The Diffusion and Permeability of Political Violence in North and West Africa

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ABSTRACT
This article explores the spatial and temporal diffusion of political violence in North and West Africa by endeavoring to represent a group leader’s mental landscape as he contemplates strategic targeting. We assume that this representation is a combination of the physical and social geography of the target environment, and the mental and physical cost of following a seemingly random pattern of attacks. Focusing on the distance and time between attacks and taking into consideration the transaction costs that state boundaries impose, we wish to understand what constrains a group leader to attack at a location other than the one that would yield the greatest overt payoff. We leverage functional data from the Armed Conflict Location and Event Data project (ACLED) dataset that catalogs violent extremist incidents in North and West Africa since 1997 to generate a network whose nodes are administrative regions. These nodes are connected by edges of qualitatively different types: undirected edges representing geographic distance, undirected edges incorporating the costs of crossing borders, and directed edges representing consecutive attacks by the same group. We analyze the resulting network using spectral embedding techniques that are able to account fully for the different types of edges. The result is a representation of North and West Africa that depicts its empirical permeability to violence. A better understanding of how location, time, and borders condition attacks enables planning, prepositioning, and response.

KEYWORDS
Political violence; terrorism; borders; spectral embedding; North and West Africa

Introduction
The study of how crime and political violence diffuse across time and space has greatly benefited from the increasing availability of geo-referenced data and the use of spatial statistical analysis. In urban policing, for example, the design and use of hot-spot analysis based on historical data makes it possible to anticipate when and where various kinds of crime are most likely to occur, and to pre-position policing assets accordingly. In this limited sense, predictive modeling of crimes has been remarkably effective. The urban environment lends itself to this kind of analysis: criminals are creatures of habit, they tend to travel limited distances, and some areas are naturally more target-rich than others.

There are some obvious difficulties in adapting this approach to attacks by armed non-state actors (ANSAs) in North and West Africa. As in urban settings, some natural targets...
attract repeated attacks, for example, foreign workers in West African capitals or government forces stationed on military bases. Most victims of recent conflicts in the region are, however, civilians, killed in a rather unpredictable manner by armed groups whose main objective is ethnic or tribal homogeneity.³

In such wars, a multiplicity of state and non-state actors build a complex ecosystem of affiliated and opposing groups that also constrain when and where an attack by a particular group might occur.⁴ Attacks also reflect competition between traffickers and violent extremist groups struggling to control trans-Saharan criminal networks, who often clash far from inhabited areas.⁵ Furthermore, many violent groups in the region do not limit their attacks to a particular “turf” as urban gangs might; instead, they move relatively freely across the region, including across state boundaries.

Insurgent groups have limited resources and compensate by striking at locations that maximize impact, even abstractly via publicity, while minimizing cost. They avoid head-on confrontation, blurring the line between zones of war and zones of peace. Naval battles are a more apt analogy. Notwithstanding strategic constraints (such as the need to blockade enemy fleets at Trafalgar and Jutland), the precise locations at which naval battles occur are not contingent on the terrain in the way in which many engagements on land are.

This article explores the spatial and temporal diffusion of political violence in North and West Africa. It models the strategic landscape in a group commander’s mind taking into account that, far from being clinically abnormal, most violent extremists pursue collective goals rather than personal fantasies.⁶ In that sense, most violent extremists can be seen as rational actors that make choices based on costs and benefits, although their goals and actions are clearly not normal in a moral sense. The location of an attack requires a complex calculus that combines properties of the comparative appeal of targets, the physical geography of the terrain between the current location and potential targets, the obstacles and impediments to movement between the current location and targets, including borders that must be crossed, the difficulty of operating close to targets, and the need to maintain an element of surprise. We wish to understand what motivates or constrains a group leader to attack at a location other than the one that would yield the greatest overt payoff.

The article proceeds as follows. The next section outlines existing literature on the geographic features that are most likely to influence how attacks are conducted across space and time: the distance between places, and the impediment of state boundaries. Section 3 presents the statistical properties of the attacks in this region, and how they differ from conventional distributions of conflict. Section 4 describes our spectral embedding methodology, with several extensions that allow qualitatively different similarity relationships to be represented consistently. Section 5 shows the results of applying this methodology to the ACLED dataset, and describes the inferences that can be drawn from them. The last section discusses the main implications of our work before concluding.

Networks, space, and borders

Space is now widely recognized as a fundamental dimension for ANSAs that often conduct operations from a territorial base, leverage geographic havens, compete with sovereign states, and fight for control over aspirational homelands.⁷ As a result, an increasing number of scholars are working to integrate social network analysis and spatial
analytical techniques. As Carley argues: “If we look only at the social network then the focus of attention is on hierarchies, communication, and other social relations. The addition of events and locations facilitates the course of action analysis and enables linkage to various strategic planning tools.”

Recent conceptual and technical developments related to the spatiality of social networks have primarily been applied to case studies located in the U.S., Middle East, Afghanistan and Pakistan, and Southeast Asia. The availability of large databases has also stimulated new approaches that use events, targets, and locations to forecast, predict or explain terrorism at the global level. Desmarais and Cranmer, for example, use Exponential Random Graph Models (ERGMs) to estimate the probability of new terrorist ties from one state to another between 1968 and 2002. Moon and Carley develop a multiagent model that simulates how terrorists interact with each other and where they relocate. The model shows that social networks and spatial patterns are closely related: while terrorist networks become more geographically dispersed over time, critical actors tend to occupy the same location. More recently, Campedelli et al. have built a model to predict future terrorist attacks based on previous targets for the period 1997–2016.

