Trapped-ion qutrit spin molecule quantum computer

D Mc Hugh\textsuperscript{1} and J Twamley\textsuperscript{1,2,3}

\textsuperscript{1} Department of Mathematical Physics, National University of Ireland
Maynooth, Maynooth, Co. Kildare, Ireland
\textsuperscript{2} Centre for Quantum Computer Technology, Macquarie University,
Sydney, NS 2109, Australia
E-mail: Jason.Twamley@nuim.ie

Abstract. We present a qutrit quantum computer design using trapped ions in
the presence of a magnetic field gradient. The magnetic field gradient induces
a ‘spin–spin’ type coupling, similar to the J-coupling observed in molecules,
between the qutrits which allows conditional quantum logic to take place. We
describe in some detail how one can execute specific one and two qutrit quantum
gates, required for universal qutrit quantum computing.

Quantum information and quantum computing [1] have made huge advances both theoretically
and experimentally in recent times. There are many different proposals [2]–[11] for the physical
implementation of a quantum computer, all of which are specified by the physics of their qubit
systems and the nature of the interactions between qubits. The latter is necessary for the execution
of conditional qubit logic, a requirement for universal qubit quantum computation. The qubit
is of course the obvious unit of a quantum computer, given our classical computer historical
dependence on binary logic. Quantum algorithms and protocols for quantum communication
and cryptography have been studied extensively with the qubit as the information storage and
transport medium. Recently $d$-level quantum systems, or qudits, have started to be considered
seriously in terms of generalizing and improving qubit-based quantum algorithms and protocols.
Interesting results have emerged in the qutrit ($d = 3$) case. It has been shown that quantum
cryptography protocols are more robust against eavesdropping attacks when qutrits are used
[12]–[14]. Quantum bit commitment and coin-flipping protocols are more secure with entangled
qutrits than with qubits [15]. Indeed, it is expected that qutrit-based quantum information
processing will be more powerful than other qudit implementations since they optimize the
Hilbert space dimensionality [17]. More speculative is the possibility of a higher error-tolerance

\textsuperscript{3} Author to whom any correspondence should be addressed.
The qutrit levels \(\{|0\rangle, |1\rangle, |2\rangle\}\) under consideration with \(\omega_{12}\) and \(\omega_{01}\) the resonant frequencies for the \(|1\rangle \leftrightarrow |2\rangle\) and \(|0\rangle \leftrightarrow |1\rangle\) transitions respectively.

Figure 1. The qutrit levels \(\{|0\rangle, |1\rangle, |2\rangle\}\) under consideration with \(\omega_{12}\) and \(\omega_{01}\) the resonant frequencies for the \(|1\rangle \leftrightarrow |2\rangle\) and \(|0\rangle \leftrightarrow |1\rangle\) transitions respectively.

for fault-tolerant qutrit quantum computation [18]. However, there have not been many qudit-based quantum computer proposals. As far as we are aware there has been only one qutrit quantum computer proposal using trapped ions, which generalizes the original Cirac–Zoller design [19].

Here, we describe a modification to previous work [20]–[22], on ion trap quantum computers, where now qutrits store the quantum information. An axial magnetic field gradient is applied across an ion chain that allows the three hyperfine Zeeman energy levels of each ion, forming the qutrit, to be individually frequency addressed. It also introduces an inter-qutrit coupling that facilitates conditional quantum logic between qutrits. Previously the operation of an ion trap quantum computer in the presence of a magnetic field gradient has been discussed with qubits as the unit of quantum information. In [20, 21], it is shown that all quantum gate operations, normally requiring optical irradiation, can be implemented using long wavelength radiation due to the effects of the magnetic field gradient and trapping potential. The gradient also introduces a term in the Hamiltonian that is analogous to the spin–spin coupling observed between nuclei in molecules in NMR. This coupling can be used to perform quantum logic. This idea is investigated further for ions in a linear array of microtraps [22]. In the only other qutrit ion trap quantum computer proposal [19], the quantized collective vibrational motion of the linearly trapped ions is used as a quantum bus to perform quantum logic.

