Neutrino Masses with “Zero Sum” Condition: $m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0$

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Abstract

It is well known that the neutrino mass matrix contains more parameters than experimentalists can hope to measure in the foreseeable future even if we impose CP invariance. Thus, various authors have proposed ansatzes to restrict the form of the neutrino mass matrix further. Here we propose that $m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0$; this “zero sum” condition can occur in certain class of models, such as models whose neutrino mass matrix can be expressed as commutator of two matrices. With this condition, the absolute neutrino mass can be obtained in terms of the mass-squared differences. When combined with the accumulated experimental data this condition predicts two types of mass hierarchies, with one of them characterized by $m_{\nu_3} \approx -2m_{\nu_1} \approx -2m_{\nu_2} \approx 0.063$ eV, and the other by $m_{\nu_1} \approx -m_{\nu_2} \approx 0.054$ eV and $m_{\nu_3} \approx 0.0064$ eV. The mass ranges predicted is just below the cosmological upper bound of 0.23 eV from recent WMAP data and can be probed in the near future. We also point out some implications for direct laboratory measurement of neutrino masses, and the neutrino mass matrix.
I. INTRODUCTION

There are abundant data [1–6] from solar, atmospheric, laboratory and long baseline neutrino experiments on neutrino mass and mixing. The present experimental data, including recent results from KamLAND [5] and K2K [6], on neutrino masses and mixing angles can be summarized as follow [8–10]. The 99.3% C.L. level allowed ranges for the mass-squared differences are constrained to be: $1.5 \times 10^{-3} \text{eV}^2 \leq \Delta m_{\text{atm}}^2 \leq 5.0 \times 10^{-3} \text{eV}^2$, and $2.2 \times 10^{-5} \text{eV}^2 \leq \Delta m_{\text{solar}}^2 \leq 2.0 \times 10^{-4} \text{eV}^2$, with the best fit values given by $\Delta m_{\text{atm}}^2 = 3.0 \times 10^{-3} \text{eV}^2$, and $\Delta m_{\text{solar}}^2 = 7.0 \times 10^{-5} \text{eV}^2$. The mixing angles are in the ranges of $\sin^2 2\theta_{\text{atm}} > 0.85$ and $0.18 \leq \sin^2 \theta_{\text{solar}} \leq 0.37$. Also the CHOOZ experiment [4] gives a bound of 0.22 on the $\nu_e - \nu_x$ (where $\nu_x$ can be either $\nu_\mu$ or $\nu_\tau$ or a linear combination) oscillation parameter.

Present data can be explained by oscillations between three active neutrinos [11] * with the atmospheric neutrino and K2K data explained by oscillation between the muon and the tauon neutrinos, and the solar neutrino and KamLAND data explained by oscillation between the electron and muon neutrinos. Neutrino oscillations provide direct evidence of non-zero neutrino masses and mixing between different species of neutrinos.

We will assume that the neutrinos are Majorana as favored by some theoretical considerations [7]. The relevant term in the Lagrangian describing Majorana neutrino masses is $\mathcal{L} = \nu_L^T C M \nu_L + H.C.$, where $\nu_L = (\nu_e L, \nu_\mu L, \nu_\tau L)$ are the left-handed neutrinos and $C$ is the charge conjugation operator. The mass matrix $M$ is symmetric due to Fermi statistics. A convenient basis to study neutrino masses and mixing is the weak basis where all charged leptons are already diagonalized. The unitary mixing matrix $V$ and the mass eigenvalues in this basis are related by

$$D = V^T M V,$$

*There are additional evidences for oscillation between electron and muon neutrinos from LSND experiment [12]. If confirmed more neutrinos are needed to explain all the data.
where $D$ is a diagonal matrix. The diagonal entries $m_i$ of $D$ are the mass eigenvalues which can always be made real by an appropriated choice of phase convention. For three generations of neutrinos, the mixing matrix can be described by three rotation angles and three phases. Note that in general $TrM \neq TrD$.

