Controllable splitting dynamics of a doubly quantized vortex in a ring-shaped condensate

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Abstract

We prepared a ring-shaped Bose–Einstein condensate with quantized circulation by imprinting a linear azimuthal phase. A doubly quantized vortex (DQV) can be generated and exist stably in the middle of the condensate cloud after the system is released into a rotationally symmetric harmonic trap. For an asymmetric trap with a small degree of anisotropy the generated DQV initially splits into two singly quantized vortices and revives again but eventually evolves into two unit vortices due to the dynamic instability. For the degree of anisotropy above a critical value, the DQV is extremely unstable and decays rapidly into two singlet vortices. The geometry-dependent lifetime of the DQV and vortex-induced excitations are also discussed intensively.

Keywords: multiply quantized vortex, ring-shaped Bose–Einstein condensate, quantum dynamics

(Some figures may appear in colour only in the online journal)
rather than one s-quantized vortex. Multiply quantized vortices are also dynamically unstable by transferring the kinetic energy to coherent excitation modes which is driven by interatomic interactions. Therefore, it will be a challenge and interesting issue to create a stable multiply quantized vortex with relatively long lifetime and investigate some complex dynamics associated with such quantum circulation in a controllable way. It is shown that a stable giant vortex may be realized in condensates with a localized pinning potential [16], in a quartic potential [17], or oblate condensates [18]. In harmonically trapped three-dimensional (3D) condensates the stability of a giant vortex depends strongly on the trap anisotropy and the strength of interatomic interaction. The splitting instability of multiply quantized vortices can be suppressed in some narrow parameter range [18–20].

Ring-shaped BECs are, as a multiply connected structure, more suitable for the superfluid behavior than singly connected BECs. They are widely employed to investigate a variety of issues, including persist current [21–23], weak connection [24–27], generation of quasi-stable dark-soliton structure [28], and mimicking cosmic expansion [29]. Recent experiments have achieved to realize multiply quantized circulation in a ring-shaped BEC [21, 30, 31]. The long life-time of the hole structure makes it applicable to produce multiply quantized vortex in a rotating toroidal BEC by releasing it into some certain geometric traps. The multiply quantized vortex would be stabilized by the toroidal BEC because it costs too much energy for the vortex core to move from the center of the torus, where the density is zero, through the high density atomic cloud [32–34]. On the other hand, due to the inward motion of the ring-shaped BEC, the cloud around the hole may bound the vortex core and suppress the dynamical instability. Thus, it is highly promising to explore the dynamics of a multiply quantized vortex in the ring-shaped BEC.

In this paper, we study the dynamics of a doubly quantized vortex (DQV) in a ring-shaped BEC released in a harmonic trap varying from rotationally symmetric geometry to asymmetric one. The stability of the DQV and its lifetime with respect to the trap asymmetry and the interatomic interaction are explored. The associated characteristic dynamics are also shown. Moreover, the higher-fold vortex structures excited by the splitting process of the initial DQV with increasing interatomic interaction of the condensate are also discussed.

2. Simulation schemes and numerical methods

Our general protocol involves preparing a ring-shaped cloud with doubly quantized circulation, of which wave function can be described in the cylindrical coordinate system by

$$\Psi(r, \phi, z) = \begin{cases} A \left[ \frac{\mu - 1}{2} m_0^2 n^2 f(r, z) \right] e^{ix} & \mu \geq \frac{1}{2} m_0^2 n^2 r^2 \\ 0 & \text{otherwise} \end{cases}$$

where $A$ is the normalization coefficient. The distribution function

$$f(r, z) = \left[ 1 - e^{-\frac{r^2}{2\omega_0^2}} \right] e^{\frac{z^2}{2\omega_z^2}},$$

with $\sigma = \sqrt{\hbar/m_0^2 c^2}$. This wave function is employed in some experiments to map the ring-shaped cloud. Such a BEC is created by the toroidal trap potential, which can be produced by using a blue-detuned laser beam to make a repulsive potential barrier in the middle of a harmonic magnetic trap [21]. The toroidal trap potential has the general form

$$V_0(r, z) = \frac{1}{2} m_0^2 n^2 r^2 + \frac{1}{2} m_0^2 n^2 z^2 + V_0 \exp \left[ -\frac{2r^2}{d_0^2} \right].$$

where $\omega_0$ and $\omega_z$ are the harmonic trapping frequencies along $r$ and $z$ direction, respectively. $V_0$ is the maximum optical potential, proportional to the laser intensity. The density distribution of a BEC with a hole in the center is created by the plug beam, if the chemical potential, $\mu$, is less than $V_0$. The parameter $a$ in the distribution function (2) is tunable according to the intensity of the laser beam to determine the size of the hole. We choose such experimental setup because the toroidal trap can be easily changed into a harmonic traps by deceasing the laser power. The harmonic trap frequencies can also be changed by adjusting the magnetic field.

