Electroweak baryogenesis in the $\mathbb{Z}_3$-invariant NMSSM

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Abstract: We calculate the baryon asymmetry of the Universe in the $\mathbb{Z}_3$-invariant Next-to-Minimal Supersymmetric Standard Model where the interactions of the singlino provide the necessary source of charge and parity violation. Using the closed time path formalism, we derive and solve transport equations for the cases where the singlet acquires a vacuum expectation value (VEV) before and during the electroweak phase transition. We perform a detailed scan to show how the baryon asymmetry varies throughout the relevant parameter space. Our results show that the case where the singlet acquires a VEV during the electroweak phase transition typically generates a larger baryon asymmetry, although we expect that the case where the singlet acquires a VEV first is far more common for any model in which parameters unify at a high scale. Finally, we examine the dependence of the baryon asymmetry on the three-body interactions involving gauge singlets.

Keywords: Supersymmetry, Baryogenesis, NMSSM, Electroweak phase transition
1 Introduction

A cosmological history that includes a period of inflation inevitably washes out any possible primordial baryon asymmetry in the Universe (BAU). Yet, currently we observe an asymmetry between baryons and anti-baryons, quantified by the ratio of the average baryon and entropy densities

\[ Y_B = \frac{n_B}{s} = \begin{cases} 
8.2 - 9.4 \times 10^{-11} & \text{(95\% CL) BBN [1]}, \\
8.65 \pm 0.09 \times 10^{-11} & \text{PLANCK [2]}. 
\end{cases} \]  

Various mechanisms have been proposed to create the observed asymmetry after inflation and many of these scenarios rely on a thermodynamic phase transition that may have happened before Big Bang Nucleosynthesis (BBN). The only known cosmic phase transition that occurred before BBN is the electroweak phase transition (EWPT). The non-equilibrium condition created by EWPT is utilised by the electroweak baryogenesis (EWBG) \cite{3, 4} mechanism to produce the baryon asymmetry. The presence of the electroweak scale suggests that tests of EWBG may be within reach \cite{5–14}. Aesthetically attractive that EWBG features a common origin for the breaking of both the baryon and electroweak symmetry.

EWBG requires that the EWPT be strongly first order. In the Standard Model (SM) the Higgs boson is too heavy to allow for a strongly first order EWPT \cite{15}. The order of the EWPT can be boosted by new weak scale particles that interact with the Higgs. In supersymmetry, the stops can catalyse a strongly first order EWPT \cite{16}. Search results
from the LHC, however, impose severe constraints on such a possibility [17–23]. Even if a stop is light enough to boost the order of the EWPT, electric dipole moment (EDM) constraints render the charge and parity (CP) violating phase present in the stop-Higgs coupling\textsuperscript{1} to be insufficient [26] to produce the observed baryon asymmetry. These issues in conjunction with the little hierarchy problem [27–34], which manifests from a combination of the Higgs mass measurement and null searches for supersymmetric particles, give strong motivation for looking at extensions to the minimal supersymmetric scenario\textsuperscript{2} [35, 36].

Adding a gauge-singlet scalar superfield to the superpotential introduces extra degrees of freedom that couple to the Higgs, which can boost the strength of the phase transition [37–39] and relax the need for the stop mass to be close the EWPT scale\textsuperscript{3}. Additionally, experimental constraints on gauge singlets are not very onerous [41, 42] and there are strong motivations for considering gauge singlets beyond baryogenesis. In the case where there is a discrete $Z_3$ symmetry between the singlet and Higgs sector, the Next-to-Minimal Supersymmetric Standard Model (NMSSM) is a proposed solution to the $\mu$ problem [43–55]. Further, gauge singlets naturally arise in GUTs [56] as well as in string theory [57]. Moreover, the NMSSM can simultaneously accommodate inflation, baryogenesis and dark matter [58]. Finally, the singlet can serve to boost the variation of $\beta$ (where $\tan \beta$ is the ratio of the two Higgs VEVs) during the phase transition by an order of magnitude compared to the MSSM [37]. Since the BAU is proportional to $\Delta \beta$ there is a possibility that CP violating phases can be smaller, further evading EDM constraints, and still produce enough BAU.

The production of the BAU has been explored in the NMSSM via the WKB approach\textsuperscript{4} [61]. This approximation, however, can miss substantial “memory effects” that can result in resonant enhancements of the BAU of up to several orders of magnitude when the masses are near degenerate\textsuperscript{5} [64]. Non-equilibrium quantum field theory has also been used [65, 66], utilizing the closed time path (CTP) formalism [67–72] to calculate the BAU in the presence of the resonances. Ref. [65], however, invokes the fast rate approximation, which can differ from more precise methods by two orders of magnitude [73] in its determination of the BAU.

In this work we derive the transport equations for the most relevant particle species in the NMSSM for the cases where the singlet acquires a VEV before and during the electroweak phase transition. We then solve them without assuming that three body Yukawa, triscalar, strong sphaleron or supergauge interactions in the Higgses and Higgsino sector are large using the semi-analytic methods described in Refs. [74, 75]. We also seek to answer the question as to whether three body interactions involving gauge singlets can in

\textsuperscript{1}For recent work on CP-violation in the MSSM and other MSSM extensions see [24, 25].

