Dephasing Effect in Photon-Assisted Resonant Tunneling through Quantum Dots

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We analyze dephasing in single and double quantum dot systems. The decoherence is introduced by the Büttiker model with current conserving fictitious voltage leads connected to the dots. By using the non-equilibrium Green function method, we investigate the dephasing effect on the tunneling current. It is shown that a finite dephasing rate leads to observable effects. The result can be used to measure dephasing rates in quantum dots.

Recent progress in quantum computing devices has renewed interest in quantum physics as well as potential applications as possible quantum computing devices. For the purpose of quantum computing, phase coherence plays an important role. However, in the previous studies of double-dot systems, the dephasing effect, caused by the electron-electron or electron-phonon interaction, has been ignored.

In this paper we study the decoherence effect on I-V characters in single and double dot systems. In a single-dot system, we find that dephasing causes minor changes in the tunneling current. On the other hand, in the pumping set-up of a double-dot system, in which the chemical potentials on the left and right measurement leads are equal, we find that the photon-assisted resonant tunneling current is sensitive to the dephasing rate. Thus, it provides a possible way to measure the dephasing rate in double-dot systems.

To introduce the dephasing effect into the system, we use the Büttiker model\textsuperscript{4}. In this approach, a system is connected to virtual electron reservoirs through fictitious voltage leads. With certain possibility, electrons are scattered into these reservoirs, lose their phase memories, then re-injected into the system. The chemical potential of the reservoir is chosen such that there is no net current flow between the system and the reservoir in order to satisfy the current conservation in the system. In the original Büttiker model, only \textit{dc} transport has been considered. In our case, the gate voltage is modulated by injected microwave. Consequently, we have to extend the constraint such that each Fourier component of the current through a fictitious voltage lead vanishes.

**Single Quantum Dot.** Before discussing the more interesting case of double-dot systems, we first consider the simpler system of a single quantum dot. The dot is connected to two reservoirs (\(l\) and \(r\)) and a fictitious voltage probe (\(\phi\)). The potential on the dot is controlled by a side-gate voltage. The Hamiltonian can be written as

\[ H = H_0 + H_\phi, \]

where

\[ H_0 = \sum_{\alpha k} \epsilon_{\alpha k}(t) c_{\alpha k}^+ c_{\alpha k} + \epsilon(t) d^+ d + \sum_{\alpha k} [t_{\alpha k} c_{\alpha k}^+ d + h.c.], \]

and

\[ H_\phi = \sum_k \epsilon_k(t) c_{\phi k}^+ c_{\phi k} + \sum_k [t_{\phi k} c_{\phi k}^+ d + h.c.]. \]

\(H_0\) is the Hamiltonian of the system without the fictitious probe and \(\alpha = l, r\) are the indices for the left and right leads. \(c_{\alpha k}^+ (c_{\alpha k})\) creates (annihilates) an electron in lead \(\alpha\) while \(d^+ (d)\) creates (annihilates) an electron in the dot. \(H_\phi\) is the Hamiltonian of the fictitious probe, and \(c_{\phi k}^+ (c_{\phi k})\) is the electron creation (annihilation) operator in the fictitious probe. The microwave injection is modeled as \textit{ac} side-bias which imposes a time-dependent site energy of the quantum dot \(\epsilon(t) = \epsilon_0 + \epsilon_{ac} \cos \omega_{ac} t\). The chemical potential and the time-dependent voltage of the fictitious probe are determined by the condition that the total current flowing through the probe \(\phi\) vanishes.

The time-dependent current from reservoir \(\alpha\) to the dot can be expressed as

\[ I_\alpha(t) = -\frac{2e}{\hbar} \int_{-\infty}^{\infty} dt_1 \int \frac{dc}{2\pi} \text{Im} \left\{ e^{i(t-t_1)} \Gamma_\alpha(\epsilon) \times \exp \left[ i \int_{t_1}^t d\tau \Delta_\alpha(\tau) \right] \left[ G^{<}(t, t_1) + f_\alpha(\epsilon) G^>(t, t_1) \right] \right\}, \]

