Theoretical Update on the
Model-Independent Determination of $|V_{cb}|$
Using Heavy Quark Symmetry

Matthias Neubert
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

In view of new precise measurements of the $\bar{B} \to D^*\ell\bar{\nu}$ decay rate near zero recoil, we reconsider the theoretical uncertainties in the extraction of $|V_{cb}|$ using heavy quark symmetry. In particular, we combine our previous estimate of $1/m_Q^2$ corrections to the normalization of the hadronic form factor at zero recoil with sum rules derived by Shifman et al. to obtain a new prediction with less theoretical uncertainty. We also summarize the status of the calculation of short-distance corrections, and of the slope of the form factor at zero recoil. We find $\mathcal{F}(1) = \eta_A \xi(1) = 0.93 \pm 0.03$ and $\bar{\alpha}^2 = 0.7 \pm 0.2$. Combining this with the most recent experimental results, we obtain the model-independent value $|V_{cb}| = 0.040 \pm 0.003$.

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1 Introduction

With the discovery of heavy quark symmetry (for a review see Ref. [1] and references therein), it has become clear that the study of exclusive semileptonic $\bar{B} \to D^* \ell \bar{\nu}$ decays close to zero recoil allows for a reliable determination of the CKM matrix element $V_{cb}$, which is free, to a large extent, of hadronic uncertainties [2]–[4]. Model dependence enters this analysis only at the level of power corrections, which are suppressed by a factor of at least $(\Lambda_{QCD}/m_c)^2$. These corrections can be investigated in a systematic way using the heavy quark effective theory [5]. They are found to be small, of the order of a few per cent.

Until recently, this method to determine $|V_{cb}|$ was limited by large experimental uncertainties of about 15–20%, which were much larger than the theoretical uncertainties in the analysis of symmetry-breaking corrections. However, three collaborations have now presented results of higher precision [6]–[8]. It is thus important to reconsider the status of the theoretical analysis, even more so since the original analysis of power corrections in Ref. [9] has become the subject of some controversy [10].

Besides reviewing some of the existing calculations, the main purpose of this note is to propose a “constructive synthesis” of the two approaches that have been suggested to obtain an estimate of the power corrections to the decay form factor at zero recoil. These corrections are parametrized by a quantity $\delta_{1/m^2}$. The “exclusive approach” of Falk and myself [9] has the advantage that it provides an exact expression for $\delta_{1/m^2}$ involving five hadronic parameters, which are defined in terms of matrix elements of higher-dimensional operators in the heavy quark effective theory. The final numerical estimate is model-dependent, since four of these five parameters are not precisely known. The “inclusive approach” of Shifman et al. [10] provides an upper bound for $\delta_{1/m^2}$ in terms of only two parameters; however, it is not clear to which extent this bound is saturated. We shall combine the two approaches and derive non-trivial constraints on the hadronic parameters in the formula for $\delta_{1/m^2}$. These constraints help to reduce the theoretical uncertainty.

Let us start with a short discussion of the decay kinematics [1]. The hadronic matrix element describing the decay process $\bar{B} \to D^* \ell \bar{\nu}$ can be parametrized by invariant helicity amplitudes corresponding to transverse and longitudinal polarization of the $D^*$ meson. As kinematic variable, we choose the product of the meson velocities, $w = v_B \cdot v_{D^*}$, which is related to the momentum transfer $q^2$ to the lepton pair by

$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}. \quad (1)$$

The differential decay rate $d\Gamma/dw$ is proportional to the sum over the squared helicity amplitudes, which up to a kinematic factor defines the square of a function
The resulting expression is
\[
\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^2} (m_B - m_{D^*})^2 m_{D^*}^3 \sqrt{w^2 - 1} (w + 1)^2 
\times \left[ 1 + \frac{4w}{w + 1} \frac{m_B^2 - 2wm_Bm_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2} \right] |V_{cb}|^2 F^2(w).
\]

The heavy quark effective theory allows the factorization of the short- and long-distance contributions to \( F(w) \) into a perturbative coefficient \( \eta_A \) and a hadronic form factor \( \hat{\xi}(w) \):
\[
F(w) = \eta_A \hat{\xi}(w).
\]

