Generalized Landauer formula for time-dependent potentials and noise-induced zero-bias dc current

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Abstract

Using a new developed single-electron approach, we derive the Landauer-type formula for electron transport in arbitrary time-dependent potentials. This formula is applied for randomly fluctuating potentials represented by a dichotomic noise. We found that the noise can produce dc-current in quantum system under zero-bias voltage by breaking the time-reversal symmetry of the transmission coefficient. We show that this effect is due to decoherence, produced by the noise, which can take place in many different systems.

Keywords: quantum transport, noise, decoherence, zero bias, dc current

(Some figures may appear in colour only in the online journal)

1. Introduction

Directed particle flow in system at equilibrium (zero-bias voltage), induced by an external periodical force, represents one of the most interesting and important effects in non-equilibrium quantum transport [1–3]. The same effect, but induced by the isotropic randomly fluctuating environment could be even more important, especially for biological systems [4, 5].

These and similar phenomena can be investigated by electron transport in mesoscopic systems under external time-dependent field. For that study one can use different approaches, like non-equilibrium Green’s function (NEGF) [6], hybrid Floquet-NEGF treatments [7, 8], time-dependent scattering state methods [9] and others. Most of them are quite complicated for applications. For this reason, a more convenient, Markovian Master equations approach,
has been widely used. However, this approach is valid for large bias limit \[ \Gamma/V \ll 1 \] where \( \Gamma \) is the energy levels width and \( V \) is the bias-voltage. Therefore it would not be suitable for zero-bias case.

In contrast, the Landauer approach to non-interacting electron transport \[11\] is free from this restriction. Indeed, the celebrated Landauer formula, relating the steady-state current to quantum transmission coefficients, is valid for any bias. However, its generalization to driven quantum transport is not straightforward. For instance, it cannot be performed without taking into account the many-particle effects \[13\]. Such an extension has been done for periodically driven quantum systems in a framework of the Floquet theory, by using the NEGF technique \[12, 13\]. As a result, the static transmission coefficients are replaced by the time-dependent transmissions in the modified Landauer formula. However, such a treatment cannot be easily adapted for arbitrary time-dependent drive, in particular for the case of randomly fluctuated energy levels and tunneling barriers.

In the present paper we use a recently proposed single-electron approach (SEA) \[14\]. This approach has been derived directly from the time-dependent Schrödinger equation by using the single-electron Ansatz for the many-electron wave function. The SEA is very different from standard NEGF techniques, and it does not involve the Floquet expansion. As a result, it yields a new generalized Landauer formula for the transient current in any time-dependent potentials. This formula is equally suitable for study electron transport in periodically modulated or in randomly fluctuating potentials \[14, 15\].

In this paper we extend our previous result to a general, non-Markovian environment. Then we concentrate on the zero-bias dc-current through single and double-dot systems, driven by an external dichotomic (telegraph) noise. A different behaviour of the both systems displays quantum nature of the zero-bias current. We show that this is a generic quantum-mechanical effect, associated with decoherence.

The plan of this paper is as following. Section 2 deals with an electron motion through quantum dot, coupled to two leads with arbitrary spectral densities. The single-electron wave-function is obtained by solving time-dependent Schrödinger equation with the time-dependent Hamiltonian.

In section 3 we introduce pure and mixed (finite temperature) many-particle states of an entire system and evaluate the time-dependent charges and currents from the many-particle wave-function. Section 4 deals with a single quantum dot under the telegraph noise and the time-dependent ensemble averaged current, flowing through this system.

Section 5 considers the electric current through a double-dot under the telegraph noise, by concentrating on the zero-bias current in the steady-state limit. Simple analytical expression is obtained for this quantity, which is compared with the exact numerical solution. Section 6 discusses generic quantum-mechanical mechanism of the zero-bias current, induced by the noise. The results and their experimental meaning are discussed in sections 4 and 7.

Derivation of the Shapiro–Loginov differential formula for a finite temperature noise is presented in appendix A. Appendix B contains some details of derivation of the single-electron Master equations used in our treatment.

2. Single-electron motion through quantum dot

Consider a single quantum dot coupled with two reservoirs, figure 1, and described by the following Hamiltonian
dependence of the energy level $E_l$ for the quantum dot. The tunneling coupling of the left (right) lead with the dot, applied only to the dot, gate voltage applied to each of the barriers and to the dot. Note that when the gate voltage is 

$$H(t) = \sum_l E_l \hat{c}_l^\dagger \hat{c}_l + \sum_r E_r \hat{c}_r^\dagger \hat{c}_r + E_0(t) \hat{c}_0^\dagger \hat{c}_0$$

$$+ \left( \sum_l \Omega_l(t) \hat{c}_l^\dagger \hat{c}_0 + \sum_r \Omega_r(t) \hat{c}_r^\dagger \hat{c}_0 + \text{H.c.} \right). \tag{1}$$

Here $\hat{c}_{l,r}(t)$ denotes the electron annihilation operator in the left (right) lead and $\hat{c}_0$ is the same for the quantum dot. The tunneling coupling of the left (right) lead with the dot, $\Omega_{l,r}(t)$, are real valued. These couplings and the energy level of the dot, $E_0(t)$, are time-dependent, where the energy levels of the leads, $E_{l,r}$, are time-independent.

The time-dependent Hamiltonian (1) can be realized experimentally via time-dependent gate voltage applied to each of the barriers and to the dot. Note that when the gate voltage is applied only to the dot, $E_0 \rightarrow E_0(t)$, it generates time-dependence of the tunneling coupling $\Omega_{l,r}$, as well. It can be seen from the Bardeen formula [16, 17]. Indeed, in the semiclassical limit it gives $\Omega_{l,r} \propto \exp \left( - \kappa_{L,R} L_{L,R} \right)$, where $L_{L,R}$ is the barrier width, $\kappa_{L,R} = \sqrt{2m[v_{L,R} - E_0(t)]}$ and $v_{L,R}$ is the barrier height. However, in the limit $\delta E_0/[v_{L,R} - E_0] \ll 1$, where $\delta E_0$, is a variation of the energy level with time, the induced time-dependence of tunneling couplings can be neglected.

Similarly, when the voltage is applied to the barrier only, $v_{L,R} \rightarrow v_{L,R}(t)$, it generates time-dependence of the dot’s energy level. It can be seen from matching of logarithmic derivative at the dot’s boundary. However, if $\delta v_{L,R}/[v_{L,R} - E_0] \ll 1$ (high barrier), the induced time-dependence of the energy level $E_0$ can be disregarded. All this implies that for high barriers, the energy dependence of tunneling couplings remains the same as for time-independent barriers. As a result, one can write

$$\Omega_{l,r}(t) \equiv \Omega_{l,r} w_{L,R}(t), \tag{2}$$

where $w_{L,R}(t)$ accounts variation of the barrier height with time.

### 2.1. Single-electron wave function

It is demonstrated in [14, 15] that the time-dependent electron current of non-interacting electrons, flowing through the quantum dot, figure 1, is totally determined by a single-electron wave function, $|\psi^{(l)}(t)\rangle$. The latter is obtained from the time-dependent Schrödinger equation,

$$i \frac{d}{dt} |\psi^{(l)}(t)\rangle = H(t) |\psi^{(l)}(t)\rangle, \tag{3}$$

where the index $k = \{l, r, 0\}$ denotes the electron’s initial state, corresponding to the occupied level $E_{l(r)}$ in the left (right) lead, or the level $E_0$ in the quantum dot, figure 1, at $t = 0$. 

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**Figure 1.** Quantum dot coupled with two leads, where $E_{l(r)}$ denote the energy levels in the left (right) lead. A single electron occupies the energy level $E_l$ in the left lead at $t = 0$. 


In order to solve equation (3), we represent the single-electron wave function, $|\psi(t)\rangle$, in the basis of the Hamiltonian (1),

$$|\psi(t)\rangle = \hat{\Phi}(t)|0\rangle$$

with

$$\hat{\Phi}(t) = \sum_i b_I^{\dagger}(t)c_i^\dagger + \sum \delta b_0^{\dagger}(t)c_0^\dagger + \sum_r b_r^{\dagger}(t)c_r^\dagger$$

where $b_{i(r)}^{\dagger}(t)$ and $b_0^{\dagger}(t)$ are probability amplitudes of finding the electron in the left (right) lead at the level $E_{i(r)}$, or inside the dot at the level $E_0$, respectively, for the initial conditions

$$b_{i(r)}^{\dagger}(0) = \delta_{iI}, \quad b_0^{\dagger}(0) = \delta_{0I}, \quad b_0^{\dagger}(0) = \delta_{00}.$$

(5)

Substituting equations (4) into (3) we obtain the following set of coupled linear differential equations for amplitudes $b(t)$,

$$i\frac{db_I^{\dagger}}{dt} = E_b b_I^{\dagger} + \Omega_I b_0^{\dagger}, \quad (6a)$$

$$i\frac{db_0^{\dagger}}{dt} = E_0 b_0^{\dagger} + \sum \Omega_I b_I^{\dagger} + \sum \Omega_R b_R^{\dagger}, \quad (6b)$$

$$i\frac{db_R^{\dagger}}{dt} = E_R b_R^{\dagger} + \Omega_R b_0^{\dagger}.$$

(6c)

Equations (6a) and (6c) for the amplitudes $b_{p \dagger}(t)$, where $p = \{l, r\}$, can be solved explicitly, thus obtaining

$$b_{p}(t) = e^{-iE_p t} \left[ \delta_{p \alpha} - \int_0^t i\Omega_p(t') b_{p}^{\dagger}(t') e^{iE_p t'} dt' \right].$$

(7)

It is useful to represent the amplitude $b_0^{\dagger}(t)$ by

$$b_0^{\dagger}(t) = b_0^{\dagger}(E_k, t) e^{-iE_k t},$$

(8)

where $\alpha = L, R$ denotes the lead, occupied initially by electron at the level $E_k$. Substituting equations (7) into (6b) and using equation (8) we find,

$$\frac{d}{dt} b_{0 \alpha}(E_k, t) = i \left[ E_0 - E_k(t) \right] b_{0 \alpha}(E_k, t) - i\Omega_k(t)$$

$$- \int_0^t \left[ G_L(t, t') + G_R(t, t') \right] e^{iE_k(t-t')} b_0^{\dagger}(E_k, t') dt',$$

(9)

with $k \in \alpha$ and

$$G_{\alpha \alpha'}(t, t') = \sum_{p \in \alpha'} \Omega_p(t) \Omega_p(t') e^{iE_p(t'-t)}.$$

(10)

Here $\alpha' = L, R$. In the continuous limit, $\sum_{p \in \alpha'} \rightarrow \int g_{\alpha'}(E_p) dE_p$, one can write

$$G_{\alpha \alpha'}(t, t') = \int_{-\infty}^{\infty} \Omega_p(t) \Omega_p(t') e^{iE_p(t'-t)} g_{\alpha'}(E_p) dE_p,$$

(11)

where $g_{\alpha'}(E_p)$ is the density of state of a lead $\alpha'$. 

4
2.2. Charges and currents

Solving the integro-differential equation (9), we obtain the amplitude \( b_0^{(\alpha)}(E_k,t) \) and therefore the probability of finding the dot occupied,

\[
q_0^{(\alpha)}(E_k,t) = \langle \psi_k(t)|\hat{c}_0^\dagger \hat{c}_0|\psi_k(t)\rangle = |b_0^{(\alpha)}(E_k,t)|^2. \tag{12}
\]

Using equations (9) and (12) one can derive the following useful relation for time-derivative of this quantity

\[
\frac{d}{dt} q_0^{(\alpha)}(E_k,t) = -2i\text{Im}[b_0^{(\alpha)}(E_k,t)]\Omega_\alpha(t)
\]

\[
-2\text{Re} \int_0^t b_0^{(\alpha)*}(E_k,t)b_0^{(\alpha)}(E_{k'},t')G(t,t')e^{iE_k(t-t')}dt'. \tag{13}
\]

where \( k \in \alpha \) and \( G(t,t') = G_L(t,t') + G_R(t,t') \).

Consider now the single-electron current in the lead \( \alpha' = L,R \) (in units of the electron charge), given by

\[
I_{\alpha'}(E_k,t) = i \langle \psi_k(t)|\left[ H(t), \sum_{p \in \alpha'} \hat{c}_p^\dagger \hat{c}_p \right]|\psi_k(t)\rangle
\]

\[
= i \sum_{p \in \alpha'} \Omega_p(t)\langle \psi_k(t)|\hat{c}_p^\dagger \hat{c}_p - \hat{c}_p^\dagger \hat{c}_p|\psi_k(t)\rangle
\]

\[
= 2\text{Im} \sum_{p \in \alpha'} \Omega_p(t)b_p^{(\tilde{k})}(t)b_p^{(\tilde{k})*}(t). \tag{14}
\]

Note that in equations (12)–(14) the index \( \alpha' \) denotes the lead where the current \( I_{\alpha'} \) is evaluated, whereas the indices \( \alpha \) and \( k \in \alpha \) denote the lead and energy level \( (E_k) \), occupied by electron at \( t = 0 \). We also point out that the electron’s initial state is not an eigen-state of the total Hamiltonian. Therefore the electron’s wave function in the final state is spread over different energy levels of the lead, even for the time-independent Hamiltonian.

