Supersymmetry Breaking by Type II Seesaw Assisted Anomaly Mediation

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Abstract

Anomaly mediated supersymmetry breaking (AMSB), when implemented in MSSM is known to suffer from the problem of negative slepton mass squared leading to breakdown of electric charge conservation. We show however that when MSSM is extended to explain small neutrino masses by including a pair of superheavy Higgs triplet superfields (the type II seesaw mechanism), the slepton masses can be deflected from the pure AMSB trajectory and become positive. In a simple model we present in this paper, the seesaw scale is about $10^{13} - 10^{14}$ GeV. Gauge coupling unification can be maintained by embedding the triplet to $SU(5)$ 15-multiplet. In this scenario, bino is the LSP and its mass is nearly degenerate with NLSP slepton when the triplet mass is right around the seesaw scale.

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I. INTRODUCTION

Supersymmetry (SUSY) is considered to be a prime candidate for TeV scale physics since it resolves several conceptual issues of the standard model (SM) such as (i) radiative stability of the large hierarchy between Planck and weak scale; and (ii) electroweak symmetry breaking. With additional assumptions, it develops other appealing features: for instance, if R-parity symmetry is assumed, it can provide a candidate for the dark matter of the universe and if no or specific new physics is assumed, it can lead to the unification of gauge couplings at a very high scale.

Since there is no trace of supersymmetry in current observations, it must be a broken symmetry and the question arises as to the origin of this breaking. While at the phenomenological level, it is sufficient to assume soft breaking terms to implement this, low energy observations in the domain of flavor changing neutral currents (FCNC) imply strong constraints on it i.e. the sparticle masses must be flavor degenerate. It is therefore reasonable to require that any mechanism for SUSY breaking must lead to such flavor degeneracy for slepton and squark masses. Indeed there exist at least two well known scenarios where this happens: gauge mediated SUSY breaking (GMSB) \[1, 2\] and anomaly mediated SUSY breaking (AMSB) \[3, 4\]. In both these cases in simplest examples, the FCNC effects are dynamically suppressed. Both involve unknown physics in the hidden sector which breaks supersymmetry and this SUSY breaking information is transmitted to the visible sector via certain messengers. In GMSB scenario, the messenger sector generically involves new particles and forces whereas in the AMSB scenario, SUSY breaking is transmitted via the conformal breaking induced by radiative corrections in supersymmetric field theories. They however differ in the way the SUSY breaking manifests in the low energy sector: in GMSB (as in gravity induced minimal SUGRA models), the detailed pattern of sparticle masses depend on ultraviolet physics i.e. physics at mass scales much higher than the SUSY breaking scale whereas AMSB models have the advantage that this pattern depends only on the low scale physics. They are therefore easier to test experimentally given a particular low scale theory.

It however turns out that AMSB models despite their elegance and predictive power suffer from a fatal problem when the low scale theory is assumed to be the MSSM i.e. they predict the slepton mass squared to be negative and hence lead to a vacuum state that breaks electric charge conservation (called tachyonic slepton problem henceforth). This is of course unacceptable and this problem needs to be solved if AMSB models have to be viable. There are many attempts to solve this problem by taking into account additional positive contribution to the slepton mass
squared [5] [6] [7] [8].

An important thing to realize at this point is that MSSM is not a complete theory of low energy particle physics and needs extension to explain the small neutrino masses observed in experiments. The relevant question then is whether MSSM extended to include new physics that explains small neutrino masses will cure the tachyonic slepton mass pathology of AMSB.

There are two simple extensions of MSSM which provide natural explanation of small neutrino masses: the two types of seesaw mechanisms i.e. type I [9] and type II [10]. In the first case, a reasonable procedure is to extend the gauge symmetry of MSSM to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ which automatically introduces three right-handed neutrinos into the theory as well as new couplings involving the leptons which one could imagine as affecting the slepton masses. In most discussions of seesaw mechanism, it is commonly assumed that the seesaw scale is very high ($\geq 10^{13}$ GeV or so); so one would expect the associated new physics interactions to decouple. Such a generic scenario will not solve the tachyonic slepton problem. However, it has recently been pointed out [11] that there exists a class of minimal SUSY left-right symmetric models with high scale seesaw where left-handed weak iso-triplets with $B-L=+2$ and doubly charged Higgs fields with $B-L=+2$ coupling to right-handed leptons have naturally weak scale mass because of higher symmetries of superpotential. Their couplings to leptons contribute to the slepton mass squared and can solve the tachyonic slepton mass problem [11].

