Quantum criticality and correlations in the cuprate superconductors

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A description of the electronic correlations contained in the Hubbard model on the square-lattice perturbed by very weak three-dimensional uniaxial anisotropy in terms of the residual interactions of charge $c$ fermions and spin-neutral composite two-spinon $s_1$ fermions is used to access further information on the origin of quantum critical behavior in the hole-doped cuprate superconductors. Excellent quantitative agreement with their anisotropic linear-$\omega$ one-electron scattering rate and normal-state linear-$T$ resistivity is achieved. Our results provide strong evidence that the normal-state linear-$T$ resistivity is indeed a manifestation of low-temperature scale-invariant physics.

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The interplay between quantum critical behavior \cite{1-5} and the mechanism underlying the pairing state of the high-temperature superconductors \cite{6,7,9-11} perturbed by very weak three-dimensional uniaxial anisotropy provides the simplest realistic description of the role of correlations effects in the properties of the hole-doped cuprate superconductors. Recent experiments on these systems \cite{3,4,12-19} impose new severe constraints on the mechanisms responsible for their unusual properties.

The virtual-electron pair quantum liquid (VEPQL) \cite{11} describes the above toy model electronic correlations in terms of residual $c$ - $s_1$ fermion interactions. Alike the Fermi-liquid quasi-particle momenta \cite{20}, those of the $c$ and $s_1$ fermions are close to good quantum numbers \cite{10,11}. The results of Ref. \cite{11} provide evidence that for a hole concentration domain the VEPQL short-range spin order coexists with a long-range $d$-wave superconducting order consistent with unconventional superconductivity being mediated by magnetic fluctuations \cite{13}. The $U(1)$ phase symmetry broken below $T_c$ refers to the hidden $U(1)$ symmetry recently found in Ref. \cite{4}. Each virtual-electron pair configuration involves one $c$ fermion pair of charge $-2e$ and one spin-singlet two-spinon $s_1$ fermion whose spin-$1/2$ spinons are confined in it.

The magnitudes of the basic parameters appropriate to YBa$_2$Cu$_3$O$_{6+\delta}$ (YBCO 123) and La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) used in this Letter are within the VEPQL scheme the effective interaction and transfer integral ratio $U/4t \approx 1.525$ where $t \approx 295$ meV and $T = 0$ critical hole concentrations $x_c \approx 0.05$ and $x_s \approx 0.27$ for both such systems, lattice spacing $a \approx 3.9$ Å, average separation between CuO$_2$ planes $d_0 \approx 5.9$ Å, maximum $s_1$ fermion pairing energy per spinon $\Delta_0 \approx 84$ meV, and coefficient $C_{s_1} = 1$ for YBCO 123 and $a \approx 3.8$ Å, $d_0 \approx 6.6$ Å, $\Delta_0 \approx 42$ meV, and $C_{s_1} = 2$ for LSCO \cite{11}. The VEPQL predictions achieve a good agreement with the cuprates universal properties \cite{11} and those of their parent compounds \cite{10} and consistency with the coexisting two-gap scenario \cite{12}: A pseudogap $2|\Delta| \approx (1-x_s)x_s2\Delta_0$ and superconducting energy scale $2|\Omega| \approx 4k_BT_c/(1-[x_c/x_s](T_c/T_{c\text{\,max}}))$ over the whole dome $x \in (x_c,x_s)$, where $T_c \approx \gamma_d [(x-x_c)/(x_s-x_c)](1-x/x_s)[\Delta_0/2k_B]$ and $(1-x_c/x_s) \geq \gamma_d \geq 1$. Those are the maximum magnitudes of the spinon pairing energy and superconducting virtual-electron pairing energy, respectively.

