Quantum Anomalous Hall Effect

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Introduction to quantum anomalous Hall (QAH) effect

Realization of QAH effect in magnetically doped topological insulators (TIs)

QAH effect observed in other systems

High Chern number QAH effect in magnetic TI/TI multilayers

Summary
Hall effect

Discovered in 1879 by Edwin Hall

\[ \rho_{yx} = \frac{V_H}{I_{xx}} \]

Edwin Hall's original data on the Au leaf

Edwin H. Hall's Data from November 12th 1878

Hall coefficient \( R_H = \frac{1}{ne} \)
Quantum Hall effect

At high magnetic field

\[ \rho_{xx} = 0 \]

\[ \rho_{yx} = \frac{h}{ve^2} \]

Discovered in 1980 by Klaus von Klitzing

At low temperature and high magnetic field

Quantized Hall resistance; vanishing longitudinal resistance

In 2D electron gas

At high magnetic field

\[ \rho_{yx} = \frac{h}{ve^2} \]

\[ \rho_{xx} = 0 \]
Anomalous Hall (AH) effect

Hall effect in ferromagnetic (FM) materials

Edwin Hall (1855 ~1938)

Chang, J. Phys. Condens. Matter. 28, 123002 (2016)
Chang et al., arXiv:2202.13902 (2022)
Intrinsic AH effect: induced by energy band

\[
\sigma_{xy} = \frac{e^2}{h} \sum_{\text{occupied}} \frac{1}{2\pi} \oint_{BZ} \Omega d\vec{k} = \frac{e^2}{h} C
\]

\[
\oint_{BZ} \Omega d\vec{k} = 2\pi C
\]

\[
\sigma_{xy} = \frac{e^2}{h} C
\]

If the Chern number \( C \) is nonzero, \( \sigma_{xy} \) can be quantized.

At zero magnetic field

\[
\rho_{yx} = \frac{h}{Ce^2}
\]

\[
\rho_{xx} = 0
\]

Haldane, Phys. Rev. Lett. 61, 2015 (1988)
Topology: insensitive to details

\[ \int_{s} \kappa dA = 4\pi (1 - g) \]

\( \kappa \): Gaussian curvature

\( g \): Genus = number of holes
Topological insulator (TI)

An insulator that conducts (the bulk is insulating but the edge/surface is metallic)

2D TI (QSH insulator)

3D TI

With the inverted band structure

HgTe quantum wells

Bernevig et al., Science 314, 1757 (2006)
König et al., Science 318, 766 (2007)

InAs/GaSb quantum wells

Liu et al., Phy. Rev. Lett. 100, 236601 (2008)
Knez et al., Phy. Rev. Lett. 107, 136603 (2011)

1T’ monolayer WTe$_2$

Fei et al., Nat. Phys. 13, 677 (2017)
Tang et al., Nat. Phys. 13, 683 (2017)
Wu et al., Science 359, 76 (2018)

Bi$_x$Sb$_{1-x}$ compound

Fu & Kane, Phy. Rev. B 76, 045302 (2007)
Hsieh et al., Nature 452, 970 (2008)

Bi$_2$Se$_3$ family

Zhang et al., Nat. Phys. 5, 438 (2009)
Xia et al., Nat. Phys. 5, 398 (2009)
Chen et al., Science 325, 178 (2009)
From TI to QAH effect

Topological Insulators

Conduction Band

Valence Band

+ Spin Orbit Coupling

Conduction Band

Valence Band

+ Ferromagnetic order

E_F

QAH effect
How to reach QAH effect

**TI thin films**

- MBE growth of high-quality films

**FM state**

- Diluted magnetic doping

**Tuning chemical potential**

- Band engineering
- Bottom gate fine tuning

Graphical representation with nodes MBE, ARPES, STM, and Transport.
MBE growth of TI thin films

ARPES band maps of MBE-grown Bi$_2$Se$_3$

Zhang et al. Nat. Phys. 6, 584 (2010)
Cr-doped 3D TI films

Introducing FM order into TIs

- Magnetic proximity effect
  ➢ Has not been achieved

- Intrinsic magnetic order
  ➢ MnBi$_2$Te$_4$

- Diluted magnetic doping

![Graphs showing $\rho_y$ vs $\mu_0 H$ for different temperatures]

**Paramagnetism**

Liu et al., *Phy. Rev. Lett.* **108**, 036805 (2012)
Zhang J, et al. *Science* **339**, 1582 (2013)

**Long-range ferromagnetism**

Chang et al., *Adv. Mater.* **25**, 1065(2013)
Chang, et al.. SCI CHINA PHYS MECH **59**, 1 (2016)
Tuning $E_F$ through band engineering

Bi$_2$Te$_3$: $n$-type, Dirac point in the valence band

Sb$_2$Te$_3$: $p$-type, Dirac point in the band gap

Bi$_2$Te$_3$ + Sb$_2$Te$_3$: Insulating state

Dirac point in the gap

Zhang et al., *Nat. Commun.* 2, 574 (2011)
Band engineering in Cr-doped TI

$p$-type

$x = 0$
$x = 0.1$
$x = 0.25$
$x = 0.6$

$E_F$ Binding Energy (eV)

➢ No change in band topology
➢ Charge neutral at $\sim x = 0.25$

5QL Cr-doped $(\text{Bi}_{x}\text{Sb}_{1-x})_2\text{Te}_3$

Chang et al., *Adv. Mater.* 25, 1065(2013)
Band engineering in Cr-doped TI

5QL Cr-doped (Bi$_x$Sb$_{1-x}$)$_2$Te$_3$

- Well-defined long-range FM order independent of carrier density and type
- Crossover between $x = 0.2$ and $x = 0.25$

