Placement and Implementation of Grid-Forming and Grid-Following Virtual Inertia and Fast Frequency Response

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Abstract—The electric power system is witnessing a shift in the technology of generation. Conventional thermal generation based on synchronous machines is gradually being replaced by power electronics interfaced renewable generation. This new mode of generation, however, lacks the natural inertia and governor damping which are quintessential features of synchronous machines. The loss of these features results in increasing frequency excursions and, ultimately, system instability. Among the numerous studies on mitigating these undesirable effects, the main approach involves virtual inertia emulation to mimic the behavior of synchronous machines. In this work, explicit models of grid-following and grid-forming virtual inertia (VI) devices are developed for inertia emulation in low-inertia systems. An optimization problem is formulated to optimize the parameters and location of these devices in a power system to increase its resilience. Finally, a case study based on a high-fidelity model of the South-East Australian system is used to illustrate the effectiveness of such devices.

I. INTRODUCTION

The past decade has seen a concerted focus on alternate sources of energy to replace conventional synchronous machine-based generation. A majority of the concerns forcing such a shift—namely greenhouse emissions, safety of nuclear generation and waste disposal, etc., are effectively addressed by cleaner alternatives, primarily-wind turbines and photovoltaics. These sources are interfaced by means of power electronic converters. Their large-scale integration, however, has raised concerns [1]–[3] about system stability and especially frequency stability [4]–[6]. The inherent rotational inertia [7]–[9] of the synchronous machines and the damping provided by governors assures system stability in the event of faults such as loss of generators, sudden fluctuation in power injections due to variable renewable sources, tie line faults, system splits, loss of loads, etc. In case of a frequency deviation, the inertia of synchronous machines acts as a first response by providing kinetic energy to the system (or absorbing energy). In contrast, converter interfaced generation fundamentally offers neither of these services, thus, making the system prone to instability.

Several studies have been carried out to propose control techniques to mitigate this loss of rotational inertia and damping. One extensively studied technique relates to using power electronic converters to mimic synchronous machine behavior [10]–[13]. These methods rely on concepts ranging from simple proportional-derivative to more complex controls under the name of, e.g., Virtual Synchronous Generators. All these strategies rely on some form of energy storage such as batteries, super-capacitors, flywheels, or the residual kinetic energy of wind turbines [14], which acts as a substitute for the kinetic energy of machines.

These investigations have established the efficacy of virtual inertia (VI) and fast frequency response (FFR), i.e., primary frequency control without turbine delay, as a short-term replacement for low-inertia power systems. Also, as power converters operate at a much faster time scales compared to conventional generation, it is plausible to foresee future power systems based on predominantly converter-interfaced generation, without a major distinction between different time-scale controls such as inertia and fast frequency response, and primary frequency control provided by synchronous machines [9], [15], [16]. Here, we exclusively focus on power systems with reduced inertia due to loss of synchronous machines and utilize virtual inertia and fast primary frequency control as a remedy.

Conventionally, the total inertia and primary frequency control in the system are the main metrics utilized for system resilience analysis [3]. However, the authors in [17] showed that not only is virtual inertia and primary frequency control vital, but its location in the power system is equally crucial and there can be a degradation in the performance due to ill-conceived spatial inertia distributions [18], even if the total virtual inertia added to the power system is identical. Other commonly used performance metrics to quantify power system robustness include frequency nadir, RoCoF (Rate of change of frequency), and power system damping ratio [19]. In the literature, the problem of optimally tuning and placing the virtual inertia and primary frequency controllers based on system norms [17], [20]–[22] has been explored for small-scale test cases with linear models. In [23]–[25] time-domain and spectral metrics such as RoCoF, nadir, and damping ratios are considered. In [23], [24] a sequential linear programming approach is used to optimize the allocation of grid-following virtual inertia and primary frequency control. This method directly optimizes the frequency nadir and RoCoF. In [25] the power system is reduced to a single swing equation with first order turbine dynamics and the damping ratio and peak overshoot are optimized subject to an economic cost.

As contributions, this paper develops explicit models of
converter-based virtual inertia devices that capture the key dynamic characteristics of phase-locked loops (PLLs) used in grid-following virtual inertia devices and of grid-forming controls such as virtual synchronous machines, droop control, and machine matching control, that can be used to provide virtual inertia. In addition, these models are suitable for integration with large-scale, non-linear power system models, thus allowing for parameter tuning through tractable optimization problems.

Moreover, the applicability of system norms as a performance metric for power system analysis is established beyond the prototypical swing equation, by considering detailed models. To this end, we propose a computationally efficient \( H_2 \) norm based algorithm to optimally tune the parameters and the placement of the VI devices in order improve the resilience of low-inertia power systems. The key idea of this algorithm is to exploit the interpretation of VI devices as feedback controllers. Though the algorithm is applicable for a broader class of services offered by power electronic devices, we concentrate our analysis on virtual inertia and fast frequency response.

