Do finite-size neutrally buoyant particles cluster?

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Abstract

The turbulent mixing of small and heavy particles presents a striking feature known as preferential concentration or clustering. We investigate here the preferential concentration of particles that are neutrally buoyant but with a diameter significantly larger than the dissipation scale η of the carrier flow (4.4η–17η). Such particles are known to not behave as flow tracers (Qureshi et al 2007 Phys. Rev. Lett. 99 184502) but it remains an open question whether they do cluster or not. For this purpose, we produce homogeneous and isotropic turbulence in a closed water flow, and seed the flow with neutrally buoyant particles spanning a range of Stokes numbers from 1.6 to 24.2 depending on the rotation frequency. The spatial structuration of these inclusions is then investigated by Voronoï tesselation analysis, as proposed recently by Monchaux et al (2010 Phys. Fluids 22 103304), from images of the particle concentration field taken in a laser sheet at the center of the flow. No matter what the rotation frequency and the Reynolds and Stokes numbers are, the particles are found to not cluster. Finite-size neutrally buoyant particles are therefore not inertial. We also conclude that the Stokes number per se is an insufficient indicator of the clustering trend in particles-laden flows.

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1. Introduction

The mixing of small particles dispersed in a turbulent flow is omnipresent in nature and industry, be it pollutant dispersion in the atmosphere, volcanic or classical cloud formation and dispersion, sedimentation in rivers or the mixing of liquid droplets or solid particles in various man-made processes (e.g. chemical reactors, internal combustion engines, turbine engines or rocket motors). The study of them is therefore of great interest and has many fundamental aspects, issues and limits so far. A striking feature of these flows is the trend of the particles to concentrate on preferentially sampled regions of the carrier flow. This has been observed and investigated for long in both experiments [1–3] and simulations [4, 5], and is still being widely studied [6, 7]. The focus is usually on small heavy particles (that is, particles with a high density ratio as compared to the fluid), especially in numerical studies. Because of their high specific density, the dynamics of such small and heavy inertial particles deviates from that of the carrier flow. Clustering phenomena are then one of the many manifestations of this departure from tracer behavior. Some other studies were conducted for light particles as well, exhibiting the same trend to cluster but with different cluster geometries [5]. Finally, tracers (ought to be both neutrally buoyant and much smaller than the dissipative scale of the carrier flow) are usually used to characterize the flow dynamics. The case of finite-size neutrally buoyant particles, however, has never been treated to our knowledge in the context of the preferential concentration phenomenon. Such finite-size particles (with a diameter significantly larger than the dissipation scale of the carrier flow) are known experimentally [8] and numerically [6] to differ from tracers. However, existing studies have focused on the dynamics of isolated particles, but not on the spatial structuration of laden flows. It remains an open question whether they cluster or not.

This paper is organized as follows. In section 2, we describe the experimental setup and the data processing used for carrying out this investigation. Section 3 describes the results on preferential concentration of finite-size neutrally
buoyant particles. The paper ends with a brief discussion and the conclusions (section 4).

2. Experimental setup and postprocessing

2.1. Turbulent flow generation

In order to study the behavior of neutrally buoyant particles, a straightforward solution is to create a turbulent water flow, with the particle density matching that of water (neutrally buoyant cloud can be obtained in air, for instance with soap bubbles inflated with helium [8, 9]; however, while it is easy to produce such bubbles individually, dense seeding with numerous particles is difficult). Many experimental apparatuses creating turbulent water flow with small mean flow velocities exist, the best-known example being the von Kármán flow. This von Kármán flow is a high-Reynolds-number turbulent flow created between two counter-rotating discs. Near the center of the apparatus, the mean velocities are much weaker than the fluctuations. This is of particular interest for the investigation of particles in turbulence as the mean central stagnation point eases the acquisition of long particle trajectories, which has made the von Kármán flow the natural choice for several pioneering experiments on Lagrangian particle tracking [10, 11]. However, von Kármán flows exhibit statistical inhomogeneity and anisotropy that may render the interpretation of the results difficult, in particular when it comes to discriminating between the effects associated with large anisotropic structures and small turbulent scales.

