Dirac point formation revealed by Andreev tunneling in superlattice-graphene/superconductor junctions

Shirley Gómez Páez, Pablo Burset, Camilo Martínez, William J. Herrera, and Alfredo Levy Yeyati

Departamento de Física, Universidad Nacional de Colombia, Bogotá, Colombia
Departamento de Física, Universidad el Bosque, Bogotá, Colombia
Department of Applied Physics, Aalto University, 00076 Aalto, Finland
Departamento de Física Teórica de la Materia Condensada and Condensed Matter Physics Center (IFIMAC) and Instituto Nicolás Cabrera, Universidad Autónoma de Madrid, E-28049 Madrid, Spain

A graphene superlattice is formed by a one-dimensional periodic potential and is characterized by the emergence of new Dirac points in the electronic structure. The group velocity of graphene’s massless Dirac fermions at the new points is drastically reduced, resulting in a measurable effect in the conductance spectroscopy. We show here that tunnel spectroscopy using a superconducting hybrid junction is more sensitive to the formation of Dirac points in the spectrum of graphene superlattices due to the additional contribution of Andreev processes. We examine the transport properties of a graphene-based superlattice–superconductor hybrid junction and demonstrate that a superlattice potential can coexist with proximity-induced superconducting correlations. Both effects contribute to change graphene’s spectrum for subgap energies and, as a result, the normalized tunneling conductance features sharp changes for voltages proportional to the energy separation between the original and the newly generated Dirac points. Consequently, the superconducting differential conductance provides an excellent tool to reveal how the new Dirac points emerge from the original band. This result is robust against asymmetries and finite-size effects in the superlattice potential and is improved by an effective doping comparable to the superconducting gap.

I. INTRODUCTION.

Graphene is a versatile material that can be modified to be metallic or semiconducting, owing to its gapless, linear low-energy spectrum. This duality can be exploited in graphene superlattices, formed by a periodic, one-dimensional electrostatic potential on the graphene sheet, making graphene a promising candidate for designed electronic circuits. It is theoretically established that charge carriers in graphene behave like chiral Dirac fermions. Under a one-dimensional superlattice potential of amplitude \( U \) and period \( L \), see Fig. 1, chirality forbids the opening of a band gap, instead creating new Dirac points (DPs) when the product \( U \cdot L \) reaches a critical value.

The propagation of chiral Dirac fermions under superlattices is highly anisotropic and can be controlled by varying the superlattice potential and Fermi energy. The resulting carrier velocity can be completely suppressed along one direction, but remains barely unchanged in the opposite, allowing for collimated electron beams. Experimental realization of high-quality periodic superlattice potentials on graphene has been achieved using boron nitride encapsulation. Dirac point formation was measured as resistivity peaks, paving the way for novel and exotic physics in graphene-based superlattices.

The recent discovery of unconventional superconductivity in bilayer graphene superlattices has pointed to the interesting connection between superconducting correlations and superlattice potentials in graphene. However, to the best of our knowledge, the interplay between superconductivity and Dirac point formation by one-dimensional superlattices has not been explored yet. Graphene–superconductor hybrids can now be fabricated in high-quality transparent junctions that work in the ballistic regime. In such hybrid junctions, electrons and holes from the conduction band of the normal lead combine to form Cooper pairs in the superconductor by means of a microscopic process known as Andreev reflection. When the doping is smaller than the applied voltage and the superconducting gap, graphene’s peculiar gapless dispersion allows for an unusual Andreev reflection where conduction band electrons are converted into holes belonging to the valence band. Here, we demonstrate that these inter-band Andreev processes provide unique signatures of the formation of Dirac points, which could substantially facilitate their experimental detection.

![FIG. 1. Semi-infinite superlattice (SL) coupled to a superconductor (S) on top of a graphene layer, with an sketch of the system’s energy diagram.](image-url)
Advances in experimental control of graphene devices are leading to a series of remarkable works reporting inter-band Andreev reflections, spectroscopy of Andreev bound states in Josephson junctions, splitting of Cooper pairs, and proximity-induced unconventional superconductivity. Since a superlattice potential can be seen as a series of p-n junctions, an example of the potential applications of superlattice–superconductor hybrids is the recently measured focusing of beams of Dirac fermions in graphene-based p-n junctions. Existing theoretical works extend such effects to graphene junctions involving ferromagnets and superconductors.

In this paper, we analyze the interplay between a superlattice potential and proximity-induced superconductivity on graphene. We focus on the effect of the emergence of new DPs on the transport properties of graphene. We consider an infinite graphene plane where a superlattice potential is created on the superlattice potential. Since a superlattice potential \( V(x) \) is created on the superlattice potential, \( \hat{H}_{\text{DBdG}} = \left( \hat{h} + V(x) \right) \hat{\sigma}_0 \), the conserved component of the wave vector parallel to the interface, \( v_F \) the Fermi velocity, and \( \hat{\sigma}_0 \) the Pauli matrices in sublattice space. The superlattice potential consists of a periodic repetition of potential barriers and wells with height \( m \hat{u}_{n,p} \) and width \( W_{n,p} \), cf. Fig. 1. Explicitly,

\[
V(x) = \begin{cases} 
E_F + U_n, & x \in [mL, mL - W_n] \\
E_F - U_p, & x \in [mL - W_n, (m - 1) L] 
\end{cases},
\]

with \( m = 0, -1, -2, \ldots \), and \( L = W_n + W_p \) the period of the superlattice potential.

We denote the Fermi energy in the superconducting regions as \( E_{F,S} \). We consider rigid boundary conditions for a conventional proximity-induced s-wave pairing \( \Delta(x) = \Delta \hat{\sigma}_0 \Theta(x) \), with \( \Delta > 0 \) the pairing amplitude and \( \Theta(x) \) the Heaviside step function. Approximating the spatial dependence of the pairing potential by a step function is valid as long as the Fermi wavelength of the quasi-particles in the superconductor is much smaller than the Cooper pairing coherence length \( \xi_0 = \hbar v_F / \Delta \), i.e., \( E_{F,S} \ll \Delta \).

