Does the QGP fire ball really exist?

G.A.Kozlov

Bogoliubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research
141980 Dubna, Moscow Region, Russia

Abstract

The strongly interacting matter under extreme conditions of temperature for the conjectured deconfined phase of the quark-gluon plasma is investigated. A systematic study of the form of the thermal ratio of the disorder deviation (TRDD) is presented with the emphasis on the degree of deviation of the TRDD-function from unity. The space-time evolution of quarks and gluons is studied in the framework of two-particle correlation functions to predict the space-time size of the closed deconfined phase.

PACS 12.38.Mh

1 Introduction

In the near future a lot of efforts will be needed in searching for the quark-gluon plasma (QGP). The existence of a deconfined phase of gluons and quarks has been predicted by Quantum Chromodynamics (QCD) [1]. Heavy-ion experiments at the CERN Large Hadron Collider (LHC) with the program ALICE [2] will start next century, aimed at possible experimental confirmation of the hypothetical theoretical predictions of QGP. It is supposed that in heavy-ion collisions the hadron matter occurs as a strong thermal squeezed state for a short time period. A deconfined phase, QGP, is possible at the energy density $\sim 2.4 \, GeV/fm^3$ in this thermalized matter [3]. However, up to now there are no fixed and guaranteed signals which
would make it possible to identify the occurrence of QGP. Matsui and Satz [4] pointed out that the signature of the phase transition from the nuclear matter to QGP would be the reduction of heavy vector meson yield. There is a suppression of a heavy charmonium state $J/\Psi$ in heavy ion collisions via the Debye color screening effect in a special mode phase of quarks and gluons—the deconfined phase. It is pointed out that charmed quarks and antiquarks would leave the zone of deconfinement before forming a hadron composed of charmed quarks.

Following the fruitful idea of Matsui and Satz [4] we suppose that
- at multi-TeV ($\sim 5 - 10$ TeV) collider energies the gluon luminosity is far larger than the quark luminosity for charmed or beauty hybrid intermediate states, where their constituents are distorted temporarily by the deconfined environment during the short time scale just after the fusion of two gluons coming from colliding ions;
- the charmed or beauty hybrid mesons turn into the s-wave charmonium or bottomonium and $\gamma$ quantum.

Here, in the naive picture, the deconfined state of QGP is occupied by an arbitrary number of gluons and/or quarks (g/q). It is clear that there are no interactions between gluons and quarks for a short time $\tau$ if the time scale for the QGP existence $\tau = \tau_{QGP}$ is larger than the characteristic time scale $\tau_V$ ($\tau_{QGP} > \tau_V$) to form the quark bound state with the average radius $<r^2>^{1/2} = 2\tau_V/m_q$ for $m_q$ quark mass. Otherwise, there are no unbounded quarks if $\tau_{QGP} < 0.32$ fm and $0.2$ fm for $J/\Psi$ and $\Upsilon$-particles, respectively. The radiative decay of charmed hybrid mesons into $J/\Psi$ with the decay width $\sim 4.0\alpha_s$ keV has been estimated in [5]. There is a very popular point of view in literature that a (de)confinement phase transition is predicted to occur at the typical energy scale involved, the temperature $T_c \sim$ QCD scale $\Lambda_{QCD} \sim 200$ MeV. This critical temperature $T_c$ is close to the limiting one in hadron interactions firstly indicated by Hagedorn [6]. The only remaining problem which has been transferring from one paper to another for a long time is the puzzle of rough equality of $T_c$ to other three fundamental quantities in QCD: the scale $\Lambda_{QCD}$, the pion mass, and the current mass of a strange quark.

But, among the issues related to QGP, we attract attention to the problem of deconfined phase through the calculation of correlation and distribution functions [7] in the thermal theory of quantized fields. In this paper, we consider the semiphenomenological model for the QGP existence. To do this, we have to use the standard theory of quantized fields replacing:
1. the asymptotic field operators and
2. the vacuum expectation values
by
1. the thermal field operators and
2. the thermal statistical averages,
respectively, in order to formulate correlation and distribution functions of
produced particles.

We assume that the heavy-ion collision (ex., Pb-Pb [2]) produces a thermalized quark-gluon gas because of the two-gluon fusion in the local chemical and thermodynamic equilibrium. The hot as well as dense gas of particles will expand into the surrounding vacuum, thereby cooling down, and the particles will interplay with each other and dilute until free gluons and/or quarks, g/q, as well as particles composed of quarks leave this equilibrium. Corrections to the free two-particle correlation functions (CF) taking account of the interaction of g/q with their source medium are very actual. This allows one to get information about the geometry of the emitter-source. For convenience, one can distinguish the fase of g/q in the equilibrium from the free particles by the freeze-out hypersurface.

