We investigate the properties and evolution of accretion tori formed after the coalescence of two compact objects. At these extreme densities and temperatures, the accreting torus is cooled mainly by neutrino emission produced primarily by electron and positron capture on nucleons ($\beta$-reactions). We solve for the disk structure and its time evolution by introducing a detailed treatment of the equation of state which includes photodisintegration of helium, the condition of $\beta$-equilibrium, and neutrino opacities. We self-consistently calculate the chemical equilibrium in the gas consisting of helium, free protons, neutrons, and electron-positron pairs and compute the chemical potentials of the species, as well as the electron fraction throughout the disk. We find that, for sufficiently large accretion rates ($M \gtrsim 10 \ M_\odot \text{yr}^{-1}$), the inner regions of the disk become opaque and develop a viscous and thermal instability. The identification of this instability might be relevant for GRB observations.

Subject headings: accretion, accretion disks — black hole physics — gamma rays: bursts — neutrinos
number of simplifying assumptions for the composition and the equation of state (EOS) of the accreting matter. In this paper we improve on our earlier results in several ways. Besides the requirement of time-dependent calculations, the high density and temperature regime in which the accreting gas lies, implies that both multidimensional numerical and semi-analytic calculations for such flows need to include the detailed microphysics. This includes photodisintegration of nuclei, the establishment of statistical equilibrium, neutronization, and the effects of neutrino opacities in the inner regions. Here, we introduce a detailed treatment of the equation of state, and calculate self-consistently the chemical equilibrium in the gas that consists of helium, free protons, neutrons, and electron-positron pairs. We compute the chemical potentials of the species, as well as the electron fraction throughout the disk using the assumption of the equilibrium between the $\beta$-processes. Our EOS equations include, self-consistently, the contribution of the neutrino trapping to the $\beta$-equilibrium. Another important addition compared to our previous work (Janiuk et al. 2004) is the inclusion of photodisintegration of helium. The presence of this term can affect the energy balance in the inner, opaque (to neutrinos as well as photons) region of the flow and, as it will be shown, it eventually produces a thermal and viscous instability in those regions. This is especially relevant since the GRB phenomenology requires a variable energy output.

Other time-dependent disk studies of binary mergers or collapsars have been performed in two dimensions (2D) using hydrodynamical simulations (e.g., MacFadyen & Woosley 1999; Ruffert & Janka 1999; Lee & Ramirez-Ruiz 2002; Rosswog et al. 2004; Lee et al. 2005) and, most recently, in three-dimensional (3D) simulations (Setiawan et al. 2004, 2006; Rosswog 2005). Also, MHD simulations of the GRB central engine have been performed, showing that the magnetic field possibly plays an important role in the generation of a GRB jet (Proga et al. 2003; Fujimoto et al. 2006). The advantage of our calculations is that, while including all the relevant physics to calculate the equation of state, the structure and stability of the accretion disk, we are able to study a much larger range of parameter space and allow our calculations to evolve beyond what can be reached in higher dimensional calculation and comparable to at least the short-burst durations.

The paper is organized as follows. In § 2 we describe the basic assumptions of the model and the method used in the initial stationary and subsequent time-dependent numerical simulations. In § 3 we discuss the structure of the hyperaccreting disk for various values of the initial accretion rate and study the time evolution of its density and temperature, as well as the resulting neutrino light curve. We also discuss the physical origin of the instabilities in the disk, and we compare our model and results with the recent 2D and 3D simulations. We summarize our results in § 4.

2. NEUTRINO-COOLED ACCRETION DISKS

In this section, we describe how we improve on our previous time-dependent calculation (Janiuk et al. 2004) by computing self-consistently the equation of state of the extremely dense matter by solving the balance of the $\beta$-reaction rates. This allows us to determine the chemical potentials of electrons, protons, and neutrons, as well as the electron fraction, in the initial disk configuration and throughout its evolution.

2.1. Initial Disk Configuration: 1D Hydrodynamics

We start by considering a steady state model of an accretion disk around a Schwarzschild black hole formed as a remnant structure either after a compact binary merger, or in a collapsar after the birth of a black hole (for a recent calculation in Kerr spacetime see Chen & Beloborodov 2007). Throughout our calculations we use the vertically integrated equations and hence derive a vertically averaged disk structure. We write the surface density of the disk as $\Sigma = H \rho$, where $\rho$ is the density and where the disk half-thickness (or disk height) is given by $H = c_s/\Omega_K$. Here the sound speed is defined by $c_s = (P/\rho)^{1/2}$ and $\Omega_K = (GM/R^3)^{1/2}$ is the Keplerian angular velocity with $P$ the total pressure. We note that, at very high accretion rates, the disk becomes moderately geometrically thick ($H \sim 0.5r_s$) in regions where neutrino cooling becomes inefficient and advection dominates. Our “slim disk” approximation neglects terms $\sim (H/r_s)^2$ and assumes that the fluid is in Keplerian rotation. For the disk viscous stress we use the standard $\alpha$ viscosity prescription of Shakura & Sunyaev (1973), where the stress tensor is proportional to the pressure:

$$\tau_{\alpha} = -\alpha P.$$  

We adopt a value of $\alpha = 0.1$.

We set the inner radius of the disk at $3R_s$, while the outer radius is at $50R_s$. The initial mass of such a disk is about 0.35 $M_\odot$ for an accretion rate $\dot{M} = 1 M_\odot$ s$^{-1}$. Throughout the calculations we adopt a black hole mass of $M = 3 M_\odot$.

2.2. The Equation of State

We assume that the torus consists of helium, electron-positron pairs, free neutrons, and protons. The total pressure is contributed by all particle species in the disk, and the fraction of each species is determined by self-consistently solving the balance of the $\beta$-reaction rates. In the equation of state we take into account the pressure due to the free nuclei and pairs, helium, radiation, and the trapped neutrinos:

$$P = P_{\text{nucl}} + P_{\text{He}} + P_{\text{rad}} + P_{\nu}.$$  

The component $P_{\text{nucl}}$ includes free neutrons, protons, and the electron-positron pair gas in $\beta$-equilibrium:

$$P_{\text{nucl}} = P_{e^-} + P_{e^+} + P_n + P_p,$$  

with

$$P_i = \frac{2\sqrt{2}}{3\pi^3} \frac{(m_i e^2)^4}{(hc)^6} \beta_i^{5/2} \left[F_3(\eta_i, \beta_i) + \frac{1}{2} \beta_i F_5(\eta_i, \beta_i)\right],$$  

where $F_i$ are the Fermi-Dirac integrals of the order $k$, and $\eta_i, \eta_p$, and $\eta_n$ are the reduced chemical potentials of electrons, protons, and neutrons in units of $kT$, respectively (where $\eta_i = \mu_i/kT$, also known as the degeneracy parameter, where $\mu_i$ the standard chemical potential), calculated from the chemical equilibrium condition (§ 2.3). The reduced chemical potential of positrons is $\eta_{p+} = -\eta_p - 2/\beta_i$ and the relativity parameters of the species $i$ are defined as $\beta_i = kT/m_i c^2$.

Under the physical conditions in the torus, helium is generally nonrelativistic and nondegenerate; therefore, its pressure is given by

$$P_{\text{He}} = n_{\text{He}}kT,$$  

where $n_{\text{He}}$ is the number density of helium. This is defined as

$$n_{\text{He}} = \frac{1}{4} \eta_0 (1 - X_{\text{nucl}}),$$
and the fraction of free nucleons is given by
\[ X_{\text{nucl}} = 295.5 \rho_{10}^{-3/4} T_{11}^{9/8} \exp (-0.8209/T_{11}), \tag{7} \]
where \( T_{11} \) is the temperature in unit of \( 10^{11} \) K (e.g., Qian & Woosley 1996; Popham et al. 1999).

The radiation pressure is given by
\[ P_{\text{rad}} = \frac{1}{3} \frac{\pi^2}{15} \left( \frac{kT}{hc} \right)^4. \tag{8} \]

When neutrinos become trapped in the disk, the neutrino pressure is nonzero. Following the treatment of photon transport under the two-stream approximation (Popham & Narayan 1995; Di Matteo et al. 2002), we have
\[ P_{\nu} = \frac{7}{8} \frac{\pi^2}{15} \left( \frac{kT}{hc} \right)^4 \sum_{i=e,\mu,\tau} \left( \frac{1}{2} \left( \tau_{\nu,i} + \tau_{\bar{\nu},i} \right) + 1/\sqrt{3} \right) = \frac{7}{8} \frac{\pi^2}{15} \left( \frac{kT}{hc} \right)^4 b, \tag{9} \]
where \( \tau_i \) is the scattering optical depth due to the neutrino scattering on free neutrons and protons and \( \tau_{\nu,i} \) and \( \tau_{\bar{\nu},i} \) are the absorptive optical depths for electron and muon neutrinos, respectively (see § 2.3). The contribution from tau neutrinos is the same as that from muon neutrinos. These optical depths and neutrino absorption processes (which are the reverse of the emission processes) are discussed in more detail in the Appendix.

