THE ROLE OF SUPERLUMINAL ELECTROMAGNETIC WAVES IN PULSAR WIND TERMINATION SHOCKS

TAKANOBU AMANO1 AND JOHN G. KIRK2

1 Department of Earth and Planetary Science, University of Tokyo, Tokyo 113-0033, Japan; amano@eps.s.u-tokyo.ac.jp
2 Max-Planck-Institut für Kernphysik, Postfach 103980, D-69029 Heidelberg, Germany

ABSTRACT

The dynamics of a standing shock front in a Poynting-flux-dominated relativistic flow is investigated by using a one-dimensional, relativistic, two-fluid simulation. An upstream flow containing a circularly polarized, sinusoidal magnetic shear wave is considered, mimicking a wave driven by an obliquely rotating pulsar. It is demonstrated that the wave is converted into large-amplitude electromagnetic waves with superluminal phase speeds by interacting with the shock when the shock-frame frequency of the wave exceeds the proper plasma frequency. The superluminal waves propagate in the upstream, modify the shock structure substantially, and form a well-developed precursor region ahead of a subshock. Dissipation of Poynting flux occurs in the precursor as well as in the downstream region through a parametric instability driven by the superluminal waves. The Poynting flux remaining in the downstream region is carried entirely by the superluminal waves. The downstream plasma is therefore an essentially unmagnetized, relativistically hot plasma with a non-relativistic flow speed, as suggested by observations of pulsar wind nebulae.

Key words: plasmas – pulsars: general – stars: winds, outflows – waves

Online-only material: color figures

1. INTRODUCTION

Poynting-flux-dominated relativistic outflows are thought to emerge from compact objects in several high-energy astrophysical environments, including pulsar winds, jets in active galactic nuclei, and gamma-ray bursts. A key issue in this context is how the energetically dominant Poynting flux is converted into particles, and, subsequently, into the observed radiation; this problem, known as the \( \sigma \)-problem, was originally formulated for the Crab pulsar wind (Rees & Gunn 1974; Kennel & Coroniti 1984; Melatos 1998), but is now recognized in a more generic context (e.g., Thompson 1994; Drenkhahn & Spruit 2002).

A rotation-powered pulsar presumably emits most of its spin-down luminosity in the form of a relativistic wind that powers the surrounding pulsar wind nebula (PWN), in which relativistic electrons and positrons emit synchrotron radiation. These pairs are believed to be produced by a cascade that occurs close to a rotating, strongly magnetized neutron star, where an enormous electric field is induced (Sturrock 1971; Ruderman & Sutherland 1975; Hibschman & Arons 2001). Although the efficiency of pair production is rather uncertain, all models predict a Poynting-flux-dominated wind, i.e., the parameter \( \sigma \), defined as the ratio of the Poynting flux to the particle kinetic energy flux, is believed to be much greater than unity \( \sigma \gg 1 \). In a supersonic, radially expanding ideal MHD wind, \( \sigma \) is constant, so that the wind remains Poynting flux dominated until it reaches the termination shock. Beyond the termination shock, however, most of the energy flow seems to be carried by particles \( (\sigma \ll 1) \) according to arguments based on the morphology of the PWN and the spectrum of its radiation (Rees & Gunn 1974; Kennel & Coroniti 1984). This apparent contradiction has been the subject of considerable debate over the years. It suggests the existence of an unknown dissipation mechanism that converts the dominant electromagnetic energy into particle kinetic energy inside the wind zone with substantial efficiency, although the constraint might be partially relaxed by an MHD instability in the inner part of the PWN (Begelman 1998; Porth et al. 2013; Mizuno et al. 2011).

One approach to resolve the problem is to invoke an oblique rotator, whose rotation axis is not parallel to the magnetic axis (Coroniti 1990; Michel 1994). Such an obliquely rotating pulsar will continuously launch a wave along with the wind because of the time-varying electromagnetic fields of the magnetosphere. The plasma populating the magnetosphere is dense enough to screen out the electric fields of such a wave, which therefore propagates essentially as an MHD wave. Being an MHD wave, it is conceivable that only a quasi-static structure survives in the wind zone far from the star. The striped wind model, first proposed by Coroniti (1990), is based on such an idea. In such a wind, toroidal magnetic fields of alternating polarity, separated by a neutral sheet, are embedded in the flow. The particles inside the neutral sheet provide the pressure to balance the magnetic pressure outside the sheet. As the wind expands radially, the plasma pressure tends to fall off adiabatically and more rapidly than the magnetic pressure, resulting in the compression of the sheet. There is obvious a minimum thickness of the sheet, below which the MHD approximation is violated. A reasonable estimate of this is the Larmor radius of the pairs defined with the temperature inside and the magnetic field outside the sheet. If the sheet were to be compressed beyond this limit, then non-ideal MHD effects would start to play a role. It has been suggested that magnetic reconnection may occur because of enhanced anomalous resistivity in such a thin neutral sheet, an idea that is supported by particle-in-cell (PIC) simulations (Zenitani & Hoshino 2001, 2005, 2007; Jaroschek et al. 2004). In order to maintain pressure balance, the plasma inside the sheet is heated by annihilating magnetic energy. As the flow propagates farther out, this process may proceed in a quasi-static manner until the stripe structure is completely erased. However, the heating of the plasma by dissipation exerts a pressure gradient on the plasma, which leads to substantial acceleration of the flow (Lyubarsky & Kirk 2001; Kirk & Skjæraasen 2003), and the apparent dissipation rate decreases because of the relativistic time dilation effect. Consequently, Lyubarsky & Kirk (2001) concluded that the flow remains Poynting flux dominated until it reaches the termination shock.
The failure of damping mechanisms to dissipate the wave before it reaches the location of the termination shock has motivated a more detailed study of the interaction between current sheets and a relativistic shock (Lyubarsky 2003; Péri & Lyubarsky 2007; Nagata et al. 2008; Sironi & Spitkovsky 2011). Lyubarsky (2003) suggested that the dissipation of the Poynting flux may occur due to driven magnetic reconnection triggered at current sheets encountered by the termination shock. By conducting two- and three-dimensional PIC simulations, Sironi & Spitkovsky (2011) have recently confirmed that magnetic reconnection is indeed driven by the interaction with the shock, and that the resulting distribution in energy of the accelerated particles agrees with that observed in PWNe. However, the pair production rate assumed in these simulations is much larger than suggested previously (Arons 2012), and, according to the estimates of Kirk & Skjæraasen (2003), would result in substantial damping of the striped wind before it encounters the shock.

The above picture is based on an MHD description; although non-ideal MHD effects cause the dissipation, the structure (or wave) itself can be described in the language of MHD with appropriate transport coefficients. Since the frozen-in condition requires that the electric field be smaller than the magnetic field $E < B$, the phase velocities of MHD waves are always subluminal ($E/B$ gives a phase velocity). But when one moves beyond the MHD model, additional degrees of freedom open up that allow a wave with $E > B$ to propagate. Although the wave amplitude under pulsar conditions is strongly nonlinear, such a wave is essentially an electromagnetic wave in vacuum modified by the presence of a plasma (e.g., Akhiezer & Polovin 1956; Max & Perkins 1971), and has long been studied in the context of pulsar physics (e.g., Kennel & Pellat 1976; Melatos & Melrose 1996; Kirk 2010; Arka & Kirk 2012). We will call this electromagnetic wave superluminal because its phase speed distinguishes it from subluminal MHD waves (which also involve electromagnetic fluctuations). An obvious advantage of superluminal modes is their expected large damping rates through instabilities (e.g., Max 1973a; Asseo et al. 1978; Lee & Lerche 1978; Sweeney & Stewart 1975), which suggests that they might efficiently dissipate either before or at the termination shock (Skjæraasen et al. 2005).

There exists a cutoff frequency below which a superluminal wave cannot propagate due to screening of the wave electric field by the plasma particles, suggesting that these modes can play a role only at a sufficiently large distance from a pulsar. More specifically, nonlinear solutions for superluminal waves have been found both for circularly and linearly polarized modes, and have been used to construct and solve the jump condition between a subluminal (or striped) mode and superluminal modes that carry the same particle, momentum, and energy fluxes (Kirk 2010; Arka & Kirk 2012). These studies formulate necessary conditions for the conversion to a superluminal wave to occur, and found it to be possible beyond a critical radius that is located well inside the termination shock in the equatorial region for an isolated pulsar. However, they did not address the question of how and where the conversion actually takes place. This could occur spontaneously when the magnetic reconnection proceeds too slowly to maintain the pressure balance required by the radial evolution of the wind. Alternatively, it may be triggered by the interaction with the termination shock. It is this latter possibility that we study here.

