Interpreting QPOs from Accreting Neutron Stars

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Abstract. The high time resolution and large area of the Rossi X-ray Timing Explorer have been essential in the detection and characterization of high-frequency quasi-periodic variability in the flux from neutron stars in low-mass X-ray binaries. An unknown phenomenon prior to RXTE, kilohertz quasi-periodic oscillations (QPOs) have now been detected from more than twenty systems. Their high frequencies (up to 1330 Hz) imply that they are generated close to the neutron star, where general relativistic effects are expected to play an important role. I summarize current models for the kilohertz QPO phenomenon. In particular, I show that there is a significant domain of agreement among the models that can be used to constrain neutron star structure and look for signatures of highly curved spacetime in the properties of the QPOs.

INTRODUCTION

Neutron stars are important laboratories for physics at high densities. Unlike the matter in relativistic heavy-ion colliders, the matter in the cores of neutron stars has a thermal energy that is much less than its rest-mass energy. Various researchers have speculated whether neutron star cores contain primarily nucleons, or whether degrees of freedom such as hyperons, quark matter, or strange matter are prevalent (see Lattimer & Prakash 2001 for a recent review of high-density equations of state). In addition, the strongly curved spacetime around neutron stars implies that we could observe predicted effects of strong gravity, such as frame-dragging or signatures of an innermost stable circular orbit.

The fast timing phenomena observed from accreting neutron stars with the Rossi X-ray Timing Explorer (RXTE) offer outstanding opportunities for us to probe the regimes of high density and strong gravity. In particular, the kilohertz quasi-periodic brightness oscillations (kHz QPOs) observed from more than twenty neutron stars in low-mass X-ray binaries are promising, because their high frequencies imply an origin near the star, where general relativity must play a role.

Here we discuss briefly some of the proposals for the origin of these QPOs, and their implications for strong gravity and dense matter. Although currently no first-principles magnetohydrodynamic simulations produce sharp QPOs (a situation expected to change in the next few years as computers become faster and more effects can be included), we show that current observational constraints on models are significant enough to allow fairly confident inferences. In § 2 we show that general relativity must inevitably influence the QPO phenomenon, independent of any detailed models. In § 3 we describe the constraints on models that follow from the observations, and the constraints on stellar mass and radius that follow from the viable options. We conclude in § 4 by discussing what discoveries about dense matter and strong gravity can be expected with a future ∼ 10 m² timing instrument.

THE EFFECTS OF GENERAL RELATIVITY

Some of the most exciting potential implications of QPOs involve the effects of general relativity. These include signatures of unstable circular orbits, and general relativistic frame-dragging. However, if general relativity does not influence QPOs then the interpretations and implications are less clear.

From the observational standpoint, a potential link to nonrelativistic systems was made by Mauche (2002), following the work of Psaltis, Belloni, & van der Klis (1999). These authors have shown that particular pairs of QPOs from sources (sometimes selected from more than two QPOs in a given system) follow a trend that, on a log-log plot, appears to link neutron star systems with black hole and white dwarf systems. Since white dwarfs are not significantly relativistic, Mauche (2002) concludes that the phenomenon as a whole cannot involve general relativity, and hence favors a model such as the one proposed by Titarchuk and collaborators, in which the QPOs arise from classical disk oscillations (e.g., Titarchuk & Osherovich 1999; Osherovich & Titarchuk 1999; Titarchuk & Osherovich 2000; Titarchuk 2002, 2003). Abramowicz and Kluźniak have also proposed a mechanism of nonlinear disk resonances to explain black hole sys-
There are still many unknowns about the brightness oscillations from disks in binary systems, and it could be that there are some underlying mechanisms in common between black holes, neutron stars, and white dwarfs. However, whatever the detailed mechanism is, basic physical considerations require that general relativity will alter the basic picture. For example, consider the highest frequency brightness oscillation ever observed, $v_{\text{QPO}} = 1330$ Hz, from 4U 0614+091 (van Straaten et al. 2000). The frequency of the innermost stable circular orbit is $\lesssim 1500$ Hz for $M \gtrsim 1.6M_\odot$ (see below). This means that all frequencies (e.g., orbital, vertical epicyclic, and radial epicyclic) are altered by the curved spacetime. In turn, oscillation modes of the disk are altered as well. Even radiative transfer is affected subtly, by light deflection effects. Thus, regardless of the underlying mechanism, general relativity will alter the basic picture.

