The decay $\mu \to e\gamma$ selecting scalar leptoquark solutions for the $(g - 2)_{e,\mu}$ puzzles

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Abstract

We demonstrate that the chirality enhancement required to simultaneously explain the $(g - 2)_e$ and $(g - 2)_\mu$ data with a single scalar leptoquark is in direct conflict with the existing constraint on the induced lepton flavor violation as given by the limits on the branching ratio for the $\mu \to e\gamma$ process. We furthermore investigate all potentially viable combinations of two scalar leptoquarks that can simultaneously address the $(g - 2)_{e,\mu}$ discrepancies only to find that the current experimental bound on $\mu \to e\gamma$ represents an unsurmountable obstacle to that appealing prospect, apart from the trivial scenario in which $S_1$ leptoquark would generate chirality enhanced contributions solely to $(g - 2)_e$ while $R_2$ leptoquark would only affect $(g - 2)_\mu$ or the other way around. Our results with regard to the simultaneous explanation of observed discrepancies are very robust as they either do not depend on the scale of new physics or, if they do, exhibit only a mild dependence.

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1 Introduction

The observed anomalous magnetic moments of electrons and muons exhibit tension with the corresponding Standard Model (SM) predictions. In particular, the experimental results \(a_{e,\mu}^{\text{exp}}\) for the electron [1] and muon [2] anomalous magnetic moments deviate from the Standard Model (SM) predictions \(a_{e,\mu}^{\text{SM}}\) roughly at the \(3\sigma\) [3] and \(4\sigma\) [4–8] levels, respectively. More precisely, the observed discrepancies that are of opposite signs currently read

\[
\Delta a_e = a_{e}^{\text{exp}} - a_{e}^{\text{SM}} = -(8.7 \pm 3.6) \times 10^{-13}, \\
\Delta a_\mu = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (2.79 \pm 0.76) \times 10^{-9}.
\]

Various sources of new physics are known to be capable to substantially alter the SM values for \(a_e = (g - 2)_e/2\) and \(a_\mu = (g - 2)_\mu/2\). One might hope that both of these experimentally observed discrepancies have a common and presumably simple new physics origin, especially one that could be tested with ease. (For a sample of studies that analyse effects of the new physics sources on anomalous magnetic moments see, for example, Refs. [9–38].) We are accordingly interested in the effects of scalar leptoquarks on these observables. There are only four scalar leptoquark multiplets one needs to consider, as we demonstrate later on, if the new physics scenarios for \(a_e\) and \(a_\mu\) are based on the SM fermionic content and include up to two leptoquarks. These are \(S_3(3,3,1/3), R_2(3,2,7/6), \tilde{R}_2(3,2,1/6), \) and...
$S_1(\bar{3}, 1, 1/3)$, where we specify the transformation properties of leptoquarks under the SM gauge group $SU(3) \times SU(2) \times U(1)$. (For the reviews on the leptoquark physics see, for example, Refs. [39, 40].)

It is already common knowledge that it is possible to address discrepancies between predicted and observed values for $(g - 2)_e$ and $(g - 2)_\mu$ at the individual level with $S_1$ and $R_2$ leptoquarks, where $S_1$ could generate chirality enhanced contributions solely to $(g - 2)_e$ while $R_2$ would only affect $(g - 2)_\mu$ or the other way around. It is mandatory that these one-loop level contributions are the quark mass chirality enhanced to be of sufficient strength and the SM gauge symmetry dictates the presence of the up-type quarks in the loop [41]. The only significant difference between the $S_1$ and $R_2$ mediation is that the former is due to the $Q = 1/3$ leptoquark and the latter is due to the $Q = 5/3$ one, where $Q$ denotes electric charge in units of absolute value of the electron charge. The consensus in the community is that one needs the top quarks in the $(g - 2)_\mu$ loop if one is to address the observed discrepancy at the 1$\sigma$ level and still be in agreement with the ever more stringent combination of constraints from the flavor physics experiments and LHC [42]. The possibility to address the $(g - 2)_\mu$ discrepancy with the top quark chirality enhanced loop contribution is phenomenologically viable even if one resorts to a scenario when two leptoquarks of the same electric charge couple to the top quark-muon pairs of opposite chiralities while mixing with each other through the Higgs field in order to close the loop [43]. This corresponds to the scenario when $S_1$ mixes with the $Q = 1/3$ component of $S_3$. The chirality enhanced loops for $(g - 2)_e$ that are generated by either $S_1$ or $R_2$, on the other hand, can be closed not only with the top quarks but also with the charm quarks without any conflict with the existing experimental limits such as those due to the $D$-$\bar{D}$ oscillation and/or atomic parity violation measurements for the leptoquark masses that are allowed by the LHC data analyses. Moreover, in the two leptoquark scenario based on the $R_2$&$\tilde{R}_2$ combination it is possible to close the $(g - 2)_e$ loops in a phenomenologically viable way with the bottom quarks [43] as well.

