A Variable-Free Dynamic Semantics

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I propose a variable-free treatment of dynamic semantics. By “dynamic semantics” I mean analyses of donkey sentences (Every farmer who owns a donkey beats it) and other binding and anaphora phenomena in natural language where meanings of constituents are updates to information states, for instance as proposed by Groenendijk and Stokhof [3]. By “variable-free” I mean denotational semantics in which functional combinators replace variable indices and assignment functions, for instance as advocated by Jacobson [6, 7].

The new theory presented here achieves a compositional treatment of dynamic anaphora that does not involve assignment functions, and separates the combinatorics of variable-free semantics from the particular linguistic phenomena it treats. Integrating variable-free semantics and dynamic semantics gives rise to interactions that make new empirical predictions, for example “donkey weak crossover” effects.

1. Decomposing dynamism

Dynamic semantics combines nondeterminism, input, and output to interpret discourse fragments such as (1) in a process informally described in (2).

(1) A man walks in the park. He whistles.

(2) Nondeterministically select a man $x$.
    Output $x$ as a candidate antecedent for future anaphora.
    Check to make sure that $x$ walks in the park; if not, abort execution.
    Input a previously encountered candidate antecedent $y$.
    Check to make sure that $y$ whistles; if not, abort execution.

In this section, I analyze each aspect in turn. In subsequent sections, I will then review the empirical and theoretical advantages gained in my variable-free treatment.

1.1. Nondeterminism. In the first half of (1), a man nondeterministically selects a man, who is then tested for the property walks in the park. If any choice of a man passes the test, the sentence is true; otherwise, it is false. Denotationally, I model nondeterminism by letting phrases denote sets of what they traditionally denote in Montague grammar. For example, I let noun phrases denote not individuals but sets of individuals: John denotes the singleton set containing John, and a man the set of all men. Formally, I assign the type Set($e$) (or $e \rightarrow t$), rather than $e$, to noun phrases such as John and a man. Here $e$ is the base type of individuals and $t$ is the base type of truth values.

Now write 1 for the unit type, the identity for the binary type constructor $\times$ for product types. This type can be thought of as a singleton set, say $\{\ast\}$. Note that $\{\ast\}$ has two subsets, namely $\{\ast\}$ and $\{\}$.

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where the “composition” function \( G \) is follows Jacobson in introducing a type-shift operation 

\[ G : (\tau \rightarrow \alpha') \rightarrow (\tau \rightarrow \alpha) \rightarrow (\tau \rightarrow \alpha') = \lambda g. \lambda f. \lambda v. g(f(v)), \]

My type for properties \( Set(e \rightarrow Set(1)) \), for example, is precisely \( Set([e \rightarrow 1]) \). Once every type is transformed, it is straightforward to specify how semantic values compose, either by adding a composition method (as in Hamblin’s interrogative semantics [4]) or by adding a type-shift operation. In programming language terms, I am using a lambda calculus that is impure because it incorporates call-by-value nondeterminism, as has been detailed by others [14, §8].

For brevity, I will henceforth write untransformed types in place of transformed ones. For example, I will write \( e \) rather than \( Set(e) \) for the type of an individual, \( e \rightarrow 1 \) rather than \( Set(e \rightarrow Set(1)) \) for the type of a property, and \( e \rightarrow e \rightarrow 1 \) rather than \( Set(e \rightarrow Set(e \rightarrow Set(1))) \) for the type of a two-place relation. Intuitively, a type \( \tau_1 \rightarrow \tau_2 \) is henceforth to be interpreted as a relation between \( \tau_1 \) and \( \tau_2 \), or equivalently, a function from \( \tau_1 \) to the power set of \( \tau_2 \). Accordingly, if \( f \) is of type \( \tau_1 \rightarrow \tau_2 \) and \( x \) is of type \( \tau_1 \), then the term \( f(x) \), of type \( \tau_2 \), is to be interpreted as the image of the set \( x \) under the relation \( f \).

