Nonlinear Vibration of Manipulator Induced by Coupling Time Delay and Control Strategy

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Abstract. This article uses the theory of Hopf bifurcation to study the stability\textsuperscript{[1-2]} of nonlinear manipulator system with time delay built by Spong. The period vibration, almost periodic vibration and chaos motion are stabilized by the controllers designed in the article, which is achieved by changed the polynomial $u$ in the dynamic of manipulator. It is proved that the controllers in this article are useful by calculating the eigenvalues of the linear form of the dynamic of manipulator, which are expressed in the complex plane, according to the stability theory\textsuperscript{[2]}.

1. Introduction
Manipulator is the integration of mechanical and electronic engineering, who gets the power to production from engine by transmutation accessories. Transmutation can lose their position accuracy and stiff, which may increases error in positioning between theory and reality. Sensors feedback the position error to the control unit. The control unit gives orders after complicated calculation. Not only the feedback procedure, but also the complicated calculation needs time. That is why there stands delay in manipulator system.

There are so many articles\textsuperscript{[3-5]} ascertain that time delay can cause the unstable motion in nonlinear dynamic system. The dynamic model of manipulator system with time delay is nonlinear. There is the possibility of unstable motion caused by time delay in manipulator. But it is still a big problem waiting for solution that how to stabilize the motion of manipulator under the influence of time delay.

Now, researchers try to find the strategic to reduce the unexpected vibration based on two different ways. The first way is to build a new controller added to the dynamic formulation of the system\textsuperscript{[8]}, listed as sliding mode control\textsuperscript{[9]} and fuzzy control\textsuperscript{[10-11]}. Another way\textsuperscript{[12]} is to adjust the stiffness of machine structure or installed some kind of damper.

However, we can see that few of articles\textsuperscript{[13-15]} control the vibration from the side of time delay. By studying the dynamic of manipulator system with time delay in this article, a new way based on designing new polynomial $u$ to stabilize the motion of nonlinear system under different parameter of time delay is advertised. Our article is going to be arranged as following: The first section talks about how to build the dynamic formulation of manipulator with time delay; the second section analyzes the stability manipulator from different parameter of time delay; the third section shows the new way to control the vibration of manipulator with time delay; the final section concludes the result of the study.

2. Dynamic formulation of manipulator with time delay
Manipulator used to be assembled with servomotor, reducer, coupling and other accessories. Servomotor gets order from control unit, and product velocity and force. Velocity and force can be
transferred by reducer so as to drive the tool against the load. Motors, reducers and tools are linked to
each other by coupling. Structure and dynamic parameter of a simple manipulator with only one
freedom on the plane is shown.

As figure 1 shows, the structure of manipulator in this article contains rotor marked by 2 and rod
marked by 1. A model of spring and damp is used to describe the complicated function of force
between rotor and rod. The stiffness of coefficient of model is assumed as \( k \). The damp coefficient
between rotor and rod is assumed as \( c_1 \), and the damp coefficient between support and rotor is
assumed as \( c_2 \). The length, mass, and dynamic moment of inertia of rod can be assumed as \( 2L \), \( M \), \( J \).

The mass and dynamic moment of inertia of rotor is assumed as \( m \) and \( I \). The rotor is driven by force
\( U(t) \). The analysis of load is shown in figure 1.

The kinetic energy of rod is expressed as
\[
T = \frac{1}{2} J \dot{\theta}_1(t)^2 + \frac{1}{2} I \dot{\theta}_2(t)^2 \tag{1}
\]

The potential energy of rod is expressed as
\[
V = \frac{1}{2} k (\theta_1(t) - \theta_2(t))^2 + MgL \cos \theta_1(t) \tag{2}
\]

The Lagrange function of rod is expressed as
\[
L = T - V = \frac{1}{2} J \dot{\theta}_1(t)^2 + \frac{1}{2} I \dot{\theta}_2(t)^2 - \frac{1}{2} k (\theta_2(t) - \theta_1(t))^2 - MgL \cos \theta_1(t) \tag{3}
\]

The preparation calculation of Lagrange equation of rod is listed as
\[
\frac{\partial L}{\partial \theta_1(t)} = J \dot{\theta}_1(t) \tag{4}
\]
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_1(t)} = J \ddot{\theta}_1(t) \tag{5}
\]
\[
\frac{\partial L}{\partial \dot{\theta}_2(t)} = k (\theta_2(t) - \theta_1(t)) + MgL \sin \theta_1(t) \tag{6}
\]

The generalized force of the dynamic system is listed as
\[
f_1 = c_1 (\dot{\theta}_2(t) - \dot{\theta}_1(t)) \tag{7}
\]
\[
f_2 = c_2 \ddot{\theta}_2(t) \tag{8}
\]
\[
f_3 = U(t) \tag{9}
\]

