Extended phase space thermodynamics and \( P - V \) criticality of black holes with Born-Infeld type nonlinear electrodynamics

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In this paper, we take into account the black hole solutions of Einstein gravity in the presence of logarithmic and exponential forms of nonlinear electrodynamics. At first, we consider the cosmological constant as a dynamical pressure to study the analogy of the black hole solutions with the Van der Waals liquid-gas system in the extended phase space. We plot \( P - v \), \( T - v \) and \( G - T \) diagrams and investigate the phase transition of adS black holes in the canonical ensemble. Moreover, we discuss about the effect of nonlinear electrodynamics on the critical values and the universal ratio \( P_c v_c/T_c \).

I. INTRODUCTION

In recent years, there has been an increasing interest in asymptotically anti-de Sitter black holes. This interest is mainly based on the fact that there is an equivalence of string theory on asymptotically anti-de-Sitter spacetime and the quantum field theory living on the boundary of it \([1]\). Although Einstein had inserted the cosmological constant as a fixed parameter in the gravitational field equations, in an increasing number of recent papers it might be regarded as a variable \([2]\). In other words, following the recent idea of including the cosmological constant in the first law of black hole thermodynamics, the cosmological constant is no longer a fixed parameter, but rather a thermodynamic variable.

Regarding the extended phase space of black holes thermodynamics, one may treat the cosmological constant as a dynamical pressure \([3]\). The results are much richer thermodynamics than heretofore, and interestingly, it is shown that the critical behavior of black holes is analogous to the Van der Waals liquid-gas phase transition. So, investigation of the mentioned extending phase space and its phase transition have gained a lot of attention recently \([3]\). Phase transition plays an important role in describing different phenomena in thermodynamics and quantum point of views. Thermodynamic properties and phase transition of the asymptotically anti-de Sitter black holes was studied by Hawking and Page \([4]\). Witten \([5]\) reconsidered the Hawking-Page phase transition in the context of gauge theory and AdS/CFT correspondence.

On the other side, although Maxwell’s theory is capable of describing different phenomena in electrodynamics domain, it fails regarding some important issues (for various limitations of the Maxwell theory see Ref. \([5]\) for more details). In order to solve these problems, one may regard the nonlinear electrodynamics \([7-10]\). In recent years, study of nonlinear electrodynamics has got a new impetus. Strong motivation comes from developments in string/M-theory, which is a promising approach to quantum gravity \([11]\). It has been shown that the Born-Infeld \([12]\) (BI) type theories are specific in the context of NLED models and naturally arise in the low-energy limit of heterotic string theory \([13]\).

In recent years, other BI types of NLED have been introduced, in which can remove the divergency of the electric field of point charge near the origin. The Lagrangians of logarithmic and exponential forms of NLED theories were, respectively, proposed by Soleng \([9]\) and Hendi \([10]\) with the following explicit forms

\[
\mathcal{L}(\mathcal{F}) = \begin{cases} 
\beta^2 \left( \exp\left( -\frac{\mathcal{F}}{\beta^2} \right) - 1 \right), & \text{ENEF} \\
-8\beta^2 \ln \left( 1 + \frac{\mathcal{F}}{\beta^2} \right), & \text{LNEF}
\end{cases}
\]

(1)

where \( \beta \) is nonlinearity parameter. In this paper, we consider asymptotically anti-de Sitter black hole solutions of the Einstein gravity in the presence of the recent BI type NLED to investigate the extended phase space thermodynamics and \( P - v \) criticality of the solutions.

Let us begin with the following \( d \)-dimensional spherically symmetric line element

\[
d\sigma^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2d\Omega_{d-2}^2,
\]

(2)

where \( d\Omega_{d}^2 \) denotes the standard metric of \( d \)-dimensional sphere \( S^d \) with the volume \( V_d \). In what follows, we consider Einstein gravity coupled with the mentioned BI type NLED models \([14, 15]\). It was shown that, regardless of gravitational sector, one may use the following electromagnetic field equation to obtain related gauge potential

\[
\nabla_{\mu} \left( \frac{d\mathcal{L}(\mathcal{F})}{d\mathcal{F}} F^{\mu\nu} \right) = 0,
\]

