Mixed MSW and Vacuum Solutions of Solar Neutrino Problem *

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Abstract

Assuming three flavour neutrino mixing takes place in vacuum, we investigate the possibility that the solar $\nu_e$ take part in MSW transitions in the Sun due to $\Delta m^2_{31} \sim (10^{-7} - 10^{-4}) \text{ eV}^2$, followed by long wave length vacuum oscillations on the way to the Earth, triggered by $\Delta m^2_{21}$ (or $\Delta m^2_{32}$) $\sim (10^{-12} - 10^{-10}) \text{ eV}^2$. The solar $\nu_e$ survival probability is shown to be described in this case by a simple analytic expression. New ranges of neutrino parameters which allow to fit the solar neutrino data have been found. The best fit characterized by the minimum $\chi^2$ is extremely good. This hybrid (MSW+vacuum oscillations) solution of the solar neutrino problem leads to peculiar distortions of energy spectrum of the boron neutrinos which can be observed by the SuperKamiokande and SNO experiments. Other flavour scheme (e.g. 2 active $\nu$s + 1 sterile $\nu$) can provide MSW+vacuum solution also.

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1. Introduction

Solar neutrino experiments [1–6] indicate an existence of solar neutrino problem. Scientists have tried several different solutions, for example, astrophysical solutions (which implies the puzzle comes from our lack of the knowledge of nucleon inside the Sun. But this solution doesn’t fit data well); spin and spin-flavour oscillations (caused by magnetic field) [7] and neutrino decay (can not fit experimental data well). In particular, there are two solutions which give good fit of data: First based on the old idea of Pontecorvo [8] that the solar $\nu_e$ take part in vacuum oscillations when they travel from the Sun to the Earth. The second based on the more recent hypothesis [9,10] of the solar $\nu_e$ undergoing matter-enhanced (MSW) transitions into neutrinos of a different type when they propagate from the central part to the surface of the Sun.

Both of MSW and vacuum oscillation mechanisms may simultaneously happen in our nature. That is, MSW effect take place when the neutrino travel from the centre of the Sun to the surface of the Sun, long wavelength effect occur afterwards during the travel from the surface of the Sun to the surface of the Earth. Thus, $\nu_\odot$-problem can be solved by a hybrid solution [11,12].

This requires three flavour neutrinos scheme which is also a motivation for the mixed solution. There are two independent $\Delta m^2$ now. For solving the $\nu_\odot$-neutrino problem, there can be sets of magnitudes of two $\Delta m^2$ [12]. The interesting one is that a $\Delta m^2$ lies in the MSW mass interval and the other lies in the long wavelength interval, corresponding to

\[
10^{-6} eV^2 \lesssim (\Delta m^2)_{\text{big}} \lesssim 2 \times 10^{-4} eV^2, \quad (1a)
\]

\[
10^{-12} eV^2 \lesssim (\Delta m^2)_{\text{small}} \lesssim 5 \times 10^{-10} eV^2 \quad (1b)
\]

Detailed studies of the vacuum oscillation or MSW transition solution of the solar neutrino problem under the more natural assumption of three flavour neutrino mixing are still lacking [13], partly because of the relatively large number of parameters involved. In this hybrid solution, two possible mass spectra are shown in FIG. 1. In first case, one $m$ is heavy but the other two are light. In the second case, two neutrino masses are heavy and the other one is light. Both two cases are
FIGURES

1) First mass case  
2) Second case (Zee mass)

FIG. 1. Different mass spectra, two light plus one relative heavy neutrinos 1) and one light with two heavy neutrinos 2), which can have mixed solutions to solar neutrino problem.

Independent from the absolute value of $m$ but the difference.

All the different patterns of neutrino masses mentioned above can arise in gauge theories of electroweak interactions with massive neutrinos, and in particular, in GUT theories [14].

