Banana orbits in elliptic tokamaks with hole currents

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Abstract. Ware Pinch is a consequence of breaking of up-down symmetry due to the inductive electric field. This symmetry breaking happens, though up-down symmetry for magnetic surface is assumed. In previous work Ware Pinch and banana orbits were studied for tokamak magnetic surface with ellipticity and triangularity, but up-down symmetry. Hole currents appear in large tokamaks and their influence in Ware Pinch and banana orbits are now considered here for tokamaks magnetic surfaces with ellipticity and triangularity.

1. Introduction

Banana orbit are very important for particles confinement in Tokamaks \cite{1,2}. The two most important parameters in banana orbits are the width and the amplitude. In this paper we are studying how this banana width depends on the geometry of the magnetic surface, as the elliptical, triangular, amplitude and other Tokamak characteristic, as the hole currents. The hole currents appear in large tokamaks and their influence in Ware Pinch and banana orbits have not been studied previously, which is the reason reason we have also considered how they could modify the banana width. The present treatment is performed using a general kind of coordinates previously developed \cite{3,4}.

2. Theoretical Treatment and Results

For trapped particles with parallel velocity \( u \ll v \), the curvature drift will be a small correction to the \( \nabla B \) drifts. Considering also the low \( \beta \) approximation, the drift velocity will be

\[
\bar{v} = \frac{\mu e}{e_i B^2} \nabla B \times B
\]

where \( \mu \) is the adiabatic invariant \( \mu = \frac{mv^2}{2B} \), \( e \) is the charge of the particle and \( B \) is the total magnetic field. The magnetic field has only two components toroidal and poloidal, that is, \( B = B_\phi \hat{\varphi} + B_\gamma \hat{b}_\gamma \), and \( B = \sqrt{B_\phi^2 + B_\gamma^2} \), where \( \hat{\varphi} \) is the unitary vector along the toroidal angle and \( \hat{b}_\gamma \) is the tangent unitary vector along the meridian magnetic curves, denoted as S-curves, in this work the tri-orthogonal frame of the curvilinear coordinates, will be denoted as \( (\hat{N}, \hat{b}_\gamma, \hat{\varphi}) \), where \( \hat{N} \) is the unit vector normal to the surface, this vector is equal to the \( -\hat{n} \), used in previous work. The unit vector along the magnetic lines is denoted here with \( \hat{l} \).

\[
v_d = -\frac{\mu}{e_i B^2} \left( B_\phi \frac{\partial B}{\partial \phi} \hat{b}_\gamma - B_\phi \frac{\partial B}{\partial \theta} \hat{N} - B_\gamma \frac{\partial B}{\partial \phi} \hat{\varphi} \right)
\]
\[
\Delta = -2 \int_0^{\tau/4} \mathbf{v}_d \cdot \hat{N} dt = 2 \frac{\mu}{e_i} \int_0^{\tau/4} \frac{B_\phi}{\mathcal{B}^2} \frac{\partial B}{\partial S} dt
\]

Where \(\tau\) is the period of the banana orbit and \(\mathbf{v}_d \cdot \hat{N}\) measures the displacement perpendicular to the magnetic surface, and \(\tau/4\) denotes the time for the trapped particle to move from the mid plane to the reflection point with major radius \(R_l\) and \(R_b\) respectively. The time zero correspond to the mid plane position.

A long and cumbersome calculation leads to

\[
\Delta = \sqrt{2 \frac{\mu m_i}{e_i}} \left( \frac{R_b}{R_i(\lambda)B_p(\lambda)} \right)^{1/2} f(\lambda) \left\{ \int_0^{\theta_b} \frac{\mu \kappa_0 dS}{G(\lambda, \theta) \sqrt{\mu^2 + f(\lambda)^2}} - \int_0^{\theta_a} \frac{\sin(\theta) dS}{G(\lambda, \theta) R_\mu} \right\}
\]

where

\[
f(\lambda) = \frac{2\pi q(\lambda)}{\oint \frac{ds}{R_c}}
\]

and

\[
G = G(\lambda, \theta) = \sqrt{\mu_b^2 + f(\lambda)^2} - \frac{R_b}{R} \sqrt{\mu^2 + f(\lambda)^2}
\]

Here \(q(\lambda)\) is the safe factor.

