Functional superconductor interfaces from broken time-reversal symmetry

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The breaking of time-reversal symmetry in a triplet superconductor Josephson junction is shown to cause a magnetic instability of the tunneling barrier. Using a Ginzburg-Landau analysis of the free energy, we predict that this novel functional behaviour reflects the formation of an exotic Josephson state, distinguished by the existence of fractional flux quanta at the barrier. The crucial role of the orbital pairing state is demonstrated by studying complementary microscopic models of the junction. Signatures of the magnetic instability are found in the critical current of the junction.

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The Josephson effect between superconductors separated by a tunneling barrier continues to be of fundamental interest, in particular as a phase-sensitive test of the pairing symmetry of unconventional superconductors and in the study of the interplay of superconductivity with magnetism. Although the qualitative features of the Josephson effect are determined by the quantum nature of the superconductors, the modification of the superconducting state at the junction interface must often be included in the proper description of the supercurrent transmission, the so-called proximity effect. In contrast, the properties of the barrier are usually regarded as fixed. Recently, however, it has been shown that a thin ferromagnetic layer on a singlet superconductor can display novel behaviour. In particular, the presence of the superconductor can suppress the uniform magnetization or stabilize a domain structure in the ferromagnet. These effects result from the competition between singlet superconductivity and ferromagnetism, and indicate that the tunneling barrier in a Josephson junction is not necessarily independent of the superconductors.

In this letter, we consider the possibility of using the presence of the two superconductors to induce a magnetic instability of a non-magnetic tunneling barrier in a Josephson junction. In particular, by both general phenomenological arguments and solution of specific microscopic models, we show that such a novel functionality of the barrier can develop for time-reversal symmetry breaking configurations of two spin-triplet superconductors on either side. The Josephson coupling across the tunneling barrier is essential to this effect, which manifests itself as an exotic state distinguished, for example, by the existence of fractional flux quanta at the barrier. Moreover, such a junction displays an anomalous temperature dependence of the critical current.

The intrinsic spin structure of the Cooper pairs in a triplet superconductor (TSC) requires a quasiparticle gap with three components \( \Delta_S \), for each \( z \)-component of spin \( S_z = -1, 0, +1 \). Each of these gaps has odd orbital parity, i.e. \( \Delta_S(z) = -\Delta_S(-z) \). The pairing state is conveniently described by the so-called \( \mathbf{d} \)-vector, defined in spin-space \( \mathbf{d} = \frac{1}{2}(\Delta_{-1} - \Delta_{+1}) \mathbf{x} - \frac{1}{2}(\Delta_{-1} + \Delta_{+1}) \mathbf{y} + \Delta_0 \mathbf{z} \), which also serves as the order parameter for the TSC. Although a multitude of different triplet pairing states are allowed by symmetry, only a few examples of TSCs have been discovered so far, e.g. \( \text{Sr}_2\text{RuO}_4 \) and \( \text{UGe}_2 \). In \( \text{Sr}_2\text{RuO}_4 \) the spin pairing state has been identified as unitary and equal-spin-pairing, i.e. the spins of the triplet Cooper pairs lie in the same plane in spin space perpendicular to \( \mathbf{d} \), but the condensate does not have a net spin. Restricting ourselves to such pairing states, we write \( \mathbf{d} = e^{i\phi} \mathbf{d} \) where \( \mathbf{d} \) is a real vector. The vector character of the TSC order parameter provides a novel degree of freedom in Josephson junction physics: in addition to the phase difference between the condensates to the left and right of the barrier, which controls the Josephson supercurrent as in the familiar spin-singlet case, there is also the mutual alignment of the left (\( \mathbf{d}_L \)) and right (\( \mathbf{d}_R \)) vectors, which controls the magnetic aspects of the transport. In particular, when \( \mathbf{d}_L \times \mathbf{d}_R \neq 0 \), a Cooper pair tunneling across the barrier undergoes a reconstruction of its spin state, producing an effective spin through the TSC breaking combination \( \langle S \rangle = i\mathbf{d}_L \times \mathbf{d}_R^* + \text{H.c.} \) at the junction interface.

The violation of TRS at the tunneling barrier by the misaligned \( \mathbf{d} \)-vectors allows the TSCs to directly couple to a magnetization \( \mathbf{M} \) of the interface. As we will see, the TSCs may in fact change the electronic properties of the interface so as to stabilize a spontaneous ferromagnetic order. This novel behavior can be understood on a phenomenological level by a Ginzburg-Landau analysis of the free energy. Introducing the TSC order parameters for each side of the interface as \( \mathbf{d}_L = \mathbf{d}_L e^{i\phi_L} \) and \( \mathbf{d}_R = \mathbf{d}_R e^{i\phi_R} \), we write the free energy of the junction \( F \) to lowest order in \( \mathbf{M} \) and \( \mathbf{d}_L, R \) as

\[
F = \frac{|\mathbf{M}|^2}{2\chi} - t\mathbf{d}_L \cdot \mathbf{d}_R \cos(\phi) + 2\gamma \mathbf{M} \cdot (\mathbf{d}_L \times \mathbf{d}_R) \sin(\phi) \tag{1}
\]