One can also look to observations to find that there are important differences between the types of sources. Abramowicz & Kluźniak (2001) made the prescient suggestion that for black hole QPOs there would be small integer ratios between frequencies, before the first detections of such a ratio (see Table 1). There may also be sources with small integer frequency ratios different than 3:2 (e.g., the possible 4:1 ratio found in 4U 1630–47 by Klein-Wolt, Homan, & van der Klis 2003). This insight may well prove to be a key to understanding this phenomenon. However, neutron star systems do not have a similar preference for any particular frequency ratio. For example, the most recent data for Sco X-1 (kindly provided by Mariano Méndez) are plotted in Figure 1. As can be seen, the 3:2 ratio that is prominent in black hole sources is not evident here (the apparent clustering found by Abramowicz et al. 2003b used older, less precise data for Sco X-1).

In addition, there is strong evidence that the spin frequency plays a role in generating at least one of the two strong kHz QPOs observed in neutron stars. This evidence comes from a comparison of the frequency separation $\Delta v$ between kHz QPOs with the spin frequency $v_{\text{spin}}$, in cases where both are known. In Table 2 we compare the range of observed frequency separations with the spin frequency as inferred from the persistent oscillations in SAX J1808–3658 (Wijnands & van der Klis 1998; Chakrabarty & Morgan 1998) and from the frequency of burst brightness oscillations in the other sources. The identification of the frequency of burst brightness oscillations with the spin frequency (or close to it) is well-established by observations of burst oscillations in SAX J1808–3658 (Chakrabarty et al. 2003) and XTE J1814–338 (Strohmayer et al. 2003) that are extremely close to the frequency of the oscillations during persistent emission. From Table 2 we see that all sources have $\Delta v$ close to $v_{\text{spin}}$ or $v_{\text{spin}}/2$, and indeed 4U 1702–429, SAX J1808–3658, and KS 1731–260 have $\Delta v$ consistent with exactly $v_{\text{spin}}$ or $v_{\text{spin}}/2$, within the uncertainties. As indicated by Table 2, the frequency difference does vary in several sources, requiring modification of basic spin modulation ideas (see, e.g., Lamb & Miller 2001), but the importance of the spin is clear.

The involvement of the spin, e.g., through radiation effects or stellar magnetic fields, indicates that there are at least some processes affecting neutron star QPOs that are different from those that generate black hole QPOs. No

### Table 1. Upper peak frequencies and frequency ratios for black hole QPO pairs.

| Source       | $v_{\text{upper}}$ (Hz) | $v_{\text{upper}}/v_{\text{lower}}$ |
|--------------|-------------------------|------------------------------------|
| GRS 1915+105*| 67±5                    | 1.63±0.13                          |
| XTE J1550–564†| 272±20                  | 1.48±0.24                          |
| GRO J1655–40**| 450±20                  | 1.5±0.13                           |

* Strohmayer 2001b
† Miller et al. 2001; Remillard et al. 2002
** Strohmayer 2001a; Remillard et al. 2002

### Figure 1. Ratios of upper to lower QPO peak frequencies for the neutron star LMXB Sco X-1 (data kindly provided by Mariano Méndez). The location of a 3:2 ratio is indicated by the dashed line. Unlike in the case of black hole QPO pairs, there is no preference in neutron star sources for a particular ratio or set of ratios. Similar trends are evident in other neutron star sources.
TABLE 2. Spin frequency and frequency separation for neutron star LMXBs. 4U 1702, SAX J1808, and KS 1731 have single measurements of $\Delta \nu$, with uncertainties indicated.

| Source         | $\nu_{\text{spin}}$ (Hz) | $\Delta \nu$ (Hz) |
|----------------|--------------------------|-------------------|
| 4U 1916–053*   | 270                      | 290–348           |
| 4U 1702–429†   | 329                      | 333±5             |
| 4U 1728–34‡    | 363                      | 342–363           |
| SAX J1808–3658¶ | 401              | 195±6             |
| KS 1731–260§   | 524                      | 260±10            |
| 4U 1636–536¶   | 581                      | 250–323           |
| 4U 1608–52†    | 620                      | 225–313           |

