Atom resonant tunnelling through a moving barrier

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Abstract. We study the one-dimensional dynamical behaviour of an atomic wavepacket moving across an opaque and spatially oscillating optical barrier. The tunnelling probability shows a dramatic dependence on the frequency of the barrier oscillation. Transmission of the atomic wavepacket is, in fact, dominated by inelastic or elastic effects, depending on whether the oscillation period is comparable to or shorter than the atom–barrier interaction time. In the elastic regime, in particular, we find that full transparency becomes possible at energies where the stationary barrier would otherwise be essentially opaque. We predict that both inelastic and elastic resonant tunnelling regimes could be observed with atomic condensates impinging on a spatially oscillating optical barrier. A few applications will also be discussed.
1. Introduction

The classically forbidden process of tunnelling through a barrier is one of the most striking manifestations of the wave-like behaviour of matter predicted by quantum mechanics. Tunnelling through a time-dependent barrier has attracted much interest for two main reasons. The first is the explanation of a variety of phenomena, e.g., photoinduced dynamics in strong laser fields [1], high frequency field impurity ionization [2], transport in superlattices under terahertz fields [3], quantum chaos and dynamical tunnelling [4, 5], or diffusion and relaxation processes [6], where tunnelling through a fluctuating barrier plays a key role. The second is the paradigmatic use of periodically modulated barriers to characterize the controversial notion of tunnelling time [7]–[9].

In most instances, only the height of the barrier is assumed to depend on time so that the barrier potential can be separated as $V_S(x, t) = U(x) f(t)$ [7, 10]. We are here instead interested in the case of atoms tunnelling across a moving barrier described by a potential of the form $V(x, t) = U(x - lf(t))$ [11, 12], where $f(t)$ is an adimensional periodic function of time with frequency $v$. Assuming $|f(t)| \leq 1$, the barrier maximum displacement is given by $l$. The specific physical barrier that we envision here consists of a repulsive optical dipole potential created by an off-resonant blue-detuned sheet of light [13] and set to move orthogonal to the atomic velocity. The spatial position of such an optical barrier along the atom’s trajectory can be controlled by an appropriate modulation of the barrier light path, e.g. via an oscillating mirror.

The atomic dynamics, which we take here for simplicity to be one-dimensional, is described by the time-dependent Schrödinger equation:

$$i\hbar \frac{d}{dt} \psi(x, t) = \left( -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x, t) \right) \psi(x, t)$$

$$= \left( -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x, t) + U(x - lf(t)) \psi(x, t) \right).$$ (1)
Figure 1. Basic description of the process under consideration. Density profiles of the reflected (left arrow) and transmitted (right arrow) matter waves after an atomic beam with energy $E_k = 0.2V_0$ near resonance and spread $\Delta E$ has impinged on the barrier with height $V_0$ and width $d$, spatially oscillating at high frequency $\nu = 10$ kHz and with amplitude $l = 2d$. Note the large portion of the transmitted matter wave well below the barrier height $E_k \ll V_0$. The fraction of the matter wave still remaining in the barrier region is visible and zoomed ($\times 10$) in the inset. Here, the time-averaged potential $V_{av}$ corresponding to the $\nu \rightarrow \infty$ limit is also displayed. The system parameters are listed in table 1.

In the high-frequency limit, i.e. when the barrier oscillation period $T = \nu^{-1}$ is shorter than the atom–barrier interaction time, the relevant physics can be easily understood considering a time-averaged potential as seen by the atom wavepacket in the form $V_{av}(x) = \int_T^0 dt \, V(x, t) / T$. If the barrier spatial motion has a large enough elongation $l$, the averaged potential $V_{av}(x)$ exhibits a double barrier structure, as shown in the inset in figure 1. In such a potential we may have one or more quasi-bound metastable states. Therefore, for sufficiently large values of $\nu$, the transmission of atoms with energy corresponding to that of a metastable state is expected to be strongly enhanced, through a mechanism of elastic resonant tunnelling, with respect to the transmission through the same barrier at rest.