In this paper, we would like to propose new features of formulation withing the framework of the Langevin-type equation. The method of Langevin equation and its extensions to the quantal case have been suggested and considered in papers [8-10] and [11-13], respectively. We propose that rather complicated real physical processes to happen in the QGP formation should be replaced by a one-constituent (ex., gluon or quark) propagation provided by a special kernel operator (in the evolution equation) to be considered as an input of the model and disturbed by the random force $F$. We assume $F$ to be the external source proposed as both a c-number function and an operator. Our evolution equation is an operator one, so that there appear new additional problems about the commutation relations and the ordering of operators, which do not exist in the classical case [13].

Based on the thermal operator-field technique, in Sec. 2 we introduce a thermal ratio of the disorder deviation (TRDD), reflecting the degree of deviation, from unity, of the ratio of the two-particle thermal momentum-dependent distribution to two one-particle thermal distribution functions of produced particles, gluons and/or quarks in a partly deconfined phase state. We study the four-momentum correlations of two identical particles, g/q, which can be both useful and instructive in heavy ion collisions to infer the
shape of the particle emitter-source (Sec.3). The sensitivity of the TRDD-functions to the size of the emitter will be given in Sec. 3. Within these features, the canonical formalism in a stationary state in the thermal equilibrium (SSTE) is formulated, and a closed structural resemblance between the SSTE and standard quantum field theory is revealed.

2 Correlation and distribution functions

To clarify the internal structure of the disordering of particles produced in heavy ion collisions, we have to use the consistent approach based on the evolution of dynamical variables as well as the extension to different modes provided by virtual transitions.

Let us consider a hypothetical system of the quark-gluon excited local thermal phase in QCD where a canonical operator $a(\vec{k}, t)$ and its Hermitian conjugate $a^+(\vec{k}, t)$ occur. In the naive representation, we suppose that in this phase there are independent particle sources localized at the points $x_\mu$ and all the particles do not interact after their emission. We formulate the distribution functions (DF) of produced particles (gluons and quarks) in terms of point-to-point equal time temperature-dependent thermal CF of two operators

$$ w(\vec{k}, \vec{k}', t; T) = \langle a^+(\vec{k}, t) a(\vec{k}', t) \rangle = Tr[a^+(\vec{k}, t) a(\vec{k}', t)e^{-H\beta}]/Tr(e^{-H\beta}). $$

Here, $\langle ... \rangle$ means the procedure of thermal statistical averaging; $\vec{k}$ and $t$ are, respectively, momentum and time variables, $e^{-H\beta}/Tr(e^{-H\beta})$ stands for the standard density operator in the equilibrium and the Hamiltonian $H$ is given by the squared form of the annihilation $a_p$ and creation $a^+_p$ operators for bose- and fermi-particles, $H = \sum_p \epsilon_p a^+_p a_p$ (the energy $\epsilon_p$ and operators $a_p$, $a^+_p$ carry some index $p$ [14], where $p_\alpha = 2\pi n_\alpha/L, n_\alpha = 0, \pm1, \pm2, \ldots; V = L^3$ is the volume of the system considered); $\beta$ is the inverse temperature of the environment, $\beta = 1/T$. We assume that $w(\vec{k}, \vec{k}', t; T) = 0$ if $|\vec{k} - \vec{k}'| < \mu_k$ for the characteristic thermolized massive scale $\mu_k$, where the temperature of the environment $T$ does not change. One can suppose that the real physical process is divided into two stages:

1. Thermolization of the hadron matter system or even the quark-gluon
formation because of the heavy-ion collision. There is an expansion of the
thermolized space-time as well as a freeze-out process.

2. Upon cooling to the critical temperature, $T_c \sim 200^\circ C$, the thermolized
hadron system or the phase with free g/q is decaying into secondary particles
which could be observed. The first stage, characterizing free particles in
SSTE, allows one to make any field operator using its expansion like (ex., for
bose particles)

$$\Phi_b(x_\mu) = \varphi(x_\mu) + \varphi^+(x_\mu),$$

where

$$\varphi(x_\mu) = \int d^3 k \; v_k \; a(\vec{k}, t), \quad v_k = \frac{e^{i\vec{k}\vec{x}}}{\sqrt{[(2\pi)^3 2 \Delta(\vec{k})]^{1/2}}},$$

and $\Delta(\vec{k})$ is the element of the invariant phase volume which will be defined
later. The standard canonical commutation relation (CCR)

$$[a(\vec{k}, t), a^+(\vec{k}', t)]_{\pm} = \delta^3(\vec{k} - \vec{k}')$$

at every time $t$ is used as usual for bose (-) and fermi (+)-operators. The next
step is to introduce the TRDD-function $D(\vec{k}, \vec{k}', t) = R(\vec{k}, \vec{k}', t) - 1$ reflecting
the deviation from unity of the R-ratio

$$R(\vec{k}, \vec{k}', t) = W(\vec{k}, \vec{k}', t)/[W(\vec{k}, t) \cdot W(\vec{k}', t)].$$

