A Lattice Chiral Gauge Theory with Multifermion Couplings

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Abstract

Analyzing an \(SU_L(2)\)-chiral gauge theory with external multifermion couplings, we find a possible scaling region where doublers decouple by acquiring chiral-invariant masses and \(\psi_R\) is free mode owing to the \(\psi_R\)-shift-symmetry, the chiral continuum theory of \(\psi_L\) can be defined. This is not in agreement with the general belief of the failure of theories so constructed.

September, 1995
PACS 11.15Ha, 11.30.Rd, 11.30.Qc

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It is a long standing problem to regularize chiral gauge theories (CGT) on a lattice and it seems that none of the methods proposed has been consistently and completely demonstrated both to ensure that an asymptotically chiral gauge theory in the continuum limit really exists and to provide a framework for doing nonperturbative calculation in these theories \footnote{SU(2) chiral symmetry.}. It is generally believed that the constructions \footnote{Symmetry shift-symmetry.} of CGT on the lattice with external multifermion couplings fail to give chiral gauged fermions in the continuum limit for the reason\footnote{Symmetry shift-symmetry.} that the theories so constructed undergo spontaneous symmetry breaking and their phase structure is similar to that of the Smit-Swift model\footnote{Symmetry shift-symmetry.}, which has been very carefully studied and shown to fail. Nevertheless, we believe that further considerations of constructing CGT on a lattice with external multifermion couplings and careful studies of the spectrum in each phase of such a constructed theory are necessary. In fact, we find the possible scaling region of defining continuum chiral fermion in such a formulation of CGT on the lattice.

Let us consider the following fermion action of the $SU_L(2)$ CGT on a lattice with two external multifermion couplings.

\begin{equation}
S = \frac{i}{2a} \sum_x \left( \bar{\psi}_L^i(x) \gamma_\mu D_\mu^i \psi_L^i(x) + \bar{\psi}_R(x) \gamma_\mu \partial_\mu \psi_R(x) \right) + \sum_x \left( g_1 \bar{\psi}_L^i(x) \cdot \bar{\psi}_R(x) \cdot \bar{\psi}_R(x) \cdot \psi_L^i(x) + g_2 \bar{\psi}_L^i(x) \cdot \partial^2 \psi_R(x) \partial^2 \bar{\psi}_R(x) \psi_L^i(x) \right),
\end{equation}

where "$a$" is the lattice spacing; $\psi_L^i$ ($i = 1, 2$) is an $SU_L(2)$ gauged doublet, $\psi_R$ is an $SU_L(2)$ singlet\footnote{Symmetry shift-symmetry.} and both are two-component Weyl fermions. The second multifermion coupling $g_2$, where $\partial^2 \psi_R(x) = \sum_\mu \left[ \psi_R(x + \mu) + \psi_R(x - \mu) - 2 \psi_R(x) \right]$, is a dimension-10 operator relevant only for doublets $p = \bar{p} + \pi_A, (\bar{p} \simeq 0$ and $\pi_A$ runs over fifteen lattice momenta $\pi_A \neq 0$), but irrelevant for normal modes $p = \bar{p}$ of the $\psi_L^i$ and $\psi_R$. In addition to the exact local $SU(2)$ chiral gauge symmetry and the global flavour symmetry $SU_L(2) \otimes U_R(1)$, the action \footnote{Symmetry shift-symmetry.} possesses a $\psi_R$-shift-symmetry\footnote{Symmetry shift-symmetry.}, $\psi_R(x) \rightarrow \psi_R(x) + \text{const.},$ when $g_1 = 0$. The Ward identity corresponding to this $\psi_R$-shift-symmetry is

\begin{equation}
\frac{i}{2a} \gamma_\mu \partial^\mu \phi'_k(x) + g_1 \left( \bar{\psi}_R^i(x) \psi_R(x) \psi_L^i(x) \right) + g_2 \left( \bar{\psi}_L^i(x) \partial^2 \psi_R(x) \psi_L^i(x) \right) + \Gamma_R^{(1)}(x) = 0,
\end{equation}

