WEAK-SCALE IMPLICATIONS OF THERMAL LEPTOGENESIS IN SUSY *

SACHA DAVIDSON
Department of Physics
Durham University
Durham, DH1 3LE, GB
E-mail: sacha.davidson@durham.ac.uk

Thermal leptogenesis is an attractive mechanism for generating the baryon asymmetry of the Universe. However, in supersymmetric models, the parameter space is severely restricted by the gravitino bound on the reheat temperature $T_{RH}$. Using a parametrisation of the seesaw in terms of left-handed inputs, which are related to weak-scale observables in mSUGRA, the low-energy footprints of thermal leptogenesis are discussed.

1. Introduction

Neutrinos are observed to have small mass differences, of order $10^{-5} - 10^{-2}$ eV$^2$. Sadly, this is not a prediction of Supersymmetry. However, the seesaw mechanism is a natural way to generate such small majorana neutrino masses, and, as an added bonus, it provides “for free” a way to make the cosmological baryon asymmetry (by leptogenesis$^2$). The seesaw can easily be supersymmetrized.

These proceedings summarise (my) attempts to relate leptogenesis to weak-scale observables. The approach is “bottom-up”: I want to avoid inputting a GUT/texture/theoretical model for the structure of the Yukawa couplings and mass matrices. They are based on work with Alejandro Ibarra and Ryuichiro Kitano, who I thank for illuminating and productive collaborations. This proceedings aims to be “bedtime reading”; the paper is certainly longer and I hope more careful.

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*see 6 for more complete references.
Small neutrino masses and the observed baryon asymmetry of the Universe (BAU) are evidence for Beyond-the-Standard-Model physics. Can they both be explained by the seesaw? And if yes, does this have observable consequences?

The answer to the first question is “yes”. Unfortunately, there are no observable consequences of generating the BAU by leptogenesis in the Standard Model seesaw. This is expected from parameter counting: the high scale seesaw model has 18 parameters, whereas the effective light neutrino mass matrix has only 9. An interesting approach, which has been followed by many people, is therefore to construct theoretically motivated models for the neutrino Yukawa matrix $Y_\nu$ and the majorana mass matrix $M$ of the heavy right-handed neutrinos.

The aim here is different. We wanted to study leptogenesis and the seesaw from a more phenomenological “bottom-up” perspective—which is possible in the supersymmetric (SUSY) version. There are nine additional low-energy parameters in the sneutrino mass matrix, which receives contributions from $Y_\nu$ and $M$ through the Renomalisation Group Equations (RGEs). It is therefore possible in principle to reconstruct the high scale seesaw parameters $Y_\nu$ and $M$ from the neutrino and sneutrino mass matrices. This “reconstruction” is in practise impossible—it would require unrealistically accurate measurements—but it is a useful parametrisation of the seesaw.

Using this parametrisation, we can express the baryon asymmetry as a function of weak scale inputs, and study the low-energy footprints of thermal leptogenesis.

2. Notation and Assumptions

The leptonic superpotential in the seesaw can be written

$$W = W_{MSSM} + \nu_R^T Y_\nu L \cdot H_u - \frac{1}{2} \nu_R^T M \nu_R,$$

where $L_i$ are the left-handed lepton doublets, $H_u$ is the hypercharge $+1/2$ Higgs doublet, $Y_\nu$ is the neutrino Yukawa matrix, and $M$ is a Majorana mass matrix, with heavy eigenvalues which are assumed hierarchical: $M_1 \ll M_2 < M_3$.

Two relevant bases for the $\nu_R$ vector space are the one where the mass matrix $M$ is diagonal ($= D_M$), and where the Yukawa matrix $Y_\nu Y_\nu^T$ is diagonal ($= D_Y^2$). The unitary matrix $V_R$ transforms between these bases,
so in the mass eigenstate basis

\[ Y_\nu Y_\nu^\dagger = V_R^\dagger D_Y^2 V_R \]  

(2)

At low energies, well below the \( \nu_R \) mass scale, the light (LH) neutrinos acquire an effective Majorana mass matrix \( [m_\nu] \). In the vector space of LH leptons, there are three interesting bases—the one where the charged lepton Yukawa \( Y_e^\dagger Y_e \) is diagonal, the one where the neutrino Yukawa \( Y_\nu^\dagger Y_\nu \) is diagonal, and the basis where \( [m_\nu] \) is diagonal. The first \( (D_{Y_e}) \) and last \( (D_{m}) \) are phenomenologically important, and are related by the MNS matrix \( U \): \( [m_\nu] = U^* D_m U^\dagger \) in the \( D_{Y_e} \) basis. The second \( (D_{Y_\nu}) \) relative to the first \( (D_{Y_e}) \) can be important for phenomenology in SUSY models, where \( Y_\nu^\dagger Y_\nu (\equiv V_L^\dagger D_Y^2 V_L \text{ in the } D_{Y_e} \text{ basis}) \) induces flavour violation via its appearance in the slepton RGEs. The second \( (D_{Y_\nu}) \) and third \( (D_{m}) \) are useful to relate LH and RH seesaw parameters, so are appropriate for connecting weak-scale observables with leptogenesis.

