Ambiguity searching algorithm via coordinate function constrain in radar dynamic calibration

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Abstract. Aiming at the difficulty of ambiguity resolution in GPS dynamic positioning, the radar calibration route is transformed into coordinate function, to compress the search space of Ambiguity Function Method (AFM). The simulation results show that this algorithm not only has the advantage of insensitivity to cycle slip, but also overcomes the shortcomings of large searching space and long searching time of AFM. It is an ambiguity searching algorithm suitable for high dynamics and high precision GPS positioning.

1. Introduction

At present, the commonly used method of shipboard radar dynamic calibration is that UAVs carry cooperative target, and DGPS is used to get real-time and high precision target position information, as a reference of radar calibration. For DGPS dynamic positioning, its accuracy depends on the correct fast ambiguity solution. But it is not easy to get, especially in the dynamic environment. Then, integrating other information to assist calculating is a kind of effective and feasible method. Literatures [1-6] study the application of other measurement information in ambiguity calculation, and reliable results are obtained. Literature [7] uses GPS and inertial navigation system, and the ambiguity solution got improved. In the shipboard radar calibration, UAVs routes usually can be planned in advance, and they can be used to constraint the ambiguity searching and speed up the searching process.

AFM is not sensitive to cycle slip, and is suitable for high dynamics and high precision GPS positioning. But at the same time it has the disadvantage of large searching space and long searching time. Next, the ambiguity searching algorithm via coordinate constraint will be discussed by using a certain shipboard radar calibration route as an example.

2. The establishment of the mathematical model

2.1. AFM model

The definition of ambiguity function is:

\[
A(x, y, z) = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{l=1}^{n_l} \cos \left[ 2\pi \left( \nabla \Delta \phi^o_i - \nabla \Delta \phi^o_j (x, y, z) \right) \right]
\]  

(1)

there, \( \nabla \Delta \phi^o_i \) is the double difference observation data; \( \nabla \Delta \phi^o_j (x, y, z) \) is the theoretical double difference value; \( n_f = 1, 2 \) is the two carrier frequency L1 and L2; \( n_i \) is the number of double difference observation in epoch \( i \); \( n \) is the epoch number.
Set \((x_0, y_0, z_0)\) as the real coordinates of mobile station. Because cosine function is not sensitive to integer times of \(2\pi\), when the mobile station coordinates \((x, y, z)\) is \((x_0, y_0, z_0)\), the ambiguity function value \(A\) gets the maximum 1. The calculating principle of AFM as shown in figure 1:

2.2. The coordinate functions in radar dynamic calibration
Take the calibration route of radar azimuth error for example: in order to eliminate the effects of pitching angle of target to azimuth error and measure the relationship between the azimuth and azimuth error, the route is an uniform circular motion taking \(r\) (1-5 km) as the radius. The route is shown in figure 2:

In this case there are two forms of constraint equation. One is the same elevation constraints:
\[(2)\]
Another is the same distance constraints:
\[(3)\]
AFM algorithm based on coordinate function constraint treats coordinate function equations as the known quantity and plug them directly into the ambiguity function, in order to reduce the unknown quantity and the calculating burden.

3. AFM algorithm based on coordinate constraint
At a certain moment, the double difference equation in GPS dynamic positioning is:
\[(4)\]
where, $P$ is the baseline vector between radar and target; $e^i$ can be obtained through the coordinates of satellite $i$ in the navigation message and base station coordinates.

Set $\beta^i$, $\alpha^i$, $\beta^j$ and $\alpha^j$ as the pitching angles and azimuth angles of satellite $i$ and $j$, and $\theta$, $\psi$ and $R$ as the azimuth angle, pitching angle and distance of the baseline vector $P$. Take equation (3) as the example, that is, $|P| = R$ is known.

