Coupled channels calculation of a $\pi\Lambda N$ quasibound state

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(Dated: May 21, 2010)

We extend the study of a $J^P = 2^+, I = \frac{3}{2}$ $\pi\Lambda N$ quasibound state [Phys. Rev. D 78, 014013 (2008)] by solving nonrelativistic Faddeev equations, using $^3S_1 - ^3D_1$, $\Lambda N - \Sigma N$ coupled channels Chiral Quark Model local interactions, and $\pi N$ and coupled $\pi\Lambda - \pi\Sigma$ separable interactions fitted to the position and decay parameters of the $\Delta(1232)$ and $\Sigma(1385)$ resonances, respectively. The results exhibit a strong sensitivity to the $p$-wave pion-hyperon interaction, with a $\pi\Lambda N$ quasibound state persisting over a wide range of acceptable parametrizations.

PACS numbers: 13.75.Ev, 13.75.Gx, 14.20.Pt, 11.80.Jy

Keywords: hyperon-nucleon interactions, pion-baryon interactions, dibaryons, Faddeev equations

I. INTRODUCTION

The success of the nonrelativistic quark model (QM) during the 1970s in reproducing the SU(3) octet and decuplet baryon masses in terms of $3q$ configurations was followed by QM studies of $6q$ configurations that aimed particularly at elucidating the baryon-baryon short-range dynamics and making related predictions for dibaryon bound or quasibound states. It is remarkable that decades of experimental searches for dibaryons have so far yielded no unambiguous evidence for a dibaryon state. In the nonstrange sector, where the quark cluster calculations for $L = 0$ $6q$ configurations suggest only a weakly bound $\Delta\Delta$ dibaryon with $(J^P, I) = (3^+, 0)$, there is a recent indication from $np \rightarrow d\pi\pi$ reactions at CELSIUS-WASA for a resonance structure at $M_R \approx 2.36$ GeV and $\Gamma_R \approx 80$ MeV that might suggest a $\Delta\Delta$ dibaryon bound by about 100 MeV, but still about 200 MeV above the $d\pi\pi$ threshold [2]. In the strange sector, Jaffe’s dibaryon $H [3]$ with strangeness $S = -2$ and quantum numbers $(J^P, I) = (0^+, 0)$ which was predicted as a genuinely bound state well below the $\Lambda\Lambda$ threshold, perhaps the most cited ever prediction made for any dibaryon, has not been confirmed experimentally to date in spite of several extensive searches [4]. Another equally ambitious early prediction was made by Goldman et al. [5], also using a variant of the MIT bag model, for $(J^P; I) = (1^+, 2^+; \frac{1}{2})$ $S = -3$ dibaryons dominated by $\Omega N$ structure and lying below the $\Xi\Lambda$ threshold. More realistic quark cluster calculations by Oka et al. [6], applying resonating group techniques, did not confirm Jaffe’s deeply bound $H$, placing it just below the $\Xi N$ threshold as a resonance about 26 MeV above the $\Lambda\Lambda$ threshold. The underlying binding mechanism common to all of these orbital angular momentum $L = 0$ configurations is the dominance of the color-magnetic interaction for gluon exchange between quarks, a feature emphasized by Oka [7] who systematically studied $L = 0$ dibaryon configurations.
that may benefit from a short-range attraction. Following earlier quark cluster calculations, these calculations resulted in no strange dibaryon bound states, and for the $\Omega N$-dominated $S = -3$ bound-state configurations predicted in Ref. 3, in particular, only a $(J^P, I) = (2^+, \frac{1}{2})$ quasibound state resulted.

For strangeness $S = -1$, which is the focus of the present work, no $L = 0$ dibaryons have been suggested for the lowest energy $I = \frac{1}{2} \pi \Lambda N - \Sigma N$ coupled channels, where the long-range pion exchange interaction is dominant, particularly for the $^3S_1 - ^3D_1$ system through the tensor component. Although old $K^- d \to \pi^- p$ data had suggested resonant $\Lambda p$ structures at the $\Sigma N$ threshold and 10 MeV above it, a $(J^P, I) = (1^+, \frac{1}{2}) \Sigma N$ quasibound state is not necessarily required in order to reproduce the general shape of the $\Lambda p$ spectrum as shown by multichannel Faddeev calculations. Several low-lying $L = 1 \Lambda N$ resonances were predicted in singlet and triplet configurations in a QM study by Mulders et al., but negative results, particularly for the singlet resonance, were reported in dedicated $K^-$ initiated experiments, near the $\Sigma N$ threshold. At higher energies, Oka’s analysis drew attention to a $(J^P, I) = (2^+, \frac{1}{2})$ dibaryon resonating about the $\Sigma \Delta(1232)$ threshold, approximately 100 MeV above the lower $\Sigma(1385)N$ threshold. We note that these two channels are substantially higher in mass, by about 300 MeV, than the $S = -1$ thresholds of $\Lambda N$ and $\Sigma N$.

