Luminosity Function of Faint Sporadic Meteors measured with a Wide-Field CMOS mosaic camera Tomo-e PM

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Abstract

Imaging observations of faint meteors were carried out on April 11 and 14, 2016 with a wide-field CMOS mosaic camera, Tomo-e PM, mounted on the 105-cm Schmidt telescope at Kiso Observatory, the University of Tokyo. Tomo-e PM, which is a prototype model of Tomo-e Gozen, can monitor a sky of 3 × 10^4 deg^2 at 2 Hz. The numbers of detected meteors are 1514 and 706 on April 11 and 14, respectively. The detected meteors are attributed to sporadic meteors. Their absolute magnitudes range from +4 to +10 mag in the V-band, corresponding to about 8.3 × 10^{-2} to 3.3 × 10^{-3} g in mass. The present magnitude distributions we obtained are well explained by a single power-law luminosity function with a slope parameter r = 3.1 ± 0.4 and a meteor rate log10 N0 = −5.5 ± 0.5. The results demonstrate a high performance of telescopic observations with a wide-field video camera to constrain the luminosity function of faint meteors. The performance of Tomo-e Gozen is about two times higher than that of Tomo-e PM. A survey with Tomo-e Gozen will provide a more robust measurement of the luminosity function.

Keywords: meteors, meteoroids, interplanetary medium

1. Introduction

The solar system is composed of a variety of objects with different sizes: the Sun, the planets and their moons, other small bodies including dwarf planets, asteroids and comets, and dust grains in interplanetary space. The size of the small bodies in the solar system covers a wide dynamic range from 1,000 km (e.g., Ceres) down to about 10 nm (β-asteroids). The size distribution of the small bodies provides important information to understand the origin and the evolution of the solar system. The amount of the interplanetary materials incoming the Earth is about 10^9 kg day^{-1}. The particles in the mass range of 10^{-9}–10^{-2} g are the major contributors of the incoming mass flux [13]. Properties of such small particles are essential to understand what kind of materials fall into the Earth. Particles smaller than about 10^{-6} g have been detected in place with dust counters on satellites and spacecrafts [11][12][14][15], while the number density of the large particles is so small that their size distribution is difficult to measure by in-situ observations. The small particles incoming the Earth are observed as meteors, interacting with the Earth’s atmosphere and convert part of their kinetic energy into emission lines [11][17][16][28][37]. Thus, the
mass of the incoming particle can be derived from measuring the brightness of the meteor with an assumption of an energy conversion efficiency. The Earth works as an extremely wide-area dust detector, providing an indirect measurement of the size distribution of interplanetary dust around the Earth. Meteors in the range between −5 and +4 mag. have been detected by large meteor survey networks, for example, Cameras for All-sky Meteor Surveillance (CAMS), and SonotaCo network [25]. CAMS obtained more than 135,000 meteor orbits in 2017. In this magnitude range, about 36% of meteors belong to some meteor showers [26]. The luminosity function of meteors in this magnitude range was intensively investigated by Hawkins and Upton [20] based on a large survey with Super-Schmidt cameras. They showed that the meteor luminosity function was well-approximated by a power-law function. The meteors detected by these surveys, however, are mostly caused by particles of larger than ~1 g. Observations of meteors fainter than +5 mag are required to investigate the size distribution of interplanetary dust. Cook et al. [9] obtained more than 2,000 sporadic meteors in four nights using a 10-m optical reflector and a photo multiplier. The photographic magnitudes of the detected meteors ranged between +7 and +12 mag. Comparing with literature, they concluded that the meteor luminosity function was well approximated by a power-law function between −2.4 to +12.0 mag. They observed meteors as light pulses since they used the photo multiplier instead of a camera. Thus, the information on the meteor trajectories was not obtained and the atmospheric correction to compensate a possibly unstable weather condition was not applied in real time. Clifton [8] and Hawkes and Jones [17] observed faint meteors down to about 8 and 6 mag, respectively, in television observations. Imaging observations of faint meteors using a wide-field video camera are favored to examine the meteor luminosity function in detail. Radar observations detect meteors caused by particles in the mass range of about $10^{-6}$–$10^{-2}$ g. The Canadian Meteor Orbit Radar has detected more than 3,000,000 meteors down to $10^{-4}$ g in seven years [4, 5]. Large aperture radars can observe meteor head echoes. Interplanetary dust grains of about $10^{-6}$ g can be detected by these radars. Kero et al. [29] obtained more than 100,000 meteors using the Shigaraki Middle and Upper atmosphere radar in Japan. Both the trajectories and the radar cross-sections of meteors were obtained simultaneously. This is a great advantage of radar head echo observations. On the other hand, the radar cross-sections are not easy to convert to the brightness of the meteors, or the mass of the meteoroids. Still, there is a need for optical observations of faint meteors. The interplanetary dust grains in the mass range of $10^{-6}$–$10^{-3}$ g correspond to about +7–14 mag in optical observation. To obtain images of such faint meteors, we need a large photon-collecting mirror, a wide-field optics, and a high-sensitive video camera [31, 36]. There are only a handful of studies on meteors detected with large telescopes. Pawlowski and Hebert [35] detected 151 Leonid meteors down to about 13 mag with the Liquid Mirror Telescope in Cloudcroft Observatory, to constrain the mass distribution index of the Leonid shower. Iye et al. [22] serendipitously detected sporadic meteors with Subaru/SuprimeCam and constrained the diameter of the collisionally excited tubes caused by the meteors. Bektešević and Vinković [21] developed a new method to discover faint meteors obtained in large optical surveys, such as the Sloan Digital Sky Survey. Currently, no facility conducts regular observations of faint meteors with a telescope larger than 1 m. We carried out observations of faint meteors with a wide-field mosaic CMOS camera Tomo-e PM in 2016. This paper summarizes the results of the observations. This paper is organized as follows: details of observations are summarized in Section 2; data reduction and observed meteors are presented in Section 3; the luminosity function of faint meteors is discussed in Section 4; we summarize the paper in Section 5.

