Review of high energy diffraction in real and virtual photon–proton scattering at HERA

G Wolf

Deutsches Elektronen-Synchrotron, DESY, Hamburg, Germany

Received 30 June 2009, in final form 1 December 2009
Published 26 October 2010
Online at stacks.iop.org/RoPP/73/116202

Abstract

The electron–proton collider HERA at DESY opened the door for the study of diffraction in real and virtual photon–proton scattering at centre-of-mass energies $W$ up to 250 GeV and for large negative mass squared $-Q^2$ of the virtual photon up to $Q^2 = 1600$ GeV$^2$. At $W = 220$ GeV and $Q^2 = 4$ GeV$^2$, diffraction accounts for about 15% of the total virtual photon–proton cross section, decreasing to $\approx 5\%$ at $Q^2 = 200$ GeV$^2$. An overview of the results obtained by the experiments H1 and ZEUS on the production of neutral vector mesons and on inclusive diffraction up to the year 2008 is presented.

(Some figures in this article are in colour only in the electronic version)

This paper is dedicated to Volker Soergel, Gustav A Voss, to the memory of Björn H Wiik, and to Antonino Zichichi, who steered HERA from a plan to reality.

This article was invited by Guido Barbiellini.

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References
1. Introduction

Diffraction phenomena in particle physics are well known since the sixties of the last century. In hadron–hadron scattering these processes are phenomenologically described by the exchange of a virtual, colourless and flavourless neutral object carrying no quantum numbers [1], called the Pomeron, see also [2, 3]. The Pomeron seemed to couple to quarks [4, 5]. The possibility that the Pomeron may have a partonic structure has been discussed [6–8]. This point was stressed again [9–11] based on results from UA8 [12, 13] which were obtained in pp collisions at a centre-of-mass (c.m.) energy of 630 GeV.

An extensive study of physics issues, such as diffraction, that could be addressed with a large electron–proton collider had been presented, e.g. by [14]. The prominent features of diffractive production of vector mesons by real and virtual photons with mass squared $-Q^2$, $\gamma, \gamma^* p \rightarrow V^0 p$, observed at lower energies, were the near constancy of the cross section with total c.m. energy, and the exponential decrease of the differential cross section as a function of the four-momentum-transfer squared ($-t$) between photon and proton. These properties were similar to those observed for elastic hadron–hadron interactions. They could be understood within the model of vector meson dominance (VDM), see, e.g., [15–17], where the photon couples directly to one of the neutral vector mesons $\rho^0, \omega, \phi, J/\Psi, \Upsilon$, which in turn scatters elastically on the target proton, see figure 1.

At the electron (positron)–proton storage ring HERA, electrons of 27.6 GeV collided with 820 (920) GeV protons, which provided photon–proton collisions at centre-of-mass energies up to 300 GeV and allowed to study diffraction from $Q^2 = 0$ up to several thousand GeV$^2$.

In an early analysis of events from virtual photon–proton scattering at large c.m. energies, the ZEUS collaboration [18], and subsequently the H1 collaboration [19], observed a special class of events, where a hadronic system is produced close to the direction of the virtual photon, and is separated by a large rapidity gap from a proton or low-mass nucleonic system emitted along the direction of the incoming proton. For comparison, standard events from deep-inelastic scattering exhibit uniform particle production in the rapidity space between the directions of the incoming virtual photon and the proton. The production characteristics of these large rapidity-gap events are those of a diffractive process in which no quantum numbers are exchanged between photon and proton, and where the cross section is found to be constant or rising logarithmically with the c.m. energy.

The discovery of events with a large rapidity gap in deep-inelastic electron–proton scattering opened the door for a systematic study of diffraction in a new domain of c.m. energy and photon virtuality. The real or virtual photon is transformed into a vector meson or a massive hadronic system. The mass squared ($-Q^2$) of the photon emitted by the incoming electron provides a hard scale which could be varied over a large range.

Overviews of the theoretical developments for diffraction at high energy and different photon virtualities can be found, e.g., in [20, 21]. Previous summaries of experimental results on diffraction from HERA have been provided by [22–24].

This paper reviews the results on diffraction in high energy photon–proton and deep-inelastic electron–proton scattering.
obtained until 2008 by the experiments H1 and ZEUS at HERA, which were in operation between 1992 and 2007.

In order to put the results on diffraction into perspective, the report starts with a brief review of the experimental results for the proton structure function $F_2$ and the total $\gamma p$ and $\gamma^* p$ cross sections.

2. Kinematics of inelastic electron–proton scattering

The reaction

$$e(k) \ p(P) \to e(k') + \text{anything},$$  \hspace{1cm} (1)$$

see figure 2, is described in terms of the four momenta $k$, $(k')$ of the incident (scattered) lepton ($e^+$ or $e^-$) and proton ($P$), with beam energies $E_e$ and $E_p$, respectively. At fixed squared centre-of-mass energy, $s = (k + P)^2 \approx 4E_eE_p$, deep-inelastic scattering is described in terms of $Q^2 \equiv -q^2 = -(k - k')^2$ and Bjorken-$x = Q^2/(2P \cdot q)$. The fractional energy transferred to the proton in its rest system is $y \approx Q^2/(sx)$. The centre-of-mass energy of the hadronic final state, $W$, is given by $W^2 = (P + q)^2 = m_p^2 + Q^2(1/x - 1) \approx Q^2/x = ys$, where $m_p$ is the mass of the proton.

In diffraction, proceeding via

$$\gamma^*p \to X + N,$$  \hspace{1cm} (2)
where $\gamma^* = e^− e^+$, a hadronic system $X$ is produced, see figure 3. The incoming proton undergoes a small perturbation by the emission of a Pomeron, which in lowest-order QCD can be represented by a two-gluon system. The proton emerges either intact ($N = p$), or as a low-mass nucleonic state $N$ with mass $M_N$, in both cases carrying a large fraction, $x_N$, of the incoming proton momentum. Diffraction is described in terms of $W, Q^2, M_X, M_N$ and of $t$, the four-momentum transfer squared between the incoming proton and the outgoing system $N$, $t = (P - N)^2$. Alternatively, diffraction is parametrized in terms of $xp$, the fraction of the proton momentum carried by the Pomeron, and $\beta$, the momentum fraction of the struck quark within the Pomeron:

$$x_p = \frac{(P - P') \cdot q}{P \cdot q} = \frac{M_X^2 + Q^2 - t}{W^2 + Q^2 - M_p^2} \approx \frac{M_X^2 + Q^2}{W^2 + Q^2},$$

$$\beta = \frac{Q^2}{2(P - P') \cdot q} = \frac{x}{x_p} = \frac{Q^2}{M_X^2 + Q^2 - t} \approx \frac{Q^2}{M_X^2 + Q^2}.$$ (3) (4)

3. The H1 and ZEUS detectors

For the study of diffraction and of the total photon–proton cross section at HERA it was essential that the detectors of H1 [25, 26] and ZEUS [27, 28] covered the full solid angle (up to the beam pipe) with calorimetry. Figures 4 and 5 show side views of the two detectors. The H1 detector had tracking detectors around the interaction region and a large liquid argon calorimeter equipped with lead plates as absorber for particle detection. In addition, hadronic energy in the pseudorapidity range $5.0 < \eta < 6.6$ (where $\eta = \ln \tan(\theta)/2$ and $\theta$ is the polar angle in radians) might have produced charged particles by secondary interactions in the collimators, beam pipe, etc, which were then detected in the muon chambers.

The ZEUS detector employed tracking detectors and a large, almost hermetic calorimeter consisting of uranium plates interspaced with scintillator for signal collection, which provided equal response to electrons, photons and hadrons of the same momentum. For a substantial fraction of the data taken by ZEUS, the forward area close to the outgoing proton beam ($\eta = 4 - 5$) was instrumented with the forward-plug calorimeter (FPC), which had been inserted in the uranium-scintillator calorimeter leaving a hole of only 3.15 cm radius for the passage of the beams. The FPC was a lead-scintillator sandwich calorimeter with 5.4 nuclear absorption lengths that provided equal response to electrons and hadrons of the same energy [29].

Close to the direction of the outgoing proton beam, at a distance of about 100 m from the central detector, H1 and ZEUS were equipped with spectrometers and calorimeters (not shown) for the detection of leading protons and neutrons.

4. The structure function $F_2(x, Q^2)$ of the proton

The differential cross section for inclusive ep scattering, (equation (1)), is represented in terms of the structure functions $F_2$ and $F_L$ of the proton by

$$\frac{d^2 \sigma^{e^- p}}{dx \ dQ^2} = 2\pi \alpha^2 \int [Y F_2(x, Q^2) - y^2 F_L(x, Q^2)](1 + \delta_t(x, Q^2)), \quad (5)$$

where $Y = 1 + (1 - y)^2$. In general, the structure function $F_2$ represents the main component of the cross section. In the deep-inelastic scattering (DIS) factorization scheme, $F_2$ corresponds to the sum of the momentum densities of the quarks and antiquarks weighted by the squares of their electric charges; $F_L$ is the longitudinal structure function and $\delta_t$ is a term accounting for radiative corrections. The contribution of $F_L$ to the cross section relative to that from $F_2$ is given by $(y^2/Y) \cdot (F_L/F_2)$. For the determination of $F_2$ by ZEUS [30, 31], the $F_L$ contribution was taken from the ZEUS next-to-leading order (NLO) QCD fit [32]. The contribution of $F_L$ to the cross section in the highest $y$ (lowest $x$) bin of this analysis was 3.2%, decreasing to 1.3% for the next highest $y$ bin. For the other bins, the $F_L$ contribution is below 1%. The resulting uncertainties on $F_2$ are below 1% [30, 31].

Figure 5. Side view of the ZEUS detector with the FPC integrated in the ZEUS calorimeter. The proton (electron) beam enters from the right (left). Figure provided by ZEUS.
In the $Q^2$ range considered here ($Q^2 \leq 500$ GeV$^2$), the contributions from $Z^0$ exchange and $Z^0–\gamma$ interference were estimated to be at most 0.4% [30, 31] and were ignored.

The structure function $F_2$ is shown in figure 6 as a function of $x$ for fixed $Q^2$. The results by H1 [33] and ZEUS [30, 31] are presented together with the results from the fixed-target experiments [34], E665 [35], NMC [36] and BCDMS [37]. For $Q^2$ between 0.11 and 0.65 GeV$^2$ and $x < 0.01$, the structure function $F_2$ exhibits a modest rise as $x \to 0$. At larger values of $Q^2$, starting from $x = 0.7$, $F_2$ is rising with decreasing $x$, reaching a plateau around $x = 0.1$. Below $x \approx 0.1 - 0.01$ and $Q^2 \geq 1.3$ GeV$^2$ the HERA measurements show $F_2$ growing rapidly as $x \to 0$; the growth accelerates as $Q^2$ increases from 1.3 to 320 GeV$^2$.

The most recent measurement of the longitudinal structure function $F_L$ at HERA, as provided by H1 [38], is shown in figure 7. Within errors, $F_L = 0.2–0.4$ for the $Q^2$ values measured.

The leading-order (LO) and NLO QCD diagrams for deep-inelastic electron–proton scattering are shown in figure 8. The QCD cascade which develops at large energies is illustrated by figure 9. The solid curves in figure 6 show the ZEUS NLO QCD fit [32] to data from ZEUS and fixed-target experiments.
with $W$ in units of GeV, $A = 57 \pm 5 \mu b$, $B = 121 \pm 13 \mu b$, $\delta = 0.200 \pm 0.024$; $\eta$ had been fixed to the value $0.716 \pm 0.030$ [46, 47]. The dot–dashed curve shows a fit which includes a soft and a hard Pomeron trajectory [48].

5.2. Virtual photons, $Q^2 > 0$

The total cross section for virtual photon–proton scattering, $\sigma_{\gamma p}^{tot} = \sigma_T(x, Q^2) + \sigma_L(x, Q^2)$, where $T(L)$ stands for transverse (longitudinal) photons, was extracted from the measurement of $F_2$ using the relation

$$\sigma_{\gamma p}^{tot} = \frac{4\pi^2\alpha}{Q^2(1-x)}F_2(x, Q^2),$$

which is valid for $4m_p^2x^2 \ll Q^2$ [49]. The total cross section for virtual photons is shown in figure 12 as a function of $W$ for fixed $Q^2$ between zero and 320 GeV$^2$. The lines show the power fit presented above for the structure function $F_2$, see equation (6). The power $\delta$, describing the rise $\sigma_{\gamma p}^{tot} \propto W^\delta$, is increasing with $Q^2$ from $\delta = 0.2$ at $Q^2 = 0$ to $\delta \approx 0.8$ at $Q^2 = 320$ GeV$^2$; the latter value of $\delta$ implies an almost linear rise with $W$. Such a rise with $W$ has not been seen in hadron–hadron interactions. The $W$-dependence can be expressed in terms of the Regge trajectory of the Pomeron, $\alpha_P = 1 + \delta/4$; the data yield for $Q^2 = 0$: $\alpha_P = 1.05$ and for $Q^2 = 320$ GeV$^2$: $\alpha_P = 1.2$. The NLO QCD fit presented above for $F_2(x, Q^2)$ also gives a good description of $\sigma_{\gamma p}^{tot}$.

Following the discussion of [50], $\sqrt{Q^2}$ determines the transverse size which the photon can ‘resolve’ in the proton, $d \approx 2.10^{-14}$ (cm)/$Q$(GeV). At small $Q^2$, the behaviour is
still very much the same as in hadron–hadron scattering: the photon resembles a hadron, e.g. a $\rho^0$ meson. With increasing $Q^2$, the photon shrinks and becomes pointlike: the rise in $W$ becomes stronger, and the overall magnitude of the cross section decreases.

6. Diffraction

First evidence for a substantial contribution from diffraction in deep-inelastic scattering was reported by ZEUS [18], observing events in which no energy is deposited close to the direction of the proton beam, i.e. at large angle $\theta$, see figure 13. Described in terms of pseudorapidity $\eta$, where

$$\eta = -\ln(\tan \theta/2), \quad (8)$$

nondiffractive events show uniform particle production in $\eta$, see figure 14(a), while diffractive events exhibit a sizeable gap between the most forward calorimeter cells ($\theta = 1.5^\circ$, $\eta = 4.3$) and the pseudorapidity of the hadron(s) observed closest to the proton direction, see figure 14(b): in diffractive scattering particle emission is concentrated around the directions of the incoming photon and proton.

Figure 15 illustrates the different topologies of nondiffractive events (bottom) and diffractive events (top), as registered in the ZEUS detector. Figure 16 shows the distribution of the maximum rapidity ($\eta_{\text{max}}$) observed in the calorimeter for events from deep-inelastic scattering with $Q^2 > 10$ GeV$^2$. There is a large excess of events at $\eta_{\text{max}} < 2$ compared with the expectation from nondiffractive deep-inelastic scattering (shaded histogram).

Different methods have been employed by H1 and ZEUS for isolating diffractive contributions. In the case of vector meson production, $\gamma^* p \rightarrow V N$, resonance signals in the decay mass spectrum combined with the absence of other substantial activity in the detector have been used. The contribution from inclusive diffraction has been extracted using the presence of a large rapidity gap, the detection of the leading proton or the hadronic mass spectrum observed in the central detector ($M_X$ method [51, 52]). Event samples selected on the basis of a large rapidity gap or a leading proton may include additional contributions from Reggeon exchange. Such contributions are exponentially suppressed when using the $M_X$ method.

