Lamb wave interactions through dispersion 2D filters

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Abstract. Acoustic surface waves are widely used to sense and map the properties of the propagation media. In order to characterise local space-time waves, methods such as Gabor analysis are powerful. Nevertheless, knowing which wave is observed, extracting its full bandwidth contribution from the others and to map it in the signal domain is also of great interest. In the Fourier domain, the acoustic energy of a wave is concentrated along the wave-number frequency (k-ω) dispersion curve; a way to extract one wave from others is to filter the signals by mean of k-ω band-pass area that keeps only the selected surface wave. The objective of the present paper is to propose 2D Finite Impulse Response (FIR) filters based on an arbitrary area shape designed to extract selected waves. FIR filtering is based on convolving the impulse response of the filter with the signals. Impulse responses derived from using k-ω elliptical areas (E-FIR) are presented. The E-FIR filters are successfully tested on three experimental space-time signals corresponding to the propagation of Lamb waves measured by standard transducers on a cylindrical shell, by laser Doppler on a plate and generated by a circular pulse and observed by shearography on a rectangular plate.

1. Introduction

Acoustic surface waves are widely used to sense and map the properties of propagation media. For decreasing both the acquisition time of such signals along the region of interest (ROI) and reach useful space-time resolutions, impulse generation for wideband acquisitions are required. This is at the cost of increasing the post-signal processing complexity [1-4]. Methods such as 2D and 3D Gabor analysis are powerful for localizing and identifying both transient and surface waves, as they need no prior knowledge upon the involved signals [5-8]. Nevertheless, knowing which wave is observed, extracting the full bandwidth contribution of one surface wave from the others and to map it in the signal domain is also of great interest [9-11]. In the Fourier domain, the acoustic energy is concentrated along the wave number-frequency (k-ω) dispersion curves. A way to extract one wave from others is to filter the signals by mean of a (k-ω) band-pass area that keeps only the selected surface wave. The aim of the present paper is to propose 2D Finite Impulse Response (FIR) filters based on an elliptical area (k-ω) shape to perform the identification of surface waves.
2. Context
Laser detection methods allow the investigation of ultrasonic transient phenomena in both space and time dimensions. Used along a two dimensional surface, laser ultrasonic leads to three dimensional (3D) space–time signal collections. The advantage of collecting impulse response transient signals is to acquire a wide frequency range from one experiment. However, post-processing depends on what goal is aimed at. For extracting stationary propagation aspects, 3D Fourier analysis or classical high resolution signal processing methods can be used to identify the wave propagation information such as dispersion curves [1-4]. However these methods are not adapted for identifying where and when the waves are generated (Figure 1).

![Figure 1](image1.png)

**Figure 1.** Advantages and drawbacks of space-time or modal representation.

Localizing wave packets through time was introduced in 1947 by Gabor [5]. It gave the principle of the time–frequency analysis by short-time Fourier transform (STFT), with the help of a sliding gaussian window [6-9]. The early applications dealt with audio signals and some efficient compression methods rose from the understanding of their frequencies components time shape. The 2D Gabor transform were introduced in acoustics [9] in order to localize ultrasonic phonons in space–time domains and their conversions on a cylindrical shell. The 3D Gabor analysis [10-11] is both an extension to three dimensions and an improvement of the early 2D (Figure 2).

![Figure 2](image2.png)

**Figure 2.** 3D Gabor transform principle.

Having the dispersion curves from Fourier analysis and the transient aspects from Gabor analysis, both methods are complementary and enable ones to identify waves properties from experimental signals. One of the goals of the 2D Finite Impulse Response (FIR) filters presented here is to go to the reverse direction: knowing dispersion curves, how to extract selected surface acoustic waves from wide-band data sets containing several waves? This kind of wideband wave selection can be done by zero-filling unwanted surface waves from their Fourier space, and recover the filtered signals by reverse Fourier transform. Straightforward for 1D signals, this method lacks of simplicity for implementing 2D trajectories in Fourier domain and controlling Gibbs oscillations in signal domain. In the following proposed method, wave selection is done in the signal domain by convolution with the
impulse response of the filter: the advantage is that for elliptical Fourier pattern, such response is an analytical expression of the filter parameters.