where \(\Gamma_\alpha(\epsilon) = 2 \pi \rho_\alpha(\epsilon) |t_{\alpha}|^2\) and \(\rho_\alpha\) is the density of states of the reservoir \(\alpha\). \(\alpha\) is the index for the left, right, or fictitious voltage lead. For simplicity, we use the wide-band approximation, i.e., treating the \(\Gamma_\alpha(\epsilon)\) as a constant, \(\Gamma_\alpha(\epsilon) = \Gamma_\alpha\). Within the wide-band approximation, the retarded Green function \(G^r\) takes the form

\[ G^r(t, t') = -i \theta(t - t') \exp \left\{ -i \left[ \epsilon_0 - i \frac{\Gamma_0 + \Gamma_\alpha}{2} \right] (t - t') - i \int_{t'}^t d\tau \Delta_{ac}(\tau) \right\}. \]
where $\Delta_{ac}(\tau) = e v_{ac} \cos \omega_0 \tau$, and $\Gamma_0 = \Gamma_l + \Gamma_r$ is the energy broadening of the quantum dot due to the left and right leads, and $\Gamma_0$ is the broadening due to the fictitious voltage lead. The spectral density, which relates to $G^\alpha$ through

$$A_\alpha(\epsilon, t) = \int_{-\infty}^{t} dt' e^{i(r(t-t')} e^{i\int_{t'}^{t} d\tau \Delta_{ac}(\tau)} G^\alpha(t, t'),$$

is given by

$$A_\alpha(\epsilon, t) = \sum_{m=\pm\infty} \frac{J_m(\tilde{a}_\alpha)}{\tilde{\epsilon}_m(\epsilon)} e^{-i\tilde{a}_\alpha \sin \omega_0 t} e^{i\epsilon \omega_0 t},$$

where $\tilde{\epsilon}_m(\epsilon) = \epsilon - \epsilon_0 - m \hbar \omega_0 + i(\Gamma_0 + \Gamma_\phi)/2$, $\tilde{a}_\alpha = e v_{\alpha}/\omega_0$, and $\tilde{\epsilon}_\phi = e(v_{\alpha} - v_{ac})/\omega_0$.

The Keldysh Green function is related to the retarded and advanced Green functions through the Keldysh equation,

$$G^<(t, t') = \int dt_1 dt_2 G^r(t, t_1) \Sigma^<(t_1, t_2) G^a(t_2, t').$$

Again, using the wide-band approximation, we obtain the Keldysh self-energy

$$\Sigma^<(t_1, t_2) = i \sum_\alpha f_\alpha(\epsilon) \Gamma_\alpha + f_\phi(\epsilon) \Gamma_\phi \delta(t_1 - t_2).$$

The Keldysh Green function is related to $A_\alpha(\epsilon, t)$ through

$$2 \int_{-\infty}^{t} dt_1 \text{Im} \left\{ e^{i(r(t-t') \int_{t_1}^{t} d\tau \Delta_{ac}(\tau)} G^<(t, t_1) \right\} = \sum_\alpha f_\alpha(\epsilon) \Gamma_\alpha |A_\alpha(\epsilon, t)|^2.$$

After some algebra, the current flowing through the fictitious probe is found to be

$$I_\phi(t) = -\frac{e}{h} \Gamma_\phi \sum_{m, n=-\infty}^{\infty} \int \frac{de}{2\pi} \frac{e^{i(m-n)\omega_0 t}}{\tilde{\epsilon}_m(\epsilon) \tilde{\epsilon}_n(\epsilon)} T_0(\phi),$$

where

$$T_0(\phi) = f_\phi(\epsilon) J_m(\tilde{a}_\phi) J_n(\tilde{a}_\phi) [i(m-n)\omega_0 - \Gamma_0]$$

+ $[f_\phi(\epsilon) \Gamma_l + f_\phi(\epsilon) \Gamma_r] J_m(\tilde{a}_\phi) J_n(\tilde{a}_\phi)$.

To satisfy the condition that the total current flowing through the fictitious voltage probe vanishes, one gets

$$f_\phi(\epsilon) J_m(\tilde{a}_\phi) J_n(\tilde{a}_\phi) = \mathcal{I}(\epsilon) \Gamma_0 J_m(\tilde{a}_\phi) J_n(\tilde{a}_\phi) / \Gamma_0,$$

where $\mathcal{I}(\epsilon) = (f_\phi \Gamma_l + f_\phi \Gamma_r) / \Gamma_0$ is an effective Fermi function of the dot without the fictitious voltage probe.

