Synchronous concentration and purification schemes of arbitrary unknown hyperentangled mixed states

Kun Du\textsuperscript{1}, Qiu-Cheng Song\textsuperscript{1} and Cong-Feng Qiao\textsuperscript{1,2*}

\textsuperscript{1}School of Physics, University of Chinese Academy of Sciences
YuQuan Road 19A, Beijing 100049, China
\textsuperscript{2}Collaborative Innovation Center for Particles and Interaction
USTC, HeFei 230026, China

Abstract

We present two efficient schemes which can simultaneously accomplish hyperentanglement concentration and purification for two-photon four-qubit systems in an unknown partially hyperentangled mixed states. The first can correct errors in the polarization entanglement and extract maximal hyperentanglement in polarization and spatial mode with additional partial frequency entanglement. The second uses additional maximal frequency entanglement to purify and concentrate hyperentanglement in polarization and spatial mode deterministically. Both of the two schemes are only based on existing optical devices and cross-Kerr nonlinearities.

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1 Introduction

Entanglement is viewed as a kind of raw resource of quantum information science, such as measurement-based quantum computing [1], quantum teleportation [2], quantum dense coding [3], and quantum cryptography [4,5]. Meanwhile, to go further in the manipulation of more entangled qubits, hyperentanglement, namely making the quanta, e.g. photons, to
be entangled simultaneously in multiple degrees of freedom (DOFs) has received more and
more attention for quantum information process [6–8]. For example, in recent years, it has
been applied in quantum key distribution (QKD) protocol [9], Bell-state analysis [10–12],
entanglement purification protocol (EPP) [13–15] and quantum repeater protocol [16].

Although at present the preparation of hyperentanglement is high-efficient and high-
quality, the entangled photon pairs are usually locally produced, thus decoherence in the
long-distance quantum communication channel is unavoidable, which will significantly re-
duce the quality of photon pairs and decrease their entanglement. Therefore the efficiency
and fidelity of quantum communication protocols between distant locations will be greatly
decreased. The main methods to overcome decoherence in quantum communication process
are entanglement purification and entanglement concentration. The former is a method by
which one can obtain a smaller set of high-fidelity entangled pairs from a large number of
less-entangled pairs in a mixed state. The latter is used to distill maximally entangled pairs
out of a set of partially entangled pairs in a pure state. In 1996, Bennett et al. proposed
the first entanglement purification protocol (EPP) for two-photon systems in mixtures of
the four Bell states, resorting to quantum controlled-not (CNOT) gates and local unitary
operations [17]. In 2002, Simon and Pan presented an EPP with parametric down-conversion
(PDC) source and currently available linear optical elements [18]. In 2010, a deterministic
EPP with hyperentangled state was proposed by Sheng et al. [19]. Meanwhile, many sig-
ificant entanglement concentration protocols (ECPs) have been presented. For example,
in 2012, Deng proposed an ECP for photon systems with known parameters based on pro-
jection measurements [20]. In 2001, an ECP with unknown polarization entangled states
was proposed by Zhao et al. [21]. Recently, Ren et al. put forward a hyperentanglement
concentration protocol (hyper-ECP) for the systems in partially hyperentangled state [22].

In this work, we investigate the methods of simultaneously correcting errors and distilling
maximal hyperentanglement in both the polarization and spatial mode DOFs with two-
photon systems in the nonlocal partially hyperentangled mixed states. First, we only correct
the bit-flip error and phase-flip error of the polarization entanglement, and extract both the
maximally polarization and spatial mode entangled states at the cost of additional partial
frequency entanglement. Subsequently, we simultaneously correct errors of polarization and
spatial mode entanglement and extract the maximal hyperentanglement by dint of additional maximal frequency entanglement.

