A NEW APPROACH FOR SOLVING INTEGER INTERVAL TRANSPORTATION PROBLEM WITH MIXED CONSTRAINTS

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Abstract:
In this paper, we suggest an interval Transportation problem where the source and destination parameters are chosen as interval numbers. The main objective of the problem is to minimize the total transportation cost and total delivery time. Algorithm is proposed (VAM and MODI) to determine initial allocation to the basic cells and the corresponding interval transportation cost. The optimal solution is obtained without converting the problem to its classical version. Numerical example is obtained which will be easy to understand the solution and procedure of the proposed method.

Keywords: Interval numbers, Interval Transportation problem, Interval VAM method, Interval MODI Method.

1. Introduction
Transportation problem is one of the most important and special type of linear programming problem, where the main objective of the problem is to minimize the transportation cost of distributing a product from a number of sources to a number of destinations. In actual life we cannot predict the shipping cost precisely due to unpredictable and unchangeable life conditions, so shipping cost will be high, therefore normal classical problem will not give a proper solution, hence it can be solved by using Intervals which gives the solution more precisely. Many researchers have proposed different types of interval transportation problem. A. Akilbasha et al. [1] studied about integer interval transportation problem in rough nature. P. Pandian et al. [2] proposed optimal solution is obtained by using intervals by non-fuzzy method. A. Akilbasha et al. [3] Introduced Fully rough integer interval transportation problem is solved by using rough slice sum method, which gives the appropriate solution for rough nature. A. Akilbasha et al. [1,3] studied solution is obtained by using intervals in rough environment. Source and destination is considered to be uncertain and rough cost are assigned in model proposed by Subhakanta Dash et al. [4]. Pradipkundu et al. [5] proposed solid transportation problem unit transportation cost with all parameters are considered as rough variables. Li et al. [6] proposed a Fuzzy compromise programming approach to a multi objective transportation problem. G. Ramesh et al. [7] proposed A method for solving interval number linear programming problem without converting it to a classical linear programming problem. K. Ganesan [8] proposed Arithmetic operations on interval numbers. That is discussed to solve the system of interval linear equations Sundaresan et al. [9] (2013) Resource Management Techniques OR. Ramne Moore et al. [10] Introduction to Interval Analysis (1-15).

2. Preliminaries
The aim of this section is to present some notations and results which are useful for our further consideration.
2.1. Interval numbers

Let \( \tilde{a} = [a_1, a_2] = \{x \in \mathbb{R} : a_1 \leq x \leq a_2 \} \) be an interval on the real line \( \mathbb{R} \). If \( \tilde{a} = a = a = a \), then \( \tilde{a} = [a, a] = a \) is a real number (or a degenerate interval). The mid-point and width (or half-width) of an interval number \( \tilde{a} = [a_1, a_2] \) can be defined as

\[
\mu(\tilde{a}) = \frac{a_1 + a_2}{2} \quad \text{and} \quad \omega(\tilde{a}) = \frac{a_2 - a_1}{2}.
\]

The interval number \( \tilde{a} \) can be expressed in terms of its midpoint and width as \( \tilde{a} = [a_1, a_2] = \langle \mu(\tilde{a}), \omega(\tilde{a}) \rangle \).

2.2. Ranking of Interval Numbers

Sengupta and Pal [11] proposed a simple and efficient index by comparing any two intervals on \( \mathbb{IR} \) through decision maker’s satisfaction.

**Definition 2.1.** Let \( \preceq \) be an extended order relation between the interval numbers \( \tilde{a} = [a_1, a_2] \), \( \tilde{b} = [b_1, b_2] \) in \( \mathbb{IR} \), then for \( \mu(\tilde{a}) < \mu(\tilde{b}) \), we construct a premise \( \tilde{a} \preceq \tilde{b} \) which implies that \( \tilde{a} \) is "inferior to \( \tilde{b} \)" (or \( \tilde{b} \) is superior to \( \tilde{a} \)).

