Correlated versus Uncorrelated Stripe Pinning: the Roles of Nd and Zn Co-Doping

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(March 21, 2022)

We investigate the stripe pinning produced by Nd and Zn co-dopants in cuprates via a renormalization group approach. The two dopants play fundamentally different roles in the pinning process. While Nd induces a correlated pinning potential that traps the stripes in a flat phase and suppresses fluctuations, Zn pins the stripes in a disordered manner and promotes line meandering. We obtain the zero temperature phase diagram and compare our results with neutron scattering data. A good agreement is found between theory and experiment.

The existence of stripes in doped Mott insulators has engendered a great debate recently. While the presence of stripes in the manganites and nickelates has been firmly established, uncertainties remain concerning whether they are present in the cuprates. In manganese and nickelates stripes are static and can be easily observed. In cuprates, on the other hand, they form a collective fluctuating state and their detection is more subtle. Co-doping of cuprates has been extremely important for unveiling the modulated charge states. However, the inclusion of co-dopant usually reduces the critical superconducting temperature, \(T_c\), raising doubts about the coexistence of superconductivity and the striped phase.\footnote{The first experimental detection of stripes in the cuprates was achieved in a Nd co-doped compound \(\text{La}_{2-x-y}\text{Nd}_x\text{Sr}_y\text{CuO}_4\). For \(y = 0.04\) and \(x = 0.12\) Tranquada et al.\footnote{Ref. 10} found that the commensurate magnetic peak at \(Q = (\pi/a, \pi/a)\) splits by a quantity \(\delta\), giving rise to four incommensurate peaks. In addition, the Bragg peaks split by \(2\delta\), indicating that the charges form domain walls and that the staggered magnetization undergoes a \(\pi\)-phase shift when crossing them.\footnote{Ref. 11} The study of co-doped cuprates has also involved other elements, such as Zn, Ni, Fe, Co, etc.\footnote{Ref. 12} }\footnote{In this paper we study the problem of co-doping within the stripe scenario by performing a renormalization group calculation on a model of quantum elastic strings under the influence of lattice and disorder potentials. We determine the zero temperature pinning phase diagram and compare it with experimental data. The different roles played by rare earth (Nd, Eu) and planar (Zn, Ni) impurities led us to establish a parallel between the stripe- and vortex-pinning problem in high-\(T_c\) superconductors.\footnote{Ref. 13} By doping the antiferromagnetic insulator \(\text{La}_2\text{CuO}_4\) with Sr, i.e., by replacing \(\text{La}^{3+}\) with \(\text{Sr}^{2+}\), charge carriers are introduced into the \(\text{CuO}_2\) planes. In the stripe scenario the carriers, instead of forming a homogeneous quantum fluid, arrange themselves into a highly anisotropic charge-modulated state with one-dimensional (1D) characteristics. The ionized Sr dopants are a source of disorder since they are located randomly in the neighborhood of the \(\text{CuO}_2\) planes. Hence, the number of holes is intrinsically connected with the number of pinning centers, and the stripes can be collectively pinned by these point-like impurities. Despite the correspondence between the number of impurity pinning centers (Sr) and the charge carriers (holes) within the \(\text{CuO}_2\) planes, it is experimentally possible to control these two parameters independently. By co-doping the superconducting material with Nd or Zn, for instance, one can alter the disorder without modifying the number of charge carriers.\footnote{Ref. 14} On the other hand, by growing the superconducting film over a ferroelectric substrate and using an electrostatic field as a control parameter, the number of charge carriers in the plane can be increased for a fixed doping concentration.\footnote{Ref. 15} Hence, the treatment of these two parameters independently is an important theoretical problem.\footnote{Ref. 16} Calculations of the pinning energy within a model in which stripes are regarded as elastic strings have shown that the problem can be described by the Collective Pinning Theory,\footnote{Ref. 17} with a critical Larkin pinning length \(L_c \sim 100\text{Å}\) for doping of order \(x \approx 10^{-2}\) (Ref. 10). Another possible source of pinning for the stripes is lattice distortion such as the tilt of the oxygen octahedra. We have recently studied the role played by lattice and dopants and have generated a phase diagram in terms of the incommensurability, \(\delta\), and the ratio between kinetic and elastic stripe energy, \(\mu\) (this parameter measures the strength of quantum fluctuations).\footnote{Ref. 18} Three different phases were identified: at large values of \(\mu\) and \(\delta\), the stripes form a collective fluctuating state or quantum membrane phase; as \(\mu\) is reduced the stripes become pinned by the underlying lattice and decoupled from each other leading to the so-called flat phase; finally, at small values of \(\delta\) and \(\mu\) disorder becomes relevant and the system can be described in terms of a disordered phase.\footnote{Ref. 19} In this paper we will generalize our earlier approach in order to incorporate the differences between different co-dopants.\footnote{Ref. 20} In general, impurities will pin the stripes, leading to the formation of a static charge order, which is usually accompanied by a reduction of \(T_c\). This statement holds for co-doping with several types of impurities, such as...}
Zn, Ni, Nd, Eu, independently of the intrinsic characteristics of each dopant. Moreover, a special reduction of $T_c$ takes place when the effective number of charges in the CuO$_2$ plane is $n \sim 1/8$ (Refs. 5-7). At this doping value, the striped structure becomes commensurate with the underlying lattice and the effective pinning potential for collective motion of the stripes is at a maximum. A second important feature of doping within the stripe model is that the average separation $L$ between neighboring stripes is not expected to change upon co-doping if the substitution element has the same valence as the replaced one. Therefore, co-doping will simply pin the stripes without changing their overall number or separation. By replacing La$^{3+}$ with Nd$^{3+}$, for instance, one does not change the number of holes introduced into the plane. The same argument holds if one replaces Cu$^{2+}$ by Zn$^{2+}$ or Ni$^{2+}$. Hence, the average stripe separation $L$ and consequently the incommensurability $\delta = a/2L$ are not altered by the introduction of the co-dopant, as is experimentally observed (and trivially inferred). We classify the pinning generated by co-dopants as uncorrelated or correlated. In the former case the statistical mechanics of the stripes is characterized by line wandering, whereas in the latter case the characteristic feature is localization. The situation here is analogous to the case of a vortex line pinned by weak point-like impurities (uncorrelated disorder) or by extended defects, like 1D screw dislocations or artificially produced columnar defects (correlated disorder). For extended defects the pinning energy grows linearly with the distance along the vortex for the case in which the vortex system is properly aligned with the defect structure. This strong anisotropic pinning is in contrast with the weak isotropic pinning produced by point-like defects that compete with one another, leading to a square-root growth of the pinning energy along the vortex line or the stripe. We consider the transverse motion of stripes embedded in an antiferromagnetic background with lattice constant $a$. This is possible because the longitudinal and transverse motions decouple due to magnetic confinement. We restrict our studies to the underdoped regime, where the stripe-stripe interaction is weaker than the interaction of each stripe with the lattice and disorder pinning potentials. Hence, we assume that the stripe-stripe interaction is merely restricting the motion of one stripe to a “box” of size $2L$ limited by the next neighboring stripes. This assumption simplifies the analysis to the case of a single stripe which interacts with the lattice and impurity potentials. The phenomenological Hamiltonian describing the system is

