A New Approach to Fourth-Order Quadrature Signal Generation for a Fast and Noise-Free PLL Output Under Non-Ideal Grid Voltage Conditions

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ABSTRACT Dual quadrature signal generator (QSG) along with a positive sequence calculator (PSC) eliminates the negative sequence components from unbalanced grid voltages. The QSGs also offer attenuation to harmonics and dc-offset present in the sensed input voltages. Fourth-order QSGs offer superior harmonic attenuation with complete dc-offset rejection compared to lower-order QSGs. Therefore, dual fourth-order QSG + PSC is an apt solution for pre-filtering in phase-locked loops (PLLs) under non-ideal grid voltage conditions. Two fourth-order QSGs (FO-QSGs) namely, second-order SOGI-QSG (SO-SOGI-QSG) and cascaded SOGI-QSG have been proposed in the literature. Parameter selection for these FO-QSGs to achieve a faster dynamic response is challenging due to the higher order of the transfer functions. The response time of the QSGs directly impact the response time of the PLL. This paper proposes a new approach to fourth-order quadrature signal generation that achieves a lower settling time for the QSG compared to existing FO-QSGs. The proposed method achieves a faster response time by setting the QSG parameters using equations that directly relate to the system settling time. Using the proposed method, the QSG settles within 25.4 ms while maintaining a low total harmonic distortion.

INDEX TERMS Phase-locked loop, non-ideal grid, second-order generalized integrator, quadrature signal generator.

I. INTRODUCTION
A phase-locked loop (PLL) provides voltage magnitude, frequency, and phase angle information of the utility grid for synchronization and reference current generation in grid-tie converter applications [1]. The commonly used PLL structure is the synchronous reference frame based PLL (SRF-PLL). The SRF-PLL utilizes a synchronously rotating direct-quadrature axis (dq axis) along with a proportional-integral (PI) controller for its implementation [2], [3]. The PI controller of the SRF-PLL is designed to achieve a required bandwidth (BW) that gives desired transient and steady-state response.

Under non-ideal grid voltage conditions, the fundamental positive-sequence voltage of the utility appears as a dc component in the dq frame while the abnormalities like harmonics, unbalance and dc-offset appear as ac components [2]. The fundamental positive-sequence voltage is the information of interest for synchronization and controls. Two broad approaches are used in literature to handle the effect of abnormalities [4]–[6], [9]–[13]. While [4] and [5] handle the abnormalities inside the PLL, [6], [9]–[13] handle the same outside the PLL using a pre-filter.

For a three-phase SRF-PLL, the pre-filter approach based on dual quadrature signal generator (QSG) with a positive sequence calculator (PSC) implemented in the $\alpha - \beta$ reference frame [6], [9], [10], [13] provides better results [14] as compared to [4], [5]. Fig. 1 shows the block diagram of a pre-filter with a dual-QSG and PSC. The QSG can be implemented using transfer functions of varying orders. While the QSGs eliminate the dc component and provide attenuation to the harmonics of the sensed grid voltage, the PSC handles the harmonics and dc-offset.
In [13] two SOGI-QSGs with identical parameters are cascaded to obtain a FO-QSG named cascaded SOGI-QSG (CSO-QSG). The first SOGI-QSG of the cascaded structure is used to eliminate the dc-offset from the sensed grid voltage while the two cascaded SOGI-QSGs together are used to give fourth-order attenuation to harmonics. The parameter selection outlined in [13] is primarily based on harmonic attenuation. The harmonic attenuation of the overall cascaded structure is set to be the same as that of a single SOGI-QSG. This is expected to lead to a faster dynamics for the individual SOGI-QSGs of the cascaded structure as compared to that of a single SOGI-QSG. The method is good in terms of harmonic attenuation performance. However, it suffers from certain flaws which are not addressed in [13]. Since the harmonic attenuation of the cascaded SOGI-QSGs and the single SOGI-QSG are equated, the damping factor of SOGI-QSGs of the cascaded structure has an empirical relation with the damping factor of a single SOGI-QSG. If the damping factor of the single SOGI-QSG is not selected carefully, the SOGI-QSGs of the cascaded structure can become overdamped and result in a slower response than the single SOGI-QSG, negating the basic premise of the design. Secondly, even though the settling time of each SOGI-QSGs is known, the combined effect of the cascaded structure on the settling time of the overall system has not been examined in [13]. The method is useful if a high attenuation is required and the settling time is of secondary importance.

In order to achieve a lower settling time for the dual-QSG + PSC structure, a new approach is proposed for the FO-QSG in this paper. The proposed approach ensures faster transients while maintaining a good steady-state performance. Further, the proposed approach helps to closely predict the settling time of the QSG at the design stage.

