Predicting submerged hydraulic jump characteristics using machine learning methods

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ABSTRACT

Hydraulic jump typically occurs downstream of hydraulic structures by converting the supercritical to subcritical flow regimes. If the tail-water depth is greater than the secondary depth of the hydraulic jump, the jump will be submerged (SHJ). In these conditions, the momentum equations will not have an analytical solution and a new solution is required. In this study, after dimensional analysis, an experimental study was conducted in a rectangular flume with a length of 9 m, a width of 0.5 m and a depth of 0.45 m in a wide range of Froude numbers (Fr = 3.5 to 11.5) and submergence ratios (Sr = 0.1 to 4). The data were then normalized and divided into two parts of training and testing. A new technique, DGMDH, was used to predict the submerged hydraulic jump characteristics. The results were then compared with the GMDH model. The results showed that the DGMDH model estimated the relative submergence depth, jump length, and relative energy loss with accuracy of $R^2 = 0.9944$ and MAPE = 0.038, $R^2 = 0.9779$ and MAPE = 0.0387, and $R^2 = 0.9932$ and MAPE = 0.0192, respectively. While the accuracy of the GMDH model for relative submergence depth, jump length, and relative energy loss was respectively $R^2 = 0.9923$ and MAPE = 0.043, $R^2 = 0.9779$ and MAPE = 0.0387, and $R^2 = 0.9932$ and MAPE = 0.0192. Due to superiority of the DGMDH model over the GMDH model, it is recommended to use this model to estimate the submerged hydraulic jump characteristics.

Key words: artificial intelligence, data mining, experimental study, submergence hydraulic jump

HIGHLIGHT

The results showed that the DGMDH model has more accurate results than the GMDH model in predicting the relative submergence depth, jump length, and relative energy loss.

INTRODUCTION

Hydraulic jump is a phenomenon that occurs by converting the supercritical to subcritical flow regimes downstream of hydraulic structures. Hydraulic jumps mostly occur after sluice gates because the flow velocity downstream of these structures is very high and the flow reaches a supercritical regime (Gumus et al. 2016; Jesudhas et al. 2017). In these conditions, the flow has enough energy to erode the channel and river bed; therefore, the channel is designed to force the flow to be subcritical (Akan 2011). These structures are usually designed in such a way that the hydraulic jump is formed just after the sluice gate; however, if the tail-water depth is greater than the secondary depth, the hydraulic jump is moved upstream so that the jump may be submerged (Habibzadeh et al. 2011a). Since this type of hydraulic jump has different conditions, the flow characteristics should be investigated to design the channel walls and the stilling basin properly to prevent overflow and scouring of the outer wall. Figure 1 shows a submerged hydraulic jump downstream of a sluice gate (Nasrabadi et al. 2010).

However, there is no suitable theoretical and analytical approaches to solve the momentum equation to obtain the submerged jump characteristics. Therefore, in the present study, to solve this problem, intelligent methods are utilized as a suitable solution to estimate the submerged hydraulic jump characteristics instead of using experimental and physical models. Todays, with the advancement of human science and increasing the speed of calculations, new methods have been introduced under the name of intelligent or meta-mental systems. Application of these methods can be a more appropriate option than the traditional methods for modeling hydraulic phenomena.
Recently, machine learning-based methods have been used to model many engineering phenomena (Khozani et al. 2019). These methods can also be utilized for modelling submerged hydraulic jumps. These methods have such advantages as low cost, high processing speed, and appropriate accuracy in hydraulic phenomena. The group method data handling (GMDH)-based algorithm is self-organizing and can be used in the neural network because it performs better than standard regressions. One of the characteristics of this type of intelligent model is the ability to detect unnecessary parameters in the simulation; therefore, the partial models made with the most optimal mode are selected (Mehri et al. 2019). Until now, many studies have been conducted on characteristics of classic/free hydraulic jump (sequent depth ratio, relative energy loss, and jump length) (e.g. Kindsvater 1944; Bradley & Peterka 1957; Rao & Rajaratnam 1963; Rajaratnam 1967; Rajaratnam 1968; Hager et al. 1990; Mahtabi et al. 2020); however, the studies on submerged hydraulic jump characteristics are limited. Long et al. (1990) studied a submerged hydraulic jump downstream of a sluice gate in a rectangular channel with a flat bed. They examined the submerged hydraulic jump characteristics including water surface profile, velocity distribution, shear stress due to turbulence, and turbulence intensity. They divided the submerged jump in terms of flow development into three parts: developing region, developed region and recovering region (Figure 2). In this figure, $b_0$ is the gate opening, $y_1$ is initial depth of the free jump, $y_2$ is secondary depth of the free jump, $y_3$ is the submergence depth at the gate, $y_4$ is the tail-water depth, $g$ is the gravity acceleration, and $L_{sj}$ is the submerged jump length.