In a recent paper, we studied within three-body Faddeev calculations the possible existence of a $\pi \Lambda N$ quasibound state, driven by the two-body $(2S+1)LJ, I = (2P_{\frac{3}{2}}, \frac{3}{2}) \pi N$ resonance $\Delta(1232)$ and the $(2P_{\frac{1}{2}}, 1) \pi \Lambda$ resonance $\Sigma(1385)$, for a $\Lambda N$ $(^3S_1, \frac{1}{2})$ configuration, all of which were represented by means of single-channel separable potentials. The coupling to the pionless $\Sigma N$ channel, with threshold about 60 MeV below the $\pi \Lambda N$ threshold, was disregarded. It was felt that this coupling was mostly responsible for the width of the $\pi \Lambda N$ quasibound state. The three-body channel $(J^P, I) = (2^+, \frac{3}{2})$ was selected since all the angular momenta, spins, and isospins in this channel have maximum values and, therefore, is likely to benefit from maximal attraction of both $\Delta(1232)$ and $\Sigma(1385)$ resonances. This opportunity is unique to strange and charmed systems: a similar choice of $(J^P, I) = (2^+, 2)$ for $\pi NN$, with each $\pi N$ pair interacting in the $(\frac{3}{2}^+, \frac{3}{2})$ resonating channel, implies a $(1^+, 1)$ Pauli-forbidden $NN$ configuration. In terms of dibaryons, the $\pi \Lambda N$ $(J^P, I) = (2^+, \frac{3}{2})$ quasibound state is a deeply bound $\Sigma(1385)N - \Lambda \Delta(1232)$ $L = 0$ dibaryon, at energy considerably below the $(J^P, I) = (2^+, \frac{3}{2}) \Sigma(1385)N - \Sigma \Delta(1232)$ $L = 0$ dibaryon suggested by the quark cluster model of Ref. 4.

Whereas the interactions in the pion-baryon resonating channels in first approximation are adequately represented by rank-one attractive separable potentials, the baryon-baryon interaction requires a rank-two separable potential to simulate both the attraction and repulsion that meson-exchange models normally yield. Indeed, we found a strong dependence of the calculated $\pi \Lambda N$ binding energy on the balance between repulsion and attraction in the $^3S_1 \Lambda N$ channel. It is therefore suggestive to consider a more realistic hyperon-nucleon interaction in the $J^P = 1^+$ channel. In the present work we used the hyperon-nucleon $(YN)$ Chiral Quark Model (CQM) interaction described in Refs. 13, 16, in terms of $^3S_1 - ^3D_1$, $\Lambda N - \Sigma N$ coupled channels local potentials. For consistency, we also generalized our previous single-channel model of $\Sigma(1385)$ as a $\pi \Lambda$ resonance to a family of pion-hyperon ($\pi Y$) interaction models, in terms of $\pi \Lambda - \pi \Sigma$ coupled channels separable interactions fitted to the position, width and decay branching ratios of $\Sigma(1385)$. Furthermore, we studied the dependence of the calculated $\pi \Lambda N$ binding energy on the $\pi Y$ interaction.

In our previous work, based on separable potentials, we considered both a nonrelativistic and a relativistic three-body formalism from which we deduced that the nonrelativistic results do not change much when the relativistic
formalism is used instead. This is relevant for the validity of the results of the present work which are based on the hyperon-nucleon interaction derived from the CQM within a nonrelativistic formalism. Therefore, in the present calculation we consider only a nonrelativistic framework. The results of the present three-body Faddeev calculations leave wide room for the existence of a \((2^+, \frac{2}{3})\) \(\pi \Lambda N\) quasibound state indicating, however, a strong dependence on the short-range behavior of the least known \(\pi Y\) and \(YN\) two-body subsystems.

The plan of the paper is as follows. In Sec. II we describe the two-body interactions in the pion-nucleon, pion-hyperon and hyperon-nucleon subsystems. In Sec. III we derive the Faddeev equations of the pion-nucleon-hyperon system. Finally, we discuss our results in Sec. IV and summarize the work in Sec. V.

II. THE TWO-BODY SUBSYSTEMS

We will denote the hyperon, nucleon, and pion as particles 1, 2, and 3, respectively, and refer to the two-body subsystems by a subscript for the spectator particle. Thus, pion-nucleon is subsystem 1, pion-hyperon is subsystem 2, and hyperon-nucleon is subsystem 3. The conventional reduced masses are given by

\[
\eta^\alpha_i = \frac{m_j m_k}{m_j + m_k}, \quad \nu^\alpha_i = \frac{m_i (m_j + m_k)}{m_i + m_j + m_k},
\]

(1)

where a superscript \(\alpha = \Lambda, \Sigma\) has been added to indicate whether particle 1 is a \(\Lambda\) or a \(\Sigma\) hyperon and, obviously,

\[
\eta_1^\Lambda \equiv \eta_1^\Sigma \equiv \eta_1 = \frac{m_\pi m_N}{m_\pi + m_N}.
\]

(2)

However, an average hyperon mass

\[
m_Y = \frac{m_\Lambda + m_\Sigma}{2}
\]

(3)

was used in the following reduced masses:

\[
\nu_2 = \frac{m_N (m_\pi + m_Y)}{m_N + m_\pi + m_Y}, \quad \nu_3 = \frac{m_\pi (m_N + m_Y)}{m_\pi + m_N + m_Y}.
\]

(4)

The \(\pi Y\) and \(YN\) amplitudes are given by \(2 \times 2\) matrices, to account for the coupling between \(\pi \Lambda\) and \(\pi \Sigma\) and between \(\Lambda N\) and \(\Sigma N\), respectively. The \(\pi N\) amplitude in the three-body system is also given by \(2 \times 2\) matrix, since the energy dependence of the two-body subsystem depends on whether the spectator particle is a \(\Lambda\) or a \(\Sigma\).

