Procedure with Massive Neutrinos for the Standard Model Processes with Negligible Lorentz Invariance Violation

Josip Šoln
Army Research Laboratory (ret.), JZS Phys-Tech Vienna, Virginia 22182, USA
soln.phystech@cox.net

January 26, 2009

Abstract
For the electroweak interactions, the massive neutrino perturbative kinematical procedure is developed in the massive neutrino Fock space; The perturbation expansion parameter is the ratio of neutrino mass to its energy. This procedure, within the Pontecorvo-Maki-Nakagawa-Sakata modified electroweak Lagrangian, calculates the cross-sections with the new neutrino energy projection operators in the massive neutrino Fock space, resulting in the Standard Model mass-less flavor neutrino cross-sections, plus the Lorentz non-invariant neutrino oscillation cross-sections which are proportional to the squares of neutrino masses and, as such, practically unobservable in the laboratory. This scheme reinforces the notion that the mass-less flavor neutrino can be considered as the superposition of three massive neutrinos.

Introduction
The flavor changing neutrino oscillations experiments, such as, The Super-Kamiokande [1], SNO [2], KAMLAND [3] as well as Homestake Collaboration [4], clearly require massive neutrinos as have been exhibited, for example, by Bilenky, Giunti and Grimus [5], Giunti and Laveder [6] and Kayser [7]. In discussing the neutrino oscillations, one assumes that the left-handed flavor mass-less neutrino fields \( \nu_{\alpha L} \), with \( \alpha = e, \mu, \tau \), are unitary linear combinations of the massive neutrino fields \( \nu_{iL} \) and analogously for the states (see [5-10]) and references therein,

\[
\nu_{\alpha L} = U_{\alpha i} \nu_{iL}, \quad |\nu_{\alpha}\rangle = U_{\alpha i}^\dagger |\nu_i\rangle \quad (i = 1, 2, 3; \alpha = e, \mu, \tau)
\]  

(1,2)
$U$ is the unitary Pontecorvo-Maki-Nakagawa-Sakat (PMNS) matrix and $\nu_{iL}$ is a left-handed neutrino field associated with mass $m_i$ (see, for example [5-11]). In the study of flavor neutrino oscillations, the flavor state from (2) is used, via the Schrödinger equation, to calculate the oscillation probability $P_{\alpha\beta}(L)$ for the flavor neutrino oscillation transition $\nu_\alpha \rightarrow \nu_\beta$ at a very large distance $L$ (\~{} thousands of km) (see, for example [5-11]).

However, it is well known that the standard model (SM) with mass-less flavor neutrinos has been remarkably successful in describing the laboratory experimental data, such as the neutrino scattering, at low and medium energies (see, for instance, M. Fukugita and T. Yanagida [8]; and C. Giunti and C. W. Kim [9]). So one can then ask whether the SM cross-sections can be derived when starting, instead with the mass-less flavor neutrino fields $\nu_{\alpha L}$, with the massive neutrino fields $\nu_{iL}$. In this article the answer to this question is in affirmative.

To proceed in this direction, as suggested by (1), the application of the PMNS substitution rule (3) transforms the SM Lagrangian density with the mass-less neutrino fields into the one with the massive neutrino fields (4):

The PMNS substitution: $\nu_{\alpha L} \rightarrow U_{\alpha i} \nu_{iL}$

(3)

$$
\begin{align*}
\alpha, \beta, \ldots, \epsilon &= e, \mu, \tau; i, j, a, \ldots, b = 1, 2, 3 : \\
l_{\alpha L} &= \left( U_{\alpha i} \nu_{iL} \right) ; \epsilon_{L,R} = P_{L,R} \epsilon, P_{L,R} = \frac{1}{2} \left( 1 \mp \gamma^5 \right) \\
L_{\text{Lepton}}^{W_{\text{int}}} &= \frac{g}{2\sqrt{2}} \sum_{\epsilon=e,\mu,\tau, i=1,2,3} \left[ \overline{\nu}_{\epsilon L}(x) U_{\epsilon i}^\dagger \gamma^\mu \epsilon_L(x) W^\mu_{\epsilon}(x, +) \\
+ \overline{\nu}_L(x) \gamma^\mu U_{\gamma j} \nu_{jL}(x) W^\mu_{\gamma}(x, +) \right], \\
W^\mu(x, \pm) &= \frac{1}{\sqrt{2}} \left[ W^\mu(x, 1) \pm i W^\mu(x, 2) \right] \\
L_{\text{Lepton}}^{Z_{\text{int}}} &= \frac{g}{c_W} \sum_{\epsilon=e,\mu,\tau} \left[ \overline{\nu}_{\epsilon L}(x) \frac{s_W^2}{2} \gamma^\mu \nu_{\epsilon L}(x) - s_W^2 (-1) \overline{\nu}(x) \gamma^\mu \epsilon(x) \right] \\
&= \frac{g}{4c_W} \left[ \overline{\nu}_{\epsilon L}(x) U_{\epsilon i}^\dagger \gamma^\mu \left( 1 - \gamma^5 \right) U_{\epsilon b} \nu_{bL}(x) \\
+ \overline{\nu}(x) \gamma^\mu \left( 4s_W^2 - 1 \right) \epsilon(x) \right], \\
s_W &= \sin \theta_W, c_W = \cos \theta_W
\end{align*}
$$

Since the Lagrangian densities (4) contains the massive neutrino fields, all the calculations are now done formally in the massive neutrino Fock space. The mass-less neutrinos will be the mass state neutrinos in the limit of negligible masses as a result of the perturbative neutrino kinematical procedure.

