Competition between Kondo and RKKY correlations in the presence of strong randomness

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Abstract

We propose that competition between Kondo and magnetic correlations results in a novel universality class for heavy fermion quantum criticality in the presence of strong randomness. Starting from an Anderson lattice model with disorder, we derive an effective local field theory in the dynamical mean-field theory approximation, where randomness is introduced into both hybridization and Ruderman–Kittel–Kasuya–Yosida (RKKY) interactions. Performing the saddle-point analysis in the $U(1)$ slave-boson representation, we reveal its phase diagram which shows a quantum phase transition from a spin liquid state to a local Fermi liquid phase. In contrast with the clean limit case of the Anderson lattice model, the effective hybridization given by holon condensation turns out to vanish, resulting from the zero mean value of the hybridization coupling constant. However, we show that the holon density becomes finite when the variance of the hybridization is sufficiently larger than that of the RKKY coupling, giving rise to the Kondo effect. On the other hand, when the variance of the hybridization becomes smaller than that of the RKKY coupling, the Kondo effect disappears, resulting in a fully symmetric paramagnetic state, adiabatically connected to the spin liquid state of the disordered Heisenberg model. We investigate the quantum critical point beyond the mean-field approximation. Introducing quantum corrections fully self-consistently in the non-crossing approximation, we prove that the local charge susceptibility has exactly the same critical exponent as the local spin susceptibility, suggesting an enhanced symmetry at the local quantum critical point. This leads us to propose novel duality between the Kondo singlet phase and the critical local moment state beyond the Landau–Ginzburg–Wilson paradigm. The Landau–Ginzburg–Wilson forbidden duality serves the mechanism of electron fractionalization in critical impurity dynamics, where such fractionalized excitations are identified with topological excitations.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The interplay between interactions and disorders has been one of the central issues in modern condensed matter physics [1, 2]. In weakly disordered metal the lowest order interaction correction was shown to modify the density of states at the Fermi energy in the diffusive regime [3], giving rise to non-Fermi liquid physics particularly in low dimensions less than $d = 3$ while further enhancement of electron correlations was predicted to cause ferromagnetism [4]. In an insulating phase, spin glass appears ubiquitously, where the average of the spin moment vanishes on the long time scale, but local spin correlations become finite, making the system move away from equilibrium [5].

An outstanding question is that of the role of disorder in the vicinity of quantum phase transitions [6, 7], where
effective long range interactions associated with critical fluctuations appear to cause non-Fermi liquid physics [7, 8]. Unfortunately, the complexity of this problem has prevented a comprehensive understanding until now. In the vicinity of the weakly disordered ferromagnetic quantum critical point, an electrical transport coefficient has been studied, where the temperature of crossover from the ballistic regime to the diffusive regime is much lowered due to critical fluctuations, compared with the disordered Fermi liquid case [9]. Generally speaking, the stability of the quantum critical point should be addressed, as given by the Harris criterion [10]. When the Harris criterion is not satisfied, three possibilities are expected to arise [7]. The first two possibilities are the emergence of new fixed points, associated with either a finite randomness fixed point satisfying the Harris criterion at this new fixed point or an infinite randomness fixed point exhibiting activated scaling behaviors. The last possibility is that quantum criticality can be destroyed, replaced with a smooth crossover. In addition, even away from the quantum critical point the disordered system may show non-universal power-law physics, called the Griffiths phase [11]. The effects of rare regions are expected to be strong near the infinite randomness fixed point and the disorder-driven crossover region [7].

This study focuses on the role of strong randomness in the heavy fermion quantum transition. Heavy fermion quantum criticality is believed to result from competition between Kondo and RKKY (Ruderman–Kittel–Kasuya–Yosida) interactions, where larger Kondo couplings give rise to a heavy fermion Fermi liquid while larger RKKY interactions lead to an antiferromagnetic metal [7, 8, 12]. Generally speaking, there are two competing viewpoints for this problem. The first direction is to regard the heavy fermion transition as an antiferromagnetic transition, where critical spin fluctuations appear from heavy fermions. The second viewpoint is that the transition is identified with breakdown of the Kondo effect, where Fermi surface fluctuations are critical excitations. The first scenario is described by the Hertz–Moriya–Millis (HMM) theory in terms of heavy electrons coupled with antiferromagnetic spin fluctuations, the standard model for quantum criticality [13]. There are two ways to realize the second scenario depending on how one describes Fermi surface fluctuations. The first way is to express Fermi surface fluctuations in terms of a hybridization order parameter, called a holon in the slave-boson context [14, 15]. This is usually referred to as the Kondo breakdown scenario. The second one is to map the lattice problem into a single-site one, resorting to the dynamical mean-field theory (DMFT) approximation [16], where order parameter fluctuations are critical only in the time direction. This description is called the locally critical scenario [17].

Each scenario predicts its own critical physics. Both the HMM theory and the Kondo breakdown model are based on the standard picture wherein quantum criticality arises from long wavelength critical fluctuations while the locally quantum critical scenario has its special structure, that is, locally (space) critical (time). Critical fluctuations are described with \( z = 2 \) in the HMM theory due to ordering at finite wavevector [13] but with \( z = 3 \) in the Kondo breakdown scenario associated with uniform ‘ordering’ [15], where \( z \) is the dynamical exponent expressing the dispersion relation for critical excitations. Thus, the quantum critical physics is characterized by scaling exponents differing from each other. In addition to qualitative agreements with experiments depending on compounds [7], these two theories do not allow the \( \omega/T \) scaling in the dynamic susceptibility of their critical modes because both theories live above their upper critical dimensions. On the other hand, the locally critical scenario gives rise to the \( \omega/T \) scaling behavior for the dynamic spin susceptibility [17] while it seems to have some difficulties associated with some predictions for transport coefficients.

We start by discussing an Ising model with Gaussian randomness for its exchange coupling, called the Edwards–Anderson model [5]. Using the replica trick and performing the saddle-point analysis, one can find a spin glass phase when the average value of the exchange interaction vanishes, characterized by the Edwards–Anderson order parameter without magnetization. Applying this concept to the Heisenberg model with Gaussian randomness, quantum fluctuations should be incorporated to take into account the Berry phase contribution, carefully. It was demonstrated that quantum corrections in the DMFT approximation lead to the spin glass phase being unstable at finite temperatures, resulting in a spin liquid state when the average value of the exchange coupling vanishes [18]. It should be noted that this spin liquid state differs from the spin liquid phase in frustrated spin systems in the respect that the former state originates from critical single-impurity dynamics while the latter phase results from nontrivial spatial spin correlations described by gauge fluctuations [19]. The spin liquid phase driven by strong randomness is characterized by its critical spin spectrum, given by the \( \omega/T \) scaling local spin susceptibility [18].

Introducing hole doping into the spin liquid state, Parcollet and Georges examined the disordered \( t-J \) model within the DMFT approximation [20]. Using the \( U(1) \) slave-boson representation, they found marginal Fermi liquid phenomenology, where the electrical transport is described with \( T \)-linear resistivity, resulting from the marginal Fermi liquid spectrum for collective modes, here the \( \omega/T \) scaling in the local spin susceptibility. They tried to connect this result to the physics of high \( T_c \) cuprates.

In this study we introduce random hybridization with conduction electrons into the spin liquid state. Our original motivation was to explain both the \( \omega/T \) scaling in the spin spectrum [21] and the typical \( T \)-linear resistivity [22] near the heavy fermion quantum critical point. In particular, we try to reveal the mechanism for the direct continuous transition between the antiferromagnetic phase and the heavy fermion Fermi liquid state in YbRh\(_2\)Si\(_2\) [22, 23], usually forbidden in the Landau–Ginzburg–Wilson framework because such a continuous transition is unlikely to occur between two symmetry-unrelated orders [24]. Although the DMFT approximation is difficult to justify in clean heavy fermion quantum criticality, the presence of strong disorder leads us to the DMFT framework naturally [25], expected to result in the \( \omega/T \) scaling for the spin spectrum [18].
Starting from an Anderson lattice model with disorder, we derive an effective local field theory in the DMFT approximation, where randomness is introduced into both hybridization and RKKY interactions. Performing the saddle-point analysis in the $U(1)$ slave-boson representation, we reveal its phase diagram which shows a quantum phase transition from a spin liquid state to a local Fermi liquid phase. In contrast with the clean limit case for the Anderson lattice model [14, 15], the effective hybridization given by holon condensation turns out to vanish, resulting from the zero mean value of the hybridization coupling constant. However, we show that the holon density becomes finite when the variance of the hybridization is sufficiently larger than that of the RKKY coupling, giving rise to the Kondo effect. On the other hand, when the variance of the hybridization becomes smaller than that of the RKKY coupling, the Kondo effect disappears, resulting in a fully symmetric paramagnetic state, adiabatically connected to the spin liquid state of the disordered Heisenberg model [18].

We focus on the competition between the Kondo effect and RKKY magnetism, which gives rise to quantum phase transitions in the heavy fermion system with strong randomness. The previous works [26] discussed how the non-Fermi liquid physics can appear in the Kondo singlet phase away from quantum criticality. A huge distribution of the Kondo temperature $T_K$ turns out to cause such non-Fermi liquid physics, originating from the finite density of unscreened local moments with almost vanishing $T_K$, where the $T_K$ distribution may result from either the Kondo disorder for localized electrons or the proximity of the Anderson localization for conduction electrons. Since RKKY interactions are not taken into account seriously in these studies, there always exist finite $T_K$ contributions. On the other hand, the presence of RKKY interactions gives rise to breakdown of the Kondo effect, making $T_K = 0$ identically in the strong RKKY coupling phase.

