**Light vector-like fermions in a minimal SU(5) setup**

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The Standard Model fermion sector is enlarged by either one light singlet vector-like down-type quark or one light vector-like lepton doublet, which might be accommodated within a five-dimensional representation of SU(5). At low energies the inclusion of these states affects precisely measured observables in flavor physics, as well as electroweak precision measurements. These experimental results strongly constrain couplings of vector-like states to the Standard Model particles. Having these bounds, we investigate the impact of vector-like fermions on the mass matrices for down-type quarks and charged leptons in an SU(5) setting. We find that unitary transformations relating an arbitrary flavor basis to the mass eigenstate one depend only on three free parameters. Then we discuss the parameter space constrained by low-energy data assuming vector-like quark and vector-like lepton masses to be 800 GeV and 400 GeV, respectively. The proposed setup uniquely determines proton decay widths. A further improvement of experimental bounds on proton decay modes would accordingly differentiate the allowed parameter space.

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I. INTRODUCTION

The LHC discovery of the Higgs boson has finally confirmed the correctness of the Standard Model (SM) picture of fundamental interactions [1, 2]. There is, however, a number of open issues that still point towards potentially new physics. These issues concern, for example, the origin of neutrino masses, the nature of dark matter, a hierarchy problem, the question of vacuum stability and the so-called flavor puzzle. One class of proposals, among many, that aims to address some of these issues, calls for the presence of vector-like fermions. These have been introduced either as a part of effective theories [3, 4] or within more elaborate frameworks such as Little Higgs models [5, 6], composite Higgs models [7, 8] and grand unified theory (GUT) models [10–23].

Vector-like fermions are primarily introduced in GUT models to modify mass relations among quarks and leptons [24]. Namely, the original $SU(5)$ model [25] predicts masses of down-type quarks and charged leptons to be degenerate at the scale of gauge coupling unification, i.e., the GUT scale $m_{\text{GUT}}$. However, the running of these masses to low energies yields substantially different values from experimentally observed ones (for a review see Ref. [26]). There are different ways to correct these erroneous mass predictions. One possibility is to add extra scalar multiplet(s) within $SU(5)$ [27]. The main drawback of this approach is the loss of predictive power due to the fact that these extra scalar multiplets have large dimensions [28–33]. Another possibility is the introduction of higher-dimensional operators in the Yukawa sector of the theory. Again, the price to pay is the loss of predictive power due to the presence of many possible terms. Needless to say, both of these possible modifications spoil the simplicity of the original setup. A third approach is to add vector-like fermions to the theory. This is the direction we plan to pursue.

We add one vector-like down-type quark and one vector-like lepton doublet that comprise a pair of five-dimensional representations of $SU(5)$ in order to obtain correct values for the masses of down-type quarks and charged leptons. A number of studies to include such fermions already exists for the $SU(5)$ GUT models with [17, 18, 20, 21] and without supersymmetry (SUSY) [22, 23]. However, all of the existing studies take the masses of the vector-like states to be at the GUT scale. In our work, on the other hand, we assume vector-like fermions to be light enough to be accessible at LHC and study current experimental constraints on the couplings of these states to the matter fields. Recent phenomenological analyses of vector-like fermions in different representations can be found for example in [34–41].

The minimal vector-like fermion extension of the original $SU(5)$ proposal requires very few new couplings, preserves renormalizability and reproduces mass relations among charged leptons and down-type quarks in accordance with experimental data. Our study demonstrates that the setup is very predictive with regard to proton decay signatures through both scalar and gauge boson mediation.

Light vector-like fermions necessarily affect low-energy observables. However, the low-energy flavor phenomena involving quarks are rather well described within the SM framework. This, therefore, strongly restricts the effective couplings of vector-like quarks with the matter fields and the gauge bosons (see [34] and references therein). Direct searches at LHC have also produced bounds on the masses of vector-like quarks [42–46]. The only dedicated study that provides direct bounds on the masses of vector-like leptons we consider is provided by the LEP L3 collaboration [47]. The effective couplings of these states to the SM fields are constrained through experimental data on processes such as $\mu-e$ conversion in nuclei and lepton number violating decays.

Our work is organized as follows. In Section II we present current constraints coming from the low-energy phenomenology on the presence of a vector-like quark that is an isosinglet with charge $-1/3$. That section also contains a discussion of constraints on the presence of a light vector-like charged lepton. We allow these new vector-like quarks (leptons) to mix with all three generations of matter fields in order to have the most general scenario. Section III contains the study of the impact the vector-like quarks and leptons, accommodated in one pair of five-dimensional representations of $SU(5)$, have on the mass relations between the down-type quarks and charged leptons. Section IV is devoted to consequences of the presented scenario on two-body proton decay due to gauge boson exchange after we impose the relevant low-energy constraints. We conclude briefly in Section V.

II. CONSTRAINTS

The new degrees of freedom we plan to introduce in the $SU(5)$ framework include one leptonic vector-like isodoublet and one down-type quark vector-like isosinglet. Both of these states can couple to the SM fermions through Yukawa and Dirac interactions, affecting the phenomenology of the electroweak sector. We parametrize the consequences from low-energy flavor and electroweak precision observables in terms of matrices in flavor space that originate from an additional mismatch between gauge and mass basis, triggered by the presence of these new fermions. In this framework, we assume the vector-like matter to be the lightest new degrees of freedom, and to consequently present the leading effects on low-energy interactions. We study two limiting cases, namely the one in which we investigate effects coming solely from the additional quark-like states assuming the vector-like leptons to be heavy enough in order to be neglected in the calculations, and vice versa.
write the branching ratio in the form entries of $Y$ interactions between the SM charged leptons can be written as the issue of neutrino masses, i.e., we do not add any field that would allow for the generation of neutrino mass terms. state manifests itself primarily in the right-handed fermion sector. It must be noted that we do not take into account added, the leading effects occur among the left-handed states. The modified interactions among the $Z$ boson and the to the flavor conserving neutral currents of the SM. In this case, i.e., when an isosinglet vector-like representation is representations differ in one of their chiralities, tree level flavor changing neutral interactions are generated, as opposed to the flavor conserving neutral currents of the SM. In this case, i.e., when an isosinglet vector-like representation is being performed through unitary rotations in the left- and right-handed sector. Since standard and vector-like diagonalization of the quark mass matrices, on top of the standard Cabbibo-Kobayashi-Maskawa (CKM) mechanism, new physics contribution. The elements of $X$ electromagnetic quark current, while the matrix $X$ and $c$ –pole physics, while the off-diagonal interactions are severely restricted by measurements on mesonic FCNC decays. The numerical upper limits on the entries of $X$, obtained in [34], are displayed in Table I and are subsequently used in Section III.\[49], [39]. The muon capture rate observables. All upper bounds are given at 95% C.L..\[49], [39]. The muon capture rate Table I. Phenomenological upper bounds on $Z$ couplings to the SM quarks (see Eq. (1)) from precision flavor and electroweak observables. All upper bounds are given at 95% C.L.

