Thermalization of gluons at RHIC including $gg \leftrightarrow ggg$ interactions in a parton cascade

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Employing a newly developed pQCD inspired parton cascade we simulate the space time evolution of gluons which are produced initially in a heavy ion collision at RHIC energy. The inelastic $gg \leftrightarrow ggg$ interactions are for the first time implemented obeying full detailed balance. The numerical results show that thermalization of gluons is mainly driven by the inelastic gluonic interactions and reaches equilibrium at $1 \sim 2$ fm/c. In simulations for noncentral collisions considerable partonic elliptic flow $v_2$ is generated being comparable with the experimental data.

1. INTRODUCTION

It was shown that the measured momentum anisotropy parameter $v_2$ at RHIC energy can be well described by ideal hydrodynamics \[1\]. This indicates that the quark-gluon matter produced seems to behave like a perfect fluid which represents a strongly interacting and thus locally thermal system. On the other hand, the initial situation of the quark-gluon system is far from thermal equilibrium. It is therefore important to understand how and which partonic interactions can thermalize the system within a short timescale.

A convenient way to study thermalization of particles is to carry out microscopical transport simulations which, however, need large computational power. There are several such numerical realizations \[2\] currently applied for investigating the space time evolution of partons. In these models only elastic $gg \leftrightarrow gg$ interactions are considered and no thermal equilibrium can be realized in Au+Au collision at RHIC when using reasonable pQCD cross sections. It is thus essential to study the contribution of multiple interactions to the thermal equilibration. In addition, the possible importance of the inelastic scatterings on thermalization was raised in the “bottom up thermalization” picture \[3\]. Recently we have developed a new 3 + 1 dimensional Monte Carlo cascade solving the kinetic on-shell Boltzmann equations for partons including inelastic $gg \leftrightarrow ggg$ pQCD processes \[4\]. Detailed balance is fulfilled in a consistant manner.

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The three-body gluonic interactions are described by the matrix element [5]

\[
|M_{gg\rightarrow ggg}|^2 = \left(\frac{9g^4}{2} \frac{s^2}{(q_\perp^2 + m_D^2)^2}\right) \left(\frac{12g^2q_\perp^2}{k_\perp^2 [(k_\perp - q_\perp)^2 + m_D^2]}\right) \Theta(k_\perp \Lambda_g - \cosh y), \tag{1}
\]

where \(g^2 = 4\pi\alpha_s\). \(q_\perp\) and \(k_\perp\) denote, respectively, the perpendicular component of the momentum transfer and that of the momentum of the radiated gluon in the c.m. frame of the collision. We regularize the infrared divergences by using the Debye screening mass \(m_D\) which is calculated locally over the actual particle density obtained from the simulation. The suppression of the radiation of soft gluons due to the Landau-Pomeranchuk-Migdal effect, which is expressed via the step function in Eq. (1), is modeled by the consideration that the time of the emission, \(\sim \frac{1}{k_\perp} \cosh y\), should be smaller than the time interval between two scatterings or equivalently the gluon mean free path \(\Lambda_g\). This leads to a lower cutoff of \(k_\perp\) and to an effective increase of the collision angles.

Until now only gluonic dynamics is considered in the cascade. In the future quarks will be included. A special interest is put on the investigation of elliptic flow of heavy quarks [6].

2. INITIAL CONDITIONS

The production of the primary partons at the very onset of a heavy ion collision is based on the picture of a free superposition of minijets being liberated in the individual semihard nucleon-nucleon interactions. Minijets denote here on-shell partons with transverse momentum being greater than a certain cutoff \(p_0\). Their production is controlled by perturbative QCD for sufficient high \(p_0\) [7]. On the other side, the smaller the cutoff \(p_0\) is, the denser will be the initial minijet system, which may accelerate thermalization. Phenomenologically the cutoff \(p_0\) can be chosen in a way to fit the (final) \(dE_T/dy\) as seen in experiment. The space time configuration of the produced partons will be determined by applying the Glauber symmetry with a Woods-Saxon nuclear distribution.

We have also considered the conditions of the initial partons according to color glass condensate [8]. First results can be found in [9].

3. RESULTS

To study possible thermalization of gluons we concentrate on the local central region which is taken as an expanding cylinder with a radius of 1.5 fm and within a unit interval of space time rapidity \(\eta\) around the collision center \(\eta = 0\). Figure 1 shows the varying transverse momentum spectrum with time obtained in the central region. The boldfaced histogram with a lower cutoff at \(p_0 = 2\) GeV (a very conservative setting) denotes the spectrum of the primary gluons (or minijets). In the simulation including inelastic \(gg\leftrightarrow ggg\) scatterings (left panel of Fig. 1) the curves from second upper to lowest depict, respectively, the spectrum at \(t = 0.2, 0.5, 1, 2, 3,\) and \(4\) fm/c. We see that the spectrum reaches an exponential shape at \(2\) fm/c and becomes increasingly steeper at late times. This is a clear indication for the achievement of thermal equilibrium and the onset of hydrodynamical evolution. In contrast, without including inelastic collisions (right panel of Fig. 1) one has no hints for equilibration.
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Figure 1. Transverse momentum spectrum in the central region at different times obtained from the simulation with $gg \leftrightarrow ggg$ (left panel) and from the simulation without $gg \leftrightarrow ggg$ collisions (right panel).

Figure 2 shows the cross sections of various gluonic interactions and the corresponding transport cross sections. The latter might be taken as a characteristic for momentum degradation. While $\sigma_{gg \rightarrow gg}$ is significantly larger than $\sigma_{gg \rightarrow ggg}$, its transport cross section is smaller than that of typical $gg \rightarrow ggg$ collision. Inspecting the fraction of the transport cross sections to the total cross sections we realize that the distribution of the collision angle in $gg \rightarrow ggg$ processes is almost isotropic, while the $gg \rightarrow gg$ collisions are much more forward peaked. Taking the contribution of $ggg \rightarrow gg$ collision to the equilibration into account, the total inelastic interactions are the dominant processes compared to the elastic collisions. Besides the kinetic scatterings plasma instabilities of the gluon field may also have contribution to a very early and fast momentum equilibration [10]. The latter should be further quantified.

We have also performed simulations when the momentum cutoff for the initial minijets is taken smaller. It turns out that for the more dense system at $p_0 = 1.3 - 1.5$ GeV (being in line with the measured $dE_T/dy$) the full equilibrium comes slightly sooner at $1 - 2$ fm/c. The timescale tends to saturate at smaller $p_0$. These results will be presented in a sequent paper.

Taking $p_0 = 1.4$ GeV for the initial minijets we simulate the parton evolution for noncentral collisions at RHIC energy in order to calculate the elliptic flow parameter $v_2$. Figure 3 shows the time evolution of $v_2$ extracted in the central rapidity for various impact parameter $b$. These calculations are still preliminary and no exhaustive tests have been finished. We see that $v_2$ increases with time and saturates at late times, $3 \sim 5$ fm/c. The larger the initial space anisotropy is, the larger is the generated $v_2$. The results give us strong indication that an early pressure is being built up. The symbols in Fig. 3 mark the time from which the energy density in the central region decreases below 1 Gev/fm$^3$. Therefore after this time the system can be hardly described by the dynamics among partons. If we take the $v_2$ values at the marked times as the contribution from the partonic phase, one realizes that they lie well in the region covered by the experimental
Figure 2. Cross sections averaged in the central region. Results are obtained from the simulation including \( gg \leftrightarrow ggg \) collisions.

Figure 3. Time evolution of the elliptic flow \( v_2 \) for various impact parameter \( b \).

data with the systematic errors [11].

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