Superallowed Fermi transitions in RPA with a relativistic
point-coupling energy functional

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Abstract

The self-consistent random phase approximation (RPA) approach with the residual interaction derived from a relativistic point-coupling energy functional is applied to evaluate the isospin symmetry-breaking corrections $\delta_c$ for the $0^+ \rightarrow 0^+$ superallowed Fermi transitions. With these $\delta_c$ values, together with the available experimental $ft$ values and the improved radiative corrections, the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is examined. Even with the consideration of uncertainty, the sum of squared top-row elements has been shown to deviate from the unitarity condition by 0.1% for all the employed relativistic energy functionals.

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I. INTRODUCTION

In the standard model, the Cabibbo-Kobayashi-Maskawa (CKM) matrix relates the quark eigenstates of the weak interaction with the quark mass eigenstates, and therefore it should be unitary. Examination of the unitarity of the CKM matrix provides a rigorous test for the standard model description of electroweak interactions. Usually, much attention has been paid to the top row of the CKM matrix. The unitarity requires, \( \sum_{f=d,s,b} |V_{uf}|^2 = 1, \) where the values of \( |V_{us}| \) and \( |V_{ub}| \) can be taken from the Particle Data Group reviews. The value of matrix element \( |V_{ud}| \) can be determined in nuclear physics through the following ways: nuclear \( 0^+ \rightarrow 0^+ \) superallowed Fermi transition, neutron decay, pion \( \beta \) decay, and nuclear mirror transition. Among them, the first method provides the most precise determination of \( |V_{ud}| \), given by \( |G_V/G_F| \), where \( G_F \) is the Fermi coupling constant for purely leptonic decays. The vector coupling constant of semileptonic weak interactions \( G_V \) is determined by the nucleus-independent \( \mathcal{F}t \) value and transition-independent part of radiative corrections. Therefore, many efforts are devoted into the study of the nucleus-independent \( \mathcal{F}t \) value.

On the theoretical side, there are several nuclear structure models that have been adopted to calculate the nucleus-independent \( \mathcal{F}t \) value, including the shell model and the self-consistent charge-exchange random phase approximation (RPA) based on both non-relativistic and relativistic energy density functionals, as well as the isospin-projection scheme on top of the Skyrme-DFT approach. It has been shown in Ref. that the constancy of the \( \mathcal{F}t \) values can be satisfied for all the employed relativistic meson-exchange energy functionals.

Recently, as the counterpart of meson-exchange energy functional, the relativistic point-coupling energy functional has attracted more and more attention because of its simplicity and success. In the point-coupling functional, there is no mesonic degree of freedom that makes its practical applications more feasible and the numerical effort considerably reduced. In particular, the point-coupling functional can be easily extended to study the effects beyond the mean-field approximation that are important for a proper description of the nuclear low-lying collective excited states.

In view of these facts, recently, the model of change-exchange RPA with the residual interaction derived from the relativistic density-dependent point-coupling functional has
been developed \[17\]. In this work, we will adopt the similar model, but we’ll start from the non-linear version of point-coupling functional to study the isospin symmetry-breaking corrections $\delta_c$ in the $0^+ \rightarrow 0^+$ superallowed transitions. With these $\delta_c$ values, together with the most recent experimental $ft$ values \[18\] and the improved radiative corrections \[8, 19\], the unitarity of the CKM matrix will be examined.

This paper is arranged as follows. The model is introduced briefly in Sec. II. The results and corresponding discussions are given in Sec. III. A brief summary of the present investigation is presented in Sec. IV.

