Command Filtering-Based Neural Network Control for Fractional-Order PMSM With Input Saturation

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ABSTRACT Command filtering-based neural network control is investigated in this paper for fractional-order input saturated permanent magnet synchronous motor (PMSM). First, the fractional-order command filter is introduced to cope with the “explosion of complexity” problem caused by the repeated derivatives of virtual signals in backstepping. Next, a compensation mechanism related to error is investigated to decrease the filtering errors under fractional calculus framework. Then, a neural network with its weight being updated online is accepted to eliminate restrictions on the uncertain nonlinear functions. Besides, the minimal learning parameterization technique is introduced to construct fractional-order adaptive law for the parameters of the neural network. Finally, the simulation results testify the availability and advantage of the designed approach.

INDEX TERMS Command filter, input saturation, fractional-order, PMSM.

NOMENCLATURE

- $\alpha$: fractional-order
- $\delta$: approximation error
- $\lambda_i$: compensating signal
- $\psi_r$: magnet flux linkage (Wb)
- $\psi_i$: virtual signal
- $i_{ic}$: filtered signal
- $B$: viscous damping coefficient (N/(rad/s))
- $E_{\beta,\gamma}(\cdot)$: Mittag-Leffler function
- $h_i$: center of receptive field
- $i_a, i_b, i_c$: real stator currents
- $i_d$: $d-$axis stator current
- $i_q$: $q-$axis stator current
- $J$: moment of inertia (Kg.m$^2$)
- $k_i$: control gain
- $L_d$: $d-$axis inductance (H)
- $L_q$: $q-$axis inductance (H)
- $n_p$: number of pole pairs
- $q_i$: width of Gaussian function
- $R$: stator resistance (\Omega)
- $S_i(\cdot)$: basis function vector
- $v_d$: $d-$axis control signal (V)
- $v_i$: compensated signal
- $v_q$: $q-$axis control signal (V)
- $x_d$: desired value
- $z_i$: tracking error
- $T_L$: load torque (Nm)
- $u_d$: $d-$axis voltage (V)
- $u_q$: $q-$axis voltage (V)

I. INTRODUCTION

Fractional calculus has a history of about 300 years. It has become an active field of study and received increasing attention in recent years [1]–[4]. Many real problems can be modeled due to the application of fractional calculus, such as fractional stochastic systems, diffusion processes, signal processing, control processing, etc. Meanwhile, various significant researches of fractional calculus have already been made [5]. For example, a synchronization criteria combined with an impulsive gain is developed in [6] to address the global synchronization problem for fractional-order complex networks. In [7], an adaptive control scheme via backstepping is proposed for fractional-order time delayed systems.
with uncertain parameters. Compared with integer-order system, the motion states of the fractional-order one may be changed under the same conditions [8]. Furthermore, many studies illustrate that the description of permanent magnet synchronous motor (PMSM) can be more accurate with the help of fractional-order models [9]–[11]. Therefore, it is worthwhile to investigate the fractional calculus in practice.

PMSM is widely accepted in industrial applications due to its high efficiency, large torque, low loss and excellent accuracy [12]. However, the PMSM suffers from highly nonlinear, numbers of disturbances and parameter uncertainties in practice [13]. To earn a good performance of the PMSM, various research achievements have been discovered. In [14], a direct torque control approach based on model predictive is presented to gain better steady-state performance for PMSMs with torque ripples. One major drawback of this scheme is that it’s not easy to find an adjustable parameter. In [15], a predictive speed controller is designed to obtain high-speed control dynamics for PMSM electric drives system. However, the parameter uncertainties of the PMSM are not considered in these works. A compensating control scheme combined with model reference adaptive technique is introduced in [16] to compensate unknown model parts. The main limitation of this approach, however, is that the boundedness of the uncertain part is imperative to achieve. Neural network (NN) approximation approach is deemed to be a basic tool for uncertain nonlinear systems [17]–[19]. By using NN approximation, a mapping-based nonlinear feedback technology is studied in [20] to control the PMSM systems. In [21], an adaptive robust backstepping control method is proposed for high-precision speed control of PMSM. The main limitation of classical adaptive backstepping, however, is the existence of “explosion of complexity” issue.

In recent years, dynamic surface control (DSC) [22], [23] approach is treated as a better choice to work out the “explosion of complexity” issue in backstepping. By using DSC, an adaptive NN-based output feedback control method is introduced in [24] to stabilize a class of nonstrict-feedback nonlinear systems with input dead-zone and unmeasurable states. For triangular structured nonlinear systems with unknown functions, a singularity-free fuzzy NN control method based on DSC design technique is investigated in [25]. However, these strategies involve potential filtering errors created by DSC. Luckily, command filtered backstepping technique combined with the error compensation mechanism is developed to conquer this problem [26]. In [27], an error compensation method fused with command filtered backstepping is presented for PMSM with iron losses. For nonlinear unknown systems with dead zone, a fuzzy control approach associated with adaptive technology is designed in [28] to handle the tracking issue by utilizing command filtering technique. However, all the studies reviewed so far, suffer from the fact that it does not consider the fractional-order model.

Within the framework of the fractional calculus, the stability problem is figured out in [29] by using Mittag-Leffler functions and Gronwall-Bellman’s inequality for fractional-order time delayed PMSM systems. But this approach relies on a precise model. For fractional-order nonlinear lower triangular structured systems with unknown function and uncertain disturbances, an adaptive fuzzy NN method via backstepping is developed in [30] to solve the stability problem. However, the repeated derivative problem remains unsolved. For fractional-order input time delayed systems, an observer-based disturbance rejection approach is proposed in [31] to examine the output tracking control issue. In order to address the robust tracking control issue, the fractional Lyapunov direct method is applied in [32] for a class of fractional-order multiagent systems with unknown input. It is recognized that the adaptive backstepping method is treated as a good strategy for solving the tracking control issue of nonlinear systems. Difficulties arise, however, how to extend the command filtered backstepping technique for nonlinear systems with actuator constraint under fractional calculus framework remains unsolved.