The initial wave function employed in our simulation originates from the experimental data\textsuperscript{10} which refers to a ring-shaped condensate with 12 $\mu$m radial length and 6.5 $\mu$m hole size. To realize such a $^{87}$Rb ring-shaped BEC with the total atom number $N = 10^5$, the radial and axial frequencies are chosen to be $\omega_0 = 2\pi \times 47$ Hz and $\omega_z = 2\pi \times 238$ Hz. A typical density profile of the ring-shaped condensate is given in figure 1(a).

In the subsequent process, the prepared cloud is released into a harmonic trap with different symmetry, varying from isotropic geometry in the radial direction to anisotropic geometry. Then we examine the role of the geometry of the trap on the dynamics of the system, which is governed by the Gross–Pitaevskii (GP) equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi + g|\Psi|^2 \Psi,$$

where $V = \frac{1}{2} \omega_0^2 r^2 |1 - \eta \sin^2 \phi| + \frac{1}{2} \omega_z^2 z^2 + \omega_0^2 r^2$ with $\omega_0 = 2\pi \times 79$ Hz. The parameter $\eta$ indicates the degree of anisotropy in the trap geometry. We set $0 \leq \eta \leq 1$ where $\eta = 0$ accounts for the isotropic trap and $\eta = 1$ for the quadratic order.

9 We also use the trial wave function, $\psi(r, \phi, z) = A \left[ \frac{1}{2} m_0^2 n^2 (r - \rho_0)^2 \exp[-z^2/(2\sigma_0^2)] \right]$ for $\mu \geq \frac{1}{2} m_0^2 n^2 (r - \rho_0)^2$ and otherwise 0, to match the ring-shaped BEC, which is created by a order-1 Laguerre–Gaussian beam with the form $V_0(r, z) = -\lambda_0 \left( \frac{z}{z_0} \right)^2 \exp \left[ -\frac{2r^2}{d_0^2} \right] + \frac{1}{2} \omega_z^2 z^2$. For a typical laser field, $\lambda_0 = 4\lambda_0 / \pi \rho_0^2$ and $K = 6072 \text{kHz} \omega_0^2 \rho_0^2$. $\rho_0$ is the beam waist and can be focused to as small as a few $\mu$m under diffraction limit. $\omega_0$ is the minimum point of trap potential along the radius direction, where the first part of the order-1 Laguerre–Gaussian beam can be treated approximately as $\frac{1}{2} \omega_0^2 (r - \rho_0)^2$ with $\omega_0 = \sqrt{\frac{\omega_0^2}{2\omega_z^2}}$. We emphasize that no qualitative difference in the dynamics is found in our simulation for two types of initial wave functions.\textsuperscript{10} Private communication.
potential along $x$ direction while uniform along $y$ direction. Here the interatomic coupling coefficient $g$ can be tuned from $0.1g_0$ to $50g_0$ where $g_0 = 4\pi\hbar^2a_s/m$. $a_s$ is set to be $5.4\text{nm}$ and $m = 1.44 \times 10^{-25}\text{kg}$, appropriate to a $^{87}\text{Rb}$ condensate.

To explore time evolution of the DQV created by a ring-shaped BEC, the traditionally prevalent way is to numerically solve the 3D GP equation (4) in the Cartesian coordinate system. Note that at the merging point around $r = 0$, the momentum of the ring-shaped BEC is relatively high and also the DQV formed in the merging area is strongly sensitive to the asymmetric-geometry potential. This indicates that a sufficiently small grid size is desired to avoid the distortion arisen from the square lattice in the Cartesian coordinate system. Thus a very demanding computational task is required. Taking the ring-shaped geometry into account and to save the calculation time, we solve the GP equation by using the split-step Crank-Nicolson algorithm on the 3D cylindrical coordinate system with $r$, $\phi$, $z$ spatial grid of $320 \times 200 \times 100$ points. Then we transform the results in the cylindrical coordinate system into ones in the Cartesian coordinate system for more intuitive understanding. We stress that to achieve the stable results for the dynamics of DQV produced in the ring-shaped BEC, our calculation in the $r$, $\phi$, $z$ space is much less time-consuming than the one in the $x$, $y$, $z$ space, e.g. at least five times less than the calculation time in the Cartesian coordinate system for one dynamical process. To decrease the error induced by the square lattice in the dynamics of giant vortices, the latest work employs discrete exterior calculus with tetrahedral tiling [18].

