T-duality Covariance of SuperD-branes

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Abstract

T-duality realized on SuperD-brane effective actions probing in constant $G_{mn}$ and $b_{mn}$ backgrounds is studied from a pure world volume point of view. It is proved that requiring T-duality covariance of such actions "fixes" the T-duality transformations of the world volume dynamical fields, and consequently, of the NS-NS and R-R coupling superfields. The analysis is extended to uncover the mapping of the symmetry structure associated with these SuperD-brane actions. In particular, we determine the T-duality transformation properties of kappa symmetry and supersymmetry, which allow us to prove that bosonic supersymmetric world volume solitons of the original theory generate, through T-duality, the expected ones in the T-dual theory. The latter proof is generalized to arbitrary bosonic backgrounds. We conclude with some comments on extensions of our approach to arbitrary kappa symmetric backgrounds, non-BPS D-branes and non-abelian SuperD-branes.

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1 Introduction

Many aspects of string physics in ten and eleven dimensions can be understood from a world volume point of view using D-branes and M-branes. In particular, duality symmetries of the full string theory admit a field theory realization on the world volume effective actions describing the low energy dynamics of these branes. Besides that, solutions of the classical equations of motion of the latter actions (world volume solitons) do admit a space time interpretation in terms of intersections of branes due to the gauge invariant character of the scalars describing these actions \[1, 2, 3\]. In this paper we will study the realization of T-duality on SuperD-brane effective actions probing in constant $G_{mn}$ and $b_{mn}$ backgrounds, on their symmetry structures and on supersymmetric bosonic world volume solitons. Given the physical equivalence between bosonic commutative D-brane gauge theory actions on such backgrounds and non-commutative gauge theory \[4\], our analysis can be seen as a first step towards the supersymmetric extension of such an equivalence, giving a full detailed analysis of the commutative side.

It is well known that T-duality admits a field theory realization in the zero slope limit of closed string theory giving rise to the T-duality rules among the NS-NS and R-R massless fields and mapping $N = 2 \, D = 10$ IIA Supergravity into $N = 2 \, D = 10$ IIB Supergravity, or viceversa \[5\]. One can ask whether such a realization exists in the same limit for the open string sector. This is answered by studying the T-duality properties of D-brane effective actions, which describe the low energy dynamics for the massless open string fields including their interactions with the massless closed string sector \[6, 7\]. In \[8\] it was proved that the double dimensional reduction of a D$p$-brane action yields the direct dimensional reduction of a D$(p-1)$-brane. Their approach was based on the already known T-duality rules mapping type IIA/IIB backgrounds derived in \[5\]. It was later proved in \[9\] that the latter set of transformations could be derived from a pure world volume perspective, by requiring $T$-duality covariance of the Dirac-Born-Infeld (DBI) and the Wess-Zumino (WZ) terms appearing in the D-brane effective action.

$T$-duality covariance is the most natural requirement having in mind the conformal field theory description of D-branes in terms of open strings \[10\]. D$p$-branes appear as hyperplanes on which open strings can end. The dimensionality $(p + 1)$ depends on the number of scalar fields satisfying Neumann boundary conditions (b.c.). Since under a longitudinal T-duality a Neumann b.c. is transformed into a Dirichlet b.c., we are left with an open string whose end points are constrained to move in a $p$-dimensional hyperplane i.e. D$(p-1)$-brane. Although the number of bosonic massless states in the open string spectrum remains invariant \[8\], the number of bosonic scalar ones increases by one, while the number of bosonic vectorial ones decreases by the same amount. In other words, while the original bosonic massless open string spectrum fits into a vector supermultiplet in $(1, p)$ dimensions, the T-dual one fits into a vector supermultiplet in $(1, p - 1)$ dimensions \[11\]. Thus, any effective field theory description of the initial and T-dual open string sectors should be a field theory realization of such vector multiplets.

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1 By a longitudinal T-duality, we mean a T-duality along a direction parallel to the hyperplane defined by the initial D$p$-brane.
in the corresponding dimensions. Since both of them are known to be of DBI + WZ type form \[3, 4\], the requirement of "\text{T-duality covariance}" is certainly justified.

The analysis done in \[3, 4\] shows that the right way to realize a longitudinal T-duality on D-brane effective actions is to apply a double dimensional reduction, which requires the existence of an isometric direction, but without rewriting the ten dimensional background fields in terms of the nine dimensional ones\[4\]. Such a reduction consists of a partial gauge fixing of the world volume diffeomorphisms to fix in which direction the original D-brane is wrapping, and a functional truncation that discards the non-zero modes of the dynamical fields in the infinitely massive \(R \to 0\) limit. This is again consistent with the conformal field theory picture, because whenever the radius \(R\) of the circle along which we are T-dualizing becomes much more smaller than the string scale \(\sqrt{\alpha'}\), physics have a much more natural description in terms of the T-dual theory, which in our case is a \(p\) dimensional field theory; the T-dual D\((p-1)\)-brane effective action.

The extension of the analysis done in \[4\] to the supersymmetric case is conceptually straightforward. When describing superD-branes, one must also include fermionic scalar fields \(\theta_i (i = 1, 2)\) having different ten dimensional chiralities in type IIA, and \(\theta'_i\) with the same chiralities in type IIB. Being world volume scalars, we will just keep their zero modes along the direction of dualization. We will show how the requirement of T-duality covariance fixes the necessary chirality changing mapping between the fermionic degrees of freedom describing type IIA/IIB D-branes in addition to the one for bosonic fields. Furthermore, this mapping of dynamical degrees of freedom indeed maps the original DBI and WZ terms into the T-dual ones, thus generalizing not only previous bosonic analysis but also the supersymmetric one \[13\] in which kappa gauge symmetry was fixed.

Our proof is not only concerned with effective actions but also uncovers their gauge and global symmetry structures, thus generalizing the corresponding bosonic analysis done in \[4\]. In particular, we will show how kappa symmetry and supersymmetry transformations of D-branes probing in constant \(G_{mn}\) and \(b_{mn}\) are mapped under T-duality. Since we will always be concerned with T-duality performed along an isometric direction of the background it ensures the preservation of supersymmetry under the dualization \[14\]. In this way, we will find out the T-duality transformation properties of the \(\Gamma_\kappa\) matrix appearing in kappa symmetry transformations.

We shall also study the effect of T-duality on bosonic supersymmetric world volume solitons. For any super-brane action in any background compatible with kappa symmetry, such configurations must satisfy

\[
\Gamma_\kappa \epsilon = \epsilon
\]

which is from now on called kappa symmetry preserving condition \[13\]. Here \(\epsilon\) is a linear combination of Killing spinors of the background and the number of supersymmetries preserved by the combined background/brane configuration is the number of linearly-independent solutions of \(1.1\). Such an equation involves, generically, a set of constraints\[3\]It would also be interesting to study transverse T-duality on abelian D-brane effective actions, generalizing the approach followed in Matrix theory compactifications \[12\], but this is beyond the scope of the present paper.
among the excited dynamical fields or \textit{BPS equations} and a set of supersymmetry projection conditions $\mathcal{P}_i \epsilon = \pm \epsilon$ determining the type of branes described by the configuration. We will argue that the functionally truncated and partially gauge fixed BPS equations are the corresponding BPS equations describing the \textit{supersymmetric T-dual configuration}. The supersymmetric projection conditions $\mathcal{P}'_i \epsilon' = \pm \epsilon'$ are obtained from the initial ones $\mathcal{P}_i \epsilon = \pm \epsilon$ by rewriting them in terms of the T-dual Killing spinors $\epsilon = \epsilon(\epsilon')$. This mapping having the same form as the one among dynamical fermionic fields $\theta = \theta(\theta')$ which would have already been fixed by \textit{T-duality covariance} of the SuperD-brane effective action.

It is interesting to remark that such mapping of supersymmetric world volume solitons we have described is nothing but a world volume realization of a well known algebraic mapping. D-brane effective actions are supersymmetric field theories, being ten dimensional target space covariant, whose group of global isometries contains the isometry supergroup of the background $\left[16\right]$. In our case, they provide a field theory realization of $N = 2$ $D = 10$ IIA(IIB) SuperPoincare algebras for $p$ even(odd), which are known to be related by some transformation of their generators, reminiscent of T-duality $\left[17\right]$. Given such a relation, BPS states in string theory admit different realizations. One is purely algebraic and is based on the saturation of the BPS bound in the supersymmetry algebra. Such a bound is exactly the same one derived from a hamiltonian analysis of brane effective actions $\left[3\right]$, giving rise to some set of BPS equations, this being the field theoretical description of such states. These BPS equations derived from the phase space formulation of D-branes are entirely equivalent to the resolution of $\left[11\right]$ due to the connection between the supersymmetry algebra and the structure of the kappa symmetry projector $\left[18\right]$.

The mapping of BPS equations and supersymmetry projection conditions will be illustrated by some examples. To begin with, we will study the effect of T-duality on BIons and dyon solutions of D-brane effective actions probing SuperPoincaré ($b_{mn} = 0$) background. BIons are mapped among themselves, in agreement with the conformal field theory picture, while dyons are mapped to a non-threshold bound state of a D2-brane and a fundamental string parallel to it intersecting in a point with a D2-brane, altogether giving a $\nu = \frac{1}{4}$ threshold bound state. Later, we concentrate on solitons in $b_{mn} \neq 0$ constant backgrounds. In particular, we will study T-duality on tilted dyons and tilted BIons on non-threshold bound states of D-strings and D3-branes. We will show that, generically, just as constant flux of magnetic field on the D-brane is seen as D-branes at angles in the T-dual picture, constant electric field (induced by the electric components $b_{0a}$) boosts the configuration in the direction along which we are T-dualizing.

Having proved the mapping of supersymmetric world volume solitons under T-duality for constant $G_{mn}$ and $b_{mn}$ backgrounds we extend the proof for an arbitrary bosonic background, relying on the $\theta = 0$ condition characterizing any bosonic configuration and the standard T-duality rules mapping bosonic backgrounds, from which we can derive the T-duality transformation properties of the bosonic kappa matrix $\Gamma_{\kappa}|_{\theta=0}$. In this way, we show the generating solution character of T-duality transformations in the low energy description of the open string sector, in close analogy with such generating character already known in type IIA/IIB supergravities describing the massless closed string sector.

In sections 2-4 we discuss the T-duality covariance of the D-brane actions and their
symmetry properties. Some details are given in appendices. In section 5 the mapping of world volume solitons is discussed with some examples. The extension to arbitrary bosonic background is examined in section 6. In the last section we comment on possible generalizations and/or extensions of our present work. They include T-duality of D-brane effective actions for arbitrary kappa symmetric backgrounds, non-BPS D-branes and non-abelian SuperD-branes.

## 2 Effective action and symmetry structure

The effective Lagrangian density of a type IIA D\(p\)-brane is a sum of DBI and WZ terms [19, 20, 21]

\[
\mathcal{L} = \mathcal{L}^{DBI} + \mathcal{L}^{WZ},
\]

\[
\mathcal{L}^{DBI} = -T_p \sqrt{-\det(G_{\mu \nu} + F_{\mu \nu})},
\]

\[
\mathcal{L}^{WZ} = [L^{WZ}]_{p+1}, \quad L^{WZ} = -T_p C e^\mathcal{F},
\]

where \(T_p\) is the D\(p\)-brane tension scaling as \(T_p \propto (g_s \alpha'^{p+1}/2)^{-1}\). Due to the fact that we are considering constant backgrounds all the dependence of the constant dilaton background is included in the string coupling constant through \(g_s = e^{\phi_0}\).

The DBI term depends on the world volume induced metric

\[
G_{\mu \nu} = E_{\mu}^{\, \underline{A}} E_{\nu}^{\, \underline{B}} \eta_{\underline{A} \underline{B}}\],
\]

where \(E_{\mu}^{\, \underline{A}}\) stand for the components of the supersymmetric invariant one forms

\[
E_{\mu}^{\, \underline{A}} \equiv dZ^M e_{M}^{\, \underline{A}} = d\sigma^\mu \partial_\mu Z^M e_{M}^{\, \underline{A}} \equiv d\sigma^\mu E_{\mu}^{\, \underline{A}}\],
\]

which for the backgrounds considered in this paper take the form

\[
E^{\underline{A}} = d\tilde{x}^{\underline{A}} + \tilde{\theta} \Gamma_{\underline{A}} d\tilde{\theta} \equiv \Pi^{\underline{A}}, \quad \tilde{x}^{\underline{A}} \equiv x^m e_{m}^{\, \underline{A}},
\]

\[
E^{\underline{A}} = d\tilde{\theta}^{\underline{A}}, \quad \tilde{\theta}^{\underline{A}} \equiv \theta^\alpha e_{\alpha}^{\, \underline{A}}.
\]

e_{m}^{\, \underline{A}} and \(e_{\alpha}^{\, \underline{A}}\) are constant components of the supervielbeins and \(Z^M \equiv (x^m, \theta^\alpha)\) parametrize the target superspace. It also depends on the supersymmetric invariant

\[
\mathcal{F} = dV - B,
\]

where \(B\) stands for the NS-NS two form, containing additional constant bosonic components \((b_{mn})\)

\[
B = \frac{1}{2} dZ^M dZ^N B_{MN} = -\tilde{\theta} \Gamma_{11} \Gamma_{\underline{A}} d\tilde{\theta} (d\tilde{x}^{\underline{A}} + \frac{1}{2} \tilde{\theta} \Gamma_{\underline{A}} d\tilde{\theta}) + \frac{1}{2} dx^m dx^n b_{mn},
\]

but still satisfying the supergravity constraint

\[
H = dB = -E^{\underline{A}} (C \Gamma_{11} \Gamma_{\underline{A}})_{\underline{A} \underline{B}} E^{\underline{B}} E^{\underline{A}} \equiv -(\overline{E} \Gamma_{11} \Gamma E),
\]

4
where $\Pi = \Gamma_0 \Pi$. For type IIB $D(p-1)$-branes, the dynamical fields will be indicated by “primes” and $\Gamma_{11}$ must be replaced by $\tau_3$.