In [27] the role of random RKKY interactions was examined, where the Kondo coupling is fixed while the chemical potential for conduction electrons is introduced as a random variable with its variance $W$. Increasing the randomness of the electron chemical potential, the Fermi liquid state for $W < W_c$ turns into the spin liquid phase for $W > W_c$, which displays the marginal Fermi liquid phenomenology due to random RKKY interactions [27], where the Kondo effect is suppressed due to the proximity of the Anderson localization for conduction electrons [26]. However, the presence of finite Kondo couplings still gives rise to Kondo screening, although the $T_K$ distribution differs from that in the Fermi liquid state, associated with the presence of random RKKY interactions. In addition, the spin liquid state was argued to be unstable against the spin glass phase at low temperatures. On the other hand, we do not take into account the Anderson localization for conduction electrons, and introduce random hybridization couplings. As a result, the Kondo effect is completely destroyed in the spin liquid phase; thus quantum critical physics differs from the previous study of [27].

We would like to point out another perspective [28, 29] for the interplay between the Kondo effect and RKKY magnetism with strong randomness, where this approach is based on a different fixed point, compared with the DMFT description. In particular, the existence of the Griffiths effect has been intensively discussed, where dissipation in the dynamics of magnetic domains, which originates from itinerant electrons, has been argued to play an important role for the Griffiths effect.

We investigate the quantum critical point beyond the mean-field approximation. Introducing quantum corrections fully self-consistently in the non-crossing approximation [30], we prove that the local charge susceptibility has exactly the same critical exponent as the local spin susceptibility. This is quite unusual because these correlation functions are symmetry-unrelated on the lattice scale. This reminds us of deconfined quantum criticality [24], where the Landau–Ginzburg–Wilson forbidden continuous transition may appear with an enhanced emergent symmetry. Actually, the continuous quantum transition was proposed between the antiferromagnetic phase and the valence bond solid state [24]. In the vicinity of the quantum critical point the spin–spin correlation function of the antiferromagnetic channel has the same scaling exponent as the valence bond correlation function, suggesting an emergent $O(5)$ symmetry beyond the symmetry $O(3) \times Z_4$ of the lattice model [31] and confirmed by Monte Carlo simulation of the extended Heisenberg model [32]. Tanaka and Hu proposed an effective $O(5)$ nonlinear $\sigma$ model with the Wess–Zumino–Witten term as an effective field theory for the Landau–Ginzburg–Wilson forbidden quantum critical point [31], expected to allow fractionalized spin excitations due to the topological term. This proposal can be considered as a generalization of an antiferromagnetic spin chain, where an effective field theory is given by an $O(4)$ nonlinear $\sigma$ model with the Wess–Zumino–Witten term, which gives rise to fractionalized spin excitations called spinons, identified with topological solitons [33]. Applying this concept to the present quantum critical point, the enhanced emergent symmetry between charge (holon) and spin (spinons) local modes leads us to propose novel duality between the Kondo singlet phase and the critical local moment state beyond the Landau–Ginzburg–Wilson paradigm [34]. We suggest an $O(4)$ nonlinear $\sigma$ model in a nontrivial manifold as an effective field theory for this local quantum critical point, where the local spin and charge densities form an $O(4)$ vector with a constraint. The symmetry enhancement serves the mechanism of electron fractionalization in critical impurity dynamics, where such fractionalized excitations are identified with topological excitations.

In this study we do not touch on the spin glass phase, which occurs ubiquitously in several heavy fermion compounds [35, 36], well discussed in [37]. It is indispensable for understanding the interplay between the Kondo effect and the RKKY correlation in the spin glass phase, in order to reveal the mechanism of non-Fermi liquid physics in actual heavy fermion compounds. In particular, the emergence of the spin glass phase at low temperatures may open the possibility of exotic quantum states of matter, where some types of spin chiral order can arise, giving rise to interesting
signatures in magnetoresistance and the anomalous Hall effect. Recently, one of the authors investigated an interesting system as the first step in this direction, where the non-Fermi liquid transport in magnetoresistance and the scaling behavior in uniform spin susceptibility have been interpreted in the Kondo–Griffiths scenario [38], basically described by the disordered Anderson lattice model without the random RKKY interaction term. It will be our important future project to introduce the spin glass phase into this Kondo disorder system.

This paper is organized as follows. In section 2 we introduce an effective disordered Anderson lattice model and perform the DMFT approximation with the replica trick. Equation (4) is the main result in this section. In section 3 we perform a saddle-point analysis based on the slave-boson representation and obtain the phase diagram showing breakdown of the Kondo effect driven by the RKKY interaction. We show spectral functions, self-energies, and local spin susceptibility in the Kondo phase. Figures 1–3 and equations (18)–(21) and (23)–(24) are the main results in this section. In section 4 we investigate the nature of the impurity quantum critical point based on the non-crossing approximation beyond the previous mean-field analysis. We solve self-consistent equations analytically and find power-law scaling solutions. As a result, we uncover the marginal Fermi liquid spectrum for the local spin susceptibility. We propose an effective field theory for the quantum critical point and discuss the possible relationship with the deconfined quantum critical point. In section 5 we summarize our results.

2. An effective DMFT action from an Anderson lattice model with strong randomness

We start from an effective Anderson lattice model

\[ H = - \sum_{ij,\sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + E_d \sum_{i\sigma} d^\dagger_{i\sigma} d_{i\sigma} + \sum_{ij} J_{ij} S_i \cdot S_j + \sum_{i\sigma} (V c^\dagger_{i\sigma} d_{i\sigma} + h.c.), \]

where \( t_{ij} = \frac{J}{M^{1/2}} \) is a hopping integral for conduction electrons and \( J_{ij} = \frac{J}{\sqrt{2M}} \) is the hopping integral for conduction electrons and

\[ \sum_{i\sigma} \frac{V}{\sqrt{M}} \varepsilon_i = 0, \quad \sum_{i\sigma} \frac{V}{\sqrt{M}} \varepsilon_i = \delta_{ij}. \]
Anderson localization for conduction electrons. Actually, this results in the metal–insulator transition at the critical disorder strength, suppressing the Kondo effect in the insulating phase. Previously, the Griffiths phase for non-Fermi liquid physics has been attributed to the proximity effect of the Anderson localization [26]. In this work we do not consider the Anderson localization for conduction electrons.

We observe that the disorder average neutralizes spatial correlations except for the hopping term of conduction electrons. This leads us to the DMFT formulation, resulting in an effective local action for the strong random Anderson lattice model:

\[
\mathcal{S}_n^{\text{eff}} = \int_0^\beta d\tau \left\{ \sum_{\sigma a} c_{\sigma a}^\dagger(\tau)(\partial_\tau - \mu)c_{\sigma a}(\tau) + \sum_{\sigma} d_{\sigma a}^\dagger(\tau)(\partial_\tau + E_\sigma)d_{\sigma a}(\tau) \right\} - \frac{V^2}{2M} \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\sigma a \, \sigma b} [c_{\sigma a}^\dagger(\tau)e_{\sigma a}(\tau) + d_{\sigma a}^\dagger(\tau)e_{\sigma a}(\tau)]^2 - \frac{f_0^2}{2M} \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\sigma a \, \sigma b} \sum_{ab} S_{ab}^{\sigma a}(\tau) \times R_{\Phi_{ab}}(\tau - \tau') \delta_{\Phi_{ab}}(\tau', \tau') + \frac{\lambda^2}{M^2} \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{a b a'} \sum_{\sigma} c_{\sigma a}^\dagger(\tau)G_{c\sigma a}(\tau - \tau')c_{\sigma a'}(\tau'),
\]

where \( G_{c\sigma a}(\tau - \tau') \) is the local Green’s function for conduction electrons and \( R_{\Phi_{ab}}(\tau - \tau') \) is the local spin susceptibility for localized spins, given by

\[
G_{c\sigma a}(\tau - \tau') = -(T_a[\tau^\dagger_{a\sigma}(\tau)e_{a\sigma}(\tau')]), \quad R_{\Phi_{ab}}(\tau - \tau') = (T_b[S_{ab}(\tau)e_{b\sigma}(\tau')]),
\]

respectively. Equation (4) with (5) serves as a completely self-consistent framework for this problem. The derivation of equation (4) from (3) is shown in appendix B.

This effective model has two well known limits, corresponding to the disordered Heisenberg model [18] and the disordered Anderson lattice model without RKKY interactions [26], respectively. In the former case a spin liquid state emerges due to strong quantum fluctuations, while a local Fermi liquid phase appears at low temperatures in the latter case as long as the \( T_K \) distribution is not too broadened. In this respect it is natural to consider a quantum phase transition driven by the ratio between variances for the RKKY and hybridization couplings.

### 3. The phase diagram

#### 3.1. The slave-boson representation and mean-field approximation

We solve the effective DMFT action on the basis of the \( U(1) \) slave-boson representation

\[
d^\dagger_{\sigma} = \hat{b}^\dagger e_{\sigma}(0), \quad S^\mu_{\sigma a} = f^\mu_{\sigma a} - f_{\sigma a}^0 \delta_{\mu a'},
\]

with the single-occupancy constraint \( |b^0|^2 + \sum_{a\sigma} f_{\sigma a}^0(\tau)f_{\sigma a}^0(\tau) = 1 \), where \( f_{\sigma a}^0 = \sum_{\sigma'} f_{\sigma a}^{\sigma\sigma'}/M \).

