Damage Detection of Transmission Tower Based on Stochastic Subspace and Statistic Model

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Abstract: During the long service period of the transmission tower, the severe environment, material aging and fatigue effects result in the damage accumulation and resistance fading of the transmission tower, which even lead to the structural failure. In this paper, a stochastic subspace-based damage detection algorithm combined with statistical model is proposed for the damage identification of transmission tower structures through processing and analysing its vibration signals. Based on the structural modal parameters identified from the structural vibration response, the nominal modal parameters are defined. Considering that the Hankel matrixes constructed by the system observability matrix and the system output covariance matrix have the same kernel space, the damage index sensitive to the modal parameters is constructed. In the manuscript, firstly, the damage index establishment method and process of the algorithm are presented; and then the vibration data of transmission tower structure simulation under different damage conditions are obtained through numerical simulations; finally, above data are analysed to obtain the damage identification results of transmission tower. The results show that the method proposed in this work can effectively identify the local damage of transmission tower structure.

1. Introduction

As an important part of electrical power supply system, transmission tower structure has the functions of supporting transmission lines and transporting electric power. The safe operation of transmission tower structure has a direct and important influence on the whole power transmission system. However, transmission towers will encounter various natural disasters during long-term service [¹], and the aging of materials and fatigue effect under dynamic load will aggravate the damage accumulation and weaken the structural resistance, even in extreme cases it will lead to the destruction of transmission tower structure [²]. In order to ensure the safety of long-term structure operation, it is urgent to adopt effective means to assess its safety status and repair initial damage in time [³⁻⁷]. Among
the various of damage identification methods, the vibration-based structural damage identification method is the most widely used method because of its convenience. Firstly, Lifshitz and Rotem [8] applied modal analysis based on vibration signal on structural damage identification in 1969, and after over 50 years of development, lots of other methods have been proposed and investigated.

In 1996 and 2001, Doebling and Sohn et al. [9, 10] conducted a detailed review on hundreds of damage identification methods. In the early stage, the modal parameters including frequency, mode shape and transfer function are directly used to analyze the structure, and calculate the structural damage according to the modal parameter change. Compared with other modal parameters, frequency information is easier to obtain with less sensors, and the position of measurement points is relatively independent. Damage recognition using frequency changes was firstly proposed by Vandiver in 1975, who applied this method to the structural state assessment of offshore platform [11]. In contrast, mode shapes contain more damage information and are more sensitive to local changes of structures. Therefore, the use of mode shapes to further construct damage indexes can accurately identify local damage of structures [12,13]. But damage identification technique based on modal shape change has problems in further application that the measuring mode is not complete and it is easily affected by noise, resulting in inaccurate structural damage identification. Among many damage identification methods, statistic-based method can effectively eliminate noise effects and improve the accuracy of damage identification, which takes into account the influence of randomness of monitoring data [14-17].

In this paper, a damage-sensitive identification index is established by combining the frequency and mode parameters based on the statistical model. When using this index to identify damage in unknown conditions, there is no need to repeat the modal analysis, and the identification error introduced by multiple modal analysis is eliminated. Finally, the effectiveness and reliability of this method are verified by an example, which provides reference for further application in damage identification and health monitoring of transmission tower structures in the future.

2. Structural damage identification method based on random subspace algorithm

Assuming that the structure won’t enter the nonlinear stage during its service, its dynamic response can be described by the following steady-state linear dynamic equation:

\[ M\ddot{Z} + C\dot{Z} + KZ = v, \quad Y = LZ \]  

(1)

Where, \( M, C \) and \( K \) are the mass, damping and stiffness matrix of the system, \( Z \) is the displacement vector of all degrees of system freedom, \( Y \) is the displacement vector of the measuring points, and the matrix \( L \) refers to the location of the sensor.

