Solar Type III Radio Bursts: Directivity Characteristics

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Abstract. Type III radio bursts are a group of fast drifting radio emissions associated with solar flares. These radio emissions are believed to be excited at the fundamental and second harmonic of the electron plasma frequency, $f_{pe}$ by the electron beam excited Langmuir waves through a mechanism called the plasma mechanism. This mechanism attributes the dipole and quadrupole beam patterns for the fundamental and harmonic emissions. To verify these predictions, we analyze the simultaneous observations of type III radio bursts by the STEREO A, B and Wind spacecraft located at different vantage points in the ecliptic plane, and determine their normalized peak intensities (directivity factors) at each spacecraft using their time profiles. Assuming that the sources of these bursts are located on the Parker spiral magnetic field lines emerging from the associated active regions, we estimate the angles between the magnetic field directions and the lines connecting the sources to the spacecraft (viewing angles). Based on the plots of the directivity factors versus the viewing angles, one can divide these bursts into (1) intense bursts emitted into a narrow cone centered around the tangent to the magnetic field, and (2) relatively weaker bursts emitting into a wider cone centered around the tangent to the magnetic field. We compute the distributions of ray trajectories emitted by an isotropic point source and show that the refraction focuses the fundamental and harmonic emissions into narrow and wider cones, respectively. The comparison of these distributions with observations indicates that the intense bursts visible to a narrow range of angles around the tangent to the magnetic field probably correspond to the fundamental, and the relatively weaker bursts visible to a wide range of angles probably are the harmonic emissions.

1. Introduction
Solar type III radio bursts are produced by the Langmuir waves excited by the flare accelerated electron beams. The partial conversion of Langmuir waves excited by these supra-thermal electrons into escaping radiation at the fundamental and second harmonic of the electron plasma frequency, $f_{pe}$ (plasma mechanism) is believed to be the emission mechanism of these bursts. As shown by [18], this mechanism attributes the dipole pattern

$$\Psi_{f_{pe}} = \sin^2 \theta$$  \hspace{1cm} (1)

for the fundamental and quadrupole pattern

$$\Psi_{2f_{pe}} = \sin^2 \theta \cos^2 \theta$$  \hspace{1cm} (2)
Figure 1. Directivity patterns of the fundamental and harmonic emissions

for the harmonic emission. Here $\theta$ is the angle between the wave vector $\vec{k}$ of the electromagnetic wave and the wave vector $\vec{k}_L$ of the Langmuir wave. The dipole and quadrupole nature of the fundamental and harmonic emissions is clearly depicted in Figure 1. The observational confirmation of the directivity patterns is an important element for the acceptance of the plasma mechanism as the emission mechanism as well as for understanding of many observations at $f_{pe}$ and at $2f_{pe}$.

Some single spacecraft observations indicate a small but systematic asymmetry in the occurrence rate of type III bursts from east to west [7, 5], which has been attributed to the beam pattern of the radio emission oriented along the average magnetic field direction. Some multi-spacecraft observations appear to support these claims [6, 1]. However, there exist other observational studies, which show the number of occurrences of type III bursts peaking at the center of the disk and steadily falling at high longitudes [4]. According to [10], multi-spacecraft observations of two events show beaming patterns approximately directed along the tangent to the spiral magnetic field line at the source. [17] analyzed the stereoscopic observations obtained by the STEREO A, B and WIND spacecraft and showed that the type III emission is very intense along the tangent to the spiral magnetic field. These authors did not find east-west asymmetry. The Monte Carlo simulation techniques are also used to investigate the visibility and directivity of these bursts [12, 13, 14, 15, 16].

In this paper, we present the analysis of a relatively larger data set and extend the study conducted by [17]. We have developed a three dimensional ray tracing code for this study. We
Figure 2. Simultaneous observations of a type III radio burst obtained by the STEREO A, B and WIND spacecraft. The WIND is separated from the Stereo A and B by 81.372 and -74.076 degrees, respectively. The active region associated with this event is N19W43.

compute the ray trajectory distributions emitted by an isotropic point source and compare with observations. This shows that the intense bursts visible to a narrow range of angles around the tangent to the magnetic field probably correspond to the fundamental, and the relatively weaker bursts visible to a wide range of angles probably are the harmonic emissions. In section 2, we describe the observations, and in section 3 we present the discussion and conclusions.

