The $q\bar{q}$ spectra and the structure of the scalar mesons

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Abstract. The $q\bar{q}$ spectrum is studied within a chiral constituent quark model. It provides with a good fit of the available experimental data from light (vector and pseudoscalar) to heavy mesons including some recent results on charmonium. The $0^{++}$ light mesons and the new $D$ states measured at different factories cannot be described as $q\bar{q}$ pairs and a tetraquark structure is suggested.

INTRODUCTION

Since Gell-Man conjecture, most of the meson experimental data were classified as $q\bar{q}$ states according to $SU(N)$ irreducible representations. Nevertheless a number of interesting issues remains still open as for example the understanding of the structure of the scalar mesons or the new $D_s$ states measured on B factories.

The theoretical study of charmonium and bottomonium made clear that heavy-quark systems are properly described by nonrelativistic potential models reflecting the dynamics expected from QCD [1]. The light meson sector has been studied by means of constituent quark models, where quarks are dressed with a phenomenological mass and bound in a nonrelativistic potential, usually a harmonic oscillator [2]. Quite surprisingly a large number of properties of hadrons could be reproduced in this way [3]. In this talk we present the meson spectra obtained by means of a chiral constituent quark model in a trial to interpret some of the still unclear experimental data in the scalar sector.

SU(3) CHIRAL CONSTITUENT QUARK MODEL

Since the origin of the quark model hadrons have been considered to be built by constituent (massive) quarks. Nowadays it is widely recognized that because of the spontaneous breaking of chiral symmetry in the light quark sector at some momentum scale a constituent quark mass $M(q^2)$ appears. Once a constituent quark mass is generated such particles have to interact through $SU(3)$ Goldstone modes (pion, kaon, eta and sigma (which simulates the two-pion exchange)). Explicit expressions of these potentials can be found elsewhere [4]. In the heavy quark sector, chiral symmetry is explicitly broken and therefore these interactions will not appear.

For higher momentum transfer quarks still interact through gluon exchanges. Following de Rújula et al. [5] the one-gluon-exchange (OGE) interaction is taken as a standard color Fermi-Breit potential. In order to obtain a unified description of light, strange
and heavy mesons a scale dependent strong coupling constant has to be used \cite{6}. We parametrize this scale dependence by

\[ \alpha_s(\mu) = \frac{\alpha_0}{\ln \left( \frac{\mu^2 + \mu_0^2}{\Lambda_0^2} \right)} , \]

(1)

where \( \mu \) is the reduced mass of the \( q\bar{q} \) system and \( \alpha_0, \mu_0 \) and \( \Lambda_0 \) are fitted parameters \cite{4}. This equation gives rise to \( \alpha_s \sim 0.54 \) for the light quark sector, a value consistent with the one used in the study of the nonstrange hadron phenomenology \cite{7}, and it also has an appropriate high \( Q^2 \) behavior \cite{8}, given a value of \( \alpha_s \sim 0.127 \) at the \( Z_0 \) mass \cite{9}. The \( \delta \) function appearing in the OGE has to be regularized in order to avoid an unbound spectrum from below. To solve numerically the Schrödinger equation with this potential we use a flavor-dependent regularization \cite{4}.

The other nonperturbative property of QCD is confinement. Lattice QCD studies show that \( q\bar{q} \) systems are well reproduced at short distances by a linear potential that is screened at large distances due to pair creation \cite{10}. One important question which has not been properly answered is the covariance property of confinement. While the spin-orbit splittings in heavy quark systems suggest a scalar confining potential \cite{11}, a significant mixture of vector confinement has been used to explain the decay widths of \( P \)-wave \( D \) mesons \cite{12}. Such property, being irrelevant for the central part of the interaction, determines the sign and strength of the spin-orbit Thomas precession term which is important for the scalar mesons. Therefore, we write the confining interaction as an arbitrary combination of scalar and vector terms \( V_{CON}^{SO}(\vec{r}_{ij}) = (1 - a_s)V_{V}^{SO}(\vec{r}_{ij}) + a_s V_{S}^{SO}(\vec{r}_{ij}) \) where \( V_{V}^{SO}(\vec{r})[V_{S}^{SO}(\vec{r})] \) is the vector (scalar) spin-orbit contribution.