A. The pion-nucleon subsystem

Since the \(\pi N\) subsystem is dominated by the \(\Delta(1232)\) resonance, a rank-one separable interaction is considered sufficient:

\[
< p_1 | V_1 | p'_1 > = \gamma_1 g_1(p_1) g_1(p'_1),
\]

(5)

so that the corresponding two-body \(t\)-matrix is given by

\[
< p_1 | t_1(E) | p'_1 > = g_1(p_1) \tau_1(E) g_1(p'_1),
\]

(6)
where \( E = p_0^2/2\eta_1 \) with \( p_0 \) the correct relativistic \( \pi N \) center of mass (c.m.) momentum and

\[
\tau_1^{-1}(E) = 1/\gamma_1 - \int_0^\infty p_1^2 dp_1 \frac{g_1^2(p_1)}{E - p_1^2/2\eta_1 + i\epsilon}.
\] (7)

The form factor \( g_1(p_1) \) was obtained from a very good fit of the \( P_{33} \) phase shift \( ^{17} \) for \( 0 \leq T_{\text{lab}} \leq 250 \text{ MeV} \) in the form

\[
g_1(p_1) = p_1[\exp(-p_1^2/\beta_1^2) + A_1p_1^2\exp(-p_1^2/\alpha_1^2)],
\] (8)

with \( \gamma_1 = -0.03317 \text{ fm}^4, A_1 = 0.2 \text{ fm}^2, \beta_1 = 1.31 \text{ fm}^{-1}, \) and \( \alpha_1 = 3.2112 \text{ fm}^{-1} \).

In the three-body calculation, when the \( \pi N \) subsystem is embedded in the \( \pi YN \) system, the energy argument of the isobar propagator \( \tau_1(E) \) depends on whether the spectator hyperon is a \( \Lambda \) or a \( \Sigma \), so that the separable \( \pi N \) amplitude \( ^{45} \) takes the form

\[
t_1 = |g_1 > \left( \begin{array}{cc} \tau_\Lambda(q_1) & 0 \\ 0 & \tau_\Sigma(q_1) \end{array} \right) < g_1|,
\] (9)

where

\[
\tau_\alpha(q_1) = \tau_1(E - \delta_{\alpha\Sigma}\Delta E - q_1^2/2\eta_1^2); \quad \alpha = \Lambda, \Sigma,
\] (10)

with \( q_1 \) the relative momentum between the hyperon and the \( \pi N \) subsystem and

\[
\Delta E = m_\Sigma - m_\Lambda.
\] (11)

### B. The pion-hyperon subsystem

The \( \pi Y \) subsystem is dominated by the \( \Sigma(1385) \) \( p \)-wave resonance which decays mainly into \( \pi \Lambda \) and \( \pi \Sigma \) with branching ratios of \( (87.0 \pm 1.5)\% \) and \( (11.7 \pm 1.5)\% \), respectively \( ^{18} \). To account for the coupling \( \pi \Lambda - \pi \Sigma \), we assume a coupled channels separable interaction:

\[
< p_2|V_{2}^{\alpha\beta}|p_2' > = \gamma_2 g_2^\alpha(p_2)g_2^\beta(p_2'); \quad \alpha, \beta = \Lambda, \Sigma,
\] (12)

so that the corresponding two-body \( t \)-matrix is given by

\[
< p_2|t_2(E)|p_2' > = g_2^{\alpha}(p_2)\tau_2(E)g_2^{\beta}(p_2'); \quad \alpha, \beta = \Lambda, \Sigma,
\] (13)

with

\[
\tau_2^{-1}(E) = 1/\gamma_2 - \int_0^\infty p_2^2 dp_2 \frac{|g_2^{\alpha}(p_2)|^2}{E - p_2^2/2\eta_2^2 + i\epsilon} - \int_0^\infty p_2^2 dp_2 \frac{|g_2^{\beta}(p_2)|^2}{E - \Delta E - p_2^2/2\eta_2^2 + i\epsilon}.
\] (14)

Again, \( E = p_0^2/2\eta_2^\Lambda \) where \( p_0 \) is the correct relativistic \( \pi \Lambda \) c.m. momentum and \( \Delta E \) is chosen such that the \( \pi \Lambda \) momentum at the \( \pi \Sigma \) threshold has its correct value, that is

\[
\Delta E = \frac{[(m_\Sigma + m_\pi)^2 - (m_\Lambda + m_\pi)^2][(m_\Sigma + m_\pi)^2 - (m_\Lambda - m_\pi)^2]}{8\eta_2^\Lambda(m_\Sigma + m_\pi)^2}.
\] (15)
TABLE I: Five choices (A-E) of form factor parameters for the coupled channels \(\pi Y\) subsystem, Eqs. (12) and (18). The last line lists a single-channel \(\pi \Lambda\) sixth model (F) with \(c_2 = 0\). The last column lists values of the r.m.s. momentum [see Eq. (19)].

| Model | \(A_2\) (fm\(^2\)) | \(\alpha_2\) (fm\(^{-1}\)) | \(\gamma_2\) (fm\(^4\)) | \(c_2\) | \(\sqrt{< p_2^2 >_{g_2}}\) (fm\(^{-1}\)) |
|-------|----------------------|------------------|------------------|------|------------------|
| A     | 0.8                  | 2.41372          | -0.00604931      | 0.890227 | 4.11             |
| B     | 1.0                  | 2.29039          | -0.00552272      | 0.925591 | 3.91             |
| C     | 1.2                  | 2.20024          | -0.00501334      | 0.956818 | 3.76             |
| D     | 1.5                  | 2.10192          | -0.00433208      | 0.997300 | 3.60             |
| E     | 1.8                  | 2.03076          | -0.00375829      | 1.03166 | 3.48             |
| F     | 3.21                 | 2.20024          | -0.00149204      | 0     | 3.79             |