We, in this study, are interested whether it is possible to simultaneously explain both discrepancies with the new physics that is generated by only one or, at the most, two scalar leptoquarks, barring the aforementioned trivial possibility of using $S_1$ to address one and $R_2$ to address the other anomalous magnetic moment. We accordingly analyse the ability of $S_1$ to simultaneously accommodate both the $(g - 2)_e$ and $(g - 2)_\mu$ discrepancies only to find that the current experimental bound on $\mu \to e\gamma$ represents an unsurmountable obstacle to that appealing prospect. The same conclusion is also applicable to the case when $R_2$ is used instead of $S_1$. Our findings hold regardless of whether the $(g - 2)_e$ contribution is generated with the top or charm quark loops. (Our conclusions with regard to the viability of the single leptoquark solution differ somewhat with respect to the results presented in Ref. [44].)
As for the potential viability of the solutions with the two leptoquark scenarios we find only two non-trivial cases that one needs to study. These are the $S_1 \& S_3$ and $R_2 \& \tilde{R}_2$ combinations, where the leptoquarks in question need to couple to each other via the SM Higgs boson. We demonstrate that both of these two leptoquark scenarios also fail to simultaneously address the $(g - 2)_e$ and $(g - 2)_\mu$ discrepancies for the same reason as for the two single leptoquark cases. In all four instances it is always necessary to turn on at least four Yukawa couplings between the leptoquark(s) and the relevant quark-lepton pairs if one is to generate chirality enhanced contributions of sufficient strength. We summarise in Table I all the possibilities that arise from these four cases that we consider in order to simultaneously address the $(g - 2)_{e,\mu}$ discrepancies.

|       | $S_1$ | $S_1$ | $R_2$ | $R_2$ | $S_1 \& S_3$ | $S_1 \& S_3$ | $R_2 \& \tilde{R}_2$ |
|-------|------|------|------|------|-------------|-------------|----------------------|
| $(g - 2)_e$ | $t$, $1/3$ | $c$, $1/3$ | $t$, $5/3$ | $c$, $5/3$ | $t$, $1/3$ | $c$, $1/3$ | $b$, $2/3$ |
| $(g - 2)_\mu$ | $t$, $1/3$ | $t$, $1/3$ | $t$, $5/3$ | $t$, $5/3$ | $t$, $1/3$ | $t$, $1/3$ | $t$, $5/3$ |

Table I: The leptoquark scenarios that have potential to simultaneously address the $(g - 2)_{e,\mu}$ discrepancies but fail to pass the $\mu \rightarrow e\gamma$ test. We specify the quark that is behind the chirality enhanced contribution and the electric charge of the leptoquark in the loops.

The paper is organized as follows. In Sec. 2 we elaborate on the scalar leptoquark contributions towards anomalous magnetic moments of electrons and muons and discuss the associated effect on the $\mu \rightarrow e\gamma$ process that we find to be the origin of the most relevant flavor constraint. We then proceed to discuss shortcomings of four different scenarios to simultaneously explain the $(g - 2)_e$ and $(g - 2)_\mu$ discrepancies with scalar leptoquarks. The first two scenarios that rely on the single leptoquark contributions towards both anomalous magnetic moments in question are discussed in Sec. 2.1 while the remaining two possibilities are addressed in Sec. 2.2. We summarize our findings in Sec. 3.

## 2 Addressing $(g - 2)_{e,\mu}$ with scalar leptoquarks

We first present an overview of the scalar leptoquark effects on $(g - 2)_{e,\mu}$ and $\mu \rightarrow e\gamma$ process using the $S_1$ scenario for concreteness. The Yukawa couplings of $S_1$ are [39]

$$
\mathcal{L} \supset y^L_{ij} \bar{Q}_i \sigma_2 S_1 L_j + y^R_{ij} \bar{u}_R^i S_1 \ell_R^j + \text{h.c.},
$$

where $Q_i = (u_{Li}^T \ d_{Li})^T$ and $L_j = (\nu_{Li}^T \ \ell_{Li})^T$ are the left-handed quark and lepton $SU(2)$ doublets, $u_{Ri}$, and $\ell_{Rj}$ are the right-handed up-type quarks and charged leptons, respectively, $\sigma_2$ is the Pauli matrix, and $i, j = 1, 2, 3$ are flavor indices. The Yukawa coupling matrices $y^L$
and $g^R$ are a priori arbitrary $3 \times 3$ matrices in the flavor space. The $S_1$ diquark couplings have been omitted to ensure proton stability.

To calculate the flavor observables, it is convenient to rewrite the Lagrangian of Eq. (3) in the SM fermion mass eigenbasis, to which end we make the following transformations of the fermion fields: $d_L \to d_L$, $u_L \to V^* u_L$, $\ell_L \to \ell_L$, and $\nu_L \to U \nu_L$. $U$ and $V$ are the Pontecorvo-Maki-Nakagawa-Sakata and Cabibbo-Kobayashi-Maskawa matrices, respectively. With these redefinitions, the part of Lagrangian presented in Eq. (3) takes the following form:

$$L_{S_1} = -(y^U)^i_{ij} \overline{d}_{Li} S_1 \nu_{Lj} + (V^* y^L)^i_{ij} \overline{u}_{Li} S_1 \ell_{Lj} + y^R_{ij} \overline{u}_{Ri} S_1 \ell_{Rj} + \text{h.c.}.$$  (4)

It is now possible to write the $S_1$ contributions towards $(g-2)^{\mu\mu}$. We will accomplish this within a scenario when $S_1$ couples to the top quark-electron and top-quark muon pairs. The Yukawa couplings that we switch on in Eq. (3) are $y^L_{32}$, $y^L_{31}$, $y^R_{32}$, and $y^R_{31}$ to find

$$\Delta a_e = -\frac{3 m_e^2}{8 \pi^2 M^2} \left[ \frac{m_t}{m_e} \text{Re} \left( V_{tb}^* y^L_{31} (y^R_{31})^* \right) \left( \frac{7}{6} + \frac{2}{3} \ln x_t \right) - \frac{1}{12} \left( |y^R_{31}|^2 + |y^L_{31}|^2 \right) \right],$$  (5)

$$\Delta a_\mu = -\frac{3 m_\mu^2}{8 \pi^2 M^2} \left[ \frac{m_t}{m_\mu} \text{Re} \left( V_{tb}^* y^L_{32} (y^R_{32})^* \right) \left( \frac{7}{6} + \frac{2}{3} \ln x_t \right) - \frac{1}{12} \left( |y^R_{32}|^2 + |y^L_{32}|^2 \right) \right].$$  (6)