Finally, a high-fidelity model of the South-East Australian power system is modified to replicate a low-inertia scenario and used for an extensive case study. This modified system is augmented with VI devices to study their impact on system stability and validate the optimal tuning obtained by applying the proposed optimization algorithm. Moreover, through extensive simulations, we validate the linearized models used in the \( H_2 \) optimization algorithm and study the impact of both grid-forming and grid-following virtual inertia on the disturbance responses of the non-linear power system. Lastly, time-domain simulations are presented to study system stability and to compare the response of grid-following and grid-forming virtual inertia in detail.

The remainder of the paper is structured as follows: In Section II the power system model, converter models in both grid-following and grid-forming implementations are presented. The key performance metrics for grid stability and design constraints are identified and suitably defined in Section III. In Section IV a computational approach to identify the location of the inverters to improve post-fault response of the low-inertia power systems is proposed. In Section V the low-inertia model based on the South-East Australian system is presented. A case study based on the two implementations of virtual inertia is presented in Section VI and suitable metrics are investigated to quantify the improvements in system stability. Finally, Section VII concludes the paper.

II. SYSTEM MODEL

We consider a high-fidelity, non-linear power system model, consisting of synchronous machines with governors, automatic voltage regulators (AVR), power system stabilizers (PSS), constant impedance and constant power loads, and renewable generation that is abstracted by constant power sources on the time scales of interest. The dynamic model of the power system is given by a differential-algebraic equation

\[
\dot{x}_s = f_s(x_s, z_s), \tag{1a}
\]

\[
0 = g_s(x_s, z_s, i), \tag{1b}
\]

where \( 0 \) is a vector of zeros, \( x_s \in \mathbb{R}^{n_x} \) is the state vector and contains (but is not limited to) the mechanical states of the generators, their controllers and the states of other devices (i.e., non-linear dynamic loads and renewable feed). The three-phase transmission network is modeled by the algebraic equation in current-balance form Sec. 7.3.2. The vector \( z_s \in \mathbb{R}^{n_z} \) comprises the AC signals of the transmission network, such as transmission line currents and bus voltages \( v_k \in \mathbb{R}^3 \). Moreover, the vector \( i \in \mathbb{R}^{n_i} \) contains the three-phase currents \( i_k \in \mathbb{R}^3 \) injected at each bus \( k \in \{1, \ldots, n_b\} \).

In this section, we will use the input \( i \) to incorporate explicit models of converter-based virtual inertia devices and disturbances into the power system model. We note that, \( z_s \) can be expressed as a function of the states \( x_s \) and the current injections \( i \). Moreover, after combining the power system model with suitable models of power converters, we shall use the control inputs of the power converters to provide virtual inertia and fast frequency response. We now elaborate on the dynamics of the VI devices and the disturbance model.

A. Modelling of virtual inertia devices

The VI devices are power electronic devices that mimic the inertial response of synchronous generators. In the following we consider the two most common implementations - grid-following and grid-forming. A grid-following virtual inertia device is controlled to inject active power proportional to the frequency deviation and rate of change of frequency estimated by a phase-locked loop (PLL). In contrast, the grid-forming virtual inertia device is a voltage source that responds to power imbalances by changing the frequency of its voltage.

In this paper, we model both types of VI devices as local dynamic feedback controllers. Even though we focus on two prototypical implementations of virtual inertia, the approach proposed in this paper can be used for any other arbitrary controller transfer function.

![Fig. 1. Grid-following VI device.](image)
where $v_{q,k}$ is the q-axis component of the bus voltage $v_k$ in a dq-frame with angle $\theta_k$ and $v_{g,k} \approx \dot{\theta}_k - \hat{\omega}_k v_k$ for small angle differences. Moreover, $\tau_k$, $K_{P,k}$, and $K_{I,k}$ are the filter time constant, proportional gain, and integral synchronization gain. With $\tau_k = 0$ the model (2) reduces to the standard synchronous reference frame phase locked loop (SRF-PLL) with a PI loop filter (i.e., $\dot{\theta}_k = -K_{P,k}v_{g,k} - K_{I,k}\int v_{g,k}$) commonly used in control of power converters [26], [33]. However, the SRF-PLL with a PI loop filter does not provide an explicit RoCoF estimate. In contrast, by incorporating a filter with time constant $\tau_k$ into the loop filter of the SRF-PLL allows us to obtain an explicit RoCoF estimate.

Remark 1 Extensive simulations indicate that using a standard SRF-PLL in combination with a realizable differentiator results in worse control performance than integrating the RoCoF estimation into the PLL. Furthermore, the input into the PLL (i.e., $v_k$) is often subject to pre-filtering and $\tau_k$ can also be interpreted as moving the pre-filter into the loop filter (see the discussion in [33]).

