ON VASSILIEV INVARIANTS NOT COMING FROM SEMISIMPLE LIE ALGEBRAS

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Abstract

We prove a refinement of Vogel’s statement that the Vassiliev invariants of knots coming from semisimple Lie algebras do not generate all Vassiliev invariants. This refinement takes into account the second grading on the Vassiliev invariants induced by cabling of knots. As an application we get an amelioration of the actually known lower bounds for the dimensions of the space of Vassiliev invariants.

Keywords: knots, Vassiliev invariants, Lie superalgebras

Introduction

Vogel found a primitive element in the Hopf algebra of chord diagrams that cannot be detected by semisimple Lie superalgebras with Casimir ([11]). This element is shown to be non-trivial thanks to an injection of a certain algebra Λ into the space of Chinese characters with two univalent vertices. As a consequence of this result not all Vassiliev invariants of knots come from semisimple Lie superalgebras with Casimir.

In this paper we will use Vogel’s methods to construct Chinese characters that cannot be detected by semisimple Lie algebras. In contrast to Vogel’s element, these Chinese characters may have any given even number of univalent vertices. To prove that they are non-trivial, we evaluate them with a weight system associated to the Lie superalgebra $D(2, 1, α)$. For these computations we will make use of the work of Kricker ([7]) that simplified and generalized a part of Bar-Natan and Garoufalidis’s proof of the Melvin-Morton-Rozansky conjecture ([3]). As a consequence of our result for every even $k$ there is a Vassiliev invariant that is an eigenvector for the $n$-th cabling of knots modulo invariants of lower degree with eigenvalue $n^k$, but that is not a linear combination of Vassiliev invariants coming from semisimple Lie algebras.
1 The Main Result

Let the ground field always be the field $\mathbb{C}$ of complex numbers. We refer to [1] for the definition of the space $\mathcal{A}$ generated by Chinese character diagrams and the so-called (STU)-relations, of the space $\mathcal{B}$ generated by Chinese characters and the (IHX)- and (AS)-relations, and of the isomorphism $\chi : \mathcal{B} \rightarrow \mathcal{A}$. Let us denote the universal weight system associated to a Lie superalgebra $L$ with Casimir element $\omega$ by $W_{L,\omega} : \mathcal{A} \rightarrow U(L)$. It takes values in the center of the universal enveloping algebra $U(L)$ of $L$. If $\rho : U(L) \rightarrow \text{End}(M)$ is an irreducible finite-dimensional representation of $L$, then we get a $\mathbb{C}$-algebra morphism $W_{L,\rho,\omega} : \mathcal{A} \rightarrow \mathbb{C}$ such that $\rho \circ W_{L,\omega}(.) = W_{L,\rho,\omega}(.)\text{id}_M$. Important classes of weight systems consist of linear combinations of the maps $W_{L,\rho,\omega}$ where $L$ is a Lie (super)algebra with Casimir.

Now we can formulate our main result.

**Theorem 1** For $d = 15$ or $d \geq 17$ and all even $k \geq 2$ there exists a non-zero linear combination $D_{k+d,k} \in \mathcal{B}$ of connected Chinese characters of degree $k + d$ and with $k$ univalent vertices such that $w(D_{k+d,k}) = 0$ for all weight systems $w$ coming from semisimple Lie algebras.

The theorem implies a conjecture on marked surfaces stated in Remark 2 of [2]. Therefore it allows to improve the actually known lower bounds of the space of Vassiliev invariants ([2], [6]). The special case $k = 2$ of Theorem 1 is implied by [11], Theorem 7.4 (the bounds for $d$ come from Proposition 2 of this paper). In the case $k = 2$, the result holds more generally for all semisimple Lie superalgebras with Casimir. It would be nice to prove Theorem 1 in this generality also for $k \geq 4$. Let us give an equivalent formulation of Theorem 1 on the level of Vassiliev invariants.

