Modeling and simulation of surface topography in five-axis ball end milling

Juan Wei*, Xiaodong Hou, Chao Sun
School of Mechanical Engineering, Xi’an University of Science and Technology, Xi’an 710054, China
Email: 18205216093@stu.xust.edu.cn

Abstract. Computer simulation can be used to predict the machined surface topography and roughness, which can provide a basis for the reasonable selection of cutting parameters and tools. To do so, the motion trajectory equation of ball end milling cutter was established based on homogeneous coordinate transformation matrix. On this basis, a 3D surface topography simulation algorithm based on workpiece meshing is constructed. Finally, the influence laws of feed rate per tooth, path internal and initial phase angle on surface topography were analyzed by simulation. The conclusion has a good guiding significance for the surface quality control and adjustment of machining parameters in actual production.

1. Introduction
Five-axis milling is widely used in machining complex free-form surface parts in aerospace, power plant, automobile and other industries due to its adjustable posture[1]. Ball milling is often used as the last processing procedure to directly achieve the required surface quality of the product. As an important aspect of surface quality, surface morphology directly affects the surface friction, fatigue resistance and assembly accuracy of parts. Besides, the simulation and prediction of surface morphology before processing eliminate the trial and error methods used to achieve the expected surface quality, thereby reducing processing costs and improving efficiency.

The modeling and simulation of machined surface topography have received great attention in recent years. Mizugaki et al. [2]established a theoretical estimation model of the surface topography on the basis of a simplified hemispherical cutting edge, and improved the model by introducing the parameter of the inclination of the tool axis, considering the effect of the inclination of the tool axis on the surface texture and residual height. But the accuracy of the model is limited by the constraint of the simplified hemispherical cutting edge. Arizmendi et al. [3]deduced the tool cutting edge and its sweep surface parameter equation considering the runout error of tool. Through the prediction model containing runout error, the effect of tool eccentricity on the surface topography was deeply explored. Wang et al. [4]proposed a five-axis side milling surface topography prediction model considering tool runout. This model uses an elliptical trajectory as cutting edge trajectory, which is inconsistent with the actual machining topography formation mechanism.

To analyze the formation mechanism and influencing factors of the surface topography in five-axis ball end milling, this paper established a prediction model for the surface topography of the ball end cutter, and simulated the surface topography with the MATLAB software, and finally based on the simulation result analyzed the influence law of different influencing factors on the machined surface morphology.
2. Model of the motion ball end milling cutter

To facilitate modeling, the coordinate system is set up as shown in Fig. 1. The workpiece coordinate system \( O_WX_WY_WZ_W \) is the reference coordinate system fixed on the workpiece, denoted \( \{O_W\} \). The spindle coordinate system \( O_AX_AY_AZ_A \), denoted \( \{O_A\} \), its coordinate axis \( O_AZ_A \) is consistent with the rotation axis of the machine tool spindle and moves with the spindle relative to the workpiece. The cutter coordinate system \( O_TC_Y_TW_Z_W \), which is fixed on the cutter, denoted \( \{O_T\} \), its coordinate origin coincides with the center of the ball head part, translation movement along the feed direction relative to the workpiece with the spindle, not rotating with the cutter.

![Fig. 1 Coordinate system of tool tooth movement trajectory](image)

Spiral edge ball milling cutter is selected as the research object, as shown in Fig. 2. The number of teeth of ball milling cutter is assumed to be \( n_t \), then the homogeneous coordinate matrix of any point \( P \) on the curve of the \( k \)-th tooth edge in the cutter coordinate system \( \{O_T\} \) is as follows:

\[
\begin{bmatrix}
    x_k^T \\
    y_k^T \\
    z_k^T \\
    1
\end{bmatrix} = \begin{bmatrix}
    R \sin \alpha \cos(\eta + \phi_k) \\
    R \sin \alpha \sin(\eta + \phi_k) \\
    -R \cos \alpha \\
    1
\end{bmatrix}
\]

(1)

Where \( R \) is the radius of the tool, \( \alpha \) is the axial immersion angle of the point \( P \), \( k \) is the serial number of cutter edge, \( n_t \) is the number of cutter edges, \( \phi_k \) is the angle between the \( k \)-th cutter edge and the first cutter edge, the angle \( \eta \) is caused by helix angle \( \beta \) of the tool. \( \phi_k \) and \( \eta \) can be expressed as follows[5]:

\[
\eta = \tan \beta \ln \left( \cot \frac{\alpha}{2} \right)
\]

\[
\phi_k = 2\pi(k - 1)/n_t
\]

Formula (1) is the static coordinate of point \( P \) in the cutter coordinate system. In the actual machining process, the ball end cutter moves along the feed direction in the state of rotation about its own axis.

