Forced shift vibrations of stamp on the border of composite half-space with interfacial partially detached thin inclusions from the matrix

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Abstract. This article considers shear stationary oscillations of an absolutely rigid stamp with a flat base on the boundary plane of a composite half-space, formed by junction of homogeneous layer and half-space with interphase stripe thin absolutely rigid inclusions, one of the long sides of which is detached from the matrix. It is assumed that the half-space is deformed under the influence of periodically changing concentrated linear loads applied to the stamp and inclusions. A governing system of SIE of the second kind with respect to the amplitudes of the contact stresses acting under the stamp and inclusions, as well as the dislocation of the points of the crack edges, was obtained. Simple formulas are written for determining the intensity coefficients at the end points of cracks and crack opening.

1. Introduction
A number of research studies have been devoted to the study of problems of forced stationary vibrations of multilayer systems with interfacial defects such as cracks and absolutely rigid inclusions, which are very relevant from a practical point of view. The development of this field of mechanics of a deformable solid is associated with the Rostov School of Mechanics, headed by Academician of the RAS V.A. Babeshko. He and his students developed and proposed effective methods for solving dynamic problems for layered media with interfacial defects [1-4], as well as posed and solved a number of topical and interesting problems in this direction. However, few studies have discussed the stress-strain state of layered structures simultaneously containing interfacial defects of various types and studied the regulations of their mutual influence. On the other hand, from the point of view of seismology, seismic construction and exploration and defectoscopy, one of the important tasks is to identify the patterns of mutual influence of different types of stress concentrators depending on the physical, mechanical and geometric parameters of the problems, as well as the frequency of the forced vibrations. In this regard, we note the work [5], where the problem of forced shear vibrations of a stamp at the boundary of a compound half-space with interfacial non-intersecting cracks and absolutely rigid, thin strip inclusions is considered, and the interaction of these stress concentrators is studied. We also note the works [6–8], which are directly related to the present work.
2. Statement of the problem and derivation of governing equations

Let a compound elastic half-space in the Cartesian coordinate system Oxyz and made up of a layer with thickness h and half-space made of heterogeneous materials with shear moduli \( \mu_1 \) and \( \mu_2 \) correspondingly filling the regions \( 0 \leq y \leq h \) and \( y \leq 0 \), in the plane of their joint \( y = 0 \), be weakened by main interfacial cracks occupying the region \( \{ y = 0, -\infty < z < \infty, x \in L = \cup_{n=1}^{N} (a_n, b_n) \} \) the upper banks of which are free of loads and thin, strip absolutely rigid inclusions are soldered to the lower banks. It is assumed that the compound half-space is deformed under action of an absolutely rigid strip stamp with a flat base applied to the layer in the free surface \( \{ y = 0, a \leq x \leq b, -\infty < z < \infty \} \) of the layer and subjected to periodically varying tangent concentrated load \( Q_n e^{i\omega t} \), as well as tangential concentrated loads with intensity \( Q_n e^{i\omega t} \) applied to inclusions.

The problem is: to determine the patterns of change of contact stresses acting under the stamp, the functions of the amplitudes of the difference in the displacements of the points of the edges of the cracks, and the functions of the jumps of the amplitudes of the stresses under the stamp, the functions of the amplitudes of the difference in the displacements of the layer and made up of a series of heterogeneous materials with shear moduli \( c_1 \) and \( c_2 \) respectively in the direction of the Oz axis, satisfying, each in the domain of its definition, the equation

\[
\frac{\partial^2 W_j}{\partial x^2} + \frac{\partial^2 W_j}{\partial y^2} = \frac{1}{(c_j^2)^2} \frac{\partial^2 W_j}{\partial t^2},
\]

where \( c_j^2 \) are the shear wave propagation velocities in the layer and half-space, respectively, and \( \tau_{y^2}^{(j)}(x,y,t) \) are the shear stresses acting in the layer and half-space related to displacements by the well-known formulas:

\[
\tau_{y^2}^{(j)}(x,y,t) = \mu_j \frac{\partial W_j(x,y,t)}{\partial y} \quad (j = 1, 2).
\]