The matrix \( W \) transforms between these bases. In the basis where \( Y_\nu \) is diagonal, \( [m_\nu] \) can be written

\[ [m_\nu] = D_Y M^{-1} D_Y v_u^2 = W^* D_m W^\dagger \]  

(3)

The light neutrino masses are taken hierarchical, with \( 10^{-3} m_\nu_2 < m_\nu_1 < 0.1 m_\nu_3 \). It is assumed that the largest eigenvalue of \( Y_\nu \), \( y_3 \simeq 1 \), and that there is a steeper hierarchy in the eigenvalues of \( Y_\nu \) than in those of the light neutrino mass matrix \( [m_\nu] \).

Twenty-one parameters are required to fully determine the Lagrangian of eqn (1). If \( Y_e \) is neglected, only 9 real numbers and 3 phases are required. These can be chosen in various ways. To relate the RH parameters relevant for leptogenesis to the LH ones, many of which are accessible at low energy, it is useful to consider the following possibilities:

1. “top-down”—input the \( \nu_R \) sector: \( D_M, D_{Y_e} Y_e^\dagger \), and \( V_R \).
2. “bottom-up”—input the \( \nu_L \) sector: \( D_\kappa, D_{Y_\nu} Y_\nu^\dagger \), and \( W \).

We assume gravity-mediated SUSY breaking, with universal soft masses at some scale \( m_X \gg M_i \), and nothing but SUSY and the seesaw between the electroweak scale and \( m_X \) (so we know the RGEs). The leading log approximation for the slepton mass matrix is used to relate angles of \( V_L \) to \( \ell_j \to \ell_i \gamma \) branching ratios. So \( W \) can be “calculated” from \( \nu \) and \( \nu \) mixing matrices.
3. Leptogenesis

The baryon asymmetry produced via leptogenesis depends on the $\nu_R$ number density, the $\mathcal{C}\mathcal{P}$ asymmetry in the $\nu_R$ decay, and whether the decay is out of equilibrium. A cosmology-independent way to produce the $\nu_R$ is by scattering in the thermal plasma after inflation. For hierarchical right-handed neutrinos, this “thermal leptogenesis” scenario can be described by 4 parameters:\footnote{The decay rate can be rescaled to be comparable to a light neutrino mass. The usual\footnote{4} leptogenesis parameter is $\bar{m}_1 = 8\pi \Gamma (H_u^0)^2 / M_1^2$.}:

- The lightest $\nu_{R1}$ mass $M_1$, its decay rate $\Gamma$, which controls the $\nu_{R1}$ production and decay processes,
- The $\mathcal{C}\mathcal{P}$ asymmetry $\bar{\epsilon}$ in the decay,
- An average neutrino mass $\bar{m}$ (which I do not discuss here).

There is an upper bound on $\bar{\epsilon}$:\footnote{4} (but see\footnote{8}): \(\bar{\epsilon} = \frac{\Gamma(\nu_R \to H\ell) - \Gamma(\nu_R \to H\bar{\ell})}{\Gamma + \bar{\Gamma}} = \frac{8\pi M_1 m_{\nu_2}}{3(H_u^0)^2} \delta, \delta \leq 1\)\ (4)

The BAU produced in thermal leptogenesis can be written

\[ Y_B = d(\Gamma) \bar{\epsilon} = \begin{cases} 3 - 9 \times 10^{-11} & BBN \\ 7.5 - 1.0 \times 10^{-11} & CMB \end{cases} \] \ (5)

where $d(\Gamma)$ is the ratio of the $\nu_R$ number density to the entropy density, times the fraction of the produced lepton asymmetry which survives as a baryon asymmetry today. $d(\Gamma)$ depends on the interactions of the $\nu_R$ in the plasma, and has been numerically calculated\footnote{9} to have a maximum value of $\sim 3 \times 10^{-4}$. A large enough BAU can be obtained if

\[ \left( \frac{6 \times 10^{-11}}{Y_B} \right) \left( \frac{d(g_*, \Gamma)}{3 \times 10^{-4}} \right) \left( \frac{M_1}{10^9 \text{GeV}} \right) \delta \gtrsim 1 \] \ (6)

