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Magneto-electric momentum transfer to atoms and molecules

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We report the first observation of mechanical momentum transferred to atoms and molecules upon application of crossed electric and magnetic fields. We identify this momentum as the microscopic analogue of the classical Abraham force. Several predictions of additional magneto-electrically induced mechanical momentum are addressed. One of them, proposed to result from the interaction with the quantum vacuum, is experimentally refuted, others are found to be currently below experimental detection.

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It has been shown that in crossed electric and magnetic field $E$ and $B$, the optical and electrical properties of matter become anisotropic along the axis $E \times B$ [1] [2] [3]. As this anisotropy manifests itself in the dispersion law and thus in the momentum of the photons and the charge carriers respectively, one may wonder whether a similar anisotropy can exist in the mechanical momentum of particles in crossed fields.

Invariance under time, charge and parity reversal straightforwardly shows it to be symmetry allowed for a particle to acquire a mechanical momentum $p$ upon applying a crossed electric and magnetic field $E$ and $B$

$$p = \alpha E \times B$$

If we assume the particles to be in a gaseous phase in a container, and the collisions between the particles to be elastic, the momentum imparted to each of the particles by the application of the fields will be conserved within the gas as a whole, and ultimately transferred to the wall perpendicular to $E \times B$. If we apply a harmonically oscillating electric field $E(t) = E \sin \omega t$ and a static magnetic field $B$, each particle will contribute a force on this wall given by

$$F_p = \frac{dp}{dt} = \alpha \omega E \times B$$

Such a force exerted on individual atoms would be the microscopic equivalent of the so-called Abraham force density which was first formulated for macroscopic media, and has been the subject of a long-standing controversy [4–7]. In the so-called Abraham version one finds a macroscopic force density (in SI units) $f_A = \varepsilon_0 (\varepsilon_r - 1/\mu_r) E \times B$ [8], whereas in the Minkowski version $f_M = \varepsilon_0 (\varepsilon_r - 1) E \times B$ ($\varepsilon_r$ and $\mu_r$ are the relative dielectric permittivity and magnetic permeability respectively). This can be compared to the quantum-mechanical conserved pseudo-momentum of a neutral atom in a homogeneous magnetic field, $K = \sum_i m_i \dot{r}_i + \sum_i q_i B \times r_i$ [9]. An additional electric field creates a finite polarization $< P > = \langle \sum_i q_i r_i \rangle = \alpha E$ ($\alpha$ is the static electric polarizability of the particle with SI unit $C m^2/V$) in the ground state so that $0 = \dot{K} = \sum_i m_i \dot{r}_i - \alpha \dot{E} E \cdot B$.

This would lead to a force density $f = N \alpha \dot{E} E \times B$ (where $N$ is the particle density) and which is consistent with the Minkowski version, since $\alpha (\varepsilon - 1) = N \alpha$, and we deduce $\alpha = \alpha$ in Eq. (1). Note that the pseudo-momentum in this model equals neither the conjugated momentum $P = \sum_i m \dot{r}_i + \frac{1}{2} \sum_i q_i B \times r_i$, nor the kinetic momentum $P_k = \sum_i m \dot{r}_i$. Both were proposed by Barnett [7] to solve the Abraham-Minkowski controversy.

The observation of the Abraham force due to a crossed oscillating electric field and a static magnetic field was reported by James [10] and by Walker et al [11] [12] in solid dielectrics. It should be noted that the Abraham force due to a static electric field and an oscillating magnetic field was reported not to be observed, against all expectation [13–15]. For a discussion of these and related experiments, see [4] and [6].

Feigel was the first to consider the interaction of a macroscopic magneto-electric material with the quantum vacuum [16]. The so-called Feigel effect implies that momentum from the vacuum fluctuations can be transferred to matter by the intermediary of the optical magneto-electric anisotropy and that therefore an QED contribution exists to the classical Abraham force, corresponding to a 'Feigel' momentum $p_F$:

$$p_F = \frac{1}{32 \pi^2} \Delta n_{MEA} \hbar \left( \frac{\omega_c}{c} \right)^4$$

where $\Delta n_{MEA} = \chi_{MEA} EB$ is the magneto-electric optical anisotropy [1] [2]. In order to avoid the notorious UV catastrophe, Feigel was obliged to introduce an empirical cut-off frequency $\omega_c$ for the material's response. Particularly this cut-off procedure was contested by several groups, since it is widely believed that the UV catastrophe should somehow be absorbed in the parameter values.
attributed to bulk media [17]. It was shown [18–20] that such a transfer would then only occur in a geometry of finite size, similar to that of the Casimir effect, albeit with much smaller values than obtained by Feigel. Obukhov and Hehl [21] also argued that no net momentum transfer from vacuum fluctuations to bulk media can exist. However, very recently, Croze has forwarded new theoretical support in favour of Feigel’s claim [22], correcting in the process a minor numerical error. Kawka and Van Tiggelen have proposed a nonrelativistic quantum theory of a harmonic oscillator in crossed electric and magnetic fields [9], in which the UV catastrophe was shown to be absorbed in a mass renormalisation of the oscillator. Applying this model to a hydrogen atom predicts a reduction of \( \pi c/\omega \) to be absorbed in a mass renormalisation of the oscillatory fields \( \chi \), in which the UV catastrophe was shown.

