Mixing dynamics and group imbalance lead to degree inequality in face-to-face interaction

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Uncovering how inequality emerges from human interaction is imperative for just societies. Here we show that the way social groups interact in face-to-face situations can enable the emergence of degree inequality. We present a mechanism that integrates group mixing dynamics with individual preferences, which reproduces group degree inequality found in six empirical data sets of face-to-face interactions. We uncover the impact of group-size imbalance on degree inequality, revealing a critical minority group size that changes social gatherings qualitatively. If the minority group is larger than this ‘critical mass’ size, it can be a well-connected, cohesive group; if it is smaller, minority cohesion widens degree inequality. Finally, we expose the under-representation of social groups in degree rankings due to mixing dynamics and propose a way to reduce such biases.

Introduction

Face-to-face interaction is a fundamental human behavior, shaping how people build and maintain social groups by segregating themselves from others (1–4). This segregation can generate or exacerbate intergroup inequality in networks, producing unequal opportunities in different aspects of people’s lives, such as education, employment, and health (5–9), especially when
individuals tend to interact with similar others (10). Though crucial to a just society, however, our understanding of the interplay between group dynamics and the emergence of inequalities in social gatherings remains limited and quantitatively unexplored.

With deep roots in sociology and anthropology (1–3), the study of face-to-face interaction has advanced considerably in recent years, mainly because of new tracking devices providing fine-grained data on human interaction (11–15). This data has enabled researchers to uncover several properties in the way people interact with others (16–21). For example, in a social gathering, individuals tend to interact with an average number of individuals, a quantity that depends on the social occasion (21). Though this number exhibits a trend across individuals in a gathering, the duration of each interaction lacks a central tendency—it might last from a few seconds to a couple of hours (17–19). These two properties have been found universally in many distinct social situations, such as schools and workplaces, indicating the existence of fundamental mechanisms underlying face-to-face interaction.

Simple social mechanisms can explain these properties as a result of local-level decisions based on individuals’ attributes (22–25). One crucial attribute in face-to-face situations is the so-called attractiveness. People with high attractiveness are more likely to stimulate interaction with others. This principle, together with individuals’ social activity, constitutes a mechanism that explains well the properties found in empirical data (23) and has been extended to describe other features in face-to-face interaction, such as people’s tendency to engage in recurrent interaction (22, 25). However, focusing solely on individuals’ attractiveness neglects the crucial role of pairwise interactions and social groups. For example, someone with a high intrinsic attractiveness might feel unwelcome in a community if this individual does not belong to that particular social group or fail to share the same social traits. In this paper, we refer to social groups as a group of people who share similar characteristics.

How social groups interact can define the position of individuals in networks, particularly
when groups have unequal sizes. For instance, the smallest group (i.e., the minority group) in a network can have a systemic disadvantage of being less connected than larger groups, depending on the group mixing (26). Having a lower number of connections poses several disadvantages to individuals, such as low social capital (27), health issues (28), and perception biases (29). Yet, the mechanisms underlying group dynamics and their relation to degree inequality in social gatherings are still unexplored.

Here we show numerically, empirically, and analytically that degree inequality in face-to-face situations can emerge from group imbalance and group mixing dynamics. We present a mechanism—the attractiveness–mixing model—that integrates mixing dynamics (i.e., pairwise preferences) with individual preferences, expanding the established attractiveness paradigm. While attractiveness is an intrinsic quality of the individual, mixing dynamics manifest between pairs of individuals; together, they form what we call social attractiveness. The mechanism reproduces the intergroup degree inequality found in six distinct data sets of face-to-face gatherings. With the analytical derivation of our model, we further demonstrate the impact of group imbalance on degree inequality, finding a critical minority group size that changes the system qualitatively. When the minority group is smaller than this critical value, higher cohesion among its members leads to higher inequality. Finally, we expose the underrepresentation of minorities in degree rankings and propose a straightforward method to reduce bias in rankings. With this work, we show how and when group imbalance leads to inequality in social gatherings, building new opportunities to understand and alleviate inequality in society.

Results

We study the dynamics of social gatherings using six different data sets, four from schools (11, 19, 30) and two from academic conferences (14). Each data set consists of the individuals’ interactions captured via close-range proximity directional sensors that individuals wore during
each gathering. With these data sets, we construct the social network of each gathering, where a node is an individual and an edge exists if two individuals have been in contact (i.e., face-to-face interaction) at least once during the study (Table 1). In these networks, the degree distributions follow a semi-normal distribution and exhibit a single peak at the center (see section 1.1 in Supplementary Material). The data further contain gender information on individuals, which allows us to define two groups in each network. Thus, the notion of group refers to individuals who share a similar characteristic with regards to gender.