II. THE MODEL

For the charge-exchange channels of both the $\tau_+$ and $\tau_-$, the relativistic RPA equation has the following form \[11\],

$$
\begin{pmatrix}
A_{n\bar{p},n'\bar{p}'}^J & B_{n\bar{p},n'\bar{p}'}^J \\
-B_{n\bar{p},n'\bar{p}'}^J & -A_{n\bar{p},n'\bar{p}'}^J
\end{pmatrix}
\begin{pmatrix}
U_{p'\bar{n}}^{J,J\nu} \\
V_{n'\bar{p}}^{J,J\nu}
\end{pmatrix}
= \omega_{\nu}
\begin{pmatrix}
U_{p\bar{n}}^{J,J\nu} \\
V_{n\bar{p}}^{J,J\nu}
\end{pmatrix}.
$$

(1)

In the above equation, $\bar{p}$ and $p$ ($\bar{n}$ and $n$) denote occupied and unoccupied proton (neutron) states, where the unoccupied states include the states above the Fermi surface and those in the Dirac sea. $U^{J,J\nu}$ and $V^{J,J\nu}$ are the amplitudes corresponding to the RPA energy $\omega_{\nu}$. The matrices $A$ and $B$ are given by,

$$
A_{12,34} = (E_1 - E_2)\delta_{12,34} + \langle 14|V_{ph}|32 \rangle,
$$

(2)

$$
B_{12,34} = -\langle 13|V_{ph}|42 \rangle,
$$

(3)

where $E_1, E_2$ are the single-particle energies from mean-field calculations.

For the sake of the self-consistency, the particle-hole residual interaction $V_{ph}$ is derived from the same effective Lagrangian density as the mean-field Dirac single-nucleon Hamiltonian that determines the mean-field for the nuclear ground-state. The spin-isospin-dependent interaction terms are generated by the isovector terms. Although the direct one-$\pi$ contribution to the nuclear ground-state vanishes at the Hartree level because of parity conservation, the pion nevertheless must be included in the calculation of spin-isospin excitations \[20\].
The derivative type of the pion-coupling necessitates the inclusion of the zero-range Landau-Migdal term, which accounts for the contact part of the nucleon-nucleon interaction,

\[ V_{\delta \pi}(1, 2) = g' \left[ \frac{f_\pi}{m_\pi} \gamma_0 \gamma_5 \gamma_\tau \right]_1 \left[ \frac{f_\pi}{m_\pi} \gamma_0 \gamma_5 \gamma_\tau \right]_2 \delta(r_1 - r_2), \]  

(4)

where \( f_\pi^2/4\pi = 0.08 \), \( m_\pi = 138 \text{ MeV} \). The parameter \( g' \) in principle has to be adjusted to reproduce the experimental Gamow-Teller resonance excitation energy \([20]\). However, the direct contribution from the \( \pi \)-meson field vanishes for \( 0^+ \to 0^+ \) transition. Therefore, the self-consistency is still maintained in the RH+RPA study of superallowed Fermi transition and we take \( g' = 1/3 \) as Ref. \([11]\).

In the calculations, the relativistic point-coupling energy functional of both PC-F1 \([21]\) and PC-PK1 \([13]\) is adopted, where the scalar-isovector coupling terms do not exist. As a result, the particle-hole residual interaction \( V_{\text{ph}} \) that gives non-zero contribution to the charge-exchange RPA matrix is composed of three parts: the vector-isovector coupling terms \([22]\),

\[ V_{TV}(1, 2) = [\gamma_0 \gamma^\mu \vec{\tau}]_1 (\alpha_{TV} + \delta_{TV} \Delta) [\gamma_0 \gamma_\mu \vec{\tau}]_2 \delta(r_1 - r_2), \]  

(5)

the Landau-Migdal term \([4]\), and \( \pi \)-meson field term \([11, 20]\),

\[ V_{\pi}(1, 2) = -\left[ \frac{f_\pi}{m_\pi} \gamma_0 \gamma_5 \gamma \cdot \nabla \vec{\tau} \right]_1 \left[ \frac{f_\pi}{m_\pi} \gamma_0 \gamma_5 \gamma \cdot \nabla \vec{\tau} \right]_2 D_\pi(r_1, r_2), \]  

