Weak lensing correlations in open and flat universes

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ABSTRACT
Correlations between the magnification or polarization of background sources, induced by gravitational lensing due to the large-scale structure, and the positions of foreground galaxies are investigated. We found that their amplitude is enhanced with respect to correlations for a single population. We analyze the dependence of the correlations with the density parameter \( \Omega \) considering a nonlinear evolution of the matter power spectrum. The contribution of the linear evolution is negligible at scales below several arcminutes. Detailed results on the dependence of the correlations on the redshift of the foreground and background populations for different cosmological models are presented. The effect reaches its maximum amplitude for foreground populations with relatively small redshifts due to the fast increase of the nonlinear matter power spectrum at recent times.

Key words: cosmology: theory - gravitational lensing - large-scale structure

1 INTRODUCTION
The gravitational deflection of photons can be used as a probe of the matter distribution along the line of sight to the sources. The latter may be at the last scattering surface \((z \approx 10^3)\), in the case of the cosmic microwave background (Seljak 1996; Martínez-González et al. 1997; Bernardke 1997), or high-\( z \) objects as QSOs or galaxies (Blanford et al. 1991; Kaiser 1992; Kaiser & Squires 1993; Bartelmann 1995; Villumsen 1995b; Villumsen 1996; Bernardde, van Waerbeke & Mellier 1996; Kaiser 1996; Jain & Seljak 1996). Information about the matter fluctuations can be obtained on different scales ranging from galaxy haloes to the large-scale structure of the universe.

Many of the theoretical studies on this subject have dealt with the polarization or ellipticity produced on background galaxies by the large-scale structure of the universe, and there are currently several ongoing observational projects trying to detect and quantify this effect. Nevertheless, measuring shear amplitudes as the ones predicted by the above mentioned calculations is very difficult from a technical point of view (although see Mould et al. 1991; Villumsen 1995a), and it is not totally clear if such a precision would be routinely achievable in the near future (Ringberg workshop 1997).

However, there is another observable phenomenon produced by gravitational lensing of background sources by foreground mass distributions which may have been already detected: QSO-galaxy associations due to the magnification bias effect (Canizares 1981). The surface density of a sample of flux-limited background sources behind a lens which magnifies them by a factor \( \mu \) is changed in the form \( N'(S) \propto \mu^{-1}N(S\mu^{-1}) \), where \( N(S) \) is the unperturbed background source density. If \( N(S) \propto S^{-\alpha} \) (or \( N(< m) \propto 10^{\alpha m} \)), the change in the density can be characterized by the factor \( q = N'/N = \mu^{\alpha-1} \). Thus, depending on the slope \( \alpha \) there may be an excess of background sources \((\alpha > 1)\), a depletion \((\alpha < 1)\), or the density may remain unchanged \((\alpha = 1)\). If we properly choose the background source population, so that it has a slope \( \alpha \) considerably different from 1, there would be a correlation (or anticorrelation) between the position of the matter overdensities acting as lenses and the background sources. Now, these matter perturbations will be traced, up to a bias factor, by galaxies and thus, there will be a correlation between these foreground galaxies (or any other tracers of dark matter) and the background sources.

There are several reported associations between foreground galaxies and high redshift, background AGNs (see Schneider, Ehlers & Falco (1992); Narayan & Bartelmann (1996) or Wu (1996) for reviews), but only a few of these studies extend to relatively large scales. Bartelmann & Schneider (1994) found a strong association between galaxies selected from the IRAS Faint Source Catalogue and high-\( z \) AGN from the 1Jy catalogue. In Benítez & Martínez-González (1995) it was found that red APM galaxies tracing large scale structures were correlated with 1Jy QSOs. Another sample of radio loud QSOs, extracted from the PKS...
we shall explore the behavior of correlation scale of several arcmin. Other studies considering the correlation between galaxy clusters and high-z QSOs (Seitz & Schneider 1995, Wu & Han 1996) have also found positive results.

