LORENTZ SYMMETRY VIOLATION, VACUUM AND SUPERLUMINAL PARTICLES

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Abstract

If textbook Lorentz invariance is actually a property of the equations describing a sector of the excitations of vacuum above some critical distance scale, several sectors of matter with different critical speeds in vacuum can coexist and an absolute rest frame (the vacuum rest frame) may exist without contradicting the apparent Lorentz invariance felt by “ordinary” particles (particles with critical speed in vacuum equal to \(c\), the speed of light). Sectorial Lorentz invariance, reflected by the fact that all particles of a given dynamical sector have the same critical speed in vacuum, will then be an expression of a fundamental sectorial symmetry (e.g. preonic grand unification or extended supersymmetry) protecting a parameter of the equations of motion. Furthermore, the sectorial Lorentz symmetry may be only a low-energy limit, in the same way as the relation \(\omega\) (frequency) = \(c_s\) (speed of sound) \(k\) (wave vector) holds for low-energy phonons in a crystal. We show that, in this context, phenomena such as the absence of Greisen-Zatsepin-Kuzmin cutoff and the stability of unstable particles at very high energy are basic properties of a wide class of non-causal models where local Lorentz invariance is broken introducing a fundamental length. Then, observable phenomena are produced at the wavelength scale of the highest-energy cosmic rays or even below this energy, but Lorentz symmetry violation remains invisible to standard low-energy tests. We discuss possible theoretical, phenomenological, experimental and cosmological implications of this new approach to matter and space-time, as well as prospects for future developments.

1 Introduction

“The impossibility to disclose experimentally the absolute motion of the earth seems to be a general law of Nature”

H. Poincaré

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"Precisely Poincaré proposed investigating what could be done with the equations without altering their form. It was precisely his idea to pay attention to the symmetry properties of the laws of Physics."

R.P. Feynman

"The interpretation of geometry advocated here cannot be directly applied to submolecular spaces... it might turn out that such an extrapolation is just as incorrect as an extension of the concept of temperature to particles of a solid of molecular dimensions."

A. Einstein

Is relativity the result of a symmetry of the laws of Nature (Poincaré, 1905), therefore necessarily broken at some deeper level (Einstein, late period), or does it reflect the existence of an absolute space-time geometry that matter cannot escape (Einstein, early papers on relativity)? Most textbooks teach "absolute" relativity (early Einstein papers) and ignore the possibility of a more flexible formulation (Poincaré, late Einstein thought) that we may call "relative" relativity (relativity is a symmetry of the laws of Nature expressed by the Lorentz group: whether this symmetry is exact or approximate must be checked experimentally at each new energy scale). In the first case, ether does not exist: light just propagates at the maximum speed allowed by the "absolute" space-time geometry; in the second case, the question of ether remains to be settled experimentally at any new small-distance scale. By introducing important dynamics into the vacuum structure, particle physics has operated a return to ether: it would be impossible for the $W^{\pm}$ and the $Z^0$ to be gauge bosons with nonzero masses if they did not propagate in a medium where the Higgs fields condense; similarly, modern theories of hadron structure conjecture that free quarks can exist only inside hadrons, due to non-trivial properties (e.g. superconducting) of the non-perturbative QCD vacuum. It could still be argued that this new "ether" does not necessarily have a preferred rest frame, and that special relativity is an exact symmetry which prevents us from identifying such a frame. However, this hypothesis does not seem to naturally fit which general physics considerations. Modern dynamical systems provide many examples where Lorentz symmetry (with a critical speed given by the properties of the system) is a scale-dependent property which fails at the fundamental distance scale of the system (e.g. a lattice spacing). In practical examples, the critical speed of the apparently relativistic dynamical system is often less than $10^{-5} \, c$ and "relativity", as felt by the dynamical system, would forbid particle propagation at the speed of light. Light would appear to such a system just like superluminal matter would appear to us. Furthermore, high-energy physics has definitely found cosmic-ray events with energies above $10^{20} \, eV$ (e.g. Hayashida et al., 1997). This energy scale is, in orders of magnitude, closer to Planck scale ($10^{28} \, eV$) than to the electroweak scale ($10^{11} \, eV$). Therefore, if Lorentz symmetry is not an exact symmetry of nature and is instead broken at $\approx 10^{-33} \, cm$ length scale, the parameters of Lorentz symmetry violation observed (if ever) in the analysis of the highest-energy cosmic-ray events will provide us with direct and
unique information on physics at Planck scale. This may be the most fundamental physics outcome of experiments such as AUGER (AUGER Collaboration, 1997) devoted to the study of cosmic rays at $E \approx 10^{20}$ eV: Lorentz symmetry violation at these energies would unravel phenomena originating at higher energy scales, including the possible existence of a fundamental length scale. In the vacuum rest frame, particles of the same type moving at different speeds are different physical objects whose properties cannot be made identical through a Lorentz transformation. This essential property remains true in any other frame, but parameters measured in the vacuum rest frame have an absolute physical meaning. Indeed, assuming that the laboratory frame moves slowly with respect to the vacuum rest frame (which may be close to that suggested by the study of cosmic microwave background radiation), the observed properties of particles at $E \approx 10^{20}$ eV may look closer to physics at Planck scale than to physics at electroweak or GeV scale (e.g. the failure of the parton model and of standard relativistic formulae for Lorentz contraction and time dilation; see Gonzalez-Mestres, 1997h), and basic parameters of Planck-scale physics may become measurable through $E \approx 10^{20}$ eV cosmic-ray events if Lorentz symmetry is violated (with exact Lorentz symmetry, collisions of very high-energy cosmic rays would on the contrary be exactly equivalent to collider events at much lower laboratory energies).

