The Question of Expressiveness in the Generation of Referring Expressions

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Abstract

We study the problem of generating referring expressions modulo different notions of expressive power. We define the notion of \( L \)-referring expression, for a formal language \( L \) equipped with a semantics in terms of relational models. We show that the approach is independent of the particular algorithm used to generate the referring expression by providing examples using the frameworks of (Areces et al., 2008) and (Krahmer et al., 2003). We provide some new complexity bounds, discuss the issue of the length of the generated descriptions, and propose ways in which the two approaches can be combined.

1 Generating referring expressions

The generation of referring expressions (GRE) – given a context and an element in that context generate a grammatically correct expression in a given natural language that uniquely represents the element – is a basic task in natural language generation, and one of active research (see (Dale, 1989; Dale and Haddock, 1991; Dale and Reiter, 1995; Stone, 2000; van Deemter, 2002) among others). Most of the work in this area is focused on the content determination problem (i.e., finding a collection of properties that singles out the target object from the remaining objects in the context) and leaves the actual realization (i.e., expressing a given content as a grammatically correct expression) to standard techniques\(^1\).

However, there is yet no general agreement on the basic representation of both the input and the output to the problem; this is handled in a rather ad-hoc way by each new proposal instead.

\(^1\)For exceptions to this practice see, e.g. (Horacek, 1997; Stone and Webber, 1998)

(Krahmer et al., 2003) make the case for the use of labeled directed graphs in the context of this problem: graphs are abstract enough to express a large number of domains and there are many attractive and well-known algorithms for dealing with this type of structures. Indeed, labeled directed graphs are nothing other than an alternative representation of relational models, used to provide semantics for formal languages like first and higher-order logics, modal logics, etc. Even valuations, the basic models of propositional logic, can be seen as one point labeled graphs. It is not surprising then that they are well suited to the task.

In this article, we side with (Krahmer et al., 2003) and use labeled graphs as input, but we argue that an important notion has been left out when making this decision. Exactly because of their generality graphs do not define, by themselves, a unique notion of sameness. When do we say that two nodes in the graphs can or cannot be referred uniquely in terms of their properties? This question only makes sense once we fix a certain level of expressiveness which determines when two graphs, or two elements in the same graph, are equivalent.

Investigating the GRE problem in terms of different notions of expressiveness is the main goal of this paper. In §2, we will show alternative (but equivalent) ways in which different degrees of expressiveness can be defined, and discuss how choosing the adequate expressiveness has an impact on the number of instances of the GRE problem that have a solution (less expressive logics can distinguish fewer instances); the computational complexity of the GRE algorithms involved; and the complexity of the surface realization problem.

We maintain that this perspective is independent of the particular GRE algorithm being used. Our work fits naturally with the approach of (Areces et al., 2008) as we show in §3, where we also answer an open question concerning the complexity
of the GRE problem for the language $EL$. In §4 we turn to the subgraph based algorithm of (Krahmer et al., 2003), and show how to generalize it for other notions of sameness. In section §5 we show how one can combine the approaches of the previous two sections and in section §6 we discuss the size of the referring expressions relative to the expressiveness employed.

2 Measuring expressive power

Relational structures are notably appropriate for representing situations or scenes. A relational structure (also called “relational model”) is a nonempty set of objects –the domain– together with a collection of relations, each with a fixed arity.

Formally, assume a fixed and finite (but otherwise arbitrary) vocabulary of $n$-ary relation symbols. Define a relational model $M$ as a tuple $\langle \Delta, \| \cdot \| \rangle$ where $\Delta$ is a nonempty set, and $\| \cdot \|$ is a suitable interpretation function, that is, $\| r \| \subseteq \Delta^n$ for every $n$-ary relation symbol $r$. We say that $M$ is finite whenever $\Delta$ is finite. The size of a model $M$ is the sum $\#\Delta + \#\| \cdot \|$, where $\#\Delta$ is the cardinality of $\Delta$ and $\#\| \cdot \|$ is the sum of the sizes of all relations in $\| \cdot \|$.

Figure 1 shows the representation of a scene with the relational model $S = (\Delta, \| \cdot \|)$:

- $\|\text{dog}\| = \{a, b, d, e\}$
- $\|\text{cat}\| = \{c, e\}$
- $\|\text{beagle}\| = \{d\}$
- $\|\text{sniffs}\| = \{(a, a), (b, a), (c, b), (d, e), (e, d)\}$

Intuitively, $a$, $b$, and $d$ are dogs, while $c$ and $e$ are cats; $d$ is a small beagle; $b$ and $c$ are also small. We read $\text{sniffs}(d, e)$ as “$d$ is sniffing $e$”.

