A general revealed preference test for quasilinear preferences: theory and experiments

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Abstract
We provide a generalized revealed preference test for quasilinear preferences. The test applies to nonlinear budget sets and non-convex preferences as those found in taxation and nonlinear pricing contexts. We study the prevalence of quasilinear preferences in a laboratory real-effort task experiment with nonlinear wages. The experiment demonstrates the empirical relevance of our test. We find support for either convex (non-separable) preferences or quasilinear preferences but weak support for the hypothesis of both quasilinear and convex preferences.

Keywords Revealed preferences · Quasilinear preferences · Lab experiments

JEL Classification D12 · D11 · C91

1 Introduction

It is difficult to overstate the importance of the assumption of quasilinear (QL) preferences in both theoretical and empirical economics. The assumption plays a crucial role in mechanism design, the theory of the household, and applied welfare analysis. It is, for instance, a necessary assumption for the Revenue Equivalence theorem (Myerson, 1981; Krishna, 2009), the existence of the truth-revealing dominant strategy mechanism for public goods (Green & Laffont, 1977), and the Rotten Kids

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theorem (Becker, 1974; Bergstrom & Cornes, 1983). It is also a frequently invoked assumption in applied welfare analysis (Domencich & McFadden, 1975; Allcott & Taubinsky, 2015). It is also a crucial assumption in the empirical literature that uses bunching at kinks and notches of budget sets (see Kleven, 2016, for a review). The first contribution of our paper is to provide a new test for QL preferences with many empirical applications. The second contribution is to provide the first empirical test of QL preferences in a laboratory setting using nonlinear budget sets. The third contribution is to show the empirical relevance of QL, albeit not convex preferences.

**Contribution** We provide criteria for a set of observed choices on nonlinear budget sets to be consistent by quasi-linear preferences that are not necessarily convex. Our criteria are based on the observation that when preferences are QL, they must also satisfy the property of cyclical monotonicity (see, e.g., Rochet, 1987). Our contribution demonstrates that cyclical monotonicity holds for a larger class of choice environments than previously established. As a special case, we show that Forges and Minelli (2009)’s generalization of revealed preferences tests to nonlinear sets extends to tests of QL. This allows testing for QL preferences in some strategic environments. When the choice sets are linear, however, the condition we identify is equivalent to the definition of cyclical monotonicity in Brown and Calsamiglia (2007). These conditions are necessary and sufficient for choices to be rationalized by a QL utility function. We provide examples that show why it is essential to consider nonlinear budget sets.

We conduct a real-effort labor supply experiment with nonlinear wages. We allow subjects to choose the terms of their contracts, where a contract specifies a wage and the number of tasks to complete. We generate nonlinear convex budgets sets to test for QL preferences and the convexity of preferences. We find that 77 percent of subjects are consistent with locally non-satiated utility (LNU), of those 86 percent are also consistent with QL and 82 percent are consistent with non-separable convex preferences and 56 are consistent with QL convex preferences. While both assumptions of convex or QL preferences have significant empirical support, less is so for QL convex preferences.

**Related Literature** Revealed preference theory is attractive because of its robustness to functional form assumptions regarding preferences. Beginning with the work of Richter (1966) and Afriat (1967), revealed preference theory has been used to test both individual and collective decision-making (see Chambers & Echenique, 2016, for a comprehensive overview of the results). It has also been used to test theories of consumer behavior in the lab. Applications include tests for social (e.g., Andreoni & Miller, 2002; Fisman et al., 2007; Porter & Adams, 2015), risk (e.g., Choi et al., 2007), and time preferences (e.g., Andreoni & Sprenger, 2012).

Recent experimental research has used real-effort task to study time-preferences (Augenblick et al., 2015) and self-control Toussaert (2018). Our experiment shows that such an environment is useful to test other properties of labor-leisure decisions. The labor supply setting is particularly relevant in the analysis of household

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1 These are the percentages of participants for whom we can reject random behavior at a 5% significance level. Details of these calculations are in Sect. 4.
decisions (see e.g. Cherchye et al., 2012, 2017). Our experimental design can be adapted to study individual and group labor supply behavior.

Recently, there has been interest in revealed preference tests for QL preferences in the context of linear budgets. Brown and Calsamiglia (2007) propose a revealed preference test for the case of concave preferences, while Nocke and Schutz (2017) discuss the corresponding integrability problem without assuming concavity. Cherchye et al. (2015) extends Brown and Calsamiglia (2007)’s test to generalized QL while maintaining the concavity of the utility and the linearity of budgets. Allen and Rehbeck (2018) provide a measure for QL misspecification and use scanner data to evaluate how this misspecification varies with the level of aggregation of the data. They find that while quasilinearity fails at the individual level, it represents the data well from the perspective of a representative agent. Chambers and Echenique (2017) show that QL preferences in the setting of combinatorial demand (with linear pricing) are equivalent to the law of demand, a condition that is simpler than cyclical monotonicity. As we will discuss, our approach provides the least restrictive test for QL preferences, and our experiment provides direct evidence that quasilinearity has empirical relevance at the individual level. Our experiments, therefore, complement these approaches nicely.

**Structure** The remainder of this paper is organized as follows. Section 2 presents our theoretical framework, Sect. 3 presents our experimental design and Sect. 4 presents the empirical results. Section 5 concludes the paper.

## 2 Theoretical framework

In this section, we discuss the necessary and sufficient conditions for choices to be rationalized with QL preferences.