Perturbative kinematical procedure for calculating the neutrino differential cross-sections
A free neutrino spinor field with the mass \( m_i \), \( i = 1, 2, 3 \), is written generally with the creation and annihilation operators as

\[
\nu_i(x) = \frac{1}{(2\pi)^\frac{3}{2}} \int \frac{d^3q}{q^0} \sum_s e^{iqx} \hat{u}(q,s)a(q,s) + e^{-iqx} \hat{v}(q,s)b^\dagger(q,s)
\]

\[
\hat{q}^0 = \left( q^2 + m_i^2 \right)^\frac{1}{2}
\]  

(5)

The perturbative kinematics is based on the fact that the neutrino mass \( m_i(m_i(1eV)) \) is generally much smaller than its absolute momentum value \( |\vec{q}| \). Therefore it is convenient to start with the “mass-less” four-component neutrino four-momentum \( q_{\gamma}^a \) with fixed flavor parameter \( \gamma \)

\[
q_{\gamma}^a = \left( -\vec{q}_{\gamma}, q_{\gamma}^0 \right); q_{\gamma}^2 = 0, \gamma = e, \mu, \tau
\]  

(6)

Next, one assumes that under this flavor parameter \( \gamma \) are grouped together three massive neutrinos, say, \( \nu_i \) with masses \( m_i ; i = 1, 2, 3 \) then the difference among their energies \( \Delta q_{0}^2 \gamma_{i,\gamma} = \left( q_{0}^2_{i,\gamma} - q_{0}^2_{0,\gamma} \right) \) is considerable much smaller than the quantum-mechanical uncertainty of the energy [12]. As a consequence, in this case with fixed \( \gamma \) it is impossible to distinguish the emission of neutrinos with different masses in the neutrino processes [12]. Hence, the three massive neutrinos, satisfying these quantum mechanical conditions, can be viewed as superposing themselves to form the flavor neutrino \( \nu_\gamma \) [11,12] as depicted by relations (1) and (2). With this in mind, with \( q_{i,\gamma}^a \) as the four-momentum of the massive neutrino with mass \( m_i \), the perturbative kinematics can be presented as

\[
q_{i,\gamma}^a \approx q_{\gamma}^a + q_{0}^a + m_i^2 2q_{\gamma}^2 q_{i,\gamma}^0 \approx q_{\gamma}^a + \frac{m_i^2}{2q_{\gamma}^2} q_{i,\gamma}^2 \simeq -m_i^2
\]  

(7)

In (7) the terms with \( O(m_i^3) \) have been neglected and the fixed parameter \( \gamma \), as already established is the neutrino flavor. Taking these relations into account, within the massive neutrino Fock space the differential cross-sections with flavor neutrinos are calculated. The question, of course is: is the result consistent with the SM?

To continue, in analogy to \( q_{i,\gamma}^a \), one now introduces \( s_{i,\gamma} \) and \( \hat{s}_{i,\gamma} \) to denote respectively, the helicity operators and eigenvalues for \( i = 1, 2, 3 \) massive neutrinos comprising the mass-less flavor neutrino \( \nu_\gamma \); The helicity operator and eigenvalue of the mass-less flavor neutrino \( \nu_\gamma \) are denoted, respectively, as \( \hat{s}_{(\gamma)} \) and \( s_{(\gamma)} \). And, the effects of the massive to mass-less-neutrino kinematical relation (7) on these helicity eigenvalues are simply, what one can call, the ordinary massive to mass-less neutrino helicity relation.
\[
\tilde{s}_{(i,\gamma)} = \vec{q}(i,\gamma) \cdot \vec{\sigma} / |q_{(i,\gamma)}| = \vec{q}(\gamma) \cdot \vec{\sigma} / |q_{(\gamma)}| = \tilde{s}_{(\gamma)},
\]
\[
\tilde{s}_{(\gamma)} = \tilde{s}_{(k,\gamma)} \Rightarrow s_{(i,\gamma)} = s_{(\gamma)} = s_{(k,\gamma)}, \text{etc};
\]
i or \( k, \ldots = 1, 2, 3; \gamma(\text{fixed}) = (e \text{ or } \mu \text{ or } \tau)
\]