2. Observations

2.1. Tomo-e PM

Tomo-e Gozen is a wide-field camera being developed in Kiso observatory, the University of Tokyo, which will be the world largest video camera for astronomy. Tomo-e Gozen, mounted on the 105-cm Kiso Schmidt Telescope [4] in Kiso observatory, will continuously monitor a 20 deg² area at 2 Hz with 84 CMOS sensors developed by Canon Inc. The designed pixel scale is about 1.19”×1.19”. The limiting magnitude is estimated to be about 18.5 mag. in the V-band observation. The limiting magnitude for meteors will be about 13 mag. in the V-band, on the assumption that meteors moves at $10^5$ s⁻¹, which corresponds to the angular velocity of a meteor at zenith, at 100 km above an observer, at a velocity of 36 km s⁻¹, and at an incident angle of 30 deg. As a pilot project, we developed a prototype of Tomo-e Gozen (hereafter, Tomo-e PM), equipped with the 8 CMOS sensors [38]. Tomo-e PM is mounted on the 105 cm Schmidt telescope at Kiso Observatory, the University of Tokyo. The pixel scale is about 1.19”×1.19”. The detectors are aligned in a line along the direction of the right ascension with some gaps (see, Sako et al. [38]). The total area of the field-of-view is about 1.98 deg². All the detectors are synchronously operated by the same control signal. The maximum frame rate is 2 Hz. The overhead time due to readout is almost negligible thanks to rolling shutter. A NTP synchronized time is recorded as a time-stamp in a FITS header, although the time-stamp is not synchronized with a shutter timing. The time-stamp is as accurate as ~1 s. The size of the imaging area is 2000×1128 pixels. The experimental observations were successfully completed, to confirm that Tomo-e PM achieved the designed sensitivity.

2.2. Observations

Meteor observations with Tomo-e PM were carried out on April 11 and 14, 2016 (UT). Details of the observations are summarized in Table 1. The sky was dark and clear on April 11, while part of the observations were affected by clouds on April 14. Meteors are detected as a streak in the 2 Hz observation, while artificial satellites and space debris are also detected.

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1 Kiso Observatory is located in 35°47′38.7″ N and 137°37′42.2″ E, and at the altitude of 1130 m.
as streaks. To eliminate these contaminations, the observations were conducted in UT 12–18, ±3 hours around midnight in JST, and the observed regions were set within the Earth’s shadow. Since the shadow moves during a night, the observed regions were relocated in every two hours. The captured movie data were compiled into FITS data cubes in every 3 minutes (hereafter, an observation unit). Thus, each FITS data cube was composed of 360 frames. In total, 808 observation units (290,880 frames) were obtained on April 11, while 580 observation units (316,800 frames) were obtained on April 14.

3. Results

3.1. Data Reduction

Streak-like signals were extracted from the movie data cube with the slant-and-compress algorithm developed by Ohsawa et al. [34]. First, background emission was subtracted and stellar signals were masked. A frame was slanted by a certain angle and compressed into a one-dimensional array with the normalization such that the noise level should be uniform. A set of one-dimensional arrays were stacked into a two-dimensional array. The streak-like signals in the original image were converted as compact sources in the converted array. The signal-to-noise ratio of faint meteors in the original image was enhanced by about $\sqrt{n}$, where $n$ was the number of pixels along the streak, by integrating signals along with the streak. Refer to Ohsawa et al. [34] for details of the detection algorithm. A number of meteor candidates were detected by the algorithm. Then, genuine meteor events were counted by humans. The numbers of the real meteor events were 2002 and 926 on April 11 and 14, respectively. Although no prominent meteor shower activity was predicted at that time, there could be some contributions on April 14 was simply attributed to the poor weather conditions in the observation unit. The small number of the detections on April 14 was poorer than that on April 11, the meteors as faint as about 12 mag in the V-band were detected on April 11. Although the weather condition on April 14 was poorer than that on April 11, the meteors as faint as about 11.0 mag were successfully detected.