Using a newly developed, one-dimensional, relativistic, two-fluid simulation code for pair plasmas, we investigate the roles played by superluminal waves in a relativistic shock. A circularly polarized sinusoidal magnetic shear wave is adopted to mimic a pulsar-driven wave, and is forced to interact with a termination shock. We demonstrate that superluminal waves are indeed generated, strongly modify, and, eventually, dominate the shock structure. A well-developed precursor region is formed ahead of a subshock, in which the flow is decelerated and substantial plasma heating occurs due to efficient dissipation of the Poynting flux. We argue that the basic process that leads to the dissipation and modification of the shock is a parametric instability: the scattering of superluminal waves off a longitudinal density perturbation causes the energy and momentum of the wave to be transferred to the longitudinal mode. When these perturbations grow to large amplitudes, they steepen to form small-scale shocks that eventually cause the required dissipation.

In Section 2, the basic equations and the simulation setup used in this study are presented. The results are discussed in Section 3, in which a detailed analysis of the shock structure, as modified by superluminal waves, is given. Section 4 contains a discussion of the astrophysical application and points out some remaining open issues, and Section 5 presents a summary.

2. MODEL

2.1. Basic Equations

The simplest possible model that is capable of describing the superluminal electromagnetic waves on which we focus in our study is that of two relativistic fluids (electrons and positrons), supplemented by Maxwell’s equations. Unlike most previous studies, which considered cold fluids, we include a finite pressure for each species that is determined by a polytropic equation of state. This leads to the following set of equations:

$$\frac{\partial}{\partial t} (\gamma_s n_s) + \nabla \cdot (n_s u_s) = 0, \quad (1)$$

$$\frac{\partial}{\partial t} \left( \frac{w_s}{c^2} \gamma_s u_s \right) + \nabla \cdot \left( \frac{w_s}{c^2} u_s u_s + I_p \right) = q_s \gamma_s n_s \left( \frac{E}{\gamma_s c^2} \times B \right), \quad (2)$$

$$\frac{\partial}{\partial t} \left( w_s \gamma_s^2 - p_s \right) + \nabla \cdot (w_s \gamma_s u_s) = q_s n_s u_s \cdot E, \quad (3)$$

$$\frac{1}{c} \frac{\partial}{\partial t} E = \nabla \times B + \frac{4\pi}{c} J, \quad (4)$$

$$\frac{1}{c} \frac{\partial}{\partial t} B = -\nabla \times E, \quad (5)$$

$$\nabla \cdot E = 4\pi \rho, \quad (6)$$

$$\nabla \cdot B = 0, \quad (7)$$

where $n_s$, $(\gamma_s, c, u_s)$, and $p_s$ are the proper number density, four-velocity, and proper pressure of particle species $s$, respectively. The enthalpy density $w_s = n_s m_s c^2 + \Gamma/(\Gamma - 1) p_s$ is written in terms of the ratio of specific heat $\Gamma (\Gamma = 4/3$ is used throughout...
in this study). The charge density $\rho = \gamma_+ q_+ n_p + \gamma_- q_- n_e$ and current density $\mathbf{J} = q_+ n_p \mathbf{u}_p + q_- n_e \mathbf{u}_e$ introduce coupling between the fluids and electromagnetic fields. Note that the subscripts ($p$ for positrons, $e$ for electrons) for particle species are omitted for brevity whenever no confusion arises. Other notations are standard: the speed of light $c$, electron mass $m_e$, elementary charge $e$ (thus $q_+ = -q_- = e$). In the current one-dimensional simulation, Equations (6) and (7) are automatically satisfied throughout the simulation, provided they are satisfied by the initial and boundary conditions. Note that we solve the fluid equations without any assumption on symmetry or antisymmetry so that Langmuir waves arising from oscillations in charge density are included in the model. However, the initial conditions are such that the longitudinal (transverse) components of the fluid velocities are symmetric (antisymmetric) and there are no charge-density perturbations. There is, therefore, no Langmuir wave activity initially. In fact, Langmuir waves remain unimportant throughout the simulations, for reasons we discuss in Section 4. This allows us to concentrate on the dynamics of the positron fluid in our discussion.

One effect introduced by the finite temperature, $T = p/n$, is the effective increase of particle inertia by a factor of $h = w/(nmc^2) = 1 + \Gamma/(\Gamma - 1)\Gamma/mc^2$. This has an important consequence when considering superluminal waves. In Appendix A, we present an exact solution of a circularly polarized superluminal wave whose cutoff frequency is determined by an effective proper plasma frequency

$$\omega_p \equiv \sqrt{\frac{8\pi n e^2}{mh}}, \quad (8)$$

with $n = n_p = n_e$. Thus, the cutoff frequency of a superluminal wave is reduced because of the increased effective inertia in a relativistically hot plasma $T \gg mc^2$.

In our numerical implementation, a different, but mathematically equivalent, set of equations is used, which enables us to solve the time evolution without violating important conservation laws. A detailed description is presented in Appendix B.

2.2. Simulation Setup

We are interested in quasi-steady-state solutions in which a relativistic, strongly magnetized flow dissipates its Poynting flux. Therefore, we attempt to initialize the system so that it will quickly reach such a solution, if it exists. We initially divide the one-dimensional simulation box into two regions. The left-hand (upstream) side contains a cold, highly magnetized plasma flowing to the right with supersonic speed, and the right-hand (downstream) side contains an unmagnetized, relativistically hot plasma.

The upstream plasma carries only an oscillating component of the magnetic field, i.e., its phase-independent component of the magnetic field tend to zero (in his notation $\eta \to \infty$); its radial evolution has been re-examined recently in the context of blazar jets (Kirk & Mochol 2011a, 2011b). In the context of laboratory plasma physics, this mode is known as a “sheet-pincho” (Li et al. 2003) configuration. We employ this substantial simplification because we believe that the most important parameter in the problem is the ratio of the wave frequency $\omega$ to the proper plasma frequency $\omega_p$, a view that is supported by the PIC simulations of Sironi & Spitkovsky (2011). This implies that the details of the wave mode, such as polarization and functional form, will not have an important qualitative influence on the results. One advantage of using circular polarization is that it greatly simplifies the theoretical analysis.

In the following, the upstream proper positron density, bulk flow Lorentz factor in the simulation frame, and proper temperature are denoted, respectively, by $n_0$, $\gamma_0$, and $T_0$. The upstream magnetization parameter is defined as $\sigma_0 = B_0^2/(4\pi n_0 m_e c^2)$, where $B_0$ is the magnetic field strength in the upstream.

To measure the dissipation in simulation results, we use a more accurate and general definition of the magnetization given by $\sigma = (\mathbf{E} \times \mathbf{B})_t/(4\pi \sum \gamma_i w_i u_i_{x,t})$, which includes the finite temperature correction and does not assume any relationship between $\mathbf{E}$ and $\mathbf{B}$.

The functional form of the upstream magnetic field in the simulation frame can be given as

$$B_y = +B_0 \sin (k_0 x - \Omega t), \quad (9)$$

$$B_z = -B_0 \sin (k_0 x - \Omega t), \quad (10)$$

where $\Omega > 0$ is the frequency and the wavenumber is $k_0 = \Omega/V_0 > 0$, with $V_0 = \sqrt{1 - \gamma_0^2}$ being the upstream three-velocity. The electric field is written according to the frozen-in condition $\mathbf{E} = -V_0 \mathbf{e}_x \times \mathbf{B}/c$ ($\mathbf{e}_x$ is the unit vector in the $x$-direction). The transverse four-velocity is determined by Ampere’s law to be

$$u_y = +u_{\perp,0} \cos (k_0 x - \Omega t), \quad (11)$$

$$u_z = -u_{\perp,0} \sin (k_0 x - \Omega t), \quad (12)$$

for positrons with $u_{\perp,0} = k_0 B_0/8\pi n_0 e$. The density, temperature, and the bulk flow velocity are not affected by the presence of this wave. This magnetic shear structure is an analytic equilibrium solution to the two-fluid equations, which is simply convected by the flow in the far upstream.

