The ‘Tip’ of the Red Giant Branch as a distance indicator: theoretical calibration and the value of $H_0$

SANTI CASSISI$^{1,2,3}$, MAURIZIO SALARIS$^3$

$^1$Osservatorio Astronomico di Collurania, I-64100, Teramo, Italy
$^2$Università degli studi L’Aquila, Dipartimento di Fisica, I-67100, L’Aquila, Italy
$^3$Max-Planck-Institut für Astrophysik, D-85740, Garching, Germany

ABSTRACT.

Updated theoretical relations for the run of the bolometric and I magnitude of the Tip of the Red Giant Branch (TRGB) with respect to the metallicity of the parent stellar population are provided. An analogous relation for the V magnitude of the Zero Age Horizontal Branch (ZAHB) at the RR Lyrae instability strip is also provided.

A comparison has been performed among our ZAHB and TRGB distances, the Cepheid distance scale by Madore & Freedman (1991) and the HIPPARCOS distance set by local subdwarfs with accurate parallax determinations. The ZAHB, TRGB and HIPPARCOS distances are in satisfactory agreement, whereas the comparison between TRGB and Cepheid distances discloses a systematic discrepancy of the order of 0.12 mag, the TRGB distances being systematically higher. This result supports the case for a revision of the zero point of the Cepheid distance scale.

The application of our TRGB distance scale to NGC3379 provides a distance to the Leo I group that is about 8% higher than the one obtained by Sakai et al. (1997) adopting the TRGB brightness calibration by Da Costa & Armandroff (1990). Our distance to the Leo I group, coupled with the relative distance Coma cluster-Leo I determined differentially by means of secondary distance indicators, provides a determination of $H_0$ at the Coma cluster: $H_0=64^{+10}_{-9}$ Km s$^{-1}$ Mpc$^{-1}$.

1. Introduction.

The Tip of the Red Giant Branch (TRGB) method has been recently used for estimating the distances to several nearby galaxies (see, e.g, the list in Salaris & Cassisi 1997; Lee, Freedman & Madore (1993 - hereinafter LFM93) and Madore & Freedman (1995) assessed the reliability and intrinsic accuracy of this method, and demonstrated that the TRGB can be successfully used for determining distances accurate within 0.2 mag for galaxies out to 3 Mpc by using ground-based telescopes, and out to 12 - 13 Mpc by using the HST.

The underlying physical mechanism which allows to use the TRGB as a standard candle is the following: the TRGB marks the Helium ignition inside the degenerate core of low-mass stars, and its brightness depends on the He core mass, which is remarkably constant for ages larger than a few billions of years.

A fundamental ingredient for using the TRGB as a distance indicator is the calibration of its bolometric magnitude ($M_{bol}^{TRGB}$) as a function of [M/H]. The relation generally used until now is the semiempirical one by Da Costa & Armandroff (1990 - hereinafter DA90), based on the observational database by Frogel, Persson & Cohen (1983 - hereinafter FPC83) of bolometric magnitudes for RGB stars in a sample of
galactic globular clusters (hereinafter GC). The observed $m_{\text{bol}}$ of the most luminous red
giants are converted into absolute magnitudes adopting distance moduli for the parent
GC obtained by using the RR Lyrae distance scale by Lee, Demarque & Zinn (1990),
and these absolute magnitudes fix the zero point of the TRGB distances. As discussed
in Salaris & Cassisi (1997) this procedure provides a zero point too faint (independently
of the accuracy of the Lee, Demarque & Zinn RR Lyrae distance scale); the reason is the
small sample of stars observed by FPC83. Taking into account the evolutionary times
along the RGB and the observed sample of stars, it is possible to compute statistically
the probability that the most luminous observed star is actually at the TRGB; the re-
sult is that this probability is very small, indicating clearly that the TRGB brightness
is systematically underestimated.

