The analysis of Archimedean spiral slots using the theory of characteristic modes is presented in this article. Purely magnetic characteristic modes are used to study the behavior of a spiral slot etched in an infinite ground plane. The effect of the spiral geometry parameters (slot width, separation between spiral slot arms, number of spiral turns, and inner spiral spacing) on the characteristic modes are investigated using the theory of characteristic modes. Based on the results of the characteristic mode analysis, guidelines are presented for selecting the appropriate slot dimensions for realizing a compact spiral slot assembly (miniaturization), multimode behavior and for improving the bandwidth of the purely magnetic characteristic modes. Additionally, analytical equations are presented for accurately predicting the resonant frequencies of the purely magnetic characteristic mode of the spiral slot antenna.

**INDEX TERMS** Characteristic modes, magnetic modes, modal analysis, planar antenna, slot antenna, spiral antenna, Archimedean spiral antenna, antenna design.

**I. INTRODUCTION**

Planar spirals were first designed to achieve frequency independent, circularly polarized antennas with bidirectional radiation [1], [2]. Since then, they have been used in a wide range of applications. Planar metallic spirals have been used as radiating elements in phased arrays [3] to achieve broadband behavior, in satellite communications [4] and for achieving radiation pattern reconfigurability [5]. Spirals in the form of narrow slots have been used in planar antennas to achieve miniaturization [6]. Recently, spiral slots have been used in RFID applications to achieve the impedance characteristics required for conjugate matching with RFID chipsets [7] and for designing placement-insensitive tag antennas [8].

The radiating behavior of planar metallic spirals has been widely studied using fullwave numerical analysis techniques [9], [10]. While fullwave numerical analyses help establish the general radiating properties of spirals, such analyses are often presented for a spiral antenna assembly containing two or more arms and a feeding mechanism included. It is thus difficult to dissociate the effects of the feedline from the behavior of the metallic spiral antenna, especially at higher frequencies. Additionally, the design guidelines that have been proposed using fullwave numerical techniques have often been based upon the knowledge of the total current distribution of the antenna and for a narrow range of frequencies. Lumped circuit models [6] and transmission line models [11] have also been used to study the behavior of wire and slot spirals. Most of available literature on full wave analysis of slots has been focused on the analysis of slots in finite ground planes in conjunction with a feeding mechanism. In such cases, the radiation from the structure is a function of the currents around the slot due to the feed and also the currents on the finite ground plane. Thus, the true behavior of the slot is masked both by the feeding mechanism and the finite ground plane. While fullwave analysis techniques help predict the first few resonances of the spiral, they do not fully capture the modal behavior of the spiral. In addition, fullwave analyses cannot be used to accurately predict the behavior of the slot modes at higher frequencies.

The theory of characteristic modes can be used as an alternative technique to analyze the radiating behavior and resonances of arbitrarily shaped structures [12]. Characteristic mode theory can be used to study both metallic objects.
Due to the physical insight provided by the technique regarding the resonances and radiating behavior of arbitrarily shaped structures, characteristic mode analysis has been widely used in the design of electrically small antennas, planar metallic antennas, and vehicular antennas among others. Characteristic modes can also be used to design the feed mechanism and to evaluate how well a given mode is excited. In recent years, characteristic mode analysis has been used to investigate the modal behavior of Archimedean spiral antennas in an infinite ground plane. The observations presented in this article can be used as preliminary guidelines for selecting the design variables and to inform the design process of a single-arm Archimedean spiral slot in an infinite ground plane. The observations presented in this article can be used as preliminary guidelines for selecting the design variables and to inform the design process of a single-arm Archimedean spiral slot or a metallic wire spiral counterpart. In addition, the analysis can be used as a basis to further explain the modal behavior of multi-arm spirals and combinations of spirals for a wide range of frequencies.

II. DUALITY BETWEEN ELECTRIC AND MAGNETIC CHARACTERISTIC MODES

The characteristic modes of PEC structures can be found by solving the eigenvalue problem using the impedance operator $Z$ to solve for characteristic electric current modes $J_n$ as shown in (1).

$$X(\tilde{J}_n) = \lambda_n R(\tilde{J}_n)$$  \hspace{1cm} (1)

where $\lambda_n$ are the real electric eigenvalues, $\tilde{J}_n$ are the characteristic electric currents and $R$ and $X$ are the real and imaginary parts of the impedance operator $Z$ defined as:

$$Z = R + jX$$  \hspace{1cm} (2)

For aperture problems, the characteristic modes can be solved using Babinet’s principle and using duality of field quantities. The characteristic mode solutions for apertures can be obtained by using the admittance operator $Y$ and solving the eigenvalue problem given in (3) to get magnetic characteristic current modes $M_n$.

$$B(\tilde{M}_n) = b_n G(\tilde{M}_n)$$  \hspace{1cm} (3)

where $b_n$ are the real magnetic eigenvalues, $\tilde{M}_n$ are the characteristic magnetic currents and $G$ and $B$ are the real and imaginary parts of the admittance operator $Y$ defined as:

$$Y = G + jB$$  \hspace{1cm} (4)

A given characteristic mode is resonant when its eigenvalue is equal to zero ($\lambda_n = 0$ for $\tilde{J}_n$ and $b_n = 0$ for $\tilde{M}_n$). When a given mode is resonant, the corresponding modal significance (5) is equal to one and the mode characteristic angle (6) is equal to $180^\circ$.

$$MS = \left| \frac{1}{1 + jv_n} \right|; v_n = \{\lambda_n, b_n\}$$  \hspace{1cm} (5)

$$CA = \alpha_n = 180^\circ - \tan^{-1}(v_n); v_n = \{\lambda_n, b_n\}$$  \hspace{1cm} (6)

A given characteristic mode is resonant ($\lambda_n = 0$) at a given frequency after which the mode eigenvalue is a small positive value. The slope of the characteristic angle trace of a given mode determines the bandwidth of the mode. The steeper the slope, the narrower the bandwidth of the mode. In order to understand the behavior of a given characteristic
mode around the resonant frequency, we can define modal bandwidth as shown in (7).

$$BW = \frac{f_H - f_L}{f_r}$$  \hspace{1cm} (7)

where $f_r$ is the resonant frequency ($MS = 1$) and $f_H$ and $f_L$ are the frequencies where the modal significance is equal to 0.8 for a given mode. The effect of the spiral geometry variations on the resonant frequencies and modal bandwidths will be presented in the following section.