\textsuperscript{2}Especially, when one considers the motivations for supersymmetry: radiative electroweak symmetry breaking leading to a light SM-like Higgs, dark matter, solution of the gauge hierarchy problem, gauge coupling unification in Grand Unified Theories (GUTs), and string theory.

\textsuperscript{3}A singlet can also ease the little hierarchy problem [29, 40].

\textsuperscript{4}For a derivation of the WKB approach from first principles see [59, 60].

\textsuperscript{5}Recent work [62] using Wigner functionals for a toy model suggested the interesting possibility that the resonance might be severely dampened when the masses are exactly degenerate. A more precise treatment in [63] however seemed to indicate there was indeed a resonance.
principle compete with resonant relaxation terms arising from CP-conserving interactions with the bubble wall. In the MSSM, such an effect provides a counter-intuitive boost to the BAU by several orders of magnitude near resonance despite these three body rates being relaxation terms. Since we are principally concerned with the plausibility of EWBG within the NMSSM being driven by interactions with the singlino, we root our results in constraints on the Higgs mass and LHC searches, performing a scan of the relevant parameter space. Finally we look at how different types of phase transitions can affect the baryon asymmetry. Whether the singlet acquires a VEV before or during the electroweak phase transition have a large impact on the determination of the BAU: if the singlet phase transition occurs before EWPT, then outside of the bubble, the singlino and Higgsinos have non-zero masses, without thermal corrections. This affects not only the CP-violating source terms and CP-conserving relaxation terms, but the thermal decay widths of the singlino. In fact, the two scenarios change the structure of the transport equations as fluctuations around the VEV are real by definition (and therefore possess no asymmetry).

The structure of this paper is as follows. In section 2 we review the NMSSM and its motivations with a particular emphasis on the structure of the effective potential. Section 3 contains our derivation of the coupled transport equations for both phase transition structures mentioned above outlining our assumptions. In section 4 we discuss the input parameters of the model with particular emphasis on the thermal widths. We then solve these equations in section 5 scanning over the NMSSM parameter space. We examine the effect of both cases: the singlet acquiring a VEV before or during the EWPT. We also examine the effect of the three body gauge singlet transport coefficients in section 6, before a final discussion and conclusion in section 7.

2 The Next-to-Minimal Supersymmetric Standard Model

We calculate the baryon asymmetry within the scale invariant, $Z_3$-conserving, NMSSM defined by [35]

$$W_{\text{NMSSM}} = W_{\text{MSSM}}|_{\mu = 0} + \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{1}{3} \kappa \hat{S}^3 .$$

(2.1)

The MSSM superpotential $W_{\text{MSSM}}|_{\mu = 0}$, less the $\mu$ term, is defined in [76], and $\hat{H}_{u,d}$ and $\hat{S}$ are SU(2) doublet and singlet Higgs superfields, respectively. In addition to the MSSM soft supersymmetry breaking terms (with $B$ set to zero [76]), the scalar potential contains

$$V_{\text{soft}}^{\text{NMSSM}} = m_s^2 |S|^2 - \lambda A_\lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.} ,$$

(2.2)

where $\hat{H}_{u,d}$ and $S$ are the scalar components of the Higgs doublet (singlet) superfields. The neutral components of these scalars acquire a non-zero VEV during electroweak symmetry breaking. Relative to the MSSM, the new terms in the Higgs potential are

$$V_{\hat{H}}^{\text{NMSSM}} = \lambda^2 |S|^2 (|\hat{H}_u|^2 + |\hat{H}_d|^2) + \frac{1}{3} \lambda A_\lambda \hat{S} \hat{H}_u \hat{H}_d + \lambda S^2 \hat{H}_u^* \hat{H}_d^* + \text{h.c.} .$$

(2.3)

To boost the baryon asymmetry we assume complex values for $\lambda, \kappa, A_\lambda$ and $A_\kappa$. Once both the Higgs doublets and the singlet scalar have acquired a vacuum expectation value, one
can write three re-phasing invariants that appear in the tree level potential \[65, 77\]
\[
\phi_\lambda - \phi_\kappa, \quad \phi_\lambda + \phi_{A_\kappa}, \quad \phi_\lambda + \phi_{A_\lambda}.
\] (2.4)

Here \(\phi_x\) is the complex phase associated with parameter \(x \in (\lambda, \kappa, A_\kappa, A_\lambda)\). One can then use the CP-odd tadpole conditions to write two of the rephasing invariants in terms of \(\phi_\lambda - \phi_\kappa\). Therefore there is in reality only a single independent rephasing invariant \[78\]. For more details on the NMSSM see Refs. [35, 58, 76].

3 Transport equations

In this section we derive the set of coupled transport equations that govern the behaviour of number densities throughout the phase transition. In the NMSSM there are two possible phase histories that qualitatively change the transport equations\(^6\)

- Singlet first phase transition (SFPT) phase transition: The singlet acquires a VEV before the EWPT.
- Singlet spontaneous phase transition (SSPT): The singlet acquires a VEV during the EWPT.

We make use of the closed time path formalism [67, 68, 70, 72, 80] following the procedure given in [64] to derive the set of transport coefficients and CP violating source terms. We will use the usual VEV insertion approximation (VIA) which assumes that the physics responsible for producing the BAU is dominated by the region in front of the bubble wall where the Higgs VEV is small compared to the nucleation temperature and the relevant mass differences. The NMSSM includes a resonant source of CP violation in addition to the CP violating interactions present in the MSSM due to singlino-Higgsino interactions with the space time varying vacuum.