Following [5], we turn to the amplitudes of displacements \( W_j(x,y) \) and stresses \( \tau_{y^2}^{(j)}(x,y) \) \( (j = 1, 2) \). Next, we introduce the functions of the amplitudes of the contact stresses acting under the stamp, the functions of the amplitudes of the difference in the displacements of the points of the edges of the cracks, and the functions of the jumps of the amplitudes of the stresses on the edges of the cracks:

\[
\begin{align*}
\tau_{y^2}^{(1)}(x,h) &= \tau(x) \quad (a < x < b), \\
W_1(x,0) - W_2(x,0) &= W(x) \quad (x \in L), \\
\tau_{y^2}^{(1)}(x,0) - \tau_{y^2}^{(2)}(x,0) &= T(x) \quad (x \in L).
\end{align*}
\]

Using the integral Fourier transformation, we solve the auxiliary boundary problem consisting of the first three conditions of problem (1) and conditions (4) and determine the amplitudes of the
displacement functions under the stamp and inclusions, as well as the amplitudes of the stress functions on the upper crack bank through the introduced unknown functions $W(x)$, $\tau(x)$, and $T(x)$. We get:

$$
W_1'(x, h) = \frac{1}{\pi \mu_1} \int_a^b \frac{\tau(s) ds}{s - x} + \int_a^b K_{11}(s - x) \tau(s) ds \\
+ \int_L K_{12}(s - x) W'(s) ds + \int_L K_{13}(s - x) T(s) ds \quad (-\infty < x < \infty),
$$

$$
\tau_{yz}^{(1)}(x, 0) = \frac{T(x)}{1 + \mu} + \frac{\mu_2}{\pi(1 + \mu)} \int_L \frac{W'(s) ds}{s - x} + \int_a^b K_{21}(s - x) \tau(s) ds \\
+ \int_L K_{22}(s - x) W'(s) ds + \int_L K_{23}(s - x) T(s) ds \quad (-\infty < x < \infty),
$$

$$
W_2'(x, 0) = -\frac{W'(x)}{1 + \mu} - \frac{1}{\pi \mu_1(1 + \mu)} \int_L \frac{T(s) ds}{s - x} + \int_a^b K_{31}(s - x) \tau(s) ds \\
+ \int_L K_{32}(s - x) W'(s) ds + \int_L K_{33}(s - x) T(s) ds \quad (-\infty < x < \infty).
$$

Here, the results obtained in [5] are used and the kernels $K_{ij}(t)$ have the same values as in [5]. We also note that, in solving (2), after passing to the amplitudes, using the Fourier integral transform, those branches of the multi-valued function $\chi_j(s) = \sqrt{s^2 - k_j^2}$, $(k_j = \omega/c_j^2$, $j = 1, 2)$, which behave like $|s| (j = 1, 2)$ [9] at infinity, are selected.

Now, using representations (5), we satisfy the last three conditions (1) by initially going over to the amplitudes and differentiating the last two of these conditions with respect to $x$. As a result, to determine the unknown contact stresses $\tau(x)$ under the stamp, the functions of dislocation $W'(x)$ of the points of the crack edges and the contact stresses $T(x)$ under inclusions, the following governing system of singular integral equations of the second kind is obtained:

$$
\frac{1}{\pi \mu_1} \int_a^b \frac{\tau(s) ds}{s - x} + \int_a^b K_{11}(s - x) \tau(s) ds \\
+ \int_L K_{12}(s - x) W'(s) ds + \int_L K_{13}(s - x) T(s) ds = 0 \quad (a < x < b),
$$

$$
-\frac{W'(x)}{1 + \mu} - \frac{\mu_2}{\pi(1 + \mu)} \int_L \frac{W'(s) ds}{s - x} + \int_a^b K_{21}(s - x) \tau(s) ds \\
+ \int_L K_{22}(s - x) W'(s) ds + \int_L K_{23}(s - x) T(s) ds = 0 \quad (x \in L),
$$

$$
T(x) + \frac{1}{\pi \mu_1(1 + \mu)} \int_L \frac{T(s) ds}{s - x} + \int_a^b K_{31}(s - x) \tau(s) ds \\
+ \int_L K_{32}(s - x) W'(s) ds + \int_L K_{33}(s - x) T(s) ds = 0 \quad (x \in L).
$$

