CHARM PHYSICS: HINTS FOR A MATURE DESCRIPTION
OF HADRONS

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Abstract

The physics of charm has become one of the best laboratories exposing the
limitations of the naive constituent quark model and also giving hints into
a more mature description of hadron spectroscopy. Recent discoveries are a
challenge that have revolutionized our understanding of the hadron spectra.
In this talk we address the study of many-quark components in charmonium
spectra. To make the physics clear we also discuss exotic many-quark systems.

More than thirty years after the so-called November revolution[1], heavy meson
spectroscopy is being again a challenge. The formerly comfortable world
of heavy meson spectroscopy is being severely tested by new experiments[2].
This challenging situation arose in the open-charm sector with the discovery of
the \( D_{sJ}^*(2317) \), the \( D_{sJ}(2460) \) and the \( D_0^*(2308) \) mesons. All of them are positive parity states with masses smaller than expectations from quark potential
models, and in the first two cases also smaller widths. In general, one could say
that the area phenomenologically understood in the open-charm meson spec-
trum extends to states where the $q\bar{q}$ pair is in relative $S-$wave. In the positive
parity sector, $P-$wave states, is where the problems arise. This has been said
as an example where naive quark models are probably too naive\(^\text{[3]}\). Out of
the many explanations suggested for these states, the unquenching of the naive
quark model has been successful\(^\text{[4]}\). When a $q\bar{q}$ pair occurs in a $P-$wave but
can couple to hadron pairs in $S-$wave the latter will distort the $q\bar{q}$ picture. In
the examples mentioned above, the $0^+$ and $1^+ c\bar{s}$ states predicted above the
$DK(D^* K)$ thresholds couple to the continuum. This mixes $DK(D^* K)$ com-
ponents in the wave function. This idea can be easily formulated in terms of a
meson wave-function described by

$$|\psi\rangle = \sum_i \alpha_i |q\bar{q}\rangle_i + \sum_j \beta_j |qq\bar{q}\rangle_j$$

(1)

where $q$ stands for quark degrees of freedom and the coefficients $\alpha_i$ and $\beta_j$
take into account the possible admixture of four-quark components in the standard
$q\bar{q}$ picture.

This explanation has open the discussion about the presence of compact
four-quark states in charmonium spectroscopy. This is an old idea long ago
advocated to explain the proliferation of light-scalar mesons\(^\text{[5]}\). In the case of
charmonium spectroscopy, some members of the new hadronic zoo may fit in
the simple quark model description as $q\bar{q}$ pairs ($X(3940)$, $Y(3940)$, and $Z(3940)$
may fit into the $\chi_{c0}$, $\chi_{c1}$, and $\chi_{c2}$ quark model structure) others appear to be
more elusive ($X(3872)$ and $Y(4260)$).

The debate has been open with special emphasis on the nature of the
$X(3872)$. Since it was first reported by Belle in 2003\(^\text{[6]}\) it has gradually
become the flagship of a new armada of states whose properties make their
identification as traditional $q\bar{q}$ states unlikely. In this heterogeneous group we
could include states like the $Y(2460)$ reported by $BABAR$, and the aforemen-
tioned $D_{sJ}(2317)$ and $D_{sJ}(2460)$ reported by $BABAR$ and CLEO. An aver-
age mass of $3871.2\pm0.5$ MeV and a narrow width of less than $2.3$ MeV have
been reported for the $X(3872)$. Note the vicinity of this state to the $D^0\overline{D}^{*0}$
threshold, $M(D^0\overline{D}^{*0}) = 3871.2\pm1.2$ MeV. With respect to the $X(3872)$ quan-
tum numbers, neither $D_0$ nor $BABAR$ have been able to offer a clear predic-
tion. Its isovector nature has been excluded by $BABAR$ due to the negative
Table 1: $c\bar{c}n\bar{n}$ results.

| $J^{PC}(K_{\text{max}})$ | CQC | BCN |
|--------------------------|-----|-----|
| $0^{++}$ (24)            | 3779 +34 | 3249 +75 |
| $0^{+-}$ (22)            | 4224 +64 | 3778 +140 |
| $1^{++}$ (20)            | 3786 +41 | 3808 +153 |
| $1^{+-}$ (22)            | 3728 +45 | 3319 +86  |
| $2^{++}$ (26)            | 3774 +29 | 3897 +23  |
| $2^{+-}$ (28)            | 4214 +54 | 4328 +32  |
| $1^{--}$ (19)            | 3829 +84 | 3331 +157 |
| $1^{--}$ (19)            | 3969 +97 | 3732 +94  |
| $0^{-+}$ (17)            | 3839 +94 | 3760 +105 |
| $0^{--}$ (17)            | 3791 +108| 3405 +172 |
| $2^{++}$ (21)            | 3820 +75 | 3929 +55  |
| $2^{+-}$ (21)            | 4054 +52 | 4092 +52  |