Again, it is always necessary to switch on at least four Yukawa couplings to simultaneously affect $(g-2)^e$ and $(g-2)^\mu$ with the chirality enhanced contributions. One pair enters $(g-2)^e$ and the other $(g-2)^\mu$. Note that we define, for convenience, $x_t = m_t^2/M^2$, where $m_t$ is the top quark mass and $M$ is the mass of $S_1$ leptoquark.

The current limit on the branching ratio for $\mu \to e\gamma$ process is $Br (\mu \to e\gamma) < 4.2 \times 10^{-13}$ [45]. We find it to be the most severe obstacle to simultaneous explanation of the $(g-2)^{e,\mu}$ discrepancies with the scalar leptoquark physics. For example, if the leading parts of the $(g-2)^{e,\mu}$ loops are proportional to the top quark mass, as given in Eqs. (5) and (6), the new physics contribution towards $\mu \to e\gamma$ is [46]

$$Br(\mu \to e\gamma) = \frac{9 \alpha \tau_\mu m_\mu^5}{1024 \pi^4 M^4} \left( |A_1|^2 + |B_1|^2 \right),$$  (7)

where

$$A_1 = -\frac{1}{12} \left[ y_{32}^L (y_{31}^L)^* + \frac{m_e}{m_\mu} y_{32}^R (y_{31}^R)^* \right] + \frac{m_t}{m_\mu} \left( \frac{7}{6} + \frac{2}{3} \ln x_t \right) V_{tb}^* y_{32}^L (y_{31}^L)^*,$$  (8)

$$B_1 = -\frac{1}{12} \left[ y_{32}^R (y_{31}^R)^* + \frac{m_e}{m_\mu} y_{32}^L (y_{31}^L)^* \right] + \frac{m_t}{m_\mu} \left( \frac{7}{6} + \frac{2}{3} \ln x_t \right) V_{tb}^* y_{32}^L (y_{31}^L)^*. $$  (9)

There are also new physics contributions to other processes such as $\mu - e$ conversion, $\mu \to eee$, and $Z \to \ell \ell'$, to name a few, that are generated once one tries to simultaneously address $(g-2)^e$ and $(g-2)^\mu$ with scalar leptoquarks. We find them to always be subdominant with respect to the $\mu \to e\gamma$ constraint. We have confirmed this explicitly with the numerical analysis that has prompted us to omit them from this study altogether.
2.1 Single leptoquark scenarios: $S_1$ and $R_2$

2.1.1 $S_1$ with the top quark loops

Let us start with the $S_1$ case, when the $(g−2)_e$ and $(g−2)_μ$ loops are both top quark induced, and with real Yukawa couplings $y_{31}^L$, $y_{32}^L$, $y_{31}^R$, and $y_{32}^R$, as defined in Eq. (3), switched on. The leading chirality enhanced contributions towards $Δa_e$ and $Δa_μ$ are

$$Δa_e = -\frac{3}{8π^2} \frac{m_t m_e}{M^2} y_{31}^L y_{31}^R \left(\frac{7}{6} + \frac{2}{3} \ln x_t\right),$$  \hspace{1cm} (10)

$$Δa_μ = -\frac{3}{8π^2} \frac{m_t m_μ}{M^2} y_{32}^L y_{32}^R \left(\frac{7}{6} + \frac{2}{3} \ln x_t\right),$$  \hspace{1cm} (11)

while the $μ → eγ$ contribution is

$$Br(μ → eγ) = \frac{τ_μ α m_μ^3}{4} \left|\frac{3m_t}{16π^2 M^2} \left(\frac{7}{6} + \frac{2}{3} \ln x_t\right)\right|^2 \left[|y_{31}^L y_{31}^R|^2 + |y_{32}^L y_{32}^R|^2\right].$$  \hspace{1cm} (12)

If we define $x = y_{31}^R / y_{32}^R$ and rearrange Eqs. (10), (11), and (12), we obtain the following expression for $Br(μ → eγ)$ in terms of $Δa_e$ and $Δa_μ$:

$$Br(μ → eγ) = \frac{τ_μ α m_μ^3}{16} \left(\frac{Δa_e^2}{m_e^2} \frac{1}{x^2} + \frac{Δa_μ^2}{m_μ^2} x^2\right) = 5.2 × 10^{-6} \frac{1}{x^2} + 1.2 × 10^{-3} x^2,$$  \hspace{1cm} (13)

where the central values for $Δa_e$ and $Δa_μ$, as given in Eqs. (1) and (2), are inserted for convenience. We also use $m_μ = 105.65$ MeV and $m_e = 0.5109$ MeV [47].

An especially nice feature of the prediction for $Br(μ → eγ)$, as given in Eq. (13), is that it does not exhibit dependance on the scale of new physics. It also makes it transparent that it is impossible to reconcile the current limit on $Br(μ → eγ)$ with required shifts in $Δa_e$ and $Δa_μ$ for any value of $x$, where $x$ is the only new physics parameter that is featured in the $Br(μ → eγ)$ prediction.

