Similarity Calculation of 3D Model By Integrating Improved ACO Into HNN

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ABSTRACT It is important for 3D model retrieval to compute similarity between two models accurately. In order to calculate two models’ similarity, their face matching scheme need be found. Hopfield neural network (HNN) and ant colony optimization (ACO) algorithm can be used to match source faces and target ones. But, they all drop into local optimum easily and can not get optimal face matching scheme. This paper proposes a new method of computing model similarity by integrating improved ACO algorithm into HNN. Shape similarity between source face and target one is calculated based on the difference of face’s edge number. Structure similarity is computed according to adjacency correspondence relationship between source face and target one. Two faces’ similarity is calculated based on shape similarity and structure similarity, which is introduced into transfer probability. Indirect expectation heuristic is defined to improve ACO algorithm. Then, improved ACO algorithm is integrated into HNN to search for optimal face matching scheme. Model similarity is computed based on optimal sequence of face pairs. Experimental results show that compared with improved ACO algorithm, the proposed method improves the ranking effect of 14.29% of models, which can measure two models’ difference effectively.

INDEX TERMS 3D model, Hopfield neural network, ant colony optimization, shape similarity, structure similarity, indirect expectation heuristic, sequence of face pairs.

I. INTRODUCTION

3D models have been widely used in computer-aided design, 3D animation and virtual reality. It is a time-consuming process to construct a complex 3D model. With the rapid development of 3D modeling technology, there are a large amount of 3D models available in Web and database. If the needed 3D model could be retrieved from Web and database, design efficiency will be improved dramatically by reusing these models. User can acquire 3D models according to his needs from Web and database. In process of developing new products, reusing those existing CAD models could avoid designing from scratch and decrease user’s cost. User modifies these models to meet his design intent or designs incrementally based on these models. It will reduce design cost and shorten design time. At the same time, it will effectively improve design quality. It is key for user to retrieve his needed 3D model accurately from Web and database. Search engines for 3D models are developed to meet user’s need [1]. Now, 3D model retrieval is research focus in computer-aided design and computer aided industrial design. Model similarity calculation is an important research problem in 3D CAD model retrieval, which directly influences retrieval efficiency. There are many methods of 3D model retrieval including geometry-based approach, graph-based one, view-based one, semantic-based one and shape distribution-based one.

In geometry-based approach, feature vector is extracted from 3D model’s shape and topology. Then, feature vector is used to compute model similarity [2], [3]. In graph-based approach, graph is applied to describe how 3D model’s shape components are linked together. The problem of 3D model retrieval is transformed into graph matching problem [4]. In view-based approach, 3D model is denoted as a group of 2D views from multiple viewpoints. Views can contain 3D
model’s spatial and structure information. Based on these views, 3D model’s similarity is computed and the needed model can be searched [5]. In semantic-based approach, ontology is constructed to describe CAD model’s functional semantics and represent conceptual design space. Semantic information in high level is used to make search much closer to engineers’ intent [6]. In shape distribution-based approach, shape distribution of 3D model is extracted to evaluate model similarity, which contains shape information and topological one [7]–[9].

Now, metaheuristic algorithms are widely applied to find optimal face matching scheme between source model and target one, based on which model similarity is computed and the needed 3D model is retrieved [10], [11].

This paper proposes a new method of calculating 3D model’s similarity. Component information of edges in face is used to evaluate face’s shape similarity. Topological information of faces in 3D model is applied to compute adjacency correspondence relationship between source faces and target ones. Face’s structure similarity is calculated through combining shape similarity and adjacency correspondence similarity. Face’s similarity is calculated based on shape similarity and structure similarity. Indirect expectation heuristic is defined and introduced into transfer probability. The purpose is to improve ACO algorithm and find better face matching scheme. Based on face matching scheme gotten by improved ACO algorithm, HNN is applied to find optimal sequence of face pairs. Experimental results show that the proposed method performs better on the task of calculating model similarity. The main contributions of this paper are summarized as follows:

- Shape similarity of face is computed based on edge number. Structure similarity of face is calculated based on shape similarity and adjacency correspondence relationship.
- Face’s similarity is evaluated based on shape similarity and structure similarity.
- Indirect expectation heuristic is defined based on face’s similarity and introduced into transfer probability. Improved ACO algorithm is given.
- Improved ACO algorithm is integrated to HNN to search for optimal sequence of face pairs.

II. RELATED WORK

Semantic information is often used to retrieve 3D model. Wang constructs 3D model’s functional semantic ontology and proposes an approach to represent CAD model as attributed adjacency graph. He proposes a model retrieval algorithm that supports multi-function extended retrieval to explore more creative design knowledge in semantic level [12]. Qin extracts layered feature ontology and ontology mapping. He gives a uniform description for heterogeneous CAD model as semantic descriptor. Meanwhile, he constructs heuristic rules with SWRL language and ontology reasoning semantic descriptors [13]. Huang divides CAD model into several subsections. He appends semantic descriptors and annotations to subsections. Model’s semantic ontology similarity is used to realize model retrieval [14]. Jeon uses domain ontology and shallow natural language processing technology to express design documents. He applies design documents to retrieve CAD models. He extracts semantic representation and hidden design information from text in CAD model [15]. However, it is a very complicated work to describe CAD model’s functional semantics and represent conceptual design space. At the same time, it is also not easy to design good semantic descriptors.

