DAMPING OF ALFVÉN WAVES BY TURBULENCE AND ITS CONSEQUENCES: FROM COSMIC-RAY STREAMING TO LAUNCHING WINDS

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ABSTRACT
This paper considers turbulent damping of Alfvén waves in magnetized plasmas. We identify two cases of damping, one related to damping of cosmic-ray streaming instability, the other related to damping of Alfvén waves emitted by a macroscopic wave source, e.g., a stellar atmosphere. The physical difference between the two cases is that in the former case the generated waves are emitted with respect to the local direction of the magnetic field, and in the latter, waves are emitted with respect to the mean field. The scaling of damping is different in the two cases. We explore effects of turbulence in the regimes from sub-Alfvénic to super-Alfvénic to obtain analytical expressions for the damping rates and define the ranges of applicability of these expressions. In describing the damping of the streaming instability, we find that for sub-Alfvénic turbulence, the range of cosmic-ray energies influenced by weak turbulence is unproportionally large compared to the range of scales where weak turbulence is present. On the contrary, the range of cosmic-ray energies affected by strong Alfvénic turbulence is rather limited. A number of astrophysical applications of the process ranging from launching of stellar and galactic winds to propagation of cosmic rays in galaxies and clusters of galaxies is considered. In particular, we discuss how to reconcile the process of turbulent damping with the observed isotropy of the Milky Way cosmic rays.

Key words: radio continuum: general – cosmic rays – magnetohydrodynamics (MHD) – radiation mechanisms: non-thermal – turbulence

1. INTRODUCTION
Astrophysical plasmas are magnetized and turbulent (see McKee & Ostriker 2007; Sharma et al. 2009, Brandenburg & Lazarian 2013). The propagation of Alfvénic waves in MHD turbulence is a problem of great astrophysical significance with applications to key astrophysical processes (see Uhlig et al. 2012; Wiener et al. 2013b; Lynch et al. 2014; van der Holst et al. 2014). Waves in astrophysical environments can arise from instabilities, e.g., from the cosmic-ray (CR) instability due to streaming (Lerche 1967; Kulsrud & Pearce 1969; Wentzel 1969; Skilling 1971). They can also be induced in the environment by macroscopic sources, e.g., they can be the result of vibrations of the stellar surface or arise from magnetic reconnection (see Königl 2009 and references therein; Suzuki 2013). The most well-known consequences of wave propagation and damping range from heating of coronal gas in the solar atmosphere (e.g., Arber et al. 2016; Reep & Russell 2016), acceleration and scattering of CRs (e.g., Jokipii 1966; Schlickeiser 2002, 2003), and launching of solar and stellar winds (e.g., Suzuki & Inutsuka 2005; van Ballegooijen & Asgari-Targhi 2016). The damping of waves induced by CR streaming is important for the confinement and acceleration of CRs in interstellar, intergalactic, and interplanetary environments (see Emslin et al. 2011; Badruddin & Kumar 2016; Kulsrud 2005). In more general terms, the damping must be understood in order to answer the long-standing question of the relative importance of waves and turbulence in various astrophysical settings (see Petrov & Lazarian 2012).

Initial studies of Alfvén wave damping in turbulent plasmas were done in Similon & Sudan (1989) using a model of isotropic turbulence that does not correspond to the present understanding of magnetized turbulence (see Section 2 for more discussions on MHD turbulence). More recently, turbulent damping of Alfvén waves was suggested by Yan & Lazarian (2002, henceforth YL02) as a process for suppressing CR streaming instability. This process was quantified in an important study by Farmer & Goldreich (2004, henceforth FG04) for the original Goldreich & Sridhar (1995, henceforth GS95) model of Alfvénic turbulence and was tested numerically in Beresnyak & Lazarian (2008b).

FG04 employs the original GS95 model of strong Alfvénic turbulence, which assumes that turbulence is injected isotropically with velocity \( u_b \) equal to the Alfvén velocity \( V_A \). Although important, this regime does not cover the full variety of astrophysical conditions. Moreover, the work deals only with the Alfvén waves generated by CR streaming instability and does not cover Alfvén waves from an external macroscopic source. At the same time, the propagation of Alfvén waves in turbulent plasmas is important for understanding the heating of stellar atmospheres, galactic halos, and intracluster media (see Zhuravleva et al. 2014). The momentum deposited by Alfvén waves in plasmas is important for launching stellar and galactic winds.

First of all, as we discuss further in this paper, the damping of Alfvén waves generated by streaming instability is different from the damping of Alfvén waves generated by a microscopic source. Moreover, the strong isotropically injected MHD turbulence discussed in FG04 is one of the types of MHD turbulence that is relevant to astrophysical settings. Depending on the injection scale, injection velocity, and media magnetization, magnetized turbulent motions can demonstrate regimes of isotropic super-Alfvénic turbulence, extremely anisotropic weak turbulence, and strong sub-Alfvénic turbulence with anisotropic injection. Turbulent motions at the injection scale can also damp Alfvén waves.

A quantitative study of the damping of the Alfvén waves injected with respect to (a) local magnetic fields and (b) the global mean field by MHD turbulence in the different aforementioned regimes, as well as with the turbulence at...
scales larger than the turbulence injection scale, is the goal of this paper.

In particular, in the present paper we address the damping of Alfvén waves by turbulence taking into account both the spatial structure of the front of the Alfvén wave and the actual properties of MHD turbulence with which these waves are interacting. The former depends on how the Alfvén waves were generated, the latter depends on the properties of MHD turbulence in different regimes. As we mentioned earlier, the FG04 study addresses the damping of Alfvén waves generated by the streaming instability of CRs (see Kulsrud 2005; Longair 2011), which corresponds to the waves generated with respect to the local direction of the magnetic field that is sampled by the streaming particles. The distinction between the local and global systems of reference (Lazarian & Vishniac 1999, henceforth LV99; Cho & Vishniac 2000; Maron & Goldreich 2001; Cho et al. 2002) plays a crucial role in the present day theory of MHD turbulence and, as we will show later, the damping of Alfvén waves differs in the case of Alfvén waves launched with respect to the local magnetic field and with respect to the mean magnetic field. This important distinction has not been addressed in the earlier studies. Similarly, we are not aware of quantitative studies of Alfvén wave damping by MHD turbulence in different regimes.

We believe that studying the Alfvén wave damping for a wide range of possible turbulent astrophysical settings is very important. Indeed, magnetic turbulence in stellar and pulsar atmospheres, in halos of spiral galaxies MHD is sub-Alfvénic with Alfvén Mach number $M_A = V_l/V_A < 1$. In spiral arms, different parts of the interstellar media exhibit different degrees of magnetization (Draine 2011). For instance, molecular clouds may be super-Alfvénic (see Lunttila et al. 2008), although this is not a universally accepted opinion (see Li & Henning 2011). Turbulence in clusters of galaxies is generally accepted to be super-Alfvénic (see Brunetti & Lazarid 2007; Brunetti & Jones 2014; Brunetti 2016).

In what follows, in Section 2 we derive the relations for different regimes of the MHD turbulence that we employ in this paper. By providing this derivation we clarify the points that are essential for our further explanation of the nature of Alfvén-wave damping. Then we provide a physical picture of damping in Section 3. Historically, the damping of streaming instability by turbulence was the first to be addressed (YL02, FG04). This type of damping is associated with Alfvén waves generated with respect to the local magnetic field. Therefore, in Section 4 we quantify the damping of waves generated in the local system of reference for both sub-Alfvénic and super-Alfvénic turbulence. There we also define the ranges of wavelengths for which the damping by weak and strong sub-Alfvénic turbulence is applicable as well as define the range of applicability of damping by super-Alfvénic turbulence. We later consider the second case of damping of Alfvén waves, i.e., the case of damping of waves generated by a macroscopic source. Therefore, for the aforementioned variety of MHD turbulence regimes, we consider the case of damping of Alfvén waves that are injected with respect to the mean field in Section 5. We compare the turbulent damping of Alfvén waves with the nonlinear Landau damping of Alfvén waves in Section 6 and briefly outline some of the astrophysical implications of our study in Section 7. We compare our results with those of earlier works in Section 8. Our discussion is provided in Section 9, and our summary is given in Section 10.

2. REGIMES OF ALFVÉNIC TURBULENCE

2.1. Cascading of Turbulence

MHD theory is applicable to astrophysical plasmas at sufficiently large scales, and for many astrophysical situations the requirement for Alfvénic turbulence to be applicable is that the turbulence is studied at scales substantially larger than the ion gyroradius $\rho_i$ (see more in Kulsrud 2005; Eyink et al. 2011). In what follows we consider this criterion satisfied. For Alfvénic turbulence, velocity and magnetic field fluctuations have the same scaling and thus below we focus on velocity scaling.

MHD turbulence can be decomposed into the cascades of Alfvén, slow, and fast modes (GS95, Lithwick & Goldreich 2001), a concept that has been elaborated and proven numerically (Cho & Lazarid 2002; Kowal & Lazarid 2010; Takamoto & Lazarid 2016). For nonrelativistic turbulence, the Alfvénic cascade is marginally affected by two other fundamental MHD modes (see Cho & Lazarid 2002, Takamoto & Lazarid 2016, cf. Stone et al. 1998) and therefore we focus on Alfvénic turbulence, which is responsible for nonlinear Alfvén-wave damping.

The pioneering studies of Alfvénic turbulence were done by Iroshnikov (1964) and Kraichnan (1965) using a hypothetical model of isotropic MHD turbulence. Later studies (see Montgomery & Turner 1981; Matthaeus et al. 1983; Shebalin et al. 1983; Higdon 1984) discovered the anisotropic nature of the energy cascade and paved the way for further advancements in the field. The breakthrough work by GS95 provided the theory of trans-Alfvénic turbulence, i.e., turbulence corresponding to injection velocity $V_l$ at the injection scale $L$ equal to Alfvén velocity $V_A$, which corresponds to the Alfvén Mach number

$$M_A = \frac{V_l}{V_A},$$

equal to unity. The generalization of the GS95 theory for $M_A < 1$ and $M_A > 1$ was obtained later (LV99, Lazarid 2006). Note that the original GS95 theory was also augmented by the concept of local systems of reference (LV99, Cho & Vishniac 2000; Maron & Goldreich 2001; Cho et al. 2002), which specifies that turbulent motions should be viewed not in the system of reference of the mean magnetic field, but in the local system of reference of the turbulent eddies. This is quite a natural concept, which, however, was missed by the earlier studies. Indeed, for the small-scale turbulent motions, the only magnetic field that matters is the magnetic field in their vicinity. Thus, this local field, rather than the mean field, should be considered. Keeping this in mind, in what follows we describe turbulent motions not using parallel and perpendicular wave numbers, but using the scale parallel to the local magnetic field $l_l$ and the scale perpendicular to the local magnetic field $l_l$. The distinction between the local system of reference related to the local field and the global system of reference related to the mean field is also important for the two cases of Alfvén-wave damping that we consider in this work.

In Table 1 (see Section 9) we present various regimes of Alfvénic turbulence and the ranges for which those are applicable. Below we consider first the sub-Alfvénic turbulence corresponding to the Alfvén Mach number $M_A < 1$. For Alfvénic perturbations, the relative perturbations of velocities
and magnetic fields are related in the following way:

$$\frac{\delta B_i}{B} = \frac{\delta B_L}{B_L} = \frac{u_l}{u_i} \frac{\nu_A}{V_A},$$  \hspace{1cm} (2)$$

where $B_i$ is the perturbation of the magnetic field $B$ at the scale $l$, $B_L$ is the perturbation of the magnetic field at the injection scale $L$, while $u_i$ is the velocity fluctuation at the scale $l$ in the turbulent flow with energy injected with velocity $u_i$.

To understand the nature of nonlinear damping of Alfvén waves, it is useful to get insight into a detailed picture of cascading by Alfvén turbulence (see Cho et al. 2003, p. 56 for more details). Consider colliding Alfvénic wave packets with parallel scales $l_{\parallel}$ and perpendicular scales $l_{\perp}$. The change of energy per collision is

$$\Delta E \sim (du_l^2/dt) \Delta t,$$  \hspace{1cm} (3)$$

where the first term represents the change of the energy of a packet in time as it interacts with the oppositely moving wave packet. Naturally, the time of the interaction is the time of the passage of the given wave packet through the oppositely moving wave packet of size $l_{\parallel}$, and thus the interaction time $\Delta t \sim l_{\parallel}/V_A$. To estimate the characteristic rate of cascading, one should accept that the cascading of our wave packet is happening due to the change of the structure of the oppositely moving wave packet which is happening at the rate $u_i/l_{\parallel}$. Indeed, more structures are being created in the passing package as the background field affected by the opposite package evolves. Therefore Equation (3) becomes

$$\Delta E \sim u_i \cdot \Delta \mathbf{u} \Delta t \sim (u_l^3/l_{\parallel})(l_{\parallel}/V_A).$$  \hspace{1cm} (4)$$

The fractional energy change per collision is the ratio of $\Delta E$ to $E$,

$$f \equiv \frac{\Delta E}{u_l^2 \Delta t} \sim \frac{u_i l_{\parallel}}{V_A l_{\perp}},$$  \hspace{1cm} (5)$$

which is the measure of the strength of the nonlinear interaction. $f$ is the ratio of the rate of shearing of the wave packet $u_i/l_{\parallel}$ to the rate of the propagation of the wave packet $V_A/l_{\perp}$. If the shearing rate is much smaller than the propagation rate, for $f \ll 1$ the cascading is a random walk process, which means that

$$\kappa = f^{-2}$$  \hspace{1cm} (6)$$

steps are required for the cascading, i.e., the cascading time is

$$t_{\text{cas}} \sim \kappa \Delta t.$$.  \hspace{1cm} (7)$$

For $\kappa > 1$, the turbulence is called weak and if $\kappa \approx 1$ the turbulence is called strong.