7. Deeply virtual Compton scattering

In deeply virtual Compton scattering, DVCS, the incoming electron emits a virtual photon with mass squared $-Q^2$, which in turn scatters on the proton and is emitted as a real photon:

$$e p \rightarrow e' + \gamma^* + p \rightarrow e' + \gamma + p. \quad (9)$$
Figure 15. Event topologies of diffractive (top) and nondiffractive deep-inelastic scattering (bottom) as seen in the ZEUS detector. The top figure shows also the leading proton spectrometer (LPS) which detected forward scattered protons. Shown are the tracks of charged particles in the central tracking chamber and the energy deposits in the uranium-scintillator calorimeter. Figure provided by ZEUS.

Figure 16. Deep-inelastic scattering, ep → eX: distribution of $\eta_{\text{max}}$, the maximum rapidity of a calorimeter cluster in an event, for data, and for Monte Carlo events generated for nondiffractive scattering (shaded).

This process can be regarded as the (quasi) elastic scattering of a virtual photon off the proton, namely, $\gamma^* + p \rightarrow \gamma + p$. Figure 17 shows the diagram for DVCS together with the diagrams for the Bethe–Heitler contribution, which is purely electromagnetic and leads to the same final state.

Measurements of DVCS at HERA were reported by H1 [53, 54] and ZEUS [55, 56]. The combined data presented in figure 18 show that for fixed $W = 82$ GeV the DVCS cross section, $\sigma_{\text{DVCS}}$, falls rapidly with increasing $Q^2$, while for fixed $Q^2$ and $W$, $\sigma_{\text{DVCS}}$ decreases exponentially with $|t|$, the four-momentum transfer squared between incoming and outgoing proton: $d\sigma/d|t| \propto e^{-bt}$ (see figure 19). A fit to the H1 data at average values of $Q^2 = 8$ GeV$^2$ and $W = 82$ GeV, yielded $b = 6.02 \pm 0.35(\text{stat.}) \pm 0.39(\text{syst.})$ GeV$^{-2}$. The slope $b$ is shown below (figure 27) as a function of $(Q^2 + M_V^2)$ (where $M_V^2 = 0$ for DVCS) together with the $b$-values for the production of vector mesons with mass $M_V$: within errors, the $b$-values from DVCS and from the production of vector mesons lie on a universal curve. The H1 data show that the DVCS cross section scales as a function of $\tau = Q^2/Q_s(x)$ where $Q_s(x) = Q_0 (x_0/x)^{-\lambda/2}$ with $Q_0 = 1$ GeV, $\lambda = 0.25$ and $x_0 = 2.7 \times 10^{-5}$, see figure 20.

The $Q^2$ dependence of the DVCS cross section for fixed $W$, as well as its $W$ dependence for fixed $Q^2$, have been compared with QCD calculations by [57] which assume that the virtual photon fluctuates into a q$\bar{q}$ system, which in turn interacts with the target proton. The observed increase in the DVCS cross section with $W$ at fixed $Q^2$ is due to the increase in the gluon content of the proton with decreasing $x$. 

Figure 17. Diagrams for deeply virtual Compton scattering (DVCS) (a) and for photon production via the Bethe–Heitler process (b), (c).
The QCD calculations are based on twist-2 generalized parton distributions (GPDs) which have been extracted from experimental data. Comparison with the H1 and ZEUS data shows that LO parton distributions (MRST2001 [58]) lead to a good description of the data, see figure 18: here the calculated DVCS cross section (curves) is compared with the HERA data for fixed \( W = 82 \text{ GeV} \) as a function of \( Q^2 \), and for fixed \( Q^2 = 8 \text{ GeV}^2 \) as a function of \( W \).

8. Production of vector mesons by real and virtual photons

8.1. \( \rho^0 \) production

8.1.1. \( \gamma p \rightarrow \rho^0 p \). First results from HERA on \( \rho^0 \) production by real photons, \( \gamma p \rightarrow \rho^0 p \), were presented in [41, 59]. The second analysis is based on 6000 events of the type \( \gamma p \rightarrow e\pi^+\pi^- \) at an average energy \( W = 70 \text{ GeV} \), obtained with an integrated luminosity of 0.55 pb\(^{-1}\). The final-state electron and proton were not detected. Figure 21(top) shows the \( \pi^+\pi^- \) mass spectrum in terms of the differential cross section \( d\sigma/dM_{\pi\pi} \) for events with four-momentum transfer squared between incoming photon and proton, \( |t| < 0.5 \text{ GeV}^2 \). The mass spectrum is skewed compared with a Breit–Wigner distribution which can be understood in terms of the interference between the resonant \( \pi^+\pi^- \) production and a non-resonant \( (\gamma p \rightarrow \pi^+\pi^- p) \) Drell-type background [60], as explained by Söding [61].

The \( W \) dependence of the \( \rho^0 \) cross section is shown in figure 21(bottom) which includes measurements from fixed-target experiments [62–67]. The ZEUS data indicate a substantial rise of \( \sigma_{\gamma p \rightarrow \rho^0 p} \) for \( W > 60 \text{ GeV} \). The differential cross section, \( d\sigma/d|t| \)—see figure 22 (top)—is proportional to \( e^{-bt} \) with \( b = 10.4 \pm 0.6 \text{(stat.)} \pm 1.1 \text{(syst.)} \text{ GeV}^{-2} \) for \( |t| < 0.15 \text{ GeV}^{-2} \). In a measurement of \( \rho^0 \) production by ZEUS, where the four-momentum of the scattered proton was determined directly, within errors the same value was obtained, namely \( b = 9.8 \pm 0.8 \text{(stat.)} \pm 1.1 \text{(syst.)} \text{ GeV}^{-2} \) [68]. A similar behaviour of \( d\sigma/d|t| \) was observed by H1 [69] where \( b = 10.9 \pm 2.4 \text{(stat.)} \pm 1.1 \text{(syst.)} \text{ GeV}^{-2} \) was measured. The
value of the slope $b$ is comparable to what is found for elastic $\pi p$ scattering.

The decay angular distribution of the $\rho^0$ mesons has been used to determine the orientation of the $\rho^0$ spin [70, 71]. The exclusive electroproduction of $\rho^0$ mesons and their decay into $\pi^+\pi^-$ are described by three angles: $\Phi_h$—the angle between the $\rho^0$ production plane and the electron scattering plane in the $\gamma^* p$ c.m. system and $\theta_h, \phi_h$—the polar and azimuthal angles of the $\pi^+ \pi^-$ w.r.t. the proton direction, all determined in the $\rho^0$ rest system. The $\rho^0$ decay angular distribution, $W(\cos \theta_h, \phi_h, \Phi_h)$, can be parametrized by the $\rho^0$ spin-density matrix elements, $\rho^{0i}_{jk}$, with $i, k = -1, 0, 1$ and $\alpha = 0, 1, \ldots, 6$. If no separation is made between the contributions from transverse and longitudinal photons, the $\rho^0$ s-channel helicity decay angular distribution w.r.t. $\theta_h$ and $\phi_h$ can be expressed in terms of the matrix elements $\rho^{0i}_{jk}$ as follows [71]:

$$\frac{1}{N} \frac{dN}{d\cos \theta_h} = \frac{3}{4} \left[ 1 - r^{04}_{00} + (3r^{04}_{00} - 1) \cdot \cos^2 \theta_h \right],$$

$$\frac{1}{N} \frac{dN}{d\phi_h} = \frac{1}{2\pi} \left[ 1 - 2r^{04}_{1-1} \cos 2 \phi_h \right].$$

where the density matrix element $r^{04}_{04}$ represents the probability that the $\rho^0$ is produced longitudinally polarized.

The distributions of $\cos \theta_h$ and $\phi_h$ for $\rho^0$ production by real photons [59] are shown in figure 22. The dominant contribution is proportional to $\sin^2 \theta_h$ which shows that the $\rho^0$ mesons are mostly transversely polarized, as verified by the small value of $r^{04}_{00} = 0.055 \pm 0.028$. The $\phi_h$ distribution is constant, yielding a value of $r^{04}_{1-1} = 0.008 \pm 0.014$. The values of $r^{04}_{00}$ and $r^{04}_{1-1}$ are compatible with zero as expected.
for s-channel helicity conservation (SCHC): the $\rho^0$ in the final state has the same helicity as the photon in the initial state.

8.1.2. $\gamma^* p \to \rho^0 p$. The first observation at HERA of $\rho^0$ production by virtual photons, $\gamma^* p \to \rho^0 p$ (see figure 23) leading to the final state $\pi^+ \pi^- p$ [72], is shown in figure 24. The $M_{\pi^+ \pi^-}$ mass spectrum is completely dominated by the production of $\rho$ mesons. The cross section for $\gamma^* p \to \rho^0 p$ shows a fast decrease with $Q^2$, namely $\sigma(\gamma^* p \to \rho^0 p) \propto Q^{−4−0.8^{+1.5}}$. For fixed $Q^2 > 8\text{ GeV}^2$, the cross section rises by about a factor of 5 from $W = 12\text{ GeV}$ to $W = 60\text{ GeV}$.

The distribution of the polar angle for the decay $\rho^0 \to \pi^+ \pi^0$ is shown in figure 24(c). The dominant $\cos^2 \theta$-type distribution shows that the $\rho$-mesons are mostly longitudinally aligned. Further results on $\gamma^* p \to \rho^0 p$ from HERA were reported in [73–75].

A first precise measurement of the helicity amplitudes for $\gamma^* p \to \rho^0 p$ at HERA was provided by H1 [77]. It showed that the helicity is conserved in the s-channel except for a small but non-zero helicity-flip amplitude at the level of $8 \pm 3\%$ of the non-flip amplitudes. This is evidence for a (small) contribution of longitudinal $\rho^0$ mesons produced by transverse photons. The finding was corroborated by results from ZEUS [78]. For comparison, in a fixed-target experiment at low $W = 1.7–2.8\text{ GeV}$ and low $Q^2 = 0.3–1.4\text{ GeV}^2$, the ratio of single flip to non-flip amplitudes had been found to be of order 15–20% [79].

An extended study of $\gamma^* p \to \rho^0 p$ was presented by ZEUS [80], based on an integrated luminosity of 120 pb$^{-1}$. The $\pi^+ \pi^−$ mass distribution (figure 25) is dominated by $\rho^0$ production$^1$. The cross section for $\gamma^* p \to \rho^0 p$, as measured by H1 and ZEUS, is presented in figure 26 as a function of $W$ for $Q^2$ values between 2 and 19.5 GeV$^2$. For fixed $W$, $\sigma(\gamma^* p \to \rho^0 p)$ is falling rapidly with $Q^2$. For fixed $Q^2$, the cross section rises with $W$. A parametrization of the form $\sigma(\gamma^* p \to \rho^0 p) \propto W^4$ yielded the $\delta$ values shown in figure 27 as a function of $(Q^2 + M_V^2)$: the power $\delta$ rises from around 0.15 at $Q^2 \approx 0$ to about 0.8 at $(Q^2 + M_V^2) = 30\text{ GeV}^2$.

$^1$ The skewing of the mass distribution is explained by the Söding model, see above. The excess of events observed at masses below 0.65 GeV is due to background from $\phi \to \pi^+ \pi^- \rho^0$, where the $\rho^0$ was not detected, and from $\phi \to K^+ K^−$, where the K’s were misidentified as $\pi$’s.

Figure 24. First observation of $\gamma^* p \to \rho^0 p$ for $30 < W < 130\text{ GeV}$, $7 < Q^2 < 25\text{ GeV}^2$, as measured by ZEUS. (Top): (a) $\pi^+ \pi^−$ mass distribution; (b) cross section for $\gamma^* p \to \rho^0 p$ as a function of $Q^2$: solid points from ZEUS; open triangles: results for $9 < W < 19\text{ GeV}$ from the fixed-target experiment NMC-D; (c) $\cos^2 \theta$ distribution for the decay-$\gamma^*$, in the s-channel helicity system; (d) $\rho^0$ density matrix element $\rho_{00}$ as a function of $Q^2$, compared with results from fixed-target experiments; (bottom): cross section for $\gamma^* p \to \rho^0 p$ as a function of $W$ for different values of $Q^2$: from fixed-target experiments and from ZEUS. Figures reprinted with permission from ZEUS [72].

The $Q^2$ dependence of $\sigma(\gamma^* p \to \rho^0 p)$ is shown in detail [80] in figure 28 for $W = 90\text{ GeV}$. There is an almost exponential decrease from $\sigma \approx 700\text{ nb}$ at $Q^2 = 2\text{ GeV}^2$ to 0.04 nb at $Q^2 = 100\text{ GeV}^2$. The measurements were compared with several models which are based on the dipole representation of the virtual photon: the photon fluctuates into...
Figure 25. The $\pi^+\pi^-$ acceptance-corrected invariant-mass distribution. The line represents a fit of the Söding form for $0.65 < M_{\pi^+\pi^-} < 1.1$ GeV. The vertical lines indicate the mass range used for the analysis. Figure reprinted with permission from ZEUS [80].

Figure 26. Cross section for $\gamma^* p \rightarrow \rho^0 p$ as a function of $W$ for different values of $Q^2$. Figure reprinted with permission from ZEUS [80].

Figure 27. The power $\delta$ determined from a fit of the cross section $\sigma(\gamma^* p \rightarrow V p) \propto W^\delta$ as a function of $(Q^2 + M^2_V)$, where $M_V$ is the mass of the vector meson indicated; for DVCS: $M_V = 0$. The data stem from H1 and ZEUS. Figure reprinted with permission from ZEUS [80].

Figure 28. Cross section for $\gamma^* p \rightarrow \rho^0 p$ as a function of $Q^2$ for $W = 90$ GeV, from ZEUS; the data presented in (a) and (b) are the same. Shown are also the predictions of several models, see text. Figure reprinted with permission from ZEUS [80].

Figure 29. The dependence of $b$ on $(Q^2 + M^2_V)$, where $M_V$ is the mass of the vector meson, is shown in figure 30. For $(Q^2 + M^2_V) \lesssim 1$ GeV$^2$, $b$ is of the order of $10$–$12$ GeV$^{-2}$, similar to what has been measured for elastic hadron–hadron scattering at high energy; e.g. for pp scattering at $W = 31$–$62$ GeV, $b \approx 11$ GeV$^{-2}$ [85]. At larger values of $(Q^2 + M^2_V)$ the slope $b$ decreases rapidly, as predicted by [86], approaching a constant value of $b \approx 5$ GeV$^{-2}$ for $(Q^2 + M^2_V) > 10$ GeV$^{-2}$. In an optical model, $b$ is proportional to the sum of the radii squared of the virtual photon and a $q\bar{q}$ pair—the colour dipole—which in turn interacts with the gluon cloud of the proton to produce the $\rho^0$, see, e.g., [81]. For $Q^2 > 1$ GeV$^2$, the calculations of [82] (DF) and [83] (KMW) give a good description of the data while those of FSS [84] are somewhat low for $Q^2$ above $10$ GeV$^2$.

