Reason for T2K to run in dominant neutrino mode for detecting CP violation

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The long-baseline experiment T2K in Japan has collected data in the neutrino mode corresponding to an exposure of $6.57 \times 10^{20}$ POT (Protons On Target) and currently it is running in the antineutrino mode. The main aim of the antineutrino run is to measure the leptonic phase $\delta_{CP}$ which may help to understand the matter-antimatter asymmetry of the universe. In this work we show that in T2K, antineutrinos are required only for removing the wrong octant solutions which in turn improves the CP sensitivity. If however the octant is known then pure neutrino run is capable of giving the maximum CP sensitivity. If we divide the total true parameter space into eight sets, then we find that T2K antineutrino run helps in improving the CP sensitivity for only two sets while for the remaining six combinations pure neutrino run gives the best CP sensitivity. Thus if the neutrino run is replaced by the antineutrino run then it causes a reduction in the CP sensitivity in 75% of the true parameter space due to lesser statistics. Thus it is worthwhile to study if the T2K antineutrino run can be reduced by the antineutrino runs of the other experiments, so that T2K can run in dominant neutrino mode to extract the best CP sensitivity. In this work we explore the possibility of the antineutrino component of NOνA and the atmospheric neutrino experiment ICAL@INO for compensating the antineutrino run of T2K.

PACS numbers:

I. INTRODUCTION

The discovery of non zero $\theta_{13}$, from the recent reactor experiments Daya Bay [1], Double Chooz [2], and RENO [3], bring the remaining unknowns in neutrino oscillation physics within a reach. Among the six parameters that describe the phenomenon of neutrino oscillation (i.e., three mixing angles $\theta_{12}, \theta_{13}$ and $\theta_{23}$, two mass squared differences $\Delta_{21}(m_2^2 - m_1^2)$ and $\Delta_{31}(m_3^2 - m_2^2)$, and the Dirac type CP phase $\delta_{CP}$) , the still undetermined parameters are: i) the neutrino mass orderings which tell if neutrinos are normal hierarchical (NH) i.e., $\Delta_{31} > 0$ or inverted hierarchical (IH) i.e., $\Delta_{31} < 0$, ii) the octant of $\theta_{23}$ which defines if $\theta_{23}$ belongs to the lower octant (LO) i.e., $< 45$ or higher octant (HO) i.e., $> 45$ and finally iii) the exact value of $\delta_{CP}$. There are several current and future generation oscillation experiments which are dedicated to measure these remaining unknowns. Among these, the long-baseline experiments like T2K [4], NOνA [5], DUNE [6], ESS/SP [7] will use neutrinos from the accelerator to observe the flavour transition, on the other hand different atmospheric neutrino experiments like Super-Kamiokande [8], Hyper-Kamiokande [9], ICAL@INO [10], PINGU [11] will make use of the neutrinos coming from the Earth’s atmosphere to detect neutrino oscillation. It is to be noted that the optimal design of any current experiment requires the understanding of the parameter space to which the experiment is sensitive to and simultaneously how the data from other experiments can help in improving its sensitivity [12, 13].

T2K is an ongoing accelerator based long-baseline experiment in Japan. The neutrino flux is generated at the J-PARC site on the East coast of Japan and directed to the Super-Kamiokande neutrino detector in the mountains of western Japan. The narrow band flux peaks at 0.6 GeV which coincides with the T2K appearance channel oscillation maxima ($\nu_\mu \rightarrow \nu_e$ oscillation). As the baseline of this experiment is only 295 km, it has very limited sensitivity to neutrino mass hierarchy and octant but on the other hand can provide good CP sensitivity. T2K has already given the first set of data providing a hint at $\delta_{CP} = -90^\circ$ [14]. This hint comes from a run of 7.8% of the proposed total exposure in the neutrino mode [16]. Currently T2K is taking data in antineutrino mode to establish CP violation in leptonic sector on a firm footing. At this stage it is important to ask if T2K should run in a dominant neutrino mode or it should run in equal neutrino-antineutrino mode. To answer this question one needs to understand the underlying physics of how antineutrinos actually help in improving the CP sensitivity of a particular experiment. The determination of $\delta_{CP}$ in long-baseline experiments is constrained by the parameter degeneracy [17–21]. In particular, the limited hierarchy and octant sensitivity of T2K, give rise to hierarchy-$\delta_{CP}$ degeneracy (degeneracy of type I) and octant-$\delta_{CP}$ degeneracy (degeneracy of type II). The behaviour of type I degeneracy is similar in the neutrino and antineutrino oscillation probability [22] but the type II degeneracy behaves differently in neutrinos and antineutrinos [23]. So while determining $\delta_{CP}$, addition of antineutrinos over neutrinos can help in removing the wrong octant solutions but not the wrong hierarchy solutions. Apart from the synergy described above, the role played by antineutrinos also depends on the baseline and flux profile of a particular experiment. In this work we will quantify this point by showing that if octant is known then antineutrino run is not at all required for any true combination for T2K. This implies that because of the T2K baseline and the flux profile there is no other contribution of the antineutrinos in T2K apart from removing the type II degeneracy. On the contrary we show that the situation is different in NOνA.