2. The Solar $\nu_e$ Survival Probability

For the first mass case, the averaged survival probability (FIG. 2) can be written

$$\bar{P}(\nu_e \rightarrow \nu_e; t_E, t_0) = \bar{P}_{2MSW}^{(31)}(\Delta m_{31}^2, \theta_{13}) - V,$$

here $V$ is the loss of $\nu_e$ caused by vacuum oscillation during the way from surface of the Sun to the Earth. It can be written in several forms:

$$V = \frac{2|U_{e1}|^2|U_{e2}|^2}{1 - |U_{e3}|^2} (1 - \cos 2\pi \frac{R}{L_{21}})\left[\frac{1}{2} + \left(\frac{1}{2} - \bar{P}_{jump}^{(31)}\right)\cos 2\theta_{31}\right]$$

$$= \frac{1 - |U_{e3}|^2}{1 - 2|U_{e3}|^2} \left[1 - P_{2VO}^{(21)}(\Delta m_{21}^2, \theta_{12})\right] [U_{e3}]^2 - \bar{P}_{2MSW}^{(31)}(\Delta m_{31}^2, \theta_{13})$$

$$= \frac{1}{2} \sin^2 2\theta_{12} (1 - \cos 2\pi \frac{R}{L_{21}}) \cos^2 \theta_{13}\left[\frac{1}{2} + \left(\frac{1}{2} - \bar{P}_{jump}^{(31)}\right)\cos 2\theta_{13}(t_0)\right].$$

Some comments are given below for these formulas:

- the term $\bar{P}(\Delta m_{31}^2, \theta_{13})$ is 2-neutrinos MSW transition $\nu_e \leftrightarrow \nu'$ survival probability with that

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1 When $\theta_{13}$ is very small, an approximate formula is given in [12].
$\nu'$ is the mixed state of $\nu_\mu$ and $\nu_\tau$:

$$\nu' = \frac{U_{\mu 3}}{\sqrt{U_{\mu 3}^2 + U_{\tau 3}^2}} \nu_\mu + e^{-i\frac{\varphi}{2}} \frac{U_{\tau 3}}{\sqrt{U_{\mu 3}^2 + U_{\tau 3}^2}} \nu_\tau,$$

where $\varphi$ is the phase of $U_{e 3}$

- $P_{2VO}^{(21)}$ is the 2-neutrinos vacuum oscillation probability with parameters $\Delta m^2_{21}$ and $\theta_{12}$.
- The mixing angles are defined as

$$\sin^2 \theta_{13} = |U_{e 3}|^2;$$
$$\sin^2 2\theta_{12} = \frac{4|U_{e 1}|^2|U_{e 2}|^2}{(1 - |U_{e 3}|^2)^2}$$

which coincide with the definition below

$$\nu_e = \cos \theta_{13} \nu^{(12)} + \sin \theta_{13} \nu_3;$$
$$\nu^{(12)} = \cos \theta_{12} \nu_1 + \sin \theta_{12} \nu_2,$$

where $\nu_1$, $\nu_2$ and $\nu_3$ are three neutrino mass eigenstates.

![Image](image.png)

**FIG. 2.** The survival probabilities $\bar{P}(\nu_e \rightarrow \nu_e)$ of all types of solar neutrinos for the first mass case. Differences is because of different production area inside the Sun.
Let us consider the properties of the survival probability $\bar{P}$. For module of term $V$, its coefficient is

$$F_1 \equiv \frac{2|U_{e1}|^2|U_{e2}|^2}{1 - |U_{e3}|^2} = \frac{1}{2} \sin^2 2\theta_{12} \cos^2 \theta_{13},$$

here $\cos^2 \theta_{13}$ is in interval $[0.5, 1]$, so $0 \leq F_1 \leq \frac{1}{2}$, i.e., the module of $V$ can vary from 0 to 1. The upper bound of $\bar{P}$ is $\bar{P}_{2MSW}$. Thus for same MSW parameters, the suppression of mixed scheme is stronger than that of 2-$\nu$ MSW. In general, in order to recover the same theoretical predictions, $\Delta m_{31}^2$ should shift to a bigger value than 2-$\nu$ MSW’s [15,16]. So for the same $\sin^2 2\theta_{13}$, the new allowed solution regions in MSW parameter plane must be always above the 2-$\nu$ MSW’s (FIG. 4).

