Cutting force prediction considering force–deflection coupling in five-axis milling with fillet-end cutter

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Abstract
With the increasing demand for higher quality and performance of equipment and assembly in aerospace, shipbuilding, medical and other fields, the machining accuracy of parts is facing higher requirements. It is particularly important to predict the cutting force accurately, which is the main physical quantity in the machining process and basis of process inspection and quality control. In this paper, the cutting force model in five-axis milling with fillet-end cutter is proposed, which reveals the law of force–deflection coupling. Firstly, the initial cutter deflection model induced by the ideal cutting force ignoring the effect of deflection is built based on analysis of the geometric characteristics of fillet-end cutter. Then, the undeformed chip thickness model is educed considering the cutter posture and cutter deflection. Further, the iterative method is utilized to resolve the coupling relationship between cutter deflection and cutting force. Finally, the cutting force model under force–deflection coupling is established for achieving more accurate prediction. To verify the effectiveness of the proposed cutting force model, the milling experiment is carried out on a five-axis milling center. The measured cutting force values are utilized to inspect the accuracy of prediction models considering and not considering force–deflection coupling, respectively. The results show that the proposed method could improve the prediction accuracy of cutting force, which show the effectiveness of taking force–deflection coupling law into consideration clearly. The influence of cutter posture on cutting force is analyzed using the proposed cutting force model. The cutting force decreases with the increase in lead angle or tilt angle, and the influence of tilt angle is greater than that of lead angle under experimental conditions of five-axis milling using the set process parameters.

Keywords Cutting force · Force–deflection coupling · Five-axis milling · Fillet-end cutter

1 Introduction
With the rapid development of the industry such as aerospace and shipbuilding equipment medical the requirements of equipment service performance are improved, which puts forward an urgent demand for the machining accuracy of key parts. These parts usually have complex free-form surface with characteristics of nonlinear contour, large surface curvature changes and overlapping machining features. For example, the nonlinear multi-blade propeller applied in underwater ships is a typical difficult-to-machine part which has the characteristics of complex blade shape and narrow flow channel and limits that the cutter must have a slender feature to match narrow feasible space. The cutter deflection produced under action of cutting force becomes an issue that cannot be ignored as the slender cutter has poor stiffness. The taking of deflection into account makes the accurate prediction
of cutting force becomes much more difficult because the cutter–workpiece engagement becomes more complex. The free-form surface of the part makes the cutter posture change in five-axis machining, so the lead angle and tilt angle should be considered in the cutting force model.

The research of prediction method of cutting force has experienced a process of gradual deepening in theory. Martellotti [1] proposed to discretize the cutting edge part as the cutting edge element and defined the cutting force of the element as the product of load area of the chip element and unit cutting force coefficient, known as the mechanical model of cutting force widely used by later scholars. Then Malekian et al. [2] established the minimum chip thickness model initially, which promoted the development of mechanical model. As an important part of cutting force model, distribution and accumulation phenomenon of the chip thickness were studied [3, 4], which provided the foundation for accurate calculation of cutting force. Applications of emerging new materials, such as ceramic matrix composites and CFRP composite, derived cutting force prediction models for machining of those new materials [5, 6], in which the auxiliary process such as ultrasonic vibration machining was used and expanded the application range of cutting force model.

Cutting geometry, kinematics and mechanics were all important links in analysis of material removal mechanism and should be taken into consideration [7]. As a key aspect of cutting geometry, the calculation of cutter workpiece engagement (CWE) determines whether the cutting force prediction is accurate or not. Zhang et al. [8] and Li and Zhu [9] established an accurate calculation model of CWE boundary meshing process. Guo et al. [10] presented a new mechanistic model of cutting forces based on cutter motion analysis. Jing et al. [11] studied the effect of elastic recovery on cutting force and established a mechanical model of cutting force, but without considering their coupling effects. Salehi et al. [13] proposed cutting forces prediction model using Bayesian inference for the Merchant and Kienzle force models.

The cutting force is the sum of the forces on all cutting edges working under different conditions. It is meaningful to analyze the action of per tooth with varying wear [12] and tool run-out effect [14, 15]. With increasing application of five-axis milling driven by the needs of free-form surface machining, the prediction of five axis cutting force has been widely studied [16, 17], in which the effects of lead and tilt angles on cutting forces were investigated. The cutting force coefficient is an important component in the cutting force model, whose calibration accurately attracts many scholars. Wang et al. and Ozturk et al. calibrated the coefficients based on particle swarm optimization algorithm [18], the tool–workpiece material couple [19] and the influence of helix angle [20], respectively.

