Improving the numerical model for high temperature coated conductors using the Hall-probe measurement

Mykola Solovyov, Milan Polak, Michal Vojenciak, Fedor Gömöry

1 Institute of Electrical Engineering, Slovak Academy of Sciences, Dubravska cesta 9, 841 04 Bratislava, Slovakia
E-mail: mykola.solovyov@savba.sk

Abstract. Electric power applications of coated conductor tapes (e.g. power transmission cables, transformers, fault current limiters) are developed nowadays. Optimal design of these applications requires a realistic model of the electromagnetic behaviour of a single tape. We compare here four models that could represent the tape in electromagnetic calculations. The simplest model assumes a rectangular cross-section of superconducting layer and constant critical current density. Models that are more complex consider the critical current density depending on magnetic field and its orientation. Furthermore, one could introduce a non-uniformity of the superconducting layer across the tape width. We checked the predictive power of these models by comparing the agreement between the calculated values of magnetic field above the tape and the experimental data obtained by Hall-probe experiments. In the case of commercial coated conductor tape on non-magnetic substrate the most representative model incorporated the critical current density dependence on magnetic field as well as the assumption of worsening the layer properties towards the tape edges. The latter conclusion was confirmed by the experiment in which longitudinal strips cut from the tape were tested.

1. Introduction

Simple and realistic model representing the properties of a superconducting conductor is essential for the calculation of AC losses by analytical or numerical method. AC dissipation in superconductor is controlled by the interaction of magnetic flux with superconductor resulting in a gradual change of electrical current distribution during the AC cycle. Because of weak influence of the AC frequency (as a first approximation the hysteretic behaviour is assumed by the critical state model) experiments in DC could provide valuable additional information.

In this work we use the analysis of data obtained in several DC experiments to improve the model representing a coated conductor tape in numerical simulations of AC transport. The dependence of critical current, $I_c$, on the value and orientation of magnetic field has been determined. In the following these are called the $I_c(B,\alpha)$ data. Mapping of the magnetic field around the tape transporting DC current $I$ have been performed with the help of a Hall sensor. Two sets of these Hall-probe data are used in the current analysis, performed for $I = I_c$, and $I = 0$, respectively. Also, after splitting the tape in few parallel strips, $I_c$ has been determined for each of them, in order to investigate the spread of superconductor’s properties across the tape width.
2. Experimental details
For our investigation, we used the commercial high temperature superconductor (HTSc) YBCO tape from SuperPower [1]. This tape has the following parameters: width 4.2 mm, thickness < 0.1 mm (including the stabilization layer - schematic structure shown on the figure 1), and the self-field $I_C = 120 \pm 5 \, \text{A}$ that slightly changed along the tape length. For the determination of $I_C(B, \alpha)$ dependence the sample of total length 100 mm was taken. The criterion of $1 \, \mu \text{V/cm}$ was used in analysing the signal taken from the voltage taps separated by 50 mm and placed symmetrically around the tape centre. The same criterion was used when critical currents of longitudinal strips cut from the tape had to be determined.

In the Hall probe mapping experiment the magnetic field was measured by the Hall-probe with the active area of $50 \, \mu \text{m} \times 50 \, \mu \text{m}$, which is prepared from n-InSb [2]. The probe signal was pre-processed by an instrumentation amplifier and noise filter [3]. The component perpendicular to the wide face of the tape was registered in distance of 0.4 mm from the tape surface. The probe holder movement across the tape width was performed with stops every 0.05 mm during which the Hall voltage was recorded. Total length of the scan was 40 mm.

![Figure 1. Schematic structure of the HTSc tape [1].](image)

3. Development of the model representing CC tape
We compare the results of experiments to the predictions obtained by numerical calculations of electrical current and magnetic field distributions. The finite element method was developed utilizing the commercial software – COMSOL Multiphysics (with AC/DC module) [4]. In order to perform calculation in series and automatize the postprocessing we used a numerical computing environment – MATLAB [5]. The details of the FEM calculation method are described in [6].