For the sake of convenience, we define our terminology for the polarization and helicity of transverse waves, which basically follows standard plasma physics conventions. Note, however, that the standard definition cannot be used in a strict sense because it defines the sense of rotation with respect to the static magnetic field, which is absent in our case. Instead, we simply define the sign with respect to the $x$-axis. The helicity describes the sense of rotation of a fluctuating vector in the $y$-$z$ plane, as $x$ increases at a fixed time; when viewed along the positive $x$-direction, it is counterclockwise for positive and clockwise for negative helicity. Similarly, right-hand ($R$) and left-hand ($L$) polarizations indicate that the rotation of a vector in time measured at fixed $x$ is clockwise and counterclockwise, respectively. With this definition, the injected magnetic shear
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wave is right-hand polarized \((R)\) and has positive helicity \((+),\) which we denote by \(R^+\). Note that the sign of the superscript does not indicate the wave propagation direction: \(R^+\) and \(L^-\) waves propagate in the positive, while \(R^-\) and \(L^+\) waves propagate in the negative \(x\)-direction. Defining a complex quantity for a vector field, e.g., \(B_x - i B_y\), it is easy to confirm that the first, second, third, and fourth quadrants of its \(k=\omega\) Fourier space correspond to \(R^+\), \(R^-\), \(L^+\), \(L^-\), respectively. Consequently, a given vector quantity (say \(E_{\nu,k}\)) can be decomposed into positive and negative helicity components by Fourier transforming the complex quantity \(E_k - i E_k\) into \(k\)-space, extracting only the \(k > 0 \ (k < 0)\) Fourier components, and inverse transforming back into configuration space to find the positive (negative) helicity component (Terasawa et al. 1986). This decomposition is used below in the analysis of the simulation results.

The initial conditions in the downstream (right-hand) side are found from the Rankine–Hugoniot relations under the assumption that the Poynting flux has completely dissipated. The conservation laws in this case give

\[
2n_1 u_{x,1} = 2n_2 u_{x,2} \tag{13}
\]

\[
2w_1 \frac{u_{x,1}^2}{c^2} + 2p_1 + \left(1 + \frac{u_{x,1}^2}{\gamma_1^2 c^2}\right) \frac{B_1^2}{8\pi} = 2w_2 \frac{u_{x,2}^2}{c^2} + 2p_2 \tag{14}
\]

\[
2w_1 \gamma_1 u_{x,1} + \frac{u_{x,1}}{\gamma_1} \frac{B_1^2}{4\pi} = 2w_2 \gamma_2 u_{x,2} \tag{15}
\]

where subscripts 1 and 2 denote quantities in the upstream and downstream regions. We retain a (small) contribution from the shear wave to the upstream Lorentz factor, so that \(\gamma_1^2 = \gamma_0^2 (1 + u_{x,0}^2)\) and \(u_{x,1} = \sqrt{1 + u_{x,0}^2} \gamma_0 V_0\). The other upstream quantities \((w_1, p_1,\text{ and } B_1)\) are the same as those with subscript 0. For a strong shock with an upstream Lorentz factor \(\gamma_0 \gg 1\) in a cold plasma, the downstream flow speed \(u_2\) and temperature \(T_2\) may be approximated as

\[
u_2 \approx \frac{\Gamma - 1}{\sqrt{\Gamma(2 - \Gamma)}} (16)
\]

\[
T_2 \approx (\Gamma - 1) \sqrt{\frac{2 - \Gamma}{\Gamma}} \gamma_0 (1 + \sigma_0) \tag{17}
\]

in the shock rest frame. The proper density is determined by the mass flux conservation. The formal difference from a strong shock in an unmagnetized plasma is the factor \(1 + \sigma_0\) in the downstream temperature. This additional temperature increase comes from the dissipation of the Poynting flux. We set up the initial downstream state by numerically solving Equations (13)–(15) without additional approximation.

In between the upstream and downstream regions, there exists, at the initial instant, a buffer region in which the upstream magnetic field smoothly decreases to zero. Since the physical quantities in the upstream and downstream regions are initialized to be in pressure balance, the resulting “shock” structure would remain stationary in the simulation frame if such a quasi-steady solution really exists.

The upstream wave is continuously injected from the upstream boundary during the whole run. We choose the location of the upstream boundary so that any perturbations generated by the interaction (propagating with the speed of light) do not reach the boundary. On the other hand, electromagnetic waves may reach the downstream boundary. We assume \(\partial / \partial x = 0\) at this boundary. Thus, in principle, there would be small but finite reflection of waves. As we will see below, however, the Poynting flux of electromagnetic waves reaching the boundary is found to be very small. Therefore, we believe that the effect of boundary conditions does not affect our discussion.

The following normalizations are used throughout the discussion: \(\omega_{p0}^{-1}\) for time, \(c / \omega_{p0}\) for space, \(c / \nu_0\) for velocity, \(n_0\) for density, \(B_0\) for electric and magnetic fields, and \(\mu_0 = \gamma_0^2 (1 + \sigma_0) n_0 mc^3\) for energy flux density, respectively. Here, \(\omega_{p0} = \sqrt{8\pi n_0 e^2 / m}\) is the proper plasma frequency in the upstream but without the relativistic temperature correction. We use a constant upstream temperature \(T_0 / mc^2 = 0.01\), so that the correction is not important in the upstream. The grid size and time step are chosen to be \(\Delta x = 0.05 c / \omega_{p0}\) and \(\Delta t = 0.005 / \omega_{p0}\), and are kept constant.

3. SIMULATION RESULTS

3.1. Overview

First of all, we contrast two simulation results, one with a high and the other with a low frequency \(\Omega\), in Figure 1. We call the high-frequency simulation with \(\Omega / \omega_{p0} = 1.2\) “run A”
and the low-frequency simulation with $\Omega/\omega_p = 0.4$ “run B.” In each case, the upstream Lorentz factor is $\gamma_0 = 40$ and the magnetization is $\sigma_0 = 10$. One immediately sees that the shock structures are completely different.

Run A exhibits fluctuations of substantial amplitude in all the fluid quantities, whereas the structure of run B is uniform except for a discontinuity, very much like a solution typically found in standard fluid simulations. Although a discontinuity is also present, run A shows a precursor region characterized by a substantial deceleration of the flow and an associated increase in density and temperature. The proper density of the plasma in the precursor is compressed by a factor of up to $\sim 10$. The corresponding increase in temperature is actually greater than that expected from adiabatic heating alone. This additional heating comes from the dissipation of the Poynting flux, which already starts at the leading edge of the precursor. Below, we attribute such a shock structure to the effects of superluminal electromagnetic waves. We call this novel structure an electromagnetically modified shock, in close analogy with the so-called cosmic-ray modified shock (e.g., Drury & Völk 1981). Following the convention established in that field, the shock-like discontinuity will be called a subshock.

Note, however, that a substantial amount of Poynting flux dissipation is evident not only in run A but also in run B (the bottom panel of Figure 1). Clearly, the shock-like discontinuity observed in run B is not just an ordinary MHD shock, at which the dissipation of the Poynting flux would be strongly suppressed. Figure 2 shows a close-up view of the electromagnetic fields around the shock-like discontinuity in run B. In the bottom panel, non-MHD electric fields defined as $\tilde{E}_y = E_y - u_x B_z/\gamma c$, $\tilde{E}_z = E_z + u_x B_y/\gamma c$ are shown. Both upstream and downstream of the discontinuity, $\tilde{E}_y$ is essentially zero, indicating that MHD is a fairly good approximation in these regions. Around the discontinuity, however, non-MHD electric fields have large amplitudes comparable to MHD-type electric fields. These non-MHD electric fields provide a sort of anomalous resistivity, as is apparent from the generalized Ohm’s law (see Appendix B),

$$E + \frac{V}{c} \times B = \sum_s \frac{1}{q_s N} \left( \frac{\partial}{\partial t} \left( \frac{w_s}{c^2} \gamma_s u_s \right) \right) + \nabla \cdot \left( \frac{w_s}{c^2} u_s u_s + p_s I \right),$$  

where $N = \sum_s \gamma_s n_s$, $V = \sum_s n_s u_s/N$. This expression is a generalization to warm fluids of the Ohm’s law found by Melatos & Melrose 1996.) A non-zero left-hand side of Equation (18) introduces two-fluid effects that cause dissipation of the electromagnetic energy and heating of the plasma.

