Thermomagnetic Effects in Vortex Liquid: Transport Entropy Revisited

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Traditionally the Nernst and Ettingshausen effects in the vortex liquid are described in terms of the "transport entropy" of vortices, $S_d$. According to current theories, the main contribution to $S_d$ is originated from the electromagnetic free energy, $F^{em}$, which includes kinetic and magnetic energy of superconducting currents circulating around vortex cores. However, this concept contradicts the London postulate, according to which a supercurrent consists of macroscopic number of particles in a single quantum state and does not transfer any entropy. Here we resolve this contradiction and show that the transport entropy is just ordinary thermodynamic entropy transferred by cores. Only in this form the theory becomes simultaneously consistent with the London postulate and the Onsager principle. The revised theory explains measured temperature dependence $S_d$. The linear increase of $S_d$ at low temperatures is determined by the entropy of electrons in the core, then $S_d$ reaches a maximum at roughly $T_c/2$ and then vanishes due to increase of the background entropy.

In recent years, extensive experimental studies of high-$T_c$ cuprates reveal a significant region of the phase diagram, in which a large Nernst effect and diamagnetism exist without the long-range phase coherence. These important observations are associated with the vortex liquid formation. The central issue of this concept is the Abrikosov’s notion of the quantized flux line, which consists of a normal core with the size of the coherent length, $\xi$, and superconducting currents circulating around the core in the area of the order of the magnetic penetration length, $\lambda$. Counting the free energy of the vortex liquid from the level of superconducting state, the free energy of cores $F^{core}$ may be presented as a sum of the condensation energy and the energy related to gradients of the order parameter. The free energy of superconducting currents, defined as the electromagnetic free energy $F^{em}$, includes the kinetic and magnetic energy of the currents.

While it contradicts thermodynamics, current theories of thermomagnetic vortex transport associate $F^{em}$ with the thermal energy (for a review see Refs. 8 and 14). In other words, they attribute "transport entropy" $S^{em}_d$ to the free energy of supercurrents. According to this concept, the superconducting currents transfer the heat in the Ettingshausen effect and create the net moving force proportional to $-\nabla T$ in the Nernst effect. Moreover, according to the current theories, the term $S^{em}_d$ significantly prevails over the core entropy. In limiting cases of low and high magnetic field, $H-H_{c1} \ll H_{c1}$ and $H-H_{c2} \ll H_{c2}$, the transport entropy $S^{em}_d$ was calculated using the Ginzburg-Landau (GL) formalism. At the intermediate fields, $S^{em}_d$ was obtained in the London model which was developed for extreme type-II superconductors ($\xi \ll \lambda$), where cores are treated as point singularities of the magnetic flux, i.e. $F^{core} = 0$. The London-type models are widely used for numerical studies of thermomagnetic effects in high-$T_c$ cuprates.

Starting from famous works by Thomson (Lord Kelvin), the general theory of thermoelectricity shows that the transport entropy of thermal carriers should coincide with their thermodynamic entropy counted from the background level. At the same time, the supercurrents as any superfluid do not transfer thermodynamic entropy. This is a direct consequence of the London postulate, according to which the supercurrent is formed by macroscopic number of particles moving coherently in a single quantum state. Thus, the electromagnetic free energy $F^{em}$ related to macroscopic degrees of freedom does not consist the entropy term.

In this work we revise the theory of thermomagnetic vortex transport and resolve contradiction between the theory and the London postulate. We show that the entropy $S_d$ is ordinary thermodynamic entropy transferred solely by vortex cores. In agreement with the London concept, the entropy of superconducting currents is zero and they do not transfer the heat in the Ettingshausen effect and do not produce the moving force in the Nernst effect. In this way we reach an agreement with both the Onsager principle and the London postulate.

