Multiparticle Clusters and Intermittent Fluctuations

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Abstract

An approach for understanding the behavior of multiplicity distributions in restricted phase-space intervals derived on the basis of global observables is proposed. We obtain a unifying connection between local multiparticle clusters and the scale-invariant power-law behavior of normalized factorial moments. The model can be used to describe multiparticle processes in terms of a decomposition of the observed intermittent signal into contributions from clusters with varying number of particles.

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1 Introduction

An inverse power-law dependence of normalized factorial moments (NFMs) $F_q(\delta)$ on the size of a phase-space bin $\delta$ (intermittency phenomenon) obtained from multiparticle production in high-energy experiments can serve as a signal for dynamical local fluctuations with self-similar structure \([1]\). For a single bin, this power-law behavior can be written as

$$
F_q(\delta) \equiv \left\langle \frac{n^{[q]}}{n^q} \right\rangle \propto \left( \frac{\Delta}{\delta} \right)^{\phi_q}, \quad n^{[q]} = n(n-1)\ldots(n-q+1),
$$

where $n$ is the number of particles in the phase-space bin of size $\delta$ in which these local multiplicity fluctuations are investigated. The angular brackets imply an averaging over all events in the sample and $\Delta$ is a full phase-space interval defined in rapidity, azimuthal angle, transverse momentum or a combination of these variables. The intermittency exponents $\phi_q$ are related to the so-called anomalous fractal dimensions $d_q$ and the Rényi dimensions $D_q$ as follows

$$
d_q = \frac{\phi_q}{q-1}, \quad D_q = D(1-d_q),
$$

where $D$ is the topological dimension of phase space. If particles are randomly distributed in phase space, then $\phi_q = 0$ and $D_q = D$. For monofractal multiplicity distributions, $d_q = \text{const}$. For a multifractal behavior, $d_q$ is a function of the rank $q$ of the NFMs.

The scale-invariant fluctuations manifest themselves as “spikes” - clustering of many particles in small phase-space bins for single events - and are common to many areas of physics (see [2–4] for reviews).

The factorial-moment method and the concept of intermittency in high-energy physics have been borrowed from the theory of turbulence. There, continuous probability densities are used as the mathematical tool for a theoretical description of fluctuations [1]. The main problem in such an approach is the comparison with experiments having a finite multiplicity of particles for single events. However, if statistical noise has a Poissonian nature, then the method of NFMs follows immediately due to its suppression of the statistical fluctuations caused by the finiteness of the number of particles in restricted phase-space bins. Then, the values of the NFMs are equal to the usual normalized moments obtained from probability densities. That is why, from a theoretical point of view, attempts have been made to understand intermittency also via the analysis of usual moments obtained from models borrowed from hydrodynamics [1,5–7]. However, little attention has so far been devoted to a systematic treatment of dynamical models involving a finite number of particles in phase space.

In this paper, we develop the method for calculation of the NFMs making use of discrete probability distributions, i.e., \textit{a priori} taking into account the finiteness of the
number of particles in a phase-space interval. Such an approach allows us to derive a multiplicity distribution in small phase-space bins from global characteristics of samples with correlations between particles. In this context, this paper may be considered as a continuation of the study of the projection method for the transition in probability distributions from full to restricted phase-space intervals [8, 9].

The method is based on a natural assumption about the existence of multiparticle clusters in phase space and, for a given cluster-size distribution, can lead to an exact solution for the rise of NFM with decreasing $\delta$. Usually, some degree of arbitrariness exists in the definition of clusters. Here, a cluster in an individual event is considered in a general and traditional sense - as a bunch of many particles with a very small extension in the phase space under investigation. Let us note that such concept of spikes is wider than that of clusters, since occurrence of the cluster is caused by dynamical reasons while the spike may have a purely statistical nature.

