Spectrum sharing with secure transmission

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Abstract

In this paper, we propose a cooperative scheme between primary and secondary networks. In this scheme, a secondary user (SU) accepts a lease of a part of the licensed band according to a cooperative request of a primary user (PU). Hence, the SU helps the PU in transmitting primary signals and has an opportunity to access the frequency spectrum. On the other hand, the SU owns important data and may mistrust the intention of a primary transmitter. Physical layer security is applied to evaluate the cooperative efficiency in terms of the data transmission and data security. The numerical and simulation results show that the system performance of the primary network in the cooperative scheme is improved in terms of the outage probability and the symbol error rate and is compared with that in the direct transmission scheme. In addition, the secure performance of the secondary network, as expressed by the outage secrecy probability and the average secrecy capacity, is maintained under the eavesdropping of the primary transmitter.

Keywords: Spectrum sharing; Cooperative communication; Physical layer security; Decode-and-forward; Outage probability; Symbol error rate; Outage secrecy probability; Average secrecy capacity

1 Introduction

Cognitive radio (CR) is a concept that enhances the utilization efficiency of the licensed frequency spectrum in that wireless devices are continuously developed for modern applications, but the spectrum bands are limited [1]. In cognitive radio networks, unlicensed users, which are known as secondary users (SUs), only access the licensed bands without causing interference to licensed users, which are known as primary users (PUs). To do this, SUs always opportunistically sense the presence or absence of the licensed bands to make a transmission decision. When the quality of service (QoS) of the primary network is very high, the transmission of the secondary network depends on the cooperative request of the PU. In this scheme, the PU leases a part of the licensed band to the SU to increase the QoS of the primary network as well as the spectrum access capacity of the secondary network [2-5]. At this time, a SU has the right to use the licensed band, but the SU does not know the artifice of PUs which PUs want to eavesdrop on the information of the SU.

Due to the broadcast characteristic of the wireless environment, the secrecy of data transmissions is very important. Physical layer security is proposed to solve the security problem in which the secrecy principle is based on the exploitation of the physical characteristics of the wireless channel [6-24]. In [6], Wyner showed that the important data is secure if the quality of the main channel is larger than that of the wiretap channel. Physical layer security has been investigated in the Gaussian channel [7], the broadcast channel [8], and the wireless fading channels [9] and has been extended to the relay-eavesdropper channel [10], the parallel relay-eavesdropper channel [11], and the multi-access channel [12,13,15]. The achievable secrecy rate (ASR) is the metric used to measure the amount of secure data that is successfully received at the intended user [17]. In [17,19], global channel state information (CSI) is available in which Dong et al. regarded a practical assumption that the eavesdropper nodes are active and their CSIs are monitored by the center network [17]. Kim et al. assumed that the source and relay nodes can detect the CSI based on frequent response messages of the eavesdropper nodes. These messages will be used to inform the source node that the eavesdropper nodes want to be served in the next phases [20]. Exploitation of relays has been proposed to overcome the fading environment as well as to enhance secrecy capacity [10,11,17-24]. Optimal relay selection methods [18,23] and weight optimization methods of the cooperative relays [17,19] have been studied to maximize the ASR and satisfy a total transmit power.
constraint. In [21-24], a utilization of jamming signals has been investigated in which friendly wireless nodes or relays operate as jammers to create artificial interference to reduce the quality of the eavesdropping channel. The authors assumed that the jamming signals are known at both the destination nodes and the relays; hence, the quality of the desired channel cannot be affected by the interference signals. A special case in which the cooperative relay does not behave as a trusted node is proposed [25].

In [4,5], the SUs have opportunity to wiretap the primary data of the PUs, and then these SUs are mistrusted as eavesdropper nodes. In [16-18,20], the eavesdropper nodes are assumed as special wireless nodes, whereas the primary data are wiretapped by nodes as relays in [19,25]. Motivated by the above issues, in this paper, we propose a cooperative scheme between the primary and secondary networks in which the PU leases a part of the licensed band to the SU to improve the performance of its system. In this proposed scheme, the SU uses the licensed band to perform two jobs as follows: First, the SU helps the PU by decoding and using the amount of power needed for forwarding the primary signals. Second, the SU uses the remaining power to transmit the important data against the eavesdropping of the primary transmitter. The cooperative efficiency of the primary network is obtained in terms of the outage probability and symbol error rate (SER) and is compared with the direct transmission scheme. For the secondary network, the outage secrecy probability and the average secrecy capacity are presented to evaluate the efficiency of the spectrum lease.

The rest of the paper is organized as follows: In Section 2, we present the cooperative system model and operation principle. The performance analysis of the primary and secondary networks is presented in Section 3. Section 4 discusses the numerical and simulative results using the Monte Carlo method and theoretical expressions. The conclusions are summarized in Section 5.

2 System model

Figure 1 presents a system model of cooperative communication between primary and secondary networks. The primary network includes the primary transmitter (PT) and primary receiver (PR) and has a licensed spectrum band. On the other hand, the primary network does not share this spectrum band, because its QoS is very high. The secondary network has two nodes, denoted as a secondary transmitter (ST) and a secondary receiver (SR), where the ST only transmits its data when receiving a spectrum lease contract from the PT. We assume that each node has a single antenna and the same transmit power P and that all nodes suffer the zero-mean additive white Gaussian noise (AWGN) with the same variance $N_0$.

In Figure 1, $(h_1, d_1)$, $(h_2, d_2)$, $(h_{31}, d_{31})$, $(h_{32}, d_{32})$, $(h_4, d_4)$, and $(h_5, d_5)$ denote the Rayleigh fading channel coefficients and the link distances of ST-SR, ST-PR, PT-ST, ST-PT, PT-SR, and PT-PR, respectively. Therefore, the channel gains $g_i = |h_i|^2$ and $g_j = |h_j|^2$ are exponential distribution random variables (RVs) with parameters $\lambda_i$ and $\lambda_j$, respectively, which are given as $\lambda_i = d_i^\beta$ and $\lambda_j = d_j^\beta$ where $\beta$ is the path loss exponent, $i \in \{1, 2, 4, 5\}$, $j \in \{1, 2\}$. We note that links PT-ST and ST-PT have the same distance but different direction.

Before transmitting a data packet, a connection establishment phase is performed using the media access control (MAC) layer protocol, which is similar to the CoopMAC in [26]. First, PT broadcasts a request-to-send (RTS) message to nodes ST, SR, and PR. Hence, ST can estimate the fading channel coefficient $h_{31}$ when receiving the RTS message. Because of the transmission desire of the secondary network on the licensed band, ST will immediately reply to the RTS message by sending a helper-ready-to-send (HTS) to nodes PT, PR, and SR. Relying on the receipt of the RTS and HTS messages, nodes PR and SR can estimate the fading channel coefficient pairs $(h_5, h_2)$ and $(h_4, h_1)$, respectively, and then these nodes will broadcast clear-to-send (CTS) messages that include their estimated fading channel coefficients. If PT wants to cooperate with ST, it will send a RTS message which includes $h_{32}$ estimated from the HTS message to notify a cooperative acceptance to ST. Based on the above steps of the setup phase, ST will know the fading channel coefficients $(h_{31}, h_{32})$ from the RTS messages, $(h_4, h_1)$ from the CTS message of SR, and $(h_5, h_2)$ from the CTS message of PR. Therefore, ST can select a power sharing parameter as well as evaluate the secrecy performance of the secondary network. Similarly, PT will know the fading channel coefficients $h_{31}, h_{32}$, and $h_5$ for selecting the cooperative method and analyzing the system performance of the primary network.