Although there is a lot of data on neutrinos, more data are needed to determine detailed properties of neutrinos. Oscillation experiments, as indicated above, cannot measure the values $m_i$, but only the mass-squared differences $\Delta m^2_{21} = m^2_{\nu_2} - m^2_{\nu_1}$ and $\Delta m^2_{32} = m^2_{\nu_3} - m^2_{\nu_1}$ and some mixing angles. In analyzing the present data, it is generally assumed that $\Delta m^2_{21} \equiv \Delta m^2_{\text{solar}}$, and $\Delta m^2_{32} \equiv \Delta m^2_{\text{atm}}$ and thus $|\Delta m^2_{21}| \ll |\Delta m^2_{32}|$. The sign of $\Delta m^2_{21}$ has now been determined to be positive from matter effect in the solar neutrino data [2,13], but the sign of $\Delta m^2_{32}$ is not determined yet.

There is at present certainly no information on any of the three CP violating phases, and in the foreseeable future no set of experiments can fully determine all the parameters in the neutrino mass matrix. Certain theoretical inputs have to be employed to reconstruct the neutrino mass matrix [14–20]. Several proposals have been made to reduce the parameters, such as texture zero [16] and determinant zero requirement [17] for the mass matrix.

Here we propose another way of reducing the number of unknown parameters, by imposing a condition on the mass eigenvalues,

$$m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0. \quad (2)$$

This “zero sum” condition that the neutrino masses sum to zero implies that some of the eigen-masses must have opposite signs. These signs of course can always be arranged by making chiral phase redefinition of the neutrino fields.

If there is no CP violation in the neutrino mass matrix $M$, the mass matrix can always be made real and it can be diagonalized by an orthogonal transformation, namely $V^TV = I$. In this case the “zero sum” condition (2) is equivalent to the traceless condition, $TrM = Tr(V^*V^*D) = TrD = 0$. If CP is not conserved, the “zero sum” and traceless conditions are different. One needs to be careful about the phase definitions [21]. In this paper we simply
explore the phenomenological consequences of requiring the neutrino masses to satisfy the “zero sum” condition with CP conservation without speculating on its theoretical origin. We note, however, that it holds if \( M = [A, B] \), that is, the mass matrix can be expressed as a commutator\(^\dagger\) of two matrices \( A \) and \( B \).

**II. “ZERO SUM” CONDITION AND NEUTRINO MASSES**

Direct measurement of neutrino masses is a very difficult experimental task. If the “zero sum” condition \( m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0 \) or equivalently \( TrM = 0 \), is applied, all the neutrino masses are determined in terms of the mass-squared differences. We have

\[
\begin{align*}
    m_{\nu_1}^2 &= -\frac{1}{3} \left[ 2\Delta m_{21}^2 + \Delta m_{32}^2 \pm 2\sqrt{(\Delta m_{32}^2)^2 + \Delta m_{21}^2 \Delta m_{32}^2 + (\Delta m_{21}^2)^2} \right], \\
    m_{\nu_2}^2 &= \frac{1}{3} \left[ \Delta m_{21}^2 - \Delta m_{32}^2 \pm 2\sqrt{(\Delta m_{32}^2)^2 + \Delta m_{21}^2 \Delta m_{32}^2 + (\Delta m_{21}^2)^2} \right], \\
    m_{\nu_3}^2 &= \frac{1}{3} \left[ \Delta m_{21}^2 + 2\Delta m_{32}^2 \pm 2\sqrt{(\Delta m_{32}^2)^2 + \Delta m_{21}^2 \Delta m_{32}^2 + (\Delta m_{21}^2)^2} \right].
\end{align*}
\]

(3)

The choice of the signs in front of the square root is decided by the requirement that all \( m_{\nu_i}^2 \) must be larger or equal to zero. From the expression for \( m_{\nu_3}^2 \), one can determine that the “+” has to be chosen for the expressions for \( m_{\nu_{2,3}} \), and “−” for the expression for \( m_{\nu_1} \). The relative signs of the eigen-masses \( m_i \) are determined by the condition (2). We will use a convention such that \( m_{\nu_3} \geq 0 \) in our later discussions.