In another study front line, input saturation problem is considered to be a common constraint of actuators in actual physical systems. It may reduce the control performance of the system and even lead the closed-loop system to an unstable state. Therefore, many researchers have devoted to controlling the nonlinear systems incorporating input saturation. A number of meaningful results have been reported for the control of input saturated systems [33]–[37]. However, to our best knowledge, few articles are available for command filtering-based NN control approach of fractional-order PMSM with input saturation.

Inspired by the literature above, this paper considers the tracking control problem for fractional-order input saturated PMSM with uncertain nonlinear functions and load disturbance. We use the backstepping technique as a basic method to control the system. The fractional-order command filter is introduced to cope with the “explosion of complexity” problem. By utilizing the compensation mechanism, the filtering errors can be decreased. Meanwhile, the NN is accepted to eliminate the restrictions of uncertainty. Compared with the current researches, the contributions of this work are list as follows

1) The command filter method combined with an error compensation technique is investigated to solve the “explosion of complexity” problem and eliminate the filtering errors under fractional calculus framework.

2) The approximation-based NNs are developed to estimate uncertain parts. Meanwhile, a fractional-order adaptive law is designed to update the adaptive parameter by utilizing the minimal learning parameterization method.

3) The desired tracking performance for fractional-order input saturated PMSM is obtained by using the designed controller which integrates command filter, NN, fractional-order error compensation mechanism, backstepping, fractional-order adaptive law and minimal learning parameterization technique.
The remainder parts of this brief are given as follows. Section II shows fractional-order PMSM dynamic and preliminaries. The command filtering-based NN controller is presented in Section III. Stability analysis and simulation results are shown in Section IV and Section V, respectively. Section VI concludes our works.

II. PROBLEM STATEMENT AND PRELIMINARIES

The mathematical model of fractional-order PMSM under $d-q$ frame can be described as follows [9]

\[
\begin{aligned}
\frac{d^\alpha \Theta}{dt^\alpha} &= w \\
\frac{d^\alpha w}{dt^\alpha} &= \left( \frac{3}{2} n_p (L_d - L_q) i_d i_q + \psi_r i_q \right) - B w - T_L / J \\
\frac{d^\alpha i_q}{dt^\alpha} &= (-R i_q - L_d w i_d - \psi_r w + u_d) / L_q \\
\frac{d^\alpha i_d}{dt^\alpha} &= (-R i_d + L_q w i_q + u_d) / L_d
\end{aligned}
\]

where $\Theta$ represents the rotor position, $w$ denotes the electrical rotor speed.

The model of PMSM can be simplified by defining the following notations

\[
\begin{aligned}
a_1 &= \frac{3}{2} n_p \psi_r, \quad a_2 = \frac{3}{2} n_p (L_d - L_q), \quad b_1 = -\frac{R}{L_q} \\
b_2 &= -\frac{n_p L_d}{L_q}, \quad b_3 = -\frac{n_p \psi_r}{L_q}, \quad b_4 = \frac{1}{L_q} \\
c_1 &= -\frac{R}{L_d}, \quad c_2 = \frac{n_p L_q}{L_d}, \quad c_3 = \frac{1}{L_d}
\end{aligned}
\]

The fractional derivative (or the Caputo derivative) $\frac{d^\alpha}{dt^\alpha} f(t)$ of order $\alpha$ of a function $f : R^+ \rightarrow R$ is defined below [30]

\[
\frac{d^\alpha}{dt^\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+m-1}} d\tau
\]

where $\Gamma(\cdot)$ represents the Euler’s Gamma function, $t \in [0, \infty)$, $\alpha \in (m-1, m)$, $m \in Z^+$.  

Define $x_1 = \Theta$, $x_2 = w$, $x_3 = i_q$ and $x_4 = i_d$, then the fractional-order PMSM can be described as [29]

\[
\begin{aligned}
\frac{d^\alpha}{dt^\alpha} x_1 &= x_2 \\
\frac{d^\alpha}{dt^\alpha} x_2 &= \frac{a_1}{J} x_3 + \frac{a_2}{J} x_3 x_4 - \frac{B}{J} x_2 - \frac{T_L}{J} \\
\frac{d^\alpha}{dt^\alpha} x_3 &= b_1 x_3 + b_2 x_2 x_4 + b_3 x_2 + b_4 u_d \\
\frac{d^\alpha}{dt^\alpha} x_4 &= c_1 x_4 + c_2 x_2 x_3 + c_3 u_d
\end{aligned}
\]

where the fractional-order $\alpha \in (0, 1)$.

It is known that the actual control signal cannot be infinite. Considering the real control signals $u_d$ and $u_q$ have saturation restriction in practical applications, the scalar input $u$ can be written as [34]

\[
u = \text{sat}(v) = \begin{cases} u_{\text{min}}, & v \leq u_{\text{min}} \\ v, & u_{\text{min}} < v < u_{\text{max}} \\ u_{\text{max}}, & v \geq u_{\text{max}} \end{cases}
\]

where $u_{\text{min}} < 0$ and $u_{\text{max}} > 0$ are assumed to be unknown constants. $v$ denotes the control signal. In order to avoid the sharp corner when $v = u_{\text{min}}$ or $v = u_{\text{max}}$, a smooth function is introduced

\[
g(v) = \begin{cases} u_{\text{min}} \cdot \text{tanh}(v), & v < 0 \\ u_{\text{max}} \cdot \text{tanh}(v), & v \geq 0 \\ u_{\text{max}} \cdot \frac{e^{v_{\text{max}}} - e^{-v_{\text{max}}}}{e^{v_{\text{max}}} + e^{-v_{\text{max}}}}, & v < 0 \\ u_{\text{min}} \cdot \frac{e^{v_{\text{max}}} - e^{-v_{\text{max}}}}{e^{v_{\text{max}}} + e^{-v_{\text{max}}}}, & v \geq 0
\end{cases}
\]

\[
u = \text{sat}(v) = g(v) + d(v), \text{ then we can obtain that } |d(v)| = |\text{sat}(v) - g(v)| \leq \max\{|u_{\text{min}}(\text{tanh}(1) - 1), u_{\text{max}}(1 - \text{tanh}(1))\} = D.
\]

