Dissipative tunneling in presence of classical chaos in a mixed quantum-classical system

Bidhan Chandra Bag$^1$, Bikash Chandra Gupta$^2$ and Debshankar Ray$^1$

$^1$Indian Association for the Cultivation of Science
Jadavpur, Calcutta 700 032, INDIA.

$^2$Institute of Physics, Sachivalaya Marg,
Bhubaneswar 751 005, INDIA.

Abstract

We consider the tunneling of a wave packet through a potential barrier which is coupled to a nonintegrable classical system and study the interplay of classical chaos and dissipation in the tunneling dynamics. We show that chaos-assisted tunneling is further enhanced by dissipation, while tunneling is suppressed by dissipation when the classical subsystem is regular.

PACS number (s) :05.45.+b, 03.65 Bz
I. Introduction

The systems with mixed quantum-classical description have been the subject of considerable interest in recent years [1-9]. The validity of this description essentially rests on whether the quantum effects of one subsystem is negligible compared to the others. The well-known example is the Maxwell-Bloch equations [3, 4] which describe a model two-level system (quantum mechanical) interacting with a strong single mode classical electromagnetic field. The others comprise the models involving nuclear collective motion [5], one dimensional molecule [6] where the motion of an electron is described by an effective potential provided by the nuclei and other electrons within adiabatic approximation scheme, etc. The mixed description has also been employed by Bonilla and Guinea [7] for the measurement processes. Taking recourse to an average description in terms of an effective classical Hamiltonian, Pattanayak and Schieve [8] have studied some interesting aspects of semi-quantal chaos.

Very recently the interplay of classical and quantum degrees of freedom in a special class of systems with system-operators pertaining to a closed Lie algebra with respect to system Hamiltonian has been demonstrated to have acquired special relevance in connection with dissipation in quantum evolution[9]. While the quantum degree of freedom is responsible for the generic quantum features the implication of classicality is two-fold. First, it has been shown that if the classical degree of freedom is assigned the type of evolution as prescribed by the classical treatment of dissipative systems it is possible to realize dissipation for such composite systems via an indirect route without any violation of quantum rule. Second, if the classical subsystem is nonintegrable then the quantum motion by virtue of this nonintegrability may be profoundly affected. Our object here is to study this dissipative evolution of a quantum system coupled to a nonintegrable classical system.

Before going into the further details let us elaborate this issue a little further in somewhat general terms.

We first consider the coupling of a quantum system to a classical system in terms of the following Hamiltonian

\[ \hat{H} = \hat{H}_q + H_{cl} + \hat{H}_{q-cl} \],

where \( \hat{H}_q \) and \( H_{cl} \) refer to quantum and classical subsystems of the total Hamiltonian, respectively. \( \hat{H}_{q-cl} \) is the interaction potential which contains both classical and quantum degrees of freedom. If the quantum operators \( \{ \hat{R}_i \} \) form a closed algebra with respect to the Hamiltonian
\( \hat{H} \) of the system, i.e., if we have relations of the type
\[
\left[ \hat{H}(t), \hat{R}_i \right] = i\hbar \sum_{j=1}^{n} g_{ji}(t) \hat{R}_j \quad i = 1, 2...n ,
\]
(2)
where \( g_{ji} \) are the elements of a \( n \otimes n \) matrix, then one can have, by virtue of the equations of motion
\[
\frac{d\hat{R}_i}{dt} = \frac{i}{\hbar} \left[ \hat{H}, \hat{R}_i \right] \quad i = 1, 2...n ,
\]
(3)
a set of linear first order differential equations
\[
\frac{d\langle \hat{R}_i \rangle}{dt} = -\sum_{j=1}^{n} g_{ji} \langle \hat{R}_j \rangle ,
\]
(4)
which describes the temporal evolution of the expectation values.

It is important to note that Eq. (4) depends crucially on the elements \( g_{ji} \) which are determined by the classical dynamical variables as appeared in the Hamiltonian \( \hat{H} \) (Eq.1). In other words, \( g_{ji} \) is dependent on classical subsystem \( H_{cl} \). If the energy is taken to be the quantum expectation value of the Hamiltonian \( \hat{H} \), then one can easily generate the temporal evolution of classical conjugate variables, coordinate \( x \) and momentum \( p \) using \( \langle \hat{H} \rangle \) as follows;
\[
\frac{dx}{dt} = \frac{\partial \langle \hat{H} \rangle}{\partial p} ,
\]
(5)
\[
\frac{dp}{dt} = -\frac{\partial \langle \hat{H} \rangle}{\partial x} .
\]
(6)
Eqs. (4-6) comprise the complete dynamical evolution corresponding to Hamiltonian \( \hat{H} \) in the dissipation-free case.

