Quantum information processing offers great advantages over classical information processing, both for efficient algorithms[1, 2, 3] and for secure communication[4]. It can be accomplished by universal gates, such as Fredkin gate, Toffoli gate and the controlled-not(CNOT) gate. Of them Fredkin gate is of interest because it is not only a universal gate for classical reversible computation[5], but also has direct applications in error correcting[6], polarization transfer in NMR[7] and some quantum algorithms[8]. Various schemes have been proposed to implement Fredkin gate. In principle any multi-qubit gate can be build up from combination of the CNOT gates and the single-qubit gates[9, 10]. Chau and Wilczek have given a construction of Fredkin gate with six specific gates[11]. Then Smolin and DiVincenzo have suggested a scheme of implementation of Fredkin gate with fewer pulses has yet been reported up to now. In addition, the simple structure of our scheme makes it easy to be implemented in experiments.

The three-qubit conditional swap gate(Fredkin gate) is a universal gate that can be used to create any logic circuit and has many direct usages. In this paper, we experimentally realized Fredkin gate with only three transition pulses in solution of alanine. It appears that no experimental realization of Fredkin gate with fewer pulses has yet been reported up to now. In addition, the simple structure of our scheme makes it easy to be implemented in experiments.

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In this paper, we report a practical experimental realization of Fredkin gate, which is accomplished by only three transition pulses in NMR QIP. The molecule that we use is alanine(Fig. 1). The effect of undesired coupling on the H nuclei which is also spin$^{\frac{1}{2}}$ is removed by common selective decoupling technique in NMR. Our experimental results show that this scheme is convenient and easy to be realized. And it appears that no experimental realization of Fredkin gate with fewer pulses has yet been reported up to now.

Fredkin gate is also called controlled-swap gate(Fig. 2a), that is, when the controlling qubit(the first qubit) is in the state $|0\rangle$, the two controlled qubits (the second and third qubits) exchange their states after the action of Fredkin gate. Otherwise, if the controlling qubit is in state $|1\rangle$, the two controlled qubits remain their original states. Smolin and DiVincenzo have suggested a scheme to implement Fredkin gate(Fig. 2b)[12, 13]. Their scheme depends greatly on implementation of two-qubit quantum gates and the combination of them. But in NMR QIP implementing two-bit gate in a multi-spin system is more complex than implementing in two-spin system. In other words, in NMR QIP, for multi-spin system combining two-qubit gates is not always an efficient way to implement bigger gates. The reason is that considering two-qubit gates as the basic elements is mainly for mathematics convenience, and can help us calculate and understand the general questions in quantum information. However, in NMR QIP considering two-qubit gates as

![Chemical shifts and J-coupling constants](Image)

**FIG. 1:** The structure of alanine and the table of the chemical shifts and J-coupling constants, the chemical shifts are given with respect to reference frequency 125.76MHz(carbons) on Bruker Avance DMX500 spectrometer. The three weakly coupled spin$^{\frac{1}{2}}$ carbon-13 nuclei served as three qubits, which are labeled by $C_1$, $C_2$, $C_3$. 

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the basic gates is not a reasonable choice, a better choice would be fewer pulses and the pulse sequences which have simple structure and are easier to be realized.

So we reconsider Smolin and DiVincenzo’s scheme to implement Fredkin gate that is composed of two CNOT gates and one Toffoli gate (FIG. 2b). We do not keep on decomposing the Toffoli gate into two-qubit gates. However, inspired by the method Du and his collaborators suggested, we notice that Fredkin gate may be realized with only three transition pulses. Transition pulses are deliberately designed to perturb transverse magnetic field to sway only a small area of spectra. The pulse sequence is shown in FIG. 3, and the parameters of the transition pulses are shown in TABLE I. TPi-cont3-2(i = 1, 3) is a π transition pulse to implement the CNOT gate (qubit-3 as controlling qubit and qubit-2 as controlled qubit).

\[
U_{TP1} = \exp[i\pi/4\sigma_y^2(1 - \sigma_z^3)]
\]

\[
= \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[
U_{TP3} = \exp[-i\pi/4\sigma_y^2(1 - \sigma_z^3)]
\]

\[
= \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

The phases of TP1 and TP3 are set to be inverse with each other. It would help to greatly conceal undesired effects of the chemical shifts evolution during the length of TP2-tof-12-3 and the time needed to switch the on-resonance frequencies \(O_1\). TP2-tof-12-3 is a π transition pulse to implement the Toffoli gate (qubit-1,2 as controlling qubit, qubit-3 as controlled qubit),

\[
U_{TP2} = \exp[-i\pi/8(1 - \sigma_z^1)(1 - \sigma_z^2)\sigma_z^3]
\]

\[
= \begin{pmatrix}
I_6 & 0 & 0 \\
0 & -i\sigma_x & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\]

