Joint Channel and Queue Aware Scheduling for Wireless Links with Multiple Fading States

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Abstract—In this work, we address the delay optimal scheduling problem for wireless transmission with fixed modulation over multi-state fading channels. We propose a stochastic scheduling policy which schedules the source to transmit with probability jointly based on the buffer and channel states, with an average power constraint at the transmitter. Our objective is to minimize the average queuing delay by choosing the optimal transmission probabilities. Using Markov chain modeling, we formulate a power-constrained delay minimization problem, and then transform it into a Linear Programming (LP) one. By analyzing its property, we can derive the optimal threshold-based scheduling policy together with the corresponding transmission probabilities. Our theoretical analysis is corroborated by simulation results.

I. INTRODUCTION

Wireless networks accommodate various multimedia traffics with different Qualities of Service (QoS) for mobile users. For high-speed real-time applications, the average delay packets experience and energy efficiency become more critical considerations [1], when data packets are delivered over time-varying wireless fading channels. Therefore, it is important to schedule data transmissions to minimize the average delay given precious system resources (e.g., average power and energy). This naturally leads to a cross-layer design issue, since the average power and delay are metrics of physical and Medium Access Control (MAC) layers, respectively.

In wireless networks, cross-layer design on power-efficient and delay-minimal transmission has been an ever-present important issue [2]–[4]. A cross-layer scheduling policy was firstly proposed in [2] to transmit data over a discrete-time two-state channel under the constraints of the average delay and peak transmitter power. In [3], Berry and Gallager considered a general cross-layer model where the user’s transmission power and data rate were allocated according to the current buffer state and the channel state in each slot. The asymptotic optimal power-delay tradeoff was derived for the large-delay and small-delay regimes in [3] and [4], respectively. The existence of stationary average delay optimal policy was shown and some structural results were obtained in [5]. In these works, the cross-layer scheduling problems were formulated using the theory of Markov decision processes and the optimal solutions were numerically computed using the dynamic programming technique.

Recently, different optimization techniques were applied to find the optimal power/energy-efficient scheduling policies under various constraints on rate, delay and maximum transmission power [6]–[9]. Meanwhile, the attempts to find analytical solutions have been made for pursuit of deep insights in protocol designs. In [8], an energy efficient scheduling problem was studied for transmitting the packets within a certain amount of time. Through probabilistic analysis, the authors derived the optimal offline scheduling algorithm with an infinite delay constraint and Poisson data arrival. In [9], we considered a cross-layer scheduling problem for a two-state wireless channel. The transmission power is adjusted according to the channel quality to achieve the target Bit Error Rate (BER). To exploit the power efficiently, the source transmits one packet in each slot if the channel state is “good”, and otherwise transmits with probability based on the buffer length. The optimal policy was to transmit based on a critical threshold on the queue length.

In this paper, we generalize the work in [9] to the scenario where data packets of real-time traffics are delivered over multi-state wireless fading channels. The transmission power is adapted in response to the channel state and fixed modulation is applied to reduce the complexity of the transceivers and delay jitter. We propose a stochastic scheduling policy where the source transmits with probability based on both the channel and buffer states. Using Markov chain modeling, we formulate a power-constrained delay minimization problem, and then transform it into a Linear Programming (LP) one. By exploiting its properties, we are able to obtain an elegant closed-form expression for the optimal solution, from which we can finally determine the optimal probabilistic transmission parameters. We show that there is a critical threshold imposed on the queue length associated with each channel state and vice versa. For example, the transmitter of the source is inactive if the data queue length is below the critical threshold, and active otherwise. We also validate the optimality of the proposed policy in [9] for a two-state wireless channel.

The rest of this paper is organized as follows. Section II introduces the system model and the stochastic scheduling scheme. In Section III, a discrete-time Markov chain model is constructed and an LP problem is formulated. The optimal scheduling policy is analyzed in Section IV. Section V demonstrates the simulation results and Section VI concludes this paper. Throughout this paper, the proofs are omitted due to limited space.
A. System Description

We consider a wireless link, where a source node transmits to its destination over a time-varying channel. The source employs a buffer to store the data packets randomly generated from higher-layer applications, as shown in Fig. 1. The system is assumed to be time-slotted.

