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Measurements of Tidal Tail Populations Designed to Optimize Their Use as Potential Probes

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Abstract. The launching of up to four astrometric missions in the next decade will enable us to make measurements of stars in tidal streamers from Galactic satellites of sufficient accuracy to place strong constraints on the mass distribution in the Milky Way. In this paper we simulate observations of debris populations in order to assess the required properties of any data set chosen to implement this experiment. We apply our results to find the desired target properties of stars (e.g., accuracy of velocity and proper motion measurements) associated with the dwarf spheroidal satellites of the Milky Way.

1. Introduction

Stars lost from a satellite galaxy in orbit around the Milky Way will gradually drift ahead or behind their parent system to form streams of debris along its orbit (Tremaine 1993; Johnston 1998). Assuming we know the phase-space coordinates of the parent satellite, the unique properties of tidal debris streams can be exploited to measure the potential of the Milky Way in two ways:

**MD Algorithm:** Integrate the satellite’s orbit backwards and forwards and try to fit any available observations to the orbit, with the “best” potential corresponding to the closest fit (e.g., Murali & Dubinski 1999).

**JZSH Algorithm:** Integrate the stars and satellite backwards and choose the “best” potential to be the one in which most stars recombine with the satellite (e.g., Johnston et al. 1999, hereafter JZSH99).

The MD algorithm can be applied with limited observations (e.g., just angular positions and line-of-sight velocities) of any accuracy. However, in reality debris in trailing/leading streamers are not on exactly the same orbit as their parent satellite, but rather in orbits offset in energy from the satellite’s orbit by $\sim \pm \epsilon$ where

$$\epsilon = r_{\text{tide}} \frac{\partial \Phi}{\partial r} \approx s v_{\text{circ}}^2.$$  

Here, $r_{\text{tide}} = rs$ is the tidal radius of the satellite, $\Phi(r)$ is the Galactic potential, $v_{\text{circ}}$ is the circular velocity of the Milky Way and $s$ is the *tidal scale*

$$s = \left( \frac{m_{\text{sat}}}{M_{\text{Gal}}} \right)^{1/3},$$  

where $m_{\text{sat}}$ is the satellite’s mass and $M_{\text{Gal}}$ is the mass of the Galaxy enclosed within the satellite’s orbit. Hence, once the observational errors are small enough
to detect the offset of the particles from the satellite’s orbit the MD algorithm will be biased by this systematic offset, rather than limited by the errors themselves. In contrast, the JZSH algorithm exploits the fact that debris stars are on orbits with different time periods than the satellite’s own and hence will only work with observations of five of the phase-space coordinates of sufficient accuracy that the offset from the orbit can be detected (the sixth coordinate, if poorly known, can always be found by applying the principle of energy conservation). The dividing line between useful application of these two approaches will therefore occur where the velocity measurement errors $\Delta v$ satisfy

$$v_{\text{circ}} \Delta v \approx \epsilon \Rightarrow \frac{\Delta v}{v_{\text{circ}}} \approx s.$$  

(3)

Once measurements of proper motions and velocities of debris can be made with accuracies of order 3-30 km/s the second approach will be more accurate for all Milky Way satellites (depending on the mass and orbit of the satellite — see Table 1) Such accuracies will become possible with the next generation astrometric missions: NASA’s Space Interferometry Mission (SIM) and ESA’s Global Astrometric Imager for Astrophysics (GAIA).

JZSH99 have already demonstrated that the JZSH algorithm can in principle recover both the mass and geometry of the Milky Way with few percent accuracies using just 100 stars. In this paper, we examine how these uncertainties grow with the limitations likely to be present in a real data set.

2. Methods

We test how well we can recover the Milky Way potential by “observing” debris populations at the end of simulations of satellite destruction and then applying the JZSH algorithm to our data sets. We illustrate our results in this paper with tests on simulations referred to as Models 1/2/3/4, which had satellite masses $5.1e6 / 2.9e7 / 6.9e7 / 1.0e8 \, M_\odot$, on orbits with pericenters at $43 / 43 / 26 / 30$ kpc and radial time periods of $2.5 / 2.5 / 2.0 / 1.0$ Gyears. These parameters corresponded to tidal scales $s = 0.026 / 0.046 / 0.075 / 0.081$ (see equation [3]). The simulation technique and Galactic model are described in Johnston, Spergel & Hernquist (1995).

We restrict ourselves to asking how well we can recover the circular velocity of the halo component that was used in the simulations, assuming we know the other parameters in the potential. As an indication of the error, we attempt to recover $v_{\text{circ}}$ from ten data sets, each containing a different group of particles or a different error realization. We take the “error” to be the dispersion in the recovered $v_{\text{circ}}$ in units of the true circular velocity ($= \sigma_{v_{\text{circ}}}/v_{\text{circ}}$).