In this work, we consider \(N\) ions in a linear ion trap in the presence of a magnetic field gradient. Three unequally spaced hyperfine Zeeman levels serve as our qutrit (figure 1). We will refer throughout to the \(^{171}\text{Yb}^+\) ion as an example qutrit, particularly the \(F = 1\) hyperfine Zeeman levels shown in figure 2. Our computational basis is now \(\{|0\rangle, |1\rangle, |2\rangle\}\) written as \(|0\rangle = (0, 0, 1)^T\), \(|1\rangle = (0, 1, 0)^T\) and \(|2\rangle = (1, 0, 0)^T\). We denote by \(\omega_{01}^{(n)}\) and \(\omega_{12}^{(n)}\) the frequencies resonant with the \(|0\rangle \leftrightarrow |1\rangle\) and \(|1\rangle \leftrightarrow |2\rangle\) transitions for the \(n\)th ion and let \(\Delta_n = \omega_{01}^{(n)} + \omega_{12}^{(n)}\) and \(\delta_n = \omega_{01}^{(n)} - \omega_{12}^{(n)}\). For each ion these frequencies, due to the spatial dependence of the magnetic field, are a function
of the ions’ positions. The internal electronic Hamiltonian describing the spin degrees of freedom of a single ion is given by $H_{\text{sp},n} = \frac{1}{2} \hbar Z_n$ with $Z_n$ expressed in the above computational basis as

$$Z_n = \begin{pmatrix} \Delta_n & 0 & 0 \\ 0 & \delta_n & 0 \\ 0 & 0 & -\Delta_n \end{pmatrix}.$$  \hspace{1cm} (1)

As in [20, 21], the ions sit in an effectively 1D harmonic oscillator potential along the trap axis and feel their mutual Coulomb repulsion. Expanding this potential around their equilibrium positions allows their vibrational motion to be treated collectively and the Hamiltonian for the motional degrees of freedom of the ions in normal coordinates is

$$H_{\text{vib}} = \frac{1}{2m} \sum_n p_{Q,n}^2 + \frac{m}{2} \sum_n v_n^2 Q_n^2.$$  \hspace{1cm} (2)

Figure 2. The hyperfine Zeeman levels of an $^{171}$Yb$^+$ ion in a spatially varying magnetic field.
The local and normal coordinates are related by \( q = D Q \), with \( D \) the unitary transformation that diagonalizes the Hessian, \( A \), of the ions potential evaluated at the equilibrium positions of the ions, \( z_{0,n} \) [see \( [21, 23] \)].

The magnetic field gradient, \( d B / dz \equiv b \), means the ions feel a spatially varying magnetic field, \( B(z) = B_0 + b \cdot z \). This introduces a new term into the spin part of the Hamiltonian,

\[
H_{\text{el},n} = \frac{1}{2} \hbar Z_n|_{z_{0,n}} + \frac{\hbar}{2} \frac{d Z_n}{d z} |_{z_{0,n}} \cdot q_n.
\]  

(3)

Setting \( M_n = d Z_n / d z |_{z_{0,n}} \) for easier notation, as long as \( \langle 1/2 \hbar M_n q_n \rangle \) is much smaller than the ground-state energy of the collective vibrational motion, then this new term will have a negligible effect on the normal modes and can be treated as a perturbation. This places a limit on the size of the magnetic field gradient, but is no more stringent than other constraints discussed later. The resultant Hamiltonian is

\[
H = \frac{\hbar}{2} \sum_n Z_n + \frac{\hbar}{2} \sum_n \hat{M}_n \sum_l D_{ln} Q_l + \frac{1}{2m} \sum_n \bar{P}^2_{Q,n} + \frac{m}{2} \sum_n v^2_n Q^2_n
\]

\[
= \frac{\hbar}{2} \sum_n Z_n + \frac{1}{2m} \sum_l \bar{P}^2_{Q,n} + \frac{m}{2} \sum_l v^2_l \left( Q_l + \frac{\hbar}{2m v^2_l} \sum_n \hat{M}_n D_{ln} \right)^2
\]

\[
- \frac{m}{2} \sum_l v^2_l \left( \frac{\hbar}{m v^2_l} \sum_n \hat{M}_n D_{ln} \right)^2.
\]  