In this paper, we shall study the effects of weak gravitational lensing by foreground matter fluctuations on a population of background sources at high-$z$. We consider different values of $\Omega$ and model the fluctuations assuming CDM with a power spectrum whose evolution in time follows a standard ansatz (Hamilton et al. 1991, Peacock & Dodds 1996, linear and non-linear contributions are considered). We assume that these matter perturbations are traced, up to a global bias parameter $b$ by galaxies. More specifically, we shall explore the behavior of $C_{ij\delta}$, i.e. the large-scale correlation between the ellipticity of background galaxies and the position of foreground ones, which apparently has not been considered in the literature. We shall also consider in detail other correlations (in particular their dependence on $\Omega$) such as $C_{i\delta\delta}$, $C_{i\mu\mu}$ i.e. magnification-foreground galaxies and magnification-magnification. $C_{i\mu\delta}$ can be indirectly estimated through the galaxy-galaxy correlation function (Villemusen 1995b). However, measuring $C_{i\mu\delta}$ offers several advantages over $C_{i\mu\mu}$ from the observational point of view. In the first place, $C_{i\mu\delta}$ has an amplitude several times higher than $C_{i\mu\mu}$. Besides, if the foreground and background galaxy populations are properly selected so that there is no redshift overlap between them (e.g. high-$z$ QSOs and bright galaxies), one does not have to bother about intrinsic correlations: any measured effect should be caused by gravitational lensing.

Section 2 develops the formalism dealing with weak gravitational lensing for a flat and open cosmological model, the concepts of magnification and polarization (or ellipticity) and the different correlations. In section 3 appear the main theoretical results as well as comments on different observational perspectives. Finally, in section 4 we give the conclusions of the paper.

2 FORMALISM

2.1 Geodesics in a perturbed Friedmann universe

We will consider the propagation of photons from a source at redshift $z$ to the observer ($z_0 = 0$), the universe being a perturbed Friedmann model with vanishing pressure. For scalar perturbations, the metric in the conformal Newtonian gauge is given in terms of the scale factor $a(t)$ and a single potential $\phi(x, t)$, that satisfies the Poisson equation, as follows (Martínez-González et al. 1997)

$$ds^2 = a^2(t)[-(1 + 2\phi)dt^2 + (1 - 2\phi)\gamma^{-2}d\lambda^i d\lambda^i],$$

$$\gamma = 1 + \frac{k}{4}\tau^2,$$

we take units such that $c = 8\pi G = a_o = 2H_o^{-1} = 1$ and $k/(4 + 1 - \Omega) = 0, -1, +1$ denote the flat, open and closed Friedmann background universe.

Assuming a perturbation scheme ("weak lensing"), the null geodesic equation for the previous metric can be integrated in the form $x = \lambda n + \epsilon$, where $n$ is the direction of observation and $\lambda$ is the distance to the photon in the background metric, i.e.

$$\lambda = \tau_0 - \tau \quad (k = 0),$$

$$\lambda = (1 - \Omega)^{-1}\tanh[(1 - \Omega)(\tau_0 - \tau)] \quad (k = -1).$$

The perturbation $\epsilon$ can be decomposed in a term parallel to the direction of observation $n$ and a term, $\alpha_\perp$, orthogonal to such a direction. The last term is given by

$$\alpha_\perp = 2\int_0^\lambda d\lambda W(\lambda, \lambda')\nabla_\perp \phi(\lambda, x = \lambda' n)$$

where $W(\lambda, \lambda')$ is a window function

$$W(\lambda, \lambda') = (\lambda - \lambda') \{ 1 + 4k\lambda^2/4 \}.$$

For photons that are propagated from a source at redshift $z$ (distance $\lambda$) to the observer ($z_0 = 0$ or $\lambda_0 = 0$), the lensing vector $\beta$ is defined in the usual way:

$$\beta \equiv n - \frac{x - x_o}{|x - x_o|}.$$ 

Thus we find

$$\beta = -\frac{1}{\lambda}\alpha_\perp(\lambda).$$

Once we have obtained the expression for the trajectory of the photon in the conformal Newtonian gauge, it is easy to calculate everything in the conformal synchronous-comoving gauge (Martínez-González et al. 1997). The lensing vector in such a gauge (that is the appropriate one from the point of view of observations) is given by the expression (5) plus some additional terms that can be interpreted as Doppler contributions at the source and observer and an acceleration term at the observer. The last two terms can be estimated from the Doppler velocity respect to the cosmic microwave background and from our local infall towards the Virgo cluster (or Great Attractor). These extra contributions are very small, so the lensing vector $\beta$ in the synchronous-comoving gauge is approximately given by $\beta$, as defined by equations (3, 5).

