Critical behavior of three-dimensional frustrated antiferromagnets with a collinear spin ordering and with an additional twofold degeneracy of the ground state is studied. We consider two lattice models, whose continuous limit describes a single phase transition with a symmetry class differing from the class of non-frustrated magnets as well as from the classes of magnets with non-collinear spin ordering. A symmetry breaking is described by a pair of independent order parameters, which are similar to order parameters of the Ising and O(N) models correspondingly. Using the renormalization group method, it is shown that a transition is of first order for non-Ising spins. For Ising spins, a second order phase transition from the universality class of the O(2) model may be observed.

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I. INTRODUCTION

A complicated tensor structure of an order parameter differing from the case of the usual vector O(N) model is realized in many physical systems. During the last several decades, such models are investigated intensively (see [1] for a review). In context of magnetic systems, interesting models are the matrix $O(N)$-vector cubic models with O(N)-symmetric couplings, and the $O(N_1) \oplus O(N_2)$ model comprises two interacting vector models. The last model describes a multicritical point and an intersection (or junction) of two critical lines corresponding to different vector order parameters. Such a multicritical point arises in antiferromagnets in an external field [2, 3], in the $O(5)$ theory of high temperature superconductors [4] etc. The critical behavior at the multicritical point can be studied by the general Ginzburg-Landau-Wilson (GLW) functional

$$F = \int d^3 x \left( (\partial_\mu \phi)^2 + (\partial_\mu \psi)^2 + r(\phi^2 + \psi^2) + u_1 \phi^4 + u_2 \psi^4 + 2w \phi^2 \psi^2 \right), \quad (1)$$

where $\phi$ and $\psi$ are N-component vector field. This model has been investigated within the $4 - \epsilon$ expansion [5, 6], the perturbative renormalization group (RG) in $d = 3$ [7], and also using the non-perturbative RG (NPRG) [7]. In addition, the NPRG approach has been used in [8] to study the case $N_1 = N_2 = 1$ of the model [11], with $O(1) \equiv \mathbb{Z}_2$ corresponds to a symmetry of the Ising model. The last case is of special interest in statistical physics in the context of the Ashkin-Teller model [9] and its critical line corresponding to a singular transition in the both Ising order parameters. In three dimensions, the Ashkin-Teller model has been studied numerically in [10]. Intensive numerical studies have been also performed in recent works [11] for the multicritical point of a $N = 3$ antiferromagnet in a magnetic field. This point is described by the $\mathbb{Z}_2 \oplus O(2)$ model.

In the present work, we consider another symmetry breaking scenario realized in certain models of frustrated antiferromagnets [12]. Two such models have been considered for quantum spins in [13, 14], and numerically for classical spins in [15]. The first model is a ferromagnet on a body-centered cubic lattice with the additional antiferromagnetic exchange interaction between next-nearest neighbor spins, and the second one is a ferromagnet on a simple cubic lattice with the additional interaction in layers. We are mainly interested in the continuum limit of these lattice models, their thermal and critical behavior with a $\mathbb{Z}_2 \oplus O(N)$ structure of the order parameter. The proposed model describes a single phase transition with breaking of $\mathbb{Z}_2 \oplus SO(N)/SO(N-1)$ symmetry.

In the particular case $N = 2$ corresponding to XY spins, $\mathbb{Z}_2 \oplus SO(2)$ symmetry is broken. Noteworthy, the same symmetry is broken in XY antiferromagnets with a planar ordering, such as helimagnets (see [10] and references therein). The static critical phenomena in such systems are described by the GLW functional

$$F = \int d^3 x \left( (\partial_\mu \phi)^2 + (\partial_\mu \psi)^2 + r(\phi^2 + \psi^2) + u (\phi^2 + \psi^2)^2 + 2w ((\phi \psi)^2 - \phi^2 \psi^2) \right), \quad (2)$$

with $w > 0$. This model (called as $O(N) \otimes O(2)$ model) has been also discussed extensively in recent decades in the context of a canted and sinusoidal phases of magnets [17] as well as superfluid $^3$He [18], some types of superconductors [19], Josephson junction arrays in a magnetic field at zero temperature [20] etc. Critical phenomena related to this model are investigated in the framework of several approaches as the $4 - \epsilon$ expansion [21], perturbative [22], and non-perturbative RG [24, 25], $1/N$ expansion [26], and numerical studies of corresponding lattice models. For a review including numerical and experimental results, see [26].

