Effects of T- and P-odd weak nucleon interaction in nuclei: renormalizations due to residual strong interaction, matrix elements between compound states and their correlations with P-violating matrix elements

V. V. Flambaum and O. K. Vorov

School of Physics, University of New South Wales, Sydney 2052, New South Wales, Australia

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Abstract

Manifestations of P-, T-odd weak interaction between nucleons in nucleus are considered. Renormalization of this interaction due to residual strong interaction is studied. Mean squared matrix elements of P-, T-odd weak interaction between compound states are calculated. Correlators between P-, T-odd and P-odd, T-even weak interaction matrix elements between compound states are considered and estimates for these quantities are obtained.

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I. INTRODUCTION

Spatial parity nonconserving weak interaction of nucleons is now subject for extensive experimental and theoretical investigations. In these studyings, quantitative comparison of experimental results and theoretical predictions is possible. Developments in experimental techniques and interpretations of the results thus obtained allow one to rise questions going far from the scope of theory of weak interaction [1]-[11].

Much less is known, both in experimental and theoretical aspects, about the component of nucleon weak interaction that violates both spatial parity (P) and time reversal invariance (T) (T-,P-odd weak interaction). The problem of possible T-violation has been of interest for a long time [12]-[20]. In the context of nuclear physics, the T-,P-odd interaction, if exists, induces T-,P-odd nuclear moments [21]-[27] (electric dipole, magnetic quadrupole moments, “Schiff” moment etc). Experimental data exist only for upper limits of these quantities. At the same time, theoretical values of constants of this interaction are not unambiguously known, varying by several orders of magnitude from one model to another (see e.g., [23], [27]). In most cases, the scale of T-,P-odd interaction is predicted to be very small.

In this situation, possible sources of enhancement of the effects caused by this interaction, which allow experimental investigation, are crucial for the further studies of T-,P-odd interaction. Apparently, the compound nuclear resonances providing a large statistical enhancement of small perturbations, are very convenient in this case.

We are considering here the weak T-,P-odd nucleon interaction in nuclei beginning at the single-particle level. The effects of the residual strong interaction on the T-,P-odd potential was considered in Ref. [9]. In present work, we focus attention on renormalization of the two-body T-,P-odd interaction due to the residual strong interaction, which is important for description of the T-,P-odd effects in nuclear states at excitation energies near or lower than that of neutron separation threshold, $B_n$. We have calculated mean squared T-,P-violating matrix elements between compound states and have considered possible correlations of these matrix elements with the matrix elements of P-odd, T-even weak interaction [8].

The structure of the paper is following. In section II we consider T-,P-odd potential, acting on a nucleon that arises in mean field approximation for the initial two-body weak interaction. We calculate the single-particle matrix elements of this potential and discuss their properties in comparison with the single-particle matrix elements of P-odd, T-even weak
interaction. In section III we consider renormalization of the T-,P-odd weak interaction by residual strong nucleon interaction. In section IV the equations of this renormalization are solved for the Landau-Migdal parametrization of the residual strong interaction and explicit analytical results for the effective two-body T-,P-odd weak interaction between the nucleons in heavy nuclei are derived. Numerical results are obtained for the matrix elements expressed through the constants of the initial weak interaction. It is shown that, in contrary to the case of P-odd, T-even weak interaction, renormalization due to strong interaction does not result in enhancement of matrix elements, though this renormalization is important for quantitative results.

In section V we calculate mean squared matrix elements of the P-,P-odd interaction between nuclear compound states of opposite parity within statistical model. Section VI is devoted to discussion of correlations of P-,T-odd and P-odd, T-even matrix elements between compound states. Calculation of the correlator in the statistical model yields the value about 10 per cent.

The results are summarized in section VII.

II. T-,P-ODD WEAK NUCLEON INTERACTION. T-,P-ODD POTENTIAL

The nuclear Hamiltonian $H$ with account for the T-,P-odd weak interaction can be written in the form

$$H = H_0 + V_S + W_{P,T} + F,$$

Here, the first term $H_0 = \frac{p^2}{2m} + U_S(r, \vec{\sigma})$ is the single particle Hamiltonian of the nucleons moving in the strong mean field $U_S(r, \vec{\sigma})$ including the spin-orbit interaction, $V_S$ stands for the residual two-body strong interaction (it will be considered in section IV), and $W_{P,T}$ describes P-,T-odd weak interaction between nucleons, $F$ denotes other possible interactions, e.g., coupling to electromagnetic field. The two-body weak P-,T-odd interaction $W_{P,T}$ can be written as follows (see e.g. [22], [23]):

$$\hat{W}_{P,T}(1, 2) = \frac{G}{\sqrt{2} 2m} \left( (\eta_{12} \vec{\sigma}_1 - \eta_{21} \vec{\sigma}_2) \cdot \vec{\nabla}_1 \delta(\vec{r}_1 - \vec{r}_2) + \eta_{12}' [\vec{\sigma}_1 \times \vec{\sigma}_2] \cdot \{\vec{p}_1 - \vec{p}_2, \delta(\vec{r}_1 - \vec{r}_2)\}\right)$$

where $G = 10^{-5} m^{-2}$ is the Fermi constant, $m$ is the nucleon mass, $p$ and $\vec{\sigma}$ are the nucleon momentum and doubled spin respectively. Hereafter, $\vec{a} \times \vec{b}$ means exterior vector product,
and \(\{a, b\}\) denotes anticommutator. The dimensionless constants \(\eta_{1,2}, \eta'_{1,2}\) which determine the scale of the T-,P-odd effects, are predicted to be very small, e.g., within Kobayashi-Maskawa model (see e.g., [22], [23], [26]).

The analysis of the P-,T-odd effects in nuclei is similar to that in the case of the P-odd weak interaction (see e.g. [8], [10]). It is convenient to introduce P-,T-odd “weak potential” \(w_{P,T}^{P,T}\) acting on a valence nucleon 1, that arises from summation \(W_{P,T}(1, 2)\) over the states of the nucleon 2 (see, e.g. Ref. [23]):

\[
w_{P,T} = \frac{G}{2\sqrt{2}m} \eta(\vec{\sigma} \vec{\nabla}) \rho(r)
\]  

(3)

where \(\rho\) is the nucleon density, the dimensionless constants \(\eta_p, \eta_n\) characterize the strength of T- and P-odd potential for proton (neutron); they are connected to the parameters of the initial two-body interaction \(W_{P,T}(1, 2)\) by the relations

\[
\eta_p = \frac{Z}{A} \eta_{pp} + \frac{N}{A} \eta_{pn}, \quad \eta_n = \frac{N}{A} \eta_{nn} + \frac{Z}{A} \eta_{np},
\]

(4)

where \(Z, N, A\) are the nuclear charge, a neutron number and its mass number respectively. The limits on these constants \((\eta_p, \eta_n)\) were obtained from the atomic [24] and molecular [25] electric dipole moment measurements. Being a single-particle operator, the T-,P-odd weak potential \(w_{P,T}^{T,P}\) obeys the same selection rules as the P-odd, T-even weak potential \(w^P\):

\[
\Delta l = \pm 1, \quad \Delta j = 0.
\]

(5)

The values of the matrix elements of the P-,T-odd weak potential (3) between single-particle nuclear states calculated for \(^{209}Pb\) are presented in Tables I,II. The numerical calculations have been performed with the use the of single-particle basis of states obtained by numerical solution of the eigenvalue problem in the Woods-Saxon potential with spin-orbital interaction in the form

\[
U_S(r, \vec{\sigma}) = -U_0 f(r) + U_s(\sigma 1)(h/(m\pi c))^2 \frac{1}{r} \frac{df}{dr} + U_c
\]

(6)

with \(f(r) = (1 + exp((r - R)/a))^\ominus\). Here, \(I\) is the orbital angular momentum, \(U_c\) means Coulomb correction for protons, \(U_c = 3Ze^2/(2R)(1-r^2/(3R^2)), r \leq R\) and \(U_c = Ze^2/r, r > R\), for \(R, a\), and \(r\) being the nuclear radius, diffusity parameter and radial variable correspondingly. The parameter values were used in accordance with Bohr-Mottelson formulas (see Ref.
for the case of $^{233}$Th: they are close to those established for heavy nuclei like lead (Ref. 23) to reproduce single-particle properties.