Logical languages are fitting for the task of (formally) describing elements of a relational structure. Consider, e.g., the classical language of first-order logic (with equality), $FL$, given by:

$$\begin{align*}
\top & | x_i \neq x_j | r(\bar{x}) | \neg \gamma | \gamma \land \gamma' | \exists x_i, \gamma
\end{align*}$$

where $\gamma, \gamma' \in FL$, $r$ is an $n$-ary relation symbol and $\bar{x}$ is an $n$-tuple of variables. As usual, $\gamma \lor \gamma'$ and $\forall x. \gamma$ are short for $\neg(\neg \gamma \land \neg \gamma')$ and $\neg \exists x. \neg \gamma$, respectively. Formulas of the form $\top$, $x_i \neq x_j$ and $r(\bar{x})$ are called atoms.\(^3\) Given a relational model $M = (\Delta, \| \cdot \|)$ and a formula $\gamma$ with free variables\(^4\) among $x_1 \ldots x_n$, we inductively define the extension or interpretation of $\gamma$ as the set of $n$-tuples $\|\gamma\|^n \subseteq \Delta^n$ that satisfy:

$$\begin{align*}
\|\top\|^n & = \Delta^n \\
\|x_i \neq x_j\|^n & = \{\bar{a} \mid \bar{a} \in \Delta^n, a_i \neq a_j\} \\
\|r(\bar{x}_1 \ldots \bar{x}_k)\|^n & = \{\bar{a} \mid \bar{a} \in \Delta^n, (a_1, \ldots, a_k) \in \| r \|\} \\
\|\neg \delta\|^n & = \Delta^n \setminus \|\delta\|^n \\
\|\delta \land \theta\|^n & = \|\delta\|^n \cap \|\theta\|^n \\
\|\exists x_i \delta\|^n & = \{\bar{a} \mid \bar{a} \in \Delta^n \in \|\delta'\|^{n+1}\}
\end{align*}$$

where $1 \leq i, j, l, \ldots, k \leq n$, $\bar{a} = (a_1 \ldots a_n)$, $\bar{a}_{n+1} = (a_{n+1} \ldots a_{n+1})$ and $\delta'$ is obtained by replacing all occurrences of $x_i$ in $\delta$ by $x_{n+1}$. When the cardinality of the tuples involved is known from context we will just write $\|\gamma\|\,$ instead of $\|\gamma\|^n$.

With a language syntax and semantics in place, we can now formally define the problem of $L$-GRE for a target set of elements $T$ (we slightly adapt the definition in (Areces et al., 2008)):

| $L$-GRE Problem |
|------------------|
| **Input:** a model $M = (\Delta, \| \cdot \|)$ and a target set $T \subseteq \Delta$. |
| **Output:** a formula $\varphi \in L$ such that $\|\varphi\| = T$, or $\bot$ if such a formula does not exists. |

In case the output is not $\bot$, we say that $\varphi$ is an $L$-referring expression ($L$-RE) for $M$.

2.1 Choosing the appropriate language

Given a model $M$, there will be an infinite number of formulas that uniquely describe a target (e.g., if $\varphi$ describes a target $T$, then $\varphi \land \varphi$ trivially describes $T$ as well; even formulas which are not logically equivalent might have the same interpretation once the model is fixed). Despite having the same interpretation in $M$, they may be quite different with respect to other parameters.

To start with, and as it is well known in the automated text generation community, different realizations of the same content might result in expressions more or less appropriate. Although, as

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\(^2\)We do not consider constants or functions as they can be represented as relations of adequate arity.

\(^3\)Notice that we include the inequality symbol $\neq$ as primitive. Equality can be defined using negation.

\(^4\)W.l.o.g. we assume that no variable appears both free and bound, that no variable is bound twice, and that the index of bound variables in a formula increases from left to right.

\(^5\)Similarly we can use formulas with two free variables to describe binary relations, etc. Throughout this paper though, we only discuss the GRE for single elements.
we mentioned in the introduction, we will only address the content determination (and not the surface realization) part of the GRE problem, generating content using languages with different expressive power can have an impact in the posterior surface generation step.

Let us consider again the scene in Figure 1. Formulas $\gamma_1$–$\gamma_4$ shown in Table 1 are all such that $\gamma_i$ uniquely describe $b$ in model $S$.

Arguably, $\gamma_1$ can be easily realized as “the small dog that sniffs a dog”. Syntactically, $\gamma_1$ is characterized as a positive, conjunctive, existential formula (i.e., it contains no negation and uses only conjunction and existential quantification). Expressions with these characteristics are, by large, the most commonly found in corpora as those compiled in (Viethen and Dale, 2006; van Deemter et al., 2006; Dale and Viethen, 2009). $\gamma_2$ on the other hand contains negation, disjunction and universal quantification and could be realized as “the small dog that only sniffs things that are not cats” which sounds very unnatural. Even a small change in the form of $\gamma_2$ turns it more palatable: rewrite it using only $\exists$, $\neg$, and $\land$ to obtain “the small dog that is not sniffing a dog”. Similarly, formulas $\gamma_3$ and $\gamma_4$ seem computationally harder to realize than $\gamma_1$: $\gamma_3$ because it contains an inequality (“the dog sniffing another dog”) and $\gamma_4$ because the quantified object appears in the first argument position in the binary relation (“the dog that is sniffed by a small cat”).

Summing up, even without taking into account fundamental linguistics aspects that will make certain realization preferable — e.g., saliency, the cognitive capacity of the hearer (can she recognize a beagle from another kind of dog?), etc. — we can ensure during content determination certain properties of the generated referring expression.