The Lagrange equation of rod is expressed as
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_1(t)} - \frac{\partial L}{\partial \theta_1(t)} = J \ddot{\theta}_1(t) - k (\theta_2(t) - \theta_1(t)) - MgL \sin \theta_1(t) = c_1 (\dot{\theta}_2(t) - \dot{\theta}_1(t)) \tag{10}
\]
The preparation calculation of Lagrange equation of rotor is listed as
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} = I \ddot{\theta}
\] (11)
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = I \ddot{\theta} + \frac{c_2}{I} \ddot{\theta} - U(t)
\] (12)
\[
\frac{\partial L}{\partial \theta} = -k \left( \theta_2(t) - \theta_1(t) \right)
\] (13)

The Lagrange equation of rotor is expressed as
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} = I \ddot{\theta} + \frac{c_2}{I} \ddot{\theta} - \frac{c_1}{I} \ddot{\theta} - \frac{c_2}{I} \ddot{\theta} - U(t)
\] (14)

Apply the method of normalization with equation (10) and equation (14).
\[
\frac{\partial}{\partial \theta_1} \left( \frac{\partial L}{\partial \theta_1} - \alpha_1 \left( \dot{\theta}_1(t) - \dot{\theta}_1(t-\tau) \right) - \alpha_2 \left( \theta_1(t) - \theta_1(t-\tau) \right) \right) - \beta \sin \theta_1(t) = 0
\] (15)

The coefficients of equations (15) are listed as
\[
\alpha_1 = \frac{c_1}{I}, \quad \alpha_2 = \frac{c_2}{I}, \quad \alpha_3 = \frac{k}{I}, \quad \beta = \frac{MgL}{J}, \quad \alpha_5 = \frac{c_2}{I}, \quad \alpha_4 = \frac{MgL}{J}
\] (16)

Considering the dynamic moment of inertia of rotor is much less than rod, An is added into the function. And considering the influence of time delay, a parameter is added into the variable. Then the equations is transformed into delay differential equations, which are expressed as
\[
\varepsilon^2 \ddot{\theta}_1(t) - \alpha_1 \left( \varepsilon \dot{\theta}_1(t) - \dot{\theta}_1(t-\tau) \right) - \alpha_2 \left( \theta_1(t) - \theta_1(t-\tau) \right) - \beta \sin \theta_1(t) = 0
\] (17)

This is the model described as Spong dynamic model in article [15]. According to the article [15], the coefficients of equations is listed as \( \alpha_1 = 0.1, \alpha_3 = 0.1, \beta = 1, \varepsilon = 0.9, \alpha_5 = 1, \alpha_4 = 1 \). It is really complicated to analysis the stability of dynamic formulation of manipulator with time delay. And our most important target is to find out the way to reduce the vibration of manipulator. We are going to analysis the model by numerical method on computer.

3. Stability analysis of manipulator with time delay

We analysis the stability of manipulator under the different parameter of time delay for preparation to design the polynomial \( u \), which can stabilize the manipulator motion. The diagram of history of manipulator is obtained by numerical method.

![Figure 2 Diagram of history of manipulator at \( \tau = 0.1 \)](image-url)
Figure 2-5 shows the different results from numerical method. The vertical axis in the figure is known as value of $\theta_1(t)$, and the horizontal axis is known as value of time. The manipulator is stable at
At $\tau=0.1$, the manipulation became unstable and had period vibration. At $\tau=0.17$, the manipulator had almost period vibration. The manipulation went to chaos at $\tau=0.21$. When parameter of time delay varies from 0.1 to 0.17, the manipulator will become unstable and have Hopf bifurcation. Next, numerical method is going to be used to prove that the strategy used in this article is valid.

4. The strategy of reducing vibration of manipulator

The vibration is going to be controlled by changing the form of polynomial $u$. Polynomial $u$ is assumed as combination of $\theta_1(t)$ and $\dot{\theta}_1(t)$. Then diagram of history of manipulator is obtained by numerical method on computer.

4.1. The result analysis of controller under period vibration. Polynomial $u$ is expressed as

$$u=c_1\theta_1(t)+c_2\dot{\theta}_1(t)+c_3\dot{\theta}_1^3$$

New controller is going to be designed by changing the coefficients of $u$.

Case 1

$$u=0.25\dot{\theta}_1(t)$$

Case 2

$$u=0.85\theta_1(t)+0.2\dot{\theta}_1(t)$$

Case 3

$$u=0.85\theta_1(t)+0.2\dot{\theta}_1(t)+3\dot{\theta}_1^3$$

Different case of controller is used to stabilize the manipulator. The diagram of history of manipulator under different case of controller is obtained by numerical method on computer.

Figure 6 Diagram of history of manipulator under the controller of case 1 at $\tau=0.17$
Compared with figure 3, the manipulator which had period vibration before had went to stable. But the controller in case 1 stabilized system faster than case 2 and case 3. To explain the viability of our strategy, eigenvalues of the system under different controller is going to be calculated.

