Abstract

This paper presents the models for determining the optimal percentage of spending of savings and reinvestment from the point of view of an individual investor, taking into account his labor income in the future. Usual expenses are required due to unfavorable market conditions (for example, spending the funds in excess of 4% of GDP).

Analysis of optimal consumption and reinvestment depends on the level of the risk-free interest rate, which can be determined on the basis of discount rates for large infrastructure projects. The current Norwegian budget rule is set in such a way that the current generation of Norwegian citizens will receive more support for future generations. The article proposes a new annuity model of spending sovereign funds, taking into account the risk-adjusted interest rate of return.

The main conclusion is that the risk-free (bond) part of the portfolio should not change after an unexpected fall in market value. The risky share of the portfolio is adjusted after each change in the quotations of the securities.

INTRODUCTION

Sovereign fund investment strategies and income spending should be interrelated. There is a practice of redistributing assets in the investment portfolios of a fund after a significant change in prices, which is not always justified when the fund is obliged to smooth the flow of current expenditures.

Unfavorable market conditions can lead to the fact that portfolio risk must be reduced to ensure smoothing budget expenditures.

In 1990, Norway decided to start sending its oil and gas revenues to a special oil fund. The fund name is the Government Pension Fund Global (GPFG). The Norwegian government uses GPFG funds to increase government spending with fixed tax revenues.

Despite its name, it is not associated with the pension system. As GPFG actively invests in infrastructure projects, the fund’s net cash flow has been negative for some periods. As a result of the growth in oil production in Norway and the persistence of high oil prices until 2015, the fund’s assets rose to NOK 7.4 trillion (USD 845 billion). In this connection, cumulative return of the fund since 1998 has amounted to about 280% (Figure 1).
Figure 1. Fixed portfolio relative yield of GPFG (solid line) versus benchmark (dotted line) since 1998

Source: Norges Bank Investment Management (http://www.nbim.no/en/).

Figure 2. Equity portfolio relative yield of GPFG (solid line) versus benchmark (dotted line) since 1998

Source: Norges Bank Investment Management (http://www.nbim.no/en/).

Figure 3. GPF relative cumulative yield since 1998

Source: Norges Bank Investment Management (http://www.nbim.no/en/).
The Parliament of Norway has established the following limits on investment of GPFG: 60% – shares, 5% – real estate, the rest – bonds. To maintain the limits, the fund’s portfolio is regularly adjusted (Figures 2-4).

The main purpose of the foundation is to preserve savings for future generations. In 2001, a budget rule was developed:

1. Expenditure of GPFG funds should smooth out the flow of government spending.
2. The annual deficit of the state budget of Norway (excluding oil revenues), which is covered by GPFG, should not exceed 4% of the value of GPFG assets at the beginning of the year.
3. Temporary deviations are allowed when it is necessary to smooth out cyclical fluctuations of budget revenues.

1. LITERATURE REVIEW

Thus, smoothing fluctuations in budget revenues is a fundamental motivation of the Norwegian fiscal rule, which is consistent with the opinion of smoothing budget revenues (Backus & Crucini, 2000).

The problem is that in the unstable economic situation, the value of the assets of the fund may fluctuate significantly (Amano & Van Norden, 1998). The third paragraph of the rule shows that smoothing should not interfere with discrete fiscal policy as a counter-cyclical tool.

Budget revenues should not strongly depend on oil prices. Many researchers find the relationship between macroeconomic indicators and oil price shocks in different countries (Olomola & Adejumo, 2006; Singer, 2007, Huang & Guo, 2007; Farzanegan & Markwardt, 2009; Iwayemi & Fowowe, 2011; Mikhaylov, 2018).

The methodological basis of the analysis is the continuation of modern portfolio theory, taking into account the works on the financial theory of optimal consumption and investment (60s of the XX century).

In the early 90s of last century, it was replaced by the postmodern portfolio theory. Of course, it cannot be argued that the fund spending policy can be based only on post modern portfolio theory.
It should also apply the methods for determining the investor’s risk appetite (Buetzer, Habib, & Stracca, 2012). The Merton theory was proved by empirical calculations (Fama, 1984; Fama & French, 1993).

The theory of time preferences suggests that the optimal risk should positively depend on the normal rate of return. The budget rule does not consider risk (although it should be taken into account from the theory). The annual expenses of the fund should depend on the normal rate of return adjusted for risk (Morozko et al., 2018a; Morozko et al., 2018b; Meynkhard, 2019).

2. METHODS

We use two implications of Merton model (Baumeister & Peersman, 2008): optimal costs and investments for an individual agent with an infinite horizon of investment. Like Merton, we use the following formula:

\[ U = E^\gamma_0 \int_0^\infty e^{-\rho t} \frac{c(t)^{-\gamma}}{1-\gamma} dt, \]  

where \( \rho \) is subjective rate of time preference and \( \gamma \) is relative risk aversion, \( E^\gamma_0 \) is the coefficient of elasticity, \( e \) is the Euler number, \( t \) is time, \( c(t) \) is an arbitrary function (series) of \( t \); \( c(0) = 0 \).