On the downstream side of the shock-like discontinuity, an interface is formed that separates the magnetized and unmagnetized regions (not visible in Figure 1, since it has already left the region shown, although still contained within the simulation box). This interface separates the plasma which was initially upstream, from that which was initially downstream, and can be called a contact-like interface in analogy with hydrodynamics. As in hydrodynamics, this interface travels with the local fluid speed, which is non-relativistic. This severely limits the Poynting flux carried by the downstream flow, so that a quasi-steady state can be achieved only if the incoming Poynting flux is dissipated in the shock-like discontinuity. Otherwise, a magnetized MHD shock would form and move rapidly into the upstream region. Obviously, the dissipation is made possible because of the additional degree of freedom provided by the oscillation of the electromagnetic fields. However, we will not analyze this run in-depth here, but instead focus on the high-frequency regime $\Omega/\omega_p > 1$ in which formation of the novel precursor structure is observed. As we show in Section 4, this is motivated by the fact that the low-density regime exemplified by run A is more likely to correspond to a realistic pulsar wind termination shock than the high-density regime of run B.

### 3.2. Time Evolution

Figure 3 shows the time evolution for run A. Already at the very beginning of the simulation when the magnetized upstream plasma starts to interact with the downstream unmagnetized hot plasma region, the incoming Poynting flux is dissipated at the subshock. (Here, we use the term “subshock” for the discontinuity in density rather loosely, since the shock has not yet developed into a modified one.) As time progresses, the dissipation front starts to propagate upstream, while the position of the subshock stays almost constant. The advance of the dissipation front ceases at around $\Omega/\omega_p \sim 1000$, and the system relaxes to a quasi-stationary state.

Since the upstream flow is supersonic, the only way to transfer any information upstream against the flow is through superluminal waves. Such waves may easily be generated when the magnetic shear waves transmit through the subshock because the oscillation frequency measured in the shock (or downstream) frame is higher than the local proper plasma frequency. While these waves tend initially to propagate in the positive $x$-direction, this is actually prevented by the downstream plasma. As can be seen in Figure 3, there is a region behind the
subshock in which finite Poynting flux remains. Beyond that, a region occupied by an unmagnetized plasma exists (identified by essentially zero Poynting flux). These two regions in the downstream are separated by a contact-like interface as in the case of run B. This interface is characterized by a large density hump, making it overdense, and prohibiting the penetration of superluminal waves. Here, as in run B, the contact-like interface propagates with the non-relativistic local flow speed. Consequently, superluminal waves are forced to reflect and propagate in the negative $x$-direction. This is the reason why backward superluminal waves are generated, which leak out through the subshock and interact with the upstream. They affect the upstream plasma in two ways: they decelerate the flow and increase its temperature into the relativistic regime. This forms a precursor region ahead of the subshock. Reflection from the contact-like interface is clearly an artifact of the initial conditions. However, once the shock has evolved into a quasi-stationary state, the mechanism by which backward-propagating superluminal waves are generated changes into one related to a parametric instability in the downstream region, as we discuss below.

It is interesting to note that the initial development resembles the evolution of a Riemann problem in fluid dynamics, or, in fact, in any set of hyperbolic partial differential equations. In this analogy, the propagation of the dissipation front upstream corresponds to a rarefaction fan. A rarefaction fan in hydrodynamics is a smooth transition of an unperturbed fluid to a rarefied (low pressure) state. In our case, the electromagnetic energy density in the upstream region decreases across a smooth transition layer. This similarity is not a mere coincidence. The carrier of information in our case is a superluminal wave, whose group velocity exceeds the flow speed. The upstream flow is thus “subsonic” with respect to this wave, resulting in the formation of a rarefaction-like smooth transition. We conjecture that this transient phase is eventually terminated when the system relaxes into a state in which the smooth transition of the precursor, followed by the subshock, satisfies the boundary condition imposed by the downstream state.

### 3.3. Modified Shock Structure

An enlarged view of the precursor region of run A is shown in Figure 4, at $\omega_{pe} t = 1800$, after the system has reached a quasi-steady state. The leading edge is located at around $x \sim 1850 c/\omega_{pe}$, beyond which the dissipation of the Poynting flux starts. Although the bulk flow decelerates substantially in the precursor, its speed remains relativistic. The subshock can be clearly identified as a discontinuous jump in the flow speed. The Poynting flux gradually decreases in the precursor and then starts to fluctuate with large amplitude around the subshock $x \sim 2100 c/\omega_{pe}$.

There is no obvious jump in the Poynting flux across the subshock, which satisfies the Rankine–Hugoniot relations for a relativistic shock in an unmagnetized plasma; i.e., the electromagnetic fields remain unchanged across it. This is a clear demonstration of non-MHD behavior, since it implies $E \neq -V/c \times B$. Thus, the plasma in the precursor is no longer magnetized, and the waves are essentially superluminal in nature. This is confirmed in the second panel from the bottom of Figure 4, which clearly shows $E/B > c$ in the precursor.

To reveal the nature of waves in more detail, in Figure 4, we decompose the transverse electromagnetic fields into positive and negative helicities (as defined in Section 2). Although MHD- and non-MHD-type electric fields of the same helicity are not
clearly separated by this procedure (unless the magnitude of $E$ and $B$ are appreciably different), the generation of waves with a helicity opposite to that of the injected wave may easily be identified. Hereafter, positive and negative helicity field components are, respectively, denoted by $+$ and $-$ superscripts. Recall that the injected magnetic shear wave has a positive helicity ($R^+$) in the simulation.

The second and third panels from the top in Figure 4 show the negative and positive components of $E_y$ and $B_z$, respectively. The amplitude of the injected positive helicity wave is modulated in the precursor, but shows an overall decreasing trend. Coherent negative helicity waves are also seen in the precursor, although they are essentially absent in the upstream. The anti-correlation in phase of $E_y^-$ and $B_z^-$ implies that this component carries a negative Poynting flux, and that the waves are propagating against the flow. This can also be seen in the top panel, which shows a $t$–$x$ diagram of the negative helicity component $E_y^-$. These waves, however, disappear when they approach the leading edge of the precursor. This can be understood from the spatial profile of the proper plasma frequency $\omega_p$ shown in the bottom panel. Because the plasma in the precursor is heated to relativistic temperatures, $\omega_p$ decreases despite the fact that plasma is compressed in the decelerating flow. This effect can occur only in a relativistically hot plasma, where, because the particle rest-mass is negligible, one has $w \propto p$ and therefore $\omega_p \propto T^{-1/2}$. Since $\omega_p$ coincides with the cutoff frequency of a circularly polarized superluminal wave, lower frequency waves are permitted in the regions of relativistically hot plasma, but are excluded from the cooler upstream plasma. As we show in the next section, the negative helicity waves have a frequency lower than $\omega\rho_0$, the cutoff frequency in the far upstream region. (This might also be expected from the longer wavelengths that are apparent in the second panel from the top of Figure 4.) These waves are therefore unable to propagate into the far upstream. It is difficult to decide whether they are reflected or absorbed when they encountered the overdense region. Nevertheless, it is clear that these waves are the only agents that can carry information from the downstream to the precursor, and trigger the mode conversion from an entropy mode to a superluminal wave. We conclude that they are essential for the formation of the modified shock.

3.4. Wave Spectra

The wave properties may be further clarified by looking at their Fourier space representations. Figure 5 shows the power spectral density of (in the left panel) $E_y^+ - iE_z^+$, (center) $B_y^+ - iB_z^+$, and (right) $n$ in $k$–$\omega$ space. The spectra are calculated for the time interval $1544 \leq \omega\rho_0 t \leq 1800$ using the Blackman window to remove edge effects and improve the dynamic
Figure 5. Power spectra of (left) $E_y - i E_z$, (center) $B_y - i B_z$, (right) $n$ in $k$-$\omega$ space obtained from run A. The spectra are calculated for the time interval $1544 \leq \omega_p t \leq 1800$ and spatial intervals corresponding to (a) the precursor edge $1850 < x/c/\omega_p < 1952$, (b) deep in the precursor $2000 < x/c/\omega_p < 2102$, and (c) downstream $2150 < x/c/\omega_p < 2252$.

(A color version of this figure is available in the online journal.)
range. As mentioned earlier, we do not observe Langmuir-like waves (or charge-density fluctuations) in the simulation. The density fluctuation is thus solely attributable to sound-like waves (positron and electron densities oscillating in phase). In each panel, the driving frequency \( \Omega/\omega_p = 1.2 \), and a theoretical dispersion relation is shown as dashed lines. In the left and center panels, the dispersion relation is that of a circularly polarized superluminal mode \((A7)\), with the value of the proper plasma frequency computed as an average over the appropriate space and time interval. In the right panel, the dispersion relation is that of sound waves moving along \( x \) with speed \( \pm c_s = \pm \sqrt{\Gamma p/\rho} \) in a frame moving at speed \( u_s/\gamma \):

\[
\omega = k^2 \frac{u_s \pm c_s \gamma}{\gamma \pm c_s u_s},
\]

(19)

with the fluid quantities again computed as averages over the appropriate domain. Note that the forward- and backward-propagating (with respect to the comoving frame) sound waves are almost identical when the flow speed is relativistic, whereas the dispersion relation of the superluminal mode is independent of the plasma streaming speed, and depends only on the local proper density and temperature (see Appendix A). Thus, in the upper right panel (the density panel of row (a)), the two sound waves are almost indistinguishable. In this panel, we plot an additional (dash-dotted) curve, which we discuss below.

