Synthesizing Short-Circuiting Validation of Data Structure Invariants

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Abstract
This paper presents incremental verification-validation, a novel approach for checking rich data structure invariants expressed as separation logic assertions. Incremental verification-validation combines static verification of separation properties with efficient, short-circuiting dynamic validation of arbitrarily rich data constraints. A data structure invariant checker is an inductive predicate in separation logic with an executable interpretation; a short-circuiting checker is an invariant checker that stops checking whenever it detects at run time that an assertion for some sub-structure has been fully proven statically. At a high level, our approach does two things: it statically proves the separation properties of data structure invariants using a static shape analysis in a standard way but then leverages this proof in a novel manner to synthesize short-circuiting dynamic validation of the data properties. As a consequence, we enable dynamic validation to make up for imprecision in sound static analysis while simultaneously leveraging the static verification to make the remaining dynamic validation efficient. We show empirically that short-circuiting can yield asymptotic improvements in dynamic validation, with low overhead over no validation, even in cases where static verification is incomplete.

Categories and Subject Descriptors D.2.4 [Software Engineering]: Software/Program Verification; F.3.1 [Logics and Meanings of Programs]: Specifying and Verifying and Reasoning about Programs

Keywords short-circuiting validation; incremental verification-validation; subtraction-directed synthesis; data structure invariants; separation logic; shape analysis

1. Introduction
We consider the problem of efficient and safe dynamic validation of deep, program-specific data structure invariants. Such invariants combine pointer-shape properties (e.g., “1 is an acyclic, singly-linked list”) with data-value properties (e.g., “the integer data elements of 1 are sorted in non-decreasing order”). Combined shape-data properties are both difficult for the programmer to maintain and challenging for fully-automatic static analysis to verify. While modern shape analyzers can reason effectively about pointer-shape properties—especially for structures with limited sharing, such as lists and trees—shape-data analyzers are limited by their dependency on and interaction with base data-value abstract domains or solvers [6, 7, 16, 17].

Static verification has always held the promise to improve software by definitively ruling out entire classes of bugs before execution. But this promise holds for a particular program of interest only if the developer is able to obtain a full verification for it. Dynamic validation compensates for the shortcomings of static verification by trapping unsafe operations at run time (e.g., by halting execution or raising an exception). Several systems [4, 11, 23] have made run-time checks easier to apply; however, run-time enforcement of data structure invariants, in particular, have remained problematic for two main reasons:

Checking Overhead. The overhead of run-time checking can be significant because a data structure invariant is a universal property over a set of objects—and thus checking the invariant is usually linear-time in the size of the structure. Since adding dynamic validation can change the asymptotic complexity of data structure operations, validation can cause dramatic slowdowns. Figure 1 shows such slowdowns for a workload of binary search tree operations on increasing data structure sizes. The dynamic validation variant (dynV) can only reach a binary search tree with 300 nodes in 15 seconds, while the no validation variant (noV) can reach 20,000,000 nodes in 5 seconds. Because of the linear-time complexity of validation, the dynV variant is $O(n)$, while the noV variant is $O(lgn)$ where $n$ is the size of the tree. Thus, frequent checking of data structure invariants can be infeasible even in debugging scenarios.

Additional Risk of Memory Faults. Run-time checking may introduce new, unexpected pointer bugs. Traversing pointers in a malformed data structure could lead to unanticipated bad dereferences (e.g., on null or freed pointers) or even non-termination (e.g., on unexpected cyclic structures). Thus, such checking may introduce unacceptable risk in deployment scenarios.

In this paper, we address these concerns by applying static shape analysis to synthesize short-circuiting dynamic data structure validation efficiently.
validation to reduce run-time checking overhead and (2) statically prove separation properties to eliminate the risk of memory faults.

Contributions. This paper is a response to the inevitable imprecision of static shape-data analysis. While a static analysis can always be made precise enough to verify any given set of benchmarks (with an appropriate amount of research, development, and specification effort), no analysis can ever be precise enough for an ever growing set. Our approach is a step towards providing the benefit of incremental verification-validation even in the face of imprecise invariant inference or insufficient specification. To clearly distinguish when some part of a data structure invariant is checked, we reserve verification for static checking and validation for dynamic checking.

In Section 2, we discuss the notion of short-circuiting invariant checkers: validation code that leverages a hypothetical fact to short-circuit dynamic validation of a data structure invariant. Short-circuit validation is the result of a simple but key observation: when asked to prove $P(x)$, we can decompose this proof into proving, for example, $P(y)$ for some $y$ and separately that $x = y$. This observation enables us to connect facts that we verify statically (e.g., $P(y)$) and facts that we need to validate dynamically (e.g., $x = y$). In particular, we get the benefit of short-circuiting validation when we carefully choose the facts to validate dynamically to be fast and cheap to check yet helpful in making the static verification more feasible. A significant challenge in this work is that $P$ is inductive and effective short-circuiting can only be obtained when static verification and dynamic validation are interleaved in the recursion.

To do so, we make the following technical contributions:

- We describe a static shape-data analysis needed to synthesize short-circuiting validation (Section 3) driven by the aforementioned proof decomposition. In particular, we introduce the validated view abstract domain, which augments the static analysis with the ability to track arbitrarily complex, uninterpreted data-value facts (e.g., to statically prove $P(y)$). Then, we describe a code instrumentation that reifies, as programs variables, the existential logic variables in the statically-inferred shape-data invariants. Conceptually, this logic-variable reification provides hooks at run time to the proof constructed by the static shape-data analysis (e.g., to dynamically validate $x = y$).

- We introduce a subtraction-directed synthesis approach to generate both short-circuiting checkers and the calls to them, using invariants inferred by the static shape-data analysis and refined logic variables (Section 4). The result is incrementalized dynamic validation of inductive data structure invariants with no run-time memoization.

- We empirically evaluate our approach by applying a prototype tool (called IVVA) to verification and validation of operations with complex shape-data invariants (Section 5). We see evidence of asymptotic improvements in dynamic validation cost—for example, the IVV line in Figure 1. The improvements hold even for invariants with data properties beyond the reach of most static verifiers (e.g., involving hashing and bit masking).

Our mixed static-dynamic technique differs from prior fully dynamic, run-time approaches to reducing checking overhead. Ditto [27] incrementalizes data structure validation checks by merging validation calls and then only executing subsequent checks for the objects for which the memoized checks have been invalidated (e.g., by writes). This fully dynamic approach essentially trades off the execution overhead of validation checks for the memory overhead of maintaining a memo table in a shadow heap. In contrast, our approach leverages a static proof of separation properties to synthesize incremental dynamic data structure validation without the memory overhead of memoization or the execution overhead of write barriers.

2. Goal: Short-Circuiting Validation

In this section, we consider a simple but illustrative example of a data structure operation with an assertion of its invariant. We illustrate that this assertion can be incrementally verified and validated in a sound manner.

We consider binary search trees here, as the data structure requires a well-known intertwined shape-data invariant. Our technique also applies to (and is even more important for) more complex data invariants. In Section 5, we evaluate our implementation on a hash trie, which is a tree-structured hash map that uses bit-blocks from hash keys to form a trie [3]. This structure employs bit-masking, hash codes, and rehashing on collision. Even for state-of-the-art static verifiers, such as Thor [16], this data structure invariant is challenging and likely out of reach for current techniques [15]. Another important feature of our approach is the ability to obtain incremental dynamic validation even when the pre-condition specification is insufficient (not strong enough) to statically prove the post-condition, which we demonstrate here with the setroot example in Figure 2.

We consider a C type declaration for a binary tree $bt$:

```c
typedef struct node { val v; struct node* l; struct node* r; } *bt;
```

Each node contains three fields: $v$, which stores a data value, and fields $l$ and $r$ that contain pointers to left and right sub-trees. We leave unspecified the type of data values ($val$). In Figure 2a, we give an inductive definition for a binary search tree in a standard separation logic notation. As a convention, we use hatted letters like $\hat{t}$ for symbolic logic variables used in static analysis and write inductive definitions like $bst$ with a distinguished recursion parameter as in $\hat{t} \cdot bst(\ldots)$. A binary search tree is a binary tree where the data values obtained from the in-order traversal are ordered—and thus the search tree invariant specification $bst$ includes the allowable range for the data values in that sub-tree. The $\hat{t} \cdot bst(\hat{min}, \hat{max})$ predicate states that $\hat{t}$ points to a binary tree with allowable range $[\hat{min}, \hat{max}]$; we emphasize with shading the parts of the $bst$ definition capturing the search data property. As is well-understood, using separating conjunction $\ast$ between the root node pointed to by $\hat{t}$ and the sub-trees pointed to by $\hat{t}$ and $\hat{r}$ specifies that $\hat{r}$ is structurally a tree (i.e., does not have sharing or cycles); in other words, the non-shaded parts of the definition correspond the binary tree shape property.

In Figure 2b, we rewrite the inductive definition $bst$ in a shorthand that suggests an executable interpretation. Disjunctions are expressed as if-then-else conditionals and existential variables correspond only to values obtained from dereferencing the recursion parameter $\hat{t}$. Note that this definition is simply a shorthand and has the same meaning as the definition in Figure 2a expressed in a more standard notation.

Now consider the `setroot` procedure shown in Figure 2c. The programmer states the assumption (i.e., pre-condition) that $t$ on entry points to a binary search tree satisfying $t \cdot bst(\neg \infty, \infty)$ (line 1) and is non-null (line 2). We write $\infty$ and $\neg \infty$ for the compile-time constants corresponding to the minimum and maximum values in the $val$ type (e.g., `INT_MIN` and `INT_MAX`), respectively. In this procedure, she then sets the data value at the root node to the passed-in parameter $v$ (line 3) and then wants to assert that the modified tree is still a binary search tree with the `assert` at line 4. First, note that the `assert` at line 4 is not provable statically because the search property may be violated after the assignment on line 3, as there are no constraints on the value of $v$. Thus, it would be unsound for any static verifier to eliminate this `assert`. Second, observe that a dynamic validation of $t \cdot bst(\neg \infty, \infty)$ is a linear-time operation in the number of nodes of the binary tree $t$ and furthermore has the non-trivial obligation to dynamically validate the separation constraints—even though the binary tree shape is unchanged and the only data value that was modified was at the root node.
(a) An inductive definition \( \text{bst} \) for binary search trees in separation logic.

\[
\begin{align*}
\text{\text{bst}(min, max)} & \triangleq \text{emp} \land \neg \text{root} \\
\text{\text{bst}(t, min, max)} & \triangleq \text{if } \text{root} \text{ then } \text{assert}(t, min, max) \\
& \quad \land \text{min} \leq t \land t < \text{max}
\end{align*}
\]

(b) The inductive definition \( \text{bst} \) from (a) written in a shorthand for checkers. A dereference expression \( \ast \rightarrow v \) corresponds to the value of an implicitly separated points-to predicate (e.g., the referencing expression \( t \ast \rightarrow v \) corresponds to \( v \) in the points-to predicate \( \hat{v} \rightarrow \hat{v} \) from (a)).

\[
\begin{align*}
\text{void } \text{setroot}(bt \, t, \text{val } v) \{ \\
\text{assume}(t \ast \text{bst}(\infty, \infty)); \\
\text{assume}(t \neq \text{null}); \\
t \ast \rightarrow v = v; \\
\text{assert}(t \ast \text{bst}(\infty, \infty)); \\
\text{bstsc}(t \ast \rightarrow l, t \ast \rightarrow v, \text{max, oldv}); \\
& \text{bstsc}(t \ast \rightarrow r, t \ast \rightarrow v, \ast \rightarrow \text{min}, oldv, v); \\
\}
\end{align*}
\]

(c) With full verification.

\[
\begin{align*}
\text{bool } \text{bstsc}(bt \, n, \text{val } \text{min}, \text{val } \text{max}, \text{val } \text{bmin}, \text{val } \text{bmax}) \{ \\
& \text{assume}(n \ast \text{bst}((\text{bmin}, \text{bmax}))); \\
& \text{if } (\text{min} = \text{bmin} \land \text{max} = \text{bmax}) \text{ return true; } \\
& \text{return } n == \text{NULL} \lor \text{true} : \text{min} \leq n \ast \rightarrow v \land n \ast \rightarrow v < \text{max} \land \\
& \quad \text{bstsc}(n \ast \rightarrow l, n \ast \rightarrow v, \text{max, newv, bmin, bmax}); \\
& \quad \text{bstsc}(n \ast \rightarrow r, n \ast \rightarrow v, \text{min, newv, bmax}); \\
\}
\end{align*}
\]

(d) With short-circuiting validation.

\[
\begin{align*}
\text{void } \text{setroot}(bt \, t, \text{val } v) \{ \\
& \text{assume}(t \ast \text{bst}(\infty, \infty)); \\
& \text{assume}(t \neq \text{null}); \\
& \text{val } \text{oldv} = t \ast \rightarrow v; \\
& t \ast \rightarrow v = v; \\
& \text{assert}(t \ast \text{bst}(\infty, \infty)); \\
& \text{bstsc}(t \ast \rightarrow l, \text{oldv}, \text{min}, t \ast \rightarrow v, \text{newv, bmin, bmax}); \\
& \text{bstsc}(t \ast \rightarrow r, t \ast \rightarrow v, \infty, \text{oldv, v}); \\
\}
\end{align*}
\]

(e) Compilation of the short-circuiting invariant checker \( \text{bstsc} \) from Figure 3c to C code (once separation has been proven statically).

