DECONFINEMENT PHASE TRANSITION IN NEUTRON STARS
AND $\delta$-MESON FIELD IN RMF THEORY

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The Maxwell and Glendenning construction scenarios of deconfinement phase transition
in neutron star matter are investigated. The hadronic phase is described within the
relativistic mean-field (RMF) theory, if also the scalar-isovector $\delta$-meson field is taken into
account. The strange quark phase is described in the frame of MIT bag model, including
the effect of perturbative one-gluon exchange interactions. The influence of the $\delta$-meson
field on the deconfinement phase transition boundary characteristics is discussed.

Keywords: Neutron stars; strange quark phase; mean-field; delta-meson.

1. Introduction

The modern concept of hadron-quark phase transition is based on the feature of
such transition that there are two conserved quantities in this transition: the baryon
number and the electric charge. It is known, that depending on the value of surface
tension, $\sigma_s$, the phase transition of nuclear matter into quark matter can occur in
two scenarios: either the ordinary first order phase transition with a density jump
(Maxwell construction-MC), or formation of a mixed hadron-quark matter with a
continuous variation of pressure and density (Glendenning construction-GC). The
uncertainty of the surface tension values does not allow to determine the phase
transition scenario, which takes place in reality. In this paper we investigate the
influence of $\delta$-meson effective field on the phase transition boundary characteristics
and compare two alternative deconfinement phase transition scenarios.

2. Glendenning and Maxwell constructions for deconfinement
phase transition.

In order to study the deconfinement phase transition it is necessary to have the mod-
els describing the hadronic matter and the quark matter. The relativistic mean-field
(RMF) theory has been effectively applied to describe the EOS of neutron and pro-
toneutron star matter, as well as the structure of finite nuclei. For hadronic phase
we use the relativistic Lagrangian density of the many-particle system consisted in nucleons, $p, n$, and exchanged mesons $\sigma$, $\omega$, $\rho$, $\delta$:

$$ L_{\sigma\omega\rho\delta}(\sigma, \omega, \rho, \delta) = L_{\sigma\omega}(\sigma, \omega, \rho, \delta) - U(\sigma) + L_\delta(\delta), \quad (1) $$

where $L_{\sigma\omega\rho\delta}$ is the linear part of Lagrangian density without $\delta$-meson field[7], $U(\sigma) = \frac{4}{3}m_N (g_\sigma \sigma)^3 + \frac{4}{3} (g_\omega \omega \sigma)^4$ and $L_\delta(\delta) = g_\delta \bar{\psi}_N \vec{r}_N \psi_N + \frac{i}{2} \left( \partial_\mu \bar{\psi}_N \gamma^\mu \delta - m_\delta \delta^2 \right)$ are the $\sigma$-meson self-interaction term and the contribution of the $\delta$-meson field, respectively. This Lagrangian density[1] contains the meson-nucleon coupling constants, $g_\sigma$, $g_\omega$, $g_\rho$, $g_\delta$, and also the parameters of $\sigma$-field self-interacting terms, $b$ and $c$. To examine the influence of the meson field, $\delta$, on the deconfinement phase transition characteristics, we use the model parameter sets obtained in our recent work (see Ref. [8] for details): $a_\sigma = (g_\sigma / m_\sigma)^2 = 9.154 \text{ fm}^2$, $a_\omega = (g_\omega / m_\omega)^2 = 4.828 \text{ fm}^2$, $a_\rho = (g_\rho / m_\rho)^2 = 13.621 \text{ fm}^2$, $a_\delta = (g_\delta / m_\delta)^2 = 2.5 \text{ fm}^2$, $b = 1.654 \cdot 10^{-2} \text{ fm}^{-1}$, $c = 1.319 \cdot 10^{-2} \text{ fm}^{-1}$. If we neglect the $\delta$ channel, we get $a_\delta = 0$ and $a_\rho = 4.794 \text{ fm}^2$. The standard QHD procedure allows to obtain the energy density, $\varepsilon(n, \alpha)$, and pressure, $P(n, \alpha)$, as a function of the baryon number density $n$ and the asymmetry parameter $\alpha = (n_n - n_p)/n$.

To describe the quark phase the MIT bag model is used, in which the interactions between the $u, d, s$ quarks are taken in one-gluon exchange approximation[5]. We choose $m_u = 5 \text{ MeV}$, $m_d = 7 \text{ MeV}$ and $m_s = 150 \text{ MeV}$ for masses, and $B = 100 \text{ MeV/fm}^3$ for bag parameter and $\alpha_s = 0.5$ for the strong interaction constant.

The chemical potentials of the constituents of the $npe$-plasma in $\beta$-equilibrium are expressed through the two independent potentials, $\mu^{(NM)}_b$ and $\mu^{(NM)}_c$, corresponding to conserving of baryonic and electric charges:

$$ \mu_n = \mu^{(NM)}_b, \quad \mu_p = \mu^{(NM)}_b - \mu^{(NM)}_c, \quad \mu_e = \mu^{(NM)}_c. \quad (2) $$

In this case, the pressure, $P^{(NM)}$, energy density, $\varepsilon^{(NM)}$ and baryon number density, $n^{(NM)}$, are functions of potentials, $\mu^{(NM)}_b$ and $\mu^{(NM)}_c$. The particle species chemical potentials for $udse$-plasma in $\beta$-equilibrium are expressed through the chemical potentials, $\mu^{(QM)}_b$ and $\mu^{(QM)}_c$:

$$ \mu_u = \frac{1}{3} \left( \mu^{(QM)}_b - 2 \mu^{(QM)}_c \right), \quad \mu_d = \mu_s = \frac{1}{3} \left( \mu^{(QM)}_b + \mu^{(QM)}_c \right), \quad \mu_e = \mu^{(QM)}_c. \quad (3) $$

In this case, the pressure, $P^{(QM)}$, energy density, $\varepsilon^{(QM)}$ and baryon number density, $n^{(QM)}$, are functions of chemical potentials $\mu^{(QM)}_b$ and $\mu^{(QM)}_c$.