In addition we emphasize that the continuous limit corresponds to the leads size increasing to infinity. Otherwise the leads spectrum remains discrete, so that the electron’s wave function cannot reach the steady-state limit at \( t \to \infty \). Thus, the steady-state limit always implies the following order of the limits: first, the size of leads goes to infinity and then \( t \to \infty \).

Using equations (7)–(11), we can rewrite the single-electron current as

\[
I_{\alpha'}(E_k,t) = 2\text{Im}[b_0^{(\alpha)}(E_k,t)]\Omega_k(t)\delta_{\alpha\alpha'}
\]

\[
+ 2\text{Re} \int_0^t b_0^{(\alpha)*}(E_k,t)b_0^{(\alpha)}(E_{k'},t')G_{\alpha'}(t,t')e^{iE_k(t-t')}dt'. \tag{15}
\]

where \( \tilde{k} \in \alpha \). Hence, for \( \alpha' \neq \alpha \), the current \( I_{\alpha'}(E_k,t) \) is given by the last term of equation (15) only. Such a single-electron current represents time-dependent extension of the quantum-mechanical transmission probability (transmission coefficient), defined as

\[
T_{L \rightarrow R}(E_i,t) = 2\pi g_L(E_i) I_R(E_i,t). \tag{16a}
\]

\[
T_{R \rightarrow L}(E_i,t) = 2\pi g_R(E_i) I_L(E_i,t). \tag{16b}
\]

The first one, \( T_{L \rightarrow R} \), is transmission probability from the level \( E_i \) in the left lead to the right lead. Respectively, the second one, \( T_{R \rightarrow L} \), is transmission probability from the level \( E_i \) in the right lead to the left lead. It is demonstrated below that for time-independent Hamiltonian,
where $T(E)$ is the standard quantum mechanical transmission coefficient. Note that equation (17) displays the time-reversal symmetry of the transmission probability, valid for any time-independent Hamiltonian.

Let us consider the single-electron currents, $I_R(E_r, t)$ and $I_L(E_l, t)$, when the initial state belongs to the same lead, $\alpha = \alpha'$ in equation (15). In this case, the current is given by two terms of equation (15). Using equation (13) we obtain

$$I_R(E_r, t) = -\frac{d}{dt} \langle \alpha' | \langle E_r | t \rangle \langle E_r | t \rangle \rangle_{\alpha} - I_L(E_l, t).$$

The same expression is found for $I_L(E_l, t)$, by interchanging $R \leftrightarrow L$ and $r \leftrightarrow l$.

### 2.3. Wide-band limit

In the Markovian case (wide-band limit), the spectral-density function, $\Omega_{\alpha'}(t)\varrho(E_p)$ is independent of energy. This implies that $\Omega_{\alpha'}(t)\varrho(E_p) = \Omega_{\alpha'}(t)\varrho(E_p)$, in equation (11). As a result,

$$G_{\alpha'}(t, t') = 2\pi \Omega_{\alpha'}(t)\Omega_{\alpha'}(t')\varrho(t', t).$$

Then using $\int^{t'}_{t} b_0^{(\alpha)}(E_k, t')\delta(t' - t)dt' = b_0^{(\alpha)}(E_k, t)/2$, we reduce equation (9) to the following equation

$$\frac{d}{dt} b_0^{(\alpha)}(E_k, t) = \left[ E_k - E_0(t) + i \frac{\Gamma(t)}{2} \right] b_0^{(\alpha)}(E_k, t) - i\Omega_{\alpha}(t),$$

(20)

where

$$\Gamma(t) \equiv \Gamma_L(t) + \Gamma_R(t) = 2\pi \Omega_L^2(t)\varrho_L + 2\pi \Omega_R^2(t)\varrho_R,$$

(21)

is total time-dependent width of the level $E_0(t)$. The time-dependent transmission coefficients, equations (16a) and (16b) are given by

$$T_{L\to R}(E_f, t) = 2\pi q_0^{(L)}(E_k, t)\Gamma_R(t),$$

$$T_{R\to L}(E_f, t) = 2\pi q_0^{(R)}(E_k, t)\Gamma_L(t)$$

(22)

where $q_0^{(\alpha)}(E_k, t) = |b_0^{(\alpha)}(E_k, t)|^2$ is probability of finding the dot occupied, equation (12), by an electron coming from the lead $\alpha$.

Equation (20) can be solved straightforwardly. One finds [14, 15],

$$q_0^{(\alpha)}(E_k, t) = \left| \int^{t'}_{t} \Omega_{\alpha}(t')e^{i[\mathcal{E}_0(t') - E_k(t') - i\mathcal{E}_0(t')]dt'} \right|^2,$$

(23)

where

$$\mathcal{E}_0(t) = \int^{t'}_{t} \left[ E_0(t') - i \frac{\Gamma(t')}{2} \right] dt'.$$

(24)
If the electron is initially inside the dot, \( \vec{k} = 0 \), then
\[
q_0^{(0)}(t) = q_0^{(0)}(0)e^{-\int_0^t \Gamma'(t')dt'}.
\] (25)

In the case of time-independent Hamiltonian, one easily obtains from equations (23) and (24) that \( T_{L \rightarrow R} = T_{R \rightarrow L} \equiv T(E, t) \), where
\[
T(E, t) = \frac{\Gamma_L \Gamma_R}{(E - E_0)^2 + \Gamma^2} \left[ 1 - 2 \cos(Et)e^{-\Gamma t/2} + e^{-\Gamma t} \right].
\] (26)

In the asymptotic limit, \( t \to \infty \), this expression reproduces a well-known Breit–Wigner formula for resonant transmission, \( \bar{T}(E) = \frac{\Gamma_L \Gamma_R}{(E - E_0)^2 + \frac{\Gamma^2}{4}} \).

2.4. Finite-band leads

Consider now the leads of a finite band-width \( W_a \), centered at \( \epsilon_a \), where \( a = \{L, R\} \). For simplicity, we assume high barrier height, so \( \Omega_{L,R}(t) \) are represented by equation (2), where \( \Omega_{L,R} \equiv \Omega_{L,R}(E_{L,R}) \) is the tunneling coupling for a static barrier. The lead’s spectral function is usually parameterized as
\[
\Omega_a^2(E_p)\rho_a(E_p) = \frac{\Gamma_a}{2\pi} \sqrt{1 - \left( \frac{E_p - \epsilon_a}{W_a} \right)^2},
\] (27)
corresponding to a semi-infinite lead, consisted of periodic one-dimensional chain of quantum wells with the nearest-neighbor coupling.

Substituting equations (2) and (27) into (11) we can evaluate \( G_a(t, t') \), and then insert it into the integro-differential equation (9). Unfortunately, despite its simple form, the spectral density (27) does not allow us to solve equation (9) analytically. For this reason, we approximate equation (27) by a Lorentzian
\[
\Omega_a^2(E_p)\rho_a(E_p) = \frac{\Gamma_a}{2\pi} \frac{\Lambda_a^2}{(E_p - \epsilon_a)^2 + \Lambda_a^2},
\] (28)
with \( \Lambda_a = \sqrt{2} W_a \). The latter provides the same curvature at the band center, making the Lorentzian (28) a very good approximation for a finite range spectral function (27) (see [18]).

Now we demonstrate that the Lorentzian form of spectral-density function (28) allows us to reduce the integro-differential equation (9) to a system of coupled linear differential equations. Substituting equations (28) into (11), we integrate over \( E_p \), thus obtaining
\[
G_a(t, t') = g_a(t)g_a(t')e^{i(\epsilon_a - i\Lambda_a)(t' - t)}
\] (29)
where \( t' \leq t \) and
\[
g_a(t) = \sqrt{\frac{\Gamma_a \Lambda_a}{2W_a}} w_a(t).
\] (30)

Substituting equations (29) into (9) and introducing the auxiliary amplitude
\[
b_a^{(\alpha)}(E_k, t) = -ig_a(t) \int_0^t e^{i(\epsilon_k - i\Lambda_a)(t' - t)} g_0^{(\alpha)}(E_k, t') dt'
\] (31)
where \( \alpha = L, R \) denotes the reservoir, occupied by electron at \( t = 0 \), we can rewrite equation (9) as a system of coupled equations.
dot coupled to two Markovian leads with density of states
The same expression is obtained for (31), we find
schematically in figure 2. Two of the dots (left and right) are fictitious, which account for the
In this basis, the original Hamiltonian, equation (1) can be mapped to the following one,
Equation (32) have simple interpretation. They describe electron transport through a triple-
Figure 2. Quantum dot coupled to two fictitious wells, which incorporate the non-
Solving these equations, we can evaluate the single electron current, equation (15) and the
time-dependent transmissions, equations (15) and (16). For instance, using equations (29) and
we find

\[
\frac{d}{dt} b^{(\alpha)}_L(E_k, t) = i \left[ E_k - E_0(t) \right] b^{(\alpha)}_L(E_k, t) - i g_L(t) b^{(\alpha)}_L(E_k, t) - i \Omega_L(t)
\]

(32a)

\[
\frac{d}{dt} b^{(\alpha)}_R(E_k, t) = i \left[ E_k - \epsilon_L + i \Lambda_L \right] b^{(\alpha)}_L(E_k, t) - i g_L(t) b^{(\alpha)}_L(E_k, t)
\]

(32b)

\[
\frac{d}{dt} b^{(\alpha)}_R(E_k, t) = i \left[ E_k - \epsilon_L + i \Lambda_L \right] b^{(\alpha)}_L(E_k, t) - i g_R(t) b^{(\alpha)}_L(E_k, t)
\]

(32c)

The same expression is obtained for \( T_{L \rightarrow R}(E_i, t) \), equation (16b), with \( R \leftrightarrow L, \tilde{r} \leftrightarrow \tilde{r} \).
Equation (32) have simple interpretation. They describe electron transport through a triple-
dot coupled to two Markovian leads with density of states \( \rho_{L,R} \), by a coupling \( \Omega_{L,R} \), as shown schematically in figure 2. Two of the dots (left and right) are fictitious, which account for the non-Markovian (Lorentzian) component of the corresponding lead’s spectrum. It implies that the original reservoir basis \( |l\rangle, |r\rangle \) is split into two components [18, 19]

\[
|l\rangle \langle l| = |L\rangle \langle L| + \sum_{l'} |l'\rangle \langle l'|
\]

\[
|r\rangle \langle r| = |R\rangle \langle R| + \sum_{r'} |r'\rangle \langle r'|
\]

(35)

In this basis, the original Hamiltonian, equation (1) can be mapped to the following one,

\[
H_1(t) = \sum_{l} E_l \hat{c}^\dagger_l \hat{c}_l + \sum_{r} E_r \hat{c}^\dagger_r \hat{c}_r + \epsilon_L \hat{c}^\dagger_L \hat{c}_L + \epsilon_R \hat{c}^\dagger_R \hat{c}_R
\]

\[
+ E_0(t) \hat{c}^\dagger_0 \hat{c}_0 + \left( g_L(t) \hat{c}^\dagger_L \hat{c}_0 + g_R(t) \hat{c}^\dagger_R \hat{c}_0 \right)
\]

\[
+ \sum_{l} \Omega^L_{L} \hat{c}^\dagger_l \hat{c}_0 + \sum_{r} \Omega^R_{R} \hat{c}^\dagger_r \hat{c}_0 + \text{H.c.} \right)
\]

(36)
One can check that the equation of motion for the wave-function, produced by this Hamiltonian, coincides with equation (32), obtained from the Hamiltonian (1), with the spectral function (28), providing that $\pi \Omega_{LR}^2 \delta_{LR} = \Lambda_{LR}$. The latter can be considered as level-widths of the fictitious dots, figure 2. Note that the same mapping, as given by equation (36), has been recently obtained for a rather general case, but using a different method [20].