In the heavy quark limit, this hadronic form factor agrees with the Isgur–Wise function \( \xi(w) \) \([3, 11]\). We use the notation \( \hat{\xi}(w) \) to indicate that the two functions differ by terms suppressed by inverse powers of the heavy quark masses. Luke's theorem determines the normalization of \( \hat{\xi}(w) \) at zero recoil \( w = 1 \) up to second-order power corrections \([12]\):
\[
\hat{\xi}(1) = 1 + \frac{\delta_{1/m^2}}{m^2}.
\]

The strategy proposed in Ref. \([4]\) is to measure the product \( |V_{cb}| F(w) \) as a function of \( w \), and to extrapolate it to \( w = 1 \) to extract
\[
|V_{cb}| F(1) = |V_{cb}| \eta_A (1 + \delta_{1/m^2}) = |V_{cb}| \left\{ 1 + O(\alpha_s(m_Q), 1/m_Q^2) \right\},
\]
where we use \( m_Q \) as a generic notation for \( m_c \) or \( m_b \). The task of theorists is to provide a reliable calculation of the symmetry-breaking corrections contained in \( \eta_A \) and \( \delta_{1/m^2} \) in order to turn this measurement into a precise determination of \( |V_{cb}| \). In Sect. \([4]\) we briefly review the status of the calculation of short-distance corrections. A new theoretical analysis of power corrections is given in Sect. \([3]\). In Sect. \([4]\), we give a theoretical prediction for the slope of the form factor \( \hat{\xi}(w) \) at zero recoil. This parameter is important for the extrapolation of experimental data to \( w = 1 \). Section \([3]\) contains a summary of the numerical results and some conclusions.

## 2 Calculation of \( \eta_A \)

The short-distance coefficient \( \eta_A \) takes into account a finite renormalization of the axial vector current in the region \( m_b > \mu > m_c \). Its calculation is a straightforward application of QCD perturbation theory. At the one-loop order, one finds \([2, 13, 14]\)
\[
\eta_A = 1 + \frac{\alpha_s}{\pi} \left( \frac{m_b + m_c}{m_b - m_c} \ln \frac{m_b}{m_c} - \frac{8}{3} \right).
\]

The scale of the running coupling constant is not determined at this order. Choosing \( \alpha_s \) between \( \alpha_s(m_b) \approx 0.20 \) and \( \alpha_s(m_c) \approx 0.32 \), and using \( m_c/m_b = 0.30 \pm 0.05 \),
one obtains values in the range $0.95 < \eta_A < 0.98$. The scale ambiguity leads to an uncertainty of order $\Delta \eta_A \sim (\alpha_s/\pi) \ln(m_b/m_c)^2 \sim 2\%$.

The calculation can be improved by using the renormalization group to resum the leading and next-to-leading logarithms of the type $[\alpha_s \ln(m_b/m_c)]^n$, $[\alpha_s \ln(m_b/m_c)]^n$, and $(m_c/m_b) [\alpha_s \ln(m_b/m_c)]^n$ to all orders in perturbation theory \[14\]–\[18\]. A consistent scheme for a next-to-leading-order calculation of $\eta_A$ has been developed in Ref. \[19\]. The result is

$$\eta_A = x^{6/25} \left\{ 1 + 1.561 \frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi} - \frac{8\alpha_s(m_c)}{3\pi} + \frac{m_c}{m_b} \left( \frac{25}{54} - \frac{14}{27} x^{-9/25} + \frac{1}{18} x^{-12/25} + \frac{8}{25} \ln x \right) + \frac{2\alpha_s(m)}{\pi} \frac{m_c^2}{m_b(m_b - m_c)} \ln \frac{m_b}{m_c} \right\},$$

where $x = \alpha_s(m_c)/\alpha_s(m_b)$, and $m_b > m > m_c$. The numerical result is very stable under changes of the input parameters. Using $\Lambda_{\overline{MS}} = (0.25 \pm 0.05)$ GeV (for four flavours) and $m_c/m_b = 0.30 \pm 0.05$, one obtains $\eta_A = 0.985 \pm 0.006$. The uncertainty arising from next-to-next-to-leading corrections is of order $\Delta \eta_A \sim (\alpha_s/\pi)^2 \sim 1\%$.

