Multi-Party Quantum Computation has attracted a lot of attention as a potential killer-app for quantum networks through its ability to preserve privacy and integrity of the highly valuable computations they would enable. Contributing to the latest challenges in this field, we present a composable protocol achieving blindness and verifiability even in the case of a single honest client. The security of our protocol is reduced, in an information-theoretically secure way, to that of a classical composable Secure Multi-Party Computation used to coordinate the various parties. Our scheme thus provides a statistically secure upgrade of such classical scheme to a quantum one with the same level of security.

In addition, (i) the clients can delegate their computation to a powerful fully fault-tolerant server and only need to perform single qubit operations to unlock the full potential of multi-party quantum computation; (ii) the amount of quantum communication with the server is reduced to sending quantum states at the beginning of the computation and receiving the output states at the end, which is optimal and removes the need for interactive quantum communication; and (iii) it has a low constant multiplicative qubit overhead compared to the single-client delegated protocol it is built upon.

The main technical ingredient of our paper is the bootstrapping of the Multi-Party Quantum Computation construction by Double Blind Quantum Computation, a new composable resource for blind multiparty quantum computation, that demonstrates the surprising fact that the full protocol does not require verifiability of all components to achieve security.

1 Introduction

The development of quantum computing and quantum communication builds upon decades of analysis of their classical counterparts. The recent path taken by start-ups and large technological corporations points toward quantum technologies being mostly cloud-accessible. While, as of now, it is possible to access experimental machines through classical networks, several initiatives are working on building quantum networks that would allow remote quantum access to quantum computers.

A wealth of questions arise in this setting from a security perspective, one of which being that clients and servers, reflecting real-life situations, do not necessarily trust each other. Protecting the clients’ inputs and computations that are run on dishonest servers is
therefore central, and even more so in the quantum domain, as quantum computers are constructed precisely for the purpose of tackling sensitive and/or high added-value computations. Several protocols providing privacy with or without verifiability have been proposed and proven secure (see e.g. [10], and [16] for a review). Among these works, most consider the measurement-based model of quantum computation introduced by Raussendorf and Briegel [32] as a natural way of allowing (almost) powerless quantum clients to delegate computations to powerful quantum servers.

In the context of networked machines, Secure Multi-Party Computation is another vital functionality whose purpose is to allow several players to collaboratively compute a joint function on private data. The parties are required to exchange messages as no single party possesses the full data required to perform the computation. However, they want to do so while minimising the exposure of their private information. The classical case has been considered first by Yao [35] and has been developed extensively in various settings (see [4] and references therein), and in particular in the quantum realm under the name of Multi-Party Quantum Computation (MPQC).

Several lines of research regarding MPQC have been followed during the past two decades. The very first one was initiated by Crépeau, Gottesman and Smith in [5]. Along with the introduction of the concept itself, they provided a concrete protocol for performing such computations in the quantum circuit model. It guarantees the security of the computation as long as the fraction of malicious parties does not exceed $\frac{1}{6}$. This work has been later extended in [2], lowering the minimum number of honest players required for security to a strict majority.

The second line of research was focused on the interesting edge case of two-party quantum computations. Several constructive results have been proposed in the circuit model. In [12], a new protocol was introduced and proven secure for quantum honest-but-curious adversaries (also called specious). This restriction on the adversaries was removed in [13] which proved security in the fully malicious setting and with negligible security bounds. The measurement-based model of quantum computation has also been considered for constructing secure two-party quantum computations as it provides a different set of tools than the circuit model. Verifiable Blind Quantum Computation first was introduced in [15] in this model and an optimized protocol was provided in [25] and in [21]. In [24] a protocol was proposed in this setting and proven secure against specious adversaries. In [22] this result was extended to fully malicious adversaries with inverse-polynomial security using the Quantum Cut-and-Choose technique.

Aside from the previous approaches focused on concrete protocols, a third line of research was initiated regarding the possible composability of such protocols, as earlier results didn’t satisfy this property. Bit commitment was shown to be complete in the Quantum Universal Composability framework of [33], meaning that it is sufficient for constructing quantum or classical Secure Multi-Party Computation (SMPC) if parties have access to quantum channels and operations. This result was later extended in [14, 11], which give a full analysis of feasibility and completeness of cryptographic primitives in a composable setting.

More recently, building on these previous works, new concrete protocols have been proposed to decrease the restrictions on adversaries and provide composable security. First, [23] described a protocol that is composable, can tolerate a dishonest majority and allows the clients to delegate the quantum computation to a powerful server. Its security is an information-theoretic upgrade of a classical SMPC primitive used for constructing

\[\text{Note that by the very definition of this functionality, any protocol constructing it is automatically blind and verifiable.}\]
the protocol. It is however limited by the absence of verifiability of outputs and the impossibility to tolerate client-server collusion. Reference [19] provides a generic recipe to turn any composable two-party delegated blind and verifiable quantum computation protocol into a multi-party delegated version. It can cope with a dishonest server only, requires a complete quantum and classical communication graph and is limited in terms of implementable computations as each client chooses only local computations and cannot coordinate with others. In the circuit model, a composably-secure protocol has been introduced recently in [9]. It is an extension of [13] that is able to cope with a dishonest majority, but which relies on a complete graph for quantum communication architecture that imposes a large number of quantum communication rounds together with powerful quantum participants. A stand-alone secure protocol is proposed in [26] as an extension on previous results of [2] based on error-correcting codes. Its aim is to lower the amount of quantum memory of participants but suffers from the same drawback as [9]: powerful quantum parties and complete quantum and classical communication graph.

1.0.1 Paper Methods and Contribution.

The current work proposes an efficient protocol for achieving delegated MPQC secure against a malicious collusion between any number of clients and the server in the demanding Abstract Cryptography framework [28, 29]. Based on the single-client construction of [25], the clients only need to be able to manipulate single qubits and perform two rounds of quantum communication with the server (reduced to a single round if the outputs are classical), which is in fact optimal. It inherits also the delegated nature of this protocol, meaning that only a single party needs to perform elaborate quantum operations while the others need only to be able to do single qubit operations and state generation. To achieve these results, we have introduced several new techniques.

We use an approach based on protocol deconstruction-reconstruction. Starting from the single client Verifiable Blind Quantum Computation (VBQC) protocol, we identified smaller key functionalities that needed to be replaced to successfully turn the initial protocol into a multi-party one that could be driven by a classical SMPC. This has led us to define and construct a new quantum resource, called Double Blind Quantum Computation (DBQC). It allows a single trusted classical party to orchestrate a delegated multi-party computation in a way that is blind to the powerful server but also to the dishonest majority of clients that collude with it. It can therefore be used to securely prepare the same types of states as those that are used in the single-client version of VBQC. These states then form the basis upon which the rest of the multi-party protocol is built.

Our protocol achieves a simultaneous optimization of the number of communication rounds, the Client-side memory size, and the operation complexity in a way that goes beyond the usual MPQC trade-off. More precisely, symmetric protocols such as [9] can be modified using gate teleportations into asymmetric versions where one party plays the role of a server while the rest are its clients. This allows to modify the topology of the network required to run the protocol as well as the number of rounds. Yet, in doing so, the complexity of the operations that need to be performed by these clients remains the same. They still need to have local fault-tolerant computation capabilities, whereas our scheme reduces this to single qubit gates. In addition, the overhead of our protocol with respect to the amount of qubits that need to be sent by each client compared to the single-client version is only a constant multiplicative factor of 9.

Furthermore, this highly modular approach to protocol design makes improving the efficiency of our protocol simpler since any improvement in one of the sub-components immediately translates into an improvement on the global protocol so long as the conditions
for composability are met. Hence, our protocol is well adapted to the currently foreseen development of quantum computing services accessible through a quantum internet.

Second, as DBQC provides blindness but not verifiability (and therefore not all components of the full protocol are verifiable), it was required to prove that it was nonetheless sufficient to bootstrap security and obtain a verifiable protocol in a composable cryptographic framework. This led to the introduction of “good-enough” sets of states that are the correct ones up to a deviation by the malicious parties that is independent on the inputs and parameters of the honest parties.

This implies an affirmative answer to the possibility of using the MBQC model to delegate MPQC in the dishonest majority setting. In fact, the inability to cope with client-server collusions in [23] outlined the essential role played by verification: it secures the key release in MBQC computations which could otherwise be vulnerable as the key depends on possibly corrupted qubits measured in the previous rounds. However, direct approaches of uplifting the trap-based technique for verifying a delegated MBQC computation to the multi-client case based on letting each client trappify sub-graphs of the computation graph were not successful. This is due to the necessity to ensure that there always exists a computation path that performs the desired computation which in turn requires collaboration between the clients. Avoiding leaks about the location of the traps during these collaboration steps was a long-standing open question that we solve using DBQC to prepare non-verified states that would nonetheless be sufficient to verify the subsequent computation.

1.0.2 Outline.

After introducing notations, the security and computation frameworks in Section 2, an iterative construction of the protocol and highlevel presentation of it is given in Section 3. Section 4 details the construction of the DBQC resource and proves its blindness. Section 5 proves that it can be used to bootstrap verifiability of the whole protocol. The latter is then formally presented in Section 6 along with its security statement and proof. While we use DBQC to implement a multi-party VBQC client-encrypted state preparation, it is in fact more general and we believe it will find other applications in the future.

Table 1 below gives a comparison of our protocol with the peer-to-peer protocols of [9] and [26], and with the more recent semi-delegated protocol of [1]. Appendix K gives a more in-depth analysis, showing that our protocol closes gaps left as open questions in previous results. In the table, $N$ is the number of parties, $d$ the depth of the computation (MBQC for our paper, circuit for [26] and $\{T, CNOT\}$-depth for [9]), $g$ the number of gates in the computed circuit, $t$ the number for $T$ gates, $c$ the number of $CNOT$ gates, $C_{\text{dist}}$ the code distance used in [26] and $\eta$ a statistical security parameter. Note that we list below the optimal parameters for the protocol’s execution. While the network topology of [9] and [26] can also be star-shaped – with one player acting as a router – this would degrade their performance in terms of quantum communication rounds, so that we kept the value of the parameters for the Complete Graph as it is more natural for these protocols.

2 Preliminaries

We write $[n] = \{1, \ldots, n\}$ for $n \in \mathbb{N}$ and $\#X$ for the size of $X$ irrespective of the nature of $X$ – it can be the number of qubits in a quantum register, a string length, set size – while $\varphi(S)$ is the set of subsets of set $S$. We write $\in_R$ to say that a value was sampled from a set uniformly at random. We suppose the existence of dummy input $\lambda$ and special
symbol **Abort** (by convention, an honest party receiving or sending the message **Abort** halts immediately after). We say that a function $\epsilon(\eta)$ is **negligible** in $\eta$ if, for all polynomials $p(\eta)$ and $\eta$ sufficiently large, we have $\epsilon(\eta) \leq \frac{1}{p(\eta)}$. On the other hand we say that $\mu(\eta)$ is **overwhelming** in $\eta$ if there exists a negligible $\epsilon(\eta)$ such that $\mu(\eta) = 1 - \epsilon(\eta)$. For a set $V$ and $O \subseteq V$, we write $O^c = V \setminus O$ (it should depend on the set $V$ in all generality but will be clear from context).

For unitary $U$ and quantum state $\rho$, we note $U(\rho) = U\rho U^\dagger$ and QOTP$_k(\cdot) = X^{k_X}Z^{k_Z}(\cdot)$ for a random Quantum One-Time-Pad (or Q-OTP) key $k = (k_X, k_Z)$, where $X^{k_X} = \bigotimes_i X(i)^{k_X(i)}$ and similarly with $Z$ (see [30] for further quantum definitions).

As we will manipulate quantum states $\rho$ involving many qubits and from different origins, it will be convenient to label the reduced state for a given qubit indexed by $i$ and being provided by Client $j$ as $\rho_j(i)$. By extension, index $j$ might be replaced by a set of clients indices (typically $H$ for honest clients and $M$ for malicious ones). We will apply the same notation for classical variables denoting secret parameters for qubit $i$ and for Client $j$, such as $\theta_j(i)$ and $r_j(i)$. When a state or classical variable depends collectively on all Clients, the subscript $j$ will usually be dropped.

### 2.1 Abstract Cryptography

#### 2.1.1 General Framework

Abstract Cryptography is a framework for defining and proving the security of cryptographic protocols, first introduced in [29, 28]. Its main advantage is that any system that follows the structure defined by the framework is inherently composable, in the sense that if two protocols are secure separately, the framework guarantees at an abstract level that their sequential or parallel composition is also secure. We refer the reader to [10] for a more in-depth presentation.

In this framework, the purpose of a secure protocol $\pi$ is, given a number of available resources $R$, to construct a new resource – written as $\pi R$. This new resource can be itself reused in a future protocol.

The actions of all honest players in a given protocol are represented as a sequence of efficient CPTP maps acting on their internal quantum registers (which may contain communication registers, both classical and quantum). An $N$-party quantum protocol
is therefore described by $\pi = (\pi_1, \ldots, \pi_N)$ where $\pi_i$ is the aforementioned sequence of efficient CPTP maps executed by party $i$ (called the converter of party $i$). A resource $R$ is described as a sequence of CPTP maps with an internal state. It has input and output interfaces describing which party may exchange states with it. It works by having the party sending it a given state at one of its input interfaces, applying the specified CPTP map after all input interfaces have been initialised and then outputting the resulting state at its output interfaces in a specified order. Classical resources are modelled by considering that the input state is measured upon reception and the output is a computational basis state.

In order to define the security of a protocol, we need to give a pseudo-metric on the space of resources. The security analysis then consists of considering a special type of converters called distinguishers which have as many input interfaces as the resources and which outputs a single bit. The distinguisher’s aim is to discriminate between an execution with a resource $R_1$ and another resource $R_2$, each having the same number of input and output interfaces. It prepares the input, interacts with the resource according to its own possibly adaptive strategy, and decides whether it was interacting with this one or the other. Two resources are said to be indistinguishable if no distinguisher can make this guess with good probability.

**Definition 1** (Indistinguishability of Resources). Let $\epsilon(\eta)$ be a function of security parameter $\eta$ and $R_1$ and $R_2$ be two resources with same input and output interfaces. The resources are $\epsilon$-statistically-indistinguishable if, for all distinguishers $D$, we have:

$$
\left| \Pr[b = 1 \mid b \leftarrow D R_1] - \Pr[b = 1 \mid b \leftarrow D R_2] \right| \leq \epsilon
$$

We then write $R_1 \approx_{\text{stat}, \epsilon} R_2$.

The construction of a given resource $S$ by the application of protocol $\pi$ to resource $R$ is captured by the fact that these resources are indistinguishable. More specifically, this captures the correctness of the protocol. The security is captured by the fact that the resources remain indistinguishable if we allow some parties to deviate in the sense that they are no longer forced to use the converters defined in the protocol but can use any other CPTP maps instead. This is done by removing the converters for those parties in Equation 1 while keeping only $\pi_H = \prod_{i \in H} \pi_i$ where $H$ is the set of honest parties. The security is formalised as follows in Definition 2.

**Definition 2** (Construction of Resources). Let $\epsilon(\eta)$ be a function of security parameter $\eta$. We say that an $N$-party protocol $\pi$ $\epsilon$-statistically-constructs resource $S$ from resource $R$ against adversarial patterns $P \subseteq \wp([N])$ if:

1. It is correct: $\pi R \approx_{\text{stat}, \epsilon} S$
2. It is secure for all subsets of corrupted parties in the pattern $M \in P$: there exists a simulator (converter) $\sigma_M$ such that $\pi_M R \approx_{\text{stat}, \epsilon} S \sigma_M$

The computational versions of these definitions are obtained by quantifying over quantum polynomial time (QPT) parties. Composing a statistically-secure protocol with a computationally-secure protocol is possible provided that the simulator for the statistically-secure one runs in expected polynomial time. The resulting protocol is of course only computationally-secure.
2.1.2 Definition of Ideal Resources.

We define below three ideal resources that will play an important role in this paper. We start by the Multi-Party Quantum Computation Ideal Resource which will be emulated by our delegated MPQC protocol. The second one is the Verifiable Blind Quantum Computation Ideal Resource which is emulated by the single-client VBQC protocol of [25] and which will be used as a building block in our protocol. The third one is the Double Blind Quantum Computation Ideal Resource which allows a classical party called a Orchestrator to delegate a quantum computation whose inputs are provided by several Clients to a Server in such a way that the Clients and the Server are blind with respect to each others’ inputs and to the computation performed.

All three allow to leak a value \( l_\rho \) about the parties’ intended computation and input, sent to all malicious players upon request. The exact value of \( l_\rho \), as a function of inputs and computation, is specified by each protocol and is public. In this paper it is an upper-bound on the size of the quantum circuit implementing the unitary and the length of the parties’ input.

**Multi-Party Quantum Computation Ideal Resource.** This resource has \( N + 1 \) interfaces, one for each Party and the last one for an Eavesdropper. Its purpose is to allow \( N \) Clients to perform a collectively defined computation \( U \) over their private quantum inputs with the guarantee that their computation is either executed properly or it is aborted altogether. Additionally, they know that the Eavesdropper didn’t receive more than the permitted leakage \( l_\rho \). Resource 1 is identical to the one from [9] apart from the explicit introduction of an Eavesdropper, i.e. a party with no input and no output. This allows for an easier account and analysis of the information the Server might obtain when the computation is delegated: despite being active in the protocol the Server has no input and no output regarding the ideal resource. It is impossible to have fairness of output distribution in the case of a dishonest majority (the malicious parties can always choose to receive their output before the honest players). This is modelled by a filtered bit \( o_j \) at each player’s interface, indicating that it receives the output before others.

**Resource 1 Multi-Party Quantum Computation**

**Inputs:**
- \( N \) players send each a quantum register \( X_j \) which contains their respective part of a collectively possessed \( \rho_{inp} \), and the classical description of a joint unitary \( U \) they wish to apply to \( \rho_{inp} \). They can each input two bits \( o_j \) and \( c_j \) as a filtered interface.
- The Eavesdropper can input two bits \( e \) and \( c \) as a filtered interface.

**Computation by the resource:**
1. If \( e = 1 \), the Resource sends the leakage \( l_\rho \) to the Eavesdropper’s interface.
2. If \( c = 1 \) or there exists \( j \) such that \( c_j = 1 \), the Resource sends **Abort** to all players.
3. Otherwise it computes \( U(\rho_{inp}) \).
4. If there exists \( j \in [N] \) such that \( o_j = 1 \), it sends qubit \( j \) to the interface of player \( j \).
5. It then sends the output qubits to all other players in a similar fashion.

**Verifiable Delegated Quantum Computation Ideal Resource.** Resource 2 allows a single Client to run a quantum computation on a Server so that the Server cannot corrupt
the computation and doesn’t learn anything besides the leakage $l_\rho$. It was introduced by [10] and we show in Lemma 1 that the Protocol from [25] implements it.

**Resource 2 Verifiable Delegated Quantum Computation**

**Inputs:**
- The Client inputs a quantum state $\rho$ and the classical description of a unitary $U$.
- The Server chooses whether or not to deviate. This interface is filtered by two control bits $(e, c)$ (set to 0 by default for honest behaviour).

**Computation by the resource:**
1. If $e = 1$, the Resource sends the leakage $l_\rho$ to the Server’s interface and awaits further input from the Server; if it receives $c = 1$, the Resource outputs $|\text{Abort}\rangle\langle\text{Abort}|$ at the Client’s output interface.
2. Otherwise it outputs $U(\rho)$ at the Client’s output interface.

**Double Blind Quantum Computation Ideal Resource.** Resource 3 introduces a new functionality as it allows $N$ Clients to submit their part of a collectively possessed input quantum state $\rho_{\text{inp}}$ and an Orchestrator to input a classical description of a unitary transformation $U$ to apply to $\rho_{\text{inp}}$. A coalition of malicious parties may induce a deviation from the unitary specified by the Orchestrator by way of a CPTP map $E$, applied to the input state and adversarially-chosen ancillary quantum state instead of the legitimate transformation. In both cases, the Server receives from the Resource the output quantum state which is encrypted by a Q-OTP while the Orchestrator gets the corresponding randomly chosen key $k$, which guarantees the blindness of the scheme.

**Resource 3 Double-Blind Quantum Computation**

**Inputs:**
- Each Honest Client $j \in [N]$ has a quantum register $X_j$ which contains their respective part of a collectively possessed $\rho_{\text{inp}}$.
- The Orchestrator inputs the classical description of a unitary $U$.
- The Server has no honest input.
- All Clients and the Server have filtered interfaces controlled by $\{c_j\}_{j \in [N] \cup \{S\}}$ (set to 0 in the honest case). When a coalition cheats, they collectively send quantum state $\rho_M$ and the classical description of a CPTP map $E$.

