Disk-to-halo mass ratio evaluations based on the numerical models of collisionless disks

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Abstract.

We propose that the lower bound of the stellar radial velocity dispersion \( c_r \) of an equilibrium stellar disk is determined by the gravitational stability condition. We compared the estimates of stellar velocity dispersion at radii \( r > (1.5 - 2) \cdot R_0 \) (where \( R_0 \) is the photometric radial scalelength of a disk), found in the literature, with the minimal values of \( c_r \) necessary for the disk to be in a stable state, using the results of numerical simulations of 3D collisional disks. This approach enables to estimate an upper limit of the local surface density and (if \( R_0 \) is known) a total masses of a disk and a dark halo. We argue that the old stellar disks of spiral galaxies with active star formation usually have the velocity dispersions which are close to the expected marginal values. A rough values of disk-to-total mass ratios (within the fixed radius) are found for about twenty spiral galaxies. Unlike spirals, the disks of the “red” S\( \alpha \)-S0 galaxies are evidently “overheated”: their radial dispersion of velocities at \( r \approx 2R_0 \) exceeds significantly the marginal values for gravitational stability.

Keywords: galaxies, numeric simulations, dynamics

1. Introduction

It is usually accepted that substantial amounts of dark matter are needed to explain flat rotation curves in the outer regions of disk galaxies. However the fraction of a dark matter belonging to a disk and to a halo is still an open question. Several different methods were proposed how to get a separate estimate of a mass or a local density of a disk. One may try, for example,

- To decompose a rotation curve of a galaxy into spherical and disk—related components;
- To assume that the mass-to-luminosity \( M_d/L \) ratio of stellar disk is known from the models of stellar population and to convert \( L \) into \( M_d \);
- To model either the formation or the kinematic properties of such structural features as a bar or spiral arms;

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− To compare the observed scaleheight of gaseous layer with the expected one for different $M_d$, assuming that the velocity dispersion of gas is known;

− To use the available information about the velocity dispersion of an old stellar disk population (in addition to the rotation curve).

The first two approaches suffer from the well known ambiguities and usually cannot get the unique solution. The third method needs the development of some theoretical background which will allow to connect the observational data with theoretical expectations. The fourth method may be applied (with some inevitable assumptions) to edge-on galaxies only. The applicability and the effectivity of the last method is still an open question. In this work we tried to verify to what extent the observed velocity dispersion of an old disk population may be used to estimate the local density or total mass of a disk.

Galactic disks, consisting mostly of old stars, may be considered as collisionless systems in quasi-stationary equilibrium with a very slow evolution (if to exclude the cases of a strong interactions or mergings with neighbor systems). The disk in equilibrium is characterized by certain radial distribution of stellar velocity dispersions in the plane of a disk ($c_r, c_\phi$), that ensures its stability to gravitational perturbations. If the stability criteria were known, it would allow to develop a self-consistent model for the disk of real galaxy and to estimate its density when both the rotational velocities and stellar velocity dispersions are measured. It is convenient to describe a local radial velocity dispersion $c_r$ in terms of dimensionless Toomre parameter (which equals to unit for the uniformly rotating thin disk, marginally stable to radial perturbations):

$$Q_T = c_r \cdot \alpha / 3.36 G \sigma,$$

where $\alpha$ is the epicyclic frequency, and $\sigma$ is a non-perturbed local density of a disk. If the local value of $c_r$ is known from observations, one may estimate the local density from (1). To follow this way, one may either a) to assume that Toomre stability parameter $Q_T$ is known from some theoretical stability criterium (see Zasov, 1985, Bottema, 1993 and more recent papers, cited in Khoperskov et al., 2003), or to apply N-body models to galaxies to verify that the observed velocity dispersion satisfies the condition of stable equilibrium (e.g. Zasov, 1985, Khoperskov et al., 2001, Zasov et al., 2002). This method is rather cumbersome, but it may be used in a more simple way if to find a minimum value of $Q_T$ of marginally stable disks which may be applied to models with a wide range of parameters.
2. THE CHOICE OF $Q_T$ FOR MARGINALLY STABLE DISKS.

