Exploring reversible thinking of preservice mathematics teacher students through problem-solving task in algebra

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Abstract. Thinking reversible supports mathematical reasoning and flexibility of thinking. It affects the success of students in solving mathematical problems. Eventually, mathematical problem solving is the core of mathematical learning. Therefore, the teacher must have thinking reversible skills to develop students' thinking reversible optimally. This study aims to explore the reversible thinking of preservice mathematics teacher students. The design of this research is qualitative and incorporating the test method. The test is in the form of an algebra solving problem which contains an algebra equation. Students are assigned to create a new equation, which is equivalent to the equation in the test as many as possible to identify the aspect of students' forwarding process. Then, students are assigned to reverse the new equation to the initial equation form. This is to identify the aspect of the reversing process. The forwarding process and reserving process is the core of reversible thinking. Therefore, the result of the student task is analyzed based on these characteristics. The result showed that there are four different strategies implemented by the students to create a new equation and reversing the new equation to the previous equation. It is elaborated in detail in the result of this article.

1. Introduction
Thinking reversible was developed at first by Piaget [1]. Then, thinking reversible was observed and linked with mathematic education by Krutetskii [2], which was then followed by an Education observer. This is aimed at investigating deeper about thinking reversible in Mathematics education. Why does thinking reversible must be observed and developed?

One of the main reasons for the importance of thinking reversible among teacher and student is it is needed in problem-solving [2][3][4][5][6][7][8][9][10]. Furthermore, problem-solving is the core of mathematical learning. Problem-solving is the soul of mathematical learning [5][6]. Problem-solving is a part that cannot be separated from all Mathematics learning [11]. The mathematical solving problem must be placed as central competence, which is appropriate with the level of the students, and it can be increased in terms of the complexity [12]. Students' competence in solving a mathematical problem is one of the purposes of Mathematics Education [13].

Therefore, to optimize the students' competence in solving problems, it is supported by thinking reversible skills. Flanders [14] also stated that by thinking reversible, students are obliged to have balance reasoning from two opposite directions so that it can strengthen the scheme of student knowledge.
Maf'ulah et al. [14] believed that thinking reversible can minimize fallacy in making decisions. Through thinking reversible, someone is prosecuted to think twice based on two opposite points of view. It means that thinking reversible can minimize student mistakes in solving a mathematical problem. So, thinking about the reversible of students should be developed. As stated by Maf'ulah et al. [5] and Maf'ulah et al. [14] that thinking reversible can minimize mistakes among the students' answers due to the thinking reversible process, the student can recheck the answers that they got from the base problem.

Students' reversible thinking skills can be elevated through learning math, and this is part of the teacher's responsibility. If the teacher is responsible for developing students' reversible thinking skills, the teacher's reversible thinking skills should also be maximized to elevate student's thinking reversibly. Therefore, the subject is preservice mathematics teacher students. If the reversible thinking skill of the preservice teacher is being a concern, so it is expected that they would be able to expand their students' reversible thinking skills in solving a mathematical problem when they have become teachers.

The purpose of this research is to explore the reversible thinking of math preservice through the problem-solving task in algebra. The solving problem task contains algebra, which must be solved by the Math preservice teacher. Through the result of this research, the researcher can identify the basic image of thinking reversible of preservice mathematics teacher-student. Then, the researcher, as a lecture in the Mathematics Education Department, will think about how to develop their thinking reversibly.

Thinking reversible is a mental activity which prosecuted someone to think logically from two opposite direction. Piaget stated that thinking reversible is the mental ability of someone to change his/her thinking directly to the previous position [15]. Thinking reversible is a mental activity that involves the process of building two-direction relations, which are reversible from two spots [1][2].