**Computation by the Resource:**
1. It samples uniformly at random a Q-OTP key $k = (k_X, k_Z)$ of the size of the output state.
2. If all $c_j = 0$, the state $\text{QOTP}_k \circ U(\rho_{\text{inp}})$ is produced at the Server’s interface.
3. If there are parties $j$ with $c_j = 1$, the Ideal Resource first sends them the leakage $l_\rho$. Then, the malicious parties send their additional inputs to the Resource. The state $\text{QOTP}_k \circ E(\rho_{H,M,U})$ is produced at the Server’s interface where $\rho_{H,M,U}$ includes the inputs of the honest Clients ($H$), the malicious parties ($M$), and the Orchestrator’s description of $U$.
4. The Q-OTP key $k = (k_X, k_Z)$ is output at the Orchestrator’s interface.
**Other Ideal Resources.** In the course of this paper, we use common ideal resources such as the Authenticated Classical Channel (Ideal Resource 5), available between any two parties, the Secure Classical Channel (Ideal Resource 6), available between all Clients and the Orchestrator, and the Insecure Quantum Channel (Ideal Resource 7), available between all Clients and the Server. Their precise definition can be found in Appendix A.

We also suppose that all participants have access to a Classical Secure Multi-Party Computation Ideal Resource 4. It allows \( N \) Clients to provide their private inputs and perform a collectively defined computation \( C \) on them with the guarantee that the computation is performed properly. We assume that it keeps an internal state between calls. Since it is impossible to guarantee fairness in output distribution with a dishonest majority, some participants may receive their output before others (and chose to abort, preventing the rest of the participants from obtaining theirs). This is modelled by allowing each participant to set a single bit \( o_j \) to 1 whenever they want to receive their output before others. We describe the uses of this resource in Section 3 during the construction of our protocol.

| Resource 4 Classical Secure Multi-Party Computation |
|---------------------------------------------------|
| **Inputs:** The \( N \) parties each input a value \( x_j \) and a common classical computation \( C \). They each have a filtered input bit \( o_j \) (set to 0 in the honest case). |
| **Computation by the resource:** The Resource computes \( y = C(x_1, \ldots, x_N) \). It first sends \( y \) to all \( i \) such that \( o_j = 1 \). Then it sends \( y \) to all other parties. |

2.2 Quantum Computation Model

The quantum computation model used in the present paper is Measurement-Based Quantum Computing [32] (or MBQC). In this section, MBQC is introduced along with protocols for blind as well as verifiable blind quantum computation [3, 25].

2.2.1 Measurement-Based Blind Quantum Computing.

The MBQC model of computation emerged from the gate teleportation principle. It was shown in [32] that, using a certain type of entangled states (represented by graphs) and one-qubit measurements, it can implement universal quantum computing and that it thus has the same power as the circuit model. The correspondence between one and the other is described with the tools of measurement calculus [8]. MBQC works by choosing an appropriate entangled state, then by measuring single qubits and, depending on the outcomes, apply correction operators to the rest. This makes it particularly adapted to delegation of quantum computations where the Client only has to provide its quantum inputs and instruct the Server with the right measurements to perform, while the Server takes on the creation of a large entangled state.

We define \( Z(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \) and \( |+\theta\rangle = Z(\theta) |+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{i\theta} |1\rangle) \). While the discussions below hold for angles in \([0, 2\pi)\), if we settle for approximate universality, it is sufficient to restrict ourselves to the set of angles \( \Theta = \left\{ \frac{2\pi k}{7} \right\}_{k \in \{0, \ldots, 7\}} [3] \).

The computation is determined by a graph \( G = (V, E) \), input and output vertices \( I, O \subseteq V \), a set of default measurement angles \( \{\phi(i)\}_{i \in O^c} \) for non-output qubits and a flow function \( f : O^c \rightarrow I^c \). Vertices in \( V \) correspond to qubits in state \( |+\rangle \) while edges in \( E \) correspond to entangling operators \( CZ \). Let \( S_X(i) \) and \( S_Z(i) \) be respectively
the $X$ and $Z$ dependency-sets for qubit $i$.\footnote{These sets and the order of measurement are given by the flow, see [18, 7] for details.} The measurement result $s(j)$ for qubit $j \in S_X(i) \cup S_Z(i)$ induces Pauli corrections on qubit $i$ which are equivalent to measuring qubit $i$ with corrected angle $\phi'(i) = (-1)^{s_X(i)}\phi(i) + \pi s_Z(i)$, where $s_X(i) = \bigoplus_{j \in S_X(i)} s(j)$ and $s_Z(i) = \bigoplus_{j \in S_Z(i)} s(j)$. After measuring all qubits in $O^c$ according to these angles, the Server returns all output qubits $i \in O$ to the Client on which the Client performs the final Pauli correction $Z^{s_Z(i)}X^{s_X(i)}$.

Universal Blind Quantum Computation (or UBQC) \cite{3} is an upgrade of any MBQC computation providing blindness in the sense that the Server does not learn anything about the computation besides an upper bound on the size of the computation.

The Client hides the computation by sending rotated states $\ket{+_{\theta(i)}}$ instead of $\ket{+}$ for each qubit $i$. The effect of this rotation is cancelled by a corresponding rotation of the measurement angle. An additional parameter $r(i)$ adds an extra $r(i)\pi$ rotation to the measurement angle in order to hide the obtained outcome. This can be classically accounted for by the Client by setting $s(i) = r(i) \oplus b(i)$, where $s(i)$ is used as above in MBQC while $b(i)$ is the outcome returned by the Server. Quantum inputs $i \in I$ are protected by a Quantum One-Time Pad operation $Z(\theta(i))X^{a(i)}$ for $a(i) \in R \{0, 1\}$ applied by the Client before being sent to the Server. The measurement angles are further adapted to account for $a(i)$: $s_X'(i) = a(i) \oplus s_X(i)$ and $s_Z'(i) = a(f^{-1}(i)) \oplus s_Z(i)$. Here, $a$ has been extended so that $a(i) = 0$ for $i \in I^c$ and $a(f^{-1}(i)) = 0$ for $i \notin \text{range}(f)$. The measurement angle sent by the Client is then $\delta(i) = \phi'(i) + \theta(i) + r(i)\pi$ (for the new $s_X'(i)$ and $s_Z'(i)$).\footnote{The initial UBQC presentation [3] applies the Q-OTP in the reversed order $X^{a(i)}Z(\theta(i))$, which is a typo since the operations would need to be commuted so that $Z(\theta(i))$ can be cancelled by the $\theta$ contained in the $\delta$. Without this correction, the commutation of $X^{a(i)}$ with $Z(\theta(i))$ would turn $\theta$ into $(-1)^a\theta$, which would not cancel out.}

2.2.2 Verifiable Blind Quantum Computing.

In UBQC, the Server is not forced to follow the instructions and the Client cannot verify whether the computation is done correctly or not. However, a modified version of the protocol allows for such verification. Introduced in [15] and later optimised in [21, 25], the central idea is to include isolated trap qubits that do not affect the computation and have deterministic outcome if measured in the correct basis, while remaining undetectable by the Server. The Client accepts the output of the computation if all trap measurements yield the expected outcomes.

We give now a brief outline of the VBQC Protocol we will use as the basis for our construction\footnote{We became aware recently of a new protocol [34] achieving the VDQC Ideal Resource 2 with better overhead and believe it could be combined with the same techniques as the ones presented here to obtain a Multi-Party Quantum Computation Protocol.}. We refer the reader to [25] for a more in-depth presentation. The Protocol works by uplifting the MBQC base-graph $G$ to a Dotted-Triple Graph $DT(G)$. This construction amounts to enlarging the initial graph so that it is possible to randomly choose one of the sub-graphs corresponding to the to-be-performed computation, isolate it from the rest of the graph using dummy qubits -- qubits in the $|0\rangle$ or $|1\rangle$ state -- and to place trap qubits on remaining positions.

The Dotted-Triple Graph is constructed from the base-graph as follows. Each vertex $v \in V$ is replaced with a set $P_v$ of three vertices called primary vertices. Each edge $e = (v_i, v_j) \in E$ is replaced by a set of nine edges $E_e$ such that every vertex in $P_{v_i}$ is connected to every vertex in $P_{v_j}$. Every edge in the resulting graph is replaced with a new...
vertex (called added vertex) connected to the two vertices originally joined by that edge. The edge or vertex of the initial graph that each vertex \( v \in DT(G) \) belongs to is called its base-location.

Sending a dummy qubit at a vertex, together with the appropriate \( Z \) corrections on the neighbouring vertices, breaks the graph at this vertex, removing it along with any attached edges. This allows us to break the \( DT(G) \) in three copies of the same base-graph (one for computation and the other two for traps). This is done by first choosing uniformly at random for each primary set one computation, one trap and one dummy vertex. For added sets, two primary computation vertices are joined by a computation vertex, two primary dummy vertices are joined by a trap vertex and the rest are dummies. Here, computation vertices are those that belong to the copy of the base-graph that will be used for performing the computation once the \( DT(G) \) is broken into three copies. Trap qubits are those qubits that are prepared in a known \(|+\theta\rangle\) state, that belong to the other two copies of the base-graph and which are, by construction, linked only to dummy qubits and hence disconnected from all other qubits. As a consequence, they yield a deterministic result when measured in the \(|\pm\theta\rangle\) basis. The states resulting from different choices are indistinguishable to the Server. This trap-colouring (Definition 3 of [25]) can be chosen in advance by the Client. The result for a specific colouring is shown in Figure 1.

![Graph Diagram](image1)

(a) Trap-colouring, chosen by the Client

(b) Computation graph and isolated traps, reduced from (a) after entanglement by the Server

Figure 1: Dotted-Triple-Graph for one-dimensional base-graph of four qubits. Circles: primary vertices; Squares: added vertices. Vertices are coloured in either green (computation), white (traps), or red (dummies).

By setting \( \phi(i) = 0 \) for added qubits \( i \), the flow can be extended to the Dotted-Triple Graph construction while also yielding deterministic results for the trap qubits. The protocol then implements the original MBQC computation on the sub-graph of \( DT(G) \) that is used for computation, while the Server learns nothing about the input/output nor computation. Moreover, the random placement of the traps guarantees that a deviating Server will be detected with non-zero probability by the Client from the corruption of trap measurement outcomes. Exponential amplification of this probability can be achieved by incorporating fault-tolerant techniques as described in [15].

Given the Dotted-Triple-Graph corresponding to the base-graph \( G \), the VBQC Protocol can be summarised as follows (below \( N_G(i) \) is the set of qubits in the neighbourhood of \( i \) in \( G \)). A full description of the protocol is given in Appendix C.

1. The Client chooses the trap-colouring of the Dotted-Triple Graph, keys for its inputs
and secret parameters for the rest of the qubits in the $DT(G)$.

2. It encrypts its inputs as in UBQC, prepares the resource single qubit states corresponding to the description chosen above and sends them along with its inputs to the Server in a predefined order. These states are (with $D$ the set of dummy qubits in $DT(G)$):

- For all $i \in D$: $|d(i)\rangle$, with $d(i) \in_R \{0,1\}$.
- For all $i \notin D$: $\prod_{j \in N_G(i) \cap D} Z^{d_j} |+\rangle_{\theta(i)}$, with $\theta(i) \in_R \Theta$.

3. The Server entangles them according to the publicly known structure of the graph (one CZ per edge in $DT(G)$).

4. The Client adaptively instructs the Server to measure all non-output qubits as in UBQC, who returns the measurement outcomes. The measurement angle of dummies is chosen uniformly at random.

5. The Server returns to the Client all qubits that correspond to vertices at output base-locations to the Client. The Client measures the output trap qubits $t$ with angle $\delta(t) = \theta(t) + r(t)\pi$.

6. The Client checks that the traps have been measured correctly (if $b(t) = r(t)$) and aborts if any failed. It computes encryption keys corresponding to Quantum One-Time-Pad of the output as in UBQC.

7. It decrypts the Quantum One-Time Pad and accepts the final quantum states as its output.

The security of the protocol is captured by comparing it with the Verifiable Delegated Quantum Computation Resource 2 from [10] (Lemma 1). We only give a sketch of the proof in Appendix C and refer to [25, 10] for more details and security bounds.

**Lemma 1** (Composable Security of VBQC). The VBQC Protocol $\epsilon_V$-statistically-constructs the VDQC Ideal Resource 2 from Insecure Quantum Channels for negligible $\epsilon_V$.

### 3 High-Level Construction of a Delegated MPQC Protocol from VBQC

The main purpose of our paper is to build a Protocol constructing a Multi-Party Quantum Computation Resource in a way that is composable secure, that can be delegated by relatively powerless Clients to a powerful quantum Server, that tolerates Client-Server collusion and requires only a single honest Client to run securely. We tackle this by first deconstructing and analysing the single-client VBQC Protocol. We determine the steps which need to be updated to transform it into a multi-client setting, together with the conditions that these replacement steps need to satisfy. Having defined these, we construct a Delegated Multi-Party Quantum Computation (DMPQC) Protocol by making use of the elegant composability property of the AC framework. We show later in Section 4 how to instantiate these new multi-party subroutines. The formal presentation of the DMPQC Protocol and associated security guarantees can be found in Section 6.

#### 3.1 Deconstructing the VBQC Protocol

The single-client VBQC protocol of Ref. [25] can be decomposed in the following parts: choosing secret parameters (graph colouring, encryption keys); preparing the VBQC client-encrypted state (comprising encrypted quantum inputs, dummies and encrypted $|+\rangle$ states sent by the Client to the Server during the initialization step of Protocol 5); sending single qubits; applying entangling operations and classically-driven measurement; receiving
quantum outputs; aborting or decrypting. Transforming this protocol into a multi-party one requires modifying each of these individual steps so that they can be performed collectively by several clients without compromising the blindness, even in the situation where some Clients and the Server collude.

First, we remark that the protocol is perfectly equivalent to one where a Trusted Third Party receives from the Client the classical parameters (chosen at random within their allowed range) that are used when preparing the corresponding states and encrypted inputs for the Server. Second, the knowledge of these classical parameters and of the unitary $U$ to apply is indeed sufficient for the Trusted Third Party to drive the VBQC computation and also verify that the traps have been measured correctly, outputting either the keys to the Client in case of success or instructing it to abort if any trap failed. In this modified setup, the only operations that the Client would still need to perform are encrypting its inputs and preparing dummies and encrypted $|+\rangle$ states to generate the VBQC client-encrypted state, sending the encryption key to the Trusted Third Party, sending the state to the Server and, if there is no abort, recovering the output state from the Server, the keys from the Trusted Third Party and decrypting to get the final state. This change is pictured in Figure 2.

3.2 Reconstructing a DMPQC Protocol with the DBQC Ideal Resource

Using the remarks above, we conclude that the VBQC Protocol can be driven by a Trusted Third Party independent of the Client – or equivalently by a Classical SMPC in the context of multiple Clients – provided that the Server receives a state that corresponds to the VBQC client-encrypted state. In that case, the verifiability of the VBQC protocol by itself guarantees that the probability a party cheated without the honest parties noticing is negligible. When following this path, two steps need attention: we need to (i) find a way to collaboratively prepare the VBQC client-encrypted state; and (ii) specify the procedure performed by each Client after receiving the quantum output from the Server.

We start with the easier case of outputs. Indeed, the only point that needs to be specifically addressed is the verification of the trap qubits placed in the output layer before receiving the keys and decrypting. These traps need to be measured and pass the test. But in doing so, the Server should not learn which of the output layer qubits of the honest
Clients are computational ones. The solution is to first send the qubits of the output layer to the Clients – each Client receiving the qubits corresponding to the base-locations of its own output –, then having the Classical SMPC reveal to each Client where the traps are among its qubits, and have them measured. The Classical SMPC then verifies that all the traps are correct which guarantees with overwhelming probability that the Server has sent to each honest Client their intended quantum states of the output layer, and in particular their quantum output. Hence, under the condition that this verification passes, the Classical SMPC can safely send the decryption keys for each Client individually as the malicious coalition cannot access the quantum states of the honest Clients. The Clients can then decrypt their outputs.

Regarding the inputs, if the Clients had access to a VBQC Client-Encrypted State Preparation Ideal Resource that would securely provide the Server with the single-client VBQC client-encrypted state and provide the Classical SMPC with the secret parameters of this state, this would solve the problem entirely. The Classical SMPC would just have to instruct the Server according to the VBQC protocol. Unfortunately, such Ideal Resource would be hard to construct as it would require to find a protocol that is already performing some kind of MPQC – albeit a simple one. The problem arises from the fact that each Client should in that case be able to verify that the output state of the protocol implementing this State Preparation Ideal Resource is indeed a correctly generated VBQC client-encrypted state (due to the fact that an MPQC Resource is usually defined so that it either produces the correct output or aborts).

What we can aim for instead is to ask the resource to produce a correct VBQC client-encrypted state up to a deviation chosen by the Server (independent on any of the secret parameters), while maintaining the blindness about the state prepared, i.e. the position, type and encryption of each qubit. This would be sufficient for our purpose as the deviation on the VBQC client-encrypted state could then be treated as a deviation by the malicious parties during the execution of the VBQC Protocol. In turn, this looser objective can be accomplished by using our DBQC Ideal Resource and we describe in the following the inputs of each party to this Resource. Clients will be assigned to each base-location of the Dotted-Triple Graph used in the VBQC Protocol (how this choice is performed is described later) and supply rotated $|+\theta\rangle$ states for these positions (three for primary locations and nine for added locations). Naturally, the input base-locations are assigned to the Client who is supposed to provide the associated input in the VBQC computation. For these input base-locations, the Client chooses one position at random to send its encrypted input qubit, and for the other two sends $|+\theta\rangle$ states. It communicates the position of its input to the Classical SMPC, which then chooses a colouring of the Dotted-Triple Graph that satisfies these positioning requirements. The Classical SMPC (acting as the Orchestrator) can choose a Clifford unitary $U$ so that it turns some of the $|+\theta\rangle$ states into dummies and applies identity on the others, keeping them for the traps and computations.

Section 4 gives an explicit DBQC Protocol for implementing the DBQC Ideal Resource and it will be proved in Section 5 that, by carefully choosing the measurement pattern of DBQC, the output state can be forced to be any VBQC client-encrypted state up to a deviation by the Server that does not depend on the secret parameters of the state. This DBQC Protocol will consist of two phases: a state preparation step – during which each Client sends both the qubits that will become part of the VBQC Dotted-Triple Graph and auxiliary qubits that are only used during the DBQC Protocol – and a computation step corresponding to an execution of UBQC on the collaboratively-generated state resulting from this state-preparation. The MPQC Protocol resulting from these modifications is represented Figure 3.
Figure 3: Replacing the classical steps of the single Client in VBQC with a Classical SMPC and the VBQC client-encrypted preparation with a possibly-deviated multi-party state preparation with the participation of Clients $C_i$. The state $\rho'_{DT(G)}$ corresponds either to the honestly prepared VBQC client-encrypted state if there was no deviation, or a deviated state $E(\rho_{in,S})$ where $\rho_{in,S}$ corresponds to a state containing the honest Client’s input and VBQC state qubits and an auxiliary state supplied by the malicious coalition. At the end, each Client $C_i$ receives its part of the encrypted output state and its associated Q-OTP key $K_i$ (unless there was an abort $Ab$).

We need to make a final modification to the VBQC Protocol: we will assume that the MBQC base graph used to perform the joint unitary has degree 1 for all input qubits.\(^5\) Then, after the entanglement operation on the $DT(G)$ has been performed, the Server applies an encrypted bridge operation – see Appendix B – instructed by the Classical SMPC on the added vertices corresponding to the inputs’ only edge. This is motivated by our security analysis from Section 5.2 which provides constraints on DBQC computations so that even though they are not verifiable, they can still be used to bootstrap the security of the full protocol.

To summarise, the Delegated MPQC Protocol consists of the following steps:

1. The Clients send qubits for the DBQC state preparation. These include qubits that will be part of the $DT(G)$ and qubits for realising the DBQC Protocol, all are either their encrypted input qubits or rotated qubits $|+\rangle$. The inputs are placed by each Client uniformly at random among the qubits of their associated base-location. The corresponding secret parameters are sent to the Classical SMPC.

