Coupled Resonator Calorimeter for Particle Detection

Ling Hao, Giuseppe Lorusso, and John C. Gallop

Abstract—We describe a novel coupled microwave dielectric resonator system (an unusual form of parametric amplifier), which is under development, aimed at realizing a high-resolution single X-ray photon energy detector. Unlike other high-resolution detectors, it operates at a relatively high cryogenic temperature in the range of 7–30 K. Construction of the cooler system and thermal characterization system is completed. All aspects of the novel calorimeter performance have been evaluated, including demonstration of single-particle detection, the required rapid thermal time constant of a coupled resonator, a high $df/dT$ response, and promising development of a low-noise microwave detection system.

Index Terms—Coupled systems, cryogenic calorimeter, microwave dielectric resonators, X-ray detection.

I. INTRODUCTION

The requirement for more precise measurement of single-particle energies is converging with the needs of quantum technologies, to drive the development of improved cryogenic calorimeters [1]–[4]. In the case of relatively low-energy particles (<0.1 MeV), high efficiency of close to 100% and a low-energy threshold of ~20 eV are required. With low-energy particles, the absorber dimensions must be large enough to absorb virtually all the particles’ kinetic energy. In the case of X-rays or gamma rays, this will require dimensions of approaching 1 mm$^3$. There has been considerable development of superconducting quantum interference device (SQUID)-based temperature sensors (e.g., metallic magnetic sensors), in which the temperature dependence of a metallic paramagnetic sample is sensed by an attached SQUID sensor. These sensors, when operated in the mK temperature range, can offer extremely high temperature sensitivity. For the application focused here, SQUID sensors have the disadvantage that they must be closely thermally linked to a sufficiently large absorber to ensure near-total particle energy absorption. The attachment makes for limited thermal response time, due to phonon mismatch and increased thermal diffusivity time in sub-kelvin absorbers. As a consequence, the particle count rate is limited. Our proposed coupled microwave resonator detector is an alternative approach, in which the absorber and temperature sensing functions are combined in a single object.

Thus, the heat transfer problem may be solved. A further advantage of our method is that temperature rise values below 1 nK can be detected at operating temperatures at least a factor of 10 higher than that at which magnetic calorimeters operate [5]. This would lead to much simpler and cheaper refrigeration requirements.

Our proposed calorimeter is based on low-loss dielectric microwave resonators. Microwave resonance has a long history of applications in precise measurement. More recently, coupled resonators have also demonstrated important applications in fields as diverse as quantum computing and remote charging. In this article, we describe two coupled microwave dielectric resonators where one resonator is made from a high-permittivity perovskite material, having much smaller size (and therefore much smaller heat capacity) than the other, larger, dielectric resonator. The latter is made of single-crystal sapphire, with a relatively temperature-stable permittivity of ~10.5. The bolometer can be viewed as an unusual form of parametric amplifier, for which the absorption of energy at the input causes a parametric change in the absorber’s permittivity, in turn producing a measurable change in the resonant frequency of the coupled system. Fixed frequency bias of the system then results in a potentially large change in the output amplitude signal at this frequency.

II. COUPLED RESONATOR SENSITIVITY

By analogy with the single photon coupling strength for a mechanical oscillator, we define a single photon coupling strength $G$. This is the product of the rate of change in a resonator’s eigenfrequency $f$ with temperature $T$, multiplied by the mean Nyquist noise temperature fluctuations of that resonator. It was pointed out long ago by Landau and Lifshitz [6] and Day et al. [7] that a system with heat capacity $C$ maintained at a temperature $T$ will exhibit a Gaussian distribution of temperature fluctuations given by $\Delta T$, where

$$\Delta T^2 = \frac{k_B T^2}{C}. \quad (1)$$

Thus, the single photon coupling strength $G$ becomes

$$G = \frac{df}{dT} \Delta T = T \frac{df}{dT} \left( \frac{k_B}{C} \right)^{1/2}. \quad (2)$$

To develop a thermal noise-limited bolometer based on a microwave resonator, we require to maximize $G$ which implies maximizing $df/dT$ and minimizing $C$, being aware that both of the latter parameters will themselves be dependent on $T$. Our approach to both maximizing the former and minimizing the latter is based on the use of coupled dielectric microwave resonators [8]. A high value of $df/dT$ requires a low-loss
dielectric material which has a strong temperature-dependent permittivity. Here the existence of quantum paraelectric materials such as the oxide perovskites SrTiO$_3$ and CaTiO$_3$ is immediately attractive. These insulating materials are incipient ferroelectrics, having relative permittivity $\varepsilon_r$ which rises rapidly with decreasing temperature. However, the transition to the ordered state is inhibited by the presence of quantum fluctuations [9]. Thus, single-crystal samples of SrTiO$_3$ can have relative permittivity $\varepsilon_r$ as high as 170,000 at 4 K [10] and temperature-dependent values $d\varepsilon/dT$ of $7 \times 10^3$ at 4 K.

