Bounds on the compactification scale of two universal extra dimensions from exclusive $b \rightarrow s\gamma$ decays

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Abstract  
The exclusive radiative $B \rightarrow K^{*}\gamma$, $B \rightarrow K_{S}^{*}\gamma$, $B_{s} \rightarrow \phi\gamma$ and $\Lambda_{b} \rightarrow \Lambda\gamma$ decays are studied in a new physics scenario with two universal extra dimensions compactified on a chiral square. The computed branching fractions depend on the size $R$ of the extra dimensions, and a comparison with the available measurements allows us to put bounds on such a fundamental parameter. From the mode $B^{0} \rightarrow K^{*0}\gamma$ we obtain the most stringent bound: $\frac{1}{R} > 710 \text{ GeV}$.

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Among the various new physics (NP) scenarios proposed to extend the standard model (SM), those with extra dimensions (ED) are particularly interesting, as they might be able to solve some of the problems affecting the SM without invoking the existence of new interactions, changing the geometry of the space-time. Indeed, although the attempt of T. Kaluza and O. Klein, as well as of G. Nordström, to unify the electromagnetism with gravity introducing one extra dimension was unsuccessful [1], the idea of the possible existence of additional space-like dimensions resulted to be fruitful, and has been pursued for sectors of fundamental interactions with the formulation of different settings involving ED. Such models can provide a unified framework for gravity and other interactions, hints on the hierarchy problem, connections with the string theory, interesting dark matter phenomenology and, remarkably, they can be at the origin of observable phenomena at the colliders and at the flavour factories [2]. A common feature is that the compactification of the extra dimensions implies the existence of Kaluza-Klein (KK) partners of SM fields in the four-dimensional description of the higher dimensional theory, together with KK modes without SM correspondents. Models in which all the SM fields propagate in the extra dimensions are usually denoted as universal extra dimension (UED) scenarios.

A simple scenario is the Appelquist, Cheng and Dobrescu (ACD) model [3], a minimal extension of SM in $4+1$ dimensions, with the extra dimension compactified to the orbifold $S^1/Z_2$ and the fifth coordinate $y$ running from 0 to $2\pi R$, $y = 0$ and $y = \pi R$ being fixed points of the orbifold. In this model the SM particles correspond to the zero modes of fields propagating in the compactified extra dimension, and the choice of the extra dimension topology is dictated by the need of having chiral fermion zero modes. In addition to the zero modes, towers of KK excitations are predicted to exist, corresponding to the heavier modes of the fields in the extra dimension; such fields are imposed to be even under a parity transformation in the fifth coordinate $P_5: y \rightarrow -y$. On the other hand, fields which are odd under $P_5$ propagate in the extra dimension without zero modes, and correspond to particles without SM partners. The masses of KK particles depend on the radius $R$ of the extra dimension orbifold, the only new parameter with respect to SM. For example, the masses of KK bosonic modes are given by $m_n^2 = m_0^2 + \frac{n^2}{R^2}$, $n = 1, 2, \ldots$, with $m_0$ the mass of the zero mode, so that for small values of $R$, i.e. at large compactification scales, the KK particles decouple from the low energy sector. Another property of the ACD model is the conservation of the KK parity $(-1)^j$, $j$ being the KK number. This prevents tree level contributions of Kaluza-Klein states to low energy ($\mu \ll 1/R$) processes, forbidding the production of a single KK particle off the interaction of standard particles. As a consequence, precise electroweak measurements can be used to derive a lower bound to the compactification scale: $1/R \geq 250 - 300$ GeV [4]. Moreover, this leads to the possibility that the lightest KK particles, for instance the $n = 1$ Kaluza-Klein excitations of the photon and of the neutrinos, are among the dark matter components [5, 6]. KK modes could be produced at colliders, or indirectly detected through investigations of loop-induced processes getting contributions from KK modes. This possibility has been explored considering Flavour Changing Neutral Current (FCNC) transitions in the ACD scenario [7, 9, 10], with a stringent constraint on $1/R$ obtained from the exclusive radiative $B \rightarrow K^* \gamma$ decay [8] and from the inclusive $B \rightarrow X_s \gamma$ mode [11].

