Lattice dynamics and structural stability of ordered Fe$_3$Ni, Fe$_3$Pd and Fe$_3$Pt alloys using density functional theory

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We investigate the binding surface along the Bain path and phonon dispersion relations for the cubic phase of the ferromagnetic binary alloys Fe$_3$X ($X = \text{Ni, Pd, Pt}$) for L1$_2$ and D0$_2$ ordered phases from first principles by means of density functional theory. The phonon dispersion relations exhibit a softening of the transverse acoustic mode at the M-point in the L1$_2$-phase in accordance with experiments for ordered Fe$_3$Pt. This instability can be associated with a rotational movement of the Fe-atoms around the Ni-group element in the neighboring layers and is accompanied by an extensive reconstruction of the Fermi surface. In addition, we find an incomplete softening in [111] direction which is strongest for Fe$_3$Ni. We conclude that besides the valence electron density also the specific Fe-content and the masses of the alloying partners should be considered as parameters for the design of Fe-based functional magnetic materials.

I. INTRODUCTION

Ferrous alloys are well known for a wide variety of anomalous structural and magnetic properties. Around 1185 K, bulk iron undergoes a structural transition from a face centered cubic (fcc) high-temperature phase to a low-temperature body centered cubic (bcc) phase, which is ferromagnetic with a Curie temperature of 1043 K. The structural transition temperature is systematically lowered by alloying elements from the right side of Fe in the periodic table, which effectively lowers the valence electron concentration $e/a$. The transition is displacive and diffusionless between a high-temperature high-symmetry austenitic phase and a low-temperature martensitic phase, in which the symmetry is usually lowered, e.g., by tetragonal distortions. In ferromagnetic Fe-alloys with $e/a$ between 8.7 and 8.5, the martensitic transition disappears and fcc austenite becomes the stable ground state structure. Iron alloys with Ni-group elements at this valence electron concentration are at the center of scientific interest for a long time, as they show, in addition to the martensitic instability, a multitude of magneto-structural anomalies that are of technological importance. The probably most prominent is the so-called Invar effect, describing the over-compensation of thermal expansion in a wide temperature range, which has been intensively studied for more than one century. The best known representative is Fe$_{95}$Ni$_{15}$, but similar behavior is also observed in Fe$_{70}$Pd$_{30}$ and Fe$_{72}$Pt$_{28}$. Although most explanations proposed so far rely on magneto-volume coupling, the vicinity to the martensitic transformation has been well noticed and discussed. The relevant literature on this topic is too numerous to be summarized at this point – for an introduction and further references, see, e.g., Refs. 1-4.

During the past three decades, the so-called magnetic shape memory (MSM) effect, which is again present in the alloys with all three Ni-group elements, was brought to the attention of the scientific community. The MSM behavior allows very large magnetic-field-induced strains of up to three percent to be achieved by externally applying moderate magnetic fields in the (sub-)Tesla range. The effect is not as pronounced as for the Ni-based Heusler alloys like Ni$_2$MnGa, but its origin is believed to be related. One common key ingredient is the appearance of intermediate or modulated martensites – 5-fold and 10-fold modulated pseudo-tetragonal or orthorhombic in the case of Ni-Mn-Ga and (slightly distorted) face centered tetragonal (fct) in the case of Fe-Pt and Fe-Pd – between the high symmetry austenite and the low-temperature martensite, which is body centered tetragonal in the case of the ferrous alloys.

For the above mentioned alloys, the relevant electron concentration range is achieved at compositions which may allow the formation of ordered stoichiometric compounds. However, Fe$_3$Pt is the only alloy where the ordered phase is reproducibly realized in experiment. Nevertheless, measurements are predominately made for slightly off-stoichiometric ordered Fe$_{72}$Pt$_{28}$ which does not transform martensitically, even if order is not fully complete. Therefore, most data are available only for disordered or partially ordered alloys. Despite this fact, we will restrict our study to stoichiometric, ordered compounds. These are much easier to handle from the technical point of view and allow deeper insight into electronic and vibrational properties with respect to their physical origin, because their simulation cells are sufficiently small while side-effects as statistical broadening can be largely avoided.

Another important feature of these alloys is the marked softening of the transversal acoustic phonons in [110] di-
resection, which has subsequently been related to any of the three above mentioned anomalies in the past. Our central aim is thus to investigate the influence of electron-phonon coupling in Fe₃Ni, Fe₃Pd and Fe₃Pt as one of the important mechanisms for structural transformations of Fe-based binary alloys. This complements the experimental information by providing a link between electronic properties and structural distortions, which will help to clarify the microscopic origin of these instabilities. We accomplished this aim by comparing features of the phonon dispersion with the electronic structure of these alloys. A similar approach has been used in the past to explore the nature of electron-phonon coupling in conventional and magnetic shape memory alloys.

During our preoccupation with this topic, we noticed that investigations providing a structural and dynamical view of the field on the basis of first principles calculations are sparse and performed with varying methodology, hindering a comparison of all three isoelectronic alloys on equal footing. We therefore decided to provide first a systematic survey of structural properties and lattice dynamics of ordered Fe-rich alloys with elements of the Ni-group, thereby filling the above mentioned gap in existing literature. The experimentally observed order in Fe₃Pt is of L₁₂ type, see Fig. 1. In our calculations, we will alternatively consider D₀₂₂ order. For lattices with bcc coordination D₀₂₂ becomes equivalent with D₀₃ order, which is related to the L₂₁ structure of ternary Heusler alloys.

