Weak processes in astrophysical nucleosynthesis

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Abstract. Neutron capture and neutrino absorption are of fundamental importance in the synthesis of elements. Both processes can be affected by the extreme conditions of the matter undergoing nucleosynthesis. In this work, we present results of neutron capture rates on $^{46}\text{Mg}$ and several Sn isotopes. We find that the high degeneracy of neutron matter, a result of the large pressures found in a neutron star, changes the captures rates by orders of magnitude compared to those of non-degenerate neutron matter. We also study the neutrino opacity above the symmetry axis of a black hole accretion disk. We find that there is a region, where matter circulates around the black hole in stable orbits, with potentially low neutrino opacity. This motivates further studies of neutrino emission in the vicinity of a black hole.

1. Introduction

Up to this point, the origins of elements throughout the nuclear chart are not well understood. Theoretically speaking, both uncertainties on the astrophysical site and on nuclear reactions leave many open questions. The recent observation of the merger of two neutron stars [1] and the correlated kilonova observation [2] have confirmed this environment as a site for the synthesis of neutron-rich elements. Similarly, X-ray observations (for a compilation see e.g. [3]) have pointed at accretion disks around neutron stars as a possible site for the synthesis of proton-rich nuclei. As more observational data become available, new insights will light up our path to the understanding of the nucleosynthesis puzzle.

The synthesis of elements depends on diverse nuclear reactions at extreme thermodynamic conditions, with occurrences across hundreds of unstable nuclei in different mass regions of the nuclear chart. Experimental facilities around the world pursue exciting efforts to extend their limits and reach conditions and/or nuclides similar to those found in stellar environments. Among other reactions, neutrino captures on protons and neutrons, and neutron captures on nuclei, play a key role. Neutrino absorption is important in the setting of the electron fraction, determining how neutron-rich the material undergoing nucleosynthesis is. On the other hand, neutron captures naturally occur in neutron-rich environments like Supernovae and mergers. However, neutron captures could also be important in accreting neutron stars, where the highly dense neutron-rich crust material can react with the companion star’s matter or the ashes of nucleosynthetic products formed in outer layers [4, 5].

In this contribution, we discuss the role of neutron degeneracy on neutron captures, and some concepts on the of neutrino scattering in accretion disks around black holes.
2. Neutron degeneracy in accreting neutron stars

When a neutron star is in the presence of a companion star, stellar atoms such as hydrogen and helium are pulled into the neutron star’s envelope. This hot matter is captured, heated, and predicted to undergo thermonuclear burning, producing X-ray bursts and Superbursts [6]. Due to a neutron star’s immense gravitational strength, incoming matter reaches velocities fast enough to overcome radioactive decay. Proton captures on nuclei then occur faster than beta decay. This is known as a rapid-proton capture process (rp-process). The composition of the rp-process ashes varies according to the accretion conditions [5, 7]. These ashes can be buried further in the crust where neutrons are abundant and undergo neutron captures. As densities inside the neutron star can reach massive values, comparable to that of the nuclear density [8], neutrons are subject to massive pressures that are responsible for very strong quantum effects governed by the Pauli exclusion principle, i.e they are degenerate. This degeneracy will affect the rates at which neutrons are captured. Thus, a logical improvement for nucleosynthesis studies in this environment is to evaluate neutron capture rates with neutrons obeying a Fermi-Dirac distribution instead of a Maxwell-Boltzmann distribution as is customary in stellar nucleosynthesis studies (see e.g. [9, 10]).

The thermodynamical conditions of a neutron star are not reproducible on earth, and so capture rates must be evaluated theoretically. Neutron degeneracy further complicates this task. Specifically, cross sections would be needed for a wide range of mass numbers, the vast majority of which have not been experimentally measured. Thus, one needs to rely on theoretical cross sections. Additionally, at the temperatures and chemical potentials relevant to a neutron star’s crust, the behaviour of the Fermi-Dirac distribution can present abrupt changes.