In the disk we have to consider both the neutrino transparent and opaque regions, as well as the transition between the two. In the transparent case, the neutrinos are not thermalized and the chemical potential of neutrinos is negligible. On the other hand, when neutrinos are totally trapped, the chemical equilibrium condition yields \( \mu_e + \mu_p = \mu_n + \mu_p \). The chemical potential of neutrinos is a parameter that depends on how many neutrinos and antineutrinos are trapped, and assuming that the number densities of the trapped neutrinos and antineutrinos are the same, \( \mu_e \), can be set to zero. In order to determine the distribution function of the partially trapped neutrinos, in principle one should solve the Boltzmann equation. To simplify this problem, we use here a “graybody” model, and we introduce a blocking factor \( b = \sum_{i=e,\mu,\tau} b_i \) to describe the extent to which neutrinos are trapped (see, e.g., Sawyer 2003). In terms of this factor, we write the distribution function of neutrinos as
\[ f_{\nu,i}(p) = \frac{b_i}{\exp \left( \frac{pc}{kT} \right) + 1} = b_i f_{\nu,i} \text{ for } 0 \leq b_i \leq 1. \tag{10} \]
This simplified assumption is consistent with the two-stream approximation which we adopt here (eq. [9]).

### 2.3. Composition and Chemical Equilibrium

The equilibrium state of the gas in the accreting torus is completely determined by the chemical potentials of neutrinos, protons, and electrons \((\eta_e, \eta_n, \eta_p)\), and the trapping factor of neutrinos \((b)\) which is related to the optical depths of neutrinos (see eq. [9]).

For a given baryon number density, \( n_b \), temperature \( T \), a value for accretion rate \( \dot{M} \), and viscous constant \( \alpha \), the chemical potentials, or equivalently the ratio of free protons \( x = n_p/n_b \), are determined from the condition of equilibrium between the transition reactions from neutrinos to protons and from protons to neutrinos. These reactions are
\[ p + e^{-} \rightarrow n + \nu_e, \tag{11} \]
\[ p + \bar{\nu}_e \rightarrow n + e^{+}, \tag{12} \]
\[ p + e^{-} \rightarrow \bar{\nu}_e + n, \tag{13} \]
\[ n + e^{+} \rightarrow p + \nu_e, \tag{14} \]
\[ n \rightarrow p + e^{-} + \bar{\nu}_e, \tag{15} \]
\[ n + \nu_e \rightarrow p + e^{-}. \tag{16} \]
Therefore, we have to calculate the ratio of protons that will satisfy the balance:
\[ n_p \left( \Gamma_{p+e^{-}\rightarrow n+\nu_e} + \Gamma_{p+\bar{\nu}_e\rightarrow n+e^{+}} + \Gamma_{p+e^{-}+\bar{\nu}_e\rightarrow \bar{\nu}_e} \right) = n_n \left( \Gamma_{n+e^{+}\rightarrow p+\nu_e} + \Gamma_{n-p+e^{-}+\nu_e} + \Gamma_{n+\nu_e\rightarrow p+e^{-}} \right). \tag{17} \]

The reaction rates are the sum of forward and backward rates and are given in the Appendix (see also Kohri et al. 2005).

These are supplemented by two additional conditions: the conservation of the baryon number, \( n_b = n_p + n_e \), and charge neutrality (Yuan 2005),
\[ n_e = n_e^{-} - n_e^{+} = n_p + n_0^{e}, \tag{18} \]
which says that the net number of electrons is equal to the number of free protons plus the number of protons in helium:
\[ n_0^{e} = 2 n_{he} = \frac{1}{2} \left( 1 - X_{\text{nucl}} \right)^{n_h}. \tag{19} \]

The number density of fermions under arbitrary degeneracy is determined by
\[ n_i = \sqrt{\frac{2}{\pi}} \frac{m_i c^2}{h} \left( \frac{\mu_i}{\hbar c} \right)^{3/2} \left[ F_{1/2}(\eta_i, \beta) + \beta_i F_{3/2}(\eta_i, \beta) \right]. \tag{20} \]
Finally, the electron fraction is defined as
\[ Y_e = \frac{n_e^{-} - n_e^{+}}{n_h}. \tag{21} \]
(Note that this is different from \( Y_e = 1/(1 + n_e/n_p) \), which is only valid for free \( n_p-e \) gas.)

### 2.4. Neutrino Cooling

The processes that are responsible for the neutrino emission in the disk are electron-positron pair annihilation \((e^{-} + e^{+} \rightarrow \nu_e + \bar{\nu}_e)\), bremsstrahlung \((n + n \rightarrow n + n + \nu_e + \bar{\nu}_e)\), plasmon decay \( (\gamma \rightarrow \nu_e + \bar{\nu}_e) \), and URCA process (reactions 11, 14, and 15). The first two processes produce neutrinos of all flavors, while the other produce only electron neutrinos and antineutrinos.

The cooling rate due to pair annihilation is expressed as
\[ q_{e^{-}e^{+}} = q_{\nu_e} + q_{\bar{\nu}_e} + q_{\nu_e}, \tag{22} \]
where the cooling rates for all three neutrino flavors are calculated by means of Fermi-Dirac integrals and are given in the Appendix (see also Itoh et al. 1996).

The cooling rate due to nucleon-nucleon bremsstrahlung (in erg cm\(^{-3}\) s\(^{-1}\)) is given by
\[ q_{\text{brem}} = 3.35 \times 10^{27} \rho_{10}^{2} T_{11}^{5.5}, \tag{23} \]
where \( \rho_{10} \) is the baryon density in units of \( 10^{10} \text{ g cm}^{-3} \) and \( T_{11} \) is temperature in units of \( 10^{11} \text{ K} \).

The cooling rate due to the plasmon decay (in erg cm\(^{-3}\) s\(^{-1}\)) is

\[
q_{\text{plasmon}} = 1.5 \times 10^{32} \rho_{10}^6 \gamma_p (1 + \gamma_p) \left( 2 + \frac{\gamma_p^2}{1 + \gamma_p} \right),
\]  
\( q_{\text{plasmon}} \)

where \( \gamma_p = 5.565 \times 10^{-2} \left\{ \left( \frac{\pi^2}{4} + 3\left( \frac{m_e}{kT} \right)^2 \right) / 3 \right\}^{1/2} \).

The cooling rate due to the URCA reactions is given by the three emissivities:

\[
q_{\text{urca}} = q_{\text{e}^- + \alpha + \nu_e} + q_{\text{e}^- + \alpha - \nu_e} + q_{\alpha - \nu_e + \bar{\nu}_e}.
\]

The emissivities are given in the Appendix.

Note that the blocking factor of the trapping neutrinos is used only for the emissivities of the URCA reactions. For simplicity, we neglect the blocking effects of neutrinos when calculating the emissivities for the electron-positron pair annihilation. Two reasons make this approximation reasonable: the emissivities for the electron-positron pair annihilation is much smaller than those of the URCA reactions, and the electron-positron pair annihilation does not change the electron fraction which sensitively affects the EOS.

Each of the above neutrino emission process has a reverse process, which leads to neutrino absorption. These are given by equations (12), (13), and (16). Therefore, we introduce the absorptive optical depths for neutrinos given by

\[
\tau_{a,i} = \frac{H}{4(7/8)\sigma T^4 q_{a,i}},
\]

where absorption of the electron neutrinos is determined by

\[
q_{a,\nu_e} = q_{\text{e}^- + \alpha + \nu_e} + q_{\text{e}^- + \alpha - \nu_e} + q_{\alpha - \nu_e + \bar{\nu}_e} + \frac{1}{3} q_{\text{brems}},
\]

and for the muon neutrinos,

\[
q_{a,\nu_\mu} = q_{\text{e}^- + \alpha + \nu_e} + \frac{1}{3} q_{\text{brems}}.
\]

In addition, the free escape of neutrinos from the disk is limited by scattering. The scattering optical depth is given by

\[
\tau_s = \tau_{s,\nu_e} + \tau_{s,\nu_\mu} = 24.28 \times 10^{-5} \left[ \frac{kT}{m_c c^2} \right]^2 \left( C_{s,\nu_e} + C_{s,\nu_\mu} \right),
\]

where

\[
C_{s,\nu_e} = 2(1 - 2) + 5 \alpha^2 / 24, \ C_{s,\nu_\mu} = (1 + 2) / 2 \sin^2 \theta_C, \ \alpha = 1.25, \ \sin^2 \theta_C = 0.23.
\]

The neutrino cooling rate is then given by

\[
Q_\nu = \frac{(7/8)\sigma T^4}{3/4 \sum_{i=e,\mu} \left( \tau_{a,\nu_e} + \tau_{a,\nu_\mu} \right)} / (2 + 1/\gamma_p + 1/3 \tau_s),
\]

2.5. Energy and Momentum Conservation

The hydrodynamic equations we solve to calculate the disk structure are the standard mass, energy, and momentum conservation. Making use of the standard disk equations, the vertically integrated viscous heating rate (per unit area) over a half-thickness \( H \) is given by

\[
F_{\text{tot}} = \frac{3GM\dot{M}}{8\pi R^3} f(r),
\]

where the Newtonian boundary condition is assumed: \( f(r) = 1 - (r_{\text{min}}/r)^{12} \). Note that in the time-dependent calculations, instead of equation (31), we will solve the viscous diffusion equation (eq. [44]).

