Ancilla-Driven Universal Blind Quantum Computation

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Blind quantum computation is a new quantum secure protocol, which enables Alice who does not have enough quantum technology to delegate her computation to Bob who has a fully-fledged quantum power without revealing her input, output and algorithm. So far, blind quantum computation has been considered only for the circuit model and the measurement-based model. Here we consider the possibility and the limitation of blind quantum computation in the ancilla-driven model, which is a hybrid of the circuit and the measurement-based models.

I. INTRODUCTION

Traditionally, quantum computation has been studied in the circuit model [1], where the quantum register which stores quantum information consists of many qubits, and a quantum gate operation is performed by directly accessing one or two qubits in the quantum register. Another canonical model of quantum computation is the one-way model [2] or more general measurement-based models [3–13], where the universal quantum computation is performed by adaptive local measurements on a highly entangled resource state. Recently, a mixture of those two models, which is called the ancilla-driven quantum computation, was proposed in [14, 15]. In this model, the quantum register is a set of many qubits like the circuit model, whereas a quantum gate operation is, like the one-way model, performed by adaptive local measurements: one or two register qubits are coupled to a single mobile ancilla, and the ancilla is measured after establishing the interaction between the ancilla and register qubit(s). The backaction of this measurement provides the desired gate operation, such as a single qubit rotation or an entangling 2-qubit operation, on register qubit(s). In the ancilla-driven model, the universal quantum computation is performed with only a single type of interaction (CZ or SWAP+CZ) between the ancilla and register qubit(s). It is a great advantage for experiments, since in many experimental setups, implementing various different types of interactions at the same time is very difficult (such as the solid-based quantum computation). Furthermore, the roles of the register and the information carrier are clearly separated, and no direct action on the register is required. Therefore, it is also useful for experimental systems where measurements destroy quantum states, such as photonic systems. In short, this model is a natural theoretical model of the “hybrid quantum computer” where the flying ancilla mediates interactions between static qubits (such as the chip-based quantum computation [16, 17]) or the hybrid system of matter and optical elements [18, 19].

In a future when a scalable quantum computer is realized, the quantum computation should be done in the “cloud” style, since only limited number of people would have enough money and technology to create and maintain quantum computers. Blind quantum computation [20–29] ensures the privacy of the client in such a cloud quantum computing. In protocols of blind quantum computation, Alice, the client, does not have enough quantum technology. On the other hand, Bob, the server, has a fully-fledged quantum power. Alice asks Bob to perform her computation on his quantum computer in such a way that Bob cannot learn anything about her input, output, and algorithm. Blind quantum computation was initially considered by using the circuit model [20–22]. However, in this case, Alice needs a quantum memory. Recent new ideas of blind quantum computation which use measurement-based models have succeeded to exempt Alice from a quantum memory [23–29].

In terms of the computational power, measurement-based models do not offer any advantage over the circuit model, since the circuit model can be simulated by measurement-based models and vice versa. However, measurement-based models have provided new points of view for studying quantum computation, and in fact such new viewpoints have enabled plenty of successes which have never been done in the circuit model, such as the high-threshold fault-tolerance [10–12, 30–36], clarifying roles of entanglement played in quantum computation [3, 37–40], and relations to condensed-matter physics [3, 7, 13, 31, 44]. Therefore it is important to explore the possibility of blind quantum computation on other models than the circuit model and measurement-based models.

II. ANCILLA-DRIVEN QUANTUM COMPUTATION

We first define several notations for the basis and for the basic transformations as follows: $|\pm\rangle$ = $\cos(\frac{\theta}{2})|0\rangle + e^{i\varphi}\sin(\frac{\theta}{2})|1\rangle$, $|\mp\rangle$ = $\sin(\frac{\theta}{2})|0\rangle - e^{i\varphi}\cos(\frac{\theta}{2})|1\rangle$, $R_z(\theta) = e^{-i\frac{2\pi}{\theta}}$ and $R_z(\theta) = e^{-i\frac{2\pi}{\theta}}$. We conventionally use the notations $\{|\pm\rangle\}$ and $\{|0\rangle,|1\rangle\}$ to denote the bases along $X$ and $Z$ axes in the Bloch sphere, respectively. Measurement outcome is represented by $s \in \{0,1\}$, associated with $\pm$. We denote the $i$th measurement outcome by $s_i$. 

