The continuum limit of quark number susceptibilities

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We report the continuum limit of quark number susceptibilities in quenched QCD. Deviations from ideal gas behaviour at temperature $T$ increase as the lattice spacing is decreased from $T/4$ to $T/6$, but a further decrease seems to have very little effect. The measured susceptibilities are 20\% lower than the ideal gas values, and also 10\% below the hard thermal loop (HTL) results. The off-diagonal susceptibility is several orders of magnitude smaller than the HTL results. We verify a strong correlation between the lowest screening mass and the susceptibility. We also show that the quark number susceptibilities give a reasonable account of the Wroblewski parameter, which measures the strangeness yield in a heavy-ion collision.

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I. INTRODUCTION

With the RHIC now in its second year of running, it is important to pin down lattice predictions for the high temperature phase of QCD. In this context, fully non-perturbative measurements of quark number susceptibilities are important for four reasons. Firstly, they are directly related to experimental measurements of event-to-event fluctuations in particle production. Secondly, earlier results showed a strong jump in the susceptibility across the phase transition, but indicated a significant departure from weak-coupling behaviour, and it is important to check whether this persists into the continuum. Third, resummed perturbative computations of this quantity have now become available, thus making it possible to accurately test the importance of non-perturbative contributions to this quantity. Finally, with continuum extrapolated results in hand, one can address the question of whether the strangeness production seen in heavy-ion collisions can be quantitatively explained as a signal of the quark-gluon plasma.

We have recently presented systematic results for quark number susceptibilities in quenched QCD as well as for QCD with two flavours of light dynamical quarks. In these studies a large range of temperatures, $T$, was covered at a series of different quark masses, $m$. These computations were done at a fixed cutoff with the lattice spacing, $a = T/4$ while keeping the finite volume effects under control so that the thermodynamic limit could be taken reliably. There was a 3–5\% difference between the quenched and dynamical computations. In this paper we examine the cutoff dependence of the quenched results and report their continuum (zero lattice spacing) limit.

The partition function of QCD with three quark flavours is

$$Z(T, \mu_u, \mu_d, \mu_s) = \int DU \exp[-S(T)] \prod_{f=u,d,s} \det M(T, m_f, \mu_f),$$

(1)

where the temperature $T$ determines the size of the Euclidean time direction, $S(T)$ is the gluonic part of the action and the determinants of the Dirac matrices, $M$, contain as parameters the quark masses, $m_f$, and the chemical potentials $\mu_f$ for each of the flavours $f = u, d, s$. We also define the chemical potentials $\mu_0 = \mu_u + \mu_d + \mu_s$, $\mu_3 = \mu_u - \mu_d$ and $\mu_8 = \mu_u + \mu_d - 2\mu_s$, which correspond to the diagonal flavour $SU(3)$ generators. Note that $\mu_0$ is the usual baryon chemical potential and $\mu_3$ is an isovector chemical potential.

Quark number densities are defined as

$$n_i(T, \mu_u, \mu_d, \mu_s) = \frac{T \partial \log Z}{V \partial \mu_i},$$

(2)

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and the susceptibilities as

$$\chi_{ij}(T, \mu_u, \mu_d, \mu_s) = \frac{T}{V} \frac{\partial^2 \log Z}{\partial \mu_i \partial \mu_j},$$

(3)

where $V$ denotes the spatial volume. The subscripts $i$ and $j$ are either of the index sets $f$ or $\alpha = 0, 3$ and 8. We lighten the notation by writing the diagonal susceptibilities $\chi_{ii}$ as $\chi_i$. We determine the susceptibilities at zero chemical potential, $\mu_f = 0$. In this limit, all $n_i(T) = 0$. Since we work with $m_u = m_d = m < m_s$, we also have $\chi_{03} = 0$.