Fig. 2 Geometric model of ball end mill

(a) Perspective view (b) Front view (c) Top view

![Fig. 2 Geometric model of ball end mill](image)
Where the rotation angle of the cutting edge in the $i$-th feed is represented by $\theta_i$, the rotation transformation matrix $M_i$ of the cutter coordinate system $\{O_T\}$ relative to the spindle coordinate system $\{O_A\}$ is as follows:

$$
M_i = \begin{bmatrix}
\cos \theta_i & \sin \theta_i & 0 & 0 \\
-\sin \theta_i & \cos \theta_i & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(2)

In the formula, $\theta_i = \varphi_i - wt\cdot \varphi_{i,1}$ is the initial phase angle of the $i$-th feed, $w$ is the angular velocity of the tool rotating with the machine tool spindle, $t$ is the time elapsed from the beginning of the $i$-th feed to the current moment.

Suppose that the feed motion equations of the tool position point in the workpiece coordinate system along the $x$, $y$ and $z$ directions are $x(t)$, $y(t)$, $z(t)$ respectively, the translation transformation matrix $M_2$ is as follows:

$$
M_2 = \begin{bmatrix}
1 & 0 & 0 & x(t) \\
0 & 1 & 0 & y(t) \\
0 & 0 & 1 & z(t) \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(3)

In five-axis machining process, the transformation of $P$ point from the spindle coordinate system $\{O_A\}$ to the workpiece coordinate system $\{O_W\}$ requires two coordinate rotation transformations: (1) the rotation angle $\gamma_3$ around the X-axis, (2) the rotation angle $\gamma_2$ around the Z-axis. $M_3$ and $M_4$ represent the rotation transformation matrix around the X-axis and Z-axis respectively. The expressions of $M_3$ and $M_4$ are shown in formula (4).

$$
M_3 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \gamma_1 & -\sin \gamma_1 & 0 \\
0 & \sin \gamma_1 & \cos \gamma_1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad
M_4 = \begin{bmatrix}
\cos \gamma_2 & \sin \gamma_2 & 0 & 0 \\
-\sin \gamma_2 & \cos \gamma_2 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(4)

After the geometric model of the ball end cutter edge is established and the coordinate system transformation matrix is calculated, the trajectory equation of the tool edge in the workpiece coordinate system $\{O_W\}$ can be obtained as follows:

$$
\begin{bmatrix}
x_i^w \\
y_i^w \\
z_i^w \\
1
\end{bmatrix} = M_4 M_3 M_2 M_1 \begin{bmatrix}
x_i^T \\
y_i^T \\
z_i^T \\
1
\end{bmatrix}
$$

(5)

According to Eq. (5), the simulation diagram of sweeping surface of cutting edge can be obtained, as shown in Fig. 3. In the figure, one tooth of the ball cutter is involved in cutting, there is feed residue in the feed direction and inter-row residue in the radial direction.

Fig. 3 Sweeping surface of the cutter edge
3. Numerical simulation algorithm
After the mathematical model of ball end milling cutter movement is established, the simulation algorithm of machined surface topography based on workpiece meshing is adopted. The simulation steps are as follows:

STEP 1: Input the initial processing conditions, including tool geometry parameters, cutting parameters, workpiece mesh model and workpiece height matrix \( H \) before the simulation of the surface topography. The detailed method of meshing the workpiece is that the length \( L_y \) along the feed direction (y direction) and the length \( L_x \) along the radial direction (x direction) of the workpiece is divided into \((m - 1)\) and \((n - 1)\) parts, and workpiece grid spacing is \( dx \) and \( dy \) respectively. The matrix \( H[i,j](i = 1,2, ..., m; j = l, 2, ..., n) \) is used to store the height of each grid point on the workpiece surface, where all elements values of the initialization matrix \( H \) are the axial cutting depth \( a_p \).

STEP 2: Set the time step and the number of discrete points according to the accuracy of the workpiece grid to ensure that micro-unit of cutter edge can sweep at most one workpiece grid point in a unit time step.

STEP 3: Calculate the coordinate value \((x_i, y_i, z_i)\) of the discrete point \( P \) of cutter teeth selected at the previous time in the workpiece coordinate system. If \( 0 < x_i < L_x, 0 < y_i < L_y \) is satisfied, it means that the point has entered the workpiece cutting area. Otherwise, the next discrete point calculation is performed. Then according to the corresponding relationship between \((x_i, y_i)\) and the subscript \((i, j)\) of the workpiece height matrix \( H \), the workpiece grid points closest to points \( P(i, j) \) are found. The \( z_i \) value of the discrete point \( P \) in the workpiece coordinate system at the current moment is compared with the stored value of the corresponding matrix element \( H[i, j] \). If \( z_i \) is smaller, it means the tool cuts into the workpiece and the stored value of \( H \) is updated with \( z_i \) value. Otherwise, no processing will be done.

STEP 4: Repeat step 3 for each discrete point on each cutter tooth.

STEP 5: According to the registered data in the \( H \) matrix, the three-dimensional topography of the machined surface is drawn with MATLAB.