(6)

where \( D_\pi(r_1, r_2) \) denotes the \( \pi \)-meson propagator,

\[ D_\pi(r_1, r_2) = \frac{1}{4\pi} \frac{e^{-m_\pi |r_1 - r_2|}}{|r_1 - r_2|}. \]  

(7)

\( \alpha_{TV} \) and \( \delta_{TV} \) are the coupling constants in the vector-isovector channel of point-coupling energy functional \([13, 21]\), and \( \Delta \) is the Laplace operator. \( \gamma^\mu \) are the four-component Dirac matrices. At the Hartree level, the Coulomb term does not contribute to the charge-exchange RPA matrix element.

The eigenvectors of the RPA Eq. \([11]\) are separated into two groups, which respectively represent the excitations of the \( \tau_- \) and \( \tau_+ \) channels with the following normalization conditions

\[ \sum_{p\bar{n}} (U_{pn}^J)^2 - \sum_{n\bar{p}} (V_{n\bar{p}}^J)^2 = +1, \text{ for } \tau_- \text{ channel } , \]  

(8)

\[ \sum_{p\bar{n}} (U_{pn}^J)^2 - \sum_{n\bar{p}} (V_{n\bar{p}}^J)^2 = -1, \text{ for } \tau_+ \text{ channel } . \]  

(9)
The excitation energies $\Omega_{\nu}$ and the corresponding forward $X^{J\nu}$, backward $Y^{J\nu}$ amplitudes in the $\tau_-$ and $\tau_+$ channels are given by,

$$\Omega_{\nu} = +\omega_{\nu}, \quad X^{J\nu}_{p\bar{n}} = U^{J\nu}_{p\bar{n}}, \quad Y^{J\nu}_{n\bar{p}} = V^{J\nu}_{n\bar{p}}, \quad \text{for } \tau_- \text{ channel }, \quad (10)$$

$$\Omega_{\nu} = -\omega_{\nu}, \quad X^{J\nu}_{n\bar{p}} = V^{J\nu}_{n\bar{p}}, \quad Y^{J\nu}_{p\bar{n}} = U^{J\nu}_{p\bar{n}}, \quad \text{for } \tau_+ \text{ channel }. \quad (11)$$

Subsequently, it is straightforward to calculate the $0^+ \rightarrow 0^+$ superallowed transition probabilities between the ground-state and excited states with the forward and backward amplitudes for the $\tau_-$ and $\tau_+$ channels as follows,

$$B^{-}_{J\nu} = \left| \sum_{p\bar{n}} X^{J\nu}_{p\bar{n}} \langle p \parallel \tau_- \parallel \bar{n} \rangle + \sum_{n\bar{p}} \kappa_{n\bar{p}} Y^{J\nu}_{n\bar{p}} \langle \bar{n} \parallel \tau_- \parallel n \rangle \right|^2, \quad (12)$$

$$B^{+}_{J\nu} = \left| \sum_{n\bar{p}} X^{J\nu}_{n\bar{p}} \langle n \parallel \tau_+ \parallel \bar{p} \rangle + \sum_{p\bar{n}} \kappa_{p\bar{n}} Y^{J\nu}_{p\bar{n}} \langle \bar{n} \parallel \tau_+ \parallel p \rangle \right|^2, \quad (13)$$

with $\kappa_{ab} = (-)^{j_a+j_b}$.

### III. RESULTS AND DISCUSSIONS

For simplicity, the pairing correlations are neglected and the filling approximation is used. Moreover, the calculation is presently restricted to preserve spherical symmetry. As a result, the Dirac equation for nucleons can be solved easily in coordinate space using the numerical techniques in Ref. [23], where the box size $R = 15$ fm and the mesh size $\Delta r = 0.1$ fm. The solutions of Dirac equation, including single-particle wave functions and energies, are used as inputs of the RPA equation. As usual, the single-particle energy truncation is introduced. We find that with the choice of the truncations $[-940, 1100]$ MeV for the PC-F1 and $[-940, 1160]$ MeV for the PC-PK1, the model-independent sum rule

$$\sum_{\nu} B^{-}_{\nu} - \sum_{\nu} B^{+}_{\nu} = N - Z \quad (14)$$

can be fulfilled up to $10^{-5}$ accuracy, and the isospin symmetry-breaking corrections $\delta_c$ are stable with respect to these numerical inputs at the same level of accuracy.

