Non-universal GUT corrections to the soft terms and their implications in supergravity models

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Abstract

Potentially large non-universal corrections to the soft supersymmetry breaking parameters arise from their evolution between the Planck and the grand-unification scales. We detail typical patterns of non-universality in GUT models, as well as elaborate on their propagation to the weak scale and on their low-energy implications. Possible corrections to the different scalar quark and lepton masses and the Higgs and the gaugino-Higgsino sector parameters are described in detail, and new allowed regions of the parameter space are pointed out. In particular, the patterns studied often lead to heavier Higgsinos and $t$-scalar. One-loop GUT threshold corrections to the soft parameters are also discussed and shown to be important.

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I. INTRODUCTION

It was recently pointed out \cite{1} that in minimal supergravity type models \cite{2}, the model-dependent renormalization of the soft mass parameters between the Planck scale \( M_P \approx 2 \times 10^{18} \) GeV and the grand-unification scale \( M_G \approx 2 \times 10^{16} \) GeV, can significantly modify the boundary conditions for these parameters at \( M_G \). In particular, contrary to the standard working assumption of universality at \( M_G \) (e.g., see Ref. \cite{3,4}), specific patterns of non-universality are induced at \( M_G \) even when the soft parameters have universal boundary conditions at \( M_P \). Hence, low-energy predictions and constraints in the minimal super-symmetric extension of the standard model (MSSM) \cite{2} are subject to model-dependent modifications. Regions of the parameter space that are of interest to present and future collider experiments may change and/or be smeared, requiring one to assign uncertainties to the MSSM low-energy predictions. On the other hand, the discovery of superpartner and Higgs particles could provide new and exciting hints on the physics near the Planck scale.

In general, as a result of the GUT effects the soft masses can be different at \( M_G \) for fields which are in different representations of the unified gauge group. This is due to (i) the different charges and (ii) the different Yukawa interactions of the different fields. Indeed, if one assumes that the corrections are proportional to only \( (\alpha_G/\pi) \ln(M_P/M_G) \sim 5\% \), then the effects are negligible. However, the above argument does not hold, regardless of the size of the Yukawa interactions, if large representations are present. In such a case, the unified coupling \( \alpha_G \) is multiplied by a large number and the effect of Planck to GUT scale evolution can be significant \cite{5,1}. Furthermore, in Ref. \cite{1} it is shown that top and bottom Yukawa couplings, as well as GUT-scale Yukawa couplings, which have to be large to avoid a too rapid proton decay \cite{6}, can also induce large deviations from universality at \( M_G \). In particular, the soft supersymmetry breaking (SSB) parameters related to the light Higgs fields can be significantly different from those related to the matter fields due to the different Yukawa interactions. (It should be emphasized that universality of the soft mass parameters was assumed, but at \( M_P \).) Moreover, once non-universality exists at \( M_G \), and if the rank of the gauge group is higher than that of the standard model (SM) group, then non-vanishing \( D \)-terms \cite{7} exist \cite{8}. The \( D \)-terms, which are charge dependent, induce a secondary breaking of universality.

Given the above, one may then question the motivation to assume universality at any scale: Relaxation of that assumption is subject to strong constraints from flavor changing neutral currents (FCNC) \cite{9}. Also, any predictive power is lost. On the other hand, the identification of the universality scale with \( M_G \) is convenient but ad hoc, and the consequences of its relaxation need exploration. Allowing \( M_P - M_G \) renormalization of the parameters leads to restricted patterns of non-universality at \( M_G \). In particular, the super-GIM mechanism suppressing FCNC in the MSSM \cite{10} need not be altered if Yukawa couplings of the first and second families remain negligible at all scales. In addition, the predictive power is altered but not lost. In SU(5) models, one has to introduce at least two more Yukawa couplings, and for higher-rank groups, e.g., SO(10), a new parameter is needed to account for the magnitude and sign of the \( D \)-terms. The deviations from universality at \( M_G \) are not arbitrary but are calculable in terms of the new parameters. However, the interference between the different corrections, e.g., those from Yukawa interactions and those from \( D \)-terms (and in particular, if higher order gravitational corrections are not negligible), can render it difficult
to disentangle traces of the high-scale theory in the low-energy physics.

Below, we will again resort to the assumption of universality of the SSB parameters at $M_P$ (the obvious choice in minimal supergravity theories [1]), i.e., a common scalar mass $m_0$, a common gaugino mass $M_{1/2}$, and common dimension-one trilinear and bilinear scalar couplings $A_0$ and $B_0$. (In fact, our discussion is independent of any assumptions regarding universality of the bilinear couplings $B_i$.) We will then assume a grand unified theory (GUT) between $M_P$ and $M_G$. ($M_G$ is determined by gauge coupling unification.) Note that the universality of the gaugino masses above $M_G$ is trivial if the GUT group is simple. For simplicity, we assume in most of our calculations the minimal SU(5) model [12,10]. However, extended models, including models with non-vanishing $D$-terms, are studied qualitatively. The effective theory below $M_G$ is assumed to be the MSSM with the appropriate $M_G$ matching conditions. We then study the different patterns of non-universality which are induced at $M_G$, their propagation to the weak scale, low-energy implications and consequences for model building. In particular, we will examine two points in the parameter space,

(a) $m_t^{pole} = 160$ GeV, $\tan \beta = 1.25$;
(b) $m_t^{pole} = 180$ GeV, $\tan \beta = 42$;

for the $t$-quark pole mass $m_t^{pole}$ and for $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$. We choose these points because of the large $t$ and $b$ quark Yukawa couplings, i.e., $h_t \approx 1 \gg h_b$ and $1 \gtrsim h_t \gtrsim h_b$, respectively. Also, because of the large Yukawa couplings, points (a) and (b) are consistent with bottom-tau unification and with minimal SU(5), e.g., see Ref. [13]. For intermediate values of $\tan \beta$ the effects are a superposition of those for points (a) and (b).

It was recently suggested that non-universality of the soft terms is a typical signature of some string models [14]. In string models one often has a direct unification at the string scale $M_S < M_P$, i.e., $M_G = M_S$, and the universal or non-universal boundary conditions are derived from the string theory at $M_S$. However, the string scale is typically $M_S \approx 5.2 \times g_S \times 10^{17}$ GeV [15], and the relation $M_G = M_S$ fails for the MSSM. String inspired non-universality is studied in Ref. [16]. In particular, Kobayashi et al. suggest a solution to the $M_S/M_G \approx 20$ discrepancy. If one assumes that the string theory leads to an intermediate GUT, then our analysis applies, but the results should be scaled down by $\sim \ln (M_S/M_P)$.

We previously found [1] that some low-energy parameters such as the $\mu$ parameter and the $t$-scalar mass, can be significantly modified while others, e.g., the SM-like Higgs boson mass, are nearly invariant under minimal SU(5) type corrections. We also pointed out the $\tan \beta$-dependent modification of the allowed parameter space and of correlations between different observables. Here, we will further elaborate on the above observations and expand our previous work. We review the possible patterns of non-universality and compare their generation via radiative corrections to the radiative symmetry breaking mechanism in section [14]. We also demonstrate that the same rule of thumb holds for the $M_P - M_G$ and $M_G - M_Z$ evolutions of the soft parameters and their resulting hierarchy: It is determined by the competition between the gauge charge and Yukawa interactions of the relevant field and

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1 That choice maximizes the effects. Universality at a lower scale would lead to lesser but similar effects. We do not consider gravitational effects other than those which induce the universal soft terms. Higher order gravitational effects could add to the uncertainty.
by the asymptotic freedom of the model. In section II we describe our numerical routines and discuss the weak-scale phenomena. In particular, we elaborate on the propagation of non-universal corrections from \(M_G\) to the weak scale; on the distinction between physical (bottom-up approach) and model-building (up-down approach) parameters; and on the implications to the first and second family scalars, the \(\mu\) parameter, the Higgs sector, and third family scalars. Future observations of the signatures described could support the existence of an intermediate GUT. Their absence, on the other hand, could indicate direct unification, but also interference between different effects or small values for the new parameters. Our conclusions are given in Section IV. For completeness, the minimal SU(5) model is defined in Appendix A. Threshold corrections (due to the ambiguity in \(M_G\)) in that model are described and discussed in Appendix B.

Rather than restrict oneself to the patterns described in section II, one could adopt a more phenomenological approach to non-universality by postulating certain universality breaking patterns. For example, Dimopoulos and Georgi considered different boundary conditions for the matter and Higgs bosons \([10]\). Similar approaches were adopted recently by several authors. In Ref. \([17]\) a split at \(M_G\) between the light Higgs and matter fields is considered [but mainly in the context of SO(10) scenarios]. In Ref. \([18]\) patterns of non-universality desired in certain SO(10) models are studied. A general discussion of non-universal scalar potentials at \(M_G\) was recently given in Ref. \([19]\). Other recent studies of non-universality were carried out in Ref. \([20]\). Where relevant, the conclusions of these authors agree with ours.

II. PATTERNS OF NON-UNIVERSALITY AT \(M_G\)

Before discussing the way in which the Planck to GUT scale evolution of the soft mass parameters can induce large deviations from universality at \(M_G\), it is useful to recall the more familiar way in which GUT to weak scale evolution can render the weak-scale scalar potential consistent with a spontaneously broken SU(2)\(_L\)\(\times\)U(1)\(_Y\) symmetry \([21]\) [often called the radiative symmetry breaking (RSB) mechanism]. In both cases large Yukawa couplings play a similar role and lead to similar behavior of the SSB parameters. The RSB mechanism is easily understood if one writes the approximate renormalization group equations (RGEs) of the soft scalar masses \([22]\):

\[
\frac{d}{d \ln Q} \begin{bmatrix} m_{H_1}^2 \\ m_{H_2}^2 \\ m_T^2 \\ m_Q^2 \end{bmatrix} = \frac{h^2}{8\pi^2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 3 & 3 \\ 0 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} m_{H_1}^2 \\ m_{H_2}^2 \\ m_T^2 \\ m_Q^2 \end{bmatrix} - \frac{2g^2}{3\pi^2} M_g^2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix},
\]

where \(Q\) is the renormalization scale, \(g_3\) is the SU(3)\(_c\) coupling and

\[
-\mathcal{L}_{\text{soft}} = \sum_i m_i^2 |\phi_i|^2 + [B\mu H_1 H_2 + A_t h_t Q H_2 U + \frac{1}{2} \sum_{\lambda} M_\lambda \lambda_\alpha \lambda_\alpha + h.c.],
\]

with \(i\) (\(\lambda\)) summing over all scalars (gauginos) and \(H_1, H_2, U = \hat{t}_R, Q = (\hat{t}_L, \hat{b}_L)\), and \(\tilde{g}\) are the down and up type Higgs doublets, right-handed \(t\)-scalar, left-handed scalar-quark doublet and the gluino, respectively. (Below, we will also refer to the \(t\)-scalar, scalar quarks,
enables one to rewrite the EWSB minimization conditions \([21]\) as

\[\lambda_{\tilde{A}}\]:

\[M\]

The stop masses grow with the gluino mass \(m_{\tilde{g}}\) \(\sim O(M_{\tilde{Z}}^2)\). \((m_{\tilde{g}})_{\text{A}}\) is not renormalized in this approximation.) The global minimum of the weak-scale Higgs potential is consistent with electroweak symmetry breaking (EWSB) provided that \((m_{\tilde{H}_1}^2 + \mu^2) \leq \beta^2 \mu^2\) (and that \(m_{\tilde{H}_1}^2 + m_{\tilde{H}_2}^2 + 2\mu^2 \geq 2|B\mu|\)), which is indeed the situation for \(O(M_{\tilde{Z}}^2)\) (possibly negative) values of \(m_{\tilde{H}_2}^2 + \mu^2\). Also, there are no tachions in the theory. The situation is, of course, more complicated when including all terms, but is qualitatively similar to the above approximation.

For future reference, observe that \(\mu\) is independent of either \(\mu\) or \(B\). The decoupling of \(\mu\) from the SSB parameters and of \(B\) from all other SSB parameters holds in general. This enables one to rewrite the EWSB minimization conditions \([21]\)

\[
\mu^2 = \frac{m_{\tilde{H}_1}^2 - m_{\tilde{H}_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_{\tilde{Z}}^2, \quad (3a)
\]

\[
B\mu = -\frac{1}{2} \sin 2\beta \left[ m_{\tilde{H}_1}^2 + m_{\tilde{H}_2}^2 + 2\mu^2 \right]. \quad (3b)
\]

Hence, \(B_0\) can be traded for \(\tan \beta\) and \(\mu\) is predicted as a function of the SSB parameters and of \(\beta\). (Triviality limits give \(\tan \beta > 1\) for \(m_{\text{pole}} \gtrsim 140\) GeV.)

To summarize, the gauge term dominates the \(M_G - M_Z\) evolution for the colored scalar soft masses (but their spectrum also bears traces of their Yukawa interactions) while the Yukawa term dictates \(m_{\tilde{H}_2}^2\) evolution (in practice, it dominates over the \(H_2\) weak and hypercharge gauge terms). Thus, a large hierarchy is generated at the weak scale, even when assuming universal scalar masses as the GUT scale boundary condition for \([1]\). Let us now write in a similar fashion the RGEs in the minimal SU(5) model, which is assumed to dictate the \(M_P - M_G\) evolution [the complete RGEs in that model are given in \([1]\) and in Appendix \([A]\)]:

\[
\frac{d}{d \ln Q} \begin{bmatrix} m_{\tilde{H}_1}^2 \\ m_{H_2}^2 \\ m_{10}^2 \end{bmatrix} = \frac{h_t^2}{8\pi^2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 6 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} m_{\tilde{H}_1}^2 \\ m_{H_2}^2 \\ m_{10}^2 \end{bmatrix} + \frac{3\lambda_2^2}{5\pi^2} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_{\tilde{H}_1}^2 \\ m_{H_2}^2 \\ m_{10}^2 \end{bmatrix} - \frac{g_\tilde{g}^2}{\pi^2} M_5^2 \begin{bmatrix} 6 \\ 6 \\ 9 \end{bmatrix}, \quad (4)
\]

where \(m_{\tilde{H}_1}\) and \(m_{\tilde{H}_2}\) are the soft masses of the \(\mathbf{5}\) and \(\mathbf{5}\) SU(5) Higgs bosons (that contain \(H_1\) and \(H_2\), respectively); \(m_{10}\) and \(m_5\) (that we use later) are the soft masses of the \(\mathbf{10}\) (that contains \(Q\) and \(U\)) and \(\mathbf{5}\) matter superfields; \(g_\tilde{g}\) and \(M_5\) are the SU(5) gauge coupling and gaugino mass, and the Yukawa coupling \(\lambda\) is defined in Appendix \([A]\). Note that the introduction of larger representations typically implies larger numerical coefficients. The GUT effects in the minimal SU(5) model can be read from \([1]\) and are described by the following patterns: If \(h_t \approx \lambda \approx 1\) at \(M_G\) (which is consistent with these couplings being \(\lesssim 2\) near \(M_P\), so the one-loop approximation is reasonable) then the \(\lambda\) term diminishes the
$H_1$ and $H_2$ squared soft masses significantly while the $h_t$ term lifts their degeneracy, further diminishing $m_{H_2}^2$. The latter also diminishes $m_{H_1}^2$. In comparison to (i), the additional Yukawa term, the larger numerical coefficients and $\alpha_G \ll \alpha_3(M_Z)$ compensate for the shorter evolution interval, leading to a similar pattern (aside from the effect on $H_1$), but at $M_G$ rather than at $M_Z$, and we observe the pattern

(i) $m_5^2 \geq m_{10}^2 \gg m_{H_1}^2 \geq m_{H_2}^2$.

First and second family scalar masses are renormalized to a very good approximation only by gauge interactions, and hence are slightly heavier than the third family $m_5^2$ (which is renormalized, in practice, by the $h_b$ term). More importantly, in this scenario the gauge interactions do not lift the degeneracies between the first and second family SSB parameters and the MSSM super-GIM mechanism is still intact. Pattern (i) would now serve as a non-universal boundary condition to (i). As can be seen in Fig. 1a of Ref. \[1\] and in Fig. [2] here, $m_{H_2}^2$ can be driven to near zero values already at $M_G$, and in principle, RSB could be achieved for small values of $h_t$ (this is, of course, irrelevant for $m_{t}^{pole} \gtrsim 100$ GeV). Before proceeding, let us stress that the hierarchy between the different SSB parameters indeed depends on the gauge charges and on the size of the Yukawa couplings. However, whether the parameters grow or diminish depends roughly on the ratio $[g_C^2 M_{1/2}^2]/[\max(h_t^2, \lambda^2) \times m_0^2]$. If the ratio is larger than unity, then typically all the parameters grow with decreasing energy. This is true in general and is seen [for pattern (i)] in Fig. 1b of Ref. \[1\] [where $g_C^2 M_{1/2}^2 \gg h_t^2 m_b^2 \Rightarrow m_{10}(M_G) \gtrsim m_0(M_G)$] and in Fig. [2] here. (This is always the case for the first and second family scalars.) If both, $h_t$ and $\lambda$, are large, as in pattern (i), then for small and moderate values of $M_{1/2}$ only $m_5^2$ grows. (Fig. [2] corresponds to the no-scale assumption $m_0 = A_0 = 0$.)

The GUT effects leading to pattern (i) are a mismatch of effects due to the large $\lambda$ and to the large $h_t$. Different assumptions regarding the Yukawa couplings lead to different patterns. If $h_t(M_G) \ll 1$ (i.e., $m_t^{pole} \sim 180$ GeV and $\tan \beta \gg 1$) and also $h_b(M_G)$ is small (i.e., $\tan \beta \lesssim 40$), then a simpler pattern arises,

(ii) $m_{10}^2, m_5^2 > m_{H_1}^2, m_{H_2}^2$.

The splitting between the Higgs and matter sectors depends, as before, on the size of $\lambda$. (We comment on the case $h_b > h_t$ below.) Regarding $\lambda$, in the minimal SU(5) model one has $\lambda = g_C M_{H_2}/M_V$ (see Appendix A). In this model, proton decay non-observation requires that the colored triplet is heavy $M_{H_2} \gtrsim M_V$ \[3\] and hence $\lambda \approx 1$. Thus, patterns (i) and (ii) completely characterize that model. However, if we ignore the proton decay constraint, assume $\lambda \ll 1$ and again take $h_t \gg h_b$, we find (e.g., see Fig. [3])

(iii) $m_5^2, m_{H_1}^2 > m_{10}^2, m_{H_2}^2$.

Finally, if all Yukawa couplings are small one has

(iv) $m_{10}^2 > m_5^2, m_{H_1}^2, m_{H_2}^2$.

It is convenient to relate the different patterns (or GUT effects) to values of $\tan \beta$. Patterns (i) and (iii) correspond to low $\tan \beta \approx 1 - 2$, i.e., choice (a). Patterns (ii) and (iv) correspond to moderate values of $\tan \beta$. An interesting scenario arises in the special
case $h_t \approx h_b \approx 1$ (and $\lambda \approx 1$), i.e., large $m_{t1}^{pole} \gtrsim 180$ GeV and large tan $\beta \approx 50 - 60$. Eq. (4) now has an additional Yukawa term previously neglected: The $h_b$ term (see Appendix A). This again leads to pattern (ii), however, the corrections are now enhanced because the three Yukawa couplings, $h_t$, $h_t$, and $h_b$, are large, and unless one assumes a small $\lambda$ and a specific mechanism to suppress proton decay (or $M_{1/2} \gg m_9/g_C$), one has a special case of (ii), i.e., $m_5^2 \gg m_2^2$, $m_1^{20} > m_1^{H_1} \sim m_1^{H_2}$). That situation is similar to the one in the minimal SO(10) model where the Higgs $\mathbf{5 + 5}$ of SU(5) are embedded in a single $\mathbf{10}$ of SO(10) (and $\mathbf{10 + 5}$ in a $\mathbf{16}$). Yukawa unification requires in that case $h_t \approx h_b \approx 1$. Our choice (b) with $h_t(M_G) \sim 0.5$ and $h_b(M_G) \sim 0.3$ corresponds to a moderate version of this scenario. A scenario corresponding to (b) is illustrated in Fig. 4. Of course, in SO(10) the RGEs have slightly different slopes than in our case, and the magnitude of the splittings calculated in SU(5) can only approximate the actual splittings in SO(10). The different patterns and their dependence on tan $\beta$ are summarized in Table I.

We now turn to examine the situation in some extended GUT models. When considering extended models one could assume a higher rank group, additional (and/or larger) representations, or both. As an example of the latter, in non-minimal SU(5) (and other) models $H_1$ and $H_2$ couple with different strength to the other Higgs superfields [below we refer to this scenario as non-minimal SU(5)]. One could now arrange for a large and arbitrary splitting at $M_G$ between $m_{H_1}^2$ and $m_{H_2}^2$, e.g.,

$(v) \ m_5^2 > m_{10}^2 > m_{H_1}^2 \gg m_{H_2}^2,$

where $i \neq j = 1, 2$. This can provide a caveat for the general rule that no RSB is possible (assuming universality) for $h_b > h_t$. In fact, we confirmed that the splitting in $(v)$ can be arranged (for $i = 1$) so that RSB is possible in that case. For example, in the SU(5) missing partner model (MPM) [23], the superpotential reads $W = \lambda_1 H_1 \Sigma(\mathbf{75}) \Phi(\mathbf{50})^2 + \lambda_2 H_2 \Sigma(\mathbf{75}) \Phi(\mathbf{50})^2 + \ldots$ Proton decay constraints in that model [24] can be approximated as $\lambda_1 \lambda_2 \sim 5 \eta g_C$ with $\eta \sim M_5/(\mathbf{75})$ in the range $0.1 \lesssim \eta \lesssim 10$. However, some caution is in order. One has to keep in mind a possible breakdown of perturbation theory when large representations (or a large number of small representations) are present. For example, the MPM is not asymptotically free and the gauge coupling typically diverges below $M_P$. (Texture models often assume large representations when trying to explain the light fermion spectrum and may not be asymptotically free as well.) In general, one needs to develop non-perturbative techniques in order to calculate the GUT effects to the SSB parameters. In non-asymptotically free but still perturbatively valid models one has $M_5(M_G) \ll M_{1/2}$.

