Vortex Formation and Quantum Turbulence in a Binary Bose-Einstein Condensates

Subrata Das,1,* Koushik Mukherjee,1 and Sonjoy Majumder1,†

1Department of Physics, Indian Institute of Technology Kharagpur, India

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Motivated by the recent observation of a gigantic vortex cluster in a two-dimensional Bose-Einstein condensate [Science 364, 1264 (2019)], we conduct a theoretical study of the creation and dynamics of vortices in a binary Bose-Einstein condensate with a mass imbalance between the species. To initiate the dynamics, we use one or two rotating paddle potentials in one species, while the other species is influenced only via the interspecies interaction. In both species, the number as well as the dominant sign of the vortices are determined by the rotation frequency of the paddle potential. Clusters of positive and negative vortices form at low rotation frequency comparable to that of the trap when using the single paddle potential. In contrast, vortices of the same sign tend to dominate as rotation frequency increases. When the rotation frequency is sufficiently high, the rapid annihilation of vortex-antivortex pairs significantly reduces the number of vortices in the system. For two paddle potentials rotating in the same direction, the vortex dynamics phenomenon is similar to that of a single paddle. However, when the paddle potentials are rotated in the opposite direction, both positive and negative signed vortices occur at all rotational frequencies. At low rotational frequencies, the cluster of like-signed vortices produces the $k^{-5/3}$ and $k^{-3}$ power laws in the incompressible kinetic energy spectrum at low and high wavenumbers, respectively, a hallmark property of the quantum turbulent flows.

I. INTRODUCTION

Vortices [1] render Bose-Einstein condensate (BEC) an excellent platform for examining various scaling aspects of classical turbulence [2–4] and, as a result, revealing their quantum counterpart [5–7]. The renowned Kolmogorov’s $5/3$’ law is one of the most well-known of these scaling laws among them [8, 9]. Several strategies are available to the current state-BEC experiments [10] to generate non-linear defects such as vortices and solitons [11–16]. These include laser stirring [17, 18], rotating the confining potential [19–21], interaction with the optical vortex [22–25], quenching through the phase transition [26], and counter-flow dynamics [27–30], just to name a few [31–34]. Theoretically, numerous intriguing aspects of three-dimensional [8, 35–38] and two-dimensional (2D) quantum turbulence (QT) [39–49] have been examined. Moreover, very recently developed machine learning techniques can be utilized to detect and classify quantum vortices [50, 51]. The incredible tunability of atom-atom interaction via Feshbach resonance [52, 53], as well as the outstanding maneuverability of dimension [54], have also resulted in the significant development of the QT experiment in BEC. In that regard, Ref. [39] shows a turbulence tangle of vortices formed by oscillating potential. Spontaneous clustering of the same circulation vortices has also been demonstrated experimentally [55, 56]. It is worth noting that clustering of vortices [57, 58] implies the transfer of energy from small to large length scales, illustrating the so-called inverse energy cascade [36, 59], a well-known phenomenon that occurs in classical 2D turbulence [60, 61]. The experiment in [55], for instance, employs a paddle that swifts around the edge of the BEC, causing randomly distributed vortices that fast assemble into Onsager point vortex clusters, a notion that has also been theoretically studied by White et al. [62].

Given that optical paddle potential is a dependable way to create 2D QT, we attempted to conduct a detailed examination of the production of vortex complexes by rotating paddle potential in this work. Furthermore, we use a more complicated system with 2D binary BECs [63–65], where only one species is exposed to the rotating paddle. The system of 2D binary BECs, which exhibits a variety of instability phenomena [66–70] and non-linear structures [71–74] is intriguing on its own right. Using the so-called tune-out technique [75], the previously mentioned species selective interaction resulting in the formation of optical paddle potential can be experimentally performed. In this tune-out method, when one species interacts with the laser light, the other becomes just an observer. Furthermore, we investigate a wide range of rotating frequency of the paddle potential, allowing us to pinpoint the domain in which clustering of the same circulation vortices arises, exhibiting the well-known scaling rule of 2D QT [48]. We also look at a region that is dominated solely by identical signed multiple vortices arranged in a lattice form. Furthermore, when the paddle rotates more vigorously, the vortex content of the system drops due to the generation of a high amount of sound waves [40, 76–78]. When there is finite interspecies contact interaction, vortex formation can occur even in the second component. Most importantly, the vortex in one component is connected by a complementary structure, referred to as a vortex-bright soliton [79], in the other. Furthermore, we demonstrate the effect of double paddle potentials, which can rotate either in the same or opposite directions.

