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Exotic bulk viscosity and its influence on neutron star r-modes

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Abstract We investigate the effect of exotic matter in particular, hyperon matter on neutron star properties such as equation of state (EoS), mass-radius relationship and bulk viscosity. Here we construct equations of state within the framework of a relativistic field theoretical model. As hyperons are produced abundantly in dense matter, hyperon-hyperon interaction becomes important and is included in this model. Hyperon-hyperon interaction gives rise to a softer EoS which results in a smaller maximum mass neutron star compared with the case without the interaction. Next we compute the coefficient of bulk viscosity and the corresponding damping time scale due to the non-leptonic weak process including $\Lambda$ hyperons. Further, we investigate the role of the bulk viscosity on gravitational radiation driven r-mode instability in a neutron star of given mass and temperature and find that the instability is effectively suppressed.

Keywords Neutron stars, dense matter, r-mode oscillation

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1 Introduction

The investigation of spin frequencies from burst oscillations of eleven nuclear-powered millisecond pulsars showed that the spin distribution had an upper limit at 730 Hz (Chakrabarty et al. 2003; Chakrabarty 2005). The fastest rotating neutron star discovered recently has a spin frequency 716 Hz (Hessels et al. 2006). In this respect, the study of r-modes in rotating neutron stars has generated great interest in understanding the absence of very fast rotating neutron stars in nature. The r-modes are subject to Chandrasekhar-Friedman-Schutz gravitational radiation instability in rapidly rotating neutron stars (Andersson 1998, 2003; Friedman & Morsink 1998; Lindblom, Owen & Morsink 1998; Andersson, Kokkotas & Schutz 1999; Stergioulas 2003) which may play an important role in regulating spins of young neutron stars as well as old, accreting neutron stars in low mass x-ray binaries (LMXBs). Another aspect of the r-mode instability is that this could be a possible source for gravitational radiation (Andersson 2003; Gondek-Rosinska, Gourgoulhon & Haensel 2003; Nayyar & Owen 2006) which may shed light on the interior of a neutron star.

It was realised that the r-mode instability could be effectively suppressed by bulk viscosity due to exotic matter in neutron star interior when the compact star cools down to a temperature $\sim 10^9$ K. The coefficient of bulk viscosity due to non-leptonic weak processes involving exotic matter such as hyperon and quark matter was calculated by several authors (Jones 2001; Lindblom & Owen 2002; Haensel, Levenfish & Yakovlev 2002; van Dalen & Dieperink 2004; Drago, Lavagno & Pagliara 2005; Madsen 1992, 2000).

In this paper, we investigate the effect of hyperon matter including hyperon-hyperon interaction on bulk viscosity coefficient and the r-mode stability. In Sec. 2 of this paper, we discuss the model used to calculate equations of state, bulk viscosity coefficient and the corresponding time scale. Results of our calculation are analysed in Sec. 3. Section 4 gives the outlook.
2 Model

2.1 Equation of State

We describe the β-equilibrated and charge neutral hyperon matter within the framework of a relativistic field theoretical model where baryon-baryon interaction is mediated by the exchange of scalar and vector mesons and hyperon-hyperon interaction is taken into account by two strange mesons - scalar $f_0$ (hereafter denoted as $\sigma^*$) and vector $\phi$ (Schaffner et al. 1993; Mishustin & Schaffner 1996).

The Lagrangian density for hyperon matter including non-interacting electrons and muons is written as

$$\mathcal{L}_B = \sum_B \bar{\Psi}_B (i\gamma_\mu \partial^\mu - m_B + g_\sigma B \sigma + g_\sigma^* B \sigma^*) - g_\omega B \gamma_\mu \omega^\mu - g_\phi B \gamma_\mu \phi^\mu - g_\omega B \gamma_\mu \omega^\mu - \frac{1}{4} (\sigma \cdot \omega) - U(\sigma) + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{2} (\partial_\mu \sigma^* \partial^\mu \sigma^* - m_\sigma^{*2} \sigma^{*2})$$

where $\sigma^*$ and $\phi$ mesons. The scalar self-interaction term (Boguta & Bodmer 1977) is given by

$$U(\sigma) = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_4 \sigma^4.$$ (2)

