Prediction of helicopter rotor noise in hover using FW-H analogy

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Abstract. The integral formulation of Ffowcs Williams and Hawking (FW-H) analogy developed by Farassat is implemented to understand the sound generation and propagation of a rotating slender body like the helicopter rotor blade in hover. Using the linear acoustic theory, the thickness and loading noise terms are implemented in a flexible, new, in-house aero-acoustic tool. The pressure distribution on the blade surface is obtained from CFD simulation which sufficiently captures the correct flow characteristics of the rotor blade. Although, there is an improved estimation of the acoustic pressure signal in comparison to existing numerical noise signal results, there is still an under prediction of the peak amplitude compared to experimental values. The code is validated against the experiment for the UH-1H model helicopter rotor equipped with two rectangular blades having NACA0012 section.

1. Introduction

Noise emission is a major concern in most industries such as automotive, aerospace, wind energy and turbomachines. The global initiative to reduce noise to meet environmental targets brings focus to the study and development of aero-acoustics. This has led to noise control now becoming a design parameter rather than a post production fix. Taking advantage of the advancements made in computers, development of a new, in-house acoustic tool to study and investigate sound generated by rotating bodies such as helicopter rotors and wind turbine rotors is the main objective here. The sound producing mechanisms are far too many and complicated for rotating bodies. For example, in the case of helicopter rotor in subsonic motion with no shocks in the flow field, there exists sound generated due to thickness, loading, blade-vortex interaction, turbulence, blade-wake interaction and high-speed impulsive noise. For a preliminary study, linear noise theory is implemented to study thickness and loading noise. Since there exist experimental datasets for the sound produced by the UH-1H model rotor in [1], it was used as one of the validation cases for the tool developed. Acoustic fields can be analyzed by either the direct method where the flow equations are classically solved or through hybrid methods where the sound source region is solved through classical CFD simulations and the far-field sound propagation is solved through acoustic analogies. Due to the criticality imposed by the acoustic scales which are at least a magnitude lower than the flow scales, the direct methods are cumbersome and expensive. Hybrid methods offer elegant solutions by separating sound generation and sound propagation mechanisms. This has led to development of several formulations over the years with a varying degree of success in predicting the sound signal matching the experimental data. This depends
on the non-linearity effects of the sources being included in the model or not. However, they
give useful insights into the specific areas of major noise generation on a rotor and help in
devising noise control methods. Since there is no clear consensus on which formulation to be
considered while studying different noise mechanisms, it is necessary to understand the physics
and governing principles of sound generation and propagation.

2. Theoretical modelling

Sound generation is captured by the non-zero source terms in the right hand side of the
in-homogeneous wave equation. This linear wave equation is obtained by re-arranging the
conservation of mass and momentum equations. Lighthill first developed a theory to predict
noise produced by a turbulent flow field in an unbounded medium during his study of jet noise [3].
Ffowcs Williams and Hawking (FW-H) developed an equation to predict noise while including
the presence of solid bodies having arbitrary motion in the flow region [4].

This equation is an exact rearrangement of flow governing equations. FW-H derived the
equation using the idea of embedding the flow problem in a larger unbounded domain using
generalized Green’s functions to solve the original acoustic flow case. They used the ordinary
derivatives as generalized derivatives while keeping the conservation laws in differential form
valid. The moving surface is represented as $f(\vec{x}, t) = 0$ such that $\nabla f = \hat{n}$, where $\hat{n}$ is the unit
outward normal as shown in figure 1.

