Internal Dynamics of a Free-Surface Viscoplastic Flow Down an Inclined Channel

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This paper presents preliminary experimental results concerning the internal dynamics of a free surface viscoplastic flow down an inclined channel. Experiments are conducted in an inclined channel whose bottom is constituted of an upward-moving conveyor belt with controlled velocity. Carbopol microgel was used as a homogeneous transparent viscoplastic fluid. This experimental setup allows generating and observing stationary gravity-driven surges in the laboratory frame. We used PIV technique (Particle Image Velocimetry) to obtain velocity fields both in the uniform zone and within the front zone where flow thickness is variable and where recirculation takes place. Experimental velocity profiles and determination of plug position are presented and compared to theoretical predictions based on the lubrication approximation.

**Key words:** debris flow, experimental fluid mechanics, Carbopol, PIV, shallow water

1. INTRODUCTION

Debris flows constitute one of main natural hazards in mountainous regions of the world because of their great destructive power when flowing on a vulnerable zone. Consequences can be both heavy economic losses and fatalities. Debris flows are commonly composed of mixtures of particles of all sizes in water. The rheological studies conducted on materials sampled in natural muddy debris flows, rich in fine particles (< 40 µm), have shown that these materials behave as non-Newtonian viscoplastic fluids [Coussot et al., 1998; Bardou, 2002; Ancey, 2007]. In particular, these materials are characterized by a critical stress threshold, called yield stress, which results, in the flows, in the existence of unsheared regions as soon as the stress drops below the yield stress.

An important challenge resides in developing models that are able to accurately predict hydraulic properties of debris flows. These complex flows are generally represented using models based on a momentum integral approach that consists in assuming a shallow flow and in depth-averaging the local conservation equations [Takahashi, 1991; Piau, 1996; Huang and Garcia, 1998; Ancey, 2007; Ancey and Cochard, 2009]. These models take into account closure terms depending on the shape of the velocity profile inside the flow. But to date, knowledge concerning the shape of velocity profiles and the position of the interface between sheared and unsheared regions, in particular in the vicinity of the front, remains poor [Andreini et al., 2012]. It is then necessary to obtain reliable and well documented experimental data about the velocity field in such flows, in order to test, validate and choose the most appropriate model. To this end, the aim of this paper is to present experimental results documenting the internal dynamics of a free surface viscoplastic flow down an inclined channel.

Experiments were conducted in an inclined channel whose bottom is constituted of an upward-moving conveyor belt. This setup allows generating and observing gravity-driven surges that remain globally stationary with respect to the laboratory reference frame. We used Carbopol as a homogeneous transparent viscoplastic material. Carbopol has already been used as model viscoplastic material in various studies [Robert and Barnes, 2001; Coussot et al., 2009; Piau, 2007; Oppong and De Bruyn, 2011; Ovarlez et al., 2012]. Its rheological properties were determined independently using a rotational parallel plate rheometer. Surges obtained in the channel are characterized by a steep front followed by a long zone where the flow height is flow both in the uniform zone and within the front.
2. EXPERIMENTAL METHODS

2.1 Conveyor belt channel

The experimental setup consists of a 3-m-long inclined channel whose bottom is formed by an upward-moving conveyor belt with controlled velocity $u_0$ (Fig. 1). Channel section is rectangular, with a width varying from 0 to 0.5 m. The belt is driven by an electrical motor allowing to set velocities from 0 to 1.5 m.s$^{-1}$. Velocity is monitored by an optoelectronic sensor with a relative accuracy of 1%. The inclination of the whole set up can be varied from 0 to 30°.

The two lateral walls and the upper wall (Fig. 1) are fixed in the laboratory frame. A system of sealings prevents fluid leakage below the upper wall and the banks while minimizing friction with the moving belt. The conveyor belt is made of 2 mm thick PVC. Its surface is macroscopically smooth, but it provides good adhesion with the used fluids.

2.2 Viscoplastic fluid

The experiments reported here were performed with a viscoplastic fluid: a polymeric aqueous microgel named Carbopol ETD 2623 supplied by Noveon. The samples of Carbopol were prepared at 0.1% mass concentration by slowly adding 30 g of the polymeric powder in 30 L of deionized water under vigorous stirring (700 rpm). After leaving the solution at rest, pH neutralization was performed using a NaOH solution at 1 mol.L$^{-1}$ under slow stirring (100 rpm) during half an hour for homogenization and elimination of bubbles.

