M/String Theory, S-branes and Accelerating Universe

Michael Gutperle
Department of Physics and Astronomy, UCLA, Los Angeles, CA and
Department of Physics, Stanford University, Stanford, CA 94305-4060, USA

Renata Kallosh and Andrei Linde
Department of Physics, Stanford University, Stanford, CA 94305-4060, USA

Recently it was observed that the hyperbolic compactification of M/string theory related to S-branes may lead to a transient period of acceleration of the universe. We study time evolution of the corresponding effective 4d cosmological model supplemented by cold dark matter and show that it is marginally possible to describe observational data for the late-time cosmic acceleration in this model. However, investigation of the compactification $11d \rightarrow 4d$ suggests that the Compton wavelengths of the KK modes in this model are of the same order as the size of the observable part of the universe. Assuming that this problem, as well as several other problems of this scenario, can be resolved, we propose a possible solution of the cosmological coincidence problem due to relation between the dark energy density and the effective dimensionality of the universe.

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I. INTRODUCTION

The problem of understanding the origin of a dark energy in the framework of the fundamental M/string theory after the recent WMAP data \([1]\) became even more urgent than before: the basic conclusion from all previous observations that $\sim 70\%$ of the energy density of the universe is in a dark energy sector has been confirmed. This implies the existence of a stage of late-time acceleration of the universe. Moreover, the possibility of the early universe acceleration (inflation) is also supported by the data. Thus one would like to derive the accelerated 4-dimensional universe from the fundamental 11/10-dimensional M/string theories.

Recently models of the compactified non-perturbative string theory have been found which have de Sitter space vacua \([2]\). These models give us a possibility to describe the late-time acceleration in the non-perturbative string theory: the corresponding equation of state $w = -1$ reflects the fact that a de Sitter minimum is attained and we have a small positive cosmological constant.

Another interesting new development is based on the study of the time-dependent M/string theory solutions, S-branes \([3-7]\) with hyperbolic compactification of the internal 7/6-dimensional space \([3-14]\). It has been discovered in 8 that some solutions of 11d supergravity describe a period of accelerated 4-dimensional cosmology. It was pointed out in 7 that S-brane solutions in a more general case may also lead to accelerating cosmologies. A natural question arises: what is the equation of state $w = p/\rho$ for the dark energy in these models. Here it is important to stress that this function $w$ may depend on time $t$, or redshift $z$. The most recent observational constraint on $w$ from WMAP+ supernova requires that $w < -0.78$. However, this observational constraint is valid only under assumption that $w = \text{const.}$

An interpretation of the S-brane solutions with accelerating cosmologies was given in 12: a 4-dimensional effective action was presented there whose time dependent solutions are exactly the time dependent solutions of the 11-dimensional supergravity with time-dependent compactification. The corresponding 4-dimensional cosmology was studied in 11 in the setting where the total cosmological evolution at all time is affected only by the dark energy stress-tensor, $\Omega_D = 1$. We would like to make such models more realistic, i.e. to add matter to describe the observed universe which was matter dominated in the past, and which presently has $\Omega_D \equiv \rho_D / \rho_{\text{total}} \approx 0.7$ and $\Omega_M \equiv \rho_M / \rho_{\text{total}} \approx 0.3$. For this purpose one should find out where the matter comes from in the compact hyperbolic compactifications. One may think of several possibilities:

1. There is a brane world construction, where one adds a space filling 3-brane which carries the matter fields, as suggested in 15.

2. The compact hyperbolic space $H_n/\Gamma$ is basically given by a freely acting orbifold (no fixed points). One could also consider orbifolds which have fixed points. In the discussion of M-theory on $G_2$ manifolds, matter is localized at singularities of the $G_2$ manifold \([10-13]\). The problem is that in the hyperbolic context in M-theory one does not have any clear idea what happens at the orbifold singularities.

