Contact representation of short range correlation in light nuclei studied by the High-Momentum Antisymmetrized Molecular Dynamics

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The high-momentum antisymmetrized molecular dynamics (HMAMD) is a new promising framework with significant analytical simplicity and efficiency inherited from its antisymmetrized molecular dynamics in describing the high momentum correlations in various nuclear states. In the aim of further improving the numerical efficiency for the description of nucleon-nucleon correlation, we introduce a new formulation by including a new Gaussian weighted basis of high momentum pairs in the HMAMD wave function, with which very rapid convergence is obtained in numerical calculation. It is surprising that the very high-momentum components in the new HMAMD basis are found to be almost equivalent to the contact representation of the nucleon-nucleon pairs with very small nucleon-nucleon distance. The explicit formulation for the contact term significantly improves the numerical efficiency of the HMAMD method, which shows the importance of the contact correlation in the formulation of light nuclei.
I. INTRODUCTION

In recent years, significant advances have been obtained in the ab initio studies of many body physics in atoms, molecules, nuclei and hadrons. In nuclear physics, the ab initio theories provide the correct descriptions of the nucleon-nucleon (NN) correlations induced by the NN interactions, which are usually unfeasible in other nuclear models. In the bare NN interactions, which is determined phenomenologically by fitting NN scattering observables, there exist strong short-range repulsion and strong tensor interaction inducing D-wave components in nuclei [1–4]. To describe the complex NN-correlations in nuclear many-body problems, Jastrow constructs a trial function which is a product of correlation functions for each NN pair [3]. This function is used in the Green’s Function Monte Carlo (GFMC) studies of finite nuclei as an input trial function [3]. Another important approach is the Unitary Correlation Operator Method (UCOM), where short-range correlations are described by unitary transformations in exponential form [4–6].

In recent works, a new variational approach is proposed for the ab initio calculation of light nuclei, namely the tensor-optimized antisymmetrized molecular dynamics (TOAMD) [7–11]. In this method, the correlation functions are introduced and multiplied to reference wave function of Antisymmetrized Molecular Dynamics (AMD). Instead of the product form in Jastrow approach, the power series expansion for the correlation functions is used in TOAMD. It is found that the TOAMD wave function provides better numerical accuracy comparing to the Variational Monte Carlo (VMC) method, which uses Jastrow type correlation functions [11]. In ab initio calculations using the AV8’ bare interaction, the GFMC results of $^3$H and $^4$He are correctly reproduced with the TOAMD method including first two orders of power expansion [3]. Furthermore, similar correlation functions have been formulated for Fermi sphere [12] and successfully applied to the nuclear matter calculation [13]. The detailed formation of the TOAMD approach can be found in Refs. [7–10].

On the basis of the same power expansion scheme for the correlation functions, another description of NN correlations has been invented by introducing high-momentum (HM) nucleon-nucleon pairs into the AMD reference states, namely the “high-momentum antisymmetrized molecular dynamics (HMAMD)” [14, 15]. In calculations using the AV4’ central interaction where the strong short-range repulsion is included, the HMAMD is found to provide the same numerical accuracy as the TOAMD and GFMC methods [15]. The tensor correlation is also nicely described in the HMAMD approach, in which two-particle two-hole excitations are completely described [14]. The major advantage of the HMAMD approach is its analytical simplicity comparing to other ab initio frameworks which makes it a very promising method to be extended to general nuclear systems. However, the basis number of HMAMD method depends on the number of HM pairs and is relatively large, as comparing to the TOAMD method [10]. To reduce the model space, in Refs. [16, 17] a hybridized approach “tensor-optimized high-momentum antisymmetrized molecular dynamics (TO-HMAMD)” is proposed to consider the balance between the analytical and numerical efforts. On the other hand, the HMAMD method itself can become even more successful if its efficiency in describing the NN correlations can be further improved.

