Creation of a theoretical simulation model of orbital referencing of lunar objects’ optical observations taken by space lunar satellite to the selenocentric coordinate system

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Abstract. This paper aims at the creation of a selenocentric catalogue of lunar objects’ positions (SCLOP) and development of a theoretical simulation model of orbital referencing of lunar objects taken by space lunar satellite to the selenocentric coordinate system defined by SCLOP. The data produced by the modern space lunar missions serve as the basis for multi-parameter and highly accurate digital simulation model. The results obtained in the work are important and relevant for the development of navigation technologies on creation and orienting of coordinate systems and the study of celestial bodies’ figures. To solve the problems stated in this work, the following new results are produced: a) reliability of coordinate data in modern systems of coordinate and time support for lunar objects is investigated by the robust analysis; b) a method and a software complex for transforming selenographic coordinates (TSC) are developed; c) the software package is tested for work with navigation data and determining planetary parameters; d) a method of adaptive regression modeling (ARM) for transforming coordinate systems and assessing structures and parameters of the selenocentric system (SS) is created; e) an algorithm of multi-parameter identification of SS based on ARM-approach is developed to carry out works on expansion and estimation of the reference selenocentric system; f) a global dynamic selenocentric system is constructed on the basis of optical observations taken at “Clementine”, “LRO”, and “Apollo” missions; g) as a result, a simulation digital model of orbital referencing of lunar objects optical observations, taken by star sensors of lunar satellite and on-board laser interferometer, to the selenocentric coordinate system is developed.

1. Introduction

At the moment, despite the significant progress in determining selenographic parameters by means of spacecrafts, the problem of highly accurate provision of navigation support in the lunar orbit is not yet solved. [1]. This follows from the fact that the data from satellite missions are referenced to the quasi-dynamic coordinates system, as the spacecrafts in most cases have orbital referencing [2, 3]. According to [4], the topography and gravimetry produced during the “LRO” and “GRAIL” space missions [5], being currently the most accurate, are referenced neither to the terrestrial nor to the celestial coordinate systems and are only oriented in relation to our natural satellite. The correctness of the results produced at the missions mentioned above was assessed by the value of a spacecraft’s
orbits shift in relation to the intersection points of these orbits [6]. Thus, it was the reliability of the spacecraft’s orbit model that was estimated, but the referencing was performed to different orbits of the spacecraft. In this connection, the need for a data of another kind is quite obvious. One of the options is highly accurate referencing to stars [7]. The approach considered in this work, within which different topography data are brought to the single system and a catalogue of lunar reference objects based on the results of modern space projects and referenced to stars is developed, allows creating the selenocentric dynamic reference system. The first results on coordinate referencing to the coordinate system, as which a catalogue of craters on both sides of the Moon was used, were obtained by means of photogrammetry methods at the “LRO” project [8]. This paper considers the opportunity to implement coordinate referencing of the near-Moon spacecrafts to the reference system using the coordinates of objects from the reference catalogue.

When there are the basic selenocentric catalogue of coordinates of reference objects from the near side of the Moon (dynamic selenocentric catalogue - DSC) and a number of catalogues of objects from the librational zone and the far side of the Moon, construction of an unified coordinate system with a center, coinciding with the lunar center of mass, and axes, coinciding with the lunar principal axes of inertia, includes the following steps:
- investigation of systematic and random errors in DSC catalogue;
- compression and expansion of DSC for the near and the far sides of the Moon as well as for the librational zone.

2. Creation global catalogue for the full sphere of the Moon in DSC system

Catalogue DSC was constructed in the system with the lunar axes, coinciding with the lunar axes of inertia and center of mass. The method for extension DSC to the whole lunar sphere is coordinates transformation matrixes. DSC is based on large-scaled lunar astroplates in stars system obtained by the unique approach of separated images [7]. The method for extension DSC to the whole lunar sphere is coordinates transformation matrixes. For transformed into DSC system the displacement vectors and elements of the matrixes should be obtained from common points for researched catalogue and DSC.

