Collusion Resistant Federated Learning with Oblivious Distributed Differential Privacy

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Abstract

Privacy-preserving federated learning enables a population of distributed clients to jointly learn a shared model while keeping client training data private, even from an untrusted server. Prior works do not provide efficient solutions that protect against collusion attacks in which parties collaborate to expose an honest client’s model parameters. We present an efficient mechanism based on oblivious distributed differential privacy that is the first to protect against such client collusion, including the “Sybil” attack in which a server preferentially selects compromised devices or simulates fake devices. We leverage the novel privacy mechanism to construct a secure federated learning protocol and prove the security of that protocol. We conclude with empirical analysis of the protocol’s execution speed, learning accuracy, and privacy performance on two data sets within a realistic simulation of 5,000 distributed network clients.

1 Introduction

Modern practitioners of machine learning often need to train models from large data sets distributed across many devices. In the past, such data would be centralized for analysis, but the practice has given rise to serious concerns around user privacy, lack of permission to transfer data, and far-reaching consequences when centralized data stores are breached.

Federated learning is a recent technique that addresses these concerns by training on each local data segment individually, then transmitting and combining only the resulting model parameters. [Bonawitz et al., 2017; Kairouz et al., 2019] A typical approach is: The trusted server selects some users to train a new model on their local data, starting from the most recent shared model. Each user sends their local model weights to the server, which computes an average-weight shared model. The new shared model is sent to all users, and the process repeats. In some cases, however, individual user privacy can still be compromised by using the trained model to infer certain details of the training data set. [Shokri et al., 2017; Nasr et al., 2018].

Two key approaches have been proposed to address this problem. The first is differential privacy, which perturbs values to guarantee statistical indistinguishability for individual inputs. [Dwork et al., 2006a] This can be applied to federated learning by having each client modify its local model weights by adding randomly-generated values (potentially reducing model accuracy), with the result that a party obtaining the transmitted weights will still have uncertainty over the original weights. The second approach, which does not compromise accuracy, is secure multi-party computation (MPC). [Goldreich et al., 1987] An MPC protocol, which allows parties to collaboratively compute a common function of interest without revealing their private inputs, is considered secure if the parties learn the computational output and nothing else.

We build on a recent line of research that combines differential privacy and MPC to produce a secure federated learning protocol. [Bonawitz et al., 2017; Jayaraman et al., 2018] These prior works provide strong protection against undesired inference by the server, but the collusion of enough clients can reveal the noisy weights of an honest client, and the scale of that noise is limited by the need for an accurate model.

We propose a novel, efficient mechanism that protects against any attempt to undermine differential privacy by collusion of $n-1$ out of $n$ total clients. Unlike prior works, we offer a protocol where the noise for each party is added in an oblivious way. Obliviousness can be achieved by running the noise generation inside the MPC, but such solutions are based on heavy cryptography machinery involving a significant amount of public key operations or incur increased communication complexity. [Jayaraman et al., 2018; Champion et al., 2019] In this work we focus on the concretely efficient aggregation protocol of Bonawitz et al. without dropout parties which does not involve any public key operations in the learning phase. [Bonawitz et al., 2017] We therefore provide the first practical protection against $n-1$ attacks by constructing an efficient oblivious distributed differentially private aggregation protocol.

2 Background

2.1 Secure Multiparty Computation

Consider $n$ parties $P_1,...,P_n$ that hold private inputs $x_1,...,x_n$ and wish to compute some arbitrary function $(y_1,...,y_n) = f(x_1,...,x_n)$, where the output of $P_i$ is $y_i$. Secure Multi-Party Computation (MPC) enables the parties to compute the function using an interactive protocol such that
As is common in the federated learning setting, we opt for a randomized mechanism which preserves \( \epsilon \)-differential privacy [Dwork et al., 2006b]. A randomized mechanism \( A \) preserves \( \epsilon \)-differential privacy (\( \epsilon \)-DP) if for any two neighboring datasets \( D_1, D_2 \) that differ by one element, and for all subsets of possible answers \( S \subseteq \text{Range}(A) \), \( \Pr \left[ A(D_1) \in S \right] \leq e^\epsilon \Pr \left[ A(D_2) \in S \right] \).

**Definition 1.** (\( \epsilon \)-differential privacy [Dwork et al., 2006b]) A randomized mechanism \( A \) preserves \( \epsilon \)-differential privacy (\( \epsilon \)-DP) if for any two neighboring datasets \( D_1, D_2 \) that differ by one element, and for all subsets of possible answers \( S \subseteq \text{Range}(A) \), \( \Pr \left[ A(D_1) \in S \right] \leq e^\epsilon \Pr \left[ A(D_2) \in S \right] \).