In this paper we will present a purely theoretical calibration of the TRGB brightness
as a function of $[\text{M}/\text{H}]$ as obtained from updated evolutionary computations, and we will
compare the distance scale set by our TRGB luminosities with RR Lyrae, HIPPARCOS
and Cepheid distances. Once assessed the reliability of our calibration, we will apply
the TRGB method for the determination of the Hubble constant.

2. The TRGB distance scale

2.1. Theoretical stellar models

We have determined the TRGB luminosities for stellar populations with age $t=15$ Gyr
(but, as discussed before and in Salaris & Cassisi 1997, the precise value of $t$ does
not influence the TRGB luminosities for ages larger than a few Gyr) and metallicity
$-2.35 \leq [\text{M}/\text{H}] \leq -0.28$ (the He abundance is set to $Y=0.23$ with the exception of
the case with $[\text{M}/\text{H}]=-0.28$, where we adopted $Y=0.255$), by computing evolutionary
tracks of low-mass stars without chemical elements diffusion. As far as it concerns the
physical inputs adopted in computing the stellar models, the interested reader is referred
to Salaris & Cassisi (1997). Assuming for the Sun $M_{\text{Bol,}\odot} = 4.75$ mag, we obtain the
following relation:

$$M_{\text{TRGB}} = -3.949 - 0.178 \cdot [\text{M}/\text{H}] + 0.008 \cdot [\text{M}/\text{H}]^2$$  \hspace{1cm} (1)

This relation takes also automatically into account the enhancement of the $\alpha$ elements
observed in galactic field halo and GC stars when considering the global metallicity
$[\text{M}/\text{H}]$ (see Salaris, Chieffi & Straniero 1993). It is moreover possible to correct this
relation for different He contents around the values we have adopted, taking into account
that on average $\frac{\partial M_{\text{tip}}}{\partial Y}$ is $\approx 1.0$ in the metallicity range covered by Equation 1.

In order to estimate the internal accuracy of the current theoretical scenario, in
Figure 1 we have compared our Equation 1 with similar relations derived by independent
updated stellar models. We compared our results with the ones by Cassisi et al. (1997 -
their 'step8', with and without He and heavy elements diffusion), Caloi et al. (1997 -
no diffusion) and Straniero et al. (1997 - no diffusion). In the same figure the relation
provided by DA90 is shown after correcting for the slightly different $M_{\text{Bol,}\odot}$ adopted by
the quoted authors.
It is important to note that the agreement between the different recent evolutionary results is quite good. All the theoretical relations lie within ≈ ±0.05 mag with respect to Equation 1. Moreover, the change of the TRGB brightness due to the inclusion of atomic diffusion - adopting the same physical inputs as in standard models - results to be quite negligible (see the results corresponding to the Cassisi et al. 1997 models). Finally, it exists a systematic difference by about 0.15 mag in the zero point between our relation and the relation provided by DA90 (while the slopes are in good agreement), our TRGB brightnesses being higher.

2.2. The Bolometric Correction scale

Before going on, we wish to briefly review the iterative procedure suggested by LMF93 in order to derive distances by means of the TRGB method from observations in the VI Johnson-Cousins bands of resolved galaxies. The first step consists in fixing a preliminary distance modulus; with this fixed distance modulus one determines the metallicity by measuring the dereddened \((V - I)\) color at \(M_I = -3.5\) mag ((\(V - I\))\(_{0,-3.5}\)) and using a relation between this color and the metallicity of the parent stellar population (see section 3.1). As a second step, with this estimate of \([\text{M/H}]\) and the observed I magnitude of the TRGB (corrected for the interstellar extinction), one redetermines the distance modulus by adopting a relation for both the TRGB bolometric magnitude as a function of metallicity and the bolometric correction to the I magnitude \((BC_I)\). At this point, one iterates the procedure until convergency is achieved. Due to the weak dependence of \(M_I^{\text{tip}}\) on the metallicity, convergence is generally achieved after one iteration.

