Controllable tunability of a Chern number within the electronic-nuclear spin system in diamond

Junghyun Lee, Keigo Arai, Huiliang Zhang, Mark J. H. Ku, and Ronald L. Walsworth

Chern numbers characterize topological phases in a wide array of physical systems. However, the resilience of system topology to external perturbations makes it challenging experimentally to investigate transitions between different phases. In this study, we demonstrate the transitions of a Chern number from 0 to 3, synthesized in an electronic-nuclear spin system associated with the nitrogen-vacancy (NV) centre in diamond. The Chern number is characterized by the number of degeneracies enclosed in a control Hamiltonian parameter space. Topological transitions between different phases are realized by varying the radius and offset of the sphere such that the Chern number changes. We show that the measured topological phase diagram is consistent with numerical calculations and can also be mapped unto an interacting three-qubit system. The NV system may also allow access to even higher Chern numbers, which could be applied to exploring exotic topology or topological quantum information.

INTRODUCTION

Currently, extensive research is being conducted on the topologically invariant Chern number, which is defined as the integral of the Berry curvature of the system of interest, and the application of its robust topological properties to quantum metrology, next-generation electronics, spintronics, and quantum computation. The Berry curvature and Chern number are used to characterize the topological properties of the Hamiltonian parameter space for complex and dynamic Hamiltonian models in condensed matter systems. These platforms have many, well-controlled experimental degrees of freedom that can be employed as powerful quantum simulators for quantum technology.

Chern numbers characterize topological phases in a wide array of physical systems. However, the resilience of system topology to external perturbations makes it challenging experimentally to investigate transitions between different phases. In this study, we demonstrate the transitions of a Chern number from 0 to 3, synthesized in an electronic-nuclear spin system associated with the nitrogen-vacancy (NV) centre in diamond. The Chern number is characterized by the number of degeneracies enclosed in a control Hamiltonian parameter space. Topological transitions between different phases are realized by varying the radius and offset of the sphere such that the Chern number changes. We show that the measured topological phase diagram is consistent with numerical calculations and can also be mapped unto an interacting three-qubit system. The NV system may also allow access to even higher Chern numbers, which could be applied to exploring exotic topology or topological quantum information.

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Fig. 1  Schematic of Chern number measurement with an NV centre in diamond. a Upper: Trajectory of the Larmor vector $\mathbf{H}(t)$ is represented by a thick blue arrow in the spherical control Hamiltonian parameter space $(H_x, H_y, H_z)$. The solid black arrow indicates the direction of the sweep of the Larmor vector. The yellow circles represent the degeneracy points of the system Hamiltonian. Lower: Time evolution of the NV electronic spin Bloch vector $\mathbf{σ}(t)$ is represented by the red arrow on the Bloch sphere. The Bloch vector path deviates from the Larmor vector path due to a nonadiabatic response. The red filled area indicates the amount of deviation, a summation of which over the path is related to the Chern number. b NV centre energy level diagram. The NV ground states consist of $|0\rangle$, $|\pm 1\rangle$ electronic spin sublevels, which are further split by hyperfine interactions with the $^{14}$N nuclear spin. Three hyperfine transitions between $|0\rangle$ and $|-1\rangle$ electronic spin sublevels (yellow double-arrows) define three degeneracy points in the rotating frame with angular speeds of $\omega_\mathbf{σ} = A_1$, $\omega_\mathbf{σ}_1$, $\omega_\mathbf{σ} + A_1$, where $A_1$ is the parallel component of the hyperfine tensor. c Experimental pulse sequence. An optical initialization pulse polarizes the electronic spin $\mathbf{σ}$ (red dashed line). An offset field $H_r(t)$ along the z-axis from the origin:

$$H(t) = (H_z \sin(\theta) \cos(\phi), H_y \sin(\theta) \sin(\phi), H_x \cos(\theta) + H_0), \quad (1)$$

where $\theta$ is the time-varying polar angle and $\phi$ is the azimuthal angle fixed at 0, without loss of generality. When the Larmor vector traverses this trajectory at a finite speed, the qubit’s Bloch vector $\mathbf{σ}(t)$ follows the Larmor vector $\mathbf{H}(t)$, but with a small deviation along the $\phi$ direction at each polar angle location owing to a nonadiabatic response2-4. For a first-order approximation, this deviation is related to the $\phi$ component of the Berry curvature $F_\phi$ through the following linear relation:

$$F_\phi(\theta) = \frac{H_z \sin(\theta) \cos(\phi)}{2v_\mathbf{σ}} , \quad (2)$$

where $\langle \phi \rangle$ is the expectation value of the y component of the Bloch vector and $v_\mathbf{σ} \equiv d\mathbf{σ}/dt$ denotes the angular speed about its polar axis. Integration of this Berry curvature over the polar angle of the trajectory yields the Chern number as follows:

$$C = \int_0^\pi F_\phi(\theta) d\theta . \quad (3)$$

Here, the Chern number depends on the number of degeneracy points of the static internal Hamiltonian enclosed in a control Hamiltonian sphere drawn by the Larmor vector. Every degeneracy point can be regarded as a synthetic magnetic monopole. These monopoles produce radial synthetic magnetic fields that exert a torque on the Bloch vector.

In this work, we apply the above protocol to experimentally observe the transition of the Chern number from 0 to 3, using three degeneracy points associated with the ground-state spin energy levels of a single NV centre in diamond (Fig. 1b). In this electronic-nuclear spin hybrid system, parameter space degeneracy points correspond to on-resonance conditions between the hyperfine-split energy levels. The NV electronic spin ground-state has three sublevels $|-1\rangle$, $|0\rangle$, and $|+1\rangle$, out of which only $|-1\rangle$ and $|0\rangle$ are used here as a two-level system, represented by $\mathbf{σ}$ in the following measurements. The NV host nuclear spin $^{14}$N, with a spin quantum number of $I = 1$, induces a hyperfine coupling. The associated internal Hamiltonian takes the form $H_0 = (\hbar/2)A_i \sigma_i^I$, where $A_i/2\pi = 2.2$ MHz is the coupling strength of the longitudinal component of the hyperfine interaction and $I_z$ denotes the $z$ component of the nuclear spin. This electronic-nuclear spin system contains three degeneracy points, allowing us to access

the north pole to the south pole with a radius $H_r$ and by introduction of an offset $H_0$ along the z-axis from the origin:

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As a measure of the degree of adiabaticity, an adiabaticity parameter \( a \) is introduced as follows:

\[
a = \frac{\Omega_0 T_{\text{ramp}}}{2n}.
\]

This parameter represents the fractional change in the Larmor vector. Recalling the extra second-order term \( O(\nu^{2}) \) in the Berry curvature formula (Supplementary Eq. 3), \( a \) characterizes the accuracy of the measured Chern number. In the nonadiabatic limit \( (a \gg 1) \), the first-order approximation of the Berry curvature in Eq. (2) breaks down. Subsequently, the effects of higher-order terms contaminate the signal in our measurements. Conversely, in the adiabatic limit \( (a \ll 1) \), the NV electronic spin remains in the instantaneous ground state; and the spin Bloch vector is approximately parallel to the direction of the control field, following the meridian. However, the deviation signal \( \langle \sigma^y \rangle \) becomes smaller and eventually lies buried in measurement noise. For the three-level NV system, the appropriate range reflecting an optimum signal-to-noise ratio is found to be \( 2 \leq a \leq 10 \) (see Supplementary Fig. 1). The adiabaticity parameter is set to \( a = 2 \) for the remainder of the work.

**RESULTS**

**Chern number transition from 0 to 3**

As a benchmark experiment, we first characterize a case with the expected Chern number of \( C = 0 \) (Fig. 2a). This case is realized by choosing a small control Hamiltonian sphere with a normalized radius of \( H_r/A_1 = 0.2 \) and a normalized detuning of \( H_0/A_1 = 0.23 \), which does not contain any of the three degeneracy points. Although these degeneracy points are expected to make the Berry-curvature zero for any \( \theta \), numerical simulations based on a time-dependent Schrödinger equation (see Methods) predict a deviation from zero. This deviation can be attributed to the nonadiabatic effect, which limits the accuracy of this quasi-static Chern number measurement approach. The measured Berry curvature is consistent with the simulation results, including the nonadiabatic effect. The resulting Chern number, obtained by integrating the measured Berry curvature over \( \theta \), converges to \( C = -0.07 \pm 0.04 \). Measurement error is evaluated from the photon-shot noise (1\( \sigma \)).
The effect is found in the case of calibration at low Rabi frequencies (see Methods). One notable case.

Numerically simulated values. The Chern numbers are determined more pronounced behaviour for the Berry curvature. The increase in the number of enclosed degeneracy points, indicating predict a larger deviation in the path of the Bloch vector with an.

