Chiral Symmetry and the Nucleon Spin Structure Functions

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Abstract

We carry out a systematic investigation of twist-two spin dependent structure functions of the nucleon within the framework of the chiral quark soliton model (CQSM) by paying special attention to the role of chiral symmetry of QCD. We observe a substantial difference between the predictions of the longitudinally polarized distribution functions and the transversity distribution ones. That the chiral symmetry is responsible for this difference can most clearly be seen in the isospin dependence of the corresponding first moments, i.e. the axial and tensor charges. The CQSM predicts
\[ \frac{g_A(0)}{g_A(3)} \approx 0.25 \]
and
\[ \frac{g_T(0)}{g_T(3)} \approx 0.46 \]
for the ratio of the isoscalar and isovector axial charges, and the ratio of the isoscalar to isovector tensor charges, respectively. These should be compared with the prediction of the naive (non-chiral) MIT bag model:
\[ \frac{g_A(0)}{g_A(3)} = \frac{g_T(0)}{g_T(3)} = \frac{3}{5}. \]

Key-words: Spin Dependent Quark Distributions of the Nucleon, Chiral Symmetry

1. Introduction

Undoubtedly, “Nucleon Spin Crisis” caused by the EMC measurement in 1988 is one of the most exciting topics in the field of hadron physics [1]. The recent renaissance of nucleon structure function physics is greatly owing to this epoch-making finding. Here we recall that the physics of nucleon structure functions has two different aspects. One is a perturbative aspect. Because of the asymptotic freedom of QCD, the \( Q^2 \)-evolution of quark distribution functions can be controlled by the perturbative QCD at least for large enough \( Q^2 \). However, the perturbative QCD is totally powerless for predicting distribution functions themselves. Here we need to solve nonperturbative QCD in some way. Unfortunately, we have no reliable analytical method yet. We are then left with two tentative choices.

One is to rely upon lattice QCD, while the other is to use effective models of QCD. If one takes the first choice, one must first calculate infinite towers of moments of distribution functions, since the direct evaluation of distribution functions does not match this numerical method. Here we take the second choice, which allows us a direct evaluation of quark distribution functions. Naturally, there are quite a lot of effective model of baryons. But let me shortly explain merits of our model, i.e. the chiral quark soliton model (CQSM) over the others. The CQSM is an effective model of baryons incorporating \( \chi_{SB} \) of QCD vacuum [2,3]. The nucleon in this model is a composite of 3 valence quarks and infinitely many sea quarks moving in a slowly rotating M.F. of hedgehog shape. It automatically simulates cloud of pions surrounding the core of three valence quarks.
quarks. Noteworthy here is that our pion fields are not independent fields of quarks, but they are rather $q\bar{q}$ composites. Since everything is described in terms of effective quark fields, we need not worry about a double counting of quark and pion degrees of freedom. This also means that we have no necessity of convoluting pion structure functions with pion probability function inside the nucleon [4-6].

2. CQSM and quark distribution functions

We start with a definition of quark distribution function given as a Fourier transform of the nucleon matrix element of bilocal quark operator [7]:

$$q(x) = \frac{1}{4\pi} \int_{-\infty}^{\infty} dz^0 e^{ixz^0}$$

$$\times <N(P=0) | \psi^\dagger(0) O \psi(z) | N(P=0)> \mid z^3=-z^0, z_\perp=0 .$$

Here the constraint $z^3=-z^0, z_\perp=0$ for the space time coordinates means that we need to evaluate quark-quark correlation function with light-cone separation. In the present talk, let me confine to spin dependent distribution functions of leading twist 2. There are two kinds of twist-2 spin dependent distribution functions [8]. One is the familiar longitudinally polarized distribution functions. The other is the less familiar chiral-odd distribution functions, which are sometimes called the transversity distribution functions. Both distribution functions consist of isoscalar part and isovector part. They are obtained by inserting the following operator into the above general expression of quark distribution functions:

$$\Delta u(x) + \Delta d(x) : O = \gamma_5 (1 + \gamma^0\gamma^3)$$

$$\Delta u(x) - \Delta d(x) : O = \tau_3 (1 + \gamma^0\gamma^3) \gamma_5 ,$$

for the longitudinally polarized distributions, and

$$\delta u(x) + \delta d(x) : O = \gamma_5 (1 + \gamma^0\gamma^3) \gamma_5 ,$$

$$\delta u(x) - \delta d(x) : O = \tau_3 (1 + \gamma^0\gamma^3) \gamma_5 .$$

for the transversity distributions. The basis of our theoretical analysis is the following path integral representation of the nucleon matrix element of bilocal quark operator:

$$<N(P) | \psi^\dagger(0) O \psi(z) | N(P)>$$

$$= \frac{1}{Z} \int d^3x d^3y e^{-iP.x} e^{iP.y} \int D\pi \int D\psi D\psi^\dagger$$

$$\times J_N(T \frac{1}{2}, \mathbf{x}) \cdot \psi^\dagger(0) O \psi(z) \cdot J_N(T \frac{1}{2}, \mathbf{y}) e^{i \int d^4x L_{CQM}} ,$$

with

$$L_{CQM} = \bar{\psi} (i \not{\partial} M e^{i\gamma_5 \mathbf{\tau} \cdot \mathbf{\pi}(x)/f_\pi}) \psi ,$$

being the basic lagrangian of the CQSM. We start with a stationary pion field configuration of hedgehog form:

$$\mathbf{\pi}(x) = f_\pi \mathbf{r} F(r) .$$

Next we carry out a path integral over $\mathbf{\pi}(x)$ in a saddle point approximation by taking care of two zero-energy modes, i.e. the translational zero-modes and rotational zero-modes.
Under the assumption of “slow rotation” as compared with the intrinsic quark motion, the answer can be obtained in a perturbative series in the collective rotational velocity $\Omega$. Without going into the detail, here I just mention that the isoscalar distribution receives only the $O(\Omega^1)$ contribution, while the isovector distribution contains both of $O(\Omega^0)$ and $O(\Omega^1)$ contributions [9]:

$$\Delta u(x) + \Delta d(x) \sim 0 + O(\Omega^1), \quad (10)$$

$$\Delta u(x) - \Delta d(x) \sim O(\Omega^0) + O(\Omega^1). \quad (11)$$

Since the model contains ultraviolet divergences, it needs a regularization. This can most easily be seen from the effective meson action obtained from our basic lagrangian:

$$S_{\text{eff}}[U] = -i N_c \text{Sp} \log [i \not\partial - MU\gamma^5]$$

$$= \frac{4N_c}{f_{\pi}^2} I_2(M) \cdot \frac{1}{2} (\partial_\mu \pi)^2 + \cdots, \quad (13)$$

with

$$I_2(M) \equiv i \int \frac{d^4k}{(2\pi)^4} \frac{M^2}{(k^2 - M^2)^2}. \quad (14)$$

The coefficient of pion kinetic term is logarithmically divergent. To regularize it, here we make use of the Pauli-Villars subtraction scheme [4]. That is, we subtract from the original action another action in which the constituent quark mass $M$ is replaced by a Pauli-Villars mass $M_{PV}$:

$$S_{\text{eff}}^{\text{reg}} \equiv S_{\text{eff}}^{M} - (\frac{M}{M_{PV}})^2 S_{\text{eff}}^{M_{PV}}. \quad (15)$$

This subtraction makes the coefficient of pion kinetic term finite. If we further require this kinetic term has a correct normalization, the Pauli-Villars mass is uniquely fixed from the equation:

$$\frac{N_c}{4\pi^2} M^2 \log \left(\frac{M}{M_{PV}}\right)^2 = f_{\pi}^2. \quad (16)$$

Other observables like quark distribution functions, which contain logarithmic divergence, can similarly be regularized by using the same subtraction:

$$<O>^{\text{reg}} \equiv <O>^{M} - (\frac{M}{M_{PV}})^2 <O>^{M_{PV}}. \quad (17)$$

3. Results and Discussion

Before showing the results of our numerical calculation, we briefly mention parameters of the model. The most important parameter of the CQSM is the dynamical quark mass $M$, which plays the role of the quark-pion coupling constant. Here we simply use the value $M = 375$ MeV established from our previous analyses of baryon observables. As already mentioned, the Pauli-Villars mass as a regularization parameter is uniquely fixed to be $M_{PV} = 562$ MeV from the normalization condition of pion kinetic term. As for the nucleon mass, we prefer to using a theoretical soliton mass instead of the physical
mass, since it respects the energy-momentum sum rule at the initial energy scale. To compare our predictions with the high energy data, we must take account of the $Q^2$-evolution of distribution functions. The initial energy scale necessary here is simply taken to be $Q^{2\text{init}} = (0.5 \text{ GeV})^2$, which is close to the Pauli-Villars mass square. For solving the DGLAP evolution equation at the NLO, we use the Fortran Program provided by Saga group [10].