(4)

We move to a rotating frame, where the spin and motional degrees of freedom are decoupled via the unitary transformation \( U = \exp [i(\hbar/2m v^2_l \sum_n \hat{M}_n D_{ln}) P_{Q,l}] \), yielding \( \tilde{H} = U H U^\dagger \), with

\[
\tilde{H} = \frac{\hbar}{2} \sum_n Z_n + \sum_n \hbar \nu_n a_n^\dagger a_n - H_{\text{MM}},
\]  

(5)

where \( H_{\text{MM}} = m/2 \sum_l v^2_l (\hbar/m v^2_l \sum_n \hat{M}_n D_{ln})^2 \) and the position and momentum operators have been expressed in terms of creation and annihilation operators in the usual way. \( H_{\text{MM}} \) can be expressed as \( H_{\text{MM}} = \hbar/2 \sum_{n,m} J_{nm} M_n M_m \), where

\[
J_{nm} = \frac{\hbar}{2m} \sum_l \frac{1}{v^2_l} D_{ln} D_{lm}.
\]  

(6)

The Hamiltonian \( \tilde{H} \) in (5), describes \( N \) individually addressable qutrits coupled through \( H_{\text{MM}} \), a ‘spin–spin’ type interaction, which we will presently show can be used to perform conditional quantum logic between qutrits. There are no extra experimental requirements compared to the setup proposed in \([20, 21]\).

It is necessary to demonstrate how single qutrit operations and conditional logic, the most basic ingredients of a universal quantum computation, can be implemented in this modified design.
In order to perform single qutrit operations, we use the fact that the two transitions $|0\rangle \leftrightarrow |1\rangle$, and $|1\rangle \leftrightarrow |2\rangle$, have different resonant frequencies allowing the operations $U_{01}$ and $U_{12}$,

$$U_{12}(\theta, \phi) = \begin{pmatrix} \cos \frac{\theta}{2} & i e^{i\phi} \sin \frac{\theta}{2} & 0 \\ ie^{-i\phi} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{01}(\theta, \phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \frac{\theta}{2} & i e^{i\phi} \sin \frac{\theta}{2} \\ 0 & ie^{-i\phi} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix},$$

(7)

to be carried out. Since the $|0\rangle \leftrightarrow |2\rangle$ transition is forbidden, any rotation between these two states requires a $\pi$-pulse between $|0\rangle$ and $|1\rangle$, $U_{01}(\pi/2, 0)$, the required gate on $|1\rangle \leftrightarrow |2\rangle$, $U_{12}(\theta, \phi)$, followed by another $\pi$-pulse on $|0\rangle \leftrightarrow |1\rangle$. Also required is the unitary differential phase operation,

$$U_D = \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{-i(\sigma+\rho)} \end{pmatrix},$$

(8)

where this gate is a composition of $\sigma$’ operations on the $|0\rangle \leftrightarrow |2\rangle$ and $|0\rangle \leftrightarrow |1\rangle$ transitions that are themselves compositions of $U_{12}$ and $U_{01}$ operations. In particular, $U_D = (Z_{02})_\rho (Z_{01})_\sigma$ with $(Z_{ij})_\rho = H_{ij} U_{ij}(\rho, 0) H_{ij}^\dagger$ and $H_{ij} = U_{ij}(\pi/4, \pi/2)$ is the Hadamard gate. Thus, the unitary operations given in (7) allow us to generate any operation in $SU(3)$ [24].

