Modeling and dynamics of growth and yield of tree species in *Mimosa scabrella* stands

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**Abstract:** "Bracatingais" are common forest formations formed by bracatinga (*Mimosa scabrella*) and secondary species, which have replaced bracatinga over time; these forests are an important source of income for small farmers. The objective of this study was to model the growth and yield volume of firewood per unit of area and to evaluate the dynamics of the stock across the years. Data from 320 plots were used to fit 12 mathematical models separately addressing the data for bracatinga and secondary species and total species. The Clutter model presented better results for bracatinga (IA = 0.954 and Syx% = 8.54) and for total species (IA = 0.917 and Syx% = 11.56). The modified Clutter model was the best for the secondary species, with IA = 0.952 and Syx% = 25.08. The volumetric estimation of these equations was used to compare the estimated volume of the bracatinga with that of the secondary species, identifying the age as between 13 and 14 years when the volume of the bracatinga is supplanted by the volume of the secondary species. Furthermore, 8 years of age was ideal for clearcutting the bracatingais.

**Key words:** Evolution the stock, *Mimosa scabrella*, mathematical models, secondary species.

**INTRODUCTION**

Stands of bracatinga are forest types composed, in their initial phase, almost exclusively of *Mimosa scabrella* Benth., popularly known as bracatinga. For this reason, these stands are called “bracatingais” (bracatinga stands). Silva et al. (2016) noticed that bracatinga is a pioneer species and exist only for a short time, eventually being replaced by secondary species. After 11 years, the stands have the characteristics of a secondary forest, with the predominance of several secondary species (species in the most advanced successional stages).

These bracatingais have great social and economic importance in the region and constitute a kind of green savings for small farmers that manage them through a short-cycle agroforestry system. Considering that the bracatingais, similarly to other forest types, are dynamic biological systems that continually modify themselves over time, it is necessary to quantify this change to aid in forest management decisions. Thus, mathematical modeling has played a fundamental role because it provides information regarding the appropriate techniques for the management forests. Growth and yield models are important tools for the description of the dynamics of trees and forest stands.

Literature on forest growth and yield modeling in Brazil focused mainly on commercial plantations of *Eucalyptus* and *Pinus*. Although the bracatingais have great socioeconomical
important in southern Brazil, there is little research or technical information related to the growth of this species. Machado et al. (2002b) studied the effects of density on growth variables, including diameter, total height, basal area, and volume per hectare, using data from 20 permanent plots of 325 m² with stand age up to 7.6 years.

Because of the importance of bracatinga, given the large number of farmers and multiple users of firewood and charcoal in the metropolitan region of Curitiba (the state capital with over 3.2 M inhabitants), some studies have been conducted on wood volume stock quantification. Machado et al. (2008a) modeled firewood volume of *Mimosa scabrella* stands using a 229 plots database distributed between the ages of 3 and 18 years. Additionally, Machado et al. (2008b), using 384 cubed trees, developed equations to estimate stem and firewood volume and evaluated the accuracy of the estimates additional 55 trees obtained using the xilometer cubic scaling method. The equation from Ahrens (1981) was tested, and it was found that it generated underestimates of the results. Recently, Urbano et al. (2017a, b, 2018) modeled the volume per unit of area of *Mimosa scabrella*, all secondary-forest species together, and all forest tree species together.

There are no data on the quantification of secondary species of the bracatingais. As an alternative, Santos et al. (2006) used individual stem volume equations fitted for broadleaved species of the Mixed Ombrophilous Forest in Paraná State and Figueiredo Filho et al. (2014) used the individual stem volume equation fitted for *Araucaria* by the “Fundação de Pesquisas Florestais do Paraná” (FUPEF) (1978).

Based on the above information and considering that one of the main objectives of forest management is the maximization of its productive capacity, there are gaps in our knowledge regarding volume quantification over time, both for bracatinga and the secondary species, as well as the total group of species that compose the bracatingais.

To improve of wood volume stock quantification process, it is important to develop equations that reliably estimate the growth and yield, to provide the basis to the proper management regimes applied to the bracatingais.

