The influence of thermal contact between medium on the surface waves propagation in a prestressed thermoelastic layered half-space

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Abstract. In the framework of the linearized theory of the propagation of thermoelastic waves, a dynamic mixed coupled problem on the oscillations of an inhomogeneous medium under the action of a thermal load oscillating on its surface is considered. The medium is a thermoelastic prestressed layer rigidly coupled with a thermoelastic prestressed half-space. At the boundary of the medium, two types of thermal conditions are considered: ideal thermal contact and heat insulation. Using operational calculus methods, the boundary value problem with mixed thermal boundary conditions is reduced to an integral equation of the first kind with respect to the unknown distribution function of the heat flux in the contact zone. We studied the distribution of the poles of the Green's function of a medium, taking into account the effect of initial strains and preheating. It is shown that thermal conditions at the interface between the medium have little effect on the distribution of the vertical displacement of the layer surface in the natural state. At the same time, the condition of thermal insulation between the layer and half-space allows to compensate for the effect of body pre-heating on the formation of the surface wave field.

1. Introduction
The development of modern technologies causes a considerable interest in the processes of excitation of mechanical vibrations due to the effect of laser radiation [1]. A fairly comprehensive overview of the work in this direction is given in [2, 3]. In the majority of works, various approaches are used for the study, which make it possible to efficiently analyse various aspects of dynamic processes in thermoelastic medium. These are, for example, the propagation features Rayleigh waves [4, 5]. In [6] consecutive linearization of nonlinear equations of a thermoelastic medium was performed and equations of motion and the governing relations of the dynamics of a prestressed thermoelastic medium were constructed. The equations are constructed in tensor form. Of considerable interest are issues of contact interaction between thermoelastic bodies [7, 8]. In paper [7], questions of the contact interaction of a thermoelastic layer were investigated. The equations of motion constructed in [8] and the defining relations of a prestressed thermoelastic medium in [8] are generalized to the class of a pre-stressed thermoelastic medium with a non-uniform coating. Of particular interest are the problems of propagation of Rayleigh waves in piezoactive thermoelastic medium. A rather complete review of the works is given in [9]. In [14] was considered problem of one-dimensional unsteady thermoelastic diffusion in homogeneous isotropic multicomponent half-space. In the present work, in the framework of the linearized theory of propagation of coupled thermoelastic waves [6], the mixed problem of oscillation of a layered half-space under the action of a thermal load specified on the surface of the medium, simulating the effect of a frequency-modulated laser beam, is considered. The effect of a
non-uniform thermoelastic half-space on the interface between the layer and half-space on the features of the wave field formation is investigated.

2. Problem formulation

We consider a layered thermoelastic body subjected to a uniform initial stress, due to mechanical stress and temperature effects. The body is represented by a layer rigidly coupled with the half-space. At the boundary of the layer with a half-space, two kinds of conditions are considered: ideal thermal contact; heat insulation. Vibration \( u^{(n)} = \{u_1^{(n)}, u_2^{(n)}, u_3^{(n)}, u_4^{(n)}\} \) - are components of the vector of mechanical displacements, \( u_4^{(n)} \) - is temperature) is caused by the temperature \( \tau_0 e^{i\omega t} \) distributed in the area \( \Omega = \{x_1 \leq 1, |x_2| \leq \infty\} \) on the layer surface. Outside the load area, the body surface is assumed to be thermally insulated, free from mechanical stresses. At infinity the damping condition is satisfied. For convenience, the notation \( \omega \) is the circular oscillation frequency, \( n = 0 \) are the parameters of the half-space, \( n = 1 \) are the parameters of the layer. Oscillations are assumed to be steady, occurring according to the harmonic law, so all functions are represented as \( f = f_0 e^{i\omega t} \). Further exponents can be omitted.

3. Basic equations

In the general case the vibrations of a prestressed layered thermoelastic medium are described by the equations of motion and thermal conductivity [6, 7, 10]:

\[
\nabla \cdot \Theta^{(n)} = \rho_0 \left( \frac{\partial^2 u^{(n)}}{\partial t^2} \right) - \beta^{(n)} \left( \frac{\partial^{(n)} \Theta^{(n)}}{\partial t} \right) - \beta^{(n)} u_4^{(n)}
\]

\[
\lambda^{(n)} u_{4,ik}^{(n)} = \frac{\lambda^{(n)}}{\rho_0} \frac{\partial \Theta_i^{(n)}}{\partial t} + \theta_0 (\theta_1 - \theta_0) \beta^{(n)} u_{4,ik}^{(n)}, \quad n = 0, 1, i, j, k, l = 1 + 3.
\]