Above studies promoted the development of cutting force prediction from different aspects such as the cutting edge element, the undeformed chip thickness model and the cutter geometry, which made many theoretical accumulations to reveal the mechanism of milling process.

The elastic flexibility of the cutter would cause the three-dimensional shape error of the workpiece surface [21, 22]. The requirement of machining accuracy becomes higher as the improvement of service performance demands and the theoretical research. The cutting force-induced cutter deflection gradually become an important factor to be considered, and also a hot topic in research of cutting force prediction.

Huo et al. [23] and Zhang et al. [24] built the mathematical model considering cutter elasticity and runout to calculate the cutting force distribution on the cutter, and obtained the cutter deflection caused by the cutting force. Many studies focus on monitoring and compensating the cutter deflection to inhibit the effect of the force-induced deflection on the machining quality. Zeroudi and Fontaine [25] and Altintas et al. [26] proposed the cutter deflection calculation and compensation methods based on the cutting force prediction model of free-form surface milling. The accuracy and productivity of part manufacturing is improved by monitoring and minimizing cutter deflection error [27]. Those scholars had explored the relationship between cutter deflection and cutting force.

In the whole machining process, deflection and cutting force affect each other. The current research mainly focuses on the prediction of cutting force or cutter deflection. However, there were few cutting force prediction considering force–deflection coupling.

This paper proposed a cutting force prediction model to solve the force–deflection coupling caused by the cutting force and deflection feedback. In this model, the geometry model of fillet-end cutter and the cutter deflection model are established. The undeformed chip thickness model according to cutter posture and cutter deflection is presented. Compared with previous models, the force–deflection coupling considering the feedback between force and cutter deflection is analyzed, and the iterative calculation of the cutting force considering the force–deflection coupling is carried out. The simulation results show that this model is more accurate than the previous cutting force prediction model.

The structure of this paper is organized as: In Sect. 2, the cutting force prediction model considering force–deflection coupling is introduced, which includes the cutter geometry model, the cutter deflection model, the undeformed chip thickness model and the iterative method of force–deflection. In Sect. 3, the experimental setup is introduced, and the experimental results are discussed. The conclusions are drawn in Sect. 4.
2 Prediction of cutting force considering force–deflection coupling

2.1 Geometric modeling of fillet-end cutter

The fillet-end cutter is adopted in the cutting force modeling, because it can represent various types of cutters such as flat-end cutter and ball-end cutter. For example, when the fillet radius is 0, the fillet-end cutter becomes a flat-end cutter; when the fillet radius is equal to the cutter radius, the cutter becomes a ball-end cutter. As shown in Fig. 1a, \( R_r \) represents the radius of the center circle of the sweep surface of the fillet-end cutter edge, and \( r \) represents the fillet radius of the arc edge. As shown in Fig. 1b, each cutting edge is discrete into many cutting edge elements along the axis of the tool, and each cutting edge element could be represented by the axial position \( z \) and the radial position angle \( \psi_j(z) \).

The cutting edge element \( P \) in the figure could be expressed as:

\[
P = \begin{bmatrix} r(z) \sin \psi_j(z) & r(z) \cos \psi_j(z) \end{bmatrix}^T
\]

where \( r(z) \) is the distance between the cutting edge element \( P \) and the cutter axis. Then, \( r(z) \) could be expressed as:

\[
r(z) = \begin{cases} R_r + \sqrt{r^2 - (r - z)^2} & , z < r \\ R_r + r & , z \geq r \end{cases}
\]

where \( \psi_j(z) \) represents the radial position angle of the cutting edge element of the \( j \)th cutting edge at the height of \( z \), and its geometric position is shown in Fig. 1b which could be calculated by the following formula:

\[
\psi_j(z) = \theta + (j - 1) \frac{2\pi}{N} - \phi(z)
\]

where \( \theta \) is the rotation angle of the spindle. The rotation angle of the spindle at time \( t \) could be expressed as \( \theta_t = \omega t \). \( N \) is the number of cutting edges of the cutter, and \( \phi(z) \) represents the radial lag angle of the current cutting edge element relative to the first cutting edge element on the same cutting edge, which could be calculated by the following formula:

\[
\phi(z) = \begin{cases} z \cdot \tan(\arctan(\tan \beta_1 \cdot \sqrt{r^2 - (r - z)^2} / r)) / R_r + r & , z < r \\ z \cdot \tan \beta_1 / R_r + r & , z \geq r \end{cases}
\]

In the above formula, \( \beta_1 \) is the nominal helix angle of the cutter. When the cutter is right-handed, \( \beta_1 \) is negative.