This system should be considered under conditions of equilibrium of the stamp, inclusions and continuity of displacements at the end points of the crack:

$$
\int_a^b \tau(x) dx = T_0, \quad \int_{a_j}^{b_j} T(x) dx = Q_n, \quad \int_{a_j}^{b_j} W'(x) dx = 0 \quad (j = 1, \ldots, N).
$$

To solve the obtained system of governing equations (6) we bring it to the canonical form. For this purpose, we multiply the last equation (6) by $\pm \sqrt{\mu_1/\mu_2}$ and add it to the second equation.
As a result, introducing new dimensionless unknown functions by the formulas
\[ \varphi_1(x) = \frac{\tau(x)}{\mu_2}, \quad \varphi_j(x) = W'(x) + (-1)^{j+1} \frac{\sqrt{\mu} T(x)}{\mu_2} \quad (j = 2, 3), \]
we come to the following system of singular integral equations:
\[
\begin{aligned}
\varphi_1(x) + \frac{1}{\pi} \int_a^b \frac{\varphi_1(s)}{s-x} ds + \int_a^b R_{1,1}(s-x)\varphi_1(s) ds + \sum_{i=2}^3 \int_{L_i} R_{1,i}(s-x)\varphi_i(s) ds &= 0 \quad (a < x < b), \\
\varphi_j(x) + \frac{(-1)^{j+1}}{\pi} \int_L \frac{\varphi_j(s)}{s-x} ds + \int_a^b R_{2,1}(s-x)\varphi_1(s) ds + \sum_{i=2}^3 \int_{L_i} R_{2,j}(s-x)\varphi_i(s) ds &= 0 \quad (x \in L, \ j = 2, 3).
\end{aligned}
\]

Conditions (10) can be written in the form:
\[
\int_a^b \varphi_1(x) dx = \frac{T_0}{\mu_2}, \quad \int_{b_j}^{b_i} \varphi_i(x) dx = (-1)^i \frac{\sqrt{\mu} Q_n}{\mu_2} \quad (i = 2, 3, \ j = 1, \ldots, N).
\]

The following notation is introduced here:
\[
\begin{align*}
R_{11}(t) &= \frac{\mu_2 K_{11}(t)}{\mu}, \quad R_{1j}(t) = \frac{1}{2\mu} \left[ K_{12}(t) + (-1)^{j+1} \frac{\mu_2 K_{13}(t)}{\sqrt{\mu}} \right] \quad (j = 2, 3), \\
R_{21}(t) &= -(1+\mu)\sqrt{\mu} K_{21}(t) + \mu_2 K_{31}(t), \\
R_{2j}(t) &= -\frac{1}{2} \left[ \frac{\sqrt{\mu} K_{22}(t)}{\mu_2} + K_{32}(t) + (-1)^{j+1} \left( K_{23}(t) + \frac{\mu_2 K_{33}(t)}{\sqrt{\mu}} \right) \right], \\
R_{31}(t) &= (1+\mu)\sqrt{\mu} K_{21}(t) - \mu_2 K_{31}(t), \\
R_{3j}(t) &= -\frac{1}{2} \left[ K_{32}(t) - \frac{\sqrt{\mu} K_{22}(t)}{\mu_2} - (-1)^{j+1} \left( K_{23}(t) - \frac{\mu_2 K_{33}(t)}{\sqrt{\mu}} \right) \right].
\end{align*}
\]

3. Anti-plane stationary stamp oscillations at the boundary of a compound half-space with an interfacial crack and absolutely rigid inclusion

Let us consider a special case of the problem when the half-space is weakened by a single main interfacial crack occupying the region \( \{ y = 0, -\infty < z < \infty, x \in (a_1, b_1) \} \), lower edge of which is reinforced by an absolutely rigid inclusion and is deformed with an absolutely rigid strip stamp with a flat base applied to the free surface of the layer in the region \( \{ y = h, a \leq x \leq b, -\infty < z < \infty \} \), i.e. when \( L = (a_1, b_1) \) (figure 1).