According to the mean-value theory, one has

\[
g(v) = g(v_0) + g_{\nu}(v - v_0)
\]

where $g_{\nu}(v) = \frac{g'(v)}{\nu}|_{\nu = v_0}, v_0 = \mu + (1-\mu)\nu_0$ with $0 < \mu < 1$.

The equation (7) can be rewritten as $g(v) = g_{\nu}(v)$ by selecting $v_0 = 0$, then we have

\[
u = g_{\nu}(v) + d(v)
\]

Then, the $d-q$ axis voltages can be expressed as

\[
\begin{aligned}
&u_d = g_{\nu_d} v_d + d(v) \\
&u_q = g_{\nu_q} v_q + d(v)
\end{aligned}
\]

where $g_{\nu_d}$ and $g_{\nu_q}$ are the $d-q$ axis notations of $g_{\nu}$, respectively.

In the engineering application, the control system may be unstable if we do not consider input saturation problem. Meanwhile, parameter uncertainties and load torque perturbation also influence the performance of system. Previous studies [33]–[37] regarding input saturated nonlinear system with uncertainties cannot be used under fractional calculus framework. Therefore, it is necessary to propose a novel control method to solve this problem.

The objective of this paper is to design a command filtering-based NN controller for fractional-order input saturated PMSM such that the position tracking issue is addressed. We will design the control signals $v_d$ and $v_q$ to guarantee the state variable $x_1$ tracks the desired signal $x_{\text{d}}$, and all the closed-loop signals are bounded.

To help with the designing of the control signals, some basic theories of fractional derivative and primary Lemmas are introduced.

The definition of Mittag–Leffler function can be expressed as

\[
E_{\beta,\gamma}(\rho) = \sum_{k=0}^{\infty} \frac{\rho^k}{\Gamma(\beta k + \gamma)}
\]

where $\rho$ denotes a complex number, $\beta$ and $\gamma$ are positive constants.
Lemma 1 [32]: If \( \beta \in (0, 2), \gamma \in R \) and \( \frac{\beta \pi}{2} < \gamma < \min(\pi, \beta \pi) \), then we have

\[
E_{\beta, \gamma}(\rho) \leq \frac{C}{1 + |\rho|}, \quad \gamma \leq |\text{arg}(\rho)| \leq \pi 
\]

where \( C > 0 \).

Lemma 2 [30]: If \( \beta \in (0, 1) \) and \( \frac{\beta \pi}{2} < \gamma \), then the following equation is obtained

\[
E_{\beta, \gamma}(\rho) = -\sum_{i=1}^{n} \frac{1}{\Gamma(\gamma - \beta)|\rho|^i} + o(1/|\rho|^{|i+1})
\]

when \( \rho \to \infty \), \( \gamma \leq |\text{arg}(\rho)| \leq \pi \) and \( n \geq 1 \).

Lemma 3 [3]: Consider a smooth function \( g(t) \), we have

\[
\frac{1}{2} \int_{0}^{\infty} g^T(t)g(t) \leq g^T(0)g(0) \quad \forall t \in [0, +\infty)
\]

Lemma 4 [38]: Let \( f(z) : R^n \to R \) be the unknown smooth nonlinear function. Then, a radial basis function (RBF) NN can approximate \( f(z) \) as \( f(z) = W^TS(z) \), where \( z \in \Omega \subset R^n \) denotes the input vector, \( n \) is the NN input dimension. \( W = [W_1, \ldots, W_m]^T \in R^m \) stands for the weight vector with \( m > 1 \) being the NN node number. \( S(z) = [s_1(z), \ldots, s_m(z)]^T \) is the basis function vector and \( s_i \) are selected as Gaussian functions which have the following form

\[
s_i(z) = \exp \left( \frac{-(z-h_i)^T(z-h_i)}{q_i^2} \right), \quad i = 1, 2, \ldots, m
\]

where \( q_i \) and \( h_i = [h_{i1}, h_{i2}, \ldots, h_{in}] \) are the width of \( s_i(z) \) and the center of receptive field, respectively. For any positive scalar \( \epsilon \), \( \epsilon \) is a radial basis function (RBF) NN \( W^TS(z) \) shown in Fig.1, which satisfies

\[
f(z) = W^TS(z) + \delta(z)
\]

where \( W \) stands for the ideal weight vector. The approximation error \( \delta(z) \) satisfies \( |\delta(z)| \leq \epsilon \). \( W \) is selected to minimize \( |\delta(z)| \) for \( z \in \Omega \).

\[
W := \arg \min_{W \in R^m} \left\{ \sup_{z \in \Omega} |f(z) - W^TS(z)| \right\}
\]

Assumption 1: The desired trajectory \( x_d \) and its fractional derivative \( D_0^\alpha x_d \) are considered to be known, smooth and bounded.

Assumption 2 [39]: There exists a positive number \( g_m \) such that \( 0 < g_m < g_{\nu_1} \leq 1 \).

III. COMMAND FILTERING-BASED NEURAL NETWORK CONTROL DESIGN

In this section, the command filtering-based NN control is addressed.