The operation principle and related formulations of the system model are expressed in two time slots as follows:

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{system_model.png}
\caption{System model of cooperative communication between primary and secondary networks.}
\end{figure}
In the first time slot, PT broadcasts a signal $x_P$ to ST, SR, and PR by using a RTS message [26]. The received signals at ST, SR, and PR are given, respectively, as

$$y_{PT-ST} = \sqrt{P}h_{31}x_P + n_{ST} \quad (1)$$

$$y_{PT-SR} = \sqrt{P}h_{4x}x_P + n_{SR} \quad (2)$$

$$y_{PT-PR} = \sqrt{P}h_{5x}x_P + n_{PR} \quad (3)$$

where $n_{ST}$, $n_{SR}$, and $n_{PR}$ are the zero-mean AWGN at nodes ST, SR, and PR, respectively.

In this case, ST can successfully decode the signal $x_P$. Then, if the PR cannot successfully decode the signal $x_P$, it will send a CTS message. After that, PT receives the CTS message, and it will send a RTS message to ST to require cooperation. Because the secondary network cannot use the licensed spectrum band of the primary network, ST will accept the cooperation request of PT by leasing a part of the licensed spectrum band. This procedure is performed at ST by combining the signal $x_P$ with its signal $x_S$ based on the power allocation strategy, which is proposed in [4,5] as

$$x_C = \sqrt{P}x_P + \sqrt{1 - \alpha} P x_S \quad (4)$$

where $\alpha$ ($0 < \alpha < 1$) is the fraction of the transmit power allocated to the signal $x_P$ and $\alpha P$ is the money of the secondary network for payment of a leased spectrum band.

Then, ST broadcasts the combined signal $x_C$ to nodes ST, SR, and PR, and even PT. The received signals at ST, PR, and PT in the second time slot can be given, respectively, as

$$y_{ST-SR} = \left(\sqrt{\alpha} P x_P + \sqrt{1 - \alpha} P x_S\right) h_1 + n_{SR} \quad (5)$$

$$y_{ST-PR} = \left(\sqrt{\alpha} P x_P + \sqrt{1 - \alpha} P x_S\right) h_2 + n_{PR} \quad (6)$$

$$y_{ST-PT} = \left(\sqrt{\alpha} P x_P + \sqrt{1 - \alpha} P x_S\right) h_{32} + n_{PT} \quad (7)$$

where $n_{PT}$ is the zero-mean AWGN at node PT.

At node PR, the well-known maximal ratio combining (MRC) technique is employed from two signals that are received from PT (the first time slot) and ST (the second time slot) to decode $x_P$ [4], Equation 23, so the received signal-to-noise ratio (SNR) at PR can be expressed as

$$\gamma_{MRC} = \frac{P|h_3|^2}{N_0} + \frac{\alpha P|h_2|^2}{(1 - \alpha) P|h_2|^2 + N_0} = \gamma g_3 + \frac{\alpha \gamma g_2}{(1 - \alpha) \gamma g_2 + 1} \quad (8)$$

where $\gamma = P/N_0$ is the transmit SNR.

Because SR also receives the signal $x_P$ in the first time slot, there are two cases for decoding the signal $x_P$ from the received signal (2) as + SR successfully decodes $x_P$, and then SR can cancel the interference component $x_P$ in (5). The received signal after canceling the interference at SR is given as

$$y'_{ST-SR} = \sqrt{(1 - \alpha) P h_3 h_1} + n_{SR} \quad (9)$$

and the corresponding received SNR $\gamma_1$ at SR is given to decode $x_S$ as

$$\gamma_1 = \frac{(1 - \alpha) P|h_1|^2}{N_0} = (1 - \alpha) \gamma g_1 \quad (10)$$

+ SR does not successfully decode $x_P$, and then the received SNR $\gamma_1$ at SR is expressed as

$$\gamma_1 = \frac{(1 - \alpha) P|h_1|^2}{\alpha P|h_1|^2 + N_0} = \frac{(1 - \alpha) \gamma g_1}{\alpha \gamma g_1 + 1} \quad (11)$$

In addition, the secondary network has important data and suspects the help request of the primary network, so physical layer security is applied to the secondary network in evaluating the secure performance. In this system model, we assume that PT wants to eavesdrop the signal $x_S$ of ST while PR only receives the signal $x_P$ of PT in the second time slot. The received ASR at SR is given as [17]

$$\gamma_{ASR} = |R_{ST-SR} - R_{ST-PT}|^+ \quad (12)$$

where $\{x\}^+$ is noted for max(0, x) function; $R_{ST-SR}$ and $R_{ST-PT}$ are the achievable data rates of links ST-SR and ST-PT, respectively, and are given as

$$R_{ST-SR} = \frac{1}{2} \log_2 (1 + \gamma_1) \quad (13)$$

$$R_{ST-PT} = \frac{1}{2} \log_2 (1 + \gamma_{32}) \quad (14)$$

where the factor (1/2) means that the signals are transmitted in two time slots, and $\gamma_{32}$ is the received SNR at PT from ST to decode the signal $x_S$.

From (7), $x_P$ is the transmitted signal of the eavesdropper PT. The $x_P$ can be considered as a side information that is available at the transmitter ST and the eavesdropper PT [27]. Hence, the eavesdropper PT can cancel the self-interference $x_P$. As a result, the received SNR $\gamma_{32}$ at PT is given as

$$\gamma_{32} = \frac{(1 - \alpha) P|h_{32}|^2}{N_0} = (1 - \alpha) \gamma g_{32} \quad (15)$$

### 3 Performance analysis

#### 3.1 The primary network: outage probability and symbol error rate

#### 3.1.1 Outage probability

In this paper, it is assumed that a receiver drops an outage event in the decoding operation if the achievable data rate is less than a target data rate. The achievable data rate
between PT and ST is given in the first time slot from (1) as

\[ R_{PT-ST} = \frac{1}{2} \log_2 (1 + \gamma g_{31}) \]  

(16)

If the achievable data rate \( R_{PT-ST} \) is less than the target data rate \( R_P \) of the primary network, ST cannot successfully decode \( x_P \). The outage probability at ST in this case is given as

\[ P_{out}^{PT-ST} = \Pr[R_{PT-ST} < R_P] = \Pr[g_{31} < \theta / \gamma] \]

(17)

where \( \theta = 2^{2R_P} - 1 \).

We note that \( g_{31} \) in (16) and \( g_{32} \) in (15) have different meanings because these symbols are calculated in the different time slots and directions. Hence, we can assume that \( g_{31} \) and \( g_{32} \) are independent exponential RVs with the same parameter \( \lambda_3 \). When ST successfully decodes the signal \( x_P \), ST accepts the help request of PT and combines \( x_P \) with its signal \( x_S \) as \( x_C \) in (4). After applying the MRC technique, the achievable data rate at PR is given as

\[ R_{MRC_{PT-PR}} = \frac{1}{2} \log_2 (1 + \gamma_{MRC}) \]  

(18)

The outage probability at PR is expressed as

\[ P_{out}^{MRC_{PT-PR}} = \Pr[R_{MRC_{PT-PR}} < R_P] = \Pr[\gamma_{MRC} < \theta] = F_{\gamma_{MRC}}(\theta) \]  

(19)

where \( F_{\gamma_{MRC}}(x) \) is the cumulative distribution function (CDF) of the RV \( \gamma_{MRC} \).