Concretely, let $\mathcal{FL}^-$ be the fragment of $\mathcal{FL}$-formulas where the operator $\neg$ does not occur. By restricting content determination to $\mathcal{FL}^-$, we ensure that formulas like $\gamma_3$ will not be generated. If we also (or, alternatively) ban $\approx$ from the language, $\gamma_3$ is precluded. And we need not restrict ourselves to explicit fragments of first-order logic: many logical languages are known to be expressively equivalent to fragments of first-order logic. For example, the language of the description logic $\mathcal{ALC}$ (Baader et al., 2003), given by:

$$\mathcal{T} \mid p \mid \neg \gamma \mid \gamma \land \gamma' \mid \exists r. \gamma$$

(where $\gamma, \gamma' \in \mathcal{ALC}$) corresponds to a syntactic fragment of $\mathcal{FL}$ without $\approx$, as shown by the standard translation to first-order logic $\tau_x$:

$$\tau_{x_i}(T) = T$$
$$\tau_{x_i}(p) = p(x_i)$$
$$\tau_{x_i}(\neg \gamma) = \neg \tau_{x_i}(\gamma)$$
$$\tau_{x_i}(\gamma_1 \land \gamma_2) = \tau_{x_i}(\gamma_1) \land \tau_{x_i}(\gamma_2)$$
$$\tau_{x_i}(\exists r. \gamma) = \exists r(x_i, x_{i+1}) \land \tau_{x_{i+1}}(\gamma)$$

Hence, by restricting content generation to $\mathcal{ALC}$ we would avoid formulas like $\gamma_3$ (no equality) and $\gamma_4$ (quantified element appears always in second argument position).

(Areces et al., 2008) discuss generation in terms of different description logics like $\mathcal{ALC}$ and $\mathcal{EL}$ ($\mathcal{ALC}$ without negation). In this article, we will extend the results in that paper, considering for instance $\mathcal{EL}^+$ ($\mathcal{ALC}$ with negation allowed only in front of unary relations) but, more generally, we argue that the primary question is not whether one should use one or other (description) logic for content generation, but rather which are the semantic differences one cares about. This determines the required logical formalism which, in turn, impacts both content determination and surface realization.

We have mentioned several logic languages (and there are many more alternatives like allowing disjunctions, counting quantifiers, etc.). Each language can be seen as a compromise between expressiveness, realizability and computational complexity. Therefore, the appropriate selection for a particular GRE task should depend on the actual context. Moreover, as we will see, the move from one logical language to another impacts not only on the shape of formulas that can be generated but also on the computational complexity of the generation problem, and on its success, i.e., when it will be possible to uniquely identify a given target.

2.2 Defining sameness

For any given logical language $\mathcal{L}$, we say that $u$ is distinguishable (in $\mathcal{L}$) from $v$ whenever there ex-
exists an $L$-formula $\gamma$ such that $u$ makes $\gamma$ true while $v$ makes it false. Formally, let $M_1 = \langle \Delta_1, \cdot, \cdot \rangle$ and $M_2 = \langle \Delta_2, \cdot, \cdot \rangle$ be two relational models with $u \in \Delta_1$ and $v \in \Delta_2$; we follow the terminology of (Areces et al., 2008) and say that “$u$ is $L$-similar to $v$” (notation $u \sim L v$) whenever $u \in \gamma \Delta_1$ implies $v \in \gamma \Delta_2$, for every $\gamma \in L$. $L$-similarity is reflexive for all $L$, and symmetric for languages that contain negation.

Observe that $L$-similarity captures the notion of ‘indistinguishability’ (in $L$). One can take $M_1$ and $M_2$ to be the same model and in that case, if $u \sim L v$ for $u \neq v$, the $L$-content determination problem for $u$ will not succeed (since for every $\gamma \in L$, $\gamma \Delta_1 \neq \{u\}$).

Fortunately, one need not consider infinitely many $L$-formulas to decide whether $u$ is $L$-similar to $v$. We can reinterpret $L$-similarity in terms of standard model-theoretic notions like isomorphisms or bisimulations which describe structural properties of the model, instead. We will use the term $L$-simulation to refer to the suitable notion for $L$; in the case of the languages we are considering they can be defined in a modular way. Given a relation $\sim L \subseteq \Delta_1 \times \Delta_2$, it may or may not possess the properties we call $\text{ATOM}_{L/R}, \text{REL}_{L/R}, \text{INJ}_{L/R}$ (see below), Table 2 defines various $L$-simulations in terms of these.

$\text{ATOM}_L$: If $u \sim L v$, then $u \in [p]_1 \Rightarrow u \in [p]_2$.

$\text{ATOM}_R$: If $u \sim L v$, then $v \in [p]_2 \Rightarrow u \in [p]_1$.

$\text{REL}_L$: If $u \sim L v$ and $(u_1, v_1) \in [p]_1$, then $v_1 \sim L v_2$ and $(u_2, v_2) \in [p]_2$, for some $v_2$.

$\text{REL}_R$: If $u \sim L v$ and $(u_2, v_2) \in [p]_2$, then $u_1 \sim L v_1$ and $(u_1, v_1) \in [p]_1$, for some $v_1$.

$\text{INJ}_L$: $\sim L : \Delta_1 \rightarrow \Delta_2$ is an injective function.

$\text{INJ}_R$: $\sim^{-1} : \Delta_2 \rightarrow \Delta_1$ is an injective function.

