Fast compensation of DC bus voltage drops using modular multilevel converters

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Abstract: For a DC grid using modular multi-level converters (MMCs) fast control of the DC bus is an important requirement. A control design is presented where a sudden DC voltage drop is compensated without any influence on the AC side. Using a suitable pattern in the MMC internal circular currents, the DC current can be adjusted and the MMC can be brought to a new steady state with symmetrised arm energies within few milliseconds.

1 Objectives and context
We consider one station of a DC grid, i.e. a system consisting of a modular multi-level converter (MMC) [1] which links a long DC transmission line with a long three-phase AC transmission line. Due to the topology of the MMC with its six arms of many identical sub-modules, each one equipped with its own operation point such that the fully symmetrised capacitance state is guaranteed at the end of each AC period [3–6]. Faster control techniques achieving the same goal in a shorter time, nevertheless, are of importance. Hence, the aim of this paper is to design a new control approach for the two dynamics (a) and (b) which, after a sudden voltage drop on the DC side, restores the power flow and manages the capacitance symmetrisation below one AC cycle. This control algorithm is being executed at the MMC itself where the information of all external voltages, currents, and capacitance voltages is known at each time step. In order to compensate large DC voltage drops, the MMC is built with full-bridge sub-modules [7].

Initially, the system operates at a steady state (ss1) with a perfect flow of active power from the DC to the AC side. The latter operates at a frequency \(f_{AC}\), a voltage amplitude \(u_{AC}\), a current amplitude \(i_{AC}\), and an active power \(P_{AC} = P_{DC}\). The initial DC voltage level is \(u_{DC}^{0}\), and the DC current level is determined according to the AC power demand. At time \(t_0\), the DC voltage suddenly drops by a large percentage, here assumed to be equal to 25%. This voltage drop is to be compensated by the MMC such that the AC side stays in its initial steady state and does not even notice the drop on the DC side. Thus, for a transition time \(T_{c}\), the MMC has to supply the missing active power from its internal capacitances. During this time \(T_{c}\), the MMC has to manage a smooth transition to a new steady state (ss2) where the lowered DC voltage is compensated by an increased DC current in order to meet the AC power demand. Since our study is concentrated on the capabilities of the MMC and its control, we do not specify the origin of the DC voltage drop in detail.

2 Equations of motion

The MMC consists of six internal arms (or valves) labelled by \(j = 1, \ldots, 6 \equiv p1, p2, p3, n1, n2, n3\), each containing \(N_{sb}\) full-bridge sub-modules with a sub-module capacitance \(C\) (Fig. 1). The voltage \(u_{j}\) provided by all the sub-modules within the \(j\)th arm is equal to \(u_{j} = \sum_{k=1}^{N_{sb}} s_{jk} n_{k} u_{k, jp}\), where \(n_{k}\) denotes the switching state \((-1, 0, +1)\) of the \(k\)th sub-module and \(u_{k, jp}\) is the voltage of the corresponding sub-module capacitance. The equation of motion for each of these capacitance voltages reads \(\frac{d}{dt} u_{k, jp} = \frac{1}{C} s_{jk} n_{k} i_{j}\). Let us assume that an underlying and fast sorting control [2] ensures that all the sub-modules within each arm have nearly the same voltage: \(u_{k, jp} \approx \frac{1}{N_{sb}} u_{AC, j}\) for \(k = 1, \ldots, N_{sb}\), where \(u_{AC, j}\) (without index \(l\)) is the sum of all capacitance voltages in the \(j\)th arm. In such a case, the total electrostatic energy stored in the arm capacitances

\[ W_{AC, j} = \frac{C}{2} \sum_{k=1}^{N_{sb}} n_{k} u_{k, jp}^{2} = \frac{C}{2N_{sb}} u_{AC, j}^{2} \]

has as an equation of motion

\[ \frac{d}{dt} W_{AC, j} = i_{j} \sum_{k=1}^{N_{sb}} n_{k} s_{jk} i_{j} = i_{j} \sum_{k=1}^{N_{sb}} n_{k} s_{jk} i_{j} = i_{j} i_{j} \]  \hspace{1cm} (1)

