Rain attenuation in millimeter wave, centimeter wave and visible light ranges

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Abstract. The paper presents the results of theoretical simulation of mmWave, cmWave and visible light attenuation in rain based on the Mie scattering theory. Specific rain attenuation has been estimated from the gamma drop size distribution for a spherical drop. Rain attenuation of electromagnetic waves in the visible range, used by video cameras and lidars of «smart» city vehicles, and at the beginning of the millimeter wave range used by radars and positioning systems, have similar values averaging 3-5 dB/km. The attenuation levels of electromagnetic waves in mmWave, cmWave and Visible light ranges must be taken into account when calculating the energy budget of vehicle communications.

1. Introduction

Modern control systems for vehicles, including unmanned vehicles, contain ultrasonic sensors, video cameras, radio detection and ranging (radar) and light detection and ranging (lidar) systems [1]. This sensor data is critical to recognition algorithms in advanced driver assistance systems (ADAS) for self-driving vehicles and is often combined with other sensors and global and local positioning systems data. A fundamental limitation on the use of video cameras and lidars is their significant dependence on weather conditions, such as rain, snow and fog. It is known that rain seriously increases the attenuation of electromagnetic waves, especially in the optical range, which leads to serious errors in driving vehicles.

Distortions of signals propagating in various environments, including the weather conditions of rain, snow and fog, are actively studied in [1-4]. In the [1] the impact of rain on the lidar system by considering the raindrop distributions of different regions are considered. The simulation results demonstrate that the signal power received by a lidar attenuates, which is modeled using Mie scattering theory, and the amount of attenuation clearly differs in the regional models. In [2], rain attenuation was investigated using the raindrop size distribution model in the K-band (24 GHz) and E-band (77 GHz) based on the Mie scattering theory. Simulation of multiple scattering when light passes through mixed aerosols using the Monte Carlo method was carried out in [3]. In [4], a technique was proposed for determining the radii and refractive indices of aerosol particles using angular light scattering and resonance spectroscopy Mie by capturing individual aerosol particles in a cell with controlled relative humidity. The Mie resonance positions are extracted from sharp peaks observed in cavity-enhanced Raman scattering spectra. Dynamic light scattering during electrophoretic motion of biomolecules in a colloidal solution was studied in [5, 6]. The analysis of correlation function allows one to obtain the size distribution, translational diffusion coefficients and the molecular weight of biomolecules.
Of particular interest in the field of light scattering are works on the condensation of a hardly volatile substance on individual impurity molecules \[7-11\]. Due to such condensation \[7, 8\], an aerosol particle of micron size is formed, in the center of which there is a detected impurity molecule \[9\]. The impurity concentration is determined from the light scattering by an aerosol particle \[10\]. This so-called molecular condensation nuclei (MCN) method is based on the measurement of Mie light scattering by aerosol particles. It has a uniquely high sensitivity and can be used for environmental and structural health monitoring \[11\]. The further development of this method is a promising direction for solving the problem of early detection of COVID-19 disease.

An integral part of modern vehicle control systems, including unmanned vehicles, are global and local positioning systems \[12-13\], Millimeter wave (mmWave) and Centimeter wave (cmWave) radars \[14-20\]. Both are susceptible to weather conditions, including rain, snow and fog.

The aim of this work is a comparative analysis of the attenuation of electromagnetic waves by rain in the visible range used by video cameras and vehicle lidars, and in the millimeter and centimeter wavelength ranges used by radars and vehicle positioning systems.

2. Light scattering by aerosol particles

Let us recall the Mie solution to the problem of scattering of a plane electromagnetic wave by a uniform sphere \[21\], as shown in figure 1.

