Enhancement of disoriented chiral condensate domains with friction

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Abstract

We investigate the effect of friction on domain formation in disoriented chiral condensate. Including a friction term, we solve the equation of motion of the linear sigma model fields, in the Hartree approximation. With boost-invariance and cylindrical symmetry, irrespective of friction, on average, we donot find any indication of domain like formation with quenched initial condition. However, with or without friction, some events can be found with large instabilities, indicating possible DCC domain formation in those events. With friction time scale during which instabilities grows increases. Correspondingly, with friction, it is possible to obtain large sized domains in some particular events.

25.75.+r, 12.38.Mh, 11.30.Rd
The possibility of forming disoriented chiral condensate (DCC) in relativistic heavy ion collisions has generated considerable research activities in recent years. The idea was first proposed by Rajagopal and Wilczek [1–4]. They argued that for a second order chiral phase transition, the chiral condensate can become temporarily disoriented in the nonequilibrium conditions encountered in heavy ion collisions. As the temperature drops below $T_c$, the chiral symmetry begins to break by developing domains in which the chiral field is misaligned from its true vacuum value. The misaligned condensate has the same quark content and quantum numbers as do pions and essentially constitute a classical pion field. The system will finally relaxes to the true vacuum and in the process can emit coherent pions. Since the disoriented domains have well defined isospin orientation, the associated pions can exhibit novel centauro-like [5–8] fluctuations of neutral and charged pions [9–12].

Most dynamical studies of DCC have been based on the linear sigma model, in which the chiral degrees of freedom are described by the real $O(4)$ field $\Phi = (\sigma, \Pi)$, having the equation of motion,

$$[\Box + \lambda(\Phi^2 - v^2)]\Phi = Hn_\sigma$$  \hspace{1cm} (1)

The parameters of the model can be fixed by specifying the pion decay constant, $f_\pi = 92$ MeV and the meson masses, $m_\pi = 135$ MeV and $m_\sigma = 600$ MeV, leading to $\lambda = (m_\sigma^2 - m_\pi^2)/2f_\pi^2 = 20.14$ and $v = [(m_\sigma^2 - 3m_\pi^2)/(m_\sigma^2 - m_\pi^2)]^{1/2}f_\pi = 86.71$ MeV and $H = (120.55 MeV)^3$ \hspace{1cm} [13]. It is apparent from eq.1 that the vacuum is aligned in the $\sigma$ direction $\Phi_{\text{vac}} = (f_\pi, 0)$ and at low temperature the fluctuations represent nearly free $\sigma$ and $\pi$ mesons. At very high temperature well above $v$, the field fluctuations are centered near zero and approximate $O(4)$ symmetry prevails.

It is instructive to decompose the chiral field,

$$\Phi(r, t) = <\phi(r, t)> + \delta\phi(r, t)$$ \hspace{1cm} (2)

where $<\phi>$ is the mean field and $\delta\phi$ are the semiclassical fluctuations around $<\phi>$ and can be identified with quasi-particle excitations. Using eq.2 and taking the average of eq.1, the equation of motion for the mean fields in the Hartree approximation can be obtained as \hspace{1cm} [13–14],

$$\frac{\partial^2 <\phi>}{\partial t^2} - \nabla^2 <\phi> = \lambda(v^2 - <\phi>^2 - 3 <\delta\phi_\parallel^2> - <\delta\phi_\perp>) <\phi> + Hn_\sigma$$ \hspace{1cm} (3)

where $<\phi> = <\phi_i >$, $\delta\phi_\parallel$ is the component of the fluctuation parallel to $<\phi>$ and $\delta\phi_\perp$ is the orthogonal component. This equation imply that the motion of the mean field is determined by the effective potential,

$$V(<\phi>) = \frac{\lambda}{4}(<\phi>^2 + 3 <\delta\phi_\parallel^2> + <\delta\phi_\perp^2> - v^2)$$ \hspace{1cm} (4)

