Basic principles of the TEVY index for the quantification of temperature variability within a year

Theodoros Kalyvas 1, Stella Manika 1 and Efthimios Zervas 1,2

1 Hellenic Open University, Parodos Aristotelous 18, 263 35, Patras

2 zervas@eap.gr

Abstract: In the context of climate change, there is a need for the determination of appropriate indexes for the quantification of temperature variability. A new index (TEVY index) is proposed in this work. This index uses the deviation of the observed temperature values from those estimated from a Fourier harmonic analysis. For this purpose, a nearly 50-year time series data from 4 stations in Greece, with very different climatic conditions, are used. One station is located in the colder northern region of Greece, another one is in the warmest southern part, while the 2 other stations are representative of continental and Mediterranean climatic features. A Fourier harmonic analysis is carried out to obtain the Fourier series which simulates the observed data time series. Fourier harmonic analysis, which is relied on the Fourier transform, is a well-established method for time series analysis, particularly for modelling periodic data. Using this procedure, an index of temperature variability is proposed, as the sum of the divergence of the above-mentioned Fourier series from the observed data. The index results are analysed as a function of the different climatic features of each station.

1. Introduction

Climate change leads to changes in the intensity, duration and frequency of weather extremes [1]. The weather extreme events are related to the extremes of the climate variables, such as temperature, wind and precipitation. Temperature extremes, for example heat waves and cold spells, can have important impacts on human health, on the physical environment and on the ecosystems [1]. A method for monitoring changes in weather extremes is through the use of indices. There is an established system of indices that is used for this purpose, proposed from the Expert Team on Climate Change Detection and Indices (ETCDII) [2]. The ETCCDI set of indices, which consists of 16 temperature-related and 11 precipitation-related indices, is used to calculate the presence of extreme weather events in meteorological time series. The ETCCDI indices have been widely used by scientists in the context of weather extremes and climate variability research, for specific locations or broader areas [3,4,5].

One limitation of the above indices is that they used only observed data, without always taking into account the deviation between the observed and the expected temperature. An alternative method to study temperature variability is to use Fourier harmonic analysis [6,7].

As temperature has a sinusoidal trend within each year, the values of the Fourier transform can be used to estimate an “expected” temperature within a year. The deviation of the observed values from this “expected” temperature can be used to quantify the variability of the temperature within a year. In the present work, we propose an index for the study of climate variability and climate extremes. This index, TEVY, (TEVY=temperature difference within a year) is the sum of the difference between the
recorded temperature values and those derived using Fourier harmonic analysis. A first application of this new index is performed on the temperature data of 4 meteorological stations in Greece, which belong to different climatic zones of the country.

2. Materials and Methods

In this study, temperature data from 4 stations of the Hellenic National Meteorological Service (HNMS) network are used. One station is located in the colder northern region of Greece (Florina), another one is in the warmest southern part of the country (Tymbaki, on the south coast of Crete), while the two other stations, Larissa and Nea Filadelfeia, are representative of continental and Mediterranean climatic features. Nea Filadelfeia is located within the capital of Greece, Athens.

The data used cover a nearly 50-year period. Some years have a high percentage of missing data and were completely eliminated for this analysis. The years used are: Florina: 1961-2016 (except the years 2005 and 2007-2010), Tymbaki: 1959–2012, (except 1991, 1993, 1995), Larissa: 1955–2016, Nea Filadelfeia: 1955–2009 (except 2005). For the years used, less than 1% of the values were missing per year and they were replaced by the average of the value of the previous and the next day. Temperature is measured every 3 hours (8 measurements per day) for each one of the four stations.

The calculation of the proposed TEVY index is carried out through the following procedure. First, the daily means are calculated for all year and for all four stations. These values consist the observed temperature values: \( y_{\text{obs}} \). Next, the mean annual temperature and the annual standard deviation are calculated for all years and for all stations, using the daily mean values.