![Archimedean spiral geometry](image)

**FIGURE 1.** Archimedean spiral geometry (wire spiral of negligible thickness).

A given spiral geometry is defined by the distance of the first spiral arm from its origin ($r$), the width of the spiral arm ($s$), the separation between the spiral arms ($m$) and the number of turns ($t$) as shown in Fig. 1. The Archimedean spiral geometry is shown in Fig. 1 in polar coordinates ($R, \theta$) can be defined as :

$$R = r + b\theta$$  \hspace{1cm} (8)

where $b$ is the rate of increase of the spiral given by $b = \frac{\pi \cdot r}{2 \pi t}$.

For planar spiral geometries (see Fig.2a), the spiral geometry is made up of two spiral traces, inner and outer, having equal growth rates ($b$). The total arc lengths of the inner and outer spiral traces can be calculated as shown in (9) and (10).

$$L_s^{in} = \int_0^{2\pi t} \sqrt{(r + b\theta)^2 + b^2} \, d\theta$$  \hspace{1cm} (9)

$$L_s^{out} = \int_0^{2\pi t} \sqrt{(r + s + b\theta)^2 + b^2} \, d\theta$$  \hspace{1cm} (10)

For thin planar metallic spirals (Fig.2a) and planar spiral slots having narrow slot widths (Fig.2b), the effective length of the spiral assembly is determined by its perimeter as given by (12).

$$L_s = P / 2$$  \hspace{1cm} (11)

where the perimeter of the spiral assembly ($P$) can be calculated as:

$$P = L_s^{out} + L_s^{in} + s$$  \hspace{1cm} (12)

In order to establish the duality between magnetic and electric characteristic modes of Archimedean spiral slot antennas, different spiral geometries are considered. For a single turn metallic Archimedean spiral shown in Fig. 2a, the dual slot has the same geometry etched on an infinite ground plane (see Fig. 2b where $L = W = \infty$). For a given pair of electric and magnetic characteristic modes ($J_n, M_n$) for any $n$ to be duals of each other, the pair must have the same resonant frequency, similar current distributions and radiation patterns. For simplicity, the vector characteristic mode currents ($\vec{J}_n, \vec{M}_n$) will be referred to by their modal currents ($J_n, M_n$) for the remainder of this article. The modal currents ($J_n, M_n$) are obtained using the characteristic mode solver in FEKO [26]. For brevity, the results of the first four resonant characteristic modes are presented in this article.

**FIGURE 2.** Geometry of Archimedean spirals: metallic and equivalent slot aperture in an infinite ground plane (with $L = W = \infty$).
TABLE 1. Resonant frequencies for the dual characteristic modes of a single-arm single-turn Archimedean spiral.

| Modal resonant frequencies (GHz) | Metallic spiral | Spiral slot |
|---------------------------------|----------------|------------|
| $J_1$                           | 8.8            | 8.76       |
| $J_2$                           | 20.5           | 20.34      |
| $J_3$                           | 29.8           | 29.67      |
| $J_4$                           | 40             | 39.69      |

omitted in Fig. 4 for brevity. It can be observed that the characteristic electric currents of the planar metallic spiral and the characteristic magnetic currents of the spiral aperture on an infinite ground plane have similar spatial distributions. In additions, the far-field radiation patterns of the characteristic mode pairs ($J_n$, $M_n$ for any $n$) are identical (with $E$ and $H$ fields interchanged, as expected). These results establish the duality between the spiral slot aperture and its complimentary form for a single-arm single-turn spiral. Duality between purely electric and purely magnetic modes also exists for an Archimedean spiral geometry with two turns as shown by their characteristic angle traces in Fig. 5. For brevity, only the characteristic angle traces are shown in Fig. 5 but it can be shown that far-field radiation patterns and current distributions are similar for a given characteristic mode pair for the two-turn spiral geometries. Thus, it can be concluded that the characteristic electric currents ($J_n$) of a planar metallic spiral are dual to the characteristic magnetic currents ($M_n$) of an Archimedean spiral slot (having the same aperture as that of the metallic spiral) in an infinite ground plane. Additionally, the duality of modal far-field quantities as outlined in [14] is also established for spiral apertures and their planar metallic counterparts. Due to this duality between purely electric and purely magnetic characteristic modes, the analysis of the spiral slot aperture presented in this article can also be used to predict the behavior of the planar metallic counterparts by using Babinet’s principle [23].

The resonant frequencies of the electric and magnetic characteristic modes depend upon the Archimedean spiral geometry which is discussed in the following section.

III. PARAMETRIC STUDY OF THE BEHAVIOR OF ARCHIMEDEAN SPIRAL SLOTS

The variation of the eigenvalues with frequency of any given magnetic characteristic mode of an Archimedean spiral slot depends upon the antenna’s geometry. The changes in spiral geometry parameters such as the number of turns ($t$), distance of the first spiral arm from its origin ($r$), slot width ($s$) and the separation between the spiral slot arms ($m$) can change the overall arc length of the spiral geometry. The arc length of the spirals can alter the the modal eigenvalues and hence the resonant frequencies of the purely magnetic modes. In order to establish the effect of the spiral geometry on the eigenvalues, a single-arm spiral slot with one turn was considered as the basis for the parametric studies. The dimensions for the spiral are given in Table 2. Parametric
studies were carried out using FEKO [24] by varying the spiral geometry parameters and changes in resonant frequencies and modal bandwidths (7) were observed for the first four purely magnetic characteristic modes.

### TABLE 2. Geometry details for Archimedean spiral slot used in the parametric studies.

| Parameter                        | Variable | Value (mm) |
|----------------------------------|----------|------------|
| Spiral arm width                 | s        | 1          |
| Spiral distance from its origin  | r        | 1          |
| Separation between spiral arms   | m        | 1          |

1) Effect of turns

The number of turns for a given spiral geometry determines the total arc length of the spiral and hence determines the resonant frequencies of the purely magnetic modes ($M_n$) of the spiral slot. For this study, a single-arm spiral slot as shown in Fig. 2b was considered. In order to understand the effect of the number of turns (t) on the modal resonant frequencies and bandwidths, the number of turns were varied while other spiral geometry parameters listed in Table 2 were held constant. The current distributions for the first four magnetic characteristic modes for a single-arm spiral are shown in Fig. 2b. Similarly current distributions for the spiral assembly with two turns ($t = 2$) and three turns ($t = 3$) turns are shown in Fig. 6.

