Moving medium electrodynamics approach for description of the electric and magnetic static polarizable properties of the nucleon at low energy.

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Abstract

Using the relativistic electrodynamics of continuous media formalism and main relativistic quantum field theory principles the covariant Lagrangian of electromagnetic field interaction with polarizable 1/2-spin particles have been obtained. This Lagrangian let us to determine canonical and metric energy-momentum tensors as well as low-energy Compton scattering amplitude. The application of this Lagrangian for the calculation of the radiative correction to the imaginary part of double virtual Compton scattering is demonstrated.

1 Introduction

At present time, the description of the Compton scattering off hadrons is performed by nonrelativistic Hamiltonian function [1, 2]. However for the extraction of the more essential experimental and theoretical information about hadron polarizabilities it is necessary to develop the Lagrangian of electromagnetic field interaction with polarizable particle in covariant form.

The such Lagrangian was constructed by the phenomenological formfactor-based approach in ref. [3]. In the present report for the construction of the similar Lagrangian developed in ref. [4] approach is used in more consistently and completely way that allows us to determine not only the Lagrangian itself but the energy-momentum tensor of electromagnetic field interaction with polarizable particles as well as to take into account introduced in ref. [4, 5] spin polarizabilities of hadrons.

One of the most interesting topic in Compton scattering investigation is the measurement of $Q^2$-dependence of the forward polarizabilities [6] that can be presented as the imaginary part of doubly virtual Compton scattering (VVCS) amplitude [7].

Performing a such kind of the experiments it will be also important to take into account radiative effects correctly. Notice that FORTRAN codes have been already developed for

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estimation of the radiative effects from lepton legs in elastic (MASCARAD [8]) and deep-inelastic (POLRAD [9]) scattering. Basing on these codes Monte Carlo generators for simulation of radiative events in elastic (ELRADGEN [10]) and deep-inelastic (RADGEN [11]) scattering was developed.

Another interesting source of radiative effect contributed to the imaginary part of VVCS amplitude that considered in the present report is photon emission from nucleon line. Using the standard Feynman technique on the lowest order this process can be presented in the frame of the static polarizability contributions.

2 Lagrangian

The responsible for the interaction of electromagnetic field with a structural particle Lagrangian we present in a following way:

\[ \mathcal{L}_I = -j^{(M)}_\mu(x)A^\mu(x), \]  

where \( j^{(M)}_\mu \) is the current density due to a motion of constituents, \( A^\mu \) is electromagnetic field four-dimensional potential.

The current density \( j^{(M)}_\mu(x) \) have to satisfy conservation law

\[ \partial^\mu j^{(M)}_\mu = 0, \]

that allows us to to present it in the form

\[ j^{(M)}_\mu = -\partial^\rho G^I_{\rho\mu}, \]

where \( G^I_{\rho\mu} \) is an antisymmetric tensor.

Let us introduce the relativistic generalization of the electric

\[ D_\nu = G^I_{\nu\rho} U^\rho \]  

and magnetic

\[ M^\sigma = \varepsilon^{\sigma\rho\kappa\mu} G^I_{\rho\kappa} U^\mu \]

dipole moment of the system. In the expressions (2) and (3) four-dimensional velocity of structural particle \( U \) has components

\[ U^\mu \{ \gamma, \mathbf{v} \gamma \}, \]

where \( \gamma = \frac{1}{\sqrt{1 - v^2}} \), and \( \mathbf{v} \) is moving media velocity.

