Testing Parity with Atomic Radiative Capture of $\mu^-$

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The next generation of “intensity frontier” facilities will bring a significant increase in the intensity of sub-relativistic beams of $\mu^-$, allowing us to study parity-violating interactions of muons with nuclei via direct radiative capture of muons into atomic $2S$ orbitals. Since atomic capture preserves longitudinal muon polarization, the measurement of the gamma ray angular asymmetry in the single photon $2S_{1/2} \rightarrow 1S_{1/2}$ transition will offer a direct test of parity. We calculate the probability of atomic radiative capture taking into account the finite size of the nucleus to show that this process can dominate over the usual muonic atom cascade, and that the as yet unobserved single photon $2S_{1/2} \rightarrow 1S_{1/2}$ transition in muonic atoms is detectable in this way using existing muon facilities.

**Introduction.**—The standard model of particles and fields (SM) has shown tremendous vitality under an onslaught of new TeV-scale data from the Large Hadron Collider (LHC). Stringent limits are derived on new hypothetical vector particles $Z'$ that mediate interactions between light quarks and charged leptons. For a sequential SM $Z$-like $Z'$ particle such limits extend to 2 TeV, rendering low-energy parity-violating tests not competitive with the LHC in the search for new heavy resonances with large couplings to SM particles. However, an alternative possibility—light and very weakly coupled particles—may easily escape the high-energy constraints while inducing some nontrivial effects at low energy [1]. In recent years the interest in this type of physics has intensified, largely due to the accumulation of various anomalous observations that such light particles may help to explain. (For a possible connection between light vectors and dark matter physics see, e.g., Ref. [2].) In parallel with this, attempts to detect such new states at “intensity frontier” facilities are becoming more frequent and more systematic [3].

Muon physics, and its study with new high intensity muon beams, is a natural point of interest because of the lingering discrepancy between calculations and measurements of the muon anomalous magnetic moment ([4] as well as the recent striking discrepancy of the proton charge radius extracted from the muonic hydrogen Lamb shift [5] as compared to other determinations of the same quantity [6].) While it is far from clear that these discrepancies are not caused by some poorly understood SM physics or experimental mistakes, it is still important to investigate models of New Physics (NP) that could create such deviations. Models with light vector particles (see, e.g., [7]) are particularly interesting as they can remove the $g - 2$ discrepancy quite naturally [8], or be responsible for extra muon-proton interactions that can be interpreted as a shift of the proton charge radius [9,10].

As was argued in Ref. [10], a lepton flavor-specific muon-proton interaction in combination with constraints in the neutrino sector may imply that right-handed muon number is gauged, leading to new parity-violating muon-proton neutral current interactions. We take this model as a representative example of new physics at the sub-GeV energy scale that can create stronger-than-weak effects in the interaction of muons with nuclei. In this Letter, we revisit the idea of searching for parity violation in the muon sector using muonic atoms, keeping in mind that no direct tests of the axial vector muon coupling have been performed at low energy, and that the NP contribution could dominate over the SM [10]. To be specific, we consider a low-energy effective neutral current Lagrangian, that includes the sum of the SM and NP contributions,

$$\mathcal{L}_\mu = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{NP}}$$

where SM vector couplings to nucleons are given by $g_V = -\frac{1}{2}, g'_V = \frac{1}{2} - 2\sin^2\theta_W$. In the model with gauged right-handed muon number, the least constrained points in the parameter space correspond to the mass of the mediator gauge boson of $m_{\gamma'} \sim 30\text{ MeV}$. In that case, the fit to the proton charge radius suggests [10]

$$\frac{4\pi\alpha g_{\mu}^\text{NP} g_{p}^\text{NP}}{m_{\gamma'}^2} \simeq \frac{2 \times 10^{-5}}{(30\text{ MeV})^2} \gg G_F,$$

which should be considered as perhaps the most optimistic value for the strength of the muon-proton interaction. In what follows we suggest a new way to search for the manifestation of [11] and [2] in muonic atoms using the process of atomic radiative capture (ARC) to the $2S$ state: $\mu^- + Z \rightarrow (\mu^- Z) 2S + \gamma$. We show that probing $\mathcal{L}_{\text{NP}}$ of maximal strength is possible with existing muon line facilities, while the SM values can eventually be tested at the next generation of high-intensity muon sources.