3. One could consider CY spaces with metrics which have negative curvature.

At present none of these possibilities is clear and moreover there are conceptual problems in introducing 4d cold dark matter starting with higher dimensional theory \([12]\). Here we propose to reinterpret the S-brane models of \([8-14]\) as follows: from the M/string theory we find out the relevant exponential scalar field potentials for the effective 4-dimensional cosmologies. Such potentials serve as a part of the effective cosmological model describing the late-time accelerating universe. Then we introduce
density of matter phenomenologically, by adding to the Friedmann equations the energy density $\rho_M$ decreasing as $a^{-3}(t)$, where $a(t)$ is the scale factor. The resulting Friedmann equations are different from the pure dark energy evolution (or S-brane solutions of 11-dimensional theory). We will find the solutions of these equations numerically and identify “today” with the point in time when $\Omega_D \sim 0.7$. This will also allow us to calculate the relevant evolution of the equation of state for the M-theory type dark energy, see Section III.

In fact, the calculations of precisely this type have been already performed in \[19\], see Section V on “M-theory and Dark Energy with Exponential Potentials”. Now the new S-brane type models give a specific combination of the exponential potentials. We will study these models following the methods developed in \[19\] and we will find equation of state $w(t) = p_D/p_D$ for the new models.

This approach can be quite reasonable from the point of view of the effective 4d theory. However, as we will see, investigation of the compactification $11d \to 4d$ suggests that the Compton wavelengths $m_{\text{KK}}^{-1}$ of the Kaluza-Klein modes in this model are of the same order as the size of the observable part of the universe. If this is the case, our universe would be effectively 11d, see Section II. On the other hand, if one finds a way to make the KK modes heavy, one may encounter large quantum corrections $O(m_{\text{KK}}^{-1})$ to the cosmological constant. These problems may completely rule out this model.

Nevertheless, the models of this type have some interesting features which may deserve further investigation. In particular, in Section IV we will show that if the problems mentioned above can be solved, one can simultaneously find a solution of the cosmological coincidence problem due to relation between the dark energy density and the effective dimensionality of the universe.

II. THE MODEL

For M-theory case one starts with 11-dimensional Lagrangian

$$I = \frac{1}{16\pi G_{11}} \int d^4x \sqrt{-G} \left( R[G] - \frac{1}{2} \times 4! F_4^2 \right).$$

The time dependent solution is a warped product of a four-dimensional spacetime and an internal compact space with $R_{ab}(\Sigma) = -6g_{ab}r_c^{-2}$, where $r_c$ is the radius of curvature of the internal space:

$$ds^2 = e^{-7M(x)} g_{\mu\nu}(x) dx^\mu dx^\nu + e^{2M(x)} dx^2.$$  

The field strength is taken as $\ast F_{[4]} = b \text{vol}(\Sigma)$ and $g_{\mu\nu}$ is the Einstein metric in four dimensions. Upon the dimensional reduction we get \[12\]

$$I = \frac{1}{16\pi G_4} \int d^4x \sqrt{-g} \left( R[g] - \frac{63}{2} (\partial M)^2 - 2V(M) \right),$$

where

$$V(M) = \frac{b^2}{4} e^{-21M} + 21 e^{-9M} r_c^{-2}.$$  

The relation between the gravitational Newton constants in 11 and 4 dimensions is

$$G_{11} = V_7 G_4,$$

where $V_7$ is a constant time independent volume of the internal compact space $\Sigma$. The 4-dimensional metric defining the 11-dimensional solution is given by

$$g_{\mu\nu}(x) dx^\mu dx^\nu = -S^6(\xi) dx^2 + S^2(\xi) dx^2.$$  

Using $t$ as a proper time of the 4d observer with $dt = \pm S^3(\xi) d\xi$ it becomes

$$g_{\mu\nu}(x) dx^\mu dx^\nu = -dt^2 + S^2(\xi(t)) dx^2.$$  

Note that we are using a notation $t$ for the proper time of the 4d observer to avoid a confusion which notation of $t$ may cause, where the proper time of the 4d observer is called $\eta$. In standard FRW cosmology $\eta$ is reserved for a conformal time $d\tau^2 = a^2(\eta)(-d\eta^2 + dx^2)$.