which clearly differs from the zero temperature one in presence of fluctuations. By varying the fluctuations, chiral symmetry can be restored or spontaneously broken. It is also evident that the evolution of the mean field critically depends on the initial values of the fluctuations. When $\delta^2 \equiv (3 <\delta\phi_\parallel^2> - <\delta\phi_\perp^2>) / 6$ is large enough the chiral symmetry is approximately (as $H \neq 0$) restored. If the explicit chiral symmetry breaking term is neglected, the phase
transition takes place at the critical fluctuations \( \delta^2_c \equiv v^2/6 \). For \( \delta^2 < \delta^2_c \), the effective potential takes its minimum value at \( \langle \phi \rangle = (\sigma_e, 0) \), where \( \sigma_e \) depends on \( \delta^2 \). When the mean fields are displaced from this equilibrium point to the central lump of the Mexican hat (\( \langle \phi \rangle \approx 0 \)), the effective mass square

\[
m_{\text{eff}}^2 = \lambda (v^2 - \langle \phi \rangle^2 - 3 \langle \delta \phi_\parallel^2 \rangle - \langle \delta \phi_\perp^2 \rangle)
\]

will become negative and DCC can form. Since the domain size is directly related to the time scale, during which the effective mass remains negative, it strongly depends on the initial condition of the system. By varying the \( \langle \phi_i \rangle \) and \( \delta^2 \), quench or annealing like initial condition can be obtained [14]. In an important paper Asakawa et al [14] studied eq.3 with initial condition corresponding to quench and annealing. They found that domains of disoriented chiral condensate with 4-5 fm in size can form through a quench. Annealing on the other hand, leads to smaller sized domains.

In the present paper, we solve eq.3 with a different motivation. Our interest is to investigate the effect of friction on DCC domain formation. Dissipative effects like friction damp the motion of the fields, inhibiting large oscillations. Naively, one would expect reduction of DCC domain formation with friction. Recently, Biro and Greiner [15], using the Langevin equation for the linear sigma model, investigated the interplay of friction and white noise on the evolution of the order parameter. While noise greatly diminishes the possibility of DCC domain formation, in some orbits, large instabilities can result, producing DCC domains. We have also studied the effect of friction on DCC domain formation using the Langevin equation [16]. There we found that for one-dimensional expansion on average large DCC domain can not be formed. However, in some particular orbit large instabilities can occur. This possibility also reduces with introduction of friction. However, if the friction is large, the system may be overdamped and then there is a possibility of DCC domain like formation. Present paper is an extension of the above study, the spatial part, which had been integrated out in our earlier analysis is being studied here.

Appropriate coordinates for heavy ion collisions are the proper time (\( \tau \)) and the rapidity (Y). For a d-dimensional expansion, the change can be effected by the following replacement,

\[
\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} \rightarrow \frac{d}{\tau} \frac{\partial}{\partial \tau} \tau \frac{\partial}{\partial \tau} - \frac{1}{\tau^2} \frac{\partial^2}{\partial Y^2}
\]

(6)

It can be seen from the above equation that with the introduction of proper time and rapidity, a dissipative term comes into effect in the equation of motion. To illustrate the role of friction, we further introduce a dissipative term (\( \eta \)) in the equation. To simplify our calculation, we assume boost-invariance as well as cylindrical symmetry in the system. With boost-invariance and cylindrical symmetry, eq.3 can be written as,

\[
\frac{\partial^2}{\partial \tau^2} + (\frac{d}{\tau} + \eta) \frac{\partial}{\partial \tau} = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \lambda (v^2 - \langle \phi \rangle^2 - T^2/2) \langle \phi \rangle + H n_\sigma
\]

(7)

where \( \eta \) is the friction coefficient and we have replaced the fluctuation terms by their counterpart (\( T^2/2 \)) in the finite temperature field theory [17,18]. The equation of motion of fields then depend sensitively on the initial temperature of the system. In the following, we
assume initial temperature to be $T_c = \sqrt{2f_\pi^2 - 2m_\sigma^2/\lambda} = 123$ MeV at the initial time $\tau_i = 1$ fm. The cooling of the system is described by the equation,

$$\frac{\dot{T}}{T} + \frac{d}{3\tau} = 0$$  \hspace{1cm} (8)