(f) Illustrating short-circuiting validation, executing lines 4–6 of (d), on a binary search tree where the data value at the root node was changed from 8 to 9 by \text{setroot}. The four-tuple labels correspond to the last four arguments of calls to \text{bstsc}.

Figure 2. Validating the binary search tree invariant in \text{setroot} after the data value of the root node is set to \( v \).

**Incremental Verification-Validation.** Even if full static verification of a data structure check is not possible (as with the \text{assert} on line 4 in the \text{setroot} procedure), surprisingly we can still use the incomplete static reasoning to incrementalize the check. Specifically, our incremental verification-validation approach on an \text{assert} consists of two phases. First, it tries to statically verify the assertion using invariants derived by a static shape analysis. If it is successful, no dynamic validation is needed and the \text{assert} can be fully eliminated. If it is partially successful in that the separation properties are statically proven, then it proceeds to synthesize a short-circuiting, incremental assertion to replace the original, full-structure assertion. This short-circuiting, incremental assertion is easy to express in C code because it does not need to check separation properties that were verified statically. If the static verifier is not able to prove separation, our tool simply raises an alarm and does not proceed to synthesis.

Returning to the \text{assert} at line 4 in the \text{setroot} procedure, we can reason about the binary search tree invariant to see that a change to the data value at the root node could only possibly invalidate the invariant along the right spine of the left sub-tree (i.e., these are the only nodes whose allowable range \((\text{min}, \text{max})\) could have changed). An incremental validation—one that takes advantage of the knowledge that \( t \) was a binary search tree before the update—needs to traverse at most these two paths and thus can re-validate the invariant while traversing only a logarithmic number of nodes. We diagram these two paths in Figure 2f with squares for the traversed nodes and with triangles for the skipped sub-trees (ignore the caption and the number labels for the moment). As we will see, lines 4–6 of Figure 2d indeed traverse at most the two paths discussed above by replacing the assertion to the \text{bst} predicate with calls to the synthesized short-circuiting invariant checker \text{bstsc} shown in Figure 2e. The rest of this section will focus on describing how one would define \text{bstsc} to obtain this short-circuiting validation.

As alluded in Section 1, the basic idea behind a short-circuiting checker like \text{bstsc} is to make explicit a hypothesis of which it can take advantage. This idea can expressed in the following correctness condition for \text{bstsc} that we call its safe replacement criterion:

**Criterion (Safe Replacement with a Short-Circuiting Invariant Checker).** For \text{bstsc}, it is the case that

\[
\begin{align*}
& \text{if } \text{bst}(h\text{min}, h\text{max}) \\
& \quad \text{then } \text{bstsc}(\text{min, max}, h\text{min}, h\text{max}) \Rightarrow \text{bst}(h\text{min}, h\text{max})
\end{align*}
\]

For clarity, we first consider deriving the short-circuiting checker \text{bstsc} in separation logic and then subsequently consider the compilation to C code. Observe that the short-circuiting checker \text{bstsc} takes two additional parameters \( h\text{min} \) and \( h\text{max} \) (standing for hypothesis \text{min} and \text{max}, respectively) compared to the original checker \text{bst}. The validation hypothesis is explicitly \( \text{bst}(h\text{min}, h\text{max}) \). This validation hypothesis may be used to return early (i.e., short-circuit checking) when validating the desired property \( \text{bst}(\text{min, max}) \). In other words, a short-circuiting checker does not need to validate the desired property directly but can do so under the assumption that the validation hypothesis holds.

A trivial implementation of \text{bstsc} that satisfies the safe replacement criterion is to ignore the validation hypothesis and be defined as \( \text{bst}(\text{min, max}) \)—of course this offers no benefit over \text{bst} itself. A slightly smarter variant, \text{bstsc}_a in Figure 3a, explicitly assumes the validation hypothesis on line 1 and then checks a short-circuiting condition on line 5 (shown shaded): if the goal is exactly the hypothesis then it can soundly short circuit and return true early without further checking. This short circuiting is, however, shallow—if the short-circuiting condition is not satisfied, the entire tree rooted at \( n \) is traversed by \text{bst}.

To arrive at a deep short-circuiting, let us consider \text{bstsc}_b in Figure 3b, which is semantically equivalent to \text{bstsc}_a. In \text{bstsc}_b, we have inlined the calls to \text{bst}. In other words, logically, we have unfolded the inductive predicate given by \text{bst} in both the hypothesis and the goal. By unfolding the hypothesis, we see that in the case that \( \neg \text{bst} \), we also have that \( \neg t \rightarrow v \ast \text{bst}(\text{null, } \text{min}, \text{max}) \). We use these hypotheses (shown shaded) to derive the deep short-circuiting checker \text{bstsc} by rewriting the recursive \text{bst} checks on lines 7–8 on the left and right sub-trees with recursive calls to the short-circuiting checker. The validation hypotheses of these recursive checks, \( \neg t \rightarrow v \ast \text{bstsc}(\text{null, } \text{min}, \text{max}, \text{null, } \text{max}) \) and \( \neg t \rightarrow v \ast \text{bstsc}(\text{null, } \text{min}, \text{null}, \text{max}) \), are satisfied by the hypotheses about
n→1 and n→r, respectively. This deep short-circuiting checker \( \text{bstsc} \) allows many more opportunities to apply a short-circuiting hypothesis and return early (because the short-circuiting is checked on each recursive call), thus avoiding a full traversal of the tree.

Intuitively, the previous paragraph describes synthesizing the deep short-circuiting checker \( \text{bstsc} \) by following the proof of the safe replacement criterion:

\[
\begin{align*}
\text{if } & \min = \text{hmin} \land \max = \text{hmax} \text{ then } \text{true} \\
\text{else } & \text{hbst(min, max)}
\end{align*}
\]

and abducting a definition for \( \text{bstsc} \). In Section 4, we see that \( \text{bstsc} \) is synthesized by attempting to prove

\[
\begin{align*}
\text{n·bstc(hmin, hmax)} & \implies \text{n·bstc(min, max)} \\
& \text{(†)}
\end{align*}
\]

while permitting abduction of pure, data constraints. We call this process subtraction-directed synthesis because the proof strategy follows the standard approach of unfolding and subtraction to prove separation. When \( \text{bstsc} \) is successfully synthesized, we have that the spatial part of the above implication (†) holds. Thus, it is straightforward to compile the \( \text{bstsc} \) definition from Figure 3c to C code using the \&\&-conjunction operator as shown in Figure 2e because we have guaranteed that the separation properties hold statically (and thus do not need them to be validated dynamically).

**Short-Circuiting Validation of setroot.** We now return to our claim that the check on lines 4–6 in the short-circuiting validation version of setroot shown in Figure 2d traverses at most two paths in tree \( t \). This is illustrated in Figure 2f, which considers an example where a call to setroot changes the data value at the root node from the value 8 to the value 9. The four-tuple labels on each node indicate the \((\min, \max, \text{hmin}, \text{hmax})\) arguments passed to \( \text{bstsc} \) at that node from the initiating calls on lines 5 and 6. This short-circuiting version skips sub-trees that are not on the aforementioned two paths because their roots satisfy the short-circuiting guard condition (i.e., that \( \min \equiv \text{hmin} \&\& \max \equiv \text{hmax} \)).

Automatic synthesis of short-circuiting checkers like \( \text{bstsc} \) is vastly preferable, for the developer, to manual checker transformations. While it is perhaps possible for a clever developer to rewrite the validation routines themselves by hand to be short-circuiting, rewriting \( \text{asserts} \) to make appropriate calls to these routines is challenging and error-prone because of the need to modify code to expose particular data values. An automated technique is needed to statically prove the validation hypothesis of the short-circuiting invariant checker (including the separation properties) and determine the appropriate arguments to it. The goal of this paper is to automate this process of incrementalization—eliminating the tediousness and guaranteeing the safety of the transformation.

**Soundness: Assumptions and Guarantees.** The expected input for our approach is a program with \( \text{asserts} \) of data structure invariants expressed in separation logic like \( \text{bst} \). If modular verification-validation is desired, pre-conditions can be expressed additionally with \( \text{assumes} \). Like with any modular verification technique, we assume that user-specified \( \text{assumes} \) are sound pre-conditions (which could be guaranteed at each call site with corresponding \( \text{asserts} \)). The verification-validation problem that we address is to try to prove all \( \text{asserts} \). If successful, the output of our technique is a static verification of the separation properties (e.g., a proof that \( t \) is a binary tree) and a rewriting of the \( \text{asserts} \) to use short-circuiting dynamic validation (e.g., an incremental run-time check that the binary tree \( t \) has the search property). If unsuccessful, the output is an alarm for each assert where it was unable to statically verify the separation properties. That is, like any other static verifier for separation logic, it soundly rejects programs where it cannot prove the separation properties. As an alternative to rejecting such programs, one could consider dynamically validating such unproven separation properties using, for example, global heap coloring techniques [20].

**Challenges: Synthesizing Short-Circuiting Validation.** As we have seen, short-circuiting validation checkers can potentially reduce the asymptotic complexity of validation while providing the same benefits as whole-structure checkers. Automatic generation of short-circuiting checkers requires addressing two key challenges:

1. **Challenge 1** Synthesis to transform whole-structure assertions (like \( \text{bst} \)) into calls to short-circuiting checkers (like \( \text{bstsc} \)). This process must generate both the additional hypothesis arguments for such calls and add the necessary scalar assertions. For example, the \( \text{assert} \) on lines 4–6 in Figure 2d correspond to unfolding the original assertion \( t \cdot \text{bst}(-\infty, \infty) \) once to account for the change at the root node.

2. **Challenge 2** Synthesis of deep short-circuiting invariant checkers (like \( \text{bstsc} \)) for general inductive data structure invariants (like \( \text{bst} \)).

3. **Challenge 3** A technical challenge that arises to support the above is determining how to connect statically verified invariants with dynamic validation. For example, the new variable \( \text{old}v \) must be instrumented into the program on line 3 so that it can be used in the \( \text{assert} \) on lines 4–6 in Figure 2d.

The rest of this paper describes how we address these challenges with a synthesis approach based on inductive shape-data analysis.

**Overview.** Our approach consists of three processes that we show schematically in Figure 4:

1. **Static shape-data analysis** infers loop invariants in separation logic. It determines when previously established data-value facts have been preserved—and can thus be relied upon, at run time, during validation. A validated view is a constraint we introduce to track arbitrary data constraints in materialized memory regions (Section 3.1).
To describe how our validated view abstraction enriches a shape-data abstraction, we first discuss an example shape-data abstraction. We realize this precise tracking of unmodified portions by augmenting a shape-data analysis with what we call a validated view abstraction (Section 3.1).

2. Logic-variable reification transforms the program to explicitly connect logic variables in the static analysis invariants to concrete values at run time by constructing a programmatic valuation (Section 3.2). This code instrumentation enables run-time assertion checking to rely upon invariants discovered during static analysis.

3. Subtraction-directed synthesis of short-circuiting validation automatically transforms expensive whole-data-structure validation asserts into cheaper, short-circuiting asserts that can soundly skip portions of the data structure at run time (Section 4). To synthesize the code for short-circuiting checkers, we fix a template (i.e., a sketch [29]) similar to the shallow short-circuiting checker bstsc in Figure 3a and then reuse our synthesizer on the template code to fill it in with deep short-circuiting, as in bstsc (Figure 3c). Short-circuiting synthesis can raise an alarm if the spatial part of the safe replacement criterion cannot be proven statically to soundly synthesize short-circuiting checkers like bstsc.

4. Assertion execution with the rewritten asserts in a normal execution environment realizes incremental invariant validation whenever the short-circuiting condition is triggered. If a data property is violated on execution, an assertion failure is raised.