The $R$-function is defined as the probability to find two particles, gluons
or quarks, with momenta $\vec{k}$ and $\vec{k}'$ in the same event at the time $t$ normalized
to the single spectrum of these particles. Here, $W(\vec{k}, t)$ stands for the one-
particle thermal DF

$$W(\vec{k}, t) = \langle b^+(\vec{k}, t) \; b(\vec{k}, t) \rangle,$$

$$b(\vec{k}, t) = a(\vec{k}, t) + \phi(\vec{k}, t)$$

of the particles emitted (gluons or quarks) under an assumption of occurrence
of the random source-function $\phi(\vec{k}, t)$ being an operator, in general. The two-
particle DF $W(\vec{k}, \vec{k}', t)$ looks like

$$W(\vec{k}, \vec{k}', t) = \langle b^+(\vec{k}, t) \; b^+(\vec{k}', t) \; b(\vec{k}, t) \; b(\vec{k}', t) \rangle.$$
This would allow one to estimate the possibility to find any constituents (gluons and quarks) in the excited QCD matter. In order to be applied to future experimental search for QGP, it should be important to compare the measured TRDD function $D$ for heavy-ion collisions at the TeV scale with those calculated in a spherically symmetric model which describes the transverse momentum spectra of g/q. Based on the theoretical model considered here one can calculate and give the prediction for the space-time size of the deconfined system. As a starting point, let us consider, for simplicity, the random source-function $\phi(\vec{k}, t)$ in (5) as a c-number one. The R-ratio (3) of the DF (6) and (4) can easily be calculated with the result leading to

$$R_b(\vec{k}, \vec{k}', t) = 1 + \frac{\sigma_b(\vec{k}, \vec{k}', t)}{\sum_{i=1}^{4} \alpha_i}$$

for bose-particles, while for fermi-ones we obtain

$$R_f(\vec{k}, \vec{k}', t) = R_b(\vec{k}, \vec{k}', t) - 2 \frac{\alpha_1}{\sum_{i=1}^{4} \alpha_i}$$

under the condition $\sigma_b(\vec{k}, \vec{k}', t) > 2\alpha_1$, where

$$\sigma_b(\vec{k}, \vec{k}', t) = w(\vec{k}, \vec{k}', t) \cdot w(\vec{k}', \vec{k}, t) + w(\vec{k}, \vec{k}', t) \cdot \phi^+(\vec{k}', t) \phi(\vec{k}, t) +$$

$$+ w(\vec{k}', \vec{k}, t) \cdot \phi^+(\vec{k}, t) \phi(\vec{k}', t),$$

$$\alpha_1 = w(\vec{k}, \vec{k}) \cdot w(\vec{k}', \vec{k}') , \alpha_2 = w(\vec{k}, \vec{k}) \cdot |\phi(\vec{k}')|^2 ,$$

$$\alpha_3 = w(\vec{k}', \vec{k}') \cdot |\phi(\vec{k})|^2 , \alpha_4 = |\phi(\vec{k})|^2 \cdot |\phi(\vec{k}')|^2 .$$

Thus, the deviation from unity, reflecting the naive result, is defined by the positive TRDD-functions $D_{b/f}$ ranging from 0 to 1:

$$D_b = \frac{\sigma_b}{\sum_{i=1}^{4} \alpha_i} , \quad D_f = \frac{\sigma_b - 2 \alpha_1}{\sum_{i=1}^{4} \alpha_i} .$$

The complete correlation is provided by the condition $D_{b/f}=1$, while $D_{b/f}=0$ corresponds to the naive mode with zeroth correlation.

The formal structure of the $D_{b/f}$- functions does not allow one to reveal the origin of the particle disordering. We have indicated the formal scheme only in order to obtain a nontrivial result ($D \neq 0$) to reveal the correlation
effect with the random source represented by the c-number function \( \phi(\vec{k}; t) \). To avoid the formal representation of the result and to do some translation into an operator space, one has to go into some evolution scheme for the set of all operators to be used.

One of our aims is to give a useful tool reflecting the evolution properties of propagating particles in a randomly distributed environment. Let us consider an elementary particle ”moving” with the momentum \( \vec{k} \) in the quantum equilibrium phase space under the influence of the random force coming from surrounding particles. The evolution equations for these particles described in terms of the operators \( b(\vec{k}, t) \) and \( b^+(\vec{k}; t) \) are

\[
i \frac{\partial}{\partial t} b(\vec{k}, t) + A(\vec{k}, t) = F(\vec{k}, t) + P.
\]

(7)

\[
-i \frac{\partial}{\partial t} b^+(\vec{k}, t) + A^*(\vec{k}, t) = F^+(\vec{k}, t) + P.
\]

(8)

