Degenerate Dirac Neutrinos In An $SU(2)_L \times U(1)_Y$ Model With $S_3 \times Z_3 \times Z_4$ Symmetry

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Abstract

We demonstrate that almost degenerate Dirac neutrinos of mass of the order of a few eV and transition magnetic moment of the order of $10^{-11} \mu_B$ can be obtained in an $SU(2)_L \times U(1)_Y$ model with $S_3 \times Z_3 \times Z_4$ discrete symmetry and appropriate Higgs. Transition magnetic moment of the Dirac neutrino arises from the contribution of leptons and charged Higgs fields at the one loop level.
Three possible scenarios of neutrino masses have been proposed by Caldwell and Mohapatra [1] which can simultaneously reconcile the solar [2] and atmospheric [3] neutrino deficits on earth as well as the apparent need for neutrino as a hot dark matter component [4]. Two of them contain a sterile neutrino besides the three neutrinos $\nu_e, \nu_\mu$ and $\nu_\tau$. The scenario, which does not contain any sterile neutrino, requires the three neutrinos $\nu_e, \nu_\mu$ and $\nu_\tau$ to be almost degenerate in mass of the order of 2-3 eV. Furthermore, the recent results from the chromium ($^{51}$Cr) source experiment [5] carried out by the GALLEX collaboration implies interesting limits on the parameters $\Delta m^2$ and $\sin^2 2\theta$ describing the neutrino flavour oscillation. Values of $\Delta m^2 > 0.17 eV^2$ for maximal mixing and of $\sin^2 2\theta > 0.38$ for $\Delta m^2 > 1 eV^2$ are ruled out (at 90 %c.l.) [6]. Another result from Bugey [7] sets the minimum excluded values of $\Delta m^2$ and $\sin^2 2\theta$ are $1 \times 10^{-2} eV^2$ and $2 \times 10^{-2}$ (at 90 % c.l.) respectively. Although the recent experimental results have strengthened the conjecture of the neutrino flavour oscillation, but the phenomenon is yet to be established [8].

Assuming the neutrino flavour oscillation, the hierarchy of the masses of the three neutrinos is proposed on the phenomenological basis [9, 10] as $m_{\nu_1}^2 = m_0^2$, $m_{\nu_2}^2 = m_0^2 + \Delta_{21}$, and $m_{\nu_3}^2 = m_0^2 + \Delta_{32} + \Delta_{21}$ with $m_0^2 >> \Delta_{21}, \Delta_{32}$ where $\Delta_{ij} = m_{\nu_i}^2 - m_{\nu_j}^2$. Moreover, if $\nu_e \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_\tau$ oscillations are assumed to be responsible for the solar and atmospheric neutrino deficits respectively, then $|\Delta_{32}| >> |\Delta_{21}|$. The degeneracy between the three generations is lifted due the presence of $\Delta_{ij}$ terms which are expected to be small. The small Majorana neutrino mass of the order of few eV can
be realized through the introduction of intermediate or large (GUT scale) mass scale [11] emerging from the symmetry breaking of the model through the standard see-saw mechanism [12]. However, the Dirac neutrinos are expected to be heavy (as the masses of the Dirac neutrinos are proportional to the electroweak symmetry breaking scale) unless the associated Yukawa couplings are extremely small.

Another interesting aspect of neutrino physics is to accommodate large magnetic moment of the order of $10^{-11} \mu_B$ as suggested by Okun, Voloshin and Vysotsky [13] in order to explain the anticorrelation between the Sun spot activity and the observed solar neutrino flux on earth. The bound obtained for the neutrino magnetic moment from the laboratory experiments of $\bar{\nu}_e e$ scattering ($< 1.5 \times 10^{-10} \mu_B$) [14], which has been corrected to ($< 4 \times 10^{-10} \mu_B$) [15], is just below the above mentioned value. From the astrophysical consideration of steller cooling, a much more stronger limit is derived ($\mu_{\nu_e} < (0.3 - 1.0) \times 10^{-11} \mu_B$) [16]. In order to explain the solar neutrino deficits, two types of oscillations are suggested [17, 18]. The first one $\nu_{eL} \rightarrow \nu_{eR}$, is supported due to the apparent anticorrelation between the Sun spot activity and the observed neutrino flux on earth, however, this is in conflict with the observation of SN1987 A data for the energy loss of the supernova burst [19]. The other scenario, which assumes the oscillation of $\nu_{eL} \rightarrow (\nu_\mu^R)^c$ seems to be a much more reasonable choice, since it has no severe bounds from the observation of SN1987 A as well as nucleosynthesis.