### 3. Results and discussion

#### 3.1. Stable DQV

To get first insight, and see the stability of a DQV, we begin by discussing the oscillation of the ring-shaped BEC with the symmetric density envelope in an isotropic trap, i.e. $\eta = 0$. Figure 1 shows that the initial ring-shaped BEC with $s = 2$ circulation (see (a)) expands toward the center and forms a DQV (see (b)). In a typical time scale $1/\omega = 2\text{ms}$, the BEC experiences the largest compression in the $r$ direction and the largest extension in the $z$ direction (see (c)). Correspondingly, the core size of the DQV oscillates and never decays into two unit quantized vortices, even though the interference pattern manifests in the density distribution at relatively long time (see (f)) and the BEC experiences quadrupole-like excitation motion, which means that the condensate expands in one direction while squeezes in the other direction. This is contrast to the well known situation where a DQV imprinted in a harmonically trapped BEC is unstable and decay rapidly into two singly quantized vortices [6, 35, 36], which is mainly a consequence of dynamical instability. This indicates that the hole-induced inward and outward motion of the ring-shaped BEC can stabilize the DQV in a symmetric trap. In what follows, we demonstrate how the asymmetric geometry of the harmonic trap affects the vortices.

#### 3.2. Splitting and revival

The initial state of the ring-shaped condensate is chosen to be identical to the one shown in figure 1. We calculate the

![Figure 1](image1.png)

**Figure 1.** Temporal density isosurface of a ring-shaped condensate with the circulation $s = 2$ in a symmetric trap, i.e. $\eta = 0$ for 1% of the maximum density and $g = g_0$. Top row refers to the top views of the density isosurface and bottom row shows the corresponding side view of the density isosurface. Different columns correspond to different time: $t = 0\text{ms}$ (a), $0.57\text{ms}$ (b), $1.42\text{ms}$ (c), $2.0\text{ms}$ (d), $2.6\text{ms}$ (e), $45.5\text{ms}$ (f). Horizontal bar shows scale.

![Figure 2](image2.png)

**Figure 2.** Temporal density isosurface of the ring-shaped condensate in an asymmetric trap, i.e. $\eta = 0.1$ for 1% of the maximum density ((a)–(h)) and 0.1% of the maximum density ((i)) with $g = g_0$. Different panels correspond to different times: $t = 0\text{ms}$ (a), $0.85\text{ms}$ (b), $1.1\text{ms}$ (c), $1.7\text{ms}$ (d), $2.6\text{ms}$ (e), $3.1\text{ms}$ (f), $3.7\text{ms}$ (g), $6\text{ms}$ (h), $9.4\text{ms}$ (i).
dynamics of the condensate after it is released into an asymmetric trap. Some typical density distributions are shown in figure 2, where the value of the isosurface are kept being 1% of the maximum density of the condensate cloud. In distinction to the evolution of the ring-shaped BEC in the rotationally symmetric trap, the DQV here splits into a pair of straight vortex lines with unit circulation (see figures 2(c) and (d)), indicating that the dynamical instability is arisen from the asymmetric geometry of the trap. In a later time, the rapidly reducing atoms around the z axis due to the breathing motion of the BEC and the resulting local imbalance of the interaction force on the vortices cause two singly quantized vortices to recombine into a giant vortex (see figure 2(e)), which exhibits a split-and-revival effect. We note that the splitting process is very different from those observed in previous work [19, 36], where a pair of singly charged vortices evolved from a DQV tends to twist around each other in cigar-shaped BECs along the longitudinal direction. We find that for an elongated ring-shaped BEC, the twist and intertwining of the splitting vortex lines will not happen either, implying that the collective excitation of the ring-shaped condensate suppresses the higher unstable frequencies and only the lowest one is left, so the splitting takes place very rapidly along the z axis. The split-and-revival process is also different from the one in [18], where the giant vortex splits with intertwining but returns nearly to its initial state in an isotropic harmonic trap.