Using the daily mean temperature values, a Fourier harmonic analysis is then performed. Fourier series represents an approximation of the observed time series (“observed” temperature) and, through it, an “estimated” value of temperature can be calculated for each day: \( y_{\text{est}} \). For the Harmonic Analysis, the standard formula of Fourier transform is used:

\[
y_n = \frac{\alpha_0}{2} + \sum_{m=1}^{k} \left( a_m \cos \frac{2\pi mn}{N} + b_m \sin \frac{2\pi mn}{N} \right)
\]

where:

- \( \frac{\alpha_0}{2} \) = the arithmetic mean of the daily mean values,
- \( a_1, b_1, a_2, b_2, \ldots, a_k, b_k \) = the amplitudes,
- \( k \) = the number of harmonic terms,
- \( n \) = the number of the day of year (from 0 to 364 or 365 for the leap years), and
- \( N \) = period (365 or 366 days for the leap years).

From the results of harmonic analysis, the Fourier series which contains the 1\(^{\text{st}}\) harmonic term is taking into account, because the 1\(^{\text{st}}\) harmonic term explains the highest percentage of the variance. Apart from this fact, the 1\(^{\text{st}}\) harmonic is very important because it has the same period as the observed data (one year). The Fourier series, which contains the 1\(^{\text{st}}\) harmonic, is composed from the Fourier results for each day of all years and represents the estimated values of temperature.

The TEVY index value is calculated for each year as the sum of the square of the difference between the observed and estimated values:

\[
\text{Index} = \sum_{i=1}^{N} (y_{\text{obs},i} - y_{\text{est},i})^2
\]

with \( N = 365 \), or \( N = 366 \) for the leap years.

In order to analyse more deeply the TEVY index results, a frequency distribution diagram of this index is also constructed for each year. This diagram shows the number of days with the difference between observed and estimated values \( \leq 1\degree C \), from 1\degree C to \( \leq 2\degree C \), from 2\degree C to \( \leq 3\degree C \), etc. Finally, the distribution for each month of the mean values, for all years, is also calculated.
3. Results

3.1. Analysis of the mean annual temperature for the 4 stations

Figure 1 shows the evolution of yearly mean temperature for the 4 stations. The mean temperature of Tymbaki is higher than that of the other stations. The second warmer station is Nea Filadelfeia and the next one is Larissa. Florina station is the coldest one among the 4 stations. The yearly mean temperature is constantly increasing in both Tymbaki and Nea Filadelfeia stations, without the cooling trend usually observed until the 70s. This cooling trend is clear in the case of Florina and Larissa; the yearly mean temperature constantly increases the next years but, in the case of Florina, it shows another quite constant decrease after year 2000.

Figure 2 shows the evolution of the standard deviation of the yearly mean temperature values. Florina, with lower mean annual temperatures, shows higher standard deviation values from the other stations, while Tymbaki, having the highest temperatures, has the lower standard deviation values among the four stations. A decreasing trend of standard deviation values is observed in the cases of Florina and Larissa until the late 70s, then the trend is increasing, except an anomaly observed during the last few years. Nea Filadelfeia and Tymbaki show a more stable, slightly increasing trend, for the whole period. These results show that, for the 4 stations, there is, in general, a constantly increased variability of temperature values within a year.

![Figure 1. Evolution of yearly mean temperature for the 4 stations.](image1)

![Figure 2. Evolution of yearly standard deviation of temperature for the 4 stations.](image2)

3.2 Application of the TEVY index in the 4 selected stations

Figures 3-6 show, for the 4 stations, the evolution of the daily mean temperature values within a year, for two years selected as examples (1975 and 2006). These figures clearly show, for the four stations, the variability in temperature, occurring during the entire year. However, it can be observed from Figures 3 and 5 that, in the case of Florina and Larissa which are the stations in the colder regions, this variability is higher during winter than during summer, while Figure 4 shows that, in the case of Tymbaki, which is located in the warmest region of Greece, the variability is higher during summer than during winter. Nea Filadelfeia (Figure 6) shows patterns of high temperature variability during the whole year. Moreover, it can be noticed that, for the 4 stations, the temperature fluctuations are more pronounced in the recent year 2006 than in 1975. These results indicate that a deeper analysis is necessary to reveal these changes.
The next step is to apply a Fourier harmonic analysis to the data of the four stations. Figures 7-10 show, for the four stations, and for the year 2002 taken as an example, the evolution of the observed values and the values estimated from Fourier analysis, using the methodology described previously.
For this particular year, the values of temperature deriving from the Fourier series fit quite well on the observed temperature data of the four stations. Larissa and Florina, with continental climate, show more prominent fluctuations of temperature during winter, while Tymbaki and Nea Filadelfeia, with warmer climate, show fluctuations of temperature mostly in summer.

