The Fermionic Density-functional at Feshbach Resonance

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We consider a dilute gas of neutral unpolarized fermionic atoms at zero temperature. The atoms interact via a short-range (tunable) attractive interaction. We demonstrate analytically a curious property of the gas at unitarity. Namely, the correlation energy of the gas, evaluated by second order perturbation theory, has the same density dependence as the first order exchange energy, and the two almost exactly cancel each other at Feshbach resonance irrespective of the shape of the potential, provided \((\mu r_s) \gg 1\). Here \((\mu)^{-1}\) is the range of the two-body potential, and \(r_s\) is defined through the number density, \(n = 3/(4\pi r_s^3)\). The implications of this result for universality is discussed.

I. INTRODUCTION

Consider a dilute gas of \(N \gg 1\) neutral fermionic atoms (mass \(M\)) at \(T = 0\) interacting with a short-range attractive potential. In general, the properties of the dilute gas are determined by the number density \(n\), and the scattering length \(a\). The Hamiltonian of this \(N\)-particle system reads

\[
\hat{H} = -\frac{\hbar^2}{2M} \sum_{i=1}^{N} \nabla_i^2 + \sum_{i<j} v(|\mathbf{r}_i - \mathbf{r}_j|).
\]

(1)

Not written explicitly here, there is also an external potential \(v_{\text{ext}}(\mathbf{r})\) that forces the \(N\) atoms to stay within a large box with volume \(\Omega\) [where \(v_{\text{ext}}(\mathbf{r}) \equiv 0\)]. The attractive interaction potential is assumed to have the 2-parameter form

\[
v(r) = -v_0 f(\mu r)
\]

(2)

where \(v_0 > 0\) is the strength of the interaction, \(R_0 = \frac{1}{\mu}\) is its range, and \(f(x)\) is a dimensionless function.

In the true ground state of the Hamiltonian (1) the attractive atoms may form dimers or even clusters. We are, however, looking for a metastable state where there is a dilute gas of separated atoms with uniform density \(n\), satisfying the condition \((\mu r_s) \gg 1\) where \(n = \frac{N}{\Omega} = \frac{\pi r_s^3}{4}\). Even then, for a weak \(v_0\), there will be BCS-type pairing, followed by dimer formation as the strength of the interaction increases. This was predicted long back by Leggett [1], and has been observed experimentally [2]. For the density functional analysis of the uniform gas at Feshbach resonance, we shall disregard the BCS condensed pairs in this paper.

To study the effect of the attractive interaction \(v(r)\), we consider the corresponding atom-atom scattering problem in the relative s-state. Separating the center-of-mass motion, we are left with the relative Hamiltonian

\[
\hat{H}_{\text{rel}} = -\frac{\hbar^2}{M} \left( \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} \right) - v_0 f(\mu r).
\]

(3)

Keeping the range of the potential small enough such that \((\mu r_s) \gg 1\), the strength \(v_0\) is adjusted such that the potential can support a single bound state at zero energy. This happens when the scattering length \(a \to \infty\), leaving no length scale from the interaction. Such a tuning of the interaction is possible experimentally, and gives rise to Feshbach resonance [3]. The scattering cross section in the given partial wave (s-wave in our case) reaches the unitary limit, and the gas is said to be at unitarity. It is then expected to display universal behavior [4]. Note that at Feshbach resonance, there is no length scale left other than the inverse of the Fermi wave number \(k_F\), where \(k_F = (3\pi^2 n)^{1/3}\). The energy per particle, \(E/N\), as a function of the density \(n\), should therefore scale the same way as the noninteracting kinetic energy, \(\frac{1}{2}\hbar^2 k_F^2 / 2M \propto n^{2/3}\). There has been much interest amongst theorists to calculate the properties of the gas in the unitary regime \((k_F/a) \gg 1\). In particular, at \(T = 0\), the energy per particle of the gas is calculated to be

\[
\frac{E}{N} = \xi \frac{3}{5} \frac{\hbar^2 k_F^2}{2M},
\]

