The cognitive process of students in understanding the triangles in Geometry of Riemann through local content

W Widada*, D Herawaty, Y Widiarti, Herawati, S Aisyah, R Tuzzahra, K Berlinda
Postgraduate Mathematics Education Program, Universitas Bengkulu. Jl. WR. Supratman, Kandang Limun Bengkulu, Indonesia 38371
E-Mail: w.widada@unib.ac.id

Abstract. Learning geometry is one that students fear. They have difficulty understanding the concepts of geometry. Students experience problems with Lobachevsky's axiom alignment, as well as Riemann's axiom alignment. Therefore, we need learning that makes it easy for students to imagine these axioms more concrete, although geometry cannot always be carried in a concrete world. The purpose of this study is to describe students' cognitive processes in understanding the properties of triangles in Riemann Geometry through local content. This is a part of development research. We conducted a needs analysis of the ability of students to understand new concepts in geometry. The subject of this research was chosen from students of mathematics education at one of the tertiary institutions in Bengkulu City, Indonesia. Subjects were interviewed in-depth through task-based interviews. His job is to solve the problem of the properties of triangles in Riemann Geometry. Interview data were analyzed through genetic decomposition analysis. The results of this study were subjects able to analyze the interrelationship between concepts, axioms, and basic properties of geometry through orange peel media. He can attribute and organize. The attributes are obtained from three segments that meet at the ends. Subjects can organize the elements of the triangle so that the nature of a large number of angles in the triangle is more than 180. The conclusions of this study are the subjects can achieve the cognitive process of analysis, and can arrange the correct population.

1. Introduction
Learning geometry is one that students fear, they have difficulty understanding the concepts of geometry [1,2]. When studying flat fields through Euclid's axiom, his logic was still able to reason well. However, students experience problems with Lobachevsky's axiom alignment, as well as Riemann's axiom alignment. Therefore we need learning that makes it easy for students to imagine these axioms more concrete, although geometry cannot always be carried in a concrete world [3–5].

Students will be able to easily understand the concepts of geometry when brought near concrete objects [6]. Learning to get up and down in Euclid's Geometry is easier than understanding Riemann's Geometry objects. As in the plane, the sum of the angles in a triangle is 180 degrees [7]. However, students experience accommodation when faced with a sum of large angles in a triangle of more than 180 degrees. This makes students disequilibrium. As a result, they experience errors in solving questions about Riemann Geometry.

Students learn geometry to improve spatial ability and geometrical reasoning. In the beginning, they understood geometry through direct interaction with concrete objects. Therefore, learning about the attributes of concrete objects in the environment becomes the starting point for learning geometry. It is
close to their culture [8,9]. Geometry has relevance for every student. The world becomes a large classroom. When students see, touch, and manipulate shapes, they begin to develop spatial reasoning skills [10]. Furthermore, we need learning geometry that makes students learn more meaningfully. It needs media assistance that is close to students’ minds and culture [11,12].

Students need help in learning Geometry [13]. Because in learning geometry, sharing coherent proof of ideas, cognitive development of evidence is dependent on the cognitive structure and representations available in students. Proofing other traits and principles in a logical deduction is a difficult one for students [14]. It requires the ability to understand concepts and principles properly. Therefore, simplification is needed in starting learning, namely through contextual media [15–17]. Learning in the context of local culture becomes a reasonable approach.

Local culture-based learning models are better than conventional learning to improve pedagogical understanding skills [8,18–20]. The local culture-based learning framework helps students overcome the accommodation process of structuralistic learning. Through a local cultural approach is one way that makes it easier for students to start learning mathematics (often called the ethnomathematics approach) [21–24]. Ethnomathematics and mathematical literacy are the two main ideas about understanding mathematics based on our living environment [25]. Ethnomathematics emphasizes different cultural groups in the daily lives of students. Mathematical literacy primarily focuses on the mathematical and societal needs of people's competencies. Therefore teachers should read and understand cultural contexts that are close to student life. That will make it easier for students to reach the mathematical concepts they learn.