The transport properties of the SL-S junction are encoded in the retarded Green function \( \hat{g}_0(x, x') = \int dq e^{i q(y - y')} \hat{g}_0(q, x', y, y') \), which satisfies the nonhomogeneous DBdG equation

\[
[(E + i0^+) \mathcal{I} - \hat{H}_{\text{DBdG}}] \hat{g}_0(x, x') = \delta(x - x') \mathcal{I},
\]

with \( \mathcal{I} \) the identity matrix. A solution of Eq. (3) is obtained combining asymptotic solutions of Eq. (1) that obey the boundary conditions at the edges of a finite length graphene sheet, following a generalization of the method developed in Refs. 36, 38–46. The Green function for the SL can then be written as

\[
\hat{g}_{\text{SL}}(0^+, 0^-) = \frac{i}{2 \hbar v_F C_n} \begin{pmatrix} 0 & 2C_n \\ 0 & M + \sqrt{J + M^2} \end{pmatrix},
\]

with

\[
M = C_p C_n - D_{pn}^2 - 1, \quad J = 4C_n C_p,
\]

\[
C_{p(n)} = \frac{C_p(n) \left( C_{p(n)}^2 + D_{n(p)}^2 \right) + C_n(n) + 1}{C_p(n) + 1}, \quad D_{pn} = \frac{i \delta_{p(n)} d_{n}}{C_p(n) + 1},
\]

and

\[
c_{n(p)} = \frac{e^{-i \alpha \chi(x)}}{1 + e^{-i \alpha \chi(x)}} \frac{C_{n(p)}^2}{C_p(n(p)) \left( 1 + e^{-2i \alpha \chi(x)} \right)}.
\]

Here, \( e^{\pm i \alpha \chi(x)} = \hbar v_F (k_x^{(h)}[x] \pm i q) / (\epsilon \pm E) \) and \( k_x^{(h)}[x] = \sqrt{[(\epsilon + E)/\hbar v_F]^2 - q^2} \), with \( \epsilon \) and \( E \) the excitation and potential energies, respectively. We consider a transparent coupling between the superlattice and the superconductor. For more details of the calculations, we refer the reader to Appendix A.

The differential conductance depends on the potential difference between SL and S, \( V = V_{\text{SL}} - V_S \), and can be written as

\[
\sigma_S(V) = \frac{\partial I}{\partial V} = \sigma_Q(V) + \sigma_A(V),
\]

where \( \sigma_Q(V) \) is the quantum conductance and \( \sigma_A(V) \) is the Andreev conductance.
where $\sigma_A$ (or $\sigma_Q$) represents the contribution of the Andreev (quasiparticle) processes. Here,

$$\sigma_Q(E) = 8\pi^2 \frac{e^2}{\hbar} \int dq \tilde{\sigma}_Q(E, q),$$

with $\tilde{\sigma}_Q(E)$ defined in terms of the Green functions in Appendix B. We normalize our results using the conductance for $\Delta = 0$, $\sigma_N$.

### III. IDEAL SUPERLATTICE

We analyze the transport properties of a SL-S junction to illustrate how the emergence of Dirac points by the superlattice potential is neatly captured in the subgap differential conductance. We start considering an ideal superlattice potential, i.e., semi-infinite, created around the charge neutrality point, $E_F = 0$, and symmetric with $U_p = U_n \equiv U$ and $W_p = W_n = L/2$, see Fig. 1. The normalized barrier strength is thus given by $u = UL/\hbar v_F$. Under these conditions, a new set of DPs is created when $u = 2\pi n$, with $n$ a positive integer.

As shown in Fig. 2(a,b), the superlattice potential can coexist with proximity-induced superconducting pairing in the superlattice region close to the interface with the superconductor. Indeed, the energy-momentum plots in Fig. 2(a), calculated for several values of the superlattice strength, clearly show how the band dispersion relation for subgap energies is modified after the first (panels A, B, C) and second (D, E) generation of DPs. The condition for the formation of DPs is thus not altered by the presence of the superconductor. Setting $E = 0$, we plot in Fig. 2(b) the spectral conductance $\sigma(E = 0, q)$ as a function of the transverse momentum $\hbar q$ and the barrier strength $u$. The first pair of DPs is formed at the critical value $u = 2\pi$, and the second when $u = 4\pi$ is reached.

The spectral density $\sigma(E, q)$ has been calculated in the SL region close to the interface with the superconductor. We identify the menorah-like structure of Fig. 2(b) as the fingerprint of the superlattice. In the absence of the superlattice potential, a small trace of the original Dirac cone centered around $q = 0$ remains for energies below the superconducting gap $\Delta$, cf. Refs. 47–49. The superlattice potential changes the Fermi velocity, thus widening graphene’s cone-like spectrum, even before creating new DPs. Importantly, when coupled to a superconductor, the spectral density of states for subgap energies is higher compared to the non-superconducting case with $\Delta = 0$, which can be seen as a sharp change in color for energies below and above the gap in Fig. 2(a).

Finally, as $u$ increases beyond the critical value [panels A-C in Fig. 2(a)], a new pair of Dirac points appears. The original DP and the newly created ones are disconnected at zero energy, but they merge for finite energies over a characteristic value $\varepsilon$, indicated by white arrows in Fig. 2(a). The parameter $\varepsilon$ becomes finite for $u > 2\pi$ and increases with $u$ until it approaches $\Delta$ as a new pair of DPs is completely formed when $u = 4\pi$. 

