

EV-Fleet Power Forecasting via Kernel-Based Inverse Optimization

Ricardo Fernández-Blanco, Juan Miguel Morales, Salvador Pineda, Álvaro Porras

Abstract—This paper considers an aggregator of Electric Vehicles (EVs) who aims to forecast the aggregate power of her fleet. The forecasting approach is based on a data-driven inverse optimization (IO) method, which is highly nonlinear. To overcome such a caveat, we use a two-step estimation procedure which requires to solve two convex programs. Both programs depend on penalty parameters that can be adjusted by using grid search. In addition, we propose the use of kernel regression to account for the nonlinear relationship between the behaviour of the pool of EVs and the explanatory variables, i.e., the past electricity prices and EV fleet’s driving patterns. Unlike any other forecasting method, the proposed IO framework also allows the aggregator to derive a bidding curve, i.e. the tuple of price-quantity to be submitted to the electricity market, according to the market rules. We show the effective performance of the proposed method against state-of-the-art machine-learning techniques.

Index Terms—Data-driven approach, electric vehicles, inverse optimization, kernel regression, short-term forecasting.

NOMENCLATURE

The main notation used throughout the text is stated below for quick reference. Other symbols are defined as required.

A. Sets and Indices

\[ \mathcal{B} \] \hspace{1cm} \text{Set of energy blocks, indexed by } b. \\
\[ \mathcal{B}^{c/d} \] \hspace{1cm} \text{Set of energy blocks associated with the charging/discharging power, indexed by } b. \\
\[ \mathcal{R} \] \hspace{1cm} \text{Set of regressors, indexed by } r. \\
\[ \mathcal{T} \] \hspace{1cm} \text{Set of time periods, indexed by } t. \\
\[ \mathcal{X} \] \hspace{1cm} \text{Set of time periods belonging to the set } X = \{tr,v,test\} \text{ where } tr, v, test \text{ refer to the training, validation, and test set, in that order.}

B. Parameters

\[ E_{b,t}, E_{b,t}^{c/d} \] \hspace{1cm} \text{Width for the aggregate discharging/charging power block } b \text{ in time period } t \text{ [kW].} \\
\[ H \] \hspace{1cm} \text{Feasibility penalty parameter.} \\
\[ K_{t,r} \] \hspace{1cm} \text{Value of the kernel on two feature vectors at time periods } t \text{ and } r.

C. Decision Variables

\[ m_{b,t} \] \hspace{1cm} \text{Marginal utility of block } b \text{ of the aggregate power in time period } t \text{ [€/kWh].} \\
\[ p_{b,t} \] \hspace{1cm} \text{Power in block } b \text{ and time period } t \text{ [kW].} \\
\[ P_{t}, P_{t}^{\mathcal{R}} \] \hspace{1cm} \text{Lower and upper bound for the aggregate power in time period } t \text{ [kW].} \\
\[ \alpha_{t}, \sigma_{t} \] \hspace{1cm} \text{Coefficient relative to the kernel regression of the lower/upper power bounds in period } t \in \Omega^{tr} \text{ [kW].} \\
\[ \epsilon_{t}, \bar{\epsilon}_{t} \] \hspace{1cm} \text{Duality gap in time period } t \in \Omega^{tr} \text{ [€].} \\
\[ \nu_{b}, \bar{\nu}_{b} \] \hspace{1cm} \text{Intercept for the lower/upper power bounds [kW].} \\
\[ \xi_{t}^{+}, \xi_{t}^{-} \] \hspace{1cm} \text{Slack variables associated with the lower power bound in time period } t \text{ [kW].} \\
\[ \xi_{t}^{+}, \xi_{t}^{-} \] \hspace{1cm} \text{Slack variables associated with the upper power bound in time period } t \text{ [kW].} \\
\[ \rho_{t} \] \hspace{1cm} \text{Coefficient relative to the kernel regression of the marginal utility in time period } t \in \Omega^{tr} \text{ [€/kWh].}

I. INTRODUCTION

According to the White Paper on transport of the European Union [1], one of the main goals to achieve a sustainable transport system is to halve the use of ‘conventionally fuelled’ cars in urban transport by 2030; phase them out in cities by 2050; achieve essentially CO2-free city logistics in major urban centres by 2030. This will spur the use of electric vehicles (EVs) across Europe. Although nowadays the penetration of EVs in the European market is slow albeit steady, the estimated electricity demand from all EVs worldwide was 54 TWh in 2017 [2]. Thus, the growing electrification of the road transport will impact the power system operation and planning of the future and new actors, e.g. aggregator agents, will come into play.

