Warm inflation with an oscillatory inflaton in non-minimal kinetic coupling model

Parviz Goodarzi* and H. Mohseni Sadjadi †

1Department of Science, University of Boroujerdi
2Department of Physics, University of Tehran

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Abstract

Inflation with an oscillatory inflaton in the non-minimal derivative coupling model is considered. Radiation generation during this era is taken into account. Cosmological perturbations for thermal fluctuation and the temperature at the end of warm oscillatory inflation are computed.

1 Introduction

In the standard inflation model, the accelerated expansion and the reheating epochs are two distinct eras [1, 2, 3]. But in the warm inflation, relativistic particles are produced during slow roll, therefore the warm inflation explains the slow roll and onset of the radiation dominated era in a unique framework [4, 5]. Warm inflation is a good model for large scale structure formation, in which the density fluctuations arise from thermal fluctuation [6, 7]. Various models have been proposed for warm inflation, e.g. tachyon warm inflation, warm inflation in loop quantum cosmology, etc. [8, 9, 10].

Oscillating inflation was first introduced in [11], where it was proposed that the inflation may continue, after the slow roll, during rapid coherent oscillation in the reheating era. An expression for the corresponding number of e-fold was obtained in [6]. A brief investigation about adiabatic perturbation in the oscillatory inflation can be found in [12]. The formalism used in [11] was extended in [13], by considering a coupling between inflaton and the Ricci scalar curvature. The shape of the potential, required to end the oscillatory inflation, was investigated in [14]. Rapid oscillatory phase provides

*goodarzi@ut.ac.ir
†mohsenisad@ut.ac.ir
a few e-folds number so we cannot ignore the slow roll era in this formalis. Due to the few e-folds number, a detailed study about evolution of quantum fluctuations has not been performed. To cure this problem one can consider nonminimal derivative coupling model. The cosmological aspects of this model has been widely studied in the literature [15].

The oscillatory inflation in the presence of a nonminimal kinetic coupling, was studied in [16] and there was shown that in high friction regime, the nonminimal coupling increases the e-folds number and so can remedy the fewness e-fold number problem arisen in [11]. Scalar and tensor perturbations and power spectrum and spectral index for scalar and tensor modes in oscillatory inflation, were derived in [16], in agreement with Planck 2013 data. However, it is not clear from this scenario that how reheating occurs or the universe becomes radiation dominated after the end of inflation. For nonminimal derivative coupling model, the reheating process after the slow roll and warm slow roll inflation are studied in [17, 18] and [19] respectively.

In the present work, inspired by the above mentioned models, we will consider oscillatory inflation in non minimal derivative coupling model. We will assume that the inflaton decays to the radiation during the oscillation, providing a new scenario: warm oscillatory inflation.

In the second section we examine conditions for warm oscillatory inflation and study the evolution of energy density of the scalar field and radiation. In the third section, thermal fluctuation is considered and spectral index and power spectrum are computed. We will consider observational constraints on oscillatory warm inflation parameters by using Planck 2015 data [20]. In the fourth section the temperature at the end of warm inflation is calculated. In the last section we conclude our results. We use units $\hbar = c = 1$ through the paper.

2 Oscillatory warm inflation

In this section, based on our previous works [16, 17, 18], we will introduce rapid oscillatory inflaton decaying to radiation in non-minimal kinetic coupling model. We start with the action [21]

$$S = \int \left( \frac{M_P^2}{2} R - \frac{1}{2} \Delta^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) \sqrt{-g} d^4 x + S_{\text{int}} + S_r, \quad (1)$$

where $\Delta^{\mu \nu} = g^{\mu \nu} + \frac{1}{M_2^2} G^{\mu \nu}$, $G^{\mu \nu} = R^{\mu \nu} - \frac{1}{2} R g^{\mu \nu}$ is the Einstein tensor, $M$ is a coupling constant with mass dimension, $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass, $S_r$ is the radiation action and $S_{\text{int}}$ describes the interaction of the scalar field with radiation. There are not terms containing more than two times derivative, so we have not additional degrees of freedom in this theory. We can calculate energy momentum tensor by variation of action
with respect to the metric
\[ T_{\mu \nu} = T_{\mu \nu}^{(\varphi)} + \frac{1}{M^2} \Theta_{\mu \nu} + T_{\mu \nu}^{(r)}. \] (2)

The energy momentum tensor for radiation is
\[ T_{\mu \nu}^{(r)} = (\rho_r + P_r) u_\mu u_\nu + P_r g_{\mu \nu}, \] (3)
where \( u^\mu \) is the four-velocity of the radiation and \( T_{\mu \nu}^{(\varphi)} \) is the minimal coupling counterpart of energy momentum tensor
\[ T_{\mu \nu}^{(\varphi)} = \nabla_\mu \varphi \nabla_\nu \varphi - \frac{1}{2} g_{\mu \nu} (\nabla \varphi)^2 - g_{\mu \nu} V(\varphi). \] (4)