In this study, the experiments are conducted in a new turbulence-generating apparatus, the Lagrangian exploration module (LEM), developed in collaboration between the Laboratoire de Physique of the École Nationale Supérieure de Lyon and the Max Planck Institute of Dynamics and Self-Organization of Göttingen. The LEM produces turbulence in a closed water flow driven by 12 impellers evenly distributed on 12 of the 20 faces of an icosahedral vessel (see figure 1). The length of the edges of the icosahedron is 40 cm, giving a volume of 140 liters water. The 12 impellers can be independently driven. In this work, all impellers are used simultaneously and rotate at the same constant frequency \( f \), which can be increased up to 12.5 Hz, with the constraint that impellers in front of each other counter-rotate. This has been shown to achieve a statistically homogeneous and isotropic flow with almost zero mean velocity in a central region of the device of order \( 10 \times 10 \times 10 \) cm\(^3\) [12].

For the present setup, turbulence was characterized with two-dimensional (2D) particle image velocimetry (PIV) using a LaVision device on a 15 cm \( \times \) 10 cm plane at the center of the LEM. This confirms a nearly homogeneous and isotropic turbulence in a sphere of nearly 8 cm diameter at the center of the LEM. The evolution of the turbulence characteristics (defined below) for the explored range of rotation frequencies is provided in table 1. The fluctuating velocities are averaged over the entire illuminated plane as \( u' = \frac{\langle u_{x, r.m.s}^2 + u_{y, r.m.s}^2 \rangle}{2} \). We estimate the energy dissipation rate \( \varepsilon \) from scalings of the velocity structure functions in the inertial range (see, e.g., [13])

\[
\varepsilon = \frac{1}{r} \left[ \frac{D_{LL}(r)}{C_2} \right]^{3/2},
\]

and

\[
\varepsilon = \frac{1}{r} \left[ \frac{3D_{NN}(r)}{4C_2} \right]^{3/2},
\]

where \( D_{LL}(r) \) and \( D_{NN}(r) \) are the second-order longitudinal and transverse structure functions. The constant \( C_2 = 2.1 \) as obtained from a compilation of data on various turbulent flows [14]. Moreover, we have checked that the isotropic conditions \( D_{NN} = 4/3D_{LL} \) hold over the entire range of resolved scales, which confirms the good isotropy properties of the LEM. The estimations of \( \varepsilon \) from the transverse and the longitudinal scaling are in excellent agreement. The integral length scale \( L \) is defined by

\[
L = \int_0^{\infty} D_{LL}(r) \, dr.
\]

Finally, the Reynolds numbers based on the Taylor micro-scale (simply referred to as the Reynolds number later on) is defined as

\[
R_s = \sqrt{\frac{15u' L}{\nu}} = \sqrt{\frac{15u_{x, r.m.s}^2 L^{4/3}}{\nu}} = \sqrt{\frac{15u_{x, r.m.s}^2 L^{4/3}}{\nu \varepsilon}}.
\]

When the rotation velocity is changed from 2 to 12 Hz, the associated Reynolds numbers based on the Taylor micro-scale vary from 160 to 395 at the center of the LEM.
2.2. Particles characteristics

Regarding the particles used, one of the main goals of this study is to explore the behavior of finite-size neutrally buoyant particles. More precisely, the particles used must be neutrally buoyant in water, meaning that their density $\rho_{\text{particle}}$ must be close to $\rho_{\text{water}}$. Their size is also of importance: not so small that they would behave as tracers and follow the carrier flow dynamics, but small enough that the flow dynamics is not altered much.

In concrete terms, we chose polystyrene particles of diameter $d = 700 \, \mu m$—corresponding to 4.5–17 times the Kolmogorov length scale $\eta$, depending on the turbulence energy dissipation rate $\varepsilon$. This large range of the ratio between the particles diameter and the Kolmogorov length scale allows us to study what can be seen as tracers ($d/\eta \approx 4.5$) on the one hand and inertial particles ($d/\eta > 5$) on the other hand [15, 16]. The particles density $\rho_{\text{particle}}$ has been adjusted so that the ratio $\Gamma = \frac{\rho_{\text{particle}}}{\rho_{\text{water}}}$ is 1 $\leq \Gamma \leq 1.015$. These particles are obtained from small expandable polystyrene particles with original density of the order of 1.05. These particles are irreversibly altered much. In the expansion process, particles whose density matches as close as possible to that of water are then selected and sieved.