We perform the calculation in the mean field approximation. The total energy density and pressure are respectively given by

$$\varepsilon = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} m_\sigma^{*2} \sigma^{*2}$$

$$+ \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_\rho^2 \rho^2$$

$$+ \frac{2 J_B + 1}{2 \pi^2} \int_0^{K_B} (k^2 + m_B^2)^{1/2} k^2 \, dk$$

$$+ \frac{1}{3} \sum_B \int_0^{K_B} (k^2 + m_B^2)^{1/2} k^4 \, dk$$

and

$$P = \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4$$

$$- m_\sigma^* \sigma^{*2} + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_\rho^2 \rho^2$$

$$+ \frac{2 J_B + 1}{2 \pi^2} \int_0^{K_B} (k^2 + m_B^2)^{1/2} k^4 \, dk$$

$$+ \frac{1}{3} \sum_B \int_0^{K_B} (k^2 + m_B^2)^{1/2} k^4 \, dk$$

where $k_{FB}$, $J_B$, $I_{BB}$ and $n_B$ are Fermi momentum, spin and isospin projection and number density of baryon $B$ respectively. The last terms in energy density and pressure are due to leptons. The effective baryon mass is defined as $m_B^* = m_B - g_\sigma B \sigma - g_\sigma^* B \sigma^*$. The charge neutrality and chemical equilibrium conditions are given by

$$Q = \sum_B \Omega_B B - n_e - n_\mu = 0 \quad \text{and} \quad \mu_\pi = \eta \mu_\pi - g_\mu \mu_\pi$$

where $\eta$, $\mu_e$ and $\mu_\mu$ are respectively the chemical potentials of neutrons, electrons and baryon $i$ and $j$, and $q_i$ are baryon and electric charge of baryon $i$. The chemical potential of baryons $B$ is given by (Chatterjee & Bandyopadhyay 2000)

$$\mu_B = (k_B^2 + m_B^2)^{1/2} + g_\sigma B \omega_B + g_\phi B \phi_B + I_{BB} B \rho_B.$$ (4)

In this calculation we adopt nucleon-meson coupling constants of GM set (Glendenning & Moszkowski 1991). The coupling constants are obtained by reproducing properties of saturated nuclear matter such as binding energy $-16.3$ MeV, saturation density $n_0 = 0.153 \, fm^{-3}$, asymmetry energy 32.5 MeV, effective nucleon mass 0.78 MeV, $\rho_B = 770$ MeV and $\rho_{N} = 938$ MeV. On the other hand, hyperon-vector meson coupling constants are determined using SU(6) symmetry of the quark model (Mishustin & Schaffner 1996; Dover & Gal 1984; Schaffner et al. 1994) and the scalar $\sigma$ meson coupling to hyperons is calculated from hyperon potential depths in normal nuclear matter such as $U_N^\sigma (n_0) = -30$ MeV (Dover & Gal 1984; Chrien & Dover 1989), $U_N^\omega (n_0) = -18$ MeV (Fukuda et al. 1998; Khaustov et al. 2000) and a repulsive potential depth for $\Sigma$ hyperons $U_N^\Sigma (n_0) = +30$ MeV (Friedman, Gal & Batty 1994; Batty, Friedman & Gal 1997; Bart et al. 1999) as obtained from hypernuclei data. The hyperon-$\sigma^*$ coupling constants are determined from double $\Lambda$ hypernuclei data (Schaffner et al. 1993; Mishustin & Schaffner 1996).

We obtain the EoS solving equations of motion along with the expression for effective baryon mass and charge neutrality and beta equilibrium constraints (Mishustin & Schaffner 1996; Chatterjee & Bandyopadhyay 2000)

2.2 Bulk viscosity coefficient, damping time scales and critical angular velocity

Now we discuss the bulk viscosity coefficient in young neutron stars which cool down to temperatures $\sim 10^9 - 10^{10}$ K after their births. As the system goes out of chemical equilibrium due to pressure and density variations associated with the r-mode oscillations, microscopic reaction processes in particular non-leptonic weak interaction processes including exotic particles might restore the equilibrium (Navar & Owen 2006; Jones 2001; Lindblom & Owen 2002; Chatterjee & Bandyopadhyay 2000). Here we calculate the real part of bulk viscosity
Fig. 1 Mass-radius relationship of non-rotating neutron stars including nucleons-only matter (blue solid line) and hyperon matter with (dashed line) and without (magenta solid line) hyperon-hyperon interaction.