As it is seen from Tables I, II, the single-particle T-, P-odd matrix elements (column 5) are numerically suppressed (by about order of magnitude) as compared to the matrix elements of P-odd, T-even potential $\hat{w}^P(1) = \langle \hat{W}^P(1, 2) \rangle = \frac{Gq}{2\sqrt{2m}} \{ (\sigma p) \rho + \rho (\sigma p) \}$ that arises from the corresponding two-body interaction 21

$$\hat{W}(1, 2) = \frac{G}{\sqrt{2}} \frac{1}{2m} \left( (g_{12}\sigma_1 - g_{21}\sigma_2) \cdot \{(p_1 - p_2), \delta(r_1 - r_2)\} + g'_{12}[\sigma_1 \times \sigma_2] \nabla_1 \delta(r_1 - r_2) \right)$$  \hspace{1cm} (7)

in the same way as the P-, T-odd potential. The difference between these two cases is due to surficial character of the potential (3) that is proportional to the nuclear density derivative and peaked at nuclear surface. As average, the mixing of the single-particle states of opposite parity due to potential (3) that give rise to nuclear T-, P-odd nuclear moments (see 22, 23)

$$f_{ab} = w^{TP}_{ab}/(\varepsilon_a - \varepsilon_b) \sim 10^{-8} \eta_{12}. \hspace{1cm} (8)$$

Due to the selection rules (5), the following considerations, analogous to those in the case of P-odd, T-even weak interaction, take place here. It is well known 30, 31 that doublets of single-particle states with the same total angular momentum, but of opposite parity usually do not appear in the same spherical nuclear shell. Thence, the energy separation between levels in such doublets is about $5...8 MeV$, the average energy distance between different shells. Thus, the coherent single-particle P-, T-odd contribution (3) does not work effectively in mixing of any excited nuclear states (including the compound states) with the energies below $S_n...4\div 6 MeV$, the neutron separation energy, because the many-particle wave functions in this energy region are dominated by nucleon excitations within the valence shells 28. Therefore, the main P-odd effects in this energy region are to be determined by the purely two-particle “residue”, $\hat{W}(1, 2)$: of the weak interaction $\hat{W}(1, 2)$, given by the difference

$$\hat{W}^{P,T}(1, 2) : \equiv \hat{W}^{P,T}(1, 2) - \langle \hat{W}^{P,T}(1, 2) \rangle = \hat{W}^{P,T}(1, 2) - w^{P,T}(1),$$  \hspace{1cm} (9)

which does not contain coherent summation in contrast to (3).

We consider first the case with the strong interaction $V_S$ being “switched off”. Technically, it is convenient to include the corrections caused by the T,P-odd potential (5) into the single-particle wave functions using unitary transformation. As is known from Refs. 22, 23, in
the simple model with the strong potential $U(r)$ being proportional to the nuclear density $\rho$ ($\rho(r) = \rho(0)U(r)/U(0)$), it is easy to find the result of the action of the perturbation $\hat{w}(1)$

$$\tilde{\psi} = \exp(-\hat{\alpha})\psi^0 \simeq (1 - \theta\vec{\sigma}\vec{\nabla})\psi^0,$$

$$\hat{\alpha} = \theta\vec{\sigma}\vec{\nabla}$$

$$\theta = \eta G_{(0)} \rho(0) \rho(0) \approx -2 \cdot 10^{-8} \eta \cdot fm,$$  \hspace{1cm} \text{(10)}

where $\psi^0$ is the unperturbed wave function, and $\tau_z = -1(+1)$ is isospin projection for proton(neutron). To get this solution, one should also neglect spin-orbit interactions. Accordingly, the matrix elements of any operator $O$, including the Hamiltonian, can be calculated by using the unperturbed wave functions $\psi^0$ and the transformed operator $\tilde{O}$:

$$\langle \tilde{\psi}_a | O | \tilde{\psi}_b \rangle = \langle \psi^0_a | \tilde{O} | \psi^0_b \rangle = \langle \psi^0_a | e^{\hat{\alpha}}Oe^{-\hat{\alpha}} | \psi^0_b \rangle \approx \langle \psi^0_a | O + [\hat{\alpha}, O] | \psi^0_b \rangle,$$

where $e^{\hat{\alpha}} \equiv e^{i\theta(\vec{\sigma}\vec{\nabla})}$ is the operator of the corresponding unitary transformation with the single-particle anti-Hermitian $\hat{\alpha}$. This transformation compensates the single-particle P-,T-odd potential in the Hamiltonian $e^{\hat{\alpha}}He^{-\hat{\alpha}}$. The effect of this potential is now included into the renormalized operators $\tilde{O}$ rather than the wave functions $\tilde{\psi}$.

**III. RENORMALIZATION OF THE P-,T-ODD EFFECTS DUE TO RESIDUAL STRONG INTERACTION**

To take the strong interaction $V_S$ into account, let us seek now for an operator $e^{\hat{A}}$ which should play the same role as $e^{\hat{\alpha}}$ above, but will incorporate the renormalization effects due to the residual strong interaction $V_S$. Eventually, as we will see below the operator $\hat{A}$ differs from $\hat{\alpha}$ mainly due to the renormalization of the weak interaction constant by the residual strong interaction $V_S$. The transformed Hamiltonian looks like:

$$\tilde{H} = e^{\hat{A}}He^{-\hat{A}} = H_0 + V_S + \hat{F} +$$

$$+ w^{P,T} + : \hat{W}^{P,T} : + [\hat{A}, H_0] + [\hat{A}, V_S] + [\hat{A}, F]$$  \hspace{1cm} \text{(11)}

where we have used the decomposition (4) and neglected all terms above the first order in the weak interaction. To obtain the effective two-particle P,T-odd interaction acting in the valence shells we should find the operator $\hat{A}$ in such a way that the single-particle P,T-odd contribution in $e^{\hat{A}}He^{-\hat{A}}$ will be compensated. The last term in (6) is a two-body operator. We employ the same decomposition, as in (4): $[\hat{A}, V_S] \equiv \langle [\hat{A}, V_S] + [\hat{A}, V_S] :$, where the first
single-particle term is the average over the paired nucleons, and the second one, \( [\hat{A}, V_S] \), which yields zero under such averaging, is the effective induced two-particle interaction which we are seeking for:

\[
W_{ITPNCI}^{TP} = [\hat{A}, V_S] , \quad \langle W_{ITPNCI}^{TP} \rangle \equiv 0 .
\]  

(12)

Now we choose the operator \( \hat{A} \) in such a way that the “compensation equation”

\[
\hat{w}^{P,T} + [\hat{A}, H_0] + \langle [\hat{A}, V_S] \rangle = 0 ,
\]  

(13)

is fulfilled. After that, the transformed Hamiltonian (11) takes the form

\[
\tilde{H} = H_0 + V_S + F + : \hat{W}^{P,T} : + W_{ITPNCI}^{P,T} + [\hat{A}, F]
\]  

(14)

where P-,T-odd single-particle terms are canceled. The sources of symmetry violations presented in Eq.(16) can be classified as follows:

(i) the term \( [\hat{A}, F] \) that gives a direct contribution of the symmetry violating potential \( w_1^{P,T} \) to the matrix elements of an external field \( F \) \( (\langle \psi | F + [\hat{A}, F] | \psi' \rangle = \langle \tilde{\psi} | F | \tilde{\psi}' \rangle) \);

(ii) The two-body residual weak interaction \( W^{P,T} : \);

(iii) \( W_{ITPNCI}^{P,T} \), which play the same role as \( W^{P,T} : \). We note that the induced P-,T-odd interaction \( W_{ITPNCI}^{P,T} \) is not enhanced in comparison with the two-particle residual P-,T-odd interaction \( W^{P,T} : \), contrary to the case of P-odd, T-even interaction that turns out to be enhanced by \( \sim A^{1/3} \) times (see [10]).