We ignore first and second generation quarks and squarks as well as all three generations of leptons and sleptons, invoking the assumption that the rates that connect these particle species to the rest of the transport equations are small due to the fact that the relevant Yukawa couplings are small. This is an assumption that can in some parts of the parameter space be too rough [81] but we leave a thorough investigation of this to future work.

It has been demonstrated that assuming local equilibrium between third generation quarks and squarks holds well for large parts of the parameter space [82]. On the other hand, one has to be cautious in assuming supergauge equilibrium between the Higgs and the Higgsino. Fast supergauge interactions in the Higgs sector would lead to a suppression of the combination \(\mu_{H_i} - \mu_{\tilde{H}}\) whereas three body interactions involving singlets/singlinos lead to the suppression of the combination \(\mu_{H_i} + \mu_{\tilde{H}}\). If one works in the approximation that both types of rates are fast enough to set the combinations of chemical potentials given before to zero, then the baryon asymmetry vanishes. While the competition of these types

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\(^6\)This is a reduced list from the four types given in [79] as this partition is more convenient when discussing electroweak 'baryogenesis.'
of rates could indeed suppress the baryon asymmetry we take the precaution of including both rates in the transport equations rather than assuming they are large enough to result in local equilibrium relations.

As usual, number densities are defined \( n \equiv n - \bar{n} \) where \( \bar{n} \) is the anti-particle density. When the gauge singlet acquires a vacuum expectation value the fluctuations around this VEV are real and cannot hold any asymmetry. Therefore in a SFPT phase transition the number density of the singlet is zero. This means that there is one less transport equation for SFPT phase transitions. Finally we note that one cannot form a vector charge for the singlino, so we do not include a number density for the singlino\(^7\).

Under these simplifying assumptions we are able to derive a set of coupled transport equations for six charge densities for SSPT phase transitions and five for SFPT phase transitions as the number density of the singlet \( n_S \equiv n_S - \bar{n}_S \) is zero. We present the more complicated SSPT case as the SFPT case can be derived from it by setting \( n_S \) to zero and modifying transport coefficients that depend on \( v_S \). For a SFPT phase transition, the six linear combination of number densities which make up the transport equations are

\[
\begin{align*}
n_{H_1} &= n_{H^+} + n_{H^0}, \\
n_{H_2} &= n_{H^-} + n_{H^0}, \\
n_{\tilde{H}} &= n_{\tilde{H}^+} + n_{\tilde{H}^0} - n_{\tilde{H}^-} - n_{\tilde{H}^0}, \\
n_t &= n_{t_R} + n_{\bar{t}_R}, \\
n_Q &= n_{L_R} + n_{b_L} + n_{\bar{t}_R} + n_{b_L}, \\
n_S &= n_S.
\end{align*}
\]

The transport coefficients are derived using the Schwinger-Dyson equations in the closed time path formalism to relate divergences of current densities to functions of self energies. These functions of self energies can be expanded in the chemical potentials of the particles involved in each self energy interaction. We can relate chemical potentials to number densities in the usual way. Ignoring terms of \( O(\mu^3) \) we can derive the relation \[84\]

\[
\mu_x = \frac{6}{T^2} \frac{n_x}{k_x},
\]

with

\[
k_x = k_x(0) \frac{c_{F,B}}{\pi^2} \int_{m/T}^{\infty} dy y \frac{e^y}{(e^y \pm 1)^2} \sqrt{y^2 - m_x^2 / T^2},
\]

where \( c_{F,B} = 6(3) \) and the sign in the denominator is \pm \) for fermions and bosons, respectively. The factors \( k_i(0) \) are 2 for Dirac fermions and complex scalars and 1 for chiral fermions. The \( k \) factors of our composite number densities in Eq. (3.1) are the sum of the \( k \) factors for each component. We then define the linear combinations of rates that act as coefficients of these composite number densities.

Tree level interactions with space time varying VEVs have CP conserving components known as “mass terms” typically denoted by \( \Gamma_{m,(\cdot)} \) where the superscript describes the

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\(^7\)In principle one can define an axial charge as was done here for binos and \( \tilde{W}^3 \), but the contribution from doing so was found to be small \[83\].
particles involved in the interaction starting with the "in" state. The full set of relevant mass terms are\(^8\)

\[
\begin{align*}
\Gamma_t^m &= \Gamma_t^{R,L} + \Gamma_t^m, \\
\Gamma_{\tilde{t}^m} &= \Gamma_{\tilde{t}^m}^{R,L} + \Gamma_{\tilde{t}^m}^0 + \Gamma_{\tilde{t}^m}^\tilde{B} + \Gamma_{\tilde{t}^m}^\tilde{S}, \\
\Gamma_{H_uH_d}^m &= \Gamma_{H_uH_d}^m, \\
\Gamma_{H_1S}^m &= \Gamma_{H_1S}^m, \\
\Gamma_{H_2S}^m &= \Gamma_{H_2S}^m, \\
\Gamma_{\tilde{H}\tilde{S}}^m &= \Gamma_{\tilde{H}\tilde{S}}^m.
\end{align*}
\]

Note that Higgsino-Higgsino interactions with a space-time varying singlet VEV do not contribute as the masses are exactly degenerate by definition and the resonance vanishes when masses are exactly degenerate. Tri-scalar and Yukawa interactions have a general form related to the functions \(I_{F/B}(m_1,m_2,m_3)\) which are defined in Ref. [73] and for completeness are also given in appendix A.

