Clean quantum point contacts in an InAs quantum well grown on a lattice-mismatched InP substrate

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Strong spin-orbit coupling, the resulting large $g$ factor, and small effective mass make InAs an attractive material platform for inducing topological superconductivity. The surface Fermi level pinning in the conduction band enables highly transparent ohmic contact without excessive doping. We investigate electrostatically defined quantum point contacts (QPCs) in a deep-well InAs two-dimensional electron gas. Despite the 3.3% lattice mismatch between the InAs quantum well and the InP substrate, we report clean QPCs with up to eight pronounced quantized conductance plateaus at zero magnetic field. Source-drain dc bias spectroscopy reveals a harmonic confinement potential with a nearly 5 meV subband spacing. We find a many-body exchange interaction enhancement for the out-of-plane $g$ factor $|g_{\perp}| = 27 \pm 1$, whereas the in-plane $g$ factor is isotropic $|g_{\parallel}| = |g_{\perp}| = 12 \pm 2$, close to the bulk value for InAs.

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I. INTRODUCTION

A quantum point contact (QPC) is a ballistic quasi-one-dimensional constriction with a tunable conductance, quantized in multiples of $e^2/h$ [1]. First demonstrated in GaAs/Al$_x$Ga$_{1-x}$As two-dimensional electron gases (2DEGs) over three decades ago [2,3], QPCs have been incorporated into mesoscale quantum devices for tunnel spectroscopy [4], quantum dots [5], charge sensors [6,7], electron injectors [8], spin polarizers [9], electronic beam splitters [10], and more. However, demonstrations of clean QPCs in InAs heterostructures remain far fewer.

InAs-based nanostructures have come under a renewed spotlight as a potential platform for proximity-induced topological superconductivity [11,12]. InAs has a small effective mass, large spin-orbit coupling, and surface Fermi level pinning [13]. Proximitized by an s-wave superconductor and exposed to a magnetic field, a one-dimensional InAs nanostructure should host Majorana zero modes at its ends [14–16]. This makes InAs-based systems an enticing platform for observing and manipulating Majorana zero modes, toward possible eventual topological quantum information processing [17–20]. InAs 2DEGs can be top-down patterned, offering a scaling advantage over directly grown nanowires for creating complex geometries and for scaling to large numbers of devices [12,21]. The small effective mass $m^* = 0.03m_e$ [22,23] in InAs quantum wells (QWs) results in a weak temperature and bias dependence of resistivity, making it easier to decouple the background 2DEG in transport measurements of the QPC. Furthermore, a single valley degree of freedom with large bulk $g$ factor $\sim 12-15$ makes InAs QWs a promising material platform for fast control of spin qubits [26–28] and quantum simulation of many-body phases [29,30]. Clean QPCs with smoothly tunable transitions are a key building block for integrating InAs quantum dot arrays in quantum simulators and processors.

We report the investigation of quantized conductance and magnetotransport properties of a narrow gate-defined constriction, fabricated in a buried InAs 2DEG grown by molecular beam epitaxy (MBE) on an InP substrate. The 3.3% lattice mismatch [13] between InAs and InP leads to a compressive strain on the quantum well and introduces dislocation defects; we demonstrate that, despite this, our QPCs are the cleanest amongst the handful of reported works in etched defects; we demonstrate that, despite this, our QPCs are the cleanest amongst the handful of reported works in etched and gate-defined constrictions in InAs and InAs/GaAs QWs [9,31–34]. The more closely lattice-matched substrate choice of GaSb has been plagued for decades with trivial edge conduction at mesa edges [35–38], which complicates interpretation of transport measurements. Though purely gate-defined nanostructures have recently allowed circumventing
this \cite{33,39}, InP has superior insulating properties compared to GaSb, simplifying the fabrication and operation of quantum devices. The QPC featured in this paper shows eight pronounced quantized conductance plateaus with a harmonic subband spacing near 5 meV. The spin-split conductance plateaus in an applied magnetic field let us extract an isotropic in-plane effective g factor $g_\parallel = g_\perp = 12 \pm 2$ and an exchange interaction-enhanced out-of-plane $g_\parallel' = 27 \pm 1$. Our work supports the integration of QPCs into quantum dots and other nanostructures.