Within the context of restructured power industry, the aggregator agents face several challenges: (i) the forecast of the charging power of the fleet of EVs in the short-term, and (ii) the determination of a bidding curve to participate in the electricity market to maximize their profits when the fleet of EVs is large enough. Moreover, if the EVs account for bi-directional vehicle-to-grid (V2G) capabilities, the aggregator also needs to forecast the EV-fleet discharging power. Short-term load
forecasting is widely applied in power systems to predict the electricity demand (and price) for different granularity levels [3]. In the last years, EV charging load forecasting tools have been proposed in the technical literature by means of ARIMA-based models [4], [5]; machine-learning techniques [6]–[8], such as support vector regression; or big data technologies [9]. All these papers neglected the bi-directional V2G capabilities of the EVs.

In this paper, we apply inverse optimization (IO) to forecast the EV-fleet power. The goal of an IO problem is to infer the optimization model parameters given a set of observed decision variables [10]–[12]. However, few papers have implemented IO in the field of power systems [13]–[17]. Zhou et al. [18] applied IO in the context of generation expansion planning to find an effective incentive policy; Ruiz et al. [19] estimated rival marginal offer prices for a strategic producer in a network-constrained day-ahead market by using IO; Saez-Gallego et al. [20] prescribed an IO approach by using bi-level programming to infer the market bid parameters of a pool of price-responsive consumers; in [21], a novel IO approach was devised to statistically estimate the aggregate load of a pool of price-responsive buildings in the short-term; and, finally, Lu et al. [22] applied IO to estimate the demand response characteristics of price-responsive consumers, as similarly done in [15]. Unlike existing works [13]–[17], we address the EV-fleet power forecasting with an IO approach in which the prediction tool accounts for two distinctive features: (i) the pool of EVs may be equipped with V2G capabilities, and (ii) there may exist a strong nonlinear relationship between the EV-fleet power and the explanatory variables, namely past EVs’ charging/discharging patterns and past electricity prices. To capture these nonlinear relations, we endogenously introduce kernels into the proposed IO approach.

Kernels are widespread in the literature on machine learning [18], [19]. However, its use in power systems is rather limited [20]–[22]. Kekatos et al. [20] applied a kernel regression to forecast the electricity prices from the Midwest Independent System Operator day-ahead market in which the kernel itself is constructed by the product of three kernels: one for vectorial data and other two to account for non-vectorial data such as time and nodal information. This approach was generalized to low-rank kernel-based learning models in [21]. Finally, reference [22] devised a probabilistic forecast method built on the Nadaraya-Watson estimator to predict the electricity prices from the Polish balancing and day-ahead markets.

The contributions of this paper are threefold:

- From a modeling perspective, we provide an IO framework to forecast the aggregate power of a fleet of EVs with V2G capabilities. In addition, the outcome of this framework may be used to bid/offer in the electricity market by using the estimated price-quantity tuples. To the best of the authors’ knowledge, this is the first time in the technical literature that IO has been used to forecast the aggregate power of a price-responsive EV aggregator.

- A novel data-driven approach is used to approximate the solution of the generalized IO problem by solving convex optimization problems. This approach is deemed computationally inexpensive and, as a salient feature of this work, a kernel is endogenously incorporated into the regression functions.

- We thoroughly analyze the performance of the proposed methodology by using real-life data based on the National Household Travel Survey 2017 [23].

The rest of the document is organized as follows: Section II presents the IO methodology; Section III gives a general overview on the comparison methodologies; in Section IV, we analyze a case study for a residential aggregator of EVs; conclusions are duly drawn in Section V and, finally, Appendix A presents the simulator of an EV’s aggregator that we have used to generate synthetic data on the behavior of an EV fleet.

II. INVERSE OPTIMIZATION METHODOLOGY

To put the problem in context, we aim to forecast the EV-fleet power \( p_t \) (also known as aggregate power) in time period \( t \) of an aggregator who may submit a bid into the electricity market. In order to predict its aggregate power, this agent may use past observed data, which are denoted as explanatory variables, features or regressors. The regressor \( r \) in time period \( t \) can be the lagged electricity price \( \lambda_{t-1} \) or the aggregate power \( p_{t-1} \), \( \forall l = 1, 2, \ldots \). In addition, past EV driving patterns, meteorological data, or categorical data (e.g., time information) can also be used for forecasting purposes.

Within this context, this section first introduces the proposed forecasting model in Section II-A. Subsequently, Section II-B explains how we can account for past information. Finally, Section II-C thoroughly describes the two-step procedure to estimate the required parameters of the forecasting model.

