A reappraisal of two-loop contributions to the fermion electric dipole moments in R-parity violating supersymmetric models

Nodoka Yamanaka, Toru Sato and Takahiro Kubota

Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

(Dated: June 5, 2012)

We reexamine the R-parity violating contribution to the fermion electric and chromo-electric dipole moments (EDM and cEDM) in the two-loop diagrams. It is found that the leading Barr-Zee type two-loop contribution is smaller than the result found in previous works, and that EDM experimental data provide looser limits on RPV couplings.

PACS numbers: 12.60.Jv, 11.30.Er, 13.40.Em, 14.80.Ly

The supersymmetry is known to resolve many theoretical problems which have been encountered in the standard model (SM) such as the cancellation of the power divergences in radiative corrections, and its supersymmetric extension is therefore one of the promising candidates of new physics. The supersymmetric SM can be extended to allow baryon number or lepton number violating interactions, known as the R-parity violating (RPV) interactions, and they have been constrained from the analysis of various phenomena [1].

The electric dipole moment (EDM) is an excellent observable to investigate the underlying mechanisms of the P and CP violations and can be measured in a variety of systems [2]. Since the contribution of the SM to the EDM is in general small [3], it is a very good experimental observable to examine the supersymmetric models and other candidates of new physics. In the past three decades, many analyses of the supersymmetric models with [4–7] and without [8–13] the conservation of R-parity have been done using the EDMs.

In the RPV supersymmetric model with trilinear RPV interactions, it has been found that the fermion (quark or lepton) EDM does not receive any one-loop contribution [9], and the two-loop contribution has been analyzed in detail to give the Barr-Zee type diagram as the leading contribution [11]. In this paper, we reexamine the RPV Barr-Zee type contribution which turns out to be in disagreement with previous works [10, 11, 13]. We will show that the RPV Barr-Zee type diagram has actually a smaller contribution than that given in previous works.

The RPV interactions are generated by the following superpotential:

\[
W_R = \frac{1}{2} \lambda_{ijk} \epsilon_{ab} L_i^a L_j^b (E^c)_k + \lambda'_{ijk} \epsilon_{ab} L_i^a Q^b_k (D^c)_k ,
\]

with \( i, j, k = 1, 2, 3 \) indicating the generation, \( a, b = 1, 2 \) the \( SU(2)_L \) indices, \( L \) and \( E^c \) denote the lepton doublet and singlet left-chiral superfields. \( Q, U^c \) and \( D^c \) denote respectively the quark doublet, up quark singlet and down quark singlet left-chiral superfields. The RPV baryon number violating interactions are irrelevant in this analysis since they do not contribute to the Barr-Zee type diagrams, and are not included in our current analysis. Also the bilinear RPV interactions were not considered. The RPV lagrangian of interest is then given as

\[
\mathcal{L}_R = -\frac{1}{2} \lambda_{ijk} \left[ \tilde{\nu}_i \tilde{e}_k P_L e_j + \tilde{e}_{Lj} \tilde{e}_k P_L \nu_i + \tilde{e}_{Rk} \tilde{e}_i P_L e_j - (i \leftrightarrow j) \right] + \text{(h.c.)}
\]

\[-\lambda'_{ijk} \left[ \tilde{\nu}_i \tilde{d}_k P_L d_j + \tilde{d}_{Lj} \tilde{d}_k P_L \nu_i + \tilde{d}_{Rk} \tilde{e}_i P_L d_j - \tilde{e}_{Lj} \tilde{d}_k P_L e_i - \tilde{d}_{Lj} \tilde{e}_i P_L d_k + \text{(h.c.)} \right],
\]

where \( P_L \equiv \frac{1}{2}(1 - \gamma_5) \) and we also define \( P_R \equiv \frac{1}{2}(1 + \gamma_5) \) for later use. These RPV interactions are lepton number violating Yukawa interactions.

