The 2D-LC-ESPRIT algorithm based on Massive MIMO

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Abstract. This paper proposes a 2D-LC-ESPRIT localization algorithm based on Massive MIMO, aiming at the problem of high computational complexity caused by positioning accuracy caused by abundant antenna array resources. Firstly, the antenna matrix is partitioned, then the received signals are superposition and dimensioned. And then the eigenvalues and corresponding eigenvectors are obtained by solving the cross-covariance matrix. Finally, simulation results show that the performance of the proposed algorithm is improved compared with the traditional two-dimension ESPRIT algorithm.

1. Introduction

As one of the key technologies of 5G [1-3], massive multiple input multiple output (MIMO) has attracted extensive attention of scholars at home and abroad. The advantages of massive MIMO technology are mainly reflected in high frequency band utilization, large system capacity and low signal interference [4-5]. Clever use of the large-scale antenna array in Massive MIMO can achieve precise positioning of users, and then provide them with many services brought by location information, such as power control of mobile terminals, cell switching and so on. Jiang et al. proposed a beam space sample covariance matrix (BSCM) correction algorithm in [7] that considers residual components. This algorithm transforms the spectral peak search in the literature [6] into a method of solving polynomials, which reduces the computational complexity of the Music algorithm to a certain extent.

Based on the above research, it is not difficult to find that large-scale antenna arrays also bring higher computational complexity. To solve this problem, the article proposes a low-complexity positioning algorithm (2D-LC-ESPRIT) based on Massive MIMO.

2. 2D-LC-ESPRIT positioning algorithm based on Massive MIMO

2.1. System model

The low-complexity positioning model based on Massive MIMO is shown in Figure 1. From this figure, we know that the horizontal angle of the incident signal is \( \beta \), and the pitch angle is \( \alpha \). The article adopts the propagation model method to offset the influence of NLOS on positioning accuracy [5].
Figure.1  Localization model based on massive MIMO

Based on the existing research, the article also assumes that the rectangular antenna array are located in the \(XOY\) plane, and the reference antenna element is selected as the origin. The position coordinate is \((0,0,0)\), then the coordinates of the \((m,n)\)th element is \((x_m,y_n,0)\). The interval between two adjacent array elements is \(d\). So we can get \(x_m=(m-1)d\), \(y_n=(n-1)d\). The article assumes that the signal is a plane wave and its propagation speed is the speed of light \(c\). A total of \(K\) signals arrive at the array, and the \(K\) signals are independent of each other and not related to noise. The two-dimensional angle of arrival with the \(K\)th signal can be expressed as \((\alpha_k,\beta_k)\), then the time delay between the \(K\)th signal and the reference array element is

\[
\tau = \frac{\Delta d_{m,n,k}}{c} = \frac{x_m \cos \beta_k \sin \alpha_k + y_n \sin \beta_k \cos \alpha_k}{c}
\]

According to the above delay difference, the phase difference between the \(K\)th signal and the reference array element can be expressed as

\[
\phi_{m,n,k} = \frac{2\pi \Delta d}{\lambda}((m-1)\cos \beta_k \sin \alpha_k + (n-1)\sin \beta_k \cos \alpha_k)
\]

Where \(\lambda\) is the wavelength. \(k = 1,2,3,\ldots,K\).

The steering vector of the \(k\)th signal antenna array can be expressed as

\[
\alpha_k(\alpha_k,\beta_k) = [\exp(-j\phi_{1,k}) \ \exp(-j\phi_{1,2,k}) \ \cdots \ \exp(-j\phi_{M_k,M_k,k})]
\]

According to the steering vector of the \(k\)-th signal antenna array, the steering vector matrix of the Massive MIMO antenna array can be obtained as

\[
A = \begin{bmatrix}
\exp(-j\phi_{1,1}) & \exp(-j\phi_{1,2}) & \cdots & \exp(-j\phi_{1,K}) \\
\exp(-j\phi_{1,1}) & \exp(-j\phi_{1,2}) & \cdots & \exp(-j\phi_{1,K}) \\
\vdots & \vdots & \ddots & \vdots \\
\exp(-j\phi_{M_k,1}) & \exp(-j\phi_{M_k,2}) & \cdots & \exp(-j\phi_{M_k,K})
\end{bmatrix}
\]

Based on equation (4), the vector expression of the received signal from the Massive MIMO antenna array can be obtained as follows

\[
x(t) = As(t) + n(t)
\]

Among them, \(x(t)\) is the vector matrix of the signal received by the antenna array, \(s(t)\) is the arrival signal vector matrix, and \(n(t)\) is the noise vector matrix.

2.2. Principle of 2D-LC-ESPRIT positioning algorithm based on Massive MIMO

The algorithm proposed in the article uses the uniform rectangular array model assumed in Section 2.1, assuming that the number of elements on the X axis is greater than the number of elements on the Y axis. Taking the X axis as an example, the matrix is divided into blocks, as shown in Figure 2. It is not difficult to find that both sub-array \(2N_x - 1\) and sub-array \(2N_y\) contain \(N - 1\) array elements, and sub-array
2N_y is obtained by shifting the sub-array 2N_y – 1 one array element to the right along the X axis.

