Improving methodology for multi-input-single-output control system using the Robust Bode plot

Hayato KATO* and Takenori ATSUMI*

* Department of Mechanical Engineering, Chiba Institute of Technology
2-17-1 Tsudanuma, Narashino, Chiba, 275-0016, Japan
E-mail: takenori.atsumi@p.chibakoudai.jp

Received: 27 January 2020; Revised: 5 April 2020; Accepted: 15 April 2020

Abstract
In order to improve a robust performance in a Multi-Input-Single-Output (MISO) control system, we proposed a loop-shaping methodology with the Robust Bode (RBode) plot for the MISO system. By using the current RBode plot, we are able to improve an existing Single-Input-Single-Output (SISO) control system with visualized guidelines on an open-loop characteristic Bode plot. Our proposed method transforms the robust performance problem for the MISO system into that for the SISO system in order to employ the RBode plot. As a result, we are able to design an improved MISO control system that compensates for disturbances against perturbations of controlled objects. A case study with the magnetic-head positioning control system shows that the proposed loop-shaping method is able to improve the positioning accuracy under the external vibration by about 83%.

Keywords: Robust control, Loop shaping, Multi-Input-Single-Output system, Hard disk drive, Positioning control

1. Introduction
In order to build a control system in an actual mechatronic product, we have to design a controller that achieve a required performance against perturbations of a controlled object. This is well-known as “robust performance problem”, and we are able to use automated design tools for this robust performance problem based on $\mu$-synthesis, or Linear Matrix Inequalities (LMI) (Doyle et al., 1992; Hirata et al., 1996). These automated design tools are very useful for control researchers who have high-level skills and rich experiences in automatic control engineering field. However, for general engineers who usually design a PID controller based on a classical control theory.

To overcome this issue, we have proposed a loop-shaping method called “Robust Bode plot (RBode plot)” (Atsumi and Messner, 2012). By using the RBode plot, we can see the robust performance criterion as allowable and forbidden regions on the open-loop Bode plot. As a result, we can improve a Single-Input-Single-Output (SISO) control system so that the control system achieves the required performance against the perturbations of the controlled object.

On the other hand, a controller design for a multi-input system is actually much harder than that of the single-input systems. In this paper, we proposed a loop-shaping methodology for a Multi-Input-Single-Output (MISO) control system with the RBode plot. A case study with a magnetic-head positioning control system in a hard disk drive shows the utility of the proposed method.

2. Robust Bode plot for MISO system
2.1. Robust performance problem for MISO system
We assume that the MISO control system in this study is the digital control system. Hereafter, $T_s$ indicates a sampling time of the control system and $\omega$ indicates a frequency [rad/s].

Fig. 1 shows a block diagram of a general MISO control system. Here, $m$ is a number of input signals, $P_1$, $P_2$, · · ·, $P_m$ are nominal plants of controlled objects, and $\Delta_1$, $\Delta_2$, · · ·, $\Delta_m$ are multiplicative uncertainties of the controlled objects (Doyle et al., 1992). $C_1$, $C_2$, · · ·, $C_m$ are feedback controllers for the controlled objects. $r$ is a reference signal, $e$ is an error signal, $u_1$, $u_2$, · · ·, $u_m$ are input signals to the controlled objects.
$L_r(\omega)$ is an open-loop characteristic with plant uncertainties (transfer characteristic from $e$ to $y_d$) and given by

$$L_r(\omega) = \sum_{i=1}^{m} C_i(e^{j\omega T_i})P_i(\omega)(1 + \Delta_i(\omega)).$$

(1)

To simplify the robust performance problem of the MISO control system in Fig. 1, we employ an MISO system shown in Fig. 2. Here, $y_{d0}$ is a sum of the output signals from controlled objects. $\Delta_u$ is a multiple uncertainty for the MISO system and defined as

$$\Delta_u(\omega) = \frac{1}{L_u(\omega)} \sum_{i=1}^{m} C_i(e^{j\omega T_i})P_i(\omega)\Delta_i(\omega),$$

(2)

where $L_u$ is a nominal open-loop characteristic (transfer characteristic from $e$ to $y_{d0}$) and given by

$$L_u(\omega) = \sum_{i=1}^{m} C_i(e^{j\omega T_i})P_i(\omega).$$

(3)

As a result, $L_r$ in Fig. 3 can be given by the following equation:

$$L_r(\omega) = L_u(\omega)(1 + \Delta_u(\omega)) = \sum_{i=1}^{m} C_i(e^{j\omega T_i})P_i(\omega)(1 + \Delta_i(\omega)).$$

(4)

Equations (1) and (4) show that $L_r$ in Fig. 2 is equal to $L_n$ in Fig. 1. This means that we can employ Eq. (4) to define the robust performance problem in the MISO control system.

