dS Supergravity from 10d

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1. Introduction

The general flux compactifications of type II supergravity on SU(3)-structure manifolds in the presence of calibrated (supersymmetric) sources and an orientifold projection gives rise to four dimensional $\mathcal{N}=1$ supergravity theories. The details of the compactification can therefore be codified into values of the Kähler potential $K$ and the superpotential $W$ for these dS supergravities with a nilpotent multiplet and non-linearly realized local supersymmetry. In addition to KKLT and LVS with uplifting $D3$-branes, we have now new models with uplifting $D5$, $D6$, $D7$, $D9$-branes. The new uplifting contribution to the supergravity potential due to Volkov-Akulov supersymmetry is universal. As one application of our general result, we study classical flux compactifications of type IIA supergravity and find that a previously discovered universal tachyon is now absent.

2. Keep the dictionary between the standard $\mathcal{N}=1$ supergravity action and the ten dimensional supergravity/string theory model.

3. Add pseudo-calibrated anti-D-branes wrapped on supersymmetric cycles and adjust the tadpole condition accordingly.

4. Add to the four dimensional Kähler potential $K$ and superpotential $W$ new terms, which we derive in this paper. These new terms include a nilpotent multiplet and therefore lead to a four dimensional dS supergravity theory.

To explain the generality of the new results we find it convenient to use the formalism of generalized complex geometry and supersymmetry/calibration correspondence, following [3–5] and references therein. In particular, it means that we start with SU(3)-structure manifolds that admit calibrated D-branes and O-planes, which reduce the supersymmetry via the constraint

$$(1 - \Gamma_p)\epsilon = 0.$$  \hspace{1cm} (1)

Such a constraint follows from the $\kappa$-symmetric D-brane action when the local $\kappa$-symmetry

$$\delta \theta(\sigma) = (1 + \Gamma_p)\kappa(\sigma)$$  \hspace{1cm} (2)

is gauge-fixed as proposed in [15]. In this context Equation (1) is an algebraic equation defining the Killing spinor.

The condition for supersymmetry (1) is universal and applies to all types of branes, fundamental strings, NS5-branes, D-branes and M-branes. Thus we expect that the results of this paper apply beyond the case of anti-D-branes in SU(3)-structure compactifications. Supersymmetric (world-volume) configurations are solutions of the Born-Infeld field equations, which satisfy Equation (1) for some non-vanishing $\epsilon$. The part of the bulk supersymmetry preserved by such a configuration depends on the number of linearly independent solutions of Equation (1) in terms of $\epsilon$.

In [16] the Killing spinor equations associated with the $\kappa$-symmetry transformations of the worldvolume brane actions were studied. It was shown that these Dirac-Born-Infeld type systems are associated with calibrations, and that all the worldvolume solitons associated with calibrations are supersymmetric.

The norms of the internal two Killing spinors admitted by the compactification manifold are equal to each other, so that calibrated $Dp$-branes are admitted by the manifold. In such case, using these spinors one can construct some polyforms bilinear in spinors and use a language common to both type IIA and IIB theory. In particular, the existence of globally defined

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nowhere-vanishing spinors allows one to construct a globally defined real two-form $J$ and a complex three-form $\Omega$ as certain bilinears of these spinors. As a result there is a very nice dictionary between the string theory models with fluxes and localized sources based on 10d supergravity, and $K$ and $W$ of the four dimensional $\mathcal{N} = 1$ supergravity.

In [3–5] a concise way of packing this dictionary is proposed, based on pure spinors which are polyforms $\Psi_i = \Psi_i^\pm$ for IIA/IIB. These concise formulas for $K$ and $W$ depend on these two polyforms, on the properties of the compactification manifold, on the RR potentials $C$, on the NSNS 2-form $B$ and on the dilaton. The explicit formulas for $K$ and $W$ are given for example in Equations (4.40), (4.41) in [4]. They involve specific combinations of polyforms involving the Hitchin function, Mukai pairing and other objects of generalized complex geometry.

We will refer to these expressions in [4] as

$$K_{\text{IIA/IIB}} = K(z', \bar{z}'), \quad W_{\text{IIA/IIB}} = W(z').$$  \hfill (3)

When specified to the type IIA or type IIB case, these produce the well known $K$ and $W$, which we will present in detail below in appendix A. $K$ is a real function of the chiral multiplets $z'$, $\bar{z}'$ and $W$ is a holomorphic function of the chiral multiplets $z'$. This summarizes the steps shown above as 1 and 2. Now we would like to explain our step 3. This step was actively studied in string theory for the $D3$-brane, see for example [17–21]. The spontaneous breaking of supersymmetry by an $D7$-brane in the GKP background[22] was studied in [23].

Here we will include anti-$Dp$-branes as one of the ingredients of the string theory models in ten dimensions, with any $p$, not just $p = 3$. One might worry that $Dp$-branes and anti-$Dp$-branes, when wrapped on the same cycle, are moving towards each other in the compact space and could quickly annihilate. While this is somewhat model dependent, we like to stress that our general results do not require the presence of $Dp$-branes. We can satisfy the tadpole condition using $Op$-planes, fluxes and anti-$Dp$-branes only. In such a case there are certainly many examples without perturbative instabilities, like for example setups with a single anti-$Dp$-brane, potentially even placed on top of an $Op$-plane. All anti-$Dp$-branes we include are pseudo-calibrated,[17] so that

$$(1 + \Gamma_p)\epsilon = 0,$$  \hfill (4)

since the $\epsilon$-symmetry on the world-volume of the anti-$Dp$-branes has the form

$$\delta \theta(\sigma) = (1 - \Gamma_p)\kappa(\sigma).$$  \hfill (5)

This means supersymmetry is non-linearly realized on the world-volume fields and spontaneously broken. The inclusion of anti-$Dp$-branes to a string theory model in addition to $Op$-planes and maybe $Dp$-branes was viewed in the past as a compactification to $\mathcal{N} = 0$ in $d = 4$, since the anti-$Dp$-branes preserve the supersymmetry opposite to the one preserved by $Dp$-branes and $Op$-planes.

