DICKE-SUPERRADIANCE IN ELECTRONIC SYSTEMS
COUPLED TO RESERVOIRS

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We present theoretical results for superradiance, i.e. the collective coherent decay
of a radiating system, in semiconductor structures. An optically active region can
become superradiant if a strong magnetic field is applied. Pumping of electrons
and holes at a rate $T$ through coupling to external ‘reservoirs’ leads to a novel kind
of oscillations with frequency $\sim \sqrt{T}$.

1 Introduction

Spontaneous emission is one of the most basic concepts of quantum physics
that can be traced back to such early works as that of Albert Einstein in
1917. Mostly discussed in the context of an atom coupled to a radiation
field, it is one of the paradigms of quantum optics. Another paradigm is
stimulated emission, which leads for the case of a large number of atoms to
the concept of a laser. The corresponding concept in the case of spontaneous
emission of an ensemble with a large number of atoms is the superradiator.
This system has been first proposed by Dicke in 1954, but it took nearly
20 years for the first experiment to be carried out to observe the predicted
superradiant emission peak with an intensity proportional to the square of
the number of atoms (molecules) of the emitting gas. Still, this effect has never
become as popular as the laser, not to speak of a use in the form of a device.
Nevertheless, the physics of superradiance is one of the most interesting in
the field of quantum physics, because it comprises a number of fundamental
concepts such as coherence, symmetry, interaction between particles, the non-
equilibrium physics of transient processes, and the notion of a quasiclassical
limit.

In this short paper, we discuss the superradiance effect in a different
context, that is in solid state physics. In a recent calculation we predicted
a novel form of superradiance for an open system, i.e. an optical active region
that is pumped externally by electron (hole) reservoirs. In semiconductor
quantum wells, the coherent decay of electron-hole pairs leads to a peak of
the emitted light with a strong intensity that, as a function of time, shows
oscillations with a frequency
\[ \omega \simeq \sqrt{2\Gamma T}, \]  
where \( \Gamma \) is the spontaneous decay rate of a single pair and \( T \) the rate at which electrons are pumped in the conduction and holes into the valence band. The system is under a strong magnetic field which guarantees that all optical matrix elements for the recombination of individual electron-hole pairs are the same.

2 Model and results

The photon field \( H_p = \sum_Q \Omega_Q a_Q^\dagger a_Q \) with creation operator \( a_Q^\dagger \) for a mode \( Q \) gives rise to transitions which change an internal degree of freedom \( \sigma = (\uparrow, \downarrow) \) of one-particle electronic states labeled \( (i, \sigma) \) with creation operator \( c_{i,\sigma}^\dagger \). The states are degenerate with respect to \( i \) with energies \( \varepsilon_{i\uparrow} = -\varepsilon_{i\downarrow} = \hbar \omega_0/2 \). The electron–photon coupling matrix element \( g_Q \) is assumed to be independent of the electronic quantum numbers \( (i, \sigma) \). Then, the Hamiltonian of the optically active region can be written as
\[ H_D = \hbar \omega_0 \hat{J}_z + \sum_Q g_Q \left( a_Q^\dagger + a_Q \right) \left( \hat{J}_+ + \hat{J}_- \right) + H_p, \]  
where the operators \( \hat{J}_+ := \sum_i c_{i,\uparrow}^\dagger c_{i,\downarrow}, \hat{J}_- := \sum_i c_{i,\downarrow}^\dagger c_{i,\uparrow} \) and \( \hat{J}_z := \frac{1}{2} \sum_i \left( c_{i,\uparrow}^\dagger c_{i,\downarrow}^\dagger - c_{i,\downarrow}^\dagger c_{i,\uparrow} \right) \) form a (pseudo) spin algebra with angular momentum commutation relations.

We allow the number of electrons \( N \) in the active region to vary by tunneling to and from electron reservoirs \( \alpha = \text{I}/\text{O} \) (‘In’ and ‘Out’) with Hamiltonians \( H_\alpha = \sum_k \varepsilon_{k,\alpha} c_{k,\alpha}^\dagger c_{k,\alpha} \) for non-interacting electrons with equilibrium Fermi distributions \( f_\alpha \). The tunneling of electrons is described by the usual tunnel Hamiltonian \( H_T = \sum_{k\sigma\alpha} \left( t_{k\sigma\alpha}^\alpha c_{k,\sigma}^\dagger c_{k,\sigma} + \text{c.c.} \right) \) with coefficients \( t_{k\sigma\alpha}^\alpha \). The total Hamiltonian is given by the sum \( H = H_D + \sum_\alpha H_\alpha + H_T \).