At the leading edge of the precursor (panels (a), 1850 < \( x/c/\omega_p < 1952 \)) where the incoming Poynting flux starts to dissipate, a peak of the field intensities in the first quadrant \( k > 0, \omega > 0 \) (corresponding to \( R^+ \)) at the driving frequency \( \Omega/\omega_p = 1.2 \) is clearly seen. One also finds small-amplitude waves around the peak (i.e., the excitation of sidebands) as well as on the \( R \)-mode \((\omega > 0)\) theoretical dispersion branch. The density spectrum looks rather structureless. Nevertheless, it seems to consist of two parts; one on \( \omega = kc \) and another on the dash-dotted curve shown in the right panel of Figure 5(a).

Deep within the precursor (panels (b), 2000 < \( x/c/\omega_p < 2102 \)), where we observe backward-propagating negative helicity waves, the power distribution is very different. The peak at the driving frequency is not so clear, though the power is still strongest at this point. Instead, the power is distributed rather broadly. In addition, there is now substantial power in negative helicity backward-propagating waves (second quadrant, \( k < 0, \omega > 0 \) corresponding to \( R^- \)) as also seen in Figure 4. The power of these waves is concentrated in the range \( \omega/\omega_p \approx 0.4-0.8 \) (i.e., below the cutoff frequency in the far upstream), consistent with their disappearance during propagation in the precursor. The power of the density spectrum in this region is almost concentrated on \( \omega = k c \).

In the downstream region (panels (c), 2150 < \( x/c/\omega_p < 2252 \)), the power distribution is also broad and lower frequency waves have relatively large amplitudes, whereas the peak structure at the driving frequency persists. Since low-frequency waves with negative helicity (second quadrant, \( R^- \)) have upstream-directed group velocities, they can propagate to the subshock and leak out into the precursor where the cutoff frequency is even lower. The top panel of Figure 4 supports this interpretation. One can see there that the backward-propagating waves are continuously propagating from the downstream to the precursor with refraction at the subshock, consistent with the idea of leakage. We think this is the reason why there are backward-propagating superluminal waves with lower frequencies in the precursor. In the density spectrum, the forward- and backward-propagating sound waves are now clearly separated because the flow speed is non-relativistic. However, the forward branch carries much more power than the backward one.

The evolution of the power spectrum described above may be explained qualitatively in terms of parametric instabilities or nonlinear wave–wave interactions. Figure 6 depicts possible nonlinear couplings of a pump superluminal wave with different wave modes that are likely to be taking place (a) in the precursor and (b) downstream. Note that the figure shows the coupling between waves as measured in the simulation frame. It is known that nonlinear interactions among waves become strongest when the matching condition for both frequency and wavenumber is satisfied:

\[
\omega_\pm = \omega_0 \pm k \quad \text{(20)}
\]

\[
k_\pm = k_0 \pm \omega \quad \text{(21)}
\]

Here, \((\omega_0, k_0)\), \((\omega, k)\), and \((\omega_\pm, k_\pm)\) describe, respectively, the pump wave, a sound-like daughter wave, and electromagnetic-like daughter waves. Here, we consider only a sound-like wave as a longitudinal perturbation, so the process may be called stimulated Brillouin scattering. It is generally expected that the growth rate of an instability is largest when the generated daughter waves lie close to the normal modes of the unperturbed plasma. Although, in principle, coupling to a daughter wave far from the normal modes may occur (such as in the case of \((\omega_+, k_+)\) in panel (b) of Figure 6), the power is usually expected to be much smaller.

In the density spectrum measured in the precursor, sound mode waves almost coincide with \( \omega = kc \). Thus, for a superluminal pump wave with \( \omega/\omega_p \gg 1 \) in the laboratory frame, all relevant waves (i.e., electromagnetic and sound waves) are almost aligned with the straight line \( \omega = kc \) in \( k-\omega \) space.
In this case, there are many possible wave–wave couplings that satisfy the matching condition, as can be envisaged from Figure 6(a). Note that once sideband waves are generated, they may subsequently decay via exactly the same process, i.e., the instability occurs recursively, generating many sidebands, and eventually leading to a turbulent spectrum. The generation of sideband modes results in amplitude modulation (i.e., a beat wave) in real space. This characteristic feature is clearly seen in both Figures 4 and 5. In Figure 4 (third panel from the top), the positive helicity component in the precursor exhibits amplitude modulation. The sideband modes causing this modulation are seen in the power spectrum measured at the same region in Figure 5(b).

Note that sideband generation, albeit of small amplitude, has already started in Figure 5(a), supporting the idea that the incoming wave has already been converted into a superluminal wave at the leading edge of the precursor. The power in density perturbations in this region is concentrated at low frequencies, consistent with the above picture (see Figure 6(a)). However, substantial power can also be seen lying roughly on the dash-dotted curve in the density spectrum in Figure 5(a). This may be explained as follows: if one considers not only a superluminal mode wave but also the incoming shear wave as the pump waves (i.e., finite amplitude), then the coupling of these two waves may produce density fluctuations. Since superluminal waves of finite amplitude are found on the theoretical $R$-mode dispersion branch, the matching condition between the superluminal waves and the incoming shear wave implies that the frequency and wavenumber ($\omega$, $k$) of the resulting density fluctuation must obey

$$\omega = \Omega - \sqrt{\omega_p^2 + (\Omega_0 - kV_0)^2c^2/V_0^2},$$

which is plotted as the dot-dashed curve in Figure 5. Such a dispersion relation is far from normal modes of the dispersion relation of the unperturbed plasma. It can exist only when the wave is forced to oscillate because of the coupling between multiple finite amplitude waves. Thus, the presence of enhanced fluctuations around this curve implies that coupling between the injected magnetic shear wave and superluminal modes is indeed taking place, and could be the mechanism responsible for mode conversion. This interpretation is supported by the absence of such density fluctuations deep inside the precursor (b), confirming that the shear wave has been fully converted into superluminal modes at this point.

In the downstream region, the forward and backward sound waves are clearly separated (see the density power spectrum in Figure 5(c)), and it is more natural to invoke the coupling depicted in Figure 6(b) producing a backward-propagating electromagnetic-like wave. The backward-propagating low-frequency superluminal waves thus may be interpreted as a result of the back scattering of the pump wave off a sound-like mode wave, i.e., backward Brillouin scattering. This interpretation is consistent with the enhanced forward-propagating sound-like wave activity seen in Figure 5(c). These waves are seen to steepen, producing many small-scale shocks propagating in the forward direction, which ultimately dissipate energy due to the numerical viscosity inherent in fluid simulations. As mentioned above, the backward-propagating superluminal waves propagate into the precursor and trigger the conversion of the incoming wave mode at its leading edge, thus causing the modification of the shock structure.

\[3.5.\text{Dissipation Efficiency}\]

Although we have discussed some elementary processes leading to the dissipation of electromagnetic energy, it is difficult to estimate the actual dissipation efficiency and its parameter dependence from an analytical treatment alone, because this involves complicated nonlinear processes. Other simulations we have performed with different $\gamma_0$ and $\sigma_0$ for a fixed $\Omega/\omega_0$ show that the basic processes seem to be essentially the same, and the shock structure is always strongly modified. Figure 7, for example, shows the spatial profiles of flow velocity $u_x$ and $\sigma$ for three runs with different initial Lorentz factors: $\gamma_0 = 20$ (blue), $\gamma_0 = 40$ (green), and $\gamma_0 = 60$ (red). In each case, $\sigma_0 = 10$ and $\Omega/\omega_0 = 1.2$ are kept constant. In these runs (and also in others we have performed but do not show here), $\sigma$ is reduced to below unity in a precursor region and appears unaffected by the subshock transition. In the downstream region, it typically goes down to $\sim 0.1$, although it exhibits strong fluctuations. However, the incoming wave is converted into superluminal waves, implying that the remaining Poynting flux is entirely carried by these waves, and the downstream plasma is essentially unmagnetized. Therefore, the conventional MHD picture does not apply, even though a finite electromagnetic field remains. Furthermore, superluminal waves in the downstream continue to dissipate due to instabilities. Thus, the $\sigma$ values obtained in simulations may give only an upper limit.

\[4.\text{DISCUSSION}\]

Dissipation in Poynting-flux-dominated relativistic flows is an important issue in high-energy astrophysics. Although the possibility that superluminal waves may play a role has been discussed for decades (e.g., Kennel & Pellat 1976; Asseo et al.
1978; Kundt & Krottscheck 1980; Melatos & Melrose 1996), it has not been known how such waves can be generated in a self-consistent manner. In this paper, we have analyzed in-depth a specific example that shows how strong superluminal waves can be generated by a relativistic shock when it interacts with an inhomogeneous upstream plasma. We selected a situation where the inhomogeneity takes the form of a periodic magnetic shear embedded in a cold plasma. The frequency $\Omega$ of the shear wave measured in the shock frame was chosen to be greater than the proper plasma frequency $\omega_p$ in the upstream. The superluminal wave was observed to be generated as a result of mode conversion from the injected shear wave.

