Nonlinear Rolling Motion of Damaged Warship Based on Cell Mapping Method

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Abstract. Based on analyzing energy coupling between flooded ship and the water of flooded compartment in nonlinear rolling motion, the paper construct the two freedom rolling motion differential equation using the LAGRANGE equation, and acquire a single-freedom ordinary derivation equation through decoupling the coupled equations. Improving the arithmetic of cell mapping to study rolling cycle, multi-cycle and chaos response domain of the type II and type III flooded ship under excitation of various waves. The results show that the nonlinear rolling motion may be single-frequency or multi-frequency, and may be periodic or chaos with different system parameter and exterior excitation.

1. Introduction

Rolling, pitching and heave are the three most important forms that affect the use and safety of warship during warship swing in wave excitation. Especially through the study of roll motion, we can determine the conditions of warship capsizing[1], which is of great practical significance. However, in the long-term research process, the researchers found that when using linear model to study the ship rolling movement, the theoretical results are far from the actual situation[2], the main reason is that the large angle rolling itself has the nonlinearity of damping and restoring torque. Warship rolling includes a lot of complex nonlinear and chaotic phenomena. Aiming at the nonlinear dynamic model of rolling, many methods such as differential dynamical system theory, bifurcation theory and chaos theory are used to study rolling motion, which has made great progress.

In reference [3], the local bifurcation of ship rolling motion under parametric excitation is studied by using the multi-scale method, and the chaotic motion is studied by using the direct numerical integration method. In reference [4], under the condition that the attractor of the system is known, the cell mapping method is used to find the attractive region of the heave roll coupling system. When the damaged ship enters the water tank, its motion in the wave will become more complex. In dealing with the problem of the rolling motion of the damaged ship in waves, the paper[5] uses Hamilton variation principle to establish the differential equation of the rolling motion of the damaged ship with class II inlet tank in waves, and uses Lyapunov's stability theory to discuss the stability conditions of the rolling motion response of the damaged ship. Murashige of Japan Tokyo University have carried out a detailed study on the motion of ships with partial tanks flooded[6-7]. In recent years, the nonlinear motion of damaged warship is a key problem, because it will aggravate the chaos of the ship. In this paper, cell mapping method is used to analyze the nonlinear characteristics of the roll motion of the damaged warship in waves.
2. The basic principle of cell mapping

Cell mapping method is a numerical method used to analyze the global properties of nonlinear dynamical system. Its basic idea is to discrete the continuous state space into cell state space, the state variable into cell state variable marked by the whole order, and the dynamical system into cell mapping dynamical system. The generalized cell mapping dynamical system is first proposed by Hsu, which is a Markov chain probability model\cite{8}. The discrete continuous state space of the generalized cell mapping theory is the basic concept of the cell state space, and the generalized cell mapping method is used for the global analysis of the nonlinear dynamic system, which can give the self cyclic cell set, the permanent self cyclic cell set and the transient self cyclic cell set. This method is widely used in the research of the nonlinear dynamic system\cite{9,10}. In this paper, the algorithm given by reference \cite{11} is improved to study the roll characteristics of the damaged intake ship. The specific calculation steps are as follows.

**Step 1: calculate the transfer closed cell matrix** $TR_{a_1}$

Defining the transitive closed cell matrix $TR_{a_1} = [t_{ij}]$:

$$t_{ij} = \begin{cases} 1 & p_{ij}^{(n)} > 0 \\ 0 & p_{ij}^{*n} = 0 \end{cases}$$

$p_{ij}^{(n)}$ is the element of row $i$ and column $j$ of the probability transfer matrix $P^n$. The row $i$ and column $j$ elements of the transfer closed cell matrix $TR_{a_1}$ are 1, which indicates that the $i$ state cell can reach the $j$ state cell through several steps of mapping of the dynamic system.

