MICROLENSING OF GLOBULAR CLUSTERS AS A PROBE OF GALACTIC STRUCTURE

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ABSTRACT

The spatial distribution of compact dark matter in the Galaxy can be determined in a few years by monitoring Galactic globular clusters for microlensing. Globular clusters are the only dense fields of stars distributed throughout the three-dimensional halo, and hence, they are uniquely suited to probe its structure. The microlensing optical depths toward different clusters have varying contributions from the thin disk, thick disk, bulge, and halo of the Galaxy. Although measuring individual optical depths to all the clusters is a daunting task, we show that interesting Galactic structure information can be extracted with as few as 40–120 events in total for the entire globular cluster system (observable with 2–5 yr of monitoring). This experiment is particularly sensitive to the core radius of the halo mass distribution and to the parameters of the thin disk.

Subject headings: dark matter — Galaxy: fundamental parameters — Galaxy: halo — Galaxy: structure — globular clusters: general — gravitational lensing

1. INTRODUCTION

The search for compact dark matter in the mass range 10^6–10^10 M☉ using microlensing (Paczynski 1986) has come to fruition with the detection of many microlensing events toward the Galactic bulge, Large Magellanic Cloud (LMC), and Small Magellanic Cloud (SMC) (Alcock et al. 1993, 1996, 1997a, 1997b; Aubourg et al. 1993; Udalski et al. 1993). These studies have confirmed the presence of a massive bar in the Galaxy by detecting excess microlensing toward the Galactic bulge (Udalski et al. 1994; Alcock et al. 1995a; Paczynski et al. 1994; Zhao, Spergel, & Rich 1995) and have found evidence for a massive halo by detecting 8 times more events toward the Large Magellanic Cloud than can be explained by known stellar populations (Alcock et al. 1997a).

Uncertainties in the spatial distribution, kinematics, and masses of the lensing objects remain (cf. review by Paczynski 1997). One major uncertainty is our incomplete knowledge of Galactic structure. Possible lens locations include a maximal Galactic disk (Kuijken 1997), the LMC’s disk (Sahu 1994), and the Galaxy’s halo. Here we propose using microlensing of globular cluster stars to discriminate the spatial distribution of the lensing objects statistically. Because the globular clusters are distributed throughout the halo of the Galaxy, we can probe many lines of sight through varying amounts of the halo, disk, and bar of the Galaxy to determine the relative contributions of each to microlensing. Similar strategies using the ratios of Bulge, LMC, and SMC optical depths have been suggested by Sackett & Gould (1993) and Gould, Miralda-Escudé, & Bahcall (1994).

The Galactic halo may be clumpy, as expected under the Searle & Zinn (1978) picture of Galaxy formation. For example, the Sagittarius dwarf galaxy (Ibata, Gilmore, & Irwin 1994) and a possible intervening population toward the LMC (Zaritsky & Lin 1997) might raise the number of microlensing events toward some lines of sight (Zhao 1998). Monitoring microlensing events toward many Galactic directions will average over such fluctuations in halo density.

2. MICROLENSING OF A GLOBULAR CLUSTER

The optical depth to microlensing is the probability that a background source will lie inside the Einstein radius of a lens and so be amplified by a factor 1.34 (Refsdal 1964; Vietri & Ostriker 1983). It is given by

\[ \tau = \int_0^\infty \frac{4\pi G \rho L}{c^2} \left(1 - \frac{L}{D}\right) dL, \]

where D is the distance to the background source, L is the distance along the line of sight, \( \rho \) is the local mass density at distance L, and G and c are the gravitational constant and the speed of light. The optical depth to the clusters is 10^6–10^8, depending on the cluster location in the Galaxy and the Galactic model (Fig. 1).

The Galaxy has ~150 globular clusters, each containing ~10^5–10^6 stars to a limiting magnitude of V ≲ 22. This gives a total of ~10^7–10^8 stars to monitor and total optical depth of a few for the whole system. The quantity relevant to discriminate among Galactic models is the number of events observable per year, which depends on the timescale of the events. The median duration of observed events toward the bulge is 30 days (Alcock et al. 1997a). Globular cluster microlensing event timescales may be comparable or different, depending on the relative kinematics of the clusters and lenses.

A significant fraction of stars in the inner parts of globular clusters will be unresolved with a ground-based telescope. Here one can look for pixel lensing, i.e., monitor the brightness variations of pixels containing multiple stars (Crotts 1992; Colley 1995). The rate of detectable pixel lensing events for each star is lower than in the uncrowded regime, because for a fixed amplification of the total flux in a pixel, a star needs to undergo a larger magnification.