The present paper focuses on an alternative approach which uses type II seesaw mechanism for neutrino masses and to see how it affects the slepton masses. An advantage of this over the type I approach is that it does not involve extending the gauge symmetry but requires adding a pair of $Y = \pm 1 \, SU(2)_L$ triplet Higgs fields to MSSM. The $SU(2)_L$-triplets have mass close to $10^{13}$ GeV which is required to implement type II seesaw for small neutrino masses. We further assume that the triplet masses arise from the vacuum expectation value (VEV) of a light singlet field with a high VEV. We then show that in AMSB scenario, the F-component of the singlet field acquires an induced VEV, leading to new set of SUSY breaking effects. These effects are gauge mediated contributions to sparticle masses in addition to the usual AMSB contributions. We find that these contributions solve the tachyonic slepton mass problem. Thus type II seesaw in addition to solving neutrino mass problem also solves the problem of SUSY breaking by AMSB.\footnote{We note that pure gauge mediation in the presence of type II seesaw has been considered recently [12]; our model is different since AMSB effects play a significant role in the final predictions.} Of course in this

\footnote{We note that pure gauge mediation in the presence of type II seesaw has been considered recently [12]; our model is different since AMSB effects play a significant role in the final predictions.}
case one needs to assume R-parity symmetry to obtain stable dark matter.

This scenario makes prediction for the sparticles which are different from other scenarios. In particular, we find that the bino and sleptons are nearly degenerate with messenger at the seesaw scale- a situation which is particularly advantageous for understanding the dark matter abundance in the universe [13]. We also show that the model does preserve the unification of couplings.

The paper is organized as follows. In Section II, we explain the scenario of “deflected anomaly mediation” which plays a crucial role in our solution to the tachyonic slepton problem. In Section III we present a simple superpotential for the singlet field and calculate the deflection parameter. Section IV contains the general formulas of sparticle masses in the deflected anomaly mediation. In Section V we present the minimal model to solve the tachyonic slepton problem as well as generate light neutrino masses. Section VI contains the extended models which preserve the gauge coupling unification. We summarize our results in Section VII. In the Appendix A, we present the calculation of the lifetime of SUSY breaking local minimum.

II. DEFLECTED ANOMALY MEDIATION AND MESSENGER SECTOR

It is well known that in the absence of additional supersymmetry breaking, the AMSB contribution to sparticle masses is ultraviolet insensitive. It has however been proposed that presence of additional SUSY breaking effects could deflect the sparticle masses from the AMSB trajectory and lead to new predictions for sparticle spectrum. This has been called “deflected anomaly mediation” scenario [5] [7]. A key ingredient of this scenario is the presence of gauge mediated contributions arising from new interactions in the theory. Typically they involve the introduction of messengers $\Psi$ and $\bar{\Psi}$ with the following coupling:

$$W = S \overline{\Psi} \Psi.$$  

Clearly $\overline{\Psi}$ and $\Psi$ are the messenger chiral superfields in a vector-like representation under the SM gauge group, and $S$ is the singlet superfield. It is crucial for the messenger fields to be non-singlets, at least, under the $SU(2)_L \times U(1)_Y$ gauge group. In our model, the $SU(2)_L$ triplets which enforce the type II seesaw will play the role of these fields $^2$. Once the scalar component $(S)$ and the $F$ component $(F_S)$ in the singlet chiral superfield develop VEVs, the scalar lepton obtains new

\footnote{In order to implement type II seesaw in the MSSM, we only need one pair of triplets and it turns out that one pair of triplets is sufficient to lift slepton masses and leave bino as the LSP.}
contributions to its mass squared through the same manner as in the gauge mediation scenario \[1\] \[2\]. In our case, \( F_S \) is induced by the hidden sector conformal compensator SUSY breaking. The effect of non-zero \( F_S \) is to deflect the sparticle masses from the pure AMSB trajectory of the renormalization group equations, thereby solving the tachyonic slepton problem.