The Alfvénic three-wave resonant interactions are governed by relations for wave vectors that reflect momentum conservation $k_1 + k_2 = k_3$, and frequencies reflecting energy conservation $\omega_1 + \omega_2 = \omega_3$. With Alfvén wave packets interacting with oppositely moving $k$ and having the dispersion relation $\omega = V_A k$, where $k_1 \sim l_{\parallel}^{-1}$ is the component of the wave vector parallel to the background magnetic field, the increase of $l_{\parallel}$ with $l_{\parallel}$ being fixed signifies the increase of the energy change per collision. This eventually makes $\kappa$ of the order of unity and the approximation of the weak Alfvénic turbulence breaks down. For $\kappa \approx 1$ the GS95 critical balanced condition,

$$u_i l_{\parallel}^{-1} \approx V_A l_{\perp}^{-1},$$  \hspace{1cm} (8)$$

is satisfied with the cascading equal to the wave period $\sim \Delta t$. The value of $\kappa$ cannot decrease further and the turbulence evolves as strong Alfvénic turbulence. Therefore the further decrease of $l_{\parallel}$ entails the corresponding decrease of $l_{\parallel}$ to keep the critical balance satisfied. The ability to change $l_{\parallel}$ means that the frequencies of interacting waves increase, which is possible as cascading introduces uncertainty into the wave frequency $\omega$ of the order of $1/t_{\text{cas}}$.

The cascading turbulent energy flux for an incompressible fluid is (Batchelor 1953)

$$\epsilon \approx u_l^2/t_{\text{cas}} = \text{const},$$  \hspace{1cm} (9)$$

which in the hydrodynamic case gives

$$\epsilon_{\text{hydro}} \approx u_l^2/l \approx u_i^2/L = \text{const},$$  \hspace{1cm} (10)$$

where the relation $t_{\text{cas}} \approx l/u_i$ is used.

For the weak cascade $\kappa \gg 1$ gives (LV99)

$$\epsilon_{\omega} \approx \frac{u_l^4}{V_A^2 \Delta t (l_i/l_{\parallel})^2} \approx \frac{u_i^4}{V_A L},$$  \hspace{1cm} (11)$$

where Equations (9) and (7) are used. The second relation in Equation (11) follows from the assumed isotropic injection of turbulence at the scale $L$.

### 2.2. Scaling Relations

Taking into account that $l_{\parallel} = L$ and that it does not change for weak turbulence, it is easy to see that Equation (11) gives

$$u_i \sim u_i (l_i/L)^{1/2},$$  \hspace{1cm} (12)$$

which is different from the Kolmogorov $l^{1/3}$ scaling.\(^1\)

For the accepted model of isotropically injected turbulence at scale $L$, the initial $l_i = L$ and the transition to $\kappa \approx 1$, i.e., to strong turbulence, occurs (LV99)

$$l_{\text{trans}} \sim L(u_i/L)^{2/3} \equiv L M_A^2,$$  \hspace{1cm} (13)$$

Thus, weak turbulence has a limited, i.e., $[L, L M_A^2]$, inertial range and at scales less than $LM_A^2$ it transits into the regime of strong turbulence. The velocity corresponding to the transition follows from the $\kappa \approx 1$ condition given by Equations (6) and (5):

$$u_{\text{trans}} \approx V_A l_{\text{trans}}/L \approx V_A M_A^2.$$  \hspace{1cm} (14)$$

The relations for the strong turbulence in the sub-Alfvénic regime obtained in LV99 can be easily derived as follows. Indeed, the turbulence becomes strong and cascades over one wave period, which according to Equation (8) is equal to $l_{\parallel}/u_i$. Substituting the latter into Equation (9) one gets

$$\epsilon_{\omega} \approx \frac{u_i^3}{t_{\text{trans}}} \approx \frac{u_i^3}{l} = \text{const},$$  \hspace{1cm} (15)$$

which is analogous to the hydrodynamic Kolmogorov cascade in the direction perpendicular to the local direction of the

\(^1\) Using the relation $k^2 E(k) \sim u_i^2$ it is easy to see that the spectrum of weak turbulence is $E_{\omega, \text{weak}} \sim k_i^{-2}$ (LV99; Galtier et al. 2000).
magnetic field. This cascade starts at $l_{\text{trans}}$ and has the injection velocity given by Equation (14). Thus (LV99)

$$u_l \approx V_A \left( \frac{l}{L} \right)^{1/3} M_A^{4/3}. \quad (16)$$

In terms of the injection velocity $u_l$, Equation (16) can be rewritten as

$$u_l \approx u_l \left( \frac{l}{L} \right)^{1/3} M_A^{4/3}. \quad (17)$$

Substituting the latter expression into Equation (8), one gets the relation between the parallel and perpendicular scales of the eddies (LV99):

$$l_l \approx L \left( \frac{l}{L} \right)^{2/3} M_A^{-4/3}. \quad (18)$$

The relations given by Equations (18) and (16) reduce to the well-known GS95 scaling for trans-Alfvénic turbulence if $M_A \equiv 1$.

The super-Alfvénic turbulence corresponds to $u_l > V_A$, which is equivalent to $M_A > 1$. For $M_A \gg 1$ the turbulence at scales close to the injection scale is essentially hydrodynamic as the influence of magnetic forces is of marginal importance, i.e., the velocity is Kolmogorov:

$$u_l \approx u_l \left( \frac{l}{L} \right)^{1/3}. \quad (19)$$

The cascading nature changes at the scale

$$l_l \approx LM_A^{-3}, \quad (20)$$

at which the turbulent velocity becomes equal to the Alfvén velocity (Lazarian 2006). The rate of cascading for $l < l_l$ can be written as

$$\epsilon_{\text{super-A}} \approx u_l^3/l \approx M_A^3 V_A^3/L = \text{const.} \quad (21)$$

This cascading can be related to the GS95 cascade, if the scale given by Equation (21) is taken as the injection scale for the trans-Alfvénic turbulence and the corresponding scaling follows from Equations (22) and (16) for the injection of $V_A$. In other words:

$$l_l \approx L \left( \frac{l}{L} \right)^{2/3} M_A^{4/3}, \quad (22)$$

$$u_l \approx u_l \left( \frac{l}{L} \right)^{1/3} M_A^{1/3}. \quad (23)$$

The relations above are used in the discussion of Alfvén-wave damping that follows.

2.3. Locality of Cascading

The interaction of oppositely moving wave packets involves interaction of different modes, and therefore it is important to know what perpendicular scales of the oppositely moving waves are responsible for most of the cascading of the Alfvénic waves with perpendicular scale $l_\perp$. Consider the case of weak turbulence and cascading with the scale $x$ larger or equal to $l_\perp$. If the wave corresponding to $l_\perp$ has velocity $u_l$, then Equation (3) can be rewritten as

$$\Delta E_x \approx \frac{u_l^2}{x/u_x} \Delta t, \quad (24)$$

where the shearing produced by the oppositely moving Alfvénic perturbation at the scale $x$ is accounted for. The corresponding shearing velocity $u_l$ belongs to the weak cascade and is described by Equation (12). Therefore the change of the energy of the wave induced by the oppositely moving package is

$$\Delta E_x \approx \frac{u_l^2}{x^{3/2}} M_A L^{1/2}. \quad (25)$$

Integrating the energy change from $l_{\perp}$ to $L$, i.e.,

$$\int \Delta E_x/(u_l^2) \, dx/x$$

one gets $\sim M_A L^{1/2}/l_{\perp}^{1/2}$, i.e., among the oppositely moving wave packets with $x > l_{\perp}$, those with the same perpendicular scale contribute most to the cascading of the waves with $l_{\perp}$. Similarly, it can be seen that the contribution of the wave packets with $x < l_{\perp}$ to the cascading is subdominant unless $x \rightarrow l_{\perp}$. Indeed, the interactions with small oppositely moving wave packets results in the incoherent distortion of the wave, with the distortions adding in a random way in the limit of $x \ll l_{\perp}$.

Similar considerations allow us to show that the locality is also true for wave packets undergoing strong cascading. Indeed, for the cascading by wave packets with $x \gg l_{\perp}$ one can show that the change of the energy

$$\Delta E_x \approx \frac{u_l^3 l_{\perp}}{x^{2/3}} L^{1/3} M_A^{3/2}, \quad (26)$$

where it is taken into account that the interaction between longer waves corresponding to $x$ and shorter waves corresponding to $l_{\perp}$ is limited by the cascading time $l_{\perp}/u_l$ of the shorter waves. It is obvious that the considerations that we applied to weak turbulence for $x < l_{\perp}$ are also applicable.

Our considerations above correspond to the notion of diffuse locality introduced in Beresnyak & Lazarian (2010). In what follows, this notion of locality considerably simplifies the derivation of Alfvén-wave damping that we provide below. This allows us to consider only the interaction of oppositely moving waves with the same $l_{\perp}$. Note that this is only applicable for balanced turbulence, while for the imbalanced one, the picture of interactions is much more involved (see Beresnyak & Lazarian 2008a).

3. GENERAL REMARKS ON TURBULENT DAMPING OF ALFVÉN WAVES

Linear Alfvén waves propagate without inducing irreversible distortions in each other. The situation changes when Alfvén waves interact with Alfvénic turbulence. There the structure of magnetic field lines changes significantly over the time of propagation of the wave, and this causes nonlinear distortion. As a result, the waves undergo cascading and dissipate together with turbulence. The difference between this process and other wave-damping processes is that turbulent damping does not depend on plasma microphysics.

We provide calculations further in this paper, but here we provide some simple arguments explaining the physics of the two regimes of damping that we consider further on. The dynamically important magnetic field of the turbulent fluid is
aligned better locally on the scale of the small eddies. Therefore, the Alfvén waves emitted parallel to the local magnetic field, as this is the case of Alfvén waves emitted by the streaming instability, will experience the least distortion from the oppositely moving eddies. Formally, the wave moving exactly parallel to the magnetic field corresponds to a wave packet with $l_{\perp} = \infty$. Therefore the least distorted Alfvén waves can be described as the Alfvén packages having the largest value of $l_{\perp}$. The wave packages are most efficiently distorted by the oppositely moving wave packages with the same $l_{\perp}$. The larger $l_{\perp}$ is, the longer the time for the evolution of the corresponding wave packages, e.g., for the strong GS95 turbulence it corresponds to $l_{\perp}/v_{\parallel} \sim 2^{3/3}$. We will show below that for the given Alfvén-wave wavelength, the largest $l_{\perp}$ corresponds exactly to the case of emission parallel to the local direction of the magnetic field. This is the case of Alfvén-wave damping relevant to the streaming instability. The other case is Alfvén-wave emission by a macroscopic source. If Alfvén waves are emitted at an angle $\theta$ to the magnetic field, the value of $l_{\perp}$ is smaller and therefore the turbulent damping of Alfvén waves is faster. This is also the case, for instance, if Alfvén waves are emitted parallel to the mean field. There the dispersion of the magnetic field direction with respect to the mean field acts as the angle $\theta$. Naturally, the damping of Alfvén waves depends on the regime of turbulence that the waves interact with. For instance, if the turbulent perturbations are weakly nonlinear, as in the case of weak turbulence, these perturbations should induce damping that is slower compared with the strongly nonlinear perturbations of the strong turbulence.

In what follows we assume that the propagating Alfvén waves are weak enough that they do not distort the Alfvénic turbulence. In the opposite case, we expect the turbulence to get more features of the turbulence with non-zero cross helicity, which is frequently called “imbanced turbulence.” A tested model of such turbulence in Beresnyak & Lazarian (2008b) predicts a significant decrease of cascading for the strong wave packages. We may expect that the perturbations significantly stronger than the background turbulence can disturb the background turbulence, inducing its imbalance. Thus the waves can potentially propagate over longer distances compared with the estimates that we are going to obtain below.

4. DAMPING OF THE STREAMING INSTABILITY

In this section we consider the damping of Alfvén waves that are generated with respect to the local magnetic field direction. This sort of damping is associated with the damping of the streaming instability of CRs (YL02, FG04). The damping of Alfvén waves in the global system of reference relevant to the damping of Alfvén waves emitted by macroscopic sources is considered in Section 5. In what follows we employ the understanding of cascading developed in the previous section.