What is meant by two spots is two concept spots, for example, reversible relation between derivative and integral. The two spots can also mean beginning condition and final purpose. For example, in solving the Mathematics problem, the mathematics problem becomes the beginning condition, and the answer to the problem becomes the final purpose that is achieved. The reversible process that happens is a process of the students to obtain the answer. After obtaining the answer, students look back by reversing the answer that they have found to the beginning problem to clarify that the answer is correct. This is in line with Kang & Lee [16] and Krutetskii [2], who stated that thinking reversible is a mental activity of a person to revers beginning data after obtaining the result.

Implicitly, in phasing problem solving that is stated by Polya [17], he involves thinking about the reversible element that is at the end of the phase. The last phase is looking back. When looking back, someone must use thinking reversible. Inhelder & Piaget [1] stated that there are two forms, those are negation form and reciprocity form. Negation form involves understanding that one direction movement can be denied by the opposite direction movement. Reciprocity form is related to equivalent relations. Hackenberg [18] also elaborated two kinds of thinking reversible, those are inversion and compensation. Heckenberg said that inversion involves the cancellation of mental action to reverse the situation to beginning statement while compensation involves the appearance of an action that obtains an equivalent situation with the beginning situation. Hence, thinking reversible can be done in two ways, those are (1) mental activity of reversing the thinking process by involving inverse which is called inversion and (2) mental activity of reversing the thinking process by equivalent relations, for instance, given scale with two sides in balance situation. Then, one side of the scale is loaded, which then becomes heavier. To be able to revers to balance position, it can be done in two ways. The first is by taking back the load on one side of the scale. This way is called inversion because involving opposite movements, those are loading and taking back the load so that it is balanced. The second is loading the same weight on another side of the scale so that the two sides become balance. This one is called reciprocity because involving the equivalency aspect. The equivalency aspect which is obtained is both sides of the scale that are in a balance situation without any load are equivalent to the scale that both sides of which are loaded the same weight so that both sides are balanced.

Thinking reversible is the ability to reverse the direction of thinking to the beginning position after doing mental operation [2][16][19]. Maf'ulah et al. [5] elaborated that there are two essential aspects in thinking reversible, those are forward and reverse. The forward aspect is the directing aspect, which
means mental process from the beginning condition after reaching the destination. So that looking back or re-observe the final result obtained by reversing or linking back the final result with the beginning problem is also thinking reversible.

Furthermore, to explore thinking reversible, researchers give tasks to preservice mathematics teacher-student. The task contains algebra problems. Algebra problem contains an equation, then the students are asked to create an equation that is equivalent to the equation in the task as many as possible. The subject's work is analyzed by a description of thinking reversible aspect as described in table 1.

**Table 1.** The description of thinking reversible aspect in creating and reversing algebra equation to the beginning equation.

| Thinking Reversible Aspect | Description |
|----------------------------|-------------|
| The forwarding process     | Subject's process in creating equation which is equivalent to the beginning equation. |
| The reversing process      | Subject's process in reversing equation which is created to the beginning equation |

2. **Methods**

This research was qualitative. The methods were tests and interviews. The test contained an Algebra problem. Data collection was conducted in two phases by giving a different test. The first phase is aimed at selecting the subject. Subject candidates were all preservice mathematics teacher-student in the 6th semester in one of the private universities in Jombang, Jawa Timur, Indonesia. In this phase, subject candidates were given a solving problem test. The test contained a reversible Algebra problem, in which the subjects were asked to decide the variable value from an equation. Then, the researcher selected a student to become the subject of the research. The selected subject was the subject that was able to solve solving-problem test correctly and had good communication skills so that researcher can explore thinking reversible of the subject. The second phase aimed at exploring thinking reversible of the subject. This phase was the process of getting the research data. The procedure to get data were; first, the subject was given a solving problem task in algebra, this task contained an equation. Second, the subject was asked to create equations that are equivalent to the equation in the task as many as possible. Third, researchers did the interview process to dig the aspect of the reversing process. The interview used the think-aloud method. The result of the task and interview was analyzed based on the characteristic of thinking reversible as elaborated in table 1.