II. DEVICE FABRICATION AND EXPERIMENT SETUP

The device was fabricated on a heterostructure grown by MBE on a semi-insulating InP (100) substrate; the growth is characterized in detail in Ref. \cite{40} (sample B). The layer sequence is shown in the cross-sectional schematic Fig. S1(a) in the Supplemental Material (SM) \cite{23}. The active region consists of a 4 nm InAs QW sandwiched between 10.5 nm of In$_{0.75}$Ga$_{0.25}$As layers. A 900 nm step-graded buffer of In$_x$Al$_{1-x}$As helps overcome the native lattice mismatch between InP and the quantum well, and a 120 nm In$_{0.75}$Al$_{0.25}$As top barrier moves the active region away from the surface for increased mobility.

Carriers originating from deep-level donor states in the In$_{0.75}$Al$_{0.25}$As layers populate the 2DEG formed in the InAs QW \cite{41,42}. The 2DEG has a mobility $\mu = 4.55 \times 10^5$ cm$^2$/V/s at an electron density $n_e = 4.34 \times 10^{11}$ cm$^{-2}$ as measured in a 5-µm-wide Hall bar at $T = 1.5$ K, corresponding to a mean-free path of $l_{\text{mf}} = 4.9$ µm. Owing to suppressed alloy and InGaAs/InAs interface scattering in our buried deep-well heterostructure, the mobility is amongst the highest reported for InAs QWs and is limited by unintentional background impurities and native charged point defects \cite{23,40}.

Our samples are first processed with standard electron beam lithography and wet etching to define an extended Hall barlike mesa with an area of 5 µm × 60 µm between voltage probes. The etch depth is 300 nm, extending into the buffer layer to achieve electrical isolation. To improve surface and edge contact, Ti/Au ohmic contacts are deposited after a light, additional wet etch and \textit{in situ} Ar mill. A 35-nm HfO$_2$ dielectric layer is added by atomic layer deposition in an oxygen ambient, followed by additional wet etch and in situ lithographic split-gate separation. The gate-defined constriction, before completely pinching off around $-2.9$ V, has a lithographic split-gate separation of $l = 8$ nm in the 2DEG. The constriction is below the noise floor of the preamplifier $R_{\text{pinch-off}} > 10^9$Ω, implying a pinch-off resistance $R_{\text{pinch-off}} > 1$ pA.

III. RESULTS AND DISCUSSION

A. Conductance quantization

Negatively biasing the split gates with a voltage around $-1.5$ V depletes the 2DEG directly underneath, forming a quasi-one-dimensional constriction. Upon further biasing, Fig. 2(a) clearly shows eight plateaus at $G = (V_{\text{ac}}/I_{\text{ac}} - R_e)^{-1}$, where $R_e$ is the gate-independent 2DEG resistance between the voltage probes.
FIG. 2. (a) Four-terminal conductance $G$ (blue solid line) and transconductance $dG/dV_g$ (red broken line) through the QPC as the constriction width and local carrier density are modulated by the voltage applied to the split gates. Quantized conductance plateaus at even-integer multiples of $e^2/h$ are observed. The large number of quantized plateaus visible is an indication of the pristine nature of the QPC. A series resistance $R_s = 380 \Omega$ has been subtracted to adjust for the 2DEG resistance between the probes. dc bias spectroscopy showing the (b) conductance as a function of $V_{dc}$ and (d) transconductance as a function of $V_g$ and $V_{dc}$. Each trace in (b) corresponds to a fixed $V_g \in [-2.92, -2.11]$ V with a step size of 5 mV. A bunching of traces is observed at even multiples of $e^2/h$ around zero bias and at odd multiples at finite bias. The dark regions in (d) correspond to the labeled conductance plateaus in units of $e^2/h$. The bright diamond-shaped stripes of finite transconductance correspond to transitions between the plateaus. A triplet of transconductance maxima, illustrated by the white circles and a dashed horizontal line at $V_g = -2.735$ V, highlights the harmonicity of the confinement potential. (c) QPC subband spacing plotted as a function of $V_g$ for the first five subbands. Sweeping the QPC voltages up from pinch-off reduces the curvature of the confinement potential, decreasing the subband spacing. The subband spacings phenomenologically show a quadratic dependence on $V_g$.