Dynamical formulation (23) is complicated. We change the form of equations.

\[ \begin{align*}
\varepsilon \dot{\theta}_1(t) &= p_1(t) \\
\varepsilon p_1(t) &= q_1(t) + \beta \sin \theta_1(t) \\
\dot{p}_2(t) &= -\alpha_2 \left[ q_2(t) - p_1(t) \right] - \alpha_2 p_2(t) + u(t)
\end{align*} \] (22)

And the linear form of equations (22) is expressed as

\[ \phi(t) = \begin{bmatrix} \dot{\theta}_1(t) \\ \dot{p}_1(t) \\ \dot{\theta}_2(t) \\ \dot{p}_2(t) \end{bmatrix} = A \phi(t) + B \phi(t-	au) \] (23)

\[ \phi(t) = \begin{bmatrix} \theta_1(t) \\ p_1(t) \\ \theta_2(t) \\ p_2(t) \end{bmatrix} \] (24)

Figure 7 Diagram of history of manipulator under the controller of case 2 at \( \tau = 0.17 \)

Figure 8 Diagram of history of manipulator under the controller of case 3 at \( \tau = 0.17 \)
The eigenvalue of system is assumed as $\lambda$, and the resolution of dynamical formulation is expressed as

$$\phi(t) = g e^{\lambda t}$$  (27)

$$g = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix}$$  (28)

Equation (27) is substituted into the equation (21). And new equation is obtained by eliminating $g$.

$$[\lambda^2 - \lambda(c_4 - \alpha_2 \cdot c_1) + 1 - c_3] \left( \varepsilon^2 + \alpha_1 \lambda e^{\lambda t} + \frac{\beta + e^{\lambda t}}{\varepsilon} \right) = \left( \frac{1}{\varepsilon} + \alpha_1 \lambda \right) \left[ c_1 + \varepsilon c_2 \lambda + (1 + \varepsilon \alpha_2 \lambda) e^{\lambda t} \right]$$  (29)

Because equations (29) cannot be solved by analytic way, numerical method is used to obtain the approximate solution. A equation is substituted into the equation (29).

$$e^{\lambda t} = 1 - \lambda t$$  (30)

Then the approximate solutions are shown in the figure follow.

Figure 9 the position of eigenvalues in the complex plane in case 1 at $r=0.17$
From the figure 9-11, all real component of eigenvalues of manipulator under control is on the left side of vertical axis. That explains all manipulator under control is stable.

4.2. The result analysis of controller under almost period vibration. The controllers used in this situation are the same as section 3.1, and the diagrams of history of manipulator under different controllers are shown as following.
Compared with figure 4, the manipulator which had period vibration before had went to stable. But the controller in case 1 stabilized system faster than case 2 and case 3. To explain the viability of our strategy, eigenvalues of the system under different controller is going to be calculated. By changed the
coefficients of the equation (29), the eigenvalues equation in this situation can be calculated. And the eigenvalues of the manipulator in this situation are obtained as the same way as section 3.1, which are shown as following.

Figure 15 the position of eigenvalues in the complex plane in case 1 at $\tau=0.19$

Figure 16 the position of eigenvalues in the complex plane in case 2 at $\tau=0.19$

Figure 17 the position of eigenvalues in the complex plane in case 3 at $\tau=0.19$
From the figure 15-17, all real component of eigenvalues of manipulator under control is on the left side of vertical axis. That explains all manipulator under control is stable.

4.3. The result analysis of controller under almost chaos. The controllers used in this situation are the same as section 3.1, and the diagrams of history of manipulator under different controllers are shown as following.

Figure 18 Diagram of history of manipulator under the controller of case 1 at $\tau=0.21$

Figure 19 Diagram of history of manipulator under the controller of case 2 at $\tau=0.21$

Figure 20 Diagram of history of manipulator under the controller of case 3 at $\tau=0.21$
Compared with figure 5, the manipulator which had period vibration before had went to stable. But the controller in case 1 stabilized system faster than case 2 and case 3. To explain the viability of our strategy, eigenvalues of the system under different controller is going to be calculated. By changed the coefficients of the equation (29), the characteristic equation in this situation can be calculated. And the eigenvalues of the manipulator in this situation are obtained as the same way as section 3.1, which are shown as following.

![Figure 21](image1.png)  
Figure 21 the position of eigenvalues in the complex plane in case 1 at $\tau=0.21$

![Figure 22](image2.png)  
Figure 22 the position of eigenvalues in the complex plane in case 2 at $\tau=0.21$
Figure 23 the position of eigenvalues in the complex plane in case 3 at $\tau=0.21$

From the figure 21-23, all real component of eigenvalues of manipulator under control is on the left side of vertical axis. That explains all manipulator under control is stable.

5. Conclusion
The dynamical formulation of manipulator with time delay built by Spong is studied in this article. And the system is stable under the controller obtained by changing the form of polynomial $u$. From study, we have several consequence as following. First, time delay can cause chaos in the response of manipulator. Second, adding controller into the dynamic formulation can reduce the vibration of manipulator with time delay. Third, all controller can stabilize the manipulator described in assay, but different controller functions differently. If one wants to reduce the vibration of the manipulator quickly, the controller of case 1 should be considered. If one wants to reduce the vibration of manipulator in a reasonable speed, case 2 and case 3 may be used.

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