The EDM \( d_F \) of the fermion is defined as follows:

\[
\mathcal{L}_{\text{EDM}} = -\frac{d_F}{2} \bar{\psi} \gamma_5 \sigma^{\mu\nu} \psi F_{\mu\nu} ,
\]

where \( F_{\mu\nu} \) is the electromagnetic field strength. With the RPV lagrangian [2], the sneutrino exchange Barr-Zee type diagrams shown in Fig. [1] contribute to the EDM. Here the emission (absorption) of the sneutrino from fermion is accompanied by \( P_R (P_L) \) projection operator as is apparent from the Eq. [2].
At first we give the expression of the two-photon decay amplitude of annihilation and production of sneutrino with internal fermion loop shown in Fig. 2 given as

\[ iM_{\text{BZ}} = -\tilde{\lambda}_{ijk} Q_F e \epsilon_{\nu}^{*} \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p-q)\gamma_{\mu}(\not{k} - \not{q} + \not{\bar{f}} + M_F)\gamma_{\nu}}{k^2[(q+k)^2 - m_{\nu}^2]2M_F[(p-q-k)^2 - M_F^2]} \]

\[ \approx \text{Im}(\bar{\lambda}_{ijk} \tilde{\lambda}_{ijk} \alpha_{\text{em}} \frac{n_e Q_F^2}{f \left( \frac{m_{\nu}^2}{m_{\bar{f}}^2} \right) - g \left( \frac{m_{\nu}^2}{m_{\bar{f}}^2} \right)} ) \epsilon_{\nu}^{*}(q) \bar{u} \gamma_{\mu} q \gamma_{\beta} u , \]

where \( \tilde{\lambda} = \lambda \) for lepton EDM contribution and \( \tilde{\lambda} = \lambda' \) for quark EDM contribution, and \( f \) and \( g \) are defined as
\[ f(z) = \frac{z}{2} \int_0^1 dx \left( \frac{1 - 2x(1-x)}{x(1-x) - z} \ln \left( \frac{x(1-x)}{z} \right) \right), \quad (7) \]
\[ g(z) = \frac{z}{2} \int_0^1 dx \left( \frac{1}{x(1-x) - z} \ln \left( \frac{x(1-x)}{z} \right) \right), \quad (8) \]
in the notation of the original notation of Barr and Zee \[14\]. For small \( z \), we have \( g(z) \approx \frac{\pi}{2} (\frac{z^2}{2} + (\ln z)^2) \), and \( f(z) \approx \frac{\pi}{2} (\frac{z^2}{2} + 4 + 2 \ln z + (\ln z)^2) \). In the last line of Eq. (9), we have taken only the part of \( iM_{\text{BZ}} \) which contributes to the EDM, disregarding \( \text{Re}(\lambda_{ijj}^* \lambda_{ikk}^*) \). For each diagram of Fig. 1, there are also diagrams with the internal fermion loop reversed, and those with internal photon and sneutrino lines interchanged. They all give the same amplitude and Eq. (9) should be multiplied by four. The total EDM of the fermion \( F \) from the Barr-Zee type diagrams with R-parity violating interaction is then

\[ d_{F_k} = \text{Im}(\lambda_{ijj}^* \lambda_{ikk}^*) \frac{e Q_f^2 Q_F^2 e}{16\pi^2 m_{f_j}} \cdot \left\{ f(\tau) - g(\tau) \right\} \]
\[ \approx \text{Im}(\lambda_{ijj}^* \lambda_{ikk}^*) \frac{e Q_f^2 Q_F^2 e}{16\pi^2 m_{f_j}} \cdot \tau (2 + \ln \tau + \cdots) , \quad (9) \]

where \( \tau = m_{f_j}^2/m_{b_j}^2 \). The flavor index, electric charge and number of color of the fermion \( F \) are denoted respectively by \( k \), \( Q_F \) and \( n_c \) \((n_c = 3 \text{ for inner loop fermion is a quark, otherwise } n_c = 1) \), and \( j \) and \( Q_f \) are respectively the flavor index and electric charge of the inner loop fermion \( f \). The second line of the above equation is the approximated expression for small \( \tau \). Note also that the Barr-Zee type diagram gives EDM contribution only to down-type quarks, and the same property holds also for the chromo-EDM (cEDM) seen below.