**Theorem 1.** The following single-integral form expression is valid for \( F_{\gamma_{MRC}}(x) \)

\[ F_{\gamma_{MRC}}(x) = \begin{cases} 
1 - e^{-\lambda_3 x / \gamma} - \frac{x e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \left( 1 - e^{-\lambda_3 x / \gamma} - \frac{e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \right) 
& \text{if } \alpha \leq 1 - \alpha \leq \frac{e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \left( 1 - e^{-\lambda_3 x / \gamma} - \frac{e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \right) 
& \text{if } \alpha \leq 1 - \alpha \leq \frac{e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \left( 1 - e^{-\lambda_3 x / \gamma} - \frac{e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \right) 
\end{cases} \]  

(20)

**Proof.** The proof is performed in Appendix 1. \( \square \)

From Theorem 1, the outage probability at PR is obtained in the single-integral form as

\[ P_{out}^{MRC_{PT-PR}} = \begin{cases} 
1 - e^{-\lambda_3 x / \gamma} - \frac{x e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \left( 1 - e^{-\lambda_3 x / \gamma} - \frac{e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \right) 
& \text{if } \alpha \leq 1 - \alpha \leq \frac{e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \left( 1 - e^{-\lambda_3 x / \gamma} - \frac{e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \right) 
& \text{if } \alpha \leq 1 - \alpha \leq \frac{e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \left( 1 - e^{-\lambda_3 x / \gamma} - \frac{e^{-\lambda_3 x / \gamma}}{\alpha(1 - \alpha)} \right) 
\end{cases} \]  

(21)

We denote \( m(u) \) as a function of variable \( u \) and is defined as \( m(u) = \lambda_3 u + \alpha \lambda_2 / u, u \in [0, \infty) \). By performing a function survey method, \( m(u) \) is minimum at \( m(u^*) \), is a decreasing function in \([0, u^*] \), and is an increasing function in \([u^*, \infty) \), where \( u^* = \sqrt{\alpha \lambda_2 / \lambda_5} \). Hence, Theorem 2 will obtain a lower bound of \( P_{out}^{MRC_{PT-PR}} \) as

**Theorem 2.** The following lower bound expression is valid for \( P_{out}^{MRC_{PT-PR}} \)

\[ P_{out}^{MRC_{lower}} = \begin{cases} 
1 - e^{-\lambda_3 \eta / \gamma} - \frac{\alpha \lambda_3 e^{-\alpha(1 - \alpha) \lambda_2 / \gamma}}{\alpha(1 - \alpha) \lambda_5 (1 - \alpha) \gamma} & \text{if } \eta > \alpha \\
1 - e^{-\lambda_3 \eta / \gamma} - \frac{\alpha \lambda_3 e^{-\alpha(1 - \alpha) \lambda_2 / \gamma}}{\alpha(1 - \alpha) \lambda_5 (1 - \alpha) \gamma} & \text{if } \eta < \alpha 
\end{cases} \]  

(22)

where \( \Delta = \begin{cases} 
m(\alpha), \eta > \alpha \\
(\alpha^*), \eta \leq \alpha 
\end{cases} \) and

\[ m(\alpha) = \begin{cases} 
m(\alpha), \eta > \alpha \\
(\alpha^*), \eta \leq \alpha 
\end{cases} \]

(23)

Finally, based on the operation principle of cooperative communication between primary and secondary networks, the outage probability of the primary network is a combination of the outage probabilities, which are obtained in (17), (21), and (23) as

\[ P_{out}^{cc} = (1 - P_{out}^{MRC_{PT-PR}}) P_{out}^{MRC_{PT-PR}} + P_{out}^{MRC_{PT-PR}} P_{out}^{MRC_{PT-PR}} \]  

(24)

where \( P_{out}^{MRC_{PT-PR}} \), \( P_{out}^{MRC_{PT-PR}} \), and \( P_{out}^{MRC_{PT-PR}} \) are obtained in (17), (21), and (23), respectively.

In addition, from Theorem 2 and (24), the lower bound form of the outage probability of the primary network is expressed as

\[ P_{out}^{MRC_{lower}} = \begin{cases} 
1 - e^{-\lambda_3 \eta / \gamma} - \frac{\alpha \lambda_3 e^{-\alpha(1 - \alpha) \lambda_2 / \gamma}}{\alpha(1 - \alpha) \lambda_5 (1 - \alpha) \gamma} & \text{if } \eta > \alpha \\
1 - e^{-\lambda_3 \eta / \gamma} - \frac{\alpha \lambda_3 e^{-\alpha(1 - \alpha) \lambda_2 / \gamma}}{\alpha(1 - \alpha) \lambda_5 (1 - \alpha) \gamma} & \text{if } \eta < \alpha 
\end{cases} \]  

(25)

where \( P_{out}^{MRC_{lower}} \) is obtained in (22).

Next, we compare the proposed cooperative transmission protocol with the direct transmission protocol (DT protocol). In the DT protocol, PT directly transmits the signal \( x_P \) to PR without the help of ST. In this case, ST does not have opportunities to transmit its signal \( x_S \). For a fair
evaluation, PT will transmit the same signal $x_P$ during two time slots, and PR will use the MRC technique to combine these received signals to increase the decoding capacity. The received SNR at PR is given as

$$\gamma_{\text{MRC}}^{\text{DT}} = \gamma g_1 + \gamma g_2 = \gamma (g_1 + g_2)$$  \hspace{1cm} (26)

where $g_1$ and $g_2$ are the channel gains of link PT-PR in the first and second time slots, respectively, and we assume that $g_1$ and $g_2$ are independent exponential RVs with the same parameter $\lambda_5$.

**Lemma 1.** The CDF of RV $\gamma_{\text{MRC}}^{\text{DT}}$ can be expressed in the exact closed form as

$$F_{\gamma_{\text{MRC}}^{\text{DT}}}(x) = 1 - (1 + \lambda_5 x / \gamma) e^{-\lambda_5 x / \gamma}$$  \hspace{1cm} (27)

**Proof.** The CDF of $\gamma_{\text{MRC}}^{\text{DT}}$ is expressed as

$$F_{\gamma_{\text{MRC}}^{\text{DT}}}(x) = \Pr \left[ \gamma_{\text{MRC}}^{\text{DT}} < x \right] = \Pr \left[ \gamma (g_1 + g_2) < x \right] = \Pr \left[ g_2 < x / \gamma - g_1 \right] = \int_0^{x/\gamma} f_{g_1}(t) \times f_{g_2}(x/\gamma - t) dt$$  \hspace{1cm} (28)

where $f_{g_1}(t) = \lambda_5 e^{-\lambda_5 t}$ and $f_{g_2}(t) = 1 - e^{-\lambda_5 t}$ are the probability density function (PDF) and CDF of RVs $g_1$ and $g_2$, respectively.

Hence, $F_{g_2}(x/\gamma - t)$ in (28) is calculated as $F_{g_2}(x/\gamma - t) = 1 - e^{-\lambda_5 (x/\gamma - t)}$. Then, substituting $f_{g_1}(t)$ and $F_{g_2}(x/\gamma - t)$ into (28), Lemma 1 is proven.

