Third Harmonic Generation of Hermite-cosh-Gaussian laser Beam

Vinay Sharma¹, Vishal Thakur¹, Niti Kant¹

¹Department of Physics, Lovely Professional University, G.T. Road, Phagwara -144411, Punjab, India.
Corresponding Address: E-mail: nitikant@yahoo.com

Abstract

In present study we have investigated the third harmonic generation (THG) of self-focused Hermite-cosh-Gaussian (HchG) laser beam, propagating through plasma. When intense short pulse laser propagates through plasma results ponderomotive force on electrons due to which electrons move away from the axial region, creating a low density plasma region. Due to electrostatic force, electrons gain oscillatory velocity and results density perturbation. Density oscillations at frequency $2\omega_0/\gamma^2$ couple with oscillatory velocity of electrons, at the fundamental frequency of incident laser result THG. Due to phase mismatch with the incident pulse, the third harmonic pulse is of smaller amplitude. Wiggler field satisfy the resonance condition by providing additional momentum to the photon of third harmonic pulse and this result increase in the normalized amplitude of third harmonic pulse. In the present study the study of beam width parameter with normalized propagation distance at optimum values of different laser parameters has been studied. We derived the coupled differential equation for the normalized amplitude, $A_{30}/A_{10}$, of third harmonic pulse. We have analysed the efficiency of third harmonic pulse with $\xi$ at optimum values of normalized intensity of fundamental laser.

Key words: - Third harmonic generation, Hermite-cosh-Gaussian laser beam, relativistic self-focusing, plasma density, wiggler field, decenterd parameter.
1. Introduction

Propagation of high intensity laser through plasma, has been widely studied in last few decades by number of researchers. When short pulse laser propagates through plasma, it results density perturbations and nonlinearity arises. Variation in dielectric properties of plasma results important nonlinear phenomena, which have their wide range of applications like inertial confinement fusion [1], laser plasma accelerator [2], microscopic resonance imaging [3] by using second harmonic generation (SHG) and THG, laser driven accelerators [4], relativistic self-focusing [5], ponderomotive self-focusing [6], harmonic generations [7] etc. Amongst different harmonic generations the THG is a specific topic of research due to its important applications in medical science [8], telecommunication [9], optoelectronics [10] etc. Kant et al. [11] studied the THG when short-pulse laser propagates through plasma created by electron-hole pair in semiconductor medium. Wiggler field satisfied the resonant condition, results enhancement in the energy conversion efficiency. Dhaiya et al. [12] investigated the SHG and THG in plasma in the presence of density ripple. Density ripple satisfy the resonance, result increase in the efficiency of harmonic generations. Vij et al. [13] studied the resonant THG in cluster density. Presence of density ripples in cluster density, satisfy the phase matching condition, which is responsible for the enhancement in harmonic generation. They observed that the velocity of fundamental laser differ from group velocity results the pulse slippage. Thakur et al. [14] studied THG during the laser plasma interaction and due to density ramp in plasma, increase of normalized amplitude was reported. In their study they used the wiggler field for necessary phase matching condition and pulse slippage of third harmonic pulse was reported due to the difference in velocity of third harmonic pulse with fundamental laser pulse. Shibhu et al. [15] investigated the Phase matched THG of laser radiation in a plasma channel, where background density perturbation satisfy the resonance condition. Rajput et al. [16] studied the resonant THG during laser plasma interaction. They used the wiggler magnetic field to satisfy the resonance condition and observed the enhancement in the efficiency of third harmonic pulse. Sharma et al. [17] studied THG under relativistic self-focusing, when short pulse laser interacts with plasma. In their study, the wiggler field provide the additional momentum to the photons of third harmonic pulse and provide the resonant condition, results enhancement of efficiency of the third harmonic pulse. Singh & Gupta [18] studied the SHG under relativistic self-focusing, using cosh-Gaussian laser beam. They had given the shifting of peak intensity in transverse direction by changing de-centered parameter. Singh et al. [19] investigate cosh-Gaussian laser beam interacting with under-dense collisional plasma with non-linear absorption. They studied that electrons gets heated non uniformly, results self-focusing of incident laser along with generation of density gradients.