At the bottom of the MSW suppression pit, (i.e., $P_{(31)}^{jump} = 0$ and $\cos 2\theta_{13}^m = -1$), term $V$ goes to zero and there is no vacuum oscillation. Under the above conditions the $\nu_e$ state in matter at the point of $\nu_e$ production (in the Sun) essentially coincides with the heaviest of the three neutrino matter-eigenstates, which continuously evolves (as the neutrino propagates towards the surface of the Sun) into the mass (as well as energy) eigenstate $|\nu_3>$ at the surface of the Sun. As a consequence, vacuum oscillations do not take place between the Sun and the Earth, and $\bar{P}(\nu_e \rightarrow \nu_e; t_E, t_0)$ coincides with the probability to find $\nu_e$ in the state $|\nu_3>$. There are two limits. When $\sin^2 2\theta_{12} \rightarrow 0$, we get $\bar{P} \rightarrow \bar{P}_{2MSW}$; when $\sin^2 2\theta_{13} \rightarrow 0$ and $\Delta m_{31}^2 \rightarrow \infty$, then $\bar{P} \rightarrow \bar{P}_{2VO}$.

For the second mass spectrum case [17], after exchanging the indices $1 \leftrightarrow 3$, the term $V$ in probability still has some difference from that in case 1,

$$V = \frac{1}{2} \sin^2 2\theta_{23} (1 - \cos 2\pi \frac{R}{L_{v32}}) \sin^2 \theta_{13}' [\frac{1}{2} - (\frac{1}{2} - P_{(13)}^{jump}) \cos 2\theta_{13}'(t_0)],$$

where $\sin^2 \theta_{13}'$ is less than $\frac{1}{2}$. Zee model (radiative) mass matrix corresponds to this case [18], it reduces the four free parameters to three free parameters. Here module of $V$ varies from 0 to $\frac{1}{2}$. Since the coefficient $F_2 = \frac{1}{2} \sin^2 2\theta_{32}\sin^2 \theta_{13}'$ in $V$ is from 0 to $\frac{1}{4}$, thus the vacuum oscillation is suppressed by factor 2 than in the first case. Furthermore, for small $\sin^2 \theta_{13}'$, $F_2$ is small. So we shouldn’t expect new solutions in small MSW parameter region. The maximal vacuum oscillation take place at the bottom of the MSW pit. In the two “asymptotic” regions (highest energy and lowest energy regions), $|V| = \sin^4 \theta_{13}' \leq \frac{1}{4}$ (see FIG. 3).
3. Hybrid MSW Transition + Vacuum Oscillation Solutions of the Solar Neutrino Problem

We have analyzed the published data from the four solar neutrino experiments searching for solutions of the solar neutrino problem of the hybrid MSW transitions + vacuum oscillations type. Only the first case of mass spectra with $\Delta m^2_{31}$ and $\Delta m^2_{21}$ having values in the intervals (1a) and (1b) was studied.

We utilized the predictions of the solar model of Bahcall and Pinsonneault in 1995 with heavy element diffusion for the $pp$, $pep$, etc. neutrino fluxes in this study. The estimated uncertainties in the theoretical predictions for the indicated fluxes were not taken into account. The solutions fit to data are called A,B,C,D,E,F as below.

Solution A and D.

For solution A, The minimum $\chi^2$ ($\sim 0.1$) value is reached at around point $(\Delta m^2_{21}, \sin^2 2\theta_{12}, \Delta m^2_{31}, \sin^2 2\theta_{13}) \cong (5.6 \times 10^{-12} \text{ eV}^2, 0.98, 4.2 \times 10^{-5} \text{ eV}^2, 10^{-3})$.