Using Eq.\(^4\) and Eq.\(^5\), the time-dependent current flowing through lead $\alpha$ can be expressed as

$$I_\alpha(t) = \frac{e}{h} \Gamma_\alpha \int \frac{de}{2\pi} \sum_{m, n=-\infty}^{\infty} e^{i(m-n)\omega_0 t} \Phi_{mn} T_{mn}(\phi)$$

$$\times \{ i(m-n)\hbar \omega_0 f_\alpha(\epsilon) + \Gamma_\alpha [f_\alpha(\epsilon) - f_\alpha(\epsilon)] \},$$

where

$$\Phi_{mn}(\phi) = \frac{J_m(\tilde{u}) J_n(\tilde{u})}{\tilde{\epsilon}_m(\phi) \tilde{\epsilon}_n(\phi)},$$

$$\Phi_{mn} = (m-n)\hbar \omega_0 - \frac{i(\Gamma_0 + \Gamma_\phi)}{(m-n)\hbar \omega_0 - i\Gamma_0}.$$
\[
H = H_0 + H_\phi \\
H_0 = \sum_{\alpha, k} \varepsilon_{\alpha, k}(t)c_{\alpha, k}^+c_{\alpha, k} + \sum_{\alpha} \varepsilon_{\alpha}(t) d_{\alpha}^+d_{\alpha} \\
+ \sum_{\alpha, k} \left[ \eta_{\alpha k}c_{\alpha, k}d_{\alpha} + h.c. \right] + [\Delta d_{\alpha}^+d_{\alpha} + h.c.], \\
H_\phi = \sum_{\alpha, k} (\phi^{(\alpha)}_{\alpha, k}(t)c_{\alpha, k}^{(\alpha)} + (\phi^{(\alpha)}_{\alpha, k}(t)^t), \\
\text{Comparing with the Hamiltonian for a single dot, two fictitious voltage probes } c_{\alpha, k}(\alpha = l, r) \text{ are connected to the two dots. The current flowing in and out the reservoirs can still be calculated by Eq.\text{(3)} provided that } G^\tau \text{ and } G^< \text{ are now } 2 \times 2 \text{ matrices with the matrix elements associated with the left and right dots. The time-dependent current flowing through the measurement lead } \alpha (\eta = 0, \alpha = l, r) \text{ or the fictitious voltage lead } \alpha (\eta = \phi, \alpha = l, r) \text{ can be expressed as} \\
J^{(\eta)}_\alpha(t) = -\frac{e}{\hbar} \Gamma_\alpha \int \frac{de}{2\pi} \left[ \sum_{\beta} J^{(\beta)}_\alpha(\phi) |A^{(\alpha, \beta)}_{\phi, \alpha}(t)|^2 \right] \\
+ \sum_{\beta} f^{(\beta)}_\beta(\phi) \Gamma_\beta |A^{(\alpha, \beta)}_{\phi, \alpha}(t)|^2 + 2f_\alpha(\phi) \text{Im}A^{(\alpha, \alpha)}_{\phi, \alpha}(t), \tag{6}
\]
where \(A^{(\alpha, \beta)}_{\phi, \alpha}(t, \phi, \alpha, \beta) \) are the matrix elements of the spectral function. The diagonal elements are
\[
A^{(\alpha, \alpha)}_{\phi, \alpha}(t, \phi, \alpha, \beta) = e^{-i\varepsilon^{(\alpha)}_{\alpha} \sin \omega_0 t} \sum_{m=-\infty}^{\infty} J^{(\alpha)}_m(\bar{\varepsilon}^{(\alpha)}_{\alpha}) e^{im\omega_0 t},
\]
and the non-diagonal elements (\(\alpha \neq \beta\)) are
\[
A^{(\alpha, \beta)}_{\phi, \alpha}(t, \phi, \alpha, \beta) = \Delta e^{-i}\varepsilon^{(\alpha)}_{\alpha} + \bar{\varepsilon}^{(\beta)}_{\alpha} \sin \omega_0 t \times \sum_{m=-\infty}^{\infty} J^{(\alpha)}_m(\bar{\varepsilon}^{(\beta)}_{\alpha}) e^{im\omega_0 t} \sum_{k=-\infty}^{\infty} J^{(k)}_k(\bar{\varepsilon}^{(\alpha)}_{\alpha}) e^{ik\omega_0 t} \sum_{k=-\infty}^{\infty} J^{(k)}_k(\bar{\varepsilon}^{(\beta)}_{\alpha}) e^{ik\omega_0 t},