Models with universal extra dimensions may have a larger number of additional dimensions, $d \geq 2$, with further theoretical motivations than for $d = 1$. For instance, a model with two compactified UEDs allows to cancel the global SU(2) anomaly with three generations [12]; the compactification of two universal extra dimensions on the so-called “chiral square”
induces the suppression of the proton decay rate even with maximal violation of the baryon number at the TeV scale, as a result of the preservation of a discrete symmetry (which is the generator of a subgroup of the six dimensional Lorentz group) [13]. This is the model proposed in [12] and considered here: we denote it as 6UED. The two extra dimensions are flat and compactified on a square of side $L$: $0 \leq x^4, x^5 \leq L$, where $x^4$ and $x^5$ are the fifth and sixth extra spatial coordinates. The compactification is performed identifying two pairs of adjacent sides of the square: $(y, 0) = (0, y)$ and $(y, L) = (L, y)$, for all $y \in [0, L]$, which amounts to folding the square along a diagonal. The fields are decomposed in Fourier modes, in terms of effective four dimensional fields labeled by two indices $(l, k)$. Hence, the KK modes are identified by two KK numbers which determine their mass in four dimensions: zero modes correspond to SM fields.

The values of the fields in the points identified through the folding are related by a symmetry transformation. For instance, for a scalar field, the field values may differ by a phase. The choice of the folding boundary conditions (and of the constraints on such phases) is mostly important in the case of fermions, since a suitable choice allows to obtain chiral zero modes, while higher KK modes have masses determined (as for scalars) by the relation:

$$M_{l,k} = \frac{\sqrt{l^2 + k^2}}{R},$$

where $R = L/\pi$ is now the compactification radius, with $l$ and $k$ integer numbers. The theory has an additional symmetry, the invariance under reflection with respect to the center of the square, which distinguishes between the various KK excitations of a given particle. A KK mode identified by the pair $(l, k)$ of indices changes sign under reflection if $l + k$ is odd, while it remains invariant if $l + k$ is even. As a consequence, stability of the lightest KK modes is guaranteed, so that such are good candidates for dark matter constituents.

The model describes SM particles, their KK excitations, and new particles without a SM correspondent from fields whose Fourier decomposition does not contain a zero mode. As an example, two scalar fields originate from the mixing of the fourth and the fifth component of the vector gauge bosons. Then, at each KK level, a linear combination due to a further mixing of these two fields with the Higgs field is promoted to a pseudo-Goldstone mode reabsorbed to give mass to the spin-1 KK mode; another combination named the “spinless adjoint” [14] remains as a physical real scalar field. The consequence is an interesting phenomenology which manifests itself in the production cross sections and decay modes of other KK states, which differ from the $d = 1$ case: as an example, the $(1,1)$ states with mass $\sqrt{2}/R$, can be produced in the $s$–channel [12] [15] [16]. The phenomenology of KK excitations has been investigated in view of collider searches [17] [18] as well as for the cosmological and dark matter implications [19] [20].

The new particles may contribute as virtual states to FCNC processes, and actually from the inclusive $B \rightarrow X_s \gamma$ decay the bound $\frac{1}{R} \geq 650$ GeV (at 95% C.L.) has been obtained [21]. The analysis can be extended to the exclusive $b \rightarrow s \gamma$ induced modes, prompted by the example of a single UED in which such processes turned out to be the effective in constraining the compactification scale. Here we focus on the transitions $B \rightarrow K^* \gamma$, $B \rightarrow K_2^0(1430) \gamma$, $B_s \rightarrow \phi \gamma$ and $\Lambda_b \rightarrow \Lambda \gamma$. For some of these processes, precise experimental data are available and can be exploited to put bounds on $1/R$. The modes that have not been observed, yet, can be explored at the CERN LHC and at the new planned flavour factories.