| Coupling | Constraint |
|----------|------------|
| $|X_{12}^d|$ | $1.4 \times 10^{-5}$ |
| $|X_{13}^d|$ | $0.4 \times 10^{-3}$ |
| $|X_{23}^d|$ | $1.0 \times 10^{-3}$ |
| $\delta X_{11}^d$ | $4.0 \times 10^{-3}$ |
| $\delta X_{22}^d$ | $6.0 \times 10^{-3}$ |
| $\delta X_{33}^d$ | $5.7 \times 10^{-3}$ |

A. Vector-like quarks

The presence of Dirac and Yukawa interaction terms mixing SM and vector-like states requires an additional diagonalization of the quark mass matrices, on top of the standard Cabbibo-Kobayashi-Maskawa (CKM) mechanism, being performed through unitary rotations in the left- and right-handed sector. Since standard and vector-like representations differ in one of their chiralities, tree level flavor changing neutral interactions are generated, as opposed to the $\theta$ conserving neutral currents of the SM. In this case, i.e., when an isosinglet vector-like representation is added, the leading effects occur among the left-handed states. The modified interactions among the $Z$ boson and the SM down-type quarks, in the mass basis, can be written as

$$\mathcal{L}_Z \supset -\frac{g}{c_W} \left( -\frac{1}{2} X^d_{ij} \bar{d}^i \gamma^\mu P_L d^j + \frac{1}{3} s_W^2 \bar{d}^i \gamma^\mu d^j \right) Z_{\mu}, \quad (1)$$

with $i,j = 1,2,3$. Here $P_{L,R} = (1 \pm \gamma_5)/2$, $g = 2 m_W/v \simeq 0.65$ is the weak coupling, while $s_W \equiv \sin \theta_W = \sqrt{0.231}$ and $c_W = \sqrt{1 - s_W^2}$ are the sine and cosine of the weak angle, respectively. The second part contains the SM electromagnetic quark current, while the matrix $X^d$ ($X^d_{ii} \equiv 1 - \delta X_{ii}^d$) incorporates the diagonal SM part as well as the new physics contribution. The elements of $X^d$ are directly connected to the unitary transformations, relevant for the GUT framework phenomenology, and the processes which affect these parameters will give the leading constraints, whereas charged current interactions give milder bounds. (See, for example, Ref. [34].) Note that flavor changing neutral currents (FCNCs) appear solely in the down-type quark sector in this setup. Strong bounds on $\delta X_{ii}^d$ come from $Z$-pole physics, while the off-diagonal interactions are severely restricted by measurements on mesonic FCNC decays. The numerical upper limits on the entries of $X^d$, obtained in [34], are displayed in Table I and are subsequently used in Section III.

B. Vector-like leptons

Contrary to the previous case, where we have introduced a new isosinglet, the addition of an isodoublet vector-like state manifests itself primarily in the right-handed fermion sector. It must be noted that we do not take into account the issue of neutrino masses, i.e., we do not add any field that would allow for the generation of neutrino mass terms. This also implies that there are no effects on charged interactions among the SM leptons. The modified neutral interactions between the SM charged leptons can be written as

$$\mathcal{L}_Z \supset -\frac{g}{c_W} \left( -Y_{ij}^\tau \bar{e}^i \gamma^\mu P_R e^j - \frac{1}{2} \bar{e}^i \gamma^\mu P_L e^i + s_W^2 \bar{e}^i \gamma^\mu e^i \right) Z_{\mu}. \quad (2)$$

As noted before, in the fit of non-standard lepton couplings we did not include $X^d$. The constraints on the diagonal entries of $Y$ coming from $Z$-pole physics, [48], [39], are given in Table I. The $\mu-e$ conversion in nuclei sets a very severe bound on $Y_{12}^\tau$, since it occurs at tree level in this setup. One can write the branching ratio in the form

$$B_{\mu N \rightarrow e N} = \frac{8 G_F^2 |Y_{12}^\tau|^2}{\omega_{\text{cap}}} \left| (2 g_u + g_d) V^{(p)} + (g_u + 2 g_d) V^{(n)} \right|^2, \quad (3)$$

[49], [39]. The muon capture rate $\omega_{\text{cap}}$, and the overlap integrals $V^{(p)}$ and $V^{(n)}$ can be found in [49] and

$$g_u = 1 - \frac{8}{3} s_W^2, \quad g_d = -1 + \frac{4}{3} s_W^2 \quad (4)$$
into the mentioned three channels, \(Q\) states. These searches focus on the QCD pair production of the exotic states with subsequent decay into third vector-like quarks, with the result of lower bounds on the particles’ masses, due to the non-observation of such magnitude \([59]\).

\[ \mu \rightarrow e\gamma \] is planned to reach a sensitivity of \(10^{-18}\) at 95% C.L.. This is the limit we present in Table II.

Lepton flavor violating decays of the form \(l_i \rightarrow 3l_j\) give a weaker bound in the \(\mu-e\) sector, while they deliver the leading constraint on mixed couplings involving the \(\tau\) lepton in our model. At tree level in this setup, the corresponding width can be written as \([50]\)

\[
\Gamma(\tau \rightarrow l_il_i) = \frac{G_F^2}{48\pi^3} m_\tau^5 |Y_{e3}|^2 \left[\left(\frac{s^2_W - 1}{2}\right)^2 + \frac{3}{2} \left(s^2_W - Y_{ii}^e\right)^2\right],
\]

with \(l_i = e \) or \(\mu\). The result in Table II was obtained by marginalizing over the ranges of \(Y_{ii}^e\) allowed by Z-pole physics. All constraints that are summarized in Table II are used in Section III.