To make our analysis simple, the gain of $\Delta_u$ is defined by

$$|\Delta_u(\omega)| := \frac{1}{|L_u(\omega)|} \sum_{i=1}^{m} |C_i(e^{j\omega T_i})P_i(\omega)| \sup |\Delta_i(\omega)|.$$  

(5)

Here, we define a nominal sensitivity function $S_n$ and a complementary sensitivity (co-sensitivity) function $T_n$ as follows.

$$S_n(\omega) = \frac{1}{1 + L_u(\omega)} \quad T_n(\omega) = \frac{L_u(\omega)}{1 + L_u(\omega)}.$$  

(6)

In the robust performance problem, we use two weighting functions $W_u(\omega)$ and $W_i(\omega)$. $W_u$ is a weighting function for the complementary sensitivity (co-sensitivity) function which specifies the plant uncertainty. $W_i$ specifies the sensitivity function performance. $W_u$ is designed to satisfy

$$|W_u(\omega)| \geq |\Delta_u(\omega)| = \sup \left| \frac{L_u(\omega)}{L_n(\omega)} - 1 \right|, \quad \forall \omega.$$  

(7)

The control system needs to satisfy the following inequality in order to meet robust stability against $\Delta_u$ (Doyle et al., 1992).

$$|W_u(\omega)|^{-1} > |T_n(\omega)|, \quad \forall \omega.$$  

(8)

The $W_i$ is a weighting function which specifies the robust performance such that

$$|W_i(\omega)|^{-1} \geq \left| \frac{1}{1 + L_u(\omega)} \right|, \quad \forall \omega.$$  

(9)
If the open-loop transfer function \( L_n \) is stable, the control system achieves (8) and (9) when the nominal sensitivity and co-sensitivity functions satisfy the following inequality (Doyle et al., 1992).

\[
\begin{align*}
|W_u(\omega)T_n(\omega)| + |W_s(\omega)S_n(\omega)| &= \frac{|W_u(\omega)L_n(\omega)| + |W_s(\omega)|}{|1 + L_n(\omega)|} < 1, \quad \forall \omega. \\
\end{align*}
\]  

(10)

We are able to transform (10) into the following inequality.

\[
|W_u(\omega)||L_n(\omega)|^2 + 2|W_u(\omega)||W_s(\omega)||L_n(\omega)| + |W_s(\omega)|^2 < 1 + 2|L_n(\omega)| \cos(\omega L_n(\omega)) + |L_n(\omega)|^2, \quad \forall \omega.
\]

(11)

From (11), we can get the following quadratic inequalities.

\[
(1 - |W_u(\omega)|^2)|L_n(\omega)|^2 + 1 - |W_s(\omega)|^2 + 2(\cos(\omega L_n(\omega)) - |W_u(\omega)||W_s(\omega)||L_n(\omega)|) > 0, \quad \forall \omega.
\]

(12)

\[
\cos(\omega L_n(\omega)) > \frac{|W_u(\omega)||W_s(\omega)| + \frac{|W_u(\omega)|^2 - 1}{2|L_n(\omega)|} + \frac{1}{2}\left|\frac{|W_s(\omega)|^2}{2|L_n(\omega)|}\right|}{|L_n(\omega)|}, \quad \forall \omega.
\]

(13)

The RBode plots show borderline between allowable regions that meet the specific robust performance criterion and forbidden regions that do not meet the criterion on the Bode plots of the open-loop characteristics.

### 2.2. Loop-shaping methodology with the RBode plot

Fig. 3 shows a block diagram of a proposed MISO control system. Here, \( L_{n0} \) is a initial nominal open-loop and \( C_l \) is a loop-shaping filter which improves the initial control system with the RBode plot. The proposed loop-shaping methodology follows steps below.

**Step 1.** Chose the initial MISO control system which includes the feedback controllers \((C_1, C_2, \cdots, C_m)\).

**Step 2.** Define the frequency responses of the nominal plants \((P_1, P_2, \cdots, P_m)\) and upper bounds of multiple uncertainties \((\Delta_1, \Delta_2, \cdots, \Delta_m)\). Then, calculate the initial nominal open-loop characteristic \((L_{n0})\). Note that we don’t need build transfer functions for them.

