Limit on Higgs boson trilinear self-coupling in coupled technicolor models

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received 28 April 2021; accepted in final form 28 July 2021
published online 11 March 2022

Abstract – The trilinear self-coupling of the Higgs boson, in a theory in which this boson is composite, is compared to the experimental bound of this quantity obtained by the CMS experiment. In the case of a model where technicolor (TC) is coupled to QCD, we find that the experimental result already constrains the dynamics of the theory, which is represented by an expression of the technifermion self-energy (Σtc) typical of technicolor coupled models, and function of the dynamically generated technifermion mass and two other parameters that describe the technifermion dynamical mass momentum dependence. The limits imposed on this dynamics allow us to make a simple determination of pseudo-Goldstone boson masses that appear in these theories, indicating that these bosons may be expected to be quite massive.

Introduction. – The Higgs boson discovery at the LHC was one of the major breakthroughs of particle physics in the last decades [1,2]. With regard to this boson, there is great experimental interest in the possible measurement of its trilinear self-coupling (λSMHHH) [3], as well as in knowing whether it is a fundamental or composite particle [4]. Any difference between the expected value of the trilinear self-coupling predicted by the standard model (SM) and that of a future measurement of this quantity may indicate a sign of composition or new physics, although the composition or new physics may also arise with the discovery of new particles. In particular, if this boson is composed by new strongly interacting particles, the most discussed signal of a possible dynamical breaking mechanism of the SM gauge symmetry would be the presence of pseudo-Goldstone bosons [5].

Any limit on the Higgs boson trilinear self-coupling, if this is a composite boson, also means a restriction on the dynamics of the interaction that forms such a boson. This occurs because the trilinear coupling is directly proportional to the wave function of the composite state and the number of fermions that form that state. In this work we will compare the trilinear Higgs boson self-coupling computed in the case of technicolor coupled models, showing how the dynamics of the theory is constrained by the experimental data on this quantity.

We review how the dynamics of coupled strongly interacting theories are modified compared to an isolated strong interaction theory. In the sequence, based on the dynamics of these coupled theories, that we assume as QCD and a non-Abelian TC theory coupled by a non-Abelian ETC or GUT, we estimate the order of the trilinear Higgs boson coupling. With the limits on the dynamics (i.e., technifermion self-energy) originated from the comparison with the experimental data, we are able to compute pseudo-Goldstone bosons masses in a very simple approximation. The results indicate that these bosons can be quite massive.

The Lagrangian describing the SM trilinear Higgs boson self-interaction is parameterized as [3]

\[ \mathcal{L}^{SM}_{HHH} = \frac{m_H^2}{2v} H^3, \] (1)

where the SM trilinear coupling with mass dimension is

\[ \lambda^{SM}_{HHH} = \frac{m_H^2}{2v}, \] (2)

whose SM expected value

\[ \lambda^{SM}_{HHH} \equiv \frac{m_H^2}{(2v^2)} = 0.129. \] (3)

The Lagrangian describing the observed trilinear Higgs boson self-coupling can be written as

\[ \mathcal{L}_{HHH} = \kappa \lambda^{SM}_{HHH} v H^3, \] (4)
where
\[ \kappa_\lambda = \lambda_{HHH} \frac{\lambda}{\mu_{HHH}} \]  \hspace{1cm} (5)
where \( \kappa_\lambda \) is the observed coupling modifier of the trilinear Higgs boson self-coupling. Recently the CMS Collaboration reported one constraint on the observed coupling \( \kappa_\lambda \) at 95\% CL [6]
\[-3.3 < \kappa_\lambda < 8.5. \]  \hspace{1cm} (6)
This result can already constrain the dynamics of a composite Higgs boson in the context of coupled technicolor models [7–10], and can also be used to determine limits on the possible masses of pseudo-Goldstone bosons.

Dynamics of technicolor coupled models. – Technicolor coupled models are technicolor (TC) models where QCD and TC theories are embedded into a larger gauge group, such that technifermions and ordinary quarks provide masses to each other [7,8]. In ref. [7] it was verified numerically that two strongly interacting theories when coupled by another interaction, which could be an extended technicolor theory (ETC) or a grand unified theory (GUT), have their self-energies (or dynamics) modified when compared to the self-energy of an isolated strong interaction theory.

As the ETC/unified theory should also mediate the interaction of technileptons and ordinary leptons with quarks and techniquarks, these fermions also acquire smaller masses than their respective strongly interacting partners (i.e., quarks and techniquarks) [8], but as we shall see technileptons also turn out to be quite massive.

