Metal-insulator transitions and large magnetoresistance effects in diluted magnetic semiconductors

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Abstract. We have discussed the localization mechanism of hole-induced magnetic solitons and large magnetoresistance effects in diluted magnetic semiconductors, by using the effective Lagrangian and the supersymmetric sigma formula.

1. Introduction
Detailed measurements of transport very close to two metal-insulator critical points and large magnetoresistance (MR) effects in diluted magnetic semiconductors (Ga, Mn)As have been reported [1]. The first metal-insulator transition (MIT) at the dilute-side of the metallic phase is associated with the appearance of the ferromagnetism. On the other hand, the second MIT at higher concentration is purely driven by disorder and disorder-modified Coulomb interaction. In addition, interesting phenomena such as the photo-induced magnetic polaron in diluted magnetic semiconductors have been discovered [2]. These works stimulated us to the study of the carrier-induced magnetic soliton, which is an interesting and challenging subject. Recently the present author [3-5] has discussed the localization mechanism in the MIT, using the gauge-invariant Lagrangian for the hole-induced magnetic solitons. In this study, we have discussed the relationship between the MIT and large magnetoresistance effects, extending the previous formula [3-5] and the supersymmetric sigma formula.

2. A model system and the large magnetoresistance
Ferromagnetism, in hole-doped diluted magnetic semiconductors, is called "carrier-induced ferromagnetism" because hole-carriers introduced Mn incorporation mediate the ferromagnetic coupling between Mn ions. It has been suggested that the ferromagnetic interaction induced by the hole seems to be cooperative and non-linear. In order to argue in the gauge-invariant formula, we shall introduce the non-linear gauge fields (Yang-Mills fields) $A^a_{\mu}$, which mediate the effective ferromagnetic interaction induced by the hole. It has been proposed that the hedgehog-like soliton in three-dimensional system is specified by rigid-body rotation, which is related to gauge fields of SO(4) symmetry for $S^3$ [6-8]. Thus it is thought that the non-linear gauge fields $A^a_{\mu}$ introduced by the hole have a local SO(4) symmetry. Then we have assumed that the SO(4) quadruplet fields, $A^a_{\mu}$, are spontaneously broken around the doped hole.
through the Anderson-Higgs mechanism, in the III-V-based DMS with magnetic manganese ion-doping. We set the symmetry breaking \( \langle 0| \phi_a | 0 \rangle = \langle 0, 0, 0, \mu \rangle \) of the Bose field \( \phi_a \) in the Lagrangian density as follows. After the symmetry breaking \( \langle 0| \phi_a | 0 \rangle = \langle 0, 0, 0, \mu \rangle \), we can obtain the effective Lagrangian density,

\[
L_{\text{eff}} = \frac{1}{2} \left( \partial_\mu S^j - g_1 \epsilon_{ijk} A^0_i S^k \right)^2 \\
+ \phi^+ \left( i \partial_\mu - g_2 T_\mu A^0_\mu \right) \phi \\
- \frac{1}{2m} \phi^+ \left( i \nabla - g_2 T_\mu A^0_\mu \right)^2 \phi \\
- \frac{1}{4} \left( \partial_\mu A^0_\mu - \partial_\mu A^0_\mu + g_3 \epsilon_{abc} A^a_\mu A^b_\mu \right)^2 \\
+ \frac{1}{2} \left( \partial_\mu \phi_a - g_4 \epsilon_{abc} A^0_\mu \phi_a \right)^2 \\
+ \frac{1}{2} \left( A^4_\mu \right)^2 + \left( A^3_\mu \right)^2 \\
+ m_1 \left[ A^1_\mu \partial_\mu \phi_2 - A^2_\mu \partial_\mu \phi_1 \right] \\
+ m_1 \left[ A^3_\mu \partial_\mu \phi_3 - A^2_\mu \partial_\mu \phi_2 \right] \\
+ m_1 \left[ A^2_\mu \partial_\mu \phi_3 - A^3_\mu \partial_\mu \phi_2 \right] \\
+ g_4 m_1 \left\{ \phi_4 \left( A^1_\mu \right)^2 + A^2_\mu \right) + \left( A^3_\mu \right)^2 \right\} \\
- g_4 m_1 \left\{ A^4_\mu \left( \phi_1 A^1_\mu + \phi_2 A^2_\mu + \phi_3 A^3_\mu \right) \right\} \\
- \frac{m_2^2}{2} (\phi_4)^2 - \frac{m_4^2 g_4 m_1}{2 m_1} (\phi_4)^2 - \frac{m_2^2 g_4^2 m_1}{8 m_1^2} (\phi_a \phi_a)^2 ,
\]