Finally, future experiments will explore a large portion of the presently allowed parameter space, since the sensitivity to the mentioned processes is supposed to improve by various orders of magnitude. The PRISM/PRIME \([54]\) and furthermore the Mu2e \([55]\) experiments, for example, are expected to probe \(\mu-e\) conversion rates to the order of \(10^{-18}\) and \(10^{-17}\), respectively. A dedicated experiment for measuring \(\mu \rightarrow 3e\) is planned to reach a sensitivity of \(10^{-16}\) \([56]\), while an improvement in sensitivity by about an order of magnitude is foreseen by SuperKEKB regarding \(\tau \rightarrow 3\ell\) \([57]\). Moreover, several other proposals for new experiments or upgrades in the area of precision physics will affect a wide range of additional processes, which will potentially make them more relevant than the mentioned ones. At this place a comment should be made on the \(\tau\) sector, while an improvement in sensitivity by about an order of magnitude is foreseen by SuperKEKB regarding \(\tau \rightarrow 3\ell\) \([57]\), furthermore the Mu2e \([55]\) experiments, for example, are expected to probe \(\mu-e\) conversion rates to the order of \(10^{-18}\) and \(10^{-17}\), respectively. A dedicated experiment for measuring \(\mu \rightarrow 3e\) is planned to reach a sensitivity of \(10^{-16}\) \([56]\), while an improvement in sensitivity by about an order of magnitude is foreseen by SuperKEKB regarding \(\tau \rightarrow 3\ell\) \([57]\). Moreover, several other proposals for new experiments or upgrades in the area of precision physics will affect a wide range of additional processes, which will potentially make them more relevant than the mentioned ones. At this place a comment should be made on the \(\tau\) sector, while an improvement in sensitivity by about an order of magnitude is foreseen by SuperKEKB regarding \(\tau \rightarrow 3\ell\) \([57]\), furthermore the Mu2e \([55]\) experiments, for example, are expected to probe \(\mu-e\) conversion rates to the order of \(10^{-18}\) and \(10^{-17}\), respectively. A dedicated experiment for measuring \(\mu \rightarrow 3e\) is planned to reach a sensitivity of \(10^{-16}\) \([56]\), while an improvement in sensitivity by about an order of magnitude is foreseen by SuperKEKB regarding \(\tau \rightarrow 3\ell\) \([57]\).

\[
\text{Table II. Phenomenological upper bounds on } Z \text{ couplings to SM leptons (see Eq. (2)) from precision flavor and electroweak observables, where } Y_{ii}^e \text{ is the new physics contribution to } Z \text{ couplings in the right-handed sector. All upper bounds are given at 95% C.L. and the theoretical constraint } Y_{ii}^e > 0 \text{ was taken into account.}
\]

| Observable | Constraint |
|------------|------------|
| \(B_{\mu Au\rightarrow e Au}\) | \(0.7 \times 10^{-12}\) \([51]\) |
| \(B_{\mu Ti\rightarrow e Ti}\) | \(1.7 \times 10^{-12}\) \([52]\) |
| \(B(\tau \rightarrow 3e)\) | \(2.7 \times 10^{-8}\) \([53]\) |
| \(B(\tau \rightarrow 3\mu)\) | \(2.1 \times 10^{-8}\) \([53]\) |

\[
\text{Table III. Experimental upper limits on several lepton flavor violating processes at 90% C.L.}
\]

| Observable | Constraint |
|------------|------------|
| \(\mathcal{B}(\mu \rightarrow e\gamma)\) | \(5.7 \times 10^{-13}\) \([58]\) |

\[ G_F^2 = \frac{\sqrt{2} \alpha \alpha^*}{\pi} \]

Table III. Experimental upper limits on several lepton flavor violating processes at 90% C.L.

with \(l_i = e \) or \(\mu\). The result in Table II was obtained by marginalizing over the ranges of \(Y_{ii}^e\) allowed by Z-pole physics. All constraints that are summarized in Table II are used in Section III.

C. Collider phenomenology

The ATLAS and CMS collaborations at the LHC have published several dedicated studies on the direct production of vector-like quarks, with the result of lower bounds on the particles’ masses, due to the non-observation of such states. These searches focus on the QCD pair production of the exotic states with subsequent decay into third generation SM quarks and gauge bosons or the Higgs. The latest of these analyses combine the possibilities of decays into the mentioned three channels, \(Q \rightarrow Vq\), with \(V = W, Z, h\), and \(q\) a top or a bottom quark. In the large mass limit, for masses \(\gtrsim 500\, \text{GeV}\), typically half of the vector-like quarks decay through the charged current channel, while

\[
| Y_{i2}^e | = 1.6 \times 10^{-7}
\]

\[
| Y_{i3}^e | = 5.5 \times 10^{-4}
\]

\[
| Y_{23}^e | = 5.5 \times 10^{-4}
\]

\[
Y_{11}^e = 6.8 \times 10^{-4}
\]

\[
Y_{22}^e = 2.8 \times 10^{-3}
\]

\[
Y_{33}^e = 2.0 \times 10^{-3}
\]
the branching ratios in each of the neutral channels amount to $\sim 25\%$, which are usually referred to as the nominal branching fractions. The most recent CMS studies report a lower bound of around 700 GeV for the nominal branching fractions [33, 45], while a recent ATLAS study delivers a limit of 645 GeV [46] for that case, all at 95% C.L. The mentioned searches do not include couplings to first two generation quarks. Nevertheless, at the masses probed by now, this generalization would not alter the outcome significantly in the most minimal models, since low energy processes highly restrict the couplings to lighter quarks. The latter, on the other hand, becomes extremely relevant in the case of single production. This might provide a relevant channel for future studies [60], since the production rate overshadows that of single production at higher masses.