In the Nernst effect the electric response is induced by a transverse temperature gradient $\nabla T$. If the entropy $S_d$ moves from the area with the temperature $T$ to the area with the temperature $T - \Delta T$, the ratio of the work produced by thermal force, $f_{th} \cdot \Delta r$, to the thermal energy, $TS_d$, is given by the Carnot efficiency $\Delta T/T$. Therefore, the thermal force may be expressed
as $f_{th} = -S_d \nabla T$. The thermal force $f_{th}$ leads to the vortex motion with the velocity $v_T = f_{th}/\eta$, where $\eta$ is the viscosity coefficient. Magnetic flux of vortices $n\phi_0$ ($n$ is the vortex concentration, $\phi_0$ is the flux quanta) generates the Nernst EMF, which is $E_N = n\phi_0 \times v_T/c$. Finally, the voltage signal in the open circuit is given by

$$E_N = \frac{S_d}{c\eta} \nabla T \times B,$$

where the magnetic field $B = n\phi_0$. In the closed circuit the Nernst EMF generates the electric current

$$j^c = -\sigma_f \nabla E = \sigma_f E_N = -\frac{S_d}{c\phi_0} \nabla T \times E_B,$$  \hspace{1cm} (2)

where $\sigma_f = \eta/(\phi_0 B)$ is the flux-flow conductivity, and $E_B$ is the unit vector in the direction of $B$.

In the Ettingshausen effect, the heat current is induced by the transverse electric current $j^c = \sigma_f E$. The current gives rise to the Lorentz force, $f_L = (j^c \times \phi_0)/c$, which leads to the vortex motion with the velocity $v_L = f_L/\eta$. The thermal energy of a vortex is expressed in terms of the transport entropy as $\epsilon_{th} = TS_d$. Then, the heat current, $j^h = n\epsilon_{th}v_L$, may be presented as

$$j^h = \alpha \cdot E \times E_B = \frac{nTS_d}{c\eta} \nabla \cdot \phi_0 = TS_d \nabla \frac{\phi_0}{c\phi_0} \times E \times E_B.$$  \hspace{1cm} (3)

The above description of thermomagnetic effects has been developed by Stephen. Comparing Eqs. 2 and 3 we see that the Stephen formalism is in agreement with the Onsager principle: $\alpha = To$. This agreement is reached by presenting both the thermal force $f_{th}$ and the thermal energy $\epsilon_{th}$ via the transport entropy $S_d$. However, after many years of extensive theoretical and experimental research, the physical sense of $S_d$ and its relation with ordinary entropy are still unclear.

Previous theoretical works associate $S_d$ mainly with $F_{em}$. In his pioneering paper, Stephen considered thermomagnetic vortex transport near $H_{c1}$, where $B = n\phi_0 \approx 0$ and an interaction between vortices can be neglected. In this case, $F_{em}$ per a vortex can be obtained in the GL formalism,

$$F_{em} = \frac{\phi_0}{4\pi} H_{c1} = \frac{\phi_0}{4\pi} |M(H_{c1})| = \left(\frac{\phi_0}{4\pi\lambda}\right)^2 \ln \frac{\lambda}{\xi},$$  \hspace{1cm} (4)

where $M = 4\pi H_{c1}$ is the magnetization. Stephen introduced the transport entropy per vortex as $S_{d}^{St} = -\partial F_{em}/\partial T$ and from Eq. 4 he obtained

$$S_{d}^{St} = -\frac{\phi_0}{4\pi} \frac{\partial H_{c1}}{\partial T} = -\frac{\partial}{\partial T} \left(\frac{\phi_0^2}{16\pi^2\lambda^2} \ln \frac{\lambda}{\xi}\right).$$  \hspace{1cm} (5)

Note, the Stephen’s approach could be easily generalized following Ref. Within GL approach, Dorsey proved that the electromagnetic free energy may be presented as $F_{em} = n\phi_0 |M|$. Thus, in the whole GL region $F_{em} = \phi_0 |M|$, so $S_{d}^{St}$ is given by $-\phi_0 |M|/\partial T$.