We may change variables so that $\frac{63}{2} M^2 = \left( \frac{\phi}{M_{Pl}} \right)^2$ and

$$L = \frac{M_{Pl}^2}{2} R[g] - \frac{1}{2} (\partial \phi)^2 - V[\phi],$$

where

$$V[\phi] = \frac{b^2 M_{Pl}^2}{4} e^{-\sqrt{3} \phi} M_{Pl} - 21 e^{-\sqrt{3} \phi} M_{Pl}^2 \left( \frac{M_{Pl}}{r_c} \right)^{-2}.$$  

We will now fix the Planck mass $M_{Pl}^2 = (8\pi G_4)^{-1} = 1$ and find a canonical form of the 4-dimensional Lagrangian,

$$\frac{1}{2} R[g] - \frac{1}{2} (\partial \phi)^2 - \frac{b^2}{4} e^{-\sqrt{3} \phi} - 21 e^{-\sqrt{3} \phi} r_c^{-2}.$$  

The solution of equations following from this 4d Lagrangian is the same as the one from 11d.

$$S(\xi) = \left( \frac{b \cosh(3(\xi - \xi_0))}{3} \right)^{\frac{1}{3}} \left( \frac{\sqrt{\frac{3}{28} \sinh(6\sqrt{\frac{3}{28}\xi})}}{\sinh(6\sqrt{\frac{3}{28}\xi})} \right)^{\frac{3}{72}},$$

$$\phi(\xi) = \sqrt{\frac{28}{3}} \left( \ln\left( \frac{b \cosh(3(\xi - \xi_0))}{3} \right) - \ln\left( \sqrt{\frac{28}{3}} \sinh(6\sqrt{\frac{3}{28}\xi}) \right) \right).$$

The potential for this model has two exponential terms $\sim e^{\lambda \phi}$ with $\lambda^2 = 14$ and $\lambda^2 = \frac{18}{27} \approx 2.57$. One expects
to find the current value of the dark energy from the potential

\[ V(\phi) = \frac{b^2}{4} e^{-\sqrt{7}\phi} + 21 e^{-\sqrt{7}\phi} r_c^{-2} \approx 10^{-120}. \]  

(13)

The contribution from the 4-form field with \( \lambda^2 = 14 \) is way too steep. The only possibility to have a reasonable late-time cosmology is in a regime where this exponent is small, for example at very large positive values of \( \phi \) assuming that the factor \( b \), specifying the form field, is not huge. In this case, which is equivalent to the absence of the 4-form field contribution, we may now study the case \( \lambda^2 \approx 2.57 \). Note that only the hyperbolic compactification is relevant to get the positive potential in this case. The second exponent has a factor \( \sqrt{14} \approx 1.6 \). It was explained in [12, 21] that for \( \lambda \lesssim 1.7 \) a reasonable description of data is possible with fine-tuned initial conditions.

Thus it is not impossible to use M-theory motivated potentials for dark energy. This would mean that the term \( 21 e^{-9M/2}r_c^{-2} \) in Eq. (13) is of the order \( 10^{-120} \) in 4d Planck units. This raises the following concern: What prevents us to see the extra 7 dimensions? In particular, we would like to establish whether the Kaluza-Klein reduction of the 11-dimensional metric [2] and its interpretation as a 4d cosmology is internally consistent.

The relation between the curvature radius \( r_c \) and the volume of the seven dimensional manifold \( V_2 \) is given by \( V_2 = r_c^7 \alpha^6 \), where \( \alpha \) depends on the topology of the manifold [17]. The mass of the massive Kaluza-Klein modes is determined by the spectrum of Laplace operators on the seven dimensional manifold. It is believed [17] that there is a mass gap, and the masses are bounded below by

\[ m_{KK} = c e^{-9M/2} r_c^{-1}, \]  

(14)

where \( c = O(1) \) is some constant. In our model, this has an important impact. For \( V(M) \sim 21 e^{-9M/2} r_c^{-2} \), the present value of the Hubble constant is given by \( H^2 \approx V/3 \sim 7 e^{-9M} r_c^{-2} \), so the mass gap is

\[ m_{KK} \sim \sqrt{V} \sim H \sim 10^{-60}. \]  

(15)

This means that the mass gap is practically nonexistent, the Compton wavelengths of KK modes are the same as the size of the cosmological horizon, and therefore particles should be able to freely move in all 10 dimensions of space.