Particles interacting with a turbulent flow are commonly characterized by their Stokes number, a dimensionless number that quantifies the ratio between the particle viscous relaxation time and a typical time scale of the flow. The latter is generally characterized by their Stokes number, a dimensionless number whose validity can only be warranted as an approximation of their Stokes number. It has to be noted that while the particle diameter and density are kept constant, the Stokes number is varied by tuning the flow dissipation time scale in equation (5). Therefore, it cannot be varied independently of the Reynolds number of the carrier flow.

$$St = \frac{\tau_p}{\tau_\eta} = \left(\frac{d}{\eta}\right)^2 \frac{1 + 2\Gamma}{36}, \quad (5)$$

where $\tau_p$ is the particle viscous relaxation time, $\tau_\eta$ the dissipation time scale of the carrier flow and $\eta$ its dissipation length scale. The Stokes number is usually used as the key—and often the only—parameter for characterizing particle dynamics in turbulence. This is highly motivated by the simplicity of Stokesian models for modeling and numerical simulation purposes, where the dominant force acting on the particle is simply taken as the drag due to the difference between the particle velocity $v$ and the fluid velocity $u$:

$$\frac{dv}{dt} = \frac{1}{\tau_p} (\vec{u} - \vec{v}), \quad (6)$$

the only explicit relevant parameter for the particles being then the viscous response time. This minimal Stokesian model, whose validity can only be warranted as an approximation of the Maxey–Riley and the Gatignol equations [17, 18] for the case of small inertial particles much heavier than the fluid, is commonly used in numerical simulations (both direct numerical simulations and kinematic simulations) investigating the turbulent dynamics of inertial particles. An important result to be emphasized in the context of this study is the observed dependence of preferential concentration on the Stokes number. More specifically, Stokesian models suggest that particles with a non-vanishing Stokes number tend to exhibit a preferential concentration. Further clustering and segregation is maximal for particles whose Stokes number is of the order of unity [3, 19]. This trend is supported by the experimental measurements of the concentration field of small inertial particles [7].

For the particles investigated in this work, the Stokes number (as defined in equation (5)) is in the wide range of 1.6–24 as shown in table 1. Although our particles are finite-size and hence the Stokesian approximation (6) is not expected to hold by itself, the question of the influence of the Stokes number on the spatial distribution of such finite-size particles is still of much relevance. In this work, the particle concentration field was investigated as a function of their Stokes number. It has to be noted that while the particle diameter and density are kept constant, the Stokes number is varied by tuning the flow dissipation time scale in equation (5). Therefore, it cannot be varied independently of the Reynolds number of the carrier flow.

2.3. Acquisitions and postprocessing

2.3.1. Particles detection. Acquisitions are performed using 12-bit digital imaging at a resolution of 2400$\times$1800 pixels corresponding to a 15 cm $\times$ 10 cm visualization window at the center of the LEM. Images are recorded with a Phantom V10 camera (Vision Research Inc.) operated at a low repetition rate of 2.5 Hz (note that we address here only the question of particle spatial distribution and do not aim at tracking particle dynamics, which would have required a much higher repetition rate). The visualization window is illuminated by a 100 W pulsed Nd:YAG laser (Condor Serial, Quantronix) synchronized with the camera, creating a light sheet with millimetric thickness. The camera is mounted with a 90 mm macro lens (Tamron) through a Scheimpflug mount to compensate for the depth of field effects resulting from the angle between the camera and the laser sheet. Each experiment consists of 2000 uncorrelated images acquired in nearly 15 m in for a fixed rotation frequency for each motor and a constant concentration of polystyrene in the LEM. We identify the particles on the images as local maxima with light intensity higher than a threshold, assuming in a first approximation that all the particles illuminated in the laser sheet belong to one plane. The center of the particles is determined as the center of mass of all the pixels surrounding one local maxima. We have checked that changing slightly the threshold value does not significantly impact the number of detected particles, essentially because the contrast between the light diffused by the particles and the background is very strong due to the large size of the particles. At the working seeding density, the average number of detected particles is of the order of 100. No diminution of the number of detected particles is observed from the beginning to the end of an experiment. This indicates good stationarity of the seeding concentration as expected for non-settling neutrally buoyant particles.

