Milestone developments in quantum Information and No-Go theorems

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In this article we present milestone developments in quantum information from historical perspectives. The domain of quantum information is very promising to develop quantum computer and varieties of quantum applications. We start the discussion on milestone developments with the era 1970s and finish the discussion with 2018. We also give the light on experimental manifestations of major theoretical developments. Further, we present important no-go theorems frequently used in quantum information along with their respective mathematical proofs.

I. INTRODUCTION

A breakthrough in information theory has been done by Shannon in 1948\cite{1, 2}. The classical information theory has its practical manifestations in communication systems, computing devices, gaming, imaging and with several countless applications in real world. The current developments in technology is unbelievable without the existence of information theory. Before the establishment of information theory, there was also a major development in 1947; it is known for the invention of transistor\cite{3}, which changed the whole electronics industry. The breakthrough developments known for transistor and information theory became the building blocks for revolutionary changes in science technology. Recently emerging quantum information theory overwhelming its applications in many domains like quantum computation, quantum communication, quantum cryptography, quantum imaging, quantum gaming and many others. The formulation of quantum information theory is based on the postulates of quantum mechanics along with the fundamental ingredients as superposition and entanglement. The efforts to develop quantum computer are on the way by using several physical techniques. Many companies in the market are eagerly applying efforts to push this area towards commercialization and to develop quantum computer with different physical approaches like NMR, Bose-Einstein condensation, Super conducting approach, Ion trap system, Ultra cold atoms, Majorana fermion etc. So there is a race among all the big companies to capture the existence of quantum computer, but still the universal quantum computer...
is missing. For practical applications, we need physical systems to store and to process the information. The microscopic world of various types of qubits is the basic physical system which support to process the quantum information. We may fire a natural question, can we store and process the information in these physical systems more efficiently than classical one? And what are the physical constraints responsible to execute the quantum information? The efficient Information storage and its processing in the microscopic world can be handled by the principles of quantum mechanics. It is always interesting to discover the feasible and non-feasible physical situations which play the important role to execute the quantum information and designing the quantum protocols. So, investigating the situations which are not possible is also an important paradigm. These impossible physical conditions are expressed by no-go theorems. Before applying the fruitful efforts to develop any quantum application it is always good to keep in view the structure of no-go theorems; which is always helpful to tackle the feasibility and non-feasibility of physical situations. On the other hand, towards the development of quantum computer, there are always challenges to manipulate and control the qubits and to protect these from decoherence. The phenomenon of decoherence is the killer of superposition in quantum systems and restrict to perform perfect quantum computation. But gradual efforts in quantum information are on the way to tackle the problem of efficient quantum information manipulation in varieties of physical systems. Recent developments in quantum computation is very progressive and rapid than the past historical developments. So, in this direction it is very important to understand the gradual milestone developments and track these; which may be useful to perform further progress in the future. In the following sections, we present major developments with theoretical aspects and touching the experimental discussions as well. We managed the time periods of developments in slots. The first time slot is dealing with 10 years and rest of the time slots deal with 9 years each to maintain the continuity. In these slots we presented gradual milestone developments over the corresponding time period of the slot. Here it is mention that, we are emphasized to discuss the milestone developments only, however there may be many others developments in parallel during each era. The sections begin with the era of 1970s; when the emergence of computation has been modeled with the concept of reversible computation and tour the article with the important major developments till the date mentioned as 2018.

II. DURATION (1970-1980)

This period is most significant for the theoretical development in quantum information and known for producing the idea of reversible computation by C. H. Bennett and famous Holevo’s theorem. Inspiring idea of reversible computation has been carried out by Toffoli to invent the first reversible quantum gate. This gate is called as CNOT gate, which is also known as reversible XOR gate; this development became the foundation of quantum circuit
Another milestone development in the same era is the foundation of Holevo bound. Alexander Holevo has established the upper bound of an amount of information that can be contained in a quantum system by using the particular ensemble. After three years of publishing Holevo bound, one of the first attempts to create the quantum information theory is made by Roman Stanislaw Ingarden; a Polish mathematical physicist; by publishing a seminal paper entitled as “Quantum information theory” in 1976. This work generalizes Shannon’s information theory in the formalism of quantum mechanics of open systems. With the progress of quantum information theory, the idea towards quantum computing is proposed by Yuri Manin in 1980 in his book entitled as “Computable and Uncomputable”. The work done by Yuri Manin opened the further research avenues in quantum computation.

III. DURATION (1981-1990)

The time period (1981-1990) deals with the milestone development of no-cloning theorem. In 1982, a major result of no-cloning in quantum physics is discovered by William Wootters and Wojciech Zurek and independently by Dennis Dieks. The no-cloning theorem states that, it is not possible to clone an unknown quantum state. This theorem became the milestone for quantum information. We are inclined to discuss this theorem with its proof in sect (7.2). With this major development, Paul Benioff proposed a first theoretical model for quantum computation based on quantum Hamiltonian. He did first attempt to quantize the Turing machine and the framework of quantum Turing machine has taken place. The concept of entanglement has already taken birth during 1935 and 1936 with the debate of Albert Einstein and Erwin Schrödinger. The advantage of entanglement and no-cloning theorem together captured the discovery of quantum cryptography done by Artur Ekert in 1991. The development of quantum cryptography open the new file of secure quantum communication, which is very promising to this date.

IV. DURATION (1991-2000)

This era of this period extensively contributes in the development of entanglement-based quantum algorithms. In 1992, David Deutsch and Richard Jozsa proposed a deterministic quantum algorithm to test weather a function is balanced or constant by using black box model in quantum computation. With the continuation of this work, a first milestone quantum algorithm is formulated by Peter Shor at Bell Labs, New Jersey in 1994 and published in 1997. The algorithm allowed a quantum computer to factor an integer very fast and run in polynomial time. This algorithm is quite useful to break the public-key cryptographic schemes as RSA scheme. Meanwhile, to the developments on quantum algorithms, Peter Shor and Andrew Steane proposed the schemes for quantum error correc-
tions in 1995[22, 24]. Quantum error corrections protocols are used to protect the quantum information from decoherence and essentially needed for quantum computation. After the discovery of Peter Shor algorithm, Lov Grover invented the quantum database search algorithm in 1996[25] at Bell Labs. Which is the fastest database search algorithm and landmark in quantum computation. Here we mention that the period of (1990-1997) has been recognized as the golden period for theoretical as well as experimental developments in quantum computation. Beside the quantum algorithms development, there is also an important protocol discovered called quantum teleportation, which is proposed by C. H. Bennett et al. in 1993[26]. The same has been experimentally verified in 1997[27]. During 1997 onwards the scientific community strongly focused on experimental manifestations of quantum information around the world. The first experimental approach to realize the quantum gates by using nuclear magnetic resonance (NMR) technique is performed by Neil Gershenfeld and Isaac L. Chuang in 1997[28]. NMR technique came out as a useful resource to produce fruitful experimental manifestations of quantum computation. In 1998, the first execution of Deutsch-Jozsa algorithm was performed by using NMR technique, this has been done by Jonathan A. Jones and Michele Mosca at Oxford University and shortly after by Isaac L. Chuang at IBM’s Almaden Research Center together with co-workers at Stanford University and MIT[29]. In the same year Grover’s algorithm also experimentally verified with NMR quantum computation[30]. This experimental development encouraged the further investigations. Beside the theoretical and experimental manifestations it was also major interest to look into some physical situations which are not feasible like in non-cloning theorem. Towards this direction, in 2000, there is one important quantum no deleting theorem is proved by Arun K. Pati and Samuel L. Braunstein, which states that given two copies of arbitrary qubits one cannot delete a copy of an unknown qubit. This theorem has its own important implications in quantum information[31]. We are inclined to discuss this theorem in sect (7.3).