Once the conversion has taken place, superluminal waves may become unstable to various types of instabilities (e.g., Max 1973a; Lee & Lerche 1978; Asseo et al. 1978). In the present paper, we have shown that the decay process proceeds via stimulated Brillouin scattering. Sound-like waves grow to large amplitude via this instability. They then steepen to form small-scale shocks that cause strong heating of the plasma. The heating results in a pressure gradient that decelerates the flow, forming a precursor ahead of the subshock. This process requires a trigger because otherwise the upstream medium is uniform. We suggest that this role is played by backward-propagating superluminal waves. Although their amplitude in the laboratory frame is relatively small compared to the incoming wave, it is easily shown that the situation is the opposite when viewed in the rest frame of the incoming plasma. Therefore, we think they are responsible for the instability that triggers the formation of the precursor. Although this is the principal finding of this paper, our understanding of the formation mechanism remains incomplete. Important questions, such as the precursor scale length, the compression ratio, and the stability of the shock, remain open. Our simulations also leave open the question of “spontaneous” conversion (i.e., well upstream of the termination shock) of the subluminal mode into a superluminal mode.

It is well known that a strong electromagnetic wave can also suffer Raman scattering off a Langmuir-like wave, as well as Brillouin scattering off a sound-like wave. However, our simulations show no evidence of charge-density fluctuations. We attribute this to the symmetry between electron and positron motions in the presence of transverse waves. It is easy to check that the transverse velocities of two fluids have the same magnitude (and opposite directions) for both the magnetic shear and superluminal mode. A first-order longitudinal velocity perturbation is affected by the presence of a pump transverse wave through the Lorentz force $\mathbf{q}\times\mathbf{B}/\gamma e$, so that the terms are exactly the same for the electrons and positrons, both in magnitude and direction, when the above-mentioned symmetry holds. This leads to sound-like perturbations rather than charge-density perturbations. From this consideration, it appears natural that we do not observe the growth of Langmuir waves. This symmetry will be broken in the presence of a finite longitudinal magnetic field component. However, a detailed analysis of possible parametric instabilities is left for future work.

In the present paper, we analyzed the case $\Omega/\omega_{pe} = 1.2$ and $\sigma_0 = 10$, where superluminal modes of frequency $\Omega$ could, in principle, propagate in both the upstream and downstream regions, if these regions were free of other electromagnetic fields. We found that a hot precursor region is formed in which the plasma frequency is reduced, and which contains waves whose frequency lies below the cutoff in the upstream. This did not happen in the comparison case (run B) in which $\Omega/\omega_{pe} = 0.4$ and $\sigma_0 = 10$, because the downstream plasma frequency is higher than the wave frequency. Although the Poynting flux was dissipated in the structure we found in run B, there was no evidence of superluminal waves, nor of shock modification. In the absence of a survey of parameter space, we cannot reach a definite conclusion on the range of frequencies and magnetizations that will produce an electromagnetically modified shock structure. However, our interpretation suggests that the ability of the downstream shocked plasma to support superluminal waves is crucial, and that their propagation in the unperturbed upstream medium (which in any case contains other wave fields of large amplitude) is not essential.

Our approach differs in concept from that of Skjærbaasen et al. (2005), who performed PIC simulations of a shock front interacting with an injected superluminal wave of prescribed frequency and amplitude. Nevertheless, the results are superficially similar, in the sense that large-amplitude, forward-propagating, superluminal waves exist upstream of the shock and were found to decrease in amplitude as they approach it. The role of backward-propagating modes, and that of density fluctuations and stimulated Brillouin scattering are not evident in their results. This may be because of the different driving mechanisms studied, or because the additional effects captured in a 2.5-dimensional PIC simulation intervene to reduce their importance. On the other hand, it may simply be due to the difficulty of resolving these features in an approach that is intrinsically more computationally intensive. Further work is clearly needed to resolve these issues.

In the case of a pulsar wind with a termination shock whose position is approximately stationary with respect to the central object, it is fairly easy to estimate the condition for the possible existence of superluminal waves. Assuming a radial wind in which the density falls off as $1/r^2$ ($r$ is the distance from the central object), the condition for superluminal waves to propagate in the shocked wind, $\Omega\sqrt{\sigma}/\omega_p > 1$, may be written as (Arka & Kirk 2012)

$$\frac{r}{r_L} > 2.8 \times 10^6 \left(\frac{L}{10^{38} \text{ erg s}^{-1}}\right)^{-1/2} \left(\frac{N}{10^{40} \text{ s}^{-1}}\right),$$  \hspace{1cm} (23)

where $r_L$ is the radius of the light cylinder, $L$ is the wind luminosity (assuming spherical symmetry), and $N$ is the flux of electron positron pairs carried by the wind. Estimates of $N$ vary widely from pulsar to pulsar. From observations of the Crab Nebula (whose pulsar has $L = 4 \times 10^{38}$ erg s$^{-1}$), one finds $N \approx 10^{40}$ s$^{-1}$, assuming that this rate has been more or less constant over the history of the nebula. Thus, in the case of the Crab, where the termination shock identified on X-ray images (Weisskopf et al. 2000) is estimated to lie at roughly $10^8 r_L$, we expect this structure to be strongly modified by electromagnetic waves. Only in the cases where the pulsar is surrounded by a high pressure medium, such as the wind of a close companion star, can one expect the termination shock to lie sufficiently close to the pulsar to be described by an MHD model (Mochol & Kirk 2012). In the case of the double pulsar system J0737–3039, for example, the lack of orbital modulation of the observed X-ray emission (Chatterjee et al. 2007) places a constraint on the radiative efficiency of the termination shocks, which has potential implications for their structure. However, the absence of a reliable estimate of $N$ for these pulsars renders the interpretation difficult.

Particles moving in a superluminal wave emit radiation that is usually called either synchro-Compton or nonlinear inverse Compton radiation. We have ignored such radiation losses in our
of the wave frequency, we find:

$$\Omega_{\text{cool}} \approx \left( \frac{2 \mu^2 \Omega}{5mc^3} \right)^{-1} a^{-3},$$  \hspace{1cm} (24)

where the strength parameter of the wave is (e.g., Kirk & Mochol 2011a)

$$a = 3.4 \times 10^{10} \left( \frac{r_1}{r} \right) \left( \frac{L}{10^{38} \text{ erg s}^{-1}} \right)^{1/2}.$$  \hspace{1cm} (25)

Thus, for a given pulsar (fixed $\Omega$ and $L$), $\Omega_{\text{cool}} \propto r^3$. For example, in the case of the Crab, we find

$$\Omega_{\text{cool}} \approx 2.4 \times 10^{-12} (r/r_1)^3,$$  \hspace{1cm} (26)

confirming the finding by Asseo et al. (1978) that radiation damping is important for waves that propagate close to the light cylinder of this pulsar, but not at the termination shock. However, for realistic pair loss rates, superluminal waves cannot propagate close to the star. Comparing Equation (26) with Equation (23) shows that $\Omega_{\text{cool}} > 1$ everywhere in the propagation zone of a pulsar wind provided that

$$N > 4 \times 10^{36} \left( \frac{L}{10^{38} \text{ erg s}^{-1}} \right) \left( \frac{P_{\text{pulsar}}}{1 \text{ s}} \right)^{-1/3},$$  \hspace{1cm} (27)

where $P_{\text{pulsar}}$ is the pulsar period. Therefore, radiation losses can be neglected at the termination shock unless this lies close to the critical radius, and the pulsar has a rather large mass-loading parameter: $\mu = L/(Nmc^2) > 3 \times 10^7 (P_{\text{pulsar}}/1 \text{ s})^{1/3}$.