As a consequence of (7) and (8), with spinor indices suppressed, the contractions of massive neutrino free-field operators with the massive neutrino states are

\[
\langle 0 | \nu(x,l) | q_{(i,\gamma)}; s_{(i,\gamma)} \rangle = \frac{1}{(2\pi)^{\frac{4}{2}}} e^{i(q_{(i,\gamma)} \cdot x)} \delta_{i,i}(q_{(i,\gamma)}, s_{(i,\gamma)}),
\]
\[
\langle q_{(j,\delta)}; s_{(j,\delta)} | \overline{\nu}(x,k) | 0 \rangle = \frac{1}{(2\pi)^{\frac{4}{2}}} e^{i(q_{(j,\delta)} \cdot x)} \delta_{j,j}(q_{(j,\delta)}, s_{(j,\delta)})
\]

where \( s_{(j,\delta)} \) and \( \delta \) have the same kind interrelationship as \( s_{(i,\gamma)} \) and \( \gamma \) in (8), etc.

Since, as shown in (7) to (9), the superposed three massive neutrinos contain the single flavor designation, either in the initial or final state, say, \( \gamma \) and \( \delta \), the process can be denoted as \( \nu(\gamma) + \alpha(P_1) \rightarrow \nu(\delta) + \beta(P_2) \). From the Lagrangian densities (4) the amplitude and its Hermitian conjugate for the process containing massive neutrinos, are build around these respective flavor designations, \( \gamma \) and \( \delta \), so that the generic amplitudes are given, respectively, as

\[
S_{\text{amp}} \sim \sum_{i,j,\ldots,k,l,\ldots} \delta_4(q_{(i,\gamma)} + P_{(1)} - q_{(j,\delta)} - P_{(2)}) iM_{i,j,\ldots},
\]
\[
S_{\text{amp}}^\dagger \sim \sum_{k,l,\ldots} \delta_4(q_{(k,\gamma)} + P_{(1)} - q_{(l,\delta)} - P_{(2)}) (-i)M_{k,l,\ldots}^\ast
\]

Here, the momenta indicate the actual massive neutrino-lepton scattering and different Latin indices indicate possibilities of summation with the \( U \) matrices which, however, here is not necessary to be explicit. To derive the cross-section, with the help of (7), (8) and (9), one needs

\[
S_{\text{amp}}^\ast S_{\text{amp}} = \sum_{i,j,\ldots,k,l,\ldots} \{ \delta_4^2(q_{(\gamma)} + P_{(1)} - q_{(\delta)} - P_{(2)})
\]
\[+
\]
\[+ \delta_3^2 |q_{(\gamma)}| + \overline{P}_{(1)} - |q_{(\delta)}| - \overline{P}_{(2)} \delta(q_{(\gamma)}^0 + P_{(1)}^0 - q_{(\delta)}^0 - P_{(2)}^0)
\]
\[\times \delta(q_{(\gamma)}^0 + P_{(1)}^0 - q_{(\delta)}^0 - P_{(2)}^0) \frac{1}{2} \left[ m_i^2 + m_j^2 - \frac{m_i^2 + m_j^2}{q_{(\gamma)}^0} \right]
\]
\[+ O(m^4) \} (M_{i,j,\ldots}^\ast(M_{k,l,\ldots}) + O(m^4)
\]

\[= \delta_4^2(q_{(\gamma)} + P_{(1)} - q_{(\delta)} - P_{(2)}) \sum_{i,j,\ldots,k,l,\ldots} (M_{i,j,\ldots})^\ast(M_{k,l,\ldots}) + O(m^4)
\]

4
The final result in (11) is the consequence of general delta function property
\[ \delta(x) \delta'(x) = 0. \]
The terms with \( O(m^4) \), denoting the fourth power of products of variety of \( m_i, m_k, \text{etc.} \), are neglected. It follows that while the Fock space contains the massive neutrino states, the cross-section will utilize the kinematics of massless flavor neutrinos.

Next, one needs the spinor expressions, appearing in (9), to reflect respectively, the kinematical and helicity relations in order to facilitate the cross-section calculations.

\[
\begin{align*}
\text{u}(q_{i(\alpha)}, s_{i(\alpha)}) & = \frac{m_i - q_{i(\alpha)}}{\sqrt{2 \left( m_i + q^0_{i(\alpha)} \right)}} u(m_i, 0, s_{i(\alpha)}), \\
\text{u}(m_i, 0, s_{i(\alpha)}) & = \pm 1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \\
q_{i(\alpha)} & = \gamma_\mu q_{i(\alpha)}; \\
\overline{\text{u}}(q_{i(\alpha)}, s_{i(\alpha)}) & = \overline{\text{u}}(m_i, 0, s_{i(\alpha)}) \frac{m_i - q_{i(\alpha)}}{\sqrt{2 \left( m_i + q^0_{i(\alpha)} \right)}}.
\end{align*}
\]