An acceptability function

\[
A: \mathbb{IR} \times \mathbb{IR} \rightarrow [0, 1]
\]

is defined as:

\[
A(\tilde{a}, \tilde{b}) = \frac{w(\tilde{b}) - m(\tilde{a})}{w(\tilde{b}) + w(\tilde{a})}, \quad \text{where} \quad w(\tilde{b}) + w(\tilde{a}) \neq 0.
\]

\( A \) be interpreted as the grade of acceptability of the first interval number to be inferior to the second interval number. For any two interval numbers \( \tilde{a} \) and \( \tilde{b} \) in \( \mathbb{IR} \) either \( A(\tilde{a}, \tilde{b}) \geq 0 \) (or \( A(\tilde{b}, \tilde{a}) \geq 0 \)) or \( A(\tilde{a}, \tilde{b}) = 0 \) (or \( A(\tilde{b}, \tilde{a}) = 0 \)). \( A(\tilde{a}, \tilde{b}) \) is zero if and only if \( \mu(\tilde{a}) = \mu(\tilde{b}) \) and \( \omega(\tilde{a}) = \omega(\tilde{b}) \), then we say that interval numbers \( \tilde{a} \) and \( \tilde{b} \) are equivalent (non-inferior to each other) and we denote it by \( \tilde{a} \approx \tilde{b} \). Also if \( A(\tilde{a}, \tilde{b}) \geq 0 \) then \( \tilde{a} \preceq \tilde{b} \) and if \( A(\tilde{b}, \tilde{a}) \geq 0 \) then \( \tilde{b} \preceq \tilde{a} \).

2.3. A New Interval Arithmetic

Ming Ma et al. have proposed a new fuzzy arithmetic based on both location index and fuzziness index function. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which exist as the least upper bound and greatest lower bound in the lattice \( L \). That is for \( a, b \in L \) we define

\[
\begin{align*}
\tilde{a} + \tilde{b} &= \langle m(\tilde{a}), w(\tilde{a}) \rangle + \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) + m(\tilde{b}), \max \{w(\tilde{a}), w(\tilde{b})\} \rangle, \\
\tilde{a} - \tilde{b} &= \langle m(\tilde{a}), w(\tilde{a}) \rangle - \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) - m(\tilde{b}), \max \{w(\tilde{a}), w(\tilde{b})\} \rangle, \\
\tilde{a} \times \tilde{b} &= \langle m(\tilde{a}), w(\tilde{a}) \rangle \times \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) \times m(\tilde{b}), \max \{w(\tilde{a}), w(\tilde{b})\} \rangle, \\
\tilde{a} \div \tilde{b} &= \langle m(\tilde{a}), w(\tilde{a}) \rangle \div \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) \div m(\tilde{b}), \max \{w(\tilde{a}), w(\tilde{b})\} \rangle, \quad \text{provided} \quad m(\tilde{b}) \neq 0.
\end{align*}
\]
3. Main Results

Let us consider a fully interval transportation problem with m sources and n destinations involving numbers. Let \( \tilde{a}_i \geq 0 \) be the availability at source i and \( \tilde{b}_j \geq 0 \) be the requirement at destination j. Let \( \tilde{c}_{ij} (\tilde{c}_{ij} \geq 0) \) be the unit interval transportation cost from source i to destination j. Let \( x_{ij} \) denote the number of interval units to be transported from source i to destination j. Then the problem is to find a feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total interval transportation cost is minimized.

3.1. Mathematical formulation of interval transportation problem

The mathematical model of fully interval integer transportation problem is as follows

\[
\begin{align*}
\text{Minimize } & \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \\
\text{subject to } & \quad \sum_{j=1}^{n} \tilde{x}_{ij} \geq \tilde{a}_i, \quad i = 1, 2, 3, \ldots, m \\
& \quad \sum_{i=1}^{m} \tilde{x}_{ij} \geq \tilde{b}_j, \quad j = 1, 2, 3, \ldots, n \\
& \quad \sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j, \text{ where } \sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j \geq 0 \quad \text{for all } i \text{ and } j
\end{align*}
\]

And \( \tilde{a}_i, \tilde{b}_j, \tilde{c}_{ij} \in \mathbb{IR} \), where \( \tilde{c}_{ij} \) is the interval unit transportation cost from \( i^{th} \) source to the \( j^{th} \) destination. The main objective is to minimize the total fuzzy transportation cost. In this paper the fuzzy transportation problem is solved by interval version of VAM and MODI method. This interval transportation problem is explicitly represented by the following interval transportation table.