As just noted an important difference between the deflected AMSB from GMSB is that the SUSY breaking in the messenger sector is induced by the anomaly mediation, namely, \( F_\phi \), a non-zero \( F \) component of the compensator field, and \( F_\phi \) therefore is the unique source of SUSY breaking in this scenario. Therefore, we can parameterize the SUSY breaking order parameter in the messenger sector such as

\[
\frac{F_S}{S} = dF_\phi. \tag{2}
\]

Here, \( d \) is the so-called “deflection parameter” which characterizes how much the sparticle masses are deflected from the pure AMSB results. Theoretical consistency constrains it to be \( |d| < O(1) \), because \( F_S/S \) is not the original SUSY breaking sector.

We consider a simple model which provides a sizable deflection parameter \( |d| = O(1) \). Let us begin with the supergravity Lagrangian for \( S \) in the superconformal framework \[15\] \[16\] (supposing SUSY breaking in the hidden sector and fine-tuning of the vanishing cosmological constant),

\[
\mathcal{L} = \int d^4 \theta \, \phi^{\dagger} \phi S^{\dagger} S + \left\{ \int d^2 \theta \, \phi^3 W(S) + \text{H. c.} \right\}, \tag{3}
\]

where we have assumed the canonical Kahler potential (in the superconformal framework), \( W \) is the superpotential (except for Eq. (1)), and \( \phi = 1 + \theta^2 F_\phi \) is the compensating multiplet with the unique SUSY breaking source \( F_\phi \), taken to be real and positive through \( U(1)_R \) phase rotation.

The scalar potential can be read off as

\[
V = |F_S|^2 - S^{\dagger} S |F_\phi|^2 - 3F_\phi W - 3F_\phi W^{\dagger} \tag{4}
\]

with the auxiliary field given by

\[
F_S = - \left( S F_\phi + W_S^{\dagger} \right), \tag{5}
\]

where \( W_S \) stands for \( \partial W/\partial S \).

Using the stationary condition \( \partial V/\partial S = 0 \) and Eq. (5), we can describe the deflection parameter in the simple form,

\[
\frac{F_S}{S} = dF_\phi = -2 \frac{W_S}{S W_{SS}} F_\phi, \tag{6}
\]
where \( W_{SS} \) stands for \( \partial^2 W/\partial S^2 \). This is a useful formula, from which we can understand that \( S \) should be light in the SUSY limit in order to obtain a sizable deflection parameter \(|d| = \mathcal{O}(1)\) because the SUSY mass term \((W_{SS})\) appears in the denominator.

### III. SINGLET SUPERPOTENTIAL AND DEFLECTION PARAMETER

As a simple model, let us consider a superpotential

\[
W = -mS^2 + \frac{S^4}{M},
\]

where \( m \) and \( M \) are mass parameters, and we assume them to be real, positive and \( m \ll M \). The scalar potential is given by

\[
V = |S|^2 \left(-2m + 4 \frac{S^2}{M}\right)^2 + F_\phi \left(mS^2 + \frac{S^4}{M}\right) + \text{H.c.}
\]

Changing a variable as \( S^2 = xe^{i\varphi} \) with real parameters, \( x \geq 0 \) and \( 0 \leq \varphi \leq 2\pi \), the scalar potential is rewritten as

\[
V(x, \varphi) = 4x \left(m^2 - 4 \frac{m}{M} x \cos(\varphi) + 4 \frac{x^2}{M^2}\right) + 2F_\phi \left(mx \cos(\varphi) + \frac{x^2}{M} \cos(2\varphi)\right).
\]

It is easy to check that \( \varphi = 0 \) satisfies the stationary condition \( \partial V/\partial \varphi = 0 \), and we take \( \varphi = 0 \).

Solving the stationary condition \( \partial V(x, \varphi = 0)/\partial x = 0 \), we find

\[
x_{\pm} = \frac{M}{24} \left(8m - F_\phi \pm \sqrt{D}\right),
\]

where \( D = 16m^2 - 40F_\phi m + F_\phi^2 \). It is easy to show that \( x_+ \) and \( x_- \) corresponding to local minimum and maximum of the potential, respectively. For a fixed \( F_\phi \), the potential minimum exists if \( D > 0 \), in other words,

\[
m > \frac{5 + 2\sqrt{6}}{4} F_\phi.
\]

From Eq. \( (6) \), the deflection parameter is given by

\[
d = \frac{-2m + 4x_+ / M}{m - 6x_+ / M} = \frac{2(4m + F_\phi - \sqrt{D})}{3(4m - F_\phi + \sqrt{D})}.
\]