**Corollary 1** For $d = 15$ or $d \geq 17$ and all even $k \geq 2$ there exists a primitive Vassiliev invariant $v \in V_{k+d,k}$ that is not a linear combination of Vassiliev invariants coming from semisimple Lie algebras.

**Proof:** Let $D_{k+d,k}$ be the element of $\mathcal{B}$ considered in Theorem 1. Choose some complement $N$ of $\mathbb{C} \cdot D_{k+d,k}$ in $\mathcal{B}$ that respects the two gradings on $\mathcal{B}$ given by half of the number of vertices and the number of univalent vertices of Chinese characters and such that $\overline{\chi}(N)$ contains the space of decomposable elements of $\mathcal{A}$. Define a weight system $w$ that vanishes on $\overline{\chi}(N)$ and is 1 on $\overline{\chi}(D_{k+d,k})$. Then $w$ is a primitive element of $\mathcal{A}$*. So the weight system $w$ integrates to a primitive Vassiliev invariant $v$ of degree $k + d$. By Theorem 3 of [8] the element $\overline{\chi}(D_{k+d,k})$ is an eigenvector for the maps $\psi^n$ from $\mathcal{B}$ with eigenvalue $n^k$ since $D_{k+d,k}$ has $k$ univalent vertices. Now Exercise 3.14 of [1] implies $v \in V_{k+d,k}$. $\square$

The rest of the paper is devoted to the proof of Theorem 1.
2 Sequences of Weight Systems

Let $L$ be one of the following simple finite-dimensional Lie superalgebras (see [4, 5]):

\[ \text{sl}(m, n) \ (m \neq n), \text{osp}(m, 2n), E_6, E_7, E_8, F_4, G_2, D(2, 1, \alpha), F(4), G(3). \]

Choose a Cartan subalgebra $H$ of $L$ and a system of positive roots $\Delta^+ \subset H^*$. For a root $\beta \neq 0$ of $L$ let

\[ L_\beta = \{ v \in L \mid [h, v] = \beta(h)v \text{ for all } h \in H \}. \quad (1) \]

Let $x_\beta$ be a generator of the one-dimensional space $L_\beta$ ($\beta \in \Delta^+$). For $\lambda \in H^*$ let $I_\lambda$ be the left ideal in $U(L)$ generated by the elements $x_\beta$ for $\beta \in \Delta^+$ and $h - \lambda(h)$ for $h \in H$. The module $V_\lambda := U(L)/I_\lambda$ is called the Verma module of weight $\lambda$. Denote the image of 1 in $V_\lambda$ by $v_\lambda$. Let $\{y_1, \ldots, y_r\}$ be a basis of $\bigoplus_{\beta \in \Delta^+} L_\beta$. Then the Poincaré-Birkhoff-Witt basis theorem for $L$ (see [3], Section 3) implies that

\[ \{y_1^{e_1} \cdots y_r^{e_r} \cdot v_\lambda \mid \deg y_i = 0 \Rightarrow e_i \geq 0, \ \deg y_i = 1 \Rightarrow e_i \in \{0, 1\}\} \quad (2) \]

is a basis of $V_\lambda$. The vector $v_\lambda \in V_\lambda$ is a basis of the one-dimensional space of vectors of weight $\lambda$ in $V_\lambda$; it generates $V_\lambda$ as a $U(L)$-module.

By Proposition 5.3 c) of [3] there exists an invariant, supersymmetric, regular bilinear form $<,>$ on $L$. Let $\omega \in L \otimes L$ be the Casimir element associated to $<,>$. Since for $a \in \mathcal{A}$ the element $W_{L,\omega}(a)$ belongs to the center of $U(L)$, the endomorphism of $V_\lambda$ induced by $W_{L,\omega}(a)$ is a scalar multiple of the identity.