In the case of time-independent Hamiltonian, $E_0(t) = E_0$ and $w_{LR} = 1$, equation (32) can be easily solved, in particular in the steady-state limit ($t \to \infty$). Then the l.h.s of equation (32) vanishes, so these equations becomes algebraic. It is useful to introduce the amplitudes $\bar{B}_0^{(\alpha)}(E_k)$, defined as $\Omega_{\alpha} \bar{B}_0^{(\alpha)}(E_k) = b_0^{(\alpha)}(E_k, t \to \infty)$ and $\Omega_{\alpha} \bar{B}_{LR}^{(\alpha)}(E_k) = b_{LR}^{(\alpha)}(E_k, t \to \infty)$. One obtains

$$(E_k - E_0)\bar{B}_0^{(\alpha)} - g_L \bar{B}_0^{(\alpha)} - g_R \bar{B}_0^{(\alpha)} = 1$$

$$(E_k - \epsilon_L + i\Lambda_L)\bar{B}_L^{(\alpha)} - g_L \bar{B}_0^{(\alpha)} = 0$$

$$(E_k - \epsilon_R + i\Lambda_R)\bar{B}_R^{(\alpha)} - g_R \bar{B}_0^{(\alpha)} = 0.$$  

Solving these equations we find

$$\bar{B}_0^{(\alpha)}(E_k) = \frac{1}{E_k - E_0 - \frac{g_L}{\epsilon_L - \epsilon_R + i\Lambda_L} - \frac{g_R}{\epsilon_R - \epsilon_L + i\Lambda_L}}$$

where $k \in \alpha$, and

$$\bar{B}_{LR}^{(\alpha)}(E_k) = \frac{g_{LR}}{E_k - \epsilon_{LR} + i\Lambda_{LR}} \bar{B}_0^{(\alpha)}(E_k).$$

Let us evaluate the steady-state transmission coefficient $\bar{T}_{L \to R}(E) = T_{L \to R}(E, t \to \infty)$, equation (33). One finds from equations (38) and (39),

$$\text{Im} \left[ \frac{d^{(L)}_{LR}(E)}{q_{LR}(E)} \right] = \frac{\Omega_{LR}^2(E) g_{LR} \Lambda_{LR}}{(E - \epsilon_R)^2 + \Lambda_{LR}^2} |\bar{B}_0^{(L)}(E)|^2$$

where $d^{(L)}_{LR}(E) = q^{(L)}_{LR}(E, t \to \infty)$ Substituting this result into equation (33) by using equation (28), we finally obtain

$$T_{L \to R}(E) = \frac{\Lambda_{LR}^2 \Gamma_{LR} \Lambda_{LR}^2 |\bar{B}_0^{(L)}(E)|^2}{[E - \epsilon_L)^2 + \Lambda_{LR}^2][E - \epsilon_R)^2 + \Lambda_{LR}^2].$$

The same expression is obtained for $\bar{T}_{R \to L}(E)$, thus verifying the time-reversal symmetry, equation (17), for leads of finite band-width. It is easy to find that in the Markovian limit, $\Lambda_{LR} \to \infty$, equation (41) coincides with the Breit–Wigner formula, equation (26) for $t \to \infty$.

Finally, we point out that the last term in equation (32a) is originated by the initial condition, corresponding to occupied level ($E_k$) in one of the leads, figure 1. In the new basis, equation (35), it would correspond to a linear superposition of states $|\ell'\rangle$ and $|L\rangle$ (or $|r'\rangle$ and $|R\rangle$). This implies that by solving the time-dependent Schrödinger equation for the initial condition, corresponding to occupied state $|\ell'\rangle$ (or $|r'\rangle$) in a fictitious reservoir, figure 2, the time-dependence of transmission coefficient $T(E,t)$, would be different. However the steady-state limit of the transmission coefficient, $\bar{T}(E)$, does not depend on the initial state, and it is always given by equation (41).
3. Time-dependent current of non-interacting electrons

Now we are going to many-electron current, flowing between the leads. Although the electrons are treated as non-interacting particles, it is necessary to consider many-body wavefunction to account non-trivial Pauli principle effects, in particular for the time-dependent Hamiltonian [13]. Such a treatment has been done in [14, 15], concentrating on the leads at zero temperature. Here we present a general case.

3.1. Pure and mixed states

Consider the leads \((L, R)\), figure 1, which are filled at \(t = 0\) by \(N_L, N_R\) electrons, respectively. These numbers vary in time, but the total number of electrons, \(N = N_L + N_R + n_0\) remains constant. (Here \(n_0 = 0, 1\) denotes number of electrons, initially occupying the quantum dot).

In the following we consider the limit of \(N_L, N_R \to \infty\).

The wave-function of an entire system, \(|\Psi(\nu)(t)\rangle\), can be written at \(t = 0\) as

\[
|\Psi(\nu)(0)\rangle = (c_0^\dagger n_0 \prod_{\bar{k} \in \nu} c_\bar{k}^\dagger)|0\rangle
\]

where \(\nu\) denotes a particular configuration of the reservoirs energy levels \((E_{\bar{k}})\), occupied by \(N_{L,R}\) electrons at \(t = 0\). For instance, if each of the leads at \(t = 0\) is taken at zero temperature [15], then \(\nu\) comprises the states \(\bar{k} = \{l, r\}\) of the leads, corresponding to \(E_l \leq \mu_L\) and \(E_r \leq \mu_R\), where \(\mu_L, R\) denote Fermi energies of the leads.

In order to obtain the total many-body wave-function \(|\Psi(\nu)(t)\rangle\) as a solution of time-dependent Schrödinger equation, we use an Ansatz for \(|\Psi(\nu)(t)\rangle\), by taking it as a (Slater) product of single-electron wave functions,

\[
|\Psi(\nu)(t)\rangle = \prod_{\bar{k} \in \nu} \hat{\Phi}^{(\bar{k})\dagger}(t)|0\rangle
\]

where \(\hat{\Phi}^{(\bar{k})\dagger}(t)\) is given by equations (4) and (6) with the initial conditions equation (5).

If the state of quantum system is described by a wave function, then it is referred to as a ‘pure’ state. However, in general, the system can be in a ‘mixture’ of pure states, described by the density matrix \(\rho(t)\). The latter is obtained from the von Neumann equation

\[
i \frac{d}{dt} \rho(t) = [H(t), \rho(t)]
\]

uniquely defined by the initial condition

\[
\rho(0) = \sum_{\nu} p_\nu |\Psi(\nu)(0)\rangle \langle \Psi(\nu)(0)|
\]

where \(p_\nu\) is a probability of finding the system in a configuration \(\nu\). One easily finds that the density matrix

\[
\rho(t) = \sum_{\nu} p_\nu |\Psi(\nu)(t)\rangle \langle \Psi(\nu)(t)|
\]

with \(|\Psi(\nu)(t)\rangle\) given by equation (43), is indeed the unique solution of equation (44), corresponding to the initial condition (45).

Thus, instead of solving the matrix equation (44) directly, one can solve the Schrödinger equation for a single electron wave-function, equation (4), for different initial states of the
electron \((E_k)\), corresponding to a set \(\nu\). Each of the many-body wave functions of this set is given by the Slater product of the single-electron wave functions, equation (43). Finally, the density matrix of a general mixture state is an incoherent mixture of these pure many-body states \((\nu)\), equation (46).

One can easily realize that \(p_\nu\) can be replaced by the initial electron distribution, \(f_{L,R}(E_k)\), in the left/right lead, respectively. In the continuous limit \(\int_\infty^\infty f_{L,R}(E)\vartheta_{L,R}(E)\,dE = N_{L,R} \to \infty\). For the definiteness \(f_{L,R}(E)\) are represented by Fermi functions, \(f(E) = 1/[1 + e^{(E - \mu)/T}]\). However, it can be any other distribution. Note that the steady state implies the same order of limits as in the previous case of a single-electron wave function. In addition the limit \(N_{L,R} \to \infty\) takes place before the limit of \(t \to \infty\).

3.2. Charges and currents

It was shown in \([14, 15]\) that the total charge on the dot at time \(t\), \(Q_0(t)\), is an incoherent sum of single-electron probabilities, equation (12), over all the states \(k\), initially occupied by electrons,

\[
Q_0(t) = \sum_{\alpha,k} q_0^{(\alpha)}(E_k,t) \equiv Q_0^{(L)}(t) + Q_0^{(R)}(t) + q_0^{(0)}(t)
\]  

(47)

where \(\alpha = L,R,0\). Here we separated the sum over \(k = \{L,R,0\}\) into three contributions for electrons coming from the left/right leads and the quantum dot. In continuous limit one can write

\[
Q_0^{(L,R)}(t) = \int_{-\infty}^{\infty} q_0^{(L,R)}(E,t)f_{L,R}(E)\vartheta_{L,R}(E)dE
\]  

(48)

where \(E \equiv E_k\).

Similar to equations (47) and (48), it was shown in \([14, 15]\) that the total current in the leads is given by an incoherent sum of single-electron currents (transmission probabilities), equations (15) and (16), over all states \(k\), initially occupied by electrons, \(I_\alpha(t) = \sum_k I_\alpha(E_k,t)\), where \(\alpha = L,R\) denotes the lead. For instance, using equation (18), we obtain for the right-lead current

\[
I_R(t) = \int_{-\infty}^{\infty} \left[T_{L \to R}(E,t)f_L(E) - T_{R \to L}(E,t)f_R(E)\right] \frac{dE}{2\pi} - \dot{Q}_0^{(R)}(t) + \dot{Z}_0^{(0)}(t),
\]

(49)

where \(E \equiv E_k\). The left-lead current, \(I_L(t)\), is given by the same formula with \(R \leftrightarrow L\) and by reversing the sign of current. In general, the circuit current is

\[
I(t) = \beta_R I_R(t) + \beta_L I_L(t)
\]

(50)

where the coefficients \(\beta_L, \beta_R\) with \(\beta_L + \beta_R = 1\) are depending on a circuit geometry (the junction capacities) \([21]\).

The last term of equation (49), \(\dot{Z}_0^{(0)}(t)\), is given by equation (15) for \(k = 0\). It always vanishes for \(t \to \infty\) (see equation (25)) and is identically zero if the dot is initially empty, \(q_0^{(0)}(0) = 0\). The second term of equation (49), \(\dot{Q}_0^{(R)}(t)\), represents so-called ‘displacement’ current \([22]\), originated by a retardation of the current flowing through the dot to the same reservoir.
If the system reaches its steady state limit at $t \to \infty$ the displacement current vanishes. As a result, equation (49) (and equation (50)) for the steady-state current, $I = I(t \to \infty)$ reads

$$I = \int_{-\infty}^{\infty} \left[ T_{L \to R}(E)f_L(E) - T_{R \to L}(E)f_R(E) \right] \frac{dE}{2\pi}. \quad (51)$$

In the case of time-reversal symmetry, equation (17), one finds that equation (51) becomes the standard Landauer formula. Thus, equation (49) can be considered as a generalization of the Landauer formula to transient currents and time-dependent potentials.

Equation (49) looks similar to the time-dependent Landauer-type formula, (equation (44) of [13]), obtained for a periodic drive by using the Floquet approach [12, 13]. In contrast, our results are derived without any use of the Floquet expansion. Hence, despite its ‘scattering’ form, equation (49) is valid for arbitrary time-dependent drive. In addition, our time-dependent transmission probabilities, are directly related to the time-dependent occupation amplitude of a quantum system, given by equation (9). The latter is obtained from a single-particle Schrödinger equation with arbitrary initial state at $t = 0$ (or at any other time). Therefore we do not need to consider the initial state at $t \to -\infty$ (as in the standard scattering theory). This considerably simplifies the treatment.

In the following, we use the generalized Landauer formula, to study time-dependent current at zero-bias ($f_L(E) = f_R(E)$). Note, that in the case of time-independent Hamiltonian, the time-reversal symmetry of the transmission probability holds. As a result, the steady-state zero-bias current would always vanish, equation (51). Below we investigate the zero-bias steady-state current for randomly fluctuating potentials.

4. Electron current through fluctuating level

Let us return to quantum dot, coupled to two (Markovian) reservoirs, figure 1, described by the Hamiltonian (1). Now we consider the tunneling couplings as time and energy independent, $\Omega_{L,R}(t) = \Omega_{L,R}$ ($\omega_{L,R} = 1$ in equation (2)), but the energy level of the dot is time-dependent

$$E_0(t) = E_0 + \xi(t) \frac{U}{2}, \quad (52)$$

where $\xi(t) = \pm 1$ is jumping randomly from 1 to $-1$ (or from $-1$ to 1) at a rate $\gamma_+$ (or $\gamma_-$), independently of its previous history. This represents so-called ‘telegraph noise’. We denote $P_{\pm}(t)$ as probabilities for finding $\xi(t)$ at the values $\xi = \pm 1$ at time $t$, while $P_+(t) + P_-(t) = 1$. This quantity is obtained from the corresponding Markovian rate equation

$$\dot{P}_{\pm}(t) = -\gamma_{\pm}P_{\pm}(t) + \gamma_{\mp}P_{\mp}(t) = -\gamma P_{\pm}(t) + \gamma_{\mp} \quad (53)$$

where $\gamma = \gamma_+ + \gamma_-$. Solving equation (53) we find

$$P_{\pm}(t) = \frac{\gamma_{\mp}}{\gamma} + \left[ P_{\pm}(0) - \frac{\gamma_{\mp}}{\gamma} \right] e^{-\gamma t}. \quad (54)$$

Thus, in the steady-state limit, the distributions are independent of the initial condition, $P_{\pm} = P_{\pm}(t \to \infty) = \gamma_{\mp}/\gamma$. If the noise is generated by a heat bath of temperature $T$ (see for instance, [24]), then

$$\frac{\gamma_+}{\gamma_-} = \frac{P_-}{P_+} = e^{U/T}. \quad (55)$$

The average value of $\xi(t)$ is
\[ \bar{\xi} = \langle \xi(t) \rangle = \sum_{\xi=\pm 1} P_\xi \xi = \frac{\gamma_- - \gamma_+}{\gamma} = \frac{1 - e^{U/T}}{1 + e^{U/T}}. \] (56)

Therefore \( \bar{\xi} = 0 \) for infinite temperature, \( (\gamma_+ = \gamma_- = \gamma/2) \) and \( \bar{\xi} = -1 \) for zero temperature \( (\gamma_+ = \gamma, \gamma_- = 0) \). The case of infinite temperature has been considered in [15]. Here we consider a finite temperature, \( T \).