The isospin symmetry-breaking correction $\delta_c$ is determined by the superallowed transition strength $M_F^2$ as follows,

$$M_F^2 \equiv |\langle f \parallel \tau_{\pm} \parallel i \rangle|^2 = |M_0|^2(1 - \delta_c), \quad (15)$$
where $M_0 = \sqrt{2}$ for $T = 1$ states with the exact isospin symmetry, and $|f\rangle, |i\rangle$ represent the final and initial states of $\tau_{\pm}$ transitions respectively. It has to be pointed out that in practical calculations, there are several final states $0^+_i$ with different excitation energies $E_{\nu}$ and strengths $B_{\nu}$ as shown in Figure 1 for the superallowed transition, for example, $^{10}\text{C} \rightarrow ^{10}\text{B}$. As usual, the final state $|f\rangle$ is taken to be the excited state with the largest transition strength $B_{\nu}$ (close to 2). Table I presents the isospin symmetry-breaking correction $\delta_c$ in several $0^+_i \rightarrow 0^+_f$ superallowed transitions, calculated using the RPA for the relativistic point-coupling energy functional of both PC-F1 and PC-PK1. For comparison, the results in Ref. [11] from the RPA calculations but with the relativistic meson-exchange energy functionals PKO1 [24] and DD-ME2 [25] are given as well. It has been found in Ref. [11] that the significant differences in $\delta_c$ values obtained by PKO1 and by DD-ME2 are mainly due to the different treatments of the Coulomb field. The Fock terms have been included in the former case (PKO1), but not in the latter case (DD-ME2). Table I shows that the point-coupling functionals PC-F1 and PC-PK1 without the Fock terms give quite similar $\delta_c$ values, both of which are in between the results by the PKO1 and DD-ME2.

In Table II, the excitation energies $E_{\nu}$ for the $0^+_i \rightarrow 0^+_f$ superallowed transitions are shown. For comparison with the experimental values taken from the recent survey [18], the corrections due to the proton-neutron mass difference in particle-hole configurations are made for the calculated results. It is shown that all the relativistic energy functionals can reproduce the excitation energies quite well.

**FIG. 1:** Distribution of the strengths $B_{\nu}(0^+ \rightarrow 0^+_\nu)$ for the superallowed transition $^{10}\text{C} \rightarrow ^{10}\text{B}$. 
TABLE I: Isospin symmetry-breaking corrections $\delta_c$ (in %) for the $0^+ \rightarrow 0^+$ superallowed transitions obtained by RPA with the relativistic point-coupling energy functional of both PC-F1 and PC-PK1. The results are compared with those obtained with the relativistic meson-exchange energy functionals PKO1 and DD-ME2 [11].