To illustrate the way of developing a representative model of the tape, that generally leads to increased complexity and higher requirements on the computing power we present the results obtained for four models. The simplest Model 1 assumes the cross-section of superconducting layer to be rectangular, and the critical current density is calculated by dividing the self-field critical current by the area of the rectangle. It is well known, that such model could give good qualitative indications, however the assumption of a field-independent critical current density is sometimes too crude. Therefore in Model 2 the local value of critical current density, $J_C$, is assumed to depend on the magnitude and orientation of magnetic field. Thus the component parallel to the tape wide face, $B_x$, and the perpendicular one, $B_z$, are present in the empirical expression

$$ J_C = \frac{J_{CO}}{1 + \left( \frac{k_0}{B_0} \frac{B_x^2}{B_0^2} + \frac{B_z^2}{B_0^2} \right)^\beta} $$

containing four parameters $J_{CO}$, $k_0$, $B_0$, $\beta$ characterizing the properties of superconductor. These can be found in an iterative way, by comparing the results of FEM calculation with experimental data of critical currents measured for the tape in a range of magnetic fields applied at different angles, $I_C(B, \alpha)$. We used in our calculation $k_0 = 0.7$, $B_0 = 0.03 \, \text{T}$, $\beta = 0.7$, respectively. To keep the possibility of comparing different models the value of $J_{CO}$ has been adjusted separately for every Model to achieve
the same value of self-field critical current, \( I_c = 118 \text{ A} \). The latter regards also the Model 3 and Model 4, where the geometry of superconducting layer is no more rectangular.

The starting assumptions for the geometry of superconducting layer in our calculations have been the following: Width of the superconducting layer 4.16 mm obtained from the total width 4.2 mm by subtracting 0.02 mm of copper from each side. The thickness of 1.4 µm has been deduced from the data of the critical current dependence on the number of deposited layers reported in [7]. The wire is assumed infinite in \( y \) direction and its cross-sections is in \([x, z]\) plane – 2D model.

To include the inhomogeneity of the superconducting layer in the model, we checked two possibilities: variable thickness of the layer, and non-uniform \( J_{C0} \) across the tape width. The numeric simulations show, that both these methods could give equivalent results in AC losses simulations and the magnetic field distributions above the tape.

Unfortunately it is not easy to obtain the information about the layer inhomogeneity in direct way, e.g. from optical micrograph. This is because we need not only the information about the real geometry of the layer (that could be very difficult in the case of the layer \( \sim 1 \) µm thick), but also the information about local \( J_{C} \) of the layer.

![Figure 2](image.png)

**Figure 2.** Distribution of the current flow calculated by four numerical models for \( I = I_c \) and \( I = 0 \), respectively.

To check, whether the geometrical non-uniformity itself of the superconductor layer that otherwise is uniform (parameters \( k_{\alpha}, B_0, \beta \) unchanged) has an influence on the calculated distribution of magnetic field, we modified the original rectangular geometry in two ways: the Model 3 represents a superconductor with thinner layer at the edges, and the Model 4 has edges thicker than the middle part - see figure 2. When comparing the magnetic field distribution above the tape measured by Hall-probe
with that one calculated in the case of different models, we have found that the discrepancy between experimental data and simulations does not favour to develop further the Model 4. In opposite, when adjusting the geometry of Model 3 we saw that this kind of shape modification improves our result. Remember that in these calculations, the area of the cross-section was not constant, therefore $J_{c0}$ was adjusted for each model to keep the same value of critical current in self-field.

The key assumption in our investigation is that the magnetic field distribution is directly depending on current distribution in the superconducting layer. To investigate the possibility of revealing the layer non-uniformity indirectly in this way, we compared the prediction of four models with experiment. Indeed, the differences between the results are notable, as shown in figure 3 and 4.

![Figure 3](image1.png)

**Figure 3.** Comparison of the magnetic field distribution calculated for different numerical models with the Hall-probe measurement for $I = I_c$ (118A for this sample). Hall probe scanned the z-component of magnetic field along the x-direction (i.e. transversal to the tape length) in the constant height 0.4 mm above the tape surface.