For gates between more than one qutrit, we propose to use the last part of the Hamiltonian $\tilde{H}$ in (5). We now consider in more detail the hyperfine Zeeman levels for the qutrits. For intermediate magnetic field strengths $B$, such that $g_J \mu_B B \approx A$, where $g_J$ is the Landé g-factor, $\mu_B$, is the Bohr magneton and $A$, is the hyperfine constant, the energy levels are described by the Rabi–Breit formula [25]. In this case, $\Delta_n / dz = g_J \mu_B b / \hbar$ and $\delta_n / dz \approx g_J \mu_B b / \sqrt{2} \hbar$ and where $b$ is the constant gradient of the magnetic field $B(z) = B_0 + bz$. We absorb the $g_J \mu_B b / \hbar$ factor into the definition of $J_{nm}$ and write $M = dZ/dz|_{z_0}$ in the matrix form

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \sqrt{2} & -1 \end{pmatrix},$$

(9)

with now $J_{nm} = (g_J \mu_B b)^2 / 2 \hbar (A^{-1})_{nm}$ [22]. Expressing the operator $M$ in (9) in terms of the generators of $SU(3)$ we have

$$M = a_0 \mathbb{1} + a_3 \lambda_3 + a_8 \lambda_8,$$

(10)

where

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

(11)

with $a_0 = 1/3 \sqrt{2}$, $a_3 = 1/2 \sqrt{2}(\sqrt{2} - 1)$ and $a_8 = 1/2 \sqrt{6}(1 + 3 \sqrt{2})$. $H_{MM}$ is thus a two-body $N$-qutrit Hamiltonian as defined in [26] and so, as arbitrary local unitaries are possible, universal quantum computation can be performed. More specifically, it has been pointed out in [19] that
a generalized XOR gate between qutrits,

\[ \text{XOR}_{mn} |j\rangle_m |k\rangle_n = |j\rangle_m |j \oplus k\rangle, \]  

(12)

where the ‘⊕’ operation now indicates addition modulo 3, can be decomposed into three operations,

\[ \text{XOR}_{mn} = F_m P_{mn} F_m^{-1}. \]  

(13)

The generalized Fourier transform, \( F \), for one qutrit is defined by

\[ F |j\rangle = \frac{1}{\sqrt{3}} \sum_{l=0}^{2} e^{2\pi i lj/3} |l\rangle, \]  

(14)

where \( j = 0, 1, 2 \).

The phase gate for qubits, \( P_{\text{qub}} |j\rangle_m |k\rangle_n = \exp(i\pi jk) |j\rangle_m |k\rangle_n \), is sandwiched by two Hadamard gates on the target qubit to give the controlled-NOT or XOR operation. The generalization of this gate to qutrits is \( P_{mn} \). It is completely specified by

\[ P_{mn} |j\rangle_m |k\rangle_n = \exp(2i\pi jk/3) |j\rangle_m |k\rangle_n. \]  

(15)

We can generate this gate between two qutrits using a combination of single qutrit rotations and evolution under the two qutrit Hamiltonian \( H_{12} = 2\pi J M_1 \otimes M_2 \). One pulse sequence for \( P_{mn} \) is

\[
\begin{aligned}
&[Z_{12}^{(1)}]_{a_2} [Z_{12}^{(2)}]_{a_2} [Z_{12}^{(1)}]_{a_4} \cdots [MM]_{a_5} [X_{01}^{(1)}] [MM]_{a_6} [X_{01}^{(1)}] \\
&\cdots [X_{12}^{(1)}] [MM]_{a_7} [X_{12}^{(2)}] [X_{12}^{(2)}] \cdots [MM]_{a_8} [X_{12}^{(1)}] [MM]_{a_9} [X_{12}^{(1)}] [X_{12}^{(2)}],
\end{aligned}
\]  

