Time delay along a chained lumped-circuits: for the physical analogy of half-wavelength power transmission lines

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Abstract. Half-wavelength AC power transmission (HWACT) technology is a kind of three-phase AC transmission technology, which can transmit electric power over a distance close to half power-frequency wavelength, i.e. 3000 km (50Hz) or 2500 km (60 Hz). In order to implement physical analogy of HWACT lines, in general, the equivalent lumped-circuits consisting of some chained π-type circuits or T-type circuits are used in laboratory. The number of the chained circuits is the most key parameter to establish good equivalence between the lumped-circuits and the transmission line. In this paper, the time delay of the chained circuits, which is defined as the time of a sine wave propagating from the sending end to the receiving end of the chained circuits, is calculated for different number of the chained circuits and different wave frequencies. Good equivalence requires the time delay equal to 10ms (the time of electromagnetic waves propagating along 3000 km). It is shown that the time delay is dependent on the number of the chained circuits, as well as the wave frequency. For 50Hz, 4 chained π-type circuits can ensure that the relative error of the time delay is less than 2.6% and the sending-to-receiving voltage ratio is approximately 1. For frequencies below 400Hz, 30 chained π-type or T-type circuits can ensure that the relative error of the time delay is less than 3.2% and the sending-to-receiving voltage ratio is approximately 1. These works are instructive for the physical analogy of HWACT lines.

1 Introduction

According to the development strategy of the West to East power transmission in China [1], there are great demand for long distance (1000 km-3000 km) and large capacity power transmission technologies. High voltage direct current (HVDC) transmissions is one of such technologies, and there are lots of HVDC engineering (±500 kV, ±800 kV) had been established and put into operation in China. However, due to the limitation of the transmission line effect, the distance of traditional AC transmission is limited to several hundred kilometres. Fortunately, the Half-wavelength AC transmission (HWACT) can avoid the transmission line effect in the conventional AC transmission due to the advantage of point-to-point transmission: the sending-to-receiving voltage ratio is fixed at 1, no reactive power compensation required (the reactive power of the whole half-wavelength transmission line is self-balancing), etc [2-3]. In addition, HWACT also has certain economic advantages relative to HVDC at the same transmission distance of 3000 km.
In recent years, State Grid Corporation of China has launched a series of research projects on half-wavelength transmission technology \cite{4-9}. The physical analogy of half-wavelength is an important field among them. In general, the transmission line can be equivalent to the lumped-circuits consisting of some chained circuits. For conventional transmission line which usually with a length of less than 500 km (less than 1/10 wavelength), the transmission line effect is weak and hence the number of chained circuits is not a critical issue. Usually several π-type circuits or T-type circuits are enough. However, for half-wavelength lines, the number is the most key parameter to establish good equivalence between the lumped-circuits and the transmission line \cite{10,11}. The present research has carried out a series of study on the steady characteristic, transient characteristic, reactive balance, relay protection of HWACT, etc \cite{12-17}. However, the relationship between the number of chained circuits and the time delay has not been discussed. As a result, it is necessary to make a study on determining the reasonable number according to the actual needs of physical analogy.

In this paper, the EMTP (Electromagnetic Transients Program) is used to simulate the equivalent lumped-circuit of lossless HWACT line. The relationship between the number of chained circuits and the relative error of time delay is analysed by comparing propagation time between the equivalent lumped-circuits and the transmission line. It is concluded that for 50Hz, 4 chained π-type circuits can ensure that the relative error of the time delay is less than 2.6% and the sending-to-receiving voltage ratio is approximately 1; The larger the number of the chained circuits is, the smaller the relative error of time delay becomes, and the sending-to-receiving voltage ratio is closer to 1. For frequencies below 400Hz, 30 chained π-type circuits can ensure that the relative error of the time delay is less than 3.2% and the sending-to-receiving voltage ratio is approximately 1. These works are instructive for the physical analogy of HWACT lines.

