The effects of frequency and amplitude of vibration on the mixing and segregation processes in granular materials

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Abstract

The mixing and segregation process of almost identical size binary steel-glass mixture within a vertically vibrated container are investigated in this study. The container is designed to allow only one layer of granules inside. In this study, the image processing techniques and particles tracking methods are employed to track the convection flow of granular materials. Six different amounts of frequency (20, 25, 30, 35, 40, and 50 Hz) for certain amplitudes are used to study the effects of miscellaneous vibration conditions on the velocity fields of moving beads. The granular bed was divided into three horizontal parts and a separate study was provided for each one. The discrete-element method and EDEM software are employed for simulating the response of steel beads under 2D vertical vibration. Good agreements between simulated and experimental results have been presented in the current work. There are one or two convection cells generated in the vibrated bed for all tests, and the amount of amplitude was the most important factor in determining the number of these cells. The heavier particles tend to move toward the convection cells centers within the container unlike the lighter particles. The average velocity of steel beads increased with amplitude, while it decreased by increasing the frequency of vibration. In addition, the steel beads concentration in the middle region of the container increased with the vibration time.

Keywords: Segregation, Vertical vibration, Frequency, Amplitude, Granular materials

1. Introduction

The segregation occurs in particles with different properties is a very important phenomenon in areas such as geophysics, material science, agriculture, and almost all engineering branches, i.e. involving preparation of drugs, food, cosmetics, detergents, and ceramics. Transmission and processing of mixtures may be lead to undesirable segregation. E.g. pharmaceutical pills industrialization usually uses mixing of many constituent substances and a mal-blending of 1% of the components may have drastic consequences.

In industry, many manufacturing and processing materials are formed as granules. The difference of grains in terms of density, size, granular temperature, and material properties may cause the segregation of particles. Mixing and segregation of powders and grains are necessary operation in several industries.
process from pharmaceuticals to food to cement to chemicals etc. It is estimated, in the chemical industrial only, that three-quarters of the raw material and half of the products are in particulates form [1-12].

Any movement observed in a group of particles and placed in a container must be caused by an energy source. The energy can be added to the system by an external field of force, such as gravity, shearing the granular material at a boundary, or subjecting it to vibration [13]. When a granular material had been trapped in a container and subjected to an external oscillation with certain vibration conditions, the granular media can be fluidized and the grains will move with specific behaviors. These behaviors may vary depending upon the frequency and amplitude of the external oscillation. The governing mechanisms affecting the flow of grains is the random movement of particles resulted from the interacting collisions between particles. The specific fluctuation kinetic energy of particles quantifies the random movement of grains [14-18]. In a vibrated shaker, the granular materials will transmit from an intensive (solid like) state to a dilute (liquid like) state. Consider that the grains case of a small particles’ mixture has one large intruder subjected to vertical vibration, the intruder can move up [19] or down [20] in a well-known phenomenon of direct and reverse Brazil-nut effects [21].

In recent years, the world has witnessed a good development in the field of technology, electronic and photographic equipment. However, as memory costs decreased and processor speed increased, computer simulation have become an effective tool for understanding the granular materials. Simulations can model conditions that are difficult to produce in experiments. For example, some simulations are performed with a frictionless particle or with altered gravity environments [22].

The discrete-element method DEM, also known as the particle-dynamics method PDM, is a numerical technique that calculates new positions, accelerations, and velocities of all particles in granular beds by using the Euler’s and Newton’s equations of motion to analyze the force components on each particle[22]. In addition, DEM is used to simulate the interaction among individual solids particles to predict its behavior. The dynamic understanding of particle flow and modeling the moving boundaries have become much easier by using this simulation method. Cundall and Strack [23] was the first who developed the DEM, which was upgraded to other application in mechanics of granular materials. Today DEM becomes a widely effective process to solve engineering problems in granular materials, especially in powder mechanics, granular flows, and rock mechanics.

