MULTIFRAGMENTATION OF NON-SPHERICAL NUCLEI: ANALYSIS OF CENTRAL Xe + Sn COLLISIONS AT 50 MeV·A

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Abstract

The influence of shape of expanding and rotating source on various characteristics of the multifragmentation process is studied. The analysis is based on the extension of the statistical microcanonical multifragmentation model. The comparison with the data is done for central Xe + Sn collisions at 50A·MeV as measured by INDRA Collaboration.

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The multifragmentation process has been studied in a broad range of bombarding energies and for various types of projectiles. The reaction mechanism is often considered in terms of two-step scenario where the first, dynamical step results in the formation of thermalized source which then, in the second step, decays statistically into light particles and intermediate-mass fragments (IMF’s). Assuming that the thermal equilibrium is attained, various statistical multifragmentation models were employed for the second step (see [1, 2] and references quoted therein). These models were so successful that deviations between their predictions and the experimental data have been often taken as an indication for dynamical effects in the multifragmentation. However, one should be aware of several oversimplifying assumptions in the statistical calculations, such as, e.g., the spherical shape of the thermalized source. Indeed, one expects that the spherical shape is perturbed during the dynamical phase and the density evolution can give rise to complicated source forms [3]. Even more important are the angular momentum induced shape instabilities [4] which may cause large fluctuations of both the Coulomb barrier and the surface energy even for moderately high angular momenta ($L \sim 40\bar{h}$). In this paper, the non-spherical fragmenting source is considered within the statistical model [5] which is based on the MMMC method of the Berlin group [1]. The observables sensitive to the source shape are discussed and preliminary comparison with the experimental data for $E_{\text{kin}}(Z)$ is presented.

An explicit treatment of the fragment positions in the occupied spatial volume allows for a direct extension of the MMMC method to the case of non-spherical shapes [5]. The source deformation in this case is considered as an external constraint, similarly as the freeze-out volume. Below we shall discuss axially symmetric ellipsoidal configurations ($R_x = R_y \neq R_z$) which are characterized by the ratio: $\mathcal{R} = R_x/R_z$. The freeze-out volume of deformed system is the same as that of an ‘equivalent’ spherical system with the radius $R_{\text{sys}} = (R_x R_y R_z)^{1/3}$. Consequently, the statistical weights in the Metropolis scheme [1] due to the accessible volume for fragments remain the same. The Coulomb energy in our model is calculated exactly for every multifragment configuration of non-spherical nucleus. The general scheme to account for the angular momentum in the calculation of statistical weights is the same as discussed in Ref. [7]. The source deformation will change the moment of inertia of rotating system. In calculating the statistical decay of fragmenting system, the angular velocity of the source is added to the thermal velocity of each fragment. For each spatial configuration of fragments, part of the total energy goes into rotation and hence the temperature of the system will slightly fluctuate. We take also into account fluctuations of the moment of inertia.
arising from fluctuations in the positions of fragments and light particles.

In calculating all accessible states in the standard MMMC method [1], the source should be averaged with respect to spatial orientations of its axes. Some of these states will not be accessible if the angular momentum is conserved [6, 7]. We disentangle in the MMMC code the beam direction (the z-axis) and the rotation axis (the x-axis, perpendicular to the reaction plane).

The rotation energy is then: \( \frac{L^2}{2J_x} \equiv \frac{L_z^2}{2J_x} \), where \( J_x \) is the rigid-body moment of inertia with respect to the x-axis. Averaging over polar angle \( \theta \) is not consistent with the angular momentum conservation. On the contrary, averaging over \( 2\pi \) in the angle \( \phi \) corresponds to averaging over azimuthal angle of the reaction impact parameter and should be included. Averaging over rotation angle \( \psi \) around \( \mathbf{L} \) depends on the considered relationship between a rotation time: \( \tau_{\text{rot}} = \frac{J_x}{L_x} \), and a characteristic life-time of the source \( \tau_c \).

For high angular momenta when \( \tau_{\text{rot}} \ll \tau_c \), the full averaging in \( 0 \leq \psi \leq 2\pi \) should be performed. In the opposite limit when \( \tau_{\text{rot}} \gg \tau_c \), only states with \( \psi \simeq 0 \) are accessible.

