Orientifolds of type IIA strings on Calabi-Yau manifolds*

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We identify type IIA orientifolds that are dual to M-theory compactifications on manifolds with $G_2$-holonomy. We then discuss the construction of crosscap states in Gepner models.

The advent of D-branes has lead to a better understanding of dualities involving strong coupling limits. In particular, $\mathcal{N} = 1$ compactifications of the heterotic string (on Calabi-Yau manifolds) are no longer the only string theories of phenomenological interest. One such class is furnished by M-theory compactifications on seven-dimensional manifolds of $G_2$-holonomy give rise to a four-dimensional theories with $\mathcal{N} = 1$ supersymmetry. When the $G_2$ manifolds have certain kinds of singularities, both non-abelian gauge groups as well as chiral fermions can appear.

Joyce has constructed manifolds of $G_2$ holonomy as $\mathbb{Z}_2$ orbifolds of a Calabi-Yau threefold $M$: $X = (M \times S^1)/\sigma \cdot I_1$ where $\sigma$ is an anti-holomorphic involution of the $CY^3$ and $I_1$, inversion of the $S^1$ [1]. One obtains a smooth manifold when the orbifold action has no fixed points. However, when there is a fixed point set $\Sigma$, one obtains a singular manifold [2,3]. The singularity can be smoothed out when $b_1(\Sigma) > 0$. The focus of this talk will be on the cases when there are fixed points.

Our working example of a Joyce manifold is the one obtained from the Fermat quintic given by the hypersurface $z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0$, in $\mathbb{CP}^4$ ($z_i$ are homogeneous coordinates of $\mathbb{CP}^4$). The anti-holomorphic involution $\sigma$ is $z_i \rightarrow \bar{z}_i$ for $i = 1, \ldots, 5$. The fixed-point set $\Sigma$ is an $\mathbb{R}P^3$, which is a special Lagrangian(sL) submanifold of the Fermat quintic [4]. Since $b_1(\mathbb{R}P^3) = 0$, the singularity of $X$, which is locally of the form $\Sigma \times \mathbb{R}^2/\mathbb{Z}_2$, cannot be resolved. $\Sigma$ is actually one in a family of $5^4 = 625$ sL submanifolds of the Fermat quintic, all of whom are $\mathbb{R}P^3$’s. They are all fixed-points of the anti-holomorphic involutions: $z_i \rightarrow \alpha^n z_i$ with $\alpha^5 = 1$.

We will focus on obtaining the precise type IIA orientifold dual for M-theory compactification on this Joyce manifold. We then will proceed to study the orientifolding in the Gepner model corresponding to the Fermat quintic. This involves the construction of crosscap states in Gepner models which we schematically discuss postponing details to a subsequent paper [5].

Obtaining the orientifold dual

M-theory compactified on $M \times S^1$ is dual to the type IIA compactification on $M$. Since the Joyce manifold $X$ is an orbifold of $M \times S^1$, the type IIA dual can be obtained if we can identify the action of $I_1$ on the type IIA side. But, $I_1$ is not a symmetry of M-theory and thus cannot quite be identified with a symmetry on the type IIA side. However, the inversion of an even number of coordinates is a symmetry of M-theory. In the example of the quintic that we considered, $\Sigma$ is the base of the SYZ $T^3$ fibration of the quintic and $\sigma$ inverts the fibre [8]. Thus, $\sigma \cdot I_1$ corresponds to the simultaneous inversion of four circles – three from the SYZ fibre and one from the $S^1$. This uniquely fixes the type IIA orientifold to be the second choice from the following two possibilities [9,10]: ($\Omega$: worldsheet parity, $F_L$: spacetime fermion number)

$$[\sigma \cdot \Omega] \text{ or } [(-)^{F_L} \cdot \sigma \cdot \Omega].$$

It also turns out only the second choice preserves $\mathcal{N} = 1$ supersymmetry [11,12]. This is easily understood by studying the action on the vertex operators for the Ramond sector.

The spectrum of M-theory on $M \times S^1$ has $\mathcal{N} = 2$ supersymmetry in $d = 4$ and consists of: (a) the $\mathcal{N} = 2$ supergravity multiplet; (b) $h_{1,1}(M)$ abelian vector multiplets; and (c) $h_{2,1}(M) + 1$ hypermultiplets. The orbifolding breaks half the supersymmetry and the spectrum for a smooth Joyce manifold $X$ (with Betti numbers $b_3$ and $b_2$) consists of [6,7]: (a) the $\mathcal{N} = 1$ supergravity multiplet; (b) $b_2(X) = h^+_{1,1}(M)$ abelian vector multiplets; and (c) $b_3(X) = h_{2,1}(M) + h^{+}_{1,1}(M) + 1$ chiral multiplets, where $h^{+}_{1,1}(M)$ are the number of Kahler moduli that are even[odd] under $\sigma$. For the case when the orbifolding has fixed points, additional moduli appear corresponding to modes that smooth the singularity.