Fe-rich Fe-Pd and Fe-Pt alloys close to the stoichiometric composition are reported to be collinear ferromagnets at low temperatures; we will therefore solely concentrate on the collinear ferromagnetic case. Excited non-collinear, ferri- and antiferromagnetic spin structures, which have been discussed with respect to the Invar effect, would certainly be beneficial for a thorough understanding of the interdependence of lattice dynamics and finite temperature anomalies as MSM and Invar effect. These, however, are beyond the scope of the current work and are thus left open for future investigation.

II. NUMERICAL DETAILS

Our investigation of the structural and dynamic properties has been performed within the framework of den-

FIG. 1: Primitive cells of fcc L₁₂ (a) and D₀₂₂ (b) ordered alloys of A₁B stoichiometry as used in our calculations. The lower two images show both structures represented in the same supercell which consists of two primitive cells stacked along the c-axis in the case of L₁₂ (c). Exchanging the A and B components (marked by dark blue and bright orange spheres) in one of the a-b-planes yields the D₀₂₂ ordering (d). Note, that the symmetry of the D₀₂₂ structure is tetragonal. After reduction of the c/a-ratio by a factor of √1/2, the cell transforms to the bcc-type D₀₃ structure with cubic symmetry.

FIG. 2: The energy profile along the Bain path for L₁₂ Fe₃Pd obtained with VASP and WIEN2k for PBE and PW91 exchange-correlation potentials. The calculations were carried out at a fixed atomic volume of 13.08 Å³, corresponding to an fcc lattice constant of a = 3.74 Å. For better comparison, all results are given relative to the energy of the bcc state (c/a = √1/2). The labeling fcc and bcc refers to coordination of the atoms regardless of the species. With the VASP code, pseudopotentials with different numbers of semi-core electrons were tested; the reference calculations (WIEN2k) account for Fe 3s and Pd 4s as semi-core electrons. Most curves coincide along the full Bain path within 2 meV/atom.
sity functional theory (DFT). The majority of our results has been obtained using the Vienna ab initio Simulation package (VASP). This code concentrates – as pseudopotential methods generally do – for efficiency reasons on the valence electrons for the description of the electronic structure. The VASP code provides, nevertheless, an excellent compromise between speed and accuracy as the interaction with core electrons is taken into account within the projector augmented wave approach (PAW). This yields results close to what is usually expected from other all electron methods. As we are dealing with Fe-rich compositions, the use of the generalized gradient approximation (GGA) is mandatory to obtain a correct description of the structural ground state properties. For structural energies within small (four atom) cells corresponding to Fig. 1 (a) and (b) we use a k-mesh containing at least 16384 k-points \times \text{ atoms} in the full Brillouin zone. For the calculation of the density of states (DOS) the k-point density was increased to values of 131072 k-points \times \text{ atoms} and 340736 k-points \times \text{ atoms} for obtaining the Fermi surface in the 4 atom cell. The Brillouin zone integration was carried out using the tetrahedron method with Blöchl corrections. Electronic self-consistency was assumed below a threshold difference of 1 \mu eV between two iteration. The VASP code is supplied with an extensive, well tested potential library which contains more than one pre-generated pseudopotential per element. These differ by number of electrons explicitly treated as valence or semi-core states or are designed for different exchange correlation functionals. In our case, the GGA formulations of Perdew and Wang (PW91) in connection with the spin-interpolation formula of Vosko, Wilk and Nusair as well as Perdew Burke and Ernzerhof (PBE) have been used. The required cutoff of the plane wave basis depends on the exchange correlation potential and was chosen as E_{\text{cut}} = 335 \text{eV} (or larger) for PW91 and PBE together with the minimal basis explicitly describing the 3d^74s^1 for Fe and 4d^105s^1 for Pd (and correspondingly for Ni and Pt). In calculations explicitly considering the semi-core p-electrons, E_{\text{cut}} was set to 367 eV for PBE and 438 eV for the PW91 pseudopotentials. A scalar relativistic formulation of the Hamiltonian was used throughout.

A. Comparison between VASP and Wien2k

Although the above mentioned technical differences with respect to the choice of the exchange correlation functional and the basis size usually do not lead to qualitative changes of the results, we must be particularly careful in our case, because energy landscapes in the vicinity of a structural transformation can be essentially flat. This requires a higher accuracy, i.e., a better energy resolution. Therefore, we decided to perform in a first step a thorough comparison of the available potentials. As a benchmark, we evaluated the energy as a function of the tetragonal distortion c/a along the Bain path for L1_2 ordered Fe_3Pd, describing a structural deformation from a face centered cubic (fcc) lattice at c/a = 1 to a body centered cubic (bcc) lattice at c/a = \sqrt{2}/2, as shown in Fig. 2. The results were compared to high precision calculations using the full potential linearized augmented plane wave method (FLAPW) as implemented within Wien2k code, which is widely counted among the most accurate DFT codes for solid state problems. Here, the calculations included 3s and 3p of iron and 4s and 4p of palladium as semi-core. The energetic cutoff was chosen as \( R_{\text{MT}} \times K_{\text{max}} = 9.0 \). Muffin-tin radii of 2.24 a.u. were used for all atoms and angular momenta were taken into account up to \( l_{\text{max}} = 10 \). The k-mesh comprised again 32000 k-points \times \text{ atoms} in the irreducible Brillouin zone.

