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Research on Numerical Wave-making Method Based on Viscous Flow Wave

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Abstract. In this paper, the numerical method on physics-based push plate wave is studied. The wave tank is numerically simulated based on the N-S equation of viscous fluid and the standard k-ε turbulence model. The volume of fluid (VOF) is used to capture the free surface, and the wave plate is oscillated by the dynamic mesh technique. In addition, the CFD commercial software Fluent and its secondary development function are employed to establish a 3D numerical regular wave tank. After the analysis of the results, it can be found that the imitating physical push-plate wave created in this paper is of high accuracy and good stability.

1. Introduction

Wave, which is one of the most important driving forces in the marine environment, is closely related to the performance of ships. Recently the research on a numerical wave tank designed to achieve the practical engineering application has become the focus of international offshore engineering field. However, since the development of the wave theory has been stuck in the stage of semi-empirical formula and ideal fluid hypothesis, as well as the high cost of the experimental physical tank and the long cycle, the numerical wave tank emerges as the third research method. The numerical wave tank is designed to simulate the various functions of the experimental physical tank through a computer simulation, so as to complete the scientific research and engineering tasks. In the investigation, perfecting and improving the various boundary techniques of the numerical wave tank is the basis for applying numerical wave tanks to practical engineering problems. With the rapid development of computers, CFD has been widely used, and many commercial fluid dynamics softwares such as CFX, STAR-CD, and FLUENT have appeared. Although there are still some deficiencies, with the rapid development of CFD, they will play more and more important roles in the field of marine engineering.

In this paper, the numerical wave tank is simulated by solving the N-S equation of the viscous fluid. The VOF method is employed to capture the free surface and the secondary development of the fluent software is performed. The dynamic grid technology is adopted to achieve the making of push-plate wave. In the end, the three-dimensional wave field of push plate wave making is simulated.
2. Mathematical Model of the Wave Tank

2.1. Basic control equation
The instantaneous incompressible fluid control equation is averaged over time, thus, the continuous equation of time average is obtained. The time-averaged Navier-Stokes equation, i.e., the Reynolds average N-S equation is written as:

\[
\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = \rho \frac{\partial f_i}{\partial x_i} - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i}{\partial x_j} \right) + \left[ -\frac{\partial (\rho u_i u_j)}{\partial x_j} \right]
\]

(1)

Where \( \overline{\rho u_i u_j} \) is the Reynolds stress.

2.2. Turbulence model
The standard two-equation model is chosen in the study, which is as follows:

\[
\frac{\partial}{\partial t} (\rho \varepsilon) + \nabla \cdot (\rho \varepsilon \mathbf{u}) = \nabla \cdot \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \nabla \varepsilon \right] + C_{1\varepsilon} \frac{\varepsilon}{k} + (G_k + C_{3\varepsilon} \varepsilon) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} + S_\varepsilon
\]

(2)

\[
\frac{\partial}{\partial t} (\rho k) + \nabla \cdot (\rho k \mathbf{u}) = \nabla \cdot \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \nabla k \right] + G_k + G_b - \rho \varepsilon - Y_M + S_k
\]

(3)

In the equation, \( G_k \) represents the turbulent kinetic energy generated by the laminar velocity gradient, \( G_b \) is the turbulent kinetic energy generated by buoyancy, \( Y_M \) is the fluctuation caused by the diffusion in compressible turbulence, \( C_{1\varepsilon}, C_{2\varepsilon}, C_{3\varepsilon} \) are constants, \( \sigma_k \) and \( \sigma_\varepsilon \) are the PRANDTL numbers of turbulence in \( k \) and \( \varepsilon \) equations, \( S_k \) and \( S_\varepsilon \) are user-defined. \( \mu_t \) represents the turbulence speed, which is determined by the formula \( \mu_t = \rho C_{\mu} \frac{k^2}{\varepsilon} \).

