NON-GAUSSIANITY TEST FOR DISCRIMINATING GRAVITATIONAL WAVE BACKGROUNDS AROUND 0.1–1 Hz

NAOKI SETO
National Astronomical Observatory, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan
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ABSTRACT

We propose a non-Gaussianity test for gravitational wave backgrounds by combining data streams of multiple detectors. This simple method allows us to check whether a detected background is “smooth” enough to be consistent with an inflation-type background, or is contaminated by individually undetectable weak burst signals. The proposed test would be quite useful for the Big Bang Observer or DECIGO whose primary target is a background from inflation at 0.1–1 Hz where gravitational wave bursts from supernovae of Population III stars might become a troublesome foreground.

Subject headings: early universe — gravitational waves — supernovae: general

1. INTRODUCTION

Since gravitational interaction is very weak, a gravitational wave (GW) background can serve as an invaluable fossil from the early universe with almost no scattering and absorption during its propagation (Maggiore 2000). By analyzing the background, we might obtain crucial information to understand physics at an extremely high energy scale, e.g., the inflation process. However, overlap of multiple signals in data streams of detectors would become a basic aspect of GW astronomy, especially in the low-frequency regime where number and variety of astrophysical sources would increase. This is partly because GW detectors have omnidirectional sensitivity. Therefore, for detecting a background from the early universe, it is essential to disentangle diversified signals contained in observed data. For example, we need to detect and subtract individual astrophysical signals such as chirping binaries from data streams (see e.g., Arnaud et al. 2007; Harms et al. 2008 for recent progress).

The primary target for the Big Bang Observer (BBO; Phinney et al. 2003) and the DECItel Interferometer Gravitational Wave Observatory (DECIGO; Seto et al. 2001; Kawamura et al. 2006) is a background from inflation in the 0.1–1 Hz band where a relatively deep window of GWs has been expected to be opened. However, it was recently pointed out that burstlike GWs produced at supernova (SN) explosions of Population III stars might become a problem for detecting an inflation background around 0.1–1 Hz (Buonanno et al. 2005; Sandick et al. 2006; Suwa et al. 2007). While a burst signal with a large amplitude might be handled in data analysis, a potential background composed by weak undetectable bursts would be a formidable obstacle for adequately identifying an inflation background. Furthermore, in contrast to regular waveforms (e.g., from binaries), it is often difficult to accurately model waveforms for burst signals and/or to predict their characteristic amplitudes beforehand. To deal with such a situation, we certainly need to study GW backgrounds with efficient quantification methods beyond the traditional simple measure $H_{\text{int}}$, the energy density of backgrounds (see also Finn et al. 1999; Drasco & Flanagan 2003). In this Letter we propose a non-Gaussianity test for GW backgrounds in 0.1–1 Hz band, and discuss its prospects with BBO, setting GWs from the Population III SNe as our fiducial burst model.

This Letter is organized as follows: in § 2 we briefly describe data streams of BBO with summaries for notations. The basic idea behind our non-Gaussianity test is presented in § 3. In § 4 we make a statistical evaluation of our method.

2. DATA STREAMS OF BBO

For BBO, two LISA-type units (arm length $L = 5 \times 10^4$ km) would be used to form a Star-of-David-like configuration. From each unit we can obtain three data streams $(A, E, T)$ and $(A', E', T')$ (Prince et al. 2002; Corbin & Cornish 2006; Seto 2006). Among these six data streams, the $T$ and $T'$ modes are less sensitive to GWs, and we neglect them hereafter. The data streams $A, E, A'$ and $E'$ can be effectively regarded as responses of simple L-shaped detectors around their optimal frequency $f_{\text{opt}} \sim 0.3$ Hz with bandwidth $\Delta f \sim f_{\text{opt}}$ (Seto 2006). In this Letter we only deal with quantities made from the following two pairs: $A-A'$ or $E-E'$. The orientation of the arms of the former pair is aligned on the common detector plane of the two units, but the latter is misaligned by 45$^\circ$ on the plane.