There is the transformation relationship in equation (4):

\[
\begin{align*}
    e^i &= \left( \cos \beta^i \sin \alpha^i, \cos \beta^i \cos \alpha^i, \sin \beta^i \right) \\
    e^j &= \left( \cos \beta^j \sin \alpha^j, \cos \beta^j \cos \alpha^j, \sin \beta^j \right)
\end{align*}
\]

So equation (4) can be converted into:

\[
\nabla \Delta \phi^j (t) = \frac{|P|}{\lambda} \begin{bmatrix}
    \sin \theta \left( \sin \beta^j - \sin \beta^i \right) + \cos \theta \cos \beta^i \cos \left( \alpha^i - \psi \right) \\
    -\cos \theta \cos \beta^j \cos \left( \alpha^j - \psi \right)
\end{bmatrix} - \nabla \Delta N^0
\]

When $N$ satellites are observed, the double difference fitness function is:

\[
F(\psi, \theta) = \frac{1}{N} \sum_{j=1, j \neq i}^{N-1} \cos 2\pi \left[ \nabla \Delta \phi^j - \frac{|P|}{\lambda} \begin{bmatrix}
    \sin \theta \left( \sin \beta^j - \sin \beta^i \right) + \cos \theta \cos \beta^i \cos \left( \alpha^i - \psi \right) \\
    -\cos \theta \cos \beta^j \cos \left( \alpha^j - \psi \right)
\end{bmatrix} \right]
\]

Under several observation satellites, the fitness function of $\psi$ and $\theta$ is a complex nonlinear multimodal function, and the true value of $\psi$ and $\theta$ are at the peak points of $F(\psi, \theta)$. And the peak points are a bunch of parallel circle on the searching sphere, and different ambiguity values are corresponding to each circle. The circle interval is determined by the value of the double difference observation and the difference vector $e^j - e^i$. The determination of intervals between peak circles is shown in figure 3.

In figure 3, assuming that $O_1$ is the first searched peak point, $\nabla \Delta \phi^{ij}(t)$, $|P| = R$ and $\lambda$ can be got accurately at the moment $t$, and a unit vector of $O_2$ on the next peak ring is

\[
e_{o2}(t) \left[ e^i(t) - e^j(t) \right] - e_{o1}(t) \left[ e^i(t) - e^j(t) \right] = \frac{\lambda}{|P|}
\]

If three satellites are observed, composed two groups of double difference observation equation whole week ambiguity have the following relations

\[
\begin{align*}
    e \cdot e^{ij} &= \frac{\lambda}{|P|} \left( \nabla \Delta \phi^{ij} + N^{ij} \right) \\
    e \cdot e^{ij} &= \frac{\lambda}{|P|} \left( \nabla \Delta \phi^{ij} + N^{ij} \right)
\end{align*}
\]

Figure 3. Determination of intervals between peak circles

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\end{align*}
\]
Assuming that \( (h^i, p^i) \) and \( (h^j, p^j) \) are respectively the azimuth and elevation of \( e^i \) and \( e^j \), then
\[
\sin h^i \sin \theta + \cos h^i \cos \theta \cos (p^i - \varphi) = \frac{\lambda}{|P|} \left( \frac{\nabla \Delta \phi^i + N^i}{|e^i - e|} \right) (12)
\]
\[
\sin h^j \sin \theta + \cos h^j \cos \theta \cos (p^j - \varphi) = \frac{\lambda}{|P|} \left( \frac{\nabla \Delta \phi^j + N^j}{|e^j - e|} \right) (12)
\]
Solution of equation (12) is
\[
\varphi = \pm \arcsin \left( \frac{g \pm \sqrt{g^2 - 4f}}{2} \right) (13)
\]
There
\[
\begin{aligned}
g &= \frac{2(s - r)(1 - r) + 4t^2}{(s - r)^2 + 4t^2} \\
f &= \frac{(1 - r)^2}{(s - r)^2 + 4t^2} (14)
\end{aligned}
\]
\[
\begin{aligned}
r &= u \cos^2 p^i + v \cos^2 p^j + w \cos p^i \cos p^j \\
s &= u \sin^2 p^i + v \sin^2 p^j + w \sin p^i \sin p^j \\
t &= \left[ u \sin 2p^i + v \sin 2p^j + w \sin \left( p^i + p^j \right) \right] / 2 (15)
\end{aligned}
\]
\[
\begin{aligned}
u &= \frac{\tan^2 h^i \left( 1 - O_i^2 \right)}{\left( O_i - O_j \right)^2} \\
v &= \frac{\tan^2 h^j \left( 1 - O_j^2 \right)}{\left( O_i - O_j \right)^2} (16)
\end{aligned}
\]
\[
\begin{aligned}
w &= -\frac{\tan h^i \tan h^j \left( 1 - O_i O_j \right)}{\left( O_i - O_j \right)^2} \\
O_i &= \frac{\lambda \left( \nabla \Delta \phi^i + N^i \right)}{|P| \left| e^i - e \right| \sin h^i} \\
O_j &= \frac{\lambda \left( \nabla \Delta \phi^j + N^j \right)}{|P| \left| e^j - e \right| \sin h^j} (17)
\end{aligned}
\]
Work out the value of \( \varphi \), then the value of \( \theta \) can be got.
Similarly, the other calibration route coordinate function equations can be converted to the form of baseline vector. Using the character that one parameter keeps the same, the method of compression space is take into AFM algorithm to reduce the amount of calculation.