3.2. System Efficiency

The meteor-collecting power of an observing system is usually evaluated in units of area. Koschack and Rendtel [30] evaluated the effective meteor-collecting power from the area of the field-of-view projected onto a meteor level at an altitude of 100 km weighted by distance. In observation with naked eyes (visual observation), a field-of-view was as large as about 52.5°. The estimated effective meteor-collecting area (hereafter, EMCA) in visual observation is 24,400 km² when observing at zenith, whereas that area becomes 20,770 km² when observing with an elevation angle of 50°, in the case that a meteor population index $r$ is 3.5. In Koschack’s calculation, the region where meteors are observable is approximated by a thin spherical shell. Hereafter, Koschack’s formalism is referred to as the thin shell approximation. In meteor observations with telescopes, the field-of-view is usually smaller than 10°. In such cases, the thin shell approximation is not valid. Kresák and Kresáková [21] evaluated the EMCA by statistically estimating the averaged angular length of meteors. This method requires a large number of meteoroids. Since the efficiency in meteor detection with Tomo-e PM changes with elevation and Tomo-e PM has the multiple image sensors, their method is not applicable to our observations. Thus, we evaluated the EMCA of Tomo-e PM by a Monte Carlo simulation. Figure 2 schematically describes the configuration of calculation. When dust grains penetrate into the upper atmosphere, they are heated by interaction and become bright in optical wavelengths at a certain altitude, observed as meteors. Then, when the grain penetrates into a lower atmospheric layer, they cease to be bright. Here, we define an upper and lower limit of an altitude where meteors are observable in optical wavelengths by $H_2$ and $H_1$, respectively. Thus, the region where meteors are observable in optical wavelengths is approximated by a spherical shell with a depth of $H_2-H_1$ (hereafter, meteor shell). In Figure 2, the meteor shell is indicated by the blue shaded region. The region captured by ADU pix$^{-1}$, was measured by fitting the projected profile with a combination of Gaussian profiles. Since the angular velocity of the meteors were not measured by Tomo-e PM observations, the line-intensity $I$ was not directly compared with the intensities of nearby stars. Thus, we adopted the video-rate magnitude defined in Iye et al. [22]. Assuming that all the meteors moved at an angular velocity of $10^5$ s$^{-1}$, the travel distance of the meteors in 0.5 s was estimated to be 1.5x10^3 pixels, where $\theta$ was the pixel scale of Tomo-e PM in units of arcsecond. The total amount of the signals emitted by the meteor in 0.5 s was obtained by $I = 1.5x10^3/\text{ADU}$, which was transformed into the V-band magnitude by comparing with the intensities of nearby stars. The ranges of the estimated magnitudes are listed in Table 1. The meteor events are faint as about 12.5 mag in the V-band were detected on April 11. Although the weather condition on April 14 was poorer than that on April 11, the meteors as faint as about 11.0 mag were successfully detected.

$^2$The number of meteors brighter than a magnitude $m$ in visible wavelengths should follow $N(<m) \propto r^m$. 

The meteor events were counted by humans. The numbers of the real meteor events were 2002 and 926 on April 11 and 14, respectively.

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The meteor events were counted by humans. The numbers of the real meteor events were 2002 and 926 on April 11 and 14, respectively.
Figure 1: Time variation in the number of meteors and the magnitude zero-point. The periods of cloudy weather are indicated by the green hatched regions. The gray cross-hatched regions mean no data due to pointings or troubles.
indicates the meteor detectable layer. The blue cross-hatched region indicates the meteor detectable volume. The thick blue region is the test region, where meteors are randomly generated. A generated meteor with the incident angle $\theta$ is indicated by the red arrow.

a camera with a single rectangular field-of-view is defined by

$$\vec{x} \in \{ c_0 \vec{n}_0 + c_1 \vec{n}_1 + c_2 \vec{n}_2 + c_3 \vec{n}_3 \mid c_m > 0 \text{ for } m = 0, 1, 2, 3 \},$$

where $\vec{n}_m(m = 0, 1, 2, 3)$ is a unit vector to point a corner of the field-of-view. The intersection of the meteor shell and the field-of-view is defined as a meteor-collecting volume (MCV). The MCV is approximated by a truncated pyramid as shown by the cross-hatched region in Figure 2. This approximation does not affect results significantly as long as the dimension of the field-of-view is smaller than about 1°. The distance to the MCV is represented by the distance to its geometric center. For a system with multiple image sensors such as Tomo-e PM, the union of the MCVs for each sensor is defined as the MCV of the system. An incident meteor is defined by two properties: an incident point $\vec{p}$ and a pair of incident angles ($\omega, i$). The incident point $\vec{p}$ means the location of a dust grain when they enter the meteor shell. The incident angles correspond to the velocity of a grain relative to the Earth. The angle $\omega$ is an azimuthal angle, while the angle $i$ is an incident angle measured from the normal vector of the meteor shell at $\vec{p}$. Incident meteors are illustrated by the orange arrows in Figure 2. The evolution of the velocity by interaction with atmosphere is not taken into account. Thus, we assume that the direction of a meteor’s velocity is constant. Incident meteors are randomly generated on the test region, which is a sufficiently large part of the meteor shell, with a surface density of $\Sigma$ at an altitude of $H_2$. An effective meteor-collecting area (EMCA) at a nominal altitude of 100 km is defined by