In SM the effective weak Hamiltonian describing the $b \rightarrow s \gamma$ and $b \rightarrow s g$ gluon transition
can be written as \[22\]:

\[
H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left( \sum_{i=1}^{6} C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8G} O_{8G} \right).
\]  

\(G_F\) is the Fermi constant and \(V_{ij}\) are elements of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix; terms proportional to \(V_{ub} V_{us}^*\) are neglected in \([11]\) since the ratio \(\frac{V_{ub} V_{us}^*}{V_{tb} V_{ts}^*}\) is \(O(10^{-2})\). \(C_i\) are Wilson coefficients, and \(O_i\) are local operators written in terms of quark and gluon fields:

\[
\begin{align*}
O_1 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha})(\bar{c}_{L\beta} \gamma_\mu c_{L\beta}) \\
O_2 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha})(\bar{c}_{L\beta} \gamma_\mu c_{L\alpha}) \\
O_3 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha})[(\bar{u}_{L\beta} \gamma_\mu u_{L\beta}) + ... + (\bar{b}_{L\beta} \gamma_\mu b_{L\beta})] \\
O_4 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\beta})[(\bar{u}_{L\beta} \gamma_\mu u_{L\alpha}) + ... + (\bar{b}_{L\beta} \gamma_\mu b_{L\alpha})] \\
O_5 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha})[(\bar{u}_{R\beta} \gamma_\mu u_{R\beta}) + ... + (\bar{b}_{R\beta} \gamma_\mu b_{R\beta})] \\
O_6 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\beta})[(\bar{u}_{R\beta} \gamma_\mu u_{R\alpha}) + ... + (\bar{b}_{R\beta} \gamma_\mu b_{R\alpha})] \\
O_{7\gamma} &= \frac{e}{16\pi^2} \left[ m_b (\bar{s}_{L\alpha} \sigma^{\mu\nu} b_{R\beta}) + m_s (\bar{s}_{R\alpha} \sigma^{\mu\nu} b_{L\beta}) \right] F_{\mu\nu} \\
O_{8G} &= \frac{g_s}{16\pi^2} m_b \left[ \bar{s}_{L\alpha} \sigma^{\mu\nu} \left( \frac{\lambda^a}{2} \right)_{\alpha\beta} b_{R\beta} \right] G^{a\mu\nu}.
\end{align*}
\]

\(\alpha, \beta\) are color indices, \(b_{R,L} = \frac{1 \pm \gamma_5}{2} b\), and \(\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]\); \(e\) and \(g_s\) are the electromagnetic and the strong coupling constant, respectively, \(m_b\) and \(m_s\) the beauty and the strange quark masses, \(F_{\mu\nu}\) in \(O_{7\gamma}\) and \(G^{a\mu\nu}\) in \(O_{8G}\) denote the electromagnetic and the gluonic field strength tensors, and \(\lambda^a\) are the Gell-Mann matrices.

The Wilson coefficients in \([1]\) have been computed at NNLO in SM \([23]\). The most relevant contribution to \(b \to s\gamma\) comes from the operator \(O_{7\gamma}\), a magnetic penguin specific of such a transition. The term proportional to \(m_s \) contributes much less than the one proportional to \(m_b\), the reason for which the emission of left-handed photons dominates over that of right handed ones in SM. Since the coefficient \(C_{7\gamma}\) depends on the regularization scheme, it is convenient to consider, at leading order, a combination that is regularization scheme independent \([24]\):

\[
C^{(0)\text{eff}}_{7\gamma}(\mu_b) = \frac{16}{23} C^{(0)}_{7\gamma}(\mu_W) + \frac{8}{3} \left( \eta^{\frac{14}{23}} - \eta^{\frac{15}{23}} \right) C^{(0)}_{8\gamma}(\mu_W) + C^{(0)}_2(\mu_W) \sum_{i=1}^8 h_i \eta^{a_i},
\]