Figure 1: The longitudinally polarized distribution in comparison with the transversity distribution for $u$- and $d$-quarks. The theoretical distributions at $Q^2 = Q^{2\text{init}}$ are evolved to $Q^2 = Q^{2\text{evol}}$ by solving the NLO evolution equations.

Now we show in Fig.1 and Fig.2 some of our numerical results for quark distribution functions. Here $\Delta u(x)$ and $\delta u(x)$ respectively stand for the longitudinal and transversity distributions for $u$-quark, whereas $\Delta d(x)$ and $\delta d(x)$ are the corresponding quantities for $d$-quark. In our model, the difference between the two distributions are sizable even at the initial (low) energy scale. A comparison with yet-to-be-obtained high energy data must be done with care, since the way of evolution of these two distributions are totally different and the difference between these two distributions becomes larger and larger as $Q^2$ increases [11,12]. A general trend is a rapid growth of small $x$ component for the longitudinally polarized distribution due to the coupling with gluons. A similar tendency is also observed for the corresponding $d$-quark distributions. We can also give some predictions for the polarized antiquark distribution functions. They are shown in Fig.2. As one can see, even the signs are different for the longitudinal and transversity distributions for both of $u$ and $d$ quarks.

Now we compare our predictions for the longitudinally polarized distributions with the existing high energy data. Shown in the upper part of Fig.3 are the prediction of the CQSM and that of the MIT bag model for the spin dependent structure function $g_1^p(x)$ for the proton, in comparison with the SLAC data [13]. As usual, the theoretical structure functions are obtained by convoluting the QCD coefficient functions with the evolved quark distribution functions. One sees that a general trend of the data is well reproduced by the CQSM. The lower part of Fig.3 shows the same quantity for the neutron. We emphasized that the prediction of the MIT bag model for this quantity is negligibly small even after $Q^2$-evolution. On the other hand, the CQSM predicts sizable magnitude of $g_1^n$ with negative sign. Although the quality of the experimental data are not very good,
they seem to support the prediction of the CQSM.

The 1st moments of the distribution functions are the simplest but most important quantities characterizing the distributions. The 1st moments of the longitudinally polarized distribution and the transversity distribution are respectively called the axial and tensor charges:

\[ g_A^{(0,3)} = \int_0^1 dx \left\{ [ \Delta u(x) + \Delta \bar{u}(x) ] \pm [ \Delta d(x) + \Delta \bar{d}(x) ] \right\}, \]
\[ g_T^{(0,3)} = \int_0^1 dx \left\{ [ \delta u(x) - \delta \bar{u}(x) ] \pm [ \delta d(x) - \delta \bar{d}(x) ] \right\}. \]

Within the framework of the nonrelativistic theory, there is no distinction between these two charges:

\[ g_A^{(3)} = g_T^{(3)} = \frac{5}{3}, \]
\[ g_A^{(0)} = g_T^{(0)} = 1. \]

Introduction of relativistic kinematics makes some difference, however. In fact, the splitting of the axial and tensor charges are due to the different sign of the lower component contribution:

\[ g_A^{(3)} = \frac{5}{3} \cdot \int (f^2 - \frac{1}{3}g^2) r^2 dr, \quad g_T^{(3)} = \frac{5}{3} \cdot \int (f^2 + \frac{1}{3}g^2) r^2 dr, \]
\[ g_A^{(0)} = 1 \cdot \int (f^2 - \frac{1}{3}g^2) r^2 dr, \quad g_T^{(0)} = 1 \cdot \int (f^2 + \frac{1}{3}g^2) r^2 dr. \]

Shown below are the numbers obtained by using typical parameters of the bag model:

\[ g_A^{(3)} \simeq 1.06, \quad g_T^{(3)} \simeq 1.34, \]
\[ g_A^{(0)} \simeq 0.64, \quad g_T^{(0)} \simeq 0.80. \]
Figure 3: The longitudinally polarized structure functions $g_1^p(x)$ and $g_1^n(x)$ for the proton and neutron. The predictions of the CQSM and of the MIT bag model are compared with the SLAC data.

