Numerical Analysis of the Effects of Oxygen-Enriched and Different Inlet Conditions on Performance of an Indirect Reheating Furnace with Pulse Combustion

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ABSTRACT: The requirement of improving efficiency and performance leads to the continuous development of furnaces and burners. For this purpose, it is necessary to establish a model suitable for industrial production and adjust it according to industrial demand. In this paper, a comprehensive numerical model is developed to characterize the combustion, heat transfer, and slab heating in an indirect reheating furnace with pulse combustion. To realize the pulse combustion process, a pulse control approach based on a user-defined function (UDF) was proposed to control the radiant tube burner state. Indirect heat transfer in the furnace was realized by coupling the radiant tubes and the furnace as a whole. In a simulation with the eddy dissipation concept (EDC) model, results from the four-step mechanism were in close accordance with those of the GRI 3.0 mechanism, and both mechanisms could describe the combustion process in detail. However, the calculation time of the EDC model with the four-step mechanism was reduced significantly. Thus, the EDC model with the four-step mechanism was selected as the ideal combustion model used for further simulation research. Through experimental validation, the simulation results of the developed model using the EDC model with the four-step mechanism showed a good agreement with those of the GRI 3.0 mechanism, and both mechanisms could describe the combustion process in detail. However, the calculation time of the EDC model with the four-step mechanism was reduced significantly. Thus, the EDC model with the four-step mechanism was selected as the ideal combustion model used for further simulation research. Through experimental validation, the simulation results of the developed model using the EDC model with the four-step mechanism showed a good agreement with the experimental results. Additionally, with this model, the effects of oxygen-enriched combustion with 74 vol % N\textsubscript{2} and 26 vol % O\textsubscript{2} in the oxidizer and inlet-change case with a fuel inlet and a primary air inlet on the performance of an indirect reheating furnace with pulse combustion were specially studied. The maximum flame temperature and the average temperature of the furnace atmosphere increased from 2046 to 2175 K and from 1241 to 1279 K for increased oxygen concentration, respectively. Compared with air-fuel combustion, the discharging slab temperature reached a growth of 2.9% in oxygen-enriched combustion. After changing the inlet boundary of the radiant tube burners, since the excessive combustion in the burner’s combustion chamber was avoided and the full combustion of fuel in the radiant tubes was promoted, the flame intensity in the radiant tubes was enhanced and the maximum flame temperature reached 2196 K. At the same time, the mole fraction of CO at the outlet became smaller and the slab temperature in all zones of the furnace increased by more than 3.5%. This study showed that higher efficiency of an indirect reheating furnace with pulse combustion can be achieved by oxygen-enriched combustion and changing the inlet boundary of the burners.

1. INTRODUCTION

Industrial furnaces are important heating facilities widely used in the steel industry, which can heat materials or workpieces using the heat of fuel combustion or electric energy conversion. Today, two types of continuous reheating furnaces are usually used to reheat slabs. One is direct-fired reheating furnaces mainly used for the reheating process before rolling. The slabs
are directly in contact with high-temperature flue gas in the direct-fired reheating furnaces, resulting in the formation of an oxide layer on the surface of slabs. The other is indirect reheating furnaces, which reheat slabs in the heat treatment process after rolling. Due to the need to ensure the slab quality after heat treatment, indirect reheating furnaces are equipped with radiant tubes to make heat transfer between the furnace atmosphere and the flue gas after combustion in the radiant tubes occur indirectly. This avoids the oxidation on the surface of slabs. To improve combustion efficiency and control accuracy and reduce pollutant emissions, the pulse combustion technique has been widely used in indirect reheating furnaces.9

Since reheating furnaces are the second largest producers of CO2 and consumers of energy in China after thermal power generation,7 efforts are being made to improve the reheating furnaces. New techniques will reduce fuel consumption and increase furnace efficiency, which requires extensive testing in actual furnace operation to verify. However, due to the high temperature in the furnace, it is difficult to carry out experiments in actual production, and the repeatability of such experiments leads to a very high cost. A numerical simulation is a good tool for understanding fluid flow, combustion, and heat transfer phenomena. Recently, different numerical simulation approaches have been used to investigate the combustion and heat transfer behavior in reheating furnaces. This avoids expensive and complicated experimental measurements and allows more detailed information to be obtained from inside the furnace.3 Many papers on the numerical simulation of reheating furnaces have been reported.