(1)

where \( S^j \) is the spin of Mn, \( \psi \) is the Fermi field of the hole, \( m_1 = \mu \cdot g_4, m_2 = 2(2)^{1/2} \lambda \cdot \mu \). Recent study [9] shows that carriers of the hole seem to be coupled to Mn spins by an antiferromagnetic Heisenberg exchange interaction. Thus \( \hat{j} \) corresponds to the reverse direction of the spin one of the hole. The effective Lagrangian describes three massive gauge fields \( A^1_\mu, A^2_\mu, \) and \( A^3_\mu \), and one massless gauge field \( A^4_\mu \). Because masses of \( A^1_\mu, A^2_\mu \) and \( A^3_\mu \) are created through the Anderson-Higgs mechanism by introducing the hole, the fields \( A^1_\mu, A^2_\mu \) and \( A^3_\mu \) exist around the hole within the length of \( \sim 1/m_1 \equiv RC \). From the first term in Eq. (1), the spins \( S \) of Mn atoms are induced in the ferromagnet state, where the average spin is parallel to \( \hat{j} \) direction, within the length of \( \sim R_C \) around the hole. That is, the effective Lagrangian represents that the ferromagnetically aligned Mn spins form clusters, in which the hole is trapped, with the radius, \( R_C \sim 1/m_1 \). Especially Katsumoto et al. [1] indicated that the finite localization length \( l_c \) of the wave functions of holes plays a crucial role in MIT in (Ga,Mn)As. It looks like that the \( l_c \) might correspond to \( R_C \sim 1/m_1 \). We shall consider the transport property in the randomly distributed system of the hole-induced magnetic solitons in DMS, by using the effective Lagrangian of diffusion modes. In terms of the four-component supervector \( \psi[10] \), the Lagrangian in this system takes the form,

\[
L = i \int \overline{\psi}(r)(-H_0 - V(r) + \frac{1}{2}(\omega + i \delta)\Lambda)\psi(r)dr \\
H_0 = \epsilon + \frac{1}{2m} \Delta + \mu.
\]

(2)

(3)
where $V(r)$ is the random potential, and $\Lambda$ is the diagonal supermatrix

$$
\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
$$

(4)

with 1 the 4 x 4 unity matrix. The potential will be regarded as a random quantity with a Gaussian $\delta$-correlation distribution

$$
\langle V(r)V(r') \rangle = \frac{\text{Im} \Sigma(k_F,\epsilon_F)}{2\pi \nu} \delta(r-r'),
$$

(5)

where $\Sigma(k_F,\epsilon_F)$ of the self energy of the hedgehog-like solitons [3], and $\nu$ is the state density per spin. After averaging, eq(2) can be rewritten in the form,

$$
\mathcal{L}_{\text{eff}} = \int \left[ -i\bar{\psi}H_0\psi + \frac{\text{Im}\Sigma(k_F,\epsilon_F)}{4\pi \nu} (\bar{\psi}\psi)^2 - \frac{i(\omega + i\delta)}{2}\bar{\psi}\Lambda\psi \right] dr
$$

$$
+ \frac{4\pi \nu \text{Im}\Sigma(k_F,\epsilon_F)}{2} Q \left. \bar{\psi}\psi \right|_Q^2 dr
$$

(6)

with the $8 \times 8$ supermatrix $Q$ satisfying the following self-consistency equation,

$$
Q = \frac{2}{\pi \nu} \int \bar{\psi}\bar{\psi}\exp(-\mathcal{L}_{\text{eff}})D\psi \equiv \frac{2}{\pi \nu} \langle \bar{\psi}\psi \rangle_{\text{eff}}.
$$

(7)

Then the free energy $F(Q)$ can be written as

$$
F(Q) = \int \left[ \frac{1}{2} \text{str} \ln(-iH_0 - \frac{i}{2}(\omega + i\delta)\Lambda + \frac{\text{Im}\Sigma(k_F,\epsilon_F)}{2} Q) \right] dr
$$

$$
+ \frac{\pi \nu \text{Im}\Sigma(k_F,\epsilon_F)}{8} \text{str} Q^2 dr.
$$

(8)