Before turning to discuss higher rank groups, the $M_G$ matching conditions for SU(5) (minimal and non-minimal) models read

$$m_Q^2 = m_U^2 = m_E^2 = m_{10}^2, \quad (5a)$$

$$m_D^2 = m_L^2 = m_5^2, \quad (5b)$$

$$m_{H_1}^2 = m_{H_2}^2, \quad (5c)$$

where $Q = (\tilde{t}_L, \tilde{b}_L)$ and $L = (\tilde{\nu}_L, \tilde{\tau}_L)$ are the left-handed scalar quark and lepton doublets, and $U, D$ and $E$ are the right-handed $t, b$, and $\tau$-scalars, respectively. Similar relations hold
for the first and second families. As was shown above, in the minimal model, $m_{H_1,2}^2$ both depend on $\lambda$ and are correlated. In non-minimal models, e.g., the MPM, they are independent parameters. Condition (5) applies to the RG-improved tree level SSB parameters at $M_G$. However, since not all heavy fields are degenerate at $M_G$, the splitting between the heavy masses induces, at one-loop, a secondary non-universal shift in the soft parameters, regardless of their initial universality. This is similar to threshold corrections to dimensionless couplings, e.g., see Ref. [26,13,24], with the exception that corrections to $m_i^2$ due to boson-fermion mass splittings are now $\sim M_G^2 \ln[(M_G^2 + m_{\text{soft}}^2)/M_G^2] \sim m_{\text{soft}}^2$ and are not suppressed. One-loop threshold corrections could smear the above patterns, and are discussed for the minimal SU(5) model in Appendix B.

If the rank of the GUT group is higher than that of the SM group [or for that matter, of SU(5)], i.e., higher than four, non-universality at $M_G$ would, in general, trigger non-vanishing $D$-terms [7,8] that could correct the $M_G$ boundary conditions. (The $D$-terms appear at the scale at which the rank of the group is reduced.) It is important to stress that had we assumed universality at $M_G$ then the $D$-terms would have vanished leaving the universality assumption intact [3]. We consider, as an example, the minimal SO(10) model which now depends on an additional parameter – the magnitude (and sign) of the $D$-terms $M_D^2$, which can be shown to be of the order of the soft mass parameters. In SO(10), condition (5) is replaced by

$$
\begin{align}
  m_Q^2 &= m_U^2 = m_E^2 = m_{16}^2 + M_D^2 \equiv m_{10}^2, \\
  m_D^2 &= m_L^2 = m_{16}^2 - 3M_D^2 \equiv m_5^2, \\
  m_{H_1,2}^2 &= m_{H_{1,2}}^2 \pm 2M_D^2,
\end{align}
$$

and in the minimal SO(10) model $m_{H_1}^2 = m_{H_2}^2 \equiv m_H^2$ and typically $m_5^2 \gg m_{16}^2 \gg m_H^2$ (recall the very large tan $\beta$ case). $m_{H_1}^2$ and $m_{H_2}^2$ could be split at $M_G$ according to the sign of $M_D^2$. [Note the SU(5) invariance of (6).] A situation similar to (6) could arise in a SU(5)$\times$U(1) model.

Below, we discuss the minimal SU(5) model [patterns (i) – (iv)] as well as non-minimal SU(5) and minimal SO(10)-inspired extensions [pattern (v) and eq. (6), respectively]. Our choices of points (a) and (b) correspond (for $\lambda \approx 1$) to patterns (i) and (ii), respectively. Hereafter, that correspondence is understood when discussing low and large values of tan $\beta$. The latter can be used as a crude approximation of the situation in the minimal SO(10) model. Intermediate values of tan $\beta$ ($h_t$ and $h_b$ are both small) are also described by patterns (ii) [and (iv)] and typically the splittings are somewhat diminished.

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2 The tree-level SSB parameters shift the GUT-scale vacuum expectation values and redefine $\mu$, $B$ (recall that we are not required to specify these parameters at the high scale), as well as induce the D-terms [23,3,5,7].

3Note, however, that in some texture models Higgs doublets are embedded, e.g., in 10’s and in 126’s, and $m_{H_1}^2$ and $m_{H_2}^2$ could evolve very differently.
III. WEAK-SCALE PHENOMENA

In the previous section we presented four patterns of $M_G$ boundary conditions, that define uniquely non-universality in the minimal SU(5) model (up to threshold corrections). To analyze their low-energy implications one has to evolve the SSB parameters from $M_G$ to $M_Z$. The GUT corrections to the scalar soft masses at $M_G$, denoted by $\Delta m_i^2(M_G)$, do not always lead to the same magnitude of corrections at $M_Z$. For the scalar fields $H_2, Q$ and $U$ that couple with a large Yukawa coupling ($h_t$), the GUT corrections are usually diminished in the evolution from $M_G$ to $M_Z$, i.e., $\Delta m_i^2(M_Z) < \Delta m_i^2(M_G)$. For the rest of the fields, however, one has $\Delta m_i^2(M_Z) \approx \Delta m_i^2(M_G)$.

Figs. 1 – 4 show the scale evolution of the soft scalar mass parameters. The solid (dashed) line shows the evolution of the soft scalar masses when the $M_P - M_G$ evolution is included (neglected). Note that the GUT correction to $m_{H_2}^2$ (the difference between the respective dashed and solid lines) is reduced at $M_Z$ when $h_t$ is large. This is especially apparent in Figs. 1 and 3. In Fig. 2, however, the gauginos give the largest contribution to the soft scalar masses and $\square \Delta m_i^2(M_Z) - \Delta m_i^2(M_G) \propto \Delta M_5^2(M_G) \approx 20\%$. This effect is especially important for colored particles because of the gluino mass that grows at low energies with the diverging QCD coupling and roughly triples between $M_G$ and $M_\tilde{g}$. This feeds back via gluino loops to the colored scalar masses (and to the $A$ parameters) – see (1) – leading to large renormalization of those SSB parameters at the weak scale. Thus, GUT corrections from the Yukawa sector [that can split fields in equal SU(5) representations] can be washed out at $M_Z$. When $h_t$ is smaller (Fig. 4), the reduction of $\Delta m_{H_2}^2$ is less drastic and can give rise to important low-energy implications (the same is true for $\Delta m_{H_1}^2$ for any value of $\tan \beta$). The evolution of $\Delta m_{Q, U}^2$ is even more dramatic. They can change the sign in the $M_G - M_Z$ evolution and even (Fig. 3) increase their value. This is because the positive term $\propto h_t^2 m_{H_2}^2$ in the RGEs of $m_{Q, U}^2$ (that decreases them with the energy scale) is smaller when the GUT effects are considered.

Therefore, large effects are expected in those observables that depend on $m_{H_i}^2$ and $m_{Q, U}^2$: The $\mu$ parameter [see eq. (3a)] and the masses of the third family scalars. The actual magnitude of the GUT effects depends on the choice of the free parameters of the model. In section 4 we introduced a set of model-building parameters

$$m_0, A_0, B_0, M_{1/2},$$

where $B_0$ is traded for $\tan \beta$ using (3b). The parameters are assumed to be real and $A_0$ (and $\mu$) can have either sign. For fixed fermion masses [and SU(5) Yukawa couplings] this set is enough, using renormalization group techniques, to predict all the low-energy observables. This is usually called up-down approach. Applying a specific set of values to these parameters as a boundary condition at $M_P$ [i.e., to eq. (4)] or at $M_G$ [i.e., to eq. (1)] can lead to a quite different mass spectrum. This is shown in the first and second columns of Tables II – V for

\[4\]

Note that the gaugino mass is enhanced $\sim 10\%$ by the $M_P - M_G$ evolution and that leads to an additional increase in the $m_{H_2}^2(M_Z)$. In Ref. [17–20] only the scalar soft masses are modified at $M_G$, i.e., $\Delta M_5^2(M_G) = 0$.  

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the scenarios illustrated in Figs. 1 – 4. LSP stands for the lightest supersymmetric particle which is stable in the MSSM assuming $R$-parity. Tables I – V give quantitative examples of the GUT corrections. Note large deviations for observables that depend on the $\mu$ parameter (such as the Higgsino masses and components) and for the stop and sbottom masses.

Nevertheless, the free parameters of the model have to be extracted from low-energy experiments. Thus, it is more convenient to choose a set of free parameters of the model defined at $M_Z$, i.e., bottom-up approach [27]. For example, let us consider the basis

$$m_{\tilde{e}_L}, M_{\tilde{g}}, \tan \beta, A_t(M_Z).$$

(8)

This basis is also convenient because the boundary conditions at $M_G$ can easily be obtained from the parameters (8). Comparing now, for a given value of (8), the low-energy predictions when the GUT effects are included (first column in Tables I – V) with those when the GUT effects are neglected (third column), one finds more modest changes, especially in the $M_{1/2} > m_0$ region (Table III). This is because using the basis (8) one eliminates global scalings of the SSB parameters arising in the $M_P - M_G$ evolution (see Fig. 2 and comment above). Although the choice (7) is the relevant one when speculating on the origin of the SSB parameters, it will be (8) or a similar basis from which the value of the SSB parameters will be extracted once supersymmetry is established.

Below, we study in more detail the way in which the predictions from naive $M_G$ universal boundary conditions are smeared and modified by the non-universal GUT corrections. Our aim is to study the uncertainties and new regions of parameter space that could be opened by those uncertainties. For example, we already mentioned that a situation with $h_b > h_t$ can now be consistent assuming pattern (v). We focus on our choices (a) and (b) given in section I. Our numerical routines are similar to those described in Ref. [4]. In short, we follow Ref. [13] in calculating the couplings (and the unification point), and Ref. [28] in treating the one-loop effective potential correction $\Delta V$, including contributions from all sectors. The boundary conditions for $0 \leq m_0 \leq 1000$ GeV, $|A_0| \leq 3m_0$, and $50 \leq M_{1/2} \leq 500$ GeV, are picked at random, unless otherwise stated. In order to minimize residual scale dependences (of order two-loop) of the one-loop effective potential, we rescale the Higgs potential (including wave function corrections) to a typical $t$-scalar scale of 600 GeV before solving the one-loop minimization equations. All Higgs masses include one-loop corrections calculated using Ref. [24]; however, a $O(10\%)$ ambiguity in the one-loop light Higgs boson mass remains [1]. We apply the conservative constraint $m_{h^0} \gtrsim 60 \pm 5$ GeV. We also force all other relevant bounds on the mass parameters, and require the correct EWSB (i.e., a solution for $M_Z^2$), a neutral LSP and positive squared masses for all physical scalars. However, we do not minimize the full scalar potential in order to eliminate color breaking minima that survive the upper bound on $|A_0|$. That may affect the status of some points with a particularly large value of the $\mu$ parameter but is also sensitive to the choice of $M_G$.

5Although the relation between $m_{\tilde{e}_L}$ and $M_{\tilde{g}}$ and the physical observables is straightforward, this is not the case for $\tan \beta$ and $A_t(M_Z)$. Nevertheless, the latter two can always be determined as a function of physical observables such as the stop or Higgs masses (these are usually complicated functions involving other parameters of the model).
or $M_P$. For $\tan \beta = 42$ some points could induce positive corrections to the $b$-quark mass of more than $\sim 20\%$ and are omitted (smaller corrections could be compensated by other threshold effects). This effect is also sensitive to the $M_G$ or $M_P$ choice.

**A. First and second family scalars**

Since the Yukawa couplings of the first and second family of squarks and sleptons are small, they can be neglected in the RGEs, i.e., only the gauge contribution is relevant. The RGEs can be solved analytically and the physical scalar quark and lepton ($\tilde{f}$) masses in the basis $[\bar{7}]$ are given by

\[ m_{\tilde{f}_{L,R}}^2 = m_0^2 + a_{\tilde{f}_{L,R}} M_{i/2}^2 \pm M_Z^2 \cos 2\beta [T_{3\tilde{f}_{L,R}} - Q_{\tilde{f}_{L,R}} \sin^2 \theta_W] + \Delta m_{\tilde{f}_{L,R}}^2, \tag{9} \]

where $a_{\tilde{f}_{L,R}} \sim 5 - 7$ for the squarks, $\sim 0.5$ for the left-handed sleptons and $\sim 0.15$ for the right-handed sleptons. $T_{3\tilde{f}_{L,R}}$ and $Q_{\tilde{f}_{L,R}}$ are the third component of SU(2)$_L$ isospin and the electric charge of $\tilde{f}_{L,R}$, respectively. The quantity $\Delta m_{\tilde{f}_{L,R}}^2$ is the extra contribution arising from the $M_P - M_G$ evolution and depends on the GUT. For SU(5) we have $[\bar{7}]$ $\Delta m_{\tilde{f}_{L,R}}^2 = [0.1 a_{\tilde{f}_{L,R}}^u + 0.45] M_{i/2}^2$ for the left-handed squarks, $\tilde{u}_R$ and $\tilde{e}_R$ and $\Delta m_{\tilde{f}_{L,R}}^2 = [0.1 a_{\tilde{f}_{L,R}}^d + 0.3] M_{i/2}^2$ for the left-handed sleptons and $\tilde{d}_R$ (the term $0.1 a_{\tilde{f}_{L,R}}^u$ arises due to the gaugino enhancement$^4$). Note that $\Delta m_{\tilde{f}_{L,R}}^2$ can be the dominant contribution to the $\tilde{e}_R$ mass and can contribute $\sim 60\%$ to the $\tilde{e}_L$ and $\nu_L$ masses. For SO(10) D-terms, the magnitude of the GUT correction depends on $M_P^2$. It is important to note that the requirement of non-tachionic sleptons, i.e., $m_{\tilde{e}_{L,R}} > 0$, leads to strong constraints on $M_P^2$ from below and above that can be easily obtained from eqs. $[\bar{8}]$ and $[\bar{9}]$.

When we use the input eq. $[\bar{8}]$, however, the scalar masses read

\[ m_{\tilde{f}_{L,R}}^2 = m_{\tilde{e}_L}^2 + a_{\tilde{f}_{L,R}} M_\tilde{g}^2 \pm M_Z^2 \cos 2\beta [T_{3\tilde{f}_{L,R}} - Q_{\tilde{f}_{L,R}} \sin^2 \theta_W] + 0.3 M_Z^2 \cos 2\beta + \Delta m_{\tilde{f}_{L,R}}^2, \tag{10} \]

where now $a_{\tilde{f}_{L,R}} \sim 0.6 - 0.9$ for the squarks, 0 for the left-handed sleptons and $\sim -5 \times 10^{-2}$ for the right-handed sleptons; $\Delta m_{\tilde{f}_{L,R}}^2 \sim 2 \times 10^{-2} M_\tilde{g}^2$ for the left-handed squarks, $\tilde{u}_R$ and $\tilde{e}_R$ and $\Delta m_{\tilde{f}_{L,R}}^2 = 0$ for the left-handed leptons and $\tilde{d}_R$. Thus, in the basis $[\bar{8}]$ GUT effects can change the $\tilde{e}_R$ (left-handed squarks and $\tilde{u}_R$) masses by $\sim 15\%$ (1%) at most. An example is given in Table $[\bar{1}]$. The GUT effects in $m_{\tilde{f}}$ and $m_{\tilde{q}}$ are less apparent when the basis $[\bar{8}]$ is used instead of the basis $[\bar{7}]$.

**B. The $\mu$ parameter**

The $\mu$ parameter is extracted from the minimization condition eq. $[\bar{3}a]$ and depends on the values of $m_{\tilde{H}_L}$ at the weak scale. If the quantities $m_{\tilde{H}_L}$ are affected by the GUT effects, the $\mu$ parameter is modified according to

\[ \Delta \mu^2 = -\frac{\Delta m_{\tilde{H}_L}^2 - \Delta m_{\tilde{H}_L}^2 \tan^2 \beta}{1 - \tan^2 \beta}, \tag{11} \]
where $\Delta m_{H}^2$ are the shifts in the soft Higgs masses at the weak scale due to the GUT effects.

For low values of $\tan\beta$, one finds that $|\mu|$ is typically large and not affected significantly by GUT corrections, $\Delta \mu/\mu \sim 0.2$. For the GUT pattern $(iii)$, $\Delta \mu$ is small because $\Delta m_{H_2}^2$ is reduced at $M_Z$ (Fig. 3). For the GUT pattern $(ii)$, this is because of a partial cancellation between $\Delta m_{H_1}^2$ and $\Delta m_{H_2}^2 \tan^2 \beta$. In Fig. 4a we compare the predictions of $\mu$ when universality is assumed at $M_P$ with those when universality is assumed at $M_G$ for different random points of the parameter space defined by (7). The fact that $\Delta \mu$ depends on the sign of $\mu$ is due to the weak-scale threshold corrections to eq. (11) that can be substantial and have to be included. In Fig. 5 we show the distribution of the $\mu$ predictions in a sample of Monte Carlo calculations. One can see that the distribution is slightly changed. The differences in the integrated area of the histograms give an estimate of the changes in the allowed parameter space.

For large values of $\tan\beta$, we have $\Delta \mu^2 \approx -\Delta m_{H_2}^2$. For large $\lambda$, the splitting $\Delta m_{H_2}^2$ can be substantial (if $h_t$ is small, the quantity $\Delta m_{H_2}^2$ is slightly diminished in the $M_G - M_Z$ evolution – see Fig. 4 and $\mu$ can receive a large shift (Figs. 3b and 7). Note that the value of $|\mu|$ is always increased since in the minimal SU(5) model $\Delta m_{H_2}^2 < 0$.

We have noted that the minimal SU(5) effects lead generically to an increase of $|\mu|$ [for low values of $\tan\beta$ the value of $|\mu|$ can be reduced (Fig. 3a), but $|\mu|$ is very large in this regime and the effects are small]. When extended GUTs are considered, however, this interesting feature is lost. In order to have $\Delta \mu^2 < 0$ one needs $[\Delta m_{H_2}^2 \tan^2 \beta - \Delta m_{H_1}^2] > 0$, and this latter condition can be obtained in the extended GUTs considered in section II. In the large $\tan\beta$ regime, however, one has also to consider the implications to EWSB. From the minimization conditions of the Higgs potential, we have that EWSB requires at the weak scale (for large $\tan\beta$),

$$\mu^2 + m_{H_2}^2 < 0, \quad \mu^2 + m_{H_1}^2 > 0,$$

(12)

which is difficult to achieve from universal $m_{H_1}^2 = m_0^2$ at $M_G$ because $h_t \approx h_b$. The GUT effects can produce a splitting $\Delta m_{H_2}^2 - \Delta m_{H_1}^2 < 0$ such that eq. (12) is satisfied more easily. For example, in the GUTs considered in section II this splitting can be induced but it requires $\Delta m_{H_2}^2 < 0$ which leads to an increase of $|\mu|$. In order to decrease $|\mu|$ we need $\Delta m_{H_2}^2 > 0$ that in the extended GUTs considered can only be obtained from the $D$–terms eq. (8). In that case, however, $\Delta m_{H_2}^2 - \Delta m_{H_1}^2 > 0$ and the EWSB is more difficult to obtain (requiring even more fine-tuning). In fact, for a given point in the parameter space there is an upper bound on $\Delta m_{H_2}^2 - \Delta m_{H_1}^2$ (that is strengthened when $\tan\beta$ increases). Note that in cases in which (because of the $M_P - M_G$ evolution) $m_0^2 > m_{H_1}^2$, the EWSB is even more difficult to obtain since $m_{H_1}^2$ and $m_{H_2}^2$ can be both negative at $M_Z$. Thus, GUT effects can ease EWSB (for large $\tan\beta$) but, in that case, they increase the value of $|\mu|$.

The fact that $\mu$ is extracted from eq. (8a) and is usually larger than $M_Z$ implies that the lightest chargino ($\chi^+_1$) and neutralinos ($\chi^0_1$ – the LSP – and $\chi^0_2$) are mostly gauginos. As we have shown, this property is not altered by GUT effects of the minimal SU(5) model.

---

6 A situation with both $m_{H_1}^2 + \mu^2 < 0$ (when including loop corrections) leads to an unacceptable minimum. Note that we plot (in Figs. 3 – 4) the tree-level soft mass parameters.
Therefore, the masses of $\chi_1^+$ and $\chi_0^0$ depend mostly on $M_{1/2}$ and are almost independent of the soft scalar masses. The $M_P - M_G$ evolution can enhance $M_5$ and thus the gaugino masses by $\sim 10\%$. Using the basis (5), however, the gaugino masses can be written as a function of $M_5$, i.e., independent of the scale.

C. The Higgs scalars

The masses and mixing angles of the Higgs bosons in the MSSM can be written as a function of two parameters that we choose to be $\tan \beta$ and the pseudoscalar Higgs mass $m_{A_0}^2 \equiv 2\mu^2 + m_{H_1}^2 + m_{H_2}^2$. Since $\tan \beta$ is considered an input parameter, only the GUT effects in $m_{A_0}^2$ are relevant. A shift in the soft Higgs masses arising from GUT physics shifts $m_{A_0}^2$ by

$$\Delta m_{A_0}^2 = \frac{1 + \tan^2 \beta}{1 - \tan^2 \beta}(\Delta m_{H_2}^2 - \Delta m_{H_1}^2).$$

(13)

For $\tan \beta \approx 1$, the behavior of $\Delta m_{A_0}^2$ is similar to that of $\Delta \mu^2$ discussed in the previous section. For large $\tan \beta$, one has $\Delta m_{A_0}^2 \approx \Delta m_{H_1}^2 - \Delta m_{H_2}^2$. Since in the minimal SU(5) model $\Delta m_{H_2}^2 \approx \Delta m_{H_1}^2$, one has $\Delta m_{A_0}^2 \approx 0$.

The mass of the lightest Higgs $h^0$ receives large radiative correction induced by loops involving the top and stop ($m_{h^0} \to m_{h^0} + \Delta_{h^0}[m_t, m_Q, m_U, A_t, \mu, \tan \beta]$) and can be changed if either the diagonal or off-diagonal entries in the stop mass matrix are shifted by GUT effects (section III D). The effects are negligible for $\tan \beta > \sim 2$ where the Higgs boson is heavy at tree level. However, for a light tree-level Higgs boson ($\tan \beta \approx 1$), unless the different effects cancel (see Tables II and III), they can modify $m_{h^0}$ by a few GeV (see Table IV). The cancellations depend on the sign of the $\mu$ parameter. In Figs. 8 and 9 we show the distribution of the lightest Higgs mass. One can note that the distributions are only slightly sensitive to the GUT corrections so that previous calculations (e.g., see [4]) of the predictions of $m_{h^0}$ in SUSY GUTs are not altered. (Fig. 8, here, roughly corresponds to Figs. 9a and 10a in Ref. [4].)