We experimentally confirm the prediction of the magneto-electrically induced pressure on magnetic field strength, \( \propto \omega NLEB \cos \omega t \) respectively. If we define the effective length of the Feigel momentum, expressed as a fraction of the classical Abraham force, \( \frac{p_F}{p_A} \approx 7 \), i.e. the magneto-electrically induced particle momentum would be dominated by the contribution from the quantum vacuum.

For the ratio between the Feigel and Abraham momenta we find

\[
\frac{p_F}{p_A} = \frac{\pi^2 \chi MEA}{2 N \alpha} \left( \frac{1}{\lambda_c} \right)^4 \hbar
\]

where \( \lambda_c = \frac{2 \pi c}{\omega_c} \). Feigel proposed \( \lambda_c = 0.1 \, nm \) as the limit of the matter response to the vacuum fluctuations. Using the experimental results of Roth and Rikken [1] for large organometallic molecules, Croze predicts \( \frac{p_F}{p_A} \approx 7 \), i.e. the magneto-electrically induced particle momentum would be dominated by the contribution from the quantum vacuum.

\[ P(t) = \alpha \omega NLEB \cos \omega t \]

FIG. 1: Schematic setup of the experiment.

Clearly the magneto-electrical momentum, in spite of its long history, still poses fundamental problems. In this Letter we will describe a new method to accurately measure the momentum transferred to atoms or molecules in crossed oscillating electric fields and static magnetic fields. We experimentally confirm the prediction of the classical Abraham force. More specifically, we do not observe any deviations from the Abraham prediction for media where the predicted contribution from the Feigel effect should be observable.

Since the work by James and by Walker et al, no new experiments to measure the Abraham force have been reported. Very recently, a proposition was made to measure it at optical frequencies using whispering gallery modes [23]. The method used here measures directly the pressure exerted by an atomic or molecular gas on the wall of the container if it is exposed to crossed electric magnetic and magnetic fields, \( E \times B \) respectively. If we define the effective length of the \( E \times B \) region as \( L = \int E(z)B(z)dz/EB \), the momentum change due to the Abraham force exerts an oscillating pressure \( P \) on the wall perpendicular to \( E \times B \) given by

\[ P(t) = \alpha \omega NLEB \cos \omega t \]

Such a pressure can be detected by a microphone located at the wall. By tuning \( \omega \) to a longitudinal acoustic resonance of the system, the pressure can be multiplied by the Q factor of the resonance. Using values of \( N = 2,7.10^5 m^{-3} \) (1 bar ideal gas), \( E = 10^5 V/m \), \( B = 1 T \), \( \omega = 3.10^4 s^{-1} \), \( L = 2 cm \), \( Q = 10 \) and \( \alpha = 2,2.10^{-41} C^2/m/V \) we find \( P = 4.10^{-9} Pa \) and a velocity of \( 0.3 \, nm/s \). (values for He, [24]). The typical sensitivity of an electret microphone is \( S = 10 \, mV/Pa \), so microphone signal voltages of around 5 nV can be expected which are within experimental reach when using phase sensitive detection (PSD). Figure 1 shows schematically the setup used. It consists of a 3 mm diameter, 5 cm long glass tube, with commercial electret microphones butt coupled to its ends, carefully shielded in thick-walled copper housings. The electric field was supplied by a high voltage amplifier (HV amp), generating voltages up to 1000 V, and the magnetic field was provided by an electromagnet, with fields up to 1,5 T. The Q factor was determined from the acoustic resonance lineshape. The systematic inaccuracy of our setup is estimated to be 3 %, mostly due to the inaccuracy of the microphone sensitivity calibration.

