A signature of quantum gravity at the source of the seeds of cosmic structure?

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Abstract

This article reviews a recent work by a couple of colleagues and myself [1] about the shortcomings of the standard explanations of the quantum origin of cosmic structure in the inflationary scenario, and a proposal to address them. The point it that in the usual accounts the inhomogeneity and anisotropy of our universe seem to emerge from an exactly homogeneous and isotropic initial state through processes that do not break those symmetries. We argued that some novel aspect of physics must be called upon to able to address the problem in a fully satisfactory way. The proposed approach is inspired on Penrose’s ideas regarding an quantum gravity induced, real and dynamical collapse of the wave function.

1 Introduction

One of the most important advances in physical cosmology are the precision measurements of the anisotropies in the CMB together with their explanation within the context of the inflationary scenarios. However after the first glances at the explanations one notices something odd: The description of our Universe starts with an initial set of conditions which are totally homogeneous and isotropic both in the background space-time and in the quantum
state that is supposed to describe the "fluctuations", and it is quite easy to see that the subsequent evolution through dynamics that do not break these symmetries can only lead to an equally homogeneous and anisotropic universe. The arguments normally used in order to deal with this issue, are phrased in terms of “the quantum to classical transition”, without focussing on the required breakdown of homogeneity and isotropy in the state. Thinking in terms of first principles, one would start by acknowledging that the correct description of the problem at hand would involve a full theory of quantum gravity coupled to a theory of all the matter quantum fields, and that there, the issue would be whether we start with a quantum state that is homogeneous and isotropic or not?. If one chose to ignore the problem and view it as something inherent to our approximations, one could not argue that one has an understanding the origin of the CMB spectrum.

Penrose’s [2], ideas regarding the fundamental changes, that he argues are needed in quantum mechanics and their connection to quantum gravity, are used as inspiration in the treatment developed in [1]: The idea is brings up the aspect that we view as part of the quantum gravity realm, to the forefront in order to modify– in a minimalistic way– the semiclassical treatment, to deal with the unsatisfactory part of the standard inflationary accounts of the issues.

2 The quantum origin of the seeds of cosmic structure

Most colleagues who have been working in this field f take the view, that there is no problem at all in the transition from a homogeneous and isotropic early state of the universe, a late state that is neither. It is however a fact that the arguments invoked in this regard tend to differ from one inflationary cosmologist to another [3]. Very few do acknowledge that there seems to be something unclear at this point [4]. One can see that the situation at hand, is quite different from any other situation usually treated using quantum mechanics where the theory affords, at least one self consistent assignment at each time of a state of the Hilbert space to our physical system, at each time. In trying to the consider such assignment when presented with any of the proposed justifications offered to deal with the issue one must be ready to accept one of the following: i) our universe was not really in that symmetric
state (corresponding to the vacuum of the quantum field), ii) our universe is still described by a symmetric state, iii) at least at some points in the past the description of the state of our universe could not be done within quantum mechanics, iv) quantum mechanics does not correspond to the full description of a system at all times, or v) our own observations of the universe mark the transition from a symmetric to an asymmetric state. None of these represent a satisfactory alternative, in particular, if we want to claim, that we understand the evolution of our universe and its structure – including ourselves – as the result of the fluctuations of quantum origin in the very early stages of our cosmology.

Next we give a short description of this analysis. The staring point is as usual the action of a scalar field coupled to gravity.

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R[g] - \frac{1}{2} \nabla_a \phi \nabla_b \phi g^{ab} - V(\phi) \right], \]  \hspace{1cm} (1)

where \( \phi \) stands for the inflaton and for its potential \( V \). One then splits both, metric and scalar field into a spatially homogeneous ”background” part and an inhomogeneous part ”fluctuation”, i.e. the scalar field is written \( \phi = \phi_0 + \delta\phi \), while the perturbed metric can, (after appropriate gauge fixing and by focussing on the scalar perturbation) be written

\[ ds^2 = a(\eta)^2 \left[ -(1 + 2\Psi) d\eta^2 + (1 - 2\Psi) \delta_{ij} dx^i dx^j \right], \]  \hspace{1cm} (2)

where \( \Psi \) is the relevant perturbation called the ”Newtonian potential”.