**FIG. 2.** Undoped, symmetric graphene superlattice. a) Spectral differential conductance for different values of the superlattice potential $u/(2\pi) = 1.03(A), 1.11(B), 1.19(C), 2.15(D), 2.23(E)$. b) Map of the zero-energy spectral differential conductance as a function of $u$ and $q = q/q_{\text{max}}$, with $\hbar v_F q_{\text{max}} \simeq 0\Delta$. c) Differential conductance as a function of the energy for the different values of $u$ used in a). The parameter $\varepsilon$ indicates the separation of the new pair of Dirac points from the original cone. d) Differential conductance as a function of $u$ for different energies. e) Estimation of $\varepsilon$ from the second derivative of the current with respect to the voltage. For all cases $E_F = 0$ and $L = \xi_0/2$.
At this critical value, \( \varepsilon \) is set to zero, and becomes finite again when \( u > 4\pi \). The energy parameter \( \varepsilon \) thus provides a qualitative measure of the separation between the original DP and every new pair.

These three effects, namely, (i) the formation of DPs when \( u = 2n\pi \); (ii) an enhanced spectral density for subgap energies; and (iii) the separation between DPs characterized by the parameter \( \varepsilon \), can be neatly observed in the normalized differential conductance \( \sigma_S/\sigma_N \). Fig. 2(c) and (d) show the normalized differential conductance of the SL-S junction as a function of the excitation energy and superlattice strength, respectively. In the absence of superlattice potential, since \( E_F = 0 \), we recover the conductance results of Ref. 21 where inter-band Andreev reflection is dominant (gray line with \( u = 0 \)). For \( u \neq 0 \), the normalized conductance at zero energy remains fixed at \( 2\sigma_0 \), except at the critical values where DPs are formed, see Fig. 2(d). Since the normal state conductance \( \sigma_0 \) accounts for the contribution from three channels, the doubling of the conductance is due to perfect Andreev reflection taking place at the original and the new DPs.

By contrast, for finite energies we observe two effects connected to the presence of a superlattice potential. First, the conductance features a peak or a dip for energies equal to the parameter \( \varepsilon \) [see arrows in Fig. 2(c)]. Second, the superlattice increases the conductance for energies close to the gap \( (E \lesssim \Delta) \). The latter is a direct consequence of the superlattice-enhanced spectral density for subgap energies. The former can be associated to the different contribution from Andreev and normal reflections. For small values of \( \varepsilon \), the normalized conductance exhibits a peak indicating that inter-band Andreev processes become dominant when the DPs are created. As \( \varepsilon \) approaches \( \Delta \), the peak in the conductance becomes a dip since the DPs are more clearly separated thus producing more backscattering for \( q \) values between DPs. In both cases, the parameter \( \varepsilon \), which indicates the separation between the newly created DPs and the original band, is accessible through the differential conductance. By taking the derivative of the conductance with respect to the voltage (i.e., \( d^2I/dV^2 \)), the small kinks in the conductance shown in Fig. 2(c) appear as clear peaks in the derivative, cf. Fig. 2(e).

The above results show that a superlattice potential can create new DPs in the presence of superconducting correlations. As long as the strength of the potential is comparable to the superconducting gap, the zero-energy normalized conductance is completely dominated by inter-band Andreev processes and fixed to \( 2\sigma_0 \). For finite energies, the normalized conductance features a sharp change at \( E = \varepsilon \), which determines the energy separation between Dirac cones in the dispersion relation. The differential conductance of a SL-S junction and its derivative thus provide a very sensitive tool to study the creation of DPs by a superlattice potential.

![FIG. 3. Doped symmetric graphene superlattice. a) Spectral differential conductance with \( E_F = \Delta/2 \) for different values of \( u/(2\pi) = 1.03(A), 1.11(B), 1.19(C) \). In all maps \( \hbar q_{\text{max}} \simeq 16\Delta \). b) Differential conductance as a function of the energy for the different values of \( u \) in a). The arrows indicate the parameter \( \varepsilon \). c) Differential conductance as a function of \( u \) for different energies. In all cases \( L = \xi_0/2 \).](Image 323x626 to 572x746)

IV. NON-IDEAL SUPERLATTICE

We now consider three deviations from the ideal superlattice potential described above. Namely, a finite doping on the graphene layer \( E_F \neq 0 \), an asymmetry in the superlattice potential, or a SL region of finite length.

A. Electrostatic asymmetry

We start analyzing the effect of an asymmetry on the superlattice potential. First, we consider the effect of a finite doping on the graphene layer with a perfectly symmetric SL potential with \( U_n = U_p \) and \( W_n = W_p \). Doping the graphene layer with \( E_F \neq 0 \) shifts the position where the new DPs are created, cf. Fig. 3(a), but it does not change the condition for the their formation, \( u = 2n\pi \). If \( E_F < \Delta \), shifting the position of the DP to finite energies results in an enhanced suppression of the conductance, see Fig. 3(b,c), due to the vanishing density of states for hole-like excitations.\(^{21,47}\) This is a unique property of graphene’s gapless spectrum which, interestingly, is not affected by the creation of new Dirac points, in contrast to the undoped case with \( E_F = 0 \) [compare Fig. 2(d) and Fig. 3(c)]. Since the splitting of the energy band into several DPs now takes place completely in the positive energy range (if \( E_F > 0 \)), the range of energies where
normal reflections are enhanced is now $2\epsilon$. As before, an analysis of the derivative of the conductance allows us to estimate the value of $\epsilon$, corresponding to the change of slope in the conductance. A finite doping of the graphene layer, in the regime where inter-band Andreev reflections are enhanced, thus helps visualizing the formation of DPs using the normalized conductance.