\(^3\) We have checked that there are no large scalar \( S \) mass terms induced by loop corrections in the theory.
The deflection parameter reaches its maximum value \(d_{\text{max}}\) in the limit \(m \to \frac{5 + 2\sqrt{6}}{4} F_{\phi}\), and

\[
d_{\text{max}} = \frac{2(3 + \sqrt{6})}{3(2 + \sqrt{6})} \approx 0.816.
\] (13)

Squared masses of two real scalar fields in \(S = (x + iy)/\sqrt{2}\) are found to be

\[
m_x^2 = 8\frac{\sqrt{Dx_+}}{M},
\]

\[
m_y^2 = \frac{2}{3} \left(24mF_{\phi} + (2m - F_{\phi})\sqrt{D} + D^2\right),
\] (14)

which are roughly of order \(m^2\). Through numerical calculation, we find \(m_x \approx 0.24 F_{\phi}\) and \(m_y \approx 6.3 F_{\phi}\) for \(m\) very close to its minimum value leading to \(d = 0.81\).

The scalar potential of Eq. (8), in fact, has a SUSY minimum at \(S = 0\), where the potential energy is zero, and the minimum at \(x_+\) we have discussed is a local minimum. In the Appendix A, we estimate the decay rate of the local minimum to the true SUSY minimum and find it is sufficiently small for \(F_{\phi} \ll M\).

### IV. SPARTICLE MASS SPECTRUM

We first give general formulas for sparticle masses in the deflected anomaly mediation with non-zero deflection parameter \(d\). Following the method developed in Ref. [14] (see also Ref. [5]), we can extract the sparticle mass formulas from the renormalized gauge couplings \((\alpha_i(\mu, S))\) and the supersymmetric wave function renormalization coefficients \((Z_I(\mu, S))\) at the renormalization scale \(\mu\) and the messenger scale \(S\). With \(F_S/S = dF_{\phi}\), the gaugino masses \((M_i)\) and sfermion masses \((\bar{m}_I)\) are given by

\[
\frac{M_i}{\alpha_i(\mu)} = \frac{F_{\phi}}{2} \left(\frac{\partial}{\partial \ln \mu} - d \frac{\partial}{\partial \ln |S|}\right) \alpha_i^{-1}(\mu, S),
\]

\[
\bar{m}^2_I(\mu) = -\frac{|F_{\phi}|^2}{4} \left(\frac{\partial}{\partial \ln \mu} - d \frac{\partial}{\partial \ln |S|}\right)^2 \ln Z_I(\mu, S).
\] (15)

For a simple gauge group, the gauge coupling and the wave function renormalizations are given by

\[
\alpha_i^{-1}(\mu, S) = \alpha_i^{-1}(\Lambda_{\text{cut}}) + \frac{b_i - N_i}{4\pi} \ln \left(\frac{S^\dagger S}{\Lambda_{\text{cut}}^2}\right) + \frac{b_i}{4\pi} \ln \left(\frac{\mu^2}{S^\dagger S}\right),
\] (16)

\[
Z_I(\mu, S) = \sum_i Z_I(\Lambda_{\text{cut}}) \left(\frac{\alpha_i(\Lambda_{\text{cut}})}{\alpha_i(S)}\right)^{\frac{2\zeta_i}{\zeta_i - N_i}} \left(\frac{\alpha_i(S)}{\alpha_i(\mu)}\right)^{\frac{2\zeta_i}{\zeta_i}},
\] (17)
where $\Lambda_{\text{cut}}$ is the ultraviolet cutoff, $b_i$ are the beta function coefficients for different groups, $c_i$ are the quadratic Casimirs, $N_i$, the Dynkin indices of the corresponding messenger fields (for example, $N_i = 1$ for a vector-like pair of messengers of a fundamental representation under $SU(N)$ gauge group), and the sum is taken corresponding to the representation of the sparticles under the SM gauge groups. Substituting them into Eq. (15), we obtain

$$M_i(\mu) = \frac{\alpha_i(\mu)}{4\pi} F_\phi (b_i + d N_i),$$

(18)

$$\tilde{m}_I^2(\mu) = \sum_i 2 c_i \left( \frac{\alpha_i(\mu)}{4\pi} \right)^2 |F_\phi|^2 b_i G_i(\mu, S),$$

(19)

where

$$G_i(\mu, S) = \left( \frac{N_i \xi_i^2 + \frac{N_i^2}{b_i^2} (1 - \xi_i^2)}{b_i} \right) d^2 + 2 \frac{N_i}{b_i} d + 1$$

(20)

with

$$\xi_i \equiv \frac{\alpha_i(S)}{\alpha_i(\mu)} = \left[ 1 + \frac{b_i}{4\pi} \alpha_i(\mu) \ln \left( \frac{S^2}{S^2} \right) \right]^{-1}.$$

(21)

In the limit $d \to 0$, the pure AMSB results are recovered and Eq. (19) leads to the mass squared negative for an asymptotically non-free gauge theory ($b_i < 0$). This result causes the tachyonic slepton problem in the pure AMSB scenario.