Hence, the arm voltage \(u_{j}\) controls the dynamics of the arm capacitance energy \(W_{AC, j}\). Furthermore, the voltages \(u_{j}\) also determine the dynamics of the arm currents \(i_{j}\). The loop equations from (left) DC earth to (right) AC earth in Fig. 1 along the upper three arms and along the lower three arms yield six coupled equations, describing the dynamics of the arm currents \(i_{j}\) controlled by the arm voltages \(u_{j}\). By expressing the arm currents in the sense of the Clarke transformation as linear combinations of the DC current \(i_{DC} = i_{p1} + i_{p2} + i_{p3} = i_{n1} + i_{n2} + i_{n3}\), the internal circular currents \(i_{AC, \theta_{AC}}\), and the AC current components \(i_{AC, \theta_{AC}}\) defined as follows:

\[ i_{AC, \theta_{AC}} = i_{AC, \theta_{AC}} + i_{AC, \theta_{AC}} + i_{AC, \theta_{AC}} \]
and using the same linear transformation for the arm voltages

\[
\begin{pmatrix}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & -\frac{1}{3}
\end{pmatrix}
\begin{pmatrix}
i_{e,\alpha} \\
i_{e,\beta} \\
i_{AC,\alpha}
\end{pmatrix} =
\begin{pmatrix}
i_{p,1} \\
i_{p,2} \\
i_{p,3}
\end{pmatrix}
\]

(2)

and

\[
\begin{pmatrix}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & -\frac{1}{3}
\end{pmatrix}
\begin{pmatrix}
u_{\alpha,\beta} \\
u_{\alpha,\alpha} \\
u_{\alpha,\beta}
\end{pmatrix} =
\begin{pmatrix}
u_{p,1} \\
u_{p,2} \\
u_{p,3}
\end{pmatrix}
\]

(3)

The six coupled current dynamics equations become five decoupled equations of motion [3]:

\[
\frac{d}{dt}i_{e,\alpha} = \frac{3R_{\alpha} + R_{e}}{L_{\alpha} + L_{e}}i_{e,\alpha} + \left(-\frac{1}{L_{\alpha} + L_{e}}\right)v_{\alpha,\alpha} - \frac{u_{\alpha,\alpha}}{2}.
\]

\[
\frac{d}{dt}i_{e,\beta} = -\frac{R_{e}}{L_{\alpha} + L_{e}}i_{e,\beta} - \frac{1}{L_{\alpha} + L_{e}}v_{\alpha,\beta}.
\]

(4)

The small resistance \(R_{e}\) describes the losses at the switches inside the arm sub-modules. Since the AC side transmission lines are star connected, \(i_{AC, \alpha} = 0\) always holds, resulting in the algebraic constraint \(u_{\alpha, \alpha} = -2u_{\alpha, \alpha} \), instead of a sixth dynamic equation. For a symmetric AC voltage system, \(u_{\alpha, \alpha} = -2u_{\alpha, \alpha} = 0\).

The dynamic equations (1) and (4) show that the voltages \(u_{j}\) or their linear combinations \((u_{\alpha, \alpha}, u_{\alpha, \alpha})\) can be interpreted as the (effective) input for controlling the capacitor’s energy dynamics as well as the current dynamics. This input \(u_{j}\) is subsequently transformed into a switching vector \(s_{j}\) by means of a fast modulation algorithm that selects at a very fast time scale as the most appropriate sub-modules for contributing to \(u_{j}\). The time scale at which \(u_{j}\) is changed for controlling the arm energies and currents is much slower than that of the underlying switching control. In this work, a control frequency of 5 kHz (time step of \(\Delta t = 0.2\) ms) is assumed for designing the appropriate \(u_{j}\) values, whereas the low-level modulation and sorting control are working at a frequency nearly one order of magnitude higher.

3 Steady state

For a given voltage and current at the AC side \(u_{\alpha, \alpha} = u_{\alpha, \alpha}\) and \(i_{\alpha, \alpha}\) (with the corresponding power factor \(\cos \varphi_{\alpha, \alpha}\)), as well as for a given voltage \(u_{\alpha, \alpha}\) at the DC side, the steady state (ss) of the system is determined by two conditions which have to be satisfied at each time step:

\[
W_{\alpha, \alpha} = -\frac{d}{dt}\left[\frac{L_{\alpha}}{4}(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}) + \frac{3L_{\alpha}}{2} \dot{E}_{\alpha, \alpha}\right] + \frac{2L_{\alpha} + L_{e}}{16} \left(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}\right)
\]

\[
= -\frac{R_{\alpha}}{2}(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}) + (R_{D\alpha} + R_{\alpha})\dot{E}_{\alpha, \alpha}
\]

\[
+ \frac{2R_{\alpha} + R_{e}}{8} \left(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}\right)
\]

