Some consequences of the Majoron being the dark radiation

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Abstract

We discuss some phenomenological consequences in a scenario where a singlet Majoron plays the role of dark radiation. We study the interrelations between neutrino mass generation and the scalar potential arising from this identification. We find the extra scalar has to be light with a mass at or below the GeV level. The mixing of this scalar with the Standard Model Higgs impacts low energy phenomena such as the muonic hydrogen Lamb shift and muon anomalous magnetic moment. Demanding that the light scalar solves the puzzle in the muon annihilation era yields a contribution to the muon magnetic moment requires the scalar to be lighter still with mass at or below the 10 MeV level. The cross-sections for the production of heavy neutrinos at LHC14 are also given.

1. Introduction

It is well known that correlations of temperature fluctuations in the Cosmic Microwave Background (CMB) depends on the number of effective relativistic degrees of freedom, \(N_{\text{eff}}\), which is usually given in terms of the effective number of neutrino species present in the era before recombination. The expected value of \(N_{\text{eff}} = 3\) is consistent with observations thus far. However, recent measurements of CMB from the Planck satellite \([1]\) combined with that of the Hubble constant from the Hubble Space Telescope (HST) \([2]\) resulted in a higher value of \(N_{\text{eff}} = 3.83 \pm 0.54\) at 95\%CL. If one further includes data from WMAP9 \([3]\), Atacama Cosmology Telescope (ACT) \([4]\) and South Pole Telescope (SPT) \([5]\) into the analysis, the extracted value becomes \(N_{\text{eff}} = 3.62^{+0.50}_{-0.48}\) at 95\%CL. This hints at a dark radiation (DR) component beyond the expected three neutrino species at a confidence level of 2.4\sigma. The origin and nature of such DR component is not known. One possibility, as pointed out recently by Weinberg \([6]\), is that it can be naturally associated with a massless or nearly massless Goldstone boson arising the spontaneous breaking of a \(U(1)\) global symmetry. A Goldstone boson will count as 4/7 of a neutrino, and this appears to agree with observation. However, in order for the temperature of the Goldstone bosons to match that of the neutrinos, they must remain in thermal equilibrium with ordinary matter until muon annihilation. If Goldstone bosons decouple much earlier, they will contribute less than 4/7 to \(N_{\text{eff}}\) as they will not be reheated but the neutrinos do. Decoupling in the muon annihilation era yields a contribution \(\delta N_{\text{eff}} = 0.39\). Weinberg further proposed that the \(U(1)\) global symmetry be a new one associated with a dark sector with its own matter content.

In this letter we examine the possibility of taking the global \(U(1)\) symmetry to be the lepton number. The spontaneous breaking of this \(U(1)_L\) by singlet Higgs will give rise to a Majoron \([7]\), which we associate with the Goldstone boson that acts as DR. Since the singlet that breaks the \(U(1)_L\) will mix with the Standard Model (SM) Higgs boson, this allows us to connect Higgs physics and DR to neutrino physics. In particular, we are able to link constraints on the parameters of the scalar sector to that in the seesaw mechanism responsible for neutrino mass generation, and to study their interrelations. We illustrate this in Type-I seesaw \([8]\) and inverse seesaw \([9]\) scenarios in this letter.

The organisation of this letter is as follows. In Sec.\textsuperscript{2} we describe in detail the framework we use to study the interrelation between the Majorons, the neutrinos, and the scalars. In Sec.\textsuperscript{3} we discuss some consequences on the scalar and neutrino parameters from measurements of the muon magnetic moment, Lamb shift of the muonic hydrogen, and decay rate of \(\mu \rightarrow e\gamma\). In Sec.\textsuperscript{4} we evaluate the range of values of heavy neutrino masses and mixings that can be probed at the LHC. We end with a summary in Sec.\textsuperscript{5}.

