Multi-Agent Pathfinding (MAPF) with Continuous Time

Anton Andreychuk1,3, Konstantin Yakovlev1,2, Dor Atzmon4, Roni Stern4,
1 Federal Research Center "Computer Science and Control" of Russian Academy of Sciences
2 National Research University "Higher School of Economics"
3 Peoples’ Friendship University of Russia (RUDN University)
4 Ben-Gurion University of the Negev
andreychuk@mail.com, yakovlev@isa.ru, dorat@post.bgu.ac.il, sternron@post.bgu.ac.il

Abstract

MAPF is the problem of finding paths for multiple agents such that every agent reaches its goal and the agents do not collide. Most prior work on MAPF were on grid, assumed all actions cost the same, agents do not have a volume, and considered discrete time steps. In this work we propose a MAPF algorithm that do not assume any of these assumptions, is complete, and provides provably optimal solutions. This algorithm is based on a novel combination of SIPP, a continuous time single agent planning algorithms, and CBS, a state of the art multi-agent pathfinding algorithm. We analyze this algorithm, discuss its pros and cons, and evaluate it experimentally on several standard benchmarks.

Introduction

MAPF is the problem of finding paths for multiple agents such that every agent reaches its goal and the agents do not collide. MAPF has topical applications in warehouse management (Wurman, D’Andrea, and Mountz 2008), airport towing (Morris et al. 2016), autonomous vehicles, robotics (Veloso et al. 2015), and digital entertainment (Ma et al. 2017). While finding a solution to MAPF can be done in polynomial time (Kornhauser, Miller, and Spirakis 1984), solving MAPF optimally is NP Hard under several common assumptions (Surynek 2010; Yu and LaValle 2013).

Nevertheless, AI researchers in the past years have made substantial progress in finding optimal solutions to a growing number of agents and scenarios (Sharon et al. 2015; Sharon et al. 2013; Wagner and Choset 2015; Standley 2010; Felner et al. 2018; Bartáč and LaValle 2012). While it seems that research on MAPF has matured enough to provide real industry values, most prior work has made several simplifying assumptions that precluded their wide-spread application. Indeed, most prior work on optimal MAPF assumed that (1) time is discretized into time steps, (2) the duration of move actions and wait actions is one time step, and (3) in every time step each agent occupies exactly a single location. In fact, most prior work performed empirical evaluation only on 4-connected grids.

We propose the first MAPF algorithm that does not rely on any of these assumptions and is sound, complete, and provides provably optimal solutions. This algorithm is based on a novel combination of SIPP (Phillips and Likhachev 2011), a continuous-time single-agent pathfinding algorithm, and CBS (Sharon et al. 2015), a state-of-the-art multi-agent pathfinding algorithm. We call the resulting algorithm CCBS.

We are not the first to study MAPF variants that are more general than basic MAPF. Indeed, several recent works adapted existing MAPF algorithms such as ICTS (Sharon et al. 2013) and CBS (Sharon et al. 2015) to richer MAPF settings (Walker, Sturtevant, and Felner 2018; Li et al. 2019).

Table 1 provides an overview of such prior works and its relation to CCBS. See a more detailed discussion of Table 1 in the related work section.

We analyze CCBS, discuss its pros and cons, and evaluate it experimentally on several standard benchmarks. The results show that CCBS is able to solve optimally MAPF problems in practice. As expected, CCBS is slower than CBS, since the latter ignores agents’ geometry, discretizes time, and considers a smallest set of actions. For the same reasons, CCBS finds significantly better solutions in practice.

| Actions | Agent |
|---------|-------|
| N.U.    | Cont. | Ang. | Vol. | Opt. | Dist. |
| CCBS    | ✓     | ✓    | ✓    | ✓    | ✓    |
| E-ICTS  | ✓     |     |     | ✓    |     |
| MCCBS   | ✓     |     |     | ✓    |     |
| POST-MAPF| ✓   |     |     | ✓    |     |
| ORCA, ALAN, and dRRT* | ✓ | ✓ | ✓ | ✓ | ✓ |
| AA-SIPP(µ) | ✓ | ✓ | ✓ | ✓ |     |

Table 1: Overview: MAPF research beyond the basic setting.