This article is arranged as follows. Sec. II describes
our setup and delves over the GP equations. In Sec. III, we investigate the non-equilibrium dynamics of a binary system consisting of a mass-imbalanced system using both single (Sec. III A) and double paddle potential (Sec. III B). Section IV examines the incompressible and compressible kinetic energy spectra. Finally, we summarize our findings and discuss potential future extensions in Sec. V. Appendix A gives a brief description of the creation of vortex and its dynamics in a binary BEC with equal mass.

II. GROSS-PITAEVSKII EQUATION

We consider a binary BECs, referred to as species A and B, that are confined in 2D harmonic trapping potentials [80]. The species consists of $N_i$ number of atoms of mass $m_i$ ($i = A, B$). The form of the trapping potentials read $V_{\text{trap}} = \frac{1}{2}m_i(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)$, where $\omega_x, \omega_y$ and $\omega_z$ are trapping frequencies along $x, y$ and $z$, respectively. To implement a quasi-2D BEC in the $x$-$y$ plane, we consider the following criterion for the trap frequencies, namely, $\omega_x = \omega_y = \omega \ll \omega_z$. We apply single or double stirring potential $V_p$ generated by far-off-resonance blue-detuned laser beam shaped into an elliptic paddle in species A to induce vortices in the condensate [55]. The potential $V_{p,\sigma}$, with $\sigma \in \{1, 2\}$ can be expressed as [62]

$$V_{p,\sigma} = V_0 \exp \left[-\frac{\eta^2(\tilde{x}_\sigma \cos(\omega_p t) - \tilde{y}_\sigma \sin(\omega_p t))^2}{d^2} \right] - \frac{(\tilde{y}_\sigma \cos(\omega_p t) + \tilde{x}_\sigma \sin(\omega_p t))^2}{d^2},$$

(1)

where $\tilde{x}_\sigma = x - x_{p,\sigma}$ and $\tilde{y}_\sigma = y - y_{p,\sigma}$, considering the center of the paddle potential at $(x_{p,\sigma}, y_{p,\sigma})$ for the $\sigma$ paddle. Here $V_0$ is the peak strength of the potential, $\omega_p$ is rotation frequency of the paddle, and $\eta$ and $d$ determines the paddle elongation and width, respectively.

The dimensionless time-dependent Gross-Pitaevskii (GP) equation that governs the dynamics of a BEC is given by [81, 82]

$$i \frac{\partial \psi_i}{\partial t} = \left[ -\frac{1}{2} \left( \frac{m_A}{m_i} \right) \nabla^2 + \frac{1}{2} \left( \frac{m_i}{m_A} \right) (x^2 + y^2) \right] \psi_i + V_{p_1} + V_{p_2} + \sum_{j=A,B} g_{ij} N_j |\psi_j|^2 \psi_i,$$

(2)

where $i = A,B$. Here, the non-linear interaction coefficient is determined by the term $g_{ij} = 2\pi a_{ij} \sqrt{\lambda_{m_A}} / (\sqrt{2\pi} m_i)$ with $a_{ij}$ being the scattering length, $l = \sqrt{\hbar/(m_A \omega)}$ is the oscillator length, $m_j = m_im_j / (m_i + m_j)$ denotes the reduced mass, and the aspect ratio $\lambda = \omega_z/\omega$ is the ratio of the axial to the radial trapping frequency. To make Eq. (2) dimensionless length scale $l$, time scale $1/\omega$ and energy scale $\hbar\omega$

are used. The $i$-th species wavefunction is normalized to $\int |\psi_i|^2 d^2r = 1$.