2.2 Differential Privacy

Differential privacy states that if there are two databases that differ by only one element, they are statistically indistinguishable from each other. In this work we use the Laplacian mechanism which preserves \( \epsilon \)-differential privacy [Dwork et al., 2006a]. (See Appendices A.2, A.3, A.4 for further detail.)

Our approach combines secure multi-party aggregation with oblivious distributed differential privacy to better secure federated learning against \( n - 1 \) collusion attacks. In this work, we consider logistic regression as the local learning method, and each client update includes the weights of that logistic regression. The server receives the weights from all clients at each iteration and computes a new global model using the average of the client updates for each weight. Recall from the Introduction the literature demonstrating that private client data can be inferred from the trained model weights, which is clearly undesirable. The general task, then, is to secure each client’s locally trained model weights against discovery while still learning an accurate shared model. We note that the collusion problem can be solved using generic MPC, but such generic solutions are impractical due to computational inefficiency. Our contribution is a practical and efficient solution to this problem using lightweight cryptographic tools.

3.1 Eliminating weight leakage

We use a secure weighted average protocol running across \( n \) clients to hide each client’s model weights from the server where each weight is sent to the server encrypted/masked. The underlying secure aggregation protocol for online/non-drop-out clients we use appeared in the work of Bonawitz et al. [Bonawitz et al., 2017], in which clients send individual updates to the server in an encrypted manner.

3.2 Eliminating weighted average leakage

Using the secure aggregation protocol of [Bonawitz et al., 2017] hides all information about client weights from the server, but the final shared model can still reveal information about individual client weights and subsequently a client’s local data set. Given the output which is the average of each model weight, \( n - 1 \) clients working together can remove their weights to discover the exact model weights of the remaining “honest” client.

Previous approaches augment the secure aggregation protocol with differential privacy to mitigate the impact of client data exposure. Under these protocols, each client independently generates and adds random noise to each model weight prior to transmission, so even in the case of \( n - 1 \) client collusion, only “noisy” weights can be recovered. This is a definite improvement, but unlike MPC it is a lossy one, and the scale of the added noise is limited by a trade-off against model accuracy.

We introduce a novel and efficient oblivious distributed differentially private mechanism. In prior works, each client...
Picks its own local noise. By contrast, we offer a protocol where the noise for each party is added in an oblivious way. More specifically: For each weight, each client receives a tuple of encrypted noise terms from each other client and adds only a subset of them. Thus, a party $P$ does not know the clearest noise added to its weight and the other parties do not know which noise term is chosen by $P$.

We show that the information leakage on the honest client’s weights after the collusion attack is smaller than previous approaches. Our task is to enable the parties to calculate the sum of their inputs (i.e., $W = \sum_{i=1}^{n} w_i$), while ensuring privacy for an honest party in the presence of a collusion attack given $W$. In prior works if $n-1$ parties collude, they subtract their weights and noise terms from $W$ and then the final noise remaining in the transmitted weight of the honest party $w_h$ is a single value chosen by the honest party. In our case, if $n-1$ parties collaborate then the final noise remaining in the transmitted weight of the honest party $w_h$ is $n-1$ times larger, because the corrupted parties cannot subtract their noise terms.

At a high level, in our scheme, each client sends two encrypted noisy terms (permuted and randomized by the server) per model weight to the other clients, but each receiving client chooses only one of the two to add to weight. Thus even if parties collude they cannot subtract a significant number of noise terms since they do not know which noise terms the honest client chose.

4 Secure Weighted Average Protocol

4.1 Our Protocol

We formally describe our weighted average protocol $\Pi_{\text{PPFL}}$, depicted in Protocol 1, for secure logistic regression performed by a set of clients $(P_1, \ldots, P_n)$ and a server $S$.

During setup, every pair of parties $P_i$ and $P_j$ will share some common randomness $r_{i,j}$. In the online weighted average phase, client $P_i$ sends its weights masked with these common random strings, adding all $r_{i,j}$ for $j > i$ and subtracting all $r_{i,k}$ for $k < i$. That is, $P_i$ sends to server $S$ the following message for its data $w_i$: $y_i := (w_i + \sum_{j=i+1}^{n} r_{i,j} - \sum_{k=1}^{i-1} r_{k,i}) \mod p$. Each weight received by the server is masked by $n-1$ large random numbers $r$, so it cannot accurately reconstruct any client’s true model weight $w_i$. Because the total randomness applied to the weights sums to zero once the server computes $W$ in round 2 of $\Pi_{\text{PPFL}}$, the averaged final model $\bar{W}$ will be identical to one calculated without security.