NOνA is also an accelerator based long-baseline experiment at Fermilab which has a baseline of 812 km. At this

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\footnote{The recent results from the NOνA also prefer $\delta_{CP}$ near to $-90^\circ$ [15].}
baseline the first oscillation maximum occurs at 1.6 GeV whereas the NUMI off axis flux peaks at slightly higher energy which is 2 GeV. NOνA has already given its first results [24] and it is planned to run in equal neutrino-antineutrino mode. In our study we find that there exists a synergy between neutrinos and antineutrinos in NOνA which helps in improving the CP sensitivity even when the octant is known. This difference between T2K and NOνA for measuring δCP, motivates us to study if the antineutrino run of T2K can be compensated by antineutrino runs of other experiments and NOνA in particular. If T2K runs in dominant neutrino mode then the measurement of δCP for the true parameter space that does not suffer from any octant degeneracy can be done without compromising on the statistics which gets reduced if one adds equal amount of antineutrinos as that of neutrinos. For the parameter space where antineutrino run is required the antineutrino component of the other experiments can provide the required sensitivity. In this way T2K, with the help of other experiments can contribute to getting the first signature of δCP using its maximum statistics by running in dominant neutrino mode. This point has been also raised in [25] in the context of T2HK [26], which considers antineutrinos from a muon decay at rest (DAR) source. In this work we have focused on the T2K experiment using the existing/funded facilities.

We have considered the options of adding NOνA and the atmospheric neutrino experiment ICAL@INO to T2K. The India-based Neutrino Observatory (INO) Project is a multi-institutional effort aimed at building a world-class underground laboratory for detecting muon events arising from the flavour oscillation of the atmospheric neutrinos. It will use Iron Calorimeter (ICAL) detector, consisting of a 50 kiloton magnetized iron plates arranged in stacks with gaps in between where Resistive Plate Chambers (RPCs) would be inserted as active detectors. The magnetic field will allow to analyze the neutrino and antineutrino events separately. As sensitivity of the atmospheric neutrino gets contribution from both neutrinos and antineutrinos, it is also relevant to study the combined sensitivity of atmospheric data and T2K.

For numerical simulation we have considered a total exposure of 8 × 10²¹ POT for T2K. We have divided the exposures in the units of 10²¹ POT and generated our results for different neutrino-antineutrino exposures. For NOνA we have taken an equal 3 years run in both neutrino and antineutrino mode unless otherwise stated. The T2K and NOνA results are simulated using GLoBES [27–36]. For ICAL we have used an in-house code with a fixed resolution of 10% in energy and 10° for the direction and generated the data for a total exposure of 500 kt-yr.

The plan of the paper goes as follows. In Section II, we briefly discuss the degenerate parameter space of T2K in terms of oscillation probability and also discuss the role of antineutrinos in solving these degeneracies at the probability level. In Section III, we will quantify the role of antineutrinos in T2K for determining the leptonic CP phase δCP in terms of CP violation discovery (CPV). We will also identify the parameter space where the antineutrino runs are necessary. In Section IV, we will study the role of antineutrinos in NOνA to discover CPV. Then in Section V, we study the possibility of reduce the antineutrino runs of T2K by the addition of NOνA and ICAL data and try to find the optimal configuration for T2K. Then finally in Section VI, we will summarize our results and then conclude.