At the nominal steady-state, we have $\dot{\theta}_k \to \hat{\theta}_k$ and $\hat{\omega}_k \to 0$. This is because, we consider a reference frame rotating at the nominal grid frequency.

With the frequency and RoCoF estimates in (2), the VI device is modeled as

$$P_{VI,k}^* = K_{fol,k}[\hat{\omega}_k \dot{\omega}_k]^T, \quad Q_{VI,k}^* = 0,$$

where $K_{fol,k} = [\hat{d}_k \hat{m}_k]$ are the control gains and $P_{VI,k}^*$ and $Q_{VI,k}^*$ are the set-points for the power injection of the grid-following VI device. The elements $\hat{m}_k \geq 0$ are referred to as virtual inertia (reacts proportional to the derivative of the measured frequency), and $\hat{d}_k \geq 0$ as the virtual damping (reacts proportional to the measured frequency itself).

The VI device utilizes a current source that injects the three-phase current $i_k$ at node $k$ (see Figure 1) and tracks the power references $P_{VI,k}^*$ and $Q_{VI,k}^*$ with time constant $\tau_{fol} = 100$ ms. Figure 2 shows the overall control strategy.

b) Grid-forming: The grid-forming VI device uses a voltage source connected to the grid via an LC filter with parasitic losses (see Figure 3) that generates a voltage $v_{VI,k}$ with an angle $\theta_{VI,k} = \hat{\omega}_{VI,k} t$ that is a function of the power in-feed of the VI devices. The device is modeled via

$$\dot{\theta}_{VI,k} = \omega_{VI,k},$$

$$\hat{m}_k \omega_{VI,k} = -\hat{d}_k \omega_{VI,k} - P_{VI,k},$$

where $\theta_{VI,k}$, $\omega_{VI,k}$ are the angle and frequency of voltage generated by the grid-forming VI device. $P_{VI,k}$ is the active power from the grid-forming VI device into the grid, $\hat{m}_k > 0$ is the virtual inertia constant, and $\hat{d}_k \geq 0$ the virtual damping constant. The amplitude of the voltage is regulated at the nominal operating voltage of the bus to which the device is connected. The overall signal flow for the grid-forming VI device is shown in Figure 3. The second-order active power-frequency droop characteristics (4) are the core operating principle of a wide range of grid-forming control algorithms for power converters. For instance, under the assumption that the controlled internal dynamics of the converter are sufficiently fast, droop control with a low pass filter in the power controller (see Figure 5 of [34]) is equivalent to (4) (see Lemma 4.1 of [35]). Similarly, (4) can be explicitly recovered for a wide range of grid-forming controls by applying model reduction techniques that eliminate the fast controlled internal dynamics of power converters [36]. This includes virtual synchronous machines that directly enforce a second-order frequency droop behavior (see Section II-A of [27] and Section II-B of [10]) as well as controls based on matching the dynamics of power converters to that of a synchronous machine [11], [29], [30], [36] (see Remark 2 and Section 3.3 of [11]).

B. Disturbance model

We consider a general class of disturbance signals $\eta_k(t)$ that act at the voltage buses of the power system (1) through the current injection $i_k$. This approach can be used to model a wide range of faults such as load steps, fluctuations in renewable generation, or generator outages (i.e., by canceling the current injection of a generator). For brevity of presentation, we focus on faults that map changes in active power injection at every bus (i.e., changes in demand or generation) to current injections $i_k$ at every voltage bus $k$. We denote by $\eta = (\eta_1, \ldots, \eta_n)$ the disturbance vector that corresponds to, e.g., changes in load/generation or fluctuations of renewables.
In this section, we discuss several performance metrics typically utilised in stability analysis of power systems. As an alternative to these conventional metrics, we propose system norms as a tool to assess transient stability and for optimizing the allocation of VI devices. Further, we discuss the constraint specifications on the virtual inertia and damping gains arising from limits maximum power output of the VI devices.

### A. Performance metrics

Based on the model presented in Section II, we now formally define a set of performance metrics that we shall use to assess the frequency stability of the grid, when subjected to disturbances. Using the response of the system following a disturbance input \( \eta(t) \) several time-domain metrics can be defined. In particular, given a negative step disturbance, e.g., a sudden load increase or generation drop, at time \( t = 0 \), we define the following indices on the time-domain evolution of the frequency vector \( \omega = (\omega_1, \ldots, \omega_n) \) that collects the frequencies at different buses. Let the frequency nadir \( |\omega_k|_\infty \), maximum RoCoF \( |\dot{\omega}_k| |\omega_k|_\infty \) for each bus \( k \) be given by

\[
|\dot{\omega}_k|_\infty := \max_{t \geq 0} |\dot{\omega}_k(t)|, \quad (5)
\]

\[
|\omega_k|_\infty := \min_{t \geq 0} \omega_k(t), \quad (6)
\]