where \(\eta = \frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)}\) and \(C^{(0)}_2(\mu_W) = 1\) (the superscript \((0)\) stays for leading log approximation), with

\[
\begin{align*}
& a_1 = \frac{14}{23}, & a_2 = \frac{16}{23}, & a_3 = \frac{6}{23}, & a_4 = -\frac{12}{23}, \\
& a_5 = 0.4086, & a_6 = -0.4230, & a_7 = -0.8994, & a_8 = 0.1456, \\
& h_1 = 2.2996, & h_2 = -1.0880, & h_3 = -\frac{3}{7}, & h_4 = -\frac{1}{14}, \\
& h_5 = -0.6494, & h_6 = -0.0380, & h_7 = -0.0185, & h_8 = -0.0057.
\end{align*}
\]
The ACD and the 6UED models belong to the class of minimal flavour violation models, therefore the only modification with respect to the SM consists in a different value of the Wilson coefficients in the effective weak Hamiltonian [1], without new operators. The explicit expression of $C_{ij}^{\text{eff}}$, in the case of two extra dimensions can be found in Ref. [21]. It should only be mentioned that the sums over the KK modes entering in the expression of the Wilson coefficients in the extra dimensional framework diverge logarithmically, and should be cut in correspondence of some values of $N_{KK} = l + k$, viewing this theory as an effective one valid up to a some higher scale. Following [21], the condition $N_{KK} \simeq 10$ can be chosen.

To consider the contribution of the effective weak vertex $O_{\gamma i}$ to the transitions of interest here we need the hadronic matrix elements

$$
<V(p', \eta) | \bar{s}_\mu q_\nu b | B(s)(p)> = \frac{i\epsilon_{\mu\nu\alpha\beta}}{2} \eta^{*\alpha} p'^\beta \frac{M_B}{2} T_{B(s)\rightarrow V}^B(q^2)
$$

$$
< V(p', \eta) | \bar{s}_\mu q_\nu \gamma_5 b | B(s)(p)> = \eta^{*\mu}(M_B^2 - m_s^2) - (\eta^* \cdot q)(p + p')_\mu \frac{M_B}{2} T_{B(s)\rightarrow V}^B(q^2)
$$

$$
+ (\eta^* \cdot q) \left[ q_\mu - \frac{q^2}{M_B^2} (p + p')_\mu \right] T_{B(s)\rightarrow V}^B(q^2)
$$

$V$ stays for $K^*$ and $K^*_2(1430)$ in the case of $B$ decays, and for $\phi(1020)$ in the case of $B_s$ decays; $q = p - p'$ is the photon momentum and $\eta$ the $V$ meson polarization vector. In the case of $K^*_2(1430)$, which is a spin 2 particle, the polarization vector is described by a two indices symmetric and traceless tensor, therefore in (6) $\eta^\alpha = \frac{\eta^{\alpha\beta} P^\beta}{M_B}$. Relations exist among the three form factors $T_i^{B\rightarrow V}$, $i = 1, 2, 3$, in particular the condition: $T_1^{B(s)\rightarrow V}(0) = T_2^{B(s)\rightarrow V}(0)$. Due to this relation, the rate of the processes $B(s)(p) \rightarrow V(p', \eta) \gamma(q, \epsilon)$ ($\epsilon$ the photon polarization vector) can be expressed in terms of a single hadronic parameter $T_1^{B(s)\rightarrow V}(0)$ for each channel:

$$
\Gamma(B(s) \rightarrow V \gamma) = \frac{C^2}{4\pi} \left[ T_1^{B(s)\rightarrow V}(0) \right]^2 \left( m_b^2 + m_s^2 \right) \frac{M_B^3}{M_{B(s)}} \left( 1 - \frac{M_V}{M_B} \right)^3,
$$