II. THE DEGENERATE PARAMETER SPACE IN T2K PROBABILITY

For T2K, the CP sensitivity comes from the appearance channel Pµe. At this baseline, the constant density approximation holds good and the probability formula for neutrinos in terms of the small parameters sinθ13 and α (= Δ21/Δ31) can be expressed as [37]

\[ P_{\mu e} = 4s_{13}^2s_{23}^2 \sin^2(A - 1)\Delta \frac{1}{(A - 1)^2} + \alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta + \delta_{CP}) \sin(A - 1)\Delta \sin A\Delta \frac{1}{(A - 1)} + O(\alpha^2), \] (1)

where \(s_{ij}(c_{ij}) = \sin(\theta_{ij})(\cos(\theta_{ij}))\), \(A = 0.000076 \times \rho \times E/\Delta_{31}\), \(\Delta = \Delta_{31}L/4E\). E is the energy of the neutrino in GeV, L is the baseline in km and \(\rho\) is the matter density in units of gm/cc. \(\Delta_{31}\) and \(A\) is +ve for NH and -ve for IH. The expression for antineutrinos can be obtained by replacing \(\delta_{CP} \rightarrow -\delta_{CP}\) and \(A \rightarrow -A\).

Due to the comparatively smaller baseline, T2K matter oscillation maxima coincides with the vacuum peak and thus \(\Delta\) corresponds to 90°. As the T2K flux peaks around this same energy where the oscillation maxima peaks, it will be sufficient to discuss the CP property of T2K at this value \(\Delta\). In this condition, the CP dependent term goes as \(\sin \delta_{CP}\) and because of this for both neutrinos and antineutrinos ±90° are the maximum separated points in the probability for both the hierarchies. Below we are listing the degenerate parameter space of T2K which affects the measurement of \(\delta_{CP}\):

i) For a given value of \(\theta_{23}\) (ε LO or HO), NH having \(0° < \delta_{CP} < 180°\) (upper half plane or UHP) is degenerate with IH having \(-180° < \delta_{CP} < 0°\) (lower half plane or LHP) i.e., \(P_{\mu e}(\text{NH, UHP}) = P_{\mu e}(\text{IH, LHP})\). This parameter space is same for both neutrino and antineutrinos as both the hierarchy sensitive term \(A\) and \(\delta_{CP}\) flips its sign. This is the degeneracy of type I also known as hierarchy-\(\delta_{CP}\) degeneracy.
ii) For a given hierarchy (NH or IH), LO having $-180^\circ < \delta_{CP} < 0^\circ$ is degenerate with HO having $0^\circ < \delta_{CP} < 180^\circ$ in neutrinos. i.e., $P_{\mu e}(\text{HO, UHP}) = P_{\mu e}(\text{LO, LHP})$. For antineutrinos the degenerate parameter space can be found out by simply changing the sign of $\delta_{CP}$. This is the degeneracy of type II also known as octant-$\delta_{CP}$ degeneracy.

Now because of the above mentioned degeneracies, CP measurement for a given set of true hierarchy, true octant and true $\delta_{CP}$ can be affected by either wrong hierarchy solutions or wrong octant solutions or both. In the next section we will identify the true parameter space for which these wrong solutions are present/absent and the role of antineutrinos.

### III. ROLE OF ANTI NEUTRINOS FOR CPV DISCOVERY IN T2K: REASON FOR T2K TO RUN IN DOMINANT NEUTRINO MODE

The CP violation (CPV) discovery potential of an experiment is defined by its capability of distinguishing the values of $\delta_{CP}$ from $0^\circ$ and $180^\circ$. We calculate the CPV $\chi^2$ by varying $\delta_{CP}$ in its full allowed range in the true parameter spectrum and keeping its value fixed at $0^\circ$ and $180^\circ$ in the test spectrum.