Thus, this study aimed to model and estimate the growth and yield of firewood volume per hectare and to evaluate the dynamics of the volume stock across the years in the bracatingais.

**MATERIALS AND METHODS**

**Characterization of the study area**

We used data coming from native bracatingais across 14 counties belonging to the Curitiba Metropolitan Region. The bracatingais were composed of bracatinga and an additional 153 secondary tree species. Secondary species with a diameter at breast height (DBH) above 4 cm begin to appear in the bracatingais at age 4 years and with 11 years they are already more abundant than bracatinga. A list of these species can be found in Silva (2013) who studied the structure and floristics of bracatingais with a successional approach.

The study area is located on the First Paraná Plateau, which according to the Köppen classification system, has a Cfb climate type, corresponding to a temperate, humid mesothermic climate with no hydric deficit, with four well-defined seasons, including hot summers, and rainy, cold winters with occasional dry periods. The average annual temperature is approximately 17°C, with a minimum of 11°C and maximum of 24°C. The average annual precipitation is between 1400 and 1600 mm.
with a slight decrease in winter but no water deficits (Souza et al. 2014).

The geographical coordinates of the study area are limited by the latitude 24°58'11"S and 25°55'44"S and by the longitude 49°29'09"W and 49°03'58"W (Silva et al. 2016). According to Souza et al. (2013), the eastern, northern, and southern regions of the study area has a strong influence from the Ocean Mountains, whereas the western part is influenced by the Devonian Escarpment. In the northern region, where the relief is more rugged, the altitude ranges from 850 m in Agudos do Sul to 980 m in Bocaiúva do Sul.

**Origin and preparation of the database**

The data for modeling growth and yield in volume of firewood per hectare of the bracatingais are the same as those used by Silva et al. (2016) and by Urbano et al. (2017a, b, 2018), and come from temporary plots, randomly distributed in bracatingais with ages varying from 3 to 20 years. The DBH and total height of all trees with DBH ≥ 4 cm were measured in every plot. The age of bracatinga in the plots were determined by local information (owners), and checked when necessary by reading the annual growth rings (Silva et al. 2016).

The data for each plot were organized to meet the proposed objective and to develop the modeling process based on the following procedures: Calculation of the mean heights ($h$), mean diameters ($d$), and mean square diameter ($dg$) of the trees in the plots; Determination of dominant heights ($h_{dom}$) of each plot with subsequent site classification; Calculation of the individual firewood volume, the cross-sectional area of each tree, and their extrapolation to a hectare base.

Initially the observed values of dominant height ($h_{dom}$) in each plot were determined according to the ASSMANN concept and the site classes were determined for each plot based on the site curves constructed by Machado et al. (2011) for bracatinga.

The firewood volume of each bracatinga tree in the plots was estimated based on the volume equations fitted by Machado et al. (2008b). The stem volume of broadleaved species was estimated by equations fitted by Santos et al. (2006). The equation developed by FUPEF (1978), cited by Figueiredo Filho et al. (2014), was used for estimating the volume of the stems of *Araucaria angustifolia*. Because no suitable equation was found in the literature to estimate the volume of firewood (including branches ≥ 4 cm of diameter) of the secondary species, the estimated volume of the stems was multiplied by 1.3. This volume expansion factor considers the secondary species to have the same proportion of branches observed in the determination of the true volume made with xylometer for bracatinga trees, which were used by Machado et al. (2008b).

Based on the quartile method, an exploratory analysis was performed on the database to detect the presence of discrepant values of volume per hectare for each sampled age.