$$
\Gamma(B \rightarrow K^*_2 \gamma) = \frac{C^2}{32\pi} \left[ T_1^{B\rightarrow V K^*_2}(0) \right]^2 \left( m_b^2 + m_s^2 \right) \frac{M_B^5}{M_{K^*_2}} \left( 1 - \frac{M_V}{M_B} \right)^5,
$$

where $C = \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^{\ast} C_{ij}^{\text{eff}} \frac{e}{16\pi^2}$ and the first equation applies both to $B \rightarrow K^* \gamma$ and $B_s \rightarrow \phi \gamma$. In the numerical analysis we set the particle masses and lifetimes, as well as the CKM matrix elements to the PDG values; for the quark masses we use $m_b \approx 4.8$ GeV and $m_s \approx 0.130$ GeV [25]. For the form factors we use results obtained by QCD sum rules [25], in particular, for $B \rightarrow K^*$ we use the results obtained by three-point QCD sum rules [26], based on the short-distance expansion, which provide $T_1^{B\rightarrow K^*}(0) = 0.38 \pm 0.06$, and light-cone QCD sum rules (LCSR) [27], based on the light-cone expansion, which give $T_1^{B\rightarrow K^*}(0) = 0.333 \pm 0.028$. These values are larger than the lattice QCD result obtained in quenched approximation [28]. LCSR calculations of $B \rightarrow K^*_2(1430)$ and $B_s \rightarrow \phi$ form factors give: $T_1^{B\rightarrow K^*_2}(0) = 0.17 \pm 0.03 \pm 0.04$ [29] and $T_1^{B_s\rightarrow \phi}(0) = 0.349 \pm 0.033 \pm 0.04$ [27].

In the case of $\Lambda_b \rightarrow \Lambda \gamma$ we define the matrix elements

$$
<\Lambda(p', s') | \bar{s}_\mu q_\nu b | \Lambda_b(p, s)> = u_{\Lambda} \left[ f_1(q^2) \gamma_\mu + i f_2(q^2) \sigma_{\mu\nu} q^\nu + f_3(q^2) q_\mu \right] u_{\Lambda_b}
$$

(9)
Table I: PDG averages for the branching fractions of radiative $B$, $B_s$ and $\Lambda_{b}$ decay modes $^{34}$.

| mode                  | BR                  |
|-----------------------|---------------------|
| $B^+ \to K^{+} \gamma$ | $(42.1 \pm 1.8) \times 10^{-6}$ |
| $B^0 \to K^{0*} \gamma$ | $(43.3 \pm 1.5) \times 10^{-6}$ |
| $B^+ \to K^*_2(1430)^+ \gamma$ | $(14 \pm 4) \times 10^{-6}$ |
| $B^0 \to K^*_2(1430)^0 \gamma$ | $(12.4 \pm 2.4) \times 10^{-6}$ |
| $B_s \to \phi \gamma$ | $(5.7^{+2.2}_{-1.0}) \times 10^{-6}$ |
| $\Lambda_{b} \to \Lambda \gamma$ | $< 1.3 \times 10^{-3}$ (90% C.L.) |

$< \Lambda(p',s')|\bar{s}i\gamma_{\mu}q^\nu\gamma_5 b|\Lambda_{b}(p,s)> = \bar{u}_{\Lambda}[g^T_1(q^2)\gamma_\mu\gamma_5 + ig^T_2(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + g^T_3(q^2)q_{\nu}\gamma_5]u_{\Lambda_{b}}$ (10)

with $u_{\Lambda}$ and $u_{\Lambda_{b}}$ the $\Lambda$ and $\Lambda_{b}$ spinors; $s$ denotes the baryon spin. The determinations of the form factors in $^{9,10}$ are quite uncertain. However, it is possible to invoke heavy quark symmetries for the hadronic matrix elements between an initial spin=$\frac{1}{2}$ heavy baryon comprising a single heavy quark $Q$ and a final spin=$\frac{1}{2}$ light baryon; due to the heavy quark symmetries the number of independent form factors is two, since for $m_Q \to \infty$ and a generic Dirac matrix $\Gamma$ one can write $^{30}$

