Recent calculations have pointed to a 2.8 $\sigma$ tension between data on $\epsilon'_K/\epsilon_K$ and the standard-model (SM) prediction. Several new physics (NP) models can explain this discrepancy, and such NP models are likely to predict deviations of $B(K \to \pi\nu\bar{\nu})$ from the SM predictions, which can be probed precisely in the near future by NA62 and KOTO experiments. We present correlations between $\epsilon'_K/\epsilon_K$ and $B(K \to \pi\nu\bar{\nu})$ in two types of NP scenarios: a box dominated scenario and a $Z$-penguin dominated one. It is shown that different correlations are predicted and the future precision measurements of $K \to \pi\nu\bar{\nu}$ can distinguish both scenarios.

1 Introduction

CP violating flavor-changing neutral current decays of $K$ mesons are extremely sensitive to new physics (NP) and can probe virtual effects of particles with masses far above the reach of the Large Hadron Collider. Prime examples of such observables are $\epsilon'_K$ measuring direct CP violation in $K \to \pi\pi$ decays and $B(K_L \to \pi^0\nu\bar{\nu})$. Until recently, large theoretical uncertainties precluded reliable predictions for $\epsilon'_K/\epsilon_K$. Although standard-model (SM) predictions of $\epsilon'_K$ using chiral perturbation theory are consistent with the experimental value, their theoretical uncertainties are large. In contrast, calculation by the dual QCD approach\(^1\) finds the SM value much below the experimental one. A major breakthrough has been the recent lattice-QCD calculation of the hadronic matrix elements by RBC-UKQCD collaboration\(^2\), which gives support to the latter result. The SM value at the next-to-leading order divided by the indirect CP violating measure $\epsilon_K$ is\(^3\)

$$\text{Re} \left( \frac{\epsilon'_K/\epsilon_K}{\epsilon_K} \right)_{\text{SM}} = (1.06 \pm 4.66\text{Lattice} \pm 1.91\text{NNLO} \pm 0.59_{\text{IV}} \pm 0.23_{m_t}) \times 10^{-4},$$

which is consistent with $(\epsilon'_K/\epsilon_K)_{\text{SM}} = (1.9 \pm 4.5) \times 10^{-4}$ given by Buras et al\(^4\).\(^a\) Both results are based on the lattice numbers, and further use CP-conserving $K \to \pi\pi$ data to constrain some of the hadronic matrix elements involved. Compared to the world average of the experimental results\(^6\),

$$\text{Re} \left( \frac{\epsilon'_K/\epsilon_K}{\epsilon_K} \right)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4},$$

the SM prediction lies below the experimental value by 2.8 $\sigma$.

Several NP models including supersymmetry (SUSY) can explain this discrepancy. It is known that such NP models are likely to predict deviations of the kaon rare decay branching ratios from the SM predictions, especially $B(K \to \pi\nu\bar{\nu})$ which can be probed precisely in the near future by NA62 and KOTO experiments\(^5\). In this contribution, we present correlations between $\epsilon'_K/\epsilon_K$ and $B(K \to \pi\nu\bar{\nu})$ in two types of NP scenarios: a box dominated scenario and a $Z$-penguin dominated one.

\(^a\)Other estimations of the SM value are listed in Kitahara et al\(^5\).

\(^b\)The correlations between $\epsilon'_K/\epsilon_K$, $B(K \to \pi\nu\bar{\nu})$ and $\epsilon_K$ through the CKM components in the SM are discussed in Ref.\(^7\).
2 Box dominated (Trojan penguin) scenario

We first focus on the box dominated scenario, where all NP contributions to $|\Delta S| = 1$ and $|\Delta S| = 2$ processes are dominated by the four-fermion box diagrams. Such a situation is realized in the minimal supersymmetric standard model. The desired effect in $\epsilon_K'$ is generated via gluino-squark box diagrams when the mass difference between the right-handed up and down squarks exists. Such a contribution is called Trojan penguin because its effect is parameterized by the electroweak penguin operator at low energy scale. While the sizable effects in $\epsilon_K'$ are obtained by the Trojan penguin, a simultaneous efficient suppression of the supersymmetric QCD contributions to $\epsilon_K$ can be achieved. The suppression occurs because crossed and uncrossed gluino box-diagrams cancel in $|\Delta S| = 2$ process, if the gluino mass is roughly 1.5 times the squark masses. With appropriately large left-left squark mixing angle and a CP violating phase, one can reconcile the measurements of $\epsilon_K$, $\Delta M_K$ and collider searches for the colored particles with the sizable contribution to $\epsilon_K'$.