We model the data streams $s_x(t)$ ($X = A, E, A')$ in terms of GW signal $H_x$ and detector noises $n_x$ as $s_x(t) = H_x(t) + n_x(t)$. For analyzing the noises $n_x$, it is advantageous to work in Fourier space. To clarify our main points, we discuss signal analysis in the optimal band and neglect the details of frequency dependence [e.g., replacing the integral $\int \cdot \Delta f$ with the product $\int \cdot df \times \delta f$]. In practice, this situation is approximately realized by applying a bandpass filter. In order to do the Fourier transformation, we decompose the data streams (total duration $T_{\text{obs}}$) into short segments of a given period $T_{\text{seg}}(\equiv f_{\text{int}}^{-1})$, and assign a label $M(=1, \ldots, T_{\text{obs}}/T_{\text{seg}})$ for each segment with chronological order. Then we calculate

$$s_{XIF}(f) = \int_{(M-1)T_{\text{seg}}}^{MT_{\text{seg}}} s_X(t)e^{2\pi if} dt = H_{XIF}(f) + n_{XIF}(f). \quad (1)$$

The number of relevant Fourier modes in a segment is $\sim T_{\text{seg}}\Delta f \sim T_{\text{seg}}f_{\text{int}}$. In the Fourier transformations above, we implicitly assume the application of an adequate time window function to suppress the leakage of underlying frequency components to nearby modes due to the finiteness of $T_{\text{seg}}$.

Hereafter, we only use the Fourier transformed quantities, assuming that the detector noises $n_x$ are stationary, Gaussian, and have identical spectrum $S_n(f)$. The assumption of Gaussian noises is not crucial for our method. For the relevant pairs...
\((X, Y) = (A, A')\) or \((E, A')\), we also assume that their noises are independent (Phinney et al. 2003) with

\[
\langle n_{\text{HM}}(f)n_{\text{LY}}(f')^* \rangle \sim \frac{1}{2} \delta_{\text{ML}} \delta_{xy} \delta_{ij} T_{\text{seg}} S_N(f),
\]

where the notation \(\langle \cdots \rangle\) represents taking an ensemble average.

Next we discuss detectors’ responses to incoming GWs. We expand the metric perturbation due to a GW background at a time \(t\) and position \(x\) by

\[
h_\nu(t, x) = \sum_{p=-1,0} \sum_{f \in B_i} e^{2\pi i f x / c} h_n(P, f) e_{\nu}^p + \text{c.c.}
\]

(c.c.: complex conjugate) with three-dimensional wavevectors \(f\), polarization bases \(e_{\nu}^p\), and a definition \(f \equiv \| \mathbf{f} \|.\) The response of a detector \(X\) to an incident GW is characterized by the beam pattern function \(F_X(f, P)\). In this Letter, unless otherwise stated, we study simple L-shaped interferometers with the long-wave approximation. The explicit form of the function \(F_X\) is presented in the literature (Flanagan 1993; Allen & Romano 1999). For the background above, the signal \(H_{\text{HM}}(f)\) in equation (1) is expressed as

\[
H_{\text{HM}}(f) = \sum_{p=-1,0} \sum_{f \in B_i} \exp[2\pi i (f \cdot x + ft_m)] h_n(P, f) T_{\text{seg}} F_X(f, P)
\]

with a shell-like three-dimensional frequency region \(B_i\) corresponding to the observed frequency \(f\). Then we have

\[
\langle H_{\text{HM}}(f) H_{\text{LY}}(f')^* \rangle = \frac{8\pi}{5} S_{\text{GW}}(f) \gamma_{XY} T_{\text{seg}}
\]