For \( |\Delta m_{32}^2| \ll |\Delta m_{32}^2| \), the above simplify to, to leading order in \( \varepsilon \equiv \Delta m_{21}^2 / \Delta m_{32}^2 \), we have

\[
m_{\nu_1}^2 = \frac{2}{3} |\Delta m_{32}^2|(1 + \varepsilon) - \frac{1}{2} \Delta m_{32}^2 (1 + 2\varepsilon),
\]

\(^\dagger\)In the simplest versions of models proposed in ref. [14], \( M \) results from radiative correction and comes out to be the commutator of a coupling matrix and the mass-squared matrix of the charged leptons. However there are a number of variations of this model and the condition \( m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0 \) fails to hold in most of them.
TABLE I. Solutions of eigen-masses for the best fit values of $|\Delta m_{21}^2| = 7.0 \times 10^{-5}$ eV$^2$ and $|\Delta m_{32}^2| = 3.0 \times 10^{-3}$ eV$^2$.

| $\Delta m_{32}^2$ (eV$^2$) | $\Delta m_{21}^2$ (eV$^2$) | $m_{\nu_1}$ (eV) | $m_{\nu_2}$ (eV) | $m_{\nu_3}$ (eV) | $|m_{ee}|$ (eV) |
|--------------------------|--------------------------|-----------------|-----------------|-----------------|----------------|
| $3.0 \times 10^{-3}$     | $7.0 \times 10^{-5}$     | -0.0313         | -0.0324         | 0.0636           | (0.01 $\sim$ 0.032) |
| $-3.0 \times 10^{-3}$    | $7.0 \times 10^{-5}$     | 0.0541          | -0.0548         | $6.43 \times 10^{-4}$ | (0.018 $\sim$ 0.054) |

$$
m_{\nu_2}^2 = \frac{2}{3}[|\Delta m_{32}^2| (1 + \varepsilon) - \frac{1}{2} \Delta m_{32}^2 (1 - \varepsilon)],$$

$$
m_{\nu_3}^2 = \frac{2}{3}[|\Delta m_{32}^2| + \Delta m_{32}^2] (1 + \frac{\varepsilon}{2}).$$

(4)

At present the sign of the measured $\Delta m_{21}^2$ is determined to be positive, but the sign of $\Delta m_{32}^2$ is not determined, there are two possible solutions corresponding to the sign of $\Delta m_{32}^2$. In Table I we list all solutions for the best fit values of the mass-squared differences.

We see that the mass eigenvalues exhibit two types of hierarchies,

$I)$ $m_{\nu_3} \approx -2m_{\nu_1} \approx -2m_{\nu_2} \approx 0.064$ eV

$II)$ $m_{\nu_1} \approx -m_{\nu_2} \approx 0.054$ eV, and $m_{\nu_3} \approx 0.00064$ eV.

(5)

The sign of $\Delta m_{32}^2$ decides which mass hierarchy the solutions belong to. Note that the "natural" sign $\Delta m_{32}^2 > 0$ corresponds to scenario I), in which the masses are of the same order of magnitude, in contrast to scenario II), in which $m_{\nu_3}$ is two order of magnitude smaller than $m_{\nu_1}$ and $m_{\nu_2}$. We would like to suggest that I) is more favored than II).

In contrast to oscillation experiments, the contribution of the neutrinos to the energy density of the universe, $\Omega_\nu \approx \sum_i |m_i|/(90$ eV) depends on the absolute values of $|m_i|$ of course. With the neutrino masses predicted with the central values of $\Delta m_{21}^2$ and $\Delta m_{32}^2$, we would have, $\Omega_\nu \ll 1$. Even if one uses the upper bound of $\Delta m_{32}^2 = 5.0 \times 10^{-3}$, the contribution to the energy density is still much smaller than one. Other astrophysical and cosmological data can give information on the neutrino masses [22], such as the CMB power spectrum and large scale structure survey data. The absolute neutrino mass sum $|m_{\nu_1}| +$
$|m_{\nu_2}| + |m_{\nu_3}|$ predicted by the “zero sum” condition is only a factor of two smaller than the present bound [23] of 0.23 eV obtained from combining WMAP and Galaxy sky survey data, and is close to the sensitivity of 0.12 eV of the combined PLANCK CMB data with the SDSS sky survey [24]. A future sky survey with an order of magnitude larger survey volume would allow the sensitivity to reach 0.03 eV [22]. The mass ranges predicted by the condition (2) may be tested in the future.

III. MIXING AND MASSES

To obtain more information about neutrino properties, one needs to have information from mixing. Before the SNO and KamLAND data, assuming three active neutrino oscillations, one of the solutions which can account for known data is the bi-maximal mixing matrix [25] with $|V_{e2}| = |V_{\mu 3}| = 1/\sqrt{2}$. Experimental data from SNO and the recent data from KamLAND however disfavor the maximal mixing for the $V_{e2}$ entry. We were thus led to propose [26] the following mixing matrix,

$$V = \begin{pmatrix}
\frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}. \quad (6)$$

This mixing matrix (but with the first and second column interchanged) was first suggested by Wolfenstein more than 20 years ago [27]. It has subsequently been studied extensively by Harrison, Perkins and Scott [28], and Xing [29].