**V. DURATION (2001-2010)**

This period is well known for the role of quantum optics in quantum information, in parallel with another major developments towards the implementation of quantum networks. In 2001, first experimental execution of Shor’s algorithm at IBM’s Almaden Research Center and Stanford University was implemented by using NMR technique[32]. The number 15 was factored by using $10^{18}$ identical molecules in NMR. In the same year, the scenario of optical quantum computing has been started. Emanuel Knill, Raymond Laflamme and Gerard Milburn showed that optical quantum computing is possible with single photon sources, linear optical elements and single photon detectors[33]. They also have shown that quantum teleportation can be performed with beam splitters by using photonic qubits. Their contribution opened the avenues of usage of optics in quantum information. The role of
optics is very promising now a days to establish long distance quantum communication. The implementation of quantum gates with optics is an essential requirement to perform quantum computation. In this direction, quantum controlled-Not gates using linear optical elements has been developed by Todd D. Pittman and collaborators at Applied Physics Laboratory, Johns Hopkins University in 2003. The similar results have been produced independently by Jeremy L. O’Brien and collaborators at the University of Queensland. Quantum optics not only had its applications in quantum cryptography but DARPA Quantum network also became operational by using optical fibers supporting the transmission of entangled photons. Quantum networks use the protocol called quantum repeater for long distance quantum communication to overcome with the decoherence. These quantum repeaters transmit the quantum states to receiver with the help of quantum memories. The recognizable framework of quantum optics with atom-photon interaction proved to be a successful framework and assisted to develop quantum memories, which are essential to establish quantum Internet. In 2005, Harvard University and Georgia Institute of Technology; researchers succeeded in transferring quantum information between “quantum memories” from atoms to photons and back again. Along with the advancement of quantum networks, the concept of distributed quantum computing has taken place and a protocol called quantum teleportation is proposed by M. Murao et al. in 2006. This is the protocol in which the optical clones of an unknown quantum state are created and distributed over distant parties. Samuel L. Braunstein at the University of York along with the University of Tokyo and the Japan Science and Technology agency gave the first experimental demonstration of quantum teleportation in 2006. Quantum networks and quantum repeaters attracted much attention of quantum community, hence along this line of research the concept of entanglement swapping is developed by Stefano Pirandola et al. in 2006, which has its important application in quantum repeaters. Beside the developments on quantum memories by using the optical techniques, there was also interest to develop the same by using the condensed matter approach. It is done in 2007 by using the Bose-Einstein condensation. Till 2007, the experimental manifestation of two qubits entanglement is successfully performed, but entanglement in hybrid systems also attracted the attention of quantum community. Much progress has been done in 2008 to perform photonic qubit-qutrit entanglement. In the direction of implementation of quantum networks and towards the reality of quantum Internet, the logic gates have been implemented in optical fibers by Prem Kumar, which became the foundation of quantum networks. Apart from quantum networks the quantum community also shown the interest in developing quantum processors mainly by using two approaches, solid state and quantum optics. Along the line of research on quantum processors, a breakthrough is achieved for the development of spin-based electronics in silicon and a model of quantum transistor, which is inspired with the work entitled as “Single atom transistor” done in 2004. Towards the development on quantum processors, there is also one more important proposal in the year 2007 by D-Wave Systems which proposed 28 qubits quantum computer based on quantum
annealing[49]. The quantum annealer has experimental manifested now and commercially available. The race of developing quantum processors also started by using optical techniques. The ions were trapped in the optical trap and the two-photon optical chip was developed in 2009[50]. Along the line of research on quantum optics and its applications in quantum information, the experimental manifestations of quantum algorithms were still on the way by using new emerging quantum techniques. With the advancement of photonic chip in 2009, in the same year scientific community implemented Shor’s quantum factoring algorithm on a photonic chip[51]. First time the use of Deutsch’s Algorithm in a cluster state quantum computer is achieved in 2007[52]. Till 2000, the entanglement was the major quantum correlation to execute quantum information, on the other hand another important quantum correlation called quantum discord has been discovered by H.Ollivier and W. H. Zurek in 2001[53]. Quantum discord is a measurement based quantum correlation, which also has its role in quantum information and further experimental investigations in terms of its applications are on the way.

VI. DURATION (2011-2018)

The continuity of past developments in quantum information and its experimental manifestations are maintained in this era with two major center of interest; how to develop efficient quantum processors and how to increase the coherence time in quantum systems? On the other hand few past records also broken in this era. With this continuity, In 2011, the von Neumann’s architecture was employed in quantum computing with superconducting approach[54]. This work contributes in developing quantum central processing unit that exchanges the data with a quantum random-access memory integrated on a chip. There was a breakthrough in 2014 as the scientists transfer data by quantum teleportation over a distance of 10 feet with zero percent error rate, this was a vital step towards a feasible quantum internet[55]. In the same year, Nike Dattani and Nathan Bryans break the record for factoring the largest number 56153 using NMR by using 4 qubits only on a quantum device which breaks the record established in 2012 for factoring the number 143[56]. After a long journey for development in quantum computation still there are many theoretical and experimental open problems inherited in the essence of quantum information. One of the major issues is controlling entanglement and its manipulation in many quantum systems and protect it from decoherence[57]. There have been gradual efforts to increase the coherence time in 2015, the coherence time has been increased up to six hours in nuclear spins[58]. With the advancement of quantum processors, there is breakthrough in 2017 by D wave systems. The company developed commercially available quantum annealing based quantum processor, which is fully functional now and have been used for varieties of optimization problems[59] and has applications in quantum machine learning. With the connection of improving coherence time and deeper theoretical investigations on entanglement, here we
mention that entanglement is a fragile phenomenon and very sensitive to quantum measurements and environmental interactions. It may die for a finite time in a quantum system and alive again as time advances. This phenomenon is called entanglement sudden death (ESD) which is investigated by Yu-Eberly[60–67]. The phenomenon of ESD is a threat to quantum applications, so overcoming from it is again an issue and needs fruitful solutions. During the period 2011-2018, there is vast research on entanglement and related aspects such as distillable entanglement and bound entanglement in quantum information theory[68]. Quantum community has investigated various mathematical tools of entanglement detections and quantification, distillable protocols, monogamy of entanglement[69–72]. However, these aspects are lacking for higher dimensional quantum systems. The efforts of quantum community is always to search the quantum systems which can sustain long coherence time, which is an important topic of research.

