Ab initio description of Si, P, S, Cl and Ar isotopes in sd-shell

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In the present work, we have reported comprehensive study of the sd-shell nuclei in the range Z = 14 – 18 with neutron number varying from N = 9 to N = 20 using the microscopic effective valence shell interactions. These effective sd-shell interactions are developed using the ab initio no-core shell model wave functions and the Okubo-Lee-Suzuki (OLS) transformation method. The valence shell effective interactions, which are used in this project, are N3LO, JISP16 and DJ16A interactions. For comparison, we have also performed shell model calculations with the empirical USDB interaction. Theoretically calculated shell model results are compared with the experimental data, to check the predictive strength of the microscopic interactions. It is found that the binding energies of the ground states are better reproduced with the DJ16A interaction as compared to other microscopic interactions. Spin-tensor decomposition of two-body interaction is presented to understand the contributions from central, vector and tensor components into these interactions. Electromagnetic properties of these isotopic chains have been studied. Spectroscopic strengths of 23Al(d,n)24Si are calculated for the newly performed experiment at NSCL. The beta-decay properties of 32Ar are also determined for recently available experimental data.

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I. INTRODUCTION

Over the last few years nuclear structure study of p- and lower sd-shell nuclei within the framework of ab initio no-core shell model (NCSM) approach has achieved a great amount of success [1–13]. In the NCSM calculations, all the constituents of a nucleus are considered to be active particles. No concept of inert core as such is assumed in this approach. When the NCSM approach is implemented to study the properties of medium-mass nuclei, dimension of the Hamiltonian matrix becomes too large to handle with the currently accessible computational power. So, other ab initio approaches such as in-medium similarity renormalization group (IM-SRG) [14, 15], coupled cluster (CC) theory [16, 17] and symmetry-adapted no-core shell model (SA-NCSM) [18–21] are being used to describe the structure of heavier nuclei. Electromagnetic properties, spectroscopic factor strengths and Gamow-Teller (GT) strengths of sd-shell nuclei have been calculated using former two of the aforementioned ab initio approaches in Refs. [22–27]. Another ab initio approach, known as the ab initio shell model with a core, has also been developed for heavier mass nuclei in Refs. [28–30].

In the ab initio shell model with core approach, effective two-body interaction is constructed by performing ab initio NCSM calculation in Nmax model space for a nuclei followed by explicit projection onto the 0ℏΩ space using Okubo-Lee-Suzuki (OLS) transformation method. Thus, the effective Hamiltonian is separated out into inert core, one, and two-body terms. In [29], the J-matrix inverse scattering potential (JISP16) [31] and chiral next-to-next-to-next-to-leading order (N3LO) interactions [32] for sd-shell have been developed and it is demonstrated that low-lying energy spectra of 19F using the NCSM is exactly reproduced with these derived effective interactions. The same interaction has been applied on 19F and the shell model results obtained therefrom are found to be very close to the NCSM results. It has also been reported that the effective Hamiltonian has weak A dependence, which means that this interaction can be applied to the heavier sd-shell nuclei. Further, new effective interactions, Daejeon 16 (DJ16) [33] and its monopoloe corrected version Daejeon 16A (DJ16A), have been constructed for sd-shell [30] similar to the above mentioned approach. Authors have implemented these microscopic interactions on O isotopes, odd-A F isotopes, 26F, 22Na, 26Si, 32S, 38K and deformed rotor 24Mg. They obtain remarkable consistency between theoretical description and experimental data. Success of these effective microscopic interactions motivates us to fruitfully apply these interactions to study heavier nuclei in sd-shell. We extend their work to study Si, P, S, Cl and Ar isotopic chain in sd-shell region. Apart from energy spectra, we have also calculated the nuclear observables for these nuclei. Our present comprehensive study will add more information to the earlier work [30].

Nuclear structure properties of sd-shell nuclei including proton and neutron drip lines have been studied in several theoretical and experimental works. In [34], authors have implemented relativistic Hartree Bogoliubov model with density dependent meson-exchange and pairing interactions to study nuclear shapes of Si, S and Ar. They found existence of magic numbers at N = 8 and 20 with spherical shape in these isotopic chains and disappearance of N = 28 shell closure with a finite deformation. Prolate-oblate shape coexistence in some iso-
topes of Si, S and Ar isotopic chains have also been observed. In [23], it has been reported that excitation of nucleons from sd- to pf-shell plays an important role for “island of inversion” nuclei at $N = 20$. Inclusion of pf-shell was found to improve the B(E2) values for $^{30}$Ne and $^{32}$Mg. *Ab initio* results with IMSRG and CC methods were seen to be in reasonable agreement with the experimental data for the sd-shell nuclei Ne, Mg and Si except for $N = 20$. Considerable success of the USDA and USDB interactions [35, 36] in describing the nuclear structure properties of sd-shell nuclei has led to the construction of new isospin-breaking USD-type interactions viz. the USD and USDI interactions [37]. The USD, just like the USDB, is based on renormalized G-matrix and fitting of the free parameters from the experimental data. The USDI is derived from *ab initio* IM-SRG interactions. These new interactions improve predictions for separation energy in the entire sd-shell.

In [38], a new type of *ab initio* nucleon-nucleon (NN) interaction was developed for sd – pf model space, viz. the EEdf1 interaction. This interaction was derived from the fundamental chiral effective field theory (χEFT) [39–42] based on Quantum chromodynamics (QCD) [43] and extended Kuo-Krenciglowa (EKK) method [44–46]. The Fujita-Miyazawa three nucleon forces [47] were also included. It was initially implemented on medium mass nuclei Ne, Mg and Si isotopes [38]. As neutron number increases in a nucleus, exotic features start to appear. Exotic neutron rich Ne, Mg and Si isotopes were explained in the aforementioned study by particle-hole excitations across two major shells without fitted interaction. Further, this interaction has also been applied for $Z = 9 – 12$ isotopes to determine the drip line and exotic properties of “island of inversion” nuclei with $Z = 10 – 12$ and $N = 20 – 22$ [48]. This work was extended to determine the spectroscopic properties such as magnetic dipole and electric quadrupole moments, charge and matter radii for Na and Mg isotopes, in Ref. [49].

Spectroscopic factor strength describes the nature and occupancy of the single particle orbits in a nucleus that are used to determine the structure of nucleus. In Ref. [50], experimental spectroscopic factor strength for ground state (g.s.) of $^{24}$Si from $^{23}$Al($p, \gamma$)$^{24}$Si reaction was obtained. Recently, an experiment has been performed with a transfer reaction $^{23}$Al($d, n$)$^{24}$Si at National Superconducting Cyclotron Laboratory [51] to measure excited states and their spectroscopic factor strengths that might be of astrophysical interest. In the present work, we have calculated theoretical spectroscopic factor strengths for g.s. as well as excited states of $^{24}$Si for one proton capture reaction $^{23}$Al($d, n$)$^{24}$Si using microscopic N3LO, JISP16 and DJ16A interactions, and have also carried out a comparison with the USDB results.

Beta decay study is another tool that helps to understand the nuclear structure of the atomic nuclei. A recent experiment at the SPIRALI facility of GANIL [52], performed to carry out a detailed study of the decay of proton rich nucleus $^{32}$Ar, was able to deduce the level structure of $^{32}$Cl. The branching ratios, B(F), B(GT) and log $ft$ values for $^{32}$Cl from the $\beta$-decay of $^{32}$Ar were determined. The shell model study was also performed with USD type interactions and results were found to support the experimental data. We have revisited this study to calculate above properties using newly constructed microscopic effective interactions.

In the present study, we apply the nuclear shell model to study energy spectra of the nuclei lying in the range $Z = 14 – 18$ with neutron number $N = 9 – 20$. Microscopically developed sd-shell effective interactions such as chiral N3LO, JISP16 and DJ16A have been used for describing the structures of these isotopic chains. We compare our microscopic results with the shell model results obtained from empirical USDB interaction [36] and experimentally measured data. We also calculate proton effective single-particle energies for Si isotopes to determine the shell evolution. Monopole term of the effective two-body interactions is evaluated and their spin-tensor decomposition are also carried out. Apart from energy levels of Si, P, S, Cl and Ar isotopic chains, we calculate electromagnetic properties such as electric quadrupole moment ($Q$), magnetic dipole moment ($\mu$) and reduced electric quadrupole transition strength B(E2). In addition, we compute spectroscopic factor strengths for $^{23}$Al($d, n$)$^{24}$Si reaction and $\beta$-decay properties of $^{32}$Ar with all four interactions and compare the results with the experimental data.

The present work is organized as follows. In Section II, we briefly describe the methodology to develop the microscopic effective sd-shell interaction and details about the interactions used in our calculations. Next, we show the results of energy spectra of Si, P, S, Cl and Ar isotopic chains in Section III. Effective proton single-particle energies of Si isotopes have been carried out in Section IV. In Section V, monopole parts and their spin-tensor components of the effective interactions have been presented. We then discuss the electromagnetic properties, where experimental data are available, of the $Z = 14 – 18$ nuclei in Sections VI. Spectroscopic factor strengths for $^{24}$Si are presented in Section VII. The $\beta$-decay properties of $^{32}$Ar are studied in Section VIII. Finally, we draw conclusion of the present study in Section IX.

II. MICROSCOPIC EFFECTIVE sd-SHELL INTERACTIONS

To describe the nuclear structure properties of sd-shell nuclei, we have performed the nuclear shell model calculations with microscopic effective sd-shell interactions in the present work. The detailed procedure to develop these effective sd-shell interactions has been described in Refs. [29, 30], which we have briefly presented here.

In this approach, the microscopic effective valence shell interactions are derived from the *ab initio* NCSM wave functions. The NCSM Hamiltonian for a nucleus of $A$ point like nucleons interacting through realistic interac-
tion can be written as

\[
H_A = \frac{1}{A} \sum_{i<j}^{A} \left( \frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{ij}^{NN} \right),
\]

where, \( m \) is the mass of a nucleon. This Hamiltonian, which is translationally invariant, has relative kinetic energy term and two-nucleon interaction term including Coulomb interaction between protons. In these calculations three-body forces are omitted.