**Modeling growth and yield**

Some models for the whole stand were chosen from publications by Scolforo (1998) and Campos & Leite (2013). As age is a fundamental variable for making a prognosis regarding forest growth and yield and is the variable that defines the yield function, some models of the total population that considered only age were chosen from publications these same authors. All these models were fitted to estimate the growth and yield of firewood volume with bark per unit area in the native bracatingais of the metropolitan region of Curitiba. These models are presented in Table I.
Table I. Models to estimate the growth and yield of firewood volume with bark per unit area in the native bracatingais of the metropolitan region of Curitiba.

| Author                      | Model                                                                 |
|-----------------------------|----------------------------------------------------------------------|
| Clutter                     | \[
\ln (V) = \beta_0 + \beta_1 S + \beta_2 \ln (G) + \beta_3 I^{-1} + \epsilon_i
\] |
| Clutter modified            | \[
\ln (V) = \beta_0 + \beta_1 S^{-1} + \beta_2 I^{-1} + \beta_3 \ln (G) + \epsilon_i
\] |
| Burkhart                    | \[
\ln (V) = \beta_0 + \beta_1 I^{-1} + \beta_2 \ln (I) + \beta_3 N + \beta_4 I \ln (N) + \epsilon_i
\] |
| Goebel and Warner           | \[
\ln (V) = \beta_0 + \beta_1 N + \beta_2 S^{-1} + \beta_3 I \ln (N) + \beta_4 I \ln (N)^{-1} + \beta_5 I^{-1} + \epsilon_i
\] |
| Campos and Ribeiro          | \[
\ln (V) = \beta_0 + \beta_1 G + \beta_2 h_{dom} + \beta_3 I^{-1} + \epsilon_i
\] |
| Backman                     | \[
\ln (V) = \beta_0 + \beta_1 \ln (I) + \beta_2 \ln^2 (I) + \epsilon_i
\] |
| Chapman-Richards            | \[
V = \beta_0 (1 - \exp^{\beta_1 I}) + \epsilon_i
\] |
| Mitscherlich                | \[
V = \beta_0 (1 - \exp^{\beta_1 I}) + \epsilon_i
\] |
| Parabolic                   | \[
V = \beta_1 I + \beta_2 I^2 + \epsilon_i
\] |
| Weibull                     | \[
V = \beta_2 I^{-\beta_3} \beta_1^{\beta_4} \exp \left( \frac{I}{\beta_1} \right) + \epsilon_i
\] |
| Prodan                      | \[
V = \frac{I^2}{\exp^{\beta_4 I + \beta_3 I^2}} + \epsilon_i
\] |
| Logistic                    | \[
V = \frac{\beta_0}{1 + \exp^{(\beta_1 I - \beta_2)}} + \epsilon_i
\] |

\(V\) = volume of firewood with bark per hectare (m³.ha⁻¹); \(G\) = basal area per hectare (m².ha⁻¹); \(I\) = age of bracatinga (years); \(h_{dom}\) = dominant height (m); \(N\) = number of trees per hectare; \(\ln\) = Neperian logarithm; \(\beta_i\) = coefficients of the models; and \(\epsilon_i\) = random error.

The selection of the best equation was based on the following statistical criteria: highest Adjusted Schlaegel Index (IA), which is a statistic similar to the adjusted coefficient of determination (\(R^2_{aj}\)), suitable for comparisons of equations of different natures, lowest standard error of estimate in percentage (\(S_{\%}\)), and graphical residual analysis to verify heteroscedasticity and bias.

According to Machado et al. (2008a), for equations fitted with the values of the dependent variable transformed to Neperian logarithm, the estimated values were corrected to eliminate the logarithmic discrepancy and make them comparable with the arithmetic equations. In this case, the estimated volumes were multiplied by the Mayer Correction Factor (MCF) (MCF = where e = base of the neperian...
logarithms and $S_{yx} = \text{standard error of the estimate in logarithm units}$). Thus, the IA and $S_{yx}$ were recalculated.

After the choice of the best equation, it was validated using the Chi-square test ($\chi^2$) with a significance limit of $\alpha = 5\%$. The square root of the mean error in percentage (RQEM%) with the validation data was also calculated. The selection of the plots for validation followed the recommendation of Prodan et al. (1997), who suggested that the data for validation should be independent of the data used in the fitting, covering the variation of ages, sites, and management regimes.

**Evaluation of volume stock behavior across years**

For the evaluation of the volume stock dynamics in the bracatingais across years, the values estimated with the best growth and yield equation plotted on the graph were used. In this way, the volume of the bracatinga was compared to that of the secondary species, identifying the age where the volume of firewood per hectare of the bracatingas was supplanted by the volume of firewood per hectare of the secondary species.