Concerning the WZ term in (2.3), it is the $p + 1$ form part of a symbolic sum of differential forms $L^{WZ}$ satisfying

$$dL^{WZ} = - T_p \, R \, e^F,$$

(2.11)

where $R$ is the field strength of the R-R gauge potential $C$. The R-R field strength $R$ is expressed in type IIA as

$$R = E \, C_B(\Pi) \, E,$$

(2.12)

whereas in type IIB as

$$R' = - E' \, S_B(\Pi') \, \tau_1 \, E',$$

(2.13)

$$S_B(\Pi') = \sum_{\ell = 0} (\tau_3)^{\ell} (\Pi')^{2\ell + 1} (2\ell + 1)!.$$

Denoting the set of fields described by the SuperD-brane effective action (2.1) by $\{\phi^i\} = \{Z^M, V_\mu\}$, we will decompose the infinitesimal transformations $(\tilde{s}\phi^i)$ leaving the effective action invariant, into gauge $(s\phi^i)$ and global $(\Delta\phi^i)$ ones. The set of gauge symmetries involves world volume diffeomorphisms $(\xi^\mu)$, an abelian $U(1)$ gauge symmetry $(c)$ and kappa symmetry $(\kappa)$. They are given by

$$s\tilde{\phi}^a = \xi^\mu \partial_\mu \tilde{\phi}^a + \delta_\kappa \tilde{\phi}^a = \xi^\mu \partial_\mu \tilde{\phi}^a - \delta_\kappa \Gamma^a \tilde{\phi},$$

(2.15)

$$s\tilde{\theta}^a = \xi^\mu \partial_\mu \tilde{\theta}^a + \delta_\kappa \tilde{\theta}^a,$$

(2.16)

$$sV_\mu = \xi^\nu \partial_\nu V_\mu + \partial_\mu \xi^\nu + \partial_\mu c + \delta_\kappa V_\mu,$$

(2.17)

where the kappa symmetry transformation for the gauge field $\delta_\kappa V_\mu$ is determined by requiring the invariance of the gauge invariant tensor $F$ in (2.8) as

$$\delta_\kappa V_\mu = - \delta_\kappa \bar{\theta} \Gamma_{11} \Gamma_2 \tilde{\theta} \left( \partial_\mu \tilde{x}^a - \frac{1}{2} \bar{\theta} \Gamma^{a\nu} \partial_\mu \tilde{\theta} \right) + \frac{1}{2} \delta_\kappa \bar{\theta} \Gamma \tilde{\theta} \bar{\theta} \Gamma_{11} \Gamma_2 \partial_\mu \tilde{\theta} + \delta_\kappa x^m \partial_\mu x^n b_{mn},$$

(2.18)

while $\delta_\kappa \bar{\theta}$ is fully determined by

$$\delta_\kappa \bar{\theta} = \bar{\kappa} (1 - \gamma^{(p)}), \quad \gamma^{(p)} = \frac{\rho^{(p)}}{\sqrt{-\text{det}(G + F)}},$$

(2.19)

$\rho^{(p)}$ is the $(p + 1)$ world volume form coefficient of $S_A(\Pi)e^F$ for type IIA theory,

$$\rho^{(p)} = [S_A(\Pi)e^F]_{p+1}, \quad S_A(\Pi) = \sum_{\ell = 0} (\Gamma_{11})^{\ell+1} \frac{\Pi^{2\ell+1}}{(2\ell + 1)!},$$

(2.20)
while for type IIB D\((p-1)\)-brane

\[
\rho^{(p-1)} = -[C_B (H') e^{F \tau_1}]_p, \quad C_B (H') = \sum_{\ell=0}^{(\tau_3)} \frac{(\Pi')^{\ell+1}}{(2\ell)!}.
\]

(2.21)

The set of global symmetries involves supersymmetry \((\epsilon)\), bosonic translations \((a^m)\) and Lorentz transformations \((\omega^{mn})\). They act as follows:

\[
\Delta \tilde{x}^a = \delta \epsilon \tilde{x}^a + \delta a \tilde{x}^a + \delta \omega \tilde{x}^a = \tilde{\Gamma}^a \tilde{\theta} + a^a + \omega^a \tilde{x}^b;
\]

(2.22)

\[
\Delta \tilde{\theta}^a = \delta \epsilon \tilde{\theta}^a + \delta a \tilde{\theta}^a = \tilde{\epsilon}^a + \frac{1}{4} \omega^{ab} (\tilde{\Gamma}^a \tilde{\theta})^b;
\]

(2.23)

\[
\Delta V_\mu = \delta \epsilon V_\mu + \delta a V_\mu + \delta \omega V_\mu \\
= \tilde{\epsilon} \Gamma_{11} \Gamma_a \tilde{\theta} \partial_\mu \tilde{x}^a - \frac{1}{6} \left( \tilde{\epsilon} \Gamma_{11} \Gamma_a \tilde{\theta} \partial_\mu \tilde{\theta} + \tilde{\epsilon} \Gamma_{11} \Gamma_a \tilde{\theta} \partial_\mu \tilde{x}^a \right) \\
+ \Delta x^m \partial_\mu x^n b_{mn}.
\]

(2.24)

3 T-duality covariance of SuperD-branes

In this section, we will find the constraints derived from the T-duality covariance requirement on the DBI term (2.2) realizing a longitudinal T-duality transformation \((T_\parallel)\) on SuperD-brane effective actions, whose solution is given in the appendix. Afterwards, it will be proved that such a solution maps the WZ terms (2.3) of both theories.

It was argued in the introduction that \(T_\parallel\) was conveniently realized on the world volume action as a kind of double dimensional reduction. The latter consists of a partial gauge fixing of the world volume diffeomorphisms

\[
z = \rho, \quad x^m \equiv \{\tilde{x}^\bar{m}, z\}, \quad \sigma^\mu \equiv \{\tilde{\sigma}^\mu, \rho\}, \quad (3.1)
\]

saying in which direction the D-brane is locally wrapping the circle of radius \(R\), besides a functional truncation

\[
\partial_\rho \phi^\bar{i} = 0, \quad \{\phi^\bar{i}\} = \{x^\bar{m}, \theta^\alpha, V_\mu\}
\]

(3.2)

that discards all but the zero modes of the rest of dynamical fields in the limit \(R \rightarrow 0\).

As in the analysis of degrees of freedom done in [8], we will allow the following relations among \(\{\phi^\bar{i}\}\) and \(\{\phi^\hat{i}\}\)

\[
V_\rho = \eta z', \quad V_\hat{\mu} = V'_{\hat{\mu}}, \quad Z' = \{x'^{\bar{m}}, \theta'^{\alpha}\} = Z^N \Gamma_{\hat{M}'} \quad (3.3)
\]

\[
Z' = \{x'^{\bar{m}}, \theta'^{\alpha}\}, \quad (3.4)
\]

where \(\eta\) and \(\Gamma_{\hat{N}'}\) are some set of constants.

\[^3\text{Along the whole paper, we will not take into account the infinite number of non-trivial global symmetries existing for the D-string and D0-brane effective actions [23], even though our conclusions also apply to them.}\]
Requiring T-duality covariance for the DBI action (2.2),
\[- T_p \int d^{p+1} \sigma \sqrt{-\text{det} (G + F)} \xrightarrow{T} - T^\prime_{p-1} \int d^{p} \sigma \sqrt{-\text{det} (G' + F')} \] (3.5)
under the assumptions (3.1-3.4), allow us to derive a relation between brane tensions
\[T^\prime_{p-1} = T_p R, \quad R \equiv \int d\rho \sqrt{G_{\rho}^{-1}} \] (3.6)
and a set of constraints among the background superfields
\[\frac{\eta^2}{G_{zz}} = G^\prime_{zz}, \] (3.7)
\[- \frac{\eta}{G_{zz}} G_{\hat{M}z} = \Gamma_{\hat{M}}^{\hat{M}'} B^\prime_{\hat{M}'z}, \] (3.8)
\[- \frac{\eta}{G_{zz}} B_{Mz} = \Gamma_{M}^{\hat{M}'} G^\prime_{\hat{M}'z}, \] (3.9)
\[G_{\hat{M}\hat{N}} - \frac{1}{G_{zz}} \{G_{\hat{M}z} G_{\hat{N}z} - B_{\hat{M}z} B_{\hat{N}z}\} = (-)^{(M+M')N} \Gamma_{M}^{\hat{M}'} \Gamma_{\hat{N}}^{\hat{N}'} G^\prime_{\hat{M}'\hat{N}'}, \] (3.10)
\[B_{\hat{M}\hat{N}} - \frac{1}{G_{zz}} (-)^{MN} \{B_{\hat{M}z} G_{\hat{N}z} - G_{\hat{M}z} B_{\hat{N}z}\} = (-)^{(M+M')N} \Gamma_{M}^{\hat{M}'} \Gamma_{\hat{N}}^{\hat{N}'} B^\prime_{\hat{M}'\hat{N}'}, \] (3.11)
Equation (3.6) is consistent with the T-dual tension, \(T^\prime_{p-1} \propto (g'_s a^p/2)^{-1}\) since \(g'_s = \frac{2e\sqrt{\alpha'}}{R}\). The latter is equivalent to the standard T-duality transformation for the dilaton background field,
\[\phi' = \phi - \frac{1}{2} \log |G_{zz}|. \] (3.12)
Equations (3.7)-(3.11) can be interpreted as the T-duality rules for the NS-NS background superfields considered in this paper. Since we already know such superfields, equations (3.7)-(3.11) can be used to fix the set of constants introduced in (3.4). The analysis is carried out in appendix A to which we refer for further details. It is nevertheless natural to expect two different sets of constraints, due to the fact that these superfields admit an expansion in the fermionic variables \(\theta\). The first set has to do with the bosonic components of such superfields. They are the usual T-duality rules for the bosonic NS-NS background fields expressed in terms of the vielbeins
\[\frac{b_{\hat{m}z}}{\lambda} = \Gamma_{\hat{m}}^{\hat{n}'} e_{\hat{n}'z}, \] (3.13)
\[e_{\hat{m}z} = \Gamma_{\hat{m}}^{\hat{n}'} b_{\hat{n}'z}/\lambda', \] (3.14)
\[e_{\hat{m}'} = \Gamma_{\hat{m}'}^{\hat{n}'} e_{\hat{n}'}, \] (3.15)
\[b_{\hat{m}\hat{n}} = \Gamma_{\hat{m}}^{\hat{n}'} \Gamma_{\hat{n}}^{\hat{n}'} \left[ b_{\hat{n}'z} - \frac{b_{\hat{n}'z}}{\lambda'} e_{\hat{n}'z}/\lambda' \right], \] (3.16)
\(^{4}\)The derivation is left to Appendix A.
where we have splitted the flat tangent space indices as $\mathbf{a}_{z} = (\hat{a}, \mathbf{z})$. We have also made use of a local $SO(1, 9)$ rotation in both type IIA/B tangent spaces to choose

$$e_{z} \mathbf{a} = \lambda \delta_{z} \mathbf{a}, \quad e'_{z} \mathbf{a} = \lambda' \delta_{z} \mathbf{a}$$  \hfill (3.17)$$

with $\lambda$ and $\lambda'$ being some constants standing for $\sqrt{G_{zz}}$ and $\sqrt{G'_{zz'}}$, respectively.

The second set of constraints, related to the fermionic components of the NS-NS superfields, fixes the chirality change mapping of the target space spinor fields up to signs

$$\tilde{\theta}_{1} = a_{2} \tilde{\theta}_{2}', \quad \tilde{\theta}_{2} = -a_{1} \Gamma_{z} \tilde{\theta}_{1}',$$  \hfill (3.18)$$

where

$$\Gamma_{11} \tilde{\theta}_{i} = (-1)^{i+1} \tilde{\theta}_{i}, \quad \tau_{3} \tilde{\theta}_{i}' = (-1)^{i+1} \tilde{\theta}_{i}'.$$  \hfill (3.19)$$

and $a_{1}^{2} = a_{2}^{2} = 1$. From these equations and the algebra of Pauli matrices, the following set of identities can be derived

$$\overline{\delta} \Gamma \bar{\partial}_{\mu} \bar{\theta} = -\overline{\delta} \Gamma \bar{\partial}_{\mu} \bar{\theta}', \quad \overline{\delta} \Gamma_{11} \bar{\partial}_{\mu} \bar{\theta} = -\overline{\delta} \Gamma_{11} \bar{\partial}_{\mu} \bar{\theta}',$$

$$\overline{\delta} \Gamma_{11} \bar{\partial}_{\mu} \bar{\theta} = -\overline{\delta} \Gamma_{11} \bar{\partial}_{\mu} \bar{\theta}', \quad \overline{\delta} \Gamma_{3} \bar{\partial}_{\mu} \bar{\theta} = -\overline{\delta} \Gamma_{3} \bar{\partial}_{\mu} \bar{\theta}'.$$  \hfill (3.20)$$

The same form of formulas are also applied when $\theta$ ($\theta'$) and $\partial \theta$ ($\partial \theta'$) are replaced by any IIA(IIB) spinors related by (3.18).