(5)

\[
W_{\Delta, \alpha} = -\frac{d}{dt}L_{\alpha} \left(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}\right) - \frac{2L_{\alpha} + L_{e}}{2} \left(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}\right) - \left(R_{\alpha} + R_{\beta}\right) \dot{E}_{\alpha, \alpha}
\]

\[
+ L_{\alpha} \left(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}\right)
\]

(6)

Equation (5) is easily interpreted: the rate of change in the energies stored in all capacitors and in all inductances is equal to the power injected from the DC side minus the effective power stored in the MMC.

Fig. 1 Schematic structure of an MMC (unshaded) connected to long DC and AC transmission lines (shaded)

i. No effective power is being stored within the MMC, and the common average of the six arm capacitance energies \((1/6) \sum_{j=1}^{6} W_{\alpha, j}\) is constant during the steady state.

ii. The difference between the upper arm \(j = 1, 2, 3 \equiv p1, p2, p3\) and the lower arm \(j = 4, 5, 6 \equiv n1, n2, n3\) capacitance energies is kept constant, \(\sum_{j=1}^{6} (\dot{E}_{\alpha, j})^{\text{cap} \text{cap}} = 0\), which implies that an initial vertical balancing of the arm energies is maintained.

Both conditions involve the time derivatives of the arm energies \((d/dt)(\dot{W}_{\alpha, j})\) and \((d/dt)(\dot{W}_{\alpha, j})\) using the linear transformation (3), with the arm powers \(W_{\alpha, j}\), replacing the arm voltages \(u_{j}\) in the right column vector. Now, the two conditions defining the steady state read \(W_{\alpha, \alpha} = 0 = W_{\Delta, \alpha}\) where these two power components are given by

\[
W_{\alpha, \alpha} = -\frac{d}{dt}\left[\frac{L_{\alpha}}{4}(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}) + \frac{3L_{\alpha}}{2} \dot{E}_{\alpha, \alpha}\right] + \frac{2L_{\alpha} + L_{e}}{16} \left(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}\right)
\]

\[
= -\frac{R_{\alpha}}{2}(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}) + (R_{D\alpha} + R_{\alpha})\dot{E}_{\alpha, \alpha}
\]

(5)

\[
W_{\Delta, \alpha} = -\frac{d}{dt}L_{\alpha} \left(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}\right) - \frac{2L_{\alpha} + L_{e}}{2} \left(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}\right) - \left(R_{\alpha} + R_{\beta}\right) \dot{E}_{\alpha, \alpha}
\]

\[
+ L_{\alpha} \left(\dot{E}_{\alpha, \alpha} + \dot{E}_{\alpha, \beta}\right)
\]

(6)
transferred to the AC side, reduced by all dissipation losses at the resistances.

For the steady state, condition $W_{20}^{\alpha} = 0$ enforces vanishing circular current components $i_{c, a/b}$. Since these appear in (6) in each term contributing to $W_{20}$. The steady-state condition $W_{20}^{\alpha} = 0$ determines the DC current $i_{DC} = 3i_{c, 0}$ for a given DC voltage, AC voltage, and current (and a vanishing circular current)

$$\varepsilon_0 - \frac{u_{DC}^{(0)}}{2(3R_{DC} + R_{e})}\varepsilon_0 + \frac{1}{2}R_{AC} + R_{e}(\varepsilon_{AC})^2 = \frac{1}{4} \frac{u_{DC}^{(0)}C_{AC}\cos \phi_{AC}}{3R_{DC} + R_{e}} (\varepsilon_{AC})^2$$

where $^\wedge$ denotes the amplitude of the respective AC variables. Once all current components are known for the steady state, the corresponding $W_{0}^{\alpha}$ are derived from the current dynamics (4) and the resulting arm capacitance energies $W_{0}^{\alpha}$ from (1). The initial values for $W_{0}^{\alpha}$ are chosen such that all the six arm energies oscillate around a common value (called from now on ‘centre of mass’ cm).