Flavour off-diagonal susceptibilities such as

$$\chi_{ud} = \left( \frac{T}{V^3} \right) \langle \text{tr} M_u^{-1} M'_u \text{tr} M_d^{-1} M'_d \rangle$$

(4)

are given entirely in terms of the expectation values of disconnected loops. Since $m_u = m_d$, we obtain $\chi_{us} = \chi_{ds}$ with each defined by an obvious generalization of the formula above. Of the flavour diagonal susceptibilities we shall use

$$\chi_s = \left( \frac{T}{V^3} \right) \left[ \langle (\text{tr} M_s^{-1} M'_s)^2 \rangle + \langle \text{tr} (M_s^{-1} M''_s - M_s^{-1} M'_s M'_s^{-1} M'_s) \rangle \right].$$

(5)

$\chi_u = \chi_d$ are given by a generalization of this formula. Numerically, the simplest quantity to evaluate is the diagonal iso-vector susceptibility

$$\chi_3 = \frac{1}{2} \left( \frac{T}{V^3} \right) \langle \text{tr} (M_u^{-1} M''_u - M_u^{-1} M'_u M'_u^{-1} M'_u) \rangle.$$  

(6)

Two more susceptibilities are of interest. These are the baryon number and electric charge susceptibilities,

$$\chi_0 = \frac{1}{9} (4 \chi_3 + \chi_s + 4 \chi_{ud} + 4 \chi_{us})$$

and

$$\chi_q = \frac{1}{9} \left( 10 \chi_3 + \chi_s + \chi_{ud} - 2 \chi_{us} \right).$$

(7)

Note that $\chi_0$ is the baryon number susceptibility for three flavours of quarks. As a result, this expression differs from the iso-singlet quark number susceptibility for two flavours, defined in [1], both in overall normalization and by terms containing strangeness. In our numerical work we have chosen to use staggered quarks. Hence, to normalise to one flavour of continuum quarks and compensate for fermion doubling on the lattice, we have to multiply each of the traces in eqs. (4-6), and secondly in the determinant in the partition function of eq. (1) which defines the weight for the chemical potential, $\mu$. In this limit, all $n_i(T) = 0$. Since we work with $m_u = m_d = m < m_s$, we also have $\chi_{03} = 0$.

The formula in eq. (8) is normalised such that there is exactly one copy of each flavour of quarks. The values of $\chi_s$, $\chi_0$ and $\chi_q$ for an ideal gas can then be simply obtained as linear combinations of $\chi^3_{FFT}$ for different valence quark masses.
II. LATTICE RESULTS

We have previously reported measurements of $\chi_3$ in quenched QCD with 4 time slices ($N_t = 4$) at temperatures of $1.1T_c$, $1.25T_c$, $1.5T_c$, $2T_c$ and $3T_c$. We have extended these measurements to higher temperatures through two simulations on $4 \times 20^3$ lattices at $\beta = 6.6$ and 6.7. These correspond to temperatures of $4.13T_c$ and $4.7T_c$. All these computations are performed at fixed bare quark masses of $m/T_c = 0.03$, 0.3, 0.5, 0.75 and 1 (the data for $m/T_c = 0.5$ are new).

The continuum limit has been taken by going to several smaller lattice spacings. One set of computations is performed with a lattice spacing $a = 1/6T$ which is 33% smaller than the lattice spacing used earlier. We have taken data on $6 \times 20^3$ lattices at $\beta = 6.0625$, 6.3384 and 6.7 corresponding to $T/T_c = 1.333$, 2 and 3.133, and on a $6 \times 24^3$ lattice at $\beta = 6.8$ corresponding to $T/T_c = 3.532$. One further computation was made on an $8 \times 18^3$ lattice at $\beta = 6.55$. This corresponds to $T/T_c = 2$ with lattice spacing $a = 1/8T$. The quark mass $m/T_c$ is kept independent of $a$ and $T$, and hence $ma$ decreases with increasing $T$ at fixed $N_t$ or with increasing $N_t$ at fixed $T$ according to eq. (9).