4.1. Streaming Instability and Local System of Reference

Alfvén waves can be generated by different astrophysical sources. The streaming instability of CRs is an important process that generates Alfvén waves. The generation happens as particles interact with the local magnetic field, and the sampling scale for the magnetic field is the Larmor radius of the energetic particle. This is a typical situation when one must consider the local system of reference related to the local direction of the wandering magnetic field (LV99, Cho & Vishniac 2000; Maron & Goldreich 2001; Cho et al. 2002).

The growth rate of the streaming instability for the direction parallel to the local direction of the magnetic field is given by the expression (see Kulsrud & Pearce 1969):

$$\Gamma_{cs} \approx \Omega_{B} n_{cr} \left( \frac{\gamma}{n_{i}} \right) \left( \frac{V_{\text{stream}}}{V_{A}} - 1 \right).$$

(27)

where $\Omega_{B} = eB/mc$ is the particle gyrofrequency and $n_{cr}$ is the number density of CRs with gyroradius $r > \lambda = \gamma mc^2/eB$. The streaming velocity $V_{\text{stream}}$ enters Equation (27) as a ratio with $V_{A}$.

For the instability to operate, the growth rate given by Equation (27) should exceed the rate of the turbulent damping that we quantify below.

4.2. Damping by Sub-Alfvénic Strong Turbulence

We consider first the case of strong sub-Alfvénic turbulence. Using sub-Alfvénic strong turbulence as a test case, we consider different ways of deriving the result.

The first approach that we present is based on calculating the distortion of the wave by evolving turbulent fluctuations as the waves propagate along magnetic field lines. The distortion of the wavefront arises from the magnetic field lines wandering over the angle $\theta_{t}$. This angle depends on the fluctuations of the magnetic field $\delta B_{t}$ induced by turbulence with perpendicular scale $x$. Simple geometric considerations suggest that the distortion induced by a wave propagating along the magnetic field for time $t$ is

$$\delta_{t} \approx V_{A} t \sin^{2} \theta_{t} \approx V_{A} t \left( \frac{\delta B_{t}}{B} \right)^{2},$$

(28)

where the perturbation induced by turbulence evolves as

$$\left( \frac{\delta B_{t}}{B} \right)_{x} \approx \left( \frac{u_{t}}{V_{A}} \right) \left( \frac{t}{x/u_{t}} \right),$$

(29)

where $u_{t}$ is the velocity corresponding to the magnetic field fluctuation $\delta B_{t}$. The time $t$ in Equation (29) is less than the eddy turnover time $x/u_{t}$, and the ratio reflects the partial sampling of the magnetic perturbation by the wave. Substituting the scaling of strong sub-Alfvénic turbulence for $u_{t}$ in Equation (29) one can rewrite Equation (28) as

$$\delta_{t} \approx \frac{V_{A}^{3} M_{A}^{6/3} \lambda^{3}}{x^{2/3} L^{4/3}}.$$

(30)

The damping of the wave with the wavelength $\lambda$ corresponds to the “resonance condition” $\delta_{t} = \lambda$ and substituting this in Equation (30) one can express the perpendicular scale of the “resonance” magnetic perturbations that distort the wave:

$$x \approx \frac{V_{A}^{3/2} M_{A}^{9/2} \lambda^{2}}{\lambda^{2} L^{2}}.$$

(31)

The required time for the damping is equal to the turnover of the resonant eddy,

$$t \approx \frac{x}{u_{t}} \approx \frac{V_{A} r^{2} M_{A}^{2}}{\lambda L},$$

(32)
which gives the rate of turbulent damping of Alfvén waves
\[ \Gamma_{\text{sub}A} \approx t^{-1}, \]  
(33)
or
\[ \Gamma_{\text{sub}A,\perp} \approx \frac{V_A M_A^2}{l^{1/2} L^{1/2}}. \]  
(34)

For trans-Alfvénic turbulence, i.e., \( M_A = 1 \), this result transfers to the one in FG04. We point out, however, the square of the Alfvén Mach number dependence, which means a significant change of the damping rate for sub-Alfvénic turbulence. We also note that in FG04 the injection scale for turbulence was defined not as the actual injection scale, but the scale at which the turbulent velocity becomes equal to the Alfvén one. We discuss the implications of this in Section 8 where we compare our approach/results with those in FG04.

For isotropic injection of turbulence the maximum perpendicular scale of strong sub-Alfvénic motions is given by \( x_{\text{max}} = L M_A^2 \) (see Equation (13)). Therefore, if one substitutes this into Equation (31) and simultaneously uses Equations (33) and (34) to express \( t \), one gets
\[ \lambda_{\text{max},\perp} \approx L M_A^4. \]  
(35)

For the streaming instability the particles emit Alfvén waves of the order of the particle gyroradius \( r_L \). Therefore the range of \( r_L \) is limited to
\[ r_L < L M_A^4, \]  
(36)
which can be a serious limitation if \( M_A \) is sufficiently small. For the higher energy particles the interactions happen with weak turbulence. We discuss this regime of damping in Section 4.3, while below we provide another derivation of Equation (34).

Because of the significance of wave damping it is also useful to present a more intuitive derivation of the same result that is based on the notion of propagating wave packets that we employed obtaining Equation (4). Consider two oppositely moving packets with the perpendicular scale \( x' \sim k_{\perp}^{-1} \). Each packet induces a magnetic field distortion \( \theta_\perp \) of the oppositely moving waves. Consider a locally emitted Alfvén wave moving parallel to the local direction of the magnetic field with wavenumber \( k_{\parallel}^{-1} \sim \lambda \). It is easy to see that the wave gets distorted by interacting with turbulence with \( k_{\parallel} \sim k_{\parallel} \sin \theta_\parallel \). The interactions of a wave with \( k_{\perp} \) and the oppositely moving packets will be most efficient if it is “resonant” i.e., \( k_{\parallel}^2 = k_{\perp}^2 \).

This suggests the relation \( k_{\parallel} \sin \theta_\parallel = k_{\perp} \), which determines the perpendicular scale of the wave packet that will cascade the wave:
\[ \lambda \approx x \sin \theta_\parallel \approx x \frac{\delta B_z}{B}. \]  
(37)

Inserting the scaling given by Equations (29) and (16) it is possible to get the expression for the “resonant” perpendicular scale \( x \):
\[ x = L^{1/4} \lambda^{3/4} M_A^{-1}. \]  
(38)

which can then be used to find the rate of damping defined as \( \Gamma_{\text{sub}A,\perp} \approx u_\perp / x \), which reproduces the earlier result given by Equation (34). Within this approach the maximum wavelength of the Alfvénic wave that can be damped by strong sub-Alfvénic turbulence can be obtained from Equation (37) if the scale \( l_{\text{trans}} \) is used instead of \( x \), i.e.,
\[ \lambda_{\text{max},\perp} \approx \left( \frac{u_{\text{trans}}}{V_A} \right) l_{\text{trans}} \approx L M_A^4, \]  
(39)
which coincides with the result given by Equation (35). The minimum scale of waves that are being damped depend on the perpendicular scale of the smallest Alfvénic eddies \( l_{\text{min}} \). Using Equation (37) and the scaling of strong turbulence given by Equation (16), one can get the range of \( r_L \) affected by turbulent damping arising from strong sub-Alfvénic turbulence:
\[ \frac{l_{\text{min}}^{4/3}}{L^{1/3}} M_A^{4/3} < r_L < L M_A^4, \]  
(40)
which indicates that waves much smaller than \( l_{\text{min}} \) can be damped. The value of \( l_{\text{min}} \) can be large in partially ionized gas (see Xu et al. 2015). Due to the differences of \( r_L \) for protons and electrons, Equation (40) suggests that the streaming instability of electrons may not be damped, while it is damped for protons. This may result in differences of acceleration of electrons and protons from the thermal pool.

The damping of streaming instability for \( r_L < \frac{l_{\text{min}}^{4/3}}{L^{1/3}} M_A^{4/3} \) is present, but significantly reduced. An estimate of it can be obtained by considering the distortion \( \delta t \ll \lambda \) given by Equation (30) for the time period of the wave \( \lambda / V_A \), which is significantly less than the period of the eddy at the scale \( l_{\text{min}} \):
\[ t_{\text{edd}} \approx \frac{L^{1/3} \lambda}{(V_A M_A^{1/3})}. \]  
The distortions accumulate as a random walk with the time step given by \( t_{\text{edd}} \). The damping requires \((\lambda / \delta t)^2\) steps, which results in the damping rate
\[ \Gamma_{\text{sub},\perp, r_L < l_{\text{min}}} \approx \frac{M_A^2 V_A r_L^4}{l_{\text{min}}^2 L^3}, \]  
(41)
which also illustrates the inefficiency of damping by turbulence with perpendicular scale larger than the “resonant” scale.

### 4.3. Damping by Sub-Alfvénic Weak Turbulence

For waves longer than \( \lambda_{\text{max},\perp} \) the wave is cascaded through weak interactions together with the corresponding wave packets, the perpendicular wave scales for which are given by Equation (37). The difference here, however, is that the scaling of weak turbulence given by Equation (12) should be used. This gives the relation between the Alfvén-wave wavelength and the perpendicular scale of the “resonant” weak mode \( l_\perp \):
\[ \lambda = l_\perp \left( \frac{l_\perp}{L} \right)^{1/2} M_A, \]  
(42)
which provides the weak eddy perpendicular scale
\[ l_\perp \approx \lambda^{2/3} L^{1/3} M_A^{-2/3}. \]  
(43)
Unlike strong turbulence, the weak wave packets are cascading $N$ times slower (see Equations (7), (6)), with

$$N \approx \left( \frac{V_A L}{u_L} \right)^2,$$  \hspace{1cm} (44)

where it is taken into account that the parallel scale of weak-turbulence wave packets is equal to the injection scale $L$. The rate of turbulent damping of the Alfvén wave is therefore

$$\Gamma_{\text{subA,w}} \approx (N \Delta t)^{-1} \approx N^{-1} \frac{V_A}{L},$$  \hspace{1cm} (45)

which gives

$$\Gamma_{\text{subA,w}} \approx \frac{V_A M_A^{8/3}}{\lambda^{7/3} L^{1/3}},$$  \hspace{1cm} (46)

where compared to the case of the earlier discussed damping given by Equation (34) shows even stronger dependence on $M_A$ as well as a different dependence on the wavelength $\lambda$. Being applicable to weak turbulence, this result does not transfer for $M_A = 1$ to that in FG04 and therefore it is essential to define the range of its applicability in terms of wavelength $\lambda$.

The maximum wavelength of the Alfvén waves that can be cascaded by the weak cascade can be obtained by substituting $1_L = L$, i.e., using the energy injection scale, in Equation (42). This gives

$$\lambda_{\text{max, w}} \approx L M_A.$$  \hspace{1cm} (47)

Thus the particles emitting Alfvén waves of the order of their gyroradius should have the range of gyroradii

$$L M_A^3 < r_L \leq L M_A$$  \hspace{1cm} (48)

in order to interact with weak turbulence, provided that $LM_A^3$ is larger than the damping scale of turbulent motions. Otherwise the lower boundary in Equation (48) is given by $l_{\text{min}}$.

Waves with $\lambda > \lambda_{\text{max, w}}$ will interact with turbulence at the injection scale $L$. Such waves cascade by the largest wave packets whose cascading rate is $N^{-1} \frac{V_A}{L}$, i.e.,

$$\Gamma_{\text{outer}} \approx N^{-1} \frac{V_A}{L} \approx M_A^2 \frac{V_A}{L},$$  \hspace{1cm} (49)

which is valid for CRs inducing waves with Larmor radii in the range

$$L M_A < r_L < L.$$  \hspace{1cm} (50)

In the case of $\lambda \gg L$ the result in Equation (60) is reduced by another random walk factor $(L/\lambda)^2$, i.e.,

$$\Gamma_{\text{outer, extreme}} \approx N^{-1} \frac{V_A}{L} \frac{L^2}{\lambda^2} \approx M_A^2 \frac{V_A}{L} L^2 \frac{1}{\lambda^2},$$  \hspace{1cm} (51)

which can be important for the damping of Alfvénic waves by turbulence injected by the streaming instability.

### 4.4. Damping by Super-Alfvénic Turbulence

The case of super-Alfvénic turbulence for scales less than the scale of the transfer to the MHD regime, i.e., $l_A = L M_A^{-3}$, can be obtained from our earlier results through the following considerations. At $l_A$ the turbulence becomes Alfvénic, and this scale can be considered as the turbulence injection scale. The injection velocity at this scale is $V_A$ and therefore the resulting damping rate can be obtained by substituting $l_A$ as the injection scale $L$ and $V_L = V_A$ in Equation (34). As a result,

$$\Gamma_{\text{super}} \approx \frac{V_A}{l_A^2 \lambda^2} = \frac{V_A M_A^2}{L^1 \lambda^{2/3}}.$$  \hspace{1cm} (52)

In a sense this is a case of trans-Alfvénic turbulence if $l_A$ is associated with the turbulence injection scale. This case corresponds to the one in FG04 where the turbulence injection scale was defined to be the scale $L_{\text{MHD}}$ at which the injection velocity becomes equal to $V_A$. Thus, for super-Alfvénic turbulence $L_{\text{MHD}} = l_A$.

