Study of Wavelet Transform-based Image Fusion Methods

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Abstract: The main objective of this paper is to get some basic knowledge about image fusion, the methods used for fusion, and the problems that still exist in image fusion technology, and to draw the attention of the research field to image fusion technology through a combination of theory and practice.

Keywords: wavelet transform; image fusion; fusion rules

1. Introduction
From the current domestic situation, the image fusion technology started relatively late, compared to the international research pace we are obviously behind a lot. Therefore, it is urgent to increase research efforts and research investment in this direction.

Researchers found that the wavelet transform has several characteristics: it can provide horizontal, vertical, and diagonal multi-directional information, the information is distributed in each individual pixel without interfering with each other, and it can achieve lossless reconstruction in the process of image reconstruction [1]. The image fusion results obtained by these characteristics are often much better than those obtained by other methods. Therefore, the practice of image fusion based on wavelet transform is of great practical significance in this topic.

2. Related Works
The reason why the wavelet transform can replace the short-time Fourier transform is that it has the characteristics of multi-resolution analysis [2], in the time domain and frequency domain, it can show the information contained in the signal clearly, and its function is equivalent to a magnifying glass, which can magnify the part to be observed to obtain useful information.

3. Asictheory
3.1. Wavelet Transform Fundamental Theory
A wavelet, as the name suggests, is a small section of the wave, which has two characteristics: it is small because the definition domain is small, and the wave has the property of oscillating up and down. Mathematically speaking, a wavelet is a function \( \psi(t) \) with zero integral [3].

\[
\int_{-\infty}^{\infty} \psi(t)dt = 0
\]  
(1)
The principle of wavelet transformation is to select the most suitable wavelet $\psi(t)$ according to the need, and then translate and telescope it to form a series of wavelets.

The mathematical definition of a continuous wavelet transform is: if a function $\psi(t) \in L^2(R)$, and condition (2-1) is satisfied, it is called a $\psi(t)$ continuous wavelet. A function is a cluster of functions that is normalized $\psi(t)$ by $a$ expansion and $b$ translation and $|a|^{-1/2}$ energy.

$$\psi_{a,b}(t) = |a|^{-1/2} \psi(\frac{t-b}{a}) \quad (a, b \in R \ a \neq 0)$$  \hspace{1cm} (2)

The waveform itself is a wavelet, and the waveform generated by it is also a wavelet $\psi(t)$. The waveform itself is a wavelet, and $\psi_{a,b}(t)$ the waveform it generates is also a wavelet, so $\psi(t)$ it is called a basic wavelet and will be $\psi_{a,b}(t)$ collectively called a wavelet [3].

When the wavelet master function $\psi(t)$ satisfies equations (2-1) and (2-2), and the signal is $f(t) \in L^2(R)$ defined as the inner product of $\psi_{a,b}(t)$ and $f(t)$, i.e.

$$WT_f(a,b) = \int_{-\infty}^{\infty} f(t)\psi_{a,b}(t)dt = \frac{1}{|a|} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right)dt$$  \hspace{1cm} (3)

It is also called continuous wavelet transform, where $a$ is the scale factor which can control the wavelet oscillation, that is, the wavelet expansion, $a$ increase the wavelet extension, $a$ decrease the wavelet compression, and the $b$ translation factor which can control the time position (the coefficients $|a|^{-1/2}$ are introduced to normalize the transformation result [4].

3.2. Fundamental Theory of Image Fusion

Image fusion is the acquisition of images of the same scene with several different sensors, either at the same time or at different times. The decomposed image can be divided into two types of signals, high-frequency signals, and low-frequency signals. The high-frequency signal contains mainly detailed information about the image, while the low-frequency signal contains mainly energy information. The fusion criterion used in this paper is a method that combines two fusion criteria. That is, the high-frequency part uses the absolute value maximum method and the low-frequency part uses the weighted average method.

4. Implementation of wavelet transform-based image fusion

This new set of coefficients preserves more band characteristics intact. The experiments will be performed using the Laplace pyramid and wavelet-based transform algorithms for the fusion process.
Depending on the design approach, the experiment can be divided into four steps.

(1) Alignment of multi-source images.

(2) The experiment is divided into two groups, one set of multispectral images and one set of normal images. The two source images are defined as A and B.

(3) Image decomposition and then fusion according to the Laplace pyramid image fusion algorithm and wavelet transform-based image fusion algorithm proposed below, with matlab as the experimental tool.

(4) Finally, each index is calculated for the four sets of experimental images obtained, and the fusion effect is analyzed according to the data.

The first group is the multispectral image fusion, defined as image A and image B to be fused, and the second group is the ordinary image fusion, also defined as image A and image B to be fused. Afterwards, subjective and objective evaluation criteria will be used to measure the experimental results of the two different algorithms. The objective evaluation will provide a comprehensive comparison of the four aspects of standard deviation (SD), information entropy (IE), spatial frequency (SF), and average gradient (AG) to make the analysis results more convincing.

![Wavelet Transform Image Fusion Schematic Diagram](image)

**Fig.1** Wavelet Transform Image Fusion Schematic Diagram

(a) Image to be fused A  
(b) Image to be fused B  
(c) Laplace pyramid image fusion algorithm C  
(d) Wavelet transform-based image fusion algorithm D

**Fig.2** Multispectral image fusion results
Subjectively, the results of Laplace's experiments are not very good, and overall are a bit fuzzy, it is clear that the image of the multispectral images based on the wavelet transform image fusion algorithm D is a bit clearer than the image of Laplace's pyramid image fusion algorithm C, the image of the image fusion algorithm based on the wavelet transform is better quality. In the group of ordinary images, the image D based on the wavelet transform image fusion algorithm has smooth edges and looks clearer and more comfortable overall, while the image C based on the Laplace pyramid image fusion algorithm has some pixels that are more in line with the original image, but the distribution is not uniform, and there are many impurities and missing information on the edge of the plane. Results.

| algorithms                | standard deviation | information | spatial frequency | mean gradient |
|---------------------------|--------------------|-------------|------------------|--------------|
| Wavelet transform-based   | 44.8730e^2        | 6.3088e^2  | 19.6964e^2       | 10.4137e^2   |
| image fusion algorithm    |                    |             |                  |              |
| Laplace Pyramid Image     | 50.7750e^2        | 5.7806e^2  | 13.4971e^2       | 7.6873e^2    |
| Fusion Algorithm          |                    |             |                  |              |

**Fig. 4** Results of objective evaluation of multispectral maps
Fig. 5 Objective evaluation results for ordinary images

5. Summary and Outlook
Wavelet theoretical system is also maturing, it has the characteristics of the image fusion above all have great advantages, this paper is mainly a detailed study of wavelets, and reasonable application to the images needed in daily life, complete the combination of theory and practice. The advantages of wavelets are more manifested in the comparison with the pyramid decomposition fusion. A comprehensive comparison of the wavelet transformation algorithm and the pyramid decomposition fusion method is made, in which there is still a lot of room for exploration and many more aspects to be worked on.

Acknowledgments
This work was supported in part by the National Natural Science Foundation of China, grant number 72073041. Open Foundation for the University Innovation Platform in the Hunan Province, grant number 18K103; 2011 Collaborative Innovation Center for Development and Utilization of Finance and Economics Big Data Property, Universities of Hunan Province. Open project, grant number 20181901CRP03, 20181901CRP04, 20181901CRP05. 2020 Hunan Provincial Higher Education Teaching Reform Research Project under Grant HNJG-2020-1130, HNJG-2020-1124. 2020 General Project of Hunan Social Science Fund under Grant 20B16.

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