Set the new matrix $TR_{a_1}(j,i) = MR_{a_1}$, for $i (i < n)$. If all $TR_{a_1}(j,i) = 1$,

then $TR_{a_1}(j,k) = TR_{a_1}(j,k) \lor TR_{a_1}(i,k)$, where $k = 1,2,\ldots,n$ ($n$ is the total number of state cells). Where the symbol $\lor$ represents Boolean, $0 \lor 0 = 0$, $0 \lor 1 = 1$, $1 \lor 0 = 1$, $1 \lor 1 = 1$.

**Step 2: calculate the self circulating cell set vector $QS$**

The self cycle cell $Q = TR_{a_1} \otimes TR_{a_1}^T$. The superscript $T$ represents the transposition of the matrix and the symbol $\otimes$ represents the product of the elements., $Q = [q_{ij}]$, then $q_{ij} = t_{ij} \cdot t_{ij}$.

$$q_{ij} = \begin{cases} 1 & i \leftrightarrow j \\ 0 & \text{other} \end{cases}$$

For $i (i < n)$, if $A = \sum_{j=1}^{n} Q(i,j) \neq 0$, then $QS(j) = \sum_{k=1}^{n} q_{ik} q_{kj}$. If $QS(j) \neq 0$, then $QS(j) = 1$ . The information of the self cycle cell set is given by the self cycle cell set vector $QS$.

**Step 3: calculate the period of self cycle cell set**

The one-step adjacency matrix $M_{SCS}$ is obtained by adjacency matrix $M_{Ra_1}$ and self cycle cell set vector $QS$.

$$d_{M_{SCS}} = g.c.d\{i\mid traceM_{i_{SCS}}^j = 1, 1 \leq i \leq n_a\}$$

In the formula, $d_{M_{SCS}}$ represents the period of the self cycle cell set. $n_a$ is the number of all cells in the self cycle cell set. $M_{i_{SCS}}$ is the sub Boolean of $M_{SCS}$ $traceM_{i_{SCS}}^j$ is the trace of $M_{SCS}^j$ or the Boolean sum of all diagonal elements of $M_{SCS}^j$, and $g.c.d$ represents the maximum common divisor.
Step 4: determine the self circulating cell set and its attraction domain

For $i \ (i \leq n)$, $TM(i) = QS(i)$, then $ATM(i) = \sum_{j=1}^{n} M_{rs}(i,j) \wedge QS(j) \vee TM(i)$.

If $A = \sum_{i=1}^{n} TM(i) \neq 0$, then $TM(i) = ATM(i) - TM(i)$.

$\sum$ represents Boolean summation, operator '-' is performed according to the following rules:

$$a - b = \begin{cases} 1 & a - b > 0 \\ 0 & a - b \leq 0 \end{cases}$$

The vector $ATM$ contains the information of the self cycle cell set and all the attracting domain cells. Further, we can get the attracting domain and the boundaries set of the attracting domain of each cycle cell set, and thus complete the global qualitative analysis of the whole dynamic system.

3. The nonlinear rolling motion of damaged warship

3.1. The rolling motion equation of damaged warship

Based on the analysis of the energy coupling between the ship and the water in the tank when the ship is rolling in the wave, the two degree of freedom differential equation of the ship rolling with the tank is established according to the Lagrange equation, and the two degree of freedom motion equation is decoupled, so that the two degree of freedom problem can be transformed into a single degree of freedom problem. After the complicated derivation process [12, 13], the nonlinear roll motion equation of the damaged watercraft is simplified as follows:

$$[1 + a_2 \cos(2\omega t) + a_4 \cos(4\omega t) + a_6 \cos(6\omega t) + b_3 \sin(\omega t) + b_5 \sin(3\omega t) + b_7 \sin(5\omega t) + a_0 \theta^2 + \frac{e_2}{\omega} \sin(2\omega t) + e_4 \sin(4\omega t) + e_6 \sin(6\omega t)] \dot{\theta} + \left[(\pm \theta + \theta^3) + [f_2 \cos(2\omega t) + f_4 \cos(4\omega t) + g_1 \sin(\omega t) + g_3 \sin(3\omega t)] \theta + [h_2 \cos(2\omega t) + h_4 \cos(4\omega t) + i_1 \sin(\omega t) + i_3 \sin(3\omega t)] \theta^3 + [p_6 + p_2 \cos 2\omega t + p_4 \cos 4\omega t + q_1 \sin \omega t + q_3 \sin 3\omega t] \dot{\theta}^2 = j + k_2 \cos(2\omega t) + k_4 \cos(4\omega t) + l_1 \sin(\omega t) + l_3 \sin(3\omega t) + l_5 \sin(5\omega t)]$$