Using mass and momentum conservation \( \dot{M} = 4\pi \rho R \nu, \approx \theta / \nu, \rho H, \) \( \nu \approx \nu(2/\nu) \) and \( \nu = (2\alpha)/(3\Omega) \) is the kinematic viscosity. The viscous heating rate can be written in terms of \( \alpha, \)

\[
Q_{\nu,\text{visc}} = \frac{3}{2} \alpha \Omega H P.
\]

Cooling in the disk is due to advection, radiation, and neutrino emission. The advective cooling in a stationary disk is determined approximately as

\[
Q_{\nu,\text{adv}} = \sum_{\nu} \frac{dS}{dY} = q_{\nu,\text{adv}} \frac{\alpha \Omega H P}{\Omega \rho^{-1}} S,
\]

where

\[
q_{\nu,\text{adv}} \propto \frac{d \ln S}{d \ln r} \propto \frac{d \ln T}{d \ln r} \propto \frac{d \ln \rho}{d \ln r} \approx \text{constant}
\]

and we adopt the value of 1.0. The entropy density \( S \) is the sum of four components:

\[
S = S_{\text{nucl}} + S_{\text{He}} + S_{\text{rad}} + S_{\nu}.
\]

The entropy density of the gas of free protons, neutrons, and electron-positron pairs is given by

\[
S_{\text{nucl}} = S_{\text{e}^-} + S_{\text{e}^+} + S_{\text{p}} + S_{\nu},
\]

where

\[
\frac{S_{\text{e}^-}}{k} = \frac{1}{kT} (\epsilon_1 + P_i) - n_i \eta_i,
\]

and

\[
\epsilon_1 = 2 \sqrt{2} \frac{(m_e c^2)^4}{\pi^2} \frac{\delta_{1/2}[P/3/2(\eta_i, \beta_i) + \beta_{i/2}[P/5/2(\eta_i, \beta_i)]}
\]

is the energy density of electrons, positrons, protons, or neutrons; \( P_i \) is their pressure, given by equation (4); \( n_i \) are the number densities; and \( \eta_i \) are the chemical potentials.

The entropy density of helium is given by

\[
S_{\text{He}} = n_{\text{He}} \left[ \frac{5}{2} + 3 \log \left( n_{\text{He}} \frac{kT}{m_e c^2} \frac{1}{2 \pi} - \log n_{\text{He}} \right) \right],
\]

for \( n_{\text{He}} > 0 \).

The entropy density of radiation is

\[
S_{\text{rad}} = 4P_{\text{rad}} / kT,
\]

while for neutrinos we have

\[
S_{\nu} = 4P_{\nu} / kT.
\]

In the initial disk configuration we assume that \( q_{\nu,\text{adv}} \) is approximately constant and of order of unity, but in the subsequent
time-dependent evolution the advection term is calculated with the appropriate radial derivatives. For the case of photon and electron-positron pairs in the plasma the radiative cooling is equal to

\[ Q_{\text{rad}} = \frac{3P_{\text{rad}} c}{4r} = \frac{11 \pi T^4}{4 \kappa \Sigma}, \tag{41} \]

where we adopt the Rosseland mean opacity \( \kappa = 0.4 + 0.64 \times 10^{23} \rho T^{-3} \) \( \text{cm}^2 \text{ g}^{-1} \).

An important term in the cooling and heating balance in the disk is due to photodisintegration of \( \alpha \)-particles, with rate

\[ Q_{\text{photo}} = q_{\text{photo}} H, \tag{42} \]

where

\[ q_{\text{photo}} = 6.28 \times 10^{28} \rho_{10} v_r \frac{dX_{\text{nuc}}}{dr}, \tag{43} \]

and \( X_{\text{nuc}} \) is given by equation (7). Finally, in order to calculate the initial stationary configuration, we solve the energy balance to the appropriate radial derivatives. For the case of photon and time-dependent evolution the advection term is calculated with the convenient change of variables \( \nu = \nu_0 \), \( T = T_0 \), where we adopt the Rosseland mean opacity \( \kappa = 0.4 + 0.64 \times 10^{23} \rho T^{-3} \) \( \text{cm}^2 \text{ g}^{-1} \).

An important term in the cooling and heating balance in the disk is due to photodisintegration of helium now must be proportional to the energy equation via the radial derivatives. The cooling term consists of radiative and neutrino cooling, given by equations (41) and (30). Advection is included in the energy equation via the radial derivatives. The cooling term due to photodisintegration of helium now must be proportional to the full time derivative of \( X_{\text{nuc}} \) (cf. eq. [43])

\[ Q_{\text{photo}} \propto \nu v_{\text{X}_{\text{nuc}}} \frac{\partial X_{\text{nuc}}}{r} + \frac{\partial X_{\text{nuc}}}{\partial t}. \tag{46} \]

2.6. Time Evolution

After solving for the initial disk configuration, we allow the density and temperature to vary with time. We solve the time-dependent equations of mass and angular momentum conservation in the disk,

\[ \frac{\partial \Sigma}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ 3 \nu^{1/2} \frac{\partial}{\partial r} \left( \nu^{1/2} \nu \Sigma \right) \right], \tag{44} \]

and the energy equation,

\[ \frac{\partial T}{\partial t} + \nu_r \frac{\partial T}{\partial r} = \frac{T}{4} \left( \frac{\partial \Sigma}{\partial t} + \nu \frac{\partial \Sigma}{\partial r} \right) \]

\[ \quad + \frac{1}{PH} \left( \frac{\partial \Sigma}{\partial r} \right) \left( Q_+ - Q_- \right), \tag{45} \]

where \( \chi = (P - P_{\text{rad}})/P \).

The cooling term \( Q_- \) consists of radiative and neutrino cooling, given by equations (41) and (30). Advection is included in the energy equation via the radial derivatives. The cooling term due to photodisintegration of helium now must be proportional to the full time derivative of \( X_{\text{nuc}} \) (cf. eq. [43])

\[ Q_{\text{photo}} \propto \nu v_{\text{X}_{\text{nuc}}} \frac{\partial X_{\text{nuc}}}{r} + \frac{\partial X_{\text{nuc}}}{\partial t}. \tag{46} \]

2.7. Numerical Method

The initial configuration of the disk is calculated by means of the Newton-Raphson method, iterated with the hydrostatic equilibrium condition. We interpolate over the matrix of precalculated results for the equation of state (pressure and entropy) and neutrino cooling rate (the number of points is 1024 \( \times \) 1024). The Fermi-Dirac integrals are calculated using the mixture of Gaussian-Legendre and Gaussian-Laguerre quadratures (Aparicio 1998).

Having determined the initial radial profiles of density and temperature, as well as the other quantities at time \( t = 0 \), we start the time evolution of the disk. We solve the set of equations (44), (46), and (46) using the convenient change of variables \( y = 2 \nu^{1/2} \) and \( \Sigma = y^{\nu} \Sigma \), at fixed radial grid, equally spaced in \( y \) (see Janiuk et al. 2002 and references therein). The number of radial zones is set to 200, which we found to be an adequate resolution.

3. RESULTS

We first analyze the pressure, entropy, and neutrino cooling rate distributions for a given temperature and baryon density in the gas. Then, we show the disk structure for a converged static disk model and finally we show examples of time evolution of the neutrino luminosity, density, temperature, and electron fraction for given sets of parameters.

3.1. EOS Solutions for a Given Temperature and Density

In Figure 1 and 2 we plot the results of the numerically calculated equation of state for the hot and dense matter. The plots show the dependence of the electron fraction, pressure, entropy, and neutrino cooling rate on temperature and density, respectively.

In the upper panels, we show the neutrino cooling rate. At low temperatures, below \( T = m_n c^2 \approx 5 \times 10^9 \) K, there are almost no positrons and free nucleons. Therefore, the neutrino emission processes switch off, and the cooling of the gas is either due to advection, or, when the matter becomes transparent to photons, radiative cooling overtakes. For larger temperatures, the neutrino emission rate increases up to the temperature of about \( 5 \times 10^{11} \) K. For very high temperatures, the optical depths for neutrinos increase very rapidly \( (\nu \propto T^3) \), see eq. [7] in Di Matteo et al. 2002). Therefore, the neutrino cooling rate decreases at high temperatures (eq. [30]). On the other hand, for a given temperature...
(e.g., $T \sim 10^{11}$ K), the neutrino cooling rate does not sensitively depend on density. It varies by 1 order of magnitude in the range of $10^{8} \leq \rho \leq 10^{12}$ g cm$^{-3}$, where the optical depth is $\tau \sim 100$.