Thus, the eigenvalue \( \mu \) and output eigenvector \( \psi_\mu \) of the system are solutions of the following eigenvalue:

\[
\begin{align*}
\det(\mu^2M + \mu C + K) &= 0 \\
(\mu^2M + \mu C + K)\phi_\mu &= 0 \\
\Psi_\mu &= L\phi_\mu
\end{align*}
\]

(2)

The dynamic model in Equation (1) is sampled at the frequency of \( \frac{1}{\tau} \), and the state space model of the discrete system can be obtained as follows:

\[
\begin{align*}
X_{k+1} &= FX_k + V_{k+1} \\
Y_k &= HX_k
\end{align*}
\]

(3)

where

\[
X_{k+1} = \begin{cases} Z(k\tau) \\
\dot{Z}(k\tau) \end{cases}, \quad Y_k = Y(k\tau)
\]

(4)
\[
F = e^{Lt}, \quad L = \begin{pmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{pmatrix}
\]

(5)

\[
H = \begin{pmatrix}
L_a & 0 \\
0 & L_a \\
-L_aM^{-1}KZ & -L_aM^{-1}CZ
\end{pmatrix}
\]

(6)

\[F\] is the state transition matrix, \(H\) is the state observation matrix. \(\lambda, \phi_\lambda\) is the eigenvalue and eigenvector of the state transition matrix \(F\), namely:

\[
det(F - \lambda I) = 0, \quad (F - \lambda I)\phi_\lambda = 0
\]

(7)

The system modal parameters defined in Equation (2) can be calculated from the above two feature structures, as shown in Equation (8):

\[
\exp \tau \mu = \lambda, \quad L\Phi_\mu = \phi_\lambda = H\phi_\lambda
\]

(8)

Assuming that the damping of the system is proportional, then the eigenvectors of the system are real numbers. Combine the eigenvalues \(\lambda\) and eigenvectors \(\phi_\lambda\), to be specific, stack all the \(\phi_\lambda\) to compose matrices and combine them with the corresponding \(\lambda\) matrix into the form shown in Equation (9), we can get the nominal system characteristic parameter vector \(\theta\):

\[
\theta = \left[ A \begin{pmatrix}
\Lambda \\
\phi_\lambda
\end{pmatrix} \right]
\]

(9)

where \(A\) is the vector composed by eigenvalues \(\lambda\), \(\Phi\) is the matrix formed by eigenvectors \(\phi_\lambda\), and \(\begin{pmatrix}\text{vec}(\phi)\end{pmatrix}\) represents stack operation. In the following damage identification algorithm, \(\theta\) will be represent the dynamic property of the structure.

The main principle of the damage identification algorithm mentioned in this paper is to detect whether \(\theta\) will change: it is assumed that the initial nominal modal parameter of the undamaged structure is \(\theta_0\). After a period of time, a new set of data samples will be obtained. Then, the problem of structural damage identification is equivalent to determining whether the structure states described by the new data are consistent with the initial structure states described by nominal modal parameters \(\theta_0\). The damage identification method defines a residual relative to the parameter identification process, and calculates the sensitivity of the residual, the damage identification is achieved by determining whether the statistical value of the sensitivity has changed. In order to obtain the estimated system parameters by the SSI method, the controllability matrix \(C\) and observability matrix \(O\) of the system need to be constructed. It is assumed that the eigenvectors corresponding to the system’s state transition matrix \(F\) are taken as the basis vector of the state space model. In this way, the observability matrix can be expressed as a new form:

\[
O_{p+1} = \left( \Phi \quad \Phi A \quad L \quad \Phi A^p \right)
\]

(10)

where \(A = \text{diag}(\Lambda)\).