$$A = \frac{N}{\Sigma} \left( \frac{100 \text{ km}}{H_2} \right)^2 = N \frac{A_0}{N_0} \left( \frac{100 \text{ km}}{H_2} \right)^2,$$

where $N$ is the number of the meteors which enter the MCV, $A_0$ is the total area where the meteors are generated, $N_0$ is the total number of the generated meteor. We developed a code to calculate an EMCA for a given observing configuration. The shape, the elevation angle $\alpha$, and the position angle $\phi$ of pointing are given. Incident meteors are generated until $N$ exceeds a threshold $\hat{N}$. Since the detections are given by a Poisson process, the statistical error of $A$ becomes negligible for sufficiently large $\hat{N}_0$. When the field-of-view is as large as $\sim 50'$ in diameter, the depth of the meteor shell becomes negligible. Thus the EMCA calculated here becomes asymptotically identical to that in the thin shell approximation. As the simplest case, we assumed that the number density of interplanetary dust around the Earth was uniform and the velocity of the interplanetary dust grains relative to the Earth was isotropic. Thus, the distribution of $\vec{p}$ was uniform on the top surface of the meteor shell, the distribution of $\omega$ distributed uniformly in $[0, 2\pi)$, and the distribution of $i$ was set to be proportional to $\cos i$. In calculation, we adopted $H_1 = 80 \text{ km}$ and $H_2 = 120 \text{ km}$. The EMCA does not significantly depend on the choice of $H_1$ and $H_2$ (see Appendix A.3). The threshold $\hat{N}$ was set to 100,000 so that the statistical errors become negligible. Note that the effective meteor-collecting area calculated based on our assumption may con-

| Date         | 2016-04-11 | 2016-04-14 |
|-------------|------------|------------|
| Weather Condition | Clear Sky | Partly Clouded |
| Lunar Age   | 3.6        | 6.6        |
| Elevation Range | 13.7°–37.9° | 10.1°–37.1° |
| Typical Zero Magnitude | 25.5 mag | 25.0 mag |
| Total Observing Time | ~5.1 hours | ~5.5 hours |
| Number of Exposures | 290,880 | 316,800 |
| Number of Detected Meteors | 1,514 | 706 |
| Apparent Magnitudes$^\dagger$ | 4.5–12.5 mag | 5.5–11.0 mag |

$^\dagger$ Video rate magnitudes defined in Iye et al. [22].
tain a systematic error since the radiant distribution of sporadic meteor is not uniform [27], and may be incorrect especially for meteor showers. The EMCAs of Tomo-e PM estimated with a Monte-Carlo simulation are shown in Figure 5. Random errors are as small as the symbols. Figure 6 indicates that the meteor-collecting area of Tomo-e PM changed from about 1,000 km to 10,000 km with elevation. Note that the EMCAs calculated here do not include the effect of the distance between the observer and meteors. When the elevation angle is low, a typical distance to meteors becomes large and faint meteors are not detected. Thus, the number of the detected meteors tends to decrease with decreasing elevation angle. Figure 7 shows that the number of the detected meteors peaked around 15:00 UT on April 11. Detailed discussion is presented in Appendix A.4. The meteor travel distances and a possible bias in the meteor arrival directions are discussed in Appendix A.2, respectively.

4. Discussion

4.1. Absolute Magnitude Distribution

The observed video-rate magnitudes \( m_v \) were converted into absolute magnitudes \( M_V \), which correspond to V-band magnitudes at the distance of 100 km, by assuming that the distances to meteors were identical to those to the meteor detectable volume:

\[
M_V = m_V + 5.1 \log_{10} \frac{D(a)}{100 \text{ km}},
\]

Figure 4 shows the distributions of the absolute magnitudes. Meteors of \( M_V \leq 10 \) and 9 mag were detected on April 11 and 14, respectively. The histograms increase exponentially with increasing magnitudes, and then start to decrease at magnitudes of 8th or 9th. The decrease is basically attributed to sensitivity limits. Detailed discussion is given in Section 4.2. Jacchia [23] derived an equation to convert the magnitude of a meteor to the mass of a meteoroid, which were calibrated based on observations with Super-Schmidt cameras. A modified equation was given by Jenniskens [24], which derives the mass of a meteoroid from the V-band magnitude of a meteor:

\[
\log_{10} M_g = 6.31 - 0.40M_V - 3.92 \log_{10} V_\infty - 0.41 \log_{10} \sin(h_v),
\]

where \( M_g \) is the mass of the meteoroid in units of grams, \( M_V \) is the V-band magnitude of the meteor, \( V_\infty \) is the incident velocity of the meteoroid in units of km s\(^{-1}\), and \( h_v \) is the elevation of the radiant point of the meteor. By assuming that \( V_\infty \) is 30 km s\(^{-1}\) and \( h_v \) is 90°, the mass of the meteors we detected ranges from about 8.3 x 10\(^{-2}\) g (4 mag) to about 3.3 x 10\(^{-4}\) g (10 mag).