We can now derive *A man walks in the park*:

\[
\text{A} : (e \rightarrow 1) \rightarrow e = \lambda p. \{ v \mid * \in p(v) \},
\text{MAN} : e \rightarrow 1, \quad \text{WTP} : e \rightarrow 1, \quad \text{WTP}(\text{A(MAN)}) : 1.
\]

1.2. **Input.** In the second half of (1), a man *he* is determined by the discourse context, who is then tested for the property *whistles*. The central idea of variable-free semantics is to model dependence on discourse context by letting phrases denote functions from *inputs* to what they traditionally denote in Montague grammar. For example, *he* will denote the identity function over men, and *he whistles* the function mapping each man to whether he whistles.

To restate this idea formally in terms of types, I introduce a new binary type constructor \( \triangleright \) (“in”). The type \( \sigma \triangleright \tau \) is like \( \sigma \rightarrow \tau \) in that they may have the same models, namely functions from \( \sigma \) to \( \tau \). I use for both kinds of types the same \( \lambda \cdot \cdot \cdot \) notation for abstraction and \( \cdot \cdot \cdot \) notation for application, but distinguish between them so that, for example, a value of type \( (a \rightarrow b) \rightarrow c \) cannot apply directly to one of type \( a \triangleright b \). (This is equivalent to how, in Jacobson’s formulation, syntactic categories regulate semantic combination to stop *loves him* from applying to *Mary* [6, §2.2.1.1].) By convention, all binary type constructors associate to the right.

I assume that *whistles* denotes some property \( \text{WHISTLE} : e \rightarrow 1 \), and let *he* denote

\[ \text{HE} : e \triangleright e = \lambda v. v. \]

Because \( \text{HE} \) does not have type \( e \), the property \( \text{WHISTLE} \) cannot apply to it directly. I follow Jacobson in introducing a type-shift operation

\[ g^\triangleright : (\alpha \rightarrow \beta) \rightarrow (\sigma \triangleright \alpha) \rightarrow (\sigma \triangleright \beta) = \lambda f. \lambda v. \lambda s. f(v(s)). \]

We can now derive *he whistles*:

\[ g^\triangleright(\text{WHISTLE})(\text{HE}) : e \triangleright 1 = \lambda v. \text{WHISTLE}(v). \]

For phrases containing more than one pronoun, for example *he loves her*, I generalize the type-shift operation \( g^\triangleright \) to a family of operations \( g_{i,j}^\triangleright = G^i(P(g^\triangleright)) \) for non-negative integers \( i \) and \( j \), that is,

\[
g_{0,0}^\triangleright = g^\triangleright, \quad g_{0,j+1}^\triangleright = I(g_{0,j}^\triangleright), \quad g_{i+1,j}^\triangleright = G(g_{i,j}^\triangleright),
\]

where the “composition” function \( G \) and the “insertion” function \( I \) are defined by

\[ G : (\alpha \rightarrow \alpha') \rightarrow (\tau \rightarrow \alpha) \rightarrow (\tau \rightarrow \alpha') = \lambda g. \lambda f. \lambda v. g(f(v)), \]

\[ I : (\tau \rightarrow \sigma) \rightarrow (\tau \rightarrow \sigma) = \lambda f. f. \]
A man who walks in the park.

The set of all pairs \( \sigma \) is like example, outputs denote cartesian products. I will use for both kinds of types the same

\[
(g_{1,0}(\text{love}))(\text{she})(\text{he}) : e \triangleright e \triangleright 1 = \lambda u. \lambda v. \text{love}(v)(u),
\]

\[
g_{0,1}(g_{1,0}(\text{love}))(\text{she})(\text{he}) : e \triangleright e \triangleright 1 = \lambda u. \lambda v. \text{love}(v)(u).
\]

My generalization here of \( g^\prec \) to handle multiple pronouns differs from Jacobson’s, which does not posit \( g^\prec_{1,j} \) for \( i > 0 \) and only generates the second scoping. The first scoping will be crucial as we consider output and binding below—typically, we need to use \( g^\prec_{1,0} \) before the binding operation \( z \), defined in (8), can apply.

1.3. Output. Informally speaking, a man can bind he in (1) by introducing a new discourse referent, i.e., a new candidate antecedent for future anaphora. I model this kind of addition to discourse context by letting phrases denote cartesian products between outputs and what they traditionally denote in Montague grammar. For example, a man will denote the set of all pairs \( \langle v, v \rangle \) where \( v \) is a man, and a man walks in the park the set of all pairs \( \langle v, * \rangle \) where \( v \) is a man who walks in the park.