where "prime" fields $\phi'_k(x) \sim \frac{\delta W}{\delta \psi_R(x)}$, the one-particle irreducible vertex functions $\Gamma(\phi'_k)$ are the Legendre transformation of the generating functional $W = -\ln Z(\eta)$ and $\Gamma_R^{(1)}(x) = \frac{\delta}{\delta \psi_R(x)} \langle \ldots \rangle$ is an expectation value with respect to the partition function $Z(\eta)$. Based on this Ward identity \footnote{Symmetry shift-symmetry.}, one can get all one-particle irreducible vertices containing at least one external $\psi_R$. $\Gamma_{RR}^{(2)}(p) = \frac{i}{2 \mu} \sin p \mp P_R(\text{the } \psi_R \text{ is free field} \footnote{Symmetry shift-symmetry.}), \Gamma_{LR}^{(2)}(p) = \Sigma'(p) \simeq a^2 (g_1 + 2g_2 w(p)) \langle \bar{\psi}_L^i \psi_R \rangle$, where $w(p) = \sum_\mu \cos \mu - 1$, and the four-point vertex function is given by

1. $\psi_R$ is treated as a “spectator” fermion since we do not consider $U_Y(1)$ symmetry.
\[
\Gamma^{(4)}_{LLRR}(p, p', q) = g_1 + 4g_2w(p + \frac{q}{2})w(p' + \frac{q}{2}),
\]
where \(p + \frac{q}{2}\) and \(p' + \frac{q}{2}\) are the momenta of the external \(\psi_R\). All other one-particle irreducible vertices \(\Gamma^{(3)}_{LLR} = \Gamma^{(3)}_{LR} = \Gamma^{(n)}_{R} = 0 (n > 4)\) identically. We find that when \(g_1 = 0\), the \(\Gamma^{(2)}_{LR}\) and \(\Gamma^{(4)}_{LLRR}\) for the normal mode of the \(\psi_R\) are vanishing at least \(O((ma)^2)\), where \(m\) is the scale of the continuum limit. This indicates that when \(g_1 = 0\), the normal mode of the \(\psi_R\) completely decouples and does not form any bound states with other modes.

2. Our goal is to seek a possible regime, where an undoubled \(SU(2)\)-chiral gauged fermion content is exhibited in the continuum limit in the phase space \((g_1, g_2, g)\), where “\(g\)” is the gauge coupling, regarded to be a truly small perturbation \(g \to 0\) at the scale of the continuum limit we consider. In the weak coupling limit, \(g_1 \ll 1\) and \(g_2 \ll 1\) (indicated 1 in fig.1), the action (1) defines an \(SU_L(2) \otimes U_R(1)\) chiral continuum theory with a doubled and weakly interacting fermion spectrum that is not the continuum theory we seek.

Let us consider the phase of spontaneous symmetry breaking in the weak-coupling \(g_1, g_2\) limit. Based on the analysis of large-\(N_f\) (\(N_f\) is an extra fermion index, e.g., color, \(N_c\)) weak coupling expansion, we show that the multifermion couplings in (1) undergo Nambu-Jona Lasinio (NJL) spontaneous chiral-symmetry breaking. In this symmetry breaking phase (indicated 2 in fig.1) the \(\psi'_L\) and \(\psi_R\) in (1) pair up to be a massive Dirac fermion violating \(SU(2)\)-chiral symmetry. The spectrum of the theory (1) can be written as,

\[
S^{-1}_b(p) = \left( \begin{array}{cc} P_{R \alpha} \sum \gamma_i f^i_L(p) \gamma^\alpha & P_R \Sigma^i(p) P_L \\ P_L \Sigma^i(p) P_L & P_{R \alpha} \sum \gamma_i \sin p^\mu P_L \end{array} \right),
\]

where the fermion self-energy function is given by \((N_f \to \infty)\)

\[
\Sigma^i(p) = 4 \int_q \frac{\Sigma^i(q)}{\text{den}(q)} (\bar{\psi}_L + 4\bar{\psi}_L w(p)w(q))
\]

where \(f_q \equiv \int_\pi \frac{d^4\sigma}{(2\pi)^4}, \text{den}(q) \equiv \sum_\rho \sin^2 q_\rho + (\Sigma^i(q)a)^2\) and \(\bar{g}_1 \equiv g_1 N_f a^2, \bar{g}_2 \equiv g_2 N_f a^2\). Using the parametrization \(\Sigma^i(p) = \Sigma^i(0) + \bar{g}_2v^i w(p)\) and \(\Sigma^i(0) = \rho v^i\) \(\text{(3)}\), where \(\rho\) depends only on couplings \(\bar{g}_1, \bar{g}_2\), and \(v^i\) plays a role as the v.e.v. violating \(SU(2)\)-chiral symmetry, we can solve the gap-equation \(\text{(5)}\). For \(v^i = O(\frac{1}{\lambda})\), one obtains