The effects of renormalization of P-odd, T-even interaction were considered in details in Ref. [10]; below, we focus our attention on the T-,P-odd interaction.

**IV. EXPLICIT FORM OF THE RESULTING TWO-PARTICLE T-,P-ODD INTERACTION**

To solve the equation (8) and find an explicit form of the ITPNCI we use the Landau-Migdal interaction [32], [33], [29]. It is the most widely used particle-hole interaction of contact type with spin- and isospin-exchange terms which goes backwards to Landau Fermi liquid theory (Ref. [32]); for the case of a nucleus it was established in the Theory of Finite
Fermi Systems \[33,29,34\] by summation of all graphs irreducible in the particle-hole direction.

This interaction can be written explicitly as follows

\[ V(\mathbf{r}_1, \mathbf{\sigma}_1 \mathbf{r}_2, \mathbf{\bar{\sigma}}_2) = C \delta(\mathbf{r}_1 - \mathbf{r}_2) \left[ f + f' \tau_1 \tau_2 + g \sigma_1 \sigma_2 + g' \tau_1 \tau_2 \sigma_1 \sigma_2 \right], \quad (15) \]

where \( C = \frac{\pi^2}{3 p_{fm}} = 300 \text{ MeV} \times \text{fm}^3 \) is the universal Migdal constant \[33,29,34\] and the strengths \( f, f', g, g' \) are in fact functions of \( r \) via density dependence: \( f = f_{in} - (f_{ex} - f_{in})(\rho(r) - \rho(0))/\rho(0) \) (the same for \( f', g, g' \)). (Quantities subscripted by “in” and “ex” characterize interaction strengths in the depth of the nucleus and on its surface, respectively). With its parameter values listed below, this interaction has been successfully used by many authors (see Refs. \[29\]) to quantitatively describe many properties of heavy nuclei.

The conventional choice of the constants widely used for heavy nuclei is (see \[33,29,34\]): \( f_{ex} = -1.95, f_{in} = -0.075, f'_{ex} = 0.05, f'_{in} = 0.675, g_{in} = g_{ex} = 0.575, g'_{in} = g'_{ex} = 0.725 \).

It can be seen that, in the same approximation of constant density as used above, the operator \( \hat{A} \) is proportional to \( \hat{\alpha} \): \( \hat{A} = i \tilde{\theta}(\mathbf{\sigma} \nabla) \). Evaluating the commutator in (13,14), we obtain

\[
\begin{align*}
&[\hat{A}, V_S] = \tilde{\theta}_1 C (f + f' \tau_1 \tau_2) \quad \tilde{\sigma}_1 [\nabla_1, \delta(\mathbf{r}_1 - \mathbf{r}_2)] + \\
&\quad + \tilde{\theta}_1 C (g + g' \tau_1 \tau_2) \quad \tilde{\sigma}_2 [\nabla_1, \delta(\mathbf{r}_1 - \mathbf{r}_2)] \\
&\quad + \tilde{\theta}_2 C (f + f' \tau_1 \tau_2) \quad \tilde{\sigma}_2 [\nabla_2, \delta(\mathbf{r}_1 - \mathbf{r}_2)] \\
&\quad + \tilde{\theta}_2 C (g + g' \tau_1 \tau_2) \quad \tilde{\sigma}_1 [\nabla_2, \delta(\mathbf{r}_1 - \mathbf{r}_2)] \\
&\quad - iC (g + g' \tau_1 \tau_2) \tilde{\sigma}_1 \times \tilde{\sigma}_2 \{\tilde{\theta}_1 \nabla_1 - \tilde{\theta}_2 \nabla_2, \delta(\mathbf{r}_1 - \mathbf{r}_2)\}. \quad (16)
\end{align*}
\]

Contrary to the case of P-odd, T-even weak interaction \[9, 10\], averaging over the core nucleons here yields nonzero result

\[ \langle [\hat{A}, V_S] \rangle \neq 0, \]

and, consequently, gives a nonzero contribution to the “compensation equation” (14). Taking together the terms with the same operator structures, we obtain from (14) and (17) equations of type \[9\]

\[ \tilde{\theta}(\mathbf{\sigma} \nabla) U = \theta(\mathbf{\sigma} \nabla) U + \gamma \frac{\rho(0)}{U(0)} (\mathbf{\sigma} \nabla) U. \quad (17) \]

which is equivalent to a system of two linear algebraic equation relating new (renormalized) interaction strengths \( \tilde{\eta}_{1,2} \) with their initial values \( \eta_{1,2} \) (without strong interaction). The solutions for this system of equations for the constants are the following:
\[ \eta_p = \frac{1}{D} \left[ \left( 1 + \tilde{C}_{gp} g_{pp} \frac{N}{A} \right) \left( \frac{Z}{A} \eta_{pp} + \frac{N}{A} \eta_{pm} \right) - \tilde{C}_{gn} g_{pn} \frac{N}{A} \left( \frac{Z}{A} \eta_{np} + \frac{Z}{A} \eta_{pp} \right) \right], \]

\[ \eta_n = \frac{1}{D} \left[ \left( 1 + \tilde{C}_{gp} g_{pp} \frac{Z}{A} \right) \left( \frac{N}{A} \eta_{nn} + \frac{Z}{A} \eta_{np} \right) - \tilde{C}_{gn} g_{pn} \frac{Z}{A} \left( \frac{Z}{A} \eta_{np} + \frac{N}{A} \eta_{pn} \right) \right], \]

(18)

with \( D = 1 + \tilde{C}_{gp} g_{pp} + 4 \tilde{C}_{gn} g_{pn} Z N/A^2 \). Here, \( \tilde{C} = C \rho/|U| = \frac{4}{3} \frac{\varepsilon_F}{|U|} = \frac{4}{3} (1 + B_n)^{-1} \simeq 1 \) and \( \eta_0^p \) and \( \eta_0^n \) are the initial values of the constants. We have used the well known relations:

\[ C = \frac{\pi^2}{p_F m}, \quad \rho = \frac{2p_F^3}{3\pi^2}, \quad \varepsilon_F = \frac{p_F^2}{2m}, \quad |U| = \varepsilon_F + B_n, \]

(19)

where \( p_F \) is a Fermi momentum, \( B_n \) is a nucleon separation energy. The renormalized matrix elements of the T-,P-odd weak potential for \(^{209}Pb\) are presented in the last column of Tables 1, 2. It is seen that the strong residual interaction reduces the values of the T-,P-odd potential constants 1.5...2 times, as average.

To this end, from (15) with account for (17) we obtain the resulting purely two-body T-,P-odd weak interaction in a nucleus that can be written as following:

\[ W^{P,T}_{eff} = : W^{P,T} : + W^{P,T}_{IPNCI} = \frac{G}{\sqrt{2} 2m} \]

\[ \times \left[ (\eta_{12} + \tilde{\eta}_{12} g_{12} \tilde{C}) \vec{\sigma}_1 - (\eta_{21} - \tilde{\eta}_{12} g_{12} \tilde{C}) \vec{\sigma}_2 \right] \left[ \tilde{\nabla}_1, \delta(\vec{r}_1 - \vec{r}_2) \right] + \tilde{C}(\tilde{\eta}_{12} \vec{\sigma}_1 - \tilde{\eta}_{21} \vec{\sigma}_2) [\tilde{\nabla}_1, \delta(\vec{r}_1 - \vec{r}_2) f_{12}(r_1)]: \\
- i \quad \vec{\sigma}_1 \times \vec{\sigma}_2 \left\{ (\tilde{\eta}_{12} g_{12} \tilde{C} + \eta_{12}^2) \tilde{\nabla}_1 - (\tilde{\eta}_{21} g_{12} \tilde{C} + \eta_{12}^2) \tilde{\nabla}_2, \delta(\vec{r}_1 - \vec{r}_2) \right\}, \]

(20)

where the constants \( \eta_1, \eta_2 \) are given by Eq. (19). We used here the fact that spin constant of the strong interaction (16) does not depend on \( r \), while the constants \( f_{pp} = f_{nn} = f(r) + f'(r), f_{pn} = f_{np} = f(r) - f'(r) \) do.