2. Method
The focus of this research is to explore and help students understand new concepts in geometry, especially the nature of a triangle. That is the intended Riemann Geometry. This research is part of development research. We conduct a needs analysis of students' ability to understand new concepts in the Riemann Geometry system. The subject of this study was chosen by one of the students of mathematics education at one of the tertiary institutions in Bengkulu City, Indonesia. The subject of this research is Yr. Subjects were interviewed in-depth through task-based interviews. His job is to find one of the triangular properties in Riemann Geometry. We provide media for the discovery of these properties through grapefruit. He was asked to peel his skin and make toy cars that are in the tradition of children. The subject analyzes the shape and properties of the triangles from the base and roof of the cars he made. The interview focused on finding the nature of a large number of angles in the triangle. To get complete and accurate data, interviews, and discovery processes are recorded with an audiovisual recording tool. Interview data were analyzed through genetic decomposition analysis.

3. Results and Discussion
Learning geometry in a system presents its challenges for students. When faced with learning Euclidean Geometry for a flat plane, students still have a fairly good understanding, but when they start studying Non-Euclidean Geometry, they begin to have difficulty understanding it. Therefore, the researchers took the initiative to make Riemann Geometry learning media use something close to students' minds. That is grapefruit. Grapefruit is a large orange, or often called pomelo (in scientific language: Citrus grandis, C. maxima) is the largest orange producing fruit. The name "Pomelo" is now suggested by the Indonesian Ministry of Agriculture because this orange has nothing to do with Bali. These oranges include types that can adapt well to dry areas and are relatively resistant to disease [26]. Grapefruit can be seen in Figure 1.
By utilizing grapefruit skin, children make cars. Toy cars from grapefruit skin are one of the traditional toys that Indonesia has. However, the toy is unknown where it came from. It is a very environmentally friendly toy. The toy was very popular with Indonesian children in the past, but until now it is still a national heritage as environmentally friendly local wisdom. That is very close to the minds of students. Toy cars of orange peel can be seen in Figure 2.

Researchers tracked the cognitive processes of students in understanding triangles through the media of orange peel made through children's toys. These are cars with a triangular base and roof. The results of interviews with research subjects (Yr) and interviewers (Q) are listed in the following excerpt.

Q : What do you think of cars made of grapefruit skin?
Yr : Alright ... the orange peel that forms the roof and base of the toy car is in the shape of a triangle, ... that I have known since I was a child and became my toy.
Q : What do you think of the triangle?
Yr : That is a triangle that is different from what I have known so far ... usually triangles are formed from three straight line segments, but the circumference of the orange peel is formed from three line segments that are curved in shape. You can see it as shown in Figure 2.
Based on the interview excerpt and Figure 2, Yr realized that there was a triangle whose shape was outside of what he had known so far. He knew triangles in flat planes (in Euclidean Geometry). He stated that the triangle is the circumference of a triangular-shaped car roof. Next, Mr. Yr was interviewed with the following footage.

Q : What about the properties that exist in the triangle, especially the large-angle inside?
Yr : You can see the triangle shape as the circumference of the roof of a toy orange peel ... see Figure 3.

Q : Ok ...
Yr : In Figure 3 we can see that the sides of a triangle are segments of curved lines, and to determine the magnitude of the inner angles we have to with a protractor.

Q : OK ... please make five triangles using orange peel and measure the angle ... then make it in the table ...
Yr : Fine ... I will do it ... of course, the size of the inner corner is not the same as the size of the triangle whose sides are straight lines.

![Figure 3. Triangle representation of grapefruit skin by Yr](image_url)

Based on the interview excerpts with Yr and Figure 3, research subjects begin to make a new understanding that triangles are not always formed from a straight line segment. He also stated that the size of the inner angles must be different. This is a learning geometry that will have an impact on students' cognitive processes. Teaching geometry and geometric thinking of students increases the geometrical reasoning and proof of its properties [27]. In designing learning, the view that reasoning is geometrically, even in the context of trying to solve school geometry problems is non-routine, different from school algebra and arithmetic that are commonly taught in schools [28]. In this connection, we continue the interview with Yr as follows.

Q : What are your measurement results for the angle?
Yr : Yes ... it turns out that almost every triangle ... a large number of angles inside is more than 180 degrees.

Q : How can it be like that?
Yr: I will show the results of the measurements I made in the table and also see Figure 4, ... that’s also more than 180 degrees.

Figure 4. Other Triangle representation of grapefruit skin by Yr

Like Figure 4, Yr measures five ABC triangles. The following are the results of measurements made by Yr for five triangles of grapefruit skin. Students perform geometry activities that can improve their spatial ability because grapefruit is a miniature globe. We hope that they will be able to generalize the concepts and principles obtained from oranges into the geometry of the globe or more generally. Therefore, geometry is the most practical mathematical element, and teaching it correctly can result in improved performance in mathematics [29]. The following are the results of angular measurements in the ABC triangle, see Table 1.