As a function of $|t|$, the differential cross section $d\sigma/d|t|$ at $W = 90$ GeV falls exponentially with $|t|$, $d\sigma/d|t| \propto e^{-bt}$ [80], see figure 29. The dependence of $b$ on $(Q^2 + M^2_V)$, where $M_V$ is the mass of the vector meson, is shown in
the proton, $b = [(R_{p,v})^2 + (R_{p,v})^2]/4$. As $Q^2$ increases, the transverse extension of the virtual photon, which is expected to be proportional to $1/Q$, goes to zero, such that $b \rightarrow (R_p)^2/4$, as has been predicted by [87].

The $W$ dependence of $d\sigma/d|t|$ at fixed values of $|t|$ was used to determine the parameters of the Pomeron trajectory, $\alpha_0$. The same behaviour is observed for all three vector meson species: $\gamma^* p \rightarrow V p \rightarrow |t|$, as a function of $Q^2$. The data suggest a small increase of $\alpha_0$ with increasing $Q^2$.

Figure 30. The slope $b$ describing the $t$ dependence of vector meson production: $\gamma^* p \rightarrow V p \propto e^{-b|t|}$, as a function of $(Q^2 + M^2_V)$ where $M_V$ is the mass of the vector meson indicated; for DVCS $M_V = 0$. The data stem from H1 and ZEUS. Figure reprinted with permission from ZEUS [80].

The data suggest a small increase of $\alpha_0$ with increasing $Q^2$.

Figure 31. Reaction $\gamma^* p \rightarrow V p$: the density matrix element $r_{04}^p$ as a function of $(Q^2/M^2_V)$ for $V = \rho^0, \phi, J/\Psi$. Figure reprinted with permission from ZEUS [105].

The ratio $R = \sigma_L/\sigma_T$ for $\gamma^* p \rightarrow \rho^0 p$ is shown in figure 32: $R$ rises rapidly with $Q^2$, reaching unity at $Q^2 \approx 2\text{GeV}^2$ and values around 4 at $Q^2 = 20\text{GeV}^2$: for $Q^2 > 2\text{GeV}^2$ the
dominant contribution to $\gamma^* p \to \rho^0 p$ comes from longitudinal photons. The ratio of the contribution from longitudinal photons to the total cross section for $\rho^0$ production—as given by the density matrix element $r_{00}^\rho$ (see figure 33)—illustrates the preponderance of longitudinal photon contributions at large $Q^2$. The measurements are well described by the models of [84, 88], which are based on two-gluon exchange.

The complete set of density matrix elements $\rho_{ik}^\alpha$ for $\gamma^* p \to \rho^0 p$ is shown in figure 34 as a function of $Q^2$. A strong dependence on $Q^2$ is observed for $r_{00}^\alpha$, $r_{1-1}^\alpha$, $\text{Im} r_{1-1}^\alpha$ and $r_{50}^\alpha$. Within errors, $r_{1-1}^\alpha = -\text{Im} r_{1-1}^\alpha$ and $\text{Re} r_{10}^\alpha = -\text{Im} r_{10}^\alpha$, in agreement with SCHC. Under the assumption of SCHC, the dependence on $Q^2$ is driven by the dependence of $R = \sigma_L/\sigma_T$ on $Q^2$, i.e. by the rapid increase in the contribution from longitudinal relative to that from transverse photons.

8.2. $\omega$ production

The data on $\omega$ production at HERA are scanty, due mainly to the difficulty in reconstructing the $\pi^0$ of the final state $\gamma p \to \omega p \to p \pi^+ \pi^- \pi^0$. Two measurements have been published by ZEUS, one on $\omega$ production by real photons [89], based on an integrated luminosity of 0.89 pb$^{-1}$, and one on $\omega$ production by virtual photons [90] (integrated luminosity 37.7 pb$^{-1}$).

**Figure 33.** Reaction $\gamma^* p \to \rho^0 p$: the ratio $R = \sigma_L/\sigma_T$ as a function of $Q^2$: the data from H1 were determined for $W = 75$ GeV, those from ZEUS for $W = 90$ GeV. Figure reprinted with permission from ZEUS [80].

**Figure 34.** Reaction $\gamma^* p \to \rho^0 p$: the $\rho^0$ density matrix elements $\rho_{ik}^\alpha$ as a function of $Q^2$. Figure reprinted with permission from ZEUS [80].
8.2.1. $\gamma p \rightarrow \omega p$. The $\gamma\gamma$ and $\pi^+\pi^-\pi^0$ mass spectra, measured at $Q^2 = 0$, $W = 70–90 \text{ GeV}$ (figure 35), show clear signals for $\pi^0$ (figure 35(a)), $\omega$ and $\phi$ production (figure 35(b)) [89]. The cross section $\sigma(\gamma p \rightarrow \omega p)$ is presented in figure 36 as a function of $W$ together with measurements by fixed target experiments from [62, 63, 66, 67, 91–98]. It is large at low c.m. energies, $W < 2 \text{ GeV}$, due to the contribution from one-pion exchange which decreases approximately proportional to $1/(W^2 - m_p^2)^2$. For $W > 6 \text{ GeV}$ the cross section is approximately constant as a function of $W$, as expected for diffractive production. The ZEUS measurement yielded $\sigma(\gamma p \rightarrow \omega p) = 1.21 \pm 0.12(\text{stat.}) \pm 0.23(\text{syst.}) \mu\text{b}$ at $W = 80 \text{ GeV}$.

The distributions of the polar and azimuthal decay angles of the $\omega$ in the helicity system are shown in figure 37. They are of the form $dN / d\cos \theta_h \propto 1 - \cos^2 \theta$ and $dN / d\phi_h = \text{constant}$ and yield for $W \approx 80 \text{ GeV}$: $r^{0\pi}_0 = 0.11 \pm 0.08(\text{stat.}) \pm 0.26(\text{syst.})$ and $r^{0\pi}_1 = -0.04 \pm 0.08(\text{stat.}) \pm 0.12(\text{syst.})$. These values are compatible with those measured for $\gamma p \rightarrow \rho^0 p$. 
see above, and are in agreement with s-channel helicity conservation SCHC [89].

8.2.2. $\gamma^*p \rightarrow \omega p$. The production of $\omega$ mesons by virtual photons was studied [90] for the kinematic region: $3 < Q^2 < 20 \text{ GeV}^2$, $40 < W < 120 \text{ GeV}$ and $|t| < 0.6 \text{ GeV}^2$. Only events with $0.3 < M_{\pi^+\pi^-} < 0.6 \text{ GeV}$ were kept. The results are presented in figure 38. The $\pi^+\pi^-\pi^0$ mass spectrum shows clear signals for the production of $\omega$ and $\phi$ mesons. The cross section for $\gamma^*p \rightarrow \omega p$ at $Q^2 = 7 \text{ GeV}^2$ is a factor of about 10 smaller than for $\gamma^*p \rightarrow \rho^0 p$ and shows, for fixed $W = 70 \text{ GeV}$, a rapid decrease with $Q^2$, similar to the $Q^2$-behaviour of $\sigma(\gamma^*p \rightarrow \rho^0 p)$.

8.3. $\phi$ production

First observations of $\phi$ production by real and virtual photons were reported by ZEUS [99, 102] for integrated luminosities of 0.9 pb$^{-1}$ and 2.6 pb$^{-1}$, respectively.

8.3.1. $\gamma p \rightarrow K^+K^-p$. The reaction $\gamma p \rightarrow K^+K^-p$ measured at a c.m. energy $W = 70 \text{ GeV}$ [99] exhibits a clean $\phi$ signal in the $M_{K^+K^-}$ mass spectrum (figure 39). The comparison with
10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 5 10 50 100 500

Figure 40. Reaction $\gamma^* p \rightarrow \phi p$: distributions of the K$^+$K$^-$ mass and of $\cos \theta_h$, where $\theta_h$ is the decay angle, for candidate $\phi$ events (top); cross section as a function of $W$ for the $Q^2$ values indicated, as measured by fixed-target experiments and by ZEUS (bottom). Figures reprinted with permission from ZEUS [102].

results at lower energy shows that the cross section $\sigma(\gamma p \rightarrow \phi p)$ rises with $W$. The differential cross section, $d\sigma/d|t|$, falls exponentially with $|t|$. The distributions of the $\phi$-meson decay angles $\theta_h$ and $\phi_h$ measured in the helicity system are shown in figure 39 (bottom). They yield $r_{01}^0 = -0.01 \pm 0.04$ and $r_{1-1}^0 = 0.03 \pm 0.05$. These values are consistent with SCHC, and agree with those measured for $\gamma p \rightarrow \rho^0 p$ and $\gamma p \rightarrow \omega p$, see above.

8.4. $\gamma^* p \rightarrow \phi p$

First results from HERA on $\phi$ production by virtual photons, $\gamma^* p \rightarrow \phi p$, have been published by ZEUS [102], based on an integrated luminosity of 2.6 pb$^{-1}$. The K$^+$K$^-$ mass spectrum (figure 40) shows the $\phi$-signal. The distribution of the decay angle $\theta_h$ is approximately proportional to $\cos^2 \theta_h$, indicating predominant production of longitudinal $\phi$'s, in contrast to $\phi$ production by real photons for which the distribution is proportional to $\sin^2 \theta_h$, see figure 39.

Further results on $\phi$ production were presented by H1 [103] (integrated luminosity $L = 3.1$ pb$^{-1}$) and ZEUS [104, 105] ($L = 45$ and 119 pb$^{-1}$, respectively). Figure 41 shows the results from ZEUS: the K$^+$K$^-$ mass spectrum is completely dominated by $\phi$ production. The cross section for $\gamma^* p \rightarrow \phi p$ at $Q^2 = 2.4, 6.5$ and 13.0 GeV$^2$ (figure 41(a))
shows a moderate rise with $W$. The dependence on $(Q^2 + M_{\phi}^2)$ is shown in figure 41(b) for $W = 75$ GeV, separately for the contributions from longitudinal (L) and transverse photons (T). At $(Q^2 + M_{\phi}^2) = 3 \text{ GeV}^2$, $\sigma_T$ and $\sigma_L$ are approximately equal; for larger values of $(Q^2 + M_{\phi}^2)$, $\sigma_T$ falls off more rapidly than $\sigma_L$. At $(Q^2 + M_{\phi}^2) \approx 45 \text{ GeV}^2$, $\sigma_L$ is larger than $\sigma_T$ by about a factor of 10.

The $|t|$ dependence of $\phi$ production is displayed in figure 42(top) for different values of $Q^2$. The data yield $d\sigma/d|t| \propto e^{-b|t|}$, with $b \approx 5.5 \text{ GeV}^{-2}$ for $(Q^2 + M_{\phi}^2) > 5 \text{ GeV}^2$, see figure 30.

The distribution of the $\phi$ polar decay angle in the helicity system, $\cos\theta_h$, is displayed in figure 42 (bottom): it is of the form $dN/d\cos\theta_h \propto a + \cos^2\theta_h$, where $a$ tends to zero as $Q^2$ increases from 2.4 to 19.7 GeV$^2$. This implies that, at large $Q^2$, the $\phi$ mesons are predominantly longitudinally polarized, as has been observed for $\rho^0$ production (see above). Under the assumption of SCHC, $\phi$ production at $Q^2 \geq 2.4 \text{ GeV}^2$ proceeds predominantly by longitudinal-photon–proton scattering.

### 8.5. $J/\Psi$ production

#### 8.5.1. $\gamma p \rightarrow J/\Psi p$

First evidence at HERA for $J/\Psi$ production by $\gamma p$ scattering was obtained by H1 [106] from the analysis of events with two leptons in the final state (40 $e^+e^-$ and 48 $\mu^+\mu^-$ events, respectively). The $l^+l^-$ mass spectrum (figure 43) shows a peak at the mass of the $J/\Psi$. The cross section for $\gamma p \rightarrow J/\Psi p$ measured by H1 for $Q^2 < 4 \text{ GeV}^2$, $30 < W_{\gamma p} < 180 \text{ GeV}$, indicates a substantial rise with $W$ compared with the fixed-target measurements at lower energies [107–113], and an exponential decrease with the $J/\Psi$ transverse momentum squared $p_T^2$, see figure 44.

The $e^+e^-$ and $\mu^+\mu^-$ mass distributions obtained in photoproduction by ZEUS [114] are shown in figures 45 and 46; they are based on integrated luminosities of 55.2 pb$^{-1}$ and 38.0 pb$^{-1}$, providing about 11000 and 6500 $J/\Psi$ events, respectively. For $W$ values between 35 and 260 GeV, $J/\Psi$ production is observed with little background.

The $W$ dependence of the cross section $\sigma(\gamma p \rightarrow J/\Psi p)$ as obtained by ZEUS [114] and H1 [115] is shown in figure 47. There is excellent agreement between the two measurements.
The $J/\Psi p$ cross section rises proportional to $W^3$, where $\delta = 0.75 \pm 0.03$ (stat.) $\pm 0.03$ (syst.), as determined by H1.

The $|t|$ dependence of the cross section as measured by ZEUS [114] is shown in figure 48 for several bins in $W$ between 50 and 290 GeV: $d\sigma / d|t|$ is of the form $c \cdot e^{-b|t|}$. The slope $b$ rises with $W$, as shown by figure 48. A fit of the data [114] to the form

$$b(W) = b_0 + 4 \cdot \alpha_p \cdot \ln(W/(90 \text{ GeV})),$$

(12)

where $\alpha_p$ is the slope of the Regge trajectory of the Pomeron, yielded $b_0 = 4.15 \pm 0.05$ (stat.) $^{+0.10}_{-0.13}$ (syst.) GeV$^{-2}$ and $\alpha_p = 0.116 \pm 0.026$ (stat.) $^{+0.010}_{-0.025}$ (syst.) GeV$^{-2}$, see the straight line. The effective Pomeron trajectory deduced by H1 [115] from the H1 and ZEUS data is shown in figure 49 as a function of $|t|.

For $Q^2 = 0.05$ GeV$^2$:

$$\alpha_p = [1.224 \pm 0.010 \text{ (stat.)} \pm 0.012 \text{ (syst.)}]$$

$$- [0.164 \pm 0.028 \text{ (stat.)} \pm 0.030 \text{ (syst.)}] \cdot |t|$$
Figure 44. Reaction $\gamma p \rightarrow J/\Psi X$: cross section as a function of $W$ (left); differential cross section $d\sigma/dp_T^2$ as a function of $p_T^2$ (right). Figure reprinted with permission from H1 [106].