\section{General Formalism}

First, let us define notations which will be used throughout this paper. The generating function $G(z)$ for a multiplicity distribution $P_n$ and the unnormalized factorial moments $\tilde{F}_q$ are defined as follows

$$G(z) \equiv \sum_{n=0}^{\infty} P_n z^n, \quad \tilde{F}_q \equiv \langle n^{[q]} \rangle \equiv G^{(q)}(z) \bigg|_{z=1}. \quad (3)$$

The normalization condition $\sum_{n=0}^{\infty} P_n = 1$ leads to $G(z = 1) = 1$. Then, the NFM $F_q$ for the multiplicity distribution $P_n$ are given by the relation

$$F_q = \frac{\tilde{F}_q}{F_1^q}. \quad (4)$$

A capital letter $N$ will be used to specify the number of particles in the full phase space of size $\Delta$, and $n$ represents the number of particles in a restricted phase-space interval of size $\delta$, so that $n \leq N$ for $\delta \leq \Delta$.

Let us consider a collision between two particles yielding exactly $N$ final particles for each event in some full phase space of size $\Delta$ with a topological dimension $D$. Let us divide the full phase-space volume into $M^D$ non-overlapping bins of size

$$\delta = \frac{\Delta}{M^D}. \quad (5)$$

In this paper for simplicity we will consider the case of a flat phase-space distribution, i.e., each infinitely small cell of phase space is regarded equally probable. In this case, none of the quantities characterizing fluctuations depend on the position of the
phase-space bin under study. To compare the model predictions with the experimental non-flat distributions, therefore, the transformation of a given non-flat distribution into a flat one should be performed \( [10] \).

The intermittent fluctuations in physical systems may manifest themselves as localized dynamical spikes (multiparticle clusters) in individual events. Let us mention briefly two well-known extreme cases of phase-space distribution:

i) If all \( N \) particles are equally distributed in a given phase-space volume, then the multiplicity distribution for particles in \( \delta \) is a positive-binomial one \( [11] \), with a generating function \( G_N(\delta, z) \) of the form

\[
G_N(\delta, z) = (pz + g)^N, \quad g = 1 - p, \tag{6}
\]

\[
p = M^{-D} = \frac{\delta}{\Delta}. \tag{7}
\]

From (4), we have the following form of the NFMs

\[
F_q = \frac{N^{[q]}}{N^q}, \quad q = 1, 2, \ldots, N. \tag{8}
\]

The extreme case thus gives \( \phi_q = d_q = 0, D_q = D \). This means that all spikes observed in single events have a purely statistical nature. Note that \( F_q < 1 \) for all \( q \). However, for \( N \to \infty \), the distribution (6) tends to a Poissonian one with \( F_q = 1 \).

ii) Another (unlikely) situation occurs if all \( N \) particles group in one single point-like cluster for all possible events. Then the generating function is \( [9] \)

\[
G_N(\delta, z) = pz^N + g, \quad g = 1 - p, \tag{9}
\]

with \( p \) of the form (7). This distribution law emphasizes that, for a given bin size \( \delta \), only two possibilities can occur: either all \( N \) particles are found in the bin or none. For the NFMs, one gets

\[
F_q = \frac{N^{[q]}}{N^q} \left( \frac{\Delta}{\delta} \right)^{q-1}, \quad q = 1, 2, \ldots, N. \tag{10}
\]

This is a maximum possible intermittency (\( \phi_q = d_q = 1, D_q = 0 \)). From a geometric point of view, this case corresponds to a point-like object having topological dimension zero.

The two examples presented above lead to the two main theoretical questions: how do such dynamical spikes lead to the actual form of intermittency \( [11] \) with a small (but non-zero) exponent \( \phi_q \) and how does one construct the multiplicity distribution for this case. The aim of the present analysis is to derive a probabilistic scheme of the realistic intermediate situation with \( 0 < \phi_q < q - 1 \) in \( [11] \), using known global characteristics of the sample.
In our approach we restrict ourselves to the following cases:

1) All single particles (monomers) and multiparticle clusters are placed in phase space independently, without any correlations. Within the analytical model to be discussed below, all possible correlations are those between particles inside each cluster.