For the Fermat quintic, $h_{2,1} = 101$; $h^{+}_{1,1} = 0$; $h^{-}_{1,1} = 1$ and the singularity cannot be resolved. The two fixed points are of the form $\Sigma \times \mathbb{R}^3,1$ and the singularity is locally like $\mathbb{R}^4/\mathbb{Z}_2$, i.e., it is an $A_1$ singularity – expect $U(1) \times U(1)$.

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enhanced gauge symmetry in M-theory. In the type IIA dual, we expect an O6-plane with the SO-projection. The RR-charges will be equal the $\mathbb{R}^3/\mathbb{Z}_2$ orientifold plane in flat space. Based on this, we add 4 D6-branes wrapping $\Sigma \times \mathbb{R}^{3,1}$ implying a SO(4) gauge symmetry. The rest of the talk will be towards checking if this geometric intuition can be realised in the orientifold of the Gepner model associated with the Fermat quintic.

**Aspects of Orientifolding**

It is useful to understand how the M-theory spectrum on $X$ must appear from the orientifold projection in the type IIA theory on $M$. Let $\tilde{\Omega}$ denote the orientifold $\mathbb{Z}_2 = (-)^F \cdot \sigma \cdot \Omega$ in our example. Under its action, the states of the original type IIA theory fall into three representations (which we label by $\epsilon = 0, \pm 1$): Real representations:$\epsilon = +1$ These have eigenvalue +1 and survive orientifold projection. Pseudo-real representations:$\epsilon = +1$ These have eigenvalue $-1$, and are projected out. Complex representations:$\epsilon = 0$ Under the action of $\tilde{\Omega}$, one state gets mapped to another. In such cases, one linear combination is projected out. In our example, it is easy to see that states that arise from the $(c,c)$ and $(a,a)$ rings (complex moduli of $M$) have $\epsilon = \pm 1$.

The presence of orientifold planes leads to unoriented strings and hence unoriented surfaces. At ‘one-loop’, this adds a Klein bottle to the torus. The Klein bottle amplitude has two “channels” related by the modular transformation. We have assumed for simplicity that all states have multiplicity of one. Thus, the direct channel amplitude encodes required projection. One general class of solutions has been provided by Pradisi-Sagnotti-Stanev [13]. The crosscap states in the Gepner model

$$|C\rangle = \sum_i \Gamma_i \frac{P_{0i}}{\sqrt{S_{0i}}} |C : i\rangle,$$

where $|C : i\rangle$ are the Ishibashi basis for crosscap states and $P = \sqrt{TS}T^2S \sqrt{T}$. This plays the analogue of the S-matrix in Cardy’s ansatz for the boundary states. The matrices $Y_k^{ij} = \sum_m \frac{S_{0i}P_{0j}P_k}{S_{0k}}$ plays a role analogous to the fusion matrix for boundary states. They satisfy the fusion algebra: $Y_i Y_j = N_{ij}^k Y_k$ with $Y_{00}^{kk} = \epsilon_k$ determining the KB projection.

**An application: $N = 2$ minimal models**

The states in the minimal models of level $k$ are labeled by $(L,M,S)$ with $L = 0, \ldots, k$, $M = 0, \ldots, (2k+3)$ mod $(2k+4)$, $S = 0, 1, 2, 3$ mod 4 and $L + M + S = $ even. There is an additional identification: $(L,M,S) \sim (k-L, M+k+2, S+2)$. Even $S$ is the NS-sector and odd $S$ is the R-sector. The S-matrix and P-matrix are schematically given by

$$S_{LMS}^{LMS} \propto \sin(L, \tilde{L}) e^{i \pi M \delta_{S+\tilde{S}}^{(2)}} e^{-i \pi S \delta_{L+\tilde{M}}^{(2)}}$$

$$P_{LMS}^{LMS} \propto \sin \left( \frac{1}{2} (L, \tilde{L}) e^{i \alpha_{LMS}} \right) e^{i \pi M \delta_{S+\tilde{S}}^{(2)}} e^{-i \pi S \delta_{L+\tilde{M}}^{(2)}} + e^{i \alpha_{LMS}} \sin \left( \frac{1}{2} (k-L, \tilde{L}) e^{i \alpha_{LMS}} \right) e^{i \pi M \delta_{S+\tilde{S}}^{(2)}} e^{-i \pi S \delta_{L+\tilde{M}}^{(2)}}$$

where $(L, \tilde{L}) = \pi(L+1)(\tilde{L}+1)/(k+2)$ and $\alpha_{LMS}$ is a phase that one needs to introduce to take care of the identification mentioned earlier [14,15,5]. The appearance of a Kronecker delta function in P-matrix implies that only NSNS (or RR) states alone appear in the PSS crosscap state.