In the ground state there is no one-to-one correspondence between a $c$ fermion pair and a two-spinon $s_1$ fermion in that such objects may participate in several virtual-electron pairs. Specifically, the strong effective coupling of $c$ fermion pairs whose hole momenta $\vec{q}^h$ and $-\vec{q}^h$ belong to an approximately circular $c$ – $s_c$ line centered at $-\pi = [-\pi,\pi]$ results from interactions within virtual-electron pair configurations with a well-defined set of $s_1$ fermions whose two spinons momenta $\pm \vec{q}^f$ belong to a uniquely defined $s_1$ – $s_c$ line arc centered at $0 = [0,0]$. A $c$ – $s_c$ line has radius $q^{hf} = |\vec{q}^h| \in (q^{f}_{c\,ec},q^{f}_{ec})$ and $c$ fermion energy $\epsilon_c(q^{hf}) \in (0,W_{ec})$ such that $\epsilon_c(q^{f}_{c\,ec}) = 0$ and $\epsilon_c(q^{f}_{ec}) = W_{ec} = 4\Delta_0/(1-x_c/x_s)$. For Fermi angles $\phi \in (0,\pi/2)$ the corresponding $s_1$ – $s_c$ line arc can be labelled either by its nodal momentum absolute value $q^{N}_{arc} = q^{N}_{ec}(\phi) \in (q^{N}_{ec},q^{N}_{B_{s1}})$ or angular width $2\phi_{arc} = \arcsin[(q^{N}_{arc}-q^{f}_{c\,ec})/(q^{N}_{B_{s1}}-q^{f}_{ec})](\phi \in (0,\pi/2)$. Here and above $q^{N}_{B_{s1}} \approx q^{N}_{B_{s1}} - [\Delta_0/|T_{c\,max}|/(C_{s_1}/|x_s-x_c|)]^{1/2}$, $q^{f}_{ec} \approx (\sqrt{1+\Delta_0/(x_s-x_c)\pi^2})q^{f}_{ec}$, $q^{N}_{B_{s1}}$ and $q^{N}_{ec}$ are...
nodal and anti-nodal momentum absolute values, respectively, belonging to the strongly anisotropic s1 band boundary-line centered at \( \theta = 10 \), and \( q_{Fc}^s \approx \sqrt{\pi}2 \) refers to the isotropic c Fermi line centered at \(-\pi\). The energy needed for the c fermion strong effective coupling is supplied by the short-range spin correlations through the c - s1 fermion interactions within each virtual-electron pair configuration. Strong c fermion effective coupling is that whose corresponding virtual-electron pair breaking under one-electron removal excitations gives rise to sharp-feature-line arcs centered at momenta \( \pm \pi = \pm [\pi, \pi] \). Those are in one-to-one correspondence to the s1 - sc-line arcs of the virtual-electron pair s1 fermion. Such sharp-feature-line arcs have angular range \( \phi \in (\pi/4 - \phi_{arc}, \pi/4 + \phi_{arc}) \) and energy \( E \approx 2W_{sc}(1 - \sin 2\phi_{arc}) \). Hence they exist only for \( E < E_1(\phi) = 2W_{sc}(1 - |\cos 2\phi|) \). The macroscopic condensate refers to c fermion pairs whose phases \( \theta = \theta_0 + \theta_1 \) line up. The fluctuations of \( \theta_0 \) and \( \theta_1 \) become large for \( x \rightarrow x_c \) and \( x \rightarrow x_{s1} \), respectively. The dome \( E \) dependence of the critical temperature \( T_c \) is fully determined by the interplay of such fluctuations. A pseudogap state with short-range spin order and virtual-electron pair configurations without phase coherence occurs for temperatures \( T \in (T_c, T^*) \) where \( T^* \approx C_s(1 - x/x_1)|\Delta_0/2k_B| \) is the pseudogap temperature. At \( T = 0 \) a normal state emerges by application of a magnetic field aligned perpendicular to the planes of magnitude \( H \in (H_0, H_{c2}) \) for \( x \in (x_0, x_{c2}) \) and \( H \in (H_0, H^*) \) for \( x \in (x_{c2}, x_1) \). The fields \( H_0, H_{c2}, \) and \( H^* \) and the hole concentration \( x_0 < x_c \) are given in Ref. [11]. For \( x \in (x_0, x_{c1}) \) the upper magnetic field \( H_{c2}(x) \) refers to the straight line plotted in Fig. 4 of that reference where \( x \approx 0.013 \) and \( x_{c1} = 1/8 \). However, for \( x \in (x_1, x_{c2}) \) the actual \( H_{c2}(x) \) line may (or may not) slightly deviate to below the straight line plotted in that figure. If so, the hole concentration \( x_{c2} \approx 0.20 \) may increase to \( \approx 0.21 - 0.22 \). Fortunately, such a possible deviation does not change the physics discussed here.