Node in graph is used to denote face and edge is applied to represent relationship between faces. Source model and target one are expressed respectively as graphs. The problem of computing model similarity is transformed into subgraph isomorphism. Tao constructs vertex similarity matrix and edge similarity matrix between 3D CAD models based on attribute adjacency graph. He transforms 3D model retrieval into a matching optimization problem [16]. Qiao uses attribute adjacency graph to represent 3D model. He utilizes conjugate subgraph matching algorithm to retrieve source model similar with target one [17]. Tao creates face adjacency graph according to 3D model’s B-Rep. He divides face adjacency graph into convex regions, concave ones and plane ones. He uses region attribute codes to represent face region, and computes two models’ similarity by comparing region attribute codes [18]. Huangfu gives a local retrieval method of CAD models based on graph indexing and filtering mechanism. 3D model is represented based on boundary information and attribute adjacency graph. Spectral theory is used to code respectively edges and points in graph. The speed of 3D model retrieval is improved through filtering and verification framework [19]. Huang designs a multi-level feature descriptor to describe 3D model at different levels, and establishes feature similarity evaluation model. Similarity of multi-level feature descriptors is computed as model similarity [20]. Though graph describes adjacency relationship between faces of 3D model, information of component edges in face is neglected. At the same time, subgraph isomorphism is a NP-hard problem and optimal face matching scheme can not be found in reasonable time.

3D model can be projected into many 2D images from different viewpoints. 3D model is expressed as a set of 2D views. Two models’ similarity is computed based on their sets of 2D views. Nie uses OpenGL visualization tool to extract 3D model’s virtual views from different angles and applies CNN to get feature vectors of model view. Models’ similarity is computed by comparing Euclidean distance between two feature vectors [21]. Nie combines many 3D model’s rendering images into a representative view, and applies multi-layer deep network to retrieve 3D model [22]. 2D views contain 3D model’s shape and structure information partially. But, a large amount of face’s shape and topology information is lost.

Heuristic algorithms can be applied to match source faces with target ones. Soulaiman gives an algorithm based...
on match propagation and particle swarm optimization to increase the number of matching points in process of reconstructing 3D model [23]. Attribute adjacent graph is established based on topological and geometrical information in 3D CAD model. Association graph and association graph matrix are constructed with the mapping relation of vertex and edge in attribute adjacency graph. Simulated annealing algorithm is used to mine typical structure through excavating maximal clique in association graph. Then, model similarity is computed based on typical structures [24]. Li defines interior dihedral angle histogram and extracts histogram feature. Genetic algorithm is applied to optimize weights of various features. A non-rigid 3D shape retrieval algorithm based on these combined features is defined [25]. Based on B-rep information, 3D model is represented as attribute adjacency graph. Ant colony algorithm is used to detect common subgraphs between two models’ attribute adjacency graphs. Detail features with similar local CAD models are obtained. Then, CAD models are evaluated by comparing similar detail features [26]. A novel method of 3D model retrieval is given based on fish swarm, in which spatial bag of words is used as shape descriptor [27]. Liu uses a multi-view attentional convolutional neural network to retrieve 3D model, in which context among multiple views are extracted from visual and spatial domain [28]. But, heuristic algorithms are easy to get into local optimum and can not get optimal sequence of face pairs.

To deal with the above problems, this paper uses the number of edges to compute face’s shape similarity. When face’s similarity is calculated, face’s shape information and topological adjacency relationship are all considered. Therefore, the proposed method can utilize shape and topology information adequately to compute model similarity. Indirect expectation heuristic is defined to improve ACO algorithm. Improved ACO algorithm is integrated into HNN to improve the quality of face matching scheme.

### III. SHAPE AND STRUCTURE SIMILARITY

3D model is composed of faces, and faces’ difference influences two models’ similarity. A face is made up of edges, and the number of edges determines face’s shape. The process of calculating shape similarity $S_H(c_i, c'_j)$ between source face $c_i$ and target one $c'_j$ is shown in formula (2).

$$S_H(c_i, c'_j) = 1 - \frac{|N(c_i) - N(c'_j)|}{\max(N(c_i), N(c'_j))}$$  \hspace{1cm} (1)

where $N(x)$ represents the number of edges in face $x$. It can be seen from formula (1) that the smaller is the difference of edge number between face $c_i$ and face $c'_j$, the larger is $S_H(c_i, c'_j)$. Source face $c_i$ is more similar with target one $c'_j$. Source model $M_S$ contains faces $c_1, c_2, \ldots, c_m$, and target model $M_T$ contains faces $c'_1, c'_2, \ldots, c'_n$. Based on shape similarity between source face and target one, shape similarity matrix $S_H$ between $M_S$ and $M_T$ can be constructed. It is shown as follows:

$$
c'_1 \quad \ldots \quad c'_j \quad \ldots \quad c'_n
\begin{pmatrix}
c_1 & S_H(c_1, c'_1) & \vdots & \cdots & S_H(c_1, c'_n) \\
\cdots & \vdots & \ddots & \vdots & \cdots \\
c_i & \cdots & S_H(c_i, c'_j) & \vdots & \cdots \\
\cdots & \vdots & \cdots & \ddots & \vdots \\
c_n & S_H(c_n, c'_1) & \cdots & \cdots & S_H(c_n, c'_n)
\end{pmatrix}
$$

In addition, two models’ similarity is also influenced by their faces’ adjacency correspondence relationship. When similarity between $M_S$ and $M_T$ is computed, shape similarity and adjacency correspondence relationship between source face and target one are all considered. In 3D model, adjacency relationship between face $x$ and face $y$ is computed as shown in formula (2).

$$D[x, y] = \begin{cases} 
1 & \text{x is adjacent to y} \\
0 & \text{x is not adjacent to y}
\end{cases}$$  \hspace{1cm} (2)

Adjacency correspondence similarity $w(c_i, c_s, c'_j, c'_t)$ can reflect adjacency correspondence relationship between source faces $c_i$, $c_s$ and target ones $c'_j$, $c'_t$. It is necessary to consider number sequence and adjacency relationship of faces. If face number $i$ is smaller than or equal to $s$ in $M_S$, the sequence between source face $c_i$ and $c_s$ is positive. If face number $i$ is bigger than $s$, the sequence between source face $c_i$ and $c_s$ is reverse. Let $DS = D[c_i, c_s] + D[c'_j, c'_t]$. When $c_j$ is adjacent to $c_t$ and $c'_j$ is next to $c'_t$, $D[c_i, c_s]$ is 1 and $D[c'_j, c'_t]$ is 1. The value of $DS$ is 2. When $c_i$ is adjoined with $c_s$ and $c'_j$ is not close to $c'_t$, $D[c_i, c_s]$ is 1 and $D[c'_j, c'_t]$ is 0. When $c_s$ is not adjacent to $c_t$ and $c'_t$ is next to $c'_j$, $D[c_i, c_s]$ is 0 and $D[c'_j, c'_t]$ is 1. The value of $DS$ is 1. When $c_i$ is not adjoined with $c_s$ and $c'_j$ is not close to $c'_t$, $D[c_i, c_s]$ is 0 and $D[c'_j, c'_t]$ is 0. The value of $DS$ is 0.