2.3.2. Voronoi tessellation analysis. In order to study the particle concentration field, we use Voronoi diagrams. A raw acquired image, and the detected particles and the associated Voronoi diagram, are provided in figures 2(a) and (b), respectively. Such a Voronoi analysis has recently
been introduced for the investigation of the preferential concentration of small water droplets in a turbulent flow of air [7], and was shown to be particularly efficient and robust to diagnose and quantify the clustering phenomenon. These Voronoi diagrams give a tesselation of a 2D space where each cell of the tesselation is linked to one seed (one detected particle in the present case) in such a way that all the points of the cell are closer to the associated particle than to any other particle. It appears that the area of a Voronoi cell is the inverse of the local concentration of particles. Studying the Voronoi area field is thus equivalent to studying local concentration fields. The Voronoi method was chosen for two main reasons: firstly, unlike other methods, for instance using box counting as a tool to study particle concentration [2], this technique gives a measure of the local concentration field at the interparticle length scale, meaning that the measure does not depend on the field size, nor on an extrinsic length scale choice. Secondly, this method is numerically very efficient regarding the number of particles we have to process (several hundreds per image). To compare the results of different experiments performed with different amounts of detected particles per image, one has to find a normalization. This is achieved using the average Voronoi area $\overline{A}$ defined as the mean particle concentration inverse, which does not depend on the spatial organization of the particles. Therefore, we focus on the distribution of the normalized Voronoi area $\mathcal{V} \equiv A/\overline{A}$ in the rest of the study.

3. Results on the preferential concentration

Clustering properties can be investigated and quantified by comparing the probability distribution function (PDF) of Voronoi cell areas in the experiment with that of Voronoi cell areas of a synthetic random Poisson process. Note that, surprisingly, no analytical form is known for the PDF of Voronoi cell areas of such a Poisson process, although its form is known to be well approximated by a Gamma distribution [20]. As a random Poisson process reference, we use here the compact analytical expression involving the space dimension as a single parameter proposed in [20].

The PDFs of Voronoi cell areas for the different experiments described in table 1 are plotted in figure 3. As can be seen, all the PDFs collapse within statistical convergence error bars. No systematic trend is visible with different experimental configurations. We have superimposed on the same plot a Gamma distribution. Interestingly, we find that the PDFs and the Gamma distribution also collapse, meaning that the particles do not exhibit any preferential concentration (or clustering) whatever be the Stokes number.

This result can be further quantified using the standard deviation of the Voronoi areas. The standard deviations of the normalized Voronoi areas $\sigma_\mathcal{V} = \sqrt{\langle \mathcal{V}^2 \rangle - 1}$ for different Stokes numbers, to be compared with the standard deviation corresponding to a random Poisson process $\sigma_\mathcal{V}^{RPP}$. 

| $St$ | 1.6 | 4.7 | 8.6 | 13.2 | 18.4 | 24.2 |
|------|-----|-----|-----|------|------|------|
| $\sigma_\mathcal{V}$ | 0.532 | 0.530 | 0.529 | 0.530 | 0.535 | 0.533 |
| $\sigma_\mathcal{V}^{RPP}$ | $\approx$ 0.53 |

be seen, all the PDFs collapse within statistical convergence error bars. No systematic trend is visible with different experimental configurations. We have superimposed on the same plot a Gamma distribution. Interestingly, we find that the PDFs and the Gamma distribution also collapse, meaning that the particles do not exhibit any preferential concentration (or clustering) whatever be the Stokes number.