**Step 3.** Calculate the frequency response of the \(|W_u|\) from Eq. (5).

**Step 4.** Define the frequency response of \(|W_s|\).

**Step 5.** Plot the RBode plot using \( L_n = L_{n0} \), \(|W_u|\), and \(|W_s|\).

**Step 6.** In order to eliminate intersections with the forbidden regions on the RBode plot using \( L_n = C_l L_{n0} \), \(|W_u|\), and \(|W_s|\), set the loop-shaping filter \( C_l \) by shaping the new MISO open-loop characteristics. Note that this step may require iterative try and error.

### 3. Case study: magnetic-head positioning system in hard disk drive

#### 3.1. Magnetic-positioning control system

A current shipped hard disk drive (HDD) is comprised of a VCM (voice coil motor), several PZT actuators, several magnetic heads, several disks, a spindle motor, and an HSA (head-stack assembly), as shown in Fig. 4 (Atsumi, 2016a). The HSA has suspensions, a coil of the VCM, PZT elements attached to the suspensions, and the magnetic heads attached to the suspensions. The VCM moves the HSA and the PZT actuator moves a sway mode of the suspension. The magnetic-head position signal is generated from embedded information in servo sectors located at regular intervals on the disks. Therefore, the magnetic-head position signal is only available as a discrete-time signal at a sampling time determined by the rotation rate of the spindle and the number of servo sectors. This means that the controlled object in the magnetic-head positioning control system is a dual-input-single-output (DISO) sampled-data system. Note that, in this paper, a DISO control system means the control system which has the DISO controlled object.
Fig. 5 shows a block diagram of the magnetic-head positioning control system in the HDD. Here, $C_{ls}$ is the loop-shaping filter, $P_{cem}$ is a VCM, $C_{cem}$ is a feedback controller for the VCM, $N_{cem}$ is a multi-rate notch filter for the VCM, $P_{pzt}$ is a PZT actuator, $C_{pzt}$ is a feedback controller for the PZT actuator, $N_{pzt}$ is a multi-rate notch filter for the PZT actuator, $S$ is a sampler, $\mathcal{H}$ is a zero-order hold (ZOH), $\mathcal{H}_m$ is a multi-rate ZOH, and $I_p$ is an interpolator. $d_e$ is a disturbance signal for the magnetic-head positioning system, $e$ is an error signal for the magnetic-head positioning system, $u_{cem}$ is an output signal from $C_{cem}$, $u_{pzt}$ is an output signal from $C_{pzt}$, $y_{cem}$ is an output signal from $P_{cem}$, $y_{pzt}$ is an output signal from $P_{pzt}$, $y_c$ is the magnetic-head position in continuous time, and $y_d$ is the measured magnetic-head-position signal in discrete time.

### 3.2. Initial control system

In this case study, we employ an initial control system which is shown in the reference paper (Atsumi et al., 2015). A sampling time $T_s$ of $C_{cem}(s)$, $C_{pzt}(s)$, and $S$ was 23.15 µs (the sampling frequency $\omega_s$: 43.2 kHz), and a multi-rate number for $I_p(s)$, $N_{pzt}(s), N_{cem}(s)$, and $\mathcal{H}_m$ is set as two (the sampling frequency for multi-rate filter: 86.4 kHz). In this study, the interpolator $I_p(s)$ consists of an interpolation filter and an up-sampler as follows.

$$vI_p(z) = \sum_{m=1}^{2} z^{-1-m}$$

(14)

The frequency response of the VCM model $P_{cem}(s)$ is shown in Fig. 6 (solid: measurement data, dashed: mathematical model). The frequency response of the PZT model $P_{pzt}(s)$ is shown in Fig. 7. In both figures, the solid lines indicate the measurement data, and the dashed lines indicate the results with mathematical models.

The multi-rate notch filters ($N_{cem}$ and $N_{pzt}$) have to decrease the gains at the mechanical resonances beyond the Nyquist frequency of the sampler $S$. As a result, these multi-rate notch filters are given as shown in Fig. 8 (solid lines: $N_{cem}$, dashed lines: $N_{pzt}$).