2.2 Magnification and Polarization

Let us assume a population of background sources (e.g. quasars or galaxies), placed at different distances $\lambda$ with a distribution $R_b(\lambda)$ ($\int_0^1 d\lambda R_b(\lambda) = 1$). Then, we can define the integrated lensing vector $\beta(n) = \int_0^1 d\lambda R_b(\lambda)\beta(\lambda, n)$ and taking into account equations (3-5) we obtain

$$\beta(n) = DS, \quad D \equiv \left( \delta^{ij} - n^i n^j \right) \frac{\partial}{\partial n^j},$$

$$S(n) \equiv 2 \int_0^1 d\lambda T_b(\lambda)\phi(\lambda, x = \lambda n),$$

$$T_b(\lambda) \equiv \frac{1}{\lambda} \int_0^\lambda d\lambda' R_b(\lambda')W(\lambda', \lambda).$$

If we take the derivative along the plane orthogonal to the direction of observation $n$, we get the convergence tensor $\theta_{ij} \equiv D_j\beta_i = D_j D_i S$ which can be decomposed in the form

$$\theta_{ij} = 2 \int_0^\lambda d\lambda W(\lambda, \lambda')W(\lambda', \lambda)\nabla_\perp \phi(\lambda, x = \lambda' n).$$
\[ \theta_{ij} = \kappa h_{ij} + p_{ij}, \quad h_{ij} = \delta_{ij} - n_in_j, \quad \kappa(n) = \frac{1}{3} b^{ij} \theta_{ij}, \quad p_{ij}(n) = \theta_{ij} - \kappa h_{ij} \quad (8) \]

For weak lensing, the convergence scalar \(\kappa(n)\) is related to the magnification by \(\mu(n) = 1 + 2\kappa(n)\). Moreover, the polarization components in the plane orthogonal to \(n\) can be defined in the standard way \(p_1 \equiv p_{11} - p_{22}, p_2 \equiv 2p_{12}\) and the complex polarization is \(p \equiv p_1 + ip_2\). So, we can define the scalar polarization \(p^2 \equiv pp^* = 2p_{ij}p^{ij}\).

### 2.3 Background-Foreground correlations

Let us consider a second population of foreground sources (e.g. galaxies) placed at different distances \(\lambda\) with a distribution \(R_f(\lambda) (\int_0^1 d\lambda R_f(\lambda) = 1)\). If \(\delta(\lambda, x)\) represents the density fluctuation (that satisfies the Poisson equation \((\nabla^2 + 3)\delta = \frac{1}{2} \rho a^2 \delta\)) we shall assume that there is a constant bias factor \(b\) relating the number fluctuation, \(\Delta \delta\), and such an overdensity. Then, we can define the integrated overdensity \(\delta(n) = \int_0^1 d\lambda R_f(\lambda)\delta(\lambda, n)\).

Taking into account equations (3-7), we obtain

\[ \langle \beta_i(n) \delta(n) \rangle = D_i \langle S(n) \delta(n) \rangle, \quad (9) \]

\[ \langle S(n) \delta(n) \rangle = 2 \int_0^1 d\lambda R_f(\lambda) \int_0^1 d\lambda' R_f(\lambda') C_{\delta \delta}(\lambda, \lambda'; r), \quad (10) \]

where \(C_{\delta \delta}(\lambda, \lambda'; r) \equiv \langle \phi(\lambda, x)\delta(\lambda', x') \rangle, r \equiv |x - x'| / \Omega\). Using the Limber approximation (see Appendix), i.e. only a small region \(r\) is contributing with \(\lambda' \approx \lambda\), the previous equation can be approximated by

\[ \langle S(n) \delta(n') \rangle \approx 4 \int_0^1 d\lambda T_b(\lambda) R_f(\lambda) [1 - (1 - \Omega) \lambda^2] \times \]

\[ \times \int_{\theta s}^{\theta f} \frac{C_{\delta \delta}(\lambda; r)}{(r^2 - \theta s^2)^{1/2}} \frac{d\theta}{\theta^2}, \quad (11) \]

