Shear waves around an underground pipeline

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Abstract. A two-dimensional axisymmetric problem of shear wave propagation around an underground pipeline under longitudinal motion of the pipeline is solved numerically. Due to the significant difference in pipeline and soil rigidity, the underground pipeline is considered to be a rigid undeformable body and the soil medium around the pipeline is modeled by a viscoelastic law in the form of a generalized Eyring model. To solve the problem, the finite difference method modified by the Wilkins difference scheme was used. On the contact surface of the underground pipeline with soil, the conditions of complete cohesion are fulfilled. Changes over time in shear stress, velocity and soil particles motion in the direction of pipeline axis in fixed sections of soil are obtained. The process of soil displacement around the pipeline at fixed time points is shown. The attenuation of the maximum values of wave parameters in soil in the radial direction with distance from the underground pipeline was detected. The maximum value of shear stress was reduced by three times compared with the value of shear stress at the contact with the pipeline. The effect of the load action time and unloading waves on the wave parameters in soil around the underground pipeline is also obtained. Based on the results of problem solution, the viscoelastic diagram "shear stress - shear strain" is obtained. The results obtained make it possible to determine the mechanism of formation of the stress state and soil reaction to the underground pipeline, which can be used in strength and reliability calculation of the pipelines.

1. Introduction
The construction of underground pipelines is growing more and more all over the world. Trunk pipelines are the most important energy and construction objects [1]. Ensuring their reliable and trouble-free operation, especially under seismic impacts, is a paramount and urgent problem. As noted in [1], even insignificant seismic forces accelerate refusals in areas and lead to the most serious damage, especially when the underground pipeline route coincides with the direction of the seismic load vector. Transportation of hazardous substances (natural gas, oil and oil products) through pipelines is also a very serious problem requiring high reliability and stability of underground pipelines [2]. The reliability and strength of the system of underground trunk pipelines, as noted in [3,4], primarily depends on the interaction forces arising under relative motion of the pipeline and the surrounding soil. In this case, one of the important points is the stress state of soil around the underground pipeline.
When solving the problems of earthquake resistance of underground structures [1-4], the main attention is paid to the motion and vibrations of the underground pipeline itself. However, the stress-strain state of soil around the pipeline also plays a decisive role in the formation of the stress state of the pipeline [5]. Moreover, the most important factors affecting the stress and strain of the pipeline are the soil moisture-content around the pipeline [6], structural destruction of soil during its interaction with the pipeline [5, 7-10].

In [6], the influence and account for the degree of soil moisture-content on its mechanical properties were considered. The change in soil mechanical characteristics under its strain was studied on the basis of experiments in [9]. Methods for determining mechanical characteristics of soils under dynamic (seismic) loads were considered in [8,11]. Changes in strain characteristics of the contact layer of the pipeline-soil interaction were considered in [7,10].

Theoretical and experimental studies of the stress-strain state of soil medium taking into account its rheological properties [13], three-dimensional finite-element analysis of the pipeline-soil interaction [14], wave attenuation in soil surrounding the pipeline [15], the interaction of pipelines lying on the seabed of sea clay [16], the consequences of earthquakes, landslides and tsunamis in underground pipelines [17–20] and the operation features of submarine pipelines [21,22] were carried out in [11-22].

The research results [5,7] show that in the process of underground pipeline-soil interaction under dynamic (seismic) loads, shear stresses in soil play a significant role.

The aim of this work is to determine the tangential (shear) stresses in soil around the underground pipeline during its dynamic interaction with soil medium.

2. Statement and method of problem solution

In development of studies given in [6], where the process of soil destruction around an underground pipeline was considered, depending on mechanical properties and characteristics of soil, this work examines the stress-strain state of soil around an underground pipeline.

Due to significant difference in rigidity of steel pipeline and soil, the underground pipeline is considered to be an absolutely rigid undeformable body. In this case, it is believed that the pipeline strain does not affect the stress state of soil around the pipeline. This assumption allows us to simplify the statement of the problem.

The two-dimensional axisymmetric problem of motion of an underground pipeline (taken as a rigid cylindrical body) in a soil medium is considered. The longitudinal movement of underground pipeline along the $z$ axis coinciding with the pipeline axis is set in the form of velocity

$$v_z = v_z(t), \quad v_r = 0 \quad \text{at} \quad t > 0.$$  \hfill (1)

The underground pipeline is considered as long and the soil medium as unlimited one. Then the equations of motion of soil have the form

$$\rho \frac{dv_r}{dt} = \frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta \theta}}{r}, \quad \rho \frac{dv_z}{dt} = \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r},$$  \hfill (2)

where $r$ is the radial coordinate, $z$ is the coordinate along the pipeline axis; $\sigma_{rr}$ are the stress components; $v_r$ is the velocity in radial direction relative to the pipeline; $\tau_{rz}$ is the shear stress; $\sigma_{zz}$ is the stress along the $z$ axis; $\sigma_{\theta \theta}$ is the stress component along the angular coordinate.