In the rest system of the structural system the vectors \( D_\nu \) and \( M^\sigma \) can be presented as a multiple expansion concerning the center of gravity [12]

\[ D_k = \sum_{L=1}^{\infty} (-1)^{L-1} Q^{(L)}_{k_1...n} \partial_{l_1}...\partial_{l_n} \delta(\vec{x}), \]

\[ M_k = \sum_{L=1}^{\infty} (-1)^{L-1} M^{(L)}_{k_1...n} \partial_{l_1}...\partial_{l_n} \delta(\vec{x}), \]

where \( k, l = 1, 2, 3; \) \( Q^{(L)} \) and \( M^{(L)} \) are multiple tensors, which determined from a displacement of constituents with regard to center of gravity.
Directly from (2) and (3) we can find that $U^{\nu}D_{\nu} = 0$, $U^{\nu}M_{\nu} = 0$.

Taking into account the space inversion one can construct antisymmetric tensor $G_{\mu\nu}^{I}$ decomposed over $U^{\nu}$, $D^{\nu}$ and $M^{\nu}$ vectors in a following way:

$$G_{\mu\nu}^{I} = U_{\mu}D_{\nu} - U_{\nu}D_{\mu} + \varepsilon_{\mu\nu\rho\sigma}U^{\rho}M^{\sigma}. \quad (4)$$

From the other hand the effective Lagrangian (1) we can present as

$$\mathcal{L}_{I} = -j_{\mu}^{(M)}A^{\mu} = (\partial^{\rho}G_{\rho\mu}^{I})A^{\mu}. \quad (5)$$

To avoid ambiguity of the Lagrangian we set

$$\mathcal{L}_{I} = -\frac{1}{2}G_{\mu\nu}^{I}F^{\mu\nu}, \quad (5)$$

or by taking (4) into account

$$\mathcal{L}_{I} = -\frac{1}{2}(D_{\mu}e^{\mu} - M_{\mu}h^{\mu}), \quad (6)$$

where $e^{\mu}$, $h^{\mu}$ are express via the electromagnetic field tensor as $e^{\mu} = F^{\mu\nu}U_{\nu}$, $h^{\mu} = \tilde{F}_{\mu\nu}U_{\nu}$.

In the situation when

$$D_{\mu} = 4\pi\alpha_{\mu\nu}e^{\nu}, \quad (7)$$

$$M_{\mu} = 4\pi\beta_{\mu\nu}h^{\nu}, \quad (8)$$

and tensors $\alpha^{\mu\nu}$ and $\beta^{\mu\nu}$ are represented in the diagonal form by using metric tensor $g^{\mu\nu}$ as $\alpha^{\mu\nu} = g^{\mu\nu}\alpha_{0}'$, $\beta^{\mu\nu} = g^{\mu\nu}\beta_{0}'$, the Lagrangian (6) can be split into two parts:

$$\mathcal{L} = \mathcal{L}_{0} + \mathcal{L}_{I}, \quad (9)$$

where

$$\mathcal{L}_{0} = -\frac{1}{2}(e^{2} - h^{2}) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}F^{2} \text{ and}$$

$$\mathcal{L}_{I} = -2\pi \left[ (\alpha_{0}' - \beta_{0}')e^{2} + \frac{\beta_{0}'}{2}F^{2} \right]. \quad (10)$$

Inserting the Lagrangian (9) into the Euler-Lagrange equation of motion

$$\partial_{\mu}\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}A_{\nu})} - \frac{\partial \mathcal{L}}{\partial A_{\nu}} = 0 \quad (11)$$

one can find

$$\partial_{\mu}F^{\mu\nu} = j_{\mu}^{(M)} = -\partial_{\mu}G^{I\mu\nu}, \quad (12)$$

where

$$G^{I\mu\nu} = 4\pi \left[ (\alpha_{0}' - \beta_{0}')e^{\mu}U^{\nu} - e^{\nu}U^{\mu} + \beta_{0}'F^{\mu\nu} \right]. \quad (13)$$