In order to better analyze this correlation, Figures 11-14 show the correlation between the observed and estimated daily values for the four stations. These figures show a quite linear correlation between estimated and observed values; however, a S-type form appears. The upper and lower queues of these S-type curves are due to the hot and cold extremes that are found quite far from the estimated values.

---

**Figure 9.** Larissa: Observed and estimated mean daily values; year 2002.

**Figure 10.** Nea Filadelfeia: Observed and estimated mean daily values; year 2002.

**Figure 11.** Florina: Observed vs. estimated mean daily temperature values; year 2002.

**Figure 12.** Larissa: Observed vs. estimated mean daily temperature values; year 2002.

**Figure 13.** Nea Filadelfeia: Observed vs. estimated mean daily temperature values; year 2002.

**Figure 14.** Tymbaki: Observed vs. estimated mean daily temperature values; year 2002.
This correlation is performed for every year for the four stations. The evolution of the linear trend line coefficients $a$ and $b$ and of the squared Pearson correlation coefficient $r^2$, for all stations, is shown in the Figures 15-17 below. In the ideal case where the estimated values are the same as the observed ones, the best fit line $y=ax+b$ must have $a=1$, $b=0$ and $r^2=1.0$. Figure 15 shows that the coefficient $r^2$ of these correlations is quite high: the mean value of all years is 0.88 for Tympaki and Larissa, 0.86 for Nea Filadelfeia and 0.82 for Florina. With some fluctuations, these values remain relatively constant in time. It is worthy to mention that Florina, having a colder, mountainous continental climate, has less good correlation values than the other stations and this is due to the several low extreme values that cannot be estimated from Fourier analysis.

![Figure 15. Correlation between estimated and observed values for the 4 stations. Evolution of $r^2$.](image1)

![Figure 16. Correlation between estimated and observed values for the 4 stations. Evolution of slope $(a)$.](image2)

The values of slope $a$ are quite close to 1: the mean value of all years is 0.88 for Tymbaki and Larissa, 0.86 for Nea Filadelfeia and 0.74 for Florina (Figure 16). The estimated values are less close to the observed ones in the case of Florina, having a colder mountainous continental climate, than the other three stations. These values are quite constant in time for the four stations. The values of the intercept $b$ are quite close to zero, and generally quite similar for the 4 stations: 1.77 for Larissa, 2.24 for Florina, 2.34 for Tymbaki and 2.39 for Nea Filadelfeia (Figure 17). As in the case of slope $a$, the values of $b$ are quite constant in time. The previous analysis shows that, even if Fourier analysis cannot follow exactly the daily mean temperature, the values estimated using this method are quite near to the observed ones.

The next step is the calculation of the TEVY index for all years (Figure 18). Florina shows a general decreasing trend of the values of this index until the late 70s, the same period where mean annual temperature decreases. From the beginning of the 70s until the 2000s, the value of the TEVY index increases, following the general trend of mean annual temperature increase. The trend for Tymbaki is quite different. Two areas with different trends, the first one before the 70s and after 1995 and the second one between the 70s and 1995, are observed. Larissa shows a decreasing trend until about the mid-80s, then the trend is slightly increasing with scarce prominent fluctuations. Nea Filadelfeia shows a decreasing trend until the mid-70s then the trend is increasing. Considering the variability of the TEVY index for the four stations, the standard deviation of the TEVY index is 1,128 for Florina, quite higher than that of the other 3 stations, which have a standard deviation of 667 for Tymbaki, 613 for Nea Filadelfeia and 586 for Larissa. Comparing the values of the TEVY index for Florina with the values of the other three stations, Florina, with cold, mountainous continental climate, has higher TEVY index values than the other stations. Tymbaki has a warmer climate and is located near the sea, therefore the TEVY index has quite low values. Nea Filadelfeia has a Mediterranean climate with mild winters and Larissa, due to its continental climate, has quite warm summers. Moreover, the variation of the TEVY index from year to year is higher in the case of Florina. These two statements indicate that the temperature variability within a year, and also its evolution in time, is
higher in the case of cold, mountainous continental climates than in the case of the warmest ones nearby the sea or in plain continental regions.

Figure 17. Correlation between estimated and observed values for the 4 stations. Evolution of intercept (b).