3. Reifying Static Analysis Invariants

The static shape-data analysis needed to synthesize short-circuiting validation code from statically-inferred shape-data invariants must address two problems. First, it must have the ability to prove existential weakenings of shape-data invariants even when it cannot prove the original (e.g., prove $\exists u. A \cdot k(u)$ when $A \cdot k(v)$ cannot be proved). While this is challenging in general, we identify a particularly important scenario for short-circuiting validation: tracking arbitrarily complicated data-value constraints in unmodified portions of the data structure. This precise tracking of unmodified portions can be seen as a static analogue of memoization in incremental computation. We realize this precise tracking of unmodified portions by augmenting a shape-data analysis with what we call a validated view abstraction (Section 3.1).

Second, it must provide access at run time to concrete values corresponding to existentially-quantified symbolic variables in statically-proven invariants. We provide this access by reading off, from the proof, a program expression derived from the pre-condition to witness each symbolic variable (Section 3.2).

In this section, we extend our running example to both illustrate the challenges introduced in Section 2 and provide intuitions for how our analysis and synthesis techniques address them. We will focus on an example-driven discussion here, and we separately formalize the algorithms for validated views and logic-variable reification in in Appendix B and Appendix C, respectively.

3.1 Validated Views

To describe how our validated view abstraction enriches a shape-data abstraction, we first discuss an example shape-data abstraction.

Preliminaries: Inductive Shape-Data Analysis. Inductive shape analysis [7, 10] uses inductive predicates like bst to statically summarize unbounded memory regions. A memory region that satisfies bst includes the pointer-shape property (i.e., is a binary tree) but also the data-value property (i.e., that it satisfies the search invariant). Thus, such a summarized bst memory region is statically known to satisfy the binary search tree invariant. On the left side of Figure 5a, we express a shape-data invariant as a separating shape graph. This invariant says a root $t$ satisfies bst with minimum value $\min$ and maximum value $\max$. Here the nodes represent pointers (i.e., memory addresses), and the edges represent abstract memory regions. The thick arrow can be read as a static fact indicating that the memory region rooted at $t$ satisfies the inductive predicate $t \cdot \text{bst}(\min, \max)$.

In inductive shape analysis, the key operations are (1) materializing abstractions of single memory cells from summarized regions by unfolding inductive predicates (left-to-right in Figure 5a) and (2) summarizing memory regions by folding into inductive predicates, using any number of unary abstraction or binary widening operators (right-to-left). Consider statically analyzing an iteration traversing into the middle of a binary search tree with a cursor pointer $p$ in Figure 5b, we show a precisely inferred loop invariant that is crucial to be able to synthesize short-circuiting validation. Here, we indicate that the program variable $p$ contains the address $a_p$ by annotating the program variable below the node representing $a_p$. We adopt the naming convention that $a_s$ is the symbolic address of the binary search tree node pointed-to by program variable $s$, and $V_s$ is the symbolic data value of that node (i.e., $x \cdot X$). The memory region between addresses $t$ and $a_p$ is described by a bst segment [7], in a form of separating implication, corresponding to a validation tree with a hole [18] at $a_p$. The thick arrow between nodes $t$ and $a_p$ is read as knowing statically that the memory region from $t$ satisfies the validation check $t \cdot \text{bst}(\min, \max)$ up to checking $a_p \cdot \text{bst}(\min, \max)$. The pair of symbolic values $\min$ and $\max$ correspond to the lower and upper bounds needed for a binary search tree rooted at $a_p$ in order for the tree rooted at $t$ to satisfy the validation check $t \cdot \text{bst}(\min, \max)$. This bst segment summarizes the path that pointer $p$ has already traversed and is derived by folding cells materialized on the previous iteration. Shape analysis tools vary in how they represent segments—but some mechanism to do so is fundamental.

The key observation to make in the rules shown in Figure 5a is that folding (i.e., going right-to-left) requires verifying a pure, non-memory, data constraint (shaded) just to summarize a memory region into a summary predicate like bst or bst-segment. In the idealized loop invariant shown in Figure 5b, the total ordering constraints on symbolic data values (shaded) must be tracked by the base data-value domain in order to derive that the whole memory region from $t$ still satisfies $t \cdot \text{bst}(\min, \max)$. The two middle constraints $\min \leq V_p < \max$ are not difficult to derive (come from unfolding at the current binary search tree node $a_p$), but the two outer equality constraints (underlined) are crucially important to connect the regions before and after $a_p$. If any of these data-value constraints are lost across joins or widens from interactions with the shape domain, the inferred loop invariant will be too imprecise to witness all symbolic variables and thus to synthesize short-circuiting validation. Imagine the likelihood that some such imprecision creeps in with arbitrarily complex data-value constraints!

Uninterpreted Shape-Data Predicates. The goal of the validated view abstract domain is to compensate for such lost precision...
by tracking uninterpreted shape-data binary predicates. While it will be uninteresting by itself, when it is combined in a reduced product [9] with a shape-data abstraction, our validated view abstract domain enables two key operations: (1) folding of non-invalidated memory regions regardless of any imprecision (in the base data-value domain) when tracking data-value constraints and (2) precise tracking of equalities that connect data-value parameters across memory regions. To illustrate what a validated view provides, consider in more detail the excisemin function shown in Figure 6.

The excisemin binary search tree operation excises the node in the tree with the minimum data value. Ignore the shaded lines of code for the moment. The first assume (on line 1) says that the input tree t is a binary search tree, while the second assume (on line 2) requires that we are in the case where the root t and the immediate left child t->l are non-null. For presentation, we focus on this case, as it is corresponds to when excisemin loops and thus requires loop invariant inference. The for loop on line 3 walks down the left spine of the binary search tree until p points to the parent node of the minimum data value node (i.e., the leftmost leaf) as terminated by the guard on line 6. We write the loop in this somewhat non-idiosyncratic way for presentation purposes to explicitly expose the join point of the loop on line 4; a more standard while (p->l->l != NULL) loop is equivalent and does not affect the program analysis. On line 10, the programmer saves the data value of the minimum node in vmin and excises the node. Finally, the code checks that tree t is still a binary search tree with the assert on line 12. Our overall goal is to replace this whole-data-structure operation with a synthesized short-circuiting validation check. Together with setroot from Section 2, we have the helper operations for deleting a node from a binary search tree.

Now consider the static invariant shown on line 9 right after the minimum-finding loop. To obtain this invariant, the shape analysis must infer a loop invariant at the head of the for loop on line 3 similar to the invariant discussed previously in Figure 5b. The shape graph on line 9 is the same, except that the exit condition p->l->l == NULL is reflected.

The additional annotations in the invariant on line 9 correspond to validated view predicates that have form \( \kappa \bowtie \varepsilon \) where \( \kappa \bowtie \alpha \cdot k(v) \) is an instance of an inductive predicate \( k \) and \( \varepsilon \bowtie e_1 \cdot e_2 | \emptyset | \kappa \) is a memory region composed of disjoint instances of inductive predicates. The semantics of this predicate \( \kappa \bowtie \varepsilon \) is a memory region that satisfies \( \kappa \) up to endpoints given by \( \varepsilon \). The predicate \( \kappa \bowtie \varepsilon \) implies the separating implication \( \kappa \bowtie \varepsilon \) meaning that \( (\kappa \bowtie \varepsilon) \bowtie \varepsilon \Rightarrow \kappa \). An important point is that a validated view concretizes to a concrete memory, just like the shape graph. The interpretation of the product of a shape graph and a validated view is that the validated view constrains a sub-store described by the shape graph. These details are described further in our formalization (Appendix B).

The reason to use validated views is that on an unfolding, the data constraints specified in the inductive predicate are always satisfied on the materialized cells corresponding to the unfolded node—at least until a modification to these materialized cells. We can see a validated view as an uninterpreted shape-data predicate that is applied to a materialized region until the analysis interprets an update that could invalidate the predicate. The view at \( \tilde{a}_p \) shown on line 9 in Figure 6

\[
\tilde{a}_p \cdot \text{bst}(\tilde{u}_1, \tilde{u}_2) \bowtie (\tilde{a}_1 \cdot \text{bst}(\tilde{u}_1, \tilde{v}_p) \bowtie \tilde{a}_4 \cdot \text{bst}(\tilde{v}_p, \tilde{u}_2))
\]

governs the materialized points-to edges from \( \tilde{a}_p \) up to \( \tilde{a}_1 \) and \( \tilde{a}_4 \) (that is, the fields corresponding to \( p->l \), \( p->r \), and \( p->v \)). This view predicate says that the materialized region consisting of these three cells satisfies whatever data constraint is specified in the “inductive step” of the inductive predicate \( \text{bst} \) in this case, that \( \tilde{a}_1 \leq \tilde{v}_p \leq \tilde{a}_2 \). By tracking the data-value constraints of a materialized region in this uninterpreted manner, we are able to infer when the shape-data invariants are preserved regardless of the complexity of the data-value constraints.

Hypothetically, consider the worst case for data reasoning where we have no ability to capture any data-value constraints (e.g., there is no part of our abstraction that captures the inequality constraints of \( \text{bst} \)) or more realistically, that important data-value constraints were lost in widening. Even in this case, the validated view domain enables our analysis to capture the key facts needed for folding: (1) that the \( \text{bst} \) allowable range parameters for \( \tilde{a}_p \) are indeed \( \tilde{u}_1, \tilde{u}_2 \) (and...
thus they correspond to the allowable range at the segment endpoint \( a_p \); and (2) the data value at \( a_p \) (that is, \( \vec{v}_p \)) is in this allowable range \([a_l, a_r]\), as is required to fold into the bst predicate. Without fact (1) maintained by validated views, the allowable range at the segment hole (marked with \( \bullet \) in Figure 6) could be inferred as some other allowable range \([a_l', a_r']\). While this invariant is sound, it is too weak to imply that the entire memory region reachable from \( \tau \) is a binary search tree (i.e., \( a_t \cdot \text{bst}(\vec{v}_{\infty}, \vec{v}_0) \)).

Validated views can be seen as a combination of ideas drawing from separation logic [24] and predicate abstraction [12], where a set of inductive-predicate labels further constrains the heap. The validated view domain itself can be seen as a predicate abstraction that two assignments join with the validated view constraint. The first component of the expression from what we call the programmatic valuation is at join points, such as at line 4. We write the assignments hold the symbolic value \( \rho \). The key challenge we address is that the static analysis must be precise enough to derive a witness to any newly introduced symbolic variable. For presentation purposes, we conflate symbolic variable names with expressions from what we call the programmatic valuation.

The programmatic valuation is generated by following transfer functions of the static analysis, specifically for assume, assignment, and join. Here, we describe how the programmatic valuation is computed by following the excisemin example. We give details in a more formal manner in Appendix C.

The complicated case for obtaining program expression witnesses is at join points, such as at line 4. We write the assignments here using standard SSA \( \phi \) notation, where the first component is the value from the loop entry edge, and the second component is the value from the loop back edge. On line 4, the \( \phi \)-assignment to \( a_p \) shows that \( a_0 \) (the value of the pointer \( p \)) is \( a_0 \) on entry and \( a_1 \) on the loop back edge. This \( \phi \)-assignment to \( a_p \) is because \( p \) is advanced via \( p = p + 2 \) and can be witnessed, along with the others on this line, by following the join of the shape graphs.

The \( \phi \)-assignments \( a_l = \phi(\vec{v}_{\infty}, \vec{v}_1) \), \( a_r = \phi(\vec{v}_{\infty}, \vec{v}_0) \) on line 5 (also at the same join point) crucially come from following the join with the validated view constraint. The first component of the two assignments \( \vec{v}_{\infty} \) and \( \vec{v}_0 \) are derived from the meaning of an empty segment between \( a_0 \) and \( a_p \); the second component comes from the validated view constraint at \( a_1 \) (stored in \( p = 1 \)). This view constraint \( a_1 \cdot \text{bst}(a_l, a_r) \equiv \cdots \) (shown on line 9) allows the analysis to derive that \( a_l \) and \( a_r \) (the allowable range at \( a_p \) in the previous loop iteration), respectively. Without this view constraint at \( a_1 \), the inferred static invariant—while sound—is not precise enough to find program expressions that witness \( a_l \) and \( a_r \) with concrete values at run-time.

If \( \phi \)-assignments come from computing the join of shape invariants. As symbolic variables correspond to existential variables, the join computes mappings from the symbolic variables of the resulting shape invariant to the variables of the input invariants. These mappings witness the existential variables in the resulting invariant in terms of variables of the input invariants and thus are reified as these \( \phi \)-assignments.