Further, let \( \omega_G = (\omega_{G,1}, \omega_{G,2}, \ldots) = [\omega_{G,1}^T, \omega_{G,2}^T, \ldots]^T \), \( \dot{\omega}_G = (\dot{\omega}_{G,1}, \dot{\omega}_{G,2}, \ldots) \), \( P_G = (P_{G,1}, P_{G,2}, \ldots) \), and \( P_{VI} = (P_{VI1}, P_{VI2}, \ldots) \) collect the generator frequencies, the RoCoF, the mechanical power injections, and the active power injections from VI devices. For the same step disturbance as \( \omega \), the mechanical power injections, and the active power injection constraint can be approximated by the magnitude and the damping response do not attain their peak values simultaneously (see Section III-C of [18], see also the scatter plot in Figure 3 of [18]). Therefore, the inertia response and the damping response do not attain their peak values simultaneously. Based on this observation, the maximum power injection constraint can be approximated by the magnitude constraints

\[
\dot{m}_k \leq \frac{P_{\text{max},k}}{|\omega_{\text{max}}|}, \quad \dot{d}_k \leq \frac{P_{\text{max},k}}{|\omega_{\text{max}}|},
\]
where $P_{\text{max},k}$ is the power rating of the $k$-th converter and $|\dot{\omega}|_{\text{max}}$ and $|\omega|_{\text{max}}$ are a priori estimates of the maximum RoCoF and frequency deviation. In addition, we limit the individual damping and inertia gains to be non-negative.

IV. CLOSED-LOOP SYSTEM AND $\mathcal{H}_2$ OPTIMIZATION

In this section we present a computational approach to answer the question of “how and where to optimally use virtual inertia and damping?” via appropriate tuning of the gain matrices $K_{\text{foll}}$ and $K_{\text{form}}$ in order to improve the post-fault response of a low-inertia power system.

A. Closed-loop system model and linearization

The placement and tuning of VI devices can be recast as a system norm (input-output gain) minimization problem for a linear system. To this end, we combine the system model (1) with the disturbance model presented in Section II-B and either grid-following (3) or grid-forming (4) virtual inertia device models. Next, we define inputs and outputs of the system interconnected with the tuning gains of the grid-forming and grid-following devices that allow us to optimize $\tilde{m}_k$ and $d_k$.

For each of the grid-following devices, we define $y_{\text{form},k} = (\omega_{\text{VI},k}, P_{\text{VI},k})$ as the output, collecting its internal frequency variable and its active power injection; and $u_{\text{form},k} = \tilde{\omega}_{\text{VI},k}$ as the control input. Choosing $K_{\text{form},k} = -[\tilde{d}_k \tilde{m}_k^{-1} \tilde{m}_k^{-1}]$, (6) can be re-expressed as

$$\dot{\tilde{\omega}}_{\text{VI},k} = u_{\text{form},k} = K_{\text{form},k} y_{\text{form},k}.$$  

The resulting overall system with input $u_{\text{form}}$, output $y_{\text{form}}$, and gain matrix $K_{\text{form}}$ is shown in Figure 5.

![Figure 5. Closed-loop system interconnection for the grid-following VI with tuning parameters $K_{\text{form}}$, where VSC is the voltage source converter.](image)

Similarly, for each grid-following device we define $y_{\text{foll},k} = (\tilde{\omega}_k, \hat{\omega}_k)$ as the output, collecting the frequency and RoCoF estimates from the PLL; and the active power set-point as the control input. Choosing $K_{\text{foll},k} = [\tilde{d}_k \tilde{m}_k^{-1}]$, (5) can be re-expressed as

$$P_{\text{VI},k}^* = u_{\text{foll},k} = K_{\text{foll},k} y_{\text{foll},k}.$$  

The resulting overall system with input $u_{\text{foll}}$, output $y_{\text{foll}}$, and gain matrix $K_{\text{foll}}$ is shown in Figure 6.

![Figure 6. Closed-loop system for the grid-following VI with tuning parameters $K_{\text{foll}}$, where CPS is the controllable power source.](image)

Next, we linearize these dynamics around a nominal operating point. In this process, the algebraic equation (12b) can be eliminated by arguing that its Jacobian with respect to $x$ has full rank at operating points that do not correspond to voltage collapse. Likewise, we can also remove the unobservable mode corresponding to absolute angles and zero eigenvalue to obtain a linearization

$$\Delta \dot{x} = A \Delta x + B \Delta u + G \eta,$$

$$\Delta y = C \Delta x, \quad \Delta y_p = C_p \Delta x,$$

where $\Delta x$, $\Delta y$, $\Delta y_p$, $\Delta u$ are the resulting deviation states, measurement outputs, performance outputs, control inputs; and $G = B \Pi$ is the disturbance gain matrix which encodes (via $\Pi = \text{diag}(\pi_1, \pi_2, \ldots)$) the location and (relative) strengths of the disturbances $\eta$. The states $x$ and outputs $y$ are different for both VI implementations. For grid-following implementation, these correspond to $x = (x_s, x_{\text{PLL}})$, $y_{\text{foll}} = (\tilde{\omega}, \hat{\omega})$ whereas, $x = (x_s, x_{\text{VI}})$, $y_{\text{form}} = (\omega_{\text{VI}}, P_{\text{VI}})$ refer to grid-forming implementation.