The proposed approach uses a cascaded structure in [13] but with non-identical parameters for the individual SOGI-QSGs, and is referred to as cascaded non-identical SOGI-QSGs (CNISOGI-QSG) in this paper. The non-identical parameters of the SOGIs offer higher degree of freedom for achieving better settling time as compared to [13]. Further, a novel parameter selection procedure that closely predicts the overall settling time of the cascaded system is developed. Conventionally, dominant closed-loop pole method is used to analyse higher-order systems. However, it requires the poles to be well separated in the frequency plane so that a simplifying assumption of neglecting the far away poles can be employed. This leads to a slower response for the system. The proposed method utilises an approximation which ensures that the actual and the approximated transfer functions have a close match of the transients only around the 2% settling limit. In this manner, unlike the dominant closed-loop pole method, the proposed approach is able to achieve...
faster dynamics since it allows the ratio of the real part of the poles to be lower than 5. Further, the method leads to a simple equation relating the settling time of the fourth-order system with its parameters which allows for a simple design procedure.

The proposed CNISOGI-QSG gives fourth-order attenuation to harmonics with complete dc-offset elimination while improving the dynamic response. The error between the expected and obtained settling times for the proposed approach is 1.7 ms for a damping factor choice of 0.7 and 0.645 for the two SOGI-QSGs of the CNISOGI-QSG. In comparison, the error for the SO-SOGI-QSG is 35.6 ms for a damping factor of 0.707 when designed using the method in [11]. Also, unlike CSOGI-SQG in [13], the proposed method is capable of estimating the settling time of the overall structure at the design stage. The proposed CNISOGI-QSG offers better overall speed of response when compared to existing FO-QSGs.

Finally, the performance of the CNISOGI-QSG with the proposed design method is tested along with a three-phase SRF-PLL. A fixed-frequency QSG based three-phase SRF-PLL is employed to ensure perfect decoupling between the QSGs and the SRF-PLL. In this manner, since the dynamics of the QSGs are unaffected by the dynamics of the PLL, the proposed design method of the CNISOGI-QSG is valid even with a SRF-PLL. The fixed-frequency QSG based three-phase SRF-PLL used in this work follows [15], where the SRF-PLL is shown to be capable of error-free tracking of the grid phase angle even when the grid frequency is different from the QSG’s corner frequency. The hardware result for a representative operating condition showed a settling time of around 25 ms for the CNISOGI-QSG with $K_1 = 1.452$ and $K_2 = 1.8$, while the frequency tracking of the same CNISOGI-QSG based SRF-PLL shows a settling time of 26 ms.

II. STRUCTURE OF THE PROPOSED CNISOGI-QSG

The proposed structure consists of two SOGI-QSGs like the one proposed in [13] but with non-identical parameters $K_1$ and $K_2$ as shown in Fig. 2. The transfer functions relating the outputs $v'$ and $qv'$ to the input $v$ are represented as $G_1(s)$ and $H_1(s)$ respectively as

$$G_1(s) = \left( \frac{K_1\omega_o s}{s^2 + K_1\omega_o s + \omega_o^2} \right) \left( \frac{K_2\omega_o s}{s^2 + K_2\omega_o s + \omega_o^2} \right)$$

$$H_1(s) = \left( \frac{K_1\omega_o s}{s^2 + K_1\omega_o s + \omega_o^2} \right) \left( \frac{K_2\omega_o^2}{s^2 + K_2\omega_o s + \omega_o^2} \right).$$

Transfer functions (1) and (2) are fourth-order band-pass filters with corner-frequency as $\omega_o$ rad/s. The damping factors $\zeta_1$ and $\zeta_2$ of the constituent transfer functions of (1) and (2) are related to $K_1$ and $K_2$ as $K_1 = 2\zeta_1$ and $K_2 = 2\zeta_2$ and are different for SOGI-QSG-I and SOGI-QSG-II.

![Block diagram of the proposed CNISOGI-QSG.](image)

III. PARAMETER SELECTION FOR THE PROPOSED CNISOGI-QSG

The parameter selection procedure aims at reducing the settling time of the CNISOGI-QSG. It is desirable to obtain a settling time equation for the system in terms of its parameters $K_1$ and $K_2$ such that the system can be tuned during the design stage to attain a chosen settling time.

A. CONVENTIONAL DOMINANT CLOSED-LOOP POLE METHOD

Closed form expression for settling time is not defined for fourth-order systems. A conventional approach is to reduce the fourth-order system into an equivalent second-order system using the dominant closed-loop pole approach and use the settling time equation of a SOS to design the fourth-order system [17]. To apply the dominant closed-loop pole method, the transfer function (1) is rearranged as

$$G_1(s) = K \left( \frac{s}{s^2 + 2\zeta_1\omega_o s + \omega_o^2} \right) \left( \frac{1}{s^2 + 2\zeta_2\omega_o s + \omega_o^2} \right)$$

where

$$K = \frac{2\zeta_1\zeta_2\omega_o}{\zeta_2 - \zeta_1}; \quad 0 < \zeta_2 < 1 \quad \text{and} \quad 0 < \zeta_1 < 1.$$  