Consider a space of alternatives \( Y = X \times \mathbb{R} \), where \( X \subseteq \mathbb{R}^n_+ \). We denote an element of this set as \((x, m) \in Y\), where \( x \in X \) and \( m \in \mathbb{R} \). Intuitively, \( x \) is a bundle of consumption goods, and \( m \) is an amount of money left after purchasing \( x \). In a labor supply setting, as in our experiment, \( x \) is leisure time (total time budget minus the time needed to complete the task), and \( m \) is the compensation for the task. In what follows, we will use the consumption analogy due to its greater generality. A QL utility function takes the following form:

\[
v(x, m) = u(x) + m,
\]

where \( u : X \to \mathbb{R} \) is a real valued sub-utility function.

Let \( E = ((x^t, m^t), B^T) \) be a consumption experiment, where \((x^t, m^t)\) is the bundle chosen from budget \( B^t \subseteq Y \) and \( T \) is the total number of decisions made. We assume that budgets are downward closed\(^2\) and can be described by a continuous and decreasing function \( \mu^t : X \to \mathbb{R} \),\(^3\) such that

\(^2\) That is, if \( y \in B^t \), then \( y' \in B^t \) for every \( y' \leq y \).

\(^3\) A function \( \mu : X \to \mathbb{R} \) is decreasing if \( x \geq x' \) implies \( \mu(x') \geq \mu(x) \) and \( x > x' \) implies \( \mu(x') > \mu(x) \).
The assumptions we make guarantee that the border of the budget set is downward sloped and represents a closed set. These assumptions allow for nonlinear budget sets.

Figure 1 illustrates how one can construct \( \mu_t(x) \) using the depicted border of the budget set. Note that \( \mu_t(x) \) can be negative; indeed, for a high enough value of \( x \), it will be negative. For the case of linear budgets, \( \mu_t(x) = \frac{l-p_x}{p_m} \times \), where \( l \) stands for income, \( p_x \) the price of the good(s) \( x \), and \( p_m \) the price of money. We assume that \( m_t = \mu_t(x_t) \) in order to avoid a violation of monotonicity in \( m \). Since the budget is downward closed for every \( x \in X \), \( (x, \mu_t(x)) \in B_t \), we allow \( \mu_t(x) \) to be negative, as this technical assumption simplifies the analysis. Intuitively one can think of \( \mu_t(x) \) as the amount of money left after purchasing bundle \( x \in X \) given budget \( B_t \).

Note that we require \( \mu_t(x) \) to be defined for every \( x \in X \). If we consider \( m_t \) to be money, then a negative amount of money can be interpreted as borrowing, and budgets being downward closed can be interpreted as the absence of a binding liquidity constraint.

**Definition 1** A consumption experiment \( E = ((x_t, m_t), B_t)_{t=1}^{T} \) can be rationalized with QL preferences if there is a function \( v(x, m) = u(x) + m \) such that

\[
u(x') + m' \geq u(x) + m \quad \text{for every } (x, m) \in B'.
\]

**Definition 2** A consumption experiment \( E = ((x_t, m_t), B_t)_{t=1}^{T} \) satisfies cyclical monotonicity if for every sequence of observations \( k_1, \ldots, k_n \in \{1, \ldots, T\} \),

\[
B' = \{(x, m) \in Y : \mu_t(x) \geq m\}.
\]
Cyclical monotonicity was introduced by Rockafellar (1970), and its importance in characterizing QL was established by Rochet (1987), Brown and Casamiglia (2007) and Nocke and Schutz (2017). The version of cyclical monotonicity we use is equivalent to the standard definition up to reordering the terms.

Figure 2 shows an example of a violation of QL preferences although this set of data is rationalizable with a locally non-satiated utility function (LNU). To see this, suppose that the agent makes decisions according to QL preferences. Since \( y^s \) is chosen from \( B^s \), it must be the case that the agent prefers \( y^s = (x^s, \mu^s(x^s)) \) to \( (x^t, \mu^s(x^t)) \), then using quaslinearity we can add \( \mu^s(x^t) - \mu^t(x^t) \) to \( y^s \) and to \( (x^t, \mu^s(x^t)) \) preserving the relation between them. Thus, \( (x^s, \mu^s(x^s) + [\mu^s(x^t) - \mu^t(x^t)]) \) is preferred to \( y^t = (x^t, \mu^t(x^t) + [\mu^t(x^t) - \mu^t(x^t)]) \). Given that \( \mu^t(x^t) - \mu^s(x^s) \leq \mu^t(x^t) - \mu^t(x^t) \) we can conclude that \( (x^s, \mu^s(x^s) + [\mu^t(x^t) - \mu^t(x^t)]) \) is in the interior of \( B' \). Thus \( y' \) is strictly revealed preferred to \( (x^t, \mu^s(x^s) + [\mu^t(x^t) - \mu^t(x^t)]) \), which is a contradiction. We now establish the main result for this section, whose proof is provided in “Appendix 1”.

**Theorem 1** A consumption experiment \( E = ((x^t, m^t), B'^T)_{t=1} \) can be rationalized with QL preferences if and only if it satisfies cyclical monotonicity.

We provide the intuition for the proof before we proceed further. The main step of the proof is to assign utility numbers to observed choices that are then
extended to the whole choice space using techniques developed in Afriat (1967). Since the necessity of cyclical monotonicity is clear, we concentrate on why cyclical monotonicity allows finding the proper utility values. We start by defining some graph-theoretic notions needed for the argument.