We see from Eq. (9) (see also Fig. 1) that the chirality structure of the scalar exchange between internal loop and external line (RPV vertices with sneutrino exchange) has the form \( P_L \otimes P_R \) and \( P_R^\dagger \otimes P_L^\dagger \), which is a consequence of the lepton number conservation of the whole EDM process. This gives as a result the structure

\[ f(\tau) - g(\tau) \approx \tau (2 + \ln \tau) , \quad (10) \]
in the final formula (11). This is consistent with the result obtained in the analysis of the Barr-Zee type diagram analogues with the exchange of Higgs bosons in the two Higgs doublet model, done originally by Barr and Zee \[14\] (see also \[12\]). In the two Higgs doublet model, there are also additional contributions with the structures \( P_L \otimes P_L \) and \( P_R \otimes P_R \) which yield contribution proportional to

\[ f(\tau) + g(\tau) \approx \tau \left( \frac{\pi^2}{3} + 2 + \ln \tau + (\ln \tau)^2 \right) , \quad (11) \]

which is absent in the RPV supersymmetric models.

| \( m_{\nu} \) | \hline
| \text{tau lepton} | \text{bottom quark} |
| \hline
| \text{[TeV]} | \text{f - g (ours)} | \text{f + g} | \text{f - g (ours)} | \text{f + g} |
| \hline
| 0.1 | 3.14 | -32.1 | 1.80 | -15.9 |
| \hline
| 1 | 5.50 \times 10^{-2} & -7.89 \times 10^{-1} & 3.64 \times 10^{-2} & -4.64 \times 10^{-1} |
| \hline
| 5 | 2.87 \times 10^{-3} & -4.99 \times 10^{-2} & 1.98 \times 10^{-3} & -3.12 \times 10^{-2} |

**TABLE I:** The electron EDM \( d_e \) \([10^{-27} \text{e cm}]\). The coupling constants of RPV interactions are set to \( \text{Im}(\lambda_{ijj}^* \lambda_{ikk}^*) = \text{Im} (\sum_i \lambda_{33i}^* \lambda_{i11}^*) = 10^{-5} \) and the masses of b-quark and tau lepton are set to \( m_b = 4.2 \text{ GeV} \) and \( m_\tau = 1.78 \text{ GeV} \). For comparison, we have shown the EDM calculated by replacing \( f - g \) by \( f + g \).

The small \( \tau \) behaviour in Eq. (9) is in contradiction with the result presented in Refs. \[10, 11, 12\], where the RPV Barr-Zee type diagrams give the leading contribution proportional to \( \tau (\ln \tau)^2 \). If one would replace \( f(\tau) - g(\tau) \) in Eq. (9) by \( f(\tau) + g(\tau) \), the formula given in Refs. \[10, 12\] would be obtained. The difference between these two results is large. For example, if we consider the Barr-Zee type diagram with the bottom quark and tau lepton in the loop, the electron EDM evaluated from our formula in Eq. (9) is one order of magnitude smaller than those of Refs. \[11, 12\] and even sign of the electron EDM is different as shown in Table I. By using our correct formula, the experimental upper bounds on RPV interactions given from the RPV Barr-Zee type contribution is loosened by one order of magnitude.

We can also evaluate Barr-Zee type diagrams which contribute to the quark cEDM. The lagrangian of the cEDM interaction is given by

\[ \mathcal{L}_{c\text{EDM}} = -i \frac{e}{2} \bar{c} \gamma_5 \sigma^{\mu\nu} T_a \psi F_{a \mu\nu} , \quad (12) \]

where \( F_{a \mu\nu} \) is the gluon field strength. The Barr-Zee type contribution of the down-type quark \( q_k \) is then

\[ d_{q_k} = \text{Im}(\lambda_{ijj}^* \lambda_{ikk}^*) \frac{e Q_f^2 Q_F^2 e}{32\pi^2 m_{q_j}^3} \cdot \left\{ f(\tau) - g(\tau) \right\} , \quad (13) \]

where \( \tau = m_{q_j}^2/m_{b_j}^2 \). The flavor indices of the quark \( q_k \) and the quark of the inner loop are denoted respectively by \( k \) and \( j \).