To find the minimum radial velocity dispersion sufficient to ensure stability of the disk against perturbations of arbitrary shapes is highly important if, as some authors have suggested, real galaxies may be in a state of threshold (marginal) stability. Note that in the general case, the old stellar population of a galactic disk can have an excess velocity dispersion in the presence of other factors heating the disk (which may act both from inside and from outside), which are not related directly to the gravitational instability. However, even in this case, the conditions for marginal stability still provide a valuable information by yielding an upper limit for the mass of the disk that enables it to be stable.

There are two different approaches used to find the threshold value of velocity dispersion: either to seek for analytical solution of the problem or to use the numerical models to reproduce the observed properties of real galaxies assuming different mass distribution in the disk, bulge and halo. Together with certain advantages over numerical simulations (the mathematical rigorousness of the solutions in the framework of the problem formulated), the analytical approach to the dynamics of perturbations in a disk and the conditions for its stability has the drawback that it can be implemented only for very simple models (usually 2D disks and the local perturbations are considered based on the analysis of dispersion equation in the epicyclical approximation) and can yield only coarse estimates for the parameters of the disk when applied to real objects. Numerical simulations of collisionless systems are more flexible in terms of the choice of model. They make it possible to go beyond simple two-dimensional models and directly follow the development of perturbations in a disk that is initially in equilibrium. However, this approach has drawbacks of its own. The most serious problems of $N$-body simulations include (1) certain inevitable mathematical simplifications, and (2) the dependence of the final state of the system (after it reaches quasi-equilibrium) on the initial parameters, which are poorly known for real galaxies.

Both numerical and analytical estimates lead to conclusion that the marginal stability of a disk corresponds to $Q_T > 1$, rising to $Q_T \geq 3$ in the outer parts, although different approaches may give different values for the same disk parameters. We refer to our paper (Khoperskov et al., 2003) for the more detailed discussion. We analyzed there the conditions for the gravitational stability of a three-dimensional collisionless disk with exponential density profile, embedding in the gravitational field of two rigid spherical components — a bulge and a halo, whose central concentrations and radial scales were varied over
wide ranges. The initially weakly unstable disks in our models started their evolution from the subcritical equilibrium state. The results of dynamical simulations allowed to determine the disk parameters at the stability limit (when the velocity dispersion ceases to change, and the disk reaches a quasi-steady state after 5 - 20 rotations of the outer edge of the disk). The stability of the solutions against the choice of computational method was verified by comparing results for several models obtained by two very different methods of computing the gravitational force: the TREE-code method and direct “particle-to-particle” (PP) integration, in which each particle interacts with each of the others, for $N = (20 - 80) \times 10^3$. A comparison of the two results revealed no significant differences between the final disk states.

We constructed different numerical models (the number $N$ of equal mass particles was up to $500 \times 10^3$), where the ratios of halo-to-disk masses inside of a disk radius varied between 0.5 and 3. In the case of a low-mass or nonexistent halo, the evolution of the disk is determined by the bar mode, and the disk is heated due to the formation of an non-axisymmetric bar and associated two-armed spiral. Models with sufficiently compact bulge or massive halos do not show any enhancement of the bar mode, however they develop a complex transient system of small-scale spiral waves. The decrease of the amplitudes of these waves with time is accompanied by a transfer of rotational kinetic energy to the chaotic component of the velocity, resulting in heating of the disk. In turn, the increase of the radial-velocity dispersion $c_r$ slows down with decreasing wave amplitude. The heating virtually ceases after the decay of the transient spiral waves. If the disk is initially cool ($Q_T \leq 1$), its heating is very efficient, and its dynamical evolution clearly demonstrates that the wave-decay process has a certain inertia: the velocity dispersion is already high enough to maintain the stability of the disk, however the spiral waves have not yet decayed (as is confirmed by Fourier analysis of the density perturbations in the disk) and continue to heat the disk. Therefore, to obtain the minimum velocity dispersion required for disk stability, we used an iterative algorithm, seeking for a subcritical starting point to make the initial velocity dispersion approach the stability limit.