Consider a weighted directed complete graph where each vertex corresponds to an observation. The weight assigned to each edge corresponds to the difference in the relative cost of bundles $x_t$ and $x_s$ in the budget $B_t$, that is $\mu_t(x_t) - \mu_t(x_s)$. Cyclical monotonicity guarantees that this graph does not contain negative cycles. Hence, we can assign a utility number to the point $x_t$ that is equal to the weight of the shortest walk passing through $x_t$ at the first transition. This construction guarantees that the chosen point is better than any other point in the budget. For this, we need to show that chosen values $u(x_t)$ satisfy

$$u(x_t) + \mu_t(x_t) \geq u(x_s) + \mu_t(x_s) \iff u(x_t) - u(x_s) \geq \mu_t(x_s) - \mu_t(x_t).$$

By construction $u(x_t)$ cannot exceed $u(x_t) + (\mu_t(x_t) - \mu_t(x_s))$, since both $u(x_t)$ and $u(x_s)$ are defined as shortest walks. This observation guarantees that QL rationalizability is satisfied for the space of chosen points. “Appendix 1” shows that $u(x)$ can be assigned using the technique developed in Afriat (1967).

Before concluding this section, we make several remarks that illustrate the difference between our test for quasilinearity and existing ones. We start from showing that the assumption of convexity of preferences is restrictive, and show how to modify the test by Brown and Calsamiglia (2007) for the setting we consider.

Figure 3 illustrates how restrictive the assumption of concavity of the utility function is when budgets are nonlinear. We use nonlinear budgets in this case because,

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4 A graph is said to be complete if there is an edge between every pair of vertexes. The construction we use is similar to that in Piaw and Vohra (2003).
as proven by Afriat (1967), the concavity of the utility function has no empirical content when budgets are linear. Figure 3a shows that a consumption experiment can satisfy cyclical monotonicity, i.e., be QL rationalizable, but fail GARP for the linearized version of the budgets (see Matzkin 1991). To proceed with this example we need to make two additional assumptions. We assume that $\mu'(x)$ is differentiable and concave. Then, assuming that preferences are convex (without assuming quasi-linearity) implies bundles worse than $y'$ are below the supporting hyperplane given by $\nabla \mu'(x')$. Linearized budgets are defined as follows:

$$\mu'_V(x) = \mu'(x') + \nabla \mu'(x')(x - x'),$$

where $\nabla \mu'(x')$ is the gradient of $\mu'(x)$ evaluated at $x'$. By assuming $\mu'(x)$ to be concave we ensure that $\mu'(x) \leq \mu'_V(x)$, and then passing QL in the linearized experiment implies passing QL in the underlying non-linear experiment. Figure 3b shows that a consumption experiment can satisfy cyclical monotonicity, i.e., be QL rationalizable, but fail to have a concave QL rationalization. Cyclical monotonicity fails in the linearized budgets in the example since $\mu'_V(x') - \mu'_V(x^s) > \mu'_V(x^t) - \mu'_V(x')$, where $\mu'_V$ is the amount of $m$ computed using the gradient of the border of the budget evaluated at the chosen point as the fixed price. Hence, we can provide the following result in spirit of Forges and Minelli (2009).

**Remark 1** A consumption experiment $E = ((x^t, m^t), B^t)_{t=1}^T$ can be rationalized by a concave QL utility function if and only if corresponding linearized experiment satisfies cyclical monotonicity.

In sum, our test significantly generalizes previous results. Before proceeding, we make a couple of remarks regarding existing results that do not deal with quasi-linearity directly. Quasilinear preferences are a special case of additively separable preferences, and revealed preference theory has addressed separability since Fishburn (1990) seminal result. The work of Varian (1983) considers the concave case of additive separability and exploits standard Afriat (1967) techniques in order to obtain a test for it. Varian’s test relies crucially on the concavity of each separable component of utility. Recent work of Polisson (2018) relaxes this assumption and provides a test for additive separability without concavity. However, this test requires each separable component to be one-dimensional albeit unknown. Moreover, Echenique (2014) shows that the general test for separability of preferences cannot be computationally feasible, while the test we provide is. We allow the first separable component to be multidimensional, while the second component has to be known and one-dimensional. Hence, while quasi-linearity is a specific case of additive separability, the test we develop is not a special case of any existing tests for separable preferences.

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5 In this sense the quasilinear analog of the Varian (1983) is the test developed by Brown and Calsamiglia (2007).
6 The crucial idea behind the requirement that each component is one-dimensional is that the corresponding set is fully ordered. Hence, we treat these notions as synonyms throughout the discussion.
Another relevant theory is the augmented utility (AU) introduced by Deb et al. (2018), who develop the Generalized Axiom of Revealed Price Preferences (GAPP). Augmented utility is close to quasilinearity in the sense that agents have preferences over the monetary costs of consumption goods. GAPP imposes acyclicity (as in GARP) over the revealed price preference relation. Thus, AU is (in general) not a theory nested within LNU, while QL is nested within both these theories. However, given that the space of alternatives includes expenditure, then notions of LNU and AU coincide. Our results relate to GAPP insofar QL is a restriction to LNU in an extended space of alternatives.

3 Experimental design

To test for QL, we implement a labor supply experiment with a real effort task: data entry. Our design bears resemblance with recent experimental research that uses real-effort tasks to study time-preferences (Augenblick et al., 2015; Augenblick and Rabin 2019) and self-control Toussaert (2018). More recent research has augmented this experimental paradigm to elicit willingness to pay for future tasks (e.g., Carrera et al., 2022). Our experiment significantly extends this paradigm by allowing non-linear incentives. This allows to test for additional features of preferences that are germane to this literature as well as the literature on labor supply in general (see e.g. Cherchye et al., 2012, 2017). Relatedly, there is a long tradition in applied welfare economics which exploits nonlinearities in budget sets, quasi-linearity and convexity to estimate labor supply responses to taxation (see Kleven, 2016). Our experimental provides the ideal setting to tests these maintained assumptions. Our design not only closely follows our theoretical results but it is implemented in a highly relevant setting.