In this work, we introduce a new formulation for the high-momentum part of the short range correlation in the framework of HMAMD. We calculate the $^3$H nucleus with this new formation and show the importance of contact correlation of NN pairs, where two nucleons contact at almost the same spatial position. Using an effective wave function, the nuclear contacts have been extracted from experimental observables in Ref. [18]. In ab initio calculations, the contact correlation is mathematically included in the model space, but it is generally treated as a component of short-range correlation. In this work, we show that the contact correlation appears as a characteristic feature in the variational calculation of light nuclei for the treatment of the short-range correlation. Furthermore, the numerical efficiency of HMAMD can be significantly improved in this work because of the explicit formulation of the contact correlations.

This paper is organized as follows. In Sec. II we introduce our new formulation of the HMAMD wave function. In Sec. III this new formulation is applied to the $^3$H nucleus using the AV4’ interaction. In Sec. IV we show the importance of the contact correlation using an analytical derivation. The last Sec. V contains conclusions.

II. FORMULATION

We start writing the AMD (Antisymmetrized Molecular Dynamics) basis to construct the wave function for a nucleus of $A$ nucleons:

$$|\Phi_{\text{AMD}}\rangle = |\mathcal{A}\{\phi_1(r_1)\ldots \phi_A(r_A)\}\rangle,$$  \hspace{1cm} (1)

where the single-nucleon wave functions $\phi(r)$ are expressed in the Gaussian wave packet with the range parameter $\nu$ and the centroid position $Z$ multiplied by the spin-isospin wave function $\chi_{\tau,\sigma}$.

$$\phi(r) = \left(\frac{2\nu}{\pi}\right)^{3/4} e^{-\nu(r-Z)^2} \chi_{\tau,\sigma}.$$  \hspace{1cm} (2)
For the s-shell nuclei \(^3\text{H}\) and \(^4\text{He}\), the centroids are optimized to be \(Z = 0\), and the corresponding single nucleon wave functions reduce to the s-wave states. This is the result of the energy variation in TOAMD \(^{10, 11}\).

Then we introduce the HMAMD bases as in Refs. \(^{14, 17}\). For the ground states of s-shell nuclei, the real parts of the centroid position \(Z\), which represent the spatial positions of the Gaussian centroid, are set zero. The imaginary part of \(Z\) represents the mean momentum of Gaussian as
\[
(p) = 2\hbar \text{Im}(Z). \tag{3}
\]

By assigning the imaginary components to the Gaussian centroids of two nucleons, \(Z_1\) and \(Z_2\) with the same magnitude in opposite directions, we introduce the high-momentum excitation of di-nucleon pairs \(^{19}\) into AMD basis as
\[
Z_1 = iD, \quad Z_2 = -iD. \tag{4}
\]

which is the HMAMD basis \(|\Phi_{\text{HMAMD}}(D_\alpha)\rangle\) with a shift parameter \(D_\alpha\). With these imaginary shifts, the correlation between a two-nucleon pair is introduced to the AMD basis in the momentum space, while the center-of-mass momentum remains to be zero. For the \(^3\text{H}\) nucleus, we prepare three kinds of high-momentum pairs for different spin-isospin components in Eq. \((\alpha)\), which we name as "discrete HMAMD" \(^{14}\). When bases with double high-momentum pairs are included in model space, the corresponding method is named as "double HMAMD", which could reproduce nicely the results of GFMC in the calculation of the \(^3\text{H}\) nucleus \(^{13}\). In this study, we adopt the single HMAMD approach to study the short-range correlations between nucleons.

In our previous studies \(^{14, 15}\), the total wave function is formulated as a linear combination of different HMAMD basis states in the following form,
\[
|\Psi_{\text{AMD+Dis}}\rangle = C_0 |\Phi_{\text{AMD}}\rangle + \sum_\alpha C_\alpha |\Psi_{\text{HMAMD}}(D_\alpha)\rangle. \tag{6}
\]

The first term in Eq. \((\alpha)\) is the AMD basis which describes the \((0s)^3\) configuration. The second term is the superposition of the HMAMD bases with sets of the discrete shift parameters \(|D_\alpha\rangle\), which we name as "discrete HMAMD basis" (AMD+Dis). In this term, the label \(\alpha\) denotes the shift parameter \(D_\alpha\) and spin-isospin components in Eq. \((\beta)\) to identify the HMAMD basis states. The total energy and the coefficients \(C_\alpha\) are obtained by solving the eigenvalue problem with respect to the Hamiltonian. Theoretically, superposing large number of discrete HMAMD basis is sufficient to obtain the exact solution, as in Refs. \(^{14, 15}\).