The complex pole pairs of the transfer-function (3) are

$$-\zeta_1\omega_o \pm j\omega_o \sqrt{1 - \zeta_1^2} \quad \text{and} \quad -\zeta_2\omega_o \pm j\omega_o \sqrt{1 - \zeta_2^2}.$$  

To reduce (3) using the dominant closed-loop pole method, it is required to have $\zeta_1 = \frac{\zeta_2}{\alpha}$ with $\alpha > 5$ [17]. Therefore (3) can be reduced to

$$G_1'(s) = \frac{2\zeta_1\omega_o s}{s^2 + 2\zeta_1\omega_o s + \omega_o^2}$$

where $G_1'(s)$ is approximated equivalent of (3) with settling time

$$t' = \frac{4.4}{\zeta_1\omega_o}.$$  

Fig. 3 shows the step response of (3) [$sG_1(t)$] and (5) [$sG_1'(t)$] for an example setting of $\zeta_2 = 0.9$ ($K_2 = 1.8$) and $\zeta_1 = 0.15$ ($K_1 = 0.3$) considering $\alpha = 6$. A close match can be observed between $sG_1(t)$ and $sG_1'(t)$.

Table 1 shows the estimated ($t_1'$) and the simulated ($t'_{\text{sim}}$) settling times for the fourth-order system (3) using the dominant closed-loop pole method for practical values of
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FIGURE 3. Step response comparison of actual transfer function \( G_1(s) \) and dominant pole based reduced transfer function \( G'_1(s) \) for \( \zeta_2 = 0.9, \alpha = 6 \).

FIGURE 4. Step response of \( G_1(s) \) for \( K_2 = 1.8 \) and \( \sigma = 2 \) along with the envelops of SOGI-QSG-I and SOGI-QSG-II.

TABLE 1. Comparison of estimated (\( t'_e \)) and simulated (\( t'_{e,\text{sim}} \)) settling time for proposed CNISOGI-QSG when designed using dominant closed-loop pole method.

| \( \zeta_2 \) | \( \alpha \) | \( \zeta_1 = \zeta_2/\alpha \) | \( t'_e \) (ms) | \( t'_{e,\text{sim}} \) (ms) | Error (ms) |
|----|----|----|----|----|----|
| 0.9 | 6  | 0.15 | 93.4 | 57.6 | 35.8 |
| 0.8 | 6  | 0.13 | 105.0 | 66.4 | 38.6 |
| 0.7 | 6  | 0.12 | 120.0 | 67.3 | 52.7 |
| 0.6 | 6  | 0.1  | 140.0 | 76.6 | 63.4 |
| 0.5 | 6  | 0.08 | 168.1 | 86.1 | 82.0 |

\( \zeta_2 \) (0.5 to 0.9) and a fixed \( \alpha = 6 \). The system response can be observed to be slow. The slow response is due to lower value of \( \zeta_1 \) as a result of the requirement of the dominant closed-loop pole method. Also, as seen from Table 1, the error between the actual and estimated settling time is high. Therefore, the parameter evaluation of the proposed CNISOGI-QSG using the dominant closed-loop pole method becomes ineffective in terms of the settling time. An alternative approach is proposed in this work to improve the settling time.

B. PROPOSED METHOD

In the dominant pole approach, the reduced-order transfer function tries to match the original transfer function at every instant. To achieve this, the poles need to be well separated. The separation ensures that except for the small initial duration when the transients due to the ignored poles are still effective, the response of the reduced-order transfer function closely matches the original transfer function. Since only the 2% settling time instant of interest in the parameter selection of the CNISOGI-QSG, the proposed approach is to allow lesser separation between the poles. The suggested separation of the poles is such that the fast decaying transient reaches negligible magnitude before the slower transient reaches its 2% value. This helps achieve a overall faster transient response for the system while simplifying the analysis of the settling time by considering only the slow transient envelope.

For the proposed design method, \( K_1 (2\zeta_1) \) and \( K_2 (2\zeta_2) \) are selected such that the constituent SOGI-QSGs of the CNISOGI-QSG are always under-damped. Associating \( K_1 \) with the slower transient, we have,

\[
K_1 = \frac{K_2}{\sigma}; \quad 0 < K_2 < 2 \text{ and } 1 < \sigma < 5. \tag{7}
\]