Generating programmatic valuations enables an instrumentation that creates a “run-time shadow” of the static analysis state. Crucially, even though this shadow refers to the heap, it tracks all symbolic variables in stack locations—the instrumentation does not add any storage on the heap. This approach allows our short-circuiting validation to effectively incrementalize checking of data structure invariants with a constant-space overhead rather than the linear-space overhead of a shadow heap with dynamic memoization.

### 4. Synthesizing Short-Circuiting Validation

The final step of our approach is to synthesize short-circuiting checks that incrementally validate data structure invariants. In this section, we describe a proof-directed synthesis algorithm that generates short-circuiting validation checks. There are two parts to this process: (1) synthesizing calls to the short-circuiting validation checkers at assertion sites (such as the call to \texttt{bstac} in Section 2); and (2) synthesizing the code for the short-circuiting validation checkers themselves (such as the code for \texttt{bstac} itself).

**Synthesis Overview.** Our approach employs a static verification to drive synthesis. Figure 7a shows the \texttt{assert} at the end of the \texttt{excisemin} procedure from Figure 6 and the inferred shape-data invariant (now labeled \( \rho \)) at that program point. From a static verification perspective, the \texttt{assert} is a request to prove that the fact \( \phi(\vec{v}_{\infty}, \vec{v}_0) \) holds in a sub-heap at that program point, where \( \rho \) is the symbolic value stored in program variable \( \tau \). In other words, the analysis is asked to prove the abstract inclusion \( \rho \subseteq \phi(\vec{v}_{\infty}, \vec{v}_0) \). In separation-logic-based shape analysis, abstract inclusion is verified by unfolding and subtraction [5, 10, 25]. Subtraction-based inclusion checking \( \rho_1 \subseteq \rho_2 \) works by unfolding the righthand-side \( \rho_2 \) to match the shape structure (i.e., spatial formulas) of the lefthand-side \( \rho_1 \) and “subtracting” matching shapes until the inclusion is trivial. (A base data-value domain, decision procedure, or SMT-solver can then be used to try to discharge any pure, data-value constraints for inclusion.)

**Part 1) Synthesizing Calls to Short-Circuiting Checkers.** We adapt the standard unfolding-subtraction inclusion algorithm to synthesize short-circuiting validation calls. We describe this algorithm subsequently in Section 4.1 and Section 4.2. For the moment, consider Figure 7c, where we present the calls to short-circuiting checkers that our approach synthesizes to replace the \texttt{assert} from Figure 7a.

Intuitively, our approach is to synthesize this code that corresponds to an unfolding of the assertion \( \tau \cdot \texttt{bst}(-\infty, \infty) \) to match the shape structure of the inferred static invariant \( \rho \). Here we write \texttt{bstsegac} and \texttt{bstac} as calls to short-circuiting validation checkers corresponding to full, non-short-circuiting validation checkers for a binary search tree segment and a complete binary search tree \texttt{bst}, respectively. We synthesize calls to short-circuiting checkers at assertion sites by (a) unfolding the assertion to match the inferred static invariant and (b) replacing calls to non-short-circuiting checkers with calls to their short-circuiting variants.

**Part 2) Synthesizing Short-Circuiting Checkers.** We can also use this subtraction-directed synthesis to generate the deep short-
circuiting validation checkers themselves. The key observation is that we can represent the safe replacement criterion from Section 2 in the form of a generic code template—which we show in Figure 7b for an arbitrary non–short-circuiting checker c (here, type da is a generic placeholder for the data structure type in question). We can then apply the same substitution-directed synthesis approach that we use in Part 1 to generate deep short-circuiting. Note the similarity of the template to the shallow short-circuiting checker bstsc_a from Figure 3a in Section 2. We write shortify to indicate the location where we apply short-circuiting synthesis to generate the recursive calls.

4.1 Shortify: Subtraction-Directed Synthesis

In this subsection, we formalize our subtraction-directed short-circuiting synthesis algorithm, which is an adaption of a standard inclusion over stores. We assume a programmatic valuation (Section 3.2) to convert from a symbolic variable to code that, at run time, retrieves a concrete value represented by that variable. So, we will, for clarity, only use symbolic variables in the remainder of this section.

In Figure 7c, we define a judgment of the form \( \hat{\rho} \vdash \rho' \), where \( \hat{\rho} \) is a formula. The judgment form is a three-place relation between the statically proven store \( \hat{\rho} \), the store to be dynamically validated \( \rho' \), and the assertion store \( \rho' \). With this judgment form, we want to derive when it is sound to optimize \( \text{assert}(\rho') \) in the original program to \( \text{assert}(\rho') \) under the assumption of the sound statically-inferred invariant \( \hat{\rho} \). In other words, we have inferred the invariant \( \hat{\rho} \) and are asked to prove an assertion \( \rho' \), so we abduct a hypothesis \( \rho' \) that allows us to prove the following implication:

\[
\hat{\rho} \Rightarrow (\rho'^* \Rightarrow \rho')
\]

We have thus reduced our synthesis problem to an abduction problem. The syntactic form of stores \( \hat{\rho} \) is given in Figure 7d, which are separation logic formulars with inductive summaries \( \kappa \) and \( \kappa' \Rightarrow \kappa \).

The SHORTIFY-PROVEN rule describes a degenerate case of short-circuiting synthesis: if the analysis can prove the inclusion statically, then there is no need to check anything dynamically and thus it can eliminate the check (i.e., no store constraints true). The remaining rules express the power of our approach: they enable us to go beyond the false “all-or-nothing” dichotomy between static proof and dynamic validation.

The SHORTIFY-EMP rule expresses the axiom for subtraction-based inclusion—the empty store emp is contained in emp—except that the analysis is not obligated to prove the pure constraint \( \hat{\rho} \) assuming \( \hat{\rho} \). Instead, it is permitted to leave a residual dynamic check \( \hat{\rho} \).

The next two rules describe the basic decomposition of the store for subtraction. Rule SHORTIFY-UNFOLD permits unfolding of the assertion store \( \rho' \). That is, it can unfold \( \hat{\rho} \) in order to match the structure of statically-inferred invariant \( \hat{\rho} \). Existential variables introduced as part of unfolding the \( \hat{\rho} \) are unified with the corresponding value in \( \hat{\rho} \). To check inclusion of matching disjoint regions, rule SHORTIFY-SEP uses the statically-inferred invariant \( \hat{\rho} \) to prove the separation constraint in assertion store \( \rho' \) true. The synthesized valuation for the two regions \( \rho'_1 \) and \( \rho'_2 \) is combined with \( \rho'_1 \) and \( \rho'_2 \), which is our shorthand for \( (\rho'_1 \land \rho'_2) \) true meaning that \( \rho'_1 \) and \( \rho'_2 \) can be disjoint or overlapping. While the rule would be sound with \( \rho'_1 \land \rho'_2 \), this more general rule emphasizes that the separation constraint in the assertion store is proven by the statically-inferred invariant, so the dynamic validation need not check separation.

The SHORTIFY-INDUCTIVE rule is the key rule that leverages the statically-inferred invariant. It attempts to synthesize short-circuiting validation for \( \hat{\rho} \) when the statically-inferred invariant has shown \( \hat{\rho} \). The synthesis is guided by the statically-inferred invariant in that the root address \( \hat{\rho} \) and the inductive checker \( k \) match but permits the additional arguments to be different. This rule motivates the safe replacement criterion with the short-circuiting checker bstsc given in Section 2. Stated in terms of ksc, the short-circuiting checker ksc should satisfy:

\[
\hat{\rho} \Rightarrow (\hat{\rho} \Rightarrow \rho')
\]

An application of this rule means that when asked to shortify \( \hat{\rho} \), we rewrite it to \( \hat{\rho} \). This rewriting is sound because we have statically proven the assumption \( \hat{\rho} \) and checking \( \hat{\rho} \) gives us the desired condition \( \hat{\rho} \).

Separately, we need to synthesize code for the short-circuiting invariant checker ksc itself and ensure that it satisfies its safe replacement criterion. In Figure 7b, we show the synthesis template for ksc. The shortify directive indicates application of this synthesis routine to generate short-circuiting validation of \( \hat{\rho} \). Without this directive, we have a shallow short-circuiting invariant checker that satisfies its safe replacement criterion. The shortify then yields a deep short-circuiting invariant checker.

The last rule SHORTIFY-VALIDATE is not allowed in our implementation, but we include it here for discussion. This is a degenerate case of short-circuiting synthesis that is dual to SHORTIFY-PROVEN: it simply gives up on using the static information \( \hat{\rho} \) and simply
checks the assertion $\hat{p}'$ dynamically. By disallowing this rule, we can observe that there are no separation constraints that need to be validated in $\hat{p}'$.

**Shortifying Asserts.** The final output is a code transformation that rewrites all asserts to perform short-circuiting validation at run time—we apply shortify to every assert(\hat{p}'). To see intuitively why this transformation is sound, consider such an assert(\hat{p}'). Before this synthesis phase, the static shape analysis will have derived the following program assertion:

$$\{ \hat{p} \} \quad \text{assert(\hat{p}')} \quad \{ \hat{p} \land (\hat{p}' \land \text{true}) \}$$

where $\hat{p}$ is the inferred invariant (i.e., the proven store) at the program point before this assert. Now, shortification infers a predicate $\hat{p}'$ such that the judgment $\hat{p} \vdash (\hat{p}') \leq \hat{p}'$ holds, which means the implication $\hat{p} \Rightarrow ((\hat{p}') \land \text{true}) \Rightarrow (\hat{p}' \land \text{true})$ holds. Because the post-condition of the transformed assert is

$$\{ \hat{p} \} \quad \text{assert(\hat{p}')} \quad \{ \hat{p} \land (\hat{p}' \land \text{true}) \} ,$$

we get the original post-condition using the above implication.

To state the soundness condition for $\lesssim$, we consider concrete semantic domains for a concrete memory $\sigma \vdash \text{Addr} \rightarrow_{\text{fin}} \text{Val}$ that maps addresses to values where we assume the set of addresses are contained in values (i.e., $\text{Addr} \subseteq \text{Val}$) and a valuation $\pi : \text{Val} \rightarrow \text{Val}$ that maps symbolic variables to concrete values. Symbolic variables are existential variables naming heap addresses and values, and they form the coordinates for the base pure, data-value domain. The concretization of an abstract store $\hat{p}$ thus yields a pair of a concrete memory and a valuation (i.e., $(\sigma', \pi) \in \gamma(\hat{p})$).

**Theorem 1 (Shortify Soundness).** If $(\sigma, \pi) \in \gamma(\hat{p})$ and $\hat{p} \vdash (\hat{p}') \leq \hat{p}'$ and $(\sigma', \pi) \in \gamma(\hat{p}')$ where $\sigma' \subseteq \sigma$, then $(\sigma', \pi) \in \gamma(\hat{p}')$ for some $\sigma' \subseteq \sigma$.

This statement is a formalization of the implication shown in (9); a proof is given in Appendix D.

### 4.2 Synthesizing Short-Circuiting Segments

The previous subsection describes a procedure for synthesizing short-circuiting validation sufficient for the $\text{HORTIFY}$ example in Figure 2 and for most recursive procedures over data structures. In particular, the previous subsection focuses on the case where the static shape analysis need only summarize complete data structures (e.g., a entire binary search tree). An additional challenge for static shape analysis on iterative programs versus recursive programs is the need to summarize prefixes of data structures. For example, in the shape invariant shown in Figure 7a before the final assert in $\text{excsin}$, there is an inductive segment from $\hat{a}_0$ to $\hat{a}_p$. Recall that we write this segment as the formula

$$\hat{a}_0 \cdot \text{bst}(\hat{v}_0, \hat{v}_0) \Rightarrow \hat{a}_p \cdot \text{bst}(\hat{a}_1, \hat{a}_2) ,$$

which describes a binary search tree at $\hat{a}_0$ with allowable range $(\hat{v}_0, \hat{v}_0)$ except with a hole at $\hat{a}_2$ such that the binary search tree could be completed if the hole is filled with a binary search tree with allowable range $(\hat{a}_1, \hat{a}_2)$. With such inductive segments, the shape analysis has a different kind of summary form to summarize the binary search tree prefix from $t$ to $p$.