In the present work we demonstrate that, within the framework of Stan-
standard $SU(2)_L \times U(1)_Y$ model with $S_3 \times Z_3 \times Z_4$ discrete symmetry, right-handed neutrinos and appropriate Higgs fields, almost degenerate Dirac neutrinos and a large transition magnetic moment $10^{-11} \mu_B$ can be achieved through the incorporation of the two widely different lepton number symmetry breaking scales. Our present model admits light degenerate Dirac neutrinos of mass of the order of 1 eV, contrary to the expectation of heavy Dirac neutrinos, through the effective coupling of the Higgs fields. The first and second generations are completely degenerate in mass and the third generation can also be made almost degenerate with the others through some appropriate fine tuning of the model parameter. The large transition magnetic moment $(\nu_{eL} \rightarrow \nu_{\mu R})$, consistent with the phenomenological bounds from SN1987 A and nucleosynthesis as mentioned earlier, arises due to the quartic couplings of the Higgs fields through the charged Higgs exchange at the one loop level.

We concentrate on the leptons and Higgs fields of the model. The lepton content in the present model is as usual

$$l_{iL}(2, -1, 1) = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_{iL}, \nu_{iR}(1, 0, 1)$$

(1)

where $i=1, 2, 3$ is the generation index. The following Higgs fields are considered with the Vacuum Expectation Values (VEV) as indicated

$$\phi_i(2, 1, 0) = \begin{pmatrix} 0 \\ \nu_i \end{pmatrix}, \eta_j(1, 0, x) = k_j$$

(2)

where $i = 1, \ldots 5$ and $j = 1, \ldots 4$ are the number of the Higgs doublets and singlets respectively. The last digit in the parenthesis represents lepton number $L(= L_e + L_\mu + L_\tau)$. The lepton number of the singlet Higgs fields is
arbitrary non-zero number since, the singlets do not couple with the leptons at the tree level due to the discrete \( Z_3 \times Z_4 \) symmetry incorporated in the model. In this sense, our present model is different from the well known CMP majoron model [20]. However, the present model also admits spontaneous lepton number violation through the development of non-zero VEV’s of the singlet Higgs fields like the CMP majoron model.

The leptons and the Higgs fields transform under discrete \( S_3 \times Z_3 \times Z_4 \) symmetry as follows:

i) \( S_3 \) symmetry :

\[
(l_{1L}, l_{2L}) \rightarrow 2, l_{3L} \rightarrow 1, (\nu_{\mu R}, \nu_{\tau R}) \rightarrow 2, \nu_{\tau R} \rightarrow 1, \\
(\nu_{e R}, \nu_{\tau R}) \rightarrow 2, \tau_R \rightarrow 1, \phi_1 \rightarrow 1, \phi_2 \rightarrow 1, \phi_3 \rightarrow 1, \\
(\phi_4, \phi_5) \rightarrow 2, \eta_1 \rightarrow 1, \eta_2 \rightarrow 1, \eta_3 \rightarrow 1, \eta_4 \rightarrow 1
\]  

(3)

ii) \( Z_3 \times Z_4 \) symmetry:

\[
(l_{1L}, l_{2L}) \rightarrow (l_{1L}, l_{2L}), l_{3L} \rightarrow l_{3L}, (\nu_{\mu R}, \nu_{e R}) \rightarrow (\nu_{\mu R}, \nu_{e R}) \\
\nu_{\tau R} \rightarrow \nu_{\tau R}, (\nu_{e R}, \mu R) \rightarrow i\omega(\nu_{e R}, \mu R), \tau_R \rightarrow i\omega\tau_R, \phi_1 \rightarrow \phi_1, \\
\phi_2 \rightarrow \omega \phi_2, \phi_3 \rightarrow -i\omega^* \phi_3, (\phi_4, \phi_5) \rightarrow -i\omega^*(\phi_4, \phi_5), \\
\eta_1 \rightarrow i\omega \eta_1, \eta_2 \rightarrow i\omega^2 \eta_2, \eta_3 \rightarrow -\omega \eta_3, \eta_4 \rightarrow -i\omega^* \eta_4
\]  

(4)

where \( \omega = e^{\frac{2\pi i}{3}} \).