An asymmetry in the superlattice potential has an important effect on the formation of new DPs. The asymmetry may be induced by changing the relative height ($U_p \neq U_n$) or width ($W_p \neq W_n$) of the potential barriers and wells. We can thus parametrize it defining $\alpha = W_n/W_p$ and $\beta = U_n/U_p$. For an asymmetric superlattice, it is useful to define the average potential as the integral over a period $\beta$,

$$\langle \tilde{U} \rangle = \frac{1}{\beta} \int_{0}^{\beta} \tilde{U} \, dx$$

where the doping was adjusted so that $E_F/\Delta = (1 + \alpha)/2$. The position of the original DP under an asymmetric superlattice is given by $E_F^\star$, so we henceforth refer to it as the effective Fermi energy. It is now possible to redefine the potential in Eq. (2) as

$$V(x) = \begin{cases} E_F^\star + \omega \frac{1 + \beta}{\alpha \beta - 1}, & x \in [mL, mL - W_n] \\ E_F^\star - \omega \frac{\alpha + \beta}{\alpha \beta - 1}, & x \in [mL - W_n, (m - 1) L] \end{cases}$$

(6)

For the case where the potential barriers and wells have the same width ($\alpha = 1$) but different heights ($\beta \neq 1$), following the sketch of Fig. 1, the barriers and wells take the values $E_F^\star(1 + \beta)/2$, respectively, with $(V) = U_p(1 + \beta)/2$. The main effect of the asymmetry is to shift the Fermi energy to the effective one, $E_F^\star$, where the original Dirac cone is and the new DPs are created, cf. Fig. 4(a). For reference, we also plot in Fig. 4(a) a case with a symmetric SL potential taken from Fig. 3(b) (blue line). By changing the value of $\beta$, the conductance features a dip at different energies below the gap, as long as $E_F^\star < \Delta$ (green line). If the asymmetry is such that $E_F^\star > \Delta$, the conductance does not exhibit any dip and approaches the case of a heavily-doped graphene layer, even if $E_F < \Delta$ (dashed gray line). The doping $E_F$ of the graphene layer thus provides an experimentally controllable parameter that can rectify any asymmetry in the height of the barriers and wells of the SL potential.

### B. Spatial asymmetry

When the asymmetry on the SL potential affects the widths of the barriers and wells ($\alpha \neq 1$), the impact on the conductance is more pronounced. In this situation, the position of the original DP is still given by $E_F^\star$, but the new DPs appear at different energies. For example, the real doping $E_F$ in the three panels of Fig. 4(b) has been adjusted so that all cases have $E_F^\star = \Delta/2$. However, the barrier’s width is bigger than the corresponding for the wells in the left panel ($\alpha > 1$), while it is smaller in the other panels ($\alpha < 1$). As a result, the new DPs are created for energies below or above the effective Fermi energy $E_F^\star$, respectively. In the right panel, the superlattice is so asymmetric that the new DPs merge back into the original Dirac cone, recovering the result for a heavily-doped graphene layer.

The spatial asymmetry drastically changes the conductance, as we show in Fig. 4(c). As a reference, we show a symmetric result with finite doping $E_F = \Delta/2$ (blue line) and compare it to the asymmetric cases from Fig. 4(b), where the doping was adjusted so that $E_F^\star = \Delta/2$. When $\alpha < 1$ (green line), the new DPs appear for energies bigger than the effective Fermi energy and their impact on the conductance is diminished. On the other hand, if $\alpha > 1$ (red line), the new DPs appear for smaller energies and can be clearly seen as kinks in the conductance. The case with a high asymmetry (gray line) approximates the conductance of a doped graphene layer with dominant intra-band Andreev reflection.

Our results thus show the importance of a regularly spaced superlattice potential, while the asymmetry in the electrostatic barriers can be compensated by an uniform change in the doping level of the graphene layer.
FIG. 5. Finite superlattice potential. (a) Spectral differential conductance for \( N = 50 \), with \( E_F = 0 \) (left) and \( E_F = \Delta/2 \) (right). In both cases \( \hbar v_F q_{\text{max}} \approx 14\Delta \). (b,c) Differential conductance for different number of n-p junctions \( N \) of the finite superlattice potential, with \( E_F = 0 \) (b) and \( E_F = \Delta/2 \) (c). The gray line recovers the semi-infinite superlattice potentials in Fig. 2 and Fig. 3. In all cases, \( L = \xi_0/2 \).

C. Finite-length superlattice

The emergence of new Dirac points and their separation from the original cone in a semi-infinite SL potential can be discerned in the differential conductance by the parameter \( \varepsilon \). We now analyze the impact of a finite length SL potential on the previous results. We consider a finite-size graphene SL contacted on one side (\( x = 0 \)) to a superconductor, and on the other side (\( x = -NL \)) to a normal state reservoir, which we model as a heavily-doped graphene semi-infinite layer\(^{36,37,47}\). Here, \( L \) is the size of a n-p junction and \( N \) the total number of n-p junctions in the finite SL region. Nano-scale hybrid junctions where the reservoirs and the intermediate scattering region are built from different materials present very different interface transmissions\(^{31,52}\). For simplicity, we only consider here transparent couplings between the finite SL and the normal and superconducting leads and a symmetric SL potential.

The finite length of the intermediate region results in the splitting of the continuous Dirac cone into energy bands, cf. Fig. 5(a). In the presence of a SL potential, the condition for the emergence of new DPs, \( u = 2n\pi \), is roughly maintained even for a very small SL. A menzel-like pattern similar to that of Fig. 2(b) emerges for small lengths, although the resonances at the positions of the DPs are broadened and not so well defined. The broadening of the dispersion relation for subgap energies is shown in Fig. 5(a) for the undoped (left panel) and doped cases (right panel). The blurring of the DPs results in an increased probability of normal backscattering. We show the finite-length effect in the differential conductance in Fig. 5(b) and (c) for \( E_F = 0 \) and \( \Delta/2 \), respectively. For reference, the gray lines show the behavior of a semi-infinite superlattice potential with the same parameters. We confirm the robustness of the effect of the SL in the differential conductance for all energies different than \( E_F \). At the charge neutrality point \( E_F \), the semi-infinite case is qualitatively reproduced for superlattice lengths \( N \gtrsim 50 \), with \( L = \xi_0/2 \). However, to estimate the value of \( 2\varepsilon \) for a doped superlattice, only a few p-n junctions (\( N \sim 20 \)) are needed.