Since CCBS considers agents’ geometric shape and continuous time, the cost of collision detection in CCBS is significantly higher than in CBS. To mitigate this, we propose a history-based heuristic, that attempts to avoid some collision detection checks by guessing which pair of agents are likely to have a conflict. We discuss the relation between

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this heuristic and the concept of cardinal conflicts [Boyarski et al. 2015], and propose a simple hybrid heuristic that combines these methods and works well.

Problem Definition

The problem we address in this work is fundamentally the MAPF variant called MAPF$_{R}$, introduced by Walker et al. [2018]. A MAPF$_{R}$ is defined by a weighted directed graph $G = (V, E)$, and a set of agents indexed by $1, \ldots, k$. Each agent $i$ has a geometric shape, an initial location $s_i$, and a goal location $g_i$. While the algorithm we propose in this work can handle agents of any geometric shape, we will assume that all agents are open-disks of some radii, to avoid the need to reason about agents’ orientation.

The set of vertices $V$ represents a set of locations that the agents can occupy. We assume that any two agents, when standing still, can occupy any two vertices in the graph without colliding, i.e., their bodies will not overlap. For graphs based on grids and disk-shaped agents, this means the radius of every agent is less than or equal to half of the cell size. When an agent is at a location $v \in V$, it can either perform a move action or a wait action. A move action moves the agent along an edge $(v, v') \in E$, and a wait action means the agent stays in $v$. Every action has a duration. The duration of a move action is the weight of the edge the agent is traversing. The duration of a wait action can be any positive real value. Thus, every agent has an infinite number of wait moves in every location.

A plan for an agent $i$ is a sequence of actions $\pi_i$ such that if $i$ executes this sequence of actions then it will reach its goal. A set of plans, one for each agent, is called a joint plan. A solution to a MAPF$_{R}$ is a joint plan such that if all agents start to execute their respective plans at the same time, then all agents will reach their goal locations without colliding with each other. In this work we focus on finding cost-optimal solutions. To define cost-optimality of a MAPF$_{R}$ solution, we first define the cost of a plan $\pi_i$ to be the sum of the durations of its constituent actions. Several forms of solution cost-optimality have been discussed in MAPF research. Most notable are makespan and SOC, where the makespan is the max over the costs of the constituent plans and SOC is their sum. The problem we address in this work is to find a solution to a given MAPF$_{R}$ problem that is optimal w.r.t its SOC, that is, no other solution has a lower SOC.

Conflict-Based Search with Continuous Times

In this section, we introduce CCBS. Since CCBS is based on the CBS algorithm, we first provide relevant background on CBS.

Conflict Based Search (CBS)

CBS [Sharon et al. 2015] is a complete and optimal MAPF solver, designed for standard MAPF, i.e., where time is discretized and all actions have the same duration. It solves a given MAPF problem by finding plans for each agent separately, detecting conflicts between these plans, and resolving them by replanning for the individual agents subject to specific constraints.

The typical CBS implementation considers two types of conflicts: a vertex conflict and an edge conflict. A vertex conflict between plans $\pi_i$ and $\pi_j$ is defined by a tuple $(i, j, v, t)$ and means that according to these plans agents $i$ and $j$ plan to occupy $v$ at the same time $t$. An edge conflict is defined similarly by a tuple $(i, j, e, t)$, and means that according to $\pi_i$ and $\pi_j$, both agents plan to traverse the edge $e \in E$ at the same time, from opposite directions.

A CBS vertex-constraint is defined by a tuple $(i, j, v, t)$ and means that agent $i$ is prohibited from occupying vertex $v$ at time $t$. CBS edge-constraint is defined similarly by a tuple $(i, j, e, t)$, where $e \in E$. To guarantee completeness and optimality, CBS runs two search algorithms: a low-level search algorithm that finds paths for individual agents, and a high-level search algorithm that chooses which constraints to add.