We define the composite transport coefficients which make up our transport equations as

\[
\begin{align*}
\Gamma_Y^{H_1H_2S} &= \frac{12|\lambda A|}{T^2}I_B(m_{H_1},m_{H_2},m_S), \\
\Gamma_Y^{H_1S} &= \frac{12|\lambda A|}{T^2}I_B(m_{H_1},m_S,m_S), \\
\Gamma_Y^{H_2S} &= \frac{12|\lambda A|}{T^2}I_B(m_{H_2},m_S,m_S), \\
\Gamma_Y^{\tilde{H}\tilde{S}} &= \frac{12|\lambda A|}{T^2}I_B(m_{\tilde{H}},m_{\tilde{S}},m_S),
\end{align*}
\]

where \(c \equiv \cos \alpha, s \equiv \sin \alpha\) and \(\alpha\) is the Higgs mixing angle in the unbroken phase. We also need to include estimates of the four body scattering terms which in general are small but may become the dominant contribution when three body decays are kinematically suppressed. For all cases our four body scattering rate is given by \(0.19|x|^2T/(6\pi)\) where \(x \in \{g_3,\lambda,\lambda A\lambda\}\) is the appropriate coupling constant. In some regions of parameter space the BAU will be sensitive to the precise value of these four body scattering rates so future work should consider a full numerical treatment of these coefficients.

\(^8\)These mass terms typically come in two flavours \(\Gamma_{m}^\pm\) however the negative type is typically much larger so the positive type we ignore.
Our transport equations can then be shown to have the form

\[ \partial_t \mu^m = -\Gamma^t_{QH_1} U^m_{QH_1} - \Gamma^t_{QH_2} U^m_{QH_2} - \Gamma^t_{QH_2} U^m_{QH_2} + \Gamma^t_{SS} U_5 + S_5^{CP}, \]

\[ \partial_t Q^m = \Gamma^m_{QH_1} U^m_{QH_1} + \Gamma^m_{QH_2} U^m_{QH_2} + \Gamma^m_{QH_2} U^m_{QH_2} - 2\Gamma^t_{SS} U_5 - S_5^{CP}, \]

\[ \partial_t \mu^H_1 = -\Gamma^H_{H2} U^m_{H1,H2} - \Gamma^H_{H2} U^m_{H1,H2} + \Gamma^H_{H1} U^m_{H1,H1} - \Gamma^H_{H2} U^m_{H1,H2} - \Gamma^H_{H1} U^m_{H1,H1} - \Gamma^H_{H2} U^m_{H1,H2}, \]

\[ \partial_t \mu^H_2 = -\Gamma^H_{H2} U^m_{H1,H2} - \Gamma^H_{H2} U^m_{H1,H2} + \Gamma^H_{H1} U^m_{H1,H1} - \Gamma^H_{H2} U^m_{H1,H2} - \Gamma^H_{H1} U^m_{H1,H1}, \]

\[ \partial_t \mu^H_1 = -\Gamma^H_{H2} U^m_{H1,H2} - \Gamma^H_{H2} U^m_{H1,H2} + \Gamma^H_{H1} U^m_{H1,H1} - \Gamma^H_{H2} U^m_{H1,H2} - \Gamma^H_{H1} U^m_{H1,H1}, \]

\[ \partial_t \mu^H_2 = -\Gamma^H_{H2} U^m_{H1,H2} - \Gamma^H_{H2} U^m_{H1,H2} + \Gamma^H_{H1} U^m_{H1,H1} - \Gamma^H_{H2} U^m_{H1,H2} - \Gamma^H_{H1} U^m_{H1,H1}. \]

(3.6)

Here we have defined the combinations of chemical potentials as follows

\[ U^m_L = \left( \frac{t}{k_t} - \frac{Q}{k_Q} \right), \quad U^m_{QH_1} = \left( \frac{t}{k_t} - \frac{Q}{k_Q} - \frac{H_1}{k_{H_1}} \right), \quad U^m_{QH_2} = \left( \frac{t}{k_t} - \frac{Q}{k_Q} - \frac{H}{k_H} \right), \]

\[ U^m_{H1,H2} = \left( \frac{H_1}{k_{H_1}} + \frac{H_2}{k_{H_2}} \right), \quad U^m_{H1,H2} = \left( \frac{H_1}{k_{H_1}} + \frac{S}{k_S} \right), \quad U^m_{H1,H2} = \left( \frac{H_1}{k_{H_1}} + \frac{H_2}{k_{H_2}} + \frac{S}{k_S} \right), \]

\[ U^m_{H1,H2} = \left( \frac{H_1}{k_{H_1}} + \frac{H_2}{k_{H_2}} \right), \quad U^m_{H1,H2} = \left( \frac{H_1}{k_{H_1}} - \frac{H}{k_H} \right), \quad U^m_{H1,H2} = \left( \frac{S}{k_S} \right). \]

(3.7)

As usual, the axial chemical potential, \( \mu_5 \), is given by \( \mu_L - \mu_R \). Since only the left handed quark doublet and the right handed top is sourced, this reduces to

\[ \mu_5 = \frac{2Q}{k_Q} - \frac{t}{k_t} + \frac{9(Q + t)}{k_b}. \]

(3.8)

Finally, the strong sphaleron rate is taken to be \( \Gamma_{SS} \approx 16a^4_5 T \) [85].