The characteristic flow curves of our Carbopol samples were determined using a rotational parallel-plate rheometer (Bohlin CVOR). As shown in Fig. 2, the flow curves can be represented by Herschel-Bulkley constitutive relation:

$$
\begin{align*}
\dot{\gamma} &= 0 & \text{if } \tau < \tau_c \\
\tau &= \tau_c + K \dot{\gamma}^n & \text{if } \tau \geq \tau_c
\end{align*}
$$

where $\tau_c$ is the yield stress, $K$ the consistency and $n$ the flow index. The rheological properties of the Carbopol samples used are summarized in Table 1.

Uncertainties on the order of ± 5% on parameters $\tau_c$ and $K$ are unavoidable despite careful testing. In addition, recent results with the same material [Chambon et al., 2014] concerning the flow properties (height, velocity profile) in the uniform zone showed a systematic discrepancy between experimental results and theoretical predictions based on the measured rheological parameters. This discrepancy can be accounted for by applying an adjustment of the rheological parameters measured with the rheometer according to the following relations:

$$
\begin{align*}
\tau_{c,corr} &= \tau_c \times 1.11 \\
K_{corr} &= K \times 1.19
\end{align*}
$$

Possible explanations for this discrepancy are discussed in [Chambon et al., 2014]. It seems that the small dimension of the Carbopol samples tested in the rheometer (thickness typically on the order of 1 mm) is not sufficient to correctly represent the mechanical behavior of the material at the flume scale. The above correction, which is unique for all tested Carbopol samples, appears to account for this
Table 1 Properties of the Carbopol samples and characteristics of channel flow experiments performed: density \( \rho \) (kg.m\(^{-3}\)), slope angle \( \theta \) (\(^{\circ}\)), yield stress \( \tau_c \) (Pa), consistency \( K \) (Pa.s\(^n\)), flow index \( n \), imposed belt velocity \( u_0 \) (mm.s\(^{-1}\)).

| exp | \( \rho \) | \( \theta \) | \( \tau_c \) | \( K \) | \( n \) | \( u_0 \) |
|-----|-----|-----|-----|-----|-----|-----|
| C_a | 1000 | 14.6 | 6.00 | 6.65 | 0.405 | 52.7 |
| C_b | 1000 | 11.9 | 21.49 | 11.3 | 0.396 | 138 |

Fig. 3 Example of velocity field for free-surface flow of Carbopol in the front zone computed using DPIVsoft. The length of the image is about 10 cm.

Fig. 4 Thin layer model: schematic representation of notations used in text. \( \xi \) denotes the distance from the front.

height measured through image processing. The height of the flow in the front region is measured only with image processing.

3. THEORETICAL FRAMEWORK

Here we consider a viscoplastic fluid flowing down an inclined plane whose behavior can be modelled by the Herschel Bulkley law (Eq. (1)). We compare the experimental velocity profiles to the theoretical profiles obtained in the frame of the lubrication approximation, i.e. neglecting inertia terms. This approximation is at the base of most shallow-flow formulations used to simulate debris flow propagation [Coussot, 1994]. Let us denote \( \theta \) the slope angle, \( \rho \) the fluid density, and \( h \) the thickness of the flowing layer.

We define a 2D coordinate system in which the \( x \) direction is the direction of the flume, and the \( y \) axis is oriented upwards (Fig. 4). The position \( y = 0 \) correspond to the bottom of the flow.

Applying the lubrication approximation, the shear stress in the layer evolves as:

\[
\tau = \rho g \sin \theta \left( 1 - \cot \theta \frac{dh}{dx} \right) (h - y) \tag{3}
\]

Consequently, an unyielded layer, called plug, exists at the top of the flow where \( \tau < \tau_c \). The thickness of the plug layer denoted \( h_p \) is given by:

\[
h_p = \frac{\rho g \sin \theta \left( 1 - \cot \theta \frac{dh}{dx} \right)}{\tau_c} \tag{4}
\]