2) We shall consider the simplified case of treating the clusters as “point-like objects”. In fact, we suppose that the probability that the cluster is emitted on the boundary of the restricted phase-space interval is very small, i.e., \( \delta \gg \delta_{\text{min}} \sim \Omega \), where \( \Omega \) is the cluster size. Since intermittency is defined via the scale-invariant ratio \( \Delta / \delta = M^D \), we can rewrite this condition as

\[
\frac{\Delta}{\Omega} \sim \frac{\Delta}{\delta_{\text{min}}} \gg M^D.
\]

Hence, for a given \( M \), a cluster can be considered as a point-like object if 1) the size \( \Omega \) of the cluster is very small; 2) the total phase-space volume \( \Delta \) of multiparticle production is large. For real experiments, in fact, condition (11) means that we consider the case when the number of phase-space bins \( M \) is not very large.

Consider the following structure of a fluctuation. Let us assume that not only single particles but also point-like multiparticle clusters can exist in single events. Let each of them contains exactly \( m \) particles (\( m \)-particle clusters), where \( m = 2, 3, 4 \ldots \). Then, only the two cases are possible: a cluster is inside \( \delta \) or outside of this interval. We assume that the cluster phase-space distribution is flat. If only multiparticle clusters with a fixed \( m \) exist, then the multiplicity distribution to have \( n \) particles (belonging to clusters) inside \( \delta \) has the binomial-like form:

\[
P_n(m, N_m) = \begin{cases} 0, & \text{if } n \neq m s, s = 1, 2 \ldots \\ C_n^{N_m} p_m^i g^{N_m - i}, & \text{otherwise,} \end{cases}
\]

where \( n = 0, 1, 2 \ldots mN_m, p \) is given by (4) and \( N_m \) is the total number of \( m \)-particle clusters in full phase space. \( C_n^{N_m} \) are the binomial coefficients. The \( N_m \) is a constant for the sample of events and will be weighted over all possible samples below. The generating function for (12) is

\[
G_{N_m}^{(m)}(\delta, z) = \sum_{i=0}^{N_m} z^i C_N^{i} p_m^i g^{N_m - i} = (pz^m + g)^{N_m}.
\]

As mentioned in the introduction, we will consider the case of no correlations between different clusters and monomers. Then, the generating function for the probability of having \( n \) particles in \( \delta \), if \( m \)-particle clusters (\( m = 2, 3 \ldots \)) and monomers (\( m = 1 \)) exist, is the product of the generating functions (13)

\[
G_R(\delta, z, N) = \prod_{m=1}^{N} G_{N_m}^{(m)}(\delta, z),
\]

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where $G_{N_1}^{(1)}$ is the generating function \[3\] for uncorrelated monomers. Note that the generating function \[4\] satisfies the normalization condition $G_R(\delta, z = 1, N) = 1$ by construction. The set $R \equiv \{N_m\} \equiv (N_1, N_2, \ldots, N_N)$ represents a cluster configuration (cluster-size distribution) which satisfies the following constraint

$$N = \sum_{m=1}^{N} m \cdot N_m. \quad (15)$$

Note that the extreme case $N_N = 1$, when all $N$ particles form a single cluster-monomer, is possible only if $N_1 = N_2 = \ldots N_{N-1} = 0$.

A real situation is expected to be more complicated, because, for a fixed $N$, the events can differ one from another by the cluster configuration $R$. In this case, $R$ is a random variable. Then, one averages the $G_N(\delta, z)$ over all possible configurations, i.e.,

$$\bar{G}(\delta, z, N) = \sum_{R=1}^{\infty} P_{N}^{\text{conf}} G_R(\delta, z, N) = \langle G_R(\delta, z, N) \rangle_R, \quad (16)$$

where $P_{N}^{\text{conf}}$ is a probability distribution depending on $N$ and controlling the relative weight of the multiplicity distributions for different cluster configurations.