It should be noted that the induced T-,P-odd interaction \( W^{TP}_{TPNCI} \) has the same operator structure as the initial two-body T-,P-odd interaction \( W^{TP} \). Thus, \( W^{TP}_{TPNCI} \) differs from \( W^{TP} \) only due to renormalization of strength constants which turns out to be weak, because response of the nucleus to the T- and P-odd potential (4) as a function of the interaction constants has poles (\( D = 0 \)) at \( g = \tilde{C}^{-1} \simeq -1 \) and \( g' \simeq \tilde{C}^{-1} \simeq -1 \) (for \( N \simeq Z \)), while the actual nuclear strong interaction “drives” the solution of the renormalization equations (18) to the direction opposite to poles. As a result, induced T-,P-odd interaction does not play especial role in the present case and causes renormalization of order 1 ÷ 2. Thus there is essential difference
with the case of the P-odd, T-even weak interaction \[10\] where the analogous induced P-odd interaction is enhanced by about \(A^{1/3}\) times and practically dominates the results.

In practical calculations, it is convenient to treat \(W_{T\bar{P}}^{*}\) in the secondly quantized version using multipole expansion in the particle-hole channel:

\[ W_{T\bar{P}}^{*} = \frac{1}{2} \sum J ((a^+b)_J W_{T\bar{P}}^{*}^J_{abcd} (c^+d)_J) \]

where \((...)_J\) means the coupling of nucleon creators \(a^+\) and destructors \(a\) to a given angular momentum \(J\) \[28\]. The numerical results for some reduced matrix elements of \(W_{T\bar{P}}^{*}\) as compared to those of the initial interaction : \(W_{T\bar{P}}\) : between valence shell states for Th-U region are presented in the Table 3.

V. T-,P-ODD MATRIX ELEMENTS BETWEEN COMPOUND STATES

In the work (Ref. \[8\]) we have introduced a method to calculate Mean Squared Matrix Elements (MSME) of operators between compound states and have obtained the results for P-odd, T-even weak interaction. Here, we apply this method to calculation of MSME of P-,T-odd interaction. Consider the mean squared value of this matrix element:

\[ \overline{W_{T\bar{P}}^2} = \frac{\langle p|W_{T\bar{P}}|s\rangle\langle s|W_{T\bar{P}}|p\rangle}{\langle p| : W_{T\bar{P}} : + W_{T\bar{P}} \Gamma_{SP}^{NCI} |s\rangle \langle s| : W_{T\bar{P}} : + W_{T\bar{P}} \Gamma_{SP}^{NCI} |p\rangle} \] (21)

We can expand now the compound states \(|C^{J\pi}\rangle = |s\rangle, |p\rangle\) in terms of their simple components (multiparticle excitations) \(|\alpha^{J\pi}\rangle\) of the same quantum numbers of angular momentum \(J\) and parity \(\pi\),

\[ |C\rangle = \sum \alpha C_{\alpha} |\alpha\rangle, \] (22)

having for the MSME the expression

\[ \overline{W_{T\bar{P}}^2} = \sum_{\alpha\beta} C_{\alpha} C_{\beta} \frac{\langle p| : W_{T\bar{P}} : + W_{T\bar{P}} \Gamma_{SP}^{NCI} |\alpha\rangle \langle \alpha| : W_{T\bar{P}} : + W_{T\bar{P}} \Gamma_{SP}^{NCI} |p\rangle}{\langle p| : W_{T\bar{P}} : + W_{T\bar{P}} \Gamma_{SP}^{NCI} |s\rangle \langle s| : W_{T\bar{P}} : + W_{T\bar{P}} \Gamma_{SP}^{NCI} |p\rangle} \] (23)

The number of different terms in the Eq.(22), \(N\), is very large \(\sim 10^5 \div 10^7\). The main contribution in Eq.(22) is dominated by the set of \(\bar{N}\) “principal components” \(|\alpha\rangle\) with shell-model energies \(E_{\alpha}\) close to the energy of a compound state \(E\). We can make use of the statistical independence of the coefficients \(C_{\alpha}\) to take their second moments in the form (Ref. \[28\], \[35\]):

\[ C_{\alpha} C_{\beta} = C_{\alpha}^2 \delta_{\alpha\beta} = \delta_{\alpha\beta} \frac{1}{N} \Delta(\Gamma_{\text{spr}}, E - E_{\alpha}). \] (24)
Bar means the averaging over a rather broad set of the compound states. Here, the spreading width $\Gamma_{\text{spr}}$ is related to the number of principal components $N^{-1/2} \approx \sqrt{\frac{2d}{\pi \Gamma_{\text{spr}}}}$ and $d$ is the average energy distance between the resonances. The Breit-Wigner-type factor $\Delta$, describing cutting off of weights before states distanced in energy,

$$\Delta(\Gamma_{\text{spr}}, E - E_\alpha) = \frac{\Gamma_{\text{spr}}^2 / 4}{(E - E_\alpha)^2 + \Gamma_{\text{spr}}^2 / 4},$$

(25)

may be treated as a “spread” $\delta$-function. It is normalized as to be of order unity for $|E - E_\alpha| \leq \Gamma_{\text{spr}}/2$ and with conventional limit $\Delta(\Gamma_{\text{spr}}, E - E_\alpha) \rightarrow \frac{\pi \Gamma_{\text{spr}}}{2} \delta(E - E_\alpha)$ for $\Gamma_{\text{spr}} \rightarrow 0$. For the principal components, $|E_\alpha - E| \lesssim \Gamma_{\text{spr}}$, expression (25) reflects “chaotic” nature of a broad mixture of the simple components in the compound state due to the strong interaction. For the small (energy distanced) components it reduces to the perturbation theory result. From (23)-(25), we obtain for the MSME

$$\overline{W_{P,T}^{P,T}} = \sum_\alpha \frac{1}{N} \Delta(\Gamma_{\text{spr}}, E - E_\alpha)$$

(26)

\[
(p|: W_{P,T} : + W_{ITPNCI}^{P,T}|\alpha > < \alpha |: W_{P,T} : + W_{ITPNCI}^{P,T}|p).
\]

The argument of the function $\Delta$ here is the change of the energy: $E - E_\alpha = \epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d$, and $\tilde{V}$ is given by Eq.(6). Summation over $\alpha$ in (26) is equivalent to summation over different components of the operator $W_{P,T}^{P,T}$ in Eq.(5), i.e. the problem is reduced to the calculation of $(p|W_{P,T}^{P,T}W_{P,T}^{P,T}|p)$. The coefficients before the “principal” components $\tilde{C}_\alpha$ in (22) are governed by the microcanonical ensemble rule $^{28,33}$. Then, to calculate the averaging over p-resonance “principal” components $\overline{(p|...|p)}$ in $\overline{W_{P,T}^{P,T}}$, we use, instead of the present microcanonical ensemble, an equivalent canonical one. The latter can always be introduced for a system with a large number degrees of freedom by introducing the effective nuclear temperature $T$ and chemical potentials $\lambda_n, \lambda_p$. In the second quantization representation, the average expectation value in (26) is reduced to a canonical ensemble average with the standard contractor rules $(p|a^{\dagger}b|p) = \delta_{ab} \nu_a^T$, for $\nu_a^T$ being the finite temperature Fermi occupation probabilities, $\nu_a^T = \{\exp[(\epsilon_a - \lambda)/T] + 1\}^{-1}$. The canonical ensemble parameters $T, \lambda_{\tau}$ ($\tau$ means isospin projection) are to be determined from conventional “consistency” equations $E = \sum_a \nu_a \epsilon_a$, $Z = \sum_p \nu_p$, and $N = \sum_n \nu_n$ for the excitation energy $E$ (being equal the to neutron separation energy, $B_N$), nuclear charge $Z$, and neutron number $N$ correspondingly.