As mentioned before, oscillation experiments can not determine the relative signs of the mass eigenvalues which implies that one can multiply a phase matrix $P = \text{Diag}(e^{i\rho}, e^{i\sigma}, 1)$ from the right on $V$. With CP invariance, $\sigma$ and $\rho$ can take the values of zero or $\pm \pi/2$.

With the above information on the mixing matrix, let us estimate two observables related to neutrino mass measurements, the effective mass electron neutrino mass $\langle m_e \rangle^2$ measured in tritium beta decay end point spectrum experiments, and the effective Majorana electron neutrino mass $m_{ee}$ in neutrinoless double beta decays.
The effective mass measured in tritium end point spectrum experiments is given by,\
\[ \langle m_e \rangle^2 = (m_{\nu_1}^2 |V_{e1}|^2 + m_{\nu_2}^2 |V_{e2}|^2 + m_{\nu_3}^2 |V_{e3}|^2). \] Using the values for the neutrino masses in Table I, we find that \( \langle m_e \rangle \) is below the sensitivity of 0.12 eV for the proposed experiment KATRIN [30]. However, neutrinoless double decay experiments may be sensitive to the predicted ranges. The amplitude of neutrinoless double beta decay is proportional to the effective electron-neutrino Majorana mass \( m_{ee} \), namely \( M_{11} \), given by

\[ |m_{ee}| = |m_{\nu_1} V_{e1}^{*} e^{-2i\rho} + m_{\nu_2} V_{e2}^{*} e^{-2i\sigma} + m_{\nu_3} V_{e3}^{*}|. \] (7)

From the above expression we see that the value \( m_{ee} \) depends on the Majorana phases \( \rho \) and \( \sigma \). If the phases \( \rho \) and \( \sigma \) are all zero, the neutrinoless double beta decays would have the smallest rate because the cancellation imposed by the “zero sum” condition. If the phases take values such that both \( m_{\nu_1} e^{-2i\rho} \) and \( m_{\nu_2} e^{-2i\sigma} \) are positive, the neutrinoless double beta decays would have the largest rate possible. To have an idea of what possible effects of the Majorana phases can have on neutrinoless double beta decays, we calculated the range for \( |m_{ee}| \) for the best fit values of the mass squared differences with \( V \) given in eq.(6) for arbitrary \( \rho \) and \( \sigma \). The allowed ranges are also listed in Table I at the last column. The ranges obtained are well below the present upper bounds of 0.4 eV [31], but can almost be fully covered by future experiments [32], such as GENIUS, MAJORANA, EXO, MOON or COURE, where sensitivity as low as 0.01 eV seems possible.

### IV. NEUTRINO MASS MATRIX

We now study implications of the “zero sum” neutrino mass condition on the neutrino mass matrix \( M \). If the neutrino masses and mixing matrix are known to a good precision, one can invert the eigen-masses according to eq. (1) to obtain the mass matrix \( M \). To have some feeling how this may provide important information about the mass matrix, we present some details of the mass matrices which produce the mixing matrix \( V \) in eq. (6).

The most general mass matrix which can produce the mixing matrix \( V \) in eq. (6) can be specified in the following form by the mass eigenvalues,
\[
M = \frac{m_{\nu_1}}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_{\nu_2}}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_{\nu_3}}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} .
\] (8)

Being real symmetric (and so a fortiori Hermitian) the above three matrices generate a \(U(1) \otimes U(1) \otimes U(1)\) subgroup of \(U(3)\).