In fact, the telegraph noise may not be related to thermal fluctuations. For instance, it can be generated by a fluctuator, representing by a quantum dot at high bias voltage [25, 27–29], (the upper dot in figure 3). There an electron from the left lead enters the dot with tunneling rate \( \gamma_- \) and leaves it to the right lead with tunneling rate \( \gamma_+ \). As a result the charge inside the dot is fluctuating in time, creating fluctuation of the energy level \( E_0 \) of a nearby quantum dot via the Coulomb interaction \( U \).

If the upper dot is under large bias, there is no back action of the lower dot on charge fluctuations inside the upper dot [25, 26]. Indeed, if \( \epsilon_0 \) and \( \epsilon_0 + U \) are deeply inside the bias, the level’s shift \( U \) due to the electron–electron interaction does not affect the charge-correlator of the upper dot. Thus, the effect of the upper dot on the lower one is entirely accounted for by an external telegraph noise fluctuating the level \( E_0 \).

An absence of the back action can be verified experimentally by measuring the current \( I \) in the upper dot, when the lower dot is set at zero bias \( \mu_L = \mu_R = \mu \). If the current \( I \) and its noise spectrum remain the same for \( \mu \ll E_0 \) (empty dot) and \( \mu \gg E_0 \) (occupied dot), then the modified Landauer formula, equation (49), for the quantum-mechanical (ensemble averaged) current, \( I(t) \), can be applied.

In the case of noise, equation (49), must include an additional ensemble average, \( T(E, t) \to \langle T(E, t) \rangle \) and \( Q(t) \to \langle Q(t) \rangle \), over all particular histories of the noise. As follows from equations (16), (48) and (49), it can be done by averaging the probability, \( q^{(\alpha)}_0(E, t) \to \langle q^{(\alpha)}_0(E, t) \rangle \), given by equation (23). In fact, it is more convenient to average the corresponding probability amplitude, \( b^{(\alpha)}_0(E, t) \), for finding the dot occupied, equation (20) with \( E_0(t) \) given by equation (52). It reads

\[ \frac{db^{(\alpha)}_0(E, t)}{dt} = i \left[ E - \xi(t) \frac{U}{2} + i \frac{\Gamma}{2} \right] b^{(\alpha)}_0(E, t) - i \Omega_{\alpha}, \] (57)
where \( E \equiv E_k \) and \( \Gamma = 2 \pi (\Omega_k^2 \partial_t + \Omega_k^2 \partial_R). \) (We choose the scale where \( E_0 = 0.) \) The average probability \( \langle q_0^{(a)}(E, t) \rangle = \langle b_0^{(a)}(E, t)^2 \rangle \), can be determined from equation (13), which in the wide-band limit, equation (19), becomes

\[
\frac{d}{dt} q_0^{(a)}(E, t) = -\Gamma q_0^{(a)}(E, t) - 2\Omega_\alpha \text{Im}[b_0^{(a)}(E, t)]. \tag{58}
\]

Solving this differential equation and then averaging over the noise we obtain [15]

\[
\langle q_0^{(a)}(E, t) \rangle = -2\Omega_\alpha \int_0^t e^{\Gamma(t'-t)} \text{Im}\left[ b_0^{(a)}(E, t') \right] dt'. \tag{59}
\]

This equation directly relates \( \langle q_0^{(a)}(E, t) \rangle \) to \( \langle b_0^{(a)}(E, t) \rangle \). Note that \( \langle q_0^{(a)}(E, t) \rangle = (\langle b_0^{(a)}(E, t) \rangle)^2 \neq |\langle b_0^{(a)}(E, t) \rangle|^2. \)

For averaging equation (57) over the noise, we need to evaluate the term \( \langle \xi(t) b_0^{(a)}(E, t) \rangle. \) In order to do it, we multiply equation (57) by \( \xi(t) \), taking into account that \( \xi^2(t) = 1. \) As a result,

\[
\langle \xi(t) \frac{d}{dt} b_0^{(a)}(E, t) \rangle = \left[ E + \frac{\Gamma}{2} \right] \langle \xi(t) b_0^{(a)}(E, t) \rangle - i \frac{U}{2} \langle b_0^{(a)}(E, t) \rangle - i \Omega_\alpha \xi \tag{60}
\]

where \( \xi = (\gamma_- - \gamma_+)/\gamma, \) equation (56).

In the case of exponential noise-correlator, \( \langle \xi(t_1) \xi(t_2) \rangle \propto \exp(-|t_1 - t_2|) \), one can use the following ‘differential formula’, representing an extension of Shapiro and Loginov result [30] (amended for a non-symmetric noise in appendix A),

\[
\frac{d}{dt} \langle \xi(t) R[\xi(t), t] \rangle = \langle \xi(t) \frac{d}{dt} R[\xi(t), t] \rangle - \gamma \langle \xi(t) R[\xi(t), t] \rangle + \gamma \xi R[\xi(t), t] \tag{61}
\]

\( R[\xi(t), t] \) is an arbitrary functional of the noise. In our case \( R[\xi(t), t] \equiv b_0^{(a)}(E, t). \) Substituting equations (61) into (60) and average over the noise equation (57), we obtain a system of two coupled differential equations for two functions \( \langle b_0^{(a)}(E, t) \rangle \) and \( \langle b_0^{(a)}(E, t) \rangle = \langle \xi(t) b_0^{(a)}(E, t) \rangle),

\[
\frac{d}{dt} \langle b_0^{(a)}(E, t) \rangle = \left[ E + i \frac{\Gamma}{2} \right] \langle b_0^{(a)}(E, t) \rangle - i \frac{U}{2} \langle b_0^{(a)}(E, t) \rangle - i \Omega_\alpha \xi, \tag{62a}
\]

\[
\frac{d}{dt} \langle b_0^{(a)}(E, t) \rangle = \left[ E + i \left( \frac{\Gamma}{2} + \gamma \right) \right] \langle b_0^{(a)}(E, t) \rangle + \left( \gamma \xi - i \frac{U}{2} \right) \langle b_0^{(a)}(E, t) \rangle - i \Omega_\alpha \xi. \tag{62b}
\]

Equation (62) represent a non-trivial extension of our previous work [15], considered only symmetric noise, \( \xi = 0 \) (corresponding to infinite noise temperature, \( T \to \infty \) in equation (55)). These equations can be simplified furthermore by introducing the following variables,

\[
b_{0_{b,a}}^{(a)}(E, t) = \langle b_0^{(a)}(E, t) \rangle \pm \langle b_0^{(a)}(E, t) \rangle. \tag{63}
\]
Then equation (62) becomes
\[
\frac{d}{dt} b_{0\pm}^{(\alpha)}(E, t) = i \left( E + \frac{U}{2} + i \frac{\Gamma}{2} \right) b_{0\pm}^{(\alpha)}(E, t) \\
- \gamma_{\pm} b_{0\pm}^{(\alpha)}(E, t) + \gamma_{\mp} b_{0\mp}^{(\alpha)}(E, t) - 2i \frac{\gamma_{\mp}}{\gamma} \Omega_{\alpha}.
\] (64)

Respectively, the probability of finding the dot occupied, averaged over the noise, equation (59), can be written as
\[
\langle q_{0}^{(\alpha)}(E, t) \rangle = \frac{1}{2} [q_{0\pm}^{(\alpha)}(E, t) + q_{0-}^{(\alpha)}(E, t)]
\] (65)

where \( q_{0\pm}^{(\alpha)}(E, t) \) is given by equation (59) with replacements \( q_{0}^{(\alpha)}(E, t) \rightarrow q_{0\pm}^{(\alpha)}(E, t) \) and \( b_{0\pm}^{(\alpha)}(E, t) \rightarrow b_{0\pm}^{(\alpha)}(E, t) \). Finally, the time-dependent transmission, averaged over the noise, \( \langle T(E, t) \rangle \), is given by equation (22).

Equations (64) and (65) can be considered as describing an electron with a pseudo-spin 1/2, traveling through a quantum dot, when the electron interacts with a fictitious field. The latter splits the energy level of the dot into two sub-levels, \( E_{0} \rightarrow E_{0} \pm U/2 \), depending on direction of the pseudo-spin. The noise produces jumps between these components with rates \( \gamma_{\pm} \), accounted for by the ‘loss’ and ‘gain’ terms, \( -\gamma_{\pm} b_{0\pm}^{(\alpha)} \) and \( \gamma_{\mp} b_{0\mp}^{(\alpha)} \), in equation (64). The last (source) term in this equation accounts the electron jump from its initial state (in the leads) to the quantum dot.

4.1. Steady-state limit

In steady-state limit, \( \langle d/dr \rangle b_{0\pm}^{(\alpha)}(E, t, t \rightarrow \infty) \rightarrow 0 \), equation (64) become a system of algebraic equations
\[
\left( E + \frac{U}{2} + i \frac{\Gamma}{2} \right) \bar{b}_{0\pm}^{(\alpha)}(E) \\
+ i \gamma_{\pm} \bar{b}_{0\pm}^{(\alpha)}(E) - i \gamma_{\pm} \bar{b}_{0\mp}^{(\alpha)}(E) = 2 \frac{\gamma_{\mp}}{\gamma} \Omega_{\alpha}.
\] (66)

Here we denoted \( \bar{T}_{0\pm}^{(\alpha)}(E) \equiv b_{0\pm}^{(\alpha)}(E, t \rightarrow \infty) \). Respectively, in the same limit, equation (58), averaged over the noise, becomes
\[
\bar{T}_{0}^{(\alpha)}(E) = - \frac{\Omega_{\alpha}}{\Gamma} \text{Im} \left[ \bar{T}_{0+}^{(\alpha)}(E) + \bar{T}_{0-}^{(\alpha)}(E) \right]
\] (67)

where \( \bar{T}_{0}^{(\alpha)}(E) = \langle q_{0}^{(\alpha)}(E, t) \rangle_{t \rightarrow \infty} \).

Finally, using equations (22) and (67), we find for the steady-state transmission probability, averaged over the noise, \( \bar{T}_{L \rightarrow R}(E) = \bar{T}_{R \rightarrow L}(E) = \bar{T}(E) \)
\[
\bar{T}(E) = - \frac{\Gamma_{L} \Gamma_{R}}{\Gamma} \text{Im} \left[ \bar{B}_{0+}(E) + \bar{B}_{0-}(E) \right]
\] (68)

where \( \bar{B}_{0\pm}(E) = \bar{b}_{0\pm}^{(\alpha)}(E)/\Omega_{\alpha} \) is independent of \( \alpha \).

Solving equation (64), we find
\[
\bar{T}(E) = - \frac{2 \Gamma_{L} \Gamma_{R}}{\Gamma} \text{Im} \left[ E + \frac{U}{2} \xi + i \left( \frac{\xi}{D_{0}(E)} + \gamma \right) \right]
\] (69)
where
\[ D_0(E) = \left( E + i \frac{\Gamma + \gamma}{2} \right)^2 + \frac{\gamma^2}{4} - \frac{U}{2} \left( \frac{U}{2} + i \gamma \xi \right) \]
and \( \xi \equiv \bar{\xi}(T) \), equation (56), is an average value of the noise.

Substituting equations (69) into (49) and taking into account that the displacement current vanishes in the limit of \( t \to \infty \), we arrive to equation (51), for the steady-state current. For infinite noise-temperature, \( \xi = 0 \), the transmission coefficient \( T \), equation (69), coincides with that, found in [15, 23]. For zero noise-temperature, \( \xi = -1 \), equation (56), one easily finds that equation (69) reproduces the Breit–Wigner formula, given by equation (26) for \( t \to \infty \) and \( E_0 = -U/2 \). This result is quite expectable, since at zero noise-temperature, the electron always occupies the lower level inside the well.

Thus, we found that the steady-state current through fluctuating energy level is given by the Landauer-type formula, equation (51), where the corresponding transmission coefficients, averaged over the noise, are symmetric under the time-reversal, equation (17). Obviously, the zero bias current would be vanished in this case.

It is interesting that in contrast with the electron current, the energy flux carried by electrons through a single dot, coupled to Markovian leads, is not zero for the asymmetric dot \( \Gamma_L \neq \Gamma_R \). This was obtained in [31], by using the Keldysh formalism for time-dependent nonequilibrium Green’s functions\(^2\). Such a difference with the electron current can be understood as a heating of the leads by an external noise via fluctuations of the dot’s energy. In this case the energy currents always flow from the dot to the two leads, proportional to the corresponding rates, \( \Gamma_L, \Gamma_R \) [31].