4.2. Luminosity Function Analysis

Generally, a luminosity function of visible meteors is well approximated by an exponential function [20]:

\[
\log_{10} N(<M) = \log_{10} N_0 + M \log_{10} r,
\]

where \( N(<M) \) and \( N_0 \) are the event rates of meteors brighter than \( M \)-th and zero-th magnitude, respectively. The present results are a composite of the observations in different conditions in terms of the meteor-collecting area and the limiting magnitudes. Thus, a naive fitting of the apparent magnitude distribution to derive luminosity functions may bring biased results. We introduce a statistical model to robustly estimate the slope parameter \( r \) and the meteor rate \( \log_{10} N_0 \). We assume that the luminosity function of meteors exactly follows Equation (5) and observational conditions are constant within the observation unit. The number of detectable meteors per observation unit is expected by

\[
\begin{align*}
\log_{10} N_0 & = \log_{10} N_0 + \eta(T' - 15:00\text{UTC}) , \\
\log_{10} N_i & = \log_{10} N_0 + \eta(T' - 15:00\text{UTC}) .
\end{align*}
\]

where \( \beta \) is a shape parameter of the exponential distribution, given by \( \beta = \log r \). The parameters are optimized to match \( n' \sim n \) and \( M' \sim M \). The optimization is carried out with Stan [6], which is a software for the Bayesian statistical inference with MCMC sampling [6, 21]. The posterior probability distributions are shown in Figure 5. The posterior mean values and the 95% confidence intervals of the parameters are listed in Table 2. No significant differences are detected between the results on April 11 and 14. The results suggest that the slope parameter is \( \sim 3.1 \pm 0.4 \) and the meteor rate is about \( \sim 5.5 \pm 0.5 \). The data on April 11 marginally suggest a positive \( \eta \) value, while the increase in the meteor rate is not confirmed on April 14. The present result, \( \eta \) of \( \sim 1 \times 10^{-2} \) dex hours\(^{-1}\), corresponds the increase by 30% in 12 hours. The increase was much smaller than reported in previous studies [29, 33]. This could be in part attributable to our assumption in calculating the EMCA: the radiant distribution of meteors is uniform. Further observations are required to confirm the non-detection of the diurnal variation with Tomo-e PM. We compare the present result with luminosity functions in literature in Figure 6. The purple solid line indicates the luminosity function derived by Hawkins and Upton [20] based on surveys with Super-Schmidt cameras. Hawkins and Upton [20] suggested that the \( r = 3.4 \pm 0.2 \) and \( \log_{10} N_0 = -5.2 \) using meteors brighter than about 4 mag. The green dashed line indicates the luminosity function provided by Cook et al. [9] using a 10-m reflector and phototubes. The slope parameter they derived was about 3 \( \pm 1 \). We assume that the luminosity function re-calibrated by Cook et al. [9], we use the luminosity function re-calibrated by Cook et al. [9]. Hawkes and Jones [17] suggested that \( r \sim 2.5 \). The red triple-dotted-dashed line and orange double-dotted-dashed line respectively show the luminosity functions in Clifton [8] and Hawkes and Jones [15]. They observed meteors using television systems. Since the brightness of meteors were not directly measured in Clifton [8], we use the luminosity function re-calibrated by Cook et al. [9]. Hawkes and Jones [17] suggested that \( r \sim 3.4 \). The red triple-dotted-dashed
Figure 3: Time variation in the meteor-collecting areas on April 11 (the red circles) and April 14 (the blue triangles).

Table 2: Statistical Inference with the Simple Model

| Parameter | 2016-04-11 Mean 2.5% 50% 97.5% | 2016-04-14 Mean 2.5% 50% 97.5% |
|-----------|---------------------------------|---------------------------------|
| $r$       | 3.24 3.01 3.23 3.50             | 3.08 2.74 3.06 3.50             |
| $\log_{10} N_0$ | $-5.59 -5.91 -5.59 -5.30$ | $-5.34 -5.82 -5.33 -4.93$ |
| $10^2\eta$ | 1.16 $-0.30$ 1.16 2.62          | 0.50 $-1.92$ 0.50 2.91          |
Figure 4: Distributions of the absolute magnitudes of the meteors detected by Tomo-e PM. The top and bottom panels respectively shows the distributions on April 11 and 14, 2016.
Figure 5: Probability density functions of the fitting parameters, $r$, $\log_{10} N_0$, and $\eta$, are shown in the top, middle, and bottom panels, respectively. The vertical lines with numbers indicate the posterior mean values.
line shows the luminosity function of fireballs [19], suggesting that \( r \approx 2.5 \). Here, we tentatively assume that the conversion factor from photographic to visual magnitude is +1.0 mag. The present result is shown by the blue region. The height of the region represents the uncertainties of the parameters. The present observations constrain the luminosity function between about +4 to +10 mag, which is the deepest among the imaging observations. The slope parameter in the present result is roughly consistent with the other observations except for Hawkins [19]. The meteor rate \( N_0 \) is by a factor of \( \sim 2.5 \) lower than the value reported in Hawkins and Upton [20]. This can be attributed to a seasonal variation in the sporadic meteor rate; The number of sporadic meteors in March and April is about a half of the annual average rate [18, 29, 33]. The meteor rate of the present result is by a factor of \( \sim 30 \) lower than that of Cook et al. [9]. This difference could be in part attributable to the difference in the assumed EMCAs. Cook et al. [9] assumed that the EMCA was 3 km\(^2\), which was basically derived with the thin shell approximation and could be underestimated. Generally, the present result is consistent with the luminosity functions in literature. Cook et al. [9] suggested that the luminosity function of meteors was well approximated by a single slope power-law function from \( -2.4 \) to \( +12 \) mag. The present result is in line with the Cook’s suggestion.