We now take into consideration two different aspects of classical motion of the subsystem \( H_{cl} \).

First, the classical dynamical variable is made dissipative in an ad hoc fashion by adding \(-\gamma p\) to Eq.(5), i.e.,
\[
\frac{dp}{dt} = -\left( \frac{\partial \langle \hat{H} \rangle}{\partial x} + \gamma p \right) .
\]
(7)
This allows \[9\] an indirect route to dissipation in the quantum evolution because of \( \hat{H}_{q-cl} \) term in Eq.(1). It is easy to check that this quantum description is quite valid since no quantum rule is violated in the process. Thus Eqs. 4, 5 and 7 govern the dissipative dynamical evolution. We note, in passing, that this method has a distinct advantage over some other approaches [10-12] to quantum theory of damping which relies on explicit time-dependent Hamiltonian, such
as, Kanai Hamiltonian [10] etc., since these approaches lead to contradiction with uncertainty principle.

The second aspect of classicality of the subsystem concerns the nature of motion generated by $H_{cl}$. If the classical subsystem is non-integrable then the quantum evolution is affected through the properties of $g_{ji}$ matrix. It is important to emphasize a pertinent point at this stage. There are well-known cases where exponential instability occurs in a system where the overall dynamical system is non-integrable. For example, we refer to the case of optical chaos described by Maxwell-Bloch equations [3, 4]. Similar consideration of quantum-classical mixed description has been given to explore quantum chaos within Born-Oppenheimer approximation in a typical molecular system where the fast electronic motion and the slow nuclear motion (classical) separate in a very natural way. In contrast to these cases the present paper concerns the nonintegrability due to the subsystem $H_{cl}$ only.

The studies of dissipative effects in quantum systems are usually based on a traditional system-reservoir linear coupling model. There are two standard ways of formulating the problem[13-15, 21, 25]. The first one is based on density operator approach and leads to the so-called master equation while the second one is based on the Heisenberg picture and leads to the noise operators. In the latter case one replaces the reservoir by damping terms in the Heisenberg equations of motion for a dissipation-free system and adds fluctuating forces as the driving terms which impart fluctuations to the system. The operator forces are such that, first, the system has the correct statistical properties to agree in the classical limit and second, they maintain the commutation properties of the Boson operators to ensure that uncertainty principle is not violated. These consideration are taken care of in the treatments of density based or operator based theories (e. g., see Leggett and co-workers[13] and Krive and coworkers [14] and others [15]. The spiritual root of quantum statistical approach to damping lies in the fluctuation-dissipation theorem, which illustrates a dynamical balance of inward flow of energy due to fluctuations from the reservoir into the system and the outward flow of energy from the system to the reservoir due to dissipation of the system mode. Such a dynamical balance is automatically maintained in the present treatment (and one need not add a stochastic term in Eq.(7)) through a feedback from quantum to classical subsystem since here one deals with a finite number of degrees of freedom in the quantum plus classical subsystems as compared to the infinite number of degrees of freedom of the reservoir in the traditional system-reservoir description. The closure property of the algebra pertaining to the quantum system thus plays an important role in the feedback mechanism. Apart from being simple to implement a decisive advantage of the approach is that it may be carried over to the nonlinear systems (e. g., a
Morse oscillator described by SU(2) or SU(1, 1) Lie algebra [16]) in a straightforward manner, while the master equations are strictly valid for the linear models.

The consideration of the two aspects of classicality as mentioned earlier therefore leads us to dissipative chaotic evolution of classical degrees of freedom. Our object is to explore the influence of classical chaos on quantum evolution, dissipation being realized indirectly in the overall dynamics through classical friction. The generic quantum feature that we study here is tunneling. It is extremely important to assess what influence if any classical chaos have on it, particularly in presence of dissipation. Although the interplay of chaos and tunneling [17-19] or tunneling and dissipation [14, 20, 21] have been the subject of many researchers separately over the last decade, it is not clear how dissipative tunneling is influenced by classical chaos. We are therefore concerned here with all three interplaying aspects of evolution, e.g., classical chaos, dissipation and tunneling in a typical model with mixed quantum-classical description.

