Extremely large matter radii in $^{49-51}$Ca isotopes and the $0^+$ breathing mode states of $^{48}$Ca

Syed Afsar Abbas  
Centre for Theoretical Physics, Jamia Millia Islamia University, New Delhi-110025, India  
(email: drafsarabbas@gmail.com)

Anisul Ain Usmani, Usuf Rahaman  
Department of Physics, Aligarh Muslim University, Aligarh-202002, India

Abstract

Through inelastic scattering cross section measurements for $^{42-51}$Ca on a carbon target at 280 MeV/nucleon recently, Tanaka et al. [arXiv:1911.05262 [nucl-ex]], in a very significant experiment, have demonstrated large swelling of doubly magic $^{48}$Ca core in calcium isotopes beyond N=28. The matter radii observed in these experiments, are surprisingly much larger than the corresponding, already amazingly large charge radii of the same calcium isotopes, by Garcia et al. [Nat. Phys. 12 (2016) 594]. Here we propose a novel solution, wherein the breathing mode states $0^+$ of $^{48}$Ca, provide a global and consistent solution of this matter radii conundrum.

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Recently, Tanaka et al. [1] have performed measurements of interaction
cross section, $\sigma_I$ for $^{42-51}$Ca, on a neutral carbon target at 280 MeV/nucleon,
in a groundbreaking first time experiment. Interaction cross section provides
informations regarding RMS radius of nucleon density distributions $< r_m^2 >^\frac{1}{2}$,
which is referred to as the “matter radius” here. This experiment comple-
ments the recent determination of charge radii of the same calcium isotopes
(plus $^{52}$Ca also), recently done by Garcia et al. [2]. Surprisingly large charge
radii were observed in calcium isotopes $^{49}$Ca to $^{52}$Ca by Garcia et al. [2].
That has presented a great challenge to theoretical physics. Already a co-
mundrum as it was, in addition, Tanaka et al. [1] have found that the matter
radii of the corresponding isotopes $^{49}$Ca to $^{51}$Ca, are amazingly, even much
larger than the corresponding charge radii. This presents a most challenging
problem to nuclear structure physics.

Through meticulous calculations, Tanaka et al. [1] have demonstrated
that these very large matter radii, cannot be taken as being due to one-,
two- and three-neutron haloes. They have also shown that this cannot be
considered as a deformation effect on these nuclei. What they have convinc-
ingly demonstrated however is, that this is due to substantial swelling of the
bare $^{48}$Ca core in these calcium isotopes.

So what can be the cause of this swelling of $^{48}$Ca core in these isotopes?
Getting motivation from the concept of E2 effective charge in nuclei, here
we propose a model wherein the breathing mode states $0^+$ provide a con-
sistent understanding of this puzzling situation made explicit by Tanaka et al.
[1]. One may ask, will this large enhancement in matter radii continue
unabated as we add more and more neutrons to $^{48}$Ca core? And whether
this will present us with a new calcium-radii-catastrophe? Our model makes
prediction that experimentalists shall find continuous large swelling of radii
in $^{52}$Ca, $^{53}$Ca and $^{54}$Ca nuclei. After which there shall be no swelling. Hence,
we shall show that our model is not only able to provide a microscopic un-
derstanding of what has already been demonstrated by Tanaka et al., but
provide new challenges for experimentalists to confirm in the future.

The idea of effective charge stems from a desire to explain electromag-
netic transitions and moments totally within the framework of the shell
model. For example, we renormalize the quadrupole operator $\sum_p r^2 Y_{2,0}$ to
$(1 + e_p) \sum_p r^2 Y_{2,0} + e_n \sum_n r^2 Y_{2,0}$ where the effective charge correction for
proton and neutron are $e_p$ and $e_n$ respectively. Note that the total effec-
tive charge of proton is $(1 + e_p)$. It has been emperically determined that
$e_n \approx +\frac{1}{2}$ and $e_p \approx +\frac{1}{2}$ [3, 5]. One has to visualize, for example, that in $^{17}$O, the 9th valence neutron polarizes the $^{16}$O core so strongly, that it gets highly deformed. Then we transfer this large deformation, phenomenologically by treating the uncharged valence neutron to become a highly charged entity of magnitude $+\frac{1}{2}$, while still treating the core $^{16}$O as being a ground state spherical nucleus [3, 5].