In the mean-field approximation we replace the holon operator \( \hat{b}^\dagger \) with its expectation value \( \langle \hat{b}^\dagger \rangle \equiv b^0 \). Then, the effective action equation (4) becomes

\[
\mathcal{S}_n^{\text{eff}} = \int_0^\beta d\tau \left\{ \sum_{\sigma a} c_{\sigma a}^\dagger(\tau)(\partial_\tau - \mu)c_{\sigma a}(\tau) + \sum_{\sigma} f_{\sigma a}(\tau)f_{\sigma a}(\tau) - \sum_{\sigma a} \sum_{a'} \lambda_{\sigma a a'} \delta(\tau_1 - \tau_2) \right\} - \frac{V^2}{2M} \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\sigma a \, \sigma b} [c_{\sigma a}^\dagger(\tau)e_{\sigma a}(\tau) + d_{\sigma a}^\dagger(\tau)e_{\sigma a}(\tau)]^2 - \frac{f_0^2}{2M} \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\sigma a \, \sigma b} \sum_{ab} S_{ab}^{\sigma a}(\tau) \times R_{\Phi_{ab}}(\tau - \tau') \delta_{\Phi_{ab}}(\tau', \tau') - \frac{\lambda^2}{M^2} \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{a b a'} \sum_{\sigma} c_{\sigma a}^\dagger(\tau)G_{c\sigma a}(\tau - \tau')c_{\sigma a'}(\tau'),
\]

where \( \lambda^a \) is a Lagrange multiplier field, to impose the constraint, and \( d_{\sigma a}^0 = \langle \hat{b}^\dagger e_{\sigma a}(0) \rangle \).

Taking the \( M \to \infty \) limit, we obtain self-consistent equations for self-energy corrections:

\[
\Sigma_{c\sigma a}(\tau) = \frac{V^2}{M} G_{c\sigma a}(\tau)(b^0)^* b^0 + \frac{f_0^2}{M} \delta_{\sigma a a'} G_{c\sigma a}(\tau), \quad \Sigma_{f\sigma a}(\tau) = \frac{V^2}{M} G_{f\sigma a}(\tau)(b^0)^* b^0 + \frac{f_0^2}{M} \sum_{\sigma a a'} G_{c\sigma a}(\tau)[R_{\Phi_{ab}}(\tau)(\tau_{\sigma a}(\tau) + R_{ba}(\tau))](\tau).
\]

\[
\Sigma_{c\sigma a}(\tau) = -\delta_{ab\sigma a'} \delta(\tau_1 - \tau_2) \frac{V^2}{M} \sum_{\tau} |f_{\tau^\dagger_{\sigma a}}(\tau)|^2 b^0 + \text{c.c.} (b^0)^*, \quad \Sigma_{f\sigma a}(\tau) = -\delta_{ab\sigma a'} \delta(\tau_1 - \tau_2) \frac{V^2}{M} \sum_{\tau} \sum_{\tau'} |f_{\tau^\dagger_{\sigma a}}(\tau)|^2 b^0 + \text{c.c.} b^0 a^a.
\]

\[
\Sigma_{c\sigma a}(\tau) = -\delta_{ab\sigma a'} \delta(\tau_1 - \tau_2) \frac{V^2}{M} \sum_{\tau} \sum_{\tau'} |f_{\tau^\dagger_{\sigma a}}(\tau)|^2 b^0 + \text{c.c.} b^0 a^a, \quad \Sigma_{f\sigma a}(\tau) = -\delta_{ab\sigma a'} \delta(\tau_1 - \tau_2) \frac{V^2}{M} \sum_{\tau} \sum_{\tau'} |f_{\tau^\dagger_{\sigma a}}(\tau)|^2 b^0 + \text{c.c.} b^0 a^a,
\]

respectively, where the local Green’s functions are given by

\[
G_{c\sigma a}(\tau) = -\langle T_c c_{\sigma a}^\dagger(\tau)e_{\sigma a}(0) \rangle, \quad G_{f\sigma a}(\tau) = -\langle T_f f_{\sigma a}^\dagger(\tau)e_{\sigma a}(0) \rangle, \quad G_{c\sigma a}(\tau) = -\langle T_c c_{\sigma a}^\dagger(\tau)f_{\sigma a}(0) \rangle, \quad G_{f\sigma a}(\tau) = -\langle T_c c_{\sigma a}^\dagger(\tau)f_{\sigma a}(0) \rangle.
\]
Green’s functions are diagonal in the spin and replica indices, i.e., $G_{\sigma \sigma'}^{ab}(\tau) = \delta_{\sigma \sigma'} G_{\sigma}(\tau)$ with $x = c, f, cf, fc$. Then, we obtain the Dyson equation
\[
\begin{pmatrix}
G_c(\text{i}\omega) & G_{cf}(\text{i}\omega) \\
G_{fc}(\text{i}\omega) & G_f(\text{i}\omega)
\end{pmatrix}
= \begin{pmatrix}
\text{i}\omega + \mu - \Sigma_c(\omega) & -\Sigma_{cf}(\omega) \\
-\Sigma_{fc}(\omega) & \text{i}\omega - E_d - \lambda - \Sigma_f(\omega)
\end{pmatrix}^{-1},
\]
(17)
where $\omega = (2l + 1)\pi T$ with $l$ integer. Accordingly, equations (9)–(12) are simplified as follows:
\[
\begin{align*}
\Sigma_c(\text{i}\omega) &= \frac{V^2}{M} G_f(\text{i}\omega)|b|^2 + \frac{\rho^2}{M^2} G_c(\text{i}\omega), \\
\Sigma_f(\text{i}\omega) &= \frac{V^2}{M} G_c(\text{i}\omega)|b|^2 + \frac{f^2}{2M} \sum_s \sum_{\nu_m} G_f(\text{i}\omega - \nu_m) \\
&\quad \times [R_{\sigma \sigma'}(\nu_m) + R_{\sigma' \sigma}(\nu_m)], \\
\Sigma_{cf}(\text{i}\omega) &= \frac{V^2}{M} G_{fc}(\text{i}\omega) (b^2)^* - \frac{V^2}{M} (b^*)^2 \\
&\quad \times \sum_s \left|f_s^c c_s + f_s^f c_s^f \right|^2,
\end{align*}
\]
(18)
(19)
(20)
in the frequency space. Note that $n$ is the replica index and the last terms in equations (20) and (21) vanish in the limit of $n \to 0$. $R_{\sigma \sigma'}(\nu_m)$ is the local spin susceptibility, given by
\[
R_{\sigma \sigma'}(\tau) = -G_{\sigma \sigma'}(-\tau)G_{\bar{\sigma}\bar{\sigma}}(\tau)
\]
(22)
in the Fourier transformation.

The self-consistent equation for boson condensation is
\[
b \left[ \lambda + 2V^2T \sum_{\omega} G_c(\text{i}\omega) G_f(\text{i}\omega) + V^2T \\
\times \sum_{\omega} \left| G_{fc}(\text{i}\omega) G_{fc}(\text{i}\omega) + G_{cf}(\text{i}\omega) G_{cf}(\text{i}\omega) \right| \right] = 0.
\]
(23)
The constraint equation is given by
\[
|b|^2 + \sum_{\sigma} \left| f_{\sigma}^c c_{\sigma}^c \right|^2 = 1.
\]
(24)

The main difference between the clean and disordered cases is that the off diagonal Green’s function $G_{fc}(\text{i}\omega)$ should vanish in the presence of randomness in $V$ with its zero mean value while it is proportional to the condensation $b$ when the average value of $V$ is finite. In the present situation we find $b^2 = \langle f_{\sigma}^c c_{\sigma}^c \rangle = 0$ while $(b^2)^* b^b = \langle f_{\sigma}^c c_{\sigma}^c c_{\sigma}^f \rangle = 0$. As a result, equations (20) and (21) are identically vanishing on both the left- and right-hand sides. This implies that the Kondo phase is not characterized by the holon condensation but described by a finite density of holons. It is important to notice that this gauge invariant order parameter does not cause any kind of symmetry breaking for the Kondo effect, as should be the case.

3.2. Numerical analysis

We use an iteration method in order to solve the mean-field equations (18)–(21), (23), and (24). For a given $E_d + \lambda$, we use iterations to find all Green’s functions from equations (18)–(21) with equation (22) and $b^2$ from equation (23). Then, we use equation (22) to calculate $\lambda$ and $E_d$. We adjust the value of $E_d + \lambda$ in order to obtain the desirable value for $E_d$. Using the $\lambda$ and $b^2$ obtained, we calculate the Green’s functions in the real frequency by iterations. In the real frequency calculation we introduce the following functions [39]:
\[
\alpha(t) = \int_{-\infty}^{\infty} d\omega \rho(\omega) f(\omega/T),
\]
(25)
where $\rho(\omega) = -\text{Im}G_f(\omega + \text{i}0^+)/\pi$ is the density of states for $f$ electrons, and $f(x) = 1/(\exp(x) + 1)$ is the Fermi–Dirac distribution function. Then, the self-energy correction from spin correlations is expressed as follows:
\[
\begin{align*}
\Sigma_f(\text{i}\omega) &\equiv \frac{f^2}{2M} \sum_s \sum_{\nu_m} G_f(\text{i}\omega - \nu_m) \left[ R_{\sigma \sigma'}(\nu_m) + R_{\sigma' \sigma}(\nu_m) \right] \\
&\quad + \text{i}j^2 \sum_{\omega} \text{tr} \left[ \left( \alpha(t) \right)^2 \alpha^{-1}(t) \right].
\end{align*}
\]
(26)
Performing the Fourier transformation, we calculate $\alpha(t)$ and obtain $\Sigma_f(\omega)$. We perform the numerical calculations with a finite value of $M$ (in particular $M = 2$). Our approach is based on the $1/M$ expansion, and it is exact in the limit $M \to \infty$. The system with a finite value of $M$ is adiabatically connected to the one in the limit $M \to \infty$. Basically, in the limit $M \to \infty$ one has to rescale $V$ and $t$ with $M$ so that $V^2/M$ and $t^2/M^2$ remain constant. This procedure is equivalent to the taking a finite value of $M$ in solving the self-consistent equations.