2.1.2 $S_1$ with the top and charm quark loops

One might entertain a possibility to address $(g−2)_e$ with the charm quark loops and $(g−2)_μ$ with the top quark loops. This would correspond to the scenario when real Yukawa couplings $y_{31}^L$, $y_{32}^L$, $y_{21}^R$, and $y_{32}^R$, as defined in Eq. (3), are switched on. If we neglect the subleading contributions towards $Δa_e$ and $Δa_μ$ we obtain the following expressions:

$$Δa_e = -\frac{3m_e m_c}{8π^2 M^2} \left(\frac{7}{6} + \frac{2}{3} \ln x_c\right) V_{cs} y_{21}^L y_{21}^R,$$  \hspace{1cm} (14)

$$Δa_μ = -\frac{3m_μ m_t}{8π^2 M^2} \left(\frac{7}{6} + \frac{2}{3} \ln x_t\right) V_{tb} y_{32}^L y_{32}^R,$$  \hspace{1cm} (15)

$$Br(μ → eγ) = \frac{9ατ_μ m_μ^5}{1024π^4 M^4} \left|V_{ts}\right|^2 \left(\frac{m^2}{m^2_μ} \left(\frac{7}{6} + \frac{2}{3} \ln x_t\right)\right)^2 \left(y_{32}^L y_{32}^R\right)^2.$$  \hspace{1cm} (16)
The part of the Lagrangian is

\[ + |V_{cb}|^2 \frac{m_c^2}{m_{\mu}^2} \left( \frac{7}{6} + \frac{2}{3} \ln x_c \right)^2 \left( y_{21}^L y_{21}^R \right)^2, \]

(16)

where we introduce \( x_c = m_c^2/M^2 \) with \( m_c \) being the charm quark mass.

If we rearrange Eqs. (14), (15), and (16), and define \( x = y_{21}^R/y_{21}^R \), we obtain

\[ Br(\mu \to e\gamma) = \frac{\tau_{\mu e} y_{\mu e}^2}{16} \left( \frac{\Delta a_{\mu}^2}{m_{\mu}^2} \right) \left( \frac{\Delta a_{e}^2}{m_{e}^2} \right) \left( |V_{ts}|^2 \right) \left( |V_{tb}|^2 \right), \]

(17)

where

\[ \Delta a_{\mu} = \frac{m_{\mu}^2}{m_{e}^2} \frac{(7/6 + 2/3 \ln x_e)^2}{(7/6 + 2/3 \ln x_c)^2}. \]

(18)

This time around the expression for \( Br(\mu \to e\gamma) \), as given in Eq. (17), exhibits logarithmic dependence on the new physics scale. If we insert \( m_t = 173 \text{ GeV}, m_c = 1.275 \text{ GeV}, V_{cs} = 0.9735, V_{tb} = 1, V_{cb} = 0.0416, \) and \( V_{ts} = 0.0409 \) [47], and take as a benchmark point \( M = 1.5 \text{ TeV} \), we obtain that \( \Delta a_{\mu} = 792 \) and, more importantly, we find that

\[ Br(\mu \to e\gamma) = 7.3 \times 10^{-6} \frac{1}{x^2} + 2.7 \times 10^{-9} x^2. \]

(19)

It is clear from Eq. (19) that this particular \( S_1 \) scenario also fails to reconcile required shifts in \( \Delta a_e \) and \( \Delta a_{\mu} \) with the current bound on \( Br(\mu \to e\gamma) \).

### 2.1.3 \( R_2 \) with the top quark loops

The analysis of this \( R_2 \) scenario will mirror that of the \( S_1 \) case as we show next. The relevant part of the Lagrangian is

\[ \mathcal{L} \supset -y_{ij}^L \bar{u}_{R_i} R_2 i\sigma_2 L_{L_j} + y_{ij}^R \bar{Q}_{L_i} R_2 \ell_{R_j} + h.c., \]

(20)

where \( y^L \) and \( y^R \) are the Yukawa coupling matrices associated with \( R_2 \). If we go to the mass and electric charge eigenstate basis we have that

\[ \mathcal{L}_{R_2} = -y_{ij}^L \bar{u}_{R_i} \ell_{L_j} R_2^{5/3} + (V y^R)_{ij} \bar{u}_{L_i} \ell_{R_j} R_2^{5/3} \]

(21)

\[ + (y^L U)_{ij} \bar{u}_{R_i} \nu_{L_j} R_2^{2/3} + y_{ij}^R \bar{Q}_{L_i} \ell_{R_j} R_2^{2/3} + h.c., \]

(22)

where \( R_2^{5/3} \) and \( R_2^{2/3} \) are \( Q = 5/3 \) and \( Q = 2/3 \) components of \( R_2 \) multiplet, respectively. We will assume that both components of \( R_2 \) are degenerate in mass and denote the corresponding mass with \( M \) in what follows.