The Aims of this paper is to present an experimental and simulation study for the phenomena of segregation in a two-dimensional 2D rectangular box with steel and glass beads under different vibration intensities. The image processing technology and particle tracking method were used to investigate the velocity and flow behaviors of the vibrated beads. Simulation of the experimental side using the theory of DEM and the EDEM software to study the effect of vibration conditions on the kinetic energy and volume fraction for the vibrating particles.

2. Experimental Setup
Figure (1) illustrates the experimental apparatus. The vibrated table serves as a vertical shaker with variable frequencies and amplitudes. The system was driven vertically by sinusoidal-signals which was produced by a function generator. The vibrational frequency $f$ was adjusted by Proportional Integral Derivative (PID) controller. The granules are placed in the granular bed and its motion recorded by the digital camera. The recording video was transferred to computer for post-processing.

The case study of this research is a mixture of granular materials. The grains used were steel beads with a diameter of 3.2 mm (with deviation of 3%) and glass beads with a range of diameter 3.5-4.5 mm, in all EDEM simulations 1000 glass and 500 steel beads were poured in a container. Since the beads diameters are close, the size difference between the components of the granular mixture is not considered as the cause of segregation in this study. The densities of glass and steel beads were 2490 $kg/m^3$ and 7822 $kg/m^3$ respectively, the angle of repose of steel and glass beads are 26° and 22° respectively.

The container used was made of transparent glass plates as a front and back wall and with glossy tape as the bottom and side walls. The inside width, height and depth of the container were 170, 280, and 5 mm respectively. Thus, there was only one particles layer existed in the container and the motion generated in the vibrated container was two dimension 2D (x: represent the horizontal axis; y: represent the vertical axis; the center point of the bed floor represent the origin point). Since the container is transparent and has a single layer of particles, it is therefore easy for the camera to record all the particles inside the container at any time.

The different cases of experiments and their frequency, vibration amplitude, and dimensionless acceleration are listed in table 1.
Table 1. Different cases of experiments and their frequency, amplitude, and dimensionless acceleration

| case | Frequency $f$ Hz | Vertical Amplitude $A$ mm | Dimensionless acceleration $\Gamma_y$ |
|------|------------------|--------------------------|-------------------------------------|
| 1    | 20               | 7                        | 11.27                               |
| 2    | 25               | 2.3                      | 5.78                                |
| 3    | 30               | 1.5                      | 5.43                                |
| 4    | 35               | 1.38                     | 6.8                                 |
| 5    | 40               | 7                        | 41.85                               |
| 6    | 50               | 1.2                      | 12.08                               |

The radian frequency of vibration $\omega$ and amplitude $A$ could be calculated from the relations [26]

$$\omega = 2\pi f$$  \hspace{2cm} \text{...........................................(1)}

$$A = a/\omega^2$$  \hspace{2cm} \text{...........................................(2)}

The dimensionless acceleration of vibration $\Gamma$ was defined as [26]:

$$\Gamma = a/g$$  \hspace{2cm} \text{...........................................(3)}

Where $g$ is the gravity acceleration.

The image processing technique is a method to carry out some operations on images, to get an enhanced images or to extract some helpful information from it. It is a signal processing type in which the input is an image, while the output may be image or features/characteristics associated with that image. Image processing, nowadays, is among rapidly develops technologies.

Image processing mainly includes the following steps:

- Importing the image by means of image acquisition equipment;
- Analyzing and manipulating it;
- Output result, which can be reported or altered image depending on image analysis.

3. Simulation Work

In the DEM, the Euler’s equations and Newton’s (Eq. 4, and eq. 5 respectively) [27] are used to analyze the new velocities, accelerations, and positions of all particles.
\[ m \frac{d\vec{\omega}}{dt} = \vec{r}_c - m\vec{g} \] \hspace{1cm} \text{.......................... (4)}

\[ I_p \frac{d\vec{\omega}}{dt} = \vec{T}_p \] \hspace{1cm} \text{.......................... (5)}

Where \( m \) and \( I_p \) are the mass and centroidal mass moment of inertia of each particle, respectively; \( \omega \) and \( \nu \) are the rotational and translational velocities of every particle respectively; and \( \vec{T}_p \) and \( \vec{r}_c \) are the couple and resultant contact force applied to every particle respectively; \( g \) is the acceleration due to gravity.

