Damage identification based on incomplete modal data and constrained nonlinear multivariable function

S. S. Kourehli
Department of Civil Engineering, Ahar Branch, Islamic Azad University, Ahar, Iran

E-mail: s-kourehli@iau-ahar.ac.ir

Abstract. Damage detection and estimation in structures using incomplete modal data is presented. In the proposed approach, damage location and severity is determined by solving an optimization problem using the constrained nonlinear multivariable function of Matlab (so-called fmincon) to perform constrained minimization. The feasibility of the presented method is validated with a three-story plane frame as numerical example containing one or several damages with different value of damage severity. The obtained results indicated that proposed method is effective and robust in detection and estimation of damage.

1. Introduction
Damage detection is one of the branches of structural health monitoring (SHM) which has recently attracted many scientific efforts. Health monitoring refers to a process of measuring and interpreting data from a system of sensors distributed about a structural system to objectively quantify the condition of the structure [1, 2]. Damage detection techniques have been successfully applied to several real-world problems. Based on the performance of structures, damage detection methods can be categorized into four levels [3]. The first level is devoted to detection of existing damages in a structure, and the second and third levels focus on the determination of the location and severity of damage in structures, respectively. The last level is a complete study that includes the estimation of the residual life of a structure, reaching a point that requires more information from fracture mechanics and structural reliability.

Among numerous methods, approaches that are based on the observation of the dynamic behavior of a structure have been developed [4-7]. Many of these techniques use the identified modal parameters like mode shapes and natural frequencies for structural damage detection and estimation. The identification of alteration in mode shapes and natural frequencies at the damaged system in comparison with the undamaged system is one of the popular methods in the structural damage detection. Carvalho et al. [8] presented a direct method for model updating with incomplete modal data. The proposed method uses an algorithmic way without requiring any model reduction or modal expansion techniques. Huajun et al. [9] extended the CMCM method to simultaneously update the mass, damping and stiffness matrices of a finite element model when only few spatially incomplete, complex-valued modes are available. The results reveal that applying the CMCM method, along with an iterative Guyan reduction scheme can yield good damage detection in general. Also, Chen [10] presented an approach for detecting local damage in large scale frame structures by utilizing...
regularization methods with incomplete noisy data. A system of linear basic equations for determining the damage indicators has been developed by directly adopting the measured incomplete modal data.

In this research, a new method for localizing and estimating the severity of structural damage is introduced. The damage identification is carried out through fmincon to minimize an objective function derived from incomplete dynamic characteristics of damaged structure. Numerical example shows that the proposed method can be considered as a flexible and robust approach in damage identification of structures.

2. Problem formulation

The modal characteristics of a structure without damage are described by the following equations:

\[
([K^{ud}] - \lambda_i[M])\{\Phi_i\} = 0 \quad i = 1, 2, \ldots, n
\]  

(1)

where, \(K^{ud}\) and \(M\) are undamaged stiffness and mass matrices, respectively; \(\lambda_i\) is the square of the natural frequency corresponding to the mode shape \(\Phi_i\); and \(n\) is the total number of obtained mode shapes.

One of the simplest techniques to determine damage-induced alteration stiffness is the degradation in Young’s modulus of an element as follows:

\[
E^d_j = E^{ud}_j(1 - d_j)
\]

(2)

where, \(E^d_j\) and \(E^{ud}_j\) are the damaged and undamaged Young’s modulus of the \(j\)th element in the finite element model, respectively; and \(d_j\) indicates the damage severity at the \(j\)th element in the finite element model whose values are between 0 for an element without damage and 1 for a ruptured element.

Moreover, it is assumed that no change would occur after damage in the mass matrix, which seems to be reasonable in most real problems.

Thus, as it was mentioned above, the eigenvalue equations for a damaged structure became:

\[
([K^d] - \lambda_i^d[M])\{\Phi_i^d\} = 0 \quad i = 1, 2, \ldots, n
\]

(3)

where, \(K^d\) is the damaged stiffness matrix; \(\lambda_i^d\) and \(\Phi_i^d\) are the square of the \(i\)th natural frequency and the \(i\)th mode shape of the damaged structure, respectively.

