Hubble tension bounds the GUP and EUP parameters

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Abstract In recent years, the discrepancy in the value of the Hubble parameter has been growing. Recently, there are works supporting the proposal that the uncertainty principles can solve the Hubble tension. Motivated by this proposal, we work with an isotropic and homogeneous FRW universe, obtain its Hamiltonian equations, and thus, the Hubble parameter through the first Friedmann equation. In the context of GUP and EUP models, the Hubble parameter is modified. Since the fingerprints of quantum gravity are imprinted on the CMB, we consider the GUP/EUP-modified Hubble parameter in the first Friedmann equation to be the one measured by the Planck collaboration which uses the CMB data. The unmodified Hubble parameter in the first Friedmann equation is considered to be the one measured by the HST group which uses the SNeIa data. Therefore, upper bounds for the dimensionless parameters of GUP and EUP are obtained.

1 Introduction

The unification of quantum theory and general relativity is an important issue of contemporary physics. If quantum mechanics and general relativity are both present in a theory, a critical scale named Planck length $\ell_p = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35}$ m, is introduced and considered as the minimum measurable scale of this theory [1]. Furthermore, the existence of such a minimal measurable length is supposed to modify the Heisenberg uncertainty principle (HUP) as

$$\Delta X \Delta P \geq \frac{\hbar}{2} \left[ 1 + \beta (\Delta P)^2 + \eta (\Delta X)^2 + \cdots \right]$$

(1)

where $\beta$ and $\eta$ denote the generalized uncertainty principle (GUP) and extended uncertainty principle (EUP) parameters, respectively [2–6]. There are also other types of GUP introduced and investigated, for instance, see Refs. [7–12]. Different GUPs and EUPs are derived in different contexts such as string theory [13,14], extra dimensions [15], black hole physics [16,17], and propose deep connections with generalized statistics [18,19].
On a different note, one of the most significant problems in modern cosmology is the Hubble tension. This is a disagreement on the current value of Hubble parameter, i.e., $H_0$, between the Planck collaboration which estimates a value of $H_{\text{CMB}} = 67.40 \pm 0.50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ \cite{20} and the Hubble space telescope (HST) group which gives $H_{\text{SN}} = 74.03 \pm 1.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$ \cite{21}. It is noteworthy to state that the HST group utilizes SNeIa data, while the Planck collaboration employs the cosmic microwave background (CMB) data. This difference has not been eliminated but, on the contrary, during the last years, it has been grown to the 4.4$\sigma$ level \cite{22}.

Despite the numerous attempts to solve this discrepancy, its mechanism remains still unclear. There are various proposals and different approaches for solving this problem in cosmology \cite{23,24}. In particular, the $H_0$ tension is proposed as the result of factors such as the curvature, the neutrino masses, and the effective number of neutrino species \cite{25–29}. As already mentioned, until now there is not a comprehensive agreement on the various proposals of eliminating this discrepancy, and therefore, this motivates physicists to think about other possibilities. In this direction, it is considered that during the Planck epoch, quantum fluctuations were produced. These quantum fluctuations propagating in spacetime generated the primordial fluctuations in the inflation era. These primordial fluctuations which are quantified by a power spectrum are encoded in the anisotropies of the CMB \cite{30,31}. So, observing the anisotropies of the CMB, we may “see the fingerprints of quantum gravity \cite{32–34}. Therefore, since the quantum features of gravity are considered to be stored on CMB \cite{34–40}, it has been claimed that GUP could solve the Hubble tension problem \cite{41,42}.

In the present work, motivated by the idea that the quantum gravity effects can solve the Hubble tension, we study the consequences of the discrepancy in the value of the Hubble parameter on the GUP and EUP. Specifically, we focus on the modifications of the Hamiltonian of a Friedmann–Robertson–Walker (FRW) universe due to the different types of GUP and EUP. Therefore, utilizing the different values for the Hubble parameter provided by the Planck collaboration and the HST group, we obtain some bounds on the GUP and EUP parameters. Our results show that the Hubble tension problem, if it stems from the signals of quantum gravity encoded in the observations, can provide information about the GUP/EUP parameters. In Section II, we briefly present the Hamiltonian of the FRW universe, derive the Hamiltonian equations in the context of HUP, and thus, find the standard Hubble parameter. In Section III, we generalize the previous analysis in the context of GUP. We obtain the GUP-modified Hamiltonian equations, and thus utilizing the $H_0$ tension, we derive a bound for the GUP parameter for two different types of the GUP model. In Section IV, we work in the context of EUP. We apply a similar to Section III approach for the EUP model. We find the corresponding EUP-modified Hamiltonian equations and find a bound for the EUP parameter for two different types of the EUP model. Finally, Section V is devoted to briefly summarize our results and make some concluding remarks.