The outward normal vector at the point \( P \) on the cutting edge element could be expressed by the following formula:

\[
n(z) = \left[ \sin \kappa \sin \psi_j(z) \sin \kappa \cos \psi_j(z) - \cos \kappa \right]^T
\]

where \( \kappa \) is the axial contact angle, when \( z < r, \kappa = \arccos((r-z)/r) \), when \( z \geq r, \kappa = \pi/2 \).

2.2 Modeling of deflection induced by cutting force

Two adjacent cutter location points (CLP) \( P_{L(n)} \) and the corresponding cutter axis vectors \( V_n \) could be extracted from the cutter location file to determine the relative position and posture of the cutter in the workpiece coordinate system (WCS). As shown in Fig. 2, the two adjacent tool sites of \( t \) at a certain time are \( P_{L(n)} \) and \( P_{L(n+1)} \), respectively, and the corresponding two cutter axis vectors are \( V_{n(n)} \) and \( V_{n(n+1)} \), respectively. According to linear vector interpolation and angle vector interpolation, the cutter location point \( P_{L(t)} \) and cutter axis vector \( V_{n(t)} \) in the workpiece coordinate system at time \( t \) could be obtained. The lead angle \( \alpha \) is the rotation angle between the cutter axis
vector $V_{n(t)}$ and the normal vector $n$ of the surface. The tilt angle $\beta$ was the rotation angle from feed direction $a$ to the projection of $V_{n(t)}$ on the tangent plane at the cutter contact point [28].

In five-axis milling, the Z-axis of the cutter coordinate system (CCS) is consistent with the direction of the cutter axis vector. The Y-axis of the CCS is perpendicular to the direction of the cutter axis vector and the direction of the feed vector. The X-axis of the CCS is determined by Cartesian right hand rule. The three vectors of the CCS could be expressed as:

\[
\begin{align*}
    x_L &= \frac{y_L \times z_L}{\|y_L \times z_L\|} \\
    y_L &= \frac{z_L \times v_L}{\|z_L \times v_L\|} \\
    z_L &= \frac{v_n}{\|v_n\|}
\end{align*}
\]

where $v_L$ is the corresponding cutter location feed vector, $v_{L(t)} = P_{L(t+1)} - P_{L(t)}$. The rotation transformation matrix from the CCS to the WCS at time $t$ is expressed as:

\[
W_{L(t)}^R = \begin{bmatrix}
    x_{L(t)} & y_{L(t)} & z_{L(t)}
\end{bmatrix}
\]

However, when the cutter deflection is induced, the CCS still conforms to Eq. (6). The rotation transformation matrix between the deflected CCS and the undeflected CCS be expressed as:

\[
R_R = \begin{bmatrix}
    \cos \delta & 0 & \sin \delta \\
    0 & 1 & 0 \\
    -\sin \delta & 0 & \cos \delta
\end{bmatrix}
\]  

Since the cutter deflection is considered, the deflected and undeflected cutter center circle are not coincidence. It is assumed that two element sections remain to be in a plane before and after the cutter deflection. But the two sections rotate at a deflection angle $\delta$ around center axis, respectively, which is defined as the angle between the tangent line of a point on the axis after deflection and the axis before deflection. As shown in Fig. 3, the angle between the projection of the deflected cutter tip on the CCS and the positive direction of Y-axis is called deviation angle which denoted as $\tau$. The displacement from the point on the axis after deflection to the axis before deflection is called $\rho$. $m$ represents the projected length of the deflected length in the undeflected Z-axis direction, and $L$ represents the length of the overhanging part. Due to the uneven distribution of the force loaded on the cutter during the cutting process, this paper regards the cutting force as concentrated on a certain point of the
cutting edge of the cutter for the convenience of analysis [29]. The radial bending deflection and axial bending deflection caused by the element force (\(df_r\) and \(df_a\)) as shown in Fig. 2. The calculation formula could be described as:

\[
\begin{align*}
\text{df}_{\text{radial}} &= \sqrt{(df_r)^2 + (df_a)^2} \\
\text{df}_{\text{axial}} &= df_a 
\end{align*}
\] (9)

In the process of five-axis milling, the small diameter cutter is deflected due to the chip reaction force, which is mainly manifested in the radial and axial bending deflection of the cutter. The radial bending deflection of the cutter would change the cutting trajectory of the cutting edge; thus, the calculation of the instantaneous micro-deflection cutting thickness of the cutting edge element is affected. To describe the deflection of the cutter, two coordinate systems are set up. As shown in Fig. 3, the origin of the WCS is fixed on the workpiece. The CCS moves along the feed direction of the cutter relative to the WCS, and moves in the direction of translation and rotation relative to the CCS.