**RESULTS**

With the quark-quark interaction described above we have solved the Schrödinger equation for the different \( q\bar{q} \) systems. Most of the parameters of the Goldstone boson fields are taken from the \( NN \) sector. The eta and kaon cutoff masses are related with the sigma and pion one as explained in Ref. \cite{13}: \( \Lambda[u(d)s] \simeq \Lambda(ud) + m_s \), \( m_s \) being the strange quark current mass. The confinement parameters \( a_c \) and \( \mu_c \) are fitted to reproduce the energy difference between the \( \rho \) meson and its first radial excitation and the \( J/\psi \) and the \( \psi(2S) \). The parameters involved in the OGE are obtained from a global fit to the hyperfine splittings well established in the Particle Data Group (PDG) \cite{14}. Finally, the relative strength of the scalar and vector confinement is fitted to the energy of \( a_1(1260) \) and \( a_2(1320) \), ordering that cannot be reproduced with a pure scalar confinement. We obtain \( a_s = 0.777 \).

The spectra for the light pseudoscalar and vector mesons and for heavy mesons have been reported in Ref. \cite{15}. The agreement with experimental data is remarkable. Let us emphasize that with only 11 parameters we are able to describe more than 110 states.

Recently Belle and BaBar \cite{16, 17, 18} collaborations have reported new experimental measurements for the mass of the \( \eta_c(2S) \). The average value from the different measurement is significantly larger than most predictions of constituent quark models and
the previous experimental value of the PDG: $M[\eta_c(2S)] = 3594 \pm 5$ MeV. Such value cannot be easily explained in the framework of constituent quark models because the resulting $2S$ hyperfine splitting (HFS) would be smaller than the predicted for the $1S$ states \cite{19,20}. Based on this fact some authors have claimed for an $\alpha_s$ coupling constant depending on the radial excitation.

We predict a value $M[\eta_c(2S)] = 3627$ MeV, within the error bar of the last two Belle measurements, the ones obtained with higher statistics. Moreover the ratio $2S$ to $1S$ HFS is found to be 0.537, in good agreement with the experimental data, 0.479. The reason for this agreement can be found in the shape of the confining potential that also influences the HFS, the linear confinement being not enough flexible to accommodate both excitations \cite{21}.

Other puzzling state is a narrow resonance around 2317 MeV, known as $D_s^*J(2317)$ reported by BaBar \cite{22}. This state has been confirmed by CLEO \cite{23} together with another possible resonance around 2460 MeV. Both experiments interpret these resonances as $J^P = 0^+$ and $1^+$ states. This discovery has triggered a series of articles presenting alternative hypothesis \cite{24}. The most striking aspect of these two resonances is that their masses are much lower than expected. We obtain a mass of 2470.9 MeV for the $J^P = 0^+$ and 2565.5 MeV for the $J^P = 1^+$. They are far from the experimental data although the rest of the states ($1^+, 2^+, 1^-$ and $0^-$) agree reasonably well with the values of the PDG for both for the $D$’s and $D_s$’s states.

Concerning the scalar sector our results are shown in Table 1(column three). We observe that for isovector states, there appears a candidate for the $a_0(980)$, the $3P_0$ member of the lowest $3P_J$ isovector multiplet. The other candidate, the $a_0(1450)$, is predicted to be the scalar member of a $3P_J$ excited isovector multiplet. This reinforces the predictions of the naive quark model, where the $LS$ force makes lighter the $J = 0$ states with respect to the $J = 2$. The assignment of the $a_0(1450)$ as the scalar member of the lowest $3P_J$ multiplet would contradict this idea, because the $a_2(1312)$ is well established as a $q\bar{q}$ pair. The same behavior is evident in the $c\bar{c}$ and the $b\bar{b}$ spectra, making impossible to describe the $a_0(1450)$ as a member of the lowest $3P_J$ isovector multiplet without spoiling the description of heavy-quark multiplets. However, in spite of the correct description of the mass of the $a_0(980)$, the model predicts a pure light-quark content, what seems to contradict some experimental evidences. The $a_0(1450)$ is predicted to be also a pure light quark structure obtaining a mass somewhat higher than the experiment.

In the case of the isoscalar states, one finds a candidate for the $f_0(600)$ with a mass of 402 MeV, in the lower limit of the experimental error bar and with a strangeness content around 8%. The $f_0(980)$ and $f_0(1500)$ cannot be found for any combination of the parameters of the model. It seems that a different structure rather than a naive $q\bar{q}$ pair is needed to describe these states. The $f_0(1500)$ is a clear candidate for the lightest glueball \cite{25} and our results support this assumption. For the $f_0(1370)$ state (which may actually correspond to two different states \cite{26}) we obtain two almost degenerate states around this energy, the lower one with a predominantly nonstrange content, and the other with a high $s\bar{s}$ content. Finally a state corresponding to the $f_0(1710)$ is obtained.