The present model admits only the Dirac neutrino mass terms. The discrete symmetry prohibits the Majorana mass terms in the lepton-Higgs
Yukawa interactions as well as gives rise to some vanishing elements in the Dirac neutrino mass matrix (at the tree level). We also discard any explicit lepton number violating terms in the present model. The purpose of incorporation of $S_3$ permutation symmetry is to generate the equality between the Yukawa couplings and VEV’s of the Higgs fields in order to get degenerate neutrino masses. Several other interesting applications of $S_3$ symmetry have been investigated in the context of quark mass matrices and CP phenomenology [21]. The Higgs field $\phi_1$ couples with the neutrinos and is prohibited to couple with the charged leptons. The $\phi_2$ Higgs field, although neither couples with the neutrinos nor with the charged leptons directly, but, gives rise to small degenerate neutrino masses through its coupling with the $\phi_1$, $\eta_1$ and $\eta_2$ Higgs fields. The purpose of incorporation of $\phi_3$ Higgs field, which couples with the charged leptons, is to generate large transition magnetic moment through the coupling with $\phi_1$, $\eta_3$ and $\eta_4$ Higgs fields similar to the diagram given in Ref.18. The doublet ($\phi_4$, $\phi_5$) is necessary to achieve non-degenerate charged lepton mass matrix. It is to be noted that, without the doublet ($\phi_4$, $\phi_5$), the charged lepton mass matrix will also becomes degenerate, which is unphysical. The four Higgs singlets are necessary to achieve the lepton number invariant terms in the Higgs potential and we will show that they play an important role to generate small Dirac neutrino masses and large transition magnetic moment.

Another interesting feature in our present model is that the absence of coupling at the tree level between the majorons (generating due to the development of non-zero VEV’s of the singlets through spontaneous lepton
number violation) and the right-handed neutrinos forbids the neutrino decays \((\nu_h \rightarrow \nu_l J)\) and/or annihilation of neutrinos \((\nu\nu \rightarrow JJ)\) which leads to a longer life time of the neutrinos so that the neutrinos could be a relevant part of the dark matter [22].

The most general renormalizable Higgs potential, respecting \(S_3 \times Z_3 \times Z_4\) discrete symmetry, in the present model can be written as

\[
V = V(\phi_i) + V(\eta_j) + V(\phi_i, \eta_j)
\]  

Explicitly the terms are given as follows:

\[
V(\phi_i) = -\sum_{i=1}^{5} m_i^2 (\phi_i^\dagger \phi_i) + \sum_{i=1}^{5} \mu_i (\phi_i^\dagger \phi_i)^2 - m_4^2 (\phi_4^\dagger \phi_4 + \phi_5^\dagger \phi_5) + \mu_4 (\phi_4^\dagger \phi_4)^2 + (\phi_5^\dagger \phi_5)^2 + \lambda_1 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_2 (\phi_1^\dagger \phi_1)(\phi_3^\dagger \phi_3) + \lambda_3 (\phi_4^\dagger \phi_4 + \phi_5^\dagger \phi_5) + \lambda_4 (\phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3) + \lambda_5 (\phi_2^\dagger \phi_2)(\phi_4^\dagger \phi_4 + \phi_5^\dagger \phi_5) + \lambda_6 (\phi_3^\dagger \phi_3)(\phi_4^\dagger \phi_4 + \phi_5^\dagger \phi_5) + \lambda_7 (\phi_4^\dagger \phi_4 \phi_4^\dagger \phi_5 + \phi_5^\dagger \phi_5 \phi_4^\dagger \phi_4 + \phi_2^\dagger \phi_2 \phi_3^\dagger \phi_3 + \phi_5^\dagger \phi_5 \phi_4^\dagger \phi_4 + \phi_3^\dagger \phi_3 \phi_4^\dagger \phi_5) \quad (6)
\]