**B. Finite-bias spectroscopy**

The level spectrum of the constriction can be probed by applying a dc bias voltage $V_{bias}$ across the source and drain electrodes of the device. The dc voltage drop across the QPC, $V_{dc}$, is obtained by subtracting the voltage drop across the bare 2DEG: $V_{dc} = V_{meas} - R_s \times I_{dc}$, where $V_{meas}$ is the four-terminal dc voltage difference measured across the QPC, $I_{dc}$ is the dc current through the Hall bar, and $R_s = 380 \Omega$ is a series resistance arising from the mesa 2DEG resistance. Figure 2(b) plots $G$ as a function of $V_{dc}$, where each trace corresponds to a particular $V_g$, as the QPC is opened from pinch-off. A bunching of traces is observed at conductance plateaus, which are even multiples of $e^2/h$ at low bias and odd multiples at high bias. The transconductance $dG/dV_g$ is shown in Fig. 2(d) as a function of $V_{dc}$ and $V_g$, with the dark regions corresponding to conductance plateaus and bright regions representing transitions.

The extent of the transconductance diamond for $G = n \times 2e^2/h$ along $V_{dc}$ is a common measure [45] of the energy spacing $\Delta E_n(V_g^*)$ of QPC subbands $\{n, n + 1\}$ at the gate voltage $V_g^*$ corresponding to the diamond end points. Opening the QPC from pinch-off decreases the curvature of the confinement potential, decreasing the subband spacing with $V_g$ as shown in Fig. 2(c). The harmonicity of the confinement
FIG. 3. In-plane magnetic field spectroscopy showing (a) transconductance $dG/dV_g$ as a function of QPC gate voltage $V_g$ and a magnetic field $B_x$ applied in the plane of the sample and parallel to the transport direction and (b) as a function of chemical potential $\mu$ estimated from the capacitive lever arm (see the SM [23]). The dark regions correspond to conductance plateaus labeled in units of $e^2/h$. The transitions between conductance plateaus are visible as bright regions. The appearance of additional dark regions at high $B_x$ ($\gtrsim 3$ T) is a signature of a Zeeman-induced spin splitting of the subbands. (c) The Zeeman energy for the first three subbands, extracted from a linear fit to the spin-split transitions [red dotted lines in (b)]. (d) The effective in-plane $g$ factor parallel ($g_x^*$) and perpendicular ($g_y^*$) to the transport direction estimated from the slopes of the Zeeman energy in (c) for the first three subbands. Within the error bars, the in-plane effective $g$ factor is isotropic and close to the bulk value for InAs $|g| = 13$. (e) Conductance as a function of $V_g$ at various fixed $B_x$. The traces in (e) are offset along the horizontal axis for clarity and display a progressive development of conductance plateaus at $1e^2/h, 3e^2/h$, and $5e^2/h$.

potential in a particular gate voltage range can be probed by considering a triplet of transconductance maxima circled in Fig. 2(d). Since they occur at approximately the same gate voltage, we infer $\Delta E_1 \simeq \Delta E_2$ [45]. Similar horizontal lines can be drawn connecting diamond vertices at higher conductances, implying a harmonic confinement potential, albeit a function of $V_g$.