2 Cosmological model

The Hamiltonian of an isotropic and homogeneous FRW universe, in natural units $\bar{h} = c = 16\pi G = 1$, is written in the form \cite{43–49}

$$\mathcal{H}_{FRW}(p_a, a) = N \frac{p_a^2}{24 a} + 6 N k a - N \rho a^3 + \kappa \Pi$$

(2)

where $a$ and $p_a$ are the scale factor (as the generalized coordinate operator) and the generalized momentum conjugate to the scale factor, respectively. Additionally, $N$ is the lapse function.
that has no dynamical role, and $\Pi$ is its conjugate momentum, while $\kappa$ is its corresponding coefficient. It should be noted that for the case of constant $N$, we get $\Pi = 0$. Furthermore, $k$ and $\rho$ denote the geometrical parameter of the FRW universe and the density of the universe, respectively.

Here, we will use the scale factor $a$ instead of the position operator $x$ in the uncertainty relation. Therefore, based on the HUP for the operators $a$ and $p_a$, the corresponding Dirac bracket will be written as [43,44]

$$[a, p_a] = i\hbar. \quad (3)$$

At this point, for simplicity reasons, we set $N = 1$ [43,44,48,49]. Thus, by employing the Hamiltonian equations, one obtains

$$\dot{a} = \frac{\partial \mathcal{H}_{FRW}}{\partial p_a} = \frac{1}{12} \frac{p_a}{a},$$

$$\dot{p}_a = -\frac{\partial \mathcal{H}_{FRW}}{\partial a} = \frac{1}{24} \frac{p_a^2}{a^2} - 6k + 3\rho a^2 + \frac{d\rho}{da} a^3,$$

$$\dot{\Pi} = -\frac{1}{24} \frac{p_a^2}{a} - 6ka + \rho a^3 = 0. \quad (4)$$

Now, we combine the first and third equations of Eq. (4), and so the first Friedmann equation reads

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{12^2} \frac{p_a^2}{a^4} = \frac{1}{6} \rho - \frac{k}{a^2},$$

which yields

$$H = \sqrt{\frac{1}{6} \rho - \frac{k}{a^2}}. \quad (5)$$

It is evident that if we consider the case $k = 0$ (a flat FRW universe in agreement with the WMAP data [50]) and set $\rho = 2\Lambda$ with $\Lambda$ to be the cosmological constant, then the Hubble parameter for a cosmological constant-dominated universe will be in the form $H = \sqrt{\frac{\Lambda}{3}}$ [51].

3 GUP and Hubble parameter

As already mentioned, fingerprints of quantum gravity are stored in CMB [34–40]. Therefore, if CMB data is used to estimate the $H_0$ value, as is the case for the Planck collaboration, then quantum gravity corrections have to be taken into consideration. Therefore, following the analysis of the previous section, Eq. (2) should be rewritten in the context of GUP. As already mentioned in the Introduction, there are several versions of GUP which are generalizations of HUP [13–17]. One of the first, and general form, types of GUP is

$$\Delta X\Delta P \geq \frac{\hbar}{2} \left(1 + \lambda f(\Delta P)\right) \quad (7)$$

where $\lambda$ denotes the GUP parameter and $f(P)$ is a function of the particle momentum, i.e., $P$, and/or its uncertainty, i.e., $\Delta P$. For instance, a well-known type of GUP with only a term quadratic in momentum was given by Kempf, Mangano, and Mann (KMM type) [2]

$$\Delta X\Delta P \geq \frac{\hbar}{2} \left(1 + \beta \Delta P^2\right). \quad (8)$$
An improved version of KMM type of GUP was introduced in Ref. [52]
\[ \Delta X \Delta P \geq \frac{\hbar}{2} e^{\beta \Delta P^2}. \] (9)

