Thermal lens effect model of Ti:sapphire for use in high-power laser amplifiers

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We mathematically model the thermal lens effect of Ti:sapphire for use in a high-power laser pulse amplifier. The model enables more accurate prediction with new interpretations and offers simplified equations for the optical path difference and thermally induced focal length. Our model is validated through comparisons with measurements of existing high-power laser facilities. Further, we apply the model to a 2 PW, 10 Hz Ti:sapphire laser amplifier design. © 2018 The Japan Society of Applied Physics

Ti:sapphire, which is characterized by its excellent thermal conductivity, high mechanical durability, and broad amplification bandwidth, is widely used as the medium for chirped-pulse amplification (CPA) techniques because its properties render it advantageous for generation of an ultrahigh-power laser pulse.1,2 Researchers at several advanced laboratories, for example, ELI, J-KAREN, Vulcan at the RAL, SIOM, PEARL, the Apollon project, and APRI at GIST,3–8 have already generated a few-petawatt laser pulse, and some scientists are targeting powers of tens of petawatts or higher. Laser pulses with powers above 10 PW have applications in fields such as laser material interaction, particle acceleration, and extreme scientific research, along with medical applications. Upon realization of these high-power laser applications, the exploration of a new scientific frontier will be made possible.9 For high-power lasers employed in scientific research and industrial applications, it is essential to achieve a high repetition rate,10,11 that is, the highest that can be achieved for the highest-power pulses with high spatiotemporal stability.12–14

Existing petawatt laser pulses can generate a few shots per minute; however, the target applications, including particle acceleration and radiation therapy, require a repetition rate of tens of hertz.1,15 An increase in the pulse repetition rate necessarily entails an increase in the thermal energy accumulated in the medium. That is, to generate high-power pulses with a high repetition rate, the thermal lens effect inevitably deteriorates.16–18 Consequently, research is needed to achieve a stable spatiotemporal shape and high repetition rate, in addition to merely generating petawatt output. As part of this research, the effects of thermal distortion in crystals such as Ti:sapphire are investigated. Accurately predicting the thermal distortion of the amplification medium is more important in a complex system consisting of many optical components such as high-power amplifiers. Unless the thermal distortion is compensated, it is difficult to obtain high power output with the high repetition rate, and the stability of the system cannot be guaranteed.14 Ultimately, to achieve the spatiotemporal uniformity and high repetition rate necessary for practical application, exact prediction of the thermal effect and a compensation method should first be studied.

This letter can facilitate the design and application of high-power laser systems by introducing a method for more accurate prediction of distortion effects due to accumulated thermal distortion in the medium compared to existing methods.18 The thermal lens effect is induced by a change in the refractive index caused by the temperature of the medium, surface curvature deformation caused by thermal expansion of the medium, and the photoelastic (PE) effect.18 This has already been studied extensively in laser systems using YAG host media; however, to the best of our knowledge, these phenomena have not been studied for the Ti:sapphire medium, and only the change in the refractive index caused by the temperature has been considered to cause the thermal lens effect to date.18–22 The reason seems to be the insignificance of the thermal effect, which results from both the excellent thermal stability of Ti:sapphire and the complexity of interpreting the hexagonal structure. However, to achieve higher than PW pulse powers and high repetition rates, the thermal lens effect in Ti:sapphire should not be ignored but rather seriously considered. To develop applications of high-power lasers using Ti:sapphire, it is necessary to study the properties of Ti:sapphire and the thermal lens characteristics arising from its crystal structure.14 The thermal lens expression in this study was derived in that context and was verified by comparing it with experimental results for known high-power laser amplifiers.19–21

The design of a laser amplifier is dependent on its target specifications such as the target intensity, energy, and spatiotemporal shape of the generated pulses. To generate a high power of a few tens of petawatts for a laser pulse using CPA, Ti:sapphire in the shape of a circular disk is used as the amplification medium.2,23 As a flat-top pump beam is used to obtain a spatially uniform energy distribution and a shorter pulse width, heat is uniformly generated throughout the gain medium.23 The heat generated during the laser pulse amplification process is generally discharged through the rim of the gain medium. An index-matched refrigerant encloses the rim to prevent amplified spontaneous emission. The medium is cooled by both cooling by the phase-matched refrigerant surrounding the rim and conductive cooling by surrounding air in contact with both sides of the medium.18–21 The orientation of the Ti:sapphire was assumed to be [1120], and the polarized light in the C-axis direction [Figs. 1(a) and 1(b)] with the highest amplification cross section was used.8 The thermal lens induced in the medium includes the refractive index change depending on the temperature of the medium, the curvature of the surface due to thermal expansion, and the

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PE effect.\textsuperscript{14,18} Many conventional studies considered only changes in the refractive index with temperature.\textsuperscript{19–21} In contrast, we considered all three factors to provide a more accurate model. The optical path difference (OPD) and the focal length of each element were derived, and the focal length was combined with the thin lens assumption to derive the focal length of the entire thermal lens. In these assumptions, the relationship between the heat, $P_h$, generated by the stored pump beam and the temperature, $T(x, y)$, of the medium are given by

$$\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = -\frac{P_h}{AL} + \frac{h}{L} [T(x, y) - T_{amb}],$$

where $T(x, y)$ is the temperature, $A$ is the area of the medium, $L$ is the thickness of the medium, $h$ is the thermal conductivity, and $\kappa$ is the thermal diffusivity, which is an anisotropic value.