Approximating the lateral confinement as a harmonic potential with a gate voltage-dependent angular frequency $\omega_0(V_g)$, the length scale $L_n(V_g^*)$ of the transverse real-space extent of the subbands at $V_g = V_g^*$ can be estimated as

$$\frac{1}{2} m^* \omega_0^2 L_n^2 = \hbar \omega_0(n - \frac{1}{2}),$$

where $m^* = 0.03m_e$ [22,23] is the effective mass and $m_e$ is the bare electron mass. Taking $\hbar \omega_0(V_g^*) = \Delta E_n(V_g^*)$ for the $n$th subband spacing as determined above, the corresponding length scales can be estimated as $L_1 = 22.7 \pm 0.9$ nm, $L_2 = 45.8 \pm 1.1$ nm, and $L_3 = 64.5 \pm 0.8$ nm for the first three subbands, consistent with expectations from the lithographic width $W_{\text{litho}} = 325$ nm $\gg L_n$.

C. In-plane magnetic field

Spin-resolved transport through the QPC can be studied by applying a magnetic field $B_x$ in the plane of the sample and parallel to the transport direction. Figure 3(a) shows the transconductance $dG/dV_g$ as a function of $B_x$ and $V_g$. 


as the gate voltage is swept up from pinch-off. The dark, diamond-shaped regions at low $B_{\perp}$ ($\lesssim 2$ T) correspond to the spin-degenerate even-integer conductance plateaus. At higher applied $B_{\perp}$, the spin splitting by the Zeeman effect dominates over the subband linewidths, resulting in the appearance of odd plateaus as additional dark regions interleaved with the spin-degenerate diamonds. Conductance traces as a function of $V_g$ for different $B_{\perp}$ are shown in Fig. 3(e). As expected, conductance plateaus at $1e^2/h$ and $3e^2/h$ emerge as $B_{\perp}$ is increased and the width of the even-integer plateaus correspondingly decreases.

Figure 3(b) elucidates the spin-split subband spectrum by translating $V_g$ to a chemical potential $\mu$, using the split-gate lever arm $a = d\mu/dV_g$ extracted from Fig. 2(d) (see the SM [23] for details on the conversion). A linear fit to the transconductance maxima for each spin-split subband pair is also shown for different $B_{\perp}$, with an enhancement for $B_{\perp} = 0$ compared to $B_{\perp} \gtrsim 2$ T (red trace).

The resultant magnetoelectric subbands have a spacing that emerges as additional dark regions interleaved with the spin-degenerate diamonds. Conductance traces as a function of $V_g$ around the 1st subband edge, $\bar{V}_g$, are shown in Fig. 3(d), with error estimates based on fitting parameter variances. Figure 3(d) also shows the in-plane $g$ factor measured in a magnetic field $B_{\perp}$ perpendicular to the direction of transport, revealing negligible anisotropy $g_{\parallel}^s \approx g_{\perp}^s$ (see the SM [23]). This is consistent with previous measurements in (In,Ga)As [46], InSb [47], and n-type GaAs [48] QPCs. The isotropic in-plane $g$ factor points to a weak Rashba spin-orbit coupling in the conduction [49].

D. Out-of-plane magnetic field

As a next step in investigating the QPC, we study the effect of electrostatic confinement on magnetic subbands by applying a magnetic field $B_{\parallel}$ perpendicular to the plane of the sample. The conductance $G$ as $V_g$ is swept up from pinch-off for $B_{\parallel} \in [0, 4]$ T is shown in Fig. 4(a). The cyclotron energy of the electrons, $\hbar\omega_c = \hbar eB_{\parallel}/m^*$ where $e$ is the electron charge, adds in quadrature to the QPC confinement energy. The resultant magnetoelectric subbands have a spacing that initially grows quadratically with field ($\omega_c \gg \omega_x$) before transitioning into a linear increase as they line up with the 2DEG Landau levels for $2\hbar c < W_{\text{qpc}}$ [56], where $c = \hbar k_F/eB_{\parallel}$ is the cyclotron radius of the classical electron trajectory in the 2DEG, $k_F = \sqrt{2\pi n}$ is the Fermi wave number in the 2DEG, and $W_{\text{qpc}}(V_g)$ is the gate voltage-dependent conduction width (see Sec. S6 in the SM [23]). The increase in subband spacing and suppression of backscattering through the Hall bar with $B_{\perp}$ results in broader and more pronounced conductance plateaus. Furthermore, the Zeeman effect of the applied field lifts spin degeneracy and results in the emergence of odd-integer conductance plateaus. Because of thermal ($k_B T \sim 130$ μeV) and disorder broadening in our measurements, we observe spin-split plateaus only at $B_{\perp} \gtrsim 2$ T (red trace).