Suppose that data packets arrive at the source buffer according to a Bernoulli process [10] with probability $\alpha$. This simple yet widely adopted traffic model allows tractable analysis [11], and provides insights for further study. The data buffer has a capacity of $Q$ ($Q \in \mathbb{Z}^+$). $Q = \infty$ and $Q < \infty$ mean that the buffer capacity is infinite and finite, respectively. Let $q[t] \in \mathbb{Q} = \{0, 1, 2, \ldots, Q\}$ be the number of backlogged data packets in the buffer at the end of slot $t$, updated as

$$q[t] = \min\{q[t-1] + a[t], Q\} - v[t],$$

(1)

where $a[t]$ and $v[t]$ denote the number of data packets arriving and delivered in each time slot $t$, respectively. The probability mass function of $a[t]$ can be expressed as $\Pr\{a[t] = 1\} = \alpha$, $\Pr\{a[t] = 0\} = 1 - \alpha$.

The channel is assumed to experience $M$-state block fading, as shown in Fig. 2. The channel state in slot $t$ is denoted by $h(t)$ ($h(t) \in \mathcal{M} = \{1, \ldots, M\}$). Assuming independent and identically distributed (i.i.d.) channel fading model, we denote by $\eta_m$ the probability that the channel stays at state $m$. The probability mass function of $h[t]$ is given by $\Pr\{h[t] = m\} = \eta_m$ ($m \in \mathcal{M}$), which satisfies $\sum_{m=1}^{M} \eta_m = 1$. If the channel state is $m$, the source will transmit with power $P_m$ to achieve a target BER. In practice, the transmission power can be adjusted based on the received signal-noise ratio (SNR) at the destination. Such Channel State Information (CSI) can be fed back to the source over control channel. Intuitively, more power is required to transmit one packet per slot when the received SNR is smaller. Therefore, we naturally assume $P_1 \leq P_2 \leq \cdots \leq P_M$ in accordance with deteriorating channel conditions. In this sense, $M$-state channel model is reasonable. Transmission schemes with adaptive modulation and coding will be considered in our future work.

B. Stochastic Scheduling

To improve the energy efficiency, the source is willing to wait for better channel conditions, since it can thus spend less power on each data transmission. However, the waiting time could be undesirably long if the channel stays at very poor states for a long time. To reduce the latency, the source may have to transmit its backlogged data packets when the current channel state is not so good. This will certainly cost more valuable power resource. Hence, there exists a delay-power tradeoff in the wireless transmissions.

Our objective is to find a scheduling policy that minimizes the average queueing delay under the constraint of a maximum average transmission power. To this end, we propose a stochastic scheduling scheme which decides whether to transmit in slot $t$ according to the current channel state $h(t)$ and data queue state $q[t-1]$. We define two sets of probabilistic parameters: $\{g_{i,m}\}$ and $\{f_{i,m}\}$ ($g_{i,m}, f_{i,m} \in [0, 1]$). Specifically, with $q[t-1] = i$ and $h(t) = m$, if there is new data arrival in this slot, i.e., $a[t] > 0$, the source node transmits one data packet with probability $g_{i,m}$ and holds from transmission with probability $1 - g_{i,m}$, respectively; If no new data packet arrives, i.e., $a[t] = 0$, it transmits with probability $f_{i,m}$ and holds with probability $1 - f_{i,m}$, respectively.

According to the proposed scheduling policy, the service process $v[t]$ depends on the queue status $q[t-1]$ and the arrival process $a[t]$, as described below.

- Case 1: $q[t-1] = 0$ and $h(t) = m$

In this case, the source transmits a newly arriving data packet with probability $g_{0,m}$ in the current time slot $t$, and the service process can be expressed as

$$v[t] = \begin{cases} 1 & w.p. g_{0,m}, \quad a[t] = 1, \\ 0 & w.p. (1 - g_{0,m}), \quad a[t] = 1, \\ 0 & w.p. 1, \quad a[t] = 0, \end{cases}$$

(2)

where $w.p.$ means ’with the probability’.