2 Model

In this paper, the transmission line investigated is a 1000kV three-phase half-wavelength lossless transmission line of 3000km, and its operating frequency is 50 Hz. The geometric parameter of the half-wavelength line is shown in Table 1. The positive sequence component of the impedance per unit length is \( Z_0 = 0.25828 \times 10^3 \) (\( \Omega/m \)), the positive sequence component of the admittance per unit length is \( Y_0 = 0.42459 \times 10^{-8} \) (S/m), the inductance per unit length is \( L_0 = 8.2213 \times 10^{-7} \) (H/m), the capacitance per unit length is \( C_0 = 1.3515 \times 10^{-11} \) (F/m), the characteristic impedance is \( Z_C = 246.64 \Omega \), the propagation constant is \( \gamma = 1.0472 \times 10^{-6} \) (rad/m).

| Line type     | 8xLGJ-500/35 |
|---------------|--------------|
| conductor diameter | 30 mm       |
| splitting distance | 400 mm      |
| horizontal distance | the upper conductor: 0 m |
| (to the centre line) | the lower conductor (two-phase): 16 m |
| tower height | the upper conductor: 65 m |
| sag          | the lower conductor (two-phase): 45 m |

Figure 1: Line arrangement of the half-wavelength line

\[ \text{Figure 1: Line arrangement of the half-wavelength line} \]
Figure 2(a) is the equivalent lumped-circuit of the multi-stage π-type for 3000 km lossless transmission line, Figure 2(b) is the equivalent lumped-circuit of the multi-stage T-type. As shown in Figure 2, the 3000 km transmission line is evenly divided into N segments, the length of each segment is \( l = \frac{3000}{N} \). Each segment is equivalent to a π-type lumped-circuit with the inductance is \( L_{0l} \) and the capacitance is \( C_{0l}/2 \). The EMTP is used to simulate the lumped-circuits, and the time step is \( 1 \times 10^{-5} \) s. At \( t = 5 \) ms, the sine wave voltage source is added to the sending end, and the voltage amplitude is 10 kV. The characteristic impedance of the transmission line is added to the receiving end, that is the load impedance is equal to the characteristic impedance (i.e. the line is matched).

![Figure 2: A 3000 km power transmission line and its chained π-type equivalent circuit (a) and chained T-type equivalent circuit (b).](image)

### 3 Results and discussion

In this section, the time domain simulation results of the lumped equivalent circuit are presented in Figure 3 to Figure 6. Where, the green line represents the voltage waveform of the sending end, and the red one represents the receiving end. The definitions of the time delay \( t_l \) and the receiving end voltage amplitude \( U_N \) are described in Figure 3. The receiving end voltage needs several cycles to reach the steady state. The time delay and the receiving end voltage amplitude are calculated at the steady state. Find the first steady peak of receiving end voltage and the peak of the corresponding sending end voltage, the time difference between the zero-crossings of the two peaks is \( t_l \), and voltage amplitude of the first steady peak of receiving end is \( U_N \).

![Figure 3: The voltage waveforms of the sending end (green line) and the receiving end (red line). The time delay \( t_l \) is defined as the time difference between the sending end and the receiving end. The receiving end voltage amplitude \( U_N \) is the receiving end voltage amplitude at the steady state. For the chained π-type lumped-circuits with 50 Hz and different values of \( N \); Table 2 gives the time delay, the receiving end voltage amplitude and their relative error (\( \epsilon_l, \epsilon_U \)); Figure 4 respectively shows](image)
waveforms of the sending end and the receiving end. $\varepsilon_t$ and $\varepsilon_U$ are calculated by Equation (1) and Equation (2)

\[
\varepsilon_t = \frac{t_L - 10}{10} \times 100\%
\]

\[
\varepsilon_U = \frac{U_N - 10}{10} \times 100\%
\]

Where, the unit of $t_L$ is ms and the unit of $U_N$ is kV.

Table 2: Results of the chained $\pi$-type circuits with 50 Hz and different $N$

| $N$ | $t_L$ (ms) | $\varepsilon_t$ (%) | $U_N$ (kV) | $\varepsilon_U$ (%) |
|-----|------------|----------------------|------------|---------------------|
| 4   | 10.26      | 2.6                  | 10.11      | 1.1                 |
| 8   | 10.08      | 0.8                  | 10.03      | 0.3                 |
| 16  | 10.02      | 0.2                  | 10.01      | 0.1                 |
| 30  | 10.00      | 0.0                  | 10.01      | 0.1                 |
| 60  | 10.00      | 0.0                  | 10.00      | 0.0                 |

As can be seen from Table 2 and Figure 4, for 50Hz, 4 chained $\pi$-type circuits can ensure that the relative error of the time delay is less than 2.6% and the sending-to-receiving voltage ratio is approximately 1. The larger the number of the chained circuits is, the smaller the relative error of time delay becomes, and the sending-to-receiving voltage ratio is closer to 1. However, when the number of the chained circuits is larger than 30, the number no longer affects the time delay and the receiving end voltage amplitude.