From the equations (12) for media in rest the definitions of charge and current densities for bound charges are follow

$$\rho^{(M)} = -4\pi\alpha_{0}'(\nabla \mathbf{E}) = -4\pi\alpha_{0}' \text{ div } \mathbf{E}, \quad (14)$$

$$j^{(M)} = 4\pi(\alpha_{0}'\partial_{\nu}\mathbf{E} - \beta_{0}' \text{ rot } \mathbf{H}), \quad (15)$$
while the Maxwell’s equations for media have the form:

\[
\text{rot } \mathbf{E} = -\partial_t \mathbf{H}, \quad \text{div } \mathbf{H} = 0, \\
\text{rot } \mathbf{B} = \partial_t \mathbf{D}, \quad \text{div } \mathbf{D} = 0,
\]

(16)

where \( \mathbf{D} = \hat{\varepsilon} \mathbf{E} \) and \( \mathbf{B} = \hat{\mu} \mathbf{H} \) are vectors of electric and magnetic induction respectively.

The equations (14) and (15) for media in rest can be presented in the covariant form by the introduction of the following polarizability tensor of media [13]

\[
\mathbf{\tilde{M}} = \begin{pmatrix} 0 & -\mathbf{P} \\ \mathbf{P} & \mathbf{M} \end{pmatrix},
\]

(17)

where \( \mathbf{P} \) and \( \mathbf{M} \) are electric and magnetic polarizability vectors of media. By the definition they are dipole moments of volume scale.

As a result the charge and current densities transform into

\[
\mathbf{j}^{(M)} = \partial_t \mathbf{P} - \text{rot } \mathbf{M}, \quad \rho^{(M)} = -\text{div } \mathbf{P},
\]

(18)

while the equation of motion reads as

\[
\partial_{\mu}(F_{\mu\nu} + G^{I\mu\nu}) = \mathbf{j}^{\nu}
\]

(19)

for moving media and

\[
\partial_{\mu}(F_{\mu\nu} + M^{I\mu\nu}) = \mathbf{j}^{\nu},
\]

(20)

for media in rest. Here \( M^{I\mu\nu} \) is the polarizability tensor of medium (17) and \( \mathbf{j}^{\nu} \) is the current density of free charges.

By introduction of the tensor [12, 14, 15]

\[
G^{I\mu\nu} = d^{\mu} U^{\nu} - d^{\nu} U^{\mu} + \varepsilon^{\mu\nu\rho\sigma} U_{\rho} b_{\sigma},
\]

(21)

with \( d^{\mu} = \varepsilon^{\mu\sigma} e_{\sigma}, \quad b_{\rho} = \mu_{\rho\sigma} h^{\sigma} \), the Lagrangian (2) for moving media can be presented as

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} G^{\mu\nu} = -\frac{1}{2} (\hat{\varepsilon} \hat{\varepsilon} e - h \hat{\mu} h),
\]

(22)

where \( \hat{\varepsilon} = I + 4\pi \hat{\alpha}, \quad \hat{\mu} = I + 4\pi \hat{\beta} \).

### 3 The energy-momentum tensor and equation of motion

The Lagrangian (2) helps us to determine the canonical energy-momentum tensor. Indeed from the Noether’s theorem follows

\[
T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\rho)} (\partial_\nu A^\rho) - g^{\mu\nu} \mathcal{L}.
\]

(23)

Inserting (22) into (23) one can find that

\[
T^{\mu\nu} = -G^{\mu\rho} (\partial_\nu A_\rho) - g^{\mu\nu} \frac{1}{4} (F_{\rho\sigma} G^{\rho\sigma}).
\]

(24)
Now we determine the metric energy-momentum tensor as
\[ \tilde{T}^{\mu\nu} = -G^{\mu\rho}(\partial^\nu A_\rho) - g^{\mu\nu}L + \partial_\mu(A^\nu G^{\mu\rho}). \] (25)

After some recombination of (25) using the equation of motion (19) we find
\[ \tilde{T}^{\mu\nu} = F_\rho^{\nu\mu}G^{\mu\rho} + \frac{1}{4}\delta^{\mu\nu}(F_{\rho\sigma}G^{\rho\sigma}). \] (26)