Figure 18. Evolution in time of the proposed index for the 4 stations.

Generally, the trends of the extreme temperatures are reflected in the trends of the TEVY index; however, a more detailed analysis will be performed in a future work.

In order to better analyze the behaviour of the TEVY index, the frequency distribution of the daily divergences between the observed and the estimated values is calculated for the 4 stations. For a temperature difference up to 1°C, the index class is 1, for 2°C the class is 4, for 3°C the class is 9, etc.

As the number of days of the first class is too high, and the differences between estimated and observed values of 1°C are too low, this class is not included in the analysis. Figure 19 shows the mean, for all years, number of days of each class for the 4 stations. The number of days decreases smoothly with the TEVY index class for all stations. A small anomaly is observed in the case of Tymbaki station, where the number of days in class 9 is lower than expected. Also, Figure 19 shows that classes 4 and 9 have lower number of days in the case of Tymbaki than in the case of Florina. Nea Filadelfeia station has the lowest number of days for the classes from 16 to 100. Each station has its own trend and the use of the TEVY index frequencies distribution can be served as a comparison between different stations.

Figure 19. Number of days of each TEVY index class for the 4 meteorological stations. Mean value of all years.

Figure 20. Mean, for all years examined here, of the TEVY index value for each month, for the 4 stations.

To better clarify the evolution of the TEVY index within a year, the next step is to calculate, for each month, the mean value of this index for all years (Figure 20). Florina shows a U shape, indicating that the TEVY index has higher values during winter time. The same trend is observed for Nea Filadelfeia and Larissa, although in the cases of these 2 stations, the difference between winter and summer values is smaller than in the case of Florina. Tymbaki shows a hybrid behavior, the trend of
the monthly mean index value is slightly increasing during the 1st half of the year, while in the second half of the year a U shape is shown, as in the cases of the other stations.

4. Conclusions
This work proposes a new index (TEVY index) for the quantification of temperature variability within a year. This index uses the sum of the deviation of the observed temperature values from those estimated from a Fourier transform. Four meteorological stations in Greece are used as pilots for the evaluation of the TEVY index. The results of this work showed that this index can be used as a metric for the assessment of the temperature variability within a year. It can also be used to compare the variability of different stations and for different time periods.

Of course, more work is necessary to establish this index: the use of data of meteorological stations located at very different climatic zones, the use of other meteorological parameters (relative humidity, precipitation etc.), the close comparison with low and high extremes, the analysis of the classes of this index within each year etc.

References
[1] Seneviratne S I, Nicholls N, Easterling D, Goodess C M, Kanae S, Kossin J, Luo Y, Marengo J, McInnes K, Rahimi M, Reichstein M, Sorteberg A, Vera C and Zhang X 2012 Changes in climate extremes and their impacts on the natural physical environment Managing the Risks of Extreme Events and Disasters to Advance Climate Change Adaptation eds C B Field, V Barros, T F Stocker, D Qin, D J Dokken, K L Ebi, M D Mastrandrea, K J Mach, G K Plattner, S K Allen, M Tignor and P M Midgley (Cambridge University Press: Cambridge UK and New York NY USA) pp 109-230
[2] Internet site of CLIVAR (Climate and Ocean: Variability, Predictability and Change) project. Retrieved 22/04/2021 http://etccdi.pacificclimate.org/list_27_indices.shtml
[3] Kostopoulou E and Jones P D 2005 Assessment of climate extremes in the Eastern Mediterranean Meteorology and Atmospheric Physics 89 69-85
[4] Degefie D T, Fleischer E, Klemm O, Soromotin A V, Soromotina O V, Tolstikov A V and Abramov N V 2014 Climate extremes in South Western Siberia: past and future Stochastic Environmental Research and Risk Assessment 28 2161-73
[5] Wang H, Chen Y, Xun S, Lai D, Fan Y and Li Z 2013 Changes in daily climate extremes in the arid area of northwestern China Theoretical and applied climatology 112 15-28
[6] Tarawneh Q Y 2018 Harmonic analysis of solar irradiation and rainfall data in the context of various climatic indicators in Saudi Arabia Arabian Journal of Geosciences 11 75
[7] Nastos P T and Zerefos C S 2010 Cyclic modes of the intra-annual variability of precipitation in Greece Advances in Geosciences 25 45-50