3. **Results and discussion**

In this part, two points will be discussed. The first is about the subject selecting process and the second is the result of data analysis of the research. The first is related to the subject selection process. As explained in the method, the subject was selected based on the result of the test of reversible problem solving (TRPS). The test contains the equation \( \frac{5+5i}{3-4i} + \left( \frac{m}{4+3i} \right) = 3-i \). Furthermore, the subject candidate was asked to determine the value of \( m \). This test was developed by researcher since the inspiration came from Algebra equation \( 14 - \frac{15}{7-x} = 9 \) delivered by Adi [8] to reveal the thinking reversible of the student. The result of TRPS was only 8.3% of students answered perfectly correctly, both in terms of the process and result. 62.5% of students had a finishing process which was close right but the result was wrong. The rest 29.2% of students answered wrongly, both in terms of the process and result. One of the factors that caused the mistake in solving the problem was they did not involve their thinking reversible maximally. If thinking reversible is involved in solving the problem, after finding the final result (or when they found the value of \( m \)) the students will recheck the final result by substituting \( m \) to the beginning equation. When it is done, the reversing process had happened.
When the finishing result of a problem is perfectly correct, both process and result, there was a possibility thinking reversible was not involved. It could happen because, accidentally, students did not do any mistake during counting until they found the value of \( m \), so the final result is correct. But when the reversing process was not done, the weakness happened was the students would never know that the final result from the finishing problem was correct or wrong. If the reversing process was done, it would be detected that the final result was correct or wrong. If it was wrong, it could be revised to become correct. This is one of the reasons about the importance of thinking reversible, that is minimizing mistake in solving a problem as stated by Maf'ulah et al. [5], Maf'ulah et al. [14], and Ramful [3].

The reversing process, in this case, is nearly similar to looking back, as proposed by Polya [19]. He defined looking back as a mental activity to recheck the result that was obtained after several phases of problem-solving. Looking back is the last phase of solving problem phases. Looking back can be done in many ways, one of them is substituting the final result to the initial problem or the starting problem or by recounting process. If looking back is done with recounting, it means that it is not categorized as a reversing process. Furthermore, the researcher selected a preservice mathematics teacher-student as a research subject with certain criteria such as 1) being able to solve problems in TRPS correctly, 2) involving the reversing process when solving problems, and 3) having a good communication skill.

Second, about the result of research data analysis. Data was obtained through the process of the task to the subject. The task contains an equation \( \frac{5 + 5i}{3 - 4i} + \left( \frac{m}{4 + 3i} \right) = 3 - i \), then the subject was asked to make an equivalent equation as many as possible to the equation on the task. After that, the subject was asked to return the equation made to return to the initial equation. The result was, for around 30 minutes, the subject was able to make 11 equivalent equations to the initial equation. Figure 1 showed the result of the equation made by the subject.

\[
\begin{align*}
1. \quad & \frac{5 + 5i}{3 - 4i} = (3 - i) - \left( \frac{m}{4 + 3i} \right) \\
2. \quad & \frac{m}{4 + 3i} = (3 - i) - \left( \frac{5 + 5i}{3 - 4i} \right) \\
3. \quad & \frac{5 + 5i}{3 - 4i} + \frac{m}{4 + 3i} = (3 - i) = 0 \\
4. \quad & \frac{5 + 5i}{3 - 4i} = 15 + 5i - m \\
5. \quad & \frac{m}{4 + 3i} = -\frac{20i}{3 - 4i} \\
6. \quad & \frac{5 + 5i}{3 - 4i} - \left( \frac{15 + 5i - m}{4 + 3i} \right) = 0 \\
7. \quad & \frac{m}{4 + 3i} + \frac{20i}{3 - 4i} = 0 \\
8. \quad & \frac{5 + 5i}{3 - 4i} = (3 - i) = -\left( \frac{m}{4 + 3i} \right)
\end{align*}
\]

**Figure 1.** Task answer sheet by subject.

Based on the subject's task answer, researchers did the interview process with a think-aloud method to dig the aspect of the reversing process. The process of making and returning a new equation to the
initial equation was presented in the following table 2. The new equation was the equation made by the subject.