The energy momentum tensor corresponding to non-minimal coupling term is as follows
\[ \Theta_{\mu \nu} = -\frac{1}{2} G_{\mu \nu} (\nabla \varphi)^2 - \frac{1}{2} R \nabla_\mu \varphi \nabla_\nu \varphi + R^\alpha_\mu \nabla_\alpha \varphi \nabla_\nu \varphi \\
+ R^\beta_\nu \nabla_\alpha \varphi \nabla_\mu \varphi + R_{\mu \nu \beta \alpha} \nabla_\alpha \varphi \nabla_\beta \varphi + \nabla_\mu \nabla_\alpha \varphi \nabla_\nu \nabla_\alpha \varphi \\
- \nabla_\mu \nabla_\nu \varphi \Delta \varphi - \frac{1}{2} g_{\mu \nu} \nabla^\alpha \nabla^\beta \varphi \nabla_\alpha \nabla_\beta \varphi + \frac{1}{2} g_{\mu \nu} (\Delta \varphi)^2 \\
- g_{\mu \nu} \nabla_\alpha \varphi \nabla_\beta \varphi R_{\alpha \beta}. \] (5)

Energy transfer between the scalar field and radiation is assumed to be
\[ Q_\mu = -\Gamma u^\nu \partial_\mu \varphi \partial_\nu \varphi, \] (6)
where
\[ \nabla^\mu T_{\mu \nu}^{(r)} = Q^\nu \quad \text{and} \quad \nabla^\mu (T_{\mu \nu}^{(\varphi)} + \frac{1}{M^2} \Theta_{\mu \nu}) = -Q_\nu. \] (7)

The scalar field equation of motion, in Friedmann-Lemaître-Robertson-Walker (FLRW) metric, and in presence of the dissipative term is
\[ (1 + 3 \frac{H^2}{M^2}) \ddot{\varphi} + 3H (1 + \frac{3H^2}{M^2} + \frac{2 \dot{H}}{M^2}) \dot{\varphi} + V'(\varphi) + \Gamma \dot{\varphi} = 0, \] (8)
where \( H = \frac{\dot{a}}{a} \) is the Hubble parameter, a ”dot” is differentiation with respect to cosmic time \( t \), prime denotes differentiation with respect to the scalar field \( \varphi \). \( \Gamma \dot{\varphi} \) is the friction term which describes the decay of the \( \varphi \) field to radiation. \( \Gamma \), in general, is a function of \( \varphi \) and temperature [5][22]. The Friedmann equations are given by
\[ H^2 = \frac{1}{3 M_P^2} (\rho_\varphi + \rho_r) \]
\[ \dot{H} = -\frac{1}{2 M_P^2} (\rho_\varphi + \rho_r + P_\varphi + P_r). \] (9)
The energy density and the pressure of inflaton can be expressed as
\begin{equation}
\rho_\varphi = (1 + \frac{9H^2}{M^2}) \frac{\dot{\varphi}^2}{2} + V(\varphi),
\end{equation}
and
\begin{equation}
P_\varphi = \left(1 - \frac{3H^2}{M^2} \right) \frac{\dot{\varphi}^2}{2} - V(\varphi) - \frac{2H\dot{\varphi}^2}{M^2},
\end{equation}
respectively. Energy density of radiation is \(\rho_r = \frac{3}{4} T S\). \(S\) is the entropy density and \(T\) is the temperature. The equation of state parameter for radiation is \(\frac{1}{3}\), hence the rate of radiation production is given by
\begin{equation}
\dot{\rho}_r + 4H\rho_r = \Gamma \dot{\varphi}^2.
\end{equation}
We assume that the potential is even, \(V(-\varphi) = V(\varphi)\), and consider rapid oscillating solution to (8) around \(\varphi = 0\), with time dependent amplitude \(\phi(t)\). The period of oscillation is
\begin{equation}
T(t) = 2 \int_{-\varphi(t)}^{\varphi(t)} \dot{\varphi}(t).
\end{equation}
The rapid oscillation is characterized by \(H \ll \frac{1}{T}\) and \(\frac{H}{T} \ll \frac{1}{T}\). The inflaton energy density may estimated as \(\rho_\varphi = V(\phi(t))\). In this epoch \(\rho_\varphi\) and \(H\) change insignificantly during a period of oscillation [17].

In rapid oscillatory phase, the time average of adiabatic in dex, defined by \(\gamma = \frac{\rho_\varphi + P_\varphi}{\rho_\varphi}\), is given by \(\gamma = \frac{\rho_\varphi + P_\varphi}{\rho_\varphi}\), where bracket denotes time averaging. For a power law potential \(V(\varphi) = \lambda \varphi^q\), and in high friction limit \(\frac{H^2}{M^2} \gg 1\), adiabatic index becomes [16]
\begin{equation}
\gamma \approx \frac{2q}{3q + 6}.
\end{equation}

By averaging the continuity equation, we obtain [17]
\begin{equation}
< \rho_\varphi > + 3H\gamma < \rho_\varphi > + \frac{\gamma M^2}{3H^2} < \rho_\varphi > = 0.
\end{equation}

In the high friction limit \(\frac{H^2}{M^2} \gg 1\) and \(\Gamma M^2 \ll 3H^3\), the average of energy density of scalar field is derived as
\begin{equation}
\langle \rho_\varphi \rangle \propto a(t)^{-3\gamma}.
\end{equation}