The other essential initial parameter is the disk thickness or vertical-to-radial velocity dispersion ratio. The thicker is the disk initially, the lower is the minimum radial velocity dispersion $c_r$, which determines its stability. This circumstance also shows that the minimum critical dispersions in both coordinates $z$ and $r$ are reached if the disk begins to evolve from a subcritical state for both radial and bending perturbations, slowly increasing radial and vertical velocity dispersion at the initial state of evolution. As expected, the minimum radial-velocity
dispersion at the end of the simulations (expressed in units of the circular velocity) is higher in the models where the relative mass of the halo is lower, the initial disk thickness is less, and the degree of differential disk rotation is higher.

The radial dependences of $Q_T$, calculated for our models, are different, being determined primarily by the relative mass and degree of concentration of the spherical components. Yet it is essential that in all cases we considered the run of $Q_T(r)$ along the radius passes through a minimum $Q_T \simeq 1.2 - 1.6$ just beyond the region controlled by a bulge, at a galactocentric distance of $(1 - 2) \cdot R_0$, where $R_0$ is the radial scale of a disk (see Figs 1a and b), and this behavior depends only slightly on the choice of model. If a bulge is of low mass absent the low value of $Q_T$ keeps down to the very centre (Fig. 1 b) This property can be used for a rough estimate of the density (and, consequently, the mass) of a galactic disk (or to put limits on these quantities) from the observed $c_r$ at these radial distances without the use of numerical simulations or analytical stability criteria.

3. THE APPLICATION OF THE METHOD

The measurements of stellar velocity dispersion obtained by absorption line spectroscopy are usually restricted by the central parts of galaxies due to steep brightness gradient of stellar disks and the difficulties of analysing the low intensity absorption line spectra. There are not so many galaxies where the line-of-sight dispersion is measured at $r \geq 2R_0$. At this distance the brightness of a bulge is usually negligible or at least is not overwhelming (some exceptions may exist however among the early type disky galaxies $Sa$–$S0$). For our purpose we took the data obtained for spiral and lenticular galaxies from the papers listed below:
Figure 2. A comparison of velocity dispersion at $r \approx 2R_0$ with the velocity of rotation of galaxy. Filled circles are for the observed velocity dispersion $c_{\text{obs}}$, open circles — for the estimates of radial component $c_r$. Error bars are given for $c_{\text{obs}}$ only.

- Shapiro et al (2003): NGC 1068, NGC 2460, NGC 2775, NGC 4030;
- Bershady et al (2002): NGC 3982;
- Bottema (1993): NGC 1566, NGC 2613, NGC 3198, NGC 5247, NGC 6340, NGC 6503;
- Heraudeau et al. (1999): IC 750;
- Beltran et al., (2001): NGC 470, NGC 4419, NGC 7782;
- Simien, Prugniel (2000, 2002): NGC 2962, NGC 3630, NGC 4143, NGC 4203, NGC 4578, NGC 5273;
- Neistein et al. (1999): NGC 584, NGC 2549, NGC 2768, NGC 3489, NGC 4251, NGC 4649; NGC 4753, NGC 5866.

The radial scalelengths $R_0$ (reduced to $H_0 = 75$ km/s/Mpc, if necessary) were taken from the cited papers or, if they are absent there, from Baggett, 1998 or Grosbol, 1985. For our Galaxy, which was added to the list above, we came from $c_r = 38$ km/s in the solar vicinity (Dehnen, Binney, 1998), at radius $r_\odot \simeq 2.7 R_0$. In most cases the estimations of the line-of-sight velocity dispersion $c_{\text{obs}}$ are related to the major axis of a galaxy. The radial velocity dispersion $c_r$ was calculated from $c_{\text{obs}}$ using the equation