(16)

where \([Z_{ij}^{(n)}] \) represents a \( \pi \)-pulse on the \( |k\rangle \leftrightarrow |l\rangle \) transition of the \( n \)th ion i.e. \([X_{ij}^{(n)}] = \mathcal{U}_{ij}(\pi/2, 0)\) and \([MM]_{ij}\) represents a period of evolution under the Hamiltonian \( H_{12} \) for a time \( t \) such that \( 2\pi J t = \theta \). While this is not a unique decomposition, we strongly suspect the number of periods of evolution under \( H_{MM} \) is minimal. The sequence is generated from diagonal operators. The single qutrit operators \([Z_{ij}^{(i)}]_{a} \) and \([Z_{ij}^{(i)}]_{b} \) on the \( i \)th qutrit plus five other operators containing \([MM]_{ij}\) or its permutations give us nine diagonal operators and nine parameters, which can be varied to generate the above sequence. These parameters \( \{\alpha_i\}_{i=1}^{9} \) have been numerically determined and are shown in table 1.

Refocusing techniques [1] developed for NMR quantum computing are necessary here given the ‘always-on’ nature of \( H_{MM} \). For qubits the interaction is \( H_{SS} = 2\pi J \sigma_z^{(1)} \sigma_z^{(2)} \). The relations \( \sigma_a \sigma_b \sigma_a = -\sigma_z \) for \( a = x, y \), where \( \{\sigma_i\}_{i=x,y,z} \) are the Pauli operators, are used to reverse the evolution under \( H_{SS} \)

\[ e^{iH_{SS} t} = e^{-i\pi \sigma_z^{(1)}} e^{-iH_{SS} t} e^{-i\pi \sigma_z^{(1)}}. \]  

(17)

Essentially the diagonal elements of \( \sigma_z \) are permuted by the two \( \pi \)-pulses. Combined with another period of evolution under \( H_{SS} \), the trace-less property of \( \sigma_z \) is exploited so that nothing happens. In fact, all qubits coupled to the first qubit are refocused by this.
Table 1. The angles $\alpha_i$ written in multiples of $\pi$ required in order to execute the pulse sequence for the phase gate for qutrits given in equation (16).

| $\alpha_i$ | Value  |
|------------|--------|
| $\alpha_1$ | $0.5628$ |
| $\alpha_2$ | $0.2604$ |
| $\alpha_3$ | $1.9045$ |
| $\alpha_4$ | $2.4299$ |
| $\alpha_5$ | $16.5854$ |
| $\alpha_6$ | $19.1630$ |
| $\alpha_7$ | $0.2738$ |
| $\alpha_8$ | $5.3918$ |
| $\alpha_9$ | $0.3045$ |

For qutrits the de-coupling procedure is basically the same. The spin–spin term $H_{MM} = 2\pi J M_1 M_2$ is composed of nine terms,

$$M_1 M_2 = a_0 I \otimes M + (a_3 \lambda_3 + a_8 \lambda_8) \otimes M. \tag{18}$$

By applying pulses, which permute the entries of $\lambda_3$ and $\lambda_8$ in three ways similar to the qubit case, the combined evolution under the second term in (18) is removed. The sequence to do this is

$$e^{i\phi} U_1 U_2 U_3 R_1^{(2)} R_2^{(2)} = I, \tag{19}$$

where

$$U_1 = (MM)_{\theta}, \quad U_2 = X_{01}^{(1)} X_{12}^{(1)} (MM)_{\theta} X_{12}^{(1)} X_{01}^{(1)}, \quad U_2 = X_{12}^{(1)} X_{01}^{(1)} (MM)_{\theta} X_{01}^{(1)} X_{12}^{(1)} \tag{20}$$

and

$$R_1^{(2)} = \exp(6a_0 a_3 \pi J t \lambda_3^{(2)}), \quad R_2^{(2)} = \exp(6a_0 a_8 \pi J t \lambda_8^{(2)}). \tag{21}$$

The global phase factor, $\phi = 6a_0^2 \pi J t$, is irrelevant while the two single-qutrit pulses on the second qutrit are due to the first term in (18) and can be easily reversed. The three periods of evolution required are expected since the matrices are 3-dimensional.

