Accelerating and decelerating cosmology from spinor and scalar fields non-minimally coupled with f(R) gravity

Yu.A. Rybalov, A. N. Makarenko, K.E. Osetrin

Department of Theoretical Physics, Tomsk State Pedagogical University, Tomsk, 634041, Russia.

In this paper we investigate the accelerating and decelerating cosmological models with non-linear spinor fields and non-minimal interaction of \( f(R) \) gravity with a scalar field. We combine two different approaches to the description of dark energy: modified gravity theory and introduction of the additional fields. Solutions for the FRW universe with power-law scale factor are reconstructed for the model under consideration with specific choice for scalar and spinor potentials. It is explained the role of scalar and spinor potentials as well as \( f(R) \) function for emergence of accelerating or decelerating cosmology.

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I. INTRODUCTION

The problem of the dark energy and dark matter is one of the main challenges of modern cosmology. Astrophysical data indicate that the observed universe is in an accelerated phase [1]. This acceleration could be induced by the so-called dark energy (see Ref. [2] for a recent review). On the other hand, astrophysical observations provide evidence [3] for the existence of a non-baryonic, non-interacting and pressure-less component of the Universe, dubbed dark matter. This leads us to the need to revise the standard cosmology.

The cosmological constant models are the simplest candidates for the solution of the problem of the universe acceleration. However, these models have still problems with the consistent description of the different evolution stages of the Universe. Scalar theory is most popular to describe the current accelerating expansion and early-time inflation. However, to describe the dark matter we have to introduce additional fields. One can consider a model with two scalar fields [4] (or scalar field and Lagrange multiplier(s) [5]), or, for example, models with additional spinor field to describe dark energy and dark matter.

The study of spinor fields in curved spacetime has a long history. The Dirac equation was investigated for massless spinor fields in curved space-time more than 50 years ago [6]. Spinor fields can be used to describe the primordial inflation [7] and current expansion [8]. However, the exact solutions in the presence of the spinor field is difficult to build (for example, see [9]).

A significant number of attempts have been made to construct the cosmological models with a spinor field for description of dark energy, where a non-canonical kinetic term was considered, such as k-inflation and k-essence models [10]. In [11], the properties of one of the foregoing models with self-interacting spinor with the noncanonical kinetic term were studied.

It should be noted that the models involving the squared classical Dirac Lagrangian can be considered as a special case of the k-essence model [12]. The scalar invariant constructed from two spinor fields dynamically develops a nonvanishing value in Quantum Chromodynamics (QCD) theory [13]. In this case, the chiral symmetry for the spinor field is broken. This is of significance for the evolution of the Universe. Only in a few papers the dynamic symmetry was assessed with the aim of interrupting non-static behavior of spacetime.

There is another way to solve the problem of dark energy that does not require the introduction of the dark component. The modified theory of gravity may be quite realistic to describe the different phase of evolution of the Universe (see recent review [14]). A simple model describing the unified description for primordial inflation and current accelerating expansion was presented by Nojiri-Odintsov in [15]. It also shows the viability of the modified gravity [16,17], which describes the \( \Lambda CDM \) epoch, like the standard theory with cosmological constant. In addition, such models satisfy the tests of solar system (see [20]), as well as to adequately describe all the stages of development of the universe, starting with the early inflation until late accelerated expansion, with a correct description of the intermediate stage [17, 18].

In order to merge two approaches, we could use non-local gravity and present its local (scalar-tensor) formulation. Such theories also naturally lead to the unification of inflation with late-time cosmic acceleration.

In the recent paper we considered a cosmological model with a spinor field and a scalar field couples with an arbitrary function of the curvature. Of course, such models are not standard ones, in the sense that they are not multiplicatively renormalizable in curved spacetime [22]. Hence, they should be considered as kind of effective theories (without clear understanding of their origin and their relation with more fundamental string/M-theory). The present paper is devoted to study of non-minimally coupled scalar theory introduced in refs. [21] with self-interacting spinor field. We study the FRW equations of motion for such non-linear and non-minimal system with scalar and spinor fields. Specific choice of scalar and spinor potentials is made in the process of the search of explicit accelerating/decelerating cosmological
solutions. Several power-law solutions for current dark energy epoch are constructed. It is known that these cosmologies are quite realistic and pass the observational bounds.

