Tests of Pauli Exclusion Principle Violations from Non-commutative quantum gravity

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Abstract

We review the main recent progresses in non-commutative space-time phenomenology in underground experiments. A popular model of non-commutative space-time is $\theta$-Poincaré model, based on the Groenewold-Moyal plane algebra. This model predicts a violation of the Spin-statistic theorem, in turn implying an energy and angular dependent violation of the Pauli Exclusion principle. Pauli Exclusion Principle Violating transitions in nuclear and atomic systems can be tested with very high accuracy in underground laboratory experiments such as DAMA/LIBRA and VIP(2). In this paper we derive that the $\theta$-Poincaré model can be already ruled-out until the Planck scale, from nuclear transitions tests by DAMA/LIBRA experiment.

1 Introduction

It is a quite common believing that no any bounds to quantum gravity effects may be provided from next future experiments. The energy-scales probed by current and future collider experiments are far below the Planck scale. It is worth to remind that the Large Hadron Collider (LHC) tests energy scales of about $1 - 10$ TeV or so, i.e. 15th – 16th order of magnitude down to the Planck energy scale. Certainly, this may inspire a certain pessimism to any serious attempts of quantum gravity phenomenology.

However, new recent progresses opened the way to a new exciting possibility building a bridge from experiments to quantum gravity physics. In Ref. \cite{1, 2}, we propose to search for exotic transitions in nuclei or atoms, induced by non-commutative space-time, which violates the Pauli Exclusion Principle. Certainly, a possible detection of a PEP violating transition has the wonderful potentiality to change our conceptions.

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of space and time. The Pauli Principle is a direct consequence of the Spin Statistic theorem (SST), in the Standard Model of particle physics. In turn, SST is valid under assumptions of: Minkowski’s space-time, causality, locality and the Poincaré symmetry group. The detection of Pauli Exclusion Principle Violations (PEPV) may be lead to indirect quantum gravity smoking guns in underground experiments of rare processes physics. A non-commutative quantum gravity model related to PEPV transitions is the \( \theta \)-Poincaré. The \( \theta \)-Poincaré is based on a deformation of the Poincaré symmetry. This model entails a dual reformulation, in terms of non-commutative space-time coordinates.

The \( \theta \)-Poincaré co-algebra can be obtained from the Poincaré algebra thanks to a mathematical map, known as the Groenewold-Moyal (GM) map \([3, 4, 5, 6]\). The same GM product map is applied to every quantum field theory operators such as creation/annihilation particle operators and every fields (electro-weak, chromo-strong and Higgs fields). In other words, a deformed version of the Standard Model of particle physics may be obtained as a Groenewold-Moyal Standard Model (GSSM). In GSSM, one can easily obtain all GM Feynman diagrams from the standard ones. However, most of the amplitudes are corrected by harmonic functions, which are dependent on the particles four-momenta.

In \( \theta \)-Poincaré model, the Poincaré algebra is deformed by the GM map as follows. The space-time translations \( x^\mu \rightarrow x^\mu + a^\mu \) are undeformed by GM map:

\[
\text{translation} \rightarrow \text{translation}.
\]

The action on the Lorentz group — namely \( \mathfrak{so}(3, 1) \) — generators is less trivial:

\[
\mathfrak{so}(3, 1) \rightarrow \text{“deformed” } \mathfrak{so}(3, 1).
\]

As aforementioned, not only space-time generators, but also quantum operators related to particle fields are deformed as follows:

\[
(\text{creation/annihilation ops.}) \rightarrow (\text{GM – phase})(\text{creation/annihilation ops.}),
\]

\[
(\text{fields}) \rightarrow (\text{GM – phase})(\text{fields}),
\]

The GM provides a non-ambiguous map among the standard second quantization in the Standard Model and the quantization in \( \theta \)-Poincaré.
The most interesting aspect of θ-Poincaré is that it is an example of quantum gravity model which, surprisingly, not only can be tested, but, even more surprisingly, is already ruled out in its democratic implementation by several underground experiments’ data NSF [2]. The θ-Poincaré models can be distinguished in two classes: the democratic and the despotic cases. The democratic θ-Poincaré models assume that all the Standard Model fields interact with the non-commutative space-time background with the same gravitational coupling, while the despotic case relaxes such a hypothesis. The main point is that θ-Poincaré can induce very tiny but testable Pauli forbidden transitions [1]. Contrary to effective PEP violating models proposed in Refs. [10, 11, 12, 13, 14, 15, 16, 17, 18], such transitions are: i) energy dependent from the particular PEPV process considered; ii) suppressed with the non-commutative energy scale; iii) highly motivated by quantum gravity.