In this particular case, we construct a solution to system (9) using the mechanical quadrature method [10]. In this order, by changing variables \( x = pt + q \) (\( p = (b - a)/2, q = (a + b)/2 \)) in the first of equations (9) and \( x = p_1 t + q_1 \) (\( p_1 = (b_1 - a_1)/2, q_1 = (a_1 + b_1)/2 \)), in the other two equations (9), we formulate them on the interval \( (-1, 1) \) and write the system of governing equations and conditions (10) in the form:
\[
\begin{aligned}
\frac{1}{\pi} \int_{-1}^{1} \psi_1(s) ds / s-x + \sum_{i=1}^{3} \int_{-1}^{1} R_{1i}^{*}(s-x)\psi_i(s) ds &= 0 \quad (-1 < x < 1; j = 2, 3), \\
\psi_j(x) + \frac{(-1)^{j+1}}{\pi} \sqrt{\mu} \int_{-1}^{1} \psi_j(s) ds / s-x + \sum_{i=1}^{3} \int_{-1}^{1} R_{ji}^{*}(s-x)\psi_i(s) ds &= 0, \\
\int_{-1}^{1} \psi_1(x) dx &= T_0^*, \quad \int_{-1}^{1} \psi_j(x) dx = Q_{1j}^* \quad (j = 2, 3).
\end{aligned}
\]
ψ \gamma \text{ are bounded function at the both ends of crack.}

ψ \text{ difficult to restore the functions to the values of the functions } n \text{ according to the standard procedure, we get a system of 3}

\text{this relation on the interval (} -1, 1 \text{). For this, we use the second of relations (8) when (} i = 2, 3 \text{).}

\psi_j(x) = \varphi_j(px + q), \quad \psi_j(x) = \varphi_j(p_1x + q_1) \quad (j = 2, 3) \quad T^*_0 = \frac{T_0}{\mu_2}, \quad Q^*_1 = \sqrt{\mu}Q_1 \over p_1 \mu_2

R_{11}^*(s,x) = pR_{11}(p(s - x)), \quad R_{1i}^*(s,x) = p_1 R_{1i}(p_1 s + q_1 - px - q) \quad (i = 2, 3)

R_{ji}^*(s,x) = pR_{ji}(ps + q - p_1 x - q_1), \quad R_{ji}^*(s,x) = p_1 R_{ji}(p_1(s - x)) \quad (i, j = 2, 3).

\text{Studying the main parts of the equations in (11), it is easy to see that the unknown functions at the points } \pm 1 \text{ have a power-law singularity and can be represented in the following forms [10]:}

\psi_1(t) = \frac{\psi_1^*(t)}{\sqrt{1 - t^2}}, \quad \psi_2(t) = \frac{\psi_2^*(t)}{(1 + t)\gamma(1 - t)^{1-\gamma}}, \quad \psi_3(t) = \frac{\psi_3^*(t)}{(1 + t)^{1-\gamma}(1 - t)^\gamma}, \quad (13)

\text{Here } \gamma = 1/(2\pi) \arctan[2\sqrt{\mu}/(1 - \mu)] \quad (\mu < 1), \quad \gamma = 1/2 - 1/(2\pi) \arctan[2\sqrt{\mu}/(\mu - 1)] \quad (\mu > 1), \quad \text{and } \psi_j^*(x) \text{ are continuous functions bounded up to the ends of the interval } [-1, 1].\text{ Substituting the values of the functions } \psi_j(x) \quad (j = 1, 2, 3) \text{ in (11)–(12) and using the relations given in [10] according to the standard procedure, we get a system of } 3n \text{ algebraic equations with respect to the values } \psi_j^*(\xi_i) \quad (j = 1, 2, 3; i = 1, \ldots, n). \text{ After determining the functions } \psi_j^*(\xi_i), \text{ it is not difficult to restore the functions } \psi_j(x) \quad (j = 1, 2, 3) \text{ using the Lagrange interpolation polynomial and determine all the necessary values characterizing the stress state in the layer and half-space. We write a formula for determining the intensity coefficient of destructive stresses at the end points of a crack. For this, we use the second of relations (8) when (} |x| > a \text{). Next, we formulate this relation on the interval } (-1, 1) \text{ using functions } \psi_j^*(\xi_i) \text{ and represent it in the form:}