We review and comment here recent work by the author on Lorentz symmetry violation and possible superluminal sectors of matter. Non-tachyonic superluminal particles (superbradyons) have been discussed in previous papers (Gonzalez-Mestres, 1995, 1996, 1997a and 1997b) and other papers have been devoted to Lorentz symmetry violation (Gonzalez-Mestres, 1997c, 1997d and 1997e) as well as to its astrophysical consequences (Gonzalez-Mestres, 1997f and 1997g), its application to extended objects (Gonzalez-Mestres, 1997h) and its relevance for future accelerator programs (Gonzalez-Mestres, 1997i).

2 Lorentz symmetry as a low-energy limit

Lamoreaux, Jacobs, Heckel, Raab and Forston (1986) have set an experimental limit from nuclear magnetic resonance measurements which, when suitably analyzed (Gabriel and Haugan, 1990), amounts to the bound $|c_{\text{matter}} - c_{\text{light}}| < 6.10^{-21} c_{\text{light}}$ in the $T\emptyset\mu$ model of Lorentz symmetry violation (e.g. Will, 1993). However, the $T\emptyset\mu$ model assumes a scale-independent violation of Lorentz invariance (through the non-universality of the critical speed parameter) which does not naturally emerge from dynamics violating Lorentz symmetry at Planck scale or at some other fundamental length scale, where we would naturally expect such an effect to be scale-dependent and possibly vary (like the effective gravitational coupling) according to a $E^2$ law ($E =$ energy scale). The $E^2$ law is indeed a trivial and rather general consequence of phenomena such as nonlocality, as can be seen from the generalized one-dimensional Bravais lattice equation (Gonzalez-Mestres, 1997d):

\[
\frac{d^2}{dt^2} \left[ \phi (n) \right] = - K \left[ 2 \phi (n) - \phi (n-1) - \phi (n+1) \right] - \omega_{\text{rest}}^2 \phi \quad (1)
\]
where \( n \) (integer) stands for the site under consideration, \( \phi(n) \) is a complex order parameter, \( K \) an elastic constant and \( (2\pi)^{-1} \omega_{\text{rest}} \) the frequency of the chain of oscillators in the zero-momentum limit. In the limit where the lattice spacing vanishes but \( K a^2 \) remains fixed, (1) becomes a two-dimensional dalembertian equation of the Klein-Gordon type, with two-dimensional Lorentz symmetry and critical speed parameter \( K^{1/2} a \). In terms of the wave vector \( k \) and the frequency \( (2\pi)^{-1} \omega \), equation (1) leads to the dispersion relation:

\[
\omega^2 (k) = 2K [1 - \cos (ka)] + \omega_{\text{rest}}^2 = 4K \sin^2 (ka/2) + \omega_{\text{rest}}^2
\]

equivalent to:

\[
E^2 = 2K [1 - \cos (2\pi h^{-1} p a) + (2\pi)^{-1} h \omega_{\text{rest}}^2] = 4K \sin^2 (2\pi h^{-1} p a) + (2\pi)^{-1} h \omega_{\text{rest}}^2
\]

where \( p \) stands for momentum and \( h \) is the Planck constant. The same procedure can be extended to a wide class of nonlocal models, including those with continuous space, giving:

\[
E = (2\pi)^{-1} h c a^{-1} e (ka)
\]

where \([e (ka)]^2\) is a convex function of \((ka)^2\) obtained from vacuum dynamics. We have checked that this is also a fundamental property of old scenarios breaking local Lorentz invariance (f.i. Rédei, 1967), although such a phenomenon seems not to have been noticed by the authors. Expanding equation (4) for \( ka \ll 1 \), we can write:

\[
e (ka) \simeq [(ka)^2 - \alpha (ka)^4 + (2\pi a)^2 h^{-2} m^2 c^2]^{1/2}
\]

where \( \alpha \) is a model-dependent constant, in the range \( 0.1 - 0.01 \) for full-strength violation of Lorentz symmetry at the fundamental length scale (\( \alpha = 1/12 \) for the Bravais-lattice model and its isotropic extension to three dimensions), and

\[
E \simeq p c [1 + \alpha (ka)^2/2] + m^2 c^3 (2p)^{-1}
\]

and the new term \( \Delta E = - p c \alpha (ka)^2/2 \) in the right-hand side of (6) implies a Lorentz symmetry violation in the ratio \( E p^{-1} \) varying like \( \Gamma (k) \simeq \Gamma_0 k^2 \) where \( \Gamma_0 = - \alpha a^2/2 \). Such an expression is not incompatible with a possible gravitational origin of Lorentz symmetry violation, where the effective gravitational coupling would rise like \( E^2 \) below Planck energy. However, other interpretations are possible (e.g. Gonzalez-Mestres, 1997d) where all presently known "elementary" particles and gauge bosons would actually be composite objects made of superluminal matter at Planck scale.

More generally, a \( k^2 \) law for the parameters of Lorentz symmetry violation, as suggested by the above formulae, would lead to substantial changes with respect to conventional models (e.g. Will, 1993). In particular, Lorentz symmetry would remain unbroken at \( k = 0 \).
With such a law, an effect of order 1 at $p = 3.10^{20}$ eV $c^{-1}$ (the estimated momentum of the highest-energy observed cosmic-ray event) would become of order $\approx 10^{-25}$ at $p = 100$ MeV $c^{-1}$ (the highest momentum scale involved in nuclear magnetic resonance tests of special relativity). Therefore, very large deviations from special relativity at the highest observed cosmic-ray energies would be compatible with a great accuracy of this theory in the low-momentum region. Thus, the main and most fundamental physics outcome of very high-energy cosmic-ray experiments involving particles and nuclei may eventually be the test of special relativity. If Lorentz symmetry is violated at Planck scale, the highest-energy cosmic ray events may, if analyzed closely and with the expected high statistics from future experiments, provide a detailed check of different models of deformed relativistic kinematics and of the basic physics behind the kinematics.

