Abstract. We calculate the fluxes of radio, hard X-rays and gamma ray emission from clusters of galaxies, in the context of a secondary electron model (SEM). In the SEM the radiating electrons are produced by the decay of charged pions in cosmic ray (CR) interactions with the intracluster medium, while gamma ray emission is mainly contributed by the decay of neutral pions. We specifically applied our calculations to the case of the Coma cluster, and found that the combined radio and hard X-ray fluxes can be explained in the SEM only if very small values of the intracluster magnetic field ($B \sim 0.1 \mu G$) are assumed, which in turn imply a large energy density of the parent CRs. The consequent gamma-ray fluxes easily exceed the EGRET limit at 100 MeV. This conclusion can be avoided only if most of the hard X-ray emission from Coma is not produced by Inverse Compton Scattering (ICS).

1. Introduction

Clusters of galaxies have recently revealed themselves as sites of high energy processes, resulting in a multiwavelength emission which extends from the radio to the gamma rays and probably beyond. In this paper we refer to the Coma cluster, because of the wide evidences now accumulated for the presence of these non-thermal phenomena.

The high energy processes which produce the observable radiations are due to the presence of a non thermal population of particles originating most likely from the cosmic ray sources in the cluster. The important role of cosmic ray electrons in Coma and in a few other clusters of galaxies is known since a long time because of the diffuse radio emission which extends over typical spatial scales of order $\sim 1$ Mpc. This radiation can be interpreted as synchrotron emission of relativistic electrons in the intracluster magnetic field. However, the combination of energy losses and diffusive propagation of these electrons makes their motion from the sources extremely difficult, so that it becomes a challenge to explain the spatial extent of the diffuse radio emission, unless electrons are continuously reaccelerated in the intracluster medium (ICM) (see [1] for a recent review). To solve this problem the SEM was first proposed in [2, 3]: in this model CRs (protons) diffuse on large scales because their energy losses are negligible and can produce electrons in situ as secondary products of $pp$ interactions with the production and decay of charged pions.

The production of charged pions is always associated with the production of neutral pions which in turn result into gamma rays mainly with energies above $\sim 100$
MeV. The flux of gamma radiation and high energy neutrinos due to the cosmological distribution of clusters of galaxies was calculated in \[4\] and \[5\], where the resulting diffuse neutrino background was also evaluated. Fluxes from single clusters were also compared with the upper limits on the gamma ray emission from the EGRET instrument onboard the CGRO satellite.

More recently UV \[6\] and hard X-ray \[7\] observations of the Coma cluster have led to the first detection of large fluxes at these wavelengths. Their interpretation based on ICS of relativistic electrons off the photons of the microwave background radiation requires an intracluster magnetic field of \(B \sim 0.1 \mu G\) \[7\].

In this paper we calculated the multifrequency spectrum of the Coma cluster in the context of the SEM and compared our predictions with the results of recent observations. We find that also for secondary electrons, the radio and hard X-ray observations imply an intracluster magnetic field \(B \sim 0.1 \mu G\) and large energy densities in CRs. As a consequence, the flux of gamma rays above 100 MeV exceeds the EGRET upper limit \[8\]. We also discuss alternative scenarios which do not imply large CR energy requirements.

The plan of the paper is as follows: in section 2 we describe the propagation of CRs in clusters of galaxies; in section 3 we calculate the fluxes of secondary radio, X and gamma radiation and we present our conclusions with application to the case of the Coma cluster in section 4.

### 2. Cosmic ray propagation in clusters of galaxies

The propagation of CRs in clusters of galaxies was considered in previous works \[4, 9, 5\] where the effect of diffusive confinement of CRs was investigated. We summarize these results in the following.

The CRs produced in a cluster of galaxies propagate diffusively in the intracluster magnetic field with a diffusion time, over a spatial scale \(R\), that can be estimated as \(\tau \approx R^2/4D(E)\), where \(D(E)\) is the diffusion coefficient. Very little is known about the diffusion coefficient in clusters, but as argued in \[4, 5\], for the bulk of CRs the diffusion time at large distances from the cluster center (\(R \geq R_c\), with \(R_c \sim 1\) Mpc the radius of the cluster) exceeds the age of the cluster for any reasonable choice of \(D(E)\). Assuming the following form of the diffusion coefficient, \(D(E) = D_0 E^n\), the maximum energy for which CRs are confined in the cluster is \(E_c \approx (R_c^2/D_0 t_0)^{1/n}\), where \(t_0\) is the age of the cluster, comparable with the age of universe. Despite of the large diffusion times in the cluster, CR protons do not suffer appreciable energy losses in the interesting energy range, so that the propagation can be simply described by a transport equation with no energy loss term (see, e.g., \[10\]). Let us first consider the case of a single point source in the center of the cluster, injecting CRs with a rate \(Q_p(E)\). As shown in \[10\] the equilibrium number density of CRs with energy \(E\) which solves the transport equation can be written in the form