2. The Server performs the rest of the DBQC Protocol with the Classical SMPC, which instructs it to measure some qubits in a rotated basis that depends on the secret parameters above. The aim of this DBQC Protocol is to transform some of the encrypted $|+\rangle$ states into dummies while keeping the rest of the state as is. The Server returns the measurement results from the DBQC Protocol to the Classical SMPC. At the end of this step, the Server is in possession of a VBQC client-encrypted state whose parameters are known to the Classical SMPC (up to a final deviation).

3. The Classical SMPC instructs the Server to perform the VBQC Protocol performing the Clients’ computation on this prepared state, with the only difference being that the first step after entangling the $DT(G)$ graph state consists of encrypted bridge operations for computation qubits for input edges (as described in Appendix B). At the end, the Server returns to each Client all the qubits in their assigned output.

\(^5\)The universal brickwork state satisfies this property, meaning that our assumption does not restrict the computations that can be performed using our scheme.
3.3 Usage of the Classical SMPC Ideal Resource

We have seen above that the Classical SMPC allows the Clients to perform collaborative tasks in a secure way. We detail below how it mediates the interactions between the Clients themselves and between the Clients and the Server.

**Initialisation SMPC.** This first call to the SMPC prepares its own internal state (there is no output at this stage): it receives from each Client a classical description of the secret parameters of all qubits that the Clients have sent to the Server along with the location of its input among them. Some of those qubits correspond to the ones that will be used as inputs to produce the VBQC client-encrypted state upon which the actual computation will take place, while others are used to perform the DBQC Protocol which will either apply a Hadamard operation on qubits that are destined to become dummies or Identity to the rest (so that the combined qubits form a Dotted-Triple Graph). The Classical SMPC chooses the colouring of the Dotted-Triple Graph associated with the base graph (the one computing the unitary chosen by the Clients). This colouring must be compatible with the positions of the Clients’ inputs (i.e. the computation qubit in the input base-locations is determined by each Client’s choice). The Classical SMPC then defines the unitary applied through the DBQC Protocol: the colouring deterministically defines at which positions to apply the $H$ and $I$ operations.

**Trusted Orchestrator SMPC.** The second execution of the Classical SMPC is called during the DBQC Protocol used to apply these operations. Protocol 2 requires an honest Orchestrator for it to construct the DBQC Resource 3. Since there is no such party in the actual DMPQC Protocol below, the aim of this call is to implement this trusted party. During its first call, the Classical SMPC has stored in its internal state the transformation that should be applied (the location of the $H$ and $I$ gates required to turn some qubits into dummies while leaving the others unaffected). It then transforms it into an MBQC computation. It receives classical information from the Server during the DBQC State Preparation protocol (corresponding to measurement results) and, knowing the state of the qubits sent by the Clients for the DBQC State Preparation, deduces the state of the qubits in the computation graph after the DBQC State Preparation step. It can then instruct the Server with the measurement angles defined by the DBQC Protocol as an honest Orchestrator would. Since the unitary that is being applied consists only of Clifford operations, the SMPC can send all the measurement angles at once, the corrections will simply translate into a different Q-OTP key on the final state. The Server returns the measurement outcomes of all measured qubits to the Classical SMPC.

**Classical Instructions SMPC.** Finally, since it knows all the parameters of the Dotted-Triple Graph and the encryption of the quantum state after the DBQC protocol, it can also drive the VBQC computation by instructing the Server to measure all non-output qubits of the DTG in a certain angle. For the last layer, it tells to each Client where its computation, trap and dummy qubits are. It sends them the measurement angle for the traps and recovers the measurement outcomes. It knows internally the correct value of the traps and so can verify that they have been all correctly measured (the ones measured
by the Server during the computation and the ones now measured by the Clients. If the verification passes, it sends to each Client the keys for decrypting its output.

4 Constructing the Double-Blind Quantum Computation Resource

High-Level View of the Protocol and Proof. Similarly to the construction of the MPQC Ideal Resource, the DBQC Ideal Resource of Section 2.1.2 will be assembled from a DBQC State Preparation (Protocol 1) followed by a regular single-client UBQC Protocol. The goal of Protocol 1 is to produce an encrypted qubit for each non-output vertex in the UBQC computation graph $G$, and to provide the Orchestrator with the corresponding key. The Orchestrator then chooses the computation to be performed and instructs the Server’s measurement by following the single-client UBQC protocol. The only difference with regular UBQC is that the output is then left encrypted at the Server’s interface while the Orchestrator keeps the corresponding decryption key.

DBQC State Preparation. In order to satisfy the blindness condition against coalitions of the Server and up to $N - 1$ Clients, the DBQC State Preparation (Protocol 1) must ensure that no coalition of less than $N$ Clients has enough information about the encryption keys of the qubits used in the UBQC Protocol. This imposes that the encryption is performed collaboratively by all the Clients, each sending at least one qubit per non-output location of the UBQC computation graph $G$. This way, each qubit used in the UBQC Protocol would always receive random bits from at least one honest Client. In the Protocol, each Client encrypts its inputs and provides encrypted $|+\rangle$ states as if it was running the UBQC protocol all by itself replacing the inputs held by other Clients by $|+\rangle$ states – i.e. these additional qubits are provided for all locations of the computation graph except for those corresponding to the Client’s input. Upon receiving the qubits of each Client, the Server applies a series of CNOT’s between the qubits corresponding to a given location in $O^c$ followed by a computational basis measurement of all but one qubit. By properly choosing the control and targets of the CNOT’s this results in a single qubit per location whose encryption depends on the secret parameters of all the Clients.

We denote with $\Omega$ a function from the set of input qubits $I$, to the set of Clients $[N]$, that determines which Client “owns” the input qubit at location $i$ and is thus supposed to provide it. The domain of $\Omega$ is extended to all the vertices in $O^c$, so that $\Omega$ is unchanged on $I$ and is constant and equal to $N$ on $I^c \cap O^c$. The ownership of non-input qubits can be symmetrised across Clients - for example by choosing the owner of these qubits at random - but here the last Client is always chosen as their owner for simplicity of presentation (this can be done without loss of generality since these qubits are prepared the same way regardless of the computation or input: in any case, one client’s qubit remains unmeasured for each location and it does not change anything in the protocol or proof to fix it in advance for non-inputs.). The formal version of the DBQC State Preparation is given in Protocol 1. We denote $X_j$ the input register of Client $j$.

This DBQC State Preparation is represented for one final qubit and eight Clients in Figure 4 for input state $\rho(i)$. More precisely, after the protocol, the state of unmeasured qubit at location $i$ is $Z(\theta(i))X^{\rho(i)}Z(\theta(i))$ for $i \in I$, and $|+\theta(i)\rangle$ for $i \in O^c \setminus I$, where $\rho(i)$ is the reduced input state for qubit $i \in I$ and

$$\theta(i) = \theta_{\Omega(i)}(i) + \sum_{j \neq \Omega(i)} (-1)^{l(j)(i)}\theta_j(i). \quad (2)$$
**Protocol 1 DBQC State Preparation**

**Client** \( j \) For each non-output qubit location \( i \in O^c \), prepares a state in the following way, sends the qubit to the Server and the classical values to the Orchestrator (through a classical secure channel):

- If \( i \in I \) and \( \Omega(i) = j \), it chooses \( a(i) \in \{0,1\} \) and \( \theta_{\Omega(i)}(i) \in \Theta \) and applies \( Z(\theta_{\Omega(i)}(i))\mathcal{X}^{a(i)} \) to its input register \( X_j \);
- Otherwise, it chooses \( \theta_j(i) \in \Theta \) and prepares \( |+_{\theta_j(i)}\rangle \).

**Server** For each output qubit location \( i \in O \), it prepares a \( |+\rangle \) state. Otherwise (for \( i \in O^c \)):

1. For each Client \( j \neq \Omega(i) \), the Server applies a CNOT between the state received from Client \( \Omega(i) \) (control) and the qubit received in state \( |+_{\theta_j(i)}\rangle \) from Client \( j \) (target);
2. It measures the (target) qubit sent by Client \( j \neq \Omega(i) \) in the computational basis and defines \( t(i) = (t_j(i))_{j\neq \Omega(i)} \) as the vector of outcomes of the measurements \( t_j(i) \) whose coordinates are labelled by the Client \( j \) providing the measured qubit. It publicly broadcasts \( t(i) \).

To prove this, let \( \rho \) and \( |+_{\theta}\rangle \) with \( \theta \in \Theta \) be two quantum states. Then, we apply a CNOT gate with the first state as control and the second as target, followed by a measurement of the second qubit in the computational basis. A simple calculation shows that the resulting state of the first qubit, depending on the outcome \( t \) of the measurement, is \( Z((-1)^t\theta)\rho Z^\dagger((-1)^t\theta) \). Replacing this in the sequence of CNOT’s and measurements performed by the Server at each location \( i \) in \( V \) where the control is the qubit sent by Client \( \Omega(i) \) and the target qubits are those sent by the Clients \( j \neq \Omega(i) \) yields the result.

Note that after the DBQC State Preparation, the state of the qubits held by the Server could have been obtained by having each Client send its inputs to a quantum-enabled Orchestrator and let it perform the state preparation of a single-client UBQC Protocol provided it chose to Q-OTP the inputs by \( Z(\theta(i))\mathcal{X}^{a(i)} \) and to rotate the other qubits by \( \theta(i) \) along the \( Z \) axis. What is important here, is that the computed angles \( \theta(i) \) from equation 2 will appear random to any party as long as at least a single \( \theta_j(i) \) is unknown. This property will grant the security of our Protocol against collusions of up to \( N - 1 \) Clients.

**Double Blind Quantum Computation Protocol.** Using the above notations, and for a given \( U \) to be implemented on a graph state defined by \( G \) and flow \( f \), Protocol 2 performs a Double-Blind Quantum Computation. It first applies Protocol 1, followed by the entanglement step given by the edges of \( G \). Before the computation step starts, the Clients privately send to the Orchestrator some random binary values \( r_j(i) \) for \( i \in O^c \) that will be combined into \( r(i) = \bigoplus_j r_j(i) \) to OTP the measurement outcomes obtained by the Server. Given \( \theta(i) \) and \( a(i) \), the Orchestrator then instructs the Server to perform single qubit measurements so that at the end of the computation step, the Server has a Q-OTP version of the quantum output of the computation whose encryption key \( s \) is given by the values of \( s_X(i) \) and \( s_Z(i) \), which are defined in Section 2.2.1. The termination step simply consists of the Orchestrator setting its key \( k \) to be equal to \( s \).

We now state the main result of this section, which we prove in Appendix E.
(a) The Server receives the qubits and applies CNOT gates (the central qubit is the control, the rest are targets).

(b) The Server measures all qubits but the central one in the computational basis and gets outcomes $t_j(i) \in \{0, 1\}$.

Figure 4: DBQC State Preparation for qubit $i$. The owner $\Omega(i) \in [8]$ of the qubit supplies the middle one in state $\tilde{\rho}(i) = Z(\theta_{\Omega}(i))X^{a(i)}(\rho(i))$, every other Client $j$ supplies $|+\theta_j(i)\rangle$. The resulting rotation angle on the central qubit after the protocol is completed depends on the values of $\theta_j(i)$ and $t_j(i)$ of all the Clients and is given by Eq. (2).

Protocol 2 Double Blind Quantum Computation

**State preparation:** The Clients and Server perform the DBQC State Preparation (Protocol 1).

**Entanglement:** The Server entangles the qubits obtained at the end of the State Preparation according to the edges of the computation graph $G$ with CZ gates.

**Computation:** For $i \in O^c$:

1. Each Client $j$ sends $r_j(i) \in_R \{0, 1\}$ to the Orchestrator, who recombines them as $r(i) = \bigoplus_j r_j(i)$;
2. As in UBQC, the Orchestrator computes the measurement angle $\delta(i) = \phi'(i) + \theta(i) + r(i)\pi$ and sends this angle to the Server;
3. The Server measures location $i$ along the $\{|+\delta(i)\rangle, |-\delta(i)\rangle\}$ basis and returns the measured outcome $b(i)$ to the Orchestrator.

**Termination:** The Orchestrator sets $k = (s'_X(i), s'_Z(i))_{i \in O}$.

**Theorem 1** (Composability of Double-Blind Quantum Computing). Under the assumption that at least one Client is honest and that the Orchestrator is honest, Protocol 2 $\epsilon_D$-constructs the Double-Blind Quantum Computing Ideal Resource 3 for $\epsilon_D = 0$ (i.e. perfect security).

5 Using DBQC to Bootstrap Verification

In this section, we show how the DBQC Protocol can be used to prepare VBQC client-encrypted states. Subsection 5.1 give one way of implementing the necessary transformations – i.e. applying $H$ gates on dummies while leaving the other qubits untouched – while subsection 5.2 gives constraints on the DBQC computation so that, although it is not verified, any deviation occurring at that stage is independent of the chosen VBQC client-encrypted state and thus will be handled by the VBQC protocol itself. It also shows that our proposal satisfies these additional constrains.
5.1 VBQC Client-Encrypted State Preparation Using DBQC

5.1.1 Subtleties Regarding DBQC Input Qubit Encryptions.

We start by emphasizing some fine points regarding how the DBQC Protocol implements the unitaries used to transform the inputs of all Clients into the VBQC client-encrypted state.

The input states to the DBQC sent by the Clients are of two types: either they correspond to their actual inputs to the DMPQC Protocol, in which case they are Q-OTPed with $Z(\theta)X^a$, or they are states of the form $|+\rangle$ for a random $\theta$ of their choice. The latter are destined to become the non-input qubits of the VBQC client-encrypted state prepared by the DBQC Protocol: either dummies, traps or computation qubits.

Hence, some qubits need to be turned into dummies, while the others must remain either $|+\rangle$ states or encrypted inputs. While this is the goal of the application of DBQC, precautions need to be taken regarding the proper implementation of the operations that are performed during this step. We begin by noting that, since the DBQC Protocol requires an Q-OTP encryption of its inputs using operators of the form $Z(\theta)X^a$, it is possible to view a $|+\rangle$ state in two different mathematically equivalent ways: it is either a state $|+\rangle$ encrypted with $Z(\theta)X^a$ or a state $|+\theta\rangle$ encrypted with $Z(\theta - (-1)^a \theta')X^a$, for an arbitrary choice of $\theta' \in \mathbb{R} \Theta$ and $a \in \mathbb{R} \{0, 1\}$. It is thus possible for the Orchestrator to arbitrarily choose to change the encryption of the states for free by picking a different rotation angle $\theta'$.

Specifically, to prepare the dummies, we consider the $|+\theta\rangle$ states as encrypted $|+\rangle$ states, upon which the DBQC computation must have the effect of applying $H$ to obtain a state in the computational basis. For the computation and trap qubits on the other hand, we remark that using the angles as in the original UBQC Protocol has the effect of striping away the $Z(\theta)$ encryption and replacing it with a standard Q-OTP. This is useful for the dummies but must be avoided for other qubits as they have to remain rotated to serve as computation and trap qubits. However, the Orchestrator can at will choose $\theta'$ and consider the $|+\theta\rangle$ state as an encrypted $|+\theta'\rangle$ state. Then the result of the operations performed through DBQC on all trap and non-input computation qubits with this encryption must be equivalent to applying an $I$ gate on these states $|+\theta'\rangle$. By doing so, the output states retain a $Z(\theta')$ encryption that can be chosen by the Orchestrator as required to apply the VBQC protocol.\(^6\)

For input qubits, using the angles as in the original UBQC Protocol also has the effect of striping away the $Z(\theta)$ encryption and replacing it with a standard Q-OTP. Yet, at the beginning of the DBQC Protocol, the DMPQC inputs start off encrypted using $Z(\theta)X^a$ and they must be similarly encrypted at the beginning of the VBQC execution. We can then use the same trick as above for the computation and trap qubits and consider them as encrypted with $Z(\theta - \theta')X^{a \oplus a'} Z((-1)^{a \oplus a'} \theta')X^{a'}$, for a random choice of $\theta'$ and $a'$. Then the parameters $\theta - \theta'$ and $a \oplus a'$ are used in the computation phase of DBQC, while $(-1)^{a \oplus a'} \theta'$ and $a'$ are used in the VBQC execution (these need to be further updated first to account for the additional encryption generated through the DBQC Protocol execution).

5.1.2 H/I-Gadget.

We show below how the Orchestrator can choose to apply the identity or a Hadamard gate blindly to a quantum state using DBQC. We will construct a measurement pattern

\(^6\)Note that this trick is necessary because, in the DBQC Protocol, the last layer of qubits are prepared by the Server in the state $|+\rangle$, which strips the $Z(\theta)$ encryption.
such that it is the same for each qubit in the DT($G$) (regardless of whether it is an input, computation, trap or dummy) up to the first measurement angle. This is required so that no possible attack in the DBQC depends on secret parameters defining the final VBQC client-encrypted state (see Section 5.2 for details).

To this end, we start with the circuit from Figure 5, where $\rho$ is the arbitrary quantum state that we wish to modify, and $|\psi\rangle$ is used to control which operation is performed.

$$\rho \rightarrow Z(-\pi/2)HZ(-\pi/2)X \rightarrow \pm$$

**Figure 5:** Circuit implementation of the $H/I$-Gadget

Above, the first gate $Z(-\pi/2)HZ(-\pi/2)$ is a rotated Hadamard gate. It maps the $\{|0\rangle, |1\rangle\}$ eigenvectors of $Z$ onto the $\{|+\rangle, |\rangle\rangle\rangle/2\}$ eigenvectors of $Y$ and vice-versa. Following the sequence of gates above, for $|\psi\rangle = \alpha |+\rangle + \beta |\rangle\rangle/2\rangle$, the state of the first and second qubits before the measurement is:

$$\alpha Z(\rho) |0\rangle - i\beta X(\rho) |1\rangle$$

As a consequence, when $|\psi\rangle$ is $|+\rangle$ it applies $Z$ to the first qubit, irrespectively of the measurement outcome on the second. Similarly, when $|\psi\rangle = |\rangle\rangle/2\rangle$, it applies $X$ to the first qubit.

The same computation can be carried out for $|\psi\rangle = |\rangle\rangle/2\rangle$, which reduces the state of the first qubit to $1/\sqrt{2} (Z - X)(\rho) = ZXH(\rho)$.

Finally, when $|\psi\rangle = |\rangle\rangle/2\rangle$, the 0-outcome corresponds to applying $ZXH$ to the first qubit state while the 1-outcome applies $H$.

Hence, by properly implementing Pauli corrections, the circuit from Figure 5 applies the identity or a $H$ depending on the state of the input for the second qubit. If we further simplify the circuit by commuting the second $X$ gate for the second qubit through the controlled-NOT gate and absorb the resulting $X$ gate on the first qubit into corrections, and the one on the second qubit into the measurement, we obtain the corrections of Figure 6.

| Bottom qubit | Outc. | Corr. | Effect | Bottom qubit | Outc. | Corr. | Effect |
|--------------|------|-------|--------|--------------|------|-------|--------|
| $|+\rangle$   | 1    | Y     | 1      | $|+\rangle$   | 0    | X     | H      |
| $|\rangle\rangle/2\rangle$ | 0    | Y     | 1      | $|\rangle\rangle/2\rangle$ | 0    | Z     | H      |
| $|\rangle\rangle/2\rangle$ | 0    | I     | 1      | $|\rangle\rangle/2\rangle$ | 0    | Z     | H      |
| $|-\rangle\rangle/2\rangle$ | 0    | I     | 1      | $|\rangle\rangle/2\rangle$ | 0    | Z     | H      |

**Figure 6:** Corrections for the $H/I$-Gadget.

We can now turn this last version of the circuit into a measurement pattern by recalling that for a graph with 2 connected vertices, when the first qubit in state $\rho$ is measured along an angle $\theta$, the output qubit (if it is initially in the $|+\rangle$ state) is in the state $Z(\theta)H(\rho)$. 21
Additionally, we use the identity $XZ(−\pi/2)HZ(−\pi/2) = Z(\pi/2)HZ(\pi/2)$. We then obtain the measurement pattern from Figure 7.

Instead of considering that the second input qubit is in one of the states of the set
\[\{\langle +\rangle, \langle −\rangle, \langle +\rangle, \langle −\rangle\},\]
we can fix it to be an encrypted $|+\rangle$ state. Changing the first measurement angle of the bottom line to $−\pi/2$ can then be seen as applying first a $Z(\pi/2)$ rotation to $|+\rangle$, thus transforming it to $|+\rangle$, before measuring in the $|\pm\rangle$ basis. Hence, by properly choosing the measurement angle of the first qubit of the bottom line, the Orchestrator can choose whether to apply $H$ or $I$.