Let $\rho : U(L) \rightarrow \text{End}(V)$ be a finite-dimensional irreducible representation with highest weight $\lambda$. Then $V$ is a quotient of $V_\lambda$ and we may determine $W_{L,\rho,\omega}(a)$ with computations in $V_\lambda$. This enables us to compare weight systems when the weight of the representation varies because we may use one and the same index set for the bases of the involved modules. Lemma 3.9 of [3] implies the following lemma.

**Lemma 1** Let $\rho_n$ be a sequence of finite-dimensional irreducible representations of $L$ with highest weight $n\lambda$. Then for a Chinese character diagram $D$ the values $W_{L,\rho_n,\omega}(D)$ depend polynomially on $n$. If $D$ has $k$ vertices lying on the oriented circle, then the degree of this polynomial is $\leq k$.

The Alexander-Conway weight systems of degree $k$ are those that vanish on diagrams that have at most $k - 1$ vertices on the oriented circle. By Lemma 3 the coefficient of $n^k$ in $W_{L,\rho_n,\omega} : \mathcal{A}_k \rightarrow \mathbb{C}$ is in the algebra of Alexander-Conway weight systems. The goal of [3] was to find an explicit expression for this highest coefficient.

For $v \in L_\beta$ we will denote $\deg v \in \{0, 1\}$ by $\deg \beta$. Since the restriction of the bilinear form $<,>$ to $H$ is nondegenerate, we identify $H$ and $H^*$ and get a bilinear form on $H^*$, also denoted by $<,>$. Then Theorem 3.11 of [3] states the following.

**Theorem 2 (Kricker)** For even $k \geq 2$ let $T_k$ be the diagram of degree $k$ from the right-hand side of Figure 3. Let $\rho_n$ be a sequence of finite-dimensional irreducible representations of $L$ with highest weight $n\lambda$. Then the coefficient of $n^k$ in
Because of frequent use we will denote the Lie superalgebra \( D_\alpha \). Let us give an explicit description of it. For \( (a,b,c) \) then

\[
\begin{align*}
\pi & = \sum_{(\epsilon_i) \in \{\pm 1\}^3} \epsilon_1 \epsilon_2 \epsilon_3 v_{\epsilon_1 \epsilon_2 \epsilon_3} \otimes v_{-\epsilon_1 - \epsilon_2 - \epsilon_3}. \\
\omega_\alpha &= (1 + \alpha)\omega_1 - \omega_2 - \alpha\omega_3 + \pi
\end{align*}
\]

3 Weight Systems Coming from \( D(2, 1, \alpha) \)

Because of frequent use we will denote the Lie superalgebra \( D(2, 1, \alpha) \) simply by \( D_\alpha \). Let us give an explicit description of it. For \( \alpha \in \mathbb{C} \setminus \{0, -1\} \) there exists a basis \( (E_i, H_i, F_i) \) \( (i = 1, 2, 3) \) of the even part \( (D_\alpha)_e \) and a basis \( (v_{\epsilon_1 \epsilon_2 \epsilon_3}) \) \( (\epsilon_i \in \{\pm 1\}) \) of the odd part \( (D_\alpha)_o \) such that

\[
\begin{align*}
[1][H_i, E_j] &= 2 \delta_{ij} E_j, \quad [H_i, F_j] = -2 \delta_{ij} F_j, \quad [H_i, H_j] = 0, \quad [E_i, F_j] = \delta_{ij} H_j, \\
[1][H_i, v_{\epsilon_1 \epsilon_2 \epsilon_3}] &= \epsilon_i v_{\epsilon_1 \epsilon_2 \epsilon_3}, \\
[E_i, v_{\epsilon_1 \epsilon_2 \epsilon_3}] &= \delta_{i, -1} \epsilon_i v_{\gamma_1 \gamma_2 \gamma_3} \quad \text{with} \quad \gamma_\nu = \epsilon_\nu \text{ for } \nu \neq i \text{ and } \gamma_i = 1, \\
[F_i, v_{\epsilon_1 \epsilon_2 \epsilon_3}] &= \delta_{i, 1} \epsilon_i v_{\gamma_1 \gamma_2 \gamma_3} \quad \text{with} \quad \gamma_\nu = \epsilon_\nu \text{ for } \nu \neq i \text{ and } \gamma_i = -1, \\
[v_{\epsilon_1 \epsilon_2 \epsilon_3}, v_{\gamma_1 \gamma_2 \gamma_3}] &= (\alpha + 1)\beta_2 \beta_3 G_i(\epsilon_1, \gamma_1) - \beta_1 \beta_3 G_2(\epsilon_2, \gamma_2) - \alpha \beta_1 \beta_2 G_3(\epsilon_3, \gamma_3),
\end{align*}
\]