In this review, we will show estimations of PEPV atomic/nuclear level transitions induced by θ-Poincaré. We will show how that underground experiments can rule out θ-Poincaré models up to non-commutative length scales, beyond the Planck scale. Rare processes, in nuclear and atomic physics, can provide indirect probes of the same structure of space and time. We will show how BOREXINO, KAMIOKANDE and DAMA still exclude θ-Poincaré coupled to hadrons, beyond the Planck scale, as a phenomenological tombstone for the democratic scenario.

2 Atomic and Nuclear transitions

Let us consider the one-particle state

$$|\alpha\rangle = \langle a^\dagger, \alpha|0\rangle = \langle c^\dagger, \alpha|0\rangle = \int \frac{d^dp}{2p_0} \alpha(p)c^\dagger(p) ,$$

opportunely normalized as

$$\langle \alpha|\alpha\rangle = 1, \quad \int \frac{d^dp}{2p_0} |\alpha(p)|^2 = 1 .$$

From the definition in Eq. (2.2), we may construct a two-identical-particles state that, in θ-Poincaré, reads

$$|\alpha,\alpha\rangle = \langle a^\dagger, \alpha\rangle\langle a^\dagger, \alpha|0\rangle =$$

$$= \int \frac{d^dp_1}{2p_{10}} \frac{d^dp_2}{2p_{10}} e^{-\frac{i}{2}\theta(p_1\alpha^\dagger p_2\alpha)} \alpha(p_1)\alpha(p_2)c^\dagger(p_1)c^\dagger(p_2)|0\rangle ,$$
where the $e^{-\frac{i}{\hbar}p_{1\mu}\theta_{\mu\nu}p_{2\nu}}$ provides the GM deformation to the Standard Model case (for $\theta \to 0$ we reobtain the Standard two particle state). The two particle state must be normalized with the following norm:

$$N = \langle \alpha, \alpha | \alpha, \alpha \rangle = \int \frac{d^d p_1}{2p_{10}} \frac{d^d p_2}{2p_{20}} (\bar{\alpha}(p_1)\alpha(p_1))(\bar{\alpha}(p_2)\alpha(p_2))(1 - e^{-ip_{1\mu}\theta_{\mu\nu}p_{2\nu}})$$

We can redefine the two-particles state as follows:

$$|\alpha, \alpha \rangle \rightarrow \frac{1}{N(\alpha, \alpha)} |\alpha, \alpha \rangle, \quad \langle \alpha | \alpha \rangle = 1.$$  

(2.5)

Now let us come to the crucial point: the transition amplitude for the overlap probability that a two-different-particles state evolves into a two-identical-particles state. In the case of fermions, the amplitude is as follows:

$$\langle \beta, \gamma | \alpha, \alpha \rangle = \frac{1}{N} \int \frac{d^d p_1}{p_{10}} \frac{d^d p_2}{p_{20}} (\bar{\beta}(p_1)\alpha(p_1))(\bar{\gamma}(p_2)\alpha(p_2))\left[1 - e^{-ip_{1\mu}\theta_{\mu\nu}p_{2\nu}}\right]$$

(2.6)

$$= \frac{1}{N} \int \frac{d^d p_1}{p_{10}} \frac{d^d p_2}{p_{20}} (\bar{\beta}(p_1)\alpha(p_1))(\bar{\gamma}(p_2)\alpha(p_2))\left[1 - \cos(p_{1\mu}\theta_{\mu\nu}p_{2\nu})\right].$$

It is trivial to check that, for $\theta \to 0$, the overlap amplitude vanishes out. But, for $\theta \neq 0$, the Pauli principle is violated, if the states are composed of fermions. A two-fermions state has a non zero probability to transit into a state in which fermions are identical.

Let us consider indeed the GM effective Hamiltonian density, which is expressed by

$$H_{GM,ij} = \langle \Psi_i^\theta | V_\theta | \Psi_j^\theta \rangle = \langle \Psi_i^0 | \mathcal{H}_E | \Psi_j^0 \rangle = V_0 \left\{ \cos(\phi_{PEPV}) - \cos(\phi_{PEPV} + p_{1\mu}\theta_{\mu\nu}p_{2\nu}) \right\},$$

(2.7)

and for a central interaction potential,

$$2\phi_{PEPV} = p_1 \wedge p_2 - p'_1 \wedge p'_2 - p'_1 \wedge p_1 + p_2 \wedge p_2,$$

(2.8)

where $p \wedge q = p^\mu q_\mu q^\nu$. The PEPV phases are provided both by Eq. (2.6) and the central interaction potential, where $p_{1,2}$ are the initial momenta of interacting particles while $p'_{1,2}$ are the out ones.