Adjacency correspondence similarity $w(c_i, c_s, c'_j, c'_t)$ between source faces $c_i, c_s$ and target ones $c'_j, c'_t$ is calculated as shown in formula (3).

$$w(c_i, c_s, c'_j, c'_t) = \begin{cases} 
1 & DS = 2 \text{ and } (i-s)(j-t) > 0 \\
0.5 & DS = 0 \text{ and } (i-s)(j-t) > 0 \\
0.25 & DS = 1 \text{ and } (i-s)(j-t) < 0 \\
0 & \text{others}
\end{cases}$$  \hspace{1cm} (3)

When $DS$ is 2 and product of $i-s$ and $j-t$ is positive, source face $c_i$ is adjacent to $c_t$ and target face $c'_j$ is also next to $c'_t$. At the same time, number sequence of $c_i$ and $c_s$ is consistent with that of $c'_j$ and $c'_t$. Probability of adjacency correspondence between $c_i, c_s$ and $c'_j, c'_t$ is the highest, and the value is 1. When $DS$ is 0 and product of $i-s$ and $j-t$ is positive, number sequence of $c_i$ and $c_s$ is consistent with that of $c'_j$ and $c'_t$. Source face $c_i$ is not adjoined with $c_s$ and target face $c'_j$ is also not close to $c'_t$. Probability of adjacency correspondence between $c_i, c_s$ and $c'_j, c'_t$ is bigger, and the value is 0.5.
When $DS$ is $1$ and product of $i - s$ and $j - t$ is negative, number sequence of $c_i$ and $c_j$ is not consistent with that of $c_i'$ and $c_j'$. There are two cases. One is that $c_i$ is adjacent to $c_j$ and $c_j'$ is not next to $c_i'$. The other is that $c_j$ is not adjoined with $c_i$ and $c_j'$ is close to $c_i'$. Probability of adjacency correspondence between $c_i$, $c_j$ and $c_i'$, $c_j'$ is smaller, and the value is $0.25$. In other cases, probability of adjacency correspondence between $c_i$, $c_j$ and $c_i'$, $c_j'$ is the smallest, and the value is $0$.

Structure similarity $S_T(c_i, c_j')$ between $c_i$ and $c_j'$ is computed as shown in formula (4).

$$S_T(c_i, c_j') = \sum_{t=1}^{n} \sum_{s=1}^{m} S_H(c_i, c_j') \ast w(c_i, c_j, c_j', c_i')/mn$$  \hspace{1cm} (4)

Similarity between $c_i$ and $c_j'$ is related to their shape similarity and structure similarity, and $S(c_i, c_j')$ is calculated as shown in formula (5).

$$S(c_i, c_j') = S_H(c_i, c_j') \ast S_T(c_i, c_j')$$  \hspace{1cm} (5)

Formula (5) is used to compute similarity between source face and target one. Then, face similarity matrix $S$ between source model and target one is constructed. By searching face similarity matrix $S$, an optimal sequence of face pairs can be obtained.

Source model $M_S$ and target one $M_T$ are shown in Figure 1.

![Figure 1. Source model $M_S$ and target one $M_T$.](image)

For source model $M_S$ and target one $M_T$ as shown in Figure 1, face similarity matrix $S$ is constructed as follows:

$$S = \begin{bmatrix} 0.4480 & 0.3620 & 0.1290 & 0.1290 & 0.1120 & 0 \\ 0.3920 & 0.3380 & 0.1410 & 0.1410 & 0.1680 & 0 \\ 0.1225 & 0.1337 & 0.1700 & 0.1700 & 0.1618 & 0 \\ 0.1225 & 0.1337 & 0.1700 & 0.1700 & 0.1618 & 0 \\ 0.1925 & 0.2262 & 0.1697 & 0.1697 & 0.3325 & 0 \\ 0.1050 & 0.1888 & 0.1885 & 0.1885 & 0.4200 & 0 \end{bmatrix}$$

In matrix $S$, row represents face number in source model, and column denotes face number in target one. If the number of faces in $M_S$ is not equal to that in $M_T$, 0 row vector or 0 column vector is added into $S$. The purpose is to make face similarity matrix $S$ become a square matrix.

### IV. MATCH FACES BASED ON HOPFIELD NEURAL NETWORK

#### A. SEARCH OPTIMAL SEQUENCE OF FACE PAIRS BASED ON HNN

HNN is a single-layer feedback neural network where neurons are fully connected with each other, as shown in Figure 2. Each neuron is both input and output. One neuron transmits its output to all other neurons. At the same time, it also receives outputs from all other neurons. All neurons’ outputs are restricted with each other to get better results. When HNN is used to search optimal sequence of face pairs, each neuron represents a face pair.

![Figure 2. Match between source faces and target ones based on HNN.](image)
where, $S$ is face similarity matrix between source model $M_5$ and target one $M_T$. In face pair $(c_{f(i)}, c'_{f(i)})$, source face $c_{f(i)}$ is matched with target one $c'_{f(i)}$, $f(i)$ is number of $c_{f(i)}$ in $M_5$. The process of searching for optimal sequence of face pairs is to find a solution which makes objective function $F(X)$ maximum. The solution process should satisfy that for target faces $c'_{j}$ and $c'_{j}(i \neq j, i, j = 1, 2, \ldots, n)$, corresponding source faces $c_{f(i)}$ and $c_{f(j)}$ are two different ones, where $f(i)$ is not equal to $f(j)$ ($f(i), f(j) = 1, 2, \ldots, m$). Output of HNN is transposition matrix $V$. In matrix $V$, each element corresponds to a neuron in HNN. In order to ensure that two target faces are corresponded with different source ones, there is only 1 in each row of matrix $V$ and the rest elements are all 0. For source model $M_5$ and target one $M_T$ in Figure 1, a sequence of face pairs is $\{(c_2, c'_1), (c_4, c'_2), (c_2, c'_3), (c_6, c'_4), (c_1, c'_5), (c_5, c'_6)\}$. Its transposition matrix $V$ is shown in Table 1.