By relation [17] and Friedmann equation \(H^2 \approx \frac{1}{3M_P^2} \rho \varphi\), in the \(\varphi\) dominated era, we can easily obtain
\begin{equation}
a(t) \propto t^{\frac{q}{q+2}} \propto t^\gamma.
\end{equation}
Therefore the Hubble parameter in the inflaton dominated era can be estimated as $H \approx \frac{2}{3\gamma t}$. In the rapid oscillation phase and with the power law potential (14) we can write the amplitude of the oscillation as

$$\phi(t) \propto a(t)^{-\frac{2}{3\gamma}} \propto t^{-\frac{2}{\gamma}}.$$  

(19)

Therefore at high friction limit

$$\langle \dot{\phi}^2 \rangle \approx \gamma M_P^2 M^2.$$  

(20)

This relation shows that for non-minimal derivative coupling model and in the rapid oscillation phase, $\langle \dot{\phi}^2 \rangle$ is approximately a constant. By inserting (20) into equation (12) we obtain

$$\rho_r = \frac{3\Gamma\gamma^2 M^2 M_P^2}{(8 + 3\gamma)^2} \left[ t - \left( \frac{t_0}{t} \right)^{1+\frac{1}{3\gamma}} \right],$$  

(21)

where $t_0$ is the time at which $\rho_r = 0$. The number of e-folds from a specific time $t_\ast \in (t_0, t_{RD})$ in inflation until radiation dominated epoch, is given by

$$N_I = \int_{t_\ast}^{t_{RD}} H dt \approx \int_{t_\ast}^{t_{RD}} \frac{2}{3\gamma t} dt \approx \frac{2}{3\gamma} \ln \left( \frac{t_{RD}}{t_\ast} \right).$$  

(22)

where $t_{RD}$ is the time at which the universe becomes radiation dominated and inflation ceases. At this time

$$\rho_r(T_{RD}) \approx \rho_\varphi(t_{RD}).$$  

(23)

We can calculate temperature at the end of warm inflation with

$$\rho_r(t_{RD}) = g_{RD}\frac{\pi^2}{30} T_{RD}^4,$$  

(24)

where $g_{RD}$ is number of degree of freedom and $T_{RD}$ is the temperature of radiation at the beginning of radiation dominated era.

3 Cosmological perturbations

In this section we study the evolution of thermal fluctuation during oscillatory warm inflation. To investigate cosmological perturbations, we split the metric into two components: the background, and perturbations. The background is described by homogeneous and isotropic FLRW metric with oscillatory scalar field and the perturbed sector of the metric determines anisotropy. We assume that the radiation is in thermal equilibrium during warm inflation. We consider the evolution equation of the first order cosmological perturbations for a system containing inflaton and radiation. In the longitudinal gauge scalar the metric can be written as

$$ds^2 = -(1 + 2\Theta) dt^2 + a^2(1 - 2\Theta) \delta_{ij} dx^i dx^j.$$  

(25)
As mentioned before, the energy momentum tensor splits into radiation part $T_{\mu\nu}^r$ and inflaton part $T_{\mu\nu}^\phi$ as

$$T_{\mu\nu} = T_{\mu\nu}^r + T_{\mu\nu}^\phi.$$  

(26)

The unperturbed parts of four velocity components of the radiation fluid satisfy $\mathbf{u}_{ri} = 0$ and $\mathbf{u}_{r0} = -1$. By using normalisation condition $g^{\mu\nu} u_\mu u_\nu = -1$, the perturbed part of the time component of the four velocity becomes

$$\delta u^0 = \frac{h_{00}}{2}. $$  

(27)

The space components $\delta u^i$, are independent dynamical variables and $\delta u_i = \partial_i \delta u$.  

Energy transfer is described by

$$Q_{\mu} = -\Gamma u^\nu \partial_\mu \delta \phi.$$  

(28)

(28) gives $Q_0 = \Gamma \dot{\phi}^2$ and the unperturbed equation (29) becomes $Q_0 = \dot{\rho}_r + 3H(\rho_r + P_r)$ which is continuity equation for the radiation field. In the same way the equation (30) becomes $-Q_0 = \dot{\rho}_\phi + 3H(\rho_\phi + P_\phi)$. Perturbations to the energy momentum transfer are described by

$$\delta Q_0 = -\delta \Gamma \dot{\phi}^2 + \Phi \Gamma \dot{\phi}^2 - 2 \Gamma \dot{\phi} \delta \dot{\phi}$$  

(31)

and

$$\delta Q_i = -\Gamma \delta \phi \partial_i \delta \phi.$$  

(32)

The variation of the equation (29) is $\delta (\nabla_\mu T_{\mu\nu}^r) = \delta Q^\nu$, so its (0-0) component is

$$\delta \rho_r + 4H \delta \rho_r + \frac{4}{3} \rho_r \nabla^2 \delta u - 4 \dot{\Phi} \rho_r = -\Phi \Gamma \dot{\phi}^2 + \delta \nabla \delta \phi.$$

(33)

Similarly, for the $i-th$ component we derive

$$4 \rho_r \dot{\delta u}^i + 4 \rho_r \dot{\delta u}^i + 20H \rho_r \delta \dot{u}^i = -[3 \Gamma \delta \phi \partial_i \delta \phi + \partial_i \delta \rho_r + 4 \rho_r \partial_i \Phi].$$  

(34)

The equation of motion for $\delta \phi$, computed by variation of (31), is $\delta (\nabla_\mu T_{\mu\nu}^\phi) = -\delta Q^\nu$. The zero component of this equation is

$$\left(1 + \frac{3H^2}{M^2}\right) \ddot{\delta \phi} + \Gamma \dot{\delta \phi} + 3H \dot{\delta \phi} + \delta V'(\phi) + \dot{\phi} \ddot{\phi}.$$  