As the number of sensors used to measure modal data is normally limited and usually is less than the number of DOFs in the finite element model, either the model reduction method should be used to match with incomplete measured mode shapes or the measured mode shapes must be expanded to the dimension of the analytical mode shapes. Because of no convergence in the proposed optimization method using the modal expansion, the first option has been adopted using the Guyan [11] static reduction method. This method is employed to condense the mass and stiffness matrices. In this method, the mass and stiffness matrices, and the displacement and acceleration vectors in Eq. (1) are partitioned into a set of master and slave DOFs:

\[
\begin{bmatrix}
M_{mm} & M_{ms} \\
M_{ms} & M_{ss}
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_m \\
\ddot{x}_s
\end{bmatrix}
\begin{bmatrix}
K_{mm} & K_{ms} \\
K_{ms} & K_{ss}
\end{bmatrix}
\begin{bmatrix}
x_m \\
x_s
\end{bmatrix}
= \begin{bmatrix}0 \\ 0\end{bmatrix}
\]

(4)

In which, \(x\) and \(\dot{x}\) are the displacement and acceleration vectors, respectively and the subscript \(m\) and \(s\) are the master and slave coordinates, respectively. To eliminate the slave DOFs, the inertia terms for the second set of equations are neglected that leads to:
where:

\[ \begin{bmatrix} x_m \\ x_s \end{bmatrix} = [T] \{x_m\} \]  

where:

\[ [T] = \begin{bmatrix} I \\ -K_{rr} T K_{sm} \end{bmatrix} \]  

Substitution of Eq. (5) into Eq. (4), followed by permultiplication by \([T]^T\) and using incomplete mode shapes, yields the reduced eigenproblem as:

\[ ([K_d] - \lambda_i^d [M_r]) \{\Phi_i^d\} = 0 \quad i = 1, 2, \ldots, n \]  

where:

\[ [K_d] = [T]^T [K] [T] \]  
\[ [M_r] = [T]^T [M] [T] \]

in which, \(K_d\) and \(M_r\) are the reduced stiffness and mass matrices of damaged structure, respectively; \(\lambda_i^d\) and \(\Phi_i^d\) are the square of the \(i\)th natural frequency and the \(i\)th incomplete mode shape of the damaged structure in the reduced state, respectively.

Applying the incomplete mode shapes and natural frequencies of damaged structure to Eq. (7) leads to the inverse problem of determining the damage severity parameter. The definition of a local damage severity parameter \(d\) in the finite element model allows estimating damage quantity and location together, since damage identification is then carried out at the element level. The problem can be formulated as an optimization problem of objective function, while using some transforms as a direct inversion to obtain solution is impossible most of the time.

The general statement for the objective function is:

\[ F = f(d_1, d_2, \ldots, d_{N_d}) \]  

In the process of substituting the incomplete measured modal parameters of the damaged structure in Eq. (10), a dynamic residue vector can be defined over each measured mode as follows:

\[ [R_i(d)] = ([K_i^d] - \lambda_i^m [M_r]) \{\Phi_i^m\} \quad i = 1, 2, \ldots, m \]  

where, \(\lambda_i^m\) and \(\Phi_i^m\) are the square of the \(i\)th natural frequency and the \(i\)th incomplete mode shape from measurements, respectively; and \(m\) is the number of available mode shape for damage detection.

Then, if structural damages are determined correctly, the residue vector would be next to 0 in Eq. (11). Therefore, the problem of damage detection can be formulated as an optimization problem. So, the first objective function can be formulated as follows:

\[ f_i(d) = \sum_{i=1}^{m} \|R_i(d)\|^2 \]  

\[ 0 \leq d_1 \leq 1, \quad 0 \leq d_2 \leq 1, \quad \ldots, \quad 0 \leq d_{N_d} \leq 1 \]

where, \(\| \|\) represents the Euclidean length of \(R_i(d)\).
The optimization technique used in this paper was a constrained nonlinear minimization, `fmincon`, which is available in the MATLAB Optimization Toolbox. This routine implements Sequential Quadratic Programming (SQP) to minimize the nonlinear cost function subjected to linear and nonlinear equality and inequality constraints. SQP converts a nonlinear minimization to a linear minimization using a Hessian matrix of cost function and gradient of nonlinear constraints. In addition to the use of `fmincon`, multiple start points were used to ensure that the global minimum was reached. The global minimum can be seen with repeated answers in the final output.