Readout of the final state of the qutrit register takes two steps. The entire ion chain is illuminated with optical radiation and the observed fluorescence spatially resolved. The radiation frequency is chosen, so that if the qutrits are projected on to $|2\rangle$ this is then detected. A $\pi$-pulse is then applied on the $|1\rangle \leftrightarrow |2\rangle$ transition of each ion and the ion string illuminated again. Fluorescence now indicates projection on to $|1\rangle$, while its absence means the qutrit is in $|0\rangle$.

We now describe an explicit example using the $^{171}$Yb$^+$ ion. The hyperfine constant for $^{171}$Yb$^+$ is $A = 2\pi \times 12.6$ GHz. In a magnetic field its levels are split as shown in figure 2. In fields of around 0.45 T, the Rabi–Breit region, the good quantum numbers are $F$ and $M_F$ and our logical states are $|0\rangle = |6S_{1/2} F = 1, M_F = -1\rangle$, $|1\rangle = |6S_{1/2} F = 1, M_F = 0\rangle$ and $|2\rangle = |6S_{1/2} F = 1, M_F = 1\rangle$. The transition frequencies $\omega_{01}$, and $\omega_{12}$, are about 3.7 and 8.9 GHz respectively, while the differences between the resonance frequencies of two neighbouring ions in a trap of frequency $2\pi \times 200$ kHz and a magnetic field gradient of $b = 100$ T m$^{-1}$
are $\delta \omega_{01} \approx 11$ MHz and $\delta \omega_{12} \approx 2$ MHz. The operator $M_n$ in (9) has an element, which is approximately $1/\sqrt{2}$. This approximation introduces a constraint on the size of the magnetic field gradient, if we require $M_n$ to be constant over the ion chain within an accepted error $\epsilon_M$. Numerical calculations in [27, 28], give the minimum distance between ions in a chain as $\Delta z_{\text{min}}(N) = 2.018 \gamma N^{0.559}$, where $\gamma = (q^2/4\pi\epsilon_0 \nu^4)$. As a conservative estimate, let us say that the ions are equally spaced at $\delta z = 1.5 \Delta z_{\text{min}}(N)$. This implies that $b < 0.03 N^{-0.441} \nu_{1}^{2/3}$, limiting the size of the magnetic field gradient. On the other hand, we need to be able to frequency discriminate between qutrit transitions on neighbouring ions and be far enough away that no vibrational motion will be excited i.e. $(d\omega_{01}/dz)\delta z \leq 2\nu_{N} + \nu_{1}$. Using the numerical result [20] that $\nu_{N} = (2.7 + 0.5N)\nu_{1}$, this imposes a minimum size on the magnetic field gradient of $b \geq 1.5 \times 10^{-9} \nu_{1}^{5/3} (3.2N^{0.559} + 0.5N^{1.559})$. For an axial trap frequency of $\nu_{1} = 2\pi 200$ kHz containing 10 ions and $\epsilon_M = 0.01$, the magnetic field gradient is limited by $30 \leq b \leq 200$ Tm$^{-1}$. These limits also determine the maximum number of ions we can place in the trap, assuming other considerations related to the ratio of the axial and radial trapping frequencies are satisfied [23]. For an axial trap frequency of $\nu_{1} = 2\pi 200$ kHz, the maximum number of ions is about 30 satisfying the above limits, when the magnetic field gradient is 120 T m$^{-1}$. The expression for the J-couplings in (6) is the same as that in the qubit case. For $10^{171}$Yb$^+$ ions at a trap frequency of $2\pi 200$ kHz and magnetic field gradient 120 T m$^{-1}$ gives a nearest-neighbour coupling of about $J = 2\pi 1.2$ kHz.