Step 1: For a given desired trajectory \( x_d \), the tracking error is defined as \( z_1 = x_1 - x_d \). Then, construct a new variable as follows

\[
v_1 = z_1 - \lambda_1
\]

where \( \lambda_1 \) is an error compensating signal which will be given later.

\[137814\]

FIGURE 1. Structure of RBF NN.

The fractional derivative of variable \( v_1 \) is calculated by

\[
C_0 D_t^\alpha v_1 = x_2 - C_0 D_t^\alpha x_d - C_0 D_t^\alpha \lambda_1
\]

Let the Lyapunov function candidate be chosen by

\[
V_1 = \frac{1}{2} v_1^2
\]

By using Lemma 3, its fractional derivative is achieved as

\[
C_0 D_t^\alpha V_1 \leq v_1 (x_2 - C_0 D_t^\alpha x_d - C_0 D_t^\alpha \lambda_1)
\]

Design the virtual signal \( a_1 \) as

\[
a_1 = -k_1 z_1 + C_0 D_t^\alpha x_d
\]

where \( k_1 \) is a positive constant.

In order to address the repeated derivative issue in backstepping for the virtual signals, the fractional-order command filter [7] is employed

\[
\begin{align*}
C_0 D_t^\alpha \varphi_1 &= w_1 \varphi_2 \\
C_0 D_t^\alpha \varphi_2 &= -\zeta w_1 \varphi_2 - w_n (v_1 - a_1)
\end{align*}
\]

where \( w_n > 0 \) and \( \zeta \in (0, 1] \). In addition, \( a_1 \) is the input of the command filter and \( a_{1c} \) is the output with \( a_{1c} = \varphi_1 \).

Remark 1: An adaptive backstepping control method is proposed in [4] for fractional-order nonlinear systems. However, this approach does not consider the "explosion of complexity" problem in classical backstepping. We introduce a fractional-order command filter to solve this problem.

Remark 2: Compared with the command filtered backstepping methods designed in [24], [33], [34], this paper mainly focuses on fractional-order system.

Construct the compensating signal \( \lambda_1 \) as follows

\[
C_0 D_t^\alpha \lambda_1 = -k_1 \lambda_1 + \lambda_2 + a_{1c} - a_1
\]

Substituting (21) and (23) into (20), one has

\[
C_0 D_t^\alpha V_1 \leq -k_1 v_1^2 + v_1 v_2
\]

where \( v_2 \) will be given later.
Step 2: Construct tracking error variable \( z_2 = x_2 - a_{1c} \), then the compensated variable is given by

\[
v_2 = z_2 - \lambda_2
\]

where \( \lambda_2 \) will be given later.

By differentiating \( v_2 \), it yields

\[
\mathcal{C}_0 D_t^\alpha v_2 = \frac{a_1}{T} x_3 + \frac{a_2}{J} x_2 x_4 - \frac{B}{J} x_2 - \frac{T_L}{J} - \frac{C}{0} D_t^\alpha a_{1c} - \frac{C}{0} D_t^\alpha \lambda_2
\]

Consider a Lyapunov function candidate \( V_2 \) as

\[
V_2 = \frac{1}{2} v_2^2 + V_1
\]

The time derivative of \( V_2 \) is

\[
\mathcal{C}_0 D_t^\alpha V_2 \leq v_2^2 \mathcal{C}_0 D_t^\alpha v_2 + \frac{C}{0} D_t^\alpha V_1 = v_2(x_3 + f_2(Z_2) - \frac{C}{0} D_t^\alpha \lambda_2) + \frac{C}{0} D_t^\alpha V_1
\]

where \( f_2(Z_2) = \frac{a_1}{T} x_3 + \frac{a_2}{J} x_2 x_4 - \frac{B}{J} x_2 - \frac{T_L}{J} - \frac{C}{0} D_t^\alpha a_{1c} \) and \( Z_2 = [x_1, x_2, x_3, x_4, q_2] \).

Remark 3: It can be noted that the function \( f_2 \) contains unknown parameters \( B, J \) and uncertain load torque \( T_L \). We cannot design the controller unless we introduce extra adaptive laws. To overcome this issue, a RBF NN can be employed to estimate the nonlinear function \( f_2 \).

Remark 4: Compared with the adaptive fuzzy command filter approach proposed in [40], our method does not require to know the information of the load torque \( T_L \).

By utilizing the approximation property of RBF NN and Lemma 4, there exists a RBF NN \( W_t^T S_t(Z_t) \), for a given \( \varepsilon_2 > 0 \) such that

\[
f_2(Z_2) = W_t^T S_t(Z_2) + \delta_2(Z_2)
\]

where \( \delta_2(Z_2) \) is the approximation error and it satisfies \( |\delta_2(Z_2)| \leq \varepsilon_2 \).

Then we can get the following formula by using Young’s inequality

\[
v_2 f_2 = v_2(W_t^T S_t + \delta)
\]

where \( \delta = 2W_t^T S_t + \frac{1}{2} (v_2^2 + l_2^2 + \varepsilon_2^2) \)

By taking (24), (28), (30), (31) and (32) into account, we have

\[
\dot{C}_0 D_t^\alpha V_2 \leq -\sum_{i=1}^{2} k_i v_i^2 + v_2 + \frac{1}{2l_2^2} v_2^2 (||W_2||^2 - \hat{\theta} S_2^2 S_2^T)
\]

where \( v_1 \) will be given later.

Step 3: Introduce tracking error variable \( z_3 = x_3 - a_{2c} \), then the compensated variable is constructed by

\[
\lambda_3 = \lambda_2 - \lambda_3 - a_{2c}
\]

where \( \lambda_3 \) will be given later.