In the case of low values of $\omega$, the free energy for all $Q$’s satisfying the condition $Q^2 = 1$ differs little from the minimum value. All such zero-trace matrices can be written in the form, $Q = V\Lambda V^\dagger$, where $V$ is an arbitrary unitary supermatriv, $VV^\dagger = 1$. In order to discuss the spin dynamics and electron hopping, we envisage an effective hamiltonian, $H$, for the magnetic-soliton, $O(r_{\tilde{i}})$, which is introduced in eq.(1),

$$
H = -J \sum_{\langle i,j \rangle} \cos(\theta_{\tilde{i}\tilde{j}}/2)O(r_{\tilde{i}}) \cdot O(r_{\tilde{j}})
$$

$$
+ K \sum_{i \neq j} \frac{O(r_{\tilde{i}}) \cdot O(r_{\tilde{j}})}{|r_{\tilde{i}} - r_{\tilde{j}}|}
$$

(9)

and the first sum taken only over nearest neighbor (the distance between each magnetic soliton is $\leq 2R_c$) and the second taken over all pair ($i \neq j$ means $|r_{\tilde{i}} - r_{\tilde{j}}| \gg 2R_c$) [4]. $S_{\tilde{i}} \equiv \sum_{i \in (4/3)\pi R_c^2(\tilde{i})} S_i$.

That is, $S_{\tilde{i}}$ is the summation of the ferromagnetic spin, $S_i$, of Mn within $\sim (4/3)\pi R_c^2(\tilde{i})$ around the photo-induced hole at the site $r_{\tilde{i}}$. $S_{\tilde{i}}$ represents the effective spin of the soliton $O(r_{\tilde{i}})$. $\theta_{\tilde{i}\tilde{j}}$ is the angle between $S_{\tilde{i}}$ and $S_{\tilde{j}}$. The first term corresponds to short-range ferromagnetic ordering interaction and the second corresponds to long-range frustration. Although the first term of the effective Hamiltonian in eq.(9) cannot be derived immediately from the effective Lagrangian in eq.(1), this term can be introduced approximately as follows. When the magnetic soliton, $O(r_{\tilde{i}})$,
with the effective spin $S_\tilde{i}$ is located in the nearest neighbors of the magnetic soliton, $O(r_j)$, with the effective spin $S_\tilde{j}$, holes are hopping between two solitons $O(r_j)$ and $O(r_j)$. If $S_\tilde{i}$ is parallel to $S_\tilde{j}$, p-d exchange interaction induces much reduction of the kinetic energy. This hopping term between the nearest neighbors of magnetic solitons (clusters) leads to an additional term in the $\sigma$ - model describing a coupling of the supermatrices, $Q_\tilde{i}$, corresponding to different magnetic solitons (clusters). Approximately, we get the following free energy, using the formula of the model of a granulated metal [11,12].

$$\tilde{F}(Q) = str(- \sum_{\langle \tilde{i},\tilde{j}\rangle} J_{\tilde{i}\tilde{j}}Q_\tilde{i}Q_\tilde{j} + \frac{i}{4}(\omega + i\delta) \sum_{\tilde{i}} \Delta^{-1}_\tilde{i}\Lambda Q)$$ (10)

$$J_{\tilde{i}\tilde{j}} = J\cos(\theta_{\tilde{i}\tilde{j}}/2)\frac{1}{\Delta_{\tilde{i}\tilde{j}}^{-1}}.$$ Where $\Delta_{\tilde{i}}$ is the mean energy level spacing at the magnetic soliton (cluster) $O(r_\tilde{i})$ and $J > 0$. Then we get the effective diffusion coefficient $D_{\text{eff}}$ approximately as follows,

$$D_{\text{eff}} \sim \frac{p \cdot \exp[-s(\alpha - \alpha_c)^{1/2}]}{(\alpha - \alpha_c)^{3/2}}.$$ (11)

Where $\alpha \equiv 8J_{\tilde{i}\tilde{j}}$, and $p$ and $s$ are constant parameters. $D_{\text{eff}}$ becomes to be zero when $\alpha$ decreases to $\alpha_c$. When high magnetic field imposes on this system, $\theta_{\tilde{i}\tilde{j}}$ decreases and then $\alpha$ increases remarkably. As a result, the effective diffusion coefficient $D_{\text{eff}}$ increases.

3. Conclusion
We have given the large magnetoresistance mechanism in diluted magnetic semiconductors, by using the magnetic solitons and the supersymmetric sigma formula.

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