To account for both resolved and unresolved stars consistently, we calculate the mean effective optical depth for an entire cluster using the pixel microlensing formalism. We use the cluster M15 as an example because stars in the interior core (down to 0.3) have been resolved by HST (Guhathakurta...
resolved and the only one contributing to the flux in that pixel, where et al. (1996). The crowding ranges from 130 stars arcsec$^{-2}$ for $V < 22$ in the inner 0.3 to resolved stars in the outer regions.

Suppose a microlensing event is claimed for a fixed fractional increase in the brightness of a pixel. Following Colley (1995),

$$\Delta m = -2.5 \log \frac{A f_s + f_{\text{pix}}}{q f_s + f_{\text{pix}}} ,$$

where $\Delta m$ is the brightness change in magnitudes, $A$ is the amplification, $q$ is the fraction of the amplified star’s light falling in the peak pixel of the point spread function, $f_s$ is the flux of the star, and $f_{\text{pix}}$ is the flux due to other sources in the pixel. The maximum radius $u$ that produces amplification $A$ is given by (Refsdal 1964) $A = (u^2 + 2)/[u (u^2 + 4)^{9/2}]$, where $u = R_\bullet R_\odot$, and $u^d$ gives the corresponding lens plane cross section as a fraction of the Einstein ring area $\pi R_\odot^2$. If the star were resolved and the only one contributing to the flux in that pixel, $u = 1$ and $A = 3/\sqrt{5}$. Using the luminosity function and surface brightness profile of M15 (Guhathakurta et al. 1996; Trager, King, & Djorgovski 1995), we calculate the reduced efficiency $\xi(r)$, which is the mean value of $u^2$ for stars in the pixel. This is a function of the surface brightness of the pixel and the brightness of the star. Summing over the stellar luminosity function $f(m)$ for M15 (Guhathakurta et al. 1996) in the V band,

$$\xi(r) = \int_{m_1 = 12}^{m_1 = 22} \int_{m_1 = 12}^{m_1 = 22} f(m) u(m)^2 dm \int_{m_1 = 12}^{m_1 = 22} f(m) dm ,$$

where $u$ depends on the local surface brightness and hence on $r$. $\xi = 6.4 \times 10^{-4}$ at a projected distance $r = 0.1$ pc from the center and approaches 1 at $r = 500$ pc when stellar density reaches 0.12 stars arcsec$^{-2}$. With $3 \times 10^5$ stars to a radius of 600 pc and a mean efficiency of $(\xi) = 65.9\%$, the cluster can be treated like a cluster with $2 \times 10^5$ fully resolved stars.

For other globular clusters, we retained the radial profile and luminosity function models from M15. The total numbers of stars were scaled relative to M15 according to each cluster’s V-band luminosity. The projected distribution on the sky (and hence crowding) and the number of stars above our $V < 22$ cutoff were adjusted for the distances to the individual clusters. The mean number of effectively resolved stars inside a 600$^\circ$ radius of a globular cluster is $5.7 \times 10^4$ per cluster.

3. DISTINGUISHING BETWEEN GALACTIC MODELS

We performed a series of simulations to determine how well microlensing observations of globular clusters can distinguish among plausible Galactic models. In each test, the microlensing optical depth toward each cluster is calculated under two assumed Galactic models, and the probability of inferring the correct model is determined for a given number of events. The results are then inverted to calculate the number of microlensing events needed to infer the correct Galactic model with 90% confidence.

The models generally consisted of three mass components: The thin disk, the halo, and the central bar. A fourth component, the thick disk, was added in one model. All the mass is assumed to be in compact objects. The optical depth to microlensing is calculated using equation (1), substituting the density $\rho = \rho_{\text{disk}} + \rho_{\text{bar}} + \rho_{\text{halo}} + \rho_{\text{thick disk}}$.

The models were required to obey three constraints. (1) Rotation curve: The potential of the Galaxy consisting of these components was required to fit the rotation curve in the inner Galaxy (Malhotra 1994, 1995) excluding the bar region ($2.5$ kpc $< R < R_\odot$) and a flat rotation curve with circular speed $220$ km s$^{-1}$ in the outer Galaxy ($R_\odot < R < 16.5$ kpc) (Knapp, Tremaine, & Gunn 1978; Fich, Blitz, & Stark 1989). The Sun–Galactic center distance $R_\odot$ is taken to be 8.5 kpc throughout this work. (2) The local surface mass density of the thin disk, $\Sigma_\odot$, lies in the interval $40 M_\odot$ pc$^{-2} \leq \Sigma_\odot \leq 80 M_\odot$ pc$^{-2}$ (Kuijken & Gilmore 1989, 1991; Bahcall, Flynn, & Gould 1992; Flynn & Fuchs 1994). We enforce this by either fixing the local surface mass density or adjusting other parameters to ensure that the final value lies in the observationally acceptable range. (3) A third, implicit, constraint is the mass profile of the thin disk. From gas dynamics it is demonstrated to be an exponential with roughly the same scale length as the light (Knapp 1990; Malhotra 1994, 1995).