By means of the same considerations, we obtain the following result for $\sqrt{\overline{W_{P,T}^{P,T}}}$:
\[ \sqrt{\mathcal{W}_{P,T}^2} = \sqrt{\frac{2d}{\pi \Gamma_{spr}} \left( \frac{1}{2} \sum_{abcd} \nu_a^T (1 - \nu_b^T) \nu_c^T (1 - \nu_d^T) \right)} \]

\[ | \mathcal{W}_{eff \ ab,cd}^{P,T \ a} |^2 \Delta(\Gamma_{spr}, \epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d) \frac{1}{2}. \] (27)

Here, \( \Delta(\Gamma_{spr}, \epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d) \) can be viewed as an approximate energy conservation law with the accuracy up to width of states.

The numerical calculations for \(^{233}\)Th have been performed with the use of single-particle basis of states obtained by numerically (see Sec.II, Eq.(7) and below).

The value of temperature \( T = 0.6 \text{MeV} \) was used in accordance to the consistency condition for excitation energy. The result for the mean squared matrix elements of T-,P-odd interaction between compound states is

\[ \sqrt{\mathcal{W}_{P,T}^2} = 0.20 \eta_0 \text{meV}. \]

The ratio of the P,T-odd matrix elements to the P-odd ones is \( \sqrt{\mathcal{W}_{P,T}^2}/\sqrt{\mathcal{W}_{P}^2} = 0.1 \eta/g \).

Here, we use equal values of constants \( \eta_{12} \) in (2), \( \eta_{12} = \eta_0 \). The corresponding mixing coefficient for compound states \( |F_{sp}| \) is

\[ |F_{sp}| \simeq \frac{\sqrt{\mathcal{W}_{P,T}^2}}{|E_s - E_p|} \simeq 1 \cdot 10^{-5} \eta_0 \]

that is about \( 10^3 \) times larger than single-particle mixing \( f_{12} \simeq 10^{-8} \eta_0 \) (Eq.(8)). We assumed in this estimate that \( |E_s - E_p| = D_s \) where \( D_s \) is the average energy interval between compound resonances in s-wave.

**VI. CORRELATIONS BETWEEN T-,P-ODD AND P-ODD, T-EVEN MATRIX IN COMPOUND STATES**

The question on possible correlations between matrix elements of P-,T-odd weak interaction and those of P-odd, T-even weak interaction is very interesting. Knowing the correlator

\[ C(W^P, W^{P,T}) = \frac{(p|W^P|s)(p|W^{P,T}|s)}{\sqrt{\mathcal{W}_{P,T}^2} \sqrt{\mathcal{W}_{P}^2}} \] (28)

\( (0 < |C(W^P, W^{P,T})| < 1) \), one can make predictive estimates on the values and signs of the P-,T-odd effects in compound states basing on the information about the corresponding quantities for P-odd effects (the latter are much easier to be measured) in the case when the
quantity $C(W^P, W^{P,T})$ differs considerably from zero. $C(W^P, W^{P,T})$ can be calculated, in principle, by the same technique \cite{39} as mean squared matrix element \cite{30}. We can employ for the calculation of the numerator of Eq.(29) $c(W^P, W^{P,T}) = \langle p | W^P | s \rangle (s | W^{P,T} | p)$ the method of reducing to averaging over the ensemble

$$
c(W^P, W^{P,T}) = \frac{(p | W^P | s)(s | W^{P,T} | p)}{(p | W_{IPNCI}^P | s)(s | W^{P,T} : +W_{ITPNCI}^{P,T} | p)} \tag{29}
$$

However, in this case more careful treatment is needed. The present thermodynamical approach makes no difference between the cases when “external” averaging (canonical) goes over p-states either s-states provided the mean squared matrix elements are not very sufficient to change $\langle p | W^P | s \rangle (s | W^P | p) \rightarrow (s | W^P | p)(p | W^P | s)$. The latter is not the case for the quantity $c(W^P, W^{P,T})$. The reason is that the matrix elements of P-odd weak operator $W^P$ are imaginary and change sign when substituting final states instead of initial states. On the contrary, the matrix elements of P-,T-odd weak operator $W^{P,T}$ are real and symmetric under such substitution. Two-body matrix elements of $W^P$ and $W^{P,T}$ obey the following symmetry rules respectively:

$$
W^P_{ab,cd} = -W^P_{ba,dc} = -W^P_{dc,ba} = W^P_{cd,ab} \\
W^{P,T}_{ab,cd} = W^{P,T}_{ba,dc} = W^{P,T}_{dc,ba} = W^{P,T}_{cd,ab} \tag{30}
$$

As a result, we have some cancellations of the different terms in the sum of the products $W^{P,T}W^P$. Thus, a symmetrization should be done when the quantity $\bar{C}(W^P, W^{P,T})$ is calculated by the present method of reduction to the ensemble averaging:

$$
c(W^P, W^{P,T}) =
\frac{1}{2} \left[ (p | W_{eff}^{P,T} | s)(s | W_{IPNCI}^P | p) + (s | W_{IPNCI}^P | p)(p | W_{eff}^{P,T} | s) \right] =
\frac{1}{2} \left[ (p | W_{IPNCI}^P | s)(s | W_{eff}^{P,T} | p) - (s | W_{IPNCI}^P | p)(p | W_{eff}^{P,T} | s) \right] \tag{31}
$$

As a result of symmetrization and the negative sign before the second term in the last line, some cancellations of similar terms in the large sum of the same type as in Eq.(23) are possible. From the last equation, it is seen that we can not pretend to obtain the correct sign of the correlator within present statistical method, because the compound states of positive and negative parity are treated on the same footing. Without having an additional information.
about occupancies of particular single-particle levels with a given total angular momentum and parity, only absolute value of the correlator can be estimated.

After evaluation of commutator in Eq.(30) and thermal averaging we obtain the following expression for the numerator in (32)

\[
|c(W^P, \mathcal{W}^{P,T})| = \frac{d}{\pi \Gamma_{spr}} \left| \sum_{abcd} \nu_a^T (1 - \nu_b^T) \nu_c^T (1 - \nu_d^T) W_{ab,cd} \mathcal{W}_{eff} \right|_{dc,ba} \delta^T \times \Delta(\Gamma_{spr}, \epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d). \tag{32}
\]

Using this result and equations (24),(25),(26) we obtain, for the same value of temperature and the same single-particle basis as in the calculations of mean squared matrix elements, the following absolute value of the correlator (26) for the $^{233}$Th:

\[
| C(W^P, \mathcal{W}^{P,T}) | \approx 0.1.
\]

It means that correlations in matrix elements are weak. Of course, the present statistical calculation is estimative, and to obtain more definite result for correlator more refined technique is needed.

**VII. SUMMARY**

To conclude, we have considered the T-,P-odd nucleon interaction in heavy nuclei. Effects of the renormalization of this interaction are considered. An effective two-body T,P-odd interaction acting near the Fermi surface is obtained and its matrix elements are calculated. This interaction accumulates the effects of the distant states admixtures. We obtained the results for means squared values of T- and P-violating interaction between compound states of opposite parity. As well as in the case of P-odd, T-even weak interaction, statistical enhancement of T-,P-odd effects in neutron resonances take place. The enhancement is about $10^3$ times for the mixing between compound states of opposite parity as compared to the single-particle mixing.

Correlations between matrix elements of T-,P-odd and P-odd, T-even interactions in compound states are found to be weak within the statistical model.
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REFERENCES

[1] J.D. Bowman, C.D. Bowman, J.E. Bush, P.P.J. Delheij, C.M. Frankle, C.R. Gould, D.G. Haase, J. Knudson, G.E. Mitchel, S. Penttila, H. Postma, N.R. Robertson, S.J. Seestrom, J.J. Szymansky, V.W. Yuan, and X. Zhu, Phys. Rev. Lett. 65, 1192 (1990).

[2] C.M. Frankle, J.D. Bowman, J.E. Bush, P.P.J. Delheij, C.M. Frankle, C.R. Gould, D.G. Haase, J. Knudson, G.E. Mitchel, S. Penttila, H. Postma, N.R. Robertson, S.J. Seestrom, J.J. Szymansky, S.H. Yoo, V.W. Yuan, and X. Zhu, Phys. Rev. Lett. 67, 564 (1991).