The three transition pulses are all 180-degree pulses. This is also help to reduce undesired effects of the chemical shifts during the length of transition pulses and J-couplings evolution during the length of transition pulses. Combining the three transition pulses we get

\[
U_{Fred} = U_{TP3} U_{TP2} U_{TP1}
\]

\[
= \begin{pmatrix}
I_5 & 0 & 0 \\
0 & -i\sigma_x & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\]

To confirm whether Fredkin gate is correctly realized, We act the pulse sequence on the state

\[
\rho_{in} = \frac{1}{25} I_{8\times8} + \epsilon(I_x^1 + I_z^2 + I_z^3)
\]

where \(I_k^\alpha(k = 1, 2, 3, \alpha = x, y, z)\) is the matrix for the \(\alpha\)-component of the angular momentum of the spin-\(k\). The relation between \(I_k^\alpha\) and Pauli Matrix \(\sigma_k^\alpha\) is \(I_k^\alpha = \sigma_k^\alpha/2\). This state can be obtained from the equilibrium state

\[
\rho_{eq} = \frac{1}{25} I_{8\times8} + \epsilon(I_x^1 + I_z^2 + I_z^3)
\]

by applying two selective pulse \(R_y^1(\pi)\) and \(R_y^3(\pi)\). Calculation shows that the final state should be

\[
\rho_{out} = U_{TP3} U_{TP2} U_{TP1} \rho_{in} U_{TP1}^\dagger U_{TP2}^\dagger U_{TP3}^\dagger
\]

\[
= \frac{1}{25} I_{8\times8} + \epsilon (I_x^I + I_z^I + I_x^I + I^I + I_x^I + I_z^I)
\]

\[
+ 2I_x^I I_z^I - 2I_x^I I_z^I + 2I_x^I I_z^I - 2I_x^I I_z^I + 4I_x^I I_z^I I_z^I + 4I_x^I I_z^I I_z^I - 4I_x^I I_z^I I_z^I - 4I_x^I I_z^I I_z^I
\]
FIG. 4: The experiment spectra, the solid-lined spectra are from state $\rho_{\text{out}}$ without any readout pulse (the spectra of Spin-C$_{2}$ with 90 degree phase adjusting); the dot-lined spectra are from the state $\rho_{\text{eq}}$ with readout pulse $R_{y}^{2.3}(\frac{\pi}{2})$.

The experimental spectra corresponding to the state of $\rho_{\text{out}}$ without applying any readout pulse are shown in FIG.4. There are two inner peaks in the spectra of qubit-1(Spin-C$_{1}$), which is the expected spectra form of $I_{z}^{2} + 4I_{x}^{2}I_{y}^{2}$; two left peaks(inverse phase) of $J_{12}$ in the spectra qubit-2(Spin-C$_{2}$), which is the expected spectra form of $-2I_{y}^{2}I_{x}^{2} + 4I_{x}^{2}I_{y}^{2}$ with 90 degree phase adjusting (Notice that $J_{12}$ has a different sign with other J); two left peaks of $J_{13}$ in the spectra qubit-3(Spin-C$_{3}$), which is the expected spectra form of $I_{z}^{3} + 2I_{x}^{2}I_{y}^{2}$. These spectra together show that we gain the expected state $\rho_{\text{out}}$ after applying the three transition pulses to the state $\rho_{\text{in}}$. We have also tried other states as the input state and also gained the corresponding expected output states. Experimental results implies that the scheme is working for all 8 basic product basis states. Hence we conclude that Fredkin gate is successfully realized by the three transition pulses.

In conclusion, with a solution of alanine, we have experimentally demonstrated that Fredkin gate can be realized with only three transition pulses in NMR QIP. The pulse sequence that we use has a excellent symmetry. Such symmetry makes the realization of Fredkin gate easier to accomplish in experiment, and also makes the realization be more accurate. We notice that there are small bumps in the spectra of qubit-1(Spin-C$_{1}$) and the spectra qubit-2(Spin-C$_{2}$). These distortions are really small if we notice that the resolution of our Bruker Avance DMX500 spectrometer(0.5Hz) and $J_{12}$(1.27Hz) are comparable. However small, they show that our realization of Fredkin gate is not perfect. Reasons are: first, the magnetic field is not homogeneous; second, $\pi$ transition pulses rotate not precisely 180 degree in experiment; third, although chemical shifts evolution during the length of TP2-tof-12-3 and the time needed to switch the on-resonance frequencies($O_{i}$) are greatly concealed, there still are a little undesired effects of the chemical shifts left during the time of pulse sequence. Our realization of Fredkin gate depend on the realization of transition pulses. For other molecule whose $J_{12}$ is bigger, realizing TPi-cont3-2 will need to implement two single-line selective pulses. With the technique presented by Steffen, Lieven and Chuang in paper [22] it can be accomplished with satisfying accuracy at the same time. So our scheme is also practicable in other molecules, and in principle it is ready to use in NMR QIP.

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