- Case 2: $q[t-1] = i$ ($i > 0$) and $h(t) = m$

In this case, the source transmits a packet with probability $g_{i,m}$ or $f_{i,m}$ depending on whether there is a new data arrival or not. The service process can be expressed as

$$v[t] = \begin{cases} 1 & w.p. g_{i,m}, \quad a[t] = 1, \\ 0 & w.p. (1 - g_{i,m}), \quad a[t] = 1, \\ 1 & w.p. f_{i,m}, \quad a[t] = 0, \\ 0 & w.p. (1 - f_{i,m}), \quad a[t] = 0. \end{cases}$$

(3)

III. Problem Formulation

A. Markov Chain Model

In our system, the queueing system can be modeled as a discrete-time one-dimensional Markov chain, each state of which represents the buffer status, as shown in Fig. 3.
Let $\gamma_{i,j} = \Pr[q[t+1] = j | q[t] = i]$ denote the one-step transition probability of the Markov chain, which is homogeneous by the scheme description. The index $t$ can be omitted below if no confusion will be caused. According to the case when one data packet newly arrives while no transmission takes place, the transition probability that the queue length is increased by one is obtained as

$$
\lambda_i = \gamma_{i,i+1} = \alpha \sum_{m=1}^{M} \eta_m (1 - g_{i,m}) \quad (0 \leq i \leq Q - 1).
$$

(4)

When one data packet is transmitted with no new data arrival, the transition probability that the queue length is decreased by one is given by

$$
\mu_i = \gamma_{i,i-1} = (1 - \alpha) \sum_{m=1}^{M} \eta_m f_{i,m} \quad (1 \leq i \leq Q).
$$

Thus, the probability that the queue status remains the same can be expressed as

$$
\gamma_{i,i} = \begin{cases} 
1 - \lambda_0, & i = 0, \\
1 - \lambda_i - \mu_i, & 1 \leq i \leq Q - 1, \\
1 - \mu_Q, & i = Q.
\end{cases}
$$

(6)

Let $\pi_i$ denote the steady-state probability that the data queue length is equal to $i$. The steady-state probability vector $\pi = [\pi_0 \pi_1 \cdots \pi_Q]$ satisfies $\pi P = \pi$ and $\pi e = 1$. In particular, the local balance equation at state $q[t] = i$ is given by

$$
\pi_i \lambda_i = \pi_{i+1} \mu_{i+1}, \quad (0 \leq i \leq Q - 1).
$$

(7)

Hence, the steady-state probability of the Markov chain can be computed as

$$
\pi_0 = \left(1 + \sum_{i=1}^{Q} \prod_{n=0}^{i-1} \frac{\lambda_n}{\mu_{n+1}}\right)^{-1}, \quad \pi_i = \pi_0 \prod_{n=0}^{i-1} \frac{\lambda_n}{\mu_{n+1}} \quad (i > 0).
$$

(8)

Thus, given the transmission parameters $\{g_{i,m}\}$ and $\{f_{i,m}\}$, we can compute the stationary distribution of the buffer state and further analyze the system performance.

### B. Queueing Delay and Power Consumption

The system performance is measured in terms of the average queueing delay and the average power consumption. By the Little’s law, the average queueing delay is related to the average buffer occupancy $[3]$, and can be computed as

$$
\bar{D} = \frac{1}{\alpha} \sum_{i=0}^{Q} i \pi_i.
$$

(9)

Let $c[t]$ denote the transmission power in slot $t$. Let $\omega_{i,m}(x) = \Pr[c[t] = x | q[t-1] = i, h(t) = m]$ denote the conditional probability that the transmission power is $c[t] = x$ ($x \in \{0, P_m\}$) given the data queue state $q[t-1] = i$ and the channel state $h(t) = m$. In this case, the source transmits at a power $c[t] = P_m$ with probability $g_{i,m}$ if one data packet newly arrives, and with probability $f_{i,m}$ if no data packet arrives, respectively. Hence, conditioned on $q[t-1] = i$ and $h(t) = m$, the probability that the transmission power is equal to $P_m$ can be expressed as

$$
\omega_{i,m}(P_m) = \begin{cases} 
\alpha g_{i,m}, & i = 0, \\
\alpha g_{i,m} + (1 - \alpha) f_{i,m}, & 0 < i \leq Q.
\end{cases}
$$