One can identify both \( b \) and \( b^+ \) with the special mode operators of the quark and gluon fields or their combinations [15] having the dependence on the thermolized QCD matter fields occurring due to heavy-ion collision. In the classical picture, equations (7) and (8) are just the Langevin-type ones for the Brownian motion of particles. Here, \( P \) and \( F(\vec{k}, t) \) stand for the stationary external force and the random one, respectively, both acting from the environment. The only operator \( F \) has a zeroth value of the statistical average, \( \langle F \rangle = 0 \). The correlation between two operators \( F(\vec{k}, t) \) represents the fluctuation dissipation theorem of the second kind [16]. The interaction of the particles considered with the surrounding ones as well as providing the propagation is given by the operator \( A(\vec{k}, t) \) which can be defined as the one closely related to the dissipation force. A most simple form of the operator \( A(\vec{k}, t) \) should be the following:

\[
A(\vec{k}, t) = \int_{-\infty}^{+\infty} K(\vec{k}, t - \tau) b(\vec{k}, \tau) \, d\tau.
\]

(9)

Here, an interplay of quarks and gluons with surrounding particles is embedded into the interaction complex kernel \( K(\vec{k}, t) \), while the real physical transitions are provided by the random source operator \( F(\vec{k}, t) \) (see eq. (7)). The random evolution field operator \( K(\vec{k}, t) \) in (14) stands for the
random noise and it is assumed to vary stochastically with a \( \delta \)-like equal time correlation function
\[
\langle K^+(\vec{k}, \tau) K(\vec{k}', \tau) \rangle = 2 (\pi \alpha)^{1/2} \kappa \delta(\vec{k} - \vec{k}') .
\] (10)

In the case of the large value of the correlation scale squared \( \alpha \) one can get
\[
\langle K^+(\vec{k}, \tau) K(\vec{k}', \tau) \rangle \rightarrow \kappa \exp[-\vec{z}^2/(4\alpha)] \quad \text{as} \quad \alpha \rightarrow \infty ,
\] (11)
where \( \vec{z}^2 = (\vec{k} - \vec{k}')^2 \) while \( \kappa \) is the strength of the noise characterized by the Gaussian distribution function \( \exp[-\vec{z}^2/(4\alpha)] \). Hence, both \( \kappa \) and \( \alpha \) in (10) and (11) define the effect of the Gaussian noise on the evolution of \( g/q \) in the thermolized environment.

The formal solution of (7) in the operator form in \( S(\mathbb{R}^4) \) (\( k^\mu = (\omega = k^0, k_j) \)) is
\[
\tilde{b}(k^\mu) = \tilde{a}(k^\mu) + \tilde{\phi}(k^\mu) ,
\] (12)
where the operator \( \tilde{a}(k^\mu) \) is expressed via the Fourier transformed operator \( \tilde{F}(k^\mu) \) and the Fourier transformed kernel function \( \tilde{K}(k^\mu) \) (coming from (9) as \( \tilde{a}(k^\mu) = \tilde{F}(k^\mu) \cdot [\tilde{K}(k^\mu) - \omega]^{-1} \),
\] (13)
while the function \( \tilde{\phi}(k^\mu) \) is provided by the function \( \sim P \cdot [\tilde{K}(k^\mu) - \omega]^{-1} \). In order to obtain the solution (12), we have used the fact that at large \( t \) the terms in the Fourier transformed evolution equation in \( S(\mathbb{R}^4) \), containing the fast oscillating factors, weakly tend to zero as \( t \rightarrow +\infty \) as well as \( t \rightarrow -\infty \).

The random force operator \( F(\vec{k}, t) \) can be expanded by using the Fourier integral
\[
F(\vec{k}, t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \psi(k^\mu) \hat{c}(k^\mu) e^{-i\omega t} ,
\]
where the form \( \psi(k^\mu) \cdot \hat{c}(k^\mu) \) is just the Fourier operator \( \tilde{F}(k^\mu) = \psi(k^\mu) \cdot \hat{c}(k^\mu) \) and the canonical operator \( \hat{c}(k^\mu) \) obeys the commutation relation
\[
[\hat{c}(k^\mu), \hat{c}^+(k'^\mu)] = \delta^4(k^\mu - k'^\mu) .
\]
The function \( \psi(k^\mu) \) is determined by the condition
\[
\Delta(\vec{k}) = \int_{-\infty}^{+\infty} \frac{d\omega \cdot \omega}{2\pi} \left[ \frac{\psi(k^\mu)}{\tilde{K}(k^\mu) - \omega} \right]^2 ,
\] (14)
where \( \Delta(\vec{k}) \) comes from the field expansion (\[1\]), and the CCR (\[2\]) is used. The requirement \( \Delta(\vec{k}) = (\vec{k}^2 + m^2)^{1/2} \) in (\[14\]) immediately leads to the condition (see also [15])

\[
\int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{|\psi_{(\mu)}|}{K_{(\mu)} - \omega} = 1.
\]

The time correlation of the random force operator is defined by

\[
\langle F^+(\vec{k}, t) F(\vec{k}, t + \tau) \rangle = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{d\omega'}{2\pi} e^{it(\omega - \omega')} e^{-\omega'\tau} \times \langle \psi^+(\omega, \vec{k}) \hat{c}^+(\omega, \vec{k}) \psi(\omega', \vec{k}) \hat{c}(\omega', \vec{k}) \rangle.
\]