To summarise, for all positions in the $DT(G)$ used in the DMPQC Protocol, the measurement pattern in Figure 7 is applied by considering the following possible cases (using the considerations in Section 5.1.1):

- If the output qubit in the measurement pattern is a dummy in the $DT(G)$, then the upper input qubit is considered to be an encrypted $|+\rangle$ state and the measurement angle of the lower input qubit is chosen by the Orchestrator to be $0$.
- If the output qubit is a trap, computation or input, the upper input qubit is treated as either an encrypted $|+\rangle$ state (trap or non-input computation) or an encrypted input state $Z(\theta)X^a(\rho_i)$ and the measurement angle of the lower input qubit is $−\pi/2$.

Finally, anticipating on the result proven in the next Subsection, we remark that the graph used for the computation is invariant with respect to a change of $DT(G)$ coloring. Additionally, the measurement angles are in \(\{0, \pi/2, \pi, −\pi/2\}\) which implies that, as a Clifford computation, all the measurements during the DBQC Protocol can be performed non-adaptively and sent in a single round of communication. The corrections are taken into account by updating the Q-OTP key encrypting the states after all measurements have been carried out.

5.2 Effect of Adversarial Deviation during DBQC on Prepared State

Proving the security of our DMPQC Protocol requires the DBQC Protocol to produce a VBQC client-encrypted state up to a CPTP deviation which is independent of the secret parameters of the state, which consist of the encryption of the state and the colouring of the $DT(G)$. Plugging these deviated states in the VBQC Protocol of [25] preserves its original security properties, as its proofs of verifiability and blindness begin precisely with such a good-enough VBQC execution state. We show in this Section that applying the DBQC Protocol using the MBQC pattern defined above yields precisely such a state (through Theorem 2 and Lemma 2).

We start by defining good-enough VBQC execution states in Definition 3.

**Definition 3** (Good-Enough VBQC Execution States). We say that a quantum state is a good-enough VBQC execution state if it is of the form $E(\rho_V \otimes \rho_{aux})$, where $E$ is a CPTP map depending only on the public and dishonest party parameters, $\rho_{aux}$ is an auxiliary
adversarially-chosen state and \( \rho_V \) is the state of quantum systems comprising quantum input, computation, trap, dummy and rotation qubits such that:

1. Quantum inputs are encrypted by operator \( Z(\theta(i))X^{a(i)} \), for \( \theta(i) \in \{0, 1\} \).
2. Quantum computation and trap qubits are initialized in \( \ket{\pm \theta(i)} \), for \( \theta(i) \in \Theta \).
3. Quantum dummy qubits are of the form \( \ket{d(i)} \), for \( d(i) \in \{0, 1\} \).
4. Traps, computation, input and dummy qubits are placed according to a verifiable graph \( G \), e.g. the Dotted-Triple Graph \([25]\), which is defined as follows:
   - The probability, taken over all possible auxiliary states, for the Server to guess that any qubit in the graph is a trap is lower-bounded by a constant value \( p_t \).
   - Trap vertices are linked only to dummy vertices.
5. Rotation qubits are in classical states of the form \( \ket{\delta(i)} \) represented as computational basis states on three qubits (as multiples of \( \pi/4 \)).

In the above, \( \delta(i) = (-1)^{a(i)} \phi'(i) + \theta(i) + r(i)\pi + \bigoplus_{i \in N_G(i) \cap D} d(\tilde{i})\pi \), for \( r(i) \in \{0, 1\} \), \( \phi'(i) \) the flow-corrected angle on qubit \( i \) for a branch of computation corresponding to the Server’s measurements \( b \) (in \( \Theta \) for computation qubits and \( 0 \) for trap qubits) and \( D \) the set of dummy qubits in the verifiable graph \( G \) and \( N_G(i) \) the neighbours of qubit \( i \) in \( G \).

This definition captures the idea of a state that, at the end of an execution of a VBQC Protocol (i.e. for a branch of the computation fixed by the Server’s measurements), allows an honest party to verify the correct application of this VBQC computation by checking the traps using secret parameters \( a(i) \) and \( r(i) \). While this definition is given here for the specific framework used in this work, it could potentially be generalised to other verifiable protocols.

Theorem 2 shows that the state prepared by the DBQC Protocol 2 is indeed a good-enough VBQC execution state. More generally, it gives a framework for DBQC computations that lead to good-enough VBQC execution states using Clifford computations. Its proof can be found in Appendix F. On the other hand we conjecture that this cannot be extended to all QPT computations, because it would imply we can reduce any verification protocol to a protocol where only the last layer is trappified (by doing the actual computation in the preparation phase).

**Theorem 2** (Constraints for Good-Enough VBQC Execution State Preparation through DBQC). The output of DBQC Protocol 2 using an MBQC pattern defined by a fixed graph \( G = (V, E, I, O) \) with flow \( (f, \preceq) \) and base angles \( \{\phi(i)\}_{i \in V} \) is a good-enough VBQC execution state for the Clients’ desired computation if all the following hold:

- For an honest input to the DBQC Protocol, the honest application of the measurement pattern produces a correct VBQC client-encrypted state \( \rho_G \) along with measurements angles \( \delta(i) \) that correspond to the correct desired computation for any fixed computation branch.
- All measurement angles for the DBQC computation are Clifford, i.e. \( \phi(i) \in \{k\pi/2\}_{0 \leq k \leq 3} \).
- The graph \( G = (V, E, I, O) \), the flow \( (f, \preceq) \) and all angles associated to vertices that have an \( X \)-dependency according to the chosen flow are all invariant under a change of colouring of the Dotted-Triple Graph.

Theorem 2 above only guarantees that the attack depends on public and adversarial parameters. The public parameters consist of the graph and flow but may or may not
include additional information about the input to this computation or measurement angles. The following Lemma (proven in Appendix G) shows that the computation described in Section 5.1.2 both satisfies the constraints of this theorem and also does not leak additional information about the honest player’s input which would allow the Adversary to attack it.

**Lemma 2 (Secure Multi-Client VBQC State Preparation).** Assume that the base graph chosen to perform an MBQC computation through VBQC has degree 1 on all input vertices. Apply the DBQC Protocol using the MBQC pattern from Figure 7 for each qubit of the final DT(G) as described in Section 5.1.2 on an input state constructed as: (i) MBQC input states and |+⟩ states for non-input positions in the DT(G) and (ii) the position of the MBQC input of honest players is randomly chosen in its associated input base-location. The final state then consists of E(ρV ⊗ ρaux), where ρV is a correctly-prepared VBQC client-encrypted state, ρaux is an arbitrary auxiliary state prepared by the Adversary and the deviation E is independent of ρV.

Using Lemma 2 allows us to reuse the security analyses of [25] for our composed protocol, therefore proving the full security of the DMPQC Protocol 3. For completeness sake, we give an analysis of how these states are used in the proof of verifiability of [25] in Appendix H. Furthermore, the general formulation of Theorem 2 gives conditions to satisfy for designing other, possibly more efficient, preparation protocols for good-enough VBQC execution states.

6 Full Delegated MPQC Protocol and Security Statement

We now present the DMPQC Protocol 3 constructing Resource 1 in more detail. As mentioned in Section 3, we assume that the base graph chosen by the Clients for performing the initial MBQC computation of the joint unitary has degree 1 on all input vertices of set I.

**Theorem 3 (Composable Security of the DMPQC Protocol).** If the VBQC Protocol of [25] ϵV-statistically-constructs the Verifiable Delegated Quantum Computation Ideal Resource 2 and the DBQC Protocol 2 ϵD-statistically-constructs the Double-Blind Quantum Computation Ideal Resource 3 from Insecure Quantum Channels, then the DMPQC Protocol 3 (ϵV + ϵD)-statistically-constructs the Multi-Party Quantum Computation Ideal Resource 1 from Insecure Quantum Channels and a Classical SMPC Ideal Resource.

We finally note that, since the DBQC Protocol perfectly emulates the DBQC Ideal Resource (i.e. ϵD = 0), the DMPQC Protocol ϵV-emulates the MPQC Ideal Resource.

7 Conclusion

The purpose of any cryptographic protocol is to mimic the ideal scenario where all players send their inputs to a trusted third party and later recover their outputs. This minimal setting implies that at least two rounds of communication are needed even when

---

7See Protocol 3: for classical outputs, the Server measures all qubits, sends the results to the Classical SMPC, that either aborts or sends the decrypted outputs to the Clients depending on the trap verification procedure.
Protocol 3 Delegated Multi-Party Quantum Computation

Inputs: The N Clients have collaboratively chosen a unitary $U$ acting on $M = \#I$ qubits, as an MBQC measurement pattern over graph $G$ (measurement angles $\{\phi(i)\}_{i \in O^c}$ for non-output qubits in the graph and a flow). Each honest Client $j \in [N]$ has a quantum register $\mathcal{X}_j$ which contains their respective part of a collectively possessed state $\rho_{inp}$ over $M$ qubits.

The Protocol:

1. For each input qubit $i$ such that $\Omega(i) = j$, Client $j$ chooses an index $p_i \in R \{1, 2, 3\}$, indicating in which position in its input base-location it will send its input qubit $i$. It sends this value to the Classical SMPC. For this input base-location, it prepares two $|+\rangle$ states for positions $\{1, 2, 3\} \setminus p_i$. The Client $N$ also prepares $|+\rangle$ states for the non-input qubits of the Dotted-Triple Graph. The Classical SMPC chooses uniformly at random a colouring of the Dotted-Triple Graph compatible with these input positions and deduces the unitary applying Hadamard gates to dummies.

2. The Clients and the Server perform the Double-Blind Quantum Computation Protocol 2 with the Classical SMPC as the Orchestrator, which uses this transformation above as the DBQC unitary.

3. The Server entangles using $\text{CZ}$ gates the qubits obtained at the end of DBQC Protocol according to the edges of the Dotted-Triple Graph $DT(G)$.

4. The Classical SMPC instructs the Server to perform bridge operations on each qubit in the base-locations of the edges linked to the input layer by measuring them according to the following angles:
   - Traps and dummies are measured as in the VBQC Protocol (random angle for dummies, correct angle for traps up to a random $\pi$ rotation).
   - The computation qubit is used as a bridge and the angle $\delta$ sent to the Server uses the base angle $\phi = \pi/2$. Let $b$ be the measurement result.
   - The $\theta$ angles of the primary computational qubits in the two base locations linked to this edge are updated by adding $(-1)^{1+b}\pi/2$.

5. The Classical SMPC instructs the Server to measure each remaining qubit $i \in O^c$ with an angle $\delta(i)$ computed according to the VBQC Protocol. The Server measures the qubit $i$ along the $\{ |+\delta(i)\rangle, |\delta(i)\rangle \}$ basis and returns the measurement result $b(i)$ to the Classical SMPC.

6. Quantum Output Key-Release Procedure:
   - (a) The Server sends to each Client the three qubits from the Dotted-Triple-Graph that correspond to their output base-locations.
   - (b) Each Client receives from the Classical SMPC the position of the computation, trap and dummy qubit and measurement angle $\delta(i)$ for the trap.
   - (c) Each Client measures its output trap qubit in the basis $\{ |+\delta(i)\rangle, |\delta(i)\rangle \}$ and sends the result to the Classical SMPC.
   - (d) The Classical SMPC verifies that all traps have been measured correctly using the same verification procedure as in the VBQC protocol and sends $\text{Abort}$ to all Clients if it fails. Otherwise, it sends to each Clients the Q-OTP key for the output computation qubit.
   - (e) Each Client undoes the Q-OTP on its output computation qubit and sets this quantum register as its output.
one assumes that the third participant is indeed honest. In this respect, we achieve an almost-perfect delegated multi-party quantum computation: the quantum communication between all Clients and the Server is round-optimal, while at the same time removing all trust requirements between participants.

This is accomplished through a deconstruction-reconstruction process of a single Client protocol where a new blind but not verifiable protocol is introduced to allow multiple Clients to prepare a good-enough VBQC execution state collaboratively. This new protocol is highly versatile and can benefit to situations where a unitary must be applied but remain unknown to all participants. In our case, by adding straight-forward conditions on the form of the input and the unitary (namely that they do not leak information about the final good-enough VBQC execution state and that the unitary is Clifford), we are able to use the generated state to perform a fully verifiable computation. This depth-independent state preparation step effectively bootstraps the verifiability of any computation by using blindness alone, thus breaking the oft-believed principle that verifiability of the full protocol cannot be achieved unless all sub-components are also verifiable.

**Open Questions.** This paper leaves a few questions open. The first one is whether it is possible to perform a Delegated Multi-Party Quantum Computation with strictly the same number of qubits per Client as the single Client VBQC. Lowering the classical communication requirement to a constant number of rounds is also an interesting problem. Optimal protocols exist in the classical case with only four rounds of communications [17], yet no Quantum Secure Multi-Party Computation Protocol has sub-linear classical round-complexity as of now. Yet another question is whether it is possible to construct a protocol ensuring blindness without verifiability even in the presence of client-server collusion (i.e. extending [23] to arbitrary corruptions). Finally, the Double-Blind Quantum Computation Protocol and DBQQC State Preparation that are used to prepare the state for the VBQC computation are interesting in and of themselves, implementing functionalities never defined before. Other protocols could benefit from using these same functionalities as subroutines.

**Acknowledgements**

We acknowledge support of the European Union’s Horizon 2020 Research and Innovation Program under grant agreement number 820445 (QIA).

**References**

[1] Alon, B., Chung, H., Chung, K.M., Huang, M.Y., Lee, Y., Shen, Y.C.: Round efficient secure multiparty quantum computation with identifiable abort (Nov 2020), https://eprint.iacr.org/2020.1464

[2] Ben-Or, M., Crepeau, C., Gottesman, D., Hassidim, A., Smith, A.: Secure multiparty quantum computation with (only) a strict honest majority. In: Proceedings of the 47th Annual IEEE Symposium on Foundations of Computer Science. pp. 249–260. FOCS ’06, IEEE Computer Society, Washington, DC, USA (2006). https://doi.org/10.1109/FOCS.2006.68

[3] Broadbent, A., Fitzsimons, J., Kashefi, E.: Measurement-Based and Universal Blind Quantum Computation, pp. 43–86. Springer Berlin Heidelberg, Berlin, Heidelberg (2010). https://doi.org/10.1007/978-3-642-13678-8_2
[4] Cramer, R., Damgrd, I.B., Nielsen, J.B.: Secure Multiparty Computation and Secret Sharing. Cambridge University Press, USA, 1st edn. (2015), https://dl.acm.org/doi/book/10.5555/2846411

[5] Crépeau, C., Gottesman, D., Smith, A.: Secure multi-party quantum computation. In: Proceedings of the Thirty-fourth Annual ACM Symp. on Theory of Computing. p. 643. STOC ’02, ACM, New York, NY, USA (2002). https://doi.org/10.1145/509907.510000

[6] Dankert, C., Cleve, R., Emerson, J., Livine, E.: Exact and approximate unitary 2-designs and their application to fidelity estimation. Phys. Rev. A 80, 012304 (Jul 2009). https://doi.org/10.1103/PhysRevA.80.012304

[7] Danos, V., Kashefi, E.: Determinism in the one-way model. Phys. Rev. A 74, 052310 (Nov 2006). https://doi.org/10.1103/PhysRevA.74.052310

[8] Danos, V., Kashefi, E., Panangaden, P.: The measurement calculus. J. ACM 54(2) (Apr 2007). https://doi.org/10.1145/1219092.1219096

[9] Dulek, Y., Gritli, A.B., Jeffery, S., Majenz, C., Schaffner, C.: Secure multi-party quantum computation with a dishonest majority. In: Canteaut, A., Ishai, Y. (eds.) Advances in Cryptology – EUROCRYPT 2020. pp. 729–758. Springer International Publishing, Cham (2020). https://doi.org/10.1007/978-3-030-45727-3_25

[10] Dunjko, V., Fitzsimons, J.F., Portmann, C., Renner, R.: Composable security of delegated quantum computation. In: Sarkar, P., Iwata, T. (eds.) Advances in Cryptology – ASIACRYPT 2014. pp. 406–425. Springer Berlin Heidelberg, Berlin, Heidelberg (2014). https://doi.org/10.1007/978-3-662-45608-8_22

[11] Dupuis, F., Fehr, S., Lamontagne, P., Salvail, L.: Adaptive versus non-adaptive strategies in the quantum setting with applications. In: Robshaw, M., Katz, J. (eds.) Advances in Cryptology – CRYPTO 2016. pp. 33–59. Springer Berlin Heidelberg, Berlin, Heidelberg (2016). https://doi.org/10.1007/978-3-662-53015-3_2

[12] Dupuis, F., Nielsen, J.B., Salvail, L.: Secure Two-Party Quantum Evaluation of Unitaries against Specious Adversaries, pp. 685–706. Springer Berlin Heidelberg, Berlin, Heidelberg (2010). https://doi.org/10.1007/978-3-642-14623-7_37

[13] Dupuis, F., Nielsen, J.B., Salvail, L.: Actively secure two-party evaluation of any quantum operation. In: Advances in Cryptology–CRYPTO 2012, pp. 794–811. Springer (2012). https://doi.org/10.1007/978-3-642-32009-5_46

[14] Fehr, S., Katz, J., Song, F., Zhou, H.S., Zikas, V.: Feasibility and completeness of cryptographic tasks in the quantum world. In: Sahai, A. (ed.) Theory of Cryptography. pp. 281–296. Springer Berlin Heidelberg, Berlin, Heidelberg (2013). https://doi.org/10.1007/978-3-642-36594-2_16

[15] Fitzsimons, J.F., Kashefi, E.: Unconditionally verifiable blind quantum computation. Phys. Rev. A 96, 012303 (Jul 2017). https://doi.org/10.1103/PhysRevA.96.012303

[16] Gheorghiu, A., Kapourniotis, T., Kashefi, E.: Verification of quantum computation: An overview of existing approaches. Theory of Computing Systems 63(4), 715–808 (May 2019). https://doi.org/10.1007/s00224-018-9872-3

[17] Halevi, S., Hazay, C., Polychroniadou, A., Venkitasubramaniam, M.: Round-optimal secure multi-party computation. In: Shacham, H., Boldyreva, A. (eds.) Advances in Cryptology – CRYPTO 2018. pp. 488–520. Springer International Publishing, Cham (2018). https://doi.org/10.1007/978-3-319-96881-0_17
[18] Hein, M., Eisert, J., Briegel, H.J.: Multiparty entanglement in graph states. Phys. Rev. A 69, 062311 (Jun 2004). https://doi.org/10.1103/PhysRevA.69.062311

[19] Houshmand, M., Houshmand, M., Tan, S.H., Fitzsimons, J.: Composable secure multi-client delegated quantum computation arXiv:1811.11929 (Nov 2018)

[20] Kapourniotis, T., Datta, A.: Nonadaptive fault-tolerant verification of quantum supremacy with noise. Quantum 3, 164 (Jul 2019). https://doi.org/10.22331/q-2019-07-12-164

[21] Kapourniotis, T., Dunjko, V., Kashefi, E.: On optimising quantum communication in verifiable quantum computing arXiv:1506.06943 (Jun 2015)

[22] Kashefi, E., Music, L., Wallden, P.: The quantum cut-and-choose technique and quantum two-party computation (Mar 2017)

[23] Kashefi, E., Pappa, A.: Multiparty delegated quantum computing. Cryptography 1(2), 1–20 (7 2017). https://doi.org/10.3390/cryptography1020012

[24] Kashefi, E., Wallden, P.: Garbled quantum computation. Cryptography 1(1) (2017). https://doi.org/10.3390/cryptography1010006

[25] Kashefi, E., Wallden, P.: Optimised resource construction for verifiable quantum computation. Journal of Physics A: Mathematical and Theoretical 50(14), 145306 (mar 2017). https://doi.org/10.1088/1751-8121/aa5dac

[26] Lipinska, V., Ribeiro, J., Wehner, S.: Secure multiparty quantum computation with few qubits. Phys. Rev. A 102, 022405 (Aug 2020). https://doi.org/10.1103/PhysRevA.102.022405

[27] Mantri, A., Demarie, T.F., Menicucci, N.C., Fitzsimons, J.F.: Flow ambiguity: A path towards classically driven blind quantum computation. Phys. Rev. X 7, 031004 (Jul 2017). https://doi.org/10.1103/PhysRevX.7.031004

[28] Maurer, U.: Constructive cryptography – a new paradigm for security definitions and proofs. In: Mödersheim, S., Palamidessi, C. (eds.) Theory of Security and Applications. pp. 33–56. Springer Berlin Heidelberg, Berlin, Heidelberg (2012). https://doi.org/10.1007/978-3-642-27375-9_3

[29] Maurer, U., Renner, R.: Abstract cryptography. In: Innovations in Computer Science. pp. 1 – 21. Tsinghua University Press (jan 2011), https://conference.iis.tsinghua.edu.cn/ICS2011/content/papers/14.html

[30] Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information: 10th Anniversary Edition. Cambridge University Press (2000). https://doi.org/10.1017/CBO9780511976667

[31] Rains, E.M.: Nonbinary quantum codes. IEEE Transactions on Information Theory 45(6), 1827–1832 (1999). https://doi.org/10.1109/18.782103

[32] Raussendorf, R., Briegel, H.J.: A one-way quantum computer. Phys. Rev. Lett. 86, 5188–5191 (May 2001). https://doi.org/10.1103/PhysRevLett.86.5188

[33] Unruh, D.: Universally composable quantum multi-party computation. In: Gilbert, H. (ed.) Advances in Cryptology – EUROCRYPT 2010. pp. 486–505. Springer Berlin Heidelberg, Berlin, Heidelberg (2010). https://doi.org/10.1007/978-3-642-13190-5_25

[34] Xu, Q., Tan, X., Huang, R.: Improved resource state for verifiable blindquantum computation. Entropy 22(9), 996 (2020). https://doi.org/10.3390/e22090996
We define here some additional Ideal Resources that will be used as building blocks in the rest of the paper.

