Linearity analysis of a hemispherical resonator gyro (HRG) based on force-rebalance mode

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Abstract: Linearity is one of the most important static indexes of hemispherical resonator gyro (HRG). Linearity directly affects the measurement error of the HRG in the full range, thus determining the effective range of the HRG. Through finite element analysis of the linearity of the HRG under force-rebalance mode, we can provide a theoretical basis for compensating the linearity by circuit or software, improving the linearity of the gyro combination and increasing the maximum effective range of the gyro combination.

1. Introduction
With the development of inertial navigation technology, space, aviation, navigation, precision-guided weapons and other fields of inertial navigation system life and accuracy requirements continue to improve, and long-life, high-precision gyro technology achieve rapid development. The HRG is considered as one of the ideal choices for high precision inertial navigation systems in the early and mid 21st century due to its long life, high accuracy, relatively small size and independence from space irradiation.

Since the beginning of the 21st century, there has been a boom in the research of HRG in China. The theoretical analysis of the working principle of HRG was carried out based on the theoretical assumptions such as thin shell model and ring model, and the working principle of HRG under force-rebalance mode and full-angle mode and some corresponding error models were further analysed. However, the results of the published research so far show a linear relationship between the waveform progression of the HRG under force-rebalance mode, which is not consistent with the actual test phenomena in engineering and cannot explain the non-linearity of the HRG measurements. The measurement linearity of a HRG combination directly determines its maximum effective range and thus its application and range. Therefore, it is of practical importance to investigate the non-linearity of HRG under force-rebalance mode in order to provide a theoretical basis for improving the linearity of the gyroscope combination and increasing the maximum effective range of the gyroscope combination by compensating for the linearity through circuitry or software.

2. Working principle of HRG under force-rebalance mode
The HRG consists of a hemispherical resonant oscillator, an excitation electrode and a signal detection unit. According to the different excitation modes, there are closed-loop force-rebalance mode and full-angle mode.
The working principle of the force-rebalance mode HRG is shown in Figure 1.

In Figure 1, C1 and C2 represent the amplitude detection electrode and excitation electrode at the excitation point, C3 and C4 represent amplitude detection electrode and force-rebalance electrode at the node point. The closed loop circuit formed by C1 and C2 controls the amplitude of the excitation point to maintain a fixed value, while the closed loop circuit formed by C3 and C4 inhibits the vibration waveform from moving, so that the vibration waveform works near the "zero" position.

When the gyroscope rotation makes the resonant oscillator vibration waveform relative to the shell in the ring direction into the movement, real-time change the force-rebalance control electrode excitation force size, so that the four wave belly vibration waveform relative to the shell does not occur deflection. The magnitude of the force-rebalance control electrode excitation force is proportional to the input angular velocity of the gyro, and the input angular velocity of the gyro can be calculated by detecting the control electrode voltage signal.

When there is an angular velocity input, the Coriolis effect caused by the hemispherical gyro deflects the vibration waveform. In this process, the Coriolis force needs to overcome the internal damping of the resonator material, elastic hysteresis and elastic after-effects, coating stresses, etc., in addition to physical defects such as uneven distribution of resonator stiffness. Disregarding these random influences, for a hemispherical resonator with an ideal structure, a HRG operating in force-rebalance mode has two factors affecting its linearity: the non-linear influence of the resonator oscillation deflection itself in fixed position excitation mode, and the influence of the force-rebalance control mode on linearity.

3. FEA of the hemispherical resonator under Excitation and Node point
In the following analysis, the diameter of the surface in the hemispherical resonator is taken as 30 mm, and the thickness of the hemispherical shell is taken as 0.9 mm. Figure 2 shows the finite element model of the hemispherical resonator simulating the excitation load applied by the excitation electrode. In order to reduce the number of finite element nodes and to improve the symmetry and uniformity of the meshing of the working part of the hemispherical resonator, the part of the hemispherical resonator involved in the resonance is meshed with a hexahedral mesh. As the support rod is connected to the hemisphere by a rounded transition, the use of hexahedral meshing for the support rod part is likely to cause mesh distortion and cell degradation, so adaptive meshing is used for the resonator support rod. The Coriolis effect was switched on in ANSYS and the harmonic response analysis was carried out with the following results.
Figure 3 to Figure 7, the high amplitude curve represent the excitation point. In Figure 7, when the angular velocity reaches a certain value, the excitation point amplitude will appear waveform interference, can not guarantee stable four-wave belly vibration.
Table 1 Angular velocity of gyro versus amplitude at two points

| Angular Velocity | Amplitude |
|------------------|-----------|
|                  | 0 rad/s   | 5 rad/s   | 10 rad/s  | 20 rad/s  | 40 rad/s  |
| Excitation (μm)  | 11.4655   | 11.4265   | 11.3126   | 11.0191   | 10.155    |
| Node (μm)        | 1.77638   | 2.4155    | 3.07272   | 4.24928   | 5.79182   |

Fig.8 Amplitude at two points with different Angular velocity

In Figure 8, as the angular velocity increases, the excitation point and wave node amplitude will show a significant non-linear change. However, within a range of 10 rad/s (approximately 573°/s), the wave node amplitude can still maintain a good linearity (0.1%), which is sufficient to meet the needs of engineering applications.