Zhang et al.4 proposed a simplified steady-state simulation method for regenerative furnaces. The slabs on the furnace floor were simplified as a sheet. They modified the energy equation inside the slab to make the simulation run in a steady state. Huang et al.5 proposed a variation of this method, and this new method was recently used by Mayr et al.6 and Garcia and Amell.7 The slabs in the reheating furnace were simplified again as a continuous plate at a constant speed. However, by treating the plate as a high viscosity laminar flow without wall shear stress, the mass and energy transport were modeled. In the steady-state simulation, this method has a low calculation cost and adequate accuracy. Based on the developed model, Garcia et al.8 studied the effect of the burner position on furnace performance and found that the staggered configuration shows the maximum useful efficiency. Compared with the steady-state simulation, Han et al.9 proposed a transient simulation method for an industrial walking beam direct-fired furnace without simplifying the slabs. The movement of slabs in the furnace was processed by the developed UDF to reach a periodically transient solution. Han and Chang10 applied this model to further study the optimal slab residence time in the furnace. It was found that 7427 s residence time is the most efficient and satisfies the requirements for heating temperature and homogeneity. Han et al.11 also analyzed the efficiency of the oxy-fuel and the air-fuel combustion in the furnace. Compared with air-fuel combustion, the efficiency of oxy-fuel combustion reached a growth of about 50%. Based on the model developed by Han et al.,10 Wang et al.12 studied the influence of burner arrangement in the reheating furnace and discovered that reducing the number of side active burners from 13 to 6 has a positive effect on the thermal efficiency of the furnace. Later, Wang et al.13 examined the effect of different inlet conditions on furnace performance. The result showed that the slab temperature in the reheating furnace increases by more than 28.5% by changing the inlet boundary. A new numerical method different from the traditional heat transfer simulation was proposed by Prieler et al.13 Steady-state gas-phase combustion in the walking hearth direct-fired furnace and transient simulation of the billets was carried out, respectively. The numerical and experimental results were in good agreement. With this model, Prieler et al.14 showed that the efficiency of the furnace is increased from 57.6 to 61.4% in oxygen-enriched combustion. Landfahrer et al.15 developed a model to investigate the combustion and heat transfer in a rotary hearth direct-fired furnace using a numerical method similar to that used by Prieler et al.13 The model revealed the existing problems of this type of furnace and provided reasonable suggestions. Further investigations on the impacts of different oxidizers in a direct-fired furnace were performed by Landfahrer et al.16 They considered various oxidizers and fuels in these parts of the furnace by dividing the whole domain into several parts. Hu et al.17 studied the effect of flameless oxy-fuel combustion on the heating efficiency of the furnace. From a technical and environmental perspective, they thought that reforming the reheating furnaces with oxy-fuel combustion was a promising choice. Hajaliakbari and Hassanpour18 established a mathematical model for a roller hearth indirect reheating furnace and analyzed the factors affecting the overall efficiency of the furnace. The governing equations in the furnace were solved by a computer code written in FORTRAN language. Vanitha and Padmavathi19 developed a numerical model to analyze the impacts of radiant tube materials in an indirect reheating furnace. Without taking into account the combustion in the radiant tube, they found that the ceramic radiant tube has the maximum heat transfer rate by changing the material of the radiant tube. Liu et al.20 applied the pulse combustion technology to the numerical simulation of the direct-fired reheating furnace by the developed UDF program to study the slab heating process in the furnace. The simulation results were in accordance with the experimental data. Liu et al.21 developed a numerical model to study the proportional control and the pulse control on the performance of a direct-fired regenerative reheating furnace. They found that pulse control is beneficial to strengthen the mixing of fuel and air, increase the combustion rate, and improve the uniformity of heat flux distribution on the surface of billets.

Additionally, there have been many works related to the numerical simulation of radiant tubes in the literature. Elmabrouk and Wu22 and Tsioumanis et al.23 studied the combustion process in an industrial single-ended radiant tube. It was found that heat recovery and the multistage combustion technique can improve the combustion process and increase energy efficiency. Ahanj et al.24 investigated the combustion, heat transfer, and flow characteristics in a U-type radiant tube. The predictions of the model and the experimental results showed good agreement. Xu and Feng25,26 improved the nozzle characteristics to optimize the combustion efficiency of the double P-type radiant tube. They found that when the nozzle is asymmetric, the radiant tube has the best thermal performance.27 Increasing the airspeed of the branch tube nozzle or the main tube nozzle can make the surface temperature of the radiant tube more uniform.29 Hellenkamp and Pfeifer27 developed a numerical model for a P-type radiant tube and analyzed the temperature and stress distribution on the radiant tube. Garcia et al.30 established two-dimensional (2D) and three-dimensional (3D) models to evaluate the combustion process in the single-ended nonrecirculating radiant tube and discovered that the EDC model and the flamelet-generated
manifold (FGM) model have good prediction ability in the radiant tube.

In the previous publications, the combustion, heat transfer, and flow characteristics in the direct-fired reheating furnaces and radiant tubes, as well as their performance improvement, have been well understood. However, there is little research on the numerical simulation of an indirect reheating furnace with pulse combustion. This is because the complexity of the structure of the radiant tube itself makes it difficult to model the furnace and radiant tubes as a whole and correctly represent such big furnaces. It is also difficult to realize the pulse input of fuel based on a radiant tube burner state during pulse combustion. Moreover, in terms of indirect reheating furnaces with pulse combustion, efforts to reduce energy consumption are insufficient.

This paper aims to investigate the effects of oxygen-enriched combustion and different inlet boundaries on the performance of an indirect reheating furnace with pulse combustion. A novel 3D model was developed for a roller hearth indirect reheating furnace with pulse combustion. The fluid domain within the radiant tubes and the furnace atmosphere were coupled as a whole to achieve indirect heat transfer between the two domains. At the same time, a pulse control approach based on UDF was proposed to control the radiant tube burner state. With experimental validation, the model showed good prediction ability. Furthermore, the temperature distribution and slab heating effectiveness in the furnace under oxygen-enriched and inlet-exchange conditions were reasonably evaluated. Compared with the original scheme used in production, the two optimized schemes improved the heating efficiency of the furnace. This provides good suggestions for the improvement of the current situation and the future design of reheating furnaces.