|                  | PC-F1 | PC-PK1 | PKO1 [11] | DD-ME2 [11] |
|------------------|-------|--------|-----------|-------------|
| $^{10}$C $\rightarrow$ $^{10}$B | 0.109 | -      | 0.082     | 0.150       |
| $^{14}$O $\rightarrow$ $^{14}$N | 0.150 | -      | 0.114     | 0.197       |
| $^{18}$Ne $\rightarrow$ $^{18}$F | 0.309 | 0.297  | 0.270     | 0.430       |
| $^{26}$Si $\rightarrow$ $^{26}$Al | 0.202 | 0.180  | 0.176     | 0.252       |
| $^{30}$S $\rightarrow$ $^{30}$P | 0.420 | 0.488  | 0.497     | 0.633       |
| $^{34}$Ar $\rightarrow$ $^{34}$Cl | 0.379 | 0.378  | 0.268     | 0.376       |
| $^{38}$Ca $\rightarrow$ $^{38}$K | 0.347 | 0.325  | 0.313     | 0.441       |
| $^{42}$Ti $\rightarrow$ $^{42}$Sc | 0.400 | 0.375  | 0.384     | 0.523       |
| $^{26}$Al $\rightarrow$ $^{26}$Mg | 0.159 | 0.141  | 0.139     | 0.198       |
| $^{34}$Cl $\rightarrow$ $^{34}$S | 0.316 | 0.314  | 0.234     | 0.307       |
| $^{38}$K $\rightarrow$ $^{38}$Ar | 0.294 | 0.275  | 0.278     | 0.371       |
| $^{42}$Sc $\rightarrow$ $^{42}$Ca | 0.345 | 0.322  | 0.333     | 0.448       |
| $^{54}$Co $\rightarrow$ $^{54}$Fe | 0.339 | 0.301  | 0.319     | 0.393       |
| $^{66}$As $\rightarrow$ $^{66}$Ge | 0.522 | 0.488  | 0.475     | 0.572       |
| $^{70}$Br $\rightarrow$ $^{70}$Se | 0.935 | 0.998  | 1.140     | 1.268       |
| $^{74}$Rb $\rightarrow$ $^{74}$Kr | 0.668 | 0.597  | 1.088     | 1.258       |

The nucleus-independent $\mathcal{F}t$ can be obtained by the experimental $ft$ value, isospin symmetry-breaking correction $\delta_c$ and transition-independent part of radioactive corrections [4],

$$
\mathcal{F}t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_c),
$$

(16)

where $\delta'_R$ is the part of nucleus-independent radiative correction, which is a functional only of the electron’s energy and the charge of daughter nucleus $Z$ while $\delta_{NS}$ is the part of radiative correction that depends on the details of nuclear structure.

With the recent theoretical results of $\delta'_R$ and $\delta_{NS}$ [8] and the recent experimental $ft$
TABLE II: Excitation energies (in MeV) for the $0^+ \rightarrow 0^+$ superallowed transitions measured by taking the ground-state of the corresponding even-even nuclei as reference. For comparison with the experimental values taken from the recent survey [18], the corrections due to the proton-neutron mass difference in particle-hole configurations are made for the calculated results.

|                | Expt. | PC-F1 | PC-PK1 | PKO1 [11] | DD-ME2 [11] |
|----------------|-------|-------|--------|-----------|-------------|
| $^{10}$C $\rightarrow$ $^{10}$B | -1.908 | -2.217 | -       | -1.698     | -2.236      |
| $^{14}$O $\rightarrow$ $^{14}$N | -2.831 | -2.967 | -       | -2.420     | -3.081      |
| $^{18}$Ne $\rightarrow$ $^{18}$F | -3.402 | -3.400 | -3.419  | -3.195     | -3.451      |
| $^{26}$Si $\rightarrow$ $^{26}$Al | -4.842 | -4.956 | -5.055  | -4.531     | -5.110      |
| $^{30}$S $\rightarrow$ $^{30}$P | -5.460 | -5.295 | -5.330  | -4.845     | -5.395      |
| $^{34}$Ar $\rightarrow$ $^{34}$Cl | -6.063 | -5.975 | -5.964  | -5.559     | -6.278      |
| $^{38}$Ca $\rightarrow$ $^{38}$K | -6.612 | -6.582 | -6.576  | -6.035     | -6.775      |
| $^{42}$Ti $\rightarrow$ $^{42}$Sc | -7.000 | -6.833 | -6.869  | -6.661     | -6.964      |
| $^{26}$Al $\rightarrow$ $^{26}$Mg | 4.233  | 4.211  | 4.304   | 3.908      | 4.350       |
| $^{34}$Cl $\rightarrow$ $^{34}$S | 5.492  | 5.292  | 5.288   | 5.062      | 5.561       |
| $^{38}$K $\rightarrow$ $^{38}$Ar | 6.044  | 5.905  | 5.905   | 5.557      | 6.083       |
| $^{42}$Sc $\rightarrow$ $^{42}$Ca | 6.426  | 6.207  | 6.242   | 6.118      | 6.333       |
| $^{54}$Co $\rightarrow$ $^{54}$Fe | 8.244  | 8.016  | 8.122   | 7.720      | 8.240       |
| $^{66}$As $\rightarrow$ $^{66}$Ge | 9.579  | 9.367  | 9.434   | 9.044      | 9.677       |
| $^{70}$Br $\rightarrow$ $^{70}$Se | 9.970  | 9.735  | 9.806   | 9.632      | 9.852       |
| $^{74}$Rb $\rightarrow$ $^{74}$Kr | 10.417 | 10.246 | 10.299  | 10.005     | 10.437      |