For $E \geq 5 \text{ MeV}$ and $\Delta m^2_{21} \leq 8.0 \times 10^{-12} \text{ eV}^2$ most of the $8B$ neutrinos undergo only MSW transitions. The MSW transitions of the $8B$ neutrinos having this energy are adiabatic for

\[ \Delta M^2_{31} = 1 \times 10^{-5} \]
\[ \Delta M^2_{32} = 1 \times 10^{-9} \]
\[ \sin^2 2\theta_{13} = 0.64 \]
\[ \cos 2\theta_{13} = 0.6 \]
values of $\Delta m_{31}^2 \approx (1.1 - 1.3) \times 10^{-4} \text{ eV}^2$ and $\sin^2 2\theta_{13} \gtrsim (3.0 - 4.0) \times 10^{-3}$ from the “horizontal” region of the solution (see Fig. 4a). They are nonadiabatic for values of $\Delta m_{31}^2$ and $\sin^2 2\theta_{13}$ from the remaining part of the allowed region.

In contrast to the main fraction of $^8\text{B}$ neutrinos, the $pp$ and the high energy line of $^7\text{Be}$ neutrinos do not undergo resonant MSW transitions but take part in vacuum oscillations between

with $E \geq 2.5 \text{ MeV}$. 

FIG. 4. Fits to the solar neutrino data: (a) in MSW parameters plain, and (b) in vacuum’s plain. For solutions B, F in (a), SuperK result is included.
the Sun and the Earth. Actually, the $^7$Be neutrino energy of 0.862 MeV is in the region of the first minimum of $P_{2V0}^{(21)}$ as $E$ decreases from the “asymptotic” values at which $P_{2V0}^{(21)} \approx 1$, while the interval of energies of the $pp$ neutrinos, relevant for the current Ga–Ge and the presently discussed future solar neutrino experiments (HELLAZ, HERON), 0.22 MeV $\lesssim E \leq 0.41$ MeV, is in the region of the first maximum of $P_{2V0}^{(21)}$ as $E$ decreases further.

Let us note that the solution A region in the $\Delta m_{21}^2 - \sin^2 \theta_{12}$ plane of the vacuum oscillation parameters is quite similar to the region of the “low” $^8$B $\nu_e$ flux 2$\nu$ vacuum oscillation solution found in ref. [20]. The latter is possible for values of the $^8$B neutrino flux which are lower by a factor of 0.35 to 0.43 (of 0.30 to 0.37) than the flux predicted in [21] (in ref. [19] and used in the present study). Note, however, that the $\chi^2_{min}$ for the indicated purely vacuum oscillation solution, $\chi^2_{min} = 4.4$ (2 d.f.) [20], is considerably larger than the value of $\chi^2_{min}$ for solution A. The two solutions differ drastically in the way the $^8$B neutrino flux is affected by the transitions and/or the oscillations.

Let us mention only here that in the case of solution A: i) the spectrum of $^8$B neutrinos will be strongly deformed, ii) the magnitude of the day-night asymmetry in the signals in the indicated detectors can be very different from that predicted in the case of the 2$\nu$ MSW solution (see,e.g., [22,23]), and iii) the seasonal variation of the $^8$B $\nu_e$ flux [24,25] practically coincides with the standard (geometrical) one of 6.68%.

Solution D are similar to A but $\Delta m_{21}^2$ is bigger than which in A. It has as a $\theta_{13} \to 0$ limit the second “low” $^8$B $\nu_e$ flux 2$\nu$ vacuum oscillation solution discussed in ref. [20]: the regions of values of the vacuum oscillation parameters of the two solutions practically coincide. This 2$\nu$ solution was found to be possible for values of the initial $^8$B $\nu_e$ flux which are by a factor $\sim (0.45 - 0.65) \sim (0.39 - 0.56)$ smaller than the flux predicted in ref. [21] (in ref. [19]). The MSW + VO effects and correspondingly the purely VO effects on the $^7$Be and/or $^8$B neutrino fluxes in the cases of the two solutions are also very different.

Solution B.

This solution (see FIG. 4) can be regarded as an “improved” MSW transitions + vacuum oscillations version of the purely 2$\nu$ MSW nonadiabatic solution.