**TABLE 1. Transposition Matrix V.**

| $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ |
|-------|-------|-------|-------|-------|-------|
| 0     | 0     | 0     | 0     | 1     | 0     |
| 1     | 0     | 0     | 0     | 0     | 0     |
| 0     | 0     | 1     | 0     | 0     | 0     |
| 0     | 1     | 0     | 0     | 0     | 0     |
| 0     | 0     | 0     | 0     | 1     | 0     |
| 0     | 0     | 0     | 0     | 0     | 1     |

Assume that $n$ is less than or equal to $m$. At this time, target function $F(X)$ is shown in formula (7).

$$F(X) = \sum_{i=1}^{n} S[f(i), i] = \sum_{x=1}^{m} \sum_{y=1}^{n} V_{xy} \ast S_{xy} \quad (7)$$

when $V_{xy}$ is 1, it means that source face $c_x$ is matched with target one $c'_y$. When $V_{xy}$ is 0, it means that $c_x$ is not matched with $c'_y$. Energy function $E$ is defined as shown in formula (8).

$$E = \frac{A}{2} \ast \sum_{x} \sum_{j \neq i} V_{x,i} V_{x,j} + \frac{B}{2} \ast \sum_{x} \sum_{y \neq x} V_{x,i} V_{y,i}$$

$$+ \frac{C}{2} \ast \left( \sum_{x} \sum_{i} V_{x,i} - n \right)^2$$

$$+ \frac{D}{2} \ast \left[ \sum_{x=1}^{m} \sum_{i=1}^{n} V_{x,y} \ast (1 - S_{x,y}) \right] \quad (8)$$

where A, B, C and D are all constants. $V$ is transposition matrix that HNN outputs. $S$ is face similarity matrix between source model and target one. The first term of energy function $E$ constrains every row in matrix $V$. When there is only 1 in every row, the first term is 0. The second term of $E$ constrains every column in matrix $V$. When there is only 1 in every column, the second term is 0. The third term of $E$ constrains matrix $V$. When the number of 1 in $V$ is less than or equal to $m$, the third term is 0. The fourth term is negative number of objective function $F(X)$. When $E$ is the smallest, value of $F(X)$ is the largest.

Dynamic equation of HNN is shown in formula (9).

$$\frac{dU_{x,i}}{dt} = -\frac{\partial E}{\partial V_{x,i}}$$

$$-A(\sum_{j=1}^{N} V_{x,j} - 1) - A \sum_{y=1}^{N} V_{y,i}$$

$$-D \sum_{i=1}^{N} ((1 - S_{x,y}) V_{y,i+1}) \quad (9)$$

**B. HNNSearch ALGORITHM**

The algorithm HNNSearch for searching a sequence of face pairs based on hopfield neural network is shown as follows:

1. $m \ast m$ neural network is constructed, where $m$ is the number of faces in source model $M_5$. Iteration number $t$ is initially set to 0. Initialize maximum iteration number $T_{max}$, and disturbance factor $u$. Initialize input value $U$, $U_{x,i}(0) = U ln(m - 1) + u$, transposition matrix $V(0) = \frac{1}{2} \left( 1 + \tanh \left( \frac{U_{x,i}(0)}{u} \right) \right)$.

2. First-order euler equation and formula (9) are used to compute $U_{x,i}(t + 1) = U_{x,i}(t) + \frac{dU_{x,i}}{dt} \Delta t$.

3. Calculate transposition matrix $V_{x,i}(t) = \frac{1}{2} \left( 1 + \tanh \left( \frac{U_{x,i}(t)}{u} \right) \right)$.

4. Compute energy function $E$ according to formula (8).

5. If there is only one non-zero element in each row and column of $V(t)$, go to Step 6. Otherwise, exit and process error.

6. If $t > T_{max}$, output optimal sequence of face pairs according to matrix $V(t)$. Otherwise, $t = t + 1$, go to Step 2.

**V. INTEGRATE IMPROVED ACO ALGORITHM INTO HNN TO MATCH FACES**

A target face corresponds to any source one. But, two target faces should be corresponded respectively with different source ones. Adjacency relationship of target faces is consistent with that of source ones. It is a combinatorial optimization problem to search for optimal face matching scheme, where edge number and adjacency correspondence of faces are considered. ACO is a heuristic algorithm, which solve successfully combinatorial optimization problems through simulating the process that ants search for food. In this paper, ACO algorithm is selected and face’s similarity are integrated into its heuristic strategy to match target faces with source ones.

When ACO algorithm is used to search for optimal face matching sequence, similarity between source face and target one is used as pheromone. When ant $k$ is searching for a sequence of face pairs, pheromone accumulates on target face $c'_j$ matched with source one $c_i$. If pheromone on $c'_j$ is more, probability that $c'_j$ is matched with $c_i$ is larger.
When ant $k$ determines face pair $(c_i, c_j')$ randomly in the $t$th iteration, it will search for a next one $(c_i, c_j')$ according to transfer probability as shown in formula (10).

\[
p(c_i, c_j')(k) = \begin{cases} 
\frac{\tau_{c_j'}(k)\alpha[S(c_i, c_j')]^\beta}{\sum_{j'}\tau_{c_j'}(k)\alpha[S(c_i, c_j')]^\beta}, & \text{ant } k \text{ hasn’t chosen } (c_i, c_j') \\
0, & \text{otherwise}
\end{cases}
\]