Here we will show that, in fact, one should view this step as a general way of relating string theory models, with calibrated and pseudo-calibrated branes, to four dimensional $dS$ supergravity.[6–11] It means that via such compactifications we obtain a supergravity action, which in addition to unconstrained multiplets has also a nilpotent one. The nilpotent multiplet represents non-linearly realized Volkov-Akulov supersymmetry.[24] The action of $dS$ supergravity interacting with matter has a local non-linearly realized supersymmetry.

Our step 4 is to give the modifications of $K$ and $W$ due to the presence of the nilpotent multiplet. The new action has a non-linearly realized $\mathcal{N} = 1$ supersymmetry, which is a hallmark of $dS$ supergravity. Our main results are the new $K$ and $W$, which depend also on a nilpotent multiplet $S$, in addition to unconstrained chiral multiplets $z'$. They are generically of the form

$$K^{\text{new}}(z', z'; S, \bar{S}) = K(z', z') + K_S(z', z')S\bar{S},$$

$$W^{\text{new}}(z', S) = W(z') + \mu^2 S.$$  \hfill (6)

We will show that the nilpotent field metric, $K_S(z', z')$ is computable: for each set of ingredients in the so-called ‘full-fledged string theory models’ one can compute $K_S(z', z')$ as function of the overall volume, the dilaton and the volume moduli of the supersymmetric cycles on which the anti-$Dp$-branes are wrapped. In IIB we will have four cases

$$D\bar{S}$$ on a 6-cycle, $D\bar{S}$ on 4-cycles,

$$D\bar{S}$$ on 2-cycles, $D\bar{S}$ on a 0-cycle.  \hfill (7)

In type IIA for SU(3) structure manifolds there are no non-trivial closed 1-forms.[31] Serre duality then implies that there are no 5-forms either. Poincare duality then implies that there are no non-trivial 1- and 5-cycles that can be wrapped by a $Dp$-branes. Thus, from all potential cases

$$D\bar{S}$$ on 5-cycles, $D\bar{S}$ on 3-cycles, $D\bar{S}$ on 1-cycles,  \hfill (8)

only one survives

$$D\bar{S}$$ on 3-cycles.  \hfill (9)

Since the nilpotent multiplet does not have a scalar component, the new potential has an additional term but still depends on the same closed string moduli. The new F-term potential acquires an additional nowhere vanishing positive term, as always associated with Volkov-Akulov non-linearly realized supersymmetry

$$V^{\text{new}}(z', z') = V(z', z') + e^{K(z', z')}|D_5 W|^2,$$  \hfill (10)

where

$$|D_5 W|^2 \equiv D_5 W K_S(z', z') D_5 \bar{W}.$$  \hfill (11)

The positivity of the new term in the potential is due to the positivity of $e^{K(z', z')}$ and the positivity of the nowhere vanishing $|D_5 W|^2$.

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1. We are setting the open string moduli on the anti-$Dp$-branes to zero for simplicity. They can be included into the general $dS$ supergravity using additional constrained multiplets, see for example[25–26] as well as [20].
signifying the non-linear realization of the Volkov-Akulov supersymmetry.

It is rather gratifying to see that dS supergravity might be associated with string theory models in case of all pseudo-calibrated $Dp$-branes, which should be wrapped on supersymmetric cycles of the compactification manifolds. The well-known case of the $D3$-brane uplift $^{27}$ is not unique anymore.

2. Type II Compactifications with Calibrated Sources

In this section we review (classical) flux compactifications of type II supergravity on SU(3)-structure manifolds in the presence of calibrated (i.e. supersymmetric) sources, in particular D-branes and O-planes (see for example [1,2] for an overview). Compactifications on SU(3)-structure manifolds give rise to four dimensional theories, which preserve linear $\mathcal{N} = 2$ supersymmetry that is explicitly broken to linear $\mathcal{N} = 1$ by performing in type IIA an $O6$ orientifold projection and in type IIB by performing an $O3/07$ or $O5/09$ orientifold projection.

The theories that lead upon compactification on an SU(3)-structure manifold to a standard 4d $\mathcal{N} = 1$ theory have an action that consists of three parts, the closed string type II action, the $Op$-plane action and the $Dp$-brane action $^{2}$

$$S = S_{II} + N_{OP} S_{OP} + N_{DP} S_{DP},$$

(12)

We will now split each of the above three terms into two parts, the second of which is relevant for the tadpole cancellation condition

$$S = \tilde{S}_{II} + \int C_{p+1} \wedge (d F_{6-p} - H \wedge F_{6-p}) + N_{OP} (S_{DBI}^{OP} + S_{CP}) + N_{DP} (S_{DBI}^{DP} + S_{CP})_0.$$  

(13)