For a process with \( \gamma \) and \( \delta \) flavor designations, \( \nu(\gamma) + \alpha(P_1) \rightarrow \nu(\beta) + \beta(P_2) \), in cross-section evaluations, one will deal with the neutrino energy projection operator over the positive energy states. Furthermore, rather than averaging over, one simply sums over the massive neutrino helicity degrees of freedom. Consistent with the ordinary neutrino helicity relation (8), the sum is carried over only the equal helicity eigenvalues:

\[
\begin{align*}
s_{i(\alpha)} & = s_{k(\alpha)} = s_{(\alpha)} : \sum_{s_{i(\alpha)}, s_{k(\alpha)}} u(q_{i(\alpha)}, s_{i(\alpha)}) \otimes \overline{\text{u}}(q_{k(\alpha)}, s_{k(\alpha)}) \\
& = \sum_{s_{(\alpha)}} u(q_{i(\alpha)}, s_{(\alpha)}) \otimes \overline{\text{u}}(q_{k(\alpha)}, s_{(\alpha)}) \equiv \frac{1}{2} \left[ q_{i(\alpha)}, q_{k(\alpha)}; +, c \right], \\
\left[ q_{i(\alpha)}, q_{k(\alpha)}; +, c \right] & = \left( m_i - q_{i(\alpha)} \right) (1 + \gamma^0) \left( m_k - q_{k(\alpha)} \right), \\
& \frac{2 \left[ \left( m_i + q^0_{i(\alpha)} \right) \left( m_k + q^0_{k(\alpha)} \right) \right]^{1/2}}{\sqrt{2 \left( m_i + q^0_{i(\alpha)} \right)}}.
\end{align*}
\]

where the + sign refers to the positive energy states and \( c \) refers to the fact that the equal helicity eigenvalues in the sum yield the coherent result. (The incoherent projection operators \( [q_{i(\alpha)}, q_{k(\alpha)}; +, i] \) with unequal helicity eigenvalues \( s_{i(\alpha)} \neq s_{k(\alpha)} \) are not dealt here.) The relation (13) defines the spinorial massive neutrino to mass-less neutrino helicity relation and it is consistent with the
ordinary helicity relation (8). Carrying out the indicated operations in relation (13) as a power series over the neutrino masses /energy, one obtains for the neutrino energy projection operator over the positive energy states the following

\[
\begin{align*}
[q(i,\alpha), q(k,\alpha); +, c] &= 2 \sum_{n=0}^{\infty} [q(i,\alpha), q(k,\alpha); +, c]_n, \\
[q(i,\alpha), q(k,\alpha); +, c]_0 &= -q_{(\alpha)}, \\
[q(i,\alpha), q(k,\alpha); +, c]_1 &= m_k + \frac{(m_k - m_i) \gamma^0 q_{(\alpha)}}{2q^0_{(\alpha)}}, \\
[q(i,\alpha), q(k,\alpha); +, c]_2 &= -\frac{(m_k - m_i)^2 q_{(\alpha)}}{8q^0_{(\alpha)}} + m_i m_k \gamma^0/2q^0_{(\alpha)}.
\end{align*}
\]

The coherent energy operator \([q(i,\alpha), q(k,\alpha); +, c]\) generates the electroweak interactions that are the same as the SM interactions plus the LIV neutrino oscillation processes that are negligible since their cross-sections are proportional to the squares of neutrino masses and, as such, are essentially zero (LIV = Lorentz invariance violating and LI = Lorentz invariant). Relation (14) is in essence the procedure for calculating the cross-sections for the processes requiring only the neutrino energy projection operators over the positive energy states.

**Applications to the differential cross-section calculations**

As established earlier and consistent with (11), the quasi-elastic electroweak process with massive neutrinos present, to \(O(m^2)\), can be denoted with the kinematics that uses just the mass-less flavor neutrinos.

\[
\nu(q(\gamma)) + \alpha(P_{(1)}) \rightarrow \nu(q(\delta)) + \beta(P_{(2)}); \quad y = \frac{q^0_{(\gamma)} - q^0_{(\delta)}}{q^0_{(\gamma)}} = \frac{P^0_{(2)} - P^0_{(1)}}{q^0_{(\gamma)}}
\]

where \(y\) is the momentum transfer. Now, although working in the massive neutrino Fock space, relation (11) says that the kinematics for the cross-sections for the quasi-elastic scattering of the massless flavor neutrinos is determined with flavor neutrino momenta according to \(\delta_{4}(q_{(\gamma)} + P_{(1)} - q_{(\delta)} - P_{(2)})\). Furthermore, since (see also [8])

\[
\int dy = \frac{1}{2\pi} \int d\sigma(q_{(\delta)}) d\sigma(P_{(2)}) \delta_{4}(q_{(\gamma)} + P_{(1)} - q_{(\delta)} - P_{(2)}), d\sigma(q) = \left(\frac{d^3q}{q^0}\right)
\]

the normalized neutrino energy transfer \(y = \left(q^0_{(\gamma)} - q^0_{(\delta)}\right)/q^0_{(\gamma)}\) cannot affect Lorentz invariance of any of the differential cross-sections.
Also, in view of (11), the cross-section normalization factor is defined with respect to the massless flavor neutrino momenta.