From Lemma 1, the outage probability of the DT protocol is given as

$$P_{\text{out}}^{\text{DT}} = \Pr \left[ \frac{1}{2} \log_2 \left( 1 + \gamma_{\text{MRC}}^{\text{DT}} \right) < R_p \right] = F_{\gamma_{\text{MRC}}^{\text{DT}}}(0) = 1 - (1 + \lambda_5 \theta / \gamma) e^{-\lambda_5 \theta / \gamma}$$  \hspace{1cm} (29)

□

### 3.1.2 Symbol error rate

The average SER of link PT-ST in the first time slot can be calculated [28, Eq. (27)] as

$$\text{SER}_{\text{PT-ST}} = \frac{a \sqrt{b}}{2 \sqrt{\pi}} \int_0^{\infty} e^{-bx} F_{\gamma_{g_1}}(x) \sqrt{x} dx$$  \hspace{1cm} (30)

where $a$ and $b$ are constants that depend on the modulation type, e.g., $a = b = 1$ for BPSK modulation, $\gamma g_1$ is the received SNR at ST, and $F_{\gamma_{g_1}}(x)$ is the CDF of the RV $\gamma g_1$ and is given as $F_{\gamma_{g_1}}(x) = 1 - e^{-\lambda_5 x / \gamma}$.

Then, (30) is solved as

$$\text{SER}_{\text{PT-ST}} = \frac{a}{2} \left( 1 - \frac{1}{\sqrt{1 + \lambda_5 / (by)}} \right)$$  \hspace{1cm} (31)

When ST successfully decodes $x_P$, PR receives two signal paths such as the direct path from PT and the cooperative path from ST at the different time slots. To restore $x_P$, PR performs the MRC decoding by storing the received signal $y_{\text{PT-PR}}$ (3) at the first time slot and then waiting to the second time slot to combine with the received signal $y_{\text{ST-PR}}$ (6). Two received signals $y_{\text{PT-PR}}$ and $y_{\text{ST-PR}}$ in the two-phase transmission can be seen as an equivalent single-input multi-output (SIMO) [5]. As described in [4,5], the MRC decoding is easier to implement. With the help of the equation [28, Eq. (27)], the average SER at PR after applying the MRC technique is expressed as

$$\text{SER}_{\text{PR-PT}}^\text{MRC} = \frac{a \sqrt{b}}{2 \sqrt{\pi}} \int_0^{\infty} e^{-bx} F_{\gamma_{\text{MRC}}}(x) \sqrt{x} dx$$  \hspace{1cm} (32)

Note that the regeneration of $x_P$ can be performed by the minimum mean square error decoding (MMSE decoding) [29,30]. The analysis of SER in the MMSE decoding is complex because it relates resolution of posterior distributions. The MMSE decoding will be our future work.

To solve $\text{SER}_{\text{PR-PT}}^\text{MRC}$ in the formula (32), Theorem 3 is obtained as follows:

**Theorem 3.** The following representation is valid for $\text{SER}_{\text{PR-PT}}^\text{MRC}$

$$\text{SER}_{\text{PR-PT}}^\text{MRC} = a \left( \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-bx} F_{\gamma_{\text{MRC}}}(x) \sqrt{x} dx \right)$$

$$\times \left[ \int_0^{a/(1-a)} e^{-b(x+\lambda_5)x / \gamma} \sqrt{1 + \lambda_5 x / (by)} dx \right]$$

$$\times \left[ \int_0^{a/(1-a)} e^{-b(x+\lambda_5)x / \gamma} \sqrt{1 + \lambda_5 x / (by)} dx \right]$$

where $\text{Erf}[x]$ is the complementary error function [31, Eq. (8.250.4)].

**Proof.** Substituting $F_{\gamma_{\text{MRC}}}(x)$ in Theorem 1 into (32), we obtain

$$\text{SER}_{\text{PR-PT}}^\text{MRC} = a \sqrt{b} \left[ \int_0^{\infty} e^{-bx} F_{\gamma_{g_1}}(x) \sqrt{x} dx \right]$$

$$- \frac{a \lambda_5 e^{(a+\lambda_5 + b)/(1-a)} / \gamma}{2 \gamma (1-a) \sqrt{\pi}}$$

$$\times \left[ \int_0^{a/(1-a)} e^{-b(x+\lambda_5)x / \gamma} \sqrt{1 + \lambda_5 x / (by)} dx \right]$$

$$\times \left[ \int_0^{a/(1-a)} e^{-b(x+\lambda_5)x / \gamma} \sqrt{1 + \lambda_5 x / (by)} dx \right]$$

(34)
After some manipulations of (34), the proof of Theorem 3 is completed.

From the result of Theorem 3, the integral component \( \frac{a}{\alpha} \int_0^\infty e^{-\frac{\lambda x}{\alpha u}} du \) depends on variable \( x \) and the function \( m(u) = \lambda s u + a \lambda_2 / u \) has a complex form. Hence, the analysis of \( \text{SER}_{\text{PT-PR}}^\text{MRC} \) in the lower bound form as \( \text{SER}_{\text{PT-PR}}^\text{MRC} \) in Theorem 2 is not feasible.

When node ST does not successfully decode \( x_P \), PR only directly receives the signal \( x_P \) in the first slot. The average SER at PR in this case is given as

\[
\text{SER}_{\text{PT-PR}} = \frac{a \sqrt{b}}{2 \sqrt{\pi}} \int_0^\infty \frac{e^{-bx} F_{\gamma S}(x)}{\sqrt{x}} dx
\]

\[
= \frac{a}{2} \left( 1 - \frac{1}{\sqrt{1 + \lambda S / (b \gamma)} \right) \tag{35}
\]

where \( F_{\gamma S}(x) \) is the CDF of the exponential RV \( \gamma S \) and is expressed as \( F_{\gamma S}(x) = 1 - e^{-\lambda S x / \gamma} \).

Using the law of total probability, the average SER of the cooperative communication between the primary and secondary networks is given as

\[
\text{SER}_{\text{cc}} = \left( 1 - \text{SER}_{\text{PT-ST}} \right) \text{SER}_{\text{PT-PR}} + \text{SER}_{\text{PT-ST}} \text{SER}_{\text{PT-PR}} \tag{36}
\]

where \( \text{SER}_{\text{PT-ST}}, \text{SER}_{\text{PT-PR}}, \) and \( \text{SER}_{\text{PT-PR}}^\text{MRC} \) are obtained in (31), (35), and Theorem 3, respectively.