[3] L.V. Inzhechik et al. Yad. Fiz. 44, 1370 (1986) [Sov. J. Nucl. Phys. 44, 890 (1986)], Zh. Eksp. Theor. Fiz. 93, 800 (1987) [Sov. Phys. JETP 66, 450 (1987)]; Zh. Eksp. Theor. Fiz. 93, 1569 (1987) [Sov. Phys. JETP 66, 897 (1987)]; V.G. Tsinoev, Talk at the International Symposium on Weak and Electromagnetic Interactions in Nuclei, Dubna 1992.

[4] V.V. Flambaum, I.B. Khriplovich, and O.P. Sushkov, Phys. Lett. 146B, 367 (1984); V.F. Dmitriev, V.V. Flambaum, O.P. Sushkov, and V.B. Telitsin, Phys. Lett., 125B, 1 (1983); V.V. Flambaum, V.B. Telitsin and O.P. Sushkov, Nucl. Phys. A444, 611 (1985).

[5] E.G. Adelberger and W.C. Haxton, Ann. Rev. Nucl. Part. Sci. 35, 501 (1985).

[6] N. Auerbach and J.D. Bowman, Phys. Rev. C46, 2582 (1992).

[7] O.P. Sushkov and V.B. Telitsin, Phys. Rev. C48, 1069 (1993).

[8] V.V. Flambaum and O.K. Vorov, Phys. Rev. Lett. 70, 4051 (1993).

[9] V.V. Flambaum and O.K. Vorov, Phys. Rev. C49, 1827 (1994).

[10] V.V. Flambaum and O.K. Vorov, submitted to Phys. Rev. C.

[11] J.F. Shriner, Jr., and G.E. Mitchel, Phys. Rev. C49, R616 (1994).

[12] P.A. Moldauer, Phys. Rev. 165, 1136 (1961).

[13] J.B. French, V.K.B. Kota, A. Pandey, and S. Tomsovic, Phys. Rev. Lett. 54, 2313 (1985).

[14] D. Boose, H.L. Harney, and H.A. Weidenmüller, Phys. Rev. Lett. 56, 2012 (1986).

[15] E. Blanke, H. Driller, W. Glöckle, H. Genz, A. Richter, and Schrieder, Phys. Rev. Lett. 51,
16] G.A.Lobov, Phys.At.Nucl. 56, 1330 (1993) [Sov. Yad.Fiz. 56, 53 (1993)].

[17] V.M.Khatseymovsk, I.B.Khirplovich, and A.S.Yelkhovsky, Ann.Phys.(NY) 186, 1 (1988).

[18] A.Schafer and E.G.Adelberger, Z.Phys. A339, 305 (1991).

[19] E.M.Henley and I.B.Khirplovich, Phys.Lett. B289, 223 (1992).

[20] A.Barroso and R.J.Blin-Stoyle, Phys.Lett. 45B, 178 (1973).

[21] W.C.Haxton and E.M.Henley, Phys.Rev.Lett. 51, 1937 (1983).

[22] O.P.Sushkov, V.V.Flambaum and I.B.Khirplovich, Sov.Phys.JETP 60(5), 873 (1984) [Zh.Eksp.Teor.Fiz. 87, 1521 (1984)].

[23] V.V. Flambaum, I.B. Khriplovich, and O.P. Sushkov, Nucl. Phys. A449, 750 (1986).

[24] S.K.Lamoreaux et al, J.P.Jacobs, B.R.Heckel,F.J.Raab and E.N.Fortson, Phys.Rev.Lett. 59, 2275 (1987).

[25] D.Cho, K.Sangster, and E.A.Hinds, Phys.Rev. A44, 1783 (1991).

[26] V.V.Flambaum, In. Proceedings of The International Symposium “Modern Developments in Nuclear Physics”, Novosibirsk, 1987 World Scientific, Singapore 1988, p.556.

[27] I.B.Khriplovich. Parity nonconservation in Atomic Phenomena, Gordon and Breach, New York, 1991; Ya.B.Zeldovich, ZhETFiz. 33, 1531 (1957).

[28] A. Bohr and B. Mottelson, Nuclear Structure (Benjamin, New York, 1969), Vol. 1.

[29] G.E.Brown, Rev.Mod.Phys., 43, 1 1971); V.Klemt, S.A.Moszkowski, and J.Speth, Phys.Rev. C14, 302 (1976); J.Speth, E.Werner, and W.Wild, Phys.Rep. 33, No.3, 127(1977) and references therein; G.Bertsch, D.Cha, and H.Toki, Phys.Rev. C24, 533 (1981); J.W.Negele, Rev.Mod.Phys. 54, 913 (1982); R.De Haro, S.Krewald, and Speth, Nucl.Phys., A388, 265 (1982); K. Goeke, and J.Speth, Annu. Rev. Nucl. Part. Sci. 32, 65 (1982); F.Osterfeld, Rev.Mod.Phys., 64, 491 (1992), and references therein.

[30] D.F. Zaretsky and V.I. Sirotkin, Yad. Fiz. 37, 607 (1983) [Sov. J. Nucl. Phys. 37, 361 (1983)]; 45, 1302 (1987) [45, 808 (1987)].
[31] S.G.Kadmensky, V.P. Markushev, and V.I. Furman, Yad. Fiz. 37, 581 (1983) [Sov. J. Nucl. Phys. 37, 345 (1983)].

[32] L.D.Landau, Zh.Eksp.Theor.Fiz. 30, 1058 (1956) [Sov.Phys.JETP 30, 920 (1956)]; Zh.Eksp.Theor.Fiz. 32, 59 (1957) [Sov.Phys.JETP 5, 101 (1957)]; Zh.Eksp.Theor.Fiz. 35, 97 (1958) [Sov.Phys.JETP 8, 70 (1959)].

[33] A.B.Migdal, Theory of Finite Fermi Systems and Applications to atomic Nuclei (John Wiley & Sons, New York, 1967).

[34] V.A.Khodel and E.E.Sapershtein, Phys.Rep. 92, 183 (1982), and references therein.

[35] O.P. Sushkov and V.V. Flambaum, Pis’ma Zh. Eksp. Teor. fiz. 32, 377 (1980) [JETP Lett. 32, 353 (1980)]; INP Reports Nos. 80-148, 1980 and 81-37, 1981; Usp. Fiz. Nauk 136, 3 (1982) [Sov. Phys. Usp. 25, 1 (1982)]; 16th LINP Winter School Proceedings, p. 200, (Leningrad, 1981); Nucl. Phys. A412, 13 (1984); S.G.Kadmensky, V.P. Markushev, and V.I. Furman, Yad. Fiz. 37, 581 (1983) [Sov. J. Nucl. Phys. 37, 345 (1983)].

[36] V.V.Flambaum, in Proc. of the Int. Conf. on Time Reversal Invariance and Parity Violation in Neutron Reactions, C.R.Gould, J.D.Bowman and Yu.P.Popov eds., Wrld.Sci., Singapore, 1994, p.39.
TABLE I. Single-particle matrix elements of P,T-odd weak interaction for protons. The levels closest to the Fermi energy are marked by asterisks.