We can synthesize short-circuiting validation for this different summary form by introducing a new short-circuiting template and a new $\text{HORTIFY}$ rule that introduces calls to this new short-circuiting template. In Figure 7a, the inductive segment from the root $\hat{a}_0$ to the end point $\hat{a}_p$ is incrementialized with a call to a synthesized short-circuiting segment checker $\text{btssegsc}$ in Figure 7c. To see how the call to $\text{btssegsc}$ is synthesized, consider the new rule $\text{SHORTIFY-SEGMENT}$ shown in Figure 8b. This rule synthesizes a call to a short-circuiting segment checker $k_1k_2\text{segsc}$ even when the additional arguments differ. That is, when asked to synthesize short-circuit validation for the specific inductive segment $\hat{a}_0 \cdot k_1(\hat{v}_1) \Rightarrow \hat{a}_2 \cdot k_2(\hat{v}_2)$, we require only a proof of $\hat{a}_0 \cdot k_1(\hat{v}_1) \Rightarrow \hat{a}_2 \cdot k_2(\hat{v}_2)$ that may differ in the additional arguments. Requiring this weaker property (in the additional arguments is analogous to the difference permitted in $\text{HORTIFY-INDUCTIVE}$).

The difference that we permit in the $\text{SHORTIFY-SEGMENT}$ rule leads to a safe replacement criterion that the short-circuiting segment checker $k_1k_2\text{segsc}$ should satisfy:

$$\begin{align*}
\text{if } & \hat{a}_0 \cdot k_1(\hat{v}_1) \Rightarrow \hat{a}_2 \cdot k_2(\hat{v}_2) \\
& \text{then } \hat{a}_0 \cdot k_1k_2\text{segsc}(\hat{a}_0, \hat{v}_1, \hat{v}_1, \hat{v}_2, \hat{v}_2) \Rightarrow (\hat{a}_0 \cdot k_1(\hat{v}_1) \Rightarrow \hat{a}_2 \cdot k_2(\hat{v}_2))
\end{align*}$$

based on its use in that rule. This safe replacement criterion motivates the short-circuiting segment checker template given in Figure 8a (we have annotated the parameters with subscripts and primes to more readily see related components). It is captured in the first disjunct of the $\text{return}$ condition on line 4 and the first disjunct of the assume on line 1. If we ignore the assignment to $\hat{v}_1$ on line 3 and the second disjuncts for now, we have the analogous template as for inductives $\text{ksc}$—line 2 checks the short-circuiting condition.

To see where the assignment to $\hat{v}_1$ comes from, let us consider how the symbolic variables $\hat{v}_1$ arise when the $\text{SHORTIFY-SEGMENT}$ rule is applied. Because segments are not directly specified in assertions, this segment with symbolic variables $\hat{v}_1$ must result from unfolding during substitution (using the $\text{HORTIFY-UNFOLD}$ rule) to match the shape of the inferred invariant. For this reason, they are not present in the analysis state and thus not in the programmatic valuation. However, we observe that we can instrument the short-circuiting segment checker template to find concrete witnesses for these symbolic variables. In the case that the short-circuiting condition applies (line 2), it must be the case that $\hat{v}_1 = \hat{v}_2$, that is, the arguments at the end point $\hat{a}_2$ correspond to the statically-inferred invariant. Our synthesis procedure will thus initialize the contents of $\hat{v}_1$ to $\hat{v}_2$ at the top-level call site, as we have shown with the assignments to $\hat{v}_1$ and $\hat{v}_2$ at $\hat{a}_1$ and $\hat{a}_2$ respectively, in Figure 7c. In the case that we reach the endpoint $\hat{a}_2$ on line 3, we witness $\hat{v}_2$ with $\hat{v}_1$ using the assignment $\text{gst} = \gamma$ using the semantics of the empty segment.

This template design enables additional precision in dynamic validation. Alternatively, we could strengthen the static obligation so that $\hat{v}_1 = \hat{v}_2$ (i.e., disallow a difference between the additional arguments at $\hat{a}_2$) at the cost that short-circuiting applies less often.

Finally, the second disjuncts of the return condition on line 4 and the assume on line 1 are another optimization in our template design. An inductive segment $k_1 \Rightarrow k_2$ is defined by unfolding $k_1$ until it either terminates in a base case or matches $k_2$ (cf. Section 3.1). The second disjuncts generalize the replacement criterion so that
this short-circuiting checker can also be used in the context the static analysis has proven \( n \cdot k(n) \), that is, where there is a full validation from \( n \) with no endpoint. This additional assumption permits this same short-circuiting checker to be used in the recursive calls generated by \texttt{shortify} at line 4, regardless of whether the recursive check is a full validation or a segment.

**Discussion: Limitations and Generalizations.** Fundamentally, synthesizing short-circuiting validation checkers from templates requires proving implications in inductive separation logic and thus is necessarily incomplete. To synthesize from the templates shown in Figures 7b and 8a, we unfold once in the \texttt{assume} on line 1 and in the \texttt{return} on line 4. This strategy is effective at proving the implication of interest for common inductive definitions, like \texttt{bst}, that unfold “one-node-at-a-time”—but, in general, is not guaranteed to succeed.

As we can observe from the above discussion about inductive segments, there is a design trade-off between the \texttt{shortify} rules and the short-circuiting templates. The \texttt{shortify} rules specify the desired weaker static verification obligation, which dictates the remaining obligation that must be satisfied by the synthesized short-circuiting checker. In this paper, we have asked the static verification to prove the same structure (cf. \texttt{shortify-inductive} and \texttt{shortify-segment}) with potentially different additional arguments. This design decision leads to short-circuiting templates that are both effective in realizing short-circuiting validation and reasonable to synthesize. But clearly, this choice is simply one in a potentially large design space. The dynamic validation of different kinds of summaries may be shortifted with a similar approach using different \texttt{shortify} rules but potentially requiring different synthesis strategies for short-circuiting checker templates.

5. **Empirical Evaluation**

We test our short-circuiting validation approach using a prototype implementation called \texttt{DIVVA}. In particular, we seek to test the following expectations: (1) Theoretical asymptotic improvements from short-circuiting translate to dramatic empirical speedups in runtime validation on workloads even assuming worst-case inference imprecision in the static verifier. (2) Short-circuiting validation is effective even when the pre-conditions are too weak to imply the validated post-condition (i.e., when it is not sound to eliminate the post-condition). (3) The absence of a shadow heap in \texttt{DIVVA} results in incremental validation with low overhead.

**Experimental Setup and Static Analysis Time.** We use \texttt{DIVVA} to incrementally verify-validate the set of workloads shown in Figure 9a. Each data structure has an invariant validation checker that requires numerical constraints on the data structure contents. All workloads consist of repeatedly applying a data structure operation assuming the data structure invariant holds on input and then asserting that the data structure invariant again holds for the output.

We consider both classic data structures, such as ordered singly-linked lists (\texttt{olist}) and binary search trees (\texttt{bst}), as well as structures less commonly considered by static verifiers: treaps (\texttt{treap}) and hash tries (\texttt{hashtrie}). A treap is a randomized version of a binary search tree that is heap-ordered on a set of randomly-generated priorities; a hash trie is a tree-structured hash map that uses bit-blocks from hash keys to form a trie [3]—challenging even for state-of-the-art static verification. And even for binary search trees—where the invariant seems amenable to current static reasoning techniques—certain operations like deletion (that have non-local modification) can be surprisingly challenging to verify statically compared to simpler operations like insertion [15]. Our validated views domain handles these structures for short-circuiting synthesis because it can preserve uninterpreted data constraints on unmodified regions.

| workload | \( m \) (K) | iVV (s) | slowdown wrt noV | speedup wrt dynV |
|----------|-------------|---------|----------------|-----------------|
| \texttt{olist} m concats | 1,180 | 1.163 | 1.07x | 2.1x |
| \texttt{olist} m drops | 600 | 0.053 | 1.01x | 230000 x |
| \texttt{olist} m inserts | 24 | 1.185 | 1.12x | 2.8x |
| \texttt{olist} m deletes | 23 | 1.087 | 1.08x | 2.9x |
| \texttt{olist} 50-50 | 27 | 1.195 | 1.10x | 2.8x |
| \texttt{bst} m inserts | 200 | 0.075 | 1.40x | 5600 x |
| \texttt{bst} m deletemons | 460 | 0.087 | 1.66x | 83000 x |
| \texttt{bst} m exciseroots | 380 | 0.157 | 2.80x | 26000 x |
| \texttt{bst} m deletes | 170 | 0.097 | 1.76x | 3300 x |
| \texttt{bst} 50-50 | 170 | 0.079 | 1.41x | 360 x |
| \texttt{treap} m deletes | 900 | 1.171 | 1.12x | 3.9x |
| \texttt{treap} m inserts | 970 | 1.399 | 1.35x | 3.3x |
| \texttt{treap} 50-50 | 660 | 1.151 | 1.09x | 2.3x |
| \texttt{hashtrie} m writes | 140 | 0.230 | 4.35x | 9300 x |
| \texttt{hashtrie} 50-50 | 280 | 0.305 | 5.78x | 1000 x |

- \texttt{geometric mean} 1.54x

(a) Execution time of data structure workloads with short-circuiting incremental validation-validation (iVV); no validation (noV); and full dynamic validation (dynV). The workloads execute \( m \) data structure operations—with a dynamic validation check between each operation in the dynV and iVV variants. We chose the number of operations \( m \) to ensure that the workloads run long enough to accurately measure elapsed time. For most workloads, the fastest variant (that with no validation, noV) takes at least 1 second. In cases where the slowest, fully dynamic validation variant (dynV) would not finish within hours on that size, we chose \( m \) to ensure that the variant without validation (noV) takes a minimum of 0.05s. These workloads are compiled with \texttt{gcc} -O2 and measured on an Intel Core 2 Quad CPU Q6600 2.40GHz with 4G RAM. The time reported is averaged over 12 runs.

(b) Short-circuiting incremental verification-validation does not change the apparent complexity of \texttt{bst} \texttt{m} insert deletes. The iVV line shows the cost of \( m \) validated operations on a data structure of size \( n \). The noV line shows the cost on the original, unvalidated code.

**Figure 9.** Effectiveness of incremental verification-validation.

To stress test our approach, we intentionally instantiate the shape-data analyzer in \texttt{DIVVA} with the “most lossy” data domain. This domain can only keep constraints in straight-line code and loses all of them (i.e., goes to \( \top \)) at any join point. Complex operations, like bit operations, are treated as uninterpreted functions. This imprecision represents the scenario in which the data property is out of reach of static verification.

We measured the static analysis time to infer invariants for our examples. As our primary concern is not the efficiency of the static analysis, our data-value domain is not particularly optimized. It calls the Z3 SMT-solver for inclusion checking. The total analysis time for all our examples was 640.2s. It is likely that improved engineering can lower the analysis time by making fewer SMT calls. If we exclude the Z3 time, then the remaining analysis time was 27.9s.
**Incrementalized Run-Time Validation Cost on Workloads.** In Figure 9a, we report the cost and benefit of incrementalized run-time validation for a variety of data structure workloads. The measurements under 'IVV' show the execution time with synthesized short-circuiting dynamic validation. The subsequent columns give the slowdown of iVV over no validation (‘noV’) and speedup of iVV over dynamic validation (‘dynV’). The dynV configuration runs C-style data structure invariant checks that dynamically validate the data constraints for the complete structure but do not validate separation properties. This configuration provides more checking than noV, but it is still weaker than iVV, which checks separation properties statically. Note that strengthening dynV to dynamically validate separation can only make the iVV speedups larger. We provide only the speedups here, but we give the detailed execution times in Appendix A.

For ordered singly-linked lists olist, concat iteratively concatenates sub-lists until reaching a list of size $m$, drop drops the first element of the list $m$ times from a list of size $m$, insert performs ordered insertion $m$ times from an empty list, delete performs element deletion $m$ times from a list of size $m$. For binary search tree bst, insert performs search tree insertion, delete delete the minimum element, exciseroot excises the root node, and delete does search tree deletion. For treaps treap, search tree rotations are performed on insertion and deletion to restore the heap-ordering invariant; we perform validation after each rotation. For hash tries hastrie, write writes to the hash trie, and we validate that the hash key of each element is along the correct path in the trie for the last 14 levels; this dynamic validation limit is because of the lack of arbitrary precision integers in C. The 50-50 workloads consist of a randomly-selected sequence of insert and delete operations with 50-50 split of each kind; the data structure size for these workloads was 10K nodes, except for the treap workloads, which were at 1M nodes. The code for hastrie write along with its invariant checker is given in Appendix E.

In most cases, our iVV approach brings the slowdown with respect to the noV configuration back down to within a factor of 1.76x. One exception is exciseroot: this workload is a bad case for short-circuiting because the operation itself requires just one $O(\log n)$ walk from the root, while the theoretically ideal short-circuiting validation has to perform three $O(\log n)$ walks. This larger slowdown is thus consistent with our expectations. We discuss the exceptions in the hastrie workloads further below.