Note, that the absence of zero-bias current through a single dot in the case of noise is predicted only for the Markovian leads (wide-band limit), adjacent to the drag system. If the dot is coupled to leads of finite bandwidths, the situation can be different. As we demonstrated above, this corresponds to electron transport through a coupled-dot system, connected to Markovian leads, figure 2. In this case the time-reversal symmetry of the transmission probability can be broken due to noise. As a result, the zero-bias current would appear. This case will be investigated below.

With respect to experiments on Coulomb-coupled quantum dots [28, 29], figure 3, a non-zero bias current (Coulomb drag) had been observed. We assume that this current is due to non-Markovian leads of the drag subsystem or due to a back-action from the ‘drag’ to the ‘drive’ subsystems. In the latter case, we anticipate that the drag current would disappear with increase of the bias voltage, applied to the drive system. This can be verified experimentally.

Regarding the theoretical analysis of the Coulomb drag problem by using Markovian Lindblad Master equations (with time-independent rates) [29, 32], the problem is that we need large bias limit \( (\Gamma/V \ll 1) \) to derive these equations [10]. Therefore, when the drag system is at zero voltage, the corresponding Lindblad-type Master equations would not be valid, in general. In contrast, the SEA approach, used in our calculation, is valid for any bias, applied for the drag system, whenever the drive system can be replaced by an external noise.

5. Double-dot system

Let us consider electron current through a double-dot coupled to two leads, figure 4. The leads are Markovian (infinite band-width). As we demonstrated above, such a system can also

\(^2\)Our calculations of the energy current within the SEA (not presented in this paper), are in full agreement with the results of [31].
correspond to a single dot, coupled to a non-Markovian reservoir (for instance, as shown in figure 2 with \( \Lambda_L \to \infty \)).

Similar to the previous case, equation (1), the system is described by the following tunneling Hamiltonian.

\[
H(t) = \sum_i E_i \hat{c}_i^\dagger \hat{c}_i + \sum_r E_r \hat{c}_r^\dagger \hat{c}_r + E_1(t) \hat{c}_1^\dagger \hat{c}_1 + E_2(t) \hat{c}_2^\dagger \hat{c}_2 \\
+ \left( \Omega \hat{c}_2 + \sum_r \Omega_r \hat{c}_r^\dagger \hat{c}_r + \sum_r \Omega_{r \gamma} \hat{c}_r^0 + \text{H.c.} \right).
\] (71)

We considered the energy levels inside the dots, \( E_{1,2}(t) \), as time-dependent. However, the tunneling couplings are time-independent.

Let us apply the generalized Landauer formula, equation (49) for the time-dependent current through this system. As in a previous case, we need to evaluate the penetration coefficient, \( T(E, t) \). It is done by solving the time-dependent Schrödinger equation (3) for a single-electron wave-function, with the initial condition corresponding to occupied energy level \( E_{\tilde{k}} \) in one of the leads. Similar to equation (4), the wave-function can be written as

\[
|\Phi^{(\tilde{k})}(t)\rangle = \left[ \sum_i b^{(\tilde{k})}_i(t) c_i^\dagger + b^{(\tilde{k})}_1(t) c_1^\dagger \right. \\
+ \sum_r b^{(\tilde{k})}_r(t) c_r^\dagger + \left. \sum_r b^{(\tilde{k})}_r(t) c_r^\dagger \right] |0\rangle,
\] (72)

where the index \( \tilde{k} \equiv \{ l, r \} \) denotes the initial condition\(^3\), \( b^{(\tilde{k})}_l(0) = \delta_{\tilde{g}l} \delta_{rr} \) and \( b^{(\tilde{k})}_r(0) = \delta_{\tilde{g}r} \delta_{lr} \) (see equation (5)).

Substituting equation (72) into the Schrödinger equation (3). We find the following system of linear equations for the probability amplitudes \( b^{(\tilde{k})}(t) \)

\[
i b_1^{(\tilde{k})}(t) = E_1 b_1^{(\tilde{k})}(t) + \Omega_L b^{(\tilde{k})}_1(t) \quad (73a)
\]

\[
i b_1^{(\tilde{k})}(t) = E_1(b_1^{(\tilde{k})}(t) + \sum_i \Omega_i b^{(\tilde{k})}_i(t) + \Omega b^{(\tilde{k})}_2(t)) \quad (73b)
\]

\[
i b_2^{(\tilde{k})}(t) = E_2(b_2^{(\tilde{k})}(t) + \sum_r \Omega_r b^{(\tilde{k})}_r(t) + \Omega b^{(\tilde{k})}_1(t)) \quad (73c)
\]

\[
i b_1^{(\tilde{k})}(t) = E_1(b_1^{(\tilde{k})}(t) + \Omega_L b^{(\tilde{k})}_1(t)) \quad (73d)
\]

These equations can be solved in the same way as in the previous case, equation (6), namely, by resolving equations (73a) and (73d) with respect to \( b^{(\tilde{k})}_l(t) \)

\(^3\)We do not consider the initial state, corresponding to occupied double-dot, since it does not contribute to steady-state current.
\[ b_{1}^{(L)}(t) = e^{-iE_{t}t} \left[ \delta_{kl} \delta_{ll} - \int_{0}^{t} i \Omega_{L} b_{1}^{(k)}(t') e^{iE_{t'}t'} dt' \right] \] (74a)

\[ b_{r}^{(R)}(t) = e^{-iE_{t}t} \left[ \delta_{kl} \delta_{rr} - \int_{0}^{t} i \Omega_{R} b_{2}^{(k)}(t') e^{iE_{t'}t'} dt' \right] \] (74b)

and then substituting the result into equations (73b) and (73c). One obtains

\[ \dot{b}_{1}^{(L)}(t) = E_{1}(t) b_{1}^{(L)}(t) + \Omega_{L} \delta_{ll} e^{-iE_{t}t} 
+ \Omega_{R}^{2} \left[ \dot{b}_{1}^{(L)}(t') - i \sum_{r} \Omega_{L}^{2} \int_{0}^{t} \dot{b}_{1}^{(L)}(t') e^{iE_{t'}t'} dt' \right] \] (75a)

\[ \dot{b}_{2}^{(R)}(t) = E_{2}(t) b_{2}^{(R)}(t) + \Omega_{R} \delta_{rr} e^{-iE_{t}t} 
+ \Omega_{L}^{2} \left[ \dot{b}_{2}^{(R)}(t') - i \sum_{r} \Omega_{R}^{2} \int_{0}^{t} \dot{b}_{2}^{(R)}(t') e^{iE_{t'}t'} dt' \right]. \] (75b)

We emphasise that \( \sum_{l,r} \) is extended over all reservoir states \( (E_{i,r}) \) without any Pauli principle restrictions.

Now we replace \( b_{1}^{(L)}(t) = b_{1}^{(L)}(E_{t}, e^{-iE_{t}t}) \) (see with equation (8)), where \( \alpha = \{L, R\} \) denotes a lead occupied by the electron at \( t = 0 \), and \( E = E_{t} \equiv E_{i,r}. \) In continuous limit equation (75) become

\[ \dot{b}_{1}^{(L)}(E, t) = i \left( E - E_{1}(t) + \frac{i}{2} \Gamma_{L} \right) b_{1}^{(L)}(E, t) 
- \frac{i}{2} \Omega_{L}^{2} b_{1}^{(L)}(E, t) - i \Omega_{L} \delta_{oL} \] (76a)

\[ \dot{b}_{2}^{(R)}(E, t) = \frac{i}{2} \left( E - E_{2}(t) + \frac{i}{2} \Gamma_{R} \right) b_{2}^{(R)}(E, t) 
- \frac{i}{2} \Omega_{R}^{2} b_{2}^{(R)}(E, t) - i \Omega_{R} \delta_{oR}. \] (76b)

\[ \Gamma_{L(R)} = 2\pi \Omega_{L(R)}^{2} \theta_{L(R)}. \]

Solving these equations we obtain the probability amplitudes for occupation of first and second dot by an electron, coming from left or right reservoir. Then using equations (7), (14) and (15), we obtain for the time-dependent transmission probabilities, equation (16),

\[ T_{L \rightarrow R}(E, t) = 2\pi q_{L}(E, t) \Gamma_{R}. \]

\[ T_{R \rightarrow L}(E, t) = 2\pi q_{R}(E, t) \Gamma_{L}. \] (77)

where \( q_{1/2}^{(\alpha)}(E, t) = |b_{1/2}^{(\alpha)}(E, t)|^{2} \) is probability of finding the corresponding dot occupied (see with equation (22)).

Let us take the energy levels of the dots time-independent, \( E_{1}(t) = 0 \) and \( E_{2}(t) = \epsilon. \) Consider the steady-state limit of equation (76), corresponding to \( (d/dt) b_{1/2}^{(\alpha)}(E, t \rightarrow \infty) \rightarrow 0. \) Then equation (73) becomes a system of algebraic equations that can be easily solved. One finds for the steady-state transmission probabilities \( T(E) = T(E, t \rightarrow \infty) \), equation (77)

\[ T_{L \rightarrow R}(E) = T_{R \rightarrow L}(E) = \Gamma_{L} \Gamma_{R} \left( \frac{\Omega}{D(E)} \right)^{2}. \] (78)
where
\[
D(E) = \left( E + i \frac{\Gamma_L}{2} \right) \left( E - \epsilon + i \frac{\Gamma_R}{2} \right) - \Omega^2.
\]  
(79)

This result coincides with equation (41) when one of the bandwidths, \( \Lambda_{L,R} \) is infinity.

5.1. Telegraph noise

Consider now the energy level of the first dot, figure 4, is randomly fluctuating in time, \( E_1(t) = (U/2) \xi(t) \), equation (52), and \( E_2(t) = \epsilon \). Then equation (76), averaged over the noise, read

\[
\frac{d}{dt} \langle b_1^{(\alpha)}(E,t) \rangle = i \left( E + i \frac{\Gamma_L}{2} \right) \langle b_1^{(\alpha)}(E,t) \rangle
\]
\[
- \frac{U}{2} \langle \xi(t) b_1^{(\alpha)}(E,t) \rangle - i \Omega \langle b_2^{(\alpha)}(E,t) \rangle - i \Omega_\alpha \delta_{\alpha L}.
\]  
(80a)

\[
\frac{d}{dt} \langle b_2^{(\alpha)}(E,t) \rangle = i \left( E - \epsilon + i \frac{\Gamma_R}{2} \right) \langle b_2^{(\alpha)}(E,t) \rangle
\]
\[
- i \Omega \langle b_1^{(\alpha)}(E,t) \rangle - i \Omega_\alpha \delta_{\alpha R}.
\]  
(80b)

In order to evaluate the average \( \langle \xi(t) b_1^{(\alpha)}(E,t) \rangle \) we multiply equation (76) by \( \xi(t) \), taking into account that \( \xi^2(t) = 1 \). As in the case of single dot, we apply the Shapiro–Loginov differential formula equation (61), for the terms \( \langle \xi(t) \frac{d}{dt} b_1^{(\alpha)}(E,t) \rangle \). Finally we obtain (see with equation (62)),

\[
\frac{d}{dt} \langle b_1^{(\alpha)}(E,t) \rangle = i \left( E + i \frac{\Gamma_L + 2\gamma}{2} \right) \langle b_1^{(\alpha)}(E,t) \rangle
\]
\[
+ \left( \gamma \bar{\xi} - i \frac{U}{2} \right) \langle b_1^{(\alpha)}(E,t) \rangle - i \Omega \langle b_2^{(\alpha)}(E,t) \rangle - i \Omega_\alpha \bar{\xi} \delta_{\alpha L}.
\]  
(81a)

\[
\frac{d}{dt} \langle b_2^{(\alpha)}(E,t) \rangle = i \left( E - \epsilon + i \frac{\Gamma_R + 2\gamma}{2} \right) \langle b_2^{(\alpha)}(E,t) \rangle
\]
\[
- i \Omega \langle b_1^{(\alpha)}(E,t) \rangle + \gamma \bar{\xi} \langle b_2^{(\alpha)}(E,t) \rangle - i \Omega_\alpha \bar{\xi} \delta_{\alpha R}
\]  
(81b)

where \( b_1^{(\alpha)}(E,t) = \xi(t) b_1^{(\alpha)}(E,t) \).