\[
V(\phi, \eta) = \sum_{i=1}^{j=4} \lambda_{ij} (\phi_i^\dagger \phi_i)(\eta_j^* \eta_j) \quad (with \lambda_{4j} = \lambda_{5j}) + \lambda' (\phi_1^\dagger \phi_2 \eta_2^* \eta_1 + \eta_1^* \eta_2 \phi_2^\dagger \phi_1) + \lambda'' (\phi_3^\dagger \phi_3 \eta_3^* \eta_3 + \eta_3^* \eta_3 \phi_3^\dagger \phi_3) \quad (7)
\]

where we have neglected \(V(\eta)\) part of the potential, since, it is not necessary for our present analysis. Substituting the VEV’s of the Higgs fields in Eqn.(6)
and (7) and minimizing with respect to \( v_1 \), we get

\[ v_1 = -\frac{B}{A} \]  

(8)

with

\[ B = (\lambda' v_2 k_2 k_1 + \lambda v_3 k_3 k_4) \]

\[ A = -m_1^2 + \lambda_1 v_2^2 + \lambda_2 v_3^2 + \lambda_3 (v_4^2 + v_5^2) + \lambda_{11} k_1^2 + \lambda_{12} k_2^2 + \lambda_{13} k_3^2 + \lambda_{14} k_4^2 \]

where we have neglected \( \mu_1 \) term for simplicity.

On simplification of Eqn.(8), we obtain

\[ v_1 = \frac{\lambda' v_2 k_2 k_1 + \lambda v_3 k_3 k_4}{m_1^2} \]  

(9)

assuming \( m_1^2 \) is much larger than all other \( \lambda \)'s appeared in the denominator of Eqn.(8). It is to be noted that the above assumptions do not lead to any drastic changes in the result of our present analysis. Similarly, minimizing the Higgs potential with respect to \( v_2 \) and \( v_3 \), we get

\[ v_2 = \frac{\lambda'' v_3 k_2 k_3 + \lambda' v_1 k_2 k_1}{m_2^2} \]  

(10)

and

\[ v_3 = \frac{\lambda v_1 k_3 k_4 + \lambda'' v_2 k_2 k_3}{m_3^2} \]  

(11)

We further consider that \( v_3 = 0 \), and, thus, from Eqn.(9) we get

\[ v_1 = \frac{\lambda' v_2 k_1 k_2}{m_1^2} \]  

(12)
It is to be noted that the quartic coupling $\lambda'$, which generates large magnetic moment, will also contribute to the neutrino masses unless we choose $v_3=0$. However, our choice is not unnatural [18], in fact, by an orthogonal transformation between the $\phi_3$, $\phi_4$ and $\phi_5$ Higgs fields, the vacuum structure can be arranged consistently with the choice $v_3 = 0$. Furthermore, we infer the hierarchy of the VEV’s of the Higgs fields as

$$k_3, k_4 \gg v_i > k_1, k_2$$

(13)

because of the fact that the VEV’s of the singlets associated with $\lambda$ term should be much more larger than the VEV’s of the singlets associated with $\lambda'$ term to yield large magnetic moment and small neutrino masses. Thus, it is necessary to incorporate two widely different lepton number symmetry breaking scales, two of them ($\sim 1 TeV$) are much above the electroweak symmetry breaking scale and the other two scales ($\sim 100 GeV$) are near to the electroweak scale. Moreover, all the lepton number violating processes at low energy (such as $\mu \rightarrow eee$, $\mu \rightarrow e\gamma$, $K_L \rightarrow \mu e$ etc.) are highly suppressed due to the small mass squared differences of neutrinos. Such type of scenario has been investigated in Ref. 23 in the context of Baryogenesis.

The most general $S_3 \times Z_3 \times Z_4$ discrete symmetry invariant lepton-Higgs Yukawa interaction, in our present model is as follows

$$-L_Y = [f_1(l_1^L \nu_{\mu R} + \bar{l}_2^L \nu_{e R}) + f_2(l_3^L \nu_{\tau R})\tilde{\phi}_1 + g_1(l_1^L \nu_{e R} + \bar{l}_2^L \nu_{\mu R})\phi_3 +$$
$$g_2(l_1^L \tau R \phi_4 + \bar{l}_2^L \tau R \phi_5) + g_3(l_3^L \tau R \phi_3 + g_4(l_3^L \nu_{e R} \phi_4 + l_3^L \nu_{\mu R} \phi_5) +$$
$$g_5(l_1^L \nu_{\mu R} \phi_4 + \bar{l}_2^L \nu_{e R} \phi_5) + h.c]$$.  