CBS: Low-Level Search

The low-level search in CBS can be any pathfinding algorithm that can find an optimal plan for an agent that is consistent with a given set of CBS constraints. To adapt single-agent pathfinding algorithms such as A* to consider CBS constraints, the search space must also consider the time dimension since a CBS constraint $(i, j, v, t)$ blocks location $v$ only at a specific time $t$. For MAPF problems, where time is discretized, this means that a state in this single-agent search space is a pair $(v, t)$, representing that the agent is in location $v$ at time $t$. Expanding such a state generates states of the form $(v', t+1)$, where $v'$ is either equal to $v$, representing a wait action, or equal to one of the locations adjacent to $v$. States generated by actions that violate the given set of CBS constraints, are pruned. Running A* on this search space will return the lowest-cost path to the agent’s goal that is consistent with the given set of CBS constraints, as required. This adaptation of textbook A* is very simple, and indeed most papers on CBS do not report it and just say that the low-level search of CBS is A*.

CBS: High-Level Search

The high-level search algorithm in CBS works on a constraint tree (CT). The CT is a binary tree, in which each node $N$ contains:

1. $N$.constraints: A set of CBS constraints imposed on the agents ($N$.constraints)
2. $N$.II: A joint plan consistent with these CBS constraints.
3. $N$.cost: The SOC of $N$.II.

Generating a node $N$ in the CT means finding $N$.II for $N$.constraints and setting $N$.cost to be the SOC of $N$.II. If the joint plan does not contain any conflict, then $N$ is a goal. Expanding a non-goal node $N$ in the CT means choosing a CBS conflict $(i, j, x, t)$ that exists in $N$.II (where $x$ is either a vertex or an edge), and generating two nodes $N_i$ and $N_j$. Both nodes have the same set of constraints as $N$, plus a new constraint that is added to resolve the conflict: $N_i$ adds the constraint $(i, x, t)$ and $N_j$ adds the constraint $(j, x, t)$. CBS searches the CT in a best-first manner, expanding in every iteration the CT node $N$ with the lowest $N$.cost.
From CBS to CCBS

CCBS follows the CBS framework: it has a low-level search algorithm that finds plans for individual agents, and a high-level search algorithm that imposes constraints on the low-level search. The main differences between CCBS and CBS are:

- For conflict detection, CCBS uses a geometry-aware collision detection mechanism.
- To resolve conflicts, CCBS imposes constraints over action-time pairs instead of location-time pairs.
- For the low-level search, a pathfinding algorithm is used that considers continuous time and agents’ shape.

Next, we explain these differences in details.

Conflict Detection in CCBS  Since actions in standard CBS implementation have unit duration, identifying conflicts is relatively straightforward: iterate over every time step $t$ and check if there is a vertex (or an edge) that more than one agent is planning to occupy in time $t$. By contrast, in CCBS actions can have any duration and thus iterating over time steps is meaningless.

Also, CCBS considers the shape agents. This means that agents may conflict even if they do not occupy the same vertex/edge at the same time. For example, consider the graph depicted in Figure 1. Agents $i$ and $j$ occupy locations $A$ and $C$. If at the same time $i$ moves along the edge $AD$ and $j$ moves along the edge $CB$, then a collision will occur. Such a “criss-cross” conflict is not considered in standard CBS.

CCBS addresses all the above by defining CCBS conflicts as conflicts between actions.

Definition 1 (CCBS Conflict). A CCBS conflict w.r.t. a pair of plans $\pi_i$ and $\pi_j$ is defined by a tuple $\langle a_i, t_1, a_j, t_2 \rangle$, representing that if agent $i$ executes $a_i$ at time $t_1$ and agent $j$ executes $a_j$ at time $t_2$ then they will collide.