When the angular velocity of the gyro is 0, the wave node amplitude is usually assumed to be zero in the theoretical analysis, but it is not consistent with the actual engineering. As can be seen from the simulation results, the node point is actually involved in the vibration. For the hemispherical resonator shown in Figure 2, when the excitation point amplitude is 11.4655μm, the wave node amplitude is 1.77638μm, and the doubled frequency signal measured in engineering is verified through finite element simulation analysis.

4. Analysis of non-linearity under force-rebalance control mode

Ideally, the HRG under the force-rebalance control mode rotates so that the resonator subvibrational pattern is in the ring direction, and the magnitude of the force-rebalance control electrode excitation force is changed in real time so that the wave belly vibration pattern does not deflect relative to the shell, and the magnitude of the force-rebalance control electrode excitation force is proportional to the gyroscope input angular velocity. However, in fact, there is a certain amount of deflection of the wave belly oscillator relative to the shell, but the angle of deflection of the wave belly oscillator relative to the shell under force-rebalance mode is much smaller than the angle of deflection in open-loop mode.
The thin solid line in Figure 9 is the elliptical curve when the resonator is not fed, the thick dashed line is the ideal elliptical curve when the resonator is fed, and the thick solid line is the actual elliptical curve when the resonator is fed. Under force-rebalance mode, the amplitude of the excitation point remains constant due to the amplitude control loop, i.e. the mass point on the excitation axis (X-axis) in Figure 10 maintains a constant amplitude, so when the wave belly deflects, the actual elliptical curve is shown as the thick solid line in the figure, i.e. the amplitude of the wave web becomes larger under the action of the amplitude control loop.

Assuming that the resonator wave web deflection angle is θ, the long semi-axis of the ellipse after the resonator wave web deflection is OB, the long semi-axis of the ellipse when the resonator wave web is not deflected is \(|OA|\), in the direction of the wave node point (45° orientation in Figure 9), the above three ellipses corresponding to the mass point distance from the centre are \(|OC|\), \(|OD|\), \(|OE|\), after the vibration type deflection θ angle, the wave node point amplitude ideally changes to \(|CE|\), the actual amount of change is \(|CD|\).

Let the long semi-axis of the ellipse be a and the short semi-axis be b when the oscillation is not deflected, and the long semi-axis of the ellipse be m and the short semi-axis be n when the oscillation is deflected by θ angle.

The length of the line segment formed by the intersection of the actual ellipse and the X-axis after deflection as:

\[
|OA|^2 = \frac{m^2 n^2 (1 + \tan^2(\theta))}{n^2 + m^2 \tan^2(\theta)} = a^2
\]  

The value of the transverse coordinate of the actual ellipse at the node after the deflection of the vibration pattern as:

\[
X^* = \frac{m^2 n^2}{m^2 \left(\frac{1 - \sin(\theta)}{1 + \sin(\theta)}\right)^2 + n^2}
\]

The value of the ellipse at the nodal point when the vibration pattern is not deflected as:

\[
X_0^* = \frac{a^2 b^2}{a^2 \left(\frac{1 - \sin(\theta)}{1 + \sin(\theta)} + \sin(\theta)\right)^2 + b^2 \left(\frac{\sin(\theta) - \sin^2(\theta)}{1 + \sin(\theta)}\right)^2}
\]

Similarly, the value of the corresponding transverse coordinate of the ideal ellipse after deflection can be obtained as:
\[ X_1^{'2} = \frac{a^2b^2}{a^2\left(1 - \sin(\theta)\right)^2 + b^2} \]  

Then the line segment

\[ |CD| = \frac{1}{\cos(45^\circ - \theta)} \left\{ \frac{n^2}{m^2 - \frac{1 - \sin(\theta)}{1 + \sin(\theta)}} \right\} + \frac{a^2b^2}{a^2\left(1 - \sin(\theta)\right)^2 + b^2} - \frac{a^2b^2}{a^2\left(1 + \sin(\theta)\right)^2 + b^2\left(1 - \frac{\sin(\theta) - \sin'(\theta)}{1 + \sin(\theta)}\right)} \]  

\[ |CE| = \frac{1}{\cos(45^\circ - \theta)} \left\{ \frac{a^2b^2}{a^2\left(1 - \sin(\theta)\right)^2 + b^2} - \frac{a^2b^2}{a^2\left(1 + \sin(\theta)\right)^2 + b^2\left(1 - \frac{\sin(\theta) - \sin'(\theta)}{1 + \sin(\theta)}\right)} \right\} \]  

Then the output error is

\[ \Delta = \frac{|CE| - |CD|}{|CE|} \]  

Let the amplitude of the resonant oscillator be 10μm at rest, and the ellipse circumference is constant, according to the calculation of equation above, it is known that when the resonant oscillator wave ventral rotation angle is about 1.126°, the output error reaches 5×10^{-4}.

5. Conclusion

Through finite element simulation analysis of the hemispherical resonator feed characteristics, the results show that the resonator feed characteristics are not the main factor affecting the non-linearity of the HRG in the range of angular velocities commonly measured in the field of inertial navigation, and the doubled frequency signal measured in engineering is verified through finite element simulation analysis. In order to increase the effective measurement range of the HRG, the wave belly deflection angle can be controlled within 1.126° by improving the resonant oscillator amplitude detection resolution and optimising the control loop parameters, or by taking corresponding compensation measures through the above principle, so as to meet the 0.05% linearity index usually required in current engineering, while effectively increasing the measurement angular velocity range.

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