An important observation is that the ratios of the isoscalar to isovector charges are $3/5$ for both of the axial and tensor charges:

$$g_A^{(0)}/g_A^{(3)} = g_T^{(0)}/g_T^{(3)} = 3/5.$$  

This is the case for both of the NRQM and the MIT bag model. Now we shall argue that this may be a limitation of valence quark models without chiral symmetry. In fact, there is another relativistic effects, which differentiate the axial and tensor charges. It is a dynamical sea quark effect.

Table 1: The axial and tensor charges of the nucleon

|                | CQSM | MIT-bag | Lattice QCD$^*$ | Experiment                      |
|----------------|------|---------|-----------------|---------------------------------|
| $g_A^{(3)}$    | 1.41 | 1.06    | 0.99            | $1.254 \pm 0.006$ (Q$^2$-indep.)|
| $g_A^{(0)}$    | 0.35 | 0.64    | 0.18            | $0.31 \pm 0.07$ (Q$^2 = 10$ GeV$^2$) |
| $g_T^{(3)}$    | 1.22 | 1.34    | 1.07            | –                               |
| $g_T^{(0)}$    | 0.56 | 0.80    | 0.56            | –                               |
| $g_A^{(0)}/g_A^{(3)}$ | 0.25 | 0.60    | 0.18            | 0.24                            |
| $g_T^{(0)}/g_T^{(3)}$ | 0.46 | 0.60    | 0.52            | –                               |

To see the importance of this dynamical sea quark effect, we compare the predictions of the CQSM with those of the MIT bag model in Table.1. A noteworthy observation is that the ratio of the isoscalar to isovector charges are much smaller for the axial one than for the tensor one in sharp contrast to the predictions of the MIT bag model. It is interesting to see that this tendency is also reproduced by a lattice calculation by
Kuramashi [14]. In our opinion, this indicates an importance of chiral symmetry as a generator of dynamical sea quark effects, and the predicted feature is expected to be confirmed by future measurement of tensor charges.

The axial and tensor charges are generally scale dependent. Fig.4 figure shows the next-to-leading order $Q^2$-evolution of axial and tensor charges. As is widely known, the isovector (or flavor-nonsinglet) axial charge is scale independent due to the current conservation, while the flavor-singlet axial charge has a very weak $Q^2$ dependence originating from the coupling with gluons. Aside from about 12\% overestimate of isovector axial charge, the theoretical predictions for the axial charges are qualitatively consistent with the experimental data. Contrary to the axial charge, the tensor charges are known to have strong $Q^2$ dependence [11,12]. Unfortunately, we must wait for future experiments for obtaining any information of these interesting observables. Fig.5 shows the NLO evolution of the quark and gluon polarization [15]. Here, we have assumed zero gluon polarization at the initial energy scale. One sees that the gluon polarization grows rapidly as $Q^2$ increases. This means that a polarized quark is preferred to radiate a gluon with helicity parallel to the quark spin polarization. We hope that future experiments will give some direct information on the size of the gluon polarization in the nucleon.

Figure 4: The NLO $Q^2$ evolution of axial and tensor charges.

Figure 5: The NLO $Q^2$ evolution of quark and gluon polarization.

4. Summary

In summary, the CQSM naturally explains qualitative behavior of the experimentally measured longitudinally polarized structure functions of the proton and the neutron. Although I could not mention in the present talk, the model also reproduces an excess of $\bar{d}$ sea over the $\bar{d}$ sea in the proton very naturally [6]. It also predicts qualitative difference between the transversity distribution functions and longitudinally polarized distribution functions. For example, in simple valence quark models like the NRQM or the MIT bag model, the ratios of the isoscalar to isovector charges are just the same for both of the axial charges and the tensor charges. On the contrary, in the CQSM, this ratio turns out to be much smaller for the axial charges than for the tensor charges. We have pointed out that this characteristic is also shared with the result of lattice gauge theory. This
observation then indicates that nonperturbative QCD dynamics due to the $\chi$SB would manifest itself in the isospin (or flavor) dependence of high energy spin observables.

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