Treating $l_A$ as the effective injection scale one can easily get from Equation (35) the maximum wavelength up to which the above treatment of the nonlinear damping is applicable:

$$\lambda_{\text{max, super}} \approx l_A = L M_A^{-3}.$$  \hspace{1cm} (53)

For the streaming instability we associate $\lambda$ with the gyroscale $r_L$ and therefore define the corresponding gyroscale range as

$$\frac{l_{\text{min}}^{4/3} M_A}{L^{1/3}} < r_L < L M_A^{-3},$$  \hspace{1cm} (54)

provided that $l_{\text{min}} < L M_A^{-3}$. In the opposite case of $l_{\text{min}} > l_A$ the turbulence does not become Alfvénic even at the smallest scales.

For wavelengths larger than those given by Equation (53) and therefore for $r_L > L M_A^{-3}$, the damping is induced by Kolmogorov-type isotropic hydrodynamic turbulence which folds magnetic fields over the scale of eddies. The characteristic damping rate in this case is expected to coincide with the turnover time of the corresponding eddies, i.e.,

$$\Gamma_{\text{hydro}} \approx \frac{u_L \lambda}{\gamma} \approx \frac{V_A M_A}{L^{1/3} \lambda^{2/3}},$$  \hspace{1cm} (55)

where we used Equation (19).

### 4.5. Other Forms of Presenting Our Results

Emission of Alfvén waves by energetic particles moving along magnetic field lines presents the most important case of Alfvén wave emission in the local system of reference. The resonant emission along the local magnetic field direction corresponds to the condition

$$\lambda = r_L,$$  \hspace{1cm} (56)

where $r_L = \gamma mc^2/eB$ is the Larmor radius of the resonant particle with a relativistic factor $\gamma$. We shall use Equation (56) in expressions below.

Expressing wave damping through the cascading rate is another way of presenting our results. Cascading of turbulent energy is a source of media heating. This can provide upper limits on the level of turbulence in astrophysical environments, which is valuable when the scales of the turbulent motions and injection rates are difficult to estimate. The cascading rate of the weak turbulence given by Equation (11) can be rewritten as

$$\epsilon_{\text{w}} \approx \frac{V_A^2 M_A^2}{L}.$$  \hspace{1cm} (57)

3 The situation is changing with the development of new techniques that obtain the injection scale and injection velocity from observations (see Chepurnov et al. 2010, 2015; Lazarian & Pogosyan 2012).
which shows a decrease of the energy dissipation by a factor $M_A^4$ compared with the case of trans-Alfvénic turbulence. For $n_s < L M_A^4$ the damping rate for waves can be obtained by expressing $M_A$ from Equation (57) and substituting it in Equation (34):

$$\Gamma_{\text{sub}_A,s} \approx \frac{\epsilon_{\text{w}}^{1/2}}{V_{L}^{1/2} L_{A}^{1/2}},$$

which differs from the expression in FG04 by the use of the cascading rate for weak turbulence $\epsilon_{\text{w}}$ instead of the cascading rate for strong turbulence. Thus, the obtained damping rate for sub-Alfvénic turbulence is $M_A^4$ times less than in the case of trans-Alfvénic turbulence (see also Equation (34)).

For $L M_A^4 < \lambda < L M_A$ we get the expression that is significantly different from its form in FG04. Indeed, expressing $M_A$ from Equation (57) and substituting it in Equation (46) we can get

$$\Gamma_{\text{sub}_A,w} \approx \frac{\epsilon_{\text{w}}^{1/3} L_{A}^{1/3}}{V_{L}^{1/2} L_{A}^{1/2}} \approx \frac{\epsilon_{\text{w}}^{1/3} M_A^{4/3}}{r_{L}^{2/3}},$$

The expression given by Equation (59) has a slower dependence on the dissipation rate compared to Equation (58). The suppression of damping rate by the factor $M_A^{4/3}$ (see Equation (46)) is important and for $M_A \ll 1$ it explains the smooth transition to the regime of insignificant Alfvén-wave damping that is present for marginally perturbed magnetic fields.

Dealing with the damping of Alfvén waves emitted by particles with larger $L$, one can obtain the expression for the damping for $L M_A < n_s < L$ (see Equation (42)) which corresponds to the damping by the outer scale of turbulent motions:

$$\Gamma_{\text{outer}} \approx \frac{\epsilon_{\text{w}}^{1/2}}{L^{1/2} V_{L}^{1/2}}.$$

For super-Alfvénic strong MHD turbulence, if one expresses $M_A$ from Equation (21) and substitutes it in Equation (52) it is easy to get

$$\Gamma_{\text{super}} \approx \frac{\epsilon_{\text{w}}^{1/2}}{V_{L}^{1/2} L_{A}^{1/2}},$$

which has the same form as the expression for the damping for sub-Alfvénic strong turbulence given by Equation (58). The cardinal difference between the two expressions, assuming that the injection scale $L$ is the same, stems from the differences in the cascading rates in super-Alfvénic and sub-Alfvénic turbulence. The sub-Alfvénic turbulence induces a significant reduction of the cascading rate compared to the trans-Alfvénic turbulence; the super-Alfvénic strong MHD turbulence induces a significant increase of dissipation compared to the trans-Alfvénic case. Thus, for the same $L$, the damping of Alfvén waves by super-Alfvénic turbulence is more efficient than damping by the sub-Alfvénic one. The damping rate for $n_s > \lambda_{\text{max,super}}$, where the latter is given by Equation (53), is produced by hydrodynamic turbulence and therefore is

$$\Gamma_{\text{hydro}} \approx \frac{\epsilon_{\text{hydro}}^{1/3}}{r_{L}^{2/3}}.$$}

In view of the astrophysical importance of damping in sub-Alfvénic turbulence, it is useful to rewrite the expressions given by Equations (34) and (46) in terms of $\lambda_{\text{max,s}}$ given by Equation (39), namely,

$$\Gamma_{\text{sub}_A,s} \approx \frac{V_{A}^{1/2}}{L_{A}^{1/2}} \left( \frac{\lambda_{\text{max},s}}{r_{L}} \right)^{1/2}, \quad r_{L} < \lambda_{\text{max},s},$$

and

$$\Gamma_{\text{sub}_A,w} \approx \frac{V_{A}^{1/2}}{L_{A}^{1/2}} \left( \frac{\lambda_{\text{max},s}}{r_{L}} \right)^{2/3}, \quad r_{L} > \lambda_{\text{max},s}.$$}

Expressed in this form, Equation (63) explicitly shows that the damping by strong turbulence $\Gamma_{\text{sub}_A,s}$ is faster than the Alfvén crossing rate of the injection scale eddies, while in the case of weak turbulence Equation (64) shows that $\Gamma_{\text{w}}$ is slower than the aforementioned rate.

For $L M_A < n_s < L$, the damping rate can be written as (see Equations (60) and (39))

$$\Gamma_{\text{outer}} \approx \Gamma_{\text{sub}_A,w} \frac{r_{L}}{L},$$

which presents another form for the Alfvén-wave damping by turbulence at the outer scale.

5. DAMPING OF ALFVÉN WAVES GENERATED IN THE GLOBAL SYSTEM OF REFERENCE

Below we consider the damping of Alfvén waves generated by an outside source that is not related to the magnetic field structure. It is important to realize that such waves should be viewed as being in the global system of reference and therefore our earlier treatment of damping is not applicable. This is a separate case of damping relevant to many astrophysical settings, e.g., the emission of waves by stellar surface activity (see Section 8.5).

5.1. Case of Strong Sub-Alfvénic Turbulence

Consider first an Alfvén wave moving at an angle $\theta \gg \delta B/B$ with respect to the mean magnetic field. In this situation one can disregard the dispersion of propagation angles that arises from turbulent magnetic wandering. For this purpose we use $\sin \theta$ instead of $\sin \theta_L$ in Equation (37) and obtain for the perpendicular scale of eddies

$$x \approx \frac{\lambda}{\sin \theta}.$$}

The rest goes along the same line of reasoning that we employed in Section 3. Indeed, the rate of the wave damping is equal to the turnover rate of strong sub-Alfvénic eddies. Therefore, using Equation (66) it is easy to get

$$\Gamma_{\text{sub}_A,L} \approx \frac{V_{A} M_A^{1/3} \sin^{2/3} \theta}{X^{2/3} L^{1/3}},$$

which provides the nonlinear damping rate of an Alfvén wave moving at the angle $\theta$ with respect to the mean field.
In terms of the cascading rate of weak turbulence $\epsilon_w$ (see Equation (11)), the above damping rate for the wave can be rewritten as

$$\Gamma_{\text{subA},x,0} \approx \frac{\epsilon_{w}^{1/3} \sin^{2/3} \theta}{L^{1/3}}. \quad (68)$$

The turbulent damping given by Equation (68) is applicable in

$$l_{\text{min}} \sin \theta < \lambda < L^2 \Lambda \sin \theta,$$  

(69)

where $l_{\text{min}}$ is the minimal scale, i.e., the perpendicular damping scale, and $L^2 \Lambda = l_{\text{trans}}$ is the maximal scale for the extent of the turbulent cascade.

Naturally, for this expression our approximation $\theta \gg \delta B/B$ fails if the wave is launched parallel to the mean magnetic field. The directions of the local magnetic field experience dispersion, and this makes the actual $\theta_0$ not zero. In the global system of reference, the dispersion is determined by the magnetic field variations at the injection scale (see Cho et al. 2002). Therefore

$$\theta_0 \approx \frac{B_\perp}{B} \approx M_\Lambda, \quad (70)$$

Substituting this into Equation (67) we get

$$\Gamma_{\text{subA},x,0} \approx \frac{\epsilon_{w}^{1/3} M^{2/3}_\Lambda}{L^{1/3}}, \quad (71)$$

which is different from our expression for the damping of Alfvénic waves moving along the local direction of the magnetic field (see Equations (34), (58)). The difference stems from the difference in Alfvén waves generated with respect to the local system of reference and the global system of reference. The rate given by Equation (71) is applicable in the range

$$l_{\text{min}} M_\Lambda < \lambda < L^2 \Lambda,$$  

(72)

which trivially follows from Equations (69) and (70).

### 5.2. The Case of Weak Sub-Alfvénic Turbulence

For weak sub-Alfvénic turbulence in the case $\theta \gg \delta B/B$, we shall use Equation (66) to relate the wavelength $\lambda$ to the scale of perpendicular motions that the wave interacts with while cascading, as well as Equation (45) to get the damping rate corresponding to such motions. As a result,

$$\Gamma_{\text{wea},x,0} \approx \frac{V_{\Lambda} \sin \theta M^{3}_\Lambda}{\lambda} \approx \frac{\epsilon_{w}^{1/2} L^{1/2} \sin \theta}{V^{3/2}_\Lambda}, \quad (73)$$

wherein the damping is expressed through the weak cascading rate $\epsilon_w$.

The applicability of this type of damping is applicable to

$$L^2 \Lambda \sin \theta < \lambda < L \Lambda \sin \theta,$$  

(74)

where the last inequality is obtained by substituting the maximum perpendicular scale of eddies $L \Lambda$ for $x$ in Equation (66).

For the propagation along the mean magnetic field one should take into account Equation (70) which results in

$$\Gamma_{\text{wea},x,0} \approx \frac{V_{\Lambda} M^{3}_\Lambda}{\lambda} \approx \frac{V_{\Lambda} \epsilon_{w}^{3/4} L^{3/4}}{\lambda V^{3/2}_\Lambda}. \quad (75)$$

The range of the applicability of this damping rate is

$$L^2 \Lambda < \lambda < L^2 \Lambda,$$  

(76)

where Equations (70) and (74) were used.

#### 5.3. Other Cases

For strong super-Alfvénic turbulence, i.e., for damping by turbulent motions at scales less than $L \Lambda$, one can still use our approach above and consider damping of Alfvén waves with $\lambda < l_\Lambda$ (see Equation (55)). The damping by eddies less than $l_\Lambda$ happens within one eddy turnover time. If the wave is at an angle $\theta$ to the magnetic field within a magnetic eddy $< l_\Lambda$, then the damping happens over one turnover time for motions of size $x$ defined by Equation (66). The procedures analogous to those we employed above provide

$$\Gamma_{\text{super},x,0} \approx \frac{V_{\Lambda} M^{3}_\Lambda}{\lambda^{1/2} L^{1/2} \Lambda}, \quad (77)$$

where in super-Alfvénic turbulence, the angle $\theta$ changes from one strong turbulence eddy of size $l_\Lambda$ to another. Therefore an averaging over such changing directions should be performed, which for a random distribution of directions gives the damping rate of $\langle \sin^{2} \theta \rangle = 3/5$.