$$H = \sum_n \left[ \frac{J}{2a^2} (\hat{u}_{n+1} - \hat{u}_n)^2 - 2t \cos \left( \frac{\hat{p}_n a}{\hbar} \right) + V_n(\hat{u}_n) \right],$$

where $\hat{u}_n$ denotes the displacement of the $n$-th hole from its equilibrium position, $|\hat{u}_n| < L$, $\hat{p}_n$ is the canonically conjugate momentum, $t$ is the hopping parameter, $J$ is the stripe stiffness, and $V_n$ is a random pinning potential with Gaussian average over the disorder ensemble. $\langle V_n(u)V_m(u') \rangle = D\delta(u-u')\delta_{mn}$. The parameter $D$ measures the strength of disorder.

The calculations can be simplified by going to the continuum limit and introducing replicas. The replicated zero temperature action reads

$$S^r = \sum_i S_0[\phi^i] + \frac{G}{a} \sum_i \int_0^\infty dt \int dy \cos (2\sqrt{\pi} \phi^i) + \frac{D}{2aL^2} \sum_{i,i'} \int dy \int_0^\infty dt dt' \cos \left[ 2\sqrt{\pi} \delta \left( \phi^i(y, \tau) - \phi^{i'}(y, \tau') \right) \right],$$

with the Gaussian action $S_0$ given by

$$S_0[\phi(y, \tau)] = \frac{\hbar}{2\pi a} \int_0^\infty dt \int dy \left[ \frac{1}{c} (\partial_\tau \phi)^2 + c (\partial_y \phi)^2 \right].$$

The stripes are oriented along the $y$-direction and $\tau$ is imaginary time. The free stripe velocity is $c = a\sqrt{2t/J}/\hbar$ and the dimensionless parameter $\mu = \sqrt{2t/J}$ measures the competition between kinetic and confining energies. The parameter $G$ accounts for the lattice effects and $i$ counts the replicas. The one-loop RG equations for $G$, $D$, and $\mu$ were obtained in Ref. 11. The phase diagram can be divided into a flat phase (correlated pinning), a disordered phase (uncorrelated pinning) and a membrane phase. Here we focus on the pinned phases only. In these two phases the single stripe approach is expected to be a reasonably good approximation and we can obtain quantitative results on the effects of the different co-dopants.