In summary, we have presented a modification of previous designs for ion quantum computation with magnetic field gradients but where the quantum information is now manipulated and stored in qutrits. A magnetic field gradient allows for individual qutrit addressing and introduces a qutrit–qutrit coupling for quantum logic. The scheme requires no additional physical resources.

Acknowledgments

We gratefully acknowledge C Wunderlich for helpful comments on this paper. DM kindly acknowledges support from Enterprise-Ireland Basic Research Grant SC/1999/080. The work was also supported by the EC IST FET project QIPDDF-ROSES IST-2001-37150 and Science Foundation Ireland.

References

[1] Nielsen M A and Chuang I L 2000 Quantum Computation and Quantum Information (Cambridge: Cambridge University Press)

Gruska J 1999 Quantum Computing (Maidenhead: McGraw-Hill)

[2] Cirac I J and Zoller P 1995 Phys. Rev. Lett. 74 4091

[3] Gershenfeld N and Chuang I L 1997 Science 275 350

Cory D G, Fahmy A F and Havel T F 1997 Proc. Natl Acad. Sci. 94 1634

[4] Loss D and DiVincenzo D P 1998 Phys. Rev. A 57 120

[5] Kane B E 2000 Fortschr. Phys. 48 1023

[6] Twamley J 2003 Phys. Rev. A 67 052318

[7] Davidovich L, Brune M, Raimond J M and Haroche S 1987 Phys. Rev. A 36 3771

[8] Vion D, Aassime A, Cottet A, Joyez P, Pothier H, Urbina C, Esteve D and Devoret M 2002 Science 296 886

[9] Platzman P M and Dykman M I 1999 Science 284 1967

New Journal of Physics 7 (2005) 174 (http://www.njp.org/)
[10] Knill E, Laflamme R and Milburn G 2001 Nature 409 46
[11] Yu Kitaev A 2003 Ann. Phys. 303 2
[12] Bruss D and Macchiavello C 2002 Phys. Rev. Lett. 88 127901
[13] Cerf N J, Bourennane M, Karlsson A and Gisin N 2002 Phys. Rev. Lett. 88 127902
[14] Durt T, Cerf N J, Gisin N and Zukowski M 2003 Phys. Rev. A 67 012311
[15] Spekkens R W and Rudolph T 2002 Phys. Rev. A 65 012310
[16] Terhal B M, Chuang I L, DiVincenzo D P, Grassl M and Smolin J A 1999 Phys. Rev. A 60 881
[17] Greentree A D, Schirmer S G, Green F, Hollenberg L C L, Hamilton A R and Clark R C 2004 Phys. Rev. Lett. 92 097901
[18] Knill E 2004 Fault-tolerant postselected quantum computation: schemes Preprint quant-ph/0402171
[19] Klimov A B, Guzman R, Retamal J C and Saavedra C 2003 Phys. Rev. A 67 062313
[20] Mintert F and Wunderlich C 2001 Phys. Rev. Lett. 87 257904
[21] Wunderlich C 2001 Laser Physics at the Limit ed H Figger, D Meschede and C Zimmermann (Berlin: Springer) pp 261–71
[22] Mc Hugh D and Twamley J 2005 Phys. Rev. A 71 012315
[23] Sasura M and Buzek V 2003 J. Mod. Opt. 49 1593
[24] Arvind K, Mallesh S and Mukunda N 1997 J. Phys. A: Math. Gen. 30 2417
[25] Woodgate G K 1980 Elementary Atomic Structure (Oxford: Oxford University Press)
[26] Nielsen M A, Bremner M J, Dodd J L, Childs A M and Dawson C M 2002 Phys. Rev. A 66 022317
[27] James D F V 1998 Appl. Phys. B 66 181
[28] Steane A 1997 Appl. Phys. B 64 623