Warped brane-world compactification with Gauss-Bonnet term

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Abstract

In the Randall-Sundrum (RS) brane-world model a singular delta-function source is matched by the second derivative of the warp factor. So one should take possible curvature corrections in the effective action of the RS models in a Gauss-Bonnet (GB) form. We present a linearized treatment of gravity in the RS brane-world with the Gauss-Bonnet modifications to Einstein gravity. We give explicit expressions for the Neumann propagator in arbitrary \( D \) dimensions and show that a bulk GB term gives, along with a tower of Kaluza-Klein modes in the bulk, a massless graviton on the brane, as in the standard RS model. Moreover, a non-trivial GB coupling can allow a new branch of solutions with finite Planck scale and no naked bulk singularity, which might be useful to avoid some of the previously known “no–go theorems” for RS brane-world compactifications.

Keywords: Braneworld, warped extra dimensions, Gauss-Bonnet interaction, no–go theorem

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1 Introduction

In recent years, there has been considerable interest in the dynamics of brane interactions and brane-worlds with warped extra dimensions. The interest was motivated by the ideas coming out of the Randall-Sundrum (RS) brane-world scenario with a warped fifth dimension [1, 2]. The RS brane-world scenario promptly received many generalization in higher dimensions [3, 4, 5, 6, 7] which have subsequently attracted much interest in gravity and cosmology research. A necessary ingredient of the brane-world models is that the space-time metric contains a warp factor $e^{-2A(z)}$ that depends non-trivially on the extra dimension(s), and, as a result, a brane model of two 3-branes with opposite brane tensions [1], known as RS1 model, may provide a geometrical resolution of hierarchy problem. This proposal was made more concrete in the scenario pioneered further by Randall and Sundrum, known as the RS single brane (or RS2) model [2], where the fifth dimension is non-compact. The latter model, viewed as an alternative to Kaluza-Klein compactification [2], gives an illustrative example of localized gravity on a singular 3-brane.

To localize gravity to the RS brane, one considers a bulk space-time with negative cosmological constant. The low energy effective action in five space-time dimensions is taken to be

$$S = \int_{\mathcal{B}} d^5 X \sqrt{|g|} \left( \frac{R}{\kappa_5} - 2\Lambda \right) + \sum_i \int_{\partial \mathcal{B}_i} d^4 x \sqrt{|h|} \left( \mathcal{L}_m - T_i \right),$$

(1)

where $\kappa_5 = 16\pi G_5 = M_5^{-3}$, with $M_5$ being the five-dimensional mass (energy) scale, and $h$ is the determinant of the induced metric on the 3-brane. In a single brane setup, the brane tension $T$ is positive ($T > 0$), and the anti-de Sitter length scale $\ell$ is set by a relation $\ell^2 = -6M_5^3/\Lambda$. One of the interesting features of the RS action (1) is the presence of four-dimensional gravity as the zero-mode spectrum of a five-dimensional theory. One also finds correct momentum and tensor structures for the graviton propagator due to the “brane-bending” mechanism [4, 7, 8, 9].

The action (1) describes the dynamics of a background metric field for sufficiently weak curvatures and sufficiently long distances. To further explore the general properties of brane-worlds, it is more natural to consider the leading order curvature corrections as predicted by string (bulk) theory. With warped space-time metrics in the bulk, however, one should take the curvature corrections in the RS action to be no larger than the second derivatives of the metric. Thus one can introduce the higher order corrections only in a special Gauss-Bonnet (GB) combination. The GB term arises as $\alpha'$ corrections in bosonic string theory [10, 11] and in heterotic M-theory scenario of Horova-Witten type [12]. Such corrections might be crucial in space-time dimensions $D \geq 5$, in particular, when the brane-world scenario is viewed as a low energy limit of string/M theory.

There are now growing interest in the RS brane-world models modified by higher derivative corrections. Such corrections in a Gauss-Bonnet form, for constant dilaton fields, had been considered earlier in Refs. [13, 14], see also Ref. [15] which give some realizations of brane-world inflation due to quantum correction. It is learnt that brane-world configurations with a GB term and several co-dimensions one
branes can induce brane junctions of non-trivial topology \[16, 17\]. Furthermore, the presence of a GB term coupled to a bulk scalar in the effective action leads to interesting physics in a variety of context, ranging from gravity localization \[18, 19, 20, 21, 22, 23, 24, 25, 26\] to FRW type cosmology on a brane \[27, 28, 29\]. It is worth noticing that a five-dimensional brane-world model with a GB term reproduces all essential properties of the original RS models, including the tensor structure of a massless graviton \[9\]. In this paper, we extend the work in Ref. \[9\] to the \(D\) dimensional space-times, and also examine the general properties of brane-world solutions without and with a bulk scalar.

A fine tuned relation between the bulk cosmological constant and the brane tension is required in the RS model to maintain flatness of the 3-brane \[30\]. This problem has been known as "no-go theorem" \[30, 31, 32\] for non-singular RS or de Sitter compactifications based in Einstein gravity. It is therefore of interest to know whether these arguments can be changed to cover alternatives in the brane-world actions. With a GB correction to Einstein gravity, fine tuning is required in the RS models to get a singularity free solution with the finite Planck scale \[33\]. Nevertheless, in the presence of such interaction, there exists a new branch of the solutions, for which it might be possible that the only bulk singularity occurs when the warp factor \(e^{-A(z)}\) vanishes at the anti-de Sitter horizon, \(z \rightarrow \infty\). Presumably, a GB term might smooth out the bulk singularities and hence avoid some of the previously given no-go arguments for the RS compactifications.

The rest of the paper is organized as follows. In Sec. 2, we introduce the brane-world action and find intersecting brane backgrounds with more than one uncompactified extra dimensions. We give the basic expressions of the linearized Einstein equations with a warped metric, and generalize results of the RS2 model when there are two transverse directions. We then discuss in Sec. 3 some interesting features of the RS solutions modified by a GB term. In Sec. 4, we find the Neumann propagators in \(D\) dimensions, where the background is described as a 3-brane embedded in AdS space of dimensions \(D \geq 5\). In Sec. 5 we analyze the energy conditions and compare the RS solutions in five dimensions with those arising due to a non-trivial GB coupling. In Sec. 6 we give some insights on the nature of singularities or no-go theorem for a class of brane-world gravity models coupled to a bulk scalar field and a Gauss-Bonnet self-interaction term. Section 7 contains discussion and outlooks. In the Appendices, we give some useful derivations of the linearized equations for a class of higher derivative gravity in brane backgrounds.

## 2 Gravity in Brane Backgrounds

We shall begin with the following \(D\)-dimensional gravitational action

\[
S = \int_{B} d^{D}x \sqrt{-g_{D}} \left\{ \frac{R}{\kappa_{D}} - 2\Lambda + \alpha \left( R^{2} - 4R_{pq}R^{pq} + R_{pqrs}R^{pqrs} \right) + \mathcal{L}_{m}^{\text{bulk}} \right\} + \sum_{k} \int_{\partial B} d^{D-1}x \sqrt{-g_{D-1}} \left( \mathcal{L}_{m}^{\text{brane}} - \Lambda_{k} \right) + \int d^{d}x \sqrt{-g_{(d)}^{(31, 22, \cdots = 0)}} (-\mathcal{T}) ,
\]

(2)
where $\partial B$ represents the $(D - 1)$-dimensional boundary. The gravitational coupling $\kappa_D = 16\pi G_D = M^{2-D}$, with $M$ and $G_D$ being, respectively, the $D$ dimensional mass scale and Newton constant. The indices $(p, q, \ldots, s) = (0, 1, 2, 3, \ldots, D - 1)$, $\Lambda$ is a $D$ dimensional bulk cosmological term, $\Lambda_k (k = 1, 2, \ldots, (D - d))$ represent the vacuum energy (brane tension) of the $(D - 2)$-branes, and $T$ is the $d$ dimensional brane tension at a common brane junction. For practical purposes we shall take $d = 4$, so $\alpha$ takes a mass dimension $M^{D-4}$. The second action in (2) is the effective action for $(D - 2)$ branes, while the last term introduced at the common intersection of higher dimensional branes characterizes a four-dimensional brane action. This term will be in effect only if $D \geq 6$, because in $D = 5$ the second action in (2) has already represented the sum of 3-brane actions. For vacuum branes, one has $\mathcal{L}_m^{brane} = 0$, and also $\mathcal{L}_m^{bulk} = 0$, since the matter degrees are supposed to be confined on the branes.

2.1 Choice of background and RS tunings

Let us consider a smooth version of the multidimensional patched $AdS$ space, introduced in Ref. [3] to study intersecting brane-world models, with metric

$$ds^2 = e^{-2A(z)} \left( \eta_{\mu \nu} dx^\mu dx^\nu + \bar{g}_{ij} dz^i dz^j \right),$$

(3)

where $\mu, \nu = 0, 1, \cdots (d - 1)$ with $N \equiv (D - d)$ being the number of extra spatial dimensions. The Einstein field equations ($\alpha = 0$) give a solution $A(z_i) = \log(\sum_{i=1}^{D-4} |z_i|/\ell + 1)$, with $\ell$ being the curvature radius of $AdS$ space, by satisfying the RS relations

$$\Lambda = -\frac{(D - 1)(D - 2)(D - 4) M^{D-2}}{2\ell^2}, \quad \Lambda_k = \frac{4(D - 2) M^{D-2}}{\ell}. \quad (4)$$

One has $T = 0$ for $\alpha = 0$. The four-dimensional brane tension $T$ at the common intersection of two 4-branes is non-zero for $\alpha > 0$ [16, 17]. The two expressions in (4) imply the RS fine-tuned relation

$$\Lambda_k^2 = -\frac{32(D - 2)}{(D - 1)(D - 4)} M^{D-2} \Lambda. \quad (5)$$

Therefore, since $\Lambda_k^2 \geq 0$, the bulk space-time is anti-de Sitter ($\Lambda < 0$). For $D = 5$, $\Lambda_k$ can be replaced by $T$, and hence $\Lambda_k$ defines a 3-brane tension in an $AdS_5$ space.