(4)

where \(\xi \approx 0.44\) [5]. The experimental value of \(\xi\) is about 0.5, but with large error bars [6]. Recently, there have been two Monte Carlo (MC) finite temperature calculations [7, 8] of an untrapped gas at unitarity, where various thermodynamic properties as a function of temperature have been computed. It is clear that at unitarity, the kinetic and potential energies should scale the same way. This has been assumed \(a\) priori in a previous density functional treatment of a unitary gas [9]. However, such a scaling behavior is not evident from the density functionals for the direct, exchange and correlation energies [10] (see sects. II and III). The aim of the present paper is to examine this point in some detail. In particular, we are able to show analytically that the leading contribution of the correlation energy (calculated in second order perturbation theory), cancels the first order exchange energy almost exactly at Feshbach resonance. This happens irrespective of the shape of the potential as specified by \(f(\mu r)\), provided the condition \((\mu r_s) \gg 1\). We show that...
our general Eq. (24) (derived later in the text) that ensures such a cancellation is satisfied at unitarity for a variety of 2-parameter potentials, including the square well and the delta-shell, as well as the smoothly varying \( \cosh^{-2}(\mu r) \) and Gaussian potentials. This is the main result of the present work. The implications of this result for universality is marginal. This is because these potential energy terms, in the limit of \((\mu r_s) \gg 1\), are very small compared to the kinetic energy [4]. For a moderately large value like \((\mu r_s) \simeq 3\) however, these terms are comparable in magnitude to the kinetic energy (sect. IV). Even then, the cancellation of the first order exchange, and the second order perturbative terms leave the direct first order term in tact. In the electron gas, this (repulsive) term got cancelled by the interaction of the electrons with the positive ionic background. There is no such mechanism of cancellation here, unless we assume, rather arbitrarily, that the short-range interatomic repulsion cancels this direct (attractive) contribution. Even without any such assumptions, however, our main result (Table I), applicable at Feshbach resonance, is interesting from the angle of potential theory.

II. PERTURBATION EXPANSION

Treating the interaction (2) as a weak perturbation in the Hamiltonian (1), the unperturbed energy \( E^{(0)} \) is the kinetic energy of a non-interacting Fermi gas,

\[
E^{(0)} = N \frac{3}{2} \frac{\hbar^2 k_F^2}{M}.
\]

Here, \( k_F = \frac{1}{\alpha r_s} \) and \( \alpha^3 = \frac{4}{9 \pi} \). The corresponding ground state \( |\Phi_0\rangle \) is a Slater determinant of plane waves.

In terms of dimensionless coordinates \( x_i = \mu r_i \), the Hamiltonian (1) can be written as

\[
\frac{M}{\hbar^2 \mu^2} \hat{H} = -\frac{1}{2} \sum_{i=1}^{N} \nabla_i^2 - \lambda \sum_{i<j} f(|x_i - x_j|), \quad \lambda = \frac{M v_0}{\hbar^2 \mu^2}.
\]

This suggests that the perturbation parameter is not really small at unitarity. For example, for the square-well potential, the zero-energy single bound state occurs when \( \lambda = \frac{\pi^2}{4} \) (see sect. III). Nevertheless the low-order terms can point to important information, even when the expansion is divergent [11]. In our problem, there are three parameters, \( \mu \), \( v_0 \), and \( r_s \). The unitarity condition relates \( \mu \) and \( v_0 \), so two independent parameters are left. One of these may be taken to be the small parameter \( \zeta = (\mu r_s)^{-1} \). The remaining free parameter \( v_0 \) may be chosen independently of \( \zeta \) to fulfill the unitarity condition.