2. The framework

The simplest Majoron model extends the Standard Model (SM) by three generations of singlet righthanded (RH) neutrinos, \(N_R^i\), and a singlet complex scalar, \(S\) \([7]\). The relevant Yukawa interactions read

\[
L \rightarrow -y_1 \bar{L}_R H N_R - y_2 N_R^c N_R S + h.c. \tag{1}
\]

where \(L = (n_L, e_L)^T\) is the left-handed (LH) SM lepton doublet, and \(H = i\sigma_2 H^\dagger\) with \(H\) the SM Higgs; the generation indices are suppressed for clarity. Note that there is an accidental global \(U(1)\) symmetry associated with the conservation...
of lepton number \((L)\) before electroweak symmetry breaking (EWSB) if \(S\) is defined to have \(L = -2\). After EWSB, we can write \(H = (0, (\nu + h)/\sqrt{2}, h^T)\) in the unity gauge, where \(v = 246.221\) GeV, and
\[
S(x) = \frac{1}{\sqrt{2}} (v_S + s(x)) e^{2\alpha (s(x)).}
\]
The Yukawa interactions then give rise to neutrino masses, which take the form
\[
\mathcal{L} \supset -\left(\begin{pmatrix} N_L \ N_R \ 0 \end{pmatrix} \frac{m_D}{M} \begin{pmatrix} n^c_L \ n^c_R \ 0 \end{pmatrix} + h.c.,
\]
where \(m_D = 2^{-3/2} y_1 v, M \sim y_2 v_S / \sqrt{2}\), and we have redefined the lepton fields, \(\psi_i \rightarrow e^{i \alpha} \psi_i\), to remove the \(e^{i \alpha}\) phase factor from the Majorana mass terms. Note that this induces the interactions \(\partial_\mu \bar{\psi} \gamma^\mu \psi\) from the lepton kinetic terms \(\bar{\psi} \gamma^\mu \partial_\mu \psi\).

If \(v = m_D / M \ll 1\), the standard Type-I seesaw is operative. It is well known in this case that the light active neutrinos have masses \(m_{\nu} \approx m_D\). To have \(m_{\nu} \lesssim 0.1 \) eV, we require
\[
y_1 = 2^{3/4} \left(\frac{y_2 y_S v}{v}\right)^{1/2} \lesssim 3.05 \times 10^{-6} \left(\frac{y_S v}{\text{TeV}}\right)^{1/2}.
\]

As a benchmark, take \(y_S = 1\) TeV and \(y_2 = 1\). Then acceptable light neutrino masses can be obtained with \(y_1\) the size of the electron Yukawa couplings, \(y_e\). We shall refer to couplings with sizes smaller than \(y_e\) as excessively fine tuned.

Type-I seesaw is not the only way neutrino masses can be generated, however. A phenomenologically more interesting case is the inverse seesaw. The inverse seesaw can be implemented by adding – in addition to the three RH singlet neutrinos – three more LH singlet neutrinos. The relevant Yukawa interactions given in Eq. (1) are now augmented to
\[
\mathcal{L} \supset -y_1 \sum H N_K - y_2 N^c_L R^c - y_2 N^c_L S - M_D N_L N^c_R + h.c.
\]
where \(N_{L,R}^c\) are the LH and RH singlet neutrinos, and \(M_D\) is a Dirac mass parameter. Note that a Yukawa coupling of the form \(\bar{L} \nu H N^c_L\) is forbidden by the global \(U(1)\) lepton number. As above, the scalar phase can be removed by the appropriate lepton field redefinitions.