Shternin et al [11] suggested the use of a power-law behaviour for the relevant cross sections and use properties of the Fermi-Dirac integrals to perform an analytic evaluation of neutron captures on some nuclei. These authors compared their analytical results to numerical integration and found that the results agreed well. However, the results can depend on the energetics of the reaction (and so on the isotopes). In this work, we numerically evaluate the Fermi-Dirac integrals and use theoretical cross sections obtained in the framework of the statistical Hauser-Feshbach model from the specialized nuclear reaction code TALYS [12]. In this context, the total cross section of the reaction \( X + n \rightarrow Y + \gamma \), in local thermal equilibrium is \( \sigma^* \), where the asterisk, *, highlights the inclusion of thermally excited nuclear levels.

The physical reaction rates are realized by the average \( \langle \sigma^*v \rangle \) of the total cross section [11]. This statistical average is calculated by weighting the cross section, \( \sigma^* \), over the velocity distribution functions of the target and incident particles (see e.g. [13]). By use of a unitary transformation, to the center of mass and relative coordinates, and integrating out the center of mass term we get

\[
\langle \sigma^*v \rangle = \sqrt{\frac{2}{\mu N}} \int_0^\infty E \sigma^*(E) f(E) dE,
\]

where \( N \) is the normalization factor given by

\[
N = \int_0^\infty \sqrt{E} f(E) dE,
\]

and the collision energy is \( E = \mu v^2 / 2 \), where \( \mu \) is the reduced mass. Assuming a non-relativistic collision then the reduced mass, the average velocity, \( v \), and the distribution function, \( f(E) \) correspond to those of neutrons. The distribution function, \( f(E) \), will be either the Maxwell-Boltzmann distribution, \( f(E)_{MB} = e^{-E/T} \), where \( T \) is the temperature in MeV, or the Fermi-Dirac distribution, \( f(E)_{FD} = [1 + e^{(E - \mu_n)/T}]^{-1} \), where \( \mu_n \) is the neutron chemical potential. To measure the deviation from the Maxwell-Boltzmann averaged rates caused by the degeneracy,
Figure 1. The R ratio for the $^{46}\text{Mg}(n,\gamma)^{47}\text{Mg}$ reaction at neutron chemical potential $\mu_n=1, 2, 3, 4$ and 5 MeV

we follow the methods in [11] and define a ratio

$$ R = \frac{\langle s^* v \rangle_{FD}}{\langle s^* v \rangle_{MB}}. $$

The ratio, $R$, is the cornerstone of this investigation and in principle should be calculated over a range of temperature and chemical potential for different nuclei predicted to be in a neutron star. Large values of $R$ indicate a significant deviation in capture rate, and will be further investigated. Plasma effects, as reported in [11], can also contribute to $R$, but for now we will focus on the effects of degeneracy.

2.1. Preliminary results

Figure 1 shows our $R$ numerical results the reaction $^{46}\text{Mg}(n,\gamma)^{47}\text{Mg}$. These results reproduce those of Shternin et al [11] who used a power law cross section for the same reaction, which agreed with experimental results. The dependence on the chemical potential is apparent. Larger chemical potentials result in a difference of many orders of magnitude, which become even more prominent at lower temperatures.

Figure 2 shows the ratio $R$ for different Sn isotopes ($^{126,132,138,140,165}\text{Sn}$) with a chemical potential $\mu_n=5\text{MeV}$. The trend in this case is to have larger changes of neutron captures as the isotope’s neutron number, $N$, increases. However, there is a drop in $R$ when $N$ is quite large, as can be seen when comparing $R$ for $^{132}\text{Sn}$ with $^{165}\text{Sn}$. Further studies are needed to understand this behaviour. For now, it is worth noting that in the two cases shown here, neutron degeneracy can change neutron capture rates by orders of magnitude. We aim to investigate many of the exotic nuclei found inside a neutron star’s crust.
Figure 2. The R ratio for neutron captures on Sn isotopes at a neutron chemical potential $\mu_n=5$ MeV. The lines indicate the isotope’s mass number, $A$.

