A Contrastive Study of Two Heaviside-based Morphological Filters

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Abstract. Morphological filters can be used to eliminate the numerical instabilities in topology optimization results, including the checkerboard patterns, mesh-dependence and intermediate densities. In this paper, two morphological filters, namely Type 1 and Type 2 filters are constructed according to the morphology-based restriction schemes. Type 1 filters are constructed using the Heaviside filter and the improved Heaviside filter proposed by Sigmund and, Type 2 filters are constructed using the Heaviside filter and the improved Heaviside filter proposed by Guest. The optimization effects of the two kinds of morphological filters are investigated, including the convergence, the ability to obtain discrete designs, iteration times and compliances. Test results show that open and close filters exhibit better performances than the advanced filters, and the advanced filters are inclined to produce instabilities during the optimization process.

1. Introduction

Topology optimization is an advanced design tool for identifying optimal distributions of materials within a domain [1-3]. It has become an important approach for structural innovation design because it provides novel and high quality structures. Many topology optimization methods have been developed since the introduction of the homogenization approach, including density method [4], evolutionary approach [5], level set method [6], moving morphable component [7] and several others.

Density method gets widely used due to its ease of implementation and high solving efficiency [8]. The basic idea of density method is to treat the design variables as the relative densities between zero and one. Wherein density one represents the solid phase material and density zero represents the void phase material [9]. An inevitable drawback of the density method is that the transitions from void to solid (zero to one) occur over several elements leaving intermediate densities (densities between zero and one) in the final topologies. At the same time, checkerboard patterns and mesh-dependence often appear in the topology optimization results. The morphological filters are proposed to deal with these problems [10]. Morphological filters include six restriction filters, which are dilation, erosion, open, close, open-close and close-open, respectively [11].

Based on the construction principle of morphological filters, the dilation and erosion filters are two basic filters. The advanced filter, i.e. the open, close, open-close and close-open filters can be formed by sequentially applying these two filters [10-11]. In this paper, the Heaviside filter [12] and two improved Heaviside filters [10, 13] are first used to construct the Heaviside-based morphological filters [14]. Then, a contrastive study is carried out to investigate the performances of the two
Heaviside-based morphological filters, including the convergence, the ability to obtain discrete designs, iteration times and compliances.

2. Construction of Two Heaviside-based Morphological Filters

2.1 Heaviside and Two Improved Heaviside Filters

Heaviside filter, which is proposed by Guest [12], is a three-field approach for obtaining black and white solutions in density-based topology optimization [8]. In the Heaviside filter, the elements with densities below one are projected to one, and the elements of densities zero are projected to zero:

\[
\bar{x}_e = H(\tilde{x}_e) = \begin{cases} 
1 & \text{if } \tilde{x}_e > 0 \\
0 & \text{if } \tilde{x}_e = 0 
\end{cases}
\]  

(1)

where \( \bar{x}_e \) represent the filtered design variables of density filter, which is given by:

\[
\tilde{x}_e = \frac{1}{\sum_{i=1}^{N_e} W_{ei}} \sum_{i=1}^{N_e} W_{ei} x_i
\]  

(2)

where \( x_i \) represent the original design variable, \( N_e \) is the neighbourhood of element \( e \), which is shown in Figure 1, \( W_{ei} \) is a weighting factor whose expression is:

\[
W_{ei} = r_{\min} - l
\]  

(3)

Figure 1 displays the neighbourhood of element \( e \) with \( r_{\min} = 2 \). Wherein the element \( e \) is the centre element, the neighbourhood elements are marked in green.

In order to take use of the gradient-based algorithms, the continuous form of equation (1) is usually used:

\[
\bar{x}_e = 1 - e^{-\beta \lambda} + \tilde{x}_e e^{-\beta}
\]  

(4)

where \( \beta \) is the smoothness parameter.

