Towards the use of asteroseismology to investigate the nature of dark matter

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ABSTRACT
The annihilation of huge quantities of captured dark matter (DM) particles inside low-mass stars has been shown to change some of the stellar properties, such as the star’s effective temperature or the way the energy is transported throughout the star. While in the classical picture, without DM, a star of 1 M\textsubscript{☉} is expected to have a radiative interior during the main sequence, the same star evolving in a halo of DM with a density $\rho_\chi > 10^8$ GeV cm\textsuperscript{-3} will develop a convective core in order to evacuate the energy from DM annihilation in a more efficient way. This convective core leaves a discontinuity in the density and sound-speed profiles that can be detected by the analysis of the stellar oscillations. In this paper we present an approach towards the use of asteroseismology to detect the signature produced by the presence of DM inside a star, and we propose a new methodology to infer the properties of a DM halo from the stellar oscillations (such as the product of the DM density and the DM particle-nucleon scattering cross-section).

Key words: dark matter - asteroseismology - stars: interiors - stars: low-mass - stars: fundamental parameters - Galaxy: centre

1 INTRODUCTION
Different observations in a wide range of scales, from galactic to cosmological, suggest the existence of a new kind of matter, called Dark Matter (DM), formed by unknown particles. Among the possible constituents of DM, the WIMPs, massive particles with non-negligible scattering cross-section with baryons, are considered one of the best candidates (Bertone, Hooper & Silk 2003).

Soon was realised that, if WIMPs exist, they will accumulate inside stars (Press & Spergel 1985) and their annihilation may lead to significant changes in the classical picture of stellar evolution if the halo where the stars evolve has a very high density of DM particles (Bouquet & Salati 1989; Salati & Silk 1989; Dearborn et al. 1990). In this context, the effects of the capture of WIMPs by the Sun were studied, addressing the prospects of helioseismology to test models that solved the old solar neutrino problem (Dappen et al. 1984; Faulkner et al. 1986), and to give constraints to the nature of DM particles (Lopes, Bertone & Silk 2002; Lopes & Silk 2002; Lopes, Bertone & Silk 2002; Bottino et al. 2002; Cumberbatch et al. 2010; Frandsen & Sarkar 2010; Taoso et al. 2010).

Recently, particular attention has been given to the first stars formed in the early Universe due to the high DM content that epoch (Spolyar, Freese & Gondolo 2008; Iocco 2008; Freese, Spolyar & Aguirre 2008; Taoso et al. 2008; Schleicher, Banerjee & Klessen 2009; Natarajan et al. 2009; Ripamonti et al. 2010; Sviretsson & Gondolo 2010), including the prospects for their detection with the JWST telescope (Freese et al. 2010; Zackrisson et al. 2010). Similarly, other authors focused on the DM effects on stars in the local Universe, either on compact stars (Moskalenko & Walter 2007; Bertone & Fairbairn 2008; Isern et al. 2008, 2010; de Lavallaz & Fairbairn 2010; Kouvaris & Tinyakov 2010; Perez-Garcia, Silk & Stone 2010) or on low-mass stars (Fairbairn, Scott & Edsjö 2008; Scott, Fairbairn & Edsjö 2008; Casanellas & Lopes 2009).

The purpose of this paper is to pave the way for the use of asteroseismology to provide an evidence of the footprint left by DM annihilation on the stellar oscillations. To do this, we will concentrate on solar-mass stars that evolve in haloes with very high DM densities, and we will show how asteroseismology may tell us about the properties of such DM haloes.
2 STELLAR EVOLUTION WITHIN DENSE DARK MATTER HALOES