From the relation (26) one could find the energy density for media in rest
\[ \tilde{T}^{00} = \omega = \frac{1}{2}(\varepsilon E^2 + \mu H^2). \] (27)

Now using the correspondence principle [16, ?] let us consider the quantum-mechanical
description of the electromagnetic field interaction with polarizable particles. The electric
and magnetic polarizabilities extracted from the Lagrangian (22) can be presented in the
form:
\[ \mathcal{L}_I = -2\pi(\alpha^0_0 F_\mu F_\sigma^\mu - \beta^0_0 \tilde{F}_\mu \tilde{F}_\sigma^\mu)U^\rho U^\sigma, \] (28)

where \( \tilde{F}_\mu = \frac{1}{2}\varepsilon_{\mu\rho\sigma\kappa}F^{\rho\sigma\kappa} \), \( \varepsilon_{0123} = -1. \)

To pass on from the Lagrangian (28) to the Lagrangian of electromagnetic field interaction
with polarizable 1/2-spin particle we perform a following transition based on the
correspondence principle between classical and quantum mechanics. First of all instead
of \( P_\mu \) we introduce a momentum operator acting on the wave function of particle \(-i\partial_\mu \psi\).

Taking into account that \( \bar{\psi}\gamma^\mu \psi \) is a current density of particles, making symmetrization
of operators and providing hermiticity requirement, relativistic and gauge invariance the
Lagrangian of electromagnetic field interaction with polarizable 1/2-spin particle whose
mass is equal to \( M \) can be presented as:
\[ \mathcal{L}_I = \frac{2\pi}{M} \left[ \alpha_0 F_\mu F_\sigma^\mu - \beta_0 \tilde{F}_\mu \tilde{F}_\sigma^\mu \right] \bar{\Theta}^{\rho\sigma}, \] (29)

where \( \bar{\Theta}^{\rho\sigma} = \frac{1}{2}(\Theta^{\rho\sigma} + \Theta^{\sigma\rho}) \) is the energy-momentum tensor of spinor field and:
\[ \Theta^{\rho\sigma} = \frac{i}{2}\bar{\psi}\gamma^\rho \gamma^\sigma \psi. \] (30)

The polarizabilities \( \alpha_0 \) and \( \beta_0 \) introducing in the expression (29) are used in hadronic
phyisc and connect with the polarizabilities of expression (28) as
\[ \alpha'_0 = \rho \alpha_0, \quad \beta'_0 = \rho \beta_0, \]

where \( \rho \) is the density of particles.

To verify that the Lagrangian (29) is correct it is sufficient to define Hamiltonian
for interaction of electromagnetic field with polarizable particle and Compton scattering amplitude off this particle.

Now to define the moving of the charged, polarizable, spinor particle in the electro-
magnetic field write out the total Lagrangian in the following form:
\[ \mathcal{L} = \frac{i}{2} \bar{\psi} \gamma^{\mu} \partial_\mu \psi - M\bar{\psi}\psi - e\bar{\psi}A^\mu \psi - \frac{1}{4}F^2 - \frac{1}{4}F_{\mu\nu}G^{(S)\mu\nu}. \] (31)