The filter impulse response design takes advantage from a method derived by Shepp and Logan and used in Magnetic Resonance Imaging [12-14]. Shepp and Logan method generates elliptical patterns in image domain, by using analytical expression in Fourier domains (Figure 3). Compared to MRI domain, Fourier and signal domains are permuted, as for the proposed application, elliptical patterns are required in Fourier domain, not in signal domain or image as in MRI. Elliptical areas transfer function Finite Impulse Response filters are investigated (E-FIR).

![Figure 3. The Fourier reconstruction of a head section ([12]).](image)

### 3. Filter design

The method lays on the convolution of the FIR impulse response with the data (Figure 4).

![Figure 4. 2D FIR filter scheme.](image)

The filter pattern is a frequency \((k-\omega)\) Region Of Interest (ROI) with an elliptical shape, chosen for its versatility and its known analytical inverse Fourier transform. The main axis of the elliptical area is oriented along the selected surface wave dispersion curve (Figure 5). As surface waves dispersion curves \((k-\omega)\) can be split in linear segments, the elliptical function parameters are well suited to keep surface waves of interest and eliminate the others.
Figure 2. Filter transfer function and parameters.

The Impulse Response (IR) of such a filter is:

\[
IR(x,t) = e^{-i(K_0 x + \omega_0 t)} \frac{AJ_1(B\sqrt{(u'AB^{-1})^2 + v'^2})}{\sqrt{(u'AB^{-1})^2 + v'^2}}
\]

where \( u' = x\cos\alpha + t\sin\alpha \) and \( v' = -x\sin\alpha + t\cos\alpha \).

In this equation, \( J_1 \) is the first order Bessel function of the first kind, \((K_0,\omega_0)\) is the center of the ellipse, \( \rho \) the intensity, \( \alpha \) its orientation and \( A \) and \( B \) are the lengths of the horizontal and vertical axes. In order to recover real valued filtered signals or select the direction of propagation, a great care must be taken into account to mirror or not this pattern in the three other quadrants of the \((k-\omega)\) spaces. The finite length of the filter is carefully chosen to optimize resolution in the space-time and in the frequency domain.

4. Experimental validation: Fluid filled cylindrical shell

4.1. Experimental set-up.

The propagation of surface waves is investigated on an aluminium cylindrical shell of external radius \( a=20.6\) mm and of thickness \( e=0.62\) mm (internal radius \( b=19.98\) mm). The characteristic ratio internal radius to external radius is \( b/a=0.97 \). This shell is immersed in water.

A pulse of 0.1\( \mu \)s long with 200V of amplitude is sent to a broadband transducer of central frequency 2.25MHz. In the enlightened zone of the shell, the plane bulk wave generates Lamb waves that propagate with continuous energy losses (Figure 6). The space-time signals collection \( s(\theta,t) \) is collected for 720 angular positions equally spaced between 0 and 360 degrees. Each time signal is 2000 points long, for a 100\( \mu \)s time FOV.

Figure 5. Filter transfer function and parameters.

Figure 6. Surface wave coupling with internal waves.
4.2. Experimental results.

Wideband signals collected around the cylindrical shell are presented in Figure 7.

![Figure 7](image1.png)

**Figure 7.** Experimental signals collected around the cylindrical shell.

Using the Elliptical FIR filters, the $S_0$ mode low frequency and high frequency dispersion curves are extracted (Figure 8). The waves are fully isolated from the other waves seen on Fig. 8 (4). As shown in Gabor analysis [9], the $S_0$ mode is clearly generated again at points M2 and M3 from acoustic bulk waves transmitted inside the shell. As the echoes M1, M2 are resolved, a Fourier analysis of each echo can give access to the mode dispersion curve and attenuation.