The same kind of deformed relativistic kinematics arises naturally in soliton models (Gonzalez-Mestres, 1997h). Starting from the equation:

$$c^{-2} \frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial x^2} = 2 \Delta^{-2} \psi (1 - \psi^2)$$

(7)

where $\Delta$ is the distance scale characterizing the soliton size ($x =$ space coordinate, $t =$ time coordinate), and writing down the one-soliton solution of this equation:

$$\psi(x, t) = \Phi(y) = \tanh (\lambda_0 y)$$

(8)

where $y = x - v t$, $v$ is the speed of the soliton, $\lambda_0 = \Delta^{-1} \gamma_R$ and $\gamma_R$ is the standard relativistic Lorentz factor $\gamma_R = (1 - v^2 c^{-2})^{-1/2}$, we can introduce a perturbation to the system by adding to the left-hand side of (7) a term $- (a^2/12) \partial^4 \psi / \partial x^4$ which corresponds to the lowest-order correction to the continuum limit when the Bravais-lattice version of (7) is expanded in powers of $a^2$. This new term in the equation will be compensated at the first order in the perturbation by the replacement:

$$\Phi \to \Phi + \epsilon \Phi (1 - \Phi^2)$$

(9)

where $\epsilon \propto a^2$. We furthermore replace $\lambda_0$ by a new coefficient $\lambda$ to be determined from the perturbed equation. To first order in the perturbation, we get the solutions:

$$\epsilon \simeq 1 - \lambda^2 \gamma_R^{-2} \Delta^2$$

(10)

$$\lambda^2 \simeq [3 \pm \sqrt{1 - 4 a^2 \Delta^{-2} \gamma_R^4/3}]^{1/2} (1 + a^2 \Delta^{-2} \gamma_R^4/6)^{-1} \Delta^{-2} \gamma_R^2/4$$

(11)

leading for $\epsilon \ll 1$ to $(\Delta \lambda)^{-2} \simeq \gamma_R^{-2} + a^2 \Delta^{-2} \gamma_R^2/3$. Thus, relative corrections to standard relativistic Lorentz contraction and time dilation factors are proportional to $a^2 \Delta^{-2} \gamma_R^4$ and dominate when this variable becomes $\approx 1$. Above this value of $\gamma_R$, we expect departures from special relativity to occur at leading level in many phenomena. Deformed kinematics can be obtained at the lowest order in the perturbation. A simplified calculation, valid for a wide class of soliton models, could be as follows. At the first order
in the perturbation with respect to special relativity, we start from an effective lagrangian for soliton kinematics:

\[ L = -m c^2 \gamma_R^{-1} (1 - \rho \gamma_R^4) \]  

(12)

where \( \rho \) is a constant proportional to \( a^2 \Delta^{-2} \), according to (10) and (11). From this lagrangian, we derive the expression for the generalized momentum:

\[ p = m \gamma_R v (1 + 3 \rho \gamma_R^4) \]  

(13)

from which we can build the hamiltonian:

\[ H = pv - L = m c^2 (\gamma_R + 3 \rho v^2 c^{-2} \gamma_R^5 - \rho \gamma_R^3) \]  

(14)

which leads to a deformed relativistic kinematics defined by the relation:

\[ E - pc = m c^2 \gamma_R^{-1} (1 + v c^{-1})^{-1} - \rho m c^2 [3 v c^{-1} (1 + v c^{-1})^{-1} + 1] \gamma_R^3 \]  

(15)

and, when expressed in terms of \( p \) at \( v \approx c \) and for small values of \( \rho \gamma_R^4 \), can be approximated by:

\[ E - pc \approx m c^2 (2p)^{-1} - 5 \rho p^3 (2m^2 c)^{-1} \]  

(16)

where the deformation term \( 5 \rho p^3 (2m^2 c)^{-1} \) differs from that obtained from phonon mechanics in the Bravais lattice only by a constant factor \( \eta \propto 2 \hbar^2 (2\pi m c \Delta)^{-2} \).

Looking at the low-speed limit of (15), we find a renormalization of the critical speed parameter \( c, \delta c \), such that \( \delta c^{-1} \approx (\Delta/a)^{-2} \approx 10^{-40} \) for hadrons if \( a \approx 10^{-33} \) \( cm \) and \( \Delta \approx 10^{-13} \) \( cm \). This effect (the only one which survives at \( k = 0 \)) is much smaller than the effects contemplated by other authors (e.g. Coleman and Glashow, 1997 and references therein) and, even assuming that it would be different for different particles, it cannot be excluded by existing data which can only rule out values of \( \delta c^{-1} \) above \( \approx 10^{-20} \).

