A study on flow characteristics around a hemispherical dome

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Abstract. Large eddy simulation (LES) is used to investigate the unsteady flow around a hemispherical dome. The Reynolds number based on the diameter of the hemisphere is $2.4 \times 10^4$. The mean velocity field and fluctuation velocity field were systematically studied. By means of the distribution of mean pressure coefficient, the vulnerable position on the hemispherical dome was found. By means of mean velocity profile, three regions of different properties were divided, and their effects on other creatures and buildings were considered. The probability density function (PDF), joint probability density function (JPDF) and Lumley triangle are further studied. The different intensity of the intermittency of turbulence, the different proportion of ejection (Q2) and sweeping (Q4) events and the anisotropy of Reynolds stress are explained respectively in these three regions.

1. Introduction

Compared with the square building, the hemispherical dome can save materials, and at the same time contain a lot of space, which is also very beautiful from the appearance. Hemispherical domes are a good choice for large buildings. In the past, hemispheric domes have distorted or collapsed under the influence of strong winds, due to the strength of the materials and unreasonable design. Savory and Toy[1][2][3] studied the flow around the hemisphere in three different turbulent boundary layers through wind tunnel experiments (WTE). The Reynolds number varied from $1.31 \times 10^4$ to $1.31 \times 10^5$. Tavakol et al.[4] studied the air flow field near the installed hemisphere in two different boundary layers by experimental and numerical methods. Studies are performed for $Re = 3.2 \times 10^4$. Kharoua and Khezzar[5] investigated large eddy simulation (LES) study of turbulent flows around smooth and rough hemispherical domes and analyzed the profile of mean pressure coefficient and mean velocity at $Re = 1.4 \times 10^5$. Fu et al.[6] studied the aerodynamic characteristics of a hemispherical dome with $Re = 2 \times 10^6$ in smooth and turbulent boundary layer flows according to WTE and LES. Wood et al.[7] investigated experimental and numerical studies of turbulence passing through hemispherical obstacles in detail, including analysis of unsteady flow characteristics observed near hemispheres and discussion of time-averaged flow fields. Cheng and Fu[8] studied Reynolds number effect from $5.3 \times 10^4$ to $2 \times 10^6$ on a hemispherical dome in smooth and turbulent boundary layer flow through WTE. It is found that, in the former condition, the transition of separation flow occurs between $Re = 1.8 \times 10^5$~$3 \times 10^5$ ;the pressure distributions become Reynolds number independent at $Re > 3 \times 10^5$;For the latter, $Re < 1.1 \times 10^5$ and $Re = 10^5$~$2 \times 10^5$, respectively. Recently, Cao and Tamura[9] investigated Reynolds number effect from $7 \times 10^4$ to $7 \times 10^5$ on a wall-mounted hemisphere by using LES. The results show that the frequency ratio of arch vortex shedding to spanwise...
oscillation was approximately 4 at subcritical Re but decreased to 3 at supercritical Re. Here the critical value of the two Reynolds Numbers depends on the drag crisis.

In summary, previous studies have focused on the effects of hemispheric surface roughness and Reynolds number. The current study also focused on the rear of the domed building. This article will do the research from three parts. Section 2 briefly introduces the numerical simulation method and model. Section 3 analyzes the results in detail. Section 4 summarizes the current work and findings.

2. Numerical simulation method and model

2.1. Large eddy simulation method

The incompressible Navier-Stokes equation is spatially filtered and combined with the continuity equation according to Eq. (1). \( u_i (1,2,3) \) denote streamwise, vertical and spanwise velocity components, respectively, and are equivalent to the velocity components (u, v and w). The corresponding Cartesian coordinates are \( x_1, x_2 \) and \( x_3 \) (x, y and z), respectively.

\[
\begin{align*}
\frac{\partial \vec{u}_i}{\partial t} + \frac{\partial \vec{u}_i \vec{u}_j}{\partial x_j} &= -\frac{1}{\rho} \frac{\partial \vec{p}}{\partial x_i} + \nu \frac{\partial^2 \vec{u}_i}{\partial x_i \partial x_j} \\
\frac{\partial \vec{u}_i}{\partial x_i} &= 0 \\
\vec{u}_i \vec{u}_j &= \vec{u}_i \vec{u}_j + \left( \vec{u}_i \vec{u}_j - \vec{u}_i \vec{u}_j \right) \\
\tau_{ij} &= \left( \vec{u}_i \vec{u}_j - \vec{u}_i \vec{u}_j \right) = 2\left( \nu \Delta \right) S_{ij} \left( 2S_{ij} S_{ij} \right) + \frac{1}{3} \tau_{kk} \delta_{ij}
\end{align*}
\]

(1)

(2)

(3)

For Eq. (2), the term in brackets is subgrid stress. The subgrid stress model used in this paper is the Smagorinsky[10] eddy viscosity model according to Eq. (3).

2.2. Computational domain and hemisphere model

Hemispheric model location and computational domain are shown in figure 1, where \( d = 60mm \) is the diameter of a hemisphere and the domain is \( X \times Y \times Z = 2000mm \times 200mm \times 250mm \). The left side is the inlet, the right side is the outlet, and all around is the wall. The origin of coordinates is 560 mm away from the inlet. Free-stream velocity \( U_\infty = 0.4m/s \). The Re based on hemispheric diameter is \( 2.4 \times 10^4 \). The turbulence intensity is increased by 5 % at the inlet.

![Figure 1](image1.png)

Figure 1 Computational domain and hemisphere model.

![Figure 2](image2.png)

Figure 2 The grid system.