The speedup column shows the improvement of incrementalized verification-validation (IVV) over non-short-circuiting, whole-structure dynamic validation (dynV). We expect short-circuiting to obtain asymptotic improvements over dynV (or rather, recover the asymptotic losses of the dynV variant over the unchecked noV configuration). Our expectation is supported by the measurements shown in Figure 9a: the cases with single-digit factor speedups are those where there is no change in asymptotic complexity between noV and dynV, while the cases with multiple order-of-magnitude speedups are those where there is a change in asymptotic complexity (the exact factors are thus not meaningful). Recall that dynamic validation does not check separation properties—these performance improvements are on top of the additional separation assurances provided by our static verifier.

We obtain these speedups with relatively unoptimized instrumentation. We never eliminate any asserts entirely—we only replace non-short-circuiting, whole-structure assertions (like on line 4 in Figure 2c) with calls to short-circuiting checkers (like on line 4 in Figure 2d). To focus our measurements on short-circuiting, we do not eliminate data invariant asserts (e.g., $x < y$) even when they are known to hold statically. Removing such statically-proven asserts would especially improve the hastrie workloads. For the other data structures, the data invariant at each node is a constant-time check. But for hastrie, the data invariant check involves recomputing the hash key of an element, which is a $O(\log n)$ operation (adding a $O(\log n)$ factor over noV but better than the $O(n)$ factor added by dynV).

Overall, these workload performance measurements are quite promising, particularly given the two intentional worst-case choices of (1) no assert eliminations even when they are statically known; and (2) a worst-case base data-value domain. Applying static verification techniques to data-value reasoning, instead of hamstringing it, would only improve the run-time performance of iVV.

**When Pre-Conditions are Insufficient to Verify.** Not only can DIVVA incrementalize to make up for inference imprecision in static analysis, but it can also tolerate “insufficient” pre-condition specifications. Insufficient pre-condition specifications are ones that are not strong enough for any static verifier to soundly eliminate checking the post-condition (like the setroot example from Figure 2). Here, the concat example and both treap examples use insufficient pre-conditions. For the concat example, the pre-condition for each input is that they are both ordered lists with a lower bound parameter of $-w$. For the treap examples, the pre-condition to the rotation operation is that both sub-trees have binary search tree bounds of $(w, \infty)$. DIVVA generates a short-circuiting validation that checks that the observed concrete instances satisfy the post-condition. This synthesized validation soundly fails when passing concrete inputs that would violate the data structure invariant (e.g., when passing two individually ordered lists to concat whose concatenation is not an ordered list).

**Low Overhead Validation.** As we have seen, incrementalizing dynamic validation checks can improve their asymptotic complexity—and, in some cases, even recover the complexity of the original, unvalidated operation while providing safety guarantees. Recall the binary search tree example from Figure 1—there, adding whole-data-structure validation to insert and delete operations changed the asymptotic complexity from $O(\log n)$ to $O(n)$, but applying DIVVA’s short-circuiting approach retained the benefits of dynamic validation without changing the complexity of the operation.

In Figure 9b, we show a plot of IVV and noV execution times on a workload of $m$ binary search tree insertdelete operations (this is the plot from Figure 1 with the dynV curve removed). An insertdelete performs a deletion of a random element followed immediately by the re-insertion of the same element (this keeps the data structure size $n$ constant over the $m$ operations); here we fix $m = 1$ million so that the workload execution time is easily measurable (on the order of 1 second) on even the smallest data structure size. We performed $k = 12$ trials at each data structure size $n$. The error bars (though negligible) show two standard deviations. Over this range, the slowdown of iVV over noV ranges from 1.11x (for the largest insertdelete workloads) to 2.55x (for the smallest). We use this same data to estimate the asymptotic slowdown—the slowdown in the limit, as the number of nodes in the tree increases—by fitting to equations of the form $\text{time} = b + c \times \log(\text{size})$. The asymptotic slowdown is then $c_{IVV}/c_{noV}$—which we estimate to a 95% confidence interval to be $4.7\% \pm 1.4$. (For completeness, we provide both a plot of slowdown vs. workload size and the details of this fitting in Appendix A.)

This asymptotic slowdown of $4.7\%$ compares quite favorably with purely dynamic approaches to validation. While not a direct comparison, the overhead for Ditto ranges from 2.5x to 50x, depending on the data structure [28]. Such dynamic techniques are quite general but must maintain sizable run-time structures in a shadow heap. In contrast, when static shape analysis is possible—and it often is—then DIVVA’s combined verification-validation approach allows for a much lower overhead.
6. Related Work

Several ways of combining pointer-shape reasoning and data-value reasoning have been proposed, including [6, 7, 17]. Magill et al. [16] proposes a sequence of two analyses: a shape analysis followed by an off-the-shelf scalar value analysis. The kind of invariants that we consider require a tight integration of pointer-shape and data-value reasoning, which may be difficult to realize in a pipelined approach. Incremental verification-validation can be seen as supplementing these approaches with the ability to discharge complex data-value constraints on heap contents at run-time in a fine-grained manner (whereas simpler data-value constraints can still be discharged statically). There is a wealth of static verification techniques for data structure properties that combine pointer-shape and data-value constraints, including [19, 21, 22, 31] amongst many others, that use user-supplied loop invariants and try to discharge proof obligations using solvers and decision procedures. Zee et al. [31] aim to verify not only rich shape-numerical data structure properties but also functional properties of algorithms that operate on them. Our technique is complementary; it offers more automation and the opportunity to discharge the hardest constraints dynamically. In another line of work, Christakis et al. [8] proposed using dynamic validation to study unsound assumptions made by static analysis tools. In contrast, our approach uses dynamic validation to check \textit{unverified assertions} from a sound static analysis (i.e., assertions that the analysis could not prove statically).

Prior approaches to run-time checking of expressive heap assertions includes work that parallelizes validation [30], allows user-specified overhead budgets [2], checks separation properties [20], and piggybacks on the GC with an expressive assertion language [23]. In this space, our work is most related to Ditto [27] where our bottom-line goal is the same: to incrementalize data structure validation checks. We present a complementary approach that offers different trade-offs. For example, our memory overhead is dictated by the size of static shape invariants that are independent of the size of the concrete data structure instances. Incremental computation [1, 13, 14] has been used for improving execution efficiency in general, beyond data structure validation checks. The basic principle is to memoize intermediate results and reuse them when a dependency is discovered. Our approach can be seen as a static analogue to these principles, applying static analysis to “statically memoize” validation checks so that only a single short-circuiting check at the root is needed to reuse the results for the entire sub-tree.

7. Conclusion

We have presented a proof-directed approach to synthesizing short-circuiting dynamic validation checks for data structures with complex data invariants. A short-circuiting checker stops checking whenever it detects \textit{at run time} that an assertion for some \textit{sub-structure} has been proven \textit{statically}. The key insight of our approach is to obtain a static verification for an existential weakening of the original property of interest and then to use this proof to synthesize short-circuiting validation. To achieve this technically, we first defined an enrichment of inductive shape analysis with validated views. This abstract domain enables run-time reification of static logic variables and precisely tracks uninterpreted data invariants in unmodified regions. Finally, we described a subtraction-directed synthesis technique that abducts short-circuiting validation checks for data constraints using the statically-inferred invariants.

The result is a hybrid technique that strikes a unique balance of static verification and dynamic validation for data structure invariants. On one hand, our technique provides benefit with efficient dynamic validation when faced with the inevitable imprecision of statically inferred data invariants on inductive structures. On the other hand, our technique leverages static verification to provide additional static assurances (i.e., separation properties) and substantially lower the run-time overhead of dynamic validation—in many cases recovering the complexity of the original unsafe and unvalidated code.

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Table 1. The execution overhead of running data structure workloads with dynamic validation (dynV) compared with no validation (noV).

| workload        | m (K) | noV (s) | dynV (s) | slowdown  |
|-----------------|-------|---------|----------|-----------|
| olist m concats | 1,180 | 1.086   | 2.399    | 2.2x      |
| olist m drops   | 600   | 0.053   | 12472.260| 236109.1x |
| olist m inserts | 24    | 1.059   | 3.294    | 3.1x      |
| olist m deletes | 23    | 1.007   | 3.147    | 3.1x      |
| olist 50-50     | 27    | 1.084   | 3.390    | 3.1x      |

| workload        | m (K) | noV (s) | dynV (s) | slowdown  |
|-----------------|-------|---------|----------|-----------|
| bst m inserts   | 200   | 0.053   | 415.414  | 7782.3x   |
| bst m deletemins| 460   | 0.052   | 7262.621 | 138725.5x |
| bst m exciseroots| 380   | 0.056   | 4101.548 | 73395.5x  |
| bst m deletes   | 170   | 0.055   | 318.750  | 5809.1x   |
| bst 50-50       | 170   | 0.056   | 28.600   | 508.5x    |

| workload        | m (K) | noV (s) | dynV (s) | slowdown  |
|-----------------|-------|---------|----------|-----------|
| treap m deletes | 900   | 1.046   | 4.595    | 4.4x      |
| treap m inserts | 970   | 1.039   | 4.568    | 4.4x      |
| treap 50-50     | 660   | 1.052   | 2.629    | 2.5x      |

| workload        | m (K) | noV (s) | dynV (s) | slowdown  |
|-----------------|-------|---------|----------|-----------|
| hashrtrie m writes | 140   | 0.053   | 2131.922 | 40326.6x  |
| hashrtrie 50-50 | 280   | 0.053   | 316.613  | 6000.3x   |

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A. Data Structure Workloads

Table 1 gives the execution times over our workloads from Section 5 on the no validation (noV) and the full, non-incrementalized (dynV) variants. These workloads are compiled with gcc -O2 and measured on an Intel Core 2 Quad CPU Q6600 2.40GHz with 4G RAM. The time reported is averaged over 12 runs.

A.1 Analysis of BST insertdelete Workload

In this subsection, we provide additional details for the measurements of the overhead of iVV over noV that we reported in Section 5.

Figure 10. Ratio of insertdelete execution times under incremental verification-validation (iVV) and for the original, unsafe code (noV).

Figure 11. We estimate the asymptotic overhead of our approach on the BST insertdelete workload by fitting lg-transformed curves.

Behavior Over Measured Range. In Figure 10, we show the ratio of iVV execution times over noV executions for the BST insertdelete over the range of tree sizes that we measured (up to 50 million nodes). The slowdown of iVV for our validation (over the original unsafe, unvalidated code) ranges from around 1.11x (for the largest insertdelete workloads) to 2.55x (for the smallest).
The asymptotic slowdown is then $C_{IVV}/C_{noV}$—which we estimate to a 95% confidence interval to be $4.7\% \pm 1.4$. We show the fitted lines in Figure 11a. The coefficients of determination ($R^2$) for the fit for IVV is 0.984; for noV is 0.995. The lack of a trend in the spread of the residuals (Figure 11b) for these fits provides evidence for a linear relationship between execution time and $\log(size)$.

### Definition 1 (Concretization of a View-Constraint Store)

If $\langle \sigma, \pi \rangle \in \gamma(\hat{\rho} \circ \hat{\omega})$ if $\langle \sigma, \pi \rangle \in \gamma(\hat{\rho})$ and $\langle \sigma', \pi' \rangle \in \gamma(\hat{\omega})$ such that $\sigma' \subseteq \sigma$.

In other words, using the syntax and semantics of connectives from separation logic, we can see $\hat{\rho} \circ \hat{\omega}$ as being equivalent to $\hat{\rho} \land (\hat{\omega} \land \hat{\omega})$—we weaken $\hat{\omega}$ to the strongest intuitionistic assertion weaker than $\hat{\omega}$ before conjuncting it with $\hat{\rho}$.

### Example 1 (Concretization of a Validated View)

For the following example validated view, we give one element of its concretization (i.e., $\langle \sigma, \pi \rangle \in \gamma(\hat{\omega})$). Other elements include memories with further unfoldings of the left or right sub-trees of $\pi(\hat{u}_1)$: there are simply two holes somewhere because of the arbitrary allowable ranges given the symbolic variables $\hat{u}_1-\hat{u}_6$. Observe that if there are some equality constraints on $\hat{u}_1-\hat{u}_6$, then that can have an effect on where the holes are allowed in the concretization.

$$\hat{\omega} : \hat{u}_1 \land [\hat{u}_1, \hat{u}_6] \Rightarrow (\hat{u}_2 \land \hat{u}_2, \hat{u}_7] \land \hat{u}_1 \land \hat{u}_2)$$

$s_1 : \pi(\hat{u}_1) = \pi(\hat{u}_2) \land \pi(\hat{u}_3) \land \pi(\hat{u}_4) \land \pi(\hat{u}_5) \land \pi(\hat{u}_6) \land \pi(\hat{u}_7) \land \pi(\hat{u}_8)$

$s_2 : \pi(\hat{u}_9) = \pi(\hat{u}_{10}) \land \pi(\hat{u}_{11}) \land \pi(\hat{u}_{12})$.