There are additional constraints on the thermal leptogenesis scenario in SUSY models. In gravity-mediated SUSY-breaking, gravitino production imposes an upper bound on the reheat temperature of the Universe after inflation: $T_{RH} \lesssim 10^9 - 10^{12} \text{ GeV}$. The canonical bound is $T_{RH} \lesssim 10^9 \text{ GeV}$, and

\[ M_1 \lesssim T_{RH} \] \ (7)

is required to produce enough $\nu_R$.\footnote{9}
4. low-energy footprints

In the parametrisation of $Y_\nu$ and $M_i$, in terms of $[m_\nu]$ and the sneutrino mass matrix $[m_{\tilde{\nu}}^2]$, there is an analytic approximation for the leptogenesis parameters $M_1, \Gamma$ and $\delta$, in terms of the light neutrino masses $m_\nu$, a matrix $W = V_L U$ which rotates from the $\nu_i$ mass eigenstate basis to the basis where $Y_\nu$ is diagonal (asymmetry rotation from the neutrino to sneutrino mass eigenstate bases), and the smallest eigenvalue $y_1$ of $Y_\nu$.

The low energy consequences of thermal leptogenesis can be found by requiring eqns (6) and (7) be satisfied. This constrains $M_1$ to sit in a narrow range around $10^9$ GeV, and $\epsilon$ to be maximal. $M_1 \sim 10^9$ GeV determines $y_1$ as a function of $W$ and the $m_\nu$. Since $y_1$ is effectively unmeasurable in our parametrisation (it affects the first generation slepton masses via the RGEs, which for $y_1 \sim 10^{-3} - 10^{-4}$ is a negligible effect), this has no observable consequences at low energy. The $\nu_R$ decay rate $\Gamma$ naturally falls within the desirable range, so the low energy consequences of eqn (6) correspond to $\delta \to 1$.

For $M_1 \sim 10^9$ GeV, $\delta$ must be $O(1)$ (and $d_1$ maximal) to obtain a baryon asymmetry at the lower end of the BBN range. This arises for $W$ near the identity, which corresponds to mixing angles in the sleptons sector of order the neutrino mixing angles. This suggests that the branching ratios for $\tau \to \mu \gamma$ or $\tau \to e \gamma$ should be observable. From a model-building perspective, $W \sim I$ could arise if the large MNS angles arise from diagonalising the charged lepton Yukawa $Y_e$.

For $M_1 \sim 10^{10}$ GeV, a large enough baryon asymmetry can be obtained for $W \sim U$, provided that $W_{13} \sim 0.04$. This corresponds to an observable CHOOZ angle $\theta_{13} \sim 0.04$, or observable $\tau \to e \gamma$, ...or to no observable consequences at all (It is unfortunately possible to have $W_{13} \sim 0.04$ with arbitrarily small CHOOZ angle and lepton flavour violating branching ratios). The case $W \sim U$ arises in many models, where the large mixing angles of the neutrino sector come from diagonalising $[m_\nu]$ in the “texture” basis.

Figure 1 shows contours of constant $Y_B$, labelled by $f = 1, 3, 6$ and 9. $Y_B \gtrsim 2 \times 10^{-11}$ inside the curve, for $M_1 = f \times 10^9$ GeV. The variables on the axes are chosen to provide as “physical” a measure on parameter space as possible. They are vaguely related to logarithms of measurable quantities: $\omega_{13} \sim \theta_{13} + \sqrt{10^8 BR(\tau \to e\gamma)} + \text{something unmeasurable}$, and $\chi_{12} \sim \sqrt{10^8 BR(\tau \to e\gamma)} + \sqrt{10^8 BR(\tau \to \mu\gamma)}$. 
Figure 1. Contour plot of $Y_B$, as a function of $\omega_{13} \approx \log[W_{13}]$, and $\chi_{12} \approx \log[V_{L12} + V_{L13}]$. The contours enclose the area when $Y_B > 2 \times 10^{-11}$, for $M_1 = f \times 10^9$ GeV, central values of $m_{\nu_3}$ and $m_{\nu_2}$, and $m_{\nu_1} = m_{\nu_2}/10$. In the direction of increasing area, the lines correspond to $f = 1, 3, 6$ and 9.

5. Summary

Thermal leptogenesis can work in supersymmetric seesaw models. It makes low energy predictions because the available parameter space is restricted. Observing lepton flavour violating decays, such as $\tau \rightarrow \ell\gamma$, or a CHOOZ angle $\sim 0.04$ would lend support to this scenario.

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