where now appears the correlation at a single time and \(s\) is given by \(s = \sqrt{1 - (1 - \Omega) \lambda^2}\). Introducing the power spectrum \(P(a, k)\) of the matter density fluctuations defined by

\[ \langle \delta_k(a) \delta_{k'}(a) \rangle \equiv P(a, k) \delta^3(k - k'), \quad (12) \]

and the relation \(P_{\delta \delta}(\lambda, k, a) = -6\Omega P(\lambda, k) a^{-3}\) (obtained via the Poisson equation for small scales, the last expression (11) is

\[ \langle S(n) \delta(n) \rangle \simeq \frac{6\Omega}{\pi} \int_0^1 d\lambda T_b(\lambda) R_f(\lambda) \frac{\lambda^2}{s^2(1 - \lambda)^2} \times \]

\[ \times \int_{\theta s}^{\theta f} dkk^{-3} P(\lambda, k) J_0(k\theta s) \quad (13) \]

Finally, taking another \(D_i\) derivative on the equation (9), we get the following correlations

\[ C_{\mu \delta}(\theta) \equiv 2 \langle \kappa(n) \delta(n') \rangle = A_0, \quad (14) \]

\[ C_{pp}(\theta) \equiv \langle p(n) \delta(n') \rangle = A_2, \quad (15) \]

\[ A_i \equiv \frac{6\Omega}{\pi} \int_0^1 d\lambda T_b(\lambda) R_f(\lambda) \frac{\lambda}{1 - \lambda}^2 \times \]

\[ \times \int_{\theta s}^{\theta f} dkk P(\lambda, k) J_i(k\theta s) \quad (16) \]

and \(J_i\) is the Bessel function of 1st kind. These are the basic formulas to be applied when two different populations are correlated. In particular, the background-foreground correlation function is given by Bartelmann (1995).

\[ \xi_{b-f}(\theta) = b(\alpha - 1) C_{\mu \delta}, \quad (17) \]

where \(\alpha\) is the slope of the background source number counts.

If the background population is concentrated at a certain redshift \(z_b\) corresponding to a distance \(\lambda_b\), having a Dirac delta distribution \(R_b = \delta(\lambda - \lambda_b)\), then the window \(T_b\) is given by

\[ T_b(\lambda) = 0, \lambda \geq \lambda_b \]

\[ T_b(\lambda) = \frac{1}{\lambda - \lambda_b} \frac{1 - (1 - \Omega) \lambda_b}{1 - (1 - \Omega) \lambda^2} \lambda \leq \lambda_b \]

(18)

If the foreground population has also a Dirac delta distribution form \(R_f = \delta(\lambda - \lambda_f)\), then the window \(T_b\) in equation (16) is \(T_b = T_b(\lambda_f)\) and

\[ A_i = \frac{6\Omega \lambda_b (\lambda_b - \lambda_f) [1 - (1 - \Omega) \lambda_b]}{\pi \lambda_b (1 - \lambda_f)^2 [1 - (1 - \Omega) \lambda_f]} \times \]

\[ \times \int_{\theta s}^{\theta f} dkk P(\lambda, k) J_i(k\theta s) \quad (19) \]

where \(s_f = \lambda_f [1 - (1 - \Omega) \lambda_f^2]^{-1}\).

### 2.4 Background autocorrelations

Let us consider a population of background sources (e.g. galaxies) placed at different distances \(\lambda\) with a distribution \(R_b(\lambda) (\int_0^1 d\lambda R_b(\lambda) = 1)\). We are now interested in the gravitational lensing properties induced by the population itself. The magnification-magnification \(C_{\mu \mu}\), polarization-polarization \(C_{pp}\) and magnification-overdensity \(C_{\mu \delta}\) correlations are defined by

\[ C_{\mu \mu}(\theta) \equiv 4 \langle \kappa(n) \kappa(n') \rangle = D^i D_i D^{i'} D_{i'} \langle S(n) S(n') \rangle, \quad (20) \]

\[ C_{pp}(\theta) \equiv 2 \langle p^{ij}(n) p^{ij}(n') \rangle = D^i D_i D^{i'} D_{i'} \langle S(n) S(n') \rangle - \frac{1}{2} C_{\mu \mu}(\theta), \quad (21) \]

\[ C_{\mu \delta}(\theta) \equiv 2 \langle \kappa(n) \delta(n') \rangle = D^i D_i \langle S(n) \delta(n') \rangle, \quad (22) \]