In a similar manner, we treat here the 29th valence neutron in $^{49}$Ca, to sit outside the magical core nucleus $^{48}$Ca and have an analogous strong deforming effect on it. This "deforming" effect, as ascertained by Tanaka et al. [1], is not like that of $^{16}$O in the case of effective charge of $^{17}$O, but to “polarize” it so that the core nucleus $^{48}$Ca "breathes" out from the ground state $0^+$, to the first excited $0^+_1$ collective state of $^{48}$Ca. As a consequence we treat the 29th valence neutron to just sit smugly on the surface, while the core has expanded out substantially to mock up the $0^+_1$ collective state. This is its ground state now. Thus this is the source of large matter radius as observed by Tanaka et al. in $^{49}$Ca.

Next, in $^{50}$Ca, now two neutrons sit smugly on the surface of the core $^{48}$Ca nucleus, which then gets even more strongly "polarized" by these (two neutrons), which then breathes out to the next $0^+_2$ collective state in $^{48}$Ca, thereby providing a much larger matter radius. Similarly in $^{51}$Ca, the valence neutrons sitting smugly on the surface of $^{48}$Ca would have induced still larger polarization of the core, to breath out to the next collective $0^+_3$ state of $^{48}$Ca.

So having explained very large matter radii of $^{49}$Ca, $^{50}$Ca, $^{51}$Ca, we ask as to how long would this large expansion continue? That is, will this expansion continue in uncontrolled manner in $^{52}$Ca, $^{53}$Ca, $^{54}$Ca, ..., $^{59}$Ca, $^{60}$Ca? If so, we would have at hand, a genuine calcium-radii-catastrophe! One may be reminded of, the end 19th century Ultraviolet Catastrophe of the black-body radiation.

As per the phenomenological model proposed here, the valence neutrons on top of N=28 in $^{48}$Ca would continue to polarize this core strongly, so as to go to higher and higher breathing mode states of $^{48}$Ca, as long as these states are available. As per data displayed below in Table 1, we give the $0^+_n$ excited breathing mode states of $^{48}$Ca, which have been determined experimentally [6, 8].

The breathing mode states $0^+_n$ of $^{48}$Ca have been taken from Ref. [6, 7]. Note the $0^+_3$ and $0^+_3'$ at 11.945 MeV and 11.967 MeV respectively, appear as almost being degenerate. The role of these two as being one, is made
Table 1: Experimental breathing mode states $0^+$ of $^{48}$Ca

| State  | Energy (MeV) |
|--------|--------------|
| $0_1^+$ | 4.284        |
| $0_2^+$ | 5.461        |
| $0_3^+$ | 11.945       |
| $0_3^{+'}$ | 11.967     |
| $0_4^{+'}$ | 12.318     |
| $0_5^{+'}$ | 12.565     |
| $0_6^{+'}$ | 12.869     |

sharper by the fact that in Burrows [8], $\sigma(\theta)$ in $(p, p')$, $(\alpha, \alpha')$ show oscillatory pattern and are well fitted by DWBA by assuming and using $0_3^{+'}$, $0_4^{+'}$, $0_5^{+'}$, $0_6^{+'}$ states (while $0_3^+$ is not used). Note that it is possible that Giant Quadrupole Resonance states and Giant Monopole Resonance states (breathing mode states) may mingle with each other in a wide range of excitation energy in $^{48}$Ca [6]. Thus if $0_3^+$ and $0_3^{+'}$ states are treated as one, the total number of breathing mode states in $^{48}$Ca are six.

Thus the three observed large matter radii as obtained by Tanaka et al. [1] in $^{49}$Ca, $^{50}$Ca, $^{51}$Ca, would be explained by the three breathing mode states in $^{48}$Ca at $0_1^+ = 4.284$ MeV, $0_2^+ = 5.461$ MeV, $0_3^+ = 11.945/11.967$ MeV. Given the fact that we have three more states $0_4^+ = 12.318$ MeV, $0_5^+ = 12.565$ MeV, $0_6^+ = 12.869$ MeV, our model here predicts that one would observe still larger matter radii in $^{52}$Ca, $^{53}$Ca, $^{54}$Ca. Thus this expansion will stop at N=34, and after that there shall be no further swelling, as per our phenomenological model prediction here.
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