The following is a fundamental model-theoretic result (Ebbinghaus et al., 1996; Kurtonina and de Rijke, 1999; Blackburn et al., 2001):

**Theorem 1.** If $M_1$ and $M_2$ are finite models, $u \in \Delta_1$ and $v \in \Delta_2$, then $u \sim L v$ if $u \sim L v$.

The right to left implication does not hold in general on infinite models. Notice that $\sim L$ corresponds to relational model isomorphism while $\sim L$ corresponds to the notion of bisimulation.

| $L$       | $u \sim L v$ if $u \sim L v$ for a relation $\sim$ that satisfies: |
|-----------|---------------------------------------------------------------|
| $\mathcal{FL}$ | $\text{ATOM}_L \text{ATOM}_R \text{REL}_L \text{REL}_R \text{INJ}_L \text{INJ}_R$ |
| $\mathcal{FL}^-$ | $\text{ATOM}_L \text{REL}_L \text{INJ}_L$ |
| $\mathcal{ACL}$ | $\text{ATOM}_L \text{ATOM}_R \text{REL}_L \text{REL}_R$ |
| $\mathcal{EL}$ | $\text{ATOM}_L \text{REL}_L \text{INJ}_L$ |
| $\mathcal{EL}^+$ | $\text{ATOM}_L \text{ATOM}_R \text{REL}_L$ |

Table 2: Simulations for various logics.

$L$-simulations allow us to determine, in an effective way, when an object is indistinguishable from another in a given model with respect to $L$.

For example, it is easy to see that $a \sim L b$ in the model of Figure 1 (simply verify that the $\sim$ such that $a \sim b$ and $x \sim x$ (for $x \in \Delta$) satisfies $\text{ATOM}_L$ and $\text{REL}_L$). Using Theorem 1 we conclude that, intuitively, no $\mathcal{EL}$-formula can distinguish “a dog sniffing itself” from “a dog sniffing (another) dog sniffing itself”. Similarly, no $\mathcal{FL}^-$ formula will distinguish two $\mathcal{FL}^-$-similar (isomorphic) objects.

There are well-known algorithms to compute certain kinds of $L$-simulations (Hopcroft, 1971; Paige and Tarjan, 1987; Henzinger et al., 1995; Dovier et al., 2004). We will discuss this in more detail in the next section. While polynomial time algorithms for many languages like $\mathcal{ACL}$, $\mathcal{ACL}$ with inverse relations, $\mathcal{EL}^+$ and $\mathcal{EL}$-simulation, etc. can be obtained, no polynomial time algorithms for $\mathcal{FL}$ and $\mathcal{FL}^-$-simulation are known (actually, even their actual complexity class is not known (Garey and Johnson, 1979)).

### 3 GRE via simulator sets

Given a model $M = \langle \Delta, \cdot, \cdot \rangle$, Theorem 1 tells us that if two distinct elements $u$ and $v$ in $\Delta$ are such that $u \sim L v$ then every $L$-formula that is true at $u$ is also true at $v$. Hence there is no formula in $L$ that can uniquely refer to $u$. From this perspective, knowing whether the model contains an element that is $L$-similar but distinct from $u$ is equivalent to that of whether there exists an $L$-RE for $u$.

We begin by considering this task. Assume fixed a language $L$ and a model $M$. Suppose we want to refer to an element $u$ in the domain of $M$. We would like to compute the simulator set of $u$ defined as $\text{sim}_L(v) = \{u \mid v \sim L u\}$. If $\text{sim}_L(v)$ is not the singleton $\{v\}$, we cannot $L$-refer to $v$.

(Henzinger et al., 1995) propose an algorithm to compute the set $\text{sim}_{L}^{E_+}(v)$ for each element $v$ of a given finite model $M = \langle \Delta, \cdot, \cdot \rangle$ in time

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7For $\gamma \in \mathcal{ACL}$ and its sublanguages, $\|\gamma\| = \|\tau_1(\gamma)\|$.

8For the rest of the article, we will focus on relational models with only unary and binary relational symbols. These are the usual models of interest when describing scenes as the one presented in Figure 1. Accommodating relations of higher arity poses only notational problems.

9Actually the algorithm proposed in (Henzinger et al., 1995) is over labeled graphs, but it can be adapted to compute $\text{sim}_{L}^{E_+}$ by appropriately labeling the model.
\(O(\#\Delta \times \#\| \cdot \|)\). Intuitively, this algorithm defines \(S(v)\) as the set of candidates for simulating \(v\) and successively refines it by removing those elements which do not simulate \(v\). At the end, \(S(v) = sim_{\mathcal{EL}^+}(v)\).

The algorithm can be adapted to compute \(sim_{\mathcal{L}}\) for other \(\mathcal{L}\). In particular, we can use it to compute \(sim_{\mathcal{EL}}\) in polynomial time which will give us the basic algorithm for establishing an upper bound to the complexity of the \(\mathcal{EL}\)-GRE problem (this will answer an open question of (Areces et al., 2008)).

Let us first introduce some notation. We fix \(\mathcal{P}\) as the set of all unary relations of the language \(\mathcal{EL}\). For \(v \in \Delta\) let \(P(v) = \{p \in \mathcal{P} \mid v \in \|p\|\}\) and let \(suc_{r}(v) = \{u \in \Delta \mid (v, u) \in [r]\}\) for any binary relation \(r\) present in the language.