| a     | b     | $e_a - e_b$ (MeV) | $w_{ab}^P$ eV | $w_{ab}^{TP}$ eV | $\tilde{w}_{ab}^{TP}$ eV |
|-------|-------|------------------|---------------|------------------|------------------|
| $2p_{3/2}$ | $2d_{3/2}^*$ | -8.554 | 0.513$g_{pp}$ + 0.748$g_{pn}$ | 0.080$\eta_{pp}$ + 0.098$\eta_{pn}$ | 0.053$\eta_{pp}$ + 0.065$\eta_{pn}$ +0.003$\eta_{pp}$ + 0.003$\eta_{pn}$ |
| $1g_{9/2}$ | $1h_{9/2}^*$ | -11.054 | 0.599$g_{pp}$ + 0.842$g_{pn}$ | 0.112$\eta_{pp}$ + 0.129$\eta_{pn}$ | 0.074$\eta_{pp}$ + 0.086$\eta_{pn}$ |
| $2p_{1/2}$ | $3s_{1/2}^*$ | -8.443 | -0.500$g_{pp}$ -0.722$g_{pn}$ | -0.066$\eta_{pp}$ -0.078$\eta_{pn}$ | -0.044$\eta_{pp}$ - 0.052$\eta_{pn}$ |
| $2p_{1/2}$ | $3s_{1/2}^*$ | -8.443 | -0.500$g_{pp}$ -0.722$g_{pn}$ | -0.066$\eta_{pp}$ -0.078$\eta_{pn}$ | -0.044$\eta_{pp}$ - 0.052$\eta_{pn}$ |
| $1g_{7/2}$ | $2f_{7/2}$ | -9.745 | -0.517$g_{pp}$ -0.720$g_{pn}$ | -0.068$\eta_{pp}$ -0.071$\eta_{pn}$ | -0.045$\eta_{pp}$ - 0.047$\eta_{pn}$ |
| $2d_{5/2}$ | $2f_{5/2}$ | -10.084 | 0.553$g_{pp}$ + 0.812$g_{pn}$ | 0.078$\eta_{pp}$ + 0.107$\eta_{pn}$ | 0.052$\eta_{pp}$ + 0.071$\eta_{pn}$ |
| $2d_{3/2}$ | $2p_{3/2}$ | 8.554 | -0.513$g_{pp}$ + 0.748$g_{pn}$ | 0.080$\eta_{pp}$ + 0.098$\eta_{pn}$ | 0.053$\eta_{pp}$ + 0.065$\eta_{pn}$ +0.003$\eta_{pp}$ + 0.003$\eta_{pn}$ |
| $2d_{3/2}$ | $3p_{3/2}$ | -8.732 | -0.558$g_{pp}$ -0.803$g_{pn}$ | -0.050$\eta_{pp}$ -0.073$\eta_{pn}$ | -0.033$\eta_{pp}$ - 0.048$\eta_{pn}$ |
| $3s_{1/2}^*$ | $3p_{1/2}$ | -9.186 | 0.549$g_{pp}$ + 0.806$g_{pn}$ | 0.055$\eta_{pp}$ + 0.091$\eta_{pn}$ | 0.037$\eta_{pp}$ + 0.060$\eta_{pn}$ +0.002$\eta_{pp}$ + 0.003$\eta_{pn}$ |
| $1h_{9/2}^*$ | $1g_{9/2}$ | 11.054 | -0.599$g_{pp}$ -0.842$g_{pn}$ | 0.112$\eta_{pp}$ + 0.129$\eta_{pn}$ | 0.074$\eta_{pp}$ + 0.086$\eta_{pn}$ |
| $1h_{9/2}^*$ | $2g_{9/2}$ | -9.417 | -0.575$g_{pp}$ -0.789$g_{pn}$ | -0.055$\eta_{pp}$ -0.064$\eta_{pn}$ | -0.037$\eta_{pp}$ - 0.042$\eta_{pn}$ |
| $2f_{7/2}$ | $1g_{7/2}$ | 9.745 | 0.517$g_{pp}$ + 0.720$g_{pn}$ | -0.068$\eta_{pp}$ -0.071$\eta_{pn}$ | -0.045$\eta_{pp}$ - 0.047$\eta_{pn}$ |
| $2f_{5/2}$ | $1d_{5/2}$ | 26.505 | 0.096$g_{pp}$ + 0.134$g_{pn}$ | -0.067$\eta_{pp}$ -0.066$\eta_{pn}$ | -0.044$\eta_{pp}$ - 0.044$\eta_{pn}$ |
| $2f_{5/2}$ | $2d_{5/2}$ | 10.084 | -0.553$g_{pp}$ -0.812$g_{pn}$ | 0.078$\eta_{pp}$ + 0.107$\eta_{pn}$ | 0.052$\eta_{pp}$ + 0.071$\eta_{pn}$ |
| $3p_{3/2}$ | $1d_{3/2}$ | 25.840 | -0.054$g_{pp}$ -0.063$g_{pn}$ | 0.036$\eta_{pp}$ + 0.036$\eta_{pn}$ | 0.024$\eta_{pp}$ + 0.024$\eta_{pn}$ |
| $3p_{3/2}$ | $2d_{3/2}^*$ | 8.732 | 0.558$g_{pp}$ + 0.803$g_{pn}$ | -0.050$\eta_{pp}$ -0.073$\eta_{pn}$ | -0.033$\eta_{pp}$ - 0.048$\eta_{pn}$ |
TABLE II. The same as in Table 1, but for neutrons.

| a     | b     | $e_a - e_b$ (MeV) | $w^P_{ab}$ eV | $w_{ab}^{TP}$ eV | $\tilde{w}_{ab}^{TP}$ eV |
|-------|-------|------------------|--------------|-----------------|-----------------|
| $3p_{3/2}^*$ | $2d_{5/2}$ | 7.784            | 0.541$g_{nn} + 0.778g_{np}$ | -0.071$\eta_{nn} - 0.048\eta_{np}$ | -0.040$\eta_{nn} - 0.027\eta_{np}$ |
| $3p_{3/2}^*$ | $3d_{3/2}$ | -8.733           | 0.446$g_{nn} + 0.661g_{np}$ | 0.060$\eta_{nn} + 0.026\eta_{np}$ | 0.033$\eta_{nn} + 0.015\eta_{np}$ |
| $2f_{5/2}^*$ | $2d_{5/2}$ | 10.055           | -0.536$g_{nn} - 0.790g_{np}$ | 0.108$\eta_{nn} + 0.078\eta_{np}$ | 0.060$\eta_{nn} + 0.044\eta_{np}$ |
| $2f_{5/2}^*$ | $3d_{5/2}$ | -6.633           | -0.539$g_{nn} - 0.773g_{np}$ | -0.044$\eta_{nn} - 0.017\eta_{np}$ | -0.024$\eta_{nn} - 0.009\eta_{np}$ |
| $2f_{5/2}^*$ | $2d_{5/2}$ | 10.055           | -0.536$g_{nn} - 0.790g_{np}$ | 0.108$\eta_{nn} + 0.078\eta_{np}$ | 0.060$\eta_{nn} + 0.044\eta_{np}$ |
| $2f_{5/2}^*$ | $3d_{5/2}$ | -6.633           | -0.539$g_{nn} - 0.773g_{np}$ | -0.044$\eta_{nn} - 0.017\eta_{np}$ | -0.024$\eta_{nn} - 0.009\eta_{np}$ |
| $3p_{1/2}^*$ | $1s_{1/2}$ | 34.531           | 0.004$g_{nn} - 0.003g_{np}$ | 0.023$\eta_{nn} + 0.022\eta_{np}$ | 0.013$\eta_{nn} + 0.012\eta_{np}$ |
| $3p_{1/2}^*$ | $2s_{1/2}$ | 23.964           | 0.037$g_{nn} + 0.050g_{np}$ | -0.044$\eta_{nn} - 0.042\eta_{np}$ | -0.025$\eta_{nn} - 0.023\eta_{np}$ |
| $3p_{1/2}^*$ | $3s_{1/2}$ | 8.811            | -0.528$g_{nn} - 0.775g_{np}$ | 0.090$\eta_{nn} + 0.054\eta_{np}$ | 0.050$\eta_{nn} + 0.030\eta_{np}$ |
| $3p_{1/2}^*$ | $4s_{1/2}$ | -6.645           | -0.452$g_{nn} - 0.660g_{np}$ | -0.035$\eta_{nn} - 0.012\eta_{np}$ | -0.020$\eta_{nn} - 0.007\eta_{np}$ |
| $2g_{9/2}$ | $1h_{9/2}$ | 7.590            | 0.561$g_{nn} + 0.770g_{np}$ | -0.058$\eta_{nn} - 0.048\eta_{np}$ | -0.033$\eta_{nn} - 0.027\eta_{np}$ |
| $3d_{5/2}$ | $1f_{5/2}$ | 24.931           | 0.023$g_{nn} + 0.039g_{np}$ | 0.012$\eta_{nn} + 0.004\eta_{np}$ | 0.006$\eta_{nn} + 0.002\eta_{np}$ |
| $3d_{5/2}$ | $2f_{5/2}^*$ | 6.633           | 0.539$g_{nn} + 0.773g_{np}$ | -0.044$\eta_{nn} - 0.017\eta_{np}$ | -0.024$\eta_{nn} - 0.003\eta_{np}$ |
| $4s_{1/2}$ | $1p_{1/2}$ | 37.016           | -0.038$g_{nn} - 0.040g_{np}$ | 0.001$\eta_{nn} + 0.006\eta_{np}$ | 0.000$\eta_{nn} + 0.003\eta_{np}$ |
| $4s_{1/2}$ | $2p_{1/2}$ | 23.364           | 0.039$g_{nn} + 0.053g_{np}$ | 0.009$\eta_{nn} - 0.004\eta_{np}$ | 0.005$\eta_{nn} - 0.002\eta_{np}$ |
| $4s_{1/2}$ | $3p_{1/2}^*$ | 6.645           | 0.452$g_{nn} + 0.660g_{np}$ | -0.035$\eta_{nn} - 0.012\eta_{np}$ | -0.020$\eta_{nn} - 0.007\eta_{np}$ |
| $2g_{7/2}$ | $1f_{7/2}$ | 28.563           | 0.034$g_{nn} + 0.068g_{np}$ | -0.066$\eta_{nn} - 0.061\eta_{np}$ | -0.037$\eta_{nn} - 0.034\eta_{np}$ |
| $2g_{7/2}$ | $2f_{7/2}$ | 11.326           | -0.518$g_{nn} - 0.770g_{np}$ | 0.101$\eta_{nn} + 0.061\eta_{np}$ | 0.057$\eta_{nn} + 0.034\eta_{np}$ |
| $3d_{3/2}$ | $2p_{3/2}$ | 25.118           | -0.021$g_{nn} - 0.020g_{np}$ | -0.028$\eta_{nn} - 0.015\eta_{np}$ | -0.016$\eta_{nn} - 0.008\eta_{np}$ |
| $3d_{3/2}$ | $3p_{3/2}^*$ | 8.733           | -0.446$g_{nn} - 0.661g_{np}$ | 0.060$\eta_{nn} + 0.026\eta_{np}$ | 0.033$\eta_{nn} + 0.015\eta_{np}$ |
TABLE III. Reduced matrix elements of T-,P-odd weak interaction $\mathcal{W}_{abcd}^{TP,J}$ for states of valence shells in the U-Th region in terms of weak constants $\eta_{12}$ (Eq.\((...)\)). $a \equiv \{p(n)l_{a}l_{a}l_{a}\}, p(n)$ means proton (neutron) states.