Number of research workers has analysed the self-focusing of HchG laser beam using different laser parameters. Nanda et al. [20] studied the Hermite-cosh-Gaussian laser beam in plasma for relativistic self-focusing, under density transition and studied the effect of different laser parameters on beam width parameter, where plasma density transition played vital role in self-focusing. Self-focusing of HchG laser beam in magneto-plasma with density ramp profile was studied by Nanda et al. [21], where they showed that for m = 0, 1 and 2, the diffraction overcomes nonlinear term for b = 0, where b is the decentred parameter. Wani et al. [22], under density transition investigated the self-focusing of HchG laser beam in plasma, where plasma density ramp reduces the defocusing and smaller spot size is obtained upto several Rayleigh length. Kaur et al. [23] used the HchG beam to study the relativistic self-focusing for m = 0, using a particular set of de-cantered parameter. At different values of decentred parameter the relativistic self-focusing of HchG laser beam in plasma becomes stronger for higher mode indices, was studied by Nanda et al. [24]. They observed stronger self-focusing for m=1, 1 and 2. Patil et al. [25], in collision less magneto plasma, studied interaction of HchG laser beam and their study comprises the combined effect of nonlinearity and spatial diffraction. Kant et al. [26] presented the stronger self-focusing of HchG laser beam in plasma and analyzed the behaviour of beam width parameter at optimum values of decentred parameter and normalized frequency of incident beam.
In the present study we are analyzing the THG at the optimum values of different laser parameters, taking intense HchG laser propagating through plasma. The expression of normalized amplitude for THG using HchG laser beam has been given in section II, results have been analyzed graphically in section III and sections IV contain the conclusion.

2. Methodology

HchG beam is considered to be propagating along z-axis in plasma with the field distribution as

\[ \vec{E} = A(x, y, z) \exp[i(\alpha_1 t - k_0 z)] \]
\[ \vec{B}_w = \vec{y} B_0 \exp(i k_0 z), \]

Where amplitude of the incident pulse is given by \( A \), \( k_0 \) is the wiggler wave number and \( \vec{B}_w \) is the wiggler magnetic field.

Electrons experience the ponderomotive force and attain oscillatory velocity

\[ \vec{v}_i = e \frac{\vec{E}_i}{me(\alpha_1 + iv)} \]

incident beam exerts ponderomotive force on electrons due to which electrons acquire oscillatory velocity at \( 2\alpha_1 \) which coupled with density ripple produces density perturbation at \( 2\alpha_1 \) and due to beating of perturbed density with oscillatory velocity, at the frequency of incident laser, results third harmonic current density \( \vec{J}_3 = \vec{J}_3^L + \vec{J}_3^{NL} \). Where \( \vec{J}_3^L \) and \( \vec{J}_3^{NL} \) [16] are given as

\[ \vec{J}_3^L = \frac{n_0 e^2 \vec{E}_3}{m 3i \alpha_1} \]

\[ \vec{J}_3^{NL} = -\frac{n_0 e^5 B_0 k_0 \vec{E}_1^3}{16 cm \alpha_1^4(\alpha_1 + iv)} \left[ \frac{5k_1 + k_1 + k_0}{18 \alpha_1^3} \right] \]

For third harmonic field, the wave equation is written as

\[ \nabla^2 \vec{E}_3 = \frac{4\pi \vec{\partial}_0}{c^2} \frac{\partial \vec{J}_3^L}{\partial t} + \frac{4\pi \vec{\partial}_0}{c^2} \frac{\partial \vec{J}_3^{NL}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \vec{E}_3}{\partial t^2} \]

where \( \vec{E}_3 = A_3 e^{i(\omega_3 t - k_0 z)} \) and \( A_3 = A_{30}(r, z)e^{-\delta R(r, z)} \)

\[ A_{30}^2 = \frac{E_0^2}{f^2(z)} \frac{H_m(\sqrt{2r}}{r_0 f(z)} \left[ \frac{(b^2)}{2} \right] \exp \left[ -2 \left( \frac{r}{r_0 f(z)} + \frac{b}{2} \right) \right] + \exp \left[ -2 \left( \frac{r_0 f(z)}{r_0 f(z) = \frac{b}{2}} \right) \right] + 2 \exp \left[ -2 \left( \frac{r_0 f(z) = \frac{b}{2}}{r_0 f(z) = \frac{b}{2}} \right) \right] \]