For LES, a fine grid is necessary. The hemisphere is divided into several regions and a fine mesh is defined for each region as shown in figure 2. The area around the hemisphere is also encrypted for fine simulation, with a grid of about four million.
3. Results and discussion

3.1. Time-averaged characteristics of flow field

Figure 3 shows the distribution of time-averaged pressure coefficient $C_p = \frac{1}{2} \rho U_0^2 \cdot \frac{d}{\delta} \cdot \theta_{(c)}$ on Z=0 plane.

It can be found that the maximum value of the mean pressure coefficient is about $20^\circ$ on the surface of the hemisphere. It can also be found that the minimum mean pressure coefficient occurs at about $80^\circ$ on the surface of the hemisphere. These two locations need to be reinforced with more material to avoid collapse.

![Figure 3 Time-averaged pressure coefficient distribution on Z=0 plane.](image)

Figure 4 Select research objects on Z=0 plane.

It is necessary to study not only the surface of the hemisphere but also the rear of the hemisphere. Because hemispherical domes also affect the lives of surrounding creatures and other structures. The study of the rear of the hemisphere is of great interest. In figure 4, the seven lines from X/d=1 to X/d=7 are selected as the research objects.

![Figure 4 Select research objects on Z=0 plane.](image)

Figure 5 first analyzes the average velocity profile. Figure 5(a) shows the mean streamwise velocity profile $U^*$. At X/d=1, the mean streamwise velocity was negative near the wall due to the influence of the backflow area behind the hemisphere. According to the distribution of mean streamwise velocity, it can be divided into three regions. Region 1 is the region Y/d is less than 0.1. In this region it can be seen that the mean streamwise velocity quickly recovers without being affected by the hemisphere. And it can be found that the closer to the wall, the faster the recovery of the mean streamwise velocity. This may be due to the fact that small scale structures near the wall react faster. Region 2 is the region where Y/d is less than 0.9 and greater than 0.1. This region is most affected by the hemisphere, and the effect can last far downstream. The recovery of mean streamwise velocity in this region is slow. This may be because the region is dominated by large scale structures, which are slow to react. Region 3 is the region where Y/d is greater than 0.9. In this region, the mean streamwise velocity remained basically unchanged and was not affected by the hemisphere. Figure 5(b) shows the mean vertical velocity profile. It can be divided into three regions equal to the mean vertical velocity. The only difference is region 3. The mean vertical velocity was affected by the hemisphere, but recovered quickly.
3.2. Fluctuation characteristics of flow field

The fluctuation velocity field is analyzed in the three regions divided by the mean velocity profile in figure 6. In figure 6 (a), the probability density function (PDF) of streamwise fluctuation velocity in region 1 shows two peaks, which may be hairpin leg and hairpin neck. All the PDF of streamwise fluctuation velocity deviate from the gaussian distribution, which is caused by the intermittency of turbulence. It can be seen from figure 6(b) that the PDF of the vertical fluctuation velocity deviates from the gaussian distribution to different degrees. From region 1 to region 3, the greater the distance from the wall, the greater the turbulence intermittency. This may be because the disturbance caused by the hemisphere reduces the intermittency of turbulence.

Then the joint probability density function (JPDF) of the fluctuation velocity is analyzed. It can be seen from figure 7(a) that the near-wall area is dominated by ejection events in quadrants II (featuring $u' < 0$ and $v' > 0$ associated with Q2) and sweeping events in quadrants IV (featuring $u' > 0$ and $v' < 0$ associated with Q4). This also indicates that the Reynolds stress $-\langle u'v' \rangle$ is positive in the near-wall region. As can be seen from region 2 in figure 7(b), Q4 accounts for a little more than the other three quadrants. From region 3 in figure 7(c), it can be seen that the proportions of the four quadrants are consistent, possibly because this region is not affected by the wall and hemisphere and develops peacefully.
Through the above study, it is found that the results under these three regions are different, and then the Reynolds stress anisotropy is studied by means of the Lumley triangle in figure 8. Two streamwise direction positions $X/d=3$ and $7$ are selected as the research objects in figure 8(a) and (b). Very close to the wall in region 1 ($Y/d < 0.025$) the turbulence is essentially two-component. The anisotropy reaches a peak at $Y/d=0.025$ in figure 8(b). The remainder of the region 1 the Reynolds stress is close to being axisymmetric with $\zeta$ positive. In region 2 and 3, the Reynolds stresses are close to axisymmetric.

4. Conclusion

In the present study, the unsteady flow around a hemispherical dome with $Re = 2.4 \times 10^4$ is studied by LES method. The time-averaged characteristic and fluctuation characteristic of the flow field are studied systematically. The main findings and conclusions are as follows:

1) By studying the distribution of the mean pressure coefficient in the hemisphere, it is found that its maximum and minimum values are at 20 degrees and 80 degrees respectively, and these two positions need to be strengthened to avoid collapse.

2) Through the study of average velocity profile, three regions were found. In region 1 and 3, the mean velocity recovered quickly, while in region 2, the mean velocity needed a longer distance to recover.

3) The further away from the wall, the more intermittent the turbulence was, according to the PDF study of these three regions. According to the study of JPDF, region 1 is dominated by ejection (Q2) and sweeping (Q4) events, while region 2 is slightly dominated by sweeping (Q2) events, and the proportion of four quadrants in region 3 is consistent.

4) By studying the anisotropy of Reynolds stress, it is found that the turbulence near the wall in region 1 is essentially two-component, while the Reynolds stresses in other regions are nearly axisymmetric.
Acknowledgments

The research is sponsored by Shuguang Program supported by Shanghai Education Development Foundation and Shanghai Municipal Education Commission in China via project 18SG53, the National Natural Science Foundation of China project 91952102 and 12032016.

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