(10)

where $\tau_{c_j'}(k)$ is pheromone left by ant $k$ on $c_j'$, $\alpha$ and $\beta$ are respectively information heuristic factor and expectation heuristic factor. $\tau_{c_j'}(k)$ is calculated as shown in formula (11).

\[
\tau_{c_j'}(k) = (1 - \rho)\tau_{c_j'}(k) + \sum_{k=1}^{M} \Delta \tau_{c_j'}(k)
\]

(11)

where $\rho$ is pheromone volatilization coefficient, and $0 < \rho < 1$. $M$ is the number of ants. $\Delta \tau_{c_j'}(k)$ is variation of pheromone produced by ant $k$ when $c_j'$ is selected. $\Delta \tau_{c_j'}(k)$ is computed as shown in formula (12).

\[
\Delta \tau_{c_j'}(k) = \begin{cases} 
Q, & \text{ant } k \text{ hasn’t chosen } c_j' \\
0, & \text{otherwise}
\end{cases}
\]

(12)

When ant $k$ selects face pair $(c_i, c_j')$, it is put into tabu table $taub$. The purpose is to avoid $(c_i, c_j')$ to be selected repeatedly in process of searching face matching scheme. In process of searching for optimal face matching sequence, pheromone concentration in local optimal path becomes high with the increase of iterations.

With the increase of iterations, pheromone concentration and face’s similarity have great influence on transfer probability. In order to increase diversity of face matching schemes and avoid local optimal solution, improved ACO algorithm is given.

In ACO algorithm, expectation heuristic is only related to similarity between source face $c_i$ and target one $c_j'$. When ant $k$ determines current face pair, it can not consider the next face pair. Current situation of matching faces is only considered, but a chain reaction is ignored. At current iteration, ant only considers face pair with the highest similarity and neglects the risk that face’s similarity becomes little when the next one is determined. At this time, ant that selects current optimal path may only get suboptimal solution.

When transfer probability $p(c_i, c_j')(k)$ is computed, pheromone accumulates with the increase of iterations. It makes the influence of pheromone $\tau_{c_j'}(k)$ increase gradually, and the influence of expect heuristic decrease gradually. ACO algorithm converges to local optimization and can not find new face matching schemes. Therefore, it is necessary to enhance the influence of face’s similarity on transfer probability.

This paper defines indirect expectation heuristic which is introduced into transfer probability, so as to improve optimization ability of ACO algorithm for searching face matching schemes.

When ant $k$ selects face pair $(c_i, c_j')$, we consider not only face’s similarity $S(c_i, c_j')$, but also similarity between source face $c_i$ and other target ones. Here, $z(c_i, c_j')$ is average similarity between source face $c_i$ and other $n - 1$ target ones which does not include $c_j'$, as shown in formula (13).

\[
z(c_i, c_j') = \frac{1}{n-1} \sum_{l=1}^{n} (S(c_i, c_j') - S(c_i, c_j'))
\]

(13)

Transfer probability is calculated as shown in formula (14).

\[
p(c_i, c_j')(k) = \begin{cases} 
\frac{[\tau_{c_j'}(k)\alpha[S(c_i, c_j')]^\beta[z(c_i, c_j')]^\gamma}{\sum_{j'}[\tau_{c_j'}(k)\alpha[S(c_i, c_j')]^\beta[z(c_i, c_j')]^\gamma]}, & \text{ant } k \text{ hasn’t chosen } (c_i, c_j') \\
0, & \text{otherwise}
\end{cases}
\]

(14)

where, $\gamma$ is indirect expectation heuristic factor.

The algorithm of searching optimal face matching sequence for ant $k$ is shown as follows:

ACOSearch(int $k$, table* $taub$){
(1)Initialize $taub = \Phi$;
(2)Initialize $Q[i][j] = 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.
(3)Ant $k$ is randomly distributed in face similarity matrix $S$ and initial face pair $(c_u, c_v)$. $\tau_{c_j'}(k)$ is determined. Compute transfer probability $p(c_i, c_j')(k)$ according to formula (14), $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.
(4)If there is 0 element in $Q$, do the following steps:
\(\oplus\)Search for face pair $(c_i, c_j')$ satisfying that value of $Q[i][j]$ is 0 and $p(c_i, c_j')(k).w(c_u, c_i, c_v, c_j')$ is the highest.
\(\oplus\)Calculate $\Delta \tau_{c_j'}(k)$ according to formula (12).
\(\oplus\)Put face pair $(c_i, c_j')$ into $taub$. Elements in the $i$th row and the $j$th column of $Q$ are all set to 1.
\(\oplus\)If all elements in $Q$ are 1, go to (4). Otherwise, go to (5).
(5)Update pheromone $\tau_{c_j'}(k)$ according to formula (11).}

Improved ACO algorithm is used to find initial sequence of face pairs between source model $M_S$ and target one $M_T$. Based on initial sequence, HNNSearch algorithm is applied to search optimal sequence of face pairs. The searching process is shown as follows:

(1)According to formula (5), construct face similarity matrix $S$ between $M_S$ and $M_T$.
(2)Initialize taboo table $taub1 = \Phi$, $taub2 = \Phi$, the number of ants $q$, iteration number $iter = 1$, maximum iteration number $iter_{max}$.
(3)ACOSearch(1, $&taub2$).
(4)While ($iter < iter_{max}$) {
\(\oplus\)for($k = 1; k <= q; k ++$){
ACOSearch($k$, $&taub1$).
\(\oplus\)Get a sequence of face pairs $G1$ from $taub1$;
\(\oplus\)According to formula (15), compute model similarity $s1$ based on $G1$;
\(\oplus\)Get a sequence of face pairs $G2$ from $taub2$;
\(\oplus\)According to formula (15), calculate model similarity $s2$ based on $G2$;
if(s1<s2)
    taub2=taub1;
}
iter++;
}

Step 5. Construct transposition matrix $V$ based on $G2$;

Step 6. Take $V$ as initial transposition matrix $V(0)$ of HNNSearch. Use HNNSearch to search face similarity matrix $S$ to obtain optimal sequence of face pairs $G = ((g_1, 1), (g_2, 2), \ldots, (g_n, n))$. Here, source face $c_{gi}$ is matched with target one $c_i$.