$$c_r = c_{\text{obs}} \cdot \left[ (c_{\phi}/c_r)^2 \sin^2 i + (c_z/c_r)^2 \cos^2 i \right]^{-1/2}.$$  \hfill (2)
It was accepted that $c_ϕ/c_r = α/2Ω$ (epicyclic approximation), and $c_z/c_r \simeq 1/2$. To get $α$ and $Ω$ we used the rotation curves obtained from gas emission lines. These curves were taken either from the original papers (see the papers cited above) or from the references presented in the Catalogue of kinematically resolved data (see HYPERLEDA database). If only the “stellar” rotation curve is available, we applied to it the approximate correction for the asymptotic drift at $r = 2R_0$ (in the original paper by Neistein et al., 2002 the corrected velocities are already given).

Fig. 2 reproduces the estimates of the observed velocity dispersion at $r \simeq 2R_0$ (filled symbols) and the corresponding values of $c_r$ (open symbols) in comparison with the maximal velocity of rotation of galaxies. The correlation between the velocity of rotation and the velocity dispersion is practically absent. Error bars are given only for the observed values. These bars are rather of illustrative nature, being taken by eye from $c_{obs}(r)$ diagrams given in the original papers. They demonstrate rather low accuracy of the estimates, especially for galaxies with low velocity dispersion, for which the errors of $c_{obs}$ and, hence the local density estimates $σ(2R_0)$, in some cases may exceed a factor of two. It means that the results we obtain from these data for individual galaxies may be considered only as rather preliminary ones. Nevertheless it makes a sense to compare them with those expected for the marginally stable disks.

In Fig. 3 we show the local surface densities of the marginally stable disks of chosen galaxies at $r = 2R_0$, calculated for the adopted parameter $Q_T = 1.5$. They are plotted against the corrected values of color.
indices of galaxies, taken from HYPERLEDA database. It is worth to remember that if the disks are overstable, that is if the velocity dispersion of stars exceeds the threshold value for the gravitational stability, these estimates may be considered as an upper bound of the density at a given radius. The candidates to galaxies with the overheated disks are several “red” galaxies (most of them are lenticulars) which stand out in Fig. 3 by their incredibly high marginal surface densities exceeding $400 \, M_\odot/pc^2$, whereas most of the other galaxies have $\sigma(2R_0) \simeq (50 - 200) \, M_\odot/pc^2$. For comparison, in solar neighborhood the column density of the galactic disk does not exceed $60 \, M_\odot/pc^2$ (Khoperskov, Tyurina, 2002).

A similar conclusion about the disk “overheating” of some galaxies follows from the estimates of the ratio of disk masses to total masses of galaxies, which we describe as the indicative mass inside of the sphere with radius $r = 4R_0$:

$$\frac{M_d}{M_t} = \frac{\int_{0}^{4R_0} 2\pi r \sigma(r) \, dr}{4V^2 R_0/G}.$$  \hspace{1cm} (3)

The galaxies of the “red group” mentioned above have unphysical estimates of the ratio $M_d/M_t > 1$, which means that their radial velocity dispersions definitely exceed the marginal values even if to admit that the whole mass of a galaxy contains in a disk. Hence either such galaxies have very “overstable” stellar disks, or the measurements of dispersion were influenced by the light of the bulge, which caused the overestimations of $c_{obs}$. Both versions are possible and in principle the situation has to be analyzed separately for every single galaxy. It is worth noting however that for three of the presumably overheated galaxies (NGC 4251, NGC 4578 and NGC 5273) the existing estimates of the velocity dispersion extend to $r > 2R_0$, reaching the distances $r/R_0 = 3.6, 3.3$ and $3.1$ correspondingly, that is their dispersion is obtained at radii where the influence of a bulge is much lower than at $r = 2R_0$. Although the uncertainty of $Q_T$ becomes more severe there, we may admit that its value still does not exceed 3, as numerical models demonstrate, which allows to obtain the upper limit of the disk mass using $c_{obs}$ at large radial distances. However even in this case the ratio $M_d/M_t$ remains unphysically high ($M_d/M_t = 1.3, 2.0$ and $1.2$ correspondingly for the galaxies in question), that is the conclusion about the “overheating” of their disks is confirmed.