\[
\rho = \frac{\bar{g}_1 \bar{g}_2 I_1}{1 - \bar{g}_1 I_0}, \quad \rho = \frac{1 - 4\bar{g}_2 I_2}{4 I_1},
\]

where the functions \(I_n = \int_q \frac{w^n(q)}{\text{den}(q)}\). Eq.\((\text{3})\) leads to \(\bar{g}_1 = 0, \rho = 0\) and \(\Sigma^i(0) = 0\), this result is due to the Ward identity \(\text{(2)}\). This means that on the line \(g_1=0, g_2 \geq 0\),

\[
= \Gamma^{(4)}_{LLRR}(p, p', q) = g_1 + 4g_2w(p + \frac{q}{2})w(p' + \frac{q}{2}),
\]
normal modes $p = \tilde{p} \simeq 0$ of the $\psi^i_L$ and $\psi^i_R$ are massless and their 15 doublers $p = \tilde{p} + \pi_A$ acquire chiral-variant masses $\Sigma^i(p)$ through the multifermion coupling $g_2$ only $(1 - 4\tilde{g}_2 I_2 = 0)$.

As for the function $f^i_L(p)^{ij}$ in eq. (6), it depends on the dynamics of the left-handed Weyl fermion $\psi^i_L$ in this region. In large-$N_f$ calculation at weak couplings we are able to evaluate the function $f^i_L(p)^{ij} = \delta_{ij} \sin p_\mu Z_2(p)$ (see fig.2) and the wave function renormalization is given by

$$Z_2^{-1}(p) = 1 - \frac{2}{N_f} \int_{k,l} \left( \tilde{g}_1 + 4\tilde{g}_2 w(k) w(k - \frac{l}{2}) \right)^2 \frac{\sum_{\mu \nu} \gamma_\mu \gamma_\nu \sin(p - k)^\mu \sin p^\nu}{\sum_\lambda \sin^2(p - k)^\lambda \sin^2 p_\rho} R(k,l)$$

$$R(k, l) = \frac{\sum_{\sigma} \sin \left( k - \frac{l}{2} \right)^\sigma \sin \left( k + \frac{l}{2} \right)^\sigma}{\sum_{\sigma'} \sin^2 \left( k - \frac{l}{2} \right)^\sigma \sin^2 \left( k + \frac{l}{2} \right)^{\sigma'}}. \tag{7}$$

Assuming the symmetry breaking takes place in the direction 1 in the 2-dimensional space of the $SU(2)$-chiral symmetry ($v^1 \neq 0, v^2 = 0$), we find the following fermion spectrum, containing a doubled Weyl fermion $\psi^2_L$ and a undoubled Dirac fermion made by the Weyl fermions $\psi^1_L$ and $\psi_R$,

$$S_{b1}^{-1}(p) = \frac{i}{a} \sum_\mu \gamma_\mu \sin p_\mu Z_2(p) P_L + \frac{i}{a} \sum_\mu \gamma_\mu \sin p^\mu P_R + v^1(\rho + \tilde{g}_2 w(p)) \tag{8}$$

$$S_{b2}^{-1}(p) = \frac{i}{a} \sum_\mu \gamma_\mu \sin p_\mu Z_2(p) P_L. \tag{9}$$

The $SU_L(2) \otimes U_R(1)$ chiral symmetry is realized to be $U_L(1) \otimes U(1)$ with three Goldstone modes and a massive Higgs mode that are not presented in this short report. As $v^1 \to 0$, eq. (8) gives a critical line $\tilde{g}_1(\tilde{g}_2^c)$ of characterizing NJL spontaneous chiral symmetry breaking and $\tilde{g}_1^c = 0.4, \tilde{g}_2^c = 0; \tilde{g}_{1}^c = 0, \tilde{g}_{2}^c = 0.0055$ (indicated 2 in fig.1). These critical values are sufficiently small even for $N_f = 1$.