V. CONCLUSIONS

We have analyzed the transport properties of a graphene SL-S junction. We have demonstrated that the superlattice potential can create new DPs even in the presence of superconducting correlations. Moreover, the changes that the SL potential causes on the graphene’s spectrum are enhanced for subgap energies. Therefore, the emergence of new DPs can be easily monitored through the differential conductance of the junction. The normalized conductance features sharp changes for energy values equal to a parameter, \( \varepsilon \), determined by the separation between newly created DPs and the original cone. Further, the superconducting conductance is always enhanced over the normal state one for energies below but close to the gap \( \Delta \). These effects are robust in the presence of asymmetry in the superlattice potential and finite-size, as long as the formation of new DPs is possible. Our results thus suggest that superconducting hybrid junctions are useful to experimentally observe the modification of the band dispersion relation due to a superlattice potential on a graphene layer.

ACKNOWLEDGMENTS

We acknowledge funding from DIEB, Universidad Nacional de Colombia project No. 34916, the Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie Grant No. 743884, Academy of Finland (project No. 312299), and Spanish MINECO via grants FIS2015-74472-JIN (AEI/FEDER/UE), FIS2014-55486P, FIS2017-84860-R and through “María de Maeztu” Programme for Units of Excellence in R&D (MDM-2014-0377).

Appendix A: Superlattice Green function.

To analyze the transport properties of the SL-S junction, we calculate the Green function of the system. The
building block for a semi-infinite graphene in the super-lattice space is the Green function of an isolated zigzag graphene layer finite in the $x$-direction:

\[
\tilde{g}_0^{(>)}(x, x') = -i \frac{\pi}{\hbar v_F} \left( 1 - i \frac{p}{L} e^{i k_x |x' - x|} \right) \cos \alpha \\
\times \left( e^{i k_x |x' - x|} f_\pm + \frac{p}{R} e^{-i k_x |x' - x|} f_\pm \right) \\
+ \frac{p}{L} e^{i k_x (x + x')} h_\pm + \frac{p}{R} e^{-i k_x (x + x')} h_\pm,
\]

(A1)

with

\[
\hat{f}_\pm = \begin{pmatrix} 1 & \pm e^{\pm (\pm i \alpha)} \\ \pm e^{\pm (\pm i \alpha)} & 1 \end{pmatrix}, \quad \hat{h}_\pm = \begin{pmatrix} e^{\pm (\pm i \alpha)} & \mp 1 \\ \mp 1 & e^{\pm (\pm i \alpha)} \end{pmatrix},
\]

and

\[
\hat{r}_L^A = -e^{i \alpha} e^{-2ik_x x_L}, \quad \hat{r}_R^A = -e^{-i \alpha} e^{2ik_x x_R}, \\
\hat{r}_L^B = e^{i \alpha} e^{-2ik_x x_L}, \quad \hat{r}_R^B = e^{-i \alpha} e^{2ik_x x_R}.
\]

Here, $v_F$ is the Fermi energy and velocity, respectively, of the graphene sheet. The symbol < (>) indicates $x < x'$ ($x > x'$). The explicit calculation of Eq. (A1) is given in Ref. 36.

One superlattice period (n-p junction) is composed of two finite graphene regions with different Fermi energies, and widths $W_{p,n}$. The Green function for each region is given by Eq. (A1) and they can be coupled using Dyson’s equation to obtain the Green function of the finite n-p junction. Assuming that the coupled region extends from $x_L = -L$ to $x_R = 0^-$, we obtain the Green functions

\[
\hat{g}_n^p (-L, -L) = -i \frac{\hbar v_F}{\pi} \begin{pmatrix} C_p & 0 \\ 0 & 1 \end{pmatrix}, \quad (A2a)
\]

\[
\hat{g}_n^p (-L, 0^-) = -i \frac{\hbar v_F}{\pi} \begin{pmatrix} 0 & -D_{pn} \\ 0 & 0 \end{pmatrix}, \quad (A2b)
\]

\[
\hat{g}_n^p (0^-, 0^-) = -i \frac{\hbar v_F}{\pi} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad (A2c)
\]

\[
\hat{g}_n^p (0^-, -L) = -i \frac{\hbar v_F}{\pi} \begin{pmatrix} 0 & 0 \\ 0 & -D_{pn} \end{pmatrix}, \quad (A2d)
\]

The Green function for a finite superlattice of length $N_L$, with $L$ the period of a n-p junction, is obtained coupling Eq. (A2) to each other $N$ times. The resulting Green function reads

\[
\tilde{g}_{nl} = -i \frac{\hbar v_F}{\pi} \begin{pmatrix} 0 & -C_{nn} \\ -C_{nn} & 0 \end{pmatrix}, \quad (A3)
\]

\[
C_{nn} = \frac{C_n^{N-1} 2(C_{pp}^{N-1} - D_{pp}^{N-1}) + C_p^{N-1}}{C_n^{N-1} C_p^{N-1} - 2(C_p^{N-1} D_{pp}^{N-1})}.
\]

The Green function of the semi-infinite superlattice is calculated using the self-similarity of a semi-infinite chain. Adding one block to a semi-infinite number of graphene n-p blocks results in the same semi-infinite chain. Therefore, it is possible to analytically calculate the Green function for the complete superlattice evaluated at one edge, $\hat{g}_{sl}(0^-, 0^-)$, by imposing that it is equal the Green function of the superlattice when a new block $\hat{g}_{np}$ has been added. Using Dyson’s equation we find

\[
\hat{g}_{sl}(0^-, 0^-) = \hat{g}_{np}(0^-, 0^-)
\]

\[
-\hat{g}_{np}(0^-, -L) \hat{\Sigma} \hat{M}^{-1} \hat{g}_{sl}(-L, -L) \hat{\Sigma} \hat{g}_{np}(-L, 0^-),
\]

with

\[
\hat{M} = I - \hat{g}_{sl}(-L, -L) \hat{\Sigma} \hat{g}_{np}(-L, -L) \hat{\Sigma}.
\]

Solving Eq. (A4), we obtain Eq. (4) of the main text.