Typical results for nitrogen gas are shown in Figure 2, confirming the linear dependencies of the magneto-electrically induced pressure on magnetic field strength, gas pressure ( \( \propto \) particle density) and electric field oscillation frequency. The linear dependence on electric field strength is intrinsic because of the phase sensitive detection of the pressure signal. The dashed lines in the two top panels are the theoretical predictions, based on Eq. 5. Within the experimental accuracy, the experimental results agree with the theory. The slope in the bottom panel of Figure 2 allows to determine \( \alpha \), using Eq. 5. Figure 3 shows the results for \( \alpha \) obtained this way for several gasses, as a function of the literature value for \( \alpha \). All gasses were measured at room temperature and atmospheric pressure, except furan, which was measured at its room temperature liquid pressure. Table 1 summarizes these results, and shows also the calculated contribution of the Feigel momentum, expressed as a fraction of the Abraham momentum, and based on experimental or theoretical values for \( \chi MEA \). Only the value for nitrogen is experimental [25], but it is in good agreement with the calculated value [26], giving confidence in the other values.
FIG. 2: Magneto-electric acoustic pressure observed in nitrogen gas. Top panel; 6,15 kHz, 1 atm, E = 370 kV/m, Middle panel; 6,15 kHz, 1 atm, B = 1,14 T. Bottom panel; 1 atm. Solid lines are linear fits to the data, dashed lines theoretical predictions.

Calculated by the same authors. For the two molecules in the table with the highest magneto-electric anisotropy, the predicted contributions for the Feigel effect are much larger than the experimental uncertainties on $\alpha_{MEMT}$, up to 7,5 times for furan. As $\alpha_{MEMT}$ and $\alpha_{LIT}$ agree within the experimental uncertainties, we conclude from these results that the prediction for the Feigel effect as expressed by Eq. 3 is not observed. Note that the Feigel prediction contains one adjustable parameter, the response cutoff wavelength $\lambda_c$, and that increasing its value to 0,17 nm decreases the prediction of the Feigel momentum contribution to below our experimental uncertainty. However, strong magneto-electric anisotropy was still reported at 0,16 nm wavelength [30], the shortest wavelength at which its observation was ever attempted. Our experimental results therefore unambiguously contradict Feigel’s prediction. Recent theoretical work on simple models suggests that $\chi_{MEA}$ decays algebraically as $\omega^{-2}$ at high frequencies, in much the same way as the dynamic electrical polarizability [31]. This makes the UV catastrophe in the macroscopic description as proposed by Feigel, unavoidable and unrepairable.

In a QED version of the Feigel effect by Kawka and Van Tiggelen [9], this UV catastrophe was removed by mass regularization. Our current experimental accuracy does not allow to make quantitative statements concerning this prediction, but our setup could be improved to attain the 1% accuracy estimated to be necessary for the observation of this regularization. We hope that this perspective will stimulate realistic calculations of this regularization, beyond the harmonic oscillator approximation and in a relativistic context.

In order to make a contribution to the Abraham-Minkowski debate, our experiment would have detect the difference between 1 and $1/\mu_r$. The gas with the largest $\mu_r$ to our knowledge is oxygen, with $\mu_r - 1 = 3,4 \times 10^{-3}$ at room temperature and 1 atm. [24]. Attaining such a precision is a considerable experimental challenge, but going to lower temperatures or higher pressures could increase $\mu_r - 1$ to accessible values.

In summary, we have reported the first observation of mechanical momentum transferred to atoms and molecules by applying crossed time-varying electric fields and static magnetic fields. We quantitatively identify this momentum as the microscopic analogue of the classical Abraham force. We exclude the existence of additional magneto-electrically transferred momentum, as proposed by Feigel to result from the optical magneto-electric anisotropy interacting with the quantum vacuum fluctuations. Other predictions for additional contributions to the Abraham force are currently beyond our experimental resolution, but the new method described in this Letter has potential to successfully address these issues.

FIG. 3: Polarizability for different atoms and molecules as deduced from magneto-electrically generated pressure, as a function of the literature values. Solid line corresponds to the classical Abraham force prediction.

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TABLE I: Polarizabilities deduced from magneto-electric momentum transfer, the corresponding literature values, the experimental or calculated magneto-electric anisotropy, and the calculated ratio of Feigel and Abraham momenta for the gases studied.

| gas  | $\alpha_{MEMT}$ | $\alpha_{LIT}$ | $\chi_{MEA}$ | $p_F/p_A$ |
|------|-----------------|----------------|-------------|-----------|
| He   | $0.20\pm10\%$  | $0.22^{[24]}$ | $0.017^{[26]}$ | $1.5\%$    |
| N$_2$ | $1.9\pm5\%$   | $1.89^{[24]}$ | $0.47^{[25]}$  | $4.8\%$    |
| C$_2$H$_4$ | $4.4\pm5\%$ | $4.4^{[24]}$ | $3.7^{[26]}$    | $16\%$     |
| furan | $7.8\pm4\%$  | $7.9^{[29]}$ | $12^{[28]}$     | $29\%$     |

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