Then the tensor (13) is determined by
\[ G^{(S)\mu\nu} = -\frac{4\pi}{M} \left\{ (\alpha_0 - \beta_0) \left[ F^{\mu\sigma} \bar{\Theta}^\nu_\sigma - F^{\nu\sigma} \bar{\Theta}^\mu_\sigma \right] + \beta_0 \bar{\Theta}^{\rho\sigma} F_{\rho\mu} \right\}. \] (32)
Using the Lagrangian (31) and antisymmetric tensor (32) the metric momentum-energy tensor we shall define as
\[
\bar{T}^{\mu\nu} = \bar{\Theta}^{\mu\nu} + F_\rho^{\ \nu} F^{\mu\rho} + \frac{1}{4} g^{\mu\nu} F^2 - \frac{e}{2} \bar{\psi} (\gamma^\mu A^\nu + \gamma^\nu A^\mu) \psi + \bar{T}_I^{\mu\nu},
\]
where
\[
\bar{T}_I^{\mu\nu} = F_\rho^{\ \nu} G_1^{(S)\mu\rho} + \frac{1}{4} g^{\mu\nu} (F_\rho G_1^{(S)\mu\rho}).
\]
One can see from the expression (34) that for the particle in a rest its interaction energy appearing from polarizability will be have the form [2]
\[
H_I = -2\pi (\alpha_0 B^2 + \beta_0 H^2).
\]
The equation of motion that follows from the Lagrangian (31) is given by
\[
\partial_\mu F^{\mu\nu} = e \bar{\psi} \gamma^\nu \psi + j^{(M)}\nu,
\]
where \(j^{(M)}\nu = -\partial_\mu G^{(S)\mu\nu}\).

Taking into account (35) the scattering amplitude within second order over photon energy will give the contribution for electric and magnetic polarizabilities as
\[
T_{\text{pol}}^{\mu\nu\mu\nu} = \frac{8\pi M \omega}{N(t)} \left[ e^* \cdot e \alpha_0 + s^* \cdot s \beta_0 \right],
\]
where \(s = n \times e\); \(s^* = n' \times e'\), where \(n\) and \(e\) are the polarization vectors, \(\omega_1\) and \(\omega_2\) are the energies of the incident and scattered photons, \(n = k/|k|, n' = k'/|k'|\).

### 4 Static polarizability vertex

For application of the obtained Lagrangian to the calculations in quantum field theory it is necessary to define the vertex of photon-nucleon interaction. Following the notations of Appendix B of [17], our Lagrangian (29) can be presented as:
\[
L^{\text{pol}}_{\text{eff}} = \int \prod_{i=1}^4 \left[ d^4 x_i \delta^4 (x - x_i) \right] \alpha_{\sigma\delta}^{r'r'} (x_1, x_2, x_3, x_4) \bar{\psi} (x_3) \psi (x_1) \times A^\sigma (x_4) A^\delta (x_2),
\]
where \(r'r'\) are the four-spinor indexes (that usually dropped). Taking into account that
\[
\alpha_{\sigma\delta}^{r'r'} (x_1, x_2, x_3, x_4) = \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \tilde{\alpha}_{\sigma\delta}^{r'r'} (p_1, q_1, p_2, q_2) \times \exp \left( i (p_1 x - x_1) + q_1 (x - x_2) - p_2 (x - x_3) - q_2 (x - x_4) \right),
\]
where \(p_1\) and \(q_1\) (\(p_2\) and \(q_2\)) are the incoming (outgoing) nucleon and photon momenta respectively (see Fig. 1) it is easy to show that
\[
\tilde{\alpha}_{\sigma\delta}^{r'r'} (p_1, q_1, p_2, q_2) = -\frac{\pi}{M} (p_1^r \gamma_{\rho}^r (\alpha_0 - \beta_0) [q_2^\rho \delta_\rho^\mu - q_2^\mu \delta_\rho^\rho] (q_1^\rho g_\rho^\delta - q_1^\delta g_\rho^\rho) \delta_\rho^\sigma - \frac{1}{2} \delta_\rho^\sigma \beta_0 [q_2^\rho \delta_\rho^\sigma - q_2^\sigma \delta_\rho^\rho] (q_1^\rho g_\rho^\delta - q_1^\delta g_\rho^\rho)).
\]
Figure 1: Polarizability vertex

Adding crossing symmetry state and multiply the result on \(i\) we receive the final expression for polarizability vertex that presented on Fig. 1

\[
\Gamma_{\sigma \delta}^{\text{pol}}(p_1, q_1, p_2, q_2) = i(\tilde{\alpha}_{\sigma \delta}(p_1, q_1, p_2, q_2) + \tilde{\alpha}_{\delta \sigma}(p_1, -q_2, p_2, -q_1)).
\] (41)