B. Virtual inertia as output feedback

The control input is constructed via a static output feedback as in (13). For the feedback matrix $K_{\text{form}}$, the control input $u_{\text{foll}}$ for the grid-following implementation is

$$\Delta u_{\text{foll}} = \begin{bmatrix} \Delta \tilde{m}_1 & \Delta \tilde{d}_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \Delta \tilde{m}_n_v & \Delta \tilde{d}_n_v \end{bmatrix} \begin{bmatrix} \Delta \tilde{\omega}_1 \\ \Delta \hat{\omega}_1 \\ \vdots \\ \Delta \tilde{\omega}_n_v \\ \Delta \hat{\omega}_n_v \end{bmatrix},$$

Similarly, the grid-forming implementation yields

$$\Delta u_{\text{form}} = \begin{bmatrix} \Delta \alpha_1 & \Delta \beta_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \Delta \alpha_{n_v} & \Delta \beta_{n_v} \end{bmatrix} \begin{bmatrix} \Delta \omega_{\text{VI},1} \\ \Delta P_{\text{VI},1} \\ \vdots \\ \Delta \omega_{\text{VI},n_v} \\ \Delta P_{\text{VI},n_v} \end{bmatrix},$$

where $K_{\text{form}}$ is the feedback matrix with $\tilde{\alpha}_k = -\tilde{d}_k \tilde{m}_k^{-1}$, $\beta_k = \tilde{m}_k^{-1}$, $n_v$ is the number of virtual inertia devices, and $u_{\text{form}}$ is the corresponding control input.

For our analysis, the performance output is selected as

$$\Delta y_p = C_p \Delta x = \left( r_2^v \Delta \tilde{\omega}_G, r_2^v \Delta \hat{\omega}_G, r_2^v \Delta P_G, r_2^v \Delta P_{\text{VI}} \right),$$
where the states $x$ depend on the VI implementation. This choice of outputs reproduces the infinite horizon integral of quadratic penalties on frequency deviations, RoCoF, as well as power injections from VI devices and generators, i.e.,

$$J_{\infty, \delta}(t) = \int_0^\infty \Delta y_p^T \Delta y_p \, dt,$$

(16)

the system energy imbalance for impulse disturbances. With $A_{\text{CL}} = A + BKC$, and combining (13) and (14) (respectively, (15)) the resulting dynamic system $G$ is given by

$$\Delta \dot{x} = A_{\text{CL}} \Delta x + G \eta, \quad \Delta y_p = C_p \Delta x.$$  

(17)

C. $H_2$ norm optimization

To compute the $H_2$-norm between the disturbance input $\eta$ and the performance output $y_p$ of the system (17), let the so-called observability Gramian $P_K$ denote the positive definite solution of the Lyapunov equation

$$PA_{\text{CL}} + A_{\text{CL}}^T P + C_p^T C_p = 0,$$

(18)

parameterized in $K$ for the given system matrices $A$, $B$, $C$, and $C_p$. Based on the observability Gramian $P_K$, the $H_2$ norm $\|G\|_2$ is given by

$$J_{\infty, \delta}(t) = \|G\|_2^2 = \text{trace}(G^T P_K G).$$

(19)

Thus, the optimization problem to compute the optimal allocation with respect to the $H_2$ norm $\|G\|_2^2$ is

$$\min_{K \in \mathcal{S} \cap \mathcal{C}} J_{\infty, \delta}(t).$$

(20)

The set $\mathcal{S}$ is used to encode the structural constraint on $K$, i.e., the purely local feedback structure of the virtual inertia control in (14) and (15). Hereafter, we consider $\mathcal{C}$ to be the set of constraints on the control gains discussed in Section III-B.

The optimization problem (20) can tune the gain of any VI device in the system. Sparse allocations (i.e., with few VI devices with significant contribution) can be obtained by including an $\ell_1$-penalty in the optimization (17) Sec. 3.5.

Note that evaluating the cost function requires solving the Lyapunov equation (18), which is non-linear in $P$ and $K$. In general, the optimization problem (20) is non-convex and may be of very large-scale. However, by exploiting the feedback structure of the problem, the gradient of $\|G\|_2^2$ with respect to $K$ can be computed efficiently (see Appendix A) and can be directly used to solve (20) via scalable first order methods (e.g., projected gradient) or to speed up higher order methods.