The lattice scale has been set using the plaquette measurements of $[8]$ and the analysis performed in $[9]$. The conversion of $\beta$ to $T$ using data for $N_t = 4$ and 8 give the two curves in Figure 1. The difference between the two curves is within the statistical uncertainty of 5% on the scale assignment. There is the same degree of statistical uncertainty in the bare quark masses and in all other scales. We shall not display this inherent scale uncertainty in the measurements, but it should be kept in mind.

The simulations have been performed with a Cabbibo-Marinari pseudo-heatbath technique with 3 $SU(2)$ subgroups updated on each hit. An initial 1000 sweeps have been discarded for thermalisation. On the $N_t \leq 6$ lattices we have used 80 configurations separated by 1000 sweeps for the measurements. On the $N_t = 8$ lattice we have used 55 configurations separated by 500 sweeps.

We have previously checked that the spatial lattice size, $L = N_s a_s$, is quite irrelevant to the values of $\chi$ measured as long as it is sufficiently large compared to the inverse pion mass $[2,4]$. In fact, at all the couplings and quark masses we have used, $m\pi L \geq 5$. The most important other constraint on $L$ comes from the requirement that it should be large enough to prevent spatial deconfinement $[4]$. This is ensured by taking $N_s/N_t \geq T/T_c$ in all our simulations.

FIG. 1: The temperature for any value of $N_t$ for a given coupling $\beta$. Crosses denote data for $N_t = 4$ lattices and pluses for $N_t = 8$ lattices. The scale has been set by a corrected QCD 2-loop formula $[9]$. The difference between the two sets of data is a measure of scale breaking by residual power-law corrections and is negligible within statistical errors.
FIG. 2: $\chi_3 / \chi_{3,\text{FFT}}$ at $T = 2T_c$ shown as a function of $a^2 \propto 1/N_t^2$ for $m/T_c = 0.03$. The ratio reaches its continuum limit when evaluated on lattices with $N_t = 6$.

The traces are evaluated by the usual stochastic technique,

$$\text{Tr} A = \frac{1}{2N} \sum_{i=1}^{N} R_i^\dagger A R_i,$$

where $R_i$ are a set of $N$ uncorrelated vectors with components drawn independently from a Gaussian ensemble with unit variance. Each vector has three colour components at each site of the lattice. Since we use a half lattice version of the Dirac operator for staggered Fermions, the number of components of each vector $R_i$ is one and a half times the number of lattice points. $(\text{Tr} A)^2$ is obtained by dividing the set of $R_i$ into disjoint blocks, constructing $\text{Tr} A$ in each block, taking all possible distinct pairs of such estimates, multiplying them and then averaging over the pairs. For $N_t = 4$ it is possible to get accurate results with $N \approx 10$, although we have chosen to use $N = 80$ for each gauge configuration.

It is easy to check that the variance of the estimator above is proportional to $\text{Tr}(A^2)$. Since the diagonal element of the Fermion matrix is proportional to the quark mass $ma$, whereas the off-diagonal elements are bounded by unity, with increasing $T$ or decreasing $a$, $\text{Tr} A$ decreases linearly, while its variance remains constant. Hence, for sufficiently small $ma$ the number of vectors has to increase quadratically with $ma$. We found that $N_v = 100$ was sufficient for $N_t = 8$ at $T = 2T_c$, although at larger $N_t$ or $T$ significantly larger values of $N_v$ are needed. A further numerical problem arises from the cancellation of the two matrix elements which give $\chi_3$ (see eq. 6). While increasing $N_t$ and keeping the physical size of the lattice fixed, each of these terms increases quadratically but mutually cancel to give a number which decreases quadratically with $N_t$. At fixed word length, this implies a reduction in accuracy by a factor of $N_t^4$.