\[
n_p(E, r) = \frac{Q_p(E)}{D(E)} \frac{1}{2\pi^{3/2}r} \int_{r/r_{max}(E)}^{\infty} dy e^{-y^2},
\]

where \(r\) is the distance from the source and \(r_{max}(E) = \sqrt{4D(E)t_0}\). It is interesting to note that for \(r \ll r_{max}(E)\) the CR distribution tends to the well known time independent form: \(n_p(E, r) = Q_p(E)/(4\pi r D(E))\) (see \[4, 5\] for details).

In the case of CRs injected homogeneously in the ICM, the equilibrium distribution
of CRs can be written as
\[ n_p(E) = K \frac{\epsilon_{\text{tot}}}{V} p^{-\gamma}, \tag{2} \]
where \( V \) is the volume of the cluster, \( \epsilon_{\text{tot}} \) is the total energy in CRs injected in the cluster and we assumed that the injection spectrum is a power law in the CR momentum \( p \). The constant \( K \) is a normalization constant determined by energy conservation. Clearly this solution breaks down close to the boundary of the cluster and at high energies, where CRs are no longer confined in the cluster.

3. Cosmic ray interactions and secondary electron emission

The main interaction channel of CR protons in clusters of galaxies is represented by \( pp \) collisions with pion production. The decay of neutral pions produces gamma rays with energy above \( \sim 100 \) MeV, while the decay of charged pions results in electrons and neutrinos. The production of gamma rays and neutrinos from clusters of galaxies was recently investigated in \[4, 5\]. The secondary electrons produced by charged pions can play a fundamental role in the explanation of not thermal emission in clusters of galaxies, and here we describe this point in a greater detail.

The ‘primary electrons’ models proposed as an explanation of radio halos in Coma-like clusters have serious problems due to the severe energy losses that make difficult the propagation of the relativistic electrons out to Mpc scales, where the diffuse radio halo emission is observed. This problem would be solved if electrons were produced or accelerated \textit{in situ}. As initially proposed in \[2, 3\] this is the case for secondary electrons, generated in CR interactions with the thermal gas in the ICM. The same electrons would also produce X-rays by inverse compton scattering (ICS) off the photons of the microwave background.

The calculation of the production spectrum of secondary electrons is explained in detail in \[14\], and will be summarized here. For both radio and X-ray emission, the relevant electrons have energies above \( \sim 1 \) GeV, and it is important to have a good description of the \( pp \) interaction in a wide range of energies. For \( pp \) collisions at laboratory energy less than \( \sim 3 \) GeV, the pion production is well described by a isobar model, in which the pions are the result of the decay of the \( \Delta \) resonance \[12, 13\]. For energies larger than \( \sim 7 - 10 \) GeV we use a scaling approach. In the latter the cross section for \( pp \) collisions depends only on the ratio of the pion energy \( E_{\pi} \) and the incident proton energy \( E_p \) \((x = E_{\pi}/E_p)\) and can be written in the following form
\[ \frac{d\sigma(E_p, E_{\pi})}{dE_{\pi}} = \sigma_0 f_x(x), \tag{3} \]
where \( f_x(x) \) is the scaling function given in \[14\] for the case of charged and neutral pions. We refer to \[14\] for a detailed expression of the cross section in the low energy case.