By contrast, North and West Africa have received little attention from network science, despite the fact that the continent faces some of the deadliest terrorist groups in the world, including Boko Haram, Al Qaeda, Al Shabaab, and the Islamic State. Most of the studies conducted so far have apprehended the spatiality of social networks based on actors whose location or territory was well known, whether in conflict studies or international relations.

This article seeks to address territories where fixed targets are the exception, focusing on the effect of two fundamental geographic features: distance and borders. Distance constrains locations of attacks in two ways. First, when attacks involve the same people or resources, these must be transported from one location to another, which takes time and costs money. Second, a distant location imposes transaction costs: unfamiliarity with the physical and social terrain, different languages, and so on.

Borders are one important aspect of the effect of distance. As Engel and Rogers and Borraz et al. showed, borders introduce price distortions that are equivalent to adding an extra distance between locations. Borders also limit social exchanges—even when people use social media—and are a major impediment to labor market integration, despite formal agreements that promote the mobility of labor. In addition to hindering the mobility of goods and people, borders also have strong effects on political violence, which often occurs at the subnational level. The effect of borders on the decision by a group to carry out attacks in more than one country can, therefore, be modeled as obstacles to be surmounted. The practical cause of the obstacle might be the overhead of the crossing, either overtly or covertly; differences in culture on the other side; or the increased risk associated with operating away from “home turf,” where, for example, it may be less obvious who can or should be bribed. (Other kinds of border-like effects exist, based on differences in ethnicity and language, and these could be integrated into our approach, but the data is harder to obtain at scale).
This article proposes a novel approach to modeling the impact of borders: as their effect increases, the perceived distance structure becomes less and less planar, and so a simple map representation becomes less and less accurate – borders behave like mountain ranges on the surface of the earth, making some movements harder and funneling others in non-obvious directions. The modification we suggest is to represent the locations where attacks have taken place as a network where the nodes are attacked locations and the edges represent perceived distances (increasingly modified) between them. This network will not be planar, because perceived distances will differ from physical distances, but our modeling technique allows them to be mapped back into two dimensions for visualization.

The uneven geography of attacks in North and West Africa

As we would expect, statistics show that the distribution of the 29,272 attacks by 921 groups at 1831 locations from 1997 to 2015 is not random. The location where the most attacks took place is near Benghazi in Libya, where 1230 attacks are recorded. However, the mean number of attacks per location is sixteen, and the mean number of attacks for the least-attacked 1600 locations is only 4.7; so, the distribution is highly skewed. Histograms of the attack frequencies are shown in Figure 1. If we consider instead how many organizations have carried out attacks at each location, the highest score is a location near Tripoli where sixty different groups have carried out an attack. However, the mean number of groups attacking at a given location is 3.7; so, again, the distribution is skewed. Of course, these highly skewed distributions mean that conventional hot-spot analysis can, and should, be carried out. However, in the North and West Africa setting it cannot be enough, because of the constantly shifting set of actors, allegiances, and motivations.

These observations make it clear that this setting is far removed from conventional warfare where the ground is taken and held; and that participants do not form large, stable blocs. Rather, interactions are fluid and consist of smaller, constantly shifting members and alliances.21 The decision about where to carry out an attack is constrained by two sets of factors: properties of the target, and properties of the attacking group. Properties of the

Figure 1. Frequency histogram of the locations of all attacks (left) and the least-frequent 1600 attacks, showing how skewed the distribution is.
target can be used to analyze risk; targets are attacked for a reason, and their appeal at any given moment can be assessed. The more difficult properties are those of the attacking groups, whose internal processes may be opaque, and who are fundamentally motivated to do the unexpected. However, such groups cannot attack at will—they are constrained by resources, broadly interpreted. We focus on the constraints imposed by distance (which matter in a non-urban environment) and the way borders distort distance.

Figure 2 shows the distribution of all attacks by latitude and longitude from 1997 to 2015. The main clusters of violence, in decreasing order of fatalities reported in the ACLED data, are located in Nigeria, Northern Algeria, Northern Libya, the Chad-Sudan border, and along the Gulf of Guinea. Nigeria is especially affected, with 50,144 fatalities, most of them resulting in either from ethnic violence, fights to control oil production in the Niger Delta, or from attacks by Boko Haram. In West Africa, the border between Chad and Sudan remains a focus of conflict due to persistent fighting between the Sudanese government and rebels in Darfur. The portion of the Gulf of Guinea that extends from Abidjan to Banjul has suffered from a succession of civil wars in Ivory Coast, Liberia, Sierra Leone, and Guinea-Bissau.