The \(\pi Y\) \(t\)-matrix (13) in the \(\pi Y N\) system may be written in compact notation as a 2 \(\times\) 2 matrix

\[
t_2 = \begin{pmatrix} g_2^\Lambda \gamma_2^\Sigma \end{pmatrix} \tau_N(q_2) \begin{pmatrix} < g_2^\Lambda | < g_2^\Sigma > \end{pmatrix},
\]

where

\[
\tau_N(q_2) = \tau_2(E - q_2^2/2\nu_2).
\]

The form factors \(g_2^Y(p_2)\) of the separable \(\pi Y\) \(p\)-wave potentials were taken in the form

\[
g_2^\Lambda(p_2) = p_2(1 + A_2 p_2^2)e^{-p_2^2/\alpha_2^2}, \quad g_2^\Sigma(p_2) = c_2 g_2^\Lambda(p_2).
\]

Solutions exist for all values of \(A_2\) between 0 and \(\infty\). Therefore, in order to fit the position, width and decay branching ratios of \(\Sigma(1385)\), we have at our disposal four free parameters: \(A_2, \alpha_2, \gamma_2\) and \(c_2\), which provide for varying one of these while adjusting the other three to the three pieces of data. We thus constructed five models (models A-E) by considering five values of the parameter \(A_2\), as shown in Table I. We also constructed a sixth model (model F) which shares the same range parameter \(\alpha_2\) with model C but which neglects the coupling to the \(\pi \Sigma\) channel \((c_2 = 0)\), as was done in our previous calculation [14]. It is instructive to classify the various \(\pi Y\) interaction form factors \(g_2^Y(p_2)\) according to their root-mean-square (r.m.s.) momentum, using the following expression for the mean-square momentum \(< p_2^2 >_{g_2}\):

\[
< p_2^2 >_{g_2} = \frac{\int_0^\infty g_2(p_2) p_2^4 d^3 p_2}{\int_0^\infty g_2(p_2) d^3 p_2} = 3\alpha_2^2 A_2 \alpha_2^2 + \frac{1}{2} \approx 3\alpha_2^2,
\]

where the approximation owes to \(2A_2 \alpha_2^2 \gg 1\). The resulting values of the r.m.s. momentum, listed in the last column of Table I, are close to \(\sqrt{< p_2^2 >_{g_2}} \approx 3.8\) fm\(^{-1}\) \(\approx 750\) MeV/c. For comparison, \(\sqrt{< p_1^2 >_{g_1}} = 5.55\) fm\(^{-1}\) \(\approx 1100\) MeV/c for the \(\pi N\) form factor \(g_1(p_1)\) of Eq. (3),\(^1\)

\(^1\) This high-momentum value for \(g_1\) does not rule out a spatial size of order 1 fm for \(\Delta(1232)\). Indeed, if \(\tilde{g}_1(r_1)\) is the Fourier transform of \(g_1(p_1)\), for \(\ell = 1\), then \(\sqrt{< r_1^2 >_{\tilde{g}_1}} = 0.875\) fm.
C. The hyperon-nucleon subsystem

The $YN$ interaction derived from the chiral quark model is a local potential obtained by application of the Born-Oppenheimer approximation to the chiral quark-quark interaction (consisting of confinement, one-gluon exchange, pseudovector-meson exchange, and scalar-meson exchange) with a fully antisymmetrized six-quark wave function \cite{15,16,19}. In the case of the $J^P = 1^+, I = \frac{1}{2}$ channel, it leads to the following system of coupled equations:

$$
t_{\alpha\beta}^{\ell\ell'}(p_3, p_3''; E) = V_{\alpha\beta}^{\ell\ell'}(p_3, p_3'') + \sum_{\gamma=\Lambda, \Sigma} \sum_{\ell'\ell''} \int_0^\infty p_3' dp_3' V_{\gamma\ell'\ell''}^{\alpha\gamma}(p_3', p_3'') \frac{1}{E - \gamma E - p_3''^2 / 2\eta_3' + i\epsilon} t_{\gamma\ell'\ell''}^{\alpha\beta}(p_3'', p_3''; E) : \alpha, \beta = \Lambda, \Sigma, \tag{20}\end{equation}$$

with $\alpha, \beta = \Lambda, \Sigma$, $\ell, \ell'' = 0, 2$ and $E = p_3^0 / 2\eta_3$, where $p_0$ is the correct relativistic $\Lambda N$ c.m. momentum, and $\Delta E$ is chosen such that the $\Lambda N$ momentum at the $\Sigma N$ threshold has its correct value, that is

$$\Delta E = \frac{[(m_\Sigma + m_N)^2 - (m_\Lambda + m_N)^2][2(m_\Sigma + m_N)^2 - (m_\Lambda - m_N)^2]}{8\eta_3^3 (m_\Sigma + m_N)^2}. \tag{21}\end{equation}$$

The $YN$ $t$-matrix \cite{20} may be written in compact notation as a $2 \times 2$ matrix

$$t_3 = \begin{pmatrix}
t_\Lambda^\Lambda & t_\Lambda^\Sigma \\
t_\Sigma^\Lambda & t_\Sigma^\Sigma
\end{pmatrix}, \tag{22}\end{equation}$$

where each $YN$ $t$-matrix $t^{\alpha\beta}$ includes, in addition, a coupling between $S (\ell = 0)$ and $D (\ell = 2)$ waves.