When the timing of $a_i$ and $a_j$ is clear from context, we omit $t_1$ and $t_2$ and define a conflict as a pair $\langle a_i, a_j \rangle$. There are various ways to detect collisions between agents with volume in a continuous space. There are standard methods to do this by analyzing the geometric properties of the agents’ movement and shape (Guy and Karamouzas 2015). CCBS is agnostic to the particular collision detection mechanism that is used.

Resolving Conflicts in CCBS  The high-level search in CCBS is runs a best-first search like regular CBS, selecting in every iteration a leaf node $N$ in the CT that has the smallest $N.cost$. If $N$ is not a goal node, it means it has at least one CCBS conflict. The high-level search expands $N$ by choosing one of the CCBS conflicts $(a_i, a_j)$ and generating two new CT nodes, $N_i$ and $N_j$: $N_i$ adds a constraint to agent $i$ and applies the low-level search to find a new plan for $i$, and $N_j$ adds a constraint to agent $j$ and applies the low-level search to find a new plan for $j$.

A constraint in CCBS is defined by a tuple $\langle i, a_i, [t_1, t_2] \rangle$, and represents that agent $i$ cannot start to perform action $a_i$ in the time range $[t_1, t_2]$. Note that $a_i$ can be either a move action or a wait action. Next, we describe which constraints to add to $N_i$ and to $N_j$. Let $t_{i,1}$ be the point in time when $a_i$ starts according to $\pi_i$. Then it computes the first point in time after $t_{i,1}$ that $i$ can perform $a_i$ without conflicting with $a_j$. We denote this point by $t_{i,2}$. Computing $t_{i,2}$ can be done by analyzing the kinematics and geometry of the agents. For simplicity, we computed $t_{i,2}$ by applying the conflict detection mechanism multiple times, for different values of $t_{i,2}$, starting from $t_{i,1}$ and incrementing by some small $\Delta > 0$. Let $t_{j,1}$ and $t_{j,2}$ denote the corresponding time points for agent $j$ and action $a_j$. CCBS adds the constraint $\langle i, a_i, [t_{i,1}, t_{i,2}] \rangle$ to $N_i$ and $\langle j, a_j, [t_{j,1}, t_{j,2}] \rangle$ to $N_j$.

For example, assume that we are running CCBS and the high-level search chooses to expand a CT node by resolving the conflict depicted in Figure 1. Agent $i$ plans to start moving along $AD$ at time 5 and agent $j$ plans to start moving along $CB$ at time 5.5. Thus, $t_{i,1} = 5$ and $t_{j,1} = 5.5$. Assume that the duration required to traverse $AD$ and to traverse $CB$ is the same. Therefore, $t_{i,2}$ will be smaller than $t_{j,2}$. This is because agent $j$ needs to wait a smaller amount of time to avoid agent $i$ starting at $t_{i,1}$ compared to the amount of time agent $i$ needs to avoid $j$ starting at time $t_{j,1}$, since $i$ starts earlier and their respective move actions have the same duration. For our example, assume that $t_{i,2} = 8$ and $t_{j,2} = 7.5$. Using these values of $t_{i,1}, t_{i,2}, t_{j,1}$, and $t_{j,2}$, CCBS will generate two new CT nodes: one with the additional constraint $\langle i, AD, [5, 8] \rangle$ the other with the additional constraint $\langle j, CB, [5.5, 7.5] \rangle$.

The CCBS Low-Level Search  The main challenges when implementing the low-level search of CCBS is that time continuous, and thus adding the time dimension results in a continuous search space. That is, the number of wait actions in every location is infinite. Moreover, CCBS constraints may specify that a wait action cannot be performed in some given time interval, which may require reducing the duration of a wait action.

To resolve this, we use the SIPP algorithm (Phillips and Likhachev 2011) as the low-level search algorithm. SIPP is an algorithm for single-agent path finding with dynamic moving obstacles. The core idea of SIPP is to identify collision-free time intervals for every location $v \in V$. Using time intervals instead of specific time points allows using a discrete search algorithm.