The projection of reduction of cutter length along Z axis caused by the deflection could be calculated by

\[
m =\frac{df_{\text{axial}} \cdot r}{EA} + \frac{df_{\text{axial}} \cdot (L - r)}{EA}.
\] (10)

Since the following deviation occurs below the action point of the element force, the deflection law of the upper and lower parts of the action point of the element force is different in the CCS. Therefore, the displacement in the cutter axial direction under the radial element force could be calculated by

\[
\begin{align*}
\rho(z) &= \frac{df_{\text{radial}}}{EI} \left( -\frac{1}{6} z^3 + \frac{\Delta z}{2} z^2 + \frac{L^2}{2} z - L \Delta z z + \frac{L^2}{2} \Delta z - \frac{1}{3} L^3 \right), \quad z > \Delta z \\
\delta(z) &= \frac{df_{\text{radial}}}{EI} \left( \frac{1}{3} \Delta z^3 - L \Delta z^2 + \frac{L^2}{2} \Delta z - \frac{1}{3} L^3 \right) + (\Delta z - z) \sin \delta(z), \quad z < \Delta z \\
\rho(z) &= \frac{df_{\text{radial}}}{EI} \left( \frac{3}{2} \Delta z^2 + \frac{1}{2} L^2 - L \Delta z \right), \\
\delta(z) &= \frac{df_{\text{radial}}}{EI} \left( \frac{3}{2} \Delta z^2 + \frac{1}{2} L^2 - L \Delta z \right)
\end{align*}
\] (11)

2.3 Modeling of undeformed chip thickness

According to the definition of deflection and deflection angle of cutter, the translate transformed matrix between two coordinate systems before and after cutter deflection could be expressed as

\[
T_R = \begin{bmatrix}
-\rho(z) \sin \tau \\
-\rho(z) \cos \tau \\
m
\end{bmatrix}
\] (12)

In the cutting process shown in Fig. 4, to describe relative position and posture of the cutter in the WCS, \(P_n\) \(V_{A(n)}\) represents the CLP and cutter axis vector at time \(t\). Then, according to the transformation in Eq. (12), the CLP \(P_n\) and cutter axis vector \(V_{A(n)}\) after deflection could be calculated as

\[
P_t' = P_n + \frac{w}{L} R : T_R \\
V_t' = \frac{w}{L} R [\frac{w}{L} R]^{-1} V_t
\] (13)

Considering the cutter deflection, the rotation transformation matrix \(WL'\) from CSL to CSW at time \(t\) is

\[
\frac{w}{L(t)} R' = R_{R(t)} \cdot \frac{w}{L(t)} R
\] (14)

So the composition transformation matrix from CSL to CSW at time \(t\) is

\[
\frac{w}{L(t)} T_C = \left( \frac{w}{L(t)} R' \frac{p'_n(t)}{1} \right)
\] (15)

At time \(t\), the descriptions of the cutting edge element \(p_{n(t)}\) in CSL and CSW, \(L_{p_{n(t)}}\) and \(W_{p_{n(t)}}\) have the following mapping relationship:

\[
\left( \begin{array}{c}
\frac{w}{L(t)} p'_{n(t)} \\
1
\end{array} \right) \Rightarrow \frac{w}{L(t)} T_C \left( \begin{array}{c}
L p'_{n(t)} \\
1
\end{array} \right)
\] (16)

In the WCS, based on the two position vectors of the cutting edge element in the adjacent cutting edge periods, the feed vector of the cutting edge element in the WCS at time \(t\) could be expressed as

\[
\frac{w}{L(t)} V_{t(t)} = \frac{w}{L(t)} p'_{n(t+\Delta t)} - \frac{w}{L(t)} p'_{n(t)}
\] (17)

The expression of the cutting edge element feed vector in the WCS mapping to the spindle deflection coordinate system is

\[
\frac{L}{w(t)} V_{t(t)} = \left[ \frac{w}{L(t)} R' \right]^{-1} \frac{w}{L(t)} V_{t(t)}
\] (18)

After deflection, the outward normal vector of cutter sweep \(n'(z)\) is calculated as
In this way, the feed vector of the cutting edge element could be obtained at time \( t \). The undeformed chip thickness of the cutting edge element is defined as the projection length of the minimum feed vector of the cutting edge element in the previous tooth cycle, which is on the normal vector of the swept surface of the cutter. The calculation formula for the undeformed chip thickness of the cutting edge element is as follows.

\[
h_t(z) = I_vL(t) \cdot n'(z)
\]

To predict the cutting force model accurately, cutter workpiece engagement (CWE) needs to be obtained. The method to determine the CWE was presented by defining two basic conditions and five types of cutting edge elements which is detailed shown in Table 1 [30].