In Figure 2 we show how the continuum limit of $\chi_3$ is reached. There is an 8% decrease in $\chi_3 / \chi_{3,\text{FFT}}$ in going from $N_t = 4$ to $N_t = 6$ lattices. However, within measurement errors, the continuum limit is reached already at $N_t = 6$. This implies a cancellation of the $N_t$-dependence of $\chi_3$ and $\chi_{3,\text{FFT}}$. The reason for the atypical behaviour for $N_t = 4$ is not in the numerator, which is non-perturbative, but in the denominator, which is the ideal gas.

For $N_t = 4$, the spectrum of $p_0$ consists of $\{\pm \pi/4, \pm 3\pi/4\}$. As a result, $\sin^2 p_0 = 1/2$ and the second term in the sum on the right of eq. (5) vanishes identically. This accident occurs only for $N_t = 4$, and for all other values of $N_t$ this term contributes a non-vanishing value. It is easy to see that in the limit $N_t \to \infty$ this term contributes to the leading part of $\chi_{3,\text{FFT}}$. This is the reason the values for $\chi_3 / \chi_{3,\text{FFT}}$ for $N_t = 4$ lie well away from the continuum limit.
FIG. 3: The ratio $\chi_3/\chi_{\text{FFT}}$ in quenched QCD, shown as a function of $T$ for the values of $m/T_c$ indicated. Pluses denote data for lattice spacing $a = 1/4T$ and circles for $a = 1/6T$. The lines are cubic spline fits to the data.

| $T/T_c$ | $\chi_3/\chi_{\text{FFT}}$ | $\chi_{ud}/T^2$ | $\chi_3/\chi_{\text{FFT}}$ | $\chi_0/\chi_{\text{FFT}}$ | $\chi_q/\chi_{\text{FFT}}$ |
|---------|-------------------------------|------------------|---------------------------|-------------------------|-------------------------|
| 1.333   | 0.70 (3)                      | $(2 \pm 4) \times 10^{-6}$ | 0.455 (5)                 | 0.66 (2)                | 0.68 (3)                |
| 2.000   | 0.761 (3)                     | $(-1 \pm 2) \times 10^{-7}$ | 0.656 (3)                 | 0.740 (3)               | 0.751 (3)               |
| 3.133   | 0.801 (2)                     | $(-1 \pm 1) \times 10^{-7}$ | 0.766 (2)                 | 0.794 (2)               | 0.798 (2)               |
| 3.532   | 0.807 (2)                     | $(1 \pm 2) \times 10^{-7}$  | 0.789 (2)                 | 0.803 (2)               | 0.805 (2)               |

TABLE I: Results for the continuum limit of quark number susceptibility in quenched QCD. We have taken $m_s/T_c = 1$ as appropriate to full QCD. Our $N_t = 4$ results indicate that at higher $T$ the ratios $\chi/\chi_{\text{FFT}}$ remain flat.

Our complete results for the dependence of $\chi_3/\chi_{\text{FFT}}$ on $T/T_c$ at several different quark masses for $N_t = 4$ and 6 are shown in Figure 3. In view of the data shown in Figure 2 our results for $N_t = 6$ are the continuum limit for the quenched theory: these are summarised in Table I. Note that even at temperature as high as $3.5T_c$ the result is far below the ideal gas limit. Closer to $T_c$ there is an even more dramatic fall in the value of $\chi_3/\chi_{\text{FFT}}$. We have also measured the off-diagonal susceptibility $\chi_{ud}$. As already seen for $N_t = 4$ this is consistent with zero. In Table I we show that the continuum limit of $\chi_{ud}$ also vanishes within errors of $10^{-7} T^2$. It is therefore much smaller than the 3-loop perturbative result given in [6].

Note that the continuum limit of our measurements lie 20–30% below the ideal gas result. They also differ significantly from HTL as well as the skeleton graph resummed results of [6]. At $T = 3T_c$, HTL predicts $\chi_3/\chi_{\text{FFT}} = 0.90–0.94$ on varying the scale of $\alpha_s$ between $\pi T$ and $4\pi T$, and the resummed computation gives $\chi_3/\chi_{\text{FFT}} = 0.95–0.97$. Our measurement shows $\chi_3/\chi_{\text{FFT}} = 0.80$ at $3T_c$ (see Table I). A second computation of the HTL result is available for $N_f = 2$ [10]. Since this agrees with the HTL result of [6] for $N_f = 2$, we believe the HTL result for $\chi_3/\chi_{\text{FFT}}$ is under control. There is thus a genuine discrepancy between these lattice results and existing perturbative computations.

