QUASI-BIENNIAL OSCILLATIONS IN THE SOLAR TACHOCLINE CAUSED BY MAGNETIC ROSSBY WAVE INSTABILITIES

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ABSTRACT

Quasi-biennial oscillations (QBOs) are frequently observed in solar activity indices. However, no clear physical mechanism for the observed variations has been suggested so far. Here, we study the stability of magnetic Rossby waves in the solar tachocline using the shallow water magnetohydrodynamic approximation. Our analysis shows that the combination of typical differential rotation and a toroidal magnetic field with a strength of \( \gtrsim 10^5 \) G triggers the instability of the \( m = 1 \) magnetic Rossby wave harmonic with a period of \( \lesssim 2 \) years. This harmonic is antisymmetric with respect to the equator and its period (and growth rate) depends on the differential rotation parameters and magnetic field strength. The oscillations may cause a periodic magnetic flux emergence at the solar surface and consequently may lead to the observed QBO in solar activity features. The period of QBOs may change throughout a cycle, and from cycle to cycle, due to variations of the mean magnetic field and differential rotation in the tachocline.

Key words: magnetic fields – magnetohydrodynamics (MHD) – Sun: oscillations – waves

Online-only material: color figures

1. INTRODUCTION

Apart from the well-known 11 year cycle, solar activity shows quasi-periodic variations on shorter timescales. Two different timescales have been frequently observed in many solar activity indicators: several months and a few years. The oscillations with periods of several months (mostly with 150–160 days) are known as Rieger-type periodicities (Rieger et al. 1984; Lean & Brueckner 1989; Carbonell & Ballester 1990; Oliver et al. 1998; Krivova & Solanki 2002; Kane 2005; Knaack et al. 2005; Chowdhury et al. 2009). The oscillations with periods of \( \sim 2 \) years are known as quasi-biennial oscillations (QBOs) and they modulate almost all indices of solar activity (Sakurai 1981; Gigolashvili et al. 1995; Knaack et al. 2005; Kane 2005; Danilovic et al. 2005; Forgács-Dajka & Borkovits 2007; Badalyan et al. 2008; Javaraiah et al. 2009; Laurenza et al. 2009; Vecchio & Carbone 2009; Vecchio et al. 2010; Šyko & Rybak 2010).

The source(s) of these periodicities is still unclear. Several mechanisms have been suggested to drive the Rieger-type periodicities: interaction between the typical differential rotation and a toroidal magnetic field with a strength of \( \gtrsim 10^5 \) G favors the growth of a magnetic Rossby wave harmonic with a period of \( \sim 2 \) years. This harmonic is antisymmetric with respect to the equator and its period (and growth rate) depends on the differential rotation parameters and magnetic field strength. The oscillations may cause a periodic magnetic flux emergence at the solar surface and consequently may lead to the observed QBO in solar activity features. The period of QBOs may change throughout a cycle, and from cycle to cycle, due to variations of the mean magnetic field and differential rotation in the tachocline.

2. SHALLOW WATER MHD EQUATIONS AND UNSTABLE MAGNETIC ROSSBY WAVE HARMONICS

In the following we use a spherical coordinate system \((r, \theta, \phi)\) rotating with the solar equator, where \(r\) is the radial coordinate, \(\theta\) is the colatitude, and \(\phi\) is the longitude.

The solar differential rotation law in general is

\[
\Omega = \Omega_0(1 - s_2 \cos^2 \theta - s_4 \cos^4 \theta) = \Omega_0 + \Omega_1(\theta),
\]

where \(\Omega_0\) is the equatorial angular velocity and \(s_2, s_4\) are constant parameters determined by observations.

In the solar tachocline, the magnetic field is predominantly toroidal, \(\vec{B} = \Xi \hat{\phi}\), and we take \(\Xi = B_\phi(\theta) \sin \theta\), where \(B_\phi\) is in general a function of colatitude. Then, the linear SWMHD equations (Gilman 2000) can be rewritten in the rotating frame (with \(\Omega_0\)) as follows:

\[
\frac{\partial u_\theta}{\partial t} + \Omega_0 \frac{\partial u_\theta}{\partial \phi} - 2\Omega \cos \theta u_\phi = -g \frac{\partial h}{\partial \theta} + \frac{b_\phi}{R_0} \frac{\partial b_\phi}{\partial \phi} - \frac{2}{4\pi \rho R_0} B_\phi \cos \theta b_\phi,
\]