2.2 Linearized Einstein gravity

Next we consider a linearized theory. With warped space-time metric (3) in the bulk, the linearized $D$-dimensional Einstein field equations take the following form:

$$-\frac{1}{2} \partial^2 h_{pq} + \frac{1}{2} \eta_{pq} \partial^r \partial_r h^{d}_d - \frac{D - 2}{2} \partial^m A \left[ \partial_p h_{qm} + \partial_q h_{pm} - \partial_m h_{pq} \right] - \frac{(D - 2)}{2} \left[ 2\partial_m \partial^m A - (D - 3) \partial^m A \partial_m A \right] h_{pq} + \frac{(D - 2)}{2} \delta_{mn} \left[ 2\partial^m \partial^p A - (D - 3) \partial^m A \partial^p A \right] \eta_{pq}$$

$$= -\kappa_D \left[ \Lambda h_{pq} e^{-2A(z)} + \Lambda_k \left( h_{\mu \nu} - \frac{1}{2} \eta_{\mu \nu} h_m \right) \delta^\mu_p \delta^\nu_q e^{(D-6)A(z)} \right], \quad (6)$$
where we have used a harmonic gauge $\partial^a h_{pq} = \frac{1}{2} h^a$. Since the warp factor $A(z)$ is only a function of the extra spatial coordinates, the indices $m$ and $n$ take face values from $z_1$ to $z_{D-d}$. When $d = 4$, the number of extra dimensions $N$ is $(D - 4)$. The $(\mu\nu)$ components of (6) largely simplify in using the RS fine-tuned relation (4). The second term in (6) would not show up in using transverse-traceless gauge $\nabla^a h_{pq} = 0 = h^a\tilde{\rho}$, instead of the harmonic gauge.

It is more convenient to perturb the metric background (3) in the following form

$$ds^2 = e^{-2A(z)} \left( (\eta_{\mu\nu} + h_{\mu\nu}) \, dx^\mu \, dx^\nu + dz^i dz^j \right).$$

However, in this gauge, other than the standard graviton $h_{\mu\nu}$, there may be some additional gravitational degrees of freedom coming from $h_{\mu m}$ and $h_{mm}$. With more than one (conformal) extra dimensions there is a general problem for diagonalizing the linearized fluctuations of all metric fields, which is computationally difficult. To this end, since a gravitational potential on the brane is mediated by effective 4D-gravitons, we find reasonable to analyze only the linearized field equations for graviton $h_{\mu\nu}$ with the gauge $h_{\mu r} = 0$, $\partial^r h_{\mu r} = 0$. One may use an approximation [3, 6], where the tensor modes either decouple from the off-diagonal components of the perturbed equations like $h_{\mu m}$ and diagonal components $h_{mm}$, or only the nonzero components of the fluctuations are $h_{\mu r}$. We comment upon the decoupling of the scalar modes of the metric perturbations in the discussion section.

In our analysis we follow the background subtraction technique introduced in Ref. [6], where one considers vacuum branes and subtracts out the background fields from the variations of $G_{ab}$ and $H_{ab}$ (the second order Lovelock tensor). For a gravity action of the form (2), one may define $\delta T_{ab} = \bar{T}_{ac} h^c_b$, where $\bar{T}_{ab} = \kappa_D \bar{G}_{ab}$ is taken about the background. This approach extends in an obvious way to developing the perturbative expansion to higher order curvature terms. An additional benefit of this approach is that the RS fine tunings of the previous subsection will be only implicit. Furthermore, it is reasonable to impose the gauge $\partial_m h^{\mu r} = 0$, $h^{\mu r}_m = 0$. With these approximations, the Einstein equations $\delta G_{ab} = \kappa_D \delta T_{ab}$ linear in $h_{\mu r}$ take a remarkably simple form [6]

$$-\partial_a \partial^a h_{\mu r} + (D - 2) \partial^m A \partial_m h_{\mu r} = 0.$$  

We define $h_{\mu r} = e^{(D-2)A(z)/2} \bar{h}_{\mu r}$ and $\Box_4 \bar{h}_{\mu r} = m^2 \bar{h}_{\mu r}$, where $\bar{h}_{\mu r} = \epsilon_{\mu r} e^{ip \cdot x} \psi(z)$, $\epsilon_{\mu r}$ is the polarization tensor, and arrive at the following analog non-relativistic Schrodinger equation

$$\left( -\partial^2_{z_i} + \frac{D(D-2)}{4(|z_i| + \ell)^2} \sum_{i=1}^{D-4} \text{sgn}(z_i)^2 - \frac{(D-2)}{(|z_i| + \ell)} \delta(z_i) \right) \psi(z_i) = m^2 \psi(z_i).$$

We are considering here the case where warp factor $e^{-A(z)}$ is a function of all transverse coordinates $z_i$, where $i = 1, 2, \ldots, (D - 4)$, so that $\sum_{i=1}^{D-4} \text{sgn}(z_i)^2 = (D - 4)$ other than at the origin (brane-junction) in the transverse space. In order to normalize four-dimensional metric at the origin, one has to assume that $A(0) = 0$. For $D = 5$, Eq. (9) gives the RS one-dimensional Schrödinger equation, and this was extensively studied, for example, in Refs. [4, 7, 6]. So we shall be interested only in the $D = 6$ case.
Define a set of new coordinates \( x_\pm \equiv |z_1| \pm |z_2| \), so that the bulk part of (9) takes the form

\[
\left[ -\partial^2_{x_-} - \partial^2_{x_+} + \frac{6}{(|x_-| + |x_+| + \ell)^2} \right] \hat{\psi}(x_-, x_) = \frac{1}{2} m^2 \hat{\psi}(x_-, x_+),
\]

where \( \hat{\psi}(x_-, x_+) \equiv \psi(z_1, z_2) \). We can separate this equation into

\[
-\partial^2_{x_-} \varphi(x_-) = m_-^2 \varphi(x_-), \quad \text{and} \quad -\partial^2_{x_+} + \frac{6}{(|x_-| + |x_+| + \ell)^2} \varphi(x_+) = m_+^2 \varphi(x_+),
\]

where we have defined \( m^2 = 2(m_-^2 + m_+^2) \), and \( \hat{\psi} = \varphi_{m_-}(x_-) \times \varphi_{m_+}(x_+) \). Since \( m_- > 0 \), \( m_-^2 \) and \( m_+^2 \) each can have only positive eigen energy. The solution for each continuum wavefunction is a linear combination of Bessel functions

\[
\varphi_{m_-}(x_-) = N_1(m_-) \sqrt{|x_-|} (\sin(m_- x_-) + A_m \cos(m_- x_-)),
\]

\[
\varphi_{m_+}(x_+) = N_2(m_+ \sqrt{(|x_+| + \ell)} \left[ J_{5/2}(m_+ (|x_+| + \ell)) + B_m Y_{5/2}(m_+ (|x_+| + \ell)) \right].
\]

These solutions must satisfy the Neumann type boundary conditions at the brane junction \( x_\pm = 0 \):

\[
\left( x_\pm \frac{\partial x_+ \pm \partial x_-}{2} + \frac{5}{2} \right) \psi(x_\pm) = 0.
\]

The coefficients \( A_m \) and \( B_m \) can easily be read off

\[
A_m = \cot(m_- x_-), \quad B_m = -\frac{Y_{5/2}(m_+ \ell)}{J_{5/2}(m_+ \ell)}.
\]

Thus \( \psi_{m_+}(0) \sim (m_+ \ell)^{\alpha - 1} \), where \( \alpha + 1/2 = 5/2 \). There are 4 \((= 2^{D-4})\) sectors in the mass spectrum spanned by (i) \( \varphi_0(x_-) \varphi_0(x_+) \), (ii) \( \varphi_{m_-}(x_-) \varphi_0(x_+) \), (iii) \( \varphi_0(x_-) \varphi_{m_+}(x_+) \), and (iv) \( \varphi_{m_-}(x_-) \varphi_{m_+}(x_+) \). The state (i) gives the four-dimensional graviton, and the set of continuum states (ii) or (iii), which is localized in \( x_- \) or \( x_+ \) direction, contributes as an integral over the single eigenvalue \( m_- \) or \( m_+ \) [6]. The set of continuum states (iv) contributes to Newton’s law. For two point masses \( m_1 \) and \( m_2 \) placed at a distance \( |x - x'| = r \) on the four-dimensional brane intersection, the Newtonian potential is

\[
-\Delta_4 V(r) \simeq \frac{m_1 m_2}{M^4(6)} \frac{1}{\ell} \int_{m_0}^{\infty} dm_- \int_{m_0}^{\infty} dm_+ e^{-\hat{m} r} \frac{1}{r} \left| \varphi_{m_-}(0) \right|^2 \left| \varphi_{m_+}(0) \right|^2,
\]

where \( \hat{m} = \sqrt{m_-^2 + m_+^2} \). A qualitative feature of this potential can be known by specializing to the case where \( m_- \) would extend down to \( m_0 = 0 \), and \( \psi_{m_-}(0) = 1 \). Hence

\[
-\Delta_4 V(r) \simeq G_N^{(4)} \frac{m_1 m_2}{r} c_1 \left( \frac{\ell}{r} \right)^3,
\]

where \( 1/G_N^{(4)} \sim \ell^2 M^4(6) \), \( c_1 \) is a number of order one. For large \( D \) and \( \ell < r \), the Kaluza-Klein mode corrections \((\ell/r)^{D-3}\) are more suppressed, so gravity becomes weaker as the number of transverse directions increases. Similar results are known for gravity localized on a four-dimensional string-like defect in \( D = 6 \) [34], a difference here is that now we have two (non-compact) transverse directions, and this is itself more interesting in the scenario of Refs. [35] and [3].
3 Corrections to Einstein Gravity

Next we investigate the brane-world solutions which occur due to a non-trivial Gauss-Bonnet coupling \( \alpha \). The exact metric solution for the modified Einstein equations is given by \( A(z) = \log(\sum_k |z_k|/L+1) \), where \( L \) is the AdS curvature radius which has a contribution from the GB coupling. We should note that \( L^2 \) is defined by the bulk solution [17]

\[
\frac{1}{L^2} = \frac{1}{2(D-4)^2(D-3) \alpha \kappa_D} \left[ 1 \pm \sqrt{1 + \frac{8(D-3)(D-4) \alpha \Lambda \kappa_D^2}{(D-1)(D-2)}} \right]. \tag{18}
\]

The space-time metric is therefore

\[
ds^2_D = \frac{1}{(\sum_{i=1}^n |z_i|/L+1)^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + \sum_{j=1}^n (dz_j)^2 \right). \tag{19}
\]

One may view the above metric background as a four-dimensional intersection of higher dimensional branes or as a 3-brane embedded in space of dimensions \( D \geq 5 \). One has \( L^2 = 2(D-4)(D-3) \alpha \kappa_D \) when the two branches of the bulk solution (18) coincide. The RS type background relations

\[
\Lambda = -\frac{(D-1)(D-2)(D-4)}{2L^2 \kappa_D} \left( 1 - \frac{\gamma/2}{D} \right), \quad \Lambda_k = \frac{4(D-2)(1-\gamma/3)}{L \kappa_D}, \tag{20}
\]

may be assumed implicitly for \( \gamma < 1 \). Here \( \gamma = 2(D-3)(D-4) \alpha \kappa_D/L^2 \) and \( \alpha \) is the usual Gauss-Bonnet coupling, so \( \gamma = 0 \) when \( \alpha = 0 \). In this limit, one finds from (20) the RS fine-tuned relations. We may define \( \alpha = M^{D-5} \alpha_1 \), with \( \alpha_1 \) being a dimensionless GB coupling. If we define \( l = L/\sqrt{(D-4)(1-\gamma/2)} \), then \( \Lambda \) takes the usual form \( \Lambda = -(D-1)(D-2)/(2 \kappa_D l^2) \).

