NON – THERMAL ACCELERATION FROM RECONNECTION SHOCKS

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ABSTRACT

Reconnection shocks in a magnetically dominated plasma must be compressive. Non-thermal ion acceleration can occur across built-in slow shocks, and across outflow fast shocks when the outflow is supermagnetosonic and the field is line-tied. Electron acceleration may be initiated by injection from the dissipation region. Reconnection and shock acceleration thus cooperate and non-thermal acceleration should be a characteristic feature.
Magnetic reconnection converts magnetic energy into particle energy as oppositely magnetized flows merge across a thin dissipation region (DR). An upper limit to the (2-D) reconnection rate comes from the Petschek model (PK) originally proposed to explain rapid solar flare energy release [1]. In PK, the DR length $\ll$ field gradient length and magnetic tension in the outflow thrusts the plasma from the DR. Slow shocks are built explicitly into PK, across which the plasma flow changes abruptly from inflow to outflow. The PK tension thrust provides faster reconnection than the Sweet-Parker (SP) type models for which the DR length $\sim$ field gradient length [2].

Though slow shocks are built into PK, similar structures are seen numerically even when the DR $>>$ than that of PK [3]. In addition, fast shocks can be present when the outflow is supermagnetosonic and the outflow boundary field is kept fixed [4]. Although real astrophysical plasmas are compressive, most work has employed the incompressible approximation to facilitate a global solution. Compressible reconnection models have also focused on perturbative solutions and the rate [5], not on the particle spectrum. Here we consider the effect of compressive shocks near a DR on the spectrum.

Slow shocks, unlike fast shocks, have not been extensively simulated, and their potential for power law acceleration has not been addressed as an important outcome of reconnection. The shocks may accelerate ions directly from a thermal distribution and the DR may provide the injection electrons required for non-thermal electron acceleration. Since much of the flow of a reconnecting region passes through the shocks, reconnection may possibly explain sustained non-thermal features in magnetically dominated astrophys-
ical phenomena.

We first solve the jump conditions across the built-in slow shock for the compression ratio, given a magnetically dominated inflow. We find the parameter space for which the outflow must be supermagnetosonic and relate the the compression ratio across the slow shock, $r_s$, to that across the fast one, $r_f$. We then discuss the implications for shock acceleration.

The non-relativistic MHD jump conditions for mass, momentum, and energy are [6]

\begin{align}
\rho_1 v_{1n} &= \rho_2 v_{2n}, \quad (1) \\
\rho_1 v_{1n}^2 + P_1 + B_{1n}^2/8\pi &= \rho_2 v_{2n}^2 + P_2 + B_{2n}^2/8\pi, \quad (2) \\
\rho_1 v_{1n} v_{1t} - B_{1n} B_{1t}/4\pi &= \rho_2 v_{2n} v_{2t} - B_{2n} B_{2t}/4\pi, \quad (3) \\
(1/2)\rho_1 v_{1n}^2 + \Gamma(\Gamma - 1)^{-1} P_1 v_{1n} + (B_{1n}^2/4\pi)v_{1n} - v_1 \cdot B_1 B_{1n}/4\pi &= (1/2)\rho_2 v_{2n}^2 \quad (4)
\end{align}

where $B$ is the magnetic field, $v$ is the velocity, $P$ is the pressure, $\rho$ is the density and $\Gamma$ is the adiabatic index. The subscript $1(2)$ refers to the up(down)stream region, and the subscript $n(t)$ refers to the normal (tangential) components. The electromagnetic jump conditions for an ideal plasma are given by

\begin{align}
B_{1n} &= B_{2n} \quad (5) \\
(v_{1n} B_1) &= (v_{2n} B_2). \quad (6)
\end{align}
The shock is \( \perp \) to the \( \hat{\mathbf{n}}, \hat{\mathbf{y}} \) plane as shown in Fig. 1. We assume the switch-off condition, \( B_{2y} = 0 \), and also that \( v_1/|v_1| \cdot \hat{\mathbf{y}} < 1 \) [8]. Define \( \tilde{c} \equiv \cos \theta, \tilde{s} \equiv \sin \theta \) and \( \tilde{t} \equiv \tan \theta \) where \( \theta \) is the angle between the downstream flow and the shock normal. Define \( c_1 \equiv \cos \phi_1, s_1 \equiv \sin \phi_1 \) and \( t_1 \equiv \tan \phi_1 \) where \( \phi_1 \) is the angle between the upstream field and the shock normal. The configuration of Fig. 1 is then described by

\[
v_{1n} = -v_1, \quad B_{1n} = -B_1 c_1, \quad B_{1y} = -B_1 s_1, \quad v_{2n} = -v_2 \tilde{c}, \quad v_{2y} = v_2 \tilde{s}, \quad B_{2n} = -B_2, \quad v_{1y} = B_{2y} = 0. \tag{7}
\]