Into DSC system have been transformed 12 catalogues: AMS, ACIC, Baldwin, ARTHUR, Goloseevo-1 and -2, MILLS-2, Kiev, SCHRUTKA-1 and -2, ULC N 2005 [4] and the Valeev catalogue [9] using displacement vectors X0 (regression model (1) under conditions (4)) and orientation of A matrices use the orthogonality conditions and the numerical approach. The efficiency of the orthogonal model TS has been obtained by comparing the results from the model (1) without orthogonality consideration. The model (1) is the regression approximating transformation as algebraic polynomial having the first order.

When solving a problem, the following steps have been made:
1) analysis and study of ULCN basic network accuracy;
2) determination of common objects for the coordinate systems being processed;
3) development of mathematical support for TSC software package.

Regression method for creating the global system of selenographic coordinates. When developing a method for bringing selenographic data to the single system the following approaches were used. The observational data produced during the processing was estimated using the regression model [10]:

\[
AX = AY + X_0 + \epsilon, \tag{1}
\]

where \(X\) – matrix of the final transformed system of different observations; \(Y\) – matrix of the initial observational system; \(A\) – orientation matrix; \(X_0\) – shifts matrix; \(\epsilon\) – errors matrix.

The system of equations (1) was solved within regression adaptive modeling (RAM) at which LSM-conditions compliance is checked, and the applicability of numerical adaptive procedures is studied if the conditions are not observed. The robustness of the estimations produced was provided by the presence of a spectrum of regression models and by taking into account multiple scenarios of data processing depending on quality measures of the constructed models.

The transformation of the reference network and increase in the number of reference objects are implemented according to the expression as follows:
\[ A \times \theta + \vec{\varepsilon} = Z, \quad (2) \]

where \( A(A_{i0}) \) – orientation matrix, \( \theta (A\zeta, A\eta, A\zeta) \) – origin’s shift matrix in relation to \( \vec{\varepsilon} \).

Within RAM approach it is postulated that the type of coordinate transformation for model (2) is unknown for each pair of coordinate systems and could be determined according to the competing characteristics being set [11]. For the general case, the relation (2) could be brought to the regression type:

\[ Y = X\vec{\beta} + \vec{\varepsilon}, \quad (3) \]

where \( \vec{\varepsilon} \) – errors matrix, \( \vec{\beta} \) – the first row of the orientation matrix \( A \).

The orientation matrix \( A \) in many cases does not correspond to the orthogonality transformation criteria from \( Y \) to \( X \) due to errors arising when coordinates of the objects from both systems are determined. At the same time, the following condition must be satisfied:

\[ A^T A = E, \det A = 1. \quad (4) \]

The relation (3) together with the condition (4) allows performing the general deterministic coordinates transformation. This problem may be solved by the method of mathematic programming.

The desired parameters \( \theta (A\zeta, A\eta, A\zeta) \) are determined as:

\[ \theta = (A^T PA)^{-1}(A^T PZ), \quad (5) \]

and their errors are determined by the expression:

\[ D(\theta) = \frac{V^T PV}{2n - 3(A^T PA)^{-1}}, \quad (6) \]

where \( V \) – residual deviations matrix.

3. The method of orbital spacecraft selenographic referencing to the selenocentric coordinate system

The implementation of the method of orbital spacecraft selenographic referencing to the selenocentric coordinate system on the basis of DCLRO was performed within the simulation model of navigation support. The simulation model represents a software package providing an opportunity to form a digital model and compare it with the digital map of the Moon. The developed method is based on the assumption that reference points located close to each other are characterized by a higher similarity. The search for the altimetry component of the investigated point is based on the analysis of neighboring points located in the close vicinity of the desired point. Taking into account the uneven distance between neighboring points is implemented by assigning them weight coefficients:

\[ \hat{Z}(S_0) = \sum_{i=1}^{N} \lambda_i Z(S_i), \quad (7) \]

where \( \hat{Z}(S_0) \) – the investigated altimetry component of the point \( S_i \);
\( \lambda_i \) – points’ weight coefficients decreasing when moving away from the investigated points;
\( Z(S_i) \) – measured value of altimetry component at the point \( S_i \);
\( N \) – the number of points located in the vicinity of the investigated point and involved in calculation.