Starting from the lattice models and acting in a stan-
standard manner, we obtain the GLW functional (section II). This functional contains three quartic coupling constants and includes models (1) and (2) as particular cases. This fact is very useful because it allows to use the known solutions for models (1) and (2). Possible scenarios of symmetry breaking in the \( \mathbb{Z}_2 \otimes O(N) \) models as well as cases where the symmetry of the functional enlarges are discussed in section III. And in section IV, the RG analysis of this model is performed. It turns out that a significant part of information about fixed points and critical behavior at them is reduced to the solution of models (1) and (2). Although new additional non-trivial fixed points are present in \( \mathbb{Z}_2 \otimes O(N) \) model, they do not respond to \( \mathbb{Z}_2 \otimes SO(N)/SO(N-1) \) symmetry breaking. So we find that a phase transition from the Ising-Heisenberg universality class is of first order for \( N \geq 2 \). The exception is the case \( N = 1 \) corresponding to Ising spins. In this case, a transition is of the second order from the \( O(2) \) universality class.

II. LATTICE MODELS AND CONTINUUM LIMIT

A scenario of \( \mathbb{Z}_2 \otimes SO(N)/SO(N-1) \) symmetry breaking is realized in frustrated antiferromagnets with a collinear spin ordering and twofold degeneracy of the ground state. Among three-dimensional models, such a structure of the ground state is present in an antiferromagnet on a body-centered cubic lattice with the Hamiltonian

\[
H = J_1 \sum_{ij} S_i S_j + J_2 \sum_{kl} S_k S_l, \tag{3}
\]

where the sum \( ij \) runs over pairs of nearest-neighbor spins, and the sum \( kl \) runs over pairs of next-nearest-neighbor spins. A spin \( S \) is a classical unit vector, \( J_1, J_2 > 0 \). At \( J_2 < 2J_1/3 \), the ground state is two embedded to each other ferromagnetic sublattices interacting antiferromagnetically, without a frustration. But at \( J_2 > 2J_1/3 \), sublattices become antiferromagnetic, and the ground state acquires the desired structure \([27]\).

Another model, discussed also in two dimensions \([28]\), is the stacked two-exchange model (stacked-J \( 1 \)-J \( 2 \) model) on a simple cubic lattice with the Hamiltonian

\[
H = -J_1 \sum_{ij} S_i S_j + J_2 \sum_{kl} S_k S_l, \tag{4}
\]

where the sum \( ij \) runs over pairs of nearest-neighbor spins, and the sum \( kl \) enumerates pairs of next-nearest-neighbor spins in layers (see fig. 1). At \( J_2 < J_1/2 \), the ground state is the ferromagnetic order. At \( J_2 > J_1/2 \), the ground state is one of two spin configurations with the wave-vectors \( q \) = (\( \pi,0,0 \)) or \( q \) = (\( 0,\pi,0 \)) (see fig. 2). This model is convenient and more expository for the derivation of the GLW functional and the continuous limit of these lattice models.

![Figure 1](image1.png)

**Figure 1.** Two non-equivalent ground states of the stacked-J \( 1 \)-J \( 2 \) model on a simple cubic lattice, which cannot be reduced to each other through global spin rotations.

![Figure 2](image2.png)

**Figure 2.** Energy of the stacked-J \( 1 \)-J \( 2 \) model as function of wave-vector components. The dark regions correspond to two non-equivalent minima of the Hamiltonian (4).

Using the Hubbard-Stratonovich transformation \([29]\) in a standard manner, we obtain an equivalent model without the constrained field \( S \), but with an additional potential and coupling constants defining the length of the new field \( \varphi \). For the GLW-approach, it is reasonable to hold just up to quartic terms in field of this additional potential \( U(|\varphi|) = m\varphi^2 + \lambda\varphi^4 \). Further, to obtain the continuum limit of the lattice model, one should expand the expression in the vicinity of both minimum \( (\pi,0,0) \) and another minimum \( (0,\pi,0) \). We introduce the fields

\[
\begin{align*}
\phi &= \varphi|_{q=(\pi,0,0)} + \varphi|_{q=(0,\pi,0)}, \\
\psi &= \varphi|_{q=(\pi,0,0)} - \varphi|_{q=(0,\pi,0)},
\end{align*}
\]