Similarly, differentiating \( v_3 \) results in

\[
\mathcal{C}_0 D_t^\alpha v_3 = s_2 z_3 + b_2 x_2 x_4 + b_2 x_2 + b_4 u_q
\]

Select the Lyapunov function as

\[
V_3 = \frac{1}{2} v_3^2 + V_2
\]

Then the time derivative of \( V_3 \) is computed by

\[
\mathcal{C}_0 D_t^\alpha V_3 \leq v_3 s_2 \mathcal{C}_0 D_t^\alpha v_3 + \frac{C}{0} D_t^\alpha V_2 = v_3(s_2 u_q + f_3(Z_3) - \frac{C}{0} D_t^\alpha \lambda_3) + \frac{C}{0} D_t^\alpha V_3
\]

where \( f_3(Z_3) = s_2 z_3 + b_2 x_2 x_4 + b_2 x_2 + b_4 u_q \) and \( Z_3 = z_3 \).

Consider \( f_3(Z_3) = f_3(Z_3) + v_3 - \frac{C}{0} D_t^\alpha a_{2c} \). Similarly, there exists a RBF NN \( W_t^T S_t(Z_3) \) such that \( f_3(Z_3) = W_t^T S_t(Z_3) + \delta_2(Z_3) \) and \( |\delta_2(Z_3)| \leq \varepsilon_3 \). Then we can get

\[
v_3 f_3 = v_3(W_t^T S_t + \delta)
\]

where \( \delta = 2W_t^T S_t + \frac{1}{2} (v_3^2 + l_3^2 + \varepsilon_3^2) \)

From the beginning, one has

\[
u_q = (s_3 u_q + d(v_3))
\]

Construct the control law \( v_q \) as

\[
v_q = -k_3 z_3 - \frac{1}{2l_3^2} v_3 \bar{\theta} S_2 S_3
\]

Remark 5: An adaptive NN control scheme is proposed in [7], [13] for fractional-order PMSM. However, the input saturation problem remains unsolved. This paper introduces virtual control laws for fractional-order PMSM with input saturation.

Define the compensating signal \( \lambda_3 \) as

\[
\mathcal{C}_0 D_t^\alpha \lambda_3 = 0
\]

According to Young’s inequality, it can be derived that

\[
b_4 v_3 d(v_q) \leq \frac{1}{2} v_3^2 + \frac{1}{2l_3^2} \bar{\theta} D_3^2
\]

From the Assumption 2, we know that \( 0 < g_m < g_{v_q} \leq 1 \). Then there exists a constant \( b_q \) such that
where
\[
\sum_{i=1}^{2} k_i v_i^2 - (k_3 b_4 g_{v_{sat}} - 1)v_3^2 - \frac{1}{2l_3^2} v_3^2 b_q \hat{\theta} S_3^T S_3 + \frac{1}{2} v_3^3 + \frac{1}{2} b_4^2 D_d^2
\]
(43)

Substituting (38), (41) and (43) into (37), one has
\[
C_0 \frac{\partial v}{\partial t} V_3 \leq -\sum_{i=1}^{2} k_i v_i^2 - (k_3 b_4 g_{v_{sat}} - 1)v_3^2 + \frac{1}{2l_2^2} v_2^2 (||W_2||^2 - \hat{\theta} S_2^T S_2 + \frac{1}{2} \sum_{i=2}^{3} q_i^2 + \epsilon_i^2)
\]
\[
+ \frac{1}{2l_3^2} v_3^2 (\frac{1}{b_q} ||W_3||^2 - \hat{\theta}) S_3^T S_3 + \frac{1}{2} b_4^2 D_d^2
\]
(44)

**Step 4:** Define the variable \( v_4 = v_4 = z_4 = x_4 \). Then, let the Lyapunov function candidate be chosen by
\[
V_4 = \frac{1}{2} v_4^2 + V_3
\]
(45)
The time derivative of \( V_4 \) can be expressed by
\[
C_0 \frac{\partial v}{\partial t} V_4 \leq v_4 C_0 \frac{\partial v}{\partial t} v_4 + C_0 \frac{\partial v}{\partial t} V_3
= v_4 (f_4(Z_4) + c_3 u_d) + C_0 \frac{\partial v}{\partial t} V_3
\]
(46)
where \( f_4(Z_4) = c_1 v_4 + c_2 v_2 v_3 \) with \( Z_4 = Z_3 \). Similarly, there always exists a RBF NN \( W_i^T S_i(Z_4) \) such that \( f_4(Z_4) = W_i^T S_i(Z_4) + \delta_4(Z_4) \) and \( |\delta_4(Z_4)| \leq \epsilon_4 \). By exploiting Young’s inequality it yields
\[
v_4 f_4 = v_4 (W_i^T S_i + \delta_4)
\leq \frac{1}{2l_4^2} v_4^2 ||W_4||^2 S_i^T S_i + \frac{1}{2} (v_4^2 + l_4^2 + \epsilon_4^2)
\]
(47)
The real control law \( u_d \) has the following form
\[
u_d = g_{v_{sat}} v_d + d(v_d)
\]
(48)
According to Young’s inequality, it can be derived that
\[
c_3 v_4 d(v_d) \leq \frac{1}{2} v_4^3 + \frac{1}{2} c_2^2 D_d^2
\]
(49)
The control signal \( v_d \) can be designed as
\[
v_d = -k_4 z_4 - \frac{1}{2l_4^2} v_4 \hat{\theta} S_i^T S_i
\]
(50)
From \( 0 < g_{m2} < g_{v_{sat}} \leq 1 \), there exists a constant \( b_4 \) such that \( b_4 \leq c_3 g_{v_{sat}} \). According to (48), (49) and (50), it can be derived as
\[
c_3 v_4 u_d \leq -k_4 c_3 g_{v_{sat}} v_3^2 - \frac{1}{2l_4^2} v_3^2 b_q \hat{\theta} S_3^T S_3 + \frac{1}{2} v_3^3 + \frac{1}{2} c_2^2 D_d^2
\]
(51)