Let us include a matter source \( T^{(m)}_{\mu\nu} \) on the brane, and study linearized equations for the effective four-dimensional gravitational fluctuations. We consider only the scalar wave equation for each of the components \( h_{\mu\nu} \) in the background (19). The linearized field equations for \( h_{\mu\nu} \) read

\[
\left[ 1 - \frac{2(D-3)(D-4)^2 \alpha \kappa_D}{L^2} \right] \left( -\partial_\lambda^2 - \partial_z^2 + \frac{(D-2)}{L} \text{sgn}(z_i) \partial_{z_i} \right) h_{\mu\nu}(x,z) + \frac{2(D-4) \alpha \kappa_D}{L} \sum_{i\neq j} \delta(z_i) \left[ -\partial_\lambda^2 - \partial_z^2 + \frac{(D-3)}{L} \sum_k \text{sgn}(z_k) \partial_{z_k} \right] h_{\mu\nu}(x,z) = -\kappa D T^{(m)}_{\mu\nu}. \tag{21}
\]

One of the important features of the modified solutions due a non-trivial \( \alpha \) is that a Gauss-Bonnet term contributes with a delta-function term to the linearized equations, which implies a non-trivial topology at the brane junction, like that a non-zero four-dimensional brane tension at the common brane-intersection with several co-dimension one branes. In Eq. (21), terms involving Dirac delta functions vanish for \( |z| > 0 \). Therefore, for \( z \neq 0 \), one has a RS type bulk equation but multiplied by the factor \( (1-2(D-3)(D-4)^2 \alpha \kappa_D/L^2) = 1-\gamma \). This apparently implies that the effect of a GB term may be removed by redefining the Einstein constant. But this is not the case, rather a non-trivial GB coupling \( \alpha \) alters the physics of the brane-world by modifying the Neumann propagator in the bulk.
If one allows the parameters to take values such that the first square bracket term in (21) vanishes, the solutions become solitonic. This is because what survives after this setting precisely gives a RS equation multiplying with an additional delta function term, but in one co-dimension lower. As in the RS solution, which is given by \( \gamma = 0 \), there always exist brane-world solutions for \( 0 < \gamma \leq 1 \). So the physical relevance of the GB coupling \( \alpha \) is two fold. If one has solutions satisfying \( 0 < \gamma < 1 \), such solutions will be fairly similar to the RS solutions except with some corrections, like in the Newtonian potential. But if one is allowed to take \( \gamma \sim 1 \), one finds a new branch of solution with finite effective gravitational constant without finite distance bulk singularity. In the next section, we shall be interested in the \( \gamma < 1 \) solution.

4 Green Function in \( D \) Dimensions

In this section we analyze the Green functions by expressing a \( D \)-dimensional propagator \( \mathcal{G}_D(x; z; x', z') \) in terms of the Fourier modes such that

\[
\mathcal{G}_D(x, z; x', z') = \int \frac{d^{D-1}p}{(2\pi)^{D-1}} e^{ip(x-x')} \mathcal{G}_p(z, z'),
\]

(22)

where the Fourier components \( \mathcal{G}_p(z, z') (\equiv \tilde{h}(z, z') \sim e^{ip\cdot x}\psi(z, z')) \) satisfy

\[
(1 - \gamma) \left( \frac{\partial^2}{\partial z^2} - p^2 - \frac{D-2}{z} \frac{\partial}{\partial z} \right) \mathcal{G}_p(z, z') = e^{(D-2)A(z)} \delta(z - z').
\]

(23)

When one of the arguments is at \( z' = L \), the Neumann propagator is calculated in Appendix D to be

\[
\mathcal{G}_D(x, z; x', L) = (1 - \gamma)^{-1} \left( \frac{z}{L} \right)^{\nu} \int \frac{d^{2\nu}p}{(2\pi)^{2\nu}} e^{ip(x-x')} \frac{1}{q} \left[ \frac{H^{(1)}_{\nu}(qz)}{H^{(1)}_{\nu-1}(qL)} + \chi(qL) H^{(1)}_{\nu}(qL) \right],
\]

(24)

where \( \nu = (D - 1)/2 \), \( \chi = 2\gamma/((D - 3)(1 - \gamma)) \), and \( H^{(1)} = J + iY \) is the first Hankel function and \( q^2 = -p^\mu p_\mu = m^2 \). For both the arguments of the propagator at \( z, z' = L \) (on the brane), using the Bessel recursion relations, one can find a two point correlator of the effective theory. For example, when \( D = 5 \), a two-point correlator is

\[
\langle \phi(\vec{p})\phi(-\vec{p}) \rangle = \frac{1}{(1 + \gamma)^2} \left[ \frac{2}{q^2L} - \frac{1}{q} \frac{(1 - \gamma) H^{(1)}_0(qL)}{(1 + \gamma) H^{(1)}_1(qL) - \gamma H^{(1)}_0(qL)} \right].
\]

(25)

For arbitrary \( D \), by using Bessel expansions in Eq. (24), we obtain the scalar Neumann propagator

\[
\mathcal{G}_D(x, L; x', L) = (1 + \gamma)^{-1} \left[ \frac{(D - 3)}{L} \mathcal{G}_{D-1}(x, x') + \mathcal{G}_{KK}(x, x') \right],
\]

(26)

where

\[
\mathcal{G}_{D-1}(x, x') = \int \frac{d^{D-1}p}{(2\pi)^{D-1}} \frac{e^{ip(x-x')}}{q^2},
\]

(27)
\[ G_{KK}(x, x') \simeq - \int \frac{d^{D-1}p}{(2\pi)^{D-1}} e^{ip(x-x')} \left[ \frac{(D-4 + (D-6)\gamma)}{2(D-5)} \frac{1}{1 + \gamma} qL \right. \\
\left. + \frac{1 - \gamma}{1 + \gamma} \frac{(qL)^{D-4}}{q C_1} \ln \left( \frac{qL}{2} \right) \right], \tag{28} \]

where \( C_1 \) is a dimension dependent number (see App. D). For \( \gamma = 0 \), the above results reproduce the scalar Neumann propagator found in Ref. [7], after a substitution \( d = D - 1 \), and the results in Ref. [9] with \( D = 5 \). The first term in (26) is the standard propagator of a massless scalar field.

The long distance behavior, \( r >> L \), of the propagator is governed by a small \( q \) behavior of the Fourier modes. Thus, for \( qL << 1 \), a leading order contribution to the propagator comes from the logarithm. For \( D = 5 \), and \( |x - x'| >> L \), \( qL << 1 \), we find, to the leading order in \( q \),

\[ G_4(x, x') = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-x')}}{q^2} \propto \frac{1}{|x - x'|^2}. \tag{29} \]

Thus \( G_4(x, x') \) is just the ordinary massless scalar propagator in four-dimensions. Eq. (26) implies that a four-dimensional massless graviton propagator is contained in the full five-dimensional propagator. This is one of the plausible results of the RS2 model supplemented by a GB term. By the same token, for the KK modes, one has

\[ G_{KK}(x, x') \simeq \frac{1 - \gamma}{1 + \gamma} \frac{L^{D-4}}{q} \int \frac{d^{D-1}p}{(2\pi)^{D-1}} e^{ip(x-x')} q^{(D-5)} \ln(qL/2) \]

\[ \propto \frac{1 - \gamma}{1 + \gamma} \frac{L^{D-4}}{|x - x'|^{2D-6}}. \tag{30} \]

For \( 1 > \gamma > 0 \), we expect some correction in Newton’s law. At large distance scales on the brane, \( |x - x'| = r >> L \), the contribution of the KK modes (30) is very small compared to the zero mode contribution (26). Like the \( \gamma = 0 \) solution [7], for \( D \geq 5 \) and odd \( D \), a leading behavior of \( G_{KK} \) goes like \( \sim r^{6-2D} \). When \( D \) is even, there are no logarithmic terms and a leading behavior of \( G_{KK}(x, x') \) goes like \( \sim L r^{1-D} \), which is further more suppressed compared to the zero-mode contribution. Therefore, the effect of gravity becomes weaker when the number of extra (transverse) dimensions grows. One also notes that the Newtonian potential for a point source of mass \( m \), and \( D = 5 \), is

\[ V(r) \simeq - \frac{G_4 m}{r} \left[ 1 + \left( \frac{1}{1 + \gamma} - \frac{2\gamma}{2} \frac{1}{r^2} \right) \frac{L^2}{r^2} \right]. \tag{31} \]

In this formula, the contribution of the brane-bending mode is not included, which might change the factor \( 1/2 \) into \( 2/3 \) [4, 9]. It is seen that the GB term contributes to Newtonian potential with the opposite sign to that of the Einstein term or scalar curvature. One can expect a small correction to Newton’s law with \( \gamma < 1 \) and \( r >> L \), but such a correction is almost trivial for \( r >> L \) or/and \( \gamma \sim 1 \).