where the spectrum \(S_{\text{GW}}(f)\) for the background has dimension \(\text{Hz}^{-1}\) as for the detector noise spectrum \(S_n(f)\), and is written with the normalized energy density \(\Omega_{\text{GW}}\) as \(S_{\text{GW}}(f) = (3H_0^2/32\pi^5 G) f^{-3} \Omega_{\text{GW}}(f)\) \((H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1}: \text{the Hubble parameter})\). The overlap function \(\gamma_{XY}(f)\) characterizes the magnitude of common responses of two detectors \((X, Y)\) to isotropic and unpolarized backgrounds (Flanagan 1993; Allen & Romano 1999). It is defined by

\[
\gamma_{XY}(f) \equiv \frac{5}{8\pi} \int_{\text{sphere}} d\mathbf{n} (F_X^* F_Y + F_X F_Y^*) e^{2\pi i (\mathbf{n} \cdot (x'-x))}.
\]

We have \(\gamma_{XY} = 0\) due to geometrical symmetry and also \(\gamma_{xy} \sim 1\) around the optimal band \(f \sim 0.3\ \text{Hz}\) of BBO (Seto 2006).

3. NON-GAUSSIANITY TEST

If the signals \(H_{\text{HM}}(f)\) in equation (4) are made from superpositions of a vast number of incoherent waves, they can be regarded as Gaussian variables from the central limit theorem. As a result, they are characterized only by second moments, and we have

\[
\langle (H_{\text{HM}}(f) H_{\text{LY}}(f'))^2 \rangle = 2 \langle H_{\text{HM}}(f) H_{\text{LY}}(f')^* \rangle^2 \propto \gamma_{XY}^2
\]

from the properties such as \(\langle H_{\text{HM}}(f) H_{\text{HM}}(f) \rangle = 0\).

For a “coarse background” made by a relatively small amount of freedom (e.g., popcorn noise due to supernova bursts), the responses of the detectors would deviate from Gaussian. Therefore, using the BBO pair \(E-A’\) with the overlap function \(\gamma_{E-A'} = 0\), we can check granularity of an isotropic background through the quantity \(\langle (H_{\text{EM}}(f) H_{\text{EOM}}(f'))^2 \rangle - 2 \langle H_{\text{EM}}(f) H_{\text{EOM}}(f')^* \rangle^2\) that should vanish for a Gaussian background, e.g., that generated at inflation. This is a key point in this Letter. For detectors with a finite overlap \(\gamma_{XY} \neq 0\), we can generalize this method by introducing combinations such as \(\langle (H_{\text{EM}}(f) H_{\text{EOM}}(f'))^2 \rangle - 2 \langle H_{\text{EM}}(f) H_{\text{EOM}}(f')^* \rangle^2\) to subtract the Gaussian component (similar to the definition of the Kurtois parameter \(k_4\) for the standard characterization of non-Gaussianity (see, e.g., Racine & Cutler 2007). We do not pursue this direction further. But the underlying approach proposed in this Letter would be applicable to a network of ground-based detectors. For general detector configurations such as the LIGO-VIRGO pair, we need the subtraction scheme described above.

For statistical analysis with BBO, we introduce the following two quantities made from the two pairs \(A-A’\) and \(E-A’\) respectively:

\[
C_2 = \sum_{m \neq 0} \sum_{f \neq 0} s_{AM}(f) s_{AM}(f)^* \text{,}
\]

\[
C_4 = \sum_{m \neq 0} \sum_{f \neq 0} (s_{EM}(f) s_{EM}(f)^*)^2.
\]

Here the second summations \(\sum_{f \neq 0}\) are for the Fourier modes (total number: \(T_{\text{seg}}\delta f\)) within a segment, and the first ones \(\sum_m\) are for the segment label \(M = (1, \ldots, T_{\text{seg}}/T_{\text{seg}})\). The combination \(C_2\) is used for traditional correlation analysis to measure \(\Omega_{\text{GW}}\), while \(C_4\) is our new probe for non-Gaussianity of a GW background. We evaluate their signal-to-noise ratios in the next section.