Another point to note is that if the repetition rate is increased for practical application of ultrahigh-power lasers, the amount of heat applied to the medium increases proportionally with the repetition rate. Equation (2) is derived for a thermally steady-state condition and a rim-cooling condition; many laser amplifier models include both conditions. We changed to the polar coordinate system for convenience in the following formula:

$$T(r, \theta) = T_0 - \frac{Q r^2 \cos^2 \theta}{4 \kappa_x (1 - h R^2 / 4 \kappa_x L)} - \frac{Q r^2 \sin^2 \theta}{4 \kappa_y (1 - h R^2 / 4 \kappa_y L)}.$$ \hfill (2)

where $R$ is the radius of the medium, and $Q$ is the heat per volume, $P_h/AL$.

Note that the thermal conductivity of sapphire varies depending on its direction; it can be expected that the temperature of the sapphire medium has an elliptical slope at the center. However, it is difficult to confirm the anisotropy of the temperature in Fig. 2. The reason is that the eccentricity of the temperature distribution gradient is negligibly small, only 0.04. The OPD due to the change in refractive index ($\Delta n$) is proportional to the temperature distribution, whereas the OPDs due to thermal expansion ($\Delta \tau$) and the PE effect ($\Delta PE$) of the medium are not exactly proportional to the temperature distribution, but are influenced by the crystalline structure and its orientation and the shape of the medium.\textsuperscript{25,26} More detailed research and measurement of sapphire, which has a hexagonal structure, are needed as it is developed for high-power applications.\textsuperscript{19} The OPDs were derived as shown in Eqs. (3)–(5) using known properties, and the physical properties used are summarized in Table I.\textsuperscript{22,27}

$$\Delta (r, \theta) = \Delta T + \Delta \tau + \Delta PE = (\chi_T + \chi_\tau + \chi_{PE}) T(r, \theta)L$$

$$\chi_T = \frac{\partial n}{\partial T}$$

$$\chi_\tau = \alpha (n_e - 1) [1 + \nu - \Omega(\theta)]$$

$$\chi_{PE} = -\frac{\alpha E}{16(1 - \nu)} n_e^3 C_{PE}(\theta)$$

$$\Omega(\theta) = \frac{(1 - \nu^2) - n[1 - \nu^2 + S(\theta)E]}{(1 - n)(1 - \nu)}$$

$$S(\theta) = \frac{1}{8} [3s_{11} + 3s_{12} - 4s_{13}] \cos(2\theta)$$

$$= -\frac{1}{4} [5s_{11} + 5s_{12} + 4s_{13} + 2\sqrt{3}s_{14} \sin(2\theta)]$$

$$C_{PE} = (10\pi_{31} + 4\pi_{33}) + (-3\pi_{31} + 2\pi_{31}) \cos(2\theta)$$

Detailed properties of sapphire are summarized in Table I.\textsuperscript{22,27}
where \( \alpha \) is the thermal expansion coefficient, \( E \) is the Young’s modulus, \( \nu \) is the Poisson’s ratio of the medium, \( \Omega \) is the thermal expansion resistance of the medium, and \( C_{\text{PE}} \) is the PE coefficient. A new variable \( (\Omega) \) was introduced to express the thermal expansion of the hexagonal crystal. A similar expression for other crystal classes has been studied; however, to the best of our knowledge, this is the first research on Ti:sapphire as a hexagonal crystal. The \( \Omega \) values derived by calculating the \( z \)-axis strain, \( \varepsilon_z = S_m \sigma_m + \alpha T \),\(^1\)\(^2\)\(^6\)\(^7\)\(^8\) and \( C_{\text{PE}} \) are shown with the relative dielectric impermeability \( (B_{m}) \). The PE coefficient, \( C_{\text{PE}} \), can be written as \( C_{\text{PE}} = \frac{\Omega}{\alpha T} \Delta B_m \) and \( \Delta B_m = \frac{\rho}{m_{\text{eff}}} \)\(^{1,18,22} \).