Figure 45. Reaction $\gamma p \rightarrow e^+e^−p$: $e^+e^−$ mass distributions for the $W$ intervals (a) 20–35 GeV, (b) 35–50 GeV, (c) 50–90 GeV, (d) 90–140 GeV, (e) 140–200 GeV, (f) 200–260 GeV and (g) 260–290 GeV. Figure reprinted with permission from ZEUS [114].

and for $Q^2 = 8.9$ GeV$^2$:

$$\alpha_p = [1.183 \pm 0.054({\text{stat.}}) \pm 0.030({\text{syst.}})]$$

$$- [0.019 \pm 0.139({\text{stat.}}) \pm 0.076({\text{syst.}})] \cdot |t|.$$  

The $J/\Psi$ polar and azimuthal angular distributions for the $l^+l^−$ decay ($l = \mu$ or $e$) in the helicity system are shown in figure 50 as measured by ZEUS [114]. They yield $r_{00}^\mu = −0.017 \pm 0.015({\text{stat.}}) \pm 0.009({\text{syst.}})$ and $r_{-1}^\mu = 0.027 \pm 0.013({\text{stat.}}) \pm 0.005({\text{syst.}})$. If the $J/\Psi$ retains the helicity of the photon (SCHC) then $r_{00}^\mu$ and $r_{-1}^\mu$ should both be zero: the data show that both matrix elements are small.
8.5.2. $\gamma^* p \rightarrow J/\psi p$. The first signal for $J/\psi$ production in deep-inelastic scattering was reported by H1 [73]: figure 51 shows the lepton–lepton mass distribution, the differential cross section $d\sigma/|t|$ and the cross section as a function of $W$. The cross-sections at $Q^2 = 10(20)$ GeV$^2$ are a factor of about 4(10) below the measurements at $Q^2 = 0$.

Further results were reported by ZEUS [75, 116] and H1 [76, 115]. The final results stem from about 1200 (ZEUS) and 600 (H1) $J/\psi p$ events with $Q^2 > 2$ GeV$^2$, see figures 52 and 53. The $W$ dependence of the cross-section $\sigma(\gamma^* p \rightarrow J/\psi p)$ is shown in figure 54 for $Q^2$ values between 3.2 and 22.4 GeV$^2$. The parametrization $\sigma(\gamma^* p \rightarrow J/\psi p) \propto W^\delta$ yields the $\delta$ values shown as a function of $Q^2$ in figure 54(bottom); within errors, $\delta \approx 0.7$.

The differential cross section $d\sigma(\gamma^* p \rightarrow J/\psi p)/|t|$ (figures 55(a)–(d)) is proportional to $e^{-H|t|}$, where
corresponds to the radius of the proton. Hence, the radius of
angles $|t|$ as a function of $|t| = b$.

Reaction $\gamma p \rightarrow J/\Psi p$, the decay $\gamma p \rightarrow J/\Psi p$, and of $\phi_1$ (rad) are shown in figures 56(a)–(f) for $Q^2$ between 2 and 100 GeV$^2$. The full set of $r_{ij}^0$ is shown in figure 57 as a function of $Q^2$ (left) and $|t|$ (right). The element $r_{04}^0$ and the combinations $r_{04}^0 + 2r_{15}^0$ and $r_{00}^0 + 2r_{04}^0$ are zero, within errors, in agreement with SCHC. Assuming SCHC, the ratio of the longitudinal to transverse cross sections, $R = \sigma_L/\sigma_T$, was determined from $r_{04}^0$. $R = (1/\epsilon)(\epsilon_{04}^0/1 - \epsilon_{00}^0)$, where $\epsilon$ is the polarization parameter. The ratio $R$ is shown in figure 56(g): it rises proportional to $Q^2$: $R = \zeta(Q^2/M_{J/\Psi}^2)$ with $\zeta = 0.52 \pm 0.16$. For $Q^2 = 20$ GeV$^2$, $\sigma_L$ and $\sigma_T$ are approximately equal.

The production of $\Psi(2S)$ in $\gamma p$ collisions was reported by H1 [117, 118]. In the first analysis, based on an integrated luminosity of 6.3 pb$^{-1}$, the $\Psi(2S)$ was identified by the decays $\Psi(2S) \rightarrow \ell^+\ell^-$, $\ell = e, \mu$ and $\Psi(2S) \rightarrow J/\Psi \pi^+\pi^-$, $J/\Psi \rightarrow \ell^+\ell^-$, see figure 58. A total of 60 $\Psi(2S)$ events were observed. In the second analysis, performed with 77 pb$^{-1}$, about 300 $\Psi(2S)$ could be isolated (figure 59). Figure 60 shows the cross section ratio $R = \sigma(\Psi(2S))/\sigma(J/\Psi)$. When $R$ is assumed to be

\[ b = 4.5 \pm 0.2 \text{ GeV}^{-2}, \text{ see figure 55(e). The measured } b \text{-value corresponds to the radius of the proton. Hence, the radius of the virtual photon must be small.} \]

The decay $J/\Psi \rightarrow \ell^-\ell^-$ was studied in terms of the decay angles $\theta_\ell$ and $\psi_\ell = \phi_\ell = \phi_\ell$, where $\theta_\ell$ and $\phi_\ell$ are the polar and azimuthal angles of the positively charged decay lepton and $\phi_\ell$ is the angle between the $J/\Psi$ production plane and the lepton scattering plane. The distributions of $\cos \theta_\ell$ and of $\psi_\ell$ are

\[ 0.9 \text{ GeV}^2 \]
Figure 51. Reaction $\gamma^* p \rightarrow J/\Psi p$ at $Q^2 > 8$ GeV$^2$, $W = 30$–150 GeV: mass distribution of $m_{l\ell^-}$, $l = e, \mu$ (left), $d\sigma/d|t|$ (middle) and cross section versus $W$ (right). Figures reprinted with permission from H1 [73].

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Figure 52. Reaction $\gamma^* p \rightarrow J/\Psi p$: distributions of $M_{l\ell^-}$, $l = e, \mu$, for the $W, Q^2$ regions indicated. Figure reprinted with permission from ZEUS [116].
Figure 53. Reaction $\gamma^* p \to J/\Psi$: distribution of $M_{\mu\mu}$ for $Q^2 = 2-80\text{GeV}^2$. Figure reprinted with permission from H1 [115].

Figure 54. Reaction $\gamma^* p \to J/\Psi$: cross section as a function of $W$ for different values of $Q^2$, as measured by H1 and ZEUS (top); parametrization $\sigma (\gamma^* p \to J/\Psi p) \propto W^\delta$: the power $\delta$ as a function of $Q^2$ as determined by H1 (bottom). Figure reprinted with permission from H1 [115].

Figure 55. Reaction $\gamma^* p \to J/\Psi$: (a)–(d) differential cross section $d\sigma/dt$ as a function of $-t$ for the $Q^2$-bins indicated; (e) assuming $\sigma (\gamma^* p \to J/\Psi p) \propto W^\delta$: $\delta$ as a function of $Q^2$. Figure reprinted with permission from ZEUS [116].

where the last error accounts for the uncertainty of the $\Psi(2S) \to J/\Psi$ branching ratio. The dashed and dashed–dotted lines show predictions [119, 120]: in these models the photon fluctuates into a $c\bar{c}$ colour dipole which interacts with the proton via the exchange of two gluons. The different wave functions of $J/\Psi$ and $\Psi'$ are taken into account. The model of Nemchik et al [119] reproduces the data somewhat better.

8.5.4. $\gamma p \to \Upsilon p$. The first observation of the reaction $\gamma p \to \Upsilon p$ was reported by ZEUS [121], see figure 61. Based on an integrated luminosity of 43.2 pb$^{-1}$, a total of 57 events were observed in the mass range 8.9–10.9 GeV of which 17 events were estimated to come from $\Upsilon$ production (assuming that the relative production cross sections for the $\Upsilon(nS)$ states are the same as those measured in $pp$ collisions by CDF [122]). For a mean $W = 120\text{GeV}$ the cross section is

$$\sigma_{\gamma p \to \Upsilon p} = 375 \pm 170\text{(stat.)}^{+75}_{-64}\text{(syst.) pb},$$

and the ratio of $\Upsilon$ to $J/\Psi$ is

$$\sigma_{\gamma p \to \Upsilon(nS)} / \sigma_{\gamma p \to J/\Psi} = [(4.8 \pm 2.2\text{(stat.)}^{+0.7}_{-0.6}\text{(syst.)}) \times 10^{-3}].$$

A further measurement of $\Upsilon$ photoproduction was presented by H1 [123]. Based on an integrated luminosity
Figure 56. Reaction $\gamma^* p \rightarrow J/\Psi p$: (a)–(f) decay angular distributions of the $J/\Psi$ in the s-channel helicity system for the $Q^2$ bins indicated; (g) ratio $R = \sigma_L/\sigma_T$ as a function of $Q^2$. Figure reprinted with permission from ZEUS [116].

Figure 57. Reaction $\gamma^* p \rightarrow J/\Psi p$: spin-density matrix elements as a function of $Q^2$ (left) and $|t|$ (right), with data from H1 and ZEUS. Figure reprinted with permission from H1 [115].
Figure 58. Reaction $\gamma p \rightarrow \Psi(2S)p$: mass distributions for the four-track sample: four-track effective mass; $l^+l^-$ effective mass; $\Delta M = (M_{l^+l^-} - M_{\Psi(2S)})$ of the two-track sample. Figure reprinted with permission from H1 [117].

Figure 59. Reaction $\gamma p \rightarrow \Psi(2S)p$: mass distributions of $\mu^+\mu^-$ and $e^+e^-$ (top); of $\mu^+\mu^-\pi^+\pi^-$ and $e^+e^-\pi^+\pi^-$ (middle); distribution of $(M_{l^+l^-} - M_{\Psi(2S)})$ (bottom). Figure reprinted with permission from H1 [118].

Figure 60. Cross section ratio $R(W_{\gamma p}) = \sigma(\Psi(2S))/\sigma(J/\Psi)$ as a function of the $\gamma p$ centre-of-mass energy $W_{\gamma p}$: solid line shows a fit to the data with $R \propto W_{\gamma p}^{\Delta \delta}$ where $\Delta \delta = 0.25$; dashed and dashed–dotted lines show predictions, see text. Figure reprinted with permission from H1 [118].

Figure 61. Mass distribution of $\mu^+\mu^-$ pairs: the histogram represents the simulated Bethe–Heitler background. Points in the $J/\Psi$ region are connected by a dotted line. The inset shows the signal in the $\Upsilon$ region after background subtraction. Figure reprinted with permission from ZEUS [121].

of 27.5 pb$^{-1}$, a total of 12.2 ± 6.3 signal events were detected, see figure 62(a). The cross section $\sigma_{\gamma p \rightarrow \Upsilon(1S)p}$ as measured by ZEUS and H1 is shown in figure 62(b): it is of the order of 0.3–0.6 nb for $W = 120 – 150$ GeV.

8.6. $\gamma, \gamma^* p \rightarrow Z^0 p$

The authors of [124] have considered the possibility of producing Z-bosons by diffractive $\gamma p$ and $\gamma^* p$ scattering. The
Figure 62. (a) Mass distribution of $\mu^+\mu^-$ pairs from H1: the grey histogram represents the simulated background; (b) cross section for $\gamma p \to \Upsilon (1S)p$ versus $W$ as measured by H1 and ZEUS; in both measurements, 70% of the signal in the $\Upsilon$ mass region is assumed to be due to $\Upsilon (1S)p$ production. Figure reprinted with permission from H1 [123].

Figure 63. Photoproduction: total cross section and cross sections for the production of vector mesons, $\gamma p \to Vp$, $V = \rho, \omega, \phi, J/\Psi, \Psi(2S)$, $\Upsilon$, as a function of $W$; inset shows the difference between data and the fitted curve; results from H1 and ZEUS.

calculations indicate that, integrated over the running time of HERA, of the order of five to ten $Z_0 \to e^+e^-, \mu^+\mu^-$ events at $Q^2 < 500 \text{GeV}^2$ could have been detected by each of the two experiments. So far, no experimental results have been reported.

8.7. The $W$ dependence of the photoproduction of vector mesons

The cross sections for photoproduction of vector mesons, $\gamma p \to V_0p$, are shown in figure 63 as a function of $W$ together with the total $\gamma p$ cross section [125]. The parametrization of the cross sections by the form $\sigma \propto W^\delta$ gives the $\delta$-values indicated: for the total cross section $\delta = 0.16$; for vector meson production $\delta$ increases with the mass of the vector meson from $\delta = 0.22 (\rho^0)$ to $\delta = 0.8 (J/\psi)$ and $\delta \approx 1.6 (\Upsilon)$.

8.7.1. Estimate of cross sections for elastic vector meson proton scattering.

Assuming vector meson dominance (VDM), the cross sections measured for photoproduction of vector mesons, $\gamma p \to V_0p$, can be used to estimate the cross sections for elastic scattering of transverse vector mesons on protons, $V_{T}^0 \to V_{T}^0p$:

$$\sigma(V_{T}^0 \to V_{T}^0p) = \frac{4 \gamma_{V}^2}{\alpha 4\pi} \cdot \sigma(\gamma p \to V_{T}^0p),$$ (15)

where $|\gamma_{V}^2/4\pi| = \alpha^2 M_V/12 \Gamma_{V\infty} = 0.489, 5.8, 3.56, 2.48$ for $V = \rho, \omega, \phi, J/\Psi$, respectively [126]. The resulting cross...
Figure 65. Cross section \( \sigma(\gamma^*p \rightarrow Vp) \) with \( V = \rho^0, \phi, J/\Psi \), as a function of \( (Q^2 + M_V^2) \) for \( W = 75 \text{ GeV} \): results from H1 and ZEUS. The curves have been computed with the \( b \)-Sat and \( b \)-CGC models. Figure reprinted with permission from Watt and Kowalski [130].

sections for \( W = 70 - 80 \text{ GeV} \) are
\[
\sigma(\rho_0^0 Tp \rightarrow \rho_0^0 Tp) = 4.0 \pm 0.5 \text{ mb},
\]
\[\text{(16)}\]
\[
\sigma(\omega Tp \rightarrow \omega Tp) = 3.8 \pm 1.0 \text{ mb},
\]
\[\text{(17)}\]
\[
\sigma(\phi Tp \rightarrow \phi Tp) = 1.9 \pm 0.6 \text{ mb},
\]
\[\text{(18)}\]
\[
\sigma(J/\Psi Tp \rightarrow J/\Psi Tp) = 0.8 \pm 0.1 \text{ mb}.
\]
\[\text{(19)}\]
The elastic \( \rho_0^0 p \) and \( \omega p \) cross sections are of the same magnitude as the cross sections measured for elastic \( \pi^p \) and \( \pi^- p \) scattering—albeit at a somewhat lower energy of \( W \approx 26 \text{ GeV} \): \( \sigma(\pi^- p \rightarrow \pi^- p) = 3.6 \text{ mb} \) and \( \sigma(\pi^+ p \rightarrow \pi^+ p) = 3.3 \text{ mb} \) [127].

8.8. The \( Q^2 \) and \( t \) dependences of vector meson production

Figure 64 shows the cross sections for \( \gamma^*p \rightarrow Vp \) at \( W = 75 \text{ GeV} \), as a function of \( (Q^2 + M_V^2) \), where \( V = \rho^0, \omega, \phi, J/\Psi, \Upsilon \) [103]. The cross sections lie on a universal
of the proton: in reactions of the type $\gamma^* p \rightarrow V p$, the transverse extension of the virtual photon becomes negligible when $Q^2 \gg 5 \text{ GeV}^2$.

The differential cross sections $d\sigma/d|t|$ for $\gamma^* p \rightarrow V p$ are shown in figure 67 as a function of $|t|$, where $V = \rho^0, J/\Psi$ ($W = 90 \text{ GeV}$) and $V = \phi$ ($W = 75 \text{ GeV}$); $Q^2$ is fixed with values between 2.4 and 19.7 $\text{ GeV}^2$. The $|t|$-dependences of the three reactions are consistent with an exponential fall-off, $d\sigma/d|t| \propto e^{-|t|/\lambda}$.