The transfer function from $u_{cem}$ to $y_d$ in Fig. 5 at $\omega_0$, which is a transfer function of the VCM in discrete time, can be given as Fig. 9 (Atsumi 2016b). In this figure, the solid lines indicate $P_{dcm}$, and the dashed lines indicate $P_{dpz}$.

The open-loop characteristics for the VCM and the PZT actuator at $\omega_0$ in discrete time can be given as follows.

$$L_{cem}(\omega_0) = C_{cem}(e^{j\omega_0T_s})P_{dcm}(\omega_0), L_{pzt}(\omega_0) = C_{pzt}(e^{j\omega_0T_s})P_{dpz}(\omega_0).$$

(15)

Note that, the loop-shaping filter $C_{ls}$ is not used ($C_{ls}$ is set as 1) in the initial control system because we employ $C_{ls}$ for improving the initial control system with the RBode plot.

The transfer function from $e$ to $y_p$ in Fig. 5 at $\omega_0$, which is the nominal open-loop characteristic for this DISO control system in a discrete-time system, can be given as follows:

$$L_{e}(\omega_0) = L_{cem}(\omega_0) + L_{pzt}(\omega_0).$$

(16)

Fig. 10 shows a frequency response of $C_{cem}$ (the feedback controller for the $P_{dcm}$). $C_{cem}$ has to stabilize a rigid-body mode of the VCM by a phase-lead element and provide an integral action. It also has to stabilize mechanical resonances at 5.3 kHz by a phase-delay element.
Fig. 11 shows a frequency response of $C_{pzt}$ (the feedback controller for the $P_{dps}$). $C_{pzt}$ has to compensate for the unstable characteristics in the $L_{cm}$. To do so, the gain of $L_{cm}$ should be canceled by the $L_{pzt}$ from 2 to 4 kHz. It must also provide the low-pass effect so that the gain of $L_{pzt}$ has low gain in the high-frequency range.

Fig. 12 shows the frequency responses of the open-loop characteristics (dashed lines: $L_{cm}$, dot-dashed lines: $L_{pzt}$, solid lines: $L_n$). Fig. 13 shows the gain frequency responses of $S_n$ and $T_n$ with the initial control system (solid line: $S_n$, dashed line: $T_n$).

3.3. Loop-shaping design with RBode plot

To demonstrate an effectiveness of the proposed method, we design a loop-shaping filter which improves a positioning accuracy of a track-following control under an external vibration in HDDs (Yabui, 2019). Here, the reference signal $r$ is set as 0, and the disturbance signal $d_c$ is set as a measured external vibration by using acceleration sensors in an actual file server. Fig. 14 shows an amplitude spectrum of the $d_c$.

The proposed loop-shaping methodology follows steps below.

**Step 1.** From the above mentioned initial DISO control system, we can set $C_1 = C_{cm}$ and $C_2 = C_{pzt}$.

**Step 2.** From the above mentioned initial DISO control system, we can set $P_1 = P_{dcm}$ and $P_2 = P_{dps}$. In order to compensate for discretization errors around the Nyquist frequency, we chose multiple uncertainties $|\Delta_1|$ and $|\Delta_2|$ to be

$$
|\Delta_1(\omega)| = |\Delta_2(\omega)| = \begin{cases}
0.03, & \omega \leq 2\pi \cdot 15000 \\
3, & \omega > 2\pi \cdot 15000
\end{cases} \quad (17)
$$

These multiple uncertainties indicate that error in the estimate of frequency response of the real plant is less than 3% below 15 kHz, but may be as large as 300% from 15 kHz.

**Step 3.** We calculated $|W_a|$ by using the following equation:

$$
|W_a(\omega)| = |\Delta_a(\omega)| = \frac{|C_1(\omega)P_1(\omega)|\Delta_1(\omega) + |C_2(\omega)P_2(\omega)|\Delta_2(\omega)|}{|C_1(\omega)P_1(\omega)| + |C_2(\omega)P_2(\omega)|}, \quad \forall \omega. \quad (18)
$$

The dashed line in Fig. 15 shows the frequency response of $W_a$.