2 Hyper-ECP with additional partial frequency entanglement

Our mission is to transmit the maximally Bell hyperentangled state $|\varphi\rangle = \frac{1}{2}(|H\rangle|H\rangle + |V\rangle|V\rangle) \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$ to the parties Alice and Bob. Using the SPDC source presented by Du et al. [23], we can produce pairs of photons entangled in three DOFs:

$$|\psi\rangle = \frac{1}{2\sqrt{2}}(|HH\rangle + |VV\rangle) \otimes (|a_1b_1\rangle + |a_2b_2\rangle) \otimes (|\omega_1\omega_2\rangle + |\omega_2\omega_1\rangle),$$

where $H$ and $V$ denote horizontal and vertical polarization, $\omega_1$ and $\omega_2$ signify different frequency and $a_1$, $b_1$, $a_2$, $b_2$ label different spatial modes, and suppose $\omega_2 > \omega_1$. As the spatial mode and frequency are more stable than polarization [24–27], we assume that there are no bit-flip errors and phase-flip errors in the spatial mode and frequency DOFs, and they only become partially entangled states after transmission through noisy channels:

$$|\psi_s\rangle = \gamma|a_1b_1\rangle + \delta|a_2b_2\rangle,$$

$$|\psi_f\rangle = \varepsilon|\omega_1\omega_2\rangle + \eta|\omega_2\omega_1\rangle.$$  

But the polarization state changes into a mixed one:

$$\rho_p = F_1|\psi_{p1}\rangle\langle\psi_{p1}| + F_2|\psi_{p2}\rangle\langle\psi_{p2}| + F_3|\psi_{p3}\rangle\langle\psi_{p3}| + F_4|\psi_{p4}\rangle\langle\psi_{p4}|,$$

where $F_1+F_2+F_3+F_4=1$, and

$$|\psi_{p1}\rangle = \alpha|HH\rangle + \beta|VV\rangle,$$

$$|\psi_{p2}\rangle = \alpha|HV\rangle + \beta|VH\rangle,$$

$$|\psi_{p3}\rangle = \alpha|HH\rangle - \beta|VV\rangle,$$

$$|\psi_{p4}\rangle = \alpha|HV\rangle - \beta|VH\rangle.$$  

We consider two pairs of photons AB and CD in the above mixed state. The photons A and C are transmitted to Alice, and the photons B and D belong to Bob. The six
parameters $\alpha$, $\beta$, $\gamma$, $\delta$, $\varepsilon$, and $\eta$ are unknown to Alice and Bob, and they satisfy the relation $|\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = |\varepsilon|^2 + |\eta|^2 = 1$.

As the two pairs are both initially in the mixed state $\rho = \rho_p \rho_s \rho_f$, so there are 16 kinds of cases. We will discuss the following situation as an example, that is the initial state of the four-photon system can be written as:

$$|\phi\rangle = |\psi_{p_1}\rangle_{AB}|\psi_s\rangle_{AB}|\psi_{f_1}\rangle_{AB} \otimes |\psi_{p_2}\rangle_{CD}|\psi_s\rangle_{CD}|\psi_{f_2}\rangle_{CD}$$

$$= (\alpha^2|HHHV\rangle + \beta^2|VVVH\rangle + \alpha\beta|HHVH\rangle + \alpha\beta|VVHV\rangle)$$

$$\otimes (\gamma^2|a_1b_1c_1d_1\rangle + \beta^2|a_2b_2c_2d_2\rangle + \gamma\delta|a_1b_1c_2d_2\rangle + \gamma\delta|a_2b_2c_1d_1\rangle)$$

$$\otimes (\varepsilon^2|\omega_1\omega_2\omega_1\omega_2\rangle + \eta^2|\omega_2\omega_1\omega_2\omega_1\rangle + \varepsilon\eta|\omega_1\omega_2\omega_1\omega_2\rangle + \varepsilon\eta|\omega_2\omega_1\omega_1\omega_2\rangle). \quad (6)$$

Figure 1: $QND_1$. Extracting the parts of maximal entanglement in spatial mode and frequency with cross-Kerr nonlinear medium performed by Alice. OD and OM denote an optical demultiplexer and an optical multiplexer respectively which are parts of wavelength division multiplexer (WDM).