In the case when the constituents (gluons or quarks) are “moving” inside the closed system being in the equilibrium state, the magnitude of the initial time \( t \) does not have any priority compared to other initial time points. Hence, the time correlation \( \rho(\vec{k}, t) = \langle F^+(\vec{k}, t) F(\vec{k}, t + \tau) \rangle \) should not carry the dependence of the time variable. This fact can be realized by putting

\[
\langle \psi^+(\omega, \vec{k}) \hat{c}^+(\omega, \vec{k}) \psi(\omega', \vec{k}) \hat{c}(\omega', \vec{k}) \rangle = 2\pi \langle \hat{c}^+(\vec{k}_{(\mu)}) \hat{c}(\vec{k}_{(\mu)}) \rangle |\psi_{(\mu)}|^2 \delta(\omega - \omega').
\]

Thus, the time correlation is translated into the following one:

\[
\rho(\vec{k}, t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} |\psi_{(\mu)}|^2 \langle \hat{c}^+(\vec{k}_{(\mu)}) \hat{c}(\vec{k}_{(\mu)}) \rangle e^{-i\omega t}.
\]

In fact, the quantity \( |\psi_{(\mu)}|^2 \langle \hat{c}^+(\vec{k}_{(\mu)}) \hat{c}(\vec{k}_{(\mu)}) \rangle \) is the spectral function characterizing the random force operator \( \tilde{F}(\vec{k}_{(\mu)}) \).

### 3 Distribution functions in \( S(\mathbb{R}_4) \) and space-time size.

In order to study the enhanced probability for emission of two identical particles, g/q, and since we are interested in the finite size of the QGP formation, the useful information comes from the consideration of DF in \( S(\mathbb{R}_4) \). We introduce the ratio \( R \) of DF as follows:

\[
R_{bf}(k_{(\mu)}, k'_{(\mu)}; T) = \frac{\tilde{W}(k_{(\mu)}, k'_{(\mu)}; T)}{W(k_{(\mu)}) \cdot \tilde{W}(k'_{(\mu)})},
\]

(15)
where $\tilde{W}(k_\mu, k_\mu'; T)$ stands for two-particle probability density subject to Bose/Fermi symmetrization and defined as

$$\tilde{W}(k_\mu, k_\mu'; T) = \langle \tilde{b}^+(k_\mu) \tilde{b}^+(k_\mu') \tilde{b}(k_\mu) \tilde{b}(k_\mu') \rangle,$$

while

$$\tilde{W}(k_\mu) = \langle \tilde{b}^+(k_\mu) \tilde{b}(k_\mu) \rangle$$

is related to the single-particle quantity for a particle with momentum $k_\mu$. Using the Fourier solution of equation (7) in $S(\Re_4)$, one can get R-ratios for DF obeying to Bose-

$$R_b(k_\mu, k_\mu'; T) = 1 + D_b(k_\mu, k_\mu'; T)$$

and Fermi-particles

$$R_f(k_\mu, k_\mu'; T) = R_b(k_\mu, k_\mu'; T) - 2 \frac{\Xi(k_\mu) \cdot \Xi(k_\mu')}{\tilde{W}(k_\mu) \cdot \tilde{W}(k_\mu')}$$

where

$$D_b(k_\mu, k_\mu') = \frac{\Xi(k_\mu, k_\mu') \Xi(k_\mu', k_\mu) + \tilde{\phi}^+(k_\mu') \tilde{\phi}(k_\mu)}{\tilde{W}(k_\mu) \cdot \tilde{W}(k_\mu')} \cdot \langle \tilde{c}^+(k_\mu) \tilde{c}(k_\mu') \rangle.$$  

(18)

In order to estimate the thermolized space-time size of QGP we need to have the detailed knowledge of CF $\Xi(k_\mu, k_\mu')$. Using the general approach, one cannot do this estimation but suppose the formal expansion to be applied to the function $\Xi(k_\mu, k_\mu')$, namely

$$\Xi(k_\mu, k_\mu') = \sum_\zeta \langle k' | \zeta \rangle \langle \zeta | k \rangle ,$$

where the sum runs over all possible $\zeta$-states of the environment while the $f$-function stands for the temperature-dependent one. Using (13) and the Fourier operator $\hat{F}(k_\mu) = \psi(k_\mu) \cdot \hat{c}(k_\mu)$ the two-particle CF $\Xi(k_\mu, k_\mu')$ looks like

$$\Xi(k_\mu, k_\mu') = \langle \hat{a}^+(k_\mu) \hat{a}(k_\mu') \rangle$$

$$= \frac{\psi^*(k_\mu) \cdot \psi(k_\mu')}{[\hat{K}^*(k_\mu) - \omega] \cdot [\hat{K}(k_\mu') - \omega']} \cdot \langle \hat{c}^+(k_\mu) \hat{c}(k_\mu') \rangle.$$

