A torsion balance device for measuring the gravity gradient

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Abstract. Based on the principle of torsion balance measuring gravity gradient, designed a torsion balance gravity gradiometer, consisting of a torsion balance system, an angle measurement system, an amplitude attenuation system, a turntable system, a vacuum container system. Considering the stable time of torsion balance suspension system was too long, which had greatly impacted the efficiency of the gravity gradient measurement, designed an amplitude attenuation system in combination with PID control technology. Simulation results show that, the curvature and horizontal gravity gradient could be measured, the accuracy was as high as 0.5 E(1E=1 × 10^{-9}m/s^2), and the limit efficiency of measuring a group of gravity gradients was better than an hour.

1. Introduction
Second order derivative of gravity potential can well describe the characteristics of the earth's gravitational field, and be suitable for detecting the partial small geologic body with higher space resolution ratio compared with gravity measurement. Gravity gradiometer is an instrument for measuring gradient of gravity field[1-2]. Due to traditional torsion balance gravity gradiometer has disadvantages of the backward extraction technique of gravity gradient, low measurement efficiency and heavy structure, it’s gradually replaced by other gravity gradient devices. But it has advantages of simple structure, low cost and high measuring precision, the method of torsion balance measuring gravity gradient doesn’t disappear[3-6].

We designed a new type of torsion balance gravity gradiometer, and used the total least squares method considering the errors of coefficient matrix and observation vector simultaneously which could accurately extract the gravity gradient data, and give the corresponding precision estimation. Using PID control technology, we designed a system of amplitude attenuation which could realize the rapid attenuation and greatly improved the torsional balance efficiency through the simulation of the system.

2. The principle of torsion balance
Eotvo’s torsion balance consists of a fine torsion wire, carrying a balance beam which supports at its extremities two test masses, at different vertical heights (see Figure 1). O-xyz was the laboratory coordinate system; O-ξηz was the coordinate system fixed with balance beam. the origin O at the centre of gravity of the balance beam, Oξ along the length direction, Oη along the width direction, Oz directed vertically downwards. Due to the different locations of two test masses, which felt different
horizontal gravity acceleration. Torsion wire twisted a certain angle to record the torque to be measured in the horizontal direction. The torque to be measured included two parts: the first part was related to the curvature gravity gradient, and the second was related to the horizontal gravity gradient. Selecting the appropriate coordinate system could establish the relation between the gravity gradients and the torque to be measured.

Figure 1. Schematic diagram of torsion balance system

In the static measurement, the relationship between twisted angle of the torsion wire and external constant torque can be written as follows:

$$k\theta = \theta_0 + \frac{J}{2} \sin 2\alpha \omega_\alpha + J \cos 2\alpha \omega_{\alpha y} + mlh \cos \alpha \omega_{\alpha z} - mlh \sin \alpha \omega_{\alpha z}$$

(1)

Where $\alpha$ is azimuth, $J$ is moment of inertia, $l$ is half length of the balance beam, $m$ is quality of the test mass, $h$ is length of the stick of light metal filament hanging one test mass, $\omega_{\alpha x}$ and $\omega_{\alpha y}$ are horizontal gravity gradient, $\omega_{\alpha x}$ and $\omega_{\alpha y}$ are curvature gravity gradient, $k$ is torsion coefficient of torsion wire, $\theta_0$ is the twisted angle of the torsion wire when it is not affected by any external force. $\theta_0$ is regarded as an unknown quantity to handle on the upper equation because of no place with uniform gravity field on earth. There are five unknown quantities. So it’s required to be measured by five times on the same point at different directions to get the gravity gradients.

3. Experimental equipment

The key of the torsion balance gravity gradiometer is the design of torsion balance. Eotvo’s torsion balance is the most traditional device to measure gravity gradient. Therefore, we designed a new torsion balance gravity gradiometer by learning from its principle (see Figure 2).

Figure 2. Schematic diagram of torsion balance gradiometer structure.
The corresponding mechanical equilibrium equation could be written as follows:

\[ k\theta = k\theta_0 + \frac{1}{2} f_{iW} \sin 2\alpha - f_{iWc} \sin \alpha + f_{iWc} \cos 2\alpha + f_{iWc} \cos \alpha \] (2)

Where \( f_i = \int (\xi^2 - \eta^2) \, dm \) and \( f_s = \int (\xi z) \, dm \) are the structural constant of the torsion balance.

