Universality of comb structures in strong-field QED

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We demonstrate a new mechanism for coherent comb generation in both radiation and matter domains, which is by using strong-field quantum electrodynamics processes. Specifically, we demonstrate this for nonlinear Compton scattering which has the potential to generate frequency combs extending towards the γ-ray spectral region as well as combs of ultra-relativistic electrons extending towards the MeV regime. Moreover, we show that coherent energy combs of positrons (or electrons) emerge in laser-induced pair creation processes such as the Breit-Wheeler process, proving the universality of the proposed mechanism.

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Historically, strong-field quantum electrodynamics (QED) is the oldest strong-field-physics area, as investigations of fundamental QED processes in laser fields began in the 1960s [1] (for recent reviews see, [2–4]). At that time, the problem was of purely theoretical interest and the main focus was on a qualitative understanding of effects imposed by intense and mostly monochromatic electromagnetic radiation. With the rapid development of high-power laser technology, that we have encountered over the last decades, the laser radiation in the visible spectrum with intensities as high as $10^{22}$ W/cm$^2$ can be produced in the laboratory [2, 5]. At the same time, the synthesis of laser pulses with well-controlled properties, such as the carrier-envelope phase or the envelope shape, is currently feasible. Due to those recent advances, it became urgent to quantitatively investigate more subtle effects of the laser-field interaction with matter than was originally pursued.

For now, the explorations of laser field interaction with matter in the area of strong-field QED are mostly carried out theoretically, paving the ground for future experimental investigations in the rapidly progressing areas of relativistic optics and information theory. To name only a few of the related studies let us mention: the vacuum polarization induced double-slit Young-type interference [6], the generation of entangled states of high-energy photons by means of the double or triple Compton process [7], the phase control of laser-induced or laser-modified fundamental QED processes [8]–[11], the light diffraction and spin dynamics induced by a strong standing electromagnetic wave (related to the Kapitza-Dirac effect) [9], and spin effects in the electron-positron pair creation [12]. Surprisingly, the strong-field QED vacuum instability and pair production have found their analogues in solid-state physics [13].

In this Letter we propose a novel method for generating γ-ray frequency combs, which is by nonlinear Compton scattering. In addition, we demonstrate the formation of coherent particle combs, which can be achieved not only via nonlinear Compton scattering but also via other strong-field QED processes as well. In this sense, the universality of comb structures in strong-field QED is emerging. As an illustration, we demonstrate the generation of positron energy combs via the nonlinear Breit-Wheeler process.

The optical frequency comb has become an indispensable tool for high precision spectroscopy [14]. Also experiments in the field of ultrafast physics rely on the frequency comb technique to generate precisely controlled attosecond optical pulses by means of the high-order harmonic generation (see, e.g., Ref. [15]). However, in order to generate even shorter laser pulses or to apply this technique in investigations of nuclear structure, combs of frequencies of the order of MeV are necessary. It seems that it may not be possible to achieve such photon energies by high-order harmonic generation (the generation of soft x-ray harmonic combs by relativistic electron spikes has been realized recently [16], but not for such high frequencies). In this context, we study the possibility of the generation of Compton-scattering-based frequency combs.

One of the most intensively studied fundamental QED processes is the laser-induced or laser-modified nonlinear Compton scattering [10]–[17]. It allows for the generation of intense electromagnetic radiation in the photon energy domain comparable to, or even exceeding, the electron rest energy. The differential energy distribution of Compton photons emitted in this process adopts the form (see, Eq. (51) in Ref. [10], where all symbols and physical units used in this Letter are defined)

$$\frac{d^3E_C(K;\sigma;\mathbf{p};\lambda)}{d\omega_K d^2\Omega_K} = \sum_{\lambda_0} \frac{\alpha(m_e c)^2 k^0 (K^0)^2 |A_C|^2}{(2\pi)^3 p_i^0 (k \cdot p_i)^2},$$