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We review the ancilla-driven quantum computation (ADQC) proposed in [14, 13]. ADQC is performed with a 1-qubit ancilla, only on which we can make measurements, and (a single or a few) 2-qubit entangle operator(s) \( E_{as} \). As in Fig.1(a), \( E_{as} \) can be decomposed into

\[
E_{as} = (V_x \otimes V_y')D_{as}(V_x' \otimes V_y) \quad \text{by the Cartan decomposition} \quad [45],
\]

where \( V_x, V_x', W_y, W_y' \) are 1-qubit local unitaries and \( D_{as} \) is a 2-qubit non-local unitary. Fig.1(a) can be rewritten to Fig.1(b) by applying \( V_y' \) to the prepared ancilla state \( |+\rangle_{a} \) and \( W_y' \) to the measurement basis \( \{|0\rangle, |1\rangle \} \). 

\[
D_{as} = e^{-i(\alpha_x X_0 \otimes X_2 + \alpha_y Y_0 \otimes Y_2 + \alpha_z Z_0 \otimes Z_2)}
\]

by using non-symmetric parameters \( 0 \leq \alpha_x, \alpha_y, \alpha_z \leq \frac{\pi}{2} \) due to the Weyl chamber [16]. 

\[
\rho_s - V_x - D_s - \rho^z_s \\
\rho_s' - V_y' - D_s' - \rho^z_s'
\]

(a) \( E_{as} \)

(b) \( \rho^z_s \)

FIG. 1. ADQC. A rectangle box with bold line represents a measurement and the inside represents a basis for the measurement.

For universal quantum computation, we should choose all the parameters appropriately. To this end, Anders et al. in [13] derive sufficient conditions for (i) Unitarity, (ii) One-step Correctable Branching, (iii) Standardization and (iv) Universality. Especially, we will discuss about One-step Correctable Branching, which states that the generalized Pauli correction according to the measurement outcome after “one” execution in ADQC enables the Kraus operator acting on the system deterministic. In order to fulfill these conditions, it is shown that the entangle operator \( E_{as} \) must be locally equivalent to either SWAP+\( CZ \) \((\alpha_x = \alpha_y = \frac{\pi}{4}, \alpha_z = 0)\) or \( CZ \) \((\alpha_x = \frac{\pi}{4}, \alpha_y = \alpha_z = 0)\).

### III. ANCILLA-DRIVEN UNIVERSAL BLIND QUANTUM COMPUTATION

ADQC of SWAP+\( CZ \) type can be considered as an extension of one-way quantum computation because the measurements are made on the system instead of on the ancilla if we exclude the SWAP and this case is exactly one-way quantum computation. So we can perform universal blind ADQC of SWAP+\( CZ \) type as in [22]. In this paper, we focus only on universal blind ADQC of \( CZ \) type. Requiring all the conditions (i)-(iv) is too strong for universal blind ADQC of \( CZ \) type, since ADQC of \( CZ \) type satisfying all the conditions cannot be blind (in the sense of [23]) as is shown in the following. The system Kraus operator for \( E_{as} \) is specified as \( K_{as}^{\pm} = V_sK_{s}^{\pm}W_s \) and \( K_{as}^{\pm} = a(\pm \theta, \phi)D_{as}|+\gamma, \delta\rangle/a \). As in [15], Unitarity and One-step Correctable Branching require that the parameters for the ancilla satisfy

\[
sin \theta \cos \gamma \sin \phi = \cos \theta \sin \sin \phi \text{ and the Kraus operator } K_s^{\pm} = f_{\pm} I + i(1-\gamma) g_{\pm} X,
\]

where \( n_{as} \) are integers that differ in the parity. These coefficients are rewritten as

\[
f_{\pm} = \frac{\cos \gamma}{2} \pm \sqrt{1 + \sin \gamma \sin \phi \cos \phi - \sin \phi \cos \phi - \sin \phi \cos \phi - \sin \phi \cos \phi}
\]

\[
g_{\pm} = \frac{\sin \gamma}{\sqrt{2}} \pm \sqrt{1 - \cos \gamma \sin \phi \sin \phi - \cos \phi \sin \phi - \cos \phi \sin \phi - \cos \phi \sin \phi}
\]

Moreover, the parameter \( \alpha_x \) for \( D_{as} \) and the parameters for the ancilla have the following relation:

\[
\tan^2 \alpha_x = \frac{1}{2} \pm \frac{\cos \gamma \sin \phi - \sqrt{1 + \sin \gamma \sin \phi \cos \phi - \sin \phi \cos \phi - \sin \phi \cos \phi - \sin \phi \cos \phi}}{2}
\]

Therefore, Unitarity and One-step Correctable Branching imply that admissible parameters of the ancilla are classified into the following four cases.

| prepared ancilla | measurement basis | Kraus operator |
|------------------|-------------------|---------------|
| \gamma = 0      | \phi = 0          | \gamma = X^rI |
| \gamma = 0      | \phi = any        | \gamma = X^rI |
| \gamma = \pi/2  | \phi = any        | \gamma = X^rI |
| \gamma, \delta = any | \phi = 0        | \gamma, \delta = any |

For “universal” blind computation, a rotation operator \( R_x(\theta) \) is indispensable. Thus, we consider only the second case. In that case, the prepared ancilla should be fixed to \( |0\rangle \) since \( \gamma = 0 \). This means that we cannot use a random ancilla restricted on some plane in the Bloch sphere to make the computation blind similarly in [23].