* Boirin et al. 2000; Galloway et al. 2001
† Markwardt, Strohmayer & Swank 1999
‡ Strohmayer et al. 1996
§ Wijnands et al. 2003; Chakrabarty et al. 2003
¶ Smith, Morgan, & Bradt 1997; Wijnands & van der Klis 1997
¶¶ Di Salvo, Méndez, & van der Klis 2003; Jonker, Méndez, & van der Klis 2002; Méndez, van der Klis, & van Paradijs 1998; Wijnands et al. 1997
|| Berger et al. 1996; Méndez et al. 1998; Méndez et al. 1999; Yu et al. 1997

such relation is evident for white dwarfs (Mauche 2002), suggesting again that even if there is a master underlying mechanism at work, there are important differences between the different classes of objects. General relativity does play a role for neutron stars and black holes, hence full understanding of the QPOs is promising for our understanding of strong gravity effects.

As an aside, the demonstration that $\Delta \nu \approx \nu_{\text{spin}}/2$ in SAX J1808–3658 is an indication of how the superb data available from RXTE, combined with sophisticated analysis (e.g., Chakrabarty et al. 2003, Wijnands et al. 2003), are still facilitating qualitative leaps in our understanding. Prior to the SAX J1808 analysis, I and many other researchers argued that $\Delta \nu \approx \nu_{\text{spin}}$ in all cases. The new results have forced modifications of the original models (for a recent proposal, see the sonic point and spin resonance beat frequency model of Lamb & Miller 2003), proving again the importance of high-quality timing data.

**IMPLICATIONS OF QPOS**

As discussed in, e.g., Lamb & Miller (2003), observations of kHz QPOs in neutron star LMXBs give a number of clues to their physical origin.

- The spin frequency is involved in producing the observed frequency differences. The difference can be close to $\nu_{\text{spin}}$ or $\nu_{\text{spin}}/2$. So far, sources with $\nu_{\text{spin}} > 400$ Hz always have $\Delta \nu \approx \nu_{\text{spin}}/2$, whereas sources with $\nu_{\text{spin}} < 400$ Hz always have $\Delta \nu \approx \nu_{\text{spin}}$ (Muno et al. 2001).

- This appears to be a single sideband phenomenon. That is, if the spin frequency modulates some other frequency, only one additional strong QPO is produced (additional QPOs have been found in 4U 1608–52, 4U 1728–34, and 4U 1636–536 by Jonker, Méndez, & van der Klis 2000, but these are much weaker than the primary peaks). This restricts models significantly; for example, amplitude modulation of one frequency by another produces two sidebands of equal strength.

- An excellent candidate for the other frequency is the orbital frequency or something close to it. The requirements are that the frequency be in the right range, while also being able to change frequency by hundreds of Hertz (for a review of the observational properties see, e.g., van der Klis 2000 or J. Swank, these proceedings). The orbital frequency has these properties. If the orbital frequency is involved, then because one expects accretion to align the stellar spin with the sense of the accretion disk over a time short compared to the accretion lifetime, the orbital and spin directions are the same and hence the orbital frequency is expected to be close to the upper peak frequency.

Detailed models need to identify a mechanism that produces the QPOs, selects a particular orbital radius among many, and allows this radius to change significantly (as indicated by the changing QPO frequencies). The current leading candidates include some variant of a beat frequency model (e.g., Lamb & Miller 2003), or possibly a resonance with the spin, modulating other frequencies (D. Psaltis, presented at the “Neutron Stars on Fire” conference, Princeton, NJ, 11-13 May 2003). In the former case, the upper peak frequency $\nu_{\text{upper}}$ is identified with a frequency close to an orbital frequency $\nu_{\text{orb}}$ at some special radius (e.g., the sonic radius; see Miller, Lamb, & Psaltis 1998), and in the latter case it is identified with a vertical epicyclic frequency of a nearly circular orbit $\nu_{\text{vertical}}$ (e.g., Abramowicz et al. 2003a). For constraints on neutron star structure these amount to the same thing because $\nu_{\text{vertical}} \approx \nu_{\text{orb}}$ outside a neutron star (for a discussion see Lamb & Miller 2003). There are details of the observations (e.g., the conditions under which $\Delta \nu \approx \nu_{\text{spin}}/2$ instead of $\nu_{\text{spin}}$) that are not obvious from first principles (for some ideas see Lamb & Miller 2003), but the general constraints on models suffice to constrain masses and radii as long as $\nu_{\text{upper}} \approx \nu_{\text{orb}}$ at some radius.