\tau_{g2}^{(j)}(p_1 t + q_1, 0) = \frac{\mu_2}{2\pi(1 + \mu)} \int_{-1}^{1} \frac{[\psi_1(\xi) + \psi_2(\xi)] d\xi}{\xi - t} + F_j(t) \quad (|t| > 1, j = 1, 2), \quad (14)

\text{where}

F_j(t) = \mu_2 \int_{-1}^{1} K_{21}(p_\xi + q - p_1 \xi - q_1)\psi_1(\xi) d\xi

+ \frac{1}{2\sqrt{\mu}} \int_{-1}^{1} [\sqrt{\mu}K_{22}(p_1(\xi - t)) - \mu_2 K_{23}(p_1(\xi - t))]\psi_2(\xi) d\xi

+ \frac{1}{2\sqrt{\mu}} \int_{-1}^{1} [\sqrt{\mu}K_{22}(p_1(\xi - t)) + \mu_2 K_{23}(p_1(\xi - t))]\psi_3(\xi) d\xi \quad (j = 1, 2),

\text{are bounded function at the both ends of crack.}
Substituting in (14) the value of the functions $\psi_1(t)$ and $\psi_2(t)$ from (13) and taking into account the relation [11]

$$
\int_a^b \frac{(x-a)^{\alpha-1}(b-x)^{-\alpha}}{x-y} \, dx = \frac{\pi}{(b-y)\sin\pi\alpha} \left| \frac{a-y}{b-y} \right|^{\alpha-1} \quad (0 < \text{Re} \alpha < 1, y < a < b, a < b < y),
$$

dimensionless stresses $\tau_{*}^{(j)}(x)$ can be represented as:

$$
\tau_{*}^{(j)}(x) = \frac{\tau_{yz}^{(j)}(p_1 t + q_1, 0)}{\mu_2} = \frac{\text{sgn}(x)}{2(1 + \mu)\sin\gamma} \left\{ \frac{\psi_3^{*}(\pm 1)}{|1 + x|^\gamma|1 - x|^{1 - \gamma}} + \frac{\psi_3^{*}(\pm 1)}{|1 + x|^{1 - \gamma}|1 - x|^\gamma} \right\} + F_j^*(t),
$$

where $F_j^*(t)$ be the bounded function at both ends of the crack.

Since $\gamma < 1/2$ the singularity of the function $\tau_{yz}^{(j)}(x, 0)$ at the point $x = b_1$ ($t = 1$) is determined by the first term, and at the point $x = a_1$ ($t = -1$) by the second term. Therefore, the complex intensity factors of the fracture stresses at the end points of the crack will be given by the formulas:

$$
K_{III}^{*}(a_1) = \frac{K_{III}^{(1)}(a_1) + iK_{III}^{(2)}(a_1)}{\mu_2} = \sqrt{2\pi} \lim_{t \to -1+0} |t + 1|^{-1 - \gamma} \tau_{*}^{(j)}(t) = \frac{2^{-\gamma-1/2} \sqrt{\pi} \psi_3^{*}(1)}{(1 + \mu)\sin(\pi\gamma)},
$$

$$
K_{III}^{*}(b_1) = \frac{K_{III}^{(1)}(b_1) + iK_{III}^{(2)}(b_1)}{\mu_2} = \sqrt{2\pi} \lim_{t \to 1+0} |t - 1|^{-1 - \gamma} \tau_{*}^{(j)}(t) = \frac{2^{-\gamma-1/2} \sqrt{\pi} \psi_3^{*}(-1)}{(1 + \mu)\sin(\pi\gamma)}.
$$