When GUT effects from extended models are considered, the value of $m_{A_0}^2$ can increase (decrease) if a splitting $\Delta m_{H_1}^2 - \Delta m_{H_2}^2 > 0$ ($< 0$) is induced. Again, when EWSB and non-tachionic particles are required, $\Delta m_{A_0}^2$ can be bounded from below and above.

D. Third family scalars

As explained above, the masses of the scalars of the third family can be largely modified by the GUT effects due to a large $h_t^7$. The GUT effects can modify (i) the soft mass $m_{10}^2$, (ii) $m_{H_2}^2$ and hence the evolution of $m_Q^2$ and $m_U^2$, and (iii) the $\mu$ parameter (section III B) and $A_{t,b,\tau}$ that enter in the left-right mixing term of the scalar masses. These three effects compete with each other to increase or decrease the masses.

7There are also gauge GUT effects that are the same as those to the first and second family scalars.
The lightest stop, $\tilde{t}_1$, has recently received much attention. Its mass, $m_{\tilde{t}_1}$, is usually smaller than the mass of the other squarks and can induce significant one-loop effects in low-energy processes such as $Z \to b\bar{b}$ and $b \to s\gamma$. In the minimal SU(5) model, the dominant GUT effect in $m_{\tilde{t}_1}$ arises from (ii). Since $m_{H_2}^2$ is diminished by GUT effects, $m_Q^2(M_Z)$ and $m_U^2(M_Z)$ are larger. Thus, we find that $m_{\tilde{t}_1}$ is always enhanced. It follows that some points of the parameter space which correspond to a tachionic $t$-scalar and are excluded when the $M_P$ to $M_G$ evolution is neglected, can be allowed. In Fig. 10 we compare the predictions of $m_{\tilde{t}_1}$ with and without the $M_P - M_G$ evolution. We find significant corrections in the low $\tan\beta$ regime, especially for small values of $m_{\tilde{t}_1}$. This implies that one-loop corrections induced by $\tilde{t}_1$ to low-energy processes can be significantly reduced. Note that $m_{\tilde{t}_1} \gtrsim 200$ GeV when including the $M_P - M_G$ evolution.

The GUT effects from non-minimal SU(5) models [pattern (v)] usually enhance $m_{\tilde{t}_1}$ since (ii) is still dominant. In the minimal SO(10), however, the splittings eq. (6) can lead to a lighter stop. For $M_D^2 < 0$, one has $\Delta m_{10}^2 < 0$ and $\Delta m_{H_2}^2 > 0$, and both effects (i) and (ii) decrease $m_{\tilde{t}_1}$. Since, as we said in section III A, these splittings can lead to tachionic sleptons, $m_{\tilde{t}_1}$ cannot be reduced significantly.

For the sbottom and stau we find that the effects (i) and (ii) can be equally important and their masses can increase or decrease depending on the point in the parameter space (see tables).

### E. Possible implications

To conclude our survey of the weak-scale phenomena, we summarize the most interesting implications of the GUT effects for experiment. We have shown that assuming the minimal SU(5) model and universality at $M_P$ instead of $M_G$ one typically predicts heavier particles. For example, the scalar leptons could be substantially heavier (see Table III). More interestingly, correlations between the different parameters are modified (see also II), i.e., correlations calculated assuming universality at $M_G$ could be misleading and should not be used to constrain the parameter space. Smearred correlations imply less EWSB-related fine-tuning and that a larger parameter space may be available. For example, the $\tilde{t}_1$ mass is shown [for choice (a)] in Figs. 11 and 12. It is typically larger when considering the GUT effects and its correlation with the $\chi^+_1$ mass (or for that matter, with the gluino mass – see Fig. 3 of Ref. [1]) is smeared while that with the $\tilde{t}_2$ mass is strengthened. For choice (b) the $\tilde{t}$ masses are only slightly altered but $\chi^+_1$ could be heavier. In Fig. 13 we examine the Higgsino fraction of the LSP for choice (b), which is relevant, e.g., for relic abundance calculations. The larger Higgsino mass implies smaller Higgsino fractions of the gaugino-like $\chi^+_1$, $\chi^0_1$ and $\chi^0_2$. That and the heavier $\tilde{t}_1$ lead to a stronger decoupling of the supersymmetric particles from low-energy processes, e.g., from $Z \to b\bar{b}$.

In extended GUT models the restrictions on the parameter space from EWSB are somewhat relaxed. In particular, values of $m^\text{pole}_t$ and $\tan\beta$ which imply $h_u > h_t$ may be consistent with EWSB. Non-vanishing $D$-terms can lead to a very different spectrum in comparison to the situation with vanishing $D$-terms. For example, they could lead to a lighter Higgsino which, as discussed above, is an interesting possibility phenomenologically. However, when combined with the $M_P - M_G$ evolution, the effects are diminished (see Table V). Also, in the models studied the amount that the $\mu$ prediction can be diminished by GUT effects is
strongly constrained (unlike in some ad hoc cases studied in Ref. [17–20]). We conclude that if able to observe the GUT effects, e.g., from correlation measurements, collider experiments could directly probe the GUT scale physics. That is a non-trivial task and it would depend on the experimental resolution as well as on the region of parameter space nature chooses.

IV. CONCLUSIONS

Above, we examined the effects of a grand-unified symmetry between the Planck and GUT scales in the SSB parameters. Our only assumptions were coupling constant unification at $M_G \approx 2 \times 10^{16} \text{ GeV}$; the MSSM as the effective theory below that scale; and universal SSB parameters at the minimal supergravity scale $M_P \approx 2 \times 10^{18} \text{ GeV}$. In addition, we had to specify the GUT. We previously analyzed these assumptions in Ref. [1] assuming the minimal SU(5) model and found potentially large deviations from universality for the SSB parameters at $M_G$. In particular, we emphasized the role of large Yukawa couplings which are generic in such models. Here, we further cataloged the possible patterns of non-universality in that model and examined in great detail their implications to the weak scale phenomena. We found potentially large corrections (in comparison with the working assumption of universality at $M_G$) to the allowed parameter space, the $\mu$ parameter and to the third-family squark spectrum. These are all related primarily to the $M_P – M_G$ evolution of the light Higgs fields. In the gaugino-dominated cases (e.g., in no-scale models) large corrections to the scalar-lepton masses are also possible. The Higgs and the first and second family squark sectors are relatively insensitive to the GUT effects. A different situation may arise in extended models. For example, we discussed the situation in non-minimal SU(5), where $m^2_{H_1}(M_G)$ and $m^2_{H_2}(M_G)$ are independent parameters, as well as the appearance of non-vanishing $D$-terms in SO(10). In both models correlations are further diminished and EWSB constraints are more easily satisfied. However, EWSB still plays an important role in constraining GUT effects, e.g., effects that could diminish $\mu$. Implications of the above to experiment were already summarized in section III E.

Finally, let us comment on the predictive power of the MSSM. Assuming minimal SU(5), two additional parameters are needed (only one of which plays an important role). In non-minimal SU(5), SU(5)$\times$U(1) and SO(10) three or more new parameters are needed. The more parameters the model has, the larger role GUT effects could play in weak-scale phenomena, but the less predictive is the model. On the other hand, when adding only a small number of new parameters, the effects in different SSB parameters are correlated, and thus, constrained (e.g., by EWSB). The predictive power can be further altered when considering threshold corrections. These are described in detail for the minimal SU(5) model in Appendix B [eqs. (B13)–(B15)] and, in general, they do not significantly modify the tree-level patterns described in section I. In extended models threshold corrections could be more important if more and larger representations are present. Also, perturbation theory could break down in these models and one would need non-perturbative methods to calculate the GUT effects.

GUT effects in the SSB parameters are generic, leading to non-universal patterns different than those, e.g., in string theory, and could probe the GUT-scale physics. However, they are model-dependent and lead to uncertainties in any model-independent analysis, which typically assumes universality at $M_G$. Until supersymmetry is established and characterized,
the effects have to be considered as uncertainties to supergravity GUT model predictions.

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APPENDIX A: THE MINIMAL SU(5) MODEL

The Higgs sector of the model consists of three supermultiplets, \( \Sigma(24) \) in the adjoint representation [which is responsible for the breaking of SU(5) down to SU(3)\(_c\) \( \times \) SU(2)\(_L\) \( \times \) U(1)\(_Y\)], \( H_1(\bar{5}) \) and \( H_2(5) \):

\[
\Sigma \equiv \sqrt{2} T_a w_a , \quad H_1 = \left( \frac{H_{C_1}}{H_1} \right) , \quad H_2 = \left( \frac{H_{C_2}}{H_2} \right),
\]

where \( H_{C_i} \) and \( H_i \) are the color triplets and SU(2)\(_L\) doublets, respectively, and \( T_a \) are the SU(5) generators with \( \text{tr}\{T_a T_b\} = \delta_{ab}/2 \). The matter superfields are in the \( \bar{5} + 10 \) representations, \( \phi(\bar{5}) \) and \( \psi(10) \). The superpotential is given by

\[
W = \mu_\Sigma \text{tr} \Sigma^2 + \frac{1}{6} \lambda' \text{tr} \Sigma^3 + \mu_H H_1 H_2 + \lambda H_1 \Sigma H_2 \\
+ \frac{1}{4} h_t \epsilon_{ijklm} \psi^{ij} \psi^{kl} H_2^m + \sqrt{2} h_b \psi^{ij} \phi_i \phi_j + \frac{1}{2} \lambda_\alpha \lambda_\beta + h.c.,
\]

where we have omitted family indices and \( h_t \) and \( h_b \) are the Yukawa couplings of the third generation (we neglect the other Yukawa couplings). In the supersymmetric limit \( \Sigma \) develops a vacuum expectation value \( \langle \Sigma \rangle = \nu_\Sigma \text{diag}(2, 2, 2, 3, -3) \) and the gauge bosons \( X \) and \( Y \) receive a mass \( M_V = 5g_G \nu_\Sigma \). In order for the Higgs SU(2) doublets to have masses of \( \mathcal{O}(M_Z) \) instead of \( \mathcal{O}(M_G) \), the fine-tuning \( \mu_H - 3\lambda \nu_\Sigma \lesssim \mathcal{O}(M_Z) \) is required and one obtains \( M_{H_C} = \frac{\lambda}{g_G} M_V \). Dimension-five operators induced by the color triplets give large contributions \( \propto 1/M_{H_C}^2 \) to the proton decay rate [8]. To suppress such operators, the mass of the color triplets has to be large, \( M_{H_C} \gtrsim M_V \), implying \( \lambda \gtrsim g_G \approx 0.7 \).

Below \( M_P \), the effective lagrangian also contains the SSB terms (note that we keep the same notation for the superfields and their corresponding scalar fields)

\[
-\mathcal{L}_{\text{soft}} = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_\Sigma \text{tr} \{ \Sigma^\dagger \Sigma \} + m_\phi |\phi|^2 + m_\psi \text{tr} \{ \psi^\dagger \psi \} \\
+ [B_{\Sigma} \mu_\Sigma \text{tr} \Sigma^2 + \frac{1}{6} A_{\Sigma} \lambda' \text{tr} \Sigma^3 + B_{H} \mu_H H_1 H_2 + A_{\lambda} \lambda H_1 \Sigma H_2 \]
+ \frac{1}{4} A_t h_t \epsilon_{ijklm} \psi^{ij} \psi^{kl} H_2^m + \sqrt{2} A_b h_b \psi^{ij} \phi_i \phi_j + \frac{1}{2} M_5 \lambda_\alpha \lambda_\beta + h.c.,
\]

where \( \lambda_\alpha \) are the gaugino fields.

The SU(5) RGEs for the SSB parameters and Yukawa couplings are given by
\[
\begin{align*}
\frac{dm^2_{10}}{dt} &= \frac{1}{8\pi^2} \left[ 3h^2 \left( m^2_{H_u} + 2m^2_{10} + A^2_t \right) + 2h^2 \left( m^2_{H_u} + m^2_{10} + m^2_b + A^2_b \right) - \frac{72}{5} g_G^2 M_5^2 \right], \\
\frac{dm^2_{1\Sigma}}{dt} &= \frac{1}{8\pi^2} \left[ 4h^2 \left( m^2_{H_u} + m^2_{10} + m^2_b + A^2_b \right) - \frac{48}{5} g_G^2 M_5^2 \right], \\
\frac{dm^2_{H_u}}{dt} &= \frac{1}{8\pi^2} \left[ 4h^2 \left( m^2_{H_u} + m^2_{10} + m^2_b + A^2_b \right) + \frac{24}{5} \lambda^2 \left( m^2_{H_u} + m^2_{H_d} + m^2 \Sigma + A^2 \lambda \right) - \frac{48}{5} g_G^2 M_5^2 \right], \\
\frac{dm^2_{H_d}}{dt} &= \frac{1}{8\pi^2} \left[ 3h^2 \left( m^2_{H_d} + 2m^2_{10} + A^2_t \right) + \frac{24}{5} \lambda^2 \left( m^2_{H_u} + m^2_{H_d} + m^2 \Sigma + A^2 \lambda \right) - \frac{48}{5} g_G^2 M_5^2 \right], \\
\frac{dm^2_{\Sigma}}{dt} &= \frac{1}{8\pi^2} \left[ \frac{21}{20} \lambda^2 \left( 3m^2_{\Sigma} + A^2 \lambda \right) + \lambda^2 \left( m^2_{H_u} + m^2_{H_d} + m^2 \Sigma + A^2 \lambda \right) - 20 g_G^2 M_5^2 \right], \\
\frac{dA_t}{dt} &= \frac{1}{8\pi^2} \left[ 9A_t h^2_t + 4A_b h^2_b + \frac{24}{5} A \lambda \lambda^2 - \frac{96}{5} g_G^2 M_5 \right], \\
\frac{dA_b}{dt} &= \frac{1}{8\pi^2} \left[ 10A_b h^2_b + 3A_t h^2_t + \frac{24}{5} A \lambda \lambda^2 - \frac{84}{5} g_G^2 M_5 \right], \\
\frac{dA_\lambda}{dt} &= \frac{1}{8\pi^2} \left[ \frac{21}{20} A \lambda \lambda^2 + 3A_t h^2_t + 4A_b h^2_b + \frac{53}{5} A \lambda \lambda^2 - \frac{98}{5} g_G^2 M_5 \right], \\
\frac{dA_{\lambda'}}{dt} &= \frac{1}{8\pi^2} \left[ \frac{63}{20} A \lambda \lambda^2 + 3A \lambda \lambda^2 - 30 g_G^2 M_5 \right], \\
\frac{dh_t}{dt} &= \frac{h_t}{16\pi^2} \left[ 9h^2_t + 4h^2_b + \frac{24}{5} \lambda^2 - \frac{96}{5} g_G^2 \right], \\
\frac{dh_b}{dt} &= \frac{h_b}{16\pi^2} \left[ 10h^2_b + 3h^2_t + \frac{24}{5} \lambda^2 - \frac{84}{5} g_G^2 \right], \\
\frac{d\lambda}{dt} &= \frac{\lambda}{16\pi^2} \left[ \frac{21}{20} \lambda^2 + 3h^2_t + 4h^2_b + \frac{53}{5} \lambda^2 - \frac{98}{5} g_G^2 \right], \\
\frac{d\lambda'}{dt} &= \frac{\lambda'}{16\pi^2} \left[ \frac{63}{20} \lambda^2 + 3\lambda^2 - 30 g_G^2 \right],
\end{align*}
\]  

(A4)

where \( t = \ln Q \). The RGE for the gauge coupling is \( d\alpha_G/dt = -3\alpha_G^2/2\pi \), and similarly \( dM_5/dt = -3\alpha_G M_5/2\pi \). We can omit the RGEs for \( \mu_\Sigma, \mu_H, B_\Sigma \) and \( B_H \), which are arbitrary parameters that decouple from the rest of the RGEs.

Below \( M_G \), the effective theory corresponds to the MSSM:

\[
W = \mu H_1 H_2 + h_t Q H_2 U + h_b Q H_1 D + h_\tau L H_1 E,
\]

(A5)

where \( Q \) and \( L \) are, respectively, the quark and lepton \( SU(2)_L \) doublets, and \( U, D \) and \( E \) are, respectively, the quark and lepton \( SU(2)_L \) singlets.

**APPENDIX B: GUT-SCALE THRESHOLD CORRECTIONS**

Even if the scale where universal SSB terms are generated is assumed to be \( M_G \), there is some arbitrariness in the value of \( M_G \) due to the mass-splitting between the particles at the GUT scale, \( i.e., \) threshold effects. These GUT effects to the SSB terms (to the best of our knowledge) have never been considered before. As we will show, they can be as important as the low-energy (supersymmetric) threshold effects, which are the only threshold effects to the SSB parameters considered in the literature.
We will only consider GUT threshold corrections to the scalar SSB parameters. Corrections to the gaugino masses have been computed in Ref. [30], where they were shown to be small. There are two ways to compute the threshold correction to the scalar SSB parameters. One way consists of calculating explicitly the one-loop diagrams that contribute to the scalar SSB terms. A second way, which is much simpler, consists of obtaining the one-loop SSB terms from the one-loop effective potential that in the Landau gauge and in the dimensional-reduction (DR) scheme reads
\[
\Delta V = \frac{1}{64\pi^2} \sum_i (-1)^{2s_i}(2s_i + 1)M_i^4 \left[ \ln \frac{M_i^2}{Q^2} - \frac{3}{2} \right],
\]
where \( M_i \) and \( s_i \) are, respectively, the field-dependent squared mass and spin of the particle \( i \). In this case, one only has to compute the masses \( M_i \). We will present the GUT threshold correction using (B1) although we have checked the results with those from the explicit diagramatic calculation.

Let us start with the one-loop correction to the SSB squared-mass \( m_i^2 \) of a scalar \( \Phi_i \) induced by a heavy chiral supermultiplet, which consists of a fermion field with mass \( M_F \) and two (real) scalars with masses \( M_{S_1} \) and \( M_{S_2} \). In the supersymmetric limit \( M_F = M_{S_1,2} \equiv M_0 \) where \( M_0 \), is of the order \( M_G \). We separate the GUT effects into logarithmic corrections and finite corrections:

1. Logarithmic corrections: The logarithmic term of eq. (B1) gives the one-loop contribution to \( m_i^2 \)
\[
\Delta m_i^2 (log) = \frac{1}{32\pi^2} \left\{ \frac{\partial M_{S_1}^2}{\partial |\Phi_i|^2} M_{S_1}^2 + \frac{\partial M_{S_2}^2}{\partial |\Phi_i|^2} M_{S_2}^2 - 2 \frac{\partial M_F^2}{\partial |\Phi_i|^2} M_F^2 \right\} \ln \frac{M_F^2}{Q^2},
\]

2. Finite corrections: The non-logarithmic contribution to \( m_i^2 \) from (B1) is given by
\[
\Delta m_i^2 (finite) = -\frac{1}{32\pi^2} \left[ \frac{\partial M_{S_1}^2}{\partial |\Phi_i|^2} M_{S_1}^2 + \frac{\partial M_{S_2}^2}{\partial |\Phi_i|^2} M_{S_2}^2 - 2 \frac{\partial M_F^2}{\partial |\Phi_i|^2} M_F^2 \right].
\]
Note that this contribution depends on the renormalization scheme. Eq. (B4) has been obtained in the DR scheme. The heavy squared-masses \( M_{S_{1,2}}^2 \) and \( M_F^2 \) can be written using a \( 1/M_0 \) expansion as
\[
M_{S_{1,2}}^2 = M_0^2 \pm aM_0 + b + \sum_i \left[ b_i \pm \frac{c_i}{M_0} + \frac{d_i}{M_0^2} \right] |\Phi_i|^2 + \ldots,
\]
\[
M_F^2 = M_0^2 + \sum_i b_i |\Phi_i|^2 + \ldots,
\]
where the coefficients $a - d_i$ depend on the SSB parameters\(^8\), and we have only kept the relevant terms for our analysis. Substituting eq. (B5) in eqs. (B2), (B3) and (B4), we get

$$
\Delta m_i^2(\text{log}) = \frac{1}{16\pi^2} [bb_i + ac_i + d_i] \ln \frac{M_F^2}{Q^2} \tag{B6}
$$

$$
+ \frac{1}{32\pi^2} [b_i(2b + a^2) + 2ac_i] + \ldots, \tag{B7}
$$

$$
\Delta m_i^2(\text{finite}) = -\frac{1}{16\pi^2} [bb_i + ac_i + d_i] + \ldots. \tag{B8}
$$

Notice that the contribution from (B3) [the term (B7)] gives a correction to $m_i^2$ not suppressed by powers of $O(m_{soft}^2/M_0^2)$, although it turns out to be non-logarithmic. From eqs. (B6)–(B8), the GUT-scale corrections can easily be obtained if the dependence of the heavy masses on the light scalar fields (the $a - d_i$ coefficients) is known. In the minimal SUSY SU(5) model (see Appendix A), the GUT spectrum consists of the 12 vector superfields $V = X, Y$, the color triplets $H_{C_i}$, and the $\Sigma$ superfield. With respect to $SU(3)_c \times SU(2)_L$, the $\Sigma$ supermultiplet decomposes into $(3, 2) + (\bar{3}, 2) + (8, 1) + (1, 3) + (1, 1)$. In the supersymmetric limit, the $(3, 2)$ and $(\bar{3}, 2)$ components are degenerate with $X$ and $Y$. The $(1, 3)$ and $(8, 1)$ components, $\Sigma_3$ and $\Sigma_8$, respectively, have a common mass $10\mu_\Sigma$, while the mass of the singlet, $\Sigma_1$, is $2\mu_\Sigma$. When the SSB terms are considered, a boson-fermion mass splitting within every supermultiplet is induced.