Volume strain of soil, i.e. the change in the ball part of stress tensor for the soil medium is described by viscoelastic equations:

$$\frac{1}{K_D} \frac{dP}{dt} + \mu \frac{P}{K_S} = \frac{1}{V} \frac{dV}{dt} + \mu \ln V,$$  \hfill (3)

where $P$ is the pressure, $V = \rho_0/\rho$ is the relative volume, $K$ is the volume modulus of soil compression, $K_D$ at $dV/dt \to \infty$, $K_S$ at $dV/dt \to 0$, $\mu$ is the volume viscosity parameter. The stress
tensor deviator (shear stress) is related to the components of shear strain by the relationship for viscoelastic strain of soil

$$\frac{1}{G_D} \frac{dS_{rr}}{dt} + \mu_e \frac{S_{rr}}{G_S} = e_{rr} + \mu_s e_{rr}, \quad \frac{1}{G_D} \frac{dS_{zz}}{dt} + \mu_e \frac{S_{zz}}{G_S} = e_{zz} + \mu_s e_{zz}, \quad \frac{1}{G_D} \frac{dS_{\theta\theta}}{dt} + \mu_e \frac{S_{\theta\theta}}{G_S} = e_{\theta\theta} + \mu_s e_{\theta\theta},$$

(4)

where the relationship of shear strains, through particle velocities, is determined by the formula

$$\frac{de_{rr}}{dt} = \frac{\partial v_r}{\partial r} - \frac{1}{3V} \frac{dv}{dt}, \quad \frac{de_{zz}}{dt} = \frac{\partial v_z}{\partial z} - \frac{1}{3V} \frac{dv}{dt}, \quad \frac{de_{\theta\theta}}{dt} = \left( \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right) \frac{v_r}{r}.$$  

(5)

In relationship (4), $G_D$ is the dynamic shear modulus and $G_S$ is the static shear modulus, $\mu_s$ is the shear viscosity parameter. From relationships (5) we obtain continuity equations to determine relative volume:

$$\frac{1}{V} \frac{dV}{dt} = \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r}.$$  

(6)

The system of equations (2) - (6) with boundary conditions (1) and zero initial conditions is solved numerically by the finite difference method according to the Wilkins scheme [23].

3. Results, their analysis and discussion

The results of numerical calculations are presented in the form of graphs for the following initial data: the outer radius of the pipeline $R_{ext}=0.2$ m, the initial soil density $\rho_0=2000$ kg·m$^{-3}$, the dynamic modulus of elasticity $E_D=200$ MPa, the Poisson's ratio $\nu=0.3$, the ratio of dynamic and static modules $\gamma=E_D/E_S=2$, the value of dynamic viscosity of soil $\eta=10^6$ Pa·s, where the parameters of volume and shear viscosity are calculated using the formulas

$$\mu = \frac{K_D K_S}{(K_D - K_S)\eta}, \quad \mu_s = \frac{G_D G_S}{(G_D - G_S)\eta} = \frac{G_D}{(\gamma - 1)\eta},$$

underground pipeline movement (1) $v_x = v_{max}/t_*$ at $t \leq t_*$, $v_z = v_{max}$ at $t > t_*$, $v_{max}=0.5$ m·s$^{-1}$, $t_* =0.002$ s.

Figures 1-3 show the changes in shear stresses, particle velocities, and soil displacement over time in fixed sections $r=0.2$ m, 0.205 m, 0.225 m, 0.255 m, 0.305 m, 0.405 m and 0.70 m (curves 1-7, respectively). Shear stresses in soil occur at the points in time when longitudinal motion of underground pipeline begins. At linear increase in pipeline velocity, the shear stresses also increase in a linear manner, then, at constant velocity of the pipeline, a decrease in intensity growth in shear stress is observed. With distance from the pipeline, the maximum stresses decrease, for example, shear stresses at a distance of 0.5 m from the pipeline (curve 7) are 3.5 times less than shear stresses that occur directly on the contact surface (curves 1,2). Curve 1 in figure 2 corresponds to a given pipeline velocity. The soil particles motion begins with the pipeline motion, when shear waves occur in soil. Behind the wave front, near the pipeline, the velocity of soil particles increases to a maximum, then it remains almost constant. The displacement of soil particles increases approximately linearly (curves 1–7 in figure 3). Note that the maximum shear stress in the vicinity of the pipeline at the contact layer of soil of 0.01 m thick does not exceed 1 MPa. The shear stress at a distance of 0.5 m from the outer surface of the pipeline decreases by about 3 times compared to the stress on the contact surface.
Figure 1. Change in shear stresses over time.