| $J$ | $a$     | $b$     | $c$     | $d$     | $\mathcal{W}_{abcd}^{TP,J}$ (eV) |
|-----|---------|---------|---------|---------|----------------------------------|
| 2   | $p1h_{9/2}$ | $p1h_{9/2}$ | $n1j_{15/2}$ | $bn1i_{11/2}$ | 0.021$\eta_{pn}'$ |
| 3   | $p1h_{9/2}$ | $p1h_{9/2}$ | $n1j_{15/2}$ | $n1i_{11/2}$  | $-0.003\eta_{pn} + 0.014\eta_{pn}'$ |
| 4   | $p1h_{9/2}$ | $p1h_{9/2}$ | $n1j_{15/2}$ | $n1i_{11/2}$  | 0.010$\eta_{pn}'$ |
| 5   | $p1h_{9/2}$ | $p1h_{9/2}$ | $n1j_{15/2}$ | $n1i_{11/2}$  | $-0.006\eta_{pn} + 0.009\eta_{pn}'$ |
| 6   | $p1h_{9/2}$ | $p1h_{9/2}$ | $n1j_{15/2}$ | $n1i_{11/2}$  | 0.006$\eta_{pn}'$ |
| 7   | $p1h_{9/2}$ | $p1h_{9/2}$ | $n1j_{15/2}$ | $n1i_{11/2}$  | 0.004$\eta_{pn}'$ |
| 8   | $p1h_{9/2}$ | $p1h_{9/2}$ | $n1j_{15/2}$ | $n1i_{11/2}$  | $-0.024\eta_{pn} + 0.005\eta_{pn}'$ |
| 9   | $p1h_{9/2}$ | $p1h_{9/2}$ | $n1j_{15/2}$ | $n1i_{11/2}$  | $-0.003\eta_{nn} - 0.009\eta_{nn}'$ |
| 2   | $n1i_{11/2}$ | $n1j_{15/2}$ | $n2g_{9/2}$ | $n2g_{9/2}$ | $0.001\eta_{nn} - 0.006\eta_{nn}'$ |
| 3   | $n1i_{11/2}$ | $n1j_{15/2}$ | $n2g_{9/2}$ | $n2g_{9/2}$ | $0.002\eta_{nn} - 0.004\eta_{nn}'$ |
| 4   | $n1i_{11/2}$ | $n1j_{15/2}$ | $n2g_{9/2}$ | $n2g_{9/2}$ | $0.002\eta_{nn} - 0.004\eta_{nn}'$ |
| 5   | $n1i_{11/2}$ | $n1j_{15/2}$ | $n2g_{9/2}$ | $n2g_{9/2}$ | $-0.001\eta_{nn} - 0.003\eta_{nn}'$ |
| 6   | $n1i_{11/2}$ | $n1j_{15/2}$ | $n2g_{9/2}$ | $n2g_{9/2}$ | $-0.004\eta_{nn} - 0.003\eta_{nn}'$ |
| 7   | $n1i_{11/2}$ | $n1j_{15/2}$ | $n2g_{9/2}$ | $n2g_{9/2}$ | $-0.001\eta_{nn} - 0.002\eta_{nn}'$ |
| 8   | $n1i_{11/2}$ | $n1j_{15/2}$ | $n2g_{9/2}$ | $n2g_{9/2}$ | $0.009\eta_{nn} - 0.002\eta_{nn}'$ |
| 9   | $n1i_{11/2}$ | $n1j_{15/2}$ | $n2g_{9/2}$ | $n2g_{9/2}$ | $-0.003\eta_{nn} - 0.010\eta_{nn}'$ |
| 2   | $n1j_{15/2}$ | $n1i_{11/2}$ | $n1i_{11/2}$ | $n2g_{9/2}$ | $0.002\eta_{nn} + 0.002\eta_{nn}'$ |
| 3   | $n1j_{15/2}$ | $n1i_{11/2}$ | $n1i_{11/2}$ | $n2g_{9/2}$ | $-0.003\eta_{nn} - 0.003\eta_{nn}'$ |
| 4   | $n1j_{15/2}$ | $n1i_{11/2}$ | $n1i_{11/2}$ | $n2g_{9/2}$ | $0.003\eta_{nn} + 0.001\eta_{nn}'$ |
| 5   | $n1j_{15/2}$ | $n1i_{11/2}$ | $n1i_{11/2}$ | $n2g_{9/2}$ | $-0.004\eta_{nn} - 0.001\eta_{nn}'$ |
| 6   | $n1j_{15/2}$ | $n1i_{11/2}$ | $n1i_{11/2}$ | $n2g_{9/2}$ | $0.003\eta_{nn}'$ |
| 7   | $n1j_{15/2}$ | $n1i_{11/2}$ | $n1i_{11/2}$ | $n2g_{9/2}$ | $-0.005\eta_{nn} - 0.001\eta_{nn}'$ |
| 8   | $n1j_{15/2}$ | $n1i_{11/2}$ | $n1i_{11/2}$ | $n2g_{9/2}$ | $0.003\eta_{nn}'$ |
| 9   | $n1j_{15/2}$ | $n1i_{11/2}$ | $n1i_{11/2}$ | $n2g_{9/2}$ | $0.003\eta_{nn}'$ |