III. THE THREE-BODY EQUATIONS

The Faddeev equations for the bound-state problem

$$T_i = \sum_{j \neq i} t_i G_0 T_j; \quad i, j = 1, 2, 3, \tag{23}\end{equation}$$

couple the amplitudes $T_1$, $T_2$ and $T_3$ together. Eliminating the amplitude $T_3$ in favor of $T_1$ and $T_2$, one obtains

$$T_1 = t_1 G_0 t_3 G_0 T_1 + (t_1 + t_1 G_0 t_3) G_0 T_2, \tag{24}\end{equation}$$

$$T_2 = t_2 G_0 t_3 G_0 T_2 + (t_2 + t_2 G_0 t_3) G_0 T_1, \tag{25}\end{equation}$$

where, in order to allow for the $Y = (\Lambda, \Sigma)$ specification, one has

$$G_0 = \begin{pmatrix}
G_0^\Lambda & 0 \\
0 & G_0^\Sigma
\end{pmatrix}, \tag{26}\end{equation}$$

Since the two-body amplitudes $t_1$ and $t_2$ are separable [see Eqs. \cite{20} and \cite{16}], the three-body amplitudes $T_1$ and $T_2$ are of the form

$$T_1 = |g_1 > \begin{pmatrix}
x_\Lambda \\
x_\Sigma
\end{pmatrix}, \tag{27}\end{equation}$$

$$T_2 = |g_2 > \begin{pmatrix}
x_\Lambda \\
x_\Sigma
\end{pmatrix}. \tag{28}\end{equation}$$

where the subscript of the amplitude $X$ indicates which particle is the spectator. Substitution of (24) and (25) into (24) and (25) leads to

\[
\begin{pmatrix}
X_{\Lambda} \\
X_{\Sigma}
\end{pmatrix} =
\begin{pmatrix}
\tau_{\Lambda} & 0 \\
0 & \tau_{\Sigma}
\end{pmatrix} < g_1 < \begin{pmatrix}
G_{0}^{0} t^{\Lambda} & G_{0}^{0} t^{\Lambda \Sigma} G_{0}^{0} \\
G_{0}^{0} t^{\Sigma} & G_{0}^{0} t^{\Sigma \Sigma} G_{0}^{0}
\end{pmatrix} > \begin{pmatrix}
X_{\Lambda} \\
X_{\Sigma}
\end{pmatrix}
+ \begin{pmatrix}
\tau_{\Lambda} & 0 \\
0 & \tau_{\Sigma}
\end{pmatrix} < g_1 < \begin{pmatrix}
G_{0}^{0} + G_{0}^{0} t^{\Lambda \Lambda} G_{0}^{0} & G_{0}^{0} t^{\Lambda \Sigma} G_{0}^{0} \\
G_{0}^{0} t^{\Sigma \Lambda} G_{0}^{0} & G_{0}^{0} + G_{0}^{0} t^{\Sigma \Sigma} G_{0}^{0}
\end{pmatrix} > \begin{pmatrix}
|g_{1}^{2}| > \\
|g_{2}^{2}| >
\end{pmatrix} X_{N},
\]

(29)

\[
X_{N} = \tau_{N} \left( < g_{1}^{2} > < g_{2}^{2} > \right) \begin{pmatrix}
G_{0}^{0} + G_{0}^{0} t^{\Lambda \Lambda} G_{0}^{0} & G_{0}^{0} t^{\Lambda \Sigma} G_{0}^{0} \\
G_{0}^{0} t^{\Sigma \Lambda} G_{0}^{0} & G_{0}^{0} + G_{0}^{0} t^{\Sigma \Sigma} G_{0}^{0}
\end{pmatrix} > \begin{pmatrix}
|g_{1}^{2}| > \\
|g_{2}^{2}| >
\end{pmatrix} X_{N},
\]

(30)

which are integral equations in one continuous variable given explicitly by

\[
X_{\alpha}(q_{1}) = \tau_{\alpha}(q_{1}) \sum_{\beta=\Lambda,\Sigma} \int_{0}^{\infty} q_{1}{'}^{2} dq_{1}{'} K^{\alpha \beta}(q_{1}, q_{1}{'}) X_{\beta}(q_{1}{'})
+ \tau_{\alpha}(q_{1}) \int_{0}^{\infty} q_{2}{'}^{2} dq_{2}{'} K^{\alpha N}(q_{1}, q_{2}) X_{N}(q_{2}); \quad \alpha = \Lambda, \Sigma
\]

(31)

\[
X_{N}(q_{2}) = \tau_{N}(q_{2}) \sum_{\alpha=\Lambda,\Sigma} \int_{0}^{\infty} q_{1}{'}^{2} dq_{1}{'} K^{N \alpha}(q_{2}, q_{1}) X_{\alpha}(q_{1})
+ \tau_{N}(q_{2}) \int_{0}^{\infty} q_{2}{'}^{2} dq_{2}{'} K^{NN}(q_{2}, q_{2}{'}) X_{N}(q_{2}{'}).  
\]

(32)

The kernels of these integral equations are given in the Appendix.

### IV. RESULTS AND DISCUSSION

We applied the formalism of the previous section, using six different versions of the $YN$ interaction obtained from the CQM, all of which reproduce equally well the experimental low-energy $YN$ data. Results are listed in Table II from where it is clear that the $\pi \Lambda N$ binding energies are substantial for $\pi Y$ models with $A_{2} < 1 \text{ fm}^{2}$. Generally, the higher the r.m.s. momentum of the $\pi Y$ form factor $g_{2}$, the stronger is the binding, as demonstrated in the table. Irrespective of which $\pi Y$ model is chosen, the $YN$ interaction always produces repulsion, thus lowering the calculated binding energy, as demonstrated by the results listed in the last line which corresponds to switching off the $YN$ interaction. This repulsive $YN$ effect owes its origin to the high-momentum components of the $\pi B$ form factors which within the three-body calculation highlight the short-range repulsive region of the $YN$ interaction.