As shown in Figure 2(a), assuming $L = L_j \cdot C = C_j / 2$.

For the case of an open load (i.e. $Z_L = \infty$), the input impedance of the $\pi$-type circuit is calculated by Equation (3)
\[
Z'' = \frac{1}{j\omega C} \left( \frac{1}{j\omega L} + \frac{1}{j\omega C} \right) \cdot \frac{1}{1 - j\omega LC}  
\]

(3)

For the case of a shorted load (i.e. \( Z_c = 0 \)), the input impedance is calculated by Equation (4)

\[
Z''_{m} = \frac{1}{j\omega C} \frac{j\omega L}{1 + j\omega L} \cdot \frac{1}{1 - j\omega LC}  
\]

(4)

The characteristic impedance \( Z_{c}^{\pi} \) of the \( \pi \)-type lumped-circuits is calculated by Equation (5)

\[
Z_{c}^{\pi} = \sqrt{Z''_{m} Z''_{n}} = \left( \frac{L}{2C} \right) \cdot \frac{1}{1 - j\omega LC}  
\]

(5)

\[
= \sqrt{\frac{L}{2C} 1 - j\omega LC}  
\]

(6)

\[
= \sqrt{\frac{L}{2C} \mu_0 \epsilon_0} = \frac{1}{c}  
\]

(7)

In Equation (7), \( c \) is the speed of light. Substituting Equation (6) and Equation (7) into Equation (5) results in

\[
Z_{c}^{\pi} = Z_{c} \cdot \frac{1}{\sqrt{1 - \left( \frac{\omega L}{\omega C} \right)^2}} = Z_{c} \cdot \frac{1}{\sqrt{1 - \left( \frac{\pi L}{\lambda} \right)^2}}  
\]

(8)

Where \( t = l/c \) is the time of direct wave propagating along the transmission line of \( l \). Similarly, the characteristic impedance \( Z_{c}^{T} \) of the T-type lumped-circuits can be expressed as

\[
Z_{c}^{T} = Z_{c} \cdot \sqrt{1 - \left( \frac{\omega L}{\omega C} \right)^2} = Z_{c} \cdot \sqrt{1 - \left( \frac{\pi L}{\lambda} \right)^2}  
\]

(9)

Next, the relationship between the time delay and the wave frequencies is discussed. The number of the chained circuits is fixed at 30, the sine wave voltage source of different frequencies is added at the sending end. For \( \pi \)-type circuit, the characteristic impedance is \( Z_{c}^{\pi} \), while for \( T \)-type circuit, the characteristic impedance is \( Z_{c}^{T} \). For \( \pi \)-type lumped-circuits, Equation (8) can make the load impedance and the circuits to achieve matching, and the receiving end voltage waveform is relatively regular, make it easy to see the time delay from the waveform. Similarly, the characteristic impedance of the \( T \)-type lumped-circuits is calculated by Equation (9). From Equation (8) and Equation (9), it is easy to find that \( Z_{c}^{\pi} \) is larger than \( Z_{c} \), while \( Z_{c}^{T} \) is smaller than \( Z_{c} \), and when \( l \leq \lambda \), \( Z_{c}^{\pi} \approx Z_{c}^{T} \approx Z_{c} \).

Table 3 gives the time delay, the receiving end voltage amplitude and their relative error of the equivalent \( \pi \)-type lumped-circuits at 50Hz, 100Hz, 200Hz, 400Hz, 600Hz, 800Hz and 1000Hz. Figure 5 respectively shows the time domain waveform of the sending end and the receiving end at above frequency.

The time delay of the receiving end voltage is discussed firstly. As can be seen from Table 3 and Figure 5, for frequencies below 400Hz, 30 chained \( \pi \)-type circuits can ensure that the relative error of the time delay is less than 3.2%; If the transmission frequency is less than 600Hz, the maximum relative error is 8%. The greater frequency is, the larger the relative error of time delay becomes, which indicates that 30 chained \( \pi \)-type circuits are not always equivalent to the 3000km transmission line, and the number of chained circuits should be increased correspondingly.