Mass terms arise after EWSB:
\[
\mathcal{L} \supset \left(\begin{pmatrix} N_L \ N_R \ 0 \end{pmatrix} \frac{m_D}{M} \begin{pmatrix} n^c_L \ n^c_R \ 0 \end{pmatrix} + h.c.,
\]
where \(m_D = 2^{-3/2} y_1 v, M_{\mu,L} \ll M_D\), it is useful to first go to a basis where large quantities are on the diagonal:
\[
\left(\begin{pmatrix} N_L \ N_R^c \end{pmatrix} = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ -1 \end{array}\right) \left(\begin{pmatrix} N_L \ N_R \end{pmatrix}.
\]
The mass matrix in the basis \(n_L, n_c, n_R\) is then
\[
\begin{pmatrix} 0 & -m_D / \sqrt{2} & -M_D + \mu_c \\ -m_D / \sqrt{2} & 0 & \mu_c \\ -M_D + \mu_c & \mu_c & 0 \end{pmatrix},
\]
where \(\mu_c = (\mu_L + \mu_R) / 2\). To leading order in \(m_D / M_D\) and \(\epsilon_\alpha / M_D\), the mass eigenstates are given by
\[
\begin{align*}
\nu_L &= n_L - \frac{m_D}{\sqrt{2}} N_L - \frac{\epsilon_\alpha}{\sqrt{2}} N^c_R, \\
\eta_{1L} &= n_L + \frac{\epsilon_\alpha}{\sqrt{2}} n^c_L - \frac{m_D}{\sqrt{2}} N^c_R, \\
\eta_{2L} &= N_R + \frac{\epsilon_\alpha}{\sqrt{2}} n^c_R + \frac{m_D}{\sqrt{2}} N^c_L,
\end{align*}
\]
with mass eigenvalues \(\epsilon^2_\alpha \mu_L, M_D - \mu_c\), and \(M_D + \mu_c\), respectively (after appropriate phase rotations).

The interactions between the Majoron and neutrinos arise from the neutrino kinetics terms. To leading order in \(\epsilon_\alpha / M_D\), they read
\[
\frac{\partial_{\mu} \chi}{2 s} \left(\eta_1 \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right) - \frac{\epsilon_\alpha}{\sqrt{2}} \left(\eta_1 \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right) - \frac{1}{2} \left(\eta_1 \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right) \left(\eta_1 \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right),
\]
where \(\bar{\nu}_L \eta_1 \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right) \left(\eta_1 \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right) \left(\eta_1 \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right)
\]
Consider now the scalar sector. The most general renormalizable Lagrangian for it is given by
\[
L_{\text{scalar}} = -\frac{g_\nu Z_\nu}{2 \cos \theta_{\nu}} \left(\bar{\nu}_L \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right) (\eta_1 + \eta_2) + \frac{\epsilon_\alpha}{\sqrt{2}} \left(\bar{\nu}_L \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right) (\eta_1 + \eta_2) + \frac{1}{2} \left(\bar{\nu}_L \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right) \left(\eta_1 \gamma^\mu \bar{\nu}_L \eta_1 + \eta_2 \gamma^\mu \bar{\eta}_1 \right)
\]
\[
- \mu_\nu^2 S^+ \tilde{S} + \lambda_3 (S^+ S)^2 + \lambda_5 (H^T H)^2
\]
\[
- \mu_\nu^2 S^+ \tilde{S} + \lambda_3 (S^+ S)^2 + \lambda_5 (H^T H)^2
\]
After EWSB, the kinetic term for \(S\) takes the form
\[
\partial_{\mu} S^\dagger \partial^\mu S = \frac{1}{2} \partial_{\mu} S^\dagger \partial^\mu S + 2 (\bar{\nu}_S + s^2) \partial_{\mu} \partial^\mu \alpha
\]
\[
= \frac{1}{2} \partial_{\mu} S^\dagger \partial^\mu S + \frac{1}{2} \partial_{\mu} \partial^\mu \alpha + \frac{1}{2} \partial_{\mu} \partial^\mu \alpha + \frac{1}{2} \partial_{\mu} \partial^\mu \alpha
\]
\[
and we identify the canonically normalized Goldstone boson \(\chi \equiv 2 \nu_S \alpha\) as the Majoron.