**RESULTS**

**Preparation and analysis of data**

After analyzing the discrepant data for volume per hectare, there were 277 plots with bracatinga, 230 plots with secondary species, and 293 plots with all species. These plots were used for the volumetric modeling processes of the sampled stands.

The correlation analysis for the data for bracatinga showed that basal area ($G$) was the variable with the highest correlation, both in the volume (0.85) and the logarithm of the volume (0.87). For site ($S$), both volume and log volume, the correlation was approximately 0.45. Age ($I$) did not present a good correlation with volume variables.

The correlation analysis for the secondary species showed that basal area ($G$) had a strong correlation with both the volume (0.99) and its logarithm of the volume (0.87), being the variable with the highest correlation with the volumetric variables. Oliveira et al. (2005), in forest fragments in the county of Viçosa, observed that the basal area presented a correlation of 0.90 with the total volume per hectare and 0.88 with the stem volume per hectare. The correlation of the site ($S$) with volume was -0.30 for the secondary species and 0.73 for bracatinga. It was also observed that the correlation of age with volume of the secondary species was 0.79. The fact that the variable age had a better correlation with volume when analyzed for the secondary species is explained by Silva et al. (2016), where bracatinga was replaced by secondary species across the years; that is, as the number of other species increase across years, the volume follows this trend, generating a positive correlation.

Preliminary analyses showed a low correlation between age and volume per unit area of the bracatinga ($r = -0.25$). However, the correlation with these two variables only for the secondary species was good ($r = 0.73$). The correlation of age with the volume of all the species in the bracatingais (bracatinga plus the secondary species) presented a mean value of $r = 0.65$.

**Growth and yield fitted equations**

The 12 models of growth and yield chosen from the literature were fitted. For fit, the data were approached separately, fitting bracatinga, then the secondary species, and finally all species together. The best equations were selected by applying the selection criteria based on the IA, $S_{yx}$, and graphical analysis of residuals.
For bracatinga, the equation based on the Clutter model performed best, presenting an IA = 0.945 and Syx% = 8.54. The independent variables in this model had a good correlation with the volume per hectare, resulting in an equation with better statistics and greater predictive power. The equation with the worst performance was that of Chapman and Richards, which presented a Syx% = 36.71 and IA = 0.053, denoting low performance of the models that consider only age as an explanatory variable for the volume. This fact is explained by the low correlation that the volume per hectare of the bracatingas had with variable age.

According to Campos & Leite (2013), models that use only age as an explanatory variable are appropriate for small and homogeneous areas with a well-defined growth trend, without great variation in density and site. This is not the case of the bracatinga in the bracatingais. The poor results of the equations that used only age made it impossible to use them to estimate the growth and yield of the firewood volume per hectare of the bracatinga.

The equation from the modified Clutter model for the secondary species showed the best performance among all fitted growth and yield equations, with Syx = 25.08% and IA = 0.952. Although the Syx of this equation presented a high value, it must be considered that there were more than 150 native species of secondary succession, and that among them, there was very high structural variability, as observed by Silva et al. (2016) in the native bracatingais of the metropolitan region of Curitiba. The equation from the Campos and Ribeiro model yielded the poorest results, with Syx% = 92.51 and IA= 0.335.

For the set of all species, the equation from the Clutter model presented the best results with Syx% = 11.56 and IA = 0.910. This performance is explained by the good correlation of the variables basal area (G) and age (I) have with the logarithm of the volume per hectare, resulting in an equation with good estimates. The least-performing equation was that from the Weibull model with Syx% = 35.48 and IA 0.252.

The best-chosen equations are presented in sequence, by approach, with their respective coefficients and Mayer Correction Factor. The graphical distribution of the residuals is shown in Figure 1.

