Mathematical mindsets: the abstraction in mathematical problem solving

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Abstract. This research aims to analyze the students’ mathematical mindsets theoretically and empirically in context. The research analyzes the students’ answer on mathematical problem solving. Forty-seven 8th grade students of a junior high school participated in this research. The researchers collected data in mixed methods research design with explanatory sequential design. The result shows that the essential cores of mathematical mindsets model are: abstraction, conceptual and procedural knowledge, that operate simultaneously in constructing the mathematical mindsets.

1. Introduction

Abstraction is a fundamental process in both mathematics and mathematics education. Abstraction has been known as something that plays an important role for the success of learning mathematics when it is viewed from a cognitive point of view. However, abstraction is also one of the main reasons for the failure of the learning process in mathematics [1]. This phenomenon is not without reason. If it is viewed from the characteristic of mathematics where the object of the study is an abstract concept, then it requires a knowledge that can bring students to understand that abstract concept in the learning process. Simply put, in the context of mathematics education, abstraction can be interpreted as a process of studying ideas, thoughts, concepts and their relationships [2]. The process of abstraction takes place through a series of learning activities involving various aspects of knowledge [3], among which are conceptual and procedural knowledge.

The student's conceptual and procedural knowledge during the learning process may involve himself in the process of learning ideas, thoughts, concepts and the relationships, in order to recognize what and why something is true when he knows how to get to know something is truly right [4,5]. In processing mathematical problem solving, the conceptual knowledge is essential to understand the basic concepts in solving them. During the process of mathematical problems solving, the procedural knowledge is required to point out the steps to solve it [6]. This is in line with Principles and Standards for School Mathematics, that understanding mathematical concepts and procedures is very significant for students in the learning process [7], so as to build the necessary mathematical mindsets in a mathematical problem solving. It is because a mathematical mindset may reflect an active learning method to learn mathematics knowledge, where the students place themselves to understand and make sense of what is being learned [8].
Based on the aforementioned reason, the purpose of this article is to explore the abstraction process based on students’ mindset in mathematical problem solving.

2. Method
In this research, researchers analysed the answers of mathematical problem solving test where forty-seven 8th grade students of a junior high school voluntarily participated, selected by purposive sampling. Researchers analysed students' mathematical mindsets abilities in applying their conceptual and procedural knowledge during the process they write their answers in the answer sheets provided. The test instruments (table 1) was designed to contain the framework of modelling, problem solving and integrating concepts [9], the development of conceptual and procedural knowledge [10,11,12], as well as in accordance with the Assessment for a Growth Mindsets [8], which aim to encourage students to develop mathematical mindsets by utilizing prior conceptual and procedural knowledge in the abstraction process. This study used a mixed method with sequential explanatory design, where the empirical data from the analysis of our students' answers are reviewed by theoretical studies.

Table 1. Test instruments.

| Domain              | Example of the Concept                                      | Assessment                                                                 |
|---------------------|-------------------------------------------------------------|-----------------------------------------------------------------------------|
| Numbers & Counting  | One-to-one; stable order; cardinality; abstraction; growing  | Students recognize incorrect assertions about principles or incorrect execution of procedures. |
|                     | shapes; order irrelevance principles                        |                                                                            |
| Algebra             | Representing relationships mathematically                   | Students can use and interpret units when solving formulas; perform unit conversions; identify part an expression; interpret solution in the context of the situation modelled and decide if they are reasonable; verify that any point on graph will result in a true equation when their coordinates are substituted into the equation; compare properties of two functions graphically, in the table form, and algebraically. |
| Geometry            | Rectangles & polygons; cube, cuboid, pyramid, prism, cylinder, sphere & cone | Students can identify, give ideas, communication, and representation, by using the visual and intuitive mathematical, connected with numerical thinking |

3. Results and Discussion
In this section, the researchers will present the findings and the actions that have been done. From the results of the action, the researchers examined the results of the students' answers on how the knowledge of the concepts and procedures that had been studied was used in mathematical problems solving, and drew conclusions on what had been achieved about the relationship between the two concepts. From the collected data, there appears a useful micro theory, developed to
explain how mathematical mindsets were formed from abstraction processes based on the conceptual and procedural knowledge they already possessed.

![Figure 1. Example of a student’s answer in the numbers and counting.](image1)

![Figure 2. Example of a student’s answer in the geometry.](image2)

In the figures 1 and 2, students whose answers are as in (a) and (e) indicate that the students’ answering process is similar to what the teacher had taught regardless of the operating properties of integers and the properties of the flat wake being learnt. While students whose answers are shown in (b), (c), (d), (f), and (g) demonstrate the students’ answering process based on the previously learned concepts and procedures. This shows that conceptual and procedural knowledge plays an important role in the students’ mindset in problems solving.

| Mathematics Strategy | Domain       |
|----------------------|--------------|
|                       | Numbering & Counting | Algebra | Geometry |
| Memorization         | 26 (55.31%)   | 34 (72.34%) | 18 (38.29%) |
| Big Idea & Connections | 21 (44.69%)  | 13 (27.66%) | 29 (61.71%) |

Table 2. Data collected based on students’ answers.
Based on the data in table 2, most students have not been able to actualize the conceptual and procedural knowledge in building new mindsets in solving the given problems. So the researchers argue that the strategies used by the majority of students in writing the answers are only based on the memorization of the conceptual knowledge and metacognitive reflection on the procedural knowledge that have been learned before; both of which indicate the abstraction process in building the mathematical mindset has not been optimally achieved.

Abstraction is a central process in learning mathematics [13,14]; however, it is very difficult to observe [14]. Nevertheless, the abstraction process that takes place in the learning process can be seen from the activity in reorganizing the mathematical knowledge that is built vertically into the new structure [14,15]. Meanwhile, Noss and Hoyles [16] put the abstraction in relation to their conceptual and procedural knowledge, and recognise it as a practice of delivering from the previous context to a new context. This shows that the mathematical mindset is formed based on an abstraction process that depends on one's ability to apply conceptual knowledge and procedures.

This is in line with the development of psychology, which articulates how procedures and concepts interact to each other in order to understand how the development of mindset is established [11,17,18,19]. Piaget explained that learning basically begins with an action on existing conceptual knowledge, and the individual's ability to internalize procedural knowledge is an important element of learning. However, for Piaget, the relationship between procedural and conceptual knowledge is much more complex, since, according to Piaget, once a person obtains skills and internalizes the procedural knowledge, he begins to reflect on this abstract process, and as a result, acquires new conceptual knowledge [20,21].

According to Byrnes and Wasik [10], conceptual and procedural knowledge are integral parts of a single cognitive scheme, where both of which cannot be separated. Thus, with Piaget-based learning models, the concepts are assimilated into the cognitive schemes. Furthermore, this assimilation process takes place in the later stages of cognitive development, characterized by metacognitive and abstract reflections; conceptual knowledge and the internalization of procedural knowledge [18,22]. This is in line with the opinions of Hiebert and Lefevre [18] which define conceptual and procedural knowledge as knowledge that is rich in relationships. It can be thought of as connected to the web of knowledge, a network in which the linking relationships are as prominent as discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some networks.

4. Concluding Remarks
Based on the study and findings of the action given, the researchers may conclude that the cores of the mathematical mindsets model are formed based on the result of the abstraction process in actualizing the conceptual and procedural knowledge that a person has.

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