$< \Lambda(p',s')|\bar{s}G\beta|\Lambda_{b}(p,s)> = \bar{u}_{\Lambda}(p',s')\{F_1(p' \cdot v) + \gamma F_2(p' \cdot v)\}\Gamma u_{\Lambda_{b}}(v,s)$ (11)

where $v = \frac{p'}{M_{\Lambda_{b}}}$ is the $\Lambda_{b}$ four-velocity. The form factors $F_{1,2}$ depend on the invariant $p' \cdot v = \frac{M_{\Lambda_{b}}^2 + M_{\Lambda}^2 - q^2}{2M_{\Lambda_{b}}}$, but we refer to their $q^2$ dependence for convenience. Using Eq.(11) we can relate the form factors in Eqs.(9)-10 to the functions $F_{1,2}$ in Eq.(11),

\begin{align*}
   f_2^T &= g_2^T = F_1 + \frac{M_{\Lambda}}{M_{\Lambda_{b}}} F_2 \\
   f_1^T &= g_1^T = q^2 \frac{F_2}{M_{\Lambda_{b}}} \\
   f_3^T &= - \left(1 - \frac{M_{\Lambda}}{M_{\Lambda_{b}}} \right) F_2 \\
   g_3^T &= \left(1 + \frac{M_{\Lambda}}{M_{\Lambda_{b}}} \right) F_2,
\end{align*}

at momentum transfer close to the maximum value $q^2 \simeq q^2_{\text{max}} = (M_{\Lambda_{b}} - M_{\Lambda})^2$. We assume their validity to the whole phase space, introducing a model dependence in the predictions. The $\Lambda_{b} \to \Lambda \gamma$ decay width reads in terms of the form factors $F_1$ and $F_2$:

$$
\Gamma(\Lambda_{b} \to \Lambda \gamma) = \frac{C^2}{4\pi} \left( F_1(0) + F_2(0) \frac{M_{\Lambda}}{M_{\Lambda_{b}}} \right) (m_b^2 + m_s^2) M_{\Lambda_{b}}^3 \left(1 - \frac{M_{\Lambda}^2}{M_{\Lambda_{b}}^2} \right)^3.
$$

A determination of $F_1$ and $F_2$ has been obtained by three-point QCD sum rules in the $m_Q \to \infty$ limit $^{31}$. In the following we use $F_1$, $F_2$ worked out in $^{10}$ updating some of the parameters used in $^{31}$: $F_1(0) = 0.322 \pm 0.015$, and $F_2(0) = -0.054 \pm 0.020$ $^{10}$. Other determinations of the form factors in the transition $\Lambda_{b} \to \Lambda \gamma$ have been performed in $^{32}$.
Figure 1: Predicted branching fractions of $B^+ \rightarrow K^{*+}\gamma$ (upper panels) and $B^0 \rightarrow K^{*0}\gamma$ (lower panels) as a function of the compactification parameter $1/R$ (in units of GeV), using the form factor in [26] (left) and in [27] (right). The horizontal bands correspond to SM theoretical expectations with $1\sigma$ uncertainties (yellow [light]) and experimental measurements with $2\sigma$ uncertainties (blue [dark]) respectively.

