Motivated by recent experimental measurements of the tunneling characteristics of high $T_c$ materials using scanning tunneling spectroscopy, we have calculated the IV and differential conductance curves in the superconducting state at zero temperature. Comparing the two results obtained via BCS-like d-wave pairing and the $SU(2)$ slave boson approach, we find that the slave-boson model can explain the asymmetric background observed in experiments. The slave-boson model also predicts that the height of the conductance peak relative to the background is proportional to the hole doping concentration $x$, at least for under-doped samples. We also observe the absence of the van Hove singularity, and comment on possible implications.

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I. INTRODUCTION

Tunneling spectroscopy has been one of the fundamental tools in studying the superconducting state of the high $T_c$ materials. In recent years it has been possible to use the scanning tunneling microscope (STM) to perform reproducible experiments on single crystal Bi$_2$Sr$_2$CaCu$_2$O$_{8+δ}$ cleaved in ultra high vacuum \cite{1,2}. In contrast to photoemission experiments, which are local probes in wave-vector space, the STM is local in real space. Thus it does not provide any information that depends on momentum, however, it has much higher energy resolution. The density of states (DOS) obtained from the $\Sigma$ energy resolution.

The STM spectra for the superconducting phase exhibit unusual structure in the DOS when viewed in the light of BCS even if effects such as energy dependence of the normal state density of states of sample and/or tip, existence of bandwidth cutoffs, unequal work functions of tip and sample and energy-dependent transmission probabilities are included \cite{3}. One notable feature in the tunneling spectra is the asymmetric background with an enhancement for hole-tunneling into the sample. This feature is reproduced very well in our calculation based on the slave-boson theory. The slave-boson model also predicts that the strength of the background in the tunneling spectra does not scale with the doping $x$ while the sharp conductance peak scales linearly with $x$. Thus by measuring the relative strength of the background and the sharp conductance peak as a function of doping $x$, one can distinguish the BCS theory and the slave boson theory experimentally.

We also observe the absence of the van Hove singularity, and comment on possible implications. One possible implication is particularly intriguing and consistent with Photoemission results. That is the quasiparticles have a long lifetime, $\tau > 0.5/\text{meV}$, below the superconducting gap, and a very short lifetime, $\tau < 0.05/\text{meV}$, (spin-charge separation) above the superconducting gap.

II. D-WAVE BCS

The differential conductance $\frac{dI}{dV}$ displays in the simplest case of constant DOS in the tip and energy independent transition probability the single electron DOS in the sample. This reflects the ability of the material to accommodate an extra electron or hole depending on the sample bias. Within the tunneling Hamiltonian formalism the tunneling current is given by

$$j_T = 4\pi e \Gamma^2 \sum_{k,p} \int d\omega [A_{L-}(\omega, p)A_{R+}(\omega + V, k)$$

$$- A_{L+}(\omega, p)A_{R-}(\omega + V, k)]$$

where the $A_{LS}$ and $A_{RS}$ are the spectral functions for the single electron Green’s functions in the tip (L) and sample (R) respectively. $V$ denotes the bias of the sample with respect to the tip and $\Gamma$ is the tunneling matrix element assumed independent of energy. Notice that positive $V$ corresponds to $e^-$ tunneling into the sample.

For a free fermion system which we suppose to represent the tip material we have the standard form for the spectral function at zero temperature $A_{L+}(\omega, p) = \Theta(\omega)\delta(\omega - \xi_p)$ and $A_{L-}(\omega, p) = \Theta(-\omega)\delta(\omega - \xi_p)$, were $\xi_p = \epsilon_p - E_F$ denotes the particle spectrum in the tip with $E_F$ the Fermi energy. $\Theta(\omega)$ is the Heaviside step function where $\omega$ is measured with respect to the Fermi energy.