It is well-known that the suppressed M1 single photon $2S_{1/2} \rightarrow 1S_{1/2}$ transition in combination with the small energy difference between the $2S$ and $2P$ states enhances...
the parity-violating asymmetry in M1-E1 interference. This idea has received a significant amount of theoretical and experimental attention, summarized in the review [11]. The most promising scheme for the detection of parity violation to date was identified as a slow muon forming a highly excited atomic state with a nucleus followed by a cascade ending with

\[ \ldots \rightarrow 2S_{1/2} \stackrel{M1-E1}{\rightarrow} 1S_{1/2} + \gamma; (\mu^-)_{1S} \rightarrow e^- \nu_\mu \bar{\nu}_e, \]

with parity violation being encoded in the correlation between the directions of the outgoing \( \gamma \) and the muon decay electron. In Fig. 1, we show a level diagram for a typical muonic atom.

Despite considerable efforts, the single photon 2S–1S transition itself has never been detected in any muonic atoms. In light atoms, \( Z \leq 10 \), this transition cannot be distinguished from the far more dominant 2P–1S, as the difference between gamma ray energies in this case is much smaller than the energy resolution of \( \gamma \)-detectors. Combining this with the tiny branching ratio of the one-photon decay of the 2S\(_{1/2} \) state in light elements, and the fact that it gets scarcely populated, \( O(1\%) \), during the cascade, makes the measurement of parity violation very challenging in light muonic atoms, even though the value of parity-violating asymmetries could be as large as few percent [11].

Heavier muonic atoms, \( Z \sim 30 \), have been suggested as promising candidates to test parity [12], because the 2S–1S and 2P–1S transitions can be easily resolved, as the energy difference between the 2S and 2P states reaches

\[ \Delta E \equiv E_{2S} - E_{2P} = \frac{(Z \alpha)^4 m_\mu (m_\mu R_e)^2}{12} \simeq 210 \text{ keV} \times (Z/36)^4 \times (R_e/4.2 \text{ fm})^2, \]

where we have normalized the nuclear charge, \( Z \), and the nuclear charge radius, \( R_e \), on the values for krypton, and suppressed total \( J \) indices, effectively neglecting the splitting between 2P\(_{3/2} \) and 2P\(_{1/2} \) states. Unfortunately, as in the case of lighter elements, the 2S–1S transition was never detected in heavier atoms, because of the dominance of the background created by quanta from \( nP–1S \) transitions, \( n \geq 3 \), whose energies have been degraded [11].

To elaborate on this, one can estimate the signal-to-background ratio of the single photon 2S–1S transition during the atomic cascade. The signal, \( S \sim N_{2S} B_{1\gamma} \), is proportional to the fraction of cascade muons \( N_{2S} \) that end up in the 2S state, where \( N_{2S} \) is typically on the order of 10\(^{-2} \) [13], and the branching of M1 single photon transition from 2S states, which for \( Z \sim 30 \) [12] is given by

\[ \frac{B_{1\gamma}}{B_{2S-2P}} \simeq \frac{\Gamma_{2S-1S+\gamma}}{\Gamma_{2S-2P}} \sim 2 \times 10^{-3}. \]

For smaller \( Z \), \( Z \leq 28 \), the single photon branching is strongly suppressed by Auger processes [14] and by the two photon transitions. The cascade-related background consists of the number of energy-degraded \( nP–1S \) \( (n \geq 3) \) photons \( (i.e. \) those that do not deposit their full energy in the detector) that fall into the energy resolution interval \( \Delta E \) centered at the energy of the 2S–1S transition. From experimental studies [15] one can conclude that \( O(20\%) \) of muons undergoing a cascade generate \( nP–1S \) transitions. For realistic \( \gamma \)-detectors, the number of energy-degraded photons is \( \sim 50\% \), and the number of photons under the 2S–1S peak within the energy resolution window of \( \Delta E \sim 2 \text{ keV} \) can be estimated as \( B \sim 0.2 \times \Delta E/(2E_\gamma) \sim 10^{-4} \) for \( E_\gamma \sim 2 \text{ MeV} \). Therefore, one arrives at the following estimate of signal-to-background:

\[ \left[ \frac{S}{B} \right]_{\text{cascade}} \leq 0.2. \]

The actual ratio is smaller than this upper bound because of additional photon backgrounds caused by other sources, which explains why the 2S–1S transition has not been detected [11].