Equation (7) is an exact result to a given order in perturbation theory. We stress that, since next-to-leading effects are properly included, it is not only valid for large values of $\ln(m_b/m_c)$. Therefore, we disagree with the criticism of this calculation by the authors of Ref. \[10\]. Of course, it would be desirable to know the non-logarithmic terms of order $\alpha_s^2$ in $\eta_A$, but we see no reason why these terms should be unusually large. Taking this usual perturbative uncertainty into account, we believe it is conservative to increase the error by a factor 2.5 and quote

$$\eta_A = 0.985 \pm 0.015.$$  \hspace{1cm} (8)

### 3 Anatomy of $\delta_{1/m^2}$

Hadronic uncertainties enter the determination of $|V_{cb}|$ at the level of second-order power corrections, which are expected to be of order $(\Lambda_{QCD}/m_c)^2 \sim 3\%$. For a precision measurement of $|V_{cb}|$, it is important to understand the structure of these corrections in detail. In our discussion (as in all previous analyses), we will investigate the $1/m_Q^2$ corrections at the tree level, thereby neglecting effects of order $\alpha_s(m_Q)/m_Q^2$. In particular, we will not discuss the running of the hadronic parameters of the effective theory. In view of the theoretical uncertainty in the estimate of these non-perturbative parameters, this is a safe approximation.

Using the technology of the heavy quark effective theory, Falk and myself have
derived the exact expression \[9\]
\[
\delta_{1/m^2} = -\left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) \left( \frac{\ell_V}{2m_c} - \frac{\ell_P}{2m_b} \right) + \frac{1}{4m_c m_b} \left( \frac{4}{3} \lambda_1 + 2\lambda_2 - \lambda_{G^2} \right),
\]
which depends upon five hadronic parameters that are independent of the heavy quark masses. They have the following physical significance: \(\ell_P\) and \(\ell_V\) parametrize the deficit in the “wave-function overlap” between \(b\)- and \(c\)-flavoured pseudoscalar \((P)\) and vector \((V)\) mesons. For instance, \(\ell_P\) is defined as
\[
\langle D(v) | c^\dagger b | B(v) \rangle = 2v^0 \eta_V \left( 1 - \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \ell_P + O(1/m_Q^3) \right),
\]
where \(\eta_V \approx 1.03\) is a short-distance correction factor \[1\], and we use a mass-independent normalization of meson states. The corresponding relation for vector mesons defines \(\ell_V\). The parameter \(\lambda_1 = -\langle p_0^2 \rangle\) is proportional to the kinetic energy of the heavy quark inside a heavy meson, and \(\lambda_2 = \frac{1}{4}(m_V^2 - m_P^2)\) determines the vector–pseudoscalar mass splitting arising from operators in the effective Lagrangian that break the heavy quark spin symmetry. From the observed mass splitting between \(B\) and \(B^*\) mesons, one obtains \(\lambda_2 \approx 0.12\) GeV\(^2\). Finally, \(\lambda_{G^2}\) parametrizes certain matrix elements containing two insertions of operators that break the spin symmetry. In our analysis below, we will assume that this parameter is small, i.e. of a magnitude similar to \(\lambda_2\) or smaller. This assumption is supported by QCD sum rule calculations of other spin-symmetry-breaking corrections to heavy quark decay form factors \[20, 21\].

With the exception of \(\lambda_2\), estimates of these hadronic parameters are model-dependent. In Ref. \[9\], we made the simplifying assumptions that \(\ell_P = \ell_V\), and that the corrections represented by \(\lambda_{G^2}\) are negligible. Using then reasonable values such as \(\ell_P = \ell_V = (0.35 \pm 0.15)\) GeV\(^2\) and \(-\lambda_1 = (0.25 \pm 0.20)\) GeV\(^2\), one obtains \(\delta_{1/m^2} = -(2.4 \pm 1.3)\%\). Here and in the following, we take \(m_b = 4.80\) GeV and \(m_c = 1.45\) GeV for the heavy quark masses. In Ref. \[9\], the error in the estimate of \(\delta_{1/m^2}\) has been increased to \(\pm 4\%\) in order to account for the model dependence and higher-order corrections. A very similar result, \(-5\% < \delta_{1/m^2} < 0\), has been obtained by Mannel \[22\].

