Inflation model with lower multipoles of the CMB suppressed

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The recent observation of the cosmic microwave background anisotropy by the WMAP confirmed that the lower multipoles are considerably suppressed. From the standpoint of the cosmic variance, it is nothing but a statistical accident. Alternatively, one can attribute the deficit of fluctuation on the large scale to the cosmic history, which might be explained in the context of the inflationary physics. In this paper, we show that it is possible to explain the suppressed lower multipoles in the hybrid new inflation model.

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The recent observational results of the cosmic microwave background (CMB) anisotropy by the WMAP strongly support the inflationary scenario, on the ground that the WMAP data are well described by pure adiabatic fluctuations [1]. Although the result agrees with the generic predictions of inflationary scenario within a statistical error, it still suggests two unusual features. One is the running spectral index, and the other is anomalously low value of the quadrupole moment of the CMB. In this paper we would like to focus on the latter feature. The former was investigated in the context of supergravity in Ref. [2], and it was discussed the implication of the deficient lower multipoles in Refs [3, 4, 5, 6]. Of course both features might be just statistical accidents. However, alternatively, it could be argued that such features give us an insight into the very early universe. In the following we show that the hybrid new inflation model is ideal for explaining the low quadrupole, since the density fluctuation generated during the new inflation is naturally larger than that during the preceding hybrid inflation.

First we review the hybrid inflation model [7], which sets the appropriate initial condition for the following new inflation. Hereafter we set the gravitational scale to be unity. The superpotential for the hybrid inflation is given as

\[ W_H = -\mu^2 S + \lambda S \overline{\Psi} \Psi, \] (1)

which is based on the \( U(1)_R \) symmetry, and the \( R \) charges of \( S, \Psi \) and \( \overline{\Psi} \) are 2, 1 and \(-1\), respectively. Assuming the minimal Kähler potential, the inflationary path is identified with \( \Psi = \overline{\Psi} = 0 \) and \( \sigma > \sigma_c \equiv \sqrt{2} \mu / \sqrt{\lambda} \), where \( \sigma \) is the real part of \( S \)

\[ \sigma \equiv \sqrt{2} \text{Re}[S]. \]

Since we are concerned with the last a few e-folds of the hybrid inflation, \( \sigma \) is relatively small. In a region of small \( \sigma \), radiative corrections cannot be negligible, so the potential for \( \sigma \) at one-loop order is approximated as [8]

\[ V_H = \mu^4 \left( 1 + \frac{\lambda^2}{8 \pi^2} \ln \frac{\sigma}{\sigma_c} \right), \] (2)

where we omit the non-renormalizable term since \( \sigma \) is relatively small. The number of e-folds during the hybrid inflation, \( N_H \), is related to the inflaton \( \sigma \) as

\[ N_H = \int_{\sigma_c}^\sigma d\sigma \frac{\sqrt{6} V_H}{V_H'} \simeq \frac{4 \pi^2}{\lambda^2} (\sigma^2 - \sigma_c^2), \] (3)

The amplitude of metric perturbations at the horizon crossing is given by

\[ k \dot{\Phi}_{k,\text{hybrid}} = \left. \frac{V_H^2}{\sqrt{6} V_H'} \right|_{k=aH} = \left. \sqrt{\frac{32 \pi^2 \mu^2}{3}} \frac{\lambda^2}{\sigma} \right|_{k=aH}, \] (4)

where \( \Phi \) is the gravitational potential, \( k \) is a comoving wave number, \( a \) is the scale factor and \( H \) is the Hubble parameter.

Next we consider the new inflation following the hybrid inflation. The detailed scenario and application can be found in Refs [9, 10, 11]. The superpotential for the new inflation is given by

\[ W_N = v^2 \phi - \frac{g}{n+1} \phi^{n+1}, \] (5)

where \( \phi \) has an \( R \) charge \( 2/(n+1) \) and \( U(1)_R \) symmetry is dynamically broken down to \( Z_{2n} \) at a scale \( v \). Hereafter \( v < \mu \) is assumed. If we identify the real part of \( \phi \) as the inflaton, the potential of \( \varphi \equiv \sqrt{2} \text{Re}[\phi] \) is obtained from the
above superpotential as

\[ V_N = v^4 - \frac{g}{2n/2-1}v^2\varphi^n + \frac{g^2}{2n}\varphi^{2n}, \]

(6)

where we assume the minimal Kähler potential for simplicity. One of the advantages of this hybrid new inflation is that the initial value of \( \varphi \) is dynamically set due to the supergravity effect, and it is given by

\[ \varphi_i = \sqrt{\frac{2}{\lambda}v^3}. \]

(7)

Thus the e-fold number \( N_{\text{new}} \) is calculated as

\[ N_{\text{new}} = \int_{\varphi_i}^{\varphi_f} d\varphi \frac{V_N}{V_N'} \simeq \frac{\lambda^{n/2-1}\mu^{2n-4}v^{-3n+8}}{n(n-2)g}, \]

(8)

where \( \varphi_f \) is the value of \( \varphi \) when the slow-roll conditions break, and given by

\[ \varphi_f = \sqrt{2} \left( \frac{v^2}{n(n-1)g} \right)^{\frac{1}{n-2}}. \]