where \( G_i(1, 1) = -E_i, \ G_i(1, -1) = G_i(-1, 1) = H_i/2, \ G_i(-1, -1) = F_i \) and \( \beta_i = \epsilon_i \delta_{i, -\gamma_i}. \)

In \( \mathbb{Z} \) a Lie superalgebra \( \tilde{D}(2, 1) \) over the ring \( R = \mathbb{Z}[a, b, c]/(a + b + c) \) is considered. Mapping \( (a, b, c) \) to \( (-\alpha - 1, 1, \alpha) \) we turn \( \mathbb{C} \) into an \( R \)-module and recover \( D_\alpha \) as \( \tilde{D}(2, 1) \otimes_R \mathbb{C} \). Let us choose generators

\[
\begin{align*}
e_1 = v_{-1, -1, -1}, \quad h_1 &= ((\alpha + 1)H_1 + H_2 + \alpha H_3)/2, \quad f_1 = v_{-111}
\end{align*}
\]

and \( e_i = E_i, \ h_i = H_i, \ f_i = F_i \) for \( i \in \{2, 3\} \) of \( D_\alpha \). Then one can check that our definition of \( D_\alpha \) agrees with the one given in Section 5 of \( \text{[1]} \).

We may choose a bilinear form \( \langle , \rangle_\alpha \) on \( D_\alpha \) as in Proposition 5.3 of \( \text{[2]} \), such that the matrix of the restriction of \( \langle , \rangle_\alpha \) to the Cartan subalgebra \( H \) with basis \( (H_1, H_2, H_3) \) is \( \text{diag}(2(1 + \alpha)^{-1}, -2, -2\alpha^{-1}) \). Let \( \omega_i = E_i \otimes F_i + H_i \otimes H_i/2 + F_i \otimes E_i \) and let

\[
\begin{align*}
\pi & = \sum_{(\epsilon_i) \in \{\pm 1\}^3} \epsilon_1 \epsilon_2 \epsilon_3 v_{\epsilon_1 \epsilon_2 \epsilon_3} \otimes v_{-\epsilon_1 - \epsilon_2 - \epsilon_3}. \\
\omega_\alpha &= (1 + \alpha)\omega_1 - \omega_2 - \alpha\omega_3 + \pi
\end{align*}
\]
is the Casimir element in $D_\alpha \otimes D_\alpha$ corresponding to $\langle \cdot, \cdot \rangle_\alpha$. The Casimir element $\Omega$ from Lemma 6.12 of [11] is mapped to $\omega_\alpha$ by the specialization of the parameters $a, b, c$.

Let $(H_1^*, H_2^*, H_3^*)$ be the basis of $H^*$ dual to $(H_1, H_2, H_3)$. We choose

$$\Delta^+ = \{2H_1^*, 2H_2^*, 2H_3^*\} \cup \left\{ \sum_{i=1}^3 \epsilon_i H_i^* \mid (\epsilon_i) \in \{(1, \pm 1, \pm 1)\} \right\}$$

(6)

as a positive root system of $D_\alpha$.