2. FURNACE AND RADIANT TUBE DESCRIPTION

The 3D full-scale model of the indirect reheating furnace with pulse combustion established in this work is presented in Figure 1.

Table 1. Fuel and Air Compositions

| species | air (mol) | fuel (mol) |
|---------|-----------|-----------|
| CH₄     | 95.97%    | 0.01765   |
| C₂H₆    | 2.25%     | 0.00953   |
| CO      | 0.01%     | 0.01896   |
| CO₂     | 1.12%     | 0.36092   |
| O₂      | 21%       | 0.01082   |
| N₂      | 79%       | 0.19517   |
| O₂      | 1.12%     | 0.795     |

Table 2. Boundary Conditions for Fuel and Air Inlets

|                     | per 3250 mm radiant tube | per 1760 mm radiant tube |
|---------------------|--------------------------|--------------------------|
| fuel flow (kg/s)    | 0.01765                  | 0.00953                  |
| primary air flow (kg/s) | 0.01896               | 0.01082                  |
| secondary air flow (kg/s) | 0.36092               | 0.19517                  |
| fuel temperature (K) | 298                      | 298                      |
| preheating temperature (K) | 795                  | 720                      |
| equivalence ratio   | 0.87                     | 0.87                     |

The dimensions of the indirect reheating furnace with pulse combustion are a length of 87.06 m, a width of 4.9 m, and a height of 3.036 m. The furnace is divided into sixteen zones along the length, and each zone is subdivided into upper and lower layers according to the arrangement of radiant tubes. Consequently, there are a total of thirty-two temperature control sections in the furnace. In production, the heating zone for heating the slabs is located in the first two-thirds of the reheating furnace. The remaining third of the reheating furnace is the
soaking zone to make the temperature distribution within the slab more uniform. There are thirteen slabs with dimensions of 5000 mm × 2500 mm × 30 mm in the furnace. The slabs are rapidly charged from the furnace entrance and pass through the furnace at a constant speed via the roller system. In the furnace, the slab residence time and the space between the adjacent slabs are 3600 s and 1.14 m, respectively. Nitrogen as the furnace atmosphere is used to prevent the oxidation on the surface of slabs during reheating.

The slabs on the rollers are heated by 320 radiant tubes located on both sides of the furnace wall. Two sizes of radiant tubes are arranged alternately in an indirect reheating furnace, since the furnace was equipped with soaking fans to enhance the gas flow, while these soaking fans were ignored during modeling, the gas flow in the radiant tube burner toward the outlet through the annular space between the inner and outer tubes. Natural gas was used as fuel in the model. Table 1 lists the compositions of fuel and air. The fuel and air inlets of the radiant tube burners were modeled as mass-flow-inlet conditions.7,20,30 The inlet turbulent intensity is 10%. More conditions for the fuel and air inlets are listed in Table 2.

### 3. NUMERICAL MODELS

#### 3.1. Flow and Turbulence

In this study, an indirect reheating furnace with pulse combustion was simulated using commercial software ANSYS FLUENT 17.0. For the indirect reheating furnace, since the furnace was equipped with soaking fans to enhance the gas flow, while these soaking fans were ignored during modeling, the gas flow in the furnace was assumed to be turbulence. The flue gas in the radiant tubes and the furnace atmosphere were modeled as an incompressible ideal gas. The density change caused by temperature was calculated using the ideal gas law. The gas flow in the furnace and radiant tubes was characterized by solving the Reynolds Averaged Navier–Stokes (RANS) equations (eqs 1 and 2).

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u_i) = 0
\]

Table 3. Reaction Mechanisms

| reactions | activation energy E (J/mol) | pre-exponential factor A (1/s) |
|-----------|-----------------------------|-------------------------------|
| CH₄ + 1/2O₂ → CO + 2H₂ | 1.26 × 10⁶ | 4.40 × 10¹⁰ |
| CH₄ + H₂O → CO + 3H₂ | 1.26 × 10⁶ | 3.00 × 10⁹ |
| H₂ + 1/2O₂ → H₂O | 1.46 × 10⁶ | 5.69 × 10¹⁰ |
| CO + H₂O → CO₂ + H₂ | 8.36 × 10⁷ | 2.75 × 10⁸ |
| C₆H₆ + 7/2O₂ → 3CO₂ + 3H₂O | 1.26 × 10⁶ | 6.19 × 10⁸ |

To describe turbulent fluctuations, a turbulence model is needed to close the RANS equations. Rezazadeh et al.31-studied the effects of three different turbulence models on the load and gas temperature distribution in the furnace. Compared with the RNG k-ε model and the Standard k-ε model, the Realizable k-ε model has higher prediction accuracy. Thus, the Realizable k-ε model12 was used for all calculations in this paper. Two equations for the Realizable k-ε model are shown in eqs 3 and 4, where \( \mu_t \) is the eddy viscosity. The pressure-velocity coupling and pressure interpolation were taken into account using the algorithms SIMPLE and PRESTO, respectively. The under relaxation factor for momentum was set to 0.3. Energy, radiation, and species were set to 0.99, while 0.7 was used for turbulence, body forces, density, and pressure. The convergence was determined by low residuals. The residuals of the energy and radiation equations were below 10⁻⁶. The other residuals were below 10⁻⁵. The operating pressure was defined by 101 325 Pa and the viscous region near the walls was treated by the standard

Figure 2. Temperature distribution near the radiant tube for simulations with the EDC model: (a) the four-step mechanism and (b) the GRI 3.0 mechanism.