The main goals of this Letter are: i) The study of the one-electron scattering rate and normal-state \( T \)-dependent resistivity within the VEPQL; ii) Contributing to the further understanding of the role of scale-invariant physics in the unusual scattering properties of the hole-doped cuprates. Our results refer to a range \( x \in (x_A, x_{c2}) \) for which \( V_{B_{s1}}^{\Delta}/V_{Fc} \ll 1 \). Here \( x_A \approx x_{c1}/2 = 0.135 \) and the s1 boundary line and c fermion velocities read \( V_{Fc} \equiv V_c(q_{Fc}^s) \approx |\sqrt{\pi}2/m^*_c| \) and \( V_{B_{s1}}^{\Delta} \equiv V_{s1}^{\Delta}(q_{B_{s1}}^d) \approx |\Delta/\sqrt{2}| \sin 2\phi_1 \), respectively, where \( m^*_c \) is the c fermion mass. For \( x \in (x_{c2}, x_s) \) that inequality is also fulfilled but there emerge competing scattering processes difficult to describe in terms of c - s1 fermion interactions. Elsewhere it is shown that the VEPQL predictions agree quantitatively with the distribution of the LSCO sharp photoemission spectral features of Figs. 3 and 4 of Ref. [18]. As predicted, they occur for energies \( E(\phi) < E_1(\phi) \) and the corresponding sharp-feature line arcs angular ranges agree with the theoretical magnitudes. This reveals experimental spectral signatures of the VEPQL virtual-electron pairing mechanism.

Here we start by using a Fermi’s golden rule in terms of the c - s1 fermion interactions to calculate for small \( \hbar \omega \) the one-electron inverse lifetime. Upon removal of one electron, two holes emerge in the s1 and c bands, respectively. For low transfer energy \( \hbar \omega \) and small transfer momentum \( \tilde{p} \) the c - s1 fermion inelastic collisions conserve the doublility \( d = \pm 1 \), which refers to one-electron excited states with the same energy and momentum but different electron velocity [16] [11]. Within such processes one s1 fermion moves to the single hole in the s1 band. One must then integrate over all particle-hole or hole-particle processes in the c fermion band that conserve energy and momentum. For low \( \hbar \omega \) and small \( \tilde{p} \) the one-electron inverse lifetime can then be written as,

$$
\frac{\hbar}{\tau_{c1,d}} = 2\pi \int \frac{dq \hbar^2}{2\pi^2} |W_{c,s1}(q^h, q_{B_{s1}}^d, \tilde{p})|^2 N_c(q^h)N_c^2(q^h + \tilde{p})N_{s1}(q_{B_{s1}}^d - \tilde{p}) \delta(e_c(q^h + \tilde{p}) - e_c(q^h) + e_s(q_{B_{s1}}^d - \tilde{p}) - e_{s1}(q_{B_{s1}}^d))
$$

where \( d = \pm 1 \). The number of c fermions equals that of spin-up plus spin-down electrons, so that there is no additional factor 2 in this expression. The c and s1 fermion energy dispersions and momentum distribution functions appearing here are introduced in Refs. [10] [11] and \( W_{c,s1}(q^h, q_{B_{s1}}^d, \tilde{p}) \) is the matrix element of the c - s1 fermion effective interaction between the initial and final states. It can be estimated for the hole concentration range \( x \in (x_A, x_{c2}) \) for which \( r_{c1} = V_{B_{s1}}^{\Delta}/V_{Fc} \ll 1 \). Indeed, then the single heavy s1 fermion hole plays mainly

$$
\frac{\hbar}{\tau_{c1,d}} = 2\pi \int \frac{dq \hbar^2}{2\pi^2} |W_{c,s1}(q^h, q_{B_{s1}}^d, \tilde{p})|^2 N_c(q^h)N_c^2(q^h + \tilde{p})N_{s1}(q_{B_{s1}}^d - \tilde{p}) \delta(e_c(q^h + \tilde{p}) - e_c(q^h) + e_s(q_{B_{s1}}^d - \tilde{p}) - e_{s1}(q_{B_{s1}}^d))
$$

