Kaon properties and cross sections in nuclear medium

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Abstract. Results for the $\pi^+ + N \rightarrow \Lambda, \Sigma + K$ reactions in nuclear matter of Ref. [1] are presented. To evaluate the in-medium modification of the reaction amplitude as a function of the baryonic density we introduce relativistic, mean-field potentials for the initial, final and intermediate mesonic and baryonic states in the resonance model. These vector and scalar potentials were calculated using the quark meson coupling model. Contrary to earlier work which has not allowed for the change of the cross section in medium, we find that the data for kaon production at SIS energies are consistent with a repulsive $K^+$-nucleus potential.

1. Introduction

The properties of kaons in nuclear matter have recently attracted enormous interest because of their capacity to signal chiral symmetry restoration or give information on the possibility of kaon condensation in neutron stars [2]. Studies with a variety of models [3, 4] indicate that the antikaon potential is attractive while the kaons feel a repulsive potential in nuclear matter. However, the analysis of available data on $K^+$ production from heavy ion collisions at SIS energies [5, 6] contradicts the predictions that the kaon potential is repulsive. The comparison between the heavy ion calculations and the data [6, 7, 8] indicates that the $K^+$-meson spectra are best described by neglecting any in-medium modification of the kaon properties. Furthermore, the introduction of even a weakly repulsive $K^+$-nucleus potential results in a substantial underestimate of the experimental data on kaon production.

Since in heavy ion collisions at SIS energies [5, 6] the $K^+$-mesons are predominantly produced by secondary pions, we investigate the kaon production reactions, $\pi + N \rightarrow \Lambda, \Sigma + K$, in nuclear matter. We calculate the in-medium reaction amplitudes, taking into account the scalar and vector potentials for incident, final and intermediate mesons and baryons.

2. Mean-field potentials for mesons and baryons

We use the quark-meson coupling (QMC) model [10] which has been successfully applied to the various problems of nuclear physics [11] and the studies of meson and hyperon properties in a nuclear medium [4, 12]. The Dirac equations for the quarks

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and antiquarks in the hadron bags \((q = u, \bar{u}, d, \bar{d})\), hereafter, neglecting the Coulomb force, are given by \([4]\):

\[
\left[ i\gamma \cdot \partial_x - (m_q - V_q) \right] \gamma^0 \left( \frac{V_q^2}{2} \right) \left( \psi_u(x) \right) = 0, \quad \left( \frac{1}{2} V_q^2 \right) \left( \psi_{\bar{u}}(x) \right) = 0.
\]

The mean-field potentials for a bag in nuclear matter are defined by \(V_q^2 = g_q^2 \sigma, V_q^4 = g_q^4 \omega\) and \(V_q^3 = g_q^3 \rho\), with \(g_q^2, g_q^3\) and \(g_q^3\) the corresponding quark-meson coupling constants.

The hadron masses in symmetric nuclear matter relevant for the present study are calculated by:

\[
m^*_h = \frac{(n_q + n_q) \Omega^*_q + (n_s + n_s) \Omega^*_s - z_h}{R^*_h} + \frac{4}{3} \pi R^*_h B, \quad \text{with} \quad \frac{\partial m^*_h}{\partial R_h} \bigg|_{R_h = R^*_h} = 0,
\]

where \(\Omega^*_q = \sqrt{x_q^2 + (R^*_h m^*_q)^2}\), with \(m^*_q = m_q - g_q^2 \sigma\) and \(\Omega^*_s = \sqrt{x_s^2 + (R^*_h m^*_s)^2}\), and \(x_q\) and \(x_s\) being the bag eigenfrequencies of the corresponding quarks. In Eq. (3), \(n_q, n_s\) are the lowest mode light quark (antiquark) and strange (antistrange) quark numbers in the hadron, \(h\), respectively, and the \(z_h\) parametrize the sum of the center-of-mass and gluon fluctuation effects, and are assumed to be independent of density. The parameters are determined in free space to reproduce their physical masses. We chose the values \(m_q = 5\) MeV and \(m_s = 250\) MeV for the current quark masses, and \(R_N = 0.8\) fm for the bag radius of the nucleon in free space \([4]\). We stress that only three coupling constants, \(g_q^2, g_q^3\) and \(g_q^3\), are adjusted to fit nuclear data – namely the saturation energy and density of symmetric nuclear matter and the bulk symmetry energy. Exactly the same coupling constants, \(g_q^2, g_q^3\) and \(g_q^3\), are used for the light quarks in the mesons and hyperons as in the nucleon. However, for the kaon system it was phenomenologically necessary to increase the strength of the vector coupling to the non-strange quarks in the \(K^+\), and thus, we will use the stronger vector potential, \(1.42 V_q^3\), for the \(K^+\)-meson \([4]\). The scalar \((U^s_h)\) and vector \((U^v_h)\) potentials felt by the hadrons, \(h\), in nuclear matter are self-consistently calculated by:

\[
U^s_h \equiv U_s = m^*_h - m_h, \quad U^v_h = (n_q - n_q) V^2_q - I_3 V^3_q, \quad (V^2_q \rightarrow 1.42 V^3_q \text{ for } K^+),
\]

where, \(I_3\) is the third component of isospin projection of the hadron, \(h\), and the \(\rho\) meson mean field potential, \(V^3_q\), is zero in symmetric nuclear matter. Then, within the approximation that the mean field potentials are independent of momentum, the four-momentum of the hadron is modified by, \(p_h^\mu = (\sqrt{p^2 + m_h^2} + U^v_h(p)\), which modifies not only the kinematical factors such as the flux, the phase space and the threshold, but also modifies the reaction amplitudes.

Furthermore, we include the in-medium modification of the resonance masses, which appear in the reaction amplitudes in the resonance model. In view of its numerous successful applications elsewhere, we base our estimate on the QMC model \([4, 12]\), and use those estimated values \([4]\) for the in-medium resonance masses.

### 3. \(\pi + N \rightarrow \Lambda + K\) in nuclear matter

Now we apply the resonance model \([4, 12]\) to calculate the in-medium amplitudes focusing on \(\pi + N \rightarrow \Lambda + K\). We consider kaon and hyperon production processes in \(\pi N\) collisions shown in Fig. 1. (See Refs. \([13, 1]\) for \(\pi + N \rightarrow \Sigma + K\).)
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We extend the model by including medium modification of the hadron properties, not only in the kinematic factors such as the flux and the phase space, but also in the reaction amplitudes, as discussed in the previous section. (See also Ref. [1].)

We found that the maximal downward shift of the reaction threshold in nuclear matter occurs at baryon densities around $\rho_B \simeq 0.6\rho_0$ ($\rho_0 = 0.15$ fm$^{-3}$), and the maximum of the downward shift of the $\pi + N \rightarrow \Lambda, \Sigma + K$ reaction threshold amounts to roughly 30 MeV. We also found that at baryon densities $\rho_B > 0.2$ fm$^{-3}$ thresholds for the $\pi + N \rightarrow \Lambda, \Sigma + K$ reactions are higher than those in free space.

We show the energy dependence of the total $\pi^- + p \rightarrow \Lambda + K^0$ cross section in Fig. 2. The calculations for free space are in reasonable agreement with the data, as shown by the solid line. The dashed line in Fig. 2 shows the results obtained for nuclear matter at $\rho_B = \rho_0$, while the dotted line is the calculation at $\rho_B = 2\rho_0$.

4. Impact on heavy ion studies

It is expected that in relativistic heavy ion collisions at SIS energies nuclear matter can be compressed up to baryonic densities of order $\rho_B \simeq 3\rho_0$. The calculation of the time and spatial dependence of the baryon density distribution is a vital aspect of dynamical heavy ion simulations. The baryon density $\rho_B$ available in heavy ion collisions evolves with the interaction time, $t$, and is given by the dynamics of the heavy ion collision. In the following estimates we investigate the density dependence of the production cross section for central central heavy ion collisions.

To calculate the $K^+$-meson production cross section averaged over the available

![Figure 1. K and Λ production processes included in the resonance model][1]

![Figure 2. Energy dependence of the total cross section, $\pi^- + p \rightarrow \Lambda + K^0$, as a function of the invariant collision energy, $\sqrt{s}$, calculated for different baryon densities. The data in free space are taken from Ref. [14]. The lines indicate our results for free space (solid) and for nuclear matter at baryon density $\rho_B = \rho_0$ (dashed) and $\rho_B = 2\rho_0$ (dotted) ($\rho_0 = 0.15$ fm$^{-3}$).][2]
density distribution we adopt the density profile function obtained by dynamical simulations \[13\] for \(Au + Au\) collisions at 2 AGeV and at impact parameter \(b = 0\). This can be parametrized as

\[
\rho_B(t) = \rho_{\text{max}} \exp \left( \frac{(t - \bar{t})^2}{\Delta t^2} \right),
\]

(5)

where the parameters, \(\rho_{\text{max}} = 3 \rho_0\), \(\bar{t} = 13\) fm and \(\Delta t = 6.7\) fm, were fitted to the heavy ion calculations \[13\].