Considering only the two large Yukawa couplings, $\lambda$ and $h_t$, the coefficients $a - d_i$ for the $\Sigma_3$ are given by

\[
\begin{align*}
    a &= B_\Sigma, \quad b = m_\Sigma^2, \\
    b_{H_1} &= b_{H_2} = \lambda^2, \\
    c_{H_1} &= c_{H_2} = \frac{\lambda^2}{2} [2A_\lambda - B_\Sigma], \\
    d_{H_1} &= \frac{\lambda^2}{2} \left[ m_{H_2}^2 - m_\Sigma^2 + A_\lambda + B_\Sigma^2 - 2A_\lambda B_\Sigma \right], \\
    d_{H_2} &= \frac{\lambda^2}{2} \left[ m_{H_1}^2 - m_\Sigma^2 + A_\lambda + B_\Sigma^2 - 2A_\lambda B_\Sigma \right].
\end{align*}
\tag{B9}
\]

The coefficients for $\Sigma_1$ can be obtained from eqs. (B9) by replacing $\lambda^2 \to \frac{2}{9} \lambda^2$. For the color triplets $H_{C_\alpha}$ ($\alpha$ being the color index) we have, assuming $M_{H_C} > M_V$,

\[
\begin{align*}
    a &= B_H, \quad b = (m_{H_1}^2 + m_{H_2}^2)/2, \\
    b_{H_1} &= b_{H_2} = \lambda^2, \\
    c_{H_1} &= c_{H_2} = \frac{\lambda^2}{2} [2A_\lambda - B_H],
\end{align*}
\]

\(^8\) The coefficients $a - d_i$ also depend on the mass parameters of the superpotential (such as the $\mu$ parameter), but this dependence has to be discarded since we are only interested in the corrections to the SSB parameters.
\[ d_{H_1} = \frac{\lambda^2}{2}[m_{H_1}^2 - m_{\tilde{H}_1}^2 + A^2_H + B^2_H - 2A\lambda B_H], \]
\[ d_{H_2} = \frac{\lambda^2}{2}[m_{H_2}^2 - m_{\tilde{H}_2}^2 + A^2_H + B^2_H - 2A\lambda B_H], \]
\[ b_{U_\alpha} = b_{Q_\beta} = b_E = h_i^2, \quad \beta \neq \alpha, \]
\[ c_{U_\alpha} = c_{Q_\beta} = c_E = \frac{h_i^2}{2}[2A_i - B_H], \]
\[ d_{U_\alpha} = d_{Q_\beta} = d_E = \frac{h_i^2}{2}[m_{10}^2 - m_{\tilde{H}_1}^2 + A^2_i + B^2_H - 2A_i B_H]. \] (B10)

The rest of the heavy fields decouple from the light scalar fields (gauge contributions have not been considered).

There are also one-loop corrections coming from the wave-function renormalization constants that cannot be obtained from the one-loop effective potential. These corrections have to be calculated from the explicit one-loop diagrams, and give a contribution to the Higgs soft masses

\[ m_{H_i}^2(Q) = m_{H_i}^2(Q) + \frac{\lambda^2}{8\pi^2}m_{H_i}^2 \left[ \frac{3}{4} \ln \frac{M_{\Sigma_3}^2}{Q^2} + \frac{3}{20} \ln \frac{M_{\Sigma_1}^2}{Q^2} + \frac{3}{2} \ln \frac{M_{H_C}^2}{Q^2} - \frac{12}{5} \right], \] (B11)

and to the sleptons and squarks soft masses

\[ m_{i}^2(Q) = m_{10}^2(Q) + \frac{n_i h_i^2}{16\pi^2}m_{10}^2 \left[ \ln \frac{M_{H_C}^2}{Q^2} - 1 \right], \] (B12)

where \( n_i = 1, 2, 3 \) for \( i = U, Q, E \), respectively. Inserting eqs. (B9) and (B11) in eqs. (B6)–(B8) and incorporating eqs. (B11) and (B12), we get the one-loop matching conditions at \( M_G \)

\[ m_{H_i}^2(M_G) = m_{H_i}^2(M_G) + \frac{\lambda^2}{4\pi^2}(m_{H_1}^2 + m_{H_2}^2 + m_{\Sigma}^2 + A^2_H) \left[ \frac{3}{4} \ln \frac{M_{\Sigma_3}^2}{M_G} + \frac{3}{20} \ln \frac{M_{\Sigma_1}^2}{M_G} + \frac{3}{2} \ln \frac{M_{H_C}^2}{M_G} - \frac{6}{5} \right] + \frac{\lambda^2}{4\pi^2} \left[ \frac{9}{10}(m_{\Sigma}^2 + A\lambda B_{\Sigma}) + \frac{3}{4}(m_{H_1}^2 + m_{H_2}^2 + 2A\lambda B_H) \right], \] (B13)

\[ m_i^2(M_G) = m_{10}^2(M_G) + \frac{n_i h_i^2}{8\pi^2}(2m_{10}^2 + m_{H_2}^2 + A^2_i) \left[ \ln \frac{M_{H_C}^2}{M_G} - \frac{1}{2} \right] + \frac{n_i h_i^2}{16\pi^2}(m_{H_1}^2 + m_{H_2}^2 + 2A_i B_H), \quad n_i = 1, 2, 3 \] for \( i = U, Q, E \). (B14)

Eq. (B13) corresponds to the one-loop corrected eq. (5c). For \( \mu_{\Sigma} \ll M_V \approx 2 \times 10^{16} \) GeV, the logarithmic term of eq. (B13) can lead to a large deviation from eq. (5c). For example, taking \( M_G = M_V \sim M_{H_C} \sim 10^3 \mu_{\Sigma}, \lambda \approx 1 \) and assuming \( m_0^2 = A^2 = m_{0}^2 \) at tree-level, we have \( m_{H_i}^2(M_G) \approx 0.6m_0^2 \). The non-logarithmic terms of eq. (B13) are smaller (~10%) and tend to cancel out for equal SSB parameters. It is interesting to note that in the regions of the MSSM parameter space where the EWSB requires a high degree of fine tuning \[ [34], a 10% GUT correction to the soft Higgs masses can destabilize the minimum. The GUT threshold corrections to \( [53] \) [given in eq. (B14)] are typically small since \( M_{H_C} \) is forced to be close to \( M_V \) \( (M_{H_C} \approx M_V \) from proton decay and \( M_{H_C} \approx 2M_V \) to stay in the perturbative
regime \[\boxdot\]). Nevertheless, it is important to stress that, unlike other GUT effects, such corrections contribute to the mass-splitting between light fields embedded in the same SU(5) representation.

Finally, the one-loop contribution to the trilinear term can be computed in the same way, and is given by

$$\Delta A_i = \frac{\lambda^2}{4\pi^2} A_\lambda \left[ \frac{3}{4} \ln \frac{M_{\Sigma_1}}{Q} + \frac{3}{20} \ln \frac{M_{\Sigma_2}}{Q} + \frac{3}{2} \ln \frac{M_{HC}}{Q} - \frac{6}{5} \right]$$

$$+ \frac{n_i h_i^2}{8\pi^2} A_t \left[ \ln \frac{M_{HC}}{Q} - \frac{1}{2} \right],$$

where \(n_i = 3, 2, 3\) for \(i = t, b, \tau\).
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TABLE I. Patterns of non-universality in the minimal SU(5) model. $\lambda \approx 0$ could lead to a too rapid proton decay via dimension-five operators.

| $\tan \beta$ range            | Pattern ($\lambda \approx 1$) | Pattern ($\lambda \approx 0$) | choice |
|-------------------------------|-------------------------------|-------------------------------|--------|
| low ($1 - 2$)                 | (i)                           | (iii)                         | (a)    |
| intermediate and large ($\gtrsim 50$) | (ii)                         | (iv)                          | (b)    |
TABLE II. The low-energy spectrum is calculated for $m_t^{pole} = 160$ GeV and $\tan \beta = 1.25$, assuming universality at the listed scale. We list, respectively, the universal gaugino and scalar masses, the $D$-term parameter and the trilinear parameter, along with the weak-scale predictions for the $\mu$ parameter, the gluino, chargino and LSP masses, the LSP eigenvector bino, wino and two Higgsino components, the heavier neutralino masses, the light and heavy Higgs boson masses, first and second family scalar quark $\tilde{q}$ and lepton $\tilde{l}$ masses, the $t$ and $b$-scalar masses and mixing $\tilde{t}_1 = -\sin \theta \tilde{t}_R + \cos \theta \tilde{t}_L$, and third family scalar lepton masses. $\lambda = 1$ (and $\lambda' = 0.1$) at $M_G$. The $\sim$ implies a rough average of the relevant masses. Equal values of the model building [low-energy] parameters eq. (7) [eq. (8)] are used in the first and second [first and third] columns. The first and second columns correspond to Fig. 1. All masses are in GeV.

| scale         | $M_P$ | $M_G$ | $M_G$ |
|---------------|-------|-------|-------|
| $M_{1/2}$     | 165   | 165   | 180   |
| $m_0$         | 987   | 987   | 991   |
| $M_D^2$       | 0     | 0     | 0     |
| $A_0/m_0$     | $-2$  | $-2$  | $-0.64$ |
| $\mu$         | $-1755$ | $-2038$ | $-1988$ |
| $M_3$         | 499   | 460   | 499   |
| $m_{\chi_1^+, 2}$ | 151, $-1758$ | 138, $-2041$ | 150, $-1991$ |
| $m_{\chi_1^0}$ | 76    | 69    | 75    |
| $a_{11}$      | $-0.9988$ | $-0.9990$ | $-0.9990$ |
| $a_{12}$      | 0.0256 | 0.0243 | 0.0228 |
| $a_{13}$      | 0.0420 | 0.0368 | 0.0375 |
| $a_{14}$      | $-0.0055$ | $-0.0047$ | $-0.0048$ |
| $m_{\chi_2^0}$ | 151   | 138   | 150   |
| $m_{\chi_{3, 4}}$ | 1755, $-1759$ | 2038, $-2041$ | 1988, $-1992$ |
| $m_{h^0}$     | 75    | 75    | 79    |
| $m_{H^0, A^0, H^+}$ | $\sim 2435$ | $\sim 2945$ | $\sim 2895$ |
| $m_{\tilde{q}}$ | $\sim 1080$ | $\sim 1060$ | $\sim 1078$ |
| $m_{\tilde{t}}$ | $\sim 997$ | $\sim 992$ | $\sim 997$ |
| $m_{\tilde{t}_1, 2}$ | 517, 811 | 250, 858 | 395, 887 |
| $\sin \theta_{\tilde{t}}$ | 0.87   | 0.93   | 0.94   |
| $m_{\tilde{b}_1, 2}$ | 735, 1075 | 794, 1059 | 834, 1075 |
| $\sin \theta_{\tilde{b}}$ | 0.01   | 0.02   | 0.02   |
| $m_{\tilde{\tau}_1, 2, \tilde{\nu}_\tau}$ | 729, $\sim 999$ | 986, $\sim 995$ | 993, $\sim 999$ |
TABLE III. Same as in Table II except it corresponds to Fig. 2.

| Scale | $M_p$ | $M_G$ | $M_G$ |
|-------|-------|-------|-------|
| $M_{1/2}$ | 400 | 400 | 436 |
| $m_0$ | 0 | 0 | 219 |
| $M_D^0$ | 0 | 0 | 0 |
| $A_0/m_0$ | 0 | 0 | 0.67 |
| $\mu$ | $-1558$ | $-1348$ | $-1539$ |
| $M_G$ | 1155 | 1065 | 1155 |
| $m_{\chi_{1,2}^+}$ | 362, $-1562$ | 333, $-1352$ | 362, $-1542$ |
| $m_{\chi_1^0}$ | 178 | 163 | 178 |
| $a_{11}$ | 0.9996 | 0.9995 | 0.9996 |
| $a_{12}$ | $-0.0099$ | $-0.0122$ | $-0.0100$ |
| $a_{13}$ | $-0.0256$ | $-0.0293$ | $-0.0258$ |
| $a_{14}$ | 0.0036 | 0.0042 | 0.0036 |
| $m_{\chi_2^0}$ | 362 | 333 | 362 |
| $m_{\chi_{3,4}^0}$ | $1558, -1563$ | $1348, -1353$ | $1539, -1544$ |
| $m_{h^0}$ | 64 | 62 | 63 |
| $m_{H^0, A^0, H^+}$ | $\sim 2097$ | $\sim 1808$ | $\sim 2081$ |
| $m_{\tilde{\ell}}$ | $1040 - 1090$ | $937 - 976$ | $1038 - 1079$ |
| $m_{\tilde{\ell}^i}$ | $318 - 384$ | $157 - 290$ | $278 - 384$ |
| $m_{\tilde{\ell}^1,2}$ | $822, 1001$ | $754, 907$ | $814, 993$ |
| $\sin \theta_{\tilde{\ell}}$ | 0.97 | 0.97 | 0.98 |
| $m_{\tilde{b}_{1,2}}$ | $982, 1037$ | $888, 937$ | $974, 1038$ |
| $\sin \theta_{\tilde{b}}$ | 0.01 | 0.01 | 0.01 |
| $m_{\tilde{\tau}_{1,2}, \tilde{\nu}_e}$ | $309, \sim 382$ | $157, \sim 289$ | $277, \sim 382$ |
|        | $M_P$  | $M_{G}$ | $M_{G}$ |
|--------|--------|---------|---------|
| $M_{1/2}$ | 175    | 175     | 191     |
| $m_0$   | 375    | 375     | 387     |
| $M_D^2$ | 0      | 0       | 0       |
| $A_0/m_0$ | 1      | 1       | 0.73    |
| $\mu$   | 969    | 946     | 992     |
| $M_3$   | 527    | 486     | 527     |
| $m_{\chi_1^+, \chi_2^0}$ | 149, 977 | 136, 953 | 149, 1000 |
| $m_{\chi_1^0}$  | 77     | 70      | 77      |
| $a_{11}$ | $-0.7025$ | $-0.7024$ | $-0.7028$ |
| $a_{12}$ | $-0.0718$ | $-0.0726$ | $-0.0700$ |
| $a_{13}$ | 0.7068  | 0.7067  | 0.7068  |
| $a_{14}$ | 0.0423  | 0.0436  | 0.0411  |
| $m_{\chi_2^0}$ | 149    | 136     | 149     |
| $m_{\chi_3, \chi_4}$ | $-969, 979$ | $-946, 956$ | $-992, 1002$ |
| $m_{H^0, A^0, H^+}$ | 64     | 57      | 66      |
| $m_{\tilde{q}}$ | $\sim 1363$ | $\sim 1327$ | $\sim 1392$ |
| $m_{\tilde{l}}$ | $\sim 610$ | $\sim 570$ | $\sim 607$ |
| $m_{\tilde{l}_1, \tilde{l}_2}$ | 189, 631 | 145, 613 | 202, 644 |
| $\sin \theta_{\tilde{l}}$ | 0.80   | 0.81    | 0.82    |
| $m_{\tilde{b}_1, \tilde{b}_2}$ | 498, 601 | 482, 565 | 518, 601 |
| $\sin \theta_{\tilde{b}}$ | 0.06   | 0.07    | 0.07    |
| $m_{\tilde{t}_{1, 2}, \tilde{e}_r}$ | 340, $\sim 410$ | 381, $\sim 395$ | 394, $\sim 410$ |
TABLE V. Same as in Table II except $m_t^{pole} = 180$ GeV, $\tan \beta = 42$, and it corresponds to Fig. 4. The last two columns list scenarios with non-vanishing $D$-terms, e.g., in SU(5) $\times$ U(1). The latter two also provides a crude approximation of the minimal SO(10) scenario. Note that $M^2_D \neq 0$ splits $\tilde{q}, \tilde{l}$, etc. according to their SU(5) embedding. The value of $M^2_D$ which is used in the last column is the minimal value still consistent with EWSB for the given set of parameters.

| scale | $M_P$ | $M_G$ | $M_G$ | $M_P$ | $M_P$ |
|-------|-------|-------|-------|-------|-------|
| $M_{1/2}$ | 89 | 89 | 96 | 89 | 89 |
| $m_0$ | 977 | 977 | 978 | 977 | 977 |
| $M^2_D$ | 0 | 0 | 0 | $+0.16m_0^2$ | $-0.05m_0^2$ |
| $A_0/m_0$ | 0 | 0 | 0.07 | 0 | 0 |
| $\mu$ | $-645$ | $-214$ | $-225$ | $-849$ | $-563$ |
| $M_{\tilde{g}}$ | 280 | 259 | 280 | 280 | 280 |
| $m_{\chi^+_{1,2}}$ | 80, $-646$ | 74, $-215$ | 80, $-226$ | 80, $-849$ | 80, $-563$ |
| $m_{\chi^0_1}$ | 40 | 35 | 38 | 40 | 40 |
| $a_{11}$ | 0.9976 | 0.9709 | 0.9739 | 0.9986 | 0.9968 |
| $a_{12}$ | 0.0107 | 0.1408 | 0.1279 | 0.0049 | 0.0153 |
| $a_{13}$ | $-0.0469$ | $-0.1221$ | $-0.1162$ | $-0.0362$ | $-0.0532$ |
| $a_{14}$ | 0.0504 | 0.1504 | 0.1452 | 0.0379 | 0.0582 |
| $m_{\chi^0_2}$ | 79 | 65 | 71 | 79 | 78 |
| $m_{\chi^0_{3,4}}$ | $-651, 652$ | $-230, 239$ | $-240, 249$ | $-854, 854$ | $-569, 571$ |
| $m_{\tilde{b}^0}$ | 114 | 113 | 114 | 114 | 114 |
| $m_{H^0, A^0, H^+}$ | $\sim 492$ | $\sim 567$ | $\sim 568$ | $\sim 920$ | $\sim 220$ |
| $m_{\tilde{q}}$ | $\sim 1007$ | $\sim 1000$ | $\sim 1007$ | $\sim 1080, 744$ | $\sim 980, 1077$ |
| $m_{\tilde{l}}$ | $\sim 979$ | $\sim 977$ | $\sim 979$ | $\sim 715, 1055$ | $\sim 1055, 954$ |
| $m_{\tilde{\ell}_{1,2}}$ | 691, 790 | 598, 731 | 601, 737 | 795, 881 | 653, 757 |
| $\sin \theta_{\tilde{t}}$ | 0.96 | 0.98 | 0.97 | 0.97 | 0.96 |
| $m_{\tilde{b}_{1,2}}$ | 743, 869 | 708, 814 | 712, 818 | 482, 875 | 725, 939 |
| $\sin \theta_{\tilde{b}}$ | 0.44 | 0.16 | 0.17 | 0.98 | 0.20 |
| $m_{\tilde{\tau}_{1,2}, \nu_\tau}$ | $818, \sim 910$ | 803, $\sim 869$ | 804, $\sim 897$ | $\sim 592, 919$ | 793, $\sim 985$ |
FIGURES

FIG. 1. Evolution of the soft parameters \( m_i^2 \) corresponding to the two Higgs doublets \( H_1 \) and \( H_2 \) and third family scalar quark fields \( Q, U \) and \( D \) [assuming minimal SU(5)] when \( M_P - M_G \) evolution is considered (solid lines) and neglected (dashed lines), plotted vs. the logarithm of the energy scale. The universal SSB parameters (taken at \( M_P \) or \( M_G \), respectively) are \( M_{1/2} = 165 \) GeV, \( m_0 = -\frac{1}{2}A_0 = 987 \) GeV, and choice (a): \( m_t^{pole} = 160 \) GeV and \( \tan \beta = 1.25 \). Also, in the SU(5) case we take \( \lambda(M_G) = 1 \) and \( \lambda'(M_G) = 0.1 \).

FIG. 2. Same as in Fig. 1 except \( M_{1/2} = 400 \) GeV and the no-scale assumption \( m_0 = A_0 = 0 \).

FIG. 3. Same as in Fig. 1 except \( M_{1/2} = 175 \) GeV, \( m_0 = A_0 = 375 \) GeV and \( \lambda(M_G) = \lambda'(M_G) = 0.1 \).

FIG. 4. Same as in Fig. 1 except \( M_{1/2} = 89 \) GeV, \( m_0 = 977 \) GeV, \( A_0 = 0 \), and choice (b): \( m_t^{pole} = 180 \) GeV and \( \tan \beta = 42 \).

FIG. 5. The prediction for the Higgsino mass parameter \( \mu \) (in GeV) is compared assuming universality at \( M_G \) and at \( M_P \) for (a) \( m_t^{pole} = 160 \) GeV, \( \tan \beta = 1.25 \) and for (b) \( m_t^{pole} = 180 \) GeV, \( \tan \beta = 42 \) (note the different scales). \( \lambda(M_G) = 1, \lambda'(M_G) = 0.1 \), and the initial values for \( m_0, A_0 \) and \( M_{1/2} \) and the sign of \( \mu \) are picked at random (see above).

FIG. 6. The \( \mu \) parameter prediction (in GeV) in a sample of Monte Carlo calculations for \( m_t^{pole} = 160 \) GeV and \( \tan \beta = 1.25 \) and assuming (i) universality at \( M_G \) and (ii) universality at \( M_P \) and \( \lambda(M_G) = 1, \lambda'(M_G) = 0.1 \).

FIG. 7. Same as in Fig. 6 except \( m_t^{pole} = 180 \) GeV and \( \tan \beta = 42 \).

FIG. 8. Same as in Fig. 6 except for the light CP-even Higgs boson mass \( m_{h^0} \) prediction.

FIG. 9. Same as in Fig. 7 except for the light CP-even Higgs boson mass \( m_{h^0} \) prediction.

FIG. 10. Same as Fig. 5 except the prediction for the light \( t \)-scalar mass \( m_{\tilde{t}_1} \).

FIG. 11. Scatter plot of the light chargino \( \chi_1^+ \) vs. the light \( t \)-scalar \( \tilde{t}_1 \) masses (in GeV) within the allowed parameter space (see above) and for \( m_t^{pole} = 160 \) GeV and \( \tan \beta = 1.25 \). Filled triangles [circles] correspond to universality at \( M_P \) [\( M_G \)] and \( \lambda(M_G) = 1, \lambda'(M_G) = 0.1 \).

FIG. 12. Same as in Fig. 11 except the light \( \tilde{t}_1 \) vs. heavy \( \tilde{t}_2 \) masses.
FIG. 13. Same as in Fig. 11 except the LSP mass $m_{\chi_1^0}$ (in GeV) vs. its Higgsino fraction $a_{13}$ and for $m_t^{pole} = 180$ GeV and $\tan \beta = 42$. 
Non-universal GUT corrections to the soft terms and their implications in supergravity models

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Abstract

Potentially large non-universal corrections to the soft supersymmetry breaking parameters arise from their evolution between the Planck and the grand-unification scales. We detail typical patterns of non-universality in GUT models, as well as elaborate on their propagation to the weak scale and on their low-energy implications. Possible corrections to the different scalar quark and lepton masses and the Higgs and the gaugino-Higgsino sector parameters are described in detail, and new allowed regions of the parameter space are pointed out. In particular, the patterns studied often lead to heavier Higgsinos and t-scalar. One-loop GUT threshold corrections to the soft parameters are also discussed and shown to be important.