An isolated strong non-Abelian interaction is known to generate a dynamical fermion mass indicated by \( \mu \), which is of the order of \( \Lambda \), that is the characteristic scale of the strong interaction. The dynamical fermion self-energy of this strong interaction theory has the following infrared behavior (IR) [11,12]:
\[ \Sigma(p^2 \to 0) \propto \mu, \]  \hspace{1cm} (7)
and the ultraviolet behavior (UV) is [13]
\[ \Sigma(p^2 \to \infty) \propto \mu \left( \frac{\mu^2}{p^2} \right). \]  \hspace{1cm} (8)

We can now consider two coupled strong interactions, QCD and TC, through an ETC or GUT theory, where the Schwinger-Dyson equations (SDE) for the coupled system is depicted in fig. 1. The IR behavior of both theories is not changed from the one of eq. (7), where now \( \mu \) for technifermions will be indicated by \( \mu_{tc} \) and for quarks by \( \mu_c \), respectively, the TC and QCD dynamical fermion masses. However, as shown in refs. [7,8], the effect of QCD and TC to technifermions and quarks is to provide “bare” masses to each other. We stress this effect, that is promoted by the second diagram in the SDE of fig. 1 for technifermions. Actually, the effect of this diagram is exactly to change the boundary conditions of the SDE in the differential form, just as it would have if we had introduced a bare mass [9]. In this case the UV behavior of the dynamical self-energy with a “bare” mass \( \mu_0 \) is given by [13]
\[ \Sigma(p^2 \to \infty) \propto \mu_0 \left[ \ln \left( \frac{p^2}{\Lambda^2} \right) \right]^{-\gamma}, \]  \hspace{1cm} (9)
where \( \gamma \) for a \( SU(N) \) non-Abelian gauge theory with fermions in the fundamental representation is
\[ \gamma = \frac{3(N^2 - 1)}{2N(11N - 2n_f)}, \]  \hspace{1cm} (10)
and \( \Lambda \) is the characteristic scale of the theory. The logarithmic behavior of eq. (9) is connected to the running of the non-Abelian gauge coupling constant.

Going back to the coupled SDE system we can notice that the IR behavior of the technifermion self-energy is still proportional to \( \mu_{tc} \), as long as we assume no other new strong interaction above the TC scale, and the technifermion bare masses generated by QCD are very small when compared to \( \mu_{tc} \). The actual TC self-energy UV behavior is a combination of a \( 1/p^2 \) component typical of an isolated TC theory, with the UV logarithmic behavior given by eq. (9) as soon as we have momenta larger than \( \mu_{tc} \), characterized by the domination of the QCD diagram to the dynamical technifermion mass. Therefore, the full TC dynamical self-energy can be roughly described by
\[ \Sigma_{tc}(p^2) \approx \mu_{tc} \left[ 1 + \delta_{1} \ln \left( \frac{p^2 + \mu_{tc}^2}{\mu_{tc}^2} \right) \right]^{-\delta_{2}}, \]  \hspace{1cm} (11)
Equation (11) is the simplest interpolation of the numerical result of ref. [7], describing the infrared (IR) dynamical mass equal to \( \mu_{tc} \) (also proportional to the technicolor characteristic scale), and a logarithmic decreasing function of the momentum in the ultraviolet (UV) region originated by another (QCD, for instance) strong interaction. It is clear that in the IR region the logarithmic term of eq. (11) is negligible, and as the momentum increases above \( \mu_{tc} \), the logarithmic term controls the UV behavior.

It is worth remembering that at leading order the fermionic SDE has the same behavior of the scalar Bethe-Salpeter (BS) equation, which was explicitly shown in ref. [14]. However, the full BS amplitude is subjected to a
normalization condition, which, considering eq. (11), imposes the following constraint on \( \delta_2 \) [13,15,16]:

\[
\delta_2 > \frac{1}{2}. \tag{12}
\]

On the other hand, just assuming that \( \Sigma(p^2 = \mu_{tc}^2) \approx \mu_{tc} \), and that the self-energy starts decreasing smoothly for \( p^2 > \mu_{tc}^2 \), we can assume

\[
\delta_1 \leq 1. \tag{13}
\]

This value is also consistent with the expansion of a dynamical self-energy (e.g., eq. (9)) at large momentum, where \( \delta_1 \) would be proportional to the running gauge coupling constant. Ultimately \( \delta_1 \) may have contributions proportional to \([bg^2]_0\) where \( b \) and \( g \) are, respectively, the first coefficient of the \( \beta \) function and the coupling constant of the strong interaction (\( si \)) that provides the “bare” mass to the technifermions (see the appendix of ref. [17] to verify the determination of this quantity in the case of an isolated theory).