In all considered data sets, there were fewer female participants than male participants. Throughout this article, we refer to the smallest group as the minority group, and we denote 0 as the label for the minority group and 1 for the majority.

Table 1. The social networks from six different studies. The number of nodes \( N \), the minority fraction \( f_0 \), the number of edges \( E \), and the average degree of the minority \( \langle k_0 \rangle \), majority \( \langle k_1 \rangle \), and overall \( \langle k \rangle \).

| Data set    | \( N \) | \( f_0 \) | \( E \)   | \( \langle k_0 \rangle \) | \( \langle k_1 \rangle \) | \( \langle k \rangle \) |
|------------|--------|---------|---------|----------------|----------------|----------------|
| School 1   | 242    | 0.46    | 8,317   | 64.85          | 72.08          | 68.74          |
| School 2   | 126    | 0.33    | 1,710   | 24.71          | 28.32          | 27.14          |
| School 3   | 180    | 0.27    | 2,220   | 23.46          | 25.11          | 24.67          |
| School 4   | 327    | 0.44    | 5,818   | 36.48          | 34.87          | 35.58          |
| Conference 1 | 115   | 0.43    | 5,508   | 95.35          | 96.12          | 95.79          |
| Conference 2 | 202  | 0.30    | 11,412  | 118.61         | 110.56         | 112.99         |

Degree inequality and mixing in face-to-face interaction

To characterize the connectivity of the groups in the networks, we measure the average degree of individuals in each group, finding a systematic degree inequality among groups (Fig. 1A). The minority group exhibits a lower average degree than the majority group in School 1, School 2, and School 3, whereas the opposite occurs in Conference 2 and School 4, and both groups have the same average degree in Conference 1. Though this degree inequality arises in face-to-face situations, an intrinsic-attractiveness model of face-to-face interaction (23) fails to explain the
group differences (dashed line in Fig. A), since it neglects group mixing in social gatherings.

To understand how groups mix in the networks, we examine the inter- and intragroup ties by comparing them with the configuration model (see Methods). We find that individuals were more likely to interact with individuals from the same group, indicating that homophily (31) plays a significant role in face-to-face situations. In most of the networks, our results show that intragroup ties are more frequent than what one would expect by chance (Fig. B).

Though fundamental in shaping networks, such group mixing dynamics are unable to emerge in the intrinsic-attractiveness paradigm because it lacks relational attributes. In this paradigm,
systematic variations in individuals’ attributes can lead to group differences, but they fail to form mixing patterns as observed in data. For example, consider a social gathering in which the members of the minority group have lower intrinsic attractiveness than the majority group. In this scenario, according to the intrinsic-attractiveness paradigm, the minority group would tend to interact with the majority group, boosting the majority group average degree. This setting would explain group degree inequality. However, in this same setting, the members of the minority group would be less prone to interact with their own group—the opposite of what occurs in the data, where intragroup mixing is significant (Fig. 1B; see also section 1.2 in Supplementary Material). In other words, variation in individuals’ attractiveness is not enough to explain the degree inequality observed in reality. To uncover the underlying mechanism of these social dynamics, we need to disentangle the individuals’ intrinsic attractiveness and the relational attributes in face-to-face interaction.

**Modeling mixing in social interaction**

We present the *attractiveness–mixing* model that incorporates (i) intrinsic attractiveness of individuals and (ii) relational attributes between groups. We show that these ingredients are sufficient to explain the degree inequality observed in social dynamics with minority groups. In the model, each individual has an intrinsic attractiveness and belongs to a group. The members of a group share the same mixing tendency, which regulates the pairwise interactions. In general, individuals move across space and form social ties depending on their group membership and the composition of their surroundings (see Fig. 2A).