In this Letter we have set GWs from Population III SNe as our fiducial burst model. Here we comment on other models. Recently, several cosmological scenarios were proposed to produce intrinsically non-Gaussian GW background in the early universe. With a typical cosmological mechanism (e.g., through preheating phases), GW background is generated by causal processes, when wavelengths were comparable to or smaller than the horizon size in the order-of-magnitude sense (see, e.g., Easther & Lim 2006). Therefore, even if the generated GWs have a correlation structure, we have a large number \([(fH_0^2)^2 \sim 10^{30} (f/1 \text{ Hz})^2]\) of independent emission regions for GWs currently observed at frequency \(f\), and it would be difficult to directly probe the intrinsic non-Gaussianity for these typical models with BBO, due to the central limit theorem. But a background made by sparse cosmological events, such as GW bursts from cusps of cosmic strings, might be an interesting target (Damour & Vilenkin 2005).

4. BURST BACKGROUND

4.1. Derivation of Formulas

In this section we analyze a GW background made by a superposition of burst events from single-species sources that have an event rate \(R\) and a characteristic duration time \(T_b\) for a GW signal in the optimal band. We start our discussion in a somewhat general way and derive useful expressions for \(C_2\) and \(C_4\). Our fiducial model (GWs from Population III SNe) will be examined later.

We assume a smooth spectral profile \(A(f)^2\) \((A > 0)\) for the GW emission throughout a burst event, and do not deal with regular waveforms with sharp frequency structures (e.g., mono-
We define the ratio for \( \ast \) from expression (5) and the corresponding mean magnitude to-noise ratio for the quantity under the condition \((S/N)\) for individual bursts with a single detector is evaluated as (see, 1/2

\[ H_{\text{XM}} \propto A[F_y(\theta, \phi)\alpha_\ast(I) + F_x(\theta, \phi, \psi)\alpha_\ast(I)]. \]

(10)

Since these four geometric parameters are randomly distributed for extra-Galactic events, we define an averaging operator \( \langle \cdot \rangle \) with respect the direction and orientation of sources, and obtain

\[ [H_{\text{XM}}H_{\text{XM}}^\text{\dagger}]_m = \frac{A^3}{5} \gamma_{XX}. \]

(11)

We define the ratio \( Q \equiv [(H_{\text{XM}}H_{\text{XM}}^\text{\dagger})^2]_m /([H_{\text{XM}}H_{\text{XM}}^\text{\dagger}]_m)^2 \) for quantitative evaluation of the probe \( C_x \). Its numerator is explicitly given as

\[ [(H_{\text{XM}}H_{\text{XM}}^\text{\dagger})^2]_m = \frac{A^3}{630} \int_0^\pi \{ 9(|a|^4 + |a_\ast|^4) \]

\[ -34|a_\ast|^2 + 52\text{Re}(a^\ast a_\ast^2) \} \sin d\ell. \]

(12)

Unless a weird cancellation occurs, the ratio \( Q \) becomes of order unity. At the end of this subsection, we will explicitly demonstrate this for our fiducial Population III SNe model.

With a relation \( \gamma_{XX} = 1 \) for a self-correlation, equation (11) shows that the angular average of the response function is \( 1/\sqrt{5} \). Therefore, the characteristic signal-to-noise ratio \((S/N)_{\text{Bat}}\) for individual bursts with a single detector is evaluated as (see, e.g., Segalis & Ori 2001; Sago et al. 2004)

\[ (S/N)_{\text{Bat}} = \frac{2}{(S/N)_{\text{Bat},C2}} \frac{(A^2)(\delta f)^{1/2}}{S_N^{1/2}}. \]

(13)