**Lemma 2** Let $\rho_{\alpha,n}$ be a sequence of finite-dimensional irreducible representations of $D_\alpha$ with highest weight $n\lambda$. Then for a Chinese character diagram $D$ the values $W_{D_\alpha, \rho_{\alpha,n}, \omega_\alpha}(D)$ depend polynomially on $n$ and $\alpha$. If $D$ has $k$ vertices lying on the oriented circle, then the degree in $n$ of this polynomial is $\leq k$.

**Proof:** The coefficients in $\omega_\alpha$ and the coefficients of the bracket of two basis vectors of $D_\alpha$ are polynomials of degree $\leq 1$ in $\alpha$. Now arguments from the proof of [7], Lemma 3.9 allow to conclude. □

Now consider the Chinese character $S_k$ with $k$ univalent vertices from the left-hand side of Figure 1. By definition the element $\chi(S_k)$ is the sum over the $k!$ diagrams in $\mathcal{A}$ that arise when the univalent vertices of $S_k$ are glued into an oriented circle in a permuted way. It follows from the (STU)-relation in $\mathcal{A}$, as shown in Figure 2, that the element $\chi(S_k)$ can be written as

$$\chi(S_k) = k! T_k + \text{diagrams that have } k - 1 \text{ vertices on the oriented circle.} \quad (7)$$

Figure 2: The STU-relation

We will now state an application of Theorem 2 that will be needed later.

**Proposition 1** Let $k \geq 4$ be even and let $S_k$ be the Chinese character of degree $k$ shown on the left-hand side of Figure 4. Then there exist finite-dimensional irreducible representations $\rho_\alpha$ of the Lie superalgebras $D_\alpha$ such that for all but finitely many values of $\alpha$ we have

$$W_{D_\alpha, \rho_\alpha, \omega_\alpha}(\chi(S_k)) \neq 0.$$
by $k$. Let us denote the coefficient of $n^k$ by $d_{\alpha,k}$. By Lemma 2 and Formula \((\ref{formula1})\) we have $d_{\alpha,k}(\overline{\chi(S_k)}) = k!d_{\alpha,k}(T_k)$.

The bilinear form $<\cdot, \cdot>_{\alpha}$ induces a bilinear form on $H^*$ with matrix

$$\text{diag}\left((1+\alpha)/2, -1/2, -\alpha/2\right).$$  \(\text{(8)}\)

Now we use Theorem $2$ to compute for even $k \geq 2$ and $\alpha = 1$ that

$$d_{1,k}(T_k) = 2 \sum_{\beta \in \Delta^+} (-1)^{\deg \beta} <\lambda_1, \beta >^k = 2(6^k + 2 - 4^k - 2(3^k) - 2^k).$$  \(\text{(9)}\)

Since we can show that $d_{1,k}(T_k) > 0$ for $k \geq 4$, the coefficient $k!d_{\alpha,k}(T_k)$ of $n^k$ in $W_{D_{\alpha},\rho,\alpha,\omega_n}(\overline{\chi(S_k)})$ does not vanish as a polynomial in $\alpha$. Thus we may choose $n_0$ such that $W_{D_{\alpha},\rho,\alpha,\omega_n,\omega_n}(\overline{\chi(S_k)})$ does not vanish as a polynomial in $\alpha$. So with the choice $\rho_\alpha := \rho_{\alpha,n_0}$ we only have to exclude finitely many values of $\alpha$ in the statement of the proposition. \(\square\)

### 4 The Elements $D_{k+d,k}$

Vogel defined a commutative graded algebra $\Lambda$ equipping the space $P(A)_{\geq 2}$ of primitive elements of degree $\geq 2$ with the structure of a graded $\Lambda$-module. Let $L$ be a simple Lie superalgebra with Casimir $\omega$. Then by Theorem 6.1 of [11] there exists a homomorphism $\chi_{L,\omega} : \Lambda \rightarrow \mathbb{C}$ such that for all $\lambda \in \Lambda$, for all representations $\rho$ of $L$ and for all $a \in P(A)_{\geq 2}$ we have

$$W_{L,\omega,\rho}(\lambda a) = \chi_{L,\omega}(\lambda)W_{L,\omega,\rho}(a).$$  \(\text{(10)}\)