In the attractiveness–mixing model, an individual $i$ has three attributes: a group label $b_i \in [0, B - 1]$, where $B$ is the number of groups; an intrinsic attractiveness $\eta_i \in [0, 1]$; and an activation probability $r_i \in [0, 1]$. The mixing patterns in this system are encoded in the $B \times B$ mixing matrix $H$. Each row of $H$ can be seen as a probability mass function that weighs the
likelihood of group interaction. In this model, \( N \) individuals perform random walks in a two-dimensional \( L \times L \) periodic space and move based on the composition of their vicinity. We define \( \mathcal{N}_i(t) \) as the set of individuals who are within radius \( d \) of individual \( i \) at time \( t \). We denote \( n_b \) as the size of a group \( b \) and \( f_b = n_b/N \) as the group fraction. The individuals move only probabilistically. At each time step \( t \), each individual \( i \) moves with probability 

\[
\alpha_i(t) = 1 - \max_{j \in \mathcal{N}_i(t)} \{\eta_j\}
\]  

(1)

and a step of constant length \( v \) along a random direction of angle \( \xi \in (0, 2\pi] \). With the comple-

![Fig. 2. The attractiveness–mixing model: the micro- and macro-level behavior.](image)

(A) A schematic description of the micro-level interaction among individuals in the attractiveness–mixing model. Each individual moves across space and forms social ties based on their group membership and the composition of their vicinity. In the figure, nodes represent individuals, and the labels are their intrinsic attractiveness; nodes’ color represents group membership, where gray indicates inactive nodes. In this example, the probability of same-group mixing is \( h = 0.8 \). At this time step \( t \), individual \( k \) moves with probability \( \alpha_k(t) = 0.6 \) because of their vicinity; with the complementary probability, this individual does not move and interacts with their neighbors with probability \( \beta_k(t) = 0.8 \). Similarly, individual \( j \) moves with probability \( \alpha_j(t) = 0.2 \); otherwise this individual stays and interacts with probability \( \beta_j(t) = 0.2 \). (B) Two examples of the model’s macro-level behavior. When the probability of same-group mixing is low (\( h = 0.2 \)), the minority group tends to connect to the majority group, and vice versa. In this scenario, group imbalance leads minority members to interact with more individuals (i.e., higher degree) than the majority members. High same-group mixing probability (\( h = 0.8 \)) leads minority members to attract individuals from the minority group. In this case, majority members have a higher degree centrality than the minority group.
mentary probability, individual $i$ does not move and has the chance to interact with individuals in the vicinity depending on the group mixing likelihood. Precisely, individual $i$ interacts with her neighbors of highest mixing likelihood with probability

$$\beta_i(t) = \max_{j \in \mathcal{N}_i(t)} \{ h_{b_i b_j} \},$$

(2)

where $h_{b_i b_j}$ is an element of $H$ and denotes the mixing probability between $b_i$ and $b_j$ (see section 3 in Supplementary Material for pseudocode). Overall, an individual interacts with other individuals depending on their intrinsic attractiveness and social group mixing; together these two ingredients form the social attractiveness. Finally, individuals can be active or inactive; they only move and interact with others if they are active. An inactive individual $i$ becomes active with probability $r_i$, whereas an active but isolated individual $i$ becomes inactive with probability $1 - r_i$. In this study, we assume that the intrinsic attractiveness $\eta_i$ and the activation probability $r_i$ come from a continuous uniform distribution in $[0, 1]$.

The mixing matrix $H$ and the group sizes have a significant impact on the model dynamics, affecting individuals’ connectivity, especially when groups have different sizes. For example, in a system having two groups, a minority group with proportional size $f_0 = 0.2$ and a majority group with $f_1 = 0.8$, the mixing dynamics lead the system to be in different regimes that influence average group degree (Fig. 2B). When intragroup interaction is less likely than intergroup interaction (i.e., $h_{00} < 0.5$, $h_{11} < 0.5$), the system is in a heterophilic regime, and the minority group has a higher degree than the majority. This degree disparity arises because of the majority group favoring interaction with a small number of people (i.e., the minority group), thereby reducing the majority group connectivity. The opposite occurs in a homophilic regime, where intragroup interaction is more likely than intergroup interaction (i.e., $h_{00} > 0.5$, $h_{11} > 0.5$), which results in the minority having a low average degree.

To investigate group degree inequality, we derive the model analytically to uncover the
impact of mixing dynamics and group sizes on inter- and intragroup edges. Without loss of
generality, we focus on the case of two groups, \( B = 2 \), finding the closed-form expressions for
the normalized group edge matrix, \( e_{rs} = E_{rs}/E \), and the mixing matrix \( H \) (see Methods for
details). The normalized intragroup edges are given by

\[
e_{00} = \frac{f_0^2 (1 - h_{01}^2)}{f_0^2 (1 - h_{01}^2) + 2f_0f_1(1 - h_{00}h_{11}) + f_1^2(1 - h_{10}^2)}
\]

(3)

and

\[
e_{11} = \frac{f_1^2 (1 - h_{10}^2)}{f_0^2 (1 - h_{01}^2) + 2f_0f_1(1 - h_{00}h_{11}) + f_1^2(1 - h_{10}^2)}
\]

(4)

and similar expressions exist for the intergroup edges (see Methods). With these expressions,
not only can one study the model dynamics, but one can also estimate the mixing matrix from
empirical networks and assess the model’s ability to explain data.