\[ B = \frac{1}{(2\pi)^6} \left| (P_{(1)} \cdot q_{(\gamma)})^2 - P_{(1)}^2 q_{(\gamma)}^2 \right|^{\frac{1}{2}} = \frac{1}{(2\pi)^6} \left| (P_{(1)} \cdot q_{(\gamma)}) \right| \]

In explicit evaluations, one uses the following short-hand notations:

\[ m_{\alpha \beta} = \sum_i U_{\alpha i} m_i U_{i\beta}^* \]
\[ m_{\alpha \beta}^2 = \sum_i U_{\alpha i} m_i^2 U_{i\beta}^* \]

(17)

Deriving the differential cross-sections with new energy projection operators for the flavor neutrino processes within the massive neutrino Fock space—-

\[ \frac{d\sigma_{W}}{dy} \]

From the Lagrangian density in (4), the free neutrino field (5), the kinematical relation (7), the relations (8) and (9), one derives in the usual way the \( W \)–exchange \( S_W \) and \( S_W^\dagger \) matrix elements for the process in (15). Specifically, with the Fierz rearrangement and repeated indices summing up, one has,

\[ S_W = \sum_{i,j} \delta_{ij} \left( q_{(i,\gamma)} + P_{(1)} - q_{(j,\delta)} - P_{(2)} \right) \delta_{\alpha \beta} U_{\delta j} U_{j\gamma} U_{\alpha i} \frac{ig^2}{(2\pi)^2} 8M_W^2 \]
\[ \times \pi \left( q_{(j,\delta)}, s_{(j,\delta)} \right) \gamma^\mu \left( 1 - \gamma^5 \right) u \left( q_{(i,\gamma)}, s_{(i,\gamma)} \right) \pi \left( P_{(2)}, r_2 \right) \gamma_\mu u \left( P_{(1)}, r_1 \right) \]

(18)

and \( S_W^\dagger \) is obtained from (18) as shown in (10). The contribution to the process (15) due to the \( W \)–exchange from (18), after taking into account (11), (17), \( \sqrt{2}g^2 = 8M_W^2 G \), and the fact that \( s_{(i,\gamma)}, s_{(j,\delta)}, ... \) obey, respectively, the ordinary and spinorial helicity relation, (8) and (13), the standard procedure gives,

\[ \frac{d\sigma_{W}}{dy} (m) = \frac{d\sigma_{W}}{dy}^c (m) = G^2 \left( \frac{\delta_{\alpha \beta}}{4\pi \left| \left( P_{(1)} \cdot q_{(\gamma)} \right) \right|} \right)^{\frac{1}{2}} \sum_{i,j,g,h} \left( U_{i\alpha}^* U_{g\gamma} U_{j\beta} U_{h\alpha}^* \right) \left( U_{\delta j} U_{\gamma h} U_{\alpha i} \right) \]
\[ \times \left[ Tr \left( M_1 - P_{(1)} \right) \gamma_\nu \left( 1 - \gamma^5 \right) \left( M_2 - P_{(2)} \right) \gamma_\mu \left( 1 - \gamma^5 \right) \right] \]
\[ \times \left[ Tr \left[ q_{(i,\gamma)}, q_{(j,\gamma)}; +, c \right] \gamma^\nu \left( 1 - \gamma^5 \right) \left[ q_{(h,\delta)}, q_{(j,\delta)}; +, c \right] \gamma_\mu \left( 1 - \gamma^5 \right) \right] \]

(19)

where \( m \) symbolically denotes dependence on \( m_{1,2,3} \). Next, the coherent energy operator expansion according to (14), with gamma matrices traces carried out, yields
The processes are: Flavor conserving are: One can summarize the neutrino flavor transitions for the mass limits. One can notice that, while the negligible $LIV$ in the usual way the $Z$ neutrino field (5), the kinematical relation (7), the contractions (9), one derives in (15). Specifically, one has

\[
\delta_{\alpha\beta} \delta_{\gamma\delta} \frac{m_{\alpha\beta}}{4} \left[ \frac{1}{m_t^2 (1) \cdot q_{(\gamma)}} \left( P_{(1)} \cdot q_{(\gamma)} \right) \left( P_{(2)} \cdot q_{(\delta)} \right) + \frac{2}{m_t^2 (2)} \left( P_{(1)} \cdot q_{(\gamma)} \right) \left( P_{(2)} \cdot q_{(\delta)} \right) \right] + O(m^4),
\]