Recently, Shifman et al. have suggested an alternative approach to obtain an estimate of \(\delta_{1/m^2}\) \[10\]. The idea is to apply an operator product expansion to the \(\bar{B}\)-meson matrix element of the time-ordered product of two flavour-changing heavy quark currents, and to equate the resulting theoretical expression to a phenomenological expression obtained by saturating the matrix element with physical intermediate states. This leads to sum rules, which can be used to derive inequalities for the \(\bar{B} \to D^{(*)}\) transition form factors at zero recoil. In Ref. \[10\], such bounds have been obtained for the parameters \(\ell_P\) and \(\delta_{1/m^2}\). They are
\[
\ell_P > \ell_P^{\text{min}} > 0,
\]
\[ \delta_{1/m^2} < -\left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) \left( \frac{\ell_{P \min}}{2m_c} - \frac{\ell_{P \min}}{2m_b} \right) + \frac{1}{4m_c m_b} \left( \frac{4}{3} \lambda_1 + 2 \lambda_2 \right) \]

\[ < -\frac{\lambda_2}{2m_c^2} \simeq -2.9\% , \]

where

\[ \ell_{P \min} = \frac{1}{2}(-\lambda_1 - 3 \lambda_2) , \quad \ell_{V \min} = \frac{1}{2}(-\lambda_1 + \lambda_2) . \] (12)

The first relation in (11) implies that

\[ -\lambda_1 > 3\lambda_2 \simeq 0.36 \text{ GeV}^2 , \] (13)

excluding some of the values for the parameter \( \lambda_1 \) used in previous analyses of \( \delta_{1/m^2} \). It implies that the average heavy quark momentum inside the heavy meson is quite large, of order 600 MeV. Ball and Braun have calculated \( \lambda_1 \) using QCD sum rules and find \(-\lambda_1 = (0.5 \pm 0.1) \text{ GeV}^2 \) [23]. Below we shall use \( \lambda_1 = -0.4 \text{ GeV}^2 \). We will comment on the (weak) dependence of our results on the value of \( \lambda_1 \) later. The upper bound for \( \delta_{1/m^2} \) in (11) implies that

\[ \eta_A \tilde{\xi}(1) < 0.956. \]

Of course, a crucial question is to what extent this inequality is saturated. The authors of Ref. [10] make an “educated guess” that \( \eta_A \tilde{\xi}(1) = 0.89 \pm 0.03 \) corresponding to \( \delta_{1/m^2} = -(9.6 \pm 3.0)\% \). However, the arguments presented to support this guess are not very rigorous.

It seems more appealing to us to use the sum rules to constrain the hadronic parameters in (9). We first note that it is possible to derive two additional relations by interchanging pseudoscalar with vector meson states, corresponding to transitions of the type \( \bar{B}^* \rightarrow D^{(*)} \). Repeating the derivations of Ref. [10] for this case, we find

\[ \ell_{V \min} > \ell_{V \min} > 2\lambda_2 \simeq 0.24 \text{ GeV}^2 , \]

\[ \tilde{\delta}_{1/m^2} < -\left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) \left( \frac{\ell_{V \min}}{2m_c} - \frac{\ell_{V \min}}{2m_b} \right) + \frac{1}{4m_c m_b} \left( \frac{4}{3} \lambda_1 + 2 \lambda_2 \right) , \] (14)

where \( \tilde{\delta}_{1/m^2} \) is obtained from \( \delta_{1/m^2} \) in (9) by interchanging \( \ell_P \) with \( \ell_V \). The first relation in (14) puts a bound on the parameter \( \ell_V \). To obtain further constraints, we use the fact that the above relations are valid for an arbitrary value of the mass ratio \( m_c/m_b \). Comparing the second relation in (14) with (9) in the limit \( m_b = m_c \), we find that

\[ \lambda_G > 0 . \] (15)

\[ ^1 \text{In Ref. [10], this number is quoted as 0.94.} \]
Figure 1: Allowed regions in the $l_P-l_V$ plane for $\lambda_{G^2} = 0.01$ GeV$^2$ (solid), 0.05 GeV$^2$ (dashed), and 0.15 GeV$^2$ (dash-dotted). We use $\lambda_1 = -0.4$ GeV$^2$, in which case $l_P^{\text{min}} = 0.02$ GeV$^2$ and $l_V^{\text{min}} = 0.26$ GeV$^2$.