| Sources | 1 | 2 | … | N | Supply |
|---------|---|---|---|---|--------|
| 1       | \( \tilde{c}_{11} \) | \( \tilde{c}_{12} \) | … | \( \tilde{c}_{1n} \) | \( \tilde{a}_1 \) |
| 2       | \( \tilde{c}_{21} \) | \( \tilde{c}_{22} \) | … | \( \tilde{c}_{2n} \) | \( \tilde{a}_2 \) |
| …      | … | … | … | … | … |
| M       | \( \tilde{c}_{m1} \) | \( \tilde{c}_{m2} \) | … | \( \tilde{c}_{mn} \) | \( \tilde{a}_n \) |
| Demand  | \( \tilde{b}_1 \) | \( \tilde{b}_2 \) | … | \( \tilde{b}_m \) |        |

The necessary and sufficient condition for the existence of an interval feasible solution to the fully interval transportation problem (3.1)
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} b_{j}, \quad i = 1, 2, 3, ..., m \text{ and } j = 1, 2, 3, ..., n
\]

(Total supply \( \approx \) Total demand). \hfill (3.2)

**Proof: ( Necessary condition)**

Let there exist an interval feasible solution to the fully interval transportation problem

Minimize \( \tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \)

subject to \( \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{\alpha}_i, \quad i = 1, 2, 3, ..., m \).

\( \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{\beta}_j, \quad j = 1, 2, 3, ..., n \).

and \( \tilde{x}_{ij} \geq 0 \) for all \( i \) and \( j \). \hfill (3.3)

From \( \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{\alpha}_i, (i = 1, 2, 3, ..., m) \), we have \( \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{x}_{ij} = \sum_{i=1}^{m} \tilde{\alpha}_i \) \hfill (3.4)

Also from \( \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{\beta}_j, \quad (j = 1, 2, 3, ..., n) \), we have \( \sum_{j=1}^{n} \sum_{i=1}^{m} \tilde{x}_{ij} = \sum_{j=1}^{n} \tilde{\beta}_j \) \hfill (3.5)

From equations (3.4) and (3.5), we have \( \sum_{i=1}^{m} \tilde{\alpha}_i = \sum_{j=1}^{n} \tilde{\beta}_j \).

**Sufficient condition**

Since all \( \tilde{\alpha}_i \) and \( \tilde{\beta}_j \) are positive, \( \tilde{x}_{ij} \) must be all positive. Therefore equation (3.2) yields a feasible solution.

**Theorem 3.2**. The dimension of the basis of an \((m \times n)\) fully interval transportation are \((m+n-1)\). That is a fully interval transportation problem has only \((m+n-1)\) independent structural constraints and its basic feasible solution has only \((m+n-1)\) positive components.

**Proof**: Consider a fully interval transportation problem with \( m \) sources and \( n \) destinations,

\[
\text{Minimize } \tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}
\]

subject to \( \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{\alpha}_i, \quad i = 1, 2, 3, ..., m \).

\( \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{\beta}_j, \quad j = 1, 2, 3, ..., n \).

and \( \tilde{x}_{ij} \geq 0 \) for all \( i \) and \( j \).

Let us consider the fully interval transportation has \( m \) rows (supply constraint equations) and \( n \) columns (demand constraint equations). There are totally \((m+n)\) constraint equations.
This is due to the condition that \( \sum_{i=1}^{m} \bar{a}_{i} + \sum_{j=1}^{n} \bar{b}_{j} = \frac{1}{g_{166}} \); which is the last requirement constraint. Therefore one of \((m+n)\) constraints can always be derived from the remaining \((m+n-1)\). Thus there exists only \((m+n-1)\) independent constraints and its basic feasible solution has only \((m+n-1)\) positive components.

**Theorem 3.3.** The values of the interval basic feasible solution are all differences between the partial sum of \( \bar{a}_{i} \) and the partial \( \bar{b}_{j} \); that is \( \bar{x}_{ij} = \pm \sum_{i=1}^{m} \bar{a}_{i} \mp \sum_{j=1}^{n} \bar{b}_{j} \), where \( \bar{r} \) and \( \bar{s} \) are either \( \bar{r} = \bar{1}=\bar{[1,1]} \) or \( \bar{0} = \bar{0} = \bar{[0,0]} \).

**Theorem 3.4.** The fully interval transportation problem has a triangular basis.

**Proof:** We note that every equation has a basic variable; otherwise, the equation cannot be satisfied for \( \bar{a}_{i} \neq 0, \bar{b}_{j} \neq 0 \). Suppose every row and column equations has atleast two basic variables, since there are \( m \) rows and \( n \) columns, the total number of basic variables in row equations and column equations will be at least \( 2m \) and \( 2n \) respectively. Suppose if the total number of basic variables is \( D \), then obviously \( D \geq 2m, D \geq 2n \).