Moreover, for actual multihadronic systems produced in high-energy experiments, one needs to average over all events with different $N$. Then,

$$G(\delta, z) = \sum_{N=0}^{\infty} P_N \bar{G}(\delta, z, N) = \langle G_R(\delta, z, N) \rangle_{R,N}. \quad (17)$$

Here, $P_N$ is the known multiplicity distribution for full phase space.

Thus, within the framework of the considered model, we have found the multiplicity distribution in restricted bins from the following global observables containing dynamical information on the fluctuations:

1) The multiplicity distribution $P_{N}^{\text{conf}}$ for the cluster configurations $R$.
2) The multiplicity distribution $P_N$ for particles in the full phase space $\Delta$.

Of course, both quantities have to be defined on the basis of physical knowledge on the dynamical structure of the particle clustering.

According to \[3\] and \[4\], the NFMs for \[17\] have the following form

$$F_q(\delta) = \frac{\langle \tilde{F}_q(\delta) \rangle_{R,N}}{\langle \tilde{F}_1(\delta) \rangle_{R,N}^q}. \quad (18)$$

The quantities inside the angular brackets define (usual) factorial moments for a fixed cluster configuration $R$ and a fixed total number of particles in the full phase space:

$$\tilde{F}_q(\delta) = \sum_{q_1=0}^{q} \sum_{q_2=0}^{q_1} \ldots \sum_{q_{N-1}=0}^{q_{N-2}} C_{q_1}^{q_1} C_{q_2}^{q_2} \ldots C_{q_{N-1}}^{q_{N-2-N_1}} \tilde{F}_1^{(1)}_{q-q_1} \tilde{F}_2^{(2)}_{q_1-q_2} \ldots \tilde{F}_N^{(N-1)}_{q_{N-2}-q_{N-1}}. \quad (19)$$
Here, we have introduced the following definitions of the moments obtained from 
\( G_{N_m}^{(m)}(\delta, z) \):

\[
\tilde{F}_q^{(1)} = N_1^{[q]} p^q, \quad (20)
\]

\[
\tilde{F}_q^{(m)} = \sum_{i=q}^{N_m} (m i)^{[q]} C_{N_m}^{[i]} p^i g^{N_m-i}, \quad m > 1. \quad (21)
\]

It is convenient to introduce, instead of \( N_m \), the probability \( W_m \) of any particle (chosen at random) to belong to an \( m \)-particle cluster. This probability can be written as

\[
W_m = m \frac{N_m}{N}, \quad \sum_{m=1}^{N} W_m(N) = 1. \quad (22)
\]

The set of \( W_m \) for \( m = 1, \ldots, N \) forms the probability distribution which we shall call the cluster configuration distribution. Hence, \( R \) used above represents a particular form of the cluster configuration distribution \( W_m \).

In fact, the quantities \( W_m \) are identical to those used in the theory of percolation clusters (see [12] for review). There, the cluster is defined as a group of occupied lattice sites connected by nearest-neighbor distances. Then \( N_m/N \) can be treated as the average number (per lattice site) of \( m \)-particle clusters and \( W_m \) defined from (22) is the probability of any lattice site to belong to a \( m \)-particle cluster.

### 3 Examples

Though the theoretical attempt to describe a sample in terms of discrete distributions is still in a very early stage, one may already interested in its possible observable consequences.

To illustrate the intermittent properties of the model, let us consider a specific system in which the particle multiplicity \( N \) is fixed for all events and the \( P_{R}^{\text{conf}} \) has a sharp maximum near \( R \approx \bar{R} \), i.e.,

\[
P_{R}^{\text{conf}} \approx \delta_{R, \bar{R}} = \begin{cases} 
1, & R = \bar{R}, \\
0, & R \neq \bar{R}. 
\end{cases} \quad (23)
\]

Then,

\[
\bar{G}(\delta, z, N) \approx \prod_{m=1}^{N} G_{N_m}^{(m)}(\delta, z). \quad (24)
\]

where \( N_m \) is the most probable cluster configuration. For a given \( N \), expression (24) permits us to calculate the NFM\( s if we know the form of \( \bar{N}_m \) (or the corresponding \( \bar{W}_m \)). Then, from (24), we have

\[
F_q(\delta) = \frac{\tilde{F}_q(\delta)}{\tilde{F}_1^{(1)}(\delta)}, \quad (25)
\]