To establish common randomness $r$, each pair of parties run the standard Diffie-Hellman Key exchange protocol from the literature [Diffie and Hellman, 1976] communicating via the
We prove that our protocol protects the privacy of honest users. For a detailed explanation of how the parties locally update their $r$ masks to be used in the next weight and next iteration of the protocol, see Section C of the Appendix. We generate the Laplacian noise in a distributed way by the use of gamma distributions $\mathcal{G}$ given that the Laplace distribution $\mathcal{L}$ can be constructed as the sum of differences of i.i.d. gamma distributions. To run machine learning algorithms and the DP mechanism which computes on rational values, we use field elements in a finite field $\mathbb{Z}_q$ to represent the fixed-point values. Concretely, for a fixed-point value $\bar{x}$ with $k$ bits in the integer part and $p$ bits in the decimal part, we use the field element $x := 2^l \cdot \bar{x} \mod q$ in $\mathbb{Z}_q$ to represent it.

\section{Security of our Protocol}

We prove that our protocol protects the privacy of honest users in the semi-honest setting given the topology in Section 2.4. In particular we show the following theorem.

\textbf{Theorem 1.} Suppose $n$ clients $P_1, \ldots, P_n$ each hold private input $w_i$, and they wish to rely on a server $S$ to compute the sum $f(w_1, \ldots, w_n) = \sum_i w_i$. There exists a protocol $\Pi_{DPFL}$, returning the sum which does not leak any information about the other clients' inputs except what can be inferred from the sum and offers collusion-privacy against a coalition of up to $t \leq n - 1$ clients.

In Appendix B.1, we further formalize and prove our theorem, and consider security against $t \leq n - 1$ semi-honest clients and a curious server, and against $t$ malicious users.

Next we argue that the error term on the honest client’s inputs after the collusion attack of $t$ parties is larger than previous approaches. For this, we require the following additional property. Consider the case of $n - 1$ collusion; we define collusion privacy as follows:

\textbf{Collusion-Privacy:} An $n$-party protocol provides Collusion-Privacy, for an aggregation function $f$ and a probability distribution $\mathcal{D}$, if any adversary, who controls all parties except client $P_h$, learns no more than the honest party’s values $w_h + \eta$ where $\eta \leftarrow \mathcal{D}$ and $f(w_1, \ldots, w_n)$.

In prior works if $n - 1$ parties collude then the final noise left in the weight of the honest party $w_h$ is a single value from $\mathcal{D}$. In our case, if $n - 1$ parties collude then the final noise left in the weight of the honest party $w_h$ is $n - 1$ times larger than $D$ since the corrupted parties cannot subtract their exact noise terms.

To measure the error, we quantify the difference between $f(D)$ and its perturbed value $\hat{f}(D)$ which is the error introduced by the differential private mechanism of the secure aggregation protocol.

\textbf{Definition 2.} (Error function) Let $D \in \mathcal{D}$, $f : \mathcal{D} \rightarrow \mathbb{R}$, and let $\delta = \frac{|f(D) - \hat{f}(D)|}{\Pr[D]}$ (i.e., the value of the error). The error function is defined as $\sigma = \mathbb{E}(\delta)$. The expectation is taken on the randomness of $\hat{f}(D)$. The standard deviation of the error is $\sigma = \sqrt{\text{Var}(\delta)}$.

After the execution of Protocol 1, parties receive the noisy sum of their inputs, i.e., $W = \sum_{i=1}^n w_i$. In prior non-oblivious works if $n - 1$ parties collaborate and remove their weights from $w$ then the final noise added to the weight of the honest party $w_h$ is a value from $\mathcal{L}(\lambda)$, and hence, the error is $\mu = \frac{1}{|W|+1} \mathbb{E}[\mathcal{L}(\lambda)] = \frac{\lambda}{|W|+1}$.

In our oblivious case, if $n - 1$ parties collaborate then the final noise added to the weight of the honest party $w_h$ is $n - 1$ times larger than $\mathcal{L}(\lambda)$, and hence, the error is $\mu = \frac{\lambda}{|W|+1} \mathbb{E}[\sum_{i=1}^{n-1} \mathcal{L}(\lambda)] = \frac{(n-1)^2 \lambda}{|W|+1}$.

However, in practice even if the parties cannot subtract their exact noise terms they can still try to subtract the average of the noise terms, or one of the two noise terms, or use the leakage $L$ to reduce the amount of error. In our protocol we consider the leakage $L$ learned from the difference of the noise terms $\eta_0, \eta_1$. Note that this leakage does not affect the error function given later in Definition 2. In Section 5.4 and Figure 5 we empirically show that such an attack is little better than the attack of subtracting nothing.