- For the bracatinga

\[
\hat{V} = \exp\{1.7677 + 0.0446 \cdot S + 1.0045 \cdot \ln (G) - 2.9512 \cdot I^3\} \cdot 1.0036
\]

- For the secondary species

\[
\hat{V} = \exp\{2.1291 - 1.9604 \cdot S^2 + 0.4537 \cdot I^3 + 1.0434 \cdot \ln (G)\} \cdot 1.0084
\]

- For the set of all species

\[
\hat{V} = \exp\{1.5043 + 0.0378 \cdot S + 1.1199 \cdot \ln (G) - 2.6684 \cdot I^3\} \cdot 1.0049
\]

Figure 1 shows that the equations have a good distribution of residuals, with low dispersion and without bias along the ages and the DBH. The comparison of the adjustment and precision statistics, as well as the graphical analysis of residuals, showed that the equations resulting from the fit of the Clutter models were higher than the equations from the other tested models for the three approaches. According to Campos & Leite (2013), the Clutter model is the most widespread growth and yield model used in the forest environment. Souza et al. (2013) fitted this model to estimate the carbon weight of bracatinga per unit of area, being classified as the one with the best statistical performance among those tested, with $R^2_{aj} = 0.917$ and Syx% = 10.25. After the selection process, the best equations were then submitted to the validation process.
Validation of the selected equations

The Chi-squared test was used for the validation process of the selected equations; the null hypothesis considered was that there was no statistical difference between the observed values and the values estimated by the selected equations. The RQEM% was calculated to compare it with the $S_x\%$ obtained by the fitted equations, as these two statistics were similar. The obtained results for the validation process for bracatinga, the secondary species, and all species are presented in Table II.

In the validation process, the Chi-square test of adherence ($\chi^2$) presented a calculated value lower than the table value for the three tested approaches; thus, there was no statistical difference between the estimated values and those observed.

The RQEM% calculated with the validation data for bracatinga, presented a value 0.13% lower than the value obtained with $S_x\%$ in the model fitting. For the secondary species, the RQEM% was 0.9% lower than the value obtained with the $S_x\%$ in the model fitting, and for the set of all species the RQEM% was 1.57% lower than the value obtained with the $S_x\%$ in the fitting of the equation.

The Chi-square test and the RQEM% statistics showed that the selected equations obtained satisfactory results in the validation process and could be used in the estimation of the growth and yield in volume per hectare in the bracatingais, with age ranging from 3 to 20 years, and for the three established site conditions.
Yield and increment curves
To observe and compare the behavior of the yield curve and increments curves (Mean Annual Increment - MAI and Current Annual Increment - CAI) of the volume of firewood per hectare across the years, the estimated yield values and estimated increment values were calculated using the selected equations.

Variation in basal area along time was noticed in the three approaches; thus, the estimated volume curve per hectare did not express a good way to interpret its behavior. Therefore, to aid in the construction of yield and increments curves, it was necessary to fit a generic equation for each approach to estimate the basal area \((G)\) across the years based on the variables age \((I)\) and site \((S)\). The best equations fitted for the estimation (trend curve) of basal area are presented in Table III.

It was observed that the adjustment and precision statistics were poor because of the low correlation that age and site have with basal area. In the case of bracatinga, the basal area correlated of \(-0.50\) with age, showing that the basal area decreased as age of the bracatinga increased. For the site, the correlation was even lower \((r = 0.29)\), indicating that the site had, in this case, little effect on the basal area. For the secondary species, the correlation between basal area and age was stronger, with a value of 0.76, indicating that across years the basal area increased. For site, the correlation was \(-0.43\) indicating that in sites of poor quality, the basal area of the secondary species tended to be higher. For the set of all species, the correlation of basal area with age and site was 0.28 and 0.08, respectively.

Thus, with the estimated basal area applied in the fitted volumetric equation, the yield, MAI, and CAI were estimated and the graphs were constructed with the estimated values. These yield and increments curves are shown in Figure 2, and the graphs corresponding to letter \((A)\) are for bracatinga, those of letter \((B)\) for secondary and letter \((C)\) for all species.

In general, when we observed the yield curve and the increment curves in Figure 2a, it was verified that the inflection point of the yield curves occurred near the age of 2 years, when the CAI showed its maximum value. According to Assmann (1970), the CAI in volume has the characteristic of culminating earlier in high-density stands, as in the case of bracatingais in their initial phase.