Table 4. Mean Temperature and Species Distribution at the Outlet of the Radiant Tube for Different Reaction Mechanisms in Air-Fuel Combustion

The other residuals were

\[
\frac{\partial (\rho u_i)}{\partial t} + \nabla \cdot (\rho u_i u_i) = \frac{\partial \tau_{ij}}{\partial x_j} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_l}{\partial x_l} \delta_{ij} \right) \right] - \frac{\partial (\rho u_i u_j)}{\partial x_j} \frac{\partial u_i}{\partial x_j}
\]

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### Table 5. Simulated Cases

| Case                          | Radiant tube size (mm) | Fuel flow/air flow (kg/s) | \(O_2\) in oxidizer (vol \%) | Inlet boundary conditions |
|-------------------------------|------------------------|---------------------------|-----------------------------|---------------------------|
| Oxygen-Enriched combustion   | Per 3250 mm radiat tube| 0.01765/0.37988           | 21                          | Inner-fuel/annulus-air    |
|                               | Per 1760 mm radiat tube| 0.00953/0.20599           | 21                          | Inner-fuel/annulus-air    |
| Oxygen-enriched combustion   | Per 3250 mm radiat tube| 0.01765/0.34532           | 26                          | Inner-fuel/annulus-air    |
|                               | Per 1760 mm radiat tube| 0.00953/0.18732           | 26                          | Inner-fuel/annulus-air    |
| Inlet-Exchange investigation  | Per 3250 mm radiat tube| 0.01765/0.37988           | 21                          | Inner-fuel/annulus-air    |
|                               | Per 1760 mm radiat tube| 0.00953/0.20599           | 21                          | Inner-fuel/annulus-air    |
| Inner-air conditions          | Per 3250 mm radiat tube| 0.01765/0.37988           | 21                          | Inner-air/annulus-air     |
|                               | Per 1760 mm radiat tube| 0.00953/0.20599           | 21                          | Inner-air/annulus-air     |

#### 3.2. Combustion and Radiation

In modeling combustion, an accurate description of the turbulence–chemistry interactions is crucial. The eddy dissipation model (EDM) proposed by Magnussen and Hjertager\(^3\) assumes that the reactions are very fast. The combustion can be regarded as mixing-limited in such cases, neglecting complex chemical kinetic rates. This approach is called "mixed is burnt".\(^3\) The assumption has the advantage of reducing the computational effort needed. However, only two reactions can be taken into account. The EDC model suggested by Magnussen\(^3\) is an improvement of the EDM, allowing the description of detailed chemical mechanisms. The model has been applied to several combustion simulations under oxygen-enriched and air-fuel conditions.\(^1,2,3,6,33,35,36,37\) The EDC model assumes that reactions occur in fine scales. The time scale of mass transfer between surroundings and fine scales is calculated using eq 5, and the length fraction of the fine scales is defined using eq 6.

\[
\tau^* = C_1 \left( \frac{\nu}{\epsilon} \right)^{1/2}
\]

(5)

\[
\xi^* = C_2 \left( \frac{\nu \epsilon}{k^2} \right)^{1/4}
\]

(6)

where \(C_1\) represents the time scale constant with a value of 0.4082 and \(C_2\) represents the volume fraction constant with a value of 2.1337. The source term, the mean species mass fraction, is modeled using eq 7, where \(Y_i^*\) is the small-scale species mass fraction.

\[
R_i = \frac{\rho (\xi^*)^2}{\tau^* \left[ 1 - (\xi^*)^2 \right]} (Y_i^* - Y_i)
\]

(7)

To describe natural gas combustion, the EDC model was used in this work. An in situ adaptive tabulation (ISAT)\(^38\) approach was used for reducing computation time during chemistry integration, and the ISAT error tolerance was set to 10\(^{-5}\). Table 3 displays the combustion mechanisms of natural gas. Methane is approximately 96% of the natural gas composition. Thus, a four-step global reaction mechanism proposed by Yin et al.,\(^39\) successfully applied during the air-fuel and oxygen-enriched combustion of methane, was used to describe the detailed chemical kinetics. Since the percentage of ethane is small, the one-step reaction mechanism was adopted.