The harmonic oscillator (HO) basis states (slater determine basis) up to \( N_{\text{max}} \), which is maximum number of HO quanta above the unperturbed \( A \) nucleon configuration, are used to solve the eigen value problem of \( A \) nucleon system. Several realistic NN interactions, \( i.e. \) charge-dependent Bonn 2000 (CDB2K) [53] and chiral effective field theory interactions [32] generate strong short-range correlations. To obtain convergent results, large basis states are required which is constrained by computational limitations. Thus, a renormalization method is employed to soften the standard realistic interactions. There are two renormalization techniques: OLS similarity transformation [54–57] and SRG method [58], with the former being implemented in the procedure outlined here.

To make derivation easier, a frequency dependency is introduced in the NCSM calculations. Addition of center-of-mass HO Hamiltonian to the initial Hamiltonian 1 modifies the resultant Hamiltonian as

\[
H_a + H_{\text{c.m.}} = \sum_{i=1}^{a} \left[ \frac{\vec{p}^2_i}{2m} + \frac{1}{2} m \Omega^2 \vec{r}^2_i \right] + \sum_{i<j=1}^{a} \left[ V_{ij}^{NN} - \frac{m \Omega^2}{2A} (\vec{r}_i - \vec{r}_j)^2 \right].
\]

This \( H_{\text{c.m.}} \) is subtracted out from the final calculations. If \( a = A \) then \( H_a \) becomes the initial Hamiltonian 1. The effective NN interaction for the NCSM calculations is derived from Eq. 2 in a cluster approximation. In these NCSM calculations, the \( a = 2 \) cluster approximation has been used. First OLS transformation is applied to construct a primary effective Hamiltonian for \( A = 18 \) system with NCSM parameters \( N_{\text{max}} = 4 \) and \( \hbar \Omega = 14 \) MeV. The lowest 28 eigen states are calculated to perform second OLS transformation. This 18-body Hamiltonian is projected onto \( 0 \hbar \Omega \) model space. These eigen states of \( ^{18}\text{F} \) are dominated by the configurations with \( ^{16}\text{O} \) system in the lowest possible HO orbits and two nucleons in \( sd \)-shell. The secondary effective Hamiltonian reproduces exactly same energy of the states in \( ^{18}\text{F} \), as generated by the primary effective Hamiltonian. Further, the NCSM calculations have been performed for \( ^{16}\text{O}, ^{17}\text{O} \) and \( ^{17}\text{F} \) with primary effective Hamiltonian to calculate core and one-body single-particle energies. These core and one-body components have been subtracted out from secondary effective Hamiltonian of \( ^{18}\text{F} \) to obtain residual two-body matrix elements for the \( sd \) valence shell.

In this work, three microscopic \( sd \)-valence space interactions have been used, \( viz. \) N3LO, JISP16 and DJ16A. The TBMEs of the JISP16 and N3LO potential are taken from Ref. [29] and for DJ16A from Ref. [30]. Together with these microscopic interactions, we have also used empirical USDB interaction [36] to know how much reliable description of nuclei are obtained from the microscopic potentials. Results with these interactions are compared with the experimental data. In the case of the empirical USDB interaction, single-particle energies are \( \epsilon(d_{5/2}) = -3.9257 \text{ MeV}, \epsilon(s_{1/2}) = -3.2079 \text{ MeV} \) and \( \epsilon(d_{3/2}) = 2.1117 \text{ MeV} \). These single-particle energies of the USDB interaction are taken for all the microscopic interactions similar to Ref. [30]. We have calculated only positive parity states of these nuclei. We have performed nuclear shell model calculations using the code KSHELL [59].

Experimental g.s. energies of \( sd \)-shell nucleus relative to the g.s. energy of \( ^{16}\text{O} \) with the Coulomb energy correction term is determined as:

\[
E(A, Z)^r = E(A, Z) - E(^{16}\text{O}) - E_c(Z),
\]

where, \( E(A, Z)^r \) and \( E(A, Z) \) are relative and absolute g.s. energy. \( E(^{16}\text{O}) \) is the g.s. energy of \( ^{16}\text{O} \), which have the value of -127.619 MeV. \( E_c(Z) \) is the Coulomb correction energy taken from Ref. [35].

### III. RESULTS AND DISCUSSION

We have presented a comprehensive nuclear shell model results for \( Z = 14 \) – 18 nuclei using microscopic effective interactions N3LO, JISP16 and DJ16A in the valence \( sd \)-shell. We have studied Si chain with mass number \( A = 22 - 34 \), P chain with \( A = 25 - 35 \), S chain with \( A = 26 - 36 \), Cl chain with \( A = 28 - 37 \) and Ar chain with \( A = 30 - 38 \). We have calculated low-lying energy spectra of these nuclei and compared them with the experimental data. The empirical USDB calculations have also been performed for comparison.

#### A. Si isotopes

We have determined g.s. as well as excited states energies of \( ^{22-34}\text{Si} \) isotopes with N3LO, JISP16 and DJ16A interactions along with the empirical USDB interaction. In Fig. 1, we have shown the low-lying energy spectra of Si isotopes with \( A = 26 - 34 \). Experimental spectrum is shown in the last column of each figure. Experimentally, only g.s. is known for \( ^{22}\text{Si} \). All microscopic interactions reproduce g.s. as \( 0^+ \) correctly. Experimental spin-parity of g.s. is \((5/2)^+\) for \( ^{23}\text{Si} \) which is confirmed with all four interactions. In \( ^{24}\text{Si} \), correct g.s. \( 0^+ \) is obtained with each interaction. For first excited state \( 2^+ \), DJ16A interaction gives 1.786 MeV excitation energy. USDB interaction predicts this state to be at 2.111
FIG. 1. Low-lying energy spectra for Si isotopes in the range $A = 26 - 34$. Theoretical shell model results with N3LO, JISP16, DJ16A and USDB interactions are compared with the experimental data [60].
MeV while experimental value is 1.879 MeV. This data indicates that DJ16A interaction provides better agreement for $2^+$ state of $^{24}$Si, with the experiment. In $^{23}$Si, only g.s. spin-parity is measured experimentally and spin-parity of excited states are unknown. Microscopic interactions are unable to reproduce spin of the g.s., which is correctly given by the USDB interaction. In case of $^{26}$Si, we observe from the spectrum that microscopic interactions produce lower excited states at small excitation energies than measured from experiment. Again, the USDB interaction provides better results. For $^{27}$Si, g.s. spin is correctly reproduced with DJ16A interaction, which has the configuration of $\pi(d_{5/2}^1) \otimes \nu(d_{5/2}^2)$ with probability of 16.42%. USDB interaction also predicts $5/2^+$ as the g.s. of $^{27}$Si with the same configuration as found in case of DJ16A interaction with 27.36% probability, although, N3LO and JISP16 interactions are unable to predict correct spin of the g.s. The IM-SRG and CCEI give the dominant configurations in wave functions from $\pi(d_{5/2}^2) \otimes \nu(d_{5/2}^1)$ and $\pi(d_{5/2}^2 s_{1/2}^1) \otimes \nu(d_{5/2}^2 s_{1/2}^1)$ with probabilities of 9.27% and 13.24%, respectively, as reported in Ref. [22]. With DJ16A, energy of $1/2^+_1$ is 0.711 MeV which is close to the experimental value of 0.781 MeV. The energy spectra with N3LO interaction is compressed as compared to the other interactions.

$^{28}$Si and $^{29}$Si have been studied in [30], where, lower excitation energies of $2^+_4$ and $4^+_4$ due to small $N = 14$ shell gap, and configurations of 0$^+_1$ and 0$^+_2$ states for $^{28}$Si have been described. We have added some excited states with all four interactions and have also studied spectroscopic properties (in Section VI). $^{28}$Si nucleus is the closed subshell nucleus which contains 6 valence protons and 6 valence neutrons. We obtain correct ordering of the states up to 0$^+_2$ in $^{28}$Si with each interaction. Energy difference between g.s. and first excited state is 1.068, 1.145, 1.296 and 1.932 MeV with N3LO, JISP16, DJ16A and USDB interactions, respectively, and corresponding experimental energy difference is 1.779 MeV. Similarly, excitation energy for $4^+_4$ state approaches the experimental value as we go from N3LO to JISP16 to DJ16A interaction. The energy of $3/2^+_1$ state for $^{28}$Si corresponding to N3LO, JISP16 and DJ16A interactions is 0.200, 0.669 and 0.770 MeV, respectively. Experimental value is 1.273 MeV which agrees best with the result of USDB interaction (1.285 MeV). The g.s. of $^{30}$Si corresponding to the DJ16A interaction is dominated by the closed subshell configuration $\pi(d_{5/2}^6) \otimes \nu(d_{5/2}^6 s_{1/2}^6)$ with 24.39% probability and the USDB interaction also generates the same configuration with 27.54% of probability. The $0^+_2$ has the configuration of $N = 16$ closed subshell $\pi(d_{5/2}^4) \otimes \nu(d_{5/2}^4 s_{1/2}^4)$ for DJ16A interaction with 16.65% of probability, while this state is populated using USDB interaction with configurations of $\pi(d_{5/2}^2) \otimes \nu(d_{5/2}^2 d_{5/2}^2)$ and $\pi(d_{5/2}^2) \otimes \nu(d_{5/2}^2 s_{1/2}^2)$ with probabilities of 30.81% and 17.44%, respectively. In case of $^{30}$Si, DJ16A interaction predicts $2_1^+$ state at 1.607 MeV while with N3LO and JISP16 interactions, this state is at 1.314 and 1.478 MeV, respectively. Experimentally this state is measured at 2.235 MeV excitation energy. Again, improved excitation energy is obtained with DJ16A interaction. Similarly for $2_2^+$, DJ16A predicts this state at 2.213 MeV which is lower to the experimental value than obtained from N3LO and JISP16 interactions. We obtain correct ordering up to $5/2^+_1$ with each of the interactions in case of $^{33}$Si.

![Graph](image)

FIG. 2. Comparison of experimental g.s. energy [60] with the shell model results obtained from N3LO, JISP16, DJ16A and USDB interactions for Si isotopes with respect to g.s energy of $^{16}$O.