(10)

and $\omega_{i,m}(0) = 1 - \omega_{i,m}(P_m)$. Hence, the average power is expressed as

$$
\bar{P} = \sum_{i=0}^{Q} \sum_{m=1}^{M} \pi_i \omega_{i,m}(P_m) = \sum_{i=1}^{Q} \sum_{m=1}^{M} \pi_i \omega_{i,m}(P_m) + \pi_0.
$$

(11)

The event of packet loss occurs when there is one new data arrival while the data buffer is full. Thus, the probability of buffer overflow can be given by

$$
\rho_{loss} = \Pr[q[t] = Q] = \alpha \pi_Q.
$$

(12)

Note that the average queueing delay can be appropriately defined by (9) only when the event of buffer overflow does not take place. From (12), the packet loss probability is zero if $\pi_Q$ is zero. This happens in two cases: 1) the buffer capacity is infinite with $Q \to \infty$ and the queueing system is stable; 2) the parameter $\lambda_{Q-1}$ is set to zero for a finite buffer with $Q < \infty$. We will discuss the optimal scheduling under the assumption that no buffer overflow occurs.

### C. Optimization Problem

In this work, we aim to study the optimal scheduling policy which minimizes the average delay $\bar{D}$ subject to the average power constraint $\bar{P} \leq P_{max}$ by determining the optimal transmission parameters $\{g_{i,m}\}$ and $\{f_{i,m}\}$. To this end, we formulate an optimization problem as

$$
\min_{\{g_{i,m}\}, \{f_{i,m}\}} \bar{D} = \frac{1}{\alpha} \sum_{i=0}^{Q} i \pi_i
$$

subject to

$$
\begin{align}
P & \leq P_{max}, \\
\pi_0 \lambda_0 & = \pi_{i+1} \mu_{i+1}, 0 \leq i < Q, \\
\sum_{i=0}^{Q} \pi_i & = 1,
\end{align}
$$

(13)

where the constraint (a) is the maximum average power constraint, the constraints (b) and (c) stem directly from the property of the Markov chain, and the constraint (d) points out the range of the probabilistic parameters $\{g_{i,m}\}$ and $\{f_{i,m}\}$. Note that the average power $\bar{P}$ (c.f. (11)) and the steady-state probabilities $\{\pi_i\}$ (c.f. (4), (5), (8)) are non-linear functions of the parameters $\{g_{i,m}\}$ and $\{f_{i,m}\}$. Therefore, it is rather difficult to solve the above optimization problem [13]. Motivated by the methods applied in [11], [12], we will transform the optimization problem (13) into an LP problem, and exploit its special structure to analyze the globally optimal solution in the next section.
IV. ANALYSIS OF DELAY OPTIMAL SCHEDULING

A. LP Problem Formulation

To formulate an LP problem, we introduce a set of new variables \( y_{i,m} \) as:

\[
y_{i,m} = \pi_i y_{i,m} + \xi \pi_{i+1} f_{i+1,m},
\]

where \( \xi = \frac{1 - \alpha}{\alpha} \). The variable \( y_{i,m} \) can be interpreted as the conditional probability that the queue state is equal to \( i \) after one data transmission over the wireless channel with state \( m \).

By substituting (4) and (5) into the local balance equation (7), we have \( \pi_i \alpha \sum_{m=1}^{M} \eta_m (1 - y_{i,m}) = \pi_{i+1} (1 - \alpha) \sum_{m=1}^{M} \eta_m f_{i+1,m} \), from which we can further obtain

\[
\pi_i = \sum_{m=1}^{M} \eta_m y_{i,m}.
\]

In Lemma 1 we show that the average delay \( \bar{D} \) and power \( \bar{P} \) are both linear functions of the variables \( \{y_{i,m}\} \).