4. Simulation
Setting baseline vector distance is fixed, the search area of the pitch angle is \( 10^\circ \sim 90^\circ \), the search area of azimuth angle is \( -180^\circ \sim +180^\circ \). In the process of calculation and search nine moons are captured, search step length is \( 0.1^\circ \). First adopt the original AFM classification to search, on the basis of single
epoch data, the number of times of formula (9) is 356100. Then use compression space method to search, using AFM algorithm based on the coordinate function of the strong constraints, the number of search points to is below 1400, thus the search volume has been greatly reduced.

In addition, in the discrete search points, solve the double difference combination of formula (9), fitness function values of various points in the single epoch data are got. The search results of formula (9) are finally threshold test, values below the threshold are eliminated, whereas keep them and carry further inspection, selection of threshold is 0.86. Figure 4 is the search space after the first epoch, the distribution of the points. The fitness function values of the discrete points are shown in figure 5. From figure 5 we can see that, the calculation of single epoch can not distinguish true value points completely from the discrete points, it needs the calculation and inspection of more epochs. Those drop with the observation time’ accumulating constantly, but the fitness function values of true value points will keep on.

![Figure 4. Diagram of searching space](image1)

![Figure 5. Ambiguity values on discrete points](image2)

With the increase of the number of observation epoch, points passing threshold test are gradually reduced, as shown in figure 6, after 15 epochs, points that can pass the threshold test reduced to 40. As shown in figure 7, increasing observation epoch until to 35, number of points conforming to the threshold detection is 11.

![Figure 6. Points passed value test at epoch 15](image3)

![Figure 7. Points passed value test at epoch 35](image4)

Take point into (12) to backstep position and inspection, ambiguity of the whole week gets the correct solution, generally the time is 5 to 7 seconds. According to the original ambiguity function method, the three-dimensional hierarchical search method usually needs 1 to 2 minutes to get the right solution. Thus, in this paper, AFM algorithm based on the coordinate function strong constraint greatly reduces the ambiguity of the whole week search time. According to the document [8], the improved AFM algorithm under the strong constraint adapts to the requirements of radar dynamic calibration completely.
5. Conclusion
The key of radar dynamic calibration is the GPS precise positioning, and fast and correct decomposition of GPS ambiguity of the whole week is the key to using GPS carrier phase to carry high precision positioning. From the analysis of this article, we can see that in the dynamic positioning of radar calibration, the use of unmanned aerial vehicle route information can effectively improve the whole week ambiguity of AFM search efficiency. Dynamic localization algorithm based on coordinate function strong constraint AFM, through taking the coordinate function into the ambiguity equation, the search space when the AFM calculates the whole week ambiguity is greatly compressed. From the algorithm simulation, the decoding efficiency has improved significantly, meeting the request of whole week ambiguity fast calculating for the GPS dynamic positioning.

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