The maximum value of the MAI curve were placed around 3 years of age, when it crossed the CAI curve. At this age, the bracatinga reached its maximum mean rate of increment yield, indicating the beginning of the senescence phase of the species. From 3 years onwards, the decrease in the MAI rate occurred slowly. In this context, Machado et al. (2002b) studying the evolution of basal area

| Approach    | Nº of data | \(\chi^2\) tabulated \((\alpha=5\%)\) | \(\chi^2\) calculated | RQEM% |
|-------------|------------|-------------------------------------|------------------------|-------|
| Bracatinga  | 33         | 46.19                               | 22.43\(^{ts}\)         | 8.41  |
| Secondaries | 31         | 43.77                               | 35.10\(^{ts}\)         | 24.15 |
| All species | 43         | 58.12                               | 45.77\(^{ts}\)         | 9.99  |

**Table II. Values obtained with the validation process of the selected equations to estimate the volume of firewood by hectare of the bracatingais.**
Table III. Fitted equations for basal area by approach, with their respective coefficients, Mayer correction factor, and adjustment and precision statistics.

| Approach   | Equation | IA   | Syx % |
|------------|----------|------|-------|
| Bracatinga | $G = 11.9490 -0.5491 \times I + 0.3406 \times S$ | 0.383 | 30.38 |
| Secondaries| $G = \exp\left[3.608636 -28.8028 \times (1/I) + 0.02612 \times S\right]\times1.4707$ | 0.562 | 73.47 |
| All species| $G = \exp\left[2.735121 -1.77719 \times (1/I) + 0.01524 \times S\right]\times1.0339$ | 0.153 | 25.05 |

$G =$ basal area per hectare (m².ha⁻¹); $I =$ age of bracatinga (years); $S =$ site (m); $IA =$ adjusted Schlaegel index; and $Syx =$ standard error of estimate (%).

Figure 2. Yield and increase curves (MAI and CAI) in volume of firewood per hectare for bracatinga (a), for secondary (b) and for set of all the species (c). Where: $VY =$ volumetric yield; $VI =$ volumetric increment.

and volume in native bracatingais, submitted different initial densities and from different sites with data from permanent plots, measured up to 7.6 years of age, and observed that the maximum MAI for the volume per hectare occurred at 4.1 years of age.

According to Assmann (1970), Scolforo (1998), and Campos & Leite (2013), the age at which the intersection occurs between the increment curves (MAI and CAI) is defined as the ideal for clear cutting or as a reference for application of thinning, because of the highest efficiency in volume production. However, the age at which the intersection between the increment curves (MAI and CAI) did not necessarily correspond to the technical rotation defined on the economic basis, because other factors are considered for this rotation, such as multiproducts, production value, interest rate, and other management decisions (Avery & Burkhart 1994, Campos & Leite 2013). According to Scolforo (1998), for many forest managers, maximum volume production
is not a realistic management objective, i.e., plantation management should be designed to provide a mix of products in appropriate quantity and quality and in the most efficient way from the economic point of view.

In the case of bracatinga, based on the foregoing statement, it was observed that because of the high initial density of plants in the bracatingais, growth reaches its maximum very early, even considering the variation of sites; thus, it is recommended that the number of plants be decreased to increase the spacing and decrease competition. Machado et al. (2001), studying the effects of initial density and site on the development of native bracatingais in 20 permanent plots of 325 m² measured up to 7.6 years, concluded that 4000 plants per hectare was the initial density suitable for new populations and that the differences generated by different initial densities were more expressive for the lowest quality site. Thus, the worse the site, the more important was the thinning of the DBH yield, mean volume, and volume per hectare.

For this reason, it was verified that the maximum production reached by bracatinga occurs close to 8 years of age, when the CAI curve equals zero, and the following ages presents negative values. However, Machado et al. (2002a), studying the effect of the number of trees on the growth of bracatinga, observed that the volume stocking per unit area (as well as in basal area) reached a maximum value at the age of 6.3 years and then decreased; however, in this case the measurements were made only up to 7.6 years and with a restricted database.