The middle panels of Figures 1 and 2 show the entropy and pressure as a function of temperature and density. At low temperatures, the entropy of gas is not important. The highly degenerate electrons do not give contribution to the entropy, while they are a dominant term in the pressure, which is therefore independent of temperature up to $T \sim 5 \times 10^{10}$ K. When the temperature increases, helium becomes disintegrated into free nucleons at energy comparable to the binding energy of helium, and after that the radiation (including photons and electron-positron pairs) contributes mainly to the total entropy and pressure. Therefore, both these quantities rise with temperature. At high densities, the entropy is dominated by neutrons.

Finally, the electron fraction is shown in the bottom panel of Figures 1 and 2. At low temperatures, the electron fraction is equal to 0.5, it then decreases sharply as the helium nuclei become disintegrated. As the temperature further increases, positrons appear as the electrons become nondegenerate. The positron capture again increases the electron fraction (see Fig. 1).

The electron fraction changes significantly as a function of density for $T > 10^{10}$ K (in Fig. 2, $T = 10^{11}$ K). At low densities, the torus consists of free neutrons and protons and $Y_e$ is close to 0.5 (see also eq. [7]). As density increases, $Y_e$ decreases to satisfy the $\beta$-equilibrium among the free $n$-$p$-$e$ gas. Above some density (when the temperature is high enough, e.g., for Fig. 2, $\rho_{\text{hel}} \approx 10^{13}$ g cm$^{-3}$) helium starts forming. Therefore, $Y_e$ has a kink and starts rising steeply, asymptotically approaching 0.5 as the torus consists of plenty of ionized helium and some electrons to keep charge neutrality.

3.2. The Steady State Disk Structure

In Figures 3 and 4 we show the profiles of density and temperature in the stationary accretion disk model for three accretion rates: 1, 10, and 12 $M_\odot$ s$^{-1}$. In general, the temperature and density profiles both increase inward.

However, for $M = 12 M_\odot$ s$^{-1}$, a distinct branch of solutions is reached, which appears different than the so-called “NDAF” branch (see Kohri & Mineshige 2002). The density and temperature profiles for this high accretion rate differ also from what was found in previous work (Di Matteo et al. 2002; Janiuk et al. 2004). Due to a more detailed equation of state, in which we allow for a partial degeneracy of nucleons and electrons as well as neutrino trapping, our solutions reach densities as high as $10^{12}$ g cm$^{-3}$ in
the innermost radii of the disk. The temperature in this inner disk part is in the range $4 \times 10^{10}$ to $1.25 \times 10^{11}$ K, depending on the accretion rate. For the hottest disk model, a local peak in the density forms around 7–8 $R_\odot$, while below that radius the density decreases. Between $\sim 3.5$ and 7 $R_\odot$, the plasma becomes much hotter and less dense than outside of this region. This means that the macroscopic state of the system is different here due to an abrupt change in the heat capacity. In order to check what is the reason for this transition, we investigate the pressure distribution in the disk.

The profile of the pressure is shown in Fig. 5. The dominant term in the total pressure is due to the nucleons, while the radiation pressure (including electron-positron pairs) is always several orders of magnitude smaller. The neutron pressure is large in the inner disk, once it gets optically thick to neutrinos (i.e., for $\dot{M} \gtrsim 10 M_\odot$ s$^{-1}$). A significant contribution to the pressure is due to helium at densities high enough for helium to form, albeit at temperatures low enough such that its nuclei are not fully disintegrated. For the largest accretion rate shown, in the region of the temperature excess and inverse density gradient (3.5–7 $R_\odot$), the total pressure distribution flattens. The helium pressure is now vanishingly small due to the complete photodisintegration, and the nuclear pressure is slightly decreased due to the composition change: smaller number density of neutrons and larger number density of protons. The substantial contribution to the pressure is now given by the neutrons (large optical depths; see below) and radiation pressure (increased number of electron-positron pairs). From the comparison of Figures 3 and 4 and the bottom panel of Figure 5, it can be seen that the total pressure becomes locally correlated with temperature and anticorrelated with density, thus constituting an unstable phase.

In Figure 6 we show the neutrino optical depths due to scattering and absorption. The total optical depth in the outer disk is typically dominated by scattering processes, while in the inner disk absorption processes take over for very high accretion rates. For $\dot{M} = 1 M_\odot$ s$^{-1}$ only the very inner disk radii have optical depth close to 1. For $\dot{M} = 12 M_\odot$ s$^{-1}$, in the radial strip of $\sim 3.5–7 R_\odot$ the disk is optically thick with absorptive optical depth for electron neutrinos exceeding the scattering term and reaching values of the order of 100.

### 3.2.1. Composition and Chemical Potentials

In Figure 7 we show the distribution of the reduced chemical potentials of protons, electrons, and neutrons throughout the disk. Reduced electron chemical potentials much larger than unity (indicating strong electron degeneracy) are found in the inner disk parts for $\dot{M} = 10$ and 12 $M_\odot$ s$^{-1}$, whereas for 1 $M_\odot$ s$^{-1}$ electrons are only slightly degenerate. For the highest accretion rate, the maximum degeneracies correspond to the radius of the local peak in the density (e.g., Fig. 3) and the excess of helium number density (e.g., Fig. 8). Below this radius, the species become nondegenerate again, contributing to the increase of the electron fraction (e.g., Fig. 9).

In Figure 8 we plot the mass fraction of free nucleons as a function of radius for $\dot{M} = 12$, 10, and 1 $M_\odot$ s$^{-1}$. As the figure shows, in the outer regions, $X_{\text{nuc}}$ increases as the radius decreases, while the temperature and density increase (Figs. 3 and 4). Consistent with the behavior of $Y_e$ (Fig. 2), $X_{\text{nuc}}$ subsequently turns around (decreases) at radii where the density is high enough for significant helium formation. This trend is reversed sharply for highest accretion rates, when the temperature in the disk is high enough (Fig. 4) for helium to be fully dissociated. In consequence, the number density of $\alpha$-particles increases at $\sim 7–12 R_\odot$ and sharply decreases at lower radii. A similar, but far less pronounced fluctuation in $X_{\text{nuc}}$ is seen at smaller radii for the case of $\dot{M} = 10 M_\odot$ s$^{-1}$. For smallest accretion rate, 1 $M_\odot$ s$^{-1}$, there is no helium throughout the disk.

In Figure 9 we show the radial distribution of the electron fraction throughout the disk for 1, 10, and 12 $M_\odot$ s$^{-1}$. For the case of 1 $M_\odot$ s$^{-1}$ (solid line), the electron fraction decreases inward in
the disk as the electrons are captured by protons (in neutronization reactions). Once the electrons become nondegenerate, positrons appear, and the positron capture by neutrons again increases the electron fraction. For the hotter plasma (accretion rate of $10 \ M_\odot \ s^{-1}$, dashed line), consistently with the behavior discussed for $X_{\text{nuc}}$, helium nuclei form as the density becomes high enough below $\sim 20 R_S$ and $Y_e$ increases. For the accretion rate of $12 \ M_\odot \ s^{-1}$ there is the sharp decrease in $Y_e$, at $\sim 7-8 R_S$, due to the sudden dissociation of helium. As helium is fully photodissociated, there is an almost equal number of neutrons and protons due to the balance of the electron and positron capture. This implies an electron fraction of 0.5. At the innermost radius, the temperature and density drop due to the boundary condition, which affects the behavior of both $Y_e$ and $X_{\text{nuc}}$.

3.2.2. Cooling and Heating Rates

In Figure 10 we plot the rates of viscous heating, advection, and cooling due to neutrino emission and photodissociation in the stationary disk. The accretion rates are $\dot{M} = 1 \ M_\odot \ s^{-1}$ (upper line), $\dot{M} = 10 \ M_\odot \ s^{-1}$ (middle), and $\dot{M} = 12 \ M_\odot \ s^{-1}$ (bottom). The other terms are: cooling rate due to advection (long-dashed line) and viscous heating rate (short-dashed line).
panel), $M = 10 M_\odot \text{ s}^{-1}$ (middle panel), and $M = 12 M_\odot \text{ s}^{-1}$ (lower panel). For the highest accretion rates, in the innermost disk the neutrino cooling rate decreases substantially with respect to the cooling by photodissociation. This is because the neutrinos are trapped in the disk due to a large opacity. The smaller the accretion rate, the less important is the neutrino-trapping effect. This implies that for an accretion rate of $\leq 10 M_\odot \text{ s}^{-1}$ neutrinos can escape from the innermost disk.