or

\[
\frac{\partial H_\theta}{\partial t} + \Omega_0 \frac{\partial H_\theta}{\partial \phi} - 2\Omega \cos \theta H_\phi = -g \frac{\partial h}{\partial \theta} + \frac{b_\phi}{R_0} \frac{\partial b_\phi}{\partial \phi} - \frac{2}{4\pi \rho R_0} B_\phi \cos \theta b_\phi.
\]
\[
\frac{\partial u_\phi}{\partial t} + \Omega_1 \frac{\partial u_\phi}{\partial \phi} + \frac{u_\theta}{\sin \theta} \frac{\partial [\Omega \sin^2 \theta]}{\partial \theta} = -\frac{g}{R_0 \sin \theta} \frac{\partial h}{\partial \phi} + \frac{B_\phi}{4\pi \rho R_0} \frac{\partial b_\theta}{\partial \phi} + \frac{b_\theta}{4\pi \rho R_0 \sin \theta} \frac{\partial [B_\phi \sin^2 \theta]}{\partial \theta},
\]
\[
\frac{\partial b_\theta}{\partial t} + \Omega_1 \frac{\partial b_\theta}{\partial \phi} + \frac{u_\theta}{\sin \theta} \frac{\partial b_\theta}{\partial \theta} = \frac{B_\phi}{R_0} \frac{\partial u_\phi}{\partial \phi},
\]
\[
\frac{\partial}{\partial \theta} (\sin \theta b_\theta) + \frac{\partial b_\theta}{\partial \phi} + \frac{B_\phi \sin \theta}{H_0} \frac{\partial h}{\partial \phi} = 0.
\]
where \(u_\theta, u_\phi, b_\theta,\) and \(b_\phi\) are the velocity and magnetic field perturbations, \(H_0\) is the tachocline thickness and \(h\) is its perturbation, \(\rho\) is the density, \(g\) is the reduced gravity, and \(R_0\) is the distance from the solar center to the tachocline. Equations (5) and (6) are the solenoidal conditions of a shallow water magnetic field and velocity, respectively (Gilman 2000).

Fourier analysis with \(\exp[i m (\phi - c t)]\) and the transformation of variables \(\mu = \cos \theta\) in Equations (2)–(6) lead to the equations

\[
[(c - \Omega_1)^2 - \Omega_A^2](1 - \mu^2) \frac{\partial H}{\partial \mu} - 2\mu [(\Omega(c - \Omega_1) + \Omega_A^2)H]
= -i m \Omega_A^2 h + i m (1 - \mu^2) [(c - \Omega_1)^2 - \Omega_A^2] h,
\]
\[
2\mu [(\Omega(c - \Omega_1) + \Omega_A^2)(1 - \mu^2) \frac{\partial H}{\partial \mu} - \left[ m^2(c - \Omega_1)^2 - m^2 \Omega_A^2 \right] H
+ \mu (1 - \mu^2) \frac{\partial \Omega_A^2}{\partial \mu} - \mu (1 - \mu^2) \frac{\partial \Omega_A^2}{\partial \mu} \right] +
= i m \Omega_A^2 (1 - \mu^2) \frac{\partial h}{\partial \mu} + 2i m \mu (1 - \mu^2) [(\Omega(c - \Omega_1) + \Omega_A^2) h],
\]
where
\[
\Omega_A = \frac{B_\phi}{4\pi \rho R_0}, \quad \Omega_g = \sqrt{\frac{g}{R_0}},
\]
are the Alfvén and surface gravity frequency, respectively, \(h\) is normalized by \(H_0\) and
\[
H(\mu) = \frac{b_0 \mu \sqrt{1 - \mu^2}}{B_\phi} = \frac{u_\theta(\mu) \sqrt{1 - \mu^2}}{R_0 (c - \Omega_1)}.
\]
In the following sections we use a magnetic field
\[
B_\phi = B_0 \mu,
\]
which changes sign at the equator (Gilman & Fox 1997).

We expand \(H\) and \(h\) in infinite series of associated Legendre polynomials (Longuet-Higgins 1968)
\[
H = \sum_{n=m}^\infty a_n P_n^m(\mu), \quad h = \sum_{n=m}^\infty b_n P_n^m(\mu),
\]
which satisfy the boundary conditions \(H = h = 0\) at \(\mu = \pm 1\) (i.e., at the solar poles). Then, we substitute Equation (11) into Equations (7) and (8) and, using a recurrence relation of Legendre polynomials, obtain algebraic equations as infinite series.