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I. INTRODUCTION

It was recently pointed out [1] that in minimal supergravity type models [2], the model-dependent renormalization of the soft mass parameters between the Planck scale \( M_P \approx 2 \times 10^{18} \) GeV and the grand-unification scale \( M_G \approx 2 \times 10^{16} \) GeV, can significantly modify the boundary conditions for these parameters at \( M_G \). In particular, contrary to the standard working assumption of universality at \( M_G \) (e.g., see Ref. [3,4]), specific patterns of non-universality are induced at \( M_G \) even when the soft parameters have universal boundary conditions at \( M_P \). Hence, low-energy predictions and constraints in the minimal supersymmetric extension of the standard model (MSSM) [2] are subject to model-dependent modifications. Regions of the parameter space that are of interest to present and future collider experiments may change and/or be smeared, requiring one to assign uncertainties to the MSSM low-energy predictions. On the other hand, the discovery of superpartner and Higgs particles could provide new and exciting hints on the physics near the Planck scale.

In general, as a result of the GUT effects the soft masses can be different at \( M_G \) for fields which are in different representations of the unified gauge group. This is due to (i) the different charges and (ii) the different Yukawa interactions of the different fields. Indeed, if one assumes that the corrections are proportional to only \( (\alpha_G/\pi) \ln(M_P/M_G) \sim 5\% \), then the effects are negligible. However, the above argument does not hold, regardless of the size of the Yukawa interactions, if large representations are present. In such a case, the unified coupling \( \alpha_G \) is multiplied by a large number and the effect of Planck to GUT scale evolution can be significant [5,1]. Furthermore, in Ref. [1] it is shown that top and bottom Yukawa couplings, as well as GUT-scale Yukawa couplings, which have to be large to avoid a too rapid proton decay [6], can also induce large deviations from universality at \( M_G \). In particular, the soft supersymmetry breaking (SSB) parameters related to the light Higgs fields can be significantly different from those related to the matter fields due to the different Yukawa interactions. (It should be emphasized that universality of the soft mass parameters was assumed, but at \( M_P \).) Moreover, once non-universality exists at \( M_G \), and if the rank of the gauge group is higher than that of the standard model (SM) group, then non-vanishing \( D \)-terms [7] exist [8]. The \( D \)-terms, which are charge dependent, induce a secondary breaking of universality.

Given the above, one may then question the motivation to assume universality at any scale: Relaxation of that assumption is subject to strong constraints from flavor changing neutral currents (FCNC) [9]. Also, any predictive power is lost. On the other hand, the identification of the universality scale with \( M_G \) is convenient but ad hoc, and the consequences of its relaxation need exploration. Allowing \( M_P - M_G \) renormalization of the parameters leads to restricted patterns of non-universality at \( M_G \). In particular, the super-GIM mechanism suppressing FCNC in the MSSM [10] need not be altered if Yukawa couplings of the first and second families remain negligible at all scales. In addition, the predictive power is altered but not lost. In SU(5) models, one has to introduce at least two more Yukawa couplings, e.g., SO(10), a new parameter is needed to account for the magnitude and sign of the \( D \)-terms. The deviations from universality at \( M_G \) are not arbitrary but are calculable in terms of the new parameters. However, the interference between the different corrections, e.g., those from Yukawa interactions and those from \( D \)-terms (and in particular, if higher order gravitational corrections are not negligible), can render it difficult
to disentangle traces of the high-scale theory in the low-energy physics.

Below, we will again resort to the assumption of universality of the SSB parameters at $M_P$ (the obvious choice in minimal supergravity theories [11]), i.e., a common scalar mass $m_0$, a common gaugino mass $M_{1/2}$, and common dimension-one trilinear and bilinear scalar couplings $A_0$ and $B_0$. (In fact, our discussion is independent of any assumptions regarding universality of the bilinear couplings $B_i$.) We will then assume a grand unified theory (GUT) between $M_P$ and $M_G$. (The GUT scale is determined by gauge coupling unification.) Note that the universality of the gaugino masses above $M_G$ is trivial if the GUT group is simple. For simplicity, we assume in most of our calculations the minimal SU(5) model [12,10]. However, extended models, including models with non-vanishing $D$-terms, are studied qualitatively. The effective theory below $M_G$ is assumed to be the MSSM with the appropriate $M_G$ matching conditions. We then study the different patterns of non-universality which are induced at $M_G$, their propagation to the weak scale, low-energy implications and consequences for model building. In particular, we will examine two points in the parameter space,

(a) $m_t^{\text{pole}} = 160$ GeV, $\tan \beta = 1.25$;
(b) $m_t^{\text{pole}} = 180$ GeV, $\tan \beta = 42$;

for the $t$-quark pole mass $m_t^{\text{pole}}$ and for $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$. We choose these points because of the large $t$ and $b$ quark Yukawa couplings, $i.e., h_t \approx 1 \gg h_b$ and $1 \gg h_t \gg h_b$, respectively. Also, because of the large Yukawa couplings, points (a) and (b) are consistent with bottom-tau unification and with minimal SU(5), e.g., see Ref. [13]. For intermediate values of $\tan \beta$ the effects are a superposition of those for points (a) and (b).

It was recently suggested that non-universality of the soft terms is a typical signature of some string models [14]. In string models one often has a direct unification at the string scale $M_S < M_P$, i.e., $M_G = M_S$, and the universal or non-universal boundary conditions are derived from the string theory at $M_S$. However, the string scale is typically $M_S \approx 5.2 \times 10^{17}$ GeV [15], and the relation $M_G = M_S$ fails for the MSSM. String inspired non-universality is studied in Ref. [16]. In particular, Kobayashi et al. suggest a solution to the $M_S / M_G \approx 20$ discrepancy. If one assumes that the string theory leads to an intermediate GUT, then our analysis applies, but the results should be scaled down by $\sim \ln \left( M_S / M_P \right)$.

We previously found [1] that some low-energy parameters such as the $\mu$ parameter and the $t$-scalar mass, can be significantly modified while others, e.g., the SM-like Higgs boson mass, are nearly invariant under minimal SU(5) type corrections. We also pointed out the $\tan \beta$-dependent modification of the allowed parameter space and of correlations between different observables. Here, we will further elaborate on the above observations and expand our previous work. We review the possible patterns of non-universality and compare their generation via radiative corrections to the radiative symmetry breaking mechanism in section II. We also demonstrate that the same rule of thumb holds for the $M_P - M_G$ and $M_G - M_Z$ evolutions of the soft parameters and their resulting hierarchy: It is determined by the competition between the gauge charge and Yukawa interactions of the relevant field and

\textsuperscript{1}That choice maximizes the effects. Universality at a lower scale would lead to lesser but similar effects. We do not consider gravitational effects other than those which induce the universal soft terms. Higher order gravitational effects could add to the uncertainty.
by the asymptotic freedom of the model. In section III we describe our numerical routines and discuss the weak-scale phenomena. In particular, we elaborate on the propagation of non-universal corrections from $M_G$ to the weak scale; on the distinction between physical (bottom-up approach) and model-building (up-down approach) parameters; and on the implications to the first and second family scalars, the $\mu$ parameter, the Higgs sector, and third family scalars. Future observations of the signatures described could support the existence of an intermediate GUT. Their absence, on the other hand, could indicate direct unification, but also interference between different effects or small values for the new parameters. Our conclusions are given in Section IV. For completeness, the minimal SU(5) model is defined in Appendix A. Threshold corrections (due to the ambiguity in $M_G$) in that model are described and discussed in Appendix B.

Rather than restrict oneself to the patterns described in section II, one could adopt a more phenomenological approach to non-universality by postulating certain universality breaking patterns. For example, Dimopoulos and Georgi considered different boundary conditions for the matter and Higgs bosons [10]. Similar approaches were adopted recently by several authors. In Ref. [17] a split at $M_G$ between the light Higgs and matter fields is considered [but mainly in the context of SO(10) scenarios]. In Ref. [18] patterns of non-universality desired in certain SO(10) models are studied. A general discussion of non-universal scalar potentials at $M_G$ was recently given in Ref. [19]. Other recent studies of non-universality were carried out in Ref. [20]. Where relevant, the conclusions of these authors agree with ours.

II. PATTERNS OF NON-UNIVERSALITY AT $M_G$

Before discussing the way in which the Planck to GUT scale evolution of the soft mass parameters can induce large deviations from universality at $M_G$, it is useful to recall the more familiar way in which GUT to weak scale evolution can render the weak-scale scalar potential consistent with a spontaneously broken SU(2)$_L \times$U(1)$_Y$ symmetry [21] [often called the radiative symmetry breaking (RSB) mechanism]. In both cases large Yukawa couplings play a similar role and lead to similar behavior of the SSB parameters. The RSB mechanism is easily understood if one writes the approximate renormalization group equations (RGEs) of the soft scalar masses [22]:

$$\frac{d}{d \ln Q} \begin{bmatrix} m^2_{H_R} \\ m^2_{H_L} \\ m^2_T \\ m^2_Q \end{bmatrix} = \frac{h_i^2}{8\pi^2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 3 & 3 \\ 0 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} m^2_{H_R} \\ m^2_{H_L} \\ m^2_T \\ m^2_Q \end{bmatrix} - \frac{2g_3^2}{3\pi^2} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

where $Q$ is the renormalization scale, $g_3$ is the SU(3)$_c$ coupling and

$$-\mathcal{L}_{soft} = \sum_i m_i^2 |\phi_i|^2 + [B \mu H_1 H_2 + A_i h_j Q H_j U + \frac{1}{2} \sum \tilde{\lambda} \lambda \lambda \lambda + h.c.],$$

with $i$ ($\lambda$) summing over all scalars (gauginos) and $H_1, H_2, U = i_R, Q = (i_L, b_L)$, and $\tilde{g}$ are the down and up type Higgs doublets, right-handed $t$-scalar, left-handed scalar-quark doublet and the gluino, respectively. (Below, we will also refer to the $t$-scalar, scalar quarks,
etc. as stop, squarks, etc.) \( \mu \) is the mass parameter in the MSSM superpotential, eq. (A5). For simplicity we omitted in (1) \( A_i \)-term contributions and neglected all terms aside from the QCD and \( h_t \) Yukawa terms which typically dominate the evolution. (As we discuss below, in some cases the \( h_t \) term can be equivalent to the \( h_t \) term and has to be considered as well.) The stop masses grow with the gluino mass \( M_{\tilde{g}}(Q_1) = [\alpha_3(Q_1)/\alpha_3(Q_2)]M_{\tilde{g}}(Q_2) \), a growth which is subject to some slow-down due to the Yukawa term. On the other hand, the Yukawa term diminishes \( m_{H_2}^2 \) to negative values at the weak scale such that the sum \( m_{H_1}^2 + \mu^2 \sim \mathcal{O}(M_Z^2) \). \( m_{H_1}^2 \) is not renormalized in this approximation.) The global minimum of the weak-scale Higgs potential is consistent with electroweak symmetry breaking (EWSB) provided that \((m_{H_1}^2 + \mu)(m_{H_2}^2 + \mu^2) \leq B^2 \mu^2 \) (and that \( m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 \geq 2|B\mu| \), which is indeed the situation for \( \mathcal{O}(M_Z^2) \) (possibly negative) values of \( m_{H_2}^2 + \mu^2 \). Also, there are no tachions in the theory. The situation is, of course, more complicated when including all terms, but is qualitatively similar to the above approximation.

For future reference, observe that (1) is independent of either \( \mu \) or \( B \). The decoupling of \( \mu \) from the SSB parameters and of \( B \) from all other SSB parameters holds in general. This enables one to rewrite the EWSB minimization conditions [21] as

\[
\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2, \tag{3a}
\]

\[
B\mu = -\frac{1}{2} \sin 2\beta \left[ m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 \right]. \tag{3b}
\]

Hence, \( B_0 \) can be traded for \( \tan \beta \) and \( \mu \) is predicted as a function of the SSB parameters and of \( \beta \). (Triviality limits give \( \tan \beta > 1 \) for \( m_{\tilde{t}}^{pole} \approx 140 \text{ GeV} \).

To summarize, the gauge term dominates the \( M_G - M_Z \) evolution for the colored scalar soft masses (but their spectrum also bears traces of their Yukawa interactions) while the Yukawa term dictates \( m_{H_2}^2 \) evolution (in practice, it dominates over the \( H_2 \) weak and hypercharge gauge terms). Thus, a large hierarchy is generated at the weak scale, even when assuming universal scalar masses as the GUT scale boundary condition for (1). Let us now write in a similar fashion the RGEs in the minimal SU(5) model, which is assumed to dictate the \( M_F - M_G \) evolution [the complete RGEs in that model are given in [1] and in Appendix A]:

\[
\frac{d}{d\ln Q} \begin{bmatrix} m_{H_1}^2 \\ m_{H_2}^2 \\ m_{10}^2 \end{bmatrix} = \frac{h_t^2}{8\pi^2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 6 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} m_{H_1}^2 \\ m_{H_2}^2 \\ m_{10}^2 \end{bmatrix} + \frac{3\lambda^2}{5\pi^2} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_{H_1}^2 \\ m_{H_2}^2 \\ m_{10}^2 \end{bmatrix} - \frac{g_{\tilde{t}}^2}{5\pi^2} M_5^2 \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}, \tag{4}
\]

where \( m_{H_1} \) and \( m_{H_2} \) are the soft masses of the \( \tilde{5} \) and \( 5 \) SU(5) Higgs bosons (that contain \( H_1 \) and \( H_2 \), respectively); \( m_{10} \) and \( m_{10} \) (that we use later) are the soft masses of the \( 10 \) (that contains \( Q \) and \( U \)) and \( 5 \) matter superfields; \( g_{\tilde{t}} \) and \( M_5 \) are the SU(5) gauge coupling and gaugino mass, and the Yukawa coupling \( \lambda \) is defined in Appendix A. Note that the introduction of larger representations typically implies larger numerical coefficients. The GUT effects in the minimal SU(5) model can be read from (4) and are described by the following patterns: If \( h_t \approx \lambda \approx 1 \) at \( M_G \) (which is consistent with these couplings being \( \lesssim 2 \) near \( M_F \), so the one-loop approximation is reasonable) then the \( \lambda \) term diminishes the
\( \mathcal{H}_1 \) and \( \mathcal{H}_2 \) squared soft masses significantly while the \( h_i \) term lifts their degeneracy, further diminishing \( m_{h_i}^2 \). The latter also diminishes \( m_{10}^2 \). In comparison to (1), the additional Yukawa term, the larger numerical coefficients and \( \alpha_G \ll \alpha_3(M_Z) \) compensate for the shorter evolution interval, leading to a similar pattern (aside from the effect on \( \mathcal{H}_i \)), but at \( M_G \) rather than at \( M_Z \), and we observe the pattern

\[
(i) \quad m_5^2 \geq m_{10}^2 \geq m_{h_1}^2 \geq m_{h_2}^2.
\]

First and second family scalar masses are renormalized to a very good approximation only by gauge interactions, and hence are slightly heavier than the third family \( m_5^2 \) (which is renormalized, in practice, by the \( h_i \) term). More importantly, in this scenario the gauge interactions do not lift the degeneracies between the first and second family SSB parameters and the MSSM super-GIM mechanism is still intact. Pattern (i) would now serve as a non-universal boundary condition to (1). As can be seen in Fig. 1a of Ref. [1] and in Fig. 1 here, \( m_{h_2}^2 \) can be driven to near zero values already at \( M_G \), and in principle, RSB could be achieved for small values of \( h_i \) (this is, of course, irrelevant for \( m_t^{pole} \geq 100 \text{ GeV} \)). Before proceeding, let us stress that the hierarchy between the different SSB parameters indeed depends on the gauge charges and on the size of the Yukawa couplings. However, whether the parameters grow or diminish depends roughly on the ratio \( [g_G^2 M_{1/2}^2]/[\max(h_i^2, \lambda^2) \times m_{10}^2] \).

If the ratio is larger than unity, then typically all the parameters grow with decreasing energy. This is true in general and is seen [for pattern (i)] in Fig. 1b of Ref. [1] [where \( g_G^2 M_{1/2}^2 \gg h_i^2 m_{10}^2 \Rightarrow m_{10}(M_G) > m_5(M_G) \)] and in Fig. 2, here. (This is always the case for the first and second family scalars.) If both, \( h_i \) and \( \lambda \), are large, as in pattern (i), then for small and moderate values of \( M_{1/2}^2 \) only \( m_3^2 \) grows. (Fig. 2 corresponds to the no-scale assumption \( m_0 = A_0 = 0 \).)

The GUT effects leading to pattern (i) are a mismatch of effects due to the large \( \lambda \) and to the large \( h_i \). Different assumptions regarding the Yukawa couplings lead to different patterns. If \( h_i(M_G) \ll 1 \) (i.e., \( m_t^{pole} \lesssim 180 \text{ GeV} \) and \( \tan \beta \gg 1 \)) and also \( h_i(M_G) \) is small (i.e., \( \tan \beta \lesssim 40 \)), then a simpler pattern arises,

\[
(ii) \quad m_{10}^2, m_3^2 > m_{h_1}^2, m_{h_2}^2.
\]

The splitting between the Higgs and matter sectors depends, as before, on the size of \( \lambda \). (We comment on the case \( h_i > h_t \) below.) Regarding \( \lambda \), in the minimal SU(5) model one has \( \lambda = g_G M_{H_c}/M_V \) (see Appendix A). In this model, proton decay non-observation requires that the colored triplet is heavy \( M_{H_c} \gtrsim M_V \) [6] and hence \( \lambda \approx 1 \). Thus, patterns (i) and (ii) completely characterize that model. However, if we ignore the proton decay constraint, assume \( \lambda \ll 1 \) and again take \( h_i \gg h_t \), we find (e.g., see Fig. 3)

\[
(iii) \quad m_5^2, m_{h_1}^2 > m_{10}^2, m_{h_2}^2.
\]

Finally, if all Yukawa couplings are small one has

\[
(iv) \quad m_{10}^2 > m_3^2, m_{h_1}^2, m_{h_2}^2.
\]

It is convenient to relate the different patterns (or GUT effects) to values of \( \tan \beta \). Patterns (i) and (iii) correspond to low \( \tan \beta \approx 1 - 2 \), i.e., choice (a). Patterns (ii) and (iv) correspond to moderate values of \( \tan \beta \). An interesting scenario arises in the special
FIG. 1. Evolution of the soft parameters $m_i^2$ corresponding to the two Higgs doublets $H_1$ and $H_2$ and third family scalar quark fields $Q$, $U$ and $D$ [assuming minimal SU(5)] when $M_P - M_G$ evolution is considered (solid lines) and neglected (dashed lines), plotted vs. the logarithm of the energy scale. The universal SSB parameters (taken at $M_P$ or $M_G$, respectively) are $M_{1/2} = 165$ GeV, $m_0 = -\frac{1}{2} A_0 = 987$ GeV, and choice (a): $m_i^{pole} = 160$ GeV and $\tan \beta = 1.25$. Also, in the SU(5) case we take $\lambda(M_G) = 1$ and $\lambda'(M_G) = 0.1$. 

The case $h_t \approx h_b \approx 1$ (and $\lambda \approx 1$), i.e., large $m_i^{pole} \gtrsim 180$ GeV and large $\tan \beta \approx 50 - 60$. Eq. (4) now has an additional Yukawa term previously neglected: The $h_b$ term (see Appendix A). This again leads to pattern (ii), however, the corrections are now enhanced because the three Yukawa couplings, $\lambda$, $h_t$ and $h_b$, are large, and unless one assumes a small $\lambda$ and a specific mechanism to suppress proton decay (or $M_{1/2} \gg m_0/g_G$), one has a special case of (ii), i.e., $m_0^2 \gg m_5^2$, $m_{10}^2 > m_{H_1}^2 \sim m_{H_2}^2$. That situation is similar to the one in the minimal SO(10) model where the Higgs $5 + \overline{5}$ of SU(5) are embedded in a single 10 of SO(10) (and $10 + \overline{5}$ in a 16). Yukawa unification requires in that case $h_t \approx h_b \approx 1$. Our choice (b) with $h_t(M_G) \sim 0.5$ and $h_b(M_G) \sim 0.3$ corresponds to a moderate version of this scenario. A scenario corresponding to (b) is illustrated in Fig. 4. Of course, in SO(10) the RGEs have slightly different slopes than in our case, and the magnitude of the splittings calculated in SU(5) can only approximate the actual splittings in SO(10). The different patterns and their dependence on $\tan \beta$ are summarized in Table I.

We now turn to examine the situation in some extended GUT models. When considering
extended models one could assume a higher rank group, additional (and/or larger) representations, or both. As an example of the latter, in non-minimal SU(5) (and other) models $\mathcal{H}_1$ and $\mathcal{H}_2$ couple with different strength to the other Higgs superfields [below we refer to this scenario as non-minimal SU(5)]. One could now arrange for a large and arbitrary splitting at $M_G$ between $m_{\mathcal{H}_1}^2$ and $m_{\mathcal{H}_2}^2$, e.g.,

\begin{equation}
    m_5^2 > m_{10}^2 > m_{\mathcal{H}_1}^2 \gg m_{\mathcal{H}_2}^2,
\end{equation}

where $i \neq j = 1, 2$. This can provide a caveat for the general rule that no RSB is possible (assuming universality) for $h_b > h_t$. In fact, we confirmed that the splitting in (v) can be arranged (for $i = 1$) so that RSB is possible in that case. For example, in the SU(5) missing partner model (MPM) [23], the superpotential reads $W = \lambda_1 \mathcal{H}_1 \Sigma(75) \Phi(50) + \lambda_2 \mathcal{H}_2 \Sigma(75) \Phi(50) + \ldots$. Proton decay constraints in that model [24] can be approximated

\begin{table}[h]
\centering
\caption{Patterns of non-universality in the minimal SU(5) model. $\lambda \approx 0$ could lead to a too rapid proton decay via dimension-five operators.}
\begin{tabular}{|c|c|c|c|}
\hline
$\tan \beta$ range & Pattern ($\lambda \approx 1$) & Pattern ($\lambda \approx 0$) & choice \\
\hline
low ($1 - 2$) & (i) & (iii) & (a) \\
intermediate and large ($\gtrsim 50$) & (ii) & (iv) & (b) \\
\hline
\end{tabular}
\end{table}

FIG. 2. Same as in Fig. 1 except $M_{1/2} = 400$ GeV and the no-scale assumption $m_0 = A_0 = 0$. 

\[ \begin{array}{c}
(1000)^2 \\
(800)^2 \\
(600)^2 \\
(400)^2 \\
\end{array} \]

energy scale

\[ \begin{array}{c}
0 \\
-(400)^2 \\
-(600)^2 \\
-(1000)^2 \\

M_z \\
M_G \\
M_p \\
\end{array} \]
as $\lambda_1\lambda_2 \gtrsim 5\eta g_G$ with $\eta \sim M_9/\langle 75 \rangle$ in the range $0.1 \lesssim \eta \lesssim 10$. However, some caution is in order. One has to keep in mind a possible breakdown of perturbation theory when large representations (or a large number of small representations) are present. For example, the MPM is not asymptotically free and the gauge coupling typically diverges below $M_P$.