The obtained energy variation along the Bain path is with \( \approx 8 \text{meV/atom} \) very small. However, except for the PBE potentials without semi-core p electrons, all curves coincide within an energy interval of 2...3meV/atom (23...35 K on a temperature scale), which can be taken as a measure for the methodological resolution in calculating structural energy differences along the Bain path. While the PBE potentials seem to systematically overestimate the energy variation (slightly), the agreement between both PW91 potentials (large and small basis) and the full potential curves is very good. This finding agrees well with a recent comparison of experimental and theoretical structural data for equiatomic FePt alloys. Our benchmark demonstrates that the PW91 pseudopotentials with the small basis (Fe 3p and Pd 4p as core electrons) is sufficient, which is especially helpful for expensive calculations of large systems.

B. Calculation of dynamical properties

Two standard techniques are currently applied for the investigation of lattice dynamics of crystals from first principles: The linear response method and the so-called direct method.

In the linear response method the dynamical matrix is obtained from the modification of the electron density, via the inverse of dielectric matrix describing the response of the valence electron density to a periodic lattice perturbation. The dielectric matrix is then calculated from the eigenfunctions and energy levels of the unperturbed system. Only linear effects, such as harmonic phonons, are accessible to this technique. The method has been applied with success to many alloys related to our present study. On the other hand, the direct-method is a frozen-phonon type of calculations based on a supercell calculation, which allows explicit account of any distortion of the atomic positions. The phonon frequencies are calculated from Hellmann-Feynman forces generated by the small atomic displacements, one at a time. Hence using the information of the crystal symmetry space group, the force constants are derived, the dynamical matrix is built and
diagonalized, and its eigenvalues arranged into phonon dispersion relations. In this way, phonon frequencies at selected high-symmetry points of the Brillouin zone can be calculated\textsuperscript{65} The direct approach in conjunction with the \textit{ab-initio} method has previously been used intensively by the authors to investigate phonon dispersion relations in magnetic shape memory Heusler compounds\textsuperscript{32,57–59}.

The results presented within this work are relying on the direct approach together with the VASP code for the calculation of the respective forces; the displacements being necessary to describe the phonon dispersions were generated by the PHON code written by Dario Alfè\textsuperscript{60,61}. This code was also used later on to generate the dynamical matrix and the resulting phonon dispersion relations. We used a supercell of $5 \times 5 \times 5$ primitive cells, containing 500 atoms in total. For sufficiently accurate of the forces a $k$-mesh of $4 \times 4 \times 4$ points in the full Brillouin zone was employed in connection with the Methfessel-Paxton finite temperature integration scheme (smearing parameter $\sigma = 0.1$ eV). First-principles calculations of this size are computationally very demanding and can, so far, only be performed on world leading supercomputer installations. The forces were calculated for three independent displacements of 0.02 Å in size each. From these calculations the phonon dispersions along the lines connecting the main symmetry points and the vibrational density of states (VDOS) were calculated. For the latter, a mesh of $61 \times 61 \times 61$ points in reciprocal $q$-space was used and additional Gaussian broadening with a smearing parameter of $\sigma = 0.1$ THz was applied.

We also carried out comparative calculations for D0\textsubscript{3} Fe\textsubscript{3}Ni and Li\textsubscript{12} Fe\textsubscript{3}Pt using the linear response method as implemented in the PWSCF package\textsuperscript{62}. To describe the interaction between ionic cores and valence electrons, we used for Fe\textsubscript{3}Ni ultra-soft pseudo-potentials\textsuperscript{63} generated using the exchange correlation functional of Perdew, Burke and Ernzerhof (PBE). The pseudopotentials for Fe\textsubscript{3}Pt were generated for the exchange correlation of Perdew-Zunger within local density approximation (LDA)\textsuperscript{66,67}. The technical parameters used here were, were a kinetic energy cutoff of 50 Ry, an energy cutoff for the augmentation charges of 600 Ry and $12 \times 12 \times 12$ $k$-points in the full Brillouin zone (Monkhorst-Pack). For the phonon dispersions, a mesh of $4 \times 4 \times 4$ $q$-points in the reciprocal space was used for the fcc basis, yielding 8 sets with finite weight while for the simple cubic basis a $2 \times 2 \times 2$ $q$-points mesh was used yielding 4 sets of special $q$-vectors with finite weight. Although the technical parameters are less tight, the obtained dispersions agree well on a qualitative level with the ones obtained within the direct approach (except for discrepancies which can be partially related to the use of LDA in the latter case as the typical underestimation of the lattice constant as well as an extension of the instability of the TA\textsubscript{1} branch to the $\Gamma$-point, see also Ref.\textsuperscript{53} and are therefore not explicitly presented in this manuscript.