(19)
The definition of the thermal statistical average \( \langle \hat{c}^+(k_\mu) \hat{c}(k'_\mu) \rangle \) is related to the Kubo-Martin-Schwinger (KMS) condition \[17\]

\[
\langle a(\vec{k}', t') a^+(\vec{k}, t) \rangle = \langle a^+(\vec{k}, t) a(\vec{k}', t - i\beta) \rangle \cdot \exp(-\beta \mu),
\]

(20)

where \( \mu \) is the chemical potential. Using the KMS condition (20) and the Fourier transformed solution \( \tilde{a}(k_\mu) \) (13), one can conclude that the thermal statistical averages for the \( \hat{c}(k_\mu) \)-operator should be presented in the following form:

\[
\langle \hat{c}^+(k_\mu) \hat{c}(k'_\mu) \rangle = \delta^4(k_\mu - k'_\mu) \cdot n(\omega, T),
\]

(21)

\[
\langle \hat{c}(k_\mu) \hat{c}^+(k'_\mu) \rangle = \delta^4(k_\mu - k'_\mu) \cdot [1 \pm n(\omega, T)]
\]

(22)

for Bose (+)- and Fermi (-)-statistics where \( n(\omega, T) = \{ \exp[(\omega - \mu) / (2T)] + 1 \}^{-1} \).

Inserting CF (19) into (18) and taking into account that the \( \delta^4(k_\mu - k'_\mu) \)-function should be changed by the smooth sharp function \( \Omega(r) \cdot \exp(-q^2/2) \), one can get the following expression for the \( D_b \)-function

\[
D_b(k_\mu, k'_\mu; T) = \lambda(k_\mu, k'_\mu; T) \cdot \exp(-q^2/2) \times \{ n(\bar{\omega}, T) \Omega(r) \cdot \exp(-q^2/2) + \phi^*(k'_\mu) \phi(k_\mu) + \phi^*(k_\mu) \phi(k'_\mu) \},
\]

(23)

where

\[
\lambda(k_\mu, k'_\mu; T) = \frac{\Omega(r)}{W(k_\mu) \cdot W(k'_\mu)} \cdot n(\bar{\omega}, T), \quad \bar{\omega} = \frac{1}{2} (\omega + \omega').
\]

The function \( \Omega(r) \cdot n(\omega; T) \cdot \exp(-q^2/2) \) describes the space-time size of the QGP fire-ball. Choosing the z-axis along the two-heavy-ion collision axis one can put

\[
q^2 = (r_0 \cdot Q_0)^2 + (r_z \cdot Q_z)^2 + (r_t \cdot Q_t)^2,
\]

\[
Q_\mu = (k - k')_\mu, Q_0 = \epsilon_k - \epsilon_{k'}, Q_z = k_z - k'_z, Q_t = [(k_x - k'_x)^2 + (k_y - k'_y)^2]^{1/2},
\]

\[
\Omega(r) \sim r_0 \cdot r_z \cdot r_t^2,
\]

where \( r_0, r_z \) and \( r_t \) are time-like, longitudinal and transverse "size" components of the QGP fire-ball. For estimation the 4-dimensional structure of the space-time correlators (21), (22) we have to identify \( r_0, r_z \) and \( r_t \),
resp., with the averages of the "time" component $\tau, z$ and $\sqrt{x^2 + y^2}$ on the freeze-out hypersurface with the temperature $T$. Formally, the function $D_b$ (23) is the positive one ranging from 0 to 1. Since there are large values of $\sqrt{Q^2} = \sqrt{(k_\mu - k'_\mu)^2}$ in (23), all the $k_\mu(k'_\mu)$ dependence should be changed by the average value $\bar{k}_\mu = 0.5(k + k')\mu$. The quantitative information (longitudinal $r_z$ and transverse $r_t$ components of the QGP spherical volume, the temperature $T$ of the environment) could be extracted by fitting the theoretical formula (23) to the measured TRDD function and estimating the errors of the fit parameters. Formula (23) indicates that a chaotic g/q source emanating from the thermolized g/q fireball exists. Hence, the measurement of the space-time evolution of the g/q source would provide information of the g/q emission process and the general reaction mechanism. In formula (23) for the $D_b$-function, the temperature of the environment enters through the two-particle CF $\Xi(k_\mu, k'_\mu; T)$. If $T$ is unstable the $R_{b/f}$-functions (15) will change due to a change of DF $\tilde{W}$ which, in fact, can be considered as an effective density of the g/q source. In the case when the temperature in the center of the fire-ball is higher than that of the border, this leads to the largest contribution to the g/q production. This fact could be "translated into" the possible statement that $\bar{k}_T << \bar{k}_z$. Formula (16) looks like the following expression for the experimental R-ratio using a source parametrization:

$$R_T(r) = 1 + \lambda_T(r) \cdot \exp(-r^2_t \cdot Q^2_t / 2 - r^2_z \cdot Q^2_z / 2),$$