In the experiment, subjects have to choose one option in each of 20 different choice sets. Each choice set has ten separate contracts that specify the number of data entry tasks ($x$) to complete and payment if the tasks are completed ($m$). One of the 20 sets of contracts is randomly selected to secure incentive compatibility, and the subject’s

Fig. 4 Construction of the budget
choice for that menu is implemented. Figure 4 provides a graphical representation of the budgets sets faced by subjects. Panel (a) shows a budget set generated by a non-linear wage schedule. The discretized budget is represented as the piece-wise linear budget. In this context, the number of tasks (or effort) is a “bad.” We use the slope between $x$ and $x + 1$ as the marginal price at $x$ tasks, except for the last case we use the slope between $x - 1$ and $x$. Panel (b) shows a transformed budget that mimics Fig. 1. Denote by $K$ the maximum number of tasks offered to the subject, then $K - x$ is a “good.” By keeping $K$ constant throughout the experiment we maintain all the necessary assumptions about budget set needed to test for QL preferences.

Figure 5a depicts the actual menus used in the experiment. The minimal wage used is $3 (for 1 task), and the maximal wage is about $27 (for ten tasks). Figure 5b presents a typical data entry task and the interface used by subjects. The table is an extract from the 1876 Peruvian population census. In the real effort tasks, subjects were asked to enter the data from the Peruvian Census in a data-entry grid (see Fig. 5b). Each cell in the data-entry grid corresponds to a cell in the scanned table from the census. To make sure that subjects entered the data correctly, the interface used control sums. The last row in each census table, called “Total general,” is the total of the entries above each column and is never equal to zero. Similarly, the numbers in the last column equal the sum of the numbers in two preceding columns. We use this to create control sums. If a subject submitted entries that did not match the control sums, she was told that the data entered was in error, and was informed which row and/or column had a problem. To make the task a real-effort job, a task was deemed complete, or a contract fulfilled, once errors were absent. Figure 5c shows the interface given to subjects to choose among contracts. All choice sets contain ten contracts, and contracts were always ordered from 1 task to 10 tasks.

The timeline of the experiment is as follows. First, to allow subjects to form an expectation of the time it takes to complete an average task, we implement a practice data entry task in which consistency checks are enforced. Second, subjects choose contracts from each one of the 20 budgets. Finally, one of the 20 budgets is chosen at random, and the chosen contract is implemented. All the tasks (census extracts) are randomly selected from a pool of pre-selected sections of comparable length and readability. The experiment was implemented in September–October 2021 using oTree (Chen et al., 2016) at the Texas A &M University (TAMU) experimental lab. A total of 65 undergraduate students participated in the experiment with average earnings of $15.

Table 1 presents basic summary statistics of the experiment. Subjects choose 7.03 (s.d. 2.75) tasks to complete on average. The number of choices varies across menus. The mean and median spread of the number of tasks is 4.75 and 5. Eleven out of 65 subjects always chose the same number of tasks. Decisions of individuals who do not vary their labor supply across menus can always be rationalized.

4 Experimental results

This section presents the results of the experiment. We denote the concave version of locally nonsatiated utility (LNU) as C-LNU, and the concave version of quasi-linear utility (QLU) as C-QLU. We illustrate the main results. First, we show that
concavity and quasilinearity are equally restrictive. That is, C-LNU and QLU have similar empirical support. Second, the joint assumption of concavity and quasilinearity (C-QLU) is more restrictive than either concavity alone (C-LNU) or

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**Fig. 5** Implementation of the experiment
quasilinearity alone (QLU). Third, subjects consistent with C-QLU exhibit behavioral patterns that are significantly different from subjects consistent with LNU/QLU/C-LNU. In particular, subjects consistent with C-QLU exhibit significantly less elastic labor supply.

We find that 11 out of 65 subjects are consistent with all the theories considered. Note that the subjects consistent with all four theories are the only subjects consistent with at least one theory. These are the subjects who chose the same number of tasks in all sets of contracts. To allow for decision errors, we evaluate consistency with different hypotheses on preferences using the Houtman and Maks (1985) Index (HMI). The HMI is the maximum percent of choices consistent with a theory (in our case, LNU, QLU, C-LNU or C-QLU). In our data, a perfectly consistent subject has an HMI of one, and the lowest possible HMI is 0.05 (1/20). Since inconsistent choices are counted as errors, the HMI provides an upper bound of the probability of deliberate choice. We use the HMI because it is the simplest to calculate and interpret for all of the theories we consider. For robustness, “Appendix 2” shows that similar results hold if we use the CCEI.

Figure 6 presents the distributions of the HMI for all of the theories. Panels (a), (b), (c), and (d) show the distributions for LNU, QLU, C-LNU, and C-QLU, respectively. The mean HMI for LNU is 0.77 (SD 0.14), the mean HMI for QLU is 0.73 (SD 0.19), the mean HMI for C-LNU is 0.74 (SD 0.17), and the mean HMI for C-QLU is 0.63 (SD 0.23). The mean (medians) of C-LNU, QLU, and C-QLU are significantly lower than for LNU. This is not surprising since they are restrictions on LNU. QLU and C-LNU perform similarly in rationalizing the behavior in the experiments. Finally, the mean (median) HMI for C-QLU is significantly lower than that of QLU and C-LNU. Imposing convexity to QL is costly.