**Resource 5** Authenticated Classical Channel

- **Inputs:** The Sender inputs a message $m$. The Receiver has no input. The Eavesdropper inputs $b, c \in \{0, 1\}^2$. This last interface is filtered and set to $(0, 0)$ in the honest case.
- **Computation by the resource:** If $b = 1$, it sends $m$ to the Eavesdropper. If it receives $c = 1$ from the Eavesdropper, it sends **Abort** to the Receiver. Otherwise it sends $m$ to the Receiver.

**Resource 6** Secure Classical Channel

- **Inputs:** The Sender inputs a message $m$. The Receiver has no input. The Eavesdropper inputs $(b, c) \in \{0, 1\}^2$. This last interface is filtered and set to $(0, 0)$ in the honest case.
- **Computation by the resource:** If $b = 1$, it sends $\#m$ to the Eavesdropper. If it receives $c = 1$ from the Eavesdropper, it sends **Abort** to the Receiver. Otherwise it sends $m$ to the Receiver.

**Resource 7** Insecure Quantum Channel

- **Inputs:** The Sender inputs a message $\rho$. The Receiver has no input. The Eavesdropper inputs $b \in \{0, 1\}$ and $\tilde{\rho}$. This last interface is filtered and $b$ is set to 0 in the honest case.
- **Computation by the resource:** If $b = 1$, it sends $\rho$ to the Eavesdropper. In that case it waits for a state $\tilde{\rho}$ and forwards it to the Receiver. Otherwise it sends $\rho$ to the Receiver.

**B Graph Bridge Operation**

We now describe the basic process of bridge operations on graph states, introduced in [18], which will be used in our MPQC Protocol later. The input to this operation is of the following form $CZ_{1,2}CZ_{2,3}(\rho \otimes |+\rangle \otimes \sigma)$, where $\rho$ and $\sigma$ are arbitrary qubit mixed states, the qubits are respectively indexed 1, 2 and 3, and the operation $CZ_{i,j}$ applies a $CZ$ operation to qubits $i$ and $j$. The purpose of the bridge operation is to delete the middle qubit 2 along with its corresponding edges and join qubits 1 and 3 by a new edge. The description of this operation is given by the following Protocol 4.
Measure qubit 2 in the basis \{ |+\pi/2⟩, |−\pi/2⟩ \} and record measurement result \( b \in \{0, 1\} \).

Apply operations \( Z(−\pi/2)_1 \) and \( Z(−\pi/2)_3 \), where 1 and 3 are the remaining qubits.

We now show that performing the bridge operation on the three qubit line graph is equivalent to constructing a line graph with only qubits 1 and 3.

Lemma 3 (Correctness of Bridge Operation). The output of Protocol 4 after tracing out the second subsystem is \( CZ_{1,3}(\rho \otimes \sigma) \) where the qubits are respectively numbered 1 and 3.

Proof. It is sufficient to prove the lemma in the case where \( \rho \) is a pure state of the form \( \alpha |0⟩ + \beta |1⟩ \) with \( |\alpha|^2 + |\beta|^2 = 1 \). Similarly, because the following holds for any pure state \( \sigma \), the lemma holds for any \( \sigma \).

The original line graph state is given by:

\[
\frac{1}{\sqrt{2}}(\alpha |0⟩ (|0⟩ \sigma + |1⟩ Z\sigma) + \beta |1⟩ (|0⟩ \sigma - |1⟩ Z\sigma)) \tag{4}
\]

If result of the measurement on the middle qubit is \( b = 0 \), then the remaining state is (tracing out the system containing the second qubit):

\[
\frac{1}{\sqrt{2}}(\alpha |0⟩ Z\sigma - i\alpha |0⟩ Z\sigma + \beta |1⟩ Z\sigma) \tag{5}
\]

which becomes, after the \( Z(−\pi/2)_1 \) and \( Z(−\pi/2)_3 \) operations:

\[
\alpha |0⟩ \frac{1-i}{\sqrt{2}} \sigma + \beta |1⟩ \frac{1-i}{\sqrt{2}} Z\sigma \tag{6}
\]

which corresponds to the state \( CZ_{1,3}(\rho \otimes \sigma) \) up to a global phase \( \frac{1-i}{\sqrt{2}} \).

If the result of the measurement on the middle qubit is \( b = 1 \), the remaining state is:

\[
\frac{1}{\sqrt{2}}(\alpha |0⟩ \sigma - i\alpha |0⟩ \sigma + \beta |1⟩ \sigma - i\beta |1⟩ \sigma) \tag{7}
\]

which becomes, after the \( Z(−\pi/2)_1 \) and \( Z(−\pi/2)_3 \) operations:

\[
\alpha |0⟩ \frac{1+i}{\sqrt{2}} Z\sigma - \beta |1⟩ \frac{1+i}{\sqrt{2}} \sigma \tag{8}
\]

which, after the \( Z_1 \) and \( Z_3 \) operations becomes:

\[
\alpha |0⟩ \frac{1+i}{\sqrt{2}} \sigma + \beta |1⟩ \frac{1+i}{\sqrt{2}} Z\sigma \tag{9}
\]

which corresponds to the state \( CZ_{1,3}(\rho \otimes \sigma) \) up to global phase \( \frac{1+i}{\sqrt{2}} \).

Note that, because the operations applied in Protocol 4 commute with diagonal operations, all qubits \( i \) can be pre-rotated using operations \( Z(\theta(i)) \) for arbitrary angles \( \theta(i) \).
Protocol 5 VBQC using DT(G)

- **Client’s resources and pre-computations:**
  - Client is given a base graph $G$. The corresponding dotted graph state $|D(G)|$ is generated by graph $D(G)$ that is obtained from $G$ by replacing every edge with a new vertex connected to the two vertices originally joined by that edge.
  - Client is given a sequence of MBQC measurement angles $\phi = \{\phi(i)\}_{i \in G^r}$ with $\phi(i) \in \Theta$ which when applied to the dotted graph state $|D(G)|$ performs the desired computation.
  - Client selects a trap-colouring of the dotted triple-graph $DT(G)$ according to Definition 3 of Ref. [25] by choosing independently the colours for each set $P_v$.
  - Client for all red vertices will send dummy qubits which perform break operations at these positions (let $D$ be the set of positions of dummy qubits).
  - The green graph is used for the computation and white qubits are isolated traps.
  - A binary string $s$ of length at most $3N(3c+1)$ represents the corrected measurement outcomes. It is initially set to all zeros.
  - Let $\phi'(i)$ be the measurement angle in MBQC after taking into account corrections due to previous measurement outcomes $s$ with $\phi'(i) = 0$ for trap and dummy qubits.
  - The Client chooses a measurement order on the dotted base-graph $D(G)$ that is consistent with the flow of the computation (this is known to the Server). The measurements within each set $P_v$ and $A_v$ of $DT(G)$ are ordered randomly.
  - $3N(3c + 1)$ random variables $\theta(i)$ with values taken uniformly at random from $\Theta$.
  - $3N(3c + 1)$ random variables $r(i)$ and $\#D$ uniformly random values $d(i) \in \{0,1\}$, let $d$ be the associated string.
  - $\#I$ uniformly random values $a(i) \in \{0,1\}$.
  - A fixed function $C(i, \phi(i), \theta(i), r(i), s, d, a(i)) = \phi'(i) + \theta(i) + r(i)\pi + \sum_{j \in D(i)} d(j)\pi + a(i)\pi$ that for each non-output qubit $i$ computes the angle of the measurement of qubit $i$ to be sent to the Server (where $D(i) := D \cap N_{DT(G)}(i)$ is the set of dummies that are neighbours of qubit $i$, and by convention $a(i) = 0$ if $i$ is a non-input qubit).

- **Initial Step:**
  - The Client Quantum One-Time-Pads the input qubits $\tilde{\rho}_{inp} = \bigotimes_{i=1}^l Z_{\theta(i)} X^{a(i)}(\rho_{inp})$ (where the $i$th operator acts on the $i$th input qubit). It prepares the remaining qubits as $|d(i)\rangle$ if $i \in D$ and $\prod_{j \in N_G(i) \cap D} Z^{\theta(j)}|+\theta(i)\rangle$ for $i \notin D$. It sends all the $3N(3c + 1)$ qubits to the Server, in the order of the labelling of the graph.
  - The Server receives these qubits and entangles them according to $DT(G)$.

- **For each non-output qubit:**
  - The Client computes the angle $\delta(i)$ using function $C$ and sends it to the Server.
  - The Server measures qubit $i$ in the $\delta(i)$ basis and sends the result $b(i)$ to the Client.
  - The Client sets the value of $s(i)$ in $s$ to be $b(i) \oplus r(i)$.

- **Final Step:** The Server returns the last layer of qubits (output layer) to the Client.

- **Verification:**
  - After obtaining the output qubits from the Server, the Client measures the trap qubits in the output layer with the angle $\delta(t)$ to obtain $b(t)$.
  - Client accepts if $b(i) = r(i)$ for all the trap qubits $i$.
  - Client applies corrections according to measurement outcomes $b(i)$ and secret parameters $\theta(i), r(i)$ to the output layer green qubits and obtains the final output.
C Full VBQC Protocol and Security Proof

We present here the VBQC protocol (Protocol 5) with Dotted-Triple-Graph from [25] that we use as basis for our construction. We assume that a standard labelling of the vertices of the Dotted-Triple-Graph $DT(G)$ is known to both the Client and the Server.

We now give the proof of Lemma 1, restated here for clarity.

**Lemma 1 (Composable Security of VBQC).** The VBQC Protocol $\epsilon$-statistically-constructs the VDQC Ideal Resource 2 from Insecure Quantum Channels for negligible $\epsilon$.

*Sketch.* We need to show that the VBQC Protocol satisfies the local criteria described in [10] and use their reduction result (Theorem 6.7 from that paper). There are four local criteria: correctness, blindness, verifiability and input-independence of verification. The first three are proven in [25] and the last one is satisfied if the adversary can deduce on its own whether the verification has succeeded or not without this information affecting the blindness of the protocol.

We modify the original protocol of [25] by making the Client announce at the end whether it has accepted or rejected the outcome of the computation and show that it remains secure. If the Client has not aborted due to the Server’s attack, then it has only affected non-trap qubits. This is equivalent to an attack on the UBQC protocol which is perfectly blind, meaning that this attack does not reveal any information about the Client’s input. On the other hand, if the Client aborts, then the Server’s attack has affected at least one trap qubit. This gives at most one bit of information about the state of trap qubits, which is independent of the input or the computation. This shows that the verification procedure is perfectly independent of the inputs and concludes the proof.

D Quantum One-Time Pad Security

We present here the Quantum One-Time Pad Protocol 6, which aims to construct a Confidential Quantum Channel (Ideal Resource 10) from an Insecure Quantum Channel (Ideal Resource 9) and a Secure Classical Channel (Ideal Resource 8).

The proof of security below is important as its principle is the basis for the proof of security of the DBQC Protocol. In particular, the trick of using EPR-pairs will be again crucial to transferring the deviation of a party from a state that is independent of the secret parameters and messages to another which may depend on it.

| Resource 8 Secure Classical Channel |
|-------------------------------------|
| **Inputs:** The Sender inputs a message $m$. The Receiver has no input. The Eavesdropper inputs two bits $(e, c) \in \{0, 1\}^2$. This last interface is filtered and set to $(0, 0)$ in the honest case. |
| **Computation by the Resource:** If $e = 1$, it sends $#m$ to the Eavesdropper. If it receives $c = 1$ from the Eavesdropper, it sends Abort to the Receiver. Otherwise it sends $m$ to the Receiver. |

The following statement captures the security of Protocol 6.

**Theorem 4 (Security of Quantum One-Time Pad).** Protocol 6 perfectly constructs Ideal Resource 10 (statistical security with distinguishing advantage 0).
**Resource 9** Insecure Quantum Channel

**Inputs:** The Sender inputs a quantum state $\rho$. The Receiver has no input. The Eavesdropper inputs a bit $c \in \{0, 1\}$ and quantum state $\tilde{\rho}$. This last interface is filtered and $c$ is set to 0 in the honest case.

**Computation by the Resource:** If $c = 1$, it sends $\rho$ to the Eavesdropper. In that case it waits for a state $\tilde{\rho}$ and forwards it to the Receiver. Otherwise it sends $\rho$ to the Receiver.

---

**Resource 10** Confidential Quantum Channel

**Inputs:** The Sender inputs a quantum state $\rho$. The Receiver has no input. The Eavesdropper inputs a bit $c \in \{0, 1\}$ and quantum state $\tilde{\rho}$. This last interface is filtered and $c$ is set to 0 in the honest case.

**Computation by the Resource:** If $c = 1$, it sends $\#\rho$ to the Eavesdropper. In that case it waits for a state $\tilde{\rho}$ and forwards it to the Receiver. Otherwise it sends $\rho$ to the Receiver.

---

**Protocol 6** Quantum One-Time Pad

**Inputs:** The Sender has as input a quantum register containing $n$ qubits. The Receiver and the Eavesdropper have no input.

**Protocol:**

1. The Sender samples uniformly at random a key $k = (k_Z, k_X) \in_R \{0, 1\}^{2n}$ with $\#k_Z = \#k_X = n$ and applies $Z^{k_Z}X^{k_X}$ to its quantum register (where the operation $Z^{k_Z}X^{k_X}$ is applied to qubit $i$ for single-qubit Paulis $Z^{k_Z}$ and $X^{k_X}$ controlled by the key bits $k_Z$ and $k_X$ respectively).

2. It sends the quantum register to the Eavesdropper through an Insecure Quantum Channel Ideal Resource and it sends $(k_X, k_Z)$ to the Receiver through a Secure Classical Channel.

3. The Eavesdropper transmits the received quantum state to the Receiver through another Insecure Quantum Channel Ideal Resource.

4. The Receiver applies $X^{k_X}Z^{k_Z}$ to the received quantum register and keeps the resulting state as its output.
Proof. The correctness of the protocol trivially stems from the properties of the Pauli operations (namely $\sigma^2 = I$ for any Pauli $\sigma$). The only security that needs to be proven is against a malicious Eavesdropper.

The Eavesdropper receives in the real protocol a quantum state from one Insecure Quantum Channel and sends one quantum state of the same size to another. In all generality, it applies in between these transmissions an arbitrary CPTP map on the received state and its internal state. In the ideal world, the Simulator receives the size of the state from the Confidential Quantum Channel Ideal Resource and must replicate the view of the Adversary and honest players by: sending a state to the Eavesdropper, receive another state in return and send a state to the Confidential Quantum Channel Ideal Resource.

We start by representing the real protocol as a quantum circuit for fixed keys $(k_Z, k_X)$ and a single qubit, where $E$ is a general CPTP map and $\rho_A$ is the Adversary’s internal state:

In the circuit above, the first two operations are performed by the Sender, $E$ is applied by the Eavesdropper and the final two Paulis correspond to the Receiver’s decryption. Note that it is possible, although slightly convoluted, for the Sender to perform the encryption via a quantum teleportation: it creates first an EPR-pair, applies a CNOT with its input as control and one qubit of the EPR-pair as the target followed by a Hadamard on its input and measures these two qubits in the computational basis. The Sender then defines the measurement results to be $(k_Z, k_X)$ and the state of the unmeasured qubit of the EPR-pair is $Z^{k_Z}X^{k_X}(\rho)$ for input state $\rho$. This perfectly equivalent to the initial protocol and the resulting circuit is represented as follows:

In the circuit above, the first CNOT creates the EPR-pair, the second CNOT and H along with the measurement in the computational basis perform the teleportation. We then remark that it is equivalent from the point of view of all parties if the measurements are postponed until after the Receiver has received the state from the Eavesdropper (otherwise this would violate the principle of no-communication through local operations). The Sender can simply send the third qubit to the Eavesdropper, wait for confirmation that the Receiver has obtained the state, measure its remaining qubits and transfer the result via the Secure Classical Channel:
Finally we can isolate from the circuit above the operations of the Simulator – dotted box – and the Ideal Resource – dashed box – for a single qubit (if multiple qubits are sent these operations are applied for all qubits in parallel). The Simulator prepares an EPR-pair and sends half to the Eavesdropper. After receiving the reply of the Eavesdropper, it sends the other half of the EPR-pair and the third qubit to the Ideal Resource. In the mean-time, the Sender sends its input qubit to the Ideal Resource. The Resource applies the CNOT and H and measures the first two qubits in the computational basis, obtaining $(k_Z, k_X)$. It applies $X^{k_X}Z^{k_Z}$ to the third qubit and sends this qubit to the Receiver as its output.

The indistinguishability comes from the fact that the input/output relation is preserved via the transformations above and the fact that the Eavesdropper cannot distinguish between half an EPR-pair and a qubit encrypted via a Quantum One-Time Pad since both are perfectly mixed states from its point of view.

\[ \Box \]

## E  Proof of DBQC Security

We give in this Appendix the proof of Theorem 1 from Section 4. Its principle follows a similar reasoning to the security proof of the Quantum One-Time Pad present in the previous appendix, which can be considered as a warm-up for the ideas developed here.

**Theorem 1** (Composability of Double-Blind Quantum Computing). Under the assumption that at least one Client is honest and that the Orchestrator is honest, Protocol 2 $\epsilon_D$-constructs the Double-Blind Quantum Computing Ideal Resource 3 for $\epsilon_D = 0$ (i.e. perfect security).

**Proof.** The goal of the proof is to describe Simulators attached to the interfaces of malicious parties of the Ideal Resource so that the real-world implementation is indistinguishable from the ideal-world version with the Simulator.

We start by showing that our Protocol is correct when all parties are honest. After the State Preparation and Entanglement steps, the Server holds the same resource state as the one used in the UBQC Protocol presented in Section 2.2.1. Then, the Computation step is identical to the single-client UBQC Protocol with the Orchestrator playing the role of the Client, which implies correctness after the measurements. The Termination step then correctly sets the random Q-OTP key $k$ to the value given by the single-client UBQC Protocol. Therefore, all quantum and classical data produced by the Protocol are identical to the ones returned by the Ideal Resource 3, proving the correctness of Protocol 2.

The second case to examine is when the Server, the Orchestrator and at least one Client are honest. This is a one-way protocol from the malicious Clients to the honest parties, which is always perfectly blind as shown in Theorem 7.2 of [10].

Finally, the case with a malicious Server and possibly some malicious Clients can be dealt with in the worst-case scenario, which corresponds to a single honest Client (e.g. Client $h$) and Orchestrator, while all other parties are malicious and colluding. The corresponding Simulator $\sigma$, connected to malicious Clients and Server on one side, and to the corresponding interfaces of the Ideal Resource on the other one, is presented in Simulator 1.

