Ostrogradsky’s Hamilton formalism and quantum corrections

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Abstract

By means of a simple scalar field theory it is demonstrated that the Lagrange formalism and Ostrogradsky’s Hamilton formalism in the presence of higher derivatives, in general, do not lead to the same results. While the two approaches are equivalent at the classical level, differences appear due to quantum corrections.

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1. Introduction

The higher derivative dynamics is of particular interest in the context of modern effective quantum field theories (see, e.g., [1, 2] and references therein). However, the quantization of Lagrangians involving higher derivatives is a non-trivial problem. The canonical quantization of field theories using the Hamilton formalism is a reliable method leading to a unitary scattering matrix. At the classical level, the Hamilton formalism for Lagrangians with higher derivatives was developed by Ostrogradsky [3] a long time ago. The canonical quantization based on Ostrogradsky’s Hamilton formalism can be found, e.g., in [4].

In this work we examine Ostrogradsky’s Hamilton formalism and demonstrate that this method, although equivalent to the Lagrange formalism at the classical level, may lead to wrong results due to quantum corrections.

2. A toy model

We consider the following Lagrangian of two scalar fields \( A \) and \( \Phi \):

\[
\mathcal{L}_1(A, \Phi) = \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{M^2}{2} \Phi^2,
\]

(1)

describing free massless \( (A) \) and massive \( (\Phi) \) spinless particles. For simplicity, we do not include the dependence on the partial derivatives of the fields in the list of arguments of the Lagrangian. The momenta canonically conjugated to the fields \( A \) and \( \Phi \) are defined by
resulting in the Hamiltonian
\[ p_A \partial_0 A + p_\Phi \partial_0 \Phi - L_1 = \frac{1}{2} p_A^2 + \frac{i}{2} \vec{\nabla} A \cdot \vec{\nabla} A + \frac{1}{2} p_\Phi^2 + \frac{i}{2} \vec{\nabla} \Phi \cdot \vec{\nabla} \Phi + \frac{1}{2} M^2 \Phi^2 \equiv \mathcal{H}_1. \]

Let us consider the generating functional of the Green’s functions of the field \( A \) in the canonical path-integral representation:
\[ Z[J] = \int DADpADp\Phi e^{i \int d^4x \left( p_A \partial_0 A + p_\Phi \partial_0 \Phi - L_1 + JA \right)}, \]
where \( \mathcal{H}_1 \) is given in equation (4). In the following, we will repeatedly make use of a Gaussian functional integral of the form
\[ \int Dp e^{i \int d^4x \left( -\frac{1}{2} p^2 + fp \right)} = N e^{i \int d^4x f^2}, \]
where \( N \) is an (irrelevant) multiplicative factor and \( f \) is a given function not depending on \( p \). Applying equation (6) to the \( p_A \) and \( p_\Phi \) (functional) integrations in equation (5) and omitting, as is common practice, the corresponding multiplicative factors \( N \) of equation (6), yields
\[ Z[J] = \int DADp e^{i \int d^4x \left( L_1(A, \Phi) + JA \right)}, \]
where \( L_1 \) is given in equation (1). The (full) propagator of the \( A \) field is given by
\[ i \Delta_A(p) = \frac{i}{p^2 + i0^+}, \]
and the \( A \) field describes a massless non-interacting spinless particle.

3. Field transformations

Our line of arguments relies on the principle that, for a given theory, the physical content of the theory both at the classical and the quantum level should not depend on the choice of variables for describing the physical degrees of freedom. We will make use of the free theory described by the simple Lagrangian (Hamiltonian) of equation (1) (equation (4)) and its canonical path-integral quantization described by equation (5). The path-integral representation of equation (7) is a deduced quantity in the sense that it is derived from the canonical result of equation (5). The Green’s functions obtained from equation (7), in particular the propagator of equation (8), will be taken as reference quantities. We will make use of two types of changes of field variables, namely, transformations without and with time derivatives of a field. Using the reference result of section 2, we will be able to point out that the canonical path-integral quantization applied to a Hamiltonian based on the Ostrogradsky method, at the quantum level, does not describe an equivalent theory.