Ead & Rajaratnam (2002) examined theoretically and experimentally the characteristics of wall jets at low tail-water depths. They concluded that as the tail-water depth is low, the forward momentum decreases significantly with increasing distance from the inlet gate, which is due to the return flow with a negative momentum, causing the water surface to drop.
near the gate. Abdel-Aal (2004) studied the characteristics of submerged hydraulic jumps in a rectangular channel, using a one-dimensional momentum equation. He analyzed the effective parameters of the hydraulic jump and, using multiple linear regression, presented experimental relationships to calculate relative jump depth and energy loss based on Froude number and submergence ratio. Sepúlveda Sepulveda et al. (2009) used experimental data to develop a relationship for determining the flow rate through a sluice gate in submerged conditions with acceptable accuracy. Nasrabadi et al. (2010) compared the characteristics of free and submerged jumps in smooth and rough beds, and concluded that at the same Froude numbers (more than about 4) the length of the free jump was always greater than or equal to the length of the submerged jump. In addition, in low Froude numbers, the relative energy loss of the submerged jump are always greater than or equal to the relative free jump. Habibzadeh et al. (2011b) examined the submerged hydraulic jump with a theoretical approach. They developed a regression equation for the discharge coefficient of sluice gates in the free and submerged hydraulic jumps.

Along with the experimental studies, many researches have been performed using data mining techniques. Azamathulla et al. (2012) utilized GEP to predict scour depth downstream of sills. He concluded that the proposed GEP approach gives satisfactory results compared to the existing equations developed in the literature. Houichi et al. (2013) used different Artificial Neural Network (ANN) methods to predict the roller length of the free hydraulic jump in U-shaped channels. They used Multi-layer Perceptron Neural Network (MLPNN) and General Regression Neural Network (GRNN) models to calculate the roller length and concluded that the MLPNN model is more accurate than the GRNN model. Karbasi & Azamathulla (2016) applied Support Vector Regression (SVR), Gene Expression Programming (GEP), and ANN methods to predict free hydraulic jump characteristics in the rough bed and concluded that the GEP model has better accuracy than other methods. Gumus et al. (2016) examined numerically the submerged hydraulic jump characteristics after sluice gates using different models. They first compared the experimental and numerical studies. They concluded that the Reynolds Stress Model (RSM) is more accurate than other turbulence models in calculating submerged hydraulic jump characteristics. Jesudhas et al. (2017) simulated a submerged hydraulic jump after a sluice gate using the Detached Eddy Simulation (DES) turbulence model. For this purpose, they used STAR-CCM software for modeling and concluded that this model has an acceptable accuracy for predicting velocity distribution in submerged conditions. Roushangar et al. (2017) studied the capability of the support vector machine (SVM) for predicting hydraulic jump characteristics in different sudden diverging stilling basins. They found that Froude number had the most significant effect on the modeling. Also, comparison between SVM and empirical equations indicated the great performance of the SVM. Roushangar & Ghasempour (2018) applied gene expression programming (GEP) to estimate hydraulic jump characteristics in suddenly expanding channels. They compared GEP models with existing empirical equations and it was found that the GEP models gave better results. Roushangar & Ghasempour (2019) assessed the effects of channel geometry and rough boundary conditions (rectangular, trapezoidal, and expanding channels with different rough elements) in predicting energy dissipation of the hydraulic jump using support vector machine (SVM). They concluded that the developed models for the case of simulation based on dimensional analysis yielded better predictions. Tiwari (2019) used adaptive neuro fuzzy inference system (ANFIS), ANN, and Multiple Regression Models (MRM) to predict the percentage of oxygen aeration in hydraulic jump and concluded that the ANFIS model is more accurate than other models. Pandey et al. (2018) evaluated multiple linear regression and genetic algorithm approaches to predict temporal scour depth near a circular pier in non-cohesive sediment. (Najafzadeh 2019; Pandey & Azamathulla 2021) used three models GEP, Evolutionary Polynomial Regression (EPR) and Model Tree (MT) in for estimation of hydraulic jump characteristics in circular pipes and concluded that the MT model has an acceptable agreement with the experimental results. Pandey et al. (2020) compared experimental data of maximum scour depth at equilibrium scour condition with genetic algorithm (GA) and multiple linear regression (MLR) techniques. Their results showed that GA based maximum scour depth relationship have more precise results than MLR. Saadatnejadgharahassanlou et al. (2020) use a multi-segment sharp-crested V-notch weir to evaluate the hydraulic jump length downstream of the weir. Their results indicated that the GRNN model is the best model among the studied models.