By taking (46), (47) and (51) into account, the time derivative of \( V_4 \) can be alternated as
\[
C_0 \frac{\partial v}{\partial t} V_4 \leq -\sum_{i=1}^{2} k_i v_i^2 - (k_3 b_4 g_{v_{sat}} - 1)v_3^2 - (k_4 c_3 g_{v_{sat}} - 1)v_3^2
\]
+ \frac{1}{2l_2^2} v_2^2 (||W_2||^2 - \hat{\theta} S_2^T S_2 + \frac{1}{2} \sum_{i=2}^{4} (l_i^2 + \epsilon_i^2))
\]
+ \frac{b_q}{2l_3^2} v_3^2 (\frac{1}{b_q} ||W_3||^2 - \hat{\theta}) S_3^T S_3 + \frac{1}{2} b_4^2 D_d^2
\]
+ \frac{b_4}{2l_4^2} v_4^2 (||W_4||^2 - \hat{\theta}) S_i^T S_i + \frac{1}{2} c_3^2 D_d^2
\]
(52)

**IV. STABILITY ANALYSIS**

**Theorem 1:** Consider the uncertain fractional-order PMSM system (4) with saturation nonlinearity input under the condition of Assumption 1 and 2, if the command filtering-based NN controllers (40), (50) combined with the fractional-order command filter (22), the adaptive updating law (56), the virtual control laws (21), (31) and the error compensation signals (23), (32), (41), then the convergence of the tracking error \( z_1 \) is achieved and all signals in the closed-loop system are bounded.

**Proof:** Choose the Lyapunov function \( V \) as
\[
V(t) = V_4 + \frac{b_4^2 \theta^2}{2r}
\]
(53)
where \( \hat{\theta} = \hat{\theta} - \theta \) and \( \theta = \max \{ \frac{1}{b} ||W_2||^2, \frac{1}{b} ||W_3||^2, \frac{1}{b} ||W_4||^2 \} \) with \( b = \min \{ 1, b_4, b_d \} \).

By differentiating \( V \), it follows that
\[
C_0 \frac{\partial v}{\partial t} V(t)
\leq -\sum_{i=1}^{2} k_i v_i^2 - (k_3 b_4 g_{v_{sat}} - 1)v_3^2 - (k_4 c_3 g_{v_{sat}} - 1)v_4^2
\]
+ \frac{1}{2l_2^2} v_2^2 (||W_2||^2 - \hat{\theta} S_2^T S_2 + \frac{1}{2} \sum_{i=2}^{4} (l_i^2 + \epsilon_i^2))
\]
+ \frac{b_q}{2l_3^2} v_3^2 (\frac{1}{b_q} ||W_3||^2 - \hat{\theta}) S_3^T S_3 + \frac{1}{2} b_4^2 D_d^2
\]
+ \frac{b_d}{2l_4^2} v_4^2 (||W_4||^2 - \hat{\theta}) S_i^T S_i + \frac{1}{2} c_3^2 D_d^2
\]
\]
(54)

According to the definition of \( \theta \), it can be derived that
\[
C_0 \frac{\partial v}{\partial t} V(t) \leq -\sum_{i=1}^{2} k_i v_i^2 - (k_3 b_4 g_{v_{sat}} - 1)v_3^2
\]
\[
- (k_4 c_3 g_{v_{sat}} - 1)v_4^2 + \frac{1}{2} \sum_{i=2}^{4} (l_i^2 + \epsilon_i^2)
\]
\[
+ \frac{b_q}{r} \frac{C_0 \frac{\partial v}{\partial t} \hat{\theta}}{\delta_3} - \sum_{i=2}^{4} \frac{r}{2l_i^2} v_i^2 S_i^T S_i
\]
\[
+ \frac{1}{2} b_4^2 D_d^2 + \frac{1}{2} c_3^2 D_d^2
\]
\]
(55)

Then the adaptive law can be constructed as
\[
C_0 \frac{\partial v}{\partial t} \hat{\theta} = \sum_{i=2}^{4} \frac{r}{2l_i^2} v_i^2 S_i^T S_i - m \hat{\theta}
\]
(56)
Substituting (56) into (55) it can be deduced that

\[
\frac{d}{dt} V(t) \leq -t^2 + (k_3 b_4 g_{vpa} - 1) V^2 + (k_4 c_3 g_{vpa} - 1) V^2 + b m \bar{g}^2 \theta \\
+ \frac{1}{2} \sum_{i=2}^{4} (I_i^2 + \varepsilon_i^2) + \frac{1}{2} b_3^2 D_q^2 + \frac{1}{2} c_3^2 D_d^2
\]  

(57)

Utilizing the inequality \(-\bar{g} \leq -\frac{1}{2} \bar{g}^2 + \frac{1}{2} \bar{g}^2\), (57) can be expressed as

\[
\frac{d}{dt} V(t) \leq -t^2 + (k_3 b_4 g_{vpa} - 1) V^2 + (k_4 c_3 g_{vpa} - 1) V^2 + b m \bar{g}^2 \theta \\
+ \frac{1}{2} \sum_{i=2}^{4} (I_i^2 + \varepsilon_i^2) + \frac{1}{2} b_3^2 D_q^2 + \frac{1}{2} c_3^2 D_d^2
\]  

(58)

Select the control gains \(k_1, k_2, k_3\) and \(k_4\) such that \(k_1 > 0, k_2 > 0, k_3 b_4 g_{vpa} - 1 > 0\) and \(k_4 c_3 g_{vpa} - 1 > 0\). From (58), we also have

\[
\frac{d}{dt} V(t) \leq -a_0 V(t) + b_0
\]  

(59)

where \(a_0 = \min\{2k_1, 2k_2, 2(k_3 b_4 g_{vpa} - 1), 2(k_4 c_3 g_{vpa} - 1), b_0 \} = \frac{1}{2} \sum_{i=2}^{4} (I_i^2 + \varepsilon_i^2) + \frac{1}{2} b_3^2 D_q^2 + \frac{1}{2} c_3^2 D_d^2.