(35)
\[-2V'(\phi) + 3\Gamma \dot{\phi} - \frac{6H\ddot{\phi}}{M^2}(3H^2 + 2\dot{H}) - \frac{6H^2\dddot{\phi}M}{M^2}\Phi + (1 + \frac{9H^2}{M^2})\dddot{\phi}\Phi + \frac{2H\ddot{\phi}\nabla^2\Phi}{M^2}\]

\[+3(1 + \frac{9H^2}{M^2} + 2\dot{H} + 2\frac{H\ddot{\phi}}{M^2})\ddot{\psi} + 6\frac{H\dot{\phi}}{M^2}\dddot{\psi} - 2(\dddot{\phi} + H\dot{\phi})\nabla^2\psi \frac{M^2}{a^2}.\]

The 0–0 component of the perturbation of the Einstein equation $G_{\mu\nu} = -8\pi G T_{\mu\nu}$ is

\[\begin{align*}
-3H\dot{\psi} - 3H^2\dot{\phi} + \frac{\nabla^2\Psi}{a^2} &= 4\pi G [(1 + \frac{18H^2}{M^2})\dddot{\phi}\Phi - \frac{9H^2\Phi^2}{M^2}]
\end{align*}\]

\[+\frac{\dot{\phi}^2}{M^2}\nabla^2\Phi + V(\phi)\delta\phi + (1 + \frac{9H^2}{M^2})\dot{\phi}\delta\phi - \frac{2H\dot{\phi}\nabla^2(\delta\phi)}{M^2}a^2 + \delta\rho_r],\]

and its $i-i$ component is

\[\begin{align*}
(3H^2 + 2\dot{H})\Phi + H(3\dot{\psi} + \Phi) + \frac{\nabla^2(\Phi - \Psi)}{3a^2} + \dot{\psi} = &
\end{align*}\]

\[\begin{align*}
4\pi G[(3H^2 + 2\dot{H})\frac{\dot{\phi}^2}{M^2} - \phi^2 + \frac{8H\dot{\phi}\dddot{\phi}}{M^2}\Phi + \frac{3H\dot{\phi}^2}{M^2}\Phi]
\end{align*}\]

\[\begin{align*}
+\frac{\dot{\phi}^2}{M^2}\nabla^2\Phi + (\frac{3H\dot{\phi}^2}{M^2} + 2\frac{\dot{\phi}\dddot{\phi}}{M^2})\ddot{\psi} + \frac{\dot{\phi}^2}{M^2}\ddot{\psi} + \frac{\nabla^2\Psi}{3a^2}
\end{align*}\]

\[-V(\phi)\delta\phi - (\frac{2H\dot{\phi}}{M^2})\delta\phi - \frac{2H\dot{\phi}}{M^2}\delta\phi + \frac{2H\dddot{\phi}}{M^2}\delta\phi]

\[\begin{align*}
\dot{\phi} - \frac{2H\dot{\phi}}{M^2}\delta\phi + \frac{2(\dddot{\phi} + H\dot{\phi})\nabla^2(\delta\phi)}{M^2} \frac{3a^2}{3a^2} + \delta\rho_r].\]

By using $-H\partial_i\Phi - \partial_i\dot{\psi} = 4\pi G(\rho + P)\partial_i\delta u$, we can obtain (from 0 – i component of field equation)

\[\begin{align*}
H\Phi + \dot{\psi} = & 4\pi G\frac{3H\dot{\phi}^2}{M^2}\Phi + \frac{\dot{\phi}^2}{M^2}\ddot{\psi} + (1 + \frac{3H^2}{M^2})\dot{\phi}\delta\phi - \frac{2H\dot{\phi}}{M^2}\dddot{\phi}
\end{align*}\]

\[\begin{align*}
+ (\rho + P_r)\delta u].\]

Using (33–38) we can calculate perturbation parameters.

During inflation the background have two component oscillatory scalar field and radiation. Energy density of scalar field decreases due to expansion and radiation generation. Quantities related to the scalar field in the background have oscillatory behaviors. So we replace the background quantities with their average values over oscillation. Also, we consider non minimal derivative coupling at high friction limit.

By going to the Fourier space, the spatial parts of perturbational quantities get $e^{ikx}$ where $k$ is the wave number. So $\partial_j \rightarrow ik_j$ and $\nabla^2 \rightarrow -k^2$. Also we define

\[\delta u = -\frac{a}{k} e^{ikx}.\]
So \( (33) \) becomes
\[
\dot{\delta \rho} + 4H \delta \rho + \frac{4}{3} \kappa \rho v - 4 \rho \dot{\Phi} = -\Gamma M^2 M_\nu^2 \Phi, \tag{40}
\]
and the equation \( (34) \) becomes
\[
4 \frac{\alpha}{k} (\dot{\rho} r + 4H (\rho r)) = -\delta \rho - 4 \rho \Phi. \tag{41}
\]

\( (35) \) reduces to
\[
(3H^2 \frac{M^2}{2}) \ddot{\delta \phi} + \left[ (3 H^2 + 2H \frac{M^2}{2}) + 3H \delta \phi + \delta V'(\phi) = -2V'(\phi) \Phi + 3(9H^2 + 2H \frac{M^2}{2}) \dot{\Phi}. \tag{42}
\]