### III. COMPARISON OF STRUCTURAL PROPERTIES

There are a large number of reports of first principle investigations concerning Fe-rich alloys with elements of the platinum group available in literature. These cover magnetic and magneto-elastic properties\textsuperscript{66–69} electronic structure,\textsuperscript{69–72} order and disorder,\textsuperscript{76–78} only few reports, however, are present describing the energetics of tetragonal distortions in ordered alloys. These exist, e. g., for Fe\textsubscript{2}Pt using the LMTO-ASA (linear muffin-tin orbital method within the atomic sphere approximation) method and for Fe\textsubscript{3}Ni using different approaches,\textsuperscript{69,72–75} Again, particularly the FLAPW calculations of Ref.\textsuperscript{81} agree very well with our results. A systematic \textit{ab-initio} comparison of all three alloys with respect to structural properties along the Bain path, describing the transformation from fcc-type to bcc-type structure, however, is still missing and therefore concisely presented in the subsequent section.

#### A. Binding surfaces along the Bain path

The total energy as a function of atomic volume and tetragonal distortion for all three isoelectronic alloys in Li\textsubscript{12} order is shown in Fig. 3. As is easily seen from the contour plots, the optimum $c/a$ ratio varies from close to bcc ($c/a = 0.74$) for Fe\textsubscript{3}Ni over bct ($c/a = 0.78$) for Fe\textsubscript{3}Pd to fcc ($c/a = 1$) for Fe\textsubscript{3}Pt. This trend cannot simply be interpreted in terms of the atomic volume as, especially for Fe\textsubscript{3}Pd and Fe\textsubscript{3}Pt, the equilibrium volumes nearly coincide. This is in good agreement with experimental measurements (see the collection of Okamoto, Ref.\textsuperscript{82} for an extensive overview). Especially the relation between lattice parameter and composition of Fe\textsubscript{3}Pd exhibits a strong positive deviation from Vegards law. The overall variation of the energy landscape along the Bain path is exceptionally small for all three alloys. It is with 22 meV/atom largest for Fe\textsubscript{3}Ni and smallest for Fe\textsubscript{3}Pd with only 4 meV/atom (minimum to saddle point). The shift of the minima agrees well with the fact that in the phase diagram of ordered Fe\textsubscript{3}Pt the stability range of the body centered structures at $T = 0$ K ends at higher Fe-content compared to the other alloys as well as with the trend in the martensitic transition temperatures, which are reduced on variation of the second element from Ni to Pt. The maximum change in equilibrium volume associated with a full tetragonal transformation along the Bain path is small, less than one percent for all three alloys. There is also a strong experimental indication for an unusually flat energy surface. Recently, it has been shown that thin disordered Fe\textsubscript{72}Pt\textsubscript{30} films can be grown epitaxially on different substrates inducing different lattice constants $a$ in the film plane\textsuperscript{83} The corresponding perpendicular lattice constant, $c$, however, is free to adjust, thereby realizing $c/a$ ratios covering most of the Bain path. Such a strained epitaxial growth can
only be expected if the energy associated with the lattice strain is sufficiently small along the Bain path.

The magnetic moment is largest on the bcc side, however, there is only little variation of the spin moments along the Bain path, which is of the order of 0.1 µB/atom (cf., Fig. 4). The slight variation of the magnitude between the alloys might well be attributed to the different equilibrium lattice constants.

B. L1₂ versus D0₂₂ order

For fcc alloys of stoichiometry A₃B, the D0₂₂ structure may be considered as another, possibly competing, realization of order. As can be seen from Fig. 1(d), the D0₂₂ structure emerges from the L1₂ structure by shifting one of the mixed planes halfway along the diagonal without shearing the other planes. At the c/a-ratio of the bcc lattice, the D0₂₂ structure turns into the highly symmetric D0₃ structure, which is equivalent to the Heusler-type L2₁ structure for a binary composition. For Fe₃Ni and Fe₃Pd, where order is not substantiated in experiment, the D0₂₂ becomes favored at c/a ratios slightly above one, as demonstrated in the lower panels of Fig. 4. For Fe₃Pt the D0₂₂ structure becomes favored at the bcc end. Only for this alloy, the energy of D0₂₂ becomes competitive with L2₁, taking into account the complete Bain path. The formation of a D0₃ phase in ordered near-stoichiometric Fe₃Pt might further be hindered by the slow kinetics of the necessary diffusive transformation at comparatively low temperatures. However, at the fcc side, the small energetic distance between L1₂ and D0₂₂ order indicates a low penalty and thus a comparatively high probability for anti-phase boundaries in the ordered phases.

The magnetic response of both order types to a tetragonal distortion is identical. An important difference between D0₂₂ and L2₁ order is, however, that along the Bain path a considerable energy barrier exists for D0₂₂ but not for L2₁, which would slow down the transformation kinetics, if D0₂₂ could be stabilized.

IV. LATTICE DYNAMICS AND SOFT PHONONS

Important information about the stability of structures and potential relaxation mechanisms can be obtained from the phonon dispersion relations. Accordingly, extensive discussions have been taking place concerning martensitic precursor effects in the lattice dynamics of the austenitic phase in martensitically transforming alloys, which might yield, e.g., additional information on the transformation mechanisms. For the systems under consideration, experimentally determined phonon dispersion relations obtained by inelastic neutron scattering are available for disordered Fe₇₀Ni₃₀ and Fe₆₅Ni₃₅, disordered Fe₇₂Pd₂₈, and disordered as well as ordered
Fe₃Pt: Theoretical calculations of the full phonon dispersion, on the other hand, have been reported so far mainly for ordered Fe₃Ni, Fe₇₂Pd, and Fe-Pd alloys in the Pd-rich composition range. Several of the above mentioned investigations rely on empirical or semi-empirical descriptions of the interatomic forces. Astonishingly, similar calculations for the Fe-Pt case appear to be missing. The inability of the experiment to provide a complete comparison of the ordered phases (which are easier to understand) together with the incomplete record of theoretical studies motivates our attempt to provide a comparative overview of the phonon dispersion of all three ordered isoelectronic alloys from first principles.