Fig. 7 shows the variation of the resonant frequencies of the first four purely magnetic modes with the number of turns of the different spiral geometries used in the parametric studies. Based on the results of the parametric studies, for a single-arm spiral with one turn, the first resonant mode $M_1$ is extremely narrowband (the slope of the characteristic angle is very steep) as shown in Fig. 3. This narrowband mode is resonant when the total perimeter ($P$) of the spiral is approximately one wavelength. This corresponds to the arc length of the spiral being approximately half-wavelength as shown in (13).

\[
f_{1}^{t=1} \approx \frac{0.85 \cdot c}{P} \approx \frac{0.85 \cdot c}{2L_s} \tag{13}
\]

where $c$ is the speed of light in vacuum, $P$ is the perimeter of the slot assembly and $L_s = P/2$ is the effective length of the spiral slot.

For a single turn spiral slot, the higher order modes ($M_n$ for $n > 1$) have wider bandwidths as shown in Fig. 3. From Fig. 7 it can be observed that such wideband modes are resonant when the slot length ($L_s$) is an integer multiple of half-wavelength. Thus for higher order modes, the resonant frequency can be approximated as:

\[
f_{n>1}^{t=1} = \frac{nc}{2L_s} \tag{14}
\]

Thus, for a single turn spiral slot geometry, the resonant frequencies of the resonant modes ($M_n$) can be approximated as:

\[
f_n^t = \begin{cases} 
\frac{0.85c}{2L_s} & \text{for } n = 1 \\
\frac{nc}{2L_s} & \text{for } n > 1 
\end{cases} \tag{15}
\]

When the number of turns of a given spiral geometry is increased, the overall length of the spiral increases and the resonant frequencies of all the magnetic modes decrease with the increase in the number of turns of the spiral slot assembly as shown in Fig. 7. For a given spiral geometry, the resonant
frequency of a given mode $M_n$ is the highest for the single turn geometry ($t = 1$) and decreases significantly with the increase in the number of spiral turns as shown in Fig. 7. The resonant frequencies of a given spiral assembly with a given number of turns $t$ can thus be calculated as:

$$f_{tn} = \frac{f_{1n}}{t^{2/2}}$$

where $t$ is the number of turns of a given spiral slot geometry, $f_{1n}$ is the resonant frequency of mode $M_n$ for a single turn spiral defined in (15).

Fig. 8 shows the variation in the separation between the resonant frequencies with the number of turns for the first four purely magnetic modes of the spiral geometry. It can be observed that with the increase in the number of turns of the spiral, the distance between the resonant frequencies decreases. As the number of turns increases ($t > 2$), the resonant frequencies of the modes are spaced equally apart.

In addition to the variation in resonant frequencies, the number of turns of a given spiral assembly affects the bandwidth of the resonant modes. Fig. 9 shows the fractional bandwidths (FBW) of the first four resonant modes evaluated at their respective resonant frequencies. It can be observed that for a single-turn spiral slot ($t = 1$), the lowest order resonant mode ($M_1$) has the narrowest fractional bandwidth (FBW < 5%) while the bandwidths for other resonant modes ($M_n$ for $n > 1$) have wider bandwidths (fractional bandwidths (FBW > 25%). When all other slot parameters ($r$, $s$ and $m$) are held constant and the number of of turns is increased, the fractional bandwidths for modes $M_1$ and $M_2$ fall below 5% while the higher order modes $M_3$ and $M_4$ are relatively wideband with fractional bandwidths greater than 10%. As the number of turns is increased further, the bandwidths of all the resonant modes decreases significantly and all resonant modes being extremely narrowband (FBW < 5%) as shown in Fig. 9.

2) Effect of $r$

For this study, a single-arm spiral slot as shown in Fig. 2b was considered. In order to understand the effect of increasing the spacing in the inner region of the spiral assembly on the modal resonant frequencies and bandwidths, the distance of the first spiral arm from the origin ($r$) was varied while other spiral geometry parameters listed in Table 2 were kept constant. The effect of the variation in ($r$) was studied for single-arm spiral slot geometries with different number of turns: $t = 1$, $t = 2$, and $t = 3$.

Fig. 10 and Fig. 11 show the changes in the spiral geometries when the distance from the first spiral arm from the origin ($r$) is varied. For both cases, $r = 1$ mm represents...
the first spiral slot arm from its origin \((r)\) for the characteristic modes for three spiral slot geometries are shown. As shown in Fig. 12, for a given spiral assembly with two turns \((t = 2)\), the increase in the distance of the first spiral arm from the origin \((r)\) affects the spacing between the modal resonant frequencies \((\Delta(M_n, M_{n+1})\) for a given \(n)\) differently. With the increase in \(r\), the modes become equally spaced as shown in Fig. 13b.

Fig. 14 shows the fractional bandwidths of the first four resonant modes as a function of the distance of the first spiral arm from the origin \((r)\). For brevity, only the results for a single-turn \((t = 1)\) and two-turn \((t = 2)\) spiral slot geometries are shown. It can be observed that, for the single-turn spiral geometry \((t = 1)\), with the increase in the distance of the first spiral arm from the origin \((r)\), the overall length of the spiral assembly increases and the modes shift lower in frequency and the spacing between the consecutive resonant frequencies \((\Delta(M_n, M_{n+1})\) decreases for all modes in a similar fashion. The general trend seen in the relative spacing between the modal resonant frequencies stays the same for the most compact geometry \((r = 4 \text{ mm})\) and the least compact geometry \((r = 1 \text{ mm})\) as shown in Fig. 13a. For the case for the spiral assembly with two turns \((t = 2)\), the increase in the distance of the first spiral arm from the origin \((r)\) affects the spacing between the consecutive resonant frequencies \((\Delta(M_n, M_{n+1})\) for a given \(n)\) differently. With the increase in \(r\), the modes become equally spaced as shown in Fig. 13b.

The effect of changing the distance of the first spiral arm from the origin \((r)\) on the geometry for an Archimedean spiral slot with 2 turns \((t = 2)\). The spiral slot width \((s)\) and the separation between the slot arms \((m)\) are kept constant \((s = m = 1 \text{ mm})\). For brevity, the infinite ground plane is shown.

![Image](https://example.com/image1.png)

**FIGURE 10.** The effect of changing the distance of the first spiral arm from the origin \((r)\) on the geometry for a 1-turn \((t = 1)\) Archimedean spiral slot. The spiral slot width \((s)\) and the separation between the slot arms \((m)\) are kept constant \((s = m = 1 \text{ mm})\). For brevity, the infinite ground plane is not shown.

![Image](https://example.com/image2.png)

**FIGURE 11.** The effect of changing the distance of the first spiral arm from the origin \((r)\) on the geometry for an Archimedean spiral slot with 2 turns \((t = 2)\). The spiral slot width \((s)\) and the separation between the slot arms \((m)\) are kept constant \((s = m = 1 \text{ mm})\). For brevity, the infinite ground plane is not shown.