The evolution of a star within a halo of DM depends strongly on the ability of the gravitational field of the star to capture the DM particles that populate the halo. The rate at which the DM particles are captured is given by (Gould 1987)

\[ C_X(t) = \sum_i \int_0^{R_X} 4\pi r^2 \int_0^{\infty} f_{\nu_i}(u) \frac{w}{u} w\Omega_{\nu_i}(w) du \, dr, \]  

where \( f_{\nu_i}(u) \) is the velocity distribution of the DM particles seen by the star (which is proportional to the density of DM on the host halo \( \rho_X \) and inversely proportional to the mass of the DM particles \( m_X \)) and \( \Omega_{\nu_i} \) is the probability of a DM particle to be captured after the collision with an element i (which is proportional to the scattering cross-section of the DM particle with the nucleus i, \( \sigma_{\chi_i} \)). The numerical subroutines to calculate the capture rate (equation[1]) were adapted from the publicly available DarksuSy code (Gondolo et al. 2004). Our assumptions regarding this calculation are described in Casanellas & Lopes (2010).

Once DM particles are captured, they accumulate in a small region in the core of the star (\( r_X \approx 0.1 \) R\( \star \) for \( m_X = 100 \) GeV). There, assuming that they are Majorana particles, they annihilate providing a new source of energy for the star. Capture and annihilation processes balance each other in a short time-scale, and consequently almost all captured particles will be converted to energy, contributing to the total luminosity with \( L_X = f_X C_X m_X \). The factor \( f_X \), which in this work we assumed to be 2/3 (Freese et al. 2008), accounts for the energy that escapes out of the star in the form of neutrinos. Recent Monte Carlo simulations suggest that the fraction of the energy lost in neutrinos may be even smaller (Scott et al. 2004).

Due to this new source of energy, stars will evolve differently from the classical picture if surrounded by a dense halo of DM. For very high DM densities (\( \rho_X > 3 \times 10^9 \) GeV cm\(^{-3} \) for a 1 M\( \odot \) star), the energy from DM annihilation prevents the gravitational collapse of the star, stopping its evolution in the pre-main-sequence phase, before the star could reach enough central temperature to trigger hydrogen burning. (Casanellas & Lopes 2003).

For lower DM densities (10\(^8\) GeV cm\(^{-3}\) < \( \rho_X \) < 3 \times 10\(^9\) GeV cm\(^{-3}\) for a 1 M\( \odot \) star), DM burning is a complementary source of energy for the star. As it is produced in a region much more concentrated than the nuclear burning, which normally extends up to 0.1–0.2 R\( \star \), the radiative temperature gradient \( \nabla_{rad} = \frac{d\ln T}{d\ln P} \) is much steeper in the core of the star. Consequently, as the radiative transport is not efficient enough to evacuate all the energy in the central region, the star develops a convective core which was not present in the classical scenario without DM. The radius and duration of the convective core increase when more energy from DM annihilation is produced (Scott et al. 2004); therefore, they depend on the density of DM in the place where the star evolves and on the properties of the DM particles. The balance between DM annihilation, nuclear burning, and the gravitational energy leads to a new hydrostatic equilibrium with a lower central temperature. The star consumes its hydrogen at a lower rate, extending the time that it spends in the main sequence. These new properties allow us, as it will be shown, to provide a tool to infer the DM characteristics from the stellar oscillations using the seismological analysis.

3 BASICS OF ASTEROSEISMOLOGY

With the improvement on the quality of the data, asteroseismology is now becoming a precise tool to infer the properties of stars showing solar-like oscillations (Michel et al. 2008; García et al. 2009; Bedding et al. 2010), which are driven by turbulence in the superficial layers of the star. The eigenfrequencies of solar-like oscillations can be approximated, for \( l/n \rightarrow 0 \) (where \( l \) and \( n \) are the degree and the radial order of the modes), by the asymptotic expression

\[ \nu_{n,l} = \frac{n + \frac{1}{2} + \epsilon_n}{\nu_0} + O(\nu^{-2}), \]  

where \( \nu_0 = \left[ 2 \int_0^{R} \frac{d\rho}{\rho} \right]^{-1} \) is the inverse of twice the time spent by the sound to travel between the centre and the acoustic surface of the star, and \( \epsilon_n \) is determined by the properties of the surface layers. For a more in-depth explanation of the basics of the seismological analysis, the reader is referred to (Tassoul 1981; Gough 1985; Lopes & Turck-Chieze 1994). The value of \( \nu_0 \) can be estimated through the large separation \( \Delta \nu_{n,l} \):

\[ \Delta \nu_{n,l} = \nu_{n,l} - \nu_{n-1,l} \approx \nu_0 \]  

This parameter is sensitive to the mean density of the star: \( \Delta \nu_{n,l} \propto (M/R^3)^{1/2} \) (Cox 1980), while the small separation \( \delta \nu_{n,l} \), given by

\[ \delta \nu_{n,l} = \nu_{n+l} - \nu_{n-1,l+2}, \]  

is sensitive to the temperature and chemical gradient in the deep interior.