Specifically, SIPP applies an A*-based algorithm, searching in the space of (location, time interval) pairs. The output of SIPP is a plan, i.e., a sequence of actions, that move the agent from its initial location to its goal. SIPP is complete and is guaranteed to find a time-minimal solution.
We chose SIPP as a low-level search algorithm because it already is designed to consider time intervals, and in CCBS the constraints are also over time intervals.

**Theoretical Properties**

Next, we prove that CCBS is sound, complete, and optimal. To do so, we define the notion of a sound pair of constraints in a similar way to Atzmon et al. (2018).

**Definition 2** (Sound Pair of Constraints). For a given MAPF$_R$ problem, a pair of constraints is sound iff in every optimal solution it holds that at least one of these constraints hold.

**Lemma 1.** For any CCBS conflict $(a_i, a_j)$, the pair of CCBS constraints $(i, a_i, [t_{i1}, t_{i2}])$ and $(j, a_j, [t_{j1}, t_{j2}])$ is a sound pair of constraints.

**Proof.** By contradiction, assume that there exists $\Delta_i \in (0, t_{i2} - t_{i1}]$ and $\Delta_j \in (0, t_{j2} - t_{j1}]$ such that perform $a_i$ at $t_{i1} + \Delta_i$ and $a_j$ at $t_{j1} + \Delta_j$ does not create a conflict. That is, $(a_i, t_{i1} + \Delta_i, a_j, t_{j1}, t_{j2})$ is not a conflict (Def. 1).

By definition of $t_{j2}$:  

$$\forall t \in [t_{j1}, t_{j2}) : (a_i, t_{i1}, a_j, t) \text{ is a conflict.}$$

$$\forall t \in [t_{i1} + \Delta_i, t_{j1} + \Delta_j) : (a_i, t_{i1} + \Delta_i, a_j, t) \text{ is a conflict.}$$

By definition of $\Delta_i$ and $\Delta_j$:  

$$(a_i, t_{i1} + \Delta_i, a_j, t_{j1} + \Delta_j) \text{ is not a conflict}$$

Therefore, $\Delta_i < \Delta_j$. Similarly, by definition of $t_{i2}$:

$$\forall t \in [t_{i1}, t_{i2}) : (a_i, t, a_j, t_{j1}) \text{ is a conflict.}$$

$$\forall t \in [t_{i1} + \Delta_i, t_{i2}) : (a_i, t, a_j, t_{j1} + \Delta_i) \text{ is a conflict.}$$

Therefore, by definition of $\Delta_i$ and $\Delta_j$ we have that $\Delta_j < \Delta_i$, which leads to a contradiction.

**Theorem 1.** CCBS sound, complete, and is guaranteed to return an optimal solution.

The proof Theorem 1 relies on Lemma 1 and directly follows Atzmon et al.’s proof for $k$-robust CBS (Atzmon et al. 2018).

**Conflict Detection and Selection Heuristics**

As noted above, conflict detection in CCBS is more complex than in regular CBS. Indeed, in our experiments we observed that conflict detection took a significant portion of time. To speedup the conflict detection, we only checked conflicts between actions that overlap in time and may overlap geometrically. In addition, we implemented two heuristics for speeding up the detection process. We emphasize that these heuristics do not compromise our guarantee for soundness, completeness, and optimality.

The first heuristic we used, which we refer to as the **history heuristic**, keeps track of the number of times conflicts have been found between agents $i$ and $j$, for every pair of agents $(i, j)$. Then, it checks first for conflicts between pair of agents with a high number of past conflicts. Then, when a conflict is found the search for conflicts is immediately halted. That found conflict is then stored in the CT node, and if that CT node will be expanded then it will generate CT nodes that are aimed to resolve this conflict. This implements the intuition that pairs of agents that have conflicted in the past are more likely to also conflict in the future.