Figure 1 shows the phase diagram of the strongly disordered Anderson lattice model in the plane of $(V, J)$, where $V$ and $J$ are variabilities for the Kondo and RKKY interactions, respectively. When the random RKKY interaction dominates over the Kondo effect, the Kondo effect is killed completely, and the spin liquid state appears. As discussed in the introduction, this spin liquid state will be unstable against the spin glass phase at low temperatures, which will be an important future subject for us. On the other hand, the Kondo effect wins as the ratio of $V/J$ increases, resulting in the local Fermi liquid. The phase boundary is characterized by $|b|^2 = 0$, below which $|b|^2 \neq 0$ appears to cause effective hybridization between conduction electrons and localized fermions although our numerical analysis shows that $\langle f_{\sigma}^c c_{\sigma}^c \rangle = 0$, meaning that $\Sigma_{cf}(\text{i}\omega) = 0$.
The phase diagram of the strongly disordered Anderson lattice model in the DMFT approximation for different values of $E_d (\mu = 0, T = 0.01, t = 1, M = 2)$. When the random RKKY interaction dominates over the Kondo effect, the Kondo effect is killed completely, and the spin liquid state appears. On the other hand, the Kondo effect prevails as the ratio of $V/J$ increases, resulting in the local Fermi liquid. It is natural to find that the spin liquid phase becomes wider as $-E_d$ is enhanced.

The phase diagram of the strongly disordered Anderson lattice model in the DMFT approximation for different values of $E_d (\mu = 0, T = 0.01, t = 1, M = 2)$, where the $x$-axis in figure 1 is translated into an effective Kondo temperature normalized by its value at $V = V_c$ and $J = 0$ (equation (27)).

The imaginary part of the self-energy of the conduction electrons and that of the localized electrons for various values of $J (V = 0.5, E_d = -0.8, \mu = 0, T = 0.01, t = 1, M = 2)$. The critical line does not depend on $E_d$ much when the variance of the RKKY interaction is small. On the other hand, the critical line shifts lower as the randomness in RKKY is enhanced, implying that the spin liquid region becomes wider.

In figure 3 one finds that the effective hybridization enhances the scattering rate of conduction electrons dramatically around the Fermi energy while the scattering rate for localized electrons is reduced at the resonance energy. One can see that the imaginary part of the conduction electron self-energy strongly increases as the energy approaches the Fermi level. The self-energy effect reflects the spectral function, shown in figure 4, where the pseudogap feature arises in conduction electrons while the sharply defined peak appears in localized electrons, identified with the Kondo effect. This Kondo temperature should be averaged over the disorder distribution, defined as follows:

$$T_K = \int_{-\infty}^{\infty} d\varepsilon P[\varepsilon] \tilde{T}_K \left[ \frac{V}{\sqrt{M}} \right],$$

where $P[\varepsilon]$ is the Gaussian disorder distribution in equations (2). Now, the phase diagram of figure 1 can be recast into that in the plane of $(T_K, J)$. In figure 2 we plot the phase diagram for different values of $E_d$, translating the hybridization variance with the effective Kondo temperature, where the $x$-axis is normalized by the effective Kondo temperature at $V = V_c$ and $J = 0$. It is interesting to observe that the critical line does not depend on $E_d$ much when the variance of the RKKY interaction is small. On the other hand, the critical line shifts lower as the randomness in RKKY is enhanced, implying that the spin liquid region becomes wider.

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4.1. Beyond the saddle-point analysis: the non-crossing approximation

Figure 4. Density of states of the conduction ($\rho_c(\omega)$) and the localized ($\rho_f(\omega)$) electrons for various values of $J$ ($V = 0.5$, $E_d = -0.8$, $\mu = 0$, $T = 0.01$, $t = 1$, $M = 2$).

resonance although the description of the Kondo effect differs from the clean case. Increasing the RKKY coupling, the Kondo effect is suppressed as expected. In this Kondo phase the local spin susceptibility is given by figure 5, displaying the typical $\omega$-linear behavior in the low frequency limit, nothing but the Fermi liquid physics for spin correlations [20]. On increasing $J$, incoherent spin correlations are enhanced, which is consistent with spin liquid physics [20].

One can check $V_c(J = 0)$ limit to recover the known result. In this limit we obtain an analytic expression for $V_c$ at half-filling ($\mu = 0$):

$$V_c(J = 0) = \sqrt{\frac{E_d}{2P_c}}, \quad (28)$$

$$P_c = \int_{-1}^{1} d\omega \rho_0(\omega) \frac{f(\omega/T) - f(0)}{\omega}, \quad (29)$$

where $\rho_0(\omega) = \frac{1}{2} \sqrt{1 - \omega^2}$ is the bare density of states of conduction electrons. One can check $V_c(J = 0) \to 0$ in the zero-temperature limit because $P_c \to \infty$.

4. The nature of quantum criticality

4.1. Beyond the saddle-point analysis: the non-crossing approximation

Resorting to the slave-boson mean-field approximation, we discussed the phase diagram of the strongly disordered Anderson lattice model, where a quantum phase transition appears, from a spin liquid state to a dirty ‘heavy fermion’ Fermi liquid phase, on increasing $V/J$, the ratio of variances of the hybridization and RKKY interactions. Differentiated from that of the heavy fermion quantum transition in the clean situation, the order parameter turns out to be the density of holons instead of the holon condensation. Evaluating self-energies for both conduction electrons and localized electrons, we could identify the Kondo effect from each spectral function. In addition, we obtained the local spin susceptibility, consistent with the Fermi liquid physics.

The next task will be addressing the nature of quantum criticality between the Kondo and spin liquid phases. This question should be addressed beyond the saddle-point analysis. Introducing quantum corrections in the non-crossing approximation, justified in the $M \to \infty$ limit [5], we investigate the quantum critical point, where density fluctuations of holons are critical.

Releasing the slave-boson mean-field approximation to take into account holon excitations, we reach the following self-consistent equations for self-energy corrections:

$$\Sigma_{c\sigma\sigma'}^{ab}(\tau) = \frac{V^2}{M} G_{f\sigma\sigma'}^{ab}(\tau) G_{\sigma\sigma'}^{c\sigma}(\tau) + \frac{f^2}{M^2} \delta_{\sigma\sigma'} G_{c\sigma\sigma'}^{ab}(\tau), \quad (30)$$

$$\Sigma_{f\sigma\sigma'}^{ab}(\tau) = \frac{V^2}{M} G_{c\sigma\sigma'}^{ab}(\tau) G_{\sigma\sigma'}^{f\sigma}(\tau) + \frac{f^2}{2M} \sum_{ss'} G_{f\sigma\sigma'}^{ab}(\tau)[R_{c\sigma\sigma'ss'}^{ab}(\tau) + R_{c\sigma'ss'}^{ab}(\tau) - \delta_{\sigma\sigma'}], \quad (31)$$

5 A cautious person may raise the criticism that the $M \to \infty$ limit is inconsistent with $M = 2$ because the single-impurity dynamics goes to the strong coupling fixed point for $M = 2$ while the $M \to \infty$ limit gives rise to the non-Fermi liquid fixed point instead of the local Fermi liquid state. However, it should be noted that we applied the non-crossing approximation to the quantum critical point, which shows non-Fermi liquid physics with anomalous scaling. We expect $M = 2$ not to be inconsistent with the non-crossing approximation, valid in the $M \to \infty$ limit, because both cases should result in non-Fermi liquid physics near the quantum critical point in the present problem. Actually, the true fixed point identified with the quantum critical point between the local spin liquid and the local Fermi liquid is characterized by anomalous scaling exponents, $\Delta_f = \Delta_0 = 1/2$, certainly independent of $M$, where $\Delta_f$ ($\Delta_0$) is the critical exponent for the spinon (holon) Green’s function.
With effective hybridization will be reduced, where RKKY interaction (the second term in equation (36)), the proportional to equations, called the non-crossing approximation:

\[
\langle c_{\sigma}^{\dagger} f_{\sigma} \rangle = \langle c_{\sigma} \rangle \langle f_{\sigma} \rangle + \langle c_{\sigma} \rangle \langle f_{\sigma} \rangle + \langle c_{\sigma} \rangle \langle f_{\sigma} \rangle.
\]