the role of a scattering center and the c fermion holes that of scatterers and one can evaluate the matrix element absolute value \( |W_{c,s1}(q^h, q_{B_{s1}}^d)| \) to zeroth order in \( V_{B_{s1}}^{\Delta}/V_{Fc} \ll 1 \). Provided that the small velocity \( V_{B_{s1}}^{\Delta} \) is accounted for in the physical quantities of expression 1 other than that matrix element, such a procedure leads to a good approximation for the one-electron inverse lifetime \( \hbar \omega \) dependence. For that \( x \) range we then find \( \lim_{\tilde{p} \to 0} |W_{c,s1}(q^h, q_{B_{s1}}^d)| \approx |\pi/4\rho_s(q^h)| \sin(\delta_1 - \delta_0) \) where \( q^h \approx q_{Fc}^s, \tilde{q} \approx q_{B_{s1}}^d, \) the angular-momentum
l = 0.1, phase shifts read \( \delta_0 = \pi/2 \) and \( \delta_1 = 2\phi \), respectively, and the density of states \( \rho_c(q^h) = m_0^* / 2\pi^2 \hbar^2 \) of the present effective two-dimensional c fermion scattering problem is independent of \( x \). Hence we arrive to \( \lim_{q \to 0} |W_{c,s}(q^h, \bar{q}; \bar{p})| \approx [\pi^2 \hbar^2 / 2m_0^*] \cos 2\phi \approx \pi^3 \alpha_{c,s}^* \cos 2\phi \). Fortunately, for \( x \in (x_A, x_{c2}) \) the quantities contributing to \( \rho_c \) are independent of the doublicity \( d = \pm 1 \), so that after some algebra we arrive to an inverse lifetime \( h / \tau_{c1} \approx \hbar \omega \pi \alpha_{c,s}^* \) and scattering rate \( \Gamma(\phi, \omega) = 1 / [\tau_{c1} V_F] \approx \hbar \omega \pi \alpha_0 \). Here \( V_F \approx V_{F_{c}} \approx \sqrt{\pi \alpha / 2m_0^*} \), \( \alpha_{c,s}^* = 2 \pi \alpha / (x \sqrt{x \pi k_{\text{F_{op}}}}) \) and \( x_{c0} = (x_0 + x_{c2}) / 2 = 0.16 \). (We use units of lattice constant \( a = 1 \).) Such small-\( \hbar \omega \) expressions are expected to remain valid for approximately \( \hbar \omega < E_1(\phi) \approx 2W_{ec}(1 - \cos 2\phi) \). The factor \( \cos 2\phi \) also appears in the anisotropic component of the scattering rate studied in Ref. \[19\] for hole concentrations \( x > x_{c2} \). For \( x = 0.145 \) the use of the LSCO parameters leads to the theoretical coefficient \( \alpha(\phi) \) plotted in Fig. 2 (solid line) together with the experimental points of Fig. 4 (c) of Ref. \[15\] for \( \alpha_{1}(\phi) - \alpha_1(\pi/4) \). (The very small \( \alpha_{1}(\pi/4) \) magnitude is related to processes that are not contained in the VEPQL.) An excellent quantitative agreement is obtained between \( \alpha(\phi) \) and the experimental points of \( \alpha_{1}(\phi) - \alpha_1(\pi/4) \).

In the following we provide strong evidence from agreement between theory and experiments that the linear-\( T \) resistivity is indeed a manifestation of normal-state scale-invariant physics. This requires that the \( T \)-dependence of the inverse relaxation lifetime derived for finite magnetic field, \( x \in (x_A, x_{c2}) \), and \( \omega \ll \pi k_B T \) by replacing \( \hbar \omega \) by \( \pi k_B T \) in the one-electron inverse lifetime \( 1 / \tau_{c1} \) and averaging over the Fermi line leads to the observed low-\( T \) resistivity. To access the low-\( T \) resistivity for the normal state a magnetic field perpendicular to the planes is applied, which remains unaltered down to \( T = 0 \), as in the cuprates \[3\]. The field serves merely to remove superconductivity and achieve the \( H \)-independent term \( \rho(T) \) of \( \rho(T, H) = \rho(T) + \delta \rho(T, H) \) where \( \delta \rho(T, H) \) is the magnetoresistance contribution. The \( T \)-dependent inverse relaxation lifetime derived by replacing \( \hbar \omega \) by \( \pi k_B T \) in the above one-electron inverse lifetime \( h / \tau_{c1} \approx \hbar \omega \pi \alpha_{c,s}^* \) and averaging over the Fermi line is given by,

\[
\frac{1}{\tau(T)} = \frac{2}{\pi} \left( \int_0^{\pi/2} \frac{d\phi}{\tau_{c1}} \right) \bigg|_{\hbar \omega = \pi k_B T} = \frac{1}{\hbar A} \pi k_B T, \\
A = \frac{32}{\pi^2} \sqrt{x x_{c0}}, \ x \in (x_A, x_{c2}).
\]

The hole concentration \( x_A \approx x_{c1}/2 \) is that at which \( A \approx 0.5 \) becomes of order one. The normal-state resistivity \( H \)-independent term \( \rho(T) \) of \( \rho(T, H) \) then reads,

\[
\rho(T) \approx \left( \frac{m_0^* d_\parallel}{2e^2} \right) \frac{1}{\tau(T)}; \ \ m_0^* = \frac{\hbar^2 \pi x_0^*}{2t_0}.
\]