**Crosscap states in the Gepner model**

The Gepner model is obtained by tensoring copies of $N = 2$ minimal models(MM) such that total central charge is 9. For the quintic – tensor five copies of $k = 3$ MM. Further, restrict to states that come from tensoring NS states with NS states and R with R from each minimal model and project onto states with total (including spacetime sector) $U(1)$ charge an odd integer.

This suggests the following strategy for crosscap states in the Gepner model: Take the tensor product of crosscap states in the individual minimal model. Implement the Gepner projection on this crosscap state. This is a natural guess for the crosscap state in the Gepner model. But this cannot be the crosscap that realises the type IIA orientifold! This is because PSS crosscap state has only contributions from the NSNS sector. This implies that its Ramond charge is zero. The direct channel KB amplitude is not supersymmetric.
Consider the two crosscap states in a single MM

\[ |C : NSNS\rangle \equiv P^{LMS}_{000} |C : LMS\rangle \quad \text{and} \quad |C : RR\rangle \equiv P^{LMS}_{011} |C : LMS\rangle \]

The first one is the PSS crosscap state while the second one is the PSS crosscap state associated with the simple current that is related to spacetime supersymmetry. It contains only RR Ishibashi states. Then, we propose that the correct crosscap state schematically takes the form

\[ |C\rangle_{\text{Gepner}} = \mathcal{P} \left( \prod_{i=1}^{r} |C_i : NSNS\rangle + \prod_{i=1}^{r} |C_i : RR\rangle \right) \]

\(\mathcal{P}\) imposes the \(U(1)\) charge projection of Gepner. This is the crosscap analogue of the Recknagel-Schomerus construction for boundary states in the Gepner model [16].

Now the crosscap state clearly carries RR charge. It has all the terms to provide a supersymmetric KB amplitude. For the quintic, in fact, we find a full family of 625 distinct crosscap states in agreement with the 625 anti-holomorphic involutions. More detailed checks such as the KB projection, tadpole cancellation etc. for specific examples will be discussed in the paper to appear soon [5]. A recent paper by A. Misra also discusses a type IIA orientifold of a Calabi-Yau threefold [17].

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[1] D. Joyce, J. Diff. Geom. 43 (1996) 291 and J. Diff. Geom. 43 (1996) 329.
[2] J. A. Harvey and G. W. Moore, arXiv:hep-th/9907026.
[3] H. Partouche and B. Pioline, JHEP 0103 (2001) 005 [arXiv:hep-th/0011130].
[4] K. Becker, M. Becker and A. Strominger, Nucl. Phys. B456 (1995) 130 [arXiv:hep-th/9507158].
[5] S. Govindarajan and J. Majumder (to appear).
[6] G. Papadopoulos and P. K. Townsend, Nucl. Phys. B456 (1995) 130 [arXiv:hep-th/9507158].
[7] C. Vafa and E. Witten, Nucl. Phys. Proc. Suppl. 46 (1996) 225 [arXiv:hep-th/9507050].
[8] A. Strominger, S. T. Yau and E. Zaslow, Nucl. Phys. B 479 (1996) 243 [arXiv:hep-th/9606040].
[9] A. Sen, JHEP 9709 (1997) 001 [arXiv:hep-th/9707123].
[10] J. Majumder, JHEP 0201 (2002) 048 [arXiv:hep-th/0109076].
[11] R. Gopakumar and S. Mukhi, Nucl. Phys. B 479 (1996) 260 [arXiv:hep-th/9607057].
[12] S. Kachru and J. McGreevy, JHEP 0106 (2001) 027 [arXiv:hep-th/0103223].
[13] G. Pradisi, A. Sagnotti, Ya. S. Stanev, Phys. Lett. B354 (1995) 279 [arXiv:hep-th/9503207].
[14] Y. Hikida, JHEP 0211 (2002) 035 [arXiv:hep-th/0201175].
[15] I. Brunner and K. Hori, arXiv:hep-th/0303135.
[16] A. Recknagel and V. Schomerus, Nucl. Phys. B 531 (1998) 185 [arXiv:hep-th/9712186].
[17] A. Misra, arXiv:hep-th/0304209.