The environment elastic and thermal parameters participating in equations (1) (2) with uniformly initial deformations and predefined expressions [6]:

\[
\left\{ \begin{array}{l}
\lambda^{(n)} = \frac{\delta_{ij} - \epsilon_m (\nu_m - 1)}{2} + \epsilon_{ijkl} V_m^{(n)} - \delta_{ij} (\theta_1 - \theta_0) \beta_{ij}^{(n)}, \\
\beta_{ij}^{(n)} = \frac{\nu_j}{\beta_0^{(n)}}, \\
\end{array} \right. \quad \beta_0^{(n)} = \frac{\nu_j}{\beta_0^{(n)}}, \quad \lambda_{ij} = \frac{\lambda_{ij}^{(n)}}{\lambda_{ij}^{(n)}}
\]

The problem is indicated in dimensionless quantities [7, 9-10]:

\[
x_i' = \frac{x_i}{V_i'}, \quad t' = \frac{\omega}{V_i'} t, V_i^{(n)} = \sqrt{c_{ijkl}^{(n)}} \rho_0^{(n)} \omega, u_i^{(n)} = \frac{u_i^{(n)}}{c_{ijkl}^{(n)}}, \quad \rho_0^{(n)} \omega V_p^{(n)}
\]

\[
\left\{ \begin{array}{l}
E_i^{(n)} = \frac{\theta_0 \beta_{ij}^{(n)} c_{ijkl}^{(n)}}{\rho_0^{(n)}}, u_i^{(n)} = \frac{u_i^{(n)}}{c_{ijkl}^{(n)}}, \quad c_{ijkl}^{(n)} = \frac{c_{ijkl}^{(n)}}{c_{ijkl}^{(n)}}, \\
\beta_{ij}^{(n)} = \frac{\nu_j}{\beta_0^{(n)}}, \quad \lambda_{ij} = \frac{\lambda_{ij}^{(n)}}{\lambda_{ij}^{(n)}}, \quad \alpha_{ij} = \frac{\nu_j}{\omega}, \quad \omega = \frac{\nu_j}{\omega}, \quad \omega = \frac{\nu_j}{\omega}
\end{array} \right. \quad \beta_0^{(n)}, \quad \lambda_{ij}^{(n)}, \quad \alpha_{ij}^{(n)} = \alpha_{ij}^{(n)} c_{ijkl}^{(n)}
\]

In equations (1) - (5) \( c_{ijkl}^{(n)} \), \( \lambda_{ij}^{(n)} \), \( \alpha_{ij}^{(n)} \), \( \beta_{ij}^{(n)} \) are the components of the elastic constant tensors, thermal conductivity coefficients, thermal expansion, thermoelasticity, \( \rho_0^{(n)} \) is density of a material in its natural state, \( c_e^{(n)} \) is specific heat capacity. \( \theta_0 \) and \( \theta_1^{(n)} \), respectively, are the body temperature in the undeformed state and the temperature of the nth layer in the initial deformed state, \( V_i^{(n)} = 1 + \delta_i^{(n)} \) where \( \delta_i^{(n)} \) is the relative elongation of the fibers, \( E_i^{(n)} \) is the constant of
thermoelastic relation, $\omega^*$ is the normalized frequency of the half-space, $V_p^{(n)}$ is the velocity of the longitudinal wave of the non-deformed material. Next, the zeros in the indices and strokes are omitted. The considered problem is assumed to be plane, i.e. all field quantities are independent of $x_2$ so that $f = f(x_1, x_3)$, $\frac{\partial}{\partial x_2} f \equiv 0$.

Taking into account formulas (1) - (5) the equations of motion and thermal conductivity of a layered prestressed thermoelastic half-space are written in the form:

\[
c_{11}^{(n)} u_{1,11}^{(n)} + c_{311}^{(n)} u_{1,33}^{(n)} + \omega^2 u_1^{(n)} + c_{2}^{(n)} u_{4,13}^{(n)} - \beta_1^{(n)} u_{4,1}^{(n)} = 0,
\]

\[
c_2^{(n)} u_{1,13}^{(n)} + c_{133}^{(n)} u_{1,33}^{(n)} + \omega^2 u_3^{(n)} + c_{333}^{(n)} u_{3,3}^{(n)} - \beta_3^{(n)} u_{4,4,3}^{(n)} = 0,
\]

\[
u_{4,1}^{(n)} + \lambda_3^{(n)} u_{4,3}^{(n)} + i \omega \theta_1^{(n)} u_4^{(n)} + i \omega \theta_1^{(n)} E(\beta_1^{(n)} u_{1,11}^{(n)} + \beta_3^{(n)} u_{3,3}^{(n)}) = 0. \tag{6}
\]