To generate \( \Delta a_{\ell} \) contributions with the chirality enhanced top quark loops we need to switch on \( y_{3\ell}^R \) and \( y_{3\ell}^L \), as defined in Eq. (20), where \( \ell = 1, 2 = e, \mu \). This yields

\[ \Delta a_{\ell} = -\frac{3m_\ell^2}{8\pi^2 M^2} \left[ \frac{m_\ell}{m_\ell} \Re \left((V_{tb} y_{3\ell}^R)^* y_{3\ell}^L \right) \left( \frac{1}{6} + \frac{2}{3} \ln x_\ell \right) + \frac{1}{4} \left(|y_{3\ell}^R|^2 + |y_{3\ell}^L|^2\right) \right], \]

(23)
\[ Br(\mu \rightarrow e\gamma) = \frac{9\alpha_t m_e^5}{1024\pi^4 M^4} \left( |A_2|^2 + |B_2|^2 \right), \]  

where

\[ A_2 = \frac{1}{4} \left[ y_{32}^R(y_{31})^* + \frac{m_e}{m_\mu} y_{32}^L(y_{31})^* \right] + \frac{m_t}{m_\mu} \left( \frac{1}{6} + \frac{2}{3} \ln x_t \right) (V_{tb} y_{31})^* y_{32}^L, \]  

\[ B_2 = \frac{1}{4} \left[ y_{32}^L(y_{31})^* + \frac{m_e}{m_\mu} y_{32}^R(y_{31})^* \right] + \frac{m_t}{m_\mu} \left( \frac{1}{6} + \frac{2}{3} \ln x_t \right) (y_{31})^* V_{tb} y_{32}^R. \]

The expression for \( Br(\mu \rightarrow e\gamma) \) in Eq. (24) translates into the exact same functional form, including the numerical prefactors, as that of Eq. (13), where, again, we define \( x = y_{31}/y_{32}^R \) but, this time, for the \( R_2 \) Yukawa couplings. This shows that the \( R_2 \) scenario with the chirality enhanced top quarks loops also fails to simultaneously address the \((g-2)_{e,\mu}\) discrepancies in phenomenologically viable way due to the conflict with the \( \mu \rightarrow e\gamma \) constraint.

### 2.1.4 \( R_2 \) with the top and charm quark loops

If we try to accommodate \((g-2)_{e}\) with the charm quark loops and \((g-2)_{\mu}\) with the top quark loops by switching on, in Eq. (20), \( y_{21}^R, y_{21}^L, y_{32}^R, \) and \( y_{32}^L, \) we find that

\[ \Delta a_e = -\frac{3 m_e m_c}{8\pi^2 M^2} \left( \frac{1}{6} + \frac{2}{3} \ln x_c \right) V_{cs} y_{21}^R y_{21}^L, \]  

\[ \Delta a_\mu = -\frac{3 m_\mu m_t}{8\pi^2 M^2} \left( \frac{1}{6} + \frac{2}{3} \ln x_t \right) V_{tb} y_{32}^R y_{32}^L, \]

while

\[ Br(\mu \rightarrow e\gamma) = \frac{9\alpha_t m_e^5}{1024\pi^4 M^4} \left[ |V_{ts}|^2 m_e^2 \left( \frac{1}{6} + \frac{2}{3} \ln x_t \right)^2 (y_{21}^R y_{32}^L)^2 \right. \]  

\[ + |V_{cb}|^2 m_e^2 \left( \frac{1}{6} + \frac{2}{3} \ln x_c \right)^2 (y_{32}^R y_{21}^L)^2 \]. \]

If we introduce \( x = y_{21}^R/y_{32}^L \) and combine Eqs. (27), (28), and (29), we obtain the following expression for \( Br(\mu \rightarrow e\gamma) \) in terms of \( \Delta a_e \) and \( \Delta a_\mu \):

\[ Br(\mu \rightarrow e\gamma) = \frac{\alpha_t}{16} \left( \Delta a_e^2 \frac{1}{m_e^2} \frac{1}{x^2} |V_{cb}|^2 + \Delta a_\mu^2 \frac{x^2}{m_\mu^2} \frac{|V_{ts}|^2}{|V_{cb}|^2} \right) = 9.5 \times 10^{-9} \frac{1}{x^2} + 2.1 \times 10^{-6} x^2. \]  

We see that the \( R_2 \) scenario with the chirality enhanced top quarks loops for \((g-2)_{\mu}\) and charm quark loops for \((g-2)_{e}\) is also not phenomenologically viable source of new physics.

### 2.2 Two leptoquark scenarios: \( S_1 \& S_3 \) and \( R_2 \& \tilde{R}_2 \)

The two leptoquark scenarios \( S_1 \& S_3 \) and \( R_2 \& \tilde{R}_2 \) open up additional possibilities to simultaneously address the \((g-2)_{e,\mu}\) discrepancies, apart from the trivial \( S_1 \& R_2 \) combination
when \( S_1 \) would generate chirality enhanced contributions solely to \((g - 2)_{e}\) while \( R_2 \) would only affect \((g - 2)_{\mu}\) or the other way around. For example, the \( R_2 & \tilde{R}_2 \) scenario can produce chirality enhanced contributions towards \((g - 2)_{e}\) that are proportional to the bottom quark mass. In what follows we systematically go through the various potentially viable possibilities, mirroring the analyses of the single leptoquarks scenarios, only to find that all of them fail to pass the \( \mu \to e\gamma \) test.

### 2.2.1 \( S_1 & S_3 \) with the top quark loops

We start with the analysis of the two leptoquark scenario based on the \( S_1 & S_3 \) combination. The idea is to address both discrepancies with the chirality enhanced top quark loops, where the leptoquarks in the loop will be mixture of \( S_1 \) with the \( Q = 1/3 \) state in \( S_3 \). Our objective is to determine whether this approach could ease the tension between the \( \mu \to e\gamma \) constraint and the \((g - 2)_{e,\mu}\) discrepancies that we observe in the single leptoquark scenarios.