A general direct search for exotic leptons has in turn been performed at LEP by the L3 collaboration [47], setting a lower bound on the mass of charged leptons, which are part of a vector-like isodoublet, at about 100 GeV. Moreover, studies by the LHC experiments searching for heavy leptons in the context of the Type III see-saw model have been made, with ATLAS setting the most stringent bound of 245 GeV [61]. Since the decays of the exotic states depend on their mixing with SM leptons, these were varied in the study, however without considering mixing with the $\tau$. In [62] it was shown that in a composite scenario with leptonic vector-like isodoublets, this bound is lifted to about 300 GeV. The authors of Ref. [10] performed a recast of a CMS multilepton search using the full dataset at $\sqrt{s} = 8$ TeV [63]. Assuming couplings to electrons or muons only, they obtained a bound of about 460 GeV, while in the case of mixing with the $\tau$ alone they computed a weaker limit of about 280 GeV. However, it should be noted that in the mentioned work, carried out in a composite Higgs framework, there is no contribution from decays of exotic to the SM leptons and the Higgs boson.

III. $SU(5)$ SETUP

The $SU(5)$ setup we study comprises matter fields that belong to $\mathbf{10}_i = \{e_i^C, u_i^C, Q_i\}$ and $\mathbf{5}_i = \{L_i, d_i^C\}$, $i = 1, 2, 3$, where $Q_i = (u_i, d_i)^T$ and $L_i = (\nu_i, e_i)^T$ [25]. It also contains one vector-like pair $(\mathbf{5}_4, \overline{\mathbf{5}}_4)$ of matter fields, where $\mathbf{5}_4 = \{T_4, \overline{d}_4^C\}$ and $\overline{\mathbf{5}}_4 = \{L_4, d_4^C\}$. The subscript for the vector-like pair is included to allow for more compact notation. This pair will be used to generate viable masses for down-type quarks and charged leptons.

The scalar sector of the setup, on the other hand, is made out of one 24-dimensional and one 5-dimensional representation we denote as $\mathbf{24}$ and $\mathbf{5}$, respectively. The adjoint representation $\mathbf{24}$ breaks $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$ while the fundamental representation $\mathbf{5}$ provides electroweak vacuum expectation value (VEV). We take $|\langle \mathbf{5} \rangle| = v'$ and $\langle \mathbf{24} \rangle \approx \sigma \text{diag}(2, 2, 2, -3, -3)$ to be the relevant VEVs, where $\mathbf{5}^\alpha$, $\alpha = 1, \ldots, 5$, represent components of the fundamental representation of $SU(5)$. We neglect the contribution of an $SU(2)$ triplet towards the VEV of $\mathbf{24}$. The exact value of $\sigma$ can be determined through the consideration of gauge coupling unification. It is constrained to be of the same order as the scale at which gauge couplings meet, i.e., the GUT scale, due to experimental input on proton decay. The common value of the SM gauge couplings at $m_{\text{GUT}}$ is $g_{\text{GUT}}$.

The up-type quark masses originate from a single $SU(5)$ operator $(Y^{10})_{ij} \mathbf{10}_i \mathbf{10}_j \mathbf{5}$, $i, j = 1, 2, 3$, where $Y^{10}$ is a complex $3 \times 3$ Yukawa matrix. The contraction in the space of flavor is explicitly shown for clarity. The important thing to note is that the up-type quark mass matrix comes out to be symmetric. This feature will be relevant for the proton decay predictions of this particular $SU(5)$ setup.

The down-type quark and the charged lepton masses, on the other hand, require a more elaborate structure in order to be viable. Namely, one requires the presence of two types of operators to generate realistic masses. These are $(Y^\overline{5})_{ij} \mathbf{10}_i \overline{\mathbf{5}}_j \mathbf{5}$ and $(M)_{ij} + (\eta) \mathbf{24}_i \overline{\mathbf{5}}_j$, where $i = 1, 2, 3$ and $l = 1, 2, 3, 4$. Here, $M$ is an arbitrary complex $1 \times 4$ mass matrix while $\eta$ and $Y^\overline{5}$ represent Yukawa matrices with complex entries of dimensions $1 \times 4$ and $3 \times 4$, respectively. It is the latter set of operators that breaks the degeneracy of the mass spectrum of vector-like leptons and vector-like quarks within the $(\mathbf{5}_4, \overline{\mathbf{5}}_4)$ pair.

We can redefine matter field multiplets at the $SU(5)$ level to go to the basis where the term $(Y^\overline{5})_{ij} \mathbf{10}_i \overline{\mathbf{5}}_j \mathbf{5}$ is completely removed and the remaining $3 \times 3$ part of the $Y^\overline{5}$ matrix with components $(Y^\overline{5})_{ij}$, $i, j = 1, 2, 3$, is diagonal [21], i.e., $(Y^\overline{5})_{ij} = y^\overline{5}_{ij} \delta_{ij}$. In this basis the $4 \times 4$ mass matrices $M_E$ and $M_D$ that are relevant for the charged lepton and the down-type quark sectors explicitly read

$$M_E = \begin{pmatrix} y_1 v' & 0 & 0 & M^E_1 \\ 0 & y_2 v' & 0 & M^E_2 \\ 0 & 0 & y_3 v' & M^E_3 \\ 0 & 0 & 0 & |M^E_4| \end{pmatrix}, \quad M_D = \begin{pmatrix} y_1 v' & 0 & 0 & 0 \\ 0 & y_2 v' & 0 & M^D_1 \\ 0 & 0 & y_3 v' & M^D_2 \\ 0 & 0 & 0 & M^D_3 \end{pmatrix},$$

where $M^E_l = (M)_l - 3(\eta)_l \sigma$ and $M^D_l = (M)_l + 2(\eta)_l \sigma$, $l = 1, 2, 3, 4$. Again, we neglect the contribution from the VEV of the $SU(2)$ triplet in $\mathbf{24}$ towards the mass of the vector-like leptons. Our convention is such that $M^D$ is multiplied
from the left by a $1 \times 4$ matrix $(d_1 \ d_2 \ d_3 \ \bar{d}_s)$ and from the right by a $4 \times 1$ matrix $(d^c_1 \ d^c_2 \ d^c_3 \ d^c_s)^T$.