A. Forecasting Model

The key idea of this work is to forecast the EV-fleet power by using a simple optimization (linear programming) model which may, to some extent, mimic its real behavior. The formulation of the forecasting model [1] that, we assume, represents the aggregate response of an EV fleet to the electricity prices at time period \( t \), can be mathematically expressed as:

\[
\max_{p_t} \sum_{b \in B} p_b \left( m_b - \lambda \right) \tag{1a}
\]

subject to:

\[
P \leq \sum_{b \in B} p_b \leq \bar{P} : (\beta, \bar{P}) \tag{1b}
\]

\[
0 \leq p_b \leq \bar{E}_b : (\varphi^d_{b}, \varphi^c_{b}), \quad \forall b \in B^c \tag{1c}
\]

\[
\bar{E}_b \leq p_b \leq 0 : (\varphi^d_{b}, \varphi^l_{b}), \quad \forall b \in B^d, \tag{1d}
\]

wherein the time index \( t \) has been dropped for the sake of clarity and dual variables are represented in parentheses after a colon in the respective constraints. For the sake of unit consistency, hourly time periods are considered.

The reconstruction problem [1] aims to maximize the aggregate welfare of the EV aggregator, as given by the objective function (1a). This objective function is made up of the EV fleet’s surplus, which is related to the aggregate surplus, and the EV fleet’s surplus

\[1\] This problem is also known as forward or reconstruction problem in the IO jargon.
charging power. It also accounts for the aggregate discharging power when the variable \( p_b \) is negative. We assume step-wise offer/bid price functions as depicted in Fig. 1. Constraints (1b) represent the lower and upper bounds on the aggregate power. Constraints (1c) impose the lower and upper bounds on each block of the charging power. Likewise, constraints (1d) impose the lower and upper bounds on each block of the discharging power. Note that the total power \( p = \sum_b p_b \).

As previously stated, we want to forecast the EV-fleet power response by solving (1). However, to this end, the set of parameters \( \Phi = \{ E_b, E_b', m_b, P, \mathcal{P} \} \) needs to be estimated. This can be done by using a series of observed values: power \( p' \), electricity prices \( \lambda' \), and other regressors. This fact gives rise to a generalized IO problem, which is highly nonlinear and nonconvex. To deal with such complexity, we apply a methodology that builds on the one first proposed in [16].

In that paper, however, the regression function is linear in their features and may be limited to capture nonlinear relations between the EV-fleet power and the regressors. To circumvent such a caveat, and as one of the salient features of this work, we incorporate kernels into the regression functions. Furthermore, the IO-based forecasting model we propose, i.e. problem (1), allows for power intakes and outputs, unlike the one used in [16]. This extra dose of model flexibility is critical to capture the behavior of an EV fleet with V2G capabilities.

### B. Accounting for Past Information: Kernels

In the realm of machine learning, the kernel functions are rather popular in learning algorithms [13] since they are able to capture nonlinear relationships between the dependent and the explanatory variables. Unlike in [16], where affine functions were used to model the dependence of the parameters of the forecasting model (1) on the regressors, we propose the use of kernel regressions to estimate \( P_t, \mathcal{P}_t \), and \( m_{b,t} \):

\[
P_t = \mu + \sum_{\tau \in \mathcal{T}'} \alpha_{\tau} K_{t,\tau}, \quad \forall t \in \mathcal{T} \tag{2}
\]

\[
\mathcal{P}_t = \mu + \sum_{\tau \in \mathcal{T}'} \alpha_{\tau} K_{t,\tau}, \quad \forall t \in \mathcal{T} \tag{3}
\]

\[
m_{b,t} = \nu_b + \sum_{\tau \in \mathcal{T}'} \rho_{\tau} K_{t,\tau}, \quad \forall t \in \mathcal{T} \tag{4}
\]

Many kernel functions can be used: polynomial, hyperbolic tangent, Gaussian, among others. For the sake of illustration purposes, the Gaussian kernel [19] can be defined as follows:

\[
K_{t,\tau} = K (z_t, z_\tau) = e^{-\gamma \| z_t - z_\tau \|^2_2}, \quad \forall t \in \mathcal{T}, \tau \in \Omega^r, \tag{5}
\]

where \( \gamma \) is a scale parameter inversely proportional to the variance of the Gaussian function; and \( \| z_t - z_\tau \|^2_2 \) is the squared Euclidean distance between two feature vectors at time periods \( t \) and \( \tau \). Thus, the Gaussian kernel can be interpreted as a similarity measure between two time periods, i.e., if the two feature vectors are identical \( z_t = z_\tau \), then the value of \( K_{t,\tau} = 1 \), otherwise its value ranges in the interval \((0, 1]\). For instance, let us assume that \( z_t \) comprises only one regressor, namely the electricity price in the previous time period, i.e., \( z_t = \lambda_{t-1} \). Thus, Fig. 2 provides the values of the kernel for each time period \( t \) of a day with respect to the second time period \( \tau = 2 \) for different values of parameter \( \gamma \). Moreover, the electricity prices for the 24 hours are shown in the figure. We can observe that high values of \( \gamma \) (low variance) lead to kernel values equal to 1 just when the two regressors are very close to each other (e.g., see time periods 21–23 for \( \gamma = 1 \)); conversely, low values of \( \gamma \) (high variance) lead to kernel values equal to 1 even when the regressors are very different from each other (e.g., see values for all time periods when \( \gamma = 0.001 \)).