The thin disk.—The disk density component(s) were modeled as

$$\rho_d = \Sigma_d (2z_i) \exp \left[ -1 |r - R_d| / r_i \right] \sech \left( |z| / z_i \right) \right| ^2 ,$$

where $r_i$ is the exponential scale length, $z_i$ is the scale height, and $(r, z)$ are Galactocentric cylindrical coordinates (Spitzer 1942). We took $z_i = 0.20$ kpc at the Solar radius. The scale height was usually kept constant with radius. Such constancy is indicated by most studies of external galaxies (van der Kruit & Searle 1982) and is consistent with the diffuse near-infrared light of the Milky Way (Spiergel, Malhotra, & Blitz 1996; Freudenreich 1996). However, there may be exceptions to this rule (de Grijs & Peletier 1997), so in one case (model 5B) the scale height was allowed to increase with radius.

Halo.—A power-law halo was adopted as it admits a self-consistent distribution function and an analytic potential, useful in matching rotation curve constraints (Evans 1993, 1994; Al-
cock et al. 1995b):

\[ \rho_h = \frac{v_0^2}{4\pi G q^2} \left( \frac{1 + 2q^2}{r_0^2 + r^2 + (2 - q^2)z^2} \right)^{5/2}, \]

where \( r_0 \) is the halo core radius and \( q \) is the halo-flattening parameter. We do not include halo truncation, since relatively few globular clusters lie at galactocentric radii \( \gg 10 \) kpc, while present evidence suggests the halo extends to 40–50 kpc (Little & Tremaine 1987).

**Bar/bulge.**—Of the many models of the central bar of the Galaxy (Dwek et al. 1995; Zhao et al. 1995; Stanek et al. 1997), we follow the Zhao et al. (1995) bar model:

\[ \rho_b = \frac{M_b}{8 \pi abc} e^{-r^2/b^2}, \]

where \( s' = [(x/a)^2 + (y/b)^2]^{1/2} + (z/c)^{1/2} \), \( a = 1.49 \) kpc, \( b = 0.58 \) kpc, and \( c = 0.40 \) kpc are the major axis, minor axis, and vertical scale lengths, and \( (x, y, z) \) is a Cartesian coordinate system aligned with the bar. The major axis of the bar is at angle \( \phi \) with respect to the Sun–Galactic center line, and \( z = 0 \) is the Galactic plane (so that the bar is not tilted). In the rotation curve fits, we approximated the potential of the bar as a point source potential and excluded the innermost 2.5 kpc.

**Tests.**—Seven tests were executed in all. Each determined the sensitivity of globular cluster microlensing observations to one parameter by comparing two paired models spanning a plausible range of that parameter. Other structural parameters were adjusted as needed to maintain consistency with the rotation curve. The models are summarized in Table 1. Test 1 is designed to discriminate between models with a large and small halo core radius. Other components become more massive in the case of a large core radius. Test 2 varies the halo-flattening parameter. Test 3 uses a thin disk mass but a range of disk scale length; the halo parameters also change substantially. Test 4 primarily explores local disk surface mass density. Test 5 allows the scale height of the thin disk to increase with Galactocentric radius as \( z_c \propto 1/\Sigma(r) \propto \exp(\sqrt{r/\sigma}) \). Test 6 includes a thick disk component of scale height 600 pc, surface density \( 1 M_\odot \) pc\(^{-2}\), and scale length identical to the thin disk. Finally, test 7 varies the angle of the Galactic bar with respect to the Sun–Galactic center line. Note that the rotation curve constraint couples the parameters of different Galactic components in our models. Differences between microlensing optical depths in models A and B of a test may thus be due to changes in more than one Galactic component.

The microlensing optical depth to each cluster was determined by evaluating equation (1) numerically for the density model given by equations (4)–(6). Our cluster sample consisted of 140 globular clusters whose coordinates, distances, and absolute \( V \)-band magnitudes are tabulated in Djorgovski & Meylan (1993) and Djorgovski (1993). We verified that our numerical integration code can reproduce Sackett & Gould’s optical depths toward the Magellanic Clouds and Galactic Bulge within 1%.