The weight coefficients are calculated according to the expression:

\[ \lambda_i = \frac{d_{i0}^{-p}}{\sum_{i=1}^{N} d_{i0}^{-p}}, \quad (8) \]

where \( d_{i0} \) – distance from the investigated point \( S_0 \) to the reference point with index \( i \) \( S_i \).

The exponent \( p \) influences the weight assigned to the reference object (point): when the distance to the investigated point increases, the weight coefficient decreases exponentially.

Based on the method described above one may implement navigation referencing of the desired points on the lunar surface and the position of a spacecraft to the selenocentric dynamic coordinate system based on the catalogue of reference point created within this work.
4. Summary and conclusions
For transformation and expansion of DSC the elements of the matrices $A$ and $X_0$ (1), obtained by the numerical method with considering the conditions (4), have been used. The common points for each pair of catalogues $(X, Y)$ are identified in rectangular coordinate system $(\xi, \eta, \zeta)$ by differences not exceeding 0.001, 0.001, 0.002 of the lunar radius in absolute values, respectively.

As a result, a digital catalogue of lunar reference objects (DCLRO) is created using 12 catalogues of reference objects on the lunar surface. DCLRO contains over 274 thousand Cartesian coordinates for the reference points produced by means of various observation methods and brought to the single selenographic coordinate system. To work with DCLRO, a specialized software allowing for user to select altimetry and compare it with the digital map of the lunar surface is developed in Matlab. User gets the data set being investigated as cube or sphere covering the chosen area. It should be noted that the least number of objects from DCLRO corresponds to the areas that are close to the poles, while in other areas the objects distribution is even in general.

The estimation of data produced is performed by means of the adaptive regression modeling. The accuracy of coordinate referencing of an object observed on the lunar surface is $\pm 40$ m in plane coordinates, the accuracy of radius vector is $\pm 80$ m. Currently, the estimated landing ellipse is $9 \times 13$ km. The use of the theoretical digital simulation model (DSM) of orbital referencing of lunar objects optical observations will therefore allow reducing the size of the landing ellipse by several orders of magnitude. The analysis of the method for creating a navigation network in the lunar orbit has shown that the modern selenographic models are not equally accurate and the data presented in these models refer to different reference systems [12]. The simulation model of coordinate support allows to perform modeling for the determination of navigation positions of the desired objects on the Moon’s surface using on-board goniometrical measurements with laser interferometer from the board of the spacecraft and the spacecraft itself using the measurements from the surface of the Moon [13, 14]. The comparative analysis of the regression topography models constructed on the bases of data from the modern satellite lunar missions has allowed assessing the accuracy of coordinates of the desired objects on the lunar surface. It is revealed that navigation support for the near side of the Moon is more accurate than for the far side. The obtained results and advantages of using of DSM when developing navigation support for the lunar missions are: 1) capacities on the control of spacecrafts and prediction of their physical and technical parameters over long time interval are extended; 2) the possibility of checking adaptation and stability for DSM of hybrid systems [15] for spacecraft is considered; 3) the creation of local systems for the work in complex environments remote from the Earth; 4) the recognition and use of local conditions without assistance; 5) for achieving the high level of space missions safety. The results produced will be applied for determining selenographic parameters for lunar bases and also for preparing and implementing lunar space missions [16]. The creation of stationary lunar bases is a promising direction not only for robotic exploration of the Moon but also as a launch pad for preparing manned space flights to Mars.

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