The range of \( K_2 \) and \( \sigma \) in (7) have been chosen to ensure that the SOGI-QSGs are under-damped as stated above. The upper limit on \( \sigma \) in (7) is the value at which CNISOGI-QSG becomes cascaded SOGI-QSG of [13]. An example is considered to demonstrate the proposed design method of the CNISOGI-QSG with \( K_2 = 1.8 \) (\( \zeta_2 = 0.9 \)) and \( \sigma = 2 \). The value of \( K_1 \) is calculated as 0.9 (\( \zeta_1 = 0.45 \)) using (7). This setting results in a faster settling time for SOGI-QSG-II when compared to SOGI-QSG-I of the CNISOGI-QSG. In this manner, the envelop of SOGI-QSG-II decays to zero before the CNISOGI-QSG response reaches around its 2% steady state. Therefore, the response of the CNISOGI-QSG from around 2% of its steady state is tracked completely by the envelop of SOGI-QSG-I as shown in Fig. 4. In this manner, the envelop of SOGI-QSG-I alone can be considered for determining the settling time of the proposed CNISOGI-QSG. The difference between the dominant closed-loop pole method and the proposed method can be clearly visualized from Fig. 3 and Fig. 4 respectively.

IV. SETTLING TIME EQUATION OF THE PROPOSED CNISOGI-QSG

The analytical expression for the step response of (1) is given by:

\[
s_{G_1A}(t) = \frac{2\zeta_1\zeta_2}{\zeta_2 - \zeta_1} \left[ \frac{e^{-\zeta_1\omega_d t}}{\sqrt{1 - \zeta_1^2}} \sin \omega_d t - \frac{e^{-\zeta_2\omega_d t}}{\sqrt{1 - \zeta_2^2}} \sin \omega_d t \right],
\]

where \( t \geq 0 \)

\[
\omega_d = \sqrt{1 - \zeta_1^2}; \quad \omega_d = \sqrt{1 - \zeta_2^2}
\]
and

\[ 0 < \zeta_1 < 1 \quad 0 < \zeta_2 < 1. \]

The settling time equation of the CNISOGI-QSG is obtained by identifying the expression of the slowest envelop guiding the step response of (1). It can be seen from (8) that the step response of (1) is guided by the two envelops

\[ env_1 = \frac{2\zeta_2}{(\sigma - 1)\sqrt{1 - \left(\frac{\zeta_2}{\sigma}\right)^2}} e^{-\left(\frac{\zeta_2}{\sigma}\right)\omega_o t}, \]  
\[ env_2 = \frac{2\zeta_2}{(\sigma - 1)\sqrt{1 - \left(\frac{\zeta_2}{\sigma}\right)^2}} e^{-\zeta_2\omega_o t}. \]

From (9), (10) and (7) it is clear that \( env_1 \) is slower than \( env_2 \). Therefore, \( env_1 \) is considered for deriving the settling time equation for (1). The 2% settling time of (1) is computed by equating (9) to 0.02, which is obtained as

\[ t_s = \frac{\sigma}{\zeta_2 \omega_o} \ln \left[ \frac{\zeta_2}{0.01(\sigma - 1)\sqrt{1 - \left(\frac{\zeta_2}{\sigma}\right)^2}} \right]; \quad 1 < \sigma < 5. \]

(11)

From (11), the proposed CNISOGI-QSG with a fixed \( \zeta_2 \) can be designed to achieve different settling times for \( 1 < \sigma < 5 \). An optimal value of \( \sigma (\sigma_{opt}) \) in the range \( 1 < \sigma < 5 \) for which the proposed CNISOGI-QSG attains a minimum settling time for a fixed \( \zeta_2 \) is established in the following discussion.

\( \sigma_{opt} \) for a fixed \( \zeta_2 \) is obtained graphically by plotting (11) versus \( \sigma \) as shown in Fig. 5 with \( \omega_o = \omega_o, nom = 2\pi 50 \text{ rad/s} \), considering a fixed 50 Hz corner-frequency CNISOGI-QSG. From Fig. 5 it is seen that \( \sigma_{opt} \approx 1.24 \) for \( \zeta_2 = 0.3 \) to \( \zeta_2 = 0.9 \) and \( \omega_o = \omega_o, nom \). The value of \( \sigma_{opt} \) can be found to be close to 1.24 for all \( \zeta_2 \) in the range \( 0 < \zeta_2 < 1 \). Substituting \( \sigma = \sigma_{opt} \) in (11), the expression of the minimum expected settling time \( t_{s, min} \) for \( 0 < \zeta_2 < 1 \) is obtained as

\[ t_{s, min} = \frac{\sigma_{opt}}{\zeta_2 \omega_o, nom} \ln \left[ \frac{\zeta_2}{0.01(\sigma_{opt} - 1)\sqrt{1 - \left(\frac{\zeta_2}{\sigma_{opt}}\right)^2}} \right]. \]

(12)