In central HI collisions, part of the excitation energy can be stored in the compression energy of pre-formed source which is transformed into the kinetic energy of fragments during the collective expansion. To get some insight into the influence of collective flow on the multifragmentation process, we shall mimic this effect by adding the blast velocity \( v_b \) to the thermal velocity of each particle/fragment for any event simulated by the Metropolis method. The average collective energy of expansion, similarly as the deformation energy, is not included in the value of the total excitation energy. We assume a simple scaling solution of non-relativistic hydrodynamics which yields the radial velocity profile [8, 9]:

\[
v_b(r) = v_0 \left( \frac{r}{R_0} \right)^\alpha ,
\]

where \( v_0 \) and \( R_0 \) are the strength and scale parameters respectively, and the exponent \( \alpha \) characterizes the power-low profile function. \( R_0 \) corresponds to the system size at the beginning of the scaling expansion regime. Hence, \( R_0 < R_{\text{sys}} \) and we take \( R_0 = 0.7R_{\text{sys}} \) in all studies. Strictly speaking, the profile function [10] describes radial expansion of a spherical source. For axially symmetric expansion, the velocity profile may be more complicated. But even in this case, the scaling solution [10] with \( 0.5 \leq \alpha \leq 2 \) was successfully applied for describing the velocity profile of the transverse expansion in high energy heavy-ion collisions [9, 11, 12]. In the multifragmentation case, we are dealing with a non-spherical expansion of unstable matter and therefore \( \alpha \) may be treated as a free parameter.
Let us now consider the multifragmentation of $^{197}\text{Au}$ having the angular momentum $L = 40\hbar$ and the thermal excitation energy $6\ A \cdot MeV$. These parameters characterize an equilibrized source formed in central $Xe + Sn$ collisions at $50\ A \cdot MeV$ studied by INDRA Collaboration [12, 13]. All calculations are carried out at the break-up density $\rho \approx \rho_0/6$, what gives $R_{sys} = 12.8\ fm$ for the effective radius of $^{197}\text{Au}$ source. In the following, we consider ellipsoidal forms with the ratio of axes $R = 0.6$ (the prolate shape) and $R = 1/0.6 = 1.667$ (the oblate shape). We have found that none of the observables related to the fragment-size distribution is sensitive to the source deformation at these high excitation energies. On the contrary, the c.m.s. angular distribution of the largest fragment ($Z = Z_{max}$) is a highly sensitive observable (see Fig.1). In the absence of collective expansion, the angular distribution of $Z_{max}$ is isotropic for oblate configurations and has small forward - backward peaks for prolate configurations if the $\psi$- averaging is performed in the whole available interval : $0 \leq \psi \leq 2\pi$. For the 'frozen' spatial configuration ($\psi = 0$), the shape effect is clearly seen: for the prolate form one finds strong forward - backward peaks, while for the oblate form the heaviest fragment is predominantly emitted in the sideward direction ($\theta_{cm} = \pi/2$), like in the hydrodynamic splashing scenario. The collective expansion ($\alpha = 2$) enhances the 'deformation effect'. One may notice a strong increase of forward and backward peaks in the prolate case and the appearance of a strong peak at $\theta_{cm} = \pi/2$ in the oblate case. Similar features can be seen also in the cumulative angular distributions of all IMF’s but the relative amplitude of the 'deformation effect' in that case is smaller.

Large sensitivity to the source shape can be seen in the analysis on an event-by-event basis using global variables, such as the sphericity, coplanarity, aplanarity and the flow angle $\Theta_{flow}$. In the latter case, the shape differences manifest themselves even for $v_h = 0$. The whole effect is extremely sensitive to the collective expansion and is enhanced furthermore for the 'frozen' configuration ($\psi = 0$). It should be mentioned that $\Theta_{flow}$-distribution is the only observable which explicitly depends on the angular momentum [5].