Formally, I introduce a new binary type constructor \( \otimes \) (“out”). The type \( \sigma \otimes \tau \) is like \( \sigma \times \tau \) in that they may have the same models, namely pairs between \( \sigma \) and \( \tau \). I will use for both kinds of types the same \( \langle \cdot, \cdot \rangle \) notation for pairs, but distinguish between them so that, for example, a value of type \( \langle a \times b \rangle \rightarrow c \) cannot apply to another of type \( a \otimes b \). For simplicity, I treat as equivalent the isomorphic types

\[
1 \otimes \tau \quad \text{and} \quad \tau
\]

for any type \( \tau \), and the isomorphic types

\[
(\sigma_1 \times \sigma_2) \otimes \tau \quad \text{and} \quad \sigma_1 \otimes (\sigma_2 \otimes \tau)
\]

for any types \( \sigma, \sigma_1, \) and \( \sigma_2 \).

As with input, I introduce a type-shift operation

\[
g^\prec : (a \rightarrow \beta) \rightarrow (\sigma \otimes \alpha) \rightarrow (\sigma \otimes \beta) = \lambda f. \lambda \alpha. \beta, \lambda \sigma. \alpha, \sigma, f(v).
\]

I then generalize \( g^\prec \) to a family of type-shift operations \( g^\prec_{i,j} = G^i(P^j(g^\prec)) \) for non-negative integers \( i \) and \( j \).

Recall from §1.1 that true and false are just nonempty and empty sets, respectively. Under this view, it is easy to define a concatenation function that conjoins the truth conditions of two discourse fragments:

\[
; : 1 \rightarrow 1 \rightarrow 1 = \lambda * \lambda * \lambda * .
\]

(Syntactically, I assume that the first argument to ; is the second of the two fragments to be conjoined, and vice versa.) Revising the denotation we specified earlier for \( a \), we can now derive the reading of (1) where a man does not bind he:

\[
A : (e \otimes e \rightarrow \sigma \otimes 1) \rightarrow \sigma \otimes e = \lambda p. \{ \langle s, v \rangle \mid \langle s, * \rangle \in p(\langle v, v \rangle) \},
\]

\[
g_{0,0}^\prec(\text{WHITP}) (A(g_{0,0}^\prec(\text{MAN}))) : e \otimes 1,
\]

\[
g_{1,0}^\prec(g_{0,1}(;)) (g_{0,0}^\prec(\text{WHISTLE})(\text{HE})) (g_{0,0}^\prec(\text{WHITP})(A(g_{0,0}^\prec(\text{MAN})))) : e \otimes e \triangleright 1.
\]

For binding to take place, we need to feed outputs produced by semantically higher arguments into inputs solicited by semantically lower arguments. To implement this, I define one last type-shift operation

\[
z : (\alpha \rightarrow (\sigma \otimes \beta) \rightarrow \gamma) \rightarrow ((\sigma \triangleright \alpha) \rightarrow (\sigma \otimes \beta) \rightarrow \gamma)
\]

\[
= \lambda f. \lambda v. \lambda u. f(v(s))(\langle s, u \rangle)
\]
and derive from it a family of type-shift operations \( z_{i,j} = G^i(P(z)) \) for non-negative integers \( i \) and \( j \). We can now derive the reading of (1) where a man does bind he:

\[
z(g_{1,0}^\kappa(\text{WHISTLE})(\text{HE}))(g_{0,0}^\kappa(\text{WITP})(A(g_{0,0}^\kappa(\text{MAN})))) : e \ni 1.
\]

2. Empirical payoffs

Many variable-free analyses of empirical facts carry over in spirit to the variable-free dynamic semantics presented here, with extended coverage over dynamic phenomena. In this section, I give some simple examples that center around the classical donkey sentence (9).\(^1\)

(9) Every farmer who owns a donkey beats it.