Machine learning-based methods have recently been widely used in laboratory studies in hydraulic engineering and nonlinear problems. According to previous studies, in this research, a new method called Developed Group Method of Data Handling (DGMDH) is used for modeling submerged hydraulic jump characteristics after the sluice gate. In addition, the accuracy of this model is compared with the GMDH model as well as with previous research. On the other hand, one of the advantages of the present study over the other research is the range of studied parameters. So creation of a wide range of Froude numbers and submergence ratios in the experiments has been tried. Table 1 compares the range of
Froude numbers and submergence ratios in the present study with other studies. In the studies that use experimental data for modeling and those models are calibrated or developed based on these data, the results will be more reliable and more accurate. Undoubtedly, in artificial intelligence methods, the use of a wide range of data can lead to more reliable results.

**MATERIALS AND METHODS**

**Experimental setup**

The experiments were performed in the Water Research Center of the Irrigation & Reclamation Engineering Department, University of Tehran, in a laboratory glass-walled flume with a rectangular cross-section with a length of 9 m, a width of 0.5 m, and a depth of 0.45 m (Figure 3). The experimental setup includes the supply tank, the sluice gate for generating the inlet jet, the rectangular flume, the hinged weir for adjusting the tail-water depth at a distance of 3 m from the beginning of the flume, and the water transfer system. Figure 4 shows a schematic view of the experimental setup. In this study, the flow discharge was measured by a rectangular weir installed in the upstream supply tank, which was calibrated using an electromagnetic flowmeter (Mag Ab 3000) before the experiments. The inflow rate to the system was also adjustable by means of a sliding gate valve. By installing a sluice gate at the beginning of the rectangular flume, the conditions for creating the initial depth of hydraulic jump were provided. The opening of the gate was considered constant in all experiments (equal to 2 cm). To measure the water surface profile along the jumps after the sluice gate, a mechanical point gauge was installed along the flume with an accuracy of ±0.1 mm with the ability to move by a rail (Figure 4). Besides, 12 piezometers were installed at a distance of 11 cm from each other, by which the water depth inside the flume could be measured in a panel.

**Experimental program**

First of all, according to the gate opening and its contraction ratio, the required flow rate for the desired Froude number was determined; accordingly, the flow in the flume was established. Using a point gauge, the outflow depth through the gate was read and the contraction ratio of the gate was calculated. If this value differs from the considered contraction ratio, the mentioned steps will be repeated to obtain the desired Froude number. Then, using the hinged weir installed at the end of the