In the case of $D_\alpha$ the specialization $(a, b, c) \mapsto (-\alpha - 1, 1, \alpha)$ maps the elements $\sigma_2 = ab + ac + bc$ and $\sigma_3 = abc$ to $-\alpha - \alpha^2$ and $-\alpha^2$ respectively. By [11], Theorem 6.13 there exists a morphism of graded algebras $\chi_D : \Lambda \rightarrow \mathbb{C}[\sigma_2, \sigma_3]$ (deg $\sigma_i = i$) such that

$$\chi_{D_{\alpha},\omega_n}(\cdot) = \chi_D(\cdot)(-1 - \alpha - \alpha^2, -\alpha^2).$$  \(\text{(11)}\)

Certain elements $t, x_3, x_5, x_7, \ldots$ generate a subalgebra of $\Lambda$ on which formulas for the maps $\chi_{L,\omega}$ are known. These formulas allow to prove the following proposition.

**Proposition 2** For $d = 15$ and all $d \geq 17$ there exist elements $P_d \in \Lambda$ with deg $P_d = d$ that satisfy $\chi_{L,\omega}(P_d) = 0$ for all simple Lie algebras $L$ with Casimir $\omega$, but $\chi_D(P_d) \neq 0$.

**Proof:** Let $\varphi : \mathbb{C}[T, X_3, X_5, \ldots] \rightarrow \Lambda$ be the algebra morphism defined by mapping $X_i$ to $x_i$ and $T$ to $t$. Let $\mathbb{C}[\lambda, \mu, \nu]_{\mathbb{S}_3}$ be the ring of symmetric polynomials in the indeterminates $\lambda, \mu, \nu$. There exist morphisms of algebras $\chi_0, \chi_{L,\omega}, \chi_D$ that make the following diagram commutative (see [11], proof of Lemma 7.5).
It is not difficult to see that the image of $\chi_0$ is

$$\mathbb{C}[t] \oplus (t + \lambda)(t + \mu)(t + \nu)\mathbb{C}[t, \lambda\mu + \nu\lambda, \lambda\mu\nu],$$

where $t = \lambda + \mu + \nu$. Let $I = \{\lambda, \mu, \nu\}$. Let

$$P = \prod_{a \in I} (t + a) \prod_{a \in I} (t - a) \prod_{a, b \in I} (a + 2b) \prod_{a \in I} (3a - 2t) \in \mathbb{C}[\lambda, \mu, \nu]^{S_3}.$$

For all $Q \in \mathbb{C}[\lambda, \mu, \nu]^{S_3}$ the element $PQ$ is in the image of $\chi_0$. Furthermore, we have $\chi'_L(\omega)(PQ) = 0$ for all simple Lie algebras $L$ with Casimir $\omega$ (see the computation of eigenvalues of $\Psi$ on $S^2 \mathfrak{l}/\omega$ in [11]).

Let $Q \in \mathbb{C}[\lambda, \mu, \nu]^{S_3}$ be a homogeneous element not divisible by $t$. Then we have $\chi'_D(PQ) \neq 0$. We have $\deg(PQ) = 15 + \deg Q$ and $\deg Q = 0$ or $\deg Q \geq 2$. Define $P_d = \varphi(p)$, where $p \in \chi_0^{-1}(PQ)$ and $d = \deg p = \deg(PQ)$. The element $P_d$ has the properties stated in the proposition.\)

Proposition 2 was proved by the author by making a computation with Mathematica (see [9]). The proof stated above is due to P. Vogel.