In the $I = 1/2$ sector, as a consequence of the larger mass of the strange quark as compared to the light ones, our model always predicts a mass for the lowest $0^{++}$ state
200 MeV greater than the $a_0(980)$ mass. Therefore, being the $a_0(980)$ the member of the lowest isovector scalar multiplet, the $\kappa(900)$ cannot be explained as a $q\bar{q}$ pair. We find a candidate for the $K^*_0(1430)$ although with a smaller mass.

**THE SCALAR MESONS AS TETRAQUARK**

Unlike the $q\bar{q}$ pairs tetraquark structures, suggested twenty years ago by Jaffe [27], can couple to $0^{++}$ without orbital excitation and therefore could be serious candidates to explain the structure of the scalar mesons.

In this section we study tetraquark bound states by solving the Schrödinger equation using a variational method where the spatial trial wave function is a linear combination of gaussians. The technical details are given in Ref. [28]. Due to the presence of the kaon-exchange there is a mixture among different configurations with the same isospin. In particular, in the isoscalar sector the configurations: $[(qq)(\bar{q}\bar{q})]$, $[(qs)(\bar{q}\bar{s})]$, and $[(ss)(\bar{s}\bar{s})]$ are mixed. The same happens in the isovector case for the configurations: $[(qq)(\bar{q}\bar{q})]$, and $[(qs)(\bar{q}\bar{s})]$, and in the $I=1/2$ case for the configurations: $[(qq)(\bar{q}\bar{s})]$, and $[(qs)(\bar{s}\bar{s})]$. In all cases $q$ stands for a $u$ or $d$ quark.

The results are shown in Table I (column four) where we present the lowest states for the three isospin sectors. As one can see, there appear two states, in the isoscalar and isovector sectors, with almost the same mass, although too high to be identified with the $f_0(980)$ and $a_0(980)$. In the $I=1/2$ sector, there appears a candidate to be identified with the $\kappa(900)$. It has been recently argued the possible importance of three-body forces arising from the confining interaction for those systems containing at least three quarks [29]. We have performed a calculation including a three-body confining term as the one reported in Ref. [29] fixing its strength to reproduce the mass of the $f_0(980)$. The results are shown in Table I (column five) and as can be seen the degeneracy between the isoscalar and isovector states remains although their masses are now compatible with the experimental data. The lowest state of the isoscalar and $I=1/2$ sectors are almost not affected.

**TABLE 1.** Light scalar meson masses in MeV

| $(q\bar{q})$ state $(n^{2I+1}S^{1+}L_J)$ | Meson   | $(q\bar{q})$ | $(4q)$ | $(4q) + $Three body | Experiment |
|----------------------------------------|---------|--------------|--------|---------------------|------------|
| $1^{3/2}P_0$                          | $a_0(980)$ | 983.5        | 1343   | 968                 | 984.7±1.2  |
| $2^{3/2}P_0$                          | $a_0(1450)$ | 1586.3       |        |                     | 1474±19    |
| $1^{1/2}P_0$                          | $f_0(600)$  | 402.7        | 604    | 644                 | 400−1200   |
|                                        | $f_0(980)$  | 1325         | 1007   |                     | 980±10     |
| $1^{3/2}P_0$                          |          | 1341.7       |        |                     |            |
| $2^{3/2}P_0$                          | $f_0(1370)$ | 1391.2       |        |                     | 1200−1500  |
| $2^{1/2}P_0$                          | $f_0(1710)$ | 1751.8       |        |                     | 1713±6     |
| $3^{1/2}P_0$                          | $f_0(2020)$ | 1893.8       |        |                     | 1992±16    |
|                                        | $\kappa(900)$ |              | 1026   | 922                 | 797±43     |
| $1^{3/2}P_0$                          | $K^*_0(1430)$ | 1213.5       |        |                     | 1412±6     |
| $2^{3/2}P_0$                          | $K^*_0(1950)$ | 1768.5       |        |                     | 1945±30    |

Using the same interaction and formalism we have calculated the $D_{ij}^+(2317)$ as a $[(uc)(\bar{u}\bar{s})]$ tetraquark. The result we obtain, $M=2389$ MeV, suggests that this state could
also have a significant tetraquark component.

As a summary, we have found tetraquark bound states in the region of the light scalar mesons and in the $D_S^*(2317)$. Our results suggest that some states, as it is the case of the $a_0(980)$ and $f_0(600)$, could present a significant mixture of $q\bar{q}$ and tetraquark structures, but it assigns a clear tetraquark structure to the $f_0(980)$ and the $\kappa(900)$. However, more accurate calculations including the exchange terms in the variational wave function which are negligible for the heavy-light tetraquarks and the explicit coupling to $q\bar{q}$ channels should be done before drawing any definitive conclusion.

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