Table 1. Results of measurements of angular size in triangles

| No. Triangle | m(ABC) | m(BAC) | m(ACB) | total |
|--------------|--------|--------|--------|-------|
| 1            | 60.2   | 60.1   | 60.0   | 180.3 |
| 2            | 60.5   | 60.2   | 60.1   | 180.8 |
| 3            | 60.7   | 60.0   | 60.3   | 181.0 |
| 4            | 60.1   | 60.4   | 60.5   | 181.0 |
| 5            | 60.3   | 60.3   | 60.2   | 180.8 |
|              | 60.36  | 60.2   | 60.22  | 180.78|

Based on Table 1, the large number of angles in ABC triangles is varied (angle size in degrees). Triangle-1, m(∠ABC) + m(∠ACB) + m(∠BAC) = 180.3; Triangle-2 is (∠ABC) + m(∠ACB) + m(∠BAC) = 180.0; Triangle-3: (∠ABC) + m(∠ACB) + m(∠BAC) = 181.0; Triangle-4: (∠ABC) + m(∠ACB) + m(∠BAC) = 181.0; and the 5th Triangle: (∠ABC) + m(∠ACB) + m(∠BAC) = 180.0. Of the five triangles, the average size is Triangle-3: (∠ABC) + m(∠ACB) + m(∠BAC) = 180.78. It can be concluded that m(∠ABC) + m(∠ACB) + m(∠BAC) ≥ 180°.

The above results are a finding in an experiment conducted by Yr. He stated that there are new properties of triangles in Balinese jet geometry. That is the model for Double-Elliptic Geometry, which is often called Riemann Geometry [7]. This needs to be given learning for every student because we live
on earth. We must understand the characteristics of the earth's surface. According to Rogers [30], that as an important part of students' daily lives, culture is concerned with geometry as a practical measurement and as a rule for dividing and combining shapes of various types. For everyday practical purposes students, people live on Earth that is not always flat.

4. Conclusion
We have used mathematics to solve life problems in various ways. Arithmetic and geometry are used to meet people's daily needs [31]. We use geometry for various practical purposes as well as the development of further geometry and mathematical systems. The conclusion of the research is through the media of grapefruit peel, it can be found empirically the nature of a triangle with a large number of angles of more than 180 degrees. This certainly becomes a trigger for us to bring in a formal and proven nature in the structure of Riemann Geometry in classroom learning. Expect us, the teacher can design learning that makes it easy for students to understand geometrical concepts and principles that have abstract properties through concrete media. Like the local culture revealed in this paper.

5. References
[1] Heinze, Aiso & Reiss K 2012 Mistake-Handling Activities In The Mathematics Classroom: Effects Of An In-Service Teacher Training On Students’ Performance In Geometry Uma ética para quantos?
[2] Adolphus T 2011 Problems of Teaching and Learning of Geometry in Secondary Schools in Rivers State , Nigeria Int. J. Emerg. Sci.
[3] Plomp T and Nieveen N 2013 Educational Design Research Educational Design Research Educ. Des. Res. 1–206
[4] Treffers A 1991 Didactical background of a mathematics programm for primary education L. Streefland (ed.), Realistic Mathematics Education in Primary School, CD-ß Press / Freudenthal Institute. Utrecht University, (Utrecht: Freudenthal Institute)
[5] Widada W, Efendi S, Herawaty D and Nugroho K U Z 2020 The genetic decomposition of students about infinite series through the ethnomathematics of Bengkulu , Indonesia IOP Conf. Ser. J. Phys. Conf. Ser. 1470 012078 doi10.1088/1742-6596/1470/1/012078 1470 1–9
[6] Herawaty D, Widada W, Novita T, Waroka L and Lubis A N M T 2018 Students’ metacognition on mathematical problem solving through ethnomathematics in Rejang Lebong, Indonesia J. Phys. Conf. Ser. 1088
[7] Prenowitz W and Jordan M 1989 Basic Concept of Geometry (Boston: Aedsley Hoyse Publishers. Inc.)
[8] Julita S and Anggoro A F D 2019 The Local Culture-Based Learning Model To Improve Teaching Abilities For Pre-Service Teachers J. Phys. Conf. Ser. 1179 1–6
[9] Widada W, Herawaty D, Yanti D and Izzawati D 2018 The Student Mathematical Communication Ability in Learning Etnomathematics Orieted Realistic Mathematics Int. J. Sci. Res. 7 881–4
[10] Howse T D and Howse M E 2015 Linking the Van Hiele Theory to Instruction Teach. Child. Math. 21 304–13
[11] Herawaty D, Khrisnawati D, Widada W and Mundana P 2020 The cognitive process of students in understanding the parallels axiom through ethnomathematics learning IOP Conf. Ser. J. Phys. Conf. Ser. 1470 012077 doi10.1088/1742-6596/1470/1/012077 1470 1–8
[12] Widada W, Herawaty D, Beka Y, Sari R M and Riyani R 2020 The mathematization process of students to understand the concept of vectors through learning realistic
mathematics and ethnomathematics 