Moreover, as long as $m_c < m_b$, it follows that

$$\ell_V - \ell_V^{\text{min}} > \frac{m_c}{m_b} (\ell_P - \ell_P^{\text{min}}) - \frac{m_c}{m_b - m_c} \lambda_{G^2},$$

$$\ell_V - \ell_V^{\text{min}} < \frac{m_b}{m_c} (\ell_P - \ell_P^{\text{min}}) + \frac{m_b}{m_b - m_c} \lambda_{G^2}. \quad (16)$$

We are free to choose any value of the mass ratio $m_c/m_b$ between 0 and 1 to make these relations as restrictive as possible. It is then straightforward to show that

$$\max\left\{\sqrt{\ell_P - \ell_P^{\text{min}}} - \sqrt{\lambda_{G^2}} ; 0\right\} < \sqrt{\ell_V - \ell_V^{\text{min}}} < \sqrt{\ell_P - \ell_P^{\text{min}}} + \sqrt{\lambda_{G^2}}. \quad (17)$$

For small values of $\lambda_{G^2}$, this relation implies a correlation between $\ell_P$ and $\ell_V$, which is such that $\ell_V - \ell_P \simeq \ell_V^{\text{min}} - \ell_P^{\text{min}} = 2\lambda_2 \simeq 0.24$ GeV$^2$. This is illustrated in Fig. 1 where we show the allowed regions in the $\ell_P$-$\ell_V$ plane for different values of $\lambda_{G^2}$. In total, we have thus identified three effects, which decrease $\delta_{1/m^2}$ with respect to the estimate given in Ref. [9]: a large value of $(-\lambda_1)$, a positive value of $\lambda_{G^2}$, and the fact that $\ell_V$ is likely to be larger than $\ell_P$ provided that $\lambda_{G^2}$ is small.

To proceed, it is convenient to introduce new parameters

$$\bar{\ell} = \frac{1}{2} (\ell_V + \ell_P),$$

$$S = \frac{1}{2} \left\{ (\ell_V - \ell_V^{\text{min}}) + (\ell_P - \ell_P^{\text{min}}) \right\} = \bar{\ell} + \frac{1}{2} (\lambda_1 + \lambda_2), \quad (18)$$

$$D = \frac{1}{2} \left\{ (\ell_V - \ell_V^{\text{min}}) - (\ell_P - \ell_P^{\text{min}}) \right\} = \frac{1}{2} (\ell_V - \ell_P) - \lambda_2,$$

in terms of which

$$\delta_{1/m^2} = -\left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \bar{\ell} - \left( \frac{1}{4m_c^2} - \frac{1}{4m_b^2} \right) (\lambda_2 + D)$$
\[
\frac{1}{4m_cm_b} \left( \frac{4}{3} \lambda_1 + 2\lambda_2 - \lambda_{G^2} \right).
\] 

(19)

The fact that \( S > 0 \) implies \( \bar{\ell} > \frac{1}{2}(-\lambda_1 - \lambda_2) > \lambda_2 \). The inequality (17) is equivalent to \(-D_{\text{max}} < D < D_{\text{max}}\), where

\[
D_{\text{max}} = \begin{cases} 
S & 0 < S \leq \lambda_{G^2}/2, \\
\sqrt{\lambda_{G^2} S - \lambda_{G^2}^2/4} & S \geq \lambda_{G^2}/2.
\end{cases}
\] 

(20)