The relevant parts of the new physics Lagrangian that have not been introduced in preceding sections are

\[
\mathcal{L} \supset y_{ij}^S \bar{q}_{Li} \sigma_2 (\sigma_a S_a^0) L_{Lj} + \lambda H^\dagger (\sigma_a S_a^0) H S_i^* + \text{h.c.,} \tag{31}
\]

where \( \lambda \) is a dimensionless coupling and \( \sigma_a, a = 1, 2, 3 \), are Pauli matrices. It is the second term in Eq. (31) that, after electroweak symmetry breaking, induces a mixing between \( S_1/3 \) and \( S_1 \) via the vacuum expectation value \( v \) of the SM Higgs field \( H(1, 2, 1/2) \). Note that the \( S_3 \) diquark couplings have been omitted to ensure proton stability.

The Yukawa couplings of \( S_3 \), in the mass eigenstate basis for the SM fermions, are

\[
\mathcal{L}_{S_3} = -(y^S U)_{ij} \bar{d}_{Li} S_3^{1/3} \nu_{Lj} - \sqrt{2} y_{ij}^S \bar{d}_{Li} S_3^{4/3} \ell_{Lj} + \sqrt{2} (V^* y^S U)_{ij} \bar{u}_{Li} S_3^{2/3} \nu_{Lj} - (V^* y^S)_{ij} \bar{u}_{Li} S_3^{1/3} \ell_{Lj} + \text{h.c..} \tag{32}
\]

We perform the analysis in the leptoquark mass eigenstate basis. The mixing matrix for \( S_3^{1/3} \) and \( S_1 \) is \([43]\]

\[
\begin{pmatrix}
S_- \\
S_+
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
S_3^{1/3} \\
S_1
\end{pmatrix} \equiv \begin{pmatrix}
c_\theta & s_\theta \\
-s_\theta & c_\theta
\end{pmatrix} \begin{pmatrix}
S_3^{1/3} \\
S_1
\end{pmatrix}, \tag{33}
\]

with

\[
\tan 2\theta = \frac{\lambda v^2}{M_1^2 - M_3^2}, \tag{34}
\]

\[
M_\pm^2 = \frac{M_1^2 + M_3^2}{2} \pm \frac{1}{2} \sqrt{(M_1^2 - M_3^2)^2 + \lambda^2 v^4}, \tag{35}
\]

where \( \theta \) is the mixing angle, \( M_\pm \) are masses of states \( S_\pm \) with \( Q = 1/3 \), \( M_1 \) and \( M_3 \) are masses of \( S_1 \) and \( S_3 \) multiplets, respectively, for \( \lambda = 0 \). Note that \( S_3^{1/3} \) and \( S_3^{-2/3} \) have common mass \( M_3 \) regardless of the mixing.
We switch on \( y_{31}^S \) and \( y_{32}^S \), as given in Eq. (31), as well as \( y_{31}^R \) and \( y_{32}^R \), as defined in Eq. (3), to find that the general formula for \( \Delta a_\ell, \ell = 1, 2 = e, \mu \), in this mixed scenario, reads

\[
\Delta a_\ell = \frac{3m_\ell^2}{8\pi^2} \left\{ \frac{1}{3M_3^2} |y_{3\ell}^S|^2 - \frac{1}{12M_3^2} \left[ |y_{3\ell}^R|^2 s_\theta^2 + |y_{3\ell}^S|^2 c_\theta^2 \right] - \frac{1}{12M_3^2} \left[ |y_{3\ell}^R|^2 c_\theta^2 + |y_{3\ell}^S|^2 s_\theta^2 \right] + \frac{m_\ell}{m_\ell} Re \left[ (V^* y_q^S)_{3q} y_{3\ell}^R \right] \left\{ \frac{s_2\theta}{2M_3^2} \left( \frac{7}{6} + \frac{2}{3} \ln x_\ell^+ \right) - \frac{s_2\theta}{2M_3^2} \left( \frac{7}{6} + \frac{2}{3} \ln x_\ell^- \right) \right\} \right\}.
\]

(36)

If we take all the Yukawa couplings to be real and keep only the chirality enhanced terms, we get that

\[
\Delta a_\ell = -\frac{3m_\ell m_\ell}{8\pi^2} y_{5\ell}^S y_{6\ell}^R \left[ \frac{s_2\theta}{2M_3^2} \left( \frac{7}{6} + \frac{2}{3} \ln x_\ell^+ \right) - \frac{s_2\theta}{2M_3^2} \left( \frac{7}{6} + \frac{2}{3} \ln x_\ell^- \right) \right],
\]

(37)

and

\[
Br(\mu \to e\gamma) = \frac{\tau_\mu}{\tau_\mu} \frac{m_\mu^3}{4} \frac{9m_\mu^2}{(16\pi^2)^2} \left( |y_{3\ell}^R y_{31}^S|^2 + |y_{3\ell}^R y_{32}^S|^2 \right) \times \left[ \frac{s_2\theta}{2M_3^2} \left( \frac{7}{6} + \frac{2}{3} \ln x_\ell^+ \right) - \frac{s_2\theta}{2M_3^2} \left( \frac{7}{6} + \frac{2}{3} \ln x_\ell^- \right) \right]^2.
\]

(38)

If we furthermore define \( x = y_{31}^R / y_{32}^R \) and rewrite \( Br(\mu \to e\gamma) \) in Eq. (38) using Eq. (37) we find that

\[
Br(\mu \to e\gamma) = \frac{\tau_\mu}{\tau_\mu} \frac{m_\mu^3}{16} \left( \frac{\Delta a_\ell}{m_\mu^2} \frac{1}{x^2} + \frac{\Delta a_\ell}{m^2_\mu} x^2 \right) = 5.2 \times 10^{-6} \frac{1}{x^2} + 1.2 \times 10^{-3} x^2.
\]

(39)

This expression is identical to the one for the \( S_1 \) scenario with the top quark loops. It, again, does not depend on the scale of new physics and trivially demonstrates that the \( S_{1,2} \) scenario with the top quark loops is not adequate for the simultaneous explanation of the \( \Delta a_\ell \) and \( \Delta a_\mu \) shifts.