A. First order

Formally, the first-order correction,

\[
E^{(1)} = \langle \Phi_0 | \hat{V}_{\text{int}} | \Phi_0 \rangle,
\]

has a direct contribution \( U(r_s, \mu) = Nu(r_s, \mu) \) with

\[
u(r_s, \mu) = \frac{\rho^2}{2N} \int_{\Omega} d^3r \int_{\Omega} d^3r' v(|r - r'|) = -3 \frac{v_0}{2 (\mu r_s)^3} f_2.
\]

Here, \( f_2 = \int_0^\infty dx x^2 f(x) \).

The other first-order contribution is the exchange energy \( E_x(r_s, \mu) = N e_x(r_s, \mu) \), [4]

\[
e_x(r_s, \mu) = -\frac{3 k_F}{\mu} \int_0^\infty dr j_1(k r) v(r).
\]

Here, \( j_1(z) \) is a spherical Bessel function. Since \( v(r) \) is short-range and \( k_F \) is small in a dilute gas, we can use the small-\( z \) expansion \( j_1(z) = \frac{z}{\pi} + O(z^3) \) to find

\[
e_x(r_s, \mu) = \frac{3}{4} \frac{v_0}{(\mu r_s)^3} f_2 + O(\mu r_s)^{-5}.
\]

B. Second order

1. General expressions

Also the second-order correction,

\[
E^{(2)} = - \sum_{n \neq 0} \frac{|\langle \Phi_n | \hat{V}_{\text{int}} | \Phi_0 \rangle|^2}{E_n - E_0} = N \left[ e_{\text{dir}}^{(2)} + e_{\text{ex}}^{(2)} \right],
\]

has a direct and an exchange contribution [12],

\[
e_{\text{dir}}^{(2)}(r_s, \mu) = -\frac{3}{32 \pi^5} \left( \frac{2 M}{\hbar^2 \mu^2} \right) v_0^2 \frac{k_F^4}{\mu^3} \int d^3q \hat{f} \left( \frac{k_F}{\mu} q \right)^2 \times
\]

\[
\int_D d^3k_1 d^3k_2 \frac{d^3k_1 d^3k_2}{q \cdot (q + k_1 - k_2)}.
\]

\[
e_{\text{ex}}^{(2)}(r_s, \mu) = +\frac{3}{64 \pi^5} \left( \frac{2 M}{\hbar^2 \mu^2} \right) v_0^2 \frac{k_F^4}{\mu^3} \int d^3q \hat{f} \left( \frac{k_F}{\mu} q \right)^2 \times
\]

\[
\int_D d^3k_1 d^3k_2 \frac{\hat{f} \left( \frac{k_F}{\mu} (q + k_1 - k_2) \right)}{q \cdot (q + k_1 - k_2)}.
\]

While \( v_0^2 (2 M / \hbar^2 \mu^2) \) has the dimension energy, the integration variables \( q \), \( k_1 \), and \( k_2 \) are dimensionless here. The domain of the integral over \( d^3k_1 d^3k_2 \) depends on \( q \).

\[
D: \quad |k_1|, |k_2| < 1; \quad |k_1 + q|, |k_2 - q| > 1.
\]
Furthermore, $\tilde{f}(y)$ is a dimensionless transform of $f(x)$,

$$\tilde{f}(y) = \int_0^\infty dx x^2 f(x) j_0(yx)$$

$$= \frac{1}{y} \int_0^\infty dx x f(x) \sin(yx). \quad (15)$$

To recover Eqs. (8) and (9) of Ref. [12], put $M = m_e$, $v_0 = -e^2 \mu$, and $f(x) = \frac{1}{x}$ or $\tilde{f}(y) = \frac{1}{y}$, such that $v(r) = \frac{e^2}{r}$ becomes the electronic Coulomb repulsion. (Note that Ref. [12] uses Rydberg units, $m_e e^4/2\hbar^2 = e^2/2a_B = 1$.)