Next we analyze the GW background formed by incoherent superposition of the bursts analyzed above, assuming that the typical signal-to-noise ratio \((S/N)_{\text{Bat}}\) is not larger than \( O(1) \). As the expected number of events in a segment is \( R T_{\text{seg}} \), we get the background spectrum

\[ S_{\text{GW,Bat}} = \frac{R \langle A^2 \rangle}{8\pi}. \]

(14)

from expression (5) and the corresponding mean magnitude \( \langle H_{\text{XM}}H_{\text{XM}}^\text{\dagger} \rangle = R T_{\text{seg}} \times \langle A^2 \rangle \gamma_{YY}/5. \) Now we evaluate the signal-to-noise ratio \((S/N)_{\text{Bat},C2}\) for the quantity \( C_x \) under the condition \( T_{\text{seg}} \gg T_x \). If a segment \( M \) contains a burst event, its averaged contribution to \( C_x \) is \( \langle A^2 \rangle \gamma_{YY} \delta f/5 \). After incoherent superposition of \( T_{\text{obs}}R(\gg 1) \) events during observation time \( T_{\text{obs}} \), we obtain the expectation value (signal strength) \( \langle C_x \rangle = \langle A^2 \rangle \gamma_{YY} \delta f/5 \times (T_{\text{obs}}/R) \). Meanwhile, assuming independence of detector noises and the condition \( S_N(f_{\text{opt}}) \gg S_{\text{GW}}(f_{\text{opt}}) \), the rms fluctuations for the product \( \langle A^2 \rangle \gamma_{YY} \delta f \) are given by the detector noise spectrum as \( 2^{-1/2} \times S_{\text{N}} \gamma_{YY} \). Here the additional factor \( 2^{-1/2} \) is associated with the projection operation of data to the expected phase direction (usually onto the real axis) in the complex plane (Seto 2006). Taking into account the total number of Fourier modes \( (T_{\text{obs}}T_{\text{seg}}) \times (T_{\text{seg}}\delta f) = T_{\text{obs}}\delta f \), we obtain the rms fluctuations for \( C_x \) as \( 2^{-1/2} \times S_{\text{N}} \gamma_{YY} \delta f \), and its signal-to-noise ratio is given by

\[ (S/N)_{\text{Bat,C2}} = \frac{2}{5} \frac{R \langle A^2 \rangle}{S_N} (2T_{\text{obs}}\delta f)^{1/2}. \]

(15)

We can derive the same result for \( T_{\text{seg}} \ll T_x \). Replacing the product \( \times \delta f \) with a frequency integral \( \int df \) and using the spectrum \( R \langle A^2 \rangle = 8\pi S_{\text{GW,Bat}} \), the square value of this expression exactly matches the standard formula for correlation analysis given in the literature (Flanagan 1993; Allen & Romano 1999).

In the same manner, we can derive

\[ (C_4) = \langle A^4 \rangle Q R T_{\text{seg}} T_{\text{obs}} \delta f/25 \]

for \( T_{\text{seg}} > T_x \) and

\[ (C_4) = \langle A^4 \rangle Q R T_{\text{seg}}^2 T_{\text{obs}} T_x \delta f/25 \]

for \( T_{\text{seg}} \ll T_x \). For both cases, the signal-to-noise ratio for \( C_4 \) is given as

\[ (S/N)_{\text{Bat,C4}} = [U/4\delta f \max (T_{\text{seg}}, T_{\text{obs}})] (S/N)_{\text{Bat,C2}} \times (S/N)_{\text{Bat}} \]

with a parameter \( U = \langle Q \rangle \langle A^4 \rangle /\langle A^2 \rangle^2 = O(1) \). In contrast to \( C_x \), the signal-to-noise ratio \((S/N)_{\text{Bat,C4}}\) explicitly depends on \( T_{\text{seg}} \). Note that here independence of detector noises is an important requirement, but their Gaussianity is not essential.