The mixing between the Higgs doublet and the complex singlet was already analyzed in Ref. [11]. The classical minimum is given by
\[
\nu_S^2 = \frac{4 \lambda_5 \mu^2 - 2 \lambda_5 \mu_S^2}{4 \lambda_5 S_F - \lambda_{5S}}, \quad \nu_S^2 = \frac{4 \lambda_5 \mu^2 - 2 \lambda_5 \mu_S^2}{4 \lambda_5 S_F - \lambda_{5S}}.
\]
Using this, the scalar mass-squared matrix reads in the \((h, s)\) basis
\[
\begin{pmatrix}
\bar{\nu}_S^2 \lambda_{HS} S_F \\
\frac{\lambda_{HS} S_F}{2 \lambda_S} v_S^2
\end{pmatrix}
\]
(19)
which has eigenvalues
\[ m_{1,2}^2 = \lambda v^2 + \lambda S v_S^2 = \sqrt{\left(\lambda S v_S^2 - \lambda v^2\right)^2 + \lambda_{HS}^2 v^2 v_S^2}. \tag{20} \]

The physical mass eigenstates are then
\[ \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h_1^0 \\ h_2^0 \end{pmatrix}, \tag{21} \]
with mixing angle
\[ \tan 2\theta = \frac{\lambda_{HS} v v_S}{\lambda S v_S^2 - \lambda v^2}. \tag{22} \]

We shall identify \( h_1 \equiv h_{SM} \) as the SM Higgs, which was recently discovered at the LHC to have a mass of 125 GeV. Note that for small mixing (which shall be the case below), \( m_{h_{SM}}^2 \approx 2h v^2 \) and \( m_{h_2}^2 \approx 2\lambda S v_S^2 \).

From Eqs. (20) and (22), the scalar quartic couplings can be written in terms of the mass eigenvalues and the mixing angle
\[ \lambda = \frac{1}{4v^2} \left[ m_1^2 + m_2^2 - (m_2^2 - m_1^2)c_{2\theta}\right], \tag{23} \]
\[ \lambda_S = \frac{1}{4v_S^2} \left[ m_1^2 + m_2^2 + (m_2^2 - m_1^2)c_{2\theta}\right], \tag{24} \]
\[ \lambda_{HS} = \frac{m_2^2 - m_1^2}{2v_S^2} s_{2\theta}, \tag{25} \]
and we define the short hand \( c_\varepsilon \equiv \cos \varepsilon \) etc. Classical stability of the vacuum demands that
\[ \lambda, \lambda_S > 0, \quad 4\lambda \lambda_S - \lambda_{HS} = \frac{m_1^2 m_2^2}{v^2 v_S^2} > 0, \tag{26} \]
and we see from above that these conditions are automatically satisfied for \( m_{1,2} \) real and positive.

Because of the scalar mixing, the SM Higgs can decay into a pair of Majorons. The partial width is given by
\[ \Gamma_{h_{SM} \to \chi \chi} = \frac{s_{\varepsilon}^2 m_{h_{SM}}^3}{32\pi v_S^2}. \tag{27} \]

From the LHC, the Higgs invisible decay branching ratio is about 19\% \cite{12}. With the Higgs width at about 4.1 MeV \cite{13}, this means that
\[ \Gamma_{h_{SM} \to \chi \chi} \lesssim 0.8 \text{ MeV} \implies \frac{v_S}{v_{so}} \gtrsim 4.93 \text{ TeV}. \tag{28} \]

Currently, the LHC data on Higgs gauge boson couplings is consistent with SM expectations, which suggests small mixings with possible extended scalar sectors beyond the SM. As a benchmark, we take \( s_{\varepsilon}^2 < 0.1 \), and we get \( v_S \gtrsim 1.5 \text{ TeV} \). Note that this lower bound on \( v_S \) is relaxed if the bound on the mixing angle, \( \theta \), becomes more stringent.