The pseudo-code is shown in Algorithm 1. We initialize \(S\) with the set of all elements \(u \in \Delta\) such that \(P(v) \subseteq P(u)\), i.e., the set of all elements satisfying at least the same unary relations as \(v\) (this guarantees that \(ATOM_{\mathcal{L}}\) holds). At each step, if there are three elements \(u, v, w\) such that for some relation \(r, (u, v) \in [r]\), \(w \in S(u)\) (i.e., \(w\) is a candidate to simulate \(u\)) but \(suc_r(w) \cap S(v)\) (there is no element \(w'\) such that \((w, w') \in [r]\) and \(w' \in S(v)\)) then clearly condition \(REL_{\mathcal{L}}\) is not satisfied under the supposition that \(sim_{\mathcal{EL}} = S\). \(S\) is ‘too big’ because \(w\) cannot simulate \(u\). Hence \(w\) is removed from \(S\).

Algorithm 1: Computing \(\mathcal{EL}\)-similarity

input : a finite model \(\mathcal{M} = (\Delta, \| \cdot \|)\)
output: \(\forall v \in \Delta\), the simulator set \(sim_{\mathcal{EL}}(v) = S(v)\)

foreach \(v \in \Delta\) do
\[
S(v) := \{u \in \Delta \mid P(v) \subseteq P(u)\}
\]
while \(\exists r, u, v, w: (u, v) \in [r], w \in S(u), suc_r(w) \cap S(v) = \emptyset\) do
\[
S(u) := S(u) \setminus \{w\}
\]

Of course, Algorithm 1 will only tell us whether a referring expression for an element \(v\) exists (that is, whenever \(sim_{\mathcal{EL}}(v) = \{v\}\)). It does not compute an \(\mathcal{EL}\)-formula \(\varphi\) that uniquely refers to \(v\). But Algorithm 1 is an instance of a family of well-known algorithms that compute \(\mathcal{L}\)-simulations by successively refining an over-approximation of the simulator sets. The “reason” behind each refinement can be encoded using an \(\mathcal{L}\)-formula; intuitively, nodes that do not satisfy it are being removed from the simulator set on each refinement.

Using this insight, one can transform an algorithm that computes \(\mathcal{L}\)-simulator sets into one that additionally computes an \(\mathcal{L}\)-RE for each set. (Areces et al., 2008) used this approach to derive their \(\mathcal{ALC}\)-GRE method from a well-known algorithm for computing \(\mathcal{ALC}\)-simulation (i.e., bisimulation) sets, but failed to notice they could derive one for \(\mathcal{EL}\) analogously.

Algorithm 2 shows a transformed version of Algorithm 1 following this principle. The idea is that each node \(v \in \Delta\) is now tagged with a formula \(F(v)\) of \(\mathcal{EL}\). The formulas \(F(v)\) are updated along the execution of the loop, whose invariant ensures that \(v \in \|F(v)\|\) and \(\|F(u)\| \subseteq S(u)\) hold for all \(u, v \in \Delta\).

Algorithm 2: Computing \(\mathcal{EL}\)-similarity and \(\mathcal{EL}\)-RE

input : a finite model \(\mathcal{M} = (\Delta, \| \cdot \|)\)
output: \(F\), the set of \(\mathcal{EL}\)-formulas, and \(S\), the simulator sets s.t. \(\forall v \in \Delta\), \(\|F(v)\| = sim_{\mathcal{EL}}(v)\)

foreach \(v \in \Delta\) do
\[
S(v) := \{u \in \Delta \mid F(v) \subseteq P(u)\}
\]
while \(\exists r, u, v, w: (u, v) \in [r], w \in S(u), suc_r(w) \cap S(v) = \emptyset\) do

invariant \((\forall v, u)\), \(\|F(u)\| \subseteq S(u) \cap \{v\}\)
\[
\text{if } \exists r. F(v) \text{ is not a conjunct of } F(u) \text{ then}
\]
\[
F(u) := F(u) \land \exists r. F(v)
\]

Initially \(F(v)\) is the conjunction of all the unary relations that satisfy \(v\) (if there is none, then \(F(v) = \top\)). Next, each time the algorithm finds \(r, u, v, w\) such that \((u, v) \in [r], w \in S(u)\) and \(suc_r(w) \cap S(v) = \emptyset\), it updates \(F(u)\) to \(F(u) \land \exists r. F(v)\). Again this new formula \(\varphi\) is in \(\mathcal{EL}\) and it can be shown that \(v \in \|\varphi\|\) and \(w \notin \|\varphi\|\), hence witnessing that \(v \not\sim_{\mathcal{EL}} w\) is false.

Algorithm 2 can be easily modified to calculate the \(\mathcal{EL}^+\)-RE of each simulator set \(sim_{\mathcal{EL}^+}\) by adjusting the initialization: replace \(\subseteq\) by \(=\) in the initialization of \(S(v)\) and initialize \(F(v)\) as \(\wedge (P(v) \cup \mathcal{P}(v))\), where \(\mathcal{P}(v) = \{\sim p \mid v \notin \|p\|\}\).