Superluminal waves may also be important in other high-energy astrophysical objects. In the present paper, we consider only a coherent upstream wave because of its relevance to pulsar winds and also for simplicity. However, any magnetic field irregularities in the upstream medium may be converted into superluminal modes by a shock when their spatial scales are small enough so that the corresponding “frequency” measured in the shock frame is greater than the cutoff frequency in the shocked plasma. For more general applicability, the effects of a phase-averaged magnetic field component and of different polarizations need to be investigated in more detail. Naively, we expect that it is only the oscillating magnetic field components that can be dissipated by this process. A finite, phase-averaged magnetic field, such as is expected in the off-equator regions of pulsar winds, will therefore remain undamped, so that the prominent jet–torus structure that is explained by MHD models would be reproduced (Komissarov & Lyubarsky 2004; Porth et al. 2013).

Our model obviously omits kinetic effects. This may at least be partially justified when the plasma is strongly magnetized, $\sigma \gg 1$, so that the dispersion effects inherent in the two-fluid equations set in earlier than kinetic effects such as that of a finite Larmor radius. Moreover, as far as superluminal waves are concerned, the two-fluid approximation is adequate because, by definition, they cannot suffer collisionless damping through resonant wave–particle interactions. On the other hand, if the superluminal waves generate longitudinal oscillations, then kinetic effects could play an essential role. In the two-fluid approximation, a linear sound wave is an eigenmode of the system. On the other hand, it is well known that an ion acoustic wave in a non-relativistic electron–ion plasma with the same temperature is heavily damped. The reason is that the phase speed of the ion acoustic wave is of the same order of the ion thermal velocity, so that thermal ions can absorb energy from a wave via Landau resonance. This damping is even more severe in a relativistic pair plasma because the phase velocity of a sound wave is limited by $c/\sqrt{3}$, whereas individual particles travel essentially with $c$. Therefore, the dissipation, in reality, may occur through immediate absorption of sound-like waves by relativistic particles, without producing small-scale shocks. Alternatively, in situations where Raman scattering dominates over Brillouin scattering, the dissipation may be caused by the nonlinear collapse of Langmuir waves.

Another important property of electromagnetically modified shocks is that the subshock behaves as if it were unmagnetized. This may be a particularly promising scenario for particle acceleration. Conventionally, acceleration at a strongly magnetized relativistic shock is considered to be much more difficult than at a non-relativistic shock because the former are generally superluminal (Begelman & Kirk 1990; Niemiec & Ostrowski 2004; Sironi & Spitkovsky 2009). In such a shock, the only way for a downstream particle to cross the shock front is by cross-field diffusion, which cannot compete with advection by a plasma moving at a significant fraction of the speed of light. In the present scenario, however, we conjecture that particles would more easily cross the subshock because they are no longer magnetized (Kato 2007; Spitkovsky 2008), and therefore do not have to rely on cross-field diffusion. In this case, the electromagnetically modified shock would be an efficient particle accelerator.

Finally, we mention the striking similarity between electromagnetically modified and cosmic-ray modified shocks. In our case, forward superluminal waves generated in association with the precursor eventually produce backscattered waves in the downstream. These return to the precursor, and then operate as a catalytic agent causing the mode conversion: this is how the quasi-stationary modified shock structure is maintained. In a cosmic-ray modified shock, cosmic rays play precisely the same role. They diffuse around the shock and thus can easily leak out from the downstream to the upstream due to their large thermal velocities. There, they excite Alfvénic waves that exert a pressure on the background plasma, leading to deceleration of the flow in the precursor. The injection of cosmic rays is thought to occur primarily at the subshock, whereas superluminal waves are created at the leading edge of the precursor in the case of an electromagnetically modified shock. It would be interesting to investigate how these two different shock modification concepts might be combined with each other, and what effect this would have on particle acceleration.

5. SUMMARY

We have investigated the interaction between a relativistic shock and a circularly polarized sinusoidal magnetic shear wave embedded in an upstream strongly magnetized plasma, mimicking a pulsar wind termination shock. We found that large-amplitude superluminal electromagnetic waves are generated through mode conversion, and subsequently strongly modify the shock structure, producing a precursor region ahead of a subshock. In association with the formation of the modified shock structure, a substantial amount of Poynting flux is dissipated. The dissipation mechanism is interpreted as a stimulated Brillouin scattering process. The superluminal waves suffer a
parametric instability and generate sound-like waves. These waves then steepen into shocks which subsequently heat the plasma. The process is considered to be an efficient mechanism for converting the dominant electromagnetic energy into particle kinetic energy. The remaining Poynting flux in the downstream is carried by superluminal waves rather than by a magnetized MHD flow. Moreover, dissipation can be expected to continue until the oscillating fields die away altogether. Therefore, the downstream state will essentially be an unmagnetized relativistic hot plasma. A simple estimate for the applicability of this model is given, suggesting that this process will play a crucial role at least at the termination shocks of young, rapidly rotating, isolated pulsars.

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**APPENDIX A**

**CIRCULARLY POLARIZED SUPERLUMINAL WAVE**

An analytic solution of a circularly polarized superluminal electromagnetic wave in a finite (relativistic) temperature plasma was first given by Max (1973b). Since a circularly polarized mode does not involve any perturbations in density, pressure, or longitudinal velocity, the finite temperature effect appears only as a factor increasing the inertia of the fluids. We consider here the case without phase-averaged magnetic fields, and thus the electromagnetic fields are those of waves only. Assuming a magnetic field perturbation of arbitrary amplitude in the form

\[ B_y = B_0 \cos(kx - \omega t) \]  
\[ B_z = -B_0 \sin(kx - \omega t), \]

the electric field is calculated by Faraday’s law as

\[ E_y = -\frac{\omega}{kc} B_0 \sin(kx - \omega t) \]  
\[ E_z = -\frac{\omega}{kc} B_0 \cos(kx - \omega t). \]

Then, it follows from the positron’s equation of motion

\[ u_y = \frac{e}{khmc} B_0 \sin(kx - \omega t) \]  
\[ u_z = -\frac{e}{khmc} B_0 \cos(kx - \omega t), \]

where a factor \( h = (1 + \Gamma)/(\Gamma - 1)T/mc^2 \) introduced in the text is the effective increase of fluid inertia due to relativistic temperature. Note that the electron’s transverse velocity may be obtained simply by changing sign of the charge. It is easy to verify that the above relations exactly satisfy Ampere’s law if the dispersion relation

\[ \omega^2 = \omega_p^2 + k^2c^2 \]

is satisfied. Here, the effective proper plasma frequency is defined by Equation (8). Although this is formally the same as that of a linear electromagnetic wave in a non-relativistic unmagnetized plasma, it is important to realize that the cutoff frequency is now given by the proper plasma frequency and, therefore, depends on the wave amplitude. Note that the dispersion relation is independent of the reference frame because \( \omega_p \) is defined using only proper-frame quantities. Thus, in any frame, the existence condition of such a superluminal wave is always given by \( \omega > \omega_p \), where \( \omega \) is a laboratory frame frequency.

