An atom diode

A. Ruschhaupt and J. G. Muga
Departamento de Química-Física, Universidad del País Vasco, Apdo. 644, Bilbao, Spain

An atom diode, i.e., a device that lets the ground state atom pass in one direction but not in the opposite direction in a velocity range is devised. It is based on the adiabatic transfer achieved with two lasers and a third laser potential that reflects the ground state.

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The detailed control of internal and/or translational atomic states is a major goal of quantum optics. Optical elements in which the roles of light and matter are reversed such as mirrors, gratings, interferometers, or beam splitters made of laser light or magnetic fields allow to manipulate atomic waves. Further handling of the atoms is inspired in electronic devices and integrated circuits: atom chips and atom-optic circuits have been realized recently. The aim of this letter is to propose simple models for an “atom diode”, a device built with laser light that lets the neutral atom in its ground state pass in one direction but not in the opposite direction for a range of incident velocities. A diode is a very basic circuit element and many applications are feasible in atomic trapping, or logic gates for quantum information processing.

More specifically our goal is to model an atom-field interaction so that the ground state atom is transmitted when traveling, say, from left to right, and it is reflected if coming from the right. We shall describe (effective) three-level and two-level atom models, for simplicity in one dimension, to achieve the desired behaviour. In both cases the atom is in an excited state after being transmitted and, in principle, excited atoms could cross the channel “backwards”, i.e., from right to left. Nevertheless, an irreversible decay from the excited state to the ground state, would effectively block any backward motion.

Let us denote by $R_{\beta\alpha}^{l}(v)$ ($R_{\beta\alpha}^{r}(v)$) the scattering amplitudes for incidence with (modulus of) velocity $v$ from the left (right) in channel $\alpha$ and reflection in channel $\beta$. Similarly we denote by $T_{\beta\alpha}^{l}(v)$ ($T_{\beta\alpha}^{r}(v)$) the scattering amplitude for incidence in channel $\alpha$ with velocity $v$ from the left (right) and transmission in channel $\beta$ to the right (left). We define, for incidence in the ground state,

$$\hat{R}(v) = \begin{cases} R_{11}^{l}(v) & : v > 0 \\ R_{11}^{r}(-v) & : v < 0 \end{cases}, \hat{T}(v) = \begin{cases} T_{31}^{l}(v) & : v > 0 \\ T_{31}^{r}(-v) & : v < 0 \end{cases}$$

The potential will be such that $|\hat{T}(v)|^2 \approx 1$, $|\hat{R}(v)|^2 \approx 0$ and $|\hat{T}(-v)|^2 \approx 0$, $|\hat{R}(-v)|^2 \approx 1$ ($v > 0$). The basic idea is to combine two lasers that achieve STIRAP (stimulated Raman adiabatic passage) with an additional reflecting interaction for the ground state. The STIRAP method is well known and consists of an adiabatic transfer of population between levels 1 and 3 by two partially overlapping (in time or space) laser beams, see Fig. 1. The pump laser couples the atomic levels 1 and 2 with Rabi frequency $\Omega_p$, and the Stokes laser couples the states 2 and 3 with Rabi frequency $\Omega_s$. We assume here that these two lasers are on resonance with the corresponding transitions. We shall need in addition a third laser causing an effective reflecting potential $V$ for the ground state component. It could be realized by an intense laser with a large positive detuning $\Delta$ (laser frequency minus the transition frequency) with respect to a transition with a fourth level, $V(x) = W(x)\hbar/2 = \Omega_{14}(x)^2\hbar/4\Delta$, $\Omega_{14}$ being the corresponding Rabi frequency. Due to the large detuning, there is no pumping so that this type of coupling has a purely mechanical effect. Neglecting decay, the resulting Hamiltonian for the atomic state, within the rotating wave approximation, and in the appropriate interaction picture to get rid of any time dependence, is

$$H_{3L} = \frac{\hbar^2}{2m} + \frac{\hbar}{2} \begin{pmatrix} W(x) & \Omega_P(x) & 0 \\ \Omega_P(x) & 0 & \Omega_S(x) \\ 0 & \Omega_S(x) & 0 \end{pmatrix},$$

where $p_x = i\hbar \frac{\partial}{\partial x}$ is the momentum operator. The shapes of the Rabi frequencies and the reflecting potential in the model are Gaussian, $\Omega_P(x) = \Omega (\Pi(x, x_P), \Omega_S(x) = \Omega (\Pi(x, x_S))$, $W(x) = \hat{W} (\Pi(x, x_W))$ with

$$\Pi(x, x_0) = \exp \left( -\frac{(x-x_0)^2}{2\Delta x^2} \right),$$

but similar shapes do not alter the results in any significant way. We shall also assume for simplicity that the shapes and widths of pump laser, Stokes laser and additional potential are equal. The location of the three laser beams is shown in Fig. 1.
FIG. 2: (a) Reflection probability \( |\hat{R}(v)|^2 \) and (b) transmission probability \( |\hat{T}(v)|^2 \); the mass is the mass of Neon, \( \Delta x = 15 \mu m \), \( x_S = 140 \mu m \), \( x_P = 170 \mu m \); three level atom: \( x_W = 260 \mu m \), \( \Omega = 0.2 \times 10^6 s^{-1} \), \( W = 20 \times 10^6 s^{-1} \) (thin dashed line), \( \hat{\Omega} = 1 \times 10^6 s^{-1} \), \( \hat{W} = 100 \times 10^6 s^{-1} \) (thick dashed line); two level atom: \( \hat{2} = 100 \times 10^6 s^{-1} \) (solid line, coincides with thick dashed line).