For applying this procedure, it is therefore necessary to have a relation providing the
bolometric correction to the I (Cousins) band. Following LFM93, an empirical $BC_I - (V - I)_0$ relation for RGB stars has been taken from DA90 ($(V - I)_0$ is the dereddened color of the considered RGB stars): $BC_I = 0.881 - 0.243 \cdot (V - I)_0$, independent of the metallicity. This empirical relation was derived by comparing the I magnitudes given in DA90 with the bolometric magnitudes given by FPC83 for a sample of RGB stars in 8 GC with different metallicities. By examining Figure 14 in DA90, it appears clearly that in the range of $(V - I)_0$ values typical of the stars considered by the authors ($(V - I)_0$ colors between 1.0 and 1.6) and of the TRGB stars in the sample of galaxies studied in section 3.2 ($(V - I)_0$ colors between 1.3 and 2.0), there is a dispersion of the order of 0.10 mag around the least square fit that they give. Moreover, the relation for the reddest stars is based only on a very small number of observational points. We have therefore used two other independent sets of $BC_I$ for better assessing the uncertainty in the TRGB distances due to the bolometric correction scale.

By using the theoretical bolometric corrections by Castelli, Gratton & Kurucz (1997a, 1997b - hereinafter K97) and the semiempirical ones by Green (1988 - hereinafter Yale transformations), we have obtained the following relations:

$$M_{I}^{TRGB,K97} = -3.953 + 0.437 \cdot [M/H] + 0.147 \cdot [M/H]^2 \quad (2)$$

$$M_{I}^{TRGB,Yale} = -4.156 + 0.157 \cdot [M/H] + 0.070 \cdot [M/H]^2 \quad (3)$$

All these three different sets of $BC_I$ will be used in the next sections for deriving TRGB distances to a sample of resolved galaxies.

3. Comparison among TRGB, RR Lyrae, HIPPARCOS and Cepheid distance scales

3.1. Globular Clusters

In the case of galactic GC it is possible to compare the RR Lyrae distance scale with the one derived from Equation 1. For determining the RR Lyrae distance scale we have here adopted the Zero Age Horizontal Branch (hereinafter ZAHB) models from Cassisi & Salaris (1997), homogeneous with the models adopted for deriving the TRGB luminosities. Our ZAHB models have been transformed into the observational plane by using both the Yale and K97 transformations. The relations between the ZAHB V magnitude (taken at $\log T_{eff} = 3.85$, that corresponds approximately to the average temperature of the RR Lyrae instability strip) and $[M/H]$ obtained from these two sets of transformations agree within 0.03 mag. In the following we adopt for fixing the RR Lyrae distance scale the relation (valid for $-2.35 \leq [M/H] \leq -0.57$):

$$M_{V}^{ZAHB} = 0.921 + 0.329 \cdot [M/H] + 0.045 \cdot [M/H]^2 \quad (4)$$

The TRGB bolometric magnitudes published by FPC83 for a sample of GC with accurate spectroscopic determinations of $[M/H]$ (see Salaris & Cassisi 1996) have been corrected for our ZAHB distance scale (see Cassisi & Salaris 1997 and Salaris & Cassisi 1997 for details about the procedure followed for determining the observational ZAHB
Fig. 2. The absolute bolometric magnitude of the brightest observed red giant as a function of the global metallicity, for the sample of clusters selected from the FPC83 database. The solid line shows the theoretical expectation for the bolometric magnitude of the TRGB; the dashed lines represent the same theoretical relation but shifted in steps of 0.1 mag.

level at the instability strip), so that the comparison displayed in Figure 2 between the observational $M_{Bol}^{tip}$ values and our Equation 1 is a comparison between our ZAHB and TRGB distances. The vertical error bar ($\pm 0.1$ mag) for the observational points represents an average error on the distance modulus obtained from Equation 1 while the error on the spectroscopic determination of [M/H] is typically of the order of 0.15 dex. Data in Figure 2 show quite clearly that Equation 1 constitutes an upper envelope to the distribution of the observational points (with the exception of one cluster, namely NGC6352; in this case, according to FPC83, the star considered to be at the TRGB could also be a field star. The second more luminous observed RGB star is $\approx 0.3$ mag fainter), that are all contained within 0.4-0.5 mag from Equation 1. This is exactly what expected on the basis of simple statistical arguments (see Salaris & Cassisi 1997), when the evolutionary times in the upper part of the RGB and the number of stars observed in each cluster are taken into account. This means that the theoretical TRGB and ZAHB distance scales in GCs are in agreement within the statistical uncertainties due to the small sample of red giant stars observed.