We are consider a particular case when oscillations in the body are induced by temperature $u_* = \{0, 0, \tau_0 \}$ distributed on the surface of the layer. For convenience, an extended stress vector of a thermoelastic medium $q^{(n)} = \{\theta_1^{(n)}, \theta_3^{(n)}, \ldots, \lambda_3^{(n)}, u_4^{(n)} \}$ is introduced. Taking into account the applied notation, the linearized boundary conditions take the form:

\[
x_1 = 0:
\]

\[
\begin{align*}
\{u\}^{(1)} &= u_0(x_1), \quad x_1 \in \left[-a; a\right], \\
\{q\}^{(1)} &= 0, \quad (x_1), \quad x_1 \not\in \left[-a; a\right],
\end{align*}
\]

\[
x_1 = -h:
\]

\[
\begin{align*}
\{\theta\}^{(1)} &= \theta_3^{(0)}, \quad \{u\}^{(0)} = u_0^{(0)}, \quad \theta_3^{(0)} = \theta_3^{(0)}, \\
\{u\}^{(1)} &= u_1^{(0)}, \quad u_3 = u_3^{(0)}, \quad (I)
\end{align*}
\]

\[
\begin{align*}
\{-\lambda_3^{(0)} u_{4,4,3}^{(0)} = 0, \quad (II)
\end{align*}
\]

\[
x_3 \rightarrow -\infty:
\]

\[
\{u\}^{(0)} \rightarrow 0, \quad (10)
\]

In eqs. (9) the notations for the thermal conditions at the interface between the medium are given: (I) an ideal thermal contact, (II) thermal insulation.

To construct the solution of the boundary value problem (6) - (10), we introduce an auxiliary boundary value problem. For this purpose, it is assumed that vibrations in the medium are induced by the vertical component of the heat flux $q_0 = \{0 \quad 0 \quad 0 \}$ distributed on the layer surface. Then the linearized boundary conditions are written as follows:

\[
x_1 = 0:
\]

\[
\begin{align*}
\{q\}^{(1)} &= q_0(x_1), \quad x_1 \in \left[-a; a\right], \\
0, \quad x_1 \not\in \left[-a; a\right],
\end{align*}
\]

\[
x_1 = -h:
\]

\[
\begin{align*}
\{\theta\}^{(1)} &= \theta_3^{(0)}, \quad \{u\}^{(0)} = u_0^{(0)}, \quad \theta_3^{(0)} = \theta_3^{(0)}, \\
\{u\}^{(1)} &= u_1^{(0)}, \quad u_3 = u_3^{(0)}, \quad (I)
\end{align*}
\]

\[
\begin{align*}
\{-\lambda_3^{(0)} u_{4,4,3}^{(0)} = 0, \quad (II)
\end{align*}
\]

\[
x_3 \rightarrow -\infty:
\]

\[
\{u\}^{(0)} \rightarrow 0 \quad (14)
\]
4. Solving the problem
The one-dimensional Fourier transform along the coordinate \( x_1 \) is applied. Then, we rewrote the system of motion and heat conduction equations (6) and boundary conditions (11) - (14) as ([7, 10]):

\[
\begin{align*}
-\alpha^2 c^{(n)}_{111} U_1^{(n)} + c^{(n)}_{311} U_3^{(n)} + \omega^2 U_1^{(n)} - i\alpha c^{(n)}_{222} U_3^{(n)} + i\alpha \beta_{11}^{(n)} U_4^{(n)} &= 0, \\
-\alpha^2 c^{(n)}_{133} U_3^{(n)} + \omega^2 U_3^{(n)} + c^{(n)}_{333} U_3^{(n)} - \beta_{33}^{(n)} U_4^{(n)} &= 0, \\
-\alpha^2 U_4^{(n)} + \beta_{23}^{(n)} U_4^{(n)} + i\omega \beta_{11}^{(n)} U_4^{(n)} + i\omega \beta_{11}^{(n)} (\alpha \beta_{11}^{(n)} U_1^{(n)} + \beta_{33}^{(n)} U_4^{(n)}) &= 0,
\end{align*}
\]