(Texture models often assume large representations when trying to explain the light fermion spectrum and may not be asymptotically free as well.) In general, one needs to develop non-perturbative techniques in order to calculate the GUT effects to the SSB parameters. In non-asymptotically free but still perturbatively valid models one has $M_5(M_G) \ll M_{1/2}$.

Before turning to discuss higher rank groups, the $M_G$ matching conditions for SU(5) (minimal and non-minimal) models read

$$m_Q^2 = m_U^2 = m_E^2 = m_{10}^2,$$

$$m_D^2 = m_L^2 = m_5^2,$$

$$m_{H_{1,2}}^2 = m_{H_{1,2}}^2,$$

where $Q = (\tilde{t}_L, \tilde{b}_L)$ and $L = (\tilde{\nu}_L, \tilde{\tau}_L)$ are the left-handed scalar quark and lepton doublets, and $U$, $D$ and $E$ are the right-handed $t$, $b$, and $\tau$-scalars, respectively. Similar relations hold
FIG. 4. Same as in Fig. 1 except $M_{1/2} = 89$ GeV, $m_0 = 977$ GeV, $A_0 = 0$, and choice (b): $m_t^{pole} = 180$ GeV and $\tan\beta = 42$.

for the first and second families. As was shown above, in the minimal model, $m_{H_1,2}^2$ both depend on $\lambda$ and are correlated. In non-minimal models, e.g., the MPM, they are independent parameters. Condition (5) applies to the RG-improved tree level SSB parameters$^2$ at $M_G$. However, since not all heavy fields are degenerate at $M_G$, the splitting between the heavy masses induces, at one-loop, a secondary non-universal shift in the soft parameters, regardless of their initial universality [1]. This is similar to threshold corrections to dimensionless couplings, e.g., see Ref. [26,13,24], with the exception that corrections to $m_i^2$ due to boson-fermion mass splittings are now $\sim M_G^2 \ln[(M_G^2 + m_{soft})/M_G^2] \sim m_{soft}^2$ and are not suppressed. One-loop threshold corrections could smear the above patterns, and are discussed for the minimal SU(5) model in Appendix B.

If the rank of the GUT group is higher than that of the SM group [or for that matter, of SU(5)], i.e., higher than four, non-universality at $M_G$ would, in general, trigger non-vanishing $D$-terms [7,8] that could correct the $M_G$ boundary conditions. (The $D$-terms appear at the

$^2$The tree-level SSB parameters shift the GUT-scale vacuum expectation values and redefine $\mu$, $B$ (recall that we are not required to specify these parameters at the high scale), as well as induce the D-terms [25,7,8].
scale at which the rank of the group is reduced.) It is important to stress that had we assumed universality at \( M_G \) then the \( D \)-terms would have vanished leaving the universality assumption intact \([8]\). We consider, as an example, the minimal \( \text{SO}(10) \) model which now depends on an additional parameter – the magnitude (and sign) of the \( D \)-terms \( M_D^2 \), which can be shown to be of the order of the soft mass parameters. In \( \text{SO}(10) \), condition (5) is replaced by \([7,8]\)

\[
m_Q^2 = m_U^2 = m_E^2 = m_{16}^2 + M_D^2 \equiv m_{10}^2,
\]

\[
m_D^2 = m_L^2 = m_{16}^2 - 3M_D^2 \equiv m_5^2,
\]

\[
m_{H_{1,2}}^2 = m_{H_{1,2}}^2 \pm 2M_D^2,
\]

and in the minimal\(^3\) \( \text{SO}(10) \) model \( m_{H_1}^2 = m_{H_2}^2 \equiv m_H^2 \) and typically \( m_6^2 \gg m_{16}^2 \gg m_L^2 \) (recall the very large \( \tan \beta \) case). \( m_{H_1}^2 \) and \( m_{H_2}^2 \) could be split at \( M_G \) according to the sign of \( M_D^2 \). [Note the \( \text{SU}(5) \) invariance of (6).] A situation similar to (6) could arise in a \( \text{SU}(5) \times \text{U}(1) \) model.

Below, we discuss the minimal \( \text{SU}(5) \) model [patterns (i) – (iv)] as well as non-minimal \( \text{SU}(5) \) and minimal \( \text{SO}(10) \)-inspired extensions [pattern (v) and eq. (6), respectively]. Our choices of points (a) and (b) correspond (for \( \lambda \approx 1 \)) to patterns (i) and (ii), respectively. Hereafter, that correspondence is understood when discussing low and large values of \( \tan \beta \). The latter can be used as a crude approximation of the situation in the minimal \( \text{SO}(10) \) model. Intermediate values of \( \tan \beta \) (\( h_t \) and \( h_b \) are both small) are also described by patterns (ii) [and (iv)] and typically the splittings are somewhat diminished.

**III. WEAK-SCALE PHENOMENA**

In the previous section we presented four patterns of \( M_G \) boundary conditions, that define uniquely non-universality in the minimal \( \text{SU}(5) \) model (up to threshold corrections). To analyze their low-energy implications one has to evolve the SSB parameters from \( M_G \) to \( M_Z \). The GUT corrections to the scalar soft masses at \( M_G \), denoted by \( \Delta m_i^2(M_G) \), do not always lead to the same magnitude of corrections at \( M_Z \). For the scalar fields \( H_2, Q \) and \( U \) that couple with a large Yukawa coupling \( (h_i) \), the GUT corrections are usually diminished in the evolution from \( M_G \) to \( M_Z \), i.e., \( \Delta m_i^2(M_Z) < \Delta m_i^2(M_G) \). For the rest of the fields, however, one has \( \Delta m_i^2(M_Z) \approx \Delta m_i^2(M_G) \).

Figs. 1 – 4 show the scale evolution of the soft scalar mass parameters. The solid (dashed) line shows the evolution of the soft scalar masses when the \( M_P - M_G \) evolution is included (neglected). Note that the GUT correction to \( m_{H_2}^2 \) (the difference between the respective dashed and solid lines) is reduced at \( M_Z \) when \( h_i \) is large. This is especially apparent in

\(^3\)Note, however, that in some texture models Higgs doublets are embedded, e.g., in \( 10 \)'s and in \( 126 \)'s, and \( m_{H_1}^2 \) and \( m_{H_2}^2 \) could evolve very differently.
Figs. 1 and 3. In Fig. 2, however, the gauginos give the largest contribution to the soft scalar masses and $\Delta m_i^2(M_Z) - \Delta m_i^2(M_G) \propto \Delta M_5^2(M_G) \approx 20\%$. This effect is especially important for colored particles because of the gluino mass that grows at low energies with the diverging QCD coupling and roughly triples between $M_G$ and $M_P$. This feeds back via gluino loops to the colored scalar masses (and to the $A$ parameters) – see (1) – leading to large renormalization of those SSB parameters at the weak scale. Thus, GUT corrections from the Yukawa sector [that can split fields in equal SU(5) representations] can be washed out at $M_Z$. When $h_t$ is smaller (Fig. 4), the reduction of $\Delta m_H^2$ is less drastic and can give rise to important low-energy implications (the same is true for $\Delta m_{H_1}^2$ for any value of tan $\beta$). The evolution of $\Delta m_{Q,U}^2$ is even more dramatic. They can change the sign in the $M_G - M_Z$ evolution and even (Fig. 4) increase their value. This is because the positive term $h_t^2 m_{H_2}^2$ in the RGEs of $m_{Q,U}^2$ (that decreases them with the energy scale) is smaller when the GUT effects are considered.

Therefore, large effects are expected in those observables that depend on $m_{H_1}^2$ and $m_{Q,U}^2$: The $\mu$ parameter [see eq. (3a)] and the masses of the third family scalars. The actual magnitude of the GUT effects depends on the choice of the free parameters of the model. In section I we introduced a set of model-building parameters

$$m_0, A_0, B_0, M_{1/2},$$

(7)

where $B_0$ is traded for tan $\beta$ using (3b). The parameters are assumed to be real and $A_0$ (and $\mu$) can have either sign. For fixed fermion masses [and SU(5) Yukawa couplings] this set is enough, using renormalization group techniques, to predict all the low-energy observables. This is usually called up-down approach. Applying a specific set of values to these parameters as a boundary condition at $M_P$ [i.e., to eq. (4)] or at $M_G$ [i.e., to eq. (1)] can lead to a quite different mass spectrum. This is shown in the first and second columns of Tables II – V for the scenarios illustrated in Figs. 1 – 4. LSP stands for the lightest supersymmetric particle which is stable in the MSSM assuming $R$-parity. Tables II – V give quantitative examples of the GUT corrections. Note large deviations for observables that depend on the $\mu$ parameter (such as the Higgsino masses and components) and for the stop and sbottom masses.

Nevertheless, the free parameters of the model have to be extracted from low-energy experiments. Thus, it is more convenient to choose a set of free parameters of the model defined at $M_Z$, i.e., bottom-up approach [27]. For example, let us consider

$$m_{\tilde{e}_L}, M_3, \tan \beta, A_t(M_Z).$$

(8)

---

$^4$Note that the gaugino mass is enhanced $\sim 10\%$ by the $M_P - M_G$ evolution and that leads to an additional increase in the $m_i^2(M_Z)$. In Ref. [17-20] only the scalar soft masses are modified at $M_G$, i.e., $\Delta M_5^2(M_G) = 0$.

$^5$Although the relation between $m_{\tilde{e}_L}$ and $M_3$ and the physical observables is straightforward, this is not the case for $\tan \beta$ and $A_t(M_Z)$. Nevertheless, the latter two can always be determined as a function of physical observables such as the stop or Higgs masses (these are usually complicated functions involving other parameters of the model).
TABLE II. The low-energy spectrum is calculated for $m^\text{pole}\ell = 160$ GeV and $\tan \beta = 1.25$, assuming universality at the listed scale. We list, respectively, the universal gaugino and scalar masses, the $D$-term parameter and the trilinear parameter, along with the weak-scale predictions for the $\mu$ parameter, the gluino, chargino and LSP masses, the LSP eigenvector bino, wino and two Higgsino components, the heavier neutralino masses, the light and heavy Higgs boson masses, first and second family scalar quark $\tilde{q}$ and lepton $\tilde{l}$ masses, the $t$ and $b$-scalar masses and mixing $\tilde{t}_1 = -\sin \theta_t \tilde{t}_R + \cos \theta_t \tilde{t}_L$, and third family scalar lepton masses. $\lambda = 1$ (and $\lambda' = 0.1$) at $M_G$. The $\sim$ implies a rough average of the relevant masses. Equal values of the model building [low-energy] parameters eq. (7) [eq. (8)] are used in the first and second [first and third] columns. The first and second columns correspond to Fig. 1. All masses are in GeV.

| scale | $M_P$ | $M_G$ | $M_G$ |
|-------|-------|-------|-------|
| $M_{1/2}$ | 165 | 165 | 180 |
| $m_0$ | 987 | 987 | 991 |
| $M_D^2$ | 0 | 0 | 0 |
| $A_0/m_0$ | $-2$ | $-2$ | $-0.64$ |
| $\mu$ | $-1755$ | $-2038$ | $-1988$ |
| $M_\beta$ | 499 | 460 | 499 |
| $m_{\tilde{g}_{1,2}}^\pm$ | 151, $-1758$ | 138, $-2041$ | 150, $-1991$ |
| $m_{\tilde{e}_{1,2}}^\pm$ | 76 | 69 | 75 |
| $a_{11}$ | $-0.9988$ | $-0.9990$ | $-0.9990$ |
| $a_{12}$ | 0.0256 | 0.0243 | 0.0228 |
| $a_{13}$ | 0.0420 | 0.0368 | 0.0375 |
| $a_{14}$ | $-0.0055$ | $-0.0047$ | $-0.0048$ |
| $m_{\tilde{\chi}_{1,2}^0}$ | 151 | 138 | 150 |
| $m_{\tilde{\chi}_{3,4}^0}$ | 1755, $-1759$ | 2038, $-2041$ | 1988, $-1992$ |
| $m_{\tilde{b}_{1,2}}^\pm$ | 75 | 75 | 79 |
| $m_{\tilde{c}_{1,2}}^\pm$ | $\sim 2435$ | $\sim 2945$ | $\sim 2895$ |
| $m_{\tilde{u}_{1,2}}^\pm$ | $\sim 1080$ | $\sim 1060$ | $\sim 1078$ |
| $m_{\tilde{d}_{1,2}}^\pm$ | $\sim 997$ | $\sim 992$ | $\sim 997$ |
| $m_{\tilde{t}_{1,2}}^\pm$ | 517, 811 | 250, 858 | 395, 887 |
| $\sin \theta_{\tilde{t}}$ | 0.87 | 0.93 | 0.94 |
| $m_{\tilde{b}_{1,2}}^\pm$ | 735, 1075 | 794, 1059 | 834, 1075 |
| $\sin \theta_{\tilde{b}}$ | 0.01 | 0.02 | 0.02 |
| $m_{\tilde{\nu}_{1,2}, \tilde{\nu}}$ | 729, $\sim 999$ | 986, $\sim 995$ | 993, $\sim 999$ |
TABLE III. Same as in Table II except it corresponds to Fig. 2.

| scale  | $M_P$ | $M_G$ | $M_G$ |
|--------|-------|-------|-------|
| $M_{1/2}$ | 400   | 400   | 436   |
| $m_0$   | 0     | 0     | 219   |
| $M_D^2$ | 0     | 0     | 0     |
| $A_0/m_0$ | 0     | 0     | 0.67  |
| $\mu$   | $-1558$ | $-1348$ | $-1539$ |
| $M_\tilde{\mu}$ | 1155 | 1065 | 1155 |
| $m_{\chi^+_1,2}$ | 362, $-1562$ | 333, $-1352$ | 362, $-1542$ |
| $m_{\chi^+_1}$ | 178   | 163   | 178   |
| $a_{11}$ | 0.9996 | 0.9995 | 0.9996 |
| $a_{12}$ | $-0.0099$ | $-0.0122$ | $-0.0100$ |
| $a_{13}$ | $-0.0256$ | $-0.0293$ | $-0.0258$ |
| $a_{14}$ | 0.0036 | 0.0042 | 0.0036 |
| $m_{\chi^+}$ | 362 | 333 | 362 |
| $m_{\chi^+_3,4}$ | 1558, $-1563$ | 1348, $-1353$ | 1539, $-1544$ |
| $m_{\chi^0}$ | 64    | 62    | 63    |
| $m_{H^+, A^0, H^0}$ | $\sim 2097$ | $\sim 1808$ | $\sim 2081$ |
| $m_{\tilde{\tau}}$ | 1040 $- 1090$ | 937 $- 976$ | 1038 $- 1079$ |
| $m_{\tilde{\chi}^+}$ | 318 $- 384$ | 157 $- 290$ | 278 $- 384$ |
| $m_{\tilde{\chi}^0}$ | 822, 1001 | 754, 907 | 814, 993 |
| $\sin \theta_{\tilde{\tau}}$ | 0.97 | 0.97 | 0.98 |
| $m_{\tilde{\chi}^0}$ | 982, 1037 | 888, 937 | 974, 1038 |
| $\sin \theta_{\tilde{\chi}^0}$ | 0.01 | 0.01 | 0.01 |
| $m_{\tilde{\chi}^1,2, \tilde{\tau}^r}$ | 309, $\sim 382$ | 157, $\sim 289$ | 277, $\sim 382$ |
TABLE IV. Same as in Table II except $\lambda(M_G) = 0.1$ and it corresponds to Fig. 3.

|       | $M_P$ | $M_G$ | $M_G$ |
|-------|-------|-------|-------|
| scale |       |       |       |
| $M_{1/2}$ | 175   | 175   | 191   |
| $m_0$   | 375   | 375   | 387   |
| $M_{\tilde{D}}^2$ | 0     | 0     | 0     |
| $A_0/m_0$ | 1     | 1     | 0.73  |
| $\mu$   | 969   | 946   | 992   |
| $M_{\tilde{\chi}}$ | 527   | 486   | 527   |
| $m_{\chi^{+}_{1,2}}$ | 149, 977 | 136, 953 | 149, 1000 |
| $m_{\chi^{0}_{1}}$ | 77    | 70    | 77    |
| $a_{11}$ | $-0.7025$ | $-0.7024$ | $-0.7028$ |
| $a_{12}$ | $-0.0718$ | $-0.0726$ | $-0.0700$ |
| $a_{13}$ | 0.7068  | 0.7067 | 0.7068 |
| $a_{14}$ | 0.0423  | 0.0436 | 0.0411 |
| $m_{\chi^{0}_{2}}$ | 149   | 136   | 149   |
| $m_{\chi^{0}_{3,4}}$ | $-969, 979$ | $-946, 956$ | $-992, 1002$ |
| $m_{\phi}$ | 64    | 57    | 66    |
| $m_{H^0, A^0, H^+}$ | $\sim 1363$ | $\sim 1327$ | $\sim 1392$ |
| $m_{\tilde{\chi}}$ | $\sim 610$ | $\sim 570$ | $\sim 607$ |
| $m_{\tilde{\chi}}$ | $\sim 405$ | $\sim 390$ | $\sim 405$ |
| $m_{\tilde{\chi}_{1,2}}$ | 189, 631 | 145, 613 | 202, 644 |
| $\sin \theta_{\tilde{\chi}}$ | 0.80   | 0.81   | 0.82   |
| $m_{\tilde{\phi}_{1,2}}$ | 498, 601 | 482, 565 | 518, 601 |
| $\sin \theta_{\tilde{\phi}}$ | 0.06   | 0.07   | 0.07   |
| $m_{\tilde{\chi}_{1,2}, \tilde{\phi}_{1,2}}$ | 340, $\sim 410$ | 381, $\sim 395$ | 394, $\sim 410$ |
TABLE V. Same as in Table II except $m_t^{pole} = 180$ GeV, tan $\beta = 42$, and it corresponds to Fig. 4. The last two columns list scenarios with non-vanishing $D$-terms, e.g., in $SU(5) \times U(1)$. The latter two also provides a crude approximation of the minimal SO(10) scenario. Note that $M_D^2 \neq 0$ splits $\tilde{q}$, $\tilde{l}$, etc. according to their SU(5) embedding. The value of $M_D^2$ which is used in the last column is the minimal value still consistent with EWSB for the given set of parameters.

| scale | $M_P$ | $M_G$ | $M_G$ | $M_P$ | $M_P$
|------|-------|-------|-------|-------|-------
| $M_{1/2}$ | 89 | 89 | 96 | 89 | 89
| $m_0$ | 977 | 977 | 978 | 977 | 977
| $M_D^2$ | 0 | 0 | 0 | +0.16$m_0^2$ | -0.05$m_0^2$
| $A_0/m_0$ | 0 | 0 | 0.07 | 0 | 0
| $\mu$ | -645 | -214 | -225 | -849 | -563
| $M_\beta$ | 280 | 259 | 280 | 280 | 280
| $m_{\tilde{q}^+, \tilde{b}^+, \tilde{t}^+}$ | 80, -646 | 74, -215 | 80, -226 | 80, -849 | 80, -563
| $m_{\tilde{q}^0}$ | 40 | 35 | 38 | 40 | 40
| $a_{11}$ | 0.9976 | 0.9709 | 0.9739 | 0.9986 | 0.9968
| $a_{12}$ | 0.0107 | 0.1408 | 0.1279 | 0.0049 | 0.0153
| $a_{13}$ | -0.0469 | -0.1221 | -0.1162 | -0.0362 | -0.0532
| $a_{14}$ | 0.0504 | 0.1504 | 0.1452 | 0.0379 | 0.0582
| $m_{\tilde{q}^0}$ | 79 | 65 | 71 | 79 | 78
| $m_{\tilde{q}^0, \tilde{b}^0, \tilde{t}^0}$ | -651, 652 | -230, 239 | -240, 249 | -854, 854 | -569, 571
| $m_{\tilde{q}^0}$ | 114 | 113 | 114 | 114 | 114
| $m_{H^+, A^0, H^0}$ | ~492 | ~567 | ~568 | ~920 | ~220
| $m_{\tilde{l}^+}$ | ~1007 | ~1000 | ~1007 | ~1080, 744 | ~980, 1077
| $m_{\tilde{l}^0}$ | ~979 | ~977 | ~979 | ~715, 1055 | ~1055, 954
| $m_{\tilde{q}, \tilde{b}, \tilde{t}}$ | 691, 790 | 598, 731 | 601, 737 | 795, 881 | 653, 757
| $\text{sin } \theta_{l}^\prime$ | 0.96 | 0.98 | 0.97 | 0.97 | 0.96
| $m_{\tilde{q}, \tilde{b}, \tilde{t}}$ | 743, 869 | 708, 814 | 712, 818 | 482, 875 | 725, 939
| $\text{sin } \theta_{l}^\prime$ | 0.44 | 0.16 | 0.17 | 0.98 | 0.20
| $m_{\tau, \tilde{\tau}}$ | 818, ~910 | 803, ~869 | 804, ~897 | ~592, 919 | 793, ~985

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This basis is also convenient because the boundary conditions at $M_G$ can easily be obtained from the parameters (8). Comparing now, for a given value of (8), the low-energy predictions when the GUT effects are included (first column in Tables II - V) with those when the GUT effects are neglected (third column), one finds more modest changes, especially in the $M_{1/2} > m_0$ region (Table III). This is because using the basis (8) one eliminates global scalings of the SSB parameters arising in the $M_P - M_G$ evolution (see Fig. 2 and comment above). Although the choice (7) is the relevant one when speculating on the origin of the SSB parameters, it will be (8) or a similar basis from which the value of the SSB parameters will be extracted once supersymmetry is established.