The definition criterion is shown in Equation (11) to determine whether the system state described by the given output covariance sequence \(\left(R_j\right)\) is consistent with the undamaged system state described by the nominal modal parameters \(\theta_0\), in other words:

\[
O_{p+1}(\theta_0) \quad \text{and} \quad H_{p+1,q} \quad \text{have the same nuclear space}
\]

(11)

where \(H_{p+1,q}\) is the Hankel matrix constructed for the system output covariance matrix, and the proof of the above properties is shown in the relevant paper [15]. To quantitatively express this property, the modal parameters of the structure need to be estimated first. As shown in Equation (10), a new
observability matrix $O_{p+1}(\theta_0)$ is constructed, and the SVD decomposition of $O_{p+1}(\theta_0)$ was carried out to define its core space, by obtaining the orthogonal matrix $S$, as shown below:

$$S^T O_{p+1}(\theta_0) = 0$$  \hspace{1cm} (12)

It can be seen that the matrix $S$ is related to the nominal modal parameter $\theta_0$ and not unique, but it can still be considered as a function of $\theta_0$, which can be expressed as $S(\theta_0)$. Whether the nominal modal parameter $\theta_0$ corresponds to the covariance matrix sequence $(R_j)_j$ of the system output can be judged by the following formula:

$$S^T(\theta_0)H_{p+1,q} = 0$$  \hspace{1cm} (13)

Suppose that there is a reference parameter $\theta_0$ and a new set of system response output sample $Y_1, ..., Y_n$ to detect whether the state of the system has changed, in other words, to determine whether the structure has been damaged, the $H_{p+1,q}$ based on the new sample should be calculated first and the residual vector can be defined as follows:

$$\zeta_n \rightarrow \sqrt{n} \text{ vec} \left( S^T(\theta_0) H_{p+1,q} \right)$$  \hspace{1cm} (14)

Assuming that $\theta$ is the nominal modal parameter identification value of the structure in the new state and $E_\theta$ is the expectation of the residual vector $\zeta_n$ shown as below:

$$E_\theta(\zeta_n(\theta_0)) = 0 \text{ if } \theta = \theta_0$$  \hspace{1cm} (15)

The equation means when $\theta$ does not change, the expectation of residual vector $\zeta_n(\theta_0)$ is zero, otherwise, the structure is damaged. It is necessary to know the statistical distribution of $\zeta_n(\theta_0)$ to test whether $\theta = \theta_0$ is true, but the prior knowledge of this distribution is unknown. Therefore, the following hypothesis tests are given:

(H) Healthy state $H_0$: $\theta = \theta_0$

(D) Damaged state $H_1$: $\theta = \theta_0 + \delta\theta / \sqrt{n}$  \hspace{1cm} (16)

where vector $\delta\theta$ is unknown fixed value.

If $n$ is sufficiently large, suppose $H_1$ corresponds to a small value of $\theta$. At this point, it can be proved that the residual vector $\zeta_n$ follows the approximate Gaussian distribution. Under the assumption of $H_0$ and $H_1$, it is easy to obtain:

$$\zeta_n(\theta_0) \xrightarrow{n \rightarrow \infty} \begin{cases} N(\theta, \Sigma(\theta_0)) & H_0: \text{Healthy state} \\ N(J(\theta_0)\delta\theta, \Sigma(\theta_0)) & H_1: \text{Damaged state} \end{cases}$$  \hspace{1cm} (17)

where $J(\theta_0)$ is the sensitivity of the residual vector to modal parameters, $\Sigma(\theta_0)$ is the variance of the residual vector.

When the nominal modal parameter $\theta$ of the system is slightly changed by $\delta\theta$, the results are reflected in the change of the residual vector $\zeta_n$, while $J(\theta_0)$ and $\Sigma(\theta_0)$ are independent of $n$ and $\delta\theta$.