The advective term is a couple orders of magnitude smaller than the other terms. The photodissociation term is negligible for an accretion rate of $1 M_\odot \text{ s}^{-1}$, since there is no helium in the whole disk, and $Q_{\text{photo}}$ is equal to zero by definition. For an accretion rate of $10 M_\odot \text{ s}^{-1}$ there is very little helium down to about $15-20 R_S$, and therefore $Q_{\text{photo}}$ is much smaller than other terms. For the accretion rate of $12 M_\odot \text{ s}^{-1}$, down to $6-10 R_S$ in the region of the disk of high density and maximum degeneracy, helium nuclei form. The nucleosynthesis of $\alpha$-particles leads to the plasma heating instead of cooling, and therefore the relevant term in the energy balance has a negative value. Outward, above $\sim 10 R_S$, there is some fraction of helium which can be photodissociated, so the cooling term due to this reaction is also important in the total energy balance. In the inner region helium is fully dissociated and $Q_{\text{photo}}$ is equal to zero, increasing again only near the inner boundary due to the local density increase and decrease of temperature.

3.3. Stability Analysis: Instabilities at High $\dot{M}$

The disk is thermally unstable if $\frac{d}{d \log T} \frac{d \log Q^-}{d \log \Sigma} > \frac{d \log Q^-}{d \log T}$. Then any small increase (decrease) in temperature leads to a heating rate which is more (less) than the cooling rate, and as a consequence a further increase (decrease) of the temperature. The viscous instability, which appears when $\frac{\partial M}{\partial \Sigma} < 0 |Q^- = 0$, manifests itself in a faster (slower) evolution of an underdense (overdense) region. The instabilities can be conveniently located in the surface density-temperature diagrams, in which the branch of thermal equilibrium solutions with a negative slope is not only unstable to the perturbations in the surface density, but it is also thermally unstable.

In Figure 11 we show such stability curves for several radii in the disk. The criterion for a viscously stable disk is generally satisfied throughout the whole disk for $M \leq 10 M_\odot \text{ s}^{-1}$. However, for larger accretion rates, there are unstable branches at the smallest radii. For $M = 10 M_\odot \text{ s}^{-1}$, the disk becomes unstable below $5 R_S$, while for $M = 12 M_\odot \text{ s}^{-1}$, the instability strip is up to $\sim 7 R_S$. Here helium is almost completely photodisintegrated while the electrons and protons become nondegenerate again. For this high accretion rate, the electron fraction rises inward in the disk. Under these conditions, the energy balance is affected leading to the thermal and viscous instability, as demonstrated by the stability curves. This instability will be discussed in more detail in § 4.1.

3.4. Time-Dependent Solutions

In this section, we discuss how the temperature, density, electron fraction, and disk luminosity evolve with time. In Figures 12 and 13 we show the time evolution of density and temperature, when the initial accretion rate is $1 M_\odot \text{ s}^{-1}$. These quantities exponentially decrease with time:

$$\rho = \rho_0(t) \exp(-a t), \quad (47)$$

and

$$T = T_0(t) \exp(-b t), \quad (48)$$

where $a \approx 1.9$ and $b \approx 0.085$. The normalization of these relations depends on the radius, and for example for $r = 6 R_S$ it is $\rho_0 = 2.2 \times 10^{11}$ and $T_0 = 3.5 \times 10^{10}$. The exponential behavior arises from the nature of energy equation (45).

In Figure 14 we show the electron fraction as a function of time for several exemplary radial locations in the disk, for the disk evolving from a starting accretion rate of $1 M_\odot \text{ s}^{-1}$. The fraction $Y_e$ is smaller in the inner disk radii, while outward, the electron fraction is over half an order of magnitude higher. Altogether,
during the evolution of the system, the electron fraction constantly increases with time throughout the disk.

The time-dependent neutrino luminosity of the disk is given by

\[ L_\nu(t) = \int_{R_{\text{min}}}^{R_{\text{max}}} Q_\nu(t) 2\pi r dr, \]

where \( Q_\nu \) is given by equation (30).

In Figure 15 we show an example of such a light curve, for our standard model parameters \( \dot{M} = 3 \, M_\odot \, \text{s}^{-1}, \, \alpha = 0.1, \, \) and \( R_{\text{max}} = 50R_\odot \). The starting accretion rate is \( \dot{M}_{\text{start}} = 1 \, M_\odot \, \text{s}^{-1} \). At this accretion rate neutrinos can already escape from the accretion disk at the beginning of the evolution. For higher initial accretion rates, e.g., \( 10-12 \, M_\odot \, \text{s}^{-1} \), neutrinos are trapped in the innermost disk, and, as a consequence, the neutrino luminosity is lower at the initial stages of disk evolution, until the accretion rate drops to about \( \sim 1 \, M_\odot \, \text{s}^{-1} \). This result is qualitatively similar, albeit it differs quantitatively, from what was obtained in Di Matteo et al. (2002) and Janiuk et al. (2004); in those calculations neutrino trapping was far more substantial even for a “moderate” accretion.
The rate of $\dot{M}$ is 12 $M_\odot$ s$^{-1}$. The difference arises from the fact that here we calculate the neutrino opacities using the $\beta$-reaction efficiencies, self-consistently with the equation of state.

For an accretion rate of 1 $M_\odot$ s$^{-1}$, the solution does not reach the viscously unstable branch. Initially, the disk contains almost no $\alpha$-particles (e.g., Fig. 8), which appear later on during the evolution and cooling of the plasma. The dynamical balance between the photodisintegration of helium and nucleosynthesis leads to an additional nonzero cooling/heating term in the energy equation and to only small amplitude flickering at the early stages of time-evolution.

The situation is much more dramatic when the starting accretion rate is 12 $M_\odot$ s$^{-1}$. In this case a large disk strip is viscously and thermally unstable and the most violent instability takes place around and below 7–12 $R_\ast$.

In Figure 16 we show the behavior of the local accretion rate in the unstable disk, at several chosen locations within the instability strip. Near $\sim$12$R_\ast$, the accretion rate varies due to the large and rapidly changing photodisintegration term (locally, it can become larger than the neutrino cooling rate).

This radius corresponds to the largest local value of the density of helium (e.g., Fig. 8, showing its starting model distribution), which is then being photodissociated. The photodissociation process is the cause of the local rapid accretion rate changes. Then, inside from this highly variable strip, the accretion rate grows too fast to preserve the disk structure. This kind of behavior occurs in the locally hotter and less dense region visible in the starting configuration, e.g., in Figures 3 and 4, between 3.5 and 7 $R_\ast$. In this region the helium is already totally photodissociated. Due to the growing accretion rate all the material is rapidly accreted onto the black hole, and the innermost strip of the disk empties.

After the inner strip is destroyed, the outer parts can still accrete onto the center. As they approach the black hole, their temperature and density grow and the above situation can repeat several times, until the whole disk is completely broken into rings and destroyed. These later injections of energy, with timescales dictated by the viscous timescale of each ring, can produce energy flares following the main GRB activity. Our results therefore provide another physical mechanism for the flare model recently proposed by Perna et al. (2006).

In Figure 17 we show the neutrino light curve of the unstable disk. The instabilities due to photodisintegration are reflected in oscillations of variable amplitude and millisecond timescales. This is of a particular interest if the neutrino annihilation provides the energy input for GRBs; however, it should be pointed out that the oscillations appearing in the presented light curve have a much smaller amplitude than the observed gamma-ray variability.

4. DISCUSSION

4.1. The Unstable Neutrino-Opaque Disk

In our calculations we have shown that, for large accretion rates, the accreting torus becomes viscously and thermally unstable. We now discuss the physical origin of the instability.

The unstable branch appears both in the steady state solutions and in the subsequent time-dependent evolutions. In the steady state case, for a chosen value of a constant accretion rate, this can be seen for instance by plotting the radial profiles of density and temperature (e.g., Figs. 3 and 4, where the distinct branch is found for the innermost radii), as well as by looking at the stability curves for a range of accretion rates at a chosen disk radius (e.g., Fig. 11, where the unstable inner disk radii exhibit a negative slope in the curve). In the time-dependent simulations, the unstable behavior is manifested by the highly variable accretion rate in certain strips of the disk and by the subsequent breaking of the disk inside from these variable strips (e.g., Fig. 16). The instability arises from the fact that the accretion rate rises locally too fast to prevent the disk strip from emptying, as the material is supplied from outer strips at much slower rate than it is accreted inward. The disk evolves unstably on a viscous timescale, $\tau_{\text{visc}} = 1/(\alpha\Theta)||r/H||^2$; for the radii shown in Figure 16, it is $\tau_{\text{visc}} \approx 0.05$ s (note that the disk is rather thick, $r/H \sim 2.5$, and therefore the viscous and thermal timescales are close to each other). Theoretically, in order to find again a stable solution, the disk would have to increase the local accretion rate up to about several tens of $M_\odot$ s$^{-1}$ within one viscous timescale. However, this may not be possible if there is not enough material in the system to support much higher accretion rates during such violent oscillations. Therefore, the system is unable to be stabilized and gets broken after a fraction of $\tau_{\text{visc}}$. In addition, the dynamical instability is the source of the flickering of the local accretion rate at the edge of the unstable strip.