![Figure 1. Definition of moving surface](image)

This implies that $f < 0$ is the volume enclosed by the surface. This volume is replaced by
the undisturbed medium and surface distribution of sources. The FW-H equation from [4] is
expressed as:

$$\Box^2 p' = \frac{\partial}{\partial t} [\rho_0 v_n \delta (f)] - \frac{\partial}{\partial x_i} [p n_i \delta (f)] + \frac{\partial^2}{\partial x_i \partial x_j} [H(f) T_{ij}]$$

(1)

with $T_{ij} = \rho u_i u_j - \sigma_{ij} + (p' - c^2 \rho') \delta_{ij}$. $p'$, $c$, $\rho_0$, $v_n$, $T_{ij}$, $\sigma_{ij}$, $\delta_{ij}$, $H(f)$ and $\delta(f)$ are the acoustic
pressure, speed of sound, density, normal velocity, Lighthill stress tensor, viscous stress tensor,
Kronecker delta, Heaviside and Dirac delta functions in the undisturbed medium respectively.
The solution to this equation is obtained by first confining all the flow non-linearities into a
control volume $V$ which is bounded by a control surface $S$ over which the pressure field is
integrated to get the acoustic signal. These non-linearities are preserved in the Lighthill stress
tensor. When a body is in motion, it produces pressure disturbances in the fluid region which are
felt as sound by an observer in the medium. One important approximation employed is that the
body does not disturb the medium appreciably and hence the non-linearities are neglected. The
turbulence generated is of low intensity and also is a very inefficient noise generating mechanism.
However, the fluctuating pressure produced by the turbulence is indirectly accounted for by the
loading term in the equation. Here, the viscous shear forces acting over the blade are neglected.
Thus converting a non-linear problem to a linear one. But this holds true only for low Mach
numbers. Also, the discontinuities of the flow like the shock waves are not taken into account
in the FW-H formulation. Both compact source and non-compact source theories have been
developed for this equation. The three terms on the right side of the equation are known as
monopole, dipole and quadrupole respectively. Monopole (thickness term) which is defined as the time derivative of elementary mass source, accounts for the displacement of the fluid produced by the body. Dipole (loading term) is defined as the doublet of equal elementary force sources of opposite phase that account for the unsteady loading exerted by the body on the fluid, quadrupole (volume term) is defined as the double doublet in opposite phase accounting for all non-linearities and noise due to turbulence.

Farassat [5] derived the integral solution known as Formulation 1 for the FW-H equation with surface sources in sub-sonic motion. This solution was proposed in two frames of reference: the observer space-time variable \((x, t)\) which is always kept fixed and the source space-time system \((y, \tau)\) as seen in figure 2.

![Figure 2. Trajectory of a source point \(\eta\) as seen by an observer fixed to the medium (x or y frame) and the definitions of visual and emission source positions [5].](image)

For a study of the helicopter rotor noise, the blade is represented as a surface in a frame, the \(\eta\)-frame fixed relative to the surface. The motion of the blade is given by the trajectory of a source on the surface described by the fixed \(\eta\) in a y-frame. Applying convolution to Green’s functions, \(G = \delta(g)\) where \(g = t - \tau - \frac{r}{c}\) and \(r = |x - y|\), the volume integrals are converted to surface integrals by using the properties of \(\delta\) function. After rigorous mathematical operations, the equation for retarded time \(\tau\) is obtained:

\[
g = \tau - t + \frac{|x - y(\eta, \tau)|}{c} = 0 \tag{2}
\]

For a subsonic source motion, there is only one emission time \(\tau_e\) for a given source point \(\eta\). This source point has an emission position \(y_e = y(\eta, \tau_e)\) and emission distance \(r_e, r_e \equiv c(t - \tau_e)\) to the observer position \(x\). This emission time is found usually numerically by a shooting technique described in [5]. The source position and trajectory is known at the observer time \(t\). This is known as the visual position of \(\eta\). To find the emission time, probing is done in small time steps in the backward direction of the trajectory for \(\tau < t\) so that it satisfies the retarded time equation. The Formulation 1 as proposed by Farassat in [5] is of the form:

\[
4\pi p'(x, t) = 4\pi(p'_T(x, t) + p'_L(x, t)) = \frac{\partial}{\partial t} \int_{f=0} \left[ \frac{\rho v_n}{r (1 - M_r)} + \frac{pcos\theta}{cr (1 - M_r)} \right] dS + \int_{f=0} \left[ \frac{pcos\theta}{r^2 (1 - M_r)} \right] dS \tag{3}
\]