(3)
in which the nonzero component of gauge potential is \( A_t \) \[14\]

\[
A_t = \begin{cases} 
\frac{2\beta \sqrt{L_W}}{2(d-3)(3d-7)} & [3d - 7 + (d - 2)] \sum \left( \frac{1}{2} \left( \frac{5d-11}{2(d-2)} \right), \frac{L_W}{2(d-2)} \right), \text{ ENEF} \\
\frac{2\beta^2 d - 1}{q(d-1)} & 1 - 2 F_1 \left( \left[ \frac{1}{2} \left( \frac{d-1}{2(d-2)} \right), \frac{d-3}{2(d-2)} \right], (1 - \Gamma^2) \right), \text{ LNEF} 
\end{cases}
\]  

where the LambertW function \( L_W = \text{LambertW} \left( \frac{4\beta^2}{e^{4\beta^2}} \right) \), \( \Gamma = \sqrt{1 + \frac{4\beta^2}{e^{4\beta^2}}} \) and \( q \) is an integration constant related to the total electric charge \( Q = \frac{\sqrt{1 - \frac{4\beta^2}{e^{4\beta^2}}} q} {8\pi^2} \). The electric potential \( U \), measured at infinity with respect to the event horizon, is \( U = -A_t \). It is easy to show that the electric field may be written as \[14\]

\[
E(r) = F_{tr} = \frac{Q}{r^2} \times \begin{cases} e^{-\frac{L_W}{2}}, \text{ ENEF} \\
\frac{2}{1+\Gamma}, \text{ LNEF} 
\end{cases} 
\]

II. EXTENDED PHASE SPACE AND \( P - V \) CRITICALITY IN EINSTEIN GRAVITY

Starting with the Einstein gravity in the presence of NLED, we consider the following field equation

\[
G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{2} g_{\mu\nu} \mathcal{L}(\mathcal{F}) - 2 \frac{d\mathcal{L}(\mathcal{F})}{d\mathcal{F}} F_{\mu\lambda} F^{\lambda}_{\nu},
\]

with the following solutions for the metric function \[14\]

\[
g(r) = 1 - \frac{2\Lambda r^2}{(d-1)(d-2)} - \frac{m}{r^{d-3}} + \begin{cases} 
- \frac{\beta^2 r^2}{(d-1)(d-2)} + \frac{2g\beta}{(d-2)^{d-1}} \int \frac{1-L_W}{\sqrt{L_W}} dr, \text{ ENEF} \\
- \frac{16\beta^2 r^2}{(d-1)(d-2)} - \frac{8\beta^2 \ln(2)}{(d-1)(d-2)} + \frac{8}{(d-2)^{d-1}} \int \frac{e^{-L_W}(\beta^2 + (d-4)(d-1))}{r^{d-2}} dr, \text{ LNEF} 
\end{cases}
\]

where \( m \) is an integration constant which is related to the total mass. Looking for the curvature singularity of the metric \[2\], one finds that the Kretschmann scalar diverges at \( r = 0 \). In addition, numerical analysis shows that the metric functions have at least one real positive root. Moreover, using series expansion of metric function for large values of \( r \), we find that the dominant term is related to \( \Lambda \). Consequently, we deduce that these solutions may be interpreted as asymptotically anti de-Sitter black holes.

Now, we take into account the surface gravity interpretation to obtain the Hawking temperature of the mentioned black hole solutions

\[
T = \frac{\Lambda r_+}{2\pi (d-2)} + \begin{cases} 
\frac{\beta r_+}{4\pi(d-2)} \left( 1 - \frac{\beta^2}{e^{2\beta^2}} \right) + \frac{(d-2)(d-3)-4r_+^2 e^{2\beta^2}}{4\pi(d-2)r_+}, \text{ ENEF} \\
\frac{8\beta^2 r_+^2 \ln(1 + \frac{E^2}{4\pi^2}) - (d-2)(d-3) + r_+ E^2}{4\pi(d-2)r_+}, \frac{r_+ E^2}{\pi(d-2)(1 + \frac{E^2}{4\pi^2})}, \text{ LNEF} 
\end{cases}
\]

The Arnowitt-Deser-Misner (ADM) mass of black hole can be obtained by using the behavior of the metric at large distances \[16\]

\[
M = \frac{\omega_d-2}{16\pi} (d-2) m,
\]

where one may obtain the parameter \( m \) from the fact that the metric functions vanish at the event horizon, \( r_+ \). The black hole entropy of Einstein gravity may be determined from the area law

\[
S = \frac{\omega_d-2}{4} \pi^{d-2}.
\]

Here, we regard \( \Lambda \) as a thermodynamic pressure \( P = \frac{-\Lambda}{4\pi} \) and its corresponding conjugate quantity is the thermodynamic volume which one can obtain with following relation
\[
V = \left( \frac{\partial H}{\partial P} \right)_{S,Q} = \left( \frac{\partial M}{\partial P} \right)_{S,Q} .
\]