The inequality (59) implies that

\[
\frac{d}{dt} V(t) + v(t) = -a_0 V(t) + b_0
\]  

(60)

where \(v(t) \geq 0\). By using Laplace transformation, one has

\[
V(s) = \frac{s^{-1}}{s^{a_0} + a_0} V(t) + \frac{b_0}{s^{a_0} + a_0} \cdot \frac{s^{a_0} + a_0}{s^{a_0} + a_0} - \frac{a_0}{s^{a_0} + a_0} \cdot \frac{s^{a_0} + a_0}{s^{a_0} + a_0}
\]  

\[
= \frac{s^{-1}}{s^{a_0} + a_0} V(t) + \frac{b_0}{s^{a_0} + a_0} \cdot \frac{s^{a_0} + a_0}{s^{a_0} + a_0} - \frac{a_0}{s^{a_0} + a_0} \cdot \frac{s^{a_0} + a_0}{s^{a_0} + a_0}
\]  

(61)

where \(V(s)\) and \(v(t)\) denote the Laplace transforms of \(V(t)\) and \(v(t)\), respectively. The solution of (61) can be derived as

\[
V(t) = V(0)E_{a,1}(-a_0 t^a) + b_0 t^a E_{a,1+a}(a_0 t^a) - v(t) * t^{-1} E_{a,0}(-a_0 t^a)
\]  

(62)

where \(*\) means convolution. It should be noted that the functions \(v(t)\) and \(E_{a,0}(a_0 t^a)\) are non-negative, so we have

\[
|V(t)| \leq |V(0)|E_{a,1}(-a_0 t^a) + b_0 t^a E_{a,1+a}(a_0 t^a)
\]  

(63)

According to Lemma 1, there exists a constant \(C > 0\) such that

\[
|E_{a,1}(-a_0 t^a)| \leq \frac{C}{1 + a_0 t^a}
\]  

(64)

Then it can be computed that

\[
\lim_{t \to \infty} |V(0)|E_{a,1}(-a_0 t^a) = 0
\]  

(65)

Therefore, for a given \(\sigma > 0\), there is a constant \(t_1 > 0\) such that \(t > t_1\), it means that

\[
|V(0)|E_{a,1}(-a_0 t^a) < \frac{\sigma}{3}
\]  

(66)

By utilizing Lemma 2, we obtain

\[
E_{a,a+1}(-a_0 t^a) = \frac{1}{\Gamma(1/a_0 t^a)} + o\left(\frac{1}{|a_0 t^a|^2}\right)
\]  

(67)

According to the discussion in [30], there exists a constant \(t_2 > 0\) such that

\[
b_0 t^a E_{a,a+1}(-a_0 t^a) \leq \frac{b_0}{a_0} + \frac{\sigma}{3}
\]  

(68)

for \(t > t_2\).

By choosing control gains to satisfy \(b_0/a_0 \leq \sigma/3\), one has

\[
|V(t)| < \sigma
\]  

(69)

It concludes from (69) that \(V(t)\) is bounded by \(\sigma\). Therefore, the compensated variables \(v_i\) for \(i = 1, \cdots, 4\) can be bounded. Then the tracking errors \(z_i\) will converge to a bounded region if the boundedness \(\lambda_i\) is achieved. Let us design the Lyapunov function as

\[
V_{\lambda} = \frac{3}{2} \sum_{i=1}^{n} |z_i|^2
\]  

(70)

Then we have

\[
\frac{d}{dt} V_{\lambda} \leq \text{sign}(\lambda_{1}) \frac{d}{dt} \lambda_{1} + \text{sign}(\lambda_{2}) \frac{d}{dt} \lambda_{2} + \text{sign}(\lambda_{3}) \frac{d}{dt} \lambda_{3}
\]

\[
= \text{sign}(\lambda_{1})(-k_1 \lambda_1 + k_2 + (a_{1c} - a_1)) + \text{sign}(\lambda_{2})(-k_2 \lambda_2 - \lambda_1 + (a_{2c} - a_2))
\]

\[
\leq -(k_1 - 1)|\lambda_1| - (k_2 - 1)|\lambda_2| + (a_{1c} - a_1)
\]

\[
+ (a_{2c} - a_2)
\]  

(71)

According to [7], we know that \(|a_{ic} - c_i| \leq \gamma_i\) for \(i = 1, 2\), then (71) can be rewritten as

\[
\frac{d}{dt} V_{\lambda} \leq -a_1 V_{\lambda} + b_\lambda
\]  

(72)

where \(a_1 = \min\{k_1 - 1, k_2 - 1\}\) and \(b_\lambda = \max(\gamma_1, \gamma_2)\). Similar to (59), we can obtain that \(\lambda_i\) are bounded, which means the tracking errors \(z_i\) are also bounded. The proof of Theorem 1 is completed.

The technical details regarding the tuning approaches for the control parameters are summarized as follows

1. The value of the tracking error is directly determined by the control gains. Increasing the values of the control gains \(k_1, k_2, k_3\) and \(k_4\) can improve the convergence rate. But the large gains may cause oscillation of the system. To avoid the oscillation problem, these values are not too large: \(k_i \leq 150, i = 1, \cdots, 4\).