In North Africa, Algeria has also been markedly affected by violence, principally due to activity by three organizations in conflict with the Algerian government: the Armed Islamic Group (GIA), the Salafist Group for Preaching and Combat (GSPC), and Al Qaeda in the Islamic Maghreb (AQIM). Violent Islamist groups were involved in 93 percent of the 12,050 fatalities in Algeria. With 12,610 fatalities reported, Libya is the third epicenter of violence, principally because of the overall political instability after the ouster of Colonel Gaddafi in 2011 and the subsequent civil war. In comparison, the Sahel and

Figure 2. The positions of all attacks by latitude and longitude. The Mediterranean coast can be seen at the top of the figure, the Atlantic coast on the left and the Gulf of Guinea on the lower side.
Sahara regions are less immediately affected by violence, with the notable exception of northern Mali where secessionist rebels and Islamist groups have opposed the government since 2012. More than 1,200 of the 2,761 victims of violent events reported in Mali from 1997 to 2015 died in an event involving one or several Islamist groups, including AQIM, Ansar Dine, the Movement for Oneness and Jihad in West Africa (MUJAO) and Al Mourabitoun. In Mauritania, where violent Islamist groups have also been active, the number of victims resulting from clashes with such groups is much lower, with eighty six fatalities, while in Niger, the number of victims (997) has increased rapidly due to Boko Haram.

Often organizations from one country carried out attacks in another. For example, the Islamist group AQIM, historically based in Algeria, has conducted numerous attacks in neighboring countries. Boko Haram is also responsible for attacking civilians and security forces in Niger, Chad, and Cameroon. The transnational activity of ANSAs has prompted many governments of the region to carry out attacks abroad. At the beginning of the 2010s, for example, the Mauritanian military carried out attacks in Mali to destroy military bases belonging to AQIM. More recently, Chad sent troops to both Nigeria and Cameroon to fight Boko Haram.

Methodology

Spectral embedding

While networks are a natural way to represent relationship data involving nodes and connection patterns, the conventional adjacency matrix representation is difficult to understand. Two approaches to making network data intelligible are used: graph drawing, and graph embedding. Graph drawing, on the one hand, attempts to provide the most understandable visualization of a network by placing the nodes and edges so that they do not occlude one another, while still placing nodes that are similar as close to one another as possible. It emphasizes clarity in the resulting picture. Graph embedding, on the other hand, tries to represent a network as accurately as possible, that is so that the distances between each pair of nodes are as representative of their similarity in the network as possible, at the expense of producing a picture that may be hard to understand directly. Graph drawing produces a representation that is qualitatively accurate, while graph embedding produces a representation that is quantitatively accurate.

We will use spectral embedding, the most effective of the graph embedding approaches, to represent networks in a geometric form. This requires a mathematical technique with two main steps.

First, the adjacency matrix, $A$ (of distances, say) is converted to one of a family of Laplacian matrices. We begin by using the combinatorial Laplacian, $L$, given by the matrix equation: $L = D - A$, where $D$ is the matrix whose ith diagonal entry is the total edge weight of the edges connected to node i, with all of its other entries zeros. Since $A$ is symmetric, so is $L$. In other words, the diagonal entries of $L$ are the (weighted) degrees of each node, and the off-diagonal entries are the negatives of the corresponding edge weights in the adjacency matrix.
Second, an eigendecomposition of \( L \) is computed such that: \( L = \mathbf{Q} \Lambda \mathbf{Q}' \), where \( \Lambda \) is a diagonal matrix of eigenvalues, in decreasing order, \( \mathbf{Q} \) is the \( n \times n \) eigendecomposition of \( L \), and the superscript dash indicated matrix transposition.

If the network is connected, then the final eigenvalue is zero, and we ignore the final row of \( \mathbf{Q} \). The \( k \) rows preceding it can be interpreted as the coordinates of each node of the graph in a \( k \)-dimensional space. In other words, if we take the \( n \) rows of \( \mathbf{Q} \), and the \((n-1)\)st and \((n-2)\)nd rows of \( \mathbf{Q} \) we can use these as coordinates to place points corresponding to the \( n \) nodes in a two-dimensional rendering of the network.

This spectral embedding comes with strong mathematical guarantees that it is the most faithful representation of the network structure in the chosen number of dimensions. From an intuitive viewpoint, the effect of spectral embedding is to begin with the cloud of points representing the nodes of the graph in a space of dimension \( n-1 \), in which the similarities (distances) between each pair can be represented exactly. The eigenvectors are then oriented along with orthogonal directions for which the cloud has the greatest variation: the \((n-1)\)st eigenvector along the direction of greatest variation, the \((n-2)\)nd along a direction orthogonal to this with the next greatest variation, and so on. Choosing \( k = 2 \) creates a projection of the graph into the two directions that most accurately represent the distances between every pair of nodes, given the constraints from all of the other nodes. Spectral graph embedding, as the name suggests, has connections to the Laplace-Beltrami operator, and the eigenvectors produced can be regarded as capturing vibrational modes of a graph. Thus, it generalizes forms of graph analysis based on differential and partial differential equations.

We apply this spectral approach to a network of attack locations. We want a network in which edge weights are large for nodes that are strongly connected and small for nodes that are weakly connected. Distances, however, are the exact opposite – locations that are far apart (and so weakly associated) have large values for their mutual distance. The edge weights need to be inverted so that close locations have large weights and vice versa. There are several ways to do this, but we choose to subtract each distance from the longest distance between any pair of locations in the network increased slightly by multiplying it by a factor of 1.1.