Table 2. The process of making and returning a new equation to the initial equation.

| Code | Information | Making Process | Returning Process |
|------|-------------|----------------|------------------|
| P1   | The equation number 1 | The two segments of the initial equation were subtracted $\frac{m}{4 + 3i}$ so it was obtained P1 | The two segments of initial segments were added $\frac{m}{4 + 3i}$ so it was obtained the initial equation |
| P2   | The equation number 2 | The two segments of initials were added $\left(\frac{5 + 5i}{3 - 4i}\right)$ so it was obtained P2 | The two segments of the initial equation were added $\frac{5 + 5i}{3 - 4i}$ so it was obtained the initial equation |
| P3   | The equation number 3 | The two segments of the initial equation were added $-(3 - i)$ so it was obtained P3 | The two segments of the initial equation were added $3 - i$ so it was obtained the initial equation |
| P4   | The equation number 4 | Based on P1, namely $\frac{5 + 5i}{3 - 4i} = (3 - i) - \left(\frac{m}{4 + 3i}\right)$ then the subject performed calculation operation toward the right segment of P1, namely $\left(3 - i\right) - \left(\frac{m}{4 + 3i}\right)$, the result was $15 + 5i - m$ so it was obtained P4 | The subject changes The right segment of P4 back to the same as the right segment of P2, through mathematical manipulation. The process was as follows. $\frac{5 + 5i}{3 - 4i} = \frac{15 + 5i - m}{A + 3i}$ $\frac{5 + 5i}{3 - 4i} = \frac{15 + 5i - m}{A + 3i}$ $\frac{5 + 5i}{3 - 4i} = \frac{(2 + 3)(1 + i) + m}{A + 3i}$ $\frac{5 + 5i}{3 - 4i} = \frac{(3 - i)(A + 3i)}{A + 3i} - \frac{m}{A + 3i}$ $\frac{5 + 5i}{3 - 4i} = \frac{(3 - i) - m}{A + 3i}$ so it was obtained P1. Because P1 could be changed to the initial equation through the same process as the returning process of P1 becomes the initial equation. So P4 was equivalent to P1. Because P1 was equivalent to the initial equation, then P4 was also equivalent to the initial equation. |
| P5   | The equation number 5 | Based on P2, namely $\frac{m}{4 + 3i} = (3 - i) - \left(\frac{5 + 5i}{3 - 4i}\right)$ the subject did calculation |

Changing the right segment of P5 back to the same as the right segment of P2,
| Code | Information | Making Process | Returning Process |
|------|-------------|----------------|------------------|
|      |             | operation on the right segment of P2, namely changing | \[
\frac{m}{4+3i} = \frac{20i \cdot 3}{3+4i} = \frac{20i}{3+4i}
\] so it was obtained P6. Because P6 could be changed to the initial equation through the same process as the returning process of P2 to the initial equation. |
| P6   | The equation number 6 | Based on P4, namely \[
\begin{align*}
\frac{5+5i}{3-4i} &= \frac{15+5i - m}{4+3i} \\
\frac{9-15i + 4i(3-4i) - 5-5i}{3-4i} &= \frac{5+5i}{3-4i} - \frac{20i}{3-4i}
\end{align*}
\] so it was obtained P6 | Adding the two segments of P6 by \[
\frac{15+5i - m}{4+3i}
\] so it was obtained P4. Because P4 could be changed to the initial equation, then P6 could also be changed to the initial equation through the same process as the returning process of P2 to the initial equation. |
| P7   | The equation number 7 | Based on P5, the subject added the two segments of P5 by \[
\begin{align*}
\frac{m}{3+4i} &= \frac{20i \cdot 3}{3-4i} = \frac{20i}{3-4i}
\end{align*}
\] so it was obtained P7 | Adding the two segments of P7 by \[
\begin{align*}
\frac{20i}{3-4i}
\end{align*}
\] so it was obtained P5. Because P5 can be changed to the initial equation, then P7 could also be changed to the initial equation through the same process as the returning process P5 to the initial equation. |
| P8   | The equation number 8 | Based on P3, the two segments were added by \[
\begin{align*}
\frac{m}{4+3i} &= \frac{20i \cdot 3}{3-4i} = \frac{20i}{3-4i}
\end{align*}
\] so it was obtained P8 | Adding the two segments by \[
\frac{m}{4+3i}
\] so it is obtained P3. Because P3 could be changed to the initial equation, then P8 could also be changed to the initial equation through the same process as the returning process of P3 to the initial equation. |
| P9   | The equation number 9 | Based on P3, the two segments were added by \[
\begin{align*}
\frac{m}{4+3i} &= \frac{20i \cdot 3}{3-4i} = \frac{20i}{3-4i}
\end{align*}
\] so it was obtained P9 | Adding the two segments by \[
\begin{align*}
\frac{5+5i}{3-4i}
\end{align*}
\] so it was obtained P3. Because P3 could be
Based on the making process and the returning process of the new equation to the initial equation described in table 2, the researcher can classify them into four different strategies, namely, they are presented in table 3.