For \( \Gamma = 5/3 \) and \( \beta_1 \equiv a_{1s}^2/v_{2A}^2 << 1 \), where \( a_{1s} \) and \( v_{1A} \) are the inflow sound and Alfvén speed, plugging (7) into (1)-(6) gives

\[
t_1^2 = 2(r_s - 1)(r_s - 4)/(5r_s - 2r_s^2), \tag{8}
\]

\[
\beta_2 = (5/3)[(r_s - 1)/r_s + t_1^2/2] \tag{9}
\]

\[
M_{2A}^2 \equiv v_{2A}^2/v_{2A}^2 = (1 + r_s^2 t_1^2)/r_s, \tag{10}
\]

where \( M_{2A} \) is the outflow Mach number, \( v_{2A} \) is the outflow Alfvén speed, \( \beta_2 \equiv a_{2s}^2/v_{2A}^2 \), and \( a_{2s} \) is the outflow sound speed. Fig. 2 shows \( t_1 \) and \( M_{2A} \) versus \( r_s \equiv \rho_2/\rho_1 \). Since \( t_1^2 > 0 \), (8) shows that \( 2.5 < r_s < 4 \) for a low \( \beta_1 \) switch-off shock [7], with the lower limit being a perpendicular (\( \perp \)) shock and the upper limit a parallel (\( \parallel \)) shock. As \( \beta_1 \to \infty \), \( r_s \to 1 \).

When \( \mathbf{B}_2 \cdot \mathbf{v}_2/|B_2 v_2| << 1 \), \( v_2 \) will be supermagnetosonic [6] when \( v_2^2 = v_{2n}^2 + v_{2y}^2 > a_{2s}^2 + v_{2A}^2 \). Using (7), (2), (3), and (5) this condition reduces to \( 6r_s^2 - 13r_s - 20 < 0 \), and is satisfied for \( r_s < 3.2 \) or \( t_1 > 1.25 \) from (8). A nearly uniform supermagnetosonic outflow
becomes the condition for a fast shock when the field is line-tied at the outflow boundary. The jump conditions, (1) – (6), across such a quasi-⊥ fast shock for $\Gamma = 5/3$ give the equation

$$M_{2A}^2 = 3r_f \beta_2/(4 - r_f) + (3/2)r_f(r_f - 1)/(4 - r_f).$$

(11)

Combining this with (8), (9), and (10) we obtain,

$$r_f = (12r_s - 30)^{-1}[4r_s^2 - 20r_s + 7 + (16r_s^4 - 544r_s^3 + 2952r_s^2 - 4312r_s - 431 + 2400/r_s)^{1/2}].$$

(12)

Fig 2. shows that $1 < r_f < 2$ when $3.2 > r_s > 2.5$. The inverse dependence is expected because a decrease in $r_s$ corresponds to an increase in tension force along the shock plane, and thus a larger $M_{2A}$, accounting for the larger $r_f$. For the canonical quasi-⊥ built-in slow shock with a line-tied outflow, a fast shock is likely, as the supermagnetosonic outflow condition requires only that $t_1 > 1.25$.

The compression ranges of $2.5 < r_s < 4$ for the built-in slow shock and $1 < r_f < 2$ for the fast shock are important for shock acceleration theory: For distribution functions isotropic to first order in $v/v^*_p$ where $v^*_p$ is the particle velocity in the proper frame of the bulk flow $v$, the steady state Boltzmann equation can be written as a diffusion-convection (DC) equation. We define $N(x, p_p)dp_p \equiv 4\pi p_p^2 f(x, p_p)dp_p$, where $f$ is the Boltzmann distribution function, $x$ measures position, and $p_p$ is the particle momentum. The DC equation across a general shock is then [9]

$$\partial_n[v_n N - \kappa_n \partial_n N] - (1/3)(\partial_n v_n)\partial p_p[p_n N] = 0,$$

(13)

where $v_n$ is the normal flow velocity, $\kappa_n$ is the normal diffusion coefficient, and we have assumed that gradients in the normal direction $>>$ those along the shock. The solution
of (13) across the shock when the shock thickness $\ll$ mean free path [9] shows that the outflow energy spectrum for a steeper inflow spectrum takes the power law form $N \propto p^{-w}$ with energy index $w = (r + 2)/(r - 1)$ depending only on the compression ratio, $r$. Fermi acceleration operates as the particles diffuse between scattering centers (presumably turbulence) on each side of the shock. Particles always see the centers converging, as the normal velocity is larger upstream.