Since we considered the paramagnetic and replica symmetric phase, it is natural to assume such symmetries at the quantum critical point. Note that the off diagonal self-energies, \( \Sigma_{ij}(\omega) \) and \( \Sigma_{j}(\omega) \), are just constants and proportional to \( \langle f_{\sigma}^{\dagger} c_{\sigma} \rangle \) and \( \langle c_{\sigma}^{\dagger} f_{\sigma} \rangle \), respectively. As a result, \( \Sigma_{ij}(\omega) = \Sigma_{j}(\omega) = 0 \) should be satisfied at the quantum critical point as for the Kondo phase, because \( \langle f_{\sigma}^{\dagger} c_{\sigma} \rangle = \langle c_{\sigma}^{\dagger} f_{\sigma} \rangle = 0 \). Then, we reach the following self-consistent equations, called the non-crossing approximation:

\[
\Sigma_{c}(\tau) = \frac{V^2}{M} G_{c}(\tau) G_{b}(\tau) - \frac{J^2}{M^2} G_{c}(\tau),
\]

\[
\Sigma_{f}(\tau) = \frac{V^2}{M} G_{c}(\tau) G_{b}(\tau) - J^2 G_{f}(\tau)^2 G_{b}(\tau),
\]

\[
\Sigma_{b}(\tau) = \frac{V^2}{M} G_{c}(\tau) G_{b}(\tau) - J^2 G_{f}(\tau)^2 G_{b}(\tau).
\]

The local Green’s functions are given by

\[
G_{c}(\omega) = \left[ \omega + \mu - \Sigma_{c}(\omega) \right]^{-1},
\]

\[
G_{f}(\omega) = \left[ \omega - E_{d} - \Sigma_{f}(\omega) \right]^{-1},
\]

\[
G_{b}(v_{\omega}) = \left[ v_{\omega} - \lambda - \Sigma_{b}(v_{\omega}) \right]^{-1},
\]

where \( \omega_{l} \equiv (2l+1)\pi T \) is for fermions and \( v_{l} \equiv 2l\pi T \) is for bosons.

### 4.2. Asymptotic behavior at zero temperature

For quantum criticality, power-law scaling solutions are expected. Actually, if the second term is neglected in equation (36), equations (36) and (37) are reduced to those of the multi-channel Kondo effect in the non-crossing approximation [30]. Power-law solutions are well known in the regime of \( 1/T_{K} \ll \tau \ll \beta = 1/T \rightarrow \infty \), where \( T_{K} = D [\Gamma_{c} / \pi D]^{1/2} \exp[\pi E_{d} / M \Gamma_{c}] \) is an effective Kondo temperature [40] with the conduction bandwidth \( D \) and effective hybridization \( \Gamma_{c} = \pi \rho e \overline{\Gamma} \). In the presence of the RKKY interaction (the second term in equation (36)), the effective hybridization will be reduced, where \( \Gamma_{c} \) is replaced with \( \Gamma_{c}' \approx \pi \rho e \overline{\Gamma}^2 - J^2 \).

Our power-law ansatz is as follows:

\[
G_{c} = \frac{A_{c}}{\tau^{\Delta_{c}}},
\]

\[
G_{f} = \frac{A_{f}}{\tau^{\Delta_{f}}},
\]

\[
G_{b} = \frac{A_{b}}{\tau^{\Delta_{b}}},
\]

where \( A_{c} \), \( A_{f} \), and \( A_{b} \) are positive numerical constants. In the frequency space these are

\[
G_{c}(\omega) = A_{c} C_{\Delta_{c}} \omega^{\Delta_{c}-1},
\]

\[
G_{f}(\omega) = A_{f} C_{\Delta_{f}} \omega^{\Delta_{f}-1},
\]

\[
G_{b}(\omega) = A_{b} C_{\Delta_{b}} \omega^{\Delta_{b}-1},
\]

where \( C_{\Delta_{c},f,b} = \int_{-\infty}^{\infty} dt e^{i\omega t} g_{c}(t) \).

Inserting equations (44)–(46) into (35)–(37), we obtain scaling exponents of \( \Delta_{c} \), \( \Delta_{f} \), and \( \Delta_{b} \). In appendix C.1 we show how to find such critical exponents in detail. Two fixed points are allowed. One coincides with the multi-channel Kondo effect, given by \( \Delta_{c} = 1 \) and \( \Delta_{f} = 1/2 \) from the scaling equations (C.22) and (C.23). Thus, \( G_{c}(\omega) \sim \delta(\omega) \) and \( G_{f}(\omega) \sim \delta(\omega) \) result for \( \omega \rightarrow 0 \). In this respect both spin fluctuations and holon fluctuations are critical as they have equal strength at this quantum critical point.

#### 4.3. Finite temperature scaling behavior

We solve equations (35)–(37) in the regime \( \tau, \beta \gg 1/T_{K} \) with arbitrary \( \tau / \beta \), where the scaling ansatz at zero temperature is generalized as follows:

\[
G_{c}(\tau) = A_{c} \beta^{-\Delta_{c}} g_{c}(\tau / \beta),
\]

\[
G_{f}(\tau) = A_{f} \beta^{-\Delta_{f}} g_{f}(\tau / \beta),
\]

\[
G_{b}(\tau) = A_{b} \beta^{-\Delta_{b}} g_{b}(\tau / \beta),
\]

\[
G_{a}(\omega) = \left( \frac{\pi}{\sin(\pi \alpha)} \right)^{\Delta_{a}}
\]

with \( \alpha = c, f, b \) the scaling function at finite temperatures. In the frequency space we obtain

\[
G_{c}(i\omega_{l}) = A_{c} \beta^{-\Delta_{c}} \Phi_{c}(i\omega_{l}),
\]

\[
G_{f}(i\omega_{l}) = A_{f} \beta^{-\Delta_{f}} \Phi_{f}(i\omega_{l}),
\]

\[
G_{b}(i\nu_{l}) = A_{b} \beta^{-\Delta_{b}} \Phi_{b}(i\nu_{l}),
\]

where \( \omega_{l} = (2l+1)\pi T, \nu_{l} = 2l\pi T \), and

\[
\Phi_{a}(i\omega_{l}) = \int_{0}^{1} dt e^{i\omega_{l}t} g_{a}(t).
\]
are somewhat complicated. All scaling functions are derived in appendix C.2.

4.4. Spin susceptibility

We evaluate the local spin susceptibility, given by

\[ \chi(\tau) = G_s(\tau)G_s(-\tau) = A_s^2 e^{-2\Delta_f} \left( \frac{\pi}{\sin(\pi \tau/\beta)} \right)^{2\Delta_f}. \]  

(55)

The imaginary part of the spin susceptibility \( \chi''(\omega) = \text{Im} \chi(\omega + i0^+) \) can be found from

\[ \chi(\tau) = \int \frac{d\omega}{\pi} \frac{e^{-\tau\omega}}{1 - e^{-\beta \omega}} \chi''(\omega). \]  

(56)

Inserting the scaling ansatz

\[ \chi''(\omega) = A_s^2 \beta^{1-2\Delta_f} \phi \left( \frac{\omega}{\tau} \right) \]  

(57)

into equation (56) with equation (55), we obtain

\[ \int \frac{dx}{\pi} \frac{e^{-x/\beta}}{1 - e^{-x}} \phi(x) = \left( \frac{\pi}{\sin(\pi \tau/\beta)} \right)^{2\Delta_f}. \]  

(58)

Changing the variable \( t = i(\tau/\beta - 1/2) \), we obtain

\[ \int \frac{dx}{\pi} \frac{e^{ixt}}{e^{x} - e^{-x}} = \left( \frac{\pi}{\cosh(\pi t)} \right)^{2\Delta_f}. \]  

(59)

As a result, we find the scaling function

\[ \phi(x) = 2(2\pi)^{2\Delta_f-1} \sinh(\frac{x}{2}) \times \frac{\Gamma(\Delta_f + ix/2\pi)\Gamma(\Delta_f - ix/2\pi)}{\Gamma(2\Delta_f)}. \]  

(60)

This coincides with the spin spectrum of the spin liquid state when \( V = 0 \) [20].

4.5. Discussion: deconfined local quantum criticality

The local quantum critical point characterized by \( \Delta_c = 1 \) and \( \Delta_f = \Delta_b = 1/2 \) is the genuine critical point in the spin liquid to local Fermi liquid transition because such a fixed point can be connected to the spin liquid state (\( \Delta_c = 1 \) and \( \Delta_f = 1/2 \)) naturally. This fixed point results from the fact that the spinon self-energy correction from RKKY spin fluctuations is of exactly the same order as that from critical holon excitations. It is straightforward to see that the critical exponent of the local spin susceptibility is exactly the same as that of the local charge susceptibility (\( 2\Delta_f = 2\Delta_b = 1 \)), proportional to \( 1/\tau \). Since the spinon spin-density operator differs from the holon charge-density operator in the respect of symmetry at the lattice scale, the same critical exponent implies enhancement of the original symmetry at low energies. The symmetry enhancement sometimes allows a topological term, which assigns a nontrivial quantum number to a topological soliton, identified with an excitation of quantum number fractionalization. This mathematical structure is actually realized in an antiferromagnetic spin chain [33], generalized to the two-dimensional case [24, 31].