A consequence of a self-energy like the one of eq. (11) is that TC coupled models must incorporate a family symmetry, in such a way that technifermions coupled at leading order only to the third ordinary fermion family, whereas the first fermionic family will be coupled at leading order only to QCD [7,9,10], i.e., the mass hierarchy between different ordinary fermionic generations can only be obtained through the introduction of a family (or horizontal) symmetry, as described in refs. [7,9,10]. We will not touch these aspects here, and in the following we just verify consequences of eq. (11) for the trilinear Higgs boson self-coupling and pseudo-Goldstone masses. The result will be compared with the recent experimental constraint on the trilinear Higgs boson coupling [6].

**Trilinear coupling of a composite Higgs boson.** — The trilinear composite scalar coupling is shown in fig. 2, where the double lines represent the composite Higgs boson, that is coupled to fermions (single line) through the dark (blue) blobs. In the SM the composite scalar boson coupling to fermions (the dark blob) can be determined using Ward identities to be [18]

\[
G^\alpha(p + q, p) = -ig_{\alpha W} \left[ \tau^\alpha \Sigma(p) P_R - \Sigma(p + q) \tau^\alpha P_L \right],
\]

where \( P_{R,L} = \frac{1}{2}(1 \pm \gamma_5) \), \( \tau^\alpha \) is a \( SU(2) \) generator, and \( \Sigma \) is a matrix of fermionic self-energies in weak-isodoublet space. At large momenta eq. (14) is quite well approximated by \( G(p, p) \), and in all situations in which we are interested \( \Sigma(p+q) \approx \Sigma(p) \). Therefore, the coupling given by eq. (14) that is dominated by the large momentum running in the loop of fig. 2 is reduced to

\[
\lambda_{Hff} \equiv G(p, p) \sim -\frac{g_W}{2M_W} \Sigma(p^2). \tag{15}
\]

**Fig. 2:** The trilinear composite scalar coupling: the dark (blue) blobs in this figure represent the coupling of composite Higgs bosons (H) to fermions (f). The double lines represent the composite Higgs bosons. The full diagram is the main contribution to the trilinear Higgs boson self-coupling.

The loop calculation of fig. 2, considering eq. (15) and \( n_F \) technifermions running in that loop, is given by [19]

\[
\lambda_{HHH} = \frac{3g_{W}^3}{64\pi^2} \left( \frac{3n_F}{M_W^4} \right) \int_0^\infty \frac{\Sigma_{tc}(p^2) p^4 dp^2}{(p^2 + \Sigma_{tc}(p^2))^3}. \tag{16}
\]

Note that, apart from a dependence on \( n_F \), the trilinear coupling is a function of the variables \( \delta_1 \) and \( \delta_2 \) shown in eq. (11). Of course, we do also have a dependence on the scale \( \mu_{tc} \), but we cannot forget another constraint on the technicolor dynamics that comes from

\[
M_W = \frac{1}{2} g_W F_\pi, \tag{17}
\]

where \( F_\pi \) is the technipion decay constant, \( g_W \) is the electroweak coupling constant, and \( F_\pi \) can be calculated through [20]

\[
F_\pi = \frac{N}{(2\pi)^2} \int_0^\infty dp^2 \left[ -\frac{\Sigma_{tc}(p^2) - \frac{1}{2} \frac{\Sigma_{tc}(p^2)}{dp} \Sigma_{tc}(p^2)}{(p^2 + \Sigma_{tc}(p^2))^2} \right]. \tag{18}
\]

Therefore, once the number of technicolors (\( N \)) and technifermions (\( n_F = 2n_q \)) are specified (where \( n_q \) is the number of weak doublets), the dynamics of the technicolor theory (i.e., \( \delta_1 \) and \( \delta_2 \)) can be constrained using eqs. (6), (12), (13), (16), (17) and (18).

Equation (16) was already calculated in ref. [19] with a different approximation for eq. (11). In that case the self-energy was based on a possible walking behavior [21], where a certain amount of the \( 1/p^2 \) behavior for this quantity was allowed. Moreover the parameter \( \delta_1 \) was chosen in an arbitrary way as \( bg^2 \), which in a coupled TC scenario does not make sense, due to the many corrections that may contribute to the \( \delta \) parameters.

