Numerical Method of Determination of Optimal Parameters of the Beam Orientation System

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Abstract. Lasers are the sources of radiation of electromagnetic waves of high directivity. This characteristic is used in the beam orientation systems of moving object different versions of construction of beam orientation systems are possible. The paper describes the method of circular lines of sighting, which makes it possible to determine the laser beam location in the coordinate system of a moving object. The orientation system parameters should be selected so that the accuracy of orientation is maximal. The numerical method is used to investigate the number of conditionality of the set of two equations, which enables us to determine the laser beam location in space. The orientation system parameters are determined, wherein the number of conditionality is minimal, and, thus, the errors of the orientation system are minimal.

1. Introduction

High directional effect of laser radiation as compared with electromagnetic radiation of radiofrequency region has made it possible to create narrow bands of directed radiation. Molecular and aerosol scattering of electromagnetic waves visualizes the beams of electromagnetic radiation that enables us to create the device, which determines the beam position in a certain coordinate system. The line of sight of laser beam is designated the line passing through the laser beam (intersecting it). The position in space of line of sight is given by a point and a direction vector. The equation of the line of sight is written in the form:

\[
\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c},
\]

where \(x_1, y_1, z_1\) are the point coordinates through which the line of sight of a beam passes; \(a, b, c\) are the components of the direction vector proportional to the direction cosine; \(x, y, z\) are the running coordinates of the line of sight. The line of sight position can be determined using a photodetector with a small viewing angle. Figure 1 shows the scheme of the device, which enables us to determine the circular line of sight. A photodetector with a small viewing angle is mounted on the rotation axis connected with the meter of angle of turn. At the photodetector output a signal occurs due to molecular and aerosol scattering of radiation when a laser beam enters the visual field of the photodetector. At this point a unit of signal selection supplies a pulse to a meter of angle of turn. Figure 2 shows the order of determination of components of the direction vector of the line of sight based on data of the meter of angle of turn. The visual field of the photodetector is directed along \(r\) and forms with the rotation axis the angle \(a\). If we set \(c=1\), then the components of the direction vector are determined by the expressions...
\[ a = \tan \alpha \cos \varphi; \quad b = \tan \alpha \sin \varphi. \] 

In the general case to determine the laser beam position in space we need only have knowledge of five circular lines of sight.

2. Determination of laser beam position when the plane of laser beam position is known

The laser beam position plate is designated the plate passing through a laser beam. For example, for orientation of vessels of water transport a laser beam can be positioned parallel to the water surface at the height \( h \) from the water surface (Figure 3). Two photodetectors are located at the height \( h_1 \) from the water surface and allow us to determine the circular lines of sight of laser beam. The rotation axes of photodetectors are in the horizontal plate, in this case the visual field of the first photodetector is positioned at the angle \( \alpha_1 \) to the rotation axis, and the visual field of the second photodetector is positioned at the angle \( \alpha_2 \). Now we determine the laser beam position in the coordinate system connected with an object being oriented. In this case the unit vectors of this coordinate system form the right basis. The axis \( z \) is directed along the rotation axes of photodetectors, and the axis \( y \) is directed perpendicular to the water surface. For the first and second lines of sight we can write:
where \((x,H,0)\) are the coordinates of the point of laser beam intersection with the plane \(XOY\), \((m,0,1)\) are the components of the direction vector of laser beam, \((a_1,b_1,1)\), \((a_2,b_2,1)\) are the components of the direction vector of the first and second lines of sight, \(H=h-h_1\) is the difference of heights of laser and rotation axes of photodetector.

From (3) we drive

\[
\begin{cases}
xb_2 + Hm = Ha_2, \\
xh_1 + Hm = Ha_1.
\end{cases}
\]  

We can express \(a_1, a_2, b_1, b_2\) with taking account of (2) and obtain:

\[
x = \frac{H(\cos \varphi_2 - \cos \varphi_1 \tan a_2 / \tan a_1)}{\sin \varphi_1 - \sin \varphi_2 \tan a_2 / \tan a_1},
\]

\[
m = \frac{\tan a_2 \sin(\varphi_1 - \cos \varphi_2)}{\sin \varphi_1 - \sin \varphi_2 \tan a_2 / \tan a_1},
\]

It is convenient to connect the coordinate system, in which the laser beam position is determined so that the axis \(OZ\) coincides with the course of oriented object, then the angle between the object course and the beam is \(\beta = \arctg m\), and \(x\) characterizes the object deviation from the beam.

![Figure 3. Determination of laser beam position, when the plane of laser beam position is known.](image)

Now we consider the question of a choice of parameters of the system of \(a_1, a_2, H\) at which the measurement errors \(x\) and \(\beta\) are minimal (Figure 4). The measurement of the parameter \(\varphi\) of the circular line of sight is performed with an error due to the measurement error of the angle of turn as well as due to the presence of divergence of the angle of visual field of the photodetector, as a result, the measurement error of the point \(A\) on the plane \(y=H\), through which the beam passes, is characterized by the least circle of aberration. In the same way the point \(B\), determined by the photodetector 2, can be calculated with the error characterized by the least circle of aberration. Therefore the laser beam can pass through any point of the least circle of aberration of the point \(A\) and any point of the least circle of aberration of the point \(B\).