We have compared the model predictions with the experimental data for central $Xe + Sn$ collisions at $50\ A \cdot MeV$ [12, 13, 14, 15]. Various statistical multifragmentation codes assuming the spherical source shape (the standard MMC [1], the SMM [2]) have been tried earlier to explain these data. All of them were successful in predicting fragment partitions [12, 14, 15] which, as stated above, do not provide a constraint on the source shape. On the other hand, these models failed to reproduce the kinematical observables such as the IMF’s angular distributions, the event shapes (sphericity, coplanarity,
aplanarity), the IMF’s average kinetic energies $E_{\text{kin}}(Z)$. In the latter case, the behaviour for heaviest fragments could not be reproduced even including radial expansion [12]. To understand this issue, we have applied our model for different shapes, orientations, expansion profiles (following (1)), starting from the source of $^{197}$Au having $6\,A\cdot MeV$ of thermal excitation energy and an angular momentum of $L = 40\hbar$. All calculated events have been filtered with the INDRA software replica, and then selected with the experimental centrality condition: complete events (i.e., more than 80% of the total charge and momentum is detected) and $\Theta_{\text{flow}} \geq \pi/3$. We consider first the average kinetic energies of IMF’s as a function of their charge. Results for different deformations $R = 0.6, 1.667$ and expansion parameterizations are compared in Fig. 2 with the experimental data [12, 13, 14]. The calculations have been done for the 'frozen' configuration ($\psi = 0$). One can see that the data excludes the radial velocity profile with $\alpha = 1$ and clearly favours the expanding prolate source with $\alpha = 2$ and $\psi \sim 0$. These source characteristics have been confirmed also by the observables related to the event shape, like the sphericity, coplanarity and aplanarity distributions, for which an excellent fit of the data has been obtained. Finally, we have examined the experimental IMF’s and heaviest fragment angular distributions, as well as the $\Theta_{\text{flow}}$-distribution which again clearly discriminate between prolate, spherical and oblate source shapes. On the basis of this whole set of observables, we believe to have the strong evidence in central $Xe + Sn$ collisions at $50\,A\cdot MeV$ for the formation of prolate, slowly rotating source with the orientation of the source aligned with the beam axis.

In conclusion, we have made an extension of the Berlin statistical multifragmentation code in order to include the effects of deformation, angular momentum and collective expansion. Due to the change in the Coulomb energy for deformed freeze-out configuration, the 'deformation effect' is clearly seen in the IMF’s angular distributions ($Z_{\text{max}}$- angular distribution) as well as in the $\Theta_{\text{flow}}$- distribution. A surprising interplay between effects of non-spherical freeze-out shapes and the memory effects of nonequilibrium phase of the reaction, such as the rotation and the collective expansion of the source, has been found. The influence of shape on rotational properties of the system is not only reduced to the modification of the momenta of inertia. The limits on the averaging interval over the angle $\psi$ about the rotation axis, which are defined by the time scales involved, affect strongly the angular observables and are able to enhance strongly the 'deformation effect'. These constraints may be important for certain observables used in experimental procedures of selecting a specific class of events. Other striking finding is that the collective...
expansion allows to disclosure the source shape in the analysis using global variables as well as in the study of $Z$-dependence of $E_{\text{kin}}$. All these experimental observables, including $E_{\text{kin}}(Z)$, are well reproduced assuming strongly deformed ($R = 0.6$), slowly rotating ($L = 40\hbar$) and expanding source which has the radial velocity profile (1) with $\alpha \simeq 2$. Alongside with the observables discussed here, it would be interesting to investigate the velocity and angular correlations between fragments which are sensitive to the source shape at the freeze-out. Such a work is now in progress [16].

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Fig. 1. Angular distribution of $Z_{\text{max}}$ in the c.m.s. Plots on the l.h.s. correspond to averaging over the whole available interval $0 \leq \psi \leq 2\pi$, whereas plots on the r.h.s. correspond to the 'frozen' configuration $\psi = 0$. (a),(b) $L = 40\, \hbar$, $v_b = 0$; (c),(d) $L = 40\, \hbar$, $v_b = 0.08c$, $\alpha = 2$. 
Fig. 2. Kinetic energy of IMF’s is plotted as a function of $Z$. Thick lines correspond to events satisfying experimental 'centrality' condition: $\Theta_{flow} \geq \pi/3$ and accepted by the INDRA filter. Thin lines correspond to unfiltered events and no condition on $\Theta_{flow}$. Prolate freezeout configuration is denoted with full lines and oblate configuration is denoted by the dashed lines. The experimental data [13] are shown by dots. (a) $L = 40\hbar$, $\alpha = 2$, $v_b = 0.08c$. (b) The same as in (a) but for $\alpha = 1$. (c) Comparison between the experimental data and the calculation ($\alpha = 2$) for both prolate and oblate freeze-out configurations and the blast velocities: $v_b = 0.088c$ (prolate shape), $v_b = 0.063c$ (oblate shape), chosen as to reproduce the experimental mean kinetic energy per nucleon of IMF’s: $5.2 \pm 0.1 MeV/\text{nucleon}$. (d) The same as in (c) but for $\alpha = 1$. The blast velocities are: $v_b = 0.08c$ and $v_b = 0.07c$ for prolate and oblate freeze-out configuration respectively.