Development of a high-effectiveness slotted plate cryogenic heat exchanger

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Abstract. High-effectiveness heat exchangers are a ubiquitous component of cryogenic systems, but their performance typically falls short of model-based expectations. The following paper summarizes modeling efforts of a heat exchanger designed to achieve an effectiveness in excess of 99% within a prescribed envelope of volume, weight, pressure drop, and operating conditions. Simulation efforts focus on employing minimal assumptions and accounting for many loss mechanisms to avoid overestimation of predicted effectiveness. Axial conduction and parallel passage flow imbalance are major contributing factors to heat exchanger inefficiency. Consequently, a staggered stacked slotted plate geometry, also known as a matrix heat exchanger, was chosen as the most promising type to achieve the desired effectiveness. High thermal conductivity, high surface area finned copper plates separated by low thermal conductivity, low cross-sectional area stainless steel spacers achieve high stream-to-stream heat transfer while limiting axial conduction and allowing for fluid redistribution between plates. The number of design parameters was reduced based on manufacturing limitations and conceptual reasoning. ANSYS Fluent computational fluid dynamics was used to form Nusselt number and Darcy friction factor correlations for the geometry and flow conditions. A finite difference MATLAB model accounted for axial conduction, parasitic heat loads, and material property variation with temperature and pressure. The Fluent-derived correlations were used as inputs to the MATLAB model. The MATLAB model was validated by comparing its results to an analytic, constant property effectiveness-NTU solution for stacked plate heat exchangers, as well as experimental data collected from a similar heat exchanger. Trade-offs between weight, pressure drop, and effectiveness were studied.

1. Background and motivation

Re recuperative heat exchangers are used extensively in the field of cryogenics to significantly raise the overall efficiency of cryogenic systems. The effectiveness of the recuperator directly impacts the system efficiency. Maximizing the recuperator effectiveness allows for lower ultimate temperatures at the cold end and minimizes required cooling power to the flow stream. Raising the effectiveness tends to also increase the pressure drop and size of the heat exchanger. SHI Cryogenics of America (SHI) has formulated this project as a study of these trade-offs.

Various parameters were outlined that formed the design space for the heat exchanger, including the operating conditions and size, but achieving an effectiveness of over 99% was the most important requirement. To accurately predict the performance of a heat exchanger with such a requirement necessitated computational models with minimal assumptions and simplifications. Material property variation with pressure and temperature, as well as losses due to axial conduction and parasitic heat loads, had to be included within a numerical modelling approach. Previous workers have produced applicable correlations for friction factor and Nusselt number [1], but the correlations did not cover the
full range of Reynolds numbers expected in this design study and made assumptions that could significantly impact the accuracy of the model. The goal was to arrive at a general manufacturable heat exchanger style that could achieve 99% or greater effectiveness, while simultaneously developing a computational modeling tool to predict the effects of each geometric feature on the tradeoffs between volume, weight, pressure drop, and effectiveness. The design could then be fine-tuned as desired by SHI. This paper briefly summarizes the work presented in the Ph. D. thesis, “Development of a High-Effectiveness Cryogenic Stacked Slotted Plate Heat Exchanger” [2], which delves into exacting detail.

2. Design specifications
Table 1 outlines the heat exchanger design envelope prescribed by SHI.

| Table 1. SHI heat exchanger design specifications. |
|---------------------------------------------------|
| Working fluid:                                    | Helium gas, counterflow |
| Temperature span [K]:                             | 300 to 30               |
| Operating pressures [MPa]:                        | 2.6 (supply side), 0.5 (return side) |
| Pressure drop [MPa]:                              | < 0.3 (supply side), < 0.1 (return side) |
| Burst pressure [MPa]:                             | 13.5                     |
| Mass flow rate [g/s]:                             | 6 (rating case), 12 (design case) |
| Materials:                                        | 316 stainless steel, copper |
| Spatial dimension limit [m]:                      | < 0.5 in any dimension   |
| Volume limit [m³]:                                | 0.1                      |
| Effectiveness:                                    | > 0.99                   |
| Weight [kg] (lbs):                                | < 45.4 (100)             |

Within this design space, all geometries, configurations, and fabrication methods were options. Several types of designs were explored. Performance, manufacturing techniques, and cost estimates were considered in order to determine the most promising design style that would achieve high effectiveness, small size, and low cost.