The implications for improvements to bolometry come from three aspects of this perovskite low-temperature behavior. First, the very high values of $\varepsilon_r$ imply that the size of a dielectric resonator of such a material will be much smaller than the conventional solid dielectrics which typically have relative permittivity of around 10. For a given resonant frequency, the required perovskite resonator volume scales with $\varepsilon_r^{-3/2}$, implying that the heat capacity (scaling with volume) can be much smaller than for a resonator with conventional $\varepsilon_r$ (~10). The second advantage which the perovskite behavior brings is that the strong temperature dependence of $\varepsilon_r$ means that a small change in temperature brings about a large frequency change in the resonator when its temperature is changed by the absorption of energy. In turn, this leads to a correspondingly large change in the coupled resonator frequency. Finally, the oxide-based perovskites are generally good electrical insulators (or very wide-band semiconductors), so their specific heats contain only phononic terms that freeze out rapidly with decreasing temperature. This gives a further improvement in sensitivity compared with metallic calorimeters.

III. Prototype Microwave-Coupled Resonator System

A microwave-coupled resonator (MCR) system has been modeled and designed, based on a 12-mm-diameter single-crystal sapphire resonator coupled to various sizes of perovskite single-crystal SrTiO$_3$. Both the absorption characteristics for X-rays and the thermal characteristics of the system have been modeled. Further design and modeling work on another perovskite, CaTiO$_3$ single-crystal puck has also been carried out.

The initial design issues in this deliverable required a calculation of the minimum size of the X-ray absorber, which would ensure that more than 99% of the incident X-ray energy would be absorbed within the temperature-sensitive perovskite resonator, in order to ensure that the coupled resonator system gives an accurate measure of the incident X-ray photon energy.

Fig. 1 shows a series of random Monte Carlo events, predicting the absorption of energy by a simulated sample of the perovskite CaTiO$_3$ for an incident X-ray energy of 50 keV.

These calculations indicate that the dimensions should be a few times greater than 100 $\mu$m in all three orthogonal directions. To allow for more complete absorption, we decided on an absorber with approximate size of 1 mm in all three dimensions. This should be sufficient to ensure effective total absorption for all incident angles within 30° of normal.

We have derived a COMSOL finite-element model to predict the key parameters of the coupled system. COMSOL can solve for resonant modes of a coupled system with reasonable accuracy. An example of modeling results for a typical coupled resonator system is shown in Fig. 2.

The dimensions of the modeled system are greater than the experimental prototype for reasons of visibility. Fig. 2 model has an absorber resonator made from single-crystal SrTiO$_3$ (radius $r = 3$ mm and thickness $t = 1$ mm) placed on top of a sapphire resonator ($r = 6$ mm and $t = 3$ mm). The reference resonator (sapphire puck) is supported in the copper housing on a quartz tube, and the resonators are joined by a thin layer of polymer adhesive. The cylindrical copper housing is 20 mm diameter by 20 mm height. We modeled the resonant modes and predicted the frequency of TE011 mode for the coupled system. As an example, we chose a relative permittivity of 650 for SrTiO$_3$, yielding a coupled resonant frequency of 11.426 GHz. Note that this value of the strong temperature-dependent permittivity is chosen at random, for purposes of testing the COMSOL model (see Fig. 2).

We derived in [8] the way in which the predicted energy resolution $\delta E$ depends on various parameters of the coupled system. For an absorber heat capacity $C$ and thermal time constant $\tau$ with a coupled resonant frequency $f$, which has a temperature-dependent relative permittivity $(1/\varepsilon)d\varepsilon/dT$, the predicted minimum detectable absorbed energy is $\delta E$ given by the expression

$$\delta E = \frac{2C}{f(\frac{d\varepsilon}{dT})\tau}. \quad (3)$$

Using realistic values for our system prototype design and the published values for perovskite temperature-dependent
at a range of temperatures, for example:

\[ \delta E \sim 10^{-23} \text{ J/Hz at 1 K} \]
\[ 10^{-20} \text{ J/Hz at 10 K} \]
\[ 2.5 \times 10^{-19} \text{ J/Hz at 30 K}. \]