so that its parallelism corresponds to the minimum \( (\pi,0,0) \), and another minimum \( (0,\pi,0) \) corresponds to antiparallel fields. Finally, we obtain the GLW functional corresponding to the starting lattice models

\[
F = \int d^3x \left( (\partial_\mu \phi)^2 + (\partial_\mu \psi)^2 + \nu(\phi^2 + \psi^2) + u(\phi^4 + \psi^4) + 2w\phi^2\psi^2 + 2w(\phi\psi)^2 \right), \tag{6}
\]

with some positive constants \( u \) and \( v \), and negative \( w \).
III. SYMMETRY AND MEAN-FIELD ANALYSIS

The ground state of the model strongly depends on a sign of the coupling constant $w$. When $w < 0$, the vectors $\phi$ and $\psi$ tend to be parallel. This is the case corresponding to the lattice models discussed in the previous section. The case $w = 0$ returns us to the model with $u_1 = u_2$, and $w > 0$ describes magnetic systems with a planar spin ordering. The last case has been also considered in the more general model in \[30\].

A. $w < 0$

The extremum of the free energy functional in the ordered phase ($r < 0$) attains on a homogeneous configuration satisfying to the conditions

$$\phi_0^2 = \psi_0^2 = -r \frac{2(u + v + w)}{2(u + v + w)} \equiv \kappa^2, \quad \phi_0 \parallel \psi_0.$$  

This extremum is the global minimum in the stability region

$$u > 0, \quad w < 0, \quad u + v + w > 0, \quad u - v - w > 0.$$  

For symmetry analysis, it is convenient to represent the order parameter as a $2 \times N$ matrix $\Phi = \{\phi, \psi\}$. The functional is invariant under the left action of orthogonal matrices on the order parameter $\Phi \to T\Phi$, where $T \in O(N)$. Also, it is invariant under the right action of $2 \times 2$ orthogonal matrices corresponding to the three discrete $Z_2$ symmetry generators

$$\left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \quad \left( \begin{array}{cc} -1 & 0 \\ 0 & -1 \end{array} \right), \quad \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right),$$  

and their combinations. One has just eight such matrices, including the unit, and they are elements of the group $Z_2 \otimes Z_2 \otimes Z_2_R \subset O(2)_R$. The first subgroup replaces the vectors $\phi$ and $\psi$ between themselves $\phi \leftrightarrow \psi$. The second and third ones change a direction of two or one vectors to an opposite $\phi \to -\phi$ and/or $\psi \to -\psi$. Using all of these right as well as left acting symmetries, one can read the ground state as $\phi_0 = \psi_0 = (\kappa, 0, \ldots, 0)$. The spontaneously broken symmetry is

$$O(N)_L \otimes (Z_2 \otimes Z_2 \otimes Z_2)_R \cong \frac{SO(N)}{SO(N - 1)_L \otimes (Z_2)_R \otimes (Z_2)_D} \otimes Z_2,$$  

for $N \geq 2$, and $Z_2 \otimes Z_2$ for $N = 1$, that is equivalent to the case $w = 0$. Among the discrete subgroups, the third one (relating to a change in the sign of one vector) is only spontaneously broken. The second subgroup breaking is compensated by rotations $O(N)_L$, that leads to the appearance of the unbroken diagonal group $(Z_2)_D$.

If one considers a weakly fluctuating configuration in the form $\phi(x) = \phi_0 + \alpha(x) + \beta(x)$, $\psi(x) = \psi_0 + \alpha(x) - \beta(x)$, then one finds the following mass spectrum of excitations

$$m_{\alpha_i} = 8\kappa^2(u + v + w), \quad m_{\beta_i} = 8\kappa^2(u - v - w), \quad m_{\alpha} = 0, \quad m_{\beta} = -8\kappa^2w,$$  

with $i = 2, \ldots, N$. The $N - 1$ massless modes are Goldstone modes corresponding to the breaking of the continuous $SO(N)/SO(N - 1)$ symmetry.

The submanifold $v = u - w$ is special. It corresponds to the sinusoidal phase of the model considered in \[17\]. Wherein, a length of the vectors $\phi$ and $\psi$ remains undefined, but the sum of their length square is determine from the minimum conditions. The mode $\beta_1$ becoming massless corresponds to the continuous symmetry associated with $SO(2)\beta_1$ rotations of the 2-vector $\langle |\phi|, |\psi| \rangle$. Also, when $v = u - w$ and $|\phi| = |\psi|$, the group $(Z_2 \otimes Z_2 \otimes Z_2)_R$ enlarges to $O(2)_R$. But this enlargement does not affect on the ground state degeneracy.