3.1. Field transformation without a time derivative

We first consider a change of field variables involving both spatial derivatives of a new field \( \phi \) and the product of the \( A \) field with the \( \phi \) field:
\[ \Phi(x) = \phi(x) - c \Delta \phi(x) + g \phi(x) A(x), \]
where \((x)\) stands for \((t, \mathbf{x})\). In equation (9), the real parameters \(c\) and \(g\) carry the dimensions of a squared inverse mass and an inverse mass, respectively, and \(\Delta\) denotes the usual Laplace operator. The Lagrangian density \(\mathcal{L}_2\) in the new variables is obtained by substituting \(\Phi(x)\) of equation (9) into the original Lagrangian \(\mathcal{L}_1\) of equation (1):

\[
\mathcal{L}_2(A, \phi) = \mathcal{L}_1(A, \Phi).
\] (10)

Applying the change of variables, given by equation (9), directly to the generating functional of equation (7), we obtain

\[
Z[J] = \int \mathcal{D}A\mathcal{D}\phi\det \left( \frac{\delta \Phi(y)}{\delta \phi(z)} \right) e^{i \int d^4x [\mathcal{L}_2(A,\phi) + JA]},
\] (11)

where

\[
\left( \frac{\delta \Phi(y)}{\delta \phi(z)} \right) = ((1 - c\Delta + g A(y))\delta^4(y - z))
\] (12)
denotes the Jacobian ‘matrix’ of the field transformation. Equation (11) is the generalization of the substitution rule for multiple Riemann integrals to functional integrals.

The same result as equation (11) for the generating functional is obtained by first applying the Hamilton formalism to the Lagrangian \(\mathcal{L}_2\) of equation (10) and by subsequently performing the canonical path-integral quantization. To that end we define the canonical momenta

\[
p_A = \frac{\partial \mathcal{L}_2}{\partial \delta_\partial_0 A} = \partial_0 A + g\phi \partial_0 \Phi,
\] (13)

\[
p_\phi = \frac{\partial \mathcal{L}_2}{\partial \delta_\partial_0 \phi} = (1 - c\Delta + gA)\partial_0 \Phi,
\] (14)

where

\[
\partial_0 \Phi = (1 - c\Delta + gA)\partial_0 \phi + g\phi \partial_0 A.
\] (15)

From equation (14) we obtain

\[
\partial_0 \Phi = \hat{O} p_\phi \equiv (1 - c\Delta + gA)^{-1} p_\phi.
\] (16)

Substituting equation (16) into equation (13), we can solve

\[
\partial_0 A = p_A - g\phi \hat{O} p_\phi.
\] (17)

Finally, inserting equations (16) and (17) into equation (15), we can solve

\[
\partial_0 \phi = (1 - c\Delta + gA)^{-1} [1 + (g\phi)^2] \hat{O} p_\phi - g\phi p_A.
\] (18)

Using integration by parts and omitting total divergences, the Hamiltonian takes the form

\[
\mathcal{H}_2 = p_A \partial_0 A + p_\phi \partial_0 \phi - \mathcal{L}_2
\]

\[
= \frac{1}{2} p_A^2 + \frac{1}{2} \left[ 1 + (g\phi)^2 \right] (\hat{O} p_\phi)^2 - g\phi \hat{O} p_\phi p_A + \frac{1}{2} \nabla A \cdot \nabla A + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi + \frac{1}{2} M^2 \Phi^2.
\] (19)

In terms of the Hamiltonian \(\mathcal{H}_2\) the generating functional for the Green’s functions of the \(A\) field is given by [4]

\[
Z[J] = \int \mathcal{D}A\mathcal{D}\phi\mathcal{D}p_\phi e^{i \int d^4x [p_A \partial_0 A + p_\phi \partial_0 \phi - \mathcal{H}_2 + JA]}.
\] (20)

Introducing the new variable \(\pi_\phi = \hat{O} p_\phi\), we obtain

\[
Z[J] = \int \mathcal{D}A\mathcal{D}\phi\mathcal{D}\pi_\phi\det \left( \frac{\delta \Phi(y)}{\delta \pi_\phi(z)} \right) e^{i \int d^4x [p_A \partial_0 A + \pi_\phi \partial_0 \phi - \mathcal{H}_2 + JA]},
\] (21)
where \( \hat{O}^{-1}_\pi \phi = (1 - c \Delta + gA)\pi \phi \). The Jacobian matrix is given by
\[
\begin{pmatrix}
\frac{\delta p_\phi(y)}{\delta \pi_\phi(z)}
\end{pmatrix}
= ([1 - c \Delta_y + gA(y)]\delta^4(y - z)),
\]
and coincides with the Jacobian matrix of equation (12). Finally, the Hamiltonian \( \tilde{H}_2 \) reads
\[
\tilde{H}_2 = \frac{1}{2} (p_A - gA \pi \phi)^2 + \frac{1}{2} \pi \phi^2 + \frac{1}{2} \nabla A \cdot \nabla A + \frac{1}{2} \nabla \phi \cdot \nabla \phi + \frac{1}{2} M^2 \phi^2.
\] (22)