A. Phonon dispersion relations

The central feature of the phonon dispersions is the complete softening of the transversal acoustic branch TA₁ around the M-point. This is common for all three alloys, but most pronounced in Fe₃Pt. The occurrence of imaginary frequencies reflects the instability of the lattice which may gain energy by static atomic displacements according to the corresponding phonon mode. A strong anomalous softening of the TA₁ branch is a widely discussed observation from neutron diffraction experiments on ordered Fe₃Pt. With decreasing temperature starting from the paramagnetic phase, the TA₁ mode becomes increasingly softer at the M-point, finally reaching phonon energies as low as 3 meV for T = 12 K. The microscopic nature of the corresponding displacements will be discussed in the following section. An incomplete softening of the TA₁ branch with anomalous temperature dependence is also found for the disordered alloys. However, in these cases, it is far less pronounced and the wave vector is shifted towards the Γ-point.

For Fe₃Pd one of the transversal acoustic modes in Fig. 5 starts with a negative slope from the Γ-point, which indicates that the system might immediately undergo a long-wavelength distortion without activation barrier. This is not the case for Fe₃Ni, although here, the underlying structure is not the groundstate, either. In both cases, the cubic L1₂ state describes only a saddle point. Thus the initial slope of the acoustic branches in the low energy and low wavevector regime depends on small details of the binding surfaces and might well be affected by the specific choice of the method and technical setup, as demonstrated in Fig. 2. The softening along the [111] direction is strongest for Fe₃Ni. Both TA branches are degenerate, reaching frequencies as low as 1 meV around ξ = 0.14. The corresponding eigenvectors are close to (1, −1, 0) and (1, 1, −2) and practically uniform for all four atoms in the primitive cells. This may correspond to the primary [112] shear on (111) planes, common for martensitic transformation mechanisms of Fe-based al-
FIG. 5: (color online) Calculated phonon dispersions (right panels) of L1$_2$ ordered Fe$_3$Ni (top), Fe$_3$Pd (center) and Fe$_3$Pt (bottom) along the main symmetry directions and the corresponding vibrational density of states (left panels). Imaginary frequencies are omitted from the density of states. The polarization of the phonon branches according to the atomic contributions to the respective eigenvectors is shown in the intensity (color) coding.
FIG. 6: (color online) Calculated phonon dispersions (right panel) of D0$_3$ ordered Fe$_3$Ni along the main symmetry directions and the corresponding vibrational density of states (left panel). Again, the polarization of the phonon branches according to the atomic contributions to the respective eigenvectors is shown in the intensity (color) coding.

The phonon dispersion of Fe$_3$Ni as shown in the upper panel of Fig. 6 agrees well with previous ab initio results for the same material obtained by the same method (but less restrictive accuracy). Differences are mainly present at the M-point, where the complete softening of the TA$_1$ mode is less pronounced in the present study and slightly shifted away from the Brillouin zone boundary. In addition, the softening in [111] direction is more pronounced in our study. Previous semi-empirical calculations based on a tight-binding scheme on the other hand did only show comparable softening if the strength of the electron phonon coupling was artificially enhanced. This underlines the necessity to fully take into account the coupling between electronic and lattice degrees of freedom within a first-principles approach. Still, the convergence with respect to the subdivision in reciprocal space for the electronic structure as well as for the lattice dynamics remains an important issue.

Another quantity which allows for a detailed comparison with experiment is the element resolved vibrational density of states (VDOS) which is shown in the diagrams on the left of the dispersions. For alloys with a large mass difference, we expect a clear separation between the elemental contributions. This is indeed the case for Fe$_3$Pt, where we find a pseudo-gap at phonon energies around 17 meV. Above this energy, there is only little contribution from Pt vibrations, while we find a hybridization of Pt and Fe contributions in the energy range below. In Fe$_3$Pd, the Pd states move to higher energies, filling up the pseudo-gap and there is increased hybridization of the partial contributions at the optical bands in the uppermost part. This trend consequently continues for Fe$_3$Ni, moving the central peak of the partial VDOS of Ni up to an energy of 22.5 meV. In turn, the distinct low energy peak appearing around 10 meV for Fe$_3$Pt and Fe$_3$Pd has disappeared. In all three cases, there exists a significant contribution of Fe-states in the low energy region ($\lesssim$ 8 meV). The main peaks of the calculated VDOS agree well with experimental measurements of disordered Fe$_3$Pt of Delaire and coworkers as well as on ordered Fe$_3$Pt by Wiele et al. and even recent measurements of the Fe-partial contributions of ordered Fe$_3$Pt nanoparticles by Roldan-Cuenya et al. using the Mössbauer approach. The pseudo-gap, however, is not showing up in experimental VDOS as these generally experience a stronger broadening, which may be attributed to finite temperature of the measurements, resolution of the measurement devices and – not at last – the incomplete ordering usually present in Fe$_3$Pt samples.