4.3. System efficiency comparison between Tomo-e PM and Tomo-e Gozen

The operation of Tomo-e PM was completed in 2016. Observations with Tomo-e Gozen started in February, 2018. Here, we compare the EMCAs between Tomo-e PM and Tomo-e Gozen, to evaluate the efficiency in the meteor observations with Tomo-e Gozen. The EMCAs are calculated out for three systems. The shape of their fields-of-view are illustrated in Figure 7. System 1 has a single field-of-view with dimensions of 39°.7 × 22°.4, corresponding to that of one CMOS sensor. System 2 emulates Tomo-e PM, a wide-field camera equipped with 8 CMOS image sensors mounted on 105 cm Kiso Schmidt telescope [38]. System 2 has eight image sensors, each of which has a field-of-view with dimensions of 39°.7 × 22°.4. System 3 is a counterpart of Tomo-e Gozen. The field-of-view of System 3 is composed of 84 segments. The size of each segment is the same in Systems 1 and 2. The MCVs with an elevation angle of 35°, 60°, and 85° are illustrated in Figure 8. The position angles of the field-of-view are fixed. Both the size of the MCV and the distance to the MCV increase with decreasing elevation angle. The EMCAs for the three systems are calculated for elevation angles from 10° to 90° at 5° intervals. Figure 9 shows the dependence of the EMCA on the elevation angle \( \alpha \). The EMCAs at zenith are about 43, 281, and 630 km\(^2\) for System 1, 2, and 3, respectively. At \( \alpha = 10° \), the EMCAs increase to about 1.1 × 10\(^3\), 7.8 × 10\(^3\), and 25.8 × 10\(^3\) km\(^2\) for System 1, 2, and 3, respectively. The EMCA for System 1, 2, and 3 similarly increase with decreasing elevation angle. The dependence of the EMCA on \( \alpha \) is well approximated by sin\(^{-2} \)\( \alpha \). Note that the EMCA, which do not include the effect of the distance to meteors, do not reflect the expected number of meteors detected. Refer to Appendix A.4 for detailed discussion. The EMCA for Systems 2 is about 7 times larger than that of System 1. This increase is almost proportional to the number of the image sensors. The efficiency of System 3 is about two times larger than that of System 2. Since the detectors of Tomo-e Gozen are tiled in the image circle of the telescope, a large fraction of meteors will be detected first in peripheral detectors; the detectors around the center of the field-of-view have little contribution to the meteor-collecting efficiency. To investigate the dependence of the EMCA on the field-of-view, we calculate the EMCA for simple systems which have a single square field-of-view pointing at the zenith. The side lengths of the fields-of-view are 10°, 20°, 30°, 40°, 50°, and 60°. Calculated EMCAs are listed in Table 3 as well as the areas of the fields-of-view projected on the surface at an altitude of 100 km above the sea level (hereafter, referred to as projected fields-of-view). The EMCA is larger than the area of the projected field-of-view. In the case of the 60° × 60° system, the EMCA is larger by a factor of about 30. This simply illustrates the thin shell approximation is not appropriate for systems with small fields-of-view. The EMCA is almost proportional to the side lengths of the fields-of-view, rather than the areas of the fields-of-view. The meteor-collecting efficiency, in general, depends on the surface area of the MCV. For a narrow field-of-view, the surface area of the MCV is approximately proportional to the side length of the field-of-view. This simply explains the dependence of the EMCA on the side length; The EMCA of narrower field-of-view systems does not decrease as with the area of the field-of-view. This is generally consistent with the fact that the EMCA of System 3 is about two times larger than that of System 2; The summation of the most outer circumference sides of System 2 is about 11.3°, while that of System 3 is about 19.5°. The expected number of detected meteors per hour is estimated by \( AN_0r^M \), where \( M \) is a limiting magnitude, \( r \) is a meteor index, \( A \) is an EMCA, and \( N_0 \) is a rate of meteors brighter than zeroth magnitude. Here, we adopt \( r = 3.4 \) and log\(_{10} N_0 = -5.1 \) [20]. Table 5 lists the event rate calculated for \( M = 10 \) mag in optical wavelengths. A system with a 1° × 1° field-of-view and a limiting magnitude of \( \sim 10 \) will detect more than 100 meteors per hour. This result encourages meteor survey observations with wide-field cameras mounted on large aperture telescopes (e.g.}
Figure 7: Fields-of-view of the three systems. The violet cross-hatched region indicates the field-of-view of System 1. The green empty rectangles indicate the field-of-view of System 2. The blue filled rectangles indicate the field-of-view of System 3.