By means of partial integration in the exponent of equation (21), the expression \( \hat{O}^{-1}_\pi \phi \partial_0 \phi \) is replaced by
\[
\pi \phi \hat{O}^{-1}_\pi \partial_0 \phi.
\]
Performing subsequently the \( p_A \) and \( \pi \phi \) integrations using equation (6), we obtain
\[
Z[J] = \int DA D\chi \det \left( \frac{\delta \phi_1(y)}{\delta \chi(z)} \right) e^{\int d^4x [L_2(A, \phi) + JA]},
\] (23)
which is identical to equation (11). To summarize this section, given the change of variables of equation (9) (without a time derivative), the substitution in the functional integral of equation (7) yields the same result as the application of the canonical path-integral quantization starting from the Hamiltonian \( \tilde{H}_2 \) derived from the Lagrangian \( L_2 \).

3.2. Field transformation with time derivatives

We now go one step further and consider the following change of field variables involving time derivatives:
\[
\phi(x) = \chi(x) + c \Box \chi(x) + g\chi(x)A(x),
\] (24)
where \( \Box = \partial_0^2 - \Delta \) denotes the d’Alembert operator. The Lagrangian in the new variables is obtained from
\[
L_3(A, \chi) = L_1(A, \phi).
\] (25)

Because of the d’Alembertian in the field transformation, the Lagrangian \( L_3(A, \chi) \) contains time derivatives of the field \( \chi \) up to and including third order. Performing the change of variables in the generating functional of equation (7) results in
\[
Z[J] = \int DA D\chi Dg_1 Dg_2 e^{\int d^4x [L_3(A, \chi) + JA + g_2 (1 + c \Box + gA)g_1]},
\] (26)
with the Jacobian matrix
\[
\begin{pmatrix}
\frac{\delta \phi_1(y)}{\delta \chi(z)}
\end{pmatrix}
= ([1 + c \Box_y + gA(y)]\delta^4(y - z)).
\]

We express the determinant of the Jacobian matrix in terms of a functional integral over ghost fields \( g_1 \) and \( g_2 \) (scalar Grassmann variables),
\[
\det([1 + c \Box_y + gA(y)]\delta^4(y - z)) = \int DG_1 DG_2 e^{\int d^4x [G_1(1 + c \Box + gA)G_2]},
\] (27)
In this representation the generating functional takes the following form:
\[
Z[J] = \int DA D\chi DG_1 DG_2 e^{\int d^4x [L_3(A, \chi) + G_1(1 + c \Box + gA)G_2 + JA]},
\] (28)
Using the example of the full propagator of the \( A \) field, we will illustrate that equation (28) gives rise to the same Green’s functions including quantum corrections.
3.3. Full propagator of the $A$ field at the one-loop level

We will investigate the propagator of the $A$ field resulting from the perturbative expansion of equation (28) at the one-loop level. We will explicitly see that the quantum corrections obtained from equation (28) do not modify the position of the pole, i.e. the $A$ field remains massless. The dressed propagator of the $A$ field is of the form

$$i\Delta_A(p) = \frac{i}{p^2 - \Sigma(p^2)},$$

where $-i\Sigma$ denotes the proper self-energy insertions of the $A$ field, i.e. the sum of one-particle irreducible diagrams contributing to the two-point function.

At the one-loop level, three diagrams contribute to the self-energy (see figure 1). The corresponding Feynman rules are summarized in the appendix. Using dimensional regularization, we obtain at $p^2 = 0$

$$\Sigma^{(a)}(0) = -i g^2 \int \frac{d^dk}{(2\pi)^n} \frac{1}{(1 - c k^2)^2} = -\frac{1}{2} \Sigma^{(b)}(0) = \Sigma^{(c)}(0).$$

These contributions cancel each other and, as a result, the quantum corrections do not give rise to a mass of the $A$ field. This was expected, as the field transformation cannot change the physical content of the theory.