II. THE FIELD EQUATIONS

Let us consider a model with the action in the form:

$$S = \int d^4\sqrt{-g} \left( \frac{R}{2} + f(R)\mathcal{L}_\phi + \mathcal{L}_D \right),$$  \hspace{1cm} (1)

where $R$ is scalar curvature.

The Lagrangian of a scalar field of mass $m$ is given by:

$$\mathcal{L}_\phi = \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi),$$  \hspace{1cm} (2)

where $V(\phi)$ is the potential of the scalar field. The Dirac Lagrangian $\mathcal{L}_D$ of fermion field of mass $m_f$ has the form:

$$\mathcal{L}_D = \frac{i}{2} \{ \bar{\psi} \Gamma^\mu D_\mu \psi - D_\mu \bar{\psi} \Gamma^\mu \psi \} - m_f (\bar{\psi} \psi) - \bar{\psi} \psi.$$  \hspace{1cm} (3)

In the expression (3), $V(\bar{\psi} \psi)$ describes the potential of fermion field and $\psi = \psi^0 \gamma^0$ denotes the conjugate spinor. $\Gamma^\mu = e_\mu^\sigma \gamma^\sigma$ are the Dirac matrices in a curved spacetime ($e_\mu^\sigma$ is tetrad). The covariant derivative in the equation (3) is defined by the rule:

$$D_\mu \psi = \partial_\mu \psi - \Omega_\mu \psi,$$

$$D_\mu \bar{\psi} = \partial_\mu \bar{\psi} + \bar{\psi} \Omega_\mu,$$  \hspace{1cm} (4)

where

$$\Omega_\mu = \frac{1}{4} g_{\rho\sigma} \left[ \Gamma_\rho^\nu \gamma^\sigma \gamma^\mu - e_\rho^\sigma \partial_\mu e_\sigma^\lambda \right] \Gamma^\lambda \Gamma^\delta.$$  \hspace{1cm} (5)

Here $\Gamma^\rho_{\sigma\delta}$ are Christoffel symbols.

Let us now consider a Friedmann-Robertson-Walker (FRW) universe with the flat spatial metric

$$ds^2 = dt^2 - a(t)^2 (dx^2 + dy^2 + dz^2).$$  \hspace{1cm} (6)

From Eq. (4) and (3) one can obtain equation for the spinor field

$$dL_D = \frac{dL_D}{d\bar{\psi}},$$  \hspace{1cm} (7)

or

$$i(D_\mu \bar{\psi}) \Gamma^\mu + m_f \bar{\psi} + \frac{dV}{d\psi} = 0,$$  \hspace{1cm} (8)

$$i \Gamma^\mu D_\mu \psi - m_f \psi - \frac{dV}{d\psi} = 0.$$  \hspace{1cm} (9)

Einstein’s equations can be written as

$$R_{\mu\sigma} - \frac{1}{2} g_{\mu\sigma} R = - T_{\mu\nu},$$  \hspace{1cm} (10)

where $T_{\mu\nu} = (T_f)_{\mu\nu} + (T_\phi)_{\mu\nu}$. $(T_f)_{\mu\nu}$ is the energy-momentum tensor of the fermion fields and $(T_\phi)_{\mu\nu}$ is the contribution of the variation of the scalar field which non-minimally interacts with $F(R)$. A symmetric form of the energy-momentum of the fermion field is as follows

$$(T_f)^{\mu\nu} = - g^{\mu\nu} L_D +$$

$$+ \frac{i}{4} \{ \bar{\psi} \Gamma^\nu D^\mu \psi + \bar{\psi} \Gamma^\nu D^\mu \psi - D^\nu \bar{\psi} \Gamma^\mu \psi - D^\mu \bar{\psi} \Gamma^\nu \psi \},$$

where $\Gamma^0 = \gamma^0$, $\Gamma^i = \frac{1}{4\sqrt{-g}} \gamma^0 \gamma^i$, $\Gamma^5 = \gamma^5$ and

$$\Omega_0 = 0, \; \Omega_i = \frac{1}{2} \dot{a}(t) \gamma^i \gamma^0.$$  \hspace{1cm} (12)