Once assessed the consistency between TRGB and ZAHB distance scale for GC, we have compared our GC ZAHB distances with distance moduli taken from the recent literature, derived from the Main Sequence Fitting (hereinafter MSF) technique using subdwarfs with accurate HIPPARCOS parallaxes. The sources of the GC MSF distances are Gratton et al. (1997), Reid (1997) and Chaboyer et al. (1997).

In Figure 3 (panels a-c) we display the results of this comparison; the error bars on the MSF distances are taken from the quoted papers. Due to the different procedures adopted by the various authors, the differences between the distance moduli obtained for the GCs in common among these three investigations give us a rough estimate of
the intrinsic error of the MSF technique.

It is worth noticing that on average there is a good agreement between our ZAHB distance scale and the HIPPARCOS MSF distances. For the most metal poor (and more distant) clusters displayed in the figure, the Reid (1997) data seem to disagree systematically with our ZAHB distances, but on the contrary the same M68 distance derived by Gratton et al. (1997) nicely agrees with the ZAHB distance. In particular, in the case of M5 and NGC6752 the distance moduli derived from the MSF by the three different groups are almost identical, and the agreement with the ZAHB distance scale is almost perfect. We can therefore finally conclude that for GC the HIPPARCOS, TRGB and ZAHB distance scales are in agreement one with each other within the present errors.

Once assessed the consistency between TRGB, ZAHB and HIPPARCOS distances, the distance scale set by Equation 4 can be also used for reliably calibrating a relation providing \([M/H]\) as a function of \((V-I)_{0,-3.5}\), by adopting a sample of GC for which \(V-(V-I)\) C-M diagrams (as given by DA90) and accurate spectroscopical \([M/H]\) measurements (as listed in Salaris & Cassisi 1996) are available; this relation is needed for applying the TRGB method to resolved galaxies. We obtain:

\[
[M/H] = -39.27 + 64.69 \cdot [(V-I)_{0,-3.5}] - 36.35 \cdot [(V-I)_{0,-3.5}]^2 + 6.84 \cdot [(V-I)_{0,-3.5}]^3 \tag{5}
\]

3.2. Resolved galaxies

In the case of resolved galaxies, we can compare the TRGB distance scale with the Cepheid and the RR Lyrae ones. The observational database used in this comparison is the one in Salaris & Cassisi (1997), with the additional data for Sextans B taken from
Sakai, Madore & Freedman (1997). In Table 2 we report the distance modulus determinations as obtained with the three different methods. The various columns provide the following data: (1) the name of the object; (2) the reddening; (3) the observed magnitude of the TRGB; (4) the mean RGB metallicity, as obtained by adopting Equation 6 and the distance moduli in column 8; (5) the intrinsic Cepheid distance on the scale by Madore & Freedman (1991), with the zero point set by a LMC distance modulus of 18.50 mag and E(B − V) = 0.10; (6) the true distance obtained by using the mean RR Lyrae V magnitude (for details about the conversion from mean RR Lyrae brightness to the corresponding ZAHB one see Cassisi & Salaris 1997. When only the g magnitude of RR Lyrae stars is determined, it has been transformed to V according to the relation by Kent 1985); (7) as in column (6) but for an average metallicity of the RR Lyrae population [M/H]=−1.5 (see below); (8) the distance modulus obtained by applying the TRGB method and making an average between the values obtained by using the three different bolometric correction scales (see section 2.2). The typical errors on the TRGB, Cepheids and RR Lyrae distances for the sample of galaxies in Table 2 are on average of the order of 0.15 mag.