In addition to these challenges in detecting the 2S–1S transition in muon cascades, another difficulty in implementing the scheme in [11] lies in the fact that the final step, muon decay, for these elements is very subdominant to nuclear muon capture. Because of the combination of these two factors, parity experiments with \( Z \sim 30 \) elements were deemed impractical [11].

**New proposal for a parity-violation measurement.**—Our proposal is to abandon [11] and use thin targets of \( Z \geq 30 \) elements that only decrease the \( \mu^- \) momentum, but do not stop the particle completely. This removes most of the background related to the muonic cascade. A fraction of the muons undergo ARC directly into the 2S state. The signal consists of two \( \gamma \) quanta, one from the ARC process (\( \gamma_1 \)), and the other from the single photon decay
of the $2S$ state ($\gamma_2$):
\[
\mu^- + Z \rightarrow (\mu^- Z)_{2S_{1/2}} + \gamma_1; \quad 2S_{1/2} \xrightarrow{E_1 - M_1} 1S_{1/2} + \gamma_2.
\] (8)
Here $\mu^-$ denotes the longitudinally polarized muon. While for the relevant range of $Z$ the energy of $\gamma_2$ is on the order of 2 MeV, the energy of $\gamma_1$ is dependent on the muon momentum, and for muon momentum of 50 MeV is in the 10 MeV range. The parity-violating signature is the forward-backward asymmetry of $\gamma_2$ relative to the direction of the muon spin.

To calculate the cross section for muonic ARC into the $2S$ state (the first step in (3)), we note that the analogous process involving an electron, electron-nucleus photorecombination, in the dipole approximation with a point-like nucleus is a standard textbook calculation [16, 17], as it can be obtained from the standard hydrogen-like photoelectric ionization cross section $\sigma_{PE}^{(0)}$. Here we adjust this for the muon case, which, besides the substitution $m_e \to m_\mu$, involves accounting for the finite nuclear charge radius and the departure from the dipole approximation. This can be done by introducing a correction factor to the standard formula,
\[
\sigma_{ARC} = \frac{2\omega^2}{p^2} \sigma_{PE}^{(0)}; \quad \sigma_{PE} = \eta(p, R_c, Z, n, l) \times \sigma_{PE}^{(0)}(nI),
\]
\[
\sigma_{PE}^{(0)}(2S) = \frac{2^{14/2} \pi a^2 E_2^4}{3 \omega^4} \left[ 1 + 3 E_2 \omega \right] \exp\left(-\frac{1}{pa} \cot^{-1}\frac{1}{2\omega}\right) \frac{1}{1 - \exp(-2\pi/pa)}.
\]
In these expressions, $p$ is the momentum of the incoming muon, $a$ is the Bohr radius, $\omega = (Zam_\mu)^{-1}$, $E_2 = Z^2 \alpha^2 m_\mu / 8$ is the (uncorrected) binding energy of the $2S$ muon, and $\omega = p^2/2m_\mu + E_2$ is the (corrected) energy of the photon emitted in the ARC process. The correction factor $\eta$ is calculated by numerically solving the Schrödinger equation for a muon moving in the field of the nucleus with uniform charge distribution with charge radius $R_c$. The results for the cross sections are plotted in Fig. 2 for $Z = 36$ and $R_c = 4.2$ fm. As one can see, the corrections to the simple formula are significant, and mostly come from the finite charge of the nucleus, suppressing a naive cross section by more than a factor of $\sim 3$ for $p_\mu > 60$ MeV. Moreover, at $p \sim m_\mu$, this formula will need to be further corrected by relativistic effects that thus far have been ignored in our treatment.