**Estimating the mixing matrix from data**

To compare the attractiveness–mixing model with the empirical data, we simulate the model
with the parameters as estimated from the data. Our results show that the model reproduces the
average degree inequality observed in the networks (Fig. 1A) with statistically validated Kol-
mogorov–Smirnov test (table 3 in Supplementary Material). In addition, this model reproduces
other properties found in face-to-face gatherings such as the distributions of interaction dura-
tion, inter-interaction time and weight distribution (figure 8 in Supplementary Material). We use
Eq. (3) and Eq. (4) to estimate the mixing matrix \( H \) from the data (Fig. 1C). This matrix tells
us the tendency of groups to connect among themselves in each face-to-face occasion. With the
estimated matrices, we simulate the model using the same number of nodes and group sizes in
each data set. We find that mixing dynamics and group imbalance enable the emergence of the
group disparities observed in the data.

We highlight that the degree inequality can favor the minority or majority group. For exam-
pbe, in the School 4 and Conference 2 data sets, the smallest group tends to have more connec-
tions than the largest group. The model exhibits the same tendency. From a model perspective, this phenomenon can occur because of the asymmetry in the group mixing. To understand such cases better, we delve into the model and its regimes.

Mixing dynamics, asymmetry, and minority size

To uncover the regimes and scenarios of group inequality in the model, we characterize the impact of mixing and group sizes on average group degree. First, we study the trivial symmetrical mixing when $h_{00} = h_{11}$, and then we examine the impact of mixing asymmetry (i.e., $h_{00} \neq h_{11}$) on the social dynamics. Note that in asymmetrical case, the intragroup mixing are expressed by $h_{00}$ and $h_{11}$ whereas the intergroup mixing are the complementary probabilities, $h_{01} = 1 - h_{00}$ and $h_{10} = 1 - h_{11}$.

In the symmetrical case, we find that homophily and heterophily manifest themselves as two distinct regimes determining the position of the minority group in the network. We analyze the average degree of the groups given different values of group mixing $h_{rr}$ and minority fraction $f_0$. First, we simulate the model and measure the average degree of the minority $\langle k_0 \rangle$ and majority $\langle k_1 \rangle$ groups (see Methods). Then, we separately compare $\langle k_0 \rangle$ and $\langle k_1 \rangle$ to the average degree $\langle k \rangle$ of the whole network using their $z$ scores (Fig. 3A). Our results show that the members of the minority group have an advantage or disadvantage depending on the model parameters.

Homophily ($h_{rr} > 0.5$) leads the minority group to be decoupled from a substantial part of the network. In this case, minority group members have a lower average degree compared to the average. In the heterophilic regime ($h_{rr} < 0.5$), the minority group has high visibility, which leads its members to have a higher degree than average. The existence of these two contrasting regimes implies that the very interpretation of a minority group depends on the extent of group mixing in the network. When studying a minority group, one has to account for inter- and intragroup dynamics to understand its position in a network.
Fig. 3. The minority (dis)advantage in networks. (A) The groups’ average degree (z score) with different minority fraction \(f_0\) and at varying levels of symmetrical mixing (i.e., \(h_{00} = h_{11} = h_{rr}\)). The minority members have a degree advantage or disadvantage if the system is, respectively, a heterophilic \((h_{rr} < 0.5)\) or homophilic \((h_{rr} > 0.5)\) regime. (B) The distance of the minority group degree to the overall average degree, denoted \(k_0 - \langle k \rangle\), at different levels of asymmetrical mixing (i.e., \(h_{00} \neq h_{11}\)). The majority mixing \(h_{11}\) explains much of the variance of \(k_0 - \langle k \rangle\). (C) The variation of \(k_0 - \langle k \rangle\) with changes in the minority mixing \(h_{00}\), with fixed \(h_{11} = 0.5\) and different minority fraction \(f_0\). The minority mixing can have opposite impacts on degree inequality depending on the minority fraction, which suggests a qualitative transition in the system. (D) The derivative of \(k_0 - \langle k \rangle\) as a function of \(f_0\). The zero of this function represents the critical minority fraction, denoted by \(f_0^*\), at which the qualitative transition occurs. (E) Two regimes delineated by \(f_0^*\), with fixed \(h_{00} = 0.5\). These regimes mean that the minority group degree may either increase or decrease with a raise of \(h_{00}\). (F) The parameter space of critical minority fraction. Given \(h_{00}\), the upper limit of \(f_0^*\), denoted as \(\overline{f_0}^*\), represents the smallest minority size allowing higher minority homophily without decreasing group average degree, regardless of the majority mixing.