This broken phase cannot be a candidate for a real chiral gauge theory (e.g., the Standard Model) for the reasons that (i) $\psi^2_L$ is doubled (9); (ii) the spontaneous symmetry breakdown of the $SU_L(2)$-chiral symmetry is caused by the hard breaking Wilson term $\mathcal{S}$ (dimension-5 operator), which must contribute the intermediate gauge boson masses through the perturbative gauge interaction and disposal of Goldstone modes. The intermediate gauge boson masses turn out to be $O(\frac{1}{a})$. This, however, is phenomenologically unacceptable.

3. We turn to the strong coupling region, where $g_1(g_2)$ is sufficiently larger than a certain critical value $g_1^c(g_2^c)$ (indicated 3 in fig.1). Analogously to the analysis and discussions of Eichten and Preskill (EP) [4], we can show that the $\psi^i_L$ and $\psi^i_R$ in (7) are bound up to form the composite Weyl fermions $(\tilde{\psi}_L \cdot \psi_R)\psi^i_L$ (left-handed $SU_L(2)$-neutral) and $(\psi_R \cdot \tilde{\psi}_L^i)\psi_R$ (right-handed $SU_L(2)$-charged) and these bound fermion states respectively pair up with the $\tilde{\psi}_R$ and $\psi^i_L$ to be massive, neutral.
\(\Psi_n\) and charged \(\Psi^c\) Dirac modes consistently with the \(SU_L(2) \otimes U_R(1)\) chiral symmetry.

The second multifermion coupling \(4g_2w(p + \frac{q}{2})w(p' + \frac{q}{2})\) in (3) gives different contributions to the effective value of \(g_1\) at large distance for sixteen modes of the \(\psi^i_L\) and \(\psi^i_R\) in the action (4). Let us consider the multifermion couplings of each mode “\(p\)” of the \(\psi^i_L\) and \(\psi^i_R\), namely, we set \(p = p', q = \tilde{q} \ll 1\) in the four-point vertex (3). Using strong multifermion coupling, \(\Gamma^{(4)}_{LLRR} = g_1 + 4g_2w^2(p) \gg 1\), expansion and recursion relation (4) for each mode “\(p\)” of the \(\psi^i_L\) and \(\psi^i_R\), in the lowest nontrivial order we calculate the propagators of neutral and charged Dirac modes to be

\[
S_n(p) \simeq \frac{i}{\alpha} \sum \gamma^\mu \sin p^\mu + M \left\{ \frac{1}{\alpha^2} \sum \sin^2 p_\rho + M^2 \right\}
\]

\[
S_c(p)_{ij} \simeq \delta_{ij} \frac{i}{\alpha} \sum \gamma^\mu \sin p^\mu + M \left\{ \frac{1}{\alpha^2} \sum \sin^2 p_\rho + M^2 \right\},
\]

where the chiral-invariant masses \(M^2 = 16a^2(g_1 + 4g_2w^2(p))^2\) and the spectrum is vector-like.

The critical value \(g_1^0(g_2^0)\) can be determined by considering (4) the propagators \(G^{(4)}_{LL}(q)\) of four composite scalars \(A^1_L = \frac{1}{\sqrt{2}}(\bar{\psi}^i_L \cdot \psi^i_R + \bar{\psi}^i_R \cdot \psi^i_L)\) and \(A^2_L = \frac{1}{\sqrt{2}}(\bar{\psi}^i_L \cdot \psi^i_R - \bar{\psi}^i_R \cdot \psi^i_L)\), which are the real and imaginary parts of a complex composite field \(A^i = \bar{\psi}^i_R \cdot \psi^i_L\). Again using the strong coupling, \(\Gamma^{(4)}_{LLRR} = g_1 + 4g_2w^2(p) \gg 1\), expansion and recursion relation (4) for each mode “\(p\)” of the \(\psi^i_L\) and \(\psi^i_R\), in the lowest nontrivial order we find these four massive composite scalar modes,

\[
G_{1,2}^{ij}(\tilde{q}) \simeq \frac{\delta_{ij}}{4a^2} \sum \gamma^\mu \frac{\sin^2 \frac{q_\mu}{2} + \mu^2}{1}, \quad \mu^2 \simeq 16 \left(g_1 + 4g_2w^2(p) - \frac{1}{2a^2}\right),
\]