### 3 Deformed relativistic kinematics

Assuming that Lorentz symmetry is violated at Planck scale or at some other fundamental length scale, how does the new kinematics apply to different particles, nuclei, atoms and larger objects? Other versions of deformed relativistic kinematics led in the past to controversies (Bacry, 1993; Fernandez, 1996) which can be resolved (Gonzalez-Mestres, 1997h) if the value of \( \alpha \) depends on the object under consideration. In the presence of a fundamental (super)symmetry, it may be reasonable to assume that \( \alpha \) has the same value for leptons and gauge bosons. From the above example with solitons, we conclude (Gonzalez-Mestres, 1997h) that the value of \( \alpha \) for hadrons is naturally of the same order as for "elementary" particles, although not necessarily identical. It can also be different.
for different hadrons. A crucial question is how to extend deformed relativistic kinematics to nuclei and larger objects. Two different simplified approaches can be considered:

- **Model i)**. Due to the very large size of atoms, as compared to nuclei, the transition from nuclear to atomic scale appears as a reasonable point to stop considering systems as “elementary” from the point of view of deformed relativity. $\alpha$ would then have a universal value for nuclei and simpler objects, but not for atoms and larger bodies.

- **Model ii)**. The example with $\Phi^4$ solitons suggests that hadrons can have values of $\alpha$ close to that of leptons and gauge bosons, and the transition may happen continuously at fermi scale, when going from nucleons to nuclei. Then, the value of $\alpha$ would be universal (or close to it) for leptons, gauge bosons and hadrons (solitons) but follow a $m^{-2}$ law for nuclei (multi-soliton bound states) and heavier systems, the nucleon mass setting the scale.

Experimental tests should be performed and equivalent dynamical systems should be studied. However, **Model i)** would lack a well-defined criterium to separate systems to which the deformed relativity applies with the same value of $\alpha$ as for leptons and gauge bosons from those to which this kinematics cannot be applied, and to characterize the transition between the two regimes. The above obtained $m^{-2} \Delta^{-2}$ dependence of the coefficient of the deformation term for extended objects, as described in **Model ii)**, seems to provide a continuous transition from nucleons to heavier systems, naturally filling this gap. On the other hand, a closer analysis reveals that there is indeed a discontinuity between nuclei and atoms, as foreseen in **Model i)**. As long as the deformation term in electron kinematics can be neglected as compared to the electron mass term, we can consider that most of the momentum of an atom is carried by the nucleus and **Model ii)** may provide a reasonable description of reality. But, when the electron mass term becomes small as compared to the part of the energy it would carry in a parton model of the atom, such a description becomes misleading. To have the same speed as a nucleon, the electron must then carry nearly the same energy and momentum. We therefore propose (Gonzalez-Mestres, 1997h) a modified version of **Model ii)** with $\Delta \approx 10^{-13} \text{ cm}$ from hadrons and nuclei where, for atoms and larger neutral systems, the coefficient of the deformation term would be corrected by a factor close to 1 at low momentum and to 4/9 at high momentum if the number of neutrons is equal to that of protons. This model is obviously approximate and should be completed by a detailed dynamical calculation that we shall not attempt here. It assumes that electrically neutral bodies can reach very high energies per unit mass, which is not obvious: spontaneous ionization may occur at speeds (in the vacuum rest frame) for which the deformation term in electron kinematics becomes larger than its mass term.

Then, for bodies heavier than hadrons, the effective value of $\alpha$ would decrease essentially like $m^{-2}$. Applying a similar mass-dependence to the $\kappa$ parameter of a different deformed Poincaré algebra considered by previous authors (Bacry, 1993 and references therein), i.e. $\kappa \propto m$ for large bodies, yields the relation:

$$F (M_0 , E_0) = F (M_1 , E_1) + F (M_2 , E_2)$$ (17)
with:

\[ F(m, E) = 2 \kappa(m) \sinh \left[ 2^{-1} \kappa^{-1}(m) E \right] \]  (18)

where \( M = M_1 + M_2 \), \( \kappa(m) \) is our above mass-dependent version of the \( \kappa \) parameter of the deformed Poincaré algebra used by these authors, and \( E_0 \) is the energy of a system with mass \( M \) made of two non-interacting subsystems of energies \( E_1 \) and \( E_2 \) and with masses \( M_1 \) and \( M_2 \). Defining mass as an additive parameter, the rest energy \( E_{i,\text{rest}} \) (in the vacuum rest frame) of particle \( i \) (\( i = 0, 1, 2 \)) is given by the equation:

\[ M_i c^2 = 2 \kappa(M_i) \sinh \left[ 2^{-1} \kappa^{-1}(M_i) E_{i,\text{rest}} \right] \]  (19)

and tends to \( M_i c^2 \) as \( \kappa(M_i) \to \infty \). Equations (17) and (18) lead to additive relations for the energy of macroscopic objects if the proportionality rule \( \kappa(m) \propto m \) is applied.

From our previous discussion with a different deformation scheme, such a choice seems to naturally agree with physical reality. Then, contrary to previous claims (Bacry, 1993; Fernandez, 1996), the rest energies of large systems would be additive and no macroscopic effect on the total mass of the Universe would be expected.