The beam width parameter equations [18] for different modes are given as
For $m = 0$

$$\frac{\partial^2 f(z)}{\partial \xi^2} = \left[ 4 - 4b^2 - \frac{6\alpha E_0^2}{\gamma} \left( \frac{m_0}{M} \right) \left( \frac{\omega_0^2 r_0^2}{c^2} \right) \left( \frac{\omega_p^2}{\omega_i^2} \right) \exp \left[ \frac{b^2}{2} \right] \right] \frac{1}{f^3(z)}$$

(7)

For $m = 1$

$$\frac{\partial^2 f(z)}{\partial \xi^2} = \left[ 4 - 4b^2 - \frac{12\alpha E_0^2}{\gamma} \left( \frac{m_0}{M} \right) \left( \frac{\omega_0^2 r_0^2}{c^2} \right) \left( \frac{\omega_p^2}{\omega_i^2} \right) \exp \left[ \frac{b^2}{2} \right] \left( b^2 - 2 \right) \right] \frac{1}{f^3(z)}$$

(8)

For $m = 2$

$$\frac{\partial^2 f(z)}{\partial \xi^2} = \left[ -8b^2 - \frac{24\alpha E_0^2}{\gamma} \left( \frac{m_0}{M} \right) \left( \frac{\omega_0^2 r_0^2}{c^2} \right) \left( \frac{\omega_p^2}{\omega_i^2} \right) \exp \left[ \frac{b^2}{2} \right] \left( 5 - 2b^2 \right) \right] \frac{1}{f^3(z)}$$

(9)

The solution of Eq. (6) gives the particular integral

$$\tilde{E}_3 = A_j \exp[-i(2\omega t - (2k_3 + k_0)z)]$$

(10)

Also $A_j = A_{30}(z)\psi_j$ and $\psi_j = \exp[-r^2/\alpha^2 \Phi_j^2] \exp[-2ik_j x_j]$.

$A_{30}$ is amplitude for the third harmonic pulse.

Using Eqs. (10), (11) and (6) we obtain

$$2\psi_3 \left( i(3k_1 + k_0) \right) \frac{\partial A_{30}}{\partial r} + \left[ \frac{9\omega_1^2 - 10\omega_p^2}{c^2} + \frac{9\omega_1^2 \phi(\tilde{E}, \tilde{E}^*)}{c^2} + \left( i(3k_1 + k_0) \right)^2 \right] A_{30} \psi_3$$

$$+ A_{30} \frac{\partial^2 \psi_3}{\partial t^2} + \frac{A_{30}}{r} \frac{\partial \psi_3}{\partial r} = \frac{12\pi n_0 e^5 B_m k_1}{c^2 16i^2 \alpha_1^4 \gamma^3 \omega_i^4} \left[ \frac{5k_1}{18\omega_i} + \frac{k_1 + k_0}{\omega_i + iv} \right] \frac{\partial \tilde{E}_3}{\partial t}$$

(12)
where \( \xi \) is the normalized propagation distance. Above equation is multiplied by \( \psi_0^* z r \, dr \). We integrate the expression with respect to \( r \) we obtain

\[
2i \frac{\partial A_{30}}{\partial \xi} \{2 - b^2\} + \left[ - \left( \frac{9 \omega_0^2 r_0^2}{c^2} \right) \left( 1 - \frac{\omega_r^2}{9 \omega_0^2} \right)^2 \left\{ 2 - b^2 \right\} + A_{30} \right] = \left( \frac{24}{f^2(z)} \right) \left\{ 2 - b^2 \right\} + \left( \frac{24}{2 f^2(z)} \right) + \frac{72 b^2}{f^2(z)} - \frac{27 b^4}{f^2(z)} - \frac{9 b^5}{2 f^2(z)} \exp(-2) - i \left\{ 2 - b^2 \right\} \frac{\partial f_1}{f_1 \partial \xi}
\]

\[
2i \frac{\partial A_{30}}{\partial \xi} \{2 - b^2\} + \left[ - \left( \frac{9 \omega_0^2 r_0^2}{c^2} \right) \left( 1 - \frac{\omega_r^2}{9 \omega_0^2} \right)^2 \left\{ 2 - b^2 \right\} + A_{30} \right] = \left( \frac{24}{f^2(z)} \right) \left\{ 2 - b^2 \right\} + \left( \frac{24}{2 f^2(z)} \right) + \frac{72 b^2}{f^2(z)} - \frac{27 b^4}{f^2(z)} - \frac{9 b^5}{2 f^2(z)} \exp(-2) - i \left\{ 2 - b^2 \right\} \frac{\partial f_1}{f_1 \partial \xi}
\]

