Research on Parallel Control Technology of Three-phase Inverter Based on Multiple Proportional Resonance Controller

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Abstract. The parallel operation of multiple inverters can increase the fault tolerance of the system and prevent the occurrence of major accidents. At the same time, it also has strong flexibility, which can reduce the volume and weight of the system and increase the capacity of the system, which is very attractive in the application of uninterruptible power supply. In the inverter parallel system, the circulation will be generated between the inverters due to the existence of the path. Circulation can cause current distortion, reduce system performance, and even damage equipment in severe cases. In order to effectively suppress the generation of circulation, this paper proposes a multiple proportional resonance control strategy for the parallel three-phase inverter system, that is, the voltage outer loop is controlled by PI+PR, and the current inner loop is controlled by PR. Two three-phase inverter parallel systems are built in MATLAB/Simulink simulation environment to compare and analyze the control effect of uncontrolled, traditional PI control and multiple PR control. The simulation results show that the proposed control method has a good suppression effect on the circulation generated in the inverter parallel system, even under the condition of unbalanced load.

1. Introduction

With the rapid development of society and the improvement of people's living standard, people's demand for energy is increasing day by day, and the traditional energy can no longer meet the needs of today's social development. With the development of renewable energy, more and more distributed power generation is connected to the public grid through the inverter [1]. The system composed of multiple inverters in parallel can improve the system capacity grade, which has the advantages of high redundancy, high reliability, convenient maintenance and so on, and is widely used. In the inverter parallel system, there are many problems to be solved, such as ensuring the amplitude, phase angle and frequency consistency of the output voltage of the parallel inverter [2]. If the output voltage cannot be consistent, there will be a possible circulation between the inverters, which can lead to the distortion of the output current and make the distribution of the current unbalanced between the loads, and the harmonics in the circulation will increase the power loss in the system and reduce the performance of the whole system [3]. However, in the actual system, because the physical parameters and control parameters of the inverter cannot normally be completely equal, it is impossible to...
guarantee that the parallel inverter is exactly the same. In addition, in the actual microgrid system, there are a large number of single-phase load and asymmetric load, which will cause the problem of three-phase voltage imbalance, and then make the circulation increase and affect the system balance. Therefore, in the design of parallel system, we need to adopt a certain control strategy to effectively suppress the circulation [4].

At present, there are two commonly used circulation suppression methods, one is to use the hardware method to interrupt the circulation path, the other is to suppress the circulation through the appropriate control method. Most of the traditional methods use the former one, such as the use of multi-winding isolation transformer to make the zero-sequence circulation cannot flow. But this will make the volume and weight of the inverter bigger and increase the production cost. Therefore, many scholars use the control method to restrain the circulation [5].

The traditional voltage outer-loop current inner-loop proportional integration dual-loop control method is widely used in the inverter, but the PI controller cannot control the sinusoidal quantity without difference, nor can it suppress the influence caused by the circulation in the three-phase inverter parallel system [6]. In this paper, a multiple PR control strategy is adopted, the traditional voltage outer loop is controlled by PR+PI, and the current inner loop is controlled by PR. The PR controller is composed of the proportional regulator and the harmonic oscillator. Compared with the traditional PI controller, the proportional resonance controller has an infinite gain at the fundamental frequency, so the steady-state error can be completely eliminated. Through the theoretical analysis, a more easily realized quasi-resonant controller is used in the system, which cannot only maintain the high gain of the resonant controller, but also increase the anti-disturbance ability of the system. In this paper, the structure and model of parallel inverter are first analyzed, and the principles and methods of PI control and multiple PR control are proposed. At last, the effectiveness of the proposed control method is verified by MATLAB/Simulink simulation software.

2. Modeling of Multi-Inverter Parallel System

2.1. Single Inverter System Modeling

The parallel three-phase inverter structure studied in this paper is shown in figure 1. The two inverters share a DC bus, and the AC side is connected to the three-phase power supply after passing through the LC filter. Where $U_i$ is the DC bus voltage, $U_{a_k}, U_{b_k}, U_{c_k}$ and $i_{a_k}, i_{b_k}, i_{c_k}$ ($k=1, 2$) are the output voltage and current of the k-th inverter respectively, $L_k$ is the filter inductance, $r_k$ is the equivalent resistance of filter inductance, $C_f$ is filter capacitor.