The scalar mixing at tree-level also give rise to the following effective interaction between the Majoron and the SM fermions:
\[ \mathcal{L}_{fJXX} = \frac{\lambda_{HS} m_f}{m_{h_{SM}}^2} \frac{m_f}{m_{h_2}} \bar{f} \gamma_\mu \partial_\mu \chi \chi. \tag{29} \]

As pointed out in Ref. \cite{6}, if the Majoron is to play the role of dark radiation that give rise to the fractional value of \( N_{eff} \) measured, it should stay in thermal equilibrium until roughly the time when muon annihilation happens. Then this requires the collision rate of Majorons with muons to be roughly the Hubble expansion rate:
\[ \lambda_{HS}^2 m_f^2 \rho_{\chi} \approx 1 \implies m_h \approx 9.3 \sqrt{|\lambda_{HS}|} \text{ GeV}, \tag{30} \]
where we take \( m_{h_{SM}} = 125 \) GeV. With the help of Eqs. \cite{25} and \cite{27}, we get from Eq. \cite{30}
\[ m_2^2 = \sqrt{X} \left[ 1 - Y^2 \right], \quad Y = \frac{\sqrt{X}}{m_{h_{SM}}^2}, \quad X = \frac{32\pi \Gamma_{h_{SM} \to \chi \chi} c_\varepsilon^2 m_f^2}{v_S^2}. \tag{31} \]

Then with \( \Gamma_{h_{SM} \to \chi \chi} \lesssim 0.8 \text{ MeV} \), we obtain \( m_2 \lesssim 1.05 \text{ GeV} \) from \( c_\varepsilon < 1 \), and from Eq. \cite{24} and the benchmark \( c_{\varepsilon}^2 > 0.9 \):
\[ \lambda_S \lesssim 7.82 \times 10^{-4} \left( \frac{\text{TeV}}{v_S} \right)^2. \tag{32} \]

We see that in order to have no excessive fine tuning in the couplings, the lepton number breaking scale (as given by \( v_S \)) should be in the range of 1 to 30 TeV. On the other hand, it is well known that Type-I seesaw scenarios generally prefer a much higher scale, so \( v_S \) should be very large, which then requires an excessive tuning of \( \lambda_S \). There is thus a tension between the very high scale Type-I seesaw and the identification of Majoron as DR in such scenarios.

Such tension, however, can be circumvented in the inverse seesaw scenario. As is seen in Eq. \cite{6}, there are two scales that control the size of active neutrino masses, viz. \( M_D \) and \( v_S \). Without pushing \( \lambda_S \) to the nonperturbative region, we can take \( v_S \) to be as low as \( O(10) \) GeV and still easily have \( m_2 \lesssim 0.1 \text{ eV} \) for the active neutrinos. For example, take \( v_2 \sim 10^2 \), then \( M_D \) can be as low as a few hundred GeV (as long as \( \epsilon_D \sim 10^{-3} \)). This is an interesting region for LHC to look for heavy neutrinos that mix with the active ones, which we explore below in Sec. \cite{6}.

We note that low values of \( v_S \) imply small mixings with the SM Higgs.

3. Consequences from low energy physics

3.1. Muon magnetic moment and Lamb shift

Due to scalar mixing, the light extra scalar \( h_2 \) has coupling \( c_\mu = -\frac{2^{1/2} G_F^{1/2} m_\mu}{s_\varepsilon} \) to the muon arising from the Higgs Yukawa interactions with the fermions and is directly proportional to \( s_\varepsilon \). Its contribution to the muon magnetic moment is given by \cite{14}
\[ \delta a_\mu = \frac{(c_\mu)^2}{8\pi^2} \int_0^1 \frac{dz}{z^2 + r(1-z)} = \frac{(c_\mu)^2}{8\pi^2} H_5(r), \tag{33} \]
where \( r = m_2^2/m_{h_2}^2 \), and
\[ H_5(r) = \frac{3}{2} - r + \frac{r(r-3)}{2} \log r - (r-1) \frac{\sqrt{r} - \sqrt{r-4}}{2}. \tag{34} \]
Currently, the discrepancy between the experimental and theory value of $a_\mu = (g - 2)_\mu / 2$ is [15]

$$\delta a_\mu = (249 \pm 87) \times 10^{-11}. \quad (35)$$

We show in Fig. 1 the parameter space that this is accounted for by the $h_2$ contribution. We see that only when $m_{h_2} < 0.02 \text{ GeV}$ is the benchmark bound on the mixing angle satisfied. We see also that the benchmark allowed parameter space has $\delta a_\mu$ lower than its current central value.