For $\Delta m_{21}^2$ from the domain $\sim (0.5 - 1.0) \times 10^{-10}$ $eV^2$ of the 2$\nu$ vacuum oscillation solution [20], however, solution B takes place for values of $\sin^2 2\theta_{12}$ which are systematically smaller than the values of the same parameter in the 2$\nu$ vacuum oscillation solution.

The 0.862 MeV $^7$Be neutrinos take part in adiabatic MSW transitions in the Sun, while the $^8$B neutrinos with $E \gtrsim 4$ MeV undergo nonadiabatic transitions. Both the $^7$Be and $^8$B neutrinos, as well as the $\nu_\mu$ and/or $\nu_\tau$ into which a fraction of the $\nu_e$ has been converted by
the MSW effect in the Sun, participate in vacuum oscillations after leaving the Sun. These oscillations are modulated by the MSW probability $\bar{P}_{2^{\text{MSW}}}^{(31)}$.

With respect to the predictions in ref. [19], the signal in the Kamiokande detector and the contribution of the $^8$B neutrinos to the signals in the Cl–Ar detector are smaller typically by factors of $\sim (0.43 - 0.47)$ and $\sim (0.32 - 0.36)$. The $pp$ and the 0.862 MeV $^7$Be $\nu_e$ fluxes are suppressed by factors of $\sim (0.65 - 0.90)$ and $\sim (0.11 - 0.27)$ for most of the values of the parameters from the allowed region. However, for $\sin^2 2\theta_{12} \sim 0.9$, for instance, one has $\bar{P}(\nu_e \to \nu_e; t_E, t_0) \approx 0.55$ for the $pp$ neutrinos. Even in this case the 0.862 MeV $^7$Be $\nu_e$ flux is reduced by a factor of $\sim 0.3$, but the indicated possibility is rather marginal.

The seasonal variations due to the vacuum oscillations of the signals in the Super-Kamiokande, SNO and ICARUS detectors are estimated to be smaller than the variations in the case of the 2$\nu$ vacuum oscillation solution [1], except possibly in the small region of the $\Delta m^2_{21} - \sin^2 2\theta_{12}$ plane where $\Delta m^2_{21} \gtrsim 10^{-10}$ eV$^2$ and $\sin^2 2\theta_{12} \gtrsim 0.7$. The range of the predicted values of the day-night asymmetry in these detectors is different from the one expected for the 2$\nu$ MSW solution. The seasonal variation of the 0.862 MeV $^7$Be $\nu_e$ flux caused by the vacuum oscillations is expected to be considerably smaller than in the 2$\nu$ case, while the day-night asymmetry is estimated to be somewhat smaller than the one predicted for the 2$\nu$ MSW nonadiabatic solution. The seasonal variation, nevertheless, may be observable. Obviously, the experimental detection both of a deviation from the standard (geometrical) 6.68% seasonal variation of the solar neutrino flux and of a nonzero day-night effect will be a proof that solar neutrinos take part in MSW transitions and vacuum oscillations.

**Solution C and E.**

The values of the parameters for solution C (Figs. 4a and 4b) form a rather large region in the $\Delta m^2_{31} - \sin^2 2\theta_{13}$ plane, and a relatively small one in the $\Delta m^2_{21} - \sin^2 2\theta_{12}$ plane, The $\chi^2_{\text{min}}$ for this solution is larger than for solutions A and B: $\chi^2_{\text{min}} \sim 1.5$ at $(\Delta m^2_{21}, \sin^2 2\theta_{12}, \Delta m^2_{31}, \sin^2 2\theta_{13}) \approx (1.2 \times 10^{-10}$ eV$^2$, 0.78, 4.6 $\times$ 10$^{-5}$ eV$^2$, 5.9 $\times$ 10$^{-4}$) (the black square in Figs. 4).

Solution E holds for a small region, it and most of solution C are excluded by the recent experimental spectrum data [2].