The T-duality covariance of the WZ actions follows from (B.5) in Appendix B,

$$\langle Re^{F} \rangle = -a_{1}a_{2} \lambda \langle R'e^{F} \rangle \, d\rho. $$  \hfill (3.21)$$

The WZ action of IIB D-brane is obtained by integrating the IIA one over $\rho$ if $a_{1}a_{2} = -1$,

$$\mathcal{L}_{A}^{WZ} \rightarrow_{T} \mathcal{L}_{B}^{WZ}. $$  \hfill (3.22)$$

### 4 T-duality and symmetry structure

Let us define by $\mathcal{A}$ the subspace of the field configuration space defined by the partial gauge fixing ($z = \rho$) and functional truncation ($\partial_{\rho} \phi_{i} = 0$). In general, $\mathcal{A}$ is not left invariant under $\tilde{s} \phi_{i}$, so we must require two consistency conditions. The first one ensures that $\tilde{s}z$ will not move our configuration from the gauge slice defined by the partial gauge fixing,

$$\tilde{s}z|_{\mathcal{A}} = 0 \quad \Rightarrow \quad \xi^{\rho} = - (\delta_{\kappa} z + \Delta z)|_{\mathcal{A}}. $$  \hfill (4.1)$$

The second consistency condition ensures that $\tilde{s} \phi_{i}$ will respect the functional truncation

$$\langle \partial_{\rho} \tilde{s} \phi_{i} \rangle|_{\mathcal{A}} = 0, $$  \hfill (4.2)$$
constraining the gauge and global parameters. A sufficient solution of this set of constraints is given by

$$\xi^\hat{\mu} = \xi^\hat{\mu}(\sigma^\hat{\nu}) \quad \kappa = \kappa(\sigma^\hat{\nu}) \quad c(\sigma^\hat{\mu}, \rho) = c(\sigma^\hat{\mu}) + A\rho,$$

(4.3)

$$\omega^{\hat{a}\hat{b}} = 0,$$

(4.4)

which is explicitly breaking the global symmetry group $SO(1,9)$ into the $SO(1,8)$. The fixation of the diffeomorphism with respect to $\rho$ will induce compensating transformations through (4.1) modifying the transformation property of $V^\hat{\mu}$. In this way, the symmetry structure $(\tilde{s}\phi|_A)$ of the partially gauge fixed truncated action is found. The gauge symmetries are given by

$$s\tilde{x}^\hat{a}|_A = \delta_\epsilon \tilde{x}^\hat{a} - \delta_a \tilde{x}^\hat{a} + \delta_\omega \tilde{x}^\hat{a} = 7\Gamma^\hat{a}\tilde{\theta} + a^\hat{a} + \omega^\hat{a}\frac{1}{2}\tilde{x}^\hat{a},$$

(4.5)

$$s\tilde{\theta}^\hat{a}|_A = \xi^\hat{\mu} \partial_{\hat{\mu}} \tilde{\theta}^\hat{a} + \delta_\kappa \tilde{\theta}^\hat{a},$$

(4.6)

$$sV^\hat{\rho}|_A = \xi^\hat{\nu} \partial_{\hat{\nu}} V^\hat{\rho} - \delta_\kappa \tilde{\theta} \Gamma^\hat{a}_{11} \tilde{\theta} \lambda + \delta_\lambda \tilde{x}^\hat{m}_\tilde{b}_n,$$

(4.7)

$$sV^\hat{\mu}|_A = \xi^\hat{\nu} \partial_{\hat{\nu}} V^\hat{\mu} + V^\hat{\rho} \partial_{\hat{\mu}} \xi^\hat{\rho} + \partial_{\hat{\mu}} c^* + \delta_\lambda V^\hat{\mu},$$

(4.8)

and the global symmetries are

$$\Delta \tilde{x}^\hat{a}|_A = \delta_\epsilon \tilde{x}^\hat{a} + \delta_a \tilde{x}^\hat{a} + \delta_\omega \tilde{x}^\hat{a} = 7\Gamma^\hat{a}\tilde{\theta} + a^\hat{a} + \omega^\hat{a}\frac{1}{2}\tilde{x}^\hat{a},$$

(4.9)

$$\Delta \tilde{\theta}^\hat{a}|_A = \delta_\epsilon \tilde{\theta}^\hat{a} + \delta_\omega \tilde{\theta}^\hat{a} = \tilde{\theta}^\hat{a} + \frac{1}{4}\omega^\hat{a}\tilde{\theta} (\Gamma^\hat{a}\tilde{\theta})^\hat{a},$$

(4.10)

$$\Delta V^\hat{\rho}|_A = A + 7\Gamma^\hat{a}_{11} \tilde{\theta} \lambda + \Delta \tilde{x}^\hat{m}_\tilde{b}_n,$$

(4.11)

$$\Delta V^\hat{\mu}|_A = \delta_\epsilon V^\hat{\mu} + \delta_a V^\hat{\mu} + \delta_\omega V^\hat{\mu},$$

(4.12)

where

$$\delta_i V^\hat{\mu} = \delta_i V^\hat{\mu} + \delta_\omega \partial_{\hat{\mu}} V^\hat{\rho}, \quad i = (\kappa, \epsilon, a, \omega)$$

(4.13)

and

$$c^* = c + V^\hat{\rho} \xi^\hat{\rho}. \quad (4.14)$$

In the following we will see that these transformations give the right transformation properties of the T-dual variables, that is we will prove that the whole symmetry structure of these theories is properly mapped under T-duality. To begin with, $(p-1)$ dimensional diffeomorphisms and $U(1)$ gaume symmetry $(\tilde{\xi}^\hat{\mu}, c^*)$ are trivially mapped as can be seen by inspection of equations (4.1)-(4.8). As we already know from the bosonic analysis, $V^\hat{\rho}$ becomes the new T-dual scalar, as can be seen from its diffeomorphism transformation (4.7).

5It should be understood that the transformations appearing in the right hand side of the forthcoming equations must be computed in $A$. 

9
4.1 Kappa symmetry

In Appendix C, it is proved that

$$\delta_{\kappa_{\hat{A}}} \tilde{\theta}_{\alpha} \mid_{A} \xrightarrow{T_{\parallel}} \delta_{\kappa_{\hat{A}}} \tilde{\theta}_{\alpha},$$

(4.15)

where IIA and IIB kappa parameter functions are related by the same form of equations as \theta's in (3.18)

$$\tilde{\kappa}_{1} = a_{2} \tilde{\kappa}_{2}, \quad \tilde{\kappa}_{2} = -a_{1} \Gamma_{z} \tilde{\kappa}_{1}.$$  

(4.16)

It shows that kappa symmetry transformations of the fermionic sector of the theory are correctly mapped under T-duality. Once (4.15) is known, it is straightforward to extend the proof for \delta_{\kappa_{\hat{A}}} \tilde{\theta}_{\alpha} \mid_{A}, using the identities (3.20). The first non-trivial check is the gauge symmetry analysis of the T-duality mapping \( \nu_{\rho} \mid_{A} = \eta \kappa \) in (3.4),

$$\delta_{\kappa} \nu_{\rho} \mid_{A} = -\delta_{\kappa_{\hat{A}}} \tilde{\theta} \Gamma_{11} \tilde{\theta} \lambda + \delta_{\kappa_{\hat{A}}} x^{n} b_{mnz} \xrightarrow{T_{\parallel}} \eta \delta_{\kappa_{\hat{A}}} \nu_{\rho} + \lambda \delta_{\kappa_{\hat{A}}} \Gamma \tilde{\theta}_{\rho} \delta_{\kappa_{\hat{A}}} \nu_{\rho}.$$  

(4.17)

It can be seen that \( \delta_{\kappa_{\hat{A}}} \nu_{\rho} \mid_{A} \) turns out to be the kappa symmetry transformation for the T-dual scalar \( \delta_{\kappa_{\hat{A}}} \nu_{\rho} \mid_{A} \), thus allowing us to write the kappa symmetry transformations of the bosonic scalar sector in the T-dual description in a fully ten dimensional covariant way

$$\delta_{\kappa_{\hat{A}}} x^{\hat{m}} = -\delta_{\kappa_{\hat{A}}} \tilde{\theta} \Gamma_{\hat{m}} \tilde{\theta}, \quad \text{for} \quad \hat{m} = (\hat{a}, \hat{z}).$$  

(4.18)

We are left with kappa transformations of the \( \nu_{\hat{A}} \) components. There is an additional contribution to the kappa transformation from \( \xi_{\rho} \) in (4.1). Using the identities (3.20), it can be shown that

$$\delta_{\kappa_{\hat{A}}} \nu_{\hat{A}} \equiv (\delta_{\kappa_{\hat{A}}} V_{\hat{A}} + \delta_{\kappa_{\hat{A}}} z \partial_{\hat{A}} V_{\rho}) \xrightarrow{T_{\parallel}} \delta_{\kappa_{\hat{A}}} V_{\hat{A}}' \mid_{\hat{A}},$$

(4.19)

where

$$\delta_{\kappa_{\hat{A}}} V_{\hat{A}}' = -\delta_{\kappa_{\hat{A}}} \tilde{\theta} \Gamma_{3} \Gamma_{2} \tilde{\theta} \left( \partial_{\hat{A}} x^{\hat{a}} - \frac{1}{2} \tilde{\theta} \Gamma_{\hat{a}} \partial_{\hat{A}} \tilde{\theta} \right) + \frac{1}{2} \delta_{\kappa_{\hat{A}}} \tilde{\theta} \Gamma_{2} \tilde{\theta} \partial_{\hat{A}} \tilde{\theta} \Gamma_{3} \Gamma_{2} \partial_{\hat{A}} \tilde{\theta} + \delta_{\kappa_{\hat{A}}} x^{n} m \partial_{\hat{A}} x^{m} V_{mn}.$$

(4.20)

which finishes the proof of our claim.

4.2 Supersymmetry

It is natural to apply the T-duality transformation properties of the fermionic scalar fields (3.18) for the supersymmetry parameters

$$\bar{\epsilon}_{1} = a_{2} \bar{\epsilon}_{2}, \quad \bar{\epsilon}_{2} = -a_{1} \Gamma_{\hat{z}} \bar{\epsilon}_{1}.$$  

(4.21)

In this way

$$\delta_{\epsilon} \tilde{\theta}_{\alpha} \mid_{A} \xrightarrow{T_{\parallel}} \delta_{\kappa_{\hat{A}}} \tilde{\theta}_{\alpha}.$$  

(4.22)
and the corresponding behaviour for $\delta_v \tilde{x}^a$ follows immediately. We are thus left with the supersymmetry transformations of the gauge field. To begin with,

$$\delta V_\rho = \tilde{c} \Gamma_{11} \tilde{\theta} \lambda + \delta_c \tilde{x}^a b_{\tilde{m}} \frac{\tau_1}{\tau_1} \eta \delta_v \tilde{z}' = -\lambda (\tilde{c} \Gamma_{11} \tilde{\theta}' - \tilde{c} \Gamma_{11} \tilde{\theta}' e_{\tilde{m}} \phi_{\tilde{m}})$$

(4.23)

from which the ten dimensional character of $V_\rho$ can be emphasized again and

$$\delta_v \tilde{x}^a = \delta_v \tilde{\theta}' \Gamma_{11} \tilde{\theta}'$$

(4.24)

Concerning to $V_\rho$ the susy transformation modified by $\xi^\rho$ is mapped to the IIB one as in the kappa symmetry case,

$$\delta^*_c V_\mu \equiv (\delta_c V_\mu + \delta_c \tilde{z} \partial_\mu V_\rho) \xrightarrow{\tau_1} \delta_c V'_\mu.$$  

(4.25)

### 4.3 Poincare Bosonic global symmetries

Let us concentrate on the manifest $ISO(1, 8)$ symmetry group. From the transformation of $\tilde{x}^a$ in (4.3) and $V_\rho$ in (4.11) it follows

$$\Delta \tilde{x}^a = \omega_{\tilde{a}} \tilde{x}^a, \quad \Delta \tilde{z}^a = A \lambda,$$

(4.26)

which allows us to interpret it as the corresponding $ISO(1, 8)$ infinitesimal transformations and a $\tilde{z}^a$ coordinate translation in the T-dual target space, whenever we take the constant $A$ as $A = \frac{\tilde{x}^a}{\tilde{x}^a}$ without loss of generality. It is worthwhile emphasizing, as in [1], that the origin of the translational symmetry is the original $U(1)$ gauge symmetry. Furthermore

$$\delta^*_a V_\mu + \delta^*_e V_\rho \longrightarrow \delta^*_a V'_\mu + \delta^*_e V'_\rho + \delta_\mu c(1)$$

(4.27)

does describe the $ISO(1, 8)$ transformations in the T-dual theory up to a $U(1)$ transformation, which can be absorbed in a redefinition of the T-dual $U(1)$ gauge parameter $c^*$ without loss of generality.

The latter analysis shows the existence of the $ISO(1, 8)$ symmetry group, but we know it should be enhanced to the full $ISO(1, 9)$. In fact, there is no theorem guaranteeing the equality of the full symmetry group of the T-dual effective action with the symmetry group of the partially gauge fixed truncated action. What is indeed true is that the latter group is a subgroup of the former. In other words, $H^{-1,p+1}(s|d) \subseteq H^{-1,q}(s,d|A)$, $H^{-1,q}(s,d)$ being the cohomological group at ghost number minus one characterizing the set of non-trivial global symmetries of any n-dimensional classical field theory [24]. There are examples of such an enhancement in the literature. For instance, the D-string effective action is known to have an infinite set of non-trivial global symmetries [23], while such structure is not known to exist for the D2-brane effective action, even though they are T-dual to each other. In the present case, it is certainly true that the T-dual theory is invariant under the following set of rotations,

$$\Delta \tilde{x}^a = \omega^{a|b} \tilde{x}^b, \quad \Delta \tilde{z}^a = \omega^{a|b} \frac{\tau_1}{\tau_1} \tilde{x}^b$$

(4.28)

$$\Delta \tilde{\theta}^a = \frac{1}{2} \omega^{ab} \left( \Gamma_{ab} \tilde{\theta}' \right)^a$$

(4.29)

$$\Delta V'_\mu = \Delta x^{mn} \partial_\mu x^{mn} \delta_{mn}$$

(4.30)
as is clear from the fact that the T-dual IIB action has manifest \( ISO(1,9) \) invariance.

5 Supersymmetric world volume solitons

The goal of the present section is to prove that any bosonic supersymmetric world volume soliton of a IIA \( D_p \)-brane in constant \( G_{mn}, b_{mn} \) backgrounds is mapped under T-duality into the corresponding bosonic supersymmetric world volume configuration for the T-dual theory. This will be shown in two different, but complementary, ways. First of all, the T-duality behaviour of the kappa symmetry preserving condition will be analyzed, and after that, the same analysis will be carried in the hamiltonian formalism describing D-branes [16], paying special attention into the hamiltonian constraint, giving rise to the energy density of such BPS configurations.