The quark number susceptibilities are closely related to the screening correlator of a one-link separated quark bilinear operator which corresponds to a $\rho$ meson at zero temperature. The $T > 0$ transfer matrix of the problem mixes this with a quark bilinear that corresponds to the $\pi$ [11]. We have earlier shown evidence for $N_t = 4$ that $\chi_3$ is closely related to the only known non-perturbative quantity among the screening correlators, that is the screening mass $M_S$ coming from the two degenerate correlators that descend from the $\pi$ and $\sigma$ operators of the zero temperature theory [4]. In view of the strongly non-perturbative character of $\chi_3$, as revealed by the comparison with HTL and...
resummation explained above, examining this correlation in the continuum becomes more significant. In Figure 4 we show data obtained on \(8 \times 18^3\) lattices which indicate that this correlation survives into the continuum.

### III. STRANGENESS

Measurements of hadron yields in relativistic heavy-ion collisions have been analysed extensively \([12, 13]\) and the observed enhancement of strangeness has been claimed to be evidence for quark-gluon plasma formation. The chemical composition is observed at the hadron freezeout temperature, which has been found to be very close to the value of \(T_c\) in full QCD. It has been argued that this chemical composition cannot arise due to hadronic rescattering \([14]\). It would be interesting if it could be directly determined whether or not the composition is characteristic of the plasma for \(T \geq T_c\). We argue below that this question can be answered by a lattice QCD measurement such as we have presented above.

The quantity which seems to be directly connected to lattice measurements is the Wroblewski parameter \([15]\), \(\lambda_s\), which measures the ratio of newly created primary strange to light quarks—

\[
\lambda_s = \frac{2\langle ss\rangle}{\langle uu + dd\rangle}. \tag{11}
\]

A fluctuation-dissipation theorem relates the rate of production of quark pairs by the plasma in equilibrium to the imaginary part of the full complex quark number susceptibilities \([16]\). If the inverse of the characteristic time scales of the QCD plasma are not close to the typical energy scales for the production of strange and lighter quarks, then the ratio of their production rates is just the ratio of the static susceptibilities that we have measured. Then, if the observed chemical composition is created in equilibrium, we should have

\[
\lambda_s \approx \frac{2\chi_s}{\chi_u + \chi_d} \approx \frac{\chi_s}{\chi_u}, \tag{12}
\]

where the last equality holds in the limit of equal \(u\) and \(d\) quark masses. These susceptibilities have to be evaluated on the lattice at the temperature and chemical potential, \(\mu_0\), relevant to the collision. However, it has been shown...
FIG. 5: The Wroblewski parameter, $\lambda_s$, obtained from various experimental measurements, compared with the value obtained from the lattice measurements reported here.

recently [17] that $\lambda_s$ is insensitive to $\mu_0$ for SpS and RHIC energies. Furthermore a lattice measurement shows that the screening mass, $M_S$, does not change rapidly with increasing chemical potential [18]. Then, in view of the correlation between $M_S$ and $\chi$ shown in Figure 4, we do not expect rapid changes with chemical potential in the ratio above. Both these arguments show that as a first approximation one can take the ratio in eq. (12) at zero chemical potential.