It is possible to make all matrix elements in Eq. (5) real by making suitable redefinitions of the quark and lepton fields \[21\]. We will assume that this is done and neglect in the rest of our work these phases for simplicity. With this in mind we introduce the parameters $m_i = |y_i^T v|$ and $x_i^{E,D} = M_i^{E,D} / |M_i^{E,D}|$, $i = 1, 2, 3$. These parameters will play a crucial role in our study of fermion masses and mixing parameters. We will use this basis and associated nomenclature as the starting point for our discussion. Note that the CKM phase will come exclusively from the up-type quark sector in our setup.

If one omits contributions from the $(5_1, 5_4)$ pair, one finds that the down quark mass $m_1^D$ and the electron mass $m_1^E$ are both equal to $m_1$ at the GUT scale and thus degenerate. This is in disagreement with experimental observations, once the measured masses are propagated from the low-energy scale to $m_{GUT}$. In fact, the same type of degeneracy would also hold for the masses of the second and the third generation of down-type quarks and charged leptons, i.e., $m_j^D = m_j^E$, $j = 2, 3$. This, again, does not correspond to what experimental values yield at $m_{GUT}$. It is the presence of vector-like matter that breaks this degeneracy and creates an opportunity to have a realistic setup.

It can be shown \[21\] that the charged fermion masses $m_i^E$, $i = 1, 2, 3$, are related to the parameters $m_i$ and $x_i^E$ through the following three equations

$$ (m_1^E)^2 + (m_2^E)^2 + (m_3^E)^2 = \frac{m_1^2(1 + |x_1^E|^2 + |x_3^E|^2) + m_2^2(1 + |x_2^E|^2 + |x_3^E|^2) + m_3^2(1 + |x_1^E|^2 + |x_2^E|^2)}{1 + |x_1^E|^2}. \tag{7} $$

$$ (m_1^Em_2^E)^2 + (m_1^Em_3^E)^2 + (m_2^Em_3^E)^2 = \frac{m_1^2m_2^2(1 + |x_1^E|^2) + m_2^2m_3^2(1 + |x_2^E|^2) + m_3^2m_1^2(1 + |x_2^E|^2)}{1 + |x_1^E|^2}. \tag{8} $$

$$ (m_1^Em_2^Em_3^E)^2 = \frac{m_1^2m_2^2m_3^2}{1 + |x_1^E|^2}. \tag{9} $$

where we introduce $|x_i^E|^2 = |x_1^E|^2 + |x_2^E|^2 + |x_3^E|^2$. These relations should be satisfied at the GUT scale.

The important point is that these equations are also applicable for the down-type quark sector. All one needs to do is to replace $m_i^E$ with $m_i^D$ and $x_i^E$ with $x_i^D$ in Eqs. (7), (8) and (9). The parameters $m_i$, on the other hand, are common for both sectors. Clearly, since $|x_i^E|$ and $|x_i^D|$ are not correlated, it is possible, at least in principle, to simultaneously generate viable masses for down-type quarks and charged leptons. The vector-like leptons and quarks from the $(5_1, 5_4)$ pair have masses $m_1^E = \sqrt{|M_1^E|^2 + |M_2^E|^2 + |M_3^E|^2 + |M_4^E|^2}$ and $m_1^D = \sqrt{|M_1^D|^2 + |M_2^D|^2 + |M_3^D|^2 + |M_4^D|^2}$, respectively.

We clearly do not address the origin of neutrino mass. Our study, however, and all correlations we generate next will be valid for all $SU(5)$ setups that do include neutrino mass generation but do not directly affect the charged fermion sector \[64\] \[65\].

Parameters that are a priori unknown in our setup are $m_i$ and $|x_i^{E,D}|$, $i = 1, 2, 3$. We thus have nine parameters to explain six experimentally measured masses, i.e., $m_i^E$ and $m_i^D$, $i = 1, 2, 3$. At this stage the masses of vector-like leptons and quarks contribute only indirectly through the parameters $|x_i^E|$ and $|x_i^D|$ towards the masses of the matter fields. It turns out, however, that it is not trivial to simultaneously satisfy all six equations that relate $m_i^E$ and $m_i^D$ with $m_i$ and $|x_i^{E,D}|$, $i = 1, 2, 3$. For example, there is no solution for $m_1m_2m_3 \leq m_1^Em_2^Em_3^E$. The observed masses, when propagated to the GUT scale, yield $m_1^Em_2^Em_3^E > m_1^Dm_2^Dm_3^D$. See, for example, Table \[IV\]. The charged lepton masses hence play a more prominent role than the down-type quark masses in Eq. (9). In fact, there exists a number of additional constraints that originate from Eqs. (7), (8) and (9) on the parameters $m_i$, $i = 1, 2, 3$. Here we elaborate on two of them.

Since the observed masses of charged leptons and down-type quarks exhibit a rather strong hierarchy, we can safely neglect terms that are proportional to $m_i^{E,D}$ on the left-hand sides of Eqs. (7) and (8). If we do that, we can rewrite Eq. (8) to read

$$ (m_2^Em_3^D)^2 \approx \frac{m_2^2m_3^2(1 + |x_2^{E,D}|^2)}{1 + |x_2^{E,D}|^2}. \tag{10} $$

Here we also neglect terms proportional to $m_1$ on the right-hand side of Eq. (8). This then allows us to obtain an approximate equality $(m_1^E)^2 \approx m_1^2/(1 + |x_1^{E,D}|^2)$, once we use Eq. (9). Since $m_1^D > m_1^E$ we finally get

$$ m_1^D \lesssim m_1. \tag{11} $$
This result implies that it is impossible to simultaneously solve all six equations if \( m_1 \) is below the mass of the down quark.

Let us now neglect terms that are proportional to \( m_1^{E,D} \) and \( m_2^{E,D} \) in Eq. \( 7 \). We accordingly neglect \( m_1 \) and \( m_2 \) in Eq. \( 7 \) to find that

\[
(m_3^{E,D})^2 \approx \frac{m_3^2(1 + |x_1^{E,D}|^2 + |x_2^{E,D}|^2)}{1 + |x_1^{E,D}|^2}.
\]

(12)

This result, when combined with Eq. \( 10 \), yields

\[
m_2^2 \approx (m_2^{E,D})^2 \left( 1 + \frac{|x_2^{E,D}|^2}{1 + |x_1^{E,D}|^2} \right).
\]

(13)

This approximate equality implies that it is impossible to simultaneously solve all six equations if \( m_2 \) is below the muon mass, i.e., \( m_2^E \lesssim m_2 \), since \( m_2^E > m_2^D \).