For a given spiral geometry under consideration, with the increase in the overall length of the spiral slot, the resonant frequencies for all the modes decrease with the increase in the distance of the first spiral arm from its origin \((r)\) as shown in Fig. 12. Thus, for a given spiral assembly with a given number of turns \((t)\), the inverse relationship between the modal resonant frequency and the spacing in the inner region of the spiral due to \(r\), when all other spiral geometry parameters are held constant, can be approximated as:

\[
f_n^t(r) = \frac{f_n^t}{\sqrt{r}}
\]

where \(t\) is the number of turns of a given spiral slot geometry, \(f_n^t\) is the resonant frequency of mode \(M_n\) for \(r = 1\) as defined in (16).

In addition to the changes in the modal resonant frequencies, the changes in the distance from the origin \((r)\) also affects the spacing between the modal resonant frequencies as shown in Fig. 13. For brevity, only the results for a single-turn \((t = 1)\) and two-turn \((t = 2)\) spiral slot geometries are shown. As shown in Fig. 14a, for the single-turn spiral geometry \((t = 1)\), the bandwidths of all the resonant modes decrease with the increase in the distance of the first spiral arm from the origin \((r)\) as shown in Fig. 14b, it can be observed that for a compact spiral assembly \((r = 1 \text{ mm})\), the first two lower order modes \((M_n\) for \(n \leq 2)\) are narrowband whereas the higher order modes \((M_n\) for \(n > 2)\) have wider bandwidths (FBW > 10%). With the increase in the the spacing in the inner region of the spiral slot assembly (as a result of the increase in \(r\)), the two lower
order modes ($M_n$ for $n \leq 2$) become extremely narrowband (FBW < 1%), $M_3$ becomes narrowband (FBW < 10%) and only $M_4$ has wide bandwidth (FBW > 10%). Thus, for the spiral slot assembly with two turns ($t = 2$), the increase in the spacing in the inner region of the spiral slot assembly (as a result of the increase in $r$) results in the reduction in the bandwidths of all the resonant modes but the effect is more pronounced for lower order modes.

3) Effect of slot width

For this study, a single-arm spiral slot (see Fig. 2b) was considered and of the variation in the width of the spiral slot ($s$) was studied for single-arm spiral slot geometries with different number of turns: $t = 1$, $t = 2$, and $t = 3$. In order to understand the effect of slot width on the modal resonant frequencies and bandwidths, the slot width ($s$) was increased while other spiral geometry parameters listed in Table 2 were kept constant. Fig. 15 and Fig. 16 show the changes in the spiral geometries when the slot width ($s$) is varied. Additionally, the distance from the first spiral arm from the origin ($r$) and the separation between the slot arms ($m$) were kept constant ($s = m = 1$ mm). For brevity, only the two spiral geometries with $t = 1$ turn and $t = 2$ turns are shown and the infinite ground plane used in the parametric studies has been omitted.

Fig. 17 shows the variation in modal resonant frequencies with the variation in the slot width ($s$) for Archimedean spiral slot geometries having different number of turns. The slot width ($s$) affects the size of the spiral slot assembly (see

(a) Separation between modal resonant frequencies as a function of the distance from origin ($r$) for a spiral slot with $t = 1$ turn.

(b) Separation between modal resonant frequencies as a function of the distance from origin ($r$) for a spiral slot with $t = 2$ turns.

FIGURE 13. The separation between modal resonant frequencies as a function of the distance of the first spiral arm from its origin ($r$) for Archimedean spiral slots with different number of turns. For a slot with a given number of turns, $r = 1$ mm represents the most compact geometry and $r = 4$ mm represents the least compact geometry. Other spiral slot parameters ($s$ and $m$) are kept constant.

(a) Separation between modal resonant frequencies as a function of the distance of the spiral arm from its origin ($r$) for Archimedean spiral slots with different number of turns. Other spiral slot parameters ($s$ and $m$) are kept constant.

(b) Separation between modal resonant frequencies as a function of the distance of the spiral arm from its origin ($r$) for Archimedean spiral slots with different number of turns. Other spiral slot parameters ($s$ and $m$) are kept constant.

FIGURE 12. The modal resonant frequencies as a function of the distance of the spiral arm from its origin ($r$) for Archimedean spiral slots with different number of turns. Other spiral slot parameters ($s$ and $m$) are kept constant.

3) Effect of slot width

For this study, a single-arm spiral slot (see Fig. 2b) was considered and of the variation in the width of the spiral slot ($s$) was studied for single-arm spiral slot geometries with different number of turns: $t = 1$, $t = 2$, and $t = 3$. In order to understand the effect of slot width on the modal resonant frequencies and bandwidths, the slot width ($s$) was increased while other spiral geometry parameters listed in Table 2 were kept constant. Fig. 15 and Fig. 16 show the changes in the spiral geometries when the slot width ($s$) is varied. Additionally, the distance from the first spiral arm from the origin ($r$) and the separation between the slot arms ($m$) were kept constant ($s = m = 1$ mm). For brevity, only the two spiral geometries with $t = 1$ turn and $t = 2$ turns are shown and the infinite ground plane used in the parametric studies has been omitted.

Fig. 17 shows the variation in modal resonant frequencies with the variation in the slot width ($s$) for Archimedean spiral slot geometries having different number of turns. The slot width ($s$) affects the size of the spiral slot assembly (see

(a) Spiral slot with 1 turn ($t = 1$).

(b) Spiral slot with 2 turns ($t = 2$).

(c) Spiral slot with 3 turns ($t = 3$).
P. Sharma et al.: Analysis of Archimedean Spiral Slot Antennas in an Infinite Ground Plane Using the Theory of Characteristic Modes

![Modal bandwidths for 1-turn spiral](image)

(a) Bandwidths of resonant modes as a function of the distance from origin \(r\) for a spiral slot with \(t = 1\) turn.

![Modal bandwidths for 2-turn spiral](image)

(b) Bandwidths of resonant modes as a function of the distance from origin \(r\) for a spiral slot with \(t = 2\) turns.

**FIGURE 14.** Modal bandwidths as a function of the distance of the first spiral arm from its origin \(r\) for Archimedean spiral slots with different number of turns. For a slot with a given number of turns, \(r = 1\) mm represents the most compact geometry and \(r = 4\) mm represents the least compact geometry. Other spiral slot parameters \((s, m)\) are kept constant.

![Effect of changing the slot width](image)

(a) \(s = 1\) mm.