### C. Two-step Estimation Procedure

The thrust of this work is the estimation of the set of parameters \( \Phi = \{ E_{b,t}, \mathcal{E}_{b,t}, m_{b,t}, P, \mathcal{P} \} \) and the corresponding coefficient estimates \( \mu, \alpha_t, \mu, \alpha_{\tau}, \nu_b, \rho_{\tau} \) of the regression functions described in (2–4). To do that, we use a two-step procedure based on two convex programming problems: (i) the feasibility problem, which is devoted to estimating all parameters that determine the feasibility of the observed EV-fleet power values in the forward problem (1) (i.e., the power bounds), and (ii) the optimality problem, which estimates the marginal utility of the EV’s aggregator, i.e., the parameters of problem (1) that are related to the optimality of the observed power values. The key idea of the feasibility problem is to shape the power bounds \( P_t \) and \( \mathcal{P}_t \) so that a certain percentage \( H \) of the observed EV-fleet power values are feasible for the forward problem (1). Note that the width for the aggregate power blocks \( E_{b,t} \) and \( \mathcal{E}_{b,t} \) can be easily computed from the estimated power bounds by assuming that the energy blocks are all of same length. Conversely, the optimality problem estimates the marginal utilities \( m_{b,t} \) driven by the
minimization of the duality gap of the forward problem once the power bounds are fixed. Its aim is thus to make the observed EV-fleet power values as optimal as possible for problem (1) (recall that we use \( \hat{P} \) as the forecasting model).

1) Feasibility Problem: Given a fixed value of control parameter \( H \in [0, 1] \), this problem can be formulated as:

\[
\begin{align*}
\min \mathcal{F}(f) &= \sum_{t \in \Omega^{tr}} H(\xi_t^+ + \xi_t^-) + \sum_{t \in \Omega^{tr}} (1 - H)(\xi_t^+ + \xi_t^-), \\
\text{subject to:} & \\
\mathcal{P}_t - p_t = & \xi_t^+ - \xi_t^-, \quad \forall t \in \Omega^{tr} \quad (6b) \\
p_t' - \mathcal{P}_t = & \xi_t^+ - \xi_t^-, \quad \forall t \in \Omega^{tr} \quad (6c) \\
\mathcal{P}_t \geq & \mathcal{P}_t \quad \forall t \in \Omega^{tr} \quad (6d) \\
\xi_t^+ \xi_t^+ \xi_t^- \xi_t^- \geq 0, \quad \forall t \in \Omega^{tr}, \quad (6f)
\end{align*}
\]

where \( f \) is the objective function value of \( \mathcal{F} \) and the set of variables to be optimized is \( \mathcal{P}, \mathcal{P}, \xi_t^+, \xi_t^-, \xi_t^+, \xi_t^- \). Problem (6) is a convex program.

The objective function (6a) minimizes the sum of feasibility and infeasibility slack variables associated with the power bounds. Constraints (6b)–(6c) are the power bound constraints with the feasibility and infeasibility slack variables, where \( p_t' \) is the observed EV-fleet power value at time period \( t \). Constraints (6d)–(6f) ensure that the upper bound of the aggregate power is greater than its respective lower bound. Constraints (6c) impose kernel regression functions for the power bounds wherein the coefficients to be estimated are \( \mu, \mu, \alpha, \sigma_t \). Finally, constraints (6f) declare the variables \( \xi_t^+, \xi_t^-, \xi_t^+, \xi_t^- \) as non-negative. Importantly, the higher the value of \( H \), the wider the power bounds delivered by (6) and, therefore, the more price-responsive the EV fleet is expected to be.

The use of kernels increases the flexibility of the regression function when increasing the size of the training set. However, it also tends to over-fitting. To control the risk of over-fitting, a regularization parameter \( M \in [0, 1] \) is used to factor in the sum of the squared values of the coefficient estimates \( \alpha_t \) and \( \sigma_t \), similarly to what is typically done in kernel ridge regression [19]. Thus, the objective function (6b) can be recast as:

\[
\min_{\xi_t} M \sum_{t \in \Omega^{tr}} \left( \xi_t^2 + \sigma_t^2 \right) + (1 - M) f \mathcal{F}.
\]