Given the optical depths, we then approximated the effect of an observing campaign by defining the effective number \( n \) of trials per star monitored. This is essentially the number of independent lensing events we expect to observe per unit optical depth, or equivalently, the number of event durations spanned by the observing campaign. Formally, \( n \) is defined by \( \langle E_\ast \rangle = n \tau \), where \( \langle E_\ast \rangle \) is the number of events expected per star monitored and \( \tau \) is the lensing optical depth. Accounting for inefficiencies in the monitoring program, we write \( n \approx \epsilon t \). Here \( t \) is the duration of the monitoring program, \( d \) is the mean duration of lensing events, and \( \epsilon \) is the fraction of events that will be detected given the time sampling of the observing program. The observing efficiency \( \epsilon \) can be approximated as \( \epsilon = \epsilon_1 \epsilon_2 \), where \( \epsilon_1 \) is the fraction of the year a source can be monitored and \( \epsilon_2 \) is the fraction of the events with durations longer than the time between observations and shorter than \( t \). Note, though, that a precise calculation of \( n \) would require the full six-dimensional phase space distribution function of the lenses to determine \( \epsilon \). We use the approximations \( d \sim 30 \) days and \( \epsilon \sim 0.3 \), so that \( n \sim 3.5 (d/\text{yr}) \).

A simulation consists of assuming a uniform number of trials \( n \) for all clusters, and for each cluster drawing a random number of “observed” events from a Poisson distribution with mean \( \mu = N_e n s \) under an assumed Galaxy model. The likelihood of obtaining the resulting fake data set is then computed under this model and under an alternate model. The model with the larger likelihood is taken as the inferred model. Many such simulations are run to obtain the probability of inferring the wrong model for each assumed model/alternate model pair. The test is then repeated for many values of \( n \).

The probability of inferring the wrong model from a pair can also be calculated analytically. Let \( \tau_i \) be the optical depth, \( N_j \) be the effective number of resolved stars, and \( \mu_j = n_j N_j \tau_i \) be the expected number of events for cluster \( j \) under model \( i \). If model 1 is correct, the probability of obtaining a data set whose likelihood under model 2 exceeds that under model 1 is

\[ P(\text{error}) = \frac{1}{2} + \frac{1}{2\pi} \int_{-\infty}^{\infty} k \times \sin \left[ \sum_{j=1}^{N} k (\mu_j - \mu_j) \sin [k \ln (\mu_j)] \right] \times \exp \left[ \sum_{j=1}^{N} \mu_j \right] \left[ \cos [k \ln (\mu_j)] - 1 \right] \, dk. \]

A more readily calculated (though approximate) scaling is that \( P(\text{error}) \approx \text{erfc}(h)/2 \), where \( \text{erfc}(x) = 2 \left[ \int_{x}^{\infty} e^{-t^2} \, dt \right]^{1/2} \) is the complementary error function, and where multiplying all the \( n_j \) by a factor \( a \) changes \( h \) by a factor \( a^{1/2} \). Interested readers may contact the authors for derivations.

Table 1 contains values of the effective number of trials \( n_{\text{eff}} \) required to distinguish between the two models in each test at the 90% confidence limit (i.e., with a 10% chance of inferring the incorrect model). Because this probability need not be the same for the two input models, two values of \( n_{\text{eff}} \) are tabulated for each test. To determine \( n_{\text{eff}} \), we interpolate between measured error probabilities in simulations at a few values of \( n \).

These calculations do not include the optical depth \( \tau \) due to self-lensing (i.e., lensing of one cluster star by another). We estimate \( \tau \sim 10^{-9} \), which is negligible compared to foreground optical depths (Fig. 1).

The microlensing observations of the globular cluster system are most powerful in our tests 5 (flat vs. flaring disk), 1 (halo core radius), 3 (disk scale length), and 4 (local disk surface density), where large-scale structural parameters of the Gal-
TABLE 1