3. Neutrino opacities and surfaces in black hole-neutron star mergers

There are only a few possible places where the synthesis of heavy neutron-rich elements can occur. This is because their formation is reliant on the rapid neutron capture process (r-process) which requires a high energy environment. There are few sites known that yield such high energies, one such being the hot outflows ejected from accretion disks. Such disks can be formed during a black-hole neutron star (BH-NS) merger and emit a large amount of neutrinos. The matter of these disks is highly dense, and therefore neutrinos are trapped in the stellar medium. When the density decreases neutrinos decouple from the disk matter and can reach a detector or interact with the outflowing ejected matter changing its electron fraction. Therefore, the nucleosynthetic outcome will change as well. There also exists the possibility of probing such an environment on earth by measuring the energy of the neutrino emissions (e.g. in current facilities like SK [14] and SNOLab [15] or future ones such as DUNE [16]), as the interstellar medium is transparent to neutrinos and therefore they would not be affected by the large distance traveled. In order to understand the chance of detection, as well as the setting of the electron fraction in outflowing matter, the neutrino effective flux must be determined. A key factor to this is the neutrino sphere. Such a surface is defined by the locations where the neutrinos are decoupled from the accretion disk. Roughly speaking, the disk can be understood as discriminated in two regions: the optically thick region, where neutrinos are trapped inside the neutrino surface, and the optically thin region where neutrinos are decoupled from the disk matter. The location of the neutrino surface is not sharp as it depends on the neutrino energy. To account for this fact, we find the neutrino opacity based on cross sections statistically averaged over the neutrino Fermi-Dirac flux,

$$\phi(E_\nu, T) = \frac{g_\nu c}{2\pi^2(hc)^2}\frac{E_\nu^2}{\exp(E_\nu/T) + 1}, \quad (1)$$
where \( c \) is the speed of light, \( \hbar = h/2\pi \), with \( h \) the Planck constant, \( E_\nu \) is the neutrino energy and the neutrino statistical weight, \( g_\nu \), is equal to 1. \( T \) is the local temperature which, in accretion disks, is usually larger than in supernovae and larger than the neutrino chemical potential. Therefore, is a good approximation to assume this chemical potential equal to zero (see e.g. [17, 18]). The resulting average cross-sections are functions of temperature. The following outlines a process by which the neutrino surface can be determined (see also [19]).

In the high-energy environment of the accretion disk, matter is dissociated into protons, neutrons, electrons, and neutrinos. The neutrino interactions included in our calculation will then consist of all the interactions with these fermions. The charged current reactions are given by

\[
\nu_e + n \rightarrow p + e^- ,
\]

(2)

\[
\overline{\nu}_e + p \rightarrow e^+ + n ,
\]

(3)

and the neutral current reactions for all flavours of neutrinos and antineutrinos are given by

\[
\nu + p \rightarrow \nu + p ,
\]

(4)

\[
\nu + n \rightarrow \nu + n ,
\]

(5)

\[
\nu + e^- \rightarrow \nu + e^- ,
\]

(6)

\[
\nu + \overline{\nu} \rightarrow e^+ + e^- .
\]

(7)

Assuming charge neutrality, the electron fraction, \( Y_e \), is equal to the proton fraction \( Y_p \). Therefore, the proton, \( n_p \), and neutron, \( n_n \), number densities are simply given by

\[
n_p = \rho_m N_A Y_e ,
\]

(8)

\[
n_n = \rho_m N_A (1 - Y_e) ,
\]

(9)

where \( \rho_m \) is the mass density, and \( N_A \) is Avogadro’s number. The electron and positron number densities can be evaluated numerically by integrating their phase space densities with relativistic corrections, and by using \( \mu_{e^-} + \mu_{e^+} = 0 \) as well as our charge neutrality assumption, \( n_p + n_{e^-} = n_e^- \). The opacity is given by

\[
\tau_\nu = \int_{h_\nu}^{h_{\text{max}}} \sum_k n_k \langle \sigma_k (E_\nu) \rangle dz ,
\]

(10)

where \( n_k \) is the number density of the particle with which the neutrino is interacting and \( \langle \sigma_k (E_\nu) \rangle \) is the corresponding statistically averaged cross section. The neutrino surface is defined when the opacity \( \tau_\nu = 2/3 \). Neutrinos within the surface cannot escape due to the large number of interactions, however, those outside can. The neutrino fluxes and average neutrino energies can be calculated from the local temperature at this surface and integrating over the surface area.