Table 1 Descriptive statistics of choices

| Choice Description         | Value  |
|----------------------------|--------|
| Mean Choice                | 7.03   |
| Average Standard Deviation | 2.75   |
| Min Spread                 | 0      |
| Max Spread                 | 9      |
| Mean Spread                | 4.75   |
| Median Spread              | 5.00   |
| Mode Spread                | 5.00   |
| Standard Deviation of Spread | 3.01  |

Dembo (2019) shows that this estimate is biased while the unbiased estimator is not computationally tractable.

Differences in both means and medians are significant at $p < .01$.

Differences in both means and medians are significant at $p < .01$. 
4.1 Validating the utility hypotheses

Proper tests of alternative utility hypotheses require accounting for false positives or the possibility that a subject passes a theory by chance. We start by constructing a test for LNU. The null hypothesis for this case is random behavior, and the alternative hypothesis is consistency with LNU. The critical value for this hypothesis is a pre-determined level of the HMI (see below). To estimate the distribution of the parameter under the null hypothesis, we calculate the HMI of 400,000 randomly generated subjects. In particular, for each menu of each random subject, we select a contract according to the empirical distribution of choices in the menu. We treat the distribution of HMI resulting from this procedure as the asymptotic distribution of HMI under the null hypothesis. The significance level \(\alpha\) is calculated using the \(100 - \alpha\) percentile of the distribution of HMIs of random subjects. We use a one-side

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10 For a more detailed discussion of the method used to generate random choices and conducting hypothesis testing, see Cherchye et al. (2020).
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We consider experimental subjects whose HMI is above this threshold as being consistent with LNU since the null hypothesis is rejected at the given significance level.

Next, to account for the nestedness of QLU and C-LNU theories into LNU. Conditional pass rates are calculated using the subpopulation of random subjects passing LNU at the \( \alpha \) significance level. This provides a test of restrictions on LNU, as now the null hypothesis is (random) behavior consistent with LNU. We consider two ways to generate cutoffs for C-QLU. One uses the subpopulation of random subjects consistent with C-LNU, and the other uses the subpopulation of random subjects consistent with QLU.\(^{11}\)

Table 2 presents pass rates (left panel) and thresholds (right panel) for the theories considered. The percentages are the pass rates, which are unconditional for LNU and conditional for the remaining. The numbers in parentheses are the subjects who pass a theory and their comparison group. The numbers in brackets are the 95% confidence intervals of the pass rate calculated using the Clopper-Pearson procedure.\(^{12}\)

Table 2 shows support for LNU, C-LNU, and QLU. Depending on the tolerance level of the test, the pass rate for LNU is between 51 and 77%. The pass rate for QLU is between 79 and 86%, and the pass rate for C-LNU is between 73 and 82%. The support for QLU and C-LNU holds even if we consider unconditional pass

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\(^{11}\) Of the 400000 random subjects, 20000 are consistent with LNU. These are used to construct cutoffs for QLU and C-LNU. Finally, 1,000 of random subjects consistent with QLU or C-LNU are used to generate cutoffs for C-QLU.

\(^{12}\) The pass rate is a standard Binomial variable, where “success” is rejecting the null hypothesis (subject passed) and “failure” is not rejecting the null (subject failed). The Clopper-Pearson procedure provides exact confidence intervals. Results are similar if we estimate confidence intervals using the CLT (see Table 4). The significance levels we consider are \( p \in \{.10, .05, .01\} \) corresponding to the 90%, 95%, and 99% cutoffs. The right panel presents the HMI thresholds used in the analysis.
rates, which are more demanding for nested theories. The unconditional pass rate for QLU is between 40 and 66% and for C-LNU is between 37 and 63%. Next, we consider adding a concavity restriction to QL (denoted C-QLU(Q)) or to C-LNU (denoted C-QLU(C)). The conditional pass rates decrease significantly after imposing this restriction \((p < .01)\) for thresholds \(p = .10\) and \(p = .05\). The unconditional pass rates for C-QLU(C) are 43\% ([.31, .56]), 37\% ([.25, .50]), and 29\% ([.19, .42]) for \(p = .10\), \(p = .05\), and \(p = .01\) and for C-QLU(Q) are 37\% ([.25, .50]), 37\% ([.25, .50]), and 29\% ([.19, .42]) for \(p = .10\), \(p = .05\), and \(p = .01\). Full results on unconditional pass rates are provided in “Appendix 2”.

### 4.2 Individual choices

This section presents individual-level results. Because quasilinearity is frequently used as an assumption that holds only locally, i.e. for relatively small changes in labor supply or changes due to minor changes in incentives, we will make clear when the assumption is assumed to be local or global, i.e. applicable to the entire domain of labor supply. We start by showing supply functions for different theories. To gain intuition, recall that in the case QLU, the optimal choice of tasks depends only on the slope of the budget constraint (e.g., in the differentiable case, for optimal choice \(x^*\) we have that \(u'(x^*_i) = m'(x^*_i)\) or \(x^*_i = (u')^{-1}(m'(x^*_i))\)). As mention in the previous section, we approximate the slope of budget constraint by the change in the wage per task completed. Figure 7 shows scatter plots of the change in wages as a function of the number of tasks chosen. We consider 3 subjects that illustrate well different behavioral patterns:

- Subject 33 who is consistent with LNU but not QLU;
- Subject 9 who is consistent with QLU;
- Subject 7 who is consistent with C-QLU.