To demonstrate the model dependence of the three-body calculation within a given $\pi Y$ model, we assembled in Table III several binding energy results based on model C and also on its limitation to the $\pi \Lambda$ channel (model F...
TABLE II: Binding energy of $\pi\Lambda N$ (in MeV) for five $\pi Y$ interaction models (A-E of Table I) and six CQM versions of the $^3S_1-^3D_1$ $YN$ interaction fitted to given $\Lambda N$ scattering length $a$ and effective range $r_0$ (both in fm). The momentum $p_{lab}(\delta = 0)$ is the $\Lambda$ laboratory momentum (in MeV/c) where the $^3S_1$ $\Lambda N$ phase shift changes sign. The last line corresponds to switching off the $YN$ interaction.

| $a$   | $r_0$ | $p_{lab}(\delta = 0)$ | Model A | Model B | Model C | Model D | Model E |
|-------|-------|------------------------|---------|---------|---------|---------|---------|
| -1.35 | 3.39  | 987                    | 147     | 99      | 65      | 30      | 6       |
| -1.40 | 3.32  | 1011                   | 147     | 99      | 66      | 30      | 6       |
| -1.64 | 3.09  | 1146                   | 150     | 102     | 68      | 32      | 8       |
| -1.71 | 3.03  | 1198                   | 150     | 102     | 68      | 33      | 9       |
| -1.78 | 2.98  | 1272                   | 151     | 103     | 69      | 33      | 9       |
| -1.86 | 2.93  | 1446                   | 152     | 104     | 69      | 34      | 10      |
| -      | -     | -                      | 170     | 120     | 74      | 47      | 21      |

TABLE III: Comparison of $\pi\Lambda N$ binding energies (in MeV) calculated within the $\pi Y$ model C and its $\pi\Lambda$ limit model F (see Table I) for $YN$ coupled channels and $\Lambda N$ single-channel models with $a = -1.40$ fm, and for no $YN$ interaction.

| $\pi\Lambda$, $\Lambda N$ | $\pi\Lambda$, $YN$ | $\pi Y$, no $YN$ | $\pi Y$, $\Lambda N$ | $\pi Y$, $YN$ |
|---------------------------|---------------------|------------------|---------------------|----------------|
| 93                        | 96                  | 84               | 73                  | 66             |

listed in Table I. The $YN$ models included in this table invariably give $a = -1.40$ fm, whether limited to the $\Lambda N$ $^3S_1$ single channel or extended to the $^3S_1 - ^3D_1$ $\Lambda N - \Sigma N$ coupled channels. Comparing the first two entries to each other, we conclude that the extension from a single $\Lambda N$ channel to $\Lambda N - \Sigma N$ coupled channels has very little effect (about 3 MeV additional attraction) within the $\pi\Lambda$ model F. In contrast, for the full $\pi Y$ model C, as in the last two entries, the extension from $\Lambda N$ to $YN$ models has a somewhat larger effect (about 7 MeV repulsion) and in the opposite direction. Within the $\pi Y$ model C, the full coupled channels $YN$ interaction contributes 18 MeV repulsion (third and fifth entries in Table III) to the three-body binding energy. Similar results hold for all other $\pi Y$ models.

To discuss the model dependence of the three-body calculation within a given $YN$ CQM, we follow Ref. [14] in singling out $p_{lab}(\delta = 0)$, the momentum where the $\Lambda N$ $^3S_1$ phase shift changes sign from attraction outside to repulsion inside, as a measure of the repulsive $YN$ effect. We notice in Table II that the CQM values of $p_{lab}(\delta = 0)$ are considerably larger than those obtained by other models [20–22], signifying less repulsion in the CQM. In order to test whether the apparent lack of repulsion in the CQM $YN$ interaction is responsible for the large binding energies obtained for $A_2 < 1$ fm$^2$, we added to the CQM with $a = -1.40$ fm and $r_0 = 3.32$ fm a short-range potential in the $^3S_1$ $\Lambda N$ partial wave of the form

$$V(r) = \gamma_R e^{-\beta_R r} - \gamma_A e^{-\beta_A r},$$

(33)

with $\beta_R = 10$ fm$^{-1}$ and $\gamma_R \geq 1000$ MeV fm, while the attractive term was adjusted to maintain the $\Lambda N$ scattering length $a = -1.35$ fm and the effective range $r_0$ as close as possible to 3.39 fm, so that the $YN$ observables are not changed noticeably. The overall effect of $V(r)$ is repulsive, as demonstrated in Table IV for the $\pi Y$ model A, with $A_2 = 0.8$ fm$^2$, where it is clearly seen that increase in the strength of the repulsive term lowers the value of $p_{lab}(\delta = 0)$ as well as lowering the $\pi\Lambda N$ binding energy.
TABLE IV: Binding energy of $\pi\Lambda N$ (in MeV) for the $\pi Y$ model $A$ ($A_2 = 0.8$ fm$^2$) and the CQM $YN$ interaction plus a short-range $\Lambda N$ potential $V(r)$, Eq. (53), with scattering length $a = -1.40$ fm and effective range $r_0 = 3.32$ fm. The strength parameters $\gamma$ are in units of MeV fm, the inverse range parameter $\beta_A$ is in units of fm$^{-1}$, $\beta_R = 10$ fm$^{-1}$, and $p_{lab}(\delta = 0)$ is the laboratory momentum (in MeV/c) where the $3S_1 \Lambda N$ phase shift changes sign.