This result can be further quantified using the standard deviation of the Voronoi areas. The standard deviations of the normalized Voronoi areas are given in table 2. The experimental standard deviations are all very close to 0.53, the standard deviation of the normalized Voronoi area corresponding to a random Poisson process. This reveals again the tendency of particles to distribute in a random way.
4. Conclusion and discussion on the Stokes number

We have investigated in this study the preferential concentration of finite-size neutrally buoyant particles using Voronoi diagrams. By varying the Reynolds number in the closed water flow, we have been able to vary the Stokes number (the density and the diameter of the particles being kept constant). Although such particles are known to have a different dynamics from that of the flow, we have found no preferential concentration in the spatial structuration of these inclusions regardless of the Stokes number.

This result is contrary to most of the studies describing the preferential concentration of particles as a result of inertial effects. As mentioned previously, these studies usually account for the deviation of the particle dynamics for inertial effects due to the difference in density between the particles and the fluid (hence the name inertial particles). The clustering is then regarded as an effect of the inertia of the particles, and the model used for quantifying this preferential concentration usually involves the Stokes number only. The present study, while dealing with particles whose Stokes number is high (above unity) and varies, does not exhibit any clustering. We conclude that finite-size neutrally buoyant particles are not inertial. In our case, we suspect that the pressure distribution at the surface of the particles must be preponderant in the dynamics of the particles. It also shows that the Stokes number by itself is not a sufficient indicator of the clustering trend in particle-laden flows.

In future studies, we plan to carry out the same kind of experiments with heavy particles in order to study the impact of the particle size on the clustering phenomena and their geometries. We will also investigate the influence of flow anisotropy on the spatial structuration of particles, thanks to the versatility of the LEM and its 12 independently driven impellers. This will throw new light on the possible clustering trend in mixing processes close to what can be found in nature and industry.

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References

[1] Squires K D and Eaton J K 1991 Phys. Fluids A 3 1169
[2] Fessler J R, Kulick J D and Eaton J K 1994 Phys. Fluids 6 3742
[3] Bec J, Biferale L, Cencini M, Lanotte A, Musacchio S and Toschi F 2007 Phys. Rev. Lett. 98 084502
[4] Yoshimoto H and Goto S 2007 J. Fluid Mech. 577 275
[5] Calzavarini E, Kerscher M, Lohse D and Toschi F 2008 J. Fluid Mech. 607 13
[6] Homann H and Bec J 2010 J. Fluid Mech. 651 81
[7] Monchaux R, Bourgoin M and Cartellier A 2010 Phys. Fluids 22 103304
[8] Qureshi N M, Bourgoin M, Baudet C, Cartellier A and Gagne Y 2007 Phys. Rev. Lett. 99 184502
[9] Qureshi N M, Arrieta U, Baudet C, Cartellier A, Gagne Y and Bourgoin M 2008 Eur. Phys. J. B 66 531
[10] La Porta A., Voit G A, Crawford A M, Alexander J and Bodenschatz E 2001 Nature 409 1017
[11] Mordant N, Metz P, Michel O and Pinton J-F 2001 Phys. Rev. Lett. 87 214501
[12] Zimmermann R., Xu H, Gasteuil Y, Bourgoin M, Volk R, Pinton J-F and Bodenschatz E 2010 Rev. Sci. Instrum. 81 055112
[13] Pope S B 2000 Turbulent Flows (Cambridge: Cambridge University Press)
[14] Sreenivasan K R 1995 Phys. Fluids 7 2778
[15] Brown R, Warhaft Z and Voit G 2009 Phys. Rev. Lett. 103 194501
[16] Volk R, Calzavarini E, Leveque E and Pinton J-F 2011 J. Fluid Mech. 668 223
[17] Maxey M and Riley J 1983 Phys. Fluids 26 883
[18] Gatignol R 1983 J. Méc. Théor. Appl. 1 143
[19] Coleman S W and Vassilicos J C 2009 Phys. Fluids 21 113301
[20] Ferenc J-S and Néda Z 2007 Physica A 385 518