Sigmund [10] reformulated the Heaviside filter to provide zero densities for elements with densities below one, and density one for elements with densities one. The Sigmund improved Heaviside filter is given by:

\[
\tilde{x}_e = e^{-\beta(1-x_e)} - (1 - \tilde{x}_e)e^{-\beta}
\]  

(5)

For easy differentiation, we mark this filter as the Type 1 improved Heaviside filter.
In order to achieve the multiple phase projection, Guest [13] improved the Heaviside filter to make the void phase material as the projected phase. The Guest improved Heaviside filter is given by:

\[
\tilde{x}_v = e^{\beta x} - \tilde{x}_v e^{\beta}
\]  

(6)

Again, for easy differentiation, we mark this filter as the Type 2 improved Heaviside filter.

2.2 Heaviside-based Morphological Filters: Type 1

In constructing the Type 1 morphological filters, the Heaviside filter acts as the dilation filter and the Type 1 improved Heaviside filter acts as the erosion filter. Based on the construction principle of morphological filters, the close filter is defined as Heaviside filter followed by Type 1 improved Heaviside filter:

\[
\tilde{x}_v = e^{\beta (1-x_v)} - (1-x_v)e^{\beta}
\]  

(7)

The open filter is defined as Type 1 improved Heaviside filter followed by Heaviside filter:

\[
\tilde{x}_v = 1 - e^{-\beta x_v} + \tilde{x}_v e^{-\beta}
\]  

(8)

The open-close filter is defined as close filter followed by open filter:

\[
\tilde{x}_v = 1 - e^{-\beta x_v} + \tilde{x}_v e^{-\beta}
\]  

(9)

The close-open filter is defined as open filter followed by close filter:

\[
\tilde{x}_v = e^{\beta (1-x_v)} - (1-x_v)e^{\beta}
\]  

(10)

2.3 Heaviside-based Morphological Filters: Type 2

When construct the Type 2 morphological filters, the Heaviside filter is used as the dilation filter and the Type 2 improved Heaviside filter is used as the erosion filter. The close, open, open-close and close-open filters are defined as follows:

\[
\tilde{x}_v = e^{\beta x} - \tilde{x}_v e^{\beta}
\]  

(11)

\[
\tilde{x}_v = 1 - e^{-\beta x_v} + \tilde{x}_v e^{-\beta}
\]  

(12)

\[
\tilde{x}_v = 1 - e^{-\beta x_v} + \tilde{x}_v e^{-\beta}
\]  

(13)

\[
\tilde{x}_v = e^{\beta x} - \tilde{x}_v e^{\beta}
\]  

(14)

3. Case Study

3.1 Problem Formulation

The MBB beam compliance minimization problem is tested. The load, boundary condition and design domain of MBB beam are shown in Figure 2 (due to symmetry, we only model half of the MBB beam). A shown in Figure 2, the load is applied perpendicularly to the upper left corner of MBB beam, the boundary condition are symmetrical along the left edge \( \Gamma_D \), and the MBB beam is horizontally supported at the lower right corner \( \Gamma_E \).
Figure 2. Load, boundary condition and design domain of MBB beam

The topology optimization problem of MBB beam is:

$$\min \: c(x) = \rho^T K U = \sum_{e=1}^{N} u_e^T x_e^e k_0 u_e$$

subject to:

$$K U = F$$

$$V(x) = fV_0$$

$$0 \leq x \leq 1$$

where $c$ is the objective function compliance, $K$, $U$ and $F$ are the global stiffness matrix, displacement vector and force vector, respectively, $N$ is the number of finite element meshes, $u_e$ is the element displacement vector, $x_e$ is the density of element $e$, $\rho$ is the penalty parameter, $k_0$ is the element stiffness matrix, $V(x)$ is the material volume of optimization result and $V_0$ is the material volume of design domain, $f$ is the prescribed volume restriction.

3.2 Test Examples

In this subsection, the optimization effects of Type 1 and Type 2 morphological filters are tested. Test contents include the convergence, the ability to obtain black and white designs (represented by the $M_{nd}$ value), iteration times and compliance.