If the atom is incident from the left in the ground state, it will be transferred by STIRAP to the third state so it is not affected by \( V(x) \), and will be transmitted, i.e. the transmission probability \( |\hat{T}(v)|^2 \approx 1 \), while the other reflection and transmission probabilities for left incidence in the first state will be approximately zero. If the atom is incident from the right in the ground state, it is reflected by the (high enough) potential \( V \). Therefore \( |\hat{T}(-v)|^2 \approx 0 \neq |\hat{T}(v)|^2 \) and \( |\hat{R}(-v)|^2 \approx 1 \) \((v > 0)\). The other reflection and transmission probabilities will be also approximately zero.

This behavior is indeed observed solving numerically the stationary Schrödinger equation with Eq. (1) by the invariant imbedding method [4].

The results are shown in Fig. 2. In a velocity range, the “diodic” behaviour holds, i.e. \( |\hat{R}(v)|^2 \approx 0 \), \( |\hat{T}(v)|^2 \approx 1 \) and \( |\hat{R}(-v)|^2 \approx 1 \), \( |\hat{T}(-v)|^2 \approx 0 \) \((v > 0)\). In this range the other transmission and reflection coefficients for incidence in the first state are zero. The upper velocity boundary, \( v_{\text{upper}} \), for the diode with incidence from the left is due to the breakdown of the STIRAP effect [4] (A spontaneous decay rate \( \Gamma \) from state 2 to state 1 does not alter \( v_{\text{upper}} \) significantly for \( \Omega/\Gamma \gtrsim 100 \)). This boundary can be increased by increasing \( \Omega \). The lower (negative) velocity boundary \( v_{\text{lower}} \) for right incidence, due to the inability of the reflecting laser to block fast atoms, decreases when \( \hat{W} \) increases, so that both boundaries can be adjusted independently from each other. We may define \( v_{\text{max}} > 0 \) as the minimum of \( v_{\text{upper}} \) and \( |v_{\text{lower}}| \). More precisely, it is defined by imposing that all scattering probabilities from the ground state be small except the ones that define the diode (i.e., the probability for transmission to 3 from the left and for reflection to 1 from the right), \( \sum_{\alpha=1}^{3}(|R_{\alpha 1}|^2 + |T_{\alpha 1}|^2) + \sum_{\alpha=1}^{2}(|R_{\alpha+1,1}|^2 + |T_{\alpha+1,1}|^2) + (1-|T_{\alpha 1}|^2) + (1-|R_{\alpha 1}|^2) < \epsilon \) for all \( v_{\text{min}} \leq v \leq v_{\text{max}} \) with \( v_{\text{min}} \approx 0.25 \text{ cm/s} \). In Fig. 3, \( v_{\text{max}} \) is plotted versus \( \Omega \) and \( \hat{W} \). For the intensities considered, \( v_{\text{max}} \) is in the ultracold regime below 1 m/s. In the \( |v_{\text{lower}}| \) surface, \( |v_{\text{lower}}| \) due to reflection failure is more restrictive in the hillside represented by circles, whereas \( v_{\text{upper}} \), due to STIRAP failure, is more restrictive in the hillside with triangles. Considering the scales used for \( \Omega \) and \( \hat{W} \), reflection failure is in general more problematic than STIRAP failure.

There is also a lower, positive-velocity boundary for the STIRAP effect, i.e. the STIRAP effect breaks down at extremely low velocities, \( 0 < v \ll v_{\text{min}} \), with the laser intensities (Rabi frequencies) of the numerical example. This may appear contradictory since one expects better adiabatic transfer at lower velocities. Indeed this is the case, but only as long as the semiclassical approximation is valid for the translational motion. For sufficiently low velocities the quantum aspects of translational motion become important and atomic reflection occurs.