4 Thermal parameters

The production of the BAU is resonantly enhanced when the masses of the singlino and Higgsino (calculated in the symmetric phase) are nearly degenerate. The width and height of the resonance, and therefore the width of the allowed parameter space, are controlled by the thermal widths of the singlino and the Higgsino. The magnitude of the thermal widths for various particles in the NMSSM is typically dominated by gauge interactions and are proportional to the involved coupling constant squared. This means that the thermal widths for (s)quarks tend to be quite large. However, in the absence of gauge interactions, the thermal width of the singlino is expected to be small. Indeed the only two places
where the thermal width of the singlino or the singlet can be broadened is through Yukawa interactions.

Let us divert more attention to the thermal width of the singlino as the thermal width of the singlet only weakly affects the BAU. The relevant Yukawa interactions involve the singlino, Higgsino and the Higgs with relevant couplings of $\kappa$ and $\lambda$. From Ref. [86] the thermal width that results from such Yukawa interactions is

$$\Gamma_S = b(m_H^*) 0.01(\lambda^2 + \kappa^2),$$

(4.1)

where $b(m_H^*)$ is a function which monotonically decreases with the mass of the Higgs in the symmetric phase. For a SSPT phase transition the Higgs mass is just the Debye mass under the VIA, where we assume the physics primarily responsible for the production of the BAU occurs in the symmetric phase just ahead of the bubble wall. The Higgs mass in the symmetric phase of a SFPT phase transition gets contributions from the singlet VEV and is thus boosted. The thermal width of the singlino is therefore much smaller for a SFPT phase transition.

We estimate the remaining thermal widths, diffusion coefficients and bubble wall properties in Table 1. For the diffusion coefficients we use the values given in [87] from which we also derive the rest of our thermal widths. For the parameter $\Delta \beta$ we note that a feature of the NMSSM is it can be an order of magnitude larger than its MSSM value [37]. However, since the BAU is linearly proportional to $\Delta \beta$ we just take the fiducial value of 0.05 along with a fiducial value of the CP violating phase which we set to its maximal value. Although the value of $\Delta \beta$ can be an order of magnitude greater in the NMSSM compared to the MSSM [37], the BAU is linearly proportional to $\Delta \beta$ so it is simple to translate our results to the case where $\Delta \beta$ is large. Since we assume that $\Delta \beta$ is small we assume that the mixing angle in the symmetric phase - that is near the bubble wall - is equal to its zero temperature value evaluated at the $Z$ boson mass. The BAU tends to get larger for small values of the bubble wall velocity which can also vary over an order of magnitude [37]. We take a moderate value. Finally, our thermal mass from the singlino agrees with [65] and is given by

$$\Delta T m_S^2 = \frac{\lambda^2 + 2|\kappa|^2}{8} T^2.$$ 

(4.2)

| Parameter | Value | Parameter | Value | Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|-----------|-------|-----------|-------|
| $D_Q$     | $6/T$ | $D_{H_u}$ | $110/T$ | $\Gamma_{H_u,d}$ | $0.025T$ | $\Delta \beta$ | $0.05$ |
| $D_{\tilde{b}}$ | $110/T$ | $D_{H_d}$ | $110/T$ | $\Gamma_{\tilde{W}}$ | $0.065T$ | $v_W$ | $0.05$ |
| $D_t$     | $6/T$ | $\Gamma_i$ | $0.5T$ | $\Gamma_{\tilde{H}}$ | $0.025T$ | $L_W$ | $5/T$ |
| $D_S$     | $150/T$ | $\Gamma_{\tilde{B}}$ | $0.04T$ | $\Gamma_S$ | $0.003T$ | $T_N$ | $100$ |

Table 1. The base set of parameters used for our numerical study. The diffusion constants are taken from [87]. Here $a = \{1, 0.5, 0.25\}$ for the case where the mass of the singlet is $\in [0, 0.25T], [0.25T, 0.5T], [0.5T, \infty]$, respectively, and $b = \kappa^2 + \lambda^2$. 


5 Semi-analytic solution

Neglecting the bubble wall curvature we can reduce the problem to a single dimension by solving the system in the rest frame of the bubble wall $z = |v_W t - x|$. We then use the diffusion approximation to write $\nabla \cdot \vec{J} = \nabla^2 n$ thus reducing the problem to a set of coupled differential equations in a single space time variable. To answer the question as to whether the singlino can drive the production of the baryon asymmetry we set all CP violating phases apart from the singlino-Higgsino-Higgs interaction to zero. Transport equations in this form have a closed form analytical solution in each phase [74].