Case (i). If \( m < n \), then \( m + n < n + n \Rightarrow m + n < 2n \Rightarrow m + n < D \Rightarrow m + n < D \).

Case (ii). If \( m > n \), then \( m + n > m + n \Rightarrow m + m > m + n \Rightarrow 2m > m + n \Rightarrow m + n < 2m \Rightarrow m + n < 2m \leq D \Rightarrow m + n < D \).

Case (iii). If \( m = n \), then \( m + n = m + n \Rightarrow m + m = m + n \Rightarrow 2m = m + n \Rightarrow m + n = 2m \Rightarrow m + n = 2m < D \Rightarrow m + n < D \).

Thus in all cases \( D \geq m + n \). But the number of basic variables in a fully interval transportation problem is \((m+n-1)\) which is a contradiction.

Therefore there is atleast one equation, either row or column having only one basic variable. Let the \( r^{th} \) equation have only one basic variable. Let \( \bar{x}_{rs} \) be the only basic variable in the \( r^{th} \) row and \( s^{th} \) column, then \( \bar{x}_{rs} = \bar{a}_{r} \). Eliminate \( r^{th} \) row from the system of equations and substitute \( \bar{x}_{rs} = \bar{a}_{r} \) in \( s^{th} \) column equation and replace \( \bar{b}_{s} \) by \( \bar{b}_{s} = \bar{b}_{s} - \bar{a}_{r} \). After eliminating the \( r^{th} \) row, the system has \((m+n-2)\) linearly independent constraints. Hence the number of basic variables is \((m+n-2)\).

Repeat the process and conclude that in the reduced system of equations there is an equation which has only one basic variable. But if this is in \( s^{th} \) column equation, then it will have two basic variables.

This concludes that in our original system of equations, there is an equation which has atleast two basic variables. Continue the process to get the theorem.

4. PROPOSED ALGORITHMS

4.1. Interval Version of Vogel's Approximation Method (IVAM)

**Step 1.** Express the interval transportation problem in the transportation table.

**Step 2.** Express the all interval parameters supply, demand and unit transportation cost in the transportation problem in terms of midpoint and half width. That is in the form of \( \bar{a} = [a_{1}, a_{2}] = (m(\bar{a}), w(\bar{a})) \).
Step 3. In the interval Transportation table, determine the row penalties for each row by subtracting the lowest cost from the next lowest cost of that row. In the similar way calculate the column penalties for each column. Write down the row penalties aside each row and the column penalties below each column.

Step 4. Identify the column or row with largest fuzzy penalty. In case of tie, break the tie arbitrarily. Select a cell with minimum fuzzy cost in the selected column (or row, as the case may be) and assign the maximum units possible by considering the demand and supply position corresponding to the selected cell.

Step 5. Delete the column/row for which the supply and demand requirements are met.

Step 6. Continue steps 1 to 3 for the resulting fuzzy transportation table until the supply and demand of all sources and destinations have been met.

4.2. Interval Version of MODI Method

Step 1: Start with interval basic feasible solution consisting of \((m+n–1)\) allocations in independent positions.

Step 2: Determine the values of \((m+n)\) dual variables \(\bar{u}_i\) (\(i = 1, 2, \ldots, m\)) and \(\bar{v}_j\) (\(j = 1, 2, \ldots, n\)) for all the rows and columns such that for each occupied cell \((i, j)\), the following condition is satisfied:

\[
\bar{c}_{ij} = \bar{u}_i + \bar{v}_j
\]

Step 3: Compute the opportunity cost \(\tilde{d}_{ij}\) for each unoccupied cell \((i, j)\) by using the formula:

\[
\tilde{d}_{ij} = \tilde{c}_{ij} - (\bar{u}_i + \bar{v}_j)
\]

Step 4: Check the sign of the opportunity cost \(\tilde{d}_{ij}\) for each unoccupied cell \((i, j)\) and conclude that

(i) If all \(\tilde{d}_{ij} \geq \tilde{0}\), then the current solution is optimal.

(ii) If at least one \(\tilde{d}_{ij} < \tilde{0}\), then the current solution is not optimal and an improved solution can be obtained.