(b) \(s = 4\) mm.

**FIGURE 15.** The effect of changing the slot width \(s\) on the geometry of a 1-turn \((t = 1)\) Archimedean spiral slot. The distance of the first spiral arm from the origin \(r\) and the separation between the slot arms \(m\) are kept constant \((r = m = 1\) mm\). For brevity, the infinite ground plane is not shown.

![Effect of changing the slot width](image)

(a) \(s = 1\) mm.

(b) \(s = 4\) mm.

**FIGURE 16.** The effect of changing the slot width \(s\) on the geometry of an Archimedean spiral slot with two turns \((t = 2)\). The distance of the first spiral arm from the origin \(r\) and the separation between the slot arms \(m\) are kept constant \((r = m = 1\) mm\). For brevity, the infinite ground plane is not shown.

Fig. 15 and Fig. 16) and the total arc length of the spiral slot which determines the resonant frequencies of magnetic characteristic modes. For a given spiral slot geometry under consideration, with the increase in the slot width \(s\) the overall length of the spiral slot increases slightly which causes a decrease in the resonant frequency a given mode as shown in Fig. 17. Thus, when all other spiral geometry parameters are held constant, the resonant frequency of a given characteristic mode \((M_n)\) as a function of the slot width \(s\) can be approximated as:

\[
f_n^t(s) = \frac{f_n^t}{\sqrt{s}}
\]

where \(t\) is the number of turns of a given spiral slot geometry, \(f_n^t\) is the resonant frequency of mode \(M_n\) for \(r = 1 \) and \(s = 1\) as defined in (16).

Fig. 18 shows the effect of the change in slot width \(s\) on the spacing between the modal resonant frequencies. For brevity, only the results for a single-turn \((t = 1)\) and two-turn \((t = 2)\) spiral slot geometries are shown. It can be observed that, for the single-turn spiral geometry \((t = 1)\), with the increase in the slot width \(s\), the overall length of the spiral assembly increases slightly and the modes shift lower in frequency and the spacing between the consecutive resonant frequencies \((\Delta(M_n, M_{n+1})\) for a given \(n)\) decreases for all modes in a similar fashion. The general trend seen in the relative spacing between the modal resonant frequencies \((\Delta(M_2, M_3) < \Delta(M_3, M_4) < \Delta(M_1, M_2))\) stays the same even when the slot width is varied as shown in Fig. 18a. For the case for the spiral slot assembly with two turns \((t = 2)\), the general trend seen in the relative spacing between the modal resonant frequencies \((\Delta(M_2, M_3) < \Delta(M_3, M_4) < \Delta(M_1, M_2)\) stays the same even when the slot width is varied as shown in Fig. 18b. Thus, variation in slot width \(s\) does not result in equally spaced resonant modes.

For a given spiral slot geometry, the overall arc length of the spiral determines the resonant frequencies of the purely magnetic characteristic modes and the mutual coupling be-
FIGURE 17. The modal resonant frequencies as a function of the slot width ($s$) for Archimedean spiral slot geometry with different number of turns. Other spiral slot parameters ($r$ and $m$) are kept constant.

FIGURE 18. The separation between modal resonant frequencies as a function of the slot width ($s$) for Archimedean spiral slot with different number of turns. For a slot with a given number of turns, $s = 1$ mm represents the narrowest slot width and $s = 4$ mm represents the least narrow slot. Other spiral slot parameters ($r$ and $m$) are kept constant.

FIGURE 19. Effect of slot width ($s$) on the bandwidths the first four magnetic characteristic modes of an Archimedean spiral slot geometry with two turns ($t = 2$).
between adjacent arms affects the bandwidth of a given magnetic mode. For a single-turn spiral assembly (see Fig. 15), when the slot width \((s)\) is increased, the overall size of the spiral assembly increases slightly without any appreciable change in the spacing in the inner region of the slot assembly. This causes the mutual coupling between magnetic characteristic modes to remain unaltered with the change in the slot width. The change in the mutual coupling between slot arms due to the change in slot width is more pronounced in spiral slots with more than one turn. Fig. 19 shows the effect of the spiral slot width on the bandwidths of the first four magnetic characteristic modes. For brevity, only the results for a spiral slot with two turns \((t = 2)\) with the narrowest \((s = 1 \text{ mm})\) and the widest \((s = 4 \text{ mm})\) slot widths are shown. It can be seen that when the slot width is narrow, the two lowest order modes \((M_1 \text{ and } M_2)\) are extremely narrowband \((\text{FBW} < 5\%)\) while the higher order modes \((M_3 \text{ and } M_4)\) have wider bandwidths \((\text{FBW} > 10\%)\). As the slot arms become wider (i.e. as \(s\) increases) when the separation between slot arms \((m)\) is kept constant, the mutual coupling between the magnetic currents in the slot arms increases. It should be noted that the two lowest order modes are still extremely narrowband \((\text{FBW} < 5\%)\) and the increase in the slot width produces appreciable changes in the bandwidths of higher order modes \((M_3 \text{ and } M_4)\) as shown in Fig. 11.

4) Effect of separation between spiral slot arms

For this study, a single-arm spiral slot was considered and the separation between the spiral slot arms was varied. The effect of the changes in the distance between the slot arms on the resonant frequencies and bandwidths of the first four characteristic modes were studied for single-arm spiral slot geometries with different number of turns: \(t = 1, t = 2,\) and \(t = 3\). For all spiral slot geometries under consideration, the distance from the first spiral arm from the origin \((r)\) and the slot width \((s)\) were kept constant \((r = s = 1 \text{ mm})\) and the separation between the spiral slot arms \((m)\) was increased from \(m = 1\) to \(m = 4\). Fig. 20 and Fig. 21 show the changes in the spiral geometries when the separation between the slot arms \((m)\) is varied. For brevity, only the two spiral geometries with a single turn \((t = 1)\) and two turns \((t = 2)\) are shown and the infinite ground plane used in the parametric studies has been omitted.