Given the benchmark allowed parameter space, we can work out how much the additional scalar, $h_2$, contributes to the Lamb shift in muonic hydrogen. The energy difference from the 2P-2S splitting in hydrogen is given by [14, 15, 16]

$$\Delta E = -\frac{e^2}{\alpha} \frac{m_{h_2}^2 (m_e \alpha)^3}{4 \pi (2m_e + m_e \alpha)^5}, \quad (36)$$

where $m_e = m_\mu m_\mu / (m_\mu + m_\mu)$ is the reduced mass, and $c_\mu^0 = -2\alpha G_F^2 m_\mu \xi s_\theta$ is the effective Yukawa couplings of $h_2$ to the proton, with $\xi = 0.3 \sim 0.5$ [19]. Fig. 2 shows the magnitude of this energy shift in the parameter space allowed by both $\delta a_\mu$ and our LHC benchmark. We see that the maximum Lamb shift coming from $h_2$ alone is about 210 $\mu eV$, and this requires a very light $h_2$ with mass about 1 MeV. Although this is a significant portion of the 310 $\mu eV$ needed to reconcile the current 7σ discrepancy between the Committee on Data for Science and Technology (CODATA) value on the proton charge radius [20] – which is determined purely from electron scatterings – and that measured from muonic hydrogen Lamb shift [21], it is not enough on its own. To solve the proton radius puzzle, additional ingredients besides $h_2$ is necessary.

3.2. The radiative decay $\mu \to e \gamma$

The radiative $\mu \to e \gamma$ decay here is mediated by both $W$ and heavy Majorana neutrino exchanges at one-loop. We can place limits on the neutrino mixings from the branching ratio of this decay. The gauge-invariant effective operator for $\mu \to e \gamma$ has the form $\frac{1}{192 \pi^2} e H \bar{F}_\mu \gamma^\mu$, with the heavy neutrino mass, $M$, the controlling scale here. For simplicity, we assume that the $\eta^b W^\pm$ couplings are flavor universal given by $e g_2 / \sqrt{2}$. We shall use $\eta$ and $\epsilon$ here and below to denote generically the heavy Majorana neutrinos and their mixings with the light neutrino respectively (e.g. $\epsilon = m_D / M$ for Type-I seesaw, $\epsilon = \epsilon_D / \sqrt{2}$ for inverse seesaw). The effective Lagrangian then reads

$$e g_2^2 \frac{e^2}{16 \pi^2} \frac{m_{h_2}^2}{M^2} \left( \hat{R} \hat{F}_\mu \gamma^\mu \right). \quad (37)$$

Following Ref. [10], the corresponding dipole coefficient is estimated as

$$A_L = \frac{e g_2^2 e^2}{32 \sqrt{2} \alpha l} \frac{1}{G_F M^2} = \frac{e^2}{8 \pi^2} \frac{M^2}{M^2} \leq 10^{-12}, \quad (38)$$

and thus the branching ratio of $\mu \to e \gamma$

$$Br(\mu \to e \gamma) = \frac{384 \pi^2 |A_L|^2}{112} \simeq 24 \frac{\alpha}{\pi} \frac{M_{h_2}^4}{M^4} \leq 10^{-12}, \quad (39)$$

which implies

$$\epsilon \lesssim 0.0142 \frac{M}{(1 \text{ TeV})}. \quad (40)$$

This constraint can be easily accommodated in both neutrino mass models.