**Lemma 1.** The average delay \( \bar{D} \) and the average power consumption \( \bar{P} \) can be expressed as

\[
\bar{D} = \frac{1}{\alpha} \sum_{m=1}^{M} \sum_{i=0}^{Q} i \eta_m y_{i,m}, \quad \bar{P} = \alpha \sum_{m=1}^{M} \sum_{i=0}^{Q} \eta_m P_m y_{i,m}.
\]

As a result, we can transform the problem (13) into an LP problem as follows:

\[
\begin{array}{l}
\min_{y_{i,m}} \quad \bar{D} = \frac{1}{\alpha} \sum_{m=1}^{M} \sum_{i=0}^{Q} \sum_{m=1}^{Q} i \eta_m y_{i,m} \\
\text{s.t.} \quad \bar{P} = \alpha \sum_{m=1}^{M} \sum_{i=0}^{Q} \eta_m P_m y_{i,m} \leq P_{max},
\end{array}
\]

\[
\gamma_i = \sum_{m=1}^{M} \sum_{i=0}^{Q} \eta_m y_{i,m} = 1, \quad 0 \leq y_{i,m} \leq \sum_{n=1}^{M} \eta_n y_{i,n} + \xi \sum_{n=1}^{M} \eta_n y_{i+n, m}.
\]

In (17), the power constraint (a) and the normalization constraint (b) are derived directly from the constraints (a) and (c) of (13). From (14) and (15), we can derive the constraint (c) by varying the probabilistic parameters \( \{g_{i,m}\} \) and \( \{f_{i,m}\} \) within their range \([0, 1]\). The optimal solution and the optimal value of (17) are denoted by \( \{y^*_i, m\} \) and \( \bar{D}^* \), respectively.

B. Structure of the Optimal Solution

We first consider the case when the maximum average power \( P_{max} \) is sufficiently large such that the source is able to transmit whenever its queue is not empty. This means that the power constraint (17a) can be omitted. In this case, we can obtain the minimum average delay \( \bar{D}^* = \frac{1}{\alpha} \sum_{i=0}^{Q} \sum_{m=1}^{M} \eta_m y^*_{i,m} = 0 \) by setting \( y^*_{i,m} = 1 \) and \( y^*_{i,m} = 0 \), which satisfies the constraints (17b) and (17c). Accordingly, we obtain the power threshold as \( P_{th} = \alpha \sum_{m=1}^{M} \eta_m P_m \).

Then, we focus on the case when the power constraint (17a) becomes tight, i.e., \( \bar{P} = P_{max} < P_{th} \). By exploiting the property of the LP problem (17), we can present the structure of the optimal solution in the following theorem.

**Theorem 2.** The optimal solution to (17) has a threshold structure. That is, there exists a threshold on the data queue length \( i^*_m \) associated with each channel state \( m \) such that \( 0 = i^*_1 \leq \cdots \leq i^*_M \) and the optimal solution takes the form as

\[
y^*_{i,m} = \begin{cases} 
0, & i < i^*_m - 1, \\
\sum_{n=1}^{M} \eta_n y^*_{i,n} + \xi \sum_{n=1}^{M} \eta_n y^*_{i+n, m}, & i \geq i^*_m,
\end{cases}
\]

And there is at most one \( \tilde{m} \in \{2, \ldots, M\} \) such that \( y^*_{i^*_m-1,m} > 0 \) may hold, and \( y^*_{i^*_m-1,m} = 0 \) for \( m \in \{2, \ldots, M\} - \{\tilde{m}\} \).

**Corollary 3.** \( y^*_{i,m} = 0 \) for all \( i > i^*_M \) and \( m \in M \).

For all \( i > i^*_M \) and \( m \), we have \( y^*_{i,m} = 0 \) and \( \pi^*_m = \sum_{m=1}^{M} \eta_m y^*_{i,m} = 0 \). This means that the length of the packet queue never exceeds the threshold \( i^*_M \). Therefore, no packet loss will be induced, i.e., \( p_{loss} = 0 \), as long as the finite queue capacity \( Q \) is larger than \( i^*_M \).

As demonstrated by Fig. 4, the optimal solution to problem (17) has a double threshold structure:

1. There exists a threshold on the data queue length \( i^*_m \) associated with each channel state \( m \);
2. And there is a threshold imposed on the channel state \( m^* \) given the queue state \( i \).