Note that the output of the aggregation protocol, $W + \eta$, is generated such that $\eta$ follows exactly the same distribution in both non-oblivious and oblivious cases, but the noise left after an $n - 1$ attack against the oblivious case is higher. For further discussion, see Appendix B.2 on collusion privacy.

\section{5 Experiments}

We empirically evaluated our protocol using ABIDES, an open source simulation platform originally designed for financial markets [Byrd et al., 2020] and later adapted for federated learning [Byrd and Polychroniadou, 2020].

Following the agent-based approach of these prior works, we implemented our oblivious protocol for 5,000 distributed clients and analyzed the timing, accuracy, and privacy of the empirical results.

\subsection{5.1 Experimental Dataset and Method}

We evaluated our protocol’s performance using the Adult Census Income dataset [Dua and Graff, 2017], which provides 14 input features such as age, marital status, and occupation, that can be used to predict a categorical output variable identifying whether (True) or not (False) an individual earns over $50K USD per year. We used a preprocessed version of the dataset from Jayaraman et al. following the method of Chaudhuri et al. which transformed each categorical variable into a series of binary features, then normalized both features and examples, resulting in 104 features for consideration. [Jayaraman et al., 2018; Chaudhuri et al., 2011] We added a constant intercept feature to permit greater flexibility in the regression. Of the 45,222 records in our cleaned data set, there were 11,208 positive examples (about 25%), representing a moderately unbalanced dataset.

The dataset was loaded only once per complete simulation of the protocol, after which a randomized train-test split (75% vs 25%) was taken. Once per round of federated learning, each client randomly selected 200 rows from the training data as its “local” data. The holdout test data was the same for all clients, and no client ever trained on it. All clients implemented Protocol 1 as previously described.
5.2 Protocol Timing Results

The ABIDES simulation permits construction of an arbitrary network graph with defined pairwise connectivity, minimum latency, and parameters for randomly selected “jitter” with nanosecond resolution. It captures the real elapsed runtime of each client activity and appropriately delays both sent messages and the earliest time at which a client may act again. Using these features, we have estimated the temporal load of Protocol 1. The mean time required to run the protocol simulation on a single Intel Xeon X5650 CPU core (2.6GHz) ranged from 32 seconds to 12 hours for 100 to 5,000 parties.

Figure 1 summarizes the time spent performing each section of our protocol on the adult census income data set: Diffie-Hellman Setup one time per client, Encryption of the weights and local model, Training per client per protocol iteration, and Server aggregation time per protocol iteration. Figure 2 shows the estimated time required to run the full protocol (not the simulation) for three different network graphs: Graph 1 places all participants around New York City, Graph 2 places the server in New York City and clients around London, Graph 3 places the server in New York City and clients all over the world. Tabular data is presented in Appendix D.2.

Figure 4a shows the MCC of our protocol’s final shared model predictions against the correct values for a range of $\epsilon$. As expected, smaller $\epsilon$ harms the accuracy of the learned model. Thus there is a dynamic lower bound, varying with population size, on useful values of $\epsilon$. For example when considering out of sample MCC($n$), with $n$ being the client population size, in our experiments with $\epsilon = 1e - 5$: MCC(100) = 0.005, MCC(200) = 0.254, and MCC(500) = 0.423. For all evaluated client population sizes (50 to 5,000), models trained under Protocol 1 with $\epsilon \geq 5e - 4$ had similar accuracy to unsecured federated learning. Figure 4b shows the impact $\epsilon$ can have on each round of

5.3 Protocol Accuracy Results

The secure multi-party aggregation component of Protocol 1 is lossless, because the MPC encryption elements sum to zero in each shared model. Differential privacy introduces shared model accuracy loss inversely proportional to the $\epsilon$ privacy loss parameter. Smaller selections of $\epsilon$ result in more uncertainty about a client’s local weights when other parties collude to reveal them, but increasingly confound learning. For example, in our protocol experiments with 200 clients, final model accuracy worsens dramatically once $\epsilon < 5e - 5$.

Matthews Correlation Coefficient: Because of the significant (3:1) class imbalance in our data, we assess accuracy using the Matthews Correlation Coefficient (MCC) [Matthews, 1975], a contingency method of calculating the Pearson product-moment correlation coefficient (with the same interpretation), that is appropriate for imbalanced classification problems. [Baldi et al., 2000; Pearson, 1895; Powers, 2011; Evans, 1996] (See Appendix D.1 for more detail.)
federated learning: with $\epsilon = 5e - 4$ or $\epsilon = 5e - 7$, population size does not matter because either all sizes succeed at learning or none do; but with $\epsilon = 5e - 6$, varying client population sizes learn at vastly different rates.