4. Ostrogradsky’s Hamilton formalism

We now apply the Hamilton formalism and the canonical quantization to the Lagrangian of equation (25) and derive the generating functional. As the Lagrangian $L_3$ contains time derivatives of higher orders, we use the Ostrogradsky formalism. While exactly the same results are obtained by following the procedure of [4], here we apply the method of [5] based on an auxiliary Lagrangian. We first define new independent fields

$$\psi = \partial_0 \chi, \quad \zeta = \partial_0 \psi.$$  

Introducing the Lagrange multipliers $\lambda_1$ and $\lambda_2$ in order to enforce the relations of equation (31), we obtain the auxiliary Lagrangian

$$\mathcal{L}_{aux} = \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \left[ \psi + c(\partial_0 \zeta - \Delta \psi) + g \partial_0 A \chi + g A \psi \right]^2$$

$$- \frac{1}{2} \nabla [\chi + c(\zeta - \Delta \chi) + g A \chi] \cdot \nabla [\chi + c(\zeta - \Delta \chi) + g A \chi]$$

$$- \frac{1}{2} M^2 [\chi + c(\zeta - \Delta \chi) + g A \chi]^2 + \lambda_1 (\psi - \partial_0 \chi) + \lambda_2 (\zeta - \partial_0 \psi).$$  

The momenta canonically conjugated to the degrees of freedom $A$, $\chi$, $\psi$, $\zeta$, $\lambda_1$, and $\lambda_2$ are defined as

$$
\begin{align*}
 p_A &= \frac{\partial L_{aux}}{\partial \dot{A}} = \partial_0 A (1 + g^2 \chi^2) + g \chi [\psi + c (\partial_0 \zeta - \Delta \psi) + g A \psi], \\
 p_\chi &= \frac{\partial L_{aux}}{\partial \dot{\chi}} = -\lambda_1, \\
 p_\psi &= \frac{\partial L_{aux}}{\partial \dot{\psi}} = -\lambda_2, \\
 p_\zeta &= \frac{\partial L_{aux}}{\partial \dot{\zeta}} = c [\psi + c (\partial_0 \zeta - \Delta \psi) + g A \chi + g A \psi], \\
 p_{\lambda_1} &= \frac{\partial L_{aux}}{\partial \dot{\lambda}_1} = 0, \\
 p_{\lambda_2} &= \frac{\partial L_{aux}}{\partial \dot{\lambda}_2} = 0.
\end{align*}
$$

(33)

For $c \neq 0$, the two equations for $p_A$ and $p_\zeta$ can be inverted to solve $\partial_0 A$ and $\partial_0 \zeta$, respectively:

$$
\begin{align*}
 \partial_0 A &= p_A - \frac{1}{c} g \chi p_\zeta, \\
 \partial_0 \zeta &= \frac{1}{c} \left[ 1 + (g \chi)^2 \right] p_\zeta - c g \chi p_A - c (1 - c \Delta + g A) \psi.
\end{align*}
$$

The remaining velocities cannot be solved from equations (33), i.e. the corresponding momenta need to satisfy the primary constraints

$$
\begin{align*}
 \Phi_1 &= p_{\lambda_1} \approx 0, \\
 \Phi_2 &= p_{\lambda_2} \approx 0, \\
 \Phi_3 &= p_\chi + \lambda_1 \approx 0, \\
 \Phi_4 &= p_\psi + \lambda_2 \approx 0.
\end{align*}
$$

(34)

Here, $\Phi_i \approx 0$ denotes a weak equation in Dirac’s sense, namely that one must not use one of these constraints before working out a Poisson bracket [6]. The so-called total or generalized Hamiltonian $H^{(1)}$ has the form

$$
H^{(1)} = \sum_{i=1}^{4} \Phi_i z_i + \mathcal{H},
$$

(35)

where

$$
\begin{align*}
 \mathcal{H} &= \frac{1}{2} \left[ p_A - \frac{g \chi}{c} p_\zeta \right]^2 + \frac{1}{2} \frac{p_\chi^2}{c^2} - \frac{1}{c} p_\chi (1 - c \Delta + g A) \psi + \frac{1}{2} \vec{\nabla} A \cdot \vec{\nabla} A \\
 &\quad + \frac{1}{2} \left[ \chi + c (\zeta - \Delta \chi) + g A \chi \right] \cdot \vec{\nabla} [\chi + c (\zeta - \Delta \chi) + g A \chi] \\
 &\quad + \frac{1}{2} M^2 \left[ \chi + c (\zeta - \Delta \chi) + g A \chi \right]^2 - \lambda_1 \psi - \lambda_2 \zeta.
\end{align*}
$$

(36)

