In the chirally symmetric Nambu–Jona–Lasinio model baryons are described as chiral solitons of mesonic quark–antiquark bound states. Hyperons are investigated in two complementary pictures: the bound state approach of Callan and Klebanov and the collective approach of Yabu and Ando. The latter is used to compute the strange vector form factors of the nucleon providing estimates for the strange electric mean square radius and the strange magnetic moment of the nucleon.

For a large number of colors ($N_C$) QCD reduces to an effective theory of weakly interacting mesons. Witten conjectured that within this effective theory baryons emerge as soliton solutions. Although Witten’s conjecture has never been proven the soliton picture of baryons has turned out quite successful in recent years.

The numerous attempts to derive the effective meson theory from QCD indicate that at low energies the effective meson theory is almost entirely determined by chiral symmetry. This suggests to study simpler chirally invariant models of the quark flavor dynamics. The prototype is the Nambu–Jona–Lasinio (NJL) model. It can be rewritten in mesonic degrees of freedom (bosonization). The predictions from the resulting effective meson theory are in satisfactory agreement with the low-energy data for pseudoscalar mesons. Moreover, the bosonized NJL model contains soliton solutions. The studies of the soliton transparently explain how baryons emerge as solitons in effective meson theories starting from an underlying quark theory simultaneously confirming Witten’s conjecture. The reason being that the baryon number is carried by the polarized vacuum for the self–consistent soliton when not only the pseudoscalar but also the (axial)vector mesons are incorporated.

Aiming at the description of hyperons within the soliton picture the flavor symmetry breaking has been treated in two conceptionally different ways: In the bound state approach of Callan and Klebanov (CK) and in the collective approach of Yabu and Ando (YA), cf. table 1. They not only predict the spectrum of the low–lying baryons but also yield baryon wave–functions in a certain configuration space. This in turn permits studying the effects of strangeness in the nucleon by computing the pertinent matrix elements. This is very interesting because there are many experimental indications for significant effects of strangeness in the nucleon. Especially experiments measuring parity violating asymmetries in scattering processes of polarized electrons on nuclei provide access to hadronic “observables” like $\langle N | \bar{s} \gamma_\mu s | N \rangle$, which e.g. enter the matrix elements of the neutral current between nucleon states.

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Table 1. Comparison of the collective and the bound state approach.

| Collective approach (YA) | Bound state approach (CK) |
|---------------------------|---------------------------|
| Symmetry breaking small (light strange quark) | Symmetry breaking large (heavy strange quark) |
| Strange components as collective coordinates (analogous to zero modes) | Restoring force for strange fluctuations (“harmonic” potential) |
| Collective Hamiltonian in flavor SU(3) including symmetry breaking | Bound State Energy and Wave Function |
| Exact diagonalization | Collective quantization of spin and isospin |
| HYPERONS | HYPERONS |

For these studies we will consider the NJL model with scalar ($S$) and pseudoscalar ($P$) fields only. After integrating out the quark fields in favor of these mesons the action in Euclidean space reads ($M = S + iP$)

$$A_{NJL} = N_C \text{Tr}_A \log \left[ i \phi_E + \frac{1}{2} \left( M^\dagger + M \right) + \frac{1}{2} \gamma_5 \left( M^\dagger - M \right) \right] + \frac{1}{4G} \text{Tr} \left[ \hat{m}^0 \left( M^\dagger + M \right) \right], \quad (1)$$