3. Chosen heat exchanger type: stacked, staggered-slotted plates
Two of the most significant factors that reduce cryogenic heat exchanger effectiveness are axial conduction and mass flow imbalance [3]. A stacked, staggered-slotted perforated plate heat exchanger was chosen specifically to mitigate these effects and achieve the required effectiveness in excess of 0.99. Figure 1 shows the general concept of the plates and spacers, which are stacked in the into-the-page direction. The nomenclature to describe the geometric attributes are labeled.
The alternating layers create anisotropic heat transfer. In this case, the perforated plates are constructed from relatively conductive copper with an array of slots to create large surface area density, resulting in an array of rectangular fins. Staggering the perforations between subsequent layers enhances heat transfer by creating flow impingement on the unperforated area of the plates at the cost of increased pressure drop. The spacer layers have a small cross-sectional area and are constructed from relatively low conductivity 316 stainless steel and thermally insulate adjacent plates from one another. The copper and stainless steel are compatible for diffusion bonding the stack, which is considered the preferred method of sealing by most authors [4]. The result is high stream-to-stream conduction with limited axial conduction. Additionally, after the helium gas passes through the parallel channels in the perforations, it redistributes in the spacer region, preventing large-scale flow maldistribution from developing. The unique combination of low axial conduction and flow redistribution zones made this heat exchanger style the most appealing option to achieve the 99% effectiveness sought by SHI.

4. Finite difference numerical model
A MATLAB finite difference model was developed that explicitly included non-ideal effects, including axial conduction, parasitic heat loads, and material property variation with temperature and pressure. The model was based on the approach developed by Nellis [5] [6]. In its generalized form the model discretizes a heat exchanger and is set up to receive inputs that describe the specific heat exchanger. These include the number of transfer units on the supply and return sides, the axial conduction parameter, heat capacitance ratios, and parasitic heat loads. All are non-dimensionalized and configured to evaluate each element using the material properties associated with that element’s temperature and pressure. A sparse matrix decomposition and relaxation technique solves for the metal and fluid inlet and outlet temperatures of each element. Figure 2 displays the discretization of the heat exchanger model.
The model was configured to apply heat transfer and pressure drop correlations that were developed using Fluent with the appropriate material properties and fluid velocities in each cell energy balance, one of which is depicted in figure 3.

**Figure 3.** Energy balance terms applied to a single finite difference element.

Calculations of the axial conduction parameter, NTU, and parasitic heat loads are performed for each control volume as though the entire heat exchanger were operating at the temperatures and material property values of that node, but they are scaled by the dimensionless length to account for the small contribution made by each individual control volume.

The temperature dependence of material properties was implemented by forming 6th-order polynomials based on the values generated by EES [7] property function calls, with 1000 temperature values evenly spaced over the 300 to 30 K temperature range. The properties of RRR 100 copper were used for the slotted plates, and 316 stainless steel for the spacers. For the fluid properties on the low- and high-pressure sides, the polynomial was generated assuming a constant pressure of 500 kPa and 2.6 MPa, respectively, because properties did not vary substantially over the allowable pressure range on a given side.

5. **Fluent model**

The friction factor and Nusselt number correlations used in the MATLAB model were generated from a parametric study performed in Ansys Fluent. The design study was specified in a table of design points in Ansys Workbench 19.2. Each row contained a unique combination of geometric values, mass flow rate, temperature, pressure, and the appertaining helium gas properties, which were the parametric inputs to the model. The length of the fins can be assumed to be long relative to the other dimensions. Therefore, the model was reduced to 2 dimensions and end effects were ignored.

The fin geometry was entirely parameterized. The four geometric parameters are the fin height, gap height, fin thickness, and spacer thickness, as shown in figure 4. The size of the model was further reduced by modelling just one unit cell and applying the appropriate symmetry boundary conditions.

**Figure 4.** 2-D representation of a section of the heat exchanger (left). Boundary conditions to reduce the 2-D representation to a single unit cell using symmetric and periodically symmetric boundary conditions (right).
Together with a specified mass flow rate, the periodic boundary condition constrained the inlet and outlet velocity profiles to be identical, simulating the flow conditions as if the geometry repeated infinitely in the periodic direction.

Figure 5. Velocity (left) and temperature (right) for the \(H_{\text{gap}} = 0.7625\) mm, \(T_{\text{in}} = 30\) K, \(P = 2.6\) MPa, \(\dot{m} = 10.5\) g/s case, resulting in \(Re = 2105\).

The average convective heat transfer coefficient \(\bar{h}\) was calculated by dividing the area-weighted average heat flux at the fin boundaries, \(\bar{q}^{rT}\), by the log mean temperature difference through the domain, \(\Delta T_{LM}\), evaluated using mass-flow-rate-weighted average temperatures at the inlet and outlet.

\[
\bar{h} = \frac{\bar{q}^{rT}}{\Delta T_{LM}} \quad (1)
\]

When using a periodic boundary condition, Fluent decomposes the pressure gradient into a cyclically varying component and a constant pressure gradient in the periodic direction. Looking at just the periodic component, the average pressure at the inlet and outlet are identical. The linearly varying component represents the actual pressure gradient through the single unit cell domain and is a direct output value of the solution.