### 2.2.2 \( S_{1,2} \) with the top and charm quark loops

In order to investigate viability of the \( S_{1,2} \) scenario when the chirality enhanced shift in \((g - 2)_\mu \) is generated with the top quark loops and the shift in \((g - 2)_e \) is due to the charm quark loops we switch on \( y_{21}^R \) and \( y_{32}^S \), as given in Eq. (31), as well as \( y_{21}^R \) and \( y_{32}^R \), as defined in Eq. (3), to find that

\[
\Delta a_\ell = \frac{3m_\ell^2}{8\pi^2} \left\{ \frac{1}{3M_3^2} |y_{3\ell}^S|^2 - \frac{1}{12M_3^2} \left[ |y_{3\ell}^R|^2 s_\theta^2 + |y_{3\ell}^S|^2 c_\theta^2 \right] - \frac{1}{12M_3^2} \left[ |y_{3\ell}^R|^2 c_\theta^2 + |y_{3\ell}^S|^2 s_\theta^2 \right] + \frac{m_\ell}{m_\ell} Re \left[ (V^* y_q^S)_{3q} y_{3\ell}^R \right] \left\{ \frac{s_2\theta}{2M_3^2} \left( \frac{7}{6} + \frac{2}{3} \ln x_\ell^+ \right) - \frac{s_2\theta}{2M_3^2} \left( \frac{7}{6} + \frac{2}{3} \ln x_\ell^- \right) \right\} \right\},
\]

(40)
where for index $\ell = e = 1$ one needs to set $q = c = 2$ while for $\ell = \mu = 2$ one needs to take $q = t = 3$ when and where appropriate. If Yukawa couplings are real and if we omit subleading terms, Eq. (40) translates into

$$
\Delta a_e = -\frac{3m_cm_e}{8\pi^2} V_{cs} y_{21}^S y_{21} R \left[ \frac{s_{2\theta}}{2M^2_+} \left( \frac{7}{6} + \frac{2}{3} \ln x^+_e \right) - \frac{s_{2\theta}}{2M^2_+} \left( \frac{7}{6} + \frac{2}{3} \ln x^-_e \right) \right],
$$

$$
\Delta a_\mu = -\frac{3m_cm_t}{8\pi^2} V_{tb} y_{32}^S y_{32} R \left[ \frac{s_{2\theta}}{2M^2_+} \left( \frac{7}{6} + \frac{2}{3} \ln x^+_t \right) - \frac{s_{2\theta}}{2M^2_+} \left( \frac{7}{6} + \frac{2}{3} \ln x^-_t \right) \right].
$$

The branching ratio for $\mu \to e\gamma$ is

$$
Br(\mu \to e\gamma) = \frac{\tau_\mu}{16} \frac{m_\mu^3}{(16\pi^2)^2} \left\{ m_t^2 \left| V_{ts} y_{21}^S y_{21}^R \right|^2 \left[ \frac{7}{6} + \frac{2}{3} \ln x^+_e - \frac{7}{6} + \frac{2}{3} \ln x^-_e \right]^2 
+ m_e^2 \left| V_{ce} y_{32}^S y_{32}^R \right|^2 \left[ \frac{7}{6} + \frac{2}{3} \ln x^+_e - \frac{7}{6} + \frac{2}{3} \ln x^-_e \right]^2 \right\}.
$$

Finally, the combination of Eqs. (41), (42), and (43) yields

$$
Br(\mu \to e\gamma) = \frac{\tau_\mu}{16} \frac{m_\mu^3}{m_e^2} \left( \frac{\Delta a_e^2}{m_e^2} \frac{\tilde{B}}{x^2} \left| V_{ts}\right|^2 + \frac{\Delta a_\mu^2}{m_\mu^2} \frac{x^2}{\left| V_{cb}\right|^2} \right),
$$

where $x = y_{21}^R / y_{32}^R$ and

$$
\tilde{B} = \frac{m_t^2}{m_e^2} \left( \frac{7}{6} + \frac{2}{3} \ln x^+_e - \frac{7}{6} + \frac{2}{3} \ln x^-_e \right)^2.
$$

To demonstrate the level of disagreement between the $Br(\mu \to e\gamma)$ prediction within this scenario and the associated experimental limit we take $M_1 = M_3 = 1.5$ TeV and $\lambda = 1$ to obtain $\tilde{B} = 350$. This benchmark point yields

$$
Br(\mu \to e\gamma) = 3.2 \times 10^{-6} \frac{1}{x^2} + 6.2 \times 10^{-9} x^2.
$$

Clearly, the $S_1$ & $S_3$ scenario when the chirality enhanced shift in $(g - 2)_\mu$ is generated with the top quark loops and the shift in $(g - 2)_e$ is due to the charm quark loops also fails to pass the $\mu \to e\gamma$ test.