To observe higher Chern numbers, we next examine cases with one, two, and three enclosed degeneracy points by increasing the radius up to $H_r/A_l = 2.25$ (Fig. 2b-d). Numerical simulations predict a larger deviation in the path of the Bloch vector with an increase in the number of enclosed degeneracy points, indicating more pronounced behaviour for the Berry curvature. The measured Berry curvatures for each case agree well with the numerically simulated values. The Chern numbers are determined to be $C = 0.95 \pm 0.35$, $2.20 \pm 0.39$, and $2.93 \pm 0.38$. Thus, our results demonstrate that the NV electronic-nuclear spin system can be used as a platform for synthesizing up to three Chern numbers.

Three monopole 2D phase transition diagram

The NV system can further explore the transition between the observed Chern numbers. Figure 3a presents measurements of the topological phase transition along the normalized radius axis ($H_r/A_l \in \{0.25, 2.25\}$) for various normalized offset conditions: $H_0/A_l = 2.0, 1.0, 0.23, 0.0$. In all cases, we observe a mild phase transition, which can be attributed to the finite NV electronic spin coherence ($\tau_2$) time and the limited adiabaticity parameter $\alpha$. Additionally, the observed consistency between experimentally measured and numerically simulated Chern numbers reflects this nonadiabatic effect within one standard deviation of the measurement error, except for $H_r/A_l \leq 1$. The disagreement within this small-radius region may be due to imperfect system calibration at low Rabi frequencies (see Methods). One notable effect is found in the case of $H_0/A_l = 0.0$, where the number of enclosed degeneracy points is expected to jump from one to three at $H_r/A_l = 1$. However, in the measurements, a sudden depletion of $C$ is observed below 1, and the transition is found to occur above 1. This shift in the transition point can be attributed to the nonadiabatic response of a qubit when the Larmor vector coincides with the position of the degeneracy points on the $z$-axis. Figure 3c presents the observed transition curves across the normalized offset ($H_0/A_l \in \{0.00, 2.25\}$) for various values of the normalized radius of $H_r/A_l = 0.23, 0.79, 1.36, 2.17$. Control Hamiltonian parameter spheres (grey) and degeneracy points (yellow and black) are shown for each case.

**DISCUSSION**

We next discuss the connection (i.e., mapping) between the NV system in our experiments and an interacting three-qubit system to reveal the implications of our observed two-dimensional topological phase diagram. The topological phase diagram presented in this study is constructed by varying the radius and...
three qubits distinctively contribute to the total Chern number $e$.

The coupled multiqubit Hamiltonian carries multiple degenerate ground states, which leads to the realization of a high Chern number. Here, the interaction strength, $g$, between qubits approaches 0, both $\tilde{g}'$ and $\tilde{H}_0$ become large, where $C = 0$ in the phase diagram. The Chern number behaviour for these limiting cases remains similar to that of the coupled two-qubit Hamiltonian. Meanwhile, a more complex phase structure can be found by analytically calculating the positions of the ground state degeneracy points with respect to the sweep parameter sphere manifold. The white dashed boundaries clarify four distinctive regions where the Chern number in each region corresponds to the number of monopoles enclosed by the surface. Along the $H_2$ axis, one monopole exits the surface at $H_2 = 1$ ($C = 3$ to $C = 2$) and secondly at $H_2 = 2$ ($C = 2$ to $C = 1$). Next, along the $\tilde{g}'$ axis, the two monopoles escape the surface at $\tilde{g}' = 1/\sqrt{2}$, inducing the Chern number transition from $C = 3$ to $C = 1$.

Finally, we project the NV three-monopole topological phase measurements onto the interacting three-qubit system using Eq. (7) and then compare with the three-qubit Chern number simulation results (Fig. 4c). For a fixed $H_0$, $H_r$ is swept from 0.22 to 2.2 by varying the Rabi frequency on the NV spin. The orthogonal parameter axes, $H_0$ and $H_r$, are nonlinearly transformed into $\tilde{H}_0$ and $\tilde{g}'$, which gives topological phase transition curves in radial cross-sections for $H_0 = 0$, 0.23, 0.45, 0.68, and 0.91. The three-monopole Chern number transition projection, evaluated using Eq. (8), and the simulated Chern number transition cross section of the interacting three-qubit system are consistent with each other (blue dotted line in Fig. 4b).
determines the position of the monopoles on the parameter space z-axis (see Supplementary Methods). In principle, investigating the topology of an N-interacting qubit system could simulate the topology of non-interacting 2N band models. For example, two interacting qubit systems, simulating the topology of the ground band of a four-band electronic model, together with an interacting three-qubit system could help to probe the topological structure of the half-filled eight-band model.29