After an straightforward calculation, one obtains

\[ \langle S(n) S(n') \rangle = \frac{72\Omega^2}{\pi} \int_0^1 d\lambda T_b^2(\lambda) \frac{1 - (1 - \Omega) \lambda^2}{(1 - \lambda)^3} \times \]

\[ \times \int_{\theta s}^{\theta f} dkk^{-3} P(\lambda, k) J_0(k\theta s) \quad (23) \]

where the Limber approximation has been assumed and \(T_b\) is given by equation (7). Now, taking into account the definitions for the correlations, one gets (see also Kaiser 1992,
the function $C_{\mu\delta}(\theta)$ is given by equation (18). Moreover, $C_{\mu\delta}(\theta)$ is obtained from equation (25) where $\omega(\theta)$ is given by equation (27).

$C_{\mu\delta}(\theta) = C_{\mu\mu}(\theta), \quad \omega(\theta) = \delta N(\theta) \delta N(\theta'), \quad (26)$

where $C_{\mu\delta}$ is the matter correlation function. So, gravitational lensing modifies the intrinsic correlation approximately by the magnification term.

3 RESULTS

With the formalism presented in the previous section we have calculated the correlations $C_{\mu\delta}$, $C_{\mu\delta}$ and $C_{\mu\mu}$. We assume a CDM model with a primordial Harrison-Zeldovich spectrum, a Hubble parameter $h = 0.5$ (H = 100$h$ km s$^{-1}$ Mpc$^{-1}$) and flat as well as open universe models. For the power spectrum we have used the fit given by equation (G3) of Bardeen et al. (1986) which is normalized to the cluster abundance: $\sigma_8 = 0.67\Omega^{-\frac{1}{2}}$, $F(\Omega) \equiv 0.34 + 0.28\Omega - 0.13\Omega^2$, following Viana and Liddle (1996) (see also White, Efstathiou and Frenk 1993; Eke, Cole and Frenk 1996; see Kaiser 1997 for a discussion on alternative normalizations). For the nonlinear evolution of the power spectrum we use the recently improved fitting formula given by Peacock and Dodds (1996). That formula is based on the Hamilton et al. (1991) scaling procedure to describe the transition between linear and nonlinear regimes. It accounts for the correction introduced by Jain, Mo and White (1995) for spectra with $n \leq -1$ and applies to flat as well as open universes.

For the redshift distributions of the background and foreground sources we consider a Dirac delta distribution peaked at $\lambda_b$ and $\lambda_f$ respectively, $R_b(\lambda) = \delta_D(\lambda - \lambda_b)$ and $R_f(\lambda) = \delta_D(\lambda - \lambda_f)$. These simple distributions are very useful since they reduce the calculations and the results differ only slightly when compared to other more realistic distributions (see below). $C_{\mu\delta}$ and $C_{\mu\delta}$ are computed using equations (14,15,19), $C_{\mu\mu} = C_{\mu\mu}$ is obtained from equation (25) where the function $T_b$ is given by equation (18).

$C_{\mu\delta}(\theta)$ as a function of the foreground population redshift $z_f$ and for background redshifts $z_b = 0.5, 1, 2, 5$. (a) $\Omega = 1$, (b) $\Omega = 0.3$, (c) $\Omega = 0.1$.

as follows: 

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lag and rapidly decreases for scales above a few arcmin. The amplitude is always above a few percent for scales \( \lesssim 1' \) and at these scales the linear contribution is negligible compared to the nonlinear one for all the \( \Omega \) values. Notice that \( C_{\mu\delta} \) increases when \( \Omega \) decreases whereas considering only the linear contribution the situation is reversed.