Microscopic calculations of the Ettingshausen coefficient have been done only near $H_{c2}$. Caroli and Maki calculated $S_d$ using the TDGL formalism. However, these calculations lead to the transport entropy, which diverges at $T \rightarrow 0$. To get rid of this contradiction, Maki suggested to complement the heat current with “the thermodynamic thermal flux” due to magnetization: $j_{th}^{mag} = -E \times M$. Near $T_c$, in the impure limit, $\ell \ll \xi$ ($\ell$ is the electron mean free path), the corrected result is given by

$$S_{d}^{Mc} = \frac{\phi_0 |M|}{T} = \frac{\phi_0}{4\pi T} \frac{H_{c2} - H}{\beta_A(2\kappa^2 - 1)},$$  \hspace{1cm} (6)

where $\beta_A$ is the geometrical factor. While Eq. 6 is widely used to fit experimental data, it is well-known that, the magnetization correction $j_{th}^{mag}$ leads to violation of the Onsager principle. Finally, Troy and Dorsey reproduced Eq. 4 by associating the thermal energy of a vortex $\epsilon_{th} = TS_d$ with the electromagnetic free energy $F_{em} = \phi_0 |M|$.

As we can see, the results of the phenomenological and microscopic theories are not in agreement: the Stephen’s entropy (Eq. 4) is proportional to the temperature derivative of $M$, while the Maki entropy (Eq. 6) is proportional to $M/T$. It is even more important that the conclusions of the phenomenological theory and the interpretation of the microscopic theory in terms of $F_{em}$ contradicts the London postulate.

Now we will show that the entropy of superconducting currents in fact is zero and, in agreement with the Onsager principle, the superconducting currents do not contribute to the Nernst effect as well. First, we would like to note that in the Stephen theory the nonzero entropy of supercurrents has been obtained due to misinterpretation of thermodynamic relations. If $S = 0$, the free energy, $F_{em} = U_{em} - TS$, is equal to the internal energy, i.e. $U_{em} = F_{em}$ and $U_{em} = F_{em} = \phi_0 |M|$.

Then, the entropy can be again expressed via thermodynamic relation as the temperature derivative of $F_{em}$ (without the energy of magnetic field $H^2/8\pi$) at the constant magnetization i.e.

$$S_{em} = -\left(\frac{\partial F_{em}}{\partial T}\right)_M = -n\phi_0 \left(\frac{\partial M}{\partial T}\right)_M = 0.$$  \hspace{1cm} (7)

Of course, this consideration does not add anything new beyond the London’s postulate. It just shows that, in fact, the temperature derivative in Eq. 5 should be calculated at constant $M$, which results in zero entropy. We also see that, contrary to Ref., the superconducting magnetization currents do not participate in the heat transfer and, therefore, consistent microscopic calculations of the heat current do not require any artificial thermodynamic corrections due to magnetization.

Now we consider the Nernst effect. Let us start with noninteracting vortices near $H_{c1}$. Taking into account the temperature dependence of the vortex energy $U_{em} = F_{em}$ (Eq. 4), the thermal force in the Nernst setup can
be calculated as

\[ f_{th} = -\frac{\partial U_{\phi}^c}{\partial r} = -\frac{\partial}{\partial T} \left( \frac{\phi_0^2}{16\pi^2\lambda^2 \ln \frac{\lambda}{\xi}} \right) \nabla T \]
\[ = -\phi_0 \frac{\partial |M|}{\partial T} \nabla T. \quad (8) \]

As it is shown in Fig. 1 (a), the thermal force is directed from cold to hot area, because the vortex energy \( U_{\phi}(T) \) decreases when \( T \) increases. Note, that our thermal force obtained under the assumption of \( S^e = 0 \) (Eq. 8) and the thermal force introduced by Stephen,8,9,10 have the same value, but opposite directions.