Chang et al., Adv. Mater. 25, 1065(2013)
Tuning $E_F$ by bottom gate

RHEED of SrTiO$_3$ (111)

RHEED of Cr-doped (Bi, Sb)$_2$Te$_3$

Tuning $E_F$ by a bottom gate

SrTiO$_3$ (111) substrate

Large dielectric constant $\varepsilon \sim 30000$
QAH effect in magnetic TI films

Chang et al., Science 340, 167 (2013)

5QL Cr$_{0.15}$ (Bi$_{0.1}$ Sb$_{0.9}$)$_{1.85}$ Te$_3$ film

$V_g = V_g^0 = -1.5 \text{V}$

$T=30 \text{mK}$

$\rho_{xx}$

$\rho_{xy}$

$\rho_{yx}(0)$

$\rho_{yx}(0)$

QAH effect with Chern number $C = 1$
QAH effect in V-doped TI

Band calculation exclude the QAH in V-doped TI
Impurity band in the band gap

DFT calculation of V-doped TI

High-precision QAH effect

\[ \rho_{xx}(0) \approx 0.00013 \pm 0.00007 \ \hbar/e^2 \]

\[ \rho_{yx}(0) \approx 1.0019 \pm 0.00069 \ \hbar/e^2 \]

Yu et al., Science. 329, 61(2010)

Chang et al., Nat. Mater. 14, 473 (2015)
Spin-polarized ballistic chiral edge mode in QAH effect

Conventional conductors

4 degrees of freedom

QAH insulators

ONLY 1 degree of freedom
Dissipation-free chiral edge state

From traffic jam to info-superhighway
Techniques to enhance the critical temperature

Modulation doping

- MTI at the top and bottom with high Cr concentration
- Larger magnetization exchange gap at two surfaces
- Undoped insulating TI as a spacer layer
- The bulk without magnetic ion doping can diminish conducting bulk channels

Critical temperature \( \sim 10 \text{ K} \)

Mogi et al., *Appl. Phys. Lett.* **107**, 182401 (2015)
Techniques to enhance the critical temperature

Cr-V co-doping

Critical temperature \( \sim 10 \) K

Ou et al., Adv. Mater. 30,1703062 (2018)
Intrinsic magnetic TI, MnBi$_2$Te$_4$

Septuple-layer (SL): Te-Bi-Te-Mn-Te-Bi-Te

- $T_N = 25$ K
- A-type AFM
- FM coupling in each SL
- AFM coupling between adjacent SLs

Otrokov et al., *Nature* **576**, 7787 (2019)
Gong et al., *Chin. Phys. Lett.* **36**, 07 (2019)

QAH insulator in odd number layers

Deng et al., *Science* **367**, 895 (2020)

Axion insulator in even number layers

Liu et al., *Nat. Mater.* **19**, 522 (2020)

Chern insulator in FM phase of MnBi$_2$Te$_4$

Ge et al., *Natl. Sci. Rev.* **7**, 1280 (2020)
Ovchinnikov et al., *Nano Lett.* **21**, 2544 (2021)
Ying et al., *Phys. Rev. B* **105**(8), 085412 (2022)
C = 1 QAH effect realized in moiré materials

C = 1 QAH effect
In twisted bilayer graphene

C = 2 Chern insulator state
In ABC-trilayer graphene

C = 1 QAH effect
In AB-stacked MoTe₂/WSe₂

Zero magnetic field QAH effect is limited to C = 1

Serlin et al., Science. 367, 900 (2020)

Chen G, Nature 579, 56 (2020)

Li et al., Nature. 600, 641 (2021)
$C = 1$ QAH to high Chern number QAH

$C = 1$ QAH

$\vec{B} = 0$

Only one chiral edge channel

$C = 3$ QAH

$\vec{B} = 0$

Multiple chiral edge channels

Contact resistance limited to $\frac{h}{Ce^2}$

From single lane road to multiple-lane highway
High Chern number QAH in $C = 1$ QAH/normal insulator multilayers

$C = 3$ QAH insulator

Magnetic TI/TI multilayers

Heavy Cr doping drives magnetic TI into a normal insulator
QAH effect with $C = 1$ to 5

Zhao et al., *Nature* 588, 419 (2020)
QAH effect with $C = 1$ to $5$

$C = 1$  

$C = 2$  

$C = 3$  

$C = 4$  

$C = 5$

$\rho_{xx}(0)$, $\rho_{xx}(0)(h/e^2)$  

$V_g - V_g^0$ (V)
Tuning $C$ by controlling magnetic doping concentration

$3$ QL $\text{Cr}_x(\text{Bi, Sb})_{2-x}\text{Te}_3$

$x = 0.13, 0.15 \quad C = 1$ QAH
$x = 0.24, 0.35 \quad C = 2$ QAH
Plateau phase transition from $C = 1$ to $C = 2$ QAH state

Chiral edge state coexists with bulk states in the Chern number change-induced plateau phase transition.
Tuning $C$ by controlling middle magnetic TI thickness

- $d < 2$: $C = 1$ QAH
- $d \geq 2$: $C = 2$ QAH

Graphs showing the relationship between $\rho_{xy}$ and $V_g - V_g^0$ for different values of $d$.
Summary

➢ QAH effect observed in magnetically doped TI system

➢ QAH effect observed in new systems: Intrinsic magentic TI MnBi$_2$Te$_4$ and moiré materials

➢ High Chern number QAH effect
Thanks for your attention