Nonlocal $SU(5)$ GUT

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Abstract

We show that in nonlocal generalization of standard nonsupersymmetric $SU(5)$ GUT it is possible to solve the problems with the proton lifetime and the Weinberg angle without introduction of additional particles in the spectrum of the theory. Nonlocal scale $\Lambda$ responsible for ultraviolet cutoff coincides (up to some factor) with GUT scale $M_{GUT}$. We find that in the simplest nonlocal modification of the $SU(5)$ model $M_{GUT} \approx 3 \cdot 10^{16}$ GeV. In general case the value of $M_{GUT}$ is an arbitrary and the most interesting option $M_{GUT} = O(M_{PL})$ could be realized.
The remarkable success of the supersymmetric $SU(5)$ grand unified theory (GUT) was considered by many physicists as the first hint in favour of the existence of low energy broken supersymmetry in nature. However the nonobservation of supersymmetry at the LHC is probably the opposite hint that the supersymmetry concept and in particular the supersymmetric $SU(5)$ GUT is wrong. It is well known that the standard $SU(5)$ GUT is in conflict with experimental data. So a natural question arises: is it possible to invent nonsupersymmetric generalizations of the standard $SU(5)$ GUT non contradicting to the experimental data? The answer is positive, in particular, in the $SO(10)$ GUT the introduction of the intermediate scale $M_I \sim 10^{11} \text{GeV}$ allows to obtain the Weinberg angle $\theta_W$ in agreement with experiment. In Refs. the introduction of the additional split multiplets $5 \oplus 10$ and $10 \oplus \overline{10}$ in the $SU(5)$ model has been proposed. In Ref. the extension of the standard $SU(5)$ GUT with light scalar colour octets and electroweak triplets has been considered.

In this note we point out that in nonlocal generalization of $SU(5)$ GUT it is possible to solve the problems with the proton lifetime and the Weinberg angle by the introduction of additional nonlocal terms in the Lagrangian that leads to the modification of the GUT condition $\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT})$ for the effective coupling constants. Nonlocal scale $\Lambda$ responsible for ultraviolet cutoff coincides (up to some factor) with GUT scale $M_{GUT}$. In the simplest nonlocal modification of the standard renormalizable $SU(5)$ GUT the value of the GUT scale is $M_{GUT} \approx 3 \cdot 10^{16} \text{GeV}$. In general case the value of $M_{GUT}$ is an arbitrary and the most interesting option $M_{GUT} = O(M_{PL})$ could be realized.

Let us start with the observation that in standard $SU_e(3) \otimes SU_L(2) \otimes U(1)$ gauge model the effective coupling constants $\alpha_3(\mu)$ and $\alpha_2(\mu)$ cross each other ($\alpha_3(M_{GUT}) = \alpha_2(M_{GUT})$) at the scale $M_{GUT} \approx O(10^{17} \text{GeV})$. At one-loop level the effective coupling constants $\alpha_i(\mu)$ obey the equations

$$\frac{d\alpha_i(\mu)}{d\mu} = \frac{b_i}{2\pi} \alpha_i^2(\mu),$$

where for the SM model with 3 generations $b_3 = -7$, $b_2 = -3\frac{1}{6}$ and $b_1 = 4.1$. As a consequence we find that

$$\frac{1}{\alpha_2(m_t)} - \frac{1}{\alpha_3(m_t)} = \frac{b_2 - b_3}{2\pi} \ln\left(\frac{M_{GUT}}{m_t}\right).$$

Numerically $M_{GUT} = (0.9 \pm 0.2) \cdot 10^{17} \text{GeV}$ and $\frac{1}{\alpha_3(M_{GUT})} = 46.9 \pm 0.2$. \[\text{In our estimates we use } \alpha_3(m_Z) = 0.118 \pm 0.001, \sin^2(\theta_W)(m_Z) = 0.231 \pm 0.001 \text{ and } \alpha^{-1}_e(m_Z) = 127.8 \pm 0.1.\]

The unification scale $M_{GUT} = (0.9 \pm 0.2) \cdot 10^{17} \text{GeV}$ is safe for the current proton decay bound. Really, in standard $SU(5)$ model the proton lifetime due to the massive vector exchange is determined by the formula

$$\Gamma(p \rightarrow e^+\pi^0)^{-1} = 4 \cdot 10^{29\pm0.7}\left(\frac{M_v}{2 \cdot 10^{14} \text{GeV}}\right)^4 \text{yr},$$

where $M_v \equiv M_{GUT} = \sqrt{\frac{5}{24}g_5\Phi_0}$ is the mass of vector bosons responsible for proton decay. From the current experimental limit the proton lifetime is $\Gamma(p \rightarrow e^+\pi^0)^{-1} \geq 1.67 \cdot 10^{34} \text{yr}$ we conclude that