4. Drell-Yan production of heavy neutrinos at the LHC

At the parton level, the Drell-Yan (DY) production of the heavy Majorana neutrino, $\eta$, at the LHC proceeds predominantly through two processes: $q\bar{q}(p_1) + \bar{q}(p_2) \to Z^0 \to \eta + \bar{\eta}$ and $W^\pm(p_1) + \bar{W}^\pm(p_2) \to W^- \to \eta + \bar{\eta}$. The parton level cross-sections read

$$\hat{s}_Z(\hat{s}) = \frac{g_2^2 + g_2^2}{384 \pi} \frac{g_2^2}{c_W^4} \left[ \frac{1}{4} \left( \hat{s} - M_{h_2}^2 \right)^2 \left( 1 + \frac{m^2_{\eta}}{2 \hat{s}} \right) + \frac{m^2_{\eta}}{2 \hat{s}} \right], \quad (41)$$

$$\hat{s}_W(\hat{s}) = \frac{g_2^4}{384 \pi} \left[ \frac{1}{4} \left( \hat{s} - M_{h_2}^2 \right)^2 \left( 1 + \frac{m^2_{\eta}}{2 \hat{s}} \right) + \frac{m^2_{\eta}}{2 \hat{s}} \right]. \quad (42)$$
where \( g_{LR} = T_{1R}^2 - Q_{LR} \sin^2 \theta_{11} \), \( \epsilon \) is the heavy-light neutrino mixing, and \( \delta = (p_1 + p_2)^2 = x_1 x_2 s \) with \( s \) the center-of-mass (CM) energy, and \( x_{1,2} \) the parton momentum fractions.

The production cross-section at the LHC is obtained after a convolution with the parton distribution functions. In Fig. 5 we show the inclusive production cross-section obtained using MadGraph 5 \([22]\) for the LHC at 14 TeV CM energy. The production cross-section is normalised to the heavy-light neutrino mixings magnitude squared, \( |\epsilon|^2 \). The on-shell heavy Majorana neutrino, \( \eta \), subsequently decays into \( Z \nu, W^\pm \nu, h_{SM} \), with branching fractions roughly 20%, 40%, 40% respectively. It is very unlikely to produce a heavy Majorana neutrino at LHC in the Type-I seesaw scenario, given that it would be very heavy \( m_{\eta} \gg 1 \text{ TeV} \), and the mixings involved are very small. For the inverse seesaw scenario however, since the heavy Majorana neutrino can be relatively light at a few hundred GeV and the mixing relatively large, it is possible to probe directly this scenario at LHC14, which is expected to have a luminosity of \( O(100) \text{fb}^{-1} \). In particular, we see from Fig. 5 that for \( m_{\eta} \sim 100 \text{ GeV} \), one could have \( \sigma \sim 1 \text{ fb} \) if \( |\epsilon|^2 \sim 10^{-5} \), which is easily obtainable in inverse seesaw. We leave a detailed study of the collider signals involved to future works.

5. Summary

We have studied in this letter the implications of identifying the Majoron as DR. Assuming that it goes out of equilibrium at the muon annihilation temperature, it can account for the fractional value of the effective neutrino species, \( N_{\text{eff}} \), measured recently by Planck. The consequence of this for the extended scalar sector associated with the Majoron is the presence of a very light scalar boson with mass \( \lesssim 1.05 \text{ GeV} \) that mixes with the SM Higgs. Furthermore, the scale of the extended scalar sector, \( \tau_S \), cannot be too high if excessive fine tuning of the parameters in the scalar potential is to be avoided.

The scalar sector scale, \( \tau_S \), also sets the scale for the neutrino sector. A relatively low \( \tau_S \) would however cause tension with the canonical Type-I seesaw scenario, which typically require a heavy scale above \( 10^{12} \text{ GeV} \). On the other hand, such tension would not arise in the inverse seesaw scenario. There, one can taking \( \tau_S \) to be as low as a TeV without fine tuning either the Yukawa couplings or the scalar parameters, although consistency with the current LHC data then requires the mixing between the scalar singlet and the Higgs doublet to be very small. This then implies that the corrections to the SM Higgs couplings will be not measurable at the LHC.