5 Nonperturbative Solutions with a GB Term

A non-singular Minkowski or de Sitter brane-world compactification could be difficult for Einstein’s theory [31, 32]. So one is lead naturally to hope that adding higher derivative corrections to the
brane-world action might improve this situation. As a convenient choice, we shall begin with the
Lagrangian of gravity, including a GB term and a bulk scalar field, in the form
\begin{equation}
S = \int d^Dx \sqrt{-g_D} \left( \frac{R}{\kappa_D} + \alpha \left( R^2 - 4R_{pq}R^{pq} + R_{pqrs}R^{pqrs} \right) - \frac{1}{2} \left( \partial \varphi \right)^2 - 2V(\varphi) + \cdots \right), \tag{32}
\end{equation}

The dots in (32) represent higher order terms in the scalar field ($\varphi$) and some other fields which are not turned on. We assume that $V(\varphi)$ is non-positive (bulk) cosmological potential, and $\varphi$ can only depend on extra coordinates, $\varphi = \varphi(z)$, as dictated by the Poincaré invariance on a brane. Though we will not restrict the action (32) to a particular string theory background, one may think about it like including $\alpha'$ corrections in type I string theories. In propagator correction-free Gauss-Bonnet scheme [11], the $R^2$-corrections can have a dependence on a scalar field, $\alpha \to \alpha' \lambda_0 e^{-m\varphi}$, with $m = 1/\sqrt{D-2}$, $\lambda_0 = 1/4 (1/8)$ for bosonic (heterotic) string. For a constant bulk scalar, since $V(\varphi)$ takes a bare value $\Lambda_0$, this parameterization is not so important for our analysis.

We can write the metric background in the form
\begin{equation}
ds_D^2 = \Omega^2(z) \left( dx_d^2 + \tilde{g}_{mn} dz^m dz^n \right). \tag{33}
\end{equation}
Here $\Omega(z) = e^{-A(z)}$, $dx_d^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu$ with $\tilde{g}$ being the $d$-dimensional metric, which can be Minkowski, de-Sitter or anti-de Sitter space. We study only the case of $D$-dimensional warped geometry compactified to $d$-dimensional RS type space-time ($\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$).

The equations of motion in $D$-dimensions take the form
\begin{equation}
R_{ab}(g) = \kappa_D \left( \tau_{ab} - \frac{1}{D-2} g_{ab} \tau_c^c \right) + \kappa_D T_{ab} \equiv \kappa_D T_{ab}, \tag{34}
\end{equation}
where the contribution to the stress energy of a massless scalar field and a bulk cosmological potential reads
\begin{equation}
\tau_{ab} = \partial_a \varphi \partial_b \varphi - \frac{1}{2} g_{ab} (\partial \varphi)^2 - V(\varphi) g_{ab}, \tag{35}
\end{equation}
and, the contribution of the GB interaction term reads
\begin{equation}
T_{ab} = \frac{\alpha}{D-2} g_{ab} R^2_{GB} - 2\alpha \left( R_{ab} - 2R_{acbd} R^{cd} + R_{acde} R_{b cde} - 2R_c R_{bc} \right). \tag{36}
\end{equation}

5.1 Energy conditions with a Gauss-Bonnet term

Let us take the $(\mu\nu)$ components of the $D$ dimensional Ricci tensor
\begin{equation}
R_{\mu\nu} = R_{\mu\nu}(\tilde{g}) - \tilde{g}_{\mu\nu} \left( \nabla^2 \log \Omega + (D-2) (\nabla \log \Omega)^2 \right). \tag{37}
\end{equation}
In the RS compactification, $d$-dimensional space is a Minkowski space, so $R(\tilde{g}) = 0$. In our metric convention (mostly positive), for $\alpha = 0$, the "c-theorem" of Ref. [31] reads
\begin{equation}
-(d-1) e^{2A(z)} \left( A'' + A^2 \right) = R_t^t - R_z^z = \kappa_{d+1} (\tau_t^t - \tau_z^z) = -\kappa_{d+1} e^{2A(z)} (\partial \varphi)^2 \leq 0, \tag{38}
\end{equation}
where $V(\varphi)$’s contribution to $\tau^i_\tau$ and $\tau^z_\tau$ cancels out. This is the obvious positive energy condition $\tau_{ab}\xi^a\xi^b \geq 0$, where $\xi^a$ is any arbitrary future-directed timelike or null vector. Since

\[
\mathcal{T}_{\mu\nu} = (d-2)(d-3)\alpha \left(2A'' - (d-2)A'^2\right) A'^2 e^{2A(z)} \eta_{\mu\nu},
\]
\[
\mathcal{T}_{zz} = d(d-2)(d-3)\alpha \left(2A'' + A'^2 \right) A'^2 e^{2A(z)},
\]

(39)

there arises an identical condition from

\[
\kappa_{d+1} \left(\mathcal{T}_{zz}^2 - \mathcal{T}_{z\tau}^2\right) = 2\varepsilon(d-1) e^{2A(z)} A'^2 \left(A'' + A'^2\right) e^{2A(z)} \geq 0,
\]

(40)

where $\varepsilon = (d-2)(d-3)\alpha\kappa_{d+1}$. The condition $A'' + A'^2 \geq 0$ may be equivalent to the weak energy condition (WEC) $-\rho + |p| \geq 0$, with $\rho$ and $p$ being the energy density and pressure. For $\alpha > 0$, the strong energy condition $\left(\tau_{ab} - \frac{1}{D-2} g_{ab}\tau\right) \xi^a\xi^b \geq 0$ is promoted to $\mathcal{T}_{ab}\xi^a\xi^b \geq 0$. First consider

\[
\left(\tau_{ab} - \frac{1}{D-2} g_{ab}\tau\right) \xi^a\xi^b = \left(\partial_a\varphi \partial_b\varphi - \frac{2}{D-2} V(\varphi) g_{ab}\right) \xi^a\xi^b
\]
\[
= (\partial_z\varphi)^2 \xi^z \xi_z - \frac{2}{D-2} V(\varphi) (\xi^\mu\xi_\mu + \xi^z \xi_z) \geq 0.
\]

(41)

So the strong energy condition is not violated if $\xi^a$ is a null vector and $V(\varphi) < 0$. For a future-directed null vector, $\xi^a\xi_a = e^{2A(z)} (\xi^\mu\xi_\mu + \xi^z \xi_z) = 0$, Eq. (41) implies that $-(\partial_z\varphi)^2 \xi^\mu\xi_\mu \geq 0$, so $\xi^\mu$ can be timelike ($\xi^\mu\xi_\mu = -1$) or null-like ($\xi^\mu\xi_\mu = 0$). This is consistent with the condition $R_{ab}\xi^a\xi^b \geq 0$:

\[
-(d-1) e^{2A(z)} \left(A'^2 + A''\right) \xi^\mu\xi_\mu \geq 0.
\]

(42)

We would like to see whether the second piece in (34), i.e., the contribution to stress energy from the GB term, violates the strong energy condition. For a null vector $\xi^a$, the condition $\mathcal{T}_{ab}\xi^a\xi^b \geq 0$, using (39), yields

\[
\left(-2\alpha (d-1)(d-2)(d-3) \left(A'^2 + A''\right) e^{2A(z)} A'^2\right) e^{2A(z)} \xi^\mu\xi_\mu \geq 0.
\]

(43)

Thus for all non-spacelike $\xi^\mu$, i.e., $\xi^\mu\xi_\mu \leq 0$, the strong energy condition (SEC) is intact provided that the WEC $A'' + A'^2 \geq 0$ holds. This may not be the case for any other combination of Riemann tensors or $R^2$ terms \footnote{We acknowledge fruitful correspondences with C. Núñez about the no–go theorem and energy conditions with a GB term that prompted us to add the above explanation.}. Here, validity of the SEC basically says that brane gravity is attractive.

5.2 Randall-Sundrum limit

Let us introduce a brane action for the RS singular 3-brane with a positive brane tension $\sigma > 0$:

\[
S_{\text{brane}} = 2 \int d^4x \sqrt{|g_4|} (-\sigma).
\]

(44)
For a constant scalar, the field equations following from (34) in $D = 5$, including a contribution from (44), simplify to

$$3\left(A'^2 + A''\right) = 3\left(A'^2 + A''\right) \left(2\varepsilon e^{2A(z)} A'^2\right) + \sigma\kappa_5\delta(z)$$

$$12A'^2 = -\kappa_5\Lambda_0 e^{-2A(z)} + 6\left(2\varepsilon e^{2A(z)} A'^2\right) A'^2,$$

where $\varepsilon \equiv 2\alpha\kappa_5$. For $\varepsilon = 0$, the bulk equation $A'' + A'^2 = 0$ may imply that $e^{2A(z)} A'^2 (\equiv C)$ is a constant. However, $C$ is already fixed by Eq. (46) such that $C = -\frac{\kappa_5\Lambda_0}{12} \equiv \frac{1}{\ell^2}$, where $\ell$ is a constant with dimension of length. Solving the equation $A'' + A'^2 = 0$ amounts to selecting a solution $A'(z) = (|z| + z_*)^{-1}$ of the unperturbed Einstein theory, but $z_*$ is undetermined by the bulk equations. One may fix $z_*$ using the continuity condition and normalization condition, such that $A'(0) = 1/\ell$, and arrive at [2]

$$\sigma = \frac{6}{\kappa_5\ell}, \quad \Lambda_0 = -\frac{12}{\kappa_5\ell^2}.$$  

(47)

As is well known one has some undesired features with $A'' + A'^2 = 0$ as an initial condition. For example, there is a finite-distance bulk singularity for $z_* < 0$ at $z = z_*$. So we will now investigate the case of interest, $\varepsilon > 0$. In this case, the bulk equation can be satisfied even by selecting

$$2\varepsilon e^{2A(z)} A'^2 = 1.$$  

(48)

It is obvious that this freedom is not there with $\varepsilon = 0$. Since $\varepsilon > 0$, the metric solution is

$$e^{A(z)} = \int_0^z \frac{1}{\sqrt{2\varepsilon}} dz \Rightarrow e^{-A(z)} = \frac{\sqrt{2\varepsilon}}{|z| + \sqrt{2\varepsilon}},$$  

(49)

where we have normalized the solution $A(0) = 0$, such that $e^{-A(z)}$ takes a value 1 at $z = 0$. We find it convenient to define the length scale $\sqrt{2\varepsilon} = l$, so $A'' + A'^2 = \frac{2\delta(z)}{(|z| + l)^2}$. The condition (48) therefore, unsurprisingly, also solves the bulk equation $A'' + A'^2 = 0$. It is obvious that $A'' + A'^2 = \frac{2}{l}$ on the brane ($z = 0$), as it should be in order to keep the WEC $A'' + A'^2 \geq 0$ intact. The solutions with $\varepsilon > 0$ have all essential properties of the RS solutions, such as $e^{-A(z)}$ converges as $z \to \pm\infty$, and there is no any finite distance bulk singularity. However, there is now a new length scale in the problem, $l$, and a common feature of these new solutions is that they are not analytic in the coefficient $\alpha$ of the Gauss-Bonnet interaction, so $\varepsilon > 0$ is a physical requirement. Remarkably, the RS solution with $\varepsilon = 0$ corresponds to a limit of these solutions where the singularities are pushed to infinity, $z \to \pm\infty$, so are harmless as they might have interpretations in field theory as (ultraviolet) energy cut-off scale.