Before examining its variations, a derivation of (9) itself is in order. The critical lexical items are every and who. Given the denotation of a specified above, we expect every to have the semantic type

\[
(e \ni e \rightarrow \sigma \ni 1) \rightarrow (\sigma \ni e \rightarrow \sigma' \ni 1) \rightarrow 1.
\]

The same semantic type is also expected for other strongly quantificational elements, such as most. Following standard treatment in dynamic predicate logic, I let every denote\(^2\)

\[
\text{EVERY} = \lambda p. \lambda g. \{ * | \forall s : \sigma. \forall v : e. (s, *) \in p(\langle v, v \rangle) \Rightarrow \exists s' : \sigma'. (s', *) \in q(\langle s, v \rangle) \}
\]

As for who, since I will only consider relative clauses with subject extraction in this paper, the following denotation is sufficient.\(^3\)

\[
\text{WHO} : \{ \sigma_2 \ni e \rightarrow \sigma_3 \ni 1 \rightarrow (\sigma_1 \ni e \rightarrow \sigma_2 \ni 1) \rightarrow (\sigma_1 \ni e \rightarrow \sigma_3 \ni 1) \}
\]

\[
= \lambda p. \lambda q. \lambda \langle s_1, v \rangle. \{ (s_3, *) | (s_2, *) \in q(\langle s_1, v \rangle), (s_3, *) \in p(\langle s_2, v \rangle) \}
\]

We are now ready to derive (9):

\[
\text{FARMER, DONKEY} : e \ni 1, \quad \text{OWN, BEAT} : e \ni e \rightarrow 1, \quad \text{IT} : e \ni e, \\
x = \text{WHO}(g_{1,0}^\kappa(g_{0,1}^\kappa(\text{OWN}))(A(g_{0,0}^\kappa(\text{DONKEY}))))(g_{0,0}^\kappa(\text{FARMER})) : e \ni e \rightarrow e \ni e \ni 1, \\
y = g_{1,0}^\kappa(z(g_{1,0}^\kappa(\text{BEAT}))(\text{IT})) : e \ni e \ni e \ni e \ni e \ni 1, \quad \text{EVERY}(x)(y) : 1.
\]

\(^1\)My examples assume that all farmers are male.

\(^2\)This meaning gives the donkey antecedent universal quantificational force; in other words, it makes (9) mean that every farmer who owns a donkey beats every donkey he owns. As Schubert and Pelletier [12] and others point out, sometimes the donkey antecedent seems to take existential quantificational force instead. For example, (ia) naturally means (ib).

(i) a. Every man who had a dime put it in the meter.
   b. Every man who had a dime put a dime in the meter.

I leave it for future work to account for this variation within the present framework. One possible solution is to posit alternative denotations for every. Another is to treat it as a paycheck pronoun that repeats the existential force of a dime, effectively implementing the paraphrase in (ib).

Related is the proportion problem, noted by Kadmon [8] and others: Each sentence in (ii) has a different truth condition.

(ii) a. Most farmers who own a donkey beat it.
   b. Most donkeys owned by a farmer are beaten by him.
   c. Mostly, when a farmer owns a donkey, he beats it.

The type constructor \( \kappa \) is not symmetric; it distinguishes between the individual that participates immediately in predicate-argument combination and any additional output available as candidate antecedents for future anaphora. Thus the differences in (ii) can easily be modeled here by positing a natural denotation for most.

\(^3\)It is no accident that the set comprehension notation used to specify this meaning is reminiscent of the list or monad comprehension notation used to express evaluation sequencing in programming languages [14].
2.1. Weak crossover. Variable-free dynamic semantics accounts for the “donkey weak crossover” contrasts in (10) and (11) in roughly the same way regular variable-free semantics accounts for the weak crossover contrast in (10) alone [6, §2.2.3].

(10) a. Every farmer$_i$ who owns a donkey loves his$_i$ mother.
   b. *His$_i$ mother loves every farmer$_i$ who owns a donkey.

(11) a. Every farmer who owns a donkey$_i$ loves the woman who beats it$_i$.
   b. *The woman who beats it$_i$ loves every farmer who owns a donkey$_i$.