Consider the case where there is no high temperature singlet VEV. In the broken phase the solution is

$$X(z) = \sum_{i=1}^{12} x_i A_X(\alpha_i)e^{-\alpha_i z} \left( \int_0^z dy e^{-\alpha_i y} S^0(y) - \beta_i \right),$$

(5.1)

and in the symmetric phase we have

$$X(z) = \sum_{i=1}^{12} A_{X,s}(\gamma_i)y_i e^{\gamma_i z},$$

(5.2)

where,

$$X \in \{ Q, t, H_1, H_2, \tilde{H}, S \}.$$  

(5.3)

The derivation of $\alpha_i, \beta_i, x_i, y_i, \gamma_i$ and $A_{X,(s)}(\alpha_i(\gamma_i))$ is given in [74]. From these solutions one can then define the left handed number density $n_L(z) = Q_{1L} + Q_{2L} + Q_{3L} = 5Q + 4T$. The baryon number density, $\rho_B$, satisfies the equation [88, 89]

$$D_Q \rho_B''(z) - v_W \rho_B'(z) - \Theta(-z) R \rho_B = \Theta(-z) \frac{n_F}{2} \Gamma_{ws} n_L(z),$$

(5.4)

where $n_F$ is the number of fermion families. The relaxation parameter is given by

$$R = \Gamma_{ws} \left[ \frac{9}{4} \left( 1 + \frac{n_{sq}}{6} \right) + \frac{3}{2} \right],$$

(5.5)

where $n_{sq}$ is the number of thermally available squarks and $\Gamma_{ws} \approx 120 \alpha_W^6 T$ [85, 90, 91]. The baryon asymmetry of the Universe, $Y_B$ is then given by

$$Y_B = -\frac{n_F \Gamma_{ws}}{2\kappa_D Q S} \int_{-\infty}^0 e^{-\kappa x} n_L(x) dx,$$

(5.6)

where

$$\kappa = \frac{v_W \pm \sqrt{v_W^2 + 4D_Q R}}{2D_Q},$$

(5.7)

and the entropy is

$$S = \frac{2\pi^2}{45} g_s T^3.$$
6 Numerical results

We sample the NMSSM parameter space using MultiNest v3.10 [92–94] and calculate the spectrum with SOFTSUSY v3.7.2 [95, 96] by restricting the Higgs mass to 125 GeV rather than performing a global fit. The parameter space is reduced to a few dimensions by fixing the values of the soft masses for second and third third generation sfermions to 6 TeV, third generation to 3 TeV and $M_1$ to 500 GeV. The prior distributions in the remaining free parameters are given in Table 2. The parameters $m_{H_u}$, $m_{H_d}$, $m_S$ are set by tadpole conditions and $A_\kappa$ is set to get the right Higgs mass. The range of parameters are chosen with the following considerations in mind

1. Since we are considering only the CPV source that is not present in the MSSM we require the singlino and Higgsino masses (including thermal corrections) to be relatively close before EWSB and not very heavy compared to a plausible value of the nucleation temperature. This latter concern is to avoid severe Boltzmann suppression of the CPV source and the former concern is to ensure a resonant enhancement of the CPV source.

2. The singlet mass cannot be too heavy compared to the Higgs since it must catalyse a strongly first order EWPT (we assume that the stop is too heavy to perform such a role).

3. Converse to the previous consideration, a light singlet with large mixing with the standard model Higgs will be ruled out by collider constraints.

The VEV insertion approximation leads us to take the masses and degrees of freedom in the symmetric phase as it is in this phase where the total left handed number density biases unsuppressed electroweak sphalerons producing the baryon asymmetry [64]. We sanitise the results by removing points where the VEV insertion approximation is unreliable — that is, where the mass gap between the singlino or Higgsino mass (including the Debye mass) is large enough to spuriously change the sign of CP conserving relaxation terms. We also note that when the mass gap between the singlino and the Higgsino is very small the VEV insertion may become reliable [62, 63].

We also sanitise all other mass relaxation terms (e.g. $\Gamma_m^\ell$) by setting them to a random positive infinitesimal number when the in- and virtual-state are far from degeneracy and the naive calculation of the rate yields a negative number. This avoids the spurious case where the mass relaxation terms change sign rather than decaying to zero due to the breakdown of the VEV insertion approximation and can give a spurious boost the the BAU. We set the CP violating phase to its maximal value, sin $\phi = 1$, as the BAU is linearly proportional to the sin of this phase. The BAU is also proportional to $\Delta \beta$ which we set to a value of 0.05 as it can be it can have a range of $\sim [0.01, 0.2]$ in the NMSSM [37]. Any point in our

\footnote{A random infinitesimal number is chosen rather than zero for the sake of the stability of our code but there is no discernible numerical difference in the BAU between setting these values to zero or a small number.}
 scan that has a BAU greater than the observed value for such a CPV phase and value of $\Delta\beta$ can be interpreted as a point where the correct value of the CP violating phase is