2. The adaptive parameters \(r\) and \(m\) can influence the tracking performance and system stability. \(r\) is a role of weigh between the control action and tracking performance. But it may create the opposite effect if the parameter \(r\) is too small. The parameter \(m\) can avoid the parameter drift caused by
measurement noise. But it will influence the adaption rate if the value is too large, its value is less than 0.5. Trial and error method is generally utilized to select $l_i$, $i = 2, 3, 4$, the value of $l_i$ is not too large or too small. That is, $0.01 \leq l_i \leq 2$.

3. Increasing the value of the parameter $w_n$ can reduce the filtering error, but too large value will increase the computational load. Therefore, $w_n$ should be less than 10000. The range of the value for $\zeta$ is usually $0 < \zeta \leq 1$.

V. SIMULATION RESULTS

Simulation results are given to demonstrate the availability and advantage of the designed controller in this section. The MATLAB software is utilized to carry out the simulations, and the fractional differential equations are solved under a step time of 0.005 by using the predictor-corrector method [3]. The flow chart for the closed-loop system architecture is shown in Fig.2.

A. PERFORMANCE ANALYSIS

The parameters of the fractional-order PMSM system are selected as $R = 0.72, L_d = 0.00324, L_q = 0.00262, \psi_r = 0.1354$ and $J = 0.00297$. The desired trajectory $x_d$ is $x_d = \sin(0.3t)$. The initial conditions of state variables are $x_1(0) = 0.5, x_2(0) = 0.2, x_3(0) = 0.1, x_4(0) = 0.1$ and the
load torque is taken as

\[
T_L = \begin{cases} 
1.5Nm & 0 \leq t \leq 20 \\
3Nm & t > 20
\end{cases}
\] (73)

The RBF NN \( W_2^T S_2(Z_2) \), \( W_3^T S_3(Z_3) \) and \( W_4^T S_4(Z_4) \), whose width \( q_i \) equals 1, has seven nodes, and the centers are evenly distributed in the region of \([-3, 3]\). In this case, choose the control parameters as

\[
k_1 = 100, \quad k_2 = 20, \quad k_3 = 15, \quad k_4 = 15 \\
r = 1, \quad m = 0.02, \quad l_2 = l_3 = l_4 = 0.5 \\
w_n = 6000, \quad \zeta = 0.5
\]

The saturation inputs are described by

\[
\begin{align*}
u_q &= \begin{cases} 
-15, & v_q \leq -15 \\
 v_q, & -15 < v_q < 20 \\
20, & v_q \geq 20
\end{cases} \\
u_d &= \begin{cases} 
-1, & v_d \leq -1 \\
 v_d, & -1 < v_d < 3 \\
3, & v_d \geq 3
\end{cases}
\] (74)
FIGURE 13. Tracking error.

FIGURE 14. \( x_1 \) and \( x_d \).

FIGURE 15. Tracking error.

FIGURE 16. Performance contrast between ADSC and the proposed approach. (a) ADSC. (b) The proposed (c) Tracking error comparisons.

Figs.3-8 depict the simulation results in the case of \( \alpha = 0.97 \). From Fig.3 and Fig.4, we can see that the designed controller tracks the desired trajectory accurately. Although there exists load disturbance, the tracking error falls into a small region with little time and it is less than \( 3 \times 10^{-3} \) (Rad).

Figs.5-6 are the performances of \( d - q \) axis currents. The curves of the control inputs are shown in Fig.7 and Fig.8, it can be obviously seen that the control signals \( u_q \) and \( u_d \) are bounded into the given intervals. Therefore, a conclusion is given that our approach can handle the tracking issue for the uncertain input-saturated fractional-order PMSM system.

The performances of the control scheme for PMSM under different orders \( \alpha \) are displayed in Figs.9-11. As the order decreases, the tracking performance also improves. Figs.12-13 are the simulation results when the system is under control at time \( t = 5s \). From the simulation, it is clear to know that the utilized controller is highly efficient for controlling the fractional-order PMSM.

B. ROBUSTNESS ANALYSIS

The parameter \( R \) is vulnerable to be affected by the operating environment. Fig.14 and Fig.15 are the results of trajectory...
the output side, the simulation results are shown in Fig. 17. Consider the influence of measurement noise, a disturbance of random noise with variance being equal to 0.01 is added to the signal. The form of the first-order filter is given as

$$\tau_i \frac{d}{dt} x_i + a_{ic} = a_i, \quad a_{ic}(0) = a_i(0)$$

where \(\tau_i\) stand for time constants for \(i = 1, 2\).

Fig. 16(a), (b) and (c) shows the tracking performance contrast between ADSC and our approach in the case of \(\alpha = 0.95, x_1(0) = x_2(0) = x_3(0) = x_4(0) = 0.1\). Considering the influence of measurement noise, a disturbance of random noise with variance being equal to 0.01 is added to the output side, the simulations results are shown in Fig. 17.

To illustrate the benefits of our method, a result of the simulation presents a good tracking performance. The availability of our approach are validated by simulation results.

**C. METHOD CONTRAST**

To demonstrate the closed-loop system has good robustness.

**VI. CONCLUSION**

Table 1 shows the comparison result. Apparently, our method can obtain better performance with respect to ADSC.

| Method                | Tracking error indexes (absolute value) between two control methods (unit: rad) |
|-----------------------|----------------------------------------------------------------------------------|
| without disturbance   | ADSC: max value: 0.018, average value: 0.013 | The proposed: max value: 0.002, average value: 0.0015 |
| with load disturbance | ADSC: max value: 0.023, average value: 0.023 | The proposed: max value: 0.003, average value: 0.0023 |
| with noise disturbance| ADSC: max value: 0.08, average value: 0.04 | The proposed: max value: 0.012, average value: 0.008 |

Table 1 shows the comparison result. Apparently, our method can obtain better performance with respect to ADSC.

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