From the observational point of view it is useful to calculate the average crosscorrelation (or equivalently the mean relative excess of background sources around foreground lenses) within a given radius \( \theta \), \( C_{\mu\delta}(\theta) \). The variation of \( C_{\mu\delta}(1') \) with \( z_f \) for different values of \( z_b \) and a \( 1' \) radius is shown in figures (2a,b,c). A maximum amplitude of a few percent is obtained at a \( z_f \) in the range 0.1 – 0.25 for all background populations and all cosmological models. (At this point it is important to recall that to compare with observations of background-foreground object correlations \( \xi_{\mu\delta}(\theta) \), the bias factor \( b \) of the foreground population and the slope \( \alpha \) of the background population enter in the calculation following equation 16). It is interesting to notice that there are already available large galaxy samples, like the APM or COSMOS catalogues, which peak at a redshift within that range (for the APM catalogue \( < z > = 0.16 \) for a magnitude limit \( B_J = 20.5 \), see Efstathiou 1995). Moreover, the use of realistic redshift distributions to represent the foreground and background populations (with a bell-like shape similar to that of the APM one), changes the results in only \( \approx 10\% \) for the relevant angular scales compared to the Dirac delta distribution used here. Those catalogues, which except for a bias factor are assumed to follow the large scale matter distribution, are therefore very suitable to crosscorrelate with a background source population. This has already been done by Benítez and Martínez-González (1995, 1997) for the 1 Jansky and Parkes samples of radio loud QSOs as background populations, finding clear evidences of positive crosscorrelations. In figure 3, we show \( C_{\mu\delta}(\theta) \) for \( 0.1 \leq \Omega \leq 1 \) and for \( z_f = 0.15 \) and \( z_b = 1 \), mean redshift values appropriate for the galaxy and radio QSO samples considered above. The dependence of \( C_{\mu\delta} \) with \( \Omega \) is more relevant at small angular scales. In figure 4, we also represent the average crosscorrelation \( \bar{C}_{\mu\delta}(1') \) as a function of \( \Omega \). A rough comparison between its expected amplitude and the measured value for the COSMOS-Parkes samples, as given in figure 4 of Benítez and Martínez-González 1997, shows agreement for realistic values of \( \Omega \) and the bias parameter. A detailed comparison of the theoretical calculations with the observational results will be given elsewhere. Dolag and Bartelmann (1997) have recently presented calculations of the QSO-galaxy correlation function for such QSO and galaxy populations produced by gravitational lensing due to the Large Scale Structure, following a similar theoretical scheme.

### 3.2 Background polarization-foreground matter crosscorrelations

The crosscorrelation between the polarization of a background source population peaked at \( z_b \) and the matter density fluctuations peaked at \( z_f \) is given in figure (5) for \( z_b = 1 \) and \( z_f = 0.3 \). The maximum value is in the angular range \( 0.4' – 1' \) and this scale is smaller for low \( \Omega \) models. As in the case of \( C_{\mu\delta} \), the linear contribution is negligible; however, it peaks at a much larger angle of \( \sim 10' \) as

![Figure 3. \( C_{\mu\delta}(\theta, \Omega) \) for a foreground population peaked at \( z_f = 0.15 \) and a background one at \( z_b = 1 \).](image)

![Figure 4. \( \bar{C}_{\mu\delta}(1') \) as a function of \( \Omega \) for the same values of \( z_f \) and \( z_b \) as in figure (3).](image)

![Figure 5. \( C_{\mu\delta}(\theta) \) for a foreground population peaked at \( z_f = 0.3 \) and a background one at \( z_b = 1 \) and for three values of \( \Omega \): 1 (solid), 0.3 (dotted) and 0.1 (dashed). The three bottom lines represent the linear contribution for the same \( \Omega \) values.](image)
a consequence of the much larger scales which contribute to the linear level. Including the nonlinear evolution, a correlation of $\sim 1\%$ can be expected at angular scales $\leq 1'$ for realistic models of structure formation. The use of realistic redshift distributions to represent the foreground and background populations (with a bell-like shape typical of magnitude limited samples) changes the results in only $\approx 10\%$ for the relevant angular scales compared to the Dirac delta distribution used here.