We propose the following local field theory in terms of physically observable fields:

\[ Z_{\text{eff}} = \int d\Psi^a(\tau)\delta((\Psi^a(\tau))^2 - 1)e^{-S_{\text{eff}}}, \]

\[ S_{\text{eff}} = -\frac{g^2}{2M} \int_0^\beta dt \int_0^\beta \Psi^a T(\tau) \Upsilon^{ab}(\tau - \tau'). \]  

(61)

where

\[ \Psi^a(\tau) = \left( \frac{S^a(\tau)}{\rho^a(\tau)} \right). \]  

(62)

represents an O(4) vector, satisfying the constraint of the delta function. \( \Upsilon^{ab}(\tau - \tau') \) determines dynamics of the O(4) vector, resulting from spin and holon dynamics in principle. However, it is extremely difficult to derive equation (61) from equation (4) because the density part for the holon field in equation (61) cannot result from equation (4) in a standard way. What we have shown is that the renormalized dynamics equation (61) cannot result from equation (4) in a standard way. However, it is extremely difficult to derive equation (61) from equation (4) because the density part for the holon field in equation (61) cannot result from equation (4) in a standard way.

One can represent the O(4) vector generally as follows:

\[ \Psi^a : \tau \rightarrow (\sin \theta^a(\tau) \sin \phi^a(\tau) \cos \psi^a(\tau), \sin \theta^a(\tau) \sin \phi^a(\tau) \sin \psi^a(\tau), \sin \theta^a(\tau) \cos \phi^a(\tau), \cos \theta^a(\tau)), \]  

(63)

where \( \theta^a(\tau), \phi^a(\tau), \psi^a(\tau) \) are three angle coordinates for the O(4) vector. It is essential to observe that the target manifold for the O(4) vector is not a simple sphere type, but more complicated because the last component of the O(4) vector is the charge-density field, where the three spin components lie in \([-1 \leq S^a_0(\tau), S^a_0(\tau), S^a_0(\tau) \leq 1] \) while the charge density should be positive, \( 0 \leq \rho^a(\tau) \leq 1 \). This leads us to identify the lower half-sphere with the upper half-sphere. Considering that \( \sin \theta^a(\tau) \) can be folded on \( \pi/2 \), we are allowed to construct our target manifold to have a periodicity, given by \( \Psi^a(\theta^a, \phi^a, \psi^a) = \Psi^a(\pi - \theta^a, \phi^a, \psi^a) \). This folded space allows a nontrivial topological excitation.

Consider the boundary configuration of \( \Psi^a(0, \phi^a, \psi^a; \tau = 0) \) and \( \Psi^a(\pi, \phi^a, \psi^a; \tau = \beta) \), connected by \( \Psi^a(\pi/2, \phi^a, \psi^a; 0 < \tau < \beta) \). Interestingly, this configuration is topologically distinguishable from the configuration of \( \Psi^a(0, \phi^a, \psi^a; \tau = 0) \) and \( \Psi^a(0, \phi^a, \psi^a; \tau = \beta) \) with \( \Psi^a(\pi/2, \phi^a, \psi^a; 0 < \tau < \beta) \) because of the folded structure. The second configuration shrinks to a point while the first excitation cannot, identified with a topologically nontrivial excitation.

This topological excitation carries a spin quantum number 1/2 at its core, given by \( \Psi^a(\pi/2, \phi^a, \psi^a; 0 < \tau < \beta) = (\sin \phi^a(\tau) \cos \phi^a(\tau), \sin \phi^a(\tau) \sin \psi^a(\tau), \cos \phi^a(\tau)), 0 \). This is the spinon excitation, described by an O(3) nonlinear \( \sigma \) model with the nontrivial spin correlation function \( \Upsilon^{ab}(\tau - \tau') \), where the topological term is reduced to the single-spin Berry phase term in the instanton core.
In this local impurity picture the local Fermi liquid phase is described by gapping of instantons while the spin liquid state is characterized by condensation of instantons. Of course, the low dimensionality does not allow condensation, resulting in critical dynamics for spinons. This scenario clarifies the Landau–Ginzburg–Wilson forbidden duality between the Kondo singlet and the critical local moment for the impurity state, allowed by the presence of the topological term.

If the symmetry enhancement does not occur, the effective local field theory will be given by

$$Z_{\text{eff}} = \int DS^a(\tau)d\rho^a(\tau)e^{-S_{\text{eff}}},$$

$$S_{\text{eff}} = -\int_0^\beta d\tau \int_0^\beta d\tau' \left\{ \frac{V^2}{2M} \rho^a(\tau)\chi^{ab}(\tau - \tau')\rho^b(\tau') \right\} + \frac{J}{2M} S^a(\tau)R^{ab}(\tau - \tau')S^b(\tau') + S_B$$

with the single-spin Berry phase term

$$S_B = -2\pi i S \int_0^1 du \int_0^\beta d\tau \int_0^\beta d\tau' \frac{1}{4\pi} S^a(u, \tau) \times \partial_u S^a(u, \tau) \times \partial_{\tau'} S^a(u, \tau'),$$

where charge dynamics $\chi^{ab}(\tau - \tau')$ will be different from the spin dynamics $R^{ab}(\tau - \tau')$. This will not allow the spin fractionalization for the critical impurity dynamics, where the instanton construction is not realized due to the absence of the symmetry enhancement.

5. Summary

In this paper we have studied the Anderson lattice model with strong randomness in both hybridization and RKKY interactions, where their average values are zero. In the absence of random hybridization, quantum fluctuations in the spin dynamics cause the spin glass phase to be unstable at finite temperatures, giving rise to the spin liquid state, characterized by the $\omega/T$ scaling spin spectrum consistent with the marginal Fermi liquid phenomenology [18]. In the absence of random RKKY interactions the Kondo effect arises [26], but differentiated from that in the clean case. The dirty ‘heavy fermion’ phase in the strongly disordered Kondo coupling is characterized by a finite density of holons instead of the holon condensation. But effective hybridization does indeed exist, causing the Kondo resonance peak in the spectral function. As long as the variation of the effective Kondo temperature is not too large, this disordered Kondo phase is identified with the local Fermi liquid state because the essential physics results from single-impurity dynamics, differentiated from the clean lattice model case.

Taking into account both random hybridization and RKKY interactions, we find the quantum phase transition from the spin liquid state to the local Fermi liquid phase at the critical $(V_c, J_c)$. Each phase turns out to be adiabatically connected to each limit, i.e. the spin liquid phase when $V = 0$ and the local Fermi liquid phase when $J = 0$. Actually, we have checked this physics, considering the local spin susceptibility and the spectral function for localized electrons.

In order to investigate quantum critical physics, we introduce quantum corrections from critical holon fluctuations in the non-crossing approximation beyond the slave-boson mean-field analysis. We find two kinds of power-law scaling solutions for self-energy corrections of conduction electrons, spinons, and holons. The first solution turns out to coincide with that of the multi-channel Kondo effect, where effects of spin fluctuations are sub-leading, compared with critical holon fluctuations. In this respect this quantum critical point is characterized by the breakdown of the Kondo effect while spin fluctuations can be neglected. On the other hand, the second scaling solution shows that both holon excitations and spinon fluctuations are critical, having the same strength, reflected in the fact that the density–density correlation function of holons has exactly the same critical exponent as the local spin–spin correlation function of spinons.

We argued that the second quantum critical point implies an enhanced emergent symmetry from $O(3) \times O(2)$ (spin $\otimes$ charge) to $O(4)$ at low energies, forcing us to construct an $O(4)$ nonlinear $\sigma$ model on the folded target manifold as an effective field theory for this disorder-driven local quantum critical point. Our effective local field theory identifies spinons with instantons, describing the local Fermi liquid to spin liquid transition as the condensation transition of instantons, although the dynamics of instantons remains critical in the spin liquid state rather than condensation due to the low dimensionality. This construction completes the novel duality between the Kondo and critical local moment phases in the strongly disordered Anderson lattice model.

We explicitly checked that a similar result can be found in the extended DMFT for the clean Kondo lattice model, where two fixed-point solutions are allowed [41, 42]. One is the same as the multi-channel Kondo effect and the other is essentially the same as the second solution in this paper. In this respect we believe that the present scenario works in the extended DMFT framework although it is applicable to only two spatial dimensions [17].

One may question the applicability of the DMFT framework for this disorder problem. However, the hybridization term turns out to be exactly local in the case of strong randomness while the RKKY term is safely approximated as local for the spin liquid state. This situation should be distinguished from the clean case, where the DMFT approximation causes several problems such as the stability of the spin liquid state [43] and strong dependence of the dimension of the spin dynamics [17].

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Appendix A. Derivation of equation (3) from equation (1) by the replica method

The replica trick [5] has been utilized for obtaining the disorder average, given by the following identity:

$$\ln Z = \lim_{n \to 0} \frac{Z^n - 1}{n},$$  \hspace{1cm} (A.1)$$

where $\mathcal{O}$ means the disorder average of an operator $\mathcal{O}$. $Z^n$ is the replicated partition function

$$Z^n = \int \mathcal{D}c^a d\sigma d\sigma^\dagger e^{-S_n},$$  \hspace{1cm} (A.2)$$

where the corresponding replica action is

$$S_n = \int_0^\beta d\tau \left[ \sum_{ia} c^a_{ia}(\tau)(\partial_\tau - \mu)c^a_{ia}(\tau) \right. \\
- \sum_{ij\sigma} \int_0^\tau c^a_{ij}(\tau)c^a_{ji}(\tau) \\
+ \sum_{ia} d^a_{ia}(\tau)(\partial_\tau + E_d)d^a_{ia}(\tau) + \sum_{ij\sigma} J_{ij} S^a_{\sigma}(\tau) \cdot S^a_{\sigma}(\tau) \\
+ \sum_{ia} (V c^a_{ia}(\tau)d^a_{\sigma}(\tau) + \text{h.c.})$$ \\
$$\left. + \sum_{i\sigma\sigma'} \int_0^\tau \bar{\mathcal{Z}} \sum_{\alpha\gamma\delta} J_{ij} \langle T_{\tau} S^a_{\sigma}(\tau) \cdot S^a_{\sigma}(\tau) \rangle_{\alpha\gamma\delta} \right] \hspace{1cm} (A.3)$$

with the spin index $\sigma = 1, \ldots, M$ and the replica index $a = 1, \ldots, n$.