We have found this history heuristic to be very effective in practice for reducing the time allocated for conflict detection. Using this heuristic, however, has some limitations. Prior work has established that to intelligently choosing which conflict to resolve when expanding a CT node can have a huge impact on the size of the CT and on the overall runtime (Boyarski et al. 2015). Specifically, Boyarski et al. (2015) introduced the notion of **cardinal conflicts**, which are conflict that any way to resolve them will result in increasing the SOC. **Semi-cardinal conflicts** are conflicts that resolving them by replanning for one of the involved agents will increases the solution cost, but replanning for the other involved agents do not increase solution cost.

For CBS, choosing to resolve first cardinal conflicts, and then semi-cardinals, yielded significant speedups (Boyarski et al. 2015). However, to detect cardinal and semi-cardinal conflicts, one needs to identify all conflicts, while the advantage of the heuristic is that we can halt the search for conflicts before identifying all conflicts.

To this end, we proposed a second hybrid heuristic approach. Initially, we detect all conflicts and choose only cardinal conflicts. However, if a node $N$ does not contain any cardinal or semi-cardinal conflict, then for all nodes in the CT subtree beneath it we switch to use the history heuristic. This hybrid approach worked well in our experiments, but fully exploring this tradeoff between fast conflict detection and smart conflict selection is a topic for future work.

**Experimental Results**

Following most prior work on MAPF, we have conducted experiments on grids. Agents can move from the center of one grid cell to the center of another grid cell. The size of every cell is $1 \times 1$, and the shape of every agent is a an open disk which radius equals $\sqrt{2}/4$. This specific value was chosen to allow comparison with CBS, since it is the maximal radius that allows agents to safely perform moves in which agents follow each other.

To allow non-unit edge costs, we allowed the agents to move in a single move action to every cell located in their $2^k$ neighborhood, where $k$ is a parameter (Rivera, Hernández, and Baier 2017). Moving from one cell to the other is only allowed if the agent can move safely to the target cell without colliding with other agents or obstacles, where the geometry of the agents and obstacles are considered. The cost of a move corresponds to the Euclidean distance between the grid centers. Figure 2 illustrates such a $2^k$ neighborhood. Increasing $k$ means a search space with higher branching factor, but also makes lower cost paths possible. As a heuris-

1 In particular, this is the largest agent radius that allows the following pair of actions: agent $i$ goes up from cell $X$ to cell $Y$, and at the same time agent $j$ moves to $X$ from the right. While such train-like movements are not allowed by some MAPF variants, they are assumed to be valid in most in research on CBS.
The results show clearly that indeed increasing $k$ yields solutions with lower SOC, as expected. The absolute difference in SOC when when moving from $k = 2$ to $k = 3$ is the largest, and it grows as we add more agents. For example, for problems with 16 agents, moving from $k = 2$ to $k = 3$ yields an improvement of 17.2 SOC, and for problems with 17 agents the gain of moving to $k = 3$ is 17.9 SOC. Increasing $k$ further exhibits a diminishing return effect, where the largest average SOC gain when moving from $k = 4$ to $k = 5$ is at 0.5.

Increasing $k$, however, has also the effect of increasing the branching factor, which in turns means that path-finding becomes harder. Indeed, the success rate of $k = 5$ is significantly lower compared to $k = 4$. An exception to this is the transition from $k = 2$ to $k = 3$, where we observed a slight advantage in success rate for $k = 3$ for problems with a small number of agents. For example, with 6 agents the success rate of $k = 2$ is 0.99 while it is 1.00 for $k = 3$. An explanation for this is that increasing $k$ also means that plans for each agent can be shorter, which helps to speedup the search. Thus, increasing $k$ introduces a tradeoff w.r.t. the problem-solving difficulty: the resulting search space for the low-level search is shallower but wider. For denser problems, i.e., with more agents, $k = 2$ is again better in terms of success rate, as more involved paths must be found by the low-level search.