**Proof of Theorem 1**. We turn the space $P(\mathcal{B})_{\geq 2}$ of primitive elements of $\mathcal{B}$ of degree $\geq 2$ into a $\Lambda$-module by the formula

$$\overline{\chi}(\lambda \cdot b) = \lambda \cdot \overline{\chi}(b) \quad (\lambda \in \Lambda, b \in \mathcal{B}). \quad (12)$$

An example is shown in Figure 3.

Figure 3: The element $t^3x_3x_9 \cdot S_6$.

Now we can define $D_{k+d,k} = P_d \cdot S_k$ with the elements $P_d$ of Proposition 2. By Formula (11) we have $W_{L,\omega,\rho}(\overline{\chi}(D_{k+d,k})) = 0$ for all simple Lie algebras $L$ with Casimir $\omega$ and representation $\rho$. By standard arguments using Exercise 6.33 of [1] this can be generalized to the case where $L$ is a semisimple Lie algebra. Let $\rho_\alpha$ be the representation of $D_\alpha$ from Proposition 1. Then by Formula (11) and Formula (11) we have
\( W_{\alpha, \rho, \omega}(\chi(D_{k+d,k})) = \chi_D(P_d)(-1 - \alpha - \alpha^2, -\alpha - \alpha^2)W_{\alpha, \rho, \omega}(\chi(S_k)). \) (13)

By Proposition 1, the factor \( W_{\alpha, \rho, \omega}(\chi(S_k)) \) only vanishes for finitely many values of \( \alpha \). With our choice of \( P_d \) the polynomial \( \chi_D(P_d) \in C[\sigma_2, \sigma_3] \) does not vanish and is homogeneous with respect to \( \deg \sigma_i = i \). So the value \( \chi_D(P_d)(-1 - \alpha - \alpha^2, -\alpha - \alpha^2) \) can also only vanish for finitely many choices of \( \alpha \). This implies \( D_{k+d,k} \neq 0 \) and completes the proof. \( \blacksquare \)

Acknowledgements

I thank C. Kassel and P. Vogel for carefully reading the paper and for proposing improvements. Thanks to the German Academic Exchange Service for financial support (Doktorandenstipendium HSP III).

References

[1] D. Bar-Natan, On the Vassiliev knot invariants, Topology 34 (1995), 423–472.

[2] D. Bar-Natan, Some Computations Related to Vassiliev Invariants, available at [http://www.ma.huji.ac.il/~drorbn](http://www.ma.huji.ac.il/~drorbn), May 5, 1996.

[3] D. Bar-Natan and S. Garoufalidis, On the Melvin-Morton-Rozansky conjecture, Invent. Math. 125 (1996), 103–133.

[4] J. E. Humphreys, Introduction to Lie Algebras and Representation Theory, Springer-Verlag, 1989.

[5] V. C. Kac, A sketch of Lie superalgebra theory, Comm. Math. Phys. 53 (1977), 31–64.

[6] J. A. Kneissler, The number of primitive Vassiliev invariants up to degree twelve, [q-alg/9706022](http://www.ma.huji.ac.il/~drorbn) and University of Bonn preprint, June 1997.

[7] A. Kricker, Alexander-Conway limits of many Vassiliev weight systems, J. of Knot Th. and its Ramif., Vol. 6, Nr. 5 (1997), 687–714.

[8] A. Kricker, B. Spence and I. Aitchison, Cabling the Vassiliev invariants, J. of Knot Th. and its Ramif., Vol. 6, Nr. 3 (1997), 327–358.

[9] J. Lieberum, charcomp.m, a Mathematica program to perform computations with the characters from [11], available at [http://www-irma.u-strasbg.fr/irma](http://www-irma.u-strasbg.fr/irma), 1997.

[10] J. Lieberum, Chromatic weight systems and the corresponding knot invariants, preprint I.R.M.A. Strasbourg (1997), to appear in Math. Ann.

[11] P. Vogel, Algebraic structures on modules of diagrams, Université Paris VII preprint (1995), to appear in Invent. Math.