(11)

Now, we would like to study the phase transition of Einstein black hole solutions in canonical ensemble with the mentioned NLED models. Using Eq. (8), one can obtain the following equation of state

\[
P = \frac{(d-2)}{4r_+} T - \left\{ \frac{\beta^2 \left( 1 - e^{\beta E_2} \right)}{16\pi} + \frac{(d-2)(d-3)-4r_+^2 e^{\beta E_2}}{16\pi r_+^2}, \quad \text{ENEF} \right. \\
\left. \frac{8\beta^2 r_+^2 \ln \left( \frac{1+e^{\beta E_2}}{16\pi r_+^2} \right) - (d-2)(d-3)}{4\pi} + \frac{E_2^2}{4\pi \left( 1 + \frac{2\pi}{d} \right)}, \quad \text{LNEF} \right. 
\]

(12)

where \( r_+ \) is linear function of the specific volume \( v \) in geometric unit \([3]\). In general, for these thermodynamical systems, we have the following result for volume

\[
V = \frac{\omega d-2r_+^{d-1}}{d-1},
\]

(13)

which is in agreement with the topological structure of our spacetime (spherical symmetric). This agreement is also another reason for considering cosmological constant as thermodynamical pressure.

In order to investigate phase transition and the behavior of these thermodynamical systems, we work in geometric unit and draw graphs of \( P - v \), \( T - v \) and \( G - T \) diagrams.

Now, we are in a position to analyze the plot of \( P - v \) isotherm diagram and investigate the existence of phase transition. In general, LNEF and ENEF models for large value of nonlinearity parameter have similar asymptotical behavior. Moreover, thermodynamical behavior of black hole solutions with the mentioned models are the same, globally. Therefore, for economical reason, we will plot phase diagrams for LNEF case. Left diagram of Figs. [1] - [5] indicates an analogue behavior between our plots with those of Van der Waals gas. One may use the inflection point properties of critical point to obtain

\[
\left( \frac{\partial P}{\partial v} \right)_T = 0,
\]

(14)

\[
\left( \frac{\partial^2 P}{\partial v^2} \right)_T = 0.
\]

(15)

These equations help us to obtain the critical values for the temperature, the pressure and the volume. Although we cannot, analytically, obtain the critical values of the temperature, volume and pressures, we may investigate them numerically. Taking into account the critical quantities, one can obtain the following universal ratio

\[
\rho_c = \frac{P_c v_c}{T_c}.
\]

(16)

Besides, one may look for the phase transition with the help of \( T - v \) diagrams. These graphs are also representing phase transition of black holes which is of an interest in this paper. Also, with drawing these figures one can see whether the obtained critical values are essentially critical values representing phase transition. Another reason for studying \( T - v \) diagrams is due to the fact that comparing to other graphs (\( P - v \) and \( G - T \)), that are used for studying critical behavior of the system, \( T - v \) plots give us better insight in understanding single phase regions and the effects of different parameters on these single phase regions. In our case, these single phase regions are representing small/large (unstable/stable) black holes. On the other hand, if one is interested in superconductivity that these nonlinear theories represent, the studying of these diagrams gives better description regarding conductor/superconductor regions and what modifies them. We plot the mentioned figures in the middle diagram of Figs. [1] - [5]. These plots indicate an inflection point which shows the phase transition of the system. Moreover, phase transition of a thermodynamic system can be studied by the free energy. We follow the method of the extended phase space to obtain Gibbs free energy. The behavior of the Gibbs free energy with respect to the temperature is displayed in right diagram of Figs. [1] - [5]. In these figures the characteristic swallow-tail behavior guarantees the existence of the phase transition.
FIG. 1: "LNEF branch." $P - v$ (left), $T - v$ (middle) and $G - T$ (right) diagrams for $d = 5$, $q = 1$ and $\beta = 0.5$. $P - v$ diagram, from up to bottom $T = 1.2T_c$, $T = 1.1T_c$, $T = T_c$, $T = 0.85T_c$ and $T = 0.75T_c$, respectively. $T - v$ diagram, from up to bottom $P = 1.2P_c$, $P = 1.1P_c$, $P = P_c$, $P = 0.85P_c$ and $P = 0.75P_c$, respectively. $G - T$ diagram, for $P = 0.5P_c$ (continuous line), $P = P_c$ (dot line) and $P = 1.5P_c$ (dashed line).