In a different physical context, nearly perfect transparency has been predicted to occur [14] for a particle with charge $q$ and mass $m$ tunnelling through a stationary opaque barrier $U(x)$ in the presence of a strong laser field of amplitude $E$ and angular frequency $\Omega$. In such a case, a transformation from the laboratory reference frame to an oscillating frame via the Kramers–Henneberger representation [1, 15] leads to a time-dependent Schrödinger equation having a potential $V(x, t)$ of the same form as above with $f(t) = \sin(\Omega t)$ and $l = qE / (m\Omega^2)$, which formally accounts for both the laser field driving and the barrier potential which is stationary in the laboratory frame. Enhanced tunnelling via the metastable states of the corresponding $V_{av}(x)$ in the oscillating frame is seen to take place at large values of $\Omega$ and $l$ through a resonant mechanism of field-induced barrier transparency [14].
Table 1. System parameters for the case of sodium atoms \((m_{\text{Na}} = 23)\).

| \(\Delta E/h\) (kHz) | \(d\) (nm) | \(V_0/h\) (kHz) | \(E_0/h\) (kHz) | \(\Gamma_0/h\) (kHz) |
|-----------------|---------|----------------|----------------|----------------|
| 0.21            | 827     | 4.2            | 0.90           | 0.06           |

Whereas the high-frequency limit is well understood, the intermediate-frequency range, which is when the barrier oscillation period \(T = \nu^{-1}\) becomes comparable to or larger than the atom–barrier interaction time, remains largely unexplored. We therefore focus here on the case of neutral atoms tunnelling through a barrier oscillating in the laboratory frame at frequencies spanning this new range. We will address, in particular, two interesting phenomena dealing with the onset of resonant tunnelling transmission and the concomitant inelastic processes both occurring when the barrier frequency \(\nu\) is swept across the whole intermediate-frequency range.

On the experimental side, field-induced transparency for charged particles is hard to realize and it is here worth noting that such an effect has not yet been observed, to the best of our knowledge, either in the high- or intermediate-frequency range. Owing to recent progress in manipulating and controlling ultracold atoms through optical potentials \([4, 16, 17]\), however, matter waves and moving barriers can nowadays be realized in routine experiments where wavepackets with the required kinetic energies and energy spreads can be prepared. Oscillating light barriers, on the other hand, can be generated by using appropriately detuned laser beams. The elongation and period of the barrier motion can be independently varied over a large range of values and in various ways, possibly including non-harmonic oscillations. In short, the condensate’s state-of-the-art techniques seem to favour the observation of both resonant tunnelling transmission and inelastic processes phenomena that we address in this work.

We use, in fact, theoretical and numerical methods to show that resonant tunnelling of atoms through a single barrier whose position is rapidly oscillating can be demonstrated for realistic experimental parameters, when the elongation \(l\) of the barrier motion is sufficiently larger than the barrier thickness. We provide a detailed study of how resonant transmission at the energy of a quasi-bound state develops as the barrier oscillation frequency is set to increase towards the high-frequency limit. We also analyse in this context the role played by inelastic processes during the onset of resonant tunnelling.

This paper is organized as follows. We describe the system under consideration in section 2 while the way in which the observables of interest are determined is given in section 3. For a realistic experiment involving sodium atom condensates we present the results for the transmission coefficient and the relevant inelastic processes, respectively, in sections 4 and 6 while those for the atom–barrier interaction time are given in section 5. We finally turn to our concluding remarks in section 7.

2. The physical system

Even though the main qualitative features of our results are of general validity, to be definite we tailor the relevant physical scales of the atomic wavepacket with reference to the case of sodium atoms (see table 1). Other species may be considered in a straightforward manner, after appropriate rescaling of energies.

We then consider the case of a one-dimensional Gaussian atomic wavepacket with kinetic energy \(E_k\) and energy spread \(\Delta E\) impinging on a square barrier of thickness \(d\) and height \(V_0\) whose
position undergoes a harmonic oscillation of amplitude \( l = 2d \) at frequency \( \nu \). The relevant potential can be written as \( U(x) = V_0 [\theta(x - 0.5d) - \theta(x + 0.5d)] \) with \( f(t) = \sin(2\pi \nu t) \). A typical process occurring near resonance in the high-frequency limit is depicted in figure 1, where the density profiles of the reflected (left arrow) and transmitted (right arrow) atomic beams are shown, together with a portion of matter wave still trapped in the region of the oscillating barrier [14] shown in the inset of figure 1. Tailoring of the system parameters is crucial to observe a marked effect\(^5\) and ultracold atoms are certainly promising candidates. Our choice of parameters was, in fact, guided by the features of the resulting time-averaged potential \( V_{av}(x) \) displayed in the inset of figure 1. In particular, \( l \) is large enough with respect to \( d \) to obtain a double-barrier structure, while \( V_0 \) and \( l \) are such that only one quasi-bound state lies within the double-barrier potential. The wavepacket energy spread \( \Delta E \) is also not too large compared with the linewidth of the metastable state. The position \( E_0 \) and linewidth \( \Gamma_0 \) of the quasi-bound state reported in table 1 have been calculated from the transmission coefficient obtained by a standard transfer matrix approach [18].