The disappearance of the sharp energy peak in the VDOS around 10 meV points out a qualitative difference between Fe$_3$Ni and the other two alloys. In the latter case, these states are connected with vibrational modes at the M and the R-point which are dominated by motions of the Pt or, respectively, Pd atoms as can be seen from the atomic polarization of the phonon dispersions shown in the intensity (color) coding in Fig. 6. In Fe$_3$Ni, however, the low lying modes in the vicinity of the M-point mainly involve Fe-atoms, which makes the Fe-subsystem particularly soft and susceptible to element specific distortions, e.g., due to magnetoelastic coupling. In contrast, this feature completely disappears in the body centered cubic phase of Fe$_3$Ni, which we rep-
FIG. 7: (color online) Schematic representation of the two M-point soft modes in the phonon dispersions of fcc Fe$_3$Ni, Fe$_3$Pd and Fe$_3$Pt shown in Fig. 5. The picture shows a projection of the repeated L1$_2$ unit cell along the [0 0 1] direction. The brighter (red) arrows correspond to the M2 mode, while the black arrows show vibrations of the M4 mode. The main feature here is that both vibrations are identical, but orthogonal to each other. Both vibrations match with a translation along one half of the face diagonal ([110] direction) as shown by the thick (blue) arrow, whereby the two vibrations remain the same but shifted in phase by $\pi$.

B. Identification of the soft mode

As laid out in the previous paragraph, the two lowest frequencies in the dispersion of L1$_2$ Fe$_3$Ni at the M-point involve only motions of the Fe species. Since the associated wave-vector corresponds to the Brillouin zone boundary, the amplitudes are opposite at either end of the primitive cell. Therefore the term antiferrodistortive transformation has been has been coined for the condensation of the unstable phonon mode.\textsuperscript{27} The displacements can be inferred from the corresponding eigenvectors and are sketched in Fig. 7. They belong to motions of the Fe-atoms either in direction of one of the two nearest neighbor Ni pairs, or perpendicular to this direction. If one describes the primitive cell as an Fe-octahedron encaged by Ni-atoms at the corner positions of the cube, the mode can be described in the first case as a rotational motion of the perimeter atoms in the (001) Fe-plane, while the top and bottom atoms in the mixed plane remain in place. This kind of rotational mode is well known from perovskite structures, where oxygen octahedra form such tilted structures.\textsuperscript{98,99} In the second case, it is rather described as a kind of breathing motion of the perimeter atoms in the Fe-plane, where one half of the atoms move inwards and the other half move outwards leading to an orthorhombic distortion of the Fe-octahedron. Keeping the terminology of Noda and Endoh as well as Kästner and coworkers, we will call these modes M4 and M2, respectively. Displacements according to both phonon modes lower the symmetry of the crystal and reduce the space group of the cubic L1$_2$ phase, Pm$\overline{3}m$ (number 221), to P4/mmb (number 127). Since the latter is already tetragonal, a variation of c/a does not imply another change of symmetry, which corresponds well to the experimental reports of a low temperature fct phase.

As mentioned above, the nature of the distortions leading to the softening of the phonon modes can be inferred in principle from the eigenvectors of the dynamical matrix which can be constructed, e.g., by a Born-von-Karmann fit to the experimental dispersions. This indirect procedure often leaves room for ambiguities and has lead to a controversy about the nature of the unstable mode in the past.\textsuperscript{24,27} A more direct approach is to monitor the variation of the total energy as a function of the displacement according to the (frozen) phonon
modes. This can be realized in a straight forward manner within a density functional theory approach. We construct $2 \times 2 \times 1$ supercells by doubling the primitive $L1_2$ cell in the plane of the atomic motion and apply the corresponding displacements $\delta$ as sketched in Fig. 2. The result, shown in Fig. [2] unambiguously proves that for all three alloys the $M2$ mode leads to a lower energy even for very small $\delta$, explaining the imaginary frequencies at the $M$-point. In Fe$_3$Pt and Fe$_3$Pd, the energy gain can reach values of about $5 \text{meV/atom}$ for displacements of about $0.08$ to $0.09 \AA$. In Fe$_3$Ni this energy gain is significantly smaller and the minimum energy is obtained for smaller displacements. On the other hand, the $M4$ mode is considerably softer in Fe$_3$Ni than in the other two alloys. Here, the two lowest frequencies correspond to the two pure Fe modes, while for the other two alloys the second eigenvalue has a marked contribution of the heavier element. In the latter case, the $M4$ mode is represented by the $5th$ and the $6th$ eigenvalue with frequencies of $18.7 \text{ meV}$ and $19.4 \text{ meV}$ for Fe$_3$Pd and Fe$_3$Pt, respectively, as opposed to $9.3 \text{ meV}$ for Fe$_3$Ni.

In this respect, it may be important to notice that a translation of the mixed $(001)$ plane in $[1\overline{1}0]$ direction, which leads to an exchange of the Fe and Ni-group atoms, transforms both modes, $M2$ and $M4$, in each other. Applying this transformation once every second unit cell transforms the $L1_2$ in D0$_{22}$ order, as can be seen from Fig. 1. D0$_{22}$ can thus also be regarded as an $L1_2$ structure with maximum density of anti-phase boundaries. Thus atoms in the Fe-plane will feel a superposition between both environments and displacements according to $M2$ and $M4$ modes become equivalent. Therefore, we expect that such distortions of the order will likely harden the soft phonon at the $M$-point, especially for Fe$_3$Pd and Fe$_3$Pt.