Assuming that $\hat{J}$ and $\hat{\Sigma}$ are the consistent estimates of $J(\theta_0)$ and $\Sigma(\theta_0)$, the problem of determining whether the residual vector $\zeta_n$ is zero can be expressed as the hypothesis testing problem in Equation (16). As shown in the following formula, the value of $\chi^2$ can be calculated and compared with the preset threshold value, so as to determine whether the structure is damaged or not.
\[ \chi^2 \triangleq \xi_n^+ \hat{\Sigma}^{-1} \hat{J}^T \hat{J}^{-1} \hat{J}^T \hat{\Sigma}^{-1} \xi_n \] (18)

3. Numerical model validation of damage identification method

3.1. Establishment of finite element model

In order to verify the validity of the algorithm, this paper applies the algorithm to the numerical model of a transmission tower. The transmission tower model is established based on the actual structure, and the height of the whole tower is 12.7m. The main body of the tower is composed of four main rods, the main rod and other rods are connected with steel plates, while the crossing of chord rods is connected with bolts. The main rod can be divided into two parts: the lower part is made of Q420 steel, the upper part and main chord are made of Q345 steel, and the secondary chord are made of Q235 steel. The finite element model of the transmission tower is established by ANSYS, as shown in Fig. 1(a). BEAM188 element was used to simulate the main rod and main chord of the tower body, while LINK180 element was used to simulate the secondary abdominal rod. The whole finite element model included 547 elements and 245 nodes.

Gaussian white noise signal was used to simulate the environment excitation and applied to the main nodes of the tower. The condition of simulated data includes health condition and damage condition, and the vibration signal under health condition is taken as the reference data for this algorithm. As the main pole is the main force member of the transmission tower structure, the damage mostly occurs at the connection of tower foot and tower body. Therefore, the stiffness of the main pole # 4 element is reduced by 20%, 30%, 50% and 70% respectively to simulate the different damage conditions. While the damage of the chord is simulated by removing the typical rod. The damage conditions are shown in Table 1, and the detailed damage locations are shown in Fig. 1(b). Measuring points are installed every 3~4m on the main pole of the structure, and the layout is shown in Fig. 1(c). The sampling frequency of the vibration signal is 200Hz, and the sampling time is 30s. Fig. 2 shows the time-history curve of the acceleration sensor in the x direction of 3# measurement point.

(a) finite element model   (b) location of damage elements   (c) location of the measuring points

Fig.1 The analysis model of transmission tower

| Damage Condition | Damaged element | Degree of damage |
|------------------|-----------------|-----------------|
| 1                | 4               | 10%             |
| 2                | 4               | 20%             |
| 3                | 4               | 30%             |
| 4                | 4               | 40%             |
| 5                | 4               | 50%             |

Table 1 The damage conditions of transmission tower
3.2. Damage sensitivity analysis

According to the above description of the proposed damage identification algorithm, the main process of the method can be summarized as follows:

1. Based on the acceleration response data of the healthy state of the structure, the SSI method is used to conduct modal analysis, and the nominal modal parameter $\theta_0$ of the structure is obtained;

2. Calculate the consistency estimation of $J(\theta_0)$ and $\Sigma(\theta_0)$, which refer to the sensitivity of residual vector $\xi_n(\theta_0)$;

3. Calculate the residual vector $\xi_n$ of damage state data;

4. Calculate $\chi^2$ value to judge whether the structure is damaged or not.

| Modal order | Frequency (Hz) | Dismantle |
|------------|---------------|-----------|
| 6          | 81, 85        | Dismantle |
| 7          | 85, 153, 157  | Dismantle |
| 8          | 109, 121      | Dismantle |
| 9          | 93, 105, 109, 121 | Dismantle |
| 10         | 61, 65, 73    | Dismantle |

Fig. 2 The time-history curve of typical acceleration sensor

In the process of modal recognition, the x direction acceleration data of 8 measuring points are used. Fig. 3 shows the stable figure of modal parameter identification based on the SSI algorithm. Table 2 shows the identification of structural modal parameters and the frequency comparison with the finite element analysis. It can be seen that the maximum frequency error of this method is 4.7%, which indicates the method can well identify the first three modes of the structure.

Fig. 3 The modal parameter identification results based on stochastic subspace algorithm

For the 10 damage conditions in Table 1, the SSI method is used to analyze and obtain the frequency change of the damage in each condition, as shown in Table 3.