which are degenerate owing to the exact \(SU(2)\)-chiral symmetry. A spontaneous symmetry breaking \(SU(2) \to U(1)\) occurs, where \(\mu^2 > 0\) turns to \(\mu^2 < 0\). Eq.(12) for \(\mu^2 = 0\) gives rise to the critical lines: \(g_1^1a^2 = 0.5, g_2 = 0; g_1 = 0, a^2g_2 = 0.002\) where the first binding threshold of the doubler \(p = (\pi, \pi, \pi, \pi)\) is, and \(g_1 = 0, a^2g_2 = 0.031\) where the last binding threshold of the doublers \(p = (\pi, 0, 0, 0)\) is, inbetween (indicated 4 in fig.1) there are the binding thresholds of the doublers \(p = (\pi, \pi, 0, 0)\) and \(p = (\pi, \pi, \pi, 0)\), and the binding thresholds of the different doublers \(p \neq p'\), which can be analogously calculated. Above \(g_1^1a^2\) all doublers are supposed to be bound, as indicated 5 in fig.1. As for the normal modes of the \(\psi^i_L\) and \(\psi^i_R\), when \(g_1 \ll 1\), the multifermion coupling, \(\Gamma^{(4)}_{LLRR} = g_1 + 4g_2w^2(\bar{p})\), is no longer strong enough to form the bound states \((\bar{\psi}^i_L \cdot \psi^i_R)\psi^i_L, (\bar{\psi}^i_R \cdot \psi^i_L)\psi^i_R\) and \(A^i\) unless \(a^2g_2 \to \infty\). It is conceivable that the critical line for normal modes \(g_1 + ag_2O((ma)^4) - \frac{1}{2a^2} = 0\) analytically continues to the limit \(g_2^\infty = g_1 \to 0, g_2 \to \infty\).

\(^2\) This is just a hopping parameter expansion.
Thus, as expected in ref.\textsuperscript{4}, several wedges open up as \(g_1, g_2\) increase in the NJL phase (indicated 5 in fig.1), inbetween the critical lines along which bound states of normal modes and doublers of the \(\bar{\psi}_L^i\) and \(\psi_R^i\) respectively approach their thresholds. In the initial part of the NJL phase, the normal modes and doublers of the \(\bar{\psi}_L^i\) and \(\psi_R^i\) undergo the NJL phenomenon and contribute to eq.\textsuperscript{(4)} as discussed in section 2. As \(g_1, g_2\) increase, all these modes, one after another, gradually disassociate from the NJL phenomenon and no longer contribute to eqs.\textsuperscript{(4)}. Instead, they turn to associate with the EP phenomenon and contribute to eqs.\textsuperscript{(10,11)}. The first and last doublers of the \(\bar{\psi}_L^i\) and \(\psi_R^i\) making this transition are \(p = (\pi, \pi, \pi, \pi)\) and \(p = (\pi, 0, 0, 0)\) respectively. At the end of this sequence, normal modes \((p = \tilde{p})\) make this transition, due to the fact that they possess the different effective multifermion coupling \(\Gamma_{LLRR}^{(4)} = g_1 + 4g_2w^2(p)\).

Had these critical lines separated the two symmetric phases, (strong couplings and the weak coupling symmetric phases) we would have found a threshold over which all doublers of the \(\bar{\psi}_L^i\) and \(\psi_R^i\) decouple by acquiring chiral invariant masses (\textsuperscript{(1)(1)}) and normal modes of the \(\bar{\psi}_L^i\) and \(\psi_R^i\) remain massless and free, and we might obtain a theory of massless free chiral fermions \textsuperscript{4}. However, this is not real case \textsuperscript{4}. As has been seen in eq.\textsuperscript{(12)}, turning \(\mu^2 > 0\) to \(\mu^2 < 0\) indicates a phase transition between the strong coupling symmetric phase to the spontaneous chiral symmetry breaking phase, which separates the strong coupling and weak coupling symmetric phases.

The possible resolution of this undesired situation is that we find a wedge in which the doublers of the \(\bar{\psi}_L^i\) and \(\psi_R^i\) have formed bound states \((\bar{\psi}_R \cdot \bar{\psi}_L^i)\psi_R^i\) and \((\bar{\psi}_L \cdot \psi_R^i)\bar{\psi}_L^i\) via the EP phenomenon, while the normal modes of the \(\bar{\psi}_L^i\) and \(\psi_R^i\) have neither formed such bound states yet and nor are they associated with the NJL-phenomenon.