Due to noisy behavior (HMI \(< 1\)), the scatter plots include choices that would need to be excluded to satisfy preference restrictions exactly. Subject 33, depicted on Fig. 7a, is consistent with LNU but not QL. As the graph shows, there is no clear relationship between choice of tasks and changes in wages due to income effects. Subject 9, depicted in Fig. 7b, is consistent with QLU. Since QLU implies a constant marginal disutility of effort, we expect a clustering of changes in wages at each level of effort.\(^{13}\)

Finally, to visualize how consistency with different behavioral assumption relates with labor supply elasticity, we analyze the dispersion of choices by HMI level. We

\(^{13}\) Note that subjects are choosing over discretized budgets and therefore marginal conditions are only approximations. This pattern is observed for levels 3, 4, 6, and 7 tasks, but not at level 5. Concavity further requires that the changes in wages increase in the number of tasks (the marginal disutility of effort is increasing). That is, the maximal \(\Delta(wage)\) faced while choosing \(x\) tasks should be below the minimal \(\Delta(wage)\) faced while choosing \(y\) tasks for \(y > x\). Subject 7, depicted in Fig. 7c, provides an example of this. The implication is satisfied for \(\Delta(wage)\) corresponding to choices of 8 and 10 tasks. However, none of the choices of 9 tasks can be rationalized exactly by C-QLU.
Fig. 7 Examples of the choices of subjects

(a) LNU but not QLU (subject 33)
(b) QLU (subject 9)
(c) C-QLU (subject 7)
focus on two indicators: spread and variance. That is, for every subject we compute the spread (difference between maximum and minimal) and variance of the number of tasks chosen. Figure 8 plots the average of these measures for each HMI level and each behavioral assumption. For instance, Fig. 8a shows that for subjects with an HMI = .6 for LNU the average spread is 4.5. Figure 8b shows that for subjects with an HMI = .6 for LNU the average variance is 2.5. The mean spread for LNU is the largest regardless the HMI level. Mean spreads for QLU and C-LNU are comparable to LNU and each other. The mean spread for C-QLU is the smallest. Results based on the variance are consistent with those using spreads.

5 Concluding remarks

We provide necessary and sufficient conditions for a set of observed choices to be rationalizable by quasilinear (QL) preferences. These conditions apply to choices over compact and downward closed budget sets and do not require the utility function to be concave. The test of QL that we present applies to a large class of problems. First, it applies to consumer problems in the presence of distortions due to taxes, subsidies, or nonlinear pricing. Second, it can be extended to test for QL preferences in strategic situations when QL is invoked (e.g., auction theory). It can also be used to analyze labor supply decisions, where nonlinear prices (wages for efforts or output) appear naturally. Finally, we test the assumption of QL preferences using a real-effort task experiment. This is not only the most straightforward environment in which the theory can be tested but also reproduces the labor-income decision setting in which QL is commonly assumed. Our main empirical result is that QL tests that do not allow for the possibility of non-convex preferences significantly underestimate the support for QL preferences.

The empirical support of QL in our experiment might appear to be at odds with recent research on income effects (see e.g. Cesarini et al., 2017; Giupponi, 2019; Golosov et al., 2021). These studies, however, consider large changes in
income (lotteries or significant subsidies), while our laboratory setting deals with small stakes (less than $30). QL might still be a reasonable assumption when only small changes in income are considered. The generality of our test for QL preferences allows us to expand the contexts in which this assumption can be tested. For instance, Strack and Taubinsky (2021) show that while time inconsistency cannot be identified from preference reversals alone, it can be identified if preferences are quasilinear. This is so because, under quasilinearity, a cardinal measure of preference reversals can be obtained. The same applies to contexts in which willingness to accept is used to assess the consistency of behavior. Our work provides a way to test this maintained assumption.

Appendix 1: Proof of Theorem 1

Proof ($\Rightarrow$) Consider a function $v(x, m) = u(x) + m$ that rationalizes the consumption experiment. For clarity, we maintain the notation $m^t(x^i)$, $m^t(x^i')$ and $m^t(x)$ for the rest of the proof. Then, for every sequence of observations $k_1, k_2, \ldots, k_n \in \{1, \ldots, T\}$, the following is true:

$$u(x^{k_{n+1}}) + m^{k_{n+1}}(x^{k_{n+1}}) \geq u(x^{k_1}) + m^{k_1}(x^{k_1}),$$

where $k_{n+1} = k_1$. We can simplify this and obtain the following inequalities:

$$\begin{align*}
  m^1(x^{k_1}) - m^1(x^{k_2}) &\geq u(x^{k_2}) - u(x^{k_1}), \\
  m^2(x^{k_2}) - m^2(x^{k_3}) &\geq u(x^{k_3}) - u(x^{k_2}), \\
  \vdots &\vdots \vdots \vdots \vdots \\
  m^n(x^{k_n}) - m^n(x^{k_1}) &\geq u(x^{k_1}) - u(x^{k_n}).
\end{align*}$$

Summing up these inequalities, we obtain the following:

$$m^1(x^{k_1}) - m^1(x^{k_2}) + m^2(x^{k_2}) - m^2(x^{k_3}) + \ldots + m^n(x^{k_n}) - m^n(x^{k_{n-1}}) \geq 0.$$

This is exactly the cyclical monotonicity condition.

($\Leftarrow$) This part of the proof is split in two parts. First, we construct the subutility numbers corresponding to the observed choices and show that the observed chosen point $(x', m')$ is at least as good as any other point $(x^i, m^t(x^i))$. Next, we extend the subutility function to the entire $\mathbb{R}_+^n$ and show that corresponding QLU rationalizes the data.