Table 2 shows the error between the expected \( t_{s, min} \) and actual \( t_{s, sim} \) settling time for the CNISOGI-QSG when parameters are selected using the proposed design method. The actual settling time matches very closely with the expected value. Also, from Table 1 and Table 2 it is evident that the proposed method is better than the conventional dominant closed-loop pole approach for parameter selection of the CNISOGI-QSG in terms of predicting the system’s response time and achieving a faster settling time.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\( \zeta_2 \) & \( \sigma_{opt} \) & \( \zeta_2 \) & \( t_{s, min} \) (ms) & \( t_{s, sim} \) (ms) & Error (ms) \\
\hline
0.9 & 1.24 & 0.726 & 27.6 & 25.4 & 2.2 \\
0.8 & 1.24 & 0.645 & 30.0 & 29.7 & 0.3 \\
0.7 & 1.24 & 0.564 & 33.1 & 31.4 & 1.7 \\
0.6 & 1.24 & 0.484 & 37.2 & 31.2 & 6 \\
0.5 & 1.24 & 0.403 & 42.8 & 39.4 & 3.4 \\
\hline
\end{tabular}
\caption{Comparison of expected \( t_{s, min} \) and obtained \( t_{s, sim} \) settling time for the Proposed CNISOGI-QSG using the proposed method of parameter evaluation.}
\end{table}

V. EFFECT OF CORNER-FREQUENCY VARIATION

The proposed mathematical model for the parameter selection of the CNISOGI-QSG is obtained for a fixed corner-frequency \( \omega_o = \omega_o, nom = 2\pi 50 \text{ rad/s} \). Therefore, (12) can no longer be used to estimate the settling time of the CNISOGI-QSG for a general application where \( \omega_o \) is different from 2\( \pi \) 50 rad/s.

It can be observed from (11) that the settling time of the CNISOGI-QSG for any corner-frequency \( \omega_o \) depends on \( \sigma \) for a chosen \( \zeta_2 \). Therefore, for \( \omega_o \neq 2\pi 50 \text{ rad/s} \), the expression of minimum expected settling time \( t_{s, min} \) can be obtained by determining the new optimal value for \( \sigma \) corresponding to the new \( \omega_o \). Analytically, the new optimal \( \sigma (\sigma'_{opt}) \) can be derived by setting the derivative of (11) w.r.t \( \sigma \) to zero in the interval \( 1 < \sigma < 5 \). We have

\[ \frac{dt_s}{d\sigma} = \frac{1}{\zeta_2 \omega_o} \frac{d}{d\sigma} \ln \left[ \frac{\zeta_2}{0.01(\sigma - 1)\sqrt{1 - \left(\frac{\zeta_2}{\sigma}\right)^2}} \right]. \]

(13)

Equation (13) can be rewritten as

\[ \frac{dt_s}{d\sigma} = \frac{1}{\zeta_2 \omega_o} f(\sigma) \]

(14)

where

\[ f(\sigma) = \frac{d}{d\sigma} \left[ \ln \left( \frac{\zeta_2}{0.01(\sigma - 1)\sqrt{1 - \left(\frac{\zeta_2}{\sigma}\right)^2}} \right) \right]. \]
Equating (14) to zero, we get

$$f(\sigma_{opt}) = 0.$$  \hspace{1cm} (15)

From (14) and (15) it is observed that the optimum $\sigma$ is independent of corner-frequency $\omega_o$. So, from (11) and Fig. 5, the generalized expression for the minimum expected settling time ($t_{s,min}^g$) for $0 < \xi < 1$ and $1 < \sigma < 5$ can be written as

$$t_{s,min}^g = \frac{\sigma_{opt}}{\xi_2} \ln \left[ \frac{\xi_2}{0.01(\sigma_{opt} - 1)^2} \right].$$ \hspace{1cm} (16)

**VI. COMPARISON WITH SO-SOGI-QSG [11]**

The design principle of SO-SOGI-QSG is introduced briefly in Section I. The transfer-functions describing the system are

$$G_2(s) = \frac{K_1 K_2 \omega_o^2 s^2}{(s^2 + \omega_o^2)(s^2 + K_2 \omega_o s + \omega_o^2) + K_1 K_2 \omega_o^2 s},$$ \hspace{1cm} (17)

$$H_2(s) = \frac{K_1 K_2 \omega_o^3 s}{(s^2 + \omega_o^2)(s^2 + K_2 \omega_o s + \omega_o^2) + K_1 K_2 \omega_o^2 s^2}.$$ \hspace{1cm} (18)

For a SOS with a chosen $\xi$ and settling time $t_s$, $\omega_0 = \frac{4.4}{t_s \xi}$. Using the same $\xi$ and $\omega_0$ values, [11] sets $K_1 = \frac{\omega_n}{\xi \omega_0}$ and $K_2 = \frac{4 \xi \omega_0}{\omega_0}$, This setting is expected to give SO-SOGI-QSG the same settling time as the SOS. In effect, the method approximates the fourth-order system to be of lower-order. As a design example in [11], the values of $K_1$ and $K_2$ are computed as 1.56 and 3.11 respectively for $\xi = 0.707$ and $t_s = 18$ ms, assuming $\omega_0 = 2 \pi 50$ rad/s.