3. Results

Measurement Errors of Debris Stars As noted in §1, we expect the JZSH algorithm to require knowledge of velocities such that $\Delta v/v_{\text{circ}} < s$. To test this, we observed 128 stars in Models 1-4. We then added errors chosen from gaussians of mean zero and dispersions $\Delta v$ and $\Delta v/d$ to each line of sight velocity and each
component of proper motion respectively, and ran the JZSH algorithm to recover $v_{\text{circ}}$. This was repeated 10 times with the same sample of stars (but different error realizations) for $\Delta v = 1, 5, 10, 15, 20, 25, 30 \text{ km/s}$ and the dispersion among the results from the 10 realizations calculated. The errors found, shown in the left hand panel of Figure 1, confirm the intuition outlined in §1.

**Length of Tidal Streamers** A typical debris star will drift by $\Delta \Psi = \epsilon (d\Omega/d\epsilon)(2\pi/\Omega) \sim r_{\text{tid}}(d\Omega/dr)(2\pi/\Omega)$ in angle away from the satellite in one orbit (where $\Omega$ is the angular velocity of the satellite), so a star with separation $\Delta \Psi$ from the satellite, was most likely lost $N_{\text{orb}}$ orbits ago, where

$$N_{\text{orb}} \sim \frac{\Delta \Psi}{2\pi} \frac{\Omega}{r_{\text{tid}}} \frac{dR}{d\Omega} \sim \frac{\Delta \Psi}{2\pi} \frac{1}{s}. \tag{4}$$

To test how the length of the tidal streamer sample might affect the accuracy of the recovered potential we observed 10 random samples of 128 particles in each simulation, with the samples chosen to be within $\Delta \Psi = 10, 20, 45, 90, 135$ and 180 degrees of the satellite. The right hand panel of Figure 1 plots the results for each simulation as a function of $N_{\text{orb}}$ defined from $\Delta \Psi$ via equation (4). The plot suggests that if we find $\sim 100$ stars in a streamer we would like to observe them at sufficiently large angle from the satellite to be sensitive to 1-2 orbits.

**Knowledge of Satellite Distance** We anticipate knowing the line-of-sight velocity and proper motion of the satellite with a precision at least comparable to those of the debris stars. However, the satellite’s distance $r_{\text{sat}}$ may be poorly known (or unknown). This means that though we can expect to place strong limits on the mass distribution in the Galaxy, $M_{\text{Gal}}(r/r_{\text{sat}})$, the physical scale of that distribution will be dependent on our measurement of $r_{\text{sat}}$ and uncertain at the same level.

**Dynamical Friction** Binney & Tremaine (1987) show that the frictional force on a satellite on a circular orbit in an isothermal sphere is given by (see their equation [7.23]) $F = -0.428 \ln \Lambda G m_{\text{sat}}^2 / R^2$, where $\ln \Lambda$ is the Coulomb Logarithm. The change in orbital energy per orbit will be $\Delta E_{\text{orb}}^{\text{df}} = F(2\pi R)/m_{\text{sat}}$, so

$$\frac{\Delta E_{\text{orb}}^{\text{df}}}{\epsilon} = -0.428 \ln \Lambda \frac{G m_{\text{sat}}}{R^2} \frac{2\pi R}{\tau_{\text{circ}}^2} \frac{1}{s} \approx -2.69 \ln \Lambda s^2. \tag{5}$$
Table 1. Properties of tidal tail data sets associated with each of Milky Way’s satellites needed to apply the JZSH algorithm.

| Name | $R$ (kpc) | dist mod | $m_{sat}$ $M_{\odot}$ | $s$ (km/s) | $\Delta v$ | $\Delta \mu$ (mas/yr) | $\Delta \Psi$ (degrees) | $\Delta P_{orb}$ $\epsilon$ |
|------|-----------|----------|------------------------|-------------|------------|------------------------|------------------------|--------------------------|
| Sgr  | 16.0      | 16.9     | 5.0E+08                | 0.151       | 30.3       | 252                    | 54                     | 0.393                    |
| Sex  | 86.0      | 19.7     | 2.6E+07                | 0.032       | 6.5        | 15                     | 11                     | 0.026                    |
| Car  | 86.6      | 19.7     | 1.1E+07                | 0.024       | 4.8        | 11                     | 9                      | 0.016                    |
| LeoI | 210.0     | 21.6     | 1.4E+07                | 0.020       | 3.9        | 4                      | 8                      | 0.010                    |

Evolution of the Galactic Potential The properties of debris trails will also be affected by the time-dependence of the Milky Way’s potential (e.g., Zhao et al. 1999). However, the existence of the > 7Gyear old thin disk suggests that potential evolution has not been significant within the the last $N_{orb} = 1$ orbit of any of the Galactic satellites.

4. Conclusion

Table 1 lists the required accuracy of line-of-sight velocity and proper motion measurements of tidal stream stars (columns 6 and 7), length of tidal stream population (column 8) and effect of dynamical friction (column 9) for a sample of Galactic satellites. The numbers demonstrate that the necessary measurement accuracies are within the capabilities of SIM and GAIA and that samples extending tens of degrees along a streamer to select targets will be sufficient. Dynamical friction needs only to be modeled carefully for the innermost satellites. We conclude that the JZSH algorithm, combined with data from SIM or GAIA will significantly improve our knowledge of the Galactic potential.

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