,86cD + 4,29k_s D^2. \]  \hspace{1cm} (15)

Let us check performance of a condition (4):

\[ 8V_c / D^3 + 2,86c + 8,58k_s D > 0, \]  \hspace{1cm} (16)

condition is met.

We equate the first derivative to zero:

\[ 4,29k_s D^4 + 2,86cD^3 - 4Vc = 0. \]  \hspace{1cm} (17)

The resulting equation has no analytical solution but can be solved graphically, for example, using the Mathcad software package. It is possible to determine the height of the elliptical bottom (lid) using the value \( D \):

\[ H_s = H_l = 0,25D. \]  \hspace{1cm} (18)

The total height \( H \):

\[ H = 4V / \pi D^3 + D / 6. \]  \hspace{1cm} (19)

In addition, the minimum mass of the vessel body \( m_{\text{min}} \) according to the equation (15).

Using the obtained expressions for vessels of different volumes, the dependencies of the optimal diameter (figure 1) and the optimal height (figure 2) corresponding to the minimum mass (figure 3) of the vessel were constructed (the body material is steel 10, \( [\sigma] = 117 \cdot 10^6 \) Pa, \( \varphi = 1 \), \( c = 1,5 \cdot 10^{-3} \) m, \( p = 0,5 \cdot 10^6 \) Pa, \( k_s = 2,141 \cdot 10^{-3} \)).

![Figure 1](image_url)  \hspace{1cm} Figure 1. Dependence of optimal diameter on vessel volume.
3. Optimization of dimensions of vessels operating under excessive pressure based on minimum surface area of the housing

Let us express the optimal dimensions (internal diameter $D$ and height $H$) of a cylindrical apparatus with an elliptical bottom and lid from the condition of the minimum surface area of the housing $A$. We will find the first derivative by the diameter of the previously obtained expression (11) for determining the surface area of the vessel:

$$\frac{dA}{dD} = -4V/D^2 + 2.86D.$$  \hfill (21)

Let us check performance of a condition:

$$d^2A/dD^2 > 0;$$  \hfill (22)

$$8V/D^2 + 2.86 > 0,$$  \hfill (23)

condition is met.

We equate the first derivative to zero and express the optimal diameter from the condition of the minimum surface area of the housing:
\[-4V/D^2 + 2.86D = 0; \tag{24}\]
\[D = (4V/2.86)^{1/3}. \tag{25}\]

It is possible to determine the remaining optimized dimensions of the vessel using the obtained value of the optimal diameter:
- height of elliptical bottom \(H_b\) (lid \(H_l\)) according to equation (19);
- the total height of the vessel \(H\) according to equation (20);
- the minimum surface area of the vessel body \(A\) according to equation (11).

For vessels of different volumes made of steel 10, the dependencies of the optimal diameter (figure 4) and the optimal height (figure 5) corresponding to the minimum surface area of the vessel are constructed. In this case, the optimum height slightly exceeds the optimum diameter.

![Figure 4](image1.png)

**Figure 4.** Dependences of optimal dimensions of vessel on its volume.

![Figure 5](image2.png)

**Figure 5.** Dependence of the minimum surface area on vessel volume.

4. **Results and discussion**

Figures 1 and 2 shows that when optimizing the size of vessels from the condition of minimum material capacity, the height of the body is an order of magnitude higher than the diameter. At the
same time, the area occupied by the object is reduced, but the height of the service platforms, the load on the foundation and the costs of its construction increase.

In the case of optimizing the dimensions of the vessel under the condition of a minimum surface area of the housing, the height is slightly exceeds the diameter (figure 4) and is practically comparable to it in size.

If the vertical vessel is filled with liquid or bulk material, the design pressure shall be the pressure of the liquid column increasing directly in proportion to the height of the layer:

\[ p_a = p_0 + \rho_l g h_l, \]  

where \( p_0 \) – overpressure in the apparatus, Pa; \( \rho_l \), \( h_l \) – respectively density (kg/m\(^3\)) and height of process medium (m).

Increasing the height of the column of process medium in the apparatus increases the design pressure, which requires thickening of the vessel wall in accordance with equation (12). As a result, the mass and cost of the vessel are increased [7, 8]. This is taken into account when designing large vertical tanks. In order to save money, such receptacles are usually made by welding several horizontal belts of different thicknesses.

5. Conclusion
The dependencies obtained in this work can be used to determine the optimal dimensions, mass and surface area of typical industrial vessels and apparatuses, to estimate the consumption of the coating material of the housing (thermal insulation, enamel, lining, etc.).

The most economical effect can be achieved by applying the proposed approach in the design of large-capacity production and vessels.

References
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