Here \( \phi = \phi_R - \phi_L \) is the phase difference, \( \chi \) denotes the intrinsic uniform spin susceptibility of the barrier, and \( t \) and \( \gamma \) are phenomenological parameters. The ground state of the junction is obtained by minimizing \( F \) with
normal-state height $U_P = Z \hbar v_F$ where $Z$ is a dimensionless quantity and $v_F = \hbar k_F / m$ is the Fermi velocity in the bulk TSCs, which are assumed to have spherical Fermi surfaces of radius $k_F$. If the barrier supports a magnetic moment $M \parallel |d_L \times d_R|$, the effective barrier height for spin-$\sigma$ quasiparticles with spin parallel to $M$ in dimensionless units is $Z = -\sigma M$ where $M = g_{\mu B} |M| / \hbar v_F$.

The electronic properties of the two junctions can be expressed entirely in terms of the Andreev bound state (ABS) spectrum. These subgap states are localized at the barrier interface and are formed by multiple Andreev reflection of tunneling quasiparticles within the barrier. We solve the Bogoliubov-de Gennes equations to obtain explicit expressions for the ABS energies $E_{k,\sigma}$ in the $p_z$- $p_z$ junction

$$E_{k,\sigma} = |\Delta(T) k_z| \sqrt{T_{\sigma}(k)} \cos(\phi/2 - \sigma \eta)$$

and the $p_y$-$p_y$ junction

$$E_{k,\sigma} = |\Delta(T) k_y| \sqrt{1 - T_{\sigma}(k)} \sin^2(\phi/2 + \sigma \eta)$$

The bound states are indexed by the component of spin $\sigma = \pm 1$ parallel to $\tilde{d}_L \times \tilde{d}_R$, and each state has two branches at $\pm E_{\sigma}$. The transparency of the barrier to spin-$\sigma$ quasiparticles is given by $T_{\sigma}(k) = k^2 / (k^2 + (Z - \sigma M)^2 k_F^2)$. We plot the ABS spectrum as a function of $\phi$ in Fig. 2. Note that the bound states always intersect the line $E = 0$ in the $p_z$-$p_z$ junction. These so-called zero energy states are guaranteed by the arrangement of the $p$-wave orbitals, such that all specularly-reflected quasiparticles experience a sign-change of the gap. Since the gap does not change sign for reflected quasiparticles in the $p_y$-$p_y$ junction, in contrast, zero energy states are only found here for a perfectly transparent tunneling barrier, as is also the case for $s$-wave superconductor junctions.

The electronic contribution to the free energy of the junctions can be written in terms of the ABS energies

$$F_{el} = -k_B T \sum_k \sum_{\sigma} \frac{|k_z|}{k_F} \log \left[ 2 \cosh \left( \frac{E_{k,\sigma}}{2 k_B T} \right) \right]$$

As in Eq. (1), the magnetic free energy of the barrier is included to lowest-order $F_{mag} = M^2 / 2 \chi$, where $\chi$ is given in units of $(g_{\mu B} \hbar v_F)^2 / 2 \Delta_0$. Numerically minimizing the total free energy $F_{el} + F_{mag}$ with respect to both
M and $\phi$, we find the global free energy minimum. Typical minimizing values for the $p_z$-$p_z$ and $p_y$-$p_y$ junctions are plotted as a function of temperature in Fig. 3(a) and Fig. 3(b) respectively. We find that the barrier undergoes a magnetic instability at sufficiently large $\chi$, and below a critical temperature $T_M < T_c$ such that the magnetic state appears only in the presence of superconductivity. For $Z \neq 0$ the junction is in a fractional state below $T_M$ with two degenerate free energy minima ($M, \phi_{\min}$) and ($-M, -\phi_{\min}$) (broken TRS).

In Fig. 3(c) and Fig. 3(d) we show the phase diagram as a function of $Z$ and $T$ at fixed $\chi$ for the $p_z$-$p_z$ and $p_y$-$p_y$ junctions respectively. The qualitatively different form of these phase diagrams follows from the response of the ABS spectrum to the appearance of the magnetization and the resulting change in the $T_{c}(k)$. As can be seen from Eq. 2 and Eq. 3, the ABS spectrum in the two junctions has very different dependence upon $T_{c}(k)$: due to the zero-energy states in the $p_z$-$p_z$ junction, each bound state $E_{k,\sigma}$ monotonically shifts towards the middle of the gap with decreasing $T_{c}(k)$; for the $p_y$-$p_y$ junction, in contrast, the states move towards the gap edges. From Eq. 4, the free energy contributed by $E_{k,\sigma}$ in a $p_y$-$p_y$ junction will therefore increase (decrease) as the transparency $T_{c}(k)$ decreases (increases), while the opposite is true for the $p_y$-$p_y$ junction.