From \( (36) \) we have
\[
-3H \dot{\Phi}(1 - \frac{3\gamma}{2}) - 3H^2 \Phi(1 - 3\gamma) = -\frac{1}{2M_\nu^2} (V'(\phi) \delta \phi + \delta \rho), \tag{43}
\]
and rewrite \( (37) \) as
\[
(3H^2 + 2H) \Phi(1 - \gamma) + \dot{H} \Phi(4 - 3\gamma) + \Phi(1 - \frac{1}{2}) \gamma = \frac{1}{2M_\nu^2} (-V'(\phi) \delta \phi + \delta P_r). \tag{44}
\]

The equation \( (38) \) may be written as
\[
H \Phi(1 - \frac{3\gamma}{2}) + \Phi(1 - \frac{1}{2}) \gamma = -\frac{2}{3M_\nu^2} \frac{a}{k} (\rho \nu r), \tag{45}
\]
and the time derivative of \( (38) \) gives
\[
(H \dot{\Phi} + \dot{H} \Phi)(1 - \frac{3\gamma}{2}) + \Phi(1 - \frac{1}{2}) \gamma = -\frac{2}{3M_\nu^2} \frac{a}{k} (H (\rho \nu r) + (\rho \nu r)). \tag{46}
\]

By analyzing the above equations we find
\[
[3H^2(1 - \frac{3\gamma}{2} - \frac{1}{2}) \gamma + \dot{H}(\frac{2}{3} - \frac{7}{6} \gamma)] \Phi + \frac{5}{6}(4 - 3\gamma) H \dot{\Phi} + \frac{5}{6} (1 - \frac{1}{2} \gamma) \dot{\Phi} = 0. \tag{47}
\]
During the rapid oscillation, the Hubble parameter is \( H = \frac{2}{3t} \), therefore equation \( (47) \) becomes
\[
(\frac{2}{3\gamma})[2 - \frac{13}{3} + \frac{7}{6} \gamma] \Phi + \frac{5}{9\gamma}(4 - 3\gamma) \frac{\dot{\Phi}}{t} + \frac{5}{6} (1 - \frac{1}{2} \gamma) \dot{\Phi} = 0. \tag{48}
\]
This equation has the solution \( \Phi \propto t^{\alpha \pm} \), therefore
\[
(\frac{2}{3\gamma})[2 - \frac{13}{3} + \frac{7}{6} \gamma] + \frac{5}{9\gamma}(4 - 3\gamma) \alpha + \frac{5}{6} (1 - \frac{1}{2} \gamma) \alpha (\alpha - 1) = 0. \tag{49}
\]
\( \alpha' \)s are the roots of this quadratic equation. We denote the positive root by \( \alpha_+ \). From equations (43 and 44), we deduce

\[
- \frac{1}{M_{P}^{2}} V'(\varphi) \delta \varphi = 2(3H^2(1-2\gamma)+H(1-\gamma))\Phi+(7-\frac{3}{2}\gamma)H\dot{\Phi}+(1-\frac{1}{2}\gamma)\ddot{\Phi}. 
\] (50)

It is now possible to use relation \( \Phi \propto t^{\alpha_+} \) to obtain

\[
\delta \varphi - \frac{1}{M_{P}^{2}} V'(\varphi) \delta \varphi = \left[ \frac{4}{3\gamma}(\frac{2}{\gamma} - 5 + \gamma) + \frac{2}{3\gamma}(7 - \frac{15}{2}\gamma)\alpha_+ + (1 - \frac{1}{2}\gamma)\alpha_+ (\alpha_+ - 1) \right] \Phi \frac{t^{\alpha_+ - 2}}{t^{2}}. 
\] (51)

\( \delta \varphi \) simplifies to

\[
\delta \varphi = -C \frac{M_{P}^{2+\alpha_+}}{V'(\varphi)} t^{\alpha_+ - 2} \times \left[ \frac{4}{3\gamma}(\frac{2}{\gamma} - 5 + \gamma) + \frac{2}{3\gamma}(7 - \frac{15}{2}\gamma)\alpha_+ + (1 - \frac{1}{2}\gamma)\alpha_+ (\alpha_+ - 1) \right] t^{\alpha_+ - 2} 
\] (52)

where \( C \) is a numerical constant. Thus the density perturbation, from relation (52), becomes [25]

\[
\delta H \approx \frac{16\pi}{5M_{P}^{2+\alpha_+}} 
\]

\[
\frac{V'}{\left[ \frac{4}{3\gamma}(\frac{2}{\gamma} - 5 + \gamma) + \frac{2}{3\gamma}(7 - \frac{15}{2}\gamma)\alpha_+ + (1 - \frac{1}{2}\gamma)\alpha_+ (\alpha_+ - 1) \right] t^{\alpha_+ - 2}} \delta \varphi 
\] (53)

In this relation \( \delta \varphi \) is the scalar field fluctuation during the warm inflation generated by thermal interaction with the radiation [4]

\[
\delta \varphi^2 = \frac{k_{F}T}{2\pi^2}, 
\] (54)

where \( k_{F} \) is the freeze out scale. To compute \( k_{F} \), we must determine when the damping rate of relation (44) becomes less than the expansion rate \( H \). At \( t_{F} \) (freeze out time), the freeze out wave number \( k_{F} = \frac{k}{a(t_{F})} \) is given by

\[
k_{F} = \sqrt{\Gamma H + 3H^2(1 + \frac{3H^2}{M^2})}. 
\] (55)