In this paper, we explore the turbulent phenomena that arise from the potentials formed by the rotating single paddle, $V_{p_1}$ and the double paddles, $V_{p_1} + V_{p_2}$. The paddle potentials are maintained in the condensate for the time $0 < t < \tau$. Afterward, the paddle is linearly ramped off to zero over the time $t = \Delta \tau$, during which the relation,

$$V_{p_{1(2)}} \rightarrow V_{p_{1(2)}} \left(1 - \frac{t - \tau}{\Delta \tau}\right),$$

holds in the Eq. (1). Here we consider a binary BEC of $^{133}$Cs and $^{87}$Rb elements having different masses [65]. The number of atoms in both species A and B are equal, and we take $N_A = N_B = 60000$. The harmonic trap potential is designed to have a frequency of $\omega = 2\pi \times 30.832$ rad/s and the aspect ratio $\lambda = 100$. The intra-species scattering lengths are $a_{AA} = 280 a_0$ and $a_{BB} = 100 a_0$, where $a_0$ is the Bohr radius [65]. The interspecies scattering length $a_{AB}$ is chosen to reside in the miscible regime, as the following relation of the miscibility i.e. $a_{AB}^2 \leq a_{AA}a_{BB}$ [83], is hold obeyed by the scattering lengths. We numerically solve the GP equation using the Split-step Crank-Nicolson method [84].

The ground state of the system is generated by propagating the wavefunctions of the BEC in imaginary time. In order to inspect the dynamical evolution of the condensate, we utilize the ground state generated in imaginary time as the initial state and solve the Eq. (2) in real-time. Moreover, the system’s initial state is prepared by placing a paddle-shaped stationary obstacle, as expressed in Eq. (1), in the component $A$. Our simulation runs on the spatial extent of $-20.48l$ to $20.44l$ along both $x$ and $y$ direction with $1024 \times 1024$ grid points.

III. CREATION OF VORTICES USING PADDLE POTENTIAL

As discussed in Ref. [62], using an optical paddle potential vortex in BEC can be generated in a variety of ways which include (i) rotating the paddle about a fixed center, (ii) moving the paddle about a fixed center, and (iii) both rotating and moving paddle simultaneously in the BEC. Though we have considered only the rotation of paddle potential to generate vortices in this work, we have employed both a single paddle and a double paddle potential to generate a vortex. In particular, while the single paddle potential rotates in species A with the paddle center being located at $(x_p, y_p) = (0, 0)$, the double paddle potentials can rotate either in the same or opposite directions about their center at $(x_p, y_p) = (\pm r_{GA} / 4, 0)$, respectively, where $r_A = 6.1l$ for $s_{AB} = 0$ is the root-mean-squared radius of species A. The parameters for single paddle are $\eta = 0.05$ and $d = 0.1l$; and for double paddle $\eta = 0.1$ and $d = 0.1l$, are identical for both.
These values determine the elliptical shape of the paddle according to Eq. (1). Moreover, we choose the peak strength of the paddles to be $V_0 = 10\mu_A$, where $\mu_A$ is the chemical potential of species A. As previously stated, after establishing the initial state with a stationary paddle, at $t = 0$, the paddle is rotated at a frequency of $\omega_p$ until the time $\tau = 40\omega^{-1} = 206\text{ms}$. With these parameters $\omega_p, \eta, d$, and $V_0$, the paddle potentials may be externally regulated, allowing for control of the creation of vortex or antivortex in BEC. In BEC, the presence of a vortex or an antivortex yields a finite amount of angular momentum which can be expressed as,

$$L_z = -i \int \psi^*(x \frac{\partial \psi}{\partial y} - y \frac{\partial \psi}{\partial x}) \, dx \, dy.$$  \hfill (3)

To study the dynamical formation of the vortices, we measure the vorticity of condensates as $[30, 85]$

$$\Omega = \nabla \times J,$$  \hfill (4)

with $J = \frac{i}{\mu m}(\psi^* \nabla \psi - \psi \nabla \psi^*)$ being the probability current density.

A. Single paddle

This section examines the implications of a single paddle potential rotating with frequency $\omega_p$, about the center of the species A. Although rotation orientation can be clockwise (CW) or counter-clockwise (CCW), we focus on a paddle rotating in the CW direction. We note that the results obtained for the CCW will be existentially identical to those obtained for the CW.

At first, we demonstrate the behavior of the BEC without interspecies interaction by setting $a_{AB} = 0$. Due to the absence of interspecies interactions, the paddle potential does not influence species B, and therefore the latter remains unaltered during the dynamics. When the paddle rotates in species A, vortices and antivortices form around it. The number of vortices and antivortices, in particular, is strongly dependent on the rotation frequency. Figures 1(a1)-(a2) and (b1)-(b2) shows time evolution of density and vorticity of species A at the paddle frequency $\omega_p = \omega$, the trap frequency.