Figure 4(b) depicts the transconductance $dG/dV_g$ as a function of $B_{\parallel}$ and $V_g$. The dark regions correspond to conductance plateaus, separated by bright features which represent the transitions between the plateaus. The transconductance has a local maximum whenever a subband edge is resonant with the source and/or drain chemical potential. Given that the confinement is described by a $V_g$-dependent harmonic potential, the magnetoelectric subbands can be described by theBeenakker and van Houten model [55]

$$E_{n,\pm} = E_0 + (n - 1/2)\hbar \omega_c^2(V_g) + \omega_x^2 \pm \frac{1}{2}g_{\parallel,\perp}^s \mu_B B_{\perp},$$

where $n = 1, 2, \ldots$ is the spin-degenerate subband index, $\pm$ labels the spin-split subband with spin oriented antiparallel (parallel) to $B_{\perp}$, $E_0$ is the energy offset of the conduction band edge, $\hbar\omega_c = \Delta E_m$ is the $V_g$-dependent QPC subband spacing at $B_{\perp} = 0$ T [see Fig. 2(d)], and $g_{\parallel,\perp}^s$ is the effective out-of-plane $g$ factor. The white dashed curve in Fig. 4(b) marks the contour $W_{\text{qpc}}(V_g) = 2c_r$. An agreement to Eq. (2), when translated to gate voltage, in the low-field regime ($B_{\perp} < 2h\omega_c/eW_{\text{qpc}}$) for $n \in \{1, 2, 3\}$ is shown as red dotted lines in Fig. 4(b). The spin-degenerate part of Eq. (2) used for the $n = 3$ subband edge spin splitting is not well observed for $W_{\text{qpc}} < 2c_r$.

The Zeeman energy can be measured by performing finite-bias spectroscopy of the QPC as a function of $B_{\perp}$. In a setup identical to Sec. III B, the QPC conductance is measured as a function of an applied dc voltage at a fixed $B_{\perp}$. Figure 4(c) shows the transconductance as a function of the dc voltage drop across the QPC $V_{\text{qpc}}$ and $V_g$ around the $G = 1e^2/h$ plateau at $B_{\perp} = 2.85$ T. The dark highlighted region corresponds to the $1e^2/h$ plateau, the extent of which along $V_{\text{qpc}}$ depends on the Zeeman energy $E_Z = g_{\parallel,\perp}^s \mu_B B_{\perp} = 4$ meV. Measured as a function of $B_{\perp}$, Fig. 4(d) shows the Zeeman energy evolution with field which fits a straight line constrained to pass through the origin, for $|g_{\parallel}^s| = 27 \pm 1$. The uncertainty in ascertaining the boundaries of the $G = 1e^2/h$ plateau, as evinced by broadened transconductance peaks in Fig. 4(c), results in large error bars for the Zeeman energies, defined as the width corresponding to 99% relative peak height. This can also be seen from the broad transconductance peaks in the $B_{\perp} \in \{2, 4\}$ T region of Fig. 4(b). Nevertheless, we can report a twofold enhancement of the out-of-plane $g$ factor compared to the in-plane and bulk InAs value $g_{\parallel,\perp}^s/g_{\parallel,\perp}^s \approx 2$.