In the first place, Alice uses a quantum nondemolition detector ($QND_1$) to pick out maximal entanglement in spatial mode and frequency. As shown in Figure 1, photons A and C are led to a cross-Kerr nonlinear medium [28, 29], which brings forth an adjustable phase shift to the coherent states through cross-phase modulation (XPM). We use a 50:50 beam splitter (BS) to divide the coherent state into two beams $|\alpha\rangle |\alpha\rangle$ [30], and then they are coupled to the photonic modes $a_1$ and $c_1$, $a_2$ and $c_2$ through the XPM respectively. Correspondingly, the phase shifts induced by the couplings are $\theta$ and $2\theta$ in both beams. Then we separate each path into two in terms of frequency by optical demultiplexers (OD) [31], all the upper paths correspond to frequency $\omega_1$ and can induce a phase shift of $\theta$ on the upper
coherent state, while all the under paths correspond to frequency $\omega_2$ and bring the same phase shift to the under coherent state. Through an X homodyne measurement [32], if the two coherent states have the same phase shift, namely corresponding to the last two terms in both spatial mode and frequency DOFs, the state of the four-photon system becomes

$$|\phi\rangle = \frac{1}{2}(\alpha^{|HHHV\rangle} + \beta^{|V VH H\rangle} + \alpha\beta^{|H HV H\rangle} + \alpha\beta^{|VV VH\rangle})$$

$$\otimes(|a_1 b_1 c_2 d_2\rangle + |a_2 b_2 c_1 d_1\rangle) \otimes(|\omega_1\omega_2\omega_2\omega_1\rangle + |\omega_2\omega_1\omega_1\omega_2\rangle) .$$  \hspace{1cm} (7)

Figure 2: Scheme of realizing entanglement transformation between polarization and frequency DOFs, and erasing frequency entanglement information of photon pair CD. HWP represents a half-wave plate which is used to perform a bit-flip operation in polarization.

Then we perform entanglement transformation between polarization and frequency DOFs by dint of the apparatuses shown in Figure 2. After dividing each path into two with different frequency by OD, we use polarizing beam splitters (PBSs) and half-wave plates (HWPs) to make the polarizations of photons A and D change into horizontal polarizations if their frequencies are $\omega_1$, whereas if their frequencies are $\omega_2$, they will be vertical polarizations. In contrast, the transformations in the polarizations of photons C and B are completely opposite to photons A and D. Furthermore, put an attenuator in each path corresponding to $\omega_2$, in order to turn the frequencies of the four photons all into the same, i.e. $\omega_1$. Whereupon
the state of total system can be rewritten as:

$$|\phi\rangle = \frac{1}{2}(|HHHH\rangle + |VVVV\rangle) \otimes (|a_1b_1c_2d_2\rangle + |a_2b_2c_1d_1\rangle) \otimes |\omega_1\omega_1\omega_1\omega_1\rangle.$$  \hspace{1cm} (8)

Finally, we need only to extract the hyperentanglement of AB out of the four-body hyperentanglement. After going through the devices shown in Figure 3, the corresponding state of CD in polarization and spatial mode becomes $|HH\rangle \otimes |c_1d_1\rangle$. In this way, we obtain the maximally hyperentangled state of photon pair AB, $|\phi\rangle_{AB} = \frac{1}{2}(|H\rangle|H\rangle + |V\rangle|V\rangle) \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$.

![Figure 3: Scheme of erasing entanglement informations of photon pair CD in polarization and spatial mode.](image)

Similarly, the other 15 cases also have the same result. In principle, as long as the two coherent states at Alice’s side have the same phase shift, this protocol succeeds with the probability $P = 4|\gamma\delta\epsilon\eta|^2$. Otherwise, it fails. Therefore, in practice, we only need Alice to judge whether it succeeds or fails, and do not rely on postselection from both sides.
3 Hyper-ECP with additional maximal frequency entanglement

In this section, we assume that the polarization states and spatial mode states of the two-photon systems are both turned into the following mixed forms after transmission:

\[ \rho_p = F_1 |\psi_{p1}\rangle \langle \psi_{p1}| + F_2 |\psi_{p2}\rangle \langle \psi_{p2}| + F_3 |\psi_{p3}\rangle \langle \psi_{p3}| + F_4 |\psi_{p4}\rangle \langle \psi_{p4}|, \]

\[ \rho_s = G_1 |\psi_{s1}\rangle \langle \psi_{s1}| + G_2 |\psi_{s2}\rangle \langle \psi_{s2}| + G_3 |\psi_{s3}\rangle \langle \psi_{s3}| + G_4 |\psi_{s4}\rangle \langle \psi_{s4}|, \] (9)