It is known that any bosonic supersymmetric world volume configuration must satisfy [15]

\[ \Gamma_\kappa \epsilon = \epsilon, \quad (5.1) \]

where \( \Gamma_\kappa \) is the matrix appearing in the kappa symmetry transformations \( (\delta_\kappa \tilde{\theta}^\alpha) \) while \( \epsilon \) is the Killing spinor of the corresponding background geometry, in our case a constant spinor. It will be useful to determine \( \Gamma_\kappa \) explicitly, in terms of the \( \gamma^{(p)} \) matrix appearing in our previous discussions of kappa symmetry. Due to the fact that \( \tilde{\theta} = \tilde{\theta}^t C \), where \( C \) is the antisymmetric charge conjugation matrix, it is straightforward to derive the type IIA relation

\[ \Gamma_\kappa = \Gamma_{\kappa}^{-1} \gamma^{(p)} C = \frac{1}{\sqrt{-\det(G + F)}} \sum_{\ell=0}^{p+1} \frac{\mathbb{H}^{2\ell+1} (\Gamma_{11})_{\ell+1}^{\ell+1}}{(2\ell+1)!} e^F \]

wheras for type IIB \( D(p-1) \)-brane, it reads as

\[ \Gamma_{\kappa'}' = \frac{1}{\sqrt{-\det(G' + F')}} \sum_{\ell=0}^{p} \frac{(-1)^{\ell+1} \mathbb{H}^{2\ell} (\tau_3)_{\ell+1}^{\ell+1} \tau_1 e^{F'}}{(2\ell)!} \quad (5.2) \]

5.1 BPS equations and susy projectors

It will be proved that the kappa symmetry preserving condition (5.1), when projected into the subspace \( \mathcal{A} \) and applying a T-duality transformation on the background \( (e_m^{\alpha}, b_{mn}) \), dynamical fields \( (\phi^i) \) and Killing spinor \( (\epsilon) \), is correctly mapped into the corresponding kappa symmetry preserving condition in the T-dual theory \( \Gamma_{\kappa'} \epsilon' = \epsilon' \). Consider equation (5.1) and split it into its different chiral components

\[ \epsilon_1 = \frac{1}{\sqrt{-\det(G + F)}} \sum_{\ell=0}^{p+1} \frac{(-1)^{\ell+1} \mathbb{H}^{2\ell+1}}{(2\ell+1)!} e^F \] \( e_{p+1} \epsilon_2; \]

\[ \epsilon_2 = \frac{1}{\sqrt{-\det(G + F)}} \sum_{\ell=0}^{p+1} \frac{\mathbb{H}^{2\ell+1}}{(2\ell+1)!} e^F \] \( e_{p+1} \epsilon_1 \). \quad (5.4)
Using the T-duality relations
\[ \epsilon_1 = a_2 \epsilon'_2, \quad \epsilon_2 = -a_1 \Gamma \epsilon'_1, \]  
and the T-duality transformation properties of the matrix \( \Gamma \), equations (5.4) can be rewritten as
\[ a_2 \epsilon'_2 = \frac{a_1}{\sqrt{-\det(G' + F')}} \left[ \sum_{\ell=0} \left\{ (-1)^\ell \left( \frac{[\Pi']^{2\ell}}{(2\ell)!} \right) e^{F'} \right\}_p \epsilon'_1, \right. \]  
\[ -a_1 \epsilon'_1 = \frac{a_2}{\sqrt{-\det(G' + F')}} \left[ \sum_{\ell=0} \left\{ \left( \frac{[\Pi']^{2\ell}}{(2\ell)!} \right) e^{F'} \right\}_p \epsilon'_2. \right. \]  
which are combined to give the final result
\[ \epsilon' = \frac{1}{\sqrt{-\det(G' + F')}} \left[ \sum_{\ell=0} \left\{ \left( \frac{[\Pi']^{2\ell}}{(2\ell)!} \right) \tau_{3\ell+1} \tau_1 e^{F'} \right\}_p \epsilon' = \Gamma' \epsilon'. \]  

It is important to stress that the latter proof applies to any configuration solving the kappa symmetry preserving condition in the subspace \( \mathcal{A} \). Since any solution to such a condition involves a set of BPS equations and a set of supersymmetry projection conditions, we conclude that both sets of equations are mapped to the corresponding BPS equations and supersymmetry projection conditions under T-duality, thus generating a supersymmetric world volume soliton for the T-dual theory. This is nothing but the same phenomena observed in supergravity theories describing the low energy dynamics of the massless closed string spectrum. There, T-duality is a generating solution transformation. The above proof, which will be examined in particular examples in next subsections, ensures the same generating character for the low energy dynamics of the massless open string spectrum.

### 5.2 Hamiltonian analysis

Given any world volume brane theory, and for any bosonic supersymmetric configuration solving (5.4), one can always use its phase space formulation to compute its energy density [3]. When the world volume theory is defined on a SuperPoincaré background, it gives us a field theory realization of the SuperPoincaré algebra. Since BPS states are known to saturate the BPS bound, it must always be possible to write the energy density as a sum of squares,
\[ \mathcal{E}^2 = \mathcal{E}_0^2 + \mathcal{Z}^2 + \sum_i \left( t^i f_i(\phi^i) \right)^2, \]  
for non-threshold BPS states and
\[ \mathcal{E}^2 = (\mathcal{E}_0 + \mathcal{Z})^2 + \sum_i \left( t^i f_i(\phi^i) \right)^2, \]  
for threshold BPS states.
for threshold BPS states. Here $\mathcal{E}_0$ stands for the energy density of the vacuum configuration and $f_i(\phi^j) = 0$ stand for the set of BPS equations derived from (5.1). They allow us to define a natural lower bound on the energy or BPS bound respectively,

\[
\mathcal{E} \geq \sqrt{\mathcal{E}_0^2 + Z^2}, \\
\mathcal{E} \geq \mathcal{E}_0 + |Z|
\]

being saturated precisely when $f_i(\phi^j) = 0$ are satisfied, thus justifying their qualification as BPS equations.

If our previous analysis was correct, it should be possible to prove that the hamiltonian constraint of the original theory is mapped to the T-dual hamiltonian constraint in the T-dual theory. To prove this we will study the phase space description of D-branes in constant $G_{mn}$ and $b_{mn}$ backgrounds by setting $\theta = 0$. The phase space formulation is given by [16]

\[
\mathcal{L} = P_m \dot{x}^m + E^a \dot{V}_a + V_i \partial_a E^a - s^a \left( \tilde{P}_{\hat{a}} \Pi^\hat{a} + E^b F_{ab} \right) - \frac{1}{2} v \left[ \tilde{P}^2 + E^a E^b g_{ab} + T_p^2 \det (G_{ab} + F_{ab}) \right],
\]

where $G_{ab}$ stands for the world space induced metric, $P_m$ and $E^a$ are conjugate momenta of $x^m$ and $V_a$ respectively and

\[
P_m = e_m \tilde{P}_{\hat{a}} - E^a \partial_a \dot{x}^m b_{mn},
\]

When computed in the subspace $\mathcal{A}$,

\[
\Pi^\hat{a} = \partial_{\hat{a}} x^{\hat{m}} e_{\hat{m}}, \quad \Pi^\hat{b} = \lambda
\]

\[
\Pi^\hat{a} = \partial_{\hat{a}} x^m e_{\hat{m}}, \quad \Pi^\hat{b} = 0
\]

\[
P_{\hat{m}} = e_{\hat{m}} \tilde{P}_{\hat{a}} + e_{\hat{m}} \tilde{P}_{\hat{b}} - E^a \partial_a \dot{x}^\hat{m} b_{\hat{mn}} - E^b b_{\hat{bn}^m}
\]

\[
P_{\hat{z}} = \lambda \tilde{P}_{\hat{a}} + E^a \partial_a \dot{x}^\hat{m} b_{\hat{mn}^z}.
\]

Note that $a, b$ stand for world space indices, while the underlined ones stand for background tangent space indices.

It is straightforward to derive the T-duality properties of these objects from the rules that we have already derived in the lagrangian formulation:

\[
\Pi^\hat{a} \rightarrow \Pi^\hat{a}, \quad \Pi^\hat{b} \rightarrow \partial_a x^m \frac{b_{\hat{mn}^z}}{\chi},
\]

\[
\mathcal{F}_{ab} \rightarrow \eta \chi \Pi^\hat{a},
\]

\[
\mathcal{F}_{\hat{a} b} \rightarrow \mathcal{F}_{\hat{a} b} + \partial_a x^{\hat{m}} \frac{b_{\hat{mn}^z}}{\chi} \Pi^\hat{a} - \Pi^\hat{a} \partial_b x^m \frac{b_{\hat{mn}^z}}{\chi}
\]

\[
det (G_{ab} + F_{ab}) = \chi^2 \det (G'_{ab} + F'_{ab}).
\]

\[6\]We have assumed the existence of a single $Z$ charge in the above derivation, but the extension to more general configurations is straightforward and completely analogous to the BPS bounds derived from a pure algebraic approach.
Note that $\mathcal{L}' = \int d\rho \mathcal{L}$. Let us be more specific and rewrite $\mathcal{L} = T_p \hat{\mathcal{L}}$, $\mathcal{L}' = T_{p-1} \hat{\mathcal{L}}'$. It is clear that under T-duality $\hat{\mathcal{L}} = \lambda \hat{\mathcal{L}}'$. From these considerations, we can derive the following relation between momenta

$$P_{\hat{m}} = T_p \frac{\partial \hat{\mathcal{L}}}{\partial \hat{x}^m} = \frac{\lambda T_p}{T_{p-1}} \Gamma_{\hat{m}} \hat{\mathcal{P}}'_{\hat{n}}, \quad P'_{\hat{n}} = T_{p-1} \frac{\partial \hat{\mathcal{L}}'}{\partial \hat{x}'^n}$$

$$E^\rho = \frac{\lambda T_p}{\eta T_{p-1}} P'_{\hat{z}}$$

$$E'^{\hat{a}} = \frac{\lambda T_p}{T_{p-1}} E^{'\hat{a}}$$

(5.23)

from which we can derive that $\hat{P}_{\hat{z}} = \frac{\lambda T_p}{T_{p-1}} \hat{P}'_{\hat{z}}$.

The process of partial gauge fixing ($z = \rho$) in the lagrangian formulation corresponds, in the phase space formulation, to solve the equation

$$\frac{\delta \mathcal{L}}{\delta s^\rho} = 0 \quad \Rightarrow \quad \hat{P}_z = \frac{E^b F_{bp}}{\lambda}. \quad (5.24)$$

Using the above information, one can show that the remaining diffeomorphism constraints ($\delta \mathcal{L}/\delta s^{\hat{a}} = 0$) and the hamiltonian constraint ($\delta \mathcal{L}/\delta v = 0$) are mapped to the corresponding T-dual constraints, by defining $v' = \frac{\lambda T_p}{T_{p-1}} v$,

$$\frac{\delta \mathcal{L}}{\delta s^{\hat{a}}} \rightarrow \frac{\lambda T_p}{T_{p-1}} \frac{\delta \mathcal{L}'}{\delta s'^{\hat{a}}}$$

$$\frac{\delta \mathcal{L}}{\delta v} \rightarrow \frac{\lambda T_p}{T_{p-1}} \frac{\delta \mathcal{L}'}{\delta v'}$$

(5.25)

thus indeed proving our initial claim.

### 5.3 Examples

The aim of this subsection is to show, explicitly, how the BPS equations and supersymmetry projection conditions characterizing world volume solitons are mapped into the corresponding ones under T-duality. We will, first of all, concentrate on orthogonal BIons solutions that are common to all D$p$-branes and on dyons in a D3-brane propagating in SuperPoincaré background ($b_{mn} = 0$). After that, we consider the more subtle effect of T-duality on tilted BIons ($b_{mn} \neq 0$). In the following we will be using the explicit parametrisation $\eta = a_2 = - a_1 = 1$ and $\Gamma_m^n = \delta_m^n$ which can always be done.

#### 5.3.1 BIons and dyons

Let us start our discussion with BIons. That is, we will look for classical solutions to the D$p$-brane equations of motion propagating in Minkowski space, corresponding to a
fundamental string ending on the brane. The latter configuration is known to be described by the ansatz

\[ x^\mu = \sigma^\mu, \quad x^{p+1} = y(\sigma^a), \quad V_0 = V_0(\sigma^a), \quad (5.26) \]

\(\mu = 0, \ldots, p, \quad a = 1, \ldots, p,\) the rest of bosonic scalar fields being constant and the magnetic components of the gauge field have a pure gauge configuration, corresponding to the array of branes

\[ Dp: \quad 1 \quad 2 \quad \ldots \quad p \quad \ldots \quad \text{probe} \]

\[ F1: \quad \ldots \quad \ldots \quad y \quad \ldots \quad \text{soliton}. \quad (5.27) \]

The solution to the kappa symmetry preserving condition when \( (5.26) \) is satisfied, is given by

\[ P_1 \epsilon = \epsilon \quad (5.28) \]

\[ P_2 \epsilon = \epsilon \quad (5.29) \]

\[ F_{0a} = \partial_a y, \quad (5.30) \]

where \( \epsilon \) is a constant Killing spinor. The first two conditions \( (5.28, 5.29) \) correspond to the supersymmetry projection conditions telling us that we are describing a \( Dp \)-brane and a fundamental string, \( P_2 = \Gamma_{\underline{0} \underline{1} \underline{2} \underline{3} \underline{4}} \Gamma_{11} \) in type IIA and \( P_2 = \Gamma_{\underline{0} \underline{y} \underline{\tau}_3} \) in type IIB, while equation \( (5.30) \) is the usual BPS equation, which by using the Gauss’ law \( (\partial_a E^a = \partial_a \delta^{ab} F_{0b} = 0) \) determines the harmonic character of the excited transverse scalar \( (\delta^{ab} \partial_a \partial_b y = 0) \).