**APPENDIX B**

**NUMERICAL METHOD**

By taking the sum and difference of the two-fluid equations (cf. Melatos & Melrose 1996), the governing equations can be rewritten as

\[ \frac{\partial}{\partial t} \mathbf{U} + \nabla \cdot \mathbf{F} = \mathbf{S}, \]  

where

\[ \mathbf{U} = \begin{pmatrix} \gamma_p n_p + \gamma_e n_e \\ (w_p \gamma_p u_p + w_e \gamma_e u_e)/c^2 + \mathbf{S}_{\text{EM}} \\ w_p \gamma_p^2 l_p + w_e \gamma_e^2 l_e - (p_p + p_e) + E_{\text{EM}} \\ \gamma_p n_p - \gamma_e n_e \\ (w_p \gamma_p u_p - w_e \gamma_e u_e)/c^2 \\ w_p \gamma_p^2 l_p - w_e \gamma_e^2 l_e - (p_p - p_e) \end{pmatrix}, \]

\[ \mathbf{F} = \begin{pmatrix} n_p u_p + n_e u_e \\ (w_p u_p u_p + w_e u_e u_e)/c^2 + (p_p + p_e)I - T_{\text{EM}} \\ w_p \gamma_p u_p + w_e \gamma_e u_e + \mathbf{S}_{\text{EM}} \\ n_p u_p - n_e u_e \\ (w_p u_p u_p - w_e u_e u_e)/c^2 + (p_p - p_e)I \\ w_p \gamma_p u_p - w_e \gamma_e u_e \end{pmatrix}, \]

and \( \mathbf{S}_{\text{EM}} = c(\mathbf{E} \times \mathbf{B})/4\pi, \) \( E_{\text{EM}} = (\mathbf{E}^2 + B^2)/8\pi, \) \( T_{\text{EM}} = (\mathbf{EE} + \mathbf{BB})/4\pi - (\mathbf{E}^2 + B^2)/8\pi. \) Defining the average laboratory density \( N = \sum \gamma n_s \) and three-fluid velocity \( \mathbf{V} = \sum u_s/N, \) the right-hand side may be written as

\[ \mathbf{S} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ eN(\mathbf{E} + \mathbf{V}/c \times \mathbf{B}) \\ eNV \cdot \mathbf{E} \end{pmatrix}. \]

It is easy to understand that the first three equations, when supplemented by \( \mathbf{E} = -\nabla \phi/c \times \mathbf{B} \) and the magnetic field induction equation, give the standard relativistic MHD equations when appropriate averages of the two fluids are employed. The remaining three equations, on the other hand, describe two-fluid effects. If the initial condition is such that the difference between the two fluids is small (i.e., MHD is an adequate approximation), then two-fluid effects will remain unimportant unless the frozen-in condition is violated, in which case the right-hand side of the above equations become non-zero. In particular, the fifth equation (the difference between momentum equations) can be identified as the generalized Ohm’s law Equation (18).
By using the above form of the basic equations with a conservative scheme, the conservation of total density, momentum, and energy are satisfied up to machine precision. On the other hand, when the two-fluid equations are written separately, the Lorentz force terms appear on the right-hand side as source terms, and conservation of total momentum and energy is not numerically guaranteed. This conservation property is very important, especially for the purpose of our discussion about energy conversion mechanisms. Therefore, we solve the above equations in our numerical simulation code.

The numerical scheme used in this work is based on the central scheme in semi-discrete form (e.g., Kurganov & Tadmor 2000; Kurganov & Levy 2000) with fifth-order weighted essentially non-oscillatory (WENO) reconstruction (Jiang & Shu 1996). We solve the all governing equations (including Maxwell’s equations) simultaneously without splitting. The characteristic speed of the system is thus always the speed of light $c$. The equations are discretized only in space, so that they can be written as a system of ordinary differential equations in time. We integrate this system of equations using the third-order total variation diminishing Runge–Kutta method (Shu & Osher 1988). By using the WENO methodology, the scheme automatically introduces numerical diffusion when a discontinuity is detected, which enables us to resolve ordinary discontinuities with only a few grid points. However, it is found that the simulations tend to be unstable in cases where strong instabilities are developing. We thus also add explicit numerical viscosity with a coefficient that depends on the local second-order derivative of the longitudinal component of four-velocity ($u_\gamma$), in order to avoid complete collapse of simulations.

In solving fluid equations in a conservative form, one needs to calculate primitive variables ($n, \gamma, u, p$) from the updated conservative variables ($U$). In our case, we solve a quartic equation to obtain $|u|$, from which other primitive variables are calculated. It is possible to verify that there is always a solution for $|u|$ that is positive and real under physical conditions, which is calculated algebraically by the method described in Zenitani et al. (2009; see their appendices for detail).

REFERENCES

Akhiiezer, A. I., & Polovin, R. V. 1956, Sov. Phys. JETP, 3, 696
Arka, I., & Kirk, J. G. 2012, ApJ, 745, 108
Arons, J. 2012, SSRv, 173, 341
Asseo, E., Kennel, C. F., & Pellat, R. 1978, A&A, 65, 401
Begelman, M. C. 1998, ApJ, 493, 291
Begelman, M. C., & Kirk, J. G. 1990, ApJ, 353, 66
Chatterjee, S., Gaensler, B. M., Melatos, A., Brisken, W. F., & Stappers, B. W. 2007, ApJ, 670, 1301

Coroniti, F. V. 1990, ApJ, 349, 538
Drenkhahn, G., & Spruit, H. C. 2002, A&A, 391, 1141
Drury, L. O., & Völk, J. H. 1981, ApJL, 248, 344
Hibschman, J. A., & Arons, J. 2001, ApJ, 560, 871
Jaroschek, C. H., Treumann, R. A., Lesch, H., & Scholer, M. 2004, PhPl, 11, 1151
Jiang, G.-S., & Shu, C.-W. 1996, JCoPh, 126, 202
Kato, T. N. 2007, ApJ, 668, 974
Kennel, C. F., & Coroniti, F. V. 1984, ApJ, 283, 694
Kennel, C. F., & Pellat, R. 1976, JPlPh, 15, 335
Kirk, J. G. 2010, PPCF, 52, 124029
Kirk, J. G., & Mochol, I. 2011b, ApJ, 736, 165
Kirk, J. G., & Skjæraasen, O. 2003, ApJ, 591, 366
Komissarov, S. S., & Lyubarsky, Y. E. 2004, MNRAS, 349, 779
Kundt, W., & Krotscheck, E. 1980, A&A, 83, 1
Kurganov, A., & Levy, D. 2000, SIAM J. Sci. Comput., 22, 1461
Kurganov, A., & Tadmor, E. 2000, JCoPh, 160, 241
Lee, M. A., & Lerche, I. 1978, JPPh, 20, 313
Li, H., Nishimura, K., Barnes, D. C., Gary, S. P., & Colgate, S. A. 2003, PhPl, 10, 2763
Lyubarsky, Y., & Kirk, J. G. 2001, ApJ, 547, 437
Lyubarsky, Y. E. 2003, MNRAS, 345, 153
Max, C., & Perkins, F. 1971, PhRvL, 27, 1342
Max, C. E. 1973a, PhFl, 16, 1480
Max, C. E. 1973b, PhFl, 16, 1277
Melatos, A. 1998, MmSAI, 69, 1009
Melatos, A. 2002, in ASP Conf. Ser. 271, Neutron Stars in Supernova Remnants, ed. P. O. Slane & B. M. Gaensler (San Francisco, CA: ASP), 115
Melatos, A., & Melrose, D. B. 1996, MNRAS, 279, 1168
Michel, F. C. 1994, ApJ, 431, 397
Mizuno, Y., Lyubarsky, Y., Nishikawa, K.-I., & Hardee, P. E. 2011, ApJ, 728, 90
Mochol, I., & Kirk, J. G., 2012, in AIP Conf. Proc. 1505, High Energy Gamma-Ray Astronomy Proc. 5th International Meeting, ed. F. A. Aharonian, W. Hofmann, & F. Rieger (Melville, NY: AIP), 358
Nagata, K., Hoshino, M., Jaroschek, C. H., & Takabe, H. 2008, ApJ, 680, 627
Niemiec, J., & Ostrowski, M. 2004, ApJ, 610, 851
Pétri, J., & Lyubarsky, Y. 2007, A&A, 473, 683
Porth, O., Komissarov, S. S., & Keppens, R. 2013, MNRAS, 431, 48
Rees, M. J., & Gunn, J. E. 1974, MNRAS, 167, 1
Ruderman, M. A., & Sutherland, P. G. 1975, ApJ, 196, 51
Shu, C.-W., & Osher, S. 1988, JCoPh, 77, 439
Sironi, L., & Spitkovsky, A. 2009, ApJ, 698, 1523
Sironi, L., & Spitkovsky, A. 2011, ApJ, 741, 39
Skjæraasen, O., Melatos, A., & Spitkovsky, A. 2005, ApJ, 634, 542
Sturrock, P. A. 1971, ApJ, 164, 529
Sweeney, G. S. S., & Stewart, P. 1975, A&A, 41, 431
Terasawa, T., Hoshino, M., Sakai, J.-I., & Hada, T. 1986, JGR, 91, 4171
Thompson, C. 1994, MNRAS, 270, 480
Weiβkopf, M. C., Hester, J. J., Tennant, A. F., et al. 2000, ApJL, 536, L81
Zenitani, S., Hesse, M., & Klimas, A. 2009, ApJL, 696, 1385
Zenitani, S., & Hoshino, M. 2001, ApJL, 562, L63
Zenitani, S., & Hoshino, M. 2005, PhRvL, 95, 095001
Zenitani, S., & Hoshino, M. 2007, ApJL, 670, 702