**Correlated pinning:** Let us first analyze the limit of vanishing disorder $D = 0$. The RG equations then read:

$$\frac{d}{dl} \Gamma^2 = 2(2 - \pi \mu) \Gamma^2, \quad \frac{d}{dl} \mu^{-1} = \frac{1}{2} \Gamma^2,$$

where $\Gamma = \pi^{3/2} Ga/(hc)$. Near the critical region, we can define the small parameter $\epsilon_g = 2 - \pi \mu$ which measures the distance from the critical line $\mu_{c1} = 2/\pi$, where the roughening transition (Kosterlitz-Thouless) takes place in the absence of co-doping. Hence, $\mu^{-1} \approx (\pi/2) + (\pi/4) \epsilon_g$ and $d\epsilon_g/d\mu = (4/\pi) d\mu^{-1}/d\mu$. Using we obtain $d\epsilon_g/d\mu = (2/\pi) \Gamma^2$, which can then be combined with yielding

$$\frac{d}{dl} \left( \epsilon_g^2 - \frac{2}{\pi} \Gamma^2 \right) = 0.$$

Eq. \(\ref{eq:4}\) implies that there is a transition at the critical value $\epsilon_g^c = \pm \sqrt{2/\pi} \Gamma$. The critical value of $\mu_{c1}(\Gamma)$ is thus

$$\mu_{c1}(\Gamma) = \frac{2}{\pi} + \sqrt{\frac{2}{\pi} \Gamma}.$$
For $\mu < \mu_{c1}(\Gamma)$ the stripes are pinned by the underlying lattice in the so called “flat phase”. The excitation spectrum is gaped and quantum fluctuations are strongly suppressed. On the other hand, for $\mu > \mu_{c1}(\Gamma)$ the stripes are fluctuating freely. Consider a fixed doping concentration $\delta$ for which the stripe system is in the free phase for $\Gamma = 0$. Upon increasing the lattice parameter $\Gamma$, the system moves along the thick line in Fig. 1 and eventually enters the pinned “flat phase” after crossing the surface $\mu_{c1}(\Gamma)$.

![Diagram](image)

**FIG. 1.** Pinning phase-diagram of the striped phase in the presence of correlated pinning.

This result can describe the effects of Nd co-doping of the lanthanum cuprate. The introduction of Nd (or any other rare earth element) induces a structural transition in the material from a low temperature orthorhombic (LTO) to a low temperature tetragonal (LTT) phase, corresponding to a buckling of the oxygen octahedra. Hence, although the Nd randomly replaces the La atoms which are located out of the plane, they indirectly produce a correlated pinning potential along the copper lattice that will act to pin the stripes in the so called flat phase, strongly suppressing thermal or quantum fluctuations. This is analogous to the case of pinning of vortices by artificially introduced columnar defects. Notice, however, that in the vortex-problem the columnar defects are randomly distributed, whereas here the “correlated” pinning potential is actually a periodic lattice potential, which is enhanced by the tilting of the oxygen octahedra within the LTT phase. Despite of this difference, the analogy is helpful, because it emphasizes the linear character of the pinning potential in both problems.

Since the Nd pins the stripe in an ordered configuration, we expect the width of the incommensurate (IC) peaks measured by neutron scattering to be reduced and the 1D behavior to be reinforced by co-doping. These conclusions are supported by experimental data: ARPES and Hall transport measurements of $La_{2-x-y}Nd_ySr_xCuO_4$ show strong evidence for a 1D striped structure in the underdoped regime. Besides, neutron scattering data taken for both compounds (with and without Nd) indicate a reduction of the half width at half maximum (HWHM) IC peak in the presence of Nd for all the investigated Sr compositions. This is an indication that lattice pinning and hence commensuration effects are enhanced through Nd doping.