While not strictly necessary, the remaining forms of views $\hat{\omega}$ enable us to speak about all the view predicates as a whole called to the “main” store. The unit $\hat{\omega}$: true is the view that abstracts any concrete memory (as in separation logic), while $\hat{u}_1 \land \hat{u}_2$ is the symmetric version of $\hat{\omega}$. More precisely, the concretization is

$$\langle \sigma \cup s_1 \cup s_2, \pi \rangle \in \gamma(\hat{\omega}_1 \land \hat{\omega}_2)$$

if $\langle \sigma \cup s_1, \pi \rangle \in \gamma(\hat{\omega}_1)$ and $\langle \sigma \cup s_2, \pi \rangle \in \gamma(\hat{\omega}_2)$

where we write $\cup$ to union maps with disjoint domains, that is, the sub-store with common addresses in $\hat{\omega}_1$ and $\hat{\omega}_2$ must be the same $\sigma$. Or in terms of separation logic, $\hat{\omega}_1 \land \hat{\omega}_2$ corresponds to $\hat{\omega}$ true and $\hat{\omega}_2$ true. Note that we can always weaken any store $\hat{\rho}$ to apply views at the top-level (i.e., $(\hat{\rho} \circ \hat{\omega}) \Rightarrow (\hat{\rho}_2 \circ \hat{\omega}_2)$ implies $(\hat{\rho}_1 \circ \hat{\omega}_1) \Rightarrow (\hat{\rho}_2 \circ \hat{\omega}_2)$ as desired).

**Introducing and Eliminating Views.** The key difference between a validated view $\hat{\omega}$ and a “main” store abstraction $\hat{\omega}$ is how they are treated in static analysis. While inductive summaries in the main store are unfolded to materialize fields for strong updates and then reintroduced to over-approximate loop invariants, a validated view on the store is auxiliary information that is dropped when potentially invalidated (somewhat like a predicate abstraction [12]). This distinction is why we can use inductive multi-segments for views even though we expect them to be very difficult to work with in the “main store.”
For an inductive validation checker instance $\kappa$ like $\hat{a} \cdot \text{bst}(\hat{u}_1, \hat{u}_2)$, the corresponding inductive definition yields an unfolding relation $\kappa \text{ust} \hat{\rho}$ for each disjunctive case of the inductive definition while replacing formal parameters with actual arguments (cf., Figure 5a in Section 3). For example, the following are in the unfolding relation for bst:

$$\begin{align*}
\hat{a} \cdot \text{bst}(\hat{u}_1, \hat{u}_2) & \text{ust} (\hat{a} \cdot \text{l} \rightarrow \hat{a} \cdot \text{r} \rightarrow \hat{a} \cdot \text{v} \
& \ast \hat{a}_1 \cdot \text{bst}(\hat{u}_1, \hat{v}) \ast \hat{a}_2 \cdot \text{bst}(\hat{v}, \hat{u}_2) \land \hat{u}_1 \leq \hat{v} < \hat{u}_2) \\
\hat{a} \cdot \text{bst}(\hat{u}_1, \hat{u}_2) & \text{ust} \text{emp} \land \hat{a} = \text{null}
\end{align*}$$

Similarly, we assume an unfolding relation for inductive segment instances $\kappa \ast \ast \cdot \hat{\rho}$ given by the recursive validation checkers in the definition of $\kappa$. For unfolding bst in the above, the endpoints $\varepsilon$ would correspond to the two recursive calls $\hat{a}_1 \cdot \text{bst}(\hat{u}_1, \hat{v}) \ast \hat{a}_2 \cdot \text{bst}(\hat{v}, \hat{u}_2)$ in the non-null case. The unfolding of a segment is similarly defined in UNFOLD-SEGMENT: it can result in a new segment whose begin point is $\kappa$ that becomes an endpoint of the view. On an update, our abstract transformer simply and soundly drops any view that constrains the updated field. For example, on line 11 in Figure 6, there are no remaining views because of the update to $p \rightarrow \text{a}$ and the dropping of $\hat{a}_1$. From the perspective of the graph representation, it is straightforward to track the set of views that constrain each field edge.

Inclusion, Join, and Widen. To define the standard abstract domain operations, such as the inclusion $\subseteq$, join $\sqcup$, and widen $\triangleright$ operators, we need a correspondence between the symbolic variables (i.e., the set of names) used in each abstract element. We call such a correspondence a valuation transformer

$$\Psi : \text{Val} \rightarrow_{\text{fin}} \text{Val} .$$

This map gives an instantiation of existentials in one element in terms of existentials in the other. We assume the shape-data abstraction (without views) comes equipped with an abstract inclusion relation

$$\Psi \triangleright \hat{\rho} \sqsubseteq \hat{\rho}'$$

that over-approximates inclusion under concretization with the variables in $\hat{\rho}'$ renamed to the variables in $\hat{\rho}$ using $\Psi$. Formally,

$$\gamma(\hat{\rho}) \subseteq \{ (\sigma, \pi) \mid (\sigma, \pi \circ \Psi) \in \gamma(\hat{\rho}') \} .$$

The setup is analogous for the abstract join operation

$$\Psi_1, \Psi_2 \triangleright \hat{\rho}_1 \sqcup \hat{\rho}_2 \triangleright \hat{\rho}' ,$$

except that we need a pair of valuation transformers $\Psi_1, \Psi_2$ mapping the variables in the result element to each of the input elements.

For inclusion, joins, and widen of validated views, we treat a view predicate as uninterpreted, so the domain structure is straightforward. Treating a view $\hat{\omega}$ as a finite set of view predicates interpreted conjunctively with $\wedge$, abstract inclusion is the set inclusion in the reverse direction among the set of view predicates up to renaming with the valuation transformer $\Psi$: true is the top element; join is set intersection up to renaming; widen is used because it satisfies the ascending chain property.

For the overall abstract domain combining shape-data abstraction with views, we define these operations point-wise

$$\begin{align*}
\Psi_1, \Psi_2 \triangleright \hat{\rho}_1 \sqcup \hat{\rho}_2 \triangleright \hat{\rho}' & \quad \Psi_1, \Psi_2 \triangleright \hat{\omega}_1 \sqcup \hat{\omega}_2 \triangleright \hat{\omega}' \\
\Psi_1, \Psi_2 \triangleright \hat{\rho}_1 \circ \hat{\omega}_1 \sqcup \hat{\rho}_2 \circ \hat{\omega}_2 & \triangleright \hat{\rho}' \circ \hat{\omega}'
\end{align*}$$

The key piece is that they share the same valuation transformers: that is, the join of the view domain additionally constrains the valuation transformers, which is the crucial precision needed to connect the summaries around a hole (cf., the allowable range for bst marked with $\bullet$ in Figure 6).

Reductions and Precision. The validated view domain is designed to compensate for inductive imprecision in the base data-value domain by enabling new reductions [9] (i.e., exchange of information) between the pointer-shape, data-value, and view components of an abstract store $\hat{\rho}$. A view $\hat{a} \cdot \kappa(\hat{v}) \triangleright \varepsilon$ can be used to fold the shape from $\hat{a}$ provided $\varepsilon$ can be shown to hold by either constraints in the shape or view components regardless of the data-value constraints. Also, important equalities connecting parameters in unmodified, “adjacent” regions of the shape graph (cf., Figure 5b) are easily maintained by the view domain. These equalities are typically difficult to derive when widening shape abstractions because the summarization of regions are considered independently in a separation logic-based analysis. While the design of the validated view domain is to compensate for imprecise data-value domains, on the flip side, reduction can be applied to derive a view $\hat{\omega}$ when the pointer-shape and data-value constraints in $\hat{\rho}$ can be shown to imply it (i.e., $\hat{\rho} \sqsubseteq \hat{\omega}$). This could be useful to “save” information in a view from data-value constraints derived in straight-line code that may be lost on widening.

Overall, the validated view domain remembers all programmer-assisted inductive validations of the heap that have not been possibly invalidated by a heap write. As we will see (Section 4), we can synthesize calls to short-circuiting checkers that use these remembered validated views as hypotheses (cf. Figure 2d) and thus avoid checking parts of the heap that have not changed with respect to the data structure invariant.

C. Computing Programmatic Valuations

We express a programmatic valuation as a finite map

$$\hat{\pi} : \text{Val} \rightarrow_{\text{fin}} \text{Expr} \times \text{ProgLoc} \times \text{N} .$$

A $\hat{\pi}$ is flow-insensitive (i.e., is the same within a lexical scope) and maps from symbolic variables to a triple of (1) a program expression $e \in \text{Expr}$ involving the program variables $x$ or symbolic variables $\hat{v}$; (2) a program location $\ell \in \text{ProgLoc}$ for which the SSA-assignment should be instrumented; and (3) a disjunction index $i$. Because an abstract memory $\hat{\sigma}$ is a disjunction of stores $\bigvee \hat{\rho}_i$, we assume every store $\hat{\rho}$ is indexed with a globally unique number given by $\text{disj}(\hat{\rho})$. At any location $\ell$ where we need to apply instrumentation, if there is more than one disjunct in the inferred abstract memory $\hat{\sigma}$ at $\ell$ then we generate an if-else on a special instrumentation variable that stores the disjunction of interest given by $\text{disj}(\hat{\rho})$. Thus, we can assume we are working with single stores $\hat{\rho}$.

Assuming the static analysis computes an invariant map from each program location $\ell$ to an abstract memory $\hat{\sigma}$ that over-approximates the set of possible concrete memories at $\ell$, we can express the analysis proof by annotating the program with these inferred invariants $\hat{\sigma}$. For computing the programmatic valuation, we do not need the annotated invariants at all program locations, but rather only in the cases where the analysis introduced new symbolic variables. To make these locations explicit, we augment the command language $c$ from the underlying programming language (e.g., C) with a few annotated commands $\hat{c}$ to express these analysis operations. The control-flow of the program remains unchanged (e.g., the statement language $s$ in Figure 13 has commands $\hat{c}$ labeled with a program location $\ell$, sequencing, etc.).

In Figure 13, we give the annotated commands of interest, which record the static analysis operations that introduce new symbolic variables (except memory allocation, whose instrumentation is
straightforward). These commands are annotated with the inferred post-state. The **assume** $x \cdot k(\gamma) \{ \hat{\rho} \}$ command indicates that the analysis assumed a data structure validation check $x \cdot k(\gamma)$ yielding a post-store $\hat{\rho}$ where $x, y$ are original program variables. The **unfold** $\kappa \{ \hat{\rho} \}$ command makes explicit the unfolding of an inductive predicate $\kappa$; and the join $\Psi_1, \hat{\rho}_1; \Psi_2, \hat{\rho}_2 \{ \hat{\rho} \}$ command indicates a join of two input stores $\hat{\rho}_1, \hat{\rho}_2$ along with their corresponding valuation transformers $\Psi_1, \Psi_2$.

We describe the generation of a $\hat{\pi}$ via the judgment form $\hat{\pi} \vdash \ell; \hat{\epsilon}$, which says that $\hat{\pi}$ is a programmatic valuation for analysis command $\hat{\epsilon}$ at program location $\ell$. The analogous judgment form $\hat{\pi} \vdash s$ simply walks over the program structure to constrain $\hat{\pi}$. The definition of this judgment is sketched in the last line of Figure 13 with the one rule for statement sequencing $s_1; s_2$, and we can see these rules as the checking system for a flow-insensitive fixed-point computation starting from the empty map. We assume an implicit, fixed parameter to this judgment $\hat{\eta}$ for an abstract environment mapping program variables to symbolic variables for the current static scope. For simplicity, we let the symbolic variables in the abstract environment denote the address of the corresponding program variables and let the abstract store hold the value of the variable.

For **assume**, we “initialize” the programmatic valuation $\hat{\pi}$ by finding a correspondence to the validation check $x \cdot k(\gamma)$ and a shape fact $\hat{a} \cdot k(\gamma)$ in terms of program variables. We add bindings to $\hat{\pi}$ mapping the symbolic variables $\hat{a}$ and $\hat{\gamma}$ in the inductive predicate to the program variable arguments $x$ and $y$, respectively. The mapping also remembers the program location $\ell$ and the store disjunct $i$. On **unfolding** an inductive predicate $\hat{a} \cdot k(\gamma)$, we need to extend the programmatic valuation $\hat{\pi}$ with bindings for the existentials introduced in the definition of $k$. Because all inductive definitions used in our static shape analysis are derived from executable specifications (e.g., static verification predicate $\text{bst}$ from dynamic validation checker $\text{bst}$), program expressions must exist that witness an instantiation for each existential. In other words, any existential introduced in an inductive definition must correspond to the value of an argument or from reading a field. We write this instantiation for an unfolding as $\text{bind}(\kappa \ adj \ \hat{\rho})$. For example, the bind function for the second disjunct in $\text{bst}$ from Figure 2a is

\[ \{ \hat{v} \mapsto f(2 \cdot \hat{v}), \hat{r} \mapsto (\hat{r} > 2) \cdot \hat{r} \mapsto (\hat{r} > 2) \cdot \hat{r} \}. \]

The instrumentation at line 2 and line 7 in Figure 6 come from applying this rule.