(1)

with

$$A_C = \sum_N D_N \frac{1 - e^{-2\lambda_0 P_k^0/k^0}}{\lambda_0 P_k^0},$$

(2)

and with the components $D_N$ defined by Eqs. (23) and (44) in [10]. Let us also introduce the phase of the Compton probability amplitude $\Phi_C = \arg(A_C)$ such...
More specifically, we consider a pulse composed of identical sub-pulses, which is illustrated in Fig. 1 (lower graphic) for \( N_{\text{osc}} = 16 \) sub-pulses. The four-vector \( k \) equals \( (\omega/c)(1, e_z) \), whereas the frequency \( \omega = 2\pi/T_p \) is determined by the duration of the laser pulse, \( T_p \). The shape function, \( f(k \cdot x) \), is an arbitrary differentiable real function of its argument, so that the electric field component is \( E_x(k \cdot x) = E_0 e_x f'(k \cdot x) \), where ‘prime’ means the derivative with respect to the argument, and \( E_0 = -A_0 \omega \).

As it is known from previous studies [10, 17], the Compton photons energy spectrum for intense and short driving pulses consists of broad incoherent peaks in the sense that for the peak-related frequencies the QED probability amplitudes have uncontrollable phases \( \Phi_C \). In order to improve this, we consider a strong-field QED analogue of the Young interference experiment [18]; namely, instead of a single pulse, we apply a modulated pulse. More specifically, we consider a pulse composed of \( N_{\text{rep}} \) identical sub-pulses, which is illustrated in Fig. 1 (lower graphic) for \( N_{\text{rep}} = 2 \). Each sub-pulse acts as a single slit in the Young experiment, from which a Compton photon is emitted with a particular probability amplitude. When adding these amplitudes one can expect that probabilities for emission of Compton photons are enhanced or suppressed for some particular frequencies. As we demonstrate below for a given direction of observation, this leads to the formation of frequency combs. In order to illustrate this effect we choose the shape function \( f(\phi) = N_{\text{rep}} N_{\text{osc}} f_0(\phi) \) where

\[
f'_0(\phi) = N_A \sin^2 \left( N_{\text{rep}} \frac{\phi}{2} \right) \sin(N_{\text{osc}} N_{\text{rep}} \phi),
\]

for \( 0 \leq \phi \leq 2\pi \) and 0 otherwise. Such a laser pulse consists of \( N_{\text{rep}} \) sub-pulses, each including \( N_{\text{osc}} \) field oscillations (this particular shape function is illustrated in Fig. 1 for \( N_{\text{osc}} = 16 \), and \( N_{\text{rep}} = 1 \) or 2). The normalization constant \( N_A \) is chosen such that

\[
\frac{1}{2\pi} \int_0^{2\pi} d\phi |f'(\phi)|^2 = \frac{1}{2},
\]

and the carrier frequency equals \( \omega_L = N_{\text{rep}} N_{\text{osc}} \omega \). As it was argued in Ref. [9], imposing such a condition [Eq. (4)] guarantees that the average laser field intensity contained in the pulse is fixed, no matter how many sub-pulses or cycles in each sub-pulse is included. For such definitions the averaged intensity of the laser field (given in \( W/cm^2 \)) equals \( I = a \omega_L^2 \mu^2 \), where the dimensionless and relativistically invariant parameter \( \mu \) is \( [eA_0]/m_c \), the carrier frequency \( \omega_L \) is in relativistic units of energy, and \( a = 2.3 \times 10^{29} \).

Let us consider the electron beam parameters reported in Ref. [10], i.e., the 35 MeV electron beam with the energy spread 0.1\%. For the laser pulse we choose the Ti:Sapphire laser with \( \omega_L = 1.548 \text{ eV} \). In Fig. 2 we present the differential energy distribution in the reference frame in which initial electrons are at rest. As mentioned above, for a nonperturbative laser pulse, this distribution consists of broad peaks, one of which is represented in this figure by a thin black line. Note that it scans a frequency range whose width is approximately equal to the carrier frequency \( \omega_L \). Moreover, the phase \( \Phi_C \) of the QED prob-
ability amplitude takes all values from the interval $[-\pi, \pi]$ (see, Fig. 3) which indicates that the emitted radiation is non-coherent, even though the driving pulse parameters are precisely controlled.