For $^{32}$Si, energy of first excited state is reasonably improved with DJ16A interaction (Fig.1). The $2^+_4$ and $2^+_5$ states have the configuration of $\pi(d_{5/2}^5) \otimes \nu(d_{5/2}^5 d_{5/2}^3 d_{5/2}^2)$ with probabilities of 17.98 and 48.62%, respectively, using DJ16A interaction. The same configuration is obtained using USDB interaction with 20.71 and 48.78% probabilities. In $^{33}$Si, g.s. spin-parity is well reproduced with shell model calculations using all the interactions. The excitation energy of $1/2^+_1$ is 0.743, 1.026, 1.417 and 0.807 MeV using N3LO, JISP16, DJ16A and USDB interactions, respectively. Experimental excitation energy for this state is 1.010 MeV. We have calculated excitation energies with spins up to 7/2$^+$. We can see from the spectrum of $^{33}$Si in Fig. 1 that there is a large energy gap between 1/2$^+$ and 7/2$^+$ states. We have computed both spin states with all four interactions. In $^{34}$Si, energy spectra with spins 3/2$^+$ and 5/2$^+$ are shown. For g.s., proton configuration with all four interactions is $\pi(d_{5/2}^6)$. For the $2^+_1$ and $2^+_2$, the configurations are $\pi(d_{5/2}^6 s_{1/2}^4)$ and $\pi(d_{5/2}^6 d_{5/2}^4)$ with probabilities of 89.71 and 88.77%, respectively, corresponding to DJ16A interaction. The same configuration is achieved with USDB interaction for above mentioned states with 87.61 and 89.84% prob-
FIG. 3. Low-lying energy spectra for P isotopes in the range $A = 27 - 35$. Theoretical shell model results with N3LO, JISP16, DJ16A and USDB interactions are compared with the experimental data [60].
abilities, respectively.
Energies of g.s. for Si isotopes relative to g.s. of $^{16}\text{O}$ with mass number $A = 22 – 34$ are depicted in Fig. 2. As the number of neutrons in Si isotope increases from $A = 22$ to $A = 34$, the energy difference between theory and experiment increases for all three microscopic interactions. It is expected that the N3LO results can be improved by the inclusion of three-body forces. The N3LO and JISP16 interactions overbind the g.s., while DJ16A interaction slightly underbinds the g.s. (which is discussed in Section VI). The deficiencies in results occur because the JISP16 and DJ16 interactions are fitted only up to $A = 16$ nuclei. The DJ16A results are better than the JISP16 because the former is monopole modified to DJ16 interaction. Still, non-monopole modifications for further tuning of TBMFs are needed in DJ16A interaction. It is possible that these discrepancies may be resolved by using NCSM results with larger $N_{\text{max}}$ parameter.

B. P isotopes

We have studied phosphorus isotopes with mass number $A = 24 – 35$ lying in $sd$-shell region, of which, $^{25}\text{P}$ is the least stable isotope while $^{31}\text{P}$ is the most stable. Here, energy level diagrams of $^{27–35}\text{P}$ isotopes are presented in Fig. 3. Experimental g.s. $(1^+)$ is not reproduced with any of the interactions. For $^{25}\text{P}$, only g.s. is experimentally measured with unconfirmed spin-parity $(1/2^+)$. Nuclear shell model with all four interactions confirm g.s. to be $1/2^+$. The $^{26}\text{P}$ has g.s. with tentative spin $(3)^+$. The $^{27}\text{P}$ interaction provides confirmation to g.s. spin, while other interactions fail to do so. Spin-parity of some excited states are unknown from experiments, therefore, we have calculated some excited states with each of the three microscopic and USDB interactions. In case of $^{28}\text{P}$, spin-parity of g.s. is $3^+$ experimentally, which is well reproduced by the empirical interaction, whereas microscopic interactions predict the g.s. to be $2^+$. First excited state has the spin of $(2^+)$, experimentally, which is 105.6 keV above the g.s. The energy difference between $2^+$ and $3^+$ states is 33, 266 and 229 keV corresponding to N3LO, JISP16 and DJ16A interactions, respectively. With USDB interaction, $2^+$ is 13 keV above the $3^+$ state. Analysing the configurations of $3^+$ and $2^+$ states, it seems that both states have the same configuration of $|\pi(d_5^0 s_{1/2}^1) \otimes \nu(d_3^0 s_{1/2}^2)|$ with USDB and DJ16A interactions. It is observed that the energy spectra of N3LO and JISP16 interactions are quite compressed. The first $1^+$ state with the configuration $|\pi(d_5^0 s_{1/2}^1) \otimes \nu(d_3^0 s_{1/2}^2)|$ obtained from the USDB and DJ16A interactions is found to be at high energy in the spectrum.

In $^{29}\text{P}$, dominant configuration of excited state $3/2^+$ comes from the excitation of proton from $s_{1/2}$ to $d_{3/2}$ with probabilities 32.50% and 15.14% using USDB and DJ16A interactions, respectively. Other interactions produce this configuration with very small probabilities. Energy of excited states $3/2^+$ and $5/2^+$ is best reproduced with USDB interaction followed by DJ16A interaction for $^{31}\text{P}$. The g.s. spin for odd-P isotopes is $1/2^+$ and the first excited state is $3/2^+$ except in case of $^{25}\text{P}$, for which, first excited state is not measured yet, experimentally. DJ16A and USDB interactions reproduce the g.s. and first excited state correctly for odd-P isotopes, except for $^{27}\text{P}$. For $^{27}\text{P}$, DJ16A yields the first excited state as $5/2^+$. The proton orbital configurations with USDB and DJ16A interactions are $|\pi(d_5^0 s_{1/2}^1)|$ for g.s. and $|\pi(d_5^0 d_{3/2}^1)|$ for the first excited state with maximum probabilities. As neutrons increase from $A = 25$ onwards, they sequentially occupy $d_{5/2}$, $s_{1/2}$ and $d_{3/2}$ orbitals for ground and first excited states with maximum probabilities, except for $^{31}\text{P}$. For $^{31}\text{P}$, the configurations of $3/2^+$ state are $|\pi(d_5^0 d_{3/2}^1) \otimes \nu(d_5^0 d_{3/2}^2)|$ and $|\pi(d_5^0 d_{3/2}^1) \otimes \nu(d_5^0 s_{1/2}^2)|$ for DJ16A interaction with probabilities of 10.15 and 9.29%. With USDB interaction, the same configuration is obtained with 15.80 and 15.34% probabilities.

In Fig. 4, we have shown the g.s. energy of the P isotopes relative to g.s. energy of $^{16}\text{O}$ in the range $A = 24 – 35$ with N3LO, JISP16 and DJ16A interactions and compared our microscopic results with empirical shell model results. Such a comparison is helpful in testing whether results obtained from new microscopic interactions are consistent with empirical results. Similar to Si isotopes, improvement in g.s. energy of P isotopes is achieved with DJ16A interaction.
FIG. 5. Low-lying energy spectra for S isotopes in the range $A = 28 – 36$. Theoretical shell model results with N3LO, JISP16, DJ16A and USDB interactions are compared with the experimental data [60].
C. S isotopes

In this section, we have calculated low-lying energy spectra of sulphur isotopes with mass number $A = 26–36$ in sd-shell regime. In Fig. 5, energy levels are reported for 28–36S isotopes.

For 26S, g.s. 0+ is well reproduced by shell model using all four interactions, while only USDB interaction is able to reproduce g.s. spin 5/2+ in case of 27S. Although only the g.s. and first excited states are measured for 28S experimentally, we have predicted several low-lying energy states with spins up to 4. The experimental g.s. spin-parity of 29S is unknown. The DJ16A and USDB interactions predict the spin-parity of g.s. to be 5/2+ with configuration $|\pi(d_{5/2}^{6}s_{1/2}^{2}d_{3/2}^{4}2\nu(d_{5/2}^{2}))|$. Our shell model results with the USDB and DJ16A confirm the first excited state as 1/2+. For 30S, g.s. 0+ with the configuration $|\pi(d_{5/2}^{6}s_{1/2}^{2}d_{3/2}^{4}2\nu(d_{5/2}^{2}))|$ is obtained with 27.54% probability using the USDB interaction, while DJ16A interaction predicts this configuration with 24.39% probability. The lowest 2+ state contains the configuration of $|\pi(d_{5/2}^{6}s_{1/2}^{2}d_{3/2}^{4})\otimes\nu(d_{5/2}^{2})|$ with 25.09% and 20.46% probability according to the USDB and DJ16A interactions, respectively. 32S has been already studied with these microscopic interactions in Ref. [30]. Here, we have extended calculations of energy states up to 7 MeV. According to the USDB, 3/2+, 1/2+, and 5/2+ states of 32S have the configurations of $|\pi(d_{5/2}^{6}s_{1/2}^{2}d_{3/2}^{4}2\nu(d_{5/2}^{2}))|$, $|\pi(d_{5/2}^{6}s_{1/2}^{2}d_{3/2}^{4})\otimes\nu(d_{5/2}^{2}d_{1/2}^{2}d_{3/2}^{2})|$, and $|\pi(d_{5/2}^{6}s_{1/2}^{2}d_{3/2}^{4})\otimes\nu(d_{5/2}^{2}d_{1/2}^{2}d_{3/2}^{2})|$ with 80.57%, 49.50% and 73.41% probabilities, respectively. The respective probabilities with DJ16A interaction are 72.88%, 47.57% and 62.12%. Now, we focus on 36S, which has closed neutron shell $N = 20$. Experimental energy of 2+ state is 3.291 MeV, while N3LO, JISP16, DJ16A and USDB give the excitation energy of this state as 3.585, 3.878, 2.986 and 3.382 MeV, respectively.

Theoretical and experimental results of g.s. energy for 26–36S isotopes with respect to g.s. energy of 16O are presented in Fig. 6. For 26S, DJ16A result is very close to the experimental value of g.s. energy. As we move to the heavier mass nuclei from 26S to 36S, the deviation between DJ16A results and experimentally measured energies increases. DJ16A interaction underbinds the g.s. for heavier isotopes of S chain. JISP16 and N3LO results of g.s. energy are quite far from the experimental data. We can conclude that monopole modifications in the DJ16 interaction are not enough to explain the g.s. energy of S isotopes. Thus, further modifications in the DJ16A interaction are required.