The step response of the in-phase component of the SO-SOGI-QSG with the above values of $K_1$ and $K_2$ is plotted in Fig. 6 along with the step response of the in-phase component of the proposed method for an expected settling time of 27.6 ms ($K_1 = 1.452$ and $K_2 = 1.8$). From Fig. 6 it can be observed that the actual settling time for the in-phase component of the SO-SOGI-QSG is around 54 ms against an expected value of 18 ms, however, the settling time (25.4 ms) of the in-phase component of the CNISOGI-QSG is much closer to the expected value (27.6 ms). Fig. 7 shows the same for the quadrature-phase components of the SO-SOGI-QSG and CNISOGI-QSG. Here too, the quadrature-phase of the CNISOGI-QSG settles within 23 ms which is much closer to the expected value of 25.4 ms while the quadrature-phase component of the SO-SOGI-QSG settles at 53.7 ms against an expected settling time of 18 ms.

Fig. 8 and Fig. 9 compare the bode plots of the in- and quadrature-phase components of SO-SOGI-QSG with the CNISOGI-QSG for the aforementioned values of $K_1$ and $K_2$. It is observed from Fig. 8 and Fig. 9 that the CNISOGI-QSG offers higher attenuation to harmonic components when compared to the SO-SOGI-QSG. From Fig. 6, Fig. 7, Fig. 8 and Fig. 9 it is clear that the proposed CNISOGI-QSG is better compared to SO-SOGI-QSG in terms of the dynamics, settling time estimation and harmonic attenuation.

**VII. COMPARISON WITH CSOOGI-QSG [13]**

In [13] a fourth-order QSG is proposed by cascading two identical SOGI-QSGs. The transfer-functions of the...
The combined effect of cascading on the settling time is not considered in [13]. Also, for a chosen value of \( h \) is not considered. The comparison in Table 3 is based on the settling time and THD of the in-phase \([v'(t)]\) and quadrature-phase \([q'(t)]\) components of the QSGs. The settling times are obtained by applying a step input while the THD values are obtained by applying a harmonic distorted voltage described in (24). The total harmonic distortion (THD) in the input voltage is 17.32%. Due to higher filtering ability, a fourth-order QSG may be slower than lower-order QSGs. Therefore, along with the settling time, it is important to consider the THD of the outputs as an important factor for comparing the performance of fourth-order QSGs with lower-order ones.

From Table 3, it is clear that the proposed QSG shows superior performance in terms of settling time, percentage THD and a correct estimation of system settling time when compared to other QSGs.

**IX. Experimental Results**

Experimental results are conducted on a digital signal processor (DSP) platform built around the processor TMS320F2812. The QSGs are implemented in the DSP using trapezoidal method of discretization with the computations done every 100 \( \mu s \), corresponding to a sampling frequency of 10 kHz.

**A. Dynamic Performance Comparison**

Fig. 10 and Fig. 11 compare the dynamic performance of the CNISOGI-QSG \((K_1 = 1.452 \text{ and } K_2 = 1.8)\) with SO-SOGI-QSG \((K_1 = 1.56 \text{ and } K_2 = 3.11)\) and CSOGI-QSG \((K = 2.66)\). Fig. 10 shows the dynamic response of the in-phase component \([v'(t)]\) for the QSGs to a unit step input, whereas, Fig. 11 considers the dynamic response of the output space-vector \([v'(t)] = \sqrt{(v'(t))^2 + (q'(t))^2} \) of the QSGs to a sinusoidal input of frequency 50 Hz \((2\pi 50 \text{ rad/s})\) which experiences a step change in magnitude from 0.5 p.u to 1 p.u.
TABLE 3. A quantitative comparison of the proposed CNISOGI-QSG with existing QSGs [7]–[13]. Except [7] all the other QSGs are capable of dc-offset elimination.

| Method                  | $t_s$ (ms) [expected] | Parameter $K$ | $t_s$ (ms) [obtained] | $t_{s,qv}$ (ms) [obtained] | $THD_{v'}$ (%) | $THD_{qv'}$ (%) |
|-------------------------|-----------------------|---------------|-----------------------|---------------------------|----------------|----------------|
| SOGI-QSG [7]            | 20                    | $K = 1.414$   | 21                    | 24                        | 3.7            | 0.65           |
| SOGI-QSG + LPF [8]      | 20                    | $K = 1.414, \omega_{LP} = 2\pi 10$ rad/s | 21                    | 72                        | 3.7            | 0.98           |
| MTOGI-QSG [9]           | 20                    | $K = 1.414$   | 21                    | 20                        | 3.7            | 3.71           |
| Third-order SOGI-QSG [10]| *                     | $K = 1.414, K_{dc} = 0.22$ | 39                    | 41                        | 3.67           | 0.64           |
| Novel two-phase generator [12] | *                  | $K_1 = 85.313$ | 27                    | 26                        | 2.64           | 0.46           |
| SO-SOGI-QSG [11]        | 20                    | $K_1 = 1.414, K_2 = 2.827$ | 51                    | 49                        | 1.94           | 0.36           |
| CSOGI-QSG [13]          | *                     | $K = 1.414$   | 30                    | 30                        | 0.94           | 0.18           |
| Proposed CNISOGI-QSG    | 28                    | $K_1 = 1.414, K_2 = 1.753$ | 25                    | 28                        | 1.11           | 0.21           |

* Settling time equation for the QSGs are not proposed in [10], [12], [13].

