Spectators effect in inclusive beauty decays

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I review the role of the spectator quarks effect in the inclusive beauty decays. The evaluation of the expectation values of four-quark operators between hadronic states and its consequences are discussed.

Inclusive decays of heavy hadrons are described by the heavy quark expansion (HQE), an expansion in the inverse powers of the heavy quark mass \(m\) based on the operator product expansion (OPE) in QCD and the heavy quark effective theory (HQET) assuming quark-hadron duality \([1]\). The leading order hadronic decay rate, proportional to \(m^3\), is that of the free heavy quark. Corrections appear at \(O(1/m^2)\) and beyond. They are due to the heavy quark motion inside the hadron and the chromomagnetic interaction at \(O(1/m^3)\) and the spectator quarks processes at \(O(1/m^3)\). The decay rate at order two in \(1/m\) splits up into the mesonic one on the one hand and the baryonic on the other. This is because of the vanishing chromomagnetic interaction in the baryons with an exception of \(\Omega\).

Among the predictions of the HQE for the inclusive properties which are confronted by the experimental values like \(\tau(\Lambda_b)\), semileptonic branching ratio of \(B\) and the charm counting in the final state \([2]\), we address the ratio \(\tau(\Lambda_b)/\tau(B^0)\) which is 0.9 by theory but 0.79 from experiment), the spectators effect in charmless semileptonic decay of \(\Lambda_b\) on \(Br(b \rightarrow X_u\ell\nu)\) and the validity of the assumption of quark-hadron duality.

In view of the discrepancy of the theoretical prediction with the experimental one for \(\tau(\Lambda_b)\), it is necessary to accomodate the contribution coming from the third order term in the HQE:

\[
C(\mu) < H | (\bar{b}\Gamma q)(q\Gamma b) | H >
\]

where the Wilson coefficient, \(C(\mu)\), describes the spectator quarks processes: in the decay \(Q(q) \rightarrow Q'q_1q_2(q)\), if either \(q_1\) or \(q_2\) is the same as \(q\), then both of them interfere destructively; if \(q_1\) or \(q_2\) is the antiquark of \(q\), then they weakly annihilate; and the other one is the \(W\)-scattering: \(Qq_{1(2)}W \rightarrow Q'q_{2(1)}\). These processes are found to enhance the decay rate of \(\Lambda_Q\). On the other hand, the central issue in the systematic incorporation of the spectators effect is the evaluation of the expectation values of the four-quark operators (EV\(_{\text{FQO}}\)). Traditionally, for mesons, the EV\(_{\text{FQO}}\) is obtained, with the vacuum saturation approximation, in terms of the leptonic decay constant of the hadron, \(f_H\); on the other hand, for baryons, the valence quark model is employed. This procedure and other methods \([\text{3, 4]}\] found that the FQO do not account for the discrepancy. However, we have shown in our recent works \([\text{5, 6]}\] that the FQO accounts for the difference in the lifetimes of \(\Lambda_b\) and \(B\).

The EV\(_{\text{FQO}}\) between hadronic states is related to the form factor characterising the light quark scattering off the heavy quark inside the hadron \([\text{8]}\):

\[
\frac{1}{2M_H} < H | (\bar{b}\Gamma q)(q\Gamma b) | H > = |\Psi(0)|^2 = \int \frac{d^3}{(2\pi)^3} F(q^2)
\]

In \([\text{8}]\), representing the form factor by \(e^{-q^2/4\beta^2}\), the wave function density is obtained as

\[
|\Psi(0)|^2 = \frac{\beta^3}{4\pi^{3/2}}
\]

where \(\beta\) is determined by solving the Schrödinger equation in Variational procedure for the wave function \(\frac{\beta^{3/2}}{2\pi^{3/2}} e^{-\beta^2 r^2/2}\) with the potential \(V(r)_{\text{meson}} = a/r + br + c\) and \(V(r)_{\text{baryon}} = a/r + br + \beta r^2 + c\). In this description, the baryon is considered as a two body system of a heavy quark-diquark. The \(\beta's\) for the hadrons are: \(\beta_{B^-} = 0.4, \beta_{B^+} = 0.44\) and \(\beta_{\Lambda_b} = 0.72\), all in \(\text{GeV}\) units. Using these values for the wave function density, the ratio of lifetimes of \(\Lambda_b\) and \(B\) is found to be 0.79.

*To appear in the Proceedings of the Sixth Workshop on High Energy Physics Phenomenology held at the Institute of Mathematical Sciences, Chennai, India during Jan. 3-15, 2000.
If one assumes that the HQE is an asymptotic expansion, then the expansion for the decay rate can safely be considered as converging at $O(1/m^3)$. Recently, Voloshin has analysed the relations between the inclusive decay rates of the charmed and beauty baryons triplet $(\Lambda_Q, \Xi_Q)$ [1]. The relations depend only on the HQE and on the flavour symmetry under $SU(3)_f$. In this procedure, the $EV_{FQO}$ between baryon states is obtained using the differences in the total decay rates. In [2], strongly assuming that the HQE converges at $O(1/m^3)$, we extended the Voloshin analysis to $SU(3)_f$ triplet of the $B$ mesons, $B^-, B^0$ and $B^0$. Their total decay rate splits up due to their light quark flavour dependence at the third order in the HQE. The differences in the decay rates of the triplet, are related to the third order terms in $1/m$ dependence at the third order in the HQE. The differences in the decay rates of the triplet, are related to the third order terms in $1/m$ dependence at the third order in the HQE. The differences in the decay rates of the triplet, are related to the third order terms in $1/m$ dependence at the third order in the HQE.