Figure 3 shows the tradeoff of increasing $k$ by showing the average gain, in terms of SOC, of using CCBS for different values of $k$ over CBS with $k = 2$. The $x$-axis is the number of agents, and the $y$-axis is the gain, in percentage. We only provide data points for configurations with a success rate of at least 40%. As can be seen, increasing $k$ increases the gain over CBS, where for $k = 4$ and $k = 5$ the gain was over 20%. Increasing $k$ also decreases the success rate, and thus the data series for larger $k$ value “disappears” after a smaller number of agents.

We also compared the performance of CCBS with $k = 2$ and a standard CBS implementation. Naturally, standard CBS was faster than CCBS, as its underlying solver is A$^*$ on a 4-connected grid, detecting collisions is trivial, and it has only unit-time wait actions. However, even for $k = 2$, CCBS is able to find better solutions, i.e., solutions of lower SOC. This is because in some cases, an agent can start to
move after waiting less than a unit time step.

To see this phenomenon, consider the example in Figure 4. There are three agents, 1, 2, and 3 in an open 2 × 4 grid. The left-most grid shows the initial locations of the agents, and the right-most grid shows their goal locations. The small arrows in the agents indicate the direction each agent is about to move to. Consider first the plan created by CBS, which is shown on the top row of Figure 4. In CBS, every action takes unit duration. Since agent 3 cannot move upwards at time t = 0 without colliding with agent 1, it will have to wait for time t = 1 before starting to move. By contrast, in CCBS a wait action can have an arbitrary duration, and thus agent 3 can start to move upwards safely earlier than in CBS, at time t = 0.707. See the file C-CBSvsCBS.gif in the supplementary material for an animation of this example. These cases, where CCBS with k = 2 finds a better solution compared to standard CBS, are not rare. However, the advantage in terms of SOC, in all our experiments, was very small.

Dragon Age Maps

Next, we experimented with a much larger grid, taken from the Dragon Age: Origin (DAO) game and made available in the movingai repository (Sturtevant 2012). Specifically, we used the den520d map, shown to the right of Table 3, which was also used by prior work on CBS (Sharon et al. 2015). Start and goal states were chosen randomly, and we create 250 problems for every number of agents.

Table 3 shows the results obtained for CCBS with k = 2, 3, and 4, in the same format as Table 2. The same overall trends are observed: increasing k reduces the SOC and decreases the success rate. Figure 5 shows the average runtime required to solve the instances solved by all values of k. Interestingly, here we observe that k = 3 was the fastest on average. Similar to the better success rate in the open grid experiments, we explain this by the fact that increasing k also yields shorter paths to the goals, which helps decrease runtime.

Conflict Detection and Resolution Heuristics

In all the experiments so far we have used CCBS with the hybrid conflict detection and selection heuristic described earlier in the paper. Here, we evaluate the benefit of using this heuristic. We compared CCBS with this heuristic against the following: (1) Vanilla: CCBS that chooses arbitrarily which actions to check first for conflicts, (2) Cardinals: CCBS that identifies all conflicts and chooses cardinal conflicts, and (3) History: CCBS that uses the history heuristic to choose where to search for conflicts first, and resolves the first conflict it finds.

Table 4 shows results for experiments run on the den520d DAO map. We explored the following points in the space of possible problem parameters: 20 agents with k = 2, 3, and 4, and 25 agents with k = 2 and k = 3. For every configuration we create and run CCBS on 1,000 in-
stances. The table shows the success rate (the row labelled “Success”), the average runtime in seconds over instances solved by all algorithms (“Time”), and the average number of high-level nodes expanded by CCBS (“HL exp.”). The results show that the proposed hybrid heuristic yields the best success rate. When comparing History to Cardinals, we see that History is faster but the number of high-level nodes expanded by Cardinals is smaller. This follows our motivation for the hybrid heuristic: the choice of which conflicts to resolve taken by Cardinals is important in minimizing the size of the CT, while detecting all conflicts can be too time consuming. The proposed hybrid heuristic enjoys the complementary benefits of History and Cardinals, as can be seen by its fast runtime and small number of high-level expanded nodes. Thus, we used it in all our experiments.