**Extending the spectral embedding to represent multiple relationships**

We want to construct a network in which both distances and border crossing difficulty are represented consistently. The spectral approach can be extended to represent two different similarity properties with a new, enhanced network. This network representation is built as follows: each node is replicated into two versions, a red version, and a green version. The red versions of the nodes are connected using the adjacency matrix based on one similarity property (distances) and the green versions are connected using the adjacency matrix based on the other similarity property (border crossing difficulty). We can think of these two subgraphs as forming two layers. Now we connect each of the pairs of replicated nodes by new (say, blue) edges. Thus, we have a single graph with red and green nodes, and red, green, and blue edges.

The question is now how to assign weights to the new, blue edges. The larger these weights are, the more the two layers are forced to be “aligned.” We can imagine that, in the larger graph, edges behave like springs that pull the nodes they connect with a force
proportional to their edge weight. In each layer, nodes are pulled together based on their closeness (in the red layer) or cross-border accessibility (in the green layer), but they are also pulled together by how consistent their “role” is in both layers at once. Note that the edge weights in the distance layer are much larger than those in the border layer and an adjustment must be made to compensate for this.

To see how to assign weights to the blue edges, we convert the adjacency matrices to random walk matrices by computing the sum of edge weights in each row, and dividing the entries of each row by its sum. The matrix entries are now values between 0 and 1, which can now be interpreted as probabilities. The name “random walk” comes from imagining a walker who inhabits the network, constantly moving from node to node along the edges. At any given node, the walker chooses which node to visit next by choosing among the outgoing edges in proportion to their probabilities. The random-walker view of a network is quite elegant: for example, the proportion of time that a random walker spends at each node is a measure of its importance, since important nodes tend to be well connected and so are easy to visit regularly.

A variation of a random walk matrix, with better properties, is a lazy random walk matrix. Here the entries of each row are divided by twice the sum of the row, so that the entries sum to 0.5. The remaining 0.5 is placed in the diagonal position. The interpretation is now that the random walker makes decisions among the outgoing edges as before, but may choose, with probability 0.5, to remain at the current node for the next step.

We use this as a model to motivate the new $2n \times 2n$ random walk matrix. The entries corresponding to each subgraph are mapped to values between 0 and 0.5, the diagonals are left as zeros, and the remaining 0.5 probability is assigned to the blue edges between the layers. Thus, from a random walk perspective, a lazy random walker behaves in each layer as it would before, but can move from layer to layer with probability 0.5 on each step.

The random walk matrix for the larger graph is bigger ($2n \times 2n$) but most of the extra entries in this matrix are zeros. The top left-hand corner is the random walk matrix from the distances, the lower right-hand corner is the random walk matrix of border crossing permeability, and the other two corners are diagonal matrices of the edge weights of the blue edges, and so mostly zeros. The cost of computing an eigendecomposition depends on the number of non-zero values in the matrix, so computing an eigendecomposition for the bigger graph is not much more expensive than computing one for each of the existing networks separately.

Now the standard spectral embedding algorithm can be used to convert this larger adjacency matrix to a Laplacian, compute its eigendecomposition, and embed the resulting graph in a two-dimensional space. In this random-walk embedding, each geographical location is represented by embedded red and green points. The distance between the two points corresponding to the same location, the length of the embedded blue edge between them, reflects how different their roles are from the perspective of distance and the perspective of borders. Locations for which these points are far apart are of particular interest.

**Extending the spectral embedding to represent a third property**

We also want to be able to model the sequential patterns of attacks. We do this by enriching our representation with a third layer, in which two nodes are connected by
a directed edge when an attack at one is followed immediately by an attack at the other. The fact that the edges are directed in the new third layer introduces three complications to the embedding process. First, in an undirected network, the importance of a node is proportional to the total edge weight of the edges connected to it. This is no longer true for a directed network—a node may have many heavily weighted incoming edges, but it may not be important if these upstream nodes themselves are hard to reach. Importance derives from the entire graph, rather than being a local property. Second, a random walker can become trapped at a single node that only has incoming edges (but this is easy to detect), or in a region that collectively has no outgoing edges (and this is expensive to detect). Third, a random walk matrix is not necessarily symmetric, which the standard embedding algorithm requires.

A solution to these problems was developed by Chung, but it has a number of drawbacks. Instead, we use a newly developed approach that models edge direction by replicating each node into an outgoing version and an incoming version, and connecting these in the obvious way by undirected edges. This reduces the problem of embedding an undirected graph at the expense of adding more layers. As before, new edges have to be added between the versions of the same nodes. This approach has been validated against the known social structures of the Florentine families in the time of the Medicis, and has been applied to understand the structure of criminal networks.

The construction is as follows: we combine three layers, the geodesic-distance layer, the border crossing difficulty layer, and the time-sequence layer. In the previous construction, we converted each layer into a random walk matrix, divided the total edge weights incident at each node in half, and allocated half to edges that remain in the layer (proportional to their original values) and a half to the edge to the other layer. We cannot follow this strategy for the three-layer graph because the directed adjacency matrix cannot be converted to a random walk matrix. However, we use the same intuition: the total outgoing edge weight incident at each node should be divided in half, with half remaining within the layer, and half allocated to the edges to other layers. There are now two other layers, so the amount allocated to other layers is split equally between them.