As shown by the shadow area in Fig. 4, we have \( y^*_{i,m} = \pi^*_i + \xi \pi^*_{i+1} > 0 \) for \( i^*_m - 1 < i \leq i^*_m \) for \( m \leq m^* \), and otherwise \( y^*_{i,m} = 0 \) for \( i < i^*_m \) or \( m > m^* \). The dotted area shown in Fig. 4 means that \( 0 \leq y^*_{i,m} < \pi^*_i + \xi \pi^*_{i+1} \) for \( i = i^*_m - 1 \) and \( m = \tilde{m} \). Based on the threshold based policy, the source should transmit one backlogged packet in each slot when the queue length reaches the threshold \( i^*_m \) (associated with the channel state \( m \)) so as to minimize the average delay. As plotted in Fig. 4 the thresholds \( \{i^*_m\} \) take the stair-step shape, which is the optimal way to exploit the limited power resource over wireless channels.

C. Derivation of the Optimal Transmission Parameters

Based on the property of the optimal solution \( \{y^*_{i,m}\} \) presented in Theorem 2, we will show how to derive the
optimal solution \( \{y_{i,m}^*\} \) and determine the optimal transmission parameters \( \{g_{i,m}^*\} \) and \( \{f_{i,m}^*\} \) thereafter. For ease of expression, we define two functions as:

\[
\Gamma(i) = \max_{i_{m} \leq i} m = \begin{cases} 
1, & 0 = i_{1}^* \leq i < i_{2}^*, \\
m, & i_{m}^* \leq i < i_{m+1}^*, \\
M, & i \geq i_{M}^*, 
\end{cases} \tag{19}
\]

\[
\chi(k) = \frac{1 - \sum_{m=1}^{k} \eta_{m}}{\xi \sum_{m=1}^{k} \eta_{m}}. \tag{20}
\]

We also define two series of probabilities \( \{\varphi_{1,i}\} \) and \( \{\varphi_{2,i}\} \) \((i \in \{0, 1, \cdots, Q\})\) as

\[
\varphi_{1,i} = \begin{cases} 
0, & i = 0, \\
(\chi(\Gamma(i-1)))^{i-i_{r}(i-1)} \prod_{m=1}^{i_{r}(i-1)} (\chi(m))^{i_{m+1}-i_{m}}, & i > 0,
\end{cases} \tag{21}
\]

\[
\varphi_{2,i} = \begin{cases} 
0, & i < i_{m}^*, \\
\frac{\eta_{m}^{i_{m}^*-i}}{\xi \sum^{i_{m}^*-1} \eta_{m}^{i_{m}^*-i}} (\chi(\Gamma(i-1)))^{i-i_{r}(i-1)} \prod_{m=1}^{i_{r}(i-1)} (\chi(m))^{i_{m+1}-i_{m}}, & i \geq i_{m}^*,
\end{cases} \tag{22}
\]

Based on the probabilities \( \{\varphi_{1,i}\} \) and \( \{\varphi_{2,i}\} \), we can define two more parameters as

\[
\theta_{1} = \sum_{m=1}^{M} \eta_{m} \left[ \varphi_{1,i_{m}^*} + (1 + \xi) \sum_{i = i_{m}^*+1}^{i_{m}^*} \varphi_{1,i} \right] P_{m}, \tag{23}
\]

\[
\theta_{2} = \sum_{m=1}^{M} \eta_{m} \left[ \varphi_{2,i_{m}^*} + (1 + \xi) \sum_{i = i_{m}^*+1}^{i_{m}^*} \varphi_{2,i} \right] P_{m} + \eta_{m} P_{m}, \tag{24}
\]

which are linear combinations of the power levels \( \{P_{m}\} \).