PARAMETERS FOR TESTED GALAXY MODELS

| Model | $\Sigma_0$ | $r_0$ | $v_o$ | $r_c$ | $M_s$ | $\phi$ | log$_{10}(t_{90})$ | $E_{\text{ratio}}$ |
|-------|------------|-------|-------|-------|------|------|-------------------|-----------------|
| 1A    | 40         | 3.5   | 175   | 2.5   | 0.8  | 1.00 | 25                | 0.82            |
| 1B    | 50         | 3.0   | 201   | 10.0  | 0.8  | 1.31 | 25                | 0.78            |
| 2A    | 50         | 3.0   | 183   | 6.9   | 1.00 | 1.00 | 25                | 1.60            |
| 2B    | 50         | 3.0   | 183   | 6.9   | 0.625| 1.00 | 25                | 1.63            |
| 3A    | 67         | 3.5   | 161   | 5.43  | 0.8  | 1.00 | 25                | 1.10            |
| 3B    | 50         | 2.5   | 220   | 16.15 | 0.8  | 1.00 | 25                | 0.86            |
| 4A    | 80         | 3.31  | 200   | 15.21 | 0.8  | 1.00 | 25                | 1.19            |
| 4B    | 40         | 2.65  | 200   | 8.6   | 0.8  | 1.00 | 25                | 1.21            |
| 5A    | 50         | 3.0   | 183   | 6.9   | 0.8  | 1.00 | 25                | 0.40            |
| 5B    | 50         | 3.0   | 183   | 6.9   | 0.8  | 1.00 | 25                | 0.46            |
| 6A    | 50         | 3.0   | 183   | 6.9   | 0.8  | 1.00 | 25                | 2.93            |
| 6B    | 49         | 3.0   | 183   | 6.9   | 0.8  | 1.00 | 25                | 2.93            |
| 7A    | 50         | 3.0   | 183   | 6.9   | 0.8  | 1.00 | 25                | 1.60            |
| 7B    | 50         | 3.0   | 183   | 6.9   | 0.8  | 1.00 | 25                | 1.63            |

Note.—See text for definitions of model parameters. The units are as follows: $\Sigma_0$ (M$_\odot$ pc$^{-2}$); $r_0$ (kpc); $v_o$ (km s$^{-1}$); $r_c$ (kpc); $M_s$ ($10^{10}$ M$_\odot$); $\phi$ (deg). Remaining quantities are dimensionless. The primary parameter tested in each model pair is in boldface in the table. The thin disk vertical scale height $z_v = 0.20$ kpc unless otherwise noted. log$_{10}(t_{90})$ and $E_{\text{ratio}}$ give the results of the simulations. $E_{\text{ratio}}$ is essentially the number of lensing events required to distinguish a model from its alternative at the 90% confidence level; $t_{90}$ is the corresponding effective number of trials (as defined in text). The 1σ random error in $t_{90}$ and $E_{\text{ratio}}$ is 2.8%.

a Disk scale height increases with Galactocentric radius. See text for description.
b Thick disk present. Column density 1 M$_\odot$ pc$^{-2}$ at Sun; scale height 0.60 kpc, scale length 3 kpc.

dxy’s most massive components change. They are not very sensitive to the presence or absence of a low-mass thick disk (test 6) or to changes in the bar geometry within plausible limits (test 7). The sensitivity to test 5 is in part because model 5B is an extreme flaring model.

Our simulations assumed the same observational program for each globular cluster. Different clusters can be given different priorities, leaving out the poorest clusters and selectively including rich clusters depending on what facet of Galactic structure is being probed. Illustrative simulations with restricted subsets of clusters show that the duration of a monitoring campaign need only be doubled to achieve 90% confidence discrimination between model pairs with a well chosen sample of ~10 clusters. The optimum cluster subsample will be different for different model pairs, however, and the results may be more strongly affected by halo substructure for small samples. An independent study by Gyuk & Holder (1998) examines globular cluster microlensing for one such sample and reaches conclusions similar to ours.

4. CONCLUSIONS

The simulations in this Letter show that the total number of events required to distinguish between Galactic structure models is typically ~40–120 for the whole set of globular clusters. The required observing time depends on the lensing event duration, which will depend on the masses and the spatial distribution of the lenses and on the relative kinematics of the lenses and clusters. Statistics of event durations will provide further constraints on these properties of the lenses. For events of typical duration 30 days (the median observed duration toward the bulge), one should be able to distinguish between typical Galactic models discussed here (tests 1, 3, and 4) in 2–5 yr by monitoring the globular cluster system to a magnitude limit of $V < 22$. For comparison, ongoing microlensing projects have found 45 events in a year after monitoring ~10$^7$ stars toward the bulge and eight events for ~10$^7$ stars toward LMC in 2 yr. The event rate toward the globular cluster system should be between these two rates, because the clusters are spherically distributed in the Galactic halo with a concentration toward the bulge. Two to five years is not an excessive amount of time given that the MACHO, OGLE, and EROS monitoring programs have been in operation for 5 years (1992–1997).

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