results in the search for a charged partner in the decay $B \to X(3872)^-K$, $X(3872)^- \to J/\psi\pi^-\pi^0$. CDF has studied the $X(3872)$ $J^{PC}$ quantum numbers using dipion invariant mass distribution and angular analysis, obtaining that only the assignments $1^{++}$ and $2^{+-}$ are able to describe data. On the other hand, recent studies by Belle combining angular and kinematic properties of the $\pi^+\pi^-$ invariant mass strongly favor a $J^{PC} = 1^{++}$ state, and the observation of the $X(3872) \to D^0\bar{D}^0\pi^0$ also prefers the $1^{++}$ assignment compared to the $2^{+-}$. Therefore, although some caution is still required until better statistic is obtained, an isoscalar $J^{PC} = 1^{++}$ state seems to be the best candidate to describe the properties of the $X(3872)$.

To study the possible existence of four-quark states in the charmonium spectrum we have solved exactly the four-body Schrödinger equation using the hyperspherical harmonic (HH) formalism. We have used two standard quark-quark interaction models: a potential containing a linear confinement and a Fermi-Breit one-gluon exchange interaction (BCN), and a potential containing besides boson exchanges between the light quarks (CQC). The model parameters have been tuned in the meson and baryon spectra. To make the physics clear we have solved simultaneously two different type of systems: the cryptoexotic $c\bar{c}n\bar{n}$ and the flavor exotic $cc\bar{n}\bar{n}$, where $n$ stands for a light $u$ or $d$ quark. The results are reported in Tables 1 and 2 indicating the quantum...
numbers of the state studied, \(J^P\), the maximum value of the grand angular momentum used in the HH expansion, \(K_{\text{max}}\), and the energy difference between the mass of the four-quark state, \(E_{4q}\), and that of the lowest two-meson threshold calculated with the same potential model, \(\Delta E\). For the \(cc\bar{n}\bar{n}\) system we have also calculated the radius of the four-quark state, \(R_{4q}\), and its ratio to the sum of the radii of the lowest two-meson threshold, \(R_{4q}/(r_{2q}^1 + r_{2q}^2)\). As can be seen in Table 1, in the case of the \(cc\bar{n}\bar{n}\) there appear no bound states for any set of quantum numbers, including the suggested assignments of the \(X(3872): 1^{++}\) and \(2^{-+}\). The situation is different for the \(cc\bar{n}\) where we observe the existence of bound states. It is particularly interesting the \(J^P = 1^{-}\) channel, that it is bound both with the CQC and the BCN models. For the \(cc\bar{n}\bar{n}\) system, independently of the quark-quark interaction and the quantum numbers considered, the system evolves to a well separated two-meson state. This is clearly seen in the energy, approaching the corresponding two free-meson threshold, and also in the probabilities of the different color components of the wave function and in the radius. We illustrate the convergence plotting in Fig. 1 the energy of the \(J^{PC} = 1^{++}\) state as a function of \(K\). It can be observed how the BCN \(1^{++}\) state does not converge to the lowest threshold for small values of \(K\), being affected by the presence of an intermediate \(J/\psi\omega\) threshold with an energy of 3874 MeV. Once sufficiently large values of \(K\) are considered

Table 2: \(cc\bar{n}\bar{n}\) results.

| \(I=0\) | \(J^P(K_{\text{max}})\) | \(E_{4q}\) | \(\Delta E\) | \(R_{4q}\) | \(R_{4q}/(r_{2q}^1 + r_{2q}^2)\) |
|---------|-----------------|----------|-------------|---------|----------------|
| 0\(^+\) (28) | 4441 | +15 | 0.624 | > 1 |
| 1\(^+\) (24) | 3861 | -76 | 0.367 | 0.808 |
| 2\(^+\) (30) | 4526 | +27 | 0.987 | > 1 |
| 0\(^-\) (21) | 3996 | +59 | 0.739 | > 1 |
| 1\(^-\) (21) | 3938 | +66 | 0.726 | > 1 |
| 2\(^-\) (21) | 4052 | +50 | 0.817 | > 1 |

| \(I=1\) | \(J^P(K_{\text{max}})\) | \(E_{4q}\) | \(\Delta E\) | \(R_{4q}\) | \(R_{4q}/(r_{2q}^1 + r_{2q}^2)\) |
|---------|-----------------|----------|-------------|---------|----------------|
| 0\(^+\) (28) | 3905 | +50 | 0.817 | > 1 |
| 1\(^+\) (24) | 3972 | +33 | 0.752 | > 1 |
| 2\(^+\) (30) | 4025 | +22 | 0.879 | > 1 |
| 0\(^-\) (21) | 4004 | +67 | 0.814 | > 1 |
| 1\(^-\) (21) | 4427 | +1 | 0.516 | 0.876 |
| 2\(^-\) (21) | 4461 | -38 | 0.465 | 0.766 |
the system follows the usual convergence to the lowest threshold (see insert in Fig. 1). The dashed line of Fig. 2 illustrates how the system evolves to two singlet color mesons, whose separation increases with $K$. Thus, in any manner one can claim for the existence of a bound state for the $c\bar{c}n\bar{n}$ system.