To characterize the impact of asymmetrical mixing on degree inequality, we examine the whole parameter space of the mixing matrix \(H\). We find that the majority mixing \(h_{11}\) substantially contributes to degree inequality. First, we estimate the average minority degree for different values of \(h_{00}\) and \(h_{11}\), given specific minority fraction values \(f_0\). Then, we measure the distance of the minority group degree to the overall average degree using z scores, denoted \(k_0 - \langle k \rangle\) (see Fig. 3B). Our results show that the majority mixing \(h_{11}\) explains much of the vari-
ance of $k_0 - \langle k \rangle$. While adjusting $h_{11}$ can change the position of the minorities from advantage to disadvantage, modifying $h_{00}$ can attenuate this inequality only slightly.

**Strategies for minority groups to alleviate degree inequality**

To uncover the ways the minority group can attenuate inequality, we investigate how $k_0 - \langle k \rangle$ varies with changes in the minority mixing $h_{00}$. We find that the size of the minority group modifies the system *qualitatively*, revealing that changes in the minority mixing can have opposite impacts on degree inequality. For instance, given a constant value of $h_{11} = 0.5$, increasing $h_{00}$ can reduce or accentuate degree inequality, depending on the minority fraction $f_0$ (see Fig. 3C). To characterize this transition, we examine the derivative of $k_0 - \langle k \rangle$ with respect to $h_{00}$ as a function of $f_0$ (Fig. 4D). Precisely, we are interested in the zero of this function, which tells us the critical minority fraction $f_0^*$ at which the qualitative transition occurs. We find that this transition depends on the mixing dynamics of the system; its analytical form is given in section 3.2.4 in the Supplementary Material. Because the exact analytical form is too intricate, we approximate it by assuming that the values of $h_{00}$ and $h_{11}$ are not at their extremes, finding that:

$$f_0^* = \frac{h_{11}}{2(h_{01} + h_{11})}.$$  

(5)

This equation demonstrates the control of the majority mixing over the minority group. The critical minority fraction $f_0^*$ delineates two regimes where $k_0$ may either increase or decrease with a raise of $h_{00}$, given a fixed $h_{11}$ (see Fig. 4E).

These regimes translate into two strategies to increase the minority average degree, which depends on the minority group size. First, when the minority group corresponds to more than $f_0^*$ of the network, increasing $h_{00}$ leads to an increase in the average minority degree. In this scenario, a more cohesive minority group is beneficial to its average degree. Second, in the case of a smaller minority group, increasing $h_{00}$ decreases its degree; a more cohesive minority
comes with the cost of a less connected minority. In this case, decreasing $h_{00}$ helps in increasing the average degree of the minority group.

To find when increasing minority homophily is always beneficial to the minority group, we characterize the upper limit of $f_0^*$. This upper limit, denoted as $\overline{f_0^*}$, represents the smallest minority size in which the minority group can increase its homophily without detriment to its average degree, regardless of the majority mixing. We note that $\overline{f_0^*}$ is equivalent to the critical size $f_0^*$ when $h_{11} = 1$ (see Fig. 4E), so its exact analytical form is:

$$\overline{f_0^*} = \frac{1}{2 - h_{00} + \sqrt{3 + 2(h_{00} - 2)h_{00}}}.$$  

For example, in the case of $h_{00} = 0.5$, when the minority represents more than 36.7% of the environment, the group increases its average degree by being more homophilic, regardless of how the majority group mixes.

**Emergence of ranking misrepresentation**

From the model’s viewpoint, mixing dynamics can inflate individuals’ degrees because these dynamics tune individuals’ intrinsic attributes. We show that group mixing can also lead social groups to be misrepresented in degree ranking. For instance, this misrepresentation is evident when we compare individuals’ degrees with their intrinsic attractiveness in heterophilic, neutral, and homophilic symmetrical regimes (see Fig. 4). Systematically, degree centrality disguises intrinsic attractiveness in nonneutral regimes. In a heterophilic scenario, minority members with low intrinsic attractiveness tend to have a higher degree than members of the majority group. In a homophilic situation, highly attractive minority members tend to have a lower degree than their majority counterparts. Because intrinsic attractiveness is hidden behind degree centrality, group members can be misrepresented in degree rankings.