The prediction results of the GRI 3.0 mechanism containing 53 species and 325 reversible reactions and the four-step...
mechanism were compared in air-fuel combustion. The temperature distribution near the radiant tube for different reaction mechanisms can be visualized in Figure 2. The EDC models with the GRI 3.0 mechanism and the four-step mechanism show a similar flame shape. For both cases, the species distribution of flue gas at the outlet is almost identical and the maximum deviation based on the four-step mechanism is below 1.5%, as seen in Table 4. This can be considered negligible. Furthermore, the flue gas at the outlet shows almost the same temperature for both cases and the deviation is approximately 9 K. However, as shown in Table 4, significant differences occur regarding the computing time. It takes about 3 weeks for the GRI 3.0 mechanism to obtain a convergent solution. Compared with the GRI 3.0 mechanism, the four-step mechanism requires 4 days of computing time. The validation process revealed that both the four-step mechanism and the GRI 3.0 mechanism can describe the combustion process in detail. Nonetheless, due to the lower computing time needed, the four-step mechanism was used for further investigations.

Thermal radiation is regarded as the main heat transfer mechanism in indirect reheating furnaces. To calculate the radiative heat transfer in the furnace, the widely used discrete-ordinates (DO) model was employed to solve the radiative transport equation (RTE). The RTE was solved for 128 directions, which is the spatial discretization of 4 × 4 in each octant, as recommended by several authors. In the radiant tubes, the flue gas mainly contains a high concentration of N₂ and a low concentration of CO₂ and H₂O in air-fuel combustion. The flue gas can be regarded as gray gases, and the absorption coefficient of flue gas was defined by the weighted sum of gray gases model (WSGGM) coefficients by Smith et al. Compared with the air-fuel environment, the flue gas after combustion contains a high concentration of CO₂ and H₂O in oxygen-enriched combustion. The flue gas ought to be regarded as nongrey gases owing to the strong absorption bands of CO₂ and H₂O. However, the nongray assumption will lead to a significant increase in calculation time. Recent investigations showed that for small beam length, as in the radiant tubes, the standard WSGGM is sufficient in oxygen-enriched combustion. Additionally, the furnace is filled with protective gas (nitrogen) as the furnace atmosphere. The absorbed emittance by the furnace atmosphere is negligible since nitrogen as the furnace atmosphere is considered transparent in thermal radiation.

3.3. Boundary and Initial Conditions. In the model, the radiant tube walls between the fluid domain within the radiant tubes and the furnace atmosphere were modeled as so-called “two-sided walls”. Each side of the walls is a unique infinitely thin wall zone. The wall zones on both sides were coupled as the coupled wall conditions, which take the radiative and convective heat transfer. The thickness of the radiant tube walls is 3 mm and the emissivity is 0.85. In the radiant tubes, the natural gas is burned with a 15% excess air with respect to the stoichiometric ratio. In the present work, the oxygen-enriched and inlet-exchange investigations will be discussed, as listed in Table 5. There are two cases in the oxygen-enriched investigation. One is the basic case in air-fuel combustion. The other case is a mixture of 74 vol % N₂ and 26 vol % O₂ as an oxidizer in oxygen-enriched combustion. For the inlet-exchange investigation, the fuel inlet and the primary air inlet are exchanged, but the flow rates of fuel and air do not change. In addition, the outer wall of the furnace and the slab boundary were treated as the convective boundary conditions and the coupled wall conditions, respectively. The heat transfer coefficient between the outer wall of the furnace and the external air is 10 W/(m² K) and the external air temperature is 298.15 K. The emissivity of the furnace wall is 0.7. Since the slab surface is smooth and nonoxidized, the slab wall emissivity is 0.5. The density of each slab was set to 7940 kg/m³. The thermal properties of the slab material are highly dependent on temperature, and the specific heat and thermal conductivity were calculated with JMatPro. The curves with different colors in Figure 3 correspond to the temperature function polynomials for the simulation. A constant heat flux is applied to the roller boundary conditions, which is calculated from the absorbed energy of the cooling water.

The initial conditions of transient calculation in an indirect reheating furnace were obtained from the convergent steady solutions of the temperature field in the furnace based on the initial slab temperature. The initial slab temperature was determined according to the level 2 model prediction in production, as shown in Figure 4. To clearly see the layout and temperature distribution of slabs in the furnace, the structure of radiant tubes and rollers inside the furnace was hidden. In the model, the slab transient reheating process was achieved using a dynamic mesh. During the calculation, the moving zone containing solid slab zones and gas zone moved at a constant speed, while the geometry domain, except for the moving zone, remained stationary. From Figure 5, the denser meshes were used for the moving zone. The moving zone bottom is the slab/roller and slab/air interface, and the height is 0.2 m. Based on the layering method, the meshes of the moving zone were regulated during the mesh movement and the connectivity of the internal nodes did not change.