D. Computational complexity of the gradient computation

In (17) gradient-based optimization methods are used to directly optimize the inertia constants of a linearized networked swing equation model to minimize the $H_2$ norm of a power system. For a system with $n$ buses, the gradient computation in (17) requires the solution of $n - 1$ Lyapunov equations of dimension $2n$, resulting in a complexity of $O((n + 1)n^3)$. In contrast (20) includes more realistic models of virtual inertia devices and the gradient of (19) can be computed by solving two Lyapunov equations of dimension $4n$, thereby reducing the complexity to $O(n^3)$. In (24) a sequential linear programming approach is used to optimize the allocation of grid-following virtual inertia and damping. This method directly optimizes the frequency nadir and RoCoF. However, every iteration of the optimization algorithm in (24) requires computing the eigenvalues of the linearized system which has complexity $O(n^3)$, as well as time-domain simulations and the solution of a linear program, resulting in far higher computation complexity than the proposed method.

V. TEST CASE DESCRIPTION

To illustrate our algorithms for optimal inertia and damping tuning, we use a test case based on the 14-generator, 59-bus South-East Australian system [6, 40] shown in Figure 7. It is equipped with higher order models for turbines, governors, power system stabilizers (PSSs), and voltage regulators (AVRs). This system has several interesting features, for instance its string topology and weak coupling between South Australia (area 5) and the rest of the system. The SIMULINK version [41] of the model [40] was developed for the light loading scenario. Variations of this model have also been studied as low-inertia test cases in [24], [42].

Fig. 7. South-East Australian Power System line diagram. The crossed out generators are replaced by constant power sources to mimic a low-inertia scenario, whereas the red lightning symbols are the locations where disturbances are injected. The circles with VI inscribed within indicate the virtual inertia and damping devices distributed across the power system.
For this paper the model presented in [41] was modified to obtain a low-inertia case study by replacing synchronous machines located at the buses labeled 101, 402, 403, and 502 with constant power sources that inject the same active and reactive power as the original generators. This modeling choice is based on the high penetration of renewable generation in the real-world power system (particularly in area 5) [42] that does not provide frequency support. The model was augmented with 15 VI devices across the system (see Figure 7). For brevity of the presentation we consider two scenarios, in the first scenario the VI devices are all grid-forming, in the second scenario they are all grid-following (see Section II). In the case study in [24] motor loads with non-negligible inertia are used to ensure that the notion of a frequency signal (as input the VI devices) is well defined. In this work, we do not require this assumption. Finally, we use constant power injections at six locations (indicated by a red bolt) to simulate disturbances. The SIMULINK model of the benchmark system including virtual inertia devices is available online [43].

VI. RESULTS

In this section we compare the performance of the original system with the closed-loop system equipped with virtual inertia and damping devices. We consider both the grid-following and the grid-forming modes of implementation and mainly focus on the performance metrics defined in Section III-A.

A. Validity of the linearized model

As discussed in (13), we optimize the virtual inertia and damping gains using a linearization of the system at the nominal operating point. To validate the linearized model we compare it to the non-linear model for step disturbances at the six locations shown in Figure 7 ranging from $-250 \text{MW}$ to $+250 \text{MW}$. In Figure 8 the relative linearization errors for different performance metrics are plotted- both for the grid-following and the grid-forming virtual inertia and damping implementations. The plots reveal a concentration of data points in the $-10\%$ to $+10\%$ band. This indicates that the linear approximation of the model closely resembles the non-linear model and justifies the effectiveness of our approach.

B. Optimal tuning and placement of VI devices

The optimal inertia and damping profiles for the system are computed using the optimization problem (20). We consider the same weighted performance outputs (IV-B) for both grid-forming and grid-following and set the penalties to $r_\omega = 0.1$, $r_d = 0.2$, $r_G = 0.2$, and $r_{VI} = 0.2$, thereby identically penalizing the power injections from the VI devices and the synchronous machines. Further, the disturbance gain matrix $\Pi$, introduced in Section IV-A is set to identity, i.e., $\pi_i = 1$. In other words each node is subject to equally sized disturbances. Finally, using the approach outlined in Section III-B we choose the constraints such that $\sum_k d_k \leq 420 \text{MWs/rad}$, $\tilde{d}_k \leq 40 \text{MWs/rad}$, and $\tilde{m}_k \leq 18.5 \text{MWs}^2/\text{rad}$. These constraints ensure that the total damping does not exceed realistic values, and that the power output of the converters is roughly limited to 40 MW for frequency deviations in the normal operating regime. The resulting inertia and damping allocations for the above parameters and constraints are depicted in Figures 9 (a), (b) for the grid-forming and grid-following implementations respectively. Finally, we observe that no significant performance gains can be achieved by optimizing the PLL gains beyond applying standard tuning techniques (see [26], [33]).