The background solution corresponds to the standard inflationary cosmology during the inflationary era has a scale factor \( a(\eta) = -\frac{1}{H_I \eta} \), with \( H_I^2 \approx (8\pi/3)GV \) while the scalar \( \phi_0 \) field in the slow roll regime so \( \dot{\phi}_0 = -(a^3/3\dot{a})V' \).

The perturbation of the scalar field leads to a perturbation of the energy momentum tensor, and thus Einstein’s equations at lowest order lead to

\[ \nabla^2 \Psi = 4\pi G \dot{\phi}_0 \delta \dot{\phi} \equiv s \delta \dot{\phi} \]  \hspace{1cm} (3)

where \( s = 4\pi G \dot{\phi}_0 \). This will be our main equation. Next, we write the quantum theory of the rescaled the field \( y = a\delta\phi \). We consider the field in a box of side \( L \), and write

\[ \hat{y}(\eta, \vec{x}) = \frac{1}{L^3} \sum_k e^{i\vec{k} \cdot \vec{x}} \hat{y}_k(\eta), \hspace{1cm} \hat{\pi}^{(y)}(\eta, \vec{x}) = \frac{1}{L^3} \sum_k e^{i\vec{k} \cdot \vec{x}} \hat{\pi}_k(\eta), \]  \hspace{1cm} (4)
where the sum is over the wave vectors \( \vec{k} \) satisfying \( k_i L = 2\pi n_i \) for \( i = 1, 2, 3 \) with \( n_i \) integers, and where \( \hat{y}_k(\eta) = y_k(\eta)\hat{a}_k + \overline{y}_k(\eta)\hat{a}^\dagger_k \) and \( \hat{\pi}_k(\eta) = g_k(\eta)\hat{a}_k + \overline{g}_k(\eta)\hat{a}^\dagger_k \) with

\[
y^{(\pm)}_k(\eta) = \frac{1}{\sqrt{2k}} \left( 1 \pm \frac{i}{\eta k} \right) \exp(\pm i\eta k), \quad g^{(\pm)}_k(\eta) = \pm i\sqrt{\frac{k}{2}} \exp(\pm i\eta k). \quad (5)
\]

Given that we are interested in considering a kind of self induced collapse which operates in close analogy with a “measurement”, we write the decompositions \( \hat{y}_k(\eta) = \hat{y}^R_k(\eta) + i\hat{y}^I_k(\eta) \) and \( \hat{\pi}_k(\eta) = \hat{\pi}^R_k(\eta) + i\hat{\pi}^I_k(\eta) \) where the operators \( \hat{y}^R_k(\eta) \) and \( \hat{\pi}^R_k(\eta) \) are hermitian. Next we provide a simple specification of what we mean by “the collapse of the wave function” by stating the form collapsed state in terms of its collapse time. We assume the collapse to be analogous to some sort of imprecise measurement of the operators \( \hat{y}^R_k(\eta) \) and \( \hat{\pi}^R_k(\eta) \). Let \( \{\Xi\} \) be any state in the Fock space of \( \hat{y}, \hat{\pi} \), and assign to each such state the following quantity: \( d^{R,I}_k = \langle \hat{a}^R_k | \Xi \rangle \). The expectation values of the modes of the fundamental field operators are then expressible as

\[
\langle \hat{y}^{R,I}_k \rangle_\Xi = \sqrt{2}\Re(y_k d^{R,I}_k), \quad \langle \hat{\pi}^{(y,R,I)}_k \rangle_\Xi = \sqrt{2}\Re(g_k d^{R,I}_k). \quad (6)
\]

For the vacuum state \(|0\rangle\) we have of course: \( \langle \hat{y}^{R,I}_k \rangle_0 = 0, \langle \hat{\pi}^{(y,R,I)}_k \rangle_0 = 0 \), while their corresponding uncertainties are

\[
(\Delta \hat{y}^{R,I}_k)^2_0 = (1/2)|y_k|^2(hL^3), \quad (\Delta \hat{\pi}^{R,I}_k)^2_0 = (1/2)|g_k|^2(hL^3). \quad (7)
\]

**The collapse:** In order to describe is the state \(|\Theta\rangle\) after the collapse we must specify \( d^{R,I}_k = \langle \Theta | \hat{a}^{R,I}_k | \Theta \rangle \).