4 Compensation of a DC voltage drop

The system is originally operating at the initial steady state (ss1). At time $t = t_0$ the DC voltage suddenly drops from $u_{DC}^{(ss1)}$ down to $u_{DC}^{(ss2)} \equiv 0.75u_{DC}^{(ss1)}$, staying at that voltage afterwards. It is assumed here that such drop is immediately detected. After the voltage drop, the DC current must increase to provide the AC side with unchanged real power. Thus, during a transition time of duration $T_s$ the MMC has to supply the missing active power from its internal capacitances as well as to transfer another part of its stored energy to the DC transmission line in order to shift the operation point to the new steady state defined by $u_{DC}^{(ss2)}$ and $i_{c, 0}^{(ss2)}$. This has to be done without changing the AC side which keeps operating at the initial AC voltage and current. In the new steady state to be reached, the capacitance energies ($C/2N_{arm}u_{DC}^{(ss2)}$) at each arm $j$ have new values after $t_0 + T_s$, and thus the task to be solved is to design a smooth shifting from (ss1) to (ss2) within $T_s$ (<10 ms), satisfying two conditions (jobs):

1. J1. The total energy stored in the capacitances of the MMC is the same for (ss1) as for (ss2):

$$\frac{C}{2N_{arm}} \sum_{j=1}^{6} \left[ \frac{(u_{DC}^{(ss1)})^2}{3} - \frac{(u_{DC}^{(ss2)})^2}{3} \right] = 0$$

$$\int_{t_0}^{t_0+T_s} \sum_{j} u_{j} \, dt$$

This means that the ‘centre of mass’ values of the arm energies are identical in the new and in the old steady states.

2. J2. The power flows within the MMC have to match the shifting in the six arm energies from (ss1) to (ss2):

$$\frac{C}{2N_{arm}} \int_{t_0}^{t_0+T_s} \left[ \frac{(u_{DC}^{(ss1)})^2}{3} - \frac{(u_{DC}^{(ss2)})^2}{3} \right] = \int_{t_0}^{t_0+T_s} u_{j} \, dt$$

Out of these six equations, only five are actually independent since their sum vanishes according to (J1).

Both conditions have to be satisfied by finding appropriate trajectories of the DC current and of the circular currents which no longer vanish during the shifting phase. Job (J1) corresponds to the weakening of the power condition $W_{20} = 0$, valid at any time step during the steady state, to an integral relation describing a vanishing energy change $\int_{t_0}^{t_0+T_s} W_{20} \, dt = 0$. According to (5) such an integral contains terms depending on the known values of the AC and DC side, terms defined by the initial and final steady states and terms whose single unknown signal variable is $i_{c, 0}$. The only exception is the term containing the squares of the circular currents (marked by the wavy line), which is proportional to the small internal resistance $R_e$ and can thus be neglected in a first approximation. In order to characterise the trajectory of $i_{c, 0}$ during $t_0 \leq t \leq t_0 + T_s$ by a single parameter which can be determined by the condition (J1), its signal form is predefined (see Fig. 2) as the superposition of (i) a smooth transition

$$\varepsilon_{i_{c, 0}}^{(ss1)} = \frac{6}{3} + \frac{6}{6} \left( 1 - \cos \left( \frac{t - T_s}{T_s} \right) \right)$$

between the old and new DC current, and (ii) a ‘hump’ component $\Delta i_{c, 0}$ given by $1 - \cos(2\pi(t - t_0)/T_s)$ multiplied with a (still unknown) amplitude $A_0$ to be determined by job (J1), which leads to an algebraic quadratic equation in $A_0$. By solving that equation, $A_0$ and therefore $i_{c, 0}$ during the shifting phase are determined. It is worth noting that due to the use of $1 - \cos$ with an integer multiple of $\pi(t - t_0)/T_s$, the current $i_{c, 0}$ has a vanishing time derivative at $t = t_0$ and at $t = t_0 + T_s$, thus guaranteeing a smooth connection between the old and the new steady states without inducing any transients after reaching the new steady state.

