THE E2/M1 RATIO, A STATUS REPORT

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Abstract

Recent determinations of the E2/M1 ratio are reviewed. Many aspects of these studies have been questioned. Here we focus on the selection of a database, methods used in the extraction of this ratio, and interpretation(s) of the results.

I. THE DATA: OLD VS NEW

Soon after the Baryons ’92 conference, the VPI group wrote a short note [1] claiming a value of $-1.5 \pm 0.5\%$ for the E2/M1 ratio - a value in good agreement with the results of an independent analysis from the RPI group [2]. Also in this paper was a value for the E2/M1 ratio obtained in a ‘forced fit’ to preliminary Σ data from the LEGS group. This forced fit yielded a ratio of $-2.9\%$, a value consistent with the most recent joint analysis of LEGS photoproduction and Compton scattering data ($-2.85 \pm 0.34 \pm 0.21\%$) [3].

Through the 90’s, the database for pion photoproduction benefited from experimental programs at Bonn, BNL, Mainz, SAL and Yerevan. Inverse reaction data were also measured at LANL and TRIUMF. Recent analyses from the Mainz, LEGS, and RPI groups imply that a much larger (larger than $-1.5\%$) E2/M1 ratio is required to fit these more recent measurements. The Mainz group(s) find values of about $-2.5\%$ [4,5] while the LEGS [3] and RPI [6] groups have found values closer to (and slightly above) $-3\%$.

In order to understand this data dependence, several groups (LEGS, Mainz, RPI, and VPI) have made fits using various subsets of the existing database. The results have been very interesting. The Mainz and LEGS groups have shown that an E2/M1 ratio close to the VPI value can be obtained, within their fitting schemes, if particular sets of data are used. The VPI and RPI groups have also (jointly) found that very similar E2/M1 ratios result when identical databases are fitted. This implies that much of the disagreement between previous analyses has been database dependent and not model dependent. However, one significant problem remains to be resolved. The effect of different cross section measurements, from Bonn, Mainz and BNL, is significant for both the E2/M1 ratio and the M1 amplitude itself. In particular, the LEGS cross sections [3] are considerably larger than the Bonn and Mainz values. Hopefully the reason for this difference can be understood in the near future.
II. EXTRACTING AMPLITUDES FROM DATA

The above mentioned agreement between different analyses holds only in certain cases, and the following sections will mainly serve to describe instances where results do not agree. Agreement is generally found when a full multipole analysis is carried out. This typically allows the variation of s- and p-wave (and possibly d-wave) multipoles over the resonance region, with higher multipoles described by Born terms [8]. The usually quoted E2/M1 ratio is given by \( \text{Im} \frac{E_3^{3/2}}{\text{Im} M_1^{3/2}} \) at the energy where \( \text{Re} M_1^{3/2} \to 0 \). The ‘correctness’ of this definition will be discussed in the following sections. Here we will simply adopt the ‘standard’ definition as a useful point of comparison.

Watson’s theorem and the dominance of a single \((M_1^{3/2})\) multipole allow stable fits over the delta resonance, apparently without the help of double-polarization data. An issue that has been vigorously debated is the minimal dataset and number of searched multipoles required for a reliable extraction of the E2/M1 ratio. In Ref. [4], a polynomial in \( \cos(\theta) \) was fitted to the polarized-photon \( \pi^0p \) cross sections. The cross sections were parameterized as

\[
\frac{d\sigma}{d\Omega} = \frac{q}{k} \left( A + B \cos(\theta) + C \cos^2(\theta) \right). \tag{1}
\]

For polarization parallel to the beam, the coefficients are

\[
A || = |E_{0+}|^2 + |3E_{1+} - M_{1+} + M_{1-}|^2, \\
B || = 2\text{Re} [E_{0+} (3E_{1+} + M_{1+} - M_{1-})^*], \\
C || = 12\text{Re} [E_{1+} (M_{1+} - M_{1-})^*]. \tag{2}
\]

After determining these coefficients, the E2/M1 ratio was given by the correspondence

\[
R_{\pi^0p} = \frac{1}{12} \frac{C ||}{A ||} \approx R_{\text{EM}} \tag{3}
\]

where \( R_{\text{EM}} \) is the usual E2/M1 ratio (to be evaluated at resonance)

\[
R_{\text{EM}} = \frac{\text{Re} \left( E_{1+}^{3/2} M_{1+}^{3/2*} \right)}{|M_{1+}^{3/2}|^2}. \tag{4}
\]

The simplest way to see if this is a good approximation is to just calculate \( R_{\pi^0p} \) and compare it to the exact result, as found from the various multipole solutions. We have done this using the RPI, Mainz, BNL, and several VPI analyses. In some cases (for example, using the Mainz or SM95 solutions), the difference is quite small (0.2% or less). However, in other cases (for example, using the RPI or BNL solutions), the difference can be significant (about 0.3% for BNL and about 0.5% for RPI). As a result, it is difficult to assign an uncertainty to \( R_{\pi^0p} \) in advance of a full multipole analysis.

A related issue has been raised by Sandorfi [3], who has noted that (in principle) the neglect of high partial waves can have a significant effect on the fitted lower partial waves [9]. In the BNL analysis [7], partial-waves up to \( l = 3 \) were fitted, with higher waves given by the Born terms. The effect of waves beyond \( l = 1 \) can be seen if one plots [10,11].
\[ \frac{(\sigma_\parallel - \sigma_\perp)}{\sin^2 \theta}. \] (5)

This quantity should be angle-independent if only s- and p-waves are important. Deviations from this behavior are evident in the LEGS \( \pi^0p \) data at their highest energies. This suggests that the cross sections require non-Born contributions in the higher partial waves. In contrast, the RPI fit, which searched only s- and p-waves - the remainder given by Born terms, actually predicts the LEGS \( \pi^0p \Sigma \) data (a ratio of cross section measurements) at 322.6 and 333.55 MeV with a \( \chi^2/\text{data} \) significantly less than unity. It should be mentioned that similar tests, applied to the recent Mainz \( \pi^o p \) measurements, showed no evidence for the importance of d-wave and higher multipoles [11].