Finally, we consider joins $\text{join}(\Psi_1, \hat{\rho}_1; \Psi_2, \hat{\rho}_2) \{ \hat{\rho} \}$. Our convention is that the set of symbolic variables in $\hat{\rho}$ are fresh with respect to $\hat{\rho}_1$ and $\hat{\rho}_2$, so we must extend the programmatic valuation $\hat{\pi}$ with mappings for these new variables. In particular, we should map these to the corresponding symbolic variable from each input store, which is precisely given by the valuation transformers $\Psi_1$ and $\Psi_2$. We express this mapping using the standard static single assignment $\phi(\cdot, \cdot)$ function. The key observation here is that the precision of the programmatic valuation $\hat{\pi}$ after a join point is determined by the valuation transformers computed by the static analysis.

### D. Shortify Soundness

In this section, we state in more detail the shortify soundness theorem from Section 4.1 that relates the statically-proven store with the assertion store to synthesize the short-circuiting validation store.

**Concretization.** In Figure 14, we define the concretization of abstract stores $(\sigma, \pi) \in \gamma(\rho)$ to $\text{CSTORE-SIMP}$ for some $\sigma, \pi$.

**Unfolding.** In Figure 14, we define the concretization of abstract stores $(\sigma, \pi) \in \gamma(\rho)$ to $\text{CSTORE-UNFOLD}$ for some $\sigma, \pi$. The concretization of abstract stores $(\sigma, \pi) \in \gamma(\rho)$ is the least relation satisfying the given inference rules.

**Short-Circuiting Template.** We give a more direct version of the short-circuiting checker template Figure 7b by unfolding the precondition and inlining the static shape analysis. Recall that the proof strategy for statically analyzing the template outlined at the end of Section 4 was by an unfolding of the pre-condition.

**Definition 2 (Short-Circuiting Invariant Checker Synthesis).** For an inductive validation checker $k$, a short-circuiting variant is defined as follows:

\[
\hat{a} \cdot k(\gamma) \overset{\text{def}}{=} \text{emp} \wedge \nabla \gamma \quad \text{or} \quad \forall \hat{\rho}' \wedge \nabla \gamma \iff \hat{a} \cdot k(\gamma)
\]

for all $i$ s.t. $\hat{a} \cdot k(\gamma)$ with

\[
(\sigma, \pi) \in \gamma(\rho)
\]

for all $\sigma, \pi$. For all $i$, $\langle [\sigma(\hat{\rho}) \rightarrow \sigma(\hat{\rho})] \rangle \in \gamma(\hat{\rho})$ for all $\sigma, \pi$. For all $i$, $\langle [\pi(\hat{\rho}) \rightarrow \pi(\hat{\rho})] \rangle \in \gamma(\hat{\rho})$ for all $\sigma, \pi$. For all $i$, $\langle [\sigma(\hat{\rho}) \rightarrow \sigma(\hat{\rho})] \rangle \in \gamma(\hat{\rho})$ for all $\sigma, \pi$. For all $i$, $\langle [\pi(\hat{\rho}) \rightarrow \pi(\hat{\rho})] \rangle \in \gamma(\hat{\rho})$.
Theorem 2 (Shortify Soundness). If \((σ, π) ∈ γ(\hat{ρ} ⋃ \hat{ρ})\) with \((σ, π) ∈ γ(\hat{ρ})\) and \(\hat{ρ} ⊢ [\hat{ρ}′] ≤ \hat{ρ}\) and \((σ′, π) ∈ γ(\hat{ρ′})\) where \(σ′ ⊆ σ\), then \((σ, π) ∈ γ(\hat{ρ})\) for some \(σ′ ⊆ σ\).

Proof. By induction on the derivation \(\mathcal{P}\) of \((σ, π) ∈ γ(\hat{ρ} ⋃ \hat{ρ})\) and the derivation \(\hat{ρ} ⊢ [\hat{ρ}′] ≤ \hat{ρ}\) using the lexicographic order of \(\mathcal{P}\) followed by \(\mathcal{R}\).

Assuming \(\mathcal{P}\), we consider the cases of the shortify derivation \(\mathcal{R}\). The particularly interesting case is for when \(\mathcal{R}\) follows from SHORTIFY-INDUCTIVE.

Case. \(\mathcal{R} = \hat{ρ} ⊢ [\hat{ρ}] ≤ \hat{ρ}\) SHORTIFY-VALIDATE

Case. \(\mathcal{R} = \hat{ρ} ⊢ \{ \hat{ρ}′ \} \) true SHORTIFY-PROVEN

Given \(\hat{ρ} ⊢ \{ \hat{ρ}′ \} \) true, we have that \(γ(\hat{ρ}) ⊆ γ(\hat{ρ}′)\) and \((σ, π) ∈ γ(\hat{ρ}′)\). By the i.h. on \(\mathcal{R}\), we have that \(\hat{ρ} ⊢ [\hat{ρ}′] ≤ \hat{ρ}\) and \((σ′, π) ∈ γ(\hat{ρ′})\) for some \(σ′ ⊆ σ\).

Case. \(\mathcal{R} = \text{emp} \wedge \text{emp} \wedge \text{emp}^2\) SHORTIFY-EMP

Given \(\text{emp} \wedge \text{emp} \wedge \text{emp}^2\), we have that \(γ(\hat{ρ}) ⊆ γ(\hat{ρ}′)\). From the hypotheses, we see that \(π ∈ γ(\hat{ρ}) \cap γ(\hat{ρ}′)\), and so \(\mathcal{R}\) is precise, we have that \((σ′, π) ∈ γ(\hat{ρ′})\) for some \(σ′ ⊆ σ\).

Case. \(\mathcal{R} = \hat{ρ} ⊢ \{ \hat{ρ}′ \} \) \(\hat{ρ} \) \(\hat{ρ}′\) \(\hat{ρ}\) \(\hat{ρ}′\) SHORTIFY-UNFOLD

By the i.h. on \(\mathcal{P}\) and \(\mathcal{R}\), we have that \((σ′, π) ∈ γ(\hat{ρ′})\) for some \(σ′ ⊆ σ\). Thus, we can construct the derivation

\[\hat{ρ} \vdash \{ \hat{ρ}′ \} \subseteq \hat{ρ} \vdash [\hat{ρ}′] \subseteq \hat{ρ}\]

CSTORE-UNFOLD

Case. \(\mathcal{R} = \hat{ρ} \vdash \{ \hat{ρ}′ \} \subseteq \hat{ρ} \vdash [\hat{ρ}′] \subseteq \hat{ρ}\) SHORTIFY-SEP

Given \((σ′, π) ∈ γ(\hat{ρ}′ \wedge \hat{ρ}′′)\), we have that \(σ′ = σ′ \cup σ′′\) for some \(σ′, σ′′\) (that need not be disjoint) such that \((σ′, π) ∈ γ(\hat{ρ}′)\) and \((σ′′, π) ∈ γ(\hat{ρ}′′)\).

Given \((σ, π) ∈ γ(\hat{ρ} \wedge \hat{ρ}′)\), we have that \(σ = σ \cup σ′\) for some \(σ′ \subseteq σ\) such that \((σ′, π) ∈ γ(\hat{ρ}′)\) and \((σ′′, π) ∈ γ(\hat{ρ}′′)\).

By the i.h. on \(\mathcal{P}\) and \(\mathcal{R}\), with \((σ′, π) ∈ γ(\hat{ρ}′)\), we have that \((σ′, π) ∈ γ(\hat{ρ}′)\) for some \(σ′ ⊆ σ\). Analogously by the i.h. on \(\mathcal{P}\) and \(\mathcal{R}\), we have that \((σ′, π) ∈ γ(\hat{ρ}′′)\) for some \(σ′ ⊆ σ\). Since \(σ′\) and \(σ′\) have disjoint domains, \(σ′\) and \(σ′\) must have disjoint domains. Thus \((σ′ \cup σ′) \subseteq γ(\hat{ρ}′ \wedge \hat{ρ}′): σ′\) and \(σ′\) ⊆ \(σ\).

Case. \(\mathcal{R} = \hat{ρ} \vdash \{ \hat{ρ}′ \} \subseteq \hat{ρ} \vdash [\hat{ρ}′] \subseteq \hat{ρ}\) SHORTIFY-INDUCTIVE

Because we construct \(ksc\) so that \(\hat{ρ} \cdot ksc(\hat{ρ}, \hat{ρ})\) is precise, we have that

\[\mathcal{S}_{\mathcal{R}} = \hat{ρ} \cdot ksc(\hat{ρ}, \hat{ρ}) \subseteq \hat{ρ}\]

or

\[\mathcal{S}_{\mathcal{R}} = \hat{ρ} \cdot ksc(\hat{ρ}, \hat{ρ}) \subseteq \hat{ρ}\]

for some \(\hat{ρ}\).

In the first subcase, from \((σ′, π) ∈ γ(\hat{ρ})\) and the hypothesis \((σ, π) ∈ γ(\hat{ρ} \cdot k(\hat{ρ}))\), we have that \((σ, π) ∈ γ(\hat{ρ} \cdot k(\hat{ρ}))\).

In the second subcase, we now consider the concretization of \(\hat{ρ} \cdot k(\hat{ρ})\).

The CSTORE-SIMP cannot apply because \(\hat{ρ} \cdot k(\hat{ρ})\) is an inductive summary. Thus, it must be case that

\[\mathcal{S}_{\mathcal{R}} = \hat{ρ} \cdot ksc(\hat{ρ}, \hat{ρ}) \subseteq \hat{ρ}\]

CSTORE-INDUCTIVE

\[\mathcal{S}_{\mathcal{R}} = \hat{ρ} \cdot ksc(\hat{ρ}, \hat{ρ}) \subseteq \hat{ρ}\]

for some \(\hat{ρ}\).

\[\mathcal{S}_{\mathcal{R}} = \hat{ρ} \cdot ksc(\hat{ρ}, \hat{ρ}) \subseteq \hat{ρ}\]

E. Hash Trie Benchmark

In this section, we describe some additional details about the hash trie benchmark. The hash trie data structure maps keys to values. To store this mapping, a key is hashed, and the bit blocks of the result are used to traverse down the trie to the storing node. Hash collisions are resolved by rehashing at certain levels to provide infinite bit blocks. We consider this data structure as a significant challenge for static verification, as it involves arbitrary hash functions, non-trivial rehashing, and bit-wise concatenation of the rehashed keys. DINV demonstrates the synergy with dynamic validation for these constraints.

The inductive predicate hashtrie shown in Figure 15a constrains \(\text{path}\) with an interpreted function hashtrie that constrains to a complex pure constraint. These interpreted functions enables the elimination of some infeasible states in the analysis, but constraints involving them are left to dynamic validation. In Figure 15b, we give the corresponding C-style invariant checker that validates the path from the root to every stored key equivalent to the bit blocks of the hashed key. The blocks are computed by hashblocks which potentially rehashes the key up to the current level. This function includes bit operations, a loop, and invocations to a hash function. We fixed the example to blocks of two bits and rehashing at every 12 levels.

The write operation of the hash trie is shown in Figure 15c. The main loop traverses down the trie to locate the storing node for the key. In an iteration, bit operations extract two bits to determine the direction, and rehashing is invoked at every 12 levels. Note that the rehashing function hash is different from hashblocks in invariant validation. For a static verifier to prove the assertion at the end of hashtrie_write, it must be able to reason about how the code in this function maintains the hashblocks property. For DINV, it does not need to be able to relate hash and hashblocks to synthesize short-circuiting dynamic validation. The only static reasoning that it needs to show that the trie invariant is maintained across traversal iterations, which is done so by the validated view abstract domain.

After the traversal locates a node to update or reaches an empty sub-trie, the write operation modifies the trie and asserts the invariant before returning. This modification essentially stores the key in
(a) An inductive definition for hash tries.

(b) Corresponding C-style invariant checker.

(c) The hash trie write.

Figure 15. The write operation for hash tries.