For that reason, we disregard One-step Correctable Branching for the present and derive some more admissible parameters of the ancilla as follows.

| prepared ancilla | measurement basis | Kraus operator |
|------------------|-------------------|---------------|
| \gamma = any, \delta = 0 | \phi = 0        | \gamma = X^rI |
| \gamma = any, \delta = 0 | \phi = \pi/2      | \gamma = X^rI |
| \gamma = \pi/2, \phi = 0 | \gamma = X^rI |
| \gamma = \pi/2, \phi = 0 | \phi = \pi/2      | \gamma = X^rI |

Then, we have the Kraus operators written as

\[
K_s^+ = R_x(\gamma) \text{ and } K_s^- = XR_x(-\gamma)
\]

by choosing the prepared ancilla parameters \( \gamma \) be any value and \( \delta = 0 \) and the measurement basis parameters \( \phi, \theta = 0 \). Blind ADQC of \( CZ \) type is enabled by allowing the above Kraus operators. For blind computation, it is sufficient that all the information Client sends to Server is uniformly random. This is for hiding the rotation of unitaries to Server in the computation and we show a rough sketch to incorporate this idea into ADQC. First, Client chooses a prepared ancilla parameter \( \gamma \) randomly and Client sends ancilla \( |+, \gamma, 0\rangle \) or \( |-, \gamma, 0\rangle \) with equal probabilities. Then, Server performs the Kraus operator \( R_x((-1)^\gamma) \) using this ancilla. At this time, the ancilla is \( \frac{1}{2} \sum_{\gamma \in \{0, 1\}} |+, \gamma, \pi, \pi, 0\rangle X |+, \gamma, \pi, \pi, 0\rangle = \frac{|0\rangle}{2} \) as maximally mixed state. Moreover, if we assume the input state \( |\psi\rangle = \cos \frac{\theta}{2} |+\rangle + e^{i\phi} \sin \frac{\theta}{2} |-\rangle \), the output state is
\[
\sum_{\theta \in [0, 1]} R_\theta((-1)^s(\gamma + r \pi)) |\psi\rangle |\gamma\rangle R_\gamma(((-1)^s(\gamma + r \pi)) = \\
\begin{pmatrix}
\cos^2(\theta/2) \\
n\sin^2(\theta/2)
\end{pmatrix}
\]
so this state contains no information about \(\gamma\). After that, Client sends a measurement basis parameter \(\theta\) and Server performs the Kraus operator \(R_\theta(\theta)\). Therefore, the total Kraus operator is \(R_\theta(\theta + (-1)^s\gamma)\) and the total rotation of the Kraus operator is hiding to Server because \(\gamma\) is hiding. Based on this idea, we relax One-step Correctable Branching to Multiple-step and derive a sufficient condition which can make ADQC of CZ type blind. Multiple-step means that, in “multiple” executions in ADQC, each Kraus operator needs not to be deterministic but the whole Kraus operator must be deterministic up to the correction.

For universal blind ADQC of CZ type, two types of Kraus operators are necessary. One type is an \textit{uncorrectable} Kraus operator which depends on an outcome of the measurement, such as \(V_x R_x((-1)^s\gamma) W_s\), performed using a prepared ancilla parameter \(\gamma\). The other is a \textit{correctable} Kraus operator, such as \(V_x R_x(\theta) W_s\), up to Pauli correction, performed with a measurement basis parameter \(\theta\). With respect to these Kraus operators, we consider two conditions: L-hiding and G-hiding.

L-hiding requires that \(W_s R_x(\theta) V_s W_s R_x((-1)^s\gamma) V_s = W_s R_x(\theta') V_s W_s V_s \) \(\equiv S\) holds, where \(\theta' = \theta \pm (-1)^s\gamma\). G-hiding requires that a gate pattern which can perform both \(U \otimes U'\) where \(U\) and \(U'\) are any 1-qubit unitaries and one kind of entangle operators is composable by using a unitary \(S\) and a controlled-Pauli that can be simulated. In L-hiding, we might use an \textit{assistant} Kraus operator, such as \(W_s R_x(\theta) V_s\), for satisfying universality.