2.3. VOF Method
VOF (Volume of Fluid) is put forward by Hirt and Nichols, based on the particle tracking method (MAC). It is to determine the motion interface and its changes by taking the volume fraction \( \alpha_q \) of each phase in the fluid elements. It is apt for various incompatible mixed fluids. The variable \( \alpha_q \) is the volume ratio of phases in the calculation unit. In the calculation unit, the sum of the volume fractions of all the phases is equal to 1, that is, \( \sum \alpha_q = 1 \). By solving the continuous equations of the phase volume fractions, the free surface between phases is tracked.

3. Wave making
Wave making is the key technology of numerical wave tanks. The schematic diagram of the push-plate wave maker is shown in Fig. 1. The coordinate system is established with the Z axis overlapping the initial position of the wave plate, the Z axis positively upwards, and the X axis coinciding with the still water surface. Assuming that the wave plate is doing small amplitude harmonic movement, \( h \) is the depth of the tank. The wave plate is doing reciprocating piston movement with the amplitude \( A_p \). Thus,
waves are generated by the wave plate, where $H$ is the wave height, $L$ is the wavelength, and $T$ the period.

The displacement of the wave plate movement can be expressed as:

$$X_p = A_p \sin(\omega t)$$  \hspace{1cm} (4)

Where $X_p$ the displacement of the wave plate motion is, $A_p$ is its amplitude, and $\omega$ is the circle frequency, $t$ is time.

Assuming the water is an incompressible and non-viscous fluid, then the velocity potential function of the flow field in the wave groove satisfies the Laplace equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$  \hspace{1cm} (5)

Meanwhile, the velocity potential satisfies the following boundary conditions.

1. **Water surface conditions:**

   $$\frac{\partial \phi}{\partial t} \bigg|_{z=0} + g \eta = 0$$  \hspace{1cm} (6)

   $$- \frac{\partial \phi}{\partial z} \bigg|_{z=0} = \frac{\partial \eta}{\partial t}$$  \hspace{1cm} (7)

2. **Underwater conditions:** Because the bottom of the tank is waterproof, the vertical speed of the water quality point is zero, so

   $$\frac{\partial \phi}{\partial z} \bigg|_{z=-h} = 0$$  \hspace{1cm} (8)

3. **Body surface condition:** The water quality velocity on the wave plate is equal to the flat motion speed of the wave plate. Assuming that the displacement and speed of the wave plate are both small, the following boundary conditions can be obtained approximately as follows:

   $$\frac{\partial \phi}{\partial x} \bigg|_{x=0} = A_p \omega \cos(\omega t)$$  \hspace{1cm} (9)

The above equation is solved with the discrete variable method. According to the corresponding initial and boundary conditions, the velocity potential function of the flow field is:

$$\phi(x, z, t) = \frac{g}{w} C_0 Z_0(kz) \sin(kx - \omega t) + \frac{g}{w} \sum_{n=0}^{\infty} C_n Z_n(k_n z) \cos(\omega t) e^{-k_n x}$$  \hspace{1cm} (10)
With \( Z_0(kz) = \cosh(k(z + h))/\cosh(kh) \) and \( Z_n(k_nz) = \cos(k(z + h))/\cosh(k_nh) \). In the formula, the second item is a transient wave, \( k = \frac{2\pi}{L} \) is the wave number. \( k_n \) And the angular frequency meet the dispersive relation: \( w^2 = kg \tanh(kh) \), \( w^2 = -k_n g \tanh(k_nh) \). \( C_0 \) And \( C_n \) are \( C_0 = \frac{2[\cosh(2kh) - 1]}{\sinh(2kh) + 2kh} \), \( C_n = \frac{2[\cos(2k_nh) - 1]}{\sin(2k_nh) + 2k_nh} \) \( (n = 1, 2, \cdots) \).

From equation (6), we can derive the wave height at the average position of the wave plate in motion: \( \eta_0 = -\frac{1}{g} \frac{\partial \phi}{\partial t} \). \( \eta_0 = C_0 A_p \cos(kx - wt) + \sum_{n=1}^{\infty} C_n A_p \sin(wt)e^{-k_n t} \). The first item on the right is the traveling wave, and the second item is the standing wave produced by the motion of the wave plate.