\[
2i \frac{\partial A_{30}}{\partial \xi} \{2 - b^2\} = \left[ \left\{ 1 - \frac{\omega_r^2}{9 \omega_0^2} \right\} \left( \frac{24}{f^2(z)} \right) \left\{ 2 - b^2 \right\} + \left( \frac{24}{2 f^2(z)} \right) + \frac{72 b^2}{f^2(z)} - \frac{27 b^4}{f^2(z)} - \frac{9 b^5}{2 f^2(z)} \exp(-2) - i \left\{ 2 - b^2 \right\} \frac{\partial f_1}{f_1 \partial \xi} \right]
\]

\[
- \left[ \left\{ 1 - \frac{\omega_r^2}{9 \omega_0^2} \right\} \left( \frac{24}{f^2(z)} \right) \left\{ 2 - b^2 \right\} + \left( \frac{24}{2 f^2(z)} \right) + \frac{72 b^2}{f^2(z)} - \frac{27 b^4}{f^2(z)} - \frac{9 b^5}{2 f^2(z)} \exp(-2) - i \left\{ 2 - b^2 \right\} \frac{\partial f_1}{f_1 \partial \xi} \right]
\]

\[
\frac{3}{16 \gamma^2 \left( \frac{\omega_r^2}{\omega_0^2} \right) \left( \frac{\omega_0^2 r_0^2}{c^2} \right) \left\{ 1 - \frac{\omega_r^2}{\omega_0^2} \right\}^{1/2}} \left( \frac{e^2 A_{30}}{m \omega_0 c} \right) \left( \frac{e B_m}{m \omega_0 c} \right) \left[ \left( \frac{1 - \frac{\omega_r^2}{9 \omega_0^2}}{1 - \frac{\omega_r^2}{\omega_0^2}} \right)^{1/2} \left\{ 2 - 3 b^2 \right\} \right]
\]

3. Results and discussion

Eqs (7), (8), & (9) are the coupled differential equations for beam width parameter for mode index \( m = 0, 1, 2 \) and Eq (13) is the derived differential equation for normalized amplitude of third harmonic pulse. We have solved these equations numerically at different values of different laser parameters as \( \omega_0 r_0 / c = 27 \), \( \epsilon_2 A_{30} / \epsilon_0 = 1 \), \( e A_{30} / m \omega_0 c = 5 \), \( e B_m / m \omega_0 c = 3 \) and \( \omega_r / \omega_0 = 0.8 \) and interpreted the results graphically. Fig. 1 gives the analysis of beam width parameter with normalized propagation distance at different mode indices = 0, 1 and 2 respectively for which the respective values of beam width parameter = 0.7, 0.38 and 0.1 respectively. This shows that self-focusing is stronger for \( m = 2 \). Similar study for HcH laser beam, considering magneto-plasma for \( m = 0, m = 1 \) and \( m = 2 \) was given by Patil et al. [25]. They showed the similar behavior where self-focusing is stronger for \( m = 2 \). The study of \( \frac{A_{10}}{A_{10}} \) of third harmonic pulse with linear propagation distance for optimum values of normalized intensity of incident pulse given as 1, 3, 5 is plotted in fig. 2, keeping other laser parameters unchanged. It is observed that normalized amplitude of third harmonic pulse attain its values 0.049, 0.26 0.55 at normalized intensity of incident pulse = 1, 3, 5 respectively. As incident laser becomes more intense, the electrons experience stronger ponderomotive and attain higher quiver velocity the makes the self-focusing to be stronger. Singh et al. [19] presented the similar results for SHG of Hermite-cosh-Gaussian laser beam were given by Singh et al., where the output of second harmonic pulse shows significant rise, with increase in intensity of incident laser pulse as it makes self-focusing more stronger.
4. Conclusion:

With the graphical analysis of beam width parameter, we observed that the self-focusing is weaker for \( m = 1 \) and is stronger at higher mode indices. We find that beam width parameter of \( HchG \) laser attain a very low value, nearly 0.1, is stronger at \( m = 2 \). It is due to stronger ponderomotive force due to result increase in refractive index. Gain in normalized amplitude of THG is significantly high for \( m = 2 \) as compare to \( m = 1 \), at different values of normalized intensity of incident laser. On comparison we have seen that gain in efficiency of \( HchG \) laser beam is more as compared to Gaussian or \( \cosh \)-Gaussian laser beam.