Table 3. Different strategies are performed by the subject in making and returning new equations to the initial equation based on the reversible thinking aspect.

| Code | Information | Making Process | Returning Process |
|------|-------------|----------------|-------------------|
| P10  | The equation number 10 | Based on the initial equation, the subject performs the left segment \((5 + 35i) + (3m - 4mi)\) to \(24 - 7i\) so it could be obtained P10 | Changing the right segment of P10 back to \((5 + 35i) + (3m - 4mi)\) by the following process. |
| P11  | The equation number 11 | Based on P10, the subject performed the two segments of P10 with \(24 - 7i\) so it was obtained P11 | Changing \(65 - 45i\) back to \(72 - 45i - 7i\). Then it was changed \(72 - 45i - 7i\) back to \((3 - i)(24 - 7i)\). Then dividing the two segments by \(24 - 7i\). |

Based on the making process and the returning process of the new equation to the initial equation, the subject can classify them into four different strategies, namely:

1. **The forwarding process**
   - Referring to the initial equation, then performs the two segments of the initial equation with the same operand so it appears new equation.
   - Referring to the initial equation, then changing one of the operands to another form which was equivalent to the operand of initial equation (for example another equivalent form was A), so it was obtained new equation.
   - Referring to one of the new equations which have been made

2. **The reversing process**
   - Re-performing the two segments of the initial equation with the same operand (which is used in making new equation), but it used the opposite/reverse operation, so it was regained the initial equation.
   - Changing A (namely another form which was equivalent to the operand of the initial equation) back to the original form. The original form referred was the operand (which had been changed before by the subject) in the initial equation. So it was re-formed to the initial equation.
   - Re-performing the two segments of the new equation with the same operand (which was used in making new equation) but it
The reversible thinking aspect

| The forwarding process                        | The reversing process                                                                 |
|----------------------------------------------|---------------------------------------------------------------------------------------|
| before (for example Pn) then                | used the opposite/reverse operation, so it was obtained the                          |
| performing the two segments of Pn            | previous equation. Pn (namely the equation which was made as a                       |
| with the same operand so it is              | reference in making equation).                                                       |
| formed the new equation.                    | According to the subject, Pn was made based on the initial                           |
|                                              | equation, and Pn could be changed back to the initial equation.                      |
|                                              | On the other hand, a new equation was made based on Pn.                              |
|                                              | Because Pn could be changed back to the initial equation, then the                   |
|                                              | new equation could also be changed to the initial equation                           |
|                                              | automatically, so it was regained the initial equation.                              |
| Referring to the new equation               | Changing A back to the original form so it was formed Pn. The                      |
| which had been made before                   | original form referred was the operand (which had been                              |
| (for example Pn), then changing              | previously changed by the subject) in the Pn equation.                               |
| one of Pn operand to another                 | According to the subject, Pn was made based on the initial                           |
| equivalent form (for example A)             | equation, and Pn could be changed back to the initial equation.                      |
|                                              |                                                                                      |

To facilitate the subject's reversible thinking flow, the researcher presents the following scheme in figure 2.

