Lattice calculation of the heavy quark potential at non-zero temperature

A. Bazavov\textsuperscript{a}, Y. Burnier\textsuperscript{b}, P. Petreczky\textsuperscript{c}

\textsuperscript{a}Department of Physics and Astronomy, University of Iowa, 203 Van Allen Hall, Iowa City, Iowa 52242-1479
\textsuperscript{b}Laboratory of Particle Physics and Cosmology, École polytechnique fédérale de Lausanne, BSP 730, Rue de la Sorge, CH-1015 Lausanne
\textsuperscript{c}Physics Department, Brookhaven National Laboratory, Upton, NY 11793, USA

Abstract

We calculated the real and imaginary parts of the static quark anti-quark potential at $T > 0$ in 2+1 flavor QCD using correlators of Wilson lines in Coulomb gauge and lattices with temporal extent $N_\tau = 12$. We find that the real part of the potential is larger than the singlet free energy but smaller than the zero temperature potential. The imaginary part of the potential is similar in size to the perturbative HTL result.

1. Introduction

There has been significant interest in studying quarkonium properties at non-zero temperature since the seminal paper by Matsui and Satz\textsuperscript{[1]} (for a recent review see e.g. Ref.\textsuperscript{[2]}). It has been conjectured that the thermal medium will modify the heavy quark potential, eventually leading to the dissolution of the heavy quarkonium states\textsuperscript{[1]}. The problem of dissolution of quarkonium states at high temperatures can be rigorously formulated in terms of spectral functions. Early attempts to calculate the quarkonium spectral functions on the lattice have been presented in Refs.\textsuperscript{[3]}. However, extraction of the spectral functions from lattice results on Euclidean correlation functions is quite difficult\textsuperscript{[4]} and one should also be careful with cutoff effects in the spectral functions extracted from the lattice\textsuperscript{[5]}. Furthermore it is difficult to observe the melting of bound states at high temperature in Euclidean correlators due to the fact that their time extent is limited to $< 1/(2T)$ (see e.g. discussions in Ref.\textsuperscript{[2, 6]}). Alternative methods to determine the spectral functions could be useful.

The effective field theory framework for heavy quark bound states, namely pQNRCD could be a useful tool for calculating quarkonium spectral functions\textsuperscript{[7]}. The effective field theory approach allows a rigorous definition of the concept of the static quark anti-quark potential both at zero and non-zero temperatures. One of the main outcomes of the effective field theory analysis is the finding that at non-zero temperature the potential has also an imaginary part, which has important consequences for the dissolution of the quarkonium states. While pNRQCD is formulated in the weak coupling framework it is possible to extend it to the non-perturbative regime. For example, if the binding energy is the smallest scale in the problem all the other scales, like the thermal scales, the inverse size of the bound state and $\Lambda_{QCD}$ can be integrated out. In this case the potential is identical to the energy of a static $Q\bar{Q}$ pair and can be calculated non-perturbatively on the lattice. Therefore, in what follows we will use the terms potential and static energy interchangeably. If one further neglects the dipole interactions one gets the generalization of the
simple potential model to the case of high temperatures [8]. In Ref. [9] it has been suggested to extract the energy of a static $Q\bar{Q}$ pair using the spectral decomposition of the temporal Wilson loops at non-zero temperature. In this contribution we calculate the quark anti-quark energy in 2+1 flavor QCD based on this idea using lattices with temporal extent $N_T = 12$. Previously we calculated the real part of the static energy using $N_T = 6$ lattices [10].

2. Numerical results

In lattice QCD calculations the static $Q\bar{Q}$ energy is extracted from Wilson loops $W(r, \tau)$. At large Euclidean time separations the exponential decay of the Wilson loops is governed by the static energy or potential, $W(r, \tau) \sim \exp(-V(r) \tau)$. More generally one can write a spectral decomposition for the Wilson loops [9]

$$W(r, \tau) = \int_{-\infty}^{\infty} d\omega \sigma(r, \omega) e^{-\omega \tau}.$$  (1)

At zero temperature the spectral function is proportional to $\delta(\omega - V(r))$ at sufficiently low $\omega$. At non-zero temperature the delta function becomes a skewed Lorentzian with the width related to the imaginary part of the potential [15]. Lattice calculations of the potential $T > 0$ using Eq. (1) and maximum entropy method (MEM) in SU(3) gauge theory have been presented in Ref. [9].

We would like to calculate the potential in 2+1 flavor QCD.

We use gauge configurations in 2+1 flavor QCD generated on $48^3 \times 12$ lattices using highly improved staggered quark (HISQ) action [12] using physical quark strange quark mass and light quark masses corresponding to pion mass of 160 MeV in the continuum limit. We consider two values of the gauge coupling $\beta = 10/g^2 = 7.28$ and 7.50 corresponding to temperatures $T = 250$ MeV and 305 MeV. The choice of the lattice parameters is discussed in Ref. [14].