### 4 Phenomenological implications

As initially stressed, very high-energy cosmic rays can open a unique window to Planck scale if Lorentz symmetry is violated. Contrary to standard prejudice which would suggest that energy-dependent effects of Lorentz symmetry violation at Planck scale can be detected only at energies close to this scale, it turns out that such effects are detectable at the highest observed cosmic-ray energies. As discussed in Section 2, we expect standard relativistic formulae for Lorentz contraction and time dilation for a proton to fail at energies such that \( a^2 \Delta^{-2} \gamma_{R}^2 \approx 1 \), i.e. \( E \approx 10^{19} \text{ eV} \) for \( a \approx 10^{-33} \text{ cm} \) and \( \Delta \approx 10^{-13} \text{ cm} \) (Gonzalez-Mestres, 1997h). Similarly, with the same figures and taking \( \alpha \approx 0.1 \), the proton mass term \( m^2 c^3 (2 p)^{-1} \) in the expression for the proton energy becomes smaller than the deformation term \( \Delta E = p c \alpha (k a)^2/2 \) for \( E \) above \( \approx 8 \times 10^{18} \text{ eV} \) and, even if both terms are very small as compared to the total energy, kinematical balances (which depend crucially on these nonleading terms) are drastically modified (Gonzalez-Mestres, 1997d). The standard parton picture of hadrons is equally disabled by the novel kinematics at very high energy (Gonzalez-Mestres, 1997h), due to the impossibility for “almost-free” constituents carrying arbitrary fractions of the total energy and momentum to travel at the same speed. Apart from the failure of the standard parton model for hadrons at wave vectors above \( \approx (8 \pi^2 \alpha^{-1})^{1/4} (mc \ h^{-1} a^{-1})^{1/2} \) (i.e. at energies above \( \approx 10^{19} \text{ eV} \) if \( a \approx 10^{-33} \text{ cm} \), \( \approx 10^{20} \text{ eV} \) for \( a \approx 10^{-35} \text{ cm} \) and \( \approx 3.10^{17} \text{ eV} \) for \( a \approx 10^{-30} \text{ cm} \)), the following new effects at leading level would occur assuming a universal value of \( \alpha \) for leptons, gauge bosons and hadrons:

a) The Greisen-Zatsepin-Kuzmin (GZK) cutoff on very high-energy cosmic nucleons (Greisen, 1966; Zatsepin and Kuzmin, 1966) does no longer apply (Gonzalez-Mestres,
Very high-energy cosmic rays originating from most of the presently observable Universe can reach the earth and generate the highest-energy detected events. Indeed, fits to data below \( E = 10^{20} \text{ eV} \) using standard relativistic kinematics (e.g. Dova, Epele and Hojvat, 1997) predict a sharp fall of the event rate at this energy, in contradiction with data (Bird et al., 1993 and 1996; Hayashida et al., 1994 and 1997; Yoshida et al., 1995) which suggest that events above \( 10^{20} \text{ eV} \) are produced at a significant rate. Lorentz symmetry violation from physics at Planck scale provides a natural way out.

The existence of the cutoff for cosmic nuclei will then depend crucially on the details of deformed relativistic kinematics, beyond the accuracy of the present discussion.

b) Unstable particles with at least two massive particles in the final state of all their decay channels (neutron, \( \Delta^{++} \), possibly muons, charged pions and \( \tau \)’s, perhaps some nuclei...) become stable at very high energy (Gonzalez-Mestres, 1997d and 1997f). In any case, many unstable particles live longer than naively expected with exact Lorentz invariance and, at high enough energy, the effect becomes much stronger than previously estimated for nonlocal models (Anchordoqui, Dova, Gómez Dumm and Lacentre, 1997) ignoring the small violation of relativistic kinematics. Not only particles previously discarded because of their lifetimes can be candidates for the highest-energy cosmic-ray events, but very high-energy cascade development can be modified (for instance, if the \( \pi^0 \) lives longer at energies above \( \approx 10^{18} \text{ eV} \), thus favoring hadronic interactions and muon pairs and producing less electromagnetic showers).

c) The allowed final-state phase space of two-body collisions is modified at very high energy when, in the vacuum rest frame where expressions (4) - (6) apply, a very high-energy particle collides with a low-energy target (Gonzalez-Mestres, 1997d). Energy conservation reduces the final-state phase space and can lead to a sharp fall of cross sections starting at incoming-particle wave vectors well below the inverse of the fundamental length, essentially above \( E \approx (E_T a^{-2} h^2 c^2)^{1/3} \) where \( E_T \) is the energy of the target. For \( a \approx 10^{-33} \text{ cm} \), this scale corresponds to: \( \approx 10^{22} \text{ eV} \) if the target is a rest proton; \( \approx 10^{21} \text{ eV} \) if it is a rest electron; \( \approx 10^{30} \text{ eV} \) for \( a \approx 1 \text{ keV photon} \), and \( \approx 10^{19} \text{ eV} \) if the target is a visible photon. For a proton impinging on a \( \approx 10^{-3} \text{ eV} \) photon from cosmic microwave background radiation, and taking \( a \approx 1/12 \) as in the Bravais-lattice model, we expect the fall of cross sections to occur above \( E \approx 5.10^{18} \text{ eV} \), the critical energy where the derivatives of the mass term \( m^2 c^3 p^{-1/2} \) and of the deformation term \( \alpha p (k a)^2/2 \) become equal in the expression relating the proton energy \( E \) to its momentum \( p \). With \( a \approx 10^{-30} \text{ cm} \), still allowed by cosmic-ray data (Gonzalez-Mestres, 1997e), the critical energy scale can be as low as \( E \approx 10^{17} \text{ eV} \); with \( a \approx 10^{-35} \text{ cm} \) (still compatible with data in the region of the predicted GZK cutoff), it would be at \( E \approx 5.10^{19} \text{ eV} \). Similar considerations lead to a fall of radiation under external forces (e.g. synchrotron radiation) above this energy scale. In the case of a very high-energy \( \gamma \) ray, taking \( a \approx 10^{-33} \text{ cm} \), the deformed relativistic kinematics inhibits collisions with \( \approx 10^{-3} \text{ eV} \) photons from cosmic background radiation above \( E \approx 10^{18} \text{ eV} \), with \( \approx 10^{-6} \text{ eV} \) photons above \( E \approx 10^{17} \text{ eV} \) and with
$\approx 10^{-9} \text{ eV}$ photons above $E \approx 10^{16} \text{ eV}$. Taking $a \approx 10^{-30} \text{ cm}$ would lower these critical energies by a factor 100 according to the previous formulae, whereas the choice $a \approx 10^{-35} \text{ cm}$ would raise them by a factor of 20.