Previously, the ARC process was considered theoretically in Ref. [15] for the case of muonic hydrogen, and searched for experimentally in Ref. [13] in muonic cascades in Mg and Al. The ARC process was not detected because in the case of stopped muons the cross section for forming muonic atoms via electron ejection is several orders of magnitude larger than $\sigma_{ARC}$. Because of that, one should not expect that the muon cascade experiments can be sensitive to the ARC processes.

Below, we estimate the probability for the ARC process in a thin gaseous target of Kr that decreases the momentum of the muon beam from $p_{\text{max}} = 30$ MeV to $p_{\text{min}} = 25$ MeV:
\[
P_{\text{ARC,2S}} = \frac{2 \times 10^{-7}}, \quad (9)
\]
where the momentum loss, $dp/dx$, is given by standard Bethe-Bloch theory. For a target size of $\sim 5$ cm, the number density of the krypton atoms would correspond to pressure of $p_{\text{Kr}} \sim 8$ atm.

Combining the probability of the ARC process (10) with the branching ratio of the M1 photons (6), we arrive at the emission rate of $2S$--$1S$ photons as a function of the incoming muon flux,
\[
\frac{dn_{2S\rightarrow 1S}}{dt} = P_{\text{ARC}} \times \text{Br}_{1\gamma} \times \Phi_{\mu^-} \sim \frac{1}{250 \text{ s}} \times \frac{\Phi_{\mu^-}}{10^5 \text{ s}^{-1}}. \quad (10)
\]
\[
\text{The lifetime of the 2S state is extremely small: for } Z > 30 \text{ it does not exceed } 10 \text{ fs [12] which allows for a tight timing correlation between } \gamma_1 \text{ and } \gamma_2 \text{ in (3).}
\]

We can also estimate the intrinsic background created by the $nP$--$1S$ transitions in this case. For a transparent target, one source of background consists of the bremsstrahlung process, $\mu + Z \rightarrow \mu + Z + \gamma$ that degrades the muon energy enough to trap it inside the target, with a subsequent muon cascade creating $nP$--$1S$ photons. To calculate the yield of $nP$--$1S$ photons, we estimate the probability for the process $\mu + Z \rightarrow \mu + Z + \gamma$ by taking the standard cross section [17] and modifying it by the correction coming from the finite nuclear charge. In this way we find, for the same parameters of the target,
\[
P_{\text{cascade}} \sim P_{\mu+Z\rightarrow\mu+Z+\gamma} \sim 20 \times P_{\text{ARC,2S}}, \quad (11)
\]
requiring that the bremsstrahlung photon be at least as energetic as that coming from ARC into the 2S state for
\( p_{\text{min}} = 25 \text{ MeV} \). Only a small fraction of the cascade photons, \( \sim O(10^{-4}) \), will be degraded to mimic the 2S–1S transition and we can conclude that the ratio of signal to irreducible background is

\[
\left[ \frac{S}{B} \right]_{\text{ARC}} = \frac{F_{\text{ARC}, 2S} \times B_{\gamma z}}{F_{\text{cascade}} \times 10^{-4}} \sim O(1), \quad (12)
\]

and the gain over (7) is rather significant. The contribution to the background due to direct capture on \( n \geq 3 \) orbits is even smaller. The background from bremsstrahlung and cascade photons in (11) is small enough that Ge detectors with \( \mu S \) response times can operate with muon fluxes of \( O(10^{10} \text{ s}^{-1}) \) without photons from these processes arriving within the lifetime of the 2S state. We conclude that while the signal rate is small, Eq. (10), the gain in the \( S/B \) can be substantial, making the search for the ARC processes and 2S–1S transitions worth pursuing experimentally. A further increase in \( S/B \) can be achieved by imposing a cut on the energy of \( \gamma_1 \) that can distinguish it from the lower energy bremsstrahlung \( \gamma \).