To satisfy the Onsager principle, the thermal force should be balanced by another force. The additional force overlooked in all previous works originates from the magnetization currents in the presence of \( \nabla T \):

\[ j_{\nabla T}^c = c \nabla \times M(T) = c \nabla T \times \frac{\partial M}{\partial T}. \quad (9) \]

As it is shown in Fig. 1 (b), the current \( j_{\nabla T} \) leads to the Lorentz force, which acts on a single vortex in the direction perpendicular to \( \nabla T \),

\[ f_L = -\frac{1}{c} j_{\nabla T} \times \phi_0 = -\left( \phi_0 \cdot \frac{\partial M}{\partial T} \right) \nabla T \]
\[ = \phi_0 \frac{\partial |M|}{\partial T} \nabla T. \quad (10) \]

The Lorentz force \( f_L \) is directed from hot to cold area (Fig. 1 (b)). Thus, Eqs. 8 and 10 show that the total moving force acting on vortex supercurrents is zero.

Employing the Dorsey result,25 the above conclusion can be generalized for interacting vortices (\( 1/\xi^2 < n < 1/\xi^2 \)) and even for overlapping cores (\( n \sim 1/\xi^2 \)). As we discussed above, the TDGL formalism leads to the general expression for the electromagnetic free energy \( F_{\phi}^c = U_{\phi}^c = n U_{\phi} = n\phi_0 |M| \), i.e. vortices can be considered as independent elementary excitations with energy \( U_{\phi}^c = \phi_0 |M| \). Then, Eqs. 8 and 10 expressing the thermal and Lorentz forces through the magnetization are also valid for the entire mixed state and, therefore, the balance of forces is universal. Moreover, the Dorsey result and the above proof are applicable to any pairing.

We have shown that thermomagnetic effects are absent as long as we limited our consideration by \( F_{\phi}^c \). To get nonzero effects, we should take into account contributions of normal electrons, i.e. \( F_{\phi}^c \). The transport entropy is an ordinary thermodynamic entropy counted from a background. If vortex cores do not overlap each other, i.e. \( n\xi^2 < 1 \), the background is "pure" superconducting and, therefore, the transport entropy is determined by the condensation energy, \( H^c_2/8\pi \), in the core area, which is \( \sim \pi \xi^2 \). Thus, the transport entropy per a vortex is

\[ S_{d}^c(T) \simeq -\pi \xi^2 \frac{\partial}{\partial T} \frac{H^c_2(T)}{8\pi}. \quad (11) \]

Note that close to \( H_{c2} \), i.e. in the magnetic field \( H \simeq B \approx \phi_0/\xi^2 \), the background is formed by cores of other vortices and Eq. 11 is inapplicable. Here, the transport entropy \( S_d \) decreases due to overlapping of vortex cores and goes to zero at \( H_{c2} \). Self-consistent description of the narrow region near \( H_{c2} \) requires microscopic consideration.

The exact results for \( S_d \) can be found from the GL formalism in the limit of large \( \kappa \). In this case the condensation energy and the transport entropy are

\[ F_{\phi}^c = a \left( \frac{\phi_0}{4\pi\lambda} \right)^2, \quad \frac{S_{d}^c(T)}{\partial T} = \frac{\partial F_{\phi}^c}{\partial T}. \quad (12) \]

In the original paper by Abrikosov, the constant \( a \) was found to be \( \sim 0.08 \pi^2 \). Abrikosov corrected its value to 0.497. Comparing with Eq. 5 we see that in this limiting case the correct value of \( S_d \) is approximately \( 2\ln(\lambda/\xi) \) times smaller than that predicted by Stephen.25