The edge weights in the different layers are of considerably different magnitudes. To compensate we must normalize each subgraph adjacency matrix to make the magnitudes of the edge weights comparable between the layers. We do this by dividing the entries for each layer by the mean of the non-zero entries in that layer. Almost all non-zero values are close to 1, with the exception of larger entries in the sequence layer.

A further complication arises because the sequence layer is extremely sparse; in comparison, the other two layers are fully connected. If the sequence layer was to be embedded by itself, there would be two strong clusters with a weak connection between them, and many isolated nodes. The isolated nodes would be embedded at the origin, with the two clusters in a dumbbell shape. When this layer is connected into the larger graph, the other two layers have the effect of connecting all of the nodes in the sequence layer to one another indirectly by paths of small weights. Nodes with no connections within the sequence layer are embedded close to the center, nodes with weak connections within the sequence layer are embedded further out, and nodes that are strongly connected within the sequence layer are embedded furthest out. This is, of course, the inverse of what we would want—important nodes being embedded centrally. The solution is to normalize the
edge weights in the sequence layer further by making the sum of the edge weights incident to each node constant—so that nodes without any connections in the layer are given a heavily weighted self-loop, nodes with weaker connections in the layer a less-weighted self-loop, and so on. The result is a three-layer structure, with added directed edges between copies of the same node in the different layers.

We now replicate each layer into two more sublayers, one to which the outgoing edges are attached, and one to which the incoming edges are attached. All edges are now undirected; so, we can use the standard embedding (on a $6n \times 6n$ matrix whose entries are almost all zeros). The newly replicated copies must also be connected to one another by edges whose weights depend on the incident edge weights of the pair. This construction is intricate but essentially straightforward.\textsuperscript{28}

**Applying spectral embedding to attack location data**

The Armed Conflict Location and Event Data project (ACLED) dataset catalogs violent extremist incidents in North and West Africa since 1997. Rich data for each incident is available, including timing, groups participating as attackers and victims/targets, and location, both in terms of latitude and longitude, and by administrative district.\textsuperscript{29} We restrict our attention to incidents that are clearly categorized as violent and fall into one of the following categories: battle with and without change of territory; riots and protests; violence against civilians; and remote violence. Attack locations for our purposes are at the granularity of local administrative districts.

We use this data to generate a form of “social networks”. As opposed to a conventional social network where humans are nodes and relationships are edges, in this article nodes are abstractions of locations, and edges represent distances as perceived by the actors concerned. In the resulting “social network,” then, nodes are administrative regions, an approach similar to the one described by Batagelj et al.,\textsuperscript{30} with edges between them that are of qualitatively different types: one set of undirected edges representing geographic distance, another set of undirected edges representing the costs associated with having to cross borders, and a set of directed edges representing consecutive attacks by the same group at two locations. This network represents the landscape as perceived by group leaders, balancing their perceptions of the next potential attack at a “remote” location (with an element of surprise) against the convenience of choosing instead a close, even a repeat, location.

We analyze the resulting network using spectral embedding techniques that combine these different types of edges into a representation of North and West Africa that depicts its empirical permeability to attacks from the perspective of any violent group. When distributions are highly skewed, as they are in this setting, statistical measures such as averages are useless for planning effective counterinsurgency deployment. This map of permeability reflects the impact of distance, borders, and time on violent group actions, and so provides a first step towards principled planning, prepositioning, and response.

**Modeling distances between attacks**

We begin our modeling with a network derived from the location data by building an $n \times n$ adjacency matrix, $A$, with rows and columns corresponding to locations, and whose $ij$th
entry is the weight of the edge connecting them. If the edges are undirected, then the $ij$th and $ji$th entries are the same, and the matrix is symmetric. The matrix has 1831 nodes representing the attack locations, fully connected by edges whose edge weights are the geodesic (great circle) physical distances between them. These distances were calculated from the latitudes and longitudes using the Haversine function.

The key functional property of spectral embedding is that it allows an arbitrary network to be represented in two dimensions in such a way that the distance between any pair of nodes accurately represents their dissimilarity. This is needed even for the simple network that connects locations where attacks have taken place, because these locations exist on the curved surface of the earth (which matters at this scale) and so must already be transformed to produce a flat two-dimensional representation.

In the figures that follow, the nodes of the network are color-coded by the countries in which they are located. The top left map on Figure 3 shows an embedding of the attack locations based purely on the geodesic distance between all pairs. The difference between this and Figure 2 (based on position) is that the network of location similarity draws locations closer together when attacks happen in closer proximity. In other words, hot-spots get hotter; or, from the perspective of group leaders, attacks in close proximity to previous attacks are, empirically, a popular option. They are more likely to take place close together than logistic considerations or a desire for surprise would indicate.

![Spectral embedding](image)

**Figure 3.** Spectral embedding based on geodesic distance, and then with borders modelled as equivalent to distances of 50 km, 100 km, 500 km.
National borders can be seen when the locations are color-coded by country, but would be less obvious if they were not. This indicates that there are few differences in the locations of attacks along the Gulf of Guinea, or along the Mediterranean coast; but Burkina Faso and Mali show clear separations from their southern neighbors; and there is a strong separation between attacks in the Mediterranean countries and those farther south. Mauritania, and to some extent Algeria, bridge these two regions.