(24)

where $r_t(r_z)$ is the transverse (longitudinal) radius parameter of the source with respect to the beam axis, $\lambda_T$ stands for the effective intercept parameter (chaoticity parameter) which has a general dependence of the mean momentum of the observed particle pair. Here, the dependence on the source lifetime is omitted. Since $0 < \lambda_T < 1$, one can conclude that the effective function $\lambda_T$ can be interpreted as a function of the core particles to all particles produced. The chaoticity parameter $\lambda_T$ is the temperature-dependent and the positive one defined by

$$\lambda_T(r) = \frac{[\Omega(r) \cdot n(\tilde{\omega}; T)]^2}{\tilde{W}_0(k_\mu, k'_\mu)},$$

(25)

where $\tilde{W}(k_\mu) \cdot \tilde{W}(k'_\mu)$ is replaced by $\tilde{W}_0(k_\mu, k'_\mu)$ for convenience regarding the point of view that one can distinguish different particles.
Comparing (18) and (23) one can identify
\[
\Xi(k_\mu, k'_\mu) = \Omega(r) \cdot n(\bar{\omega}; T) \cdot \exp(-q^2/2) .
\]

In the simple case, the random source function \( \tilde{\phi}(k_\mu) \) in (18) should be the following [18]
\[
\tilde{\phi}(k_\mu) = [\alpha \cdot \Xi(k_\mu)]^{1/2} ,
\]
where \( \alpha \) is a real positive parameter. Neglecting \( \tilde{\phi}(k_\mu) \)-source we immediately obtain the trivial result
\[
D_b(k_\mu, k'_\mu; T) = \tilde{\lambda}(\bar{\omega}; T) \cdot \exp(-q^2) ,
\]
where the temperature-dependent chaoticity \( \tilde{\lambda}(\bar{\omega}; T) \) is
\[
\tilde{\lambda}(\bar{\omega}; T) = \frac{n^2(\bar{\omega}; T)}{n(\omega; T) \cdot n(\omega'; T)} .
\]

Saving the random source-function we can find the formal representation for the TRDD-function \( D_b \) using the \( \alpha \)-factorization (26)
\[
D_b(q^2; T) = \frac{\tilde{\lambda}^{1/2}(\bar{\omega}; T)}{(1 + \alpha)(1 + \alpha')} e^{-q^2/2} \left[ \tilde{\lambda}^{1/2}(\bar{\omega}; T)e^{-q^2/2} + 2(\alpha\alpha')^{1/2} \right] .
\]

It is easily to see that in the vicinity of \( q^2 \approx 0 \) one can get the full correlation if \( \alpha = \alpha' = 0 \) and \( \tilde{\lambda}(\bar{\omega}; T) = 1 \). Putting \( \alpha = \alpha' \) in (27) we find the formal lower bound on the space-time dimensionless size of the fire-ball for bose-system:
\[
q^2_b \geq \ln \frac{\tilde{\lambda}(\bar{\omega}; T)}{[\sqrt{(\alpha + 1)^2 + \alpha^2} - \alpha]^2} .
\]

In the case of fermi-particles, the following restriction on \( q^2_f \) is valid (see (17))
\[
\ln \frac{\tilde{\lambda}(\bar{\omega}; T)}{[\sqrt{2\alpha(\alpha + 1) + 3 - \alpha}]^2} \leq q^2_f \leq \ln \frac{\tilde{\lambda}(\bar{\omega}; T)}{[\sqrt{\alpha^2 + 2} - \alpha]^2} .
\]
In fact, the function $D_b(k_\mu, k'_\mu; T)$ in (23) could not be observed in the heavy-ion experiment ALICE because of some model uncertainties. In the standard consideration, the TRDD-function has to contain a background contribution as well as other physical particles (resonances) which have not been included in the calculation of the $D_b$-function. In order to be close to the experimental data, one has to expand the $D_b$-function as projected on some well-defined function (in $S(\mathcal{R}_4)$) of the relative momentum of two particles produced in heavy-ion collisions

$$D_b(k_\mu, k'_\mu; T) \rightarrow D_b(Q^2_\mu; T).$$

Thus, it will be very instructive to use the polynomial expansion which is suitable to avoid any uncertainties as well as characterize the degree of deviation from the Gaussian distribution, for example. In $(-\infty, +\infty)$, a complete orthogonal set of functions can be obtained with the help of the Hermite polynomials in the Hilbert space of the square integrable functions with the measure $d\mu(z) = \exp(-z^2/2)dz$. The function $D_b$ corresponds to this class if

$$\int_{-\infty}^{+\infty} dq \exp(-q^2/2) |D_b(q)|^n < \infty, n = 0, 1, 2, \ldots.$$