where \( F_1 + F_2 + F_3 + F_4 = 1 \), \( G_1 + G_2 + G_3 + G_4 = 1 \) and

\[ |\psi_{p1}\rangle = \alpha |HH\rangle + \beta |VV\rangle, \]
\[ |\psi_{p2}\rangle = \alpha |HV\rangle + \beta |VH\rangle, \]
\[ |\psi_{p3}\rangle = \alpha |HH\rangle - \beta |VV\rangle, \]
\[ |\psi_{p4}\rangle = \alpha |HV\rangle - \beta |VH\rangle; \] (11)

\[ |\psi_{s1}\rangle = \gamma |a_1b_1\rangle + \delta |a_2b_2\rangle, \]
\[ |\psi_{s2}\rangle = \gamma |a_1b_2\rangle + \delta |a_2b_1\rangle, \]
\[ |\psi_{s3}\rangle = \gamma |a_1b_1\rangle - \delta |a_2b_2\rangle, \]
\[ |\psi_{s4}\rangle = \gamma |a_1b_2\rangle - \delta |a_2b_1\rangle. \] (12)

While the frequency state still remains maximally entangled, i.e.

\[ |\psi_f\rangle = |\omega_1\omega_2\rangle + |\omega_2\omega_1\rangle. \] (13)

We also use two pairs of photons AB and CD in the new mixed states. Similar to the protocol introduced in preceding section, the photons A and C belong to Alice, and the other two B and D belong to Bob. The four parameters \( \alpha, \beta, \gamma \) and \( \delta \) are unknown to Alice and Bob, and satisfy the relation \( |\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = 1 \).

In this case, there will be 256 kinds of combinations. We take the following generic form
as an example:

\[
\begin{align*}
|\phi\rangle &= |\psi_1\rangle_{AB}|\psi_s_1\rangle_{AB}|\psi_f\rangle_{AB} \otimes |\psi_{p_2}\rangle_{CD}|\psi_{s_2}\rangle_{CD}|\psi_f\rangle_{CD} \\
&= (\alpha|HH\rangle + \beta|VV\rangle)(\alpha|HV\rangle + \beta|VH\rangle) \\
&\otimes (\gamma|a_1 b_1\rangle + \beta|a_2 b_2\rangle)(\gamma|c_1 d_1\rangle + \delta|c_2 d_2\rangle) \\
&\otimes (|\omega_1 \omega_2\rangle + |\omega_2 \omega_1\rangle)(|\omega_1 \omega_2\rangle + |\omega_2 \omega_1\rangle). \qquad (14)
\end{align*}
\]

To begin with, we can change the spatial mode states of the two pairs into maximal entanglement respectively with the quantum circuit shown in Figure 4. Alice and Bob let the two paths of each photon link with an OD and be respectively separated into different paths again according to frequency. As shown, the paths with the same frequency will be merged into the same path by an optical multiplexer (OM). Here we also need to erase the information of frequency entanglement by attenuators. Hence, the state of the four photons becomes

\[
\begin{align*}
|\phi\rangle &= \frac{1}{2}(\alpha^2|HHHV\rangle + \beta^2|VVVH\rangle + \alpha\beta|HHVH\rangle + \alpha\beta|VHVH\rangle) \\
&\otimes (|a_1 b_1\rangle + |a_2 b_2\rangle)(|c_1 d_1\rangle + |c_2 d_2\rangle) \otimes |\omega_1 \omega_1 \omega_1 \omega_1\rangle. \qquad (15)
\end{align*}
\]

Next, as shown in Figure 5, both Alice and Bob perform H and H’ operations on the horizontal and vertical polarization of photons A and B respectively. As a result, horizontal and vertical polarization are both transformed into superposition state \(\frac{|H\rangle + |V\rangle}{\sqrt{2}}\). And for the photon C (D), we transform its polarization into H and V, according to its spatial modes.
Figure 5: Scheme of transforming the polarization of photons A and B into superposition state, and forming an EPR-like entangled state between polarization and spatial mode DOFs of photon pair CD. H (H’) represents the operation $|H\rangle(|V\rangle \rightarrow \frac{|H\rangle+|V\rangle}{\sqrt{2}}$.