VII. DEVELOPMENT ON NO GO THEOREMS

Under this section we provide the developments for important no-go theorems in quantum information. No-go theorem implies the impossibility of a particular physical situation. These theorems have the major impact on the experimental development of quantum information. All these theorems are developed by taking the linear property of quantum mechanics. Here in the following subsections we discuss important no-go theorems with their corresponding proofs.

A. Bell’s theorem

Bell’s theorem[73] is the no go theorem, which has its own beauty in quantum mechanics. This theorem states that, “No physical theory of local Hidden Variables can ever reproduce all of the predictions of Quantum Mechanics”. Bell’s theorem has strong connection with EPR paradox. As per Einstein[74], a particle must have a separate reality independent of the measurements. It means, an electron has spin, location and so forth even when it is not being measured, this is called realistic point of view. On the contrary there is another point of view called as orthodox view, which state that, the measurement is responsible to create the attribute of the particle, which do not exist before measurement. Here we express EPR paradox with an example which points towards the elements of reality. Let consider a splitting of pi meson in two anti particles so called electron (e) and positron (p). Conservation of angular momentum demands that total spin will be zero, so the electron and positron composite wave function should be in singlet configuration as below,

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle) \] (1)
Where \( |0\rangle \) is the spin down state and \( |1\rangle \) is the spin up state of particles. If one needs to perform the measurement on electron (e) and positron (p) with their respective detectors \( D^e \) and \( D^p \). Here we assume that both the detectors are in same direction for both particles. The measured state of either particle is just opposite to the another particle and vice versa. The measurement on one particle influence the state of other particle, but that influence do not travel faster than the speed of light, this is called "locality principle". EPR named it "spooky action at a distance". With this phenomenon, EPR expected that there are hidden variables \( (\lambda) \) associated with the wave function \( (\psi) \), which form the "elements of reality" [75]. But we do not have any information how to calculate these hidden variables or measure these. Bell looked into this problem in different way and concluded that hidden variables are unreasonable and this is matter of opinion which does not have any proof. As per Bell’s opinion the existence of local hidden variables needs specific requirements which is not clearly obvious. Bell’s established the famous inequality [76]; so called Bell’s inequality, which is related to electron spin. He generalized the measurement tests done in EPR experiment. He oriented the detectors \( (D^e, D^p) \) in different directions rather than a fix direction for both the particles electron (e) and positron (p). He allowed them to rotate independently and measured the average values of the product of the spins with the orientation angle between the detectors. Let we make the experimental setup of the detectors \( (D^e_1, D^p_2) \) along the directions of unit vectors \( (m, n) \) respectively. We collect the measurement data \( (d^e_i, d^p_i) \) for each measurement \( (1 < i \leq l) \) and calculate the product \( (d^e_i . d^p_i) \). Now we find the average of this product as

\[
P(m, n) = \frac{\sum_{i=1}^{l}(d^e_i . d^p_i)}{l}
\]

Here we write, the measurement data takes the following values.

\[
(d^e_i, d^p_i) = (\pm 1, \pm 1)
\]

Let suppose, both the detectors are parallel ie. \( (m = n) \), than we can have the average of the product as,

\[
P(m, m) = -1
\]

If the detectors are anti parallel \( (m = -n) \), then the average of the product is given by,

\[
P(m, -m) = +1
\]

For arbitrary orientations of the detectors, we can write as,

\[
P(m, n) = -m.n
\]

Where \( (.) \) is the dot product between the unit vectors \( (m, n) \). The above Eq. 6 is the general prediction of quantum mechanics. This prediction is disproved by Bell’s inequality and prove the impossibility of theory of hidden variables. To proceed the proof of Bell’s
inequality let assume, there exists hidden variable(s) \((\lambda)\), which may vary and may not be controlled. So there exists the functions for the measurement directions \((m, n)\) such as,

\[ M(m, \lambda) = N(n, \lambda) = \pm 1 \]

With the conditions,

\[ [M(m, \lambda)]^2 = [N(n, \lambda)]^2 = 1 \]  \hspace{1cm} (7)

If both the detectors are aligned than the results are anti-correlated, hence we can write,

\[ M(m, \lambda) = -N(m, \lambda), \quad \forall \lambda \]  \hspace{1cm} (8)

or

\[ M(n, \lambda) = -N(n, \lambda), \quad \forall \lambda \]  \hspace{1cm} (9)

Simply the average of the product of the measurements can be written as below,

\[ P(m, n) = \int g(\lambda)M(m, \lambda)N(n, \lambda)d\lambda \]  \hspace{1cm} (10)

Where \(g(\lambda)\) is the probability density distribution of the of hidden variables \((\lambda)\), which satisfy the normalization condition

\[ \int g(\lambda) = 1 \]  \hspace{1cm} (11)

By using Eq.9, the Eq. 10 can be re-written as,

\[ P(m, n) = -\int g(\lambda)M(m, \lambda)M(n, \lambda)d\lambda \]  \hspace{1cm} (12)

Let we assume another arbitrary unit vector \(r\), so we can write another measurement average equation as,

\[ P(m, r) = -\int g(\lambda)M(m, \lambda)M(r, \lambda)d\lambda \]  \hspace{1cm} (13)

Subtracting the Eq 13 from Eq. 12, we get,

\[ P(m, n) - P(m, r) = -\int g(\lambda)[M(m, \lambda)M(n, \lambda) - M(m, \lambda)M(r, \lambda)]d\lambda \]  \hspace{1cm} (14)

\[ = -\int g(\lambda)[M(m, \lambda)M(n, \lambda) - M(m, \lambda)(1)M(r, \lambda)]d\lambda \]  \hspace{1cm} (15)

\[ = -\int g(\lambda)[M(m, \lambda)M(n, \lambda) - M(m, \lambda)([M(n, \lambda)]^2)M(r, \lambda)]d\lambda \]  \hspace{1cm} (16)

\[ = -\int g(\lambda)[1 - (M(n, \lambda)M(r, \lambda)]M(m, \lambda)M(n, \lambda)d\lambda \]  \hspace{1cm} (17)