| $\gamma_R$ | $\gamma_A$ | $\beta_A$ | $p_{lab}(\delta = 0)$ | $B(A_2 = 0.8)$ |
|------------|------------|-----------|----------------------|----------------|
| 1000       | 240        | 5.371     | 873                  | 107            |
| 2000       | 530        | 5.811     | 846                  | 88             |
| 3000       | 720        | 5.749     | 822                  | 70             |
| 4000       | 990        | 5.928     | 810                  | 59             |
| 5000       | 1260       | 6.056     | 802                  | 51             |
| 6000       | 1360       | 5.921     | 788                  | 39             |
| 7000       | 1670       | 6.086     | 775                  | 34             |

V. SUMMARY

In this work, we have extended the Faddeev equations study of a $(J^P, I) = (2^+, \frac{3}{2})$ quasibound $\pi\Lambda N$ state [14] from a $3S_1 \Lambda N$ single-channel to $3S_1 - 3D_1$, $\Lambda N - \Sigma N$ coupled channels, and from a $\pi\Lambda$ single-channel description of $\Sigma(1385)$ to $\pi\Lambda - \pi\Sigma$ coupled channels description. Local interaction potentials given by the CQM were used in the $YN$ sector, whereas one-rank separable potentials were used in the $\pi B$ sectors. We have shown within a nonrelativistic version of the Faddeev equations, but using semirelativistic kinematics, that the $\pi\Lambda N$ system is bound under a wide choice of parametrizations of the $\pi Y$ interaction form factor. The form factors of the $\pi B$ subsystems are sufficiently short ranged such that the pion undergoes almost coherently attraction to both baryons. The short-ranged repulsion between the two baryons in the CQM is insufficient to overcome the attraction gained by the pion unless the CQM is modified arbitrarily at very short distances to do this job. Altogether, the acceptable model dependence of the $\pi Y$ interaction form factor, and the uncertainty of the short-range behavior of the $YN$ interaction, leave plenty of room, theoretically, for a quasibound $S = -1, (J^P, I) = (2^+, \frac{3}{2})$, $\pi\Lambda N$ dibaryon.

Before closing we list several production reactions, where the first two were already discussed in our previous paper [14], in which to search for this $S = -1$ dibaryon here denoted $D$:

$$K^- + d \rightarrow D^- + \pi^+ , \quad \pi^- + d \rightarrow D^- + K^+ , \quad (34)$$

$$p + p \rightarrow D^+ + K^+ . \quad (35)$$

Correlated with the missing mass spectrum of the $D$ dibaryon, for a forward outgoing meson, one should look for $\Sigma N$ decays that can be assigned to a $\Sigma N$ resonance with invariant mass $M_D$. Total cross sections for the associated strangeness production $pp \rightarrow \Sigma N K^+$ near the hyperon production threshold have been reported from Juelich, for $\Sigma^0 p$ by the COSY-11 Collaboration [23], for $\Sigma^+ n$, also by COSY-11 [24], and by the ANKE Collaboration [25] and the HIRES Collaboration [26], with conflicting results among all these $\Sigma^+ n$ reports. Old DISTO data for the reaction $pp \rightarrow \Lambda p K^+$ have been analysed to search for an intermediate $K^- pp$ quasibound state, with the astounding report of a broad resonance at the $\pi \Sigma N$ threshold [27]. Of course, this $I = \frac{1}{2}$ resonance cannot be assigned to a $I = \frac{3}{2} \pi \Lambda N$
quasibound state, but forthcoming data from the FOPI detector Collaboration at GSI \cite{28} could be analysed also with respect to a ΣN rather than a Λp final state.

Acknowledgments

The research of H.M. was supported in part by COFAA-IPN (México) and that of A.G. by the EU initiative FP7, HadronPhysics2, under Project 227431.

Appendix: Expressions for the kernels of the integral equations Eqs. (31) and (32)

We provide here detailed expressions for the kernels appearing in the integral equations Eqs. (31) and (32).

\[
K^{\alpha\beta}(q_1, q_1') = \frac{1}{4} \sum_{\ell, \ell' = 0, 2} \int_0^\infty q_3^2 dq_3 \int_{-1}^1 dcos \theta \int_{-1}^1 dcos \theta' \\
\times g_1(p_1^\alpha) G_0^\alpha(p_1^\alpha, q_1) b_{13}^0 A_{13,13}^{\ell\ell'}(q_1, q_3, \cos \theta) \\
\times t_{\ell\ell'}^{\alpha\beta}(p_3^\alpha, p_3^\beta; E - q_3^2/2\nu_3) b_{31}^\beta A_{31,31}^{\ell\ell'}(q_3, q_1', \cos \theta') G_0^\beta(p_1^\beta, q_1') g_1(p_1^\beta), \quad (36)
\]