In the last years, other relations between the frequencies of the oscillation modes were proposed (for a recent review, see Christensen-Dalsgaard & Houdek 2000 or Aerts et al. 2010), broadening the diagnostic potential of seismology. Among the possible diagnostic cores of convective cores and envelopes (Monteiro et al. 1994; Gough 1985; Lopes & Gough 2001), we highlight the ratios between the small separations and the large separations developed by Roxburgh & Vorontsov (2003) in order to suppress the effects of the modelling of the near-surface layers:

\[ r_{10} = \frac{d_{10}}{\Delta \nu_{n,1}} \]  

\[ r_{10} = \frac{d_{10}}{\Delta \nu_{n+1,0}}, \]  

where

\[ d_{10} = \frac{1}{8}(\nu_{n-1,1} - 4\nu_{n-1,0} + 6\nu_{n,0} - 4\nu_{n+1,0} + \nu_{n+1,1}), \]  

\[ d_{10} = -\frac{1}{8}(\nu_{n-1,1} - 4\nu_{n,0} + 6\nu_{n+1,0} - 4\nu_{n+1,1} + \nu_{n+1,1}). \]  

The mixing of elements produced in the convective regions introduces a sharp structural variation in the border with the radiative regions that can be seen in the density and sound-speed profiles. This sharp structural variation produces an oscillatory signal in the frequency spectrum (Gough 1990), whose period is related with the acoustic depth of the discontinuity inside the star. Recently, Silva Aguirre et al. (2010) proposed the use of the ratios \( r_{10} \) and \( r_{10} \) to determine the size of a convective core by fitting...
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4 ASTEROSEISMIC SIGNATURE OF DARK MATTER PARTICLES

To grasp the signature that the annihilation of captured DM particles leaves on low-mass stars we evolved a set of 1 M☉ stars, with the same initial conditions (Z=0.018), in haloes of DM with different densities ρχ and different spin-dependent (SD) WIMP-nucleon cross-sections σχ,SD. Throughout our work, we considered fiducial values for the mass of the DM particles, mχ = 100 GeV, and for their self-annihilation cross-section, <σv> = 3·10^{-26} cm^{2} s^{-1}. The evolution of the stars was computed using a well-established stellar evolution code (CESAM; Morel 1997) used to compute sophisticated solar models for helioseismology (Couvidat et al. 2003; Turck-Chièze et al. 2010) and more recently used in the context of asteroseismic studies (Kervella et al. 2004; De Ridder et al. 2006; Suárez et al. 2010). When the stars reached a luminosity equal to that of the Sun, a very precise mesh (with 1000 layers) was generated. Then, we calculated the frequencies of the oscillation modes of the stars using the ADIPLS code (Christensen-Dalsgaard 2008). The characteristics of some of these stars are shown in Table 1 and their sound-speed and density profiles, in Fig. 1.

The accretion and the annihilation of DM particles in the core of the stars may change significantly their properties. As expected, we found that the effective temperature of the stars that evolved in haloes with high DM densities is shifted to lower values (see Table 1), due to the presence of a convective core (see Fig. 1a), in agreement with previous works (Fairbairn et al. 2008; Casanellas & Lopez 2003). The lower effective temperature and the larger radius lead to a decrease in the large separation Δν_{n,l} (see Fig. 1b). For a star with a known mass, the drop in Δν_{n,l} predicted by the relation Δν_{n,l} ∝ M^{1/2} R^{-3/2}, is unmistakably related with the radius of the star. Furthermore, we also observed a drop on the small separation δν_{n,o} (see Fig. 1c), caused by a decrease in the central density. The strong dependence of the global modes on the density profile of the star is responsible for that drop.