These results can be combined with the equation for end presented in previous work

\[
R = R(\lambda, \theta) = R_m + \lambda a \cos \theta - a T(1 - \cos 2\theta) \quad \mu = \mu(\lambda, \theta) = \exp \left( \int_0^s \kappa_s ds \right)
\]

\[
R_l = R_m + \lambda a - \lambda^2 \Delta_s(a) \quad R_b = R_m + \lambda a - \lambda^2 (\Delta_s(a) + a T(a))(1 - \cos \theta_b)
\]

Where \(R_m\) is the major radius of the minor axis, \(E(\alpha)\) and \(T(\alpha)\) are the ellipticity and triangularity distortions, \(\Delta_s(a)\) and Shafranov shift, \(2a\) is the width of the plasma along the mid-plane, \(\alpha\) is the parameter denoting the magnetic surface where the path of the particle is projected and \(\theta_b\) and \(\theta'_b\) are the poloidal angle corresponding to the largest and smallest reflection points respectively. The coefficients of ellipticity and triangularity distortion, \(E(\alpha)\) and \(T(\alpha)\), are measured at the plasma on the 95% surface, and they are related to the ellipticity coefficient \(\kappa(\alpha)\) and triangularity \(\delta(\alpha)\) by expressions given in previous papers. In particular case in which the magnetic surfaces have a circular geometry the curvature \(\kappa_0 = 0\) and \(\mu(\lambda, \theta) = 1\). In this case the expression 3 and 4 have the form

\[
\Delta = \sqrt{2 \frac{\mu m_i}{e_i}} \left( \frac{R_b}{R_{lc}(\lambda)R_{pc}(\lambda)} \right)^{1/2} f(\lambda) \left\{ - \int_0^{\theta_b} \frac{\sin \theta ds}{G(\lambda, \theta) R_c} \right\}
\]

and

\[
f_c(\lambda) = 2\pi q(\lambda) / \oint \frac{ds}{R_c}, \quad G_c = 1 - \frac{R_b}{R} = \sqrt{1 + (f_c(\lambda))^2},
\]

where the sub index c is to denote circular surfaces.

Here we are interested on find the banana width effects due to current holes. In this case the Shafranov shift, triangularity and elliptic elongation are taken from the work of Yavorky et al. They definitions are

\[
\Delta(\lambda) = \Delta_0 (1 - \lambda^2), \quad \kappa(\lambda) = \kappa_0 - (\kappa_0 - \kappa_0)(1 - \lambda^2) - \frac{1}{2} \kappa'_0 \lambda^2 (1 - \lambda^2)
\]
where $\Delta_0$ denote the triangularity instead of the usual notation. The parameter $\delta(\epsilon \lambda)$ denoting as usual the magnetic surface, $\lambda$ which is also defined as $\lambda = \tilde{\delta}/a = r/a$. In the Yavorsky paper a parameter $x_m$ is introduced in order to adjust the analytic form to the experimental data. The calculation here will be carried out for the tokamak JT-60U and for this device a good value of $x_m$ is 0.6. In this tokamak the toroidal current density has the analytic form

$$j_\phi = (1 - \lambda^2)^2(1 - (1 - \lambda)^6)^2m, \quad m = \frac{3((1-x_m)^{-6} - 1)}{1 + x_m^{-1}} \quad (11)$$

The results of the application of these equations are shown in figures 4, 5, 6, 7 and 8. The two first figures can be considered as those with no hole currents and three last ones are for large Tokamaks, that is, those with hole currents. In figure 4 we see that the maximum width are almost for the same value of the ellipticity $\kappa$, which is different to one. This peculiarity also happens for the triangularity in the case of small Tokamaks, see figure 5, but it is not so clear for last Tokamaks as the JY-60U, see figure 6 and 7. The figure 8 is not related to the banana width, and it shows how triangularity, elasticity, toroidal current density and safety factors change with the location of the magnetic surfaces, that is, the value of $\lambda$. In the case of elasticity and triangularity both parameter increase with $\lambda$. In the case of toroidal current density, there is the maximum of some value of $\lambda$, and the safety factor increases rapidly to very large value nearby the magnetic axis.

![Figure 01](image-url)  
*Figure 01: cross section od a Tokamak, showing the system of coordenates used in this paper.*
Figure 02: Cross section of a Tokamak showing the magnetic surfaces for different values of \( \lambda \) and all the important radius in this work.

Figure 03: The banana orbit is shown in this figure with the most relevant parameters.
Figure 04: Normalized width of the banana orbit as a function of the ellipticity $\kappa$, for 4 different banana amplitudes. The widths are normalized with circular banana width in all the figures in this work.

Figure 05: Normalized width of banana orbits as a function of the triangularity for 4 different banana amplitudes (no hole current).
Figure 06: Normalized width of banana orbits as a function of the triangularity for ITER, in a range around the values of design.

Figure 07: As in the preceding figure here the normalized width is considered as a function of triangularity for the Japanese Tokamaks JT-60, considered a Tokamak with hole currents.
3. Conclusions
It is shown that the Ware Pinch effect can be obtained from the guided center first order theory, for tokamak magnetic surfaces with ellipticity and triangularity, and Current Hole. The Banana orbit width had been determined using general system of coordinates proposed by us in previous papers. It is also confirmed than the Ware Pinch is a consequence of a up-down simetry breaking due to the toroidal inductive electric field and dependent of the geometry properties of the magnetic surfaces. The results are in agreement with the clasical Ware Pinch. However the expressions of the new Ware Pinch are more precise and they depend of the aditional parameters of the surface as triangularity and ellipticity. There are some differences due to current hole, but the shape of the banana is similar and the differences seem to be no essentials.

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