![Fig. 4 Chip thickness before and after cutter deflection](image)

### 2.4 Cutting force without consideration of deflection

As shown in Fig. 5, the cutting edge is discretized into cutting edge elements along the cutter axis. And the cutting force of the element is defined as the product of the chip load area of the element and the unit cutting force coefficient.

\[
n'(z) = [R_R] \cdot n(z)
\]

In this way, the feed vector of the cutting edge element could be obtained at time \( t \). The undeformed chip thickness of the cutting edge element is defined as the projection length of the minimum feed vector of the cutting edge element in the previous tooth cycle, which is on the normal vector of the swept surface of the cutter. The calculation formula for the undeformed chip thickness of the cutting edge element is as follows.

\[
h_t(z) = I_vL(t) \cdot n'(z)
\]

The mechanical model of cutting force [1] in radial, tangential and axial directions of the cutting edge element at time \( t \) is

\[
\begin{align*}
df_r &= k_r h(\psi_r(z), z) db(z) \\
df_i &= k_i h(\psi_i(z), z) db(z) \\
df_a &= k_a h(\psi_a(z), z) db(z)
\end{align*}
\]

where \( k_r \), \( k_i \), and \( k_a \) represent the radial, tangential and axial cutting force coefficients of the cutting edge element, respectively. Based on the measured data from the machining experiment, the cutting force coefficients calibration are carried out with the same workpiece material and cutter using general method which is given in another paper [30].

### Table 1 Cutting edge element types and CWE analysis

| Type | Whether it is outside the workpiece surface | Whether it is inside the envelope of previous cutting edge | Whether it is below the envelope of previous cutting edge | Whether it involved in cutting process |
|------|-------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|--------------------------------------|
| 1    | Y                                          | -                                                        | -                                                        | N                                    |
| 2    | Y                                          | -                                                        | -                                                        | N                                    |
| 3    | N                                          | Y                                                        | -                                                        | N                                    |
| 4    | N                                          | -                                                        | Y                                                        | Y                                    |
| 5    | N                                          | N                                                        | N                                                        | Y                                    |
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\[
\begin{align*}
    k_1(h_j) &= 3683.7 + 6637.16 e^{-49.3h_j} \\
    k_2(h_j) &= 1942.2 + 3901.8 e^{-44.6h_j} \\
    k_3(h_j) &= -214.9 - 3444.8 e^{-123.1h_j} \\
\end{align*}
\]  

(22)

And \(db(z)\) represents the undeformed chip width at the axial position \(z\), \(db(z) = dz/\sin \kappa\).

By integrating the discrete cutting edge elements along the cutter axis and adding up the cutting forces of all discrete cutting edges, the total cutting force model of all elements integral on all cutting edges on three axes is obtained as

\[
\begin{align*}
    f_x &= \sum_{j=1}^{N} (f_1^x k_1(\psi_j, z) db(z)) \\
    f_y &= \sum_{j=1}^{N} (f_1^y k_1(\psi_j, z) db(z)) \\
    f_z &= \sum_{j=1}^{N} (f_1^z k_1(\psi_j, z) db(z)) \\
\end{align*}
\]

(23)

The rotation transformation matrix between the element coordinate system and the CCS could be described as

\[
R = \begin{bmatrix}
    -\sin \kappa \sin \psi & -\cos \psi & -\cos \kappa \sin \psi \\
    -\sin \kappa \cos \psi & \sin \psi & -\cos \kappa \cos \psi \\
    \cos \kappa & 0 & -\sin \kappa \\
\end{bmatrix}
\]

(24)

The cutting force in the CCS could be expressed as

\[
\begin{bmatrix}
    f_x \\
    f_y \\
    f_z \\
\end{bmatrix} = \begin{bmatrix}
    f_x' \\
    f_y' \\
    f_z' \\
\end{bmatrix}
\]

(25)

According to the cutter geometry parameters (cutting edge number \(N\), circular arc edge radius \(R\), arc length \(r\), cutter length of overhanging section \(L\), nominal spiral angle \(\beta\)) and the material parameters (material for 1045 steel, elastic modulus \(E\) and moment of inertia \(I\)), the deflection \(a\) of the cutting edge can be calculated by considering the influence of cutter posture and cutter deflection on the instantaneous undeformed chip thickness and cutting state of cutting edge to accurately predict cutting force.