Within the last wedge (indicated 5 in fig.1) between two the thresholds \(g_2^c,a\) and \(g_2^c,\infty\), all doublers of the \(\bar{\psi}_L^i\) and \(\psi_R^i\) acquire chiral-invariant masses and decouple (considering eqs.\textsuperscript{(1)(1)}) as the propagators for doublers \(p = \tilde{p} + \pi_A\) only) and we have the undoubled low-energy spectrum that involves only the normal modes of the \(\bar{\psi}_L^i\) and \(\psi_R^i\). However, because of the multifermion coupling \(g_1 \neq 0\), these normal modes of \(\psi_L^i\) and \(\psi_R^i\) still remain in the NJL broken phase, the \(SU(2)\)-chiral symmetry is violated by \(\Sigma^1(0) = \rho v^1\), to which only normal modes contribute. The propagators of the normal modes in this wedge should be the same as eqs.\textsuperscript{(8,9)} for \(p = \tilde{p}\). However, when \(g_1 \neq 0\), the normal mode of the \(\psi_R^i\) is not guaranteed to completely decouple from that of the \(\bar{\psi}_L^i\).

Once we go onto the line A \((g_1 = 0, g_2^c,a < g_2 < g_2^c,\infty\) as indicated in fig.1), the spectrum is undoubled for \(g_2 > g_2^c,a\) and as the results of the \(\psi_R\)-shift-symmetry of the action \textsuperscript{(1)}: (i) the normal mode of the \(\psi_R\) is a free mode; (ii) the interacting vertex \(\Gamma_{LLRR}^{(4)} = 4g_2w^2(\tilde{p}) \ll 1\) for the normal modes, which prevent the normal modes of the \(\bar{\psi}_L^i\) and \(\psi_R^i\) from binding up bound states \((\bar{\psi}_L^i \psi_R^i)\psi_R^i\) and \((\bar{\psi}_R^i \psi_R^i)\psi_R^i\) and \(\mathcal{A}^i\); (iii) the NJL mass term \(\Gamma_{LR}^{(2)} = \Sigma^1(0) = 0\) for which the \(SU_L(2) \otimes U_R(1)\)-chiral
symmetry is completely restored. In this scaling region, the spectrum consists of the doublers eq.(10,11) for $g_1 = 0$ and $p = \tilde{p} + \pi_A$, the massless normal modes eqs.(8,9) for $g_1 = 0$ and $p = \tilde{p}$,

$$S^{-1}_L(\tilde{p})^{ij} = i\gamma_\mu \tilde{p}^\mu \hat{Z}_2 \delta^{ij} P_L; \quad S^{-1}_R(\tilde{p}) = i\gamma_\mu \tilde{p}^\mu P_R,$$

which is in agreement with the $SU_L(2) \otimes U_R(1)$ symmetry. Namely, this normal mode of the $\psi^i_L$ is self-scattering via the multifermion coupling $g_2$ (see fig.2) without pairing up with any other modes. The wave function renormalization $\tilde{Z}_2$ can be considered as an interpolating constant of $Z_2(p)$ eq.(7) for $p = \tilde{p} \simeq 0$ and $g_1 = 0$.

In summary, in the scaling region for the long distance, we have the spectrum that contains the $SU_L(2)$-invariant and $U_R(1)$-covariant neutral Dirac mode $\Psi_n$ eq.(10)($p \neq \tilde{p}$); the $U_R(1)$-invariant and $SU_L(2)$-covariant charged Dirac mode $\Psi^c_\pm$ eq.(11)($p \neq \tilde{p}$); the $U_R(1)$-covariant Weyl mode $\psi_R$ and the $SU_L(2)$-covariant Weyl mode $\psi^i_L$ eq.(13)($p = \tilde{p}$) as well as the $SU_L(2) \otimes U_R(1)$ covariant scalar $A^i$ eq.(12)($q = \tilde{q}$). In order to see all possible interactions between these modes in the scaling region, we consider the one-particle irreducible vertex functions of these modes. In the light of the exact $SU_L(2) \otimes U_R(1)$ chiral symmetry and $\psi_R$-shift-symmetry, one can straightforwardly obtain non-vanishing vertex functions ($d$=dimensions) at physical momenta ($p = \tilde{p}, q = \tilde{q}$): (i) $A^j A^{j\dagger} A^i A^{i\dagger}$ ($d = 4$); (ii) $\bar{\psi}^i_L \psi^j_L A^{j\dagger}, \bar{\psi}^c_+ \psi^i_L A^{j\dagger}$ and $\bar{\psi}^c_- \psi^i_R A^{j\dagger}$ ($d = 5$), as well as $d > 5$ vertex functions. The vertex functions with dimensions $d > 4$ vanish in the scaling region as $O\left(a^{d-4}\right)$ and we are left with the self-interacting vertex $A^j A^{j\dagger} A^i A^{i\dagger}$.