(14)
Substituting the VEV’s of the Higgs fields in Eqn.(14), the Dirac neutrino mass matrix turns to be

\[ M^D_\nu = \begin{pmatrix} 0 & a & 0 \\ a & 0 & 0 \\ 0 & 0 & \xi a \end{pmatrix} \]  \hspace{1cm} (15)

where \( a = f_1 v_1, \xi = \frac{f_2}{f_1}. \)

It is to be mentioned that the mixing between the third and the other two generations has made zero (at the tree level) by choice and this is in agreement with the ansatz for \( M^D_\nu \) in Ref.1. The non-zero contribution to the vanishing elements of \( M^D_\nu \) arise at the one loop level and gives rise to a very small mixing angles consistent with the present experimental limits as mentioned earlier. Diagonalizing \( M^D_\nu \), we obtain the mass eigenvalues of the three Dirac neutrinos as

\[ m_{\nu_1} = m_{\nu_2} = a = m_0 \]  \hspace{1cm} (16)

\[ m_{\nu_3} = \xi m_0 \]  \hspace{1cm} (17)

Thus we see that the present model admits two degenerate Dirac neutrinos and the degeneracy between \( m_{\nu_3} \) and \( m_{\nu_1} \) or \( m_{\nu_2} \) is lifted due to the presence of the factor \( \xi \). We will estimate the value of \( \xi \) from the knowledge of the experimental data of atmospheric neutrino, in the following. Substituting Eqn.(12) in Eqn.(16), we obtain the mass of the two degenerate Dirac neutrinos as

\[ m_{\nu_3} = m_{\nu_2} = \frac{f_1 \lambda' v_3 k_1 k_2}{m_1^2} \]  \hspace{1cm} (18)
For a generic choice of model parameters $v_2 = 100 \, \text{GeV}$, $k_1 = k_2 = 100 \, \text{GeV}$, $m_1 \sim 1 \, \text{TeV}$ and $f_1 \sim 1$ we obtain $m_{\nu_1} = m_{\nu_2} = 1 \, \text{eV}$ for $\lambda' \sim 10^{-9}$. Such a small coupling is also consistent with the other area of investigations [24]. However, the model contains a tiny parameter space and there is not much freedom in the variation of model parameters. In particular, $v_2$ is restricted in the range $(100 - 250) \, \text{GeV}$ and, although $k_1, k_2$ can be varied in a wide range but, to yield small neutrino masses, $k_1, k_2 \geq 100 \, \text{GeV}$ [18, 22]. The Yukawa coupling $f_1$ and the coefficient of the Higgs potential $\lambda'$ are less than equal to unity in order to satisfy the unitarity bound. The mass of the neutral Higgs boson $m_1$ is restricted within the range $(65.1 \, \text{GeV} - 1 \, \text{TeV})$ where the lower bound comes from the results of four combined experiments at CERN [25] whereas the upper bound is also due to the unitarity of the theory.

The $\xi$ factor, which lifts the degeneracy between the three neutrinos, can be determined from the atmospheric neutrino problem. The hierarchy between $m_0$ and $m_{\nu_3}$ is manifested from the value of $\Delta_{32} = m_0^2(\xi^2 - 1)$. For a typical value of $\Delta_{32} \sim 4 \times 10^{-3} \, \text{eV}^2$ [7] which can explain the atmospheric neutrino deficit, we get $\xi \sim 1$ and thus, from Eqn. (17), we obtain $m_{\nu_3} \sim m_0$.