With a naive implementation Algorithm 2 executes in time \(O(\#\Delta^3 \times \#\| \cdot \|)\) providing a polynomial solution to the \(\mathcal{EL}\) and \(\mathcal{EL}^+\)-GRE problems. (Henzinger et al., 1995) show a more involved version of Algorithm 1 with lower complexity, which can be adapted in a similar way to compute \(F(v)\). We shall skip the details.

Theorem 2. The \(\mathcal{EL}/\mathcal{EL}^+\)-GRE problems over \(\mathcal{M} = (\Delta, \| \cdot \|)\) have complexity \(O(\#\Delta \times \#\| \cdot \|)\).

Theorem 2 answers a question left open by (Areces et al., 2008): the \(\mathcal{EL}\)-GRE problem can be solved in polynomial time. Note that the above result assumes a convenient representation of formulas as directed acyclic graphs (for \(O(1)\) formula
construction). In section 6 we will take a look at this in more detail.

We have not addressed the issue of preferences with respect to the use of certain relations, and moreover, we have presented our algorithms as close as possible to the original proposal of (Henzinger et al., 1995) which makes them prioritize unary relations over binary relations. The latter can be avoided by representing unary relations as binary relations (cf. §4). Certain control on preferences can then be introduced by taking them in consideration instead of the non-deterministic choice of differentiating elements made in the main loop of the algorithm. But despite these modifications, the algorithms based on simulator sets seem to offer less room for implementing preferences than the ones we will discuss in §4.

4 GRE via building simulated models

We now revisit the algorithm presented by (Krahmer et al., 2003), identify its underlying notion of expressiveness, and extend it to accommodate other notions. For reasons of space we assume the reader is familiar with this algorithm and refer her to that article for further information.

We must first note that scenes are encoded in that article in a slightly different way: there, graphs have only labels on edges, and non-relational attributes such as type or color are represented by loops (e.g., small(a,a)). While our presentation is, arguably, conceptually cleaner, it forces us to treat the atomic and relational cases separately.

The second thing to note is that the output of their algorithm is a connected subgraph H of the input graph G that includes the target u among its nodes. This means that the algorithm does not fit in the definition of L-GRE we presented in §2.

Now, H must be such that every subgraph isomorphism10 f between H and G satisfies f(u) = u. On relational models, subgraph isomorphism corresponds to FL-simulations, which makes explicit the notion of expressiveness that was used. Indeed, from the output H of this algorithm, one can easily build a FL-formula that univocally describes the target u, as is shown in Algorithm 3. Observe that if FL-simulations were used instead, we would have to include also which unary and binary relations do not hold in H.

Having made explicit the notion of sameness underlying the algorithm of (Krahmer et al., 2003) and, with it, the logical language associated to it, we can proceed to generalize the algorithm, as shown in Algorithm 4. This algorithm is parametric on L; to make it concrete, one needs to provide appropriate versions of buildFormulaL and extendL. In order to make the discussion of the differences with the original algorithm simpler, we list the code for buildFormulaFL- and extendFL- in Algorithms 3 and 6.

Algorithm 3: buildFormulaFL-(H,v)

```
// let H = {(a1,...,an), ||·||}, v = a1
γ := \( \bigwedge (x_i \neq x_j) \land \bigwedge r(x_i, x_j) \land \bigvee p(x) ;
\quad a_i \neq a_j \quad \quad (a_i,a_j) \in [r] \quad a_i \in |p| \)

return \( \exists x_2 \ldots \exists x_n, \gamma \);
```

Algorithm 4: makeREL(v)

```
vH := new node;
⟨(H,f) := (((vH),\emptyset,\emptyset),{vH \mapsto v});
H' := findL(vH,⊥,H,f);
return buildFormula\_L\( (H',v_H) \);
```

Algorithm 5: findL(vH,best,H,f)

```
if best \neq ⊥ ∧ cost(best) ≤ cost(H) then
L return best

\text{distractors} := \{n \mid n \in ∆_G \land n \neq v \land vH \not\approx n \};
if distractors = \emptyset then
L return H

\text{foreach} \( (H', f') \in \text{extend}_L(H,f) \)
I := findL(vH,best,H',f');
if best = ⊥ ∨ cost(I) ≤ cost(best) then
L best := I

return best
```

Notice that makeREL computes not only a graph H but also an L-simulation f. In the case of FL-, H is a subgraph of G and, therefore, f is the trivial identity function id(x) = x. We will see the need for f when discussing the case of less expressive logics like EL. Observe also that in extendFL- we follow the notation by (Krahmer et al., 2003) and write, for a model \( M = (\Delta, ||·||) \), \( M + p(u) \) to denote the model \( \langle \Delta \cup \{u\}, ||·|| ∪ p \rangle \) such that \( ||p'|| = ||p|| \cup \{u\} \) and \( ||q'|| = ||q|| \) when \( q \neq p \). Similarly, \( M + r(u,v) \) denotes the model \( \langle \Delta \cup \{u,v\}, ||·|| ∪ \{r\} \rangle \) such that \( ||r'|| = ||r|| \cup \{(u,v)\} \) and \( ||q'|| = ||q|| \) when \( q \neq r \). It is clear, then, that this function is returning all the extensions of H by adding a missing attribute or relation to H, just like is done in the original algorithm.