A completely different behavior is observed in Table 2. Here, there are some particular quantum numbers where the energy is quickly stabilized below the theoretical threshold. For example, the solid line in Fig. 2 illustrates how the radius of the $1^+ c\bar{c}n\bar{n}$ state is stable, and it is smaller than the sum of the radius of the two-meson threshold. We obtain $r_{4q} = 0.37$ fm compared to $r_{M_1} + r_{M_2} = 0.44$ fm for the $1^+$ state. The analysis of the color components in the wave function is involved in this case. One cannot directly conclude the presence of octet-octet components in the wave function, because the octet-octet color component in the $(c_1\bar{n}_3)(c_2\bar{n}_4)$ basis can be re-expressed as a singlet-singlet color component in the $(c_1\bar{n}_4)(c_2\bar{n}_3)$ coupling, being the same physical system due to the identity of the two quarks and the two antiquarks. The actual interest and the capability of some experiments to detect double charmed
states makes this prediction a primary objective to help in the understanding of QCD dynamics.

There is an important difference between the two physical systems studied. While for the $c\bar{c}n\bar{n}$ there are two allowed physical decay channels, $(c\bar{c})(n\bar{n})$ and $(c\bar{n})(\bar{c}n)$, for the $cc\bar{n}\bar{n}$ only one physical system contains the possible final states, $(c\bar{n})(c\bar{n})$. This has important consequences if both systems (two- and four-quark states) are described within the same two-body Hamiltonian, the $c\bar{c}n\bar{n}$ will hardly present bound states, because the system will reorder itself to become the lightest two-meson state, either $(c\bar{c})(n\bar{n})$ or $(c\bar{n})(\bar{c}n)$. In other words, if the attraction is provided by the interaction between particles $i$ and $j$, it does also contribute to the asymptotic two-meson state. This does not happen for the $cc\bar{n}\bar{n}$ if the interaction between, for example, the two quarks is strongly attractive. In this case there is no asymptotic two-meson state including such attraction, and therefore the system will bind.

Once all possible quantum numbers of the $X(3872)$ have been analyzed and discarded very few alternatives remain. If this state is experimentally proved to be a compact four-quark state this will point either to the existence of non two-body forces or to the emergence of strongly bound diquark structures.
within the tetraquark. Both possibilities are appealing, does the interaction becomes more involved with the number of quark or does the Hilbert space becomes simpler? On the one hand, some lattice QCD collaborations\(^{13}\) have reported the important role played by three- and four-quark interactions within the confinement (the $Y^-$ and $H$-shape). On the other hand, diquark correlations have been proposed to play a relevant role in several aspects of QCD, from baryon spectroscopy to scaling violation\(^{14}\). The spontaneous formation of diquark components can be checked within our formalism. The four-quark state can be explicitly written in the $(cn)(\bar{c}\bar{n})$ coupling to isolate the diquark-antidiquark configurations. In the case of $J^{PC} = 1^{++}$ only two components of the wave function have the proper quantum numbers to be identified with a diquark, being their total probability less than $3\%$. Therefore, it is clear that without any further hypothesis two-body potentials do not favor the presence of diquarks and any description of these states in terms of diquark-antidiquark components would be selecting a restricted Hilbert space.

Finally, our conclusions can be made more general. If we have an $N$-quark system described by two-body interactions in such a way that there exists a subset of quarks that cannot make up a physical subsystem, then one may expect the existence of $N$-quark bound states by means of central two-body potentials. If this is not true one will hardly find $N$-quark bound states\(^{15}\). For the particular case of the tetraquarks, this conclusion is exact if the confinement is described by the first $SU(3)$ Casimir operator, because when the system is split into two-mesons the confining contribution from the two isolated mesons is the same as in the four-quark system. The contribution of three-body color forces\(^{16}\) would interfere in the simple comparison of the asymptotic and the compact states. Another possibility in the same line would be a modification of the Hilbert space. If for some reason particular components of the four-quark system (diquarks) would be favored against others, the system could be compact\(^{17}\). Lattice QCD calculations\(^{18}\) confirm the phenomenological expectation that QCD dynamics favors the formation of good diquarks\(^{15}\), i.e., in the scalar positive parity channel. However, they are large objects whose relevance to hadron structure is still under study. All these alternatives will allow to manage the four-quark system without affecting the threshold and thus they may allow to generate any solution.

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