In equation (35), the $z_i$ are arbitrary functions which have to be determined. The constraints of equation (34) have to be conserved in time. Therefore, we demand that the Poisson brackets of $\Phi_i$ with $H^{(1)} = \int d^3 x H^{(1)}$ vanish. An explicit evaluation of the Poisson brackets yields

$$
\begin{align*}
 \{ \Phi_1, H^{(1)} \} &\approx 0 \Rightarrow z_3 = \psi, \\
 \{ \Phi_2, H^{(1)} \} &\approx 0 \Rightarrow z_4 = \zeta.
\end{align*}
$$
\[
\{\Phi_3, H^{(1)}\} \approx 0 \Rightarrow z_1 = -\frac{g}{c} p_c \left( p_A - \frac{gX}{c} p_t \right) - (1 - c\Delta + gA)((\Delta - M^2)(\chi + c\zeta - c\Delta\chi + g\chi A),
\]
\[
\{\Phi_4, H^{(1)}\} \approx 0 \Rightarrow z_2 = -\frac{1}{c}(1 - c\Delta + gA)p_c - \lambda_1.
\]

According to [4], the generating functional for the Green’s functions of the A field can be written as a path integral over canonical coordinates and momenta,

\[
Z[J] = \int D\mathcal{A}D\mathcal{X}Dp_\mathcal{A}Dp_\mathcal{X}Dp_\mathcal{X}D\mathcal{P}_\mathcal{X}D\mathcal{P}_\mathcal{A}D\mathcal{P}_\mathcal{X}Dp_\mathcal{X}Dp_\mathcal{X}
\times \delta[\Phi_1]\delta[\Phi_2]\delta[\Phi_3]\delta[\Phi_4][\text{det}(\{,\})]^\frac{1}{2} e^{S[J]},
\]

(37)

where

\[
S[J] = \int d^4x (p_\mathcal{A} \partial_\mathcal{A} A + p_\mathcal{X} \partial_\mathcal{X} X + p_\mathcal{A} \partial_\mathcal{A} \lambda_1 + p_\mathcal{X} \partial_\mathcal{X} \lambda_2 + p_\mathcal{A} \partial_\mathcal{A} \psi + p_\mathcal{X} \partial_\mathcal{X} \zeta - H + J A).
\]

(38)

The delta functionals in equation (37) take care of the constraints of equation (34). The entries of the \(4 \times 4\) matrix \((\{\Phi, \Phi\})\) are given by the Poisson brackets \(\{\Phi_i, \Phi_j\}\) and, in the present case, the determinant reduces to a constant which will be omitted in the following.

In equation (37), because of the delta functionals, the integrations over \(\lambda_i\) and \(p_{\mathcal{A}}\) are straightforward and give rise to the intermediate result

\[
Z[J] = \int D\mathcal{A}D\mathcal{X}Dp_\mathcal{A}D\mathcal{X}D\mathcal{P}_\mathcal{X}Dp_\mathcal{X}D\mathcal{P}_\mathcal{A}D\mathcal{P}_\mathcal{X}Dp_\mathcal{X}Dp_\mathcal{X}
\times e^{i \int d^4x (p_\mathcal{A} \partial_\mathcal{A} A + p_\mathcal{X} \partial_\mathcal{X} X + p_\mathcal{A} \partial_\mathcal{A} \lambda_1 + p_\mathcal{X} \partial_\mathcal{X} \lambda_2 + p_\mathcal{A} \partial_\mathcal{A} \psi + p_\mathcal{X} \partial_\mathcal{X} \zeta - \frac{1}{2}p_A^2 - \frac{1}{2}p_X^2 - \frac{1}{2}M^2(\chi + c\zeta - c\Delta\chi + g\chi A)^2 - p_\mathcal{X}^2 \psi - p_\mathcal{A}^2 \zeta + JA}).
\]

(39)

The momenta \(p_\mathcal{X}\) and \(p_\mathcal{A}\) appear linearly in the exponent of equation (39). Therefore, the integrations over \(p_\mathcal{X}\) and \(p_\mathcal{A}\) give rise to the delta functionals \(\delta[\partial_\mathcal{A} X - \psi]\), and \(\delta[\partial_\mathcal{A} \psi - \zeta]\), respectively. The \(\zeta\) integration then results in the replacement \((\zeta, \partial_\mathcal{A} \zeta) \rightarrow (\partial_\mathcal{A} \psi, \partial_\mathcal{A}^2 \psi)\), and the \(\psi\) integration in \((\psi, \partial_\mathcal{A} \psi, \partial_\mathcal{A}^2 \psi) \rightarrow (\partial_\mathcal{A} \psi, \partial_\mathcal{A}^2 \psi, \partial_\mathcal{A}^2 \psi)\). Finally, using equation (6) to perform the \(p_\mathcal{A}\) and \(p_\mathcal{X}\) integrations, and inserting equation (24), we obtain the following expression:

\[
Z[J] = \int D\mathcal{A}D\mathcal{X} e^{i \int d^4x \left\{ \frac{1}{2}(\partial_\mathcal{A} A)^2 \frac{1}{2}(\partial_\mathcal{A} \Phi)^2 - \frac{1}{2}(\partial_\mathcal{A} \bar{\Phi})^2 - \frac{1}{2}M^2(\Phi^2 + J A) \right\}}
\]

(40)

A comparison of equation (40) with equation (28) shows that the two results differ in the ghost part of the effective Lagrangian. Considering the dressed propagator of the A field generated by equation (40) we see that only diagrams (a) and (b) in figure 1 contribute and hence their contributions at \(p^2 = 0\) do not cancel. As a result the field \(A\) gains a non-vanishing mass due to the quantum corrections. This evidently contradicts the original physical content of the considered toy model.

5. Conclusions

In this work we discussed the quantization of field theories containing higher order time derivatives of the fields in the Lagrangian. We started from a model describing one massless and one massive free spinless particle. The corresponding path-integral representation of the generating functional for the Green’s functions of the massless field served as the reference point. We performed a change of field variables involving (time) derivatives resulting in a
new Lagrangian containing higher order time derivatives. To this Lagrangian we applied Ostrogradsky’s Hamilton formalism for theories with higher derivatives and subsequently quantized the obtained theory using canonical quantization. The resulting generating functional for the Green’s functions of the massless field differs from the one obtained by the change of variables in the reference generating functional. As a specific consequence, we showed that Ostrogradsky’s formalism gives rise to a non-vanishing mass contribution for the massless particle due to the quantum corrections. These findings may be understood as follows. Ostrogradsky’s formalism is equivalent to the introduction of new non-physical degrees of freedom. At the classical level, for the field variables of the original Lagrangian, Ostrogradsky’s formalism leads to equations of motion which are equivalent to the ones of the Lagrange formalism. On the other hand, the non-physical degrees of freedom result in non-trivial contributions at the level of quantum corrections. These contributions exactly cancel the ghost contributions as clearly seen from the comparison of equations (40) and (28). We conclude that Ostrogradsky’s Hamilton formalism may lead to wrong results and therefore, in general, cannot be considered as a satisfactory basis for the quantization of systems described with Lagrangians involving higher derivatives. Unfortunately, so far we failed to find a solution to this problem.

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Appendix.

Up to a total divergence, the Lagrangian of equation (25) can be written as

\[ \mathcal{L}_3(A, \chi) = \frac{1}{2} \partial_\mu A \partial_\mu A - \frac{1}{2}(\chi + c \Box \chi)(\Box + M^2)(\chi + c \Box \chi) \]

\[ - g A \chi(\Box + M^2)(\chi + c \Box \chi) + \frac{1}{2} g^2[A^2(\partial_\mu \chi \partial_\mu \chi - M^2 \chi^2) - \chi^2 A \Box A]. \]  
(A.1)

In combination with the ghost contribution, equation (A.1) results in the following Feynman rules:

1. Internal line of an \( A \) field with momentum \( k \):

\[ \frac{i}{k^2 + i0^+}. \]

2. Internal line of a \( \chi \) field with momentum \( k \):

\[ \frac{i}{(k^2 - M^2 + i0^+)(1 - ck^2)^2}. \]

3. Internal line from ghost fields with momentum \( k \):

\[ \frac{i}{1 - ck^2}. \]

4. Vertex \( \chi(p_i) \rightarrow \chi(p_f) + A \): \( ig \left[ (1 - cp_i^2)(p_i^2 - M^2) + (1 - cp_f^2)(p_f^2 - M^2) \right] \).

5. Vertex \( \chi(p_i) + A(k_i) \rightarrow \chi(p_f) + A(k_f) \): \( ig^2 \left[ 2(p_i \cdot p_f - M^2) + k_i^2 + k_f^2 \right] \).

6. Vertex \( g_1 \rightarrow g_2 + A \): \( ig \).

7. Because of the Grassmann nature of the ghost fields, a ghost loop produces an overall minus sign.
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