From the equations (12), (11), (3) and (6) we get the non-zero components of the energy-momentum tensor of the fermion field

$$(T_f)^{00} = m_f (\bar{\psi} \psi) + V,$$  \hspace{1cm} (13)

$$(T_f)^{0i} = V - \frac{\bar{\psi} dV}{2 d\psi} - \frac{dV}{d\psi} \frac{\psi}{2}.$$  \hspace{1cm} (14)

The interaction between the fermionic components is modeled by a non-equilibrium pressure ($\overline{\rho}$) in the energy-momentum tensor source.

A symmetric form the energy-momentum tensor of the scalar field can be obtained from (11) in the form

$$(T_\phi)_{\mu\nu} = -(\mathcal{L}_\phi f' R_{\mu\nu} + f' \phi_{,\mu} \phi_{,\nu} -$$

$$- 2 \left[ \square \mathcal{L}_\phi f' + 2 \mathcal{L}_\phi ; \sigma f'_\sigma + \mathcal{L}_\phi \square f' \right] g_{\mu\nu} +$$

$$+ 2 \left[ \mathcal{L}_\phi ; \mu f' + \mathcal{L}_\phi ; \nu f'_\mu + \mathcal{L}_\phi ; \nu f'_\mu + \mathcal{L}_\phi ; \mu + \mathcal{L}_\phi ; \mu f'_\mu \right].$$  \hspace{1cm} (15)

$$(T_\phi)^{00} = p_\phi = -(\mathcal{L}_\phi f' R^0_0 + f' \phi^2 -$$

$$- 2 \left[ \square \mathcal{L}_\phi f' + 2 \mathcal{L}_\phi ; \sigma f'_\sigma + \mathcal{L}_\phi \square f' \right] +$$

$$+ 2 \left[ \mathcal{L}_\phi \phi f' + \mathcal{L}_\phi ; \nu f'_\nu + \mathcal{L}_\phi ; \nu f'_\nu + \mathcal{L}_\phi f'_\nu \right].$$  \hspace{1cm} (16)

$$(T_\phi)^{0i} = - \rho_\phi =$$

$$= -(\mathcal{L}_\phi f' R^i_i - 2 \left[ \square \mathcal{L}_\phi f' + 2 \mathcal{L}_\phi ; \sigma f'_\sigma + \mathcal{L}_\phi \square f' \right]).$$  \hspace{1cm} (17)

We now write the equation of motion of the scalar field as

$$f(R) \square \phi + g^{\mu\nu} f_{,\mu} \phi_{,\nu} + V'(\phi) f(R) = 0,$$

$$V'(\phi) = \frac{dV(\phi)}{d\phi}. $$  \hspace{1cm} (18)

The consequence of the equations of motion of the spinor fields is given by:

$$\frac{d}{dx} \bar{\psi} \psi + 3H \bar{\psi} \psi = 0,$$  \hspace{1cm} (19)

or
\[
\frac{\dot{\psi}}{\psi} = -3H, \quad (20)
\]
\[
\bar{\psi}\psi = \frac{c}{a(t)^3}. \quad (21)
\]

Self-interaction potential can be written as \( V = a_n\bar{\psi}\psi^{2n} \), where \( a_n \) and \( n \) are constants. The potential \( V \) is considered as a scalar invariant.

