Multi-kaonic Hypernuclei and Kaon Condensation

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Abstract. This contribution reports on dynamical, self-consistent calculations of multi-\bar{K} hypernuclei, which were performed by adding antikaons to particle-stable nuclear configurations of nucleons, \Lambda and \Xi hyperons. Our results show a robust pattern of saturation of the \bar{K} separation energy \( \bar{B}_\bar{K} \) as a function of the number of \bar{K} mesons, with \( \bar{B}_\bar{K} \) bounded from above by approximately 200 MeV. The associated baryon densities saturate at values 2-3 times nuclear-matter density. The main reason for saturation is the repulsion induced by the vector meson fields between \bar{K} mesons, similarly to what was found for multi-\bar{K} nuclei. The calculations confirm that strangeness in finite strong-interaction self-bound systems is realized through hyperons, with no room for kaon condensation.

1. Introduction
The properties of antikaons in the nuclear medium have been extensively studied since the pioneering works \cite{1, 2} on the possibility of kaon condensation in dense matter. The most natural dense systems where kaon condensation could be realized are offered by neutron stars with densities extending over several times \( \rho_0 \) \cite{3, 4, 5}. Once the \bar{K} effective mass drops below approx. 200 MeV, the strangeness-changing weak-interaction processes allows for conversion of high-pressure electron gas into kaon condensate via \( e^- \rightarrow \nu_e \bar{K}^- \). According to some scenarios the onset of condensation could occur at \( 3-4\rho_0 \) or even higher densities depending on the presence of hyperons.

The subject of multi-\bar{K} (hyper)nuclei was studied recently by us in Refs. \cite{6, 7, 8}, where the focal question of interest was whether the kaon condensation could occur in strong interaction self-bound systems. The \bar{K} mesons could provide the physical strangeness degrees of freedom if the \bar{K} binding energy \( \bar{B}_\bar{K} \) exceeds the threshold value of \( \bar{B}_\bar{K} \gtrsim 320 \) MeV \approx \( m_\bar{K} + \mu_N - m_\Lambda \), where \( \mu_N \) is the nucleon chemical potential. This strong-binding scenario \cite{9} would allow the conversion \( \Lambda \rightarrow KN \) in matter, thus \bar{K} mesons would condense macroscopically. However, our detailed calculations showed saturation of \bar{K} binding energies upon increasing the number of antikaons. The \bar{K} binding energies were found to be generally below 200 MeV, assuming accepted range of the strength of the \bar{K} interaction, which is considerably short of the threshold value of 320 MeV required for the onset of condensation.

In Section 2 we briefly outline the model applied for the study of multi-strangeness self-bound systems. Selected results of our calculations are presented in Section 3 and conclusions are summarized in Section 4.
2. Methodology

2.1. RMF Model

The interaction of $K$ mesons with the (hyper)nuclear medium was studied using the relativistic mean-field (RMF) models. In this approach, the strong interaction between baryons $B$ and $K$ mesons is mediated by the exchange of several effective mesonic degrees of freedom - scalar fields $\sigma$, $\sigma^*$, and vector fields $\omega$, $\rho$, $\phi$:

$$\mathcal{L} = \hat{B} [i\gamma^\mu D_\mu - (M_B - g_{\sigma B} \sigma - g_{\sigma^* B} \sigma^*)] B$$

$$+ (D_\mu K) \frac{1}{m_K} (\hat{D}^\mu K) - (m_K^2 - g_{\sigma K} m_K \sigma - g_{\sigma^* K} m_K \sigma^*) K^\dagger K$$

$$+ (\sigma, \sigma^*, \omega_\mu, \rho_\mu, \phi_\mu, A_\mu, \text{free-field terms}) - U(\sigma) - V(\omega),$$

with covariant derivative:

$$D_\mu = \partial_\mu + i g_{\omega B} \omega_\mu + i g_{\rho B} \rho_\mu + i g_{\phi B} \phi_\mu + ie (I_3 + \frac{1}{2} Y) A_\mu,$$

where $I$ denotes isospin operator, $I_3$ its third component and $Y$ stands for hypercharge. The Lagrangian density (1) was parametrized by standard linear [10] and nonlinear [11, 12] RMF parameter sets as well as the density dependent meson-nucleon coupling models [13, 14, 15].

For hyperons, the coupling constants to the vector fields were fixed by the SU(6) symmetry:

$$g_{\omega \Lambda} = \frac{2}{3} g_{\omega N}, \quad g_{\rho \Lambda} = 0, \quad g_{\phi \Lambda} = -\frac{\sqrt{2}}{3} g_{\omega N},$$

$$g_{\omega \Xi} = \frac{1}{3} g_{\omega N}, \quad g_{\rho \Xi} = -g_{\rho N}, \quad g_{\phi \Xi} = -2 \frac{\sqrt{2}}{3} g_{\omega N}. $$

The coupling of the $\Lambda$ hyperon to the scalar fields ($\sigma$, $\sigma^*$) was constrained by fitting to the measured properties of single- and double-$\Lambda$ hypernuclei. For the case of $\Xi$ hyperons, $g_{\sigma \Xi}$ was fitted to yield an optical potential $V_{\Xi^-} = -14$ MeV in the center of the $^{12}\text{C}$ nucleus [16]. Since there are no experimental data on $\Xi(\Lambda)$-$\Xi$ interaction available we set $g_{\sigma^* \Xi} = g_{\phi \Xi} = 0$.