Similar to the previous case, equation (64), it is useful to introduce the new variables,

\[
b_1^{(\alpha)}(E,t) = \langle b_1^{(\alpha)}(E,t) \rangle \pm \langle b_{1\xi}^{(\alpha)}(E,t) \rangle
\]
\[
b_2^{(\alpha)}(E,t) = \langle b_2^{(\alpha)}(E,t) \rangle \pm \langle b_{2\xi}^{(\alpha)}(E,t) \rangle
\]  
(82)

that transform equations (80) and (81) to more transparent equations, which are simpler for treatment. These equations read

\[
\frac{d}{dt} b_{\pm}^{(\alpha)}(E,t) = i \left( E \mp \frac{U}{2} + i \frac{\Gamma_L}{2} \right) b_{\pm}^{(\alpha)} - i \Omega b_{\mp}^{(\alpha)}
\]
\[
- \gamma_{\pm} b_{\pm}^{(\alpha)} + \gamma_{\mp} b_{\mp}^{(\alpha)} - 2i \Omega_\alpha \frac{\gamma_{\mp}}{\gamma} \delta_{\alpha L}.
\]  
(83a)
Figure 5. Resonant tunneling through a double-dot, where the energy level of the first (left) dot is split into two sub-levels, ±U/2, corresponding to the pseudo-spin up and down.

\[
\frac{d}{dt} b_{\pm}^{(\alpha)}(E,t) = i \left( E - \epsilon + \frac{\Gamma_R}{2} \right) b_{\pm}^{(\alpha)} - \Omega b_{\pm}^{(\alpha)} - \gamma b_{\pm}^{(\alpha)} + \frac{\gamma}{\gamma} \delta_{aR}.
\]

As in the previous case of a single dot, equation (83) can be considered as describing electron with a pseudo-spin 1/2, travelling through a coupled-dot system and interacting with a fictitious field inside the first (left) dot. This field splits the first dot level into the two sub-levels, ±U/2, figure 5, where the noise produces jumps between these sub-levels with rates \(\gamma_\pm\).

Solving equation (83) we obtain the probability amplitudes of occupation the dots, averaged over the noise

\[
\langle b_{1(2)}^{(\alpha)}(E,t) \rangle = \frac{1}{2} \langle b_{1(2)}^{(\alpha)}(E,t) + b_{1(2)}^{(\alpha)}(E,t) \rangle.
\]

Next, we have to average the corresponding transmission probabilities, equation (77), given by a bi-linear form (density-matrix) of these amplitudes, \(\langle q_{1(2)}^{(\alpha)}(E,t) \rangle = \langle |b_{1(2)}^{(\alpha)}(E,t)|^2 \rangle\). Since \(\langle |b_{1(2)}^{(\alpha)}(E,t)|^2 \rangle \neq \langle |q_{1(2)}^{(\alpha)}(E,t)|^2 \rangle\), we cannot use directly equations (83) and (84) to evaluate this quantity. For this reason, we derive differential equations, relating the density-matrix of the double-dot system, averaged over the noise, to the averaged amplitudes \(\langle b_{1(2)}^{(\alpha)}(E,t) \rangle\), similar to equation (58). We find the following equations for the noise-averaged density matrix,

\[
\langle q_{12}^{(\alpha)}(E,t) \rangle = \frac{1}{2} \langle q_{12}^{(\alpha)}(E,t) + q_{12}^{(\alpha)}(E,t) \rangle
\]

(see with equation (65)), where

\[
\frac{d}{dt} q_{1\pm}^{(\alpha)}(E,t) = -\Gamma_L q_{1\pm}^{(\alpha)} + i\Omega q_{21\pm}^{(\alpha)} - 2\Omega \Re \text{Im}[h_{1\pm}^{(\gamma)}] \delta_{aL}
\]

(86a)

\[
\frac{d}{dt} q_{2\pm}^{(\alpha)}(E,t) = -\Gamma_R q_{2\pm}^{(\alpha)} - i\Omega q_{12\pm}^{(\alpha)} - 2\Omega \Re \text{Im}[h_{2\pm}^{(\gamma)}] \delta_{aR}
\]

(86b)

\[
\frac{d}{dt} q_{12\pm}^{(\alpha)}(E,t) = i\left( \epsilon + \frac{U}{2} + \frac{1}{2} \right) q_{12\pm}^{(\alpha)} + i\Omega (q_{1\pm}^{(\alpha)} - q_{2\pm}^{(\alpha)}) - \gamma q_{12\pm}^{(\alpha)} + \frac{\gamma}{\gamma} \delta_{aL} - \gamma q_{12\pm}^{(\alpha)} + \frac{\gamma}{\gamma} \delta_{aR}.
\]

(86c)
One finds that for each direction of the pseudo-spin, equation (86) represent the Lindblad-type Master equation for a single electron in a double-dot level system, coupled to leads. The noise only generates transitions between the two pseudo-spin directions. Detailed derivation of equation (86) is presented in appendix B.

Solving equations (83) and (86), we find the time-dependent average current, equations (16) and (49), through a double-dot system. Now we concentrate on steady-state.

5.2. Steady-state

Consider equations (83)–(86) in steady-state limit, corresponding to \( t \to \infty \). Since in this limit the l.h.s of these equations vanishes, these turn into the following linear algebraic equations

\[
\left( E + \frac{U}{2} + i \frac{\Gamma_L}{2} \right) \mathcal{B}^{(\alpha)}_{1\pm} - \Omega \mathcal{B}^{(\alpha)}_{2\pm} + i \gamma_\pm \mathcal{B}^{(\alpha)}_{1\pm} - i \gamma_\mp \mathcal{B}^{(\alpha)}_{1\mp} = 2 \frac{\gamma_+}{\gamma} \delta_{\alpha L} \tag{87a}
\]

\[
\left( E - \epsilon + i \frac{\Gamma_R}{2} \right) \mathcal{B}^{(\alpha)}_{1\pm} - \Omega \mathcal{B}^{(\alpha)}_{1\pm} + i \gamma_\pm \mathcal{B}^{(\alpha)}_{2\pm} - i \gamma_\mp \mathcal{B}^{(\alpha)}_{2\mp} = 2 \frac{\gamma_+}{\gamma} \delta_{\alpha R} \tag{87b}
\]

and

\[
\Gamma_L \mathcal{Q}^{(\alpha)}_{1\pm} - i \Omega \left[ \mathcal{Q}^{(\alpha)}_{1\pm} - \mathcal{Q}^{(\alpha)}_{2\pm} \right] + \gamma_\pm \mathcal{Q}^{(\alpha)}_{1\pm} - \gamma_\mp \mathcal{Q}^{(\alpha)}_{1\mp} = -2 i \text{Im} \left[ \mathcal{B}^{(\alpha)}_{1\pm} \right] \delta_{\alpha L} \tag{88a}
\]

\[
\Gamma_R \mathcal{Q}^{(\alpha)}_{2\pm} + i \Omega \left[ \mathcal{Q}^{(\alpha)}_{1\pm} - \mathcal{Q}^{(\alpha)}_{2\pm} \right] + \gamma_\pm \mathcal{Q}^{(\alpha)}_{2\pm} - \gamma_\mp \mathcal{Q}^{(\alpha)}_{2\mp} = -2 i \text{Im} \left[ \mathcal{B}^{(\alpha)}_{2\pm} \right] \delta_{\alpha R} \tag{88b}
\]

\[
\left( E + \frac{U}{2} + i \frac{\Gamma_R}{2} \right) \mathcal{Q}^{(\alpha)}_{1\pm} + \Omega \left[ \mathcal{Q}^{(\alpha)}_{1\pm} - \mathcal{Q}^{(\alpha)}_{2\pm} \right] + i \gamma_\pm \mathcal{Q}^{(\alpha)}_{1\pm} - i \gamma_\mp \mathcal{Q}^{(\alpha)}_{1\mp} = 2 \frac{\gamma_+}{\gamma} \delta_{\alpha L} - \mathcal{B}^{(\alpha)}_{1\pm} \delta_{\alpha R} \tag{88c}
\]

where \( \Omega \alpha \mathcal{B}^{(\alpha)}_{1\pm}(E) = \alpha^{(\alpha)}_{1\pm}(E, t \to \infty) \) and \( \Omega \alpha \mathcal{Q}^{(\alpha)}_{1\pm}(E) = \alpha^{(\alpha)}_{1\pm}(E, t \to \infty) \).

Solving equations (87) and (88), we obtain the steady-state transmission coefficients, \( \mathcal{T}(E) = T(E, t \to \infty) \), equation (77), given by the following expressions

\[
\mathcal{T}_{L \to R}(E) = \Gamma_L \mathcal{Q}^{(L)}_{1+} \Gamma_R = -i \Omega \left[ \mathcal{Q}^{(L)}_{12} - \mathcal{Q}^{(L)}_{21} \right] \Gamma_L \tag{89a}
\]

\[
\mathcal{T}_{R \to L}(E) = \Gamma_R \mathcal{Q}^{(R)}_{1+} \Gamma_L = i \Omega \left[ \mathcal{Q}^{(R)}_{12} - \mathcal{Q}^{(R)}_{21} \right] \Gamma_R \tag{89b}
\]

where \( \mathcal{Q}^{(\alpha)}_{1\pm}(E) = \frac{1}{2} \left[ \mathcal{Q}^{(\alpha)}_{1\pm(2+)}(E) + \mathcal{Q}^{(\alpha)}_{1\pm(2-)}(E) \right] \).

Equation (89) show that the transmission probability to a reservoir is given by occupation of a quantum dot, adjacent to that reservoir and multiplied by the corresponding tunneling rate (see with equation (22)). However for a double-dot it can be rewritten through the imaginary part of the off-diagonal occupation density, \( \mathcal{Q}_{12} \), equation (89). This relation is very remarkable. It shows that the transmission probability is given by the off-diagonal density-matrix of an electron in two dots.
Indeed, as follows from Schrödinger equation, the quantum transport between two isolated levels always proceeds through linear superposition of these states (Rabi oscillations). This is very different from the transport between two reservoirs with continuum states, or from an isolated to continuum states. Then the tracing over continuum states in the equation of motions, eliminates coherent transitions, making all the transport incoherent, see [10] and additional discussion in section 6.

Using equation (89) we obtain for the asymmetry between the left-right and right-left transmission probabilities, \( \Delta T(E) = T_{L\rightarrow R}(E) - T_{R\rightarrow L}(E) \), the following expression,

\[
\Delta T(E) = -2\Omega \text{Im} \left[ \Gamma_L \tilde{\mathcal{Q}}_{12}^{(L)}(E) + \Gamma_R \tilde{\mathcal{Q}}_{12}^{(R)}(E) \right],
\]

where \( \tilde{\mathcal{Q}}_{12}^{(\alpha)}(E) = \frac{1}{2} \left[ \mathcal{Q}_{12+}^{(\alpha)}(E) + \mathcal{Q}_{12-}^{(\alpha)}(E) \right]. \) If \( \Delta T(E) \neq 0 \), then the time-reversal symmetry of transmission coefficients is broken, leading to the zero-bias current. First we investigate this point analytically.

5.3. Analytical treatment

Let us take for simplicity symmetric double-dot, \( \Gamma_L = \Gamma_R = \Gamma/2 \), \( \epsilon = 0 \) and also symmetric noise \( \gamma_+ = \gamma_- = \gamma/2 \), corresponding to infinite noise temperature, equation (55). Although equations (87) and (88) are linear algebraic equations, their analytical solution looks rather complicated. Therefore in order to obtain simple analytical expressions, we expand \( \mathcal{B}^{(\alpha)} \) and \( \mathcal{Q}^{(\alpha)} \) in powers of \( \gamma \) by keeping only linear terms. Thus, we represent these variables as

\[
\mathcal{B}^{(\alpha)}_{1\pm(2\pm)} = \mathcal{B}^{(\alpha)}_{1\pm(2\pm)} + \gamma \mathcal{X}^{(\alpha)}_{1\pm(2\pm)} \quad \text{(91a)}
\]

\[
\mathcal{Q}^{(\alpha)}_{1\pm(2\pm)} = \tilde{Q}^{(\alpha)}_{1\pm(2\pm)} + \gamma \mathcal{Y}^{(\alpha)}_{1\pm(2\pm)} \quad \text{(91b)}
\]

\[
\mathcal{Q}^{(\alpha)}_{12\pm} = \tilde{Q}^{(\alpha)}_{12\pm} + \gamma \mathcal{Y}^{(\alpha)}_{12\pm} \quad \text{(91c)}
\]

where \( \mathcal{B}^{(\alpha)}_{1\pm(2\pm)}, \tilde{Q}^{(\alpha)}_{1\pm(2\pm)} \) and \( \mathcal{Q}^{(\alpha)}_{12\pm} \) are obtained from equations (87) and (88) for \( \gamma = 0 \). One easily finds

\[
\mathcal{B}^{(L)}_{2\pm} = \mathcal{B}^{(R)}_{1\pm} = \frac{\Omega}{D_{\pm}(E)}, \quad \mathcal{B}^{(L)}_{1\pm} = \frac{E + i\Gamma}{D_{\pm}(E)}
\]

\[
\mathcal{B}^{(R)}_{2\pm} = \frac{E \pm \frac{U}{2} + i\frac{\Gamma}{4}}{D_{\pm}(E)} \quad \text{(92)}
\]

where

\[
D_{\pm}(E) = \left( E \pm \frac{U}{2} + i\frac{\Gamma}{4} \right) \left( E + i\frac{\Gamma}{4} \right) - \Omega^2. \quad \text{(93)}
\]