With the “zero sum” condition \(m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0\), the above matrix can be written as

\[
M = \frac{m_{\nu_1}}{3} \begin{pmatrix} 2 & -1 & -1 \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{pmatrix} + \frac{m_{\nu_2}}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1/2 & 5/2 \\ 1 & 5/2 & -1/2 \end{pmatrix} .
\] (9)

Since \(|\Delta m_{21}^2/\Delta m_{32}^2| \ll 1\), it is instructive to work with the case \(\Delta m_{21}^2 = 0\) as the first approximation and to see how the obtained mass matrix can be perturbed to produce the desired \(\Delta m_{21}^2\). With this approximation the two mass hierarchies I) and II) are then \((m_{\nu_1} = m_{\nu_2} = -m_{\nu_3}/2 = 2a)\), and \((m_{\nu_1} = -m_{\nu_2} = 2a, m_{\nu_3} = 0)\), respectively. The corresponding mass matrices are given by

I) \(M_0 = a \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 3 \\ 0 & 3 & -1 \end{pmatrix} \); II) \(M_0 = \frac{1}{3}a \begin{pmatrix} 2 & -4 & -4 \\ -4 & -1 & -1 \\ -4 & -1 & -1 \end{pmatrix} .\) (10)

Note that the unperturbed mass matrix \(M_0\) looks “simpler” in the “natural” hierarchy I) than in the “inverted” hierarchy II).

The mass matrix in case I) has been studied in a previous paper by us [26]. The desired mass-squared difference \(\Delta m_{21}^2\) can be obtained by a small perturbation of the form

\[
\delta M_T = \varepsilon a \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} ,
\] (11)

with the perturbed eigenvalues given by \(m_{\nu_1} = 2a(1 - \varepsilon/2), m_{\nu_2} = 2a(1 + \varepsilon),\) and \(m_{\nu_3} = -4a(1 + \varepsilon/4)\). The parameter \(\varepsilon\) to the lowest order is given by \(\varepsilon = \Delta m_{21}^2/\Delta m_{32}^2\).
For case II), adding $\delta M_T$ to $M_0$, the eigen-masses are given by $m_{\nu_1} = 2a(1 - \varepsilon/2)$, $m_{\nu_2} = -2a(1 - \varepsilon)$, and $m_{\nu_3} = -a\varepsilon$ with $\varepsilon \approx \Delta m_{21}^2/\Delta m_{32}^2$.

The perturbation $\delta M_T$ preserves the “zero sum” condition in eq. (2). One can also consider situations where the perturbations break this “zero sum” condition, but still produce the mixing matrix $V$ in eq. (6). For example, adding a “democratic” perturbation,

$$\delta M_D = \varepsilon a \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

produces a mixing matrix of the form given by $V$ in eq. (6), but different mass eigenvalues, $(m_{\nu_1} = 2a, m_{\nu_2} = 2a(1 + 3\varepsilon/2), m_{\nu_3} = -4a, \varepsilon = \Delta m_{21}^2/\Delta m_{32}^2)$, and $(m_{\nu_1} = 2a, m_{\nu_2} = -2a(1 - 3\varepsilon/2), m_{\nu_3} = 0, \varepsilon = \Delta m_{21}^2/3\Delta m_{32}^2)$ for case I) and case II), respectively.

We would like to comment that the reconstruction of mass matrix depends crucially on knowing the mixing matrix even if all the eigenvalues for the masses are known through experimental measurements or theoretical considerations such as $m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0$ or the traceless condition for real Majorana neutrino mass matrix proposed here. Had the mixing matrix been given by the bi-maximal mixing matrix\,\cite{25} which was allowed by pre-SNO and KamLAND data, the mass matrix would be very different. We mention that the simplest version of the model proposed in ref.\,\cite{14} with all diagonal entries zero in the mass matrix can easily produce a bi-maximal mixing, but it is difficult to produce the mixing matrix given by $V$ in eq. (6).

V. CONCLUSIONS

We have studied the consequences of neutrino masses with the “zero sum” condition $m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0$. With this condition the neutrino masses can be determined from measured mass-squared differences from oscillation experiments. We find that this condition predicts only two types of neutrino mass hierarchies with one of them characterized by $m_{\nu_3} \approx -2m_{\nu_1} \approx -2m_{\nu_2} \approx 0.063$ eV, and another by $m_{\nu_1} \approx -m_{\nu_2} \approx 0.054$ eV and $m_{\nu_3} \approx 0.0064$.
eV. These masses although small, can be probed by experiments from CMB measurements and large scale structure survey, and can also be probed by neutrinoless double beta decay experiments. In conjunction with information on neutrino mixing, the “zero sum” condition also predicts simple mass matrices for neutrinos.

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Note Added

The Ansatz discussed in this paper has been proposed earlier by D. Black, A. Fariborz, S. Nasri and J. Schechter [33]. Their theoretical motivation and the values of the masses they obtained are however rather different.
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