### III. A PLETHORA OF AMPLITUDES

The value of E2/M1 ratio determined from \( \text{Im} E_{1+}^{3/2} / \text{Im} M_{1+}^{3/2} \), at the energy where \( \text{Re} M_{1+}^{3/2} \to 0 \), corresponds to the ratio of amplitudes at the K-matrix pole. A second possibility is the evaluation of an E2/M1 ratio at the T-matrix pole. Here one forms a ratio of residues of the E2 and M1 amplitudes. Both the individual residues and the resulting ratio are complex quantities. Other definitions of this ratio typically require: (a) a separation of the full E2 and M1 amplitudes into resonance and background pieces [2], or (b) a dynamical model, in which the distinction between ‘bare’ and ‘dressed’ amplitudes is made. These variations will be briefly discussed in the next section. Just how these amplitudes are related appears to be a model-dependent question. Since the separation of resonance and background pieces has also been questioned, little further progress can be made without confronting the models themselves.

### IV. MODEL DEPENDENCE IN E2 AND M1

#### A. The K-matrix Pole

Model dependence enters at a number of levels. Data are fitted to phenomenological forms which satisfy the constraints of unitarity. The precise form is not unique, and the effect of this ambiguity on the E2/M1 ratio has been extensively studied by a number of groups [2]. Many of these recipes can be written in the form

\[ M = (M_{\text{Born}} \cos \delta + M_R) e^{i\delta}. \] (6)

where \( M_{\text{Born}} \) is the Born amplitude, \( \delta \) is the corresponding \( \pi N \) phase shift, and \( M_R \) is the remainder [2][3][4]. The first term alone gives a qualitative description of the E2 amplitude; the second term is dominant in the M1 amplitude.

The most important difference, between approaches falling into this class, is the interpretation of \( M_R \). If this term is associated with the resonant part, there is no ‘background subtraction’ at the energy where the \( \pi N \) phase shift \( \to \pi/2 \) (since the first term vanishes). A significantly different view has been offered in the analysis of Aznauryan [13], which has also been discussed at this conference. In this work, a particular solution of an integral equation is associated with the background; the homogeneous solution being associated with the
resonant part. This particular solution is similar in shape to the first term in Eq. (6) but is larger in magnitude with an imaginary part passing through zero at a higher energy. As a result, the remaining ‘resonant’ part is significantly reduced. A similar ‘background’ curve has been found by T.-S.H. Lee and collaborators [16]. However, here the interpretation is quite different. In Ref. [16], the result of this background subtraction is a ‘bare’ photo-decay amplitude [17]. In Ref. [15], the resonant part was attributed to a ‘dressed’ amplitude. In another recent work, the ‘dressed’ amplitude of Lee has been related to values extracted at the K-matrix pole [18]. As a result, one cannot yet claim any clear link between these different methods.

B. The T-matrix Pole

The Mainz group has noted that, even for models and multipole fits having very different E2/M1 ratios at the K-matrix pole, the T-matrix pole ratio is quite stable [5]. This ratio of residues is determined either via the “speed plot” method or by continuing a model into the complex energy plane. Results differ slightly depending on the method. However, for fits excluding older datasets, one generally finds a (modulus, phase) of about (0.06, −125°).

The relation between this quantity and the K-matrix pole ratio can be understood, at least qualitatively, if we choose to write Eq. (6) in the form

\[ M = \alpha(1 + iT_{\pi N}) + \beta T_{\pi N} \]  

(7)

where \( T_{\pi N} \) is the \( \pi N \) T-matrix in a particular partial-wave, and \( \alpha \) and \( \beta \) are (real) functions of energy ensuring the correct threshold behavior. Comparing with Eq. (6) one should recognize that we have rewritten \( \cos \delta_1 e^{i\delta} \) in a form \( 1 + iT_{\pi N} \) more convenient at the T-matrix pole. The energy dependence of \( \alpha \) (in the neighborhood of the resonance) is essentially given by Born terms which are approximately flat for E2 and linear for M1. The additional term proportional to \( T_{\pi N} \) is required to reproduce the M1 amplitude and give a non-zero E2 at the K-matrix pole. In fitting the phenomenological multipoles, apart from kinematic factors, a good representation is possible with only a constant factor for \( \beta \).

Since much of the E2 energy variation is given by the first term in Eq. (7), it is clear that the pole residue depends largely on a quantity which goes to zero at the K-matrix pole. Thus, the K- and T-matrix pole ratios are sensitive to different parts of the full multipole amplitude. In fact, the T-matrix pole ratio will remain non-zero even in the event of a zero E2/M1 ratio at the K-matrix pole [19].

V. SUMMARY

In summary, the E2/M1 ratio evaluated at the K-matrix pole is fairly model-independent if a consistent database is used. Inclusion of some older Bonn neutral-pion production cross sections tends to lower the result, while the LEGS data appear to favor a value slightly above the Mainz result of −2.5%.

The T-matrix pole ratio is quite stable but is also less obviously related to the (generally real) model predictions. We have heard [20] that complex ratios naturally arise in ChPT
calculations. This is encouraging but at present it is not obvious that we are calculating precisely the same quantities.

The proper way to separate background and resonance contributions continues to generate debate. I expect this question will occupy our attention until we better understand the connections between quark-model and phenomenological resonance properties.

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