Figure 2. Change in particle velocity over time.

Figure 3. Change in motion of soil particles over time.
Figure 4 shows the shear stress–shear strain diagram obtained at the same fixed points in soil as in figures 1-3. Here, curve 0 corresponds to the elastic case of soil strain. As seen from figure 4, “shear stress–shear strain” diagram in this case varies nonlinearly.

Figure 5 shows the change in shear stress over time at the same fixed points in soil as in figures 1-4, at a decrease in the rise time of the maximum velocity of the underground pipeline by half \(t_r = 0.001\) s, i.e., at double increase in pipeline acceleration. Changes in time to reach the maximum velocity of the pipeline practically do not affect the values of the maximum shear stresses. Its influence is observed on the intensity of increase in shear stress. The displacements of soil particles relative to the underground pipeline at \(t=0.013\) s, for different sections of the pipeline, are shown in figure 6. The black circles in figure 6 are the positions of soil particles at \(t=0\), and the red circles - at \(t=0.013\) s. This shows that soil displacement relative to underground pipeline occurs similarly in all sections of the pipeline. This result confirms the one-dimensionality of the shear wave propagation process in soil generated by longitudinal motion of a pipeline, [8].

![Figure 4. Dependence of shear stress on shear strain.](image)

![Figure 5. Change in shear stresses over time.](image)

Of particular interest is soil behavior around the underground pipeline, taking into account the back motion of the pipeline, i.e. considering unloading waves. Let the velocity of the underground pipeline be as shown in figure 7. Here curve 1 corresponds to the motion of underground pipeline. In this case,
equation (1) takes the form $v_z = v_{\text{max}} t / t_s$ at $t \leq t_s$, $v_z = v_{\text{max}}$ at $t_s < t < t_{ss}$, $v_z = v_{\text{max}} + v_{\text{max}} (t_{ss} - t) / t_s$ at $t_s \leq t \leq (t_{ss} + t_s)$, $v_z = 0$ at $t > (t_{ss} + t_s)$. $v_{\text{max}} = 0.5 \text{ m/s}$, $t_s = 0.001 \text{ s}$, $t_{ss} = 0.003 \text{ s}$. For this case, figures 7-8 show changes in particle velocities and shear stress over time in fixed soil sections in the radial direction $r=0.2 \text{ m, 0.205 m, 0.225 m, 0.255 m, 0.305 m, 0.405 m and 0.7 m}$, which correspond to curves 1-7. As seen from these figures, with wave arrival, the values of velocities and shear stress increase rapidly. Further, at a constant value of the pipeline velocity over time, an increase in the particle velocity and shear stress occur with less intensity. There is also a decrease in the maximum values of wave parameters with distance. With a decrease in the pipeline velocity, a shear unloading wave begins to propagate, which indicates an intensive decrease in both soil particles velocity and the value of shear stress, tending to a zero value.

**Figure 6.** Soil particles motion relative to underground pipeline.

**Figure 7.** Change in soil particle velocity over time.
Next, consider the case at $t_*=0.006$ s. Figures 9–11 show changes in velocities, shear stress and soil particles displacement at the same sections as shown previously in figures. As expected, with an increase in constant velocity duration of the underground pipeline, the maximum values of shear stress in the vicinity of the pipeline increase to 30%, with a distance from the pipeline, this increase reaches 10%. At the same time, the maximum values of the velocity of soil particles remained almost unchanged compared to the previous case (figures 7 and 9). From figure 11 it is seen that after pipeline motion stops, the particle displacements do not change over time. Acquired soil displacement can be considered as residual one. The dependence of shear stress on shear strain for the latter cases is shown in figure 12 at the same fixed soil sections. These dependences show that equations (1)–(6) describe the process of shear strain of soil under translational-reciprocal motion of the pipeline.

Thus, when the underground pipeline moves in the axial direction, transverse waves occur in the surrounding soil. In this case, significant shear stresses and shear strains are formed in soil.
Figure 10. Change in shear stresses over time.

Figure 11. Change in soil particles displacement over time.