According to $G$, corresponding elements are extracted from face similarity matrix $S$ between $M_S$ and $M_T$. By accumulating similarities between source faces and target ones, model similarity $S(M_S, M_T)$ can be computed as shown in formula (15):

$$S(M_S, M_T) = \frac{1}{\min(m, n)} \sum_{i=1}^{\max(m, n)} S[g_i, i] \quad (15)$$

VI. EXPERIMENTS AND RESULT ANALYSIS

A. EXPERIMENTAL RESULTS

Experiments are conducted to measure the proposed method’s performance. Hexagonal pyramid in Figure 3 is used as target model. Target model has 7 faces including face $c_1$, $c_2$, $c_3$, $c_4$, $c_5$, $c_6$ and $c_7$. Face $c_1$, $c_2$, $c_3$, $c_4$, $c_5$ and $c_6$ have 3 edges, and $c_7$ has six edges.

![Figure 3. Target model.](image)

14 CAD models are selected as source ones as shown in Figure 4.

According to formula (2), face shape similarity matrix between source model $C$ and target one is constructed, as shown in Figure 5.

Formula (2)~(5) are used to construct face similarity matrix between source model $C$ and target one, as shown in Figure 6.

6 groups of experiments are designed. Improved ACO algorithm, HNN and the proposed method are respectively used to search face shape similarity matrix to obtain optimal sequence of face pairs. Improved ACO algorithm, HNN and the proposed method are respectively applied to search face similarity matrix to obtain optimal sequence of face pairs. The purpose is to verify that the proposed method performs better than improved ACO algorithm and HNN. Face similarity matrix provides more discriminative information than face shape similarity matrix. In order to ensure that comparison between different methods is fair, 6 experiments will be conducted on these 14 source models and target one.

Parameters of improved ACO algorithm are $\alpha$, $\beta$ and $\gamma$. Parameters of energy function $E$ in HNN are $A$, $B$, $C$ and $D$. We determine optimal values of parameters $\alpha$, $\beta$, $\gamma$, $A$, $B$, $C$ and $D$ through experiments. In HNN, $A$ is related with the first term in energy function $E$. $B$ is relevant to the second term. $C$ is related with the third term and $D$ is relevant to the fourth term. The first term, the second one and the third one all constrain transposition matrix $V$. But, the fourth term restricts objective function $F(X)$. In order to simplify the process of

![Figure 4. Source models.](image)

![Figure 5. Face shape similarity matrix between source model C and target one.](image)
determing parameters, A, B and C are set to the same value. Under different values of parameters $\alpha$, $\beta$, $\gamma$, A, B, C and D, the proposed method is applied to search face similarity matrix between source model C and target one for getting optimal sequence of face pairs. Then, formula (15) is used to compute model similarity as shown in Table 2.

**Table 2. Model similarity between source model C and target one.**

| $\alpha$ | $\beta$ | $\gamma$ | A   | D   | Model similarity |
|---------|---------|----------|-----|-----|------------------|
| 0.6     | 4.7     | 0.07     | 1.2 | 0.4 | 0.0871           |
| 0.7     | 4.8     | 0.08     | 1.3 | 0.5 | 0.0918           |
| 0.8     | 4.9     | 0.09     | 1.4 | 0.6 | 0.0969           |
| 1       | 5       | 0.1      | 1.5 | 0.7 | 0.1590           |
| 1.2     | 5.1     | 0.11     | 1.6 | 0.8 | 0.0948           |
| 1.3     | 5.2     | 0.12     | 1.7 | 0.9 | 0.0907           |
| 1.4     | 5.3     | 0.13     | 1.8 | 1   | 0.0893           |

From Table 2, we can find that model similarity is maximal when $\alpha = 1$, $\beta = 5$, $\gamma = 0.1$, $A = 1.5$, $B = 1.5$, $C = 1.5$, and $D = 0.7$. So, optimal values of parameters $\alpha$, $\beta$, $\gamma$, A, B, C and D in the proposed method are respectively $1$, $5$, $1.5$, $1.5$, $1.5$, and $0.7$.

In experiment 1-3, improved ACO algorithm, HNN and the proposed method are respectively used to search face shape similarity matrix to obtain optimal sequence of face pairs. Formula (15) is applied to calculate model similarity and results are shown in Table 3.

**Table 3. Model similarity in experiment 1~3.**

| Source model | Improved ACO algorithm | HNN | The proposed method |
|--------------|------------------------|-----|---------------------|
| A            | 0.6786                 | 0.6786 | 0.8063             |
| B            | 0.5957                 | 0.6071 | 0.8450             |
| C            | 1.0000                 | 1.0000 | 1.0000             |
| D            | 0.4420                 | 0.6314 | 0.6700             |
| E            | 0.5212                 | 0.6429 | 0.7500             |
| F            | 0.3866                 | 0.7500 | 0.7614             |
| G            | 0.3592                 | 0.5842 | 0.7157             |
| H            | 0.2960                 | 0.6557 | 0.7614             |
| I            | 0.2594                 | 0.7386 | 0.7500             |
| J            | 0.5389                 | 0.6471 | 0.6471             |
| K            | 0.6957                 | 0.6957 | 0.7400             |
| L            | 0.3814                 | 0.4660 | 0.5340             |
| M            | 0.4233                 | 0.6507 | 0.4179             |
| N            | 0.3214                 | 0.7143 | 0.6671             |

From Table 3, we can see that ranking results of improved ACO algorithm are C, K, A, B, J, E, D, M, F, L, G, H, I in experiment 1. Sorting results of HNN are C, F, I, N, K, A, H, M, J, E, D, B, G, L in experiment 2. Ranking results of the proposed method are C, B, A, F, H, I, E, K, G, D, N, J, L, M in experiment 3.