Let

$$u' = \min\{m^1(x^{k_1}) - m^1(x') + \ldots + m^n(x^{k_n}) - m^n(x^{k_{n-1}})\}$$
over all sequences in the data, including those with repeating elements. Note that all the sequences are starting with \( k_1 \), however, the real “anchoring” element of the sequence is \( k_2 = t \) for the observation \( t \in T \). We will show that the minimum is well-defined. It will be enough to show that there will be no cycles in the minimal sequence, since the rest will follow from the fact that the minimum is taken over finite sums of finite numbers. So assume, to the contrary, that the minimum sequence contains a cycle; then we have

\[
\ldots + (m^{k_n}(x^{k_n}) - m^{k_n}(x^{k_{n-1}})) + \ldots + m^{k_s}(x^{k_s}) - m^{k_s}(x^{k_{n-1}}) + \ldots.
\]

However, cyclical monotonicity implies that this term is greater or equal than zero, and hence, excluding it would make the sequence even smaller. That contradicts the original assumption that the sequence was the smallest.

Further, we show that such a construction of \( u' \) will guarantee that the following system of inequalities is satisfied. For any \( t, s \in \{1, \ldots, T\} \) we want to show that

\[
 u' - u^s \geq m'(x^t) - m'(x^r).
\]

By the construction of \( u(x) \), we can guarantee that

\[
 u^s \leq m'(x^t) - m'(x^r) + u',
\]

since

\[
 u' = m^{k_1}(x^{k_1}) - m^{k_1}(x^r) + \ldots + m^{k_s}(x^{k_s}) - m^{k_s}(x^{k_{n-1}})
\]

for some (minimal) sequence, and we construct \( u(x^t) \) using that minimal sequence. Recall that we allowed taking every sequence, including those with repeating elements, so we can add any element to the existing sequence and the utility level \( u(x^t) \) will not exceed the value of the new extended sequence. Therefore,

\[
 u' - u^s \geq u' - (m'(x^t) - m'(x^r) + u(x^s)) = m'(x^s) - m'(x^r).
\]

This concludes the first part of the proof. Next, we use the numbers constructed above to recover the entire utility function.

For every \( x \in X \), let

\[
 u(x) = \min_{t \in \{1, \ldots, T\}} \{ u' + m'(x^t) - m'(x^r) \}.
\]

Note that since \( m' \) is continuous and monotone, so is \( u(x) \).

First, we will show that for every \( t \in \{1, \ldots, T\} \), \( u(x^t) = u' \). As we have shown above for every \( s \in \{1, \ldots, T\} \) for which \( m'(x^t) \) is defined,

\[
^{14} \text{In terms of graph theory, this is equivalent to searching for the shortest walk (on a weighted directed graph), rather than a shortest path (on a weighted directed graph). However, the shortest walk is not always well-defined. A sufficient condition for it to be well-defined is the absence of negative cycles, which is guaranteed by cyclical monotonicity if we define the complete directed graph with vertexes as observations and weights as } w_{s,s'} = m'(x^s) - m'(x^r).\]
Therefore, $u(x') = u'$. Next, we show that the constructed utility rationalizes the data, that is $v(x, m) \leq v(x', m'(x'))$ for every $(x, m) \in B'$, where $v(x, m) = u(x) + m$. By the construction of $u(x)$, we know that

$$u(x) = \min_{t \in \{1, \ldots, T\}} \{ u' + m'(x') - m'(x) \} \leq u' + m'(x') - m'(x).$$

Therefore,

$$u(x) + m \leq u(x) + m'(x) \leq u(x') + m'(x').$$

Given the proof of the Theorem 1 one can immediately infer the following remark.

**Remark 2** A consumption experiment $E = ((x', m'), B')_{t=1}^T$ can be rationalized by a QLU function if and only if there is a monotone function $u : C \to \mathbb{R}$ such that

$$u(x') - u(x) \geq m'(x') - m'(x)$$

for every $t, s \in \{1, \ldots, T\}$.

where $C = \bigcup_{t \in \{1, \ldots, T\}} \{ x' \}$ is the set of chosen points.

Remark 2 crucially important for the following reason. Cyclical monotonicity is an elegant condition, it may be less computationally tractable especially once mixed with measures of distance to rationality. However, the linear programming approach provides the definitely computationally tractable way of testing QLU rationalizability.

**Appendix 2: Additional empirical results**

This appendix presents additional empirical analysis. We first present additional results using the HMI index. We then present analysis using the Critical Cost Efficiency Index (CCEI) introduced by Afriat (1973). The results are robust to alternative measures of distance to rationality.

**Table 3** Left panel: Pass rates for each theory considered. Unlike Table 2 in the main text, this table reports unconditional pass rates. The first number is the percentage pass rate; the fraction in parentheses is the numbers of subjects used to compute the pass rates; the numbers brackets are the 95% confidence intervals. Right panel: HMI cutoffs for different $p$-levels.

| Theory   | Pass Rates | HMI cutoffs |
|----------|------------|-------------|
|          | $p = .10$  | $p = .05$  | $p = .01$  | $p = .10$  | $p = .05$  | $p = .01$  |
| LNU      | 77% (50/65) [.65,.87] | 77% (50/65) [.65,.87] | 51% (33/65) [.38,.63] | .70 | .70 | .75 |
| QLU      | 66% (43/65) [.54,.74] | 66% (43/65) [.53,.74] | 40% (26/65) [.38,.53] | .65 | .65 | .75 |
| C-LNU    | 63% (41/65) [.50,.75] | 63% (41/65) [.50,.75] | 37% (24/65) [.25,.50] | .70 | .70 | .80 |
| C-QLU(C) | 43% (28/65) [.31,.56] | 37% (24/65) [.25,.50] | 29% (19/65) [.19,.42] | .65 | .70 | .80 |
| C-QLU(Q) | 37% (24/65) [.25,.50] | 37% (24/65) [.25,.50] | 29% (19/65) [.19,.42] | .70 | .70 | .80 |
Additional results for HMI

We present two additional tables. Table 3 presents unconditional pass rates. Table 4 presents an alternative construction of confidence intervals using normal approximations instead of the exact procedure.