**III. RECONSTRUCTION OF SOLUTIONS**

We consider our model for the case of power-law dependence of scale factor on the time \( a(t) = a_0 t^n \) when the function \( F(R) \) has a form \( F(R) = r_0 R^p \). Spinor field we select as (21) and limit the potential to the first three terms

\[
V(\bar{\psi}\psi) = a_1(\bar{\psi}\psi)^2 + a_2(\bar{\psi}\psi)^4 + a_3(\bar{\psi}\psi)^6. \quad (22)
\]

**A. Model 1**

Let us consider the action in the following form \( (F(R) = 1) \):

\[
S = \int d^4\sqrt{-g} \left\{ \frac{R}{2} + \mathcal{L}_\phi + \mathcal{L}_D \right\}. \quad (23)
\]

One can choose the Lagrangian of the scalar field as [2], we get the following expressions

\[
V(\phi) = -\frac{2c(a^{15}m_f + a^{12}c\alpha_1 + a^6c^3\alpha_2 + c^5\alpha_3)}{2a^{18}r_0} + \frac{-6a^{16}a^{r_0}\phi^2}{2a^{18}r_0} \quad (24)
\]

\[
0 = -\frac{2a}\phi^2 + r_0\phi^2 + \frac{a^{15}cm_f + 2a^{12}c^2\alpha_1 + 4a^6c^4\alpha_2 + 6c^6\alpha_3 + 2a^{17}a^{a^2}}{a^{18}}, \quad (25)
\]

\[
0 = -\frac{2a^{16}a^3 + 2a^{18}(ar_0\phi^2 + a^{(3)}) + 3a^{(-2)(a^{12}c^2\alpha_1 + 6a^6c^4\alpha_2 + 15c^6\alpha_3) + a^{18}r_0\phi^2)}{a^{18}}. \quad (26)
\]

Then, using that \( a(t) = a_0 t^n \), \( \phi(t) = f_0 t^k \), we get the following solutions:

1) \( n = \frac{2}{3}, \alpha_2 = \alpha_3 = 0, m_f = \frac{4a^3}{3c} \),
\( k = -1, \alpha_1 = -\frac{a^{12}f_0^{(2)}r_0}{2c^2}, \ V(\phi) = 0. \)

2) \( n = \frac{2}{5}, \alpha_1 = \alpha_3 = 0, m_f = \frac{4a^3}{3c} \),
\( k = -3, \alpha_2 = -\frac{9a^{12}f_0^{(2)}r_0}{4c^2}, \ V(\phi) = -\frac{9a^6}{4f_0^{(2)^2}} = -\frac{9}{4f_0^{(2)^2}}. \)

3) \( n = \frac{2}{5}, \alpha_1 = \alpha_2 = 0, m_f = \frac{4a^3}{3c} \),
\( k = -5, \alpha_1 = -\frac{25a^{12}f_0^{(2)}r_0}{4c^2}, \ V(\phi) = -\frac{25a^{12}}{3f_0^{(2)^2}} = -\frac{25}{3f_0^{(2)^2}}. \)

4) \( n = \frac{1}{3}, \alpha_2 = \alpha_3 = 0, m_f = -\frac{a^{6}f_0^{(2)}r_0}{4c} \),
\( k = \frac{1}{2}, \alpha_1 = \frac{a^6}{3c^2}, \ V(\phi) = \frac{3f_0^{(2)^2}}{4c} = \frac{3}{4c}. \)

5) \( n = \frac{1}{3}, \alpha_1 = \alpha_3 = 0, m_f = -\frac{9a^6r_0}{16c} \),
\( k = \frac{2}{5}, \alpha_2 = \frac{a^{6}}{12c^2}, \ V(\phi) = \frac{9f_0^{(2)^2}}{32c} = \frac{9}{32c}. \)

6) \( n = \frac{1}{5}, \alpha_1 = \alpha_2 = 0, m_f = -\frac{25a^{6}f_0^{(2)}r_0}{4c} \),
\( k = \frac{3}{5}, \alpha_3 = \frac{a^6}{25c^3}, \ V(\phi) = \frac{25f_0^{(2)^2}}{720c} = \frac{25}{720c}. \)

We have several solutions that meet the decreased expansion. This situation is obvious, because the presence of of the spinor field leads to slower the universe expansion.