3.4. Pulse Combustion. The pulse combustion process controlled by the developed UDF was conducted in the transient calculation. During pulse combustion, the radiant tube burners were not affected by changes in system flow and were switched
between fully open and fully closed states. During reheating, the mean heat demand of the whole furnace is about 50%. Therefore, the heat demand of each temperature control section of the furnace was assumed to be 50%. The pulse period of all radiant tube burners controlled in pairs on both sides of the furnace is 60 s. The heat demand of each pair of radiant tube burners in different temperature control sections of the furnace is defined using eq 8, where \( N_t \) is the number of all radiant tube burner pairs in each temperature control section and \( N \) and \( H_d \) are the number of radiant tube burner pairs being in good working order in each temperature control section and the heat demand of each temperature control section of the furnace, respectively. The heat demand of each pair of radiant tube burners is 50% due to all radiant tube burners being in good working order. In addition, in each pulse period, the ignition interval \( (t_i) \) between each pair of radiant tube burners and the burning time \( (t_w) \) of radiant tube burners are given by eqs 9 and 10, respectively. \( t_p \) is the pulse period of radiant tube burners. Figure 6 shows a scheme of the solution procedure based on UDF. The pulse combustion sequence of radiant tube burners in each temperature control section is \( A_1B_1-A_2B_2...A_nB_n \). \( H_d \), \( t_i \), \( t_w \), and \( t_p \) are calculated constants. The two working states of radiant tube burners in different temperature control sections of the furnace correspond to different pulse times, and the number of

Figure 6. Scheme of the solution procedure based on UDF.
radiant tube burner pairs in each temperature control section has an influence on the pulse time of the corresponding working state. In the first pulse period, when the sum of the ignition interval and burning time of radiant tube burners is less than or equal to the pulse period, the burning time of radiant tube burners is continuous and the ignition interval is segmented; when the sum of the ignition interval and burning time of radiant tube burners is greater than the pulse period, the burning time of radiant tube burners is segmented and the ignition interval is continuous. Other pulse periods are the same as the first pulse period. As the heating time \((t)\) changes, the radiant tube burners continuously turn on and off, and output the fuel inlet mass flow \((m_f)\), the primary air inlet mass flow \((m_p)\), and the secondary air inlet mass flow \((m_s)\) in the corresponding state.

\[
H_d = \begin{cases} 
0 & (N = 0) \\
H_{d1} \times \frac{N}{N} \left( H_{d1} < \frac{N}{N} \right) \\
1 & (H_{d1} \geq \frac{N}{N}) 
\end{cases}
\]  
(8)

\[
t_w = H_d \times t_p 
\]  
(9)

\[
t_i = \frac{t_p}{N} 
\]  
(10)

The arrangement of the radiant tubes in the upper-temperature control section of the eighth zone is shown in Figure 7a, and the pulse combustion sequence of radiant tube burners in the
temperature control section is A1B1-A2B2-A3B3-A4B4-A5B5. In each pulse period, the burning time of radiant tube burners is 30 s. The ignition interval between each pair of radiant tube burners in the upper-temperature control section of the eighth zone is 12 s, as shown in Figure 7b. In a pulse period, different colored lines correspond to different radiant tube burners and the length of the lines represents the pulse width (burning time) of radiant tube burners. During a one-h reheating process, the radiant tube burners work for a total of 60 pulse cycles.

3.5. Computational Grid. A numerical grid is very important for obtaining accurate solutions. Unstructured grids were used in the simulation, and the grids are fine in the vicinity of radiant tubes due to high-temperature gradient, as shown in Figure 8. Additionally, to conduct the grid independence test, four different numbers of grids were tested with approximately 740,000, 920,000, 1,170,000, and 1,680,000 cells in the eighth zone. Five points in the eighth zone were selected as test points. The coordinates are point 1 (42,500, -2100, 960), point 2 (42,500, 2100, 960), point 3 (45,000, 0, 960), point 4 (47,500, 2100, 960), and point 5 (47,500, -2100, 960). From Figure 9, the temperature of five test points varies slightly with the increase of the number of grids. The results showed that when the number of grids is greater than 920,000, the number of grids has little influence on the calculation results. Therefore, to satisfy the requirement of the grid independence, 920,000 cells were chosen. In the simulation, the computational grid of the full-scale furnace is more than 14 million and the calculation was performed on a 64-core high-performance computer (HPC) at Northwestern Polytechnical University.

4. RESULTS AND DISCUSSION

4.1. Experimental Validation and Energy Analysis. Since the slab heating profiles determine the heating quality of the slabs during reheating, it is considered the most important variable. In this work, the heating profiles of the slabs in actual production were measured by experiments to verify the accuracy of the model. From Figure 10, nine monitoring thermocouples labeled “#1” to “#9” were installed inside the slab and were connected to a data acquisition device (Datapaq). During reheating, the Datapaq placed in the water cooling box moved with the slab and recorded the temperature variation of the slab. In Figure 10, nine thermocouples were divided into three groups to cover the areas of the slab top surface (#3, #6, #9), the areas of the slab mid-thickness (#2, #5, #8), and the areas of the slab bottom surface (#1, #4, #7). The measurement point (#10) is the gas temperature 150 mm above the slab.

Figure 11 shows the comparison between the simulation and experimental results. From Figure 11a–c, the predictions of the model are consistent with the experimental data. The average deviation between the slab temperature predicted by the model and the experimental data is 14 K, with a maximum of 46 K, which occurs at the top surface of the slab in the late heating zone. This can be regarded as sufficient for high temperatures above 1150 K in the furnace. In Figure 11d, the gas temperature above the slab predicted by the model is significantly different from the experimental data. This difference may be due to the effect of the turbulent flow of cold air brought by the subsequent charging slab on the gas temperature variation above the trial slab, especially in the heating zone where the slab is still in the heating stage. Nevertheless, the level of agreement between the
experimental and numerical results is reasonable. Thus, the model developed in this paper is able to be used for the research of the indirect reheating furnace with pulse combustion.