FIGURE 10. Hardware result comparing the step response of $v'(t)$ of the CNISOGI-QSG ($K_1 = 1.452$ and $K_2 = 1.8$), SO-SOGI-QSG ($K_1 = 1.56$ and $K_2 = 3.11$) and CSOGI-QSG ($K = 2.66$) to a unit step input. X-axis: 10 ms/div and Y-axis: 0.2 p.u/div.

FIGURE 11. Dynamic response comparison of $v(t)$ of the CNISOGI-QSG ($K_1 = 1.452$ and $K_2 = 1.8$), SO-SOGI-QSG ($K_1 = 1.56$ and $K_2 = 3.11$) and CSOGI-QSG ($K = 2.66$) to a sinusoidal voltage with a step change. X-axis: 10 ms/div and Y-axis: 0.2 p.u/div for $v'(t)$ and 0.5 p.u/div for $v(t)$.

The input voltage $v(t)$ and steady-state in-phase output voltage $[v'(t)]$ of the QSGs are shown in Fig. 12. In fourth-order QSGs, a comparison of the bode plots would indicate that the in-phase component offers lower attenuation to harmonics when compared to the quadrature-phase component. So, Fig. 9 considers the in-phase output voltage alone for analyzing the steady-state performance of the QSGs. In this manner, the worst-case harmonic elimination ability of the QSGs can be assessed. It is observed from Fig. 12 that the harmonics are well attenuated at the outputs. A simulation result shows that the THD in $v'(t)$ and $q(t)$ is (i) CNISOGI-QSG is 1.16% and 0.22% (ii) SO-SOGI-QSG is 2.34% and 0.44% and (iii) CSOGI-QSG is 2.75% and 0.51% respectively. From the above discussion and Section IX-A, it is clear that the CNISOGI-QSG with the proposed method of parameter selection shows superior dynamic- and steady-state performance in terms of settling time and percentage THD.
It is observed that the FFDFO-QSG SRF-PLL tracks the $\pi$ from 2 $\omega$ FFDFO-QSG SRF-PLL. The grid frequency ($\omega$) is different from the QSG corner-frequency ($\omega_0$). Similar compensators are employed for the CNISOGI-QSG and SO-SOGI-QSG based FFDFO-QSG SRF-PLL too, following the design principle outlined in [15]. Since the frequency of the grid voltage ($\omega_m$) is unaffected by the QSG during the steady state, the tracked frequency of the FFDFO-QSG SRF-PLL ($\omega_{PLL}$) remains same as $\omega_m$ in the steady-state. The PI controller gains of the CNISOGI-QSG, SO-SOGI-QSG and CSOGI-QSG based FFDFO-QSG SRF-PLLs are calculated as $K_p = 2.546$ p.u and $K_i = 1019$ p.u for achieving a settling time of 10 ms with $\zeta = 0.707$ for the PLL ($V_{base} = V_m$ and $\omega_{base} = \omega_0 = 2\pi 50$ rad/s).

Fig. 14 is considered to demonstrate the phase tracking capability of a FFDFO-QSG SRF-PLL while $\omega_m$ is different from the QSG corner-frequency ($\omega_0 = 2\pi 50$ rad/s). The phase tracking is shown only for the CNISOGI-QSG based FFDFO-QSG SRF-PLL. The grid frequency ($\omega_m$) is changed from 2 $\pi$ p.u/div. to 1.8 $\pi$ p.u/div. for Fig. 14, Fig. 16 and Fig. 18. X-axis: 10 ms/div and Y-axis: 0.5 p.u/div.

Fig. 15 shows the dynamic behaviour of the CNISOGI-QSG and SO-SOGI-QSG based FFDFO-QSG + PSC structure. X-axis: 10 ms/div and Y-axis: 0.5 p.u/div.

Fig. 16 shows the dynamic response of the CNISOGI-QSG based FFDFO-QSG + PSC structure with $K_i = 1.452$ and $K_2 = 1.8$ to the input of Fig. 15. X-axis: 10 ms/div and Y-axis: 0.5 p.u/div.