Note, that the limit of \( \gamma = 0 \) describes the electron motion through the upper or lower level (\( \pm U/2 \)) of the first dot without the noise, figure 5. Therefore in this case

\[
\mathcal{Q}^{(\alpha)}_{1\pm(2\pm)} = |\mathcal{B}^{(\alpha)}_{1\pm(2\pm)}|^2, \quad \mathcal{Q}^{(\alpha)}_{12\pm} = \mathcal{B}^{(\alpha)}_{1\pm} \mathcal{B}^{(\alpha)\ast}_{2\pm}. \quad \text{(94)}
\]

The terms \( \mathcal{X}^{(\alpha)}_{1\pm(2\pm)} \), equation (91a), are obtained straightforwardly from equation (87) by taking the limit
\[ X^{(a)}_{1\pm(2\pm)} = \left( \left[ \left( \bar{B}_{1\pm(2\pm)}^{(a)} - \bar{B}_{1\pm(2\pm)}^{(a)} \right) \right] \right) \gamma \rightarrow 0. \] (95)

Substituting equation (91c) into (90) and taking into account that \( \Delta T(E) = 0 \) for \( \gamma = 0 \), we find
\[ \Delta T(E) = \gamma \Omega \Gamma \text{ Im} \left[ Y_{12}^{(a)} + Y_{12}^{(R)} \right] \] (96)
where \( Y_{12}^{(a)} = \left[ Y_{12}^{(a)} + Y_{12}^{(a)} \right] / 2 \) and \( Y_{12}^{(a)} \) are determined from the equations
\[ \Gamma Y_{12}^{(a)} + 4 \text{ Im}[Y_{12}^{(a)}] = \mp \Delta \tilde{Q}_{12}^{(a)} - 4 \text{ Im}[X_{12}^{(a)}] \delta_{aL} \] (97a)
\[ \Gamma Y_{22}^{(a)} - 4 \text{ Im}[Y_{22}^{(a)}] = \mp \Delta \tilde{Q}_{22}^{(a)} - 4 \text{ Im}[X_{22}^{(a)}] \delta_{aR} \] (97b)
\[ \left( \pm \frac{U}{2} + \frac{i}{2} \right) Y_{12}^{(a)} + \Omega(Y_{12}^{(a)} - Y_{22}^{(a)}) = \mp \frac{i}{2} \Delta \tilde{Q}_{12}^{(a)} + X_{12}^{(a)} \delta_{aL} + X_{22}^{(a)} \delta_{aR} \] (97c)
with \( \Delta \tilde{Q}_{12}^{(a)} = \tilde{Q}_{12}^{(a)}(2+) - \tilde{Q}_{12}^{(a)}(2-)- \) and \( \Delta \tilde{Q}_{12}^{(a)} = \tilde{Q}_{12}^{(a)} - \tilde{Q}_{12}^{(a)} \). Solving equation (97) we arrive to a simple analytical formula for \( \Delta T(E) \), equation (96). It becomes even more simple when we expand it in powers of \( U \), keeping the leading term \( \propto U^2 \). We find
\[ \Delta T(E) = \frac{2 \gamma \Gamma U^2 \Omega^2}{\Gamma^2 + 16 \Omega^2} \left[ \frac{E^2 + E^2 \frac{\Omega^2}{4} - \frac{1}{16} \left( \Omega^2 + 4 \Omega^2 \right)^2}{\left( E^2 + \frac{\Omega^2}{4} - \Omega^2 \right)^2 + \left( \frac{\Omega^2}{4} \right)^2} \right] + O[\gamma^2, U^4]. \] (98)

Let us use equation (98) for an analysis of the zero bias current.

First, this result confirms that time-reversal symmetry of the transmission coefficient is broken when the energy level of one of the dots is randomly fluctuating, even though, the system by itself is symmetric. In addition, one finds from equation (98), that \( \Delta T(E) \) changes its sign with increase of the energy. This happens at \( E = \pm \Omega \) (for \( \Gamma \ll \Omega \)), which are the energy eigenvalues of a symmetric double-dot.

The change of sign \( \Delta T(E) \) results in changing direction of the zero-bias current, \( I_{zb} \) given by the generalized Landauer formula, equation (51),
\[ I_{zb} = \int_{-\infty}^{\infty} \Delta T(E) f(E) \frac{dE}{2\pi}, \] (99)
where \( f(E) \) is Fermi-function of reservoirs. We can evaluate it analytically when the leads are (initially) at zero temperature, \( f(E) = \theta(\mu - E) \), where \( \mu \) in the Fermi energy. Using equation (98) one finds
\[ I_{zb} = \frac{\gamma \Gamma U^2}{16\pi \left( \Gamma^2 + 4 \Omega^2 \right)} \left[ \frac{\mu \left( \mu^2 + \frac{\Omega^2}{4} - 3 \Omega^2 \right)}{\left( \mu^2 + \frac{\Omega^2}{4} - \Omega^2 \right)^2 + \frac{\Omega^2}{4} \Omega^2} + \frac{1}{\Omega} \text{ Im} \left[ \arctan \left( \frac{4\mu}{\Gamma + 4i\Omega} \right) \right] \right] + O[\gamma^2, U^4]. \] (100)

It follows from this equation that the zero bias current changes its sign at \( \mu = 0 \). This implies that in the case of Coulomb drag (figure 3), the drag and drive currents would be of opposite directions. Then with increase (decrease) of \( \mu \), the current reaches its maximal value,
when the Fermi energy is close to the energy level of the isolated double-dot, $\pm \Omega$. Finally, $I_{zb}$ vanishes when $\mu \to \infty$, equation (100). The same takes place when the leads are at infinite temperature, as follows from equation (99).

5.4. Numerical results

Let us compare our analytical formulae equations (98) and (100) with exact numerical calculations of $\Delta T(E)$, equation (90) and $I_{zb}$, equation (99). The results are shown in figure 6 for symmetric double-dot, $\Gamma_L = \Gamma_R = \Gamma/2$, $\epsilon = 0$ and symmetric noise, $\mathcal{T} = \infty$ in equation (55), corresponding to $\gamma_+ = \gamma_- = \gamma/2$. The upper panel displays $\Delta T(E)$ as a function of electron energy (in arbitrary units), and the lower panel displays the zero-bias current, $I_{zb}(\mu)$, as a function of the Fermi energy ($\mu$). The leads are at zero temperature. The solid line (black), corresponds to exact numerical solution of equations (87) and (88), where dashed line (blue) shows approximate calculations (for small $\gamma$), obtained from equation (97). The dot-dashed line (red) corresponds to equations (98) and (100).

One finds from figure 6 that simple formulae (98) and (100) reproduce general behavior of $\Delta T$ and zero-bias current very well, in particular, change of the current direction and position of its extremal values. A similar behavior of $\Delta T(E)$ and $I_{zb}(\mu)$ is found for a non-symmetric double-dot ($\epsilon \neq 0$), and even if the noise spectral width ($\gamma$) is not narrow. It is displayed in figure 7, which shows the asymmetry of transmission coefficient (upper panel) and the zero bias current (lower panel), obtained from equations (87) and (88) for $\epsilon = \pm 1$. We find that similar to symmetric double-dot, $\Delta T(E)$ changes its sign when $E$ is close to eigen-states of the double-dot. Moreover, the zero-bias current $I_{zb}(\mu)$ reaches its extremal values at the same values of the Fermi energy $\mu$, as for symmetric double-dot. In the same way, $I_{zb}(\mu)$ vanishes for $\mu \to \infty$.

Until now, our examples presented symmetric noise, corresponding to infinite noise-temperature, $\gamma_+ = \gamma_- = \gamma/2$ equation (55). Figure 8 displays the zero bias current as a function of the inverse noise temperature $\beta = U/\mathcal{T}$. One finds from this figure that as expected, the zero-bias current vanishes, when the noise-temperature, $\mathcal{T} \to 0$, corresponding to $\gamma_+ = 0$. As a result, there are no transitions due to noise between the sub-levels, $\pm U$ of the left dot in figure 5. As a result, $\Delta T(E) = 0$.

6. Interpretation

It follows from our calculations that the zero-bias dc-current, generated by noise, would appear in double-dot coupled to Markovian leads. However, for the same conditions, the zero-bias current is not expected in single dot. What is the reason for such a different behavior of similar systems under the noise? A proper understanding of this point is very necessary. In particular, it can reveal physical origin of the zero-bias current produced by isotropic in space noise.

Consider first the resonant tunneling through a single quantum dot, figure 1, coupled to Markovian leads, where the tunneling rates to the leads are very different, for instance $\Gamma_L \gg \Gamma_R$. Then the dwell-time (occupation probability) of electron inside the dot would be always much larger for an electron traveling from the left-to-right lead, than that from the right-to-left lead. Indeed, an electron from the left lead enters the dot very fast ($\sim 1/\Gamma_L$). Then it is trapped in the dot, since tunneling from the dot to the right lead is very slow ($\sim 1/\Gamma_R$). The situation is opposite for the reversed case.

On first sight this looks as a violation of time-reversal symmetry in resonant tunneling. Yet, it is not the case, since penetration probability equals to occupation probability of the
As a result, different dwell-times for direct and reversed processes are compensated by the subsequent tunneling rate to a corresponding lead. This restores the time-reversal symmetry of the transmission probabilities.

In the case of Markovian leads, the presence of noise does not violate the time-reversal symmetry of transmission probabilities, since the tunneling rates to the leads ($\Gamma_L, \Gamma_R$) are energy independent. As a result, fluctuations of the energy level ($E_0$) do not affect the tunneling rates, so that the time-reversal symmetry would hold.

Consider now the resonant tunneling through a double-dot system, figure 4, starting from the no-noise case. Here too, the occupation probability of the left dot depends on whether the dot, multiplied by tunneling rate from the dot to the lead, equation (77). As a result, different dwell-times for direct and reversed processes are compensated by the subsequent tunneling rate to a corresponding lead. This restores the time-reversal symmetry of the transmission probabilities.

Figure 6. Break the time-reversal symmetry of transmission coefficient, $\Delta T$, (upper panel) and zero-bias current, $I_{zb}$, (lower panel) for a symmetric double-dot. Solid line (black) shows exact calculations, where dashed lined (blue) corresponds to leading terms of expansions in powers of $\gamma$. The dot-dashed lines (red) display equations (98) and (100).
electron is coming from left or from right lead, even if the double-dot is symmetric ($\Gamma_L = \Gamma_R$, $\epsilon = 0$). Indeed, an electron from the left lead is coming directly to left dot, where an electron from the right lead have to cross the middle barrier. However, as in previous case, such a difference in dwell-times is fully compensated by a subsequent tunneling rate, restoring the time-reversal symmetry of transmission probabilities.

Now we include the noise, randomly fluctuating the energy level of the left dot between two values, $\pm U/2$, figure 5. These fluctuations cannot influence the energy-independent tunneling rates from the dots to leads, $\Gamma_{L,R}$. However, they destroy the linear superposition of electron states between two dots. Note that any quantum mechanical transitions between isolated states proceed through such a superposition of these states. The latter is described by

![Figure 7](image-url)
the off-diagonal density-matrix element, \( q_{12}^{(LR)} \), equation (85). Since it depends on energy difference between the levels (see equation (86c)), the ensemble average of \( q_{12}^{(LR)} \) over the noise diminishes this quantity (decoherence), unless the noise equally affects the both dots\(^4\).

If the double-dot is isolated from leads, a single electron occupying such a system (qubit), is under the noise infinitely long time. Then its off-diagonal density matrix element vanishes due to decoherence, \( q_{12}^{(LR)}(t \rightarrow \infty) \rightarrow 0 \), (see with [25, 26]. However, in the case of electron transport through the double-dot, each electron coming from the leads, occupies the dots only by a finite time. Then the effect of decoherence would be proportional to the electron dwell-time inside the dot, which is under the noise. Hence, the decoherence effect would be different for electrons, arriving the dot from the left lead than that for electrons arriving the dot from the right lead. This produces violation of the time-reversal symmetry, resulting in zero bias current.

It follows from the above explanation that the difference in occupation of the left dot by an electron, coming from left and right lead, should be similar to the time-reversal violation of the transmission probability, \( \Delta T(E) \), equation (98). Indeed, let us evaluate the difference in occupation of the first dot,

\[
\Delta \tilde{q}_1(E) = \Gamma_L \tilde{Q}_1^{(L)}(E) - \Gamma_R \tilde{Q}_1^{(R)}(E).
\]

(101)

Since this quantity is not considerably affected by the noise, we do it analytically for the case of no-noise, \( U = 0 \). Using equations (92)–(94), one easily obtains from for a symmetric double dot, \( \Gamma_L = \Gamma_R = \Gamma/2 \) and \( \epsilon = 0 \), very simple analytical expressions

\[
\Delta \tilde{q}_1(E) = \frac{\Gamma}{2} \frac{E^2 + \frac{\Gamma^2}{16} - \frac{\Omega^2}{4}}{(E^2 + \frac{\Gamma^2}{16} - \frac{\Omega^2}{4})^2 + \frac{\Gamma^2}{4}\Omega^2}.
\]

(102)

By confronting this expression with equation (98) we find that \( \Delta T(E) \) and \( \Delta \tilde{q}_1(E) \) are inter-related. It is illustrated in figure 9. Comparing this figure with figure 6 (upper panel), we find that the behaviour of \( \Delta T(E) \) as a function of \( E \) is very similar to that of \( \Delta \tilde{q}_1(E) \). Altogether

\(^4\) Similar mechanism of decoherence is discussed in [26], for a single qubit, imbedded in the AB ring.
it confirms our understanding of the zero-bias current as the effect of decoherence, generated by an external noise.