Now we have that $S_{CP} = -2^{p-5} \int C_{p+1}$ and $S_{CP} = \int C_{p+1} + \ldots$, where ... includes other bosonic and fermionic terms (for the ease of presentation we temporarily set the $Dp$-branes tension to one). Varying the action with respect to $C_{p+1}$ leads to the following (integrated) tadpole cancellation condition

$$\int d F_{6-p} - H \wedge F_{6-p} = -2^{p-5} N_{OP} + N_{DP}.$$  

(14)

Once we satisfy this tadpole cancellation condition the remaining part of the action that gives rise to a standard 4d $\mathcal{N} = 1$ supergravity action is

$$S_{\text{Standard-SUGRA}} = \tilde{S}_{II} + N_{OP} S_{DBI}^{OP} + N_{DP} S_{DBI}^{DP}.$$  

(15)

$\text{Here } S_{OP/DP} \text{ denote the action for a single plane/brane. The O-planes or D-branes can wrap different cycles but we omit a corresponding index. In the case of } O3/07 \text{ and } O5/09 \text{ there are two different } p \text{'s and our argument goes through in the same way.}$
3. Adding Pseudo-Calibrated Anti-$d p$-Branes

In most string theory compactifications with phenomenological applications the goal was to find the ingredients of standard 4d $\mathcal{N} = 1$ supergravity, i.e. to find $K$ and $W$ for unconstrained chiral multiplets and to identify the potential (17) associated with ‘full-fledged string theory models’. In [17] an important step was made to accommodate the KKLT construction in this setting. At that time adding an anti-$D3$-brane, even pseudo-calibrated, meant that supersymmetry of the kind available in standard supergravity becomes broken down to $\mathcal{N} = 0$. The additional term in the potential, the so called uplifting term in KKLT, $V_{up} = \frac{8D}{(4\pi)^2}$, was not part of the potential in (17) and only the bosonic term $V_{up}$ was presented.

Since then the manifestly supersymmetric version of the KKLT uplifting was proposed in the form in which the anti-$D3$-brane is represented by a nilpotent multiplet $S$ with $S^2 = 0$, corresponding to VA non-linearly realized supersymmetry.[28] In this case the new $K$ and $W$ are (in the unwarped case) given by

$$K = -3 \log (T + \bar{T}) + S \bar{S},$$

$$W = W_0 + A \exp(-a T) + \mu^2 S, \Rightarrow$$

$$V_{up} = e^K |D_2 W|^2 \big|_{S=\bar{S}=0} = \frac{\mu^2}{(T + \bar{T})^2}.$$  \hspace{1cm} (20)

The reason why in the KKLT case the presence of $D3$-branes and $O3$-planes were constrained by a tadpole condition, was not leading to an uplift term, is due to the fact that these were calibrated: they preserved the same symmetry as the background, $(1 - \Gamma_3)e = 0$. Meanwhile, the anti-$D3$-branes are pseudo-calibrated, they preserve the symmetry opposite to that of the background and the $D3$-branes/$O3$-planes, $(1 + \Gamma_3)e = 0$.

The concept of calibrated $Dp$-branes and pseudo-calibrated pseudo-calibrated anti-$Dp$-branes is totally general. From this perspective, in dS supergravity constructions there is no need to restrict ourselves to anti-$D3$-branes as an exclusive source of Volkov-Akulov non-linearly realized supersymmetry. Any $D$-brane has a non-linearly realized supersymmetry and therefore one has to look at the general case of including pseudo-calibrated anti-$D$-$p$-branes, wrapped on supersymmetric cycles, as new local sources, and check the tadpole condition, as suggested in point 3 in the Introduction.

From all possible $Dp$-branes with $p \geq 3$ we can get uplift terms, i.e. positive new terms in the 4d scalar potential, if there are supersymmetric $(p-3)$-cycles on our compactification manifold. In type IIB there are 6-, 4-, 2-, 0-cycles, therefore we will have an uplift term due to anti-$D9$, anti-$D7$, anti-$D5$, anti-$D3$-branes. In type IIA on SU(3)-structure manifolds there are only 3-cycles and therefore only anti-$D6$-branes can give rise to a new positive uplift term in the scalar potential.

Let us now repeat the general derivation of the four dimensional action at the beginning of section 2 but now we also include anti-$Dp$-branes. The action is

$$S = S_{11} + N_{Dp} S_{Dp} + N_{Dp} S_{Dp} + N_{Dp} S_{Dp},$$  \hspace{1cm} (21)

We again split each of the above terms into two parts and use that $S_{Dp} = S_{DBI}^D - S_{CS}^D$

$$S = \tilde{S}_{11} + \int C_{p+1} \wedge (d F_{8-p} - H \wedge F_{6-p}) + N_{Dp} (S_{DBI}^D + S_{CS}^D)$$

$$+ N_{Dp} (S_{DBI}^D + S_{CS}^D) + N_{Dp} (S_{DBI}^D - S_{CS}^D).$$  \hspace{1cm} (22)

Varying the action with respect to $C_{p+1}$ leads now to the following (integrated) tadpole cancellation condition

$$\int d F_{8-p} - H \wedge F_{6-p} = -2^{p-5} N_{Dp} + N_{Dp} = N_{Dp}.$$  \hspace{1cm} (23)

Once we satisfy this tadpole cancellation condition the remaining part of the action that now will give rise to a new 4d $\mathcal{N} = 1$ dS supergravity action is

$$S_{dS-\text{SUGRA}} = \tilde{S}_{11} + N_{Dp} S_{DBI}^D + N_{Dp} S_{DBI}^D + N_{Dp} S_{DBI}^D.$$  \hspace{1cm} (24)

