Precise determination of the $f_0(500)$ and $f_0(980)$ parameters in dispersive analysis of the $\pi\pi$ data

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Schedule

- Theory
- Experiment
- Results
- Conclusions
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- dispersion relations,
- state of art of data,
- combined analysis of dispersion relations and data
Dispersion relations for $\pi \pi$ interactions
theory $\leftrightarrow$ experiment

Once subtracted dispersion relations "GKPY" (for the $S$ and $P$ waves):

\[ \mathcal{T}_s(s, t) = \hat{C}_{st} \mathcal{T}_t(t, s) \]

Crossing symmetry:

\[ \mathcal{T}(s, t) + \text{crossing symmetry} \rightarrow \text{dispersion relations for } 4m_\pi^2 < s < \sim (1150 \text{ MeV})^2 \]

\[
\text{Re } t^{(OUT)}_{\ell}(s) = \sum_{l'=0}^{2} C_{st}^{ll'} a_{0}^{l'} + \sum_{l'=0}^{2} \sum_{l''=0}^{4} \int_{4m_\pi^2}^{\infty} ds' K_{l'l''}^{ll'}(s, s') \text{Im } t^{(IN)}_{\ell l''}(s')
\]

$a_{0}^{l'}$ - subtraction constant = $\mathcal{T}_s(s = 4m_\pi^2, t = 0)$ - scattering lengths in $S$ wave
Dispersion relations for $\pi \pi$ interactions

Once subtracted dispersion relations "GKPY" (for the $S$ and $P$ waves):

$$\pi \quad s \quad \pi$$

\[ \begin{align*}
\text{crossing symmetry:} & \quad s \rightarrow t \rightarrow \mathbf{T}_s(s, t) = \hat{C}_{st} \mathbf{T}_t(t, s) \\
\mathbf{T}(s, t) + \text{crossing symmetry} & \rightarrow \text{dispersion relations for } 4m^2_\pi < s < \sim (1150 \text{ MeV})^2
\end{align*} \]

\[ \begin{align*}
\text{Re} \ t^{(\text{OUT})}_l(s) &= \sum_{l'=0}^{2} C^{l''}_{st} a^{l''}_0 + \sum_{l'=0}^{2} \sum_{l''=0}^{4} \int_{4m^2_\pi}^{\infty} ds' K^{l''}_{l'l'}(s, s') \text{Im} \ t^{(\text{IN})}_{l'}(s') \\
a^{l''}_0 &- \text{subtraction constant} = \mathbf{T}_s(s = 4m^2_\pi, t = 0) - \text{scattering lengths in } S \text{ wave}
\end{align*} \]

Condition for crossing symmetry:

$$\text{Re} f^{(\text{OUT})}_l(s) - \text{Re} f^{(\text{IN})}_l(s) = 0$$
Experimental data on the $\pi \pi$ interactions in the $S_0$ wave (notation $J^I$)
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- elimination of several sets of experimental data
Experimental data on the $\pi\pi$ interactions in the $S_0$ wave (notation $JI$)

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"Precise determination of the f0(500) and f0(980) pole parameters from a dispersive data analysis",
R. Garcia-Martin, R. Kamiński, J.R. Pelaez, J. Ruiz de Elvira,
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Method of combined analysis (data dispersion relations)

- input amplitudes for the $S$, $P$, $D$ and $F$ waves constructed, at the beginning, only by fit to the data,
- just simple polynomials in energy$^2$,
- no assumption on the low threshold parameters (we use NA48/2 data),
- set of dispersion relations used in the analysis:
  - once subtracted dispersion relations (GKPY),
  - twice subtracted dispersion relations (Roy),
  - Forward Dispersion Relations (FDR),
  - Olsson sum rule (SR),
- phenomenological input partial amplitudes used up to 1420 MeV,
- above 1420 MeV - Regge parameterizations
Method of combined analysis (data dispersion relations)

\[ \chi^2_{tot} = \chi^2_{data} + \bar{d}^2_{Roy} + \bar{d}^2_{GKPY} + \bar{d}^2_{FDR} + \bar{d}^2_{SR} \]

where \[ \bar{d}^2_i = \frac{1}{\text{number of points}} \sum_j^n \left( \frac{\Delta_i(s_j)}{\delta \Delta_i(s_j)} \right)^2 \]

(number of points = 28)

|                | fit only to data | fit to data and to dispersion relations |
|----------------|------------------|----------------------------------------|
| \( \bar{d}^2_{Roy} \) | 0.87             | 0.14                                   |
| \( \bar{d}^2_{GKPY} \) | 1.9              | 0.32                                   |
| \( \bar{d}^2_{FDR} \)   | 1.98             | 0.4                                    |
Constrained Fits to Data (FDR+SR+Roy+GKPY)

\[ \text{Re } t_0^{(0)}(s) \]

\[ \bar{d}^2 = 0.24 \]
fit to GKPY ($P1$ wave)