The present model admits a large transition magnetic moment due to the presence of the quartic coupling term $\lambda(\phi_1^\dagger \phi_3 \eta_3 \eta_4 + \eta_1^\dagger \eta_3 \phi_3^\dagger \phi_1)$ in the Higgs potential through charged Higgs exchange at the one loop level. Similar diagram has also been obtained in Ref.18 in which the internal fermion lines are not ordinary leptons, contrary to the present model. The contribution to $\mu_\nu$ in the present model is given by (in the weak basis)
\[ \mu_\nu \sim \frac{e}{8\pi^2} f_1 g_1 \frac{m_e k_3 k_4 \lambda}{m_{\phi_1}^2 m_{\phi_3}^2} \] (19)

where \( m_{\phi_1}^+ \), \( m_{\phi_3}^+ \) are the masses of the charged Higgs bosons. With the following choice of model parameters consistent with the present experimental limits [25] \( m_{\phi_1}^+ \sim 200 \text{ GeV}, \ m_{\phi_3}^+ \sim 100 \text{ GeV}, \ k_3 \sim k_4 \sim 1 \text{ TeV} \), present experimental bound on \( \mu_\nu < 10^{-11} \mu_B \) can be obtained for \( f_1 g_1 \lambda \sim 1 \).

It is to be noted that the \( \lambda \) term, which contributes to the neutrino magnetic moment, cannot contribute to the neutrino masses due to our choice of \( v_3 = 0 \) and, hence, the lepton number breaking scales, which are much higher than the electroweak scale, do not give rise to any contribution to the neutrino masses. Similarly, the \( \lambda' \) term of the Higgs potential which generates small neutrino masses is prohibited to contribute to the magnetic moment due to the discrete symmetry incorporated in the model. Thus, by decoupling the two sets of widely different lepton number symmetry breaking scales, it is possible to achieve small degenerate Dirac neutrinos \( \sim 1 \text{ eV} \) and large transition magnetic moment \( \sim 10^{-11} \mu_B \) in the present model. Furthermore, \( \nu_{eL} \rightarrow \nu_{\mu R} \) flipping in the magnetic field of the Sun requires the mass splitting \( \Delta_{21} \leq 10^{-4} \text{eV}^2 \) [26]. In the present model \( \Delta_{21} \) vanishes due to the degeneracy in mass between \( \nu_e \) and \( \nu_\mu \). However, a small mixing (which in turn generates non-zero \( \Delta_{21} \)) can be obtained from the one loop level and is expected to be well within the present experimental limit.

In summary, we demonstrate that Standard \( SU(2)_L \times U(1)_Y \) model with \( S_3 \times Z_3 \times Z_4 \) discrete symmetry, right-handed neutrinos and appropriate
Higgs fields can give rise to small degenerate Dirac neutrinos $\sim 1$ eV which can simultaneously reconcile the solar and atmospheric neutrino deficits as well as the candidature of the neutrino as a hot dark matter, through spontaneous lepton number violation at two widely different scales. The first two generations are completely degenerate to each other due to our choice of discrete symmetry while the third can also be made almost degenerate with the first two generations through some reasonable fine tuning of the model parameter $\xi$. The present model also admits large transition magnetic moment $\sim 10^{-11}\mu_B$ consistent with the present experimental limits due to the charged Higgs exchange at the one loop level through the oscillation of $\nu_{eL} \rightarrow \nu_{\mu R}$, which seems to be a reasonable choice for the solution of the existing anticorrelation between the Sun spot activity and the observed neutrino flux on earth but evades the constraints from the SN 1987 A data and nucleosynthesis.

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References

1. D.Caldwell and R.N.Mohapatra, Phys. Rev. D48, (1993) 3259.

2. K.Lande et.al. Proc. XXVth Int. Conf. on High Energy Physics, Singapore, ed. K. K. Phua and Y. Yamaguchi, World scientific (Singapore, 1991); K.S. Hirata et.al. Phys.Rev. Lett. 66, (1991) 9; P. Anselmann et. al. Phys. Lett. B285, (1992) 376; A. I. Abazov et.al. Phys. Rev. Lett. 67, (1991) 3332; V. Garvin, talk at Int. Conf. on HEP, Dallas 1992.

3. E. M. Beier et.al. Phys. Lett. B280, (1992) 149; D. Casper et.al. Phys. Rev. Lett. 66, (1991) 2561.

4. A. N. Taylor and M. Rowan Robinson, Nature 359, (1992) 396; M. Davis, F. J. Summers and D. Schlegel, Nature 359, (1992) 393.

5. GALLEX Collaboration, P.Anselmann et.al. Phys. Lett. B342, (1995) 440.

6. J.N.Bachall, P.I.Krastev and E.Lisi, Phys. Lett. B348, (1995) 121, E. Kh. Akhmedov, A. Lanza and S. T. Petcov, Phys. Lett. B348, (1995) 124.