The main uncertainty in evaluating (19) comes from the unknown values of the parameters \( \bar{\ell} \) and \( \lambda_{G^2} \). As a guideline, one may employ the constituent quark model of Isgur et al. [25], in which one uses non-relativistic harmonic oscillator wave functions for the ground-state heavy mesons, for instance

\[
\psi_B(r) \sim \exp\left(-\frac{1}{2} \mu \omega r^2 \right),
\]

where \( \mu = (1/m_q + 1/m_b)^{-1} \) is the reduced mass. One then obtains \( \bar{\ell} = \frac{3}{4} m_q^2 \approx 0.2 \) GeV\(^2\), where we take \( m_q \approx 0.5 \) GeV for the light constituent quark mass, corresponding to the difference between the spin-averaged meson masses and the heavy quark masses. However, this estimate of \( \bar{\ell} \) is probably somewhat too low. Lattice studies of heavy-light wave functions suggest an exponential behaviour of the form

\[
\psi_B(r) \sim \exp(-\kappa \mu r),
\]

which leads to \( \bar{\ell} = \frac{3}{4} m_q^2 \approx 0.4 \) GeV\(^2\). We believe that values much larger than this are unlikely, since we use a rather large constituent quark mass \( m_q \). In fact, adopting the point of view that the sum rules for \( \ell_P \) and \( \ell_V \) are saturated to approximately 50% by the ground-state contribution [10], one would expect \( \bar{\ell} \approx (-\lambda_1 - \lambda_2) \approx 0.28 \) GeV\(^2\), which seems a very reasonable value to us. In Fig. 2, we show the allowed regions for \( \delta_1/m^2 \) as a function of \( \lambda_{G^2} \) for two values of \( \bar{\ell} \). When \(-\lambda_1 \) is varied between 0.36 and 0.5 GeV\(^2\), the resulting values for \( \delta_1/m^2 \) change by less than 1%. For all reasonable choices of parameters, the results are in the range \(-8\% < \delta_1/m^2 < -3\%\). Hence, we quote our new value as

\[
\delta_1/m^2 = -(5.5 \pm 2.5)\%,
\]

(21)

which is consistent with the previous estimates in Refs. [9, 10, 22] at the 1\( \sigma \) level. A more precise determination of the parameter \( \bar{\ell} \) would help to reduce the uncertainty in this number.

We conclude this section with a word of caution. Recently, it has been shown [27] that the sum rules derived by Shifman et al. in Ref. [10] suffer from a renormalon ambiguity; in other words, they do not obey the renormalization-group equation if the theory is regulated with a hard momentum cutoff. This is a serious problem, which has to be solved before these sum rules can be used with confidence in phenomenological applications. Here, we assume that the renormalon problem can be cured without changing the form of the sum rules.
Figure 2: Allowed regions for $\delta_{\frac{1}{m^2}}$ as a function of $\lambda_{G^2}$ for the two cases $\bar{\ell} = 0.2 \text{ GeV}^2$ (solid) and $0.4 \text{ GeV}^2$ (dashed).

4 Prediction for the slope parameter $\hat{\rho}^2$

In the extrapolation of the differential decay rate (4) to zero recoil, the slope of the function $\hat{\xi}(w)$ close to $w = 1$ plays an important role. One defines a slope parameter $\hat{\rho}^2$ by

$$\hat{\xi}(w) = \xi(1) \left\{ 1 - \hat{\rho}^2 (w - 1) + O[(w - 1)^2] \right\}.$$  \hfill (22)

It is important to distinguish $\hat{\rho}^2$ from the corresponding slope parameter $\rho^2$ of the Isgur–Wise function. They differ by corrections that break the heavy quark symmetry. Whereas the slope of the Isgur–Wise function is a universal, mass-independent parameter, the slope of the physical form factor depends on logarithms and inverse powers of the heavy quark masses. On the other hand, $\hat{\rho}^2$ is an observable quantity, while the value of $\rho^2$ depends on the renormalization scheme.

To establish the relation between the two parameters, it is convenient to introduce in an intermediate step the axial vector form factor $h_{A_1}(w)$ defined as

$$\langle D^*(v_{D*}, \epsilon) | \bar{c} \gamma^\mu \gamma_5 b | \bar{B}(v_B) \rangle = (w + 1) h_{A_1}(w) \gamma^\mu + \ldots,$$ \hfill (23)

where the ellipses represent terms proportional to $v_B^\mu$ or $v_{D*}^\mu$. The relation between the physical form factors $\hat{\xi}(w)$ and $h_{A_1}(w)$ is given in Ref. [1]. Defining a slope parameter $\hat{\rho}^2_{A_1}$ in analogy to (22), we find