Let us investigate the possibility of generating the coherent Compton radiation. For this purpose, we consider the incident pulse which consists of a finite sequence of $N_{\text{rep}}$ sub-pulses, as it is illustrated in Fig. 1. In the case when $N_{\text{rep}} > 1$, we observe the emergence of the regular comb structure in which peak positions are independent of $N_{\text{rep}}$. It follows from this figure that the resolved comb frequencies are equally spaced. At those peak frequencies, the energy distribution scales like $N_{\text{rep}}$, whereas widths of individual peaks become increasingly narrower, with increasing $N_{\text{rep}}$. In order to demonstrate that the comb consists of coherent radiation, in Fig. 3 we plot the phase $\Phi_C$ which at the comb frequencies adopts the same value modulo $\pi$. This phase coherence makes it promising to use the proposed Young-type mechanism for generation of zepto- or even yoctosecond laser pulses.

In light of these results, an important question arises: How sensitive is the observed comb structure with respect to a change of the electron initial energy? To answer this question, in Fig. 4 we present the same comb structure as in Fig. 2 but in the laboratory frame of reference for $N_{\text{rep}} = 3$ and for two electron initial energies which differ from each other by 0.1% [19]. As we see, when changing the initial electron energy, the comb structure shifts a bit but nevertheless, after averaging over the electron initial energy distribution, the comb peaks will be clearly resolved. This shows that the coherent comb structures for the Compton process can be observed experimentally with currently available laser and electron beam parameters. Let us also note that performing the Young-type experiment with respect to nonlinear Compton scattering, which is by using a modulated pulse shown in Fig. 1 for $N_{\text{rep}} = 2$, one could produce very energetic radiation combs; for the parameters employed in Fig. 2 such a structure reaches the 100 keV regime. The same concerns the MeV regime, although the respective comb structures are smaller in magnitude.

It is worth noting that similar structures are also observed in the Thompson radiation spectrum, though for smaller frequency ranges, because of the domain of applicability of the classical theory. Analysis of the relativistic Newton-Lorentz equation shows that the pulses used in this paper do not accelerate or decelerate electrons. Note that this is in contrast to the Compton process in which scattered electrons change their energy. Therefore we looked at the energy spectrum of final Compton electrons when increasing the number of driving sub-pulses. Specifically, we investigated the behavior of the proba-
that the \( \mu = 1 \), and \( N_{\text{osc}} = 16 \) collides with a counterpropagating \( \gamma \)-photon of energy \( 10^5 m_e c^2 \) and a linear polarization along the \( x \)-axis. The positrons are created in the direction such that \( \theta_\pm = 0.99999\pi \) and \( \varphi_\pm = 0 \). The results have been summed up with respect to the spin degrees of freedom of produced particles. As in the previous cases, the coherent comb structure for created positrons (or electrons) is formed (the corresponding probability distributions are divided by \( N_{\text{rep}}^2 \)).

The above results demonstrate the universality of the comb structures in strong-field QED, as they can also be generated for electron beams. This opens new possibilities of exploring relativistic optics of matter waves. Moreover, to emphasize the general character of comb structures generated in fundamental processes of strong-field QED, we show in Fig. 6 the formation of the comb energy spectrum for positrons created via the Breit-Wheeler process \([11, 20]\). The exact same properties of the presented spectra as those discussed above are observed.

To conclude, we have demonstrated in this Letter the formation of coherent comb structures in fundamental processes of strong-field QED, that for the Compton scattering can be experimentally verified with presently available laser and electron beam parameters. These structures relate to both radiation and particle spectra, having the potential to impact a broad range of scientific studies. In particular, the radiation combs can be used for the generation of zepto- and even yoctosecond laser pulses. It appears that subtle characteristics of these structures, like the distance between the peaks, can be further controlled by delaying sub-pulses with respect to each other. Such problems are going to be presented in future publications.

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