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**Table 1** Comparing the range of parameters in present study with previous research

| Researchers        | Range of Froude numbers | Range of submergence ratio |
|--------------------|-------------------------|----------------------------|
| Long et al. (1990) | 3 to 5.5                | 0.2 to 1.7                 |
| Ead & Rajaratnam (2002) | 4 to 8               | 1.58 to 8.62               |
| Abdel-Aal (2004)  | 2.2 to 4.4              | 0.15 to 0.75               |
| Present study      | 3.5 to 11.5             | 0.1 to 4                   |

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**Figure 3** | A schematic view of the experimental set-up.
flume, the tail-water depth was adjusted so that the free hydraulic jump was formed at a distance of 10 cm from the sluice gate. The secondary depth was determined using both the point gauge and the piezometers. Water surface profiles were also recorded using a point gauge. After these steps, the tail-water depth was increased using a hinged weir to form a submerged hydraulic jump and all mentioned steps were repeated. After completing these steps, the next Froude number was adjusted and all these steps repeated. As can be seen in Table 1, in this study, by considering different Froude numbers in the range of 3.5 to 11.5, a submerged hydraulic jump with different submergence ratios \( S_r = (y_4 - y_2)/y_2 \) of 0.1 to 4 is formed. In total, 67 experiments were performed.

**Dimensional analysis**

This section discusses the dimensional analysis to find the parameters affecting the submerged hydraulic jump characteristics. In general, the characteristics of a submersible hydraulic jump in a smooth bed are a function of the following parameters:

\[
F(\rho, \mu, y_1, y_2, y_3, y_4, g, V_1, L_j) = 0
\]

in which, \( \rho \) is the fluid density, \( \mu \) is the dynamic viscosity, \( y_1 \) is initial depth of the free jump, \( y_2 \) is secondary depth of the free jump, \( y_3 \) is the submergence depth at the gate, \( y_4 \) is the tail-water depth, \( g \) is the gravity acceleration, \( V_1 \) initial flow velocity, and \( L_j \) is the submerged jump length. Using Buckingham theory and using iterative variables, the following relationships can be derived:

\[
\frac{y_3}{y_1} = f_1 \left( \frac{\rho V_1 y_1}{\mu}, \frac{V_1}{\sqrt{g y_1}}, \frac{y_4 - y_2}{y_2} \right)
\]

\[
\frac{L_j}{y_2} = f_2 \left( \frac{\rho V_1 y_1}{\mu}, \frac{V_1}{\sqrt{g y_1}}, \frac{y_4 - y_2}{y_2} \right)
\]

Equation (3) uses the ratio of the submerged hydraulic jump length to the secondary depth of the free hydraulic jump \( (L_j/y_2) \). Also, in very high Reynolds numbers, the effect of viscosity and consequently, the Reynolds number, can be neglected (the range...
of Reynolds number in the present study is Re = 17,200 to 52,200). Therefore, the above equations are written as follows:

\[
y_3 \overline{y_1} = f_3(Fr_1, S)
\]

(4)

\[
L_2 \overline{y_2} = f_4(Fr_1, S)
\]

(5)

In order to calculate the relative energy loss of the submerged hydraulic jump, the amount of specific energy at the beginning \((E_1)\) and end of the jump \((E_2)\) are calculated as follows:

\[
E_1 = y_3 + \frac{v_1^2}{2g}
\]

(6)

\[
E_2 = y_4 + \frac{v_4^2}{2g}
\]

(7)

\[
\frac{\Delta E}{E_1} = \frac{E_2 - E_1}{E_1}
\]

(8)

Using Buckingham’s theory, the following equation is considered to determine the relative energy loss of the submerged hydraulic jump:

\[
\frac{\Delta E}{E_1} = f_5(Fr_1, S)
\]

(9)

GMDH method

The GMDH algorithm generates all combinations of input variables and selects the best model from the set of generated models based on a selected criterion of error value relative to the previous layer. The hybrid algorithm is the complete basis of mathematical induction and does not consider probabilistic models. In this self-organizing algorithm, all inputs are stored in a flexible network of regression equations that are polynomial

\[
y = a_0 + \sum_{i=1}^{s} a_i x_1^{j_1} x_2^{j_2} \ldots x_k^{j_k}
\]

(10)

where \(a_i\) is a coefficient and \(s\) is the number of vector \((j_1, j_2, \ldots, j_k)\) for \(j \in N_0\) and \(j \leq p\), so that \(p\) is the maximum power of the polynomial and \(k\) is the number of inputs. This algorithm tries to make a model with optimized complexity of combination with polynomial components and estimate the coefficients using the least squares method. The following structure is also used during the modeling shown in Equation (11):

\[
q_n = \sum_{i=0}^{n} c_i^n = 2^n
\]

(11)

where, \(n\) is the number of polynomial components that are calculated using \(n = (p + k)!/p!k!\).