In the case of \( \gamma = 1 \), we are dealing with a logarithmic utility. For understandable reasons, we will consider options for \( \gamma \geq 1 \), which is consistent with the preferences of a socially oriented state like Norway and with the preferences of savings accumulation.

There is an opportunity to divide them into investments in a safe asset, generating a fixed income \( r \), and investments in a risky asset as follows:

\[ z(t) \approx N I I D(r + \mu, \sigma^2), \]

where \( N I I D \) (0,1) is a Gaussian variable with independent and equally distributed values in the considered time interval, \( r + \mu \) is income from investments in risk-free and various risky assets, \( \sigma^2 \) is standard deviation.

For simplicity, we assume that the returns of risky assets are serially uncorrelated. Thus, we allow the return of asset prices to the average, but offer some special comments on this issue.

\[ dW(t) = [(r + \alpha(t) \cdot \mu) \cdot W(t) - c(t)] dt + \alpha(t) \cdot \sigma W(t) \cdot dw(t), \]

where \( \alpha(t) \) is the share of investments in risky assets, \( r + \alpha(t) \cdot \mu \) is the expected return on the portfolio, \( w(t) \) is the Wiener process, \( c(t) \) is an arbitrary function (series) of \( t \); \( c(0) = 0 \).

We need to find \( m \) for the following expression:

\[ m = \frac{\mu}{\gamma \sigma^2}, \quad \bar{r} = r + m \mu, \quad \bar{F} = \frac{1}{2} \gamma m^2 \sigma^2. \]

If condition is like this:

\[ (1-\gamma) \bar{F} < \rho < \gamma + (1-\gamma) \bar{F}. \]

This restriction means that the expenditure of the fund is positive, but does not exceed the fund.

Solving this optimization problem implies a constant value of the share of risky assets \( \alpha(t) \) in the portfolio and a constant percentage of the fund’s expenses:

\[ \alpha(t) = m, \quad \eta = \left( \frac{1}{\gamma} \right) \rho + \left(1 - \frac{1}{\gamma}\right) \bar{F}. \]

This result allows us to make two observations related to the formulation of the budget rule. For the first observation, we note that the coefficient \( \eta \) consists of two terms. At the same time, the Norwegian budget rule has only an annuity component. If \( \gamma > 1 \), as we suggest, this combination is a weighted average of two components.

Return to average values, taking into account the risk appetite of investors in our model, the coefficient \( y \) is the magnitude of risk taking (risk appetite).
In the case of the Norwegian budget rule, a permanent component may be associated with a discount rate. Thus, the following assumptions can be made:

1. The optimal rate of spending GPFG funds may exceed the annuity value, if the state prefers to stimulate the current generation of residents of the country.

2. The annuity component of the consumption share of the fund should take into account the amendment to the risk appetite of the inhabitants of the country.

The Norwegian Ministry of Finance has calculated the annual standard deviation for GPFG yield of 9.8%. Then, at $\gamma = 2$, risk adjustment will be 1%. Thus, the model assumes a reduction in fund expenditures, taking into account the Norwegian budget rule, from 4 to 3% of GDP. For GPFG (assets of USD 800 billion), this corresponds to not less than USD 8 billion.

The process of returning to the mean (mean reversion) is studied. Modern researchers (Hooker, 1996; Ferraro, Rogoff, & Rossi, 2015) noted a return to the mean. The standard deviation of GPFG yield in this case will decrease from a 15-year period to 2.5% per annum. This will lead to a decrease in fund expenditures, taking into account the Norwegian budget rule, from 4.0 to 3.8% of GDP.

Thus, the normal rate of return should not be the only component for calculating the disbursement of the fund, the risk of the portfolio and the risk appetite are also important.

The above model assumes a risk-free rate that remains constant over time, but in reality, risk-free rates usually change over time. Head of the Bank of Norway argued that the decline in world real interest rates since the early 80s of the last century should mean a decrease in the optimal expenditure of funds of GPFG.

To take into account the effect of low interest rates, we will use the utility function (1), although the diffusion process will change as follows:

$$dW(t) = \left[ (r(t) + \alpha(t)\mu)W(t) - c(t) \right] dt + \alpha(t)\sigma W(t) dw_\gamma(t).$$

We define the diffusion process for the risk-free rate as:

$$dr(t) = \theta \left[ r^* - r(t) \right] dt + \sigma dw_\gamma(t),$$

where $\theta$ is the expected rate of return of the long-term interest rate to the normal value, expressed by the parameter $> 0$.