**Related Work**

Several prior work has also attempted to relax some of the simplifying assumptions made by most MAPF research. Yakovlev and Andreychuk (2017) proposed an any-angle MAPF algorithm, i.e., an algorithm where agents are not restricted to moving along pre-defined edges and can move in any angle they choose. Their algorithm, called AA-SIPP(\(m\)), is similar to CCBS in that agents can wait for any desired duration. Also, AA-SIPP(\(m\)) heavily relies on SIPP. Unlike CCBS, they adopted a prioritized planning approach that does not guarantee completeness or optimality.

Li et al. (2019) proposed a CBS-based algorithm for solving the Large-Agents MAPF (LA-MAPF) problem. In LA-MAPF, agents have a geometrical shape, and may have different configuration spaces. Their algorithm, called MC-CBS, is also based on CBS. However, they assumed all actions have a unit duration and did not address continuous time. We note that adapting CCBS to cases where the agents have different configuration spaces is trivial.

Walker et al. (2018) adapted the ICTS MAPF algorithm (Sharon et al. 2013) to MAPF\(_{\ell}\), that is, to consider actions with non-uniform duration and agents with a geometric shape. However, their extended ICTS does not allow agents to wait an arbitrary amount of time, and relies on discretizing the possible wait times (they called it \(\delta\)). By contrast, CCBS relies on a different MAPF framework – CBS – and do not require a-priori definition of the smallest wait action. Still, CCBS maintains optimality and completeness. Ma et al. (2016) proposed MAPF-POST, which is a post-processing step that adapts a MAPF solution to different action durations that due to kinematic constraints. MAPF-POST does not guarantee optimality as well.

dRRT\(^*\) is a MAPF algorithm designed for continuous spaces (Dobson et al. 2017). It is a sample-based technique that is asymptotically complete and optimal. CCBS is optimal and complete, and is designed to run over a discrete graph. ORCA (Van Den Berg and Overmars 2005) and ALAN (Godoy et al. 2018) are also MAPF algorithms designed for continuous space. They are fast and distributed, but do not provide optimality or completeness guarantees.

Table[1] provides a differential overview of related work on MAPF beyond the basic MAPF setting. Column “N.U.” means support for non-uniform action durations. Column “Cont.” means support for actions with arbitrary continuous duration. Column “Ang.” means support for any-angle actions, that is, beyond a predefined configuration space. Column “Vol.” means support for agents with a volume, i.e., some geometric shape. Column “Opt.” means returning a provably optimal solution, and column “Dist.” means the algorithm is distributed. Every row corresponds to a different algorithm or family of algorithms. We highlighted the row corresponding to CCBS, which illustrates the generality of CCBS.

**Conclusion and Future Work**

CCBS is an algorithm for solving MAPF problems that allows continuous time, actions with non-uniform duration, and agents and obstacles with a geometric shape. It follows the CBS framework, uses SIPP as a low-level solver, and uses unique types of conflicts and constraints. We prove that CCBS is sound, complete, and optimal. To the best of our knowledge, CCBS is the first MAPF algorithm to provide optimality guarantees for such a broad range of MAPF settings.

Our experimental results showed that CCBS can solve actual MAPF problems and indeed finds solutions significantly better than CBS. However, current results were based on grid maps that are extended by considering \(2^k\) neighborhoods. We chose grids as a domain to allow natural comparison with existing solvers, but CCBS can work on arbitrary graphs. Indeed, this is a topic for future work.

This work also highlighted that conflict detection becomes a bottleneck when solving MAPF\(_{\ell}\) problems. We suggested a hybrid heuristic for reducing these cost. However, we expect that future work can apply meta-reasoning techniques to decide when and how much to invest in conflict detection throughout the search.

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