Titarchuk and colleagues (e.g., Titarchuk 2003) have suggested instead that it is the lower peak frequency $\nu_{\text{lower}}$ that is close to $\nu_{\text{orb}}$, with consequently different implications. In their model, the upper peak frequency $\nu_{\text{upper}}$ is instead close to the hybrid frequency $(\nu_{\text{lower}}^2 + 4\nu_{\text{mg}}^2)^{1/2}$, where the magnetospheric frequency $\nu_{\text{mg}} \approx \nu_{\text{spin}}$. This is an interesting suggestion, but the re-
cent high-precision measurements of SAX J1808 present puzzles for this model. Chakrabarty et al. (2003) show that the spin frequency is 401 Hz, rather than half this value. Wijnands et al. (2003) find a pair of QPOs, at $v_{\text{lower}} = 499$ Hz and $v_{\text{upper}} = 694$ Hz; the hybrid model would predict $v_{\text{upper}} = (499^2 + 4[401]^2)^{1/2} = 945$ Hz, in conflict with the observations. Similarly, data for KS 1731–260 present difficulties. Wijnands & van der Klis (1997) find $v_{\text{lower}} = 898$ Hz and $v_{\text{upper}} = 1159$ Hz. The burst oscillation frequency is 524 Hz (Smith et al. 1997). If $v_{\text{spin}} = 524$ Hz, the hybrid model predicts $v_{\text{upper}} = 1380$ Hz. If instead $v_{\text{spin}} = 262$ Hz, the hybrid model predicts $v_{\text{upper}} = 1040$ Hz. Both appear not in accord with the data. For this reason, we will concentrate on models in which $v_{\text{upper}} \approx v_{\text{orb}}$.

In such models, measurement of $v_{\text{upper}}$ constrains the mass and radius of a neutron star. If $R_{\text{orb}}$ is the radius of a circular orbit of frequency $v_{\text{orb}}$, then clearly $R < R_{\text{orb}}$. In addition, the high quality factors of the QPOs require that they be produced outside the region of unstable circular orbits predicted by general relativity. For a nonrotating star, for which the exterior spacetime is described by the Schwarzschild geometry, the radius of the innermost stable circular orbit (ISCO) is $R_{\text{ISCO}} = 6GM/c^2$. The constraint $R_{\text{orb}} > R_{\text{ISCO}}$ places a mass-dependent limit on the radius; for example, for a nonrotating star $R < (GM/4\pi^2 v_{\text{orb}}^2)^{1/3}$ (Miller et al. 1998). The additional constraint $R_{\text{orb}} > R_{\text{ISCO}}$ places an absolute upper limit on the mass and hence on the radius. When one considers frame-dragging effects, the upper limits on the mass and radius are (Miller et al. 1998)

$$
M \ < \ 2.2 \ M_\odot \left(1000 \text{ Hz}/v_{\text{orb}}\right) \left(1 + 0.75j\right) \ \ (1)
$$

$$
R \ < \ 19.5 \text{ km} \left(1000 \text{ Hz}/v_{\text{orb}}\right) \left(1 + 0.2 j\right). \ \ (1)
$$

Here $j \equiv cJ/\mathcal{M}^2$ is a dimensionless spin parameter, where $J$ is the stellar angular momentum. If in a particular case one believes that the observed frequency is in fact the orbital frequency at the ISCO, then the mass is equal to the upper limit given in equation (1).

The highest frequency QPO so far detected with confidence has a frequency $v_{\text{QPO}} = 1330$ Hz (van Straaten et al. 2000), which would imply $M \lesssim 1.8M_\odot$ and $R \lesssim 15$ km for a system with spin parameter $j = 0.1$. These constraints essentially rule out the hardest equations of state proposed (see Figure 2). The existence of the ISCO means that the frequencies cannot be arbitrarily high. If the radius at which the QPOs are generated gets close to the ISCO, a variety of signatures are possible, including flattening in the observed relation between frequency and countrate, or sharp drops in the amplitude or coherence of the QPO (see Miller et al. 1998). Such signatures would confirm the presence of unstable orbits, a key prediction of strong-gravity general relativity, and allow a direct mass measurement. It is possible that the system 4U 1820–30 has already shown such a signal (Zhang et al. 1998), but there are complications in the spectral behavior that make this uncertain (Méndez et al. 1999).