Fig. 22 shows the variation in modal resonant frequencies with the variation in the separation between the spiral slot arms \((m)\) for Archimedean spiral slot geometries having different number of turns. The distance between slot arms \((m)\) affects the size of the spiral slot assembly (see Fig. 20 and Fig. 21). For a given spiral slot geometry under consideration, with the increase in the separation between the slot arms \((m)\), the overall length of the spiral slot increases slightly which causes a decrease in the resonant frequency a given mode as shown in Fig. 22. Additionally, it can be observed from Fig. 17 and Fig. 22 that for a spiral geometry with a given number of turns \((t)\) and distance from the center of the spiral \((r)\), the total arc length of the spiral geometry produced by varying the separation between slot arms \((m)\) is smaller than the arc length of the spiral geometry produced by varying the slot width \((s)\). However, the increase in the distance between the spiral slot arms \((m)\) significantly increases the total area of the spiral assembly and hence decreases its compactness. When the width of the spiral slot \((s)\) is kept constant and the separation between slot arms \((m)\) is increased, the decrease in the resonant frequencies of the characteristic modes follows a trend similar to (17) and (18). Thus, when all other spiral geometry parameters are held constant, the resonant frequency of a given characteristic mode \((M_n)\) as a function of the separation between the slot arms \((m)\) can be approximated as:

\[
f_n^t(m) = \frac{f_n^t}{\sqrt{m}} \tag{19}
\]

where \(t\) is the number of turns of a given spiral slot geometry, \(f_n^t\) is the resonant frequency of mode \(M_n\) for \(s = 1\) and \(m = 1\) as defined in (16).

Fig. 23 shows the effect of the change in the spacing between slot arms \((m)\) on the spacing between the modal...
resonant frequencies. For brevity, only the results for a single-turn \( t = 1 \) and two-turn \( t = 2 \) spiral slot geometries are shown. It can be observed that, for the single-turn spiral geometry \( t = 1 \), with the increase in the spacing between slot arms \( m \), the overall length of the spiral assembly increases slightly and the modes shift lower in frequency and the spacing between the consecutive resonant frequencies \( \Delta(M_n, M_{n+1}) \) for a given \( n \) decreases for all modes. From Fig. 23a, it can be observed that with the increase in the spacing between the slot arms, the distance between the resonant frequencies of modes \( M_2, M_3, \) and \( M_4 \) are nearly equal \( \Delta(M_2, M_3) \approx \Delta(M_3, M_4) \) whereas the lowest order mode \( M_1 \) is spaced further apart from other modes \( \Delta(M_1, M_2) < \Delta(M_2, M_3) \). Fig. 23b shows the effect of varying the spacing between slot arms \( m \) on the spacing between the modal resonant frequencies for an Archimedean spiral slot geometry with two turns \( t = 2 \). It can be observed that as the spacing between the slot arms is increased, the modes move closer to each other and the distance between consecutive modal resonant frequencies is nearly equal \( \Delta(M_2, M_3) \approx \Delta(M_1, M_2) \approx \Delta(M_3, M_4) \).

As discussed in the previous section, the mutual coupling between adjacent slot arms affects the bandwidth of a given magnetic characteristic mode. For a single-turn spiral assembly (see Fig. 21), when the slot width \( s \) is kept constant and the spacing between the spiral slot arms \( m \) increased, the spacing in the inner region of the slot assembly increases significantly. This leads to an increase in both the total length and the overall size of the spiral assembly. The increase in the length causes the redistribution of areas of

![FIGURE 22.](image-url)

**FIGURE 22.** The modal resonant frequencies as a function of the separation between slot arms \( m \) for Archimedean spiral slot geometry with different number of turns. Other spiral slot parameters \( r \) and \( s \) are kept constant.

![FIGURE 23.](image-url)

**FIGURE 23.** The separation between modal resonant frequencies as a function of the separation between slot arms \( m \) for Archimedean spiral slot geometry with different number of turns. For a slot with a given number of turns, \( m = 1 \) mm represents the most tightly spaced slot arms and \( m = 4 \) mm represents the least tight spacing between slot arms. Other spiral slot parameters \( r \) and \( s \) are kept constant.
current maximum and minimums along the length of the spiral slot. However, the increase in length and size of the spiral assembly is similar to that produced by a variation in $r$. Thus, the effect of varying the spacing between slot arms ($m$) on the modal bandwidths is similar to that produced by varying the spacing in the inner region of the spiral (due to variation in $r$) for a single-turn ($t = 1$) spiral slot assembly as shown in Fig. 14a.

The change in the mutual coupling between slot arms due to the change in the separation between slot arms is more pronounced in spiral slots with more than one turn. Fig. 24 shows the effect of the spiral slot width on the bandwidths of the first four magnetic characteristic modes. For brevity, only the results for a spiral slot with two turns ($t = 2$) with the narrowest spacing ($m = 1$ mm) and the widest spacing ($m = 4$ mm) between the slot arms are shown. It can be seen that when the spacing between slot arms is narrow, the two lowest order modes ($M_1$ and $M_2$) are extremely narrowband (FBW $< 5\%$) whereas the higher order modes ($M_3$ and $M_4$) have wider bandwidths (FBW $> 10\%$). As the slot arms are separated further apart (i.e. as $m$ increases) when the slot width ($s$) is kept constant, the mutual coupling between the magnetic currents in the slot arms weakens. While there is a slight improvement in the bandwidth of mode $M_2$, the redistribution of currents around the slot and decrease in mutual coupling between magnetic characteristic currents in the slot arms results in the decrease in the bandwidths of all other modes as shown in Fig. 24.

IV. SUMMARY OF RESULTS AND PRELIMINARY GUIDELINES

A parametric study of the effect of the spiral geometry parameters on the first four magnetic characteristic modes was presented in Section III. Based on the results of the parametric study, design equations were developed to relate the spiral geometry parameters and the resonant frequencies of the characteristic modes. Additionally, several important observations were made which helped establish the effect of the spiral geometry parameters on the bandwidths of the resonant characteristic modes. A summary of the observations which may be used as preliminary guidelines while designing Archimedean spiral slots is as follows:

1) Resonant frequency

- The resonant frequency of a given magnetic characteristic mode depends upon the total arc length of the spiral geometry.
- The number of turns of the spiral assembly determines the total arc length of the spiral slot and should be used as the primary variable for estimating the resonant frequencies in a characteristic mode-based design. The resonant frequency of any given characteristic mode is the highest for a single-turn spiral (given by (15)). As the number of turns increases, the resonant frequency of a given mode decreases.
- The purely magnetic characteristic modes ($M_n$ for any given $n$) are resonant when the total arc length of the spiral geometry is integer multiples of half-wavelength. The resonant frequency of a given magnetic characteristic mode can be determined by using (15) and (16).
- Once the number of turns of the spiral is fixed, the frequency of each mode may be tuned by adjusting the inner spacing of the spiral ($r$), the slot width ($s$) or the distance between the spiral slot arms ($m$). Increase in either $r$, $s$ or $m$ leads to a decrease in the resonant frequency of the magnetic characteristic modes.