It immediately follows that the $p_z$-$p_z$ junction is non-magnetic at $Z = 0$, as $M \neq 0$ would reduce the transparency for both spin orientations and hence raise the total free energy. The $M = 0$ state remains stable below some critical value of $Z$; increasing $Z$ beyond this, a magnetic moment appears as the free energy gain from decreasing the transparency in the $\sigma = -1$ sector outweighs the increase in the $\sigma = +1$ sector. Although the decrease in electronic free energy favors the indefinite growth of $M$ with increasing $Z$, the maximum magnitude of $M$ is limited by the cost in magnetic free energy. For sufficiently large $\chi$ and low temperatures, the transition back into the non-magnetic state is first order, with regions in the phase diagram where the non-magnetic and magnetic states are metastable, as shown in Fig. 3(c). At higher temperatures or smaller $\chi$, $|M|$ continuously vanishes after going through a maximum.

In contrast, the $p_y$-$p_y$ junction displays a spontaneous magnetization at $Z = 0$ for all $T < T_c$, stabilized due to the reduction in $F_{cl}$ from the decreased transparency in each spin sector. This effect is absent from the Ginzburg-Landau expansion of $F_{cl}$ in Eq. 1, as we have only kept terms to first order in $M$; from Eq. 3, however, we see that the magnetization only enters $F_{cl}$ as $|M|^2$ when $Z = 0$. Despite the spontaneous magnetization, the junction is not in a fractional state and $\phi_{\min} = 0$. Turning on a finite tunneling barrier strength ($Z > 0$) at fixed $T$, the magnetic moment of the barrier decreases to compensate for the free-energy increase from the enhanced transparency in the $\sigma = +1$ sector; $\phi_{\min}$ simultaneously takes on a fractional value. As $Z$ is further increased, the barrier moment is monotonically suppressed, while the stable phase difference passes through a stationary point before returning to its $Z = 0$ value.

The characteristic signature of the magnetic instability is the appearance of fractional flux quanta at the junction interface. In Fig. 4 we show a proposal for their observation in a Josephson junction with a tunneling barrier consisting of two materials, one of which undergoes the magnetic instability proposed here, while the other remains non-magnetic at all temperatures. The stable phase difference across the magnetic region is $\phi$, while it is 0 across the non-magnetic region. If a magnetic flux line is trapped at the boundary between the barrier segments, a line integral along the contour $C$ in Fig. 4

![FIG. 4: (color online) Proposed experiment for the observation of fractional flux quanta at the interface between magnetic and non-magnetic regions of a tunneling barrier. The contour $C$ is used in evaluating Eq. 3.](attachment:figure4.png)
As the temperature is lowered below $T_M$, the magnetic instability of the tunneling barrier results in the development of a spontaneous magnetic configuration of two TSCs in a Josephson junction. This realizes an exotic Josephson state with a TRS-breaking configuration. The magnetic instability of the tunneling barrier radially alters the supercurrent transmission through the junction. From Eq. 1, we find that the Josephson current vs phase relationship $I_J = (e/h)\partial F/\partial \phi \propto \sin(\phi - \phi_{\text{min}})$ is shifted from its usual form for a non-magnetic barrier. This is clearly seen in Fig. 5(a) for the $p_{y'}-p_y$ junction (the $p_z-p_z$ junction results are qualitatively identical). Note that the different magnitudes of $\max\{|I_J|\}$ and $\min\{|I_J|\}$ are due to higher-order harmonics in $\phi$ which are not included in the free energy expansion Eq. 1. We also find a strong enhancement of the critical current $I_J^c = \max\{|I_J|\}$ below the magnetic instability [Fig. 5(b)]. This occurs as the increased current through the spin sector with the enhanced transparency over-compensates for the decreased current through the spin sector with the lowered transparency. The increase of $I_J^c$ below $T_M$ is reminiscent of the “low-temperature-anomaly” of d-wave Josephson junctions.

For simplicity, we have neglected the suppression of the TSC state near the interface due to the proximity effect. Including this would increase $F_M$ and hence shrink the parameter space where the moment is stable. Although quantitative changes in the phase diagrams Fig. 3(c) and (d) are expected, the free energy expansion Eq. 1 and the basic experimental signatures of the fractional state remain valid. Furthermore, we do not expect a qualitatively different ABS spectrum, and so the essential role of the orbital pairing state of the two TSCs should remain in a fully self-consistent analysis.

In conclusion, we have shown that a TRS-breaking configuration of two TSCs in a Josephson junction can cause the tunneling barrier to develop a spontaneous magnetization. This realizes an exotic Josephson state with stable phase difference $0 < \phi_{\text{min}} < \pi$. The orbital part of the TSC pairing state was demonstrated to control the magnetic instability. The existence of fractional flux quanta at the barrier, and a large increase in the critical current beneath the magnetic transition temperature, are the experimental signatures of this state.

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