Table 3 replicates the results of Table 2 but uses unconditional pass rates instead. These numbers serve for purely illustrative purposes, as it is not totally correct to analyze nested theories using unconditional pass rates. Recall that the nested structure can be summarized as follows.

\[ \text{C-QLU(C)} \subseteq \text{C-LNU} \subseteq \text{LNU} \quad \text{and} \quad \text{C-QLU(Q)} \subseteq \text{QLU} \subseteq \text{LNU} \]

That is, C-QLU is nested within both C-LNU and QLU, while both C-LNU and QLU are nested within LNU. Finally, C-LNU and QLU are independent theories.

Table 4 replicates the results of Table 2 using the CLT approximations for the confidence intervals of pass rates. Note that unlike the confidence intervals generated by Clopper-Pearson procedure (exact confidence intervals), the CLT confidence intervals are symmetric. Thus, they might run out of the domain: returning values above one or below zero. Even though there are no bound violations in our data we prefer to report the exact confidence intervals. Using the CLT confidence intervals does not alter the results presented in Table 2.

Analysis using CCEI

Another measure of distance to rationality is the Critical Cost Efficiency Index (CCEI) introduced by Afriat (1973). The definition of the CCEI in the augmented consumption space we consider is, however, ambiguous. An interpretation of the CCEI is the share of wealth to be left on the table needed to make an agent rational.

### Table 4

| Theory | Pass Rates | HMI cutoffs |
|--------|------------|-------------|
|        | $p = .10$  | $p = .05$  | $p = .01$  | $p = .10$  | $p = .05$  | $p = .01$  |
| LNU    | 77% (50/65) | 77% (50/65) | 51% (33/65) | .70  | .70  | .75  |
|        | [.67, .87]  | [.67, .87]  | [.39, .63]  |      |      |      |
| QLU    | 86% (43/50) | 86% (43/50) | 79% (26/33) | .65  | .65  | .75  |
|        | [.76, .96]  | [.76, .96]  | [.65, .93]  |      |      |      |
| C-LNU  | 82% (41/50) | 82% (41/50) | 73% (24/33) | .70  | .70  | .80  |
|        | [.71, .93]  | [.71, .93]  | [.57, .88]  |      |      |      |
| C-QLU(C)| 69% (28/41) | 59% (24/41) | 79% (19/24) | .65  | .70  | .80  |
|        | [.54, .83]  | [.43, .74]  | [.63, .96]  |      |      |      |
| C-QLU(Q)| 56% (24/43) | 56% (24/43) | 73% (19/26) | .70  | .70  | .80  |
|        | [.41, .71]  | [.41, .71]  | [.56, .91]  |      |      |      |
This can be done in linear budget sets straightforwardly. Assume all prices ($p$) are set up such that for each budget set income is normalized to be equal to 1. Then $p'y \leq 1$ defines the “revealed preference relation.” The CCEI redefines this constraint by introducing $e \in [0, 1]$ such that $p'y \leq e$ defines a new revealed preference relation. In this case $1 - e$ provides the relative measure of the wealth one needs to sacrifice in order to rationalize observed choices. Further, the CCEI finds the maximal $e$ (minimizing potential wealth losses) such that observed choices are rationalizable.

However, in the augmented consumption space, “wealth” (wage in our context) is already one of the dimensions of the consumption space. Thus one option is to consider only changes in wages. This corresponds to the case described on the Fig. 9b. An alternative explanation of the CCEI is based on the idea of “thick indifference curves” or “just noticeable differences” as axiomatized by Dziewulski (2020). This

![Graph](image-url)
interpretation allows us to disregard the fact that the consumption space includes money already. This interpretation allows shifting the budget downwards as presented in Fig. 9a. The nonlinearity of the budget set, however, produces another complication. Since we are using piece-wise linear budgets, it matters if the value of \( x \) at which price regime changes varies with the level of the CCEI. The just noticeable difference interpretation of the CCEI (see Fig. 9a) implies that the “break-point” varies with \( e \). The wealth interpretation of the CCEI (see Fig. 9b) does not allow the break-point to vary with \( e \). Next we present the results for the pass rates for both interpretations of CCEI.

CCEI based on wealth

Table 5 presents the results as in Table 2 using the CCEI based on changes in wealth only (version from Fig. 9b) as the measure of distance to rationality. The pass rates for LNU and QLU are comparable to those using HMI. However, comparing C-LNU to QLU we observe that the pass rates for C-LNU are lower. This is evidence that the assumption of concavity is more restrictive than quasilinearity. Overall we see that the pass rates for C-QLU(Q) are lower than those for QLU. We observe that concavity is a restrictive assumption. However, pass rates for C-QLU(C) and C-LNU are comparable. Thus, adding the quasilinearity assumption to C-LNU does not seem to be restrictive. These results are consistent with the results presented in Table 2.

CCEI based on just noticeable differences

Table 6 presents results using the just noticeable differences interpretation of the CCEI (version from Fig. 9a). We see that the main results still hold. First, we see that pass rates for QLU condition on LNU are high. Moreover, using this version
of the CCEI we obtain higher pass rates for QLU than C-LNU. We also see that pass rates for C-LNU are weakly lower than those of C-QLU(C) for all significance levels. This means that under the assumption of concavity adding quasilinearity is costless. Finally, we see that pass rates for C-QLU(Q) are lower than those of QLU. This agrees, accounting for noise, with the results obtained with alternative measures of rationality. We conclude that our main finding of quasilinearity is not more restrictive than concavity (see Table 2) does not depend on the measure of distance to rationality utilized.

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