The only possible loophole in this argument that we are able to see is to assume that one can find spaces with \( V_7 = r_c^7 e^{\alpha} \ll r_c^7 \), i.e. with \( e^\alpha \ll 1 \). In this case it might happen that the KK masses will be given by \( V_7^{-1/7} \approx r_c^{-1} e^{-\alpha/7} \). To make this mass scale greater than 1 TeV one would need to have an incredibly small volume \( V_7 \lesssim 10^{-315} r_c^7 \). In fact, what really matters is the bound on the lowest eigenvalue of the Laplace operators on the seven dimensional compactified hyperbolic manifold. Not much is known about it at present, and the estimate \( V_7^{-1/7} \) for the KK mass in such spaces is very speculative. Anyway, it seems unlikely that the lowest level of the KK mass in such models is phenomenologically viable [22].

Even if we were able to find a way to ensure the large values for the KK masses, we would need to encounter another problem related to these masses. There can be an additional contribution to the potential for the scalar field \( \phi \) in Eq. (13) coming from the Casimir energy of the compact space \( H_n/\Gamma \) [22, 24]. For a manifold which breaks supersymmetry the contribution is generically of order \( \lambda_s^4 \), where \( \lambda_s \) is the supersymmetry breaking scale. In our case this term would be proportional to \( m_{KK}^4 \). For \( m_{KK} \gtrsim 1 \) TeV, this term would be 60 orders of magnitude greater than the present value of dark energy \( \sim 10^{-120} \).

In [22, 26] it was noted that in principle odd dimensional compact hyperbolic orbifolds could exist which preserve some Killing spinors. It would be very interesting to check this claim by constructing a concrete example. Furthermore there are no calculations of the Casimir energy, going beyond the dimensional analysis given above, for hyperbolic orbifolds in more than three dimensions. For the rest of the paper we will assume that the Casimir energy is vanishing due to existence of Killing spinors or small compared to the dominant term in the potential [13]. However, one should remember that if this term is not negligible, it can completely invalidate the model.

Thus we tend to conclude that the model is not really working for the purpose of describing current acceleration of the universe in a way, consistent with the fundamental eleven-dimensional M-theory. It has an advantage in principle over the models of eleven-dimensional supergravity with “non-compactification” [27] whose 4d cosmology was studied in [14]. This advantage, the existence of the finite gap for KK states, is invalidated by the extraordinary small value of the dark energy, which the model was designed to explain. Still, as in case of N=8 gauged supergravity studied in [19] we may study the 4d cosmology of the 4d model in eq. (10) since it is derived from M-theory and may therefore inherit some of its properties.

III. ACCELERATING UNIVERSE

A. Dark energy without matter

First we will study the S-brane dark energy models in the context of pure dark energy without additional matter, just to get the feeling about the relevant equation of state. Since we know the scale factor \( S(\xi(t)) \) we may find the equation of state of this kind of dark energy

\[ w = \frac{p}{\rho} = -\frac{1}{3} - \frac{2}{3} \frac{\dot{S}S}{S^2} = -1 - 2\eta, \]  

(16)
where \( q = -\frac{\dot{S}}{S} \) is the deceleration parameter and \( \dot{S} \equiv \frac{dS}{dt} = S^{-3} \frac{dS}{d\xi} \). It can be also given in terms of the \( \xi \)-time derivatives, \( S' \equiv \frac{dS}{d\xi} \)

\[
\frac{w}{3} = 1 - \frac{\dot{S}}{S} - \frac{2}{3} \frac{S''S}{(S')^2} \quad (17)
\]

since an analytic expression for \( S(\xi) \) is known. The function \( w(\xi) \) is plotted in Fig. 1.

![FIG. 1: Dark Energy equation of state \( w(\xi) = \frac{2}{3} - \frac{2}{3} \frac{S''S}{(S')^2} \) corresponding to the S-brane solution \( S(\xi) \) as the function of time \( \xi \).](image)

The plot of \( w \) drops from the value \(-1/3\) till \(-1\) and afterwards raises again towards \(-1/3\). This is in agreement with the observation in [11-12] that the acceleration is changing the sign from negative to positive and back. Note also that one can represent \( w \) as a function of the kinetic energy of the scalar field and a potential energy

\[
w = \frac{E_{\text{kin}} - V}{E_{\text{kin}} + V} . \quad (18)
\]

In [12] this property of the solution was interpreted as follows: the field starts with large kinetic energy and the field runs up the exponential hill, \( w \) decreases. By the time all kinetic energy is lost \( w = -1 \) is achieved when the field stops at some point of the hill. It starts rolling down and \( w \) increases. Before and after the turning point the universe accelerates for some time.