Finally, for the antikaons we adopted the SU(3) symmetry relations for the vector meson couplings:

$$2g_{\omega K} = 2g_{\rho K} = \sqrt{2} g_{\phi K} = g_{\rho\pi} = 6.04, $$

while the strength of the scalar interaction $g_{\sigma K}$ was varied to obtain several assumed $K^-$ binding energies in the range of 100-150 MeV. The coupling to the $\sigma^*$ was estimated from the $f_0 \rightarrow K \bar{K}$ decay to be $g_{\sigma^* K} = 2.65$.

For more details of the model and its parameters see e.g. Ref. [8].
3. Results

3.1. \( \{N, K^-\} \) systems

In Refs. [6, 7] we studied nuclear systems containing several \( K^- \) mesons. In our calculations we found saturation of \( K^- \) binding energies \( B_{K^-} \) upon increasing the number of antikaons embedded in the nuclear medium. This is demonstrated in Fig. 1 where the \( K^- \) binding energy \( B_{K^-} \) is plotted as a function of the number \( \kappa \) of antikaons in several \( K^- \)-nuclear systems calculated using the density dependent meson-nucleon coupling RMF model DD-ME1 (see Ref. [14] for details). The coupling constant \( g_{\sigma K} \) was chosen to yield \( B_{K^-} = 100 \) MeV in single-\( K^- \) configurations. The figure demonstrates that the larger the \( K^- \)-nuclear system is, the more antikaons is needed to saturate \( B_{K^-} \), but the number \( \kappa \) does not exceed \( \approx 15 \) even in heavier systems such as \( ^{208}\text{Pb} \).

The saturation of \( \bar{K} \) binding energies is accompanied by saturation of the associated nuclear density distributions as displayed in Fig. 2, where the nuclear density distributions \( \rho_N \) are plotted for several numbers \( \kappa \) of antikaons embedded in \( ^{16}\text{O}+\kappa K^- \). The dotted line stands for the nuclear density distribution in \( ^{16}\text{O} \) in the absence of \( \bar{K} \) mesons. The density distributions behave quite regularly with increasing number \( \kappa \) of antikaons.
3.2. \{N, Y, K^{-}\} systems

In Ref. [8] we generalized our calculations and considered SU(3) octet baryons. We built up \{N, \Lambda, \Xi\} hypernuclear systems with maximum strangeness per baryon number for selected core nuclei by first adding \Lambda hyperons up to the \Lambda Fermi level. Then we continued by adding \Xi hyperons as long as both reactions \Lambda \Lambda \leftrightarrow \Xi N were kinematically forbidden. As a result we obtained strong-interaction particle-stable multi-strangeness configuration decaying only via weak interactions.

We verified, that our previous conclusions concerning \{N, K^{-}\} systems hold also in the presence of hyperons. Fig. 3 shows the \kappa^{\text{exp}} binding energies \(B_{K^-}\) for several numbers \(\eta\) of \Lambda hyperons as a function of the number \(\kappa\) of antikaons in \(^{16}\text{O+}\eta\Lambda+\kappa K^-\) calculated using the nonlinear RMF model NL-TM2 (see Ref. [12] for details). The \(B_{K^-}\) saturates for any given number \(\eta\) of \Lambda hyperons. Moreover, the saturation of \(B_{K^-}\) holds in systems with much larger fraction of strangeness as displayed in Fig. 4 for various hypernuclear systems \(^{A}Z+\eta\Lambda+\mu\Xi+\kappa K^-\) with maximum \(|S|/B\) for selected core nuclei.

\textbf{Figure 2.} The nuclear density distributions \(\rho_N\) for several numbers \(\kappa\) of antikaons embedded in \(^{16}\text{O}\) calculated using the DD-ME1 model, the dotted line stands for the density distribution in the absence of \(K\) mesons.
Figure 3. The $K^-$ binding energies $B_{K^−}$ as a function of the number $κ$ of antikaons for several numbers $η$ of Λ hyperons in $^{16}O+ηΛ+κK^−$ calculated using the NL-TM2 RMF model [12].

4. Summary
In this contribution we reviewed our recent dynamical calculations of multi-strangeness self-bound hadronic systems containing $K^−$ mesons.

The results of our comprehensive RMF calculations show robust saturation of $\bar{K}$ binding energies as well as the associated baryon density distributions as a function of the number of antikaons embedded in the medium. This saturation phenomena is found to be present across the periodic table, is valid for any considered RMF model, and occurs for every mean-field composition containing the $\omega$ meson. The $\omega$ meson, mediating vector interaction, is the primary reason for saturation caused by strong $\bar{K}$-$K$ repulsion. In all our calculations the $K$ binding energies are well below the threshold value required for conversion $Λ \rightarrow K\bar{N}$, thus $\bar{K}$ mesons do not provide the relevant constituents of strange hadronic systems.

It is to be stressed, that our conclusion of “no kaon condensation” refers to strong-interaction self-bound configurations and does not necessarily apply to kaon condensation in neutron stars, where the composition of matter is controlled by weak-interaction processes.

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Figure 4. The $K^-$ binding energies $B_{K^-}$ as a function of the number $\kappa$ of antikaons in several hypernuclear configurations calculated using the NL-TM1(2).

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