![Parallel Inverter System Topology](image)

**Figure 1.** Parallel Inverter System Topology
When there is only one inverter, according to Kirchhoff’s law of voltage, the following formulas for voltage and current can be obtained,

\[
\begin{align*}
U_a &= L \frac{di_a}{dt} + r_i + U_{ca} \\
U_b &= L \frac{di_b}{dt} + r_i + U_{cb} \\
U_c &= L \frac{di_c}{dt} + r_i + U_{cc}
\end{align*}
\]

(1)

\[
\begin{align*}
C_i \frac{dU_{ca}}{dt} &= i_a - i_{la} \\
C_i \frac{dU_{cb}}{dt} &= i_b - i_{lb} \\
C_i \frac{dU_{cc}}{dt} &= i_c - i_{lc}
\end{align*}
\]

(2)

To facilitate the control, it is necessary to convert the AC component into a DC component [7]. The equation of state of the system can be obtained by transforming the model from a three-phase static coordinate system to a dq two-phase rotating coordinate system by means of a Parker transformation. The state equation of the system is as follows:

\[
\begin{bmatrix}
\frac{d}{dt} 
\begin{bmatrix}
U_{cd} \\
U_{cq} \\
U_{c0}
\end{bmatrix}
\end{bmatrix} = 
\begin{bmatrix}
0 & \omega & 0 \\
-\omega & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
U_{cd} \\
U_{cq} \\
U_{c0}
\end{bmatrix} + 
\frac{1}{C_i}
\begin{bmatrix}
i_d \\
i_q \\
i_0
\end{bmatrix} - 
\frac{1}{L}
\begin{bmatrix}
i_d \\
i_q \\
i_0
\end{bmatrix}
\]

(3)

\[
\begin{bmatrix}
\frac{d}{dt} 
\begin{bmatrix}
i_d \\
i_q \\
i_0
\end{bmatrix}
\end{bmatrix} = 
\begin{bmatrix}
-r_f & \omega & 0 \\
0 & -r_f & 0 \\
0 & 0 & -r_f
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
i_0
\end{bmatrix} + 
\frac{1}{L}
\begin{bmatrix}
U_d \\
U_q \\
U_{c0}
\end{bmatrix}
\]

(4)

Where,

\[
\begin{bmatrix}
[\begin{bmatrix}
U_{cd}, U_{cq}, U_{c0}
\end{bmatrix} \quad \begin{bmatrix}
U_{da}, U_{dq}, U_{d0}
\end{bmatrix} \quad \begin{bmatrix}
l_{id}, l_{iq}, l_{i0}
\end{bmatrix}
\end{bmatrix} = P \ast [U_{ca}, U_{cb}, U_{cc}]^T, \quad \begin{bmatrix}
U_{da}, U_{dq}, U_{d0}
\end{bmatrix} = P \ast [U_{a}, U_{b}, U_{c}]^T
\]

\[
[i_{id}, i_{iq}, i_{i0}]^T = P \ast [l_{id}, l_{iq}, l_{i0}]^T, \quad [i_d, i_q, i_0]^T = P \ast [i_a, i_b, i_c]^T, \quad i = 1, 2, 3, \ldots, N
\]

\[
P = \sqrt{2}
\begin{bmatrix}
\cos \theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\
-\sin \theta & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3)
\end{bmatrix}
\]

Where, \(P\) is peck transformation matrix; \(\omega\) is the grid angle frequency.
2.2. Modeling of Inverter Parallel System

For the inverter parallel system, the zero sequence current also needs to be modeled. Figure 2 shows a single bridge arm of the inverter, where $S_{up}$ and $S_{an}$ are switch functions of upper and lower bridge arms respectively and $S_{up} + S_{an} = 1$; According to the duty cycle and voltage current relationship of the switch on a bridge arm, the average model can be obtained as shown in Figure 3. Then the average equivalent model of a single three-phase inverter is shown in figure 4. The average equivalent model of the two inverters is shown in Fig.5.

Where, defined $i_z$ as zero sequence current:

$$i_z = i_{ak} + i_{bk} + i_{ck} \quad (6)$$

Compared with figures 3 and 4, it can be seen that there is no circulation in a single inverter system because there is no circulation path. But in the inverter parallel system, there is a circulation path, so there is a circulation [8].

The circulation $i_H$ of parallel inverter system is defined as

$$i_H = \frac{i_{a1} - i_{a2}}{2} \quad (7)$$

![Figure 2. One bridge arm of inverter](image)

![Figure 3. Average model of one bridge arm](image)
3. Control strategy

3.1. Traditional PI Control Algorithm

In order to simplify the design of the controller, it is necessary to transform the model from three-phase stationary coordinate system to two-phase rotating coordinate system. In a two-phase rotating coordinate system, the ac quantity is converted to dc quantity, and the PI controller can track the dc component well [9].