The disorder average for the replicated partition function is straightforward, given by

$$\overline{Z^n} = \int d\epsilon_i P[\epsilon_i] \int \mathcal{D}c^a d\sigma d\sigma^\dagger e^{-S_n},$$  \hspace{1cm} (A.4)$$

where $P[\epsilon_i]$ is the Gaussian distribution function with $\int d\epsilon_i P[\epsilon_i] = 1$. Performing integrations for random variables, we obtain an effective action equation (3).

Appendix B. Derivation of equation (4) from equation (3) by the cavity method

We solve the replicated Anderson lattice model equation (3) in the DMFT approximation. We apply the cavity method for the DMFT mapping [16]

$$\overline{S_n} = \overline{S^n_0} + \Delta S_n + \overline{S^n_0},$$  \hspace{1cm} (B.1)$$

where $\overline{S^n_0}$ is the part of the action at a particular site 0, $\Delta S_n$ is the part of the action connecting the site 0 with other sites, given by

$$\Delta S_n = -\int_0^\beta d\tau \sum_{ia} \langle c^a_{ia}(\tau)(\partial_\tau - \mu)c^a_{ia}(\tau) \rangle_{\alpha\gamma\delta}$$ \\
+ $\varepsilon^a_{ia}(\tau)\langle \epsilon^{b\ast}_{0\sigma}(\tau) \rangle_{\alpha\gamma\delta} - \frac{J^2}{2zM} \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{i\alpha\beta\gamma\delta} S^a_{\alpha\gamma\delta}(\tau) \\
\times S^b_{\beta\gamma\delta}(\tau') \cdot S^b_{\beta\gamma\delta}(\tau'),$$  \hspace{1cm} (B.3)$$

respectively, and $\overline{S^n_0}$ is the rest of the action.

The partition function can be expanded as follows:

$$\overline{Z^n} = \int \mathcal{D}c^a d\sigma d\sigma^\dagger e^{-\overline{S^n_0}}$$ \\
$\times \int \mathcal{D}c^a d\sigma d\sigma^\dagger e^{-\Delta S_n}$$ \\
$+ \frac{1}{\overline{Z^n_0}} \int \mathcal{D}c^a d\sigma d\sigma^\dagger e^{-\overline{S^n_0}} \exp(-\overline{S^n_0})$$ \\
$= \Delta S_n$$  \hspace{1cm} (B.4)$$

The nontrivial term in the first order is given by

$$\int_0^\beta d\tau \langle \Delta \mathcal{L}(\tau) \rangle_{(0)} = -\frac{J^2}{2zM}$$ \\
$\times \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{i\alpha\beta\gamma\delta} \langle T_{\tau} S^a_{\alpha\gamma\delta}(\tau) \cdot S^b_{\beta\gamma\delta}(\tau') \rangle_{(0)}$$ \\
$\times S^b_{\beta\gamma\delta}(\tau') \cdot S^b_{\beta\gamma\delta}(\tau') \rangle_{(0)}$$ \\
$\times R_{\alpha\beta\gamma\delta}(\tau - \tau') S^a_{\alpha\gamma\delta}(\tau')$$  \hspace{1cm} (B.5)$$

where $\overline{S^n_0} = \int \mathcal{D}c^a d\sigma d\sigma^\dagger e^{-\overline{S^n_0}}$ and $\Delta S_n = j_0^\beta d\tau \Delta \mathcal{L}(\tau)$. 

The second-order term is

$$\int_0^\beta d\tau \int_0^\beta d\tau' \langle T_{\tau} \Delta \mathcal{L}(\tau) \Delta \mathcal{L}(\tau') \rangle_{(0)}$$ \\
$\times \{ c^{1b}_{ij\sigma}(\tau')j^{b\ast}_{\alpha\gamma\delta}(\tau') \}$$ \\
$\times c^{1b}_{ij\sigma}(\tau')j^{b\ast}_{\alpha\gamma\delta}(\tau') \rangle_{(0)}$$ \\
\times c^{ab}_{ij\sigma}(\tau - \tau')j^{b\ast}_{\alpha\gamma\delta}(\tau')$$  \hspace{1cm} (B.6)$$

where $\overline{S^n_0} = \int \mathcal{D}c^a d\sigma d\sigma^\dagger e^{-\overline{S^n_0}}$. One can easily verify that all higher order expansions in equation (B.4) vanish in the limit $z \to \infty$, which is at the heart of the DMFT approximation [16].
For the $z \to \infty$ Bethe lattice, we perform a further simplification [16]:

\[
G_{c\sigma a}^{\text{ab}(0)}(t) = \delta_{ij}G_{c\sigma a}^{\text{ab}(0)}(t) \equiv \delta_{ij}G_{c\sigma a}^{\text{ab}(0)}(\tau),
\]

\[
R_{c\rho a}^{\text{ab}(0)}(t) = R_{c\rho a}^{\text{ab}(0)}(\tau).
\]

As a result, we reach an effective single-site action equation (4) called the DMFT approximation.

**Appendix C. Derivation of critical exponents $\Delta_c$, $\Delta_f$, and $\Delta_b$**

### C.1. At zero temperature

Inserting equations (44)–(46) into (35)–(37), we obtain

\[
\Sigma_c(\omega) = \frac{V^2}{M} A_J A_C C_{\Delta_c} + \Delta_c - i \omega \Delta_c + \Delta_c - 1
\]

\[
+ \frac{f^2}{M^2} A_C C_{\Delta_c} - i \omega \Delta_c - 1,
\]

(C.1)

\[
\Sigma_f(\omega) = \frac{V^2}{M} A_J A_C C_{\Delta_f} + \Delta_f - i \omega \Delta_f + \Delta_f - 1
\]

\[
- J^2 A_J C_{\Delta_c} - i \omega \Delta_c - 1,
\]

(C.2)

\[
\Sigma_b(\omega) = V^2 A_J A_C C_{\Delta_f} = - i \omega \Delta_f + \Delta_f - 1.
\]

(C.3)

It is naturally expected that

\[
\Sigma_f(0^+) = -E_d - \lambda,
\]

(C.4)

\[
\Sigma_b(0^+) = -\lambda
\]

(C.5)

for power-law solutions at zero temperature.

Combining this with the Dyson equations (38)–(40), we reach the following equations:

\[
\frac{1}{A_J A_C C_{\Delta_c} - 1} \omega^{1-\Delta_c} = \omega + \mu - \frac{V^2}{M} A_J A_C C_{\Delta_c} + \Delta_c - i \omega \Delta_c + \Delta_c - 1
\]

\[
- \frac{f^2}{M^2} A_C C_{\Delta_c} - i \omega \Delta_c - 1,
\]

(C.6)

\[
\frac{1}{A_J C_{\Delta_f} - 1} \omega^{1-\Delta_f} = - \frac{V^2}{M} A_J A_C C_{\Delta_f} + \Delta_f - i \omega \Delta_f + \Delta_f - 1
\]

\[
+ J^2 A_J C_{\Delta_c} - i \omega \Delta_c - 1,
\]

(C.7)

\[
\frac{1}{A_J A_C C_{\Delta_c} - 1} \omega^{1-\Delta_c} = - V^2 A_J A_C C_{\Delta_f} + \Delta_f - i \omega \Delta_f + \Delta_f - 1
\]

(C.8)

The last equation gives

\[
\Delta_c + \Delta_f + \Delta_b = 2.
\]

(C.9)

Comparing this with the first equation, we get

\[
\Delta_c = 1.
\]

(C.10)

The second equation gives two possible solutions. One is again $\Delta_c + \Delta_f + \Delta_b = 2$, and the other $\Delta_f = \Delta_b = 1/2$. We can find the first solution by equating the coefficients in equations (C.7) and (C.8) and obtain

\[
\frac{1}{C_{\Delta_c - 1} C_{\Delta_f}} = - \frac{V^2}{M} A_J A_C A_C.
\]

(C.11)

These two equations result in

\[
C_{\Delta_c - 1} C_{\Delta_f} = M C_{\Delta_c - 1} C_{\Delta_f}.
\]

Using the property of $C\Delta_1 = C_{\Delta_2}$, we obtain

\[
\Delta_f = M \Delta_b.
\]

(C.12)

As a result, the first solution is

\[
\Delta_f = \frac{M}{M + 1},
\]

(C.13)

\[
\Delta_b = \frac{1}{M + 1},
\]

(C.14)

exactly the same as those for the multi-channel Kondo effect.