The boundary condition relates $l$ to the brane tension $\sigma$. For a brane at $z = 0$ with positive tension, $\sigma > 0$, Eq. (45) implies that

$$\frac{6\delta(z)}{l + |z|} \left(1 - \frac{2\varepsilon}{l^2} \text{sgn}(z)^2\right) = \sigma\kappa_5\delta(z).$$

(50)

This after regularizing the $\delta$-function, $\delta(z)\text{sgn}(z)^2 = \delta(z)/3$, determines the brane tension

$$\sigma = \frac{1}{\kappa_5} \frac{4}{l}.$$  

(51)
The tension $\sigma$ is generally not fine-tuned because $l$ is arbitrary. However, the scale $l$ is used also to fix the bulk cosmological constant, via Eq. (46),

$$\Lambda_0 = -\frac{1}{\kappa_5} \frac{6}{l^2}. \quad (52)$$

There is a fine-tuning between $\sigma$ and $\Lambda_0$, which is required to maintain flatness of the 3-brane. This may be relaxed for more general solutions, like (anti-) de Sitter branes, with $\varepsilon > 0$.

A remark is in order. The bulk scale $l$ is somehow fixed by the coefficient $\varepsilon$, but this is not an unnatural choice from the view point of low energy effective string action, rather a common result in higher-curvature stringy gravity, see, for example, Refs. [14, 19, 20]. In the Einstein frame, the tree-level (bosonic) string action, with appropriate conformal weights for a dilaton $\phi$, reads [11, 39]

$$S_{\text{bulk}} = \frac{1}{\kappa_D} \int d^D x \sqrt{-g_D} \left( R + \lambda_0 \alpha' e^{-m\phi} \left( \mathcal{R}_{GB}^2 + m^2 \frac{D-4}{D-2} (\partial\phi)^4 \right) - \frac{2(D-10)}{3\alpha'} e^{m\phi} - m (\partial\phi)^2 + \mathcal{O}(\alpha'^2) \right), \quad (53)$$

where $m = 4/(D - 2)$. Therefore, with $\phi = \phi_0 = const$ and $D < 10$, the bulk cosmological term $\Lambda_0 \propto -1/\alpha' \sim -1/\varepsilon$. The above action diverges at $\alpha' = 0$, so one might require $\alpha' > 0$, so $\varepsilon > 0$, in order to get solutions which are free of singularities. In the context of lowest-order brane-world Einstein gravity, so $\alpha' = 0$, there arise naked bulk singularities due to the fact that both dilaton and graviton field exhibit logarithmic singularities [40]. This problem can be resolved by considering the leading order $\alpha'$ corrections as recently shown in Ref. [39].

6 Brane-World No–Go Theorem

Basically, with warped space-time metrics in the bulk, there are three different arguments given for the brane-world “no–go theorem” in the Einstein theory. They are

- No singularity free solution with a finite Planck mass is possible without a fine tuning [30]. In other words, singularities in the self-tuned solutions are generic if gravity is to be localized.
- It is impossible to have $e^{A(z)} A'(z)$ approach a positive constant as $z \to \infty$ and a negative constant as $z \to -\infty$. This monotonicity of $e^{A(z)} A'(z)$ is often called the brane-world $c$–theorem [31].
- There are no non-singular Randall-Sundrum or de-Sitter compactifications where the only possible singularities occur when the warp factor $e^{-A(z)}$ goes to zero at the singularity [32].

The first argument above is mainly related to the fine tuning of the cosmological constant, which may complement the Hawking-Penrose singularity theorem [36], rather than being intrinsic to the Randall-Sundrum models \(^3\). It is interesting to know whether any of these arguments can be avoided. It

\(^2\)In terms of the $y$ coordinate such that $dy = e^{-A(z)} dz$, one has $A'(y) = e^{A(z)} A'(z), A''(y) = e^{2A(z)} \left( A''(z) + A'^2(z) \right)$.

\(^3\)In the RS brane-world context, this implies that the generic initial conditions, such as $A''(z) + A'(z)^2 = 0$ for $z > 0$, lead to singular solutions of Einstein equations.
should be possible to circumvent some of these arguments by considering higher derivative corrections to the gravity equations [32].

6.1 No–go theorem and possible avoidance

Let us take the \((\mu\nu)\) components of (34) and contract the \(\mu\nu\)-indices to arrive at

\[
d\left[(d - 1)A'^2 - A''\right] = \kappa_{d+1} e^{-2A(z)} \tilde{T} - d(d - 2)\alpha \kappa_{d+1} [2A'' - (d - 2)A'^2] e^{2A(z)} A'^2,
\]

where

\[
\tilde{T} \equiv -\tau_{\mu}^\mu + \frac{d}{d-1} \tau_c^c = -\frac{2d}{d-1} V(\varphi) \geq 0.
\]

Here we have used the fact that \(\varphi = \varphi(z)\), and assumed \(V(\varphi) < 0\).

First we briefly review the no-go theorem advocated in Ref. [32]. With \(\alpha = 0\) and \(d = 4\), one has

\[
\Omega^3 \tilde{\nabla}^2 \Omega^3 = 3 e^{-6A(z)} \left[3A'^2 - A''\right] \geq 0.
\]

Integrating (56) over the compact internal space by parts one finds \(\int dz \sqrt{g} (\tilde{\nabla} e^{-3A(z)})^2 \leq 0\). This may be satisfied only if one allows the equality sign in (56) and \(e^{-A(z)}\) is a constant. The condition \(R_{00} = \tau_{00} - \frac{1}{d-2} g_{00} \tau_c^c = 3A'^2 - A'' = 0\), or \(R_{00} = \tilde{T} = 0\), may not generate gravitational fields in the bulk space-time [37]. Moreover, in the RS compactification \(e^{-A(z)}\) should not be a constant, rather this factor is essential to explain the warped nature of a bulk geometry and hence a RS compactification.

We will now investigate the case of interest, \(\alpha > 0\), so \(\varepsilon > 0\). For \(d = 4\), the four-dimensional parts of the curvatures read

\[
R(x) = R^{(4)}(x) + 4 e^{2A(z)} \left(2A'' - (N + 2)A'^2\right)
\]

\[
R_{GB}^2(x) = R_{GB}^{(4)}(x) + R^{(4)}(x) \left[4(N + 1)A''(z)
- 2N(N + 1)A'^2(z)\right] e^{2A(z)} + \cdots,
\]

where \(N = D - 4\). In the following we restrict our attention to \(D = 5\). The four-dimensional Planck scale is therefore [17]

\[
M_{Pl}^2 \simeq M_{(5)}^3 \int_{-\infty}^{\infty} dz e^{-3A(z)} \left[1 + 4\alpha \kappa_5 e^{2A(z)} \left(2A'' - A'^2\right)\right]
\]

\[
= M_{(5)}^3 \int_{-\infty}^{\infty} dz e^{-3A(z)} \left(1 + 4\alpha \kappa_5 e^{2A(z)} A'^2\right) + M_{(5)}^3 8\alpha \kappa_5 \left[e^{-A(z)} A'\right]^{+\infty}_{-\infty}.
\]

It is obvious that \(M_{Pl}^2\) is finite for \(A(z) = \ln (1 + |z|/l)\). From Eq. (54) one has

\[
4 \left(3A'^2 - A''\right) = -\frac{8}{3} \kappa_5 e^{-2A(z)} V(\varphi) - 8\varepsilon \left(A'' - A'^2\right) e^{2A(z)} A'^2.
\]

13
Thus, with $\alpha > 0$, the inequality (56) may hold even in reverse order without violating the positive energy condition, and the warp factor $e^{-A(z)}$ need not be a constant. In particular, when the weak energy condition $A'' \geq -A'$ saturates, we find
\[
16 e^{2A(z)} A'^2 \left( 1 - \varepsilon e^{2A(z)} A'^2 \right) = -\frac{8 \kappa_5}{3} V(\varphi).
\]
This can be satisfied when $V(\varphi) < 0$, $A'^2 > 0$, and $\varepsilon e^{2A(z)} A'^2 < 1$. We have shown that in the presence of a GB term there are some improvements over the no-go theorem.

### 6.2 Solutions with a bulk scalar

For a non-constant scalar field, the bulk equations of motion following from (34) simplify to
\[
k_{d+1} \varphi'^2 = 2(d - 1) \left( A'^2 + A'' \right) \left( 1 - 2 \varepsilon e^{2A(z)} A'^2 \right)
\]
\[
2k_{d+1} V(\varphi) = -e^{2A(z)} (d - 1) \left[ (d - 1) A'^2 - A'' \right] + (d - 1) \varepsilon e^{A(z)} \left[ (d - 2) A'^2 - 2 A'' \right],
\]
where $\varepsilon = (d - 2)(d - 3) \alpha \kappa_{d+1}$. We may express these equations in the $y$ coordinate such that $dy = e^{-A(z)} dz$. For $d = 4$, and $\varphi = \varphi(y)$, $A'(z)e^{A(z)} = A'(y) \equiv W(\varphi)$, $\partial W(\varphi)/\partial \varphi \equiv W_\varphi$, Eqs. (61), (62) take the following form:
\[
V(\varphi) = \left( \frac{3W_\varphi^2}{2\kappa_5} - \Lambda_0 \right) \left( 1 - 2 \varepsilon W^2(\varphi) \right)^2 + \Lambda_0, \quad \Lambda_0 \equiv -\frac{3}{4\alpha \kappa_5},
\]
\[
\varphi'(y) = \frac{6W_\varphi}{\kappa_5} \left( 1 - 2 \varepsilon W^2(\varphi) \right).
\]
The above (super) potential (63), named due to its resemblance with supergravity solution, for the $\varepsilon = 0$ case, was analyzed in Ref. [38]. In the following we shall be interested in analyzing bulk solutions with $\varepsilon > 0$, without retaining explicit form of $A(y)$ (see Refs. [23, 25] for some relevant discussions).

Some non-singular solutions are found in Ref. [39] by satisfying $-22.2 < \alpha \Lambda_0 < -5/12$, in the units $\kappa_5 = 1$. So in the bulk it might be possible to take $V(\varphi) = \Lambda_0$ and $(1 - 2 \varepsilon W^2(\varphi)) = 0$.