Our numerical procedure confirms these two relations. We actually find the following set of inequalities to be satisfied: \( m_1^D \lesssim m_1 \lesssim m_2^D \), \( m_2^E \lesssim m_2 \lesssim m_3^D \) and \( m_3^E \lesssim m_3 \). Note that perturbativity considerations place an upper bound on \( m_3 \). However, the flavor physics constraints keep all Yukawa couplings well below that threshold as we show later on. Note also that Eqs. \( 7 \), \( 8 \) and \( 9 \) are invariant under the exchange \( (m_i, |x_i^{E,D}|) \leftrightarrow (m_j, |x_j^{E,D}|) \), \( i, j = 1, 2, 3 \). To account for that we consider only the following numerical procedure. We first specify \( m_i^{E,D}, \ i = 1, 2, 3 \), at the GUT scale to be our input. Relevant values of \( m_i^{E,D} \) we use are summarized in Table \( IV \). We consider scenario without supersymmetry and take the GUT scale to be \( m_{GUT} = 1 \times 10^{16} \text{ GeV} \). The running of masses is performed under the assumptions specified in Ref. \[32\]. The \( m_{GUT} \) values shown in Table \( IV \) are to be understood as representative values that would change if one changes the GUT scale and/or introduces additional particles. Only central values for masses of down-type quarks and charged leptons are considered in our study.

Once the input is defined, we vary the parameters \( m_i \) until we numerically obtain particular values of \( |x_i^E| \)'s that simultaneously satisfy Eqs. \( 7 \), \( 8 \) and \( 9 \). We then fix \( m_i \)'s to these values and proceed to find viable solutions that relate \( m_i^D \) and \( |x_i^E| \). What we end up with are viable sets of \( m_i \) values and associated values of the parameters \( |x_i^{E,D}| \) that yield realistic masses for down-type quarks and charged leptons at the GUT scale. In other words, we find all possible values of \( m_i \) and \( |x_i^{E,D}| \) that satisfy Eqs. \( 7 \), \( 8 \) and \( 9 \) for a given set of \( m_i^{E,D}, \ i = 1, 2, 3 \). Finally, we test whether these solutions also satisfy low-energy constraints after we specify masses of the vector-like states. \( m_i^{E,D} \) need to be specified in order for us to determine unitary transformations that bring the \( 4 \times 4 \) matrices \( M_E \) and \( M_D \), explicitly shown in Eq. \( 6 \), into a diagonal form that corresponds to the fermion mass eigenstate basis. It is these unitary transformations that enter low-energy considerations as we demonstrate in section \( II \). Once we specify \( m_i^D (m_i^E) \), we numerically determine all entries of the matrix \( X^d (X^e) \) and test these entries against the constraints presented in Table \( III \) (Table \( II \)). For example, to find \( Y^e \) we first construct a real normal matrix \( (M_E^T M_E)^{-1} \) that diagonalizes with a congruent transformation \( E_R(M_E^T M_E)E_R^{-1} = (M_E^T M_E)^{\text{diag}} \). This then allows us to define \( Y^e_{ij} = |\sum_{k=1}^3(E_R)_{ik}(E_R)_{jk} - \delta_{ij}|)/2 \).

Note that the unitary transformations we find numerically that act on the matrices \( M_E \) and \( M_D \) are valid at the GUT scale. On the other hand, the constraints we want to impose on the entries of \( X^d \) and \( Y^e \) are valid at the low-energy scale. We, however, opt not to run numerically obtained entries of \( X^d \) and \( Y^e \) to low energies, since the angles that enter unitary transformations that define them are required to be small to satisfy low-energy constraints and should thus not change substantially through the running.

The vector-like states we consider can be either quarks or leptons. We accordingly study and present both cases separately. In particular, when we consider the scenarios with light quark and lepton vector-like states we take \( m_1^D = 800 \text{ GeV} \) and \( m_1^E = 400 \text{ GeV} \), respectively. These masses are allowed by direct searches for the vector-like states. Again, to numerically determine \( X^d (Y^e) \) we need to specify \( m_i^D (m_i^E) \).

We choose to present the outcome of our numerical analysis in form of plots of \( m_2 \) vs. \( m_3 \) for a given value of \( m_1 \). In other words, we have regions of constant \( m_1 \) in the \( m_2-m_3 \) plane that represent a phenomenologically viable parameter space. Every point within that region is associated with a unique set of values of \( |x_i^{E,D}| \)'s that were generated for a given set of \( m_i, \ i = 1, 2, 3 \), that satisfy Eqs. \( 7 \), \( 8 \) and \( 9 \).

We show in Figs. \( I \) and \( II \) the parameter space allowed by low-energy constraints in the \( m_2-m_3 \) plane for \( m_1^D = 800 \text{ GeV} \) and \( m_1^E = 400 \text{ GeV} \), respectively. The contours bound viable regions of constant \( m_1 \) in the \( m_2-m_3 \) plane that yield satisfactory fermion masses. We opt to present regions with \( m_1 = m_2^D, \ m_1 = 5m_1^E \) and \( m_1 = m_1^E \). Recall, there exists no solution for \( m_1 < m_1^E \) or \( m_1 > m_2^D \).
Table IV. Central values for masses of the SM down-type quarks $m_{D_i}^\mu (\mu)$ and charged leptons $m_{E_i}^\mu (\mu)$, $i = 1, 2, 3$, at $\mu = m_Z$ and $\mu = m_{\text{GUT}}$, where $m_{\text{GUT}} = 1 \times 10^{16}$ GeV.

| $\mu$ | $m_{D_1}^\mu (\mu)$ (GeV) | $m_{D_2}^\mu (\mu)$ (GeV) | $m_{D_3}^\mu (\mu)$ (GeV) | $m_{E_1}^\mu (\mu)$ (GeV) | $m_{E_2}^\mu (\mu)$ (GeV) | $m_{E_3}^\mu (\mu)$ (GeV) |
|-------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $m_Z$ | 0.00350                     | 2.890                       | 0.103                       | 0.000487                    | 0.092                       | 1.75                        |
| $m_{\text{GUT}}$ | 0.00105                     | 0.782                       | 0.092                       | 0.000435                    | 1.56                        |

Figure 1. Allowed parameter space in the $m_2$–$m_3$ plane for $m_{D_4}^\mu = 800$ GeV as a function of $m_1$. The contours define viable regions generated for $m_1 = m_{D_2}$, $m_1 = 5m_{E_1}^\mu$ and $m_1 = m_{D_1}$, starting with the innermost one. Regions outside of the contours are excluded.