### C.2. At finite temperatures

Inserting equations (51)–(53) into (35)–(37), we obtain

\[
\Sigma_c(i\omega_l) = \frac{V^2}{M} A_J A_C B_1 - \Delta_c - \Delta_b \Psi_{b}(i\omega_l)
\]

\[
+ \frac{f^2}{M^2} B_1 - \Delta_c \Psi_c(i\omega_l),
\]

(C.15)

\[
\Sigma_f(i\omega_l) = \frac{V^2}{M} A_J A_C B_1 - \Delta_c - \Delta_b \Psi_{eb}(i\omega_l)
\]

\[
- J^2 A_J \bar{B}_3 - \Delta_f \Psi_{bf}(i\omega_l),
\]

(C.16)

\[
\Sigma_b(i\nu_l) = V^2 A_J A_C B_1 - \Delta_c - \Delta_b \Psi_{ef}(i\nu_l),
\]

(C.17)

where

\[
\Psi_{bf}(i\omega_l) = \int_0^1 dt e^{i\omega_l t} g_f(t) g_b(-t),
\]

(C.18)

\[
\Psi_{eb}(i\omega_l) = \int_0^1 dt e^{i\omega_l t} g_e(t) g_b(t),
\]

(C.19)

\[
\Psi_{ff}(i\nu_l) = \int_0^1 dt e^{i\nu_l t} [g_f(t)]^2 g_f(-t),
\]

(C.20)

\[
\Psi_{ef}(i\nu_l) = \int_0^1 dt e^{i\nu_l t} [g_f(t)]^2 g_e(-t).
\]

(C.21)

Using the Dyson equations, we obtain the final self-consistency expressions

\[
A_c^{-1} \Phi_c^{-1}(i\omega_l) = [i\omega_l T + \mu] B_1 - \Delta_c
\]

\[
- \frac{V^2}{M} A_J A_C B_2 - \Delta_c - \Delta_b \Psi_{b}(i\omega_l) - \frac{f^2}{M^2} B_2 - 2 \Delta_c \Phi_c(i\omega_l),
\]

(C.22)

\[
A_f^{-1} \Phi_f^{-1}(i\omega_l) = i\omega_l B_1 - \Delta_f - [E_d + \lambda - \Sigma_f(i\omega_l)] B_1 - \Delta_f
\]

\[
- \frac{V^2}{M} A_J A_C B_2 - \Delta_c - \Delta_b \Psi_{c}(i\omega_l) - \Psi_{eb}(i\omega_l)
\]

\[
+ J^2 A_J \bar{B}_3 - 4 \Delta_f \Psi_{bf}(i\omega_l),
\]

(C.23)

\[
A_b^{-1} \Phi_b^{-1}(i\nu_l) = i\nu_l B_1 - \Delta_b - [\lambda - \Sigma_b(i\nu_l)] B_1 - \Delta_b
\]

\[
+ V^2 A_J A_C B_2 - \Delta_c - \Delta_b \Psi_{ef}(i\nu_l) - \Psi_{cf}(i\nu_l).
\]

(C.24)
As for the zero-temperature case, we obtain two power-law solutions, comparing the powers of $\beta$ terms. One is

$$\Delta_c = 1,$$

$$\Delta_f + \Delta_b = 1$$

with $\Delta_f > 1/2$. The other solution is

$$\Delta_c = 1,$$

$$\Delta_f = \Delta_b = 1/2.$$  

Note that equations (C.22) and (C.24) are the same for both solutions. Only equation (C.23) distinguishes these two solutions.

Inserting equation (50) into equations (54) and (18)–(19), we obtain

$$\Phi_i(i\omega_0) = i\pi \text{sgn}(i\omega_0),$$

$$\Phi_f(i\omega_i) = (2\pi)^i \delta_0 \ i (-1)^i$$

$$\times \frac{\Gamma(1 - \Delta_f)}{\Gamma(1 - \Delta_f) + \frac{\omega_i}{2\pi}} \frac{\bar{\omega}}{\Gamma(1 - \Delta_f) + \frac{\bar{\omega}}{2\pi}},$$

$$\Phi_b(i\bar{\omega}) = (2\pi)^i \delta_0 \ i (-1)^i$$

$$\times \frac{\Gamma(1 - \Delta_b)}{\Gamma(1 - \Delta_b) + \frac{\omega_i}{2\pi}} \frac{1}{\Gamma(1 - \Delta_b) + \frac{\omega_i}{2\pi}}$$

and

$$\Psi_{\phi}(i\omega_i) = i\pi \text{sgn}(i\omega_i),$$

$$\Psi_{\phi}(i\omega_i) = (2\pi)^i \delta_0 \ i (-1)^i$$

$$\times \frac{\Gamma(1 - 3\Delta_f)}{\Gamma(1 - 3\Delta_f) + \frac{\omega_i}{2\pi}} \frac{\bar{\omega}}{\Gamma(1 - 3\Delta_f) + \frac{\bar{\omega}}{2\pi}},$$

$$\Psi_{\phi}(i\bar{\omega}) = (2\pi)^i \delta_0 \ i (-1)^i$$

$$\times \frac{\Gamma(1 - \Delta_b)}{\Gamma(1 - \Delta_b) + \frac{\omega_i}{2\pi}} \frac{1}{\Gamma(1 - \Delta_b) + \frac{\omega_i}{2\pi}}.$$

Inserting these expressions into equation (C.22), we obtain the equation for $A_c$:

$$\frac{1}{\pi A_c} + \frac{1}{M^2} \pi A_c = i\mu \text{sgn}(i\omega_i) + \frac{V^2}{M} \pi A_b A_f.$$

From equation (C.24) we obtain the condition

$$\Sigma_f(i\omega_0) - \lambda = T^{1 - \Delta_f} \frac{1}{A_b \Phi_f(i\omega_0)}$$

$$= \frac{T^{1 - \Delta_b}}{A_b (2\pi)^i \delta_0 (-1)^i \Gamma(1 - \Delta_b)}$$

and the equation

$$A_f^{-1} \left[ \Phi_f(i\bar{\omega}) - \Phi_f(i\omega_i) \right]$$

$$= V^2 A_c A_f [\Psi_{\phi}(i\bar{\omega}) - \Psi_{\phi}(i\omega_i)].$$

Inserting equations (C.31) and (C.35) into equation (C.38), we obtain

$$\left[ \frac{\Gamma(1 - \Delta_f^2 + \frac{\bar{\omega}}{2\pi}) \Gamma(1 - \Delta_f^2 - \frac{\bar{\omega}}{2\pi})}{2\pi \pi} \right]^{(i-1)} \Gamma(1 - \Delta_f)$$

$$= \frac{V^2}{M} \pi A_b (2\pi)^i \delta_0 (-1)^i \Gamma(1 - \Delta_f)$$

$$= \frac{V^2}{M} \pi A_b A_f (2\pi)^i \delta_0 (-1)^i \Gamma(1 - \Delta_f).$$
One can show that

\[
\left[ \Gamma(1 - \frac{\Delta_f}{2} + \frac{l \omega_i}{2\pi}) \Gamma(1 - \frac{\Delta_f}{2} - \frac{l \omega_i}{2\pi}) \right]^{-1} (l = 0) = \left( \frac{1}{2} - \frac{\Delta_f}{2} \right) \frac{\left[ \Gamma\left( \frac{1}{2} + \frac{\Delta_f}{2\pi} \right) \right]^2}{\Gamma(1 - \Delta_f)} \times \frac{\prod_{k=1}^{l} (k + \frac{\Delta_f}{2})}{\prod_{k=1}^{l} (k - \frac{\Delta_f}{2} + \frac{\omega_i}{2\pi}) - 1}.
\]

\[\text{(C.46)}\]

From equations (C.45)–(C.47) we get

\[1 = -\frac{V^2}{M} A_f A_b (2\pi)^2 \frac{\Gamma(-\Delta_b) \Gamma(1 - \Delta_f)}{[\Gamma(\frac{1}{2} + \frac{\Delta_f}{2\pi}) \Gamma(\frac{1}{2} - \frac{\Delta_f}{2\pi})]^2} \times \frac{\prod_{k=1}^{l} (k + \frac{\Delta_f}{2})}{\prod_{k=1}^{l} (k - \frac{\Delta_f}{2} + \frac{\omega_i}{2\pi}) - 1}.
\]

\[\text{(C.47)}\]

Equations (C.42) and (C.48) result in the following equation:

\[M \Gamma(-\Delta_f) \Gamma(1 - \Delta_b) \Gamma(1 - \Delta_f) = \Gamma(-\Delta_b) \Gamma(1 - \Delta_f),\]

\[\text{(C.49)}\]

or equivalently,

\[M A_b = \Delta_f.\]

\[\text{(C.50)}\]

As a result, we obtain the solution

\[\Delta_f = \frac{M}{M + 1},\]

\[\text{(C.51)}\]

\[\Delta_b = \frac{1}{M + 1},\]

\[\text{(C.52)}\]

with the condition \(M > 1\), and \(M = 2\) actually.

### C.2.2. The second solution: \(\Delta_f = \Delta_b = 1/2\)

In this case the scaling equation (C.23) becomes

\[A^{-1}_f \Psi_{\text{hyb}}(i\omega_i) - \Phi^{-1}_f(i\omega_i) = -\frac{V^2}{M} A_f A_b [\Psi_{\text{hyb}}(i\omega_i) - \Psi_{\text{hyb}}(i\omega_i)],\]

\[+ J^2 A_f^2 \Psi_{\text{hyb}}(i\omega_i) - \Phi^{-1}_f(i\omega_i),\]

\[\text{(C.53)}\]

where the hybridization and the RKKY interactions give rise to the same order of magnitude for self-energy corrections. We can use equations (C.45)–(C.47) with \(\Delta_f = \Delta_b = 1/2\), where the RKKY term of \(\Psi_{\text{hyb}}(i\omega_i)\) will give a similar result.

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