About the influence of neglecting locking effects on the failure behavior at the interface

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In the present work, a novel cohesive discontinuous Galerkin (CDG) method is proposed to model interfacial failure of brittle materials. Before the failure, an incomplete interior penalty Galerkin (IPG) variant of discontinuous Galerkin (DG) family is applied for the linear elasticity. Once the failure criterion is met, an extrinsic cohesive zone (CZ) model captures the failure behavior of the interface. Through application of the DG method in combination with reduced integration on the boundary terms, the locking problem of the bulk elements is solved as well as a realistic propagation of the crack is obtained. In addition, due to the presence of the DG elements prior to failure, remeshing of the interface during crack propagation is not required for the proposed extrinsic CZ model. Delamination of a composite structure is simulated in a numerical example. Furthermore, the performance of the CDG element formulation in comparison to a conventional intrinsic CZ element is discussed and conclusions are drawn.

1 Introduction

Discontinuous Galerkin methods [1] are used in various engineering applications ranging from fluid and solid mechanics [2,3] to the modeling of damage. The latter is attracting significant attention in the failure modeling of the interfaces [4,5]. This is mainly due the inherent non-conformity of the DG methods. A combination of DG methods with extrinsic CZ models are advantageous when it comes to the elimination of the initial stiffness in the pre-failure regime. Unlike the extrinsic CZ models, the use of cohesive discontinuous Galerkin (CDG) methods does not necessitate dynamic mesh adaptation at the crack tip. In this work, artificial stiffening of the bulk and its notorious effect on the crack propagation are eliminated in addition to the aforementioned benefits. After introduction of the governing equations, a fiber composite structure under tensile loading is modeled. Finally, the behavior of the new model in comparison to a conventional intrinsic CZ model is discussed.

2 Governing equations

The weak form of the balance law is given in equation (1) (see [5]). In addition to the Drichlet and Neumann boundary conditions, the continuity of the displacements and tractions is prescribed in a weak form for the pre-failure regime (weak discontinuity) denoted by α = 0. As soon as the failure criterion is met, cracks (strong discontinuities) initiate and the continuity of the displacements is not valid anymore. This is denoted by α = 1. In this stage, the cohesive tractions are computed by a traction separation law (TSL) [5] which is given in equation 2.

\[
\int_{\Omega} \sigma : \varepsilon \, dV + \int_{\Gamma} (1 - \alpha) [\delta u] \cdot \{\sigma\} \, d\Gamma + \int_{\Gamma} (1 - \alpha) \theta \{\delta u\} \cdot [u] \, d\Gamma + \int_{\Gamma} \alpha \, \delta g \cdot t_{CZ} \, d\Gamma
\]

\[
\int_{\partial \Omega_t} t_p \cdot \delta u \, dA + \int_{\Omega} f \cdot \delta u \, dV,
\]

\[
t_{CZ}(g) = \frac{t_0 \, \sigma_{\text{eff}}}{\lambda} \left( \frac{\lambda_f - \lambda_{\text{max}}}{\lambda_f} \right) \left( \frac{\lambda}{\lambda_{\text{max}}} - \theta \right) \left[ \frac{0}{(g_n)^n} \right].
\]

Here, \(\sigma\) is the Cauchy stress, \(\varepsilon\) is the strain tensor and \(u\) is the displacement vector. In addition, \(n\) represents the normal vector to the discontinuity \(\Gamma\) and \(\theta\) is the penalty parameter. The gap is defined by \(g_{\text{eff}} = [\beta g_s \, (g_n)^{1/2}]^T\) with its shear and normal components. The parameter \(\beta\) controls the contribution of the shear separation. The effective separation is given by \(\lambda = ||g_{\text{eff}}|| = \sqrt{\langle g_n \rangle^2 + \beta^2 g_s^2}\) (see [6]). The effective separation at full failure is denoted by \(\lambda_f\) whereas \(\lambda_{\text{max}}\) represents the maximum separation reached before full failure. The critical traction at failure initiation is given by \(t_0\). The convexity of the drop of the traction is given by the material parameter \(m\) while \(n\) is a numerical parameter to avoid instabilities during contact. This is manifested in the last term of equation 2.

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3 Numerical example: Uniaxial tension of a 2D fiber composite

A fiber with a circular cross-section in its surrounded matrix is pulled apart. Geometry, boundary conditions, loading and discretization of the quarter of the problem are illustrated in Fig. 1. The material parameters are given in Table 1. The reaction-force displacement curves for different mesh refinement levels are plotted in Fig. 2 for two different CZ models.

![Fig. 1: A quarter of a composite structure. a Geometry, boundary conditions and loading, b discretization in the undeformed configuration, c magnified displacements in x-direction with a propagated crack at the interface.](image)

### Table 1: Parameters for the uniaxial tension of the 2D fiber composite.

|                 | $E$ [MPa] | $\nu$ | $\vartheta$ | $\lambda_0$ [mm] | $\lambda_f$ [mm] | $\beta$ | $t_0$ [MPa] | $n$ | $m$ |
|-----------------|-----------|-------|-------------|------------------|------------------|--------|-------------|-----|-----|
| matrix fiber    | 1000      | 0.4999| 500         | ---              | ---              | ---    | ---         | --- | --- |
| fiber           | 2000000   | 0.2   | 500         | ---              | ---              | ---    | ---         | --- | --- |
| extrinsic CDG [5]| 100500   | 0.3   | 100$E/h$    | ---              | 0.017            | 1.0    | 1.5         | 1.2 | 1.0 |
| intrinsic CZ [6] | 100500   | 0.3   | 10$^5$      | 10$^{-4}$        | 0.017            | 1.0    | 1.5         | 1.2 | 1.0 |

![Fig. 2: Convergence of an intrinsic CZ model [6] a compared to the CDG elements b in terms of reaction force-displacement curves.](image)

4 Conclusion

In the present work, an IIPG variant of the DG methods in combination with an extrinsic cohesive zone model was applied to model failure at the interface of a fiber composite structure. A conventional intrinsic CZ model was considered for the sake of comparison. The CDG elements with reduced integration on the boundary terms outperformed the standard intrinsic CZ element formulation in terms of the number of elements. This is due to the elimination of the locking effects. In addition, a realistic behavior of the crack initiation as well as propagation was captured.

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