DGMDH method

The GMDH intelligent algorithm has a self-organized structure. This system can be used in a neural network because it performs better than standard regression models. One of the features of this model is the ability to detect unnecessary parameters in the prediction. Afterwards, the partially constructed models are selected with the optimum conditions. This can be modeled using a set of neurons. In this model, different pairs in each layer are connected to each other through second and third degree polynomials. Then, by determining the number of appropriate neurons along with the appropriate layers, the least precision is required. Figure 5 shows the process of solving a problem using the DGMDH method (Abdel-Aal 2004). The structure of this method is a progressive multilayer network consisting of a series of supporting neurons. Supporting neurons have at least two inputs. The stimulus or transmission function of these neurons can be expressed as a linear or non-linear
polynomial, defined as follows:

\[ y = w_0 + w_1x_1 + w_2x_2 + w_3x_1^2 + w_4x_2^2 + w_5x_1x_2 \]  \hspace{1cm} (12)

where, \( w_0 \) to \( w_5 \) are polynomial’s coefficients. In the GMDH structure, each neuron performs a non-linear function of neurons. This non-linear function is as Equation (12). Suppose \( N \) is the input vector of \( X_n = (x_{n1}, x_{n2}, \ldots, x_{np}) \), in which, \( n = 1, 2, \ldots, N \) exists in the training set, each consisting of an integer \( P \) value. The desired output of \( n \) value is displayed with \( \phi_n \). A set of six coefficients must be found for each neuron so that the mean squared error between the neurons’ outputs \( (y_n) \) and the actual value \( (\phi_n) \) is minimal. Using normal Gaussian equations, these equations are obtained as follows:

\[ \phi_1 = w_0 + w_1x_{11} + w_2x_{11} + w_3x_{11}^2 + w_4x_{11}^2 + w_5x_{11}x_{11} \]

\[ \phi_2 = w_0 + w_1x_{21} + w_2x_{21} + w_3x_{21}^2 + w_4x_{21}^2 + w_5x_{21}x_{21} \]

\[ \vdots \]

\[ \phi_N = w_0 + w_1x_{N1} + w_2x_{N1} + w_3x_{N1}^2 + w_4x_{N1}^2 + w_5x_{N1}x_{N1} \]  \hspace{1cm} (13)

Figure 5 | GMDH problem solving process.
The above equations can be shown as a matrix:

$$\phi = XW$$  \hspace{1cm} (14)

The matrices of $\phi$, $X$, and $W$ have dimensions of $N \times 1$, $N \times 6$, and $6 \times 1$. Normal equations are obtained by multiplying the sides of the above equation to the transposition of matrix $X$.

$$X^T \phi = (X^T X)W \rightarrow W = (X^T X)^{-1} X^T \phi$$  \hspace{1cm} (15)

in which, $X^T \phi$ is a $6 \times 6$ matrix and the coefficients can be obtained by inverse method. The $W$ matrix consists of a set of six coefficients that are able to approximate correct outputs with a minimum mean square error. The above steps are repeated for all neurons of the first layer and for all neurons of the subsequent layer. After obtaining the coefficients based on the training data, the performance index of the obtained neurons is calculated by calculating the amount of error with the actual control data and only the neurons with the appropriate performance are used to form the next layer.

**RESULTS AND DISCUSSION**

In this study, first, with performing an experimental study and forming a submerged hydraulic jump, the data required to evaluate the performance of a new DGMDH model are examined and compared with the GMDH model and other research. In the following, the results of prediction of submerged hydraulic jump characteristics by both GMDH and DGMDH models are presented.

