Oblique interaction of a laminar vortex ring with a non-deformable free surface: Vortex reconnection and breakdown

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Abstract. Direct Numerical Simulation (DNS) is used to study the interaction of a laminar vortex ring with a non-deformable, free-slip surface at an oblique angle of incidence. The interaction leads to the well-known phenomenon of vortex reconnection. It was found that the reconnection process leads to rapid production of small-scale vortical structures. This phenomenon was found to be related to the kinematics of the reconnection process.

1. Introduction

Vortex reconnection is a ubiquitous feature of vortical and turbulent flows. It has been studied in the context of colliding vortex rings, interacting vortex pairs and the interaction of vortices with a free surface (e.g. Kida, Takaoka & Hussain, 1991; Melander & Hussain, 1989; Gharib & Weigand, 1996). The present work studies a vortex ring impinging on a non-deformable, free-slip surface at an oblique angle of incidence. This problem is equivalent to an oblique-angle collision of two vortex rings with the interaction plane coinciding with the surface. As the vortex ring impinges upon the surface, vortex filaments in the interaction region are disconnected via viscous diffusion towards the surface (or equivalently diffusion towards the oppositely-signed vorticity of the image filaments) and thereby links normally with the disconnected image filaments across the surface (see figure 3a). Subsequently the lower part of the ring convects towards and interacts with the surface, resulting in a second reconnection. Under certain conditions the reconnection results in two half-rings connected to the surface.

The main feature of vortex reconnection is the transfer of circulation from the surface-parallel direction to the surface-normal direction. Previous studies on vortex reconnection have mostly considered colliding vortex rings and vortex pairs with a growing instability and have described the mechanism of the reconnection process. Here we use direct numerical simulation (DNS) to study reconnection of a laminar vortex ring with a free-slip surface at a relatively high Reynolds number (defined as the ratio of the initial circulation of the vortex ring $\Gamma_0$ to the kinematic viscosity) of 7500. At this Reynolds number, despite the absence of the pure-ring instability, reconnection is accompanied by rapid production of a range of smaller scale vortical structures.
2. Numerical approach

The numerical method consists of DNS of the incompressible Navier-Stokes equations on a staggered grid with second-order finite differencing in space and Adams-Bashforth stepping in time. The finite-difference code has been used in previous work involving vortex rings (Archer, Thomas & Coleman, 2010).

The initial setup consists of a laminar ring of radius $R_0$ and circulation $\Gamma_0$ with a Gaussian distribution of vorticity $\omega(r) = (\Gamma_0/\pi\delta^2) \exp(-r^2/\delta^2)$ in the core (where $\delta$ defines the core thickness). The initial value of $\delta$ is taken to be $0.30 R_0$. The simulations are carried out for the initial Reynolds numbers $Re_0 = \Gamma_0/\nu = 7500$. The cubic domain size is $12R_0 \times 12R_0 \times 6R_0$ (with the length along the vertical $z$-axis being the shortest) with $1152 \times 1152 \times 576$ grid points.

The ring centre is initially located at a depth of $3.0 R_0$ below the free surface and the axis (lying in the $x-z$ plane) directed at an angle of $\theta = 7^\circ$ to the horizontal.

A free-slip boundary condition is imposed on the top and bottom surface of the cartesian domain (figure 1). A reference frame which moves in the $x$-direction along with the vortex ring is used in order to prevent the ring from encountering its wake. Appropriate inflow and outflow boundary conditions are imposed in the $x$-direction and the velocity of the co-moving frame is adjusted to keep the ring centred in the domain as in Archer, Thomas & Coleman (2008). A periodic boundary condition is used in the $y$-axis direction.

Figure 1. Cross section of computational domain at the symmetry plane, $y = 0$.

3. Results

3.1. First reconnection

Since the gaussian vorticity profile used to set the initial condition is not an exact solution of the Navier-Stokes equation, immediately after initialization the vortex ring undergoes an adjustment by shedding 4% of its circulation. The ring translates under its self-induced velocity towards the surface (figure 2a). When the ring is close to the surface, the influence of the image vortex ring causes the upper part of the vortex ring to bend back (figures 2b,c). The near-surface vortex filaments disconnect from each other and simultaneously reconnect with their image (figure 2d).
forming two bridges on either side of the symmetry plane composed of the reconnected vortex filaments. Within $20 \frac{R_0^2}{\Gamma_0}$, 94% of the circulation of the upper limb of the vortex ring is transferred to the surface normal direction.