$c_1 (d_1)$ and $c_2 (d_2)$ respectively with PBSs, HWPs and OMs. Then the state of the whole system can now be written as

$$|\phi\rangle = \frac{1}{4}(|H\rangle + |V\rangle)(|H\rangle + |V\rangle) \otimes (|a_1b_1\rangle + |a_2b_2\rangle) \otimes (|HHc_1d_1\rangle + |VVc_2d_2\rangle) \otimes |\omega_1\omega_1\omega_1\omega_1\rangle.$$ 

(16)

Subsequently, as shown in Figure 6, both Alice and Bob exploit another quantum non-demolition detector (QND2) composed of PBSs and cross-Kerr nonlinear medium. The cross-Kerr nonlinearity will make the upper coherent beam $|\alpha\rangle$ pick up a phase shift $\theta$, if the polarization of photon A (B) is $H$ ($H$) or photon C (D) is in the mode $c_2$ ($d_2$). While if the polarization of photon A (B) is $V$ ($V$) or photon C (D) is in the mode $c_1$ ($d_1$), there will be a a phase shift $\theta$ in the under coherent beam $|\alpha\rangle$. After performing homodyne measurements on both sides and transforming the state of CD into $|HH\rangle \otimes |c_1d_1\rangle$, if the differences between phase shift of upper $|\alpha\rangle$ and phase shift of under $|\alpha\rangle$ from Alice and Bob are both 0 or 2$\theta$ (homodyne detection can’t distinguish plus and minus [33, 34]), the selected terms constitute the maximally hyperentangled state of photon pair AB, $|\varphi\rangle_{AB} = $
\( \frac{1}{2}(|H\rangle|H\rangle + |V\rangle|V\rangle) \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle) \). If one of the differences is 0, the other is \( 2\theta \), the state of AB becomes \( |\phi\rangle_{AB} = \frac{1}{2}(|H\rangle|V\rangle + |V\rangle|H\rangle) \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle) \). And then we can get the ideal maximally hyperentangled state by performing the bit-flipping operation \( \sigma_x = |H\rangle\langle V| + |V\rangle\langle H| \) on the photon A or B.

Figure 6: QND2. If the results of homodyne measurements from Alice and Bob are same, the state of photon pair AB is projected into the maximally hyperentangled state \( |\phi\rangle_{AB} = \frac{1}{2}(|H\rangle|H\rangle + |V\rangle|V\rangle) \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle) \). If the results are different, only a bit-flip operation \( \sigma_x = |H\rangle\langle V| + |V\rangle\langle H| \) on the photon A or B is needed to obtain the state \( |\phi\rangle_{AB} \).

For other combinations, in the same way, we can purify arbitrary mixed states in polarization and spatial mode, and distill target systems in the maximally hyperentangled state from whole systems in the partially hyperentangled state. Alice and Bob can determinately obtain the expected nonlocal states just through local operation and classical communication (LOCC), i.e. success probability is 100%.

4 Conclusions

In summary, we have proposed two hyper-ECPs for two-photon systems in partially hyperentangled unknown state, resorting to linear optical instruments and cross-Kerr non-linearities. In the first situation, the entangled states in all the three DOFs turn to be in the form of partial entanglement, while only the polarization part may suffer from bit-flip and
phase-flip errors during transmission. The two nonlocal parties use two arbitrary photon pairs in the unknown hyperentangled mixed state to correct errors in polarization and extract maximal entanglement in polarization and spatial mode. It is obvious that whether this ECP may succeed or not only depends on the result of QND1 (see Figure 1) placed on the side of Alice. The success probability changes with the parameters of the partially entangled states in spatial mode and frequency. Besides, it is easy to see when the parameters of the partially polarization entangled states of the two pairs are different, even for non-entangled states, the ECP is also applicable. In the second situation, both of the entangled states in polarization and spatial mode are likely to suffer from bit-flip and phase-flip errors, and become less-entangled, but the frequency entanglement remains intact. The two parties can get a pair of photons with maximal entanglement in both polarization and spatial mode DOFs by LOCC and the consumption of another photon pair and their frequency entanglements determinately. Especially, for the two photon pairs, all the four parameters of the partially hyperentangled states can be arbitrary (including 0) and different from each other. Theoretically, our ECPs can be simply expanded to the situation with more photons or other particles, thus they are generally applicable. In a practical application, our ECPs can be realized with current technology and will greatly improve the efficiency and fidelity of long-distance quantum communication, which enable us to take full advantage of the superiority of hyperentanglement in the future.

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