For the DT protocol with a fair evaluation, PT also transmits the same signal \( x_P \) during two time slots, and PR uses the MRC technique. The average SER of the DT protocol is expressed as

\[
\text{SER}_{\text{DT}} = \frac{a \sqrt{b}}{2 \sqrt{\pi}} \int_0^\infty \frac{e^{-bx} F_{\gamma S}(x)}{\sqrt{x}} dx \tag{37}
\]

Using \( F_{\gamma S}(x) \) in the Lemma 1, (37) is obtained in the closed-form expression as

\[
\text{SER}_{\text{DT}} = \frac{a \sqrt{b}}{2 \sqrt{\pi}} \int_0^\infty \frac{e^{-bx} \left[ 1 - e^{-\frac{\lambda S x}{\gamma} - \frac{a \lambda_2}{(b+\lambda S / \gamma)^{1/2}} / \gamma } \right]}{\sqrt{x}} dx
\]

\[
= \frac{a \sqrt{b}}{2} \left[ 1 - \frac{2b + 3\lambda S / \gamma}{2(b+\lambda S / \gamma)^{1/2}} \right] \tag{38}
\]

\[\Box\]

### 3.2 The secondary network: outage secrecy probability and average secrecy capacity

#### 3.2.1 Outage secrecy probability

When ST successfully decodes the signal \( x_P \), it will then transmit the combined signal \( x_C \) as (4). However, the ST does not believe the help request of PT, because ST thinks that PT wants to get its information. Therefore, ST only considers the security of the important information. Physical layer security is applied at the secondary network in Figure 1, where ST wants to secure the important information against the eavesdropping of PT in the second time slot. The outage secrecy probability of link ST-SR is given as

\[
P_{\text{ASR}}^\text{out} = \Pr[\text{ASR} < R_S] = \Pr \left[ \frac{1}{2} \log_2 \left( \frac{(1 + (1 - \alpha) \gamma g_1)}{1 + (1 - \alpha) \gamma g_32} \right)^+ < R_S, R_{\text{PT-SR}} \geq R_P \right] + \Pr \left[ \frac{1}{2} \log_2 \left( \frac{(1 + (1 - \alpha) \gamma g_1)}{1 + (1 - \alpha) \gamma g_32} \right)^+ < R_S, R_{\text{PT-SR}} < R_P \right] \tag{39}
\]

where ASR is given in (12), \( R_S \) is the target secrecy rate [18], and \( R_{\text{PT-SR}} \) is the achievable data rate from PT to SR and is obtained from (2) as

\[
R_{\text{PT-SR}} = \frac{1}{2} \log_2 \left( 1 + \gamma g_4 \right) \tag{40}
\]

Substituting (40) into (39), we obtain

\[
P_{\text{ASR}}^\text{out} = \Pr \left[ g_4 \geq \frac{\theta}{\gamma} \right] \times \Pr \left[ \frac{1}{2} \log_2 \left( \frac{(1 + (1 - \alpha) \gamma g_1)}{1 + (1 - \alpha) \gamma g_32} \right)^+ < R_S \right] + \Pr \left[ g_4 \geq \frac{\theta}{\gamma} \right] \times \Pr \left[ \frac{1}{2} \log_2 \left( \frac{(1 + (1 - \alpha) \gamma g_1)}{1 + (1 - \alpha) \gamma g_32} \right)^+ < R_S \right] = e^{-\lambda S / \gamma} P_{\text{ASR}}^1 + \left( 1 - e^{-\lambda S / \gamma} \right) P_{\text{ASR}}^2 \tag{41}
\]

**Theorem 4.** The following closed-form expression is valid for \( P_{\text{ASR}}^1 \)

\[
P_{\text{ASR}}^1 = 1 - \frac{\lambda S e^{-\lambda (\rho - 1)/(1 - \alpha) \gamma}}{\lambda_3 + \rho \lambda_1} \tag{42}
\]
Proof. By expanding the expression \(|x|^+\) in \(P_{\text{ASR}}^1 (41)\), the probability \(P_{\text{ASR}}^1\) is given as

\[
P_{\text{ASR}}^1 = \Pr \left[ g_1 \leq g_{32} \right] + \Pr \left[ g_{32} < g_1 < \frac{\rho - 1}{(1 - \alpha) \gamma} + \rho g_{32} \right]
\]

\[
= 1 - \Pr \left[ g_1 > \frac{\rho - 1}{(1 - \alpha) \gamma} + \rho g_{32} \right]
\]

\[
= \int_0^\infty f_{g_{32}} (x) \times f_{g_1} \left( \frac{\rho - 1}{(1 - \alpha) \gamma} + \rho x \right) \, dx
\]

(43)

where \(f_{g_{32}} (x)\), \(f_{g_1} (x)\) are the PDF and CDF of RVs \(g_{32}\) and \(g_1\), respectively, and are given as follows: 

\[
f_{g_{32}} (x) = \lambda_3 e^{-\lambda_3 x}
\]

and \(f_{g_1} (x) = 1 - e^{-\lambda_1 x}; \rho = 2^{K_Y}\).

Hence, \(f_{g_1} (\rho - 1)/(1 - \alpha) \gamma + \rho x)\) in (43) is obtained as 

\[
f_{g_1} (\rho - 1)/(1 - \alpha) \gamma + \rho x = 1 - e^{-x / (1 - \alpha) \gamma + \rho x}.
\]

Then, substituting \(f_{g_{32}} (x)\) and \(f_{g_1} (\rho - 1)/(1 - \alpha) \gamma + \rho x)\) into (43), the proof of Theorem 4 is provided.

Likewise, the probability \(P_{\text{ASR}}^2\) is solved in the next theorem.

Theorem 5. The following single-integral form representation is valid for \(P_{\text{ASR}}^2\)

\[
P_{\text{ASR}}^2 = \left\{ \begin{array}{ll}
1 & , \rho \alpha \geq 1 \\
1 - \lambda_3 e^{-\lambda_3 x} / (1 - \rho \alpha) / (1 - \alpha) & , \rho \alpha < 1
\end{array} \right.
\]

(44)

Proof. The proof is provided in Appendix 3.

We set as \(n(y) = \lambda_3 y / \rho - \lambda_1 / y\), \(y \in [0, (1 - \rho \alpha) / (1 - \alpha)]\). By performing a function survey method, \(n(y)\) is an increasing function in \([0, (1 - \rho \alpha) / (1 - \alpha)]\). As a result of the increasing function \(n(y)\), we have an inequality as \(n(y) \leq n((1 - \rho \alpha) / (1 - \alpha))\). Hence, the lower bound of \(P_{\text{ASR}}^2\) is obtained as

\[
P_{\text{ASR}}^{2, \text{lower}} = \left\{ \begin{array}{ll}
1 & , \rho \alpha \geq 1 \\
1 - \lambda_3 e^{-\lambda_3 x} / (1 - \rho \alpha) / (1 - \alpha) & , \rho \alpha < 1
\end{array} \right.
\]

(45)

Substituting the result of Theorems 4 and 5 into (41), we have \(P_{\text{SN}}\) in hand. Finally, the outage secrecy probability of the secondary network during two time slots is given as

\[
P_{\text{SN}} = (1 - P_{\text{PT-SR}}) P_{\text{ASR}} + P_{\text{PT-SR}}
\]

(46)

where \(P_{\text{PT-SR}}\) is given in (17).

In addition, from the lower bound of \(P_{\text{ASR}}^{2, \text{lower}}\) in (45), the lower bound form of \(P_{\text{SN}}\) is expressed as

\[
P_{\text{SN}}^{\text{lower}} = (1 - P_{\text{PT-ST}}) P_{\text{ASR}}^{\text{lower}} + P_{\text{PT-ST}}
\]

(47)

where \(P_{\text{ASR}}^{\text{lower}}\) is obtained from (41) as

\[
P_{\text{ASR}}^{\text{lower}} = e^{-\lambda_1 y / (1 - \alpha) \gamma} + (1 - e^{-\lambda_1 y / (1 - \alpha) \gamma}) P_{\text{ASR}}^{2, \text{lower}}
\]

(48)

3.2.2 Average secrecy capacity

In the operation protocol of the secondary network, ST only transmits the secrecy signals when ST successfully decodes the primary signal \(x_P\) of PT. The average secrecy capacity of the secondary network is defined by calculating the expected value over \(ASR\) [32], ([33] Eq. (8)), ([34] Eq. (3)). In addition, the ASR expression in (12) depends on the decoding of the primary signal \(x_P\) at SR (10-11). Hence, the average secrecy capacity of the secondary network is formulated as

\[
C_{\text{avg}} = (1 - P_{\text{PT-ST}}) \times \left[ \begin{array}{c}
\text{ST successfully decodes } x_P \\
\text{SR successfully decodes } x_P \\
\text{SR cannot successfully decode } x_P
\end{array} \right]
\]