In this limit, a domain wall solution smoothly interpolates between two anti-de Sitter minima $\varphi_{\pm}$ of the potential $V(\varphi)$ as $y \rightarrow \pm \infty$. Because $\varphi'(y)$ vanishes in the bulk, $V(\varphi)$ takes a bare value $V(\varphi_{\pm}) = \Lambda_0$. The scalar field and warp factor simply become $\varphi = \varphi_0$ and $A(y) = \pm \sqrt{-\Lambda_0} |y|$. One notes that $1 - 2 \varepsilon W^2_\varphi \not = 0$ on the brane, rather this becomes $1 - 2 \varepsilon W^2_\varphi / 3 > 0$ due to an essential $\delta$-function regularization. There is no any bulk singularity for the (super) potential of the linear form $W(\varphi) = \lambda_1 \varphi + \lambda_2$ [14].

It might be desirable to know what would happen if $2 \varepsilon W^2(\varphi) \not = 1$ in the bulk? Then, since
\[
A''(y) = \varphi'(y) \frac{\partial W(\varphi)}{\partial \varphi} = \frac{6W_\varphi}{\kappa_5} \left( 1 - 2 \varepsilon W^2(\varphi) \right),
\]

\[
\text{(65)}
\]
the $A''(y) \geq 0$ condition holds only if $2\epsilon W^2(\varphi) < 1$. In this case, there is no improvement over the c–theorem. As we expect that $A''(y \neq 0) = 0$, $W_\varphi$ should vanish in the bulk, and hence $W(\varphi)$ becomes a constant function of scalar field, which implies that a bulk scalar is not dynamical.

In the $\epsilon = 0$ case, the condition

$$A''(y) = \frac{6W^2}{\kappa_5} \geq 0$$

is used to prove a c–theorem in Ref. [38]. With $\epsilon = 0$, the metric solution reads $A(y) \sim |y|/\ell$, where $\ell = \sqrt{6/(-\Lambda)}$. In general, a domain-wall solution should interpolate between $y \to -\infty$ (infrared region) where $A(y)$ is linear, and $y \to \infty$ (ultraviolet region or AdS horizon) where $A(y)$ is again linear. But the condition $A''(y) \geq 0$ rules out here the second possibility. So it is not possible to have $A'(y)$ approach a positive constant as $y \to +\infty$ and a negative constant as $y \to -\infty$. This monotonicity of $A'(y)$ has been known as brane-world “c–theorem” [31, 38]. However, with $\epsilon > 0$, and $2\epsilon A''(y) = 1$ as a bulk solution (c.f. [46]), warp factor gets the both signs

$$A(y) = \pm \frac{|y|}{\ell}.$$ 

Then it might be possible that $A'(y)$ approaches a positive (negative) constant as $y \to +\infty (-\infty)$. Therefore, the brane-world “c–theorem” of [31] may not be available to the $\alpha > 0$ case.

### 7 Discussion and Outlook

If the Randall-Sundrum models are to be the low energy limits of some fundamental (string) theory or yet-unknown theory of quantum gravity, it is likely that the gravity equations include higher curvature corrections such as a Gauss-Bonnet invariant. For a background of branes coupled to matter sources and a GB self-interaction term, we have presented some useful expressions for the Neumann propagator in arbitrary $D$ dimensions and analyzed the structure of graviton interaction. It is shown that the RS model with a GB term in the bulk gives a massless graviton on the brane as in the standard RS model. Perhaps this is one of the most striking results of this paper. We have shown that for a small GB coupling $\alpha$, so $\gamma < 1$, the brane-world solutions are qualitatively similar to the RS solutions. We also pointed out a possibility that the Newton’s law is exact, other than that such a behavior one would expect at large distance along the brane, if one is allowed to take $\gamma$ in the order of unity.

We have examined the general properties of the RS solutions coupled to a bulk scalar and a GB term and have found that fine-tuning is a generic feature of RS models. We analyzed the energy conditions with warped space-time metrics in the bulk and found that energy conditions are not violated by a GB term. Meanwhile, we observed a new branch of solution with finite Planck scale and no naked bulk singularity. More precisely, a bulk singularity for the $\epsilon = 0$ solution, which may complement the Hawking-Penrose singularity theorem [36], is pushed for $\epsilon > 0$ to the singularity at the anti-de Sitter horizon $z \to \pm \infty$, so is harmless as it might have field theory interpretation. The
new solutions with $\varepsilon > 0$ are also found useful to avoid some of the previously known no–go arguments for the RS brane-world compactifications.

In this paper, we have performed the calculations with an assumption that the scalar modes of the metric fluctuations decouple from the tensor modes. This is a possible one, because, at least for the flat RS branes, there are no delta function sources for the scalar and vector modes [17]. In the presence of several co-dimension one branes, such a decoupling certainly requires a moduli stabilization and it is possible that the physics responsible for this stabilization would modify the analysis for the tensor structure of the graviton propagator, which is not analyzed here due to this subtlety. This treatment might require more general assumptions that there is a bulk scalar coupled to brane gravity, and the branes are (anti-)de Sitter. We hope to return to this point in future publication.

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**Appendix A: Metric variations with quadratic curvature terms**

The starting brane-world action is the Lagrangian of gravity which has a general form in $D$ dimensions, including quadratic-order curvatures,

$$S = \int_M d^D x \sqrt{-g_D} \left[ \frac{R}{\kappa_D} - 2\Lambda + \left( \alpha R^2 + \beta R_{ab} R^{ab} + \gamma R_{abcd} R^{abcd} \right) \right]$$

$$+ \sum_{i=1}^{i'th \text{brane}} \int_{\delta M} d^{D-1} x \sqrt{-g_{(D-1)}} (L^{\text{bdry}}_m - \Lambda_i(z))$$

$$+ \int d^d x \sqrt{-g_d} (-T), \quad (A.1)$$

where $a, b, \cdots$ denote the $D$-dimensional space-time indices. For $D = 4$, the action (A.1) will be free of massive spin-2 ghost only if $\beta + 4\gamma = 0$ [41]. For $D > 4$, however, one requires that $\alpha = -\beta/4 = \gamma$, which is the Gauss-Bonnet relation. The graviton equations derived by varying the above action with respect to $g^{ab}$ may be expressed in the following form

$$\sqrt{-g_D} \left( \kappa_D^{-1} G_{ab} + H_{ab} + \Lambda_{gb} \right)$$

$$= -\frac{1}{2} \sum_{i=1}^{D-4} \Lambda_i \sqrt{-g^{(z_i = 0)}} \delta(z_i) \delta_a^\mu \delta_b^\nu \delta_{pq}^{(z_i = 0)}$$

$$- \frac{T}{2} \sqrt{-g_d} \delta(z_1) \delta(z_2) \cdots \delta(z_n) \delta_\mu^a \delta_\nu^b g^{z_1, z_2, \cdots, z_n = 0}, \quad (A.2)$$
where $H_{ab}$

$$H_{ab} = -\frac{1}{2} g_{ab}(\alpha R^2 + \beta R_{cd} R^{cd} + \gamma R_{cdef} R^{cdef}) + 2[\alpha R R_{ab} + \beta R_{acbd} R^{cd} + \gamma (R_{acde} R_b^{cde} - 2R_a^c R_{bc} + 2R_{acbd} R^{cd})] - (2\alpha + \beta + 2\gamma)(\nabla_a \nabla_b R - g_{ab} \nabla^2 R) + (\beta + 4\gamma)\nabla^2 \left( R_{ab} - \frac{1}{2} g_{ab} R \right). \quad (A.3)$$

The curvature derivatives vanish for a Gauss-Bonnet invariant $R^2_G$ (i.e., with $4\alpha = -\beta = 4\gamma$). Then the tensor $H_{ab}$ reduces to the second order Lovelock tensor

$$H_{ab} = -\frac{\alpha}{2} g_{ab} R^2_G + 2\alpha \left( R R_{ab} - 2R_{acbd} R^{cd} + R_{acde} R_b^{cde} - 2R_a^c R_{bc} \right) = 2\alpha \left( I_{ab} - 4J_{ab} + K_{ab} \right), \quad (A.4)$$

where the quantities $I_{ab}$, $J_{ab}$, $K_{ab}$ are defined below. The linearized form of the curvatures are

(i) Variation of $G_{ab}$

$$\delta G_{ab} = \delta R_{ab} - \frac{1}{2} g_{ab} \delta R - \frac{1}{2} h_{ab} \tilde{R}. \quad (A.5)$$

(ii) Variation of $I_{ab} (\equiv R \left( R_{ab} - \frac{1}{4} g_{ab} R \right))$

$$\delta I_{ab} = G_{ab} \delta R + \tilde{R} \delta R_{ab} - \frac{1}{4} h_{ab} \tilde{R}^2. \quad (A.6)$$

(iii) Variation of $J_{ab} (\equiv R_{acbd} R^{bd} - \frac{1}{4} g_{ab} R_{cd} R^{cd})$

$$\delta J_{ab} = \delta R_{apb} \tilde{R}_{b q} + \tilde{R}_{apb q} \delta R^{pq} - \frac{1}{2} g_{ab} \tilde{R}_{pq} \delta R_{pq} - (\tilde{R}_{apb} \tilde{R}_{b p q} - \frac{1}{2} g_{ab} \tilde{R}_{pq} \tilde{R}_{b q}) h_{pq} - \frac{1}{4} h_{ab} \tilde{R}_{pq} \tilde{R}_{pq}. \quad (A.7)$$

(iv) Variation of $K_{ab} (\equiv R_{acde} R_b^{cde} - \frac{1}{4} g_{ab} R_{cdef} R^{cdef} - 2R_a^c R_{bc} + 2R_{acbd} R^{cd})$

$$\delta K_{ab} = 2 \tilde{R}_{(p} R_{q) [r} ^{s} \delta R_{b]pq} \tilde{R}_{r s} - \frac{1}{2} g_{ab} \tilde{R}_{pqrs} \delta R_{pqrs} + 2 \tilde{R}^c_a \delta R_{acbd} + 2 \tilde{R}_{acbd} \delta R^{cd} - 4 \tilde{R}_{(a}^c \delta R_{b)c} - \left( \tilde{R}_{acpq} \tilde{R}_{b q} - \frac{1}{2} g_{ab} \tilde{R}_{cpqr} \tilde{R}_{d} ^{pqrs} + 2 \tilde{R}_{ac} \tilde{R}_{de} - 2 \tilde{R}_{ac} \tilde{R}_{bd} \right) h_{cd} - \frac{1}{4} h_{ab} \tilde{R}_{pqrs} \tilde{R}_{pqrs}. \quad (A.8)$$