The mechanical and chemical equilibrium conditions (Gibbs conditions) for the mixed phase are:

$$ \mu^{(QM)}_b = \mu^{(NM)}_b, \quad \mu^{(QM)} = \mu^{(QM)}_c, \quad (4) $$

$$ P^{(QM)}(\mu_b, \mu_c) = P^{(NM)}(\mu_b, \mu_c). \quad (5) $$
We applied the global electrical neutrality condition for mixed quark-nucleonic matter, according to Glendenning \cite{5},

\[
(1 - \chi) (n_p - n_e) + \chi \left( \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e \right) = 0.
\]

(6)

Here \( \chi = V_{QM}/(V_{QM} + V_{NM}) \) is the volume fraction of the quark phase, where \( V_{QM} \) and \( V_{NM} \) are the volumes occupied by the quark matter and nucleonic matter, respectively.

The baryon number density and energy density in the mixed phase are:

\[
n = (1 - \chi) (n_p + n_n) + \frac{1}{3} \chi (n_u + n_d + n_s),
\]

(7)

\[
\varepsilon = (1 - \chi) (\varepsilon_p + \varepsilon_n) + \chi (\varepsilon_u + \varepsilon_d + \varepsilon_s) + \varepsilon_e.
\]

(8)

In case of \( \chi = 0 \), the chemical potentials, \( \mu^N \) and \( \mu^Q \), corresponding to the lower threshold of the mixed phase, are determined solving Eqs. (5) and (6). This allows to find the lower boundary parameters, \( P_N, \varepsilon_N \) and \( n_N \). Similarly, we calculate the upper boundary values of mixed phase parameters, \( P_Q, \varepsilon_Q \) and \( n_Q \), for \( \chi = 1 \). The system of Eqs. (5), (6), (7) and (8) makes possible to determine EoS of the mixed phase between this critical states.

Note, that in the case of an ordinary first-order phase transition both nuclear and quark matter are assumed to be separately electrically neutral, and at some pressure, \( P_0 \), corresponding to the coexistence of the two phases, their baryon chemical potentials are equal, i.e., \( \mu_N(P_0) = \mu_Q(P_0) \). Such a phase transition scenario is known as the phase transition with constant pressure (MC).

Table 1. Threshold parameters of the deconfinement phase transition for both MC and GC scenarios with and without \( \delta \)-meson field

| Model     | \( n_N \) (fm\(^{-3}\)) | \( n_Q \) (fm\(^{-3}\)) | \( P_N \) (MeV/fm\(^3\)) | \( P_Q \) (MeV/fm\(^3\)) | \( \varepsilon_N \) (MeV/fm\(^3\)) | \( \varepsilon_Q \) (MeV/fm\(^3\)) |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| GC with \( \delta \) | 0.241           | 1.448           | 16.9            | 474.4           | 235.0           | 1889.3          |
| GC without \( \delta \) | 0.260           | 1.436           | 18.0            | 471.3           | 253.8           | 1870.8          |
| MC with \( \delta \) | 0.475           | 0.650           | 93.3            | 93.3            | 503.3           | 723.5           |
| MC without \( \delta \) | 0.551           | 0.717           | 121.3           | 121.3           | 593.4           | 810.0           |

Table 1 represents the threshold parameter sets of the quark phase transition with and without \( \delta \)-meson field. It is shown that the presence of \( \delta \)-field alters the threshold characteristics of phase transition.

In Fig.1 we plot the quark matter volume fraction, \( \chi \), as a function of pressure, \( P \). The solid lines correspond to both scenarios MC and GC with \( \delta \)-meson field, while the dashed lines correspond to this scenarios without \( \delta \)-meson field.

In Fig.2 we plot the particle species number densities as a function of the baryon density, \( n \). the upper panel corresponds to the MC scenario and the lower one corresponds to the GC scenario. The MC scenario leads to appearance of a discontinuity.
The charge neutral nucleonic matter at the baryon density $n_N = 0.475$ fm$^{-3}$ coexists with the charge neutral quark matter at the baryon density $n_Q = 0.650$ fm$^{-3}$. Thus, the density range $n_N < n < n_Q$ is forbidden. In GC scenario the quarks appear at the critical density $n_N = 0.241$ fm$^{-3}$ and the hadronic matter completely disappears at $n_Q = 1.448$ fm$^{-3}$, where the pure quark phase occurs.

3. Conclusion

We show that the scalar isovector $\delta$-meson field inclusion leads to the increase of the EOS stiffness of nuclear matter due to the splitting of proton and neutron effective masses, and also due to the increase of the asymmetry energy. The presence of $\delta$-meson field alters the threshold characteristics of deconfinement phase transition. The lower threshold parameters, $n_N, \varepsilon_N, P_N$, for GC scenario decrease, meanwhile the upper ones, $n_Q, \varepsilon_Q, P_Q$, slowly increase. In case of MC scenario the coexistence pressure, $P_0$, decreases. These alterations of the phase transition parameters can lead to the corresponding alterations of structural and integral characteristics of neutron stars with quark degrees of freedom.

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