Let us study the effect of T-duality along the world volume direction \( \rho = p \). If, as suggested by our analysis, we apply the partial gauge fixing plus functional truncation on \( (5.30) \), the only non-trivial equation that we get is the corresponding BPS equation in the T-dual description

\[ F'_{02} = \partial_2 y'. \quad (5.31) \]

Concerning supersymmetry projections, take eq. \( (5.28) \) for a D4-brane. In that case, \( P_1 = \Gamma_{\underline{0} \underline{1} \underline{2} \underline{3} \underline{4}} \Gamma_{11} \) and eq. \( (5.28) \) is equivalent to

\[ \Gamma_{\underline{0} \underline{1} \underline{2} \underline{3} \underline{4}} \epsilon_2 = -\epsilon_1 \]

\[ \Gamma_{\underline{0} \underline{1} \underline{2} \underline{3} \underline{4}} \epsilon_1 = \epsilon_2 \quad (5.32) \]

which when written in terms of the T-dual Killing spinors \( \epsilon' \) look as

\[ \Gamma_{\underline{0} \underline{1} \underline{2} \underline{3} \underline{4}} \epsilon'_1 = -\epsilon'_2 \]

\[ \Gamma_{\underline{0} \underline{1} \underline{2} \underline{3} \underline{4}} \epsilon'_2 = \epsilon'_1 \quad (5.33) \]

which is consistent with the projection \( \Gamma_{\underline{0} \underline{1} \underline{2} \underline{3} \underline{4}} i \tau_2 \epsilon' = \epsilon' \) satisfied by a D3-brane. Concerning the soliton projection \( (5.29) \), it can be splitted into

\[ \Gamma_{\underline{0} \underline{y} \underline{y}} \epsilon_1 = -\epsilon_1 \]

\[ \Gamma_{\underline{0} \underline{y} \underline{y}} \epsilon_2 = \epsilon_2 \quad (5.34) \]

\[ \Gamma_{\underline{0} \underline{y} \underline{y}} \epsilon'_1 = -\epsilon'_1 \]

\[ \Gamma_{\underline{0} \underline{y} \underline{y}} \epsilon'_2 = \epsilon'_2 \quad (5.36) \]

which are equivalent to

\[ \Gamma_{\underline{0} \underline{y} \underline{y}} \epsilon'_2 = \epsilon'_2 \]

\[ \Gamma_{\underline{0} \underline{y} \underline{y}} \epsilon'_1 = -\epsilon'_1 \quad (5.37) \]
respectively. Equations (5.37) can be joined into \( \Gamma_0 \tau_3 \epsilon' = -\epsilon' \), which is the soliton projection for a fundamental string in type IIB. Analogous discussion applies for other values of \( p \).

To sum up, we have indeed shown that BPS equations and susy projections are mapped, under T-duality, to the corresponding BPS equations and susy projections describing the T-dual configuration

\[
\begin{align*}
D(p - 1) & : 1 \quad 2 \quad p - 1 \quad \ldots \quad \text{probe} \\
F1 & : \quad \ldots \quad y \quad \ldots \quad \text{soliton.}
\end{align*}
\]

We would like to comment on the explicit solution of the harmonic equation \( \delta^{ab} \partial_a \partial_b y = 0 \). Of course, we could just restrict ourselves to a particular solution of this equation independent on the world volume coordinate \( \rho \) along which we are T-dualizing, to be consistent with the functional truncation we were discussing in previous sections. Another possibility is to consider a superposition of Dions of the same mass and charge, located periodically along the \( \hat{\rho} \) axis with period \( a = 2\pi R \)

\[
y = k_p \sum_{n \in \mathbb{Z}} \frac{1}{|\sigma - n a \hat{\rho}|^{p-2}} \quad p \geq 3
\]

\[
y = k_2 \sum_{n \in \mathbb{Z}} \log |\sigma - n a \hat{\rho}| \quad p = 2.
\]

In the limit \( R \rightarrow 0 \), which is the one we have been studying along the whole paper, the discrete sum is replaced by an integral,

\[
\sum_{n \in \mathbb{Z}} \frac{k_p}{|\sigma - n a \hat{\rho}|^{p-2}} \rightarrow \int_{-\infty}^{\infty} \frac{k_p d\rho}{(\hat{\sigma}^2 + \rho^2)^{(p-2)/2}} = \tilde{k}_p \frac{d\hat{\rho}}{\hat{\sigma}^{p-3}} \quad p \geq 4
\]

\[
\sum_{n \in \mathbb{Z}} \frac{k_3}{|\sigma - n a \hat{\rho}|} \rightarrow \int_{-\infty}^{\infty} \frac{k_3 d\rho}{(\hat{\sigma}^2 + \rho^2)^{1/2}} = \tilde{k}_2 \log |\hat{\sigma}| \quad p = 3
\]

\[
\sum_{n \in \mathbb{Z}} k_2 \log |\sigma - n a \hat{\rho}| \rightarrow \int_{-\infty}^{\infty} k_2 d\rho \log |\sqrt{\sigma^2 + \rho^2}| = \tilde{k}_1 \sigma_1 \quad p = 2
\]

which is effectively equal to ignoring all the heavy modes along the \( \hat{\rho} \) direction, giving the correct functional behaviours in the T-dual theory.

Let us now describe the effect of T-duality on dyons. We will look for classical solutions to the D3-brane equations of motion propagating in Minkowski space, corresponding to a \((p, q)\) string ending on the brane. The latter configuration is known to be described by the ansatz

\[
x^\mu = \sigma^\mu \quad , \quad x^{p+1} = y(\sigma^a) \quad , \quad V_0 = V_0(\sigma^a) \quad , \quad V_b = V_b(\sigma^a),
\]

\[\text{(5.39)}\]

\[\text{7The minus sign is related with the freedom of choosing as a BPS equation } F_{0a} = -\partial_\sigma y', \text{ instead of (5.31).}\]
where $\mu = 0, \ldots, 3$, $a, b = 1, \ldots, 3$, and the rest of bosonic fields are being constant, corresponding to the array of branes

\[
D3: \quad 1 \quad 2 \quad 3 \quad - \quad - \quad - \quad - \quad - \quad - \quad \text{probe}
\]
\[
F1: \quad - \quad - \quad 4 \quad - \quad - \quad - \quad - \quad - \quad \text{soliton}
\]
\[
D1: \quad - \quad - \quad 4 \quad - \quad - \quad - \quad - \quad - \quad \text{soliton}.
\]

The solution to the corresponding kappa symmetry preserving condition is

1. $\Gamma_{0123} i \tau_2 \epsilon = \epsilon$
2. $\Gamma_{0y} \left( \cos \alpha \tau_3 + \sin \alpha \tau_1 \right) \epsilon = \epsilon$
3. $F_{0a} = \cos \alpha \partial_a y$
4. $\frac{1}{2} \epsilon^{abc} F_{bc} = \sin \alpha \delta^{ab} \partial_b y$.

Equations (5.41)-(5.42) are the supersymmetry projection conditions for this configuration. The first one describes a D3-brane along directions 123, as expected, while the second one describes a $(p, q) - string$ along the transverse direction $y$. Equations (5.43)-(5.44) are the BPS equations for this configuration [3].

The longitudinal T-dual configuration is known to be

\[
D2: \quad 1 \quad 2 \quad - \quad - \quad - \quad - \quad - \quad - \quad \text{probe}
\]
\[
F1: \quad - \quad - \quad 4 \quad - \quad - \quad - \quad - \quad - \quad \text{soliton}
\]
\[
D2: \quad - \quad 3 \quad 4 \quad - \quad - \quad - \quad - \quad - \quad \text{soliton}.
\]

Proceeding as before, the truncated BPS equations one gets are

\[
F_{0a}' = \cos \alpha \partial_a y'
\]
\[
\epsilon^{\hat{a} \hat{b}} \partial_{\hat{b}} z' = \sin \alpha \delta^{\hat{a} \hat{b}} \partial_{\hat{b}} y'.
\]

while the supersymmetry projections become

\[
\Gamma_{012} \epsilon' = \epsilon'
\]
\[
\left( - \Gamma_{0y} \Gamma_{11} \cos \alpha + \Gamma_{0y} \sin \alpha \right) \epsilon' = \epsilon'.
\]

Equations (5.46, 5.49) describe a threshold bound state of a D2 brane and a fundamental IIA string realized on the world volume of the first D2-brane. Note that studying the particular limit, $\alpha = 0$ we recover the BIon discussion, while for $\alpha = \pi/2$, equation (5.47) is equivalent to the Cauchy-Riemann equations (when written in terms of complex world volume coordinates and the complex function $U = y + iz$) describing a $D2 \perp D2(0)$, which is the direct dimensional reduction of the $M2 \perp M2(0)$ configuration [3, 26].

### 5.3.2 World volume solitons in constant b fields

We will concentrate on world volume solitons on a D3-brane proving in Minkowski space and a constant arbitrary $b_{mn}$ field. The kappa symmetry preserving condition looks
as
\[ \sqrt{-\det (G + F)} \epsilon = \frac{1}{4!} \epsilon^{\mu_1 \ldots \mu_4} (\gamma_{\mu_1 \ldots \mu_4} i \tau_2 + 6 F_{\mu_1 \mu_2} \gamma_{\mu_3 \mu_4} \tau_1 + 3 F_{\mu_1 \mu_2} F_{\mu_3 \mu_4} i \tau_2) \epsilon. \]  
(5.50)

We will describe two different configurations solving eq. (5.50). First of all, we will generalize the BPS equations (5.43-5.44) describing dyons in the absence of a \( b_{mn} \) field. Using the same ansatz as in (5.39), the solution to (5.50) involves the same supersymmetry projectors (5.41) and (5.42), while the BPS equations are given by

\[ F_{0a} = \cos \alpha \partial_a y \]  
(5.51)

\[ B^a = \frac{1}{2} \epsilon^{abc} F_{bc} = \sin \alpha \delta^{ab} \partial_b y \]  
(5.52)

which are the straightforward generalization of the usual dyonic BPS equations in the presence of a \( b_{mn} \) field, this being the reason of the appearance of the gauge invariant tensor \( F \).

If we T-dualize along the direction 3, the BPS equations that we obtain are

\[ \partial_0 z' = -G'_{03} \]  
(5.53)

\[ F'_{0a} = \cos \alpha \partial_a y' \]  
(5.54)

\[ \frac{1}{2} \epsilon^{\hat{a} \hat{b}} F'_{\hat{a} \hat{b}} = 0 \]  
(5.55)

\[ \epsilon^{\hat{a} \hat{b}} \left( \partial_{\hat{a}} z' + G'_{\hat{a} \hat{b}} + \partial_{\hat{b}} y' G'_{\hat{b} y} \right) = \sin \alpha \partial_{\hat{b}} y' , \]  
(5.56)

where \( \hat{a}, \hat{b} = 1, 2 \). The most remarkable feature of this T-dual configuration is being non-static, see equation (5.53). Let us discuss in more detail equations (5.51) and (5.52) when the background is such that only the electric components of the \( b_{mn} \) field along the world volume are non-vanishing (\( b_{0a} \neq 0 \)), and \( \alpha = 0 \), that is, we will be concerned with BIon type solutions. In this case, eqs. (5.51) and (5.52) besides the Gauss’ law are easily integrated to give the solution

\[ y(\sigma^b) = y_h(\sigma^b) + d_a \sigma^a \]  
(5.57)

\[ V_0(\sigma^b) = -y_h(\sigma^b) + c_a \sigma^a \]  
(5.58)

\[ d_a + c_a = -b_{0a} , \]  
(5.59)

where \( y_h(\sigma^b) \) denotes the harmonic part of the solution whereas \( d_a, c_a \) is some set of constants constrained by (5.59). Notice that \( d_a \) are physical parameters, due to the gauge invariant character of the excited scalar, determining the tilting of the BIon [27]. In other words, due to the non-orthogonal character of the BIon, when we study T-duality along the \( \rho \) direction, this is seen as a T-duality at angle from the BIon perspective. As a result of that, one should expect the T-dual configuration to be one with a tilted BIon ending on a D2-brane boosted in the direction of dualization, which is what we get from inspection of eq. (5.53). To sum up, constant electric field (\( F_{0a} \)) boosts the configuration
in the direction along we T-dualize, just as constant flux of magnetic field on the D-brane \((F_{ab})\) is seen as D-branes at angles in the T-dual picture \([7]\).

Due to the generating character of the T-duality transformation, we would like to understand how the original static configuration give rise to the non-static solution in the T-dual theory. First of all, due to the functional truncation defining the \(A\) subspace, we will choose \(d_3 = 0\), thus avoiding the linear dependence in \(\sigma^3\) on the gauge invariant quantity \(y(\sigma^b)\). By (5.59), \(c_3 = -b_{03}\), or equivalently,

\[
V_0(\sigma^b) = -y_h(\sigma^b) + c_\hat{a}\sigma^{\hat{a}} - b_{03}\sigma^3. \tag{5.60}
\]

The latter has also a linear dependence in \(\sigma^3\), but this is certainly gauge dependent, since we can find a gauge parameter \(c = b_{03}\tau\sigma^3\) transforming the gauge field configuration (5.60) into

\[
\begin{align*}
V_0(\sigma^b) &= -y_h(\sigma^b) + c_\hat{a}\sigma^{\hat{a}} \\
V_3(\tau) &= b_{03}\tau,
\end{align*} \tag{5.61}
\]

which is explicitly time dependent. The latter is the most natural higher dimensional solution giving rise to the non-static T-dual configuration \([8]\).

As a second example, we will consider a non-threshold bound state of a D-string inside the D3-brane together with some BIon, which is generically tilted, due to the non-vanishing of the \(b\) field. Using the same ansatz as in (5.39), the solution to the kappa symmetry preserving condition is given by

\[
\begin{align*}
\left(\cos \alpha \Gamma_{0123} i\tau_2 + \sin \alpha \Gamma_{01} \tau_1\right) \epsilon &= \epsilon \tag{5.62} \\
\Gamma_{04} \tau_3 \epsilon &= \epsilon \tag{5.63} \\
F_{23} &= \mathcal{F} = \tan \alpha \tag{5.64} \\
F_{02} &= -\partial_2 y \quad a = 2, 3 \tag{5.65} \\
F_{01} &= \partial_1 y = 0. \tag{5.66}
\end{align*}
\]

Equations (5.62), (5.64) are a straightforward generalization of the conditions satisfied by any non-threshold bound state involving a D\((p-2)\)-brane inside a Dp-brane in the case of non-vanishing \(b\) field, for \(p = 3\). On the other hand, equations (5.63) and (5.65) describe a tilted BIon, this time being delocalized in the direction where the D-string lies along \(\sigma^1\) direction, (5.66).

We can study two different T-duality transformations, since there are two inequivalent world volume directions. Let us study T-duality along direction \(\sigma^1\). Proceeding as before, the BPS equations in the T-dual configuration turn out to be

\[
\begin{align*}
\mathcal{F}' &= \tan \alpha \tag{5.67} \\
\partial_0 z' &= -G_{03}' \tag{5.68} \\
\mathcal{F}_{\hat{a}\hat{b}}' &= -\partial_1 y' \tag{5.69}
\end{align*}
\]

\(^a\)JS would like to thank David Mateos for discussions related to this point.
which correspond to a non-threshold D0-D2 bound state with some (tilted) BIon ending on it, boosted in the compact direction.