However, there is no such cancellation in the (dominant) chargino box contribution to $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ which permits potentially large effects. We investigate the correlation between $\epsilon_K'$ and $B(K \rightarrow \pi^0\nu\bar{\nu})$ varying the following parameters:

$$|\Delta_{Q,12}|, \theta, M_3, m_{\tilde{U}}/m_{\tilde{D}},$$

with $0 < |\Delta_{Q,12}| < 1$ and $0 < \theta < 2\pi$. Here, defining the bilinear terms for the squarks as $M_{X,ij}^2 = M_X^2(\delta_{ij} + \Delta_{X,ij})$ for $X = Q, U, D$, $\theta \equiv \arg(\Delta_{Q,12})$, $M_3$ is the gluino mass. We fix the slepton mass and the lightest squark mass close to the experimental limit ($m_L = 300 \text{ GeV}$ and $m_{\tilde{q}_1} = 1.5 \text{ TeV}$) and use GUT relations among all three gaugino masses.

The main result is shown in Fig. 1 in the $B(K_L \rightarrow \pi^0\nu\bar{\nu}) - B(K^+ \rightarrow \pi^+\nu\bar{\nu})$ plane which are normalized by their SM predictions. We find that the necessary amount of the tuning in the gluino mass and the CP violating phase in order to suppress contributions to $\epsilon_K$ determines deviations of $B(K \rightarrow \pi^0\nu\bar{\nu})$ from the SM values. The current $\epsilon_K'/\epsilon_K$ discrepancy is resolved at $1\sigma$ ($2\sigma$) within the dark (light) green region. In the left (right) panel we used $m_{\tilde{D}}/m_{\tilde{U}} = 1.1$ (2) with $m_{\tilde{U}} = m_Q$ for $0 < \theta < \pi$, and $m_{\tilde{U}}/m_{\tilde{D}} = 1.1$ (2) with $m_{\tilde{D}} = m_Q$ for $\pi < \theta < 2\pi$. Numerically, we observe $B(K_L \rightarrow \pi^0\nu\bar{\nu})/B^{SM}(K_L \rightarrow \pi^0\nu\bar{\nu}) \lesssim 2(1.2)$ and $B(K^+ \rightarrow \pi^+\nu\bar{\nu})/B^{SM}(K^+ \rightarrow \pi^+\nu\bar{\nu}) \lesssim 2(1.2)$.
\[ \frac{\pi^+ \nu \bar{\nu}}{B^{SM}(K^+ \to \pi^+ \nu \bar{\nu})} \lesssim 1.4 (1.1) \] in light of \( \epsilon_K' / \epsilon_K \) discrepancy, if all squarks are heavier than 1.5 TeV and if a 1 (10) % fine-tuning is permitted.

We also observe a strict correlation between \( B(K_L \to \pi^0 \nu \bar{\nu}) \) and \( m_{U_D}/m_{D} : \text{sgn} \left( B(K_L \to \pi^0 \nu \bar{\nu}) - B^{SM}(K_L \to \pi^0 \nu \bar{\nu}) \right) = \text{sgn} \left( m_U - m_D \right) \). Thus, \( B(K_L \to \pi^0 \nu \bar{\nu}) \) can indirectly determine whether the right-handed up or down squark is the heavier one.

3 Z-penguin dominated (modified Z-coupling) scenario

Next, we focus on the Z-penguin dominated scenario. The negative dominant contribution to \( \epsilon_K' / \epsilon_K \) comes from Z-penguin diagrams in the SM. Since in the SM there is a large numerical cancelation between QCD-penguin and the Z-penguin contributions to \( \epsilon_K' / \epsilon_K \), a modified Z flavor-changing (s-d) interaction from NP can explain the current \( \epsilon_K' / \epsilon_K \) easily \( ^{11} \). Then, the decay, \( s \to d \nu \bar{\nu} \), proceeding through an intermediate Z boson, is modified by NP. Therefore, the branching ratios of \( K \to \pi \nu \bar{\nu} \) are likely to deviate from the SM predictions once the \( \epsilon_K' / \epsilon_K \) discrepancy is explained by the modified Z-coupling. They could be a signal to test the scenario.

Such a signal is constrained by \( \epsilon_K \). The modified Z couplings affect \( \epsilon_K \) via the so-called double penguin diagrams; the Z boson mediates the transition with two flavor-changing Z couplings. Such a contribution is enhanced when there are both left-handed and right-handed couplings because of the chiral enhancement of the hadronic matrix elements. An important point is that since the left-handed coupling is already present in the SM, the right-handed coupling must be constrained even without NP contributions to the left-handed one. Such interference contributions between the NP and the SM have been overlooked in the literature. We \(^{12} \) and recent work by Bobeth \( et \ al \) \(^{13} \) have revisited the modified Z-coupling scenario including the interference contributions, and found the parameter regions allowed by the indirect CP violation change significantly.