2) Optimality Problem: Once the power bounds (i.e., \( \mathcal{P}, \mathcal{P} \)) are estimated from (4), we can compute the power block limits \( \mathcal{E}_{b,t}, \forall b \in B^c \) and \( \mathcal{E}_{b,t}, \forall b \in B^d \) based on the assignments described in Table I. The optimality problem can then be derived by using results from duality theory of linear programming and it can be formulated as:

\[
\begin{align*}
\min \mathcal{F} &= \sum_{t \in \Omega^{tr}} \varepsilon_t \quad (8a) \\
\mathcal{P}_t \beta_t - & \mathcal{P}_t \beta_t + \sum_{b \in B^c} \mathcal{E}_{b,t} \mathcal{P}_b + \sum_{b \in B^d} \mathcal{E}_{b,t} \mathcal{P}_b^d - \sum_{b \in B^d} \mathcal{E}_{b,t} \mathcal{P}_b^d \quad (8b) \\
& - \varepsilon_t \sum_{b \in B} \mathcal{P}_b (m_b - \lambda_t), \quad \forall t \in \Omega^{tr} \\
& - \mathcal{P}_b (m_b - \lambda_t), \quad \forall b \in B^c, t \in \Omega^{tr} \quad (8c)
\end{align*}
\]

TABLE I

| Value of \( \mathcal{P}_b \) | \( \mathcal{P}_b \) | \( \mathcal{P}_b \) |
|-----------------|-----------------|-----------------|
| in \( B^c \)    | in \( B^d \)    | in \( B^d \)    |
| \( \mathcal{P}_b \) | \( \mathcal{P}_b \) | \( \mathcal{P}_b \) |
| \( \mathcal{P}_b \) | \( \mathcal{P}_b \) | \( \mathcal{P}_b \) |
| \( \mathcal{P}_b \) | \( \mathcal{P}_b \) | \( \mathcal{P}_b \) |

III. COMPARISON METHODOLOGIES

We compare the performance of the proposed kernel-based IO approach, hereinafter referred to as \( \text{kio} \), against (i) the state-of-the-art model to forecast the EV-fleet power, namely support vector regression (\( \text{svr} \)), (ii) a kernel-ridge regression model (\( \text{krr} \)), (iii) an IO approach with linear kernels (\( \text{llo} \)), and (iv) persistence or naive models. Note that we use a Gaussian kernel in the regression functions of the feasibility problem and a linear kernel in the regression function of the optimality problem, as this combination exhibited the best trade-off between forecasting performance and simplicity in our numerical experiments.
Regarding the svr and krr, we respectively use the epsilon-svr and the kernel-ridge regression models implemented in the scikit-learn library [24] under the Python programming language. The interested reader is referred to [25] for a detailed description of the svr. For the sake of comparison, we also use the Gaussian kernel and we tune the corresponding hyperparameters via grid search. Specifically, we tune the cost of constraints violation C and the parameter associated with the kernel γ for svr; and the penalty parameter δ and the γ parameter for krr.

Regarding the naive models, we use three different ones since the EV-fleet power may experience seasonal patterns: \( h\text{-naive}, d\text{-naive}, \) and \( w\text{-naive} \), in which the forecast value of the aggregate power at time \( t \) is equal to the observed value at time \( t-1 \), \( t-24 \), and \( t-68 \), in that order.

The performance of the methods is compared with two metrics: the mean absolute error (MAE) and the root mean square error (RMSE) on the test set.

IV. CASE STUDY

A. EV-fleet Data

To our knowledge, there is no real-life data available about an EV’s aggregator. Thus, we resort to the formulation of an optimization problem to simulate the behavior of such an EV fleet. The interested reader is referred to Appendix A for a detailed description of this simulator.

We assume a residential aggregator with 100 EVs. For the sake of simplicity, the technical parameters associated with each EV are identical: The maximum charging rate is 7.4 kW, the round-trip efficiency is 0.95, the minimum and maximum energy rates are 10 and 51 kWh, in that order, and the energy rating per kilometer is 0.137 kWh/km [26]. Due to the lack of real-life data about the parameters associated with the driving patterns (availability profiles and energy required for transportation) of EVs, we resort to the National Household Travel Survey (NHTS) 2017 [23]. From this data base, we can extract the availability status by using the departure/arrival time periods for each daily trip. Specifically, we assume that the EV is available until it begins its first daily trip and after it returns from its last daily trip for each day of the year. Otherwise the EV is unavailable and thus it may be in a motion status. The energy required for transportation \( \chi_{v,t} \) can be computed as the product of the travelled distance and energy rating per kilometer (i.e., 0.137 kWh/km).

The electricity prices are obtained from the ENTSO-e Transparency Platform [27] in year 2018 in Spain. We also assume that the load shedding cost \( C^S = 1000 \) €/kWh. We run daily simulations with 15-min time steps to build a synthetic database for a pool of EVs.

The simulations have been performed on a Linux-based server with one CPU clocking at 2.6 GHz and 2 GB of RAM using CPLEX 12.6.3 [28] under Pyomo 5.2 [29]. Optimality gap is set to 0%.