![Figure 8](image2.png)

**Figure 8.** Filtered signals for $S_0$ Lamb mode in (3) and (4) using filter function defined respectively in (1) and (2).

The Lamb mode $S_1$ is also studied near its frequency cutoff in Figure 9. In this region, the $S_1$ dispersion curve implies two branches, one with positive group velocity, the other one with a negative group velocity. The two branches are identified in Figure 9. The $S_1$ branch with a negative group velocity (M2, M4) has a strong energy contribution in the shadow side of the shell, whereas the $S_1$ branch with a positive group velocity (M1, M3) has a strong energy contribution in the insonified zone. Due to the small elliptic areas used in the filter design, the FIR filter used for Figure 9 is close to Gabor analysis [9] and point out the generation zones with a better understanding than the corresponding Gabor analysis. To the question “where are $S_1$ positive or negative group velocity branches generated?” the Figure 9 answers with a good space-time resolution.
5. Experimental validation : Lamb wave on a plate

5.1. Experimental set-up.

The propagation of the surface waves is investigated on a plane plate (length L=60 mm and thickness e=2 mm). The aluminium plate is immersed in a water tank. A pulse of 0.1 µs long with 200 V of amplitude is sent to a broadband transducer (2.25 MHz). In the enlightened zone of the plate, the plane bulk wave generates Lamb waves that propagate along the plate (Figure 10). A Polytec laser vibrometer is used for the vibration measurement. The incidence angle used is 13°.

5.2. Experimental results.

The complete signal is presented in Figure 11. Surface waves are generated mainly by the edges of the plate (x=0 and x=L).

Figure 9. $S_1$ mode with positive group velocity in (3) and negative group velocity in (4) using filter function defined respectively in (1) and (2).

Figure 10. Experimental set-up
Fluid born A-wave [4, 15] and A\textsubscript{0} mode are extracted from the complete signal using the FIR method (Figure 12). The mode conversion sequence is recovered: the A-wave is generated at both plate borders while the A\textsubscript{0} mode is generated at one border. At time t\textsubscript{1}: the A\textsubscript{0} mode is reflected at the border and part of its energy is converted in A-wave. The FIR filter has a good resolution, even with the low frequency noise included in the signals.

In the following experimental results, high power laser beam was diffracted to produce a circular acoustic source in thermo-elastic mode (10ns laser pulse). The resulting signals S(x,y,t) are 3D and reach a high spatial resolution, at the cost of a lower signal to noise ratio due to the lack averaging and optical artefacts (Figure 13).

6. Experimental validation: Surface acoustic waves from laser induced circular source
In the previous experiments, the signals were bi-dimensional, and scanned by classical transducers or laser vibrometers. The signal to noise ratio was increased by the use of time-averaging for each position. In the following experiments, the signals are acquired by grating interferometer measurement [16]. This allows the fast acquisition of the 2D images through time.
However, in the frequency domain, the artefact are localized close to zero frequency, whereas the observed surface waves $(kx-ky-\omega)$ dispersion curves are localized along trajectories far from the zero frequency origin (Figure 14).

In order to extract propagating surface waves from stationary patterns, the E-FIR has been designed following Fig. 14. The filter was applied on to successive pass: one along $x-t$ and one along $y-t$ dimensions. The resulting filtered images clearly exhibits the two waves propagating respectively towards and outwards the source centre, by nearly eliminating the non oscillating image background.

7. Conclusion
The 2D FIR filters have been successfully employed for extracting individual surface waves present in transient space time signals. Whereas classical wave localization often lays on choosing one frequency or one wave number, the proposed E-FIR filters give the opportunity to extract the wave impulse response of a target for each wave. By using wide $k$-bandwidth and narrow $\omega$-bandwidth E-FIR filter, Gabor analysis is also reached. The method is straightforward to extend to 3D signals obtained from surface scans with laser vibrometers.

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