**Limit on the trilinear coupling.** — In fig. 3 we present the 3D plot of the technipion decay constant \( F_\pi \) given by eq. (18). The plot was generated for \( F_\pi = v/\sqrt{3} \), with \( v = \sqrt{\frac{4\pi}{3}F_\pi} = 246 \text{ GeV} \) assuming \( n_d = 3(n_F = 6) \) and the following range of technicolor dynamical masses

\[
0.5 \text{ TeV} \leq \mu_{tc} \leq 2 \text{ TeV}.
\]

The dependence of the technipion decay constant on \( \mu_{tc} \) is not appreciable. However, there is a large parameter space for the quantities \( \delta_1, \delta_2, N \) that satisfy the experimental \( F_\pi \) value. The
Fig. 3: 3-dimensional plot of the technipion decay constant ($F_\pi$) given by eq. (18). This quantity is a function of ($\delta_1$, $\delta_2$, $N$), and we considered $\mu_{tc}$ in the interval $0.5 \text{ TeV} \leq \mu_{tc} \leq 2 \text{ TeV}$. The yellow region is the allowed one.

Fig. 4: The region of allowed ($\delta_1$, $\delta_2$) values obtained for the coupling modifier $\kappa_\lambda$. In this figure we consider $\mu_{tc} = 1 \text{ TeV}$ and $n_f = 2$, furthermore we assume $N = 2$ which allows the largest region of parameters bounded by eq. (18). The expected SM value is also indicated by a continuous line.

main relevant fact is the variation of this quantity with $N$ (the number associated to the technicolor gauge group). For instance, the figure above illustrates that in the region where $N \leq SU(5)_{tc}$, we still have a large volume allowed for $\delta_1$ and $\delta_2$.

Considering eqs. (5), (12), (13), (16) and (17), in fig. 4 we present the behavior obtained for eq. (5), calculated assuming the dynamics prescribed in eq. (11), $\mu_{tc} = 1 \text{ TeV}$ and $n_f = 2$. We also include in the figure the upper limit on the observed coupling modifier ($\kappa_\lambda$) of the trilinear Higgs boson self-coupling of ref. [6], which is indicated by the dot-dashed black line.

In the filled region below the dotted line the ($\delta_1$, $\delta_2$) parameter space allowed by the experimental constraint on $\kappa_\lambda$ is shown, which in this case corresponds to $\delta_1 \geq 0.074$ and $\delta_2 \geq 0.53$. In fig. 5 we consider the case where $n_f = 4$, which is a little bit more restrictive than the previous one.

Fig. 5: The allowed region of ($\delta_1$, $\delta_2$) values obtained for the coupling modifier $\kappa_\lambda$. We consider again $\mu_{tc} = 1 \text{ TeV}$, $N = 2$ and now we set $n_f = 4$.

Fig. 6: The allowed region of ($\delta_1$, $\delta_2$) values obtained for the coupling modifier $\kappa_\lambda$, with $\mu_{tc} = 1 \text{ TeV}$, $N = 2$ and setting $n_f = 6$.

corrections to the coupled non-linear SDE system may modify this quantity.

The CMS upper bound on $\kappa_\lambda$ is indicated in the above figures by a dot-dashed line and is already constraining the dynamics of composite coupled models for the Higgs boson.

We do not expect major changes in our results in the case of technifermions in higher-dimensional representations, because the parameters $\delta_1$ and $\delta_2$ are proportional to the product of the Casimir operator of a given representation times the TC coupling constant, and according to the most attractive channel (MAC) hypothesis the TC chiral symmetry breaking occurs when this product is of $O(1)$ no matter the representation.

**Pseudo-Goldstone boson masses.** – In technicolor models it is usual to have a large number of pseudo-Goldstone bosons (or technipions) resulting from the chiral symmetry breaking of the technicolor theory. In coupled models like the ones discussed in refs. [8] and [10], these technipions, besides the ones absorbed by the $W$’s and $Z$ gauge bosons, will be of the following type:

a) charged and neutral color singlets, for example,

$$\bar{U}iD^i - 3\bar{N}E,$$

$$\bar{U}iU^i - \bar{D}iD^i - 3(\bar{N}N - \bar{E}E);$$
The above results for \( m_N \) follow from the upper limit on \( \delta \) reported by CMS and \( \delta_1 \) and \( \delta_2 \) values presented in table 1. These are the \( m_N \) masses obtained in the case of \( (2 \leq n_F \leq 6) \). However, note that for a realistic ETC model, where new interactions including \( N \) and ETC bosons are taken into account, we shall obtain even higher \( m_N \) masses. It is important to stress that all other corrections to colored or charged technifermion masses are larger than this one due to the larger charges and coupling constants (basically changing \( \alpha_w \) by \( \alpha_s \) and \( M_Z \) by a dynamical gluon mass in eq. (19)).