This is done by making the following assumption about the state \(|\Theta\rangle\) after collapse:

\[
\langle \hat{y}^{R,I}_k(\eta_k) \rangle_\Theta = x^{R,I}_{k,1} \sqrt{(\Delta \hat{y}^{R,I}_k)^2_0} = x^{R,I}_{k,1}|y_k(\eta_k)|\sqrt{hL^3}/2, \quad (8)
\]

\[
\langle \hat{\pi}^{(y,R,I)}_k(\eta_k) \rangle_\Theta = x^{R,I}_{k,2} \sqrt{(\Delta \hat{\pi}^{(y,R,I)}_k)^2_0} = x^{R,I}_{k,2}|g_k(\eta_k)|\sqrt{hL^3}/2, \quad (9)
\]

where \( x^{R,I}_{k,1}, x^{R,I}_{k,2} \) are selected randomly from within a Gaussian distribution centered at zero with spread one. From these equations we solve for \( d^{R,I}_k \). We note that our universe, corresponds to a single realization of the random variables, and thus each of the quantities \( x^{R,I}_{k,1,2} \) has a single specific value. Later, we will see how to make relatively specific predictions, despite these features.
The gravitational sector is treated at the semiclassical level so basic formula Eq.(3) turns into

$$\nabla^2 \Psi = s \langle \delta \dot{\phi} \rangle.$$  \hspace{1cm} (10)

Before the collapse occurs, the expectation value on the right hand side is zero. Let us now determine what happens after the collapse: To this end, take the Fourier transform of Eq.(10) and obtain

$$\Psi_k(\eta) = -\frac{s}{k^2} \langle \delta \dot{\phi}_k \rangle \Theta = -\frac{s}{k^2} \sqrt{\hbar L^3 k} \frac{1}{2a} F(k),$$  \hspace{1cm} (11)

where

$$F(k) = (1/2) [A_k(x_{k,1}^R + ix_{k,1}^I) + B_k(x_{k,2}^R + ix_{k,2}^I)],$$  \hspace{1cm} (12)

with

$$A_k = \frac{\sqrt{1 + z_k^2}}{z_k} \sin(\Delta_k), \quad B_k = \cos(\Delta_k) + (1/z_k) \sin(\Delta_k),$$  \hspace{1cm} (13)

and where $\Delta_k = k \eta - z_k$ with $z_k = \eta_k^c k$.

Next we turn to the observational quantities. We will, disregard the changes to dynamics that happen after re-heating and due to the transition to standard (radiation dominated) evolution. The quantity that is measured is $\frac{\Delta T}{T}(\theta, \phi)$ which is a function of the coordinates on the celestial two-sphere which is expressed as $\sum_{lm} \alpha_{lm} Y_{lm}(\theta, \phi)$. The angular variations of the temperature are then identified with the corresponding variations in the “Newtonian Potential” $\Psi$, by the understanding that they are the result of gravitational red-shift in the CMB photon frequency $\nu$ so $\frac{\delta T}{T} = \frac{\delta \nu}{\nu} = \frac{\delta (\sqrt{g_{00}})}{\sqrt{g_{00}}} \approx \Psi$. Thus, the measured quantity is the “Newtonian potential” on the surface of last scattering: $\Psi(\eta_D, \vec{x}_D)$, from where one extracts $a_{lm} = \int \Psi(\eta_D, \vec{x}_D) Y_{lm}^* d^2 \Omega$. To evaluate the expected value for the quantity of interest we use (11) and (11) to write

$$\Psi(\eta, \vec{x}) = \sum_k \frac{s}{k^2} \sqrt{\frac{\hbar}{L^3}} \frac{1}{2a} F(\vec{k}) e^{i \vec{k} \cdot \vec{x}},$$  \hspace{1cm} (14)

then, after some algebra we obtain

$$\alpha_{lm} = s \sqrt{\frac{\hbar}{L^3}} \frac{1}{2a} \sum_k \frac{U(k) \sqrt{k}}{k^2} F(\vec{k}) 4\pi i^l j_l(\sqrt{\vec{k} \cdot R_D}) Y_{lm}(\vec{k}),$$  \hspace{1cm} (15)
where \( \hat{k} \) indicates the direction of the vector \( \vec{k} \). It is in this expression that the justification for the use of statistics becomes clear. The quantity we are in fact considering is the result of the combined contributions of an ensemble of harmonic oscillators each one contributing with a complex number to the sum, leading to what is in effect a 2 dimensional random walk whose total displacement corresponds to the observational quantity. Next we evaluate the most likely value for such total displacement with the help of the imaginary ensemble of universes, and the identification of the most likely value with the mean ensemble value. After taking the continuum limit and rescaling the variables of integration to \( x = kR_D \), we find