This model is much more complicated than (1). We can use the same method to obtain an approximate solution as follows:

$$\alpha = \frac{\mu}{\gamma\sigma^2} - \beta \gamma \left( 1 - 1/\gamma \right) / (\eta^* + \theta),$$

$$\eta(r(t)) = k \exp \left[ \frac{1 - 1/\gamma}{\eta^* + \theta} r(t) \right],$$

where $k$ is an insignificant constant, $\gamma$ is risk appetite, $\eta^*$ is the share of expenditure of the fund, according to the formula (5), if the risk-free interest rate is unchanged $r$, $\beta$ is theoretical regression coefficient, $\theta$ is the expected rate of return of the long-term interest rate to the normal value.

From formula (3) it can be seen that the share of risky assets in the portfolio is higher when the risk-free rate is constant. The behavior of riskier assets is similar to dynamic hedging against a fall in the risk-free asset.

### 3. RESULTS

Thus, when the risk-free rate temporarily deviates from the long-term normal value, the optimal fund-spending rate should be reduced (increased) by the natural logarithm.

The higher the rate of return rates to average values, the greater the difference. If the deviation from the long-term interest rate is instantly corrected, then $\theta \rightarrow \infty$. If the adjustment to the average value of the risk-free rate takes a lot of time, then this is equivalent to the example discussed above (6).
Table 1. Share of annual spending of the GPFG

| Date            | Fund growth, % | Growth – 4% Spending difference, % | Growth – 3% Spending difference, % |
|-----------------|----------------|------------------------------------|------------------------------------|
| December 31, 1998 | 0.143341627    | 0.103342                           | 0.113342                           |
| December 31, 1999 | 0.078362952    | 0.038363                           | 0.048363                           |
| December 31, 2000 | −0.025798347   | −0.0658                            | −0.0558                            |
| December 31, 2001 | −0.068752802   | −0.10875                           | −0.09875                           |
| December 31, 2002 | 0.048953678    | 0.008954                           | 0.018954                           |
| January 31, 2004 | 0.236418319    | 0.196418                           | 0.206418                           |
| December 31, 2004 | 0.135216135    | 0.095216                           | 0.105216                           |
| December 31, 2005 | 0.023076277    | −0.01692                           | −0.00692                           |
| December 31, 2006 | 0.143109373    | 0.103109                           | 0.113109                           |
| December 31, 2007 | 0.099133372    | 0.059133                           | 0.069133                           |
| December 31, 2008 | −0.300526344   | −0.34053                           | −0.33053                           |
| December 31, 2009 | 0.285990976    | 0.245991                           | 0.255991                           |
| December 31, 2010 | 0.097437883    | 0.057438                           | 0.067438                           |
| December 31, 2011 | −0.031402193   | −0.0714                            | −0.0614                            |
| December 31, 2012 | 0.140697255    | 0.100697                           | 0.110697                           |
| December 31, 2013 | 0.141678426    | 0.101678                           | 0.111678                           |
| December 31, 2014 | 0.007530381    | −0.03247                           | −0.02247                           |
| December 31, 2015 | −0.051789636   | −0.09179                           | −0.08179                           |
| Average         | 0.061259852    | 0.02126                            | 0.03126                            |

Table 1 shows that Norwegian budget rule satisfied the optimal spending approach only in 11 years from 18 years. If GPFG would spend less (3% for example), the assets would rise more stable temps (3% per year in average).

Any deviation from the Norwegian budget rule should mean a gradual movement back to its observance. Individuals usually earn labor income in addition to profits from their financial well-being. Similarly, the budgets of most countries (except Saudi Arabia) collect large tax revenues in those years when investment income is also high. That is, budget revenues and sovereign funds revenues are procyclical.

Empirical results proved that the risk-free rate temporarily deviates from the long-term normal value, the optimal fund spending rate should be reduced by the natural logarithm. Conclusively, governments usually try to make these revenues as stable as possible over time, maintaining constant tax rates. However, actual revenue naturally depends on the business cycle. That is, government spending is also procyclical one (Lopatin, 2019).

Thus, in the absence of a change in fiscal policy and without taking into account current contributions to sovereign funds, the budget balance has a countercyclical trend.

Therefore, European countries regularly use this mechanism to reduce the impact of business cycles on the economy as a whole. Recent studies (Singer, 2007; Iwayemi & Fowowe, 2011; Mikhaylov, 2018a) indicate that this mechanism can be applied to the United States and Russia.

Since the volume of assets of Russian sovereign funds in 2016 is only about 5% of GDP, it can be assumed that it will be exhausted in the next 2 years, and there is no sense in talking about optimal spending of funds (Figure 5).
The Norwegian budget rule provides for covering the deficit (excluding oil revenues) of no more than 4% of GDP. Although the structural deficit cannot exceed 4% of GDP, in fact the difference between the actual and structural deficit is covered by the GPFG, because the Norwegian government has no other sources of financing.

Fiscal policy is often a countercyclical tool. Currently, subject to low interest rates and the absence of effective instruments, monetary policy has lost its influence. The Norwegian budget rule allows smoothing out the business cycle due to fiscal policy.

ACKNOWLEDGEMENT

This work was supported by Hankuk University of Foreign Studies Research Fund.

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