The relative size of these cross sections at large $Q^2$ can be understood by assuming that the virtual—predominantly longitudinal—photon couples to a vector meson $V$, which scatters elastically on the proton, see figure 1. The cross section for $\gamma^*_c p \rightarrow V_L p$ is given by

$$\sigma(\gamma^*_c p \rightarrow V_L^0 p) = \frac{\alpha}{4} \cdot \frac{1}{\gamma^2/4\pi} \cdot \frac{M_V^4}{(Q^2 + M_V^2)^2} \cdot \frac{Q^2}{M_V^2} \sigma(V_L^0 p \rightarrow V_L^0 p).$$  \hspace{1cm} (20)$$

From figure 67 one obtains at $|t| = 0.5 \text{ GeV}^2$:

$$d\sigma(d|t|) = 0.06 \pm 0.05$$

$$d\sigma(d|t|) = 0.06 \pm 0.05$$

Both ratios are about a factor of 2 larger than those obtained with transverse photons at $Q^2 = 0$, see above.

8.9. $\gamma^* p \rightarrow D^{*\pm}(2010)X p$ and $\gamma^* p \rightarrow c\bar{c}p$

The contribution from diffractive production of $c\bar{c}$ pairs (see diagrams in figure 68) was determined by ZEUS [131], studying the reaction $\gamma^* p \rightarrow X p$, where the system $X$ contains at least one $D^{*\pm}(*)$. The $D^{*\pm}(*)$ mesons decaying via $D^{*\pm} \rightarrow D^{\mp \pm} \rightarrow D^{\mp \pm}$ were isolated by making use of the small mass difference between $D^*$ and $D^0$, see figure 69. For $Q^2 > 1 \text{ GeV}^2$, in total 253 ± 21 events with an identified $D^*$ meson were obtained.

The differential cross section for $\gamma^* p \rightarrow D^{*\pm}(2010)X p$ as a function of $\beta = Q^2/(Q^2 + M_V^2)$, $Q^2$ and $W$, are shown in figure 70. The ratio of diffractively produced $D^{*\pm}$ mesons to inclusive $D^{*\pm}$ production is of the order of 0.05 to 0.1, see figure 71. The charm contribution to the diffractive structure

---

2 These models are based on the so called colour glass concept [129]: the hadrons participating in the scattering process behave as a new form of matter (colour glass condensate) which is made of small-$x$ gluons. These gluons carry small fractions $x$ of the total momentum and are created by slowly moving colour sources—i.e. partons at large $x$—as for a glass where, on short time scales, the constituents appear to be frozen.
Figure 70. Reaction $\gamma^* p \rightarrow D^{*+}X p$: differential cross sections $d\sigma/d\beta$, $d\sigma/d\log(\beta)$, $d\sigma/d\log(Q^2)$ and $d\sigma/dW$. Figure reprinted with permission from ZEUS [171].

Figure 71. Reaction $\gamma^* p \rightarrow D^{*+}X p$: cross section ratio of diffractive to inclusive $D^{*\pm}$ meson production as function of $p_T$, $\eta$, $x$ of the $D^*$, and as function of $\log Q^2$ and $W$. Figure reprinted with permission from ZEUS [131].
function of the proton, $x_P f^{D(3),c\bar{c}}_2(\beta, Q^2, x_P)$—which is proportional to the diffractive cross section (see below)—is presented in figures 72 and 73 as a function of $\log(\beta)$ for $Q^2 = 4$ and 25 GeV$^2$ and $x_P = 0.004, 0.02$: the structure function rises rapidly as $\beta$ decreases. The ACTW calculation [132] with the gluon-dominated fit B reproduces the data.

A similar measurement was performed by H1 [133]. Based on an integrated luminosity of 47 pb$^{-1}$, 70 ± 13 (124 ± 15) events with $D^*(2010)$ production were detected at $Q^2 < 0.01$ GeV$^2$ ($Q^2 > 2$ GeV$^2$). Figure 73 shows for $Q^2 = 35$ GeV$^2$ the combined results from H1 and ZEUS for $x_P f^{D(3),c\bar{c}}_2 = x_P f^{D(3),c\bar{c}}_2$: within the limited statistics there is agreement between the two data sets. The fraction $f^c_2$ of the total diffractive cross section contributed by the production of charm quarks is shown in figure 74 as a function of $\beta$ for $Q^2 = 35$ GeV$^2$, $x_P = 0.004$ and 0.018. For $\beta$ around $0.03–0.1$, $f^c_2$ is of the order of 20–30%.

9. Jet production by $\gamma p$ and $\gamma^* p$ diffractive scattering

The study of jet production by virtual photon proton scattering led ZEUS to the observation of events with a large rapidity gap between the jet—or system of jets—and the direction of the outgoing proton [134]. With an integrated luminosity of 0.55 pb$^{-1}$ a total of 39,000 events were collected for
$Q^2 > 10 \text{ GeV}^2$. A fraction of these events showed a large gap between the direction of the outgoing proton and the direction of particle(s) emitted closest to the outgoing proton. The size of the gap was quantified by the variable $\eta_{\text{max}} = - \ln \tan \theta_{\text{min}} / 2$, where $\theta_{\text{min}}$ is the polar angle of the most forward going particle measured w.r.t the proton beam direction. Figure 75 shows two events with a large rapidity gap in $\eta - \phi$ space ($\phi =$ azimuthal angle), (a) with one jet and (b) with two jets (b). In both events there is a large space in $\phi$ (a) or $\eta$ (b) without energy deposition. The distribution of the total hadronic transverse energy, $E_T^\star$, as determined in the $\gamma^* p$ c.m. system, is shown in figure 76 for events with 0, 1, 2 or 3 jets and a large rapidity gap. A fraction of these events have one ($\approx 15\%$) or two jets ($\approx 0.3\%$). For $E_T^\star > 7 \text{ GeV}$, the dominant production mechanism is jet production. In the case of two-jet production the two jets are back-to-back in the $\gamma^* p$ system (not shown).

9.1. $\gamma p \rightarrow \text{two jets} + N$

An early observation of photo-produced events with a large rapidity gap and two back-to-back jets was reported by

Figure 74. Ratio of $c\bar{c}$ production to the total diffractive cross section as a function of $\beta$ for the $x_p$-values indicated. Figure reprinted with permission from H1 [133].

Figure 75. Transverse energy deposition in $\eta - \phi$ space for a large rapidity-gap event with one hadronic jet balancing the momentum of the scattered electron (left); (b) for a large rapidity-gap event with two jets (right). Figure reprinted with permission from ZEUS [134].

Figure 76. Total hadronic energy transverse to the direction of the virtual photon in $\eta - \phi$ space for DIS events with a large rapidity gap and zero, $\geq 1$ (hashed), $\geq 2$ (cross-hashed) or 3 jets (solid). Figure reprinted with permission from ZEUS [134].
Figure 77. Photoproduction of a two-jet event with a large rapidity gap. Figure reprinted with permission from H1 [135].

Figure 78. Photoproduction of two jets which are back-to-back in azimuth and separated by a pseudorapidity gap \( \Delta \eta \): gap fraction \( f(\Delta \eta) \) as a function of \( \Delta \eta \) (dots). Shown are also the expectations for colour non-singlet exchange (dotted line) and colour singlet exchange (solid line). Figure reprinted with permission from ZEUS [137].

H1 [135], see figure 77. An analysis by ZEUS [136] indicated that between 30% and 80% of the Pomeron momentum carried by partons is due to hard gluons. Photoproduction of two or more jets with jet-transverse energies above 6 GeV was studied by ZEUS [137]. For a fraction of events with two jets, the jets are separated by a large rapidity gap, \( \Delta \eta \), of up to four units, with little hadronic energy in between the jets. The fraction of such events decreases exponentially with \( \Delta \eta \), up to \( \Delta \eta \approx 3 \), where it reaches a constant value of about 0.08, see figure 78. The excess of events above the exponential fall-off gave evidence for hard diffractive scattering in photoproduction.

Further measurements on diffractive photoproduction of two jets have been reported by ZEUS [138, 139] and

H1 [140–142]. Figure 79 shows LO diagrams for direct (left) and resolved (right) processes in diffractive photoproduction of dijets via the exchange of a Pomeron (IP). Figure reprinted with permission from [139].

Figure 79. Leading-order diagrams for direct (left) and resolved (right) processes in diffractive photoproduction of dijets via the exchange of a Pomeron (IP). Figure reprinted with permission from [139].

The process is described in terms of the four-momenta \( e, e' \) of the incoming and scattered electron, the incoming and scattered proton \( p, p' \) and the outgoing system \( X \). Defining the four-momentum of the virtual photon, \( q = e - e' \), and the square of the photon–proton c.m. energy, \( W^2 = (p + q)^2 \), the fraction of the energy of the incoming electron transferred to the proton is given by

\[
y = \frac{p \cdot q}{p \cdot e} \approx \frac{W^2}{2p \cdot e}.
\]

Under the assumption that the virtual photon interacts with the Pomeron (IP) and produces a hadronic system \( X \) of mass \( M_X \), the fraction of the proton momentum carried by the Pomeron is

\[
x_p = \frac{(p - p') \cdot q}{p \cdot q}.
\]

The partons from the resolved photon and the diffractive exchange have fractional momenta

\[
x_{\gamma'} = \frac{(p \cdot u)}{(p \cdot q)} \quad \text{and} \quad z_p = \frac{v \cdot q}{(p - p') \cdot q}
\]

where \( u \) is the four-momentum of the parton in the resolved photon and \( v \) is the four-momentum of the parton in the diffractive exchange. The variables \( x_{\gamma'} \) and \( z_p \) are approximately given
Figure 80. Diffractive photoproduction of dijets as function of $y$, $M_X$, $x_{IP}$, $z_{obs}^{jet}$, $E_{jet1}^{T}$ and $\eta_{jet1}$, from ZEUS; also shown are NLO QCD predictions based on the H1 2006 DPDFs. Figure reprinted with permission from ZEUS [139].

by the energies and pseudorapidities of the two jets with the highest transverse energies $E_{jet1,2}^{T}$ in the laboratory system:

$$x_{obs}^{jet} = \frac{\sum_{jet1,2} E_{jet}^{T} e^{-\eta_{jet}}}{2 y E_{e}}$$

$$z_{obs}^{jet} = \frac{\sum_{jet1,2} E_{jet}^{T} e^{\eta_{jet}}}{2 x_{IP} E_{p}}$$

The differential cross sections as measured by ZEUS [139] are shown in figures 80 and 81 together with NLO QCD predictions [143]. These are based on diffractive parton densities, dPDFs, which have been determined from the structure function $F_{2}^{D(3)}$ for deep-inelastic inclusive
diffraction, as measured by H1 and ZEUS. The predictions lie systematically below the data by about a factor of 1.5.

9.2. $\gamma^* p \rightarrow$ two jets + $N$ and three jets + $N$

A first in-depth analysis of deep-inelastic scattering leading to the production of two jets combined with a large rapidity gap has been presented by ZEUS [144] for $160 < W < 250$ GeV, $5 < Q^2 < 185$ GeV$^2$ and $\eta_{\text{max}} < 1.8$. For this event sample, the value of $x_p$ is below 0.01; thus, dominance by Pomeron exchange can be expected. The observed multihadron final-state $X$ of mass $M_X$ is studied in its rest system which is interpreted as the $\gamma^*$-Pomeron rest system. As shown by figure 82, the system $X$ is collimated around the direction of the virtual photon. The observed features are close to those observed in $e^+e^-$ annihilation at a total c.m. energy $\sqrt{s} \approx M_X$ [145, 146].

The solid and dotted histograms show the comparison with the VBLY model [147] which is based on QCD. In this model, the Pomeron is assumed to have a pointlike coupling to $q\bar{q}$, and an additional gluon may be radiated. The diffractive parton density functions used had been determined by [148–151], from H1 and ZEUS measurements of the diffractive structure function $F_2^{\text{D}}(x, Q^2)$ [148, 149]. The prediction for $q\bar{q}$ alone agrees with the data for $\cos\theta_1 < 0.7$, but is too low for $\cos\theta_2 > 0.8$, while the prediction for $q\bar{q}$ plus $qg\bar{g}$ agrees with the data at $\cos\theta_2 > 0.8$ and overshoots the data when $\cos\theta_2 < 0.7$. RAPGAP [152], which also considers the sum of the contributions from $q\bar{q}$ plus $qg\bar{g}$, gives a fair representation of the data for the full $\cos\theta_2$ range.

The importance of multi-jet production by diffraction is also illustrated by figure 83 [153] which shows the fraction of events with $\eta_{\text{max}} < 3$ and two or more jets, as a function of the jet resolution parameter $\chi_{\text{cut}}$. For $\chi_{\text{cut}} > 0.02$, the dominant contributions are of the two- and three-jet type. QCD diagrams leading to two- and three-jet configurations by diffractive processes are shown in figure 84. The curves in figure 83 show predictions of several models: the Monte Carlo generator SATRAP [154] combined with higher-order QCD contributions [155, 156] describes the data best.

In [157], diffractive dijet production (see figure 85) for $100 < W < 250$ GeV, $5 < Q^2 < 100$ GeV$^2$ has been studied by ZEUS with an integrated luminosity of 61 pb$^{-1}$. The jet-transverse energies in the $\gamma^* p$ rest system were required to be $E_{T,1}^* > 5$ GeV and $E_{T,2}^* > 4$ GeV, respectively; their pseudorapidities had to satisfy $-3.5 < \eta_j^* < 0$ and their pseudorapidities in the laboratory frame were restricted to $|\eta_j^{\text{LAB}}|^\text{AB} < 2$. Diffractive events were selected by requiring less than 1 GeV energy deposition in a small forward calorimeter (FPC), which ensures a large rapidity gap, and by demanding $x_p^{\text{obs}} = (Q^2 + M^2_{\gamma})/(Q^2 + W^2) < 0.03$. Figure 86 shows the differential cross section as functions of $E_{T,1}^*$, $\eta_j^*$, $z_{\text{obs}}^*$ = $(Q^2 + M^2_{\gamma})/(Q^2 + M^2_J)$ and of $x_j$, the fractional momentum of the virtual photon. For $E_{T,1}^* > 6$ GeV, the jet cross section falls exponentially with $E_{T,1}^*$. The requirement of two high-$E_T$ jets suppresses the contribution at low $x_p$. The low value of the peak position in $z_{\text{obs}}$ indicates additional production of hadrons.

The data were compared with NLO calculations which include direct and resolved photon contributions and use dPDFs of [151, 158]. They have been determined by fits to inclusive diffractive DIS data. The predictions from the ZEUS LPS+charm fit [148], as well as those from the H1 2006 fits A and B [158], lie in general above the data. The MRW fit [151] is in broad agreement with the ZEUS data, except for the high $x_j^{\text{obs}}$ region, where the predictions are too high.