2) Compactness of spiral assembly

- For a spiral assembly with a given number of turns, the resonant frequency of higher order modes are integer multiples of the frequency of the lowest order characteristic mode which in turn depends upon the total length of the spiral. The resonant frequency of the lowest order characteristic mode is determined by the number of turns of the spiral. Thus, the overall size of the spiral slot should be determined by the lowest order mode.
- The total area of the spiral assembly depends upon the distance of the first spiral arm from the origin ($r$), the slot width ($s$), and the separation between the spiral slot arms ($m$).
- Even small increments in $r$ and $m$ can significantly increase the area occupied by the spiral assembly. Hence, for designs where compactness is desired, both $r$ and $m$ should be as small as possible.
- In order to design a compact spiral with equally spaced modal resonant frequencies, the spiral assembly should have at least 2 turns i.e. $t \geq 2$, the inner spacing of the spiral should be small ($r$ should be kept as small as possible), the slot width...
should be narrow and the separation between the spiral arms ($m$) should be adjusted to change the spacing between the modal resonant frequencies.

3) Modal bandwidth

- For a given spiral geometry, the lowest order characteristic mode ($M_1$) has the narrowest bandwidth. Higher order modes have wider bandwidths.
- The slot width ($s$) affects the bandwidths of the magnetic characteristic modes and should be used as the primary variable for improving the bandwidth of a given characteristic mode.
- The wider the slot width, the wider the bandwidth of a given characteristic mode.
- The separation between the spiral arms ($m$) inversely affects the bandwidth. The bigger the size of the spiral assembly, the narrower the bandwidths of the resonant modes.

4) Multimode behavior

- By changing the spiral geometry parameters, the spiral assembly can be designed for singlemode operation with one resonant mode over a wide frequency range of interest or multimode operation with multiple resonant modes within a narrow frequency range of interest.
- The relative spacing between the modal resonant frequencies of a given spiral assembly with a given number of turns is determined by the spacing between the spiral arms ($m$).
- For multimode operation of the spiral slot antenna with equally spaced resonant modes, the number of turns of the spiral should be greater than 2 and the spacing between the spiral arms ($m$) should be increased. Conversely, if singlemode operation is desired within a narrow frequency range of interest and with wide modal bandwidth ($FBW > 10\%$), a single-turn spiral ($t = 1$) should be used with the smallest inner spacing ($r$), narrow slot width and smallest possible spacing between the slot arms. In both cases, the lowest order characteristic mode ($M_1$) has narrow bandwidth and should not be excited.

V. DISCUSSION

The analysis of a single-arm Archimedean spiral slot in an infinite ground plane using the theory of characteristic modes was presented in Section III. A parametric study was carried out to understand the effect of spiral geometry parameters on the resonant frequencies, spacing between the resonant frequencies and bandwidths of the magnetic characteristic modes.

Based on the results of the parametric study, it was shown that the resonant frequency of a given mode is the highest for a single-turn spiral and decreases with the increase in the number of turns. It was shown that the magnetic characteristic modes ($M_n$ for any given $n$) for a single-turn spiral are resonant when the total arc length of the spiral geometry is integer multiples of half-wavelength. Additionally, design equations were developed to estimate the resonant frequencies of the characteristic modes for a single-turn spiral assembly and given spiral assembly with a given number of turns. The equations establish a direct relationship between the geometry parameters and the modal resonant frequencies. Furthermore, it was shown that by varying other spiral geometry parameters such as the distance of the spiral arm from its origin, slot width and the spacing between the spiral slot arms, the modal resonant frequencies can be tuned further. It was also shown that as the number of turns is increased, the modal resonant frequencies decrease and the lower order characteristic modes also become extremely narrowband with the increase in the number of turns.

It was shown that a compact spiral geometry can be realized by minimizing the spacing in the inner region of the spiral assembly (the distance between the spiral arm from its origin) and by reducing the spacing between the slot arms. Such compact spiral geometries are suitable for designs where miniaturization is the desired goal. However, as the spiral slot assembly becomes more compact, the bandwidths of the lower order characteristic modes decrease. For compact spiral slot assemblies, the lower order modes are extremely narrowband ($FBW < 5\%$) whereas higher order modes have wider bandwidths ($FBW > 10\%$). The bandwidths of the modes can be improved by increasing the slot width of a given spiral assembly. Additionally, the bandwidth of the lower order characteristic modes of the compact spiral slot antenna can be improved by increasing the separation between the slot arms. Thus, a trade-off must be made between the desired bandwidth of the resonant modes of the spiral and the overall compactness of the spiral assembly.

The analysis presented in this article details the behavior of the purely magnetic characteristic modes of an Archimedean spiral slot in an infinite ground plane. In practical applications, slots are etched on finite ground planes and excited using appropriate feeding mechanism (e.g. coplanar waveguide, coaxial or microstrip feed). The next steps would be to incorporate the analysis in the design of the Archimedean slot in finite ground planes and design the appropriate feed mechanism to excite the modes. For open-slot antennas, the magnetic modes of the slot can be excited by placing a voltage source across the slot. For maximum coupling between the voltage source and the magnetic characteristic currents, the excitation should be placed at the locations where the magnetic current is maximum. As shown in Section III, the higher order magnetic characteristic modes of the Archimedean spiral have multiple current maximums at different locations along its geometry. This presents the opportunity to excite more than one mode by placing excitations at multiple locations across the slot. For a given structure, depending upon the feed mechanism and the location of the feed on the structure, one or more characteristic modes may be excited. Thus, the next step in the design
of the Archimedean spiral slot antenna would be to leverage the knowledge about the magnetic current distribution of the purely magnetic characteristic modes in feed design and determining the location of the feed. By conducting a characteristic mode study with the feed included, a study of the modal weighting coefficients should be performed to determine how well the magnetic characteristic modes are excited by the chosen feed mechanism. A study of the modal weighting coefficients would also help determine if multiple modes can be excited to leverage the multimode behavior of the characteristic modes of the slot to achieve multiband behavior within a given frequency range of interest.