2. The limit $\mu r_s \gg 1$

For a dilute gas (small $k_F$) with short-range interaction (large $\mu$), Eqs. (12) and (13) can be evaluated in the limit $\mu/k_F = \alpha \mu r_s \gg 1$ where $\alpha^2 = \frac{1}{2\pi}$. Following Ref. [13], we choose a number $q_1$ such that $1 < q_1 \ll \mu/k_F$ and split the integrals over $d^3q$ into two parts,

$$\int d^3q = \int_{q < q_1} d^3q + \int_{q > q_1} d^3q. \quad (16)$$

In the first part with $q < q_1$, we have $q \ll \mu/k_F$ and $|q + k_1 - k_2| \ll \mu/k_F$ (note that $|k_1|, |k_2| < 1 < q_1$). Therefore, we may expand $\tilde{f}(y) = f_2 + O(y^2)$ in Eqs. (12) and (13) and keep the leading term $f_2$ only. The sum of the two resulting $q < q_1$ contributions reads

$$e_{c_q < q_1}^{(2)}(r_s, \mu) = -\frac{3}{64\pi^3} \left( \frac{2M}{\mu^2 k_F^2} \right) v_0 \frac{k_F^4}{\mu^4} f_2^2 \times$$

$$\times \int_{q < q_1} d^3q \int_0^\infty \frac{d^3k_1 d^3k_2}{q \cdot (q + k_1 - k_2)}. \quad (17)$$

The number $q_1$ can be chosen independently of $\mu/k_F \gg 1$, despite the condition $1 < q_1 \ll \mu/k_F$. Then, the integral in Eq. (17) is a finite constant and we conclude [13]

$$e_{c_q < q_1}^{(2)}(r_s, \mu) = (\mu r_s)^{-4}. \quad (18)$$

In the second part $q > q_1 \gg 1$ of the integral (16), we can put $q + k_1 - k_2 \approx q$, since $|k_1|, |k_2| < 1$. The resulting contributions to Eqs. (12) and (13) add up to

$$e_{c_q > q_1}^{(2)}(r_s, \mu) = -\frac{3}{64\pi^3} \left( \frac{4\pi}{3} \right)^2 \left( \frac{2M}{\mu^2 k_F^2} \right) v_0 \frac{k_F^4}{\mu^4} \times$$

$$\times \int_{q > q_1} \frac{d^3q}{q^2} \int_0^\infty \frac{d^3k_1 d^3k_2}{q \cdot (q + k_1 - k_2)} \left( \frac{k_F \mu}{\mu} \right)^2 \quad (19)$$

where $\int_0^\infty d^3k_1 d^3k_2 = \left( \frac{4\pi}{3} \right)^2$ has been used. Now,

$$\int_{q > q_1} \frac{d^3q}{q^2} \tilde{f}(k_F \mu) \quad (20)$$

$$= \frac{\mu}{k_F \mu} \frac{4\pi}{3} \int_{y_1}^\infty dy \tilde{f}(y)^2$$

where $y_1 = k_F q_1 / \mu \ll 1$. If $\int_{y_1}^\infty dy \tilde{f}(y)^2$ in Eq. (20) did not depend on $y_1$, expression (19) did rigorously have order $O(\mu r_s)^{-3}$. However, using the small-$y$ expansion $\tilde{f}(y) = f_2 + O(y^2)$, we have $\int_{y_1}^\infty dy \tilde{f}(y)^2 = f_2^2 y_1 + O(y_1^3)$. Consequently, shifting the lower limit $y_1$ of the integral (20) to zero does not affect the leading-order contribution to expression (19),

$$e_{c_q > q_1}^{(2)}(r_s, \mu) = (\mu r_s)^{-3}. \quad (21)$$

Therefore, the quantity (18) does not contribute to the leading order of $e_c^{(2)} = e_{c_{\text{dir}}}^{(2)} + e_{c_{\text{n}}}^{(2)}$ which is purely due to expression (19),

$$e_c^{(2)}(r_s, \mu) = -\frac{3}{4\pi} \left( \frac{2M}{\mu^2 k_F^2} \right) \frac{v_0^3}{(\mu r_s)^3} F + O(\mu r_s)^{-4} \quad (22)$$

where $F = \int_0^\infty dy \tilde{f}(y)^2$.