The Hertz-Mindlin model is the contact force model carried out in the present simulations because the magnitude of steel beads plastic deformation was negligible. In this model, the elastic and plastic deformations, of the colliding bodies, are computed for tangential and normal directions; the force due to friction is considered between the bodies in the tangential direction. Figure (2) shows two contacting spherical beads and the corresponding forces of interaction.

![Figure (2) The interaction contacts forces and the normal overlap of colliding sphere [28]](image)

The normal elastic force was calculated by using Hertz formulas (Eq.6 -Eq.9) [27].

\[ F_n^e = -k_n \delta_n^2 \] \hspace{1cm} \text{.......................... (6)}

\[ k_n = \frac{4}{3} E^* \sqrt{R^*} \] \hspace{1cm} \text{.......................... (7)}

\[ \frac{1}{R^*} = \frac{1}{R_i} + \frac{1}{R_j} \] \hspace{1cm} \text{.......................... (8)}

\[ \frac{1}{E^*} = \frac{1-v_i^2}{E_i} + \frac{1-v_j^2}{E_j} \] \hspace{1cm} \text{.......................... (9)}
Where $R$, $\nu$ and $E$, are radius of curvature, Poisson’s ratio, and Young’s modulus of each body in contact respectively. $\delta_n$ is the normal deformation (overlap) between the particles. $R^* $ and $E^*$ are the equivalent radius and the equivalent young’s modulus respectively; the indices $i$ and $j$ refers to contacting particles. Different properties were used for the walls and the particles for the wall-particle interactions. For impact of beads to the walls, the curvature radius of the walls were set to infinity in Eq.10.

\[
\frac{1}{R^*} = \frac{1}{R_i} + \frac{1}{R_j} \tag{10}
\]

The normal damping (plastic) force, $F^d_n$, was determined with having the normal coefficient of restitution $e$, normal overlap $\delta_n$ , and the relative normal velocity between particles before impact, $v_n^{rel}$ as listed in the following equations:

\[
F^d_n = -2 \sqrt{\frac{5}{6}} \beta \sqrt{S_n m^*} v_n^{rel} \tag{11}
\]

\[
\frac{1}{m^*} = \frac{1}{m_i} + \frac{1}{m_j} \tag{12}
\]

\[
S_n = 2E \sqrt{R \delta_n} \tag{13}
\]

\[
\beta = \frac{\ln e}{\sqrt{\ln^2 e + \pi^2}} \tag{14}
\]

\[
e = \frac{v_{n2} - v_{n1}}{v_{n1} - v_{n2}} \tag{15}
\]

Where $v_n$ and $V_n$ represent the normal velocity of particles after and before the collision respectively; $m^*$ is the equivalent mass of the contacting particles; $S_n$ is normal stiffness, and $\beta$ is a function of normal coefficient of restitution.

With having tangential stiffness $S_t$ and tangential overlap $\delta_t$, the tangential elastic force $F^e_t$, can be calculated (Eq. 16). The tangential overlap can be defined as the relative displacement of the particles throughout impact within the tangential direction without considering the rolling contribution.

\[
F^e_t = -S_t \delta_t \tag{16}
\]

\[
S_t = 2G \sqrt{R \delta_t} \tag{17}
\]

The damping (plastic) force in the tangential direction $F^d_t$, is determined by using the tangential restitution coefficient $e$, and the relative tangential velocity between beads before impact $v_{t1}^{rel}$, and the tangential overlap $\delta_t$.

\[
F^d_t = -2 \sqrt{\frac{5}{6}} \beta t \sqrt{S_t m^*} v_t^{rel} \tag{18}
\]
The coulomb friction force $f_s$ is limits the tangential force. It means that:

$$ F_t < f_s ; $$

where

$$ F_t = F_t^e + F_t^d $$

$$ f_s = \mu_s F_n $$

And $\mu_s$ is the static friction coefficient.