Instead, we could have T-dualized along the $\sigma^3$ direction. The truncation of the BPS equations is given by

\begin{align}
\mathcal{F}'_{01} &= 0 = \partial_1 y' \\
\partial_0 z' &= -G'_{03} \\
\mathcal{F}'_{02} &= -\partial_2 y' \\
\partial_2 z' + G'_{23} + \partial_2 y' G'_{3y} &= \tan \alpha
\end{align}

which describe a non-threshold D2-D2 bound state with some (tilted) BIon ending on it, again, boosted in the compact direction.

6 Arbitrary bosonic background

In previous sections, we showed that in constant $G_{mn}$ and $b_{mn}$ backgrounds, bosonic configurations satisfying the kappa symmetry preserving condition (5.1) are mapped under T-duality to the corresponding bosonic configurations in the T-dual picture. We would like to extend that proof for an arbitrary bosonic background.

It is well known that D-branes are kappa symmetric whenever the background satisfies the superspace constraints \cite{19, 20}. The structure of kappa symmetry transformations is always given by the requirements

\begin{align}
\delta_\kappa Z^M E^a_M &= 0 \hspace{1cm} (6.1) \\
\delta_\kappa Z^M E^a_M &= \frac{1}{2} (1 + \Gamma_\kappa)^{\alpha}_\beta \kappa^\beta. \hspace{1cm} (6.2)
\end{align}

where $E^a_M$ are the different components of the supervielbeins, which should be thought of as power expansions in the fermionic $\theta$ fields and

\begin{align}
\Gamma_\kappa &= \frac{1}{\sqrt{-\det (G + \mathcal{F})}} \sum_{l=0}^{(2l+1)} \Gamma^{l+1}_{11} \wedge e^\mathcal{F} \hspace{1cm} (type \ IIA) \hspace{1cm} (6.3) \\
\Gamma_\kappa &= \frac{1}{\sqrt{-\det (G + \mathcal{F})}} \sum_{l=0}^{(2l)} \gamma_{(2l)} t_3^l \wedge e^\mathcal{F} i \tau_2 \hspace{1cm} (type \ IIB), \hspace{1cm} (6.4)
\end{align}

where

\begin{align}
\gamma^{(1)} &= d\sigma^\mu \gamma^a_\mu = d\sigma^\mu \partial_\mu Z^M E^a_M \Gamma_a \\
The &= F - \frac{1}{2} dZ^M E^a_M \wedge dZ^N E^b_N B_{AB}. \hspace{1cm} (6.6)
\end{align}

It is nevertheless true that the condition for any bosonic configuration ($\theta = 0$) to preserve some supersymmetry is still given by $\Gamma_\kappa \epsilon = \epsilon$. The reason is that when studying
the θ = 0 limit of the supervielbein components [28],
\[ E^a_m|_{\theta=0} = e^a_m(x^n), \quad E^a_m|_{\theta=0} = 0, \quad B_{mn}|_{\theta=0} = b_{mn}(x^n) \] (6.7)
so that
\[ \delta_{\kappa}^{\theta} e^a_{\alpha} = \frac{1}{2} (1 + \Gamma_{\kappa}|_{\theta=0})^{\alpha}_{\beta} \kappa^\beta, \] (6.8)
which determines the universal condition
\[ \Gamma_{\kappa}|_{\theta=0} \epsilon = \epsilon, \] (6.9)
\( \epsilon \) being the Killing spinor of the corresponding bosonic supergravity background.

\( \Gamma_{\kappa}|_{\theta=0} \) depends on the background geometry, but since the T-duality rules for the bosonic sector of the supergravity fields are known [5]
\[ G_{zz} = \frac{1}{G_{zz}^\prime} \]
\[ b_{nz} = -\frac{G_{nz}^\prime}{G_{zz}^\prime} \]
\[ G_{nz} = -\frac{b_{n^z}^\prime}{G_{zz}^\prime} \]
\[ G_{mn} = \frac{G_{mn}^\prime}{G_{zz}^\prime} - \frac{(G_{mn}^\prime G_{nz}^\prime - G_{nz}^\prime b_{m^z}^\prime b_{n^z})}{G_{zz}^\prime} \]
\[ b_{mn} = b_{mn}^\prime - \frac{(b_{m^z}^\prime G_{nz}^\prime - G_{nz}^\prime b_{n^z}^\prime G_{m^z}^\prime)}{G_{zz}^\prime} \] (6.10)
once can indeed compute the behaviour of \( \Gamma_{\kappa}|_{\theta=0} \) under T-duality, as we did previously [1].

Using the same notation as in previous sections
\[ \gamma(1)|_{\theta=0} = dx^m e_m \tilde{\Gamma} = \tilde{\Pi} + \tilde{\Gamma}_z D\rho, \] (6.11)
where
\[ \tilde{\Pi} = \tilde{\Gamma}_z d\tilde{x}^m e_m^\tilde{a}(x^n) \]
\[ D\rho = \mu d\rho + dx^m e_m(x^n). \] (6.12)

Under T-duality, it can be checked that
\[ \tilde{\Pi} = \tilde{\Pi}^\prime \]
\[ \mathcal{F} = \mathcal{F}^\prime + D\rho \tilde{\Pi}^\prime_z, \] (6.15)
where
\[ \tilde{\Pi}^\prime_z = \lambda d\tilde{x}^\prime + dx^m e_m^\prime(x^n). \] (6.16)

Once (6.15) is known, it is straightforward to extend the techniques developed in appendix B and C to show that any bosonic configuration solving (5.3) in type IIA, is mapped to the corresponding T-dual one satisfying
\[ \Gamma_{\kappa}|_{\theta=0} \epsilon = \epsilon^\prime, \] (6.17)
where the relation among Killing spinors is given by
\[ \epsilon_1 = \epsilon_2, \quad \Gamma_z \epsilon_2 = \epsilon_1, \] (6.18)
which is consistent with the transformations found in [29], since we have used a Lorentz (gauge) rotation to set \( e_{\tilde{a}}^\prime = 0 \).

9In the following we are explicitly using the parametrisation \( \eta = a_2 = -a_1 = 1 \) and \( \Gamma_m^\prime = \delta_m^n \).
7 Discussion

We would like to finish with some discussion concerning possible natural extensions of the results presented in this paper. In particular, we will concentrate on three subjects:

- T-duality realized on D-branes coupled to an arbitrary kappa symmetric background.
- Non-BPS D-branes.
- Non-abelian D-branes.

Arbitrary kappa symmetric backgrounds. There has been some recent interest in the open problem related to T-duality in curved kappa symmetric backgrounds [29]. When trying to extend our approach to this general case, it seems rather natural to demand the relations

\[ \theta_1^\alpha E_\alpha^\alpha = a_2 \theta_2^\alpha E_\alpha^\alpha, \quad \theta_2^\alpha E_\alpha^\alpha = -a_1 \left( \Gamma_2 \right)_\alpha^\beta E_\alpha^\beta \theta_1^\alpha. \]  

Equations (7.1) deserve several remarks. First of all, they are reminiscent of the extension of the kappa symmetry transformations from the SuperPoincaré case to the arbitrary kappa symmetric background. Secondly, it is not clear which is the solution to them, that is, \( \theta_1^\alpha \theta_2^\beta = f_\alpha^\beta + \theta_1^\alpha \), since the supervielbeins appearing in both sides of them admit an expansion in the corresponding fermionic fields. Finally, the mapping between fermionic fields will be non-constant in general, so that when computing the T-duality transformation of the operators coupling to derivatives of these fermionic fields, they will involve components of the spin connection.

Irrespective of which is the real solution, the latter should certainly satisfy some constraints. First of all, it should be such that the T-duality rules for the closed string sector must map the supergravity constraints of type IIA to the ones of type IIB. This is equivalent to map the D-brane effective action and its kappa symmetry structure in type IIA to the corresponding ones in type IIB. In other words, the mapping should be T-duality covariant and satisfy

\[ \delta_\kappa Z^M E_M^\alpha = 0 \rightarrow \delta_{\kappa'} Z'^M E_M^\alpha = 0 \]  

\[ \delta_\kappa Z^M E_M^\alpha = \frac{1}{2} (1 + \Gamma_\kappa)^\alpha_\beta \kappa'^\beta \rightarrow \delta_{\kappa'} Z'^M E_M^\alpha = \frac{1}{2} (1 + \Gamma'_{\kappa'})^\alpha_\beta \kappa'^\beta. \]  

Non-BPS D-brane effective actions. It has recently been argued that the effective action describing a non-BPS D-brane probing in SuperPoincaré should be splitted into a DBI term [31]

\[ S_{\text{non-BPS}} = - \int d^{p+1} \sigma \sqrt{-\det (G + F) f(T, \partial_\mu T, \ldots G_{S}^{\mu} \tilde{G}^{\mu} A)} \]  

plus a WZ term describing the coupling of the tachyonic scalar field \( T \) to the R-R sector [31],

\[ S_{\text{WZ}} = \int_{M_{p+2}} C \wedge dT \wedge e^F. \]
Due to the scalar character of the tachyonic field, it is natural to extend the functional truncation \((\partial_\rho \phi^i = 0)\) to it, \(\partial_\rho T = 0\). In this way, it is straightforward, using our previous analysis, to check that WZ terms \((7.3)\) are indeed T-duality covariant, as they should be. When being concerned about the T-duality properties of \((7.4)\), we do appreciate an important characteristic of the T-duality covariance requirement. Indeed, T-duality covariance does not fix the effective dynamics of the open string sector by itself, it just constrains it. For example, \((7.4)\) must be T-duality covariant, which means that \(f(T, \partial_\mu T, . . . \tilde{G}^{\mu\nu}_S, \tilde{G}^{\mu\nu}_A)\) is covariant, since the usual DBI square root is. This requirement does not fix \(f\). For instance, we can not distinguish between

\[
\sqrt{-\det (\tilde{G}_{\mu\nu} + F_{\mu\nu} + \partial_\mu T \partial_\nu T)}
\]

and

\[
\sqrt{-\det (\tilde{G} + F) \sum_n a_n (\tilde{G}^{\mu\nu}_S \partial_\mu T \partial_\nu T)^n}
\]

for arbitrary constant coefficients \(a_n\), both being T-duality covariant due to the covariance of \(\tilde{G}^{\mu\nu}_S = (\tilde{G} + F)^{-1(\mu\nu)}\). See [32] for a discussion of T-duality properties of non-BPS D-brane effective actions.

Non-abelian D-branes. In [33], the approach followed in this paper was used to determine the effect of non-trivial commutators among scalar fields in the non-abelian bosonic generalization of the DBI action. The main idea there was to assume that the trace over the \(U(N)\) gauge group indices was the symmetrized one (again T-duality does not fix this possibility) and study the dimensional reduction of the D9-brane field theory where world volume diffeomorphisms had been gauge fixed (since no covariant version is known for non-abelian D-brane effective actions).

In the following, we will briefly comment on the extension of that result to non-abelian SuperD-branes propagating in SuperPoincaré. As in [33], we will assume a symmetrized prescription for the trace and replace all partial derivatives by covariant derivatives. Since the new action includes fermions, one must also gauge fix kappa symmetry. Following [21], we choose

\[
\theta_1 = 0 \quad , \quad \theta_2 = \lambda
\]

ensuring the vanishing of the WZ term, so that we concentrate on the DBI term of the effective action. The components of the tensor

\[
E_{\mu\nu} = \Pi^m_{\mu} \Pi^n_{\nu} \eta_{mn} + F_{\mu\nu}
\]

can be written after the gauge fixing as

\[
E_{\hat{\mu}\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}} - 2\tilde{\lambda} \Gamma_{\hat{\mu}} D_{\hat{\nu}} \lambda + F_{\hat{\mu}\hat{\nu}} + (\tilde{\lambda} \Gamma^m D_{\mu} \lambda)(\tilde{\lambda} \Gamma^n D_{\nu} \lambda) \eta_{mn} \quad (7.10)
\]

\[
E^i_{\hat{\mu}} = -2i\lambda \Gamma_{[i,x^i,\lambda]} - [i(\tilde{\Lambda} \Gamma^m D_{\mu} \lambda)(\tilde{\Lambda} \Gamma^n [x^i, \lambda]) \eta_{mn} + \nabla_{\hat{\mu}} x^i \quad (7.11)
\]

\[
E^i_{\hat{\nu}} = -2\tilde{\lambda} \Gamma_{i} D_{\nu} \lambda + i(\tilde{\lambda} \Gamma^m D_{\mu} \lambda)(\tilde{\lambda} \Gamma^n [x^i, \lambda]) \eta_{mn} - \nabla_{\hat{\nu}} x^i \quad (7.12)
\]

\[
E^{ij} = \delta^{ij} - (\tilde{\lambda} \Gamma^m [x^i, \lambda])(\tilde{\lambda} \Gamma^n [x^j, \lambda]) \eta_{mn} - 2i\tilde{\lambda} \Gamma^{ij} [x^j, \lambda] + i[x^i, x^j] \quad (7.13)
\]

\(^{10}\) JS would like to thank Eduardo Eyras for a discussion concerning this point.
where we used the same reduction rules as those used in [33], with the addition that $D_i \lambda = i[x^i, \lambda]$, and we splitted the initial world volume directions $\{\mu, \nu\}$ into T-dual world volume ones $\{\hat{\mu}, \hat{\nu}\}$ and transverse directions denoted by scalars $\{x^i\}$.

By introducing the matrix

$$Q'_k = \delta^i_k + i[x^i, x^j]\delta_{jk} - 2i\lambda\Gamma^i[x^i, \lambda]\delta_{jk} - (\lambda\Gamma^m[x^i, \lambda])(\lambda\Gamma^m[x^i, \lambda])\eta_{mn}\delta_{jk}$$  \hspace{1cm} (7.14)

we can rewrite $E^{ji}$ and its inverse $E_{ik}$ as

$$E^{ji} = \delta^{ji} + (Q'_k - \delta^i_k)\delta^{ki} = Q'_k\delta^{ki}$$ \hspace{1cm} (7.15)
$$E_{ik} = \delta^i_l (Q^{-1})^l_k.$$ \hspace{1cm} (7.16)

In this way, we can now compute the determinant of the ten dimensional original matrix (notice that $\det E^{ij} = \det Q'_k$):

$$\det (G + F) = \det A \det Q$$ \hspace{1cm} (7.17)
$$A_{\hat{\mu}\hat{\nu}} = E_{\hat{\mu}\hat{\nu}} - E_{\hat{\mu}}^i E_{ik} E_{\hat{\nu}}^k,$$ \hspace{1cm} (7.18)

thus generalizing the result presented in [33].