**Theorem 4.** Let \( \nu_{1} = \sum_{i=0}^{i_{m}^*} \varphi_{1,i} \) and \( \nu_{2} = \sum_{i=0}^{i_{m}^*} \varphi_{2,i} \). The optimal steady-state probability \( \pi_{i_{m}^*}^* \) can be expressed as a linear function of the optimal steady-state probability \( \pi_{0}^* \):

\[
\pi_{i_{m}^*}^* = \frac{\varphi_{2,i}}{\nu_{2}} + \left( \frac{\nu_{1}}{\nu_{2}} \varphi_{1,i} \right) \pi_{0}^*, \tag{25}
\]

where \( \pi_{0}^* \) is obtained as

\[
\pi_{0}^* = \left( \frac{p_{\text{max}}}{\alpha} - \frac{\theta_{2}}{\nu_{2}} \right) \left( \nu_{1} - \frac{\nu_{1}}{\nu_{2}} \theta_{2} \right)^{-1}. \tag{26}
\]

Once obtaining the steady-state probabilities \( \{\pi_{i_{m}^*}\} \) given by (25), we can compute the optimal solution \( \{y_{i,m}^*\} \) as

\[
y_{i,m}^* = \begin{cases} 
0, & i < i_{m}^* - 1, \\
\left( \sum_{n=m}^{M} \frac{\eta_{n}}{\eta_{m}} \right) \pi_{i_{m}^*-1}^* - \left( \sum_{n=1}^{m-1} \frac{\eta_{n}}{\eta_{m}} \right) \xi \pi_{i_{m}^*}^*, & i = i_{m}^* - 1, \\
\pi_{i_{m}^*}^* + \xi \pi_{i_{m}^*+1}^*, & i \geq i_{m}^*,
\end{cases} \tag{27}
\]

From its definition \( y_{i,m}^* = \pi_{i_{m}^*}^* g_{i,m}^* + \xi \pi_{i_{m}^*+1} f_{i_{m}^*+1,m} \) (c.f. (14)), we can further determine the optimal transmission probabilities \( \{g_{i,m}^*\} \) and \( \{f_{i,m}^*\} \).

**Theorem 5.** The pair of the optimal transmission probabilities \( g_{i,m}^* \) and \( f_{i_{m}^*+1,m}^* \) satisfy

\[
\begin{align*}
g_{i,m}^* &= f_{i_{m}^*+1,m}^* = 0, & i < i_{m}^* - 1, \\
g_{i,m}^* &= f_{i_{m}^*+1,m}^* = 1, & i \geq i_{m}^*.
\end{align*} \tag{28}
\]

for any channel state \( m \) and for \( i = i_{m}^* - 1 \)

\[
\begin{align*}
g_{i_{m}^*-1,m}^* &= f_{i_{m}^*-1,m}^* = 0, & m \neq \hat{m}, \\
\pi_{i_{m}^*-1,m}^* g_{i_{m}^*-1,m}^* + \xi \pi_{i_{m}^*}^* f_{i_{m}^*-1,m}^* &= \gamma_{i_{m}^*-1,m}, & m = \hat{m}. \tag{29}
\end{align*}
\]

**Remark:** In Theorem 5 we show that the delay optimal scheduling algorithm is a threshold-based transmission scheme. When the channel state is \( m \), the source transmits one backlogged packet with power \( P_{m} \) if the queue length reaches the threshold \( i_{m}^* \), and otherwise remains silent. Note that the optimal threshold is \( i_{1}^* = 0 \) when the channel state is \( m = 1 \). This implies that the source transmits one data packet, either newly arriving or stored, in each slot provided the channel condition is the best. There may exist multiple pairs of transmission probabilities \( g_{i_{m}^*-1,m}^* \) and \( f_{i_{m}^*-1,m}^* \) which satisfy the equation (29).

It is not trivial to derive the integer thresholds \( \{i_{m}^*\} \). Fortunately, we are able to reduce the computational complexity significantly by exploiting the monotonic property \( 0 = i_{1}^* \leq \cdots \leq i_{M}^* \). For example, nested bisection methods can be applied to find the optimal thresholds \( \{i_{m}^*\} \). Then, we show that an elegant expression of the optimal thresholds can be derived in the case with \( M = 2 \).