7
where \( \tilde{F}_q(\delta) \) is given by (19), if we substitute \( \bar{N}_m \) instead of \( N_m \). Note that (25) does not depend on the global particle density \( N/\Delta \), since, for any value of \( N \), all dependence of the NFMs on the bin size is determined by the ratio \( \Delta/\delta = M^D \), but not by \( \Delta \) itself.

To be specific, we present the results of our calculation of the NFMs for a few particular cases:

1) Contribution from two-particle clusters

To study this case, we use the following simple cluster configuration

\[
\bar{W}_1 = 0.4, \quad \bar{W}_2 = 0.6.
\]

(26)

In Fig. 1, we present the results of our calculation for \( F_2 \) to \( F_5 \) with \( N = 50 \) and \( D = 1 \) (closed symbols). For this value of \( N \), according to (22), configuration (26) approximately corresponds to \( N_1 = 20, N_2 = 15 \).

From Fig. 1 we can see that the NFMs have a tendency to grow linearly with increasing of the number of bins \( M \). The increase can be fitted by the power-law (1) with a intermittency exponent \( \phi_q \) and a coefficient of proportionality \( a_q \),

\[
\ln F_q = \phi_q \ln M + \ln a_q, \quad \phi_q > 0.
\]

(27)

Since the main aim of this section is only to illustrate a real possibility of the intermittent rise of the NFMs with increasing \( M \), we do not pursue the purpose to fit the results obtained by (27) for this (hypothetical) cluster configuration.

2) Contribution from three-particle clusters

The open symbols in Fig. 1 represent the behavior of \( F_2 \) to \( F_5 \) for a sample with three-particle clusters (D=1). Here,

\[
\bar{W}_1 = 0.4, \quad \bar{W}_3 = 0.6.
\]

(28)

This corresponds to \( N_1 = 20, N_3 = 10 \) (for \( N = 50 \)). The sample with three-particle clusters has stronger intermittent behavior than that obtained using (26).

4 Bunching-Parameter Method

In this section, we argue and numerically show that the bunching parameters \([13, 14]\) are more sensitive to the features of a cluster configuration than the NFMs discussed above. In fact, the result of this subsection complements the statement made in \([14]\), where it has been shown that the NFMs are insensitive to the structure of fluctuations, i.e. rather different behaviors of the underlying local multiplicity distribution \( P_n(\delta) \) with decreasing \( \delta \) can have the same intermittent trend of the NFMs.
The definition of the bunching parameters \( \eta_q \) is [13, 14]

\[
\eta_q = \frac{q}{q - 1} \frac{P_q(\delta)}{P_{q-2}(\delta)},
\]

(29)

where \( P_n(\delta) \) is the probability of having \( n \) particles inside a restricted phase-space bin \( \delta \). For example, for a Poisson distribution, one has \( \eta_q(\delta) = F_q(\delta) = 1 \), so that qth order BP measures the deviation of the local multiplicity distribution from the Poisson. In general, independent particle production leads to \( \eta_q(\delta) \neq 1 \) for \( \delta \to 0 \). In this case, however, the bunching parameters are \( \delta \)-independent constants. For example, if all spikes are purely statistical for events with a fixed finite multiplicity \( N \), i.e., when the cluster configuration reduces to the trivial case

\[
\bar{W}_m = \delta_{m,1},
\]

(30)

then (24) becomes a positive binomial distribution and the corresponding BPs have the following \( \delta \)-independent form

\[
\eta_q = \frac{q - 1 - N}{q - 2 - N}.
\]

(31)

The interest in the use of BPs to extract information on the cluster configuration lies in the fact that the BP of rank \( q \) is sensitive only to multiparticle clusters with \( m \leq q \) particles. That is, a given BP acts as a filter for clusters having a small number of particles. This follows directly from the definition (29). On the contrary, the NFM of rank \( q \) is sensitive to spikes with \( n \geq q \) particles and acts as a filter for clusters with large number of particles. This means that the bunching parameters have a complementary property which is very important to obtain a refined insight into various multiparticle systems with intermittent behavior.