With increasing the segment length \( T_{\text{seg}} \), from its minimum \( \sim f_{\text{opt}}^{-1} \), we have a transition from \( \langle C_4 \rangle \propto T_{\text{seg}}^{2} \) to \( \langle C_4 \rangle \propto T_{\text{seg}}^{-1} \) at the point \( T_{\text{seg}} \sim T_x \) where the signal-to-noise ratio \((S/N)_{\text{Bat,C4}}\) also starts to decrease from a constant due to dilution of power. Therefore, if the signal \( C_4 \) is detectable at \( T_{\text{seg}} \sim f_{\text{opt}}^{-1} \), we can estimate the duration time \( T_x \) by identifying the transition. In the future, we set \( T_{\text{seg}} \sim f_{\text{opt}}^{-1} \) which will provide us with the maximum value of \((S/N)_{\text{Bat,C4}}\) for a burst background.

Now we focus our discussion on a burst model with \( T_x \sim f_{\text{opt}}^{-1} \) (e.g., for our fiducial Population III SNe model at 0.1–1 Hz). In this case, we have

\[ (S/N)_{\text{Bat,C4}} = U \times (S/N)_{\text{Bat,C2}} \times (S/N)_{\text{Bat}}/4, \]

and this relation is very suggestive. Even if an individual burst has signal-to-noise ratio \((S/N)_{\text{Bat}}\) less than the detection threshold, there is an amplification factor \((S/N)_{\text{Bat,C2}} \propto T_x^{1/2} \) that increases with observational time \( T_{\text{obs}} \) and enable us to statistically study the bursts.

With the parameters related to the bursts, we obtain

\[ (S/N)_{\text{Bat,C4}} = (S_{\text{GW,Bat}}/U) \times (2T_{\text{obs}}\delta f)^{1/2}/(25S_N^{1/2}). \]

For a fixed background level \( S_{\text{GW,Bat}} \propto R \langle A^2 \rangle \), the signal-to-noise ratio \((S/N)_{\text{Bat,C4}}\) decreases for a higher event rate \( R \) (corre-
sponding to a smaller amplitude \((\mathcal{A}^2)\). This is reasonable, considering that the background would become more Gaussian-like. If the burst events are supposed to be the dominant sources of the total GW background around \(f_{\text{opt}}\) and both \(\langle C_i \rangle \propto R(\mathcal{A}^2)\) and \(\langle C_i \rangle \propto R(\mathcal{A}^2)/U/T_o\) are measured, we can roughly estimate the event rate \(R\) and the amplitude \((\mathcal{A}^2)\) separately, assuming \(U = O(1)\). In addition to the estimated duration \(T_o\), these will be basic information for disclosing the nature of the burst sources.

As we commented earlier, the ratio \(Q = [(H_{\text{EM}}H_{\text{GW}}^*)]^n/[H_{\text{EM}}H_{\text{GW}}^*]^n\) becomes of order unity for typical burst waveforms. Here we demonstrate this for our fiducial model: burst GWs from Population III SNe. In the band around 0.1–1 Hz, the emitted waves are dominated by memory effects caused by anisotropic neutrino emissions at supernova explosions (Buonanno et al. 2005), and we put the GW waveforms characterized by the background level \(\Omega_{\text{GW}}\) and their rate \(R\), \textit{BBO} has a sensitivity corresponding to

\[
(S/N)_{\text{BBO}} \sim 0.6 \left(\frac{\Omega_{\text{GW}}}{4 \times 10^{-16}}\right)^{1/2} \left(\frac{R}{0.01 \text{ s}^{-1}}\right)^{-1/2}
\]  

for individual bursts, and

\[
(S/N)_{\text{BBO,}\text{c2}} \sim 80 \left(\frac{\Omega_{\text{GW}}}{4 \times 10^{-16}}\right) \left(\frac{T_{\text{obs}}}{10 \text{ yr}}\right)^{1/2},
\]

\[
(S/N)_{\text{BBO,}\text{c4}} \sim 10 \left(\frac{\Omega_{\text{GW}}}{4 \times 10^{-16}}\right)^2 \left(\frac{R}{0.01 \text{ s}^{-1}}\right)^{-1} \left(\frac{T_{\text{obs}}}{10 \text{ yr}}\right)^{1/2}
\]

for the background with \(T_{\text{obs}} \times \delta f \sim 1, Q \sim 1\), and \(U \sim 1\). Therefore, while identification of each burst might be difficult with small \((S/N)_{\text{BBO}}\), our method has the potential to discriminate whether a background once detected is smooth enough and consistent with inflation origin.