The transfer function of the PI controller is:

$$G_p(s) = K_p + \frac{K_i}{s}$$  \hspace{1cm} (8)

Where, $K_p$ and $K_i$ are proportional and integral control parameters respectively.

From formula (3) and formula (4), we can deduce:
\[
\begin{aligned}
    i_d &= C \frac{dU_{cd}}{dt} + i_{id} - C \omega U_{cq} \\
    i_q &= C \frac{dU_{cq}}{dt} + i_{iq} + C \omega U_{cd} \\
    U_d &= L \frac{di_d}{dt} + ri_d + U_{cd} - L \omega i_q \\
    U_q &= L \frac{di_q}{dt} + ri_q + U_{cq} - L \omega i_d \\
\end{aligned}
\]  
\[ (9) \]

\[
\begin{aligned}
    C \frac{dU_{cd}}{dt} &= (K_{vp} + \frac{K_{vi}}{s})(U_{cd}^* - U_{cd}) \\
    C \frac{dU_{cq}}{dt} &= (K_{vp} + \frac{K_{vi}}{s})(U_{cq}^* - U_{cq}) \\
    L \frac{di_d}{dt} + ri_d &= (K_{ip} + \frac{K_{ii}}{s})(i_d^* - i_d) \\
    L \frac{di_q}{dt} + ri_q &= (K_{ip} + \frac{K_{ii}}{s})(i_q^* - i_q) \\
\end{aligned}
\]  
\[ (10) \]

Where, \( K_{vp} \) and \( K_{vi} \) are the proportional and integral gains of PI controller of voltage outer loop respectively; \( K_{ip} \) and \( K_{ii} \) are the proportional and integral gains of the current inner loop PI controller, respectively; \( U_{cd}^* \) and \( U_{cq}^* \) are the reference values of the voltage outer loop; \( i_d^* \) and \( i_q^* \) are the reference values of the current inner loop [10].

From formulas (9) and (10), it can be seen that at this time the d-axis and q-axis components have been decoupled. The decoupled control block diagram of the three-phase inverter is shown in figure 6.

**Figure 6.** Decoupling control block diagram of three-phase inverter

Firstly, the d-axis is taken as an example to analyze the transfer function of the control system, and the current inner loop controller is designed [11].

Laplace transformation of formula (9):

\[
\begin{aligned}
    U_d &= C \frac{dU_{cd}}{dt} + i_{id} - C \omega U_{cq} \\
    U_q &= C \frac{dU_{cq}}{dt} + i_{iq} + C \omega U_{cd} \\
    U_d &= L \frac{di_d}{dt} + ri_d + U_{cd} - L \omega i_q \\
    U_q &= L \frac{di_q}{dt} + ri_q + U_{cq} - L \omega i_d \\
\end{aligned}
\]
The current control loop is shown in figure 7.

The closed-loop transfer function of the current inner loop can be deduced as follows:

\[
G_t(s) = \frac{i_d}{i_{id}} = \frac{(K_{ip}s + K_{iu})}{Ls^2 + (r + K_{ip}s) + K_{iu}}
\]  

(11)

Compared to the standard second order equation, we can deduce the integral gain and the proportional gain of the current inner loop:

\[
\begin{cases}
K_{ip} = 2L\xi\omega_n - r \\
K_{iu} = L\omega_n^2
\end{cases}
\]

(12)

\(\omega_n\): Natural frequencies; \(\xi\): Damping coefficient.

In the same way, the proportional and integral gain of the voltage outer loop can be derived.

3.2. Proportional Resonance Control Algorithm

We present a design scheme of a quasi-PR controller to optimize the tracking performance and anti-interference performance of the system in this paper. The transfer function of an ideal quasi-PR controller is:

\[
G(s) = k_p + \frac{2k_r\omega_s}{s^2 + 2\omega_cs + \omega_r^2}
\]

(13)

In this formula, \(k_p\) is proportional gain and \(k_r\) is resonance gain. \(\omega_r\) is resonance angular frequency and \(\omega_c\) is cut-off frequency. As we can know from the transfer function:
\[
\frac{y(s)}{u(s)} = \frac{s}{s^2 + 2\omega_c s + \omega_c^2}
\]  \hspace{1cm} (14)

Split the formula (14),

\[
\begin{align*}
    y(s) &= \frac{1}{s} [u(s) - v(s) - w(s)] \\
    v(s) &= \frac{1}{s} \omega_c^2 y(s) \\
    w(s) &= 2\omega_c y(s)
\end{align*}
\]  \hspace{1cm} (15)

According to formula (15), the control block diagram of the quasi-PR controller can be obtained. Taking the \( \alpha \) axis as an example, the control system with a quasi-PR controller is shown in Figure 8:

![Figure 8. Quasi-PR controller control block diagram](image)

According to the transfer function, the controller has 3 parameters: \( k_p, k_r, \) and \( \omega_c \). In order to facilitate the analysis, it is assumed that any two parameters are unchanged, and then the effect of the third parameter change on the system performance is observed.