The reduced symmetry in quasi-2D heterostructures, as compared to the bulk, introduces anisotropy between $g_{\parallel,\perp}^s$ and $g_{\parallel,\perp}^s$ [50]. Furthermore, as previously measured [46] and analyzed [49] for (In,Ga)As QPCs, the orbital effect of the out-of-plane field strengthens many-body exchange interactions in the 2DEG, resulting in an enhanced $g_{\parallel}^s$. The depopulation of consecutive spin-split Landau levels with $B_{\perp}$ leads to an oscillatory exchange enhancement, with local maxima at odd filling factors [57–59]. Sadofyev et al. [58] measured an enhanced out-of-plane $g$ factor $\approx 60$ at high
FIG. 4. (a) Conductance $G$ of the QPC as a function of $V_g$ for a series of out-of-plane magnetic fields. The curves are offset along the horizontal axis for clarity. Above $B_\perp = 2$ T (bold red trace), odd-integer conductance plateaus emerge. (b) Transconductance $dG/dV_g$ as a function of QPC gate voltage $V_g$ and a magnetic field $B_\perp$ applied out of plane of the sample. The dark regions correspond to conductance plateaus labeled in units of $e^2/h$. The transitions between conductance plateaus are visible as bright regions and illustrate the magnetoelectric subband energy evolution with field. The white dashed curve marks the subband transition to 2DEG Landau levels, based on the gate dependent constriction width $W_{qpc}(V_g)$ and cyclotron radius $r_c$. The red dashed lines show agreement with a model by Beenakker and van Houten [55] in the low-field regime $W_{qpc} < 2r_c$. (c) Transconductance as a function of $V_{dc}$ and $V_g$ at $B_\perp = 2.85$ T. The white dashed lines highlight the extent of the $1e^2/h$ conductance plateau diamond along the $V_{dc}$ axis, a measure of the Zeeman energy $E_Z$. (d) The $B_\perp$ dependence of the Zeeman energy, as extracted from the $1e^2/h$ transconductance diamond size, similar to (c). The linear fit, weighted by inverse $E_Z$ variances, shows an effective out-of-plane $g$ factor $|g^{*}_\perp| \sim 27 \pm 1$. fields in InAs/AlSb QWs. Similar measurements for $g^{*}_\perp$ in our QW reveal an enhanced 2DEG $g$ factor $\simeq 30$ in the $B_\perp \in [2, 4]$ T field range (see the SM [23]). Consequently, we attribute the enhanced splitting of the QPC subband [Figs. 4(c) and 4(d)] to many-body exchange interactions in the 2DEG, rather than 1D confinement effects due to the constriction.

E. Shifting the confinement potential

By applying an asymmetric voltage bias to the QPC split gates, we can laterally shift the position of the confining potential in real space. This serves as a spatial map of localized disorder or other potential fluctuations which may increase backscattering in the channel or create accidental quantum dots [60]. Tuning the two gate voltages independently, the transconductance with respect to the fast sweep axis $V_{g2}$ is shown in Fig. 5. The bright features correspond to transitions between conductance plateaus and they appear consistently smooth across the entire range. Resonances caused by localized disorder would appear as additional gate voltage-dependent lines in this map; the absence of such features here suggests a clean, defect-free channel within this range. Tuning the gate asymmetry to avoid spurious resonances is a common technique in QPC operation—not needing it here will significantly simplify the operation of devices with larger numbers of gates, where cross-capacitances
must be diligently accounted for. The blue dots in the inset show a fit to the first transconductance peak in the $V_g$ range, at least four conductance plateaus are visible. The white dotted regions indicate transitions between them. In the shown gate voltage range, at least four conductance plateaus are visible. The white dotted line marks the trajectory of the symmetric gate sweep ($V_g$ and $V_g'$) used in this work. The inset shows a fit to the first transconductance maxima in the $V_g \in [-3, -2.7] V$ and $V_g' \in [-3.8, -3.4] V$ range, with discontinuities (at $V_g = -2.84 V$, for example) arising from a drift in the QPC conductance between traces along the slow sweep axis $V_g$.

We additio-
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