Then

$$
K_{III}(c, t) = K_{III}^{*}(c) e^{i\omega t} = |K_{III}^{*}(c)| e^{i(\omega t - \delta)} \quad \left( \delta = - \arctan \frac{K_{III}^{(2)}}{K_{III}^{(1)}}, \quad c = a_1, b_1 \right).
$$

To determine the amplitude of the dimensionless crack opening, we will use the formula:

$$
w_* = \frac{w(a_1 t + b_1)}{p_1} = \frac{1}{2} \int_{-1}^t [\varphi_2(\xi) + \varphi_3(\xi)] \, d\xi.
$$

In order to determine the dimensionless contact stresses acting under inclusion we use the second of formulas (8) and define the function $T(x)$ in terms of functions $\varphi_j(x)$. Then, considering that $\tau_{yz}^{(1)}(x, 0) = 0$ when $a_1 < x < b_1$, as a result of which, in the indicated interval $T(x) = -\tau_{yz}^{(2)}(x, 0)$ and writing the expression obtained on the interval $(-1, 1)$ using functions $\psi_j(t)$ ($j = 2, 3$), to determine the dimensionless contact stresses, we obtain the formula:

$$
\tau(t) = \frac{\tau_{yz}^{(2)}(p_1 t + q_1, 0)}{\mu_2} = \frac{T(p_1 t + q_1, 0)}{\mu_2} = \frac{\psi_2(t) - \psi_3(t)}{2\sqrt{\mu}} \quad (-1 < t < 1).
$$

4. Some numerical results

In some practically important particular cases, a numerical calculation was carried out and the regulations of the changes of the important mechanical characteristics for the problem depending on the frequency of the forced vibrations, the ratio of the shear moduli of the layer and half-space, and the location of the crack and stamp were studied. First, we present the results of numerical calculations of the problem in the case when the crack length is equal to the stamp width and the layer height, and the stamp is directly above the crack, i.e. when,

$$
\frac{p}{h} = \frac{p_1}{h} = 1, \quad \frac{q}{h} = \frac{q_1}{h} = 0, \quad c_2^{(1)} = 2.
$$
In this case, it is assumed that $T_0 = 0.05$, $Q_i^1 = 0$, $\mu = 2/3$ and studied the laws of changes in the modules of dimensionless intensity factors of destructive stresses and their arguments at the end points of the crack, as well as the opening of the crack and contact stresses under the stamp and inclusion, depending on the reduced frequency of the forced vibrations $\lambda = h\omega/c_2^{(1)}$. The results of numerical calculations are given in table 1 and figures 2–4. Table 1 shows the amplitude moduli of dimensionless intensity factors of destructive stresses and their arguments at the end points of the crack.

It is apparent that with an increase in the parameter $\lambda$, the intensity factors increase first and then decrease. Moreover, when $\lambda = 0.92$, as the case under consideration suggests, they take the maximum value.

Figures 2–4 show graphs, respectively, of the absolute value of the amplitude of crack opening, of the real part of the dimensionless amplitudes of contact stresses under the stamp and under inclusion in the case when $\lambda = 0, 1, 2$. From them it is clear that with an increase in the frequency of forced vibrations, the absolute value of the amplitude of crack opening increases and then...
decreases (figure 2). In this case, the amplitudes of the real parts of the contact stresses under the stamp are redistributed: they increase in the middle part of the stamp and decrease at the ends (figure 3). As for the real part of the contact stresses acting under inclusion, they change sign with increasing parameter $\lambda$ (figure 4). In the case considered above, the dependence of the intensity factors of destructive stresses at the end points of the crack on the parameter $\mu$ in the case when $\lambda = 0.5$ was also studied. The calculation results are shown in Table 2 and are presented in the form of graphs (figures 5 and 6).

Table 2 shows the values of the amplitude moduli of the stress intensity factors of the destructive stresses at the end points of the crack at various values $\mu$. It can be seen that with an increase in the parameter $\mu$, which can be interpreted as decreasing $\mu_1$ at a constant $\mu_2$, the amplitudes of the stress intensity factors of the destructive stresses at both ends of the crack increase.