Let us now discuss in more detail the physical reason driving this instability. In the inner part of the disk (below $r \sim 10R_\ast$ for $M = 12 M_\odot$ s$^{-1}$) there are two important processes, both of which are incorporated in our equation of state: photodisintegration of helium and neutrino trapping. As we already mentioned in § 3.2.1 and as can be seen from Fig. 8, below $\sim$7–8 $R_\ast$ helium in this disk is completely photodisintegrated. This part of the disk is also opaque to neutrinos, as we show in Fig. 6.

These two mechanisms competitively influence the electron fraction in the disk (cf. § 3.2, Figs. 6 and 9). Well outside the unstable strip, above $\sim 20 R_\ast$, the electron fraction smoothly decreases inward as positrons appear because of the neutronization process. Then the scattering optical depth for neutrinos becomes $\tau_{\nu} > 1$, and the electron fraction increases again. After photodisintegration, the electron fraction decreases significantly from almost 0.3 to much less than 0.1 due to electron capture. But

\[^6\] In addition to the gravitational instability in the outer parts of the disk, which was hinted by the calculations of Di Matteo et al. (2002) and confirmed by those of Chen & Beloborodov (2007).
again, when the disk becomes optically thick to absorption of electron neutrinos, the electron fraction gets higher and approaches almost 0.5.

The total pressure of subnuclear matter (e.g., Fig. 5) is mainly contributed by electrons, and therefore it is influenced by the changes in the electron fraction. In the narrow range of radii \(6.8 < r/R_S < 7.8\), the pressure decreases due to photodisintegration. The sudden decrease of the pressure might drive the dynamical instability. (This picture is somewhat similar to that of the iron core collapse in the core collapse supernova explosions: electron capture consumes most of the electrons and makes the EOS softer, consequently, it triggers the collapse of the iron core.) However, the transition from neutrino transparent to opaque disk, and the increase of the electron fraction due to the \(\beta\)-equilibrium (see also Yuan & Heyl 2005), are the reason for a steeper increase of the total pressure of the system.

The same effect can also be observed in the time-dependent plot (Fig. 18), in which we show the pressure changes in the characteristic radii of the unstable part of the disk (cf. Fig. 16). The pressure decreases with time up to a radius \(R = 6.8 R_S\), since the temperature and density gradually drop, as well as the neutrino opacities, so the electron fraction gets smaller. Then, in a strip between \(6.8\) and \(12 R_S\), the pressure rises with time; in fact, when \(\alpha\)-particles appear, the electron fraction rises and matter locally piles up, thus increasing the pressure.

At the border of these radii the disk breaks up, when the thermal-viscous instability induces an avalanche-like increase of the local accretion rate below \(\sim 8 R_S\). This happens because the increase in the pressure causes an excess in the local energy dissipation rate and the disk heats up, while at adjacent radii the pressure decreases and heating is insufficient. The system tries to compensate for these temperature gradients by decreasing/increasing the temperature in the outer/inner radius, respectively. But since in the unstable mode of the thermal balance this causes a further increase/decrease of density, the pressure drops further and the disk heats up in the outer radius, while it cools down in the inner one. As long as it cannot find any stable track of evolution, the emptying of the inner strip continues, and finally the whole material is accreted toward the black hole or blown out.

The radial extent of the unstable part depends on the initial accretion rate, and in our model for \(M = 12 M_{\odot} \text{ s}^{-1}\) it is up to \(\sim 8 R_S\), for \(M = 10 M_{\odot} \text{ s}^{-1}\) it is up to \(\sim 5 R_S\), while for \(M = 1 M_{\odot} \text{ s}^{-1}\) it is below \(\sim 3.5 R_S\). In the latter case, since the inner radius is located at \(\sim 3 R_S\), the instability hardly affects the disk. The extension of the instability strip depends also on the mass of the accreting compact object, and since for lower mass black holes the accretion disk is generally denser, it reaches a density \(\sim 10^{12} \text{ g cm}^{-3}\) around \(\sim 15 R_S\).

Above \(25-30 R_S\), where the plasma is already optically thin and the evolution is stable, both the pressure and the accretion rate smoothly drop with time. The dominant source of cooling of the disk in this region is the neutrino emission (advective cooling decreases as the disk transits from neutrino opaque to transparent). The photodisintegration term (if nonzero) is usually by 1–2 orders of magnitudes smaller than neutrino cooling, and in the inner disk, up to about \(6.8 R_S\), there are no helium nuclei and the photodisintegration term is negligible, while at \(7.5 R_S\) it has a value of about \(Q_{\text{photo}} = 10^{58} \text{ erg cm}^{-2} \text{ s}^{-1}\) with rapid fluctuations. These fluctuations induce the local accretion rate flickering (e.g., Fig. 16), on a timescale and amplitude much smaller than for the viscous instability.

4.2. Comparison with Previous Work

The neutrino-dominated accretion flow has already been studied in a number of papers, including both one-dimensional (1D) models and multidimensional simulations. The steady state 1D models (e.g., Popham et al. 1999; Kohri & Mineshige 2002) assumed the disk was optically thin to neutrinos and neglected photodisintegration cooling. Di Matteo et al. (2002) took these two effects into account and showed that the trapped neutrinos dominate the pressure in the inner region of the hyperaccreting disk; however, their equation of state did not include the numerical calculation of chemical equilibrium and did not incorporate the opacities directly in the EOS iterations (see also the time-dependent model of Janiuk et al. 2004). Kohri & Mineshige (2002) considered the neutrino opaque disk and the equilibrium between neutrons and protons and calculated the number densities of species by numerically integrating their distribution functions. However, these authors calculated the gas pressure from the ideal gas approximation and neglected the contribution of helium to the pressure. In all of these papers the disk occurred to be stable against any kind of instability.

On the other hand, in their recent work, Chen & Beloborodov (2007) find that the outskirts of the disk are gravitationally unstable. The approach used by these authors provides a detailed treatment of the microphysics which is very similar to ours; however, some differences between our work and theirs must be crucial to the development of viscous and thermal instabilities. One difference deals with the approximation made for the treatment of transition region between the neutrino-opaque and the transparent matter. In our work, we adopted a graybody model, i.e., we introduced the \(b\) factor to describe the distribution function (e.g., eq. [10]). This assumption is consistent with the two fluids approximation we have made, which has recently been studied numerically by Sawyer (2003) and shown to be appropriate for the conditions of these disks. On the other hand, Chen & Beloborodov (2007) smoothly connect the optically thin and thick regimes by means of interpolation. A further difference lies in the description of the mass fraction of free nucleons. In this work we use an expression for \(X_{\text{nuc}}\) developed by Qian & Woosley (1996), while in Chen & Beloborodov (2007) \(X_{\text{nuc}}\) is a
function of $Y_e$, which couples the nucleosynthesis to the electron fraction.

In our calculations we reach the range of densities and temperatures where the nucleons start to become partially degenerate. This is accompanied by the neutrinos being more and more trapped in the gas and helium being destroyed by photodisassociation. As a result of these calculations, we found an additional, unstable branch of solutions for the disk thermal balance.

This supports the recent results of 2D simulations by Lee et al. (2005), who found the disk that was opaque to neutrinos to be thermally unstable. Their simulations showed that large circulations develop in the accretion flow. Setiawan et al. (2006) found small fluctuations of the accretion rate and neutrino luminosity thermally unstable. Their simulations showed that large circulations develop in the accretion flow. Setiawan et al. (2006) found small fluctuations of the accretion rate and neutrino luminosity on the dynamical timescale, after the 10–20 ms of relaxation period (note that in our calculations we start from the steady state disk model at a given accretion rate, thus having no need for a relaxation to the quasi-steady configuration). The equation of state used in their work (see also Janka et al. 1999) is based on the work of Lattimer & Swesty (1991). Given the electron fraction, this EOS assumes the condition of nuclear statistical equilibrium without neutrino trapping, but the evolution of the electron fraction is affected by the asymmetric neutrino emission from the hot and dense matter, which is called “neutrino leakage scheme.” The neutrino leakage scheme focuses on the effects of the neutrino trapping on the net neutrino emissivities, not on the nuclear statistical equilibrium. The equation of state used in the work of Rosswog et al. (2004) is temperature- and composition-dependent, based on the relativistic mean field theory (Shen et al. 1998a, 1998b), and the neutrino cooling is accounted for by the multi-flavor scheme (Rosswog & Liebendoerfer 2003).