![Figure 1. Scattering of a plane electromagnetic wave by a uniform sphere.](image)

Aerosol particles can be considered as dielectric spheres. The amplitudes of the incident and scattered electric fields in the far field region are given by the following equations

\[
\begin{pmatrix}
E_{||}
E_{\perp,||}
\end{pmatrix} =
\begin{pmatrix}
S_2 & 0
0 & S_1
\end{pmatrix}
\begin{pmatrix}
E_{||}
E_{\perp,||}
\end{pmatrix},
\]

where \(k\) – the wavenumber, \(R\) – the distance from the observation point to the center of the sphere, \(S_1\) and \(S_2\) – the elements of the amplitude scattering matrix, the index \(i\) – denotes the incident wave and \(s\) – the scattered wave. For not polarized incident light the intensity of the scattered light is given by \(I_s = \frac{1}{kR^2} \cdot \frac{1}{2} \cdot \left( |S_1|^2 + |S_2|^2 \right) \cdot I_i\). Single particle scattering matrix element is \(S_{11} = \frac{1}{2} \cdot \left( |S_1|^2 + |S_2|^2 \right)\). The scattering matrix \(S_{ij}\) is a 4x4 matrix connecting the Stokes parameters of the incident and scattered light.

The elements of the amplitude scattering matrix and for a homogeneous sphere \(S_1\) and \(S_2\) were first derived by Mie.
where $a_n$, $b_n$ – the scattered field coefficients (Mie coefficients), $\pi_n$, $\tau_n$ – the angular scattering coefficients.

The scattered field coefficients are given by

$$
\begin{align*}
    a_n &= \frac{\mu m^2 j_n(mx)[xj_n(x)]'}{\mu m^2 j_n(mx)[xj_n(x)]'} - \mu j_n(mx)[xj_n(x)]'
    \quad \text{and} \\
    b_n &= \frac{\mu j_n(mx)[xj_n(x)]'}{\mu j_n(mx)[xj_n(x)]'} - \mu h_n^{(1)}(x)[xj_n(x)]'
\end{align*}
$$

(2)

where the stroke means differentiation with respect to the argument in the parenthesis;

- $x = k r_0 \frac{2 \pi \cdot N \cdot r_o}{\lambda}$ – the diffraction parameter;
- $m = \frac{k}{k_1} = \frac{N_1}{N}$ – relative refractivity index; $N_1$, $\mu_1$ and $N$, $\mu$ – refractivity index and magnetic permeability of the particle and surrounding media respectively. Other designations are traditional: $\lambda$ – the wavelength of incident light; $j_n$ – spherical Bessel function of the order $n$; $h_n^{(1)}$ – spherical Hankel function of the order $n$. The angular scattering coefficients are functions $\pi_n = \frac{P_n}{\sin \theta}$ and $\tau_n = \frac{dP_n}{d\theta}$, where $P_n$ – connected Legendre functions of the first kind. The extinction efficiency $Q_{ext}$ can be calculated from Mie scattering theory:

$$
Q_{ext} = \frac{2}{\pi^2} \sum_{n=1}^{\infty} (2n+1) \Re\{a_n + b_n\}
$$

(3)

Convergence of the series can be improved by using Ricatti-Bessel functions and their logarithmic derivatives [21].

3. Estimation and simulation of rain attenuation based on Mie theory

A practical model of rain attenuation is specified in ITU-R P.838-3 [22]. The model was obtained empirically without direct connection with the physics of radio wave propagation. The choice of values for the distance factor involved in the ITU model is based on long-range measurement datasets. If we apply the ITU model for calculating short-haul lines (≈ 350 m), the distance factor value will exceed the recommended one by several times. Thus, the ITU model is satisfactory for long lines, for example, satellite communication lines, and poorly suited for short lines, which include, among other things, radio channels of vehicle control systems. More accurate estimates of rain attenuation in radio control channels can be obtained using the extinction calculation based on the Mie theory.