The above method was applied to a 2D axis-symmetric hydrodynamical simulation of an accretion disk resulting from a BH-NS merger by O. Just et al [20]. The BH in the simulation has mass \( M = 3M_\odot \) and spin parameter \( a = 0.8 \). Cylindrical coordinates implemented on the system allow for the tracking of density, temperature, and electron fraction at each point in space defined by \( (r, \phi, z) \). The simulation is Newtonian, however, the effects of general relativity are introduced via a pseudo-Newtonian potential, specifically, the Artemova potential [21]. The results of our post-processing calculation for the opacity \( \tau_\nu \) (coloured scale), for electron neutrinos and electron antineutrinos, at a time \( t = 20 \) ms, are shown in figures 3 and 4. Here we have superimposed the neutrino surface (thick black lines). The region above the neutrino
surface is considered to be optically thin to neutrinos, whereas the region below is optically thick.

We see that the electron neutrino surface is much larger than the electron antineutrino surface. This is due to the larger number of scattering processes that electron neutrinos undergo with neutrons (see eqs.2 and 3) compared to the number of interactions that electron antineutrinos have with protons. This is to be expected as the accretion disk mainly consists of neutrons from the neutron star, and thus the charged current reaction for the electron neutrino will happen much more frequently than the corresponding of the electron antineutrino, increasing the size of the neutrino surface.

Figure 3. Electron neutrino opacity (colour) and surface (black line).

We calculate the innermost stable circular orbit radius, $R_{ISCO}$, in geometrized units for the system in the Kerr metric by [22],

$$R_{ISCO} = M \left[ 3 + Z_2 - [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{\frac{1}{2}} \right],$$

(11)

where

$$Z_1 = 1 + \left[ 1 - \left( \frac{a}{M} \right)^2 \right]^{\frac{3}{2}} \left[ (1 + \frac{a}{M})^{\frac{3}{2}} + (1 - \frac{a}{M})^{\frac{3}{2}} \right],$$

(12)

$$Z_2 = \left[ 3 \left( \frac{a}{M} \right)^2 + Z_1^2 \right]^{\frac{1}{2}},$$

(13)

$a$ is the spin parameter of the BH, and $M$ is its mass. Orbits within this radius may be bound, however, no circular orbit can be stable. As shown in the equation, $R_{ISCO}$ decreases with increasing spin parameter. This is due to the BH dragging spacetime with it as it rotates. Further, we can see the orbit increases with larger BH masses due to larger gravitational forces.
Figure 4. Electron antineutrino opacity (colour) and surface (black line).

For the values of $a$ and $M$ from the O. Just et al simulation we find that $R_{ISCO}=12.8$ km. However, the inner edge of the disk, as given in the original simulation, is at 18.5 km. This suggests that matter in the accretion disk between a radii of 12.8 to 18.5 km may emit neutrinos. The fact that the neutrino opacity is low, close to the BH on the symmetry axis, means that neutrinos emitted inside these radii are going to be strongly affected by the BH. At the same time, from the simulation we also know that those neutrinos are the most energetic neutrinos (the local temperature is high: about 10 to 15 MeV), and therefore this could potentially change the properties of the neutrino spectrum.

4. Conclusions
We have presented two weak processes that affect the synthesis elements: neutron captures on nuclei and neutrino absorption on nucleons. Neutron captures in neutron degenerate matter is important in the production of elements in accreting neutron stars. We find that the neutron degeneracy changes the rate of absorption by orders of magnitude for $^{46}$Mg and different isotopes of Sn. In future work we will study different nuclei over a broader range of temperatures and densities, as well as delve into more efficient algorithms to account for resonant peaks in the cross sections.