For solving the five equations given by (J2) for the energy shift of the arm energies, the following linear combinations are used:
The energy shift \( \Delta W_{\Sigma.0} \) has already been considered in job (J1). The energy shifts of \( \Delta W_{\Delta,0\beta} \) are exactly linear in \( i_c, i_{\beta} \) due to the subtraction between the upper and the lower arms: this can be easily seen in power equation (6) as well as in the two following equations for the power components \( W_{\Delta,0\beta} \):

\[
W_{\Delta,0\beta} = -\frac{d}{dt} \left[ \frac{1}{2} L_{AC} i_c + L_c i_c, \alpha, \beta \right] + (2L_{AC} + L_c) i_c, \alpha, \beta \] 

\[
+ (3R_{DC} + 2R_c) i_c, \alpha, \beta] 
\]

\[
- 3L_{DC} \left( \frac{d}{dt} i^{AC}_{\alpha, \beta} + u_{AC. \alpha, \beta} \right) 
\]

\[
+ L_{AC} \left( \frac{d i_{\alpha, \beta}}{dt} - i_{\alpha, \beta} \right) + \frac{i_{\alpha, \beta}}{2} \right) - u_{AC.\alpha, \beta} \right] - 2u_{AC.\alpha, \beta} \]

This results in, e.g. \( \Phi_{\alpha} = \pm \sqrt{27/3} \left( 1 - \cos(4\pi(t - t^0)T_0) \right) \) (with sign + during the first half of \( T_0 \) and - during the second one) and so forth. These five basis functions are shown in Fig. 3. The circular current components \( i_{\alpha, \beta} \) during the shifting time interval \( t_0 \leq t \leq t_0 + T_0 \) are pre-defined as a linear superposition of these functions with still unknown amplitudes \( A_i \): \( i_{\alpha, \beta} = \sum_{i=1}^{A_i} \Phi_i \) and \( i_{\alpha, \beta} = \sum_{i=1}^{A_i} \Phi_i \). Which particular basis functions \( \Phi_i \) are assigned to \( i_{\alpha, \beta} \) in (9) and thus \( \Delta W_{\Sigma, \beta} \) becomes exactly linear in the circular currents, or equivalently in the five amplitudes \( A_1, \ldots, 5 \). Only term \( f_{\alpha}^{\alpha \rightarrow \beta} (i_{\alpha} - i_{\beta}) \) remains in (9) as the only quadratic contribution in the circular currents: since it consists of the difference between the two squared circular components, whose absolute values during the shifting phase are of a similar order of magnitude, such term is small. Thus, it can be implemented in an iterative way by means of the linearisation
transition between both steady states achieved by the developed algorithm.

\[ \int_{t_0}^{t_0 + T_s} (\Phi_{\alpha, a} - \Phi_{\alpha, b}) dt \approx - \int_{t_0}^{t_0 + T_s} (\Phi_{\alpha, a}^{prov} - \Phi_{\alpha, b}^{prov}) dt \]

\[ + 2 \sum_{j=1}^{C-1} A_j \int_{t_0}^{t_0 + T_s} \Phi_{j, \alpha, a}^{prov} dt - 2 \sum_{j=1}^{C-1} A_j \int_{t_0}^{t_0 + T_s} \Phi_{j, \alpha, b}^{prov} dt \]

where \( \Phi_{\alpha, a/b}^{prov} \) denotes a provisional solution for the circular currents obtained from a previous iteration.

The following efficient algorithm for solving iteratively the unknown amplitudes \( A_0 \) and \( A_{1/\ldots/5} \) requires only very few iterations to satisfy the six nonlinear equations (J1) and (J2):

i. For the first iteration, the provisional values for the circular currents \( \Phi_{\alpha, a/b}^{prov} \) are assumed to be equal to zero.

ii. For the given circular amplitudes, amplitude \( A_0 \) is solved from the algebraic quadratic equation corresponding to job (J1)

\[ 0 = \frac{3}{8} A_0^2 + A_0 \left( \frac{\Phi_{\alpha, 1}^{(DC)} + \Phi_{\alpha, 2}^{(DC)}}{6} - \frac{4(3R_{\alpha, DC} + R_e)}{3R_{\alpha, DC} + R_e} \right) \]

\[ + \frac{1}{4} \left( \frac{\Phi_{\alpha, 1}^{(SS)} \Phi_{\alpha, 2}^{(SS)}}{3R_{\alpha, DC} + R_e} + \frac{3L_{AC} + L_e + 6R_{\alpha, DC}}{3R_{\alpha, DC} + R_e} \right) - \frac{2}{3} \left( \frac{\Phi_{\alpha, 1}^{(SS)} \Phi_{\alpha, 2}^{(SS)}}{3R_{\alpha, DC} + R_e} \right) \]