To demonstrate this point, we shall use (24) and the relation \( P_q(\delta) = G^{(q)}(z) \big|_{z=0} / q! \). Note that for this calculation we can use the expression (19), substituting \( P_q(m, N_m) \) (see (12)) multiplied by the factor \( q! \), instead of the \( \tilde{F}_q^{(m)} \).

In Fig. 2 (a),(b), we present the behavior of the NFMs for \( \bar{W}_1 = 0.4, \bar{W}_3 = 0.6 \) (only monomers and three-particle clusters exist) and \( \bar{W}_1 = 0.22, \bar{W}_2 = 0.4, \bar{W}_3 = 0.3, \bar{W}_4 = 0.08 \). The latter configuration represents a sample with two-, three- and four-particle clusters. The behavior of the NFMs is the same for these rather different samples, not only qualitatively, but also quantitatively (actually, these configurations were chosen for an illustrative purpose, because they exhibit the same behavior of the NFMs). In contrast to the NFMs, the behavior of the BPs is different for the same cluster configurations (see Fig. 3 (a) (b)). This means that the bunching parameters provide a simple and effective method to analyze models and compare them with experimental data.

This point has already been illustrated in [14] using a Monte-Carlo model, where we have shown that systems with a similar trend of the NFMs exhibit different behavior when one studies them with the help of bunching parameters.
5 Conclusion

We have presented a statistical analysis of multiparticle fluctuations in ever smaller phase-space bins for different dimensions using the characteristics of primarily observable - multiparticle clusters. This new theoretical method links the global characteristics (a cluster configuration distribution and a multiplicity distribution of particles in the full phase space) of the sample to local multiplicity fluctuations of a multiparticle process. As we have seen, the theoretical approach developed in its simplest form (no correlations between clusters) reveals many promising features. The model can lead to an intermittent rise of the NFM$s with decreasing $\delta$, even for the simplest special cases of the global characteristics taken for illustrative purposes. In our model, we “a priori” take into account the clusters, without any physical examination of the clustering phenomenon itself, which, up to now, is a standard topic in soft hadron physics where the application of QCD is very restricted.

One of the reasons for the approach presented in this paper is to give an exact method to derive the intermittent behavior in various cluster models. In particular, the general method developed here can be used to study intermittent behavior of the statistical models \[15\] of a fragmentation process in usual phase-space variables, rather then for mass distributions.

There is an interesting point connected with our approach. In fact, our prescription is to decompose the local fluctuations into clusters with varying number of particles. Apparently this situation is closely analogous to that in which the intermittent signal is decomposed into contributions from clusters living on specific scales (the so-called method of orthogonal wavelet transform used in high-energy physics \[16\]). Note, however, that the connection between that method and our approach is not trivial, since the conceptual distinction between these strategies is mathematically clear-cut.

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Figure 1: NFM as a function of the number of bins $M$. Solid lines represent the configuration $\bar{W}_1 = 0.4, \bar{W}_2 = 0.6$, dashed lines correspond to $\bar{W}_1 = 0.4, \bar{W}_3 = 0.6$. For both cases $N = 50$. 
Figure 2: Dependence of $\ln F_q(M)$ ($q = 2, \ldots, 5$) on $\ln M$. (a): $\bar{W}_1 = 0.4, \bar{W}_3 = 0.6$; (b): $\bar{W}_1 = 0.22, \bar{W}_2 = 0.4, \bar{W}_3 = 0.3, \bar{W}_4 = 0.08$. Here, we set $N = 50$ for both cases.

Figure 3: Dependence of $\ln \eta_q(M)$ ($q = 2, \ldots, 5$) on $\ln M$ for the same cluster configurations as in Fig. 2.