B. $w = 0$

In this case, the minimum conditions does not determine the relative orientation of the vectors $\phi$ and $\psi$. Therefore, the broken symmetry is

$$\frac{SO(N)}{SO(N - 1)} \oplus \frac{SO(N)}{SO(N - 1)}; \quad N \geq 2,$$  

and $Z_2 \oplus Z_2 \cong Z_2 \otimes Z_2$ for $N = 1$. There is $2N - 2$ massless modes $\alpha_i$ and $\beta_i$ (see \[11\]) in this case. When $u = v$, the symmetry enlarges to $O(2N)$ group with $2N - 1$ Goldstone modes.

The symmetry is broken only when a multicritical point is tetracritical $u > |v|$ \[31\]. A bicritical point $u < v$ describes symmetry breaking in only one order parameter, and another parameter remains zero ($u < -v$ is out of the stability region).

C. $w > 0$

A planar (canting) ordering appears in this case

$$\phi_0^2 = \psi_0^2 = -r \frac{2(u + v)}{2(u + v)} \equiv \kappa^2, \quad \phi_0 \perp \psi_0,$$  

with the stability region

$$u > 0, \quad w > 0, \quad u + v > 0, \quad u - v > 0.$$  

Using the symmetry of the GLW functional, the ground state can take the form $\phi_0 = (\kappa, 0, \ldots, 0)$ and $\psi_0 = (0, \kappa, 0, \ldots, 0)$. The group, right acting to the order parameter $\Phi$, is compensated entirely by $O(N)_L$ rotations. Thus the spontaneously broken symmetry is

$$\frac{O(N)_L \otimes (Z_2 \otimes Z_2 \otimes Z_2)_R}{O(N - 2)_L \otimes (Z_2 \otimes Z_2 \otimes Z_2)_D} \cong \frac{SO(N)}{SO(N - 2)}.$$  

for \( N \geq 3 \), and \( SO(2) \otimes \mathbb{Z}_2 \) for \( N = 2 \).

It is useful to choose a weak fluctuating field configuration in the form
\[
\phi(x) = \phi_0 + \alpha(x) + \beta(x),
\]
\( \psi(x) = \psi_0 + \tilde{\alpha}(x) - \tilde{\beta}(x) \), where \( \tilde{\alpha} = (\alpha_2, \alpha_3, \ldots, \alpha_N) \) and \( \tilde{\beta} = (\beta_2, \beta_1, \beta_3, \ldots, \beta_N) \). Now, mass spectrum of excitations is
\[
m_{\alpha_i} = 8\kappa^2(u + v), \quad m_{\alpha_2} = 8\kappa^2 w, \quad m_{\alpha_i} = 0, \\
m_{\beta_1} = 8\kappa^2(u - v), \quad m_{\beta_2} = 0, \quad m_{\beta_i} = 0,
\]
where \( i \geq 3 \). Thus, we have \( 2N - 3 \) Goldstone modes. At \( v = u - w \), the group \( (\mathbb{Z}_2 \otimes \mathbb{Z}_2 \otimes \mathbb{Z}_2)_R \) enlarges to \( O(2)_R \) again, but the spontaneously broken symmetry remains the same.

The submanifold \( v = u \) is special, when a length of the vectors \( \phi \) and \( \psi \) is undefined as in the case \( w < 0 \). The symmetry \( SO(2)_{\beta_i} \) becomes broken spontaneously but not evidently. The additional Goldstone mode \( \beta_1 \) in \( H \) corresponds to the sliding degree of freedom of the spin-density wave. Similarly to the case \( w < 0 \), this special submanifold demarcates the region of the ground state stability with the region where the minimum corresponds to only one non-zero order parameter.