where the trace includes Euclidean space–time, flavor and spin degrees of freedom. The proper–time regularization is indicated by the $O(4)$ invariant cut–off $\Lambda$ and $G$ denotes the dimensionful coupling constant of the NJL Model. The flavor symmetry breaking occurs in two instances: explicitly in the current quark mass matrix\[\hat{m}^0 = \text{diag}(m^0, m^0, m^0_s)\] and (as a consequence of the gap–equations) in the vacuum expectation values of the meson fields $\langle M \rangle = \text{diag}(m, m, m_s)$. The quantities $m$ and $m_s$ are referred to as the up and strange constituent quark masses, respectively. The parameters in the non–strange sector are fitted to the pion mass ($m_\pi = 135\text{MeV}$) and decay constant ($f_\pi = 93\text{MeV}$). The remaining undetermined parameter can be expressed in terms of $m$. In the strange sector the kaon mass ($m_k = 495\text{MeV}$) is employed to compute $m^0_s$. Then the kaon decay constant ($f_k$) is left as a prediction. For commonly accepted values of $m = 350\ldots450\text{MeV}$ the experimental value ($f_k \approx 113\text{MeV}$) is underestimated by about 10 to 15%.

In order to construct the static soliton solution to the action (1) the scalar fields are constrained to their vacuum expectation values and for the pseudoscalar fields the hedgehog ansatz is adopted

$$M_0(r) = \begin{pmatrix} m \exp(i \mathbf{r} \cdot \hat{r} \Theta(r)) & 0 \\ 0 & m_s \end{pmatrix}. \quad (2)$$

The radial function $\Theta(r)$ minimizes the static energy associated with (2).

In the CK approach the $3 \times 3$ matrix $M$ not only contains the hedgehog (2) in the isospin subgroup but also a time dependent kaon field $K(r, t)$. Furthermore collective coordinates $R_I(t)$ are introduced for the iso–rotations. The latter correspond to large amplitude

*) We assume isospin symmetry $m^0_u = m^0_d = m^0.$
fluctuations. Expanding the action (1) up to quadratic order in \( K \) yields the associated Bethe–Salpeter equation in the soliton background. This equation contains a bound solution, i.e. its energy \( \omega \) lies between zero and \( m_k \), which carries strangeness \( S = -1 \). Hence each occupation of this bound state increases the energy by \( |\omega| \) while \( S \) decreases by one unit. Canonical quantization of the fields finally removes the degeneracy of baryons with identical spin and/or isospin like \( \Sigma \) and \( \Lambda \). In the YA approach the kaon fields are treated as large amplitude fluctuations as well, i.e. \( M(r, t) \sim R_3(t) M_0(r) R_3(t)^\dagger \), with \( R_3(t) \in SU(3) \). These coordinates are canonically quantized yielding an Hamiltonian which may be diagonalized exactly despite of the fact that symmetry breaking parts occur. The predictions for the mass differences are displayed in table 2. The CK approach seems to give a somewhat better agreement with the experimental data. In both approaches the mass differences are slightly underestimated. This is inherited from the meson sector where \( f_k \) is predicted too small. Adjusting for this shortcoming in the baryon sector as well yields excellent agreement with the experiment (last column in table 2).

In the next step the Noether currents associated with the action (1) are constructed. We are interested in the matrix elements of the strange vector current between nucleon states

\[
\langle N| s \gamma_\mu s |N \rangle = \bar{u}(p') \left[ \gamma_\mu F_3(Q^2) + \frac{\sigma_{\mu\nu}Q^\nu}{2M_N} \tilde{F}_3(Q^2) \right] u(p), \quad Q_\mu = p'_\mu - p_\mu .
\]

(3)

In the YA approach we have computed the strange magnetic moment \( \mu_s = \tilde{F}_3(0) \) and the mean squared radius \( \langle r_S^2 \rangle \) of the electric form factor \( G_E = F_3 + (Q^2/4M^2) \tilde{F}_3 \). We predict

\[
-0.05 \leq \mu_s \leq 0.25 \quad \text{and} \quad -0.25 \text{fm}^2 \leq \langle r_S^2 \rangle \leq -0.15 \text{fm}^2 .
\]

(4)

The uncertainties stem from the fact that the NJL model result for the isovector magnetic moment of the nucleon is too small and pertinent adjustments (which are outside the self-consistent solution) are necessary.

We have seen that the NJL model provides illuminating inside into the description of baryons as solitons when starting from a microscopic theory of the quark flavor dynamics.

References

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