The initial state of species A, with the paddle potential being elongated along the $x$-axis, is shown in the Fig. 1(a1). At $t = 0.1s$, after the rotation of the paddle has been established [Fig. 1(b1)], both vortices (red color) and antivortices (blue color) are generated in species A. In fact, a close inspection of the Fig. 1(b2) reveals that the vortex-antivortex structures are located symmetrically with respect to the paddle. Additionally, the number of vortices exceeds that of the antivortices [Fig. 1(b2)].

![FIG. 1. Snapshot of (a1)-(c1) density ($n_A$) and (a2)-(c2) vorticity ($\Omega_A$) profiles of the species A at different instants of time (see legends). The binary BECs are made of $^{133}\text{Cs}^{87}\text{Rb}$ atoms. An elliptical paddle potential characterized by the parameters $\eta = 0.05$ and $d = 0.1l$ is rotated with the angular frequency $\omega_p = \omega$ within the species A ($^{133}\text{Cs}$) in order to trigger the dynamics. The colorbar of top row represents the number of atoms. The binary BECs are initialized in a two dimensional harmonic potential with frequencies $(\omega, \omega_z)/(2\pi) = (30.832, 30.832)\text{ Hz}$ and having following intra- and interspecies scattering lengths $a_{AA} = 280a_0$ and $a_{BB} = 100.4a_0$, and $a_{AB} = 0$. The number of atoms for both the species are $N_A = N_B = 60000.$](image)
The generation of vortices and antivortices continues until 0.206s, at which point the paddle potential vanishes. It is worth noting that the numbers of vortices and antivortices are nearly equal around this time. Following that, a considerable number of the vortex-antivortex pairs decay due to self-annihilation or drifting out of the condensate, see Fig. 1(b). However, some vortex and antivortex persist in the condensate by forming vortex dipoles (vortex antivortex pair) or vortex pairs (pairs of identical charges), as depicted in Fig. 1(b). Without being annihilated, these vortex dipoles, vortex pairs, and vortex/antivortex structures remain in the BEC for an extended period.

When the paddle frequency \( \omega_p \) increases, the vortex complexes exhibit a distinct behavior. The number of antivortices vastly exceeds that of vortices for a CW rotation of the paddle potential with frequencies \( \omega < \omega_p \). After removing the paddle potential, vortex-antivortex annihilation begins, finally eliminating all vortices from the condensate. As a result, only a sub-stantial number of vortex-antivortex persist in the condensate by forming vortex dipoles (vortex antivortex pair) or vortex pairs (pairs of identical charges), as depicted in Fig. 1(b). Without being annihilated, these vortex dipoles, vortex pairs, and vortex/antivortex structures remain in the BEC for an extended period.

Fig. 2(b), which depicts the \( \Omega_A \) for \( \omega_p = 5\omega \). As the \( \omega_p \) increases, although a large, nearly equal number of vortex or antivortex is created, only a few survive due to a higher annihilation rate. As a result, the system has almost no vortex or antivortex structure in the long-time dynamics (density profiles not shown here for brevity).

The above scenario of non-linear structures formation in species A can further be elucidated by invoking the angular momentum of species A, \( L^A_z \). The time evolution of \( L^A_z(t) \) for various \( \omega_p \) is displayed in Fig. 2(c). The \( L^A_z(t) \) remains negative throughout the time evolution indicating the surplus generation of antivortices. For \( \omega_p = \omega \), \( L^A_z(t) \) remains nearly constant within the ballpark. The magnitude of \( |L^A_z(t)| \) monotonically increases at the early stage of the dynamics to reach a maximum at a time within the time interval \( \tau \), and then show a decreasing tendency to reach a stationary value in the long time dynamics. The maximum value of \( |L^A_z(t)| \) is the largest for \( \omega_p = 3\omega \), a result which emanates from the maximum number of antivor-
tices displayed in Fig. 2(a). As \( \omega_p \) is further increased, the net angular momentum imparted to the condensate by the generated vortex-antivortex structure drastically diminishes, indicating the reduction of the number of vortices and antivortices due to the annihilation [2(b)].