The EV of $B$ mesons, $B^-, B^0$ and $B^0$, is obtained above, and the total decay rate splits up due to their light quark flavour dependence at the third order in the HQE. The differences in the decay rates of the triplet, are related to the third order terms in $1/m$ dependence at the third order in the HQE.

### Equation (4)
\[ d\Gamma_{B^+ - B^-} = -\Gamma_0(1 - x)^2 \left\{ Z_1 \frac{1}{3} (c_0 + 6) + (c_0 + 2) \right\} \langle O_0 \rangle_{B^0 - B^-} \]

### Equation (5)
\[ d\Gamma_{B^0 - B^-} = -\Gamma_0(1 - x)^2 \left\{ Z_2 \frac{1}{3} (c_0 + 6) + (c_0 + 2) \right\} \langle O_0 \rangle_{B^0 - B^-} \]

### Equation (6)
\[ d\Gamma_{B^0 - B^0} = -\Gamma_0(1 - x)^2 \left\{ (Z_1 - Z_2) \frac{1}{3} (c_0 + 6) \right\} \langle O_0 \rangle_{B^0 - B^0} \]

On the other hand, for the triplet baryons, $\Lambda_b$, $\Xi^-$ and $\Xi^0$, with $\tau(\Lambda_b) < \tau(\Xi^0) \approx \tau(\Xi^-$), we have the relation between the difference in the total decay rates and the terms of $O(1/m^3)$ in the HQE, as

### Equation (7)
\[ d\Gamma_{\Lambda_b - \Xi^0} = \frac{3}{8} \Gamma_0(c_{Q0} - 2) \langle O_0 \rangle_{\Lambda_b - \Xi^0} \]

For the decay rates $\Gamma(B^-) = 0.617 \text{ ps}^{-1}$, $\Gamma(B^0) = 0.637 \text{ ps}^{-1}$ and $\Gamma(B_s^0) = 0.645 \text{ ps}^{-1}$, the $EV_{FQO}$ are obtained for $B$ meson, as an average, from Eqs. (4-6): $\langle O_0 \rangle_B = 8.08 \times 10^{-3} \text{GeV}^3$. The $EV_{FQO}$ for the baryon $\langle O_0 \rangle_{\Lambda_b - \Xi^0} = 3.072 \times 10^{-2} \text{GeV}^3$, where we have used the decay rates corresponding to the lifetimes $1.24 \text{ ps}$ and $1.39 \text{ ps}$ of $\Lambda_b$ and $\Xi^0$ respectively. The $EV_{FQO}$ for baryon is about 3.8 times larger than that of $B$. For these values $\tau(\Lambda_b)/\tau(B) = 0.78$. Using the experimental value of $\tau(B^-) = 1.55 \text{ ps}$ along with the above theoretical value, the lifetime of $\Lambda_b$ turns out to be $\tau(\Lambda_b) = 1.20 \text{ ps}$.

We now turn up to the spectator quarks effect in $\Lambda_b \rightarrow \Lambda_u l\nu_l$ [1], in view of the ALEPH measurement [11] of $Br(b \rightarrow X_u l\nu_l)$. When $b$ decays into $ul\nu_l$, the final state $u$ quark constructively interferes with the $u$ quark in the initial state. This effect increases the decay rate leading to the ratio, using the $EV_{FQO}$ for baryons obtained above,

\[ \frac{\Gamma(\Lambda_b \rightarrow X_u l\nu_l)}{\Gamma(b \rightarrow X_u l\nu_l)} = 1.34 \]

### Equation (8)

The $b$-baryon contributes about 10% to $Br(b \rightarrow X_u l\nu_l)$. The above estimate will have effect on the branching ratio considerably if there is no compensation from elsewhere. The estimate above will increase if the spectators effect from $\Xi^0$. It seems that any compensation is absent to offset the above estimate plus that one from $\Xi^0$. This will, though modestly, effect the value of the CKM matrix element $|V_{ub}|$.

Concerning quark-hadron duality in the heavy hadron decays, we make inference that follows the results obtained above. The agreement found between theory and experiment for $\tau(\Lambda_b)$, besides consistency in the $B$ mesons case, clearly signals that quark-hadron duality holds good in the HQE. In the previous case, Eqs. (2-3), by the choice of the form factor representation, we obtained $\tau(\Lambda_b)/\tau(B)$, whereas in the latter it is the very assumption that the HQE converges at $O(1/m^3)$ which leads to the prediction for the ratio. The validity of the assumption that we made needs to be verified [12]. In the recent lattice study [13], the authors stated that the role of the FQO is significant to explain $\tau(\Lambda_b)$. We hope that their claim will throw light.

In conclusion, we make a note of warning. The evaluation of the $EV_{FQO}$ is model dependent one in the first case [5] and is subject to the validity of the assumption on the convergence of the HQE in the latter one [5]. The intriguing point is that $|\Psi(0)|^2_{\Lambda_b}$ for meson is smaller that the estimate in terms of the leptonic decay constant. The ratio $|\Psi(0)|^2_{\Lambda_b}/|\Psi(0)|^2_B$ is larger than expected.

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1Voloshin did not consider that the expansion converges at the third order which is consequential.
ACKNOWLEDGMENTS

The author is grateful to Prof. H. Yamamoto, Prof. Rahul Sinha, Dr Anjan Giri, Dr Rukmani Mahanta and Mr K. R. S. Balaji for useful discussions. I acknowledge the encouragement being shown by Prof. P. R. Subramanian and Dr. D. Caleb Chanthi Raj.

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