| $\beta$ | $v_c$ | $T_c$ | $P_c$ | $\frac{P_c v_c}{T_c}$ |
|---------|-------|-------|-------|---------------------|
| 0.1000  | 0.4012| 0.4004| 0.2449| 0.2454              |
| 0.5000  | 1.4142| 0.1744| 0.0375| 0.3044              |
| 1.0000  | 1.4765| 0.1712| 0.0360| 0.3106              |
| 1.5000  | 1.4870| 0.1707| 0.0357| 0.3117              |
| 2.0000  | 1.4907| 0.1705| 0.0356| 0.3120              |

Table (1) for $q = 1$, $d = 5$.

| $\beta$ | $v_c$ | $T_c$ | $P_c$ | $\frac{P_c v_c}{T_c}$ |
|---------|-------|-------|-------|---------------------|
| 0.1000  | 0.7380| 0.6526| 0.4410| 0.4987              |
| 0.5000  | 1.1549| 0.4769| 0.2264| 0.5483              |
| 1.0000  | 1.1944| 0.4711| 0.2207| 0.5597              |
| 1.5000  | 1.2014| 0.4698| 0.2195| 0.5612              |
| 2.0000  | 1.2037| 0.4694| 0.2191| 0.5618              |

Table (2) for $q = 1$, $d = 7$.

III. DISCUSSION ON THE RESULTS OF DIAGRAMS

As one can see, thermodynamical behavior of our systems is presented in Figs. 1-6. The obtained critical values are representing a phase transition point which is evident from swallowtail appearing in $G - T$ diagrams. On the other hand, studying $P - v$ and $T - v$ graphs for related critical values shows that obtained values are critical point that phase transition occurs in them. As nonlinearity parameter increases, the temperature, in which phase transitions take place, decreases and the value of Gibbs free energy of phase transition points increases. These results indicate that in more powerful nonlinearity parameter our thermodynamical system needs less energy in order to have a phase transition (see the middle and right diagrams of Fig. 8 for more details). This could also be interpreted from the fact that as nonlinearity parameter increases, the Enthalpy which is represented by total finite mass of black hole increases, too. Therefore, for obtained black holes we expect to phase transition occurs with absorbing less mass form surrounding.

Studying $P - v$ diagram shows that as $\beta$ increases, the critical pressure decreases, but in opposite side the horizon radius of critical points increases. Also, one can see that as the nonlinearity parameter increases the distance between two diagrams related to two different values of nonlinearity decreases which shows the fact that effect of nonlinearity in higher values do not change critical values so much (see the left diagram of Figs. 7 and 8 for more details). Also,
FIG. 2: "LNEF branch:" $P - v$ (left), $T - v$ (middle) and $G - T$ (right) diagrams for $d = 5$, $q = 1$ and $\beta = 1$.
$P - v$ diagram, from up to bottom $T = 1.2T_c$, $T = 1.1T_c$, $T = T_c$, $T = 0.85T_c$ and $T = 0.75T_c$, respectively.
$T - v$ diagram, from up to bottom $P = 1.2P_c$, $P = 1.1P_c$, $P = P_c$, $P = 0.85P_c$ and $P = 0.75P_c$, respectively.
$G - T$ diagram, for $P = 0.5P_c$ (continuous line), $P = P_c$ (dot line) and $P = 1.5P_c$ (dashed line).

FIG. 3: "LNEF branch:" $P - v$ (left), $T - v$ (middle) and $G - T$ (right) diagrams for $d = 5$, $q = 1$ and $\beta = 1.5$.
$P - v$ diagram, from up to bottom $T = 1.2T_c$, $T = 1.1T_c$, $T = T_c$, $T = 0.85T_c$ and $T = 0.75T_c$, respectively.
$T - v$ diagram, from up to bottom $P = 1.2P_c$, $P = 1.1P_c$, $P = P_c$, $P = 0.85P_c$ and $P = 0.75P_c$, respectively.
$G - T$ diagram, for $P = 0.5P_c$ (continuous line), $P = P_c$ (dot line) and $P = 1.5P_c$ (dashed line).