(49)

where \(E[X]\) is the expected operation of RV \(X\); \(P_{\text{PT-ST}}\) is the outage probability when SR cannot successfully decode the signal \(x_P\) in the first time slot and is obtained as

\[
P_{\text{PT-ST}} = \Pr [R_{\text{PT-SR}} < R_P] = \Pr [g_4 < \theta / \gamma] = 1 - e^{-\lambda_1 y / (1 - \alpha) \gamma}
\]

(50)

At a high SNR \(\gamma\) value, \(C_{\text{avg}}^{1}\) and \(C_{\text{avg}}^{2}\) in (49) can be formulated, respectively, as

\[
C_{\text{avg}}^{1} \approx E \left[ \frac{1}{2} \log_2 \left( \frac{g_1}{g_{32}} \right) \right]^{+}
\]

(51)
\[ C_{\text{avg}}^2 \approx E \left[ \frac{1}{2} \log_2 \left( \frac{g_1/(a_\gamma g_1 + 1)}{g_{32}} \right) \right]^+ \]
\[ = \frac{1}{2} E \left[ \log_2 \left( \frac{Z}{g_{32}} \right) \right]^+ \]  
(52)

where \( Z = g_1/(a_\gamma g_1 + 1) \) as defined in Appendix 3.

**Theorem 6.** The following asymptotic expressions are valid for \( C_{\text{avg}}^1 \) and \( C_{\text{avg}}^2 \) respectively

\[ C_{\text{avg}}^1 \approx \ln \left( 1 + \frac{\lambda_3/\lambda_1}{2 \ln 2} \right) \]  
(53)

\[ C_{\text{avg}}^2 \approx E^{-1/(a_\gamma)} \left( 1 - \frac{\lambda_3}{a_\gamma} \right) \]
\[ \times \left\{ \text{EulerGamma} + \Gamma \left( 0, \frac{\lambda_3}{a_\gamma} \right) + \ln \left( \frac{\lambda_3}{a_\gamma} \right) \right\} \]  
(54)

where \( \Gamma(n,x) \) is the 'upper' incomplete gamma function ([31] Eq. (8.350.2), \( \Gamma(n,x) = \int_x^\infty t^{n-1} e^{-t} dt \)), and EulerGamma is Euler’s constant (EulerGamma \( \approx 0.577216 \)[31] Eq. (8.367.1)).

**Proof.** The proof is solved in Appendix 4.

Substituting (17), (50), and the results of Theorem 6 into (49), we obtain the asymptotic form of the average secrecy capacity of the secondary network as

\[ C_{\text{avg}}^{SN} \approx E^{-1/(a_\gamma)} \left( 1 - \frac{\lambda_3}{a_\gamma} \right) \]
\[ \times \left\{ \text{EulerGamma} + \Gamma \left( 0, \frac{\lambda_3}{a_\gamma} \right) + \ln \left( \frac{\lambda_3}{a_\gamma} \right) \right\} \]  
(55)

4 Numerical and simulative results

In this section, the performance of the primary and secondary networks is analyzed by using the Monte Carlo simulation method and the theoretical expressions. The performance analyses include the outage probability and SER of the primary network and the outage secrecy probability and the average secrecy capacity of the secondary network. We note that the single-integral expression of (21) and (44) and the double-integral expression of (33) are not presented in the closed forms but are easily solved by the numerical method in Matlab. In a two-dimensional plane, the coordinates of the primary and secondary nodes are set to PT (0, 0), PR (1, 0), ST (0.5, \( y_1 \)), and SR (0.5, \( y_2 \)). Hence, the distance of the PT-PR link is 1 (\( d_5 = 1 \)) during the simulation intervals, and ST and SR move on the median line of the PT-PR line segment, which depend on the values of the variables \( y_1 \) and \( y_2 \) on the y-axis. We assume that the path loss exponent, the target data rate of the primary network, and the target secrecy rate of the secondary network are constants (\( \beta = 3, R_\rho = 1 \) bit/s/Hz, \( R_S = 0.1 \) bit/s/Hz), and the SNR symbol in the x-axis is defined as \( \text{SNR} = P/N_0 \).

Figure 2 presents the outage probability of the primary network at PR versus SNR (dB) when \( y_1 = 0.1 \) and with different values of the power allocation at ST (\( \alpha \)). In Figure 2, for the cooperative communication protocol (called the CC protocol), the outage probability of the primary network is the smallest when \( \alpha \) is the largest (\( \alpha = 0.9 \)), and the primary system performance is improved when the SNR increases. These results are explained that when the fraction of the transmit power \( \alpha \) increases, ST uses more power for forwarding the signal of PT to PR in the second time slot. For comparison purposes, the CC protocol outperforms the DT protocol when ST shares the high power to help PT (\( \alpha \geq 0.8 \)). In addition, the theoretical expressions match very well with the simulation results, and the lower bound forms converge to the exact theories when the fractions of the transmit power \( \alpha \) are small.

In addition to the outage probability, the SER is suitable for evaluating the QoS of the primary network. Figure 3 presents the SER of the primary network with the same ST location as in Figure 2. In this simulation result, we assume that the cooperative system uses the BPSK modulation (e.g., \( a = b = 1 \)) and coherent maximum likelihood detection [35]. The SER of the CC protocol is the smallest at \( \alpha = 0.9 \). For different values of \( \alpha \), the SER of the CC protocol is smaller than that of the DT protocol when SNR \( \leq -1 \) dB for \( \alpha = 0.5 \), SNR \( < 8 \) dB for \( \alpha = 0.7 \), and SNR \( < 17 \) dB for \( \alpha = 0.8 \). These results are explained as follows. In the symmetric network model, when the fraction of the transmit power \( \alpha \) is enough large (such as \( \alpha = 0.5, 0.7, 0.8 \)), the demodulation of the primary data at PR depends on the total transmit power \( P \) of ST in the second time slot. For example, if ST uses the small total transmit power \( P \), PR is difficult to demodulate the primary data when \( \alpha \) is fixed. Therefore, the SER at PR will increase. In addition, in the DT protocol, by transmitting the same primary data during two time slots, PR will increase the demodulation capacity by the MRC technique.
From Figures 3 and 4, the performance of the CC protocol is better than that of the DT protocol at high values of $\alpha$ (e.g., $\alpha = 0.9$), because the located power for helping the primary user PT is higher by far than that for transmitting the own signal of the secondary user ST.

Figure 4 presents the outage secrecy probability of the secondary network versus the SNR (dB) when $y_1 = 0.1$ and $y_2 = 0$. In Figure 4, the outage secrecy probability decreases when the SNR increases and $\alpha$ decreases (the secrecy performance is the best when $\alpha = 0.1$). These results are explained that when $\alpha$ decreases, ST uses more power for transmitting the own signal than forwarding the signal of PT to PR in the second time slot. In addition, with the high SNR (SNR $\geq$ 10 dB), the outage secrecy probability is almost independent of the values of $\alpha$. Hence, from Figures 2, 3 and 4, ST can provide the high power to help the primary network so that the performance of the cooperative primary network (the CC protocol) outperforms the DT protocol and the secure performance of the secondary network is maintained, especially in the high SNR region. For example, when SNR $\approx$ 12 dB, the outage secrecy probability of the secondary network is maintained at $5 \times 10^{-2}$, and ST can assign the parameter $\alpha = 0.9$ to improve the performance of the primary network. In addition, the theoretical expressions are valid for the simulation results, and the lower bound forms converge to the exact results when the fractions of the transmit power $\alpha$ are large.