**The effect of linear (additive) border costs**

We now examine the effect of the presence of borders as it might enter into the calculus of a group planning its next attack. A simple way to model the cost of crossing a border is as an increased distance between origin and destination. For example, the addition of 100 km to account for crossing a border captures a delay of, say, two hours (assuming typical speeds of 50 km/h for travel using local pick-up trucks) caused by the overheads of the crossing.

We compute the number of borders that must be crossed to pass between all pairs of the twenty-one countries we consider. This calculation was done based on the great circle distance between a median point in each country but when such a route would have required crossing many borders and a slightly longer route would have required crossing many fewer, the lower number of border crossings were used. For example, a direct path from Sierra Leone to Niger passes through Guinea, Ivory Coast, and Burkina Faso, but a path through Guinea and Mali crosses fewer borders without adding much actual distance. Of course, some borders are less permeable than others due to government or military policy. This can be straightforwardly modeled by altering the effective added distance for each border.

In a model where border crossings are modeled as artificial added distances, the effect of multiple crossings is linear, since crossing two borders is twice as expensive as crossing one. Given a distance equivalent for each border crossed, we add this distance to the edge weight associated with each pair of nodes before inverting distances as described earlier. Additive border crossing can be modeled in a single network by this adjustment of the edge weights.

Figure 3 shows the embedding when borders are modeled as equivalent to an increased distance of 50 km between countries. The maximum distance between attack locations in the dataset is almost 5000 km, so this is a small distortion, and indeed the differences between Figure 3 are too small to see at this resolution. However, when this distortion is increased to 100 km, the situation changes. On the one hand, attack locations in different countries now begin to separate in the visualization, indicating that they have become less similar, especially along the Gulf of Guinea. On the other hand, the border between Algeria and Tunisia shows little change, indicating how similar attack locations in these countries are. When the effect of a border is increased to be equivalent to 500 km, Figure 3 shows that locations clearly separate by country. When the cost of a border crossing is as great as this, cross-border locations seem less similar, and locations within the same country, by contrast, seem more similar to one another.

From the perspective of a group leader at a particular location and considering the location for a next attack, these results suggest that the presence of a border has little impact until the potential overhead of crossing that border is at least equivalent to the
costs of 100 km of intra-country travel. This has implications for the amount of effort a country should put into hardening its border to have any effect on the attack calculus of ANSAs.

**Non-linear border costs**

While an argument could be made for linear border costs, it seems more plausible that the perceived cost of crossing borders is non-linear. For example, suppose that the probability of interdiction at any given border is 20 percent. Then, the probability of interdiction when crossing two borders is 36 percent, since there is an 80 percent chance of successfully crossing the first border, and an 80 percent chance of success at the second border, so the probability of crossing both successfully in the sequence is $0.8 \times 0.8 = 0.64$. Thus, a group planning an attack two countries away should perceive it as substantially more costly than one in a neighboring country.

There are arguably two ways in which groups might frame non-linear border costs. On the one hand, a group with pan-national ambitions, such as AQIM, must exert its influence by carrying out attacks in countries that are far away from its center of influence. For such groups, a cost of crossing borders might be appropriately framed in terms of the rate of success, and values as large as 95 percent would be necessary for them to succeed across multiple borders. On the other hand, a group whose interests are primarily domestic, such as Boko Haram, might regard borders as substantial impediments to their choice of locations to attack, both because of the discomfort of operating in another country, and the reduced impact such an attack might have on their local agenda. For such groups, a much lower success rate associated with crossing borders, perhaps 50 percent, might appropriately frame their calculus.

A new border adjacency matrix for the 1831 locations was computed by setting the $ij$th entry to a given border success probability (between 0 and 1) raised to the power of the number of borders between the country of location $i$ and the country of location $j$. If the probability of success is 0.95, then the result is $0.95^n = 1$ for locations in the same country, 0.95 for locations in neighboring countries, $0.95^2 = 0.9$ for locations two countries apart, $0.95^3 = 0.86$ for locations three countries apart, and so on. If the probability of success is only slightly smaller, the effect becomes more pronounced. For a probability of success of 0.9 per border, the rate of success for crossing two borders sequentially is $0.9^2 = 0.81$ and for three, $0.9^3 = 0.73$.

**Borders as a separate layer**

Non-linear costs of borders cannot be represented as an addition to the representation of distances, because their impact depends on the particular pairs of locations being considered. Instead, we consider locations to be connected by two kinds of relationships: the obvious one based on how far apart they are, and the other by how many borders must be crossed on the path between them. Thus, there are two networks, with the same set of nodes, but qualitatively different types of edges. We use the two-layer approach to build a single network by connecting the two networks by adding an extra edge (“blue”) between the two versions of each nodes, and adjusting the edges weights as described above. The resulting network can then be embedded using a spectral technique. Each
location has two positions in the resulting two-dimensional embedding, reflecting their roles from distance and border perspectives. The distance between these two positions shows how much impact borders have on their perceived distance from other locations.

Figure 4 shows the full embedding of the two-layer graph with border-crossing success probability set to 95 percent. The red versions of the nodes, which capture the embedded locations based on geography, lie around the outside of the embedding, being “pushed” apart by the effect of borders; the green versions of the nodes, which capture the embedded locations based on border crossing difficulty, lie further towards the center, pulled inwards by a relatively smaller effect of border crossing costs; and the blue lines indicate the magnitude and direction of the difference for each location.