Our numerical results show that the two vortices are also driven to oscillate due to the collective excitation motion of the BEC. For a small η (the degree of asymmetry is not sufficiently large), the process of splitting and revival of the DQV will continue in a relatively long period. The size of the core region of the vortices fluctuates with the oscillation of the condensate. After several oscillations, the interference pattern of the matter wave is manifested clearly. It is shown that the splitting vortices prefer to stay in the interference valley when the interference fringes are along the direction of vortex line (see figure 2 (h)), while the vortex lines are ‘cut’ into segments when the interference fringes are perpendicular to the vortex lines (see figure 2 (i)).

We note that the DQV prefers to split along the elongated direction of the BEC. The conservation of local energy is crucial for such splitting through dynamical instability. The local energy of s-charged vortex is approximately

\[ \epsilon_v \approx \frac{s^2}{m} \frac{s^2}{b^2} \ln b \]

[35], where the spatial variation of local density with the radius b around the vortex is neglected. \( n \) is the background density around the vortex and \( \xi \) is the healing length of the BEC. After splitting, s singly quantized vortices are created and have approximately the energy

\[ \epsilon_v' \approx \frac{s^2 m}{\xi^2} \ln \left( \frac{1}{1.464} \right) \]

where the interaction energy between vortices is neglected[15]. For \( s = 2 \) in our case, two single quantized vortices prefers to stay in the area of \( n' = n \) to ensure that the local energy around them is not changed greatly in the dynamic process. This results that the 2-charged vortices split into two unit quantized vortices along the elongated direction of the condensate cloud, where the density varies more slowly than the one in the shorten direction and is closer to the density in the position of the initial vortices (see figure 3).

To explore the effect of the asymmetry trap geometry on the splitting of DQVs further, we perform a series of calculations and show the lifetime of the DQVs, \( T \), as a function of the degree of anisotropy, \( \eta \), in figure 4. For \( \eta < 0.16 \), \( T \) decays approximately exponentially with increasing \( \eta \) while it reaches its lower limit at about \( \eta = 0.16 \). It indicates that when \( \eta \) is above the critical value, the DQV is extremely unstable and rapidly splits into two singly quantized vortices if ever the DQV is formed from the merging ring-shaped BEC. However, for \( \eta \) smaller than the critical value, the lifetime of the DQV in the ring-shaped BEC is dependent of the trap geometry and becomes stable for a sufficiently small degree of anisotropy. By comparing the blue circles and the dark squares in figure 4, one can see that for a given \( \eta \) smaller than the critical value, a strong interatomic interaction tends to shorten the lifetime of the DQV, despite it speeds up the

\[ \text{Figure 3. Schematic plot of the splitting of 2-charged vortex induced by the asymmetric trap potential.} \]

\[ \text{Figure 4. The lifetime of a DQV against its decay into two singly quantized vortices, } T, \text{ as a function of the degree of anisotropy in the trap geometry, } \eta. \text{ The dark squares indicate } g = g_0 \text{ and the blue circles } g = 10g_0. \text{ The red dashed line denotes the point of time at which the ring-shaped BEC merges fully into the central region and forms a DQV.} \]

11 If the separation \( d \) of two singly quantized vortices is much larger than the healing length \( \xi \), this approximation is also valid.
merging of ring-shaped BEC and the resulting formation of the DQV in an early time. Since the critical value of \( \eta \) is not changed manifestly for different interatomic interaction, this implies that the geometry of the harmonic trap is a prominent factor for the dynamical instability of the DQV produced in the ring-shaped BEC. Moreover, we emphasize that the theoretical lifetime here should be much shorter than the realistic experimental observation when the identical parameters are taken into account. We identify the separation of two vortex lines with its inner structure of the density iso-surface for less than 0.1% of the maximum density but the outer structure near the surface of the condensate appears still as one core (see figure 4 insert). By contrast, experiments confirm the DQV splitting through the visible spatial separation of two vortex lines, e.g. similar to figure 2 (f).