A Spalart-Allmers turbulence model was used to capture the turbulent behavior of the flow that occurs at high Reynolds numbers. The Reynolds number was defined based on the gap height, with hydraulic diameter \(D_h = 2H_{\text{gap}}\), and average fluid velocity in the gap, \(\bar{V}_{\text{gap}}\).

\[
Re = \frac{\rho \bar{V}_{\text{gap}} D_h}{\mu} \quad (2)
\]

With this definition, the Reynolds number ranged from as low as 2.4 to as high as 4,000 depending on the geometry, mass flow rate, temperature, and pressure, with the onset of turbulent behaviour occurring near \(Re = 150\). Table 2 displays the values chosen for the combinations used in the parametric study.

Table 2. Numerical values chosen for each Fluent model parameter.

| Parameter | Values |
|-----------|--------|
| \(L_{\text{micro}} [\text{mm}]\) | 0.05, 0.2875, 0.525, 0.7625, 1 |
| \(\dot{m}_{2D} [\text{kg/m}^3\text{s}]\) | 0.24, 1.38, 2.52, 3.66, 4.80 |
| \(T [\text{K}]\) | 30, 97.5, 185, 232.5, 300 |
| \(P [\text{MPa}]\) | 0.5, 2.6 |

\(L_{\text{micro}}\) is equivalent to \(H_{\text{gap}}\) as will be discussed in the next section. \(\dot{m}_{2D}''\) denotes the mass flux rate in the 2-D model, which assumes 1 meter of depth into the page. The channel width was varied in the parametric study, and combined with variations in mass flow rate to produce a range of flow velocities. \(\dot{m}_{2D}''\) spans the range of equivalent 2-D mass flux rates that results in the same range of fluid flow velocities in the 2-D model as would be seen in the 3-D heat exchanger. Fluid properties were largely
independent of pressure on a given side of the heat exchanger due to the limits on allowable pressure drops, and therefore only the inlet pressures on each side were included in the parametric study.

6. Parametric study and lightest weight design

6.1. Parameter reduction

The set of parameters that were varied in the final MATLAB parametric study were deduced via a combination of logical reasoning and manufacturing constraints. The heat exchanger size will be limited by weight rather than volume. The photochemical etching manufacturing process constrains the relative sizes of the plate thickness, gap height, and fin height to be roughly equal to maximize the convective heat transfer area density [8]. The spacer thickness was set to the same value to eliminate another parameter while avoiding excessive restriction or expansion of the flow. The width of the flow channel boarder must be as thin as possible to minimize axial conduction, without being so thin that bonding is unreliable. From Kumar’s work, the width of the flow channel boarder was set to a constant 2 mm [9].

Increasing the channel width or the total height had the same effect on the pressure drop. The open cross-sectional area increased. The fluid velocity and pressure drop were reduced, which is desirable. However, there were two counteracting effects on heat transfer. The lower fluid velocity reduced the convective heat transfer coefficient. Conversely the surface area per plate increased, which increased the total heat transfer area per plate. The increase in surface area and open flow area scaled in the same manner whether \( W \text{ channel} \) or \( H \text{ total} \) was varied. For example, if \( W \text{ channel} \) was doubled, the surface area of each fin doubled and the total surface area per plate doubled. If \( H \text{ total} \) was doubled, the surface area per fin was the same, but the number of fins doubled, so the total surface area also doubled. The same was true for the open flow area cross-section.

In this regard, whether \( W \text{ channel} \) or \( H \text{ total} \) was varied was arbitrary because the effects were the same. However, the fin efficiency decreased with increasing \( W \text{ channel} \) but not with increasing \( H \text{ total} \). It was concluded that \( H \text{ total} \) should not be varied, and should instead be set to the maximum value allowed in the design specifications of 0.5 m. This ensured the highest surface area per plate for a given fin efficiency. \( W \text{ channel} \) was then varied. The viable designs would have a large enough value of \( W \text{ channel} \) to achieve a high heat transfer area per plate and a low pressure drop, without being so large that the heat transfer would be crippled by low fin efficiency.

The allowable weight was the final parameter chosen to constrain the design space. SHI had not provided a specific target value but were generally interested in designs under 45.4 kg (100 lbs). Heavier designs achieved higher effectiveness, but also suffered higher pressure drops. Below a certain weight, no combination of the other parameters resulted in a design that met the effectiveness requirement set by SHI. The number of plates was linked to the weight. For a given set of parameters, the number of plates that resulted in the specified weight was calculated.