Substituting Eq. (23) into deflection model, \(\rho_j(z)\), \(\rho_j(z)\), \(\rho_j(z)\) could be calculated as

\[
\begin{align*}
    \rho_j(z) &= \frac{df_{radial}}{EI}(-\frac{1}{6}\Delta z^3 + \frac{\Delta z^2}{2} + \frac{L_z^2}{2} - L_1\Delta z_1 + \frac{L_z^2}{2} - L_1\Delta z_1 - \frac{1}{2}L_1^2), z > \Delta z \\
    \rho_j(z) &= \frac{df_{axial}}{EI}(-\frac{1}{3}\Delta z^3 - \Delta L_2\Delta z_2 + \frac{L_2^2}{2} - \Delta z_2 - \frac{1}{3}L_2^2) + (\Delta z - \Delta z) \sin \delta, z < \Delta z \\
    \rho(z) &= \frac{df_{axial}}{EA} + \frac{df_{axial}}{EA} \cdot (L - r) \\
\end{align*}
\]

(26)
Fig. 6 Flowchart of cutting force prediction calculation considering force–deflection coupling
Through analysis, the cutter in the cutting process would be subjected to the workpiece on its reaction force, so the deflection in the X-, Y- and Z-axis is induced under the cutting force. The cutter deflection under action of cutting force is $\rho(z)$. The undeformed chip thickness is decreased as

$$h'(\gamma_j(z), z) = h(\gamma_j(z), z) - \rho(z)$$  \hspace{1cm} (28)

However, the cutter deflection makes the cutting edge of the cutter tend to be far away from the workpiece. It reduces the force of the workpiece on the cutter and the reaction force of the cutter on the workpiece. It could be seen from Eq. (27) that the deflection is proportional to the force. On the $X_WO_WY_W$ plane, when the force applying on the cutter decreases, the deflection $\rho$ of the cutter would also decrease. Similarly, when the $\rho$ becomes smaller, the force applying on the cutter would also decrease. The iterative calculation formula is as

$$
\begin{align*}
\frac{df_q(i+1)}{db(z)} &= k_q h_i(\gamma_j(z), z) db(z) \\
 h_i(\gamma_j(z), z) &= h_{i-1}(\gamma_j(z), z) - \rho_i(z)
\end{align*}
$$  \hspace{1cm} (29)

where $q = r, t, a, i = 1, 2, 3, \ldots$ and $k_q$ are the radial, tangential and axial cutting force coefficients.

Finally, the new deflection would tend to be 0, so that the difference value of force before and after calculation is close to 0. For the convenience of calculation, the difference is expressed by $em$, so there is the following relationship

$$em = \left| \frac{df_q(i+1) - df_q(i)}{df_q(i+1)} \right|$$  \hspace{1cm} (30)

In practical calculation, $em$ is taken as 1e-8, which is used to ensure both accuracy and efficiency.

### 3 Experimental setup and verification

#### 3.1 Experimental setup

To verify the effectiveness of the cutting force model with force–deflection coupling feedback, the experiments in this paper are carried out on the five-axis milling center GMC1600H (Fig. 7).

The main force measuring apparatuses used in the machining process are Kistler9257A dynamic piezoelectric dynamometer and Kistler5070 charge amplifier. The workpiece material to be cut is 1045 steel (40CrNi2Si2MoVA, a low alloy carbon martensitic reinforced ultra-high strength steel) after heat treatment. The cutting force prediction model is universal for different materials. When the material of the workpiece changes, it is needed to calibrate the cutting force coefficient for the corresponding workpiece through calibration experiment.

The used cutter is a cemented carbide integral end mill with a 10 mm diameter, and the model of the milling cutter is SANDVIK Coromant R216.24-10050EAK22H 1620. The geometric parameters of the milling cutter are shown in Table 2.

The parameters set in the five-axis milling experiment are shown in Table 3, and the milling mode is slot milling. Parameters are selected through experiments of cutter,

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**Table 2** Cutter parameters

| Parameters                  | Value | Parameters                  | Value |
|-----------------------------|-------|-----------------------------|-------|
| Cutter diameter (mm)        | 10    | Cutter fillet radius (mm)   | 2     |
| Number of cutter teeth      | 4     | Cutter helix angle (deg)    | -50   |
| Total cutter length (mm)    | 100   | Cutter clamping length (mm) | 20    |
| Length of the tooth (mm)    | 30    | Effective coefficient       | 0.8   |

---

**Table 3** Parameter setting

| Parameters                  | Value |
|-----------------------------|-------|
| Feed rate (mm/r)            | 0.05  |
| Depth of cut (mm)           | 2     |
| Spindle speed (rpm)         | 1500  |
| Cutting depth (mm)          | 2     |
| Milling width (mm)          | 10    |
| Milling length (mm)         | 100   |
| Number of passes            | 1     |
| Spindle table height (mm)   | 100   |
| Spindle table width (mm)    | 100   |
| Spindle table length (mm)   | 100   |

---

**Fig. 7** Experiment platform
workpiece material, workpiece geometry and engineering practice and many milling experiments are accomplished to keep the cutter in a good mechanical condition and make cutting forces change smoothly.