Here the factors takes the following values,

\[-1 \leq \{M(m, \lambda)M(n, \lambda)\} \leq 1\]  \hspace{1cm} (18)
and
\[ g(\lambda)[1 - (M(n, \lambda)M(r, \lambda))] \geq 0 \] (19)

Hence,
\[ |P(m, n) - P(m, r)| \leq \int g(t)[1 - (M(n, \lambda)M(r, \lambda)]d\lambda \] (20)

Or,
\[ |P(m, n) - P(m, r)| \leq 1 + P(n, r) \] (21)

Eq.21 is the famous Bell’s inequality. The quantum mechanical assumption given in Eq.6 is incompatible to this inequality. Let assume all the three vectors \((m, n, r)\) are in a same plane such that the vector \((r)\) makes 45° angle with each of the vector \(m\) and \(n\). So applying the Eq.6 we get,
\[ P(m, n) = 0, \] (22)
\[ P(m, r) = P(n, r) = -0.707, \] (23)

But on the contrary bell’s inequality present in Eq.21, gives the following result,
\[ 0.707 \leq 0.293. \] (25)

Which indicate that Einstein’s radical idea of “elements of reality” which incorporate hidden variables with the wave function, is wrong. Bell’s theorem is a landmark theorem in quantum mechanics but does not imply the existence of any nonlocality in quantum mechanics itself.

**B. No-Cloning theorem**

The no-cloning theorem states that one can not create an identical copy of an arbitrary unknown quantum state, the theorem is true for pure states. No-Cloning theorem is provided by Park in 1970, further re-investigated in 1982 by Wootters et al. and by Dieks separately[78, 79]. This is the same year in which the development on quantum computing models has been very much active. The theorem of quantum cloning is easy to prove. Here we give two proofs of this theorem by using the property of unitary operation and another by using the linearity property of the quantum mechanics.

**C. Proof 1:**

Here we present the proof of no-cloning theorem by using the property of unitary operation. Consider two pure states as \(|\psi\rangle, |\phi\rangle\) and a blank state \(|b\rangle\). Mixing each pure state with
blank state and perform the unitary operation which has the goal to copy the pure state into a blank state. So we get,

\[ U(|\psi\rangle \otimes |b\rangle) = |\psi\rangle \otimes |\psi\rangle \] (26)

\[ U(|\phi\rangle \otimes |b\rangle) = |\phi\rangle \otimes |\phi\rangle \] (27)

Taking the complex conjugate of both the sides of both the above equations, we get,

\[ (|\psi\rangle \otimes \langle b|)U^\dagger = \langle \psi| \otimes \langle \psi| \] (28)

Multiplying the left and right sides of Eq. 28 and Eq. 27, we get,

\[ (|\psi\rangle \otimes \langle b|)U^\dagger U(|\phi\rangle \otimes |b\rangle) = (|\psi\rangle \otimes \langle \psi|)(|\phi\rangle \otimes |\phi\rangle) \] (29)

We know \[ U^\dagger U = I \], which further leads,

\[ \langle \psi|\phi \rangle = \langle \psi|\phi \rangle^2 \] (30)

The equation is conflicting, hence it is true with only two cases, either \[ \langle \psi|\phi \rangle = 0 \], or \[ |\psi\rangle = |\phi\rangle \]. These conditions reveal that there is no unitary operation which can be used to clone the arbitrary quantum state (hence proved).

**D. Proof 2:**

Here we present the proof by using the linearity property of quantum mechanics. Suppose there exists a perfect cloning machine, which can copy an unknown pure quantum state \[ |\psi\rangle \], it can be defined as,

\[ |\psi\rangle|\Sigma\rangle|A\rangle = |\psi\rangle|\psi\rangle|A\rangle \] (31)

Where \[ |\Sigma\rangle \] is the blank state in which the state \[ |\psi\rangle \] is to be copied and \[ |A\rangle \] is the auxiliary state. If the state \[ |\psi\rangle \] is prepared in \[ |0\rangle \] and \[ |1\rangle \] respectively, than the following equations will takes place,

\[ |0\rangle|\Sigma\rangle|A\rangle = |0\rangle|0\rangle|M(0)\rangle \] (32)

\[ |1\rangle|\Sigma\rangle|A\rangle = |1\rangle|1\rangle|M(1)\rangle \] (33)

Lets consider the pure state as \[ |\psi\rangle = \alpha|0\rangle + \beta|1\rangle \], than the cloning machine takes place as,

\[ (\alpha|0\rangle + \beta|1\rangle)|\Sigma\rangle|A\rangle = \alpha|00\rangle|M(0)\rangle + \beta|11\rangle|M(1)\rangle \] (34)

On the other hand the Eq.34 can also be solved by using the Eq.31 as below,

\[ (\alpha|0\rangle + \beta|1\rangle)|\Sigma\rangle|A\rangle = \alpha^2|00\rangle + \beta^2|11\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle)|M(|\psi\rangle)\rangle \] (35)

Here we conclude that, Eq.34 and Eq.35 are not same and hence this result claims that cloning is not possible of pure states. In case the quantum state is mixed state rather than pure, than the generalization of no cloning theorem is treated by no-broadcasting theorem, which we are intended to deal in next section.
E. No-Broadcast theorem

The generalized framework of pure state no cloning theorem is treated with No-Broadcast theorem. The first attempt to prove that non commutating mixed state can no be broadcast is done by Barnum et al.\[80\]. Further extensions on No-broadcast theorem is done by many authors, to look into broad view of No-broadcasting and different paradigms, we suggest the reader to loon into the references [80–85] for broad spectrum. Here we present the Barnum et al. idea, as per Barnum et al., a set of quantum states \( A = \{\rho_s\} \) from a source can be broadcast to a target (\( \Sigma \)) if and only if the states in the set \( A \) commutes. The composite system of source and target \( (S \otimes T) \) can go through a physical process to broadcast the quantum state to target with the following broadcasting machine,

\[
(\rho_s \otimes \Sigma) \mapsto P(\rho_s \otimes \Sigma) = \rho_{out} \tag{36}
\]

Where \((P)\) is a physical process and \((\rho_{out})\) is the output state which satisfy the following conditions.

\[
Ptr_s(\rho_{out}) = \rho_s \tag{37}
\]

\[
Ptr_t(\rho_{out}) = \rho_s \tag{38}
\]

The operation \((Ptr)\) denotes the partial traces over the subsystems \( s \) and \( t \) respectively. Here we recall that, the core result shown by Barnum et al. is that, the set \( A \) can be broadcast if the states in this set \( A \) commutes.