The above equation is a joint equation of parametric excitation and external excitation. When the damaged ship has large area of water inflow, the original equation is Doffing Holmes type. The results show that the numerical values of damping coefficient $d_0$, parametric coefficient $i$ and $q_1$, external excitation coefficient $l_i$ are larger than those of other systems.

In order to study the global properties of nonlinear rolling of damaged ship, the above formula is studied by cell mapping. Due to the limitation of computing time and computer capacity, the region $\theta \times \dot{\theta} = [-2.0, 2.0] \times [-3.5, 3.5]$ is divided into cells of $60 \times 60$, and each cell is further divided into daughter cells of $3 \times 3$. With the centre of each cell as the initial value, the fourth-order Runge Kutta method is used for numerical integration for a period.

3.2. The research of rolling motion of damaged warship

In this paper, the cell mapping method is used to analyze the roll motion of a damaged ship under the condition of symmetrical water inflow in class II and symmetrical water inflow in class III. The length of the ship is 155m, the width of the ship is 15.6m, the draft is 4.6m and the displacement is 5280t.
A. Rolling characteristics of the ship with symmetrical water inflow of class II Tank.

In this paper, the specific parameters of the condition of symmetrical water inflow of class II tank are two tanks. The water inflow inside the tank is not connected with the water outside the tank. The depth of water inflow inside the tank is 4.6 meters, the volume of water inflow is 1269.6, and the draft of the real ship after water inflow is 5.314. The calculation results under this condition by cell mapping method are shown in Fig.1 ~ Fig.3. In the figure, the abscissa is the dimensionless angular displacement $\theta$, the ordinate is the dimensionless angular velocity $\dot{\theta}$, the blank area is the attraction domain of the self circulating cell set, and "×" is the self circulating cell set.

![Figure 1](image1.png)

**Figure 1.** Cell mapping and phase diagram of wave force of $2.31 \times 10^5$N.m and frequency of 0.41rad/s

![Figure 2](image2.png)

**Figure 2.** Cell mapping and phase diagram of wave force $2.72 \times 10^7$N. m and frequency 0.41rad/s

![Figure 3](image3.png)

**Figure 3.** Cell mapping and phase diagram of wave force $2.31 \times 10^7$N.m , and frequency 0.41rad/s

B. Rolling characteristics of the ship with symmetrical water inflow of class III Tank.

In this paper, the specific parameters of the condition of symmetrical water inflow of class III tank are the water inflow of two tanks, the water inflow in the tank is connected with the overboard water, and the actual draft of the ship is 5.49m under the state of positive floating after water inflow. The calculation results under this condition by cell mapping method are shown in Fig.4 ~ Fig.6.

![Figure 4](image4.png)

**Figure 4.** Cell mapping and phase diagram of wave force $5.24 \times 10^3$N. m and frequency 0.41rad/s
5. Conclusions

From the above results, we can get the following rules of the damaged watercraft under the action of wave force.

a). Under the action of small wave moment, the damaged ship only has periodic motion; when the wave moment increases, there will be chaotic attractor. If the moment of wave excitation continues to increase, a periodic motion with large amplitude and several times of wave period will appear. The chaotic motion of damaged ship not only has single period motion, but also has sub harmonic motion of double period and super harmonic motion of double frequency.

b). Due to large area of water inflow, the damaged ship has negative initial stability, and the ship motion control equation becomes Holmes type, which is the basic reason why the damaged ship is prone to chaotic motion.

c). The Hyper harmonic motion of the damaged ship is closely related to the coupling effect of the water in the cabin on the ship. The coupling effect of the water in the cabin on the ship is more and more significant with the increase of the external excitation. Therefore, super harmonic motion is another main feature of damaged ship motion.

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