In conclusion, we have reanalyzed the RPV supersymmetric contribution to the Barr-Zee type two-loop diagrams, and have found that the result gives smaller fermion EDM, by one order of magnitude than the previous analyses \[10, 11, 12\] \( \text{(for sneutrino mass = 1 TeV). This difference is significant, as it can alter the relative size between other contributing processes such as the 4-fermion interactions} 10, 12 \}. Nevertheless our finding does not alter the dominance of the Barr-Zee type diagrams over the other two-loop diagrams, as shown in the analysis of Chang \textit{et al.} \[11\], and their conclusion is still very important.
[1] G. Bhattacharyya, arXiv:hep-ph/9703395; H. K. Dreiner, M. Rosenberry and T. E. Chupp, Phys. Rev. Lett. 86, 22 (2001); B. C. Regan, E. D. Commins, C. J. Schmidt, D. DeMille, Phys. Rev. Lett. 88, 071805 (2002); C. A. Baker et al., Phys. Rev. Lett. 97, 131801 (2006); W. C. Griffith et al., Phys. Rev. Lett. 102, 101601 (2009); G. W. Bennett et al. (Muon $(g - 2)$ Collaboration), Phys. Rev. D80, 052008 (2009); J. J. Hudson et al., Nature 473, 493 (2011).

[2] J. Hisano and Y. Shimizu, Phys. Rev. D70, 035004 (2004); M. Endo, M. Kakizaki and M. Yamaguchi, Phys. Lett. B644, 263 (2002); O. Lebedev and M. Pospelov, Phys. Rev. Lett. 89, 101801 (2002); D. Demir, O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Nucl. Phys. B680, 339 (2004); O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Phys. Rev. D70, 016003 (2004); M. Pospelov, A. Ritz, Ann. Phys. 318, 119 (2005); J. Ellis, J. S. Lee and A. Pilaftsis, JHEP 09, 049 (2008); JHEP 02, 045 (2011); B. C. Regan, E. D. Commins, C. J. Schmidt, Baker et al., Nucl. Phys. B449, 420 (1994); R. Barbieri and A. Masiero, Nucl. Phys. B273, 348 (1992); A. Czarnecki and B. Krause, Phys. Rev. Lett. 78, 4339 (1997).

[3] J. Hisano and Y. Shimizu, Phys. Rev. D70, 035004 (2004); M. Endo, M. Kakizaki and M. Yamaguchi, Phys. Lett. B583, 186 (2004); G.-C. Cho, N. Haba and M. Honda, Mod. Phys. Lett. A20, 2969 (2005); J. Hisano, M. Nagai and P. Paradisi, Phys. Rev. D78, 075019 (2008); Phys. Rev. D80, 095014 (2009).

[4] J. Ellis, S. Ferrera and D. V. Nanopoulos, Phys. Lett. B114, 231 (1982); W. Buchmüller and D. Wyler, Phys. Lett. B121, 321 (1983); J. Polchinski and M. B. Wise, Phys. Lett. B125, 393 (1983); F. del Aguila, M. B. Gavela, J. A. Grifols and A. Mendez, Phys. Lett. B216, 71 (1983); D. V. Nanopoulos and M. Srednicki, Phys. Lett. B312, 61 (1983); M. Dugan, B. Grinstein and L. J. Hall, Nucl. Phys. B255, 413 (1985); P. Nath, Phys. Rev. Lett. 66, 2565 (1991); Y. Kizukuri and N. Oshimo, Phys. Rev. D46, 3025 (1992); W. Fischler, S. Paban and S. D. Thomas, Phys. Lett. B289, 373 (1992); T. Inui, Y. Mimura, N. Sakai and T. Sasaki, Nucl. Phys. B449, 49 (1995); T. Ibrahim and P. Nath, Phys. Rev. D57, 478 (1998); Phys. Lett. B418, 98 (1998); Phys. Rev. D58, 11301 (1998); S. Pokorski, J. Rosiek and C. A. Savoy, Nucl. Phys. B570, 81 (2000); S. Y. Ayazi and Y. Farzan, Phys. Rev. D74, 055008 (2006); JHEP 06, 013 (2007).