In our work, we use an equation of state based on the $\beta$-equilibrium, including the contribution from the trapped neutrino, and neutrino-trapping effects are accounted for by the appropriate opacities. It should be emphasized that most previous multidimensional simulations neglected the effects of neutrino trapping on the $\beta$-equilibrium, as well as the contribution of the trapped neutrinos to the thermodynamical properties of the dense matter. Another difference between our treatment of the EOS and the previous numerical simulations is that we include the cooling of the photodisintegration of helium. Even though the original EOS of Lattimer & Swesty (1991) can provide detailed information about the composition of the dense matter, this information was not considered in order to keep the table of the EOS as small as possible (see e.g., Ruffert et al. 1996) just for numerical reasons. In this way, the disintegration cooling has been investigated, although without the information on the composition. Our results indicate that photodisintegration significantly affects the energy balance.

4.3. Limitations of Our Model

We find the thermal-viscous instability to be an intrinsic property of the disk for extremely large torus densities (about $10^{12} \text{ g cm}^{-3}$) and high accretion rates ($M \geq 10 M_\odot \text{ s}^{-1}$). This is seen both in the steady state results (radial profiles of density and temperature) and in the subsequent time evolution.

Thermal and viscous instabilities have been studied in the case of standard accretion disks around compact objects (Lightman & Eardley 1974; Pringle 1976; Shakura & Sunyaev 1976). Two main physical processes that lead to disk instabilities were invoked to explain the time-dependent behavior of various objects: partial ionization of hydrogen in the disks of Dwarf Novae (e.g., Meyer & Meyer-Hofmeister 1981; Smak 1984) and domination of radiation pressure in the X-ray transients (e.g., Taam & Lin 1984). Such instabilities do not have to lead to a total disk breakdown, but rather to a limit-cycle behavior, if only an additional (i.e., upper) stable branch of solutions can be found. This might be a hot state with a temperature above $10^4 \text{ K}$, or a slim disk, dominated by advection (Abramowicz et al. 1988). In our 1D calculation the disk in the GRB central engine is not stabilized but rather breaks down into rings, as no stable solutions are reached (possibly, for even higher accretion rates again a stable part near the black hole could be formed, but these extremely high accretion rates would not be produced by any compact merger scenario). Therefore, instead of a limit-cycle activity, what we find here are several dramatic accretion episodes on the viscous timescale. The remaining parts of the torus will subsequently accrete and, while approaching the central black hole, will get hotter and denser, breaking at $\sim 7R_\odot$.

Of course, it would be interesting to study whether such a violent instability would occur also in the 2D or 3D simulations. This is indeed likely to be the case, since as the multidimensional simulations of accretion disks show, the instabilities derived first in 1D are still present in the hydrodynamical simulations of flows with non-Keplerian velocity fields (e.g., Agol et al. 2001; Turner 2004; Ohsuga 2006). Possibly, the instability region would be located at other (larger) radii if the calculations included the vertical structure of the disk; this is dependent on temperature and density, which above the meridional plane may be larger than the mean value considered in the vertically averaged model.

We need to note that our 1D calculations do not take into account the possible effects of nonradial velocity components in the fluid. For example, the inverse composition gradient that leads to the disk instability, might be stabilized by rotation (e.g., Begelman & Meier 1982; Quataert & Gruzinov 2000). In the 2D simulation of Lee et al. (2005) the neutrino opaque disk exhibits circulations in the $r$-$z$ direction. Such meridional circulations are known to be present in the Keplerian accretion disks (e.g., Siemiginowska 1988); however, it is unclear if they could always provide a stabilizing mechanism for the thermal-viscous instability. Possibly, if the nonradial motions of the flow provided a stronger stabilizing effect, the disk would exhibit oscillations in the viscous timescale without breaking, similar to the outbursts of Dwarf Nova disks.

The assumption of the $\beta$-equilibrium (justified, as the mixture of protons, electrons, neutrons, and positrons is able to achieve the equilibrium conditions) might also have an effect on this result, as the equilibrium conditions reduce the heating and entropy in the gas. In fact, the $\beta$-equilibrium condition which is satisfied in the innermost part of a hyperaccreting disk that is optically thick to neutrinos is $\mu_p = \mu_e + 2\mu_n$. Once the disk becomes transparent in its outer part, this condition is no longer valid. Analytically, it has been derived by Yuan (2005) that the condition for $\beta$-equilibrium in this case is $\mu_p = \mu_e + 2\mu_n$.

4.4. Observational Consequences

Our findings might be relevant for interpreting some recent observations. The flickering due to the photodisintegration of $\alpha$-particles may lead to a variable energy output on small (milliseconds) timescales. The consequence of this may be variability in the gamma-ray luminosity, although the changes in the local accretion rate may be spread by viscous effects [in the light curve $L_\gamma(t)$ integrated over the whole surface of the disk, the millisecond variability is somewhat smeared, and the amplitudes are not very large]. Therefore, the mass accreted by the black hole may not be varying substantially, while some irregularity in the overall outflow could help produce internal shocks.

The thermal-viscous instability, if accompanied by the disk breaking, may lead to the several episodic accretion events and several rebrightenings of the central engine on longer timescales,
possibly detected in the later stages of the evolution. A similar kind of a long-term activity is possible also if the disk was not completely broken, but exhibited some large accretion rate fluctuations on the viscous timescale.

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APPENDIX

The neutrino absorption and production rates in the $\beta$-processes for all participating particles at arbitrary degeneracy have been obtained in the previous works (Reddy et al. 1998; Yuan 2005). In the subnuclear dense matter with high temperatures, the nucleons are generally nondegenerate; therefore, the transition reaction rates from neutrons to protons and from protons to neutrons can be simplified as

$$
\Gamma_{p+e^-\to n+\nu_\alpha} = \frac{1}{2\pi^2} |M|^2 \int_0^\infty dEe E_p \rho_p(E_p - Q)^2 f_e(1 - b_e f_e),
$$

(A1)

$$
\Gamma_{p+e^-\to n+\nu_\alpha} = \frac{1}{2\pi^2} |M|^2 \int_0^\infty dEe E_p \rho_p(E_p - Q)^2 (1 - f_e) b_e f_e,
$$

(A2)

$$
\Gamma_{n+e^-\to p+\bar{\nu}_\alpha} = \frac{1}{2\pi^2} |M|^2 \int_{m_e}^\infty dEe E_p \rho_p(E_p + Q)^2 f_e(1 - b_e f_e),
$$

(A3)

$$
\Gamma_{n+e^-\to p+\bar{\nu}_\alpha} = \frac{1}{2\pi^2} |M|^2 \int_{m_e}^\infty dEe E_p \rho_p(E_p + Q)^2 (1 - f_e) b_e f_e,
$$

(A4)

$$
\Gamma_{n-p+e^-\to n+\bar{\nu}_\alpha} = \frac{1}{2\pi^2} |M|^2 \int_{m_e}^Q dEe E_p (Q - E_e)^2 (1 - f_e)(1 - b_e f_e),
$$

(A5)

$$
\Gamma_{n-p+e^-\to n+\bar{\nu}_\alpha} = \frac{1}{2\pi^2} |M|^2 \int_{m_e}^Q dEe E_p (Q - E_e)^2 f_e b_e f_e.
$$

(A6)

where $Q = (m_n - m_p)c^2$ and $|M|^2$ is the averaged transition rate which depends on the initial and final states of all participating particles; for nonrelativistic noninteracting nucleons, $|M|^2 = G_F^2 \cos^2 \theta_C (1 + 3 q_\pi^2)$, where $G_F \approx 1.436 \times 10^{-11}$ erg cm$^{-3}$ is the Fermi weak interaction constant, $\theta_C$ (sin $\theta_C = 0.231$) is the Cabibbo angle, and $q_\pi = 1.26$ is the axial-vector coupling constant. Also, $f_{e,n}$ are the distribution functions for electrons and neutrinos, respectively. The “chemical potential” of neutrinos is generally assumed to be zero. The factor $b_e$ reflects the percentage of the partially trapped neutrinos. When neutrinos completely trapped, $b_e = 1$.