(15)

\[
x_3 = 0
\]

\[
\begin{align*}
U_1^{(l)} &= U_1^{(r)}, \\
U_3^{(l)} &= U_3^{(r)}, \\
\begin{cases}
U_1^{(l)} - i\alpha \sigma_4^{(l)} U_3^{(l)} - c^{(l)}_{133} U_3^{(l)} + i\alpha c^{(l)}_{222} U_3^{(l)} + i\alpha \beta_{11}^{(l)} U_4^{(l)} = 0, \\
U_3^{(l)} - i\sigma_4^{(l)} U_4^{(l)} - \beta_{33}^{(l)} U_4^{(l)} + c^{(l)}_{333} U_3^{(l)} + i\alpha \beta_{11}^{(l)} U_4^{(l)} = 0,
\end{cases}
\end{align*}
\]

(16)

\[
x_3 = -h:
\]

\[
\begin{align*}
U_1^{(l)} &= U_1^{(r)}, \\
U_3^{(l)} &= U_3^{(r)}, \\
\begin{cases}
U_1^{(l)} - i\alpha \sigma_4^{(l)} U_3^{(l)} - c^{(l)}_{133} U_3^{(l)} + i\alpha c^{(l)}_{222} U_3^{(l)} + i\alpha \beta_{11}^{(l)} U_4^{(l)} = 0, \\
U_3^{(l)} - i\sigma_4^{(l)} U_4^{(l)} - \beta_{33}^{(l)} U_4^{(l)} + c^{(l)}_{333} U_3^{(l)} + i\alpha \beta_{11}^{(l)} U_4^{(l)} = 0,
\end{cases}
\end{align*}
\]

(17)

(18)

(19)

Here the notation is used: \( f' = df/dx_3 \), \( f^* = d^2f/dx_3^2 \).

The solution of the auxiliary boundary value problem (15) - (19) will be sought in the form [7, 10]:

\[
\begin{align*}
U_1^{(l)}(\alpha, x_3, \omega) &= -i\alpha \sum_{k=1}^{3} f_k^{(l)} \left( C_k s \sigma_k^{(l)} x_3 + C_k x_3 \chi \sigma_k^{(l)} x_3 \right), \\
U_3^{(l)}(\alpha, x_3, \omega) &= \sum_{k=1}^{3} f_k^{(l)} \left( C_k \chi \sigma_k^{(l)} x_3 + C_k x_3 \chi \sigma_k^{(l)} x_3 \right), \\
U_4^{(l)}(\alpha, x_3, \omega) &= \sum_{k=1}^{3} f_k^{(l)} \left( C_k s \sigma_k^{(l)} x_3 + C_k x_3 \chi \sigma_k^{(l)} x_3 \right) \quad -h \leq x_3 \leq 0;
\end{align*}
\]

\[
\begin{align*}
U_1^{(0)}(\alpha, x_3, \omega) &= -i\alpha \sum_{k=1}^{3} f_k^{(0)} D_k e^{\sigma_4^{(0)} x_3}, \\
U_3^{(0)}(\alpha, x_3, \omega) &= \sum_{k=1}^{3} f_k^{(0)} D_k e^{\sigma_4^{(0)} x_3}, \\
U_4^{(0)}(\alpha, x_3, \omega) &= \sum_{k=1}^{3} f_k^{(0)} D_k e^{\sigma_4^{(0)} x_3}, \quad x_3 \leq -h.
\end{align*}
\]

(20)

(21)

In (20) - (21), \( \sigma_k \) are found numerically for each value of \( \alpha \) and \( \omega \) from the characteristic equation given in [5]. To find the unknown coefficients \( C_k, D_k \), we substitute the representation of the solution (20) - (21) into the boundary conditions (16) - (19) and obtain a system of linear algebraic equations, which can be written in the matrix form:

\[
LC = Q^T,
\]

(22)