In addition, there is another type of GUP proposed by Pedram [8, 9]
\[ \Delta X \Delta P \geq \frac{\hbar}{2} \frac{1}{1 - \beta P^2}. \] (10)

At this point, a couple of comments are in order. First, at the Planck scale, this type of GUP naturally induces a UV cutoff for the momentum. Second, this type of GUP is seen in Refs. [53–55]. In all of the above-mentioned types of GUP, the momentum \( P_a \) (valid near the Planck scale) is expressed as a function of the canonical momentum, i.e., \( p_a \), as follows [2, 8, 9, 53–55]
\[ P_a = p_a \left( 1 + \lambda_1 p_a + \lambda_2 p_a^2 + \mathcal{O} \left( p_a^3 \right) \right) \] (11)

where \( \lambda_1 \) and \( \lambda_2 \) are the GUP parameters of the corresponding types of GUP under study.

From now on, without loss of generality and for the sake of simplicity, we will keep terms up to second order in momentum. Thus, by applying the “transformation” \( p_a \to P_a \) in Eq. (2), the deformed Hamiltonian of an isotropic and homogeneous FRW universe reads
\[ H_{GUP}^{FRW} (p_a, a) = \frac{1}{24} \frac{p_a^2 (1 + 2 \lambda_1 p_a + 2 \lambda_2 p_a^2 + \lambda_1^2 p_a^2)}{a} + 6ka - \rho a^3 + \kappa \Pi. \] (12)

It is evident that the corresponding GUP-modified Hamiltonian equations will now be of the form
\[ \dot{a} = \frac{\partial H_{GUP}^{FRW}}{\partial p_a} = \frac{1}{12} \frac{p_a}{a} (1 + 3 \lambda_1 p_a + 4 \lambda_2 p_a^2 + 2 \lambda_1^2 p_a^2), \] (13)
\[ \dot{p}_a = - \frac{\partial H_{GUP}^{FRW}}{\partial a} = \frac{1}{24} \frac{p_a^2 (1 + 2 \lambda_1 p_a + 2 \lambda_2 p_a^2 + \lambda_1^2 p_a^2)}{a^2} - 6k + 3 \rho a^2 + \frac{d \rho}{da} a^3, \] (14)
\[ \dot{\Pi} = - \frac{1}{24} \frac{p_a^2 (1 + 2 \lambda_1 p_a + 2 \lambda_2 p_a^2 + \lambda_1^2 p_a^2)}{a} - 6ka + \rho a^3 = 0. \] (15)

By combining Eqs. (13) and (15), we obtain
\[ H_{GUP}^2 = H^2 + 48 \lambda_1 a^2 H^3 + 864 \lambda_2 a^4 H^4 + 576 \lambda_1^2 a^4 H^4. \] (16)

It is obvious that for the case of \( \lambda_1 \neq 0 \) and/or \( \lambda_2 \neq 0 \), we obtain \( H_{GUP} \neq H \). This can be considered as the answer to the Hubble tension problem. To be more specific, on the one hand, as already mentioned, CMB carries fingerprints of quantum gravity [34–40]. Therefore, the Hubble parameter reported by the Planck collaboration (\( H_{CMB} \)) can be viewed as the GUP-modified Hubble parameter in Eq. (16), i.e., \( H_{CMB} = H_{GUP} \). On the other hand, the supernova phenomena are explained by quantum mechanics quite well, and furthermore, based on our current understanding, general relativity is adequate to study the spacetime
around supernovae. Therefore, we can view the Hubble parameter reported by the HST group as the unmodified Hubble parameter in Eq. (16), i.e., $H_{SN} = H$.