II. The model and the mixed quantum-classical dynamics

We consider a particle with fixed energy \( E_q < 0 \) penetrating an inverted potential barrier. \( -E_q \) is the energy measured from the top of the barrier (Fig. 1). The quantum subsystem \( \hat{H}_q \) which describes the penetration of the particle through the barrier is given by

\[
\hat{H}_q = \frac{\hat{p}_q^2}{2m} - \frac{1}{2}m\omega_0^2\hat{x}_q^2 .
\]  

\( \hat{x}_q \) and \( \hat{p}_q \) are the quantum mechanical operators corresponding to position and momentum of the particle respectively. \( m \) is the mass of the particle and \( \omega_0 \) refers to the frequency of the inverted well.

The Hamiltonian for the classical subsystem is given by

\[
H_{cl} = \frac{p_{cl}^2}{2M} + ax_{cl}^4 - bx_{cl}^2 + gx_{cl} \cos \omega_1 t ,
\]

which governs the motion of a classical system with mass \( M \) and characterized by its position \( x_{cl} \) and momentum variable \( p_{cl} \) in a double-well potential driven by an external field with frequency \( \omega_1 \). \( a \) ad \( b \) are the parameters of the double-well oscillator. \( g \) includes the effect of coupling of the external field to the oscillator and the strength of the field.

\( H_{cl} \) is nonintegrable and has been widely employed by various workers [17-19, 22-23] over the last few years in a variety of situations related to classical and quantum chaos.
The description of the Hamiltonian with mixed degrees of freedom is now made complete by considering the coupling of classical and quantum degrees of freedom in terms of the interaction potential \( \hat{H}_{q-cl} = \lambda x_{cl} \hat{x}_{q} \) so that we have

\[
\hat{H} = \hat{H}_{q} + \hat{H}_{cl} + \lambda x_{cl} \hat{x}_{q},
\]

\( \lambda \) represents the coupling constant.

Making use of the following rescaled dimensionless quantities

\[
\hat{x} = (m\omega_{0})^{\frac{1}{2}} \hat{x}_{q}, \quad \chi = \frac{\lambda}{(mM\omega_{0})^{\frac{1}{2}}}, \quad A = \frac{g}{M\omega}, \quad B = \frac{b}{M\omega^{2}}, \quad G = \frac{g}{(M\omega)^{\frac{7}{2}}},
\]

we rewrite the Hamiltonian (10)

\[
\hat{H} = \omega_{0} \hat{p}_{q}^{2} - \omega_{0} \hat{x}_{q}^{2} + \omega p_{s}^{2} + \omega A s^{4} - B \omega s^{2} + G \omega s \cos \omega_{1} t + \chi \omega_{0} s \hat{x},
\]

where \( \omega \) is the linearized frequency of the double-well potential.

By choosing the relevant operators belonging to the set \( \{ \hat{1}, \hat{x}, \hat{p}, \hat{x}^{2}, \hat{p}^{2}, \hat{L} = \hat{x} \hat{p} + \hat{p} \hat{x} \} \) we may construct a partial Lie algebra (\( \hat{1} \) is the unity operator). The temporal evolution of the expectation values of these operators (see Eq.4) is given by the following set of coupled differential equations

\[
\frac{d\langle \hat{x} \rangle}{d\tau} = \langle \hat{p} \rangle,
\]

\[
\frac{d\langle \hat{p} \rangle}{d\tau} = \langle \hat{x} \rangle - \chi s,
\]

\[
\frac{d\langle \hat{x}^{2} \rangle}{d\tau} = \langle \hat{L} \rangle,
\]

\[
\frac{d\langle \hat{p}^{2} \rangle}{d\tau} = \langle \hat{L} \rangle - 2\chi s \langle \hat{p} \rangle,
\]

\[
\frac{d\langle \hat{L} \rangle}{d\tau} = 2(\langle \hat{p}^{2} \rangle + \langle \hat{x}^{2} \rangle - \chi s \langle \hat{x} \rangle),
\]

where \( \tau \) denotes the scaled dimensionless time \( \tau = \omega_{0} t \).
Since \( \langle H \rangle \) coincides with the energy of the system, the classical equations of motion are

\[
\frac{ds}{d\tau} = \Omega p_s,
\]
\[
\frac{dp_s}{d\tau} = -\left[ \chi \langle \dot{x} \rangle + G \Omega \cos \Omega_1 \tau + 4A \Omega s^3 - 2B \Omega s + \Gamma p_s \right],
\]
(14)

where,
\[
\Omega = \frac{\omega}{\omega_0},
\]
and
\[
\Omega_1 = \frac{\omega_1}{\omega_0},
\]
\[
\Gamma = \frac{\gamma}{\omega_0}.
\]
(15)