B. Forecast Results without Enabling V2G Capabilities

We assume that EVs do not enable their V2G capabilities (i.e., \( B^l_0 = 0 \) in the model (9) in the Appendix A) and we will compare the results for three cases: (i) a case in which the EVs satisfy their energy needs by using a naive charging; (ii) a case in which the charging is highly synchronized, which occurs when \( C^S \) is set to 0 in (9); and (iii) a case in which the charging synchronization is avoided, which we attain by setting \( C^S = 520 \) €/MWh². Those cases are respectively denoted as naive-ch, sync, and non-sync. Note that, in the former case, i.e. naive-ch, each EV will be charged to its required maximum energy as soon as it is available, thus neglecting the dependence of the charging power on the price; whereas, the latter cases sync and non-sync are driven by the cost minimization of the EV’s aggregator wherein the electricity prices are accounted for. As an example, Fig. 3 shows the EV-fleet charging power of a certain day for the three cases along with the electricity prices. As can be seen, the choice of \( C^S \neq 0 \) is a simple albeit convenient way to avoid the undesirable charging synchronization by smoothing the aggregate power. In addition, we can observe that the charging pattern of case naive-ch is independent of the prices.

The sizes of the training, validation, and test sets are 672, 168, and 168, in that order. Fig. 4 represents the hourly electricity price versus the corresponding charging power for all periods of the \( \Omega^{tr} \) for the cases mentioned above. As can be seen, the aggregate power of the non-sync case depends linearly on the price, unlike cases naive-ch and sync. For the case naive-ch, we consider 17 regressors, namely the charging power and the total number of EVs available for the six periods previous to time \( t \), i.e., \( p_{t-1} \) and \( \sum_{v} s_{v,t-1} \), \( \forall l = 1...6 \), and 5 binary-valued categorical variables to indicate the hour of the day. For the cases sync and non-sync, we consider 12 regressors, namely the electricity price and the charging power for the six periods previous to time \( t \), i.e., \( \lambda_{t-1} \) and \( p_{t-1} \), \( \forall l = 1...6 \). We also assume six energy blocks in total. Finally, hyper-parameter \( H \) ranges in the interval [0.5, 1.0] with 0.01 steps, \( M \) ranges in the interval [0.0001, 0.0024] with 0.0001
other machine-learning techniques such as \textit{krr} the performance of fleet power and the electricity price shown in Fig. 3. Finally, \textit{pio} is able to capture the nonlinear relations between the EV-lio by reducing RMSE and MAE by 40.6% and 42.2% since As expected, we can also observe that the \textit{kio} which provides the best performance among the naive models.

Second, it should be noted that the optimal values of current electricity price on the aggregate power of the EV fleet. Expected as the marginal utilities encode the impact of the bounds for the former cases are wider than for the latter one. Therefore, the \textit{optimality problem}, which is used to estimate the marginal utility, plays a major role to forecast the aggregate response of the EV fleet for the price-driven cases. This is expected as the marginal utilities encode the impact of the current electricity price on the aggregate power of the EV fleet. Second, it should be noted that the optimal values of \(H^*\) for the models \textit{kio} and \textit{pio} are quite similar, except for the case \textit{naive-\textit{chio}}, for which \textit{pio} is unable to identify the insensitiveness of the aggregate power on the price.

The error metrics of the test set for all models are compared in Table III for the three cases. In the \textit{naive-\textit{chio}}, the least RMSE is obtained with the proposed model \textit{kio} with an error reduction of 4.4% and 17.3% compared to \textit{krr} and \textit{svr}. In the \textit{sync} case, the proposed model \textit{kio} achieves 28.3% reduction in RMSE and 15.3% reduction in MAE compared to the \textit{w-naive}, which provides the best performance among the naive models. As expected, we can also observe that the \textit{kio} outperforms \textit{lio} by reducing RMSE and MAE by 40.6% and 42.2% since \textit{kio} is able to capture the nonlinear relations between the EV-fleet power and the electricity price shown in Fig. 3. Finally, the performance of \textit{kio} is comparable to the performance of other machine-learning techniques such as \textit{krr} or \textit{svr} in the \textit{non-sync} case, the aggregator behaves as a price-responsive EV fleet with a linear dependence and thus both \textit{kio} and \textit{lio} models achieve the least errors in the \(\Omega^{\text{ext}}\) compared to the other benchmarks. Note also that, in this case, the \textit{h-naive} is the one with the least error among the naive models. However, the RMSE of the \textit{kio} is decreased by 51.3%, 25.7%, and 27.6% with respect to the one attained with the models \textit{h-naive}, \textit{krr}, and \textit{svr}, in that order. Overall, the \textit{kio} model is characterized for being versatile since it makes good predictions under any pattern of the EV-fleet power with the price.