**Step 4.** In order to compensate for the external vibration, we employed a frequency response of $|W_{d0}|$ which is a 50-point moving average of the amplitude spectrum of the disturbance signal $d_c$ in Fig. 14 and then multiplying the result by 40000 so that the initial control system satisfies inequality (11) below 100 Hz. Moreover, in order to make the $H_\infty$ norm of the sensitivity functions less than 10 dB, we defined the frequency response of $|W_s|$ as follows.

$$
|W_s(\omega)| = \begin{cases}
|W_{d0}(\omega)|, & \omega \leq 2\pi \cdot 1860 \\
-10, & \omega > 2\pi \cdot 1860
\end{cases} \quad (19)
$$
the open-loop gain at those frequencies. Therefore, we choose a notch filter frequency response at 15 and 18.3 kHz. This RBode plot also shows that we can eliminate the intersections by decreasing

\[ W_2 \]

Then, we are able to set \( C_{pkf} \) to be

\[ C_{pkf}(s) = \frac{2}{\pi} \left( \frac{s^2 + 2\zeta_{pkf1}\omega_{pkf1}s + \omega_{pkf1}^2}{\pi^2} \right) \]

(20)

where

\[ \omega_{pkf1} = 2\pi \cdot 200, \zeta_{pkf1} = 0.4, \zeta_{pkf2} = 0.15, \omega_{pkf2} = 2\pi \cdot 270, \zeta_{pkf3} = 2.1, \zeta_{pkf4} = 0.1. \]

Then, we are able to set \( C_{pkf}[z] \) that is discretized \( C_{pkf}(s) \) with a matched pole-zero method.

The RBode plot in Fig. 17 shows that there are intersections between the forbidden regions and the open-loop frequency response from 110 Hz to 1 kHz. This RBode plot also shows that the open-loop gain from 110 Hz to 1 kHz must be increased because we have no allowable region on the phase plot from 110 Hz to 1 kHz. Therefore, we choose a peak filter \( C_{pkf} \) to be

\[ C_{pkf}(s) = \frac{1}{\pi} \left( \frac{s^2 + 2\zeta_{pkf1}\omega_{pkf1}s + \omega_{pkf1}^2}{s^2 + 2\zeta_{pkf1}\omega_{pkf1}s + \omega_{pkf1}^2} \right) \]

(21)

where

\[ \omega_{pkf1} = 2\pi \cdot 15000, \zeta_{pkf1} = 0.01, \zeta_{pkf2} = 0.03, \omega_{pkf2} = 2\pi \cdot 18300, \zeta_{pkf3} = 0.01, \zeta_{pkf4} = 0.02. \]

Then, we are able to set \( C_{pkf}[z] \) that is discretized \( C_{pkf}(s) \) with the matched pole-zero method.

Fig. 18 shows the RBode plot by using \( L_n = C_{pkf}C_{ntc}L_{00s}, |W_n|, \) and \( |W_n| \). This RBode plot shows that we can eliminate the intersections from 110 Hz to 1 kHz, 15 kHz, and 18.3 kHz by using \( C_{pkf}[z] \) and \( C_{ntc}[z] \). However, there is a new intersection from 2 to 3 kHz. This RBode plot also shows that the open-loop gain or phase must be increased from 2 to 4 kHz to eliminate the intersection. In this case, a complex lead filter is well suited because it realizes a narrow band phase-lead effect by using complex poles and zeros (Messner et al. 2007). Therefore, we chose the complex lead filter \( C_{clf} \) as follows.

\[ C_{clf}(s) = \frac{1.37\bar{S}^2 + 2\zeta_{clf}\omega_{clf}s + \omega_{clf}^2}{\bar{S}^2 + 2\zeta_{clf}\omega_{clf}s + \omega_{clf}^2}, \]

(22)
Then, we are able to set robust performance criterion for all frequencies. Therefore, we defined the loop-shaping filter $C_L$ regions have been eliminated. This means that the nominal open-loop characteristics $C_L$. Fig. 20 shows the frequency response of system. To verify the effectiveness of the proposed control system, we performed a simulation of the track-following control under the measured external vibration shown in Fig. 14. In these simulations, the positioning accuracies (three standard deviations of $e$) with the initial controller is 5.24 nm, and that with the improved controller is 0.87 nm. This means that the proposed loop-shaping method is able to improve the positioning accuracy by about 83%. Where

$$\omega_{cln} = \omega_{cl} \left( -\zeta_{cl} \tan(\phi_{cl}/2) + \sqrt{-\zeta_{cl}^2 \tan^2(\phi_{cl}/2) + 1} \right), \quad \omega_{cld} = \omega_{cl} \left( \zeta_{cl} \tan(\phi_{cl}/2) + \sqrt{-\zeta_{cl}^2 \tan^2(\phi_{cl}/2) + 1} \right).$$

(23)