From our measurements of $\chi$ reported here, we can form the lattice “prediction” of $\lambda_s$ created in the plasma at $T_c$. This has been done by taking our measurements at $T/T_c = 4/3$ and 2, and extrapolating them linearly to $T_c$. For $\chi_u$ we use data taken for $m/T_c = 0.03$, which is light enough for this purpose. Since scaled quantities feel the smallest correction for unquenching [4], we take $\chi_s$ to correspond to $m/T_c = 1$, which is appropriate to full QCD. In Figure 5 the resultant prediction is compared with the data collected in [13, 17]. It can be seen that there is a fair agreement with the data. However, this is subject to several assumptions—

1. The foremost assumption is that the characteristic time scales of the plasma are not close to the inverse energy scale of the production processes. It is not possible to test this assumption in an Euclidean computation. However, it has been suggested that these characteristic time scales could be observed in real or virtual photon production as spikes in the spectrum [19]. If these are seen and they lie in the energy range for the production, the assumption would be falsified.

2. Another important assumption is that chemical equilibration takes place in the plasma. If chemical equilibrium is not achieved but the light degrees of freedom achieve energy equipartition, then the effective temperature could be higher.

3. We have assumed slow variation of the ratio $\chi_s/\chi_u$ with chemical potential. While this seems to be a reasonable estimate in view of the results of [17, 18] and our own observations about the relation between the lowest screening mass and the susceptibilities, one should keep in mind possible changes in the ratio due to violation of this assumption.

4. Our continuum results are obtained in quenched QCD, whereas the data corresponds to full QCD. As mentioned already, we have taken $m/T_c = 1$, as appropriate to full QCD, in order to correct for this. However, an additional 5–10% shift in our results could arise when unquenching [4]. This is not shown in Figure 5.
Our measurements are made away from $T_c$ and extrapolated down to this temperature. In principle this can be corrected by a computation made directly at $T_c$. However, due to the enormous cost of extracting physical quantities at $T_c$, we postpone this to the time when we attempt a full QCD simulation.

It might seem surprising that $\lambda_s$ corresponds to the chemical composition at $T_c$. However, a moment’s thought shows that this is entirely in accord with the assumption of chemical equilibrium. Even if the system is thermalized at a much higher temperature, it remains in equilibrium until it reaches $T_c$. Departure from equilibrium as the system cools through the transition is unlikely, and if it occurs, should have a visible signature such as the formation of a disoriented chiral condensate \[24\]. In the absence of such phenomena, if $\lambda_s$ does not correspond to the chemistry at $T_c$ then the second assumption above would be demonstrated to be false. Also, with increasing ion energy the system moves closer to zero baryon number, as seen at RHIC. This makes our third assumption better at the RHIC and LHC. The fourth assumption can be removed by a computation with full QCD. Such a computation is planned, where also the final assumption can be removed, and will be reported elsewhere.

IV. SUMMARY

In summary, we have presented new and precise results on quark number susceptibilities over a wide range of temperatures and quark masses in the high temperature phase of QCD. The main aim of this study was to obtain the continuum extrapolation of these susceptibilities. As shown in Figure 2 while there is a significant change in going from $N_t = 4$ to $N_t = 6$ lattices, there is no statistically significant change in the susceptibilities beyond this. The results obtained for $N_t = 6$ lattices can then be taken as the continuum limit. This limit is shown in Figure 3 and Table I.

There is a strong discrepancy between the continuum extrapolated lattice results and HTL computations for these quantities—varying between about 12% at the highest $T$ to even larger at smaller $T$. The off-diagonal susceptibility, $\chi_{ud}$, is zero, in contrast to the HTL results. There is a somewhat larger discrepancy between the continuum extrapolated lattice results and the skeleton graph resummed results. The conjecture that there is a strong non-perturbative component to the quark number susceptibilities is supported by an observed strong correlation between the smallest quark bilinear screening mass, $M_S$, and the susceptibility $\chi_3$ (Figure 4).

It is interesting to note that the continuum extrapolated results for the strange and light quark susceptibilities can be used to give a surprisingly good description of the chemical composition of hadrons at freezeout in SpS and RHIC experiments (Figure 5). This has to be treated as a preliminary estimate due to the many caveats which we have listed, and some of which we plan to check in future lattice measurements.

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