Assuming \( k_p = 0, \omega_c = 1 \), when \( k_r \) is changed, then the bode plot of formula (13) is shown in Figure 9. It can be seen that \( k_r \) only affects the gain of the controller and does not affect the bandwidth of the controller. The gain of the controller is proportional to \( k_r \).
Figure 9. Bode plot when $k_r$ changed

Assuming $k_p = 0$, $k_r = 1$, when $\omega_c$ is changed, then the bode plot is shown in Figure 10. We can know that $\omega_c$ not only affects the gain of the controller, but also the bandwidth of the controller. With the increasing of $\omega_c$, both the gain and bandwidth of the controller are increased. Assuming $k_p = 0$, we substitute $s = j\omega$ into formula (13) and then we can obtain the formula:

$$G(j\omega) = \frac{2k_r \omega_c j\omega}{-\omega^2 + 2\omega_c \omega j + \omega_c^2} = \frac{k_r}{1 + j(\omega^2 - \omega_c^2) / 2\omega_c \omega}$$

Figure 10. Bode plot when $\omega_c$ changed
We can know from the definition of broadband, when \( |G(j\omega)| = \frac{k_r}{\sqrt{2}} \) is established, the corresponding two frequency differences are the bandwidth. Assuming \( \left| \omega^2 - \omega_0^2 \right| / 2\omega_0 \omega = 1 \), the bandwidth of the quasi-resonant controller is calculated as \( \omega_c / \pi \) Hz.

Assuming the allowable fluctuation range of the grid frequency is \( \pm 0.8 \) Hz, then \( \omega_c / \pi = 1.6 \) Hz and \( \omega_c = 5 \) rad/s.

In the end, assuming \( \omega_c = 5 \), \( k_r = 100 \) and \( k_p \) changed, then the bode plot is shown in Figure 11. System harmonic impedance is affected by \( k_p \). With the increased \( k_p \), the greater the harmonic impedance of the system, the better the anti-interference performance of the system.

![Bode Diagram](image)

**Figure 11.** Bode plot when \( k_p \) changed

In summary, when designing a quasi-PR controller, we need to choose \( \omega_c \) according to the controller bandwidth, choose \( k_r \) according to the controller gain, and choose \( k_p \) according to the harmonic impedance [12].

Under the premise that the system can be stable, we need to increase \( k_r \) as much as possible to improve the tracking accuracy of the reference voltage and the ability to resist load disturbances so that we can achieve high-precision control of the output voltage. Combined with the specific simulation model, the parameters of the quasi-PR controller used are as follows: \( k_p = 1, \omega_c = 8, k_r = 100 \).

**4. Analysis of simulation results**

In order to verify the effectiveness of the proposed control method, a two-parallel inverter system model as shown in Fig. 1 was established in the Matlab/Simulink simulation.

Set the DC voltage at the bus terminal to 600V and the filter capacitor to 30uF. The load line voltage is 380V, frequency is 50Hz, and power is 20kw. In order to make the parameters of the two parallel inverters different, the filter inductance of the first inverter is set to 2mH, and the filter inductance of the second inverter is set to 2.5mH.

Circulation without control is shown in Figure 12, the circulation is about 5A. When PI control is used, the circulation is shown in Figure 13, and the circulation is about 1.1A. When dual PR control is
used, the circulation is shown in Figure 14 and the circulation is about 0.8A. It can be seen that the control strategy in this paper can suppress the circulation better.

Figure 12. The circulating current without control (two times)

Figure 13. The circulating current during PI control (two times)

Figure 14. 2 times the circulating current in dual PR control

We simulate the inverter when it has an unbalanced load, assuming R1=20Ω, R1=5Ω, R1=5Ω. The waveform of the load output voltage (line voltage) during dual PR control is shown in Figure 15, and the circulating current is shown in Figure 16. It can be seen that even when the parameters of the two inverters are different and the load is unbalanced, the output voltage can still be balanced, the three-phase voltage waveform is very symmetrical, and the circulating current is small.
Figure 15. Load output voltage when dual PR control

Figure 16. Circulating current with unbalanced load and double PR control

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