Figure 3. Energy dependence of the total cross section, \(\pi^- + p \rightarrow \Lambda + K^0\), as a function of the invariant collision energy, \(\sqrt{s}\). The data in free space are taken from Ref. \[14\]. The solid line indicates our calculation for free space. The dashed line shows the cross section calculated by averaging over the density function profile \[15\] given by the time evolution obtained for \(Au + Au\) collisions at 2 AGeV (see Eq. (5)).

The total cross section for the \(\pi^- + p \rightarrow \Lambda + K^0\) reaction integrated over the time range \(5 \leq t \leq 23\) fm and weighted by the time dependent density profile given in Eq. \(5\), is shown by the dashed line in Fig. 3. The limits of the \(t\) integration were taken from the simulations of the \(Au + Au\) collision time evolution in Ref. \[13\]. The circles and solid line in Fig. 3 show the experimental data in free space \[14\] and the calculations in free space, respectively.

One can see that the total cross section averaged over the collision time (time dependent density profile) for the \(\pi^- + p \rightarrow \Lambda + K^0\) reaction is quite close to the result given in free space integrated up to \(\sqrt{s} \simeq 1.7\) GeV. Indeed, the results shown in Fig. 3 actually explain why the heavy ion calculations with the free space kaon production cross section might quite reasonably reproduce the experimental data.

Furthermore, the total cross section averaged over the time dependent density profile, shown by the dashed line in Fig. 3, should additionally be averaged over the invariant collision energy distribution available in heavy ion reactions. The number of meson-baryon collisions, \(N_{mB}\), for the central \(Au + Au\) collisions at 2 AGeV is given in Ref. \[16\] as a function of the invariant collision energy, \(\sqrt{s}\). It can be parametrized for \(\sqrt{s} > 1\) GeV as

\[
dN_{mB}/d\sqrt{s} = N_0 \exp \left( \frac{\sqrt{s} - \sqrt{s_0}^2}{[\Delta \sqrt{s}]^2} \right),
\]

(6)

where the normalization factor \(N_0 = 6 \times 10^4\) GeV\(^{-1}\), while \(\sqrt{s_0} = 1\) GeV and \(\Delta \sqrt{s} = 0.63\) GeV. Note that, at SIS energies \(N_{mB}\) is almost entirely given by the pion-nucleon interactions, and heavy meson and baryon collisions contribute only to the high energy tail of the distribution in Eq. \(6\) – with quite small densities \(16\). Finally, if we also average the calculated, in-medium, total cross section for \(\pi^- + p \rightarrow \Lambda + K^0\), shown by the dashed line in Fig. 3, over the available energy distribution given in Eq. \(6\),
we obtain an average total kaon production cross section of $<K> = 65 \mu b$ for central $Au + Au$ collisions at 2 AGeV. This result is indeed compatible with the calculations using the free space total cross section of the $\pi^- + p \rightarrow \Lambda + K^0$ reaction, which provide an average total kaon production cross section of $<K> = 71 \mu b$ for central $Au + Au$ collisions at 2 AGeV. Note that the inclusion of even a slight modification of the $K^+$ mass because of the nuclear medium (without the corresponding changes introduced here) leads to a substantial reduction of the inclusive $K^+$ spectra (by as much as a factor of 2 or 3), compared to that calculated using the free space properties for the relevant hadrons [8].

We stress that at SIS energies reaction channels with a $\Sigma$-hyperon in the final state play a minor role, because of the upper limit of the energy available in the collisions. The downwardly shifted $\pi + N \rightarrow \Sigma + K$ reaction threshold at small densities is still quite high.

5. Conclusion

Our present study shows that if one accounts for the in-medium modification of the production amplitude (i.e., the in-medium properties of the $K^+$-meson and hadrons) correctly, it is possible to understand $K^+$ production data in heavy ion collisions at SIS energies, even if the $K^+$-meson feels the theoretically expected, repulsive mean field potential. The apparent failure to explain the $K^+$ production data if one includes the purely kinematic effects of the in-medium modification of the $K^+$-meson and hadrons, appears to be a consequence of the omission of these effects on the reaction amplitudes.

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