Now let us analyze the measured temperature dependence of \( S_d \). Detailed numerical analyses26 shows that at moderate temperatures \( 0.2 \leq T/T_c \leq 0.9 \) the radius of the vortex core \( \xi_1 \), defined by fitting the pair potential \( \Delta(r) \) by an expression \( \Delta(r) = \Delta_0 \tanh(r/\xi_1) \), just weakly depends on temperature. Therefore, according to Eq. 11 the temperature dependence of \( S_d \) is mainly determined by the dependence \( H_d(T) \propto 1 - (T/T_c)^2 \), so \( S_d \) is proportional to \( (T/T_c)[1 - (T/T_c)^2] \) and has a smooth maximum at \( T \approx 0.6 T_c \). In Fig. 2 we compare the above conclusions with the temperature dependence of the transport entropy determined by Solomon and Otter27 from the Ettingshausen effect in Pb films. As seen, we get a good agreement with the data. The linear increase of \( S_d \) at low temperatures is determined by the entropy of electrons in the core, then \( S_d \) reaches a maximum and vanishes due to increase of the background entropy. Using parameters InPb alloy26, we evaluate the maximum of \( S_d(T) \) is \( 1.2 \cdot 10^{-7} \text{ erg/cm K} \), while the experiment gives \( 2 \cdot 10^{-7} \text{ erg/cm K} \). Thus, the proposed model provides a simple explanation of the nonmonotonic temperature dependence of \( S_d \) in ordinary superconductors.

Origin of giant thermomagnetic effects in high-\( T_c \) cuprates is a key point for understanding of the nature of the ground state in these materials.1,2,3,4,5 We have shown that thermomagnetic coefficients in the Fermi liquid are always proportional to the square of the particle-hole asymmetry (PHA). It means that the giant effects cannot be explained by the interaction effects in the Fermi liquid, e.g. by superconducting fluctuations.28,29 The explanation requires strong PHA, e.g. the Fermi-surface reconstruction due to the spin density wave gap30 or a non-Fermi liquid state such as the vortex liquid.31,32 The formation of the vortex liquid is associated with the 3D analog of the Kosterlitz–Thouless transition.4,5 While the thermodynamics of this unusual phase is still under debates and various models are proposed, our conclusion that the transport entropy is the thermodynamic entropy transferred by cores is fully applicable to any vortex model.

In summary, we have shown that the superconducting
currents circulating around cores do not contribute to $S_d$, i.e. the supercurrents do not transfer the thermal energy in the Ettingshausen effect and do not produce the moving force proportional to $-\nabla T$ in the Nernst effect. Only this approach is consistent with thermodynamics of irreversible processes (i.e. the Onsager relation and the third law of thermodynamics) and the London postulate. According to the London postulate, any superconducting currents, including superconducting fluctuation currents, do not transfer entropy and thermal energy. It is surprising that all recent papers related to the fluctuation region above the mean-field transition temperature state opposite and insist on the heat transfer by fluctuation magnetization currents (i.e. see Refs.\textsuperscript{28-29} and our comment\textsuperscript{31}). For the vortex liquid, we have shown that the transport entropy of vortices is just ordinary thermodynamic entropy of cores counted from the background entropy. In this way we have connected thermomagnetic transport with thermodynamics. Our theory provides natural explanation of nonmonotonic dependence $S_d(T)$ in ordinary superconductors. It can be easy generalized for various models, which were recently suggested for the vortex liquid in cuprates.

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\textbf{FIG. 1:} Balance of two forces acting on the superconducting currents: (a) $f_{th}$ is the thermal force (Eq. 8), (b) $f_L$ is the Lorentz force due to the magnetization currents (Eq. 10).

\textbf{FIG. 2:} The temperature dependence of the transport entropy: theory (solid line) and data from Ref. 27.
Fig. 1. Balance of two forces acting on the superconducting currents circulating around core:
(a) $f_{th}$ is the thermal force, which originates from the temperature dependence of $F^{em}$,
(b) $f_L$ is the Lorentz force due to the magnetization currents in the presence of $\nabla T$.

Fig. 2. The temperature dependence of the transport entropy $S_d(T)$: theory (solid line) and experimental data from Ref. 27.