One can see from Figs. 1 and 2 that the allowed region for a given value of $m_1$ in the case of a light vector-like quark state is significantly less constrained with regard to the case when the light state is a vector-like lepton. This was to be expected, since the experimental constraints from observables that concern flavor physics effects in the charged lepton sector are much more stringent than in the down-type quark sector. The allowed region for the $m_{D_4}^\mu = 800$ GeV case, for fixed $m_1$, becomes comparable in size to the region that corresponds to the case of a light vector-like lepton state only when $m_{E_i}^\mu \approx 1.2$ TeV. It is interesting to note that $m_i$, $i = 1, 2, 3$, exist only within narrow ranges once the low-energy constraints are implemented. For example, once the vector-like leptons are taken to be light, $m_1$ and $m_3$ can change, at most, by about a factor of twenty whereas $m_2$ can change by about a factor of eight. (See Fig. 2.) It is also clear from our numerical study that $m_{E_2}^\mu \lesssim m_2 \leq m_{D_3}^\mu$ and $m_{E_3}^\mu \lesssim m_3$.

The shape of the available parameter space shown in Figs. 1 and 2 has a very simple interpretation. The size of the bounded area shrinks as $m_1$ grows simply due to the fact that any departure of $m_1$ from its “natural” value requires more substantial rotations of the $4 \times 4$ matrices $M_D$ and $M_E$ to correctly account for the masses for the first generation of down-type quarks and charged leptons, respectively. These rotations, on the other hand, need to be small if one is to satisfy existing low-energy constraints, especially when the light vector-like fermions are leptons. The same effect is evident with regard to departures of $m_2$ and $m_3$ from their preferred values that are set by the mass scales of the second and third generation of down-type quarks and charged leptons, respectively. Clearly, the most natural and hence the least constrained part of the available parameter space is the one where $m_i \sim m_{E,D_i}^\mu$, $i = 1, 2, 3$.

IV. PREDICTIONS

All viable extensions that represent minimal departures from the original $SU(5)$ setup are very predictive [64–66] with regard to proton decay. The same is true for the framework under consideration that includes only one extra vector-like pair of fields with regard to the Georgi-Glashow model [25] as we show next.
The scenario with an extra vector-like pair \((5_1, 5_4)\) yields all unitary transformations that are necessary to go from an arbitrary flavor basis to the mass eigenstate one in terms of three parameters. These parameters, i.e., \(m_1, m_2\) and \(m_3\), suffice to describe all viable redefinitions of the SM down-type quark and the SM charged lepton fields as we demonstrated in Section III. The description of all unitary transformations in the up-type quark sector, on the other hand, requires no additional parameters for the following reason. The \(SU(5)\) invariant operator that generates all up-type quark masses guarantees the symmetric nature of the relevant mass matrix. This fact allows one to relate rotations in the left-handed sector of up-type quarks to rotations in the right-handed sector of up-type quarks. Moreover, it is the CKM mixing matrix that provides the link between left-handed rotations in the up- and the down-type quark sectors. To establish this connection we use the following CKM parameters: \(\lambda = 0.22535, A = 0.811, \bar{\rho} = 0.131\) and \(\bar{\eta} = 0.345\) \cite{67}.

The setup thus yields accurate proton decay signatures for two-body decays of the proton through gauge boson exchange in terms of only three parameters that are already constrained to be within very narrow ranges. Note that the exact mechanism of the neutrino mass generation cannot affect these predictions since the relevant decay amplitudes do not refer to the neutrino mixing parameters. In any case, the fact that the transformations in the charged lepton sector are known in terms of \(m_i\), \(i = 1, 2, 3\), allows one to reconstruct unitary transformations in the neutrino sector via the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix.

The gauge coupling strength at the GUT scale is \(\alpha_{GUT} = g^2_{GUT}/(4\pi)\) and the mass of the gauge boson that mediates proton decay corresponds to \(m_{GUT}\) in \(SU(5)\). We take \(\alpha_{GUT} = 0.033\) \cite{52} and \(m_{GUT} = 4 \times 10^{15}\) GeV for definiteness in our numerical analysis. Recall, \(m_{GUT}\) can be identified with the mass of proton decay mediating gauge bosons in \(SU(5)\). The partial lifetimes we present scale with \(\alpha_{GUT}^{-1} (m_{GUT})\) to the second (fourth) power and can thus be easily recalculated for different values of these two parameters. To generate proton decay predictions we furthermore use \(\hat{\alpha} = -0.0112\) \(\text{GeV}^{3}\) \cite{68}, where \(\hat{\alpha}\) is the relevant nucleon matrix element. The leading-log renormalization corrections of the \(d = 6\) operator coefficients are taken to be \(A_{SL} = 2.6\) and \(A_{SR} = 2.4\) \cite{32} and the exact dependence of the decay amplitudes on unitary transformations is taken from Ref. \cite{69}.