Then, we are able to set $C_{cl}[z]$ that is discretized $C_{cl}(s)$ with the matched pole-zero method. Fig. 19 shows the RBode plot by using $L_n = C_{pk}/C_{ntc}C_{cl}L_{n0}$, $|W_u|$, and $|W_d|$. In this RBode plot, we can see that the intersections between the open-loop frequency response and the forbidden regions have been eliminated. This means that the nominal open-loop characteristics $L_n = C_{pk}/C_{ntc}C_{cl}L_{n0}$ satisfies the robust performance criterion for all frequencies. Therefore, we defined the loop-shaping filter $C_{lt}$ by

$$C_{lt}[z] = C_{pk}[z]C_{ntc}[z]C_{cl}[z].$$

(24)

Fig. 20 shows the frequency response of $C_{lt}$. Fig. 21 (a) shows the frequency responses of $|W_u|^{-1}$ (dashed line) and $|S_n|$ with the loop-shaping filter (solid line). Fig. 21 (b) shows the frequency responses of $|W_u|^{-1}$ (dashed line) and $|T_n|$ with the loop-shaping filter (solid line). These figures show that the control system with the loop-shaping filter meets the specific robust performance criterion. Fig. 22 shows the comparison result of the nominal sensitivity functions (solid: $|S_n|$ in the initial control system, dashed: $|S_n|$ in the improved control system).

### 3.4. Performance evaluation

In this subsection, we have shown comparison studies between the initial control system and the improved control system. To verify the effectiveness of the proposed control system, we performed a simulation of the track-following control under the measured external vibration shown in Fig. 14. Fig. 23 shows the simulation results of $y_t$ under external vibrations with the initial and improved control systems ((a): time domain, (b): frequency domain). In this figure, the dashed lines represent the results with the initial control system, and the solid lines represent the results with the improved control system. In these simulations, the positioning accuracies (three standard deviations of $e$) with the initial controller is 5.24 nm, and that with the improved controller is 0.87 nm. This means that the proposed loop-shaping method is able to improve the positioning accuracy by about 83%. [DOI: 10.1299/jamdsm.2020jamdsm0053] © 2020 The Japan Society of Mechanical Engineers
4. Conclusion

In order to improve a robust performance in an MISO control system, we proposed a loop-shaping methodology with the RBode plot. Our proposed method transforms the robust performance problem for the MISO system into that for the SISO system so that we can analyze the robust performance of the MISO control system on the RBode plot. As a result, we can easily design an improved control system that compensates for disturbances against perturbations of controlled objects. A case study with the magnetic-head positioning control system shows that the proposed loop-shaping method enables us to improve the positioning accuracy under the external vibration.

Acknowledgement

This work is supported by JSPS KAKENHI Grant Number JP18K04210.

References

Atsumi, T., Emerging Technology for Head-Positioning System in HDDs, IEEJ J. Industry Applications, Vol. 5, No. 2, (2016a), pp. 117–122.
Atsumi, T., Analysis of open-loop characteristics on SISO sampled-data positioning control system, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 10, No. 4, (2016b), DOI: 10.1299/jamdsm.2016jamdsm0061.
Atsumi, T., Suzuki, K., Nakamura, S., and Ohta, M., Vibration Control with Thin-Film-Coil Actuator for Head-Positioning System in Hard Disk Drives, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 9, No. 1, (2015), DOI: 10.1299/jamdsm.2015jamdsm0010.
Atsumi T. and Messner, W. C., Optimization of Head-Positioning Control in a Hard Disk Drive Using the RBode Plot, The IEEE Transactions on Industrial Electronics, vol. 59, no. 1, (2012), pp. 521–529.
Doyle, J., Francis, B., and Tannenbaum, A., Feedback Control Theory, Macmillan Publishing Company, (1992).
Hirata, M, Liu, K., and Mita, T., Active vibration control of a 2-mass system using µ-synthesis with a descriptor form representation, Control Engineering Practice, vol. 4, no. 4, (1996), pp. 545–552.
Messner, W. C., Bedillion, M. D., Xia, L., and Karns, D. C., Lead and Lag Compensators with Complex Poles and Zeros, The IEEE Control Systems Magazine, vol. 27, no. 1, (2007), pp. 44–54.
Yabui, S., Compensation and identification for external disturbances in head positioning systems of hard disk drives based on a data-based design method, Mechanical Systems and Signal Processing, vol. 115, (2019), pp. 434–449.