**Appendix B: Differential conductance for the superlattice–superconductor junction.**

Once the Green function for the coupled SL-S system is obtained, the electric current follows from Keldysh formalism. Following the extension of the Hamiltonian approach described in Refs. 37 and 53, the zero-temperature differential conductance reads as

\[
\hat{\Sigma}_Q = t^2 \text{Tr} \left[ \text{Re} \left( \hat{G}^r_{t,ee} \sigma_1^T \tilde{\rho}_{sc,ee} \left( \hat{I} + t \hat{G}^r_{t,ee} \rho_{sl,ee} \right) \right) \right] + t^4 \text{Tr} \left[ \text{Re} \left( \hat{G}^r_{t,eh} \tilde{\rho}_{sl,ee} \hat{G}^r_{t,eh} \sigma_1^T \tilde{\rho}_{sc,eh} \sigma_1 \right) \right],
\]

(B1)

\[
\hat{\Sigma}_Q = -i t^3 \text{Tr} \left[ \text{Re} \left( \hat{G}^r_{t,ee} \sigma_1^T \tilde{\rho}_{sc,ee} \left( \hat{I} + t \hat{G}^r_{t,ee} \rho_{sl,ee} \right) \right) \hat{G}^r_{t,ee} \sigma_1^T \right],
\]

(B2)

\[
\hat{\Sigma}_A = t^4 \text{Tr} \left[ \text{Re} \left( \hat{G}^r_{sc,eh} \tilde{\rho}_{sl,ee} \hat{G}^r_{sc,eh} \tilde{\rho}_{sl,ee} \hat{G}^r_{sc,eh} \tilde{\rho}_{sl,ee} \hat{G}^r_{sc,eh} \tilde{\rho}_{sl,ee} \right) \right],
\]

(B3)

with $\tilde{\rho}_{sl,eh} = \sigma_1 \tilde{\rho}_{sl,ee} \sigma_3$. The density of states $\rho_{sl}$ and $\rho_{sc}$ are related to the Green functions of the decoupled system at equilibrium, and are defined as $\rho(E) = \frac{1}{2\pi} \left( g - g' \right)$, where $sl$ denote the superlattice and $sc$ the superconductor at $x = 0$. 

The Green functions of the coupled junction (SL-S), $G^r_{sc,eh}$ and $G^r_{t,ee}$, are matrices in graphene sublattice space, representing elements on the Nambu space.
The coupling is realized via Dyson’s equation as follows:

\[ \tilde{G}_{sc}^{r} = \tilde{\mathcal{G}}_{sc}^{r} (0^+, 0^+) + \tilde{\mathcal{G}}_{sc}^{r} \tilde{\Sigma} \tilde{\mathcal{G}}_{sc}^{r} (0^+, 0^+) , \]
\[ \tilde{G}_{t}^{\nu} = \tilde{g}_{sc}^{\nu} (0^+, 0^+) \tilde{\Sigma} P^{-1} \tilde{g}_{SL} (0^-, 0^-) , \]
\[ P = I - \tilde{g}_{SL} (0^-, 0^-) \tilde{\Sigma} \tilde{g}_{sc} (0^+, 0^+) \tilde{\Sigma}, \]

where \( \tilde{g}_{sc}^{r} (0^+, 0^+) \) corresponds to the Green function in equilibrium for the superconducting region and is given in Ref. 36.

In the case of a finite superlattice, the differential conductance is also given by Eq. (5), substituting the Green function for the semi-infinite superlattice by a new function representing the coupling of semi-infinite, heavily-doped graphene contact, \( \tilde{g}_{sc}^{r} \), cf. Refs. 36 and 47, with the finite superlattice \( \tilde{g}_{fsl} \), Eq. (A3).