F. No-Deleting theorem

The No-Deletion theorem states that; given two copies of arbitrary quantum states[86], then it is impossible to delete one of these. The process of quantum deletion is different than quantum erasing. Let we define the quantum deleting machine as follows,

\[
U(|\psi_A\rangle|\psi_B\rangle|A_C\rangle) = |\psi_A\rangle|0_B\rangle|A_C^{x1}\rangle \tag{39}
\]

On the left-hand side of the equation, \( U \) is the unitary operation on the composited system \((ABC)\). For right-hand side of the equation, the term \(|0_B\rangle\) signify the deletion of the state \(|\psi_B\rangle\) and \(|A_C^{x}\rangle\) is the transformed auxiliary qubit. Let assume both the qubits \(|\psi_A\rangle\) and \(|\psi_B\rangle\) are in same states then the deleting machine takes place,

\[
U(|0_A\rangle|0_B\rangle|A_C\rangle) = |0_A\rangle|0_B\rangle|A_C^{x1}\rangle \tag{40}
\]

and

\[
U(|1_A\rangle|1_B\rangle|A_C\rangle) = |1_A\rangle|0_B\rangle|A_C^{x2}\rangle \tag{41}
\]
Let assume, the state of two arbitrary unknown qubits are assumed in the same state as, $|\psi_A\rangle = \alpha|0_A\rangle + \beta|1_A\rangle$ and $|\psi_B\rangle = \alpha|0_B\rangle + \beta|1_B\rangle$ respectively. Now implementing the deleting machine, we get,

$$U(\alpha|0_A\rangle + \beta|1_A\rangle)(\alpha|0_B\rangle + \beta|1_B\rangle)|A_C\rangle = \alpha^2|0_A0_B\rangle|A_C\rangle + \beta^2|1_A1_B\rangle|A_C\rangle + \alpha\beta|0_A1_B\rangle|A_C\rangle + \beta\alpha|1_A0_B\rangle|A_C\rangle$$

Using the Eqs.40 and 41 we get,

$$U(\alpha|0_A\rangle + \beta|1_A\rangle)(\alpha|0_B\rangle + \beta|1_B\rangle)|A_C\rangle = \alpha^2U(|0_A\rangle|0_B\rangle|A_C^1\rangle) + \beta^2U(|1_A\rangle|0_B\rangle|A_C^2\rangle) + (\alpha\betaU|0_A1_B\rangle + \beta\alphaU|1_A0_B\rangle)|A_C\rangle$$

$$= \alpha^2|0_A\rangle|0_B\rangle|A_C^1\rangle + \beta^2|1_A\rangle|0_B\rangle|A_C^2\rangle + \sqrt{2}\alpha\beta|\zeta_{AB}\rangle$$

Where

$$|\zeta_{AB}\rangle = (\frac{1}{\sqrt{2}})(|0_A1_B\rangle + |1_A0_B\rangle)|A_C\rangle$$

As per the deleting machine the output should be,

$$U(|\psi_A\rangle|\psi_B\rangle|A_C\rangle) = (\alpha|0_A\rangle + \beta|1_B\rangle)|0_B\rangle|A_C^x\rangle$$

So we conclude that, the output in the Eq. 45 and the output in Eq. 47 are not equal, hence the machine do not delete the arbitrary unknown qubit.

### G. No-Teleportation Theorem

In quantum information, the no teleportation theorem[87] states that neither an arbitrary quantum state can be converted in a sequence of classical bits nor the classical bits can create original quantum state. This theorem is the consequence of no-cloning theorem. If an arbitrary quantum states allow producing sequence of classical bits, then as we know the classical bits can always be copied and hence the quantum state also can be copied, which violate the no cloning theorem. So the conversion of an arbitrary quantum states in sequence of classical bits is not possible. Theorem is simple to prove. The similarity of two states is defined as; two quantum states $\rho_1$ and $\rho_2$ are identical if the measurement results of any physical observable have the same expectation value for $\rho_1$ and $\rho_2$. Let prepare an arbitrary mixed quantum state $\rho_{input}$, now perform the measurement on the state and obtain the classical measurement results. Now by using these classical measurement results the original quantum states is recovered as $\rho_{output}$. Both the input and output states are not equal, i.e.,

$$\rho_{input} \neq \rho_{output}$$

This result is irrespective to the state preparation process and measurement results outcome. Hence Eq.48 proves, one can not convert an arbitrary quantum states in a sequence of classical bits. The theorem does not have any relation with the protocol quantum teleportation.
H. No-communication Theorem

No communication theorem[88, 89] is also known as the no-signaling principle. The theorem captures the essence, such that the measurement action performed at the end of Alice is not detectable by Bob at his end. This is true in both the cases weather the composite state of Alice and Bob is separable or entangled. Let we first consider the case when composite state is separable. Assume the composite state of Alice and Bob is $\rho$ and Alice perform the measurement on his end. The measurements performed by Alice can be modeled by Kraus operators, these may not be commuting. Let suppose the Kraus operators at the end of Alice are $\{A_m\}$. Now the probability of measurement outcome $x$ can be easily written in the formalism of Kraus operators as,

$$p_x = \sum_m \text{Tr}(A_{xm}\rho A_{xm}^\dagger) = \text{Tr}[\rho V_x]$$  \hspace{1cm} (49)

where

$$V_x = \sum_x A_{xm}^\dagger A_{xm}$$  \hspace{1cm} (50)

$$\sum_x V_x = 1$$  \hspace{1cm} (51)

Let assume the Kraus operators at the end of Bob are $\{B_n\}$. The probability of measurement outcome $y$ at the end of Bob, irrespective to what Alice has found; is given as,

$$p_y = \sum_x \text{Tr}(\sum_{mn} B_{yn} A_{xm} \rho A_{xm}^\dagger B_{yn}^\dagger)$$  \hspace{1cm} (52)

The order of measurements on composite system does not matter, so the following commutation relation should satisfy,

$$[A_{xm}, B_{yn}] = 1$$  \hspace{1cm} (53)

By using the Eq. 52, the Eq. 53 can be written as,

$$p_y = \sum_x \text{Tr}(\sum_{mn} A_{xm} B_{yn} \rho B_{yn}^\dagger A_{xm}^\dagger)$$  \hspace{1cm} (54)

Using the cyclic property of trace operation and expanding the summation we obtain,

$$p_y = \text{Tr}(\sum_n B_{yn} \rho B_{yn}^\dagger)$$  \hspace{1cm} (55)