values [18], the nucleus-independent $F_t$ value and its uncertainty $\delta F_t$ for each superallowed Fermi transition are obtained with the calculated $\delta_c$ values and listed in Table III where the corresponding average value $\overline{F}_t$ and $\chi^2/\nu$ are given as well. In the calculation of $\delta F_t$, the uncertainty of $\delta_c$ is taken as zero. The average value $\overline{F}_t$ is calculated in the following way [26],

$$\overline{F}_t \pm \delta \overline{F}_t = \frac{\sum_i \omega_i F_{t_i}}{\sum_i \omega_i} \pm \left( \sum_i \omega_i \right)^{-1/2}$$

where the weight is

$$\omega_i = 1/(\delta F_{t_i})^2.$$
The $\chi^2/\nu$ is calculated by

$$\chi^2/\nu = \sum_i^N \left( \frac{F_{ti} - F_t}{\delta F_{ti}} \right)^2 / (N - 1) $$  \hspace{1cm} (19)$$

where $N$ is the number of the calculated superallowed Fermi transitions. Since the uncertainties $\delta F_{ti}$ in the superallowed Fermi transitions $^{34}\text{Ar} \rightarrow ^{34}\text{Cl}$ and $^{74}\text{Rb} \rightarrow ^{74}\text{Kr}$ are obviously larger than those in other transitions and excluded in the calculations of average value $\overline{F_t}$ and $\chi^2/\nu$.

TABLE III: Nucleus-independent $F_t$ values (in s), its average value $\overline{F_t}$ (in s) and the $\chi^2/\nu$.

|                | PC-F1       | PC-PK1       |
|----------------|-------------|--------------|
| $^{10}\text{C} \rightarrow ^{10}\text{B}$ | 3078.7(45)  | -            |
| $^{14}\text{O} \rightarrow ^{14}\text{N}$ | 3077.0(31)  | -            |
| $^{34}\text{Ar} \rightarrow ^{34}\text{Cl}$ | 3078.5(84)  | 3078.5(84)  |
| $^{26}\text{Al} \rightarrow ^{26}\text{Mg}$ | 3077.0(13)  | 3077.6(13)  |
| $^{34}\text{Cl} \rightarrow ^{34}\text{S}$ | 3081.0(15)  | 3081.1(15)  |
| $^{38}\text{K} \rightarrow ^{38}\text{Ar}$ | 3083.6(16)  | 3084.2(16)  |
| $^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$ | 3082.3(21)  | 3083.0(21)  |
| $^{54}\text{Co} \rightarrow ^{54}\text{Fe}$ | 3083.3(24)  | 3084.4(24)  |
| $^{74}\text{Rb} \rightarrow ^{74}\text{Kr}$ | 3119.6(88)  | 3120.2(88)  |
| average        | 3080.3(7)   | 3081.1(7)   |
| $\chi^2/\nu$   | 1.1         | 1.4         |

Table III shows that the $\chi^2/\nu$ is 1.1 for PC-F1 and 1.4 for PC-PK1. It indicates that the constancy of the $F_t$ values is good for both cases.