After integrating the messengers out, the scalar mass squared at the messenger scale is given by (taking $\xi_i = 1$)

$$\tilde{m}_I^2(S) = \sum_i 2 c_i \left( \frac{\alpha_i(S)}{4\pi} \right)^2 |F_\phi|^2 \left[ N_i d^2 + 2 N_i d + b_i \right];$$

(22)

where the first, the second and the third terms in the brackets correspond to pure GMSB, mixed GMSB and AMSB, and pure AMSB contributions, respectively. The sign of the second term is proportional to $d$, so that the sign of the deflection parameter results in different sparticle mass spectrum. The case $d < 0$ has been investigated in Ref. [5] and the resultant sparticle mass spectrum at the electroweak scale is very unusual and colored sparticles tend to be lighter than color-singlet sparticles. On the other hand, the case $d > 0$ examined in Ref. [7] leads to the mass spectrum similar to the GMSB scenario. In the following, we consider the case $d > 0$ based on the simple model discussed in Section [III].
V. MINIMAL MODEL

From the above discussion, it is clear that to solve the tachyonic slepton problem, we need messenger fields which are non-singlet under $SU(2)_L \times U(1)_Y$. If we now look at the way type II seesaw formula for small neutrino mass is implemented [10], we find that we need a pair of $SU(2)_L$ triplet fields, $\Delta : (3, -1)$ and $\Delta : (3, +1)$, which can play the dual role of both generators of neutrino masses as well as messenger fields.

To see their role in the neutrino sector, we add to the MSSM superpotential the following couplings of the triplets to the lepton doublets ($L_i$) and the up-type Higgs doublet ($H_u$)

$$W_{\text{seesaw}} = Y_{ij} L_i \Delta L_j + \lambda H_u \Delta \bar{H}_u,$$

(23)

where $i, j$ denotes the generation index, and $Y_{ij}$ is Yukawa coupling. If they couple to the singlet field $S$ discussed above as:

$$W_{\text{mess}} = S \text{ tr } [\Delta \Delta],$$

(24)

then once $\langle S \rangle \neq 0$, it will give heavy mass to the triplets. Integrating out the heavy messengers with mass $M_{\text{mess}} = \langle S \rangle$, this superpotential leads to light neutrino mass matrix $M_\nu \sim Y_{ij} \lambda \langle H_u \rangle^2 / M_{\text{mess}}$. This is the type II seesaw mechanism. If the messenger scale lies around the intermediate scale $M_{\text{mess}} = 10^{13-14}$ GeV, the seesaw mechanism provides the correct scale for light neutrino masses with $Y_{ij} \lambda$ of order one.

Note that since $F_S \neq 0$, the triplets can also serve as messenger superfields as in usual GMSB models and make additional contributions to slepton masses. In this minimal case, with a given $d$ and the formulas in Eq. (18)-(21), we now calculate the sparticle mass spectrum including the effects of AMSB and anomaly deflection. The beta function parameters needed for this purpose are: $(b_1, b_2, b_3) = (-33/5, -1, +3)$, $(N_1, N_2, N_3) = (18/5, 4, 0)$. Neglecting the effects of Yukawa couplings $^4$, the sparticle masses (in GeV) evaluated at $\mu = 500$ GeV are depicted in Fig. 1 as a function of the messenger scale $\log_{10}(M_{\text{mess}}/\text{GeV})$. Here, we have taken $d = 0.81$, $F_\phi = 25$ TeV, and the standard model gauge coupling constants at the Z-pole as $\alpha_1(m_Z) = 0.0168$, $\alpha_2(m_Z) = 0.0335$ and $\alpha_3(m_Z) = 0.118$. Since the Higgs triplet pair do not carry color quantum