where \( Q = \begin{bmatrix} 0 & 0 & Q_4 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \), \( Q_4 \) is the Fourier image of \( q_{40} \). Dispersion equation of the problem: \( \det L = 0 \).
\[ L = (L_1, L_2, L_3), \quad L_1 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
u_1^{(1)} s_1^{(1)} & u_1^{(1)} s_1^{(1)} & u_2^{(1)} s_1^{(1)} \\
u_3^{(1)} s_1^{(1)} & u_3^{(1)} s_1^{(1)} & u_4^{(1)} s_1^{(1)} \\
u_1^{(1)} s_1^{(3)} & u_1^{(1)} s_1^{(3)} & u_2^{(1)} s_1^{(3)} \\
u_3^{(1)} s_1^{(3)} & u_3^{(1)} s_1^{(3)} & u_4^{(1)} s_1^{(3)} \\
u_1^{(1)} s_1^{(4)} & u_1^{(1)} s_1^{(4)} & u_2^{(1)} s_1^{(4)} \\
u_3^{(1)} s_1^{(4)} & u_3^{(1)} s_1^{(4)} & u_4^{(1)} s_1^{(4)} \\
u_1^{(1)} s_1^{(1)} & u_1^{(1)} s_1^{(1)} & u_2^{(1)} s_1^{(1)} \\
u_3^{(1)} s_1^{(1)} & u_3^{(1)} s_1^{(1)} & u_4^{(1)} s_1^{(1)} \\
u_1^{(1)} e_{11} & u_1^{(1)} e_{13} & u_2^{(1)} e_{13} \\
u_3^{(1)} e_{11} & u_3^{(1)} e_{13} & u_4^{(1)} e_{13} \\
u_1^{(1)} e_{14} & u_1^{(1)} e_{14} & u_2^{(1)} e_{14} \\
u_3^{(1)} e_{14} & u_3^{(1)} e_{14} & u_4^{(1)} e_{14} \\
u_1^{(1)} e_{11} & u_1^{(1)} e_{11} & u_2^{(1)} e_{11} \\
u_3^{(1)} e_{11} & u_3^{(1)} e_{11} & u_4^{(1)} e_{11} \\
u_1^{(1)} e_{13} & u_1^{(1)} e_{13} & u_2^{(1)} e_{13} \\
u_3^{(1)} e_{13} & u_3^{(1)} e_{13} & u_4^{(1)} e_{13} \\
u_1^{(1)} e_{14} & u_1^{(1)} e_{14} & u_2^{(1)} e_{14} \\
u_3^{(1)} e_{14} & u_3^{(1)} e_{14} & u_4^{(1)} e_{14}
\end{pmatrix} \]

\[ \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
u_1^{(1)} e_{11} & u_1^{(1)} e_{11} & u_2^{(1)} e_{11} \\
u_3^{(1)} e_{11} & u_3^{(1)} e_{11} & u_4^{(1)} e_{11} \\
u_1^{(1)} e_{13} & u_1^{(1)} e_{13} & u_2^{(1)} e_{13} \\
u_3^{(1)} e_{13} & u_3^{(1)} e_{13} & u_4^{(1)} e_{13} \\
u_1^{(1)} e_{14} & u_1^{(1)} e_{14} & u_2^{(1)} e_{14} \\
u_3^{(1)} e_{14} & u_3^{(1)} e_{14} & u_4^{(1)} e_{14} \\
u_1^{(1)} e_{11} & u_1^{(1)} e_{11} & u_2^{(1)} e_{11} \\
u_3^{(1)} e_{11} & u_3^{(1)} e_{11} & u_4^{(1)} e_{11} \\
u_1^{(1)} e_{13} & u_1^{(1)} e_{13} & u_2^{(1)} e_{13} \\
u_3^{(1)} e_{13} & u_3^{(1)} e_{13} & u_4^{(1)} e_{13} \\
u_1^{(1)} e_{14} & u_1^{(1)} e_{14} & u_2^{(1)} e_{14} \\
u_3^{(1)} e_{14} & u_3^{(1)} e_{14} & u_4^{(1)} e_{14}
\end{pmatrix} \]

where

\[ l_{ik}^{(n)}(\xi) = -ia(c_{1131}^{(n)} f_{ik}^{(n)} + c_{1331}^{(n)} f_{ik}^{(n)}), \quad l_{ik}^{(n)} = -\alpha^2 c_{1131}^{(n)} f_{ik}^{(n)} + c_{3331}^{(n)} f_{ik}^{(n)} - \beta_{3}^{(n)} f_{4k}^{(n)}, \]

\[ l_{ik}^{(n)} = \sigma_k f_{ik}^{(n)}, \quad u_p^{(n)} s_p^{(n)} = f_{pk}^{(n)} sh(-\sigma_k^{(n)}), \quad u_p^{(n)} c_p^{(n)} = f_{pk}^{(n)} ch(-\sigma_k^{(n)}), \]

\[ u_p^{(n)} e_k^{(n)} = f_{pk}^{(n)} e^{-\sigma_k^{(n)}}, \quad e_k^{(1)} = ch(-\sigma_k^{(1)}), \quad s_k^{(1)} = sh(-\sigma_k^{(1)}), \]

\[ e_k^{(0)} = e^{-\sigma_k^{(0)}}. \]