The $M_{nd}$ value, which is proposed by Sigmund [10], is a measure of discreteness of the topology optimization results. The expression of $M_{nd}$ is:

$$M_{nd} = \frac{\sum_{e=1}^{N} 4x_e(1-x_e)}{N} \times 100\%$$

The lower the $M_{nd}$ value, the more discrete the optimization result. The smoothness parameter $\beta$ is updated using the continuation method: the initial value is one, and $\beta$ increases by 1.01 times per iteration until it reaches 100. Parameter settings of topology optimization model are as follows: the design domain is divided into $120 \times 40$ elements, the penalty parameter is set to 3, the filter radius is set to 3 and the prescribed volume restriction is set to 0.5. Figure 3 shows the optimization results of MBB beam and the detailed data are listed in Table 1 and Table 2. In order to distinguish whether the proposed filters are volume-preserving, each optimization result exhibits two images, the upper one represents the filtered result and the lower one represents the unfiltered result.
As seen from Figure 3, in Type 1 and Type 2 filters, both the two kind open filters keep the features of Heaviside filter, and the both of the two kind close filters keep the features of improved Heaviside filters (Type 1 and Type 2 improved Heaviside filter). Similarly, the open-close filters keep the features of open filters and the close-open filters keeps the features of close filters. See our previous work [14] for more detail.

As shown in Table 2, the optimization process of Type 2 open-close filter does not converge. The optimization result shown in Figure 3 (g) is the result of 200th iterations. The smoothness parameter $\beta$ is just increased to 7.39 at 200th iteration, so the optimization result exhibits a large amount of intermediate densities, which results in a larger $M_{nd}$ value. Although the optimization processes of Type 1 morphological filters are convergent, the optimization process of close-open filter is trapped in local minimum, which results in different topology structures (see Fig. 3(d)).

| Filter       | Convergence | $M_{nd}$(%) | Iterations | Compliance |
|--------------|-------------|-------------|------------|------------|
| Open         | Yes         | 0.937       | 473        | 190.712    |
| Close        | Yes         | 1.166       | 471        | 192.262    |
| Open-close   | Yes         | 1.729       | 741        | 195.901    |
| Close-open   | Yes         | 1.321       | 705        | 232.733    |

For iterations and compliance, comparing the data in Tables 1 and 2, it can be seen that the iteration numbers and compliance values of Type 1 and Type 2 open-close and close-open filters are higher than the Type 1 and Type 2 open and close filters, respectively. The iteration numbers and compliance values of Type 1 and Type 2 open and close filters are close.

| Filter       | Convergence | $M_{nd}$(%) | Iterations | Compliance |
|--------------|-------------|-------------|------------|------------|
| Open         | Yes         | 0.957       | 479        | 190.715    |
| Close        | Yes         | 1.959       | 464        | 193.598    |
| Open-close   | No          | 20.944      | 201        | 214.945    |
| Close-open   | Yes         | 10.162      | 573        | 204.753    |
Among the constructed filters, the open filters get better optimization effects, as shown in Tables 1 and 2, both the Type1 and Type 2 open filters get lower $M_{ad}$ values, iteration times and compliance values. This can also be viewed in Figs. 3(a) and 3(e). The topology structures are clear-cut. Although the $M_{ad}$ values and compliance values are slightly high than the open filters, the close filters show good optimization performances as well.

4. Conclusion

Based on the construction principle of morphology-based restriction filters, two types of Heaviside-based morphological filters are constructed in this paper. In each type of Heaviside-based morphological filter, the Heaviside filter works as the dilation filter and the improved Heaviside filters (Type 1 and Type 2) works as the erosion filter. The difference between the two types of morphological filters lie in the improved Heaviside filter used: Type 1 filters utilize the Sigmund improved Heaviside filter [10] to construct the morphological filters and Type 2 filters utilize the Guest improved Heaviside filter [13] to construct the morphological filters.

The performances of the two types of morphological filters are investigated, including the convergence, the ability to obtain black and white designs, iteration times and compliance. Test results shows that the Type 1 and Type 2 open and close filters exhibit better optimization effects; they get lower $M_{ad}$ values, iteration times and compliance values in the optimization results. While the Type 1 and Type 2 open-close and close-open filters get poor optimization results, and they are likely to produce instabilities during the optimization processes.

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