To show that the Ideal Resource and this Simulator are indistinguishable from the real-world Protocol 2, we need to prove that by observing the classical messages and quantum states exchanged, no Distinguisher can tell apart the two situations. This is done as in [10] by reducing Protocol 2 to one which is equivalent from the point of view of a Distinguisher,
Simulator 1

For each $i \in O^c$:

1. The Simulator creates an EPR-pair and sends half of it to the Server.
2. It receives the quantum states from the malicious Clients and forwards them to the Server (i.e. $N-1$ states for each $i \in O^c$).
3. It receives from each malicious Client $j$, the values for $r_j(i)$ and $\theta_j(i)$ for $i \in O^c$, and for $i \in I$ and $j = \Omega(i)$ it receives $a(i)$.
4. It receives the vector $t(i)$ from the Server.
5. It sends $\delta(i)$ chosen at random in $\Theta$ to the Server and receives $b(i) \in \{0,1\}$ in return.
6. It sends to the Ideal DBQC Resource (Resource 3) its remaining half-EPR-pair, along with the values $\theta_j(i)$, $r_j(i)$, $a(i)$, $t(i)$, $\delta(i)$ and $b(i)$.

and where it will be possible to identify the Ideal Resource and the Simulator. Figure 8 depicts the protocol when focusing on a single qubit $i$ owned by an honest Client $h$.

![Figure 8: Schematic circuit for implementing DBQC with 2 Clients. $C_h$ is honest and owns qubit $i$ whereas $C_j$ provides a $|+\rangle$ state. $E_G$ is the entangling operation involving the other qubits used in the computation. The Orchestrator computes the measurement angle $\delta(i)$ as a function of the target computation and the previous measurements through $\phi'(i)$, of the secret parameters defined by the Clients for qubit $i$, and of the public measurement outcomes $t(i)$.](image)

We start by replacing the quantum operations performed by Client $h$ in the DBQC State Preparation (Protocol 1). For all non-output qubits, it instead prepares and EPR-pair and sends half of it to the Server. For its non-input qubits it measure the other half in basis $|\pm_{\theta_h(i)}\rangle$ for $\theta_h(i) \in R \Theta$. For its input qubits, it first applies a CNOT between its input (as control) and the other half-EPR-pair (as target) followed by a $Z(\theta_h(i))$ gate and a Hadamard on the control qubit and then measures both in the computational basis. The measurement results are set as values for $r_h(i)$ (control) and $a(i)$ (target). The rest of the protocol remains the same.

We now analyse its effect on the information accessible to the Distinguisher. Since Client $h$ is honest and never reveals $\theta$, the qubits that it sends to the Server appear completely mixed from the Distinguisher’s perspective. Replacing them by half EPR-pairs does not change this. Additionally, the classical information sent in this reduction by Client $h$ to the Orchestrator and later used in determining the measurement angles follow the same probability distribution as in the original protocol while remaining correct. Combining these two facts establishes the equivalence of the original protocol and this first
reduction from the Distinguisher’s perspective.

The second reduction further alters the modified protocol by (i) delaying in time the measurement on the EPR-pairs for each \( i \in O_c \), which does not modify the distribution of quantum nor classical messages exchanged between the various parties; (ii) choosing the angles \( \delta(i) \) at random and adapting the angle \( \theta_h(i) \) instead of the other way around, which keeps the angle-update equation of UBQC satisfied and \( \theta_h(i) \) uniformly distributed over \( \Theta \). This measurement-induced value of \( \theta_h(i) \) is given by:

\[
\theta_h(i) = \begin{cases} 
\delta(i) - \phi'(i) - \sum_{j \neq h} (-1)^{t_j(i)+a(i)} \theta_j(i) \\
(-1)^{t_{\Omega(i)}+a(i)} \times (\delta(i) - \phi'(i) - \theta_{\Omega(i)}(i) - \sum_{j \neq \Omega(i)} (-1)^{t_j(i)+a(i)} \theta_j(i))
\end{cases}
\]  

(10)

Here again, the equivalence between the two protocols is immediate: the UBQC equations are still satisfied and no Distinguisher can detect that the measurements have been performed later. The combined modifications are presented in Reduction 1.

**Reduction 1** Replacing Q-OTP with Teleportation and Delayed Measurement

**Client** \( h \):

1. For locations \( i \in O_c \), it prepares an EPR-pair and sends half of it to the Server.
2. If \( i \in I \) and \( h = \Omega(i) \):
   (a) It performs a CNOT between its input qubit (control) and the remaining half EPR-pair (target) followed by a Hadamard on the control qubit.
   (b) It measures the target qubit in the computational basis and sends the obtained value to the Orchestrator as \( a(i) \).

**The Orchestrator:**

1. It chooses a measurement angle \( \delta(i) \in_R \Theta \) and sends it to the Server. It receives in return the measured outcome \( b(i) \).
2. It computes \( \theta_h(i) \) according to Equation 10.
3. It asks Client \( h \) to measure the control (yet unmeasured) qubit in the basis defined by \( \theta_h(i) \) and receives in return the outcome of the measurement as \( r_h(i) \).

The rest of the protocol is unchanged.

It can be checked that the Simulator’s operations can be extracted from this modified protocol by grouping the EPR-pair creation done on behalf of Client \( h \) with the operations consisting of sending and receiving quantum or classical data to the Server and to the Ideal Resource. To conclude the proof, we need to show that, using the data transmitted by the Simulator, the Ideale Resource is able to take into account the deviation induced by the malicious parties and output the correct encryption key \( k \) for the Orchestrator. This is done by isolating an explicit quantum circuit (Ideal Resource 11) from the modified protocol, which then functions as a sub-system of the DBQC Ideal Resource 3. Figure 9 depicts the circuit after the reduction and extraction of the Simulator and the Ideal Resource.

Note that there is no need for the Ideal Resource to transmit additional information to the Simulator. The Ideal Resource, through the adaptative measurements of the Deviation Teleportation sub-system, lead the Server to prepare the expected final state.
Figure 9: Schematic circuit for the reduced protocol where the Simulator (blue) and Ideal Resource (red) have been singled out. From the point of view of $C_j$ and the Server, the interaction with the simulator is the same as for the concrete implementation of the protocol. More importantly, even if a distinguisher provides all input states, comprising that of $C_h$ and $O$, no difference will be apparent between this setup and the concrete implementation.

**Resource 11 Deviation Teleportation**

The following steps correspond to the concrete implementation of step 3 of the computation by the DBQC Ideal Resource 3:

1. The Ideal Resource performs a CNOT between the input qubits of Client $h$ (control) and the corresponding half-EPR-pairs (target) provided by the Simulator $\sigma$. It measures the target qubit in the computational basis and gets $a(i)$.
2. For qubits $i \in O^c$, it computes the angles $\theta_h(i)$ according to Equation 10. It performs the corresponding measurements in the basis $|+\theta_h(i)\rangle$ in the order defined by the flow, getting the values of $r_h(i)$ and updating the measurement angles for the next qubits.
3. For qubits $i \in O$ it computes the corrections $s = (s'_X(i), s'_Z(i))$ and sends the key $k = s$ to the Orchestrator.

**Theorem 2 (Constraints for Good-Enough VBQC Execution State Preparation through DBQC)**

We prove here Theorem 2 form Section 5.2.

We prove the theorem by following the steps outlined in the previous section. The Ideal Resource role is indeed restricted to using the Deviation Teleportation Resource sub-system to output the correct Q-OTP key at the Orchestrator’s interface. This concludes the proof as the Simulator together with the Ideal Resource are shown to stem from a protocol equivalent to the real-world implementation, making both situations perfectly indistinguishable.
The output of DBQC Protocol 2 using an MBQC pattern defined by a fixed graph $G = (V, E, I, O)$ with flow $(f, \preceq)$ and base angles $\{\phi(i)\}_{i \in O}$ is a good-enough VBQC execution state if:

- For an honest input to the DBQC Protocol, the honest application of the measurement pattern produces a correct VBQC client-encrypted state $\rho_{DT(G)}$.
- All measurement angles for the DBQC computation are Clifford, i.e. $\phi(i) \in \{k\pi/2\}_{0 \leq k \leq 3}$.
- The graph $G = (V, E, I, O)$, the flow $(f, \preceq)$ and all angles associated to vertices that have an $X$-dependency according to the chosen flow are all invariant under a change of colouring of the Dotted-Triple Graph.

We will require the following Pauli Twirling Lemma from [6].

**Lemma 2** (Pauli Twirling). Let $\rho$ be a matrix of dimension $2^n \times 2^n$, and $Q$, $Q'$ two arbitrary $n$-fold tensor products of Pauli and identity operators $\{I, X, Y, Z\}$, and $\{P_k\}$ the set of all $n$-fold tensor products of Pauli operators and the identity $\{I, X, Y, Z\}$. Then:

$$\sum_{k=1}^{4^n} P_k Q P_k \rho P_k Q' P_k = 0, \text{ if } Q \neq Q'$$

(11)

**Theorem 2.** We decompose the proof in two parts: first the DBQC State Preparation Protocol 1, followed by the remaining computation steps of the DBQC Protocol. We unitarise the protocol in order to commute the Adversary’s attack and show that the resulting equivalent attack does not depend on the secret parameters. We prove that, irrespective of the adversarial scenarios, the DBQC Protocol produces good-enough VBQC execution states spanning the full range of parameters.

**Preparation Phase of DBQC.** We consider each prepared qubit $i$ separately and two Clients. The goal of this step is to decorrelate the malicious and honest parameters and their corresponding states. This is achieved by unitarising the state preparation step of the DBQC Protocol and the deviation of the Adversary, which then allows us to “extract” various unitaries from the Adversary’s deviation for each prepared qubit.

Crucially, these unitaries extracted from the attack depend only on public or leaked parameters and therefore do not create a dependency of the attack on the secrets of the honest party. They will however depend on the malicious or honest nature of the owner of the prepared qubit. This is allowed since the deviation of the Adversary may anyway depend on this factor. Note that the conditions above also mean that this extraction can be undone by the Adversary and therefore it does not change the set of possible deviations (up to a relabelling). There are three cases to consider and the same unitary can be extracted for all qubits of the protocol in each sub-case, meaning that the final attack does not depend on the positioning of the different types of qubits (computation, trap or dummy). Generalisation for more than two clients can be done by extracting the same operators for each client consecutively.

First, we examine the case where the owner of the qubit is malicious and the prepared qubit is its input to the VBQC execution. The Server receives the state $\rho$ from the dishonest owner $\Omega(i)$ of vertex $i$ (encrypted with its own parameters), and a state $\left| +_{\theta(i)} \right>$ from honest Client $j \neq \Omega(i)$. The Server then performs the collective encryption, thus acquires the classical states $t_j(i)$. It will receive the updated measurement angles $\delta(i)$ sent by the Orchestrator at the time of execution of DBQC and VBQC, where:

$$\delta(i) = (-1)^{a(i)} \phi(i) + \theta_{\Omega(i)}(i) + (-1)^{t_j(i)} \theta_j(i) + r_{\Omega(i)}(i)\pi + r_j(i)\pi$$

(12)
While a deviation on the input of the malicious Client is always allowed, it needs to be shown that a deviation from the Server during the collective encryption step can be incorporated into the input of the malicious Client. Doing so, the commutation required to incorporate any effect of the deviation into the input shows a dependency on the value of \((-1)^{t_j(i)}\theta_j(i)\) from the honest player since it is used to encrypt this input. This seemingly prevents the malicious Client from absorbing the deviation into its input as the required operation depends on the honest player’s secret parameters. Yet, the values of \(\phi(i)\), \(a(i)\), \(\theta_{\Omega(i)}(i)\) and \(r_{\Omega(i)}(i)\) are set by the malicious Client, and \(\delta(i)\) is public. Therefore \((-1)^{t_j(i)}\theta_j(i)\) is known up to a \(\pi\) rotation by a malicious Server. Having access to the honest Client’s qubit, the Server can then measure it to recover \(r_j(i)\), so that it is always possible to incorporate the effect of the deviation into the input of the malicious player using solely public or leaked parameters.\(^8\)

The next case deals with a dishonest owner of vertex \(i\) that is not its input qubit to the VBQC execution—sending the state \(|\chi\rangle\) and an honest Client \(j \neq \Omega(i)\) providing \(+\theta_j(i)\rangle\) to the Server. The Server also receives classical states \(\delta(i)\) and \(t_j(i)\), where:

\[
\delta(i) = \phi'(i) + \theta_{\Omega(i)}(i) + (-1)^{t_j(i)}\theta_j(i) + r_{\Omega(i)}(i)\pi + r_j(i)\pi
\]

The DBQC Protocol instructs the Server to perform a CNOT between the qubits of the owner and the honest party. Having unitarised the protocol and the attack of the Adversary, we can extract the unitary in the dashed box in Figure 10 from the general attack operator that follows.

\[
\begin{align*}
|+\theta_j(i)\rangle & \quad \text{---} \quad |\chi\rangle \\
|\chi\rangle & \quad \oplus \quad |\delta(i)\rangle \\
|\delta(j)\rangle & \quad \text{---} \quad |t_j(i)\rangle \\
|t_j(i)\rangle & \quad \oplus \quad |\chi\rangle
\end{align*}
\]

\[
\begin{align*}
\delta(i) = \phi'(i) + \theta_{\Omega(i)}(i) + (-1)^{t_j(i)}\theta_j(i) + r_{\Omega(i)}(i)\pi + r_j(i)\pi
\]

\[
\begin{align*}
|+\theta_j(i)\rangle & \quad \text{---} \quad |\chi\rangle \\
|\chi\rangle & \quad \oplus \quad |\delta(i)\rangle \\
|\delta(i)\rangle & \quad \text{---} \quad |t_j(i)\rangle \\
|t_j(i)\rangle & \quad \oplus \quad |\chi\rangle
\end{align*}
\]

Figure 10: The honest Client controls the first line and the owner the second one. The operation acting on the state \(|\delta(i)\rangle\) is the unitary associated with the classical (reversible) operation that adds the value \(-\theta_{\Omega(i)}(i) + r_{\Omega(i)}(i)\pi\) to any angle in \(\Theta\). We have \(\phi'(i) = \phi'(i) + (-1)^{t_j(i)}\theta_j(i) + r_j(i)\pi\).

The operations inside the dashed box are extracted from the attack operation.

In the last case, the owner of vertex \(i\) is honest, sending a qubit in state \(\rho\) with \(\rho = |+\rangle\) for non-inputs, and the other Client is dishonest. Again we have (with \(a(i) = 0\) for non-inputs):

\[
\delta(i) = (-1)^{a(i)}\phi'(i) + \theta_{\Omega(i)}(i) + (-1)^{t_j(i)}\theta_j(i) + r_{\Omega(i)}(i)\pi + r_j(i)\pi
\]

As previously, we extract a unitary from the attack (see Figure 11).

The same considerations as above apply. In all cases, the final state has the form described in properties 1, 2, 4 and 5 of a good-enough VBQC execution state (Definition

\(^8\)At first glance, it seems like knowing all these parameters would in fact yield an attack on the whole scheme by allowing a malicious coalition to recover the secret parameters of all qubits in the DTG. However, an additional crucial information is used here: the fact that the malicious Client knows that this is a base-location associated to one of its input qubits. It therefore also knows the position of its computation qubit in this base-location. Although the malicious Client also knows the value of \(\phi(v)\) for all computation and trap qubits of the Dotted-Triple Graph, the same reasoning does not apply since it does not know the nature of the qubits in base-locations that are not associated to its input.
3). The proof generalises to more than two parties by noticing that we can always extract an attack that disentangles the qubit of the owner from the qubits of the other dishonest parties and also undo their corresponding $\theta$’s and $r$’s from $\delta$.

The result at the end of this step is a set of qubits in the Server’s register that contain: (i) the inputs of all parties to the VBQC computation; (ii) all non-input qubits that will be part of the VBQC graph, some of which must be turned into dummies; (iii) all additional qubits used in the DBQC computation; (iv) for a given branch of VBQC and DBQC computations, the classical state corresponding to the measurement angles sent as instructions to the Server. All states apart from the VBQC inputs of the malicious Clients are encrypted solely with the honest player’s parameters, even in the case of a single non-deviating Client.

We can furthermore extract from the Server’s deviation the unitary entangling operation for the DBQC graph, after which we can consider the operations applied during the DBQC computation phase, i.e. blindly transforming the states $|+\rangle$ into dummies.

**Computation Phase of DBQC.** We must now consider the operations applied during the computation phase, i.e. blindly transforming $|+\rangle$ states into dummies.

**Residual Deviation on a Single Gate Teleportation.** We start by determining the residual deviation when a single gate teleportation step is extracted from a general attack.

The corresponding circuit for a single gate teleportation is represented graphically on Figure 12a, where the top qubit is the input and the bottom the output. We apply the entanglement operation followed by $Z(-\phi'(i) - \theta(i) + r(i)\pi)$ and $H$ to the first qubit. This yields a unitarised version of the DBQC computation step, where it is understood that any attack that could have been performed during the preparation phase or during this step has been commuted to after the execution of the circuit and has been reduced as a pure deviation. By simplifying and rearranging these operations we get Figure 12b. Because the state after the CZ is stabilized by $(X \otimes Z)^p(i)$ for any choice of $p(i) \in \{0, 1\}$, we get Figure 12c after commuting the stabilizer through the Z rotation and the Hadamard. Due to the commutation, the rotation angle is modified and transformed to $-(-1)^p(i)\phi'(i)$. We can then get rid of the gate $Z^p(i)$ on the second qubit because it only contributes towards a global phase (Figure 12d).

Following [20], by taking the sum over $r(i)$ and $p(i)$, we can use the Pauli Twirling Lemma 2 from [6]. This allows to rewrite the residual deviation as a convex combination of Pauli operators on the first qubit tensored with possibly different generic attacks on the output qubit. Because the first qubit is measured in the computational basis, the deviation’s effect reduces to a convex combination of no action on the measurement result $b(i)$ tensored with a generic attack on the output qubit and a classical bit flip on $b(i)$ tensored with a possibly different generic attack on the output qubit.

![Figure 11](image-url)
Note that in the original verifiability proof [15], there is no need for these extra parameters \( p(i) \). It is only required here because we want to assess the effect of a deviation on internal qubits of DBQC instead of computing the probability of obtaining a correct measurement outcome for traps. This can concludes that the most general attack on these qubits are bit flips of the measurement outcomes.

\[
\left| +\theta(i) \right\rangle \quad Z(-\phi'(i) - \theta(i) + r(i)\pi) - H
\]

(a) DBQC Computation Step.

\[
\left| +\theta(j) \right\rangle \quad Z(-\phi'(i)) - H - X^{r(i)}
\]

(b) Simplification.

\[
\left| +\theta(j) \right\rangle \quad Z(-(-1)^p(i)\phi'(i)) - H - X^{r(i)} - Z^{p(i)}
\]

(c) Stabilisation by \( X \otimes Z \).

\[
\left| +\theta(j) \right\rangle \quad Z(-(-1)^p(i)\phi'(i)) - H - X^{r(i)} - Z^{p(i)}
\]

(d) Final Circuit.

Figure 12: Elementary step in DBQC computation phase.

**Residual Deviation on the Whole DBQC Computation Phase.** The above transformation can be applied successively to the whole DBQC computation phase in order to describe the residual deviation for a generic attack. In doing so, the deviation is reduced to a probabilistic combination of bit flips on the classical measurement outcomes \( b(i) \), each tensored with (possibly different) arbitrary attacks on the output qubits. In addition, it was shown in the previous section that the residual deviation from the state preparation step was dependent only on public or leaked parameters. Because of the blindness of the scheme, the attacks that can be performed for each gate teleportation are also only depending only on public or leaked parameters. As a consequence, when the residual deviation is written as a convex combination of bit flips for the classical measurement outcomes \( b(i) \), each remaining generic attack on the output qubits depends only on public or leaked parameters.

Hence, if there is a dependency upon secret parameters, it can only come from the effect of the classical bit flips.

**Effect of the Classical Bit Flips.** Finally, we have to consider how the classical bit flips for the measurement outcomes will translate into an attack on the output qubits only. Their effects can be analysed using the flow of computation. This can be done easily by recalling how the measurement angle of a not-yet-measured qubit \( j \) depends on...
that is independent of the input/computational and trap qubits. or random for each base-location. Summing over this random choice would give an attack CPTP operator less stringent condition on the DBQC computation angles would be to impose that they are either equal of the types of qubits (input/computation and traps) is chosen at random in each base-location. Then a

the an Adversary cannot distinguish between the application of two different flows [27].

already distinguish the qubits in each base-location, but the flow condition is new since it has been proven

the graph and a measurement pattern on it that will implement a unitary for turning some

Then, after choosing a colouring for the Dotted-Triple-Graph, the Orchestrator chooses a

using DBQC. It amounts first to collectively encoding quantum inputs and

back and examine the generic procedure to prepare good-enough VBQC execution states

Independence of secret parameters. Without further constraints, the above residual deviation could very well depend on secret parameters. To understand why, we take a step back and examine the generic procedure to prepare good-enough VBQC execution states using DBQC. It amounts first to collectively encoding quantum inputs and \(|+\rangle\) states. Then, after choosing a colouring for the Dotted-Triple-Graph, the Orchestrator chooses a graph and a measurement pattern on it that will implement a unitary for turning some \(|+\rangle\) states into dummies and placing all the qubits at their required location.