1. S. Das Sarma, Shaffique Adam, E. H. Hwang, and Enrico Rossi, “Electronic transport in two-dimensional graphene,” Rev. Mod. Phys. 83, 407–470 (2011).
2. A.V. Rozhkov, G. Giavaras, Yury P. Blökh, Valentin Freilikher, and Franco Nori, “Electronic properties of mesoscopic graphene structures: Charge confinement and control of spin and charge transport,” Physics Reports 503, 77–114 (2011).
3. T.O. Wehling, A.M. Black-Schaffer, and A.V. Balatsky, “Dirac materials,” Advances in Physics 63, 1–76 (2014).
4. L. Brey and H. A. Fertig, “Emerging zero modes for graphene in a periodic potential,” Phys. Rev. Lett. 103, 046809 (2009).
5. M. Barbier, P. Vasilopoulos, and F. M. Peeters, “Extra dirac points in the energy spectrum for superlattices on single-layer graphene,” Phys. Rev. B 81, 075438 (2010).
6. P. Burset, A. Levy Yeyati, L. Brey, and H. A. Fertig, “Transport in superlattices on single-layer graphene,” Phys. Rev. B 83, 195434 (2011).
7. Cheol-Hwan Park, Young-Woo Son, Li Yang, Marvin L. Cohen, and Steven G. Louie, “Electron beam supercollimation in graphene superlattices,” Nano Letters 8, 2920–2924 (2008).
8. Régis Decker, Yang Wang, Victor W. Brar, William Regan, Hsin-Zon Tsai, Qiong Wu, William Gannett, Alex Zettl, and Michael F. Crommie, “Local electronic properties of graphene on a bn substrate via scanning tunneling microscopy,” Nano Letters 11, 2291–2295 (2011).
9. Jiamin Xue, Javier Sanchez-Yamagishi, Danny Bulmash, Philippe Jacqaud, Aparna Deshpande, K. Watanabe, T. Taniguchi, Pablo Jarillo-Herrero, and Brian J. LeRoy, “Scanning tunnelling microscopy and spectroscopy of ultra-flat graphene on hexagonal boron nitride,” Nature Materials 10, 282 (2011).
10. Matthew Yankowitz, Jiamin Xue, Daniel Cormode, Javier D. Sanchez-Yamagishi, K. Watanabe, T. Taniguchi, Pablo Jarillo-Herrero, Philippe Jacqaud, and Brian J. LeRoy, “Emergence of superlattice dirac points in graphene on hexagonal boron nitride,” Nature Physics 8, 382 (2012).
11. L. A. Ponomarenko, R. V. Gorbachev, G. L. Yu, D. C. Elias, R. Jalil, A. A. Patel, A. Mishchenko, A. S. Mayorov, C. R. Woods, J. R. Wallbank, M. Mucha-Kruczynski, B. A. Piot, M. Potemski, I. V. Grigorieva, K. S. Novoselov, F. Guinea, V. I. Fal’ko, and A. K. Geim, “Cloning of dirac fermions in graphene superlattices,” Nature 497, 594 (2013).
12. Menyoung Lee, John R. Wallbank, Patrick Gallagher, Kenji Watanabe, Takashi Taniguchi, Vladimir I. Fal’ko, and David Goldhaber-Gordon, “Ballistic miniband conduction in a graphene superlattice,” Science 353, 1526–1529 (2016).
13. Yuan Cao, Valla Fatemi, Ahmet Demir, Shiang Fang, Spencer L. Tomarken, Jason Y. Luo, Javier D. Sanchez-Yamagishi, Kenji Watanabe, Takashi Taniguchi, Efthimiios Kaxiras, Ray C. Ashoori, and Pablo Jarillo-Herrero, “Correlated insulator behaviour at half-filling in magic-angle graphene superlattices,” Nature 556, 80 (2018).
14. Yuan Cao, Valla Fatemi, Shiang Fang, Kenji Watanabe, Takashi Taniguchi, Efthimiios Kaxiras, and Pablo Jarillo-Herrero, “Unconventional superconductivity in magic-angle graphene superlattices,” Nature 556, 43 (2018), article.
15. Yuan Cao, Valla Fatemi, Shiang Fang, Kenji Watanabe, Takashi Taniguchi, Efthimiios Kaxiras, and Pablo Jarillo-Herrero, “Unconventional superconductivity in magic-angle graphene superlattices,” Nature (2018), 10.1038/nature26160.
16. Peter Rickhaus, Markus Weiss, Laurent Marot, and Christian Schonenberger, “Quantum hall effect in graphene with superconducting electrodes,” Nano Letters 12, 1942–1945 (2012).
17. V. E. Calado, S. Goswami, G. Nanda, M. Diez, A. R. Akhmerov, K. Watanabe, T. Taniguchi, T. M. Klapwijk, and L. M. K. Vandersypen, “Ballistic josephson junctions in edge-contacted graphene,” Nat Nano 10, 761–764 (2015).
18. M. Ben Shalom, M. J. Zhu, V. I. Fal’ko, A. Mishchenko, A. V. Kretinin, K. S. Novoselov, C. R. Woods, K. Watanabe, T. Taniguchi, A. K. Geim, and J. R. Prance, “Quantum oscillations of the critical current and high-field superconducting proximity in ballistic graphene,” Nat Phys 12, 318–322 (2016).
19. Gil-Ho Lee and Hu-Jong Lee, “Proximity coupling in superconductor-graphene heterostructures,” Reports on Progress in Physics 81, 056502 (2018).
20. C. W. J. Beenakker, “Colloquium: Andreev reflection and Klein tunneling in graphene,” Rev. Mod. Phys. 80, 1337–1354 (2008).
21. C. W. J. Beenakker, “Specular andreev reflection in graphene,” Phys. Rev. Lett. 97, 067007 (2006).
22. D. K. Efetov, L. Wang, C. Handschin, K. B. Efetov, J. Shuang, R. Cava, T. Taniguchi, K. Watanabe, J. Hone, C. R. Dean, and P. Kim, “Specular interband andreev reflections at van der waals interfaces between graphene and hse2,” Nat Phys 12, 328–332 (2016).
23. Travis Dirks, Taylor L. Hughes, Siddhartha Lal, Bruno Uchoa, Yang-Fu Chen, Cesar Chialvo, Paul M. Goldbart, and Nadya Mason, “Transport through andreev bound states in a graphene quantum dot,” Nature Physics 7, 386–390 (2011).
Z. B. Tan, D. Cox, T. Nieminen, P. Lähteenmäki, D. Golubev, G. B. Lesovik, and P. J. Hakonen, “Cooper pair splitting by means of graphene quantum dots,” Phys. Rev. Lett. 114, 096602 (2015).

C. Tomnion, A. Kimouche, J. Coraux, L. Magaud, B. Delsol, B. Gilles, and C. Chapelier, “Induced superconductivity in graphene grown on rhenium,” Phys. Rev. Lett. 111, 246805 (2013).

B. M. Ludbrook, G. Levy, P. Nagge, M. Zonno, M. Schneider, D. J. Dvorak, C. N. Veenstra, S. Zhdanovich, D. Wong, P. Dosanjh, C. Straßer, A. Stöhr, S. Forti, C. R. Ast, U. Starke, and A. Damascelli, “Evidence for superconductivity in Li-decorated monolayer graphene,” Proc. Natl. Acad. Sci. USA 112, 11795 (2015).

