Using «Kansky indices» to Estimate Transport and Geographical Simplexes

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Abstract. In the article, using the «γ» index as an example, we propose a way to use a number of «Kansky's indices» to estimate transport-geographical simplexes. As a rule, «Kansky Indices» are used for comparative assessment of spatial and / or temporal differences in the transport network as a whole. However, in the system of mass public transport, subject to direct passenger traffic, through the projection of general properties formed by a combination of graphs of different routes, a «woven» graph onto its individual elements - vertices, these indices can serve to characterize certain stopping points and adjacent to them territories. They will demonstrate the level of stability of the transport network, of which the stopping point is a part, to possible traffic disruption in some of its sections, and, accordingly, will act as one of the indicators that characterize the degree of convenience for passengers of a particular stopping point.

1. Introduction

In 1736, Leonard Euler outlined in letters to Giovanni Giacobo Marinoni and Karl Gottlieb Ehler the solution to the problem of the seven Konigsberg bridges, at the end expressing some doubt that this problem and its solution belong to mathematics [1; 2]. However, despite Euler's doubts, this work laid the foundation for such a branch of discrete mathematics as graph theory.

Currently, graph theory is used to solve various scientific and practical problems in many fields, including geography, especially in the geography of transport. Among the methods of graph theory that are used in the geography of transport, a special place was taken by the «Kansky indices», which got their name in honor of the American scientist K J Kansky, who adapted and introduced them into the scientific circulation of transport geography in the 60-s years of the XX century [3]. "Kansky indices" («α», «β», «γ», «ε», «χ», «π») are used to assess the connectivity, development and form of the transport network [4]. Their advantage lies in the relative ease of calculation. To date, the study of these indices is included in various training courses, and many educational publications on the use of mathematical methods in the geography and geography of transport are introduced to the methodology of their use [5; 6; 7].

As a rule, the «Kansky indices», as well as their modifications, are used to make a comparative assessment of spatial and / or temporal differences in the transport network as a whole (transport network of one of the «linear» modes of transport, or several modes of transport, but separately). In the fundamental work of S A Tarkhova "Evolutionary morphology of transport networks", we can see
the use of both the considered indicators and those associated with them, in the analysis of spatio-temporal changes in the road and rail transport networks of the countries of the world and regions of Russia. In one of his last works, "Analysis of topological defects in the land transport network of the regions of Siberia and the Far East" [8], the regions of Russia that lie east of the Ural ridge are compared. The article by O Kuzkin «Graph theory methods in analysing commuting networks of municipal electric transport» [9] provides an analysis of urban electric transport networks in a number of Ukrainian cities (Donetsk, Zaporozhye, Krivoy Rog, Lvov, Odessa). A P Bezruchenok used the «Kansky index» when he analyzed the railway transport network of the Republic of Belarus [10]. L M Laptashkina when characterizing changes in the period 1897-2007. «Road» network of Chuvashia [11]. A Erath, K Axhausen, M Löchl to characterize spatial and temporal changes in the road network in Switzerland [12]. S Derrible and C Kennedy compared transport systems of 19 cities around the world (Toronto, Montreal, Chicago, New York, Washington, San Francisco, Mexico City, London, Paris, Lyon, Madrid, Berlin, Athens, Stockholm, Moscow, Tokyo, Osaka, Seoul and Singapore) [13, 16].

2. Materials and methods
Let us ask ourselves whether it is possible to use the «Kansky indices» (or some of these indices) not only to characterize the transport system as a whole (statically or dynamically, in space or in time), but to characterize only its part, which is not a subsystem , but is a separate element, as E B Alaev «simplex» [14], a separate point with comparing it with other, the same points in the same transport system? On a separate point, which is one of the vertices in the graph of the transport system, it is possible to project some general properties of the system of which it is a part. However, it is permissible to attribute the same properties to the rest of the separate points, which are the vertices of this system. Spatial differences will not be observed if subsystems do not arise, each of which will transfer its properties to one or another separate point – a vertex or their groups. Such subsystems are most clearly manifested in the systems of public transport of many urban settlements, when the conditions for the implementation of the non-stop movement of passengers are implemented. Various public transport routes pass through a separate point – a public transport stop. Each such route connects this stop with other stops – peaks, as a result forming a “first-order graph” (figure 1.a). The set of routes of mass public transport, which pass through a particular stop, leads to the "plexus" of the "first-order graphs" formed by them into a more complex, "twisted second-order graph" (figure 1.b). The properties of a "twisted graph of the second order", including those that can be described by «Kansky indices», can be projected onto the separate point (stop) itself – the vertex. Other stops – vertices in the mass public transport system can repeat this property if the same routes pass through them (figure 1.b), or have slightly or significantly different properties if the list of passing routes changes, changing the shape of the "woven graph of the second order "(figure 1.c).