Our scheme clearly shows that a high Chern number can be deterministically simulated using a single-qubit-based monopole system, in addition to tuning the level of its transition depending on the range of \( H_0/A_1 \) and \( H_f/A_1 \) variations. In particular, the electron-nuclear spin-coupled NV system in diamond can be a versatile tool for studying a high-dimensional topology; e.g., further scaling up to a higher topologically-invariant number can be straightforwardly performed, in principle, by utilizing the intrinsic \(^{13}\)C nuclear spins near the NV electronic spin qubit, with hyperfine coupling strengths varying from a few tens of kHz to almost \(-100\) MHz45. For a higher-number symmetric monopole system, one can engineer the Chern number transition with an increment of 1, or an even number transition, \( C = 0, 2, 4 \ldots \), or an odd number transition \( C = 1, 3, 5 \ldots \), by adjusting the detuning \( H_0/A_1 \).

In this work, we simulate a high topologically-invariant number (the Chern number, \( C \)) using the experimentally accessible system of a single NV electronic spin qubit that is hyperfine coupled with the host \(^{14}\)N nuclear spin. We also demonstrate the robust tunability of the measured Chern number up to \( C = 3 \) by harnessing the control parameters of the qubit. A systematic design of the Hamiltonian parameter sphere reveals the detailed topological structures over three synthetic monopoles, as well as intriguing Chern number physics associated with the adiabaticity of the system’s evolution over time. The generality of this method can be expanded to various qubit platforms to investigate the topology of higher dimensions, such as N-interacting qubit systems, which can simulate the topology of non-interacting 2N band models in condensed-matter physics. Furthermore, the tunability of the topological invariant of a qubit system can be directly applied to explore more exotic topology, which could be applied to the field of topological quantum information science.

### METHODS

#### NV spin system with three degeneracies

The NV centre ground-state has an electronic spin with quantum number \( S = 1 \) and sublevels \( |0\rangle \) and \( |\pm 1\rangle \). Throughout this work, we use only sublevels \( |0\rangle \) and \( |\pm 1\rangle \) as a two-level system, by Zeeman splitting the \( |\pm 1\rangle \) states using a static external bias magnetic field. The NV electronic spin experiences a hyperfine interaction with the host nuclear spin \(^{14}\)N, with spin quantum number \( I = 1 \). The longitudinal component of the hyperfine interaction, with coupling strength \( A_1/2\pi = 2.2\) MHz, further splits the \( |\pm 1\rangle \) sublevel into three hyperfine levels. The internal NV Hamiltonian assumes the form \( H_0 = \hbar/2A_1|\sigma F \rangle \). Consequently, this electronic-nuclear spin system contains three degeneracy points, allowing us to simulate topological phases with a Chern number greater than 1. Additionally, the transition between different Chern numbers can be realized by introducing a common offset to these degeneracy points. The topology realized in this study corresponds to an eight-band noninteracting triangular lattice model.

#### Experimental setup

Measurements are performed using a home-built NV-diamond confocal microscope. An acousto-optic modulator (Isomet Corporation) enables time gating of a 400 mW, 532 nm diode-pumped solid-state laser (Changchun New Industries). The laser beam is coupled to a single-mode fibre, and subsequently, delivered to an oil-immersion objective (\( \times 100, 1.3\) NA, Nikon CFI Plan Fluor), and focused onto a diamond sample. The diamond sample is fixed on a three-axis motorized stage (Micos GmbH) for precise position control. NV red fluorescence (FL) is collected using the same objective and then passed through a dichroic filter (Semrock LP02–633RS-25). A pinhole (diameter 75 \( \mu \)m) is used with an \( f = 150\) mm telescope to spatially filter the FL signal, which is detected using a silicon avalanche photodetector (Perkin Elmer SPCM-ARQH-12). A signal generator (SG, Agilent E4428C) provides the carrier microwave signal. The phase and amplitude of the carrier signal is modulated with a 1 G s \(^{-1}\) rate arbitrary waveform generator (AWG, Tektronix AWG 5014 C) and an IQ mixer (Marki IQ 1545 LMP). The microwave sideband signal is amplified (Mini-circuits ZHL-16W-43-S+–) and passed through a gold coplanar waveguide, fabricated on a quartz coverslip using photolithography that is mounted directly on the diamond sample to control the NV spin qubit. The diamond sample is CVD-grown, \(^{12}\)C isotopically purified to 99.99%, with dimensions of 2 mm \( \times \) 2 mm \( \times \) 0.5 mm. The diamond is annealed at 800 °C for 8 h and then at 1000 °C for 10 h to mobilize vacancies in the diamond and, combined with existing nitrogen defects from the CVD process, to create the NV centre used in the experiment. During the measurement, the external magnetic field is aligned with the NV crystalline axis at a field strength of \( \sim 100\) G. The measured NV spin resonance lifetimes are \( T_1 \approx 3\) ms, Hahn-Echo \( T_2 \approx 400\) \( \mu\)s, and \( T_2^* \approx 40\) \( \mu\)s.