It is important to remember that the [M/H] values given in column 4 of Table 2 are derived from RGB stars, and correspond to an average metallicity of this stellar population, that for the sample of galaxies in Table 2 shows generally a spread in [M/H] (this spread does not introduce a big error on the TRGB distances, since the weak dependence of $M_{TRGB}^I$ on the metallicity). In principle this average RGB metal content could be different from the RR Lyrae one, especially for the highest and lowest values of [M/H] displayed in Table 2, due to the low probability that metal-poor and metal-rich RGB stars evolve during their He central burning phase through the RR Lyrae instability strip. For roughly estimating the uncertainty due to the unknown original metal content of the RR Lyrae population, the distance moduli obtained assuming for the RR Lyrae stars an average metallicity equal to [M/H]=−1.5 - adopted as a reasonable estimate of the average metallicity for the galactic halo RR Lyrae population - have been reported in column 7 of Table 2 (with the unique exception of the LMC; in this case we have a determination for the metallicity of the considered RR Lyrae).

When comparing TRGB distances determined with the three $BC_I$ scales presented in section 2.2, we obtain that the average difference adopting respectively the DA90 and Yale bolometric correction scales is $(m − M)_{TRGB,DA90} − (m − M)_{TRGB,Yale} = −0.06 ± 0.06$ mag, while the average difference when considering the DA90 and the K97 $BC_I$ is $(m − M)_{TRGB,DA90} − (m − M)_{TRGB,K97} = −0.08 ± 0.06$ mag. As already discussed in section 2.2, these differences can be considered as a rough estimate of the error on the TRGB distances due to the uncertainty on the bolometric correction scale. Moreover, the values of $(m − M)_{TRGB,DA90} − (m − M)_{TRGB,Yale}$ and $(m − M)_{TRGB,DA90} − (m − M)_{TRGB,K97}$ are fully compatible with the dispersion of the observational points around the empirical $BC_I$ scale by DA90 (see discussion in section 2.2). In Table 2 and in the following we have adopted, for the TRGB distance of each galaxy, the value obtained by averaging the three distance moduli corresponding to the three different $BC_I$ scales.

A comparison between TRGB and RR Lyrae distances (when one neglects the very discrepant point corresponding to NGC205; see also the discussion in LFM93 about this galaxy), considering for the RR Lyrae the same mean metallicity of the RGB stars,
gives an average difference \((m - M)_{TRGB} - (m - M)_{RRLyrae} = 0.07 \pm 0.09\) mag; when considering (with the exception of the LMC) a metallicity \([M/H]=-1.5\), one obtains an average difference \((m - M)_{TRGB} - (m - M)_{RRLyrae} = 0.02 \pm 0.09\) mag. One can therefore conclude that the RR Lyrae and TRGB distance scales agree well, at the level of less than 0.10 mag, when considering our sample of resolved galaxies.

In Figure 4, we have displayed the difference between the distance moduli obtained by adopting the TRGB and the Cepheid distance scale. The average difference \((m - M)_{TRGB} - (m - M)_{Cepheids}\) between the two scales is equal to 0.12 \pm 0.06\) mag, the TRGB distances being systematically larger, in good agreement with the difference obtained considering only the LMC. Since the good agreement between ZAHB and TRGB distances, and the agreement between ZAHB and HIPPARCOS distance scales discussed in the previous section, this systematic offset between the TRGB and Cepheid distance scales supports, within the limits of the small sample of galaxy considered, the results by Feast & Catchpole (1997), Gratton et al. (1997), Reid (1997), that point to the direction of a higher LMC distance modulus (and higher zero point of the Cepheid distance scale) with respect to the value of 18.50 mag adopted by Madore & Freedman (1991).