Because the sets \(S_Z\) and \(S_X\) above are specific to the chosen DBQC computation graph and its flow, the attack can only be independent of the secret parameters under the condition that the graph used in DBQC for all Dotted-Triple Graph qubits is the same and that the same flow of computation is chosen on this graph (i.e. they are both invariant under permutation of the qubits of the Dotted-Triple Graph in each base-location).\(^9\) By simply imposing this, we already get that the \(Z\) correction is independent of any secret parameter since it will always add a \(\pi\) rotation to the measurement angle and so this will affect similarly any qubit regardless of its type.

The \(X\) dependency induces additional restrictions. When \(i \in S_X(j)\) the effect of the bit flip on \(b(i)\) on the subsequently measured qubit \(j\) depends on the value of the angle \(\phi(j)\). As a consequence, its action might pick up a dependency on a secret parameter of the DBQC computation. We therefore need to analyse it further since this remaining Pauli attack on the output qubits that cannot be further reduced by twirling. For \(\phi(j) \in \{0, \pi\}\) a sign flip has no effect, while there is an additional \(Z\) operation when \(\phi(j) \in \{-\pi/2, \pi/2\}\). Hence, to get the independence upon the secret parameters (here the value of \(\phi(j)\)), it is necessary for the angles \(\phi(j)\) for \(j\) such that \(S_X(j) \neq \emptyset\) to be independent on the chosen graph colouring, meaning that the measurement angles of those qubits is invariant under permutation of the qubits in each base-location of the Dotted-Triple Graph.\(^10\)

Only qubits that are in the future of other qubits (in terms of flow) may get a malicious correction. This is directly reflected in the invariance condition since only qubits that are in the image set of the flow function can be affected. Therefore, for \(j\) such that \(S_X(j) = \emptyset\),

\(^9\)Intuitively, if the graph or the measurement order induced by the flow is different, the Adversary can already distinguish the qubits in each base-location, but the flow condition is new since it has been proven the an Adversary cannot distinguish between the application of two different flows [27].

\(^10\)This condition is sufficient but not necessary. Instead we can use the fact that the initial positions of the types of qubits (input/computation and traps) is chosen at random in each base-location. Then a less stringent condition on the DBQC computation angles would be to impose that they are either equal or random for each base-location. Summing over this random choice would give an attack CPTP operator that is independent of the input/computational and trap qubits.
there is no constraint on the measurement angle as the impact of the bit flips only affects dependent qubits.\footnote{Most commonly, all qubits apart from the input qubits have an X dependency, in which case the only angles that may differ according to the nature of the qubits in each base-location are the ones associated to the input positions of the DBQC graph}

This concludes our proof as it shows that the residual deviation on the output qubits – i.e. the prepared good-enough VBQC execution states – is independent of the secret parameters defining the good-enough VBQC execution states.

\[\square\]

G Proof of Secret-Independence of Residual Attack

We give here the proof of Lemma 2 from Section 5.2.

**Lemma 2** (Secure Multi-Client VBQC State Preparation). Assume that the base graph chosen to perform an MBQC computation through VBQC has degree 1 on all input vertices. Apply the DBQC Protocol using the MBQC pattern from Figure 7 for each qubit of the final DT(G) as described in Section 5.1.2 on an input state constructed as: (i) MBQC input states and $|+\rangle$ states for non-input positions in the DT(G) and (ii) the position of the MBQC input of honest players is randomly chosen in its associated input base-location.

The final state then consists of $E(\rho_V \otimes \rho_{aux})$, where $\rho_V$ is a correctly-prepared VBQC client-encrypted state, $\rho_{aux}$ is an arbitrary auxiliary state prepared by the Adversary and the deviation $E$ is independent of $\rho_V$.

**Proof.** We first need to show that the operation applied through the DBQC Protocol satisfies the conditions given by Theorem 2. The correctness of this scheme follows directly from the discussion found in Section 5.1.2: it correctly produces computation qubits, traps and dummies in positions defined by a Dotted-Triple Graph colouring, the angles of the graph are all Clifford, the full graph is obtained by applying the same graph and flow on each qubit of the DT(G). Therefore it does not depend on their nature or secret parameters. Finally the measurement angles are all the same for each of these applications apart from the lower input, which can be either 0 or $-\pi/2$ depending on the nature of the final qubit. However, this qubit is not X-dependent on any other. Hence, we can apply Theorem 2. After this step, we thus have a state of the form $E(\rho_V \otimes \rho_{aux})$, with a deviation $E$ that acts on a correctly-prepared VBQC client-encrypted state $\rho_V$ and which is guaranteed to not have picked-up any additional dependency on the secret parameters as a consequence of the application of the DBQC Protocol.

As a second step, we therefore need to assess the public or leaked information available to an Adversary in the rest of the Protocol. We treat the sub-cases corresponding to non-input and input base-locations separately.

For non-input base-locations, it is public knowledge that all qubits used as inputs to the DBQC Protocol are rotated $|+\rangle$ states. The rotation angles of all qubits in the state remain secret as a result of the blindness of the Protocol thanks to the collaborative encryption and the state provided by the honest Client.\footnote{In a sense, it is precisely because this honest state contains sufficient randomness that we are able to rewrite the final DBQC state in the form of $\rho_V$ up to the subsequent deviation.} For the same reason, this is also the case for the measurement angle of the first lower input qubit in the H/I-Gadget above. Hence, an attack cannot depend on the secrets of the honest clients, nor on type – i.e. trap, computation, dummy – chosen by the Orchestrator for each qubit in a non-input base-location.
For input base-locations, if the owner is an honest Client who chose the location of its inputs at random, and because DBQC hides the type chosen by the Orchestrator for each non-input qubit in this input base-location, the Adversary does not gain any information compared to a plain VBQC execution. The Adversary’s only additional information comes from its knowledge of the position of inputs provided by the malicious Clients. We now prove that any attack after the state has been prepared that uses this information can be expressed as a modification of the malicious party’s input.\textsuperscript{13} To this end, we use the fact that all attacks in VBQC can be reduced to classical attacks (flipping the measurement outcomes) by using the twirling lemma\textsuperscript{14} and therefore analyse the effect of these only.

These classical attacks are probabilistic mixtures of bit strings that depend on the knowledge of the Adversary. Although the malicious owner of the input qubit knows where this input qubit is placed in its input base-location, it does not know the position of the computation qubits in the second layer of primary base-locations and after that. This means that the state from the the second layer onward is exactly the same from the point of view of the Adversary as one in a normal VBQC execution. Therefore, any attack that does not affect the first layer (malicious) qubits nor the added qubits in the connecting edge is equivalent to one in the normal VBQC execution where the Adversary simply decide begin attacking after these positions have already been measured. Hence, the only attacks stemming from this information that are not already taken into account in the analysis of the VBQC Protocol are ones which have a non-trivial effect on this input base location and/or the edge base-location directly connected to it. We now analyse the effect that flipping a measurement outcome on these qubits has on the rest of the state.

A classical attack on the malicious Client’s input base location that does not trigger a trap is, by construction, equivalent to a modification of this player’s input (by applying $Z$ before performing the entangling and measurement). On the added qubits, the Adversary knows that the neighbours of the computational input qubit cannot be traps since the computation primary qubits are linked to one added computational qubit and two dummies. It can then flip the measurement outcomes of these qubits while being sure to never triggering the trap in this base-location. Flipping the measurement outcome of the dummies has no effect since it is perfectly random and does not influence the rest of the graph through corrections. However, any classical attack on the computational (bridge) qubit translates to a Pauli $Z$ operations on the computational neighbours due to the bridge operation corrections. The $Z$ operation on the input computation qubit can be directly rewritten as a modification of this input since it commutes with the $CZ$ entangling operation. The other $Z$ correction (on the computation qubit of the second layer) is equivalent to having applied an $X$ operation on the input qubit once before entangling the state and then another time after applying the $CZ$. The first $X$ operation can be integrated as an input modification and the second one is similar to multiplying the measurement angle $\phi$ associated to this input position by $-1$. Since this angle is known to the malicious Clients (they know the computation being performed), this is then equivalent to having pre-rotated the input around the $Z$-axis by $-2\phi$ first and applying the correct measurement.

Combining these steps shows that, the knowledge of the location of the malicious Client’s inputs can be incorporated into modifications of their inputs, hence showing that the final state $\mathcal{E}(\rho_V \otimes \rho_{aux})$ is effectively a correctly prepared VBQC client-encrypted state up to an arbitrary deviation which is independent of the secret parameters of the state.

\textsuperscript{13}This behaviour is always allowed as all Parties are free to choose their input arbitrarily.

\textsuperscript{14}Note that the twirling lemma only requires blindness, which is provided by the collaborative encryption.
and in particular of the corresponding coloring of the $DT(G)$. □

H Compatibility of FK Proof and Good-Enough VBQC Execution States

The state after the DBQC Protocol is proven to be equivalent to an honestly prepared VBQC client-encrypted state upon which the Server performs a deviation independent from the secret parameters (Theorem 2). This state can therefore be directly used in the VBQC Protocol and all the security properties follow since this deviation could have been performed by the Server during the VBQC Protocol as well (and such deviations are taken care of by the verifiability of the protocol).

For completeness, we show here formally that this good-enough VBQC execution state is sufficient for the proof of verifiability of the VBQC Protocol to go through. We reuse the same notations as in [25], recalled here for clarity, with the subscript denoting dependence on parameters and parenthesis denoting qubit index, e.g. $|\eta_{\nu_T}(v)\rangle$ is a state of qubit with index $v$ that depends on parameters $\nu_T$:

- $\nu$ are all random parameters, including the computational random parameters $\nu_C$ and the trap random parameters $\nu_T$. The hidden parameters include: all $\theta$’s, $r$’s, $a$’s, $d$’s of the honest Client and the positions of the traps $T$.
- $c_r$ and $C_{\nu_C,b}$ are the classical and quantum corrections on the outputs of the Server.
- $|\Psi_{\nu,b}\rangle$ is the state received by the Server from the Clients and the Orchestrator, including all classical states and quantum states from each party. In particular it contains: The Q-OTPed input qubits with parameters $\theta$ and $a$, all the $|+\theta\rangle$ states generated from each Client (including the owner for the non-input vertices) for each vertex. $|\Psi_{\nu,b}\rangle$ also contains all $\delta$’s sent by the Classical SMPC for all phases of the protocol corresponding to the fixed branch $b$. Notice that the $\delta$’s have a dependence on $T$.
- $\mathcal{P}$ is the whole protocol unitary operator applying on $|\Psi_{\nu,b}\rangle$ (the DBQC Protocol 2 and the actual VBQC computation on the latter state).
- $\Omega$ is any unitary that represents the deviation on the Server’s system, including its private register $\otimes S |0\rangle\langle 0|$. All deviations at any time step of the protocol have been gathered together. As proven in Section 5 for DBQC and since the VBQC Protocol unitary does not depend on $\nu$, there is no dependence of $\Omega$ on $\nu$ either.

The output state of the protocol, held by the Clients, is the same as Equation (C1) from [25] (with our choice of $\mathcal{P}$):

$$
\text{Tr}_S \left( \sum_b |b + c_r\rangle\langle b| C_{\nu_C;b}^\dagger \Omega \mathcal{P}(\otimes S |0\rangle\langle 0| \otimes |\Psi_{\nu,b}\rangle\langle \Psi_{\nu,b}|) \mathcal{P}^\dagger \Omega^\dagger C_{\nu_C;b} |b\rangle\langle b + c_r| \right)
$$

An important point to be made here is that the projection operators $|b\rangle$ for the measurements of the Server also include the measurements for the DBQC State Preparation (Protocol 1). Therefore all the $t$’s sent during this step are included in the global $b$’s here. The $t$’s are not encrypted, which means that $c_r = 0$ for these bits. Following the same steps as in [25], we arrive at their Equation (C7):
\[
p_{\text{fail}} \leq \sum_{k,v,i,j} a_{k,i} a^*_{k,j} p(v_T) \sum_{\nu} Tr \left( \bigotimes_{v \in T} |\eta_{\nu_T}(v)\rangle \langle \eta_{\nu_T}(v)| \otimes |b\rangle \langle b'| \right) \times \left( \sum_{i \in E_i} \sigma_i \right)
\]

In the equation above: (i) \( |\eta_{\nu_T}(v)\rangle = |r_v\rangle \) for \( v \in O^c \) and \( |\eta_{\nu_T}(v)\rangle = |\theta_v\rangle \) for \( v \in O \) (the traps in non-output base-locations have already been measured, while the ones in output positions are measured by Clients); (ii) string \( b' \) is the substring of \( b \) that does not include the measured traps; (iii) \( E_i \) is the subset of all multi-qubit Pauli operators \( \sigma_i \) that can corrupt the output of the computation (the attack unitary \( \Omega \) becomes a CPTP maps after tracing out the Server’s register, expressed in terms of Kraus operators indexed by \( k \), which are then each decomposed as linear combination of Pauli operators \( \sigma_i \) with coefficients \( a_{k,i} \)).

Before following the proof steps in [25] we need to reduce our state and our protocol to the state and the protocol in [25]. Here we use the fact that, as state before, the DBQC Protocol prepares a good-enough VBQC execution state. We can therefore apply the part of unitary protocol \( P \) that corresponds to the DBQC Protocol and subsequent entanglement of the graph by the Server. This reduces the state before the attack to the same (Dotted-Triple Graph) state as in the VBQC Protocol. Any attack on the ancillary qubits used for this preparation is automatically transferred to the final qubits by a simple re-indexing of the attack operators. Also, an attack on the bridge qubits of the first layer is Pauli twirled using the random pre-rotations of the bridge qubit and remapped as an attack on the input qubit. We then sum over the secret parameters (i.e. unknown to the adversarial coalition) for the non-trap part of the state, including the non-trap \( \delta \)'s. This gives us Equation (C8) from [25] and the rest of their proof follows:

\[
p_{\text{fail}} \leq \sum_{k,v,i,j} a_{k,i} a^*_{k,j} p(v_T) \sum_{\nu} Tr \left( \bigotimes_{v \in T} |\eta_{\nu_T}(v)\rangle \langle \eta_{\nu_T}(v)| \right) \times \left( \sum_{i \in E_i} \sigma_i \right)
\]

### I Colouring-Independence Condition of DBQC Angles is Necessary

We describe here how it is possible to perform an attack on VBQC if the last condition of Theorem 2 regarding the measurement angles is not satisfied (i.e. if some measurement angles with a non-trivial dependency set are not independent of the DT(G) colouring). For this purpose we give a description of a Hadamard/Identity gadget that does not fulfil this condition and show how to use it to break the verifiability of the VBQC Protocol.

The Hadamard or Identity can be applied by using a line graph of five qubits [32], with the first one being the input and the last on in the line being the output (which is the only one left unmeasured). Than in order to apply the Identity, the chosen measurement angles on the first four qubits are all equal to 0, while for Hadamard the first one is 0 and the other three are \( \pi/2 \). If the adversary flip the measurement outcome of the next to last qubit, in both cases the final qubit is affected by an additional \( Z \) operation (which the Adversary undoes since it may hit a trap). But only in the case of Hadamard being
applied, the final output qubit also gains an X correction. Since this operation is used to prepare dummies, this means that the final value of the dummy is flipped compared to its intended value. In short it is possible for an Adversary to apply an attack that always flips the value of a dummy qubit while leaving computation and trap qubits unaffected.

It may seem at first that this attack does not harms the protocol since it does not affect either traps or computations. However the effect of the attack on the dummies is propagated to adjacent qubits in the Dotted-Triple Graph. Through the application of the CZ gate, an application of X on the dummy is equivalent to applying Z on all qubits which are linked to this dummy in the graph. We now show how to use this fact to perform an attack which may hit computation qubits but is never detected by traps.

Consider a line graph of two qubits and its transformation in a Dotted-Triple Graph. This graph contains two primary locations with three qubits and one added location with nine qubits. It is useless to use the attack above on a primary vertex since the primary dummy is always connected to one trap but no computation qubits. We notice that if an even number of dummies that are linked to the same qubit are attacked, then this qubit is not affected by the attack since the effect of the Z operations cancel out. However if an odd number of dummies linked to a qubit are attacked, then this qubit will be corrupted. The trick then relies on the difference in the number of dummies in the neighbourhood of traps and computation qubits. Traps are only linked to dummies while a computation qubit will always have at least one other computation qubit among its neighbours. It is possible, as show in Figure 13 below, to choose added qubits to attack so that each primary vertex is linked to exactly two attacked middle qubits.

![Figure 13: Example of attack layout where each top and bottom primary qubit is attached to exactly two attacked added qubits. Qubits that have been chosen for the attack are circled in blue.](image)

In that case, since the primary trap qubits are only linked to dummies, the attack does not trigger either trap (if one of the middle qubits that is attacked is a trap, the effect of the attack on this trap is the Identity as explained above). However, the attack may either affect two dummies linked to the primary computation qubits, in which case there is no attack since the effects cancel out, or one added dummy and the added computation qubit. Then, the effect on the added computation qubit is Identity but the attacked dummies will apply a Z operation on primary computation qubits on both sides of the link. If we assume fixed (but unknown) the attack positions, whether this attack succeeds in modifying the computation depends only on the colouring that is used, while never triggering any trap. The probability of success is equal to 2/3: the attack succeeds if the computational added qubit is chosen for the attack, there are 6 possible choices of attack configuration and each
added qubit is left untouched by 2 out of the 6 attack configurations. We give in Figure 14 two possible colourings, ones in which the attack has no effect on the computation while the other corrupts it.

(a) Z attack on both primary computational qubits due to odd number of attacked added dummies.

(b) No attack on either primary computational qubit due to even number of attacked added dummies.

Figure 14: Two colourings of the previous graph (computational qubits are green, traps are white and dummies are red) for the same attacked qubits but a different effect on primary computational qubits. Attacked qubits are circled in blue, which translates to an X effect on dummies (yellow-filled circle) and no effect on added computational qubits (empty circle). The primary trap qubits are never affected by the attack since they are always attached to an even number of attacked added dummies.

Essentially, allowing an attack to depend on the nature of the qubits, even without the Adversary knowing the position of these qubits, introduces new attacks compared to those that are possible in the plain VBQC Protocol. We have shown above that it is sufficient to break the verifiability property of this composed protocol. We have shown in the proof of Theorem 2 above that using either fixed or perfectly random angles is sufficient to make this attack independent of the nature of the qubits, but it is unclear what its effect is if the randomisation is not perfect but skewed towards one angle or the other.

J Proof of Delegated MPQC Security

We prove in this Appendix the correctness and security of the DMPQC Protocol, stated as Theorem 3.

Theorem 3 (Composable Security of the DMPQC Protocol). If the VBQC Protocol of \[25\] \(\epsilon_V\)-statistically-constructs the Verifiable Delegated Quantum Computation Ideal Resource 2 and the DBQC Protocol \(\epsilon_D\)-statistically-constructs the Double-Blind Quantum Computation Ideal Resource 3 from Insecure Quantum Channels, then the DMPQC Protocol \(\epsilon_V + \epsilon_D\)-statistically-constructs the Multi-Party Quantum Computation Ideal Resource 1 from Insecure Quantum Channels and a Classical SMPC Ideal Resource.

This proof uses three other results. The first one is the security of the VBQC Protocol as emulation of the Verifiable Delegated Quantum Computation Ideal Resource found in Lemma 1. The next one corresponds to the security of the DBQC Protocol from Section 4. Finally we will use the fact that, for the choice of unitary transformations implemented by the DBQC Protocol to prepare the VBQC client-encrypted state, any deviation during its execution can be commuted to the end in a way that does not depend on the secret parameters of the honest Clients (Theorem 2 and Lemma 2 from Section 5).

Correctness. We consider here that all the parties are honest and prove that the protocol is in that case equal to the Ideal Resource (the correctness error is 0). The correctness of
the DBQC Protocol and the Classical SMPC that acts as the Orchestrator mean that the
Server’s state after step 2 in the DMPQC Protocol is a correct VBQC client-encrypted
state. The instructions in the last steps of the DMPQC Protocol correspond to an exec-
ution of the VBQC Protocol on this state, driven by the Classical SMPC that performs
the same steps as an honest VBQC Client. The outcome is therefore also correct per the
correctness of VBQC.