Then, straightforward integration, considering the Herschel Bulkley constitutive law in simple shear (\( \dot{\gamma} = \frac{\tau}{\eta} \)) leads to the expression of the velocity profile:

\[
u(y) = \begin{cases} 
    u_0 \left( 1 - \left( \frac{1}{y_0} \right)^n \right) & \text{if } y < y_0 \\
    u_0 & \text{if } y \geq y_0 
\end{cases} \tag{5}
\]

with \( u_0 = \frac{n+1}{n} \frac{\rho g \sin \theta}{K} \left( 1 - \cot \theta \frac{dh}{dx} \right)^\frac{1}{n} \), and \( y_0 = h - h_p \) denotes the thickness of the sheared layer below the plug. Note that this expression reduces to the velocity profile in steady uniform regime
[Chambon et al., 2014] by setting $\frac{\partial u}{\partial x}$ to zero. Fig. 5a presents a typical velocity profile computed according to Eq. (5). In our experiments with the conveyor belt, we expect the velocity profile in the surges to correspond to a simple shift, by the value of the belt velocity $u_b$, of this velocity profile obtained down a simple inclined plane (Fig. 5b). Hence, provided that the rheological parameters $\tau_r$, $K$ and $n$ of the fluid are known, and using the values of $u_{\infty}$ measured experimentally, Eq. (5) can be directly compared to the velocity profiles measured by PIV and shifted by $u_b$.

4. RESULTS AND DISCUSSION

4.1 Experimental results in the uniform zone

We briefly recall here the results presented by [Chambon et al., 2014]. In this study, owing to the absence of vertical velocities in the uniform zone, velocity profiles with a high vertical resolution were obtained using a linear correlation algorithm (Fig. 6). These profiles are compared with the theoretical predictions given by Eq. (5) when setting the depth gradient term to zero. An excellent agreement was found between the measured velocity profiles and the theoretical predictions based on corrected rheological parameters (Fig. 6). This result shows that in a steady regime, the macroscopic properties of viscoplastic flow can be accurately predicted by Herschel Bulkley constitutive law.

4.2 Preliminary experimental results in the front zone

The evolution of velocity profiles in the front zone is presented in Fig. 7. In the upper part of the flow and sufficiently far from the front we note the existence of a plug marked by a substantial layer of fluid in which all the tracers are moving at the same velocity. When approaching the front ($\xi < 10$ mm typically), this plug seems to disappear. However, due to the lack of resolution of the images, it is not possible to assess if the plug really disappears or rather becomes very thin. We also note that the maximum velocities of all the profiles presented in Fig. 7 seem to be approximately constant for $\xi > 10$ mm, and to progressively decrease for $\xi \leq 10$ mm. Comparison between the measured and theoretical velocity profiles at 6 different locations in the frontal region of the surge is presented in Fig. 8. The positions of the theoretical and experimentally measured yield surface are also reported. Far from the front ($\xi \geq 68$ mm), measured velocities appear to be in good agreement with velocities predicted by
Eq. (5), accounting for the corrected rheological parameters. In this region we note a maximum disagreement between experimental and theoretical plug velocities of 11%. The plug position also appears to be well predicted. For $\xi < 65$ mm, however, disagreement between experiments and theoretical predictions becomes apparent, both for the velocity values and for the plug position.

4.3 Discussion

The preliminary results presented above show a good agreement between experimental data and predictions based on the lubrication approximation in the front zone, except close to the front line itself. This indicates that, sufficiently far from the front, the lubrication approximation can be used to predict velocity profiles and plug position. However, close to the front line, the lubrication approximation appears insufficient to account for the dynamics of the flow. Unfortunately, due to insufficient image resolution, it was not possible to observe precisely the evolution of the plug in this zone. These experimental limitations also explain why velocity profiles display artifacts in the top and bottom regions of the flow (Fig. 7).

5. CONCLUSION

This paper reports on preliminary results concerning the internal dynamics in the front zone of a viscoplastic flow. We compared velocity profiles measured experimentally in the front zone to velocity profiles predicted by the lubrication approximation. Experiments were conducted with a model viscoplastic fluid (Carbopol ETD 2623).

Results show that lubrication provides a good approximation to predict velocity profiles except in the zone close to the front line. Due to insufficient images resolution, these preliminary results did not allow us to fully monitor the evolution of the velocity profiles in this zone.

Work is currently on-going to improve these results. A new experimental configuration based on thin laser sheet visualization in the central part of the flow will allow us to avoid the possible side effects due to the walls. Higher resolution images and better signal-to-noise ratio will also allow us a painstaking investigation of the internal dynamics of viscoplastic free-surface flows from the uniform zone to the front.

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