An interesting question related to our non-Gaussianity test is whether we can separate smooth and burst contributions for the total energy spectrum \(\Omega_{\text{GW}}\). To estimate the latter component, we need the combination \(R(\mathcal{A}^2)\). With our approach based on a fourth-order moment, we can obtain a different combination \(RQ(\mathcal{A}^2) = (R(\mathcal{A}^2))^2 \times U/R\). If the burst rate \(R\) is independently estimated, e.g., with optical observation of Population III SNe, we can roughly estimate the burst component \(\propto R(\mathcal{A}^2)\) in the total spectrum \(\Omega_{\text{GW}}\) by introducing a model parameter \(U = O(1)\).

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4.2. Prospects around 0.1–1 Hz

In this subsection we specifically discuss prospects of our non-Gaussianity test for Population III SNe GW background with \textit{BBO}. Since the anisotropies of neutrino emissions from Population III SNe or the formation rate of Population III stars are poorly understood, the amplitude of the Population III background is currently quite uncertain. With a parameter set for Population III SNe or the formation rate of Population III stars

\[
\Omega_{\text{GW}} \sim 4 \times 10^{-16} \text{ at } f \sim 0.3 \text{ Hz.}
\]

Meanwhile, for bursts characterized by the background level \(\Omega_{\text{GW}}\) and their rate \(R\), \textit{BBO} has a sensitivity corresponding to

\[
(S/N)_{\text{BBO}} \sim 0.6 \left(\frac{\Omega_{\text{GW}}}{4 \times 10^{-16}}\right)^{1/2} \left(\frac{R}{0.01 \text{ s}^{-1}}\right)^{-1/2}
\]  

for individual bursts, and

\[
(S/N)_{\text{BBO,}\text{c2}} \sim 80 \left(\frac{\Omega_{\text{GW}}}{4 \times 10^{-16}}\right) \left(\frac{T_{\text{obs}}}{10 \text{ yr}}\right)^{1/2},
\]

\[
(S/N)_{\text{BBO,}\text{c4}} \sim 10 \left(\frac{\Omega_{\text{GW}}}{4 \times 10^{-16}}\right)^2 \left(\frac{R}{0.01 \text{ s}^{-1}}\right)^{-1} \left(\frac{T_{\text{obs}}}{10 \text{ yr}}\right)^{1/2}
\]

for the background with \(T_{\text{obs}} \times \delta f \sim 1, Q \sim 1\), and \(U \sim 1\). Therefore, while identification of each burst might be difficult with small \((S/N)_{\text{BBO}}\), our method has the potential to discriminate whether a background once detected is smooth enough and consistent with inflation origin.

An interesting question related to our non-Gaussianity test is whether we can separate smooth and burst contributions for the total energy spectrum \(\Omega_{\text{GW}}\). To estimate the latter component, we need the combination \(R(\mathcal{A}^2)\). With our approach based on a fourth-order moment, we can obtain a different combination \(RQ(\mathcal{A}^2) = (R(\mathcal{A}^2))^2 \times U/R\). If the burst rate \(R\) is independently estimated, e.g., with optical observation of Population III SNe, we can roughly estimate the burst component \(\propto R(\mathcal{A}^2)\) in the total spectrum \(\Omega_{\text{GW}}\) by introducing a model parameter \(U = O(1)\).

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