**Corollary 6.** In the two-state channel case with \( M = 2 \), the optimal transmission probabilities can be expressed as \( g_{i_{1}^*+1,1}^* = f_{i_{1}^*+1,1}^* = 1 \) for all \( i \geq 0 \), and

\[
\begin{align*}
g_{i_{1}^*,2} &= f_{i_{1}^*,2}^* = 0, & i < i_{1}^* - 1, \\
\eta_{2}(\pi_{i_{1}^*,2}^* g_{i_{1}^*,2}^* + \xi \pi_{i_{1}^*+1}^* f_{i_{1}^*+1,2}^*) &= x_{i_{1}^*}, & i = i_{1}^* - 1, \\
g_{i_{1}^*,2} &= f_{i_{1}^*,2}^* = 1, & i \geq i_{1}^*,
\end{align*} \tag{30}
\]

where the threshold \( i_{2}^* \) is given by

\[
i_{2}^* = \min\left\{ \left\lfloor \log_{\eta_{1}/\eta_{2}} \left( \frac{1}{\pi_{0}^*} \right) \right\rfloor, \left\lfloor \log_{\eta_{1}/\eta_{2}} \left( \frac{1}{\pi_{0}^*} \right) \right\rfloor \right\} \tag{31}
\]

One can see that this result is equivalent to that in [9]. Therefore, the delay optimal scheduling policy proposed in [9] is indeed the optimal for a two-state wireless channel.

**V. SIMULATION RESULTS**

In this section, simulation results are presented to demonstrate the performance of the proposed scheduling scheme and validate our theoretical analysis. Each simulation runs over \( 10^6 \) time slots. In each slot, the packet transmissions are scheduled according to our proposed policy. In the figures, the solid lines and the marks “o” indicate theoretical and simulation results, respectively. One can see that theoretical and simulation results match well.
and the channel state is $m$. We proposed a stochastic scheduling policy: the source transmits one packet with probability $p$ depends on the largest queue length $i$. For a target BER, the source adjusts its transmission power $\alpha$ when data packets arrive more frequently with larger $m$. Thus the queueing delay is zero. We also notice that to achieve a same average delay $\bar{D}$, the source consumes more power when data packets arrive more frequently with larger $\alpha$.

In Fig. 6 we plot the optimal threshold on the queue length $i_m^*$ for the channel state $m = 1, 2, 3$, respectively, when the data arrival rate is $\alpha = 0.5$. The thresholds $\{i_m^*\}$ satisfy $i_1^* \leq i_2^* \leq i_3^*$ for any power constraint. This means that the source should exploit relatively better channel conditions to transmit as possible. When the channel quality is the best with $m = 1$, the optimal threshold is equal to $i_1^* = 0$, regardless of the average power constraint $p_{\text{max}}$. When $m = 2$ or $m = 3$, the optimal threshold $i_m^*$ steps down with the increase of the power $p_{\text{max}}$. Meanwhile, a smaller minimum average delay is observed from Fig. 5 since $\bar{D}^* = \alpha^{-1} \sum i_m i_m^* \pi_m$ highly depends on the largest queue length $i_M^*$. When the average power is sufficiently large, i.e., $p_{\text{max}} \geq P_{\text{th}}$, all the thresholds $\{i_m^*\}$ are equal to zero and the minimum average delay is zero.

VI. CONCLUSIONS

In this paper, we investigated the delay optimal scheduling problem over a $M$-state wireless channel with fixed modulation. For a target BER, the source adjusts its transmission power $P_m$ according to the channel state $m$. In this system, we proposed a stochastic scheduling policy: the source transmits one packet with probability $g_{i,m}$ or $f_{i,m}$ depending on whether there is new data arrival, when the queue length is $i$ and the channel state is $m$. By Markov chain modeling and variable substitution, we constructed an LP problem to minimize the average delay under the average power constraint. By exploiting the property of the LP problem, we revealed the structure of the optimal solution and then derived the optimal probabilities $\{g_{i,m}\}$ or $\{f_{i,m}\}$. It was found that the source should always transmit as long as the channel quality is best. Otherwise, the source holds from transmission when the queue length is below the optimal threshold $i_m^*$, and transmits with power $P_m$ when the data queue length exceeds the threshold $i_m^*$, given the channel state $m$. Simulation results confirmed our theoretical analysis.

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