\[\text{Here } \Phi_0 \text{ is the vacuum expectation value of the } SU(5) \text{ scalar 24-plet } < \Phi > = \frac{\Phi_0}{\sqrt{15}} \text{Diag}(1,1,1-3/2,-3/2) \text{ responsible for } SU(5) \rightarrow SU_e(3) \oplus SU_L(2) \oplus U(1) \text{ gauge symmetry breaking and } g_5 \text{ is the } SU(5) \text{ gauge coupling at the GUT scale } M_{GUT}.\]
 leads to the decrease of $M/\Delta$ problem with wrong Weinberg angle prediction. It was realized that this interaction allows to increase the GUT scale but can’t solve the $M$ the unification takes place at the scale $\Delta = 10.9 \pm 0.2$ and $\Lambda_{\Phi 1} \approx 2.3 \cdot M_\ell$. So we find that additional nonrenormalizable interaction (4) modifies GUT unification condition in such a way that the unification takes place at the scale $M_{\text{GUT}} \approx 10^{17} \text{ GeV}$ nondenranlizable for proton decay bound and the unification scale $M_{\text{GUT}}$ does not contradict to the experimental values of $\sin^2(\theta_W)(M_Z)$ and $\alpha^{-1}(M_Z)$. The appearance of additional arbitrary parameter $\Delta$ in the relation (5) means that we can’t predict the value of $\sin^2(\theta_W)$. Here the untrivial fact is that the unification of $\alpha_2(\mu)$ and $\alpha_3(\mu)$ effective coupling constants takes place at the scale $M_{\text{GUT}} = O(10^{17} \text{ GeV})$ which is safe for the proton lifetime bound. An account of two-loop effects for the evolution of the effective couplings $\alpha_k(\mu)$ leads [11] to the replacement

$$\frac{1}{\alpha_k(m_Z)} \rightarrow \frac{1}{\alpha_k(m_Z)} - \theta_k,$$

(7)

where

$$\theta_k = \frac{1}{4\pi} \sum_{j=1}^{3} \frac{b_{kj}}{b_j} \ln \left[ \frac{\alpha_j(M_{\text{GUT}})}{\alpha_j(m_Z)} \right].$$

(8)

Here $b_{ij}$ are the two-loop $\beta$-functions coefficients [3]. An account of two-loop corrections leads to the decrease of $M_{\text{GUT}}$ by factor 3. The parameter $\Delta$ in (5) is not small. Really, $\Delta/\langle \alpha_3(M_{\text{GUT}})^{-1} \rangle \approx 0.24$ and $\Lambda_{\Phi 1} \approx 2.3 \cdot M_\ell$. It means that at the scale $M_{\text{GUT}}$ we must have some ultraviolet cutoff(regulator) to make sense to the nonrenormalizable interaction (4) at quantum level. Probably the most promising way to deal with nonrenormalizable theories is the use of nonlocal field theory [29, 30]. The simplest nonlocal generalization of the renormalizable Yang-Mills Lagrangian

$$L_{YM} = -\frac{1}{2g_5^2} Tr(F_{\mu\nu}F^{\mu\nu}),$$

(9)

\[\text{In Refs.} [25, 26] \text{ the influence of nonrenormalizable interaction } L_{nl} = c/M_{\ell} Tr(F_{\mu\nu}F^{\mu\nu}) \text{ with } c = O(1) \text{ has been studied. It was realized that this interaction allows to increase the GUT scale but can’t solve the problem with wrong Weinberg angle prediction.}\]

\[\text{At two loop level the renormalization group equations for } \alpha_i(\mu) \text{ effective coupling constants are } \mu \frac{d\alpha_i}{d\mu} = \frac{b_i}{2\pi} \alpha_i^2 + \sum_{j=1}^{3} \frac{b_{ij}}{4\pi} \alpha_i^2 \alpha_j, \text{ see Ref.} [27, 28].\]
is \[31\]

\[ L_{YM,nl} = -\frac{1}{2g_5^2} Tr(F_{\mu\nu}V(-\Delta_\mu\Delta^\mu)F^{\mu\nu}), \quad (10) \]

where \( F_{\mu\nu} = \Delta_\mu A_\nu - \Delta_\nu A_\mu, \Delta_\mu = \partial_\mu - iA_\mu, A_\mu = A_\mu^a T_a \] and the formfactor \( V(x) \) is entire function on \( x \). The gauge propagator \( D_{\mu\nu}(p^2) \) for the nonlocal Lagrangian (10) in Feynman gauge is

\[ D_{\mu\nu}(p^2) = \frac{g_{\mu\nu}}{i g_5^2} \cdot \frac{1}{V(p^2)}. \quad (11) \]