2. Observations
The STEREO mission consists of two identical spacecraft- one ahead of Earth in its orbit (Stereo 'A'), the other trailing behind (Stereo 'B'), each of which is equipped with identical instrumentation. The helio-longitudinal separation of these two spacecraft increases during first four years of the mission. The data obtained by the WAVES experiments of these two spacecraft and by the near-Earth Wind spacecraft [2, 3] provide an unique opportunity for stereoscopic studies of various radio emissions. We have identified several type III radio bursts detected simultaneously by all these three spacecraft for which unambiguous identification of the associated flare or active region is possible. In Figure 2, we present a typical event observed simultaneously by the STEREO A, B and Wind spacecraft. Here, the yellow circle in the center corresponds to the sun and the outer circle corresponds to the orbits of the spacecraft. The dashed lines correspond to the spiral magnetic field. The type III event at STEREO A is very intense probably because it is well connected magnetically to the source.

We define the ratio of the peak intensity at a given spacecraft to the sum of the peak intensities at all three spacecraft \( R_i = \frac{I_i}{\sum I_i} \) as the directivity factor. We use the time profiles as given in Figure 3 to estimate these directivity factors. We identify the associated active regions using the Solar Geophysical Data (NOAA). We estimate the heliolongitudes of the sources of these bursts at different frequencies by assuming that they are located on the Parker spiral magnetic field lines emerging from the associated active regions. As far as the viewing angle is concerned, it is defined as the angle between the direction of the tangent to the magnetic field at the source and the line connecting the source to the spacecraft. These viewing angles are estimated using the heliolongitudes of the sources. A detailed procedure of estimating the viewing angles is given in [17]. We estimate the height of the sources using the empirical density model due to [8] \( N_e = a \rho^{-2} + b \rho^{-4} + c \rho^{-6} \text{ cm}^{-3} \).
Figure 3. The time profiles of a type III radio burst observed simultaneously by the STEREO A, B and WIND spacecraft.

Figure 4. LHS: Normalized peak intensities of type III bursts observed by the STEREO A, B and Wind spacecraft at 625 kHz as a function of viewing angle $\gamma$. The curve fitted to the data is the Gaussian. RHS: Rose diagram of the viewing angles, $\gamma$.

where $\rho$ is the radial distance in units of solar radius $R_\odot$, $a = 3.3 \times 10^5$, $b = 4.1 \times 10^6$, and $c = 8.0 \times 10^7$. The electron plasma frequency $f_{pe}$, which is related to $N_e$ as

$$f_{pe}(kHz) = \sqrt{80.6N_e(cm^{-3})}$$

is used to estimate the heights of the sources. For example, the heights of the fundamental and harmonic sources at 625 kHz are $\sim 9R_\odot$ and $\sim 16.9R_\odot$, respectively.

In the left hand side of Figure 4, we present the plot of the directivity factor $R_\iota$ as a function of the viewing angle at 625 kHz for all three spacecraft. This plot clearly shows that the directivity factor peaks around $\gamma \approx 0$ corresponding to the tangent to the ambient magnetic field, and as $\gamma$ increases to higher values the directivity factor falls. The fitted curve is the Gaussian. In the right hand side of Figure 4, we present the histogram of the viewing angles as a rose diagram at 625 kHz. In this plot, the origin coincides with the radio source and the radial direction with zero degree angle coincides with the tangent to the magnetic field at the source. From this rose diagram, it is clear that the emission of a large portion of type III events is confined to narrow range of angles around the tangent to the magnetic...
Figure 5. The distributions of ray trajectories at the fundamental (F) and harmonic (H) emissions emitted by an isotropic point source placed at the center of disk field. This implies that the emission is mostly beamed along the tangent to the magnetic field.

3. Discussion and Conclusions

According to plasma mechanism, the Langmuir waves are the source of the solar radio type III bursts. The fundamental, which is excited at a layer where the refractive index is zero can escape only within a narrow cone normal to the surface of zero refractive index. The solid angle of this emission cone can be approximated as \[ \Omega_{f_{pe}} = 3 \pi \frac{v_{Te}^2}{v_b^2}, \] (5)

where \( v_{Te} \) is the electron thermal speed and \( v_b \) is the speed of the electron beam. For typical values of \( v_{Te} \sim 1.5 \times 10^8 \text{ ms}^{-1} \) and \( v_b \sim 0.1c \), we estimate \( \Omega_{f_{pe}} \sim 2.38 \times 10^{-2} \) steradians. In the case of second harmonic, the restrictions on the escape conditions are less severe. The solid angle of emission cone in this case is \[ \Omega_{2f_{pe}} = 2 \pi [1 - \cos(\sin^{-1}(\mu_{2f_{pe}}))] \simeq \pi, \] (6)

where \( \mu_{2f_{pe}} \sim \frac{1}{2} \) is the refractive index. Since \( \Omega_{2f_{pe}} \sim 2\pi(1 - \cos \theta) \), the half widths of these emission cones can be estimated as \( \sim 5 \) and \( \sim 60 \) degrees, respectively.