D. Cl isotopes

In this section, we have performed nuclear shell model calculations of low-lying energy spectra for 28–37Cl. Low-energy spectra for 32–37Cl are shown in Fig. 7. In the case of 28Cl, only g.s. is measured with spin-parity (1+) from experiment. Shell model calculations using all four interactions predict the g.s. as 2+. USDB interaction gives the configuration $|\pi(d_{5/2}^{6}s_{1/2}^{2}d_{3/2}^{4}\nu(d_{5/2}^{2}))|$ with 48.95% probability for this state, while microscopic interactions predict the same configuration with probability in the range 27-37%. For 29Cl, g.s. has tentative spin-parity (1/2+) and first excited state (3/2+) lies 500 keV above the g.s., experimentally. Only DJ16A interaction shows correct ordering of g.s. and first excited state. The experimental spin-parity of g.s. is (3+) for 30Cl, which is predicted by USDB interaction only with configuration of $|\pi(d_{5/2}^{6}s_{1/2}^{2}d_{3/2}^{4}\nu(d_{5/2}^{2}))|$. For 32Cl, apart from N3LO, all other interactions including USDB give the correct g.s. of 1+. The experimental spin-parity of g.s. for 33Cl is 3/2+ and that of the first excited state is 1/2+, which lies 0.811 MeV above the g.s. Looking into the configurations of these calculated states, it seems that major contribution in the wavefunction comes from normal filling of single particle energy levels. The single particle configurations of 3/2+ and 1/2+ states are obtained as $|\pi(d_{5/2}^{6}s_{1/2}^{2}d_{3/2}^{4}\nu(d_{5/2}^{2}s_{1/2}^{2}))|$ and $|\pi(d_{5/2}^{6}s_{1/2}^{2}d_{3/2}^{4}\nu(d_{5/2}^{2}s_{1/2}^{2}))|\otimes\nu(d_{5/2}^{2}s_{1/2}^{2})|$, respectively, with maximum probabilities, except in the case of N3LO interaction, for which, the aforementioned configuration corresponding to the 1/2+ state does not occur with maximum probability.

Only the N3LO interaction gives correct g.s. spin for 34Cl, opposite to the case of 35Cl. Ordering of the three lowest states is correctly reproduced by DJ16A interaction for 35Cl. For 36Cl, all microscopic interactions predict correct spin of g.s. as 2+. DJ16A interaction gives reverse ordering of states 1/2+ and 3/2+, while other interactions give the correct ordering. Energy of 3/2+ state rela-
FIG. 7. Low-lying energy spectra for Cl isotopes in the range $A = 32 - 37$. Theoretical shell model results with N3LO, JISP16, DJ16A and USDB interactions are compared with the experimental data [60].

We have shown theoretical as well as experimental g.s. energy of $^{28 - 37}$Cl isotopes with respect to g.s. energy of $^{16}$O in Fig. 8. Experimental value of g.s. energy for $^{28}$Cl is -101.557 MeV. The N3LO, JISP16, DJ16A and USDB interactions give the g.s. energy as -115.782, -112.884, -101.240 and -102.437 MeV, respectively. For $^{28}$Cl, DJ16A result of g.s. energy almost matches with the experimental value. For $^{37}$Cl, energy of g.s. is -234.211 MeV, while theoretical value of g.s. energy using N3LO, JISP16, DJ16A and USDB interactions are -271.653, -259.047, -224.082 and -234.289 MeV, respectively. The data indicates that as we add more neutrons to $^{28}$Cl, DJ16A interaction results deviate from the experimental data.

tive to g.s. is 788 keV, experimentally. Shell model excitation energy of this state using N3LO, JISP16, DJ16A and USDB interactions is 622, 608, 818 and 836 keV, respectively. So, DJ16A result for this state is closer to the experimental value compared to other interactions. $^{37}$Cl lies on the boundary of sd-shell. The structures of wavefunctions for $3/2^+$ and $1/2^+$ come from one unpaired proton in $d_{3/2}$ and $s_{1/2}$ orbitals, respectively.
E. Ar isotopes

We have studied energy spectra of argon isotopes in the range \(N = 12 - 20\). In Fig. 9, the energy level diagrams of \(^{33 - 38}\)Ar isotopes are shown. The g.s. spin of \(^{39}\)Ar is correctly reproduced by all four shell model interactions. The first excited state \(2^+\), which is tentative as of yet, lies 700 keV above the g.s. Excitation energy for this state using N3LO, JISP16, DJ16A and USDB interactions are too high: 1.202, 1.278, 1.352 and 1.591 MeV, respectively. In the case of \(^{31}\)Ar, only g.s. is known experimentally and shell model calculations using all interactions give correct spin-parity of g.s. to be \(5/2^+\), which contains the major configuration of \(\pi(3^2d_{5/2}^2s_{1/2}^2d_{3/2}^2)^{-}\).

In the case of \(^{32}\)Ar, first experimental \(2^+\) state is 1.867 MeV above the g.s. \(0^+\). This state lies at 1.385, 1.524 and 1.785 MeV above the g.s. according to N3LO, JISP16 and DJ16A interactions, respectively, while USDB gives 2.053 MeV excitation energy. Thus, microscopic DJ16A interaction gives better result for this state. Only spin-parity of g.s. is confirmed and other states have tentative spin-parity assignments for \(^{33}\)Ar, experimentally. Theoretical results using each interaction predict the first excited state as \(3/2^+\). For \(^{34}\)Ar, the \(d_{5/2}\) neutron orbital occupancy increases with excitation energy according to the USDB calculations as 0.67 for \(0^+_1\), 0.87 for \(2^+_1\), 1.24 for \(2^+_2\), 1.65 for \(0^+_2\). From DJ16A calculations, the \(d_{3/2}\) neutron orbital occupancy also increases with excitation energies: 0.91 for \(0^+_1\), 1.18 for \(2^+_1\), 1.34 for \(2^+_2\), 1.70 for \(0^+_2\). The g.s. \(3/2^+\) is well reproduced by theoretical calculations for \(^{35}\)Ar. The DJ16A and USDB give the correct ordering of \(1/2^+\) and \(5/2^+\) states, while other two interactions predict reverse ordering of these states. In \(^{36}\)Ar, a state at 4.329 MeV could be either \(0^+\) or \(1^+\) or \(2^+\). Thus, shell model calculations have been performed to determine these spin states and we notice that \(1^+\) state has excitation energy around 6.7-7.0 MeV, so we have not shown this state in the figure. This state is most likely to be \(2^+\).

The g.s. of \(^{37}\)Ar has the configuration of 2/1/3/2 by the \(d^3\) interactions with probabilities of 75.11, 78.94, 70.69 and 84.51% according to N3LO, JISP16, DJ16A and USDB interactions, respectively.
The JISP16 interaction fails to reproduce the experimental first excited state $1/2^+$. Experimentally, $2^+_1$ state lies at 2.167 MeV for $^{38}\text{Ar}$. As we go from N3LO to JISP16 to DJ16A interactions, energy of first excited $2^+$ state improves from 0.987 to 1.003 to 1.040 MeV, which is still $\approx 1$ MeV away from the experimental data. Thus, it is clear that $sd – pf$ model space is essential to describe the energy spectra of $^{38}\text{Ar}$.

In Fig. 10, calculated g.s. energies of $^{30–38}\text{Ar}$ isotopes with respect to g.s. energy of $^{16}\text{O}$ are shown. We notice that the USDB interaction results are very close to the experimental data. The DJ16A interaction shows less bound g.s. of $^{30–38}\text{Ar}$ than obtained from experiment. The N3LO and JISP16 predict more bound g.s. of Ar isotopes.

In Fig. 11, we have shown the excitation energy of $2^+_1$ state for Si, S and Ar isotopes with neutrons $N = 10–20$. For $^{28}\text{Si}$, shell model predictions with all four interactions have been done. USDB interaction gives better agreement with the experimental data for Si and S isotopes, except for $^{34}\text{Si}$. For $^{34}\text{Si}$, result obtained from JISP16 interaction is in good agreement with experimental value. We can clearly observe from the experimental data that there is a shell gap at $N = 20$ for Si and S isotopes. But in the case of argon isotopes, this shell gap is weak. For $^{34}\text{Si}$, DJ16A and USDB interactions produce large excitation energy for $2^+$ state compared to that obtained from experiment by excitation of a proton from $d_{5/2}$ to $s_{1/2}$ orbit which indicates a large shell gap at $Z = 14$. This is explained in the next section. At $N = 14$ ($^{32}\text{Ar}$), energy of $2^+_1$ state from DJ16A interaction is very close to the experimental energy. The N3LO, JISP16 and DJ16A interactions do not produce sufficiently large excitation energy of $2^+$ state for $^{38}\text{Ar}$. Even USDB interaction gives small excitation energy of $2^+_1$ state at $N = 20$. This energy difference may be accounted for by the inclusion of $sd – pf$ model space.
FIG. 13. Proton effective single-particle energies of Si isotopes, calculated with the N3LO, JISP16, DJ16A and USDB interactions.

$sd - pf$ model space.

Similarly, we have plotted energy splitting between $1/2^+_1$ and $3/2^+_1$ states with respect to neutron numbers for phosphorous and chlorine isotopes in Fig. 12. In the case of $Z = 15$, excited $3/2^+_1$ state has not yet been found for $N = 10$. So, we have predicted the energy of this state with different interactions. The DJ16A interaction gives the same trend as obtained from experiment for phosphorus isotopes. $^{35}$P and $^{37}$Cl have one proton hole and one proton particle beyond $Z = 16$ with $N = 20$. Lowest $3/2^+$ or $1/2^+$ excited state is produced by the excitation of proton between orbitals $s_{1/2}$ and $d_{3/2}$. Large excitation splitting at $N = 20$ in phosphorous and chlorine nuclei indicates a shell gap at $Z = 16$. For $^{37}$Cl, we can see that all the theoretical interactions predict large energy splitting between $1/2^+_1$ and $3/2^+_1$ states as compared to the experimental splitting.