Consistency with the above \( h / \tau_{c1} \) expression validity range \( \hbar \omega < E_1(\phi) \) implies that the behavior \( \rho_c \) remains dominant in the normal-state range \( T \in (0, T_1) \). Here,

\[
T_1 \approx 2 \int_0^{\pi/2} d\phi \frac{E_1(\phi)}{k_B} = \frac{(2\pi - 4)W_{ec}}{\pi^2 k_B}.
\]

At \( x = 0.16 \) this gives \( T_1 \approx 554 \) K for LSCO and \( T_1 \approx 1107 \) K for YBCO 123. Extrapolation of expression \( \rho_c \) to \( H = 0 \) leads to \( \rho(T, 0) \approx \hbar \omega (T - T_1) \rho(T) \) for \( T < T_1 \). Here the critical low-\( T \) resistivity behavior \( \rho_c \) is masked by the onset of superconductivity at \( T = T_c \).

We now compare our theoretical linear-\( T \) resistivity with that of LSCO \[21\] and YBCO 123 \[22\] for \( H = 0 \) and \( T \) up to 300 K. Transport in the \( b \) direction has for YBCO 123 contributions from the CuO chains, which render our results unsuitable. In turn, \( \rho_{bc}(T, 0) \approx \hbar \omega (T - T_c) \rho(T) \) at \( H = 0 \) for the \( c \) direction. \( \rho_{bc}(T, 0) \) and \( \rho_{a}(T, 0) \) are plotted in Figs. 2 and 3 for the parameters for LSCO and YBCO 123, respectively. \( x \) is between \( x \approx x_{c1} \approx 0.135 \) and \( x_{c2} \approx 0.20 - 22 \) for the LSCO theoretical lines of Fig. 2. Fig. 3 for YBCO 123 refers to three \( x \) values near \( x_{c0} \), expressed in terms of the oxygen content. Comparison of the theoretical curves of Fig. 2 with the LSCO resistivity curves of Ref. \[21\] also shown in the figure confirms an excellent quantitative agreement between theory and experiments for the present range \( x \in (x_A, x_{c2}) \). In turn, for YBCO 123 our scheme provides a good quantitative description of the experimental curves near \( x_{c0} \), for \( y = 6.95, 7.00 \). The \( y = 6.85 \) experimental curve of Ref. \[22\] already deviates from the linear-\( T \) behavior. (The hole concentration that marks the onset of such a behavior reads for that material \( x_A \approx 0.15 \).)

For the present range \( x \in (x_A, x_{c2}) \), the interplay of the \( c \) Fermi line isotropy with the \( s1 \) boundary line

![FIG. 2: (a) The \( T \) dependence of the resistivity \( \rho(T, 0) = \theta(T - T_c) \rho(T) \) with \( \rho(T) \) given in Eq. (4) for \( x \in (x_A, x_{c2}) \) where \( x_A \approx 0.135 \) and \( x_{c2} \approx 0.20 - 22 \) and the parameter magnitudes for LSCO. (b) Corresponding experimental curves. Experimental curves figure from Ref. \[21\].](image)
strong anisotropy plays an important role. It is behind the -s1 fermion inelastic collisions leading to anisotropic one-electron scattering properties associated with the factor $(\cos 2\phi)^2$ in the one-electron scattering rate expression. In turn, consistently with the experimental resistivity curves of Figs. 2 and 3 the non-linear $T$ dependence of the resistivity developing for approximately $x < x_A$ for a range of low temperatures that increases upon decreasing $x$ is in part due to the matrix element $W_{c,1}(\tilde{q}^h, \tilde{q}^c; \tilde{p})$ acquiring a different form due to the increase of the ratio $\tau_\Delta = V_{B}\tau_0/V_{F\tau}$ magnitude. Our method does not apply to that regime. On the other hand, for the range $x > x_{c2}$ also not considered here a competing scattering channel emerges, leading to an additional $T^2$-quadratic resistivity contribution.