In our present study, we propose a new kind of wave function as a linear combination of different HMAMD basis states,
\[
|\Psi_{\text{AMD+Dis+Gau}}\rangle = C_0 |\Phi_{\text{AMD}}\rangle + \sum_\alpha C_\alpha |\Psi_{\text{HMAMD}}(D_\alpha)\rangle + C_\beta \int dD e^{-\frac{D^2}{2\beta^2}} |\Psi_{\text{HMAMD}}(D)\rangle. \tag{7}
\]

In order to investigate analytically the short-range correlation between two nucleons in the high-momentum pair, we introduce additionally the third term
\[
\int dD e^{-\frac{D^2}{2\beta^2}} |\Psi_{\text{HMAMD}}(D)\rangle \tag{8}
\]
of integration form in Eq. \((\gamma)\), which is named as "Gaussian HMAMD basis". The Gaussian parameter \(\beta\) is treated as the variational parameter. This term has only two parameters, i.e., the Gaussian parameter \(\beta\) and the superposition coefficient \(C_\beta\), which provide a crystal description of the NN-correlations because of its simplicity in analytical derivations.

To show the effects of this Gaussian HMAMD bases, we also perform calculations using the superposed "AMD+Gau" wave function as the reduced form of Eq. \((\delta)\).
\[
|\Psi_{\text{AMD+Gau}}\rangle = C_0 |\Phi_{\text{AMD}}\rangle + C_\beta \int dD e^{-\frac{D^2}{2\beta^2}} |\Psi_{\text{HMAMD}}(D)\rangle. \tag{9}
\]

Finally, all the total wave functions are projected on to the eigenstates of the angular momentum \(J\) by using the angular momentum projection \(^{20}\):
\[
|\Psi^J_M\rangle = \frac{2J + 1}{8\pi^2} \int d\Omega D^*_MK(\Omega)R(\Omega) |\Psi\rangle. \tag{10}
\]
The Hamiltonian adopted for the $^3$H nucleus is the Argonne V4′ $NN$ central potential, which includes a strong short-range repulsion. The range parameter $\nu$ is set to be 0.22 fm$^{-2}$ for the single-nucleon wave functions as in Ref. [15]. To simplify the calculation, we consider the imaginary shifts $D$ only in the $z$ direction and take the magnitude of $D_z$ in step of 1 fm for the discrete HMAMD bases. The numerical integration in the Gaussian HMAMD basis in Eq. (8) is performed by using the Monte Carlo technique. We also put the truncation at $D_z = 18$ fm for this integration, which is large enough for the numerical convergence.

**III. RESULTS**

We compare the numerical results of $^3$H, calculated by using the wave functions of AMD, AMD+Gaussian, AMD+Discrete+Gaussian, and AMD+Discrete. We list the corresponding total energy and the discrete HMAMD basis included in Table I. The Gaussian parameter $\beta$ is optimized for each wave function, respectively. The optimized value is 2.27 fm for the AMD+Gaussian wave function and 2.22 fm for the AMD+Discrete+Gaussian wave function.

We show in Table I the energy of $^3$H corresponding to the wave functions of AMD, AMD+Gaussian, AMD+Discrete+Gaussian, and AMD+Discrete. The AV4′ potential is adopted for the $NN$ interaction. The second row includes the total energies of $^3$H calculated by using each wave function. The third row represents the shift parameters $D_z$ of the superposed discrete HMAMD basis. The forth row includes the number of bases in each wave function.

| AMD      | AMD+Gau | AMD+Dis+Gau | AMD+Dis [15] |
|----------|---------|-------------|--------------|
| Energy (MeV) | 15.6    | −1.76       | −7.38        | −7.40        |
| Discrete bases (fm) | −       | −           | 1, 2, ..., 6 | 1, 2, ..., 14 |
| Total number of bases | 1       | 2           | 8            | 15           |