Here we dropped the four-spinor indexes. At the end of this section we show the following convolution:

\[
q_2^\sigma \Gamma_{\sigma \delta}^{\text{pol}}(p_1, q_1, p_2, q_2) = 0, \quad q_1^\delta \Gamma_{\sigma \delta}^{\text{pol}}(p_1, q_1, p_2, q_2) = 0
\] (42)

5 Static polarizabilities contributions to VVCS

The lowest order contribution of static polarizabilities to imaginary parts of VVCS amplitudes and the hadronic tensor of inclusive DIS as well are presented by Feynman graphs on Fig. 2. It should be noticed that contribution from Fig. 2 (e) is negligible (~ \(10^{-8}\) fm\(^6\)) and can be dropped. Performing cut over dash line on Fig. 2 (a-d) these graph contributions to the imaginary part of amplitude can be presented as

\[
\text{Im} \, T = \pi^2 \epsilon^*_\nu(q) \epsilon_\mu(q) \int d\Theta \bar{u}(p) \left[ \frac{\Gamma_\mu \nu VVCS \alpha^a}{(p + q)^2 - M^2} + \frac{\Gamma_\mu \nu VVCS \alpha^c}{(p - k)^2 - M^2} \right] u(p)
\] (43)

where

\[
\Gamma_\mu \nu VVCS \alpha^a = \Gamma^{\text{pol}} \nu \alpha(p + q - k, k, p, q)(\hat{p} + \hat{q} - \hat{k} + M)\Gamma^\text{el}_\alpha(-k) \\
\times (\hat{p} + \hat{q} + M)\Gamma^\text{el}_\mu(q),
\]

\[
\Gamma_\mu \nu VVCS \alpha^b = \Gamma^\text{el}_\nu(-q)(\hat{p} + \hat{q} + M)\Gamma^\text{el}_\alpha(k)(\hat{p} + \hat{q} - \hat{k} + M) \\
\times \Gamma^{\text{pol}} \alpha \mu(p, q, p + q - k, k),
\]

\[
\Gamma_\mu \nu VVCS \alpha^c = \Gamma^{\text{pol}} \nu \alpha(p + q - k, k, p, q)(\hat{p} + \hat{q} - \hat{k} + M)\Gamma^\text{el}_\mu(q) \\
\times (\hat{p} - \hat{k} + M)\Gamma^\text{el}_\alpha(-k),
\]

\[
\Gamma_\mu \nu VVCS \alpha^d = \Gamma^\text{el}_\alpha(k)(\hat{p} - \hat{k} + M)\Gamma^\text{el}_\nu(-q)(\hat{p} + \hat{q} - \hat{k} + M) \\
\times \Gamma^{\text{pol}} \alpha \mu(p, q, p + q - k, k),
\] (44)

and \(\Gamma_\mu^\text{el}(q) = -ie (F_D(-q^2)\gamma_\mu + F_P(-q^2)i\sigma_{\mu \alpha}q^\alpha / 2M)\) is the usual elastic vertex.

The phase space has a form:

\[
d\Theta = \frac{d^4k}{(2\pi)^4} \delta(k^2) \delta((p + q - k)^2 - M^2) = \frac{(S_x - Q^2)d\tau d\phi_k}{64\pi^4 \tau^2 \sqrt{\lambda_q}} = \frac{(S_x - Q^2)d\tau}{32\pi^3 \tau^2 \sqrt{\lambda_q}}.
\] (45)
where we integrate over an azimuthal photonic angle $\phi_k$. The invariants have a standard form:

$$Q^2 = -q^2, \quad S_x = 2p \cdot q, \quad \lambda q = S_x^2 + 4M^2 Q^2, \quad \tau = \frac{k \cdot (p + q)}{k \cdot p}. \quad (46)$$