(9)

Likewise, the amplitude of the metric perturbation at the horizon crossing is

\[ k^{\frac{2}{3}}\Phi_{k_{\text{hor}}^{\text{new}}} = \left. \frac{V_N^2}{\sqrt{6}V_N'} \right|_{k=aH} = \frac{2^{n/2-1}v^4}{\sqrt{6}ng} \varphi^{-n+1} \bigg|_{k=aH}, \]

(10)

and it can be also related to the e-folding number \( N_{\text{new}} \) when evaluated at comoving wave number \( k_b \) corresponding to the horizon scale at the beginning of the new inflation:

\[ k_b^{\frac{2}{3}}\Phi_{k_{\text{hor}}^{\text{new}}} = \frac{n-2}{\sqrt{12}}\lambda^{3/2}v^{-1}N_{\text{new}}. \]

(11)

Furthermore the spectral index, \( n_{\text{new}} \), is calculated as

\[ n_{\text{new}} = 1 - 2 \left( \frac{\varphi}{\varphi_f} \right)^{n-2} \simeq 1, \]

(12)

where \( \varphi \ll \varphi_f \) is assumed in the last equation. Therefore the new inflation adopted here predicts almost scale invariant power spectrum.

We would like to compare the amplitude of the metric perturbation, \( k^{\frac{2}{3}}\Phi_{k_{\text{hor}}^{\text{new}}} \), with \( k^{\frac{2}{3}}\Phi_{k_{\text{hybrid}}^{\text{new}}} \) at comoving wave number \( k_b \). Using Eqs. (11) and (10), we find that the amplitude of the metric perturbation generated during the hybrid inflation is suppressed by a factor

\[ \kappa \equiv \left. \frac{k^{\frac{2}{3}}\Phi_{k_{\text{hor}}^{\text{new}}}}{k^{\frac{2}{3}}\Phi_{k_{\text{hybrid}}^{\text{new}}}} \right|_{k=k_b} = \frac{8\pi^2bg^2\sigma_b\varphi_b^{n-1}}{2^{n/2-1}\lambda^2v^4}, \]

(13)

where we defined

\[ \varphi_b = \varphi_i, \]

\[ \sigma_b = \frac{\lambda}{2n}\sqrt{\frac{2}{3}}\ln \frac{\mu}{v}. \]

(14)

Using the expression for \( N_{\text{new}} \), it can be rewritten as

\[ \kappa = \frac{8\pi\sqrt{\ln \frac{\mu}{v}}}{\sqrt{3(n-2)}\lambda^{3/2}N_{\text{new}}} \sim \frac{v}{\lambda^{3/2}} \ll 1. \]

(15)

Since we require the transitive scale \( k_b^{-1} \) be the present horizon size, \( N_{\text{new}} \) must be close to 50. Note that the hybrid inflation needs not last for a long time, therefore \( \lambda \) does not have to be much smaller than unity. The desired amount
FIG. 1: The predicted CMB angular power spectrum for our scenario (solid line). The WMAP data and the best-fit ΛCDM model (dotted line) are also shown. We have set $k_b = 3 \times 10^{-4}$ Mpc$^{-1}$, $\Omega_b h^2 = 0.024$, $\Omega_m h^2 = 0.14$ and $h = 0.72$, where $\Omega_b$ and $\Omega_m$ are density parameters of baryon and non-relativistic matter, respectively and $h$ the Hubble constant in units of 100 km/sec/Mpc, which are suggested from the WMAP experiment [1].

Thus the power spectrum of the density perturbation is also obtained if we take $n = 4$, $g = 1$, $\lambda = 0.1$, $\mu = 3 \times 10^{-6}$ and $v = 3.8 \times 10^{-7}$, for example. As shown in Refs. [9, 12], the gravitino mass $m_{3/2}$ and reheating temperature $T_{RH}$ are related to the model parameters, and are given as $m_{3/2} \sim 170$GeV and $T_{RH} \sim 6.8$TeV for those exemplified values. To demonstrate how the CMB angular power spectrum is deformed in our scenario, we have performed numerical calculation (see Fig. 1). Thus the power spectrum of the density perturbation has the desired feature: it is strongly suppressed at the scale larger than $k_b^{-1}$, while the amount of the density perturbation at smaller scale is reasonable, reproducing the low quadrupole favored by the WMAP data. Also note that the power spectrum predicted by the inflation model adopted here is almost scale invariant, which is consistent with the WMAP results.

In this paper, we have shown that it is possible to suppress the density perturbation at large scale, adopting the hybrid new inflation model for a definite discussion. This mechanism can be responsible for the anomalously low value of the quadrupole moment of the CMB confirmed by the WMAP. Although we have considered one specific inflation model, the strategy is rather generic. That is to say, in the double inflation model, the density perturbation generated during the preceding one is somehow suppressed. For example, it can be also implemented in the curvaton scenario as follows. We assume such double inflation model that the D-term inflation follows the F-term inflation. The curvaton field is assumed to get the negative Hubble-induced mass term during the F-term inflation, while it acts as a massless field during the following D-term inflation. Thus the density perturbation can have the similar cut-off at large scale, if the D-term inflation lasts for an appropriate period.

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