3. The observables

We focus on the dependence of the transmission coefficient and the inelastic losses on the barrier oscillation frequency \( \nu \) as functions of the incident kinetic energy \( E_k \) varying in the range from 0.1\( V_0 \) to \( V_0 \). These correspond to central velocities of the atomic wavepacket ranging from approximately 2 to 18 mm s\(^{-1}\). For the sake of clarity, the effects of atomic interactions [19] are hereby neglected.

While the analysis of the time-independent tunnelling through the barrier at rest (\( \nu \to 0 \)) or in the high-frequency limit (\( \nu \to \infty \)) is amenable to standard methods, the dynamical behaviour of the wavepacket for intermediate values of the oscillation frequency \( \nu \) has been determined by solving numerically the time-dependent Schrödinger equation using a well-tested explicit time-marching algorithm [20]. Either in the high-frequency limit or for a barrier at rest, scattering from the barrier is elastic and the wavepacket remains smooth. In the case of intermediate oscillation frequencies, instead, the transmitted/reflected wavepackets may spread (requiring larger simulation boxes) and acquire fast spatial (and temporal) oscillations (requiring tiny mesh sizes). We have required an accuracy of at least \( 10^{-7} \) in the conservation of probability, leading to grids up to \( N = 24\,000 \) points, typical spacings of the order of \( dx \simeq 0.03d \) and time steps \( dt \simeq 2 \times 10^{-4} \) \( 2md^2/\hbar \). Typical simulations last up to 300 times the oscillation period of the barrier.

3.1. The transmission coefficient

The transmission coefficient is defined as

\[
S(t) = \frac{\int_{R_T} |\psi(x, t)|^2 \, dx}{\int_{R} |\psi(x, t)|^2 \, dx}.
\]

Here \( R \equiv [x_-, x_\infty] \) is the full simulation box domain, \( R_T = [x_T, x_\infty] \) is the region of the transmitted wavepacket, where \( x_T \) is a position sufficiently far away from the interaction region and in our case we take \( x_T = 30d + 5l \). As displayed in figure 2 for the case of atoms tunnelling

\(^5\) Such time-dependent resonant tunnelling was not found in [11, 12] because an appropriate choice of parameters was not made.
Figure 2. Atoms tunnelling across the time-averaged potential. Fraction of matter wave in the transmission region $R_T = [x_T, x_\infty]$ as a function of time in milliseconds in the case of atoms tunnelling across the time-averaged potential. Different curves refer to different values of $E_k/V_0$.

across the time-averaged potential $V_{av}(x)$, $S(t)$ has an S-shape: it is zero up to the time when the transmitted wavepacket starts reaching $x_T$, and then it saturates up to a constant value when the wavepacket definitely passes $x_T$. Thus, the transmission coefficient $T$ is given by the value at which $S(t)$ saturates. The reflection coefficient $R$ is computed in an analogous way and, along with $T$, they satisfy the property $T + R = 1$.

A few details that characterize the case of the moving barrier, are summarized in figure 3. For intermediate frequencies, oscillations are superimposed on the $S(t)$ profile during the atom–barrier interaction time. These oscillations are seen to smooth away at high frequencies. For oscillation frequencies definitely smaller than 0.5 kHz and larger than 10 Hz, saturation and hence the transmission coefficient depends on the phase of the barrier oscillation. In these cases, the value of $T$ and of all the other observables is determined after averaging over several phases in the interval $[0, 2\pi]$ [11].