\[
|\alpha_{lm}|^2_{\text{M.L.}} = \frac{s^2\hbar}{2\pi a^2} \int \frac{C(x/R_D)}{x^4} j_l^2(x)x^3 dx,
\]

(16)

where

\[
C(k) \equiv 1 + (2/z_k^2) \sin^2(\Delta_k) + (1/z_k) \sin(2\Delta_k).
\]

(17)

In the exponential expansion regime where \( \mu \) vanishes and in the limit \( z_k \rightarrow -\infty \) where \( C = 1 \), we find:

\[
|\alpha_{lm}|^2_{\text{M.L.}} = \frac{s^2\hbar}{2a^2} \frac{1}{l(l+1)}.
\]

(18)

which has the standard functional result. However we must consider the effect of the finite value of times of collapse \( \eta_k \) codified in the function \( C(k) \).

We note is that in order to get a reasonable spectrum there is a single simple option: That \( z_k \) be essentially independent of \( k \) that is the time of collapse of the different modes should depend on the mode’s frequency according to \( \eta_k = z/k \). This is a rather strong conclusion which could represent relevant information about whatever the mechanism of collapse is.

3 A version of ‘Penrose’s mechanism’ for collapse in the cosmological setting

Penrose has argued that the collapse of quantum mechanical wave functions is dynamical process independent of observation, and that the underlying mechanism is related to quantum gravity. More precisely, according to this suggestion, the collapse into one of several coexisting quantum mechanical
alternatives would take place when the gravitational interaction energy between the alternatives exceeds a certain threshold. A naive realization of Penrose’s ideas in the present setting could be obtained as follows: Each mode would collapse by the action of the gravitational interaction between it’s own possible realizations. In our case, one could estimate the interaction energy $E_I(k, \eta)$ by considering two representatives of the possible collapsed states on opposite sides of the Gaussian associated with the vacuum. We interpret $\Psi$, literally as the Newtonian potential and consequently, $\rho = a^{-2}\dot{\phi}_0 \delta \dot{\phi}$ should be identified with matter density. Then for the interaction energy between alternatives we would have:

$$E_I(\eta) = \int \Psi^{(1)}(\eta) \rho^{(2)}(\eta) dV = \left(\frac{a}{L^3}\right) \dot{\phi}_0 \Sigma_k \Psi^{(1)}_k(\eta) \delta \dot{\phi}^{(2)}_k(\eta),$$

where (1), (2) refer to the two different realizations chosen. Recalling that $\Psi_k = (-s/k^2)\delta \dot{\phi}_k$, we find

$$E_I(\eta) = -4\pi G (a/L^3) \dot{\phi}_0^2 \Sigma_k (1/k^2) \delta \dot{\phi}^{(1)}_k(\eta) \delta \dot{\phi}^{(2)}_k(\eta) \approx \Sigma_k (\pi hG/ak)(\dot{\phi}_0)^2.$$  

Where we have used equation (7), to estimate $\delta \dot{\phi}^{(1)}_k(\eta) \delta \dot{\phi}^{(2)}_k(\eta)$ by $|<\delta \dot{\phi}_k>|^2 = \hbar kL^3(1/2a)^2$.

This result can be interpreted as the sum of the contributions of each mode to the interaction energy of different alternatives. We view each mode’s collapse as occurring independently, so the collapse of mode $k$ would occur when this energy $E_I(k, \eta) = (\pi hG/ak)(\dot{\phi}_0)^2 = \frac{\pi hG}{9H_I}(a/k)(V')^2$ reaches the value of the Planck Mass $M_p$. Thus the condition determining the time of collapse $\eta^k_c$ of the mode $k$ becomes,

$$z_k = \eta^k_c = \frac{\pi}{9}(hV')^2(H_I M_p)^{-3} = \frac{\epsilon}{8\sqrt{6} \pi} (V)^{1/2} \equiv z^c,$$

which is independent of $k$, and thus, as we saw in the previous section leads to a roughly scale invariant spectrum of fluctuations in accordance with observations.