First, we consider the case $\lambda_2 = 0$ which can be found in Refs. [53–55]. From Eq. (16), the Hubble parameter now reads

$$H_{CMB} = H_{SN} \left( 1 + 24\lambda_1 a^2 H_{SN} \right).$$  \hfill (17)

At this point, a couple of comments are in order. First, the above expression implies that $\lambda_1$ has to be negative since $H_{CMB} < H_{SN}$ according to the recent observations. This is in agreement with Refs. [10–12] where the linear term in momentum is subtracted from unity. Second, by using Eq. (17), one can easily obtain an expression for $\lambda_1$ as

$$\frac{H_{CMB} - H_{SN}}{H_{SN}^2} = 24\lambda_1 a^2.$$  \hfill (18)

By considering the recent reports for the values of the Hubble parameter, i.e., $H_{CMB}$ and $H_{SN}$, and setting $a = 1$ for today, we find $|\lambda_1| \approx 1.5 \times 10^{40}$ GeV$^{-1}$.

At this point, we need to reinstate the units in Eq. (18) and we write $\lambda_1 = \frac{\lambda_{01}}{M_p c}$, with $\lambda_{01}$ to be the dimensionless GUP parameter and $M_p$ to be the Planck mass. Therefore, we obtain the value for the dimensionless GUP parameter $|\lambda_{01}| \approx 2.9 \times 10^{58}$ which will be an upper bound for Eq. (18) since future observations will reduce the discrepancy.

Second, we consider the case $\lambda_1 = 0$ which can be found in Ref. [7]. From Eq. (16), the Hubble parameter now reads

$$H_{CMB} \simeq H_{SN} \left( 1 + 432\lambda_2 a^4 H_{SN}^2 \right).$$  \hfill (19)

At this point, a couple of comments are in order. First, the above expression implies that $\lambda_2$ has to be negative since $H_{CMB} < H_{SN}$ according to the recent observations. This is in agreement with the negative dimensionless GUP parameter obtained in Ref. [56]. Second, by using Eq. (19), one can easily obtain an expression for $\lambda_2$ as

$$\frac{H_{CMB} - H_{SN}}{H_{SN}^3} = 432\lambda_2 a^4.$$  \hfill (20)

By considering the recent reports for the values of the Hubble parameter, i.e., $H_{CMB}$ and $H_{SN}$, and setting $a = 1$ for today, we find $|\lambda_2| \approx 3.7 \times 10^{42}$ GeV$^{-2}$.

By reinstating the units in Eq. (18), we write the GUP parameter as $\lambda_2 = \frac{\lambda_{02}}{(M_p c)^2}$, with $\lambda_{02}$ to be the dimensionless GUP parameter and $M_p$ to be the Planck mass. Therefore, we obtain the value for the dimensionless GUP parameter $|\lambda_{02}| \approx 1.3 \times 10^{79}$ which will be an upper bound for Eq. (19), namely for the dimensionless GUP parameters, since that future observations will reduce the discrepancy.

### 4 EUP and Hubble parameter

In this section, we will employ a similar to the previous section analysis but for the case of the EUP model. It is known that for cosmological models, the scale factor $a$ plays the role of the position $x$. In the context of the EUP model, the momentum operator (valid near the Planck scale) reads

$$P_a = p_a \left( 1 + \eta_1 a + \eta_2 a^2 \right) \hfill (21)$$
where $\eta_1$ and $\eta_2$ are the dimensionless EUP parameters. From now on, without loss of generality and for the sake of simplicity, we will keep terms up to second order in position. Thus, by applying the “transformation” $p_a \rightarrow P_a$ in Eq. (2), the deformed Hamiltonian of an isotropic and homogeneous FRW universe reads

$$H_{\text{EUP}}^{\text{FRW}}(p_a, a) = \frac{1}{24} \frac{p_a^2}{a} \left( 1 + 2\eta_1 a + \eta_1^2 a^2 + 2\eta_2 a^2 \right) + 6ka - \rho a^3 + \lambda \Pi.$$

(22)

It is evident that the corresponding GUP-modified Hamiltonian equations will now be of the form

$$\dot{a} = \frac{\partial H_{\text{EUP}}^{\text{FRW}}}{\partial p_a} = \frac{1}{12} \frac{p_a}{a} (1 + 2\eta_1 a + \eta_1^2 a^2 + 2\eta_2 a^2),$$