If the two conditions are satisfied, we can perform universal quantum computation by tuning the gate pattern in G-hiding regularly as in Fig.2. What unitaries the gate pattern performs depends on a parameter \(\theta'\) of each gate in L-hiding composing the gate pattern. When Client decides a parameter \(\theta'\), Client sends a measurement basis parameter \(\theta\) such that \(\theta = \theta' \mp (-1)^s\gamma\). By choosing a prepared ancilla parameter \(\gamma\) randomly, \(\theta\) also looks random to Server. This process is performed similarly to the protocol in [23]. We use the following protocol for the performing of each \(S\).

1. Client chooses a prepared ancilla parameter \(\gamma\) randomly and sends the ancilla to Server.
2. Server performs \(W_s R_x((-1)^s\gamma) V_s\) with the given ancilla. Server sends an outcome \(s\) of the measurement in this simulation to Client.
3. Client decides \(\theta'\) and calculates \(\theta = \theta' \mp (-1)^s\gamma + r \pi\) with a random bit \(r \in \{0, 1\}\) then sends \(\theta\) to Server.
4. Server performs \(W_s R_x(\theta) V_s\) and sends an outcome \(s'\) of the measurement in this simulation to Client.
5. Client inverts \(s'\) if \(r = 1\).

If we use an assistant Kraus operator in the protocol, Server performs the corresponding simulation in Step 2. In the above protocol, each ancilla state is maximally mixed and each \(\theta\) looks random to Server. Therefore, the information leaked to Server is only the upper bound on the size of the universal gate pattern, that is, the upper bounds on the input size and the depth of the computation.

In the rest of this section, we discuss relations between the compatibility of the two hiding conditions and Universality. In the case the protocol uses only one kind of entangle operators (e.g., \((H \otimes H) CZ\) used in [14, 15] and no assistant Kraus operator, L-hiding and G-hiding do not hold simultaneously. From L-hiding, it must hold that \(V_x W_s R_x((-1)^s\gamma) = R_x(\pm (-1)^s\gamma) V_x W_s\). By the similar discussion for Universality in [13], \(V_x W_s\) must be \(aY + bX\) or \(aX + bY\) up to a global phase, where \(a, b \in \mathbb{R}\). Therefore, any 1-qubit unitary \(U\) composed of \(S\) is described as \(U = W_s U V_s\) with the kernel \(U\) which moves a quantum state only in some plane of the Bloch sphere, parallel to the \(Y-Z\) plane. If \(V_x\) and \(W_s\) are determined, \(U\) becomes a unitary which moves a quantum state only in one plane of the Bloch sphere so we cannot perform any arbitrarily rotation \(U \otimes U'\) in G-hiding. In the case the

![Universal gate pattern](image)

FIG. 2. Universal gate pattern.

![Simulating HR\(_x(\theta')\) and CZ](image)

FIG. 3. Simulating \(HR_x(\theta')\) and CZ.

![Gate pattern](image)

FIG. 4. Gate pattern. (a) is for a single entangle operator and (b) for two entangle operators.
the gate pattern in G-hiding is shown in Fig. 3(b). In summary, universal blind ADQC of CZ type is possible by considering 3-step Correctable Branching in the case one kind of entangle operators is allowed in the protocol and 2-step Correctable Branching in the case two kinds of entangle operators are allowed.

\[ |\psi_\theta\rangle = \begin{cases} \frac{1}{\sqrt{2}} (|0\rangle + e^{i\theta}|1\rangle) & \text{for } \theta = 0 \\
\frac{1}{\sqrt{2}} (|0\rangle - e^{i\theta}|1\rangle) & \text{for } \theta = \pi \\
\frac{1}{\sqrt{2}} (|0\rangle - e^{i\theta/2}|1\rangle) & \text{for } \theta = 2\pi \\
\frac{1}{\sqrt{2}} (|0\rangle - e^{-i\theta/2}|1\rangle) & \text{for } \theta = 4\pi \\
\end{cases} \]

(a) \( R_x(\theta') \) such that \( \theta' = -\theta + (-1)^{s_1}\gamma \)

(b) \( R_z(\theta') \) such that \( \theta' = \theta - (-1)^{s_1}\gamma \)

FIG. 5. Simulating \( R_x(\theta) \) and \( R_z(\theta) \).

IV. CONCLUSION

In this paper, we considered the possibilities and limitations for universal blind computation in ADQC. First, we proved that if we satisfy all the conditions for universal quantum computation in [13], we can not perform universal blind computation. Therefore, we relaxed one condition and derived a sufficient condition for the blindness. Finally, we provided ways of universal blind computation in ADQC of CZ type.

Secondly, our way of universal blind ADQC needs less quantum requirements for Client than the way in [22] in the case of using quantum inputs. In our ways, Client does not need to rotate input states with respect to Z axis in the Bloch sphere but only to apply \( Z \). Extending our way to one-way model, we can also consider a way which is also less quantum requirements in one-way model.

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