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**A  Proof of T-duality covariance**

In this appendix, we will analyze the constraints derived from requiring T-duality covariance of the DBI term

$$- T_p \int d^{p+1}\sigma \sqrt{-\det (G + F)} - T'_{p-1} \int d^p\sigma \sqrt{-\det (G' + F')}.$$ \hspace{1cm} (A.1)

For this mapping to be satisfied, it is sufficient to hold

$$T'_{p-1} = T_p R,$$ \hspace{1cm} (A.2)

which is derived from operators involving no derivative of the dynamical fields $\{\phi^i\}$, and

$$G_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}} - \frac{1}{G_{\hat{\rho}\hat{\rho}}}(G_{\hat{\rho}\hat{\gamma}} + F_{\hat{\rho}\hat{\gamma}})(G_{\hat{\gamma}\hat{\rho}} - F_{\hat{\gamma}\hat{\rho}}) = G'_{\hat{\mu}\hat{\nu}} + F'_{\hat{\mu}\hat{\nu}}$$ \hspace{1cm} (A.3)
from operators involving such derivatives \(\{\partial_\mu \phi'^i\}\).

Equation (A.2) gives the correct tension for the T-dual D-brane. Notice that it is
equivalent to the usual T-duality transformation for the dilaton field

\[
\phi' = \phi - \frac{1}{2} \log |G_{zz}| \tag{A.4}
\]
when the latter is constant. Equations (A.3) are further split into their symmetric and
anti-symmetric parts

\[
\begin{align*}
\mathcal{G}_{\mu\nu} - \frac{1}{G_{pp}} (\mathcal{G}_{\mu\rho} \mathcal{G}_{\nu\rho} - \mathcal{F}_{\mu\rho} \mathcal{F}_{\nu\rho}) &= \mathcal{G}'_{\mu\nu}, \tag{A.5} \\
\mathcal{F}_{\mu\nu} + \frac{1}{G_{pp}} (\mathcal{G}_{\mu\rho} \mathcal{F}_{\nu\rho} - \mathcal{F}_{\mu\rho} \mathcal{G}_{\nu\rho}) &= \mathcal{F}'_{\mu\nu}. \tag{A.6}
\end{align*}
\]

The induced metric on the IIA Dp-brane is given by

\[
\begin{align*}
G_{\rho\rho} &= G_{zz}, \\
G_{\rho\rho} &= \partial_\mu Z^M G_{Mz}, \\
G_{\mu\nu} &= \partial_\mu Z^M \partial_\nu Z^N (-)^{MN} G_{M'N'}, \tag{A.7}
\end{align*}
\]
where we took into account the conditions (3.1)-(3.4) defining \(T_\parallel\), while the one on the
IIB D(p−1)-brane is just

\[
\begin{align*}
\mathcal{G}'_{\mu\nu} &= \partial_\mu z' \partial_\nu z' G'_{zz} + \partial_\mu Z'^M \partial_\nu z' G'_{Mz} + \partial_\mu Z'^M \partial_\nu Z^N (-)^{MN} G'_{M'N'}. \tag{A.8}
\end{align*}
\]

\((-)^{MN} = -1\) when both M and N are odd, \((-)^{MN} = 1\) for others. On the other hand,
the components of the gauge invariant tensor \(\mathcal{F}/(\mathcal{F}')\) on the IIA/(IIB) D-branes can be
decomposed as

\[
\begin{align*}
\mathcal{F}_{\mu\nu} &= \partial_\mu V_\nu - \frac{1}{2} \partial_\mu Z^M \partial_\nu Z^N B_{M'N'} \\
\mathcal{F}_{\mu\nu} &= \partial_\mu V_\nu + \partial_\mu Z^N B_{z'N'} \tag{A.9} \\
\mathcal{F}'_{\mu\nu} &= \partial_\mu V_\nu - \frac{1}{2} \partial_\mu Z'^M \partial_\nu Z'^N B_{M'N'} - \partial_\mu z' \partial_\nu z' G'_{zz} B_{z'N'}. \tag{A.10}
\end{align*}
\]

Using these decompositions in (A.5), and matching the coefficients of the different
independent operators \(\{\partial_\mu \phi^i \partial_\nu \phi^j\}\) appearing in both sides, we find

\[
\begin{align*}
\eta^2 G_{zz} &= G'_{zz}, \tag{A.11} \\
-\frac{\eta}{G_{zz}} B_{Nz} &= \Gamma^M_N G'_{z'M} \tag{A.12} \\
G_{M'N'} - \frac{1}{G_{zz}} [G_{Mz} G_{Nz} - B_{z'M} B_{z'N}] &= (-)^{(M+M')} \Gamma^M_N \Gamma^N_{M'} G'_{M'N'}. \tag{A.13}
\end{align*}
\]
Proceeding in the same way with (A.6), we obtain
\begin{equation}
\frac{-\eta}{G_{zz}} G_{\hat{M}z} = \Gamma_{\hat{M}} \hat{N} B_{\hat{N}z}
\tag{A.14}
\end{equation}

\begin{equation}
B_{\hat{M}\hat{N}} - \frac{1}{G_{zz}} \left[ (-)^{MN} \{ G_{\hat{M}z} B_{\hat{N}z} - B_{\hat{M}z} G_{\hat{N}z} \} \right] = (-)^{(M+M')N} \Gamma_{\hat{M}} \hat{N} B_{\hat{M}'\hat{N'}}
\tag{A.15}
\end{equation}

Equations (A.11)-(A.15) are the set of constraints derived from the T-duality covariance requirement. They can be interpreted as the generalization of the usual bosonic T-duality rules for the kind of superfields we are considering along the whole paper.

In the following, we will start analyzing equations (A.11)-(A.15). Before doing so, we must identify the different components of the superfields appearing in them. We can read the components of the superspace metric $G_{MN} = e_M^a e_N^b \eta_{ab}$ in type IIA from (2.6)-(2.7)
\begin{equation}
G_{mn} = e_m^a e_n^b \eta_{ab},
\end{equation}
\begin{equation}
G_{i\alpha,m} = (\bar{\theta}_i \Gamma_a)_{\alpha} \epsilon_m^a \equiv (\bar{\theta}_i \Gamma_a)_{\alpha} \epsilon_m^a,
\end{equation}
\begin{equation}
G_{i\alpha,j\beta} = (\bar{\theta}_i \Gamma_a)_{\alpha} (\bar{\theta}_j \Gamma^a)_{\beta},
\tag{A.16}
\end{equation}
and those of the NS-NS superfield $B_{MN}$ from (2.8)
\begin{equation}
B_{mn} = b_{mn},
\end{equation}
\begin{equation}
B_{i\alpha,m} = -(-1)^i (\bar{\theta}_i \Gamma_a)_{\alpha} \epsilon_m^a,
\end{equation}
\begin{equation}
B_{i\alpha,j\beta} = (-1)^i (\bar{\theta}_i \Gamma_a)_{\alpha} (\bar{\theta}_j \Gamma^a)_{\beta}.
\tag{A.17}
\end{equation}

In type IIB, $G'_{MN}$ is decomposed as
\begin{equation}
G'_{mn} = e'_m^a e'_n^b \eta_{ab},
\end{equation}
\begin{equation}
G'_{i\alpha,m} = (\bar{\theta}_i \Gamma'_a)_{\alpha} \epsilon'_m^a \equiv (\bar{\theta}_i \Gamma'_a)_{\alpha} \epsilon'_m^a,
\end{equation}
\begin{equation}
G'_{i\alpha,j\beta} = (\bar{\theta}_i \Gamma'_a)_{\alpha} (\bar{\theta}_j \Gamma'^a)_{\beta},
\tag{A.18}
\end{equation}
while the NS-NS superfield $B'_{MN}$ as
\begin{equation}
B'_{mn} = b'_{mn},
\end{equation}
\begin{equation}
B'_{i\alpha,m} = (-1)^i (\bar{\theta}_i \Gamma'_a)_{\alpha} \epsilon'_m^a,
\end{equation}
\begin{equation}
B'_{i\alpha,j\beta} = -(-1)^i (\bar{\theta}_i \Gamma'_a)_{\alpha} (\bar{\theta}_j \Gamma'^a)_{\beta}.
\tag{A.19}
\end{equation}

It is always possible to make a local $SO(1,9)$ rotation in both IIA/IIB tangent spaces to set
\begin{equation}
e_z^a = \lambda \delta_z^a, \quad e'_z^a = \lambda' \delta_z^a,
\tag{A.20}
\end{equation}
where \( \lambda \) and \( \lambda' \) are constants such that

\[
G_{zz} = \lambda^2, \quad G'_{zz} = \lambda'^2.
\]  
(A.21)

Equation (A.11) becomes

\[
\eta^2 = \lambda^2 \lambda'^2,
\]  
(A.22)

which is the analogue of the usual T-duality rules relating the radius of the original circle with the radius of the T-dual circle, \( G'_{zz}, G_{zz} = 1 \).

Equations (A.12) and (A.13) allow us to set some elements of \( \Gamma^N_M \) to zero

\[
\Gamma^j_{\beta m} = \Gamma^m_{j\beta} = \Gamma^{1\alpha}_{1\beta} = \Gamma^{2\alpha}_{2\beta} = 0
\]  
(A.23)

and

\[
s \frac{b_{mz}}{\lambda} = \Gamma^m_{n} e'_{n z}, \quad s e_{mz} = \Gamma^m_{n} b'_{n z} \lambda',
\]  
(A.24)

\[
s (\widetilde{\theta}_i \Gamma_{\hat{a}})_{\alpha} = -(1)^i \Gamma_{ia}^{j\beta} (\widetilde{\theta}_j \Gamma_{\hat{a}})_{\beta},
\]  
(A.25)

where \( s \equiv \frac{\eta}{\lambda N} = \pm 1 \) is a signature. Assuming IIB spinors \( \theta'_j \) have positive chirality, we can take

\[
\widetilde{\theta}_1 = a_2 \widetilde{\theta}_2, \quad \widetilde{\theta}_2 = a_1 \widetilde{\theta}_1 \Gamma_{\hat{a}}.
\]  
(A.26)

and

\[
s a_2 e_{\alpha}^{\hat{a}} = e'_{\beta}^{\hat{a}} \Gamma_{1\alpha}^{2\beta}, \quad -s a_1 (\Gamma_{\hat{a}})_{\alpha}^{\beta} e_{\alpha} = e'_{\beta} \Gamma_{2\alpha}^{1\beta}.
\]  
(A.27)

It follows, in addition to (A.25), for \( \hat{a} \neq \hat{a} \) components that

\[
s (\theta_i \Gamma_{\hat{a}})_{\alpha} = (\theta_j \Gamma_{\hat{a}})_{\beta} \Gamma_{ia}^{j\beta}.
\]  
(A.28)

From the equation (A.14) we get

\[
e_{m}^{\hat{a}} = s \Gamma_{m}^{\hat{a}} e_{m}^{\hat{a}}.
\]  
(A.29)

Finally (A.13) requires

\[
b_{mn} - \frac{b_{mz}}{\lambda} e_{n z} = \Gamma_{m}^{m'} \Gamma_{n}^{n'} b'_{m'n'}.
\]  
(A.30)

Thus, as we claimed in the introduction, T-duality covariance of the DBI action fixes the chirality change mapping among spinor fields (A.26) up to constant factors \((a_1, a_2)\) from the fermionic components of the background superfields, and reproduces the well known transformations for their bosonic components (A.22), (A.24), (A.29) and (A.30).
Having solved such T-duality constraints, we will next study the T-duality properties of the different supersymmetric invariant forms defined on D-branes. Let us consider \( \mathcal{H}_I = \Gamma_\alpha \Pi^\alpha \) in type IIA. It can be splitted in terms of
\[
\Pi = \hat{\Pi} + \Gamma \Pi^z, \tag{A.31}
\]
where
\[
\hat{\Pi} \equiv \Gamma \hat{a} \left( dx^m e_\alpha^m + \bar{\theta} \Gamma \bar{z} d\bar{\theta} \right) \tag{A.32}
\]
\[
D \rho \equiv \Pi^z = \lambda d\rho + dx^m e_\alpha^m + \bar{\theta} \Gamma \bar{z} d\bar{\theta}. \tag{A.33}
\]
Analogously, in type IIB,
\[
\Pi' = \hat{\Pi}' + \Gamma \Pi'_{\bar{z}}, \tag{A.34}
\]
where
\[
\hat{\Pi}' \equiv \Gamma \hat{a}' \left( dx^{m'} e_\alpha^{m'} + \bar{\theta}' \Gamma \bar{z}' d\bar{\theta}' \right) \tag{A.35}
\]
\[
\Pi_{\bar{z}} \equiv (\lambda' dz' + dx^{m'} e_\alpha^{m'} + \bar{\theta}' \Gamma \bar{z}' d\bar{\theta}'). \tag{A.36}
\]
We can write \( \hat{\Pi} \) in terms of type IIB variables by inserting (A.26), (A.28) and (A.29) into (A.32).
\[
\hat{\Pi} \equiv \Gamma \hat{a} \left( dx^m e_\alpha^m + \bar{\theta} \Gamma \bar{z} d\bar{\theta} \right) = \hat{\Pi}'. \tag{A.37}
\]
Thus, by choosing
\[
s = +1, \quad a_1^2 = 1, \quad a_2^2 = 1 \tag{A.38}
\]
we can identify both one forms
\[
\hat{\Pi} = \Gamma \hat{a} \left( dx^{m'} e_\alpha^{m'} + \bar{\theta}' \Gamma \bar{z}' d\bar{\theta}' \right) = \hat{\Pi}'. \tag{A.39}
\]
The latter equation is telling us that the supersymmetric invariant one form in nine dimensions is T-duality covariant. Furthermore, when using (A.37) in (A.26) and (A.28), it follows that
\[
\bar{\theta} \Gamma \hat{a} d\bar{\theta} = + \bar{\theta} \Gamma \hat{a} d\bar{\theta}', \quad \bar{\theta} \Gamma_{11} \hat{a} d\bar{\theta} = + \bar{\theta} \tau_3 \Gamma \hat{a} d\bar{\theta}', \tag{A.40}
\]
which are the form version of the identities (3.20), and
\[
D \rho = \lambda d\rho + dx^m e_\alpha^m + \bar{\theta} \Gamma \bar{z} d\bar{\theta} = \lambda d\rho + dx^{m'} e_\alpha^{m'} - \bar{\theta} \tau_3 \bar{z} d\bar{\theta}'. \tag{A.41}
\]
Concerning the supersymmetric invariant two form \( \mathcal{F} \)
\[
\mathcal{F} = dV + (\bar{\theta} \Gamma_{11} d\bar{\theta})(dx^m + \frac{1}{2} \bar{\theta} \Gamma m d\bar{\theta}) - \frac{1}{2} dx^m dx^n b_{mn}, \tag{A.42}
\]
29
it can be written in terms of type IIB variables as
\[
\mathcal{F} = \mathcal{F}' + D\rho \Pi \tilde{z}, \quad (A.43)
\]
where
\[
\mathcal{F}' \equiv dV - \frac{1}{2} dx^m dx^n b_{mn} + (\bar{\theta} \tau_3 \Gamma \, d\bar{\theta}')(d x^m e^a_m + \frac{1}{2} \bar{\theta} \Gamma^a d\bar{\theta}'). \quad (A.44)
\]

It is remarkable that all terms \(i_{\partial_r} \mathcal{F}\) appearing in the decomposition of the supersymmetric invariant form \(\mathcal{F}\) under the double dimensional ansatz, \(\mathcal{F} = \mathcal{F} - + d\rho \wedge i_{\partial_r} \mathcal{F}\), can be written as \(\Pi' \bar{z}\), the supersymmetric invariant one form along the T-dual circle. Furthermore, all the dependence of \(d\rho\) in supersymmetric invariant forms \(\Pi\) and \(\mathcal{F}\) is through \(D\rho = \Pi \tilde{z}\), the supersymmetric one form along the original circle. Thus, what T-duality does is to exchange both forms \(\Pi \tilde{z} \leftrightarrow \Pi' \bar{z}\). This is the supersymmetric generalization of the corresponding phenomena observed in the bosonic case [9], whose relevance may become more clear in the discussion of the T-duality transformation of the WZ term in appendix B.