Torsion balance gravity gradiometer consisted of a torsion balance system, an angle measurement system, an amplitude attenuation system, a turntable system, a vacuum container system. Where the turntable system was used to change the azimuth angle of torsion balance. The vacuum system was used to maintain the vacuum level of the working environment. Torsion balance system was the most important part of the gravity gradiometer, which was used to measure the curvature and horizontal gravity gradients. Torsion balance system mainly consisted of a torsion wire, a weighbeam (material vycor, size \( 200 \times 12 \times 14 \text{mm}^3 \)) and two titanium alloy vertical bars (size \( 15 \times 12 \times 100 \text{mm}^3 \)). In order to avoid electrostatic interference, the outer surface of the weighbeam was coated with a layer of metal film. One end of the weighbeam at the upper surface and another end at lower surface was equipped respectively with a titanium alloy vertical bar by pasting way. The torsion wire was produced by Goodfellow (99.9% purity, diameter 50um, length 500mm). One end of the torsion wire was connected with the rotating guide, and the other end was connected at the center of the upper surface of the weighbeam by the sleeve and the clamp. Rotating the rotating guide not only could change the torsion balance position, but also could adjust the amplitude of the torsion balance in the initial state to quickly make itself into the measurement state.

The angle measuring system used for monitoring the movement states of torsion balance consisted of an autocollimator, a quartz window, a mirror, a weighbeam. The front surface of the weighbeam was used as a mirror, and the autocollimator was used as an angle measuring device. If the torsion balance was free of the action of external force, the angle between mirror and autocollimator standard line was \( \theta_0 \). When torsion balance subjected to uneven gravity, the angle between them was \( \theta \), where mirror turning angle was the deflection angle of tungsten wire.

The amplitude attenuation system consisted of an autocollimator, a mirror, feedback plates (1-4), a data processing system (labview), A/D and D/A converters. By plus two pairs of feedback on both sides of the weighbeam formed the capacitor, the system used electrostatic feedback mode to control it when the torsion balance deviated from the equilibrium position.

4. Experimental simulation
4.1 Simulation of stable time of torsion balance system
In order to evaluate the stable time of torsion balance system, simulink software was used to carry out a preliminary analysis about the amplitude attenuation system. The simulation results was shown in figure 3.

![Figure 3. Step signal simulation diagram.](image-url)

When the amplitude was reduced by 10000urad, the control time required was about 7 mins. Compared with the traditional torsion balance which the stable time of the suspension system was about 20–30min, the measuring efficiency was improved by 3–4 times. Considering the motion
period of torsion balance was about 9–10 min, the limit efficiency of gravity gradient measurement of the torsion balance gravity gradiometer was about 60 mins.

4.2 Extraction simulation of gravity gradient

The design of error distribution affecting gravity gradients was shown in Table 1.

| Item           | Error distribution  |
|----------------|----------------------|
| Structural coefficient | $\delta f_1 / f_1 < 1/1000$ |
| Torsion coefficient   | $\delta k / k < 1/1000$ |
| Deflection angle      | $\delta \theta < 1\text{urad}$ |
| Azimuth              | $\delta \alpha < 100\text{urad}$ |

By using the total least square method, we carried out a large number of random error numerical simulations for the gravity gradient values\(^7\) of 21 measured points. The maximum accuracy of the curvature and horizontal gravity gradients were estimated, as shown in figure 4.

![Figure 4. Gravity gradient error distribution simulation.](image)

The maximum accuracy estimates of the gravity gradient were all in the range of 0.1E to 0.3E. The accuracy of $W_x$, $W_y$, and $W_z$ was about twice as high as that of $W_{xy}$. So the design accuracy was better than 0.5E.

5. Conclusion

A torsion balance gravity gradiometer has been designed to measure the curvature and horizontal gradient of gravity. The accuracy was as high as 0.5 E, and the limit efficiency of measuring a group of gravity gradients was better than an hour. It had the characteristics of high precision, simple structure, and simultaneous measurement of multiple gravity gradients.

6. References

[1] Tijing C et al 1999 Journal of Technology of China 71: 39-42
[2] Szabo Z et al 2016 Acta Geod Geophys 51:273–293
[3] Bod L et al 1991 Acta Physica Hungarica 69: 335-355
[4] Hansen R et al 1999 The Leading Edge 18(4):478-480
[5] Edwin H et al 2000 The Leading Edge 19(12):1296-1297
[6] Pawlowski B et al 2012 The Leading Edge 17 (1) :51-52
[7] Shaw H et al 1922 Quality Assurance in Education 22(1):88-104(17)