The end result of this process of parameter reduction was 4 total parameters: \( L \text{ micro} \), the singular length dimension used for the plate thickness, spacer thickness, fin height and gap height; \( W \text{ channel} \), which was also the fin axial length; allowable weight, which was achieved under a given set of parameters by choosing the appropriate number of plates and spacers; and the mass flow rate, which varied from 6 to 12 g/s for the rating and design cases, respectively.

To further simplify the process of identifying which designs achieved acceptable performance, a few observations that were universally true over the range of parameter values were employed. First, only the effectiveness of one side of the heat exchanger needed to be considered because the heat exchanger is nearly balanced. Second, only the low-pressure side needed to be considered because it experienced a higher pressure drop than the high-pressure side while being restricted to a lower allowable pressure drop. Third, only the highest mass flow rate had to be considered because, for any given geometry, it resulted in the lowest effectiveness and highest pressure drop.
6.2. Results and analysis

Through trial and error, the ranges were modified such that the maximum and minimum value of each parameter was just beyond the point where, no matter what combination of other parameters were paired with it, a viable design would never result. The parameter ranges were 0.4 cm ≤ W_{channel} ≤ 1 cm, 0.064 mm ≤ L_{micro} ≤ 0.5 mm, and 24.9 kg (55 lbs) ≤ weight ≤ 45.4 kg (100 lbs). A total of 1,331 unique combinations were formed by choosing 11 evenly-spaced values for each parameter. Figure 6 displays the pressure drop and effectiveness for designs within the refined range of parameters grouped by weight, W_{channel} and L_{micro}, with just 5 values for each parameter displayed for the sake of plot clarity. Viable designs occupy the north-west quadrant formed by the darkened vertical line at 10^{5} Pa, denoting the maximum allowable pressure drop, and horizontal line at \( \varepsilon = 0.99 \), denoting the minimum allowable effectiveness.

![Figure 6. Effectiveness and low-pressure side pressure drop of design permutations with \( m = 12 \text{ g/s} \), 0.4 cm ≤ W_{channel} ≤ 1 cm and 0.064 mm ≤ L_{micro} ≤ 0.5 mm, with 5 evenly spaced values for each parameter, and allowed weight ranging from 24.9 kg (55 lbs) to 45.4 kg (100 lbs), grouped by weight (left), W_{channel} (center), and L_{micro} (right).](image)

Generally, a higher allowed weight results in a larger plate number, higher effectiveness, and higher pressure drop. The highest effectiveness was 0.9927 and occurred very close to the limit of allowable pressure drop, with W_{channel} = 0.7 cm and L_{micro} = 0.1512 mm. The design with the lowest pressure drop that still barely achieved 0.99 effectiveness exhibited 40 kPa of pressure loss, with W_{channel} = 0.82 cm and L_{micro} = 0.2384 mm. Both coincided with the largest allowable weight of 45.4 kg (100 lbs).

6.3. Lightest-weight design

The high weight of all viable designs led SHI to primarily be concerned with determining the lightest-weight design that still meets the other performance requirements. The lightest possible design that can achieve 99% effectiveness and remain under 100 kPa of pressure drop weighed 27.7 kg (61 lbs), with W_{channel} = 0.55 cm and L_{micro} = 0.15 mm. The corresponding number of plates was 2403, and the total length was 0.721 m. The total volume was only 0.00618 m\(^3\), well under the maximum allowable 0.1 m\(^3\). This configuration is pictured in figure 7.
Figure 7. (Left): Rough appearance of the bulk dimensions of the lightest design with re-headering manifold. (Middle): Drawing of a single plate of the lightest design. (Right): One end of a single plate with magnified detail. All dimensions are in [mm]. This design has 1653 slots per side.

The total length exceeded the maximum allowable length in any one dimension in space of 0.5 meters. As such it would be necessary to re-header the heat exchanger to double-back upon itself.

7. Conclusions
The numerical modelling tool that has been developed can be applied to heat exchangers of varying geometries and flow conditions. Fluent was used to correlate constant wall temperature Nusselt number and Darcy friction factor with Reynolds number, which were used as inputs to the MATLAB model. The results of this study suggest that a copper-stainless steel stacked plate heat exchanger with offset rectangular slots can achieve the effectiveness and pressure drop requirements prescribed by SHI, with the lightest viable design weighing 27.7 kg (61 lbs), excluding headers. Readers are encouraged to review the thesis upon which this work is based for more detailed information [2].

8. References
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