The use of nonlocal formfactor \( V(p^2) \) with decreasing behaviour in the euclidean region at \( p^2 \to -\infty \), for instance \( V^{-1}(p^2) = \exp(p^2/\Lambda_{\Phi_1}) \) makes the Yang-Mills model superrenormalizable \[31\]. Possible nonlocal generalization of nonrenormalizable interaction (4) is

\[ \Delta L_{F\Phi F\Phi, nl} = -\frac{1}{4\Lambda_{\Phi_1}^2} \left( Tr(F_{\mu\nu}\Phi) V_{\Phi_1}(-\partial^\mu\partial_\mu)(Tr(F^{\mu\nu}\Phi)) \right) \quad (12) \]

with \( V_{\Phi_1}(p^2) \sim \exp(p^2/\Lambda_{\Phi_1}) \) The use of nonlocal formfactors \( V \) and \( V_{\Phi_1} \) cures bad ultraviolet properties of nonrenormalizable interaction (4) and make it superrenormalizable. For nonlocal Lagragian (12) the parameter \( \Delta \) in formula (5) depends on the scale \( \mu \)

\[ \Delta(\mu) = \frac{\pi \Phi_0^2}{\Lambda_{\Phi_1}^2} V_{\Phi_1}(-\mu^2) \quad (13) \]

We can use the normalization condition \( V_{\Phi_1}(-M_{GUT}^2) = 1 \). In this case formula (6) and numerical estimate for \( \Delta \) are valid.

We can also add to the \( SU(5) \) Lagrangian other nonrenormalizable term

\[ \Delta L_{F\Phi F\Phi_2} = -\frac{Tr(F_{\mu\nu}\Phi^2 F^{\mu\nu})}{4\Lambda_{\Phi_2}^2}. \quad (14) \]

Nonzero vacuum expectation value \(<\Phi> = \frac{\Phi_0}{\sqrt{15}} Diag(1,1,1-3/2,-3/2)\) of \( SU(5) \) 24-plet \( \Phi \) leads to additional contributions for coupling constants at GUT scale, namely

\[ \frac{1}{\alpha_1(M_{GUT})} \to \frac{1}{\alpha_1(M_{GUT})} + \frac{7\kappa}{120}, \quad (15) \]

\[ \frac{1}{\alpha_2(M_{GUT})} \to \frac{1}{\alpha_2(M_{GUT})} + \frac{3\kappa}{40}, \quad (16) \]

\[ \frac{1}{\alpha_3(M_{GUT})} \to \frac{1}{\alpha_3(M_{GUT})} + \frac{\kappa}{30}, \quad (17) \]

where \( \kappa = \frac{4\pi\Phi_0^2}{\Lambda_{\Phi_2}^2} \). As a consequence we find that

\[ \frac{1}{\alpha_2(M_{GUT})} - \frac{1}{\alpha_3(M_{GUT})} = \frac{5\kappa}{120}. \quad (18) \]

\(^5\)Here \( T_a \) are the \( SU(5) \) matrices with \( Tr(T_a T_b) = \frac{1}{2} \delta_{ab} \) and \( g_5 \) is the \( SU(5) \) gauge coupling constant.

\(^6\)We can consider nonlocal Yang-Mills Lagrangian (10) as a generalization of Slavnov gauge invariant regularization \[32\ \[33\] of Yang-Mills model with higher order derivatives.
It means that playing with the $\kappa$ parameter we can increase the GUT scale $M_{GUT}$. For instance, for $\kappa = 30.2(47.1)$ we obtain $M_{GUT} = m_{PL} = 2.4 \cdot 10^{18} \text{ GeV} (M_{GUT} = m_{PL} = 1.2 \cdot 10^{19} \text{ GeV})$. Nonlocal generalization of nonrenormalizable interaction (13) is

$$\Delta L_{F\Phi \Phi^2, nl} = \frac{Tr(F_{\mu \nu} \Phi \Phi^2 (\Delta_{\mu} \Delta^\nu) \Phi F_{\mu \nu})}{\Lambda_{\Phi^2}^2}. \quad (19)$$

For nonlocal Lagrangian (19) as in the case for nonlocal Lagrangian (12) the parameter $\kappa$ depends on the scale $\mu$. For the normalization condition $V_{\Phi^2}(-M_{GUT}^2) = 1$ formulae (15-18) and the numerical estimates are not changed.

Let us make our main conclusions. Additional nonlocal interactions (12) or (12,19) allow to overcome the standard $SU(5)$ GUT problems with fast proton decay and wrong Weinberg angle prediction. Nonlocal generalizations (12,19) cure the problems with ultraviolet behaviour of nonrenormalizable interactions (4,14). The price of such modification is the absence of predictive power at least for the Weinberg angle $\theta_W$. The nonlocal scales $\Lambda_{\Phi^1}$ coincide by the order of magnitude with the GUT scale $M_{GUT}$. In the simplest nonlocal extension of the standard $SU(5)$ GUT with $\kappa = 0$ the value of GUT scale is $M_{GUT} \approx 3 \cdot 10^{16} \text{ GeV}$. For general case with $\kappa \neq 0$ the GUT scale $M_{GUT}$ is an arbitrary. It is well known that quantum gravity is nonrenormalizable theory. To cure bad ultraviolet properties of quantum gravity we have to modify gravity at Planck scale, in particular, nonlocal generalization of gravity [31] leads to superrenormalizable theory [31, 34]. Therefore the most interesting and natural option is the equality at least by the order of magnitude of $M_{GUT}$, $M_{PL}$ and the nonlocal scale $\Lambda$.

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