Below, we study in more detail the way in which the predictions from naive $M_G$ universal boundary conditions are smeared and modified by the non-universal GUT corrections. Our aim is to study the uncertainties and new regions of parameter space that could be opened by those uncertainties. For example, we already mentioned that a situation with $h_b > h_t$ can now be consistent assuming pattern (v). We focus on our choices (a) and (b) given in section I. Our numerical routines are similar to those described in Ref. [4]. In short, we follow Ref. [13] in calculating the couplings (and the unification point), and Ref. [28] in treating the one-loop effective potential correction $\Delta V$, including contributions from all sectors. The boundary conditions for $0 \leq m_0 \leq 1000$ GeV, $|A_0| \leq 3m_0$, and $50 \leq M_{1/2} \leq 500$ GeV, are picked at random, unless otherwise stated. In order to minimize residual scale dependences (of order two-loop) of the one-loop effective potential, we rescale the Higgs potential (including wave function corrections) to a typical $t$-scalar scale of 600 GeV before solving the one-loop minimization equations. All Higgs masses include one-loop corrections calculated using Ref. [30]; however, a $\mathcal{O}(10\%)$ ambiguity in the one-loop light Higgs boson mass remains [4]. We apply the conservative constraint $m_{h_0} \gtrsim 60 \pm 5$ GeV. We also force all other relevant bounds on the mass parameters, and require the correct EWSB (i.e., a solution for $M_Z^2$), a neutral LSP and positive squared masses for all physical scalars. However, we do not minimize the full scalar potential in order to eliminate color breaking minima that survive the upper bound on $|A_0|$. That may affect the status of some points with a particularly large value of the $\mu$ parameter but is also sensitive to the choice of $M_G$ or $M_P$. For $\tan \beta = 42$ some points could induce positive corrections to the $b$-quark mass of more than $\sim 20\%$ and are omitted (smaller corrections could be compensated by other threshold effects). This effect is also sensitive to the $M_G$ or $M_P$ choice.

A. First and second family scalars

Since the Yukawa couplings of the first and second family of squarks and sleptons are small, they can be neglected in the RGEs, i.e., only the gauge contribution is relevant. The RGEs can be solved analytically and the physical scalar quark and lepton ($\tilde{f}$) masses in the basis (7) are given by

$$m_{\tilde{f}_{L,R}}^2 = m_0^2 + a_{\tilde{f}_{L,R}} M_{1/2}^2 \pm M_Z^2 \cos 2\beta [T_{3\tilde{f}_{L,R}} - Q_{\tilde{f}_{L,R}} \sin^2 \theta_W] + \Delta m_{\tilde{f}_{L,R}}^2,$$

where $a_{\tilde{f}_{L,R}} \sim 5 - 7$ for the squarks, $\sim 0.5$ for the left-handed sleptons and $\sim 0.15$ for the right-handed sleptons. $T_{3\tilde{f}_{L,R}}$ and $Q_{\tilde{f}_{L,R}}$ are the third component of SU(2)$_L$ isospin and the electric charge of $\tilde{f}_{L,R}$, respectively. The quantity $\Delta m_{\tilde{f}_{L,R}}^2$ is the extra contribution
arising from the $M_P - M_G$ evolution and depends on the GUT. For SU(5) we have \cite{1] \[ \Delta m_{JL,R}^2 = [0.1a_{JL,R} + 0.45]M_{I/2}^2 \] for the left-handed squarks, $\hat u_R$ and $\hat c_R$ and $\Delta m_{I,LR}^2 = [0.1a_{JL,R} + 0.3]M_{I/2}^2$ for the left-handed sleptons and $\hat d_R$ (the term $0.1a_{JL,R}$ arises due to the gaugino enhancement\cite{4}). Note that $\Delta m_{JL,R}^2$ can be the dominant contribution to the $\hat e_R$ mass and can contribute $\sim 60\%$ to the $\hat e_L$ and $\hat \nu_L$ masses. For SO(10) D-terms, the magnitude of the GUT correction depends on $M_H^2$. It is important to note that the requirement of non-tachyonic sleptons, \textit{i.e.}, $m_{\tilde{e},L,R} > 0$, leads to strong constraints on $M_H^2$ from below and above that can be easily obtained from eqs. (6) and (9).

When we use the input eq. (8), however, the scalar masses read

\[ m_{JL,R}^2 = m_{\tilde{e}}^2 + a_{JL,R}M_{\tilde{h}}^2 \pm M_Z^2 \cos 2\beta [T_{3,f_{L,R}} - Q_{f_{L,R}} \sin^2 \theta_W] + 0.3M_Z^2 \cos 2\beta + \Delta m_{JL,R}^2, \]

where now $a_{JL,R} \sim 0.6 - 0.9$ for the squarks, 0 for the left-handed sleptons and $\sim -5 \times 10^{-2}$ for the right-handed sleptons; $\Delta m_{JL,R}^2 \sim 2 \times 10^{-2}M_{\tilde{h}}^2$ for the left-handed squarks, $\hat u_R$ and $\hat c_R$ and $\Delta m_{JL,R}^2 = 0$ for the left-handed leptons and $\hat d_R$. Thus, in the basis (8) GUT effects can change the $\hat e_R$ (left-handed squarks and $\hat u_R$) masses by $\sim 15\%$ ($1\%$) at most. An example is given in Table III. The GUT effects in $m_{\tilde{t}}$ and $m_{\tilde{\nu}}$ are less apparent when the basis (8) is used instead of the basis (7).

**B. The $\mu$ parameter**

The $\mu$ parameter is extracted from the minimization condition eq. (3a) and depends on the values of $m_{\tilde{h}}^2$ at the weak scale. If the quantities $m_{\tilde{h}}^2$ are affected by the GUT effects, the $\mu$ parameter is modified according to

\[ \Delta \mu^2 = \frac{-\Delta m_{\tilde{h}}^2 - \Delta m_{\tilde{h}}^2 \tan^2 \beta}{1 - \tan^2 \beta}, \]

where $\Delta m_{\tilde{h}}^2$ are the shifts in the soft Higgs masses at the weak scale due to the GUT effects.

For low values of $\tan \beta$, one finds that $|\mu|$ is typically large and not affected significantly by GUT corrections, $\Delta \mu/\mu \sim 0.2$. For the GUT pattern (\textit{iii}), $\Delta \mu$ is small because $\Delta m_{\tilde{h}}^2$ is reduced at $M_Z$ (Fig. 3). For the GUT pattern (\textit{ii}), this is because of a partial cancellation between $\Delta m_{\tilde{h}}^2$ and $\Delta m_{\tilde{h}}^2 \tan^2 \beta$. In Fig. 5a we compare the predictions of $\mu$ when universality is assumed at $M_P$ with those when universality is assumed at $M_G$ for different random points of the parameter space defined by (7). The fact that $\Delta \mu$ depends on the sign of $\mu$ is due to the weak-scale threshold corrections to eq. (11) that can be substantial and have to be included. In Fig. 6 we show the distribution of the $\mu$ predictions in a sample of Monte Carlo calculations. One can see that the distribution is slightly changed. The differences in the integrated area of the histograms give an estimate of the changes in the allowed parameter space.

For large values of $\tan \beta$, we have $\Delta \mu^2 \approx -\Delta m_{\tilde{h}}^2$. For large $\lambda$, the splitting $\Delta m_{\tilde{h}}^2$ can be substantial (if $h_t$ is small, the quantity $\Delta m_{\tilde{h}}^2$ is slightly diminished in the $M_G - M_Z$ evolution – see Fig. 4) and $\mu$ can receive a large shift (Figs. 5b and 7). Note that the value of $|\mu|$ is always increased since in the minimal SU(5) model $\Delta m_{\tilde{h}}^2 < 0$. 

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FIG. 5. The prediction for the Higgsino mass parameter $\mu$ (in GeV) is compared assuming universality at $M_G$ and at $M_P$ for (a) $m_t^{\text{pole}} = 160$ GeV, $\tan\beta = 1.25$ and for (b) $m_t^{\text{pole}} = 180$ GeV, $\tan\beta = 42$ (note the different scales). $\lambda(M_G) = 1$, $\lambda'(M_G) = 0.1$, and the initial values for $m_0$, $A_0$ and $M_{1/2}$ and the sign of $\mu$ are picked at random (see above).

We have noted that the minimal SU(5) effects lead generically to an increase of $|\mu|$ [for low values of $\tan\beta$ the value of $|\mu|$ can be reduced (Fig. 5a), but $|\mu|$ is very large in this regime and the effects are small]. When extended GUTs are considered, however, this interesting feature is lost. In order to have $\Delta\mu^2 < 0$ one needs $[\Delta m_{H_2}^2 \tan^2 \beta - \Delta m_{H_1}^2] > 0$, and this latter condition can be obtained in the extended GUTs considered in section II. In the large $\tan\beta$ regime, however, one has also to consider the implications to EWSB. From the minimization conditions of the Higgs potential, we have that EWSB requires at the weak scale (for large $\tan\beta$),

$$\mu^2 + m_{H_2}^2 < 0, \quad \mu^2 + m_{H_1}^2 > 0,$$

which is difficult to achieve from universal $m_{H_1}^2 = m_{H_2}^2$ at $M_G$ because $h_t \approx h_b$. The GUT effects can produce a splitting $\Delta m_{H_2}^2 - \Delta m_{H_1}^2 < 0$ such that eq. (12) is satisfied more easily. For example, in the GUTs considered in section II this splitting can be induced but it requires $\Delta m_{H_2}^2 < 0$ which leads to an increase of $|\mu|$. In order to decrease $|\mu|$ we need
FIG. 6. The $\mu$ parameter prediction (in GeV) in a sample of Monte Carlo calculations for $m_t^{\text{pole}} = 160$ GeV and $\tan \beta = 1.25$ and assuming (i) universality at $M_G$ and (ii) universality at $M_P$ and $\lambda(M_G) = 1, \lambda'(M_G) = 0.1$.

$\Delta m_{H_2}^2 > 0$ that in the extended GUTs considered can only be obtained from the $D$-terms eq. (6). In that case, however, $\Delta m_{H_2}^2 - \Delta m_{H_1}^2 > 0$ and the EWSB is more difficult to obtain (requiring even more fine-tuning). In fact, for a given point in the parameter space there is an upper bound on $\Delta m_{H_2}^2 - \Delta m_{H_1}^2$ (that is strengthened when $\tan \beta$ increases). Note that in cases in which (because of the $M_P - M_G$ evolution) $m_6^2 \gg m_{H_i}^2$, the EWSB is even more difficult to obtain since $m_{H_i}^2$ and $m_{H_2}^2$ can be both negative$^6$ at $M_Z$. Thus, GUT effects can ease EWSB (for large $\tan \beta$) but, in that case, they increase the value of $|\mu|$.

The fact that $\mu$ is extracted from eq. (3a) and is usually larger than $M_Z$ implies that the lightest chargino ($\chi_1^+$) and neutralinos ($\chi_1^0$ – the LSP – and $\chi_2^0$) are mostly gauginos. As we have shown, this property is not altered by GUT effects of the minimal SU(5) model. Therefore, the masses of $\chi_1^+$ and $\chi_1^0$ depend mostly on $M_{1/2}$ and are almost independent of the soft scalar masses. The $M_P - M_G$ evolution can enhance $M_5$ and thus the gaugino masses by $\sim 10\%$. Using the basis (8), however, the gaugino masses can be written as a function of $M_5$, i.e., independent of the scale.

$^6$A situation with both $m_{H_2}^2 + \mu^2 < 0$ (when including loop corrections) leads to an unacceptable minimum. Note that we plot (in Figs. 1 - 4) the tree-level soft mass parameters.
The masses and mixing angles of the Higgs bosons in the MSSM can be written as a function of two parameters that we choose to be $\tan \beta$ and the pseudoscalar Higgs mass $m_{A^0}^2 = 2\mu^2 + m_{H_1}^2 + m_{H_2}^2$. Since $\tan \beta$ is considered an input parameter, only the GUT effects in $m_{A^0}^2$ are relevant. A shift in the soft Higgs masses arising from GUT physics shifts $m_{A^0}^2$ by

$$\Delta m_{A^0}^2 = \frac{1 + \tan^2 \beta}{1 - \tan^2 \beta} (\Delta m_{H_2}^2 - \Delta m_{H_1}^2).$$

For $\tan \beta \approx 1$, the behavior of $\Delta m_{A^0}^2$ is similar to that of $\Delta \mu^2$ discussed in the previous section. For large $\tan \beta$, one has $\Delta m_{A^0}^2 \approx \Delta m_{H_1}^2 - \Delta m_{H_2}^2$. Since in the minimal SU(5) model $\Delta m_{H_2}^2 \approx \Delta m_{H_1}^2$, one has $\Delta m_{A^0}^2 \approx 0$.

The mass of the lightest Higgs $h^0$ receives large radiative correction induced by loops involving the top and stop ($m_{h^0} \rightarrow m_{j}\Delta_{\Delta} [m_t, m_Q, m_U, A_t, \mu, \tan \beta]$) and can be changed if either the diagonal or off-diagonal entries in the stop mass matrix are shifted by GUT effects (section III D). The effects are negligible for $\tan \beta \gtrsim 2$ where the Higgs boson is heavy at tree level. However, for a light tree-level Higgs boson ($\tan \beta \approx 1$), unless the different effects cancel (see Tables II and III), they can modify $m_{h^0}$ by a few GeV (see Table IV). The cancellations depend on the sign of the $\mu$ parameter. In Figs. 8 and 9 we show the distribution of the lightest Higgs mass. One can note that the distributions are only
slightly sensitive to the GUT corrections so that previous calculations (e.g., see [4]) of the predictions of $m_{h^0}$ in SUSY GUTs are not altered. (Fig. 8, here, roughly corresponds to Figs. 9a and 10a in Ref. [4].)

When GUT effects from extended models are considered, the value of $m_{A^0}$ can increase (decrease) if a splitting $\Delta m_{A_1}^2 - \Delta m_{A_2}^2 > 0 (< 0)$ is induced. Again, when EWSB and non-tachionic particles are required, $\Delta m_{A^0}^2$ can be bounded from below and above.

D. Third family scalars

As explained above, the masses of the scalars of the third family can be largely modified by the GUT effects due to a large $h_t$. The GUT effects can modify (i) the soft mass $m_{t_0}^2$, (ii) $m_{H_2}^2$, and hence the evolution of $m_Q^2$ and $m_U^2$, and (iii) the $\mu$ parameter (section III B) and $A_{t,b,\tau}$ that enter in the left-right mixing term of the scalar masses. These three effects compete with each other to increase or decrease the masses.

The lightest stop, $\tilde{t}_1$, has recently received much attention. Its mass, $m_{\tilde{t}_1}$, is usually smaller than the mass of the other squarks and can induce significant one-loop effects in low-energy processes such as $Z \rightarrow b\bar{b}$ and $b \rightarrow s\gamma$. In the minimal SU(5) model, the dominant

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There are also gauge GUT effects that are the same as those to the first and second family scalars.
FIG. 9. Same as in Fig. 7 except for the light CP-even Higgs boson mass $m_{h^0}$ prediction.

GUT effect in $m_{\tilde{t}_1}$ arises from (ii). Since $m_{H_2}^2$ is diminished by GUT effects, $m_{Q}^2(M_Z)$ and $m_{H_2}^2(M_Z)$ are larger. Thus, we find that $m_{\tilde{t}_1}$ is always enhanced. It follows that some points of the parameter space which correspond to a tachionic $t$-scalar and are excluded when the $M_P$ to $M_G$ evolution is neglected, can be allowed. In Fig. 10 we compare the predictions of $m_{\tilde{t}_1}$ with and without the $M_P - M_G$ evolution. We find significant corrections in the low tan $\beta$ regime, especially for small values of $m_{\tilde{t}_1}$. This implies that one-loop corrections induced by $\tilde{t}_1$ to low-energy processes can be significantly reduced. Note that $m_{\tilde{t}_1} \gtrsim 200$ GeV when including the $M_P - M_G$ evolution.

The GUT effects from non-minimal SU(5) models [pattern (v)] usually enhance $m_{\tilde{t}_1}$ since (ii) is still dominant. In the minimal SO(10), however, the splittings eq. (6) can lead to a lighter stop. For $M_D^2 < 0$, one has $\Delta m_{10}^2 < 0$ and $\Delta m_{H_2}^2 > 0$, and both effects (i) and (ii) decrease $m_{\tilde{t}_1}$. Since, as we said in section III A, these splittings can lead to tachionic sleptons, $m_{\tilde{t}_1}$ cannot be reduced significantly.

For the sbottom and stau we find that the effects (i) and (ii) can be equally important and their masses can increase or decrease depending on the point in the parameter space (see tables).

E. Possible implications

To conclude our survey of the weak-scale phenomena, we summarize the most interesting implications of the GUT effects for experiment. We have shown that assuming the minimal
SU(5) model and universality at $M_P$ instead of $M_G$ one typically predicts heavier particles. For example, the scalar leptons could be substantially heavier (see Table III). More interestingly, correlations between the different parameters are modified (see also [1]), i.e., correlations calculated assuming universality at $M_G$ could be misleading and should not be used to constrain the parameter space. Smeared correlations imply less EWSB-related fine-tuning and that a larger parameter space may be available. For example, the $\tilde{t}_1$ mass is shown [for choice (a)] in Figs. 11 and 12. It is typically larger when considering the GUT effects and its correlation with the $\chi_1^+$ mass (or for that matter, with the gluino mass – see Fig. 3 of Ref. [1]) is smeared while that with the $\tilde{t}_2$ mass is strengthened. For choice (b) the $\tilde{t}$ masses are only slightly altered but $\chi_1^+$ could be heavier. In Fig. 13 we examine the Higgsino fraction of the LSP for choice (b), which is relevant, e.g., for relic abundance calculations. The larger Higgsino mass implies smaller Higgsino fractions of the gaugino-like $\chi_1^+$, $\chi_1^0$ and $\chi_2^0$. That and the heavier $\tilde{t}_1$ lead to a stronger decoupling of the supersymmetric particles from low-energy processes, e.g., from $Z \to b\bar{b}$.

In extended GUT models the restrictions on the parameter space from EWSB are somewhat relaxed. In particular, values of $m_{\tilde{t}}^{pole}$ and $\tan \beta$ which imply $h_b > h_t$ may be consistent with EWSB. Non-vanishing $D$-terms can lead to a very different spectrum in comparison to
FIG. 11. Scatter plot of the light chargino $\chi^+_1$ vs. the light $t$-scalar $\tilde{t}_1$ masses (in GeV) within the allowed parameter space (see above) and for $m^\text{pole}_t = 160$ GeV and $\tan \beta = 1.25$. Filled triangles [circles] correspond to universality at $M_P$ [$M_G$] and $\lambda(M_G) = 1$, $\lambda^\prime(M_G) = 0.1$.

the situation with vanishing $D$-terms. For example, they could lead to a lighter Higgsino which, as discussed above, is an interesting possibility phenomenologically. However, when combined with the $M_P - M_G$ evolution, the effects are diminished (see Table V). Also, in the models studied the amount that the $\mu$ prediction can be diminished by GUT effects is strongly constrained (unlike in some ad hoc cases studied in Ref. [17-20]). We conclude that if able to observe the GUT effects, e.g., from correlation measurements, collider experiments could directly probe the GUT scale physics. That is a non-trivial task and it would depend on the experimental resolution as well as on the region of parameter space nature chooses.

IV. CONCLUSIONS

Above, we examined the effects of a grand-unified symmetry between the Planck and GUT scales in the SSB parameters. Our only assumptions were coupling constant unification at $M_G \approx 2 \times 10^{16}$ GeV; the MSSM as the effective theory below that scale; and universal SSB parameters at the minimal supergravity scale $M_P \approx 2 \times 10^{18}$ GeV. In addition, we had to specify the GUT. We previously analyzed these assumptions in Ref. [1] assuming
the minimal SU(5) model and found potentially large deviations from universality for the SSB parameters at $M_G$. In particular, we emphasized the role of large Yukawa couplings which are generic in such models. Here, we further cataloged the possible patterns of non-universality in that model and examined in great detail their implications to the weak scale phenomena. We found potentially large corrections (in comparison with the working assumption of universality at $M_G$) to the allowed parameter space, the $\mu$ parameter and to the third-family squark spectrum. These are all related primarily to the $M_P - M_G$ evolution of the light Higgs fields. In the gaugino-dominated cases (e.g., in no-scale models) large corrections to the scalar-lepton masses are also possible. The Higgs and the first and second family squark sectors are relatively insensitive to the GUT effects. A different situation may arise in extended models. For example, we discussed the situation in non-minimal SU(5), where $m_{H_1}^2(M_G)$ and $m_{H_2}^2(M_G)$ are independent parameters, as well as the appearance of non-vanishing $D$-terms in SO(10). In both models correlations are further diminished and EWSB constraints are more easily satisfied. However, EWSB still plays an important role in constraining GUT effects, e.g., effects that could diminish $\mu$. Implications of the above to experiment were already summarized in section III E.

Finally, let us comment on the predictive power of the MSSM. Assuming minimal SU(5), two additional parameters are needed (only one of which plays an important role). In

FIG. 12. Same as in Fig. 11 except the light $\tilde{t}_1$ vs. heavy $\tilde{t}_2$ masses.
non-minimal $SU(5)$, $SU(5) \times U(1)$ and $SO(10)$ three or more new parameters are needed. The more parameters the model has, the larger role GUT effects could play in weak-scale phenomena, but the less predictive is the model. On the other hand, when adding only a small number of new parameters, the effects in different SSB parameters are correlated, and thus, constrained (e.g., by EWSB). The predictive power can be further altered when considering threshold corrections. These are described in detail for the minimal $SU(5)$ model in Appendix B [eqs. (B13)–(B15)] and, in general, they do not significantly modify the tree-level patterns described in section II. In extended models threshold corrections could be more important if more and larger representations are present. Also, perturbation theory could break down in these models and one would need non-perturbative methods to calculate the GUT effects.

GUT effects in the SSB parameters are generic, leading to non-universal patterns different than those, e.g., in string theory, and could probe the GUT-scale physics. However, they are model-dependent and lead to uncertainties in any model-independent analysis, which typically assumes universality at $M_G$. Until supersymmetry is established and characterized, the effects have to be considered as uncertainties to supergravity GUT model predictions.