It is evident that at \(a_2=90^\circ\) (with the availability of the measurement errors \(\varphi_2\) and \(\varphi_1\)) \(x\) will be determined with high precision, because in this case the error of determination of \(x\) decreases, and the
determination error of $m$ remains constant.

Now we represent (4) in the matrix form taking into account $\alpha_2=90^\circ$

$$
\begin{pmatrix}
\sin \varphi_2 & 0 \\
tg \alpha_1 \sin \varphi_1 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
\frac{x}{H}
\end{pmatrix} =
\begin{pmatrix}
\cos \varphi_2 \\
tg \alpha_1 \cos \varphi_1
\end{pmatrix},
$$

(7)

The matrix $A = \begin{pmatrix}
\sin \varphi_2 & 0 \\
tg \alpha_1 \sin \varphi_1 & 1
\end{pmatrix}$ is not degenerated at $\varphi_1, \varphi_2 \in (0, \pi)$.

As a degree of conditionality of the matrix $A$ we take the number $\text{cond} A$ determined as

$$
\text{cond} A = \|A\|_E \|A^{-1}\|_E^{-1},
$$

(8)

where $\|A\|_E, \|A^{-1}\|_E$ are the Euclidean norms of matrix $A$ and $A^{-1}$, respectively.

Using the equalities

$$
\|A\|_E = \sqrt{\text{Sp}(A^T A)} \quad \text{and} \quad \|A^{-1}\|_E = \sqrt{\text{Sp}(A^{-1})^T (A^{-1})^T},
$$

(9)

where $\text{Sp}(A^T A), \text{Sp}(A^{-1})^T (A^{-1})^T$, the spur of matrices $A^T A$ and $(A^{-1})^T (A^{-1})^T$, respectively; $T$ is the sign of transposition, and we can write

$$
\text{cond} A = \sqrt{2 + 2tg^2 \alpha_1 \sin^2 \varphi_1 + \sin^2 \varphi_2 + \frac{1}{\sin^2 \varphi_2} \left(2tg^2 \alpha_1 \sin^2 \varphi_1 + tg^4 \alpha_1 \sin^4 \varphi_1 + 1\right)}.
$$

(10)

The number of conditionality $\text{cond} A$ of the matrix $A$ calculated by (8) shows how many times the ratio of root-mean-square error of the unknown to the root-mean-square unknowns themselves exceeds the ratio of root-mean-square errors of coefficients of the system to the root-mean-square coefficients of the system.

Hence $\alpha_2, H$ should be chosen such as in the orientation zone of objects the number of conditionality of the matrix $A$ $\text{cond} A$ is minimal.

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**Figure 4.** The choice of optimal parameters of the orientation system.
3. Optimal parameters of the beam orientation system

In the optimal analyzer the system of equations (7) should be well stipulated, i.e., its solution should be insensitive to errors or to the uncertainty of input data. Figure 5a, b, and c show that the number of conditionality of the matrix $A$ in the range $\varphi_1=(45^\circ...140^\circ)$, $\varphi_2=(45^\circ...140^\circ)$, $\alpha_1=(10^\circ...50^\circ)$ does not exceed 5, that is a very good value and shows good conditionality of the system of equations (7) in the above-mentioned angular ranges of $\varphi_1$, $\varphi_2$, $\alpha_1$. Slight change of the number of conditionality in the above-mentioned angular ranges, $\varphi_1$, $\varphi_2$, $\alpha_1$, shows that the system of orientation has a large operation zone.

The function $\text{cond}A=f(\varphi_2)$ always reached minimum at $\varphi_2=90^\circ$. Relative to the parameter $H$ we can say that it must be increased because the angle $\varphi_2$ in this case will less differ from $90^\circ$ when moving from the laser beam. Relative to the parameter $\alpha_1$ we can say that it can be decreased because this leads to the decrease of $\text{cond}A$ in the above-mentioned angular ranges $\varphi_1$, $\varphi_2$ (Figure 6) $\text{cond}A$ increases rarely at $\varphi_1>60^\circ$.

![Figure 5](image_url)

**Figure 5.** Numerical investigation of the function $\text{cond}A=f(\varphi_1, \varphi_2, \alpha_1, \alpha_2) (\varphi_2=90^\circ)$: $a-\alpha_1=10^\circ$; $b-\alpha_1=30^\circ$; $c-\alpha_1=50^\circ$. 
Figure 6. The behavior of the function $\text{cond} A = f(\alpha_1, \alpha_2, \varphi_1, \varphi_2) \ (\alpha_2 = 90^\circ, \varphi_1 = 90^\circ, \varphi_2 = 90^\circ)$.

References
[1] Voevodin V V and Kuznetsov Yu A 1984 Matrix and calculations (Moscow, Nauka)