X. FREQUENCY-FIXED DUAL FO-QSG BASED THREE-PHASE SRF-PLL

A frequency-fixed dual FO-QSG (FFDFO-QSG) + PSC based three-phase SRF-PLL (FFDFO-QSG SRF-PLL) is implemented by replacing the QSGs of Fig. 1 with fixed corner-frequency ($\omega_0 = 2\pi 50$ rad/s) CNISOGI-QSGs as shown in Fig. 13. For performance comparison of the CNISOGI-QSG based FFDFO-QSG SRF-PLL, a SO-SOGI-QSG and CSOGI-QSG based FFDFO-QSG SRF-PLL were also implemented in the DSP. It is shown in [15] that a frequency-fixed SOGI-QSG based SRF-PLL (FFSOGI-QSG SRF-PLL) is superior when compared to frequency-adaptive SOGI-QSG SRF-PLL [18] in terms of system stability. Higher stability in FFSOGI-QSG SRF-PLL is due to the decoupling between the QSG and SRF-PLL. The decoupled nature of QSGs allows for higher bandwidth/faster response for the SRF-PLL [15]. Also, due to decoupling, the settling time of the QSGs are unaffected by the PLL dynamics.

A FFSOGI-QSG SRF-PLL requires additional compensator ($C(\theta)$) for correcting the error in tracked phase angle ($\theta'$) when the grid frequency ($\omega_m$) is different from the SOGI-QSG corner-frequency ($\omega_0$) [15]. Similar compensators are employed for the CNISOGI-QSG and SO-SOGI-QSG based FFDFO-QSG SRF-PLLs too, following the design principle outlined in [15]. Since the frequency of the grid voltage ($\omega_m$) is unaffected by the QSG during the steady state, the tracked frequency of the FFDFO-QSG SRF-PLL ($\omega_{PLL}$) remains same as $\omega_m$ in the steady-state. The PI controller gains of the CNISOGI-QSG, SO-SOGI-QSG and CSOGI-QSG based FFDFO-QSG SRF-PLLs are calculated as $K_p = 2.546$ p.u and $K_i = 1019$ p.u for achieving a settling time of 10 ms with $\zeta = 0.707$ for the PLL ($V_{base} = V_m$ and $\omega_{base} = \omega_0 = 2\pi 50$ rad/s).

Fig. 14 is considered to demonstrate the phase tracking capability of a FFDFO-QSG SRF-PLL while $\omega_m$ is different from the QSG corner-frequency ($\omega_0 = 2\pi 50$ rad/s). The phase tracking is shown only for the CNISOGI-QSG based FFDFO-QSG SRF-PLL. The grid frequency ($\omega_m$) is changed from 2 $\pi$ p.u/div. to 1.8 $\pi$ p.u/div. for Fig. 14, Fig. 16 and Fig. 18. X-axis: 10 ms/div and Y-axis: 0.5 p.u/div.
sequence space-vector experiences a 50% dip in magnitude at 40 ms. It is observed that the CNISOGI-QSG based FFDO-QSG + PSC structure tracks the change in input within 22 ms (Fig. 16) whereas, the SO-SOGI-QSG and CSOGI-QSG based FFDO-QSG + PSC structure take around 45 ms (Fig. 17) and 30 ms (Fig. 18) respectively for the same.

Fig. 19 to Fig. 21 considers the frequency tracking capability of the CNISOGI-QSG, SO-SOGI-QSG and CSOGI-QSG based FFDO-QSG SRF-PLL respectively. The parameters of the QSGs are same as discussed above. The grid frequency ($\omega_{in}$) in Fig. 19, Fig. 20 and Fig. 21 experiences a step change from $2\pi$50 rad/s (50 Hz) to $2\pi$52 rad/s (52 Hz) at 20 ms. The CNISOGI-QSG based FFDO-QSG SRF-PLL tracks the change in $\omega_{in}$ within 26 ms (Fig. 19) while the SO-SOGI-QSG and CSOGI-QSG based FFDO-QSG SRF-PLL take a relatively longer time of 50 ms (Fig. 20) and 42 ms (to Fig. 21) respectively. Ideal three-phase grid voltages are considered for Fig. 19, Fig. 20 and Fig. 21 to strictly focus on the dynamic frequency tracking ability of the PLLs. Similar results are expected under non-ideal grid scenario also.

XI. CONCLUSION

A fourth-order QSG, cascading two non-identical SOGI-QSGs is proposed. A novel method of parameter selection based on settling time is developed by identifying and utilising the slower envelop around the 2% settling time limit. This approach allows for lesser separation between the poles of the fourth-order transfer function and helps achieve a faster overall response. The proposed method is analysed for the effect of variation in corner frequency. A brief comparison with existing methods are provided. The proposed parameter selection approach allows the QSG to achieve lower setting times when compared to the existing fourth-order QSGs. The proposed method shows a faster response of 25.4 ms settling time. For a sample input voltage with a THD of 17.32%, the THD in the output voltages ($v'$ and $qv'$) are shown to be 1.16% and 0.22% respectively. The proposed method is useful in frequency-fixed QSG based single- and three-phase PLLs under non-ideal grid voltage conditions.

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