In the weak coupling limit, the friction $\eta$ is related to the on-shell plasmon damping rate, $\eta \equiv 2\gamma_{pl}$. In the standard $\phi^4$ theory, the plasmon damping rate can be calculated for the $\sigma$ and the $\Pi$ fields \cite{13,20}. Assuming that the meson masses are the same, the friction coefficient can be obtained as \cite{15},

$$\eta = 2\frac{\gamma_{pl}}{16\pi^3} \lambda^2 \frac{T^2}{\pi f^6} \int \ln \frac{t}{t-1}$$  \hspace{1cm} (9)

where $f_{\text{Sp}} = -\int_1^r dt \int_1^r f_{\text{Sp}}(1 - e^{-m/T})$ is the Spence function. At $T = T_c = \sqrt{2f_\pi^2 - 2m_\sigma^2/\lambda} = 123$ MeV, and if $m/T \simeq 1$, the friction $\eta = 2.2 \text{ fm}^{-1}$. In order to have a comparison, we also consider scenarios with $\eta=0$ and $\eta=0.5 \text{ fm}^{-1}$. We also neglect the temperature dependence of the friction coefficient.

We solve the set of partial differential equations with the quenched initial condition, for two dimensional $(d=2)$ expansion using the standard Leap frog method. Evolution of the fields upto $\tau=7$ fm are followed. As argued before, DCC formation depend critically on the initial condition. To reflect the uncertainty in the initial condition, the initial fields are randomly distributed to a Gaussian form with the following parameters \cite{14},

$$< \sigma > = (1 - f(r))f_\pi$$

$$< \pi_i > = 0$$

$$< \sigma^2 > - < \sigma >^2 = < \pi_i^2 > - < \pi_i >^2 = \frac{v^2}{6}f(r)$$

$$< \dot{\sigma} >= < \dot{\pi}_i >= 0$$

$$< \dot{\sigma}^2 > = < \dot{\pi}^2 > = 4v^2/6$$  \hspace{1cm} (13)

The interpolation function

$$f(r) = [1 + \exp(r - r_0)/\Gamma]^{-1}$$  \hspace{1cm} (15)

separates the central region where the initial field configuration is different from their vacuum expectation value. We use $r_0=5$ fm and $\Gamma=0.5$ fm.

The equation of motion was solved for 500 trajectories. For each trajectories, at each space-time we compute the effective mass $m_{\text{eff}}^2$. The phenomenon of long wavelength DCC amplification will occur whenever the effective mass squared is negative. To single out the trajectories for which maximum instabilities occur for each trajectories, we calculate the following quantity,

$$G = \int |m_{\text{eff}}| \Theta(-m_{\text{eff}}^2) d\tau dr$$  \hspace{1cm} (16)

This can be a measure of instability in a particular evolution. We call it amplification factor. This is an important parameter, as it directly relates to the size of DCC domains. In
fig.1, we have shown the distribution of the amplification factor $G$ for the 500 trajectories. Three scenarios; (i) friction free case ($\eta=0$), (ii) small friction ($\eta=0.5 \text{ fm}^{-1}$) and (iii) large friction ($\eta=2.2 \text{ fm}^{-1}$) are shown. Several interesting features emerge. Maximum amplification factor is increased with friction. However the increase is small ($G_{\text{max}}=2.19$ for $\eta=0$ and $G_{\text{max}}=2.55$ for $\eta=2.2 \text{ fm}^{-1}$). Minimum amplification factor consequently decreases with friction. As a result, distribution is broadened with friction. The result suggest that with friction, while number of orbits with minimum instabilities are increased, some orbits can be found with enhanced instabilities. Thus with friction possiblity of DCC domain like formation is increased in some orbits. This is contrary to the naive expectation that friction will inhibit DCC like formation. However, this is in agreement with our earlier study that with friction some trajectories can show appreciable instabilities [16].

In fig.2, we have shown the contour plot of $\text{sgn}(m_{\text{eff}})$ obtained by averaging the fields over the 500 events, as obtained in the three scenarios, (i)$\eta=0$, (ii)$\eta=0.5 \text{ fm}^{-1}$ and (iii)$\eta=2.2 \text{ fm}^{-1}$. Friction lowers the magnitude of $m_{\text{eff}}^2$ is evident from the figure. However, $m_{\text{eff}}^2$ remain positive throughout the space-time region in all the three scenarios. The result indicate that on averaging, quenched initial condition do not lead to DCC domain formation. Though not shown, average fields are also found to be order of magnitude lower than obtained in an individual event. The result suggests that on average DCC like phenomena will not be observed.