The production electron spectrum at distance \( r \) from the cluster center, assumed to be spherically symmetric, can be easily calculated according to the expression
\[ q_{e}(E_{e}, r) = \frac{m_{\pi}^2}{m_{\pi}^2 - m_{\mu}^2} n_H(r)c \cdot \int_{E_{\pi}}^{E_{\pi}^{\text{max}}} dE_{\pi} \int_{E_{\pi}^{\text{min}}}^{E_{\pi}^{\text{max}}} dE_{\pi} \int_{E_{\pi}(E_{\mu})}^{E_{\pi}^{\text{max}}} dE_{\pi} \int_{E_{\pi}(E_{\mu})}^{E_{\pi}^{\text{max}}} dE_{\pi} F_{\pi}(E_{\pi}, E_{\mu}) F_{e}(E_{e}, E_{\mu}, E_{\pi}) n_p(E_p, r), \tag{4} \]
where $F_\pi$ is the differential cross section for the production of a pion with energy $E_\pi$ in a $pp$ collision at energy $E_p$ (see [10] for details), $F_e$ is the spectrum of electrons generated by the decay of a single muon with energy $E_\mu$ and $n_p$ is the spectrum of CRs. The function $F_e$ depends also on the pion energy because the muons produced in the pion decay are fully polarized, and this effect is taken into account here. The gas density at distance $r$ is assumed to follow a King profile

$$n_H(r) = n_0 \left[ 1 + \left( \frac{r}{r_0} \right)^2 \right]^{-3\beta/2},$$

(5)

where, in the case of the Coma cluster we use $n_0 = 3 \times 10^{-3} cm^{-3}$, $r_0 = 400 kpc$ and $\beta = 0.75$ (we use here $H_0 = 60 km s^{-1} Mpc^{-1}$).

The equilibrium electron distribution, $n_e(E_e, r)$, is achieved mainly due to energy losses, dominated at high energy by ICS and synchrotron emission and at low energy by Coulomb scattering. The effect of losses is to produce a steepening of the electron spectrum by one power in energy, at high energy, and a flattening by one power of energy, at low energy. In the next subsections we outline the calculations of non thermal radio, X and gamma ray emission from a cluster.

3.1. The radio halo emission

Electrons with energy $E_e$ in a magnetic field $B$ radiate by synchrotron emission photons with typical frequency

$$\nu = 3.7 \times 10^6 B_e E_e^2 Hz,$$

(6)

where $B_e$ is the value of the magnetic field in $\mu G$. The emissivity at frequency $\nu$ and at distance $r$ from the cluster center can be easily estimated as

$$j(\nu, r) = n_e(E_e, r) \left( \frac{dE_e}{dt} \right)_{syn} \frac{dE_e}{d\nu},$$

(7)

where $(dE_e/dt)_{syn}$ denotes the rate of energy losses due to synchrotron emission and $dE_e/d\nu$ is obtained from eq. (6). The total fluence from the cluster is obtained by volume integration.

Some simple comments can help understanding general features of the radio halo spectrum: for this purpose let us assume that CR are confined in the cluster and that the density of intracluster gas is spatially constant. If the injected CR spectrum is $\propto E_p^{-\gamma}$ then the equilibrium CR spectrum is, within the distance $r_{max}(E)$ defined above, a power law $\propto E_p^{-(\gamma+\eta)}$. Provided the electron energy is $E_e \geq 1$ GeV, the production electron spectrum reproduces the parent CR spectrum, so that the equilibrium electron spectrum in the same energy region is $\propto E_e^{-(\gamma+\eta+1)}$. From eq. (6) it is easy to show that $j_\nu(\nu, r) \propto \nu^{-(\gamma+\eta)/2}$. The volume integration gives the observed spectrum: in the simple assumptions used here (complete confinement) the integration over the distance $r$ at each frequency $\nu$ must be limited by a maximum value $r_{max}(\nu) \propto \nu^{\gamma/4}$, so that the resulting radio halo spectrum is $\propto \nu^{-\gamma/2}$, independent of the diffusion details. Repeating the same discussion, it is easy to show that the synchrotron radiation produced by CRs non confined in the cluster volume has a spectrum as steep as $\nu^{-(\gamma+\eta)/2}$. Therefore there is, in principle, a break frequency where the spectrum steepens from a power index $\gamma/2$ to a power index $(\gamma + \eta)/2$. In
any realistic scenario, there is a smooth steepening of the spectrum. The presence of a density profile in the intracluster gas also contributes an additional small steepening of the spectrum. All these propagation effects are self-consistently considered in eq. (1).

3.2. Non thermal X-rays

The peak energy where most of the photons are produced by ICS of electrons with energy $E_e$ is

$$E_X = 2.7 E_e^2 \text{(GeV)} \text{keV}. \quad (8)$$

So, that the emissivity in the form of X-rays with energy $E_X$ at distance $r$ from the cluster center is:

$$\phi_X(E_X, r) = n_e(E_e, r) \left( \frac{dE_e}{dt} \right)_{ICS} \frac{dE_e}{dE_X}. \quad (9)$$

Here $(dE_e/dt)_{ICS}$ is the rate of energy losses due to ICS and $dE_e/dE_X$ is calculated from eq. (8). As for the radio halo emission, the observed fluence is determined by volume integration.