The existence of paddle potential in species A affects species B for non-zero interspecies interactions \( a_{AB} \). In particular, vortices and antivortices are created in species B, and their number can be controlled by \( a_{AB} \). Additionally, the null density region at the vortex or antivortex site in one species is filled by the other species’ localized density hump. Figure 3(a)-(b) show the density and vortexicity pattern at \( t = 3.5s \) for the interspecies interaction \( a_{AB} = 80a_0 \) and \( \omega_p = 3\omega \). Other parameters such as \( (\eta = 0.05, d = 0.1l \) and \( V_0 = 10\mu_B \)) are similar to the \( a_{AB} = 0 \) case. Notably, the scattering lengths explored here ensure that the condensates are miscible, allowing us to directly analyze the role of mean-field coupling. Moreover, the paddle potential in species A performs CW rotation. Both species accommodate only antivortices solely in the long-time dynamics, which are extremely similar to those of the non-interacting scenario. This behavior implies that within a particular frequency range, a cluster of identical vortices forms being entirely determined by the direction of paddle rotation, regardless of the species interaction. Furthermore, it is worth mentioning that species A possesses a smaller healing length due to the larger mass and the intraspecies interaction. This makes the vortices of species A smaller in size compared to those in species B. The creation and stability of vortex complexes in the presence of non-zero interspecies interaction can be further comprehended by evaluating the angular momentum \( L_z^i \) of both species. The time evolution of \( L_z^A \) and \( L_z^B \) as a function of \( \omega_p \) are shown in Fig. 3(c) and Fig. 3(f), respectively. A close inspection indicates that the angular momenta of both species are maximum at \( \omega_p = 3\omega \), similar to that in the \( a_{AB} = 0 \) case. For \( \omega_p > 5\omega \), \( L_z^A \) becomes almost negligible due to the higher annihilation rate of the vortex-antivortex pairs. Moreover, the \( L_z^A \) is more pronounced than \( L_z^B \), indicating that the antivortex number is always high in species A.

### B. Double Paddle

After discussing the impact of a single paddle potential, we will look at a more complex scenario involving two paddle potentials. To that intent, we consider two paddles rotating in species A and having a center at \((\pm r_A/4, 0)\). Moreover, we choose \( \eta = 0.1 \) and keep \( d \) same as the single paddle case. Depending on the relative rotational orientation of the two potentials, different dynamical behavior can emerge. When both paddles rotate in the same direction, the effects are similar to those mentioned previously for a single paddle. To substantiate the above statement, we demonstrate the fluctuation of angular momentum with time (log-scale) in Fig. 4 for CW rotation of the paddle potential with interspecies scattering length \( a_{AB} = 80a_B \). For paddle frequency close to \( \omega_p = 4\omega \), \( L_z^A \) and \( L_z^B \) are most prominent, and the corresponding antivortex structures generated in species A and species B are shown in Fig. 4(b)-(c), respectively.

![Fig. 4](image)

**FIG. 4.** (a) Time evolution (log-scale) of angular momentum \( L_z^i \) for the species \( i = A, B \) at interspecies interaction \( a_{AB} = 80a_0 \) for different paddle frequencies \( \omega_p = \omega, 2\omega, 3\omega, 4\omega \) and \( 5\omega \). Here two identical paddle rotates in species A in CW direction. The inset figures (b) and (c) depict the snapshot of vorticity profiles of species A (\( \Omega_A \)) and species B (\( \Omega_A \)), respectively, at \( t = 3.5ms \) with \( \omega_p = 4\omega \).

Surprisingly, a more interesting case occurs when one paddle rotates in the CW and the other in the CCW way [Fig. 5(a1)-5(b1)]. Because the rotational directions of the paddles are opposite, each paddle contributes an equal number of vortices of the opposite sign, see Figs. 5(a2)-(a3) and Figs. 5(b2)-(b3). In the long-term dynamics of both species, this equal distribution of vortex and antivortex leads to a high rate of annihilation, meaning that just a few vortex-anti vortex pairs survive in the long-time dynamics [Figs. 5(a4)-(a5) and 5(b6)-(b5)].