The corresponding neutrino emissivities for the URCA reactions are given by

$$
q_{p+e^-\to n+\nu_\alpha} = \frac{1}{2\pi^2} |M|^2 \int_0^\infty dEe E_p \rho_p(E_p - Q)^3 f_e(1 - b_e f_e),
$$

(A7)

$$
q_{n+e^-\to p+\bar{\nu}_\alpha} = \frac{1}{2\pi^2} |M|^2 \int_{m_e}^\infty dEe E_p \rho_p(E_p + Q)^3 f_e(1 - b_e f_e),
$$

(A8)

$$
q_{n-p+e^-\to n+\bar{\nu}_\alpha} = \frac{1}{2\pi^2} |M|^2 \int_{m_e}^Q dEe E_p (Q - E_e)^3 (1 - f_e)(1 - b_e f_e).
$$

(A9)

The emissivities due to the electron-positron pair annihilation, following the notation of Yakovlev et al. (2001), is written as

$$
q_{e^-+e^+\to \nu_\alpha+\bar{\nu}_\alpha} = \frac{Q_e}{36\pi} \left[ C_{e^+e^-}^2 \left( 8\Phi_1 U_2 + \Phi_2 U_1 \right) - 2(\Phi_- U_2 + \Phi_2 U_-) + 7(\Phi_0 U_1 + \Phi_1 U_0) \right] + \left( 5(\Phi_0 U_- + \Phi_- U_0) + 9 C_{e^+e^-}^2 (\Phi_0 U_1 + U_-) + (\Phi_- U + \Phi U_0) \right),
$$

(A10)

where

$$
Q_e = \frac{G_F^2}{\hbar} \left( \frac{m_e c}{\hbar} \right)^9 = 1.023 \times 10^{23} \text{ erg cm}^{-3} \text{ s}^{-1},
$$

(A11)
\[ C_{\nu} = C_0^2 + C_2^2, \quad \text{and} \quad C_{-\nu} = C_2^2 - C_0^2 \], where \( C_{\nu} \) and \( C_{-\nu} \) are the vector and axial-vector constants for neutrinos (\( C_{\nu} = 2\sin^2 \theta c + 0.5 \), \( C_{-\nu} = 0.5 \), \( C_{\mu} = C_{\tau} = 2 \sin^2 \theta c - 0.5 \), and \( C_{\mu} = C_{\tau} = -0.5 \)). The dimensionless functions \( U_k \) and \( \Phi_k \) (\( k = -1, 0, 1, 2 \)) in the above equation can be expressed in terms of the Fermi-Dirac functions:

\[
U_{-1} = \frac{\sqrt{2}}{\pi^2} \beta^{3/2} F_{1/2}(\eta_e, \beta_e),
\]

\[
U_0 = \frac{\sqrt{2}}{\pi^2} \beta^{3/2} [F_{1/2}(\eta_e, \beta_e) + \beta_e F_{3/2}(\eta_e, \beta_e)],
\]

\[
U_1 = \frac{\sqrt{2}}{\pi^2} \beta^{3/2} [F_{1/2}(\eta_e, \beta_e) + 2\beta_e F_{3/2}(\eta_e, \beta_e) + \beta_e^2 F_{5/2}(\eta_e, \beta_e)],
\]

\[
U_2 = \frac{\sqrt{2}}{\pi^2} \beta^{3/2} [F_{1/2}(\eta_e, \beta_e) + 3\beta_e F_{3/2}(\eta_e, \beta_e) + 3\beta_e^2 F_{5/2}(\eta_e, \beta_e) + \beta_e^3 F_{7/2}(\eta_e, \beta_e)].
\]

Replacing \( \eta_e \) with \( \eta_{e'} \) in \( U_k \), we get the corresponding expressions for \( \Phi_k \).

REFERENCES

Abramowicz, M. A., Czerny, B., Lasota, J. P., & Szuszkiewicz, E. 1988, ApJ, 332, 646
Abramowicz, M. A., & Kato, S. 1989, ApJ, 336, 304
Agol, E., Krolik, J., Turner, N. J., & Stone, J. M. 2001, ApJ, 558, 543
Aparicio, J. 1998, ApJS, 117, 627
Begelman, M. C., & Meier, D. L. 1982, ApJ, 253, 873
Berger, E., et al. 2005, Nature, 438, 988
Chen, W.-X., & Beloborodov, A. 2007, ApJ, 657, 383
Di Matteo, T., Perna, R., & Narayan, R. 2002, ApJ, 579, 706
Eichler, D., Livio, M., Piran, T., & Schramm, D. N. 1989, Nature, 340, 126
Fox, D. B., et al. 2005, Nature, 437, 845
Fujimoto, S., Kotake, K., Yamada, S., Hashimoto, M., & Sato, K. 2006, ApJ, 644, 1040
Gehrels, N., et al. 2005, Nature, 437, 851
Gu, W.-M., Liu, T., & Lu, J.-F. 2006, ApJ, 643, L87
Hjorth, J., et al. 2003, Nature, 424, 847
Itoh, N., Hayashi, H., Nikishawa, A., & Kohayama, Y. 1996, ApJS, 102, 411
Janiuk, A., Czerny, B., & Siemiginowska, A., 2002, ApJ, 576, 908
Janiuk, A., Perna, R., Di Matteo, T., & Czerny, B. 2004, MNRAS, 355, 950
Janka, H.-T., Eberl, T., Ruffert, M., & Fryer, C. L. 1999, ApJ, 527, L39
Kawamura, N., & Mineshige, S. 2007, ApJ, 662, 1156
Kohri, K., & Mineshige, S. 2002, ApJ, 577, 311
Kohri, K., Narayan, R., & Piran, T. 2005, ApJ, 629, 341
Kouveliotou, C., et al. 1993, ApJ, 413, L101
Lattimer, J. M., & Swesty, F. D. 1991, Nucl. Phys. A, 535, 331
Lee, W. H., & Ramirez-Ruiz, E. 2005, ApJ, 632, 421
Lightman, A. P., & Eardley, D. M. 1974, ApJ, 187, L1
Livio, M., & Piran, T., & Schramm, D. N. 1989, Nature, 340, 126
Marek, A., & Eichler, D. 2004, ApJ, 610, 1025
MacFadyen, A. I., & Woosley, S. E. 1999, ApJ, 524, 262
Meszaros, P. 2006, Rep. Prog. Phys., 69, 2259
Meyer, F., & Meyer-Hofmeister, E. 1981, A&A, 104, L10
Narayan, R., Paczynski, B., & Piran, T. 1992, ApJ, 395, L83
Narayan, R., Piran, T., & Kumar, P. 2001, ApJ, 557, 949
Ohsuga, K. 2006, ApJ, 640, 923
Paczynski, B. 1986, ApJ, 308, L43
———. 1991, Acta Astron., 41, 257
———. 1998, ApJ, 494, L45
Perna, R., Armitage, P., & Zhang, B. 2006, ApJ, 636, L29
Piran, T. 2005, Rev. Mod. Phys., 76, 1143
Popham, R., & Narayan, R. 1995, ApJ, 442, 337
Popham, R., Woosley, S. E., & Fryer, C. 1999, ApJ, 518, 356
Pringle, J. E. 1976, MNRAS, 177, 65
Proga, D., MacFadyen, A. I., Armitage, P. J., & Begelman, M. C. 2003, ApJ, 599, L5
Qian, Y. Z., & Woosley, S. E. 1996, ApJ, 471, 331
Quataert, E., & Gruzinov, A. 2000, ApJ, 539, 809
Reddy, S., Prakash, M., & Lattimer, J. M. 1998, Phys. Rev. D, 58, 013009
Rosswog, S. 2005, ApJ, 634, 1202
Rosswog, S., & Liebendoerfer, M. 2003, MNRAS, 342, 673
Rosswog, S., Speith, R., & Wynn, G. A. 2004, MNRAS, 351, 1121
Ruffert, M., & Janka, H.-T. 1999, A&A, 344, 573
Ruffert, M., Janka, H.-T., & Schuefer, G. 1996, A&A, 311, 532
Sawyer, R. F. 2003, Phys. Rev. D, 68, 063001
Setiawan, S., Ruffert, M., & Janka, H.-Th. 2004, MNRAS, 352, 753
———. 2006, A&A, 458, 553
Shakura, N. I., & Sunyaev, R. 1973, A&A, 24, 337
———. 1976, MNRAS, 175, 613
Shen, H., Toki, H., Oyamatsu, K., & Sumiyoshi, K. 1998a, Prog. Theor. Phys., 100, 1013
———. 1998b, Nucl. Phys. A, 637, 435
Siemiginowska, A. 1988, Acta Astron., 38, 21
Smak, J. 1984, Acta Astron., 34, 161
Stanek, K. Z., et al. 2003, ApJ, 591, L17
Surman, R., & McLaughlin, G. C. 2004, ApJ, 603, 611
Taam, R. E., & Lin, D. N. C. 1984, ApJ, 287, 761
Turner, N. J. 2004, ApJ, 605, L45
Villasenor, J. S., et al. 2005, Nature, 437, 855
Witt, H., Jarozyinski, M., Haensel, P., Paczynski, B., & Wambganss, J. 1994, ApJ, 422, 219
Woosley, S. E. 1993, ApJ, 405, 273
Yakovlev, D. G., Kaminker, A. D., Gnedin, O. Y., & Haensel, P. 2001, Phys. Rep., 354, 1
Yuan, Y. 2005, Phys. Rev. D, 72, 013007
Yuan, Y., & Heyl, J. S. 2005, MNRAS, 360, 1493
Zhang, B. 2007, Chinese J. Astron. Astrophys., 7, 1