Figure 5 shows the embedding of the locations based on distance, which represents our “red” nodes color-coded by the countries in which they are located, as before. Comparing this figure to Figure 3, where borders were represented as equivalent to distances of 100 km, shows that the spread of locations is not dissimilar – but there is a greater spread from east to west, as expected given the number of borders along the Gulf of Guinea.

Figure 6 shows the difference between locations based on distance and based on border crossings. The “blue” lines, also color-coded by country, make it clear that the effect of borders is effectively to spread locations further apart from a virtual center in Southern Algeria, a fixed point where the distance to all other locations in North and West Africa is proportional to the number of borders that have to be crossed to reach them. This point corresponds to the commune of Bordj Badji Mokhtar, in Adrar Province, Algeria. The
embedding shows that Bordj Badji is the most central place to use as a base when border crossing costs, as well as distance, are taken into account. Indeed, Bordj Badji Mokhtar and the adjacent trading town of Al-Khalil in Mali have long been known for being a haven for arm and drug traffickers and a central node in the transnational network that connects local Tuareg tribes with the Algerian military and secret police.\textsuperscript{31} Between 2008 and 2014, Bordj Badji Mokhtar saw repeated clashes between Algerian governmental forces and Islamist groups such as AQIM and MUJAO.

Figure 6 shows a similar figure but with the border-crossing success rate reduced to 80 percent. With this assumption, the distortion introduced by borders is quite different. Locations in the north and center of the region show the same radial distortion, in which locations appear further apart than they are because of the presence of borders. But, for the countries in the southwest and southeast, the distortion introduced by borders is oriented orthogonally to the previous distortion. For example, Sierra Leone and Liberia “push” one another apart rather than being influenced by distant Algeria and Tunisia; and Nigeria and Cameroon show a similar pattern.

Figure 6 shows what happens when the probability of successfully crossing borders is reduced to 50 percent, reflecting the mindset of groups with primarily local agendas. Distortions caused by borders are almost completely local, depending primarily on a few near neighbors. Because of the roughly triangular shape of North and West Africa, the net effect is that most distortions align toward the center but, since the probability of crossing a substantial number of borders drops quickly to a small value, countries are only weakly connected. Note that Cameroon sees the rest of the region through the lens of Nigeria, from which it is now indistinguishable. This situation reflects the increasing interdependence between the two countries since Boko Haram, historically active in Nigeria, spread to adjacent countries in 2014.
Time sequence as a separate layer

So far, we have ignored the dimension of time. Consider the sequence of attacks by each particular group. Unless subgroups of the group act completely independently, there are some constraints on the sequencing of attacks, and these may offer insights into the group’s constraints or strategy. For example, if the same (sub)group carries out successive attacks, perpetrators must travel from one location to another, perhaps taking or gathering material; and perhaps crossing borders as well. Even if the attacks are not carried out by the same individuals, the time sequence reflects, at some level, strategic thinking by the group’s leadership.

From the ACLED dataset, we extract a third adjacency matrix connecting successive attacks by the same group by a directed edge of weight 1. If the jth entry is 1, then the jth entry is unlikely to be, so this matrix is typically asymmetric.

Our scope includes all the armed groups for which a clear transnational activity is documented in the ACLED from 1997 to 2015. This includes Boko Haram and nine other Islamist groups affiliated with Al Qaeda that share a common historical and ideological background and form several components of a single, flexible network: Al Qaeda, Ansare...
Dine, AQIM, GIA, Al Mourabitoune, GSL, GSPC, MUJAO, and Those Who Signed in Blood.\textsuperscript{32} Note, however, that AQIM has historically occupied two very distant regions: the Kabylie region in northern Algeria, where its “national” emir Abdelmalek Droukdel supposedly resides, and the Sahel-Saharan region where several sections (\textit{katibas}) have developed since the mid-2000s.\textsuperscript{33} Only the events related to the Sahelian-Saharan section of AQIM were considered. Mapping the chronological activity of both AQIM’s Kabylian leadership and Sahelian-Saharan sections would give the impression that much of the attacks span the Saharan desert, which is probably unrealistic. To solve this issue, we consider separately all violent events south of In Amenas, Algeria. Because most of these groups have a trans-national agenda, we use a border crossing probability of 95 percent in the analysis.

Figure 7 shows the locations’ embedded distance as in Figure 3, but overlaid by black edges connecting sequential attacks by the same organization (that is, whenever there is a one in the newly constructed directed adjacency matrix). The map confirms the pan-regional ambition of the nine Islamist groups affiliated with Al Qaeda that conducted attacks from Mauritania to Chad, often across borders. Their patterns of attack diverge greatly from those of Boko Haram, the majority of whose violent attacks took place within Nigeria itself.\textsuperscript{34} It is also notable how large the distances between successive attacks can be.

We now want to extend the embedding to include the time-sequence structure, that is to incorporate the empirical similarity of each pair of sequential attacks. We do this by extending the layered model to three layers: one representing geodesic closeness, one representing border permeability, and one representing sequence in time.