Here \( \Gamma \) as defined above is the rescaled dimensionless damping constant introduced in the classical equations of motion (14) in an ad hoc fashion. We have already pointed out that this ad hoc introduction of classical friction leads to dissipation in the overall dynamics without any contradiction to uncertainty principle. Eqs. (13) and (14) thus govern the complete dynamics of the mixed system.

We now turn to the motion of the wave packet. In order that the motion of a wave packet comes close to the motion of a classical particle it is necessary that its average position and momentum follow the laws of classical mechanics. However, this condition is automatically satisfied for the inverted harmonic potential barrier we consider here.

We now describe the wave function of a particle by a Gaussian wave packet of the form

\[
\psi(x,t) = N(t) \exp \left[ -\beta(t)(x - \alpha(t))^2 \right],
\]
(16)

where \( \alpha(t) \) and \( \beta(t) \) are two complex, time-dependent parameters to be determined. \( N(t) \) is a normalization factor which is not of much interest here. The Gaussian wave packets have the decisive advantage since the ansatz (16) is a solution of the Schrodinger equation,

\[
(\hat{H} - i \frac{\partial}{\partial t}) \psi(x,t) = 0
\]
(17)

if \( \alpha \) and \( \beta \) satisfy the following two equations

\[
i \frac{d\beta}{dt} = 2\omega_0 \beta^2 + \frac{\omega_0}{2}
\]

and

\[
i \beta \frac{d\alpha}{dt} = \chi \frac{\omega_0 s(t)}{2} - \frac{\omega_0 \alpha}{2}
\]
(18)
or their scaled version (using $\tau = \omega_0 t$)

$$\frac{id\beta}{d\tau} = 2\beta^2 + \frac{1}{2},$$

$$i\beta \frac{d\alpha}{d\tau} = \frac{\chi s(\tau)}{2} - \frac{\alpha}{2}.$$ \hspace{1cm} (19)

It is easy to express the expectation values of $\hat{x}, \hat{p}$ and others operators in terms of $\alpha$ and $\beta$ as follows;

$$\langle \hat{x} \rangle = \frac{\alpha \beta + \alpha^* \beta^*}{\beta + \beta^*},$$

$$\langle \hat{p} \rangle = -\frac{2i\beta \beta^*(\alpha - \alpha^*)}{\beta + \beta^*},$$

$$\langle \hat{x}^2 \rangle = \frac{1}{\beta + \beta^*} + \langle \hat{x} \rangle^2,$$

$$\langle \hat{p}^2 \rangle = \frac{2\beta^*}{\beta + \beta^*} - \langle \hat{p} \rangle^2,$$

$$\langle \hat{L} \rangle = \frac{d\langle \hat{x}^2 \rangle}{d\tau}.$$ \hspace{1cm} (20)

The wave function of Gaussian form (16) satisfies the minimum uncertainty condition

$$\Delta p \Delta x = \frac{1}{2}.$$ \hspace{1cm} (21)

with $\Delta x^2 = \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2$ and $\Delta p^2 = \langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2$. For such a wave packet it is easy to see that

$$\beta = \beta^*.$$ \hspace{1cm} (22)

Since for a parabolic barrier the energy of classical particle is equal to the average energy of the quantum particle one can show that the energy spread for the coupling-free parabolic barrier is

$$\Delta \hat{H}_q^2 = \langle \hat{H}_q^2 \rangle - \langle \hat{H}_q \rangle^2$$

$$= \omega_0^2 \left[ \langle \hat{p} \rangle^2 \Delta p^2 + \langle \hat{x} \rangle^2 \Delta x^2 \right].$$ \hspace{1cm} (23)