Source model A and B are very similar in shape. In experiment 1, improved ACO algorithm ranks model A in the third place and sorts model B in the fourth one. In experiment 2, HNN ranks model A in the 6th place and sorts model B in the 12th one. In experiment 3, the proposed method ranks model A in the third place and sorts B in the second one. Therefore, the proposed method performs better than improved ACO algorithm and HNN.

Compared with model A and B, model K, I and N are quite different from target one. However, improved ACO algorithm ranks model K in the second place. Model K is sorted before A and B. HNN ranks model I in the third place, and model N in the fourth one. Model I and N are sorted after A and B. The proposed method ranks model K in the 8th place, model I in the 6th one, and model N in the 11th one. Model K, I and N are all sorted after model A and B. Therefore, the proposed method performs better than improved ACO algorithm and HNN.

In experiment 4-6, improved ACO algorithm, HNN and the proposed method are respectively used to search face shape similarity matrix to obtain optimal sequence of face pairs. Formula (15) is used to calculate model similarity and results are shown in Table 4.

**Table 4. Model similarity in experiment 4~6.**

| Source model | Improved ACO algorithm | HNN | The proposed method |
|--------------|------------------------|-----|---------------------|
| A            | 0.0985                 | 0.1452 | 0.0977             |
| B            | 0.1490                 | 0.1291 | 0.1090             |
| C            | 0.1967                 | 0.1968 | 0.1242             |
| D            | 0.0460                 | 0.0632 | 0.0682             |
| E            | 0.0617                 | 0.1094 | 0.0589             |
| F            | 0.0453                 | 0.0616 | 0.0291             |
| G            | 0.0166                 | 0.0299 | 0.0278             |
| H            | 0.0160                 | 0.0240 | 0.0219             |
| I            | 0.0136                 | 0.0276 | 0.0271             |
| J            | 0.0611                 | 0.0582 | 0.0684             |
| K            | 0.1116                 | 0.0722 | 0.0954             |
| L            | 0.0213                 | 0.0299 | 0.0200             |
| M            | 0.0293                 | 0.0421 | 0.0455             |
| N            | 0.0217                 | 0.0376 | 0.0255             |

From Table 4, we can see that sorting results of improved ACO algorithm are C, B, K, A, E, J, D, F, M, N, L, G, H, I in experiment 4. Ranking results of HNN are C, A, B, E, K, D, F, J, M, N, L, G, I, H in experiment 5. Sorting results of the proposed method are C, B, A, K, J, D, E, M, F, G, I, N, H, L in experiment 6.

Model C is the same with target one in shape. Improved ACO algorithm, HNN and the proposed method all rank model C in the first place.
Model A is very similar to target one in shape. HNN ranks model A in the second place in experiment 5. The proposed method sorts model A in the third place in experiment 6. But, improved ACO algorithm ranks model A in the 4th place in experiment 4. Sorting results of the proposed method and HNN are better than that of improved ACO algorithm.

Compared with model K, model A is more similar to target one in shape. Improved ACO algorithm ranks model K before model A. The proposed method sorts model A before model K. Compared with model E, model J is more similar to target one in shape. Improved ACO algorithm ranks model E before model J. The proposed method sorts model J before model E. From perspective of shape, there is obvious difference between model A and K. At the same time, there is obvious difference between model E and J. So, the proposed method improves the ranking effect of 14.29% of models at least.

Compared with model D, model J is more similar to target one in shape. HNN ranks model D in the 6th place and model J in the 8th one in experiment 5. The proposed method sorts model J in the 5th place and model D in the 6th one in experiment 6. Ranking results of the proposed method are better than that of HNN.

From Table 3 and Table 4, it can be seen that ranking results are different when the same method is respectively applied to search face shape similarity matrix and face similarity matrix.

Compared with model K, model B is quite closer to target one. Model B is sorted in the 4th place and model K is ranked in the 2nd one in experiment 1. Model B is sorted in the 2nd place and model K is ranked in the 3rd one in experiment 4. It can be seen that sorting result of experiment 4 is better than that of experiment 1. The reason is that face’s edge number and adjacency correspondence relationship are used in experiment 4, but face’s edge number is only considered in experiment 1.

In model A, F, I and N, model A is closest to target one. Compared with model I and N, model F is closer to target one. Model I and N are less similar to target one. Experiment 2 ranks model F, I, N and A respectively in the 2nd, 3rd, 4th and 6th place. Model A is sorted after F, I and N. Experiment 5 ranks model F, I and N respectively in the 7th, 13th and 10th place and sorts model A in the 2nd one. Model A is ranked before F, I and N. It can be seen that sorting result of experiment 5 is better than that of experiment 2. This is because that shape similarity and structure similarity are used in experiment 5, but shape similarity is only applied in experiment 2.

From Table 3 and Table 4, we can conclude that compared with face shape similarity matrix, face similarity matrix can provide more discriminative information to evaluate models’ difference effectively. This is because that face similarity matrix is built based on face’s edge number and adjacency correspondence relationship. But, face shape similarity matrix is only constructed based on face’s edge number.

These 14 CAD models include simple models and complex ones. There are plane faces, concave ones and convex ones in these 3D models. A face may be simple one including few edges or complicated one including many edges. They are all representative models and often used in 3D modeling process. Many complex models are always constructed by theses models. The task of computing model similarity is transformed into the problem of matching source faces with target ones. In 3D model, there are plane faces, concave ones and convex ones. It can include few faces or many ones. Whether 3D model is simple or complex, the process of matching faces is the same and face’s similarity need be calculated. Its face may include few edges or many ones. Whether face is simple or complex, the process of computing face’s similarity is the same, in which edge number and adjacency correspondence relationship need be considered.

