Can we decide whether QCD is confining or not at high temperature?

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Abstract At high temperature measurements of the Polyakov loop suggest a deconfinement transition to the (strongly interacting) quark-gluon plasma. At the same time at the infinitely large temperature the four-dimensional QCD is reduced to the three-dimensional QCD that is confining. The Polyakov loop and related $Z_3$ symmetry are strict order parameters only for infinitely heavy quarks. In such a situation the $SU(2)_{CS}$ and $SU(4)$ symmetries of confinement in the light quark sector could be helpful to distinguish between the confining and deconfining phase in a regime where $SU(2)_L \times SU(2)_R$ and $U(1)_A$ symmetries are manifest. In order to reveal a presence or absence of these symmetries one needs to measure and compare correlation functions related by these symmetry transformations.

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1 Introduction

In spite of big efforts, both experimental and theoretical, to reveal properties of QCD at high temperature, the issue about microscopical structure of the matter is still open. Experimentally a transition to the (strongly interacting) “quark-gluon plasma” is assumed, given observation of some specific properties of the matter at high temperature in heavy ion collisions. What is reliably established on the lattice - is a crossover to the chirally symmetric regime. It has recently been demonstrated by the JLQCD collaboration that at $T > T_c$ not only $SU(2)_L \times SU(2)_R$ but also a $U(1)_A$ symmetry is restored [1].

Situation with confinement is by far not clear, however. On the one hand, lattice simulations of the Polyakov loop [2] suggest a transition to the deconfinement regime approximately at the same temperatures like chiral restoration, see for a review Ref. [3]. On the other hand, the Polyakov loop and related $Z_3$ symmetry can be considered as order parameters for confinement only for infinitely heavy quarks. At the same time it is known that at the infinitely high temperature QCD becomes effectively a three-dimensional theory which is known to be confining [4].

In this short note we suggest that $SU(2)_{CS}$ and $SU(4)$ symmetries of confinement in the light quark sector [5] could serve as a confinement-deconfinement order parameter in a regime where chiral $SU(2)_L \times SU(2)_R$ and $U(1)_A$ symmetries are manifest. Then, through a study of the correlation functions that are connected by the $SU(2)_{CS}$ and $SU(4)$ transformations and not linked by the $SU(2)_L \times SU(2)_R$ and $U(1)_A$ symmetries one could judge about existence or nonexistence of the $SU(2)_{CS}$ and $SU(4)$ symmetries at high temperature.

2 $SU(2)_{CS}$ and $SU(4)$ symmetries of confinement

Consider the QCD Hamiltonian in Coulomb gauge [10]:

$$H_{QCD} = H_E + H_B + \int d^3x \bar{\psi}(x)[-i\alpha \cdot \nabla + \beta m]\psi(x) + H_T + H_C, \quad (1)$$

where the transverse (magnetic) and Coulombic interactions are:

$$H_T = -g \int d^3x \bar{\psi}(x) \alpha \cdot t^a A^a(x) \psi(x), \quad (2)$$

$$H_C = \frac{g^2}{2} \int d^3x d^3y J^{-1} \rho^a(x) F^{ab}(x,y) J \rho^b(y), \quad (3)$$

with $J$ being Faddeev-Popov determinant, $\rho^a(x)$ is a color-charge density and $F^{ab}(x,y)$ is a confining Coulombic kernel.
The fermionic and transverse parts of the Hamiltonian have the SU(2)\(_L\times\)SU(2)\(_R\) and U(1)\(_A\) symmetries. A symmetry of the confining Coulombic part is higher, however. It is not only invariant under the SU(2)\(_L\times\)SU(2)\(_R\) and U(1)\(_A\) transformations, like the fermionic and magnetic parts, but is also a singlet with respect to SU(2)\(_{CS}\) chiral spin rotations as well as SU(4) transformations [11].

The chiral spin SU(2)\(_{CS}\) transformations are defined as rotations of the fundamental vectors

\[ U = (u_L, u_R)^T \quad D = (d_L, d_R)^T \]  

in an imaginary chiral spin space:

\[ U \rightarrow U' = e^{i\frac{\Sigma}{2}} U, \quad D \rightarrow D' = e^{i\frac{\Sigma}{2}} D, \]

where \( \Sigma \) are 4 × 4 matrices

\[ \Sigma = \{ \gamma^0, i\gamma^5\gamma^0, -\gamma^5 \} \],

that satisfy the SU(2) algebra:

\[ [\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk} \Sigma^k. \]

Upon rotations in the chiral spin space the left- and right-handed components of the quark fields get mixed.

A group that contains at the same time SU(2)\(_L\) × SU(2)\(_R\) and SU(2)\(_{CS}\) ⊃ U(1)\(_A\) is SU(4) with the fundamental vector

\[ \Psi = (u_L, u_R, d_L, d_R)^T \]

and a set of generators

\[ \{(\tau^a \otimes \mathbb{1}_D), (\mathbb{1}_F \otimes \Sigma^i), (\tau^a \otimes \Sigma^i)\}. \]

3 What symmetries can we expect at high temperatures?

In the plasma (deconfining) regime one expects a priori that the system has the SU(2)\(_L\) × SU(2)\(_R\) (or SU(2) \times SU(2) \times U(1)\(_A\)) symmetries of the QCD Lagrangian. One views the deconfined plasma as a system of quarks and gluons that freely propagate through the matter. What is generic for plasma is a Debye screening of the electric gluons that freely propagate through the matter. What we would imply absence of SU(2)\(_{CS}\) and SU(4) symmetries. At the same time, their inequility would imply absence of SU(2)\(_{CS}\) and SU(4) symmetries.

In the baryon sector convenient operators could e.g. be

\[ O_{N \pm} = \epsilon^{abc}_5 u^a u^b u^c [u^T C\gamma_5 d^c] \]

and

\[ O_{N \pm} = i\epsilon^{abc}_5 u^a u^b u^c [u^T C\gamma_5\gamma_0 d^c], \]

that belong to distinct representations of SU(2)\(_L\)×SU(2)\(_R\) and U(1)\(_A\) groups and at the same time are members of the same irreducible representations of SU(2)\(_{CS}\) and of SU(4).

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