More specifically, binding is disallowed in (10b) and (11b) because $z$ forces the binder—or, in the case of donkey anaphora, the NP containing the binder—to c-command the bindee. For the disallowed binding configurations to be possible, the grammar would need an alternative binding operation

\[
\lambda f. \lambda(s, u). \lambda v. f((s, u)(v(s))).
\]

2.2. Functional questions. Regular variable-free semantics derives the functional question-answer pair in (13a) and predicts the weak crossover violation in (13b) under natural analyses of extraction [6, §3.1–2]. Variable-free dynamic semantics further derives the “donkey functional” question-answer pair in (14a) while predicting the donkey weak crossover violation in (14b). The answers are all of type $e \triangleright e$.

(13) a. Who does every farmer$_i$ who owns a donkey love? His$_i$ mother.
   b. Who loves every farmer$_i$ who owns a donkey? ?His$_i$ mother.

(14) a. Who does every farmer who owns a donkey$_i$ love? The woman who beats it$_i$.
   b. Who loves every farmer who owns a donkey$_i$? ?The woman who beats it$_i$.

2.3. Across-the-board binding. Typical compositional analyses of right node raising allow variable-free semantics to predict (15) [6, §3.4]; variable-free dynamic semantics further predicts the “donkey functional” question-answer pair in (14a) while predicting the donkey weak crossover violation in (14b). The conjuncts are all of type $(e \triangleright e) \rightarrow 1$.

(15) Every farmer$_i$ who owns a donkey loves—but no farmer$_j$ who owns a donkey wants to marry—his$_i$/j/*k mother.

(16) Every farmer who owns a donkey$_i$ loves—but no farmer who owns a donkey$_j$ wants to marry—the woman who beats it$_i$/j/*k.

2.4. Sloppy readings. Because VPs can take semantic type $e \times e \times e \rightarrow 1$ in variable-free dynamic semantics, it is straightforward to derive “sloppy” readings for sentences involving VP ellipsis (17) or association with focus (18).

(17) a. Every East Coast farmer who owns a donkey beats it, but no West Coast farmer who owns a donkey does.
   b. Every farmer who owns a donkey showed it to his mother, but no farmer who owns a horse did.

(18) Only the farmer who owns THIS donkey beats it.

2.5. Antecedent accessibility. Muskens [11] notes the contrast in (19).

(19) a. No girl walks.
   b. *No girl$_i$ walks. If she$_i$ talks, she$_i$ talks.

The second sentence in (19b) accesses a discourse referent she but is a tautology. Muskens’s semantic theory is one of many that cannot account for this contrast without introducing representational constraints.

In variable-free semantics, types keep track of the discourse referents free in each phrase. For example, the second sentence in (19b) has type $e \triangleright 1$, not 1, even though it does denote a constant function. The theory here thus blocks (19b) and accounts for Muskens’s observation while remaining non-representational.
3. Discussion

From a theoretical perspective, variable-free dynamic semantics is appealing for the same reasons variable-free semantics and dynamic semantics are appealing—because it preserves direct compositionality, eliminates assignment functions, models updates to information states, and treats donkey anaphora.

There have been numerous proposals to combine Montague grammar with dynamic semantics, including ones by Groenendijk and Stokhof [2], Muskens [11], Kohlhase, Kuschert, and Müller [9], and van Eijck [13]. The variable-free approach here clearly shares the outlook of van Eijck’s work, which uses de Bruijn indexing to eliminate variable indices from dynamic reasoning. By comparison, the theory here is more prominently guided by types, using them to record the computational side effects [10] of nondeterminism, input, and output that are incurred during dynamic interpretation. This type system seems related to but more simplistic than Fernando’s proof-theoretic semantics [1]; for example, there are no dependent types.

I have tried to draw an analogy between incoming assignments (⊲) and outgoing ones (⊳), characterizing with similar combinatorics the former with the exponential functor and the latter with the product functor. The success of the analogy—between, for example, the type-shift operations \( g^{⊲} \) and \( g^{⊳} \)—suggests that the same combinatorics may be applicable to an even wider range of linguistic phenomena. For instance, note that Hendriks’s Argument Raising operation [5, §1.4.2] follows from the functoriality of \( (– \rightarrow σ_1) \rightarrow σ_2 \) for any \( σ_1 \) and \( σ_2 \), in the same way \( g^{⊲} \) and \( g^{⊳} \) follow from the functoriality of \( σ \rightarrow – \) and \( σ \kappa – \), respectively, for any \( σ \).

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