### III. DENSITY SCALING AT UNITARITY

If the perturbation expansion is convergent [12], the total energy $E(r_s, \mu) = N \epsilon(r_s, \mu)$ of the gas can be expressed in the form

$$\epsilon(r_s, \mu) = t_s(r_s) + c_c(r_s, \mu) + \sum_{n=2}^\infty \epsilon_c^{(n)}(r_s, \mu). \quad (23)$$

At unitarity, when the relative Hamiltonian (3) has a single bound state at zero energy, the exchange plus correlation energy $c_c + \sum_{n=2}^\infty c_c^{(n)}$ should display the same density scaling as the kinetic energy, $t_s(r_s) \propto r_s^{-2} \propto \rho^{2/3}$. This is obviously not the case with any one of the present (leading-order) results (10) and (22). However, since the exchange energy (10) and the second-order correlation energy (22) have opposite signs, they can cancel each other at some value of $\mu$. This happens when

$$\frac{Mv_0}{\hbar^2 \mu^2} = \frac{pf_2}{2F}, \quad (24)$$

where $f_2 = \int_0^\infty dx x^2 f(x)$ and $F = \int_0^\infty dy \tilde{f}(y)^2$. This is the main result of our paper, and we check it by taking four different potentials. The results of this analysis, summarized in Table I, are discussed in detail below.

Generally, we need an eigenfunction $\psi(r) = \frac{u(r)}{r}$ of the relative Hamiltonian (3) with eigenvalue zero. Writing $u(r) = \phi(\mu)$, the corresponding dimensionless Schrödinger Equation reads

$$\phi''(x) = -\lambda f(x) \phi(x), \quad \lambda = \frac{Mv_0}{\hbar^2 \mu^2}. \quad (25)$$

Precisely, we wish to determine that particular value $\lambda_{\text{ut}}$ of $\lambda$ for which this zero-energy solution is the only bound state. Then, $\phi(x)$ must obey $\phi(0) = 0, \phi'(x) < 0$ for $x > 0$, and $\phi(x) \rightarrow \text{const.}$ for $x \rightarrow \infty$. In the following examples (A-D), the solution $\phi(x)$ can be found analytically or numerically.
(A) Square-well potential of radius $R_0 = 1/\mu$:
\[ v(r) = -v_0 \Theta(R_0 - r), \]
where $\Theta(z)$ denotes the Heavyside step function, $\Theta(z) = 1$ for $z > 0$ and $\Theta(z) = 0$ for $z \leq 0$. By setting the dimensionless variable $\mu r = x$, we see that $f(x) = \Theta(1 - x)$. The square-well potential (26) supports a single zero energy bound state when the LHS of Eq.(24) is $\lambda_{\text{aty}} = \pi^2/4$. It may be easily checked analytically that for the square-well potential (26), $f_2 = 1/3$ and $F = \pi/12$ so that the RHS of Eq.(24) is $\frac{5}{6}$, very close to its LHS, $\pi^2/4 = 2.47$.

(B) Rosen-Morse hyperbolic potential [5]. This potential is given by
\[ v(r) = -v_0 \text{sech}^2(\mu r), \]
which supports a single zero energy bound state when the LHS of Eq.(24) is $\lambda_{\text{aty}} = 2$ instead of $\pi^2/4$. For this potential, it is easy to check that $f_2 = \pi^2/12$. The quantity $F$, however, has to be calculated numerically, and is given by $F = 0.596$. Again, Eq.(24) is approximately satisfied, since its RHS for this potential is 2.17.