d) In astrophysical processes, the new kinematics may inhibit phenomena such as GZK-like cutoffs, decays, radiation emission under external forces (similar to a collision with a very low-energy target), momentum loss (which at very high energy does not imply deceleration) through collisions, production of lower-energy secondaries, photodisintegration of some nuclei... potentially solving all the basic problems raised by the highest-energy cosmic rays (Gonzalez-Mestres, 1997e and 1997g). Due to the fall of cross sections, energy losses become much weaker than expected with relativistic kinematics and astrophysical particles can be pushed to much higher energies (once energies above $10^{17} \text{ eV}$ have been reached through conventional mechanisms, synchrotron radiation and collisions with ambient radiation may start to be inhibited by the new kinematics); similarly, astrophysical particles will be able to propagate to much longer astrophysical distances, and many more sources (in practically all the presently observable Universe) can produce very high-energy cosmic rays reaching the earth; as particle lifetimes are much longer, new possibilities arise for the nature of these cosmic rays. Models of very high-energy astrophysical processes cannot ignore a possible Lorentz symmetry violation at Planck scale, in which case observable effects are predicted for the highest-energy detected particles.

e) If the new kinematics can explain the existence of $\approx 10^{20} \text{ eV}$ events, it also predicts that, above some higher energy scale (around $\approx 10^{22} \text{ eV}$ for $a \approx 10^{-33} \text{ cm}$), the fall of cross sections will prevent many cosmic rays (leptons, hadrons, gauge bosons) from depositing most of its energy in the atmosphere (Gonzalez-Mestres, 1997e). Such extremely high-energy particles will produce atypical events of apparently much lower energy. New analysis of data and experimental designs are required to explore this possibility. Again, the interaction properties of nuclei will depend on the details of deformed kinematics.

Velocity reaches its maximum at $k \approx \left(8\pi^2 \alpha^{-1}\right)^{1/4} \left(m c h^{-1} a^{-1}\right)^{1/2}$. Observable effects of local Lorentz invariance breaking arise, at leading level, well below the critical wavelength scale $a^{-1}$ due to the fact that, contrary to previous models (f.i. Rédei, 1967), we directly apply non-locality to particle propagators and not only to the interaction hamiltonian. In contrast with previous patterns (f.i. Blokhintsev, 1966), $s-t-u$ kinematics ceases to make sense and the motion of the global system with respect to the vacuum rest frame plays a crucial role. The physics of elastic two-body scattering will depend on five kinematical variables. Noncausal dispersion relations (Blokhintsev and Kolerov, 1964) should be reconsidered, taking into account the departure from relativistic kinematics. As previously stressed (Gonzalez-Mestres, 1997d) , this apparent nonlocality may actually reflect the existence of superluminal sectors of matter (Gonzalez-Mestres, 1996) where causality would hold at the superluminal level (Gonzalez-Mestres, 1997a). Indeed, electromagnetism appears as a nonlocal interaction in the Bravais model of phonon dynamics, due to the fact that electromagnetic signals propagate much faster than lattice vibrations.
Very high-energy accelerator experiments (especially with protons and nuclei) can play a crucial role in the test of a possible Lorentz symmetry violation. To fit with cosmic-ray events, they should be performed in the very-forward region. At LHC, FELIX (e.g. Eggert, Jones and Taylor, 1997) could provide a crucial check of special relativity by comparing its data with cosmic-ray data in the $\approx 10^{16} - 10^{17}$ eV region. VLHC experiments would be expected to lead to fundamental studies in the kinematical region which, according to special relativity, would be equivalent to the collisions of $\approx 10^{19}$ eV cosmic protons. With a 700 TeV per beam $p - p$ machine, it would be possible to compare the very-forward region of collisions with those of cosmic protons at energies up to $\approx 10^{21}$ eV. Thus, it seems necessary that all very high-energy collider programs allow for an experiment able to cover secondary particles in the far-forward and far-backward regions. A model-independent way to test Lorentz symmetry between collider and cosmic-ray data should be carefully elaborated, but the basic phenomena involved in the case of Lorentz symmetry violation can be (Gonzalez-Mestres, 1997d and 1997h):

i) failure of the standard parton model (in any version, even incorporating radiative corrections and phase transitions);

ii) failure of the relativistic formulae for Lorentz contraction and time dilation;

iii) longer than predicted lifetimes for some of the produced particles (e.g. the $\pi^0$).

The role of high-precision data from accelerators would then be crucial to establish the existence of such phenomena in the equivalent cosmic-ray events. To reach the best possible performance, cosmic-ray experiments should (if ever feasible) install in coincidence very large-surface detectors (providing also the largest-volume target) with very large-volume underground or underwater detectors and with balloon or satellite devices able to study early cascade development. It would then be possible to perform unique tests of special relativity involving violations due to phenomena at some fundamental scale close to Planck scale, and even to determine the basic parameters of Lorentz symmetry violation (e.g. of deformed kinematics) and of physics at the fundamental length scale.