[5] T. H. West, Phys. Rev. D50, 7025 (1994); T. Kadoyoshi and N. Oshimo, Phys. Rev. D55, 1481 (1997); D. Chang, W.-Y. Keung and A. Pilaftsis, Phys. Rev. Lett. 82, 900 (1999); A. Pilaftsis, Phys. Rev. B471, 174 (1999); D. Chang, W.-F. Chang and W.-Y. Keung, Phys. Rev. Lett. B478, 239 (2000); A. Pilaftsis, Phys. Rev. D62, 016007 (2000); D. Chang, W.-F. Chang, W.-Y. Keung, Phys. Rev. D66, 116008 (2002); N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice, A. Romanino, Nucl. Phys. B709, 3 (2005); D. Chang, W.-F. Chang and W.-Y. Keung, Phys. Rev. D71, 076006 (2005); Y. Li, S. Profumo and M. J. Ramsey-Musolf, Phys. Rev. D78, 075009 (2008).

[6] J. Dai, H. Dykstra, R. G. Leigh, S. Paban and D. Dicus, Phys. Lett. B237, 216 (1990); T. Falk, K. A. Olive, M. Pospelov and R. Roiban, Nucl. Phys. B560, 3 (1999); M. Bhrlhik, G. J. Good and G. L. Kane, Phys. Rev. D59, 115004 (1999); S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B606, 151 (2001); A. Pilaftsis, Nucl. Phys. B644, 263 (2002); O. Lebedev and M. Pospelov, Phys. Rev. Lett. 89, 101801 (2002); D. Demir, O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Nucl. Phys. B680, 339 (2004); O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Phys. Rev. D70, 016003 (2004); M. Pospelov, A. Ritz, Ann. Phys. 318, 119 (2005); J. Ellis, J. S. Lee and A. Pilaftsis, JHEP 10, 049 (2008); JHEP 02, 045 (2011); Y. Li, S. Profumo and M. J. Ramsey-Musolf, JHEP 08, 062 (2010); T. Fukuyama, arXiv:1201.4252 [hep-ph].

[7] J. Hisano and Y. Shimizu, Phys. Rev. D70, 095001 (2004); M. Endo, M. Kakizaki and M. Yamaguchi, Phys. Lett. B583, 186 (2004); G.-C. Cho, N. Haba and M. Honda, Mod. Phys. Lett. A20, 2969 (2005); J. Hisano, M. Nagai and P. Paradisi, Phys. Rev. D78, 075019 (2008); Phys. Rev. D80, 095014 (2009).

[8] R. Barbieri and A. Masiero, Nucl. Phys. B267, 679 (1986).

[9] R. M. Godbole, S. Pakvasa, S. D. Rindani and X. Tata, Phys. Rev. D61, 113003 (2000); S. A. Abel, A. Dedes and H. K. Dreiner, JHEP 05, 13 (2000).

[10] P. Herczeg, Phys. Rev. D61, 095010 (2000).

[11] D. Chang, W.-F. Chang, M. Frank and W.-Y. Keung, Phys. Rev. D62, 095002 (2000).

[12] K. Choi, E. J. Chun and K. Hwang, Phys. Rev. D63, 013002 (2000); Y.Y. Keum and Otto C.W. Kong, Phys. Rev. D63, 113012 (2001); C.-C. Cho, O. C. W. Kong and R. D. Vaidya, Phys. Rev. D76, 013003 (2007).

[13] A. Faessler, T. Gutsche, S. Kovalenko and V. E. Lyubovitskij, Phys. Rev. D73, 114023 (2006); Phys. Rev. D74, 074013 (2006).

[14] S. M. Barr and A. Zee, Phys. Rev. Lett. 65, 21 (1990).

[15] V. Barger, A. Das and C. Kao, Phys. Rev. D55, 7099 (1997).