Based on the analysis results, it is obtained that the subject's process of making and returning a new equation to the initial equation is through the same path or process; the difference is just the activity, which is opposite because of the forwarding process and then the reversing process. As stated by Krutetskii [2] and Maf'ulah et al. [5] that the process of forwarding and reversing does not have to go the same way. That is, it can go the same way; it can or not, and it does not matter.

According to Hackenberg [18], reversing a scheme does not only involve inversion, where someone takes the opposite action from what he has done to return to the initial state. Reversing the scheme can involve compensation, in which someone takes new actions to return to a state equivalent to the initial state. This means that reversible thinking can involve inversion or compensation (reciprocity). However, these opinions differ from the ideas or thoughts of researchers. Through this article, the researcher wants to convey that reversible thinking does not only involve aspects of negation and reciprocity but that doing an opposite mental activity is also included in the category of reversible thinking, in this case, the researcher uses the terms "forwarding process" and "reversing process". The researcher's ideas or thoughts related to reversible thinking are in line with what was conveyed by Ramful [3], but he uses the terms "working forwards" and "back working". Working forward starts from the initial situation given (initial state) to the desired final destination, from known data to unknown. Whereas working forward is analogous to the act of releasing or canceling. To work backward, one must reverse mental or physical actions to return from the results of a process to the beginning of the process.
Strategy scheme 1

Initial equation \[\rightarrow\] Operating \textsuperscript{*} both part of initial equations with the same operand \[\rightarrow\] New equation (Ne S1)

Strategy scheme 2

Initial equation \[\rightarrow\] Converting one operand of the initial equation into another equivalent form (the sample of another equivalent form is A) \[\rightarrow\] Transforming A into the previous operand \[\rightarrow\] New equation (Ne S2)

Strategy scheme 3

New equation \[\rightarrow\] Operating \textsuperscript{*} both part of initial equations with the same operand \[\rightarrow\] New equation

Strategy scheme 4

New equation \[\rightarrow\] Transforming one of the Ne S1 or Ne S2 operand into another equivalent form (the sample of another equivalent form is A) \[\rightarrow\] Transforming A into the previous operand

Notes:
\textsuperscript{*} the operation used in making a new equation was opposite/ inverse to the operation used in returning the new equation to the initial equation.

\begin{itemize}
  \item \(\bigcirc\) : Initial activity
  \item \(\square\) : Mental activity
  \item \(\blacksquare\) : Results
  \item \(\ldots\) : A series of forwarding and reverse processes
  \item \(\longrightarrow\) : plot
\end{itemize}

\textbf{Figure 2.} The scheme of the subject’s reversible thinking.

\textbf{4. Conclusion}

When the math preservice teachers were given algebraic problems that require reversible thinking, 8.3\% of students answer correctly and perfectly, both in terms of process and results. As many as 62.5\% of students whose completion process was close to correct but the final result was wrong. The remaining 29.2\% of students answered incorrectly, both in terms of process and results. Furthermore, from 8.3\% of students who answered correctly and perfectly, the researcher chose one student to explore further the process of reversing thinking through a task. The task contains an equation; then the subject is asked to make as many equations as possible equivalent to the equation in the task. It aims to reveal aspects of
the subject forwarding process. Then the subject is asked to return the equation that he made back to the initial equation. This aims to reveal aspects of the reversing process. As a result, there are 4 different strategies in the forwarding process and reversing the process that has been carried out by the subject, as presented in Table 3 and Figure 2. Forwarding process and the reversing process must be balanced so that mental reversible activities can occur optimally which finally will impact optimal problem-solving skills too.

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