3.3. The gamma ray emission

As pointed out above, the production of neutral pions in $pp$ collisions results in the generation of gamma rays with typical energy $E_\gamma \geq 100 \text{ MeV}$. The emissivity of gamma rays with energy $E_\gamma$ at distance $r$ from the center of the cluster is given by

$$j_\gamma(E_\gamma, r) = 2n_H(r)c \int_{E_{\pi}^{min}(E_\gamma)}^{E_{\pi}^{max}(E_\gamma)} \int_{E_{\text{th}}(E_\pi)}^{E_{\text{th}}(E_\gamma)} dE_p F_{\pi0}(E_\pi, E_p) \frac{n_p(E_p, r)}{(E_\pi^2 + m_\pi^2)^{1/2}}. \quad (10)$$

where $E_{\pi}^{min}(E_\gamma) = E_\gamma + m_\pi^2/(4E_\gamma)$. The function $F_{\pi0}$ is again calculated using the isobar model for proton energy less than 3 GeV and with the scaling model for energies larger than 7 GeV. The observed gamma ray flux is obtained by integration over the cluster volume.

In [10] the contribution of secondary electrons to the gamma ray flux through bremsstrahlung was also calculated. Since its contribution is small if compared with the contribution from pion decay, we neglect here the bremsstrahlung gamma ray emission.

4. Application to the Coma cluster

In this section we apply our predictions in the SEM framework to the case of the Coma cluster, for which multiwavelength observations are available.

Two extreme scenarios of CR injection are considered here: a point-like CR source in the cluster center and a uniform CR injection distributed over the cluster volume. In both cases we assume that the CR injection spectrum is a power law in momentum with power index $2.1 \leq \gamma \leq 2.4$, covering the range of values expected for first order Fermi acceleration at shocks as well as for other CR acceleration mechanisms.
The diffusion of CRs in the cluster is described by a diffusion coefficient derived from a Kolmogorov spectrum of magnetic fluctuations in the cluster, according with the procedure described in \[5\]. In this case the diffusion coefficient can be written as

\[
D(E) = 2.3 \times 10^{29}E(\text{GeV})^{1/3}B_\mu^{-1/3} \left( \frac{l_c}{20\text{kpc}} \right)^{2/3} \text{cm}^2/\text{s} \quad (11)
\]

where \(l_c\) is the size of the largest eddy in the Kolmogorov spectrum.

The procedure used to evaluate the expected fluxes is the following: we first fit the spectrum of the radio halo as given in \[14\] for \(\gamma = 2.1\) and \(\gamma = 2.4\). This allows to find the value of the absolute normalization of the injection CR spectrum in terms of an injection luminosity \(L_p\) as a function of the average intracluster magnetic field \(B_\mu\). We carried out this calculation for \(B_\mu = 0.1, 1, 2\). After calculating \(L_p\) in this way, we determine the hard X-ray flux and the gamma ray flux above 100 MeV according to the expressions given in the previous section. Our results for the radio and hard X-ray emission for the case of a single source are shown in Figs. 1 and 2 respectively. The three panels refer to values \(B_\mu = 0.1, 1, 2\), as indicated. The normalization constants and the integral fluxes of gamma rays above 100 MeV in the same cases are reported in Table 1, where the gamma ray flux is compared with the upper limit obtained by the EGRET experiment \[8\]. The last column in Table 1 contains the ratio of the flux of gamma rays from electron bremsstrahlung compared with the flux of gamma rays from pion decay, as calculated in \[10\].

**Figure 1.** Fluxes of radio radiation from the Coma cluster calculated in the SEM for \(\gamma = 2.1\) (solid lines) and \(\gamma = 2.4\) (dashed lines). The three panels are, from left to right, for \(B_\mu = 0.1, 1, 2\).