In this equation all the operators of Alice disappear, so Bob is not able to detect which statistics of measurements Alice has been performed at his end. Hence, the statistics of measurements at the end of Bob has not been effected by Alice also whatever Alice has been performed. The no communication theorem is trivially for separable case, but it can also be true if the composite state is entangled. Let consider the composite state is entangled
which is prepared in singlet state given in Eq. 1. Alice and Bob perform the measurements at his end by using the detectors \((D^A, D^B)\) respectively. Following the Bell’s experiment, the detectors are oriented initially along the z axis and rotated independently at the end of Alice and Bob. Let consider the difference between the angles of detectors is \((\alpha - \beta)\), then quantum mechanically on can calculate the following conditional probabilities of measurements outcome

\[
\begin{align*}
\{A(0), B(0)\}, & \quad p_{00} = \frac{1}{2} \sin^2\left(\frac{\alpha - \beta}{2}\right) \\
\{A(0), B(1)\}, & \quad p_{01} = \frac{1}{2} \cos^2\left(\frac{\alpha - \beta}{2}\right) \\
\{A(1), B(0)\}, & \quad p_{10} = \frac{1}{2} \cos^2\left(\frac{\alpha - \beta}{2}\right) \\
\{A(1), B(1)\}, & \quad p_{11} = \frac{1}{2} \sin^2\left(\frac{\alpha - \beta}{2}\right)
\end{align*}
\]

The following normalization condition is satisfied over the probabilities given below,

\[
P_{00} + P_{01} + P_{10} + P_{11} = 1
\]

Calculating the probabilities of measurement outcome as spin up \((|1\rangle)\) and spin down \((|0\rangle)\) at the Alice end,

\[
\begin{align*}
P_A^1 & = P_{11} + P_{10} = \frac{1}{2} \\
P_A^0 & = P_{01} + P_{11} = \frac{1}{2}
\end{align*}
\]

Similarly we can calculate in the case of Bob as \((P_B^1 = P_B^0 = \frac{1}{2})\). We observe that, the probabilities of measurement outcomes are totally independent to the difference between the angles \((\alpha - \beta)\). So the actions performed of measurements at either end of Alice or Bob are not detected at another end and vice versa. This is the essence of no-communication theorem.

I. No-Hiding Theorem

No-hiding theorem[90] is an important theorem in quantum information, which also indicates the conservation principle of quantum information. The idea of no hiding theorem comes from the one time cipher protocol. The protocol is used to send the message by adding the random key in real information. Shanon proved that the original information neither reside in encoded message nor in the key, so where the information is gone? In the process of one time cipher pad method the information is hidden in the correlations of original information and key. One can think the same scenario in quantum mechanical sense. The teleportation can be assumed as a quantum analogue of one time cipher method. In teleportation, there are two parties Alice and Bob both share an entangled state; Alice
apply few unitary operations at his end and send the measurement results to Bob. Bob apply corresponding measurements and recover the information. In this whole process, the decoherence is missed., as quantum systems are too evasive and always decoherence prone. If, one has to consider the decoherence in teleportation process than Alice might be interacting with the environment. If a quantum system is interacted with an environment in the form of ecoherence, the environment destroy the information. So a natural question arises; where the lost information from the original system has gone? In quantum mechanical case it does not reside in correlations. This idea leads to the ”No-hiding theorem”. Here the original information resides in the subspace of the environmental Hilbert space and not the part of correlation of the system and environment. To proceed the proof of no hiding theorem, let consider an arbitrary input quantum state $\rho_I$, after encoding into a larger Hilbert space. With respect to a hiding process, there exists an output state $\sigma_O$ into a subspace $O$, both the subspaces $(I, O)$ are the part of larger Hilbert space. The rest of the portion of larger Hilbert space is called as ancilla space $A$. The hiding process perform the following mapping,

$$\rho_I \mapsto \sigma_O, \quad (\sigma \text{ fixed } \forall \rho) \quad (63)$$

Here we assume the input state in the subspace $(I)$ is a pure state. A hiding process of this pure state can be considered by taking into account the sub-spaces $(O, A)$, hence it can be written simply as a map given below,

$$|\psi_I\rangle = \sum_{i=1}^{n} \sqrt{p_i} |i\rangle_O \otimes |A_n(\psi)\rangle_A \quad (64)$$

The right had side in the above equation is the Schmidt decomposition. Here $p_i$ are the positive eigenvalues of the state $\sigma_O$ and $(|i\rangle)$ are its eigenvectors. The set of basis $(|i\rangle, |A_n\rangle)$ are orthonormal basis. By imposing the restriction on the ancila and taking into account its linear property, we can write as follows,

$$|\psi_I\rangle = \sum_{i=1}^{n} \sqrt{p_i} |i\rangle_O \otimes (|g_n\rangle \otimes |\psi\rangle \oplus 0)_A \quad (65)$$

Here we see, since we may swap the state $|\psi\rangle$ with any other state in the ancilla using purely ancilla-local operations, we conclude that any information about $|\psi\rangle$ that is encoded globally is in fact encoded entirely within the ancilla. Neither the information about $|\psi\rangle$ is encoded in system-ancilla correlations nor in system-system correlations.

**VIII. CONCLUSIONS**

In this article, we discussed milestone developments in quantum information. It covers the beginning of quantum information and computation based on the idea of reversible computing and quantum Turing machine. We captured the experimental manifestations of
theoretical developments as well. In addition we discussed physical situations, which are not possible in no-go theorems with their mathematical proof. These theorems have their own important consequences for execution of quantum information and in designing in quantum protocols. Covering the broad aspects of milestone developments in this article may be useful for the quantum community.

[1] C. E. Shannon, A Mathematical Theory of Communication. Bell System Technical Journal. 27, 379 (1948).
[2] C. E. Shannon, A Mathematical Theory of Communication. Bell System Technical Journal. 27, 623 (1948).
[3] W. F. Brinkman, D. E. Haggan, W. W. Troutman, A history of the invention of the transistor and where it will lead us, IEEE Journal of Solid-State Circuits 32 (1997).
[4] R. Landauer, Information is physical, Physics Today, 44, 23 (1991).
[5] R. Landauer, Information is a physical entity. Physica A, 263, 63 (1999).
[6] A. Oldofredi, No-Go Theorems and the Foundations of Quantum Physics, Journal for General Philosophy of Science (https://doi.org/10.1007/s10838-018-9404-5).
[7] M. X. Luo, H. R. Li, H. Lai, X. Wang, Unified quantum no-go theorems and transforming of quantum states in a restricted set, Quantum Information Processing, 16, 297 (2017).
[8] C. H. Bennett, Logical Reversibility of Computation, IBM Journal of Research and Development 17, 6 (1973).
[9] A. S. Holevo, Bounds for the quantity of information transmitted by a quantum communication channel, Problems of Information Transmission, 9 177 (1973).
[10] T. Toffoli, Reversible computing, MIT Laboratory for Computer Science, 545 Technology Sq., Cambridge, MA 02130 (1980).
[11] R. Stanisław Ingarden, Quantum information theory, Math. Phys. 10, 43 (1976).
[12] C. E. Shannon, A Mathematical theory of communication, Bell Syst. Tech. J. 27, 379 (1948).
[13] Y. I. Manin, Vychislimoe i nevychislimoe (in Russian), (Moskva: Sov.Radio, 1980).
[14] W. K. Wootters, W. H. Zurek, A single quantum cannot be cloned, Nature 299, 5886 (1982).
[15] D. Dieks, Communication by EPR devices, Phys. Lett. A 92, 271 (1982).
[16] P. Benioff, Quantum mechanical Hamiltonian models of Turing machines, Journal of Statistical Physics, 29 3, 515 (1982).
[17] A. Einstein, B. Podolsky, N. Rosen, Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 47, 777 (1935).
[18] M.A. Nielsen, I.L. Chuang, Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2000).
[19] D.Phil. Thesis, Phys. Rev. Lett. 67, 661 (1991).
[20] D. Deutsch and R. Jozsa, Rapid solutions of problems by quantum computation, Proceedings
18

of the Royal Society of London A. 439 (1992).