\]
with
\[
\varepsilon^{(\alpha)}_{m}(\phi, \alpha) = \varepsilon^{(\alpha)}_{\alpha}(m) - |\Delta|^2 \sum_k J^{(k)}_k(\bar{\varepsilon}^{(\alpha)}_{\alpha})(m+k),
\]
\[
\bar{\varepsilon}^{(\alpha)}_{\alpha}(m) = \varepsilon_{\alpha} - \varepsilon_{\alpha} - m\hbar \omega_0 + \frac{\Gamma_\alpha + \Gamma^{(\alpha)}_{\phi}}{2},
\]
\[
\bar{\varepsilon}^{(\beta)}_{\alpha}(\phi) = \varepsilon_{\alpha} - \varepsilon_{\alpha} \frac{\Gamma^{(\alpha)}_{\phi}}{\hbar \omega_0},
\]
\[
\bar{\varepsilon}^{(\alpha)}_{\alpha}(\phi) = \varepsilon_{\alpha} - \varepsilon_{\alpha} \frac{\Gamma^{(\alpha)}_{\phi}}{\hbar \omega_0}.
\]
We further require the net current through each fictitious probe to vanish, i.e., \(I^{(\phi)}_\alpha(t) = 0\). From this constraint, we can obtain relations among \(f^{(\phi)}_\beta(\phi, \alpha, \beta)\) and \(A^{(\alpha, \beta)}_{\phi, \alpha}(t, \phi, \alpha, \beta)\). Using these relations to replace \(\sum_{\beta} f^{(\phi)}_\beta(\phi) \Gamma_\beta |A^{(\alpha, \beta)}_{\phi, \alpha}(t)|^2 \) in Eq.\text{(3)}, we obtain the current through lead \(\alpha\),
\[
I^{(\phi)}_\alpha(t) = \frac{e}{\hbar} \int \frac{de}{2\pi} \sum_{m,n=-\infty}^{\infty} e^{i(m-n)\omega_0 t} \Phi_{mn}(\phi) \Gamma_\alpha |A^{(\alpha, \beta)}_{\phi, \alpha}(t)|^2. \tag{7}
\]
In the above equation,
\[
T^{\phi} m n (\phi) = f^{(\phi)}_\alpha(\phi) F^{(\phi)} m n (\phi) - f^{(\phi)}_\alpha(\phi) F^{(\phi)} m n (\phi),
\]
\[
F^{(\phi)} m n (\phi) = \frac{J^{(\phi)}_m(\bar{\varepsilon}^{(\phi)}_{\alpha})J^{(\phi)}_n(\bar{\varepsilon}^{(\phi)}_{\alpha})}{\varepsilon^{(\phi)}_{\alpha}(\phi, \alpha)} \Lambda^{(\alpha, \phi)} m n (\phi, \alpha),
\]
\[
\Phi_{mn}(\phi, \alpha) = \frac{\Omega^{(l)}_{mn}(\phi, \alpha) \Omega^{(l)}_{mn}(\phi, \alpha)}{1 - |\Delta|^4 \Gamma^{(\phi)}_{\phi} \Lambda^{(\alpha, \phi)}_{mn}(\phi, \alpha)}
\]
\[
\Omega^{(l)}_{mn}(\phi, \alpha) = i(m-n)\omega_0 - \left( \Gamma_\alpha + \Gamma^{(\alpha)}_{\phi} \right) + |\Delta|^2 \left[ i(m-n)\omega_0 - \left( \Gamma_\alpha + \Gamma^{(\alpha)}_{\phi} \right) \right] \Lambda^{(\alpha)} m n (\phi, \alpha).
\]
The time average of the current \(T = \langle I(t) \rangle_t \) can be obtained
\[
T = \frac{e}{\hbar} \Gamma_{\alpha} |\Delta|^2 \int \frac{de}{2\pi} \sum_{m=-\infty}^{\infty} \Phi_{mn}(\phi) \left[ f^{(\phi)}_\alpha(\phi) F^{(l)} m n (\phi, \phi) - f^{(\phi)}_\alpha(\phi) F^{(l)} m n (\phi, \phi) \right]. \tag{8}
\]
From the above expression, one finds a non-zero tunneling current even without dc bias between the right and left leads. It is consistent with the experimental observations.