We now discuss a version of this algorithm for EL. The first thing to note is that one could, in principle, just use extendFL- also for EL. Indeed, since findEL uses an EL-simulation to
compute the set of *distractors* (in the terminology of (Krahmer et al., 2003), the output of this function would be a subgraph \( H \) of \( G \) such that for every \( \mathcal{E}L \)-simulation \( \sim \), \( u \sim v \) iff \( u = v \). The problem is this subgraph may contain cycles and, as was observed in §2, they cannot be tell apart using \( \mathcal{E}L \). The upshot is that we might be unable to realize the outcome of such function.

A well-known result establishes that every relational model \( M \) is equivalent, with respect to \( \mathcal{E}L \)-formulas\(^\text{11}\), to the *unraveling* of \( M \) (cf. (Blackburn et al., 2001)). That is, any model and its unraveling satisfy exactly the same \( \mathcal{E}L \) formulas. Moreover, the unraveling of \( M \) is always a tree, and as we show in Algorithm 7, it is straightforward to extract a suitable \( \mathcal{E}L \)-formula from a tree.

\[ \text{Algorithm 6: extend}_{\mathcal{E}L}-(H, f) \]
\[
a := \{ H + p(u) \mid u \in \Delta_H, u \in [p \in G \setminus [p]_H] \};
b := \{ H + r(u, v) \mid u \in \Delta_H, (u, v) \in [r]_G \setminus [r]_H \};
\text{return } (a \cup b) \times \{ id \}
\]

Therefore, we need \( \text{extend}_{\mathcal{E}L} \) to return all the possible extensions of \( H \) by either adding a new proposition or a new edge that is present in the unraveling of \( G \) but not in \( H \). This is shown in Algorithm 8.

Observe that the behavior of \( \text{find}_{\mathcal{E}L} \) is quite sensible to the cost function \( f \) employed. For instance, on cyclic models, an \( f \) that does not guarantee the unraveling is explored in a breadth-first way may lead to non-termination (since \( \text{find}_{\mathcal{E}L} \) may loop exploring an infinite branch).

It is also possible to use modal model-theoretical results to put a bounds check that avoids generating an unraveling of infinite depth when there is no possible referring expression, but we will not go into the details for reasons of space.

\[ \text{Algorithm 7: buildFormula}_{\mathcal{E}L}(H, v) \]
\[
\text{requires } H \text{ to be a tree }
\gamma := \{ [r]. \text{buildFormula}_{\mathcal{E}L}(H, u) \mid (v, u) \in [r] \};
\text{return } (\Lambda \gamma) \wedge (\Lambda_{u \in [r]} p);
\]

As a final note on complexity, although the set of \( \mathcal{E}L \)-distractors may be computed more efficiently than \( \mathcal{F}L^- \)-distractors, we cannot conclude that \( \text{find}_{\mathcal{E}L} \) is more efficient than \( \text{find}_{\mathcal{F}L^-} \) in general: the model built in the first case may be exponentially larger (it is an unraveling, after all).

### 5 Combining GRE methods

An appealing feature of formulating the GRE problem modulo expressivity is that one can devise general strategies that combine \( L \)-GRE algorithms. We illustrate this with an example.

The algorithms based on \( L \)-simulator sets like the ones in §3 simultaneously compute referring expressions for every object in the domain, and do this for many logics in polynomial time. This is an interesting property when one anticipates the need of referring to a large number of elements. However, this family of algorithms is not as flexible in terms of implementing preferences as those in §4.

There is a simple way to obtain an algorithm that is a compromise between these two techniques. Let \( A_1 \) and \( A_2 \) be two procedures that solve the \( L \)-GRE problem based on the techniques of §§3 and §4, respectively. One can first compute an \( L \)-RE for every possible object using \( A_1 \) and then (lazily) replace the calculated RE for \( u \) with \( A_2(u) \) whenever the former does not conform to some predefined criterion. But one can do better.

Since \( A_1 \) computes, for a given \( M = \langle \Delta, \cdot, \cdot \rangle \), the set \( \text{sim}(u) \) for every \( u \in \Delta \), one can build in polynomial time, using the output of \( A_1 \), the model \( M_L = \langle \{ [u] \mid u \in \Delta \}, \cdot, \cdot \rangle \), such that:

\[ [u] = \{ v \mid u \sim v \text{ and } v \sim u \}\]
\[ [r]_L = \{ ([u_1] \ldots [u_n]) \mid (u_1 \ldots u_n) \in [r] \}\]

\( M_L \) is known as the *\( L \)-minimization* of \( M \). By a straightforward induction on \( \gamma \) one can verify that \( (u_1 \ldots u_n) \in [\gamma] \) iff \( ([u_1] \ldots [u_n]) \in [\gamma]_L \) and this implies that \( \gamma \) is a \( L \)-RE for \( u \) in \( M \) iff it is a \( L \)-RE for \( [u] \) in \( M_L \).

If \( M \) has a large number of indistinguishable elements (using \( L \)), then \( M_L \) will be much smaller than \( M \). Since the computational complexity of \( A_2 \) depends on the size of \( M \), for very large scenes, one should compute \( A_2([u]) \) instead.