To characterize minority representation in rankings, we analyze individuals’ rankings based
Fig. 4. Same intrinsic attractiveness but different degree centrality. The intrinsic attractiveness of individuals versus their degree in different symmetrical regimes. In the neutral case \( h_{rr} = 0.5 \), individuals with higher intrinsic attractiveness have a higher degree regardless of their group membership. In nonneutral regimes, however, degree centrality disguises intrinsic attractiveness. In a heterophilic regime \( h_{rr} = 0.2 \), minority members have a higher degree compared to their majority counterparts with the same intrinsic attractiveness. In a homophilic regime \( h_{rr} = 0.8 \), highly attractive minority members do not have similar degree centrality to that of their majority counterparts.

on degree and intrinsic attractiveness. We rank individuals separately by degree and intrinsic attractiveness, and then we measure the minority percentage in each ranking as we increase the rank length (i.e., the top-\(k\) rank; see Fig. 5A for a homophilic network). In the case of the intrinsic attractiveness, we expect that the number of minority members in the top-\(k\) rank is proportional to the minority size, since intrinsic attractiveness is uniformly distributed. This ranking displays this exact behavior (Fig. 5A). In the case of the degree ranking, however, the minority group has substantially lower chance of appearing in top ranks despite what we would expect from their attractiveness. These results indicate that degree ranking incorporates the mixing dynamics of the system (see section 3.2.6 in Supplementary Material for a heterophilic case).

This effect begs the question as to how to adjust the degree ranking to accurately represent our expectation. One reason for adjusting the rank would be related in situations in which ranking algorithms are used to rank and recommend people. In this case, despite their high intrinsic attractiveness, the minority’s visibility will be disproportionately penalized in top ranks.
Adjusting ranks

To adjust the degree ranking we need to account for the group sizes and the mixing. Given that the mixing dynamics affect group’s degree, we ensure to only compare degrees of individuals from the same group excluding the group mixing. First, we calculate the $z$ score degree of each individual with respect to their average group degree; then, we rank all individuals based on their $z$ score. Our results show that the adjusted degree ranking increases the representation of the minority group members as we would expect (Fig. 5A). The adjusted ranking also

![Fig. 5. Adjusting for group mixing decreases misrepresentation in degree rankings.](image)

(A) The percentage of minority members in rankings of different lengths (i.e., the top-$k$ rank). Each curve is a different ranking of a system with minority fraction $f_0 = 0.3$ and at a symmetrical homophilic mixing $h_{rr} = 0.8$. The minority members are underrepresented in this regime; they are overrepresented in the heterophilic regime (see section 3.2.6 in Supplementary Material). The adjusted ranking accounts for individuals’ group membership, decreasing the misrepresentation of group members. (B) The adjusted ranking versus the degree ranking in different regimes. As expected, heterophily promotes minority members to higher positions in the degree ranking, whereas homophily pushes the minority down. (C) The Spearman correlation between the intrinsic attractiveness and the adjusted and degree ranking. The adjusted ranking tends to agree with the intrinsic attractiveness except in the extreme cases of $h_{rr} = 0$ and $h_{rr} = 1$. 

15
helps expose misrepresentation in degree rankings in different regimes. We compare individuals’ positions in each ranking (Fig. 5B). As expected, heterophily promotes minority members to higher positions in the degree ranking, whereas homophily pushes the minority down. To characterize this misrepresentation, we analyze the correlation between the intrinsic attractiveness and the adjusted and degree rankings. We measure the Spearman correlation between the rankings and intrinsic attractiveness of nodes in different regimes (Fig. 5C). We find that the adjusted ranking tends to agree with attractiveness except in the extreme cases of $h_{rr} = 0$ and $h_{rr} = 1$. With the adjusted degree ranking, we decrease the misrepresentation bias in rankings.

**Discussion**

Face-to-face interaction is arguably a primary mechanism for the transmission and affirmation of culture (3). When people interact with others, they construct a social world and participate in shaping the identity of social groups. In this work, we show that systemic degree inequality emerges in social gatherings from the way group members interact in imbalanced scenarios. Previous research has overlooked this inequality, suggesting that mainly the individuals’ intrinsic attractiveness governs their connectivity. Our results indicate, however, that group dynamics modulate individuals’ intrinsic attractiveness, forming what we have called social attractiveness in face-to-face situations.