### Tab. 2 - Selected parameters for a sample of resolved galaxies.

| Galaxy  | \(E(B-V)\) | \(I_{Tip}\) | \([M/H]\) | \((m - M)_0\) | \((m - M)_{Ceph}\) | \((m - M)_{RR}\) | \((m - M)_{RR^{-1.5}}\) | \(I_{Tip}\) |
|---------|-------------|-------------|----------|----------------|----------------|----------------|----------------|----------|
| LMC     | 0.10        | 14.60       | -1.0     | 18.50          | 18.54          | 18.63          | 23.62          | 23.67    |
| NGC6822 | 0.28        | 20.05       | -1.7     | 24.06          | 24.15          | 24.14          | 24.17          | 24.28    |
| NGC185  | 0.19        | 20.30       | -1.0     | 24.17          | 24.28          | 24.28          | 24.41          | 24.46    |
| NGC147  | 0.17        | 20.40       | -0.9     | 24.44          | 24.56          | 24.67          | 24.56          | 24.60    |
| IC1613  | 0.02        | 20.25       | -1.2     | 24.63          | 24.85          | 24.78          | 24.85          | 24.90    |
| M31     | 0.08        | 20.55       | -0.9     | 24.92          | 25.01          | 24.90          | 24.92          | 25.03    |
| M33     | 0.10        | 20.95       | -2.0     | 25.85          | 25.09          | 24.90          | 25.09          | 25.92    |
| WLM     | 0.02        | 20.85       | -1.5     | 25.69          | 25.79          | 24.90          | 25.79          | 25.79    |
| NGC3109 | 0.04        | 21.55       | -1.5     | 25.50          | 25.68          | 25.50          | 25.68          | 25.68    |

4. The Leo I group and Coma cluster distances, and the value of \(H_0\).

The Leo I group is a relatively nearby group of galaxies, compact, with a line-of-sight depth estimated to be \(\approx 2\%\) compared to its distance (Tanvir et al. 1995). Very recently, Sakai et al. (1997 - hereinafter SA97) detected the TRGB in NGC3379 (one of the dominant galaxies in Leo I), by means of HST WFPC2 observations. They placed the observed TRGB at \(I=26.32 \pm 0.05\) mag, assumed \(A_I=0.02\) mag, and adopted a metallicity \([M/H]=-0.68 \pm 0.40\) (see SA97 for more details).

By using the quoted values (and the associated errors) for extinction, metallicity and TRGB location, we derive a TRGB distance modulus \((m - M)_{0,3379}=30.48 \pm 0.12\) mag (other sources of errors included in the error budget are the uncertainty on the
WFPC2 photometric zero point as given by SA97, the uncertainty on the theoretical calibration of the TRGB, estimated to be of $\pm 0.05$ mag on the base of the comparison in Figure 1, and the variation of the TRGB brightness due to a variation $\Delta Y = \pm 0.03$ in the initial He content of the theoretical models, that is $\approx \pm 0.03$ mag. This distance modulus corresponds to a linear distance $d = 12.5 \pm 0.7$ Mpc, and it is $\approx 8\%$ higher than the value derived by SA97, using the DA90 calibration of the TRGB distance scale.

Once fixed the absolute distance to Leo I, we can obtain the distance to the Coma cluster using the distance ratio Coma-Leo I as determined by means of secondary distance indicators.

According to the recent analysis by Colless & Dunn (1996) the Coma cluster consists of two components: the main cluster centered around NGC4874 and NGC4889, with a mean heliocentric recession velocity $cz = 6853$ km s$^{-1}$, and a subgroup around NGC4839 characterized by a mean value of $cz = 7339$ km s$^{-1}$. The relative distance between galaxies in the main component of Coma and Leo I has been recently re-determined by Gregg (1997) by means of the diameter - velocity dispersion method, and results to be $d_{\text{Coma}}/d_{\text{LeoI}} = 8.84 \pm 0.23$. The same relative distance is also obtained when using the diameter - velocity dispersion data by Faber et al (1989) for 2 ellipticals in Leo I and 27 ellipticals in the main component of the Coma cluster. With the TRGB Leo I distance modulus previously derived, this Coma-Leo I distance ratio provides a distance to the main component of the Coma cluster $d_{\text{Coma}} = 111 \pm 9$ Mpc, and $(m - M)_{0, \text{Coma}} = 35.23 \pm 0.17$ mag (accounting in the error budget also for an uncertainty by $\pm 0.04$ mag due to the r.m.s. depth of the Leo I group as given by Tanvir et al 1995).