The limits of integration over $\tau$ read:

$$\tau_{\text{max/min}} = 1 \pm \frac{S_x \sqrt{\lambda q}}{2M^2}. \quad (47)$$

The contribution of the presented above amplitude to the hadronic tensor reads:

$$W^{VVCS}_{\mu\nu} = \frac{1}{2M^2 \pi} \text{Tr}[T_{\mu\nu}^{VVCS}] = \left( g_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2} \right) \text{Im} T_1$$

$$+ \frac{1}{p \cdot q} \left( p_{\mu} - \frac{p \cdot q}{q^2} q_{\mu} \right) \left( p_{\nu} - \frac{p \cdot q}{q^2} q_{\nu} \right) \text{Im} T_2$$

$$+ \frac{i}{M} \varepsilon^{\mu\nu\alpha\beta} q_{\alpha} \eta_{\beta} \text{Im} S_1 + \frac{i}{M} \varepsilon^{\mu\nu\alpha\beta} q_{\alpha} (p \cdot q \eta_{\beta} - \eta \cdot q p_{\beta}) \text{Im} S_2. \quad (48)$$

Using the standard projection operator technique as well as the relations between the imaginary parts of amplitudes and absorption cross sections:
where \( \nu = S_x/2M \) is a lab virtual photon energy, we can extract \( K\sigma_T, K\sigma_L, K\sigma_{TT} \) and \( K\sigma_{LT} \).

Taking into account the asymptotic limit

\[
\lim_{\nu \gg Q,M} K\sigma_T = K\sigma_T^\nu = \frac{8}{3M} \alpha_{QED}\pi^2(\alpha_0 - \beta_0)\nu^3 F_{P}(Q^2) F_{P}(0),
\]

\[
\lim_{\nu \gg Q,M} K\sigma_L = 0,
\]

\[
\lim_{\nu \gg Q,M} K\sigma_{TT} = K\sigma_{TT}^\nu = \frac{40}{3M} \alpha_{QED}\pi^2(\alpha_0 - \beta_0)\nu^3 F_{P}(Q^2) F_{P}(0),
\]

\[
\lim_{\nu \gg Q,M} K\sigma_{LT} = K\sigma_{LT}^\nu = \frac{2}{3M} \alpha_{QED}\pi^2\nu^2 Q(3F_D(Q^2) - 2F_P(Q^2))\beta_0 - 3(F_D(Q^2) - 2F_P(Q^2))\alpha_0) F_P(0) - 12(\alpha_0 - \beta_0) F_P(Q^2) F_D(0),
\]

we can define the contribution of \( \alpha_0 \) and \( \beta_0 \) to the forward polarizabilities in a following way:

\[
\alpha(Q^2) + \beta(Q^2) = \frac{1}{2\pi^2} \int_0^\infty K \hat{\sigma}_T \frac{d\nu}{\nu^4}, \quad \alpha_L(Q^2) = \frac{1}{2\pi^2} \int_0^\infty K \hat{\sigma}_L \frac{d\nu}{\nu^3},
\]

\[
\gamma_0(Q^2) = \frac{1}{2\pi^2} \int_0^\infty K \hat{\sigma}_{TT} \frac{d\nu}{\nu^4}, \quad \delta_{LT}(Q^2) = \frac{1}{2\pi^2} \int_0^\infty K \hat{\sigma}_{LT} \frac{d\nu}{\nu^3 Q}, \quad (51)
\]

where \( K \hat{\sigma}_T = K(\sigma_T - \sigma_T^\nu) \), \( K \hat{\sigma}_{TT} = K(\sigma_{TT} - \sigma_{TT}^\nu) \) and \( K \hat{\sigma}_{LT} = K(\sigma_{LT} - \sigma_{LT}^\nu) \). \( Q^2 \) dependence of these polarizabilities for the proton are presented on Fig. 3.