Security. This proof will not construct explicit simulators for all parties since we will
directly prove through a series of hybrid reductions that the DMPQC Protocol is equivalent
to the MPQC Ideal Resource using results previously proven in the paper. This Resource
acts as a black-box that takes as input the Clients’ input states and outputs either an abort
message or the correct output to all parties (with malicious parties receiving the output
first). In the Protocol on the other hand, the parties send additional states to construct
the graph state collaboratively and communicate classically, which therefore need to be
removed. The proof is driven by the goal of applying the security result for the VBQC
Protocol and we must therefore indistinguishably transform the state preparation and key
release steps into ones resembling those of the VBQC Protocol.

Using Results for DBQC to Simplify State Preparation. We must first transform
the output state of the DBQC Protocol into an honestly-generated VBQC client-encrypted
state. To this effect, we use the specific properties of our DBQC Protocol to rewrite the
potentially deviated output state in a way that separates the state preparation and the
deviation.

We remark that we can at will choose to separate the Server (and Classical SMPC)
into two entities $S_1$ and $S_2$ (respectively $T_1$ and $T_2$), so long as we authorise them to
share state (quantum for the Server, classical for the SMPC). $S_1$ act during the DBQC
Protocol while $S_2$ receives the output of the DBQC Protocol and uses it in the latter steps
of the DMPQC Protocol. The Classical SMPC $T_1$ performs the same actions as an honest
Orchestrator during the DBQC Protocol, while $T_2$ coordinates the VBQC execution and
key-release steps.

The Adversarial Clients and $S_1$ are allowed to deviate during the DBQC Protocol.
However Lemma 2 states that, up to a relabelling of the Server’s deviation, there is then
a one-to-one correspondence between the state at the end of an execution of the DBQC
Protocol followed by VBQC and that after a VBQC execution on an honestly-generated
VBQC state. It is therefore possible to rewrite the state $QOTP_k \circ \mathcal{E}(\rho_{H,M,U})$ obtained by
$S_2$ at the end of the protocol (and therefore also the one sent by the Ideal Resource since
the proof of security of DBQC show that the deviation can be replicated by the Simulator
in the ideal execution) together with the instructions $\delta(i)$ given by the Classical SMPC
$T_2$ as $\mathcal{E}'(\rho_{V} \otimes \rho_{aux})$. There $\rho_{V}$ is a state obtained by the Server from an honest Client
during an execution of the single-client VBQC Protocol upon which the Server performs
an arbitrary deviation independent of the secret parameters that were used to generate
this state. This a purely formal rewriting procedure and therefore it is indistinguishable
from the original protocol.

Using the composition property of the AC framework, the DBQC Protocol can be
replaced by the DBQC Ideal Resource 3. Since both are perfectly indistinguishable so
long as the Orchestrator is honest (and here it is instantiated with the Classical SMPC),
the output state is the same as above. This yields the hybrid presented in Reduction 2,
the distinguishing advantage with the DMPQC Protocol is $\epsilon_D$ as stated in Theorem 1.
Reduction 2 Replacing DBQC Protocol with Resource

1. The Clients and Classical SMPC perform the same initialisation procedure as in the protocol: they place their input qubits at random in their corresponding input base-located and prepare $|+\rangle$ states for the other qubits that they own. These are all encrypted and the position and encryptions are sent to the Classical SMPC $T_1$, who then chooses at random the unitary to be applied (according to its random choice of Dotted Triple-Graph).

2. The Clients send their qubits to be transformed into a VBQC client-encrypted state to the DBQC Resource, the Classical SMPC $T_1$ sends the unitary as the Orchestrator and $S_1$ can choose an input state $\rho_{aux}$ together with a malicious coalition of Clients and deviation $\mathcal{E}'$. $S_2$ recovers the output state $\mathcal{E}'(\rho_V \otimes \rho_{aux})$ as defined in Resource 3, along with a state $\rho_S$ from $S_1$. The Classical SMPC $T_2$ recovers the key $k$ and a classical message state from $T_1$ containing the description of the random parameters of the Dotted Triple-Graph.

3. $S_2$ follows the instructions given by the Classical SMPC $T_2$ according to the VBQC Protocol for non-output qubits.

4. The Clients and $S_2$ perform the Quantum Output Key-Release Procedure along with the Classical SMPC $T_2$.

Rearranging of Honest and Malicious Behaviour during VBQC State Generation. In all generality we can then assume that the deviation occurs after DBQC has been performed: equivalently, $S_2$ receives an honestly prepared VBQC client-encrypted state $\rho_V$ from the DBQC Ideal Resource, along with the deviation $\mathcal{E}'$ and state $\rho_{aux}$ from $S_1$ and applies it to $\rho_V$. Note that in that context, no Clients are malicious since all the malicious behaviour is contained in this deviation and auxiliary state (up to deviations on their input state as explained in Lemma 2, which is always allowed).

The effect is that the Clients in this new hybrid only send correct states to the Resource to be transformed into a VBQC client-encrypted state, along with their inputs (i.e. they no longer need to send additional state to be used in the DBQC Protocol). Since these state are honestly prepared, it is equivalent to having a trusted third party (modelled as a Resource), which we call the Input Super Client which receives the inputs from the Clients, encrypts them and prepares the rest of the input state to the DBQC Resource by itself.

Note that since the deviation on the malicious Client’s inputs has been incorporated as an (adversarially known) modification of their inputs, the only remaining deviation happens after the Server $S_2$ has received the output state from the DBQC Resource. Therefore the interface of the Server on the DBQC Resource is now filtered. This results in the following hybrid in Reduction 3. The distinguishing advantage compared to the previous hybrid is 0.

We notice that the operations performed by $S_1$ consist only of internal operations, therefore it can be dropped in favour of subsuming these steps into $S_2$ (i.e. the choice of deviation $\mathcal{E}'$ and the state $\rho_{aux}$ can be made by $S_2$ directly). Furthermore, we merge the DBQC Resource with the Classical SMPC $T_1$ and the Input Super Client since they are both trusted third parties (and therefore Resources) as well.\textsuperscript{15} The result is simply a new

\textsuperscript{15}Since Resources are described as CPTP maps, if an interface from one Resource is linked to another Resource they can be merged simply by composing the CPTP maps corresponding to these interfaces.
Reduction 3 Transferring Honest Operations to Super Client

1. The Clients send their input qubits to the Input Super Client, which then encrypts them, positions them and prepares additional $|+\rangle$ states. These parameters are sent to $T_1$, who chooses the unitary to be applied as previously.
2. The Input Super Client sends its qubits to the DBQC Resource, the Classical SMPC sends the unitary as the Orchestrator. $S_1$ does not input a deviation in the DBQC Resource but sends $E'$ and state $\rho_{aux}$ to $S_2$, which furthermore receives the output of the DBQC Resource $\rho_V$, an honestly generated VBQC client-encrypted state. The Classical SMPC $T_2$ receives a classical message state from $T_1$, which contains the secret parameters of $\rho_V$.
3. $S_2$ applies $E'$ to $\rho_V \otimes \rho_{aux}$ and proceeds as above.

Resource which we call Collaborative VBQC State Preparation Resource, that takes as input the input qubits of all Clients, prepares the VBQC client-encrypted state $\rho_V$ and sends it to the Server. There is one additional interface that outputs the description of the randomness associated with this state, which is plugged into the Classical SMPC $T_2$. This results in Reduction 4.

Reduction 4 Merging Input Resources and Adversaries

1. The Clients send their input qubits to the Collaborative VBQC State Preparation Resource, who outputs $\rho_V$ to Server $S_2$ and the description of the secret parameters to the Classical SMPC $T_2$.
2. The Classical SMPC $T_2$ instructs the Server to measure the non-output qubits according to the VBQC Protocol. At the end, $S_2$ sends the qubits of the output layer to the appropriate Clients.
3. The Classical SMPC $T_2$ perform the key-release step as in the DMPQC Protocol.

Simplifying Output Recovery Procedure. At this point, up to the quantum key-release step, the execution between the Server, the Collaborative VBQC State Preparation Resource and the Classical SMPC $T_2$ is exactly the same as an execution of the VBQC Protocol. We now focus on this final step, which consists of the following: (i) each Client measures the trap in their output base-location according to the instruction provided by the Classical SMPC and returns the outcome; (ii) the Classical SMPC $T_2$ either sends Abort to the Clients and the Server or sends Q-OTP keys to the Clients; (iii) if there is no abort, each Client decrypts its output.

Any deviation by the malicious Clients after receiving the qubits from their output base-location either lead to an abort (if they affect a trap) or correspond to a deviation on their outputs. Because, in the ideal execution, the malicious Clients are allowed to deviate arbitrarily on their outputs, deviations on output qubits for non-ideal executions do not correspond to attacks as they can be mapped to actions performed after the protocol has concluded. In the absence of abort, all the trap measurement outcomes are correct and we can conclude that the output state is correct up to allowed deviations on the output qubits of malicious Clients. If the deviation affects traps however, the Classical SMPC aborts similarly to an honest Client in the VBQC Protocol. This is then equivalent, up
to a rewriting of the malicious Clients’ deviations to include them as a deviation due to the Server $S_2$ during the VBQC Protocol execution, to having an Output Super Client recovering the output layer from the Server, performing the measurements on traps as instructed by the Classical SMPC $T_2$ and, if there is no abort, decrypting the output before sending to each individual Client its own output state. This results in Reduction 5.

**Reduction 5 Recovering an Honest-Client VBQC Protocol**

1. The Clients send their input qubits to the Collaborative VBQC State Preparation Resource, who outputs $\rho_V$ to the Server and the description of the secret parameters to the Classical SMPC $T_2$.
2. The Classical SMPC $T_2$ instructs the Server to measure the non-output qubits according to the VBQC Protocol. At the end, the Server sends the output qubits to the Output Super Client.
3. The Classical SMPC $T_2$ instructs the Output Super Client to measure the trap qubits. If a trap has been incorrectly measured, $T_2$ outputs `Abort` to the Output Super Client, who forwards it to the Server and the Clients.
4. Otherwise, $T_2$ sends the decryption keys to the Output Super Client, who then decrypts the output and sends to each Client its output.

**Application of VBQC Security Result.** We again use the Resource fusion property to merge the Collaborative VBQC State Preparation Resource, Classical SMPC $T_2$ and the Output Super Client. This new Collaborative VBQC Client Resource performs exactly the same steps as an honest VBQC Client using the collaborative input state provided by the individual Clients, after which it distributes the output states back to the individual Clients.

Finally, we use the result of Lemma 1 which shows the equivalence between the VBQC Protocol with an honest Client and the Verifiable Delegated Quantum Computation Resource. In fact, the Server only ever interacts in the hybrid above with honest parties (or equivalently, Resources). It is therefore possible to replace the VBQC Protocol execution between the Collaborative VBQC Client Resource and the Server with the VDQC Resource, at a cost of $\epsilon_V$.

After this change, the Collaborative VBQC Client Resource is replaced by one which simply forwards the inputs of the individual Clients to the VDQC Resource, and later recovers the output and distributes it to the individual Clients (or aborts). These three Resources – input aggregation, VDQC and output distribution – can then be further merged to exactly form the MPQC Ideal Resource 1. It is furthermore statistically equivalent, up to a combined cost of $\epsilon_D + \epsilon_V$, to the initial DMPQC Protocol. This concludes the proof.

$\square$

**K In-depth Comparison with Previous Work**

**K.1 Performance Analysis**

We now compare our result with [9, 26, 1]. Reference [9] also achieve an information-theoretic upgrade of a Classical SMPC to the quantum domain, secure against an ar-
arbitrary number of corrupted parties. On the other hand, the protocol from [1] is only computationally-secure since it relies on a Fully-Homomorphic Encryption Scheme on top of the Classical SMPC, but it is also secure against arbitrary corruptions. The protocol of [26] constructs an information-theoretically secure Quantum SMPC but suffers from an artificial blow-up in the number of participants and exchanged qubits.\footnote{It is based on error-correcting codes and the size of the code must correspond to the number of players $N$. The maximum number of cheaters tolerated by the protocol is the number of correctable errors $\lceil \frac{N-1}{2} \rceil$, which by the quantum Singleton bound [31] is at most $\lfloor \frac{N-1}{2} \rfloor$. In their example, 7 players are required for implementing a two-party computation since the code that is used is of size 7 and corrects 1 error. This leads to a situation where 5 participants that don’t have inputs nor outputs must still exchange messages and none can be malicious if one of the players with inputs is.} The protocols of [26, 1] are proven secure in the Stand-Alone Model, whereas ours and that of [9] are fully composable. On top of blindness, all protocols provide verifiability with unanimous abort apart from that of [1] who achieves the stronger notion of identifiable abort.\footnote{A protocol satisfies the unanimous abort property if all honest players abort at the same time, as compared with selective aborts where the Adversary can choose which players will abort separately. On top of that, identifiable abort means that all honest players agree on the malicious party responsible for the failure of the protocol.}

One key advantage of our protocol over the others lies in its delegated nature, where only one participant needs a full fault-tolerant quantum computer while the rest only perform very limited quantum operations, compared with the symmetric setup in [9, 26] where all participant has requires fault-tolerance. The protocol of [1] can be considered semi-delegated in the sense that the brunt of the quantum computation is performed by a single player. However, all players must have the ability to perform arbitrary Cliffords on large states and cannot do so without having at their disposal a full fault-tolerant quantum computer. This is also reflected in the network topology: whereas the best performance in [9, 26, 1] can only be reached by using a complete quantum and classical communication graph, we only need a star graph for quantum communications.

Regarding classical primitives, [26] only requires secure coin-tossing and authenticated broadcast channels (information-theoretically secure since they can rely on an honest majority). We only use our Classical SMPC to perform coin-tossing, basic string operations (selection in array) and computations in $\mathbb{Z}_8$ and $\mathbb{Z}_2$. The Classical SMPC is more complex in [9, 1] since it must be able to sample uniformly at random and perform computations on the classical descriptions of arbitrary Cliffords.

We can now quantify more precisely the number of classical rounds of communication or calls to the Classical SMPC resource, quantum rounds of communication, and size of quantum memory required by each participant in the protocol. Let $N$ be the number of parties, $d$ the depth of the computation (MBQC for our paper, circuit for [26] and $\{T, \text{CNOT}\}$-depth for [9]), $t$ the number of $T$ gates, $c$ the number of $\text{CNOT}$ gates and $\eta$ a statistical security parameter.

\cite{9} calls the Classical SMPC very often: a constant number of times for each input qubit and gate in the circuit. But the most costly part is the generation of ancillary magic states (for implementing $T$ gates via gate-teleportation), which requires $O(\eta (N + t))$ invocations of the Classical SMPC. Our protocol simply uses $d + 5$ calls to this Resource, 2 of which are made for setting up the state and 3 for the key-release step (2 for classical outputs). This is equivalent to the classical communication requirements of [26], where they only need $d + 2$ classical broadcasts per participant (one for setting up the shared randomness and another for the state preparation, while the calls during the computation can be parallelised). If all quantum communications are done in parallel in [26], it can be further parallelised to only require a constant number of classical broadcast rounds. The protocol
of [1] uses FHE (classical and quantum) to perform the computation and consequently the number of calls to the Classicl SMPC is only constant. We note that using another classical primitive called functional encryption, where a party in possession of an evaluation key can recover the clear-text of a function of the encrypted values (and only that), would allow to attain the same result for our construction by allowing the Server to compute the next measurement angle as a function of the encrypted secrets and previous measurement results.

The protocol of [9] requires numerous rounds of quantum communication as they need to send encoded states around for the verification of inputs and T and CNOT gates. After parallelisation the total cost is $O(Nd)$ quantum rounds. [1] aims to remove the circuit dependency in the number of rounds, obtaining $O(N^4)$ quantum rounds in the worst case in the case where the protocol is parallelised. [26] seeks to optimise the quantum memory requirement of players and therefore their communication is done sequentially, yielding $O(\eta^2(N + t))$ quantum rounds. Parallelisation lowers it to 3 (or 2 for classical outputs), at a higher quantum memory cost for all parties. Our protocol is optimal as there are only 2 quantum rounds (1 for classical outputs): sending to the Server the inputs and all states required for the collaborative state preparation phase and later recovering the output layer qubits from the Server.

Finally, the number of qubits required by [9] during the computation phase is $O(\eta(N + t))$ for each participant (they encode each of their input qubits, ancillae and magic states using $O(\eta)$ qubits). However they use $O(\eta^2(N + t))$ additional qubits in the offline phase to prepare the ancillary qubits (if the quantum communications are performed in parallel). On the other hand, [26] reduces the number of qubits for each participant to $O(N^2)$ for sequential quantum communication, but this blows up to $O(\eta^2 N(N + t))$ if parallelised. The construction from [1] uses a compiler that adds automatically a cost of $O(N^2)$ for each base qubit. The costly double encryptions and multiple layers of traps, in particular for the magic state distillation procedure, yields a total quantum memory cost per participant of at least $O(tN^9 \eta^2)$ (this is a weak lower bound). In our paper the Server needs $O(\eta Nd)$ qubits to perform the VBQC computation, and it must be able to apply a constant-depth MBQC computation through the DBQC Protocol to these qubits first to transform some input dummies. The graph applied contains 9 qubits per qubit in the final $DT(G)$ used in VBQC. Each qubit in these graphs must be generated using $N$ qubits, resulting in a total qubit cost of $O(\eta N^2 d)$ for parallel rounds of quantum communication but only $O(\eta Nd)$ if they are performed sequentially. However, the Clients can prepare the additional qubits on the fly, therefore each Client only requires three qubits of quantum memory which it uses to store its input at the start of the protocol and the output, dummy and trap at the end. For classical inputs and outputs, the Clients do not even need quantum memory (their inputs can be encoded into the qubits that have been sent by adding $\pi$ to the rotation angle and the outputs are classical since the Server performs the measurements).

K.2 Proof Techniques

The proof techniques used in the present work are very different from those of previous results. This is not only a consequence of the divide between the verifiable and non-verifiable segments of our protocol but this change of paradigm is also a pre-requisite for

\footnote{They send states along a path of size $N^2$ in the communication graph of the parties, and remove a party if it doesn’t deliver a packet before resending the states along a different path of the same size. In the worst case where there are $N - 1$ malicious players which do not want to get caught cheating, they can drop $(N - 1)(N - 2)/2$ packets before they get disconnected from the communication graph.}
attaining a low number of quantum communication rounds.

In previous works, in order to collaboratively encrypt the initial quantum state (comprising input and ancillary qubits), it is sent through all participants so that each one may apply its own encryption operation. This allows the Simulator for the malicious parties to intercept this state as it is forwarded to the honest party which it impersonates. The Simulator can then replace this state with another which does not contain the input of the malicious players and continue the protocol with this “fake” state. The Simulator is later free to transmit the malicious players’ inputs to the Ideal Functionality and recover the correct output of the dishonest players. At the end of the protocol there is a key-release step where the encoded output state is again transmitted through all parties. If the protocol does not abort on the fake input state, \(^{19}\) the Simulator then uses this opportunity to substitute the fake state with the (encoded) output state of the malicious parties. All technical details are there to make sure that these steps go through.

Seeing as one of our protocol’s main goals is to limit quantum communication rounds to two (one for sending inputs and another for receiving outputs), a similar technique cannot be used as it requires a number of rounds that is linear in the number of participants. Instead, for all honest players’ qubits, the simulator sends a fake state to the Server (in this case, half of an EPR-pair). The other half of the EPR-pair is sent to the Ideal Functionality. In this setting, it seems like the Ideal Functionality never receives the input of the malicious players (which the Simulator cannot extract since it never has access to it) and therefore cannot produce the ideal output in case the Simulator does not abort. However, using the half EPR-pair, it can instead perform a gate-teleportation on the input state to reproduce the correct operations even though the qubits upon which this transformation is applied remain on the Server’s internal registers. The final output is the same as in the real protocol. This is most natural in the MBQC framework since the computation is based on the same principle, but it could also be used in circuit-based models by having the participants send ancilla states for performing the encryption to a single participant.

This technique is used in a number of MBQC-based protocols such as [10], and is explicitly apparent in our DBQC Protocol. Our full protocol would use the same technique in its proof if we wanted to exhibit a simulator. We have chosen instead to use a reduction-based proof in order to leverage the composability properties of the Abstract Cryptography framework and demonstrate the proper way to bridge the verifiable and non-verifiable subroutines in such a setting.

\(^{19}\)The probability of aborting should be negligibly close to the one in the real protocol and dependent only on the actions of the malicious players.