There are two challenges when one tries to extract the potential using $N_T = 12$ lattices. First, it is not possible to obtain reliable results using MEM on $N_T = 12$ lattices. The second problem is the poor overlap of simple (unsmeread) Wilson loops with the ground state $Q\bar{Q}$ energy. To deal with the second problem smeared gauge fields are used in spatial links when constructing Wilson loops on the lattice. Alternatively, one can fix the Coulomb gauge and calculate the
correlation functions of two temporal Wilson lines separated by distance $r$ without connecting them by spatial links [11]. Both method have been used in the past at zero temperature and very recently also at $T > 0$ [13]. The $\tau$ dependence of the smeared Wilson loop and Wilson line correlators is very similar [13], and therefore, in this study we will use Wilson line in Coulomb gauge. To deal with the first problem we will make use of the analytic calculations of the spectral function of Wilson line correlators in Coulomb gauge in Hard Thermal Loop (HTL) perturbation theory [16]. In Ref. [16] the spectral functions corresponding to Wilson loops and correlators of Wilson lines in Coulomb gauge has been calculated. For the Wilson loops the spectral function is quite complicated and the ground state peak is not the dominant structure. In the case of the Wilson line correlators the spectral function is very simple, it consist of a single peak, though the peak is asymmetric and has long tails [16]. The simple structure of the spectral functions makes the correlators of Wilson lines in Coulomb gauge suitable for extraction of the potential at $T > 0$.

It is useful to analyze the $\tau$-dependence of this correlator in terms of the effective potential $V_{\text{eff}}(r, \tau) = -\ln(W(r, \tau)/\tau)$. For $T = 0$, $V_{\text{eff}}(r, \tau)$ reaches a plateau at sufficiently large $\tau$ which is given by the static $Q\bar{Q}$ potential. In Fig. 1 we show the $V_{\text{eff}}(r, \tau)$ at zero and finite temperature in lattice calculations. We see that at zero temperature the plateau is reached already for $\tau T \approx 0.5$, but for $T > 0$ the effective potential always decreases without reaching a plateau. This is due to the width of the peak, i.e. to the imaginary part of the potential. We calculated the effective potential using the results on spectral functions in HTL perturbation theory [16] which show the same qualitative $\tau$ dependence as observed in lattice calculations. Therefore, we extract the real and imaginary part of the potential by fitting the $\tau$-dependence of the lattice results on $V_{\text{eff}}(r, \tau)$ by the two parameter Ansatz $\sigma^{\text{HTL}}(\lambda(\omega - E))$ with $E$ and $\lambda$ being the fit parameters that determined the peak position and its width. The fits are shown are also shown in Fig. 1 as line-points. We do not attempt to fit the points and small $\tau$ as the HTL Ansatz is not expected to work there. The results of this analysis are are shown in Figs. 2 and 3. The errors shown in the figure are mostly systematic and arise from the variation of the fit interval. We compared the real part of the potential with the zero temperature result as well as with the singlet free energy. The real part of the potential is equal or larger than the singlet free energy but is always smaller than the zero temperature potential. Furthermore, it decreases with increasing temperature in qualitative agreement with the picture of color screening. The central value of the imaginary part of the potential turns out to be larger than the imaginary part of the potential in HTL perturbation
theory though withing errors it is the same.

3. Conclusion

We have extracted the real and imaginary parts of the potential using lattice results on the Wilson line correlators in Coulomb gauge on $N_\tau = 12$ lattices and a fit Ansatz motivated by HTL perturbation theory. Our results on the real part of the potential are in agreement with our previous findings obtained using $N_\tau = 6$ lattices and more simplistic ad-hoc form for the corresponding spectral function \[10\]. We were also able to obtain the imaginary part of the potential which turns out to be larger than in HTL perturbation theory.

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References

[1] T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416.
[2] A. Mocsy, P. Petreczky and M. Strickland, Int. J. Mod. Phys. A 28, 1340012 (2013) [arXiv:1302.2180 [hep-ph]].
[3] T. Umeda et al., Eur. Phys. J. C 39S1 (2005) 9; M. Asakawa and T. Hatsuda, Phys. Rev. Lett. 92, 012001 (2004); S. Datta et al., Phys. Rev. D 69, 094507 (2004)
[4] I. Wetzorke et al., Nucl. Phys. Proc. Suppl. 106, 510 (2002).
[5] E. Follana et al., Phys. Rev. D 75, 054502 (2007)
[6] P. Petreczky, Phys. Rev. D 85, 054502 (2012)
[7] N. Brambilla et al., Phys. Rev. D 75, 054503 (2012); Phys. Rev. D 85, 054503 (2012)
[8] P. Petreczky, C. Miao and A. Mocsy, Nucl. Phys. A 855, 125 (2011)
[9] A. Rothkopf et al, PoS LAT 2009, 162 (2009); Phys. Rev. Lett. 108, 162001 (2012); Y. Akamatsu and A. Rothkopf, Phys. Rev. D 85, 105011 (2012)
[10] A. Bazavov and P. Petreczky, Nucl. Phys. A904-905 2013, 599c (2013) [arXiv:1210.6314 [hep-lat]].
[11] C. Aubin et al., Phys. Rev. D70, 094505 (2004)
[12] C. Aubin et al., Phys. Rev. D75, 054502 (2007)
[13] E. Follana et al., Phys. Rev. D75, 054502 (2007)
[14] A. Bazavov et al., Phys. Rev. D 85, 054503 (2012); J. Phys. Conf. Ser. 230, 012014 (2010)
[15] Y. Burnier and A. Rothkopf, Phys. Rev. D 86, 051503 (2012) [arXiv:1208.1899 [hep-ph]]
[16] Y. Burnier and A. Rothkopf, Phys. Rev. D 87, no. 11, 114019 (2013) [arXiv:1304.4154 [hep-ph]].
[17] M. Laine, O. Philipsen, P. Romatschke and M. Tassler, JHEP 0703, 054 (2007) [hep-ph/0611300]