In order to test the validity of our method, we checked if classical stars with similar characteristics may mimic the properties we described for stars evolving in DM haloes. In particular, we found that a star with a mass M_*=955 M☉ and a metallicity Z = 0.04 reaches, near the end of the main sequence, the same luminosity and effective temperature as the star (iv) in our set (see Table 1). At that moment, the radius of both stars is identical, leading to very similar great separations (<Δν_{n,l}>=128 μHz for star (iv) and 126 μHz for the other). However, as the star that evolved without DM is in a later stage of evolution (X_e=0.03, while X_e = 0.38 for star (iv), the small separation, being very sensitive to the chemical gradient in the deep interior, allows us to differentiate both stars. In our case, star (iv), which evolved in a dense halo of DM, has a <δν_{n,o}>=7 μHz. This is almost double than that of the star with different M_*, and Z (<δν_{n,o}>=4 μHz in that case).

\[ dr_{0213} ≜ \frac{D_{02}}{Δν_{n,-1,1}} - \frac{D_{13}}{Δν_{n,0}} \]  

where \[ D_{l,l+2} ≜ \delta ν_{n,l}/(4l + 6) \], was suggested by Cunha & Metcalfe (2007) to measure the amplitude of the sound-speed discontinuity at the edge of a convective core.

These seismic parameters (equations 4 and 8) are sensitive to the presence of DM inside a star, given that they are uniquely dependent on the star’s core structure and almost independent of the physical processes occurring in the surface layers.

Figure 1. Sound-speed (a) and density profiles (b) of 1 M☉ stars that evolved in DM haloes with different densities ρχ and SD WIMP-nucleon cross-sections σχ,SD when they reached a luminosity L = 1 L☉ (for each star, the product ρχσχ is indicated in the legend in GeV cm^{-1}).

Figure 2. (a) Size of the convective core, and the calculated seismolgic parameters: (b) mean large separation (for l=0,1,2,3), (c) mean small separation (for l=0) and (d) slope of dr_{0213}, for 1 M☉ stars that evolved in DM haloes with different densities ρχ and SD WIMP-nucleon cross-sections σχ,SD, when the stars reached a luminosity L = 1 L☉.
In addition, one of the most promising signatures of annihilating DM in stars is the fact that it can originate the formation of a convective core (unexpected in the classical picture for stars with masses < 1.2 M⊙) whose radius grows with the DM density ρχ. The convective core leaves a peculiar footprint in the profiles of the sound speed and density (see Fig. 1) characterized by a discontinuity in the edge of the core. The presence of the convective core can be detected by the seismological analysis using a relation between the small separation of modes with different degrees (and therefore with different depths of penetration inside the star). For that purpose low-degree modes (l = 0, 1, 2, 3) are chosen, because these modes are the ones that penetrate deep into the stellar core.

In particular, we found that the seismological parameter dr0213 (see equation 8) is sensitive to the sound-speed discontinuity at the edge of the convective core and, consequently, to the characteristics of the DM halo. In Fig. 3(a) we show the behaviour of the parameter dr0213 for stars that evolve in DM haloes with different characteristics. We found that the absolute value of the slope of dr0213 at high frequencies increases with the amplitude of the sound-speed discontinuity caused by the convective core, as predicted by Cunha and Metcalfe. Therefore, the slope of dr0213 is directly related with the amount of DM in the halo where the star evolves (see also Fig. 2d).

We also tested the method recently proposed by Silva Aguirre et al. (2010) designed to estimate the size of a convective core in 1.5 M⊙ stars. We found that the period of the sinusoidal fit to the ratios r01 and r10 (see Fig. 3b) does not match exactly the acoustic radius of the convective cores (see Table 1), most probably because we are applying this method to stars of mass 1 M⊙. However, the ratios r01 and r10 have a great sensitivity to the amplitude of the sharp variation of the sound speed caused by the annihilation of DM particles inside the star. We conclude that these ratios may be used in the future as a stellar probe to confirm the presence of DM in the neighbourhood of low-mass stars.