$$\frac{\Delta\beta \sin \phi}{F} = \frac{Y_B^{\text{obs}}}{Y_B},$$

where $F = 0.05$. For each parameter point we calculate the BAU for SFPT and SSPT if the square of the Higgs mass at $H = S = 0$ is positive when one includes the Debye mass as well as corrections from the thermal functions $J_B$ and $J_F$. If the potential does not have a positive curvature at $H = S = 0$ for any reasonable range of nucleation temperature ($\lesssim 200$ GeV) the implication is that the origin of field space is a local maximum rather than a local minimum. This means that the phase transition must be either SFPT or second order for these points in the parameter space. As a GUT scale model (with soft masses on the order of the electroweak scale in magnitude) can drag $m_{H_u}^2$ to large negative values when it is evolved to the electroweak scale, it could be harder to find regions of parameter space that survive this cut; the SFPT is the more realistic case. This analysis we leave to a future study. For our prior ranges the majority of the sample does indeed survive this cut. For the points that survive this cut we calculate the baryon asymmetry. We then calculate the posterior distributions based on the mass of the Higgs and the criteria that both the singlino and Higgsino masses are less than a TeV. We colour the $1\sigma$ and $2\sigma$ credible regions in orange and blue respectively as shown in Fig. 1 for the SSPT. We perform a similar analysis in the SFPT case and find that generically this scenario produces a lower asymmetry as shown in Fig. 3. Indeed the largest value of the BAU is an order of magnitude large in the SSPT compared to the case of the SFPT. This suppression is due to the fact that the soft masses of the singlino and Higgsino are both proportional to the VEV of the singlet which tends to be quite large.

In performing the scan for SFPT we make the approximation that the singlet VEV does not change throughout the electroweak phase transition. This results in an underestimate of the baryon asymmetry if the singlet VEV is smaller in the EW symmetric phase then the EWSB phase. The reason for this is that the VEV of the singlet contributes to the masses of the particles involved in the CPV source and their contribution is usually large enough that Boltzmann suppression can become an issue. This along with the greater variability of $\Delta\beta$ really motivates future work where the dynamics of the phase transition and the calculation of the baryon asymmetry are performed simultaneously.

In Fig. 3 we show the range of zero temperature masses for the Higgsino and singlino against the BAU for a SFPT phase transition. There is a substantial proportion of the

| $A_\lambda$ | $A_\kappa$ | $M_2$ | $\tan \beta$ | $\lambda$ | $\kappa$ | $\lambda v_S$ |
|------------|------------|-------|--------------|----------|---------|-------------|
| $[-4000, 4000]$ | $[0, 200]$ | $[100, 1000]$ | $[1.1, 5]$ | $[0.001, 0.5]$ | $[0.001, 0.5]$ | $[200, 800]$ |

Table 2. Sample ranges for the scanned NMSSM parameters. All dimensionful numbers are in GeV. $M_1$ was fixed to 500 GeV. The modest range of $\tan \beta$ was to help satisfy flavour constraints. Sfermions were decoupled by setting the first and second generation squark masses to 6 TeV and the third generation to 3 TeV.
Figure 1. Two dimensional posterior probability distribution in the baryon asymmetry, produced by the maximum CP-violating phase in the SSPT (singlet simultaneous) scenario, and the thermal mass of the Higgsino (top panel) and singlino (bottom panel). We colour the $1\sigma$ and $2\sigma$ credible regions in orange and blue, respectively.

parameter space with a sufficiently large BAU. We find that the resonant enhancement that occurs when the masses of the singlino and Higgsino are near degenerate is the dominant predictor of a large BAU. This is clear from Fig. 4. Of particular interest to us is the fact
that well off resonance one can still obtain a BAU that is close to an order of magnitude below the observed rate. This shortcoming in the BAU, however, can be made up for by a sufficiently large $\Delta \beta$. This possibility opens up another avenue in which the BAU can be produced within the NMSSM off resonance. As a caveat we note that as one ventures further off resonance, more skepticism should be held toward the accuracy of the result as

**Figure 2.** Baryon asymmetry for a benchmark point with lowest value of $\chi^2$ for a SFPT (singlet first) phase transition with varying values of the three body rates. The horizontal axis varies the three body rates involving (s)tops and Higgs(inos). The vertical axis varies the three body rates involving singlets or singlinos.
In Ref. [73] it was shown that the BAU can vary by orders of magnitude with the variation of the magnitude of three body rates involving Higgsinos and stops. The VEV insertion approximation is losing its reliability.
NMSSM has new three body rates involving Higgs(inos) and singlet/singlinos. We show how the BAU varies as a function of both types of three body rates for a benchmark point which has the lowest value of $\chi^2$ (given in Table 3). We introduce the factors $\xi_H$ and $\xi_S$ to the transport equations given in Eq. (3.6) so that every three body rate $\Gamma_Y$ not involving singlets or singlinos is multiplied by $\xi_H$, and those involving singlets or singlinos are multiplied by $\xi_S$. E.g.,

$$\Gamma^t_{QH} \rightarrow \xi_H \Gamma^t_{QH}, \quad \text{and} \quad \Gamma^\tilde{H}H_1\tilde{S} \rightarrow \xi_S \Gamma^\tilde{H}H_1\tilde{S}. \quad (6.2)$$

The BAU increases with the three body rates involving (s)tops and Higgs(ino) interactions. We believe the reasons are the same as that given in Ref. [73]. For three body rates involving singlet (or singlino) interactions we find a different behaviour. When these rates are very small the BAU increases to a peak, similar to the 3 body rates involving (s)quarks. However, the BAU drops sharply after the peak. We explain the sharp drop by the fact that these three body rates relax the linear combination $\mu_{\tilde{H}} + \mu_H \approx 0$, whereas both the supergauge rates as well as all other triscalar and Yukawa rates conspire to relax the combination $\mu_{\tilde{H}} - \mu_H \approx 0$ [82]. The supergauge rate is typically a moderately large value, so if the singlino-Higgsino rate is also large then the result is that both $\mu_{\tilde{H}}$ and $\mu_H$ are relaxed to zero. Since the BAU is, in a fast rate approximation, proportional to $\mu_H$ [64], the BAU goes to zero as well.