![Figure 4](image2.png)

**Figure 4.** The same as figure 3 but for the remanent state ($I = 0$).

To quantify the agreement between the results calculated in the frame of different models and the experimental data the mean squared errors (MSE) have been evaluated using the formula:

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (B_{mi} - B_{ci})^2,$$  \hspace{1cm} (2)
where: $n$ - number of values in observed interval, $B_{mi}$ - measured value of magnetic field, $B_{ci}$ - calculated value of magnetic field in the same position as $B_{mi}$. The values in 401 points from the interval [-10, 10] mm of $x$ have been taken in this calculation, with the results given in table 1.

|        | Model 1 | Model 2 | Model 3 | Model 4 |
|--------|---------|---------|---------|---------|
| $I = I_C$ | 0.293   | 0.099   | 0.034   | 2.129   |
| $I = 0$    | 0.134   | 0.084   | 0.054   | 0.974   |

Comparing the situation at $I = I_C$ and $I = 0$, we see that the model ranking remains the same with Model 3 exhibiting the lowest error. We assume that this is because incorporating the magnetic field dependence of the critical current density (like in Model 2) as well as assuming the non-uniformity of superconducting layer. Slight changes in the relations between errors for different models at $I = I_C$ and $I = 0$ probably reflect an additional phenomena that should be subject of further study.

As already mentioned above, the Model 4 assuming larger current capability of the tape edges lead to the worst prediction – even compared to the simplest Model 1.

4. **Direct check of critical current density distribution in the tape**

The prediction that our sample has lower current carrying ability at the edges was checked by another experiment. A piece of tape 25 mm long was divided into 6 strips using longitudinal cuts [8]. Six tapes obtained in this way have the following widths: 0.65, 0.66, 0.70, 0.50, 0.66 and 0.50 mm. Five cuts caused the width reduction of $\Delta w = 4.2 - 3.67 = 0.53$ mm. The width reduction per 1 cut is $0.53/5 = 0.106$ mm. The critical currents, $I_C$ (A) at 1 microvolt per centimeter were measured between taps distanced by 1.5 cm in self-field as well as in DC field.

![Figure 5](image.png)

**Figure 5.** The critical currents of strips cut from the tape in longitudinal direction.

Figure 5 shows the corresponding critical currents, normalized to 1 mm width - $I_C\text{ norm}$ (A/mm) in the self-field and 56 mT at 77 K. This value of DC field is sufficiently higher than the characteristic
field of $J_c(B)$ dependence, $B_0 = 0.03$ T, and on the other hand the critical currents are still big enough for precise measurements.

As seen, in the self-field the lowest $I_c$ \textit{norm} values were observed at the edges. This is in good agreement with the analysis of Hall-probe experiment mentioned before. The decreasing of critical current on the edges is about 28% in self-field. However, in the field of 56 mT the critical currents distributed more uniformly, only at one edge $I_c$ \textit{norm} reduced by 20%. More detailed investigation is needed to explain the observed depression of non-uniformity with applied DC field.

5. Conclusion
We compare four models that could represent the superconducting layer of coated conductor tape in numerical simulations. Starting model (Model 1) assumed the rectangular geometry and constant critical current density. Taking the magnetic field dependence of critical current density into consideration more complex models, requiring the analysis of $I_c(B,\alpha)$ data, have been proposed. One of them assumed a rectangular cross-section of the layer (Model 2). In order to simulate the effect of layer non-uniformity across the tape width, two further models have been investigated with non-constant thickness of the layer. In Model 3 we assumed thinner layer at the tape edges, in Model 4 the opposite.

Assessment of the models has been performed by checking the prediction for the magnetic field distribution in the tape vicinity (transversal scan of the perpendicular component) with experimental data obtained by Hall probe scanning. Our conclusion is that for the best prediction one should incorporate the $J_c(B)$ dependence into the model, and also consider a non-uniformity of superconductor layer. In the case of the tape we studied, the current carrying capability at the tape edges was lower than in its centre. This was represented in the model by the tape thickness decreasing at the edges. An equivalent result would be obtained when assuming a constant thickness of the layer but reduced critical current density at the tape edges.

The deduction of worse properties of the tape edges was confirmed also by the determination of critical current distribution on the set of strips cut from the tape.

Unfortunately, the method of improving the numerical model by using the Hall-probe measurement is useful just for the tapes with nonmagnetic substrate [9], because the magnetization of the substrate material makes the data analysis very complicated.