For Alfvén waves from a macroscopic source $\gg l_\Lambda$ the turbulent volume can be considered as consisting of MHD cells with regular MHD turbulence but with the injection of trans-Alfvénic turbulence at the scale $l_{\Lambda}$. The wave damping will differ depending on the angle $\theta$ between the magnetic field in a given cell and the wave propagation direction. The rate of damping can be obtained by substituting in Equation (67) the actual angle $\theta$ as well as $M_\Lambda = 1$ and $L = l_{\Lambda}$. The minimum wavelength in this case depends on $l_{\text{min}} \sim \theta$.

At scales larger than $l_\Lambda$ the turbulence is essentially hydrodynamic. Therefore, for turbulent damping by super-Alfvénic eddies of size larger than $l_{\Lambda}$ as well as for damping by outer-scale eddies there is no difference between local and global frames. Therefore our earlier results are applicable.

#### 5.4. Finite-size Macroscopic Emitter

Our considerations obtained for an infinitely extended microscopic emitter can be generalized for a finite-size emitter. If the size of the emitter is $y$ and the wave is emitted at the angle $\theta \gg \delta B_y/B$, then our considerations in Sections 5.1–5.2 stay the same. Note, however, that $B_{\parallel}$ in this case is $\min [B_{\parallel}, B_{\text{damp}}]$, where $B_{\text{damp}}$ is the magnetic field deviation at the scale of wave damping, i.e., $l_{\text{damp}} \approx \Gamma_{\text{global},0} V_{\Lambda}$, where $\Gamma_{\text{global},0}$ is, for instance, defined for weak and strong sub-Alfvénic turbulence in Sections 5.1 and 5.2.

If the wave is emitted parallel to the local magnetic field at the scale $y$, then we have to deal with the intermediate case having features of Alfvén-wave damping in local and global systems of reference. Indeed, the variations of the magnetic field directions should be calculated at the scale of $y$ and compared with the variations of the magnetic field at the “resonant” scale. For strong sub-Alfvénic turbulence, this scale is given by Equation (31) and for weak sub-Alfvénic turbulence by Equation (43). The latter two scales depend on $\Lambda$. Therefore we expect to see the scaling corresponding to Alfvén-wave damping if $y$ is smaller than the values given by the aforementioned equations. Note that the damping of the waves emitting with respect to the mean magnetic field will be
happening inhomogeneously with patches where the local magnetic field happens to parallel to the wavefront having the ability to support Alfvén-wave propagation for a longer period of time.

6. COMPARISON WITH NONLINEAR LANDAU DAMPING

It is important to compare the turbulent damping that we study in this paper with the nonlinear Landau damping process that can also damp Alfvén waves (see Kulsrud 2005). The latter damping is inversely proportional to the square root of the CR scale-height $L_\perp$, so we may expect that this process is subdominant for weak gradients of the CR distribution. Indeed, the ratio of the rate of turbulent sub-Alfvénic damping and the rate nonlinear Landau damping $G_{NL}$ can be evaluated to give

$$\frac{\Gamma_{\text{subA},t}}{\Gamma_{NL}} \approx \frac{B_{\parallel}^2}{\nu_{\perp}} \frac{n_{\perp}^{1/2} L_{\perp,100}^{1/2} M_\ast^{1/2}}{B_{\perp,100}^{1/2} n_{\perp} \gamma^{1/2} \gamma_{100}^{1/2}}, \quad (78)$$

where $T_{4\text{keV}} = (T/4 \text{ keV})$, $B_{\parallel,100} = (B/1 \mu G)$, $L_{\perp,100} = (L_\perp/100 \text{ kpc})$, $n_{\perp,3} = (n_{\perp}/10^{-3} \text{ cm}^{-3})$, $n_{\perp,10} = n_{\perp}^{CR}(\gamma > 1)/10^{-10} \text{ cm}^{-3}$, $\gamma_{100} = \gamma/100$, and scaled to $n = 4.6$. Note that $\nu^{CR}(>\gamma) = 10^{-10} \gamma^{-1.6} \text{ cm}^{-3}$ of the order a CR energy density in equipartition with a $\sim \mu G$ B-field. Therefore, if the CR profile falls ($n_{\perp} \rightarrow 0$) and flattens ($L_\perp \rightarrow \infty$), the nonlinear Landau damping becomes subdominant.

One may wonder whether for very weak levels of turbulence $M_\ast \rightarrow 0$, the nonlinear Landau damping may become important. The latter, however, is a self-regulated process as the suppression of the streaming instability is bound to allow the CRs to spread fast, decreasing the CR gradient. Resonance scattering could potentially mitigate this spreading. However, this depends on the presence of fast modes that, in the absence of streaming instabilities, were identified in Yan & Lazarian (2002) as a major factor of CR scattering in the interstellar plasmas.\footnote{The importance of fast waves is easy to understand. One should recall that due to the extreme anisotropy of the tensor that describes the Alfvén turbulence at small scales (Cho et al. 2002), the scattering by Alfvénic modes of the MHD cascade initiated at the large injection scale $L$ is very small (Chandran 2000; Yan & Lazarian 2002).}

In many instances, e.g., galactic halos, these modes are subject to significant collisionless damping, and therefore their efficiency in controlling the CR streaming is limited. On the contrary, there is no such self-regulation for turbulent damping of CR streaming, which makes the process dominant in most astrophysical settings. In many instances when the turbulent damping fails, the nonlinear Landau damping is unlikely to damp the CR streaming either. For instance, we argue in the next section that the turbulent damping of the streaming instability is not important for the Milky Way halo due to the low level of turbulence there. Therefore we do not expect nonlinear damping to be important there due to the self-regulation which entails the increase of $L_\perp$.

7. IMPLICATIONS FOR GALACTIC CR TRANSPORT

7.1. Physics of CR Streaming

One of the simplest models of galactic CR propagation is the so-called “leaky box model” (see Longair 2011). Within this model, CRs propagate freely within the galactic disk, while they experience streaming instability as they enter a fully ionized halo surrounding the galaxy. Free zooming through the galactic disk is possible as, in the leaky box model, the galactic disk is assumed to be partially ionized and therefore the streaming instability is being suppressed by ion-neutral damping (see Kulsrud & Pearce 1969). This model is surely naive, as the galactic disk is definitely not fully filled with partially ionized gas. In fact, a significant fraction of the galactic disk is filled with hot ionized gas (McKee & Ostriker 1977, see Draine 2011). Moreover, the leaky model does not account for turbulent damping of streaming instability.

The first treatment of CR propagation that took streaming instability damping into account was by FG04. This study came to a paradoxical conclusion, namely, that turbulence suppresses streaming instability for most of the CR energies, and therefore it is really difficult to understand the observed high isotropy of CRs. Below we subject the problem to scrutiny and arrive at conclusions different from those FG04. We feel that there are two serious problems with the physical assumptions in FG04 when they evaluate the expected rate of streaming. Thus, first we deal with the problem of streaming velocity. In particular, we will show that (a) damping is produced by weak rather than strong Alfvénic turbulence and therefore is reduced, and (b) the turbulent dissipation rate assumed in FG04 to be equal to the plasma cooling rate is an overestimate of the actual dissipation rate, as this way of estimating disregards other important heating mechanisms. In addition, we disagree with FG04 in the way of relating the streaming velocity and the observed degree of anisotropy. We discuss the corresponding point at the end of this subsection.

Our analysis above shows that turbulent damping of the streaming instability changes significantly depending on whether the damping is performed by strong or weak Alfvénic turbulence. Note that it is natural to assume from the very beginning that turbulence in the halo is sub-Alfvénic rather than trans-Alfvénic or super-Alfvénic. This fact is easy to understand. Indeed, the ISM is turbulent (see Armstrong et al. 1995; Elmegreen & Scalo 2004; McKee & Ostriker 2007; Chepurnov & Lazarian 2010) and the sources of turbulence driving, whether they are related to supernovae (see MacLow 2004; Draine 2011) or magnetorotational instability (see Mac Low & Klessen 2004), are within the galactic disk. The magnetic field in the halo is expected to become more and more quiescent with increasing distance from the disk as turbulence decays diffusing from the disk. Our quantitative estimates based on the observational data that we provide below support this intuitive notion.

For sub-Alfvénic turbulence it is possible to express the streaming rate using the textbook approach to streaming instability (see Kulsrud 2005) but equating the turbulent damping rate to the streaming instability growth rate in Equation (27) (see FG04):

$$\nu_{\text{stream}} \approx V_\perp \left( 1 + \frac{\Gamma_{\perp,10} n_\gamma}{\gamma_{\perp} n_{\perp}} \right). \quad (79)$$

where we used the relation $n_\perp = \gamma c \Omega^{-1}$. Note that the rates of damping $\Gamma$ are different for weak and strong turbulence. In particular, the damping is by strong turbulence if $\Gamma = \Gamma_{\text{subA},t}$, i.e., for $n_\perp < M_\perp$, and by weak turbulence if $\Gamma = \Gamma_{\perp}$, i.e., for $n_\perp > M_\perp$. For the Milky Way galaxy, the quantities that enter Equation (79) can be estimated for the hot coronal gas of the halo, i.e., plasma with density $n_\parallel \approx 10^{-3} \text{ cm}^{-3}$, temperature
To find the streaming velocity using Equation (81) one should know both the dissipation of weak turbulence $\epsilon_w$ and the Alfvén Mach number $M_A$. Two different estimates of the cascading rate were presented in FG04. One was based on the cooling rate for the hot gas, the other was based on the supernova energy injection rate. The latter is readily available. Indeed, it is accepted that supernovae are releasing $10^{51}$ ergs of mechanical energy into the gas once every one million years in a disk area of 100 pc$^2$ (see Drainé 2011). Assuming that the resulting turbulence is trans-Alfvénic and therefore decays in one Alfvén crossing time, FG04 obtained the estimates for the turbulent dissipation rate of $\sim 25$ erg s$^{-1}$ g$^{-1}$. We believe that this is an estimate that has relevance to the galactic disk, rather than to the galactic halo. Such a significant rate of turbulent dissipation according to Equation (80) should ensure that within the media of the galactic disk, Alfvénic turbulence suppresses streaming instability, which corresponds to the disk part of the “leaky box” model.

The situation is very different for the hot plasmas in the Milky Way halo. There the second estimate in FG04 based on the gas cooling might be relevant. Indeed, the turbulent cascading rate determines hot gas heating, and this cannot be larger than the radiative cooling rate, which is about 0.06 erg s$^{-1}$ g$^{-1}$ (see Binney & Tremaine 1987). In fact, this provides the upper limit for the turbulent cascading, and the actual rate, as we discuss further, may be substantially lower. Indeed, turbulent heating is not the only way of heating the halo plasmas. For instance, we may consider heating that comes from CR streaming (see Wiener et al. 2013a). The irreversible energy transfer from streaming CRs to gas provides the volumetric heating rate (see Kulursd 2005):

$$\Gamma_{\text{heat}} \approx -V_A \nabla P_{\text{cr}}.$$  

(82)

To get the heating per unit of mass, one has to divide the heating rate given by Equation (82) by the plasma density. Taking as a rough estimate the energy density of CRs to be 1 eV per cm$^{-3}$ and the characteristic scale of the CR change to be $L_{\text{cr}} \approx 5$ kpc, one gets heating $\sim 0.06$ erg s$^{-1}$ g$^{-1}$, which coincides with the cooling rate in Binney & Tremaine (1987). This may indicate that the galactic halo is heated by CR streaming that does take place in the halo environment. As a result, the cascading rate adopted in FG04 significantly overestimates the actual turbulence cascading in the halo gas of the Milky Way.

We feel that it is most important to establish whether streaming instability really fails in the realistically turbulent Milky Way halo. Therefore, for the rest of our discussion, we concentrate in showing that the conclusions about the “streaming catastrophe” reached in FG04 are not obtained in a self-consistent basis.

6 The rate of turbulent heating by supernovae above and the upper limit of turbulent heating at hand are so different both due to the decrease of turbulent velocities in the galactic halo compared to the disk and to the decrease of the Alfvén Mach number $M_A$. The latter makes turbulence less dissipative in proportion to $M_A$.

7 We note parenthetically that the adopted cascading of $\sim 0.06$ erg s$^{-1}$ g$^{-1}$ corresponds to $M_A \approx 0.2$ if the injection scale $L = 100$ pc is adopted. This suggests that even with this cascading rate that we argue to be an overestimate, the turbulence is sub-Alfvénic.
It is well known that the anisotropy is less than 0.1% for the CRs with $\gamma < 10^6$ (see Longair 2011). As the Alfvén velocity in the hot plasmas is $\sim 0.1\%$ of $c$, FG04 assumed that the second term of Equation (80) in brackets is not larger than unity.\footnote{The difference in terms of strong turbulence cascading assumed in FG04 and the weak that is employed in Equation (80) is not important for the argument as we discussed earlier.} This provided $v_{\text{stream}} \approx V_A (1 + 0.01 \gamma^{1/3})$ for the cascading of $\sim 0.06 \text{ erg s}^{-1} \text{ g}^{-1}$. On the basis of this estimate, FG04 concluded that to avoid the contradiction with the observational data for $\gamma \sim 10^9$ one should assume that the rate for the turbulent dissipation is less than $4 \times 10^{-14} \text{ erg s}^{-1} \text{ g}^{-1}$, which is very different from the assumed $\sim 0.06 \text{ erg s}^{-1} \text{ g}^{-1}$ rate. On the basis of this FG06 came to the conclusion that streaming instability is not feasible as the solution for solving the problem of explaining the observed isotropy of cosmic rays.