3. Results and discussion
We have tested the above method. As a convenient testing polygon, we will choose the mass public transport system of the city of Novocheboksarsk of Chuvash Republic. The city of Novocheboksarsk of the Chuvash Republic is the second largest urban settlement in the region; as of January 1, 2020, the number of resident population living in it was 127,226 thousand people [15]. It is located east of the capital of Chuvashia – the city of Cheboksary. The north-easterly outskirts of the Cheboksary urban district and the north-westerly outskirts of the Novocheboksarsk urban district have a common border. The area of the city of Novocheboksarsk is 51.14 sq. km. The city has 13 permanent routes of intracity mass public transport, of which: 5 trolleybus, 1 bus and 7 route taxis. The length of the route network of the city of Novocheboksarsk is 33.99 km. The number of stopping points for mass public transport is 61 (excluding duplicated stopping points, as well as points through which permanent routes do not pass). The density of the route network to the entire area of the city is 0.67 km / sq. Km, to the area of the residential area – 2.17 km / sq. km. The average pedestrian accessibility of the route network varies from 0.5 km for the entire territory of the city to 0.15 km in the residential part. The coverage
ratio of the route network is 69.68%, the coefficient of route compatibility is 2.76. Intracity public transport routes are complemented by 10 intercity routes (electric bus (1 route), buses and minibuses). The peculiarity of the city of Novocheboksarsk is that it is a satellite city of Cheboksary, part of the Cheboksary urban agglomeration. Pendulum migration flows are very strong between the two cities. Transport connecting Cheboksary and Novocheboksarsk performs not only the functions of intercity communication, but also intracity communication in both cities.

Figure 1. An example of the formation of a "woven graph" of the second order.

With regard to the «intertwined» graphs formed by all (one, two, several) routes of mass public transport (taking into account intercity routes) that pass through certain separate – stop points of Novocheboksarsk, we calculated the indices «α», «β» and «γ». Let us dwell on the territorial differences in one of them - in the index «γ», which demonstrates the completeness of connections in the chain of the "woven" graph [5, p. 153] - the achieved level of network connectivity [4, p. 56]. «The index «γ» is the ratio of the number of edges (m) to their maximum possible number in the network, which in flat graphs is equal to 3(n - 2), where n is the number of vertices» [5, p. 153]. Calculations have shown that the values of the index «γ» for stopping points of mass public transport in the city of
Novocheboksarsk (for «sprouting» through them «intertwined» graphs of routes of mass public transport) range from 0.35 to 0.44, while theoretically the values of the index «γ» can vary from 0 to 1 [5, p. 153]. Thus, a «woven» graph with a high level of connectivity does not «sprout» through any of the stopping points in the mass public transport network of the city of Novocheboksarsk. Despite the fact that the values of the index «γ» are not high and rather close to each other, we can observe territorial differences in its values. The lowest values (0.35 – 0.38) are typical for stopping points in the northeastern (next to the treatment facilities, property complex of the river port) and southwestern (next to the large greenhouse farm «Agrofirma «Oldeevskaya», the city cemetery) outskirts of the city. Note that stopping points («intertwined» graphs «growing» through them) on the southeastern outskirts of the city, located near the main town-forming enterprise PJSC "Khimprom", are characterized by higher values of the «γ» index - 0.38 – 0.41. The highest values of the index «γ» are at stopping points located in the center of the main residential part of the city, on Vinokurov Street, stretching from the north from the city administration, the palace of culture, large sports facilities (city stadium, ice palace, sports complex) to the western outskirts of the city (Nikolskaya Church, city registry office, children's clinic, children's hospital, perinatal center, city market). The highest values of the index «γ» (0.44) are at the stopping points, where the routes of mass public transport cross, going from north-east to south-west and from north-west to south, southeast of the city (figure 2).

If, when considering the graphs formed by routes of mass public transport, focus only on intracity routes, refusing to take into account intercity routes (although they are involved in the transportation of passengers within the city), then the values of the index «γ» for stopping points located in the northwest of the city, as and stopping points in the central part of the main residential area in the west of the city will remain unchanged. Not to mention the stopping points of the industrial outskirts of the city, where intercity vehicles do not go at all. Whereas for stopping points located in the central part of the main residential zone in the north of the city, in the south of the main residential zone of the city and stopping points in the extreme west of the city, the values of the index «γ» will decrease, although not very significantly. The reasons for this are that the routes of intercity routes of mass public transport in the first zones within the urban district completely coincide with the routes of intracity routes passing here, and in the second zones there will be some differences in the passage of routes of intercity and intracity routes. That is, the use of intercity routes for intercity direct communication by people in the second zones is more justified, since it allows them to get to those parts of the city where intercity transport cannot deliver them. In the very first zones of the city, the use of intercity transport for intercity movement can only reduce the waiting time at a bus stop (although, as a rule, it is possible to use intercity transport for an intercity trip only not during rush hours).