3.2. The dwell time

The atom–barrier interaction time $\tau_D$ is calculated by adopting the standard definition of dwell time [8, 9, 21, 22]:

$$\tau_D = \int dt \ P(t) = \int dt \ \int_{R_D} dx \ |\psi(x, t)|^2$$

(3)

is the probability of finding the atom in the barrier region $R_B = [-l - 0.5d, +l + 0.5d]$. For the case of the barrier at rest the dwell time is calculated only in the region $R_B = [-0.5d, +0.5d]$. Here $P(t)$ has a bell-shape, which becomes asymmetrical in correspondence with resonant tunnelling. This is shown in figure 4 in the case of atoms tunnelling across the time-averaged potential $V_{av}(x)$.
Figure 3. Atoms tunnelling across the barrier oscillating with frequency \( \nu \). Fraction of the matter wave in the transmission region \( R_T = [x_T, x_\infty] \) as a function of time in milliseconds. The atomic beam has energy \( E_k/V_0 = 0.21 \) close to resonance. Different curves refer to different values of \( \nu \).

potential. When the atoms tunnel across the moving barrier, the same oscillations as in figure 3 also appear in \( P(t) \), as displayed in figure 5. From the inset of figure 5, it is clear that oscillations occur at the frequency of the moving barrier.

4. Transmissivity

4.1. Low- and high-frequency limit

We start from the two limiting regimes of very low and high barrier oscillation frequencies which correspond, respectively, to tunnelling through the stationary barrier \( U(x) \) and through the time-averaged potential \( V_{av}(x) \) as shown in the inset in figure 1. We compare in figure 6 the transmission coefficient as a function of the atomic energy \( E_k \) at \( \nu = 10 \) Hz with the results obtained for the barrier at rest. These are further compared with the analytical expression for a strictly monochromatic (CW) case. As expected, the three of them agree because, on the one hand, the Doppler shift at \( \nu = 10 \) Hz is negligible and, on the other hand, the analytical CW transmission is a smooth function of \( E_k \) on the scale of the atomic energy spread \( \Delta E \).

In the same figure, we also compare the transmission coefficient at \( \nu = 10 \) kHz with the one for the time-averaged potential. Again there is no significant difference. We notice, in particular, the well developed resonant tunnelling peak at the energy \( E_k = 0.214 V_0 \) of the metastable state in the corresponding time-averaged potential. We also show the CW transmission calculated with a standard transfer matrix approach [18]; this, in particular, gives the values of the metastable state energy \( E_0 \) and linewidth \( \Gamma_0 \) reported in table 1. Since the energy spread \( \Delta E \) of the atomic wavepacket is larger than the linewidth \( \Gamma_0 \) of the metastable state, the resonant peak shown
Figure 4. Atoms tunnelling across the time-averaged potential. Fraction of the matter wave in the interaction region $R_D = [-l - 0.5d, +l + 0.5d]$ as a function of time in milliseconds. Different curves refer to different values of $E_k/V_0$.

by the simulations in figure 6 is not fully resolved. In contrast, the CW transmission across the symmetric double-barrier potential $V_{av}(x)$ reaches unity at resonance. As far as the second transmission peak at $E_k \approx 0.36V_0$ is concerned, it is already near the maximum of the time-averaged potential and does not correspond to a metastable state. This is consistent with figure 4, where the curves at $E_k/V_0 \geq 0.3$ do not show any significant asymmetry.

4.2. Intermediate-frequency regime

Here we assess how the resonant tunnelling peak develops as the oscillation frequency is increased. On physical grounds one would expect the transition between the low and high frequency regime to occur when the period of the barrier oscillations becomes small compared to the atom–barrier interaction time (see figure 8 below and related discussion). From figure 7, it is evident that the resonant tunnelling peak develops about $\nu \approx 3$ kHz, with no trace of it remaining at $\nu = 1$ kHz. Finally, the high-frequency limit is already reached at $\nu = 8$ kHz.