One can notice that, while the negligible $LIV$ is associated with the neutrino mass, the $LI$ Standard Model result is formally identified with zero neutrino mass limits

\[
\frac{d\sigma_W (m)}{dy} = \frac{d\sigma_W (SM)}{dy} + O (m^2; LIV)
\]

one can summarize the neutrino flavor transitions for the $W$-exchange neutrino processes. Flavor conserving are: $LI$ to $O(m = 0)$ terms and negligible $LIV$ to $O(m^2)$ terms. Flavor violating are: negligible $LIV$ to $O(m^2)$ terms.

\[
\frac{d\sigma_W}{dy} = \frac{d\sigma_W (SM)}{dy} + O (m^2; LIV)
\]

As in the previous case, from the Lagrangian density in (4), the free neutrino field (5), the kinematical relation (7), the contractions (9), one derives in the usual way the $Z$-exchange $S_Z$ and $S_Z^\dagger$ matrix elements for the process in (15). Specifically, one has

\[
S_Z = \sum_{i,j} \delta_{ij} \left( q_{(i,\gamma)} + P_{(1)} - q_{(j,\delta)} - P_{(2)} \right) \delta_{i\beta} \delta_{j\gamma} U_{ij}^\dagger U_{i\gamma}
\]

\[
\times \frac{ig^2}{(2\pi)^2 16r_W M_Z^2} \left[ \pi \left( q_{(j,\delta)} - s_{(j,\delta)} \right) \gamma^\mu \left( 1 - \gamma^5 \right) u \left( q_{(i,\gamma)} - s_{(i,\gamma)} \right) \left( P_{(1)} \cdot r_{(1)} \right) \right]
\]

while $S_Z^\dagger$ is obtained from the $S_Z$ through the Hermitian conjugation. In what follows, one will find the following shorthand notation very useful:

\[
w_0 = s_W^2, \quad w_1 = 2s_W^2 - 1, \quad z_1 = s_W^2 \left( 2s_W^2 - 1 \right) + \frac{1}{4}, \quad z_2 = s_W^2 \left( 2s_W^2 - 1 \right), \quad z_3 = s_W^2 - \frac{1}{4}, \quad z_4 = s_W^2 \left( s_W^2 - 1 \right) + \frac{1}{4}, \quad z_1 + z_3 = 2s_W^4
\]

After taking into account that $c_W^2 M_Z^2 = M_W^2$ and the fact that the helicities, $s_{(i,\gamma)}$, $s_{(j,\delta)}$, ..., obey both the ordinary and the spinorial helicity relations (8) and (13), the standard procedure yields the general expressions.
While the terms of scattering (15) due to the overlapping LIV Here also, the negligible traces of gamma matrices, yields

\[ s_{(i,\gamma)} = s_{(k,\gamma)} = s_{(\gamma)}; s_{(i,\delta)} = s_{(j,\delta)} = s_{(\delta)} : \]

\[
\frac{d\sigma^Z(m)}{dy} = \frac{d\sigma^Z(c,c)(m)}{dy} = \frac{G^2}{4\pi |(P(1), q(\gamma))|} \sum_{i,j,k,l} \left( U_{ij} U_{k\gamma} U_{l\delta} U_{\gamma k} \right) \delta_{ij} \delta_{kl} (24)
\]

\[
\times \left[ Tr \left( M_1 - P_{(1)} \right) \gamma_\nu \left( 2z_4 + \frac{1}{2} \gamma^5 \right) \left( M_2 - P_{(2)} \right) \gamma_\mu \left( 2z_4 + \frac{1}{2} \gamma^5 \right) \right]
\]

\[
\times \left[ Tr \left[ q_{(\gamma)} + +, c \right] \gamma_\nu \left( 1 - \gamma^5 \right) \left[ q_{(\delta)} + +, c \right] \gamma_\mu \left( 1 - \gamma^5 \right) \right]
\]

The coherent energy operator expansion according to (14), with evaluating the traces of gamma matrices, yields

\[
\frac{d\sigma^Z(m)}{dy} = \frac{d\sigma^Z(SM)}{dy} \left[ 1 + \frac{m^2_{\gamma\gamma}}{4} \left( \frac{1}{q_{(\gamma)}^2} + \frac{1}{q_{(\delta)}^2} \right) \right] - \frac{G^2 m_{\gamma\delta} m_{\delta\gamma} \delta_{\alpha\beta}}{8\pi |(P(1), q(\gamma))|} \times \left\{ 2 \left( \frac{1}{q_{(\gamma)}^2} + \frac{1}{q_{(\delta)}^2} \right) \left[ M_1 M_2 \gamma_{\gamma} \left( q_{(\gamma)} \cdot q_{(\delta)} \right) + \left( P_{(1)} \cdot q_{(\delta)} \right) \left( P_{(2)} \cdot q_{(\gamma)} \right) \right] \right. \\
\left. + \left( P_{(1)} \cdot q_{(\gamma)} \right) \left( P_{(2)} \cdot q_{(\delta)} \right) \right] \right. (z_1 + z_3) \\
\left. + \frac{4}{q_{(\delta)}^2} \left[ M_1 M_2 \gamma_{\gamma} q_{(\gamma)}^0 + P_{(1)}^0 \left( P_{(2)} \cdot q_{(\gamma)} \right) \right] (z_1 + z_3) \\
\left. + P_{(2)}^0 \left( P_{(1)} \cdot q_{(\delta)} \right) \right] \right. (z_1 + z_3) \right. \\
\left. + \frac{4}{q_{(\gamma)}^2} \left[ M_1 M_2 \gamma_{\gamma} q_{(\delta)}^0 + P_{(1)}^0 \left( P_{(2)} \cdot q_{(\delta)} \right) \right] \right. (z_1 + z_3) \right. \\
\left. \right\} + O(m^4)
\]