Now, let us consider the problem of atomic level transitions. In this case, we can consider a non-relativistic quantum mechanics approach, based on perturbation theory.
The effective Hamiltonian is the 0th order standard one plus a PEPV perturbation term:

$$H = H_0 + V_{i,0} + V_{i,0}\phi_{PEPV}^2.$$ 

The 1st order perturbation coefficient is

$$\dot{c}_b^{(1)}(t) = (i\hbar)^{-1}H'_{ba}(t)e^{i\omega_{ba}t}. \quad (2.9)$$

If the perturbation is time independent, we obtain

$$\dot{c}_b^{(1)}(t) = -\frac{H'_{ba}}{\hbar\omega_{ba}}(e^{i\omega_{ba}t} - 1). \quad (2.10)$$

The transition probability is, then, found to be

$$P_{ba}(t) = |c_b^{(1)}(t)|^2 = \frac{2}{\hbar}|H'_{ab}|^2F(t, \omega_{ba}) = \frac{2}{\hbar}V_0^2\phi^2F(t, \omega_{ba}), \quad (2.11)$$

with $F = (1 - \cos \omega t)/\omega^2$. In the long-time adiabatic approximation, Eq.(2.11), by means of $F \rightarrow \pi t\delta(\omega)$, leads to

$$W = \frac{2\pi}{\hbar}|H'_{ba}|^2 = \frac{2}{\pi\hbar}V_0^2\phi_{PEPV}^2 = W_0\phi_{PEPV}^2 \quad (2.12)$$

However, such a quartic power suppression in $\theta$ powers may be an artifact, of the number of fields involved in the initial state. In presence of three particles, Eq. (2.6) leads to a linear order correction in the phase $\phi_{PEPV}$. In this latter case we obtain

$$W \simeq W_0\phi_{PEPV}, \quad (2.13)$$

where $\phi_{PEPV}$ coincides with the $\delta^2$ parameter, parametrizing the PEP deviation from creation/annihilation commutators, constrained by experimental measurements.

Now we can distinguish two cases, corresponding to different choices of the $\theta$-components. In the first case, the time-space (electric) components is set to zero:

$$\theta_{0i} = 0 \rightarrow \phi_{PEPV} = \frac{1}{2}\left(p_i^1\theta_{ij}p_j^2 - p_i^2\theta_{ij}p_j^1 - p_i^1\theta_{ij}p_j^1 + p_i^2\theta_{ij}p_j^2\right). \quad (2.14)$$

Let us consider particle 1 as an electron and the particle 2 as a nucleus. Then all terms involving $p_1$ and $p'_1$ are subleading to $p_2, p'_2$, while $|p_2|$ and $|p'_2|$ are of the order of the energy levels in the atom. Therefore, $p_1 \wedge p'_1$ and $p_2 \wedge p'_2$ are subdominant (the first for magnitude subdominance $|p_1|, |p'_1| < |p_2|, |p'_2|$, the second for $p_2 \simeq p'_2$). Therefore, the relevant terms are $p_i^1\theta_{ij}p_j^2 - p_i^2\theta_{ij}p_j^1$, which is $|p_1|\hat{p}_1 \cdot \theta \cdot |p_2|\hat{p}_2 - |p'_1|\hat{p}'_1 \cdot \theta \cdot |p'_2|\hat{p}'_2$. Introducing the cutoff UV dimension energy $\Lambda$ hidden in the dimensionful $\theta_{\mu \nu}$ and
redefining $\theta$ as an antisymmetric dimensionless tensor; the GM-Standard Model predicts the result as follows

$$\phi_{PEPV} \simeq \frac{1}{2} C \frac{\bar{E}_1 \bar{E}_1'}{\Lambda \Lambda'},$$  \hspace{1cm} (2.15)

where $\bar{E}_1, \bar{E}_1'$ are the energy levels occupied by the initial and the final electrons, while $C = \hat{p}_1 \cdot \theta \cdot \hat{p}_2$. The PEPV phase as an angular function is displayed in Fig. 1.

The second case has an extra phase with respect to the first one as follows:

$$\theta_{0i} \neq 0 \rightarrow \Delta \phi_{PEPV} = \frac{1}{2} \left( p_0^0 \theta_{0j} p_2^j - p_1^0 \theta_{0j} p_2^j' - p_1^0 \theta_{0j} p_1^j + p_2^0 \theta_{0j} p_1^j' \right) + \left( 0 \leftrightarrow j \right), \hspace{1cm} (2.16)$$

with

$$\phi_{PEPV} \simeq \frac{D}{2} \frac{E_N \Delta E}{\Lambda \Lambda'},$$  \hspace{1cm} (2.17)

where $E_N \simeq m_N \simeq Am_p$ is the nuclear energy, and $\Delta E = E_1' - E_1$ is the transition energy of the electron.