For a slot antenna on a finite ground plane and excited by a voltage source, the modal bandwidth of the magnetic characteristic modes determine the variation of the imaginary part of the input impedance [24]. The wider the modal bandwidth of the magnetic modes, the smoother the variation of the imaginary part of the input impedance around resonance. This variation is directly related to the overall bandwidth of the antenna. Thus, knowledge of the modal bandwidths of the magnetic characteristic modes can provide insight into the impedance characteristics and the overall bandwidth of the slot antenna. Additionally, purely magnetic characteristic modes of the slot will couple with one or more electric characteristic modes of the ground plane. The coupling between the two sets of modes determines the total current distribution of the slot antenna and hence its radiation pattern [24]. Naturally, the next step in the design process would be to study the electric modes of the ground plane independently and then determine which of the electric modes are excited by the slot. This can be done by evaluating the correlation between the total radiation pattern of the slot antenna with the modal far-field radiation patterns [21], [24]. Thus, a systematic procedure for designing Archimedean spiral slots on finite ground planes based on characteristic mode theory would comprise of the following steps:

1) Complete characterization of the Archimedean slot in an infinite ground plane.
   - study the current distributions and far-field radiation patterns of the magnetic characteristic modes.
   - study the effects of the spiral geometry parameters on modal resonant frequencies and modal bandwidth.
   - establish relationship between modal resonant frequencies and geometry parameters.

2) Design the appropriate feed mechanism and feed location (based on Step (1)).
   - place excitation at single / multiple locations on the slot where magnetic current is maximum.
   - evaluate the modal weighting coefficients to determine which magnetic characteristic modes can be excited for singleband / multiband operation within a given frequency range.

3) Study and determine the effects of the ground plane (based on Step (1)).
   - study the current distributions and far-field radiation patterns of the purely electric characteristic modes of the ground plane.
   - study the interaction between the magnetic modes and purely electric modes and determine the total current distribution of the slot antenna.
   - design the shape and size of the ground plane to couple with the slot modes.
   - study the impedance behavior of the slot in the finite ground plane.
   - study the radiation characteristics of the slot in the finite ground plane.

Based on the step-by-step design procedure outlined above, it can be seen that the analysis of the magnetic characteristic modes of the Archimedean slot in an infinite ground plane can not only be used to fully characterize the modal behavior of Archimedean spiral slots but also serve as a crucial first step in the design of spiral slots in finite ground planes.

VI. CONCLUSION

The analysis of spiral slot antennas using the theory of characteristic modes lays the foundation for understanding the modal behavior of Archimedean spiral slots in an infinite ground plane. A complete characterization of spiral slots was performed by evaluating the effect of spiral geometry parameters on the resonant frequencies, spacing between the resonant frequencies and bandwidths of the magnetic characteristic modes. In addition, analytical equations were developed to estimate the resonant frequencies of the characteristic modes based on the geometry parameters. In addition to the parametric study, it was also shown that duality exists between the purely magnetic characteristic modes of the Archimedean spiral slots and purely electric characteristic modes of metallic spiral assembly having the same aperture. Due to the existence of duality between the characteristic modes, the results of modal analysis presented in this article can also be extended to predict the behavior of purely electric characteristic modes of metallic wire spirals or narrow planar metallic spiral assemblies. Furthermore, the design equations presented in this article can be used to design planar metallic spiral antennas and spiral inductors.

The aim of the article is to analyze and understand the behavior of Archimedean spiral slots in isolation without the effects of the finite ground plane or the feed mechanism involved. The next logical step would be to analyze the slot in a finite ground plane and conduct an in-depth analysis of different feed mechanisms to determine which magnetic modes can be excited. Additionally, depending upon the excitation, the maximum realizable bandwidth for the structure needs to be evaluated. The analysis presented in this article can be used in designing the appropriate feeding mechanism, determining the location of the feed, and in understanding the behavior of the slot in a finite ground plane. The guidelines presented in this article can be used as preliminary step in the systematic design of Archimedean spiral slots in finite ground planes.
ground planes using characteristic mode theory. For the sake of brevity, the scope of the paper has been limited to the theoretical analysis of Archimedean slot in an infinite ground plane. Feed design, excitation of slot modes, study of ground plane effects and the impedance behavior of the slot in a finite ground plane and experimental verification thereof will be presented in a separate article in the future.

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JENNIFER T. BERNHARD ’89–M’95–SM’01–F’10 was born in New Hartford, NY, USA, in 1966. She received the B.S.E.E. degree from Cornell University, Ithaca, NY in 1988, and the M.S. and Ph.D. degrees in electrical engineering from Duke University, Durham, NC, USA, in 1990 and 1994, respectively, with support from the National Science Foundation Graduate Fellowship.

She was a McMullen Dean’s Scholar, and participated in the Engineering Co-Op Program, with the IBM Federal Systems Division, Owego, NY. From 1994 to 1995, she was a Post-Doctoral Research Associate with the Departments of Radiation Oncology and Electrical Engineering, Duke University, where she developed RF and microwave circuitry for simultaneous hyperthermia (treatment of cancer with microwaves) and magnetic resonance imaging thermometry. She was also an organizing member of the Women in Science and Engineering Project, a graduate student-run organization designed to improve the climate for graduate women in engineering and the sciences. From 1995 to 1999, she was an Assistant Professor with the Department of Electrical and Computer Engineering, University of New Hampshire, Durham, NH, USA, where she held the Class of 1944 Professorship. Since 1999, she has been with the Electromagnetics Laboratory, University of Illinois at Urbana-Champaign, Urbana, IL, USA, where she is currently the Donald Biggar Willett Professor with the Department of Electrical and Computer Engineering. In 1999 and 2000, she was a NASA-ASEE Summer Faculty Fellow with the NASA Glenn Research Center, Cleveland, OH, USA. From 2012-2019, she served as the Associate Dean for Research with the Grainger College of Engineering, University of Illinois at Urbana-Champaign. Prof. Bernhard is currently the Director of the Illinois Applied Research Institute. Her current research interests include reconfigurable and wideband microwave antennas and circuits, wireless sensors and sensor networks, high-speed wireless data communication, electromagnetic compatibility, and electromagnetics for industrial, agricultural, and medical applications. She holds six patents on technology in her research areas. She was a recipient of the NSF CAREER Award in 2000. She and her students were the recipients of the 2004 H.A. Wheeler Applications Prize Paper Award from the IEEE Antenna and Propagation Society for their paper published in 2003 issue of the IEEE Transactions on Antennas and Propagation. She served as an Associate Editor for the IEEE Transactions on Antennas and Propagation from 2001 to 2007 and IEEE Antennas and Wireless Propagation Letters from 2001 to 2005. She is a member of USNC-URSI Commissions B, C, and D (Chair 2012-2014), Tau Beta Pi, Eta Kappa Nu, Sigma Xi, and ASEE. She served as an Elected Member for the IEEE Antennas and Propagation Society’s Administrative Committee from 2004 to 2006. She was the President of the IEEE Antennas and Propagation Society in 2008 and IEEE Division IV Director on the IEEE Board of Directors (2017-2018).