\textbf{Figure 7.} Spectral embedding as in Figure 3, overlaid by lines connecting sequential attacks by the same group (for 9 groups with trans-national intentions, and Boko Haram).
The resulting embedding is shown in Figure 8: Boko Haram attacks are local to Nigeria, with occasional incursions into Chad, Cameroon, and Niger. Several locations are subject to repeated attacks. The attacks by Islamist insurgents affiliated with Al-Qaeda are concentrated in northern Algeria, but another nexus of attacks can be observed in southern Algeria, Mali, Mauritania, and Niger. These second groups of attacks are much more widespread geographically. Note that, as locations attacked sequentially appear more similar, locations where this does not happen spread further apart in the embedding.

**Conclusion**

The aim of this article is to provide a dynamic description of the structure of conflict in North and West Africa across time. While the causal inferences that can be drawn are necessarily limited, our findings significantly advance the state of knowledge in network science and conflict studies.

First, the article expands our understanding of the way structural data can be extended into the analysis of conflict through the application of spectral embedding techniques to network science. We have shown how newly developed extensions to spectral embedding techniques (with typed edges, and directed edges) that have been previously applied to conventional social networks, where humans are nodes and relationships are edges, can be extended to social networks in which the nodes are locations and edges represent distances as perceived by the actors concerned.
Second, the article suggests ways to think differently about the nexus of space and conflict: commonly we think of networks as a function of place; instead, this article inverts this convention to think of the place as a function of networks. Place is no longer simply a physical location—its positional and strategic importance define it, and these changes as the movements that give rise to networks change. Our study of political violence in Africa is an application of ideas from social networks to networks of a different kind, reflecting the fact that place is a human construct as well as a physical one.

An interesting finding of our work is to show that some of the most violent places in the region are far from inhabited areas, such as the extreme north of Mali or the north-eastern reaches of Niger. Instead of thinking of conflicts as a function of place per se, we can now think of conflicts as a function of movement. Since movement, unlike place, is not fixed, strategic consideration can now be given to ways to influence, alter, or disperse some movements while generating and encouraging others. Conflict has long been known to be dynamic; this article posits a method to model that dynamism, one that makes it possible to respond to conflict and violence in terms of strategic consideration of movement rather than simple spatial coordinates.

Finally, the spectral embedding of these networks of places provides an insight into the potential mindset of group commanders as they consider their next actions. Geodesic distance obviously plays a role, but other constraints are also significant. Borders are one such constraint. Future work might explore the effect of more complex factors as political conditions in an adjoining country, the type of regime, and state-sponsorship of armed groups on the model and results. The aim of this article, however, is limited to two effects: geodesic distance and borders. Borders are modeled from two perspectives: that of a group with pan-national aspirations, and that of a group with more local aspirations, and shown that the mental landscapes produced differ. In other words, we show that intent, capability, and opportunity affects the framing of the transaction costs imposed by crossing borders and resource constraints of operating across vaster distances, and so affect the calculus of a group’s leadership. Constraints as simple as habits also play a role: at some pairs of locations attacks occurred at the same pair in sequence sixteen times.

Our findings have implications for conflict prevention and early intervention in the region. For example, consider the practical problem of anticipating, after an attack by a particular group, the probable timing, and location of the next attack. Without extrinsic information, the conventional approach is simply to draw concentric circles around the location of the current attack, and assign a reduced probability the more distant location. However, if we know that borders represent an impediment to movement (the cost of which we can estimate), that habits play a role, or that target groups are themselves mobile, then this has implications for behavior, and the mental calculus that motivates behavioral habits: movement in some directions will be harder, movement in other directions will be easier.

Conventionally, the probability of associated behavior is represented by warping contours of space on a map—concave in shape where movement in that direction is hard, and convex when movement is easy. One novelty of our technique is that it forgoes this complex, ad hoc manipulation, and warps the space instead, so that locations that are easier to reach (physically, border-crossing wise, or strategically) are embedded closer together, and locations that are hard to reach are embedded further apart. As a result, concentric circles again demarcate regions that are equally likely to be the site of the next
attack. In other words, a concentric circle drawn on any of the embeddings in this article represents locations at a similar mental strategic distance from its center, given the cost assumptions associated with that figure.

Myriad other measures potentially modulate the role of distance in projecting the “next” place. The varying difficulty and desirability of border crossings for migrants into and within Europe is one example: some countries are attractors and so the distance to them may seem shorter than they are in the mental calculus of migrants, but some borders are harder to cross, making distances that involve them seem longer than they are. The ability to incorporate these multiple modulating factors with distance; to create a mental representation that combines all of these varying criteria into a framework that migrant populations might hold and informs their behavioral logic, enables both insight and strategic reaction in what might otherwise appear as wholly unpredictable settings.

Our approach to political violence gets us closer to a prototype of predictive modeling of conflict of the sort that is widely used by urban police to anticipate crime. Although necessarily somewhat simplistic, it nonetheless lays the groundwork for working towards building a more comprehensive model of the trade-offs that factor into the selection of potential target sets and resource allocation by weighing the costs and benefits of such decisions in the light of transaction costs, such as those imposed by borders. Here we have treated all borders as equally difficult barriers, but the methodology would also allow more detailed modeling. For example, the efforts by the Mauritanian government to enhance border security since 2011 by increasing patrols and working with local tribes make this border more difficult to cross, and this could be added into the border-cost matrix to recalibrate the mental landscape, not only of the adjacent countries, but of groups from much further afield.

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