It is required that the minimum wave packet should have a minimum energy spread $\Delta \hat{H}_q^2$ also. Then we infer from Eq.(23) that the wave packet must be prepared at the classical turning point $-x_0$, where $\langle \hat{p} \rangle = 0$ and $\langle \hat{x} \rangle^2$ has a minimum. Let us therefore assume that the center of the wave packet reaches the classical turning point at $t = 0$ with $\langle \hat{p} \rangle = p_0 = 0$. We then have [from $\hat{H}_q = \omega_0 \hat{p}^2 - \frac{\omega_0^2}{2} \hat{x}^2$, Eq.12]

$$\epsilon \omega_0 = \frac{\omega_0 x^2}{2}.$$ \hspace{1cm} (24)
so that $x_0 = \pm (2\epsilon)^{\frac{1}{2}}$, where $\epsilon(= \frac{E_{\text{q}}}{\omega_0})$ is a dimensionless parameter (see Fig 1) denoting the energy of the particle measured from the top of the barrier. We obtain

$$\alpha(t = 0) = \alpha_0 = -x_0 = -(2\epsilon)^{\frac{1}{2}}$$
$$\beta(t = 0) = \beta_0 = \frac{1}{2} .$$

The initial conditions (25) imply that the particle’s wave packet is as compact as possible when it arrives at the turning point. By choosing $\beta = \frac{1}{2}$, the wave packet satisfies the quantum-classical correspondence that the energy of the quantum particle is equal to that of the classical particle for parabolic barrier.

The above two initial conditions for $\alpha$ and $\beta$ also set the initial conditions for the others. Thus we further have from Eqs. (20),

$$\langle \hat{x} \rangle_{t=0} = -(2\epsilon)^{\frac{1}{2}} ,$$
$$\langle \hat{p} \rangle_{t=0} = 0 ,$$
$$\langle \hat{x}^2 \rangle_{t=0} = \frac{1}{2}(1 + 4\epsilon) ,$$
$$\langle \hat{p}^2 \rangle_{t=0} = \frac{1}{2} ,$$
$$\langle \hat{L} \rangle_{t=0} = 0 .$$

Eqs. (26) suggest that the initial conditions required for the dynamics can be manipulated by controlling a single parameter $\epsilon$, which denotes the average energy of the wave packet with which the particle impinges on the barrier at the left turning point.

Since the classical subsystem is coupled to the quantum degrees of freedom it is also necessary to specify the initial conditions for the classical variables for its evolution. To this end we fix the parameter set $\omega = 6.32$, $A = 0.002$, $B = 0.25$ and $G = 0.63$ for the present study and choose the initial conditions for $s$ and $p_s$ for the regular and chaotic trajectories as $s = -4.02, -4.52, -5.02$ for three regular and $s = -8.80, -9.30, -11.31$ for three chaotic trajectories, $p_s$ being chosen to be zero always. We refer to Lin and Ballentine [13] for a typical illustration of the phase space.

The initial conditions (25) and (26) along with those for the classical variables then allow us to follow the evolution of the expectation values and of the wave packet in terms of Eqs. (13-14) and Eq. 19. The relevant physical quantities of interest are determined in the next section.

Before closing this section a relevant point need to be discussed here. We have considered the regular and chaotic trajectories of the classical subsystem and referred to its phase space as
if this subsystem were not coupled to the quantum subsystem. Since the quantum subsystem considered here has a potential which is not bounded from below (a prototype model for barrier penetration), the phase space of the mixed system (quantum plus classical) is unbounded and therefore the very notion of classical chaos is far from clear in such a situation. Based on this consideration we have selected the chaotic and regular parts of the phase space corresponding to the classical subsystem only.

III. The tunneling probability and the current

We are now in a position to calculate the wave function and the corresponding probability that the incident particle penetrates beyond the position \( x = \zeta \). We write

\[
T(x \geq \zeta, \tau) = \int_{\zeta}^{\infty} dx |\psi(x, \tau)|^2 .
\]

(27)

Normalization of the wave function(16) leads us to

\[
|\psi(x, \tau)|^2 = \sqrt{\frac{a(\tau)}{\pi}} \exp[-a(\tau)(x - \langle x \rangle)^2] ,
\]

(28)

where \( a(\tau) = \beta(\tau) + \beta^*(\tau) \).