In this case, the unoccupied cell with the most negative value of \(\tilde{d}_{ij}\) is considered for the new transportation schedule.

Step 5: Construct a closed path (loop) for the unoccupied cell \((i, j)\) having largest negative opportunity cost. Mark a (+) sign in this cell and move along the rows (or columns) to find an occupied cell. Mark a (-) sign in this cell and find out another occupied cell. Repeat the process and mark the occupied cells with (+) and (-) signs alternatively. Close the path back to the selected unoccupied cell.

Step 6: Select the smallest quantity amongst the cells marked with (-) sign. Allocate this value to the unoccupied cell of the loop and add and subtract it in the occupied cells as per their signs. Thus an improved solution is obtained by calculating the total transportation cost by this method.

Step 7: Test the revised solution further for optimality. The procedure terminates when all \(\tilde{d}_{ij} \geq \tilde{0}\), for unoccupied cells.

5. Numerical Examples

Example 5.1. Consider the following interval transportation problem discussed by Akilbasha and Natarajan [3]. They proposed a new method namely, roughslice sum method for finding an optimal solution for interval transportation problems where transportation cost, supply and demand are intervals.
Table 5.1. Interval Transportation table

|       | D₁     | D₂     | D₃     | Supply |
|-------|--------|--------|--------|--------|
| S₁    | [3,4]  | [7,9]  | [6.8]  | [6.7]  |
| S₂    | [7,8]  | [5,7]  | [3.5]  | [8,10] |
| S₃    | [9,10] | [11,13]| [4.6]  | [11,13]|
| Demand| [9,10] | [12,14]| [7,9]  |        |

A. Akilbasha et al. obtained the optimum interval transportation cost of this problem as [120,195]. Let us consider the same problem and solve by VAM and MODI method. Let us define all the interval parameters as \(\bar{a} = [\bar{a}_1, \bar{a}_2] = (m(\bar{a}), w(\bar{a}))\). Now the given interval transportation problem is as follows

Table 5.2. Interval Transportation table

|       | D₁     | D₂     | D₃     | Supply |
|-------|--------|--------|--------|--------|
| S₁    | (3,5,0.5) | (8,1)  | (7,1)  | (6.5,0.5) |
| S₂    | (7.5,0.5) | (6,1)  | (4,1)  | (9,1)  |
| S₃    | (9,5,0.5) | (12,1) | (5,1)  | (12,1) |
| Demand| (9,5,0.5) | (13,1) | (8,1)  |        |

Table 5.3. Interval transportation table

To balance the above problem, now we have to add one row

|       | D₁     | D₂     | D₃     | Supply |
|-------|--------|--------|--------|--------|
| S₁    | (3.5,0.5) | (8,1)  | (7,1)  | (6,5,0.5) |
| S₂    | (7.5,0.5) | (6,1)  | (4,1)  | (9,1)  |
| S₃    | (9.5,0.5) | (12,1) | (5,1)  | (12,1) |
| S₄    | (0,0)   | (0,0)  | (0,0)  | (3,0)  |
| Demand| (9,5,0.5) | (13,1) | (8,1)  | (30,5,1) |
Applying the Interval version of Vogel’s Approximation Method (IVAM), the initial interval basic feasible solution is obtained as follows:

**Table 5.4. Interval transportation table**

|       | $D_1$    | $D_2$    | $D_3$    | Supply   | $\theta$ |
|-------|----------|----------|----------|----------|----------|
| $S_1$ | (3.5,0.5)| (8,1)    | (7.1)    | (6,5,0.5)| (3.5,1)  |
| $S_2$ | (7.5,0.5)| (6,1)    | (4.1)    | (9,1)    | (2,1)    |
| $S_3$ | (9,5,0.5)| (12,1)   | (5,1)    | (12,1)   | (4,5,1)  |
| $S_4$ | (0,0)    | (0,0)    | (0,0)    | (3,0)    | (0,0)    |
| Demand| (9.5,0.5)| (10,1)   | (8,1)    |          |          |
| $\theta$| (3.5,0.5)| (6,1)    | (4,1)    |          |          |