Shock acceleration can dominate synchrotron loss when $\tau_{\text{syn}}$, the shortest synchrotron loss timescale of the region, exceeds the longest shock acceleration timescale $\tau_{sh}$:

$$\tau_{\text{syn}} \equiv \frac{6\pi m_e c}{(\gamma_e B_1^2 \sigma_T)} \geq \tau_{sh} \sim \frac{\kappa_{n||}}{v_1^2}, \quad (14)$$

where $\sigma_T$ is the Thomson cross section, $\gamma_e$ is the electron Lorentz factor, and $\kappa_{n||}$ is the diffusion coefficient normal to the slow shock and thus $||$ to the downstream field. For particles moving at $c$, $\kappa_{n||} \sim c \lambda_{||}/3$, where $\lambda_{||}$ is the field gradient length [9] which we assume is of the same order in the inflow and outflow regions. From (14), the condition justifying the absence of a synchrotron loss term in (13) is then

$$\gamma_e \leq \sim 0.08 (v_1/ \text{cm sec}^{-1})^2 (B_1/\text{Gauss})^{-2} (\lambda_{||}/\text{cm})^{-1}. \quad (15)$$

The third term in (13) can be thought of as the 1st order correction to the rest frame equation, when measured in the lab frame. This motivates the finding [10] that (13) includes not only guiding center diffusion through pitch angle scattering, but also drift acceleration from motion along the induced electric field. The per particle energy change from the latter increases with obliquity. For slow (fast) shocks, the curvature (gradient)
drift is || to the electric field and accounts for energy gains while the gradient (curvature) drift is anti-|| to the electric field and incurs particle energy losses [11]. These contributions conspire with those from the gyromotion component along the electric field for a net per particle momentum change $dp_p/dt = -p_p \nabla \cdot v_\perp$, where $v_\perp$ is the flow velocity $\perp$ to $B$. The relevant component of this force is included in the coefficient of the $\partial N/\partial p_p$ term of (13).

The relative importance of a reconnection fast shock varies inversely with the size of the DR: When both shocks are present $3.2 > r_s > 2.5$ with $2.4 < w_s < 3$, so that $1 < r_f < 2$ with $w_f \geq 4$, where $w_{s(f)}$ is the slow (fast) shock energy index. (The range $3.2 \leq r_s < 4$ corresponds to a slow shock with no outflow fast shock and $2.4 \geq w_s > 2$.) Thus, if slow shock acceleration is effective, fast shocks cannot further steepen the already steep spectrum of particles that passed through the slow shock. However, if the DR length $\sim \lambda_||$, then more of the flow will see only the fast shock, and the spectrum from shock acceleration should have a somewhat lower energy index.

Although we have used a DC equation, we recognize that shock acceleration is a rather non-linear process. However, fast shock simulations show that the Fermi acceleration engine is very efficient, transferring $\geq 1/10$ of the inflow energy to particles [9]. These particles tend to smooth out the shock by diffusion, produce turbulence, and increase the compression ratio above the jump condition value, as their escape and acceleration change the downstream equation of state. The shock smoothing can violate the assumptions built into the simple DC scheme, but the increase in the compression ratio over the linear limit enhances the non-thermal acceleration. The efficiency is relatively insensitive to the
obliquity of the inflow field unless the Mach number exceeds $\sim 30$. For our case, outflow fast shocks would then be effective if $M_{2A} \sim r_s t_1 < 30$ from (10).

A recent hybrid simulation [12] of oblique slow shocks and the ion-ion cyclotron instability shows that steady or cyclically reforming slow shocks can develop depending on the inflow conditions. In the steady case a coherent Alfvén wave train forms downstream. In the unsteady case the shock quasi-periodically transforms from a thin sharp transition to a wide diffuse transition, and Alfvén turbulence is seen downstream. This cyclic reformation is strikingly similar to that seen in quasi-|| fast shocks [9] and is an example of an electromagnetic beam instability brought on by the interaction of backstreaming ions with the inflowing plasma. Waves produced in the upstream by such instabilities are amplified as they convect back to the shock front. The compressed waves then interact strongly with the inflow particles, scattering and slowing them, producing the entropy required for the shock. Some of the ions are scattered back upstream by the waves, and a subset of those are scattered back to the shock. The reformation of the thin structure occurs if the backstreaming particles leave the simulation region, and then no longer produce waves that are convected to the shock. The above process is in fact how non-thermal ions are extracted from an initially thermal input, initiating the Fermi process. It is the effectiveness of the Fermi acceleration which makes shock acceleration such a non-linear process.