We are now ready to investigate the feasibility of the parity violation experiment with the use of the ARC scheme in (8). The forward- backward asymmetry of the 2S–1S photon is related to the coefficient of 2S–2P mixing \( \delta \) and the ratio of E1 and M1 amplitudes [12],

\[
A_{\text{FB}} = \frac{N_{\gamma_2}(\theta > \frac{\pi}{2}) - N_{\gamma_2}(\theta < \frac{\pi}{2})}{N_{\gamma_2}(\theta > \frac{\pi}{2}) + N_{\gamma_2}(\theta < \frac{\pi}{2})} = \frac{(\text{E1})_{2P-1S}}{(\text{M1})_{2S-1S}} 
v \simeq 680 \times \left( \frac{36}{Z} \right)^3 \times \delta, \quad i \delta = \frac{(2S_{1/2} | H_{PV} | 2P_{1/2})}{\Delta E}, \quad (13)
\]

where the parity-violating Hamiltonian can be derived from (1) and (2). The size of the parity-violating admixture in the SM [12] and in the presence of non-standard interactions [10] is given by

\[
\delta_{\text{SM}} \simeq \frac{3 \sqrt{3} G_F}{8 \sqrt{2} \pi Z \alpha R_{\gamma Z}^2} \left( g_p + g_n \frac{A - Z}{Z} \right), \quad (14)
\]

\[
\delta_{\text{NP}} = \frac{3 \sqrt{3} g_{\mu \text{NP}}}{Z \alpha R_{\gamma Z}^2 m_{\gamma Z}^2 (m_{\gamma Z} + 1)^3} \left( g_p + g_n \frac{A - Z}{Z} \right).
\]

For the non-standard interaction (2), we normalize its strength to the possible size of the effect suggested by the muonic hydrogen Lamb shift discrepancy, following [10]. This way for \( Z = 30 \) we find

\[
A_{\text{FB}}[\text{SM}] \simeq 0.5 \times 10^{-4}, \quad A_{\text{FB}}[\text{NP}] = (0.5 - 11\% \). \quad (15)
\]

The lower value of the asymmetry \( A_{\text{FB}}[\text{NP}] \) is for small, \( \sim 10 \text{ MeV} \), masses of vector mediators, while larger values are for the scaling regime, \( m_V \gg 1/a \).

Using these asymmetries and a realistic efficiency factor of \( \sim 0.1 \) for the detection of a two-photon transition, we arrive at the following estimate of the time required to achieve the number of events \( N \propto 1/A_{\text{FB}}^2 \):

\[
T[\text{SM}] \sim 10^8 s \times \frac{10^{11} \text{ s}^{-1}}{\Phi_{\mu}},
\]

\[
T[\text{NP}] \sim 3 \times 10^5 s \times \frac{10^7 \text{ s}^{-1}}{\Phi_{\mu}} \times \left( \frac{0.1}{A} \right)^2. \quad (16)
\]

One can see that, while the test of a muonic parity violating \( A_{\text{FB}} \) down to the \( O(10^{-4}) \) value of the SM via the method suggested in this paper is statistically possible only with future high- intensity muon beams, tests of some NP models [10] are feasible even at existing facilities.

In conclusion, let us summarize the main advantages of possible tests of parity using the atomic radiative capture scheme in Eq. (8):

i. The muon capture onto the 2S orbit proceeds via an E1 transition and does not depolarize the muons. Therefore it is possible to capture a fully polarized muon onto the 2S orbit and study an angular asymmetry of the outgoing \( \gamma \) without the need to observe muon beta decay in the 1S state.

ii. The gain in \( S/B \) is significant, as the \( nP-1S \) \( n > 3 \) transitions of cascade muons that prevented the detection of the single photon 2S–1S decay in the past are greatly reduced. The detection of this transition can be realistically performed even with the existing sources of \( \mu^- \).

iii. The use of a transparent target allows one to study parity with muons in a "parasitic" set-up, when the dominant part of the muon flux is used for other experiments. It also appears that the ARC-based method [8] can withstand the increase of the muon beam intensity more easily than the cascade-based methods [3].

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