Since a portion of the wave packet has already tunneled through the barrier at \( \tau = 0 \), the tunneling contribution due to it should be subtracted from the tunneling probability given by (27). Thus between \( \tau = 0 \) and \( \tau = \infty \) the wave packet tunnels with the probability

\[
p = T(x \geq x_0, \tau \to \infty) - T(x \geq x_0, \tau = 0)
\]

\[
= \sqrt{\frac{a(0)}{\pi}} \int_0^{x_0} \exp[-a(0)y^2]dy - \sqrt{\frac{a(\infty)}{\pi}} \int_{x_0 - \langle x \rangle}^{\infty} \exp[-a(\infty)y^2]dy .
\]

(29)

Here \( \langle x \rangle \) at \( \tau = 0 \) is defined by \( \langle x \rangle_0 = x_0 \) and \( \langle x \rangle \) at \( \tau = \infty \) by \( \langle x \rangle_\infty \).

Another important quantity of interest is the tunneling current. The time evolution of the tunneling current

\[
j(x, \tau) = \frac{1}{2i} [\psi^* \frac{\partial \psi}{\partial x} - (\frac{\partial \psi}{\partial x})^* \psi] ,
\]

(30)

can be calculated by making use of the wavepacket(16) after normalization, together with the solution for \( \alpha(t) \) and \( \beta(t) \) in terms of Eqs(18) and (19). It is easy to see that the particle reaches the end of the tunnel at \( x = x_0 \) where it produces the following current:

\[
j(x_0, \tau) = \frac{1}{i} [\beta \alpha - (\beta^*)^* - x_0(\beta - \beta^*)] |\psi(x_0, \tau)|^2 .
\]

(31)
We would now like to discuss here the following questions:

(i) how the tunneling probability (29) depends on classical chaos due to the nonintegrability of the classical subsystem.

(ii) how long it takes a particle to tunnel through the barrier and how it depends on classical chaos.

(iii) since the present model incorporates dissipation through classical friction, it is of interest to consider how the dissipative tunneling probability and the current depend on the nature of the classical motion of the subsystem.

Let us first discuss the case of dissipation-free tunneling ($\Gamma = 0$). Fig. 2 depicts the tunneling probability for two sets of regular and chaotic trajectories, when the incident wave packet carries the average energy $\epsilon = 1$. It is evident that irrespective of the initial energy of the classical subsystem, the tunneling probability is substantially higher for the chaotic trajectories than that for the regular one. The calculation is repeated for lower incident energy (i. e. higher $\epsilon$) of the wave packet ($\epsilon = 3$) and the result is shown in Fig. 3. The difference is more marked in the region of higher values of coupling constant.

The tunneling current (31) is shown in Fig. 3 as a function of time for two trajectories (one regular and the other chaotic). For a relative comparison we have also plotted the current for the coupling-free case. We observe that a particle needs more time to tunnel through the barrier when the classical subsystem is regular.

We now turn to the case of dissipative tunneling ($\Gamma \neq 0$). In Fig. 4 we display the effect of dissipation on the tunneling probability when the classical subsystem is chaotic. A set of three chaotic trajectories corresponding to three different initial conditions ($p_s = 0.0; s = -11.31, -9.30, -8.80$) which refer to the widely different energies of the undriven oscillator were studied. Similarly the initial conditions for the set of three regular trajectories corresponding to widely seperated turning points of the undriven well ($p_s = 0.0; s = -5.02, -4.52, -4.02$) were examined. A variation of $\Gamma$ was also carried out. For the sake of brevity we have plotted here only the representative variation for one chaotic ($s = -11.31, p_s = 0.0$) and one regular ($s = -5.02, p_s = 0.0$) trajectory with and without dissipation. It is interesting to observe that dissipation increases the tunneling probability quite significantly when the classical subsystem is chaotic (solid curves) but for the regular classical subsystem (dotted curves) tunneling probability is suppressed by dissipation. Such a differential behavior of dissipative tunneling of wave packets for the chaotic and regular trajectories (which remains qualitatively the same for other sets) is also apparent in the peak height of the tunneling current (Fig. 5), although the
tunneling time does not differ too much in the two cases.

Summarizing the above numerical results we observe that (i) classical chaos of the subsystem increases the tunneling probability and decreases the tunneling time quite significantly. (ii) Dissipation enhances the tunneling when the classical subsystem is chaotic in contrast to the case when the subsystem behaves regularly.