$$\Delta \hat{\rho}^2 = \hat{\rho}^2 - \hat{\rho}^2_{A_1} = -\frac{1}{6} (R_1^2 - 1) - \frac{1}{3} \frac{m_B}{m_B - m_{D*}} (1 - R_2),$$ \hfill (24)

where $R_1$ and $R_2$ denote certain ratios of the $\bar{B} \rightarrow D^*$ decay form factors at zero recoil [20]. In the heavy quark limit, $R_1 = R_2 = 1$, and the two slope parameters coincide. The symmetry-breaking corrections to these ratios have been analysed.
in detail. Including both short-distance and $1/m_Q$ corrections, which in this case can be calculated without much model dependence, one obtains $R_1 = 1.3 \pm 0.1$ and $R_2 = 0.8 \pm 0.1 \ [1]$. From (24), it then follows that $\Delta q^2 = - (0.22 \pm 0.06)$. Recently, the form factor ratios $R_1$ and $R_2$ have been measured by the CLEO collaboration, with the result that $R_1 = 1.30 \pm 0.39$ and $R_2 = 0.64 \pm 0.29 \ [28]$. This leads to $\Delta q^2 = -(0.31 \pm 0.20)$, in nice agreement with our theoretical prediction.

The next step is to relate the form factor ratios $R_1$ and $R_2$ to the Isgur–Wise function. The matrix element that defines the Isgur–Wise function in the heavy quark effective theory is ultraviolet-divergent (for $w \neq 1$) and needs to be regularized by introducing a subtraction scale $\mu$. To leading order in $1/m_Q$, the regularized function $\xi(w, \mu)$ is related to the physical form factor $h_{A_1}(w)$ by a renormalization-group-invariant Wilson coefficient function $\hat{C}_1$, which contains the dependence on the heavy quark masses, and a universal function $K_{\text{hh}}$ containing the dependence on the renormalization scale $[19]$:

$$h_{A_1}(w) = \hat{C}_1(m_b, m_c, w) K_{\text{hh}}(w, \mu) \xi(w, \mu) + O(1/m_Q) \ .$$

(25)

These functions are known to next-to-leading order in renormalization-group-improved perturbation theory. Using the explicit expression for $K_{\text{hh}}(w, \mu)$ given in Ref. [1], we find that the physical slope parameter $\varphi^2_{A_1}$ is related to the slope parameter $\varphi^2(\mu)$ of the regularized Isgur–Wise function by

$$\varphi^2_{A_1} = \varphi^2(\mu) + \frac{16}{81} \ln \alpha_s(\mu) + \frac{8}{81} \left( \frac{94}{9} - \pi^2 \right) \frac{\alpha_s(\mu)}{\pi}$$

$$- \left[ \frac{\partial}{\partial w} \ln \hat{C}_1^\delta(m_b, m_c, w) \right]_{w=1} + O(1/m_Q)$$

$$\equiv \varphi^2 - \left[ \frac{\partial}{\partial w} \ln \hat{C}_1^\delta(m_b, m_c, w) \right]_{w=1} + O(1/m_Q) \ ,$$

(26)

where the last equation defines the $\mu$-independent slope $\varphi^2$ of the renormalized Isgur–Wise function at next-to-leading order. Using the explicit expression for the Wilson coefficient given in Ref. [19], one finds that $\varphi^2_{A_1} = \varphi^2 + (0.21 \pm 0.02) + O(1/m_Q)$. An estimate of the $1/m_Q$ corrections to this relation is model-dependent. We shall not attempt it, but instead increase the theoretical uncertainty to $\pm 0.2$. Hence, we obtain

$$\varphi^2 = \varphi^2_{A_1} - (0.22 \pm 0.06) \approx \varphi^2 \pm 0.2 \ .$$

(27)