Table 3: EMCA for Square Fields-of-View

| Side Length (arcmin) | 10×10′ | 20×20′ | 30×30′ | 40×40′ | 50×50′ | 60×60′ |
|----------------------|--------|--------|--------|--------|--------|--------|
| FOV area (deg²)      | 0.03   | 0.11   | 0.25   | 0.44   | 0.69   | 1.00   |
| EMCA (km²)           | 14.6   | 27.2   | 41.4   | 55.1   | 69.2   | 83.2   |
| Projected Areaa (km²)| 0.09   | 0.34   | 0.76   | 1.35   | 2.12   | 3.05   |
| Event Rateb (hour⁻¹) | 22     | 45     | 68     | 90     | 114    | 136    |

a The area of the field-of-view projected on the sphere at an altitude of 100 km.
b Estimated meteor event rates for meteors brighter than 10 magnitude.
Figure 8: Schematic view of meteor-collecting volumes (MCV). The top, middle, and bottom plots respectively show the MCVs for Systems 1, 2, and 3. The labels “A”, “B”, and “C” indicate the MCVs for elevation angles of 85°, 60°, and 35°, respectively. The blue dots indicate the location of the observing system. The green solid curve in Panels B indicates the sea level. The gray dashed lines in Panels B respectively show the top and bottom surface of the meteor sphere.
Figure 9: Effective meteor-collecting areas (EMCAs) in units of km$^2$ against elevation angle $\alpha$. The error bars are as small as the line width.
Subaru/HSC [32]). Tomo-e Gozen is expected to detect more than about 2,000 faint meteors a night. Since meteors will run across multiple detectors of Tomo-e Gozen, faint meteors are more robustly detected with Tomo-e Gozen than Tomo-e PM. Tomo-e Gozen will be an ideal instrument to investigate variations or structures in the luminosity function of meteors brighter than about +10 mag.

5. Conclusion

Tomo-e Gozen is a mosaic CMOS camera developed in Kiso Observatory, the University of Tokyo. Tomo-e Gozen, equipped with 84 CMOS sensors, will continuously obtain images of 20 deg$^2$ at 2 Hz. In this sense, Tomo-e Gozen will be the world largest video camera for astronomy. As a pilot project, we developed a prototype model of Tomo-e Gozen, Tomo-e PM, which has 8 CMOS sensors. Tomo-e PM can monitor a sky of about 2 deg$^2$ at 2 Hz. We carried out imaging observations of faint meteors with Tomo-e PM on April 11 and 14, 2016. We set the observation field within the Earth’s shadow to eliminate contamination from debris and satellites in the low Earth orbit. The total observing time was about 10 hours, and 2,220 meteor events were detected in total. Our observations provide a new measurement of the meteor luminosity function. The video rate magnitudes of the meteors we detected are typically from +4 to +12 mag. The corresponding mass of the meteors ranges from $8.3 \times 10^{-5}$ to $3.3 \times 10^{-4}$ g. The present results are consistent with a single power-law luminosity function. A statistical model suggest that the slope parameter $r = 3.1 \pm 0.4$ and the meteor rate $\log_{10} N_0 = -5.5 \pm 0.5$. The diurnal variation in the sporadic meteor rate is marginally confirmed only on April 11. The slope is roughly consistent with those of luminosity functions in literature. The meteor rate is lower than those in literature. The meteor rate is roughly consistent with those of luminosity functions in literature. The meteor rate is lower than those in literature. The meteor rate is lower than those in literature. The meteor rate is lower than those in literature. The meteor rate is lower than those in literature. The meteor rate is lower than those in literature.

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Appendix A. Validation of the EMCA Calculation

Appendix A.1. Travel Distances

In the present formalism, we assume that an incident meteor never fades away inside the meteor shell. This assumption may bring an overestimate of a rate of meteors with a long travel distance, and an EMCA could be overestimated accordingly. Figure [A.10] shows the distribution of the travel distances of the meteors for System 1 with $\alpha = 60^\circ$. The distribution neither depends on the systems nor elevation angles. The fraction of the meteors with a travel distance longer than 400 km was $\lesssim 10\%$. Thus, we concluded that the assumption had virtually no effect on the present results.

Appendix A.2. Position Angles

Artificial patterns are found in the distribution of position angles on the focal plane. Position angles (PA) of meteors for System 2 are summarized in Figure [A.11]. Dips around PA $\approx -90^\circ$ and $90^\circ$ in the top panel of Figure [A.11] are attributed to the alignment of the detectors. A fraction of meteors with PA $\approx \pm180^\circ$ increased with decreasing elevation angle. Thus, a large fraction of meteors run from the zenith to the nadir in a field-of-view. The observations on April 11 and 14 were carried out with elevation lower than $38^\circ$. Similar distributions of the PAs were confirmed in the observations. Our calculation is qualitatively consistent with the observations. The degree of the concentration depends on the distribution of the incident angle $i$. Since it was difficult to constrain the distribution of $i$ from the observations, we did not discuss the PA distribution of the meteors in the paper.