To further appreciate the previous argument, we calculate the time average of the angular momentum defined as \( \bar{L}_z^i = \int L_z^i dt \) for different rotation frequencies \( \omega_p \) of the binary paddle potentials. For \( \omega_p < \omega \), the \( \bar{L}_z^i \) remains zero. Within \( 2\omega > \omega_p > 5\omega \), \( \bar{L}_z^i \) shows extremely chaotic behavior with respect to the \( \omega_p \). Recall that this is also the frequency region where maximum number of vortex-antivortex creation occurs. The vortex-antivortex either annihilates each other or drifts away from the condensate, and these behaviors are extremely random. This might create a finite imbalance of vortex or antivortex number in the overall dynamics leading to the finite angular momentum of either positive or negative signs. For increasing \( \omega_p \), the annihilation mechanism becomes the dominant mechanism responsible for reducing the vortex number. Consequently, such imbalance of vortex-antivortex number gets reduced, contributing to the lesser fluctuation in the \( \bar{L}_z^i \).
and compressible energy, species A in Fig. 5 and compressible energy, species B in Fig. 6 at different instants of time. Two elliptic paddles characterized by the parameters $n_\text{A}$ and $d = 0.1f$ and rotating opposite to each other with frequency $\omega_p = 5\omega$ within species A (133Cs) are used to trigger the dynamics. The colorbars represent the number of atoms. (c) shows the variation of time-averaged angular momentum $L_a^i$ and $L_B^i$ as a function of $\omega_p$. All other parameters for the interactions (except $a_{AB}$) and the trapping potential are same as Fig. 1.

IV. ENERGY SPECTRA

To gain a better understanding of the system when it is subjected to a paddle potential, we compute the system’s kinetic-energy spectrum. In order to do so we decompose the kinetic energy into a compressible and incompressible parts [8]. The energy decomposition is performed by defining a density weighted velocity field, which reads $\sqrt{n_i}u_i$, with $u_i = \frac{1}{2m} \nabla \theta_i$, where $n_i$ and $\theta_i$ are the position dependent condensate density and phase of the $i$-th species. The velocity field is separated into a solenoidal (incompressible) part $u_i^c$ and an irrotational (compressible) part $u_i^c$ such that $u_i = u_i^c + u_i^c$ and obeying $\nabla \cdot u_i^c = 0$ and $\nabla \times u_i^c = 0$. Once these velocity field are calculated following the Refs. [8, 30, 40, 85], we can calculate incompressible energy,

$$E_i^c = \frac{1}{2} \int n_i |u_i^c|^2 \, dx \, dy$$  \hspace{1cm} (5)

and compressible energy,

$$E_i^c = \frac{1}{2} \int n_i |u_i^c|^2 \, dx \, dy.$$ \hspace{1cm} (6)

Afterwards the compressible and incompressible energy spectra for the $i$-th species can be calculated as

$$E_i^{c|c}(k) = \frac{k}{2} \sum_{q=x,y} \int 2\pi |F_q(\sqrt{n_i}u_{c,i}^{c|c})|^2 \, d\phi,$$  \hspace{1cm} (7)

where $F_q(\sqrt{n_i}u_{c,i}^{c|c})$ denotes the Fourier transformation of $\sqrt{n_i}u_{c,i}^{c|c}$, corresponding to the $q$-th component of $u_i = (u_{x,i}, u_{y,i})$.

We present incompressible energy spectra $E_A^c(k)$ of species A in Fig. 6 at various time instants and frequencies $\omega_p$. Since the interspecies interaction is zero here, species B is not impacted by the paddle potential, which allows us to focus on species A. For $\omega_p = \omega$, $E_A^c(k)$ attains a stationary state at early time ($t = 0.1s$) and maintains it till $t = 3.5s$, as evidenced from the Fig. 6(a). Moreover, $E_A^c(k)$ exhibits a very pronounced $k^{-3}$ in the region $10 \lesssim k \lesssim 60$ and $k^{-5/3}$ power law in the region $1 \lesssim k \lesssim 10$. Note that for $\omega_p = \omega$ vortex pairs and vortex dipole are noticed in Fig. 1(e1) [44, 55, 78, 86]. Surprisingly for the frequency $\omega = 3\omega_p$, where only the same sign vortices dominate, we notice that $k^{-3}$ spectrum fails to develop in our system, see Fig. 6(b). However, while $k^{-5/3}$ spectrum develops in long time dynamics, it does not emerge in early time dynamics [Fig. 6(b)]. Finally, we notice that $k^{-5/3}$ and $k^{-3}$ power laws are totally absent from the spectra for $\omega_p = 5\omega$ [Fig. 6(c)]. This behaviour is expected since the system at $\omega = 5\omega_p$ is primarily governed by the generation of a huge number of sound waves caused by the rapid annihilation of the vortices and antivortices.