The dispersion relation for the infinite number of harmonics can be obtained when the infinite determinant of the system is set to zero. In order to solve the determinant, we truncate the series at \(n = 70\), and the resulting polynomial in \(\omega\) is solved numerically. This gives the frequencies of the first 70 harmonics. The harmonics with real frequencies are stable, while those with complex frequencies are unstable (see the general technique of Legendre polynomial expansion in Longuet-Higgins 1968; Watson 1981; Gilman & Fox 1997; Zaqarashvili et al. 2010, and references therein).

The typical values of equatorial angular velocity, radius, and density in the tachocline are \(\Omega_0 = 2.7 \times 10^{-6} \text{ s}^{-1}\), \(R_0 = 5 \times 10^{10} \text{ cm}\), and \(\rho = 0.2 \text{ g cm}^{-3}\), respectively. We use a tachocline thickness \(H_0 = 0.02 R_0 = 10^9 \text{ cm}\). The ratio between angular and surface gravity frequencies \(\epsilon = \Omega_0^2 / \Omega_g^2 = \Omega_0^2 R_0^2 / (g H_0)\) is an important parameter in the shallow water theory. \(\epsilon \ll 1\) means strongly stable stratification (main part of tachocline), while \(\epsilon > 1\) considers weakly stable stratification (upper overshoot region). Here, we consider the mean part of the tachocline and thus we take the limit \(\epsilon \ll 1\).

The observed differential rotation parameters near the solar surface satisfy \(s_2 + s_4 \approx 0.28\), which may tend to \(s_2 + s_4 \approx 0.26\) near the upper part of the tachocline (Schou et al. 1998). The solar radiative interior rotates uniformly, therefore the latitudinal differential rotation parameters drop to zero from the upper part of the tachocline to its base. The radial dependence of latitudinal differential rotation through the tachocline is not clear, and \(s_2, s_4\) may also vary throughout the solar cycle (Howe et al. 2000). Therefore, \(s_2 + s_4\) may take any value between 0.26 and 0.

Figure 1 shows the real, \(m_c\), and imaginary, \(m_i\), frequencies of all \(m = 1\) unstable harmonics for different combinations of \(\epsilon\) (i.e., reduced gravity) and magnetic field strength. The differential rotation parameters are fixed to \(s_2 = s_4 = 0.11\). The plot shows that each combination of the magnetic field strength, differential rotation parameters, and reduced gravity leads to the occurrence of a particular harmonic whose growth rate is much stronger than that of other harmonics. This is similar to what happens in the two-dimensional case (Zaqarashvili et al. 2010). An increase of magnetic field strength leads to the reduction of the frequency of the most unstable harmonic. The unstable harmonics are mostly symmetric (defined by astersisks) with respect to the equator for a magnetic field strength \(< 10^4 \text{ G}\), while they become mostly antisymmetric (defined by circles) for a strength \(> 10^5 \text{ G}\). A magnetic field strength between \(10^4\) and \(10^5 \text{ G}\) yields unstable harmonics for both symmetries. This can be explained in terms of magnetic and differential rotation energies. Equi-partition between the magnetic energy and the kinetic energy of differential rotation occurs at \(< 5 \times 10^8 \text{ G}\) for \(s_2 = s_4 = 0.11\). When the magnetic field strength is smaller, then the differential rotation is the main energy source for instability and this obviously yields the symmetric harmonics as the differential rotation is symmetric around the equator. However, when the magnetic field is stronger, then the magnetic energy is the main source for the instability and the unstable harmonics are antisymmetric as the magnetic field is antisymmetric with respect to the equator.

The importance of the equi-partition value of the magnetic field strength is clearly seen in Figure 2. This figure displays the periods and growth rates (defined as \(m_c / \Omega_0\)) versus magnetic field strength. The growth rates are higher for weaker \(< 10^4 \text{ G}\) and stronger \(> 10^5 \text{ G}\) magnetic fields. However, the growth rates are much lower when the magnetic field strength is inside the interval \(10^4\)–\(10^5 \text{ G}\). The weaker field \(< 10^4 \text{ G}\) favors
Rieger-type periodicities (150–160 days), while the stronger field (>10^4 G) supports QBOs. Increasing the magnetic field suppresses symmetric harmonics as it has been shown in Paper I.