$^4$ In general, there are Yukawa mediation contributions to the $SU(2)_L$ doublet slepton mass due to the coupling $Y_{ij} L_i L_j \Delta$. In this paper, we consider the case in which $Y_{ij} \leq 0.1$ by adjusting the seesaw scale and also parameter $\lambda$, so that the Yukawa mediation contributions are negligible.
number, the gluino mass still stays on the AMSB trajectory and does not depend on the messenger scale as showed in the Fig. 1. Note that for the messenger scale $M_{\text{mess}} \gtrsim 10^{14}$ GeV, the bino becomes the lightest super particle (LSP) and the bino like neutralino would be the candidate of the dark matter in our scenario [17]. For a small $\tan \beta$, annihilation processes of bino like neutralinos are dominated by p-wave and since this annihilation process is not so efficient, the resultant relic density tends to exceed the upper bound on the observed dark matter density. This problem can be avoided, if the neutralino is quasi-degenerate with the next LSP slepton and the co-annihilation process between the LSP neutralino and the next LSP slepton can lead to the right dark matter density. It is very interesting that our results show this degeneracy happening at $M_{\text{mess}} \simeq 10^{14}$ GeV, which is, in fact, the correct seesaw scale.

In the simple superpotential of singlet discussed in Section III, the messenger scale is given by $M_{\text{mess}} = \langle S \rangle \sim \sqrt{F_\phi M}$. To obtain $M_{\text{mess}} \sim 10^{13} - 10^{14}$ GeV with $F_\phi = O(10)$ TeV, we can specify the superpotential in Eq. (7) as

$$W \sim -mS^2 + \frac{S^4}{M_{Pl}}$$

with $\eta \sim 10^{-3} - 10^{-5}$, where $M_{Pl}$ is the Planck scale.

VI. MINIMAL MODEL WITH GRAND UNIFICATION

The messengers we have introduced in the minimal model are $SU(3)_c$ singlets, and the existence of such particles below the grand unification scale $M_{\text{GUT}} \sim 10^{16}$ GeV spoils the successful gauge coupling unification in MSSM. As is well-known, the gauge coupling unification can be kept if the messenger fields introduced are in the $SU(5)$ GUT multiplets. There are two possibilities for such messengers that play two different roles in the neutrino sector by the seesaw mechanism. One is to introduce the messengers of $15 + \overline{15}$ multiplets under $SU(5)$, which include $\Delta$ and $\overline{\Delta}$ as submultiplets. The other possibility is to introduce $24$ multiplets [18].

Let us first consider the $15$ and $\overline{15}$ case in the $SU(5)$ GUT model. We introduce the superpotentials,

$$W_{\text{mess}} = S\overline{T}T,$$

$$W_{\text{seesaw}} = Y_{ij} \overline{5}_i \overline{5}_j T + \lambda 5_H 5_H \overline{T},$$

(26)

where $T$ and $\overline{T}$ are $15$ and $\overline{15}$ multiplets. After integrating the heavy messengers out, we obtain the light neutrino mass matrix as $M_\nu \sim \langle 5_H \rangle^2 / \langle S \rangle$ through the type II seesaw mechanism.
Sparticle masses can be evaluated in the same manner as before, but in this case, \( N_1 = N_2 = N_3 = 7 \). The resultant sparticle masses at \( \mu = 500 \) GeV are depicted in Fig. 2 as a function of the messenger scale \( \log_{10}[S/\text{GeV}] \). Here, we have taken \( d = 0.48 \) and \( F_\phi = 25 \) TeV. The bino becomes the LSP, degenerating with right-handed sleptons for the messenger scale \( M_{\text{mess}} \sim 10^{13} \) GeV.

In the case of 24 multiplets \((\Sigma)\), the relevant superpotential is given by

\[
W_{\text{mess}} = S \, \text{tr}[\Sigma^2],
\]
\[
W_{\text{seesaw}} = Y_i \, \overline{\mathbf{5}}_i \, \Sigma \, 5_H.
\]

After integrating out the heavy 24, the light neutrino mass matrix is given by \( M_\nu \sim Y_i Y_j \langle 5_H \rangle^2 / \langle S \rangle \). Note that the rank of this matrix is one. We need to introduce at least two 24 messengers to incorporate the realistic neutrino mass matrix. As an example, we consider two 24 messengers with the same masses. We evaluate sparticle masses with \( N_1 = N_2 = N_3 = 2 \times 5 = 10 \) in this case. The resultant sparticle masses at \( \mu = 500 \) GeV are depicted in Fig. 3 as a function of the messenger scale \( \log_{10}[S/\text{GeV}] \). Here, we have taken \( d = 0.35 \) and \( F_\phi = 25 \) TeV. The bino becomes the LSP, degenerate with right-handed sleptons for the messenger scale \( M_{\text{mess}} \sim 10^{13} \) GeV.