C. Contrasting allocations for different VI implementations

The two allocations highlight some interesting features. Note that the optimized allocations are not uniform across the system. In fact, the virtual inertia for both implementations is predominantly allocated in area 5. Incidentally, the blackout in South-Australia in 2016 was also in this area [2]. Moreover, uniform allocations, chosen as the initial guess for the optimization, are typically not optimal (see also [17]). Another facet of the allocations is that the gains for the grid-following virtual inertia devices are limited by the constraints imposed in the optimization. This may be primarily attributed to time-delays (RoCoF estimation, response time $\tau_{\text{foll}}$ of the power source, etc.) encountered for the inertial response. To compensate for these delays, the allocation for the grid-following VI relies on larger inertia and damping gains (see Figures 9 at some nodes as well as larger total damping and inertia (refer Table 1). While significant inertia and damping is allocated at all nodes in the case of grid-forming virtual inertia devices, the grid-following implementation results in negligible allocations for some nodes outside of area 5.

D. Impact of VI devices on frequency stability

To investigate the effect of the VI devices, we consider the non-linear model of the South East Australian grid with
Moreover, the maximum is typically attained during the first swing of the system after a fault. In contrast, the time-domain simulations depicted in Figure 11 show that virtual inertia devices can have significant impact on the RoCoF $\dot{\omega}_G$ after the first swing. These differences are not captured when using the maximum RoCoF $|\dot{\omega}_{G,k}|_\infty$ as performance metric, but are accurately captured by the $\mathcal{H}_2$ norm.

We conclude that the VI devices have the expected positive impact on frequency stability. Moreover, the differences between the two VI implementations appear to be mostly related to differences in the maximum power injection. In the next section we will investigate the time-domain response of the system with and without VI devices in more detail.

### E. Time-domain responses

We now simulate a load increase of 200 MW at node 508, this represents a realistic contingency in the system (see [6, Sec. II]). Broadly speaking, this disturbance could also represent a loss of 200 MW renewable generation in area 5 and is of the type considered in the $\mathcal{H}_2$ optimization. Moreover, due to the low levels of rotational inertia in area 5, the placement of this fault corresponds to the worst-case location. The system responses are illustrated in Figure 11 and underscore the efficacy of virtual inertia and damping devices in a low-inertia power system. The grid-following and grid-forming VI implementation with the optimal allocations from Figure 9 are simulated and compared with the response of the original system. Table I shows the key performance indicators discussed in Section III-A. The top panels of the time-domain plots in Figure 11 illustrate the frequencies and the RoCoF of the 10 generators in the system for the two different VI implementations and the original system. The power injections from the generators and the 15 virtual inertia and damping devices across the power system are plotted in the bottom two panels of the figure. The key insights drawn from a closer analysis of these plots are summarized below:

(a) While both VI implementations improve the frequency nadir and maximum RoCoF, the grid-forming VI implementation performs better in terms of the absolute values. Further, the total inertia and damping is also less.

(b) The maximum active power $\max_k |P_{VI,k}|$, injected by a single virtual inertia device as well as the maximum power $\max_{t>0} \left| \sum_k P_{VI,k}(t) \right|$ injected by all the virtual inertia devices combined is smaller for the grid-forming virtual inertia devices. Thus, grid-forming virtual inertia achieves a better performance with a lesser control effort in comparison to the grid-following converters.

(c) A drop in the maximum governor response $\max_k |P_{G,k}|$, by a single synchronous machine compared to the original system is observed due to the active power injections from the virtual inertia devices.

(d) A decrease in the $\mathcal{H}_2$ norm is observed for both VI implementations, i.e., the $\mathcal{H}_2$ norm is an effective proxy for other time-domain metrics relevant for analysis [18].

(e) Another difference in the implementations pertains to the computation times for solving the optimization problem. Using MATLAB on a Core i7-6600U CPU, the optimization...
for grid-forming VI takes around 60s in comparison to 160s for the grid-following VI for identical penalties.

VII. SUMMARY AND CONCLUSIONS

In this paper we considered the problem of low-inertia power systems equipped with grid-following or grid-forming VI implementation using power electronic interfaced renewable energy sources. We modeled these two implementations as dynamic feedback control loops that provide virtual inertia and damping. A system norm-based optimization approach was used to study the problem of optimal placement and tuning of these devices. Our proposed tuning algorithm was far more scalable and computationally efficient in comparison to some of the other existing approaches in the literature. Further, we showcased the capabilities of such VI devices on a high-fidelity non-linear model of the South-East Australian power system and illustrated their efficacy. For a range of disturbances both types of virtual inertia implementations improved