The layout of the basic coupled resonator system is shown in Fig. 3. The MCR system includes the coupled resonators (sapphire puck as a reference resonator and perovskite resonator as absorber on the top of the sapphire puck) supported on a low microwave loss quartz tube. The radioactive source is drop dried onto the underside of a silver disk mounted on a threaded rod allowing the separation between source and absorber to be adjusted. These components are contained inside a Cu enclosure housing with diameter 20 mm and height of 20 mm. The heater and thermometer are installed on the Cu housing to provide stable temperature control of the MCR system. Two low thermal conductivity microwave leads with TE coupling structures allow the microwave signal to couple into and out of the MCR system. The entire structure is attached to the pulse tube cooler cold plate by a low thermal conductivity support. Cooling is provided by a cryomech pulse-tube cooler which has a base temperature of 2.2 K. Temperature control to \( \sim 0.1 \) mK is achieved, using Lakeshore 340 controller with Si diode thermometers. For mode characterization, the microwave transmission \( S_{21} \) of the system is measured with a vector network analyzer (VNA).

As a first test of the coupled resonator system, we attached a single-crystal (001 oriented) SrTiO\(_3\) puck \((r = 3 \text{ mm and } t = 2 \text{ mm})\) to the center of the top surface of the sapphire puck, using minimal layer of GE7031 cryogenic adhesive.

The combined coupled puck resonator enclosed in the Cu cylindrical housing was cooled to the current base temperature of 7 K by the closed cycle dry pulse tube cooler while monitoring several microwave resonances, as the system approached base temperature. These resonance frequencies were compared with the bare sapphire resonator previously measured. The bare sapphire puck shows an overall fall in resonant frequency as the temperature is raised, reflecting both the thermal expansion of the sapphire and the positive temperature coefficient of its relative permittivity.

In contrast, coupled resonances, where a significant fraction of the electromagnetic energy of a mode is shared between sapphire and SrTiO\(_3\), tend to show frequencies which increase as the temperature is raised. However, at temperatures where a resonance in the SrTiO\(_3\) moves into coincidence with one of the predominantly sapphire resonances, the coupled frequency shifts very rapidly down and then up. Associated changes in both resonance width and insertion loss accompany this change. Even in this strong coupling regime, the temperature stability is such that the overall resonance frequency may be determined with high accuracy and repeatability (\(<1 \) kHz in a single measurement).

To illustrate the basic interaction between individual modes in the sapphire and SrTiO\(_3\) pucks, we made careful measurements of the overall resonance line shape as the temperature is adjusted through a range of 150 mK from 29.50 to 29.65 K (Fig. 4). Over this range, one mode in the SrTiO\(_3\) puck comes close to the stable sapphire mode. The frequency shift in the combined modes shows the classic avoided crossing behavior, with a closest approach of 15 MHz. Note this behavior is highly reproducible, depending only on the bulk temperature-dependent properties of the resonators.

IV. THERMAL RESPONSE OF ABSORBER PUCK

When assessing the performance of a dielectric-coupled microwave bolometer, it is not only \( df/dT \) which is of interest. The heat capacity of the absorber puck and its thermal linkage to the more massive sapphire puck are also important parameters of interest. It is the ratio of these two parameters that determines the thermal response time \( \tau \) of the system and hence set an upper limit on the speed with which a particle can be absorbed and then detected. At the time of these experiments, no source of radioactive decays was available for testing so instead a microwave calibration pulse technique was proposed and implemented. Here another microwave mode is chosen for which the stored energy in the perovskite puck is a significant fraction of the total. Applying a microwave pulse with a duration of 1 ms or less deposits a calculable amount of heat into the absorber resonator, producing a frequency shift in the coupled resonance and a corresponding change in the readout signal from the microwave interrogation system. This signal is monitored over a time period following the pulse, revealing the thermal response time of the system.
The proposed thermal response time was designed to be less than 1 ms, in order to enable a reasonable count rate for the X-ray measurement process. Consequently, a second coupled resonance system was designed to have a much shorter response time achieved by reducing the volume of the absorber by a factor of more than 50. The SrTiO$_3$ absorber was replaced by a smaller single-crystal CaTiO$_3$ puck ($r = 1\, \text{mm}$ and $t = 0.5\, \text{mm}$), again glued to the top center of the surface of the 12-mm-diameter sapphire puck. The resonant modes of this combination were measured from room temperature to 7 K.