The above action is actually related to the standard supergravity action in Equation (15) in a very simple way. Let us assume for example that we satisfy the new tadpole condition in Equation (23) by not changing the fluxes on the left-hand-side nor $N_{Dp}$, but simply by adding an additional $N_{Dp}$ $Dp$-branes so that $N_{Dp} \rightarrow N_{Dp} + N_{Dp}$. Then we find that the new action has the form

$$S_{dS-\text{SUGRA}} = S_{\text{standard-SUGRA}} + 2 N_{Dp} S_{DBI}^D.$$  \hspace{1cm} (25)

So the new action is actually related to the old one by adding twice the DBI action for the anti-$Dp$-brane. This result holds in full generality also in the absence of any $Dp$-branes. In this case one has to adjust the fluxes because of the tadpole condition in Equation (23). This adjustment of the fluxes then modifies $S_{11}$ exactly in the right way to give the new term in the dS supergravity action.

Therefore, for all anti-$Dp$-branes we find that they lead to a new contribution to the scalar potential in four dimensions that is in string frame of the form

$$V_{\text{up}} = 2 N_{Dp} T_{Dp} \int_{\Sigma_d} d^{p-1} \xi \epsilon - e^{-\phi} \sqrt{\det (G + B - 2\pi \alpha^' F)}.$$  \hspace{1cm} (26)

where $\alpha$ labels the different $(p-3)$-cycles $\Sigma_d$ that are wrapped by the anti-$Dp$-branes and $T_{Dp}$ denotes their tension. In the next two sections we will work out exactly how this new term can be included in the Kähler and superpotential via a nilpotent chiral superfield. For simplicity we do not include the worldvolume scalar fields on the anti-$Dp$-branes, like the gauge field or the position moduli in our discussion. It should be possible to include them using other constrained multiplets as in [25,26]. Note however that these moduli could be absent in some cases, if we for example place a single anti-$Dp$-brane on top of an $Op$-plane.

In all cases we will find that

$$V_{\text{new}}(z^i, \bar{z}^j) = V(z^i, \bar{z}^j) + e^{k(z^i, \bar{z}^j)} D_5 W \bar{K}^S(z^i, \bar{z}^j) \overline{D_5 W}$$  \hspace{1cm} (27)

and the dictionary between string theory models with anti-$Dp$-branes and dS supergravity with a nilpotent multiplet will be established.
3.1. Pseudo-calibrated anti-$Dp$-branes in type IIB

The calibration condition for $p = 3, 5, 7, 9$, is given in [23] in the paragraph between (2.185) and (2.186). It allows us to rewrite the new positive term in the scalar potential, given above in (26), as

$$V_{Dp} = 2N_{Dp,a}T_{Dp}\int_S d^{p-3}\xi e^{-\phi} \text{Re} \left(e^{i/2\beta}\right).$$  \hspace{1cm} (28)

Explicitly this means (see appendix A for our notation)

$$V_{D7} = 2N_{D7,a}T_{D7}\int_S d^{2}\xi e^{-\phi} J = 2N_{D7,a}T_{D7}\text{Im}(\tau),$$

$$V_{D9} = 2N_{D9,a}T_{D9}\int_S d^{4}\xi e^{-\phi} (J \wedge J - B \wedge B)$$

$$= -2N_{D9,a}T_{D9}\text{Im}(T^w),$$

$$V_{D6} = 2N_{D6,a}T_{D6}\int_S e^{\phi/2} J \wedge J - \frac{1}{2} J \wedge B \wedge B$$

$$= -2N_{D6,a}T_{D6}\text{Im}(T).$$  \hspace{1cm} (29)

So we see that there is a nice unifying description.

Now we go to the 4d Einstein frame. Above we have already identified the correct moduli in Einstein frame so that this rescaling changes all the above expressions only due to the $f d^4x\sqrt{-g^E_t} = f d^4x\sqrt{-g^E_t} = f d^4x\sqrt{-g^E_t}$ factor in the DBI action. Here we defined in the last equation the four dimensional dilaton $\phi_4 = \phi - \frac{1}{2} \text{Im}(\nu_4)$.

For the anti-D3-brane this gives the usual (unwarped) expression, if we use that for a single Kähler modulus in 4d Einstein frame we have $2\nu_6 = ie^{\phi_4}(T - \bar{T})$,

$$V_{D3} = 2N_{D3,a}T_{D3}\text{Im}(\tau)\frac{e^{\phi_4}}{V_6} = 2N_{D3,a}T_{D3}\frac{e^{\phi_4}}{V_6} = 2N_{D3,a}T_{D3}\frac{2}{i(T - \bar{T})}.$$  \hspace{1cm} (30)

For all cases we have simply

$$V_{D3} = -i T_{D3} N_{D3,g}(\tau - \bar{\tau}) e^{\phi_4},$$

$$V_{D7} = -i T_{D7} N_{D7,a}(\nu^4 - \bar{\nu}^4) e^{\phi_4},$$

$$V_{D9} = i T_{D9} N_{D9,a}(T^w - \bar{T}^w) e^{\phi_4},$$

$$V_{D6} = i T_{D6} N_{D6,a}(T - \bar{T}) e^{\phi_4}.$$  \hspace{1cm} (31)

Let us introduce the shorthand notation for all cases above

$$V_{Dp} = 2T_{Dp} e^{\phi_4} \text{Im} \Phi,$$  \hspace{1cm} (32)

where $\text{Im} \Phi = \{ N_{Dp,m} \text{Im} \nu^m, - N_{Dp,a} \text{Im} T^w, - N_{Dp,a} \text{Im} T \}$ is a positive, real linear combination of the respective complex moduli in the particular setups.