Figure 6 shows the changes in the water surface profile for different Froude numbers and different submergence ratios from free jump to the maximum submergence ratio (x is the distance along the hydraulic jump and y is the flow depth from the bottom of the flume). It should be mentioned that, in the present study, due to limited depth of the flume (45 cm), the tail-water depth increased until its value is approximately equal to the submergence depth at the gate. As can be seen in Figure 6, in each Froude number, the jump length increases with increasing submergence ratio. Unlike the free hydraulic jump, the results shows that the water surface profile in submerged hydraulic jumps does not have the same form and by changing the submergence ratio, the general form of the water surface profile changes too.

**Submergence depth at the gate**

**Experimental results**

One of the important characteristics of a submerged hydraulic jump is the submergence depth at the gate ($y_3$), which is required to calculate the initial energy of the submerged hydraulic jump. As can be seen in Figure 7, with increasing tail-water depth, the submergence depth at the gate increases. Also, in a certain submergence ratio, as the Froude number increases, the submergence depth at the gate increases.

**Machine learning results**

According to the dimensional analysis, the relative submergence depth ($y_3/y_1$) is a function of $S$ and the Froude number and these parameters were used as input to the model. After normalization, all data are considered between 0 and 1. In addition, 80% of the data were used for training and 20% of the data were used for testing. Figure 8 shows the predicted values versus the experimental data for both GMDH and DGMDH models. Table 2 shows a comparison between both studied models and the equations presented in previous studies. It can be seen that the neural network model has an effect on the structure of the GMDH model and, on the other hand, has increased the accuracy of the DGMDH model. The results showed that the DGMDH model has an appropriate performance in predicting the relative submergence depth with the accuracy of $R^2 = 0.9944$ and MAPE = 0.038 followed by the GMDH model with the accuracy of $R^2 = 0.9923$ and MAPE = 0.043. In addition, the relationships developed by other researchers are in the next ranks. As can be seen, the relationships presented in previous studies are less accurate than intelligent models. The reason is that the simple regression is involved for developing relationships, while the DGMDH intelligent model consists of a combination of two regression and intelligent methods that will have higher accuracy.
Another important SHJ characteristic is the jump length, which plays an important role in the economic design of stilling basins as well as the length of the protection downstream of the hydraulic structures. In the study, the Long et al. (Mahtabi et al. 2020) criterion was used to measure the jump length, so that the distance between the gate to the end of the developed zone (end of return flows) is considered as the submerged jump length. Figure 9 shows the changes in the relative jump length ($L_j/y_2$) versus the submergence ratio ($S$). It can be seen that the submerged jump length is not sensitive to change in the Froude number and there is a certain trend between the relative jump length and the submergence ratio in different Froude numbers, that is, as the submergence ratio increases, the submergence depth at the gate increases. Additionally, in a constant submergence ratio, the submerged jump length increases with increasing Froude number.
**Figure 7** | Changes in relative submergence depth \((y_3/y_1)\) versus \(S\) for different Froude numbers.

**Figure 8** | Experimental data versus modeling results for relative submergence depth.
Machine learning results

After normalizing the data, 80 and 20% of the data was used for training and testing, respectively. The input parameters were selected according to the dimensional analysis. The superiority of the DGMDH model over the GMDH model can be seen in Figure 10, which shows the predicted data against the experimental data in the training and testing phases. Table 3 shows the performance of the DGMDH model relative to GMDH and relationships developed by other researchers. This table also shows the superiority of the DGMDH model in predicting relative submergence depth. The DGMDH model has an accuracy of $R^2 = 0.9779$ and $MAPE = 0.0387$ and the GMDH model has an accuracy of $R^2 = 0.9671$ and $MAPE = 0.0527$.

The accuracy of the DGMDH model is higher than that of the GMDH because it uses both the Kolmogorov-Gabor polynomials and the intelligent neural network method. In this section, it can also be seen that both GMDH and DGMDH models have performed appropriately compared to regression relationships that use simple functions.