7 Discussion and Conclusion

Electroweak baryogenesis is an attractive paradigm for producing the BAU due to its testability. In fact testability is an unavoidable feature of this paradigm as any physics that is responsible for catalysing the baryon production during the EWPT must have mass scales at (or just above [9]) the weak scale and non-trivial couplings. In this paper we have examined one of the most popular extensions to the standard model - the NMSSM - and indeed we find that the scenario requires that at least some neutralinos must be relatively close to the weak scale. If the singlino and Higgsino are both light, the contribution to dark matter from a neutralino lsp tends to be smaller than the observed value. It would certainly be interesting to test whether the electroweak baryogenesis constraints derived in this paper are compatible with getting the right dark matter abundance, or whether one needs to extend the NMSSM. Apart from the constraints on the parameter space, we also examined the structure of the transport equations keeping in mind the different possibilities.
Figure 4. Baryon asymmetry produced during a SFPT against the mass difference of the singlino and Higgsino masses including the Debye masses (top panel) and a contour plot of the baryon asymmetry against the same aforementioned masses (bottom panel). The resonance is the dominant feature of the plot and it appears that in the SFPT case, one needs to be near the center of the resonance to produce the observed BAU unless $\Delta \beta$ is very large. The possibility of large $\Delta \beta$ in the NMSSM leads to the interesting possibility of off resonance baryogenesis.

of how the EWPT proceeds. As usual the most striking feature is the existence of a resonant boost in the CP violating source when the masses of the singlino and Higgsino are near degenerate. In the MSSM one can also have a surprising boost to the baryon asymmetry
in the case where three body rates involving stops and Higgs are large. We have new three body rates involving the Higgs, Higgsino and singlino which enhances the BAU up to a peak and then suppresses the baryon asymmetry when they get large. The suppression is quite severe when its size becomes large enough to compete with the supergauge rate involving Higgs and Higgsinos. This we put down to these two interactions creating approximate local equilibrium relations which conflict to set the baryon asymmetry to approximately zero.

On SSPT phase transitions we can make the qualitative comment that in general it is easier to get a resonance boost to the baryon asymmetry in a SSPT phase transition where the masses of the singlino and the Higgsino are just the Debye masses in the region just outside the bubble of broken elecroweak phase which is primarily responsible for the BAU production. There are reasons however to be skeptical on whether SSPT would frequently occur in any GUT scale model. A full statistical analysis of this we leave to future work. SSPT phase transitions also present technical challenges. The masses of the neutralinos in the broken phase might have a larger contribution from the VEV of the singlet than the Higgs. So the space-time variation of these masses during the electroweak phase transition is very large stretching faith in the VEV insertion approximation. A full Wigner functional treatment as well as a numerical study of the phase transition would shed further light on the viability of baryogenesis in SSPT phase transitions.

Finally we conclude by noting that the relationship between masses requiring a resonance in order to produce enough BAU makes electroweak baryogenesis a fairly fine tuned mechanism. The NMSSM has the attractive possibility of providing more paths to such a boost through producing a large enough $\Delta \beta$ which could potential make EWBG work even well off resonance. A detailed numerical study of $\Delta \beta$ in the NMSSM would shed light on how realistic this possibility is.

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A Transport coefficients and sources

For completeness we present the three body rates here. For a triscalar interaction one has

\[ I_B(m_1,m_2,m_3) = \frac{1}{16\pi^3} \int_{m_1}^{\infty} d\omega_1 h_B(\omega_1) \]

\[ \times \left\{ \log \left( \frac{e^{\omega_1/T} - e^{\omega_2/T}}{e^{\omega_1/T} - e^{\omega_2/T}} \right) \left[ \Theta(m_1 - m_2 - m_3) - \Theta(m_2 - m_1 - m_3) \right] \right. \]

\[ + \log \left( \frac{e^{-\omega_1/T} - e^{\omega_2/T}}{e^{-\omega_1/T} - e^{\omega_2/T}} \right) \Theta(m_3 - m_2 - m_1) \}, \quad (A.1) \]

whereas the the three body Yukawa rate is

\[ I_F(m_1,m_2,m_3) = - (m_1^2 + m_2^2 - m_3^2) \frac{1}{16\pi^3} \int_{m_1}^{\infty} d\omega_1 h_F(\omega_1) \]

\[ \times \left\{ \log \left( \frac{e^{\omega_1/T} + e^{\omega_2/T}}{e^{\omega_1/T} + e^{\omega_2/T}} \right) \left[ \Theta(m_1 - m_2 - m_3) - \Theta(m_3 - m_1 - m_2) \right] \right. \]

\[ + \log \left( \frac{e^{-\omega_1/T} + e^{\omega_2/T}}{e^{-\omega_1/T} + e^{\omega_2/T}} \right) \Theta(m_2 - m_1 - m_3) \}, \]

where

\[ h_{B/F}(x) = \frac{e^x/T}{(e^x/T \pm 1)^2}. \quad (A.2) \]
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