FIG. 4: "LNEF branch:" $P - v$ (left), $T - v$ (middle) and $G - T$ (right) diagrams for $d = 7$, $q = 1$ and $\beta = 0.5$.
$P - v$ diagram, from up to bottom $T = 1.2T_c$, $T = 1.1T_c$, $T = T_c$, $T = 0.85T_c$ and $T = 0.75T_c$, respectively.
$T - v$ diagram, from up to bottom $P = 1.2P_c$, $P = 1.1P_c$, $P = P_c$, $P = 0.85P_c$ and $P = 0.75P_c$, respectively.
$G - T$ diagram, for $P = 0.5P_c$ (continuous line), $P = P_c$ (dot line) and $P = 1.5P_c$ (dashed line).
FIG. 5: "LNEF branch:" $P - v$ (left), $T - v$ (middle) and $G - T$ (right) diagrams for $d = 7$, $q = 1$ and $\beta = 1$.

$P - v$ diagram, from up to bottom $T = 1.2T_c$, $T = 1.1T_c$, $T = T_c$, $T = 0.85T_c$ and $T = 0.75T_c$, respectively.

$T - v$ diagram, from up to bottom $P = 1.2P_c$, $P = 1.1P_c$, $P = P_c$, $P = 0.85P_c$ and $P = 0.75P_c$, respectively.

$G - T$ diagram, for $P = 0.5P_c$ (continuous line), $P = P_c$ (dot line) and $P = 1.5P_c$ (dashed line).

FIG. 6: "LNEF branch:" $P - v$ (left), $T - v$ (middle) and $G - T$ (right) diagrams for $d = 7$, $q = 1$ and $\beta = 1.5$.

$P - v$ diagram, from up to bottom $T = 1.2T_c$, $T = 1.1T_c$, $T = T_c$, $T = 0.85T_c$ and $T = 0.75T_c$, respectively.

$T - v$ diagram, from up to bottom $P = 1.2P_c$, $P = 1.1P_c$, $P = P_c$, $P = 0.85P_c$ and $P = 0.75P_c$, respectively.

$G - T$ diagram, for $P = 0.5P_c$ (continuous line), $P = P_c$ (dot line) and $P = 1.5P_c$ (dashed line).

FIG. 7: "LNEF branch:" $P - v$ (left), $T - v$ (middle) and $G - T$ (right) diagrams for $d = 5$, $q = 1$.

$P - v$ diagram, for $T = T_c$, $\beta = 0.3$ (bold line), $\beta = 0.5$ (bold dot line), $\beta = 1$ (continuous line), $\beta = 1.5$, (dot line) and $\beta = 10$ (dash line).

$T - v$ diagram, for $P = P_c$, $\beta = 0.3$ (bold line), $\beta = 0.5$ (bold dot line), $\beta = 1$ (continuous line), $\beta = 1.5$, (dot line) and $\beta = 10$ (dash line).

$G - T$ diagram, for $P = 0.5P_c$, $\beta = 0.5$ (continuous line), $\beta = 1$, (dot line) and $\beta = 1.5$ (dash line).
FIG. 8: "LNEF branch:" $P - v$ (left), $T - v$ (middle) and $G - T$ (right) diagrams for $d = 7$, $q = 1$.

$P - v$ diagram, for $T = T_c$, $\beta = 0.3$ (bold line), $\beta = 0.5$ (bold dot line), $\beta = 1$ (continuous line), $\beta = 1.5$, (dot line) and $\beta = 10$ (dash line).

$T - v$ diagram, for $P = P_c$, $\beta = 0.3$ (bold line), $\beta = 0.5$ (bold dot line), $\beta = 1$ (continuous line), $\beta = 1.5$, (dot line) and $\beta = 10$ (dash line).

$G - T$ diagram, for $P = 0.5P_c$, $\beta = 0.5$ (continuous line), $\beta = 1$, (dot line) and $\beta = 1.5$ (dash line).

FIG. 9: "LNEF branch:" $P - v$ (left), $T - v$ (middle) and $G - T$ (right) diagrams for $q = 1$, $P = 0.5P_c$, and $\beta = 1$.

$P - v$, $T - v$ and $G - T$ diagrams for $d = 5$ (continuous line), $d = 6$ (dot line) and $d = 7$ (dashed line), respectively.

one can argue that due to fact that pressure is related to cosmological constant which is related to asymptotical curvature of the background and our thermodynamical system, as nonlinearity parameter increases, the necessity of having a background with more curvature decreases.

As one can see in $T - v$ diagrams, as the nonlinearity parameter increases, temperature (horizon radius) of critical points of phase transition decreases (increases) (see the middle diagram of Figs. 7 and 8). In other words, in order to have phase transition for large $\beta$, we need less energy which is consistent with results that we find in studying Gibbs free energy diagrams. Therefore, for having phase transition in presence of higher value of $\beta$, black hole needs to absorb less mass in order to achieve stable state.