The quantities defined with bar are to be taken about their background values. In the following, we adopt a slightly different scheme of linearization, which we find more convenient to use for a warped bulk geometry. The linearized equations take the form

$$\delta \tilde{G}_{ab} + \kappa_D \delta \tilde{H}_{ab} = 0, \quad (A.9)$$

where

$$\delta \tilde{G}_{ab} = \delta G_{ab} - G_{ac} h^c_{\ b} = \delta G_{ab} - \kappa_D \delta T_{ab} = \delta R_{ab} - \frac{1}{2} g_{ab} \delta R - \tilde{R}_{ac} h^c_{\ b}, \quad (A.10)$$

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\[ \delta I_{ab} = \delta I_{ab} - \tilde{I}_{ac} h^c_{\ b} = \bar{G}_{ab} \delta R + \bar{R} \delta R_{ab} - \bar{R} \bar{R}_{ac} h^c_{\ b} \]  
(A.11)

\[ \delta J_{ab} = \delta J_{ab} - \tilde{J}_{ac} h^c_{\ b} = \delta R_{apb} g^r \bar{R}^p_{\ q} + \bar{R}_{apbq} \delta R^{pq} - \frac{1}{2} g_{ab} \bar{R}^{pq} \delta R_{pq} \]  
\[- \left( \bar{R}_{apb} \bar{R}^r_{\ q} - \frac{1}{2} g_{ab} \bar{R}^r_{\ pq} \right) h^{pq} - \bar{R}_{apbq} \bar{R}^{pq} h^c_{\ b} \]  
(A.12)

\[ \delta K_{ab} = \delta K_{ab} - \dot{K}_{ac} h^c_{\ b} = 2 \bar{R}_{(a}^{pq} \delta R_{b)pq} r - \frac{1}{2} g_{ab} \bar{R}^{pq} s \delta R_{pq}^s + 2 \bar{R}^e_{\ d} \delta R_{ac}^d \]  
\[ + 2 \bar{R}_{abcd} \delta R^{cd} - 4 \bar{R}_{(a}^c \delta R_{b)c} \]  
\[- \left( \bar{R}_{apcq} \bar{R}_{bd}^{pq} - \frac{1}{2} g_{ab} \bar{R}_{cpqr} \bar{R}_{d}^{pq} + 2 \bar{R}_{ac}^{e} \bar{R}_{de} - 2 \bar{R}_{ac} \bar{R}_{bd} \right) h^{cd} \]  
\[- \left( \bar{R}_{apcq} \bar{R}_{bd}^{pq} + 2 \bar{R}_{apcq} \bar{R}^{pq} - 2 \bar{R}_{a}^{p} \bar{R}_{cp} \right) h^c_{\ b} \]  
(A.13)

In using (A.9), the RS fine-tuned relations do not appear explicitly in the equations of motion.

**Appendix B: Useful Identities with Warped Space-Time Metrics**

Let us consider the function defined by \( A(z) = \log(k |z| + 1) \), where \( k \) in the inverse AdS curvature scale, \( k \equiv 1/L \). Since \( A(z) \) is a function of \( |z| \), we have \( \partial_z A = A' \partial_z |z| \), where \( \partial_z |z| = 2 \Theta(z) - 1 \), \( A' \) is derivative of \( A \) with respect to its argument \( |z| \), \( \Theta(z) \) is the Heaviside function. In \( D \) space-time dimensions, the warp factor \( A(z) \), as a general solution of the Einstein field equations modified by a Gauss-Bonnet term, reads

\[ A(z) = \log \left( \sum_{i=1}^{D-4} k_i |z_i| + 1 \right) . \]  
(B.1)

For simplicity we assume that \( k_1 = k_2 = \cdots = k \). A straigt forward simplifications would rise to give the following results

\[ A'^2 = e^{-2A(z)} k^2 \sum_i (\partial_z |z_i|)^2 = e^{-2A(z)} k^2 (D - 4) . \]  
(B.2)

\[ A'' = -e^{-2A(z)} k^2 (D - 4) + e^{-A(z)} 2k \sum_i \delta(z_i) . \]  
(B.3)

Some useful identities that hold among the cross terms are

\[ \partial_{z_j} \partial_{z_k} A \partial_{z_j} A \partial_{z_k} A = e^{-3A(z)} 2k^3 \sum_i \delta(z_i) - e^{-4A(z)} k^4 (D - 4)^2 . \]  
(B.4)

\[ \partial_{z_j} \partial_{z_k} A \partial_{z_j} A \partial_{z_k} A = -e^{-2A(z)} 4k^2 \sum_{i \neq j} \delta(z_i) \delta(z_j) + (A'')^2 . \]  
(B.5)

\[ \partial_{z_j} \partial_{z_k} A + \partial_{z_j} A \partial_{z_k} A = -A k \sum_i (\partial_{z_j} \partial_{z_k} |z_i|) = e^{-A(z)} 2k \delta(z_i) \delta_{z_j z_k} . \]  
(B.6)

\[ \partial_{z_k} A \partial_{z_i} h_{\mu \nu} = e^{-A(z)} k \sum_i \text{sgn} (z_i) \partial_{z_i} h_{\mu \nu} . \]  
(B.7)

\[ \sum_i (\partial_{z_j} \partial_{z_k} |z_i|) \partial_{z_i} \partial_{z_k} h_{\mu \nu} = 2 \sum_i \delta(z_i) \partial_{z_i} h_{\mu \nu} . \]  
(B.8)
Appendix C: Linear Expansions with the Gauss-Bonnet Term

In the gauge $h^\mu_\nu = 0 = \partial_\nu h^\mu_\nu$, the first order linearized equations for $h_{\mu\nu}$ take the general form

$$
\delta \hat{G}_{\mu\nu} = \delta G_{\mu\nu} - \delta T^{(0)}_{\mu\lambda} k^\lambda_{\nu} = \left( -\frac{1}{2} \partial_i^2 - \frac{1}{2} \partial_{z_i}^2 + \frac{D-2}{2} \partial_i z_i \partial_{z_i} \right) h_{\mu\nu},
$$

$$
\delta \hat{H}_{\mu\nu} = 2(D-4)\alpha e^{2A} \left[ \left( \frac{D-5}{2} \partial_{z_i} A \partial_{z_i} A - \partial_{z_i} \partial_{z_i} A \right) (\partial_i^2 + \partial_{z_i}^2)
+ \partial_k A \left( (D-3) \partial_{z_k} A - \frac{(D-3)(D-4)}{2} \partial_{z_k} A \partial_{z_k} A \right) \partial_{z_k}
+ (\partial_{z_i} \partial_{z_j} A + \partial_{z_i} A \partial_{z_j} A) \partial_{z_i} \partial_{z_j} \right] h_{\mu\nu},
$$

where as defined above $i, j = 1, 2, \cdots, (D-4)$ count the number of extra (transverse) coordinates. In using the metric solution $A(z) = \log \left( \sum_{i=1}^{D-4} k_i |z_i| + 1 \right)$, the linearized fluctuations $h_{\mu\nu}$ simplify to

$$
\frac{1}{\kappa_D} \left[ -\Box - \Box_z + (D - 2)ke^{-A(z)} \sum_{i=1}^{D-4} sgn(z_i) \partial_{z_i} \right] h_{\mu\nu}
+ 2\alpha (D - 4) \left[ (D - 3)(D - 4) k^2 (\Box_z + \Box_z) - 4ke^{A(z)} \sum_{i=1}^{D-4} \delta(z_i) \Box \right. - 4ke^{A(z)} \sum_{i \neq j}^{D-4} \delta(z_i) \partial_{z_i}^2
+ (D - 3)k^2 \left( 4 \sum_{i=1}^{D-4} \delta(z_i) - (D - 2)(D - 4)ke^{-A(z)} \right) \sum_{i=1}^{D-4} sgn(z_j) \partial_{z_j} \right] h_{\mu\nu} = 2T^{(m)}_{\mu\nu},
$$

where $\Box_z = \partial_{z_1}^2 + \partial_{z_2}^2 + \cdots + \partial_{z_{D-4}}^2$. By redefining metric fluctuations as $h_{\mu\nu} = e^{(D-2)A(z)/2} \tilde{h}_{\mu\nu}$, one can remove from the first square bracket in Eq. (C.2) the single (linear) derivative term, and the kinetic term will have a canonical form. After this rescaling, linearized equations take the form

$$
\frac{1}{\kappa_D} \left[ -\Box - \Box_z - (D - 2)ke^{-A(z)} \sum \delta(z_i) + \frac{(D-4)(D-2)D k^2}{4} e^{-2A(z)} \right] \tilde{h}_{\mu\nu}
+ (D - 4) \alpha e^{2A(z)} \left[ 2(D - 3)(D - 4) k^2 e^{-2A(z)} (\Box_z + \Box_z) - 8ke^{-A(z)} \sum \delta(z_i) \Box \right. - 8ke^{-A(z)} \sum_{i \neq j} \delta(z_i) \partial_{z_j}^2
+ 8k^2 e^{-2A(z)} \left( (D - 3) \sum sgn(z_i) \delta(z_i) \partial_{z_i} - \sum_{i \neq j} \delta(z_j) sgn(z_i) \partial_{z_i} \right)
+ 8(D - 2) k^2 e^{-2A(z)} \left( \frac{D(D-3)(D-4)}{8} k^2 e^{-2A(z)} + 2 \sum \delta(z_i) \delta(z_j)
- (D^2 - 8D + 18) e^{-A(z)} \sum \delta(z_i) \right) \right] \tilde{h}_{\mu\nu} = 2T^{(m)}_{\mu\nu}.
$$

We have implemented these expressions in the bulk part of the text.