[21] P. W. Shor, Polynomial time algorithms for prime factorization and discrete logarithms on a quantum computer, SIAM J. Comput. 26, 1484 (1997).

[22] P. W. Shor, Scheme for reducing decoherence in quantum computer memory, Phys. Rev. A 52, R2493 (1995).

[23] R. Rivest, A. Shamir, L. Adleman, A Method for Obtaining Digital Signatures and Public-Key Cryptosystems, Communications of the ACM 21 120 (1978).

[24] A. M. Steane, Error correcting codes in quantum theory, Phys. Rev. Lett. 77, 793 (1996).

[25] L. K. Grover, Quantum telecomputation, arXiv:quant-ph/9704012 (1997).

[26] C. H. Bennett, G. Brassard, C. Crpeau, R. Jozsa, A. Peres, W. K. Wootters, Teleporting an Unknown Quantum State via Dual Classical and EinsteinPodolskyRosen Channels, Phys. Rev. Lett. 70, 1895 (1993).

[27] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter and A. Zeilinger, Experimental quantum teleportation. Nature. 390, 575( 1997).

[28] N. A. Gershenfeld et al., Bulk spin-resonance quantum Computation, Science 275, 350 (1997).

[29] J. A. Jones and M. Mosca, Implementation of a Quantum Algorithm to Solve Deutsch’s Problem on a Nuclear Magnetic Resonance Quantum Computer, J. Chem. Phys., 109 1648 (1998).

[30] I. L. Chuang, N. Gershenfeld, and M. Kubinec, Experimental implementation of fast quantum searching, Phys. Rev. Lett. 80, 3408 (1998).

[31] A. K. Pati and S. L. Braunstein, Impossibility of Deleting an Unknown Quantum State, Nature 404, 164 (2000).

[32] L. M. K. Vandersypen, M. Steffen, G. Breyta, C. S. Yannoni, M. H. Sherwood and I. L. Chuang, Experimental realization of Shor’s quantum factoring algorithm using nuclear magnetic resonance, Nature 414, 883 (2001).

[33] E. Knill, R. Laflamme, G. J. Milburn, A scheme for efficient quantum computation with linear optics, Nature 409, 46 (2001).

[34] T. B. Pittman, M. J. Fitch, B. C Jacobs, and J. D. Franson, Experimental controlled-not logic gate for single photons in the coincidence basis, Phys. Rev. A 68, 032316 (2003).

[35] J. L. O’Brien, G. J. Pryde, A. G. White, T. C. Ralph and D. Branning, Demonstration of an all-optical quantum controlled-NOT gate, Nature 426, 264 (2003).

[36] Chip Elliott, The DARPA Quantum Network, arXiv:quant-ph/0412029v1, 2004.

[37] G. Brennen, E. Giacobino and C. Simon, Focus on Quantum Memory, New J. Phys. 17, 050201 (2015 ).

[38] H. J. Kimble, The quantum internet, Nature volume 453, 1023 (2008).

[39] T. Chaneliere, D. N. Matsukevich, S. D. Jenkins, S. Y. Lan, T. A. B. Kennedy and A. Kuzmich, Storage and retrieval of single photons transmitted between remote quantum memories, Nature 438, 833 (2005).
[40] L.K. Grover, e-print quant-ph/9704012; A. Ekert, S. F. Huelga, C. Macchiavello and J. I. Cirac, e-print quant-ph/9803017.
[41] M. Murao, D. Jonathan, M. B. Plenio, V. Vedral, Quantum telecloning and multiparticle entanglement, Phys.Rev.A 59 156 (1999).
[42] S. Koike, H. Takahashi, H. Yonezawa, N. Takei, Samuel L. Braunstein, T. Aoki, and A. Furusawa, Demonstration of Quantum Telecloning of Optical Coherent States, Phys. Rev. Lett.96, 060504 (2006).
[43] S. Pirandola, D. Vitali, P. Tombesi and S. Lloyd, Macropscopic Entanglement by Entanglement Swapping, Phys. Rev. Lett. 97, 150403 (2006).
[44] F. Brennecke, T. Donner, S. Ritter, T. Bourde, M. Koh and T. Esslinger, Cavity QED with a Bose Einstein condensate, Nature 450, 268 (2007).
[45] B. P. Lanyon, T. J. Weinhold, N. K. Langford, J. L. O'Brien, K. J. Resch, A. Gilchrist, and A. G. White, Manipulating Biphotonic Qutrits, Physical Review Letters 100, 060504 (2008).
[46] J. Chen, J. B. Altepeter, M. Medic, K. Fook Lee, B. Gokden, R. H. Hadfield, S. W. Nam, and P. Kumar, Demonstration of a Quantum Controlled-NOT Gate in the Telecommunications Band, Phys. Rev. Lett. 100, (2008).
[47] D. D. Awschalom and M. E. Flatté, Challenges for semiconductor spintronics, Nature Phys. 3, 153 (2007).
[48] M. Fuechsle, J. A. Miwa, S. Mahapatra, H. Ryu, S. Lee, O. Warschkow, L. C. L. Hollenberg, G. Klimeck and M. Y. Simmons, A single-atom transistor, Nature Nanotechnology 7 (2012).
[49] T. Kadowaki and H. Nishimori, “Quantum annealing in the transverse Ising model” Phys. Rev. E 58, 5355 (1998).
[50] L. DiCarlo, J. M. Chow, J. M. Gambetta, Lev S. Bishop, B. R. Johnson, D. I. Schuster, J. Majer, A. Blais, L. Frunzio, S. M. Girvin, et al., Demonstration of two-qubit algorithms with a superconducting quantum processor, Nature 460, 240 (2009).
[51] P. Alberto, C. F. M. Jonathan, L. O. Jeremy, Shors Quantum Factoring Algorithm on a Photonic Chip, Science325, 1221 (2009).
[52] M. S. Tame, R. Prevedel, M. Paternostro, P. Bhi, M. S. Kim, and A. Zeilinger, Experimental Realization of Deutschs Algorithm in a One-Way Quantum Computer, Phys. Rev. Lett. 98, 140501 (2007).
[53] H. Ollivier and W. H. Zurek, Quantum Discord, A Measure of the Quantumness of Correlations, Physics Review Letters 88, 017901 (2001).
[54] M. Mariantoni, H. Wang, T. Yamamoto, M. Neeley1, R. C. Bialczak1, Y. Chen, M. Lenander, E. Lucero, A. D. O’Connel, D. Sank, M. Weides, J. Wenner, Y. Yin, J. Zhao, A. N. Korotkov, A. N. Cleland, John M. Martinis, Implementing the quantum von Neumann architecture with superconducting circuits, Science 334, 61 (2011).
[55] W. Pfaff, B. J. Hensen, H. Bernien, S. B. van Dam, M. S. Blok, T. H. Taminiau, M. J. Tiggelman, R. N. Schouten, M. Markham, D. J. Twitchen, R. Hanson, Unconditional quantum
teleportation between distant solid-state quantum bits, Science 345, 532 (2014).