The effects of dimensionality on critical points and their behavior is another interesting issue that we discuss in this section. The obtained $G - T$ diagram for different dimensions shows the fact that as dimensionality increases, the temperature of critical points increases which indicate the necessity of more energy for having a phase transition (see the right diagram of Fig. 8 for more details). On the other hand, the gap between two different phases of our thermodynamical system increases which shows the fact that as dimensionality increases the change in energy of system in which phase transition occurs, increases too. For the $P - v$ diagrams, we have higher pressure in which phase transition takes place. In other words, as dimensions of system increase pressure, hence cosmological constant increases, which is acceptable because of the fact that cosmological constant is dimension dependent (see the left diagram of Fig. 8 for more details). Finally, for the $T - v$ diagram of Fig. 8 we can see that the temperature of critical value and the length of subcritical isobars increases which means that the single phase region of small/large black holes decreases. Therefore, as dimensionality increases the system needs to absorb more mass in order to have phase transition.

Finally, we are studying the behavior of critical values and the universal ratio of $\frac{P_c v_c}{T_c}$. As One can see, the critical
horizon radius increases, as nonlinearity parameter increases, whereas critical temperature and pressure decrease. Also, we can see the ratio of \( \frac{P_{v}}{T_{c}} \) is an increasing function of nonlinearity parameter. For higher dimensions, we have higher values of critical points which result into higher value of \( \frac{P_{v}}{T_{c}} \). Studying of these tables also confirms the results that we have derived through studying graphs. As the nonlinearity parameter increases, the thermodynamical system will be in need of less energy (in our black hole case by absorbing mass) in order to have phase transition.

IV. CONCLUSIONS

In this paper, we have considered two classes of nonlinear electromagnetic fields in presence of Einstein gravity and studied their phase structure. By considering cosmological constant as thermodynamical pressure and its conjugating variable as volume we have extended the phase space and regarded the interpretation of total mass of black hole as the Enthalpy. For spherical symmetric spacetime, obtained volume from thermodynamical equations is the same that we have been expected for our topological structure. Studying calculated critical values through three different types of phase diagrams resulted into phase transition taking place in the critical values. \( P - v, T - v \) and \( G - T \) diagrams representing similar behavior near critical points similar to their corresponding diagrams in Van der Waals gas.

Studying the effects of nonlinearity parameter on phase diagrams revealed the fact that as nonlinearity parameter increases the critical temperature decreases which indicates that for large values of nonlinearity parameter the system needs less energy (mass) absorption to have phase transition. Due to fact that for large \( \beta \) these nonlinear models reduce to Maxwell theory, one might conclude that the lowest temperature in which phase transition takes place, belongs to Maxwell theory. On the other hand, for large \( \beta \), the critical pressure decreases. Besides, we found that the universal ratio of \( \frac{P_{v}}{T_{c}} \) increases as nonlinearity parameter increases. On the other hand, studying the effects of dimensionality showed that, for higher dimensional black holes, phase transitions take place in higher temperature and in lower energy. As one can see the effect of dimensionality on critical values, gap of Gibbs free energy between two different phases and phase transition is strong. We found that as dimensionality of black holes increases, phase transition for obtaining stable state becomes more difficult and black holes need to absorb more mass in order to have phase transition.

One should take this fact into account that for small values of nonlinearity, changes in critical values are greater comparing to large values of nonlinearity parameter. This fact is evident from studying phase diagrams and tables. These differences in critical values are due to fact that as nonlinearity parameter decreases to values near zero the power of nonlinear electromagnetic fields grows stronger. This behavior indicates that as nonlinear electromagnetic field grows stronger, for having phase transition, the black hole needs to absorb more mass, hence the total mass of black hole must increase in order to have small/large (unstable/stable) black hole phase transition. Considering the Hawking radiation which decreases the total mass of black hole, one can say that this radiation shifts black hole from stable to unstable phase or it prevents black holes from having phase transition to achieve stable phase. In other words, Hawking radiation mechanism (black hole evaporation) affects the stability and phase transition of black holes fundamentally and make black holes unstable in our models.

Considering the fact that phase transition plays an important role in studying conductor/superconductor in adS/CFT correspondence, it will be constructive to study the obtained critical values in this context. On the other hand, as results showed the Hawking radiation makes black holes unstable and prevent them from having phase transition in the presence of strong nonlinear electromagnetic field, it will be interesting to consider Hawking radiation and the restrictions that it puts on phase transition in studying black holes phase transition.

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