C. Electronic origin of the antiferro-distortive transformation

While the total energy calculations supply us with the information that the imaginary phonon mode is related to an orthorhombic distortion of the Fe octahedra in the $L1_2$ unit cell, we are still left with the task to find the origin of this instability. A clue can be obtained from the comparison of the respective electronic density of states, which is shown paradigmatically in Fig. 3 for the undistorted, perfect $L1_2$ structure of Fe$_3$Pt as well as for supercells with $M2$ and $M4$ distortions. For all three cases, the majority spin density of states remains similar. The $L1_2$ DOS is in excellent agreement with the previous extensive study of Podgorny, apart from a small shift of the Fermi level in both spin channels which can be attributed to the use of an exchange correlation functional without gradient corrections in Ref. [22]. Fe$_3$Pt is at the onset of strong ferromagnetism, thus the $d$-states of the majority spin channel are filled and the respective DOS is small at the Fermi energy, $E_F$, leaving no room for changes that could account for the structural distortions. This, however, does not apply for the minority spin channel, where a considerable number of $d$-states is encountered right at $E_F$, especially for the undistorted $L1_2$ structure. The vast majority of these states comes from the Fe atoms, while the Pt DOS remains small, due to the nearly filled $d$-shell and only moderate induced magnetic polarization of $0.36 \mu_B$.

On the other hand, the condensation of the $M2$-mode opens a deep pseudo-gap right at the Fermi-level which is the indication of an extensive redistribution of states. In the spirit of a band-Jahn-Teller mechanism, lifting the degeneracy of electronic states at the Fermi level can lead to a net gain in band energy. This has been previously proposed as the main electronic mechanism leading to the tetragonal distortion in disordered magnetic shape memory Fe-Pd alloys.

Accordingly, a distortion corresponding to the $M4$ mode does not decrease the number of states at $E_F$. Nevertheless, it opens another pseudo-gap $0.25 \text{eV}$ below the Fermi level. Although this gap is very narrow and should be expected to vanish already at rather low temperatures, it appears possible that a moderate decrease of the valence electron concentration $\epsilon/\alpha$ may bring this pseudo-gap to the Fermi level and possibly causes a change the order of the modes at the $M$-point which alters the corresponding ground state structure.

The appearance of a pseudo-gap at $E_F$ implies a severe reconstruction of the Fermi surface. Such reconstructions in connection with anomalous softening of the acoustic modes are a sign of strong electron-phonon coupling and frequently related to a Kohn anomaly. This involves a significant number of nesting states at $E_F$ that can be connected with the same reciprocal vector which describes the anomaly in the phonon dispersions. Fermi-surface nesting and strong electron-phonon coupling in combination with transitions to modulated phases are commonly found in conventional and magnetic shape memory systems, as Ni-Al, Ni-Ti and Ni-Mn-Ga.

In fact, the phonon softening in disordered Fe-Ni Invar has been related previously to a Kohn anomaly in the nonmagnetic state on the basis of first principles calculations of ordered Fe$_3$Ni. Here, we consider spin-polarized minority spin Fermi surfaces, i.e., the isosurfaces formed by the Kohn-Sham eigenvalues at the Fermi level in reciprocal space. These are depicted for Fe$_3$Ni and Fe$_3$Pt in Fig. 10 and we find good agreement of the shape with previous results obtained using Wien2k.

Six bands are crossing $E_F$ in the minority spin channel making the Fermi surfaces quite complex objects. Nevertheless, it is possible to relate nesting behavior to two of these bands, the 13th and the 15th band which are shown separately in Fig. 11. The 13th minority spin band represents an extended spherical hole pocket around the $\Gamma$-point with hopper-shaped extrusions reaching the Brillouin zone boundary. Facilitated by the cubic symmetry, the flattened, horizontal walls of these extrusions can provide nesting with a vector of exactly $(1/2, 1/2, 0)$, c.f., Fig. [11].
FIG. 9: (color online) Comparison of the total and partial, site resolved electronic density of states (DOS) of Fe₃Pt with perfect L₁₂ order (left) and M₂ distorted Fe₃Pt with an displacement amplitude $\delta = 0.795$ Å (center) and M₄ distorted Fe₃Pt with $\delta = 0.748$ Å (right). The majority spin DOS are denoted by positive values, while negative values refer to the minority spin channel. In the M₂ case, a pseudo-gap opens at the Fermi level due to a redistribution of the Fe states. For the M₄ distortion, a narrower gap opens up 0.25 eV below the Fermi level, while the DOS at the Fermi level remains largely unchanged (pseudo-gaps denoted by arrows).

It should be mentioned that a thorough investigation of the relation between Fermi surface nesting and transformative processes requires a detailed statistics, e.g., an evaluation of the generalized susceptibility with respect to all contributing reciprocal vectors as well as the calculation of the electron-phonon coupling matrix elements. This is a formidable task in itself and beyond the scope of this work. Nevertheless, in our view, the purely graphical discussion above gives already sufficient arguments that a Kohn anomaly is responsible for the low temperature structural changes in the ordered alloys.