The software package EDEM 4 was used to perform DEM simulation of cases of vibration tests of granular bed. There is a set of built-in contact models in EDEM software that knows Hertz – Mindlin is the main one. The material properties of steel and glass beads, as well as the glass container used in the simulation (including Poisson ratio, shear modulus, and density) are all given in Table (2). Also restitution Coefficient, static friction and rolling friction of interactions in particles-particles and particles-container walls collisions are given in Table (3).

### Table (2): Material properties of particles and container

| Material                  | Shear modulus (GPa) | Poisson’s ratio | Density ($kg/m^3$) |
|---------------------------|---------------------|-----------------|-------------------|
| Steel particle            | 76                  | 0.23            | 7800              |
| Glass particle and container | 26                  | 0.29            | 2500              |

### Table (3) Restitution coefficients, static friction and rolling friction of interactions in particles-particles and particles-container walls collisions

| Colliding material      | Coefficients of restitution | Coefficients of static friction | Coefficients of rolling friction |
|-------------------------|----------------------------|---------------------------------|---------------------------------|
| Glass-Steel             | 0.76                       | 0.1                             | 0.02                            |
| Steel-Steel             | 0.85                       | 0.62                            | 0.02                            |
| Glass-Glass             | 0.7                        | 0.6                             | 0.02                            |
In all EDEM simulations 1000 glass and 500 steel particles with diameters of 4 mm and 32 mm, respectively, was poured in a container with dimensions of 300 mm, 170 mm and 5 mm in length, width and thickness, respectively (Figure 3).

As on the experimental side, the tests was repeated on the simulation side six times, according to the frequencies and amplitudes values shown in table (1). Then, in order to prevent a severe shock and accelerate too high in the particles, the frequency and amplitude of the vibration of the container gradually increase to the desired value. After the test conditions arrived, it takes at least 4 seconds for the system to reach a stable state.

4. Results and Discussions

The flow diagram of tracked particles of each case of experiments and simulations tests are shown in figures. (4,5). It is clearly shown that the high amplitude of vibration caused to generate two convection cells, on the other hand, the low amplitude leads to generate one cell.

The amount of vibration frequency does not affect the number of load cells generated. The velocities achieved for each case of experiments and simulation work are shown in table (2). In all of the cases, there is a relatively good agreement between the experimental and simulation results. Because of complexity and nonlinearity of the phenomena of the system studied in this work, existence of error with average of 9.14 % into experimental data is considered as a good result for simulation and also experimental measurements.

By comparing the velocities achieved for two frequencies of 20 and 40 Hz, which is shown in the result table (4), it can be seen that in a constant amplitude of the vibration, with the doubling of the frequency of vibration and hence the quadruple of the vibration acceleration, the average velocity of the movement of the steel balls in the upper and middle fields of the container increased slightly, while in the bottom of the container there is a significant increase in the average velocity. As a result, it can be seen...
that increasing frequency alone has a positive effect on increasing the velocity of the bullets and thus can play a useful role in segregation systems.

On the other hand, by comparing two frequencies of 20 and 50 Hz, it can be seen that at an about constant magnitude of dimensionless acceleration ($\dot{\gamma} \approx 12$), the velocity in the upper, middle and lower portions of the bed is higher at 20 Hz, and thus increasing the amplitude of the vibrations can have a much more impressive effect on the velocity of the beads in their flow of movement. Therefore, in a general result, it can be stated that in constant dimensionless acceleration, the vibration amplitude in the segregation systems can play a more effective role in comparison with frequency in increasing the velocity of flow.

Also, by comparing three frequencies of 25, 30 and 35 Hz, it can be seen that at an about constant magnitude of dimensionless acceleration, the velocities do not change much in the three fields of the container, but generally in these frequency and dimensionless acceleration limits. The increase in the amount of vibration frequency or amplitude causes the magnitude of the motion of the particles to increase in general, but the effect of the frequency is more.