The critical factor defined by \(r | 1 - M_r |\) seen in the above equation contains the source motion information. It indicates the difference in the time between the sound received by the observer and the time at which the same sound signal was emitted by the source. \(M_r\) represents the
source Mach number in the direction of the observer. The singularity of this formulation arises when the $M_r$ reaches 1. This leads to non-linearity which affects the quadrupole term and indirectly the dipole term. This is an important limitation of this formulation. Compact source body condition is a critical criterion of this formulation as demonstrated in [5]. Numerically, when the blade surface is divided into many small source cells, it is necessary that the compact source condition is satisfied by every single source cell. It implies that for each source cell, the acoustic field should be related to the global parameters of the rotating body, for example the thickness noise to the net rate of mass injection, loading noise to the net force on the body. If the sources are non-compact instead of the net force contributing to the noise signal, only the local pressure would be accounted for. These compact conditions also depend on the observer time and position making it quite challenging to numerically quantify them for each and every source cell and at each time interval.

3. Methodology

The integral formulation of Farassat has been implemented in the Tecplot 360 macros and scripting language. Tecplot 360 is a visualization and analysis software with built-in macros for scripting and also provides python scripting options. The helicopter rotor blade surface is discretized into small cells which act as acoustic sources. Various parameters of equation 3 are calculated for each source cell at the emission time. Then the corresponding observer time is obtained using the retarded time equation 3. The steady pressure distribution for the loading term is supplied from CFD simulation. The surface integration is obtained through a mid-point formula which is a second-order approximation. The differentiation scheme used for observer time derivatives is also a second-order scheme. The blade surface given as input is taken at time instant $t=0$ and it is rotated through small time steps to complete one period of revolution. All the parameters are computed for each and every single source cell at each time step. This is known as the acoustic potential for each source at emission time. Then their corresponding observer time is computed satisfying the retarded time equation. At given observer time instant, all the source panels with the correct emission time, contributing to the noise signal are added to get the acoustic pressure pulse. This tool provides flexibility in terms of studying the different types of noise terms individually, in terms of analyzing the individual parameters of the equation thus providing opportunities to implement noise control methods. Also enables to study noise contribution from different blade portions such as the leading edge, trailing edge and blade tips and at various observer locations aiding in better understanding of sound generation and propagation.

4. Results and discussion

The analysis was conducted for 1/7-scale model of the UH-1H helicopter rotor in hover, having two rectangular blades with NACA0012 profile with tip Mach number of 0.8 and radius of 1.045m with the observer located at a distance of 3R from the blade tip in the rotor plane, same as in the experimental investigations of Boxwell [1]. The blades are untwisted and un-tapered with aspect ratio of 13.71 as shown in figure 3. The pressure data was obtained from the CFD simulations. The case was set up similar to the CFD computational model as presented in [9]. These simulations were conducted using a cell-centered, block-structured, parallel code SPARC with a Spalart–Allmaras turbulence model with a mesh of 3 million cells. The acoustic signal obtained had to be flipped due to the reverse direction of probing of the acoustic parameters in the implemented formulation.
4.1. Grid dependency
A grid dependency test was conducted for varying number of grids from coarse 584 to 26708 source panels as seen in figure 4. The pressure distribution used for the computation of the loading term for all grids is constant steady data from the fine grid case of 26708 cells. With increasing number of cells, the peak negative amplitude decreases and also the small positive peaks on eitherside of the negative peak change nature. This is because of the varying distribution of source cells on the blade surface. Loading noise computed represents both the near-field and the far-field loading terms added together and it decreases the amplitude of the thickness noise peak. Thickness noise signal and the loading noise signal are added to get the final acoustic pressure pulse. It is seen that the peak amplitude decreases with increasing cell number rather in a counter intuitive way. This could be because of the sharp slopes that are seen in the fine grids in the loading term gets smoothed out as well as the peak thickness amplitude decreases, thereby reducing the overall peak amplitudes.