Figure 2 shows the time-averaged tunneling current \(< I > \) for different dephasing strength \(\Gamma_\phi\). \(< I > \) strongly depends on \(\Gamma_\phi\). In real systems, \(\Gamma_\phi\) is a function of temperature. Temperature has two effects on the tunneling current: the direct contribution from Fermi distribution function and the indirect contribution through \(\Gamma_\phi\). We find that at low temperatures the temperature dependence of the current is essentially determined by the temperature dependence of \(\Gamma_\phi\). The current is almost independent of temperature within \(kT \ll |\epsilon_1 - \epsilon_2|\) while the dephasing strength \(\Gamma_\phi\) is set to a constant. Thus, by measuring the temperature dependence of the current, it is possible to determine the temperature dependence of the dephasing rate in double-dot systems.
The pumping current. Thus, in a non-pumping set-up, the tunneling current is like the pumping case (Fig. 2) superimposed by a curve similar as shown in Fig. 1 which makes the dephasing effect less pronounced.

Before summary, we would like to comment on the difference between single-dot and double-dot systems. Although the dephasing effect shows up in both single-dot and double-dot systems, the most pronounced effect is under the pumping situation (Fig. 2) in double dots (There is no pumping effect in single-dot systems.). In a non-pumping situation in double dots, i.e., the chemical potentials are not equal on the left and the right leads, the dephasing effect is reduced. To see this, one can rewrite the current (Eq. (8)) into two terms, one contains \((f_i(\epsilon) - f_r(\epsilon))(\mathcal{F}^{(l)}_{m\ell}(\epsilon, \phi) + \mathcal{F}^{(r)}_{m\ell}(\epsilon, \phi))\) and the other contains \((f_i(\epsilon) + f_r(\epsilon))(\mathcal{F}^{(l)}_{m\ell}(\epsilon, \phi) - \mathcal{F}^{(r)}_{m\ell}(\epsilon, \phi))\). The first term shows a similar behavior for the current in a single-dot, whereas the second term gives a similar behavior for

It is commonly believed that the dephasing strength \(\Gamma_\phi\) is a power law function of temperature \(T\). Hence, \(\Gamma_\phi\) approaches zero as \(T\) goes to zero. However, this consensus has been challenged in a recent experiment in which \(\Gamma_\phi\) was found to saturate to finite values in many one or two dimensional systems. The question remains whether such a saturation exists in the zero-dimensional quantum dots. We suggest that this issue can be resolved in double-dot systems by studying the temperature dependence of the tunneling current. Figure 3 shows the temperature dependence of the dephasing strength \(\Gamma_\phi\). The first kind is the normal linear dependence (Other power-law dependence gives rise to qualitatively similar results.) with \(\Gamma_\phi = \Gamma_0 T\) (\(\Gamma_0 = 0.5\) and \(T_0 = 2\) in the plot.). The second kind is the abnormal temperature dependence with \(\Gamma_\phi = \Gamma_0 \tanh(T_0/T)\), in which the dephasing rate saturates at low temperatures. Figure 3 clearly shows that the average current exhibits distinct behaviors at low \(T\) for these two temperature dependence of \(\Gamma_\phi\). Thus, one can detect the possible dephasing rate saturation in a double-dot system by measuring the tunneling current.

In summary, we analyze the dephasing effects in photon-assisted tunneling in quantum dots. The dephasing effect is introduced by using fictitious voltage probes. We find that the time-averaged current is insensitive to dephasing in single-dot systems. However, in the pumping set-up of double-dot systems, dephasing has profound effect in the tunneling current which can be measured to determine the dephasing rate.

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