**Table 5.5. Interval transportation table**

|       | $D_1$    | $D_2$    | $D_3$    | Supply   | $\theta$ |
|-------|----------|----------|----------|----------|----------|
| $S_1$ | (3.5,0.5)| (8,1)    | (7.1)    | (6,5,0.5)| (3.5,1)  |
| $S_2$ | (7.5,0.5)| (6,1)    | (4,1)    | (9,1)    | (2,1)    |
| $S_3$ | (9,5,0.5)| (12,1)   | (5,1)    | (4,1)    | (4,5,1)  |
| Demand| (9.5,0.5)| (10,1)   | (8,1)    |          |          |
| $\theta$| (4,0.5) | (2,1)    | (1,1)    |          |          |

**Table 5.6. Interval transportation table**

|       | $D_1$    | $D_2$    | Supply   | $\theta$ |
|-------|----------|----------|----------|----------|
| $S_1$ | (3.5,0.5)| (8,1)    | (6,5,0.5)| (4,5,1)  |
| $S_2$ | (7.5,0.5)| (6,1)    | (9,1)    | (1,5,1)  |
| $S_3$ | (9,5,0.5)| (12,1)   | (4,1)    | (2,5,1)  |
| Demand| (3,0.5)  | (10,1)   |          |          |
| $\theta$| (4,0.5) | (2,1)    |          |          |
Table 5.7. Interval transportation table

|     | $D_1$    | $D_2$    | Supply  | $\theta$ |
|-----|----------|----------|---------|----------|
| $S_2$ | (7.5,0.5) | (6.1)    | (9.1)   | (1.5,1)  |
| $S_3$ | (9.5,0.5) | (12.1)   | (4.1)   | (2.5,1)  |
| Demand | (3,0.5)   | (1,1)    |         |          |
| $\theta$ | (2,0.5)   | (6.1)    |         |          |

Table 5.8. Interval transportation table

|     | $D_1$    | $D_2$    | Supply  | $\theta$ |
|-----|----------|----------|---------|----------|
| $S_3$ | (9.5,0.5) | (12.1)   | (3.1)   | (2.5,1)  |
| Demand | (3,0.5)   | (1,1)    |         |          |
| $\theta$ | (9.5,0.5) | (12.1)   |         |          |

Table 5.9. Interval transportation table

|     | $D_1$    | Supply |
|-----|----------|--------|
| $S_3$ | (9.5,0.5) | (3.1)  |
| Demand | (3,0.5)   |        |

Hence the initial basic feasible solution of the interval transportation problem by using interval Vogel’s approximation method is

Table 5.10. Interval transportation table

|     | $D_1$    | $D_2$    | $D_3$    | Supply  |
|-----|----------|----------|----------|---------|
| $S_1$ | (3.5,0.5)| (8,1)    | (7,1)    | (6.5,0.5)|
| $S_2$ | (7.5,0.5)| (6.1)    | (4,1)    | (9.1)   |
| $S_3$ | (9.5,0.5)| (12,1)   | (5,1)    | (12,1)  |
| $S_4$ | (0,0)    | (0,0)    | (0,0)    | (3,0)   |
| Demand | (9.5,0.5)| (13,1)   | (8,1)    | (30.5,1)|
The initial interval transportation cost
\[
= (3.5, 0.5)(6.5, 0.5) + (6, 1)(9, 1) + (9.5, 0.5)(3.1) + (12.1, 1)(1, 1) + (5.1)(8.1) + (0, 0)(3, 0)
= (0, 0) + (40.1) + (22.75, 0.5) + (54.1) + (12.1)(1, 1) + (28.5, 1)
= (157.25, 1)
\]
Now apply the interval version of MODI method for checking the optimality, we have

Table 5.11. Interval transportation table

|          |     |     |     |
|----------|-----|-----|-----|
|          |     |     |     |
| (3.5, 0.5) | (8, 1) | (7.1) | \( u_1 = (0, 1) \) |
| (6, 5, 0.5) | (6.1) | (9, 1) | \( u_2 = (0, 1) \) |
| (7, 5, 0.5) | (12, 1) | (1, 1) | \( u_3 = (6.0, 5) \) |
| (9.5, 0.5) | (0.0) | (3, 0) | \( u_4 = (6.1) \) |
|          | \( v_1 = (3.5, 0.5) \) | \( v_2 = (6.1) \) | \( v_3 = (-1, 1) \) |

Since all \( \tilde{d}_k \geq 0 \), the current solution is an interval optimal solution. Hence the optimal interval transportation cost is
\[
= (3.5, 0.5)(6.5, 0.5) + (6, 1)(9, 1) + (9.5, 0.5)(3.1) + (12.1, 1)(1, 1) + (5.1)(8.1) + (0, 0)(3, 0)
= (0, 0) + (40.1) + (22.75, 0.5) + (54.1) + (12.1)(1, 1) + (28.5, 1)
= (157.25, 1)
= [156.25, 158.25]
\]
It is to be noted that our solution is very much sharper than the solution obtained by Akilbasha et al.