Finding \( C_k \), \( D_k \) from equations (22), we can obtain the solution of the auxiliary boundary value problem. After applying the inverse Fourier transform to expressions (20) - (21) the solution is written in the form

\[ u_i^{(n)}(x_1, x_3) = \frac{1}{2\pi i} \int k_i^{(n)}(x_1 - \xi, x_3, \omega) q_{40}(\xi) d\xi, \quad i = 1, 3, 4, \]

\[ k_i^{(n)}(x_1, x_3, \omega) = \int K_i^{(n)}(\alpha, x_3, \omega)e^{-i\alpha} d\alpha, \]

where \( K_i^{(n)}(\alpha, x_3, \omega) \) are Green’s function matrix elements, which are represented in [7].

Relations (29) determine the displacement of an arbitrary point of the thermoelastic half-space under the action of given load \( q_{40}(x_i) \) in the region \( x_i \in [-a; a]. \) To solve the original problems (6) - (10) in the eqs. (24) it is necessary to set \( x_3 = 0 \) and take into account the condition (7) of the effect of temperature \( \tau_0(x_1). \) As a result, we obtain the integral equation:

\[ \tau_0(x_1) = \frac{1}{2\pi i} \int k_i^{(n)}(x_1 - \xi, 0, \omega) q_{40}(\xi) d\xi, \quad k_i^{(1)}(x_1, 0, \omega) = \int K_i^{(1)}(\alpha, 0, \omega)e^{-i\alpha} d\alpha \]

The solution of the integral equation (26) is searched by using the method of boundary elements ([11 -12]). For this purpose, the function \( q_{40}(x_i) \) is constructed in the form
\[ q_{40}(x) = \sum_{k=1}^{2M} C_k \varphi_k(x), \quad \varphi_k(x), k = 1..2M. \tag{27} \]

Then we obtain the solution of the original boundary-value problems by substituting the solution of the integral equation (27) into the expression (24).

5. Results discussion

Consider the problem of vibrations of a steel layer rigidly coupled with a half-space of magnesium oxide, subjected to the temperature distribution on the medium surface in the region \([-1,1]\) determined by the formula \( u_{40} = \text{const} = 1 \). Tasks differ in thermal conditions (I) and (II) at the interface of medium \( x_3 = -h \).

For numerical studies, the materials described in [15] were selected.

A special feature of problems for thermoelastic bodies is the presence of a countable set of complex zeroes and poles for the elements of the function \( K \), some of which have a small imaginary part [7]. The analysis showed that, along with complex poles with a small imaginary part, the function has a class of essentially complex poles with a large imaginary part [7]. In order to build a solution and efficiently study a dynamic process, a detailed study of the behavior of the poles depending on the initial stresses and preheating is necessary.

Figure 1 represents calculated in a limited frequency range the Green function of a layered thermoelastic half-space poles with a small imaginary part in the absence of initial deformations and heating (a), taking into account the initial hydrostatic stretching of the body \( \nu = 1.01 \) (b), preheating \( d\theta = 0.1 \) (c). The poles in the graphs are related to the pole values \( \alpha \), of the shear wave arising in a homogeneous half-space in its natural state.

The analysis showed that for problems with conditions (I) and (II) dispersion curves with small imaginary parts have the same value distribution. In figure 1, it can be seen that preheating contributes to a significant decrease in the phase velocities of surface waves. While pre-stretching leads to a slight increase in phase velocities.

Figures 2-3 represent the distribution curves of the difference between the displacements \( du_3 = u_{3t}^{(I)} - u_{3t}^{(II)} \) of the medium on the surface of the layered half-space for problems with boundary conditions (I) (ideal thermal contact) and (II) (thermal insulation). \( u_{3t}^{(I)} \) is vertical component of the layer displacement with presence of preheating \( d\theta = \theta_1 - \theta_0 \).

Graphs 2-3 show that under conditions of ideal thermal contact at the interface between the medium, the effect of preheating on the distribution of the medium displacement vertical component significantly exceeds its effect during thermal insulation. This means that the condition of thermal insulation between the layer and half-space can compensate for the effect of preheating on the formation of a wave field in a layered thermoelastic body.
Figure 1. The normalized poles of the layered medium Green’s function: a) natural state; b) with stretching \( \nu = 1.01 \); c) with preheating \( d\theta = 0.1 \).

Figure 2. The effect of preheating on the real part of the vertical displacement component with the boundary conditions a) thermal contact b) thermal insulation.
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