### PROSPECTS WITH A 10 M$^2$ INSTRUMENT

As discussed by Swank (these proceedings), a trend evident from RXTE observations is that as the mass accretion rate increases (inferred, e.g., from the $S_a$ index; Méndez et al. 1999), the frequencies of QPOs increase and the amplitudes of QPOs decrease. For many sources observed with RXTE, the inferred mass accretion rate continues to increase after the QPOs become unobserv-
able. This suggests that a larger area timing instrument would be able to detect higher frequencies. For example, for 4U 0614+091, projection of the amplitude versus accretion rate trends suggests that a 10 m$^2$ instrument could detect frequencies of $\sim 1500$ Hz (M. van der Klis, personal communication). Other atoll sources could yield even higher frequencies.

An important threshold is reached at $\sim 1500$ Hz, because this is at or above the orbital frequency at the ISCO for realistic masses. The orbital frequency at radius $r$ in a Kerr spacetime of spin parameter $j$ is $\Omega = M^{1/2}/(r^{3/2} + jM^{3/2})$ (e.g., Shapiro & Teukolsky 1983, equation 12.7.19), and to $\theta'(j)$, $R_{\text{ISCO}} = 6M[1 - (2/3)^{3/2}]$ (see, e.g., Miller & Lamb 1996), so the orbital frequency is

$$v_{\text{orb.ISCO}} \approx 2199 \text{ Hz}(M_{\odot}/M) \left[1 + \frac{11}{8} \left(\frac{2}{3}\right)^{3/2}j\right].$$

(2)

For $j = 0.1$, $v_{\text{orb.ISCO}} < 1500$ Hz for $M > 1.58 M_{\odot}$. It has long been suspected that neutron stars in LMXBs are more massive than the canonical $1.4M_{\odot}$ because of mass transfer, and direct evidence of this has arrived recently. Nice, Splaver, & Stairs (2003) report that the 22 ms pulsar in the 0.26 day binary J0751+1807 (with a low-mass white dwarf companion, likely a remnant after substantial mass transfer) has $M > 1.6M_{\odot}$ at better than 95% confidence, and the mass could be well above this. Measurements of QPOs above $1500$ Hz therefore have excellent prospects for stronger constraints on masses and radii, and even for detection of signatures of the ISCO. A QPO frequency as high as $1800$ Hz would be large enough to argue against all standard nucleonic or hybrid quark matter equations of state, leaving only strange stars (see Figure 3).

A qualitative advantage of a large-area timing instrument compared with RXTE is that there are a number of sources with strong enough QPOs that they could be detected in less than a coherence time. For example, in some states, the kHz QPOs in Sco X-1 could be detected within $\approx 4$ ms with a 10 m$^2$ instrument (M. van der Klis, personal communication). Because current detections are averaged over many coherence times, it is not possible to determine whether, e.g., QPOs are present at all times or whether they are superpositions of more coherent pulses. A larger area instrument would help resolve this, and if indeed there are underlying highly coherent pulses this could lead to substantial additional insights. For example, if the QPOs are caused by the orbits of radiating clumps, then observation within a coherence time would lead to detection of Doppler shifts, which when combined with the observed frequency would allow a unique solution of both the gravitational mass of the neutron star and the radius of the orbit.

**FIGURE 3.** Constraints on mass and radius for hypothetical detections of a $1500$ Hz QPO and a $1800$ Hz QPO, if they are identified with an orbital frequency. At $1500$ Hz one expects signatures of the ISCO to be present; a detection of $1800$ Hz would present strong difficulties for standard nucleonic equations of state. The equation of state curves are as in Figure 2. Rotational effects are not included in this figure.

In conclusion, RXTE observations have not only revealed a previously unsuspected phenomenon, but have constrained models of kHz QPOs significantly. With current data, we are just short of expected signatures of the innermost stable circular orbit, a crucial predicted characteristic of strong gravity. A larger-area instrument is likely to push us over this important threshold, and also to allow novel new methods of analysis that can detect qualitatively new phenomena such as periodic Doppler shifts of orbiting clumps.

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