4 Discussion

First we address a recent article [5], which is part of this volume in which colleagues reiterate, in response to [1], that the problem we have alluded to, does
not exist. It is illuminating to consider their claims and views in this regard: 1) That the situation is analogous to the spontaneous symmetry breaking in field theories, 2) that the environment selects as preferential "pointer" basis the one that diagonalizes the field operators rather than the momenta operators because of couplings of other fields to the field operators, and 3) that "the initial symmetric vacuum state evolves into a symmetric superposition of inhomogeneous states out of which one component is selected". The first point is rather complex to discuss and will be addressed elsewhere, limiting ourselves here to point out, in general, the dangers of the "arguments by analogy" so let's focus here on the last two. Point 2) seems to ignore the fact that most of the known interactions are of the gauge-theory type and couple both to the momentum and the other "spatial" field gradients rather than to the underived fields themselves. However the clearest problem lies in point 3) where the authors do acknowledge that the unitary evolution leads at late times to a "symmetric superposition of inhomogeneous states" which is, according to quantum mechanics nothing but a fully symmetric state. Then somehow, *one of the components of this state gets "selected"*. Is this a physical process or mechanism?, does this occur at some time?, if not, then what is this *get selected* supposed to represent? Is this to be regarded as just part of our subjective perceptional framework? if so what part of the treatment is not? Do or do not the states represent the physical condition of the system they describe? If they do not, why would we view the initial state as indicating the the early universe was homogeneous and isotropic? Could we say that we understand the origin of the anisotropies and inhomogeneities if we didn't claim we started with a condition that was homogeneous and isotropic? Perhaps we should think that our own actions play an active role in producing this selection?. If this is the case, are we understand the emergence of the conditions that make us possible (the inhomogeneity and anisotropy of the universe) are in part the result of our own actions? See also the issue of the assignment of a state at every time raised in the introduction. In short, it is quite clear that something strange is being called upon with the statement that "one of the alternatives gets selected" which character is not being revealed by avoiding to address all these issues. We must understand under which conditions that this selection mechanism operate. Our point of view is that it is always healthier to confront the hard issues face on, because even if one fails to find a satisfactory answer at the start, their acknowledgment is the first step to their eventual resolution. This is the posture we have taken and we find it quite remarkable that in
doing so we are able to obtain a relatively satisfactory picture. We do not
know what exactly is the physics of collapse but we were nevertheless able
to obtain some constraints on it (about the time of collapse of the different
modes), and shown that a simplistic extrapolation of Penroses ideas satisfy
this constraint.

In conclusion, we have reviewed a serious shortcoming of the inflationary
account of the origin of cosmic structure, and have given a brief account
of the proposals to deal with them which were first reported in [1]. These
lines of inquiry have lead to the recognition that something else seems to be
needed for the whole picture to work and that it could be pointing towards
an actual manifestation quantum gravity. We have shown that not only the
issues are susceptible of scientific investigation based on observations, but
also that a simple account of what is needed, seems to be provided by the
extrapolation of Penrose’s ideas to the cosmological setting. Interestingly
the scheme does in fact lead to some deviations from the standard picture
where the metric and scalar field perturbations are quantized. For one, as
explained elsewhere[6], one is lead to expect no excitation of the tensor modes
because it is only the scalar metric perturbation that gets excited by the
collapse of the quantum inflaton field. We also find new avenues to address
the fine tuning problem that affects most inflationary models, because one
can follow in more detail the objects that give rise to the anisotropies and
inhomogeneities, and by having the possibility to consider independently the
issues relative to formation of the perturbation, and their evolution through
the reheating era. That is, the present analysis offers a path to get rid of the
“fine tuning problem” for the inflationary scenarios [1, 6]. Some of these
aspects can, in principle, be tested, indicating that what initially could have
been thought to be essentially a philosophical problem, leads instead to truly
physical issues.

Our main point is however that in our search for physical manifestations
of new physics tied to quantum aspects of gravitation, we might be ignoring
the most dramatic such occurrence: The cosmic structure of the Universe
itself.

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