3.3. Vortex-induced excitation

In this part, we further discuss the influence of an increasing interatomic interaction on the dynamic excitation of a ring-shaped BEC expanding in a fixed anisotropic trap. Due to the \( s = 2 \) circulation of the whole condensate, the angular momentum of the system is much larger than the local energy of the DQV. We find that with \( g \) increasing up to \( 20g_0 \), the two singly quantized vortices with \( l = 2 \) fold rotational symmetry can excite four singly quantized vortices with approximately four-fold symmetry (see figures 5(a) and (c)). With stronger interatomic interaction, e.g. \( g = 50g_0 \), the two singly quantized vortices excite not only four vortices but also six quantized vortices with approximately six-fold symmetry (see figures 5(b) and (d)). This indicates that the stronger interatomic interaction can produce excitations with higher energy. The dynamical excitations prefer to the \( \phi \)-direction excitations rather than the \( r \)-direction and \( z \)-direction. Here we stress that under the same parameters, the ring-shaped BEC with \( s = 1 \) circulation is quite robust and no \( l > 2 \) fold excitations are produced. Note that the nonlinear excitation patterns are induced by the multiple vortex and strong interatomic interaction, in distinction to those appear in [8, 18, 37] where the low-lying excitation determines the decay pattern of the multiply quantized vortex.

4. Conclusion

We have studied the dynamics of the DQV created in a ring-shaped BEC evolving in the harmonic traps with varying degree of anisotropic geometry as well as different interatomic interactions. For an isotropic trap, the DQV can exist stably, while for the degree of anisotropy below a critical value the life time of the DQV depends strongly on the geometry of the harmonic trap. However, for the degree of anisotropy above the critical value, the DQV becomes extremely unstable and decay rapidly into two unit vortices once the DQV is formed. Moreover, the finitely large mean-field interaction has little effect on the critical value for a given ring-shaped BEC. Our simulation suggests that the ring-shaped BEC can be a desirable candidate to investigate the dynamics of multiply quantized vortex in a controllable way.

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**References**

[1] Madison K W, Chevy F, Wohlleben W and Dalibard J 2000 Vortex formation in a stirred Bose–Einstein condensate Phys. Rev. Lett. **84** 806

[2] Abo-Shaeer J R, Raman C, Vogels J M and Ketterle W 2001 Observation of vortex lattices in Bose–Einstein condensates Science **292** 476

[3] Hölty E, Hohenblumner G, Hopkins S A, Maragó O M and Foot C J 2001 Vortex nucleation in Bose–Einstein condensates in an oblate purely magnetic potential Phys. Rev. Lett. **88** 010405

[4] Fischer U R and Baym G 2003 Vortex states of rapidly rotating dilute Bose–Einstein condensates Phys. Rev. Lett. **90** 140402

[5] Leanhardt A E, Górilitz A, Chikkatur A P, Kielinski D, Shin Y, Pritchard D E and Ketterle W 2002 Imprinting vortices in a Bose–Einstein condensate using topological phases Phys. Rev. Lett. **89** 190403

[6] Shin Y, Saba M, Vengalattore M, Pasquini T A, Sanner C, Leanhardt A E, Prentiss M, Pritchard D E and Ketterle W 2004 Dynamical instability of a doubly quantized vortex in a Bose–Einstein condensate Phys. Rev. Lett. **93** 160406

[7] Kumakura M, Hirotani T, Okano M, Takahashi Y and Yabuzaki T 2006 Topological formation of a multiply charged vortex in the rb Bose–Einstein condensate: effectiveness of the gravity compensation Phys. Rev. A **73** 063605

[8] Isoshima T, Okano M, Yasuda H, Kasa K, Huhtamäki J A M, Kumakura M and Takahashi Y 2007 Spontaneous splitting of a quadruply charged vortex Phys. Rev. Lett. **99** 200403

[9] Mottönen M, Pietilä V and Virtanen S M M 2007 Vortex pump in Bose–Einstein condensates Phys. Rev. Lett. **99** 250406

[10] Andersen M F, Ryu C, Cladé P, Natarajan V, Vaziri A, Helmersen K and Phillips W D 2006 Quantized rotation of atoms from photons with orbital angular momentum Phys. Rev. Lett. **97** 170406

[11] Mel’nikov A S and Vinokur V M 2002 Mesoscopic superconductor as a ballistic quantum switch Nature **415** 60

[12] Roncaglia M, Rizzi M and Dalibard J 2011 From rotating atomic rings to quantum hall states Sci. Rep. **1** 43

[13] Abraham M, Aranson I and Galanti B 1995 Vortex dynamics in a model of superflow: the role of acoustic excitations Phys. Rev. B **52** R7018–21

[14] Aranson I and Steinberg V 1996 Stability of multicharged vortices in a model of superflow Phys. Rev. B **53** 75–8