(C) Delta-shell potential [14]. Consider the potential
\[ v(r) = -\frac{\hbar^2}{M} \delta(r - R_0), \]
\[ = -\frac{\hbar^2}{M R_0} \delta \left( \frac{r}{R_0} - 1 \right), \]
\[ = -v_0 f(\mu r). \]
Thus, we have $v_0 = \eta / M R_0$, $\mu = 1 / R_0$, and $f(x) = \delta(x - 1)$. So we get $f_2 = 1$, $f(y) = \frac{\sin y}{y}$, and $F = \frac{\pi}{2}$. Hence the RHS of Eq. (24) is unity. The LHS is $(\eta R_0)$, which is exactly unity when the s-state scattering length goes to infinity [14]. Thus Eq.(24) is exactly obeyed in this case.

(D) Gaussian Potential.
\[ v(r) = -v_0 \exp(-\mu^2 r^2) \]
For this example, $f(x) = \exp(-x^2)$ in Eq. (2). We find $f_2 = \frac{1}{4} \sqrt{\pi}$ and $F = \frac{1}{4} (\frac{\pi}{3})^{3/2} \approx 0.89$ so that the RHS of Eq. (24) becomes $\pi f_2/2F = 1.000$. Solving Eq. (25) numerically for this $f(x)$, we obtain a single bound state at zero energy when the LHS of Eq. (24) is $\lambda_{\text{aty}} = 0.949 \times 2^{3/2}$, close to $2^{3/2}$.

Note, however, that contributions $O(\mu r_s)^{-3}$ may also come from higher order terms of the perturbation expansion in section II, since that expansion is carried out with respect to the parameter $\lambda = M v_0 / \hbar^2 \mu^2$, but not $1/\mu r_s$.

### IV. DISCUSSION

The dimensionless Hamiltonian $\hat{\mathcal{H}}$ from Eq. (6) depends on the dimensionless parameters
\[ \lambda = \frac{M v_0}{\hbar^2 \mu^2}, \]
and, not written explicitly, $x_s = \mu r_s$. The perturbation expansion of the ground-state energy of $\hat{\mathcal{H}}$ reads
\[ \varepsilon(x_s, \lambda) = \sum_{n=0}^{\infty} \varepsilon_n(x_s) \lambda^n. \]

The ground-state energy of the original Hamiltonian $\hat{\mathcal{H}}$, with three independent parameters, is then given by
\[ E(r_s, \mu, \lambda) = \frac{\hbar^2 \mu^2}{M} \varepsilon(\mu r_s, \lambda) = \frac{\hbar^2 \mu^2}{M} \sum_{n=0}^{\infty} \varepsilon_n(\mu r_s) \lambda^n. \]
For $\mu r_s \gg 1$, we may expand
\[ \varepsilon_n(\mu r_s) = \sum_{m=0}^{\infty} \frac{\varepsilon_{nm}(\mu r_s)}{(\mu r_s)^m}. \]

From Eq. (5), we have $\varepsilon_{02} = N \frac{\pi}{16} \alpha^{-2}$ while $\varepsilon_{0m} = 0$ for $m \neq 2$. Eqs. (8) and (10) imply that $\varepsilon_{1m} = 0$ for $m < 3$ and $\varepsilon_{13} = N(-\frac{3}{4} + \frac{5}{12}) f_2$. Eventually, due to Eq. (22), $\varepsilon_{2m} = 0$ for $m < 3$ and $\varepsilon_{23} = N(-\frac{3}{12}) 2F$.

So far as the unitary point is concerned, we are interested in a situation where $k_F |a| \gg 1 \gg k_F R_0 \sim (\mu r_s)^{-1}$. In view of the fact that the perturbation series above does not converge at unitarity, how significant is our low order perturbation calculation in this situation ? Note that our first order direct and exchange (potential) energy terms given by Eqs. (8,10) are the same as those obtained in the Hartree-Fock calculation (see, for example, Eq.(10) of Heiselberg [4]). How big are these terms at unitarity compared to the kinetic energy per particle ? Taking the example of the square-well potential discussed earlier, it is straightforward to show that our exchange term (10) at Feshbach resonance is
\[ e_x(r_s, \mu) = \frac{\pi}{18} \left( \frac{9 \pi}{4} \right)^{1/3} E_F \frac{1}{\mu r_s}. \]
For the square-well example,

\[(k_F a) = \left(\frac{9\pi}{4} \right)^{1/3} \frac{1}{(\mu_r s)} \left[1 - \tan \sqrt{\lambda} \sqrt{\lambda} \right]. \quad (35)\]