#### Quantum state tomography

To create a hemispherical trajectory of the Larmor vector and hence the NV electronic spin Bloch vector to follow, a single-envelope-chirped microwave signal is used, with sweeping of both the detuning and Rabi frequency. To match the relative phase of the chirped signal, a tomography pulse is applied directly after the control pulse at a specific time \( T_{\text{meas}}. \) The (\( \sigma^y \)) rotation tomography pulse’s relative phase is set with respect to the end phase of the chirped control signal. The tomography pulse Rabi frequency is set to 10 MHz. During calibration of the tomography pulse, the observed dynamic phase noise contribution is highly suppressed. The final NV electronic spin state is read out by optical illumination at 532 nm and measurement of the NV fluorescence (FL) in the 640–800 nm band over an observation time of 400 ns. A change in NV FL intensity occurs due to a non-radiative decay pathway via metastable singlet states (Fig. 1b). Measurements are repeated \( \sim 10^6 \) times to establish good statistics, given finite optical collection efficiency of the apparatus. Because the NV spin qubit system has two isolated spin levels in the presence of an external bias magnetic field \( \sim 100\) G, leakage to other states can be neglected, which gives a fidelity advantage for using an NV centre for quantum simulations.

#### Adiabaticity parameter determination

We determine an optimized condition for the adiabaticity parameter \( a \), where the NV electronic spin qubit response is quasi-adiabatic. This condition is fulfilled when (i) the qubit adiabatically follows the Larmor vector trajectory; and also (ii) the observable NV electronic spin Bloch vector component \( (\sigma^z) \), which is the Lorentzian deviation from the Larmor vector trajectory, has a sufficiently large signal-to-noise ratio to be detected. Using dynamic-state preparation as a benchmark of the adiabaticity parameter calibration25, we first detect the Landau–Zener transition (Supplementary Fig. 3) by measuring \( (\sigma^y) \), the hemispherical manipulation of the NV spin qubit, and confirm that the transition probability depends on both the Rabi frequency \( \Omega \) and \( T_{\text{temp}} \) as expected. The Landau–Zener \( (\sigma^z) \) measurements, by varying \( a \), show that our system’s quasi-adiabatic boundary is approximately within the \( 2 \leq a \leq 10 \) range.
Numerical simulations

All numerical simulations of NV spin evolution in this work are performed by computing the time-ordered time evolution operator at each time step:

$$U(t_i, t_f) = \hat{T}\left\{\exp\left(-i\int_{t_i}^{t_f} \mathbf{H}(t) dt\right)\right\} = \prod_{j=1}^{N} \exp\left(-i\Delta t \mathbf{H}(t_j)\right)$$

where \(t_i\) and \(t_f\) denote the initial and final time, respectively; \(\hat{T}\) is the time-ordering operator; \(\Delta t\) is the time step size of the simulation; \(N = (t_f - t_i)/\Delta t\) is the number of time steps; and \(\mathbf{H}(t)\) is the time-dependent Hamiltonian. In the simulation, a step size of \(\Delta t = 1\) ns is used, which is sufficiently small in the rotating frame. The algorithm is implemented using MATLAB® software.

DATA AVAILABILITY

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

CODE AVAILABILITY

The codes and data used in this study are available from the corresponding author upon reasonable request.

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AUTHOR CONTRIBUTIONS

J.L. and K.A. contributed equally to this work. The project was conceived by J.L. and K.A., and supervised by R.L.W. J.L., K.A., and H.Z. designed the experimental set-up. J.L., K.A., and M.J.K. performed the measurements and analysed the data. All authors contributed to the discussion and preparation of the manuscript.
COMPETING INTERESTS
The authors declare no competing interests.

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