To illustrate the constancy of the $F_t$ values more clearly, we plot the nucleus-independent $F_t$ values as a function of the charge of the daughter nucleus $Z$ in Figure 2. The shaded horizontal band gives one standard deviation around the average $\overline{F_t}$ value.

With the nucleus-independent $\overline{F_t}$ value, the absolute value of element $V_{ud}$ in the CKM matrix can be calculated by

$$|V_{ud}|^2 = \frac{K}{2G_F^2(1 + \Delta_K^2)\overline{F_t}}$$  \hspace{1cm} (20)$$

FIG. 2: Nucleus-independent $\mathcal{F}t$ values as a function of the charge of the daughter nucleus $Z$. The shaded horizontal band gives one standard deviation around the average $\mathcal{F}t$ value.

where $K/(hc)^6 = 8120.2787(11) \times 10^{-10}$ GeV$^{-4}$ s, the purely leptonic decays Fermi coupling constant $G_F/(hc)^3 = 1.16637(1) \times 10^{-5}$ GeV$^{-2}$ and the radiative corrections’ transition-independent part $\Delta_{VR}^V = 2.361(38)\%$.

Together with the other two CKM matrix elements $|V_{us}| = 0.2255(19)$ and $|V_{ub}| = 0.00393(36)$, the unitarity of the CKM matrix can be examined by the sum of squared top-row elements of the CKM matrix. The values of $|V_{ud}|$ and $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ are listed in Table IV where we also give the values of uncertainty, which might be underestimated to some extent as the uncertainty of $\delta_c$ was assumed to be zero and the systematic errors were not taken into account.

**TABLE IV: The absolute value of matrix element $V_{ud}$ and the sum of squared top-row elements of the CKM matrix from the RPA calculations with different relativistic energy functionals.**

|          | $|V_{ud}|$     | $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ |
|----------|---------------|-------------------------------------|
| PC-F1    | 0.97290(21)   | 0.9974(10)                          |
| PC-PK1   | 0.97278(22)   | 0.9972(10)                          |
| PKO1 [11]| 0.97273(27)   | 0.9971(10)                          |
| DD-ME2 [11]| 0.97311(26) | 0.9978(10)                          |

Table IV shows that the $|V_{ud}|$ values obtained by PC-F1 and PC-PK1 are quite similar and close to the results of PKO1. In addition, it is seen that even with the uncertainty, the sum of squared top-row elements deviates from the unitarity condition by 0.1% in the
charge-exchange RPA calculations with these four relativistic effective interactions.

IV. SUMMARY

In summary, the self-consistent RPA with the residual interaction derived from the relativistic point-coupling energy functional has been applied to calculate the isospin symmetry-breaking corrections $\delta_c$ for several typical $0^+ \rightarrow 0^+$ superallowed transitions. Together with the experimental $ft$ values in the most recent survey and the improved radiative corrections, the corresponding nucleus-independent $\mathcal{F}t$ values and matrix element $|V_{ud}|$ have been calculated. It has been found that the $|V_{ud}|$ values obtained by PC-F1 and PC-PK1 are quite similar and close to the results of PKO1. However, even with the uncertainty, the sum of squared top-row elements has been shown to deviate from the unitarity condition by 0.1% for all the employed relativistic energy functionals. It indicates that other effects, including deformation and pairing correlations, would play important roles. Therefore, it is very interesting to study the $0^+ \rightarrow 0^+$ superallowed transitions in the framework of deformed QRPA with a proper pairing force, for instance a separable pairing force [27]. Of course, in this case, particle number projection and angular momentum projection are required to give good nucleon number and angular momentum for RPA states. Work along this direction is in progress.

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