Fig. 2 shows that a joint fit for the radio and hard X-ray fluxes is possible only
for low values of the magnetic field \( (B_\mu \sim 0.1) \), which in turn imply large values of \( L_p \) (see Table 1). As a consequence, the gamma ray fluxes easily exceed the EGRET upper limit from Coma. These results indicate that the SEM cannot explain the multiwavelength observations of the Coma cluster without violating the EGRET limit. This conclusion is not appreciably changed in the case of homogeneous injection of CRs. In this case, the value of \( \gamma \) is fixed by radio observations and is \( \gamma = 2.32 \). In order to fit the hard X-ray data at the same time, an ICM magnetic field \( B_\mu \approx 0.1 \) and a total energy in CRs \( \epsilon_{\text{tot}} \approx 8 \times 10^{63} \) erg are required. This value, averaged over the age of the cluster, corresponds to a typical CR luminosity of \( L_p \approx 2 \times 10^{46} \) erg/s, which implies a gamma ray flux above 100 MeV in excess of the EGRET upper limit by a factor \( \sim 3 \). Therefore, also for a homogeneous CR injection, the SEM fails in explaining the radio and hard X-ray observations at the same time, without exceeding the EGRET limit.

This conclusion has, however, further important consequences for other models too. In fact, we checked that the CR energy densities obtained in the present paper are comparable with those obtained assuming equipartition, as done in recent papers (e.g. [11]) to explain the radio, hard X-ray and UV observations. Therefore, the gamma ray limit applies to other models as well and forces the CR energy density in clusters to be some fraction of the equipartition value. Actually, as shown in [11], the present gamma ray observations put weak constraints on this fraction, but future gamma ray observations will definitely do better. We can envision at least two other arguments against equipartition of CRs and the ICM in clusters of galaxies: first of all, the most powerful CR sources typically present in clusters of galaxies (see [4, 5]) allow to achieve a CR energy density equal to a small fraction (typically \( \sim 5\% \)) of the equipartition value. Moreover, the magnetic field derived in [5] \( (B \sim 0.1 \mu G) \), which is the main reason to call for equipartition, is quite smaller than the equipartition magnetic field in the cluster, and it seems difficult to envision a scenario where CRs are in equipartition but not magnetic fields, in particular if the origin of CRs and magnetic fields in clusters are related each other.

| \( B_\mu \) | \( \gamma \) | \( L_p \) \( \times 10^{42} \) ergs \(-1\) | \( F_\gamma(E_\gamma \geq 100 \text{MeV}) \) | \( F_{\text{EGRET}}(E_\gamma \geq 100 \text{MeV}) \) | \( F^{\text{brem}}_\gamma / F^\pi_\gamma \) |
|---------|-------|----------------|----------------|----------------|----------------|
| 0.1     | 2.1   | 50             | 1.93           | 7.15           | 0.13           |
| 0.1     | 2.4   | 180            | 1.8 \cdot 10^{-2} | 4.5 \cdot 10^{-2} | 0.14           |
| 1       | 2.1   | 0.35           | 5.3 \cdot 10^{-3} | 1.1 \cdot 10^{-2} | 0.12           |
| 1       | 2.4   | 1              | 1              | 0.11           |
| 2       | 2.1   | 0.1            | 1.1 \cdot 10^{-2} | 0.095          |

So, at the present status of the debate, it is necessary to look for possible alternative explanations of the hard X-ray excess observed in Coma with the SAX satellite, since there is no stringent argument in favour of the ICS origin of such hard X-ray tail.
Figure 2. Spectrum of the diffuse X-ray emission from Coma. The three panels refer to the same values of the IC magnetic field as in Fig. 1. The shaded area shows the best fit to the HEAO1-A4 and GINGA thermal emission data (open triangles) at $T = 8.21 \pm 0.20$ keV [18]. The OSSE upper limits [17] are indicated by the open circles. The SAX data [7] are indicated by filled squares. For this data set, errors are shown at 90% confidence level. Arrows and labels show, for each panel, the energy ranges in which the three different data sets are located. Predictions of the SEM for $\gamma = 2.1$ (solid lines) and $\gamma = 2.4$ (dashed lines) are shown in each panel.

A possible alternative was proposed in [16] where it was speculated that the presence of a non thermal tail in the electron distribution could account for the X-ray excess through bremsstrahlung emission. If this turns out to be the explanation of the hard X-ray excess, then the radio and hard X-ray fluxes become unrelated and magnetic fields of order $\geq 1 \mu G$, as the ones suggested by Faraday rotation measurements, would still be allowed. As a consequence, the CR energy density required to fit the radio and hard X-ray data would be of the same order of that predicted in [4, 5]. In this case, the SEM still remains a viable option for the origin of the Coma radio halo emission.
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