Apart from the improvement in terms of RMSE and MAE of the \textit{kio} against the rest of the models to forecast the EV-fleet power, the proposed approach is able to provide a bidding curve, as imposed by rules in electricity markets. Figures 5-7 show the results for cases \textit{naive-\textit{chio}}, \textit{sync}, and \textit{non-sync}, respectively. In Fig. 5(a) and 7(a), we show the estimated marginal utility prices for the six blocks for each hour of the first day of the \(\Omega^{\text{ext}}\) and for the cases \textit{sync} and \textit{non-sync}. Correspondingly, Fig. 6(b) and 7(b) depict the estimated bounds as well as the forecast and observed EV-fleet power.

| Case       | \(kio\) | \(krr\) | \(svr\) | \(pio\) |
|------------|---------|---------|---------|---------|
| naive-\textit{chio} | \(H^* = 0.64\) | \(\delta^* = 0.01\) | \(C^* = 100\) | \(H^* = 0.91\) |
| sync       | \(M^* = 0.0002\) | \(\gamma^* = 0.1\) | \(\gamma = 0.01\) |
| non-sync   | \(H^* = 0.82\) | \(\delta^* = 0.1\) | \(C^* = 10\) | \(H^* = 0.89\) |

| Model      | \textit{naive-\textit{chio}} | \textit{sync} | \textit{non-sync} |
|------------|-------------------------------|---------------|-------------------|
| \textit{kio} | 8.6 | 3.7 | 35.2 | 13.3 | 3.8 |
| \textit{krr} | 9.0 | 3.5 | 35.5 | 15.7 | 7.4 | 5.2 |
| \textit{svr} | 10.4 | 5.7 | 41.7 | 14.7 | 7.6 | 5.0 |
| \textit{pio} | 16.8 | 6.4 | 59.3 | 23.0 | 5.9 | 3.9 |
| \textit{h-naive} | 90.3 | 29.3 | 72.7 | 25.3 | 11.3 | 7.1 |
| \textit{d-naive} | 13.2 | 4.8 | 64.8 | 22.3 | 17.3 | 13.3 |
| \textit{w-naive} | 10.8 | 4.6 | 49.1 | 15.7 | 13.0 | 9.1 |
for such a day.

In the naive-ch case, the kio provides coincident power bounds, as illustrated in Fig. 5, which means that the optimality problem (i.e., the marginal utility estimation problem, which captures the price effect) is useless and thus the aggregate charging power can be directly explained by estimating the bounds. In Fig. 6(a) and 7(a), we can observe that the kio model identifies whether the EV-fleet power is price-responsive or not by assigning different values to the marginal utility for each block. On the one hand, in Fig. 5(a), the blockwise marginal utilities are almost identical at any time period, thus suggesting that the EV fleet is almost inelastic for the sync case. In this case, the power bounds are basically shaping the EV-fleet charging forecast. On the other hand, for the non-sync case, the bounds are generally wider than those obtained for the sync case (see Fig. 7(b)). The marginal utility is thus shaping the aggregate power forecast since the kio model gives rise to a wider range of marginal utility values at any time period, as can be observed in Fig. 7(a). In short, unlike any other forecasting tool, we gain interpretability with the proposed IO approach kio due to two aspects: (i) the width of the bounds, which sheds light on the price-responsive-ness of the EV fleet; and (ii) the derivation of a bidding curve when there exists a dependence of the EV-fleet power on the price.

C. Forecast Results with V2G Services

We now assume that EVs may enable their V2G capabilities (i.e., $B^d_v \neq 0$ in the model (9)), and we will compare the results for two cases: (i) a highly-synchronized power case for $C^S = 0$; and (ii) a case in which the power synchronization is avoided for $C^S = 52$ kE/MWh^2. Those cases are denoted as sync and non-sync. The problem setup is identical to that explained in Section IV.B Table IV provides the error metrics on the $\Omega^{test}$ for all models. As can be seen, kio clearly outperforms by far the lio and naive models for both cases. Notwithstanding, the performance of lio in terms of error is closer to the proposed approach for the non-sync case because the EV-fleet power is more price-responsive. Also, the performance of kio is similar to the machine-learning techniques krr and svr in the case sync; and the RMSE (MAE) decrease by 4.8% and 5.9% (11.4% and 6.7%) compared to krr and svr, respectively, in the case non-sync.

V. Conclusions

This paper proposes a data-driven two-step estimation procedure relying on two main concepts: inverse optimization and kernel regression. This novel approach allows to capture the nonlinear relationship between an aggregate price-response and the associated explanatory variables, while deriving a bidding/offering curve, as imposed by rules in electricity markets. We apply such a framework to forecast the aggregate price-response of an EV fleet. The proposed approach attains a better performance (around 20%–40% error reduction) than naive or linear models. Moreover, it achieves a similar or better (depending on the case) performance than state-of-the-art machine-learning techniques such as support vector regression or kernel-ridge regression. Overall, the proposed approach is versatile since its performance is good regardless of the price-power relation and it increases the degree of interpretability of the prediction model.