**Uncorrelated pinning:** A completely different scenario is presented for the Zn doping case: $Zn^{2+}$ replaces $Cu^{2+}$ directly on the CuO planes. They are located randomly and act as uncorrelated point-like pinning centers in a way very similar to the pinning of vortices by oxygen vacancies. Again, they do not alter the position of the IC peaks observed in neutron scattering, since they do not change the hole density. Moreover, the randomly distributed Zn atoms induce stripe meandering and pin the stripe in a fuzzy phase, similar to Sr doping. The RG equations in the limit of negligible lattice pinning but relevant disorder are

$$\frac{d}{d\ell} \Delta = (3 - \gamma \mu)\Delta,$$

$$\frac{d}{d\ell} \mu = -\frac{1}{2} \mu^2 \Delta,$$

where $\gamma = 2\pi \delta^2$ and $\Delta = 4\pi^2 D\delta^2 q^2/(h^2 c^2 L)$. Close to the critical region we define $\epsilon_\mu = 3 - \gamma \mu$, and following a similar procedure as done for the lattice pinning case, we obtain

$$\frac{d}{d\ell} \left(\Delta - \frac{\gamma}{9} \epsilon_\mu^2 \right) = 0,$$

indicating that $\Delta - (\gamma/9)\epsilon_\mu^2$ is preserved under the RG flow. The transition then happens at the critical value

$$\mu_{c2} = \frac{3}{2\pi \delta^2} + \frac{3\sqrt{\Delta}}{(2\pi \delta^2)^{3/2}}.$$

The corresponding phase diagram is shown in Fig. 2. The thick line indicates how the system undergoes a transition for a constant $\delta$ from a “free phase” ($\mu > \mu_{c2}$) at $\Delta = 0$ to a “fuzzy phase” ($\mu < \mu_{c2}$) at finite $\Delta$. An inspection of the phase diagram indicates that uncorrelated pinning is more relevant at low doping values. Hence, we expect the effects of Zn co-doping to decrease with doping. Moreover, in contrast to Nd doping, Zn pinning destroys the 1D behavior and increases the width of incommensurate neutron scattering peaks implying that the stripes are pinned within a broader region. This is indeed observed experimentally. Neutron scattering measurements in $La_{2-x}Sr_xCu_{1-y}Zn_yO_4$ for $y = 0.012$ and $x = 0.14$ show that Zn produces no relevant effect and that the width $\kappa_a$ of the IC peaks remains practically unaltered. The IC peak width for $La_{1.85}Sr_{0.15}CuO_4$ is $\kappa_a = 0.020 \pm 0.006$ Å$(E = 8$ meV, $T = 8$ K) (see Ref. 13) and the Zn-doped compound with a similar Sr-concentration ($x = 0.14$ and $y = 0.012$) displays the same features within the experimental error bars: $\kappa_a = 0.014 \pm 0.002$ Å for $E = 5$ meV, $T = 10$ K (Ref. 18). The scenario changes quite a bit in the underdoped regime. For $x = 0.12$ and $y = 0.03$ (a composition for which superconductivity is completely suppressed) the elastic IC peaks were observed at the same
position as for the Zn-free material, but $\kappa_s$ was increased due to the doping: $\kappa_s < 0.005 \text{ Å}^{-1}$ for the Zn-free material, whereas $\kappa_s = 0.013(1) \text{ Å}^{-1}$ for $y = 0.03$ (Ref. 14), reflecting the random character of the pinning centers. Although the commensurability at $x = 0.12$ makes this point special, we expect this trend (increase of $\kappa_s$ upon Zn-doping) to continue, especially at lower values of $x$.

We emphasize that the pinning energy grows sub-linearly with the length of the stripe in the case of uncorrelated disorder. Hence, the Zn-pinned phase is analogous to the vortex-glass phase discussed in the context of vortex creep.

In conclusion, we have shown that the main experimental features of co-doping in cuprates can be understood within models of lattice-pinned or disorder-pinned stripes. We divide the co-dopants into two classes: those which produce correlated and and those which produce uncorrelated pinning. Correlated pinning is produced through rare earth co-doping. The problem is analogous to the pinning of vortices by columnar defects or screw dislocations. In this case the stripes are pinned in a flat phase and the fluctuations are strongly suppressed. The effective stripe width is reduced and consequently the IC neutron scattering peaks become sharper after the introduction of the co-dopant. On the other hand, in-plane Zn- or Ni-doping provides randomly distributed point-like pinning centers, similar to the oxygen vacancies in the vortex-creep problem. Within our model, in which the stripe is regarded as a quantum elastic string, the effect of randomness is to “disorder” the string, increasing the effective stripe width and broadening the IC peaks. We expect this kind of pinning to be relevant only at low doping, as indicated in the phase diagram shown in Fig. 2, in agreement with the experimental results.

We are indebted with G. Blatter, D. Baeriswyl, A. O. Caldeira, R. Noack, S. Uchida, and K. Yamada for fruitful discussions. N. H. is financially supported by the Graduierten Kolleg “Physik nanostrukturierter Festkörper.” A. H. C. N. acknowledges support from a LANL CULAR grant.

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12. Notice that the commensurability or incommensurability of the stripe array with the underlying copper lattice is a different issue than the incommensurability $\delta$ measured with neutron scattering. When $\delta \propto x = 1/8$ the stripe array is commensurate with the lattice.
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