As we pointed above, for realistic magnetization of the galactic halo $M_A \ll 1$ and Equation (81) rather than Equation (80) should be used to determine the streaming velocities.\footnote{In fact, for $M_A < 0.003$, the CRs with $\gamma = 10^6$ interact with the turbulence in the outer scale, which further reduces turbulent damping (see Equation (60)).} It is safe to say that in the situation where the turbulent damping in galactic halos is not being constrained observationally it is premature to be alarmed about the failure of streaming instability to explain the CR isotropy. In fact, we expect the turbulent velocity to decrease quickly with distance from the galactic plane. In addition, due to the drop in the plasma density, we also expect the exponential increase of $V_A$ with distance from the galactic plane. Therefore, in Equation (81), both $\epsilon_v$ and $M_A$ are likely to decrease exponentially, i.e., $\sim \exp(-H/h)$, where $H$ is the halo size $\sim 5 \text{ kpc}$ and $h \sim L$ is the scale-height of the galactic disk, which is one or two hundred parsecs. It is likely that at some distance from the disk $\gg L$ the second term in brackets in Equation (81) becomes small. This is the only thing required for the streaming instability to isotropize CRs.

Expressing the streaming velocity through the turbulence dissipation rate is advantageous only when this dissipation rate is readily available from observations. In the situation of the galactic halo where the turbulent heating may not be the dominant process, it seems advantageous to use the other expressions for the turbulent damping rate, e.g., for the damping by weak turbulence to substitute in Equation (79) the expression for damping given by Equation (46). This way we get

$$v_{\text{stream, w}} \approx V_A \left(1 + \frac{V_A n_B l_h^{1/3} M_A^{8/3}}{L^{7/3} \gamma^{1/3}} \right),$$

where the turbulence injection scale $L$ can be obtained from observations with statistical techniques using spectral lines (see Chepurnov et al. 2010, 2015) or synchrotron emission (see Lazarian & Pogosyan 2012, 2016), while the Alfvén Mach number $M_A$ can be obtained using anisotropy studies with spectral lines (see Esquivel & Lazarian 2005; Burkhardt et al. 2014; Esquivel et al. 2015; Kandel et al. 2016a, 2016b) or synchrotron studies (see Lazarian & Pogosyan 2012, 2016; Herron et al. 2016). In particular the variations of $L$ and $M_A$ with distance from the observer can be obtained using multifrequency polarization studies as explained in Lazarian & Pogosyan (2016). We believe that this is a promising future direction of research.

In fact, in view of our study, the “leaky box” model can be reformulated. Instead of suppression of streaming instability in the disk by ion-neutral collisions, the instability is likely to be efficiently suppressed by turbulence there. At the same time, the streaming instability can be present in the Milky Way halo, returning and isotropizing CRs. This, as we discuss below, should not be viewed as an endorsement of the leaky box model. On the contrary, we consider that this model is unrealistic and disregards a lot of what we know about turbulence and the magnetic field structure of the Milky Way galaxy.

Consider to what extent the streaming instability velocity that we obtained above can be directly related to the degree of anisotropy. In FG04 the degree of anisotropy was equated to the ratio of the streaming velocity to the speed of CR, which is approximately the speed of light, i.e., the degree of anisotropy was estimated as $v_{\text{stream}}/c$. It is evident that this estimate is valid within a very simple incarnation of the “leaky box model,” where there exists a single source of CRs and they zoom along straight magnetic field lines within the disk and experience scattering only in the hot media where they induce streaming instability. Naturally, the distribution of CR sources as well as the structure of the Milky Way magnetic fields are quite different. For instance, in the presence of CR sources distributed in space, the observed CR anisotropy will depend on the distribution and will be mitigated, in general, compared with the case of a single source. In addition, a more complex/realistic topology of magnetic field is expected to influence and, in most cases, decrease the degree of CR anisotropy. More importantly, one should take into account that MHD turbulence not only suppresses the streaming instability but also isotropizes CRs. The isotropization in magnetic turbulence comes from the combination of two effects. One of them arises from the wandering of magnetic field lines and is usually associated with the diffusion of CR perpendicular to the mean magnetic field (see Jokipii 1973). In fact, as it was shown in Lazarian & Yan (2014), for sub-Alfvénic turbulence, on scales less than the injection scale $L$, the CR dynamics induced by the magnetic field wandering is not diffusion but superdiffusion.\footnote{Magnetic-field wandering for Alfvénic turbulence was first described in LV99 and later employed in solving different problems from thermal conduction of magnetized plasmas (Narayan & Medvedev 2001, Lazarian 2006) to shock acceleration (Lazarian & Yan 2014).} Indeed, the spread of magnetic field lines scales as $s^{3/2}$, where $s$ is the distance along the magnetic field (LV99), which presents the special case of Richardson dispersion (see Eyink et al. 2011). As a result, CRs following magnetic field lines spread superbalistically in the direction perpendicular to the mean magnetic field, modifying and decreasing the anisotropies (see Lopez-Barquero et al. 2016). On scales larger than $L$, magnetic-field wandering induces diffusion of CRs.\footnote{For super-Alfvénic turbulence, superdiffusion happens on scales less than $l_A$ and is overtaken by diffusion at scales larger than $l_A$.}
degree of anisotropy even in the absence of streaming instability. In fact, the model of an extended magnetic halo for the Milky Way was originally suggested to explain the observed isotropy of CRs without appealing to the effects of streaming instability (see Ginzburg 1988). At that time, the sensitivity of radio telescopes was not sufficient to test the extent of the magnetic halos. However, modern observations confirm the existence of synchrotron-emitting regions of a few kiloparsecs around spiral galaxies (Beck 2015). Therefore, we do not think that the streaming instability alone is responsible for the observed isotropy of CRs, and to estimate the degree of isotropy, more sophisticated models of CR propagation are required. A quantitative study of such models is beyond the scope of this paper.

We should note that additional ways of CR isotropization were also mentioned in FG04. However, FG04 pointed out the magnetic mirror effect arising from dense molecular clouds, which was the idea put forward in Chandran (2000) in his attempts to reconcile the low efficiency of CR scattering by Alfvén waves and the established significant residence time of CRs in the Milky Way galaxy. This way of removing anisotropies looks questionable, however. Indeed, in view of the low filling factor of dense clouds it looks unrealistic to think that CRs have to encounter many magnetic bottles created this way prior to their leaving the galaxy. In addition, with the new data that shows that the strength of magnetic fields stays in a significant fraction of molecular clouds on a level close to the value of the field in diffuse interstellar medium (Crutcher et al. 2010), the confinement efficiency of magnetic bottles created by molecular clouds is expected to be low.12

All in all, our calculations of the streaming instability provide streaming velocities less than the FG04 estimate. This should mitigate the anisotropies of galactic CRs. However, in realistic models of CR propagation, other effects contribute to achieving CR isotropy. Therefore, we do not believe that there is any quantitative contradiction that exists between the efficiency of streaming instability damping that is calculated in this paper and the observed degree of anisotropy of the CRs.

7.3. Possible Role of Nonlinear Landau Damping

In the previous section we provided the arguments why the nonlinear Landau damping of the streaming instability is expected to be less important compared to the turbulent damping. In application to Milky way propagation, the arguments are straightforward. In the case of no turbulent damping and the gradients of CRs in the galaxy, nonlinear Landau damping can act to suppress the streaming instability. However, doing so, the action of nonlinear Landau damping will act to suppress the streaming instability. This means that the CRs can spread over a larger scale $L_{\text{CR}}$, which, in turn, decreases the efficiency of the damping. For an idealized model of CR propagation, one may expect to get a critically balanced damping at which $L_{\text{CR}}$ will readjust to keep the instability at a relatively low efficiency. This introduces another parameter for the degree of anisotropy that makes the problem highly nonlinear even in the framework of the oversimplified and unrealistic version of the leaky box model discussed above. This problem is solvable, but earlier we pointed out that the actual galactic CR propagation is a much more complicated process, and therefore we are not motivated to address this problem in this paper. The processes of diffusion and superdiffusion of CRs discussed above will act to smooth the gradients, decreasing the role of nonlinear Landau damping. Estimating this is possible only in detailed models of galactic CR propagation, which is far beyond the scope of the present study.

8. OTHER ASTROPHYSICAL IMPLICATIONS

The paradox of expected CR anisotropies presented in FG04 got the biggest resonance. However, there are many other exciting implications of the effect of Alfvén-wave damping by MHD turbulence. It is also important to stress that our study covers not only the case of streaming instability damping but also deals with the damping of Alfvén waves emitted by macroscopic sources. We sketch a few astrophysical cases to which our results are applicable, while the detailed treatment of these implications will be provided elsewhere.

8.1. Acceleration of Particles in Shocks

We also want to stress that the issues related to CR streaming are not limited to the observed CR isotropy. We believe that the CR streaming instability can also be present in the galactic disk but at places of significantly higher than average CR flux, e.g., near places of CR acceleration, such as shocks (see Bell 1978; Schlickeiser 2002) or reconnection sites (de Gouveia dal Pino & Lazarian 2005; Lazarian 2005; Drake et al. 2006; Lazarian & Opher 2009).

The acceleration of CRs in shocks is an accepted process for explaining the population of galactic CRs (Bell 1978; Krymskii et al. 1978; Armstrong & Decker 1979). To be efficient, the process of CRs returning back to the shock must also be efficient. Potentially, the streaming instability should be important for returning particles back (see Longair 2011). Turbulence, however, is likely to complicate the process. In fact, apart from pre-existing turbulence, there is turbulence that is generated both in the precursor (Beresnyak et al. 2009; del Valle et al. 2016) and the post-shock media (Giacalone & Jokipii 2007). This super-Alfvénic small-scale turbulence is expected to efficiently damp the streaming. At the same time, the same turbulence also generates a turbulent magnetic field, which can act as a magnetic mirror that returns the CRs back to the shock. Therefore, it is likely that the CR acceleration in shocks proceeds without the important contribution from the streaming instability.

8.2. Effects of Turbulent Reconnection

Streaming instability can also return particles accelerated by magnetic reconnection to the reconnection site, enhancing the first-order Fermi acceleration that arises from reconnection (de Gouveia dal Pino & Lazarian 2005; Drake et al. 2006). The corresponding reconnection can proceed both when large-scale magnetic field reconnects releasing its free energy and within multiple reconnection regions in the steady-state turbulence. The latter process was recently considered in Brunetti & Lazarian (2016). The role of streaming instability depends on the level of turbulence in the system. Generically, we expect the level of turbulence to increase in the reconnection regions as magnetic reconnection progresses (see Lazarian et al. 2016) and therefore the role of streaming instability to decrease. However, the study of the parameter space for which streaming
is important both for magnetic reconnection and shock CR acceleration is beyond the scope of the present study. We also note that magnetic reconnection can be a source of Alfvénic waves (see Kiguru et al. 2010). As the process of reconnection happens generically in turbulent fluids, it is natural that the generated Alfvén waves should experience turbulent damping. Eventually, as we discussed, this should contribute to generating more turbulence in the reconnection region. Turbulence was shown in LV99 to change the nature of magnetic reconnection, making it independent of plasma resistivity (see more in Kowal et al. 2009, 2012, Eyink et al. 2011, 2013; Eyink 2015; Lalescu et al. 2015). Turbulence is being generated by reconnection, thus inducing fast reconnection in the case when the initial state of magnetized plasmas is not turbulent (Berensnyak 2013; Lazarian et al. 2015; Oishi et al. 2015). In highly magnetized plasmas with magnetic energy significantly exceeding the thermal energy, the transition to turbulent reconnection has an explosive character, with a higher level of turbulence increasing the rate of reconnection and the higher reconnection increasing the level of turbulence (LV99; Lazarian & Vishniac 2009). Our study shows that the transition to turbulence is inevitable even if initially a significant part of energy leaves the reconnection zone in the form of Alfvén waves.

8.3. Implications for Galaxy Clusters

In WOG (see also Ensslin et al. 2011; Pinzke et al. 2015), streaming-instability suppression was invoked to explain the bimodality of the cluster radio emission, namely, the fact that the majority of clusters are radio quiet (Brunetti et al. 2007, 2009; Brown et al. 2011; Brunetti & Jones 2014, and references therein), and it is only the clusters associated with merger activity that demonstrate radio halos. The authors above suggested a way to account for this property by assuming that the CRs escape at super-Alfvénic speeds, and this fast escape turns off the radio galaxies (see Ensslin et al. 2011).

