Spin-Charge Separation is the Key to the High $T_c$ cuprates

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The most striking fact about the high-$T_c$ cuprates is that in none of the relevant regions of the phase diagram is there any evidence of the usual effects of phonon or impurity scattering. This is strong evidence that these states are in a “quantum protectorate”, to borrow Laughlin’s term. (Two striking experimental facts which demonstrate this are the absence of phonon self-energy in ARPES measurements, demonstrated recently by Johnson in BISCO, and the scattering-independent $T_c$ in YBCO, even though the superconductivity is $d$-wave.) A quantum protectorate is a state in which the many-body correlations are so strong that the dynamics can no longer be described in terms of individual particles, and therefore perturbations which scatter individual particles are not effective.

The Mott-Hubbard antiferromagnetic phase is manifestly spin-charge separated (there is a charge gap, but no spin gap), and I propose this property extends throughout the phase diagram in different guises, and is the reason for the quantum protectorate. Quasiparticles are never the exact, long-lived elementary excitations ($Z \equiv 0$ throughout) so that scattering of electrons does not necessarily disturb the excitations, especially the spinons, the Fermion-like elementary magnetic excitations with spin $1/2$ and charge 0.

I want to emphasize that this protectorate effect is completely incompatible with any perturbative theory starting from a Fermi liquid approach, as for example the spin-fluctuation theory. The experimental situation presents us with a clean dichotomy, which cannot be repaired by “summing all the diagrams”.

I propose that as we dope the antiferromagnetic state, two things happen. First, there is a first-order phase transition in the charge sector where the Mott-Hubbard gap closes, which sometimes leads to mesoscopic inhomogeneity (the stripe phenomenon). Second, and independently, we pass a critical point in the spin sector where antiferromagnetism vanishes and becomes a soft mode of the $d$-wave (or flux-phase-like) RVB.

T. Hsu, in his thesis, long ago showed that antiferromagnetism could be treated as an unstable mode of the “flux phase” i.e., the $d$-wave RVB. Hence when the latter becomes stable antiferromagnetism becomes a stable soft mode, which is seen as the notorious neutron resonance of Keimer.
This RVB, which was postulated independently by Affleck-Marston-Kotliar and by Laughlin, is the spin gap phase, stable below a crossover $T^*$ which is a rapidly decreasing function of doping. It is correct to think of this phase as analogous to the Mott insulator: where the Mott phase has a charge gap and no spin gap, this one has a spin gap—for most momenta—and no charge gap.

What is the phase above $T^*$? This is not a conventional Fermi liquid, but the original “extended s-wave” RVB I proposed in 1987, equivalent to the “tomographic Luttinger liquid” I derived for moderate densities in 1989. If there is an antiferromagnetic superexchange $J$ this phase has a Cooper instability at $T^*$ where the $d$-wave spin gap is favored.

Why does $T^*$ decrease so rapidly with doping? The Cooper instability occurs at

$$kT^* \sim p_Fv_s \exp\left(\frac{-p_Fv_s}{J_{\text{eff}}}\right)$$

where $v_s$ is the spinon velocity. When $x \to 1$, the spinon and holon velocities become equal, and equal to $t/p_F$, while as $x \to 0$, $v_s = J/p_F$, so we linearly interpolate

$$p_Fv_s \simeq J + x(t - J).$$

As for the interaction term, the antiferromagnetic interaction will tend to be compensated by a ferromagnetic double exchange term roughly proportional to $x$, which comes from loop (non-repeating) paths of the holes. For rough purposes, I estimate

$$J_{\text{eff}}(x) = J - tx.$$ 

This is adequate to account for the vanishing of $T^*$ at around $x \simeq .3$. This Cooper instability is not a true phase transition. Both the $s$-wave state and the $s + id$ retain the full symmetry of the Hamiltonian,

$$[SU(2)]_{\text{spin}} \times [U(1)]_{\text{charge}} \times \text{[Lattice Symmetry]}.$$ 

It is the conventional Fermi liquid which has an anomalous extra symmetry $Z_2$ mixing spin and charge: the Fermi liquid is itself a quantum critical point.
The RVB (spin gap) phase is not superconducting. The motivation which drives superconductivity and converts the spin gap into a superconducting gap is frustrated kinetic energy. The opening up of a gap in the spectrum at the Fermi level means that the distribution \( n(k) \) cannot approach its optimal step-function form (or step-function-like, in the case of a Luttinger liquid). This effect may be quantified by using the Ferrell-Glover-Tinkham sum rule.

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\int_0^\infty \sigma(\omega) d\omega = e^2 < K.E. >
\]

It was observed by Orenstein et al, very early, that in the underdoped (spin-gapped) cuprates a gap in the optical conductivity opens up. The magnitude of \( \int \sigma(\omega) d\omega \) is proportional to \( x \) (as noted by Sawatsky) so the relevant loss of kinetic energy is \( \propto x \). (P.A. Lee has estimated a similar effect quantitatively.) One may think of this effect as setting an upper limit to the transition temperature \( T_C < T_{KE} \propto x \). But if the spin gap \( T^* \) is smaller than this upper limit, essentially no spin gap can open without a charge gap as well, so \( T_c \) follows \( T^* \) down at high doping.

The above is far from a quantitative theory of \( T_c \). In particular, I believe there is still a role for the interlayer kinetic energy in the bilayer and multilayer cases. But contrary to my “Dogma V” there is a one-layer mechanism for superconductivity which seems to be quite effective in some cases. But in all cases the relevant mechanism is the recovery of frustrated kinetic energy. Note that in the charge channel this is not a d-wave but an s-wave condensation, hence not strongly affected by ordinary scattering.

To summarize: The two-dimensional electron gas in the cuprates is dominated by the short-range repulsive interaction which remains relevant and causes spin-charge separation. A spin gap develops in the metallic phase below a crossover temperature \( T^* \), at the Cooper instability caused by the antiferromagnetic superexchange. The extra kinetic energy required to open the spin gap is relaxed at a lower temperature \( T_c \) by making the charge fluctuations coherent, and this is the immediate cause of superconductivity.