IV. EFFECTIVE SINGLE-PARTICLE ENERGIES: MONOPOLE PROPERTIES

The Hamiltonian can be classified into monopole and multipole (pairing, quadrupole-quadrupole correlations etc.) parts. The monopole part of the Hamiltonian tells about the spherical nuclear mean fields that give information of filling of nucleons in the single particle states. It provides the position of the single particle orbitals which play important role in describing the evolution of shell gap. In this section, we have shown the proton effective single particle energies (ESPEs) corresponding to N3LO, JISP16, DJ16A and USDB interactions for Si isotopes in Fig. 13. ESPEs describe how single particles energies of $sd$ orbitals vary as we increase neutrons in the silicon nuclei. In calculation of ESPEs, monopole term of two-body interaction is needed which is given by

$$V(jj') = \frac{\sum_J (2J+1) \langle jj'; J | V | jj'; J \rangle}{\sum_J (2J+1)}.$$  \hspace{1cm} (4)

Monopole interaction shows an average effect between two nucleons in orbits $j$ and $j'$. This monopole interaction changes the single-particle energy and consequently the shell gaps as we add more nucleons. Thus, spin-tensor decomposition of the monopole interaction is carried out in next section to understand the origin of this shell evolution. Effective single-particle energy and role played by different components of the two-body interaction in the shell evolution had been discussed in several papers [48, 61–67].

The Si is a mid shell nuclei, for which states have contributions of both protons and neutrons. From Fig. 13, we can see that single particle orbitals $s_{1/2}$ and $d_{3/2}$ in-
FIG. 14. Monopole term of total, central, vector and tensor forces for \( T = 0 \) and \( T = 1 \) channels corresponding to N3LO, JISP16, DJ16A and USDB interactions.

terchange at \(^{28}\text{Si}\) for N3LO interaction and ESPs of orbitals \( d_{5/2} \) and \( s_{1/2} \) are very close to each other. For JISP16 and USDB interactions, \( s_{1/2} \) and \( d_{3/2} \) orbitals almost overlap. As mentioned in the previous section, there is high-lying \( 2^+_1 \) state for \(^{34}\text{Si}\) predicted by DJ16A and USDB interactions. This high excitation energy is due to sufficiently large energy difference between \( d_{5/2} \) and \( s_{1/2} \) orbitals. So, it can be argued that there is a shell closure at \( Z = 14 \) in Si isotopes. This shell gap increases from N3LO to JISP16 to DJ16A to USDB interaction, thus, excitation energy for this state increases accordingly. It is also observed that the shell gap at \( N = 16 \) between orbitals \( s_{1/2} \) and \( d_{3/2} \) is very small for microscopic N3LO interaction as compared to other interactions.

V. SPIN-TENSOR DECOMPOSITION OF THE TWO-BODY INTERACTION

To understand the contributions from different components of two-body interactions, we have carried out spin-tensor decomposition. A two-body interaction can be written as

\[
V = \sum_{k=0,1,2} (S^{(k)}, Q^{(k)}) = \sum_{k=0,1,2} V^{(k)},
\]

where, \( k \) runs over 0, 1, 2. The term with \( k = 0 \) indicates the central component, the term with \( k = 1 \) the vector component and the term with \( k = 2 \) the tensor component. \( S^{(k)} \) are spherical tensors of rank \( k \) in spin space, constructed for spin-1/2 nucleons and \( Q^{(k)} \) are spherical tensors of rank \( k \) in coordinate space. The 1 and \( (\sigma_1, \sigma_2) \) operators correspond to the spin part of the central component of the effective interaction. The spin part of the tensor component is given by \( [\sigma_1 \times \sigma_2]^{(2)} \). The vector component includes symmetric spin-tensor forces, which have operators \( \sigma_1^+ \sigma_2^+ \) and antisymmetric spin-tensor forces, which have operators \( \sigma_1^+ \sigma_2^- \) and \( [\sigma_1 \times \sigma_2]^{(1)} \).

To determine the spin-tensor decomposition of the two-body interaction, firstly, one has to transform two-body matrix elements to LS-coupling from jj-coupling as follows

\[
\langle n_a l_a n_b l_b : LS, JT \rangle V | n_c l_c n_d l_d : L' S', JT \rangle = \sum_{j_a j_b} \sum_{j_c j_d} N_{l_a l_b} N_{j_a j_b} \sqrt{(2L + 1)(2S + 1)(2j_a + 1)(2j_b + 1)} \left\{ \begin{array}{ccc} l_a & l_b & L \\ \frac{1}{2} & \frac{1}{2} & S \end{array} \right\} \left\{ \begin{array}{ccc} j_a & j_b & J \\ \frac{1}{2} & \frac{1}{2} & S' \end{array} \right\} \langle n_a l_a j_a n_b l_b j_b : JT \rangle V | n_c l_c j_c n_d l_d j_d : JT \rangle,
\]

where, normalization factor \( N_{l_a l_b} \) is

\[
N_{l_a l_b} = \frac{1}{\sqrt{2(1 + \delta_{l_a l_b})}}.
\]

In LS-coupling, matrix elements of each component of the two-body interaction are written as
\begin{align}
\langle n_d^k n_b^l : LS, J' T | V^{(k)} | n_c^m n_d^l : L'S', J T \rangle &= (2k + 1)(-1)^{\frac{L'}{2}} \sum_{J} (-1)^{J}(2J + 1) \\
&\times \langle L S J' | n_d^k n_b^l : LS, JT | V | n_c^m n_d^l : L'S', JT \rangle.
\end{align}

In this section, we have studied the multipole matrix elements of $T = 0$ and $T = 1$ channels for the N3LO, JISP16, DJ16A and USDB interactions. We have shown multipole terms of the total, central, vector and tensor components in Fig. 14. Total multipole term of $T = 0$ channel is more attractive for all interactions as compared to $T = 1$ channel. We can see that the central forces play a dominant role in the total interaction. The vector multipole terms of $\langle s_1/2 s_1/2 | V | s_1/2 s_1/2 \rangle$ are zero for $T = 0$ and $T = 1$ channels. The tensor multipole term of $\langle d_5/2 s_1/2 | V | d_5/2 s_1/2 \rangle$, $\langle s_1/2 s_1/2 | V | s_1/2 s_1/2 \rangle$ and $\langle s_1/2 d_3/2 | V | s_1/2 d_3/2 \rangle$ are zero corresponding to all interactions. For the central part, the multipole matrix element of $\langle s_1/2 s_1/2 | V | s_1/2 s_1/2 \rangle$ for $T = 0$ channel of DJ16A interaction is less attractive in comparison with other interactions. Both $T = 0$ and $T = 1$ vector multipole terms are relatively flat for all interactions. The $T = 0$ tensor multipole term of $\langle d_5/2 d_5/2 | V | d_5/2 d_5/2 \rangle$, $\langle d_5/2 d_3/2 | V | d_5/2 d_3/2 \rangle$ and $\langle d_3/2 d_3/2 | V | d_3/2 d_3/2 \rangle$ are stronger than $T = 1$ tensor monopole. The total, central, vector and tensor monopoles of $\langle d_3/2 d_3/2 | V | d_3/2 d_3/2 \rangle$ of $T = 0$ channel for N3LO interaction are $-2.079$, $-2.612$, $-0.116$ and $0.649$ MeV, which means tensor forces give significant contribution in total two-body interaction. The DJ16A $T = 1$ total monopole is close to those obtained with USDB interaction except for $\langle s_1/2 s_1/2 | V | s_1/2 s_1/2 \rangle$, for which, the former is attractive and latter is repulsive but with small variation.

Energies of g.s. for Si, P, S, Cl and Ar isotopes with USDB interaction are consistent with the experimental data. DJ16A interaction underbinds the g.s. of these nuclei. This underbinding is related to the central part of $T=0$ channel which is more attractive for USDB interaction than this interaction.

VI. ELECTROMAGNETIC PROPERTIES

In this section, we have presented the electromagnetic properties of Si, P, S, Cl and Ar isotopes, wherever experimental data are available. We have calculated electric quadrupole and magnetic dipole moments, reduced electric quadrupole and magnetic dipole transition strengths for $Z = 14 – 18$ isotopic chains. To find the most suitable interaction for the sd-shell, we have shown a comparison between theory and experiment. Experimental quadrupole moments are taken from [68] and magnetic moments are taken from [69]. In the calculation of quadrupole moment and B(E2), we have used effective charges as $e_p = 1.5e$ and $e_n = 0.5e$. The free spin g-factor ($g_s$) values 5.585 and -3.826, and free orbital g-factor ($g_l$) values 1.0 and 0.0 for proton and neutron, respectively, are used for computing magnetic moment and B(M1) values.

First, we discuss the electric quadrupole and magnetic dipole moments of either g.s. or (and) first excited state corresponding to other interactions show consistency with the experimental data of either quadrupole or magnetic moment or both are available, to check the predictive power of our theoretical shell model interactions. The DJ16A and USDB predict opposite sign of magnetic moment for g.s. of $27^\text{Si}$ compared to N3LO and JISP16 interactions. The sign of this magnetic moment has not been confirmed experimentally yet. Magnetic moments using all interactions for 2$^+$ state are in good agreement with the experiment in case of $28^\text{Si}$. The experimental value of $\mu$ for this state is $1.1(2)\mu_N$. With N3LO, JISP16, DJ16A and USDB interactions, this value comes out to be 1.015, 1.019, 1.025 and 1.031 $\mu_N$, respectively. Quadrupole moments for $28^\text{Si}$ are in reasonable agreement using all interactions with the experimental data. In case of $28^\text{Si}$, magnetic moment has negative sign for 1/2$^+$ state. Our microscopic interactions give the correct sign of magnetic moment for this state. For $30^\text{Si}$, quadrupole moment for 2$^+$ is -0.05(6) eb experimentally. Only DJ16A interaction reproduces correct sign of quadrupole moment for 2$^+$. The DJ16A shows small magnetic moment for 2$^+$, while results corresponding to other interactions show consistency with experimental value. We have predicted the quadrupole moment of 3/2$^+$ state with USDB, which matches with the experimental value of 1.206 $\mu_N$ for $33^\text{Si}$. DJ16A interaction provides magnetic moment of 3/2$^+$ as 1.325 $\mu_N$.