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**FIG. 1:** CPV $\chi^2$ vs $\delta_{CP}(\text{True})$ of T2K for NH with $\theta_{23}(\text{true}) = 39^\circ(51^\circ)$ for LO (HO) and $\Delta_{31}(\text{true}) = 2.4 \times 10^{-3}$. 
In Fig. 1 we have plotted the CPV discovery $\chi^2$ of T2K by taking true hierarchy as NH. The upper panels are for true $\theta_{23}$ value of $39^\circ$ which belongs to LO and lower panels correspond to the true $\theta_{23}$ value of $51^\circ$ which belongs to HO. As we are interested in understanding the role of antineutrinos in detecting CP violation and we already know that the nature of hierarchy-$\delta_{CP}$ degeneracy is same for both neutrinos and antineutrinos, we keep the test hierarchy as unknown throughout our analysis. In the left column, the octant has been assumed to be known and in the right column the octant is unknown. To see the role of antineutrinos in each panel we have taken five different sets of the neutrino and antineutrino exposures in units of POT. In all the panels we can notice a drop in the $\chi^2$ for $\delta_{CP} \in \text{UHP}$. This is because for a given value of $\theta_{23}$, NH suffers from hierarchy-$\delta_{CP}$ degeneracy for $\delta_{CP} \in \text{UHP}$ and the $\chi^2$ minima occurs with the wrong hierarchy. In top left and bottom left panels of Fig. 1, we see that in both the cases (8+0) give the best sensitivity and the sensitivity decreases as the antineutrino run is increased. In both the plots the octant has been assumed to be known and this establish the fact that in T2K if octant is known then antineutrinos do not play any role to improve the CP sensitivity. Moreover addition of antineutrino causes a decrease in the statistics and hence the sensitivity decreases. In top right panel of Fig. 1,
When the true octant is LO, we notice that, for $\delta_{CP} = -90^\circ$, pure neutrino run of T2K gives the worst sensitivity and addition of antineutrinos help in improving the CP sensitivity. For the T2K combination (5+3) the sensitivity becomes maximum and then the $\chi^2$ falls with the further addition of antineutrinos [38]. This is because, neutrinos suffer from octant degeneracy in (LO, LHP) but antineutrinos do not. Thus at this true value, addition of antineutrinos help in improving the CP sensitivity. But once the degeneracy is removed and the $\chi^2$ minima shifts into the correct octant, further addition of antineutrino causes a decrease in statistics and hence the sensitivity and as a result we see (5+3) is better than (4+4). But the situation is different when the true octant is HO (bottom right panel). As neutrinos do not have octant degeneracy at this point, pure neutrino run gives the maximum sensitivity while addition of antineutrinos only reduces sensitivity.

Now let us discuss the role of antineutrinos when the true hierarchy is inverted. In Fig. 2, we have plotted the same curves that of Fig. 1 but assuming the true hierarchy as IH. In all the panels we identify the drop in the $\chi^2$ sensitivity in the the LHP due the hierarchy-$\delta_{CP}$ degeneracy. As in NH, here it is noticed that for the cases when octant is known (top left and bottom left panels) antineutrinos do not help in the CP sensitivity but when the octant is unknown adding antineutri-
FIG. 5: combined CP sensitivity for various anti-neutrino combinations taking unknown hierarchy and unknown octant.

nos only help in the UHP when the true octant is HO. This is because for true IH, neutrinos suffers from octant degeneracy at HO-UHP but antineutrinos resolve this degeneracy. For the other combinations of true octant and CP addition of antineutrinos only reduces sensitivity.

Thus from the above discussion it is clear that if we divide the total true parameter space into eight sets based on the unknown hierarchy i.e., NH/IH, unknown octant i.e., LO/HO and unknown $\delta_{CP}$ i.e., LHP/UHP, T2K antineutrino run helps in improving the CP sensitivity for only the true combination NH-LO-LHP and IH-HO-UHP. For the remaining other six combinations, antineutrinos do not have any role to play other than reducing the sensitivity due to lack of statistics. So if T2K is decided to run in equal neutrino and antineutrino mode, then it will be at the cost of compromising the CP sensitivity of the 75% of the total true parameter space. But at the same time it is also true that for the true parameter values for which antineutrino runs are required, pure neutrino run give
the worst sensitivity. To overcome this situation, we suggest that T2K run in dominant neutrino mode, since other experiments will anyway collect antineutrino data. In this way T2K with the help of other experiments will be able to obtain the best CP sensitivity for all values in the true parameter space.

IV. ROLE OF ANTINEUTRINOS IN NO$^\nu$A

Now let us discuss the role of antineutrinos in NO$^\nu$A for detecting CP violation. To understand the role of anti-neutrinos we have considered NO$^\nu$A (6+0) which corresponds to 6 year running of NO$^\nu$A in the pure neutrino mode and NO$^\nu$A (3+3) which corresponds to 3 years equal running in both neutrino and antineutrino.