![Figure 2](image)

**Figure 2.** Isosurfaces of second invariant of velocity gradient tensor ($\lambda_2$) showing different stages of vortex reconnection at (a) 38 (b) 56 (c) 58 and (d) 64 $\frac{R_0^2}{\Gamma_0}$.

The topology of the reconnection process is depicted in figure 3a. As the filaments of the vortex ring and its image with oppositely-signed vorticity overlap in the region of interaction, viscous diffusion leads to their annihilation. The disconnected filaments simultaneously reconnect with the disconnected filaments from the image vortex ring. The evolution of the circulation of the filaments from the vortex ring undergoing disconnection which are directed parallel to the surface ($\Gamma_\parallel$) and the circulation of the reconnected filaments directed normal to
the surface ($\Gamma_\perp$) is shown in figure 3b. The magnitude of the former decreases with time and the latter increases with time while the sum is conserved. However the reconnection process does not go to completion and a small remnant of the original circulation is still present in the form of a thin sheet.

\[ \Gamma_\parallel + \Gamma_\perp = \Gamma'_0 \]

$\Gamma_\parallel$: Circulation associated with the interacting vortex core on the symmetry plane $y = 0$
$\Gamma_\perp$: Circulation at the surface ($z = 0$) on the half-plane $y > 0$
$\Gamma'_0$: Circulation of the vortex ring after its readjustment after initialisation

Figure 3. (a) Topology of the reconnection due to the collision of a vortex ring with its image. (b) Symmetry plane ($y = 0$) and half-plane at the surface ($z = 0, y > 0$) showing the two measured circulations. c) Evolution of the surface-parallel and surface-normal circulations (normalized by circulation at $t = 0$) with non-dimensional time.
The absolute rate of change of these two quantities is the same and is given by a line-integral along the projection of the symmetry plane ($y = 0$) at the surface.

\[
\frac{d\Gamma_{\parallel}}{dt} = -A, \quad \frac{d\Gamma_{\perp}}{dt} = A
\]

\[
A = \nu \int_x \frac{\partial \omega_y}{\partial z} z_{surf, y=0} \, dx
\]  \hspace{1cm} (1)

When the vortex ring core first encounters the surface, the rate of change of circulation is very small. Since the upper part of vortex ring is bent backwards, the self-induced velocity due to the curvature causes the vortex core to be flattened against the surface. The resulting strain is balance primarily by a strain in the y-direction, stretching the vortex filaments. The vorticity gradient at the surface increases due to the vortex stretching and the flattening of the vortex core leading to an increase in the rate of change of circulation. The flattening of the vortex core continues until viscous diffusion becomes dominant. During this process, the vortex core at the surface transforms into a head-tail structure as seen in figures 4a, 4b.

Figure 4. Contours of vorticity ($\omega_y$) on the symmetry plane $y = 0$ at $t = (a) 53$ (b) 56 (c) 59 and (d) 60 $R_0^2/\Gamma_0$. 


A reconnection time \( t_R \) characteristic of the timescale over which the circulation is transferred from the surface-parallel to surface-normal direction can be defined as the inverse of the maximum value attained by \( A \) through the reconnection process (normalized by \( \Gamma_0 \)). Performing an alternate set of simulations varying the fluid viscosity, it was found that for values of Reynolds number higher than a particular value, maximum rate of change of circulation remained roughly constant. Hence, from (1) the effect of increasing Reynolds number is an intensification of the vortex stretching and flattening process such that the quantity \( \nu \frac{\partial \omega_y}{\partial z} \bigg|_{z=0, y=0} \) remains roughly constant. In the present \( Re = 7500 \) simulation, the vortex core transforms into a thin sheet during the reconnection process. This flattened vortex core, then splits into two with the 'head' separating from the 'tail' (figure 4c, 4d).