Therefore the density perturbation becomes

\[
\delta_{H}^2 \approx \left( \frac{16\pi}{5M_{P}^{2+\alpha_+}} \right)^2 \frac{V'^2}{\left[ \frac{4}{3\gamma}(\frac{2}{\gamma} - 5 + \gamma) + \frac{2}{3\gamma}(7 - \frac{15}{2}\gamma)\alpha_+ + (1 - \frac{1}{2}\gamma)\alpha_+ (\alpha_+ - 1) \right]^2 t^{2\alpha_+ - 4}} \delta \varphi^2. 
\] (56)
We can rewrite this relation as

\[
\delta_H^2 \approx \left( \frac{128}{25M_P^4+2\alpha_+} \right)^2 \frac{V'^2}{\left[ \frac{4}{3\gamma} \left( \frac{2}{\gamma} - 5 + \gamma \right) + \frac{2}{3\gamma} \left( 7 - \frac{15\gamma}{2} \right) \alpha_+ + (1 - \frac{1}{2}\gamma)\alpha_+(\alpha_+ - 1) \right]^2} t^{4-2\alpha_+} \sqrt{\Gamma H + 3H^2 (1 + \frac{3H^2}{M^2}) T}.
\]

(57)

We can now calculate power spectrum from relation \( P_s(k_0) = \frac{25}{4}\delta_H^2(k_0) \) [24]. \( k_0 \) is a pivot scale. The spectral index for scalar perturbation is

\[
n_s - 1 = \frac{d\ln \delta_H^2}{d\ln k}.
\]

(58)

The derivative is taken at horizon crossing \( k \approx aH \). The spectral index may be written as

\[
n_s - 1 = \frac{d\ln \delta_H^2}{d\ln (aH)} = \left( \frac{1}{H + \frac{\Gamma}{H}} \right) \frac{d\ln \delta_H^2}{dt}.
\]

(59)

From \( H = \frac{2}{3\gamma t} \) we have

\[
n_s - 1 \approx \left( \frac{t}{\frac{2}{3\gamma} - 1} \right) \frac{d\ln \delta_H^2}{dt},
\]

(60)

therefore

\[
n_s - 1 \approx \left( \frac{4}{3\gamma} - \frac{5}{2} - 2\alpha_+ \right) \left( \frac{1}{\frac{2}{3\gamma} - 1} \right).
\]

(61)

This relation gives the spectral index as a function of \( \gamma \). From Planck 2015 data \( n_s = 0.9645 \pm 0.0049 \) (68% CL, Planck TT,TE,EE+lowP) \( \gamma \) is determined as \( \gamma = 0.55902 \pm 0.00016 \).

4 Evolution of the Universe and temperature of the warm inflation

In this section, by using our previous results, we intend to calculate temperature of warm inflation as a function of observational parameters for the power law potential (14) and a constant dissipation coefficient \( \Gamma \) in high friction limit. For this purpose we follow the steps introduced in [24], and divide the evolution of the Universe from \( t_\star \) (a time at which a pivot scale exited the Hubble radius) in inflation era until now into three parts

\( I \) – from \( t_\star \) until the end of oscillatory warm inflation, denoted by \( t_{RD} \). In this period energy density of the oscillatory scalar field is dominated.

\( II \) – from \( t_{RD} \) until recombination era, denoted by \( t_{rec} \).
from $t_{\text{rec}}$ until the present time $t_0$.

Therefore the number of e-folds from horizon crossing until now becomes

$$N = \ln \left( \frac{a_0}{a_*} \right) = \ln \left( \frac{a_0}{a_{\text{rec}}} \right) + \ln \left( \frac{a_{\text{rec}}}{a_{\text{RD}}} \right) + \ln \left( \frac{a_{\text{RD}}}{a_*} \right) = N_I + N_{II} + N_{III} \quad (62)$$

### 4.1 Oscillatory warm inflation

During the warm oscillatory inflation, the scalar field oscillates and decays to ultra relativistic particles. In this period the energy density of oscillatory scalar field is dominated and the Universe expansion is accelerated. The beginning time of radiation dominated era is determined by the condition $\rho_r(t_{\text{RD}}) \approx \rho_{\varphi}(t_{\text{RD}})$ which gives \[17, 18\]

$$t_{\text{RD}}^3 = \frac{4(8 + 3\gamma)}{9\Gamma^4 M^2}. \quad (63)$$

From equations \[63\] and \[21\] we can calculate energy density of radiation at $t_{\text{RD}}$

$$\rho_r(t = t_{\text{RD}}) \approx M_P^2 \left[ \frac{12\Gamma^2 \gamma^2 M^4}{(8 + 3\gamma)^2} \right]^{\frac{1}{3}}. \quad (64)$$

Therefore the temperature of the universe at the end of oscillatory warm inflation becomes

$$T_{RD}^4 \approx \frac{30M_P^2}{\pi^2 g_{\text{RD}}} \left[ \frac{12\Gamma^2 \gamma^2 M^4}{(8 + 3\gamma)^2} \right]^{\frac{1}{3}}. \quad (65)$$