Once the distance to Coma is known, the value of $H_0$ is derived using the heliocentric recession velocity of the main cluster component ($cz = 6853$ Km s$^{-1}$) transformed to the centroid of the Local Group, and corrected for the motion of the Local Group relative...
to the cosmic background radiation in the direction of Coma (272 Km s^{-1} according to Staveley-Smith & Davies 1989, to which we attach an error by ±100 Km s^{-1}). Moreover, we corrected for the peculiar motion (V_p) of the cluster as estimated by Han & Mould (1992), V_p=+66±428 Km s^{-1} (we have taken the median value of their three solutions for V_p). We finally obtain a cosmic recession velocity cz=7068±440 Km s^{-1}, and H_0=64±10 Km s^{-1}Mpc^{-1}.

Acknowledgements

The work of one of us (M.S.) was carried out as part of the TMR programme (Marie Curie Research Training Grants) financed by the EC.

References

Caloi V., D’Antona F. & Mazzitelli I. 1997, Astron. Astrophys. 320, 823

Cassisi S. & Salari S. M. 1997, Mon. Not. R. Astr. Soc. 285, 593

Cassisi S., Castellani V., Degl’Innocenti S. & Weiss A. 1997, Astron. Astrophys., in press

Castelli F., Gratton R.G. & Kurucz R.L. 1997a, Astron. Astrophys. 318, 841

Castelli F., Gratton R.G. & Kurucz R.L. 1997b, Astron. Astrophys. 324, 432

Chaboyer B., Demarque P., Kernan P.J. & Krauss L.M. 1997, Astrophys. J., submitted

Colless M. & Dunn A.M. 1996, Astrophys. J. 458, 435

Da Costa G.S. & Armandroff T.E. 1990, Astron. J. 100, 162

Faber S.M. et al. 1989, Astrophys. J. Suppl. 69, 763

Feast M.W. & Catchpole R.M. 1997, Mon. Not. R. Astr. Soc. 286, L1

Frogel J.A., Persson S.E. & Cohen J.G. 1983, Astrophys. J. Suppl. 53, 713

Gratton R.G., Fusi Pecci F., Carretta E., Clementini G., Corsi C.E. & Lattanzi M. 1997, Astrophys. J., in press

Green E.M. 1988, in "Calibration of Stellar Ages", A.G. Davis Philip ed. p. 81

Gregg M.D. 1997, New Astronomy, 1 363

Han M. & Mould J., 1992, Astrophys. J. 396, 453

Kent S.M. 1985, Publ. Astr. Soc. Pacific 97, 165

Lee M.G., Freedman W. & Madore B.F. 1993, Astrophys. J. 417, 553

Lee Y.-W., Demarque P. & Zinn R. 1990, Astrophys. J. 350, 155

Madore B. F. & Freedman W.L. 1991, Publ. Astr. Soc. Pacific 103, 933

Madore B. F. & Freedman W.L. 1995, Astron. J. 109, 1645

Reid I.N. 1997, Astron. J. 114, 161

Sakai S., Madore B.F. & Freedman W.L. 1997, Astrophys. J. 480, 589

Sakai S., Madore B.F., Freedman W.L., Lauer T.R., Ajhar E.A., Baum W.A. 1997, Astrophys. J. 478, 49

Salaris M. & Cassisi S. 1996, Astron. Astrophys. 305, 858

Salaris M. & Cassisi S. 1997, Mon. Not. R. Astr. Soc. 289, 406

Salaris M., Chieffi A. & Straniero O. 1993, Astrophys. J. 414, 580

Staveley-Smith L. & Davies R.D. 1989, Mon. Not. R. Astr. Soc. 241, 787

Straniero O., Chieffi A. & Limongi M. 1997, Astrophys. J. submitted,

TanvirN.R., Shanks, T., Ferguson H.C. & Robinson D.R.T. 1995, Nature 377, 27