Predicted quadrupole moment for g.s. is $\approx 0.1$ eb which matches well with experiment in $28^\text{P}$ and the experimental value of magnetic moment is 0.312(3) $\mu_N$ which is closer to the USDB magnetic moment of 0.303 $\mu_N$ for the same state. The DJ16A interaction predicts magnetic moment with negative sign for this state. We have predicted the quadrupole moment for $31,32^\text{P}$. Magnetic moment of 1/2$^+$ improves from N3LO to JISP16 to DJ16A for $29,31^\text{P}$, still it is far from experiment. Magnetic moment of 3/2$^+$ for JISP16 and DJ16A agree with experiment in case of $31^\text{P}$. The first excited state in $31^\text{P}$ and g.s. of $32^\text{P}$ are found to be oblate. The DJ16A interaction gives the value of magnetic moment as -0.239
TABLE I. Calculated electric quadrupole and magnetic dipole moments of $Z = 14 - 18$ nuclei, in comparison with the experimental data [69].

| Nuclei | $A$ | $J^+$ | Q (eb) | $\mu (\mu_N)$ |
|--------|-----|-------|--------|----------------|
|        |     |       | N3LO   | JISP16 | DJ16A | USDB | Exp. | N3LO | JISP16 | DJ16A | USDB | Exp. |
| Si     | 27  | 5/2+  | 0.082  | 0.094  | 0.136 | 0.141 | 0.063(14) | 0.477 | 0.298  | -0.261 | -0.678 | (-)-0.8652(4) |
|        | 28  | 2+    | 0.245  | 0.242  | 0.231 | 0.209 | 0.16(3) | 1.015 | 1.019  | 1.025  | 1.031  | 1.1(2) |
|        | 29  | 1/2+  | -      | -      | -     | -     | -     | -0.085 | -0.120 | -0.164 | -0.503 | -0.55529(3) |
|        | 30  | 2+    | 0.191  | 0.187  | -0.100 | 0.024 | -0.05(6) | 1.141 | 1.187  | 0.392  | 0.732  | 0.8(2) |
|        | 31  | 3/2+  | 0.122  | 0.114  | 0.093 | 0.065 | NA     | 1.510 | 1.422  | 1.325  | 1.206  | 1.21(3) |
| P      | 28  | 3+    | 0.119  | 0.096  | 0.185 | 0.148 | 0.137(14) | 2.180 | 1.192  | -0.497 | 0.303  | 0.312(3) |
|        | 29  | 1/2+  | -      | -      | -     | -     | -     | 0.582 | 0.627  | 0.732  | 1.133  | 1.2346(3) |
|        | 31  | 1/2+  | -      | -      | -     | -     | -     | 0.196 | 0.395  | 0.473  | 1.087  | 1.13160(3) |
|        | 3/2+ | -0.138 | -0.131 | -0.115 | -0.091 | NA | -0.464 | 0.387 | 0.232  | 0.164  | 0.30(8) |
|        | 5/2+ | 0.189  | 0.197  | 0.193  | 0.049 | NA | 0.706 | 1.010  | 0.987  | 2.217  | 2.8(5) |
|        | 32  | 1+    | -0.050 | -0.058 | -0.047 | -0.019 | NA | -0.657 | -0.382 | -0.239 | -0.2021(3) |
| S      | 31  | 1/2+  | -      | -      | -     | -     | -     | 0.307 | 0.128  | 0.071  | 0.441  | 0.48793(8) |
|        | 32  | 2+    | 0.097  | 0.055  | -0.138 | -0.128 | -0.16(2) | 1.011 | 1.014  | 1.015  | 1.006  | 0.9(2) |
|        | 4+   | 0.043  | 0.033  | -0.051 | -0.060 | NA | -0.2026 | 2.031 | 2.041  | 2.029  | 1.6(6) |
|        | 33  | 3/2+  | -0.134 | -0.115 | -0.121 | -0.073 | -0.0678(13) | 0.248 | 0.390  | 0.542  | 0.792  | 0.6438212(14) |
|        | 34  | 2+    | 0.039  | 0.042  | -0.059 | 0.044 | 0.04(3) | 1.062 | 1.029  | 1.297  | 1.093  | 1.0(2) |
|        | 35  | 3/2+  | 0.078  | 0.074  | 0.080  | 0.057 | 0.0471(9) | 1.163 | 1.093  | 1.015  | 1.112  | 1.00(4) or -1.07(4) |
|        | 36  | 2+    | -0.106 | -0.102 | -0.103 | -0.103 | NA | 2.482 | 2.542  | 2.516  | 2.381  | 2.6(10) |
| Cl     | 32  | 1+    | -0.049 | -0.058 | -0.049 | -0.028 | NA | 1.744 | 1.414  | 1.195  | 0.878  | 1.114(6) |
|        | 33  | 3/2+  | -0.131 | -0.121 | -0.123 | -0.099 | NA | 1.284 | 1.079  | 0.930  | 0.549  | 0.7549(3) |
|        | 35  | 3/2+  | -0.105 | -0.107 | -0.108 | -0.097 | -0.0817(8) | 0.836 | 0.855  | 0.880  | 0.589  | 0.8218743(4) |
|        | 36  | 2+    | -0.019 | -0.020 | -0.068 | -0.017 | -0.178(4) | 1.553 | 1.497  | 1.167  | 1.173  | 1.28547(5) |
|        | 37  | 3/2+  | -0.089 | -0.089 | -0.089 | -0.086 | -0.0644(6) | 0.610 | 0.606  | 0.588  | 0.352  | 0.6841236(4) |
| Ar     | 33  | 1/2+  | -      | -      | -     | -     | -     | -0.160 | -0.297 | -0.309 | -0.765 | -0.723(6) |
|        | 35  | 3/2+  | -0.111 | -0.109 | -0.113 | -0.101 | -0.084(15) | 0.580 | 0.558  | 0.553  | 0.764  | 0.6322(2) |
|        | 36  | 2+    | 0.167  | 0.152  | 0.178  | 0.152 | 0.11(6) | 0.998 | 0.989  | 0.996  | 0.973  | 1.0(4) |
|        | 37  | 3/2+  | 0.097  | 0.095  | 0.103  | 0.090 | 0.076(9) | 1.472 | 1.436  | 1.446  | 1.225  | 1.145(5) |
|        | 38  | 2+$^+$ | 0.011  | 0.009  | 0.014  | 0.029 | NA | 0.626 | 0.605  | 0.588  | 0.462  | 0.5(2) |
|        | 2+$^+$ | 0.078  | 0.078  | 0.071  | 0.059 | NA | 2.525 | 2.573  | 2.585  | 2.552  | 2.2(22) |

$\mu_N$, the experimental value is $-0.2524(3) \mu_N$ for $^{32}$P.

All microscopic interactions give correct sign of $\mu$ for $^{31}$S, while the USDB fails to do so. The quadrupole moment for first excited state of $^{32}$S is $-0.16(2)$ eb. In case of $^{32}$S, DJ16A and USDB interactions give opposite sign compared to N3LO and JISP16 for quadrupole moment of $2^+$ and $4^+$ states. Negative sign of quadrupole moment with DJ16A interaction is correctly reproduced for $2^+$ in $^{32}$S while it is unable to reproduce the sign for $^{34}$S. Magnetic moments for $^{32,34}$S corresponding to all interactions are in good agreement with experiment. Experimentally, magnetic moment for g.s. in $^{35}$S could be either 1.00(4)
TABLE II. Comparison of calculated reduced electric quadrupole transition strengths using different interactions of $Z = 14 – 18$ nuclei, with the experimental data [60]. The B(E2) values are in $e^2$fm$^4$.