FIG. 13. Same as in Fig. 11 except the LSP mass $m_{\chi^+_1}$ (in GeV) vs. its Higgsino fraction $a_{13}$ and for $m_t^{\text{pole}} = 180$ GeV and $\tan \beta = 42$. 

m_{\text{LSP}}

higgsino fraction $a_{13}$
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APPENDIX A: THE MINIMAL SU(5) MODEL

The Higgs sector of the model consists of three supermultiplets, \( \Sigma(24) \) in the adjoint representation [which is responsible for the breaking of SU(5) down to SU(3)_c \( \times \) SU(2)_L \( \times \) U(1)_Y], \( \mathcal{H}_1(\mathbf{5}) \) and \( \mathcal{H}_2(\mathbf{5}) \):

\[
\Sigma \equiv \sqrt{2} T_a w_a , \quad \mathcal{H}_1 = \left( \begin{array}{c} H_{C_1} \\ H_1 \end{array} \right) , \quad \mathcal{H}_2 = \left( \begin{array}{c} H_{C_2} \\ H_2 \end{array} \right) , \quad (A1)
\]

where \( H_{C_i} \) and \( H_i \) are the color triplets and SU(2)_L doublets, respectively, and \( T_a \) are the SU(5) generators with \( \text{tr}\{T_a T_b\} = \delta_{ab}/2 \). The matter superfields are in the \( \mathbf{5} + \mathbf{10} \) representations, \( \phi(\mathbf{5}) \) and \( \psi(\mathbf{10}) \). The superpotential is given by

\[
W = \mu_\Sigma \text{tr}\Sigma^2 + \frac{1}{6} \lambda \text{tr}\Sigma^3 + \mu_H H_1 H_2 + \lambda H_1 \Sigma H_2
\]

\[
+ \frac{1}{4} h_i \epsilon_{ijklm} \psi^{ij} \psi^{kl} H_2^m + \sqrt{2} h_b \psi^{ij} \phi_i H_1 j , \quad (A2)
\]

where we have omitted family indices and \( h_i \) and \( h_b \) are the Yukawa couplings of the third generation (we neglect the other Yukawa couplings). In the supersymmetric limit \( \Sigma \) develops a vacuum expectation value \( \langle \Sigma \rangle = \nu_\Sigma \text{diag}(2, 2, 2, -3, -3) \) and the gauge bosons \( X \) and \( Y \) receive a mass \( M_X = 5 g_G \nu_\Sigma \). In order for the Higgs SU(2) doublets to have masses of \( \mathcal{O}(M_Z) \) instead of \( \mathcal{O}(M_G) \), the fine-tuning \( \mu_H - 3 \lambda \nu_\Sigma \lesssim \mathcal{O}(M_Z) \) is required and one obtains \( M_{H_C} = \frac{\lambda}{g_G^2} M_Y \). Dimension-five operators induced by the color triplets give large contributions \( \propto 1/M_{H_C}^2 \) to the proton decay rate [6]. To suppress such operators, the mass of the color triplets has to be large, \( M_{H_C} \gtrsim M_Y \), implying \( \lambda \gtrsim g_G \approx 0.7 \).

Below \( M_P \), the effective lagrangian also contains the SSB terms (note that we keep the same notation for the superfields and their corresponding scalar fields)

\[
-\mathcal{L}_{soft} = m_{H_1}^2 \left| H_1 \right|^2 + m_{H_2}^2 \left| H_2 \right|^2 + m_{\Sigma}^2 \text{tr}\{\Sigma \Sigma \} + m_{\phi}^2 \left| \phi \right|^2 + m_{\psi}^2 \text{tr}\{\psi \psi \}
\]

\[
+ \left[ B_{\Sigma} \mu_\Sigma \text{tr}\Sigma^2 + \frac{1}{6} A_{\Sigma} \lambda \text{tr}\Sigma^3 + B_{H} \mu_H H_1 H_2 + A_{\lambda} \lambda H_1 \Sigma H_2 \right]
\]

\[
+ \frac{1}{4} A_i h_i \epsilon_{ijklm} \psi^{ij} \psi^{kl} H_2^m + \sqrt{2} A_b h_b \psi^{ij} \phi_i H_1 j + \frac{1}{2} M_5 \lambda_\alpha \lambda_\alpha + h.c. , \quad (A3)
\]

where \( \lambda_\alpha \) are the gaugino fields.

The SU(5) RGEs for the SSB parameters and Yukawa couplings are given by

\[
\frac{dm_{10}^2}{dt} = \frac{1}{8 \pi^2} \left[ \beta h_i^2 (m_{H_2}^2 + 2 m_{10}^2 + A_i^2) + 2 h_i^2 (m_{H_1}^2 + m_{10}^2 + m_5^2 + A_5^2) - \frac{7}{5} g_G^2 M_5^2 \right] ,
\]

\[
\frac{dm_{5}^2}{dt} = \frac{1}{8 \pi^2} \left[ 4 h_i^2 (m_{H_1}^2 + m_{10}^2 + m_5^2 + A_5^2) - \frac{48}{5} g_G^2 M_5^2 \right] .
\]
where \( dM \) be small. There are two ways to compute the threshold correction to the scalar SSB parameters that decouple from the rest of the RGEs. A second way, which is much simpler, consists of obtaining the quark and lepton SU(2) parameters considered in the literature.

\[
\frac{dm_{\frac{1}{2}}}{dt} = \frac{1}{8\pi^2} \left[ 4h_i^2(m_{\frac{1}{2}t}^2 + m_{10}^2 + m_{3}^2 + A_i) + \frac{24}{5} \lambda^2(m_{\frac{1}{2}t}^2 + m_{\frac{1}{2}t}^2 + m_{3}^2 + A_i) - \frac{48}{5} g_G^2 M_5^2 \right],
\]
\[
\frac{dm_{\frac{3}{2}}}{dt} = \frac{1}{8\pi^2} \left[ 3h_i^2(m_{\frac{3}{2}t}^2 + 2m_{10}^2 + A_i) + \frac{24}{5} \lambda^2(m_{\frac{3}{2}t}^2 + m_{\frac{3}{2}t}^2 + m_{3}^2 + A_i) - \frac{48}{5} g_G^2 M_5^2 \right],
\]
\[
\frac{dm_{\Sigma}}{dt} = \frac{1}{8\pi^2} \left[ \frac{21}{20} \lambda^2(3m_{\Sigma}^2 + A_\Sigma) + \lambda^2(m_{\frac{1}{2}t}^2 + m_{\frac{1}{2}t}^2 + m_{3}^2 + A_i) - 20g_G^2 M_5^2 \right],
\]
\[
\frac{dA_i}{dt} = \frac{1}{8\pi^2} \left[ 6g_G h_i^2 + 4A_b h_i^2 + \frac{24}{5} A_i \lambda^2 - \frac{96}{5} g_G^2 M_5 \right],
\]
\[
\frac{dA_b}{dt} = \frac{1}{8\pi^2} \left[ 10A_b h_i^2 + 3A_i h_i^2 + \frac{24}{5} A_i \lambda^2 - \frac{84}{5} g_G^2 M_5 \right],
\]
\[
\frac{dA_\lambda}{dt} = \frac{1}{8\pi^2} \left[ \frac{21}{20} A_\lambda \lambda^2 + 3A_i h_i^2 + 4A_b h_i^2 + \frac{53}{5} A_i \lambda^2 - \frac{98}{5} g_G^2 M_5 \right],
\]
\[
\frac{dA_\lambda'}{dt} = \frac{1}{8\pi^2} \left[ \frac{63}{20} A_\lambda \lambda^2 + 3A_i \lambda^2 - 30g_G^2 M_5 \right],
\]
\[
\frac{dh_i}{dt} = \frac{1}{16\pi^2} \left[ 9h_i^2 + 4h_i^2 + \frac{24}{5} \lambda^2 - \frac{96}{5} g_G^2 \right],
\]
\[
\frac{dh_b}{dt} = \frac{1}{16\pi^2} \left[ 10h_b^2 + 3h_b^2 + \frac{24}{5} \lambda^2 - \frac{84}{5} g_G^2 \right],
\]
\[
\frac{d\lambda}{dt} = \frac{1}{16\pi^2} \left[ \frac{21}{20} \lambda^2 + 3h_i^2 + 4h_b^2 + \frac{53}{5} \lambda^2 - \frac{98}{5} g_G^2 \right],
\]
\[
\frac{d\lambda'}{dt} = \frac{1}{16\pi^2} \left[ \frac{63}{20} \lambda^2 + 3\lambda^2 - 30g_G^2 \right],
\]

where \( t = \ln Q \). The RGE for the gauge coupling is \( d\alpha_G/dt = -3\alpha_G^2/2\pi \), and similarly \( dM_5/dt = -3\alpha_G M_5/2\pi \). We can omit the RGEs for \( \mu_\Sigma \), \( \mu_H \), \( B_\Sigma \) and \( B_H \), which are arbitrary parameters that decouple from the rest of the RGEs.

Below \( M_G \), the effective theory corresponds to the MSSM:

\[
W = \mu H_1 H_2 + h_i Q H_2 U + h_i Q H_1 D + h_\tau L H_1 E,
\]

where \( Q \) and \( L \) are, respectively, the quark and lepton SU(2)_L doublets, and \( U \), \( D \) and \( E \) are, respectively, the quark and lepton SU(2)_L singlets.

**APPENDIX B: GUT-SCALE THRESHOLD CORRECTIONS**

Even if the scale where universal SSB terms are generated is assumed to be \( M_G \), there is some arbitrariness in the value of \( M_G \) due to the mass-splitting between the particles at the GUT scale, i.e., threshold effects. These GUT effects to the SSB terms (to the best of our knowledge) have never been considered before. As we will show, they can be as important as the low-energy (supersymmetric) threshold effects, which are the only threshold effects to the SSB parameters considered in the literature.

We will only consider GUT threshold corrections to the scalar SSB parameters. Corrections to the gaugino masses have been computed in Ref. [30], where they were shown to be small. There are two ways to compute the threshold correction to the scalar SSB parameters. One way consists of calculating explicitly the one-loop diagrams that contribute to the scalar SSB terms. A second way, which is much simpler, consists of obtaining the
one-loop SSB terms from the one-loop effective potential that in the Landau gauge and in the dimensional-reduction (DR) scheme reads

$$\Delta V = \frac{1}{64\pi^2} \sum_i (-1)^{2s_i}(2s_i + 1)M_i^4\left[\ln \frac{M_i^2}{Q^2} - \frac{3}{2}\right], \quad (B1)$$

where $M_i^2$ and $s_i$ are, respectively, the field-dependent squared mass and spin of the particle $i$. In this case, one only has to compute the masses $M_i$. We will present the GUT threshold correction using (B1) although we have checked the results with those from the explicit diagramatic calculation.

Let us start with the one-loop correction to the SSB squared-mass $m_i^2$ of a scalar $\Phi_i$ induced by a heavy chiral supermultiplet, which consists of a fermion field with mass $M_F$ and two (real) scalars with masses $M_{S_1}$ and $M_{S_2}$. In the supersymmetric limit $M_F = M_{S_{1,2}} \equiv M_0$ where $M_0$, is of the order $M_G$. We separate the GUT effects into logarithmic corrections and finite corrections:

1. Logarithmic corrections: The logarithmic term of eq. (B1) gives the one-loop contribution to $m_i^2$

$$\Delta m_i^2(log) = \frac{1}{32\pi^2} \left\{ \frac{\partial M_0^2}{\partial|\Phi_i|^2} M_{S_1}^2 + \frac{\partial M_0^2}{\partial|\Phi_i|^2} M_{S_2}^2 - 2 \frac{\partial M_F^2}{\partial|\Phi_i|^2} M_F^2 \right\} \ln \frac{M_F^2}{Q^2} \quad (B2)$$

$$+ \frac{\partial M_{S_1}^2}{\partial|\Phi_i|^2} M_{S_1}^2 \ln \frac{M_{S_1}^2}{M_F^2} + \frac{\partial M_{S_2}^2}{\partial|\Phi_i|^2} M_{S_2}^2 \ln \frac{M_{S_2}^2}{M_F^2}. \quad (B3)$$

The first term (B2) gives the logarithmic contribution arising from the energy-scale difference between the mass of the superfield and the scale $Q$ (splittings between different heavy superfields). The second term (B3) arises from the boson-fermion mass splitting within the superfield. This latter type of correction to the Yukawa and gauge couplings is of $O(m_{soft}^2/M_0^2)$ and then negligible for $M_0 \approx M_G \gg m_{soft}$. However, it can be important to the $m_i^2$, as we will show below.

2. Finite corrections: The non-logarithmic contribution to $m_i^2$ from (B1) is given by

$$\Delta m_i^2(finite) = -\frac{1}{32\pi^2} \left[ \frac{\partial M_0^2}{\partial|\Phi_i|^2} M_{S_1}^2 + \frac{\partial M_0^2}{\partial|\Phi_i|^2} M_{S_2}^2 - 2 \frac{\partial M_F^2}{\partial|\Phi_i|^2} M_F^2 \right] \quad (B4)$$

Note that this contribution depends on the renormalization scheme. Eq. (B4) has been obtained in the DR scheme. The heavy squared-masses $M_{S_{1,2}}^2$ and $M_F^2$ can be written using a $1/M_0$ expansion as

$$M_{S_{1,2}}^2 = M_0^2 \pm aM_0 + b + \sum_i \left[ \beta_i \pm \frac{c_i}{M_0^2} + \frac{d_i}{M_0^2} \right]|\Phi_i|^2 + \cdots,$$

$$M_F^2 = M_0^2 + \sum_i \beta_i |\Phi_i|^2 + \cdots, \quad (B5)$$

where the coefficients $a-d_i$ depend on the SSB parameters$^8$, and we have only kept the

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$^8$The coefficients $a-d_i$ also depend on the mass parameters of the superpotential (such as the $\mu$ parameter), but this dependence has to be discarded since we are only interested in the corrections to the SSB parameters.
relevant terms for our analysis. Substituting eq. (B5) in eqs. (B2), (B3) and (B4), we get

\[
\Delta m_i^2 (\log) = \frac{1}{16\pi^2} \left[ b_i + a_i + d_i \right] \ln \frac{M_i^2}{Q^2} + \frac{1}{32\pi^2} [b_i(2b + a^2) + 2ac_i] + \ldots ,
\]

(B6)

\[
\Delta m_i^2 (\text{finite}) = -\frac{1}{16\pi^2} \left[ b_i + a_i + d_i \right] + \ldots .
\]

(B7)

(B8)

Notice that the contribution from (B3) [the term (B7)] gives a correction to \( m_i^2 \) not suppressed by powers of \( \mathcal{O}(m_{\text{soft}}^2/M_0^2) \), although it turns out to be non-logarithmic. From eqs. (B6)–(B8), the GUT-scale corrections can easily be obtained if the dependence of the heavy masses on the light scalar fields (the \( a-d_i \) coefficients) is known. In the minimal SUSY SU(5) model (see Appendix A), the GUT spectrum consists of the 12 vector superfields \( \bar{V} = X, Y \), the color triplets \( H_C, i \), and the \( \Sigma \) superfield. With respect to SU(3)_c \times SU(2)_L, the \( \Sigma \) supermultiplet decomposes into \( (3, 2) + (\bar{3}, 2) + (8, 1) + (1, 3) + (1, 1) \). In the supersymmetric limit, the \( (3, 2) \) and \( (\bar{3}, 2) \) components are degenerate with \( X \) and \( Y \). The \( (1, 3) \) and \( (8, 1) \) components, \( \Sigma_3 \) and \( \Sigma_8 \), respectively, have a common mass \( 10\mu \), while the mass of the singlet, \( \Sigma_1 \), is \( 2\mu \). When the SSB terms are considered, a boson-fermion mass splitting within every supermultiplet is induced.

Considering only the two large Yukawa couplings, \( \lambda \) and \( h_i \), the coefficients \( a-d_i \) for the \( \Sigma_3 \) are given by

\[
a = B_\Sigma, \quad b = m_\Sigma^2, \\
b_{H_i} = b_{H_2} = \lambda^2, \\
c_{H_i} = c_{H_2} = \frac{\lambda^2}{2} [2A_\lambda - B_\Sigma], \\
d_{H_i} = \frac{\lambda^2}{2} [m_{H_2}^2 - m_\Sigma^2 + A_\lambda^2 + B_\Sigma^2 - 2A_\lambda B_\Sigma], \\
d_{H_2} = \frac{\lambda^2}{2} [m_{H_1}^2 - m_\Sigma^2 + A_\lambda^2 + B_\Sigma^2 - 2A_\lambda B_\Sigma].
\]

(B9)

The coefficients for \( \Sigma_1 \) can be obtained from eqs. (B9) by replacing \( \lambda^2 \to \frac{2}{3} \lambda^2 \). For the color triplets \( H^0_{C_i} \) (\( \alpha \) being the color index) we have, assuming \( M_{H_C} > M_V \),

\[
a = B_H, \quad b = (m_{H_1}^2 + m_{H_2}^2)/2, \\
b_{H_1} = b_{H_2} = \lambda^2, \\
c_{H_1} = c_{H_2} = \frac{\lambda^2}{2} [2A_\lambda - B_H], \\
d_{H_1} = \frac{\lambda^2}{2} [m_{H_2}^2 - m_{H_1}^2 + A_\lambda^2 + B_H^2 - 2A_\lambda B_H], \\
d_{H_2} = \frac{\lambda^2}{2} [m_{H_1}^2 - m_{H_2}^2 + A_\lambda^2 + B_H^2 - 2A_\lambda B_H], \\
b_{V_\alpha} = b_{Q_\beta} = b_E = h_i^2, \quad \beta \neq \alpha, \\
c_{V_\alpha} = c_{Q_\beta} = c_E = \frac{h_i^2}{2} [2A_i - B_H], \\
d_{V_\alpha} = d_{Q_\beta} = d_E = \frac{h_i^2}{2} [m_{10}^2 - m_{H_i}^2 + A_i^2 + B_H^2 - 2A_i B_H].
\]

(B10)
The rest of the heavy fields decouple from the light scalar fields (gauge contributions have not been considered).

There are also one-loop corrections coming from the wave-function renormalization constants that cannot be obtained from the one-loop effective potential. These corrections have to be calculated from the explicit one-loop diagrams, and give a contribution to the Higgs soft masses

\[ m_{H_i}^2(Q) = m_{H_i}^2(Q) + \frac{\lambda^2}{8\pi^2} m_{H_i}^2 \left[ \frac{3}{4} \ln \frac{M_{\Sigma_3}}{Q^2} + \frac{3}{20} \ln \frac{M_{\Sigma_1}}{Q^2} + \frac{3}{2} \ln \frac{M_{H_C}}{Q^2} - \frac{12}{5} \right], \]

and to the sleptons and squarks soft masses

\[ m_{\tilde{q}}^2(Q) = m_{\tilde{q}}^2(Q) + \frac{n_i^2 h_i^2}{16\pi^2} m_{\tilde{q}}^2 \left[ \ln \frac{M_{H_C}}{Q^2} - 1 \right], \]

where \( n_i = 1, 2, 3 \) for \( i = U, Q, E \), respectively. Inserting eqs. (B9) and (B10) in eqs. (B6)–(B8) and incorporating eqs. (B11) and (B12), we get the one-loop matching conditions at \( M_G \)

\[ m_{H_i}^2(M_G) = m_{H_i}^2(M_G) + \frac{\lambda^2}{4\pi^2} (m_{H_1}^2 + m_{H_2}^2 + m_{H_3}^2 + A_t^2) \left[ \frac{3}{4} \ln \frac{M_{\Sigma_3}}{M_G} + \frac{3}{20} \ln \frac{M_{\Sigma_1}}{M_G} + \frac{3}{2} \ln \frac{M_{H_C}}{M_G} - \frac{6}{5} \right] \]

\[ + \frac{\lambda^2}{4\pi^2} \left[ \frac{9}{10} (m_{H_1}^2 + A_\lambda B_{H_1}) + \frac{3}{4} (m_{H_2}^2 + m_{H_3}^2 + 2A_\lambda B_{H_2}) \right], \]

\[ m_{\tilde{q}}^2(M_G) = m_{\tilde{q}}^2(M_G) + \frac{n_i^2 h_i^2}{8\pi^2} (2m_{\tilde{q}_{10}}^2 + m_{\tilde{q}_{12}}^2 + A_t^2) \left[ \ln \frac{M_{H_C}}{M_G} - \frac{1}{2} \right] \]

\[ + \frac{n_i^2 h_i^2}{16\pi^2} (m_{H_1}^2 + m_{H_2}^2 + 2A_t B_{H}), \quad n_i = 1, 2, 3 \text{ for } i = U, Q, E. \]

Eq. (B13) corresponds to the one-loop corrected eq. (5c). For \( \mu_\Sigma \ll M_V \approx 2 \times 10^{16} \) GeV, the logarithmic term of eq. (B13) can lead to a large deviation from eq. (5c). For example, taking \( M_G = M_V \approx M_{H_C} \approx 10^3 \mu_\Sigma, \lambda \approx 1 \) and assuming \( m_\Sigma^2 = A_t^2 = m_0^2 \) at tree-level, we have \( m_{H_i}^2(M_G) \approx 0.6m_0^2 \). The non-logarithmic terms of eq. (B13) are smaller (~10%) and tend to cancel out for equal SSB parameters. It is interesting to note that in the regions of the MSSM parameter space where the EWSB requires a high degree of fine tuning [3,4], a 10% GUT correction to the soft Higgs masses can destabilize the minimum. The GUT threshold corrections to (5a) [given in eq. (B14)], are typically small since \( M_{H_C} \) is forced to be close to \( M_V \) (\( M_{H_C} \approx M_V \) from proton decay and \( M_{H_C} \approx 2M_V \) to stay in the perturbative regime [6]). Nevertheless, it is important to stress that, unlike other GUT effects, such corrections contribute to the mass-splitting between light fields embedded in the same SU(5) representation.

Finally, the one-loop contribution to the trilinear term can be computed in the same way, and is given by

\[ \Delta A_i = \frac{\lambda^2}{4\pi^2} A_\lambda \left[ \frac{3}{4} \ln \frac{M_{\Sigma_3}}{Q} + \frac{3}{20} \ln \frac{M_{\Sigma_1}}{Q} + \frac{3}{2} \ln \frac{M_{H_C}}{Q} - \frac{6}{5} \right] \]

\[ + \frac{n_i^2 h_i^2}{8\pi^2} A_t \left[ \ln \frac{M_{H_C}}{Q} - \frac{1}{2} \right], \]

where \( n_i = 3, 2, 3 \) for \( i = t, b, \tau \).
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