Relative energy loss

Experimental results

Figure 11 presents the changes in the relative energy loss of the submerged hydraulic jump ($\Delta E/E_1$) versus the submergence ratio $(S)$ for different Froude numbers. It can be seen that as the submergence ratio increases, the energy loss decreases; however, the slope of the energy loss decreases with increasing Froude number.

Machine learning results

After normalizing the data and dividing them into training and testing data, the changes in the relative energy loss of the submerged hydraulic jump against the predicted values by both models is shown in Figure 12. Table 4 shows the performance of the DGMDH model compared to other models. This model, with an accuracy of $R^2 = 0.9994$ and $MAPE = 0.0093$ compared to GMDH with accuracy of $R^2 = 0.9932$ and $MAPE = 0.0192$, has more accuracy in predicting relative energy loss.

CONCLUSIONS

The study of submerged hydraulic jump in various experimental conditions has always been emphasized by hydraulic engineers. In addition, machine learning methods have always been developed using experimental data with the aim of reducing
costs and the time of laboratory studies. Therefore, in the present study, both experimental and machine learning methods were evaluated. To do this, an experiment was first performed on submerged hydraulic jumps to collect the data required for the development and evaluation of the two statistical methods of GMDH and DGMDH. The experimental results showed that the general form of the water surface profile is not similar in submerged hydraulic jumps and changes with changing submergence ratio. As the submergence ratio increases (increasing the tail-water depth), the submergence depth at the gate increases. The results also showed that the submerged jump length is not very sensitive to change in the Froude number and there is a certain trend between the relative jump length and the submergence ratio in different Froude numbers. By

**Table 3 | Performance evaluation of models for submerged jump length**

| Researcher/method         | MAPE   | $R^2$  |
|---------------------------|--------|--------|
| Rao & Rajaratnam (1963)   | 0.0926 | 0.9505 |
| Abdel-Aal (2004)          | 0.0569 | 0.9478 |
| GMDH                      | 0.0527 | 0.9671 |
| DGMDH                     | 0.0387 | 0.9779 |

**Figure 10 | Experimental data versus modeling results for submerged jump length.**
Figure 11 | Changes in the relative energy loss of the submerged hydraulic jump ($\Delta E/E_1$) versus the submergence ratio ($S$) for different Froude numbers.

Figure 12 | Experimental data versus modeling results for relative energy loss.
increasing the Froude number, the energy loss will increase. Also, the energy loss decreases with increasing submergence ratio, but the slope of energy loss decreases with increasing Froude number.

After a comprehensive review of previous studies and dimensional analysis, dimensionless parameters affecting submerged hydraulic jump characteristics were identified. The data were first normalized between zero and one and were divided into two parts of training and testing. Then, modeling was performed using both GMDH and DGMDH models. The results showed that the DGMDH model estimated the relative submergence depth, jump length, and relative energy loss with an accuracy of $R^2 = 0.9944$ and MAPE = 0.058, $R^2 = 0.9779$ and MAPE = 0.0587, and $R^2 = 0.9932$ and MAPE = 0.0192, respectively. While the accuracy of the GMDH model for relative submergence depth, jump length, and relative energy loss was respectively $R^2 = 0.9923$ and MAPE = 0.043, $R^2 = 0.9671$ and MAPE = 0.0527, and $R^2 = 0.9932$ and MAPE = 0.0192. According to the evaluation, the developed DGMDH model is more accurate than the GMDH model and other previous researches in predicting the relative submergence depth, jump length and relative energy loss.

**DATA AVAILABILITY STATEMENT**

All relevant data are included in the paper or its Supplementary Information.

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| Researcher/method               | MAPE  | $R^2$  |
|---------------------------------|-------|-------|
| Rao & Rajaratnam (1963)         | 0.1251| 0.9980|
| Abdel-Aal (2004)                | 0.0403| 0.9972|
| GMDH                            | 0.0192| 0.9932|
| DGMDH                           | 0.0095| 0.9994|

Table 4 | Performance evaluation of models for submerged relative energy loss
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