[15] Cividir A, dos Santos F E A, Galantucci L, Bagnato V S and Bareghchi C F 2016 Controlled polarization of two-dimensional quantum turbulence in atomic Bose–Einstein condensates Phys. Rev. A **93** 033651

[16] Simula T P, Virtanen S M M and Salomaa M M M 2002 Stability of multiquantum vortices in dilute Bose–Einstein condensates Phys. Rev. A **65** 033614

[17] Lundh E 2002 Multiply quantized vortices in trapped Bose–Einstein condensates Phys. Rev. A **65** 043604

[18] Rabinin J, Kuopanportti P, Kivioja M I, Mottönen M and Rossi T 2018 Three-dimensional splitting dynamics of giant vortices in Bose–Einstein condensates Phys. Rev. A **98** 023624

[19] Huhtamäki J A M, Mottönen M, Isoshima T, Pietilä V and Virtanen S M M 2006 Splitting times of doubly quantized vortices in dilute Bose–Einstein condensates Phys. Rev. Lett. **97** 110406

[20] Lundh E and Nilsen H M 2006 Dynamic stability of a doubly quantized vortex in a three-dimensional condensate Phys. Rev. A **74** 063620

[21] Ryu C, Andersen M F, Cladé P, Natarajan V, Helmersen K and Phillips W D 2007 Observation of persistent flow of a Bose–Einstein condensate in a toroidal trap Phys. Rev. Lett. **99** 260401

[22] Beattie S, Moulder S, Fletcher R J and Hadzibabic Z 2013 Persistent currents in spinor condensates Phys. Rev. Lett. **110** 025301

[23] Yakimenko A I, Isaieva K O, Vilechinski S I and Weyrauch M 2013 Stability of persistent currents in spinor Bose–Einstein condensates Phys. Rev. A **88** 051602(R)

[24] Ramanathan A, Wright K C, Muniz S R, Zelan M, Hill W T III, Cobb J C, Helmersen K, Phillips W D and Campbell G K 2011 Superflow in a toroidal Bose–Einstein condensate: an atom circuit with a tunable weak link Phys. Rev. Lett. **106** 130401

[25] Wright K C, Blakestad P B, Cobb J C, Phillips W D and Campbell G K 2013 Driving phase slips in a superfluid atom circuit with a rotating weak link Phys. Rev. Lett. **110** 025302

[26] Piazza F, Collins L A and Smerzi A 2009 Vortex-induced phase-slip dissipation in a toroidal Bose–Einstein condensate flowing through a barrier Phys. Rev. A **80** 021601(R)

[27] Piazza F, Collins L A and Smerzi A 2013 Critical velocity for a toroidal Bose–Einstein condensate flowing through a barrier J. Phys. B: At. Mol. Opt. Phys. **46** 095302

[28] Gallucci D and Proukakis N P 2016 Engineering dark solitary waves in ring-trap Bose–Einstein condensates New J. Phys. **18** 025004

[29] Eckel S, Kumar A, Jacobson T, Spielman I B and Campbell G K 2018 A rapidly expanding Bose–Einstein condensate: an expanding universe in the lab Phys. Rev. X **8** 021021

[30] Moulder S, Beattie S, Smith R P, Tamuz N and Hadzibabic Z. 2012 Quantized supercurrent decay in an annular Bose–Einstein condensate Phys. Rev. A **86** 013629

[31] Murray N, Krygier M, Edwards M, Wright K C, Campbell G K and Clark C W 2013 Probing the circulation of ring-shaped Bose–Einstein condensates Phys. Rev. A **88** 053615

[32] Leggett A J 2001 Bose–Einstein condensation in the alkali gases: some fundamental concepts Rev. Mod. Phys. **73** 307

[33] Bloch F 1973 Superfluidity in a ring Phys. Rev. A **7** 2187

[34] Tempere J, Devreese J T and Abraham E R I 2001 Vortices in Bose–Einstein condensates confined in a multiply connected laguerre–gaussian optical trap Phys. Rev. A **64** 023603

[35] Pethick C J and Smith H 2002 Bose–Einstein Condensation in Dilute Gases (Cambridge: Cambridge University Press)

[36] Mateo A M and Delgado V 2006 Dynamical evolution of a doubly quantized vortex imprinted in a Bose–Einstein condensate Phys. Rev. Lett. **97** 180409

[37] Kuopanportti P and Mottönen M 2010 Splitting dynamics of giant vortices in dilute Bose–Einstein condensates Phys. Rev. A **81** 033627