The behavior is changed in individual events. In fig.3, we have shown the contour plot of $\text{sgn}(m_{\text{eff}})$ for the most unstable orbit i.e. for which the amplification factor is the maximum. The three cases are labeled appropriately. For the friction free case, the effective mass remains negative over a considerable space-time. Thus from from $\tau=2-4 \text{ fm}$ over a considerable radial range (2-4 fm), $m_{\text{eff}}^2$ is negative and one may expect DCC like formation correspondingly. With small friction, the pattern remains essentially the same, but space-time region where $m_{\text{eff}}$ remains negative is increased. Entirely different behavior results when friction is high ($\eta=2.2 \text{ fm}^{-1}$). In some radial ranges, soon after the start of the evolution, $m_{\text{eff}}^2$ becomes negative and remains negative throughout the evolution. This is an indication of overdamped system. The viscous drag is large and once the system enters the unstable region viscous drag forces it remain there. This result is interesting, as it suggest possibility of forming large domain like structure with high viscous force in the system.
In fig.4, the contour plot of the $\pi_1$ field for the most unstable orbit are shown. The fields have been scaled by $f_\pi$. For the friction free case several regions can be identified where growth of negative $\pi_1$ is considerable. Thus for example around $r=6$ fm, negative $\pi_1$ grows for about 4-5 fm of (proper) time. The results for small friction case is nearly similar. We observe that for friction free and small friction case, DCC domain like structure can grow for about 4 fm. However, the radial extension is not large, it is about 1 fm. For large friction case ($\eta=2.2$ fm$^{-1}$, the pattern is again different and reminiscent of the $m^{eff}$. We find strips of positive and negative $\pi_1$ again of radial extension of about 1 fm starts growing just after the start of the evolution. Evidently, with large friction the domains can grow for a longer duration.

![Contour plot](image)

**FIG. 2.** Contour plot of evolution of $sgn(m_{eff})$ obtained by averaging the fields over the 500 events for (i)$\eta=0$, (ii)$\eta=0.5$ fm$^{-1}$ and (iii)$\eta=2.2$ fm$^{-1}$. 
FIG. 3. Contour plot of evolution of $sgn(m_{eff})$ for the event for which the amplification factor is (G) the maximum. Three scenarios (i)$\eta=0$, (ii)$\eta=0.5 \text{ fm}^{-1}$ and (iii)$\eta=2.2 \text{ fm}^{-1}$ are shown.

Before we summarise, it is important to note one apparent contradiction we observe presently. For the friction free and small friction case domain like structure of $\pi_1$ field do not exactly follow the pattern of $sgn(m_{eff})$. Thus in those space-time region where $m^2_{eff}$ is most negative, we donot find large amplitude of $\pi_1$ field. Though not shown, we have checked that this is true for $\pi_2$ and $\pi_3$ fields also. However in the large friction scenario, $\pi_1$ follows nearly the pattern of $m^2_{eff}$. The reason may be the symmetry breaking term. The assumption that phenomenon of long wavelength oscillation occur whenever $m^2_{eff}$ is negative implicitly neglects the symmetry breaking term. However, for small friction, symmetry breaking term may not be negligible. Then, long wavelength oscillations will not follow exactly $sgn(m_{eff})$. 
FIG. 4. Contour plot of evolution of $\pi_1/f_\pi$ for the event for which the amplification factor (G) is the maximum. Three scenarios (i) $\eta=0$, (ii) $\eta=0.5 \text{ fm}^{-1}$ and (iii) $\eta=2.2 \text{ fm}^{-1}$ are shown.

To summarise, we have investigated the effect of friction on the possible DCC domain formation. In the equation of motion for the linear sigma model fields, we include a friction term and solve it assuming boost invariance and cylindrical symmetry. Initial field configuration was assumed to be Gaussian random with quenched condition ($\langle \phi \rangle = \langle \dot{\phi} \rangle = 0$). At each space-time $m_{\text{eff}}^2$ was calculated. Phenomenon of long wavelength amplification occurs when it becomes negative. $m_{\text{eff}}^2$ obtained by averaging the field over 500 events remain positive throughout the space-time region investigated, indicating that on average, DCC domain like formation is not possible. However, instabilities can occur in some particular events. In those events, where the instabilities are the largest, we find evidence of domain like structure formation. For friction free and small friction, domains can grow for about 3-4 fm. But with large friction, the system become overdamped and instabilities can continue for longer duration.
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