\[
K^{\alpha N}(q_1, q_2) = \frac{1}{2} \int_{-1}^1 dcos \theta g_1(p_1^\alpha) G_0^\alpha(p_1^\alpha, q_1) b_{12}^0 A_{12,12}^{\ell\ell'}(q_1, q_2, \cos \theta) q_2^2(p_2^\alpha) \\
+ \frac{1}{4} \sum_{\ell, \ell' = 0, 2} \sum_{\beta = \Lambda, \Sigma} \int_0^\infty q_3^2 dq_3 \int_{-1}^1 dcos \theta \int_{-1}^1 dcos \theta' \\
\times g_1(p_1^\alpha) G_0^\alpha(p_1^\alpha, q_1) b_{13}^\beta A_{13,13}^{\ell\ell'}(q_1, q_3, \cos \theta) \\
\times t_{\ell\ell'}^{\alpha\beta}(p_3^\alpha, p_3^\beta; E - q_3^2/2\nu_3) b_{32}^\beta A_{32,32}^{\ell\ell'}(q_3, q_2, \cos \theta') G_0^\beta(p_2^\beta, q_2) g_2^\beta(p_2^\beta), \quad (37)
\]

\[
K^{N\alpha}(q_2, q_1) = K^{\alpha N}(q_1, q_2), \quad (38)
\]

\[
K^{N\alpha}(q_2, q_2') = \frac{1}{4} \sum_{\ell, \ell' = 0, 2} \sum_{\alpha = \Lambda, \Sigma} \int_0^\infty q_3^2 dq_3 \int_{-1}^1 dcos \theta \int_{-1}^1 dcos \theta' \\
\times g_2^\alpha(p_2^\alpha) G_0^\alpha(p_2^\alpha, q_2) b_{23}^\beta A_{23,23}^{\ell\ell'}(q_2, q_3, \cos \theta) \\
\times t_{\ell\ell'}^{\alpha\beta}(p_3^\beta, p_3^\alpha; E - q_3^2/2\nu_3) b_{32}^\beta A_{32,32}^{\ell\ell'}(q_3, q_2', \cos \theta') G_0^\beta(p_2^\beta, q_2') g_2^\beta(p_2^\beta), \quad (39)
\]

with

\[
G_0^\alpha(p_1, q_1) = \frac{1}{E - \delta_0 \Lambda \Delta - p_1^2/2\eta_1 - q_1^2/2\nu_1 + i\epsilon}, \quad \alpha = \Lambda, \Sigma. \quad (40)
\]

The orbital angular momentum recoupling coefficients \( A_{ij,\Lambda;\alpha}^{\ell_1,\ell_2,\lambda}(q_i, q_j, \cos \theta) = A_{ij,\Lambda;\alpha}^{\ell_1,\ell_2,\lambda}(q_j, q_i, \cos \theta) \), and isospin recoupling coefficients \( b_{ij}^\alpha = b_{ij}^\alpha \), are calculated by consideration of a cyclic pair \( ij \). (The spin recoupling coefficients are all equal to 1.) For isospin we have
\[ b_{ij}^\alpha = (-)^{i_j+\tau_j-i} \sqrt{(2i_i+1)(2i_j+1)} W(\tau_j \tau_i; i_i i_j), \] (41)

where \( W \) is a Racah coefficient. If \( \alpha = \Lambda \) then \( \tau_1 = 0, \tau_2 = \frac{1}{2}, \tau_3 = 1, i_1 = \frac{3}{2}, i_2 = 1, i_3 = \frac{1}{2}, \) and \( I = \frac{3}{2} \) so that \( b_{12}^\Lambda = b_{23}^\Lambda = 1 \). If \( \alpha = \Sigma \) we have instead \( \tau_1 = 1 \) so that \( b_{12}^\Sigma = \sqrt{5/6}, b_{31}^\Sigma = -\sqrt{5/3}, \) and \( b_{23}^\Sigma = -1/\sqrt{6} \).

The orbital angular momentum recoupling coefficients are given by

\[ A_{\ell_i \lambda_i \ell_j \lambda_j}^{L \alpha}(q_i, q_j, \cos \theta) = \frac{1}{2L+1} \sum_{M_{m_i}, m_j} C_{\ell_i \lambda_i \ell_j \lambda_j}^{L M_{m_i}, M_{m_j}} C_{m_j, M_{m_j}; M_{m_i}, M} \Gamma_{\ell_i, m_i} \Gamma_{\lambda_i, M_{m_i}} \cos (-M \theta - m_i \theta_i^\alpha + m_j \theta_j^\alpha); \quad \alpha = \Lambda, \Sigma, \] (42)

where \( \Gamma_{\ell m} = 0 \) for odd values of \( \ell - m \), and

\[ \Gamma_{\ell m} = (-1)^{(\ell+m)/2} \sqrt{(2\ell+1)(2\ell+1)!/[(\ell-m)!2^l][(\ell+m)!2^l]}, \] (43)

for even values of \( \ell - m \). The angles \( \theta_i^\alpha \) and \( \theta_j^\alpha \) are obtained from

\[ \cos \theta_i^\alpha = -\frac{q_i \cos \theta + q_i a_{ij}^\alpha}{p_i^\alpha}, \] (44)

\[ \cos \theta_j^\alpha = \frac{q_j \cos \theta + q_j a_{ji}^\alpha}{p_j^\alpha}, \] (45)

\[ p_i^\alpha = \sqrt{q_i^2 + (q_i a_{ij}^\alpha)^2 + 2q_i q_j a_{ij}^\alpha \cos \theta}, \] (46)

\[ p_j^\alpha = \sqrt{q_j^2 + (q_j a_{ji}^\alpha)^2 + 2q_i q_j a_{ji}^\alpha \cos \theta}, \] (47)

where

\[ a_{ij}^\alpha = \frac{m_j}{m_j + m_k}, \quad a_{ji}^\alpha = \frac{m_i}{m_i + m_k}, \] (48)

with \( m_i = m_\alpha; \alpha = \Lambda, \Sigma \). Equations (46) and (47) provide also the relative momenta appearing in Eqs. 36)-39.

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