Measurement of the thermal response time of this new coupled resonator combination has yielded much interesting information. First, at the lowest temperature ($\sim 15\, \text{K}$) at which this combination shows useful bolometric capability, the thermal response times are 0.25 and $\sim 1\, \text{ms}$ for the rise and fall times, respectively (Fig. 5). This is already more than 100 times smaller than that of the previously measured SrTiO$_3$ puck. Most of this improvement is due to the reduction in volume by a factor of 36. A further factor arises from the difference in Debye temperatures between the two perovskite materials, leading to a reduced specific heat for CaTiO$_3$ compared with SrTiO$_3$. In addition, we have shown that over a temperature range from 15 to 30 K, the thermal response times vary approximately as $T^3$. This is as expected, since at low temperatures the heat capacity of an insulator varies with this polynomial dependence and also the thermal conductivity of a single-crystal insulator is expected to be approximately temperature independent.

These observations are particularly helpful because they indicate that it will be possible to select materials and to design shapes and sizes of absorber to tailor the response time appropriately to the application under consideration (e.g., energy of X-rays and count rate). Operation at $\sim 4\, \text{K}$ should enable single-particle response times to be as short as 5 $\mu\text{s}$.

Performance of a number of coupled resonances has been investigated over a wide temperature range. The aim is to investigate the maximum value of $df/dT$ which can be achieved at low temperature where $C$ is also minimized. For several samples of SrTiO$_3$ and CaTiO$_3$ so far tested, at around $T \sim 20$–25 K the maximum value of $df/dT$ ($\sim 1\, \text{GHz/K}$) is achieved. We believe this is a result of ordering of the paraelectric state of the perovskite material at around this temperature. Below the ordering temperature, the permittivity becomes much less temperature dependent. According to the heuristic generalized Barrett theory [11,12], the permittivity $\varepsilon(T)$ varies as

$$
\varepsilon(T) = A + \frac{C}{(T_c \coth(\frac{T}{T_c}) - T)^\gamma}.
$$

For our perovskite samples, this formula gives a good fit to the experimental data, as shown in Fig. 6 for the CaTiO$_3$ resonator. Here data for the temperature variation in a coupled resonance at a frequency of around 14.85 GHz is measured over the temperature range from 8 to 25 K. For this mode, the dominant part of the electromagnetic stored energy resides in the CaTiO$_3$ puck, so that the temperature dependence of frequency is dominated by the temperature dependence of $\varepsilon(T)$. Fitting equation (4) to the observed data indicates that the transition temperature for this perovskite sample is around 15 K.

**V. PULSE DETECTION WITH MICROWAVE SYSTEM**

To demonstrate the basic operation of the bolometer, we developed a simple pulse acquisition system based on taking the output of the homodyne detection system (see Fig. 6) and converting the analog signal to a stream of pulses by digitizing and then digitally filtering the output stream. A very weak (200 Bq) $^{241}\text{Am}$ source was included within the copper housing of the coupled resonator system spaced 4 mm away from the absorber. Calculation of the solid angle subtended by the absorber of the $^{241}\text{Am}$ source predicts a detection rate of approximately 0.25 counts/s of the emitted 5-MeV alphas.

The system was cooled to a temperature of 28.5 K where the temperature-dependent $df/dT$ was maximized for this setup. A homodyne detection system was set up as shown in Fig. 7 to detect individual pulses with a resolution of
VI. Conclusion

This article first describes the basic principles of operation of a proposed new type of cryogenic calorimeter and then continuing to describe the construction and performance of a prototype. An important step along this development route has been the first detection, reported here, of single-alpha particle using this prototype. Although at this stage, the limitations of the available perovskite materials have prevented operation below 20 K, it is expected that improved material will show paraelectric behavior to around 4 K, as has been reported elsewhere. This should lead to much improved performance.

In the meantime further efforts are being made to improve the detection sensitivity by moving from a homodyne system to a heterodyne detector which should eliminate most of the low-frequency vibration noise which is visible in Fig. 8(a).

In summary, the primary advantage of this proposed calorimeter is that it is noncontacting, interrogation is done by the microwave field itself; no cables are required to contact the sensor. Also, since the absorber is an insulator, it has minimal heat capacity, due only to phonons, with no conduction electron contribution (unlike metal or semiconductor calorimeters). For $T < 5$ K, an insulating sensor can have a much smaller heat capacity than a metal one of similar mass at the same temperature. Thus, a dielectric sensor may provide higher energy resolution than a metallic one or alternatively it could achieve the same resolution but be operated at a higher temperature, reducing cryogenic complexity. Further, the physical volume of the perovskite resonator (≈1 mm$^3$) is several orders greater than that of thin film sensors, and more compatible with the absorption lengths associated with 1–10$^3$-keV X-rays (the proposed energy range of this detector), which may be as great as 1 mm [13]. The prototype work
reported here predicts an energy resolution of <10 eV and a time constant of 1 ms achievable at temperatures ∼4 K [8], thus opening up a large number of potential X-ray detection applications [1]–[3].

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