\[
\begin{align*}
 f_x^{-1} &= f_{x,y}^{-1} + f_{x,y}^{-1} + f_{x,y}^{-1} \\
 &= \frac{\partial^2}{\partial r^2} \Delta (r, 0) \\
 &= \left( \chi + \chi_{\nu} + \chi_{\text{PE}} \right) \frac{QL}{2k_1(1 - hR^2/4k_1L)} \tag{7} \\
 f_y^{-1} &= f_{x,y}^{-1} + f_{x,y}^{-1} + f_{x,y}^{-1} \\
 &= \frac{\partial^2}{\partial r^2} \Delta (r, \pi/2) \\
 &= \left( \chi + \chi_{\nu} + \chi_{\text{PE}} \right) \frac{QL}{2k_1(1 - hR^2/4k_1L)} \tag{8}
\end{align*}
\]

The focal length equations, Eqs. (7) and (8), have been validated for the measured focal lengths of Ti:sapphire laser amplifiers from other facilities. We refer to the papers on the three facilities.\(^{19,20,21} \) Table II shows the coefficients used in Eqs. (7) and (8), and Table III summarizes the specifications, predicted focal lengths, and measured focal lengths obtained using the equations in each paper. The superscript in Table III indicates the orientation; if there is no superscript, the orientation is not indicated. The OPDs induced through the medium in each condition are shown in Fig. 3. The measured wavefronts for Cases 1 and 2 are given in the respective papers, and they agree with the expected results obtained using Eq. (3). The expected change in the focal length with the laser pumping power is shown in Fig. 3.

Case 1\(^{19} \) (Fig. 3, top) was expected to have thermally induced vertical and horizontal focal lengths of 33.9 and 32.0 m, respectively, and the measured focal length was 20 m. Under the conditions used in that study, we predicted that the vertical and horizontal focal lengths of the thermal lens would be 24.2 and 18.1 m, respectively. Although there are some differences, we can confirm that the predictions obtained from the known experimental values and the proposed method are consistent. In case 2\(^{20} \) (Fig. 3, center), the astigmatism aberration of the Ti:sapphire thermal lens was also considered. As a result, the measured thermal focal lengths in the vertical and horizontal directions, including the astigmatism, are 60.6 and 45.4 m, and the expected values are 83.0 and 78.2 m, respectively. Under these conditions, the values obtained using Eqs. (7) and (8) are expected to be 59.1 and 44.3 m. In case 3\(^{21} \) (Fig. 3, bottom), the measured thermal focal length was more similar to the conventional theory than ours is. In that study, only the refractive index change caused by temperature was considered, and the expected and measured focal lengths were 138 and 135 m, respectively. The expected vertical and horizontal focal lengths obtained using the proposed method are 104.7 and 78.4 m, which are somewhat different from the measurement values. This difference can be attributed to the difference in the cooling condition of the medium and the doping concentration of titanium ions.

The thermally induced focal lengths required in compensation design or realistic applications can be calculated using only the pump beam power and length of the Ti:sapphire crystal. Therefore, this paper is useful for researchers and developers using a high-power laser amplifier. Even if a cryogenic cooling system or highly doped crystal is used, our model can be used by applying the properties under the appropriate conditions. Because the elastic compliances and piezo-optical tensor depend slightly on the temperature and doping concentration, we have applied the specifications of Ti:sapphire with a low doping concentration.\(^{14,23} \) However, because there is no information on the elastic compliance and piezo-optical tensor for various conditions, research on the relationships between the properties and the temperature and ion doping density is urgently needed.\(^{24} \)

We present a new interpretation of the thermal lens by describing the medium using a new thermal lens expression.
Because the proposed model contains all three causes of the thermal lens effect, it also allows us to evaluate the contribution of each cause. Analysis of astigmatism aberration is a typical example. The known reason for the astigmatism of Ti:sapphire is temperature anisotropy due to the anisotropy of its thermal conductivity. However, according to Fig. 3, the astigmatism is affected by the effect of thermal expansion on the OPD, and the unique OPD shape due to the PE effect can be obtained, as shown in Fig. 4. Equation (6) gives the PE coefficient as a function of $\theta$. The signs of the coefficients are different for $\theta$ values of 0 and $\pi/2$, that is, in the horizontal and vertical directions.

The model can be applied to the design and maintenance of a petawatt-class amplifier, and the beam focusing shape can be estimated by applying the results of this study. To construct a multipass amplifier for an output power of 2 PW at 10 Hz, a 25-mm-thick Ti:sapphire crystal can be irradiated by a pump beam of 100 J with a diameter of 60 mm. For this amplifier, the expected vertical and horizontal focal lengths are 34.1 and 32.0 m, respectively, for a pump beam energy of 100 J at 10 Hz. Because we can predict the thermal lens effect more accurately, we can also design the thermal lens compensation more effectively.

Research on the optical properties of Ti:sapphire and the changes in the properties depending on the temperature or the doping density of active ions is necessary to predict the spatiotemporal shape of the temporally compressed and spatially focused PW pulse more precisely.

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