Appendix D: Green Function in $D$ Dimensions

In Sec. 3, we made use of several analytic properties of bessel functions. Here we summarize some of the technical derivations that were involved. By defining in Eq. (23) $G_p = (zz'/L^2)^{(D-1)/2} \hat{G}_p$ and
\[ p^2 = -q^2, \] we arrive to
\[
(1 - \gamma) \left( z^2 \partial_z^2 + z \partial_z + q^2 z^2 - \frac{(D - 1)^2}{4} \right) \hat{G}_p(z, z') = L z \delta(z - z'). \tag{D.1}
\]

One can also derive the \( D \) dimensional boundary condition at \( z = L \), analogous to the Neumann condition on the gravitational field that \( \partial_z \mathcal{G}_D(x, x') \mid_{z=L} = 0 \), to be
\[
\left( z \partial_z + \frac{D - 1}{2} + \frac{2}{(D - 3)} \frac{\gamma}{(1 - \gamma)} q^2 z^2 \right) \hat{G}_p(z, z') \mid_{z=L} = 0. \tag{D.2}
\]

This is a boundary condition implied by continuity of the derivative fields on the brane. Equation (D.1) further implies the following matching conditions at \( z = z' \):
\[
\hat{G}_{<z=z'} = \hat{G}_{>z=z'}, \quad \partial_z (\hat{G}_{>z} - \hat{G}_{<z}) \mid_{z=z'} = (1 - \gamma)^{-1} \frac{L}{z'}. \tag{D.3}
\]

One can find the general solution of (D.1) by satisfying (D.3) and (D.2). The most general solution of (D.1), for \( z < z' \) is
\[
\hat{G}_{z<z'} = i A(z') \left[ (J_{\nu-1} + \chi (qL) J_{\nu} H^{(1)}_{\nu}(qz) - (H^{(1)}_{\nu-1}(qL) + \chi (qL) H^{(1)}_{\nu}(qz)) \right], \tag{D.4}
\]
where \( \chi = \gamma / ((\nu - 1) (1 - \gamma)) \), \( \nu = (D - 1) / 2 \), and \( H^{(1)}_{\nu}(qL) = J_{\nu}(qL) + i Y_{\nu}(qL) \) is the Hankel function of the first kind. For \( z > z' \), one can use the following boundary condition at the AdS horizon, \( z = \infty \):
\[
\hat{G}_{z>z'} = B(z') H^{(1)}_{\nu}(qz). \tag{D.5}
\]

Following [7, 9], we solve Eqs. (D.3,D.4,D.5) and arrive to a \( D \)-dimensional Neumann propagator
\[
\mathcal{G}_D(x, z; x, z') = - (1 - \gamma)^{-1} \frac{i \pi}{2 L^3 \gamma \nu} \int \frac{d^D p}{(2\pi)^{D-3}} e^{i p(x-x')} \left[ J_{\nu}(qz) H^{(1)}_{\nu}(qz) \right.
\]
\[
- \left( J_{\nu-1}(qL) + \chi (qL) J_{\nu}(qL) \right) H^{(1)}_{\nu}(qz) \left. H^{(1)}_{\nu}(qL) \right] \tag{D.6}
\]

This gives the results in Ref. [9] for \( D = 5 \), and that of Ref. [7] with \( \gamma = 0 \) and \( D = d + 1 \). When an argument of the propagator is at \( z' = L \), (D.6) can be reduced to Eq. (24) given in the text. When both arguments are on the brane, Eq.(24), after Bessel expansions and some algebraic manipulations, would simplify to yield
\[
\mathcal{G}_D(x, L; x', L) \simeq (1 - \gamma)^{-1} \int \frac{d^{D-1} p}{(2\pi)^{D-3}} e^{i p(x-x')} \left[ \frac{1 - \gamma}{1 + \gamma} \frac{(D - 3)}{qL} + \frac{[(D - 4) + (D - 6)\gamma]}{2(D - 5)} \right]
\]
\[
\times \frac{1 - \gamma}{(1 + \gamma)^2} qL \mathcal{O}((qL)^2) + \cdots + \frac{(1 - \gamma)^2}{(1 + \gamma)^2} \frac{(qL)^{D-4}}{C_1} \ln \left( \frac{qL}{2} \right), \tag{D.7}
\]
where \( C_1 \) is a dimensional constant (\( C_1 = 1, 4, 64, \cdots \), for \( D = 5, 7, 9, \cdots \)). The the last expression involving logarithmic term would be absent when the number of total dimensions \( D \) is even.
References

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370, arXiv:hep-ph/9905221.
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690, arXiv:hep-th/9906064.
[3] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, Phys. Rev. Lett. 84 (2000) 586, arXiv:hep-th/9907209.
[4] J. Garriga and T. Tanaka, Phys. Rev. Lett. 84 (2000) 2778, arXiv:hep-th/9911055.
[5] N. Kaloper, Phys. Lett. B 474 (2000) 269, arXiv:hep-th/9912125.
[6] C. Csáki, J. Erlich, T.J. Hollowood and Y. Shirman, Nucl. Phys. B 581 (2000) 309, arXiv:hep-th/0001033.
[7] S. B. Giddings, E. Katz and L. Randall, J. High Energy Physics 0003 (2000) 023, arXiv:hep-th/0002091.
[8] J. E. Kim and H. M. Lee, Nucl. Phys. B 602 (2001) 346, arXiv:hep-th/0010093; Erratum-ibid, Nucl. Phys. B 619 (2001) 763.
[9] Y. M. Cho, I. P. Neupane, and P. S. Wesson, Nucl. Phys. B 621 (2002) 388, arXiv:hep-th/0104227.
[10] D. G. Boulware and S. Deser, Phys. Rev. Lett. 55 (1985) 2656.
[11] R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. B 293 (1987) 385; D. J. Gross and J. H. Sloan, Nucl. Phys. B 291 (1987) 41.
[12] A. Lukas, B. A. Ovrut and D. Waldram, Nucl. Phys. B 532, 43 (1998), arXiv:hep-th/9710208; K. Kashima, Prog. Theor. Phys. 105, 301 (2001), arXiv:hep-th/0010286.
[13] J. E. Kim, B. Kyae and H. M. Lee, Nucl. Phys. B 582 (2000) 296, arXiv:hep-th/0004005;
[14] I. Low and A. Zee, Nucl. Phys. B 585 (2000) 395, arXiv:hep-th/0004124.
[15] S. Nojiri and S. D. Odintsov, Phys. Lett. B 484 (2000) 119, arXiv:hep-th/0004097; JHEP 0007 (2000) 049, arXiv:hep-th/0006232.
[16] J. E. Kim, B. Kyae and H. M. Lee, Phys. Rev. D 64 (2001) 065011, arXiv:hep-th/0104150.
[17] I. P. Neupane, Class. Quant. Grav. 19 (2002) 5507, arXiv:hep-th/0106100.
[18] I. P. Neupane, Phy. Lett. B 512 (2001) 137, arXiv:hep-th/0104226.
[19] N. E. Mavromatos and J. Rizos, Phys. Rev. D 62 (2000) 124004, arXiv:hep-th/0008074.
[20] I. P. Neupane, JHEP 0009 (2000) 040, arXiv:hep-th/0008190.
[21] N. Deruelle and T. Dolezel, Phys. Rev. D 62 (2000) 103502, arXiv:gr-qc/0004021.
[22] H. Collins and B. Holdom, Phys. Rev. D 63 (2001) 084020, arXiv:hep-th/0009127.
[23] M. Giovannini, Phys. Rev. D 63 (2001) 064011, arXiv:hep-th/0011153; Phys. Rev. D 63 (2001) 085005, arXiv:hep-th/0009172; Phys. Rev. D 64 (2001) 124004, arXiv:hep-th/0107233.
[24] Z. Kakushadze, Phys. Lett. B 494 (2000) 302, arXiv:hep-th/0009022; O. Corradini, A. Iglesias, Z. Kakushadze and P. Langfelder, Phys. Lett. B 521 (2001) 96, arXiv:hep-th/0108055.
[25] A. Iglesias and Z. Kakushadze, Int. J. Mod. Phys. A 16 (2001) 3603, arXiv:hep-th/0011111.
[26] K. A. Meissner and M. Olechowski, Phys. Rev. Lett. 86 (2001) 3708, arXiv:hep-th/0009122; Phys. Rev. D 65 (2002) 064017, arXiv:hep-th/0106203.
[27] S. Nojiri and S. D. Odintsov, JHEP 0112 (2001) 033, arXiv:hep-th/0107134; B. Abdesselam and N. Mohammed, Phys. Rev. D 65 (2002) 084018, arXiv:hep-th/0110143; C. Charmousis and J.-F. Dufaux, Class. Quant. Grav. 19 (2002) 4671, arXiv:hep-th/0202107.
[28] C. Germani and C. F. Sopuerta, Phys. Rev. Lett. 88 (2002) 231101, arXiv:hep-th/0202060.
[29] J. E. Lidsey, S. Nojiri and S. D. Odintsov, JHEP, 0206 (2002) 026, arXiv:hep-th/0202198; Y. M. Cho and I. P. Neupane, Phys. Rev. D 66 (2002) 024044, arXiv:hep-th/0202140; S. Nojiri, S. D. Odintsov and S. Ogushi, Int. J. Mod. Phys. A 17 (2002) 4809, arXiv:hep-th/0205187.
[30] C. Csaki, J. Erlich, C. Grojean and T. Hollowood, Nucl. Phys. B 584 (2000) 359, arXiv:hep-th/0004133;
[31] D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, Adv. Theor. Math. Phys. 3 (1999) 363, arXiv:hep-th/9904017.
[32] J. Maldacena and Carlos Nuñez, Int. J. Mod. Phys. A 16 (2001) 822, arXiv:hep-th/0007018.
[33] S. Dasgupta, R. Venkatachalapathy and S. K. Rama, JHEP 0207 (2002) 061, arXiv:hep-th/0204136.
[34] T. Gherghetta and M. Shaposhnikov, Phys. Rev. Lett. 85 (2000) 240, arXiv:hep-th/0004014.
[35] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429 (1998) 263, arXiv:hep-ph/9803315.
[36] S. W. Hawking and R. Penrose, Proc. Roy. Soc., London, A 314 (1970) 529.

[37] G. W. Gibbons, “Aspects of Supergravity Theories”, GIFT Seminar 1984, pp. 123-146, (QCD161;G2:1984), Print-85-0061 (CAMBRIDGE).

[38] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, Phys. Rev. D 62 (2000) 046008, arXiv:hep-th/9909134.

[39] N. E. Mavromatos and J. Rizos, Int. J. Mod. Phys. A 18 (2003) 57, arXiv:hep-th/0205299; P. Binétruy, C. Charmousis, S. Davis and J.-F. Dufaux, Phys. Lett. B 544 (2002) 183, arXiv:hep-th/0206089.

[40] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, Phys. Lett. B 480 (2000) 193, arXiv:hep-th/0001197; S. Kachru, M. Schulz and E. Silverstein, Phys. Rev. D 62 (2000) 045021, arXiv:hep-th/0001206; S. P. de Alwis, Nucl. Phys. B 597 (2001) 263, arXiv:hep-th/0002174.

[41] I. P. Neupane, Class. Quant. Grav. 19 (2002) 1167, arXiv:hep-th/0108194.