In social gatherings, social attractiveness entangles with space and time variables, which restricts opportunities for face-to-face interaction, leading to systemic degree inequality. To interact, individuals must have opportunity availability (i.e., available place and time) and space–time convergence (i.e., individuals must agree on where and when to interact). In confined situations, such as conferences and workplaces, spatiotemporal constraints are critical to interaction opportunities. For example, there exists a limited number of opportunities for interaction at conferences (e.g., coffee breaks). When an individual uses an opportunity to interact with
someone, fewer opportunities remain for interacting with other individuals. In imbalanced sce-
narios, when a majority member interacts with someone from the majority, fewer opportunities
remain for this individual to interact with minorities, thereby decreasing minority connectivity.
Such a decrease means that in the majority group position, creating homophilic ties comes at
the cost of promoting inequality in group connectivity.

From a group-level perspective, mitigating this inequality depends primarily on the mixing
of the majority group and its size—the minority group can only slightly reduce inequality,
exhibiting a qualitative transition in the strategy for this reduction. Our results show that the
majority group mixing explains most of the variance in connectivity inequality. To attenuate
inequality, the minority group needs to follow a strategy that depends on its size. When the
minority size is below a threshold in the network, homophilic minority interaction decreases
minority connectivity. However, if the minority group size is sufficiently large, homophilic mi-
nority interaction helps in increasing minority connectivity. In this case, the minority group has
proportionally enough individuals to interact with and decrease the disparity between groups.
This critical mass allows the minority group to be a strong, tightly connected group; without
this critical mass, a stronger minority implies higher inequality. This result is somewhat related
to the critical mass for social change, as recently shown (32), in which committed groups with
size higher than 25% are sufficient to change social conventions.

To summarize, we have investigated numerically, empirically, and analytically the emer-
gence of group inequality in social gatherings. We have focused on a binary attribute, where we
show that group mixing and intrinsic attractiveness together (i.e., social attractiveness) lead to
group degree inequality. Further research will help clarify the dynamical and temporal aspects
of inequality, as well as multiple and continuous-valued attributes. It would also be interesting
to empirically estimate the model parameters such as the activation probability and intrinsic
attractiveness.
In this article, we understand the concept of minority quantitatively: A minority group is the smallest group in a social gathering. Nevertheless, in the social sciences, the concept is often associated with the critique of inequality, deprivation, subordination, marginalization, and limited access to power and resources (33). Performing a quantitative analysis of the structural mechanisms of mixing, exclusion, and interaction by no means implies ignorance about these issues. With this work, we contribute with insights on how inequality can emerge from social interaction, building new opportunities to understand and alleviate inequality and biases in our society.

**Methods**

**Group mixing analysis**

To characterize how groups mix, we compare the inter- and intragroup edges in data with the configuration model. This approach enables us to assess whether the mixing patterns found in data would occur just by chance. With the configuration model, we generate random networks that preserve the degree of each node in a given network and reshuffle the links, which we can use to compare against the actual data (34). For each data set, first, we generate 500 random instances of the network and count the number of edges $E'_{rs}$ between groups $r$ and $s$ in each instance; then, we count the actual number of edges $E_{rs}$ in the data; finally, we compare $E_{rs}$ to $E'_{rs}$ via $z$ scores, defined as $z_{sr} = (E_{sr} - \overline{E'_{sr}})/s[E'_{sr}]$, where $\overline{E'_{sr}}$ and $s[E'_{sr}]$ are, respectively, the mean and standard deviation of $E'_{sr}$ over the 500 instances. The $z$ score $z_{sr}$ reveals the number of standard deviations by which $E_{rs}$ differs from the random case.

**Model simulation analysis**

To analyze the symmetrical case of the attractiveness–mixing model, we first simulate the model with different values of $h$ and $f_0$ and then measure the average degree of the whole network $\langle k \rangle$.
and the average group degree of the minority, denoted as $\langle k_0 \rangle$, and majority, $\langle k_1 \rangle$. We compare each group $r$ with the whole network using $z$ scores, defined as $(\langle k_r \rangle - \langle k \rangle)/s[k]$, where $s[k]$ is the standard deviation of $k$. We simulate the model with the following parameters: $L = 100$, $d = 1$, $v = 1$, and $N = 200$, and we average the results over 50 different simulations (Fig. 3).