5 Superluminal particles

Lorentz invariance can be viewed as a symmetry of the motion equations, in which case no reference to absolute properties of space and time is required and the properties of matter play the main role (Gonzalez-Mestres, 1996). In a two-dimensional galilean space-time, the equation:

$$\alpha \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = F(\phi)$$

with $\alpha = 1/c_o^2$ and $c_o = \text{critical speed}$, remains unchanged under ”Lorentz” transformations leaving invariant the squared interval:

$$ds^2 = dx^2 - c_o^2 dt^2$$
so that matter made with solutions of equation (20) would feel a relativistic space-time even if the real space-time is actually galilean and if an absolute rest frame exists in the underlying dynamics beyond the wave equation. A well-known example is provided by the solitons of the sine-Gordon equation, obtained taking in (20):

$$F(\phi) = -\left(\frac{\omega_0}{c_0}\right)^2 \sin \phi$$  \hspace{1cm} (22)

where $\omega_0$ is a characteristic frequency of the dynamical system. A two-dimensional universe made of sine-Gordon solitons plunged in a galilean world would behave like a two-dimensional minkowskian world with the laws of special relativity. Information on any absolute rest frame would be lost by the solitons, as if the Poincaré relativity principle (Poincaré, 1905) were indeed a law of Nature, even if actually the basic equation derives from a galilean world with an absolute rest frame. The actual structure of space and time can only be found by going beyond the wave equation to deeper levels of resolution. At this stage, a crucial question arises (Gonzalez-Mestres, 1995): is the speed of light the only critical speed in vacuum, are there particles with a critical speed different from that of light? The question clearly makes sense, as in a perfectly transparent crystal it is possible to identify at least two critical speeds: the speed of light and the speed of sound. It has been shown (Gonzalez-Mestres, 1995 and 1996) that superluminal sectors of matter can be consistently generated replacing in the Klein-Gordon equation the speed of light by a new critical speed $c_i \gg c$ (the subscript $i$ stands for the $i$-th superluminal sector). All standard kinematical concepts and formulas (Schweber, 1961) remain correct, leading to particles with positive mass and energy which are not tachyons. We call them superbradyons as, according to standard vocabulary (Recami, 1978), they are bradyons with superluminal critical speed in vacuum. The rest energy of a superluminal particle of mass $m$ and critical speed $c_i$ will be given by the generalized Einstein equation:

$$E_{\text{rest}} = m c_i^2$$  \hspace{1cm} (23)

Energy and momentum conservation will in principle not be spoiled by the existence of several critical speeds in vacuum: conservation laws will as usual hold for phenomena leaving the vacuum unchanged. Each superluminal sector will have its own Lorentz invariance with $c_i$ defining the metric. Interactions between two different sectors will break both Lorentz invariances. Lorentz invariance for all sectors simultaneously will at best be explicit (i.e. exhibiting the diagonal sectorial Lorentz metric) in a single inertial frame (the vacuum rest frame, i.e. the ”absolute” rest frame). If superluminal particles couple weakly to ordinary matter, their effect on the ordinary sector will occur at very high energy and very short distance (Gonzalez-Mestres, 1997c), far from the domain of successful conventional tests of Lorentz invariance (Lamoreaux, Jacobs, Heckel, Raab and Forston, 1986; Hills and Hall, 1990). In particular, superbradyons naturally escape the constraints on the critical speed derived in some specific models (Coleman and Glashow, 1997; Glashow, Halprin, Krastev, Leung and Pantaleone, 1997). High-energy experiments can therefore open new
windows in this field. Finding some track of a superluminal sector (e.g. through violations of Lorentz invariance in the ordinary sector) may be a unique way to experimentally discover the vacuum rest frame. Furthermore, superbradyons can be the fundamental matter from which Planck-scale strings would actually be built. Superluminal particles lead to consistent cosmological models (Gonzalez-Mestres, 1997d), where they may well provide most of the cosmic (dark) matter. Although recent criticism to this suggestion has been emitted in a specific model on the grounds of gravitation theory (Konstantinov, 1997), the framework used is crucially different from the multi-graviton approach suggested in our papers, where each dynamical sector would generate its own graviton.

Conventional tests of special relativity are performed using low-energy phenomena. The highest momentum scale involved in nuclear magnetic resonance tests of special relativity is related to the energy of virtual photons exchanged, which does not exceed the electromagnetic energy scale $E_{\text{em}} \approx \alpha_{\text{em}} r^{-1} \approx 1 \text{ MeV}$, where $\alpha_{\text{em}}$ is the electromagnetic constant and $r$ the distance scale between two protons in a nucleus. However, the extrapolation between the $1 \text{ MeV}$ scale and the $1 - 100 \text{ TeV}$ scale (energies to be covered by LHC and VLHC) may involve a very large number, making compatible low-energy results with the possible existence of superluminal particles above $\text{TeV}$ scale. Assume, for instance, that between $E \approx 1 \text{ MeV}$ and $E \approx 100 \text{ TeV}$ the mixing between an "ordinary" particle (i.e. with critical speed in vacuum equal to the speed of light $c$ in the relativistic limit) of energy $E_0$ and a superluminal particle with mass $m_i$, critical speed $c_i \gg c$ and energy $E_i$ is described in the vacuum rest frame by a non-diagonal term in the energy matrix of the form (Gonzalez-Mestres, 1997c):

$$\epsilon \approx \epsilon_0 \rho \left( p^2 \right)$$

(24)