7. B.Achkar et.al. Nucl. Phys. B434, (1995) 503.

8. D.R.O.Morrision, CERN-PPE/95-47, (1995).

9. A.S.Joshipura, Z.Phys. C64, (1994) 31.

10. P.Bamert and C.P.Burgess, Phys. Lett. B329, (1994) 289.
11. K. S. Babu, IV Mexican school of Particles and Fields, ed. by J. Lucio M and A. Zepeda (World Scientific, Singapore, 1992), p 104; P. Langacker, Summer school in HEP and Cosmology, Trieste 1992 ed. E. Gava et.al. (World Scientific, Singapore) p 487; M. Fukugita and T. Yanagida, in "Physics and Astrophysics of Neutrinos" ed. by M. Fukugita and A. Suzuki, (Springer Verlag, 1995).

12. M. Gell Mann, P. Ramond and R. Slansky in "Supergravity", ed. by D. Z. Freedman, (North Holland, 1979), T. Yanagida, in Proceedings of "Workshop on Unified theory and Baryon Number in the Universe" ed. by O. Sawada and A. Sugamoto, (KEK, 1979), R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, (1980) 912, Phys. Rev. D23, (1981) 165.

13. L. B. Okun, M. B. Voloshin and M. I. Vysotsky, Sov. J. Nucl. Phys. 44, (1986) 440, M. B. Voloshin and M. I. Vysotsky, Inst. for Theoretical and Experimental Physics (Moscow) Report No. 86-1, (1986), L. B. Okun, M. B. Voloshin and M. I. Vysotsky, Inst. for Theoretical and Experimental Physics (Moscow) Report No. 86-82, (1986).

14. C. L. Cowan Jr. and F. Reines, Phys. Rev. Lett. 107, (1957) 528; J. Kim, V. Mathur and S. Okubo, Phys. Rev. D9, (1974) 3050; A. V. Kyuldjiev, Nucl. Phys. B243, (1984) 387.

15. W. J. Marciano et.al. Particle Data Group, Phys. Lett. B239, (1990) 1, P. Vogel and J. Engel, Phys. Rev. D39, (1989) 3378.

16. M. Fukugita and S. Yazaki, Phys. Rev. D36, (1987) 3817, S. I.
Blinnikov, Moscow preprint, ITEP-88-19 (1988); G. G. Raffelt, Phys. Rev. Lett. 64, (1990) 2856.

17. K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 64, (1990) 1705, G. Ecker, W. Grimus and H. Neufeld, Phys. Lett. B232, (1989) 217.

18. D. Chang, W. Y. Keung and G. Senjanovic, Phys. Rev. D45, (1992) 31.

19. Y. Aharonov, I. Goldman, G. Alexander and S. Nussinov, Phys. Rev. Lett. 60, (1987) 1789, J. Lattimer and J. Copperstein, Phys. Rev. Lett. 61, (1988) 25, R. Barbieri and R. N. Mohapatra, ibid, 61 (1988) 27; D. Notzold Phys. Rev. D38, (1990) 1658.

20. Y. Chikashige, R. N. Mohapatra and R. D. Peccei, Phys. Rev. Lett. 45, (1980) 1926, Phys. Lett. B98, (1981) 265.

21. E. Ma, Phys. Rev. D43, (1991) 587, (1991) 2761; N. G. Deshpande, M. Gupta and P. B. Pal, Phys. Rev. D45, 953 (1992), N. G. Deshpande and X. G. He, OITS preprint-529/94.

22. G. Gelmini and E. Roulet, CERN-TH-7541/94.

23. G. Gelmini and T. Yanagida, Phys. Lett. B294, (1992) 53.

24. V. Berezinsky and J. W. F. Valle, Phys. Letts. B318, (1993) 360, J. T. Peltoniemi and J. W. F. Valle, Phys. Letts. B304, (1993) 147, J. McDonald Phys. Letts. B323, (1994) 339.

25. Andre Sopczak, CERN-PPE/ 95-46, hep-ph/9504300.
26. C. S. Lim and W. Marciano, Phys. Rev. D37, (1988) 1368, E. Kh. Akhmedov, Phys. Lett. B213, (1988) 64.