If enough radial modes are identified with the precision presently achieved by space-based telescopes as CoRoT (a relative error on the individual frequencies of ∼ 10⁻⁴ (Deheuvels et al. 2010)), then our method will allow the discrimination between haloes of DM with different characteristics. To illustrate this point, we plotted in Fig. 3 the error bars on dr0213, r01 and r10 for star (iii) derived from the mentioned uncertainty (10⁻⁴δν) on the determination of the frequencies, as done by Cunha & Metcalfe (2007).

5 DISCUSSION AND CONCLUSIONS

In this paper, we have presented a new methodology towards the use of asteroseismology to prove the presence of DM in the location where a star evolves. For a main-sequence star of 1 M⊙ evolving in a DM halo with a density ρχ > 10⁸ GeV cm⁻³ (assuming σχ,SD = 10⁻⁴⁸ cm²), the annihilation of captured DM particles on its interior leads to decreases in the large and small separations, when compared with the same star in the classical scenario without DM, which are related to changes in the global properties of the star. Furthermore, the highly concentrated production of energy by DM annihilation creates a convective core which is not present in the classical picture. This convective core leaves a discontinuity signature in the sound-speed and density profiles which can be detected by the analysis of the stellar oscillations.

We have shown that seismological parameters such as
The method presented in this paper is valid for haloes with very high DM densities. In Fig. 4 we show the DM densities at which a 1 M⊙ star with a luminosity 1 L⊙ are expected to show strong signatures on the seismic parameters $\Delta \nu, \delta \nu, \sigma_{0213}, r_{01}$ and $r_{10}$ (see text) due to the annihilation of DM particles with different characteristics $(m_\chi, \sigma_{\chi,p,SD})$ in their interior. In the particular case of the Galactic Centre (GC), the DM densities in the Figure (from top to bottom) are expected at a distance from the GC of 0.1 pc, 0.04 pc and 0.02 pc, following the adiabatically contracted profile of Bertone & Merritt. The grey lines are the present limits from direct detection experiments: XENON10 (dotted), PICASSO (dashed) and COUPP (solid), and the grey region is the DAMA/LIBRA allowed region.

$\sigma_{0213}$ and the ratios $r_{01}$ and $r_{10}$ are very sensitive to the size of the convective core, which is determined by the density of DM $\rho_\chi$ where the star evolved and by the scattering cross-section of the DM particles off nuclei $\sigma_\chi$. Consequently, this relationship may be used in the future to help in the determination of these parameters (or at least to their product, $\rho_\chi \sigma_\chi$) and to provide a stellar probe that identifies the presence of self-annihilating DM.

The precision required for our analysis is similar to the one achieved by present asteroseismic missions in observations of one hundred days. Nevertheless, the most likely place to find the kind of stars described here is near the centre of our Galaxy, where the distance and the presence of dust makes the observations difficult. These difficulties encourage us to extend our study to more massive and luminous stars in a future work. Future technical improvements in the observations of the GC and of the Milky Way dwarf spheroidal galaxies may open the possibility of using the method proposed here to investigate the nature of DM.

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**Figure 4.** DM densities at which 1 M⊙ stars with a luminosity 1 L⊙ are expected to show strong signatures on the seismicological parameters $\Delta \nu, \delta \nu, \sigma_{0213}, r_{01}$ and $r_{10}$ (see text) due to the annihilation of DM particles with different characteristics $(m_\chi, \sigma_{\chi,p,SD})$ in their interior. In the particular case of the Galactic Centre (GC), the DM densities in the Figure (from top to bottom) are expected at a distance from the GC of 0.1 pc, 0.04 pc and 0.02 pc, following the adiabatically contracted profile of Bertone & Merritt. The grey lines are the present limits from direct detection experiments: XENON10 (dotted), PICASSO (dashed) and COUPP (solid), and the grey region is the DAMA/LIBRA allowed region.