Constrained Fits to Data (FDR+SR+Roy+GKPY)

$\text{Re } t^{(1)}_1(s)$

$\overline{d}^2 = 0.60$

$\text{GKPY}^\text{P}_{\text{in}}$

$\text{GKPY}^\text{P}_{\text{out}}$
Constrained Fits to Data (FDR+SR+Roy+GKPY)

\[ \text{Re } t_0^{(2)}(0) \]

\[ \tilde{d}^2 = 0.11 \]

GKPY$^{S2 \text{ in}}$

GKPY$^{S2 \text{ out}}$
precise determination of $f_0(500)\ (\sigma)$ meson and threshold parameters

$f_0(500)\ (\sigma)$

- PDG 2010:
  $M = 400 \pm 1200$ MeV
  $\Gamma = 2 \times (250 \pm 500)$ MeV

- GKPY:
  $E_\sigma = 457 \pm 14 - i279^{+11}_{-7}$ MeV

threshold parameters, e.g. $a^0_0$:

- ChPT + Roy eqs (Bern group):
  $0.220 \pm 0.005 \, m^{-1}_\pi$

- GKPY:
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precise determination of $f_0(980)$ meson

\[ \delta_0^{(0)} \]

- UFD
- Na48/2
- K$\rightarrow$2 $\pi$ decay
- Old K decay data
- Kaminski et al.
- Grayer et al. Sol. B
- Grayer et al. Sol. C
- Grayer et al. Sol. D
- Hyams et al. 73

\[ s^{1/2} \text{(MeV)} \]

\[ \eta_0^{(0)}(s) \]

\[ s^{1/2} \text{(MeV)} \]
precise determination of \( f_0(980) \) meson

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\[ \eta_0^0(s) \]

\[ s^{1/2} \text{ (MeV)} \]

- Cohen et al.
- Etkin et al.
- Wetzel et al.
- Hyams et al. 73
- Kaminski et al.
- Hyams et al. 73
- Protopopescu et al.
- CFD
Model-independent analytic continuation to the complex plane gets pole at $s_{pole}^{1/2}$ on the 2nd Riemann sheet

assuming that $M = Re(s_{pole}^{1/2})$ and $\Gamma = -2 Im(s_{pole}^{1/2})$ we get:

$$M_{f_0(980)} = 996 \pm 7 \text{ MeV} \quad \text{and}$$

$$\Gamma_{f_0(980)} = 50^{+20}_{-12} \text{ MeV}$$

PDG'2010: Mass $m = 980 \pm 10 \text{ MeV}$

Width $\Gamma = 40 - 100 \text{ MeV}$
precise determination of couplings to the $\pi\pi$ channel

$$g^2 = -16\pi \lim_{s \to s_{pole}} (s - s_{pole}) t_\ell(s)(2\ell + 1)/(2p)^{2\ell}$$

where $p^2 = s/4 - m^2_{\pi}$. 

|                  | $\sqrt{s_{\text{pole}}}$ (MeV) | $|g|$          |
|------------------|-------------------------------|---------------|
| $f_0(500)^{\text{GKPY}}$ | $(457^{+14}_{-13}) - i(279^{+11}_{-7})$ | $3.59^{+0.11}_{-0.13}$ GeV |
| $f_0(500)^{\text{Roy}}$     | $(445 \pm 25) - i(278^{+22}_{-18})$    | $3.4 \pm 0.5$ GeV          |
| $f_0(980)^{\text{GKPY}}$    | $(996 \pm 7) - i(25^{+10}_{-6})$       | $2.3 \pm 0.2$ GeV          |
| $f_0(980)^{\text{Roy}}$     | $(1003^{+5}_{-27}) - i(21^{+10}_{-8})$  | $2.5^{+0.2}_{-0.6}$ GeV    |
| $\rho(770)^{\text{GKPY}}$  | $(763.7^{+1.7}_{-1.5}) - i(73.2^{+1.0}_{-1.1})$ | $6.01^{+0.04}_{-0.07}$    |
| $\rho(770)^{\text{Roy}}$   | $(761^{+4}_{-3}) - i(71.7^{+1.9}_{-2.3})$  | $5.95^{+0.12}_{-0.08}$    |
Conclusions

due to works on once and twice subtracted dispersion relations with imposed crossing symmetry condition we have in disposal very efficient set of rules for testing the partial $\pi\pi$ amplitudes in the $S$, $P$, $D$ and $F$ waves,

we also have set of model independent unitary $\pi\pi$ amplitudes in those waves in the range from $2m_\pi$ to several GeV fulfilling very well crossing symmetry below $\sim 1100$ MeV,

as an artefact we got very precise values of parameters for the $f_0(500)$ and $f_0(980)$ resonances