It was shown in WOG that nonlinear Landau damping (e.g., Felice & Kulskud 2001) is too weak to inhibit wave growth, while turbulent damping (YL02, FG04) can suppress instability. This conclusion agrees with our analysis in Section 6. Moreover, our present study allows us to express the results in WOG in terms of the actual parameters of the turbulence in galaxy clusters, e.g., their magnetization and the turbulence injection scale. This turbulence is accepted to be super-Alfvénic (see Brunetti & Lazarian 2007; Miniati & Beresnyak 2015). The Alfvén Mach number of the intracluster medium is expected to vary depending on the level of turbulence. In Brunetti & Lazarian (2016) the range of $M_A$ was estimated to be from 3 to 9. The value of $L_\Lambda$ thus may range from approximately 10 pc to 0.3 pc if we assume the injection scale of 10$^3$ pc. Our study dictates that these values of $L_\Lambda$ should be used in WOG for the $L_{\text{MHD}}$ that they employ in their study while dealing with the streaming instability by strong MHD turbulence. Adopting $L_\Lambda = n_l = 1$ pc one obtains that CRs with $\gamma < 10^5$ interact with strong turbulence, as it is assumed in WOG. At the same time the streaming induced by CRs with higher $\gamma$ is affected by the damping induced by super-Alfvénic turbulence in the hydrodynamic regime.

Our quantitative insight strengthens the conclusion in WOG that the CRs can stream rapidly in the presence of super-Alfvénic turbulence. However, the consequences of this effect for the dynamics of CRs on large scales are not easy to evaluate. Indeed, the escape of CRs is limited not only by streaming but also by turbulence scattering as well as the diffusion of magnetic field lines. In super-Alfvénic turbulence, the latter are entangled on the scale $l_\Lambda$ (e.g., Brunetti & Lazarian 2007), which produces the random walk with the scale of $l_\Lambda$. This entails the increase of the escape time by a factor $(D/l_\Lambda)^2$, where $D$ is the length of order of Mpc that the particles should cover, while our estimate of $l_\Lambda$ is of the order of 1 pc. These are the complications that should be considered in future quantitative models. The process of turning off and on can also be explained by merger-induced scenarios of turbulent reacceleration as discussed in detail by Brunetti & Jones (2014 and references therein). A synthesis of the approaches above will be presented in a future publication.

8.4. Streaming of CRs and Ionization of Molecular Clouds

Streaming of CRs into molecular clouds is an interesting process that requires further studies. For instance, in a recent paper by Schlickeiser et al. (2016), streaming instability arising as the CRs penetrate molecular clouds was described. This study, however, does not account for a possible suppression of streaming instability by ambient turbulence. For super-Alfvénic turbulence, the processes of penetration of CRs inside the clouds are going to be modified. As a result, one can imagine a situation in which the coefficient for the “along the magnetic field” diffusion is larger in the outer turbulent parts of the molecular cloud and smaller at the inner part of the molecular cloud where the streaming instability operates and creates waves scattering CRs. In this situation, the density of CRs may potentially be higher in the interior of molecular clouds than in the ambient interstellar medium. This can also be relevant to explaining observations (see McCall et al. 2003; Le Petit et al. 2004) that suggest significant variations of the CR density in molecular gas. In realistic inhomogeneous interstellar gas one can expect regions where streaming instability is suppressed and regions where it operates, creating significant variations of CR diffusion and density.

8.5. Heating of Plasmas and Launching of Winds

While the damping of Alfvén waves by turbulence has become a well-accepted process in the field of CR research, in other fields the studies of Alfvén damping frequently ignore the turbulent nature of the magnetized plasmas and focus instead on wave steepening and pure plasma effects. Therefore we would like to point out that our results are applicable to the heating of stellar corona by Alfvén waves and launching of stellar winds by damping of Alfvén waves (see Suzuki & Inutsuka 2005; Verdini et al. 2005, 2010; Evans et al. 2009; Vidotto & Jatenco-Pereira 2010; Suzuki 2015). The cascading that we consider results in efficient dissipation of Alfvén waves, and this dissipation is very robust, i.e., it does not depend on the microphysics of plasma processes. Our results show that in highly magnetized regions of solar atmosphere with low Alfvén Mach number $M_A$, Alfvén waves can propagate for longer distances than in regions with lower $M_A$. This should be accounted for in the quantitative modeling of wind launching and plasma heating. We note that the turbulent damping scenario does not require efficient coupling between Alfvén and fast-mode turbulence, which is assumed in some of the studies (see Cramer et al. 2014); it does not require having nonlinear Alfvén waves of large amplitude either (cf. Airapetian et al. 2010). For instance, we believe that the turbulent damping can be relevant to explaining the observed “unexpected” damping of Alfvén waves in the regions above the
Sun’s polar coronal holes (Hahn et al. 2012). These and other issues should be clarified by further research that takes into account the turbulent damping of Alfvén waves.

Heating by waves emitted by various sources can be an important source of heating of turbulent plasmas in galaxy clusters. Our study provides a way to quantify the distribution of heating as a function of distance from the source.

Heating by Alfvén waves emitted by processes on the stellar surface and the processes of launching of stellar winds are intrinsically connected. Alfvén waves emitted by stars carry momentum. This momentum is deposited into the plasmas as Alfvén waves dissipate, and this can be the process that launches the wind itself or contributes to the process of the wind launching, e.g., together with the radiation force (see Suzuki 2011, 2015). The efficient turbulent damping of Alfvén waves that we have demonstrated in this paper makes this process efficient.

Interestingly enough, a sufficiently strong flux of Alfvén waves can induce an instability, resulting in the formation of an area of enhanced turbulence damping. Consider a train of Alfvén waves subject to turbulent damping in magnetized plasmas where a particular region has an enhanced level of turbulence. This area will induce stronger damping of Alfvén waves. Those, as they cascade, will decrease their perpendicular scale until their parallel and perpendicular scales eventually satisfy Equation (8), corresponding to the turbulence critical balance. So the cascading Alfvén waves will create more turbulence which through inverse cascading can produce motions that can cascade the Alfvén waves more efficiently.13 The turbulence initially gets imbalanced, but in realistic turbulent media with density inhomogeneities as well as in the presence of parametric instabilities (Del Zanna et al. 2001), the scattered waves become part of the balanced MHD turbulence.

The dissipation of Alfvén waves that we discussed above happens in the global system of reference. However, this does not exhaust all the possibilities for launching the winds. For instance, CRs can launch winds getting coupled with the magnetized plasmas through streaming instability (see Recchia et al. 2016). In the latter case, turbulent damping of Alfvén waves happens in the local system of reference. Our work shows that quantitative models of CR-driven winds (see Ruskowski et al. 2016) should account for the spatial change of turbulent damping arising from the change of $M_{A}$. We expect to see near the galactic disk the super-Alfvénic CR streaming that was invoked by Ruskowski et al. (2016) in their modeling. This, however, should change to Alfvénic streaming in the galactic halo as the turbulent damping is expected to get less efficient there. The consequences of this change are difficult to evaluate without detailed calculations.

8.6. Gyroresonance Instability and Other Effects

We used the streaming instability as an example of Alfvén-wave generation in the local system of reference. However, turbulent suppression of other types of plasmas and CR instabilities can be important. For instance, when equal fluxes of particles are moving in opposite directions, gyroresonance instability can be induced (see Lazarian & Beresnyak 2006, henceforth LB07). It arises from particles having anisotropic distribution in momentum space, and it results in the generation of Alfvén waves that scatter and isotropize the distribution.

The anisotropic distribution in momentum space can arise both due to plasma/CRCR compression by MHD turbulence, e.g., by slow modes (LB07), when the perpendicular momentum of the particles increases, or it can also arise due to the Transient Time Damping (TTD) acceleration of particles, e.g., by the acceleration of CRs by fast modes (see Yan & Lazarian 2004), when the parallel momentum of particles increases.14

In the presence of external turbulence, the Alfvén waves experience damping coinciding with the damping of the waves generated by the streaming instability. The corresponding suppression of the instability provides an attractive possibility of explaining why in the solar wind plasmas we see signatures of momentum randomization by mirror and firehose instabilities, but no signatures of gyroresonance (see Santos-Lima et al. 2014).

9. COMPARISON WITH EARLIER WORKS

Similon & Sudan (1989), in their studies of Alfvén-wave damping, used models of MHD turbulence that were not supported by further research. Yan & Lazarian (2002) suggested that the streaming instability can be suppressed by turbulence, but did not provide a quantitative study of the processes. In this situation the closest study to the present one is the quantitative pioneering study of the streaming instability damping by strong Alfvénic turbulence in FG04. In view of the theory provided in this paper, this is a particular case of damping. As we identified in our paper, this is the case of Alfvén waves emitted in the local system of reference. In the present paper, we also identified the other regime of damping, i.e., the damping of Alfvén waves emitted by a macroscopic source. The latter damping happens with respect to the mean magnetic field, i.e., in the global system of reference. The scalings of the damping are different in two cases (see Table 1).

As for the streaming instability damping, our treatment is different from FG04 and we explain the differences below. The model of turbulence adopted in FG04 is based on the assumption that the turbulent energy is injected at the scale $L_{MHD}$ with velocity $V_{A}$. This is a case of trans-Alfvénic turbulence with the caveat that FG04 did not associate $L_{MHD}$ directly with the injection scale, but defines this scale as the scale at which turbulence becomes trans-Alfvénic. Thus defined, $L_{MHD}$ can be associated with the scale $L_{A}$ for the transition to the MHD regime of the trans-Alfvénic turbulence that we quantified in this paper. The extension of the FG04 approach to strong sub-Alfvénic turbulence is problematic, however. No quantitative expressions of this $L_{MHD}$ are given in FG04 but the paper contains a footnote: “Turbulence can also be injected at smaller velocities on smaller scales, in which case $L_{MHD}$ should be considered an extrapolation beyond the actual outer scale of the cascade” (p. 671). This extrapolation has not been elaborated and it faces conceptual difficulties. Indeed, as we discussed in Section 2, the sub-Alfvénic turbulence has two regimes, weak turbulence and strong turbulence. The regime of

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13 This turbulence can scatter fluctuations of wavelengths smaller than the original wavelength of the train. It can also transfer some part of the energy to large scales through the inverse cascading process and thus increase the turbulent damping of the original train of Alfvén waves. The difference between the scales at which the damping of waves occurs depends on the angle $\theta$ between the direction of Alfvén waves and the mean magnetic field as well as the Alfvén Mach number of the original turbulence. For sufficiently large $\theta$ or/and sufficiently large $M_{A}$, the scales of turbulent damping and the transfer of the energy into the energy of turbulent motions can be close, making the instability efficient.

14 The isotropization of CRs is an essential factor in the operation of TTD acceleration.
The non-linear Landau damping of the CR streaming instability is very different from the case of trans-Alfvénic turbulence. Transition to strong sub-Alfvénic turbulence happens at the scale $\ell_{\text{trans}}$ and the energy is being injected anisotropically at this scale, which is in contrast to the isotropic injection for the trans-Alfvénic turbulence. At scales larger than $\ell_{\text{trans}}$ the turbulence is no longer strong, but follows a weak turbulence cascade with a very different scaling (see Table 1). Thus there is no physically justified way of defining $\ell_{\text{MHD}}$ for the sub-Alfvénic injection of energy in the system. Nevertheless, when expressed in terms of the energy dissipation our results look for strong sub-Alfvénic turbulence similar to those in FG04, with the difference that the weak cascading rather than the strong cascading rate enters the formulae. This coincidence stems from the fact that in this particular regime the damping does depend on the dissipation rate only and not on the injection scale. This makes the case of strong turbulence special, as in other regimes both the turbulence dissipation rate and the injection scale influence the streaming instability damping.

We have quantified the streaming instability damping for a variety of different regimes of turbulence, including (a) hydro-like super-Alfvénic, (b) magnetic super-Alfvénic, (c) weak turbulence sub-Alfvénic, and (d) strong turbulence sub-Alfvénic. Case (b) coincides with the one in FG04 if we identify the $\ell_{\text{MHD}}$ there with $\ell_A$ in this paper. Damping of streaming instability in different turbulent regimes is important for different astrophysical environments. For instance, we identified weak turbulence as the major agent for streaming instability damping in sub-Alfvénic turbulence in the Milky Way halo.

In terms of astrophysical consequences, we do not share the worries of FG04 related to the crisis in explaining the observed degree of isotropy of the Milky Way CRs. We believe that any reliable estimates of the degree of anisotropy can be drawn only by considering more realistic models where the streaming instability and its suppression by turbulence is one of the factors. Other factors, e.g., CR scattering and superdiffusion, should not be ignored. We do not provide any comprehensive modeling, but, within this paper focused on the damping of the streaming instability, we show that for the parameters expected for the turbulence in the galactic halo, the turbulent damping should arise from the interaction of CRs with weak turbulence, rather than with strong turbulence as it is assumed in FG04. This reduces the damping. We provided arguments suggesting that the estimate of the turbulence dissipation rate in FG04 that is based on the cooling of the hot gas is, in fact, an upper limit, which does not constrain the actual turbulence dissipation rate. Therefore we claim that the streaming instability suppression by turbulence is significantly reduced in the Galactic halo. We also pointed out the self-regulating nature of the competing nonlinear Landau damping of the CR streaming instability. This damping shuts off as soon as freely streaming particles spread into space, decreasing the gradients in CR distribution.