### 6 On the size of referring expressions

The expressive power of a language \( L \) determines if there is an \( L \)-RE for an element \( u \). But observe that when \( u \) can be described in \( L \), it may

\(^{11}\text{Actually, the result holds even for } \mathcal{ALC} \text{ formulas.} \)
also influence the size of the shortest $\mathcal{L}$-RE. Intuitively, with more expressive power we are able to ‘see’ more differences and therefore have more resources at hand to build a shorter formula.

A natural question is, then, whether we can characterize the relative size of the $\mathcal{L}$-REs for a given $\mathcal{L}$. That is, if we can give (tight) upper bounds for the size of the shortest $\mathcal{L}$-REs for the elements of an arbitrary model $\mathcal{M}$, as a function of the size of $\mathcal{M}$.

For the case of one of the most expressive logics considered in this article, $\mathcal{FL}^-$, the answer follows from algorithm makeRE$_{\mathcal{FL}^-}$ in §4. Indeed, if an $\mathcal{FL}^-$-RE exists, it is computed by buildFormula$_{\mathcal{FL}^-}$ from a model $H$ that is not bigger than the input model. It is easy to see that this formula is linear in the size of $H$ and, therefore the size of any $\mathcal{FL}^-$-RE is $O(\#\Delta + \#\cdot \|)$.

It is not hard to see that this upper bound holds for $\mathcal{FL}$-REs too (cf. §4 for details).

Although buildFormula$_{\mathcal{FL}}$ also returns a formula that is linear in the size of the tree-model $H$, $H$ could be, in principle, exponentially larger than the input model. We can use this to give an exponential upper bound for the size of the shortest $\mathcal{EL}$-RE, but is it tight?

One is tempted to conclude from Theorem 2 that the size of shortest $\mathcal{EL}$-RE is $O(\#\Delta \times \#\cdot \|)$, but there is a pitfall. Theorem 2 assumes that formulas are represented as a DAG and guarantees this DAG is polynomial in the size of the input model. One can easily reconstruct (the syntax tree of) the formula from the DAG, but this, in principle, may lead to a exponential blow-up (the result will be a exponentially larger formula, but composed of only a polynomial number of different subformulas).

As the following example shows, it is indeed possible to obtain an $\mathcal{EL}$-formula that is exponentially larger when expanding the DAG representation generated by Algorithm 2.

**Example 3.** Consider a language with only one binary relation $r$, and let $\mathcal{M} = \langle \Delta, \| \cdot \| \rangle$ where $\Delta = \{1, 2, \ldots, n\}$ and $(i, j) \in \| r \|$ iff $i < j$. Algorithm 2 initializes $F(j) = \top$ for all $j \in \Delta$. Suppose the following choices in the execution: For $i = 1, \ldots, n - 1$, iterate $n - i$ times picking $v = w = n - i + 1$ and successively $u = n - i, \ldots, 1$. It can be shown that each time a formula $F(j)$ is updated, it changes from $\varphi$ to $\varphi \land \exists r \varphi$ and hence it doubles its size. Since $F(1)$ is updated $n - 1$ many times, the size of $F(1)$ is greater than $2^n$.

The large $\mathcal{L}$-RE of Example 3 is due to an unfortunate (non-deterministic) choice of elements. Example 4 shows that another execution leads to a quadratic $\mathcal{RE}$ (but notice the shortest one is linear).

**Example 4.** Suppose now that in the first $n - 1$ iterations we successively choose $v = w = n - i$ and $u = v - 1$ for $i = 0 \ldots n - 2$. It can be seen that for convenient choices, $F(1)$ is of size $O(n^2)$.

We are yet unable to answer whether the exponential bound for the size of the minimum $\mathcal{EL}$-RE is tight. We conjecture no polynomial bound can be given, though. In any case, it seems clear that not only existence of RE but relative lengths should be taken into account when considering the trade-off between expressive powers.

### 7 Conclusions

There is some notion of expressiveness underlying the formulation of every GRE problem. This “expressiveness” can be formally measured in terms of a logical language or, dually, a simulation relation between models. In this article we have discussed making the notion of expressiveness involved an explicit parameter of the GRE problem, unlike usual practice.

We have taken an abstract view, defining the “$\mathcal{L}$-GRE problem”; and though we considered various possible choices for $\mathcal{L}$, we did not argue for any of them. Instead, we tried to make explicit the trade-off involved in the selection of a particular $\mathcal{L}$. This, we believe, depends heavily on the given context.

By making expressiveness explicit, we can transfer general knowledge and results from the well-developed field of computational logics. This was exemplified in §3 and §4 where we were able to turn known GRE algorithms into families of algorithms that may deal analogously with different logical languages. We also applied this in §5 to devise new heuristics.

Arguably, an explicit notion of expressiveness also provides a cleaner interface, either between the content-determination and surface realization modules or between two collaborating content-determination modules. An instance of the latter was exhibited in §5.

As a future line of research, one may want to avoid sticking to a fixed $\mathcal{L}$ but instead favor an incremental approach in which features of a more expressive language $\mathcal{L}_1$ are used only when $\mathcal{L}_0$ is not enough to distinguish certain element.
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