Low energy physics can provide further constraints on the light scalar mass. By demanding that the light scalar account for discrepancy between the experimental and theory value of the muon magnetic moment while consistent with the LHC data, the light scalar mass is pushed down to below 0.02 GeV. Although not able to completely solve the proton radius puzzle on its own, the light scalar can contribute a significant amount towards the 310 \( \mu_e \text{eV} \) of the muonic hydrogen Lamb shift required if it is even light with mass at around 1 MeV.

Finally, we are hopeful that the inverse seesaw scenario may be directly probed at LHC14 given that the heavy neutrino can be relatively light and the mixing relatively large.

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References

[1] P. A. R. Abe et al (Planck Collaboration) [arXiv: 1303.5076 [astro-ph.CO]]
[2] A. G. Riess et al, Astrophys. J. 730, 119 (2011), [Erratum-ibid 732, 129(2011)] [arXiv:1103.2976 [astro-ph.CO]]
[3] G. Hinshaw et al [ arXiv: 1212.5226 [astro-ph]]
[4] J. L. Sievers et al at arXiv 1301.0824 [astro-ph.CO]]
[5] Z. Hou et al [ arXiv. 1212.6267 [astro-ph.CO]]
[6] S. Weinberg. Phys. Rev. Lett. 110. 241301 (2013) [arXiv:1305.1971 [astro-ph.CO]]
[7] Y. Chukashige, R. N. Mohapatra and R. D. Peccei, Phys. Lett. B 98, 265 (1981).
[8] P. Minkowski, Phys. Rev. B 67, 421 (1977); T. Yanagida Workshop on Unified Theories (KEK rep79-18, 95 (1979); M. Gell-Mann, P. Ramond, and R. Slansky Supergavity eds. P. van Nieuwenhuizen and D. Freedman 315 (North Holland, Amsterdam, 1979); S. L. Glashow Cargese Summer Institute on Quarks and Leptons ed. M. Levy, p687 (Plenum Press, N.Y. 1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 912 (1980)
[9] D. Wyler and L. Wolfenstein, Nucl. Phys. B218, 205 (1983) R. N. Mohapatra and J.W.F.Waff, Phys. Rev. D 34 1642 (1986)
[10] W. -F. Chang and J. N. Ng, Phys. Rev. D 71, 053003 (2005) [hep-ph/0501161]
[11] W. -F. Chang, J. N. Ng and J. M. S. Wu, Phys. Rev. D 75, 115016 (2007), [arXiv: [hep-ph]]
[12] P P. Giardino, K. Kannike, I. Masina, M. Raidal and A. Strumia, [arXiv:1303.3570 [hep-ph]]
[13] A. Djouadi, Phys. Rept. C457 1 (2008)
[14] J P. Leveille, Nucl. Phys. B137, 63 (1978); D. McKeen. Ann. Phys. 326 1501 (2011)
[15] A. Hoecker and W. J. Marciano, Phys. Rev. D 86, 010001 (2012)
[16] C. E. Carlson and B. C. Rislow, Phys. Rev. D 86, 035013 (2012)
[arXiv:1206.3587 [hep-ph]].
[17] D. Tucker-Smith and I. Yavin, Phys. Rev. D 83, 101702 (2011)
[arXiv:1011.4022 [hep-ph]].
[18] K. Pachucki, Phys. Rev. A 53, 2092 (1996).
[19] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, Front. Phys. 80, 1
(2000).
[20] P. J. Mohr, B. N. Taylor and D. B. Newell, Rev. Mod. Phys. 84, 1527
(2012) [arXiv:1203.5425 [physics.atom-ph]].
[21] R. Pohl, A. Antognini, F. Nez, F. D. Amaro, F. Biraben et al., Nature
(London) 466, 213 (2010).
[22] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer and T. Stelzer, JHEP
1106 128 (2011)