### 3.2 Results and discussions

According to the proposed cutting force model with considering force–deflection coupling in this paper, the predicted value of the cutting force model is compared with the measured value by simulation and experiment. The simulation calculation is carried out for the two cases with and without considering the force–deflection coupling, and then, the simulation diagrams of the calculated values and the experimental measured values of the two cases are drawn, respectively. Experiments were carried out on different cutter postures, as shown in Fig. 8. The lead angles of the cutter are 0°, 5°, 10° and the tilt angle is 0°, respectively. The left side of the figure shows the change rule of the predicted value and the experimental value of the cutting force model without considering the force–deflection coupling. The right side of the figure shows the change rule of the predicted value and the experimental value of the cutting force model with considering the force–deflection coupling.

Overall, Fig. 8 shows the measured and predicted cutting forces along three orthogonal directions within a cutter rotation cycle. The bottom point is the measured cutting force of X-axis, and the bottom curve is the predicted cutting force distribution of X-axis. The top point is the measured cutting force of Y-axis, and the top curve is the predicted cutting force distribution of Y-axis. The middle point is the measured cutting force of Z-axis, and the middle curve is the predicted cutting force distribution of Z-axis. It could be clearly seen that the predicted value calculated by the cutting force model with considering force–deflection coupling is closer to the measured value than that calculated by the cutting force model without considering force–deflection coupling.

The predicted value of unconsidered force–deflection coupling is generally greater than the experimental value on Y-axis and Z-axis, and the absolute value of predicted value of unconsidered force–deflection coupling on X-axis is greater than the experimental value. This is because the cutting force is rigid cutting force when the force–deflection coupling is not considered, which is greater than the actual cutting force. The cutting force considering force–deflection coupling is slightly reduced after iteration calculation of cutting force, which is more in line with the experimental measured value. The Y-axis direction is the feed direction, so the force in Y-axis direction is greater than that in X-axis, Z-axis direction.

To verify the universality of the model, the lead angle is kept at 0°, and the tilt angles are simulated at −20°, 0° and 20°, respectively, as shown in Fig. 9. The comparison results show that the cutting force without considering force–deflection coupling is greater than that of the model with considering force–deflection coupling, which further proves the accuracy of the model. In Fig. 9, the error between the simulation value and the experimental value without considering force–deflection coupling is obvious at the peak, since the cutting edge contacts the workpiece and the undeformed chip thickness reaches the largest. At the peak, the cutter deflection is most obvious, when the force–deflection coupling is considered, the effect of cutting force reduction caused by iteration is more significant, so the error between the simulation value and the experimental value without considering the cutter deflection at the peak is the largest. When the tilt angles are -20° and 20°, the angles are equal with different directions, so there is little difference for the force value in Y-axis which is the feed direction.

Figure 10 shows the maximum error between the model without considering force–deflection coupling and the experimental value, and the maximum error between the model with considering force–deflection coupling and the experimental value. It could be seen from the figure that the maximum errors of the prediction model without considering the force–deflection coupling in the X-axis, Y-axis and Z-axis are 15%, 27% and 15%, respectively. Since the Y-axis direction is close to the feed direction, the cutter deflection of the Y-axis is the main deflection, so the error between the predicted value and the experimental value in the X-axis is greater. The maximum error of the prediction model considering force–deflection coupling proposed in this paper is less than 10% in the X-axis, less than 6% in the Y-axis, and less than 8% in the Z-axis. It is concluded that the prediction model considering force–deflection coupling proposed in this paper could be accurate for different cutter postures.