The splitting of the vortex core can be understood as occurring due to the tendency of a vortex sheet to roll-up at its edge. Here, the process can only occur on one edge due to the presence of the surface. Figure 5a shows an isosurface of vorticity at a late stage of the first reconnection, showing the separated head-tail structure. The separated head is visible in the form of a rolled-up vortex and the tail as a thin vortex sheet. Both are wrapped around the two bridges of reconnected vorticity. As the bridges rotate under their self-induced velocity, the vortex sheet gets further stretched and wrapped around the bridges. This leads to a generation of small scale vortical structures at the bridges (figure 5b). Consequently, at the end of the first reconnection, the overall enstrophy of the flow rises sharply.

![Figure 5. Isosurfaces of vorticity showing late stages of first reconnection at \( t = (a) 68 \) and (b) \( 76 R_0^2/\Gamma_0 \).](image)

3.2. Second reconnection
Simultaneous with the first reconnection, the lower part of the vortex ring continues to advect towards the surface under its self-induced velocity and begins the second reconnection with the surface. The interacting vortex core again gets deformed into a head-tail structure (oriented in the opposite direction). The disconnection of vortex filaments, reconnection with the surface and accumulation of reconnected vortex filaments in the form of two bridges occurs similar to the first reconnection. The vortex core flattened into a thin sheet rolls up at its edge forming a vortex which splits away. This process repeats twice leading to three distinct vortex tubes wrapped around the reconnected bridges at their ends. These vortices being of the same sign merge into a single vortex. The remaining vorticity in the tail also rolls up into a separate vortex tube (Figures 6 a,b,c,d).
3.3. Generation of small-scale structures

At the end of the second reconnection, the flow is dominated by several thin vortex tubes. These are progressively stretched and wrapped around the two pairs of bridges formed as a result of the two reconnections. This process leads to a rapid generation of smaller scale vortical structures from the entire doubly-connected vortex ring structure extending into its wake (figure 7). In a recent numerical study involving the reconnection of a pair of vortex tubes, Hussain & Duraisamy (2011) have also observed a similar production of small scales at Reynolds numbers greater than 6000.

The energy spectrum in a windowed three-dimensional region enclosing the reconnected vortex ring is plotted at \( t = 80 \frac{R_0^2}{\Gamma_0} \) (figure 8a) and \( t = 140 \frac{R_0^2}{\Gamma_0} \) (figure 8b). Within a limited range of wavenumbers the spectrum at \( t = 140 \frac{R_0^2}{\Gamma_0} \) exhibits the \( k^{-5/3} \) inertial-range behaviour.

The spectrum at \( t = 140 \frac{R_0^2}{\Gamma_0} \) while consistent with the existence of a wide range of scales does not change significantly from the spectrum at \( t = 80 \frac{R_0^2}{\Gamma_0} \) when most of the small scale structure had not been produced. The similarity of figures 8a and b underlines the danger of making too tight of a conceptual connection between energy at a given wave number \( k \) and an individual eddy structure of size \( l \) proportional to \( k^{-1} \).
Figure 7. Isosurfaces of second invariant of velocity gradient tensor ($\lambda_2$) showing the late stages of second vortex reconnection at $t = (a) \ 80$ (b) $96$ (c) $104$ and (d) $140 \ R_0^2/\Gamma_0$.

Figure 8. Log-log plot of wavenumber and energy present at that wavenumber at $t = (a) \ 80$ and (b) $140 \ R_0^2/\Gamma_0$. Dashed line indicates $k^{-5/3}$ behaviour

4. Summary

The vortex core interacting with the surface is flattened into a vortex sheet whose thickness decreases and strength increases with increasing $Re$. The vortex sheet is wrapped around the bridges of reconnected vortex filaments on either side, and is continuously stretched and wound...
by them. The edge of the vortex sheet rolls up and separates, creating a new edge which can then undergo the same process, ultimately producing several strands of vorticity. Unlike the well-known instabilities associated with vortex rings which are due to the structure of the vortex ring, the phenomenon studied in this paper occurs solely due to the kinematics of the reconnection process, which provides a means of producing small-scale structure from the reconnected vortex ring. Future work is to involve comparing the characteristics of these spatial scales with those of typical three-dimensional fully developed 'Kolmogorov-type' turbulence.

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