### IV. RG ANALYSIS

In the one-loop approximation, beta-functions of the coupling constants are following
\[
\beta_u = -\epsilon u + \frac{1}{2} \left( u^2 (N + 8) + v^2 N + 2uv + w^2 \right), \\
\beta_v = -\epsilon v + \frac{1}{2} \left( uv (2N + 4) + 4v^2 + 2uv + w^2 \right), \\
\beta_w = -\epsilon w + \frac{1}{2} \left( w^2 (N + 2) + 4uw + 8vw \right).
\]

This system of equations predicts existence of eight fixed points (FP). Six of them turn to be well-known in the context of the models \( 1 \) and \( 2 \). Each of them belongs to at least one of three submanifolds \( w = 0 \) (the model \( 1 \)), \( v = u - w \) (the model \( 2 \)), and \( v = u \). The last flat may describe some physically interesting model, but its interpretation is unknown for the author.

- **Gaussian FP.**
- **Heisenberg FP.** This point lays on the line \( u = v, w = 0 \), and corresponds to the \( O(2N) \)-model. Together with the GFP, it belongs to the all of three submanifolds.
- **Decoupled FP.** It lays on the line \( v = w = 0 \) and describes two decoupled \( O(N) \)-models. This multicritical point is always tetracritical.
- **Biconical FP.** It is non-trivial point on the submanifold \( w = 0 \). Depending on \( N \), it can be tetracritical as well as bicritical. It describes two interacting \( O(N) \)-models. The submanifold \( w = 0 \) is stable.
- **Chiral and antichiral FPs.** These points belong to the submanifold \( v = u - w \). They appear on the RG-diagram when \( N \) is sufficient small in the sinusoidal phase. In this case they are marked as \( S_{\pm} \). With \( N \) increasing, they coincide at some \( N_{c1} \) and become complex. With a further increase of \( N \), these points appear again at some \( N_{c2} \) and are marked as \( C_{\pm} \). The chiral point \( C_+ \) describes a phase transition in the \( O(N) \otimes O(2) \)-model.
that the submanifold $v = u - w$ is not fixed for the RG-equations. Nevertheless, this pair of the FPs belongs just to this submanifold for all values of $N$.

- **Other FP $P_{1,2}$.** These points belong to the stable submanifold $u = v$.

Certainly, a position and stability of the FPs strongly depend on $N$. Qualitative diagram showing the position of the FPs in the physically interesting case $N = 1$ is shown in fig. 3. Below, we consider evolution of RG-diagram with increasing of $N$. Course, the exact critical values of $N$, when a qualitative picture changes, require knowledge of higher orders in the $\epsilon$-expansion and resummation of the series. Fortunately, such a information obtained using different approaches is known for the models (1) and (2). In addition, properties of the novel points $P_{1,2}$ as a function of $N$ is closely related to the properties of already studied points.

We find four critical values of $N$ associated with a coincidence of two or more FPs. All of these critical values of $N$ appear in the models (1) and/or (2).

1. $N < N_H$.

One observes the stable fixed point is Heisenberg FP (fig. 3). This point is also stable in all of three particular two-charge models $w = 0$, $v = u - w$...
and \( v = u \) (fig. 4). There are two FPs \( S_- \) and \( P_1 \) in the region \( w < 0 \), but both are unstable. In the one-loop approximation \( N_H = 2 \), but higher orders predict the value \( N_H \approx 1.45 \) \cite{21, 22}. Also this value is obtained as the marginal dimension of the \( O(N) \) model with a single-ion cubic anisotropy. At \( N = N_H \), HFP, BFP, the points \( S_+ \) and \( P_1 \) coincide.

2. \( N_H < N < N_{c1} \).

There are no stable FPs in this case. Above the value \( N_H \), the points \( S_+ \) and \( P_1 \) change the sign of their \( w \)-coordinate (figs. 5, 6). They are stable in the two-charge models \( v = u - w \) and \( v = u \) correspondingly. The point \( S_+ \) lays in the region \( w < 0 \). BFP becomes tetracritical and stable in the model \( w = 0 \). When \( N \) reaches to the value \( N_{c1} \), the points \( S_+ \) and \( S_- \) coincide and become complex. In the one-loop approximation, \( N_{c1} \approx 2.20 \), and \( N_{c1} \approx 1.97 \) in higher orders in \( \epsilon \) \cite{21}.

3. \( N_{c1} < N < N_{c2} \).