The problem of chaos-assisted tunneling has been addressed by various workers over many years. For example, while studying the model two-dimensional autonomous systems it has been observed that energy splitting can increase dramatically with chaos of the intervening chaotic layer in which tunneling takes place between distinct, but symmetry related regular phase space regimes separated by a chaotic layer [24]. Lin and Ballentine [17] carried out a numerical study on a driven double-well oscillator to show that as the separating phase space layer grows more chaotic with increasing driving strength the tunneling rate is enhanced by orders of magnitude over the rate of the undriven system. Utermann et. al. [15] also investigated the same system to point out the role of classical chaotic diffusion as a mediator for barrier tunneling.

The enhancement of tunneling of wave packet by classical chaos in the present model of mixed quantum-classical description can be understood in the light of this classical chaotic diffusion. Instead of considering tunneling between two regions(tori) mediated by a chaotic transport between them we consider here a single tunneling process which starts at one of the turning points of the barrier. As the wave packet evolves through the barrier its mean motion by virtue of the coupling of the inverted potential with classical subsystem is affected by classical chaotic diffusive motion of the latter. While in the former cases tunneling is reversible, the process considered here is irreversible in nature.

The role of dissipation is somewhat more intriguing since one is concerned here with three interplaying aspects of evolution, e. g, tunneling, classical chaos and dissipation. The mechanism of enhancement of tunneling probability by dissipation when the subsystem is chaotic, can be understood qualitatively in the following way; It has been shown recently [19] that tunneling may get enhanced due to virtual mixing of excited states by dissipative interaction in contrast to ground state tunneling which is suppressed due to dissipation. The more relevant to the present work is an earlier observation by Ford, Lewis and O'Connell [25] who had employed a quantum Langevin equation and found an increased tunneling rate for a particle tunneling through a parabolic barrier in a black-body radiation(incoherent) field which behaves as a standard harmonic bath. It is important to realize that in the present model the classical chaotic subsystem although a few degree-of-freedom system mimics the behaviour of a typical heat bath [22] which in course of energy exchange with the quantum subsystem acquires partial quantum
character and brings in decoherence in the dynamics. The classical chaotic subsystem is thus reminiscent of the background of a black-body radiation field reservoir and the enhancement of the resulting tunneling of the wave packet through a parabolic barrier can be understood qualitatively in the spirit of Ford, Lewis and O’Connell [25].

IV. Conclusion

In this study we have considered the tunneling of a Gaussian wave packet through a potential barrier which in turn is coupled to a nonintegrable classical system. The operators pertaining to this system with mixed quantum-classical description close a partial Lie algebra with respect to the Hamiltonian operator of the system. By introducing a phenomenological classical friction one realizes a mechanism of dissipation in the overall dynamical evolution in this model without violating any quantum rule. Because of nonintegrability the classical subsystem admits of chaotic behavior. We have studied the interplay of classical chaos and dissipation in the tunneling of wave packet through the barrier and shown that chaos-assisted tunneling is further enhanced by dissipation while tunneling is suppressed by dissipation when the subsystem behaves regularly. Dissipation thus plays a significant role in the evolution of a tunneling process in presence of classical chaos.

Acknowledgements: BCB is indebted to the Council of Scientific and Industrial Research for partial financial support. DSR is thankful to the Department of Science and Technology for a research grant.
References

1. P. M. Stevenson, Phys. Rev. D30 1712(1984); D32 1389(1985).

2. A. K. Pattanayak and W. C. Schieve, Phys. Rev. A46 1821 (1992); H. Schanz and B. Esser, Phys. Rev. A55 3375 (1997).

3. P. W. Milonni, J. R. Ackerhalt and H. W. Galbraith, Phys. Rev. Letts. 50 966 (1983); J. R. Ackerhalt and P. W. Milonni, J. Opt. Soc. Am. B1 116 (1984); P. W. Milonni, J. R. Ackerhalt and H. W. Galbraith, Phys. Rev. A28 887 (1983); P. I. Belobrov, G. P. Berman and G. M. Zaslavski, JETP 49 993 (1979).

4. A. Nath and D. S. Ray, Phys. Rev. A36 431 (1987); Phys. Letts. A117 341 (1986); Phys. Letts. A116 104 (1986); G. Gangopadhyay and D. S. Ray, Phys. Rev. A40 3750 (1989).