2.2.3 $R_2$&$\tilde{R}_2$ with the top and bottom quark loops

We consider the $R_2$&$\tilde{R}_2$ scenario when $(g - 2)_\mu$ is addressed via the top quark loops and with Yukawa couplings of $R_2^{5/3}$ state while the $(g - 2)_e$ discrepancy, on the other hand, is addressed with the bottom quark loops and with Yukawa couplings that are associated with the mixture of $R_2^{2/3}$ and $\tilde{R}_2^{2/3}$ states. The relevant parts of the Lagrangian that have not been featured in the preceding sections are

$$
\mathcal{L} \supset -\sqrt{\tau} y_{ij}^L \tilde{R} \sigma_2 L_j - \lambda \left( R_2^{5/3} \right) \left( R_2^{5/3} \right) + h.c.,
$$
where $\bar{d}_{R_i}$ are the right-handed down-type quarks and $\lambda$ is a dimensionless coupling.

The Yukawa couplings of the $\tilde{R}_2$ charge eigenstate components, in the mass eigenstate basis for the SM fermions, are

$$L_{\tilde{R}_2} = - \tilde{y}_L y_{Lj} \bar{d}_{R_i} \ell_{Lj} \tilde{R}_2^{2/3} + (\tilde{y}_L U)_{ij} d_{Ri} \nu_{Lj} \tilde{R}_2^{-1/3} + \text{h.c.}$$  \hspace{1cm} (48)$$

We judiciously switch on $\tilde{y}_L$ in Eq. (48) and $y_{L2}$, $y_{L3}$, and $y_{R2}$ in Eq. (20) to generate the loops of interest. The mixed states of electric charge $Q = 2/3$ are

$$R_2^{2/3} = c_\theta R_- - s_\theta R_+$$  \hspace{1cm} (49)$$
$$\tilde{R}_2^{-1/3} = s_\theta R_- + c_\theta R_+,$$  \hspace{1cm} (50)$$

with the mixing angle $\theta$ defined via

$$\tan 2\theta = \frac{\lambda v^2}{M^2 - M^2},$$  \hspace{1cm} (51)$$

and

$$M_\pm^2 = \frac{M^2 + M^2}{2} \pm \frac{1}{2}\sqrt{(M^2 - M^2)^2 + \lambda^2 v^4},$$  \hspace{1cm} (52)$$

where the new physics mass eigenstates, after the mixing takes place, are $R_{2}^{5/3}$, $R_\pm$, and $\tilde{R}_2^{-1/3}$ with $M = M_{R_{2}^{5/3}}$, $M_\pm = M_{R_\pm}$, and $\tilde{M} = M_{\tilde{R}_2^{-1/3}}$.

For real Yukawa couplings we obtain that

$$\Delta a_e = - \frac{m_e m_b}{32\pi^2} s_2 y_{L2} y_{R1} \left( \frac{5 + 2 \ln x_b^+}{M^2} - \frac{5 + 2 \ln x_b^-}{M^2} \right),$$  \hspace{1cm} (53)$$

$$\Delta a_\mu = \frac{m_\mu m_t}{16\pi^2} y_{L2} y_{R3} \left( \frac{1 + 4 \ln x_t}{M^2} \right),$$  \hspace{1cm} (54)$$

$$Br(\mu \to e\gamma) = \frac{\tau_\mu \alpha m_\mu^3 m_t^2}{4096\pi^4} \times$$

$$\left[ \left| y_{R3}^L y_{R2}^R \left( \frac{1 + 4 \ln x_t}{M^2} \right) \right|^2 + \left( \frac{s_2 m_b}{2 m_t} \right)^2 \left| y_{L2} y_{R3} \left( \frac{5 + 2 \ln x_b^+}{M_+^2} - \frac{5 + 2 \ln x_b^-}{M_-^2} \right) \right|^2 \right],$$  \hspace{1cm} (55)$$

where $x_b^\pm = m_b^2/M_\pm^2$ and $m_b$ is the bottom quark mass.

If we introduce $x = y_{R3}^L / y_{R2}^L$ and rewrite $Br(\mu \to e\gamma)$ in Eq. (55) using Eqs. (53) and (54) we find that

$$Br(\mu \to e\gamma) = \frac{\tau_\mu \alpha m_\mu^3}{16} \left( \frac{\Delta a_e^2}{m_e^2} \frac{1}{x^2} + \frac{\Delta a_\mu^2}{m_\mu^2} x^2 \right) = 5.2 \times 10^{-6} \frac{1}{x^2} + 1.2 \times 10^{-3} x^2,$$  \hspace{1cm} (56)$$

which demonstrates, once again, that the limit on $\mu \to e\gamma$ is an unsurmountable obstacle even for this particular scenario.

12
3 Conclusions

We investigate all possible ways in which one could potentially address discrepancies between the observed values of the electron and muon anomalous magnetic moments and the SM theoretical predictions with the new physics scenarios that introduce one or, at most, two scalar leptoquarks. Our findings are that all potentially viable scenarios to produce combined explanation for the observed shifts in \((g - 2)_e\) and \((g - 2)_\mu\) are ruled out by the current limits on the \(\mu \to e\gamma\) branching ratio, apart from the \(S_1\&R_2\) combination, where \(S_1\) would generate chirality enhanced loop contributions solely to \((g - 2)_e\) while \(R_2\) would only affect \((g - 2)_\mu\) or the other way around.

In order to be of the correct strength the chirality enhanced \((g - 2)_e\) loops should be due to propagation of top quark, charm quark, and bottom quark, while the \((g - 2)_\mu\) loops need to be generated by the top quark propagation. Our results with regard to the simultaneous explanation of observed discrepancies are very robust as they either do not depend on the scale of new physics or, if they do, exhibit only a mild dependence. The scenarios we consider require at least four Yukawa couplings to be switched on in order to generate the aforementioned loops, where one pair feeds into the electron anomalous magnetic moment and the other pair into the muon one.

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