3 Pauli violating transitions in underground experiments: atomic and nuclear processes

Let us discuss in the following the phenomenological implications of PEPV in several underground experiments.
In Fig.2, we show limits on the PEPV strength ($\phi_{PEPV} = \delta^2$) parameter determined by the searches for new exotic transition [1]. In the following, we will recall various PEP experimental results provided by different experimental techniques.

The VIP experiment searches for PEP violating atomic transitions in copper atoms [20]. The experimental technique is based on the injection of “fresh” electrons into a copper material, from circulating current. Possible PEP forbidden transitions are searched for by injecting “fresh” electrons into a copper strip and searching for the X-rays following such forbidden radiative transitions occurring when one of these electrons is captured by a copper atom and cascades down to the already-filled 1S state. The energy gap corresponds to $\Delta E_{2P\rightarrow1S} = 7.729$ keV; it should be compared with the
ordinary $K_\alpha$ transition energy (8.040 keV).

Tests of PEP forbidden electromagnetic atomic transitions, in Iodine atoms deploying NaI(Tl) detectors, have been performed by ELEGANTS V [21] and DAMA/LIBRA [22] experiments. PEPV electromagnetic transitions in Germanium atoms in PPC HPGe detectors were searched for by the MALBEK experiment [23]. These experiments exploited a different strategy than VIP: PEPV transitions emit X-rays and Auger electrons, directly by the transition itself and by the following arrangements of the atomic shell. Very high detection efficiency, almost 100%, is achieved in the DAMA/LIBRA detectors; in particular, the whole ionization energy for the considered shell is detected, shifted by a certain $\Delta E$ related to the other electrons filling the shells. The atomic K-shell provides the largest available energy emissions of X-rays or Auger-electrons; however, severe limits, from DAMA/NaI, can be achieved also for L-shell transitions ($4 \div 5$ keV radiation emission) in Iodine atoms [26] thanks to the low energy thresholds of the DAMA/NaI detectors.

It is worth noting that the most stringent constraints on PEPV, in atomic transitions, are provided by the DAMA/LIBRA experiment, searching for PEPV K-shell transitions in Iodine. DAMA/LIBRA consists of an about 250 kg array of highly radiopure NaI(Tl) detectors, hosted in the Gran Sasso National Laboratory (LNGS). The data set corresponds to 0.53 ton×yr, implying a limit on the PEPV transition characteristic time of $4.7 \times 10^{30}$ s. This limit corresponds to $\phi_{PEPV} = \delta^2 < 1.28 \times 10^{-47}$ at 90% C.L. [22]. This entails very strong constraints on the non-commutativity scale. In the magnetic-like $\theta$-Poincaré scenario, $\Lambda < 10^{18}$ GeV is excluded. In the electric-like phase, the limit is less stringent than the magnetic-like case, but still arriving to very high energy scale: $\Lambda > 5 \times 10^{16}$ GeV.

On the other hand, the most stringent bounds arrived from nuclear transitions, where the statistics can be even higher then atomic ones.

DAMA/LIBRA collaboration also sets severe limits on PEPV nuclear transitions [22]. PEPV processes in nuclear shells of $^{23}$Na and $^{127}$I are investigated, emitting protons with an energy of $E_p \geq 10$ MeV: the emission rate of protons with energy $E_p \geq 10$ MeV from PEPV transitions, in $^{23}$Na and $^{127}$I, was constrained up to $\gtrsim 1.63 \times 10^{33}$ s (90% C.L.) [22], which corresponds to $\phi_{PEPV} = \delta^2 \lesssim 4 \times 10^{-55}$ (90% C.L.). Such a strong bound rules out both the electric and the magnetic like $\theta$-Poincaré models with a non-commutative scale at the Planck scale energy.
4 Conclusions and remarks

In this paper, aspects of Pauli Exclusion Violating processes induced by non-commutative $\theta$-Poincaré quantum gravity in underground experiments, are reviewed. In the following the main conclusions are summarized:

- Pauli Violating transitions are sharp predictions of non-commutative $\theta$-Poincaré, i.e of the Groenewold-Moyal Standard Model (GMSM).

- Predicted PEPV transitions are energy and angular dependent! In particular, they depend on the momenta of the particles involved in the process.

- The PEPV Democratic scenario is already ruled out by DAMA/LIBRA experiment. The PEPV Despotic scenario, where non-commutativity is particle species dependent will be tested in atomic channels by VIP(2) experiments.

- Detectors with anisotropic response may also test the angular dependence of PEPV transitions. For example, a tempting possibility that we would like to suggest is that a good candidate for such searches is provided by [27], which was suggested for measuring the dark matter directionality (highly motivated by dark matter candidates as Mirror matter [28, 29]).

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