In analogy to the previous subsection, it is useful to calculate the average crosscorrelation within a given radius $\theta$, $C_{\mu\delta}(\theta)$. The variation of $C_{\mu\delta}(1')$ with $z_f$ for several values of $z_b$ and $\Omega$ is shown in figures (6a,b,c). Maximum amplitudes of the order of 1% are found for $z_f$ within a relatively wide range $0.2 - 0.5$. The amplitude grows appreciably with $z_b$ being a factor of $\approx 2$ difference between $z_b = 0.5$ and 2. Considering the behavior of $C_{\mu\delta}(\theta)$ from these figures, suitable populations to detect the crosscorrelation would be a foreground sample with $z_f < 1$ and a background one peaked at $z_b \gtrsim 2$.

3.3 Magnification and Polarization autocorrelations

In this subsection we concentrate on a single population of background sources peaked at a given $z_b$ and calculate $C_{\mu\mu}(\theta)$. This is done in figure (7) for $z_b = 1$. The maximum effect is at zero lag and its amplitude is relatively small $< 1\%$. The nonlinear contribution clearly dominates over the linear one but now the amplitude grows with $\Omega$, contrary to the crosscorrelations $C_{\mu\delta}(\theta)$ and $C_{\mu\delta}(\theta)$. Our results are in agreement with Jain & Seljak (1996), and with Kaiser (1997) for one degree scales (see also those papers for a more detailed analysis of $C_{\mu\mu}$), where the dominant contribution is the linear one. Considering also the slope of the population of sources following equation (26), $C_{\mu\mu}$ could be estimated from the observed autocorrelation of faint galaxies. Nevertheless, measuring $C_{\mu\mu}$ should be more feasible from the practical point of view, as we do not have to disentangle the contribution caused by lensing from the intrinsic correlations; it is usually assumed (and strongly hoped) that the intrinsic ellipticities of background galaxies are not correlated.

4 CONCLUSIONS

We have obtained the expressions for the correlations between the magnification or polarization of background sources and the foreground matter distribution as function of the nonlinear evolution of the power spectrum. These formulas are valid for flat and open universes.

For the crosscorrelation of the background magnification and foreground matter distribution, $C_{\mu\delta}(\theta)$, the maximum is at zero lag and the amplitude remains above a few percent for scales $\leq 1'$. $C_{\mu\delta}(\theta)$ increases significantly when $\Omega$ decreases. The linear contribution is negligible compared to the nonlinear one for the relevant scales below a few arcmin. Varying the redshift of the foreground population, $z_f$, a maximum amplitude of a few percent for the integrated correlation $C_{\mu\delta}(1')$ is obtained at $0.1 \lesssim z_f \lesssim 0.25$ for all background populations and all cosmological models.

The crosscorrelation of the background polarization and foreground matter distribution, $C_{\mu\delta}(\theta)$, presents a maximum of the order of 1% at a non-null angle, typically in the range $\approx 0.4' - 1'$. Fixing the redshifts $z_f$ and $z_b$ of the two populations, the angular scale of the maximum decreases with
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The correlation of the magnifications for a single population, $C_{\mu\mu}(\theta)$, has a relatively small maximum amplitude $\lesssim 1\%$ in all cases. The amplitude grows with $\Omega$, contrary to the crosscorrelations $C_{\mu\nu}(\theta)$, $C_{\nu\mu}(\theta)$. The contribution to the typical distances to the foreground and background objects in the samples. So, if we consider a double integral

$$\int \int d\lambda R_t(\lambda) T_b(\lambda) \approx 2 \int \int d\lambda' R_t(\lambda') T_b(\lambda') \int_{-\lambda}^{+\lambda} \int_{-\lambda}^{+\lambda} dt C(\lambda', \lambda; r(t)), \quad (29)$$

where $C(\lambda', \lambda; r(t)) \equiv C(\lambda', \lambda; r(t))$ and $T_b(\lambda + t) \approx T_b(\lambda)$, the latter being a smooth function. The previous integral for the variable $t$ can be approximated by $2 \int_{-\infty}^{\infty} dt$ if one assumes that the foreground sources are placed at distances far away from the observer and Hubble distance (case of practical interest). Finally, changing the variable $t$ by $r$ one gets equation (11). We remark that equation (11) is also valid when a Dirac distribution is assumed to represent the foreground redshift distribution. The Limber approximation in Fourier space has been also considered by Kaiser (1992) in the context of weak gravitational lensing.

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