The computed branching fractions are functions of $\frac{1}{R}$, through the dependence of the coefficient $C_7$ in the 6UED model. The results are depicted in figs.1, 2, where we have included the uncertainty on the form factor value at $q^2 = 0$, and a second uncertainty, intrinsic of the model, due to the choice of the matching scale in the calculation of $C_7$ and of the value of $N_{KK}$. These last uncertainty is discussed in [21], together with another one which comes from fixing two boundary couplings $h_{1,2}$ which are $\mathcal{O}(1)$ and enter in the expression of the masses of the Higgs fields in this model. Altogether these uncertainties do not exceed $+17\%$ $-8\%$ [21], and we include this range in our error on the branching fractions. The experimental data for the various branching ratios are collected in Table I and represent PDG averages [34]. For $B \rightarrow K^{*}\gamma$ modes, the results represent averages of BaBar [35], Belle [36] and CLEO [37] measurements. For $B \rightarrow K^{*0}\gamma$ the result is determined on the basis of the analysis in [38], while for $B_s \rightarrow \phi$ it stems from ref. [39]. The upper bound on the $\mathcal{B}(\Lambda_b \rightarrow \Lambda\gamma)$ has been obtained in [41]. Experimental data are represented as horizontal blue [dark] bands (at 95% c.l.) in fig.1 and in fig.2 for $B^0 \rightarrow K^{0}\gamma$ (due the large experimental uncertainty, we only show the neutral mode where the error is smaller) and $B_s \rightarrow \phi\gamma$.

In the two upper plots in fig.1 $\mathcal{B}(B^+ \rightarrow K^{*+}\gamma)$ is computed using either the form factor $T_1$ in [26] (left panel) or that derived in [27] (right panel). The same applies to the two lower plots, where the neutral channel $B^0 \rightarrow K^{*0}\gamma$ is considered. There is a model dependence,
with the resulting bounds: $\frac{1}{R} \geq 397$ GeV (charged channel, form factors in [26]), $\frac{1}{R} \geq 564$ GeV (charged channel, form factors in [27]), $\frac{1}{R} \geq 433$ GeV (neutral channel, form factors in [26]), $\frac{1}{R} \geq 710$ GeV (neutral channel, form factors in [27]). As for $B^0 \rightarrow K_s^{0} \gamma$, we obtain $\frac{1}{R} \geq 324$ GeV. The other two plots in fig. 2 refer to $B_s$ and $\Lambda_b$ decays. In a previous analysis [40] also the decays $B \rightarrow K\eta(\bar{\eta})\gamma$ has been considered in order to constrain UEDs models (with 1 and 2 extra dimensions respectively), finding for 6UED a lower bound of $\frac{1}{R} \geq 400$ GeV.

![Graphs showing predicted branching fractions of $B \rightarrow K_s^{0} \gamma$, $B_s \rightarrow \phi \gamma$ and $\Lambda_b \rightarrow \Lambda \gamma$ as a function of the compactification parameter $1/R$ (in units of GeV). The SM results (with 1 $\sigma$ uncertainties) and experimental measurements (with 2 $\sigma$ uncertainties) are represented as horizontal yellow [light] and blue [dark] bands, respectively.](image)

Figure 2: Predicted branching fractions of $B \rightarrow K_s^{0} \gamma$, $B_s \rightarrow \phi \gamma$ and $\Lambda_b \rightarrow \Lambda \gamma$ as a function of the compactification parameter $1/R$ (in units of GeV). The SM results (with 1 $\sigma$ uncertainties) and experimental measurements (with 2 $\sigma$ uncertainties) are represented as horizontal yellow [light] and blue [dark] bands, respectively.

Bounds from direct searches of KK modes have been discussed in [17], where the hadron collider phenomenology of (1,0) KK modes in the 6UED model was studied. The limit $\frac{1}{R} \geq 270$ GeV was found, set from direct searches at the Tevatron. Our bound is more restrictive than the one obtained from the inclusive radiative $B$ transition [21], as in the case of a single UED. Since the discovery signals of UEDs at the CERN LHC or at a future $e^+e^-$ linear collider, searching for leptons plus missing energy, are expected with the experimental reach of about $1/R \simeq 1$ TeV [18, 42, 43], the complementarity of the studies of indirect effects of possible UEDs in flavour observables is noticeable.
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