**Example 5.2.** Consider another problem discussed by Akilbasha et al.

For this problem, Akilbasha et al. obtained the optimum interval transportation cost as \([63, 288]\).

Table 5.12. Interval transportation table

|          | \( D_1 \) | \( D_2 \) | \( D_3 \) | Supply |
|----------|-----------|-----------|-----------|--------|
| \( S_1 \) | [2, 5] | [5, 11] | [4, 10] | [5, 8] |
| \( S_2 \) | [6, 9] | [3, 9] | [1.7] | [6, 12] |
| \( S_3 \) | [8, 11] | [9, 15] | [2, 8] | [9, 15] |
| Demand   | [8, 11] | [10, 16] | [5, 11] |

Let us define all the interval parameters \( \tilde{a} = [a_1, a_2] \) in terms of midpoint and width as \( \tilde{a} = [a_1, a_2] = (\tilde{m}(\tilde{a}), \tilde{w}(\tilde{a})) \). Now the given interval transportation problem will be as follows
To balance the above problem, now we have to add one row.

### Table 5.13. Interval Transportation table

|        | D₁    | D₂    | D₃    | Supply  |
|--------|-------|-------|-------|---------|
| S₁     | (3.5,1.5) | (8,3) | (7,3) | (6.5,1.5) |
| S₂     | (7.5,1.5) | (6.3) | (4,3) | (9,3)   |
| S₃     | (9.5,1.5) | (12,3)| (5,3) | (12,3)  |
| Demand | (9.5,1.5) | (13,3)| (8,3) |         |

### Table 5.14. Interval transportation table

|        | D₁    | D₂    | D₃    | Supply  |
|--------|-------|-------|-------|---------|
| S₁     | (3.5,1.5) | (8,3) | (7,3) | (6.5,1.5) |
| S₂     | (7.5,1.5) | (6.3) | (4,3) | (9,3)   |
| S₃     | (9.5,1.5) | (12,3)| (5,3) | (12,3)  |
| S₄     | (0,0)  | (0,0) | (0,0) | (3,0)   |
| Demand | (9.5,1.5) | (13,3)| (8,3) | (30.5,3) |

Applying the Interval version of Vogel’s Approximation Method (IVAM), the initial interval basic feasible solution is obtained as follows

### Table 5.15. Interval transportation table

|        | D₁    | D₂    | D₃    | Supply  | θ     |
|--------|-------|-------|-------|---------|-------|
| S₁     | (3.5,1.5) | (8,3) | (7,3) | (6.5,1.5) | (3.5,3) |
| S₂     | (7.5,1.5) | (6.3) | (4,3) | (9,3) | (2,3)   |
| S₃     | (9.5,1.5) | (12,3)| (5,3) | (12,3) | (4.5,3) |
| S₄     | (0,0)  | (0,0) | (0,0) | (3,0) | (0,0)   |
| Demand | (9.5,1.5) | (10,3)| (8,3) |       |        |
| θ      | (3.5,1.5) | (6,3) | (4,3) |        |        |
Table 5.16. Interval transportation table

|       | $D_1$      | $D_2$      | $D_3$      | Supply  | $\theta$ |
|-------|------------|------------|------------|---------|----------|
| $S_1$ | (3.5,1.5)  | (8,3)      | (7,3)      | (6.5,1.5)| (3.5,3)  |
| $S_2$ | (7.5,1.5)  | (6,3)      | (4,3)      | (9,3)   | (2,3)    |
| $S_3$ | (9.5,1.5)  | (12,3)     | (5,3)      | (8,3)   | (4,3)    |
| Demand| (9.5,1.5)  | (10,3)     | (8,3)      |         |          |
| $\theta$| (4,1.5)   | (2,3)      | (1,3)      |         |          |

