Twist expansion of differential cross-sections of forward Drell-Yan process

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Abstract. Forward Drell-Yan process at the LHC is a sensitive tool for investigating higher twist effects in QCD. The expansion of all Drell-Yan structure functions is performed assuming GBW saturation model and the saturation scale plays the role of the hadronic scale of OPE. We show that the Lam-Tung relation is broken at twist 4. The results open the way for a forthcoming analysis of multiple scattering and higher twist effects.

Keywords: twist expansion, forward Drell-Yan, low x

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Introduction

Large Hadron Collider (LHC) opens new kinematic regions in high energy physics. One of the most interesting of those areas is a region of small Bjorken-\(x\). The most promising process at the LHC for investigating QCD effects below \(x \approx 10^{-6}\) and moderate energy scales is a forward Drell-Yan (DY) [1]. Such a small \(x\) at parton density scale \(\mu^2 > 6\ \text{GeV}^2\) is about two orders of magnitude smaller than measurements at HERA. Due to the forward character of the process the LHCb detector is the most suitable for those measurements [1]. The differentialDY cross-section may be parametrized in terms of four structure functions that describe hadronic part of the process. One of the main goal of our work is to perform Operator Product Expansion (OPE) for these structure functions. The leading twist, namely twist 2, is well known from theoretical and experimental side, in particular it could be computed using known Parton Density Functions (PDFs). However higher twists are neither well understood theoretically nor explored experimentally yet. On the other hand, in the small \(x\) regime higher twist effects and multiple scattering corrections are important. Understanding of those effects is necessary for example to improve the determination precision of parton densities at the leading twist. Due to possible very small \(x\) in the process, analysis of higher twists in Drell-Yan should provide new valuable information.

For our purpose the most convenient description of DY process is based on so-called helicity structure functions \(W_L, W_T, W_{TT}, W_{LT}\) (see [2]). In this approach one factorizes leptonic and hadronic degrees of freedom by contracting both hadronic and leptonic tensors with virtual photon polarization vectors (PPVs). The tensor reduces to a distribution of lepton angles \(\Omega = (\theta, \phi)\) in lepton pair center-of-mass frame while the result of contraction of hadronic tensor with different PPVs are the \(W\)-structure functions. The inclusive DY cross-section is given by the formula:

\[
\frac{d\sigma}{dxFdM^2d\Omega d^2q_\perp} = \frac{\alpha^2 m^2 \sigma_0}{2(2\pi)^4 M^4} \left[ W_L(1 - \cos^2 \theta) + W_T(1 + \cos^2 \theta) + W_{TT}(\sin^2 \theta \cos 2\phi) + W_{LT}(\sin 2\theta \cos \phi) \right],
\]

where \(x_F\) is a fraction of projectile’s longitudinal momentum taken by virtual photon, \(M\) is an invariant mass of leptons pair, \(q_\perp\) is transverse momentum of virtual photon and the constant \(\sigma_0\) gives the dimension. The form of \(W\)-structure functions depends on arbitrary choice of axes (which defines PPVs) in lepton pair center-of-mass frame. In this paper we use a frame with the \(Z\) axis antiparallel to the target’s momentum and the \(Y\) axis orthogonal to the reaction plane (in [2] this frame is called \(t\)-channel helicity frame).

Inclusive cross section for forward Drell-Yan

In forward Drell-Yan scattering at the LHC there is a strong asymmetry in longitudinal momentum fractions, \(x\), of the colliding partons. We denote the proton from which comes the fast parton \((x_2 \sim 0.1)\) by \(P_2\) and call it the
and it consists of the following parts:

- $\phi(x_F/z)$ is a collinear parton distribution function for the projectile.
- $F(T, P)$ is an unintegrated gluon density describing interaction of fast quark with proton $P_1$.
- $L^{\tau\tau'}(\Omega)$ is a lepton tensor contracted with PPV which reduces to angular coefficients like in (1).
- Impact factor $\Phi_{\tau\tau'}(q_\perp, k_T, z)$ is a hard part of amplitude describing emission of virtual photon with polarization $\tau$. Indices $\lambda_1, \lambda_2$ are the helicities of incoming and outgoing quarks.