**Solution F.**

This solution (FIG. 4a) was found in paper [14]. For $\Delta m^2_{13} \sim (4 - 8) \cdot 10^{-6}$ eV$^2$, the $^7$Be-$\nu_e$ flux can be suppressed by resonance conversion. Since $\sin^2 2\theta_{e\tau} \sim (3 - 10) \cdot 10^{-4}$, the pit is narrow

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3These variations were shown [23] to be not larger than 15% for the 2$\nu$ vacuum oscillation solution.
and suppression of the high energy part of the boron neutrinos is rather weak. This flux can be suppressed by vacuum oscillations if it is placed in the first minimum of oscillatory curve. For pp-neutrinos one gets then the averaged oscillation effect. Thus we arrive at configuration with resonance conversion pit at small energies and vacuum oscillation pit at high energies.

4. Minimum $\chi^2$ value by including SuperKamiokande data

In model [19], using five experiment results (for SuperK, we use 201.6 live days data), we find the minimum $\chi^2$ of MSW+vacuum solution is 0.033, located at $(\Delta m_{21}^2, \sin^2 2\theta_{12}, \Delta m_{31}^2, \sin^2 2\theta_{13}) \approx (1.4 \times 10^{-10} eV^2, 0.48, 9.9 \times 10^{-6} eV^2, 4.0 \times 10^{-3})$, which is in the B-down solution region. Here we have four free parameters, thus it is one degree of freedom.

5. Distortion of the boron neutrino spectrum and signals in SuperKamiokande and SNO

An interplay of vacuum oscillations and MSW conversion can lead to peculiar distortion of the boron neutrino energy spectrum. For $\Delta m_{12}^2 > 10^{-11} eV^2$ there is modulation of the spectrum due to vacuum oscillations. We have studied a manifestation of such a distortion in the energy spectrum of the recoil electrons in the SuperK (SuperKamiokande) and SNO [26] experiments.

Using energy resolution function for electrons [2] we have found the ratio $R_e$ of expected (with conversion) number of events $S(E_{vis})$ to predicted (without conversion) one for different values of oscillation parameters (FIG. 5):

$$S(E_{vis}) = \int dE_e \cdot f(E_{vis}; E_e) \cdot \int_{E_e - m_e}^{E_e + m_e} dE_\nu \cdot \Phi(E_\nu) \cdot$$

$$\left[ P(\nu_e \rightarrow \nu_e) \frac{d^2 \sigma_{\nu_e}}{dE_\nu dE_\nu}(E_e; E_\nu) + \nu_\mu, \nu_\tau \text{ contribution} \right],$$

where $E_e$ is the total energy of recoil electrons and the original neutrino flux is $\Phi(E_\nu)$. The lower limit of integration is first order approximation but the precise form is $\frac{1}{2} \left( E_e - m_e + \sqrt{E_e^2 - m_e^2} \right)$. The energy resolution function can be written as [2]

$$f(E_{vis}; E_e) = \frac{1}{\sqrt{2\pi} E_e \sigma} \cdot exp \left( - \left( \frac{E_{vis} - E_e}{\sqrt{2} E_e \sigma} \right)^2 \right).$$

Experimental table of $\sigma$ (for SuperK) is used in our calculation but an approximate relation is $\sigma \propto \frac{1}{\sqrt{E_e}}$. We show also the $R_e$ measured by SuperK during 201.6 days [27]. As follows from

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4For ideal energy resolution, $f(E_{vis}; E_e)$ goes to $\delta$-function.
FIG. 5, the integration over neutrino energy and on electron energy weighted with resolution function leads to strong averaging out of the oscillatory behaviour. Indeed, for present water Cherenkov experiment like SuperK the energy resolution is typically $\sim 1.6$ MeV (at $E_e = 10$ MeV) which is bigger or comparable with “period” (in the energy scale) of the oscillatory curve.

This hybrid solutions give very rich distortions of the recoil spectrum. It not only has the pure 2-$\nu$ solution’s distortions but also give other peculiar distortions.