According to the analysis of the results in Table 3, for the damage case of single element stiffness reduction, the change of modal frequency is quite small, but gradually increases with the deepening of the damage degree. For the bar removal condition, the modal frequency will have a larger change.
Using the above damage recognition algorithm, damage identification results are shown in figure 4, y axis refers to $\chi^2$ value, namely represents the degree of structural damage.

It can be seen from the calculation results that when the stiffness of 4 # element of the main rod is reduced by 10%, 20%, 30%, 40% and 50% respectively (working conditions 1~ 5), the damage index reaches 3099097.7, 5366124.2, 6473527.9, 8196168.6 and 13913718.0. Compared with condition 1, the $\chi^2$ value increases by 73.15%, 108.99%, 164.47.62% and 348.96% respectively for damage condition 2~5, which indicates the damage index has high sensitivity for stiffness reduction. For the single chords removal condition, the damage index $\chi^2$ reaches 3.69×10^8, 2.72×10^9 and 1.21×10^9 respectively for condition 6, 8, 10. And we can see there are obvious differences in the damage index values for different chords removal condition, and the damage index $\chi^2$ is the most sensitive to the middle oblique chord. As for the damage condition 7 and 9, the damage index $\chi^2$ differs and has no clear law. The analysis results show that the damage index $\chi^2$ can measure the size of the damage to some extent, and has different sensitivity to different locations of the damage, it is still unable to accurately locate and quantify the damage.

| Table 2: The identification results of frequency and damping |
|-----------------------------------------------|
| Order | Vibration frequency(Hz) | Finite element calculation | SSI Identify | Error(%) | Damping ratio | MAC(%) |
|-------|--------------------------|-----------------------------|---------------|----------|----------------|--------|
| 1     | 7.250                    | 7.227                       | 0.323         | 0.0067   | 99             |
| 2     | 16.044                   | 15.732                      | 1.945         | 0.0087   | 99             |
| 3     | 24.903                   | 23.733                      | 4.700         | 0.0087   | 100            |

| Table 3: The frequency change of damaged structure |
|-----------------------------------------------|
| Damage condition | Order number |
|------------------|--------------|
| 1                | 0.045        | 0.000        | 0.016 |
| 2                | 0.080        | 0.000        | 0.027 |
| 3                | 0.202        | 0.000        | 0.058 |
| 4                | 0.463        | 0.001        | 0.115 |
| 5                | 0.006        | 0.580        | 0.143 |
| 6                | 0.117        | 3.503        | 0.962 |
| 7                | 0.037        | 1.128        | 0.983 |
| 8                | 0.179        | 3.712        | 2.409 |
| 9                | 0.085        | 1.560        | 1.965 |

(a) Stiffness reduction of main rod  
(b) Diagonal demolition

Fig.4 The identification results of damage
4. Conclusion
In this paper, a stochastic subspace and statistical model based damage identification algorithm is proposed. The modal parameters of healthy state are taken as reference, and the statistical characteristics of the data is conducted to form a structural damage index. The structural local damage identification can be realized without repeating modal analysis. A numerical simulation for transmission tower is carried out, and the results show that:

The modal analysis method based on SSI method can be used to identify high-precision modal frequency, mode shapes and damping information. The identification result of the healthy state is taken as the reference state, and the structural damage index $\chi^2$ value based on statistical model can effectively identify the damage.

The damage identification results of different damage condition were analyzed, for the damage condition of a single member, with deepening of the damage degree, the structure frequency did not change significantly, but the damage index $\chi^2$ increased significantly. In the condition of bar removal damage, the change of structure frequency was quite large, while the damage index $\chi^2$ had a much greater change. Therefore, it can be seen that the damage index proposed in this paper is more sensitive than the traditional modal parameters.

The damage identification method proposed in this paper will be further used to solve the damage identification problem of actual transmission tower structures.

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