The above simulation results show that the cutting force prediction model proposed in this paper has enough stability, so the cutting force prediction of different cutter postures is carried out through the prediction model, and its
Fig. 8 When the tilt angle is 0°, the predicted values of cutting force are compared with the experimental values with different lead angles. (a) The comparison between simulation and experiment of considering force–deflection coupling and without considering force–deflection coupling when the lead angle is 0° and the tilt angle is 0°. (b) when the lead angle is 5° and the tilt angle is 0°. (c) when the lead angle is 10° and the tilt angle is 0°.
Fig. 9 When the lead angle is 10°, the predicted values of cutting force are compared with the experimental values with different tilt angles. (a) The comparison between simulation and experimental of considering force–deflection coupling and without considering force–deflection coupling when the lead angle is 10° and the tilt angle is –20°. (b) when the lead angle is 10° and the tilt angle is 0°. (c) when the lead angle is 10° and the tilt angle is 20°
law is analyzed. Figure 11a shows the effect of different lead angles on cutting force when the tilt angle is 0°. Under fixed depth cutting, the lead angle increases from 0° to 20° and the contact area decreases gradually, resulting in the decrease of average cutting force. Figure 11b shows the effect of different tilt angles on cutting force when the lead angle is 10°. When the tilt angle increases from 0° to 20°, the contact area decreases gradually, and the average cutting force has a decreasing trend. Further about the change rate of cutting force, it is found that the decrease rate of average cutting force caused by the increase in lead angle is slower than that caused by the increase in tilt angle. However, the variations of cutter posture angles would directly impact on the cutter–workpiece engagement which would determine the quantity of involved cutting edge elements. It reflects that the influence of the tilt angle on the contact area is greater than the lead angle in this experimental setup. It could be seen that optimized cutter posture could effectively reduce the cutting force and thus improve the processing quality and increase the cutter life.
4 Conclusions

This paper proposed the undeformed chip thickness model considering the cutter posture and cutter deflection by analyzing the interaction between force and deflection based on the established geometric model of the fillet-end cutter according to the geometric characteristics. The calculation method of cutting force prediction based on analysis of relationship between force and deflection for five-axis machining is presented, which contains the influence of cutter posture and cutter deflection for more accurately prediction of the cutting force. The experiments were designed and conducted to verify the effectiveness of the proposed cutting force model taking the force–deflection coupling into account by contrast with model neglecting the cutter deflection. Influence of cutter posture on cutting force was studied using the verified cutting force model for better cutter orientation planning in five-axis milling under condition of machining difficult-to-cut material. Based on this research, the following conclusions could be drawn.

1. There are 17%, 9% and 7% reduced rate of prediction error of cutting force along X-axis, Y-axis and Z-axis, respectively, using the model considering force–deflection coupling than that neglecting force–deflection coupling, which shows the effectiveness of the proposed prediction method.

2. Using the same process parameters set in the experiment, the cutting force simulation is carried out applying the validated prediction model, which reflects a series of changing rules between cutting forces and cutter orientations. When the lead angle or tilt angle increases from 0° to 20°, the average cutting force would decrease gradually, which illustrates the effect of cutter orientation on the cutting force. Further, the influence of tilt angle on cutting force is greater than that of lead angle under the experimental conditions, whose reason is that the tilt angle has greater impact on the cutter–workpiece engagement which directly impacts on the cutting force.

3. The relationship between the cutting force and the cutter posture angles could be used in cutter orientation planning for optimizing the machining process. Based on the proposed prediction method, the cutting mechanism and process are further revealed, which could contribute to the industrial applications such as improving the condition monitoring and the cutter orientation planning, especially for conditions of machining difficult-to-cut material.

Abbreviations  
- $P$: Element of cutting edge;  
- $r$: Fillet radius of the arc edge;  
- $R$: Radius of the center circle of the sweep surface of the fillet-end cutter edge;  
- $z$: Axial position;  
- $x$: Axial contact angle;  
- $y(z)$: Radial position angle;  
- $j$: Cutting edge number;  
- $N$: Number of cutting edges of the cutter;  
- $\theta$: Rotation angle of the spindle;  
- $\varphi(z)$: Radial lag angle;  
- $\beta$: Nominal helix angle;  
- $\delta$: Deflection angle;  
- $\rho$: Deflection of cutter;  
- $m$: Projection of reduction of the cutter length along Z-axis direction;  
- $L$: Length of the overhanging part of the cutter;  
- $df_x$: Axial force;  
- $df_y$: Radial force;  
- $df_z$: Tangential force;  
- $O_yX_yY_yZ_y$: Workpiece coordinate system;  
- $O_xX_yY_yZ_y$: Cutter coordinate system;  
- $n(z)$: Normal vector;  
- $h$: Undeformed chip thickness;  
- $d_{O,LXLYLZL}$: Normal vector;  
- $\rho$: Undeformed chip width;  
- $k_r$, $k_t$, $k_\phi$: Radial, tangential and axial cutting force coefficients

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Availability of data and material The measuring data in our paper are available from the corresponding author by request, and other related materials can also be obtained from the corresponding author.

Code availability The code for cutting force model during the study is available from the corresponding author by request.

Declarations

Ethics approval Not applicable.

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