[56] N. S. Dattani, N. Bryans, Quantum factorization of 56153 with only 4 qubits, arXiv:1411.6758 (2014).

[57] M. Schlosshauer, Decoherence, The measurement problem, and interpretations of quantum mechanics, Rev. Mod. Phy. 76, 1267 (2005).

[58] M. Zhong, M. P. Hedges, R. L. Ahlefeldt, J. G. Bartholomew, S. E. Beavan, S. M. Wittig, J. J. Longdell and M. J. Sellars, Optically addressable nuclear spins in a solid with a six-hour coherence time, Nature 517, 177 (2015).

[59] E. Gibney, D-Wave upgrade: How scientists are using the worlds most controversial quantum computer, Nature 541 447 (2017).

[60] T. Yu, J. H. Eberly, Finite-time disentanglement via spontaneous emission, Phys. Rev. Lett. 93, 140404 (2004).

[61] T. Yu, J. H. Eberly, Sudden death of entanglement, Science 30, 598 (2009).

[62] K. K. Sharma, S. K. Awasthi, S. N. Pandey, Entanglement sudden death and birth in qubit-qutrit systems under Dzyaloshinskii-Moriya interaction. Quantum Inf. Process. 12, 3437 (2013).

[63] K. K. Sharma, S.N. Pandey, Entanglement Dynamics in two parameter qubit-qutrit states under Dzyaloshinskii-Moriya interaction. Quantum Inf. Process. 13, 2017 (2014).

[64] K. K. Sharma, S.N. Pandey, Influence of Dzyaloshinshkii-Moriya interaction on quantum correlations in two qubit Werner states and MEMS. Quantum. Info. Process. 14, 1361 (2015).

[65] K. K. Sharma, S.N. Pandey, Dzyaloshinshkii-Moriya interaction as an agent to free the bound entangled states. Quantum. Info. Process. 15, 1539 (2016).

[66] K. K. Sharma, S.N. Pandey, Dynamics of entanglement in two parameter qubit-qutrit states with x-component of DM interaction. Commun. Theor. Phys. 65, 278 (2016).

[67] K. K. Sharma, S.N. Pandey, Robustness of Greenberger-Horne-Zeilinger and W states against Dzyaloshinskii-Moriya interaction, Quantum Inf Process 15 4995 (2016).

[68] R. Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, Quantum entanglement, Rev. Mod. Phys. 81, 865 (2009).

[69] V. Coffman, J. Kundu, W. K. Wootters, Distributed entanglement, Phys. Rev. A 61, 052306 (2000).

[70] B. M. Terhal, Is entanglement monogamous?, IBM Journal of Research and Development 48, 71 (2004).

[71] M. Koashi and A. Winter, Monogamy of quantum entanglement and other correlations, Phys. Rev. A 69, 022309 (2004).

[72] T. J. Osborne and F. Verstraete, General Monogamy inequality for bipartite qubit entanglement, Phys. Rev. Lett. 96, 220503 (2006).

[73] H. P. Stapp, Bell’s Theorem and World Process, Nuovo Cimento. 29B, 270 (1975).

[74] J. Bell, On the Einstein Podolsky Rosen Paradox, Physics. 1, 195 (1964).
[75] J. S Bell, On the Einstein-Podolsky-Rosen paradox, Speakable and Unspeakable in Quantum Mechanics: Collected Papers on Quantum Philosophy, Cambridge University Press, **14** ISBN 978-0521523387 (2004).

[76] J. Clauser, H. Michael, A. Shimony, H. Richard, Proposed Experiment to Test Local Hidden-Variable Theories, Phy. Rev. Lett., **23**, 880 (1969).

[77] S. Weisner, Conjugate coding, Association for computing machinery, Special Interest Group in Algorithms and Computation Theory **15**, 78 (1983).

[78] W. Wootters, W. Zurek, A Single Quantum Cannot be Cloned, Nature, **299** 802 (1982).

[79] D. Dieks, Communication by EPR devices, Physics Letters A. **92**, 271 (1982).

[80] H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa and B. Schumacher, Noncommuting Mixed States Cannot Be Broadcast, Phy. Rev. **76** (1996).

[81] G. M. D’Ariano, C. Macchiavello, and P. Perinotti, Superbroadcasting of Mixed States, Phys. Rev. Lett. **95**, 060503 (2005).

[82] G. Lindblad, A General No-Cloning Theorem, Letters in Mathematical Physics, **47** (1999).

[83] H. Barnum, J. Barrett, M. Leifer and A. Wilce, Generalized No-Broadcasting Theorem, Phys. Rev. Lett. **99** (2007).

[84] A. Kalev, and I. Hen, No-Broadcasting Theorem and Its Classical Counterpart, Phys. Rev. Lett. **100** (2008).

[85] M. Piani, P. Horodecki and R. Horodecki, No-Local-Broadcasting Theorem for Multipartite Quantum Correlations, Phys. Rev. Lett. **100** (2008).

[86] A. K. Pati and S. L. Braunstein, Impossibility of Deleting an Unknown Quantum State, Nature **404**, 164. (2000).

[87] J. Gruska, I. Imai, Power, Puzzles and Properties of Entanglement, appearing in Machines, Computations, and Universality: Third International Conference. edited by Maurice Margetsov, Yuri Rogozhin, **25**, (2001).

[88] S. Popescu, D. Rohrlich, Causality and Nonlocality as Axioms for Quantum Mechanics, Proceedings of the Symposium on Causality and Locality in Modern Physics and Astronomy, (1997).

[89] A. Peres and D. R. Terno, Quantum information and relativity theory, Rev. Mod. Phys. **76** (2004).

[90] S. L. Braunstein and A. K. Pati, Quantum Information Cannot be Completely Hidden in Correlations: Implications for Black Hole Information Paradox, Phys. Rev. Lett. **98**, 080502 (2007).