It can be seen in this table and also in the figure (6), by increasing the frequency of vibration in a constant amplitude (7 mm), flow mean velocity of steel particles has an increasing trend in the lower, middle and upper field.
Figure (4) Flow diagram of tracked particles in 20, 25, 30, 35, 40, 50Hz respectively for the experimental work.

Figure (5) Flow diagram of tracked particles in 20, 25, 30, 35, 40, 50Hz respectively for the simulation work.
Table (4) Results of simulation for flow mean velocity of steel particles in three fields of granular bed in comparison with experimental results (E: Experiment, S: Simulation)

| case | Frequency | Amplitud e-Y (mm) | V-lower field (mm/s) | V-Middle field (mm/s) | V-Upper field (mm/s) |
|------|-----------|-------------------|----------------------|-----------------------|----------------------|
|      |           |                   | E   | S   | E   | S   | E   | S   |
| 1    | 20        | 7                 | 9.35 | 9.92 | 9.01 | 9.21 | 9.81 | 11.52 |
|      |           |                   | (+6%)| (+2.22%)| (+2.22%)| (+17.43%)|      |
| 2    | 25        | 2.3               | 5.21 | 5.33 | 5.58 | 5.57 | 6.15 | 6.8 |
|      |           |                   | (+2.6%)| (0%)| (0%)| (0%)| (10.6%)|
| 4    | 30        | 1.5               | 5.38 | 6.03 | 5.7 | 4.91 | 6.5 | 7.6 |
|      |           |                   | (+12.08%)| (-13.86%)| (+13.86%)| (+16.9%)|      |
| 6    | 35        | 1.4               | 7.6 | 7.41 | 6.23 | 7.08 | 6.75 | 7.05 |
|      |           |                   | (+12.27%)| (+13.6%)| (+13.6%)| (+4.4%)|      |
| 8    | 40        | 7                 | 19.36 | 21.75 | 16.77 | 17.23 | 15.96 | 16.45 |
|      |           |                   | (+11.5%)| (+2.7%)| (+2.7%)| (+3.07%)|      |
| 9    | 50        | 1.2               | 4.31 | 4.32 | 3.64 | 4.53 | 4.38 |
|      |           |                   | (-13.8%)| (-18.6%)| (-18.6%)| (-3.4%)|      |

Figure (6) Diagram of flow mean velocity for steel particles in different frequencies and in the lower, middle and upper fields
The results of simulations showed that the volume fraction of steel beads increased with time in the middle filed of container. This result was not effective for high magnitudes of amplitudes as shown in figure 7.

Figure (7) The volume fraction of steel beads for case 1, 2, 3, 4, 5, and 6 respectively.
5-Conclusion

In the present study, steel and glass beads of almost identical size were placed in a 2D container and vertically vibrated. It was studied the effect of vibration conditions on the motion of the beads. The image processing technique used to do the experimental side of the study, while EDEM software used to do the simulation side. The results of computer vision-based analysis of convective flow in two kinds of data, DEM simulations and experimental measurements, were compared and understood to be in a good agreement.

The dynamic behavior and particles flow within a vibrated granular bed are highly nonlinear phenomena and have a lot of complexity and are affected by variation in the frequencies and amplitude of vibration. The vibration of the granular material placed in a two-dimensional container causes to generate one pair of convection rolls when the amplitude of vibration is high enough to generate those cells. On the other hand, only one convection roll would be generated at relatively low amplitude. In addition, the number of convection cells is not highly dependent on vibration frequency. The heavier particles (steel beads) tend to move toward the convection rolls centers within the container unlike the lighter particles (glass beads).

The average velocity of steel beads decreased by increasing the frequency of vibration at a constant level of amplitude; this result was obtained in all fields of the vibrated bed tested. Conversely, as the amplitude of vibration increased, the average velocity of the vibrating steel beads increased, regardless of the bead’s height from the base of the container. In general, the steel beads concentration (volume fraction) in the middle zone of the container increased with the vibration time.

6-References

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