**B. The model including dark matter**

To make this model realistic, one should add to the energy momentum tensor of the scalar field (dark energy) the contribution of non-relativistic matter which dominated the evolution of the universe at earlier times. In the phenomenological setting, one can introduce matter by adding the energy density of matter \( \rho_M \approx a^{-3}(t) \) to the energy density of the scalar field. We will assume that the matter energy density dominated in the early universe, leading to the expansion of the universe with the Hubble constant \( H = \frac{\dot{a}}{a} \). As a result, in the early universe the friction term \( 3H\dot{\phi} \) in the equation for the field \( \phi \) (see below) was very large. Therefore the field \( \phi \) was sitting at the slope of the potential and waiting until the Hubble parameter became sufficiently small [19]. When the Hubble parameter \( H \sim \frac{1}{a^2} \) becomes small enough, the field \( \phi \) starts rolling down with vanishing initial velocity. This means that the field \( \phi \) evolves starting with \( w = -1 \) with increasing \( w \). In the language of [12], the field \( \phi \) evolves from the position on the hill downwards only.

We assume that the universe is spatially flat. Our equations are

\[
\ddot{\phi} + 3H\dot{\phi} = -V(\phi) \quad (19)
\]

\[
H^2 = \frac{1}{3} \left( \rho_M + \frac{1}{2} \dot{\phi}^2 + V(\phi) \right) , \quad (20)
\]

where \( H = \frac{\dot{a}}{a} \) and

\[
\rho_{\text{total}} = \rho_M + \frac{1}{2} \dot{\phi}^2 + V(\phi) . \quad (21)
\]

Density of matter satisfies the equation \( \dot{\rho}_M + 3H\rho_M = 0 \) which is solved by \( \rho_M = \frac{C}{a^3} \) where \( C \) is a constant. Here \( a(t) \) is a scale factor on the universe with the metric \( ds^2 = -dt^2 + a(t)^2 dx^2 \). Note that \( a(t) \) is not the same function as \( S(t) \) since eq. (20) is different from equations following from the 4d action [3] and therefore different from the equations of 11d supergravity.

![FIG. 2: The scale factor of the universe \( a(t) \) as a function of the 4d observer time \( t \). The universe evolves from the Big Bang at \( t \approx -1 \) and \( a \) close to zero. The point \( t = 0 \), defines “today” where the scale factor is normalized to 1. The future evolution of the scale factor is at \( t > 0 \). In the model with potential [4] it expands forever, approaching a Minkowski vacuum.](image)

We solved equations (20) numerically. We found an initial value \( \phi_0 \) of the field \( \phi \), such that at the time when the Hubble constant \( H = \frac{\dot{a}}{a} \) reaches its presently observed value, the fraction of the energy density concentrated in the field \( \phi \) becomes equal to \( \Omega_D = 0.7 \). The method used to find such solutions is explained in [19]. The corresponding solutions for the scale factor of the universe
0.73 which is one of the possibilities discussed in [1]. For this value of $\Omega_D$ we have also evaluated the equations of state of the dark energy. It raises today even more that the one for the 0.7 case.

The observational constraint [1] on a time-independent equation of state is $w < -0.78$. In our case $w$ remains smaller than −0.78 for $z > 0.7$, but it grows up to −0.57 at $z = 0$, which makes it rather vulnerable. The future cosmological observations may either rule out this model or show that it is compatible with the data.

One should keep in mind that only the dark energy component of our model was derived directly from M-theory in $d = 11$. We have added ‘by hand’ the phenomenological CDM component required for the consistency of any cosmological model of late-time acceleration. If not only dark energy but also dark matter were derived from M-theory, it could possibly affect some of our conclusions.