![Figure 2: Schematic diagram of X-axis block of uniform rectangular array](image)

According to the research in [9], The article selects sub-array 1 as the reference array, and in the same way, it can be obtained that there is rotation invariance between other sub-arrays and array 1. The relationship can be expressed as follows

\[ X_1(t) = Cs(t) + n_1(t) \]
\[ X_2(t) = CFs(t) + n_2(t) \]
\[ \vdots \]
\[ X_{2N_y}(t) = CG^{N_y-1}s(t) + n_{2N_y}(t) \]
\[ X_{2N_y}(t) = CG^{N_y-1}Fs(t) + n_{2N_y}(t) \]

\[ F = diag\{exp(-j2\pi d \cos \beta_1 \sin \alpha_1 / \lambda), exp(-j2\pi d \cos \beta_2 \sin \alpha_2 / \lambda), \cdots, \]
\[ exp(-j2\pi d \cos \beta_k \sin \alpha_k / \lambda)\} \]
\[ G = diag\{exp(-j2\pi d \sin \beta_1 \sin \alpha_1 / \lambda), exp(-j2\pi d \sin \beta_2 \sin \alpha_2 / \lambda), \cdots, \]
\[ exp(-j2\pi d \sin \beta_k \sin \alpha_k / \lambda)\} \]

\[ C = \begin{bmatrix}
    \exp(-j\phi_{1,1,1}) & \exp(-j\phi_{1,1,2}) & \cdots & \exp(-j\phi_{1,1,k}) \\
    \exp(-j\phi_{1,2,1}) & \exp(-j\phi_{1,2,2}) & \cdots & \exp(-j\phi_{1,2,k}) \\
    \vdots & \vdots & \ddots & \vdots \\
    \exp(-j\phi_{N_y-1,1,1}) & \exp(-j\phi_{N_y-1,1,2}) & \cdots & \exp(-j\phi_{N_y-1,1,k})
\end{bmatrix} \]

According to the same method, you can calculate the result after dividing the matrix in the Y-axis direction. This article synthesizes the above formula, and the uniform rectangular array in Figure 2 is equivalent to the L matrix shown in Figure 3. The article further constructs the direction of arrival matrix by solving the cross-correlation matrix of the axis direction and the axis direction. Then the two-dimensional angle of arrival pairing is automatically realized by the method of eigenvalue decomposition of the direction of arrival matrix.

![Figure 3: Schematic diagram of equivalent L matrix](image)
The article uses the method that the uniform rectangular array in Figure 2 is equivalent to the L matrix shown in Figure 3, reduces the original two-dimensional array to an L-shaped matrix, and then performs covariance matrix estimation, eigenvalue decomposition and inversion operation. Therefore, the computational complexity becomes $O(N_y)$. At the same time, the traditional positioning algorithm needs to consider the number of elements $N_x$ on the X axis and the number of elements $N_y$ on the Y axis at the same time, so its computational complexity is $O((N_x N_y)^2)$.

3. Simulation result

The number of simulations is set to $Num = 10^4$. If there is no special description, the simulation parameters of the system studied in this paper are as follows: assuming that the channel model is Gaussian white noise channel, the base station antenna is a uniform matrix array as shown in Figure 1, the spacing between adjacent array elements is $d = \frac{\lambda}{2}$, the number of elements of uniform array is $20 \times 20$, the number of transmitting antennas is 4, the number of transmitted signals is 4, the incident direction is $(10, 20), (15, 25), (10, 40), (70, 80)$, and the signal-to-noise ratio is 15dB.

Observing the figure 4, it is not difficult to find that the RMSE gradually decreases with the increase of azimuth and pitch. This shows that the algorithm proposed in the article can obtain good estimation accuracy when both the azimuth angle and the pitch angle are relatively large. The article uses the MSE between the correct azimuth angle, pitch angle, and the estimated azimuth angle, pitch angle, to measure the accuracy of positioning. Figure 5 shows that with the improvement of the signal-to-noise ratio, the algorithm proposed in the article still has better positioning performance than the comparative positioning algorithm. Although the positioning accuracy of the proposed algorithm does not vary greatly, it has good signal-to-noise ratio adaptability. It can be seen that the algorithm proposed in the article is not much different from the traditional algorithm when the number of elements is small. When the number of elements is large, its positioning performance is significantly better than the traditional algorithm.
4. Conclusion
The advantage of the proposed 2D-LC-ESPRIT is to process all the received signals, and the utilization rate of the original data is very high. The computational complexity of the 2D-LC-ESPRIT algorithm is $O(N_r^3)$, while that of the traditional localization algorithm is $O((N_r N_t)^3)$. Therefore, the algorithm greatly reduces the computational complexity of localization algorithm. The algorithm proposed in the article When the number of elements is large, its positioning performance is significantly better than the traditional algorithm.

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