![Graph](attachment:graph.png)

FIG. 5. The expected spectrum deformations of the recoil electrons in SuperK and SNO. Boxes together with solid lines are SuperK data.

Obviously some of distortions like in region E are already excluded by 201.6 days data. The smoothing effect is weaker in SNO experiment: The intergration over neutrino energy gives smaller averaging and energy resolution is slightly better.

6. Other case of MSW+vacuum solution.

Besides three active neutrinos, other cases can provide mixed solutions also. For instance, the mass scheme (28) below can give 2 active + 1 sterile neutrinos to solve solar neutrino problem (FIG. 6).
FIG. 6. Qualitative pattern of the neutrino masses and mixing. Boxes correspond to different mass eigenstates. The sizes of different regions in the boxes determine flavors ($|U_{ij}|^2$) of given eigenstates. Arrows connect the eigenstates involved in ATM - atmospheric oscillation, $\nu_\odot$-MSW - MSW conversion inside the Sun, $\nu_\odot$-VO - vacuum oscillation between Sun and Earth.

Suppose there are four neutrinos in nature, three independent $\Delta m^2$ have magnitudes $10^{-11} eV^2$, $10^{-5} eV^2$ and $10^{-2} eV^2$. The biggest $\Delta m^2$ gives solution to atmospheric and LSND anomalies via $\nu_\mu \leftrightarrow \nu_\tau$ vacuum oscillation. The middle $\Delta m^2$ provide MSW transition ($\nu_e \leftrightarrow \text{mixed state of } \nu_\mu, \nu_\tau$) inside the sun. And the smallest $\Delta m^2$ gives vacuum oscillation $\nu_e \leftrightarrow \nu_s'$ which is $\nu_{\text{sterile}}$ mixed slightly with $\nu_\mu$ during the travel from the surface of the Sun to the surface of the Earth.

7. Conclusions.

A general feature of the MSW + VO solutions studied by us is that the $pp$ $\nu_e$ flux is suppressed (albeit not strongly - by a factor not smaller than 0.5) primarily due to the vacuum oscillations of the $\nu_e$, the suppression of the 0.862 MeV $^7$Be $\nu_e$ flux is caused either by the vacuum oscillations or by the combined effect of the MSW transitions and the vacuum oscillations, while the $^8$B $\nu_e$ flux is suppressed either due to the MSW transitions only or by the interplay of the MSW transitions in the Sun and the oscillations in vacuum on the way to the Earth. The solutions differ in the way the $pp$, $^7$Be and the $^8$B neutrinos are affected by the $\nu_e$ MSW transitions and/or the oscillations in vacuum.

For all MSW + VO solutions we have considered, the $^8$B $\nu_e$ spectrum is predicted to be rather strongly deformed. The SuperK data on the shape of the $^8$B neutrino spectrum can be
used to further constrain the solutions we have found. Such an analysis can exclude solution E, almost all of C and some part of A,B,D,F solutions.

For the MSW + VO solutions considered by us the day-night asymmetry in the signals of the detectors sensitive only to $^{8}$B or $^{7}$Be neutrinos are estimated to be rather small, not exceeding a few percent. The seasonal variation effect caused by the vacuum oscillations can be observable for $^{7}$Be neutrinos and, for certain relatively small regions of the allowed values of the parameters, can also be observable for the $^{8}$B or for the $pp$ neutrinos if the $pp$ neutrino flux is measured with detectors like HELLAZ or HERON.

The global minimum $\chi^2$ of this hybrid solution is very small. The interplay of the resonance conversion and vacuum oscillations leads to additional peculiar distortion of the neutrino energy spectrum. In the SuperKamiokande experiment, and (to a slightly weaker extend) in the SNO, the integrations over the neutrino energy and finite energy resolution result in strong smoothing of oscillatory distortion of the electron energy spectrum.

Finally, the MSW transitions + vacuum oscillation solutions can also be obtained from the neutrino mass and mixing structure in the case of two active plus one sterile neutrinos.

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