It’s essential that for all galaxies but two early type lenticulars (NGC 2768 and NGC 4203), which do not belong to a group of red galaxies with a high marginal disk density at Fig. 3, the ratio $M_d/M_t < 1$ (Fig. 4). It means that their radial velocity dispersions at $r = 2R_0$,
Figure 4. A histogram of \(M_d\) over \(M_t\) ratio, where \(M_d\) is the mass of a marginally stable disk, \(M_t\) is the indicative total mass of a galaxy within the radius \(r = 4R_0\).

obtained from the observational data, cannot be explained without the presence of massive haloes – independently on whether the disks are marginally stable or overheated. Thence a dark matter, if exists in a galaxy, cannot be concentrated in a disk. A fraction of mass of dark matter in galaxies seems to change significantly from one galaxy to another.

However we should have in mind that \(M_d\) is no more than the crude estimate of the upper limit of the disk mass. It is worth trying to verify whether the real disk mass is close to \(M_d\). If this proposition is correct, one can expect that the mass-to-luminosity ratio for a disk is lower in galaxies with the less evolved stellar population. These galaxies should possess lower color indices due to the presence of young stars.

In Fig. 5 we compare the ratio \(M_d/L_B\) with the corrected color index \((B - V)\) for spiral galaxies of our sample, where the disk gives the main input to a total luminosity (lenticular galaxies were omitted). As the evolutionary models of stellar population show, the ratio \(M_d/L_B\) increases along the sequence of color indices, weakly depending (for a given color) on the history of star formation (Bell, de Jong, 2001). A curve drawn in Fig. 5 is model relationship, taken (without any normalization) from Bell, de Jong 2001, which was obtained for a modified Salpeter Initial Mass Function and presented in an analytical form (see their Table 3 for a “closed box” model). Two reddest galaxies are IC 750 (Sab) and NGC 2962 (S0a), the other galaxies belong to later types. In spite of the significant spread of points (which is not surprising due to the crudeness of the estimates), a general agreement between the model and the observed dependencies is evident: masses of the disks estimated under proposition of their marginal stability are close to
those expected for galaxies with the observed luminosity and colour. In enables to propose that in most spiral galaxies a radial velocity dispersions of old stars in the disks (at least for $r \simeq 2R_0$) is really close to the minimum values necessary for the disks to be stable. If this is the case, one may conclude that the mechanisms of dynamical “heating” were not too efficient for the late-type galaxies we considered after they reached a stable state.

This conclusion evidently cannot be applied to the overstable disks of early type galaxies. It is possible that high color indices (low star formation rate) and a low gas content, typical for these galaxies are caused by the same events as the dynamical heating of their stellar disks (like a merging or a capture of small galaxies followed by the fast gas consumption).

4. CONCLUSION

- Numerical modeling of 3D stellar disks evolving from the subcritical (to gravitational instability) to stationary state allows to find the minimal value of local parameter Toomre $Q_T \simeq 1.2 - 1.6$ which reaches at the radial distance $r \simeq (1 - 2) \cdot R_0$ for a wide ranges of masses and radial scales of bulges, disks and halos of model galaxies.

- The observed stellar velocity dispersion in the disks of spiral galaxies well agrees with the proposition that the radial velocity disper-
Disk-to-halo mass ratio

The observed velocity dispersion in the disks of most of spiral galaxies we considered makes the presence of a dark halo unavoidable for the disks to be gravitationally stable. Dark matter in a galaxy, if exists, cannot be entirely concentrated in a disk.

Galaxies with the high color index \((B - V)\) (most of them are lenticular galaxies) may possess strongly “overstable” disks, with the radial velocity dispersion exceeding the threshold level for gravitational instability. Hence the latter cannot be responsible for the observed velocity dispersion.

The detailed discussion may be found in Zasov et al., AstL, 2004 (to be published).

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