Figures 5–6 respectively show graphs of the absolute values of the amplitudes of crack opening and the real part of dimensionless contact stresses under inclusion, depending on the parameter $\mu$. It can be seen from them that with an increase in the parameter $\mu$, both the absolute values of the amplitudes of crack opening and the real parts of dimensionless contact stresses under inclusion also increase. The calculations also show that the contact stresses under the stamp less depend on the parameter $\mu$.

A numerical calculation of the problem is carried out in one more case, when $T_0^* = 0.05$, $Q_1^* = 0$, $\lambda = 0.5$, $\mu = 2/3$, $c/h = p_1/h = 1$, $e_2^{(1)}/e_2^{(2)} = 2$ and the laws of changing the amplitudes of the stress intensity factors of destructive stresses at the end points of the crack, their argument $\delta$, the real parts of the contact stress amplitudes under the stamp and under inclusion, and also the crack opening depending on the parameters $k = q/h$.

The results of numerical calculations are given in the form of table 3 and graphs figures 7–9. Table 3 shows the magnitudes of the amplitude moduli of the intensity factors of the destructive stresses at the end points of the crack and its argument, depending on $k$.

From table 3 it is seen that with an increase in the parameter $k$, i.e. when the stamp is removed from the crack, as in the absence of inclusion [5], the amplitudes of the stress intensity factors of the destructive stresses at the end points of the crack and its argument, depending on $k$. 

### Table 2.

| $\mu$ | 0.25 | 0.5 | 1   | 2   | 5   |
|-------|------|-----|-----|-----|-----|
| $|K_{II}(\pm1)|$ | 0.00312 | 0.00603 | 0.00962 | 0.01349 | 0.01832 |
Factors at both ends of the crack decrease, while their argument at the left end of the crack increases in absolute value, and decreases at the right end of the crack, and then also increases.

Figures 7–9 show graphs, respectively, of the real part of the dimensionless amplitudes of contact stresses under the stamp, under inclusion, and the absolute value of the amplitude of crack opening in the case when $k = 0, 1, 5$. From them it can be concluded that when the stamp is removed from the crack, both the contact stresses under inclusion and the absolute value of the crack opening amplitude decrease, tending to zero. At the same time, as one would expect, contact stresses under the stamp hardly changes.

| $k$ | 0   | 1   | 2   | 10  | 50  |
|-----|-----|-----|-----|-----|-----|
| $|K_{III}(-1)|$ | 0.00485 | 0.00654 | 0.00541 | 0.00099 | 0.00034 |
| $|K_{III}(1)|$ | 0.00485 | 0.00107 | 0.00086 | 0.00083 | 0.00032 |
| $\delta(-1)$ | 3.1215 | 3.1374 | 3.1131 | 2.0323 | 0.6233 |
| $\delta(1)$ | -3.1214 | -2.9343 | -0.4002 | -1.4458 | -2.8886 |
Conclusion
The regularities of the mutual influence of the interfacial absolutely rigid, thin inclusions partially detached from the matrix located in a piecewise-homogeneous half-space and an absolutely rigid stamp with a flat base acting on the free surface of the half-space and under the influence of harmonic in time concentrated load are studied. Using numerical calculations, it is shown that in the case when there is only one inclusion partially detached from the matrix, the length of which is equal to the stamp width and the layer height and when the stamp is directly above the crack, at the selected parameter values, the absolute value of the stress intensity factors of the destructive stresses at the end points of the crack formed at the separation of the inclusion, with increasing frequency of the forced oscillations increase and take the maximum value at $\omega = 0.92c_2^{(1)}/h$, which is approximately 12% more than in the case when the inclusion is absent [5]. With a further increase in the frequency of forced oscillations, they decrease, tending to zero. In the indicated case, it was shown that with a decrease in the parameter $\mu$, as in the absence of inclusion [5], the intensity factors of the destructive stresses at the end points of the crack decrease in absolute value.

It was also shown that when the stamp is removed from the inclusion partially detached from the matrix, as expected, both the amplitudes of the stress intensity factors of destructive stresses at both ends of the crack, the contact stresses under the inclusion, and the crack opening in absolute value decrease.

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