In the proposed method, the first part is improved ACO algorithm which is used to search for initial sequence of face pairs. Its computational complexity is \(O(n \cdot q \cdot \text{iter}_{\text{max}})\). Here, \(n\) is the number of faces in target model, \(q\) is the number of ants and \(\text{iter}_{\text{max}}\) is maximum iteration number. The second part is HNN which is applied to search for optimal face matching scheme based on initial sequence of face pairs. Its computational complexity is \(O(T_{\text{max}})\). \(T_{\text{max}}\) is maximum iteration number. So, computational complexity of the proposed method is \(O(n \cdot q \cdot \text{iter}_{\text{max}} + T_{\text{max}})\).

Runtime of computing 14 source models’ similarities for improved ACO algorithm, HNN and the proposed method is shown in Table 5. We can find that runtime of HNN is less than that of improved ACO algorithm and the proposed method. The proposed method’s runtime is highest. The reason is that computational complexity of improved ACO algorithm is bigger than that of HNN. Computational complexity of the proposed method is bigger than that of improved ACO algorithm.

| Source model | Improved ACO algorithm | HNN  | The proposed method |
|--------------|------------------------|------|---------------------|
| A            | 2.745                  | 0.128| 3.284               |
| B            | 2.41                   | 0.097| 2.819               |
| C            | 2.463                  | 0.07 | 2.783               |
| D            | 2.944                  | 0.072| 3.260               |
| E            | 1.982                  | 0.064| 2.254               |
| F            | 4.087                  | 0.067| 4.404               |
| G            | 4.122                  | 0.088| 4.437               |
| H            | 5.028                  | 0.071| 5.346               |
| I            | 7.038                  | 0.092| 7.406               |
| J            | 2.303                  | 0.091| 2.607               |
| K            | 1.722                  | 0.092| 2.062               |
| L            | 1.630                  | 0.066| 1.922               |
| M            | 3.457                  | 0.080| 3.782               |
| N            | 4.445                  | 0.070| 4.753               |

B. RESULT ANALYSIS

Experimental results show that the proposed method performs better than improved ACO algorithm and HNN on the task of calculating 3D model’s similarity. This is because that ACO algorithm and HNN all drop into local optimum easily.
and can not find optimal face matching scheme. In the proposed method, improved ACO algorithm is used to get initial sequence of face pairs, based on which transposition matrix is constructed. Then, HNN is applied to search for better face matching scheme. The proposed method uses improved ACO algorithm to make HNN jump out of local optimum as far as possible. Face’s shape similarity and face’s similarity are applied respectively to the process of matching faces. Experiments show that face’s similarity performs better than face’s shape similarity under the same method. The reason is that edge number and adjacency correspondence relationship are combined to compute face’s similarity. When face’s shape similarity is calculated, edge number is only adopted. So, it is effective for matching source faces with target ones to combine shape information and structure one.

VII. DISCUSSIONS
A target face is corresponded with any source one. But, two target faces correspond respectively to different source ones. It is a combinatorial optimization problem to search for a match between source faces and target ones. ACO is a swarm intelligence optimization algorithm to find an optimal path in graph. ACO algorithm can be applied to find sequence of face pairs. At the same time, HNN can also get face matching scheme. But, they are all trapped easily in local optimum and can not find optimal face matching scheme. An effective method of matching source face with target one is proposed to overcome this shortcoming. Edge number and adjacency correspondence relationship are adopted to evaluate face’s similarity, which is introduced into indirect expectation heuristic and transfer probability to improve ACO algorithm. Improved ACO algorithm is applied to find optimal face matching scheme, which is used to construct initial transposition matrix of HNN. Then, HNN is applied to get optimal sequence of face pairs. Experimental results show that the proposed method performs better than improved ACO algorithm and HNN on the task of comparing model similarity. When face’s shape similarity is computed, edge number is considered. When face’s structure similarity is calculated, adjacency correspondence relationship is considered. Face’s similarity is computed based on shape similarity and structure similarity. This suggests that the proposed method has a strong ability to effectively utilize shape information and structure one of face, which provides opportunities of finding optimal sequence of face pairs. It has good potential in evaluating two models’ difference. In the future, the authors will introduce more shape and structure descriptors into the process of computing face’s similarity. In improved ACO algorithm, parameters $\alpha$, $\beta$, and $\gamma$ have important influences on face matching results. In HNN, parameters $A$, $B$, $C$ and $D$ influence face matching scheme. But they are set manually in experiments. It is difficult to manually select optimal values of parameters $\alpha$, $\beta$, $\gamma$, $A$, $B$, $C$ and $D$. It is a combinatorial optimization problem to set parameters $\alpha$, $\beta$, $\gamma$, $A$, $B$, $C$ and $D$ optimally. In the future, the authors will use particle swarm optimization algorithm to search for optimal values of parameters $\alpha$, $\beta$, $\gamma$, $A$, $B$, $C$ and $D$.

VIII. CONCLUSION AND FUTURE WORKS
This paper proposes a new method of computing 3D model’s similarity by integrating improved ACO algorithm into HNN. The difference of edge number is used to compute face’s shape similarity. Adjacency correspondence similarity between source faces and target ones is defined. Structure similarity of face is calculated based on face’s shape similarity and adjacency correspondence similarity. Face’s similarity is computed based on shape similarity and structure one. At the same time, face similarity matrix is constructed. Face’s similarity is applied to define indirect expectation heuristic, which is introduced into transfer probability. The purpose is to improve the performance of ACO algorithm. Improved ACO algorithm is adopted to search face similarity matrix to get a sequence of face pairs, which is used to construct initial transposition matrix. Then, HNN is applied to search for optimal face matching scheme and similarity of 3D model is calculated. Six groups of experiments are conducted to measure the proposed method’s performance. Improved ACO algorithm, HNN and the proposed method are respectively used to search face shape similarity matrix and face similarity matrix. Experiments show that the proposed method performs better than improved ACO algorithm and HNN on the task of matching source faces with target ones, in which edge number and adjacency correspondence relationship provide more discriminative information.

In the future, particle swarm optimization algorithm is used to search parameters of improved ACO algorithm to get better face matching scheme. At the same time, more shape and structure descriptors will be integrated into the process of computing face’s similarity to find optimal sequence of face pairs.

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