The overall energy balance of the furnace based on the simulation results is shown in Figure 12. According to the operating conditions, the thermal input of fuel is 33,805 kW, representing 89.86% of the total thermal input. The energy supplied by the preheated combustion air is 3,815 kW. In the model, the energy absorbed by the slabs is 21,398 kW, which accounts for 63.29% of the energy supplied by the fuel. This value is able to be regarded as the heating efficiency of the furnace. Due to the significant temperature difference between the internal hot air and the external cold air, there is a heat loss of 497 kW at the entrance and exit of the furnace. The heat losses through the roller system, the walls, and the flue gas are 907 kW, 3,487 kW, and 11,331 kW, respectively. But it must be mentioned that the wall loss contains the radiant tube wall loss and the furnace wall loss.

4.2. Oxygen-Enriched Investigation. In each pulse period, the surface temperature of the radiant tubes will not drop quickly during the closed state of the radiant tube burners due to the radiant tube body’s heat conduction and the temperature of the furnace atmosphere near the radiant tubes will not drop significantly. Thus, the stationary simulation of the furnace was assumed to be conducted with all radiant tube burners open. In the stationary simulation, the temperature distribution on the radiant tubes in the eighth zone with different oxygen concentrations is shown in Figure 13. The surface temperature near the radiant tube end is higher. The surface averaged temperature at a certain position of the radiant tubes refers to the average temperature along the circumference of the radiant tubes at that position. The maximum surface averaged temperature of the radiant tubes occurs where the inner radiant tube ends and is approximately 1318 and 1362 K for 21 and 26 vol % O₂ in the oxidizer, respectively. This is because a large volume of flue gas begins to flow back from here, resulting in stronger convective heat transfer between the outer radiant tube and flue gas. Therefore, the high-temperature zones on the surface of radiant tubes appear here.

To analyze the influence of the oxygen enrichment, based on the pulse combustion sequence of radiant tube burners in Figure 7, the gas temperature variation for different oxygen concentrations in the first pulse period at z = 0.96 m in the upper-temperature control section of the eighth zone is shown in Figure 14. The upper radiant tubes are split by the plane z = 0.96.
For oxygen-enriched combustion, the average temperature of oxygen-enriched combustion is 1155 and 1198 K, respectively. At the same time, the temperature distribution on the slab surface becomes uniform. As oxygen concentration increases, the average temperature of the trial slab is raised from 1171 to 1205 K at the exit of the furnace, with an increase of 2.9% (34/1171 K). Oxygen enrichment shows higher heating efficiency at the same fuel mass-flow rate.

4.3. Inlet-Exchange Investigation. In this section, the primary air and fuel inlets are exchanged based on the inner-fuel conditions where the fuel inlet is located in the inner circle area, and the fuel inlet is located in the annulus area of the primary air inlet under inner-air conditions. Figure 17 shows the temperature and velocity distribution on the cross-section of the radiant tube for the inlet-exchange case. For inner-air conditions, the gas temperature in the combustion chamber of the radiant tube burner is lower. This is because when the mass-flow rate is constant, the air inlet with a small central passage area has a high flow velocity in the combustion chamber, as shown in Figure 17b. The high-velocity air quickly brings the fuel out of the combustion chamber. This avoids the local high temperature and excessive combustion in the combustion chamber. The incomplete combustion mixture and the secondary air are burned more fully in the radiant tube compared to inner-fuel conditions, which enhance the flame intensity in the radiant tube, as shown in Figure 17a. The maximum flame temperature is increased from 2042 to 2196 K after changing the inlet flow rate. Since a large volume of flue gas flows back from the annular gap between the inner and outer tubes, the flue gas velocity increases here. This leads to stronger convective heat transfer between the outer radiant tube and flue gas. The species distribution at the outlet of the radiant tube for the inlet-exchange case is listed in Table 6. By changing the inlet boundary, the mole fractions of CH$_4$ and C$_2$H$_6$ at the outlet of the radiant tube are decreased from $7.54 \times 10^{-7}$ to $3.42 \times 10^{-8}$ and from $2.16 \times 10^{-7}$ to $8.64 \times 10^{-9}$, respectively. At the same time, the mole fraction of CO$_2$ becomes larger and the mole fraction of CO becomes smaller. This indicates more complete combustion under inner-air conditions. Figure 18 presents the temperature distribution of different cross-sections in the eighth zone for the inlet-exchange case. The upper and lower radiant tubes are split by the planes $z = 0.96$ and $-0.96$ m, respectively. For inner-fuel and inner-air conditions, the temperature of the furnace atmosphere around the upper radiant tubes is higher.
than that around the lower radiant tubes. Since the rollers are water-cooled and absorb heat from the surrounding furnace atmosphere, the temperature of the furnace atmosphere around the rollers and the lower radiant tubes is lower. For inner-air conditions, the temperature of the furnace atmosphere is

Figure 14. Gas temperature variation for different oxygen concentrations in the first pulse period at $z = 0.96 \text{ m}$ in the eighth zone: (a) 21 vol $\%$ O$_2$ and (b) 26 vol $\%$ O$_2$. 