We find that similar to the previous section, the deviations of \( B(K \to \pi \nu \bar{\nu}) \) from the SM values are determined by the necessary amount of the tuning in NP contributions to \( \epsilon_K \). We parametrize it by \( \xi \): A degree of the NP parameter tuning is represented by \( 1/\xi \), e.g., \( \xi = 10 \) means that the model parameters are tuned at the 10% level.

In Fig. 2, contours of the tuning parameter \( \xi \) are shown for the simplified scenarios: LHS (all NP effects appear as left-handed), RHS (all NP effects appear as right-handed), ImZS (NP effects are purely imaginary), and LRS (left-right symmetric scenario) on the plane of the branching ratios of \( K \to \pi \nu \bar{\nu} \) which are normalized by their SM predictions. We scanned the whole parameter space of the modified Z-coupling in each scenario, and selected the parameters where
\( \epsilon'_K/\epsilon_K \) is explained at the 1σ level. The experimental bounds from \( \epsilon_K, \Delta M_K, \) and \( \mathcal{B}(K_L \to \mu^+\mu^-) \) are satisfied. In most of the allowed parameter regions, \( \xi = \mathcal{O}(1) \) is obtained. Thus, one does not require tight tunings in these simplified scenarios. In the figures, \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \) is smaller than the SM value by more than 30%. On the other hand, \( \mathcal{B}(K_0 \to \pi^+\nu\bar{\nu}) \) on the scenarios. In LHS, we obtain 0 < \( \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})/\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\text{SM}} < 1.8 \). In RHS, \( \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \) is comparable to or larger than the SM value, but cannot be twice as large. In \( \text{ImZS} \), the branching ratios are perfectly correlated and \( \epsilon_\nu \) from \( \text{Z} \) produces different correlations between two NP scenarios. In the future, measurements of \( \mathcal{B}(K \to \pi^+\nu\bar{\nu}) \) will be significantly improved. The NA62 experiment at CERN measuring \( \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \) is aiming to reach a precision of 10% compared to the SM value already in 2018. In order to achieve 5% accuracy more time is needed. Concerning \( K_L \to \pi^0\nu\bar{\nu} \), the KOTO experiment at J-PARC aims in a first step at measuring \( K_{2018} \). In order to achieve 5% accuracy more time is needed. Concerning \( K_L \to \pi^0\nu\bar{\nu} \), the KOTO experiment at J-PARC aims in a first step at measuring \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \) in the box dominated scenario and the \( Z \)-penguin dominated one. It is shown that the constraint from \( \epsilon_K \) produces different correlations between two NP scenarios. In the future, measurements of \( \mathcal{B}(K \to \pi^+\nu\bar{\nu}) \) will be significantly improved. The NA62 experiment at CERN measuring \( \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \) is aiming to reach a precision of 10% compared to the SM value already in 2018. In order to achieve 5% accuracy more time is needed. Concerning \( K_L \to \pi^0\nu\bar{\nu} \), the KOTO experiment at J-PARC aims in a first step at measuring \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \) around the SM sensitivity. Furthermore, the KOTO-step2 experiment will aim at 100 events for the SM branching ratio, implying a precision of 10% of this measurement. Therefore, we conclude that when the \( \epsilon'_K/\epsilon_K \) discrepancy is explained by the NP contribution, NA62 experiment could probe whether a modified \( Z \)-coupling scenario is realized or not, and KOTO-step2 experiment can distinguish the box dominated scenario and the simplified modified \( Z \)-coupling scenario.

4 Conclusions

We have presented the correlations between \( \epsilon'_K/\epsilon_K, \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \), and \( \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \) in the box dominated scenario and the \( Z \)-penguin dominated one. It is shown that the constraint from \( \epsilon_K \) produces different correlations between two NP scenarios. In the future, measurements of \( \mathcal{B}(K \to \pi^+\nu\bar{\nu}) \) will be significantly improved. The NA62 experiment at CERN measuring \( \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \) is aiming to reach a precision of 10% compared to the SM value already in 2018. In order to achieve 5% accuracy more time is needed. Concerning \( K_L \to \pi^0\nu\bar{\nu} \), the KOTO experiment at J-PARC aims in a first step at measuring \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \) around the SM sensitivity. Furthermore, the KOTO-step2 experiment will aim at 100 events for the SM branching ratio, implying a precision of 10% of this measurement. Therefore, we conclude that when the \( \epsilon'_K/\epsilon_K \) discrepancy is explained by the NP contribution, NA62 experiment could probe whether a modified \( Z \)-coupling scenario is realized or not, and KOTO-step2 experiment can distinguish the box dominated scenario and the simplified modified \( Z \)-coupling scenario.

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