Slow shocks in the geomagnetic tail show wave substructures with properties similar to those of Ref. [12], and also show turbulence ahead and behind the shock fronts [13]. In addition, although much of the shock acceleration goes into ions, significant non-thermal
tails in the electron spectra are seen [14], and are not modeled by hybrid simulations which assume a fluid electron population. Fast shock simulations show electron acceleration with injection electrons [15], and we expect that slow shocks could operate similarly. More simulations of slow shocks are needed which predict the spectrum of accelerated particles.

Jet plasma in AG may be largely pair plasma [16], so ion-electron simulations might not be applicable. We suggest that reconnection and its shocks are a strong candidate for solving the re-acceleration problem in jets, which requires sustaining non-thermal electron emission over distances exceeding \( c\tau_{\text{syn}} \), where \( c \) is the speed of light. Shock electron acceleration requires injection particles [16] which we now show, and argue that the DR may be able to provide such injection.

We can estimate the range of particle energies accelerated by the Fermi process for the slow and fast shocks, and in particular the range of \( \gamma_e \). First consider the slow shock: An upper limit can be found by ensuring that the particles see an ordered field. This requires \( \lambda_{||} > g_2 \) where \( g_2 \sim c^2\gamma_e m_e/(eB_2) \) is the particle gyroradius associated with the smaller field of the two flow regions. This implies \( \gamma_e < eB_2\lambda_{||}/(m_e c^2) \sim 10^{-3}(B_2/\text{Gauss})(\lambda_{||}/\text{cm}) \).

A lower limit can be found by demanding that the downstream particles be able to diffuse upstream. This requires particles of large enough energy to resonantly interact with the plasma waves which pitch-angle scatter the particles upstream. For Alfvén turbulence [16], the electron lower bound is a factor \( \sim m_p/m_e \) times that for protons and is given by \( \gamma_e > (m_p/m_e)(v_{2A}/c) \) where \( m_p \) is the proton mass and \( e \) is the charge. Thus for the slow
shock

\[(m_p/m_e)(v_{2A}/c) < \gamma_e < 10^{-3}(B_2/\text{Gauss})(\lambda_{||}/\text{cm}).\]  

(16)

Consider now the fast shock: Since the flow downstream from the fast shock is primarily \(\perp\) to the shock normal, diffusion across the shock requires [9]

\[\kappa_{n\perp}/v_3 > g_3 \sim \gamma_em_ec^2/(eB_3),\]  

(17)

where \(v_3\) is the downstream flow speed, \(B_3\) is the downstream field, \(g_3\) is the associated gyroradius, and \(\kappa_{n\perp}\) is the diffusion coefficient normal to the shock but \(\sim \perp B_3\). For \(\lambda_{||} > g_3\) we use \(\kappa_{n||} \sim c\lambda_{||}/3\), so [9] \(\kappa_{n\perp} = cg_3^2/(3\lambda_{||})\) and (17) gives \(\gamma_e > 3eB_3v_3\lambda_{||}/(m_ec^3)\). This limit must be combined with the analogous limits as described for the slow shock so that for the fast shock we have

\[
\text{Max } [(m_p/m_e)(v_{3A}/c), 10^{-3}(B_3/\text{Gauss})(\lambda_{||}/\text{cm})(v_3/c)] < \gamma_e < 10^{-3}(B_2/\text{Gauss})(\lambda_{||}/\text{cm}),
\]  

(18)

where \(v_{3A}\) is the Alfvén speed downstream from the fast shock. Note that the upper limits in (16) and (18) are less than the upper limit in (15) when \(v_1^2 > (0.01\text{ cm} \cdot \text{sec}^{-2}\text{Gauss}^{-3})B_1^2B_2\lambda_{||}\).

The highly dissipative energy conversion in the DR could provide the injection electrons. To see this, note that upon absorbing the annihilated field energy, the average \(\gamma_e\) there \(\sim (v_{1A}^2/c^2)(m_p/2m_e)\). Combining this with (17) we see that slow shocks of large obliquity favor a DR which can inject, as the condition is

\[v_{1A}/v_{2A} = r_s^{1/2}/c_1 > \sim c/v_{1A}.\]  

(19)
We have discussed non-thermal acceleration by low $\beta_1$ reconnection slow shocks and outflow fast shocks. The particle spectra across the slow shocks should be at least as flat as that given by the DC spectral index range of $2 < w_s < 3$ for sufficiently large slow shocks. The dissipation region can provide injection electrons for acceleration and the above index range is consistent with observed features of radio galaxy lobes and jets [16].

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