Rain attenuation can be estimated from the drop size distribution (DSD) [2]:

$$
\gamma = 4.343 \cdot 10^{-3} \int_0^\infty \delta_{ext} (D) N(D) dD
$$

(4)

where $\gamma$ is the specific attenuation in dB/km, $\delta_{ext} = \pi \left( \frac{D}{2} \right)^2 Q_{ext}$ is the extinction cross section (mm$^2$) for water drops of diameter $D$ (mm) and $N(D)$ (m$^{-3}$·mm$^{-1}$) is the drop size distribution. Extinction efficiency $Q_{ext}$ is determined in accordance with expression (3).

Three distribution models are applied to fit the rain DSD: the gamma distribution, the exponential distribution, and the lognormal distribution [2]. The DSD values measured by meteorological stations for the most frequent rains (rain intensity 1-5 mm/h) are best approximated by the gamma distribution with the parameters $N_0 = 1.24 \cdot 10^4$, $\mu = 2.5$ and $\alpha = 3.704$.
\[ N(D) = N_0 D^\alpha \exp(-\mu D) \] (5)

Modeling of the dependence of extinction efficiency and specific rain attenuation on the wavelength of electromagnetic radiation was carried out on the basis of expressions (3), (4), (5) in the MATLAB computing environment. In the calculations, we used the DSD model (5) with a diameter range of spherical raindrops from 0.1 to 10 mm. Expression (4) was integrated within the same limits. Figure 2 shows the dependences of the extinction efficiency \( Q_{\text{ext}} \) of a raindrop with a diameter of 3 mm on the wavelength of electromagnetic radiation in the visible range. Extinction efficiency \( Q_{\text{ext}} \) is very weakly dependent on the wavelength, therefore specific rain attenuation also weakly depends on the wavelength in the optical range. The calculated specific rain attenuation at a rain intensity of 5 mm/h is 3.29 dB/km. Figure 3 shows used in calculations DSD. The value of the refractive index of a raindrop in the visible range is assumed to be real and equal to \( m = 1.335 \). The diameter of a raindrop exceeds the wavelength by 3-4 orders of magnitude, which makes it possible to use the geometric optics approximation along with the Mie theory. The extinction efficiency dependence has a huge number of extrema caused by the interference of waves reflected from different points on the surface of a spherical particle (raindrops). These extrema are also called Mie resonances and are used, for example, to determine the size and refractive index of aerosol particles [4].
In the mmWave range the refractive index of a rain drop has a complex character, its value in calculations is taken to be $m = 3.8528 + j2.0742$ for the range 1-10 mm (exactly corresponds to the frequency 77.52 GHz, ($\lambda = 3.87$ mm) and $m = 6.45 + j2.78$ for the range 10-100 mm (exactly corresponds to the frequency of 24.0 GHz, ($\lambda = 12.49$ mm). The sizes of a raindrop in these ranges are of the same order of magnitude as the wavelength, therefore only the Mie theory is applicable for calculations.

Figures 4 and 5 show the dependences of the extinction efficiency $Q_{\text{ext}}$ of a rain drop with a diameter of 3 mm (a) and specific rain attenuation $\gamma$ at a rain intensity of 5 mm/h (b) on the wavelength of electromagnetic radiation in the mmWave (figure 4) and cmWave (figure 5) ranges.

4. Conclusion
Rain attenuation of electromagnetic waves in the visible range, used by video cameras and lidars of «smart» city vehicles, and at the beginning of the millimeter wave range used by radars and positioning systems, have similar values, averaging 3-5 dB/km. At wavelengths longer than 4 mm, a significant decrease in the attenuation value is observed, which continues in the centimeter range. Therefore, for vehicle traffic control and positioning systems rain is less dangerous in the wavelength range of 50 - 100 mm. In a rain-free atmosphere the attenuation is $\sim 0.18$ dB/km at 24 GHz ($\lambda = 12.49$ mm) and $\sim 0.4$ dB/km at 77 GHz ($\lambda = 3.87$ mm). Thus in rain conditions the received signals weaken by 3-5 dB/km, which must be taken into account when calculating the energy budget of vehicle communications.

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