\[ + \frac{1}{6} \left( \frac{\Phi_{\alpha, 1}^{(DC)} + \Phi_{\alpha, 2}^{(DC)}}{3R_{\alpha, DC} + R_e} \right) \]

with a small contribution to the square bracket from

\[ \frac{R_e}{2(3R_{\alpha, DC} + R_e)} \int_{0}^{T_s} \int_{0}^{t_0 + T_s} (\Phi_{\alpha, a}^{prov} - \Phi_{\alpha, b}^{prov}) dt \]

Note that in the case of no DC voltage drop \( (u_{DC}^{(DC)} = u_{DC}^{(SS)}) \) and thus \( (\Phi_{\alpha, 1}^{(SS)} = \Phi_{\alpha, 2}^{(SS)}) \) the previous equation yields \( A_0 = 0 \) according to the definition of a steady state given in (7).

iii. For the (provisional) \( i_{\alpha, 0} \) obtained from the just calculated \( A_0 \), the five linear equations of job (J2) – exactly linear in \( i_{\alpha, a/b} \), \( \Delta W_{\alpha, 0} \), \( \Delta W_{\alpha, 1} \), \( \Delta W_{\alpha, 2} \) and \( \Delta W_{\alpha, 3} \) – are solved to get amplitudes \( A_{1/\ldots/5} \).

iv. Improved (provisional) circular currents \( i_{\alpha, a/b} \) are obtained from these latter amplitudes, and steps 2–4 are repeated until a desired accuracy is reached, which takes only very few iterations. The full solution procedure is carried out for \( R_{\alpha, 0} = 2u_{DC, 0} \) which vanishes for an AC symmetric voltage.

Once \( i_{\alpha, a/b} \) for the shifting phase have been determined, the corresponding input arm voltages \( u_{\alpha} \) are calculated according to the first-order differential (4). It is noteworthy that the AC current components \( i_{\alpha, 0} \) maintain the whole time their steady state, without requiring any further modification: jobs (J1) and (J2) are exclusively solved by means of the dynamics in the DC and in the circular currents.

5 Results and conclusions

The parameters used for the numerical simulation are listed in Table 1. The system operates at a power level of 400 MW, and the time step for the control method explained in the previous section is equal to \( \Delta t = 0.2 \) ms. The results for a shifting phase of duration \( T_s = 9.6 \) ms after a DC voltage drop of 25% from 400 kv down to 300 kv are displayed in Fig. 4, showing that conditions (J1) and (J2) are satisfied. The arm capacitance voltages of \( u_{\alpha} \) reached the demanded (ss2) signal form (marked as dashed lines) quite accurately (J2).
Simultaneously, their common ‘centre of mass’ $u_{C,\text{cm}}$, defined by $u_{C,\text{cm}} = (1/6)\sum_{j=1}^{6}u_{C,j}$ and equal (up to a factor $3C/N_{sb}$) to the total energy stored in all arm capacitances, reaches in the new steady state the value of the old steady state as required by job (J1). As shown in Fig. 5, the only affected current components during the shifting phase are: (i) the DC current, whose dynamics is necessary to satisfy the overall power requirements and (ii) the internal circular currents, for achieving the desired symmetrised distribution of energy in the arm capacitances when reaching the new steady state. These circular currents remain $<3\, \text{kA}$ and can therefore be sustained by the MMC. The AC current components, on the other hand, are not modified at all and maintain their steady-state trajectory. If the duration of the shifting window is reduced to one-third of the AC cycle, the smooth transition in the arm capacitance energies is still achieved (Fig. 6), but at the cost of circular currents reaching $\sim 6\, \text{kA}$.

6 References

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Fig. 5 Simulation of circular currents $i_{C,\alpha}$ and $i_{C,\beta}$. DC current as $i_{DC,0} = i_{DC}/3$ and AC current components for parameters in Table 1 corresponding to the arm capacitance volages in Fig. 4

Fig. 6 Arm capacitance voltages $u_{C,ph}$, 1/2/3 and their common ‘centre of mass’ $u_{C,cm}$. Simulation for parameters in Table 1 and $T_s = 7.2\, \text{ms}$