### 4.2 Radiation dominated and recombination eras

At the end of the warm inflation the magnitude of radiation energy density equals the energy density of the scalar field. Thereafter the universe enters a radiation dominated era. During this period, the Universe is filled with ultra-relativistic particles which are in thermal equilibrium. In this epoch the Universe undergoes an adiabatic expansion where the entropy per comoving volume is conserved: $dS = 0 \ [27]$. In this era the entropy density, $s = S a^{-3}$, is \[27\]

$$s = \frac{2\pi^2}{45} gT^3. \quad (66)$$

So we have

$$\frac{a_{\text{rec}}}{a_{\text{end}}} = \frac{T_{\text{end}}}{T_{\text{rec}}} \left( \frac{g_{\text{RD}}}{g_{\text{rec}}} \right)^{\frac{1}{3}}. \quad (67)$$

In the recombination era, $g_{\text{rec}}$ corresponds to degrees of freedom of photons, hence $g_{\text{rec}} = 2$. Thus

$$N_{II} = \ln \left( \frac{T_{\text{end}}}{T_{\text{rec}}} \left( \frac{g_{\text{RD}}}{2} \right)^{\frac{1}{3}} \right). \quad (68)$$
By the expansion of the Universe, the temperature decreases via
\[ T(z) = T(z=0)(1 + z), \]
where \( z \) is the redshift parameter. Hence \( T_{\text{rec}} \) in terms of \( T_{\text{CMB}} \) is
\[ T_{\text{rec}} = (1 + z_{\text{rec}})T_{\text{CMB}}. \quad (69) \]

We have also
\[ \frac{a_0}{a_{\text{rec}}} = (1 + z_{\text{rec}}). \quad (70) \]

Therefore
\[ N_{\text{II}} + N_{\text{III}} = \ln \left( \frac{T_{\text{end}}}{T_{\text{CMB}}} \left( \frac{g_{\text{RD}}}{2} \right)^{\frac{1}{3}} \right). \quad (71) \]

### 4.3 Temperature of the warm oscillatory inflation

To obtain temperature of the warm inflation we must determine \( N \) in (62).

We take \( a_0 = 1 \), so the number of e-folds from the horizon crossing until the present time is
\[ \Delta = \exp(N), \quad (72) \]

By relations (72,71,62) we can derive
\[ T_{\text{RD}} \]
\[ T_{\text{RD}} = T_{\text{CMB}} \left( \frac{2}{g_{\text{RD}}} \right)^{\frac{1}{3}} \left( \frac{2}{3\gamma k_0} \right) \left[ \left( \frac{4(8 + 3\gamma)}{9\Gamma \gamma^4 M^2} \right)^{\frac{3}{10}} \times t_\star \left( \frac{1}{\gamma} - 1 \right). \quad (73) \]

We can remove \( \Gamma M^2 \) in this relation by (65)
\[ T_{\text{RD}} \approx \left( \frac{2}{3\gamma k_0} \right) \left( \frac{2}{g_{\text{end}}} \right) \left[ \left( \frac{4(8 + 3\gamma)}{9\Gamma \gamma^4 M^2} \right)^{\frac{3}{10}} \times t_\star \left( \frac{1}{\gamma} - 1 \right). \quad (74) \]

By using relation \( P_s(k_0) = \frac{25}{4} \delta_H^2(k_0) \) and equation (71), power spectrum becomes
\[ P_s(k_0) \approx \left( \frac{32}{M_P^{4+2\alpha_+}} \right) \times \left( \frac{V'(\phi_\star)}{2^\alpha_+} \right)^2 \]
\[ \left[ \frac{4}{\pi} \left( \frac{3}{\pi} - 5 + \gamma \right) + \frac{2}{\pi} \left( 7 - \frac{15\gamma}{2} \right) \alpha_+ + \left( 1 - \frac{1}{2} \gamma \right) \alpha_+ \left( \alpha_+ - 1 \right) \right] \]
\[ \times t_\star^{4-2\alpha_+} \sqrt{\Gamma H_\star + 3H_\star^2 \left( 1 + \frac{3H_\star^2}{M^2} \right)} T_\star. \quad (75) \]

In this relation \( T_\star \) is the temperature of the universe at the horizon crossing. By relation (21) we can calculate temperature at horizon crossing as a function of \( t_\star \)
\[ T_\star = \left[ \frac{90\Gamma \gamma^2 M^2 M_P^2}{(8 + 3\gamma) \pi^2 g_\star} \right]^{\frac{1}{4}} \left( \frac{1}{t_\star} \right)^{\frac{1}{2}}. \quad (76) \]
We can remove $\Gamma M^2$ in relation (76) by (65)

$$T_* = \left[ \frac{\pi \gamma g_{\frac{1}{2}}}{2 \sqrt{10 M_P}} \right] \frac{1}{4} T_{RD} t_*^{\frac{1}{2}}. \tag{77}$$

The time average of the potential derivative may be computed as follows

$$< V' > = q \lambda \int_{-\phi}^{\phi} \frac{\phi^{n-1} d\phi}{\int_{-\phi}^{\phi} \frac{d\phi}{\sqrt{1-x^2}}} = q \lambda \phi^{n-1} \int_{0}^{1} \frac{1}{\sqrt{1-x^2}} \int_{0}^{1} \frac{d\phi}{\sqrt{1-x^2}} = 2 q \lambda \frac{\Gamma \left( \frac{2+q}{2q} \right)}{\Gamma \left( \frac{1}{q} \right)} \phi^{q-1}. \tag{78}$$