Table 5.17. Interval transportation table

|       | $D_1$      | $D_2$      | Supply  | $\theta$ |
|-------|------------|------------|---------|----------|
| $S_1$ | (3.5,1.5)  | (6.5,1.5)  | (8,3)   | (6.5,1.5)| (4.5,3)  |
| $S_2$ | (7.5,1.5)  | (6,3)      | (9,3)   | (1.5,3)  |
| $S_3$ | (9.5,1.5)  | (12,3)     | (4,3)   | (2.5,3)  |
| Demand| (3,1.5)    | (10,3)     |         |          |
| $\theta$| (4,1.5)   | (2,3)      |         |          |

Table 5.18. Interval transportation table

|       | $D_1$      | $D_2$      | Supply  | $\theta$ |
|-------|------------|------------|---------|----------|
| $S_2$ | (7.5,1.5)  | (6,3)      | (9,3)   | (1.5,3)  |
| $S_3$ | (9.5,1.5)  | (12,3)     | (4,3)   | (2.5,3)  |
| Demand| (3,1.5)    | (1,3)      |         |          |
| $\theta$| (2,1.5)   | (6,3)      |         |          |

Table 5.19. Interval transportation table

|       | $D_1$      | $D_2$      | Supply  | $\theta$ |
|-------|------------|------------|---------|----------|
| $S_3$ | (9.5,1.5)  | (12,3)     | (3,3)   | (2.5,3)  |
| Demand| (3,1.5)    | (1,3)      |         |          |
| $\theta$| (9.5,1.5) | (12,3)     |         |          |
Hence the initial basic feasible solution for the interval transportation problem using interval Vogel’s approximation method is

\[
\begin{array}{c|c|c}
\text{D} & \text{S} & \text{Supply} \\
\hline
\text{D}_1 & (9.5,1.5) & (3,3) \\
\text{D}_2 & (6.5,1.5) & (7,3) \\
\text{D}_3 & (7.5,1.5) & (9.3) \\
\text{D}_4 & (9.5,1.5) & (12,3) \\
\text{D}_5 & (0,0) & (3,0) \\
\text{Demand} & (9.5,1.5) & (8,3) & (30.5,3) \\
\end{array}
\]

The initial interval transportation cost
\[
= (3.5,1.5)(6.5,1.5)+(6.3)(9.3)+(9.5,1.5)(3,3)+(12,3)(1.3)+(5,3)(8,3)+(0,0)(3,0)
\]
\[
= (0,0)+(40,3)+(22.75,1.5)+(54,3)+(12,3)(1,3)+(28,5,3)
\]
\[
= (157,25,3)
\]

Now by applying the interval version of MODI method for checking the optimality, we have

\[
\begin{array}{c|c|c|c|c}
\text{D} & \text{S} & \text{D} & \text{Supply} \\
\hline
\text{D}_1 & (3.5,1.5) & (8,3) & (7,3) & (6.5,1.5) \\
\text{D}_2 & (7,5,1.5) & (6.3) & (4.3) & (9.3) \\
\text{D}_3 & (9.5,1.5) & (12,3) & (5,3) & (12,3) \\
\text{D}_4 & (0,0) & (0,0) & (0,0) & (3,0) \\
\text{Demand} & (9.5,1.5) & (13,3) & (8,3) & (30.5,3) \\
\end{array}
\]

The initial interval transportation cost
\[
= (3.5,1.5)(6.5,1.5)+(6.3)(9.3)+(9.5,1.5)(3,3)+(12,3)(1.3)+(5,3)(8,3)+(0,0)(3,0)
\]
\[
= (0,0)+(40,3)+(22.75,1.5)+(54,3)+(12,3)(1,3)+(28,5,3)
\]
\[
= (157,25,3)
\]
Since all $\bar{d}_i \geq \check{0}$, the present solution is an interval optimal solution. Hence the optimal interval transportation cost is

$$= (3.5, 1.5)(6.5, 1.5) + (6, 3)(9.3) + (9.5, 1.5)(3, 3) + (12, 3)(1.3) + (5, 3)(8, 3) + (0.0)(3, 0)$$

$$= (157.25, 3)$$

$$= [154.25, 160.25]$$

It is to be noted that our solution is very much sharper than $[63, 288]$ the solution obtained by Akilbasha et al.

6. Conclusion

The Transportation problem with mixed constraints having all parameters as integer intervals is considered in this paper. A method namely VAM and MODI method is proposed to solve fully integer interval transportation problem without converting it to the classical transportation problem so that the total shipping cost is minimum. The above method is a systematic procedure which is both easy to understand and apply in real life. This method is a method provides an optimal solution which is sharper than the other methods.

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