V. CONCLUSIONS

Within the present investigation, we systematically compared structural, electronic and dynamic properties of ordered Fe-rich alloys with the elements of the Ni group. The only one of these three alloy, which can be obtained experimentally in the ordered state, Fe₃Pt, is also the only one which possesses an fcc ground-state structure. The dominant type of ordering in all three alloys is L₁₂, in accordance with experimental observation. Competing D₀₂₂ order was observed for hypothetical Fe₃Ni and Fe₃Pd for fct structures with a tetragonal distortion $c/a > 1$ and for Fe₃Pt with bcc coordination. However, for ideal fcc ($c/a = 1$), D₀₂₂ order is well within the range of thermal energies, suggesting that anti-phase
FIG. 10: Plots of the isoenergy-surface in reciprocal space of the minority spin electronic states the Fermi level (Fermi surface) of Fe$_3$Ni (left) and Fe$_3$Pt (right). The Fermi level is intersected by 6 bands (shown online in different colors). The features are largely similar and differ mainly only in size due to the varying atomic volumes. Both surfaces exhibit large nearly flat portions which may give rise to a Kohn anomaly.

boundaries are very likely to appear in the cubic L1$_2$ phases.

The phonon dispersions of the three alloys possess as a common feature a complete softening of TA$_1$ branch in [110] direction around the M-point, which is strongest for Fe$_3$Pt and Fe$_3$Pd as well as a partial softening along the [111] direction. The first leads to the freezing of the respective phonon mode (M2) which can be identified as an orthorhombic distortion of the Fe octahedra in the primitive L1$_2$ cell without involving the Ni-group elements. The second anomaly becomes increasingly pronounced with decreasing mass of the Ni-group element, as this leads effectively to an increased Fe-contribution to the low energy acoustical phonons.

It may be speculated whether the rather smeared out softening of the TA$_1$ branch in [110] direction in the disordered alloys can be interpreted in terms of a corresponding distortion with similar electronic origin, but on a larger, statistically modified period. This owes to the fact that the Fe-octahedra can be considered to be statistically distributed within the bulk alloy and result finally in a rather broadened and less pronounced anomaly. These octahedra are characteristic for the L1$_2$ structure, but also other locally Fe-enriched cluster configurations, which are more or less susceptible to deformations, may contribute. Similarly, the gross electronic structure will appear rather smeared out for the disordered crystal, but distinct features may prevail locally within suitable Fe-clusters and induce corresponding local distortions. Indeed, characteristic relaxations of the atomic positions have been reported in ab initio calculations for fcc FeNi-Invar alloys for 64-atom quasi-random structures. This also fits well with our observation that in Fe$_3$Ni not only the M2 mode but also the orthogonal M4 mode becomes rather soft at the M-point, which makes collective motions in the Fe-(001) planes very cost effective. Furthermore, own investigations considering structural relaxations of 108-atom Fe-Pd and Fe-Pt supercells with random distribution of atoms, yield a considerable energy gain from a similar relaxation process.

The sum of these observations supports the view that the origin of the anomalous features in the acoustical branches can be attributed solely to the Fe species. This is confirmed by the calculation of the electronic structure of the L1$_2$ ordered and the distorted alloys. The condensation of the M2 mode opens a pseudo-gap in the minority density of states right at the Fermi level, reducing the density of states by nearly 50%. Again, only contributions from Fe states are involved. The origin for this strong reconstruction of the Fermi surface can be related to extended flat areas which nest with a reciprocal vector of nearly (1/2, 1/2, 0) which corresponds to the softening of the TA$_1$ phonon mode at the M-point.

The subtle interdependence between electronic struc-
FIG. 11: Fermi surfaces of the 13th (a) and 15th (b) minority spin band of Fe₃Pt (with an additional semitransparent image of the 13th band in the latter case). These two bands exhibit flat parts which may be connected with a wave vector close to \((\frac{1}{2}, \frac{1}{2}, 0)\) as indicated by the arrows. The vector can either be applied tangentially between the flattened, horizontal walls of the cone-shaped extensions at the zone boundary entirely within the surface of the 13th band as shown in (a) or between the vertical walls on the left and the respective sections of the cube shaped surface centered around the R-point in (b). The eigenvalues in reciprocal space of the of the 13th and 15th bands along two horizontal planes \((k_z = 0.5\) and \(k_z = 0.43\)) are shown in (c) as contour plots. The intersection of the complete Fermi surface (all bands) with these planes are marked by black lines. The arrows denote again parallel vectors \((\frac{1}{2}, \frac{1}{2}, 0)\). The energies are given relative to the Fermi level; the difference between two contour lines is 50 meV.

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It should be mentioned at this point, that for heavy elements, as the 5d transition metal Pt, a fully relativistic treatment would be superior in general. In the present case, this also implies to go beyond the frequently employed perturbative corrections, because fully self-consistently obtained charge densities are needed for the calculation of the inter-atomic forces, which are used to determine the respective phonon dispersion relations. Within the VASP code, spin-orbit terms are only available in connection with a non-collinear representation of magnetic moments, which significantly increases the computational demands while slowing down convergence. On the other hand, the benefit would be rather limited, because the essential physics is governed by a redistribution of Fe 3d states in the vicinity of the Fermi level, as layed out in the further course of the paper. For our purposes, we therefore restrict to the scalar relativistic approach, allowing us to treat all three alloys consistently on the same level of approximation at reasonable numerical cost.
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