\[
\frac{d\sigma^Z(SM)}{dy} = \frac{G^2 \delta_{\alpha\beta} \delta_{\gamma\delta}}{\pi |(P(1), q(\gamma))|} \times \left[ M_1 M_2 \gamma_{\gamma} \left( q_{(\gamma)} \cdot q_{(\delta)} \right) + \left( P_{(1)} \cdot q_{(\delta)} \right) \left( P_{(2)} \cdot q_{(\gamma)} \right) \right] (z_1 + z_3) \\
+ \left( P_{(1)} \cdot q_{(\gamma)} \right) \left( P_{(2)} \cdot q_{(\delta)} \right) (z_1 + z_3)
\]

Here also, the negligible LIV is associated with the neutrino mass while the LI Standard Model result is identified with formally zero neutrino mass limits:

\[
\frac{d\sigma^Z(m)}{dy} = \frac{d\sigma^Z(SM)}{dy} + O(m^2; LIV), \quad (26)
\]

While the terms of \( O(m = 0) \) are LI and flavor conserving, the negligible LIV terms of \( O(m^2) \) are either flavor violating or flavor conserving.

\[
\frac{d\sigma^{W,Z}(m)}{dy} - \text{Here, the differential cross-section for the quasi-elastic neutrino scattering (15) due to the overlapping } S - \text{matrix elements from the } W - \text{and }
\]
Again, the negligible $Z$-is given as a sum of its components after taking into account relations (11), (18), and (22). Importantly, again taking into account the fact that helicities, $s_{(i,\gamma)}, s_{(j,\delta)}, \ldots$, obey both the ordinary and spinorial helicity relations, (8) and (13), the standard procedure yields the general expression

\[
\begin{align*}
\sigma_{WZ}^{(c,c)}(m) &= \frac{d\sigma_{WZ}^{(c,c)}(m)}{dy} = \frac{G^2}{8\pi \left|\left(P_{(1)}, q_{(\gamma)}\right)\right|^2} \delta_{\alpha\beta} \\
&\times \left[\text{Tr} \left(M_1 - P_{(1)}\right) \gamma_\nu (4z_3 + \gamma^5) \left(M_2 - P_{(2)}\right) \gamma_\mu (1 - \gamma^5)\right] \\
&\times \left[\sum_{g,h,c,f} \left(U_{gh}^\dagger U_{cf}U_{h\alpha}U_{j\gamma} \delta_{ef}\right)\right] \\
&\times \text{Tr} \left[q_{(c,\gamma)}, q_{(g,\gamma)}; +, c\right] \gamma_\nu (1 - \gamma^5) \left[q_{(h,\delta)}, q_{(f,\delta)}; +, c\right] \gamma_\mu (1 - \gamma^5)\right]\right]\right]
\end{align*}
\]

where one took into account the identity:

\[
\begin{align*}
\text{Tr} \left(M_1 - P_{(1)}\right) \gamma_\nu (4z_3 + \gamma^5) \left(M_2 - P_{(2)}\right) \gamma_\mu (1 - \gamma^5) \\
= \text{Tr} \left(M_1 - P_{(1)}\right) \gamma_\nu (1 - \gamma^5) \left(M_2 - P_{(2)}\right) \gamma_\mu (4z_3 + \gamma^5)
\end{align*}
\]

Of course, one cannot avoid the coherent energy operator expansion according to (14), and evaluating the traces of gamma matrices one obtains