We can then obtain the above expression $V_{Dp}$ from

$$K = K_{\text{before}} + ie^{\phi_4} \frac{e^{-\phi_4}}{(\Phi - \bar{\Phi})} S S,$$

$$W = W_{\text{before}} + \mu^2 S,$$  \hspace{1cm} (33)

where $\mu^4 = T_{Dp}$. For the particular case of an anti-D3-brane this agrees with the previously derived Equation (3.40) in [20].

3.2. Pseudo-calibrated anti-$Dp$-branes in type IIA

Spacetime filling $Dp$-branes in type IIA wrap an odd dimensional internal cycle, this leaves us only with the case of anti-D6-branes, since there are no non-trivial 1- and 5-cycles.

The calibration condition for D6-branes is given in [23] in Equation (2.184). It allows us to rewrite the new term in the scalar potential, given above in (26), as

$$V_{D6} = 2N_{D6,a}T_{D6}\int_S e^{\phi/2} \text{Re} \Omega = 2N_{D6,a}T_{D6}e^{\phi_4} \text{Im}(Z^K).$$  \hspace{1cm} (34)

We again can write this new term by including a nilpotent chiral multiplet $S$ coupled to the other fields. In particular, one finds that

$$K = K_{\text{before}} + ie^{\phi_4} \frac{e^{-\phi_4}}{N_{D6,a}K(Z^K - \bar{Z}^K)} S S,$$

$$W = W_{\text{before}} + \mu^2 S,$$  \hspace{1cm} (35)

where $\mu^4 = T_{D6}$.

4. dS Vacua in Type IIA dS Supergravity

We are now focusing on the particular case of massive type IIA flux compactification to which we can add anti-D6-branes as explained in subsection 3.2. This case is particularly simple since all moduli can be stabilized (see [30] for a review of this particular class of compactifications). However, it has never been possible to find (meta-)stable dS vacua in this context. All example of dS critical points have always had at least one tachyonic direction with large slow-role parameter $|\eta| \gtrsim O(1)$. [31] This had lead people to investigate whether there are no-go theorems in this case that forbid stable dS vacua. [31–34] Two for us important insights have emerged from these studies: 1) The obstinate tachyonic direction involves the 3-cycle moduli, [31,33,34] 2) In the limit of very small positive value of the potential, the tachyonic direction seems be connected to the sGoldstino. [31,33–35]

4 We use slightly different conventions compared to [2]. We take $i \int \Omega \wedge \bar{\Omega} = 1$ (see eqn. (2.12) of [29]) instead of [2] where $i \int \Omega \wedge \bar{\Omega} = \nu_4$. Hence we have an extra factor of $\sqrt{\nu_4}$. As discussed above, we get an extra factor $\frac{2}{N_D} \bar{\nu}_4$ from going to 4d Einstein frame.
Our new term that appears in the action does involve the 3-cycles since we can wrap them with anti-D6-branes, so the new term should have an effect on the tachyonic direction. Furthermore, since the anti-D6-branes break supersymmetry they will modify the sGoldstino direction. For dominant SUSY breaking from anti-D6-branes, the Goldstino will be the worldvolume fermion on the anti-D6-brane, which is encoded in the nilpotent field $S^2 = 0$. This Goldstino has no scalar partner and therefore there is no sGoldstino that is at the risk of being tachyonic.\footnote{The supersymmetry on the anti-D6-branes is non-linearly realized. The fermions on the worldvolume do not simply get mapped into a boson under these transformations (see for example\cite{ref36} for more details).} The explicit no-go theorem\cite{ref34} that predicts a tachyonic field with $\eta \leq -\frac{1}{2}$ in standard type IIA compactifications is circumvented in the more general dS supergravity, due to the presence of anti-D6-branes.

Given the above it might not be guaranteed that the tachyonic direction can be absent in these models, however, we will provide a simple intuitive reason for why this is actually the case. Let us restrict to the case of a model with a single 3-cycle modulus $\text{Im}(Z)$ (or more generally this $\text{Im}(Z)$ could be the linear combination of 3-cycle moduli that is tachyonic). Then near the dS saddle point at $\text{Im}(Z) = \text{Im}(Z_0)$ the potential without the anti-D6-branes has the form $V_{\text{tachyon}} \propto V_0 - (\text{Im}(Z) - \text{Im}(Z_0))^5$, for some $V_0 > 0$. The positive new term from the anti-D6-branes in Equation (34) above has an implicit $\text{Im}(Z)$ dependence from $e^{i\alpha_6} \propto 1/\text{Im}(Z)^6$, so that it scales like $V_{\text{up}} \propto 1/\text{Im}(Z)^4$. The combination of these terms then generically has a dS minimum for an appropriately chosen number of anti-D6-branes, as is shown in Figure 1.

We study this for the simplest known example and find indeed for appropriately tuned parameters that the obstructive tachyon is absent. In the truncation to left-invariant fields, there is no other tachyon, so it is possible that this is the first stable dS vacuum in this context. In order to know for sure, one has to check that there are no other light fields that have a negative mass, see section 3.1 of\cite{ref36} for a discussion of this point.

One might worry that anti-D6-branes could annihilate quickly against the background fluxes.\cite{ref37} However, the analysis in\cite{ref37} is only valid for a large number of anti-D6-branes, while for a small number a different result seems likely.\cite{ref38} So it is plausible that uplifting leads to long lived dS vacua, if one uses a single anti-D6-brane or an anti-D6-brane on top of an O6-plane.