We also point out that their CR superdiffusion on scales less than the $\min[L, \ell_A]$ (Lazarian & Yan 2014) as well as scattering by fast modes (Yan & Lazarian 2004) must be accounted for within realistic models of CR propagation. These effects increase the degree of CR anisotropy and, together with the reduction of the aforementioned turbulent damping efficiency, should result in a more isotropic distribution of CRs. Before such detailed calculations are done with realistic models of magnetic field and MHD turbulence, we feel that it is premature to claim that the observed high degree of CR isotropy presents a problem for modern theories of CR propagation (cf. FG04).

Importantly, our study goes beyond the studies of streaming instability damping. Damping of the Alfvén waves in turbulent media is a widespread phenomenon. It is associated with the damping of other instabilities, e.g., a gyroresonance instability. It is also present when waves emitted by a macroscopic source propagate in turbulent media. The expressions that we obtained for the damping of Alfvén waves emitted by macroscopic sources describe new ways for launching stellar/galactic winds and heating cosmic plasmas.

10. DISCUSSION OF RESULTS

In this paper we have presented the calculations of Alfvén-wave damping arising from Alfvénic turbulence. We have dealt both with the case of Alfvén waves generated in the local system of reference as well as by an external macroscopic source. The latter may be the oscillating part of the stellar atmosphere, while the former is related to the particles moving along magnetic field lines.

We have provided the study for a variety of possible astrophysical conditions from super-Alfvénic turbulence, i.e., for $M_A > 1$ to sub-Alfvénic turbulence, i.e., for $M_A < 1$. We have shown significant changes of wave damping depending on $M_A$ and point out the differences in Alfvén-wave damping for waves generated in the local system of reference and launched with respect to the mean magnetic field. We have demonstrated that some of the paradoxes related to the CR-observed anisotropies disappear when the variations of the turbulence magnetization and more realistic models of CR propagation are taken into account. The different regimes of damping that we have considered in this paper are applicable to various astrophysical settings and should be accounted for within the detailed modeling.

In this paper we have discussed streaming instability of CRs. However, the damping rates calculated in the paper are applicable to other types of instabilities that are related to the generation of Alfvén waves in the local system of reference, e.g., to the gyroresonance instability of CRs (see Lazarian & Beresnyak 2006). For damping of instabilities of thermal particles, our treatment should be generalized for kinetic Alfvén waves.

Some of our results are presented in a concise form in Table 1. This table describes both the regimes of turbulence and the damping rates for Alfvén waves that this turbulence entails. We see that, compared to the earlier study in FG04, a variety of different scalings are present. The table also describes the ranges of applicability of different regimes of turbulent damping. Both the damping of waves in the local system of reference, corresponding to the waves generated by streaming instability, and damping of waves emitted by external sources parallel to the mean magnetic field are presented. In particular, Table 1 illustrates that the scalings of damping in the two situations and the ranges of the waves for which damping is applicable are different (see the last two columns; the first column provides the damping of the streaming instability and the range of the CR Larmor radii $\rho_l$ for which this damping works; the second column is for the damping of the waves launched by an external source parallel to the mean magnetic field and the range of the wavelengths for which the damping acts). Other cases, e.g.,

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15 We find this way of presenting results may sometimes be confusing, as in many cases the turbulent dissipation is not directly measurable in view of multiple sources of media heating. On the contrary, the scale of the turbulence $L$ and the magnetic Mach number $M_A$ can be observationally measured as we discuss in this paper.
Alfvén waves emitted at an arbitrary angle, as well as damping of the Alfvén waves by outer-scale turbulence, are also presented in the current paper. We would like to stress the important role of weak turbulence in the suppression of the streaming instability at low $M_A$. While the weak turbulence has a limited inertial range $[LM_L, L]$, it can affect CR streaming for $n_L$ in the range $[LM_M^1, LM_M^2]$. For instance, for a moderate $M_A = 0.1$, the weak turbulence that is present over one decade range of scales can control the propagation of CRs over three decades of energy scales. The range of energies of CRs whose streaming is affected by strong sub-Alfvénic turbulence is significantly reduced. Thus, as we discussed in Section 7.1, for the Milky Way galactic halo we expect most of the CRs streaming to interact with the weak rather than strong turbulence.

Our study employs a number of simplifying assumptions, the importance of which we would like to discuss. The first is that we can consider Alfvénic turbulence separately from the turbulence induced by other modes. This issue has been studied theoretically and quantified numerically (GS95; Lithwick & Goldreich 2001; Cho & Lazarian 2002, 2003). The rates of energy transfer from Alfvén to compressible (fast and slow) modes did not exceed the 10% to 15% in the study of Cho & Lazarian (2002). This is a reasonable degree of accuracy for the approximation we employed here.

A more serious point is related to the model of turbulence that is chosen. Our study makes use of the theory of balanced MHD turbulence, i.e., when the flow of energy in the opposite direction is the same, while localized astrophysical sources and sinks of turbulent energy may make Alfvénic turbulence imbalanced, i.e., with the flow of energy in one direction exceeding the flow in the opposite direction. Solar winds up to 1 au present an example of such imbalanced turbulence. A few theories have been suggested to account for the imbalance (e.g., Lithwick & Goldreich 2007; Beresnyak & Lazarian 2008b; Chandran 2008; Perez & Boldyrev 2009). Among these theories, the one by Beresnyak & Lazarian (2008b) was shown to correspond to numerical simulations in Beresnyak & Lazarian (2009). For small imbalances, this theory smoothly transfers to GS95, while large imbalances are difficult to create in astrophysical media because of the reflection of Alfvénic perturbations in realistically compressible and inhomogeneous media. Therefore, we believe that our present study can provide a reasonable estimate for such situations.

Our treatment was presented for a single scale of energy injection. In real astrophysical situations small-scale energy injection takes place along with the large-scale energy cascade. The local energy injection may dominate the dynamics of intracluster media. Our exploration of the damping of the turbulence of the large-scale Galactic cascade. Our treatment can be generalized for such situations.

This study has important astrophysical implications. Naturally, astrophysical fluids exhibit a variety of turbulent regimes. The damping of the waves also depends on how the Alfvén waves are launched. We have quantified the whole variety of regimes of Alfvén-wave damping by strong and weak sub-Alfvénic turbulence, and turbulence at the injection scale and at the dissipation scale. We identified the difference in damping for waves emitted by streaming particles and macroscopic astrophysical sources. The latter are essential, e.g., for launching stellar and galactic winds, or for the heating of intracluster media. Our exploration of the damping of the super-Alfvénic turbulence is the closest to that described in the earlier studies. For a number of implications we have just sketched the possible physics and do not get into

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**Table 1**

| Type of MHD Turbulence | Injection Velocity | Range of Scales | Spectrum $E(k)$ | Streaming Instability Damping and $n_L$ Range | Wave From Source Damping and Wavelength Range |
|------------------------|--------------------|----------------|-----------------|---------------------------------------------|---------------------------------------------|
| Strong sub-Alfvénic    | $V_L < V_A$        | $[L_{min}, L]$ | $k_L^{-5/3}$    | $\frac{V_A m_{A}^{1/2}}{r_{L}^{1/2} L^{3/2}}$, $LM_A^1 < n_L < LM_A^2$ | $\frac{V_A m_{A}^{1/2}}{r_{L}^{1/2} L^{3/2}}$, $L M_A^1 < \lambda < LM_A^2$ |
| Hydro- like super-Alfvénic | $V_L < V_A$      | $[L_{min}, L]$ | $k_L^{-5/3}$    | $\frac{V_A m_{A}^{1/2}}{r_{L}^{1/2} L^{3/2}}$, $L M_A^1 < n_L < L$ | $\frac{V_A m_{A}^{1/2}}{r_{L}^{1/2} L^{3/2}}$, $L M_A^1 < \lambda < L$ |
| Strong super-Alfvénic  | $V_L > V_A$        | $[L_{min}, L]$ | $k_L^{-5/3}$    | $\frac{V_A m_{A}^{1/2}}{r_{L}^{1/2} L^{3/2}}$, $L M_A^1 < n_L < L$ | $\frac{V_A m_{A}^{1/2}}{r_{L}^{1/2} L^{3/2}}$, $L M_A^1 < \lambda < L$ |

**Note.** $L$ and $L_{min}$ are the injection and perpendicular dissipation scales, respectively. $M_A = \delta B/B$ and $L_{max} = LM_A^2$ for $M_A < 1$ and $L_A = LM_A^3$ for $M_A > 1$. For weak Alfvénic turbulence $l_I = L$ at all times. The waves are sent parallel to the mean field, and $\theta$ varies as discussed in Section 5.3.
the quantitative details. This is natural, as the Alfvén-wave damping in turbulent media is widely spread in astrophysical settings and this paper is focused on quantifying different regimes of the process rather than its numerous astrophysical consequences.

We would like to stress that our paper is focused on turbulent damping of Alfvén waves. We deal with the nonlinear Landau consequences. We argue that the nonlinear Landau damping acts in a self-regulating fashion, and therefore it can be in many instances subdominant compared to the damping by turbulence. Indeed, if the streaming instability is suppressed by the aforementioned mechanism, it allows the spread of the CRs, increasing the scale-height of their distribution. This, in turn, suppresses the nonlinear Landau damping. On the basis of this reasoning, we believe that in most astrophysical situations turbulent damping is more important than the nonlinear Landau damping of Alfvén waves.

The limitation of this study is the use of the model of balanced turbulence, i.e., turbulence with the opposite fluxes of Alfvénic packets in both directions being equal. We argued above that the turbulence gets balanced very quickly and therefore this type of turbulence is most important for astrophysical settings. However, imbalanced turbulence can also be important. For instance, turbulence in solar wind is imbalanced up to 1 au. Using the model in Beresnyak & Lazarian (2008a), our calculations can be generalized for the case of imbalanced turbulence.

11. SUMMARY

Our study of Alfvén-wave damping in MHD turbulence revealed a variety of damping regimes with important astrophysical consequences. We express our results through the magnetization of the media, which is given by the Alfvén Mach number $M_A$ and the turbulence injection scale $L$. Both quantities can be obtained through observations. We quantified the wave damping for different regimes of sub-Alfvénic and super-Alfvénic turbulence as well as for damping of Alfvén waves by the turbulence at the injection scale. In every case we obtained the range of wavelengths for which the damping in the particular regime is applicable. This work opens ways for studies of the consequences of Alfvén-wave damping in various astrophysical settings. Those include launching of stellar and galactic winds, heating of the media, control of the CR streaming instability, etc. Our results can be briefly summarized as follows.

1. The damping is different if Alfvén waves are generated in the local system of reference and in the global system of reference when Alfvén waves are launched with respect to the mean field. The former case takes place when, e.g., particles subject to the streaming instability generate waves with respect to the magnetic field they interact with, while the latter case takes place, e.g., when Alfvén waves are injected into turbulent media by an external macroscopic source. The structure of the Alfvén wavefronts and their interaction with turbulence in two cases are different, and this entails the difference in turbulent damping.

2. For Alfvén waves launched by streaming instability, their damping in sub-Alfvénic turbulence is different for weak and strong regimes of turbulence. In both cases, the damping is significantly slower compared to the case of the super-Alfvénic turbulence. The weak turbulence, while being present over a limited range of scales, can affect CR streaming over a significant range of energies. On the contrary, the range of the damping by strong sub-Alfvénic turbulence is significantly reduced. This results, for instance, in the CR streaming in low $M_A$ environments of galactic halos being mostly affected by weak turbulence.

3. Turbulent damping of Alfvén waves generated in the local system of reference goes beyond its application to CR streaming instability. Other instabilities, e.g., gyroresonance instability, are subject to a similar turbulent damping. Thus the expressions obtained in this study describe the physics in many essential astrophysical settings.

4. The damping of Alfvén waves launched with respect to the mean magnetic field depends on the angle between the mean field and the direction of the wave propagation. In the limiting case of Alfvén waves propagating along the mean magnetic field, the damping is different compared to that present for the waves sent along the local direction of the magnetic field by the streaming instability.

5. The study suggests many important astrophysical consequences, many of which are still to be elaborated. For instance, the efficient damping of Alfvén waves generated by astrophysical sources, e.g., stars and galaxies, heats the media and launches winds, as Alfvén waves deposit both their energy and momentum into the ambient turbulent magnetized plasmas. At the same time, we do not expect the damping of streaming instability by turbulence in the Milky Way galactic halo to be strong enough to affect the observed level of galactic CR isotropy.

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