Theoretical predictions for the renormalized slope parameter $\varphi^2$ have been obtained from QCD sum rules, including a next-to-leading-order renormalization-group improvement. These calculations are tedious, since it is necessary to include two-loop radiative corrections to resolve the issue of scheme dependence. The complete calculation of these corrections has been performed in Ref. [29]. It leads to $\varphi^2 = 0.7 \pm 0.1 \ [1]$. A similar result has been found by Bagan et al. [30]. Based on (27), we thus predict

$$\varphi^2 = 0.7 \pm 0.2 \ .$$

(28)
5 Summary

The exclusive semileptonic decay mode $\bar{B} \to D^{*}\ell \bar{\nu}$ provides for the cleanest determination of the CKM matrix element $V_{cb}$. Heavy quark symmetry can be used to calculate the differential decay rate close to zero recoil in a model-independent way, up to small symmetry-breaking corrections, which can be analysed in a systematic expansion in powers of $\alpha_s(m_Q)$ and $1/m_Q$ using the heavy quark effective theory. In this note, we have reconsidered and updated the analysis of these corrections. We find $\eta_A = 0.985 \pm 0.015$ for the Wilson coefficient of the axial vector current, and $\delta_{1/m^2} = -(5.5 \pm 2.5)\%$ for the power corrections to the normalization of the function $\hat{\xi}(w)$ at zero recoil. The latter value is new and has been obtained by combining the existing approaches to estimate these corrections in a constructive way. Using these results, we predict

$$F(1) = \eta_A \hat{\xi}(1) = 0.93 \pm 0.03$$

for the normalization of the hadronic form factor $F(w)$ at zero recoil.

Three experiments have recently presented new measurements of the product $|V_{cb}| F(1)$. When rescaled using the new lifetime values $\tau_{B^0} = (1.61 \pm 0.08) \text{ ps}$ and $\tau_{B^+} = (1.65 \pm 0.07) \text{ ps}$ [31], the results obtained from a linear fit to the data are

$$|V_{cb}| \eta_A \hat{\xi}(1) = \begin{cases} 
0.0347 \pm 0.0019 \pm 0.0020 \; ; \; \text{CLEO [6]}, \\
0.0382 \pm 0.0044 \pm 0.0035 \; ; \; \text{ALEPH [7]}, \\
0.0388 \pm 0.0043 \pm 0.0025 \; ; \; \text{ARGUS [8]},
\end{cases}$$

where the first error is statistical and the second systematic. Following the suggestion of Ref. [32], we add $0.001 \pm 0.001$ to these values to account for the curvature of the function $\hat{\xi}(w)$. Using then the theoretical result (29), we obtain

$$|V_{cb}| = 0.0399 \pm 0.0026 \text{ (exp)} \pm 0.0013 \text{ (th)} = 0.0399 \pm 0.0029,$$

which corresponds to a model-independent measurement of $|V_{cb}|$ with 7% accuracy. This is by far the most accurate determination to date.

We disagree with the conclusion of Ref. [10] that inclusive $b \to c \ell \bar{\nu}$ decays would allow for a more reliable determination of $|V_{cb}|$. In this case, one has to make an assumption about the heavy quark masses that appear in the theoretical expression for the inclusive decay rate even at leading order. Moreover, it has been demonstrated that the extraction of $|V_{cb}|$ from inclusive decays suffers from a perturbative uncertainty of about 10%, due to unknown higher-order corrections in the expansion in $\alpha_s(m_Q)$ [33]. Nevertheless, the most recent values obtained from the analysis of $b \to c \ell \bar{\nu}$ decays, which are [32]

$$|V_{cb}| = \begin{cases} 
0.039 \pm 0.001 \text{ (exp)} \pm 0.005 \text{ (th)} \; ; \; \text{measurements at } \Upsilon(4s), \\
0.042 \pm 0.002 \text{ (exp)} \pm 0.005 \text{ (th)} \; ; \; \text{measurements at } Z^0,
\end{cases}$$

\footnote{The ARGUS result has also been corrected for the new $D$ branching fractions [32].}
are in excellent agreement with (31). The theoretical uncertainty in these numbers is larger than in the extraction from exclusive decays, however, and it is harder to control.

Finally, we have related the physical slope parameter $\hat{\rho}^2$ to the slope of the Isgur–Wise function and obtain the prediction $\hat{\rho}^2 = 0.7 \pm 0.2$ based on existing QCD sum rule calculations. It is consistent with the average value observed by experiments, which is $\hat{\rho}^2 = 0.87 \pm 0.12$ [6]–[8].

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