Comparing the values of the index «γ» of stopping points («sprouting» through them «woven» graphs) of the city of Novocheboksarsk with the number of routes of the mass public transport through them, we find a fairly strong direct correlation between these two indicators. When taking into account only intracity routes, the value of the correlation connection is 0.75, when taking into account intercity routes – 0.80. That is, in the city of Novocheboksarsk, with almost every new route of mass public transport passing through this or that stopping point, the level of connectivity of the «woven» graph growing through it grows.

Using the index «γ» as an example, we see that the calculation of the Kansky index values not for the transport network as a whole, but through the network, for its individual elements - transport-geographical simplices (in our case, public transport stops) is possible. However, let's ask ourselves the question - is there any practical sense in these calculations? And let us give an affirmative answer to it – yes, there is a practical sense in these calculations. «Kansky's indices», characterizing the connectivity of the transport network, the saturation of its contacts, also demonstrate the degree of resistance of the network to disruptions in operation that may arise in its individual sections.

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3 We use this definition as going back to some peculiarities of the terminology of graph theory, in which a connected graph that does not contain cycles is called a tree. However, we note that "sprouting" "woven" graphs can be both trees and contain one, two, or several cycles.
Figure 2. Territorial differences in the values of the index "γ" at stopping points («sprouting» through them «woven» graphs) routes of mass public transport of the city of Novocheboksarsk (taking into account intercity routes).
The higher the values of a number of «Kansky indices», including the «γ» index, the more likely the transport network will remain operational if there is an interruption or difficulty in the movement of vehicles on some of its segments. Accordingly, provided that the non-stop movement of passengers is realized, those stopping points should be recognized as the most convenient ones, to which, «intertwined» graphs «sprouting» through them, formed by a combination of passing routes of mass public transport, "transmit" a higher value of the «γ» index. All other things being equal, in the event of violations in the movement of vehicles on some sections of individual routes, a passenger from stops with a higher value of the index «γ» will be more likely to achieve the purpose of the trip without resorting to transfers from one vehicle of mass public transport to another. As a rule, for this, it is enough for him, while still standing at the stop, to simply choose a different route. This property (estimated by us using the index «γ») is in demand by passengers of mass public transport at times when they expect violations in the movement of vehicles on some sections of individual routes, for example, during rush hours, in anticipation of traffic congestion, or on holidays, due to the closure of public and personal transport on certain sections of the city transport network.

The calculation of indices is not difficult, but time consuming. However, in the process of constructing «intertwined» graphs formed by the routes of public transport routes passing through one or another stop, the possibility of a non-stop passenger journey from this stop to other stops in the city will also be identified. The number of such stops, or their share in the total number of public transport stops in the city, will point us to another criterion for the ease of use of a particular stopping point by passengers – its accessibility in the urban space, and the availability of urban spaces from it. For example, in Novocheboksarsk, from the stopping point located at the children's clinic by public transport, without making transfers, you can get to 61 stops, which is 100% of the total number of public transport stops in the city, and from the stopping point located at the main entrance of the city-forming enterprise PJSC «Khimprom» only up to 36 (59%), which indicates a greater transport accessibility (when using mass public transport) of the children's clinic than the main entrance of the city-forming enterprise. It is permissible to use absolute values in assessing accessibility if stopping points and adjacent territories are compared at a particular point in time, in any city; specific values can be used when comparing the transport accessibility of equally socially significant objects in different cities.

4. Conclusion

Thus, the index «γ», like a number of other «Kansky indices», can be used in relation to transport and geographical simplexes - stopping points, to more fully assess the degree of convenience of their use by passengers, demonstrating the level of stability of the network, of which the stop is part, to a possible disruption of traffic in some of its sections. Simultaneously with the calculation of these indices, the degree of accessibility of a stopping point in the urban space and the accessibility of urban spaces from it can be determined, with a direct connection - another criterion for the convenience of passengers using a stop of mass public transport and, accordingly, the comfort for living in adjacent territories. It is permissible to use the described methods of using the «γ» index, as well as a number of other «Kansky indices», the «γ» index, not only in the study of the city's mass public transport system, but also in the study of intercity, especially railway passenger traffic.

5. References

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* as a rule, the direct movement (without changing) of passengers reduces the cost of movement for them, makes the trip more comfortable, especially for representatives of low-mobility groups of the population (disabled people, elderly people, parents with small children)

* often the passenger intuitively chooses the stopping point that provides him with the opportunity to use different public transport routes that go to the destination, but each in its own way
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