From 8 years of age, the volumetric production of bracatinga decreased continuously. The high mortality of bracatinga in the initial phase resulted in a decrease in the degree of occupation of the area (number of trees and basal area). This allowed the entry of secondary species, which were observed from 4 years of age, increasing competition, and a constant decrease in the number of trees and basal area, resulting in a volumetric decrease after 8 years of age.

Traditionally, the producers, based on their experience, already have been rotating the bracatingas at ages close to 8 years (Mazuchowski 2014). This research confirms, based on a wide sampling, that the producers adopted a correct procedure concerning to the age to cut the bracatingais.

When analyzing the estimated curves for the secondary species (Figure 2b), it was verified that from 4 to 20 years (age of the bracatingas) the volumetric production of all secondary species was increasing, with a higher rate after 8 years of age. At this age, the volumetric production of bracatinga was beginning to decrease.

From the age of 4 to 15 years, the growth of all secondary species was in the juvenile stage of development. At the age of 15 years, the inflection point of the yield curve occurred, when CAI was at its maximum value and the growth rate passed to the maturity stage.

Up to 20 years of age, the maximum value of the MAI curve did not takes place. Based on the equation constructed with this model, it will occur at close to 30 years; however, this prediction is far beyond the measured ages, and should be considered only as a hypothesis for the secondary species of the bracatingais.

The estimated curves for all species of the bracatingais (Figure 2c) showed that the inflection point of the yield curve occurred near 3 years of age, when CAI is at its maximum value. The maximum value of the MAI curve occurred closer to 5 years of age, when it crosses the CAI curve. At this age, the population reached its maximum productivity, indicating the beginning of the senescence phase.
When analyzing the set of all species, the presence of the secondary species initially results in an increase in volumetric increment values for the stand, indicate that the MAI and the CAI of the stands occur at ages higher than that observed for the volumes of bracatinga, and at ages lower than that observed for volumes of secondary species only. Because until 8 years of age the stands are strongly influenced by the bracatingas, after this age, the volume of the bracatinga begins to decrease. For the set of all species, the maximum production of the stands does not occur until 20 years of age.

**Comparison of the basal area and volume by hectare between bracatinga and secondary species**

Based on the fitted equations, the volume at basal area was estimated across the years. These values were plotted against age as can be visualized in Figure 3.

It is observed in Figure 3 that the volume of firewood per hectare of the bracatinga is supplanted by the volume of firewood of the secondary species between 13 and 14 years of age of the stand. The same behavior is observed for the basal area of the bracatinga.

Considering this, it was observed by Silva et al. (2016), who referred to the number of trees per hectare by which the bracatinga is supplanted by secondary species at 11 years of age, the reflection of this behavior on the basal area and volume per hectare is only observed between 13 and 14 years of age. It is evident that in the process of succession of the bracatingas, the secondary species replace the bracatinga over the years, until that at 20 years, the bracatinga almost disappears, with only the secondary species remaining. From this age, the bracatingas disappear naturally, giving rise to what is called a "white forest" or "capoeirão (secondary mix-species forest)."

**CONCLUSIONS**

When bracatinga is considered as the main source for firewood production, it is observed that the bracatinga reach their maximum volumetric productivity at 8 years, which is the technically recommended age for rotations.

The fitted equations for growth and yield estimate with good precision and low error the volume of firewood for bracatinga, secondary species, and all species.

The validation tests indicated that there was no statistical difference between the actual and estimated values, providing the best-fitted equations for estimating the firewood volume per hectare. These equations can be used, according to the manager’s objective, for the processing of inventories of bracatingas in the metropolitan region of Curitiba.
Comparing the evolution of the volume stock of bracatinga and that of secondary species along the years, it was verified that between 13 and 14 years of age of the bracatingais, the volume of firewood per hectare and the basal area per hectare of the secondary species surpass the volume and the basal area of the bracatinga; about 2 years before (11 years) the same behavior was observed for the number of trees.

The observed results for the behavior of the species in the bracatingais across the years, as well as the models developed, are of great importance for definitions of management regime and for the volumetric quantification techniques of the forest inventories conducted in these areas.

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