Analytical Calculation of Stiffness Characteristics of Swing Cylinder Hydro-Pneumatic Suspension

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Abstract: The swing cylinder hydro-pneumatic suspension was mainly composed of balance elbow, rocker arm and oil-gas spring, which could convert the rotary motion of the balance elbow into the linear reciprocating motion of the piston. According to the mechanical transfer relationship, the suspension theoretical modelling research had been carried out, and the analytical relationship between key parameters such as suspension stiffness, leverage ratio and balance elbow swing angle were derived. The change trend of suspension characteristics with displacement was also given through programming, which provided a theoretical method for the design of actuators.

1. Introduction
In light-to-medium-duty wheel and tracked vehicles, swing-cylinder hydro-pneumatic suspensions are widely used. It converts the rotary motion of the balance elbow into the linear reciprocating motion of the hydro-pneumatic spring through a set of rocker arm linkage mechanism. The rocker link mainly bears loads from all directions, while the oil-air spring is only responsible for the transmission of force along the axis of the cylinder. This typical structure can effectively improve the force working conditions of the hydro-pneumatic spring and improve product reliability. Therefore, it is necessary to carry out dynamic modeling research on the swing hydro-pneumatic suspension, which can effectively guide the design of the vehicle chassis.

2. Schematic diagram of Swing type hydro-pneumatic suspension
The working schematic diagram of the swing type hydro-pneumatic suspension is shown in the figure 1, in which, point O is the center of balance elbow, point A is the hinge point of oil-air spring, point B is lower hinge point of oil-air spring, and hinged with the rocker arm; point C is the center of wheel, point D is the vertical foot from point O to line AB, H is the height from the bottom of the vehicle to the ground, HL is track thickness, E is the height of the center hole of the balance elbow from the bottom surface of the vehicle bottom deck, R is the length of balance elbow, L is the distance from the upper hinge point A to the center hole of the balance elbow, \( F_f \) is the force of the road wheel, \( F_{j0}, F_J, F_{jd} \) are the loads of the road wheel at the initial installation position, static equilibrium position, and ultimate compression position; \( r \) is the distance between straight lines OB, \( \alpha \) is the angle between the balance elbow and the horizontal line, \( \alpha_0, \alpha_j, \alpha_d \) are the angles of the road wheel at the initial installation position, static equilibrium position, and ultimate compression position, \( \beta \) is the angle between the pull arm and the balance elbow, \( \gamma = \arcsin(d/L) \) is the angle between the line AB and AO, \( \Theta \) is the
angle between the straight line OA and the horizontal line, \( A_h \) is cross-sectional area of the piston, and \( f \) is road wheel stroke.

3. Swing type suspension mathematical modelling

The equation of a straight line passing through two points AB is\([1]\):

\[
\begin{align*}
1 & = x y - 1 \\
x_1 & = y_1 - 1 \\
x_2 & = y_2 - 1
\end{align*}
\]

Simplify the above formula\([2]\):

\[
A x + B y + M = 0
\]

Where

\[
A = y_1 - y_2 \\
B = x_2 - x_1 \\
M = x_1 y_2 - x_2 y_1
\]

The coordinates of point A are \( x_1 = -L \cos \theta \), \( y_1 = L \sin \theta \), The coordinates of point B are \( x_2 = r \cos(\beta - \alpha) \), \( y_2 = r \sin(\beta - \alpha) \), that is, \( x_2 \) and \( y_2 \) are functions of variable \( \alpha \).

The road wheel stroke is:

\[
f = R(\sin a_0 - \sin a)
\]

\( \alpha \) can be expressed as:

\[
\alpha = \arcsin \left( \frac{H + E - H L - D/2 - f}{R} \right)
\]

The vertical distance from point \((x_0, y_0)\) to the straight line AB is:

\[
D = \frac{|Ax_0 + By_0 + M|}{\sqrt{A^2 + B^2}}
\]

Then the vertical distance from the origin O to the straight line AB is:

\[
D = \frac{|M|}{\sqrt{A^2 + B^2}}
\]
The distance between two points AB is the length of the hydro-pneumatic spring at any position, which is recorded as:

$$le = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(x_1 - r \cos(\beta - \alpha))^2 + (y_1 - r \sin(\beta - \alpha))^2}$$  \hspace{1cm} (7)

In the initial position, the distance between the two points AB is the largest:

$$l_{\text{max}} = \sqrt{(x_1 - r \cos(\beta - \alpha_0))^2 + (y_1 - r \sin(\beta - \alpha_0))^2}$$  \hspace{1cm} (8)