**B T-Duality transformation of WZ term**

In this appendix we prove that WZ terms of type IIA SuperD-branes are mapped to WZ terms of type IIB SuperD-branes under T-duality, using the results obtained in appendix A. WZ terms of IIA D-branes are obtained from
\[
dL^{WZ}_A = - T_p \, E C_A E \, e^\mathcal{F}, \quad C_A(\mathcal{H}) = \sum_{\ell=0} \frac{(\Gamma_{11})^{\ell+1}}{(2\ell)!} \mathcal{H}^{2\ell}, \quad (B.1)
\]
where
\[
E^\alpha = d\bar{\theta}^\alpha = d\bar{\theta} \varepsilon^\alpha, \quad \bar{E}_2 = d\bar{\theta} \varepsilon C_{\alpha \beta}, \quad \mathcal{H} = \Gamma(\tilde{z} + \bar{\theta} \Gamma \bar{z} d\bar{\theta}). \quad (B.2)
\]
The latter show that \(dL^{WZ}_A\) just depends on supersymmetric invariant forms, whose T-duality properties were determined in appendix [4]. In particular, from \([A.43]\) and using \(D\rho \wedge D\rho = 0\),
\[
e^\mathcal{F} = \sum_{n=0} (\mathcal{F}' + D\rho \Pi \tilde{z})^n = (1 + (D\rho \Pi \tilde{z})) \, e^\mathcal{F}'. \quad (B.3)
\]
Next, using \([A.28]\) and \([A.38]\) besides that \(\hat{\Pi}\) and \(\Gamma \bar{z} D\rho\) are commuting one forms,
\[
E C_A E = d\bar{\theta} \sum_{\ell=0} \frac{(\Gamma_{11})^{\ell+1}}{(2\ell)!} \hat{\Pi}^{2\ell} d\bar{\theta} = d\bar{\theta} \sum_{\ell=0} \frac{(\Gamma_{11})^{\ell+1}}{(2\ell)!} \hat{\Pi}^{2\ell} + (D\rho \Pi \tilde{z}) \, d\bar{\theta}
\]
\[
= \frac{d\bar{\theta}_2}{(2\ell)!} \hat{\Pi}^{2\ell} - 1 + \frac{d\bar{\theta}_1}{(2\ell)!} \hat{\Pi}^{2\ell} + (D\rho \Pi \tilde{z}) \, d\bar{\theta}_2.
\]
In this appendix we prove that the infinitesimal kappa symmetry transformation $\delta_{\kappa} \theta$ in type IIA is mapped to $\delta_{\kappa'} \theta'$ in type IIB as claimed in \ref{A.13}. The kappa symmetry transformations for type IIB spinors are obtained from those of IIA using \ref{A.26}.

\begin{equation}
\begin{split}
&+ d\bar{\theta}_2 (\Gamma z D \rho) \frac{\hat{H}^{2\ell+1}}{(2\ell+1)!} d\bar{\theta}_1 + d\bar{\theta}_1 (\Gamma z D \rho) (-1)^\ell \frac{\hat{H}^{2\ell+1}}{(2\ell+1)!} d\bar{\theta}_2 \\
= & a_1 a_2 \{ d\bar{\theta}_1 \Gamma \frac{\hat{H}^{2\ell}}{(2\ell)!} d\bar{\theta}_2 + d\bar{\theta}_2 (\Gamma z \hat{H}^{2\ell+1}) \Gamma z d\bar{\theta}_1' \\
&+ d\bar{\theta}_1 D \rho (2\ell+1)! d\bar{\theta}_2 + d\bar{\theta}_2 D \rho (-1)^\ell \frac{\hat{H}^{2\ell+1}}{(2\ell+1)!} d\bar{\theta}_1' \}
\end{split}
\end{equation}

Joining \ref{B.3} and \ref{B.4}

\begin{equation}
\begin{split}
dL_A^{WZ} &= -T_p a_1 a_2 d\bar{\theta} \left\{ \sum_{\ell=0}^{2\ell} \frac{\hat{H}^{2\ell}}{(2\ell)!} \Gamma z \tau_3^\ell \tau_1 + (D \rho) \sum_{\ell=0}^{2\ell+1} \frac{\hat{H}^{2\ell+1}}{(2\ell+1)!} \tau_3^\ell \tau_1 \right\} d\bar{\theta} (1 + (D \rho \Pi^z)) e^{F'} \\
&= -a_1 a_2 T_p (D \rho) \left[ d\bar{\theta} \sum_{\ell=0}^{2\ell+1} \frac{\hat{H}^{2\ell+1}}{(2\ell+1)!} \tau_3^\ell \tau_1 \right] e^{F'} + \ldots \\
&= a_1 a_2 T_p \left[ d\bar{\theta} \ S_B (\Pi') \tau_1 \right] e^{F'} (D \rho) + \ldots,
\end{split}
\end{equation}

where dots stand for terms not depending on $d\rho$.

From $dL_A^{WZ}$ in \ref{B.3} we can find the WZ Lagrangian $L_A^{WZ}$, written in terms of IIB variables, by taking the $p+1$ form part of $L_A^{WZ}$ on $\sigma^\mu$. IIB (p-1) brane WZ term will be obtained by integrating it over $\rho$. It means that only the coefficient of $d\rho$ in \ref{B.3} contributes to $L_B^{WZ}$. The coefficient of $d\rho$ in \ref{B.3} gives $dL_B^{WZ}$

\begin{equation}
dL_B^{WZ} = -T_{p-1} \left[ E' S_B (\Pi') \tau_1 \right] e^{F'},
\end{equation}

if $a_1 a_2 = -1$, where $T_{p-1}$ is given in \ref{A.2}.

\section{Kappa symmetry}

In this appendix we prove that the infinitesimal kappa symmetry transformation $\delta_{\kappa} \theta$ in type IIA is mapped to $\delta_{\kappa'} \theta'$ in type IIB as claimed in \ref{A.13}. The kappa symmetry transformations for type IIB spinors are obtained from those of IIA using \ref{A.26}.

\begin{equation}
\begin{split}
a_2 \delta \bar{\theta}_2 &= \delta \bar{\theta}_1 = \delta \bar{\theta} \Gamma_+ = \bar{\kappa} (1 - \gamma^{(p)}) \Gamma_+ \\
a_1 \delta \bar{\theta}_1 &= \delta \bar{\theta}_2 \Gamma^z = \delta \bar{\theta} \Gamma_+ \Gamma^z = \delta \bar{\theta} \Gamma_+ \Gamma_+ = \bar{\kappa} (1 - \gamma^{(p)}) \Gamma_+ \Gamma_+.
\end{split}
\end{equation}

First terms in the right hand side of \ref{C.1} and \ref{C.2} are

\begin{equation}
\bar{\kappa} \Gamma_+ = \bar{\kappa}_1 \equiv a_2 \bar{\kappa}'_2, \quad \bar{\kappa} \Gamma_+ \Gamma^z = \bar{\kappa}_2 \Gamma^z \equiv a_1 \bar{\kappa}'_1.
\end{equation}
where we assumed that kappa symmetry parameters $\kappa_j$ have the same T-duality transformation as the dynamical fields $\theta$ in (A.26),

$$\tilde{\kappa}_1 = a_2 \tilde{\kappa}_2', \quad \tilde{\kappa}_2 = a_1 \tilde{\kappa}_1' \Gamma^z.$$  \hspace{1cm} (C.5)

The second term in the right hand side of (C.1) is

$$- \tilde{\kappa} (\gamma^{(p)}) \Gamma_{-} = \frac{-\tilde{\kappa}}{\sqrt{-\det(G + F)}} [S_A([\Pi] e^{F})]_{p+1} \Gamma_{-}$$

$$= \frac{-\tilde{\kappa}}{\sqrt{-\det(G + F)}} \left[ \sum_{\ell=0} (2\ell + 1)! e^{F} \right]_{p+1} \Gamma_{-}$$

$$= \frac{-\tilde{\kappa}}{\sqrt{-\det(G + F)}} \left[ \sum_{\ell=0} (2\ell + 1)! \left( \hat{\Pi}^2 \Pi \right)_{p+1} \Gamma_{-} \right]$$

$$= \frac{-\tilde{\kappa}}{\sqrt{-\det(G + F)}} \left[ \sum_{\ell=0} (2\ell + 1)! \left( \Gamma^z \right)_{p+1} \Gamma_{-} \right]$$

$$= -a_1 \tilde{\kappa}_1 ' \Gamma_{-}$$  \hspace{1cm} (C.6)

Here $[... ]_{p+1}$ means p+1 form coefficient of $[...]$, the coefficient of $d\sigma^0 d\sigma^1 ... d\sigma^p$ after taking the pullback. In the last second line p+1 form coefficient is replaced with p form coefficient (the coefficient of $d\sigma^0 d\sigma^1 ... d\sigma^{p-1}$) by dropping $d\rho$. We have also used the relation of DBI term

$$\sqrt{-\det(G + F)} = \lambda \sqrt{-\det(G' + F')}.$$  \hspace{1cm} (C.7)

Analogously for the second term in (C.1),

$$- \tilde{\kappa} (\gamma^{(p)}) \Gamma^z \Gamma_{-} = \frac{-\tilde{\kappa}}{\sqrt{-\det(G + F)}} [S_A([\Pi] e^{F})]_{p+1} \Gamma^z \Gamma_{-}$$

$$= \frac{-\tilde{\kappa}}{\sqrt{-\det(G + F)}} \left[ \sum_{\ell=0} (2\ell + 1)! e^{F} \right]_{p+1} \Gamma^z \Gamma_{-}$$

$$= \frac{-\tilde{\kappa}}{\sqrt{-\det(G + F)}} \left[ \sum_{\ell=0} (2\ell + 1)! \left( \hat{\Pi}^2 \Pi \right)_{p+1} \Gamma^z \Gamma_{-} \right]$$

$$= \frac{-\tilde{\kappa}_1}{\sqrt{-\det(G' + F')}} \left[ \sum_{\ell=0} (2\ell + 1)! \left( \Gamma^z \right)_{p+1} \Gamma_{-} \right]$$

$$= -a_2 \tilde{\kappa}_2' \Gamma_{-} \Gamma_{-} - a_1 \tilde{\kappa}_1 ' \Gamma_{-}$$  \hspace{1cm} (C.8)
Thus, joining the partial results

\[
\tilde{\delta}_1' = \bar{\kappa}'_1 - \frac{a_2}{a_1} \frac{\bar{\kappa}'_2}{\sqrt{-\text{det}(G' + F')}} \left[ \sum_{\ell=0}^{\ell+1} (-1)^{\ell+1} \frac{(\hat{M}')^{2\ell}}{(2\ell)!} e^{F'} \right]_p \Gamma_-, \tag{C.9}
\]

\[
\tilde{\delta}_2' = \bar{\kappa}'_2 - \frac{a_1}{a_2} \frac{\bar{\kappa}'_1}{\sqrt{-\text{det}(G' + F')}} \left[ \sum_{\ell=0}^{\ell} \frac{\{ (\hat{M}')^{2\ell} \} e^{F'}}{(2\ell)!} \right]_p \Gamma_. \tag{C.10}
\]

Since \(a_1^2 = a_2^2 = -a_1 a_2 = 1\) we get \(a_1/a_2 = -1\). We can express them by using \(\tau\) matrices as

\[
\tilde{\delta}' = \bar{\kappa}' + \frac{\bar{\kappa}'}{\sqrt{-\text{det}(G' + F')}} \left[ \sum_{\ell=0}^{\ell+1} (\tau_3)^{\ell+1} \frac{(\hat{M}')^{2\ell}}{(2\ell)!} e^{F'} \right]_p \tau_1 \Gamma \tag{C.11}
\]

Thus we have shown that the kappa symmetry transformation of IIA spinor \(\delta_{\kappa} \overline{\theta}\) is mapped to that of IIB spinor \(\delta_{\kappa'} \overline{\theta}'\) under \(T_\parallel\),

\[
\tilde{\delta}' = \bar{\kappa}'(1 - \gamma'^{(p-1)}), \tag{C.12}
\]

\[
\gamma'^{(p-1)} = \frac{-1}{\sqrt{-\text{det}(G' + F')}} \left[ C_B [\hat{M}] \tau_1 e^{F'} \right]_p. \tag{C.13}
\]

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