In Fig. 3 we plot the CPV $\chi^2$ of NO$^\nu$A for true hierarchy NH and $\theta_{23} = 39^\circ$. For this true combination, T2K requires antineutrino run in the LHP when octant is unknown but do not need the same when octant is known. But on the other hand for NO$^\nu$A, we can see that CP sensitivity of NO$^\nu$A (3+3) is better than NO$^\nu$A (6+0) even when octant is known. This implies that for NO$^\nu$A, antineutrinos not only removes the wrong octant solutions but also gives an extra edge to improve the overall determination of $\delta_{CP}$. This difference between T2K and NO$^\nu$A can be easily explained from Fig. 4 where we plotted the probability spectrum for both the experiments for $\theta_{23} = 39^\circ$.

Left panel is for neutrinos and the right panel is for antineutrinos. The red curves correspond to NO$^\nu$A and blue curves correspond to T2K. In both the plots the vertical lines indicate the energy where the flux peaks. The CPV sensitivity of $\delta_{CP} = \pm 90^\circ$ is determined by the distance between the $\delta_{CP} = 0^\circ$, $180^\circ$ and $\delta_{CP} = \pm 90^\circ$ curves. Due to the off-axis configuration of the experiments, it is relevant to look only at the energies where the flux peak. For T2K we see that in both neutrino and antineutrino, the $\delta_{CP} = 0^\circ$, $180^\circ$ curves are equidistant from $\delta_{CP} = \pm 90^\circ$. This is because for T2K, the flux peaks at energy very close to the oscillation maxima. At this limit $\cos(\Delta + \delta_{CP})$ goes as $\sin \delta_{CP}$ which gives the same probability for $\delta_{CP} = 0^\circ$ and $180^\circ$. For this reason the sensitivity at any true value of $\delta_{CP}$ is same in both neutrinos and antineutrinos. But for NO$^\nu$A as the flux peaks at slightly different energy than the $P_{\mu e}$ oscillation maxima, in neutrinos $-90^\circ$ is closer to $0^\circ$ and in antineutrinos it is closer to $180^\circ$. Due to this opposite nature in the probability, the combined sensitivity of neutrinos and antineutrinos give better result as compared to only neutrinos. Though we show our results for only $\theta_{23} = 39^\circ$, this conclusion is valid for other true $\theta_{23}$ values in the present allowed range. This signifies the fact that, for a combination of true $\delta_{CP}$ and true $\theta_{23}$, where there is no octant degeneracy, addition of antineutrinos from NO$^\nu$A will improve the CP sensitivity but for T2K it will reduce the sensitivity.

V. COMBINED CP SENSITIVITY OF T2K, NO$^\nu$A AND ICAL: REPLACING THE T2K ANTINEUTRINO RUN

After discussing the role of antineutrinos in detecting CP violation in T2K and NO$^\nu$A and realizing the fact that T2K antineutrinos help only in a limited true parameter space let us now see what happens if data from other experiments are combined with T2K.

In Fig. 5 we plot the CP sensitivity of T2K for different combinations of neutrino and antineutrino runs, when combined with other experiments. Here we have taken the (3+3) configuration of NO$^\nu$A and 500 kt-yr exposure of the ICAL detector. The first row correspond to the combined sensitivity of T2K and NO$^\nu$A. In 2nd and 3rd row we present our results for T2K+ICAL and T2K+NO$^\nu$A +ICAL respectively. For each row, the four panels correspond to the following combinations of true hierarchy and true $\theta_{23}$: NH-39$^\circ$, NH-51$^\circ$, IH-39$^\circ$ and IH-51$^\circ$. Both hierarchy and octant have been assumed to be unknown in all the panels. From the first row, we see that when NO$^\nu$A data is added to T2K, there is an improvement of the overall CP sensitivity. For $\theta_{23} \in$ HO, the hierarchy sensitivity of NO$^\nu$A resolves the hierarchy-$\delta_{CP}$ degeneracy significantly (which occurs at NH-UHP and IH-LHP) as compared to $\theta_{23} \in$ LO. But most importantly we notice that for the true combination NH-LO-LHP and IH-HO-UHP, there is a significant improvement of the CP sensitivity as compared to the top right panel of Fig. 1, when T2K runs in dominant neutrino mode. We see that T2K(7+1) gives the maximum sensitivity for all the combinations of true parameters when NO$^\nu$A (3+3) is added to T2K. This is in stark contrast to the scenario when T2K alone runs in dominant neutrino mode for this true combinations. This is because except for adding CP sensitivity of its own, the antineutrino component of NO$^\nu$A also provides the required sensitivity to lift the wrong octant solutions which appears due to the dominant neutrino runs of T2K. In the next row, we see that, in all the panels the best sensitivity comes from T2K(7+1) when the atmospheric data of ICAL is added to it. As atmospheric neutrinos themselves do not have much CP sensitivity by its own [39, 40], the overall CP sensitivity remains the same but from the figures we see that apart from removing the wrong octant solutions in NH-LO-LHP and NH-HO-UHP, the hierarchy sensitivity of ICAL also resolves the hierarchy-$\delta_{CP}$ degeneracy which appears in NH-UHP and IH-LHP in T2K. So here the improvement in the CP sensitivity is two fold. Finally in the third row we present the combined sensitivity of T2K, NO$^\nu$A and ICAL. In these figures we see that, all possible combinations of true hierarchy and true octant are now almost free from both type I and type II degeneracies and there is an improvement of the overall CP sensitivity. It is also clear from the figures that for every true value, it is possible to have best CP sensitivity from T2K(7+1).