**IV. WHY $\Omega_D$ AND $\Omega_M$ ARE OF THE SAME ORDER OF MAGNITUDE? A POSSIBLE SOLUTION OF THE COINCIDENCE PROBLEM.**

Finally, one should discuss the issue of fine-tuning of the initial conditions in this model. We have found cosmological solutions consistent with the present observational data implying that 14 billion years after the big bang one has $\rho_D \sim 10^{-120}$ and $\Omega_D \sim 0.7$. In order to find these solutions it was necessary to choose a proper initial value of the field $\phi = \phi_0$. One can roughly estimate $\phi_0$ assuming that the field does not change much during the cosmological evolution. This leads to relation $21 e^{-\sqrt{27/18}} \approx 10^{-120}$, and, consequently $\phi_0 \approx 174$, in Planck units. Numerical analysis shows that the field grows only by 0.6 during the cosmological evolution up to the present time, see Fig. 3, so the expression $\phi_0 \approx 174$ gives a good estimate of the numerical magnitude of $\phi_0$.

On the other hand, numerical analysis shows that if the initial value of the field was equal to $\phi_0 - 0.5$, then 14 billion years after the big bang one would have $\Omega_D \sim 0.8$. Meanwhile, increasing $\phi_0$ by 0.5 would lead to $\Omega_D = 0.54$. Both values are incompatible with cosmological observations.

Fixing initial value of the field $\phi$ with accuracy $\pm 0.5 M_{Pl}$ may not seem to be such a terrible thing to do. However, if, in the absence of a better idea, we assume that all initial values of $\phi$ are equally probable, we immediately realize that we have a serious fine-tuning problem.

Indeed, if one takes initial value of $\phi$ in any place of the semi-infinite interval $\phi \gg \phi_0 + M_{Pl}$, one finds $\Omega_D \ll 1$. Therefore one may argue that the probability to have $\Omega_D \sim 0.7$ in this model is infinitesimal small.

Another way to formulate this problem is to note that for any given initial value of the field $\phi = \phi_0$, the cosmological time when $\Omega_D$ becomes equal to 0.7 is proportional to $H^{-1}(\phi_0) \sim V^{-1/2}(\phi_0) \sim e^{-\sqrt{\phi_0}}$. Thus the
cosmological time when \( \Omega_D \) becomes equal to 0.7, and, more generally, the time when \( \Omega_D \) and \( \Omega_M \) are of the same order of magnitude, is exponentially sensitive to initial conditions. Therefore it looks very surprising that we live at the time when \( \Omega_D \) and \( \Omega_M \) are of the same order of magnitude (the coincidence problem).

This problem is quite generic for most of the models of dark energy. However, there are some models that do not suffer from this problem. The simplest (and perhaps the first) model of dark energy was proposed in \[28\]. It has a linear potential \( V(\phi) = \alpha \phi + C \). If the slope of the potential is sufficiently small (a condition that should be satisfied in all versions of models of dark energy), the field \( \phi \) practically does not move during the last 14 billions of years, so the potential acts as an effective cosmological constant. During eternal inflation, the universe becomes divided into many different exponentially large domains with different values of \( \phi \), and, correspondingly, with different values of the effective cosmological constant \[28\]. In those parts of the universe where \( V(\phi) < -10^{-119} \), the universe collapses within the time smaller than 14 billion years \[28, 29, 30\]. In those parts of the universe where \( V(\phi) \gg 10^{-119} \), the probability of formation of galaxies would be strongly suppressed \[28, 31, 32\]. Thus, in the context of this simple model, we can live and make our observations only in those parts of the universe where \( |V(\phi)| \lesssim 10^{-119} \), which corresponds to a finite range of values of the field \( \phi \) \[28\].

A different anthropic solution of the cosmological constant problem and the coincidence problem was proposed for the dark energy model based on N=8 supergravity \[14, 30\]. In that model, the universe collapses very fast, unless the present value of the effective cosmological constant is smaller than \(10^{-119}\).

One could try to use similar arguments for the dark energy model discussed above. Indeed, we would be un-

able to live in a universe with initial value of the field \( \phi < \phi_0 - 3 \) because for small \( \phi \) one has \( V(\phi) &\gg 10^{-119} \).