Appendix A.3. Depth of the Meteor Shell

We assume that the top and bottom boundaries of the meteor shell were at 120 and 80 km, respectively. They are chosen as nominal values [23]. An appearing and disappearing altitude of a meteor, however, depend on the size or composition of the meteoroid. We calculate the EMCAs for $H_1 = 60$ km and $H_2 = 140$ km, the results are almost the same. Thus, we conclude that the choice of $H_1$ and $H_2$ has little impact on the present results.

Appendix A.4. Reduced Efficient Meteor-Collecting Area

The EMCA defined in Equation (2) does not take into account the apparent brightness of incident meteors and atmospheric extinction. A practical meteor-collecting area is given by reducing $A$ as

$$A^{\text{red}} = A_0 \left(\frac{100 \text{ km}}{H_2}\right)^2 \sum_{i=3}^{N} \langle M \rangle^{i-5 \log_{10} \phi} \times \epsilon_i,$$  \hfill (A.1)

where $r$ is the population index, $d_i$ is the distance to the $i$-th meteor, and $\epsilon_i$ is the amount of extinction of the $i$-th observable meteor. The reduced effective meteor-collecting area (hereafter, referred as to RMCA) corresponds to the area defined in Equation (1) of Koschack and Rendtel [30]. For the sake of simplicity, we adopt $\epsilon_i = 0$ in the rest of the paper. We calculate the RMCAs for Systems 2. The position angle $\phi$ is fixed at zero. Figure [A.12] shows the RMCA for $r = 2.0, 2.5, 3.0, 3.5$, and 4.0 against an elevation angle. While the RMCA is almost the same as the EMCA when the elevation angle $\alpha$ is larger than about $80^\circ$, the differences between the EMCA and RMCA become significant at low elevation angles. The differences increase as $r$ becomes large. Hawkins and Upton [20] and Cook et al. [9] independently obtained $r = 3.4$ for sporadic meteorites with $M_\phi < +5$ and $+7 < M_\phi < +12$, respectively. Thus, we safely assume $r > 3.0$ for visible sporadic meteorites. In such cases, the RMCA is largest at the zenith. This does not depend on systems. In observations of sporadic meteors, a monitoring observation close to the zenith is favored. When the RMCA is larger than about 3.0, the RMCA monotonically decreased with decreasing elevation angle in contrast to the EMCA. The RMCA is well-approximated by

$$A^{\text{red}} = A r^{-5 \log_{10} \phi},$$  \hfill (A.2)

where $\langle d \rangle$ is the averaged distance to the MCVs. Since the field-of-view is sufficiently narrow, the distances to the meteors are approximately identical. This justifies Equations (6) and (A.2). Equation (A.1) indicates that an expected number of meteorites can be given by $\bar{n} = A^{\text{red}} N_0 r^M$, which is a function of the meteor index $r$, the elevation angle $\alpha$, and the limiting magnitude $M$. Figure [A.12] illustrates the dependence of $A^{\text{red}}$ on the meteor index $r$; the RMCA ratio at $\alpha = 30^\circ$ to $\alpha = 90^\circ$ decreases from 1.74 to 0.63 when the meteor index $r$ increases from 2.0 to 4.0. Thus, comparing the number of meteorites detected in different elevation angles provides an estimate of the meteor index $r$ without measuring the brightness of the meteorites. Figure [A.13] shows an application of this method. The orange crosses show the numbers of meteor events in 3 minute on April 11, 2016. The number of detections decreases with decreasing elevation angle. The lines indicate the expected number of meteor detections for different $r$ values. The limiting magnitudes are estimated from neighboring stars. The observations are well explained by the expectations with $r = 3.0$–4.0, consistent with previous studies [9, 17, 20]. This confirms that the present results are self-consistent.

\[\text{The data on April 14, 2016 are not shown, since large part of the data with } \alpha > 30^\circ \text{ were obtained in bad conditions.}\]
Figure A.10: Distribution of the travel distances of observable meteors. The filled bars show the histogram the travel distances for System 1 with an elevation angle $\alpha = 60^\circ$. The cumulative distribution is described by the solid step line.

Figure A.11: Distributions of the position angles of mock meteors. A position angle of zero corresponds to a meteor moving toward the zenith, or upward in the field-of-view.
Figure A.12: Reduced effective meteor-collecting areas (RMCAs) in units of km$^2$ against an elevation angle $\alpha$. The symbols indicate an assumed meteor index value $r$. The gray dashed line indicates the EMCA for System 3 as a reference.

Figure A.13: Deriving the meteor index $r$ by counting the number of meteors. The orange crosses shows the number of the meteors in 3 minute against the elevation angle in observation. The lines show the expected number of detections for different $r$ values.