The active region is characterized by so-called Dicke eigenstates of the total pseudo spin \( \hat{J} \) and its projection \( \hat{M} \) through \( \hat{J}^2, \hat{J}_z, \hat{M} \) with eigenvalues \( J, J, M \), \( \{\lambda\} = J(J + 1), \{\lambda\} = M \{J, M, \{\lambda\}\} \), where \( \hat{J} \) is the total pseudo spin operator. Here, \( \{\lambda\} \) denotes all additional quantum numbers apart from \( J \) and \( M \) that are necessary to characterize the eigenstates of \( H_D \). Radiative transitions obey the selection rule \( M \rightarrow M \pm 1 \) with a spontaneous emission intensity \( I_{JM} = \hbar \omega_0 \Gamma \nu_{JM} \), \( \nu_{JM} := (J + M)(J - M + 1) \). Here, \( \Gamma \) is the spontaneous emission rate of one single two-level system; for radiative transition in atoms, \( 1/\Gamma \) is in the nano second range.
In order to describe the time evolution of the active region coupled to the reservoirs, we used a master equation description in the basis of the Dicke eigenstates. The only relevant quantum numbers in this effective description are $J$, the length of the pseudo-spin, and $M$, its projection. We chose boundary conditions in analogy to electronic pumping in semiconductor laser diodes: each electron entering or leaving the active region leads to an increase of $M$. On the other hand, the collective radiative decay decreases $M$. The dynamics of the pseudo spin therefore is driven by the combined influence of two ‘forces’. In fact, in the classical limit of the master equation, we find the dynamics described by an equation in the $J$-$M$ phase space,

\[
\dot{M}(t) = -\Gamma \nu J(t) M(t) + T
\]
\[
\dot{J}(t) = T \cdot M(t)/J(t).
\] (3)

For $T > 2\Gamma$, this describes a harmonic oscillator with frequency Eq. (1) and amplitude dependent damping. For $T \to 0$, there is a smooth crossover to the conventional Dicke peak with vanishing intensity at large times and without oscillations.

3 Discussion and outlook

Our model applies to the small sample limit of the superradiant problem: reabsorption processes of photons that may lead to oscillatory behavior of the intensity do not play any role here. In fact, that kind of oscillations in superradiance has already been observed in the first experimental verification of the Dicke effect by Skribanowitz et al. They are due to a flow of energy between the atoms and the field modes and can be considered as generalized Rabi-oscillations. Their frequency $\omega$ can be estimated by solving for a mean field theory of the Maxwell-Bloch equations in a one-dimensional model with the result $\omega \sim N^{1/2}d/\ln N$, where $d$ is the modulus of the dipol matrix element.

In our model, the oscillations are due to the combination of two mechanisms (decay and pumping), where the backflow of energy from the field is irrelevant. The dependence of the frequency $\omega \simeq \sqrt{2\Gamma T}$ on the pumping (transmission $T$) should be used to identify this kind of oscillation, which is similar to relaxation oscillations in semiconductor laser diodes. We point out here, however, that the physics of superradiance is different from that of a laser because not stimulated but spontaneous emission is the basic mechanism.

To observe this kind of superradiance experimentally, we propose the system of electrons and holes in a semiconductor quantum well in a strong magnetic field. Electrons are injected into the conduction band and holes into
the valence band of the active region, either by vertical tunneling or thermal emission. The strong magnetic field is necessary to have dispersionless single electron levels \( i = X \), corresponding to the lowest Landau bands \( (n = 0) \) and guiding center \( X \) in the conduction and the valence bands. In this case, the interband optical matrix elements are diagonal in \( i \).

Superradiance in bulk semiconductors under strong magnetic fields has already been discussed theoretically in the literature \(^7\). The recombination of free electrons and holes was found to yield a short and powerful pulse of coherent light of duration 0.1-1 ps and intensities of larger than 100 MW cm\(^{-2}\), with stationary or pulsed magnetic fields of larger than 10\(^5\) G.

Our proposal is aimed at a situation which is closer to quantum-Hall conditions, where the kinetic energy of the electrons is quenched in two dimensions. Furthermore, our calculations support that the initial peak of superradiance is strongly enhanced by the electronic pumping so that in principle one should arrive at very short light pulses of high intensity. The time scale to observe this effect is limited by electron-electron scattering which, as a dephasing mechanism, has to be reduced in order to obtain the collective build-up of the large pseudo spin that decays coherently. Typical temperatures should then be in the sub-Kelvin regime, with magnetic fields of a few Tesla. The subsequent oscillations should also be observable in corresponding oscillations of the pumping current around its stationary value. A preliminary estimate of the time scales then yields an oscillation period of 1 ps for a 1 mA current, with pulse durations shorter than 1 ps and a dephasing time of 10\(^{-11}\) s.

References

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