**B. Model 2**

Let us consider our model in the absence of a spinor field

\[
S = \int d^4\sqrt{-g} \left\{ \frac{R}{2} + f(R)\mathcal{L}_\phi \right\}. \quad (27)
\]

If the Lagrangian of the spinor field has the form [2]

\[
a(t) = a_0 t^n, \ \phi(t) = f_0 t^k, \ f(R) = r_0 R^p,
\]

then we obtain the following solution

\[
V(\phi) = \frac{f_0^2 p^2 (1 - 3n + p) \phi^{2 - 2/p}}{2(-1 + p)} = \frac{f_0^2 p^2 (1 - 3n + p) t^{-2 + 2p}}{2(-1 + p)} \quad (28)
\]

\[
k = p,
\]

\[
ro = \frac{6n^{1-p}(-1 + 2n)^{(-6 + 12n)^{-p}(-1 + p)}}{f_0^2 p^2 (3 - 6n + p + 3np)}.
\]

p and n may be arbitrary, except \( n = 0, \), \( n = 1/2, \) \( p = 0, n = \frac{3(p-3)}{2(p-2)}. \)

For this model, the \( n \) can be arbitrary. If one selects the potential of the scalar field to be zero, we obtain the limit for \( n \)

\[
n = \frac{1 + p}{3}.
\]

The same solution was obtained in the review [14] as realistic cosmology satisfying observational bounds and predicted by modified gravity.
C. Model 3

We now choose the action in its original form $\mathcal{L}$, where the Lagrangian of the scalar field is (2) and $a(t) = a_0 t^n$, $f(R) = r_0 R^p$.

1.1) $V(\phi) = (434 - 38 \sqrt{73}) a_0^2 t^{1/6} (-5 - \sqrt{73})$

$= \left(11 + 7 \sqrt{73}\right) \phi^2 - \left(19 + \sqrt{73}\right)^2 (1 + a_0^2 t^{4/3})$

Substituting $\phi(t) = f_0 t^k$ we get,

$V(\phi) = -\frac{1}{144} (11 + 7 \sqrt{73}) f_0^2 t^{(17 - \sqrt{73})}$

$= \alpha_2 = 0, \alpha_3 = 0, n = 2/3, m_f = \frac{4a_0^3}{3c}$.

$k = p - 1 = -1.12867,$

$\alpha_1 = -\frac{2^{11/6 - \sqrt{73}/12} \pi^{(\sqrt{73} - 31)} (19 \sqrt{73} - 217) a_0^6 f_0^2 r_0}{\left(19 + \sqrt{73}\right)^2} = -0.27989 a_0^6 f_0^2 r_0 / c^2, p = \frac{1}{12} (7 - \sqrt{73}) = -0.12867.$

1.2) $V(\phi) = \frac{1}{4(-1+2p+6a_0^2(1-p)t^{2/3})} \times (-1)^{p-1} (-2^{1 - 2p} (-e^{ip\phi} (f_0 + 2 f_0 \phi t^{2/3} - 2(-1)^p t^{1 - 2p} \phi^2 ((1 - 2p - 6a_0^2)(-2 + 3p)t^{2/3}) \phi + 12a_0^2 \mu t^{3/2} \phi''))$, $\alpha_1 = 0, \alpha_3 = 0, n = 1/3$.

$m_f = -\frac{2p - 2 - 3p a_0^6 e^{ip\phi} f_0^2 r_0 (2p + 1)^2}{c}$.

$k = p + 1/2, \alpha_1 = \frac{a_0^6}{3c^2}$.

