Collective neutrino flavor transitions in Supernovae: Analytical and numerical aspects

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Abstract. Non-linear effects on supernova neutrino oscillations, associated with neutrino-neutrino interactions, are known to induce collective flavor transformations near the supernova core for $\theta_{13} \neq 0$. For typical electron density profiles (as taken from shock-wave simulations at a few seconds after bounce) these transformations precede ordinary matter effects, and become more amenable to both numerical computations and analytical interpretations in inverted hierarchy—while they basically vanish in normal hierarchy. We numerically evolve the neutrino density matrix in the region relevant for self-interaction effects, using thermal spectra and a representative value $\sin^2 \theta_{13} = 10^{-4}$. Our results neatly show the collective phenomena of synchronization, bipolar oscillations, and spectral split, with analytically understandable features. They also suggest that averaging over neutrino trajectories plays a minor role in the final outcome. The split/swap of (anti)neutrino spectra emerges as an unmistakable signature of the inverted neutrino hierarchy.

1. Introduction

Neutrinos emitted by core-collapse supernovae (SN) represent an important laboratory for both particle physics and astrophysics. While propagating in the dense SN environment, they can feel not only the presence of background matter (via ordinary Mikheev-Smirnov-Wolfenstein, MSW, effects) but also of the gas of neutrino and antineutrinos (via self-interaction effects). The self-interactions appear to modify the flavor evolution of SN neutrinos in a collective way, completely different from ordinary MSW effects. However, the analysis is made complicated by the non-linear nature of the flavor evolution equations, and by the fact that neutrino-neutrino interactions depend on the intersection angle $\vartheta_{ij}$ of their trajectories [1].

We have recently contributed to the analysis of such collective neutrino flavor transitions in SN in the work [2], to which we refer the reader for further details and references. In particular, we have shown that, in the context of typical SN explosion simulations, collective effects (which precede MSW effects) are not strongly affected by a detailed (multi-angle) description of the $\vartheta_{ij}$ distribution. Indeed, one of the main observable effects arising for nonzero mixing angle $\theta_{13}$ in inverted neutrino mass hierarchy—the so-called spectral swap and split—is already captured by the (single-angle) approximation where the $\vartheta_{ij}$’s are replaced by an average angle. These results will be briefly illustrated in the following. See also the talk [3] at this Conference for a study of single-angle vs multi-angle simulations with variable neutrino-antineutrino asymmetry.
2. Supernova neutrino input

Three main quantities with dimension \([T^{-1}]\) ("frequencies") characterize the flavor evolution of SN neutrinos along the radial coordinate \(r\), namely, the energy \(E\) or the associated vacuum oscillation frequency \(\omega\),

\[
\omega = \frac{\Delta m^2}{2E},
\]

(in terms of the largest squared mass difference \(\Delta m^2 = m_3^2 - m_1^2\)), the interaction energy difference \(\lambda\) in matter between \(\nu_e\) and \(\nu_x\) (where \(x = \mu, \tau\)),

\[
\lambda(r) = \sqrt{2} G_F N_{e-}(r),
\]

\(N_{e-}(r)\) being the net electron number density at the point \(r\), and the self-interaction strength parameter \(\mu(r)\),

\[
\mu(r) = \sqrt{2} G_F [N(r) + \overline{N}(r)],
\]

where \(N\) and \(\overline{N}\) are the effective neutrino and antineutrino densities, respectively.

We fix the above frequency parameters according to SN shock-wave simulations (a few seconds after bounce) as shown in Fig. 1 (energy spectra) and in Fig. 2 (interaction energies). Figure 2 also shows the expected ranges where the so-called synchronized oscillations (joint flavor evolution of neutrinos and antineutrinos at high density), bipolar oscillations (partially disjoint flavor evolution of neutrinos and antineutrinos at intermediate densities) and spectral split effects (with freeze-out of energy spectra at low density) are analytically expected to take place. Ordinary MSW effects (if any) occur only later, beyond the radial range of Fig. 2.

3. Equations of motion (EOM)

In a two-family \((\nu_e, \nu_x)\) scenario, the \(2 \times 2\) neutrino and antineutrino density matrices in flavor basis can be decomposed onto Pauli matrices via "polarization" (Bloch) vectors \(\mathbf{P}\) and \(\overline{\mathbf{P}}\), respectively, at any fixed energy. In single-angle approximation, the individual polarization vectors obey the following coupled equations of motion in flavor space,

\[
\dot{\mathbf{P}} = (\omega \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \mathbf{P},
\]

\[
\dot{\overline{\mathbf{P}}} = (-\omega \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \overline{\mathbf{P}},
\]

where \(\mathbf{D} = \mathbf{J} - \overline{\mathbf{J}}\), and \(\mathbf{J}, \overline{\mathbf{J}}\) are the integral (energy-averaged) polarization vectors. The \(z\)-component of the polarization vectors is directly linked to the \(\nu_e\) survival probability (e.g., inversion of \(z\)-component corresponds to complete flavor change). In the multi-angle case, a further dependence on the neutrino trajectory crossing angle arises in the above equations.
4. Results and conclusions

We have solved numerically the EOM in the single-angle and multi-angle cases. Many of the observed features can be understood analytically. The main results are the following.

Figures 3 and 4 show the behavior of the norm and of the $z$-component of the global polarization vectors for neutrinos and antineutrinos, assuming inverted neutrino hierarchy (i.e., $\Delta m^2 < 0$). We have fixed the mixing angle $\theta_{13} = 10^{-2}$, but its exact value has little relevance (as far as it is nonzero). The multi-angle case appears to be a “smeared” version of the single-angle one, but the average behavior and the final states at the end of collective effects ($r \sim 200$ km) are similar. Figures 5 and 6 display the energy spectra at the end of collective effects in single-angle and multi-angle calculations. In both cases, a comparison with Fig. 1 shows that the $\nu_e$ and $\bar{\nu}_x$ spectra are split and swapped above a critical neutrino energy, while the $\nu_e$ and $\bar{\nu}_e$ spectra are basically interchanged. This peculiar feature (spectral split/swap) appears to be a robust, important signature of self-interaction effects in inverted hierarchy, which cannot be mimicked by ordinary MSW effects. See [2] for further details, explanations and references.

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