Next, we turn to the scenario of finite interspecies interaction characterized by $a_s = 80a_B$ and investigate whether species B produces power law spectrum in the incompressible sector of its energy, see Figs. 7(a)-(c). We note that $k^{-5/3}$ and $k^{-3}$ power laws are manifested in a similar manner within the range $1 \lesssim k \lesssim 10$ and $10 \lesssim k \lesssim 70$, respectively, for $\omega_p = \omega$. Again, $k^{-3}$ power law disappears for $\omega_p = 3\omega$, but $k^{-5/3}$ appears in the region $2 \lesssim k \lesssim 30$. We note that in this condition the $E_A^c(k)$ does not demonstrate different behaviour with regard to $\omega_p$ when compared to that of $a_{AB} = 0$ case (hence not shown here for brevity).

We now explain the compressible kinetic energy spectra $E_A^c(k)$ of species A for a few representative cases, shown in Fig. 8. To begin, in the case of $\omega_p = \omega$, we notice that a power-law region with $E_A^c(k) \propto k$ develops in the low-$k$ region of the spectrum, a relation

![Figure 5](image-url)
that expresses the frequencies of Bogoliubov’s elementary excitations at low-wave number [Fig. 7(a)]. The spectrum reaches a maximum near $k$ ranges from 20 to 40 (the peak positions differ for different time instants until the system reaches an equilibrium) and then exhibits a power-law exponent of $-7/2$ within a very small region before rapidly dropping. Similar behavior of the compressible energy spectrum has also been reported in Ref. [48]. For larger paddle frequency such as $\omega_p = 5\omega$ or $\omega_p = 10\omega$, the relations $E_A^\omega(k) \propto k$ and $E_A^\omega(k) \propto k^{-7/2}$ [48] become more prominent and appear over a broader region of $k$-space, see Fig. 8(b)-(c). Additionally, while the maximum value of $E_A^\omega(k)$ does change significantly as one goes from $\omega_p = 5\omega$ to $\omega_p = 10\omega$ [Figs. 8(b)-(c)], it is one order of magnitude larger when compared to that of the $\omega_p = \omega$ [Fig. 8(a)].

V. CONCLUSIONS

We have explored the phenomenon of non-linear structures formations and their dynamics optical paddle potential in a binary BEC composed of two distinct atomic elements. One of the species experiences (species A) rotating single or double paddle potentials, while the other species (species B) is only influenced via the interspecies contact interaction. The paddles are rotated for a finite amount of time, resulting in the creation of vortices. In the long-time dynamics, the sign and number of the vortex are dependent on the frequency and orientation of paddle rotation. Additionally, we discussed the effect of paddle rotation on other species. We observed many diagnostics to obtain insight into the dynamics, including density, vorticity, the $z$-component of the angular momentum, and the species’ compressible and incompressible energy spectra.

Clusters of positive and negative vortices emerge within the system when a single paddle potential is rotated with a low rotational frequency. Interestingly, when the frequency is gradually increased, we observe a transition to a regime dominated by same-sign vortices, with the species A gaining the maximum angular momentum. At larger paddle frequencies, the annihilation of vortex-antivortex pairs becomes considerable, reducing the sys-
There are many research directions to be pursued as a future research endeavor. One straight would be to extend present work in the presence of finite temperature [89]. Extending the present work to the three-dimensional setup and exploring the corresponding nonlinear defect formations would be equally interesting [38, 90, 91]. Another vital prospect would be to employ dipolar BEC to inspect the impact of the long-range interaction [92]. Finally, the investigations discussed previously would be equally fascinating at the beyond mean-field level, where significant correlations between particles exist [93].