Diffractive production of dijets has also been reported by H1 [159], using data from an integrated luminosity of $51.5$ pb$^{-1}$ with $100 < W < 250$ GeV and $5 < Q^2 < 100$ GeV$^2$. Diffractive events were required to have $x_p < 0.03$, $M_N < 1.6$ GeV, where $M_N$ is the mass of the nucleonic system produced in the forward ($\approx$ proton direction) and $|t| > 1$ GeV$^2$. The jet selection required $p_{T,\text{jet1}}^* > 5.5$ GeV, $p_{T,\text{jet2}}^* > 4$ GeV and $-3 < \eta_j^* < 0$, where the index $i$ indicates evaluation relative to the collision axis in the $\gamma^*$-centre-of-mass frame. Figures 87 and 88 show the differential cross section as functions of $y$, $x_p$, $p_{T,\text{jet1}}^*$ and $\Delta^* \eta_{\text{jets}} = |\eta_j^{\text{LAB}} - \eta_{\text{jet1}}^{\text{LAB}}|$, for all $z_p$ ($z_p$ is the fractional longitudinal momentum of the diffractive exchange carried by the parton entering the hard interaction) and for $z_p < 0.4$, respectively.

The dijet data are compared with QCD predictions at NLO based on diffractive parton distribution functions obtained from a fit to these data (H1 2007 Jets dPDF fit B, based on [160, 161]), and to H1 data on inclusive diffraction in...
**Figure 82.** Multihadron final states of mass $M_X$ produced by deep-inelastic scattering with $160 < W < 250$ GeV, $5 < Q^2 < 185$ GeV$^2$, where the most forward going (in proton direction) condensate has pseudorapidity $\eta_{\text{rec}}^{\text{max}} \leq 1.8$: distribution of $\cos \theta_S$, where $\theta_S$ is the angle between the sphericity axis and the direction of the virtual photon in the $\gamma^* - IP$ rest frame, for the range of $M_X$ values indicated. Figure reprinted with permission from ZEUS [144].

**Figure 83.** Diffractive production of jets by deep-inelastic scattering: (left) jet fraction as a function of the jet resolution parameter $y_{\text{cut}}$ for $n_{\mu} = 2, 3$ and $> 3$, for the kinematic range indicated; (right) distribution of the 3-jet sample in the $\xi, x_1$ plane. The MC expectations were calculated with RAPGAP. Figure reprinted with permission from ZEUS [153].
Figure 84. Diagrams for two- and three-jet production in deep-inelastic diffractive scattering; incoming and scattered proton are not shown. Figure reprinted with permission from ZEUS [144].

Figure 85. Boson–gluon fusion for LO dijet production in diffractive DIS. Figure reprinted with permission from ZEUS [157].

Figure 86. Cross section for diffractive dijet production by deep-inelastic scattering with $Q^2 < 100$ GeV$^2$, $100 < W < 250$ GeV. Also shown are NLO predictions using the dPDFs indicated. Figure reprinted with permission from ZEUS [157].
Figure 87. Cross section for diffractive production of dijets by deep-inelastic scattering with $Q^2 = 4$–$80$ GeV$^2$, $101 < W < 266$ GeV. Also shown are NLO predictions with error bands, based on the H1 2007 DPDF fit. Figure reprinted with permission from H1 [159].

Figure 88. Cross section for diffractive production of dijets by deep-inelastic scattering with $Q^2 = 4$–$80$ GeV$^2$ and $101 < W < 266$ GeV, restricted to $z_P < 0.4$. Also shown are NLO predictions with error bands, based on H1 2006 DPDF fit A (dotted) and fit B (dashed). Figure reprinted with permission from H1 [159].
Figure 89. Diffractive quark and gluon densities as a function $z$, the fraction of the longitudinal momentum of the diffractive exchange carried by quark (gluon), for the factorization scales $\mu_f^2 = 25$ and $90$ GeV$^2$, respectively. Figure reprinted with permission from H1 [159].

Figure 90 shows, for $Q^2$ between 14 and 380 GeV$^2$, the energy flow distributions, $1/N \cdot (dE/d(\Delta \eta))$, as a function of $\Delta \eta = \eta_{\text{cell}} - \eta(\gamma_H)$, where $\eta_{\text{cell}}$ is the pseudorapidity of a calorimeter cell with energy deposit above 60 MeV (110 MeV), depending on the calorimeter section, and $\gamma_H$ corresponds to the scattering angle of the massless struck quark emerging as the current jet. Here, $\eta_{\text{cell}}$ and $\eta_{\gamma_H}$ were measured in the laboratory frame. For non-rapidity-gap events, $\eta_{\text{max}} > 1.5$ (open circles), the energy flow rises towards large $\Delta \eta$ in the direction of the incoming proton. For events with a large rapidity gap, $\eta_{\text{cell}} \lesssim 2.5$, the energy flow peaks around the direction of the virtual photon, $\Delta \eta = 0$, and is small at large $\Delta \eta$. Note the outgoing proton or low-mass nucleonic system arising from diffractive production was not detected in this analysis.

The colour dipole model (CDMBGF [163, 164, 165]) which describes standard deep-inelastic scattering without diffraction, considers the production of arbitrarily many jets in leading-log approximation for parton-shower development. It gives an excellent representation of the non-rapidity-gap events (solid histograms) but fails for events with a large rapidity gap. Diffractive scattering is modelled by POMPYT [166], where the beam proton emits a Pomeron, whose constituents take part in a hard scattering process with the virtual photon. POMPYT reproduces the distribution of events with a large rapidity gap (dashed histograms).

10.1. Inclusive diffraction by $\gamma p$ interactions

First results on inclusive photoproduction by diffraction were presented by ZEUS [167]. The selection of events with $W \approx 180$ GeV, $\eta_{\text{max}} < 2$ and $\eta_{\text{max}} > 2$, respectively, provided separate samples of diffractive and nondiffractive events. For diffraction, events with average hadronic masses reconstructed in the detector, $M_X = 5$ and 10 GeV, respectively, were selected. The $p_T$ spectra for charged particles with $-1.2 < \eta < 1.4$ from the nondiffractive and diffractive event samples are displayed in figure 91: for the diffractive samples, the $p_T$ spectra fall-off $\propto e^{-b p_T}$ with $b = 5.9(5.3)$ GeV$^{-1}$ (average $M_X = 5(10)$ GeV). The $p_T$ spectra for the nondiffractive sample show a long tail towards large $p_T$. 

![Graphs showing diffractive quark and gluon densities as a function of $z$.](image-url)
was presented by H1 [168], based on an integrated luminosity of 0.27 pb with permission from ZEUS [162]. exponentially suppressed. The dotted histograms show the nondiffractive production where large rapidity gaps represent the expectation of the colour dipole model (CDMBGF) for rapidity-gap (LRG: η = 16 and 90), and subsequently by H1 [19], were followed by deep-inelastic scattering by ZEUS [18, 162] (see figures 13, the early observations of inclusive diffractive production in the section for γ∗p cross section data used below for extracted from the differential cross section by at most 17%. The cross section data used below for F∗D have been integrated over t. The diffractive structure function will therefore be denoted by F∗D(3).

Figure 92 shows the diffractive structure function measured by H1 [168] as a function of xP for different values of β and Q2. For fixed Q2 and β, F2 ∝ xP proportional to (xP)(−n) with n = 1.19 ± 0.06(stat.) ± 0.07(syst.), independent of Q2 and β. This behaviour suggests a colourless target (T) in the proton, which carries only a small fraction of the proton momentum. In this case F2 ∝ fT/p(xP), where fT/p describes the flux of T in the proton, and the cross section factorizes into the flux f and the cross section for virtual photon–proton scattering on target T. The observed xP dependence corresponds to an effective Regge trajectory of α(t = 0) = 1 + (n − 1)/2 = 1.10 ± 0.03(stat.) ± 0.04(syst.), a value which agrees with those determined for the Pomeron from γ∗p → ρ∗p, see above. In contrast, for ρ and ω exchange in the t-channel one expects α(t = 0) ≈ 0.5. Therefore, the large rapidity-gap events observed in deep-inelastic ep scattering result (predominantly) from diffractive scattering. A similar conclusion was reached by ZEUS [169]. Furthermore, the ratio of diffractive to total γ∗p cross section for Q2 = 13 and 39 GeV2 was found to be of the order of 10 to 20%, independent of x and W. Further results were reported by H1 [170].

A novel method (MT-method [51]) in the analysis of inclusive diffractive production was introduced by ZEUS [52]. Until then, the diffractive contribution had been extracted by selecting events with a large rapidity gap (ηmax cut), or with a leading proton. The drawback of both procedures is that not

10.2. Inclusive diffraction by γ∗p interactions

The early observations of inclusive diffractive production in deep-inelastic scattering by ZEUS [18, 162] (see figures 13, 16 and 90), and subsequently by H1 [19], were followed by detailed studies of this subject.

The first measurement of the diffractive structure function was presented by H1 [168], based on an integrated luminosity of 0.27 pb−1. For unpolarized beams, the differential cross section for γ∗p → Xp can be described in terms of the diffractive structure functions, F2D(4) (β, Q2, xP, t):

\[ \frac{d^3\sigma_{\text{diff}}}{d\beta dQ^2 dxP dt} = \frac{2\pi\alpha^2}{\beta Q^4} \{ (1 + (1 − y)^2) F^2\text{(D4)}(x − y^2) F^2\text{(D4)} , \} , \]

where y = Q2/(x · s), s is the square of the ep c.m. energy and the relation x = β · xP has been used. A possible contribution from longitudinal photons, as measured by F2D(4)*, will be neglected in the following; it would increase the value of F2D(4) extracted from the differential cross section by at most 17%.

The cross section data used below for F2 have been integrated
only diffractive production (by the exchange of the Pomeron), but any $t$-channel exchange (see figure 93) of a colourless and electrical neutral particle such as the Reggeons $\rho^0$ and $\omega$ can produce events with a large rapidity gap and/or with a leading proton. The $W$ dependence of the cross section in the case of Reggeon exchange is proportional to $W^{2\alpha_R - 2}$ at $t = 0$; for instance $\rho$-exchange, with $\alpha_R \approx 0.5$ at $t = 0$, leads to $\sigma \propto W^{-1}$.

Such background is avoided by using the $M_X$-method for the extraction of the diffractive part since contributions from Reggeon exchange are exponentially suppressed. Here, $M_X$ is the mass of the hadronic system observed in the detector. The first extraction of the diffractive cross section with the $M_X$-method was performed by ZEUS [52], using data from an integrated luminosity of 0.54 pb$^{-1}$. The $M_X$ and $\ln M_X^2$ distributions are shown in figure 94 at $Q^2 = 14$ GeV$^2$ for several bins of $W$. In the $M_X$ spectra (top) the diffractive contribution is falling rapidly with increasing $M_X$, while the $\ln M_X^2$ spectra (bottom) show a plateau for the diffractive component once $M_X$ is sufficiently above the kinematical threshold. This allows the extraction of the diffractive contribution by a fit to the $\ln M_X^2$ spectrum of the form:

$$\frac{dN}{d \ln M_X^2} = D + c \cdot \exp(b \cdot \ln M_X^2).$$

(26)

The fit is applied in the interval $\ln M_X^2 \leq \ln W^2 - \eta_0$, where $\ln W^2 - \eta_0$ is the maximum value of $\ln M_X^2$ up to which the exponential behaviour of the nondiffractive part holds. The diffractive contribution is not taken from the fitted value of $D$; rather it is obtained by subtracting the nondiffractive contribution determined by the fit ($c \cdot \exp(b \cdot \ln M_X^2)$) from the observed number of events.

The cross section for diffractive scattering, $d\sigma^{\text{diff}}_{\gamma^*p \rightarrow XN}/dM_X$, is shown in figure 95 as a function of $W$ for different
Figure 94. Reaction $ep \rightarrow eX$: distribution of $M_x$ (top) and $\ln M_x^2$ (bottom) for different regions of $W$ at $Q^2 = 14 \text{ GeV}^2$. The shaded distributions show the prediction for diffractive production; the dashed histograms show the expectations for nondiffractive production as predicted by the CDMBGF model. Figure reprinted with permission from ZEUS [176].

Figure 95. Cross section $d\sigma^{\text{diff}}(\gamma^*p \rightarrow XN)/dM_x$ as a function of $W$ averaged over the $M_x$ intervals indicated, for $Q^2 = 14$ and 31 GeV$^2$. The lines show the result of a fit to all data with the form $d\sigma^{\text{diff}}/dM_x \propto (W^2)^{\bar{\alpha}_{IP} - 2}$, see text. Figure reprinted with permission from ZEUS [52].
values of $Q^2$ and $M_X$. The diffractive cross section is rising with $W$ for all $Q^2, M_X$ values. Assuming that only Pomeron exchange contributes, the $W$ dependence for fixed $Q^2$ was parametrized in terms of the Regge trajectory of the Pomeron,

$$\frac{d\sigma_{\gamma^*p \rightarrow Xp}}{dM_X} \propto (W^2)^{2\alpha_{IP} - 2}.$$  \hspace{1cm} (27)

The value for the Regge trajectory of the Pomeron \[52\],

$$\alpha_{IP} = 1.23 \pm 0.02(\text{stat}) \pm 0.04(\text{syst}),$$  \hspace{1cm} (28)

is in agreement with the result obtained by H1, see above.

The diffractive structure function determined with the $M_X$-method is shown in figure 96 together with the results from previous measurements of H1 \[168\] and ZEUS \[169\]. As indicated, the three measurements were performed at slightly different values of $\beta$ and $Q^2$. Broad agreement is observed between the H1 and ZEUS measurements. A first attempt to determine the quark and gluon distributions for diffractive scattering was made in \[171\].

The first measurement of $\gamma^*p \rightarrow Xp$, where the scattered proton was momentum analysed in a magnetic spectrometer, has been reported by ZEUS \[172\]. Figure 97 shows $d\sigma/d|t|$ as a function of the four-momentum-transfer squared $t$, for the range $5 < Q^2 < 20\text{GeV}^2$, $50 < W < 270\text{GeV}$ and $0.15 < \beta < 0.5$. A fit of the form $d\sigma/d|t| = A \cdot e^{-b|t|}$ yielded for the slope $b = 7.2 \pm 1.1(\text{stat})^{0.2}_{0.9}(\text{syst}) \text{GeV}^{-2}$. Further results on $b$ from \[148, 149\] are also presented in figure 97. Since for these $Q^2$-values the transverse size of the virtual photon is negligible compared with the radius of the proton $R_p$ then $b = (R_p^2)/4$. Elastic pp scattering at a c.m. energy of 546 GeV yields $b = 15.35\text{GeV}^{-2}$ \[173\] which is expected to
be equal to \((R^2 + R'^2)/4\). This result is in good agreement with the data from HERA.

In the following, the high-statistics results on inclusive diffraction as obtained by H1 and ZEUS are presented.

### 10.3. Inclusive diffraction with high statistics: results from H1

The diffractive structure function \(x_F^2D(3)\) has been measured by H1 [174] with integrated luminosities of 2.0 pb\(^{-1}\) (\(3 < Q^2 < 13.5\) GeV\(^2\)), 10.6 pb\(^{-1}\) (\(13.5 < Q^2 < 105\) GeV\(^2\)) and 61.6 pb\(^{-1}\) (\(Q^2 > 133\) GeV\(^2\)). Figure 98 shows the diffractive structure function as a function of \(x_F\) for \(\beta\) values between 0.01 and 0.9 and \(Q^2\) values from 3.5 to 1600 GeV\(^2\).