We simulate the escaping conditions by tracing the rays emitted by an isotropic point source in three dimensions. Here we neglect the magnetic field since \( f_{pe} \gg f_{ce} \) (\( f_{ce} \) is the electron cyclotron frequency), and write the refractive index as

\[ \mu^2 = 1 - \frac{f_{pe}^2}{f^2} = 1 - \frac{80.6 N_e}{f^2}, \] (7)

where the frequency \( f \) is in units of kHz. To trace the rays, we use the Cartesian coordinate system with origin at the center of the Sun, and \( z \)-axis connecting the center of the Sun to one of the spacecraft. The relevant equations are the set of 6 first-order differential equations

\[ \frac{d\vec{R}}{d\tau} = \vec{T}, \] (8)

\[ \frac{dT}{d\tau} = D(\vec{R}) = \frac{1}{2} \frac{\partial \mu^2}{\partial \vec{R}}, \] (9)
where
\[ T_x^2 + T_y^2 + T_z^2 = \mu \] (10)
and
\[
\vec{R} \equiv \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \& \quad \vec{T} \equiv \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix}
\]
are the position and direction vectors, respectively. This system of differential equations is integrated using the 4th order Runge-Kutta algorithm [11]

\[
R_{n+1} = R_n + T_n \Delta \tau + \frac{1}{6}(K_1 + 2K_2)(\Delta \tau)^2
\] (11)
\[
T_{n+1} = T_n + \frac{1}{6}(K_1 + 4K_2 + K_3)\Delta \tau
\] (12)
\[
K_1 = D(R_n)
\] (13)
\[
K_2 = D(R_n + \frac{1}{2}T_n\Delta \tau + \frac{1}{8}K_1(\Delta \tau)^2)
\] (14)
\[
K_3 = D(R_n + T_n\Delta \tau + \frac{1}{2}K_2(\Delta \tau)^2).
\] (15)

This algorithm can be used to trace the rays in any medium with an arbitrary density distribution. Starting from a known point \((\vec{R}_0, \vec{T}_0)\), one can generate successively \((\vec{R}_1, \vec{T}_1)\), \((\vec{R}_2, \vec{T}_2)\)......\((\vec{R}_n, \vec{T}_n)\). In the present case, we compute the distribution of ray trajectories emitted by an isotropic point source at 625 kHz located at a height of 9\(R_\odot\) and 16.9\(R_\odot\) for the fundamental and harmonic, respectively. We launch the rays with direction cosines
\[
T_{x0} = \mu_0 \sin \theta_0 \sin \phi_0
\] (16)
\[
T_{y0} = \mu_0 \sin \theta_0 \cos \phi_0
\] (17)
\[
T_{z0} = \mu_0 \cos \theta_0,
\] (18)
where the azimuthal and elevation angles are randomly sampled using
\[
\phi_0 = 2\pi \xi_1
\] (19)
\[
\cos \theta_0 = 2\xi_2 - 1,
\] (20)
where \(\xi_1\) and \(\xi_2\) are the random numbers uniformly distributed between 0 and 1. This sampling scheme represents an ideal isotropic point source. As seen from Figure 5, the rays from the fundamental source escape in a narrow cone along the radial direction. In the case of second harmonic, the rays launched in the forward direction escape directly, and the rays launched in the backward direction get reflected back and escape into a broader cone, i.e., these rays are distributed over a wide range of angles. The estimated widths of these emission cones are \(\sim 10\) and \(\sim 120\) degrees as expected by the theoretical estimates. If we compare the computed ray distribution in Figure 5 with the observed directivity diagram (LHS of Figure 4), it is clear that the intense bursts observed in the range of angles around \(\gamma = 0\) probably correspond to the fundamental, and relatively weaker emissions distributed over a wider range of viewing angles probably correspond to second harmonic.

Thus, the observed distribution of viewing angles as well as the variation of the directivity factors as functions of viewing angles are consistent with the computed ray trajectory distributions, i.e., the type III emissions are mainly peaked along the tangent to the magnetic field. The intense emissions emitted into a narrow cone along the tangent are probably emitted in the fundamental mode, whereas, the emissions emitted into a much wider cone are probably the harmonic emissions. Most of the observed directivity characteristics can be accounted for by the refractions. The emissions beyond the 60 degrees of the cone are probably due to scattering.
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