(23)

$$\dot{p}_a = -\frac{\partial H_{\text{EUP}}^{\text{FRW}}}{\partial a} = \frac{1}{24} \frac{p_a^2}{a^2} (1 - \eta_1^2 a^2 - 2\eta_2 a^2) - 6k + 3\rho a^2 + \frac{d\rho}{da} a^3,$$

(24)

$$\dot{\Pi} = -\frac{1}{24} \frac{p_a^2}{a} (1 + 2\eta_1 a + \eta_1^2 a^2 + 2\eta_2 a^2) - 6ka + \rho a^3 = 0.$$

(25)

By combining Eqs. (23) and (25), we obtain

$$H_{\text{EUP}}^2 = H^2 + 2\eta_1 a H^2 + \eta_1^2 a^2 H^2 + 2\eta_2 a^2 H^2.$$

(26)

Now, if we consider the cases $\eta_1 = 0$ and $\eta_2 = 0$, we get, respectively,

$$H_{\text{EUP}} = H (1 + \eta_1 a),$$

(27)

$$H_{\text{EUP}} \simeq H (1 + \eta_2 a^2).$$

(28)

Utilizing Eqs. (27) and (28), we can express the dimensionless GUP parameters, i.e., $\eta_1$ and $\eta_2$, in terms of the Hubble parameter as follows, respectively,

$$\frac{H_{\text{CMB}} - H_{\text{SN}}}{H_{\text{SN}}} = \eta_1 a,$$

(29)

$$\frac{H_{\text{CMB}} - H_{\text{SN}}}{H_{\text{SN}}} = \eta_2 a^2.$$

(30)

Now, by considering recent reports for the values of the Hubble parameter, i.e., $H_{\text{CMB}}$ and $H_{\text{SN}}$, and setting $a = 1$ for today, dimensionless GUP parameters are both negative, and their magnitudes are $|\eta_1| = |\eta_2| = 9.0 \times 10^{-2}$. This value can be considered as an upper bound for Eqs. (29) and (30), namely for the dimensionless EUP parameters, due to the fact that future observations will reduce the discrepancy.

5 Conclusion

In recent years, the Hubble tension has been growing. Recently, many works support the proposal that the uncertainty principles can solve the Hubble tension [41,42]. Following this proposal, we exploit the discrepancy between two different ways of measuring the rate of the universe’s expansion, to derive bounds on the dimensionless GUP/EUP parameters.
Specifically, in this work, we consider an isotropic and homogeneous FRW universe, and relying on canonical variables, i.e., $a$ and $p_a$, we obtain the first Friedmann equation. Then, it is shown that when GUP/EUP is taken into account, the Hubble parameter is GUP/EUP-modified. By adopting the proposal that quantum gravity effects are stored in the CMB, we consider the GUP/EUP-modified Hubble parameter to be the one measured by the Planck collaboration which used the CMB data, while the unmodified Hubble parameter is considered to be the one measured by the HST group which used the SNeIa data. Finally, since we expect future observations to reduce this discrepancy, the values for the dimensionless GUP/EUP parameters obtained with recent data can be the upper bounds for these GUP/EUP parameters. In particular, the dimensionless GUP parameter $\lambda_{01}$ is similar to the dimensionless GUP parameter $\lambda_0$ introduced in Refs. [10–12]. Thus, the bound obtained here, namely $|\lambda_{01}| < 2.9 \times 10^{58}$, is a not-so-tight bound compared to the ones existing in the literature, for instance, see Refs. [57, 58]. The dimensionless GUP parameter $\lambda_{02}$ is similar to the dimensionless GUP parameter $\alpha_0$ [2]. Thus, the bound obtained here, namely $|\lambda_{02}| < 1.3 \times 10^{79}$, is similar to the bounds achieved using data from our solar system, for instance, see Ref. [56].

The dimensionless EUP parameters, namely $\eta_1$ and $\eta_2$, introduce a bound $|\eta_1| = |\eta_2| < 9.0 \times 10^{-2}$ which is a very tight bound compared to the ones existing in the literature, for instance, see Ref. [59].

Data availability statement Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study. This manuscript has associated data in a data repository. [Authors’ comment: This work is purely theoretical and thus no datasets were used and/or generated.]

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