(From here on the term ‘diffractive structure function’ is used for the function \(x_F^2D(3)\) which H1 denotes by \(x_F^2\sigma_t^{D,(3)}\)) while ZEUS uses the notation \(x_F^2F_2^{D(3)}\); multiplication by \(x_F\) takes out a trivial dependence on \(x_F\) and elucidates better the \(x_F\) dependence of the diffractive structure function.)

Figure 98 shows that \(x_F^2F_2^{D(3)}\) increases as \(x_F \to 0\), provided \(\beta \geq 0.2\). In figures 99 and 100 measurements for \(x_F^2F_2^{D(3)}\) are shown as a function of \(Q^2\) at fixed values of \(x_F\) and \(\beta\). The dependence on \(Q^2\) can be summarized as follows: for \(\beta \leq 0.5\) the diffractive structure function rises with increasing \(Q^2\), while for \(\beta > 0.8\) it decreases with \(Q^2\).

H1 has performed a fit of their \(x_F^2F_2^{D(3)}\) measurements to a QCD-motivated model, which includes the contributions from Pomeron plus Reggeon exchanges in the \(t\)-channel. Diffractive parton distribution functions (DPDFs) have been determined by a next-to-leading order DGLAP analysis of the data with...
$Q^2 \geq 8.5 \text{ GeV}^2$, $\beta \leq 0.8$ and $M_X > 2 \text{ GeV}$. The fit gives a good account of the data as shown by the curves in figures 98, 99 and 100. The resulting quark and gluon distributions are presented in figure 101 as a function of $z$. Here $z$ is the longitudinal momentum fraction of the parton entering the hard sub-process, such that $z = \beta$ for the lowest-order quark–parton process, whereas $0 < \beta < z$ for higher-order processes. Figure 101 shows that a large fraction of the momentum of the Pomeron exchanged in the $t$-channel is carried by gluons (dashed curves). In the model, the rise of $x_P f_2^{D(3)}$ for $x_P > 0.1$ is due to Pomeron–Reggeon interference.

10.4. Inclusive diffraction with high statistics: results from ZEUS

Inclusive diffraction in deep-inelastic scattering has been measured by ZEUS for a wide range in $Q^2$, $W$ and $M_X$, making use of the forward-plug calorimeter (FPC). The FPC limited the mass of the nucleonic system escaping undetected in the forward (= proton) direction to $M_N < 2.3 \text{ GeV}$, on average. Data were collected in two periods for $Q^2 = 2.7–80 \text{ GeV}^2$ and $Q^2 = 25–320 \text{ GeV}^2$ with integrated luminosities of 4.2 pb$^{-1}$ [30] and 52.4 pb$^{-1}$ [31], respectively.
The diffractive cross section \( d\sigma^{\text{diff}} / dM_X \) is shown in figures 102 and 103 as a function of \( W \) for the bins of \( Q^2 \) and \( M_X \) indicated. In all \( Q^2, M_X \) bins with sufficient \( W \) coverage, the diffractive cross section shows a strong rise with \( W \).

The ratio of the diffractive contribution to the total \( \gamma^* p \) cross section is displayed in figures 104 and 105 as a function of \( Q^2 \) and \( M_X \). For fixed values of \( Q^2 \) the relative contribution of diffraction to the total \( \gamma^* p \) cross section is approximately independent of \( W \). It is substantial when \( M_X^2 > Q^2 \). The ratio \( r = \sigma^{\text{diff}} / \sigma^{\text{tot}} \) is 15.8\% at \( Q^2 = 4 \text{ GeV}^2 \), decreasing to 5.0\% at \( Q^2 = 190 \text{ GeV}^2 \), see figure 106. A fit of the data to the form \( r = a - b \cdot \ln(1 + Q^2) \) yielded \( a = 0.207 \pm 0.008 \) and \( b = 0.32 \pm 0.002 \); the fit is shown by the curve.

Since the optical theorem relates the forward amplitude for elastic scattering to the total cross section—which behaves, say, proportional to \( W^3 \)—one would naively expect that the diffractive cross section is proportional to \( W^{2d} \). However, the measured ratio of the diffractive contribution to the total \( \gamma^* p \) cross section is approximately independent of \( W \), as shown in figures 104 and 105. This apparent contradiction is understood as the result of a strong growth of the gluon density (‘colour glass condensate’) such that gluon–gluon interactions become important and damp the rise of the diffractive cross section [175].
Figure 101. Total quark singlet and gluon distributions as a function of $z$, from an NLO QCD fit by H1 to the data for diffraction with $Q^2 > 8.5 \text{ GeV}^2, \beta < 0.8$ and $M_X > 2 \text{ GeV}$; $z$ is the longitudinal momentum fraction of the parton entering the hard sub-process: for the lowest order quark–parton process $z = \beta$, whereas $0 < \beta < z$ for higher-order processes. Figure reprinted with permission from H1 [158].

Figure 102. Diffractive cross section, $d\sigma^{\text{diff}}/dM_X$, $M_N < 2.3 \text{ GeV}$, as a function of $W$ for bins of $M_X$, and of $Q^2$ between 2.7 and 25 GeV$^2$; for FPC I data (stars) and FPC II data (dots). Figure reprinted with permission from ZEUS [31].
The diffractive structure function $x_F F_D^{(3)}$ for $\gamma^* p \rightarrow XN, M_N < 2.3$ GeV, or equivalently for fixed $Q^2, \beta: x_F F_D^{(3)}$ rises approximately proportional to $\ln 1/x_F$ as $x_F \rightarrow 0$, reflecting the rise of the diffractive cross section $d\sigma/dM_X$ with increasing $W$. The rise is observed for most $Q^2$ values between 2.7 and 320 GeV$^2$. The curves show the result of the BEKW fit to the ZEUS data, see below. The contributions from transverse and longitudinal photons to the production of $q\bar{q}$ and $qg$ final states, respectively, are shown separately as well as their sum.

The $Q^2$ dependence of $x_F F_D^{(3)}$ for fixed $\beta$ and $x_F$ is shown in figure 109. The data are dominated by positive scaling violations for $x_F \beta = x < 10^{-3}$, by negative scaling violations for $x \geq 5 \times 10^{-3}$, and by constancy in between. The data contradict the assumption of Regge factorization, namely that $x_F F_D^{(3)}(\beta, x_F, Q^2)$ factorizes into a term that depends only on $x_F$, and a second term that depends only on $\beta$ and $Q^2$.

10.5. Leading protons and the contribution from diffraction

Deep-inelastic diffractive scattering has also been studied in measurements where the scattered proton was detected in a forward spectrometer. The first such measurement has been reported by ZEUS [172]. Figure 97 shows $d\sigma/d|t|$ as a function of the four-momentum-transfer squared $t$, for the range $5 < Q^2 < 20$ GeV$^2$, $50 < W < 270$ GeV and $0.15 < \beta < 0.5$. A fit of the form $d\sigma/d|t| = A \cdot e^{-b|t|}$ yielded for the slope $b = 7.2 \pm 1.1$ (stat)$^{0.7\ (syst)}$ GeV$^{-2}$. Further results on $b$ [148, 149] are also presented in figure 97. For the $Q^2$-values selected, the transverse size of the virtual photon is negligible compared with the radius of the proton.

The final measurement [176], based on an integrated luminosity of 32.6 pb$^{-1}$, provided results for the kinematic region $Q^2 > 2$ GeV$^2$, $W = 40–240$ GeV, $M_X > 2$ GeV. The diffractive peak covers the region $x_L > 0.98$, where $x_L$ is the fraction of the momentum of the incoming proton carried by the scattered proton.
The differential cross section $d\sigma^{ep\to eXp}/|t|$ is shown in figure 110 as a function of $|t|$ for $x_p = 0.0002 - 0.01$ and $x_p = 0.01 - 0.1$, respectively. The cross section falls exponentially with $|t|$, $d\sigma^{ep\to eXp}/|t| \propto e^{-|t|}$, where $b = 7.0 \pm 0.3$ GeV$^{-2}$ for $x_p = 0.0002-0.01$ and $b = 6.9 \pm 0.3$ GeV$^{-2}$ for $x_p = 0.01-0.1$. Within errors the value of $b$ is independent of $x_p$, $Q^2$ and $M_X$.

The cross section for $ep \to eXp$ was also studied in terms of the azimuthal angle $\Phi$ between the positron scattering angle and the proton scattering plane. Within errors, no significant dependence on $\Phi$ was observed for $x_p = 0.0002 - 0.01$ and $x_p = 0.01 - 0.1$, respectively. This is in contrast to $\rho^0$ production in deep-inelastic scattering, $ep \to ep\rho^0p$ [80], where a substantial dependence on $\Phi$ had been observed, namely $d\sigma/d\Phi \propto 1 + A_{LT}\cos\Phi + A_{TT}\cos2\Phi$, with $A_{LT} = \sqrt{2}(1+\epsilon)(r_{30}^0 + 2r_{31}^5) = 0.256 \pm 0.030^{+0.032}_{-0.025}$ and $A_{TT} \approx 0$.

The structure function $x_F P_{2D(3)}$, denoted by $x_F P_{2D(3)}$, is shown in figures 111 and 112 and compared with the FPCII measurements discussed before. The $x_F P_{2D(3)}$ data from the FPCII measurements have been scaled down by a factor of 0.83 to account for the extra contribution from $p \to N$ dissociation with $M_N < 2.3$ GeV. Good agreement between the FPC and LPS measurements is observed.

Figure 104. Ratio of the diffractive cross section, integrated over the $M_X$ intervals indicated, $\int_{M_X}^M dM_X \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \frac{dM_X}{dM_X}$, to the total $y^+p$ cross section, $\frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \frac{dM_X}{dM_X}$, as a function of $W$ for $Q^2$ between 2.7 and 55 GeV$^2$. Figure reprinted with permission from ZEUS [31].

Figure 105. Ratio of the diffractive cross section, integrated over the $M_X$ intervals indicated, $\int_{M_X}^M dM_X \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \frac{dM_X}{dM_X}$, to the total $y^+p$ cross section, $\frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \frac{dM_X}{dM_X}$, as a function of $W$ for $Q^2$ between 25 and 320 GeV$^2$. Figure reprinted with permission from ZEUS [31].

Figure 106. Ratio of the diffractive cross section, integrated over $0.28 < M_X < 3.5$ GeV, to the total $y^+p$ cross section at $W = 220$ GeV, as a function of $Q^2$. The line shows the result of fitting the data to the form $r = a - b \cdot \ln(1 + Q^2)$, see text. Figure reprinted with permission from ZEUS [31].
11. Models for deep-inelastic diffractive scattering

An in-depth introduction to the theoretical description of diffraction in high energy deep-inelastic scattering has been provided by [20]. A brief summary is given here.

In QCD all basic constituents carry colour. As a consequence, there is no basic constituent with the quantum numbers of the Pomeron. In lowest-order QCD, the Pomeron is represented by a gluon–gluon system. The BFKL equation [177, 178] determines the behaviour of the Pomeron in perturbative QCD. The virtual photon dissociates into q and ¯q which carry a fraction of z and (1 − z), respectively, of the photon energy. The q, ¯q pair propagates a longitudinal distance—the origin of the large gap in rapidity—of \( \Delta L \approx \frac{1}{(m_p \cdot x)} \), \( m_p \) = mass of the proton), before interacting with the target proton. For instance, if \( Q^2 = 10 \text{ GeV}^2 \) and \( W = 200 \text{ GeV} \) then, in the rest frame of the target proton, \( \Delta L \) is around \( 10^{-10} \) cm. Consider the contribution to the cross section from large size q¯q pairs, i.e. \( r \gg R \gg 1/Q \), where \( r \) is the transverse size of the Pomeron and \( R \) the radius of the proton, \( R \approx 1 \) fm. The cross section for transverse photon–proton scattering is \( \propto 1/(Q^2 R^4) \), while for longitudinal photons it is \( \propto 1/(Q^3 R^3) \). The perturbative regime for diffraction is characterized by a large four-momentum transfer to the BFKL–Pomeron. Diffractive q¯q production by transverse photons is dominated by large size configurations, where one parton carries most of the photon energy: the ‘aligned jet’ configuration [179]. For large \( \beta \gg 0.8 \), the contribution from longitudinal photons becomes important.

Models based on a two-gluon exchange picture of the Pomeron [180–182] have been formulated in the target rest frame with the virtual photon splitting into a colour dipole at a distance \( L_1 \propto 1/(m_p \cdot x) \) before the target [183]. The dipole is assumed to consist predominantly of a q¯q pair for small diffractive masses \( M_X \), or of a q¯q pair plus one or more gluons for large diffractive masses \( M_X \). The amplitude for quasi-elastic dipole-proton scattering was evaluated making an assumption on the gluon distribution of the target proton.

![Figure 107](https://example.com/figure107.png)

**Figure 107.** Diffractive structure function of the proton, \( x_{IP}F_2^{(N)} \), for \( \gamma^* p \rightarrow XN, M_N < 2.3 \text{ GeV} \), as a function of \( x_{IP} \) for different regions of \( \beta \) and \( Q^2 < 25 \text{ GeV}^2 \). The curves show the results of the BEKW(mod) fit for the contributions from (q¯q) for transverse (dashed) and longitudinal photons (dotted) and for the (q¯qv) contribution for transverse photons (dashed–dotted), together with the sum of all contributions (solid). Figure reprinted with permission from ZEUS [31].
Figure 108. Diffractive structure function of the proton, $x_IPF_2D(3)$, for $\gamma^* p \rightarrow XN$, $M_N < 2.3$ GeV, as a function of $x_IP$ for different regions of $\beta$ and $Q^2 \geq 35$ GeV$^2$. The curves show the results of the BEKW(mod) fit for the contributions from $(qq)$ for transverse (dashed) and longitudinal photons (dotted) and for the $(qqg)$ contribution for transverse photons (dashed–dotted), together with the sum of all contributions (solid). Figure reprinted with permission from ZEUS [31].

An excellent description of the data, and further insight into the behaviour of inclusive diffractive scattering within QCD, can be achieved by using the BEKW parametrization [181]. The latter assumes that the incoming virtual photon fluctuates into a $qq$ or $qqg$ dipole, which in turn interacts with the target proton via the exchange of a two-gluon system, the Pomeron. The contributions from the transitions: transverse photon to $qq$ and $qqg$, respectively, and longitudinal photon to $qq$, are taken into account. The $\beta$ spectrum and the scaling behaviour in $Q^2$ are derived in the non-perturbative limit from the wave functions of the incoming transverse (T) and longitudinal (L) photons on the light cone. The $x_IP$ dependence of the cross section is not predicted by BEKW and has to be determined by experiment. The BEKW parametrization reads as follows:

$$x_IPF_2D(3)(\beta, x_IP, Q^2) = c_T \cdot F_T^{q\bar{q}} + c_L \cdot F_L^{q\bar{q}} + c_g \cdot F_T^{q\bar{g}},$$

where

$$F_T^{q\bar{g}} = \left(\frac{x_0}{x_p}\right)^{\alpha_T(Q^2)}} \cdot \beta(1 - \beta),$$
Figure 109. Diffractive structure function of the proton, $x_F D^{(3)}_F$, as a function of $Q^2$ for different regions of $\beta$ and $x_F$. The curves show the result of the BEKW(mod) fit to the data. Figure reprinted with permission from ZEUS [31].