5. Atom–barrier interaction time

We consider then the atom–barrier interaction time $\tau_D$ as defined in section 3.2. This is shown in figure 8 as a function of the atomic energy $E_k$ for different barrier oscillation frequencies $\nu$ in the range from 1 to 10 kHz. Comparison with figure 7 shows that for $\nu > 1$ kHz the development of the resonant tunnelling peak is accompanied by a sharp increase in the dwell time. The dwell time remains nearly constant at about $\tau_D \approx 0.2$ ms in the case of a stationary barrier, as shown by the triangles in the inset of figure 8. At high frequencies, that is around $\nu = 10$ kHz (full curve in the inset) and for the time-averaged potential (full circles), we obtain $\tau_D \approx 3.5$ ms. This

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Figure 5. Atoms tunnelling across the barrier oscillating with frequency $\nu$. Fraction of the matter wave in the interaction region $R_D = [-l - 0.5d, +l + 0.5d]$ as a function of time in milliseconds. The atomic beam has energy $E_k/V_0 = 0.21$ close to resonance. Different curves refer to different values of $\nu$. Inset: enlarged view of the curve at 1 kHz, showing the oscillations.

is in agreement with the expectations for a time-independent double barrier [21], in which case $\tau_D \simeq 2\hbar/\Gamma_0$ at exact resonance and $\tau_D \simeq \hbar/\Gamma_0$ at a detuning $\Gamma_0$.

Remarkably, $\tau_D$ is seen to be very sensitive to changes in the barrier oscillation frequency, which preludes the possibility of an accurate control of the tunnelling dynamics. In fact, whereas $\tau_D$ is still smaller than the barrier oscillation period $T$ at $\nu = 1$ kHz, it becomes several times longer than $T$ already at $\nu = 4$ kHz, substantiating the above estimate of the transition frequency at which the resonant tunnelling peak shows up.

We finally discuss the very low frequency region ($\nu < 100$ Hz), where we find an enhancement of $\tau_D$ for small values of $E_k/V_0$, with an overall monotonic decreasing behaviour. This can be understood as a dynamical effect due to wavepacket accumulation in front of the slowly oscillating barrier before being reflected, as also seen in the simulational movies. Under these conditions, the definition of the atom–barrier interaction time is at least controversial and the dwell time in the region $R_D$ is no longer a meaningful quantity.

6. Elastic versus inelastic processes

In this section we address the role of elastic and inelastic processes, the latter contributing to tunnelling in the intermediate-frequency region [7, 11, 12, 23]. We here calculate the energy spectra of the reflected and transmitted atomic wavepacket by using a Fourier analysis of the wavefunction obtained from the numerical simulations.

Very low- and high-frequency limit regimes ($\nu \to 0$ and $\infty$) are examined in figure 9, where inelastic processes are absent. Tunnelling through the stationary barrier is suppressed and
Figure 6. Transmission coefficient versus $E_k/V_0$ in the limits of low $
u \to 0$ (rightmost curves) and high $
u \to \infty$ (leftmost curves) oscillation frequency, demonstrating resonant transmission below threshold. Rightmost curves: simulational data for a wavepacket (full curve) and exact result for a CW-monochromatic wave (full squares) impinging on the barrier at rest, and simulational data for a wavepacket impinging on a barrier oscillating with $\nu = 10$ Hz in the Doppler limit (broken curve). Leftmost curves: simulational data for a wavepacket (full circles) and transfer-matrix data for CW-monochromatic wave (dotted curve) impinging on the time-averaged potential. A few cases at intermediated frequencies $\nu = 1, 2$ and $10$ kHz are also shown for comparison with figure 7.

The reflected wavepacket represented by full circles does not show any spectral distortion with respect to the spectrum of the incoming matter wave (full curve).

The resonant tunnelling through the time-averaged potential, instead, is accompanied by a significant narrowing of the energy spread of the transmitted matter wave and a corresponding splitting of the spectrum of the reflected one. This is due to the filtering effect of the tunnelling resonance whose linewidth $\Gamma_0$ is about half the energy spread $\Delta E$ of the incoming wavepacket.

We proceed to report in figure 10 the energy spectra of the reflected and transmitted wavepackets in the intermediate-frequency regime. The role of inelastic processes is here rather significant and several inelastic components corresponding to the absorption of a number of energy quanta $\hbar \nu$ are clearly resolved. While the degree of inelasticity is attenuated at high frequencies, as displayed in the top panels for $\nu = 10$ kHz, at intermediate frequencies the amplitude of the sidebands may sometimes become even larger than that associated with the fundamental peak, as shown instead in the bottom and middle panels at 1 and 4 kHz. Note that, at 4 kHz, the first sideband in the transmission spectrum accounts for a fraction of about 15% of the whole atomic distribution. We also notice that, in some of the cases analysed in figure 10, filtering acts so as to amplify the sidebands of the reflected (transmitted) matter wave at the
Figure 7. Transmission coefficient versus $E_k/V_0$ for intermediate oscillation frequencies in the range from 100 to 10 kHz, demonstrating the onset of the resonant tunnelling peak. The curve at 100 Hz is obtained after averaging over the phase, and is next to the curve at 10 Hz in figure 6.