With formula (4) and the secondary development of Fluent, the motion law of the wave plate is defined. By adopting UDF function DEFINE_AFLOW_MOTION (M_push, dt, cg_vel, cg_omega, time, dtime), the program is written for the movement of push board.

4. Numerical Computation and Analysis of Simulation Results

In the study, the 3D numerical wave tank model simulation of physical is established with the pre-processing software ICEM-CFD. The pressure inlet on top of the wave tank is set to be the boundary condition. The outlet is Outlet-vent, and Loss Coefficient is 1. The appropriate parameters are set on Open Channel, the wave board is set using UDF, and other boundary conditions are set as wall boundary.

The wave plate adopts three-dimensional dynamic grid technology for wave making. Figure 2 shows the partially enlarged view of the dynamic mesh. The finite volume method is employed for the fluid control equations and turbulence models to perform discreteness (FVM). Pressure Semi-Implicit Segmentation Technology is applied for the coupling of pressure speed, and the geometric reconstruction method for capturing the free liquid surface. At the initial time \( t=0 \) on top of the free liquid surface is all air, with a density of 1.225 kg/m\(^3\). The lower part of the free fluid surface is full of water with a density of 998.2 kg/m\(^3\).

![Figure 2. Partial enlargement of the wave plate grid.](image)

The target wavelength is set as 8m and the period as 2.2678s.with the push plate the three-dimensional waveforms with different wave heights of 0.2m, 0.3m, and 0.4m are created. Wavegauges are put in different positions in the numerical wave tank to collect the wave height time history at these locations.
Figure 3 shows history chart of the wave height over time when the horizontal distances from the wave plate are respectively 1, 2, 3, or 4 times of the wavelength. From the figure, it can be seen that as time goes on, the waveform tends to be stable. The longer away from the push wave plate the horizontal distance is, the longer the development of the waveform needs, and the slower the stable wave forms. As a result, the wave height is smaller than the theoretical value. This is because the wave transmission takes time to accumulate and the wave energy is constantly attenuated during the transmission due to the effect of the viscosity. After some periods, when the energy the wavefield acquired is equivalent to the energy dissipated, the wavefield eventually reaches stable. Due to the effect of the viscous dissipation, the smallest is the average of wave height where the distance from the wave plate is 4 times the wavelength.

Figure 4. Distribution diagram of pressure field.
Figure 4 is the distribution of the pressure field, which shows the pressure gradually increases along the depth of the water. The free liquid surface is an isobaric surface, which corresponds with the theoretical result. Figures 5 and 6 indicate the instantaneous speed vector. As can be seen, the further away from the wave surface the fluid particle is, the smaller its velocity turns.

![Figure 5. Vector diagram of instantaneous speed.](image)

![Figure 6. Vector diagram of the velocity near the free liquid surface](image)

It can be seen from Table 1 that the waveform formed by the wave plate is of high stability and quality. The error of wavelength numerically simulated and theoretically adopted is 0.9063%, and the period error is 0.8863%. The wave heights recorded by the wave height meter at different positions are shown in Figure 7. It can be seen that the wave height decays faster where the distance from the wave plate is 2 times the wavelength, and then gradually stabilizes until a stable wave field is formed.

|                           | Wavelength /m | Period /s |
|---------------------------|---------------|-----------|
| Theoretical Value         | 8.0000        | 2.2678    |
| Numerical Value           | 7.9275        | 2.2879    |
| Relative Error /%         | 0.9063        | 0.8863    |

![Figure 7. Attenuation curves of different wave height](image)

5. Conclusion
In this paper, with FLUENT software as a platform, using VOF method, the software Fluent is secondary development to achieve the motion of wave plate. By solving the standard turbulence
equation the numerical simulation of the wave-making in the wave tank is achieved. From the results, we can see that the quality of the wave made by wave plate is of high quality and stability. This study can provide technology support for the numerical wave-making with CFD calculations of ships.

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