5. A. Bulgac, Phys. Rev. Letts. 67 965 (1991).

6. R. Blumel and B. Esser, Phys. Rev. Letts. 72 3658 (1994).

7. L. L. Bonilla and F. Guinea, Phys. Letts. B271 196 (1991); Phys. Rev. A45 7718 (1992).

8. A. K. Pattanayak and W. C. Schieve, Phys. Rev. Letts 72 2855 (1994).

9. A. M. Kowalaski, A. Plastino and A. N. Proto, Phys. Rev. E52 165 (1995).

10. E. Kanai, Prog. Theo. Phys. 3 440 (1948).

11. M. D. Kostin, J. Stat. Phys. 12 145 (1975).

12. D. Greenberger, J. Math. Phys. 20 762 (1979).

13. A. O. Caldeira and A. J. Leggett, Physica 121A 587 (1983).

14. I. V. Krive and A. S. Rozhavskii, Theo. and Math. Phys. 89 1069 (1991); I. V. Krive and S. M. Latinsky, Ann. Phys. 221 204 (1993).

15. W. H. Louisell, Quantum Statistical Properies of Radiation (Wiley, NY 1973).
16. D. S. Ray, Phys. Letts. A122 479 (1987); J. Chem. Phys. 92 1145 (1990); R. D. Levine and C. E. Wulfman Chem. Phys. Letts. 60 372 (1974).

17. W. A. Lin and L. E. Ballentine, Phys. Rev. Letts. 65 2927 (1990).

18. J. Plata and J. M. Gomez Llorente J. Phys. A25 L303 (1992).

19. R. Utermann, T. Dittrich and P. Hænggi, Phys. Rev. E49 273 (1994).

20. K. Fujikawa, S. Iso, M. Sasaki and H. Suzuki, Phys. Rev. Letts. 68 1093 (1992). See the references therein.

21. A. J. Leggett and A. O. Caldeira Phys. Rev. Letts. 46 211 (1981).

22. S. Chaudhuri, G. Gangopadhyay and D. S. Ray, Phys. Rev. E52 2262 (1995).

23. S. Chaudhuri, D. Majumdar and D. S. Ray, Phys. Rev. E53 5816 (1996).

24. O. Bohigas, S. Tomsovic and D. Ullmo, Phys. Rep 223 43 (1993).

25. G. W. Ford, J. T. Lewis and R. T. O’Connell, Phys. Letts. A158 367 (1991).
Figure Captions

Fig. 1. A gaussian wave packet is shown at the left classical turning point $-x_0$, of an inverted parabolic barrier.

Fig.2. The tunneling probability ($p$) is plotted as a function of coupling strength ($\chi$), for the wave packet’s average energy $E_q = \hbar \omega_0(\epsilon = 3.0)$ for different initial positions of classical subsystem. The initial positions for three classical chaotic trajectories are (a) $s=-11.31$, (b) $s=-9.30$, (c) $s=-8.80$ and for three regular trajectories and (d) $s=-5.02$, (e) $s=-4.52$ and (f) $s=-4.02$; $p_s = 0$. (The quantities are dimensionless; scale arbitrary)

Fig.3. The tunneling current ($T$) is plotted as a function of time ($\tau$) for the wave packet’s average energy $\epsilon = 1.0$, (a) for classical chaotic subsystem, (b) for classical regular subsystem and (c) uncoupled system ($\chi$)=0.

Fig.4. The tunneling probability ($p$) is plotted as a function coupling strength ($\chi$) for the wave packet’s average energy $\epsilon = 3$ for different damping values (a) $\Gamma=2.0$ and (b) $\Gamma=0.0$ when the classical subsystem is chaotic. Similarly curves (c) $\Gamma=2.0$ and (d) $\Gamma= 0.0$ are plotted for the regular classical subsystem.

Fig.5. The tunneling current ($T$) is plotted as a function time ($\tau$) for the wave packet’s average energy $\epsilon = 3$ for different damping values (a) $\Gamma=2.0$ and (b) $\Gamma=0.0$ when the classical subsystem is chaotic. Similarly curves (c) $\Gamma=2.0$ and (d) $\Gamma= 0.0$ are plotted for the regular classical subsystem.
\[ E_q = 0 \]

\[ -E_q \]

\[ V_q(x) \]

Fig. (1)
Fig. (3)
Fig. (5)
Fig. (4)

Tunneling Probability vs Coupling strength

(a), (b), (c), (d)