In our simulated network environment, with 1000 clients implementing the described protocol for federated logistic regression using privacy loss parameter $\epsilon = 5e - 4$, we found an out of sample final iteration relative accuracy loss (MSE versus learning in the clear) of $1.1e - 6$ and an out of sample final iteration relative MCC loss of 0.0018.

5.4 Adversarial Data Recovery

Prior works like Bonawitz et al. discuss attacks from a “snooping” server which attempts to infer the unencrypted weights of a particular client. [Bonawitz et al., 2017] The attacks fail since the server does not have the common random values $r$. 

**Collusion attack:** We consider the $n - 1$ attack, in which all other clients conspire to recover the unencrypted model weights of a single “honest” client. Let the honest client be $h$ and the set of $n - 1$ colluding clients be $C$. For a single model weight, let $F = w_h + W_C + T_h + T_C$ be the output of the aggregation protocol at the end of each iteration, where $w_h$ is the honest party’s original weight, $T_h$ is the honest party’s noise sum, $W_C = \sum_{c \in C} w_c$, $T_C = \sum_{c \in C} T_c$. Note that the sum of the randomness $r,s$ is removed by design from the output when the computation is performed, so MPC cannot defend against this type of attack.

Under prior non-oblivious protocols in which each party generates and adds its own noise locally, the colluding parties know $W_C$ and $T_C$ and can therefore recover the honest party’s noisy weights $W_h + T_h$. Under our oblivious protocol, the noise which cannot be subtracted in $F$ has a more dispersed distribution that cannot be much narrowed by the colluding parties since $h$ will have received from each colluding party $c$ a choice of two difference of gamma privacy noises $\hat{\gamma}_ch$ and $\hat{\gamma}_1c$ and $c$ will not know which was selected by $h$. Moreover, since the server permutes and randomizes the encrypted noise terms, $T_C$ is also not precisely known to the colluding parties. For an extreme Sybil attack, the server will have to run a secure shuffle protocol which we have not implemented in this version (see discussion in Appendix B.4).

We empirically illustrate the dramatic improvement in privacy against an $n - 1$ distributed attack by considering five cases. In each case, the colluding parties attempt to recover $W_h$ and always remove $W_C$. In the Non-Oblivious case followed by prior works, the corrupted parties can accurately remove $T_C$. In the other four cases, Protocol 1 is attacked, and the corrupted parties must decide how to deal with the unknown noise choices by other parties: under Naive they do nothing additional (i.e., they do not remove any noise terms); under Random each corrupted party $c$ removes either $\hat{\gamma}_0c$ or $\hat{\gamma}_1c$ at random; and under Diff and Mean each corrupted party $c$ removes the difference or mean of $\hat{\gamma}_0c$ and $\hat{\gamma}_1c$, respectively.

We consider the recovery attempt across 1,000 full iterations of Protocol 1 with 100 clients participating. At the end of every iteration, 99 clients share information in an attempt to recover the unencrypted model weights of the one honest client. Privacy loss parameter $\epsilon = 5e - 4$ was selected because it did not cause significant shared model accuracy loss for any tested number of parties.

Figure 5 shows a density plot of the honest party’s actual model weight versus the collaborators’ estimate of that weight. Figure 3 summarizes the distribution of the difference between estimated and actual weights for each attack scenario. The $n - 1$ attack is successful ($r^2 = 0.894$) against the prior non-oblivious protocol, but not successful ($r^2 = 0.164$ or worse) against our new oblivious protocol. (For additional discussion of the attacks, see Appendix D.3.)

To further confirm our empirical results, we present a similar attack against the protocol while learning a credit card fraud data set in Appendix D.4 and Figures D.1 and D.2.

6 Conclusion

We presented an efficient mechanism for oblivious distributed differential privacy that is the first to secure against collusion attacks on the clients’ model parameters, and leveraged that mechanism to construct a secure federated learning protocol. We also detailed the protocol and proved its security.

To empirically evaluate the protocol in a practical setting, we implemented it for a common data set with 5,000 parties in an open source simulation that has been adapted to the domain of privacy-preserving federated learning, and estimated its accuracy and running time for various client counts and values of the $\epsilon$ privacy loss parameter. We also conducted an $n - 1$ attack and showed that it is effective against prior non-oblivious protocols, but not against our new protocol.

We have left the case where clients drop off during the protocol as future work. Our mechanism is therefore well suited to cross-silo federated learning applications where clients are different organizations (e.g. medical or financial) or geodistributed datacenters, as opposed to mobile or IoT devices which can possibly go offline.
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