Next, we discuss the average secrecy capacity of the secondary network when ST cooperates with the primary network. Figure 5 presents the average secrecy capacity versus the SNR (dB) when the locations of nodes ST and SR are the same as in Figure 4. In Figure 5, the average secrecy capacity increases when $\alpha$ decreases because the allocated power for transmitting the signal of ST increases. In addition, the average secrecy capacity also converges to the asymptotic value when the transmit SNR is high, but the convergence of the average secrecy capacity is faster when $\alpha$ is smaller.

Finally, we analyze and evaluate the effects of the locations of the secondary nodes ST and SR on the performance of both networks. Figure 6 presents the outage
probability and the outage secrecy probability of the primary network and the secondary network, respectively, when the parameter $\alpha$ is set to $\alpha = 0.9$. In Figure 6, when SR is fixed at (0.5, 0) and the ST moves on the median line of the PT-PR line segment where $y_1$ ranges from 0.1 to 1, the performance of both networks decreases. These results are explained as follows. Because of the movement of ST, the distances of links PT-ST, ST-PR, and ST-SR increase, so it is difficult for ST to decode the received signal from PT, and the data rate at SR decreases. Whereas, when ST is set at the central point (0.5, 0) and SR moves on the median line of the PT-PR line segment where $y_2$ ranges from 0.1 to 1, the outage probability of the primary network does not change, while the outage secrecy probability of the secondary network increases. These results are explained by three factors as follows. First, the primary network does not depend on the action of SR. Second, because SR goes farther, SR hardly decodes the signal of PT to cancel the interference elements. Third, the distance of the ST-SR link increases. For these reasons, the performance of the secondary network is poor, while that of the primary network is stable. From Figure 6, the lower bound forms of the outage secrecy probabilities well converge to the exact results when $\alpha = 0.9$.

5 Conclusions
In this paper, the artifice of the primary user in the spectrum lease is investigated. The basis of this problem is based on the transmission desire of the secondary user. The system performance of the primary network is evaluated by using the outage probability and the symbol error rate, while that of the secondary network is analyzed in terms of the outage secrecy probability and the asymptotic average secrecy capacity. The simulation results show that the primary network outperforms the direct transmission protocol when the spectrum share coefficient of the secondary user increases and that the secrecy performance of the secondary network is better in the high SNR region. These results are important for selecting the best power sharing fraction so that the performance of the primary network is improved and the secrecy performance of the secondary network is maintained in the cooperative communication. In addition, the location of the secondary nodes is discussed to evaluate the effects on both networks.
Appendices

Appendix 1

Proof of Theorem 1

We set $U = \alpha g_2 / ((1 - \alpha) \gamma g_2 + 1)$, so the CDF of $U$ is given as

$$F_U(x) = \Pr \left[ \frac{\alpha g_2}{(1 - \alpha) \gamma g_2 + 1} < x \right] = \Pr \left[ g_2 (\alpha - x (1 - \alpha) \gamma) < \gamma \right]$$

$$= \begin{cases} 
1, & x \geq \frac{\alpha}{(1 - \alpha) \gamma} \\
1 - e^{-\lambda_2 x / (\alpha - x (1 - \alpha) \gamma)}, & x < \frac{\alpha}{(1 - \alpha) \gamma}
\end{cases} \quad (56)$$

From (8), the CDF of $\gamma_{MRC}$ is expressed as

$$F_{\gamma_{MRC}}(x) = \Pr \left[ \gamma_{MRC} < x \right] = \Pr \left[ \gamma g_5 + \gamma U < x \right]$$

$$= \Pr \left[ g_5 + U < \frac{x}{\gamma} \right] = \int_0^{x/\gamma} f_{g_5}(t) \times F_U(x/\gamma - t) dt \quad (57)$$

where $f_{g_5}(t) = \lambda_5 e^{-\lambda_5 t}$ is the PDF of the RV $g_5$.

The term $F_U(x/\gamma - t)$ is calculated from (56) as

$$F_U(x/\gamma - t) = \begin{cases} 
1, & t \leq \frac{x}{\gamma} < 1 - \frac{\alpha}{(1 - \alpha) \gamma} \\
1 - e^{-\lambda_2 (t - (1 - \alpha) \gamma)}, & t > \frac{x}{\gamma} \geq 1 - \frac{\alpha}{(1 - \alpha) \gamma}
\end{cases} \quad (58)$$

Substituting (58) into (57), the CDF of $\gamma_{MRC}$ is obtained as

$$F_{\gamma_{MRC}}(x) = \begin{cases} 
1 - e^{-\lambda_2 x/\gamma}, & x \geq \frac{\alpha}{(1 - \alpha) \gamma} \\
\frac{x}{\gamma} \int_0^{\frac{x}{\gamma}} e^{-\lambda_2 (t - (1 - \alpha) \gamma)} dt, & \frac{x}{\gamma} < \frac{\alpha}{(1 - \alpha) \gamma}
\end{cases} \quad (59)$$

Performing a variable transformation method as $u = \alpha - (\theta/\gamma - t) (1 - \alpha) \gamma$, $F_{\gamma_{MRC}}(x)$ is solved in the single-integral form as (20). Hence, Theorem 1 is proven.
Appendix 2

Proof of Theorem 2

First, we consider the condition \( \theta \geq \alpha / (1 - \alpha) \) and the integral expression in the region \( u \in [0, \alpha] \) in (21), there are two minimum values of the function \( m(u) \) as

\[
\min m(u) = \begin{cases}
  m(\alpha) & \text{when } \alpha < u^* \\
  m(u^*) & \text{when } \alpha \geq u^*
\end{cases}
\]

(60)

Substituting \( u^* = \sqrt{\alpha \lambda_2 / \lambda_5} \) into (60), we obtain as

\[
\min m(u) = \begin{cases}
  m(\alpha) & \text{when } \theta \geq \alpha / (1 - \alpha), \eta > \alpha \\
  m(u^*) & \text{when } \theta \geq \alpha / (1 - \alpha), \eta \leq \alpha
\end{cases}
\]

(61)

where \( \eta = \lambda_2 / \lambda_5 \).