3. Numerical study

Consider a three-story plane steel frame for which the finite-element model consists of nine elements (six columns and three beams) and six free nodes, as shown in Fig. 1. For the steel frame considered, the material properties of the steel include Young’s modulus $E=200$ GPa, mass density $\rho=7850$ kg/m$^3$. The mass per unit length, moment of inertia, and cross-sectional area of the columns are: $m=117.75$ kg/m, $I=3.3 \times 10^{-4}$ m$^4$ and $A=1.5 \times 10^{-2}$ m$^2$, respectively; for the beams are: $m=119.32$ kg/m, $I=3.69 \times 10^{-4}$ m$^4$ and $A=1.52 \times 10^{-2}$ m$^2$. Also, the damage severity in each element is given by the reduction factor listed in Table 1.

![Fig. 1 Three-story plane frame with the finite element model](image)

In this example, two damage scenarios are represented as the elements with reduction in Young’s modulus. The damage severity in each element is given by the reduction factor listed in Table 4.1. In this case, only 6 translational DOFs are selected as the measured DOFs in the process of damage detection and quantification.

| Table 1. Damage scenarios for three story plane frame |
|-----------------------------------------------|
| Scenario 1 | Scenario 2 |
| Element 2 | 20% | Element 2 | 20% |
| Element 4 | 35% |
| Element 9 | 25% |

Different initial values of damage severities in the proposed method have been tested to check its convergence. Figures 2 and 3 show the results of damage identification in the three story plane frame for two damage patterns with zero and 50% initial values, respectively. It depicts that the proposed
method is a robust and effective method in detecting and quantifying various damage patterns with different initial values of the damage severities.

Fig. 2 The obtained results for two damage patterns of the three story frame with 0% initial values

Fig. 3 The obtained results for two damage patterns of the three story frame with 50% initial values
4. CONCLUSIONS
In this paper, a method has been developed for detection and estimation of damage in structures based on the incomplete modal data of the damaged structure using an optimization problem. In this method, constrained nonlinear minimization, fmincon, is used to determine the damage in structures by optimizing a cost function. For damage detection and estimation, this proposed method was applied to a three story plane frame with one or several damage patterns. The obtained results indicated that the proposed method is a strong and viable method to the problem of detection and estimation of damage in the structures. The results revealed high sensitivity of the proposed method to the damage in spite of incomplete measurements.

References
[1] Johnson E A, Lam H F, Katafygiotis L S and Beck J L 2004 Phase I IASC-ASCE structural health monitoring benchmark problem using simulated data J. Eng. Mech. 130(1) 3-15.
[2] Zingoni A 2005 Structural health monitoring, damage detection and long-term performance J. Eng. Struct. 27(12) 1713-1714.
[3] Rytter, A 1993 Vibration based inspection of civil engineering structures PhD Dissertation Aalborg University, Aalborg.
[4] Doebling SW, Farrar CR, Prime MB, Schewitz DW 1996 Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review Technical Report LA-13070-MS, Los Alamos National Laboratory, Los Alamos.
[5] Kourehli, S S 2015 Damage assessment in structures using incomplete modal data and artificial neural network International Journal of Structural Stability and Dynamics Vol. 15 No. 6 1450087. DOI: 10.1142/S0219455414500874.
[6] Kourehli S S, Ghodrati Amiri G, Ghafoory-Ashtiany M., Bagheri A 2013 Structural damage detection based on incomplete modal data using pattern search algorithm Journal of vibration and control Vol.19 No.6 pp. 821-833.
[7] Kourehli S S, Bagheri A, Ghodrati Amiri, G, Ghafoory-Ashtiany, M 2013 Structural damage detection using incomplete modal data and incomplete static response KSCE journal of civil engineering Vol.17 No.1 pp. 216-223.
[8] Carvalho J, Datta BN, Gupta, A and Lagadapati M 2007A direct method for model updating with incomplete measured data and without spurious modes Mechanical System and Signal Processing, Vol. 21 No. 7 pp. 2715-2731.
[9] Huajun L, Fushun L, James HS 2008 Employing incomplete complex modes for model updating and damage detection of damped structures Sci China Ser E-Tech Sci. Vol. 51 No. 12 pp. 2254-2268.
[10] Chen H 2008 Application of regularization methods to damage detection in large scale plane frame structures using incomplete noisy modal data Engineering Structures Vol. 30 No. 11 pp. 3219-3227.
[11] Guyan RJ 1965 Reduction of stiffness and mass matrices AIAA Journal, Vol. 3, No. 2, pp. 380-387.