7. Discussion

In this paper we investigated the zero-bias current, induced by a dichotomic (telegraph) noise, fluctuating energy levels of a quantum system. Our results were obtained by application of the generalized Landauer formula for the time-dependent non-interacting electron transport, assuming the absence of back-action from the quantum system to the noise. In this case the origin of noise becomes irrelevant, so that the results can be considered as applicable for any quantum system in the fluctuating environment.

We found that the necessary condition for the steady-state zero-bias dc-current through a quantum system is a break of the time-reversal symmetry in the transmission probability. If this symmetry holds, no zero-bias current is expected in such a system. We confirm it for the case of a single quantum dot coupled to Markovian leads, where the time-reversal symmetry persists, even in the presence of noise.

However, in the case of a double-dot, we found that the noise violates the time-reversal symmetry of the transmission coefficient, leading to zero-bias current. Our detailed analysis demonstrates that such a violation is a result of decoherence, generated by the noise. This always takes place when the current proceeds through linear superposition of isolated quantum states (delocalized orbitals). Since decoherence due to noise is ubiquitous phenomenon, we assume that similar zero-bias current can be found in many other quantum systems.

For instance, it can appear even in a single dot, coupled to non-Markovian leads of finite band-width. Indeed, we demonstrated in this paper that this case corresponds to a quantum dot, directly coupled to isolated (pseudo) modes, imbedded in a Markovian spectrum. As a result, the electron traveling through the dot, appears in a linear superposition of the dot and

![Figure 9. Difference in occupation of the first dot for an electron, coming from left or right lead, in the case of no-noise (U = 0).](image)
pseudo-mode states. This superposition is affected by decoherence due to noise, resulting in zero-bias current.

Our predictions can be verified experimentally by attaching a random voltage source to the system, for instance via a plunger. Alternatively, it can be done by coupling capacitively the quantum system at zero bias to an external fluctuator, represented by an impurity (quantum dot) in equilibrium with a heat bath [24, 33, 34]. The back action can be ignored when the fluctuator’s dynamics is governed by its coupling to a thermalizing heat bath, which is much stronger than its coupling to the system. In an another (non-equilibrium) example, discussed in section 4, a current flows between two reservoirs through a quantum dot, located near the system, figure 3.

In this paper we did not consider the zero-bias electron current generating by a periodically oscillating energy level of quantum dot, for Markovian and non-Markovian leads [35]. The most interesting question is whether it exists an essential difference in directed particle flow induced by a periodic ac-field versus random forces. This problem will be discussed in a separate work.

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Appendix A. Shapiro–Loginov formula for asymmetric noise

Since the use of Shapiro–Loginov differential formula [30] makes the SEA very effective tool for an account of the noisy environment, we present here an original (Shapiro and Loginov) derivation of this formula with an amendment, suited for asymmetric telegraph noise.

Consider a functional $R[\xi(t), t]$ of a random variable $\xi(t)$. The average of this functional over all the possible trajectories $\{\xi(t)\}$ in a time-interval $(0, t)$ is denoted by $\langle R[\xi(t), t]\rangle$. By expanding $R[\xi(t), t]$ in a time-ordered functional Taylor series, we find [30]

$$
R[\xi(t), t] = R[0, t] + \sum_{n=1}^{\infty} \int_{0}^{t} dt_{1} \int_{0}^{t_{1}} dt_{2} \cdots \int_{0}^{t_{n-1}} dt_{n} \\
\times K_{n}(t, t_{1}, t_{2}, \ldots, t_{n})\xi(t_{1})\xi(t_{2})\cdots\xi(t_{n})
$$

(A.1)

where

$$
K_{n}(t, t_{1}, t_{2}, \ldots, t_{n}) = \left. \frac{\delta^{n}R[\xi(t), t]}{\delta \xi(t_{1})\delta \xi(t_{2})\cdots\delta \xi(t_{n})} \right|_{\xi \to 0}
$$

(A.2)

$\xi_{j} = x(t_{j})$ and $\delta / \delta \xi(t)$ is a functional derivative.

Let us multiply equation (A.1) by $\xi(t)$, represented the telegraph noise, equations (52)–(56) and average it over all trajectories. One obtains

$$
\langle \xi(t)R[\xi(t), t]\rangle = \langle \xi(0)R[0, t]\rangle + \sum_{n=1}^{\infty} \int_{0}^{t} dt_{1} \int_{0}^{t_{1}} dt_{2} \cdots \int_{0}^{t_{n-1}} dt_{n} \\
\times K_{n}(t, t_{1}, t_{2}, \ldots, t_{n})\langle \xi(t_{1})\xi(t_{2})\cdots\xi(t_{n})\rangle,
$$

(A.3)

where $\bar{\xi} = \langle \xi(t) \rangle = (\gamma_{-} - \gamma_{+})/\gamma$, equation (56).

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Consider the integrant of this expression. One can rewrite it explicitly as
\[ \langle \xi(t) \xi(t_1) \cdots \xi(t_n) \rangle = \sum_{\xi_{i_1}, \ldots, \xi_{i_n} = \pm 1} \xi_{i_1} \xi_{i_2} \cdots \xi_{i_n} P_{\xi_{i_1}, \xi_{i_2}, \ldots, \xi_{i_n}}(t, t_1) \times P_{\xi_{i_1}, \xi_{i_2}}(t, t_2) \cdots P_{\xi_{i_{n-1}}, \xi_{i_n}}(t_{n-1}, t_n) P_{\xi_{i_n}}(t_n) \] (A.4)
where \( P_{\xi_{i_{j+1}}, \xi_{i_j}}(t_j, t_{j+1}) \) denotes the conditional probability for finding \( \xi_{i_{j+1}} \) at time \( t_{j+1} \), where it was \( \xi_i \) at time \( t_j \). Using equation (54) we can write
\[ P_{\xi_{i_1}, \xi_{i_2}}(t, t_1) = \frac{\gamma_- \delta_{\xi_1, 1} + \gamma_+ \delta_{\xi_1, -1}}{\gamma} + \left[ \delta_{\xi_1, 1} - \frac{\gamma_- \delta_{\xi_1, 1} + \gamma_+ \delta_{\xi_1, -1}}{\gamma} \right] e^{-\gamma(t-t_1)}. \] (A.5)

Therefore
\[ \sum_{\xi = \pm 1} \xi P_{\xi_{i_1}, \xi_{i_2}}(t, t_1) = \frac{\gamma_- - \gamma_+}{\gamma} \]
\[ + \sum_{\xi = \pm 1} \xi \left[ \delta_{\xi_1, 1} - \frac{\gamma_- \delta_{\xi_1, 1} + \gamma_+ \delta_{\xi_1, -1}}{\gamma} \right] e^{-\gamma(t-t_1)}. \] (A.6)

Differentiating this expression by time \( t \) and substituting this result into equation (A.4) we find
\[ \frac{d}{dt} \langle \xi(t) \xi(t_1) \cdots \xi(t_n) \rangle = -\gamma \langle \xi(t) \xi(t_1) \cdots \xi(t_n) \rangle \]
\[ + (\gamma_- - \gamma_+) \langle \xi(t_1) \cdots \xi(t_n) \rangle. \] (A.7)
Using this result and equation (A.3) (see also in appendix of [15]), we easily arrive to equation (61).

**Appendix B. Derivation of single-electron Master equations**

Consider equation (76) for the amplitudes \( b^{(\alpha)}_{1(2)}(t) \). Multiplying these equation on \( b^{(\alpha)*}_{1(2)}(t) \) and subtracting the complex conjugated equations, one finds
\[ \frac{d}{dt} q_1^{(\alpha)}(E, t) = -\Gamma_L q_1^{(\alpha)} - 2\Omega \text{Im}[q_1^{(\alpha)}] \]
\[ - 2 \Omega_a \text{Im}[b_1^{(\alpha)}] \delta_{\alpha L} \] (B.1a)
\[ \frac{d}{dt} q_2^{(\alpha)}(E, t) = -\Gamma_R q_2^{(\alpha)} + 2\Omega \text{Im}[q_2^{(\alpha)}] \]
\[ - 2 \Omega_a \text{Im}[b_2^{(\alpha)}] \delta_{\alpha R} \] (B.1b)
\[ \frac{d}{dt} q_{12}^{(\alpha)}(E, t) = i \left[ E_2(t) - E_1(t) + \frac{\Gamma}{2} \right] q_{12}^{(\alpha)} \]
\[ + i\Omega [q_1^{(\alpha)} - q_2^{(\alpha)}] - i\Omega_a [b_1^{(\alpha)*} \delta_{\alpha L} - b_2^{(\alpha)*} \delta_{\alpha R}] \] (B.1c)
where $\Gamma = \Gamma_L + \Gamma_R$ and $q^{(a)}_{1(2)} \equiv q^{(a)}_{1(2)}(E,t) = |b^{(a)}_{1(2)}(E,t)|^2$, $q^{(a)}_{12} \equiv q^{(a)}_{12}(E,t) = b^{(a)}_1(E,t)h^{(a)*}_2(E,t)$. In the case of fluctuating level of the left dot (figure 5), $E_1(t) = \pm U/2$ and $E_2(t) = \epsilon$, equation (B.1a), averaged over the noise, read

$$
\frac{d}{dt} \langle q^{(a)}_1(E,t) \rangle = -\Gamma_L \langle q^{(a)}_1 \rangle - 2\Omega a \text{Im}[b^{(a)}_1] \delta_{aL}
$$

(B.2a)

$$
\frac{d}{dt} \langle q^{(a)}_2(E,t) \rangle = -\Gamma_R \langle q^{(a)}_2 \rangle + 2\Omega a \text{Im}[b^{(a)}_2] \delta_{aR}
$$

(B.2b)

$$
\frac{d}{dt} \langle q^{(a)}_{12}(E,t) \rangle = i\left[ \epsilon + i\frac{\Gamma}{2} \right] \langle q^{(a)}_{12} \rangle - i\frac{U}{2} \langle q^{(a)}_{12} \rangle
$$

+ i\Omega \left[ \langle q^{(a)}_1 \rangle - \langle q^{(a)}_2 \rangle \right] - i\Omega a \left[ (b^{(a)*}_2 \delta_{aL} - (b^{(a)}_1 \delta_{aR}) \right]
$$

(B.2c)

where $\langle q^{(a)}_{12} \rangle \equiv \langle \xi(t)q^{(a)}_{12}(E,t) \rangle$. To evaluate this term, we use the same procedure as in equations (80) and (81), together with the Shapiro–Loginov differential formula, equation (61), thus obtaining

$$
\frac{d}{dt} \langle q^{(a)}_1(E,t) \rangle = -\left( \Gamma_L + \gamma \right) \langle q^{(a)}_1 \rangle + \gamma \xi \langle q^{(a)}_1 \rangle - 2\Omega a \text{Im}[b^{(a)}_1] \delta_{aL}
$$

(B.3a)

$$
\frac{d}{dt} \langle q^{(a)}_2(E,t) \rangle = -\left( \Gamma_R + \gamma \right) \langle q^{(a)}_2 \rangle + \gamma \xi \langle q^{(a)}_2 \rangle + 2\Omega a \text{Im}[b^{(a)}_2] \delta_{aR}
$$

(B.3b)

$$
\frac{d}{dt} \langle q^{(a)}_{12}(E,t) \rangle = i\left[ \epsilon + i\frac{\Gamma + 2\gamma}{2} \right] \langle q^{(a)}_{12} \rangle + \left( \gamma \xi - i\frac{U}{2} \right) \langle q^{(a)}_{12} \rangle
$$

+ i\Omega \left[ \langle q^{(a)}_1 \rangle - \langle q^{(a)}_2 \rangle \right] - i\Omega a \left[ (b^{(a)*}_2 \delta_{aL} - (b^{(a)}_1 \delta_{aR}) \right]
$$

(B.3c)

where $\langle q^{(a)}_{12}(E,t) \rangle = \langle \xi(t)q^{(a)}_{12}(E,t) \rangle$ and $\langle q^{(a)}_{12}(E,t) \rangle = \langle \xi(t)q^{(a)}_{12}(E,t) \rangle$.

Now we introduce new variables (see with equation (82))

$$
q^{(a)}_{1(2)}(E,t) = \langle q^{(a)}_{1(2)}(E,t) \rangle \pm \langle q^{(a)}_{1(2)}(E,t) \rangle
$$

(4.4)

In these variables, equation (B.3a) turn to equation (86), representing the Master equations for single-electron transport through a double-dot system.

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