Neutrinos are copiously emitted from black-hole neutron star mergers. They are key in setting the ratio of neutrons to protons in these environments. They could also be detected if one of these events occurs in a nearby galaxy. We have introduced here results of neutrino opacities and neutrino surfaces for an accretion disk formed during the merger of a black-hole and a neutron star. Our results suggest that further studies of the general relativistic effects are needed to better understand the neutrino spectra close to $R_{ISCO}$ where presumably the most energetic neutrinos are emitted.
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5. References
[1] B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 119, no. 16, 161101 (2017)
[2] B. P. Abbott et al. [LIGO Scientific and Virgo and Fermi GBM and INTEGRAL and IceCube and IPN and Insight-Hxmt and ANTARES and Swift and Dark Energy Camera GW-EM and Dark Energy Survey and DLT40 and GRAWITA and Fermi-LAT and ATCA and ASKAP and OzGrav and DWF (Deepener Wider Faster Program) and AST3 and CAASTRO and VINROUGE and MASTER and J-GEM and GROWTH and JAGWAR and CaltechNRAO and TTU-NRAO and NuSTAR and Pan-STARRS and ePESSTO and GROND and Texas Tech University and TOROS and BOOTES and MWA and CALET and IKI-GW Follow-up and H.E.S.S. and LOFAR and LWA and HAWC and Pierre Auger and ALMA and Pi of Sky and DFN and ATLAS Telescopes and High Time Resolution Universe Survey and RIMAS and RATIR and SKA South Africa/MeerKAT Collaborations and AstroSat Cadmium Zinc Telluride Imager Team and AGILE Team and 1M2H Team and Las Cumbres Observatory Group and MAXI Team and TZAC Consortium and SALT Group and Euro VLBI Team and Chandra Team at McGill University], Astrophys. J. 848, no. 2, L12 (2017)
[3] Q. Z. Liu, J. van Paradijs and E. P. J. v. d. Heuvel, Astron. Astrophys. 469, 807 (2007)
[4] H. Schatz, L. Bildsten, A. Cumming and M. Wiescher, Astrophys. J. 524, 1014 (1999)
[5] S. Gupta, E. F. Brown, H. Schatz, P. Moeller and K. L. Kratz, Astrophys. J. 662, 1188 (2007)
[6] H. Schatz et al., Phys. Rept. 294, 167 (1999).
[7] R. McKinven, A. Cumming, Z. Medin and H. Schatz, Astrophys. J. 823, no. 2, 117 (2016)
[8] P. Haensel, A.Potekhin and D.Yakovlev”, "Neutron stars: Equation of state and structure”, Springer,Newyork, 2007, pages 517.
[9] C. Arlandini, F. Kaeppeler, K. Wissnack, R. Gallino, M. Lagaro, M. Busso and O. Straniiero, Astrophys. J. 525, 866 (1999)
[10] M. Arnould, S. Goriely and K. Takahashi, Phys. Rept. 450, 97 (2007)
[11] P. S. Shertonin, M. Beard, M. Wiescher and D. G. Yakovlev, Phys. Rev. C 86, 015808 (2012).
[12] A.J. Koning, S. Hilaire, M.C. Duijvestijn TALYS-1.0 O. Bersillon, F. Gunsing, E. Bauge, R. Jacqmin, S. Leray (Eds.), Proceedings of the International Conference on Nuclear Data for Science and Technology, EDP Sciences, vol. 211, April 2227, 2007, Nice, France (2008)
[13] D. D. Clayton, Principles of stellar evolution and nucleosynthesis, McGraw-Hill, New York, 1968, page 288
[14] S. Fukuda et al., Nucl. Inst. Meth. Phys. A. 501 (2003) 418.
[15] F. Descamps [SNO+ Collaboration], Nucl. Part. Phys. Proc. 265-266, 143 (2015).
[16] R. Acciarri et al. [DUNE Collaboration], arXiv:1512.06148 [physics.ins-det].
[17] R.F. Sawyer, Phys. Rev. D 68, 063001 (2003)
[18] G. M. Fuller, R. Mayler, B. S. Meyer, J. R. Wilson, Astrophys. J., 389, 517 (1992)
[19] O. L. Caballero, G. C. McLaughlin and R. Surman, Phys. Rev. D. 80, 123004 (2009)
[20] O. Just, A. Bauswein, R. Ardevol Pulipilo, S. Goriely, and H.-T. Janka. R. Astron. Soc. 448, 541-567 (2015)
[21] Artemova I. V., Bjørnsson G., Novikov I. D., 1996. Astrophys.J.,461,565
[22] J. M. Bardeen, W. H. Press and S. A. Teukolsky, Astrophys. J. 178, 347 (1972).