As neutral technifermions may have masses heavier than 100 GeV we can determine the mass of the lightest pseudo-Goldstone composed with this neutral particle (for instance, \( \Pi^N \rightarrow N\gamma_5\tau^*N \), where \( \tau \) represents electroweak indexes). This neutral pseudo-Goldstone boson will obtain a mass that may be computed with the help of the Gell-Mann-Oakes-Renner relation

\[
m_H^N \approx \sqrt{m_{HN} \langle \bar{N}N \rangle_{2F_R^H}},
\]

where \( \langle \bar{N}N \rangle \approx (\mu_{tc})^3 \text{GeV}^3 \) is the TC condensate. However, we may follow a very simple hypothesis, where the pseudo-Goldstone masses are determined just as the addition of the current masses of their constituents [22,23], which was shown to be satisfactory for QCD phenomenology. In this case, supposing that the neutral technipion (\( \Pi^N \)) is composed just by two \( N \) particles we have

\[
m_{HN} \approx 200–460 \text{ GeV}.
\]

Notice that we assumed that such neutral boson is solely composed by \( N \) technifermions. In general the composition is more complex according to the symmetries of the TC group, and this neutral boson will also be composed by charged and colored particles increasing the above estimate.

Charged and colored technifermions will not only have larger masses than the neutral technifermion, but also more radiative corrections to their masses, and we can...
expect even larger masses for colored and charged pseudo-Goldstone bosons. For instance, following the same hypothesis, the colored triplet and colored octet technipions $\Pi^{(3)}$ and $\Pi^{(8)}$ will obtain masses

$$m_{\Pi^{(3)}} \approx m_U + m_E,$$

(21)

where $m_U$ and $m_E$ are the current masses of the $U$ and $E$ techniquarks. Along the same proposal a simple estimate of the colored octet technipion of item c) would be

$$m_{\Pi^{(8)}} \approx 2m_U.$$

(22)

Changing the weak coupling by the QCD one in the calculation of the $N$ technifermion mass in order to estimate the $U$ and $E$ masses, we can predict $\Pi^{(3)}$ and $\Pi^{(8)}$ masses certainly to be above 400 GeV, only with the naive assumption the the strong coupling constant is at least twice the value of the weak one at the TC scale.

Conclusions. – In technicolor coupled models, where TC and QCD are embedded into a large gauge theory, technifermions and ordinary fermions provide bare masses to each other. In this case the self-energy dynamics of technifermions can be described by eq. (11), as verified in refs. [7,9].

With the technifermion self-energy given by eq. (11) we have computed the trilinear self-coupling of a composite Higgs boson. This calculation is compared to the recent limits on this coupling obtained by the CMS experiment. The comparison with the experimental data can constrain the trilinear coupling and consequently the dynamics of the TC theory. Once the TC scale ($\mu_{tc}$) is specified we can obtain limits on the variables $\delta_1$ and $\delta_2$ of eq. (11) describing the TC self-energy. Our main result is that the recent experimental data about the trilinear Higgs boson self-coupling is already imposing limits on the TC dynamics, although it is still far from the expected SM value for this quantity. The Higgs boson coupling has been determined with high precision in the case of heavy fermions, and it would be interesting to verify how the composite wave function (i.e., self-energy) discussed here is affected by these experimental limits, although in this case the calculation is much more dependent on the ETC/GUT masses and horizontal symmetries necessary for this type of model.

After obtaining a constraint on the parameters of the TC self-energy for one specific TC scale and number of technifermions we can calculate the technifermion bare masses. With the values of table 1, a technicolor mass scale around 1 TeV, and assuming the simple hypothesis of refs. [22,23], where the pseudo-Goldstone boson masses are roughly given by the sum of the particle masses that participate in the boson composition, we can estimate that pseudo-Goldstone boson masses. If these models are realized in Nature, the pseudo-Goldstone boson masses may be at the order or above 0.5 TeV.

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This research was partially supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under grants Nos. 303588/2018-7 (AAN) and 310015/2020-0 (AD).

Data availability statement: The data that support the findings of this study are available upon reasonable request from the authors.

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