In this scaling region, the chiral continuum limit is very much like that of lattice QCD. We need to tune only one coupling $g_1 \to 0$ in the neighborhood of $g_2^{\alpha} < g_2 < g_2^{\infty}$. For $g_1 \to 0$, the $\psi_R$-shift-symmetry is slightly violated, the normal modes of the $\psi^i_L$ and $\psi_R$ would couple together to form the chiral symmetry breaking term $\Sigma(0)\bar{\psi}^i_L \psi_R$, which is a dimension-3 renormalized operator and thus irrelevant at the short distance. We desire this scaling region to be ultraviolet stable, in which the multifermion coupling $g_1$ turns out to be an effective renormalized dimension-4 operator $[12]$.

4. The conclusion of the existence of a possible scaling region for the continuum chiral theory is plausible and it is worthwhile to confirm this scenario in different approaches. However, we are still left with several problems. Their possible resolutions are mentioned and discussed in this section, and deserve to be studied in future work.

The question is whether this chiral continuum theory in the scaling region could be the correct chiral gauge theory, as the $SU(2)$-chiral gauge coupling $g$ perturbatively is turned on in the theory $[4]$. One should expect a slight change of critical lines (points). We should be able to re-tune the multifermion couplings $(g_1, g_2)$ to compensate these perturbative changes. In the scaling regime, disre-
garding those uninteresting neutral modes, we have the charged modes including both the $SU(2)$-chiral-gauged, massless normal mode $\psi^i_L$ and the $SU(2)$-vectorial-gauged, massive doublers of the Dirac fermion $\Psi^i_c$, which is made by the 15 doublers of the $\psi^i_L$ and the 15 doublers of the bound Weyl fermion $\psi^i_R \cdot \psi^i_L \psi^i_R$. The gauge field should not only chirally couple to the massless normal mode of the $\psi^i_L$ in the low-energy regime, but also vectorially couple to the massive doublers of Dirac fermion $\Psi^i_c$ in the high-energy regime. Thus, we expect the coupling vertex of the $SU_L(2)$-gauge field and the normal mode of the $\psi^i_L$ to be chiral at the continuum limit. We are supposed to be able to demonstrate this point on the basis of the Ward identities associating with the $SU(2)$-chiral gauge symmetry that is respected by the spectrum in the scaling regime. In fact, due to the reinstating of the manifest $SU(2)$-chiral gauge symmetry and corresponding Ward identities of the undoubled spectrum in this scaling regime, we should then apply the Rome approach (which is based on the conventional wisdom of quantum field theory) to perturbation theory in the small gauge coupling. It is expected that the Rome approach would work in the same way but all gauge-variant counterterms are prohibited; the gauge boson masses vanish to all orders of gauge coupling perturbation theory for $g_1 = 0$.

Another important question remaining is how chiral gauge anomalies emerge, although in this short report the chiral gauge anomaly is cancelled by purposely choosing an appropriate fermion representation of the $SU_L(2)$ chiral gauge group. We know that in the doubled spectrum of naive lattice chiral gauge theory, the reason for the correct anomaly disappearing in the continuum limit is that the normal mode and doublers of Weyl fermion produce the same anomaly these anomalies eliminate themselves. As a consequence of decoupled doublers being given chiral-invariant mass ($\sim O(\frac{1}{a})$), the survival normal mode of the Weyl fermion (chiral-gauged, e.g., $U_L(1)$) should produce the correct anomaly in the continuum limit. We also have the question of whether the conservation of fermion number would be violated by the correct anomaly structure $trFF$ that is generated by the $SU(2)$ instanton in the continuum limit.

I am grateful to G. Preparata for advice and discussions and thank M. Testa, D.N. Petcher and M. Golterman for discussions at Melbourne Lat’95.

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**Figure Captions**

**Figure 1:** The phase diagram for the theory (1) in the $g_1 - g_2$ plane (at $g \simeq 0$).

**Figure 2:** Contributions to the wave function renormalization $Z_2(p)$ of the $\psi^i_L$. 