Therefore galaxies would not form in such a universe, in accordance with \[28, 31, 32\]. In fact, the situation here is even better than in the models studied \[28, 31, 32\]. Indeed, in these models the probability distribution was supposed to be constant with respect to the vacuum energy. Therefore the suppression of galaxy formation for \( \rho_D \gg 10^{-119} \) did not provide an entirely satisfactory explanation of the present value of the vacuum energy \( \rho_D \sim 10^{-120} \). Meanwhile, in our case the situation is somewhat better because the canonically normalized variable is not \( V(\phi) \) but \( \phi \). The energy density \( V(\phi) \) depends on \( \phi \) exponentially, so the change of \( V(\phi) \) by one or two orders of magnitude occurs within a small range of variation of the field \( \phi \). If the probability is constant with respect to the field \( \phi \) (which is a reasonable assumption in the context of inflationary cosmology \[28, 32\]), then the probability to live in a part of the universe with \( V(\phi) \gg 10^{-120} \) becomes much stronger suppressed than in the models studied in \[28, 31, 32\].

However, at the first glance, in this model we must live in a universe with \( V(\phi) \ll 10^{-120} \). Indeed, galaxies can be formed and the universe does not collapse for all of the initial values \( \phi \gtrsim \phi_0 \). The probability to live in a universe with the field \( \phi \) belonging to the finite interval \(-0.5 < \phi - \phi_0 < 0.5 \) seems to be infinitesimally small as compared with the probability to live in a universe in the infinitely large interval \( \phi \gg \phi_0 \), with dark energy \( \Omega_D \ll 1 \).

One can solve this problem if one finds a true 11d implementation of this model, as discussed in Section II. Indeed, let us remember that the mass gap for the KK modes in our model is given by

\[
m_{kk} \sim c \sqrt{V(\phi)} \sim c e^{-\sqrt{\phi} \Delta} \sim c e^{-0.8 \phi}.
\] (22)

If \( c = O(1) \), then, as we argued, the theory is effectively 11d, and the model does not work \[28\]. Let us assume for a moment that some resolution of this problem will be found, so that the KK mass gap is large enough for \( \phi \sim \phi_0 \). However, this gap exponentially rapidly disappears for \( \phi \gg \phi_0 \) and therefore our world becomes effectively 11d for \( \phi \gg \phi_0 \). For example, even if the mass gap at \( \phi \sim \phi_0 \) is as large as \( M_{Pl} = 1 \), it becomes much smaller than 1 TeV and our universe becomes effectively 11d for \( \phi \gg \phi_0 \gtrsim 45 \). If the mass gap at \( \phi \sim \phi_0 \) is about 1 TeV, the universe becomes effectively 11d for \( \phi \gg \phi_0 \).

Life as we know it can exist only under fine-tuned relations between masses and coupling constants, which would be strongly affected by appearance of new low-mass states \[28\]. This suggests that life of our type may exist only if the deviation \( \Delta \phi = \phi - \phi_0 \) of the initial value of the field \( \phi \) from \( \phi_0 \sim 174 \) was very limited, \(-3 \lesssim \Delta \phi \lesssim O(10) \). Therefore it is not very surprising that we live in the universe with \(-0.5 \lesssim \Delta \phi \lesssim 0.5 \). This provides a possible resolution of the cosmological coincidence problem discussed above.

The structure of the universe in this model looks especially interesting if one can incorporate it into the eternal inflation scenario \[32, 34\]. If the potential of the field \( \phi \) remains very flat during inflation, then inflationary quantum fluctuations of the field \( \phi \) divide the universe into infinitely large number of exponentially large domains with all possible values of \( \phi \) being approximately equally represented, just like in the dark energy models of Refs. \[28, 32\]. The domains with \( \phi \ll \phi_0 \) will be effectively 4d, but they will contain no galaxies. The domains with \( \phi \gg \phi_0 \) will be effectively 11d, unsuitable for life as we know it. We can live only in domains with \( \phi \sim \phi_0 \). In a considerable part of such domains \( \Omega_D \) and \( \Omega_M \) are of the same order of magnitude. In a distant future, the value of the field \( \phi \) in each of these domains will grow, the mass of the KK model will fall down exponentially, the 4d space will gradually decompactify and become 11d.

As we already emphasized, the model of dark energy discussed in our paper suffers from many serious problems. In this respect, this model seems much less satisfactory than the theory of cosmic acceleration in a metastable de Sitter state in a context of string theory
with stabilized moduli. It may still be interesting that the model discussed above provides a possibility to describe the universe marginally consistent with the present observational data. Some of the features of this model may have a more general significance. In particular, the link between the vacuum energy density and the effective dimensionality may appear in other models as well. It may provide a new way towards a solution of the cosmological constant problem.

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