The original BEKW parametrization also includes a higher-twist term for $q\bar{q}$ produced by transverse photons. Since the data were insensitive to this term, it was neglected. A fit of this modified BEKW form (BEKW(mod)) to the ZEUS data [30, 31] gave an excellent description of the data with the following result for the five parameters left free: $c_T = 0.118 \pm 0.002$, $c_L = 0.087 \pm 0.005$, $c_g = 0.0090 \pm 0.0003$, $n_1 = 0.062 \pm 0.002$ and $\gamma = 8.22 \pm 0.46$.

For $W^2 \gg M_X^2$, $Q^2$, the power $n(Q^2)$ implies for the $W$-dependence of the diffractive cross section, $d\sigma_{\text{diff}}/dM_X^2 \propto W^{2n(Q^2)}$ with $n = 0.20 \pm 0.01$, $0.34 \pm 0.01$, $0.41 \pm 0.01$, for $Q^2 = 10 \text{ GeV}^2$, $100 \text{ GeV}^2$, $320 \text{ GeV}^2$, respectively.

The curves in figures 107 and 108 show the individual contributions from the $(q\bar{q})_T$, $(q\bar{q})$, and $(q\bar{q}g)\gamma$ terms, and their sum. The contribution from $(q\bar{q})_L$ is important for $\beta \gtrsim 0.96$,
In general, the $x_F F_D^{(3)}$ values measured at different $Q^2$ values cluster around a narrow band of the form $\beta \cdot (1 - \beta)$. However, for $x_F > 0.005$ and low $\beta < 0.3$ or high $\beta > 0.9$ the data lie above the $\beta \cdot (1 - \beta)$ band, a feature which becomes more prominent as $x_F$ increases. The curves show the contributions from $(q\bar{q})_T$, $(q\bar{q}g)_T$ and $(q\bar{q})_L$, as determined by the BEKW(mod) fit. The dominant contribution is of the form $\beta \cdot (1 - \beta)$ as predicted for $(q\bar{q})_T$; the rise towards small $\beta \leq 0.2$ is the result of gluon emission, $(q\bar{q}g)_T$; contributions from longitudinal photons, $(q\bar{q})_L$, are important only near $\beta = 1$.

12. High-mass diffraction at the LHC—a cross section estimate

Given the results from HERA, one can attempt to estimate the contribution from diffraction by pp scattering, producing a heavy system $X$ by $pp \to ppX$ at the large hadron collider (LHC) of CERN. For the HERA data the maximum value of $x_F$, for which the contribution from diffraction is substantial,
Figure 112. Diffractive structure function of the proton, $x_p F_2^{D(3)}$, here denoted by $x_p \sigma^{D(3)}$, as function of $x_p$ for $Q^2$ values between 55 and 190 GeV$^2$. Figure reprinted with permission from ZEUS [176].

is around $x_p = 0.03 - 0.04$, and $\sigma_{\text{diff}} (M_X = 8 - 35 \text{ GeV}, W = 225 \text{ GeV}) \approx 0.06 \cdot \sigma_{\text{tot}} (\gamma^* p)$. Assume now that each of the LHC colliding beam protons emits a Pomeron, $\text{IP}_1$, $\text{IP}_2$, which carry, on average, fractions $x_{\text{IP}_1}, x_{\text{IP}_2} \approx 0.035$, respectively, of the momenta of the proton beams. The two Pomerons collide and produce a system with mass $M_{\text{IP}_1 \cdot \text{IP}_2} \approx (x_{\text{IP}_1} + x_{\text{IP}_2}) \cdot p_{\text{beam}} = 0.07 \cdot p_{\text{beam}}$, e.g. for $p_{\text{beam}} = 5 \text{ TeV}$: $M_{\text{IP}_1 \cdot \text{IP}_2} = 0.35 \text{ TeV}$. An estimate of the cross section for this process gives

$$
\sigma (\text{pp} \to \text{ppX}, X = \text{IP}_1 \cdot \text{IP}_2, M_X = 350 \text{ GeV}) / \sigma_{\text{tot}} (\text{pp})
\approx [\sigma_{\text{diff}} (\gamma^* p \to X p) / \sigma_{\text{tot}} (\gamma^* p)]^2 \approx (0.06)^2 = 4 \times 10^{-3}.
$$

This exercise indicates that at the LHC, it will be possible to investigate the diffractive production of states $X$, which carry the quantum numbers of the vacuum, up to masses $M_X$ of several hundred GeV.

Recent studies of diffractive processes that can be studied with proton–proton collisions at LHC, using input from the measurements at HERA, can be found in [184, 185].

13. Summary and conclusions

HERA has opened the door for the study of diffraction in real and virtual photon–proton scattering over a large range in centre-of-mass energy $W$, photon virtuality $Q^2$, and mass $M_X$ of the system $X$ produced by the transition $\gamma^* p \to X N$. For diffractive $\gamma^* p$ scattering, $Q^2$ provides a hard scale which could be varied from zero up to 2000 GeV$^2$; the latter value corresponds to a spatial resolution of about $4 \times 10^{-16}$ cm. For comparison, in hadron–hadron collisions, diffractive scattering reflects features of the QCD force at a resolution of about $10^{-13}$ cm, as given by the radii of the colliding hadrons.

A substantial fraction of diffractive scattering leads to the production of vector mesons, $\gamma^* p \to V p$, where $V = \rho^0, \omega, \phi, J/\Psi, \Upsilon$. For fixed $Q^2$, the cross sections rise with c.m. energy proportional to $W$. In photoproduction, $\delta \approx 0.2$ for the light vector mesons, and $\delta = 0.8$ ($\approx 1.6$) for $J/\Psi$ ($\Upsilon$). In virtual photon–proton scattering, the integrated cross sections at fixed $W (75 \text{ GeV})$ lie on a universal curve, decreasing exponentially with $(Q^2 + M_V^2)$ provided $(Q^2 + M_V^2) > 3 \text{ GeV}^2$. The differential cross-sections $d\sigma/d|t|$ decrease with $|t|$ proportional to $e^{-b|t|}$, where $b \approx 10 \text{ GeV}^{-2}$ for $(Q^2 + M_V^2) < 1 \text{ GeV}^2$. With increasing $Q^2$, $b$ decreases rapidly to a constant value of about $5 \text{ GeV}^{-2}$, consistent with the radius of the proton: the transverse extension of the $\gamma^*$ becomes negligible.

The $Q^2$ dependence of $\sigma (\gamma^* p \to \rho^0 p)$ is reproduced well by a QCD-based model where the photon fluctuates into a
Figure 113. Diffractive structure function of the proton, $x_F F_{2D}^{(3)}$, as function of $\beta$ for the $Q^2$ values indicated, (a) at fixed $x_F = 0.0012$ and (b) at $x_F = 0.0025$. The curves show the result of the BEKW(mod) fit for the contributions from transverse (dashed) and longitudinal photons (dotted) and for the $(q\bar{q}g)$ contribution from transverse photons (dashed–dotted), together with the sum of all contributions (solid). Figure reprinted with permission from ZEUS [31].

Figure 114. Diffractive structure function of the proton, $x_F F_{2D}^{(3)}$, as function of $\beta$ for the $Q^2$ values indicated, (a) at fixed $x_F = 0.005$ and (b) at $x_F = 0.01$. The curves show the result of the BEKW(mod) fit for the contributions from $(q\bar{q})$ for transverse (dashed) and longitudinal photons (dotted) and for the $(q\bar{q}g)$ contribution for transverse photons (dashed–dotted), together with the sum of all contributions (solid). Figure reprinted with permission from ZEUS [31].
The diffractive structure function of the proton, $x_F F^{(D)}_2(\beta, Q^2, x_p)$, is important for
$\beta$ and $Q^2$, $x_F F^{(D)}_2$ rises as $x_p \to 0$.

A QCD-inspired model (BEKW), in which the incoming photon fluctuates into a $q\bar{q}$ or $q\bar{q}g$ dipole that interacts with the target proton by the exchange of a two-gluon system (the ‘Pomeron’), gives an excellent description of the data for inclusive diffraction, after adjusting the free parameters. The contribution from longitudinal photons turning into a $q\bar{q}$ pair is important for $\beta > 0.96$. Transverse photons leading to a $q\bar{q}$ pair dominate the region $0.14 < \beta < 0.9$, while those leading to a $q\bar{q}g$ system are important for $\beta < 0.1$.

Finally, at the large hadron collider (LHC), it will be possible to study diffraction proceeding via double Pomeron exchange, $pp \to IPp$, and to search for states with vacuum quantum numbers, $IP$, up to masses of several hundred giga-electronvolts.

Acknowledgments

The author is most grateful to Brian Foster, Erich Lohrmann and Paul Söding for a critical review of the manuscript.

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References

[1] Pomeranchuk I 1958 Sov. Phys. JETP 7 499
[2] Chew G F and Frautschik S C 1961 Phys. Rev. Lett. 7 394
[3] Griblov V N 1961 J. Exp. Theor. Phys. 41 667
[4] Landshoff P V and Polkinghorne J C 1971 Nucl. Phys. B 32 541
[5] Jaroszkiewicz G A and Landshoff P V 1974 Phys. Rev. D 10 1170
[6] Low F E 1975 Phys. Rev. D 12 169
[7] Nussinov S 1975 Phys. Rev. Lett. 34 1286
[8] Nussinov S 1976 Phys. Rev. D 14 246
[9] Ingelman G and Schlein C E 1985 Phys. Lett. B 152 256
[10] Donnachie A and Landshoff P V 1987 Phys. Lett. B 191 309
[11] Steng K H 1987 HERA Study Group on QCD and JET Physics DESY HERA Workshop (Hamburg, Germany) ed R D Peccei, p 365
[12] UAS Coll., Brandt A et al 1988 Phys. Lett. B 211 239
[13] UAS Coll., Brandt A et al 1992 Phys. Lett. B 297 417
[129] McLerran L and Venugopalan R 1994 Phys. Rev. D 49 2233
[130] McLerran L and Venugopalan R 1994 Phys. Rev. D 50 2225
[131] Watt G and Kowalski H 2008 Phys. Rev. D 78 014016
[132] ZEUS Coll., Chekanov S et al 2007 Eur. Phys. J. C 51 301
[133] Alvero L et al 1999 Phys. Rev. D 59 074022
[134] H1 Coll., Aaron F D et al 2007 Eur. Phys. J. C 50 1
[135] ZEUS Coll., Derrick M et al 1994 Phys. Lett. B 332 228
[136] H1 Coll., Ahmed T et al 1995 Nucl. Phys. B 435 3
[137] ZEUS Coll., Derrick M et al 1995 Phys. Lett. B 356 129
[138] ZEUS Coll., Derrick M et al 1996 Phys. Lett. B 396 55
[139] ZEUS Coll., Breitweg J et al 1998 Eur. Phys. J. C 5 41
[140] ZEUS Coll., Chekanov S et al 2008 Eur. Phys. J. C 55 177
[141] H1 Coll., Adloff C et al 1999 Eur. Phys. J. C 6 421
[142] H1 Coll., Adloff C et al 2002 Eur. Phys. J. C 24 517
[143] H1 Coll., Aktas A et al 2007 Eur. Phys. J. C 51 549
[144] Klasen M and Kramer G 2004 Eur. Phys. J. C 38 93
[145] ZEUS Coll., Breitweg J et al 1998 Phys. Lett. B 421 368
[146] PLUTO Coll., Berger Ch et al 1982 Z. Phys. C 12 297
[147] TASSO Coll., Althoff M et al 1984 Z. Phys. C 26 157
[148] Vermaseren J A M et al 1997 DESY Report 97-031 and Proc. Madrid Workshop 1997, Low-x Physics at HERA p 273
[149] ZEUS Coll., Chekanov S et al 2004 Eur. Phys. J. C 38 43
[150] H1 Coll., Aktas C et al 2006 Eur. Phys. J. C 48 749
[151] Groys M, Levy A and Proskuryakov A 2005 HERA and the LHC, CERN-2005-014 and DESY-Proc-2005-001, ed A DeRoeck and H Jung, p 499
[152] Martin A D, Ryskin M G and Watt G 2006 Phys. Lett. B 644 131
[153] Jung H 1995 Comput. Phys. 86 147
[154] ZEUS Coll., Chekanov S et al 2001 Phys. Lett. B 516 3
[155] Kowalski H 1999 DESY 99-141
[156] Yamashita T 2001 PhD Thesis University of Tokyo
[157] Jung H and Kowalski H private communication
[158] ZEUS Coll., Chekanov S et al 2007 Eur. Phys. J. C 52 813
[159] H1 Coll., Aktas A et al 2006 Eur. Phys. J. C 48 715
[160] H1 Coll., Aktas A et al 2007 JHEP10(2007)042
[161] Nagy Z and Trocsanyi Z 2001 Phys. Rev. Lett. 87 082001
[162] ZEUS Coll., Derrick M et al 1994 Phys. Lett. B 338 483
[163] Bengtsson M, Ingelman G and Sjöstrand T 1988 Nucl. Phys. B 301 554
[164] L"onnblad L 1986 Comput. Phys. Commun. 39 347
[165] Sj"ostrand T and Bengtsson M 1987 Comput. Phys. Commun. 43 367
[166] Bruni P and Ingelman G 1993 DESY-Report 187
[167] ZEUS Coll., Derrick M et al 1995 Z. Phys. C 67 227
[168] H1 Coll., Ahmed T et al 1995 Nucl. Phys. B 348 681
[169] ZEUS Coll., Derrick M et al 1995 Z. Phys. C 68 569
[170] H1 Coll., Aid S et al 1996 Z. Phys. C 70 609
[171] H1 Coll., Adloff C et al 1997 Z. Phys. C 76 613
[172] ZEUS Coll., Breitweg J et al 1998 Eur. Phys. J. C 1 81
[173] UA4 Coll., Bozzo M et al 1984 Phys. Lett. B 147 385
[174] H1 Coll., Aktas A et al 2007 Eur. Phys. J. C 48 715
[175] Bartels J 2005 Eur. Phys. J. C 43 3
[176] ZEUS Coll., Chekanov S et al 2009 Nucl. Phys. B 816 1
[177] Fadin V S, Kuraev E A and Lipatov L N 1976 Sov. Phys.—JETP 44 443
[178] Balitsky Y and Lipatov L N 1978 Sov. J. Nucl. Phys. 28 822
[179] Bjorken J D 1964 SLAC-PUB-6477
[180] W"usthoff M and Martin A D 1999 J. Phys. G: Nucl. Part. Phys.—JETP R309
[181] Bartels J, Ellis J, Kowalski H and W"usthoff M 1999 Eur. Phys. J. C 7 443
[182] Martin A D, Ryskin M G and Watt G 2005 Eur. Phys. J. C 37 285
[183] Iofie B L 1969 Phys. Lett. B 30 123
[184] Bartels J, Borras K, Diehl M and Jung H (ed) 2007 12th Int. Conf. on Elastic and Diffractive Scattering (Blois Workshop) Forward Physics and QCD and DESY-Proc-2007-02
[185] Jung H and DeRoeck A (ed) 2006–2008 HERA and the LHC, 2nd Workshop on the Implications of HERA for LHC Physics, 2006–2008 (Hamburg-Geneva) (Hamburg: Verlag Deutsches Elektronen-Synchrotron)