In this simplest model the unstable dS vacua that were previously found in\cite{ref39,ref40} can be shown to all lie at small volume and large string coupling\cite{ref30} so that one expects large $\alpha'$ and string loop corrections. The anti-D6-brane contributions shift the positions of the vacua so that one has to analyze the full moduli space in this model to check whether dS vacua in a trustworthy regime could exist. There are of course also many more models that one can study in this new context. We leave a more detailed analysis to the future.\cite{ref41}

4.1. The isotropic $S^3 \times S^3/Z_2 \times Z_2$ example

Probably the simplest example of compactifications of type II string theory is the compactification on $T^6/Z_2 \times Z_2$, where one identifies the three $T^2$ in $T^6$. After this identification this model has only three complex moduli, whose imaginary parts correspond to a single volume modulus, a single complex structure modulus and the dilaton.\footnote{This model is the harmonic oscillator of compactifications and is often called ‘STU-model’ in the literature. We reserve $S$ for the nilpotent field here and also label the other moduli differently.} We compactify on this space and include in the NSNS sector H-flux, denoted by $h$ below, as well as metric fluxes. The latter are being equivalent to adding curvature and we choose them in such a way that the internal space is actually $S^1 \times S^1$. This model has been studied in\cite{ref31,ref33,ref34,ref36,ref39,ref42,ref47}. The only non-trivial fluxes we can add in the RR sector are $F_2$ and $F_5$ fluxes, whose parameter we denote by $f_2$ and $f_5$. Furthermore, we do an O6-orientifold projection and now allow for the addition of $N_{\text{D6}}K = 1, 2$ anti-D6-branes on the two even 3-cycles. In our notation the Kähler and superpotential take the form

\begin{equation}
K = -\ln \left[ -i(t - \bar{t})^3 - i\frac{S\bar{S}}{N_{\text{D6}}(Z^K - \bar{Z}^K)} \right] - \ln \left[ -i(t - \bar{t})^3 - i\frac{S\bar{S}}{N_{\text{D6}}(Z^K - \bar{Z}^K)} \right],
\end{equation}

\begin{equation}
W = \frac{1}{2}(h + 3t)Z^2 - \frac{3}{2}(h - t)\bar{Z}^2 + 3f_2t^2 - f_5t^3 + \mu^2 S. \tag{36}
\end{equation}

In this model there are no D-terms. The internal volume is $8V_0 = i(t - \bar{t})^3$ and the four dimensional dilaton is $e^{-\phi_6} = e^{-\phi_6}V_0^2 = 2\text{Im}(Z)\text{Im}(Z)^3$. We have used $S^2 = 0$ to rewrite

\begin{equation}
-3\ln \left[ -i(t - \bar{t})^3 - i\frac{S\bar{S}}{N_{\text{D6}}(Z^K - \bar{Z}^K)} \right] = -\ln \left[ -i(t - \bar{t})^3 - i\frac{S\bar{S}}{N_{\text{D6}}(Z^K - \bar{Z}^K)} \right]. \tag{37}
\end{equation}

The scalar potential of this model is not too complicated and we have actually been able to minimize it analytically in terms of

---

Figure 1. The extra term from an anti-D6-brane can lead to a stable minimum near a dS saddle point. The dashed blue potential $V_{\text{tachyon}}$ with the maximum is the standard potential when only calibrated D6-branes and O6-planes are present. The uplift due to the anti-D6-brane, $V_{\text{up}} \propto 1/\text{Im}(Z)^3$, is shown by a black dotted line. Finally, the sum of the original tachyonic potential and the uplift potential is shown as solid red line and exhibits a dS minimum.
the parameters. We have found that for suitable chosen values of the parameters we do indeed find stable dS solutions in our truncated model, i.e. the addition of anti-D6-branes has removed the tachyon, see Figure 2.

We have explicitly checked that no other of the left-invariant moduli directions are tachyonic and that there are indeed metastable dS solutions in this truncation. There is a large parameters space and we leave it to the future to map it out and check whether one finds stable dS vacua in a trustworthy regime. One concrete set of values that leads to a dS vacuum, that is however at small volume, strong coupling and does not have properly quantized fluxes and numbers of anti-D6-branes, is given by

\[
\begin{align*}
    f_0 &= -1 & f_2 &= 1 & h &\approx -4.55 \\
    \text{Re}(t) &\approx -1.67 & \text{Re}(Z_i^2) &\approx 5.30 & \text{Re}(Z_i^2) &\approx 0.804 \\
    \text{Im}(t) &\approx 1.76 & \text{Im}(Z_i^2) &\approx 4.31 & \text{Im}(Z_i^2) &\approx 0.251 \\
    N_i &= .05 & N_2 &= .05 & \mu &= 1
\end{align*}
\]

In this case the value of the scalar potential at the minimum is \( V \approx 2 \times 10^{-4} \) and the eigenvalues of the Hessian \( \partial_i \partial_j V \), for \( i, j \) running over real and imaginary part of \( t, Z_i^2 \) and \( Z_i^2 \) are approximately \( 4.7, 3.1, 0.95, 0.73, 0.024, 0.00011 \). This example is clearly not yet a full-fledged string theory solution. However, it is a proof of principle that the usefulness of uplift terms from anti-Dp-branes and the corresponding dS supergravity theories are very interesting and extend well beyond the KKLT(27) and LVS(48) scenarios.