1.3) $V(\phi) = \frac{1}{r_0} - \frac{4a_0^2 f_0^2 t^{3/2}}{32} + \left( \frac{9}{2} + 9a_0^2 t^{1/3} \phi^2 - 12a_0^2 t^{4/3} \phi' \phi'' \right)$.

$V(\phi) = \frac{1}{r_0} + \frac{49 a_0^2 f_0^2 t^{3/2}}{32} = \frac{1}{r_0} + \frac{49 a_0^2 f_0^2 t^{3/2}}{32}, \quad (31)$

$\alpha_1 = 0, \alpha_3 = 0, n = 1/6, m_f = \frac{49a_0^3 f_0^2 r_0}{24c}$.

$k = p + 3/4, \alpha_2 = -\frac{a_0^2}{12c}, p = 1.$

1.4) $V(\phi) = \frac{1}{r_0} - \frac{847 a_0^3 f_0^2 \phi^5/3 + 15 (1 + 2a_0^2 t^{1/3} \phi')^2 - 18a_0^2 \phi' \phi''}{486c}$.

$V(\phi) = \frac{1}{r_0} + \frac{121 a_0^2 f_0^2 \phi^5/3}{72} = \frac{1}{r_0} + \frac{121 a_0^2 f_0^2 \phi^5/3}{72}, \quad (32)$

$\alpha_1 = 0, \alpha_2 = 0, n = 1/9, m_f = \frac{847a_0^3 f_0^2 r_0}{486c}$.

$k = p + 15/18, \alpha_3 = -\frac{a_0^2}{27c}, p = 1.$

In this case, we see that many potentials allows us to find the explicit solutions. However, in several cases the expansion may be decelerated. We don't discuss the details of the found solutions because it is known that several versions of them satisfy the observational bounds.

IV. CONCLUSIONS

Thus, in our model we have a different types of behavior of the universal expansion. The presence of the spinor field leads to a slowing of the universe expansion, when the scale factor is positive and less than one ($1/3 < \alpha_0 < 1$). For the case of a free scalar field if $n = 2/3$ then we get the scalar field decreasing over time. Otherwise, the scalar field increases with time ($\phi \sim t^{1/2}, t^{3/4}$ and $t^{5/6}$).

If we consider the model in the absence of a spinor field the situation is changing. The presence of non-minimal interaction allows to obtain solutions for any value of the degree in the scale factor. The degree of a scalar field is arbitrary. We have only one restriction - $k = p$ (where $\phi = f_0 t^k$ and $F(R) = r_0 R^p$).

All the solutions we obtained for the power-law scalar field ($\phi \sim t^k$). Choosing a different type of fields, such as logarithmic function of time, lead us to an equation without explicit solution.

Consider as an example the case of the scalar potential field set to $1/2 m \dot{\phi}^2$. In this case, the equation (18) gets the form

$2m t^2 \phi + (3n - 2)p \dot{\phi} + t \ddot{\phi} = 0.$

The solution of this equation is

$\phi = t^{p - 2} \left( c \text{BesselY} \left[ \frac{1}{2} - \frac{3n - 2}{2} + p, \sqrt{2}\sqrt{mt} \right] \right)$.
where $c_1$ and $c_2$ are constant, BesselJ is the Bessel function of the first kind and BesselY is the Bessel function of the second kind. We see that in this case it would be difficult to check the compatibility of the solutions with the Einstein equations. For this reason, we restricted ourselves to the power dependence of the scalar field on time.

If the degree of the scale factor is positive, we get the quintessence-type universe. However, we can consider the case of a negative power. One can do a replacement $t \rightarrow t - t_s$ ($t_s$ is a constant) and we obtain the phantom universe with the singularity of the future such as the Big Rip.

We see that the presence of a spinor field of specific type does not permit the universe to expand with acceleration. The introduction of a scalar field does not change the situation. However, in the absence of a spinor field the non-minimal interaction leads us to arbitrary powers of the scale factor and the scalar (in power form), but fixing potential and function $f(R)$.

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