J. Chapman, Y. Su, C. A. Howard, D. Kundys, A. N. Grigorenko, F. Guinea, A. K. Geim, I. V. Grigorieva, and R. R. Nair, “Superconductivity in ca-doped graphene lamimates,” Sci. Rep. 6, 23254 (2016).

Anand P. Tiwari, Soohyeon Shin, Eunhee Hwang, Soon-Gil Jung, Tuson Park, and Hyoyoung Lee, “Superconductivity at 7.4K in few layer graphene by li-intercalation,” Journal of Physics: Condensed Matter 29, 445701 (2017).

A. Di Bernardo, O. Milo, M. Barbone, H. Alpern, Y. Kalcheim, U. Sassi, A. K. Ott, D. De Fazio, D. Yoon, M. Amado, A. C. Ferrari, J. Linder, and J. W. A. Robinson, “p-wave triggered superconductivity in single-layer graphene on an electron-doped oxide superconductor,” Nature Communications 8, 14024 (2017).

Gil-Ho Lee, Geon-Hyoung Park, and Hu-Jong Lee, “Observation of negative refraction of dirac fermions in graphene,” Nature Physics 11, 925 (2015).

Vadim V. Cheianov, Vladimir Fal’ko, and B. L. Altshuler, “The focusing of electron flow and a veselago lens in graphene p-n junctions,” Science 315, 1252–1255 (2007).

József Cserti, András Pályi, and Csaba Pétérfalvi, “Caustics due to a negative refractive index in circular graphene p–n junctions,” Phys. Rev. Lett. 99, 246801 (2007).

Yanxia Xing, Jian Wang, and Qing-feng Sun, “Focusing of electron flow in a bipolar graphene ribbon with different chiralities,” Phys. Rev. B 81, 165425 (2010).

Ali G. Moghaddam and Malek Zareyan, “Graphene-based electronic spin lenses,” Phys. Rev. Lett. 105, 146803 (2010).

S. Gómez, P. Burset, W. J. Herrera, and A. Levy Yeyati, “Selective focusing of electrons and holes in a graphene-based superconducting lens,” Phys. Rev. B 85, 115411 (2012).

William J. Herrera, Pablo Burset, and A. Levy Yeyati, “A green function approach to graphene-superconductor junctions with well-defined edges,” Journal of Physics: Condensed Matter 22, 275304 (2010).

Oscar E. Casas, Shirley Gómez Páez, Alfredo Levy Yeyati, Pablo Burset, and William J. Herrera, “Subgap states in two-dimensional spectroscopy of graphene-based superconducting hybrid junctions,” Phys. Rev. B 99, 144502 (2019).

W. L. McMillan, “Theory of superconductor-normal metal interfaces,” Phys. Rev. 175, 559–568 (1968).

Akira Furusaki and Masaru Tsukada, “De josephson effect and andreev reflection,” Solid State Communications 78, 299 – 302 (1991).

Satoshi Kashiwaya and Yukio Tanaka, “Tunnelling effects on surface bound states in unconventional superconductors,” Reports on Progress in Physics 63, 1641 (2000).

Bo Lu, Pablo Burset, Keiji Yada, and Yukio Tanaka, “Tunneling spectroscopy and josephson current of superconductor-ferromagnet hybrids on the surface of a 3d ti,” Superconductor Science and Technology 28, 105001 (2015).

François Crépin, Pablo Burset, and Björn Trauzettel, “Odd-frequency triplet superconductivity at the helical edge of a topological insulator,” Phys. Rev. B 92, 100507 (2015).

Pablo Burset, Bo Lu, Grigory Tkachov, Yukio Tanaka, Ewelina M. Hankiewicz, and Björn Trauzettel, “Superconducting proximity effect in three-dimensional topological insulators in the presence of a magnetic field,” Phys. Rev. B 92, 205424 (2015).

Feix Keidel, Pablo Burset, and Björn Trauzettel, “Tunable hybridization of majorana bound states at the quantum spin hall edge,” Phys. Rev. B 97, 075408 (2018).

Bo Lu and Yukio Tanaka,”Study on green’s function on topological insulator surface,” Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 376, 20150246 (2018).

Daniel Brenning, Pablo Burset, and Björn Trauzettel, “Creation of spin-triplet cooper pairs in the absence of magnetic ordering,” Phys. Rev. Lett. 120, 037701 (2018).

P. Burset, A. Levy Yeyati, and A. Martín-Rodero, “Microscopic theory of the proximity effect in superconductor-graphene nanostructures,” Phys. Rev. B 77, 205425 (2008).

P. Burset, W. Herrera, and A. Levy Yeyati, “Proximity-induced interface bound states in superconductor-graphene junctions,” Phys. Rev. B 80, 041402 (2009).

Pablo Burset, William J. Herrera, and A. Levy Yeyati, “Formation of interface bound states on a graphene-superconductor junction in the presence of charge inhomogeneities,” Graphene 2, 7 (2013).

D.P. Arovas, L. Brey, H.A. Fertig, E.-A. Kim, and K. Ziegler, “Dirac spectrum in piecewise constant one-dimensional (1d) potentials,” New Journal of Physics 12, 123020 (2010).

T. Klapwijk and S. Ryabchun, “Direct observation of ballistic andreev reflection.” Journal of Experimental and Theoretical Physics 119, 997–1017 (2014).

Jonas Wiedemann, Eva Liebhaber, Johannes Kührt, Erwan Bocquillon, Pablo Burset, Christopher Ames, Hartmut Buhmann, Teun M. Klapwijk, and Laurens W. Molenkamp, “Transport spectroscopy of induced superconductivity in the three-dimensional topological insulator hgte,” Phys. Rev. B 96, 165302 (2017).

J. C. Cuevas, A. Martín-Rodero, and A. Levy Yeyati, “Hamiltonian approach to the transport properties of superconducting quantum point contacts,” Phys. Rev. B 54, 7366–7379 (1996).