At unitarity, the RHS diverges for any finite value of \((\mu_r s)\), how ever large. Even in the neighbourhood of unitarity, it is possible to have \((k_F |a|) \gg 1\) for \((\mu_r s) \gg 1\). From Eq.(34), we note that too large a choice for \((\mu_r s)\) would make \(e_x\) negligible against \(\frac{1}{2} E_F\). Instead, taking a modestly large value, \(\mu_r s = 3\), we obtain the ration of \(e_x\) to kinetic energy per particle to be about 0.56. Noting that \(e_x\) has a different density-dependence than the kinetic energy per particle, its cancellation with the second order perturbative correlation term helps towards scale invariance, but only if there is a mechanism for the direct first order term to be cancelled.

We conclude by emphasizing that the new result in this paper is displayed in Table 1, and should be of interest from the point of view of potential theory.

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[1] A.J. Leggett, in *Modern Trends in the Theory of Condensed Matter*, Springer-Verlag Lecture Notes, Vol. 115, edited by A. Peiklaški and J. Przystawa (Springer-Verlag, Berlin, 1980), p.13

[2] C.A. Regal et al., Nature (London) **424**, 47 (2003); M.W. Zwierlein et al., Phys. Rev. Lett. **91**, 1250401 (2003); C.A. Regal et al., Phys. Rev. Lett. **92**, 040403 (2004); M.W. Zwierlein et al., Nature (London) **435**, 1046 (2005); G. B. Partridge et al., Science **311**, 503 (2006).

[3] S. Inouye et al., Nature (London) **392**, 151 (1998); Ph. Courteille et al., Phys. Rev. Lett **81**, 69 (1998).

[4] G.A. Baker, Phys. Rev. **C60**, 054311 (1999); H. Heiselberg, Phys. Rev. **A63**, 043606 (2001); T.-L. Ho. Phys. Rev. Lett. **92**, 090402 (2004).

[5] J. Carlson, S.-Y. Chang, V. R. Pandharipande, and K. E. Schmidt, Phys. Rev. Lett. **91**, 050401 (2003); A. Perali, P. Pieri, and G. C. Strinati, Phys. Rev. Lett. **93**, 100404 (2004).

[6] M. Bartenstein et al., Phys. Rev. Lett. **92**, 120401 (2004); T. Bourdel et al., Phys. Rev. Lett. **93**, 050401 (2004).

[7] A. Bulgac, J. E. Drut J.E., and P. Magierski, Phys. Rev. Lett. **96**, 090404 (2006).

[8] E. Burovski, N. Prokof’ev, B. Svistunov, and M. Troyer, Phys. Rev. Lett. **96**, 160402 (2006).

[9] T. Papenbrock, Phys. Rev. **A72**, 041603 (R) (2005); A. Bhattacharyya and T. Papenbrock, Phys. Rev. **A74**, 041602 (R) (2006).

[10] R. G. Parr and W. Yang, *Density-Functional Theory of Atoms and Molecules* (Oxford University Press, New York, 1989); W. Kohn, Rev. Mod. Phys. **71**, 1233 (1999).

[11] M. Seidl, J. P. Perdew, and S. Kurth, Phys. Rev. Lett. **84**, 5070 (2000).

[12] M. Gell-Mann, K. A. Brueckner, Phys. Rev. **106**, 364 (1957).

[13] L. Zecca, P. Gori-Giorgi, S. Moroni, and G. B. Bachelet, Phys. Rev. B **70**, 205 127 (2004).

[14] K. Gottfried, *Quantum Mechanicsvol.I*, (W. A. Benjamin, Inc., New York, 1966). See sect. (15).