\[
\begin{align*}
\frac{d\sigma_{WZ}^{(SM)}}{dy} &= \frac{\sigma_{WZ}^{(SM)}}{dy} \left[1 + \frac{m_1^2}{4} \left(1 - \frac{1}{q_{(\gamma)}^2} + \frac{1}{q_{(\delta)}^2}\right)\right] - \frac{G^2 m_1 m_2 \delta_{\alpha\beta}}{2\pi \left|\left(P_{(1)}, q_{(\gamma)}\right)\right|^2} \\
&\times \left\{ \left[M_1 M_2 w_0 \left(q_{(\gamma)} \cdot q_{(\delta)}\right) + w_1 \left(P_{(1)}, q_{(\gamma)}\right) \left(P_{(2)} q_{(\delta)}\right)\right] \\
&\times \left(\delta_{\alpha\gamma} \frac{1}{q_{(\gamma)}^2} + \delta_{\beta\delta} \frac{1}{q_{(\delta)}^2}\right) + \frac{2\delta_{\alpha\gamma}}{q_{(\gamma)}^2} \left[M_1 M_2 w_0 q_{(\alpha)}^0 + w_1 \left(P_{(1)}, q_{(\gamma)}\right) P_{(2)}^0\right] \\
&\times + \frac{2\delta_{\alpha\gamma}}{q_{(\gamma)}^2} \left[M_1 M_2 w_0 q_{(\beta)}^0 + w_1 \left(P_{(2)}, q_{(\delta)}\right) P_{(1)}^0\right]\right\} + O(m^4), \\
\frac{d\sigma_{WZ}^{(SM)}}{dy} &= \frac{2G^2 \delta_{\alpha\beta}}{\pi \left|\left(P_{(1)}, q_{(\gamma)}\right)\right|^2} \left[M_1 M_2 w_0 \left(q_{(\gamma)} \cdot q_{(\delta)}\right) + w_1 \left(P_{(1)}, q_{(\gamma)}\right) \left(P_{(2)}, q_{(\delta)}\right)\right]
\end{align*}
\]

Again, the negligible LIV is associated with the neutrino mass while the LI
Standard Model result is identified with formally zero neutrino mass limits:

\[
\frac{d\sigma_{W,Z}(m)}{dy} = \frac{d\sigma_{W,Z}(SM)}{dy} + O(m^2; LIV) \quad (29)
\]

The overlapping \( W^- \) and \( Z^- \) exchanges cross-section terms of \( O(m = 0) \) are \( LI \) and flavor conserving, while the negligible terms of \( O(m^2) \) carry the \( LIV \) terms with, both, the conserved and violate flavor.

**Discussion**— One thing that one notices right away is the fact that while the \( LIV \) is very real, because it is associated with the \( O(m^2) \) terms, it is negligible at least in the scattering-like experiments. Therefore, the "mass-less" SM is consistent with massive neutrinos whose masses are \( \leq 1 \text{eV} \). Because they are proportional to \( O(m^2) \), the neutrino oscillation scattering cross-sections derived here are not observable. However, the interesting problem to deal with would be as to how to generalize the negligible neutrino oscillation scattering cross-sections in the laboratory into the practical long baseline oscillations probabilities.

**Acknowledgements**— I wish to thank Dr. Howard E. Brandt for friendly discussions and help with the computer manipulations. To my wife Patricia Marie Stone Soln, I am deeply grateful for her expert librarian help in locating literature over the internet.

**References**—

[1] Super-KamiokandeCollaboration, Y. Ashie et al., Phys. Rev. Lett. 93, 101801 (2004); M. Shiozawa, Prog. Part. Nucl. Phys. 57, 79 (2006).

[2] SNO Collaboration, Phys. Rev. Lett. 81, 071301 (2001); 89, 011301 (2002); 89, 011302 (2002); Phys. Rev. C72, 055502 (2005).

[3] KAMLAND Collaboration, T. Araki et al., Phys. Rev. Lett. 94, 081801 (2005).

[4] Homestake Collaboration, T. Leveland et al. Astrophys. J. 496, 505 (1998); GNO Collaboration, M. Altman et al. Phys. Lett. B616, 174 (2005); SAGE Collaboration, J. N. Abdurashitov et al., Nucl. Phys. Proc. Suppl. 110, 315 (2002); Super-KamiokandeCollaboration, J. Hosaka et al., Phys. Rev. D73, 112001 (2006).

[5] S. M. Bilenky, C. Giunti and W. Grimus, Progress In Particle and Nuclear Physics, 43, 1 (1999).

[6] C. Giunti and M. Laveder, “Neutrino Mixing”; [hep-ph/0310238v2].

[7] B. Kayser, “Neutrino Oscillation Phenomenology”; Proc. of the 61st Scottish Universities Summer School in Physics, Eds. C. Frogatt and P. Soler (to appear); arXiv: 0804.1121v3 [hep-ph].

[8] M. Fukugita and T. Yanagida, “Physics of Neutrinos and Applications to Astrophysics” (Springer Verlag Berlin Heidelberg 2003).

[9] C. Giunti and C. W. Kim, “Fundamentals of Neutrino Physics and Astrophysics” (Oxford University Press, Oxford 2007).
[10] C. Giunti, “Neutrino Flavor States and the Quantum Theory of Neutrino Oscillations” (XI Mexican Workshop on Particles and Fields, 7-12 November 2007)), arXiv: 0801. 0653 v1 [hep-ph].

[11] C. Giunti, “Fock States of Flavor Neutrinos are Unphysical”; Eur. Phys. J. C39, 377-382 (2005); hep-ph/0312256v2.

[12] S. M. Bilenky, F. von Feilitzsch and W. Potzel, J. Phys. G; Nucl. Part. Phys. 34, 987 (2007), hep-ph/0611285v2.