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increased, which reveals that the heating ability of the radiant tube burners is improved by changing the inlet boundary.

To further study the influence of changing the inlet boundary, the temperature distribution on the upper surface of slabs and the average temperature of the slabs under different inlet boundaries at $t = 60$ min are shown in Figures 19 and 20, respectively. In the heating zone, the temperature on the upper surface of slabs under inner-air conditions is significantly higher.

Figure 15. Slab temperature variation for air-fuel (a) and oxygen-enriched (b) combustion.
than that under inner-fuel conditions. Compared with the 9th slab prepared to enter the soaking zone at an average surface temperature of about 1030 K under inner-fuel conditions, the average temperature on the upper surface of the 9th slab at the same position almost reached 1185 K under inner-air conditions. After entering the soaking zone, due to the small temperature difference between the slabs and radiant tubes, the slabs obtain less heat flux. The temperature increase of the slabs is mainly caused by the radiative heat flux, and the convective heat flux has little contribution. The average temperature of the slab receives an increase of 3.5% (41/1171 K) at discharge by changing the inlet boundary. Figure 21 shows the average temperature of the slabs in different zones for the inlet-exchange case. Compared with the soaking zone, the average temperature of the slabs in different zones increases by at least 3.5%. From the above, compared with the inner-fuel scheme used in production, the inner-air scheme improves the heating efficiency of the furnace.

5. CONCLUSIONS

In this work, the slab heating characteristics in an indirect reheating furnace with pulse combustion were investigated by a developed numerical model. Compared with direct-fired reheating furnaces, indirect heat transfer between the furnace atmosphere and the fluid domain within the radiant tubes was realized by coupling the radiant tubes and the furnace as a whole in the model. Furthermore, a pulse control approach based on UDF was used for controlling the radiant tube burner state. The effects of oxygen-enriched and different inlet conditions on the slab heating process and the temperature distribution in the furnace during pulse combustion were analyzed. The following conclusions can be drawn from this work.

(1) The predictions by the numerical model and the experimental data are in good agreement. The maximum deviation between the slab temperature predicted by the numerical model and the experimental results is 46 K, which can be regarded as sufficient for slab target temperature exceeding 1150 K. The energy balance of the whole furnace was discussed, and the heating efficiency of the furnace is 63.29%. With this model, the furnace operation can be optimized, resulting in better performance on energy efficiency and productivity.

(2) The EDC models with two different mechanisms were compared in air-fuel combustion. The results from the GRI 3.0 mechanism and the four-step mechanism show similar flame shape, flue gas temperature, and species distribution. Based on the four-step mechanism, the maximum deviations of flue gas temperature and species distribution at the outlet are 9 K and less than 1.5%, respectively. Consequently, in the simulation with the EDC model, two different mechanisms have the ability to accurately describe the combustion process. However, due to the lower computing time needed, the four-step mechanism is an excellent choice.

(3) For the oxygen-enriched investigation, from air-fuel combustion to oxygen-enriched combustion, the maximum surface averaged temperature of the radiant tubes
and the average temperature of the furnace atmosphere are increased from 1318 to 1362 K and from 1241 to 1279 K, respectively. This is because oxygen-enriched combustion has a stronger radiative active medium and higher medium temperature. In oxygen-enriched combustion, the average temperature of the trial slab receives...
increases of 3.7 and 2.9% at the exits of the heating and soaking zones, respectively. The temperature variation of the slab in the heating zone is mainly caused by the large temperature difference between the slab and radiant tubes. In the soaking zone, the slab temperature does not increase significantly.

(4) For the inlet-exchange investigation, the heating ability of the radiant tube burners is improved after changing the inlet boundary. This is because under inner-air conditions, excessive combustion in the combustion chamber is avoided and full combustion in the radiant tubes is promoted, which results in higher temperatures on the radiant tubes and further affects the slab heating. Compared with inner-fuel conditions, the slab temperature increases by 3.5% at discharge and the slab temperature in different zones increases by at least 3.5% after changing the inlet boundary. This reveals that the inner-air scheme can optimize the combustion process in the radiant tubes and improve the heating efficiency of the furnace.

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**Notes**

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**NOMENCLATURE**

| Symbol | Description |
|--------|-------------|
| $C_v$ | volume fraction constant |
| $C_t$ | time scale constant |
| $k$ | turbulent kinetic energy (m$^2$ s$^{-2}$) |
| $p$ | pressure (N m$^{-2}$) |
| $R_i$ | source term for species $i$ |
| $T_i$ | ignition interval between each pair of radiant tube burners (s) |
| $T_p$ | pulse period of radiant tube burners (s) |
| $t$ | time (s) |
| $u$ | velocity (m s$^{-1}$) |
| $v$ | kinematic viscosity (m$^2$ s$^{-1}$) |
| $x$ | coordinates (m) |

**GREEK SYMBOLS**

| Symbol | Description |
|--------|-------------|
| $\varepsilon$ | turbulent dissipation rate (m$^2$ s$^{-3}$) |
| $\rho$ | density (kg m$^{-3}$) |
| $\tau$ | time scale |

**SUBSCRIPTS**

$i$, $j$ indices

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