VII. CONCLUSION

In conclusion, we have pointed out that a minimal extension of MSSM needed to explain small neutrino masses via the seesaw mechanism can also cure the tachyonic slepton mass problem of anomaly mediated supersymmetry breaking. We have presented the sparticle spectrum for these models and shown that they can preserve the unification of gauge couplings. We find it interesting that the same mechanism that explains the smallness of neutrino masses also cures the tachyonic slepton problem of AMSB.

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APPENDIX A: LIFETIME OF THE LOCAL MINIMUM

The scalar potential in Section III $V(S)$ has the global SUSY minimum at the origin, and the minimum we have discussed is a local minimum. If our world is trapped in the local minimum, it will eventually decay into the SUSY minimum. The life time of the local minimum should be sufficiently long, at least, longer than the age of the universe, $\tau_U \sim 4.3 \times 10^{17}$ s for our model to be viable. Here we estimate the decay rate of the false vacuum within the parameters of our model.

In our calculation, the scalar potential is treated in the triangle approximation [19]. A schematic picture of the scalar potential is depicted in Fig. 4. Let us take the path in the direction of $\Re[S]$: climbing up from the local minimum at $\Re[S] = \sqrt{x_+}$ to the local maximum at $\Re[S] = \sqrt{x_-}$, then rolling down to the SUSY minimum at $S = 0$. In the triangle approximation, parameters characterizing the potential are

$$\Delta V_\pm, \Delta \Phi_\pm, \quad (A1)$$

where $\Delta V_\pm$ and $\Delta \Phi_\pm$ are the difference of potential height and the distances between the local and global minima and potential barrier. Following Ref. [19], we define

$$c \equiv \frac{\Delta V_- \Delta \Phi_+}{\Delta V_+ \Delta \Phi_-} \quad (A2)$$

and the decay rate per unit volume is estimated as $\Gamma/V \sim e^{-B}$ with

$$B = \frac{32\pi^2}{3} \left(1 + \frac{c}{\sqrt{1 + c - 1}}\right)^4 \Delta V_+^4. \quad (A3)$$

The consistency condition to apply the triangle approximation is given by [19]

$$\left(\frac{\Delta V_-}{\Delta V_+}\right)^\frac{1}{2} \geq \frac{2 \Delta \Phi_-}{\Delta \Phi_- - \Delta \Phi_+}. \quad (A4)$$

For the scalar potential analyzed in Section III

$$\Delta \Phi_+ = \sqrt{x_+} - \sqrt{x_-}, \quad \Delta \Phi_- = \sqrt{x_-},$$

$$\Delta V_+ = V(x_+, 0) - V(x_+, 0), \quad \Delta V_- = V(x_-, 0). \quad (A5)$$

In order to get the deflection parameter as large as possible, let us consider the case that the local minimum and maximum points are very close, namely, $\Delta \Phi_+$ and $\Delta V_+$ are very small. In this case, the condition Eq. (A4) is satisfied, and we can apply the triangle approximation. With a small parameter $0 < \epsilon \ll 1$, we parameterize

$$m = \frac{5 + 2\sqrt{6}}{4} F \phi (1 + \epsilon). \quad (A6)$$
In the limit $\epsilon \rightarrow 0$, the local minimum and maximum collide and the local minimum disappears. The deflection parameter is approximately described as

$$d \simeq d_{\text{max}} - \frac{\sqrt{12 + 5\sqrt{6}}}{3}\epsilon^\frac{3}{2} \simeq d_{\text{max}} - 1.64\epsilon^\frac{3}{2}. \quad (A7)$$