Fig. 10. Distribution of generator frequency nadirs, maximum generator RoCoF, and VI power injections for load steps ranging from $-350$ MW to $-150$ MW at the nodes indicated in Figure 7 for different converter configurations.

| TABLE I |
| --- |
| **Performance Metrics for a Load Increase of 200 MW at Node 508** |

| Metric | Original | Grid-Following | Grid-Forming |
| --- | --- | --- | --- |
| $\sum_i \dot{m}_i [\text{MW s}^2/\text{rad}]$ | - | 111.8 | 99.9 |
| $\sum_i \dot{d}_i [\text{MW s}/\text{rad}]$ | - | 420 | 375.9 |
| $\max_k |\omega_{G,k}|_\infty [\text{Hz/s}]$ | 0.34 | 0.31 | 0.27 |
| $\max_k |\omega_{G,k}|_\infty [\text{mHz}]$ | 128.6 | 112.1 | 104.3 |
| $\max_k |P_{VI,k}|_\infty [\text{MW}]$ | - | 21.98 | 15.62 |
| $\max_{i \geq 0} \sum_k P_{VI,k}(t) | [\text{MW}]$ | - | 68.1 | 41.1 |
| $\max_{i \geq 0} \sum_k P_{G,k}(t) | [\text{MW}]$ | 50.5 | 39.9 | 45.9 |
| $H_2$ norm | 11.72 | 9.92 | 9.66 |
the location of virtual inertia devices can be found in [44]. Given that our proposed tuning algorithm is computationally efficient, our approach can be used to optimize a virtual inertia allocation with respect to multiple linearized models, each modeling different dispatch points and changes in model structure (e.g., system splits, tripping of generators, etc.). Finally, in future systems operating entirely based on converter-interfaced generation further services that are provided by synchronous machines today (e.g., voltage regulation) need to be provided by grid-forming power converters. Therefore, an interesting direction for future research would involve extending the proposed framework by incorporating suitable performance metrics for ancillary services apart from inertia, damping and fast frequency response.

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APPENDIX

By using the implicit linearization technique from [45], the gradient of the norm \( \|G\|_2^2 \) with respect to \( K \) is given by

\[
\nabla_K \|G\|_2^2 = 2(B^T P_K) L_K C^T,
\]

(21)

where \( L_K \) is the positive semidefinite controllability Gramian obtained from the linear (in \( L_K \)) Lyapunov equation

\[
L_{AC}^T + A_C L + G G^T = 0,
\]

(22)

parameterized in \( K \) for the given system matrices \( A, B, C, \) and \( G \). Thus, computing the norm \( \|G\|_2^2 \) and its gradient \( \nabla_K \|G\|_2^2 \) for a given \( K \) requires solving the Lyapunov equations (18) and (22). Moreover, the number of decision variables of the optimization problem (20) can be reduced by projecting the gradient \( \nabla_K \|G\|_2^2 \) on the sparsity constraint \( S \). Using the vector of non-zero parameters \( \phi = [\hat{n}_1, d_1, \ldots, \hat{n}_n, d_n] \) for the grid-following or alternatively \( \phi = [\hat{\alpha}_1, \hat{\beta}_1, \ldots, \hat{\alpha}_n, \hat{\beta}_n] \) for the grid-forming implementation, the projected gradient is given by e.g.,

\[
\text{proj}_S (\nabla_K \|G\|_2^2) = \left( \frac{\partial}{\partial \hat{n}_1} \|G\|_2^2, \frac{\partial}{\partial d_1} \|G\|_2^2, \ldots, \frac{\partial}{\partial d_n} \|G\|_2^2 \right).
\]

Similar projections can be performed for the constraint set \( C \).

Because the \( \mathcal{H}_2 \) norm is infinite for unstable systems, both the system norm \( \|G\|_2^2 \) as well as its gradient (21) are only well defined for a stable closed-loop system (17). Thus, to optimize the control gain \( K \), an initial guess for \( K \) is required that stabilizes (17) and that satisfies the constraints \( S \) and \( C \). Considering that the inertia devices are stable, it follows that an initial guess for which the plant is stable is given by \( \hat{m}_k = 0 \) and \( \hat{d}_k = 0 \). Moreover, the \( \mathcal{H}_2 \)-norm cost is smooth and approaches infinity as the control gains \( K \) approach the boundary of the set of stabilizing gains. In other words, any sequence of control gains \( K \) with non-increasing cost is guaranteed to be stabilizing.

Assuming that the projections onto \( C \) can be efficiently computed, the projected gradient method [46] and gradient computation outlined above can be used to find a locally optimal solution to the optimization problem (20) even for systems of very large dimension. For instance this is the case when \( C \) encodes upper and lower bounds on \( \hat{m}_k \) and \( \hat{d}_k \). If the projection onto \( C \) cannot be computed efficiently, the above gradient computation can still be used to speed up the computation times of higher-order methods.