That the dependence on the Fermi angle $\phi \in (0, \pi/2)$ of the scattering-rate coefficient $\alpha = \alpha_{c,1}/V_{F\tau}$ is $(\cos 2\phi)^2/(x \Delta x_\phi \sqrt{2\pi})$ associated with that of the inverse lifetime $h/\tau_\Delta \approx h_\omega \pi \alpha_{c,1}$ is consistent with the experimental points of Fig. 1 and the evidence provided in Refs. 1, 2, 10, 11 that the VEPQL may contain some of the main mechanisms behind the unusual properties of the hole-doped cuprates and their parent compounds. That in addition the inverse relaxation lifetime $1/\tau$ $T$-dependence obtained for $h\omega \ll \pi k_B T$ and $x \in (x_A, x_{c2})$ by merely replacing $h\omega$ by $\pi k_B T$ in $1/\tau$ $T$ and averaging over the Fermi line leads to astonishing quantitative agreement with the resistivity experimental lines is a stronger surprising result. Consistently, for $h\omega \ll \pi k_B T$ and $x \in (x_A, x_{c2})$ the system exhibits dynamics characterized by the relaxation time $\tau = \hbar A/\pi k_B T$ of Eq. 2, where $A = \sqrt{2\pi}/(x \Delta x_\phi \sqrt{2\pi}) \approx 1$ for $x > x_A$. This second stronger result provides clear evidence of normal-state scale-invariant physics. It may follow from beyond-mean-field theory the $T = 0$ line $H_{c2}(x)$, plotted in Fig. 4 of Ref. 11 for $x \in (x_A, x_{c2})$, referring to a true quantum phase transition. Such a transition could occur between a state with long-range spin order regulated by monopoles and antimonopoles for $H > H_{c2}$ and a vortex liquid by vortices and antivortices for $H < H_{c2}$. That would be a generalization for $H_{c2} > 0$ of the quantum phase transition speculated to occur at $x_{c2} \approx x_0$ and $H_{c2} \approx 0$ in Ref. 8. The field $H_{0}$ marks a change from a short-range spin order to a disordered state and thus refers to a crossover. Hence the normal-state scale invariance occurring for $x \in (x_A, x_{c2})$ could result from the hole concentration $x_{c2} \approx 0.2$, where the lines of Fig. 4 of Ref. 11 associated with the fields $H_{B}(x)$ and $H^*(x)$ meet, referring to a quantum critical point [1, 2]. Such a quantum critical point may prevent the competing $T^2$-quadratic resistivity contribution to strengthen below $x = x_{c2}$.

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[1] C. Panagopoulos et al., Phys. Rev. B 66, 064501 (2002).
[2] D. van der Marel et al., Nature 425, 271 (2003).
[3] R. Daou et al. Nature Phys. 5, 31 (2009).
[4] R. A. Cooper et al., Science 323, 603 (2009).
[5] R. Jaramillo et al., Nature 459, 405 (2009).
[6] A. Damascelli, Z. Hussain, and X. Z. Shen, Rev. Mod. Phys. 75, 473 (2003).
[7] F. A. Lee, N. Nagaosa, and X. G. Wen, Rev. Mod. Phys. 78, 17 (2006).
[8] Z. Tešanović, Nature Phys. 4, 408 (2008).
[9] J. M. P. Carmelo, Stellan Östlund, and M. J. Sampaio, Annals Phys. (2010), doi: 10.1016/j.aop.2010.03.002.
[10] J. M. P. Carmelo, Nucl. Phys. B 824, 452 (2010).
[11] J. M. P. Carmelo, arXiv:1004.0923 (2010).
[12] S. Hüfner, M. A. Hussain, A. Damascelli, and G. A. Sawatzky, Rep. Prog. Phys. 71, 062501 (2008).
[13] G. Yu, Y. Li, E. M. Motoyama, and M. Greven, Nature Phys. 5, 873 (2009).
[14] Y. Kohsaka et al., Nature Phys. 5, 642 (2009).
[15] J. Chang et al., Phys. Rev. Lett. 105, 064501 (2002).
[16] M. C. Boyer et al., Nature Phys. 3, 802 (2007).
[17] L. Li, J. G. Checkelsky, S. Komiya, Y. Ando, and N. P. Ong, Nature Phys. 3, 311 (2007).
[18] J. Chang et al., Phys. Rev. B 75, 224508 (2007).
[19] M. Abdel-Jawad et al., Nature Phys. 2, 821 (2006).
[20] D. Pines and P. Nozieres, in The theory of quantum liquids (Benjamin, New York, 1996) Volume 1.
[21] S. Komiya, H.-D. Chen, S. C. Zhang, and Y. Ando, Phys. Rev. Lett. 94, 207004 (2005).
[22] K. Segawa and Y. Ando, Phys. Rev. Lett. 86, 4907 (2001).
[23] R. Liang, D. A. Bonn, and W. N. Hardy, Phys. Rev. B 73, 180505 (2006).