The straightforward calculations give the following results:

$$\Delta \Phi_+ \simeq \sqrt{\frac{12 + 5\sqrt{6}}{54 + 24\sqrt{6}}} \sqrt{F_\phi M\epsilon^\frac{3}{2}},$$

$$\Delta \Phi_- \simeq \frac{1}{2}\sqrt{\frac{9 + 4\sqrt{6}}{6}} \sqrt{F_\phi M},$$

$$\Delta V_+ = \frac{(12 + 5\sqrt{6})^{3/2}}{27} F_\phi^3 M\epsilon^\frac{3}{2},$$

$$\Delta V_- = \frac{1107 + 452\sqrt{6}}{288} F_\phi^3 M. \quad (A8)$$

Also, we find

$$B \simeq \frac{\pi^2 128 (12 + 5\sqrt{6})^{3/2}}{9(6937 + 2832\sqrt{6})} M^{\frac{3}{2}} \epsilon^\frac{3}{2} \simeq 1.21 \times \frac{M}{F_\phi}\epsilon^\frac{3}{2}. \quad (A9)$$

Recalling that the messenger scale is roughly given by $M_{\text{mess}} \sim \sqrt{F_\phi M}$ and $F_\phi \simeq 10$ TeV to obtain sparticle masses around 100 GeV - 1 TeV, we can rewrite $B$ as

$$B \simeq 1.21 \left(\frac{M_{\text{mess}}}{F_\phi}\right)^2 \epsilon^\frac{3}{2} = 1.21 \times 10^{20} \left(\frac{M_{\text{mess}}/10^{14} \text{ GeV}}{F_\phi/10 \text{ TeV}}\right)^2 \epsilon^\frac{3}{2}. \quad (A10)$$

For the parameters chosen in Fig. 1, $M_{\text{mess}} \simeq 10^{14}$ GeV, $F_\phi = 25$ TeV, $d = 0.81$ corresponding $\epsilon \simeq 1.57 \times 10^{-5}$, we find $B \simeq 1.20 \times 10^{12}$. The life time of the local minimum is extremely long.

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FIG. 1: Sparticle masses at $\mu = 500$ GeV as a function of at the messenger scale in the type II seesaw model with one pair of $SU(2)_L$ triplet messengers. Here $d = 0.81$ and $F_\phi = 25$ TeV have been taken. Each line corresponds to the left-handed squark ($m_{\tilde{Q}}$), the gluino ($M_3$), the right-handed up-squark ($m_{\tilde{u}}$), the right-handed down-squark ($m_{\tilde{d}}$), the left-handed slepton ($m_{\tilde{L}}$), the Wino ($M_2$), the bino ($|M_1|$), and the right-handed slepton ($m_{\tilde{e}}$) from above at $M_{\text{mess}} = 10^3$ GeV. Two lines of $m_{\tilde{u}}$ and $m_{\tilde{d}}$ are overlapping and not distinguishable. For the messenger scale $M_{\text{mess}} \gtrsim 10^{14}$ GeV, the bino becomes the lightest super particle.
FIG. 2: Sparticle masses at $\mu = 500$ GeV as a function of at the messenger scale in the type II seesaw model with one pair of $\mathbf{15} + \mathbf{15}$ messengers. Here $d = 0.48$ and $F_\phi = 25$ TeV have been taken. Each line corresponds to $M_3, m_{\tilde{Q}}, m_{\tilde{u}^c}, m_{\tilde{d}^c}, m_{\tilde{L}}, M_2, |M_1|,$ and $m_{\tilde{e}^c}$ from above at $M_{\text{mess}} = 10^3$ GeV. Two lines of $m_{\tilde{u}^c}$ and $m_{\tilde{d}^c}$ are overlapping and not distinguishable. For the messenger scale $M_{\text{mess}} \gtrsim 10^{13}$ GeV, the bino becomes the LSP.
FIG. 3: Sparticle masses at $\mu = 500$ GeV as a function of at the messenger scale in the model with two pairs of 24 messengers. Here $d = 0.35$ and $F_\phi = 25$ TeV have been taken. Each line corresponds to $M_3$, $m_{\tilde{Q}}$, $m_{\tilde{u}}$, $m_{\tilde{d}}$, $m_{\tilde{L}}$, $M_2$, $|M_1|$, and $m_{\tilde{e}}$ from above at $M_{\text{mess}} = 10^3$ GeV. Two lines of $m_{\tilde{u}}$ and $m_{\tilde{d}}$ are overlapping and not distinguishable. For the messenger scale $M_{\text{mess}} \simeq 10^{13}$ GeV, the bino becomes the LSP.
FIG. 4: Schematic picture of the scalar potential $V(S)$ as a function of the real part of $S$. 