### 6 Conclusion

On the basis of the relativistic electrodynamics of continuous media formalism and main relativistic quantum field theory principles the covariant Lagrangian of electromagnetic field interaction with polarizable 1/2-spin particles have been obtained. This Lagrangian let us to determine canonical and metric energy-momentum tensors as well as low-energy Compton scattering amplitude.

The present above Lagrangian was apply for calculation of the static polarizability contribution to generalized forward polarizabilities in VVCS. Performed numerical analysis shows that:

- the electric \( \alpha_0 \) and magnetic \( \beta_0 \) static polarizabilities contribute not only to \( \alpha(Q^2) + \beta(Q^2) \) but to \( \gamma_0(Q^2) \) and \( \delta_{LT}(Q^2) \) as well;

- the behavior of \( \alpha_0 \) and \( \beta_0 \) contribution to \( \alpha(Q^2) + \beta(Q^2) \) and \( \alpha_L(Q^2) \) agrees with the results obtained in other channels and presented by [7];
Figure 3: $\alpha_0$ and $\beta_0$ contribution to $Q^2$ dependence of the generalized forward polarizabilities of the proton

- the behavior of $\alpha_0$ and $\beta_0$ contribution to $\gamma_0(Q^2)$ and $\delta_{LT}(Q^2)$ are disagree only at low $Q^2$ with the results obtained in other channels and presented by [7];
- similar to [7] in real photon limit $Q^2 \rightarrow 0$ we found that $\alpha_L(0) = 0$ while $\delta_{LT}(0) \neq 0$.

In order to estimate the static polarizabilities contribution to VVCS correctly it is necessary to consider $\gamma_{E1,E2,M1,M2}$ contribution too. Now basing on the corresponding principle between the moving medium electrodynamic and quantum field theory the Lagrangian with this interaction is under construction.

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References

[1] S. Scherer, A.Yu. Korchin, J.H. Koch, Phys. Rev. C54, 904 (1996).
[2] D. Babusci, J. Jiordano, A.J. L’vov, J. Matone, A.N. Nathan, Phys. Rev. C58, 1013 (1998).
[3] A.I. L’vov, Int. J. Mod. Phys. A., 145 (1993).
[4] N.V. Maksimenko, L.G. Moroz // Proc. 11 Int. School on High Energy Physics and Relativistic Nucl. Phys. (D2-11707) Dubna, 533 (1979).
[5] S. Ragusa, Phys. Rev. D47, 3757 (1993).
[6] By Jefferson Lab E94010 Collaboration (M. Amarian et al.), Phys.Rev.Lett. 93, 152301 (2004); E97110 Collaboration
[7] D. Drechsel, B. Pasquini, M. Vanderhaeghen, Phys. Rept. 378, 99 (2003)
[8] A. Afanasev, I. Akushevich, N. Merenkov, Phys.Rev.D64:113009,2001.
[9] I. Akushevich, A. Ilyichev, N. Shumeiko, A. Soroko, A. Tolkachev, Comput.Phys.Commun.104:201-244,1997
[10] A. Afanasev, I. Akushevich, A. Ilyichev, Czech.J.Phys.53:B449-B454,2003.
[11] I. Akushevich, H. Bottcher, D. Ryckbosch, hep-ph/9906408
[12] J. D. Jackson, Classical Electrodynamics, Wiley & Sons, New York, 1975.
[13] F.I. Fedorov, Gyrotropy theory. (Minsk, 1973) (in Russian).
[14] B.V. Bokut’, A.N. Serdukov, Jour. of Appl. Spectr. 11 N4, 704 (1969) (in Russian).
[15] G. Preti, Phys. Rev. D70, 024912 (2004).
[16] H. Weyl, The theory of groups and quantum mechanics. (Dover, 1950).
[17] T.-P. Cheng, L.-F. Li, Gauge theory of elementary particle physics, Oxford (1984)