APPENDIX A

AGGREGATOR OF ELECTRIC VEHICLES

To simulate the behavior of a pool of EVs, i.e., its aggregate power, we assume an aggregator of EVs in residential districts who aims to minimize their total costs. This can be mathematically expressed as:

$$\min_{\Xi^v} \sum_{v \in \mathcal{V}} \left( \lambda_1 \Delta p_t + \sum_{v \in \mathcal{V}} \left( C^D_{v,t} + C^P_{s_v,t} \right) + C^S \Delta t^2 P^2_t \right)$$

subject to:

$$p_t = \sum_{v \in \mathcal{V}} (c_{v,t} - d_{v,t}), \quad \forall t \in \mathcal{T}$$

$$s_{v,t} = s_{v,t-1} + \Delta t \left( \eta_{c,v,t} - \frac{d_{v,t}}{\eta^{c,v,t}} \right) - \chi_{c,v,t} + s_{v,t}$$

$$\forall v \in \mathcal{V}, t \in \mathcal{T}$$

$$0 \leq c_{v,t} \leq B^c_{c,v,t}, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

$$0 \leq d_{v,t} \leq B^d_{d,v,t}, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

$$SOC_{v,t} \leq s_{v,t} \leq SOC_{v,t}, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

$$s_{v,N_T} = s_{v,0}, \quad \forall v \in \mathcal{V}$$

TABLE IV: Error Metrics – Cases with V2G Services (kW)

| Model   | sync | non-sync |
|---------|------|----------|
| RMSE    | MAE  | RMSE     | MAE   |
| kio     | 148.6| 94.3     | 35.3  | 20.9  |
| krr     | 146.9| 108.4    | 35.2  | 23.6  |
| svr     | 147.1| 92.4     | 35.6  | 22.4  |
| lio     | 172.1| 120.0    | 36.2  | 23.7  |
| h-naive | 235.4| 142.2    | 49.5  | 30.0  |
| d-naive | 261.8| 162.5    | 71.1  | 50.2  |
| w-naive | 199.5| 112.3    | 60.4  | 37.7  |
where the set of decision variables $\Xi_{\text{EV}} = \{v_{\text{v},t}, p_{\text{v},t}, \varphi_{\text{v},t}, S_{\text{v},t}, \text{SOC}_{\text{v},t}\}$. $V$ is the set of EVs in the fleet, $T$ is the set of time periods. The variable $p_{\text{v},t}$ represents the power the aggregator buys in the electricity market whereas the variables $v_{\text{v},t}$ and $d_{\text{v},t}$ represent the charging power from and discharging power to the grid of EV $v$ in period $t$. The variable $C_{\text{v},t}^D$ represents the cost of battery degradation due to motion and charging/discharging cycle of EV $v$ in period $t$. The variables $s_{\text{v},t}$ act as a load shedding term when the energy balance of the EVs cannot be satisfied. Finally, $\text{SOC}_{\text{v},t}$ is the state of charge of the battery of EV $v$ in period $t$. In addition, $\lambda_t$ is the electricity price in period $t$; $\Delta_t$ is the time step; $C^{PF}$ is the load shedding cost; $C^S$ is a penalty cost to avoid power synchronization; $\gamma_{\text{v}}$ is the charging (discharging) efficiency for the EV $v$; $\chi_{\text{v},t}$ represents the energy required for transportation of each EV throughout the time horizon; $B_{\text{v}}^u$ and $B_{\text{v}}^d$ are the maximum charging and discharging power of EV $v$, respectively; $s_{\text{v},t}$ represents the availability of the EV $v$ in period $t$; $\text{SOC}_{\text{v},t}$ and $\text{SOC}_{\text{v},t}$ are the minimum and maximum limits of the energy state of charge of EV $v$ in period $t$; $N_T$ is the number of time periods; $F_t$ is the degradation cost per kW due to charging-discharging cycles and it depends on the battery cost of EV $v$; and $A_{\text{v},t}$ is the degradation cost due to motion of EV $v$ in time period $t$.

The problem \(9\) aims to minimize the total costs as given in \(9a\), which comprise four terms: (i) the operational costs due to charging from and discharging to the grid, (ii) the degradation costs of the vehicles’ batteries, (iii) the load shedding costs when the equation associated with the energy state-of-charge evolution is violated, and (iv) the penalty costs to avoid power synchronization that may lead to overloads in the distribution network [30]. Constraints \(9b\)–\(9c\) relate the power bought in the electricity market with the charging and discharging power. Constraints \(9d\) model the energy state of charge evolution while taking into account the energy required for transportation. Expressions \(9g\) and \(9h\) impose the lower and upper bounds for the charging and discharging power, in that order. Constraints \(9i\) set the lower and upper bounds for the energy state of charge of the EVs. Expressions \(9g\) enforce boundary conditions on the energy state-of-charge of the EVs. Expressions \(9h\) model the battery degradation costs based on the motion status and the discharging energy, as described in \(9i\). Finally, constraints \(9i\) define the non-negativity character of the decision variable $s_{\text{v},t}$.

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