IOP Conf. Ser. J. Phys. Conf. Ser. 1470 012071
doi:10.1088/1742-6596/1470/1/012071 1470 1–10

[13] Sipos E R 2011 Teaching Geometry using Computer Visualization (Bolyai Institute)

[14] Widada W, Herawaty D, Jumri R and Wulandari H 2020 Students of the extended abstract in proving Lobachevsky’s parallel lines theorem IOP Conf. Ser. J. Phys. Conf. Ser. 1470 012098 doi:10.1088/1742-6596/1470/1/012098 1470 1–10

[15] Widada W 2015 Proses Pencapaian Konsep Matematika dengan Memanfaatkan Media Pembelajaran Kontekstual J. Penelit. Pendidik. Mat. dan Sains 22 31–44

[16] Widada W 2004 Pendekatan Pembelajaran Matematika Berbasis Masalah (Surabaya: Unipa Press)

[17] Syafriafdi N, Fauzan A, Arnawa I M, Anwar S and Widada W 2019 The Tools of Mathematics Learning Based on Realistic Mathematics Education Approach in Elementary School to Improve Math Abilities Univers. J. Educ. Res. 7 1532–6

[18] Supahar, Abdulfattah A, Sukardiyono and Putranta H 2020 The Development of Mobile Learning-Assisted Local Culture-Based Subject Specific Pedagogy in Realizing the Learning Outcomes of Physics Subject

[19] Soko I P, Setiawa A and Widodo A 2017 Developing a culture-based teacher education and training program model: Improving teachers’ content and pedagogical knowledge Regionalization and Harmonization in TVET

[20] Irfan M, Slamet Setiana D, Fitria Ningsih E, Kusumaningtyas W and Adi Widodo S 2019 Traditional ceremony ki ageng wonolelo as mathematics learning media J. Phys. Conf. Ser. 1175

[21] Ambrosio U D 2001 What is ethnomathematics , and how can it help children in schools ? Natl. Counc. Teach. Math. Feb

[22] Rosa M and Orey D C 2011 Ethnomathematics: the cultural aspects of mathematics Rev. Latinoam. Enomatemática 4 32–54

[23] François K 2010 The Role of Ethnomathematics Within Mathematics Education Proc. Cerme 1517–26

[24] Andriani D, Widada W, Herawaty D, Ardy H, Nugroho K U Z, Ma’rifah N, Anggreni D and Anggoro A F D 2020 Understanding the number concepts through learning Connected Mathematics (CM): A local cultural approach Univers. J. Educ. Res. 8 1055–61

[25] Wedege T 2010 Ethnomathematics and mathematical literacy: People knowing mathematics in society Math. Math. Educ. Cult. Soc. Dimens. Proc. MADIF 7. Seventh Math. Educ. Res. Semin. Stock. 7 31–46

[26] Wikipedia 2019 Jeruk Bali https://id.wikipedia.org/wiki/Jeruk_bali

[27] Jones K and Tzekaki M 2016 Research on the teaching and learning of geometry The Second Handbook of Research on the Psychology of Mathematics Education: The Journey Continues

[28] Küchemann D and Rodd M 2012 On learning geometry for teaching Math. Teach.

[29] Sunzuma G and Maharaj A 2019 In-service Teachers’ Geometry Content Knowledge: Implications for how Geometry is Taught in Teacher Training Institutions Int. Electron. J. Math. Educ.

[30] Rogers L 2011 Geometry : A History from Practice to Abstraction https://nrich.maths.org/6352

[31] Snipes V and Moses P 2001 Linking Mathematics and Culture to Teach Geometry Concepts LATM J. 1 1–17