Figure 3 displays the period of the most unstable symmetric (asterisks) and antisymmetric (circles) harmonics vs. $\epsilon$ for a magnetic field strength of $8 \times 10^4$ G and the differential rotation parameters $s_2 = s_4 = 0.11$. (A color version of this figure is available in the online journal.)

3. DISCUSSION

Our results show that the differential rotation and the magnetic field with a strength of $>10^5$ G may lead to large-scale oscillations of the tachocline with periods of $\sim$2 years. The oscillation is due to the $m = 1$ unstable mode of magnetic Rossby waves. The magnetic Rossby waves are magnetohydrodynamic counterparts to usual hydrodynamic Rossby waves (Zaqarashvili et al. 2007, 2009). The period and growth rate of the unstable harmonics depend on the magnetic field strength and the differential rotation parameters (Figures 1, 2, and 4). The unstable harmonics with periods of $\sim$2 years are antisymmetric with respect to the solar equator.

The unstable magnetic Rossby waves in the tachocline can be the reason for QBOs observed in almost all indices of the solar activity. Recent papers suggest that QBOs are not persistent...
but may vary from cycle to cycle (Vecchio & Carbone 2009) and throughout a cycle (Sýkora & Rybak 2010). Our analyses also suggest this behavior as the period of unstable harmonics depends on magnetic field strength and differential rotation parameters, which may vary in time depending on phase and strength of a particular cycle.

The antisymmetric behavior of unstable harmonics with respect to the equator may explain the recent observational results of Badalyan et al. (2008), which show that QBOs are well recognizable in the N–S asymmetry of solar activity indices.

Our analysis suggests the reduction of growth rates of unstable harmonics when the magnetic field strength is inside the interval $10^4 – 10^5$ G (see Figure 2). It is clearly seen that the relatively weak magnetic field $< 10^4$ G leads to the occurrence of Rieger-type periodicities (see the same results in Paper I), while the field of $> 10^4$ G favors QBOs. The upper overshoot region of the tachocline probably contains relatively weaker magnetic field compared to the lower stable layers. Therefore, we may speculate that the Rieger-type periodicities are formed in the overshoot layer (this was also suggested in Paper I), while QBOs are formed in lower layers with strongly stable stratification. Therefore, the both periodicities may occur simultaneously.

The magnetic field of $10^5$ G is unstable due to the buoyancy instability, which makes difficult to keep it in the tachocline. On the other hand, the emergence of magnetic flux at observed latitudes requires sufficiently strong magnetic field ($\sim 10^5$) below the convection zone (Fan 2004). Therefore, the storage of strong fields below the convection zone is still open question.

Significant simplifications in our approach is the linear stability analysis, which only describes the initial phase of instability. Intense numerical simulations are needed in the future to study the developed stage of magnetic Rossby wave instabilities.

4. CONCLUSIONS

We have shown that the unstable magnetic Rossby waves in the solar tachocline could be responsible for the observed intermediate periodicities in solar activity. The periods and growth rates of unstable harmonics depend on the differential rotation parameters and the magnetic field strength. The unstable harmonics are either symmetric or antisymmetric with respect to the equator. The latitudinal differential rotation is mainly responsible for the growth of symmetric harmonics, while the antisymmetric toroidal magnetic field favors the growth of antisymmetric harmonics. A magnetic field with a strength $\lesssim 10^4$ G leads to oscillations with shorter period (150–170 days), while a stronger magnetic field $\gtrsim 10^5$ G favors oscillations with longer periods (1–2.5 years). Hence, $\sim 2$ year oscillations can be formed in the main part of the tachocline with stronger toroidal magnetic field and strongly stable stratification. The oscillations may trigger the periodic magnetic flux emergence at the solar surface and consequently QBOs in solar activity.

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Figure 4. Periods of the most unstable symmetric (asterisks) and antisymmetric (circles) harmonics vs. $x_4$ for different values of $x_2$. Dark blue, blue, green, magenta, and red colors correspond to $x_2 = 0.13$, 0.12, 0.11, 0.10, and 0.09, respectively. The magnetic field strength equals $8 \times 10^4$ G and $\epsilon = 0.12$. (A color version of this figure is available in the online journal.)