VI. SUMMARY AND CONCLUSION

In this work we have studied the physics of antineutrino run of T2K for detecting CP violation and also studied the pos-
sibility to reduce the antineutrino run of T2K by adding the data from other experiments. We have shown that for T2K, if octant is known then antineutrinos have no role to play and addition of antineutrinos cause decrease in the CP sensitivity due to lesser statistics. This is true for any true combination of hierarchy, octant and $\delta_{CP}$. If octant is unknown, then we see that T2K antineutrino runs are required only for the true combination of NH-LO-LHP and IH-HO-UHP as these true combinations suffer from the octant-$\delta_{CP}$ degeneracy which behaves differently for neutrinos and antineutrinos. We also notice that the CP sensitivity of T2K is also compromised due to the hierarchy-$\delta_{CP}$ degeneracy for NH-HO and IH-LO and addition of antineutrinos do not help in those regions as this degeneracy is same for neutrinos and antineutrinos. We find that this situation is slightly different in NO$\nu$A . For NO$\nu$A , antineutrinos help in improving the CP sensitivity even when the octant is known. From this we can understand that unlike T2K, NO$\nu$A antineutrino can help in increasing the CP sensitivity for the true parameter space where octant degeneracy is absent. As T2K antineutrino run is useful for only 25% of the true parameter space and running T2K in equal neutrino-antineutrino mode will affect the CP sensitivity in the other 75% of the true parameter space, we ask the question if the T2K antineutrino run can be reduced by adding data from other experiments. We find that when the data from NO$\nu$A (3+3) and and ICAL are added to it, it is indeed possible to obtain the maximum CP sensitivity from the T2K(7+1) configuration. We notice that addition of only NO$\nu$A (3+3) data to T2K help in removing the wrong octant solutions in the respective true parameter combination and also removes the wrong hierarchy solutions significantly for $\theta_{23} \in$ HO. The CP sensitivity of NO$\nu$A is also added to the CP sensitivity of T2K and causes an overall improvement in the CP sensitivity. When the only ICAL data is added to T2K, we find that the overall improvement of CP sensitivity is not that much but the hierarchy and octant sensitivity of the atmospheric data resolve both the wrong hierarchy and wrong octant solutions for every values of $\delta_{CP}$. We also showed that when both NO$\nu$A and ICAL are added to T2K, we obtain an improved CP sensitivity with almost no degeneracy and the best CP sensitivity can be obtained from the T2K(7+1) combination. Though we understand that when T2K will complete its run, ICAL data will not be available but it can be noticed that the amount of sensitivity required to solve the degeneracy for the CPV discovery is not too high and it can also come from other atmospheric data which will be available at that time, for example data of Super Kamiokande experiment. Though we have only considered the value of $\theta_{21}$ as 39° for LO and 51° for IH, the physics of the antineutrinos in T2K as discussed in this work is true for every value of $\theta_{23}$, even those close to maximal value. The results presented in this work are very important for the planning of future antineutrino runs of T2K and in understanding the synergy of T2K with other different experiments.

Acknowledgments

The author would like to thank Prof. Srubabati Goswami for her encouragement and critical reading of the manuscript. The author would also like to thank Dr. Sushant Raut for checking the manuscript thoroughly in WHEPP XIV meeting. A special thanks to Dr. Pomita Ghoshal for her help in learning the atmospheric neutrino code. The author also thanks Newton Nath for useful discussion.
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