Secondly, the receiving end voltage amplitude is discussed. As can be seen from the diagram, for frequencies below 800Hz, 30 chained \( \pi \)-type circuits can ensure that the receiving end voltage...
amplitude is approximately 10kV. However, after the frequency increases to 1000Hz, the amplitude of the receiving end voltage is greatly attenuated, and the amplitude frequency characteristic is no longer approach to 1. Hence the 30 chained π-type circuits can no longer be equivalent to the half-wavelength line of 1000 Hz.

To sum up, for frequencies below 400Hz, 30 chained π-type circuits can ensure that the relative error of the time delay is less than 3.2% and the sending-to-receiving voltage ratio is approximately 1. Similar conclusions can be drawn for the T-type equivalent lumped-circuits from Table 4 and Figure 6.

Table 3: Results of the Π-type lumped-circuits with different frequency

| f (Hz) | $Z_i^\Pi$ (Ω) | $t_i$ (ms) | $\varepsilon_t$ (%) | $U_N$ (kV) | $\varepsilon_U$ (%) |
|-------|---------------|------------|---------------------|------------|-------------------|
| 50    | 246.98        | 10.01      | 0.1                 | 10.00      | 0.0               |
| 100   | 248.00        | 10.02      | 0.2                 | 10.01      | 0.1               |
| 200   | 252.23        | 10.08      | 0.8                 | 10.04      | 0.4               |
| 400   | 271.62        | 10.32      | 3.2                 | 10.01      | 0.1               |
| 600   | 317.03        | 10.80      | 8.0                 | 10.01      | 0.1               |
| 800   | 451.69        | 11.85      | 18.5                | 10.09      | 0.9               |
| 1000  | -793.43i      | 14.46      | 44.6                | 0.076      | 92.4              |

Figure 5: The voltage waveforms of the sending end (green line) and the receiving end (red line) of the π-type lumped-circuits: (a) 100 Hz; (b) 200 Hz; (c) 400 Hz; (d) 600 Hz; (e) 800 Hz; (f) 1000 Hz.

Table 4: Results of the T-type lumped-circuits with different frequency

| f (Hz) | $Z_i^T$ (Ω) | $t_i$ (ms) | $\varepsilon_t$ (%) | $U_N$ (kV) | $\varepsilon_U$ (%) |
|-------|--------------|------------|---------------------|------------|-------------------|
| 50    | 246.30       | 10.00      | 0.0                 | 10.01      | 0.1               |
| 100   | 245.28       | 10.02      | 0.2                 | 10.04      | 0.4               |
| 200   | 241.17       | 10.07      | 0.7                 | 10.04      | 0.4               |
| 400   | 223.96       | 10.32      | 3.2                 | 10.01      | 0.1               |
| 600   | 191.87       | 10.80      | 8.0                 | 10.07      | 0.7               |
| 800   | 134.67       | 11.83      | 18.3                | 10.03      | 0.3               |
| 1000  | 76.67i       | 13.88      | 38.8                | 2.20       | 78.0              |
4 Conclusion
In this paper, the relationship between the number of chained circuits and the time delay of the chained circuits is analysed. The time delay is calculated for different number and different wave frequencies. The main conclusions are as follows:

(1) For 50Hz, 4 chained π-type circuits can ensure that the relative error of the time delay is less than 2.6% and the sending-to-receiving voltage ratio is approximately 1.

(2) For 50Hz, the larger the number of the chained circuits is, the smaller the relative error of time delay becomes, and the sending-to-receiving voltage ratio is closer to 1. When the number of the chained circuits is larger than 30, the number no longer affects the time delay and the receiving end voltage amplitude.

(3) For frequencies below 400Hz, 30 chained π-type circuits can ensure that the relative error of the time delay is less than 3.2% and the sending-to-receiving voltage ratio is approximately 1.

Acknowledgements
This project is supported by laboratory opening fund project of China Electric Power Research Institute. Equivalence Analysis of transmission characteristics of half wavelength transmission lines simulated by a chained lumped-circuits.

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