where $p$ stands for momentum, $\epsilon_0$ is a constant describing the strength of the mixing and $\rho \left( p^2 \right) = p^2 \left( p^2 + M^2 c^2 \right)^{-1}$ accounts for a threshold effect with $M c^2 \approx 100 \text{ TeV}$ due to dynamics. Then, the correction to the energy of the "ordinary" particle will be $\approx \epsilon^2 \left( E_0 - E_i \right)^{-1}$ whereas the mixing angle will be $\approx \epsilon \left( E_0 - E_i \right)^{-1}$. Taking the rest energy of the superluminal particle to be $E_{i,\text{rest}} = m_i c_i^2 \approx 1 \text{ TeV}$, we get a mixing $\approx 0.5 \epsilon_0$ at $p c = 100 \text{ TeV}$, $\approx 10^{-2} \epsilon_0$ at $p c = 10 \text{ TeV}$ and $\approx 10^{-4} \epsilon_0$ at $p c = 1 \text{ TeV}$. Such figures would clearly justify the search for superbradyons at LHC and VLHC ($E \approx 100 \text{ TeV per beam}$) machines provided low-energy bounds do not force $\epsilon_0$ to be too small. With the above figures, at $p c = 1 \text{ MeV}$ one would have a correction to the photon energy less than $\approx 10^{-32} \epsilon_0^2 p c_i$ which, requiring the correction to the photon energy not to be larger than $\approx 10^{-20}$, would allow for large values of $\epsilon_0$ if $c_i$ is less than $\approx 10^{12} c$. In any case, a wide range of values of $c_i$ and $\epsilon_0$ can be explored. More stringent bounds may come from corrections to the quark propagator at momenta $\approx 100 \text{ MeV}$. There, the correction to the quark energy would be bounded only by $\approx 10^{-24} \epsilon_0^2 p c_i$ and requiring it to be less than $\approx 10^{-20} p c$ would be equivalent to $\epsilon_0 < 0.1$ for $c_i = 10^6 c$. Obviously, these estimates are rough and a detailed calculation of nuclear parameters using
the deformed relativistic kinematics obtained from the mixing would be required. It must be noticed that the situation is fundamentally different from that contemplated in the TH$\epsilon\mu$ formalism and, in the present case, Lorentz invariance can remain unbroken in the low-momentum limit, as the deformation of relativistic kinematics for "ordinary" particles is momentum-dependent. Therefore, it may be a safe policy to explore all possible values of $c_i$ and $\epsilon_0$ at accelerators (including other possible parametrizations of $\epsilon$) without trying to extrapolate bounds from nuclear magnetic resonance experiments.

The production of one or two (stable or unstable) superluminal particles in a high-energy accelerator experiment is potentially able to yield very well-defined signatures through the shape of decay products or "Cherenkov" radiation in vacuum events (spontaneous emission of "ordinary" particles). In the vacuum rest frame, a relativistic superluminal particle would have energy $E \simeq p c_i$, where $c_i \gg c$ is the critical speed of the particle. When decaying into "ordinary" particles with energies $E_\alpha \simeq p_\alpha c (\alpha = 1, \ldots, N)$ for a $N$-particle decay product), the initial energy and momentum must split in such a way that very large momenta $p_\alpha \gg p$ are produced (in order to recover the total energy with "ordinary" particles) but compensate to give the total momentum $p$. This requires the shape of the event to be exceptionally isotropic, or with two jets back to back, or yielding several jets with the required small total momentum. Similar trends will arise in "Cherenkov-like" events, and remain observable in the laboratory frame. It must be noticed that, if the velocity of the laboratory with respect to the vacuum rest frame is $\approx 10^{-3} c$, the laboratory velocity of superluminal particles as measured by detectors (if ever feasible) would be $\approx 10^3 c$ in most cases (Gonzalez-Mestres, 1997a).

The possibility that superluminal matter exists, and that it plays nowadays an important role in our Universe, should be kept in mind when addressing the two basic questions raised by the analysis of any cosmic ray event: a) the nature and properties of the cosmic ray primary; b) the identification (nature and position) of the source of the cosmic ray. If the primary is a superluminal particle, it will escape conventional criteria for particle identification and most likely produce a specific signature (e.g. in inelastic collisions) different from those of ordinary primaries. Like neutrino events, in the absence of ionization (which will in any case be very weak) we may expect the event to start anywhere inside the detector. Unlike very high-energy neutrino events, events created by superluminal primaries can originate from a particle having crossed the earth. As in accelerator experiments (see the above discussion), an incoming, relativistic superluminal particle with momentum $p$ and energy $E_{in} \simeq p c_i$ in the vacuum rest frame, hitting an ordinary particle at rest, can release most of its energy into two ordinary particles or jets with momenta (in the vacuum rest frame) close to $p_{max} = \frac{1}{2} p c_i c^{-1}$ and oriented back to back in such a way that the two momenta almost cancel, or into several jets with a very small total momentum, or into a more or less isotropic event with an equally small total momentum. Then, an energy $E_R \simeq E_{in}$ would be transferred to ordinary secondaries. Corrections due to the earth motion must be applied (Gonzalez-Mestres, 1997a) before defining the expected event con-
figuration in laboratory experiments (AUGER, AMANDA...). At very high energy, such events would be easy to identify in large-volume detectors, even at very small rate. If the source is superluminal, it can be located anywhere (and even be a free particle) and will not necessarily be at the same place as conventional sources of ordinary cosmic rays. High-energy cosmic ray events originating form superluminal sources will provide hints on the location of such sources and be possibly the only way to observe them. At very high energies, the GZK cutoff does not in principle hold for cosmic ray events originating from superluminal matter: this is obvious if the primaries are superluminal particles that we expect to interact very weakly with the cosmic microwave background, but is also true for ordinary primaries as we do not expect them to be produced at the locations of ordinary sources and there is no upper bound to their energy around $100 \text{ EeV}^{-1}$. Besides "Cherenkov" deceleration, a superluminal cosmic background radiation may exist and generate its own GZK cutoffs. However, if there are large amounts of superluminal matter around us, they can be the main superluminal source of cosmic rays reaching the earth.

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