Fig.1 Variation of beam width parameter \( f \) of the pump laser beam with normalized propagation distance \( \xi \) for different values of \( m = 0, 1 \& 2 \). The other parameters are normalized intensity = 5, \( \omega_1 r_0 / c = 27, \ e_2 A_{10}^3 / e_0 = 1, eB_w / m_0 \omega_1 c = 3 \) and \( \omega_p / \omega_1 = 0.8 \).

Fig.2 Variation of normalized third harmonic amplitude \( A_{30} / A_{10} \) with normalized propagation \( \xi \) distance for different values of normalized intensity of incident laser, normalized intensity = 1,3 \& 5. The other parameters are same as taken in Fig. 1.
5. References:

[1] Nakai S. and Takabe H. 1996, *Rep. Prog. Phys.* **59** 1071.
[2] Corde S., Phuoc K. T., Lamber G. T, Fitour R., Malka V., and Rousse A. 2013, *Rev. Mod. Phys.* **85** 1.
[3] Hernandez C. F., Ortiz G. R., Tseng S. Y., Gaja M. P. and Kippelen B., 2013, *J. Mater. Chem.* **19** 7394.
[4] Badziak J., 2018, *IOP Conf. Series: Journal of Physics: Conf. Series* **959** 012001.
[5] Sohbatzadeh F., Rabbani M. and Ghalandar M., 2012, *Opt. Commun.* **285** 3191–3194.
[6] Jafari Milani M. R., Niknam, A. R. and Farahbod A. H., 2014, *Phys. Plasmas* **21** 063107.
[7] Zhang S. J., Zhuo H. B., Zou D. B., Gan L. F., Zhou H. Y., X. Z. Li, Yu M. Y., and Yu W., 2016, *Phys. Rev. E* **93** 053206.
[8] Zhang Z. , Kuzmin N. V. , Groot M. L. and Munck J. C. D. 2017, BMC Bioinformatics **33** 1712.
[9] Sointsev S., Sukhorukov A., Neshev D. N. and Iliew R., 2011, *Appl. Phys. Lett.* **98** 23.
[10] Oaga Y. T, Votobyev A. and Guo C., 2018, *Materials* **11** 501.
[11] Kant N., Gupta D. N. and Suk H., 2012, *Phys. Plasmas* **19** 013101.
[12] Dahiya D., Sajal V. and Sharma A.K., 2007, *Physics plasmas* **14** 123104.
[13] Vij S., Kant N. and Aggarwal M., 2016, *Laser part. Beams.* **34** 171.
[14] Thakur V., Kant N., 2016, *Front. Phys.* **11** 115202.
[15] Shibu S. and Tripathi V.K., 1998, *Phys. Lett. A* **239** 99.
[16] Rajput J., Kant N., Singh H. and Nanda V., 2009, *Opt. Commun.* **282** 4614.
[17] Sharma V., Thakur V. and Kant N., 2019, *HEDP* **32** 51.
[18] Singh A. and Gupta N.,2016, *Laser part. Beams* **34** 10.
[19] Singh N., Gupta N. and Singh A., 2016, *Opt. Commun.* **381** 180.
[20] Nanda V. and Kant N., 2014, *Phys. plasmas.* **21** 72111.
[21] Nanda V., Kant N. and Wani M. A., 2013, *Phys. Plasma* **20** 113109.
[22] Wani M. A. and Kant N., 2016, *Adv. Opt. Photonic* **2014** 5.
[23] Kaur S., Kaur M., Kaur R. and Gill T.S., 2017, *Laser part. Beams* **35** (2017) 100.
[24] Nanda V., Kant N. and Wani M. A., 2013, *IEEE T Plasma Sci.* **41** 2252.
[25] Patil S.D., Takale M.V., Navare S.T. and Dongare M.B., 2010, *Laser Part. Beams* **28** 343.

[26] Kant N. and Nanda V., 2014, *OA. Lib Journal* **1** 2333.