In inflationary regime we have $H^2 \approx \frac{1}{3 M_P} \rho \phi \approx \frac{1}{3 M_P} \lambda \phi^n$ and $H^2 \approx \frac{4}{9 \gamma^2 \pi}$, therefore

$$< V' (\varphi_*) > = 12 \lambda \frac{\gamma}{2 - 3 \gamma} \frac{\Gamma \left( \frac{1}{3\gamma} \right)}{\Gamma \left( \frac{1}{3\gamma} - \frac{1}{2} \right)} \left( \frac{4 M_P^2}{3 \lambda \gamma^2} \right)^{\frac{9\gamma-2}{\gamma}} t_*^{1/3\gamma}. \tag{79}$$

Thus we can write (75) as

$$P_s (k_0) \approx M_P \left( \frac{2}{7} - \frac{3\gamma}{7} - 2 \alpha_+ \right) \lambda \left( \frac{2}{7} - 1 \right) \Gamma \left( \frac{1}{2} \right) \frac{\Gamma \left( \frac{1}{3\gamma} - \frac{1}{2} \right)}{\Gamma \left( \frac{1}{3\gamma} \right)} \frac{2 - 9\gamma}{2 - 9\gamma}. \tag{80}$$

$\beta$ is given by

$$\beta = \frac{2048 \sqrt{\pi} \left( \frac{90\gamma^2}{8 + 3\gamma} \right)^{\frac{1}{2}} \left( \frac{4}{3\gamma^2} \right)^{\frac{9\gamma-2}{\gamma}} \left( \frac{\Gamma \left( \frac{1}{3\gamma} \right)}{\Gamma \left( \frac{1}{2} \right)} \right)^{2}}{(2 - 3\gamma)^2 \left[ \frac{2}{3\gamma} \left( \frac{2}{7} - 5 + \gamma \right) + \frac{2}{3\gamma} \left( 7 - \frac{15\gamma}{2} \right) \alpha_+ + (1 - \frac{1}{\gamma}) \alpha_+ \left( \alpha_+ - 1 \right) \right]^2}. \tag{81}$$

From equation (80), we derive $t_*$ as

$$t_* = \left[ \frac{P_s (k_0) g_{RD} \frac{1}{4}}{M_P \left( \frac{2}{7} - \frac{3\gamma}{7} - 2 \alpha_+ \right) \lambda \left( \frac{2}{7} - 1 \right) \Gamma \left( \frac{1}{2} \right) \frac{\Gamma \left( \frac{1}{3\gamma} - \frac{1}{2} \right)}{\Gamma \left( \frac{1}{3\gamma} \right)} \frac{2 - 9\gamma}{2 - 9\gamma}} \right]^{1/10^{4(2-3\gamma)} - 24\alpha_+}. \tag{82}$$

By substituting $t_*$ from relation (82) into equation (73), the temperature at the end of warm oscillatory inflation or beginning or radiation domination is obtained as

$$T_{end} = T_{CMB} \left( \frac{2}{g_{RD}} \right)^{\frac{1}{2}} \left[ \frac{4(8 + 3\gamma)}{9 \gamma \Gamma^4 M^2} \right]^{-\frac{9\gamma}{9\gamma}} \times \left[ \frac{P_s (k_0) g_{RD} \frac{1}{4}}{M_P \left( \frac{2}{7} - \frac{3\gamma}{7} - 2 \alpha_+ \right) \lambda \left( \frac{2}{7} - 1 \right) \Gamma \left( \frac{1}{2} \right) \frac{\Gamma \left( \frac{1}{3\gamma} - \frac{1}{2} \right)}{\Gamma \left( \frac{1}{3\gamma} \right)} \frac{2 - 9\gamma}{2 - 9\gamma}} \right]^{1/4(2-3\gamma) - 24\alpha_+}. \tag{83}$$
The number of e-fold during warm oscillatory inflation becomes
\[
N_I \approx \frac{2}{3\gamma} \left[ \frac{1}{3} \ln \left( \frac{4(8 + 3\gamma)}{9\Gamma^{1/4}M^2} \right) \right.
\]
\[
- \frac{12\gamma}{16 - 45\gamma - 24\gamma\alpha_+} \ln \left( \frac{P_s(k_0)g^{1/2}_{RD}}{M_P^{(\frac{1}{2} + \frac{4}{11})-2\alpha_+} \chi^{(\frac{1}{2} - 1)}(x^{-1})} \right). \tag{84}
\]

At the end as an example let us take \( M = 10^{-9} M_P \) and \( \Gamma = 10^{-10} M_P \).

By setting \( g_{end} = 106.75 \), which is the ultra relativistic degrees of freedom at the electroweak energy scale, and by taking the pivot scale as \( k_0 = 0.05 M_P c^{-1} \) and \( P_s(k_0) = (2.014) \times 10^{-9} \) and \( n_s = 0.9645 \) \footnote{In equation (77), the temperature of the universe at the end of warm inflation becomes \( T_{end} \approx 8.27 \times 10^{13} \text{GeV} \). In this example the number of e-folds becomes \( N_I = 38.41 \).}

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