4.2. Time step dependency
Time step plays a crucial role in predicting noise as the acoustic potential of the sources were differentiated with respect to the time taken. Tests were conducted from a large time step such as $1^\circ$ to $0.25^\circ$ of the azimuth. The thickness peak amplitude increases with increasing time step thereby increasing the overall acoustic peak amplitude as seen in figure 5.

4.3. Higher order of differentiation
The default differentiation scheme in Tecplot 360 is a second order approximation. To improve the peak amplitude, higher order observer time derivatives - 4th and 6th order of central differencing scheme are used. The 4th order scheme improves the signal by 1 Pa but there is no difference between the 4th and 6th order schemes as seen in figure 6. Hence, 4th order scheme is preferred for further analysis.
Figure 4. Grid dependency study
a) thickness noise  b) loading noise  c) total acoustic pressure.

Figure 5. Time step dependency study
a) thickness noise  b) loading noise  c) total acoustic pressure.
4.4. Precise experimental input data

The ambient conditions like the reference pressure, density and temperature recorded during the experiment is unknown. Taking the average conditions during the time of recording in the wind tunnel, the atmospheric pressure is taken as 101400 Pa. The angular frequency ($\omega$) is extracted from the experimental plot by measuring the period of one revolution and using the expression $\omega = \frac{2\pi}{T}$, the new $\omega$ is calculated. Using this value in Mach number formula, the ambient temperature is found to be $T_0 = 285.3K$. Using the ideal gas law, density is also computed $\rho = 1.238 \text{ kg/m}^3$. These new values are then implemented and this improves the peak amplitude of the signal by about 2.3 Pa approximately as seen in figure 7.

5. Validation

The final acoustic signal is computed with higher order of differentiation scheme, with the correct experimental input data for the reference grid of 7028 surface cells and $\Delta t = 0.5^\circ$ as seen in figure 9. This signal is compared with the experimental data from [1]. It is observed that the final peak amplitude obtained using the new in-house tool has a value of 42 Pa approximately when compared to the experimental value value of 60 Pa as seen in figure 8. It is improved compared to the 30 Pa approximately achieved in the theoretical modeling in [1] and also the numerical analysis conducted in [10]. It still underpredicts the experimental value. Other factors like the twisted blade, pre-cone angle are not implemented here and the experimental investigations shows an uncertainty in the value of about +/- 10 Pa.

6. Conclusions

The correct shape of the acoustic signal is captured very well by the tool. The numerical results underpredict the experimental values which points towards the limitations of the formulation itself and the assumptions of the linear theory. The sound due to turbulence encaptured by quadrupoles is not modelled here as at subsonic motion and low Mach numbers their contribution is insignificant. The underprediction in the validation raises the question if the non-linearity effects must also be considered in the formulation. Considering the effects of quadrupoles, non-linearity in higher Mach flows or in forward motion of helicopter rotors is understandable but their significance in low speeds is questionable as pointed out in [8]. Another important criterion is the acoustic compact source conditions. It is necessary that each and every cell satisfies the
Figure 8. Acoustic pressure pulse recorded at inplane location of \( r/D = 1.5 \) \((M_T = 0.8)\) [1].

Figure 9. Acoustic pressure pulse computed at inplane location of \( r/D = 1.5 \) \((M_T = 0.8)\).

In the implementation of the equation 3, it is observed that the acoustic signal is very sensitive to the number of source cells on the blade surface and their distribution. The case with fewer cells and higher time steps seem to produce results more closer to the experiment than the fine grid with smaller time step cases. The location of the positive peaks in the signal which corresponds to sink and source strength and the time shift in the arrival of these signals towards the observer makes the discretization of the blade surface very crucial. This raises the question of the upper and the lower limits of discretization that is allowed while implementing the integral solution of the FW-H equation. The surface integration is a critical step in the computation and since the errors present in this step only gets amplified with numerical differentiation, it is imperative to reduce these numerical errors. This forms the basis of future work to understand the reasons for underprediction at low Mach number helicopter hover cases.

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