Figure 12. Dependence of shear stress on shear strain.
4. Conclusion
A statement of a two-dimensional axisymmetric non-stationary wave problem describing soil behavior around an underground pipeline under its motion in axial direction is given in the paper. An algorithm and a program for numerical solution to the problem using the finite difference method according to the Wilkins scheme are developed. The obtained numerical solutions are given in the form of graphs. Changes in shear stress, velocity and soil particles motion around the underground pipeline are determined. Attenuation of wave parameters with distance from the underground pipeline was detected. The greatest shear stresses and displacements of soil particles occur in the contact layer of soil of about 0.01 m thick. Later their maximum values decrease by about 3 times at a distance of 0.5 m from the pipeline.

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References
[1] Muravyeva L and Vatin N 2014 Risk assessment for a main pipeline under severe soil conditions on exposure to seismic forces Appl. Mech. Mater. 635-637 468
[2] Lalin V V and Kushova D A 2014 New results in dynamics stability problems of elastic rods Appl. Mech. Mater. 617 181
[3] Jung J K, O’Rourke T D and Argyrou C 2016 Multi-directional force–displacement response of underground pipe in sand Can. Geotech. J. 53 1763
[4] Gao F P, Wang N, Li J and Han X T 2016 Pipe–soil interaction model for current-induced pipeline instability on a sloping sandy seabed Can. Geotech. J. 53 822
[5] Wijewickreme D, Monroy M, Henegger D G and Nyman D J 2017 Soil restraints on buried pipelines subjected to reverse-fault displacement Can. Geotech. J. 54 1472
[6] Khusanov B and Rikhsieva B 2019 Thickness dimensions of the contact layer of soil-rigid body interaction E3S Web of Conferences 97 04040
[7] Sultanov K S and Khusanov B É 2002 Determination of the slump-type settlement of a nonlinearly deformable soil mass during wetting Soil Mech. Found. Eng. 39 81
[8] Bakhodirov A A, Ismailova S I and Sultanov K S 2015 Dynamic deformation of the contact layer when there is shear interaction between a body and the soil J. Appl. Math. Mech. 79 587
[9] Sultanov K S, Loginov P V, Ismoilova S I and Salikhova Z R 2019 Wave processes in determining mechanical characteristics of soils E3S Web of Conferences 97 04009
[10] Sultanov K S, Loginov P V, Ismoilova S I and Salikhova Z R 2019 Variable moduli of soil strain E3S Web of Conferences 97 04013
[11] Sultanov K S and Khusanov B E 2001 Slump-type soils statement equations with moistening consideration Osnovaniya, Fundamenty i Mekhanika Gruntov 3 7
[12] Sultanov K S, Loginov P V, Ismoilova S I and Salikhova Z R 2019 Quasistaticity of the process of dynamic strain of soils Magazine of Civil Engineering 85 71
[13] Mirsaidov M M, Abdikarimov R A and Khodzhaev D A 2019 Dynamics of a viscoelastic plate carrying concentrated mass with account of physical nonlinearity of material PNRPU Mech. Bull. 2 143
[14] Liyange K and Dhar A S 2018 Stresses in cast iron water mains subjected to non-uniform bedding and localised concentrated forces Int. J. Geotech. Eng. 12 368
[15] Smith A, Dixon N and Fowmes G 2017 Monitoring buried pipe deformation using acoustic emission: quantification of attenuation Int. J. Geotech. Eng. 11 418
[16] Meyer V., Langford T and White D J 2016 Physical modelling of pipe embedment and equalisation in clay Géotechnique 66 602
[17] Feng W, Huang R, Liu J, Xu X and Luo M 2015 Large-scale field trial to explore landslide and pipeline interaction Soils Found. 55 1466
[18] Matsuhashi M, Tsushima I, Fukatani W and Yokota T 2014 Damage to sewage systems caused by the Great East Japan Earthquake, and governmental policy Soils Found. 54 902
[19] Lam S Y, Haigh S K and Bolton M D 2014 Understanding ground deformation mechanisms for multi-propped excavation in soft clay Soils Found. 54 296
[20] Zhang Z and Zhang M 2013 Mechanical effects of tunneling on adjacent pipelines based on Galerkin solution and layered transfer matrix solution Soils Found. 53 557
[21] Williams E S, Byrne B W and Blakeborough A 2013 Pipe uplift in saturated sand: rate and density effects Géotechnique 63 946
[22] Chatterjee S, Randolph M F and White D J 2012 The effects of penetration rate and strain softening on the vertical penetration resistance of seabed pipelines Géotechnique 62 573
[23] Wilkins M L 1999 Computer Simulation of Dynamic Phenomena (Berlin, Heidelberg: Springer Berlin Heidelberg)