coefficient (ζ) in terms of relaxation times of microscopic processes [Landau & Lifshitz 1993; Lindblom & Owen 2002]

\[
\zeta = \frac{P(\gamma_\infty - \gamma_0)\tau}{1 + (\omega T)^2},
\]

where the difference of infinite (γ_\infty) and zero (γ_0) frequency adiabatic indices is given by [Lindblom & Owen 2002; Nayyar & Owen 2006]

\[
\gamma_\infty - \gamma_0 = -n_e^2 \frac{\partial P}{\partial n_n} \frac{d \delta_n}{d t_b},
\]

In the co-rotating frame, the angular velocity (ω) of (l, m) r-mode is related to angular velocity (Ω) of a rotating neutron star as ω = \frac{1}{l+1}Ω [Andersson 2003]. In this calculation, for l = m = 2 r-mode, it is ω = \frac{2}{3}Ω. The relaxation time (τ) for the non-leptonic process \( n + p \rightarrow p + A \),

\[
\frac{1}{\tau} = \frac{(kT)^2}{192\pi^3}k_{F_A}^2 \langle |M_A|^2 \rangle \frac{\delta\mu}{n_0 \delta x_n},
\]

where \( k_{F_A} \) is the Fermi momentum for A hyperons, \( \langle |M_A|^2 \rangle \) is the angle averaged squared matrix element and

\[
\frac{\delta\mu}{n_0 \delta x_n} = \alpha_{nn} - \alpha_{An} - \alpha_{nA} + \alpha_{AA},
\]

with \( \alpha_{ij} = \frac{\partial\mu_i}{\partial x_j} n_k k_{ij} \). We obtain expressions for \( \alpha_{ij} \) from the baryon chemical potential and equations of motion for meson fields [Chatterjee & Bandopadhyay 2006]. For example,

\[
\alpha_{An} = \frac{\partial\mu_A}{\partial n_n} = \frac{g_{\omega A}g_{\omega N}}{m_\omega^2} \frac{\partial g_{\omega A}}{\partial n_n} - \frac{m_\omega^4 g_{\sigma A}}{\sqrt{k_{F_A}^2 + m_\omega^2}} \frac{\partial g_{\sigma A}}{\partial m_n} - \frac{m_\omega^4 g_{\sigma^* A}}{\sqrt{k_{F_A}^2 + m_\omega^2}} \frac{\partial g_{\sigma^* A}}{\partial m_n}.
\]

Similarly, we compute other components of \( \alpha_{ij} \). It is to be noted here that nucleons do not couple with strange mesons, i.e. \( g_{\sigma N} = 0 \).

The hyperon bulk viscosity damping timescale (τ_h) contributes to the imaginary part of the r-mode frequency and is calculated in the following way [Navyar & Owen 2006; Lindblom, Owen & Morsink 1998; Lindblom, Mendell & Owen 1999]

\[
\frac{1}{\tau_h} = \frac{dE}{dt},
\]

where \( E \) is the energy of the perturbation in the co-rotating frame of the fluid

\[
E = \frac{1}{2}\alpha^2 r^2 \int_0^R \epsilon(r)r^2dr.
\]

The derivative of \( E \) with respect to time is given by

\[
\frac{dE}{dt} = -4\pi \int_0^R \zeta(r)\langle |\nabla \delta v|^2 \rangle r^2dr,
\]

where we need to know the energy density \( \epsilon(r) \) and bulk viscosity \( \zeta(r) \) profiles of a neutron star. Similarly, we estimate the damping time scale (τ_ξ) corresponding to the bulk viscosity due to the modified Urca process including only nucleons using the bulk viscosity coefficient as given by [Sawyer 1989]. It was noted that gravitational radiation drives the r-modes unstable. So the gravitational radiation time scale (τ_GR) has a negative contribution to the imaginary part of the r-mode frequency. Further, the overall r-mode time scale (τ_r) is defined as

\[
\frac{1}{\tau_r} = \frac{1}{\tau_{GR} + \frac{1}{\tau_B} + \frac{1}{\tau_U}}.
\]

The r-mode is stable below the critical angular velocity which is obtained from the solution of \( \frac{dE}{dt} = 0 \). The critical angular velocity depends both on the mass of the neutron star and its temperature.