Notice that the diode behaviour can also be obtained for a two level atom. It is well known [2] that the three level Hamiltonian [11] with \( W = 0 \) can be reformulated.
in the form of a two-level one, but here we use a different idea to construct directly a two-level potential with the “diodic” property. Assume first that we can neglect the kinetic term and that the motion in x direction is classical. Let us define the two position dependent eigenvectors of the two level potential $V'$ to be

$$
\zeta_1(x) = \frac{1}{\sqrt{f_p^2(x) + f_s^2(x)}} \begin{pmatrix} f_s(x) \\ -f_p(x) \end{pmatrix},
$$

$$
\zeta_2(x) = \frac{1}{\sqrt{f_p^2(x) + f_s^2(x)}} \begin{pmatrix} f_p(x) \\ f_s(x) \end{pmatrix}.
$$

With the order of $f_s, f_p \geq 0$ shown in Fig. 4 we get for Gaussian (or similar) functions $f_s$ and $f_p$ the asymptotic properties

$$
\zeta_1(-\infty) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \zeta_1(+\infty) = \begin{pmatrix} 0 \\ -1 \end{pmatrix},
$$

$$
\zeta_2(-\infty) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \zeta_2(+\infty) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.
$$

This means that ground and excited state are asymptotically swapped. $\zeta_1$ should correspond to the eigenvalue $\lambda_1 = 0$ which results in adiabatic transfer from ground to excited state if the atom impinges from the left, and $\zeta_2$ should correspond to $\lambda_2 = (\hbar/2)(f_s^2(x) + f_p^2(x)) > 0$, so there will be nearly full reflection if the atom impinges from the right. The eigenfunctions and eigenvalues define $V'$ and the two-level Hamiltonian is

$$
H_{2L} = \frac{p^2}{2m} + \frac{\hbar}{2} \left( f_p^2(x) f_p(x) f_s(x) f_s(x) / f_s^2(x) \right).
$$

(2)

We have calculated the scattering amplitudes numerically with $f_p(x) = \tilde{f} \Pi(x, x_p)$ and $f_s(x) = \tilde{f} \Pi(x, x_S)$ for right and left incidence and observed the diodic behaviour, see Fig. 2. The two-level Hamiltonian can be also used as a diode for incidence in the excited state. Then it works in the opposite direction, i.e. $|T_{13}(v)|^2 \approx 1$, $|R_{13}(v)|^2 \approx 0$ and $|T_{31}(v)|^2 \approx 0$. This is not the case for the Hamiltonian 1 unless an additional potential acting on the third level is added.

Let us return to the three-level atom to study the possible effect of decay from the third state to the first state with a relatively small decay rate $\gamma$. This is unlikely a spontaneous process but it can be forced by a laser coupling of the third state to an auxiliary state decaying to the ground state. The process may be characterized by an effective decay rate from 3 to 1 2. We examine the time-dependent case, see Fig. 3 by means of a one-dimensional master equation which includes the effect of recoil (see 3)

$$
\frac{\partial}{\partial t} \rho = -\frac{i}{\hbar} [H_{3L}, \rho] - \frac{\gamma}{2} \{ |3 \rangle \langle 3| , \rho \} + \gamma \int_{-1}^{1} du \frac{3}{8} (1 + u^2) \exp \left( i \frac{mv_{rec}}{\hbar} ux \right) |1 \rangle \langle 1| \exp \left( -i \frac{mv_{rec}}{\hbar} ux \right).
$$

(3)

The initial state at $t = 0$ is $\rho(0) = |\Psi_0 > < \Psi_0|$, namely a Gaussian wave packet with mean velocity $v_0$,

$$
\Psi_0(x) = \frac{1}{N} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \exp \left( -\frac{\Delta v_0 m}{2\hbar} (x - x_0)^2 + i \frac{v_0 m}{\hbar} x \right),
$$

where $N$ is a normalization constant. We solve the master equation by using the quantum jump technique 10. Let $t_{max}$ be a sufficient large time such that the
resulting wave packet $\Psi_j(t_{\text{max}})$ of nearly every quantum “trajectory” $j$ separates in right and left moving parts far from the interaction region but possibly with third state components (not decayed yet at $t_{\text{max}}$). By averaging over all trajectories we get

$$\hat{p}_r = \int_0^\infty dv \left( \langle v | \rho_{11}(t_{\text{max}}) | v \rangle + \langle v | \rho_{33}(t_{\text{max}}) | v \rangle \right)$$

which is plotted in Fig. 6 as a function of $v_0$ for different $\gamma$ and $v_{\text{rec}}$. The error bars, defined by the absolute difference between averaging over $n/2$ and $n$ trajectories, are smaller than the symbol size.

A value $\hat{p}_r(v_0) \approx 1$ for $v_0 < 0$ means that nearly all atoms coming from the right are reflected. The reflection probability is not affected by the decay since the reflected atoms are rarely excited during the collision.

A value $\hat{p}_r(v_0) \approx 1$ for $v_0 > 0$ means that nearly all atoms coming from the left are transmitted and will be finally in the ground state moving to the right. This is true for $v_0 \geq 8 \text{ cm/s}$ (with $\hat{p}_r(v_0) \geq 0.95$) for all examined combinations of decay rate $\gamma$ and recoil velocity $v_{\text{rec}}$. Therefore for not too low velocities a large part of theatoms will be transmitted and stay finally in the ground state, i.e. the atom diode works also with decay and recoil, with the advantage that decay prevents the backward motion of excited atoms. The decrease of $\hat{p}_r$ for low, positive velocities is due to the atom decay before passing the potential $W(x)\hbar/2$.

Summarizing, we have presented a simple model for an atom diode that can be realized with laser interactions, a device which can be passed by the atom in one direction but not in the opposite direction.

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