| Nuclei | A | $J_i^+ \rightarrow J_f^+$ | N3LO | JISP16 | DJ16A | USDB | Exp. |
|--------|---|--------------------------|------|--------|-------|------|------|
| Si     | 26 | $2_1^+ \rightarrow 0_1^+$ | 112.8| 109.4  | 85.2  | 57.4 | 70.0(68) |
|        |    | $2_2^+ \rightarrow 0_2^+$ | 8.1  | 7.1    | 14.2  | 25.4 | 7.8(18)  |
|        | 27 | $3/2_2^+ \rightarrow 5/2_2^+$ | 264.3| 239.7  | 148.5 | 91.4 | 110.7 |
|        | 28 | $2_1^+ \rightarrow 0_1^+$ | 144.9| 141.7  | 125.2 | 99.8 | 66.7(25) |
|        | 29 | $3/2_2^+ \rightarrow 1/2_2^+$ | 0.1  | 0.002  | 10.1  | 37.9 | 21.7(21) |
|        | 30 | $2_1^+ \rightarrow 0_1^+$ | 118.4| 112.4  | 37.8  | 58.6 | 47.1(61) |
|        |    | $1_1^+ \rightarrow 2_1^+$ | 0.3  | 0.01   | 11.7  | 2.8  | 8.3(61)  |
|        | 31 | $5/2_2^+ \rightarrow 3/2_2^+$ | 59.0 | 102.2  | 111.8 | 91.6 | 69(23)  |
|        | 32 | $2_1^+ \rightarrow 0_1^+$ | 96.9 | 90.5   | 74.4  | 54.3 | 26.6(78) |
|        |    | $2_2^+ \rightarrow 0_2^+$ | 0.2  | 0.7    | 1.1   | 2.2  | 1.03(36) |
|        | 34 | $2_1^+ \rightarrow 0_1^+$ | 46.9 | 46.1   | 41.6  | 43.2 | 17.0(65) |
| P      | 29 | $3/2_2^+ \rightarrow 1/2_2^+$ | 2.3  | 3.4    | 22.8  | 55.7 | 14.3(26) |
|        | 30 | $2_1^+ \rightarrow 1_1^+$ | 31.0 | 39.8   | 60.6  | 7.6  | 0.83(22) |
|        | 31 | $3/2_2^+ \rightarrow 1/2_2^+$ | 33.0 | 34.4   | 46.1  | 42.9 | 24.3(35) |
|        | 32 | $2_1^+ \rightarrow 0_1^+$ | 44.8 | 59.7   | 29.7  | 54.2 | 37.0(29) |
|        | 33 | $3/2_2^+ \rightarrow 1/2_2^+$ | 47.4 | 54.8   | 54.4  | 48.4 | 63(25)  |
|        | 34 | $5/2_2^+ \rightarrow 1/2_2^+$ | 57.5 | 58.8   | 47.1  | 38.4 | 32.1(50) |
| S      | 28 | $2_1^+ \rightarrow 0_1^+$ | 74.3 | 70.8   | 70.5  | 61.7 | 36.2(62) |
|        | 30 | $2_1^+ \rightarrow 0_1^+$ | 112.7| 103.4  | 59.0  | 72.5 | 70.0(66) |
|        | 31 | $3/2_2^+ \rightarrow 1/2_2^+$ | 45.3 | 45.7   | 57.7  | 48.3 | 40(12)  |
|        | 32 | $5/2_2^+ \rightarrow 1/2_2^+$ | 40.1 | 53.1   | 25.2  | 57.0 | 45(13)  |
|        | 33 | $2_1^+ \rightarrow 0_1^+$ | 117.3| 107.7  | 84.4  | 60.1 | NA      |
|        |    | $4_1^+ \rightarrow 2_1^+$ | 121.6| 109.0  | 121.2 | 85.5 | 84(18)  |
|        |    | $0_2^+ \rightarrow 2_2^+$ | 27.0 | 57.2   | 19.0  | 67.0 | 71.2(72) |
|        | 33 | $1/2_2^+ \rightarrow 3/2_2^+$ | 55.6 | 41.7   | 62.7  | 24.0 | 38(25)  |
|        | 34 | $5/2_2^+ \rightarrow 3/2_2^+$ | 13.1 | 5.2    | 61.2  | 61.8 | 50(19)  |
|        | 35 | $2_1^+ \rightarrow 0_1^+$ | 73.2 | 60.4   | 60.0  | 45.9 | 40.8(10) |
|        | 36 | $0_2^+ \rightarrow 2_2^+$ | 25.1 | 25.9   | 19.2  | 26.3 | 27.5(46) |
| Cl     | 33 | $5/2_2^+ \rightarrow 3/2_2^+$ | 62.1 | 57.7   | 47.7  | 39.7 | 25.8(48) |
|        | 35 | $1/2_2^+ \rightarrow 3/2_2^+$ | 24.0 | 24.9   | 24.7  | 32.6 | 48(20)  |
|        | 36 | $2_1^+ \rightarrow 0_1^+$ | 21.5 | 22.6   | 23.3  | 26.2 | 20.0(17) |
|        | 37 | $5/2_2^+ \rightarrow 3/2_2^+$ | 101.2| 89.9   | 109.0 | 82.3 | 76.2(82) |
|        |    | $7/2_2^+ \rightarrow 3/2_2^+$ | 36.8 | 33.1   | 36.7  | 24.6 | 25.2(34) |
or -1.07(4) $\mu_N$. Our results are closer to the former value. For $^{36}$S, all interactions predict similar results for $Q(2^+)$ and reproduce the experimental magnetic moment.

For $^{32,33}$Cl, we have given predictions for the quadrupole moment of g.s. using all four interactions. Results of magnetic moment are improved from N3LO to JISP16 to DJ16A interaction for $^{32}$Cl and become closer to the experimental value. The g.s. of $^{32,33,35,36,37}$Cl is found to be oblate with all interactions and agrees with experimental ones, wherever available. Shell model results of quadrupole moments are in reasonable agreement with experimental data for $^{35-37}$Ar. For $^{38}$Ar, prediction of quadrupole moments for first and second $2^+$ have been made, which are not yet experimentally measured. There is significant deviation of microscopic results from the experimental ones for magnetic moment of $1/2^+$ in $^{33}$Ar. For $^{38}$Ar, all the microscopic interactions give $\approx 0.6\, \mu_N$ for $2^+_1$ state which matches well with the experimental value of 0.5(2) $\mu_N$. From the above discussion, we can see that the USDB interaction fails sometimes in reproducing experimental data of some properties. So we are trying to better reproduce the experimental data with microscopic interactions in order to understand the actual form of the nuclear interaction.

Now, we focus on the B(E2) transition strengths for nuclei lying in the range $Z = 14 - 18$. In Table II, B(E2) transition strengths using microscopic and USDB interactions along with the experimental data for the selected transitions are reported. For $^{26}$Si, JISP16 and N3LO interactions give better result for $\text{B}(E2; 2^+_2 \rightarrow 0^+_1)$ value while other interactions predict enhanced value for this transition. The B(E2) value for $2^+_2 \rightarrow 0^+_1$ in $^{28}$Si is 66.7(25) e²fm⁴, experimentally. All the microscopic as well as empirical interactions predict large value of this transition strength. Similarly, experimental value of B(E2) transition strength for $^{32}$Si from $2^+_2$ to $0^+_1$ is 26.6(78) e²fm⁴, while theoretical interactions predict enhanced values. In case of $^{34}$Si, same pattern is seen for B(E2; $2^+_2 \rightarrow 0^+_1$). For $^{29}$Si and $^{30}$Si, transitions from $3/2^+_1$ to $1/2^+_1$ and $1^+_2$ to $2^+_1$, respectively, obtained from both JISP16 and N3LO interactions are significantly weaker than that found experimentally. Collectivity in the nuclei is measured from the B(E2) transition strength. A gradual decrease of B(E2) value for an isotopic chain shows appearance of magic number. As neutrons are increased in Si isotopes from $A = 26$ to $A = 34$, the B(E2) transition strength from $2^+_1$ to $0^+_1$ decreases in even-even nuclei, which indicates that collectivity is reduced near magic number configuration, experimentally. This trend is reproduced for the JISP16 and USDB interactions from $^{28}$Si to $^{34}$Si. The DJ16A and N3LO results deviate from the experimental results up to $^{32}$Si. For $^{33}$P, we get smaller B(E2) value than experimental data for $3/2^+_1 \rightarrow 1/2^+_1$ transition with all four interactions, while, we obtain enhanced values of B(E2) for transition from $5/2^+_3$ to $1/2^+_1$.

There is no experimental data available in the case of $^{32}$S for B(E2; $2^+_2 \rightarrow 0^+_1$). From theoretical aspect, we have given prediction for this transition using microscopic as well as empirical interactions. For sulphur isotopes, we observe that experimental B(E2) transition strength between first excited state $2^+_1$ and $0^+_1$ for even-even nuclei decreases from $^{30}$S to $^{36}$S. Reduction in the B(E2) is obtained with increasing neutrons from 16 to 20 in our calculations, which favours the appearance of $N = 20$ shell closure. For $^{36}$S, B(E2) value with all microscopic interactions are in better agreement with the experimental value, in comparison with USDB. For $^{33}$Cl, weak transition strength from $5/2^+_3$ to $3/2^+_1$ is found with JISP16 and N3LO interactions. The B(E2; $7/2^+_4 \rightarrow 3/2^+_1$) value for $^{33}$Cl using USDB (24.6 e²fm⁴) is well reproduced, corresponding experimental value is 25.2(34) e²fm⁴.

For $^{34}$Ar, B(E2) transition strength for $2^+_1 \rightarrow 0^+_1$ comes

| TABLE II. Continuation. |
|-------------------------|
| 36 | 3$^+_1 \rightarrow 2^+_1$ | 44.8 | 43.1 | 49.7 | 35.7 | 71(28) |
| 1$^+_1 \rightarrow 2^+_1$ | 58.8 | 57.2 | 57.6 | 52.6 | 3.5(13) |
| 37 | 1$^+_2 \rightarrow 3/2^+_1$ | 19.8 | 21.4 | 23.4 | 20.6 | 16.8(37) |
| 5$^+_2 \rightarrow 3/2^+_1$ | 38.9 | 40.6 | 42.9 | 44.4 | 19.0(44) |
| Ar | 32 | 2$^+_1 \rightarrow 0^+_1$ | 71.9 | 68.7 | 69.3 | 65.3 | 54(14) |
| 34 | 2$^+_1 \rightarrow 0^+_1$ | 62.5 | 58.3 | 45.7 | 56.4 | 44.5(59) |
| 2$^+_2 \rightarrow 0^+_1$ | 6.1 | 3.4 | 16.5 | 1.6 | 0.65(46) |
| 36 | 2$^+_2 \rightarrow 0^+_1$ | 71.8 | 67.2 | 75.6 | 64.8 | 57.9(35) |
| 4$^+_2 \rightarrow 2^+_1$ | 86.6 | 80.3 | 93.5 | 80.7 | 85(11) |
| 37 | 7$^+_2 \rightarrow 3/2^+_1$ | 41.0 | 36.9 | 45.9 | 38.5 | 33.7(37) |
| 38 | 2$^+_1 \rightarrow 0^+_1$ | 30.0 | 29.4 | 30.1 | 31.0 | 25.8(12) |
TABLE III. Comparison of spectroscopic factors strengths calculated from shell model with three microscopic N3LO, JISP16, DJ16A and phenomenological USDB interactions, with the experimental data. Experimental excitation energies and C$^2$S of the states of $^{24}$Si are taken from the Ref. [51]. Different $l$ and $j$ values for each state of $^{24}$Si are shown in columns III and IV.