As long as two points become complex-valued, just six fixed points are presented in the RG-diagram, but a stable FP absents again (fig. 7), as well as FPs absent in the interesting region \( w < 0 \). For the models \( w = 0 \) and \( u = v \), the value \( N = N_{c1} \) is not critical, and one observes the same picture as in the previous item (fig. 8). At \( N = N_D \), two events of coincidence occur, the point \( P_1 \) coincides with \( P_2 \), and DFP with BFP. \( N_D = 4 \) in the one-loop approximation, and \( N_D \approx 2 \) in higher orders.

4. \( N_D < N < N_{c2} \).

Still one observes six FPs (fig. 9). In the models \( w = 0 \) and \( u = v \), the points DFP and \( P_2 \) become stable. With further increasing of \( N \), the qualitative picture in these models remains similar with-
out significant changes. This picture is shown in fig. 11.

5. \( N_{c2} < N \).

At \( N = N_{c2} \), the points \( C_+ \) and \( C_- \) appear in the region \( w > 0 \) of the RG-diagram (figs. 11, 12). The first of them is stable. It describes a phase transition in the \( O(N) \otimes O(2) \) model. The value \( N_{c2} \approx 6 \) [21, 22, 29] (the one-loop result is \( N_{c2} \approx 21.8 \)). Monte Carlo simulations predict a second order transition in the \( O(N) \otimes O(2) \) model for \( N = 6 \) [22].

Summarizing, we note that a stable fixed point is present in the RG-diagram at \( N < N_H \) and \( N > N_{c2} \) located in the region \( w \geq 0 \). Thus, a phase transition from the \( \mathbb{Z}_2 \otimes O(N) \) universality class is of first order for all physically interesting values of \( N \).

V. CONCLUSION

We performed RG-analysis of the \( \mathbb{Z}_2 \otimes O(N) \) model describing in particular the critical behavior in the class of frustrated antiferromagnets with a collinear spin ordering and an additional twofold degeneracy of the ground state. In the case \( N = 1 \) interesting also in the context of the Ashkin-Teller model, one expects that a phase transition with the \( \mathbb{Z}_2 \otimes \mathbb{Z}_2 \) symmetry breaking is of second order from the universality class of the \( O(2) \) model or of first order dependently on initial values of the coupling constants. In addition, crossover critical exponents may be observed, associated with the fixed point \( P_2 \) belonging to a submanifold, which separates these two types of the critical behavior.

For \( N \geq 2 \), a first order transition is predicted for the \( \mathbb{Z}_2 \otimes O(N) \) universality class. At \( N = 2 \), this class is equivalent to the symmetry class of the \( O(N) \otimes O(2) \) model corresponding to magnets with a planar spin ordering, where \( \mathbb{Z}_2 \otimes SO(2) \) symmetry is broken. In this class, a transition must be of first order [21, 24, 25]. At the same time, one discusses a possibility that this transition is of weak first order or almost second order. This is intended to explain the pseudo-scaling and pseudo-universality observed for this symmetry class (see [26] for a review). In terms of the renormalization group, an imitation of a second order transition is possible, if the RG diagram contains a sufficient small region attractive for RG trajectories starting from a quite wide range of initial parameters, and where the RG-flow is rather slow. The existence of such a region in the \( O(2) \otimes O(2) \) model has been studied in works [24, 25].

Of cause, this region has \( w > 0 \). An almost second order transition is possible in the \( \mathbb{Z}_2 \otimes O(N) \) model. But here, a region of slow RG-flow must be in the region \( w < 0 \), so one expects that critical (pseudo-)exponents is different from the observed indices in the \( O(2) \otimes O(2) \) model. It should be noted that in \( N = 2 \) case a region of the slow RG-flow may be away from the submanifold \( v = u - w \) corresponding to the \( O(2) \otimes O(2) \) model. For \( N = 3 \) case, a local minimum of the RG-flow has been found, associated with a smallness of the imaginary part of the fixed points \( C_\pm \) coordinates, the real part of which belongs to the submanifold \( v = u - w \) [24, 27]. One can expect that the position of this RG-flow minimum remains the same in the general three-charge model [6] with \( w > 0 \). However, in \( N = 2 \) case, a non-trivial local minimum of the RG-flow is not found. And therefore, one should search a slowdown region in the more general model but not only in the submanifold \( v = u - w \).

The RG-flow geometry in the region \( w < 0 \) is such that a slowdown region is possible while \( |w| \) is sufficient small (see fig. 12), at least for not large values of \( N \). For large \( N \), we expect that a phase transition is of distinct first order.

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