In description of high energy scattering it is convenient to use the color dipole model in which the unintegrated gluon density in (2) is replaced by an (equivalent in the leading logarithmic approximation) color dipole cross-section [3]. This approach was proven to be successful in description Deep Inelastic Scattering (DIS) and diffractive DIS data from HERA (see [4]). In the application of the color dipole model to the forward DY scattering we follow Refs. [5, 6]. We shall use dipole cross section $\tilde{\sigma}$ fitted to the DIS data in our DY calculations.

The dipole cross-section is related to the unintegrated gluon density $f(\vec{x}_g, k_T^2)$ as follows:

$$\tilde{\sigma}(r) = \frac{2\pi\alpha_s}{3} \int d^2k_T \frac{f(\vec{x}_g, k_T^2)}{k_T^2} \left| 1 - e^{-i\vec{r} \cdot \vec{k}_T} \right|^2. \quad (3)$$

Inverting this formula one can rewrite (2) in transverse position space. Then the helicity structure functions (1) are easily related to the impact factors:

$$W_i = \frac{2(2\pi)^4 M^4}{\alpha_s^2} \int_{x_F}^1 dz \, \phi(x_F/z) \int d^2r \, \tilde{\sigma}(r) \Phi_i(q_\perp, r, z), \quad \text{for } i = \{L, T, TT, LT\}. \quad (4)$$

Here we introduced impact factors with definite helicity $\Phi_L, \ldots, \Phi_{LT}$ which are linear combinations of $\tilde{\Phi}_{\tau\tau'}$'s Fourier-transformed to position space.
In order to find the twist expansion of (4) we follow methods developed in Refs. [9, 10, 11] and apply the Mellin transformation to the last integral:

$$W_i = \int_{s_F}^1 dz \, \varphi(x_F/z) \int_C \frac{dz}{2\pi i} \left( \frac{z^2 Q_0^2}{M^2(1-z)} \right)^s \tilde{\sigma}(-s) \Phi_i(q_\perp, s, z),$$

(5)

where $\tilde{\sigma}(-s)$ and $\Phi_i(q_\perp, s, z)$ are Mellin transforms of $\sigma(r)$ and $\Phi_i(q_\perp, r, z)$, respectively. The contour $C$ is a straight vertical line in the complex $s$ plane which should be closed from the positive side.

Note that in (5) we have explicitly three energy scales: one soft $Q_0$ which is a saturation scale (coming from dipole cross section $\tilde{\sigma}$) and two semihard scales: $M$ and $q_\perp$. OPE is here given in terms of positive powers of the soft scale $Q_0$.

**Twist expansion and Lam-Tung relation**

In order to perform twist expansion one should choose a model of the dipole cross-section $\tilde{\sigma}$. We adopt the Golec-Biernat and Wüsthoff model [4]

$$\tilde{\sigma}(\vec{r}) = \sigma_0(1-e^{-r^2 Q_0^2/4}),$$

(6)

where $Q_0$ is a $x_1$-dependent saturation scale.

Mellin transform of such function is particularly simple: $\tilde{\sigma}(-s) = -\sigma_0 \Gamma(-s)$. Since $\Phi_i(q_\perp, s, z)$ are analytic functions of $s$ in positive half-plane, the poles of integrand in (5) come from $\Gamma(-s)$ function. These are positive integers so the integral is a sum of infinite number of residues which are proportional to $Q_0^{2k}$ with $k = 1, 2, \ldots$.