**5. Discussion**

In supersymmetry preserving compactifications of string theory, without the so-called pseudo-calibrated anti-Dp-branes, it seems difficult to find de Sitter vacua. Here we have shown that

\[
7 \text{ Similarly to the anti-D3-brane case, one probably also needs to substantially warp down the anti-brane tension, which is certainly problematic for the anti-D9-branes.}
\]
to be seen, if full-fledged string theory solutions at large volume
and small string couplings are available in this new setting.

In conclusion, arguably, the discovery of dark energy may be
viewed as a discovery of the Volkov-Akulov non-linearly realized
supersymmetry from the sky.

Appendix A: Explicit Four Dimensional Supergravity Theories

In this appendix we discuss the detailed form of type II compac-
tifications on SU(3)-structure manifolds, where we will use
the notation of [51], to which we refer the interested reader for
more details. For simplicity, we restrict ourselves here to the four
dimensional data without including open string moduli fromD-
branes (see for example [52] and references to that article for how
open string moduli appear). Our expressions are still correct in
the presence of D-branes, if one sets the world volume fields like
for example the scalar fields that control the D-brane position and
the presence of D-branes, if one sets the worldvolume fields like
the proper flux quantization and tadpole conditions.

Note, that we restrict to purely geometric compactifications so
that the corresponding type II supergravity setups and their so-
lutions should correspond to full-fledged string theory solu-
tions that are found at large volume and weak coupling.

A.1. Type IIA with O6-planes

In type IIA we have the RR-fluxes $F_0$, $F_2$, $F_4$ and $F_6$ that can thread
the six dimensions of the SU(3)-structure manifold. In this sec-
tor we get scalar fields from the RR-form $C_1$ only since SU(3)-
structure manifolds have no 1- and 5-cycles. However, they do
have 2-cycles so that we can get also Abelian vector fields from
$C_1$. The reduction of type IIA on CY$_3$ manifolds was worked out
in [33]. There it was found that the complex four dimensional
scalar fields $Z^a$ and $t^a$ are given via the expansion of

$$
\begin{align*}
\Omega &= C_1 + 2\text{i} e^{\phi_4} \text{Re}(\Omega) = 2z^K a_K, \\
J &= B + iJ = t^a\omega_a,
\end{align*}
$$

(38)

where $e^{-\phi_4}$ is related to the dilaton $\phi_4$ and the overall volume $V_3 =
\frac{1}{3} \int J \wedge J \wedge J$ via $e^{-\phi_4} = e^{-\phi} \sqrt{V_3}$.

The resulting Kähler and superpotential was worked out for
CY$_3$ manifolds in [33], while [42] was the first paper to describe
an extra term in $W$ that arises from metric fluxes and [54] showed
that also non-vanishing D-terms can arise in this case. The four
dimensional data is given by

$$
K = 4\phi_4 - \ln(8V_3),
$$

$$
W = \int \left( \Omega \wedge (H - \omega \cdot J) + F_0 + F_4 + F_6 + \frac{1}{2} F_2 \wedge J \wedge J + \frac{1}{6} F_6 J \wedge J \wedge J \right),
$$

$$
f_{a\beta} = i\kappa_{a\beta} t^a,
$$

(39)

where we have given a simple form of $K$ that is an implicit func-
tion of the complex scalars. The curvature determines $\omega$ that
maps the 2-form $J_2$ to a 3-form $\omega \cdot J$. The triple intersection
numbers $\kappa_{a\beta}$ are determined by the geometry. The D-terms are given by

$$
D_a = 2\text{i} e^{\phi_4} t^K F_K,
$$

(40)

where $F^K_a$ is determined by the curvature of the SU(3)-structure
manifold and the $F_K$ are purely imaginary functions of $\text{Re}(\Omega)$. Thus we find that type IIA reduced on SU(3)-structure manifolds
in the presence of O6-planes gives generically rise to a standard
four dimensional $\mathcal{N} = 1$ supergravity theory that has F-terms as well as D-terms.

The interesting feature of these type IIA flux compactifications
is that all moduli can be stabilized by using the classical scalar
potential (as was first observed in [29,42,43]). This means that
perturbative and non-perturbative corrections can be neglected
in solutions that are found at large volume and small coupling.

This makes this class of models particular simple. It was proven
in [53] that in the absence of curvature, i.e. if the internal space
is a Calabi-Yau 3-fold, then the scalar potential has only AdS
vacua. If one includes curvature, then one can find dS critical
points, [39,45,46] however, until now all of these dS solutions had al-
ways one tachyonic direction, i.e. they were saddle points rather
than local minima.

A.2. Type IIB with O3/O7-planes

The RR-sector for type IIB compactified on SU(3)-structure man-
ifolds gives only rise to parameters in the scalar potential via the
$F_3$-flux (since there are no 1- and 5-cycles that we could thread
with fluxes). The complex scalars that appear holomorphically in
the superpotential are the complex expansion coefficients of the
holomorphic 3-form $\Omega$ as well as $\tau$, $G^a$ and $T_a$, defined via

$$
\tau = C_0 + e^{-\phi},
$$

$$
G^a \omega_a = C_4 + \tau B,
$$

$$
T_a \mu^a = C_4 + C_2 \wedge B + \frac{1}{2} \tau B \wedge B - \frac{1}{2} e^{-\phi} J \wedge J.
$$

(41)