To show the effect of the Gaussian HMAMD basis explicitly, we present in Fig. 1 the energy curve with respect to the successive addition of discrete HMAMD bases with the imaginary shift $D_z$ from both AMD and AMD+Gaussian wave functions. In this figure, the red curve shows the AMD+Discrete calculation, where the discrete HMAMD bases are added successively increasing $D_z$ to the AMD basis, and the energy convergence is obtained at around $D_z = 12$ fm. For the blue line, the first point corresponds to the energy by superposing the AMD basis and the Gaussian HMAMD basis, and in the following points additional discrete HMAMD bases are added successively. It is found that the energy of $^3$H in each wave function converges to the same value of $−7.40$ MeV, which is obtained much faster with the Gaussian HMAMD basis with small number of discrete HMAMD basis. This result indicates that very high-momentum components with $D_z > 6$ fm in discrete HMAMD bases can be well described by the simple Gaussian HMAMD basis.

In Fig. 2, we show the contribution to the total energy and Hamiltonian components from the AMD basis and further successive addition of the Gaussian HMAMD basis and discrete HMAMD bases. We obtain very nice agreement for the total energy and each Hamiltonian component between the AMD+Gaussian+Discrete results and the AMD+Discrete results from Ref. [15]. In the blue curve, a large contribution in the central interaction energy is observed by the addition of the Gaussian HMAMD basis, which indicates the importance of the Gaussian term in describing the short-range correlation. It is also interesting that in the red curve the kinetic energy remains the same when adding the Gaussian HMAMD basis, showing that the $NN$ correlation described by Gaussian HMAMD basis, does not contribute to the kinetic energy. In the summary of these results, we clearly confirm the importance and efficiency of the Gaussian HMAMD basis in short-range correlation.
FIG. 1. Energy curves for $^3$H of the AMD+Discrete (red line) and AMD+Discrete+Gaussian (blue line) calculations with respect to the successive addition of discrete HMAMD bases with the imaginary shift $D_z$. The dotted part of red line are the energies in AMD+Discrete with $D_z > 6$, which are not necessary in the AMD+Discrete+Gaussian calculation. The first points of red and blue curves with $D_z = 0$ correspond to the AMD and AMD+Gaussian results, respectively.

FIG. 2. The total energy $E$ and the Hamiltonian components of $^3$H by adding Gaussian form basis (+Gau) and the discrete HMAMD basis (+Dis) successively to the AMD basis. The symbols K and V indicate the kinetic and central interaction energies, respectively. Dashed lines represent the corresponding results of AMD+Discrete calculation in Ref. [15].

IV. CONTACT REPRESENTATION OF SHORT RANGE CORRELATION

In previous results, it is very surprising that the simple Gaussian HMAMD basis in Eq. (8) actually provides an important contribution to the short-range correlation. In order to understand the property of the nucleon-nucleon correlation in this term, we investigate the analytical property of this Gaussian HMAMD basis. We start with the spatial part of the single particle wave function in HMAMD bases,

$$\phi(r \pm iD) = \left(\frac{2\nu}{\pi}\right)^{3/4} e^{-\nu(r \pm iD)^2}, \quad (11)$$

where the real parts of centroid position $Z$ are optimized to be zero as in Eq. (2). Then, the high-momentum pair in the HMAMD basis can be constructed as

$$\phi_1(r_1 - iD)\phi_2(r_2 + iD), \quad (12)$$
where antisymmetrization is ignored for simplicity. In the Gaussian HMAMD basis, we perform the numerical integration of Gaussian form for the shift parameter $D$:

$$\int d\mathbf{D} e^{-D^2/\beta} \phi_1(\mathbf{r}_1 - i\mathbf{D}) \phi_2(\mathbf{r}_2 + i\mathbf{D}) \propto e^{-\nu (r_1^2 + r_2^2)} \int d\mathbf{D} e^{2\nu D(r_1 - r_2) + (2\nu - \frac{1}{2}) D^2}. \quad (13)$$