4. The influence of process parameters on surface morphology
4.1. Feed rate per tooth
In the milling process of ball end cutter, the residual height in the feed direction and the radial direction is determined by feed rate per tooth and path internal respectively. Therefore, this article first studies the influence of feed rate per tooth and path internal under different combinations on surface topography. Under the condition that the tool radius \( R \) is 1.5mm, the number of cutting edges \( n_e \) is 2, the inclination of tool axis is 10°, the spindle speed \( n \) is 6000r/min, the cutting depth \( a_p \) is 0.2mm, and the initial phase angle difference is 0°, the machined surface topography is simulated. Figure 4 shows the machined surface topography when feed per rate tooth \( f \) is from 0.04mm/z to 0.08mm/z, 0.12mm/z and path internal \( p \) is 0.1mm.

![Fig. 4 Simulation of surface morphology under different feed rate per tooth](image-url)
It can be seen from the simulation results in Fig. 4 that the surface topography in the feed direction has a greater change when other cutting parameters remain unchanged, only the feed per tooth is changed. As the feed rate per tooth increases, the length of the micro-unit in the feed direction continues to increase, and the residual height value is increasing.

4.2. path internal
Figure 5 illuminates the machined surface topography when path internal $p$ is from 0.1mm to 0.2mm, 0.3mm and feed per rate tooth $f$ is 0.04mm. As can be seen from Fig. 5(a), (b) and (c), when feed per rate tooth remains constant, the residual height in the radial direction increases with the increase of path internal, the number of inter-row residue occurrences continues to decrease, while the length of the feed direction remains basically unchanged.

Combined with Fig. 4 and Fig. 5, it can be seen that the surface texture is determined by both feed rate per tooth and path internal. When feed rate per tooth is greater than path internal, the residual height in the feed direction is greater than the residual height in the radial direction, and the surface texture is determined by the residual height in the feed direction. When path internal is greater than feed rate per tooth, the residual height in the radial direction is greater than the residual height in the feed direction, and the surface texture is determined by the residual height in the radial direction.

4.3. Initial phase angle
When the ball end milling cutter cuts into the workpiece, it will enter the workpiece edge with a different initial phase angle. The change of the initial phase angle will cause the distance of the feed residue to the workpiece boundary to change. Therefore, the initial phase angle has a certain impact on surface topography. The difference between initial phase angle between two adjacent rows is defined initial phase angle difference.
Figure 6 describes the machined surface topography when initial phase angle difference $\Delta \phi$ is from $0^\circ$ to $30^\circ$, $150^\circ$ and $180^\circ$. From Fig. 6 (a) to (d), under the same processing parameters, the initial phase angle difference of the tool paths in two adjacent rows will cause the surface topography and texture to be very different. With the gradual increase of initial phase angle difference, the inclination angle of the feed texture relative to the feed direction is also constantly changing. Since initial phase angle difference is a constant value, the consistency of the feed texture remains fairly good. A texture with a $\Delta \phi = 180^\circ$ is consistent with a $\Delta \phi = 0^\circ$. Due to the number of cutter teeth being 2, the texture direction will cycle every $180^\circ$.

5. Conclusions
Cutting geometry simulation technology is a hot research topic in the field of advanced manufacturing technology. In this paper, the modeling method of surface topography in five-axis ball end milling is studied, and the influence of different cutting parameters and initial cutting phase angle on the machined surface topography is simulated and analyzed. It can be concluded that feed rate per tooth, path internal and initial phase angle have a great influence on the shape, direction and height of the surface topography. Simulation results can quickly and economically provide the basis for the reasonable selection of the actual processing parameters, which is of great significance to improve the processing efficiency and ensure the processing quality.

References
[1] L. Shujuan, Y. Dong, Y. Li, P. Li, Z. Yang, and R. G. Landers, “Geometrical simulation and analysis of ball-end milling surface topography,” Int J Adv Manuf Technol, vol. 102, no. 5–8, pp. 1885–1900, Jun. 2019.
[2] Y. M. A, K. K. A, H. T. B, M. H. C, and T. S. D, “Theoretical Estimation of Machined Surface Profile Based on Cutting Edge Movement and Tool Orientation in Ball-nosed End Milling,” CIRP Annals, vol. 52, no. 1, pp. 49–52, 2003.
[3] M. Arizmendi et al., “Model development for the prediction of surface topography generated by ball-end mills taking into account the tool parallel axis offset. Experimental validation,” CIRP Annals - Manufacturing Technology, vol. 57, no. 1, pp. 101–104, 2008.
[4] L. Wang, S. Ge, H. Si, L. Guan, F. Duan, and Y. Liu, “Elliptical model for surface topography prediction in five-axis flank milling,” Chin. J. Aeronaut., vol. 33, no. 4, pp. 1361–1374, Apr. 2020.
[5] M. A. A, A. J. a, W. E. C. A, M. E. B, and M. A. A, “Modelling of elliptical dimples generated by five-axis milling for surface texturing - ScienceDirect,” International Journal of Machine Tools and Manufacture, vol. 137, pp. 79–95, 2019.