We present results for four proton decay channels that turn out to be the most relevant in Fig. 3. These channels are \(p \to \pi^0 e^+, p \to \pi^0 \mu^+, p \to \pi^+ \bar{\nu}\) and \(p \to K^0 \mu^+\). Current experimental limits are \(\tau_{p \to \pi^0 e^+} > 1.3 \times 10^{34}\) years \cite{70}, \(\tau_{p \to \pi^0 \mu^+} > 1.1 \times 10^{34}\) years \cite{70}, \(\tau_{p \to \pi^+ \bar{\nu}} > 3.9 \times 10^{32}\) years \cite{71} and \(\tau_{p \to K^0 \mu^+} > 1.6 \times 10^{33}\) years \cite{72}. (All other two-body decay modes of the proton can be safely neglected for all practical purposes.) The parameter points we use to generate proton decay predictions correspond to the \(m_{GUT}^F = 400\) GeV case shown in Fig. 2. We plot minimal and maximal predicted partial lifetimes for four proton decay channels in Fig. 3 for all allowed values of parameter \(m_1\): \(m_1 \in (m_1^D, m_2^D)\). Note that we extract relevant unitary transformations using reduced forms of the original matrices \(M_E\) and \(M_D\) that are given in Eq. \(\[9\]\). Namely, we bring the effective \(3 \times 3\) matrices \(\hat{M}_E\) and \(\hat{M}_D\) of the SM charged

\[\begin{align*}
m_1 = m_1^D \quad m_1 = 5m_1^D \quad m_1 = m_1^D\end{align*}\]

Figure 2. Allowed parameter space in the \(m_2-m_3\) plane for \(m_{GUT}^F = 400\) GeV. The contours define viable regions generated for \(m_1 = m_1^D, m_1 = 5m_1^D\) and \(m_1 = m_1^D\), starting with the innermost one. Regions outside of the contours are excluded.
leptons and down-type quarks to diagonal form. $\hat{M}_E$ explicitly reads [21]

$$
\hat{M}_E = \begin{pmatrix}
\frac{m_1}{\sqrt{1 + |x_1^E|^2}} & 0 & 0 \\
-\frac{m_1 |x_1^E||x_2^E|}{\sqrt{1 + |x_1^E|^2|1 + |x_2^E|^2}} & \frac{m_2}{\sqrt{1 + |x_1^E|^2|1 + |x_2^E|^2}} & \frac{m_3}{\sqrt{1 + |x_1^D|^2|1 + |x_3^D|^2}} \\
-\frac{m_1 |x_1^E||x_3^E|}{\sqrt{1 + |x_1^E|^2|1 + |x_3^E|^2}} & -\frac{m_2 |x_2^E||x_3^E|}{1 + |x_2^E|^2} & \frac{m_3 |x_3^F||x_1^E|}{1 + |x_3^F|^2}
\end{pmatrix}. \tag{14}
$$

To obtain $\hat{M}_D$ from $\hat{M}_E$ all one needs to do is to replace $|x_i^E|$ with $|x_i^D|$, $i = 1, 2, 3$, in Eq. (14) and transpose the resulting mass matrix.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{Plot of $m_1$ vs. $\tau_{p \rightarrow \pi^0 e^+}$, $\tau_{p \rightarrow \pi^0 \mu^+}$, $\tau_{p \rightarrow \pi^0 \nu}$ and $\tau_{p \rightarrow K^0 \mu^+}$. The bands represent the ranges of predicted values for partial proton decay lifetimes for associated decay channels.}
\end{figure}

The bands in Fig. 3 represent predicted values for associated proton decay channels. They reflect the dependence of the decay amplitudes on unitary transformations of the quark and lepton fields that, in turn, depend on $m_i$, $i = 1, 2, 3$ parameters. The most important observation is the possibility that one can pinpoint the value of $m_1$ for a fixed value of $m_2^E$ through the proton decay signatures in this setup. For small values of $m_1$ it is the $p \rightarrow \pi^0 e^+$ signature that dominates. It is followed by $p \rightarrow \pi^+ \nu$, $p \rightarrow K^0 \mu^+$ and $p \rightarrow \pi^0 \mu^+$. For larger values of $m_1$, on the other hand, the $p \rightarrow \pi^0 \mu^+$ signature starts to dominate over $p \rightarrow \pi^0 e^+$, with $p \rightarrow \pi^0 e^+$ and $p \rightarrow \pi^+ \nu$ processes being of the same strength. Another nice feature of the setup is the constancy of the $p \rightarrow \pi^+ \nu$ decay amplitude. This is expected since the mass matrix of the up-type quark sector is symmetric and the sum over all neutrino flavors removes all dependence on the unitary transformations [69]. Finally, we see that the amplitudes for both the $p \rightarrow \pi^0 e^+$ and the $p \rightarrow K^0 \mu^+$ modes vary only slightly with regard to the parameters of the setup. Clearly, the low-energy phenomenology allows for the presence of only those transformations that correspond to small changes in the angles of rotations that enter redefinitions of the quark and lepton fields. This, again, is reflected in the narrow widths of allowed bands for predicted proton decay signatures of the setup.

We can use the predictions for $p \rightarrow \pi^0 e^+$ and $p \rightarrow \pi^0 \mu^+$ we displayed in Fig. 3 to find a conservative lower bound on the GUT scale in our setup. We present this bound in Fig. 4. Note that the bound on $m_{GUT}$ is set by two distinct proton decay channels. For small (large) values of $m_1$ parameter the bound is set by $p \rightarrow \pi^0 e^+$ ($p \rightarrow \pi^0 \mu^+$).

V. CONCLUSIONS

We study the original $SU(5)$ model extended with one vector-like down-type quark and one vector-like lepton doublet. These comprise a pair of five-dimensional representations of $SU(5)$ and help to obtain correct values of the down-type quark and charged lepton masses. All unitary transformations that relate an arbitrary flavor basis to the mass eigenstate basis of matter fields are completely described with three parameters in our setup. These, on the other hand, are limited to reside in very
narrow ranges due to existing experimental data. We accordingly find a clear correlation between these parameters, proton decay and the lightness of either vector-like quarks or vector-like leptons, with the light vector-like lepton case being more predictive than the light vector-like quark case. Representative mass scales for vector-like quarks and leptons are taken to be 800 GeV and 400 GeV, respectively.

We investigate the viability of the setup when either vector-like quark or vector-like lepton states are light, taking into account relevant low-energy constraints. These, for example, include the influence of vector-like leptons on $\mu-e$ conversion and the modification of the couplings of the SM fermions to the $Z$ boson due to the vector-like state presence. Our study demonstrates that the proposed framework is very predictive with regard to proton decay signatures through gauge boson mediation. We find the most relevant decay channels to be $p \to \pi^0 e^+$, $p \to \pi^0 \mu^+$, $p \to \pi^+ \bar{\nu}$ and $p \to K^0 \mu^+$. A lower bound on the GUT scale is generated by two proton decay channels and it turns out to be between $4.2 \times 10^{15}$ GeV and $5.7 \times 10^{15}$ GeV.

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