In the right hand side of Eq. (13), the coefficient $(2\nu - \frac{1}{2})$ for $D^2$ in the exponent depends on the parameter $\nu$, which is 0.22 fm$^{-2}$, and the variational optimized parameter $\beta$, which controls the Gaussian distribution of $D$, namely the momentum. In the results of the variational calculation, the optimized parameter $\beta$ is obtained to be 2.22 fm$^2$, and hence we have approximately the following numerical relation

$$\nu \times \beta \approx 0.5, \quad (14)$$

with which the integration in Eq. (13) reduces to

$$\int d\mathbf{D} e^{-D^2/\beta} \phi_1(\mathbf{r}_1 - i\mathbf{D}) \phi_2(\mathbf{r}_2 + i\mathbf{D}) \propto e^{-\nu (r_1^2 + r_2^2)} \int d\mathbf{D} e^{2\nu \beta D(r_1 - r_2)}$$

$$\propto \delta(\mathbf{r}_1 - \mathbf{r}_2) \phi(\mathbf{r}_1) \phi(\mathbf{r}_2). \quad (15)$$

This simple expression of the delta function shows a quite important result: the Gaussian HMAMD basis describes the $NN$ correlation with the relative position of two nucleons being zero, indicating spatial contact of two nucleons. This result represents the contact correlation, as a very specific component of the short-range correlation between nucleons.

![FIG. 3. The figure with different values of parameter $\nu$ and the values of $\nu \times \beta$. The corresponding values for parameter $\beta$ are variationally optimized and presented as the blue dots. The red dotted line denotes the 0.5 value.](image)

In order to further check the numerical relation “$\nu \times \beta = 0.5$”, we perform calculations of AMD+Discrete+Gaussian with various choices of parameter $\nu$ and obtain the corresponding optimized $\beta$ parameter with variational calculation, as shown in Fig. 3. It is clearly observed that for different choices of parameter $\nu$, the numerical relation “$\nu \times \beta = 0.5$” persists, which shows that this numerical relation is not a parameter dependent coincidence. This coincidence strongly supports the existence of the contact correlation in our Gaussian HMAMD basis.

Meanwhile, we observe slight but finite difference between optimized values of $\nu \times \beta$ and 0.5 in Fig. 3, which indicates that the nucleon-nucleon distance in the contact correlated pair is not exact zero and the $\delta$ function form in Eq. (15) originates from the approximation of $\nu \times \beta \approx 0.5$. Without this approximation, the exact solution for the integration in Eq. (13) can be obtained, as introduced in Ref. [16],

$$\int d\mathbf{D} e^{-D^2/\beta} \phi_1(\mathbf{r}_1 - i\mathbf{D}) \phi_2(\mathbf{r}_2 + i\mathbf{D}) \propto e^{-\nu (r_1^2 + r_2^2)} \int d\mathbf{D} e^{2\nu \beta D(r_1 - r_2) + (2\nu - \frac{1}{2}) D^2}$$

$$\propto e^{-r_1^2/\beta \sigma^2} \phi(\mathbf{r}_1) \phi(\mathbf{r}_2)$$

$$\propto e^{-r_1^2/2\sigma^2} \phi(\mathbf{r}_1) \phi(\mathbf{r}_2). \quad (16)$$
With parameter $\nu = 0.22 \text{ fm}^{-2}$ and optimized parameter $\beta = 2.22 \text{ fm}^2$, the width $\sigma$ in the Gaussian term of Eq. (16) has very small value of $\sigma = 0.33 \text{ fm}$, which confirms the contact of two nucleons in the $NN$ pair as concluded from Eq. (15).

Our finding of the contact representation of short range correlation reminds us the recent study of the generalized contact theory by Weiss et al. [18]. They write the nuclear wave function in a factorized wave function in the relative and center-of-mass coordinates of two nucleons in short distance. In our results, we find a simple factorized wave function to express very high momentum components of short range correlation.

$$\psi(r_1, r_2) \propto \delta(r_1 - r_2) e^{-\frac{1}{2}(r_1-r_2)^2} e^{-\frac{1}{2}(r_1 + r_2)^2}$$  (17)

It is interesting to note that the very high momentum components of the short range correlation should be expressed by the simple contact form and the intermediate range should be represented by the sum of medium range momentum components.