The stroke of the hydro-pneumatic spring piston rod is:

$$s = l_{\text{max}} - le$$  \hspace{1cm} (9)

The output force of the hydro-pneumatic spring is along the AB direction, according to the principle of torque balance [3-4]:

$$F_f \cdot R \cos \alpha = F_s \cdot D(\alpha)$$  \hspace{1cm} (10)

And then

$$F_f = i \cdot F_s$$  \hspace{1cm} (11)

$$i = \frac{F_f}{F_s} = \frac{D(\alpha)}{R \cos \alpha}$$  \hspace{1cm} (12)

Piston elastic force at static equilibrium position is:

$$F_{sj} = (p_j - p_a) \cdot A_h$$  \hspace{1cm} (13)

The comprehensive formula (12), (13) can obtain the pressure of the oil-air spring at the static equilibrium position:

$$p_j = p_a + \frac{F_{sj} \cdot R \cos \alpha}{A_h \cdot D(\alpha)}$$  \hspace{1cm} (14)

Then the pressure expression at any position is [5]:

$$p = p_j \left(\frac{h_0 - s}{h_0 - s_j}\right)^m$$  \hspace{1cm} (15)

The elastic force of the oil-air spring is:

$$F_s = (p - p_a) \cdot A_h = (p_j \left(\frac{h_0 - s}{h_0 - s_j}\right)^m - p_a) \cdot A_h$$  \hspace{1cm} (16)

Differentiate the elastic force to get the stiffness of the gas spring:

$$K_s = \frac{dF_s}{ds} = \frac{mp_0 A_h}{h_0} \left(1 - \frac{s}{h_0}\right)^{-(m+1)}$$  \hspace{1cm} (17)

The load at the road wheel is obtained through the leverage ratio as:

$$F_f = \frac{D(\alpha)}{R \cos \alpha} \cdot (p_j \left(\frac{h_0 - s}{h_0 - s_j}\right)^m - p_a) \cdot A_h$$  \hspace{1cm} (18)

The rigidity of the oil and gas suspension is:

$$K = \frac{dF_f}{df} = i^2 K_s + F_s \frac{di}{df}$$  \hspace{1cm} (19)

$$\frac{di}{df} = \frac{di}{d\alpha} \cdot \frac{d\alpha}{df}$$  \hspace{1cm} (20)
Derivation of formula (12) can be obtained:

\[
\frac{di}{d\alpha} = \frac{1}{R \cos \alpha} \cdot \frac{d(D(\alpha))}{d\alpha} + \frac{\sin \alpha}{R \cos^2 \alpha} \cdot D(\alpha) \tag{21}
\]

Where,

\[
\frac{d(D(\alpha))}{d\alpha} = \frac{d|M|}{d\alpha} \sqrt{A^2 + B^2} - \frac{d\sqrt{A^2 + B^2}}{d\alpha} |M| \\
\frac{d|M|}{d\alpha} = \frac{1}{d\alpha} \left( (x_1 y_2 - x_2 y_1) = x_1 x_2 + y_2 y_1 \right) \\
\frac{d\sqrt{A^2 + B^2}}{d\alpha} = \frac{r(A \cos(\alpha - \beta) - B \sin(\alpha - \beta))}{\sqrt{A^2 + B^2}}
\]

\[
\frac{d\alpha}{df} = -\frac{1}{R \cos \alpha}
\]

4. Suspension characteristics simulation calculation

The programming has performed analytical calculations on the characteristics of the swing hydropneumatic suspension. As shown in the following figures, it can be seen that with the increase of the road wheel stroke, the leverage ratio of the suspension system shows a non-linear downward trend. However, the load and stiffness of the oil-gas suspension show obvious upward trend, compared with the traditional linear suspension, the energy storage ratio at the end of the stroke is significantly improved, and it has better cushioning performance, thus avoid the situation of rigid impact limit, thereby improve the vehicle’s off-road maneuverability.

![Figure 2](image-url)

Figure 2 Leverage ratio and spring pressure changes with road wheel
5. Conclusion
The article carried out theoretical modelling and simulation analysis for the swing cylinder hydro-pneumatic suspension, and the mechanical transmission relationship between the road wheel and the piston was deduced, which revealed the variation of suspension system characteristics with key parameters. It provided theoretical support for the optimization of suspension system configuration.

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