When the condition \( \theta < \alpha / (1 - \alpha) \) and the integral expression in the region \( u \in [\alpha - \theta (1 - \alpha), \alpha] \), three minimum values of the function \( m(u) \) are given as

\[
\min m(u) = \begin{cases}
  m(\alpha) & \text{when } \alpha < u^* \\
  m(u^*) & \text{when } \alpha \geq u^* > \alpha - \theta (1 - \alpha) \\
  m(\alpha - \theta (1 - \alpha)) & \text{when } u^* \leq \alpha - \theta (1 - \alpha)
\end{cases}
\]

(62)

After some manipulations of (62), we obtain as

\[
\min m(u) = \begin{cases}
  m(\alpha) & \text{when } \theta < \alpha / (1 - \alpha), \eta > \alpha \\
  m(\alpha) & \text{when } \alpha / (1 - \alpha) < \eta \leq \alpha - \theta (1 - \alpha)^2 / 2 \\
  m(\alpha - \theta (1 - \alpha)) & \text{when } \theta < \alpha / (1 - \alpha), \eta \leq \alpha - \theta (1 - \alpha)^2 / 2
\end{cases}
\]

(63)

Substituting (61) and (63) into (21), the lower bound form of \( P_{\text{MRC, lower}}^{\text{PT-PR}} \) is given as

\[
P_{\text{MRC, lower}}^{\text{PT-PR}} = \left\{ \begin{array}{ll}
1 - e^{-\lambda \beta / \gamma} & \text{when } \theta < \alpha / (1 - \alpha), \eta > \alpha \\
1 - e^{-\lambda \beta / \gamma} & \text{when } \alpha / (1 - \alpha) < \eta \leq \alpha - \theta (1 - \alpha)^2 / 2
\end{array} \right.
\]

(64)

where \( \Delta = \begin{cases}
  m(\alpha), \eta > \alpha \\
  m(u^*), \eta \leq \alpha \\
  m(\alpha - \theta (1 - \alpha)), \eta \leq (\alpha - \theta (1 - \alpha)^2 / 2
\end{cases} \)

\[ \Omega = \begin{cases}
  m(\alpha), \eta > \alpha \\
  m(u^*), \eta \leq \alpha \\
  m(\alpha - \theta (1 - \alpha)), \eta \leq (\alpha - \theta (1 - \alpha)^2 / 2
\end{cases} \]
By solving the integrals in (64), the proof of Theorem 2 is completely provided.

Appendix 3  
Proof of Theorem 5  
By expanding the expression \([x]^+\) in \(P_{ASR}^2\) (41), the probability \(P_{ASR}^2\) is given as

\[
\begin{align*}
P_{ASR}^2 &= \left[ \frac{g_1}{\alpha \gamma g_1 + 1} \leq g_{32} \right] \\
&\quad + \Pr \left[ g_{32} < \frac{g_1}{\alpha \gamma g_1 + 1} < \frac{\rho - 1}{(1 - \alpha) \gamma} + \rho g_{32} \right] \\
&= \Pr \left[ \frac{g_1}{\alpha \gamma g_1 + 1} < \frac{\rho - 1}{(1 - \alpha) \gamma} + \rho g_{32} \right] \\
&= \Pr \left[ \frac{g_1}{\alpha \gamma g_1 + 1} < \frac{\rho - 1}{(1 - \alpha) \gamma} + \rho g_{32} \right]
\end{align*}
\]

(65)

We set \(Z = g_1/(\alpha \gamma g_1 + 1)\) and look at the CDF of the RV \(U\) in (56), the CDF of \(Z\) is given as

\[
F_Z(x) = \Pr \left[ \frac{g_1}{\alpha \gamma g_1 + 1} < x \right]
\]

\[
= \begin{cases} 
1 & x \geq (\alpha \gamma)^{-1} \\
1 - e^{-\lambda_1 x/(1 - \alpha \gamma)} & x < (\alpha \gamma)^{-1}
\end{cases}
\]

(66)

From (66), \(P_{ASR}^2\) is obtained as

\[
P_{ASR}^2 = \int_0^\infty f_{g_{32}}(y) \times F_Z \left( \frac{\rho - 1}{(1 - \alpha) \gamma} + \rho x \right) \, dx
\]

\[
= \begin{cases} 
1 & \rho \alpha \geq 1 \\
1 - \lambda_3 \frac{1 - \rho \alpha}{(\alpha - 1) \gamma} e^{-\frac{\rho - 1}{(1 - \alpha) \gamma} + \rho x} & \rho \alpha < 1
\end{cases}
\]

(67)

After performing a variable transformation method as \(y = 1 - ((\rho - 1)/(\alpha - 1) \gamma) + \rho x\), \(P_{ASR}^2\) is obtained in the single-integral form as (44). Hence, the proof of Theorem 5 is completed.

Appendix 4  
Proof of Theorem 6  
We set \(X = g_1/g_{32}\), and the CDF and PDF of RV \(X\) are given as

\[
F_X(x) = \Pr \left[ g_1/g_{32} < x \right] = \Pr \left[ g_1 < x g_{32} \right]
\]

\[
= \int_0^\infty f_{g_{32}}(y) \times F_{g_1}(xy) \, dy = 1 - \frac{\lambda_3}{x \lambda_1}
\]

(68)
\[
 f_X(x) = \frac{\partial F_X(x)}{\partial x} = \frac{\lambda_1 \lambda_3}{(\lambda_3 + x \lambda_1)^2}
 \] (69)

From (69) and the expression of the expected operation [36, Eq. (5.55)], \( C_{\text{avg}} \) in (51) is formulated as

\[
 C_{\text{avg}} \approx \frac{1}{2} \int_1^\infty (\log_2 x) \times f_X(x) \, dx
 = \frac{\lambda_3}{2 \lambda_1 \ln 2} \int_1^\infty \frac{\ln x}{(x + \lambda_3/\lambda_1)^2} \, dx
 \] (70)

Solving the integral in (70), \( C_{\text{avg}} \) is obtained as (53).

To solve \( C_{\text{avg}} \), we set \( Y = Z/\lambda_3 \), where \( Z \) is a RV and is defined in Appendix 3, and the CDF of RV \( Y \) is given as

\[
 F_Y(x) = \Pr \left[ \frac{Z}{\lambda_3} < x \right] = \Pr [ Z < x \lambda_3 ] = \int f_{\text{EZ}}(y) \times F_Z(xy) \, dy
 \] (71)

In (71), the CDF of \( Z \) is solved in (66), and then \( F_Y(x) \) is rewritten as

\[
 F_Y(x) = 1 - \frac{\lambda_3}{\lambda_1} \left( I_{(0)}(\alpha y x) - e^{-\lambda_3 x} \ln \frac{\lambda_3}{\lambda_1 y} x \right) \, dy
 \] (72)

Again, when \( y \) is high (\( y \to +\infty \)), the integral term in (72) goes to 1. Hence, \( F_Y(x) \) is asymptotically obtained as

\[
 F_Y(x) \approx 1 - \frac{\lambda_3 e^{\lambda_1/\alpha y} - \lambda_3/\alpha y x}{\alpha y x}
 \] (73)

From (73), the PDF of RV \( Y \) is solved as

\[
 f_Y(y) = \frac{\partial F_Y(x)}{\partial x} = \frac{\lambda_3 e^{\lambda_1/\alpha y} - \lambda_3/\alpha y x}{\alpha y x^2} - \frac{1}{\lambda_3/\alpha y x}
 \] (74)

From (74) and the expression of the expected operation [36, Eq. (5.55)], \( C_{\text{avg}} \) in (52) is formulated in the asymptotic form as

\[
 C_{\text{avg}} \approx \frac{1}{2} \int_1^\infty (\log_2 x) \times f_Y(x) \, dx
 = \frac{1}{2} \int_1^\infty (\log_2 x) \times \frac{\lambda_3 e^{\lambda_1/\alpha y} - \lambda_3/\alpha y x}{\alpha y x^2} \left( 1 - \frac{1}{\lambda_3/\alpha y x} \right) \, dx
 \] (75)

Solving the integral in (75), we obtain \( C_{\text{avg}} \) as (54). Hence, the proof of Theorem 6 is proven.

Competing interests
The authors declare that they have no competing interests.

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