These findings significantly simplify the corresponding numerical calculations in HMAMD framework. In our future works, we will study the contact correlation using the new wave function with the AV8′ interaction, where both the tensor interaction and short-range repulsion are included.

V. CONCLUSION

In conclusion, we found a contact representation of high momentum components of short range correlation between nucleons through variation in the HMAMD framework. This contact wave function can be expressed in the HMAMD formalism by a single Gaussian weighted AMD wave function with relative imaginary shift components of Gaussian wave packets representing the momentum. We perform the calculations of $^3\text{H}$ with AV4′ central potential using explicit description of this new contact wave function and compare the results with those of our previous works, in which the short-range correlation is not assumed in any specific form in the basis states. We confirm the equivalent results between two methods. It is found that our new formulation is very effective in describing short range correlation, where a very rapid convergence is obtained in the numerical calculation and the total basis number is reduced to almost the half of the previous works. The present new understanding of the contact correlation is expected to be very beneficial for future ab initio investigations of nuclear systems.

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[1] R. Jastrow, Phys. Rev. 98, 1479 (1955).
[2] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C 51, 38 (1995).
[3] S. C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci. 51, 53 (2001).
[4] H. Feldmeier, T. Neff, R. Roth, and J. Schnack, Nucl. Phys. A632, 61 (1998).
[5] T. Neff and H. Feldmeier, Nucl. Phys. A713, 311 (2003).
[6] T. Myo, H. Toki and K. Ikeda, Prog. Theor. Phys. 121, 511 (2009).
[7] T. Myo, H. Toki, K. Ikeda, H. Horiiuchi, T. Suhara, Prog. Theor. Exp. Phys. 2015, 073D02 (2015).
[8] T. Myo, H. Toki, K. Ikeda, H. Horiiuchi, T. Suhara, Phys. Lett. B 769, 213 (2017).
[9] T. Myo, H. Toki, K. Ikeda, H. Horiiuchi, T. Suhara, Phys. Rev. C 95, 044314 (2017).
[10] T. Myo, H. Toki, K. Ikeda, H. Horiiuchi, T. Suhara, Prog. Theor. Exp. Phys. 2017, 073D01 (2017).
[11] T. Myo, H. Toki, K. Ikeda, H. Horiiuchi, and T. Suhara, Phys. Rev. C 96, 034309 (2017).
[12] T. Yamada, arXiv:1808.07257 [Nucl-Th] (2018).
[13] T. Yamada, T. Myo, H. Horiiuchi, K. Ikeda, and H. Toki, arXiv:1808.08120 [Nucl-Th] (2018).
[14] T. Myo, H. Toki, K. Ikeda, H. Horiiuchi, T. Suhara, M. Lyu, M. Isaka, and T. Yamada, Prog. Theor. Exp. Phys. 2017, 11D01 (2017).
[15] T. Myo, Prog. Theor. Exp. Phys. 2018, 031D01 (2018).
[16] M. Lyu, T. Myo, M. Isaka, H. Toki, K. Ikeda, H. Horiuchi, T. Suhara, and T. Yamada, arXiv:1807.11213 [Nucl-Th] (2018).
[17] M. Lyu, M. Isaka, T. Myo, H. Toki, K. Ikeda, H. Horiuchi, T. Suhara, and T. Yamada, Prog. Theor. Exp. Phys. 2018, 011D01 (2018).
[18] R. Weiss, R. Cruz-Torres, N. Barnea, E. Peasetzky and O. Hen, Phys. Lett. B 780, 211 (2018).
[19] N. Itagaki and A. Tohsaki, Phys. Rev. C 97, 014304 (2018).
[20] P. Ring and P. Schuck, The Nuclear Many-Body Problem (Springer-Verlag, New York, 1980).