VI. ACKNOWLEDGMENT

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Appendix A: Mass-Balanced binary Bose-Einstein condensate

In the main text, we have focused our discussion on the mass-imbalanced binary BECs, since such a system is the most suitable for creating species selective potential by the tune-out approach. To examine to what extent phenomenology differs from a system of mass-balanced system, here we consider a binary BEC composed of $^87$Rb atoms with two different hyperfine levels [71, 94]. We take an equal number of atoms in both species, namely, $N_A = N_B = 60000$. The intra-species scattering lengths are $a_{AA} = 95.4a_0$ and $a_{BB} = 100.4$ [95]. All other parameters, like trapping configuration and paddle configuration, are the same as the single paddle case of Sec. III.

We examine the creation of vortices using a single rotating paddle with $a_{AB} = 0$ and $80a_0$. For paddle frequency $\omega_p = \omega$ we observe clustering of opposite sign vortices at $a_{AB} = 0$, see Fig. A.1(a). However, at higher interspecies scattering length $a_{AB} = 80a_0$, the clustering is not visible [Fig. A.1(b)], instead we observe a sparse cluster composed of same-signed vortices. The number of vortices organized into lattice structure increases as we increase the paddle frequency ($2\omega \lesssim \omega_p \lesssim 5\omega$) in species A due to direct impact of paddle rotation. And as a effect of interspecies interaction vortex lattice is also formed in species B [Fig. A.1(c)-(d)]. Since the interaction between two species in $^87$Rb-$^{85}$Rb are very similar, we find that the lattice-like structure that appeared here is more or-

![Diagram](image-url)

**FIG. 8.** Compressible kinetic energy spectra of species A,$E^c_A$, with $a_{AB} = 80a_0$ at different time instants (see legends) for different paddle frequencies $\omega_p = (a) \omega$, (b) $5\omega$ and (c) $10\omega$. Black ‘solid’ and ‘dashed’ lines represent the slopes of $k^{-7/2}$ and $k$, respectively.

system’s total vortical content. The behavior mentioned above holds for species A both in the absence or presence of interspecies interaction. Interestingly enough, when the interspecies contact is enabled, species B exhibits similar dynamical behavior. However, species B has a substantially lower vortex and angular momentum than species A. When two paddle potentials are employed, their relative orientation becomes crucial in determining the vortical content of species A. For the rotation of the paddles with the same orientation, the behavior is almost identical to single paddle applied to species A. However, when the two paddles rotate opposite to each other, due to the almost equal number of vortex-antivortex structures formed regardless of the rotation frequency of the paddles, the net angular momentum imparted to the system during long-time dynamics remains close to zero.

Following that, we explored the system’s dynamics by invoking the compressible and incompressible kinetic energy spectra. However, a key highlight of this work is its examination of various power-law scalings of the kinetic energy spectra. We observed $-5/3$ and $-3$ power-law scaling in the low and high wavenumber regimes of the incompressible energy spectrum, respectively, in the low rotation frequency regime, where we saw clusters of identical sign vortices. These scalings provide evidence for the development of quantum turbulence in our system at low frequencies. However, analogous scaling is not apparent in the incompressible energy spectrum as the rotation frequency increases.

When the two rotations are opposite to each other, due to the almost equal number of vortex-antivortex structures formed regardless of the rotation frequency of the paddles, the net angular momentum imparted to the system during long-time dynamics remains close to zero. For paddle frequencies $\omega_p = \omega$ we observe clustering of opposite sign vortices at $a_{AB} = 0$, see Fig. A.1(a). However, at higher interspecies scattering length $a_{AB} = 80a_0$, the clustering is not visible [Fig. A.1(b)], instead we observe a sparse cluster composed of same-signed vortices. The number of vortices organized into lattice structure increases as we increase the paddle frequency ($2\omega \lesssim \omega_p \lesssim 5\omega$) in species A due to direct impact of paddle rotation. And as a effect of interspecies interaction vortex lattice is also formed in species B [Fig. A.1(c)-(d)]. Since the interaction between two species in $^87$Rb-$^{85}$Rb are very similar, we find that the lattice-like structure that appeared here is more or-
FIG. A.1. (a)-(b) Vorticity and (c)-(d) density profiles of species A [(a), (c)] and species B [(b), (d)] at different paddle frequencies, $\omega_p$, and different scattering lengths, $a_{AB}$, (see the legends) at $t = 3.5\text{ms}$. The binary BECs is realized at the hyperfine levels of $^{87}\text{Rb}$ atoms. To trigger the dynamics, an optical paddle potential is rotated in species A.

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