As an example of a result, for $W_L$ we get twist 2 of the form:

$$W_L^{(2)} = \frac{Q_0^2}{M^2} \int_{s_F}^1 dz \, \varphi(x_F/z) \frac{4M^6 q_\perp^2 (1-z)^2}{[q_\perp^2 + M^2(1-z)]^3},$$

(7)

and twist 4:

$$W_L^{(4)} = \frac{Q_0^4}{M^4} \int_{s_F}^1 dz \, \varphi(x_F/z) z^2 \frac{4M^8 [7q_\perp^2 - 10M^2q_\perp^2 (1-z) + M^4(1-z)^2] (1-z)^2}{[q_\perp^2 + M^2(1-z)]^6}.$$

(8)

For experimental searches of higher twists the most interesting are quantities that vanish at the leading twist. In the DY process such a quantity may constructed using the Lam-Tung relation [7]. This relation was proven for the parton model and in terms of helicity structure functions takes the form [8]:

$$W_L^{\text{par}} - 2W_{TT}^{\text{par}} = 0.$$

(9)

At twist 2 $W_{TT}$ is given by:

$$W_{TT}^{(2)} = \frac{Q_0^2}{M^2} \int_{s_F}^1 dz \, \varphi(x_F/z) \frac{2M^6 q_\perp^2 (1-z)^2}{[q_\perp^2 + M^2(1-z)]^3},$$

(10)

and comparing it with (7) we immediately see that the Lam-Tung relation (9) is satisfied at the leading twist, as expected.

At the next-to-leading twist, namely twist 4, relation is broken:

$$W_L^{(4)} - 2W_{TT}^{(4)} = \frac{Q_0^4}{M^4} \int_{s_F}^1 dz \, \varphi(x_F/z) z^2 \frac{4M^8 (1-z)^2}{[q_\perp^2 + M^2(1-z)]^7}.$$

(11)

It is a well known fact that (9) is violated also by higher order QCD corrections, however at the very small $x$ the contribution coming from higher twists is sizeable comparing to them.
Twist expansion of $q_\perp$-inclusive cross section

In [11] the twist expansion for forward DY process was performed for cross-section inclusive in $q_\perp$ and lepton angles ($\theta$, $\phi$), then only $W_L$ and $W_T$ give the contribution. To fill this gap we performed the twist expansion for $W_{TT}$ and $W_{LT}$ integrated over $q_\perp$. This is interesting also since angular distribution inclusive in $q_\perp$ is easier to measure.

We define $\hat{W}_i = (2\pi M^2)^{-1} \int W_i d^2 q_\perp$, which for the cross-section $\frac{d\sigma}{d\theta dM^2}$ are analogues of $W_i$. At first glance the twist expansion could be performed in the same way as in the differential case, however taking residue for $s = 1$ for $W_T$ we get:

$$\frac{Q^2_0}{M^2} \int x F(x) \frac{1 + (1 - z)^2}{1 - z} \frac{\pi M^2}{3}.$$

This is clearly a divergent integral over $z$. This means that the integration over $q_\perp$ introduces double poles in expressions for $\hat{W}_i$. We shall follow [11] to obtain convergent expressions in the twist expansion. Then for example:

$$\hat{W}_{T}^{(2)} = \frac{Q^2_0}{4 M^2} \left\{ \frac{\rho(x_f)}{3} \left[ \frac{Q^2_0}{4 M^2} (1 - x_f) \right] + \frac{2}{3} \int_{x_f}^{1} dz \frac{1 + (1 - z)^2}{1 - z} \frac{\rho(x_f)}{3} \right\}.$$

Due to additional poles coming from integral over $z$ we get also nonzero odd twists — in our frame only for $\hat{W}_{LT}$:

$$\hat{W}_{LT}^{(3)} = \text{const} \frac{Q^2_0}{M^3} \rho(x_f), \quad (14)$$

where const $\approx 0.593$.

CONCLUSIONS AND OUTLOOK

The forward Drell-Yan scattering is a promising process for searching of higher twists because LHC is expected to provide data for kinematic region of very small $x$ where they are sizeable. The quantity $W_L - 2W_{TT}$ should be particularly useful for such searches since it vanishes at the leading twist. In this talk we briefly presented theoretical calculations of the twist expansion of the forward Drell-Yan cross-section. The full results and predictions of higher twist contributions for the LHC will be given in a forthcoming paper [12]. We emphasize that precise measurements of angular distribution of the forward Drell-Yan could be essential for understanding higher twists effects in QCD.

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