**Analytic derivation of the model**

Here we derive the attractiveness–mixing model analytically for the case of two groups, $B = 2$, denoted as group 0 and group 1. We show that we can calculate the normalized group edge matrix analytically. To simplify the derivation, we assume a situation of low spatial density of agents; thus, almost all situations of potential interaction involve only two individuals, and non-pairwise interactions are negligible (i.e., dilute system hypothesis). Due to the activation process, the average number of active nodes at one time $t$ is $N_a = \langle r_i \rangle N$, since we do not expect a correlation between $r_i$ and the presence of isolated individuals. Therefore, the number of active nodes in each group is $N_0 = f_0 N_a$ and $N_1 = f_1 N_a$. We note that the attractiveness–mixing model generates a temporal network in which nodes are the individuals and interactions between them are edges that are created and destroyed as time passes. Let $E$ be the number of edges created during a time step $\Delta t$. Since the interaction mechanism is time-independent, the total number of edges in the network after a time $T$ is given by $E_T = E \times T/\Delta t$.

To express the number of new edges created at time $t$ in each component, first, we focus on edges between two individuals from group 0. The probability $p_0$ to find an individual $j$ from group 0 in the vicinity of individual $i$ relates to the surface of the vicinity and the density $\rho_0$ of individuals from group 0 on the field, expressed by:

$$p_0 = \rho_0 S = \frac{N_0}{L} \pi d^2 = \frac{f_0 N_a}{L} \pi d^2 = f_0 \rho_a \pi d^2,$$

(7)

where $\rho_a$ is the density of active individuals. To have an interaction, both individuals need to be available and not move away. In our case, the probability for one individual to be available
is the attractiveness of the other individual; thus, the probability of having both is the product $\eta_i \eta_j$, which average value is a constant $\gamma = \langle \eta_i \eta_j \rangle$. We note that three situations can lead to the creation of an edge between an individual $i$ and an individual $j$: (1) only individual $i$ initiates the creation, (2) only individual $j$ initiates the creation, or (3) both individuals initiates the creation.

Therefore, the probability for the edge to appear is thus:

$$p_{ij} = p_{i \rightarrow j} + p_{j \rightarrow i} + p_{i \leftrightarrow j}. \quad (8)$$

In the case of two individuals from the group 0, this probability is on average:

$$p_{ij,00} = f_0 \rho_a \pi d^2 \times \gamma \times [h_{00}(1 - h_{00}) + (1 - h_{00})h_{00} + h_{00}^2] = f_0 \rho_a \pi d^2 \gamma (1 - h_{01}^2). \quad (9)$$

Thus, the number of intra-group edges created during one time step for group 0 is:

$$E_{00} = \frac{N_a^2 \pi d^2}{2L} \gamma f_0^2 (1 - h_{01}^2), \quad (10)$$

where the $1/2$ factor takes into account double counting. With a similar approach, we find that

$$E_{11} = \frac{N_a^2 \pi d^2}{2L} \gamma f_1^2 (1 - h_{10}^2), \quad E_{10} = \frac{N_a^2 \pi d^2}{L} \gamma f_0 f_1 (1 - h_{00} h_{11}), \quad (11)$$

and

$$E_{01} = \frac{N_a^2 \pi d^2}{L} \gamma f_0 f_1 (1 - h_{00} h_{11}). \quad (12)$$

Finally, since $E = E_{00} + E_{01} + E_{10} + E_{11}$ by definition, the share of intra-group edges for group 0 is given by:

$$e_{00} = \frac{E_{00}}{E} = \frac{f_0^2 (1 - h_{01}^2)}{f_0^2 (1 - h_{01}^2) + 2 f_0 f_1 (1 - h_{00} h_{11}) + f_1^2 (1 - h_{10}^2)}. \quad (13)$$

By following analogous procedure, we can find $e_{11}$ as written in Eq. (4). In addition, $e_{01}$ and $e_{10}$ can be written as

$$e_{10} = e_{01} = \frac{f_0 f_1 (1 - h_{00} h_{11})}{f_0^2 (1 - h_{01}^2) + 2 f_0 f_1 (1 - h_{00} h_{11}) + f_1^2 (1 - h_{10}^2)}. \quad (14)$$
We verify that these equations predict the model behavior well by comparing them with simulations (see figure 5 and figure 6 in Supplementary Material).

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