3 Results

We perform this calculation using the GM set [Glendenning & Moszkowski 1991] as it has been described in sec 2. It is found that A hyperons appear first at 2.6n_0 followed by Ξ^- hyperons at 2.9n_0. However no Σ hyperons appear in the system because of strongly repulsive Σ-nuclear matter interaction [Chatterjee & Bandopadhyay 2006]. Further, we find that after the appearance of negatively charged Ξ hyperons, number densities of electrons and muons decrease in the system. This is attributed to the charge neutrality condition. Due to the appearance of additional degrees of freedom in the form of hyperons, the EoS of hyperon matter becomes softer compared with that of nucleons-only matter. On the other hand, the interplay between the attractive σ^* field and repulsive φ field which becomes dominant with increasing density, makes the EoS softer initially and stiffer at higher densities than...
the EoS without hyperon-hyperon interaction. This behaviour of the EoS is reflected in the calculation of maximum mass and the corresponding radius of the neutron star. In Fig. 1, we plot the mass-radius relationship of non-rotating neutron stars calculated by solving Tolman-Oppenheimer-Volkoff equation for equations of state including nucleons-only matter (blue solid line) and hyperon matter with (dashed line) and without (magenta solid line) hyperon-hyperon interaction. Here the highest maximum mass corresponds to the stiffest EoS of nucleons-only matter. The maximum masses corresponding to the EoS with and without hyperon-hyperon interaction are 1.64 $M_\odot$ and 1.69 $M_\odot$ respectively. Using the rotating neutron star (RNS) model of Stergioulas (Stergioulas & Friedman 1995), we find that the maximum masses corresponding to the EoS with and without hyperon-hyperon interaction are respectively 1.95 $M_\odot$ and 2.00 $M_\odot$.

Next we discuss the hyperon bulk viscosity due to the non-leptonic process given by Eq. (7). The bulk viscosity coefficient is dependent on the EoS as it is evident from the expression in Eq. (5). The temperature dependence of the bulk viscosity enters through the relaxation time as given by Eq. (8). The bulk viscosity coefficient reaches a maximum and then drops with increasing baryon density as found by Chatterjee & Bandyopadhyay (2006). It has been also noted that the bulk viscosity increases as temperature decreases. Therefore, the large value of $\zeta$ might be effective in suppressing r-mode instability as neutron stars cool down to a few times $10^9$ K.

The neutron star which was rotating rapidly at its birth, slows down due to gravitational wave emission. A rotating neutron star has less central energy density than its non-rotating counterpart because of the centrifugal force. Consequently, hyperon thresholds are sensitive to rotation periods of compact stars. In the calculation of critical angular velocity, we consider a rotating neutron star of mass 1.6 $M_\odot$ and the corresponding central baryon density is 3.9$n_0$ which is well above the threshold of $\Lambda$ hyperons. This star is rotating at an angular velocity 2952 s$^{-1}$. It slowed down from its Keplerian angular velocity 5600 s$^{-1}$ when the central baryon density was below the $\Lambda$ hyperon threshold. The calculation of critical angular velocity requires the knowledge of energy density and bulk viscosity profiles in compact stars as it is evident from Eqs. (11)-(14). In Fig. 2, the hyperon bulk viscosity profile (dashed line) of the rotating neutron star of mass 1.6 $M_\odot$ is shown as a function of equatorial distance along with that of a non-rotating (solid line) star of same mass. We note that the peak of the profile shifts towards the center in the case of the rotating neutron star. Earlier calculation of critical angular velocity was performed using energy density and bulk viscosity profiles of non-rotating neutron stars. Recently, Nayyar & Owen (2006) studied the effect of rotation on critical angular velocity. We calculate the critical angular velocity using energy density and bulk viscosity profiles of the rotating star. In Fig. 3 the critical angular velocity is plotted versus temperature. We find that the r-mode instability is suppressed by hyperon bulk viscosity. Here we do not consider the impact of superfluidity of baryons on bulk viscosity. However, several calculations (Haensel, Levenfish & Yakovlev 2000, 2001, 2002; Nayyar & Owen 2006) showed that superfluidity of baryons might suppress the bulk viscosity and the damping of the r-mode instability.
4 Outlook

Other forms of matter such as $K^{-}$ condensed matter and quark matter may appear and compete with the $\Lambda$ hyperon threshold in neutron star interior. In an earlier calculation we showed that the condensate of $K^{-}$ mesons appeared at $\sim$ twice normal nuclear matter density followed by the appearance of $\Lambda$ hyperons [Banik & Bandyopadhyay 2001]. It would be worth investigating how the bulk viscosity coefficient due to the process (7) is modified in this situation. Also, we are studying whether the non-leptonic process involving $K^{-}$ condensate $n \leftrightarrow p + K^{-}$ could give rise to bulk viscosity coefficient as large as that of the non-leptonic process (7) and damp the r-mode instability effectively [Chatterjee, Bandyopadhyay & Schaffner-Bielich 2006].

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