The resulting four dimensional Kähler and superpotential have been worked out in [34,57] and are given by

$$
K = -\ln \left[ i \int \Omega \wedge \Omega \right] - 4 \ln \left[ -i(\tau - \bar{\tau}) \right] - 2 \ln [2V_6],
$$

$$
W = \int \left[ F_3 + \tau H_4 + \omega \cdot (C_2 + \tau B) \right] \wedge \Omega.
$$

(42)

Here we have again given $K$ as implicit function of the mod-
uli and $\omega \cdot (C_2 + \tau B)$ is a 3-form that depends on the $G^a$
moduli as well as the curvature of the SU(3)-structure manifold.
The volume is defined as in type IIA above via $V_6 = \frac{1}{3} \int J \wedge J \wedge J = \frac{1}{2} \kappa_{a\beta} t^a t^\beta t^\gamma$. The gauge kinetic function is a holomorphic
function that depends only on the complex structure moduli
contained in $\Omega$ and is given in Equation (3.11) of [51].

The resulting scalars are a 3-form $\tau$ that depends
on the complex structure moduli $\theta^a$ and $\phi$. The
Kähler potential is $K = -\ln \left[ i \int \Omega \wedge \Omega \right] - 4 \ln \left[ -i(\tau - \bar{\tau}) \right] - 2 \ln [2V_6]$.

$$
W = \int \left[ F_3 + \tau H_4 + \omega \cdot (C_2 + \tau B) \right] \wedge \Omega.
$$

(42)

The resulting four dimensional Kähler and superpotential have
been worked out in [34,57] and are given by

$$
K = -\ln \left[ i \int \Omega \wedge \Omega \right] - 4 \ln \left[ -i(\tau - \bar{\tau}) \right] - 2 \ln [2V_6].
$$

$$
W = \int \left[ F_3 + \tau H_4 + \omega \cdot (C_2 + \tau B) \right] \wedge \Omega.
$$

(42)
D-terms are only non-vanishing, if there is curvature, encoded in \( \tilde{f}_{aK} \), and they are given by

\[
D_K = -\frac{e^\phi}{2V_6} \tilde{f}_{aK} v^a.
\]

(43)

So contrary to the F-term potential that satisfies a no-scale condition and depends on the Kähler moduli \( T_a \) only via an overall factor, the D-term potential can have a more interesting dependence on the volume via the dependence on \( v^a \).

Generically, but also specifically in the case of Calabi-Yau compactifications, which have no curvature, one finds that the above classical scalar potential is not sufficient to stabilize all moduli. The flat directions are then lifted by perturbative and non-perturbative contributions. The most well studied scenarios here are KKLT\(^{12,17}\) where a non-perturbative contribution \( W_{\text{top}} \) is used and the LVS scenario,\(^{14,15}\) where \( W_{\text{top}} \) as well as a perturbative contribution to \( K \) are used. All of these perturbative and non-perturbative corrections that modify the functions in the standard 4d \( \mathcal{N} = 1 \) supergravity should be included and do not affect our general observation below that anti-D-branes give rise to a simple extra term in \( K \) and \( W \).

\[ \quad \]

### A.3. Type IIB with O5/ O9-planes

In the case of an O5/ O9 orientifold projection the story is similar to the case above. We again only have \( F_3 \) flux giving rise to parameters in the superpotential. However, the particular combination of fields that appear holomorphically in the superpotential are different. They are again given by the complex expansion coefficients of \( \Omega \) and by \( v^a \), \( L_a \) and \( T \) defined via

\[
\begin{aligned}
\imath^a \mu_a &= C_2 + \imath e^{-\phi} J, \\
L_a \bar{\phi} &= C_4 + C_2 \wedge B + \imath e^{-\phi} B \wedge J, \\
T &= \int_{\mathcal{C}_3} \left[ C_6 + C_2 \wedge B + \frac{1}{2} C_2 \wedge B \wedge B \\
&\quad + \imath e^{-\phi} \left( \frac{1}{2} J \wedge B \wedge B - \frac{1}{6} J \wedge J \wedge J \right) \right].
\end{aligned}
\]

(44)

The resulting four dimensional Kähler and superpotential have been worked out\(^{16,57}\) and are given by

\[
\begin{aligned}
K &= -\ln \left[ \int \Omega \wedge \bar{\Omega} \right] - 4 \ln \left[ e^{-\phi} \right] - 2 \ln \left[ 8V_6 \right], \\
W &= \int \left[ F_3 + \omega \cdot (C_2 + \imath e^{-\phi} J) \right] \wedge \Omega.
\end{aligned}
\]

(45)

The Kähler potential is actually the same as for the O3/ O7 orientifold projection but it is now an implicit function of the moduli given in Equation (44). The gauge kinetic function depends holomorphically on the complex structure moduli contained in \( \Omega \) and is given in Equation (3.36) of \([51]\). The H-flux does not appear in \( W \) but appears in the D-terms via its expansion coefficients \( p_i \). The D-terms are given by

\[
D_k = \frac{e^\phi}{2V_6} (r_{ak} u^a - p_k),
\]

(46)

where the \( u^a \) are the expansion coefficients of \( B = u^a \mu_a \) and \( r_{ak} \) is again determined by the curvature of the SU(3)-structure manifold.

Again one expects in this setup, like in any 4d \( \mathcal{N} = 1 \) supergravity theory, that all flat directions will receive important perturbative and/or non-perturbative corrections. It would be very interesting to investigate whether the new terms we derive in this paper are sufficient (maybe together with quantum corrections) to lead to dS vacua in this setup.

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**Conflict of Interest**

The authors have declared no conflict of interest.

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