The expansion in terms of the Hermite polynomials $H_n(q)$

$$D_b(q) = \lambda \sum_n c_n \cdot H_n(q) \cdot \exp(-q^2/2)$$

is well suited for the study of possible deviation from both the experimental shape and the exact theoretical form of the TRDD function $D_b$ (23). The coefficients $c_n$ in (28) are defined via the integrals over the expanded functions $D_b$ because of the orthogonality condition

$$\int_{-\infty}^{+\infty} H_n(x) \ H_m(x) \ \exp(-x^2/2) \ dx = \delta_{n,m}.$$

Hence, the observation of the two-particle correlation (both for bose- and fermi-symmetrization) enable to extract the properties of the structure of $q^2$, i.e. the space-time size of QGP formation.
4 Conclusion

1. In this paper we investigated the finite temperature momentum correlations (of two identical particles, gluons/quarks) which can be both useful and instructive in multi-TeV heavy-ion collisions (e.g. Pb+Pb in ALICE experimental program) to infer the shape of the gluon/quark source-emitter. In fact, we have presented the method of extracting the intercept and source parameters from the shape of the TRDD-function.

2. We used the operator form evolution equation (7) and (8) for the special mode operators $b(\vec{k},t)$ and $b^+(\vec{k},t)$ (of the gluon/quark field), respectively, in the thermolized equilibrium at the freeze-out stage.

3. The relations between the CF $\Xi(k_\mu,k'_\mu)$ and the full $R$-functions for bose (16)- and fermi (17)-particles at the stage of the freeze-out are obtained. We have shown the sensitivity of the correlation functions to the space-time geometry of the source-emitter (23). In fact, the TRDD-function $D_b$ describes the size and shape of the space-time domain where the secondary observed particles are generated.

4. We can conclude that formally, the QGP size scale can be determined by the evolution behavior of the field operators and the critical temperature $T = T_c$ (see formula (23)).

5. In fact, the full $R$-ratio (15) is the function of four-momentum difference $Q_\mu$ of two identical particles as well as the mean total momentum $\bar{k}_\mu$. Since, the TRDD-function $D_b$ is the positive one and restricted by 1, we expect that the $R$-ratio at too small values of $Q_\mu$ starts from the fixed point $R(Q_\mu \to 0) = 2 - \epsilon (\epsilon \to +0)$ and then falls down (with the Gaussian shape) up to unity over some momentum scale interval of an order of the inverse source size (see (24) and (25)).

6. A large momentum scale $Q_\mu$ determines the minimal size of the QGP fire-ball volume for which $R(r_\mu;T)$ deviates from its asymptotic value of $R(r_\mu \to 0;T) = 1$.

7. In order to use the quantitative information like longitudinal and transverse source size, lifetime, we have to fit our expressions to the measured $R$-functions.
References

[1] J. Cleymans et al., Phys. Rep. 130 (1986) 217.

[2] ALICE, the CERN report CERN/LHCC 95-71, LHCC/P3 (1995)

[3] T. Matsui, "Signatures of the quark-gluon plasma in ultrarelativistic nucleus-nucleus collision", in Proc. of RIKEN Symp. on Physics of High Energy Heavy Ion Collisions, (1992) 1 edited by S. Date and S. Ohta (Saitama, Japan).

[4] T. Matsui and H. Satz, Phys. Lett. B178 (1986) 416.

[5] S. Ishida et al., Nuovo Cim. A107 (1994) 2451.

[6] R. Hagedorn, Suppl. Nuovo Cim. 3 (1965) 147.

[7] M. Namiki and S. Muroya, "Distribution-correlation functions of produced particles in high energy nuclear collisions", in Proc. of Intern. School- Seminar'93 "Hadrons and Nuclei from QCD" (1993) 100, edited by K. Fujii et al., (World Scientific).

[8] M.C. Wang and G.E. Uhlenbeck, Rev. Mod. Phys. 17 (1945) 923.

[9] G.W. Ford M. Mac and P. Mazur, J. Math. Phys. 6 (1965) 504

[10] R. Zwanzig, J. Stat. Phys. 9 (1973) 215.

[11] H. Haken, Rev. Mod. Phys. 47 (1975) 63.

[12] R. Benguria and M. Kac, Phys. Rev. Lett. 46 (1981) 1.

[13] T. Tsuzuki, Prog. Theor. Phys. 81 (1989) 770.

[14] N.N. Bogoliubov and N.N. Bogoliubov (Jr.) "Introduction to quantum statistical mechanics", Nauka, Moscow (1984) (in Russian).

[15] M. Mizutani, S. Muroya and M. Namiki, Phys. Rev. D37 (1988) 3033.

[16] T. Arimitsu, "Non-equilibrium thermo field dynamics and thermal processes", in "Thermal Field Theory," (1991) 207, edited by H. Ezawa, T. Arimitsu, Y. Hashimoto (Elsevier Science Publishers).
[17] R. Kubo, M. Toda and N. Hashitsume, Statistical Physics II- Nonequilibrium Statistical Physics (Springer-Verlag, 1985).

[18] M. Namiki and S. Muroya, ”Theory of particle distribution-correlation in high energy nuclear collisions”, in Proc. of RIKEN Symp. on Physics of High Energy Heavy Ion Collisions, (1992) 91 edited by S. Date and S. Ohita (Saitama, Japan).