| $E_x$ (KeV) | $J^*$ | $l$ | $j$ | N3LO | JISP16 | DJ16A | USDB | Exp. |
|------------|------|----|----|------|-------|-------|------|------|
| 0          | $0_1^+$ | 2  | 5/2 | 2.49 | 2.32  | 2.92  | 3.42 | $\leq 2.8$ |
| 1874(3)    | $2_1^+$ | 0  | 1/2 | 0.07 | 0.26  | 0.28  | 0.25 | 0.6(2) |
|            |        | 2  | 3/2 | 0.01 | 0.02  | 0.009 | 0.03 | 0.07(2) |
|            |        | 2  | 5/2 | 0.0002 | 0.04  | 0.16  | 0.18 | 0.4(1) |
| 3449(5)    | $(2_1^+)$ | 0  | 1/2 | 0.45 | 0.36  | 0.36  | 0.46 | 0.7(4) |
|            |        | 2  | 3/2 | 0.0007 | 0.00  | 0.03  | 0.00 | 0.002(1) |
|            |        | 2  | 5/2 | 0.13 | 0.16  | 0.27  | 0.16 | 0.3(2) |
| 3471(6)    | $(4_1^+)$ | 2  | 3/2 | 0.009 | 0.0001 | 0.03 | 0.02 | 0.07(4) |
|            |        | 2  | 5/2 | 0.16 | 0.08  | 0.00  | 0.001 | 0.004(3) |
| $(0_2^+)$  |        | 2  | 5/2 | 0.23 | 0.47  | 0.17  | 0.24 | 0.8(4) |

VII. SPECTROSCOPIC FACTOR FOR $^{24}$Si

In the present study, spectroscopic factor strengths for g.s. and excited states of $^{24}$Si from one proton transfer reaction $^{23}$Al($d$, $n$)$^{24}$Si are calculated within the shell model using microscopic effective N3LO, JISP16, DJ16A and empirical USDB interactions. These results are compared with the extracted experimental values that are taken from Ref. [51]. One proton is captured in $l = 0$ and 2 orbitals to populate the states of $^{24}$Si. Some states of $^{24}$Si are generated by capturing one-proton in $d_{5/2}$ orbital only and some, from the mixing of $d_{5/2}$ and (or) $s_{1/2}$ and (or) $d_{3/2}$ orbitals. In Ref. [51], experimental spectroscopic factors (C$^2$S), which are shown in last column of Table III, have been extracted using the relation:

$$C^2S^i_{\text{exp}} = \frac{C^2S^i_{\text{tho}} \times \sigma^i_{\text{tho}}}{\sum_i (C^2S^i_{\text{tho}} \times \sigma^i_{\text{tho}})} \times \sigma^i_{\text{exp}},$$ (8)

where, $C^2S^i_{\text{tho}}$ were calculated corresponding to USDB-cdpn interaction using the shell model [35] and $i$ represents individual quantum numbers. Extracted experimental C$^2$S [51] for $0_1^+$ state is $\leq 2.8$ which is in good agreement with previous experimental C$^2$S of 2.7(2), which are reported in Ref. [50]. The N3LO, JISP16 and DJ16A interactions give the C$^2$S of 2.49, 2.32 and 2.92, respectively, while the C$^2$S of 3.42 is obtained with USDB which is quite far from the experimental value. The C$^2$S for g.s. of $^{24}$Si is very large which shows single-particle nature of this state. Experimental excited states of $^{24}$Si are tentative. For first 2+, experimentally extracted C$^2$S is much smaller for $d_{3/2}$ transfer than $d_{5/2}$ and $s_{1/2}$. Except for N3LO interaction, all interactions predict smallest value for $d_{3/2}$ transfer.

In the recent work [70], spin-parity assignment has been made for excited states of $^{24}$Si. The 3439 keV state of $^{24}$Si, which is important in astrophysical scenario, was confirmed to be $2_2^+$. Also, the state at 3471 keV of $^{24}$Si is confirmed as $0_2^+$ instead of $1_1^+$. 

VIII. BETA DECAY STUDY OF $^{32}$Ar

In this section, we have reported beta decay properties of $^{32}$Ar. We have studied Gamow-Teller transitions of g.s. $0^+$ of $^{32}$Ar into g.s. and excited states of $^{32}$Cl. Experimental data of B(GT) are taken from Ref. [52]. In the present work, we have used quenching factor of
0.6, which is deduced in [52]. In Fig. 15, we have shown the summed B(GT) for the $^{32}\text{Ar}$ $\beta$ decay with excitation energy up to 6 MeV for $^{32}\text{Cl}$. Experimentally, two strong transitions occur at 1.168 and 4.082 MeV. USDB interaction gives strong transitions at 1.119 and 4.132 MeV, while the DJ16A, JISP16 and N3LO interactions predict strongest transition at 3.681, 3.782 and 3.564 MeV, respectively. Experimental B(GT) value is 0.0020(4) for the weak state at 2.208 MeV. Near this energy, the B(GT) value is 0.0463 at 1.981 MeV with USDB interaction. Microscopic DJ16A, JISP16 and N3LO interactions give B(GT) strength of 0.0050, 0.0247 and 0.0327 at 2.031, 2.003 and 2.088 MeV, respectively, which are closer to the experimental value as compared to the empirical result. Overall, DJ16A results are near to the experimental data. The B(GT) values with JISP16 and N3LO interaction are smaller before the strong transition.

![FIG. 15](image)

FIG. 15. The summed Gamow-Teller transition strength up to excitation energy 6 MeV for beta decay of $^{32}\text{Ar}$. Experimental data is taken from [52].

To extract more information about the nuclear structure of $^{32}\text{Cl}$ and determine the contribution from different orbitals in total M(GT) strength, we have calculated the M(GT) strengths for the transition $^{32}\text{Ar}(0^+ \rightarrow 2^+ \text{Cl}(1^+_2))$ corresponding to different orbitals as shown in Fig. 16.

The excitation energy of $1^+_2$ is 1.168 MeV, experimentally. The N3LO, JISP16 and DJ16A interactions predict $1^+_2$ at 1.341, 1.280, and 1.086 MeV, respectively. The $1^+_2$ state is predicted to be at 1.119 MeV for USDB interaction. The notation $d_{3/2} \rightarrow d_{3/2}$ indicates that a proton of $d_{3/2}$ orbital in $^{32}\text{Ar}$ converts into a neutron in $d_{3/2}$ orbital of $^{32}\text{Cl}$. From figure, it can be seen that contribution from $s_{1/2} \rightarrow s_{1/2}$ in total M(GT) strength is maximum. Also, transitions $d_{3/2} \rightarrow d_{3/2}$, $d_{5/2} \rightarrow d_{5/2}$ and $s_{1/2} \rightarrow s_{1/2}$ are in same phase for USDB interaction. The contribution of $d_{5/2} \rightarrow d_{5/2}$ and $s_{1/2} \rightarrow s_{1/2}$ transitions in total M(GT) strength are in phase for the DJ16A interaction. Similar trend is obtained for N3LO interaction. The contribution of different orbitals in transitions $d_{5/2} \rightarrow d_{5/2}$ and $s_{1/2} \rightarrow s_{1/2}$ are in opposite phases for JISP16 interaction as compared to DJ16A interaction.

![FIG. 16](image)

FIG. 16. The M(GT) matrix elements contribution from different orbitals corresponding to N3LO, JISP16, DJ16A and USDB interactions for $^{32}\text{Ar}(0^+ \rightarrow 2^+ \text{Cl}(1^+_2))$ transition.

| Interaction       | $|\langle M(F) \rangle|$ |
|------------------|--------------------------|
| $d_{5/2} \rightarrow d_{5/2}$ | 0.664 0.286 1.050 |
| $s_{1/2} \rightarrow s_{1/2}$ | 0.589 0.361 1.050 |
| $d_{3/2} \rightarrow d_{3/2}$ | 0.397 0.499 1.104 |
| USDB             | 0.247 0.590 1.163 |

TABLE IV. Contribution from different orbitals in total M(F) matrix element obtained from N3LO, JISP16, DJ16A and USDB interactions for super-allowed $^{32}\text{Ar}(0^+; T = 2) \rightarrow 2^+ \text{Cl}(0^+; T = 2)$ transition. Here, $|\langle \sum M(F) \rangle| = 2$

In addition to Gamow-Teller transition strengths, the Fermi strength is determined for isobaric analogue state (IAS) state in $^{32}\text{Cl}$. For this super-allowed $0^+; T = 2 \rightarrow 0^+; T = 2$ transition, experimental B(F) of 3.99(93) is measured at 5.046 MeV with log $ft = 3.19$. Theoretical shell model calculations with microscopic as well as empirical interactions provide Fermi strength B(F) = 4 with log $ft = 3.186$, which is in good agreement with the experimental value. In Table IV, we have shown M(F) matrix elements contribution corresponding to different orbitals calculated with microscopic and USDB interactions. By
analysing M(F) matrix elements obtained in our calculations, we can conclude that this Fermi strength is dominated by the contribution coming from $d_{3/2} \rightarrow d_{3/2}$ transition.

IX. CONCLUSION

In the present work, we have implemented the nuclear shell model with microscopic effective interactions to study $sd$-shell nuclei in the range $Z = 14 – 18$. These microscopic NN interactions, which are obtained from the NCSM wave functions and OLS transformation method, are chiral N3LO, JISP16 and DJ16A. These microscopic results are compared with phenomenological results using USDB interaction to check the predictive power of these interactions. We have studied the energy spectra for Si, P, S, Cl and Ar isotopes lying in $sd$-shell. We found that monopole modified DJ16 interaction (DJ16A) for Si, P, S, Cl and Ar isotopes calculated by using USDB interaction to check the predictive power of these interactions. We have studied the energy spectra for Si, P, S, Cl and Ar isotopes lying in $sd$-shell. We found that monopole modified DJ16 interaction (DJ16A) results for energy spectra are better than the other microscopic interactions. Still, some corrections are needed in DJ16A to reproduce the experimental data. Our calculated results will be helpful to further investigate the modifications that should be incorporated in this microscopic interaction.

Variation of proton effective single-particle energies

for Si isotopes are presented using all the interactions to determine the shell closure. To understand the contribution from different components of the two-body matrix elements in the shell evolutions, we have also presented the spin-tensor decomposition of the effective interactions. Apart from low-lying energy spectra, electromagnetic observables for these isotopic chains have also been calculated for complete description of the nuclei. Spectroscopic factor strengths for $^{24}$Si are calculated, which is of importance in astrophysical scenario. It is reported that the $^{24}$Si of g.s. with microscopic interactions is better reproduced than that obtained from USDB interaction. Also, beta decay properties for recently available experimental data of $^{32}$Ar have been studied. Contributions from different $sd$-shell orbitals in M(F) and M(GT) strengths are also reported.

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