Figure 8. Dwell time (see text) as a function of atomic energy $E_k/V_0$ for different oscillation frequencies: in correspondence with the resonant tunnelling peak the dwell time exhibits a sharp maximum. The inset shows the low- and high frequency cases: barrier oscillating with $\nu = 10$ kHz (full curve), time-averaged potential (full circles) and barrier at rest (full triangles).
Figure 9. Energy spectra of the reflected (left panel) and transmitted (right panel) matter waves next to resonance ($E_k/V_0 = 0.21$) in the low (full circles) and high (broken curve) frequency limits. The spectra for the incoming wavepacket (full curve) are shown for reference.

frequencies at which the transmitted (reflected) part is depressed. Finally, the average width of the energy spectrum may become considerably larger, in the range of tens of oscillation quanta $h\nu$.

We choose the energy change $\Delta E_k \equiv \bar{E}_k(t_\infty) - E_k$ as a useful observable to assess the importance of inelastic processes as a function of $\nu$. Here $\bar{E}_k(t_\infty)$ is the expectation value of the kinetic energy calculated from the entire atom wavefunctions, including both reflected and transmitted parts, after the atom–barrier interaction. We display in figure 11 the fractional change of $\Delta E_k$ with respect to the oscillation quanta $h\nu$ (bottom panel), and with respect to $E_k$ (top panel). The number of exchanged quanta has a maximum of about 7 around 0.5 kHz, after which it decreases to a negligible value at 10 kHz for both cases of $E_k/V_0 = 0.21$ (open circles) and $E_k/V_0 = 0.33$ (stars). The large values attained at $\nu = 10$ and 100 Hz, where the latter are obtained after averaging over the phase, are consistent with the fact that $\Delta E_k$ corresponds to a large number of small quanta. In fact, when the change $\Delta E_k$ is compared to $E_k$, much smaller numbers are obtained, as displayed in the top panel of figure 11.

7. Concluding remarks

In conclusion, we have predicted the occurrence of resonant tunnelling of matter waves through a single moving barrier over the whole range of barrier oscillation frequencies. In particular, we have examined the possibility of observing under typical experimental conditions quite large transparencies when sodium atom condensates impinge on a spatially oscillating optical barrier.

We have found that the onset of the resonant tunnelling peak occurs with increasing frequencies of the barrier oscillation and is accompanied by a pronounced increase of the atom–
Figure 10. Energy spectra of the reflected (left panel) and transmitted (right panel) matter waves next to resonance ($E_k/V_0 = 0.21$) for intermediate frequencies. The occurrence of inelastic processes is signalled by the appearance of additional peaks at multiples of $\nu$. From bottom to top panels: $\nu = 1, 4$ and $10$ kHz.

Figure 11. Energy loss $\Delta E_k$ as a function of the barrier oscillation frequency $\nu$ for a process close to resonance ($E_k/V_0 = 0.21$, open circles) and half-way to continuum ($E_k/V_0 = 0.33$, stars). The energy loss is compared against the oscillation quanta $h\nu$ in the bottom panel and against the incoming kinetic energy $E_k$ in the top panel. The points at the two lowest $\nu$ values refer to $\nu = 10$ and 100 Hz.
barrier interaction time. The visibility of the peak turns out to be very sensitive to the oscillation frequency, thereby indicating a way for a precise control of the transmission tunnelling dynamics.

We have also addressed the relevance of inelastic processes in the regime of intermediate oscillation frequencies, determining a well defined dynamical range where strong sidebands in the energy spectra of the transmitted or reflected matter waves appear. This other effect can be exploited to further tailor and control the transport of matter waves, especially when filtering and efficient production of atom laser sidebands are required.

The inclusion of atomic interactions might limit the matter wave filtering effect or might lead to bistability as well similarly to what happens an atomic Fabry–Perot interferometer [24], an issue which we intend to address in the future.

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