For the sample, we first consider the following single particle spectral distribution at zero temperature.
\[ A_{R+}(\omega, p) = \Theta(\omega) u^2(p) \delta(\omega - E_p) \]
\[ A_{R-}(\omega, p) = \Theta(-\omega) v^2(p) \delta(\omega + E_p) \]

where \( u^2(p) = \frac{1}{2} \frac{1}{1 + \frac{\epsilon_p}{\xi_p}} \) and \( v^2(p) = \frac{1}{2} \frac{1}{1 - \frac{\epsilon_p}{\xi_p}} \) are the BCS coherence factors \( E_p = \sqrt{\xi_p^2 + \Delta^2_p} \) is the dispersion relation of the quasiparticles in the superconducting state and \( \xi_p = \epsilon_p - E_F \) with \( \epsilon_p \) the dispersion relation in the normal state. Here \( \Delta_p \) denotes the gap function which is taken to have a d-wave symmetry in reciprocal space.

Within the above approximation to the spectral functions the van Hove singularity is close to the Fermi surface and \( \Delta_0 \) is taken to have a d-wave symmetry in reciprocal space.

The coherence factors which mix particle and hole density of states lead to the van Hove singularity also showing up on the particle side of the double peak structure. The coherence factors which mix particle and hole density of states lead to the van Hove singularity also showing up on the particle side of the \( \frac{dI}{dV} \) curve albeit with much smaller amplitude.

### III. SLAVE BOSONS

Next we consider the tunneling problem in the light of the \( SU(2) \) slave boson theory of Wen and Lee. It is commonly believed that the simplest model that incorporates the strong correlation physics relevant for the high \( T_c \) cuprates is the t-J model. Due to the strong on-site Coulomb repulsion energy the doubly occupied states should not contribute to the low energy effective theory. Within the \( SU(2) \) approach this constraint is implemented via the introduction of a slave boson doublet. The physical electron operator can then be written as an \( SU(2) \) singlet. Within this representation, the mean-field electron propagator is given by the product of the boson and the fermion propagators and was calculated in [7].

For our purpose we only need the \( T = 0 \) spectral functions which can be read off from the expression for the Green’s function as

\[ A_{R+}(\omega, k) = \Theta(\omega) \left( \frac{x}{2} u^2(k) \delta(\omega - E^f(k)) \right) \]

for the particle part

\[ A_{R-}(\omega, k) = \Theta(-\omega) \left\{ \frac{x}{2} v^2(k) \delta(\omega + E^f(k)) + \frac{1}{2N} \sum_q \left[ u_b(q-k) u_f(q) + u_b(q-k) v_f(q) \right]^2 \times \delta(\omega + E^f(q) + E^b_-(q-k)) \right\} \]

for the hole part of the spectrum. Here \( N \) denotes the number of sites and \( x \) is the hole doping concentration. The remaining variables are defined as follows: \( u_{f,b}(k) = \frac{1}{\sqrt{2}} \sqrt{1 + \frac{\epsilon^b(k)}{E^b(k)}} \), \( v_{f,b}(k) = \frac{1}{\sqrt{2}} \sqrt{1 - \frac{\epsilon^b(k)}{E^b(k)}} \), \( E_f^f(k) = \sqrt{(\epsilon^f(k))^2 + (\Delta^f(k))^2} \) and \( E_b^b(k) = \pm \sqrt{(\epsilon^b(k))^2 + (\Delta^b(k))^2 - \mu_b} \), where \( \epsilon^f(k), \Delta^f(k) \) are the fermion dispersion and gap function respectively \( \epsilon^b(k), \Delta^b(k) \) are the dispersions of the bosons and \( \mu_b \) is the boson chemical potential.
Notice that besides a coherent part for the spectral functions which resembles the form of the BCS spectral weight \[ \delta \] albeit scaled by a factor of \( \frac{1}{2} \) there is also an added incoherent contribution to the hole part of the spectral function \[ \delta \).

To calculate the tunneling current we have used the fit to ARPES measurements as dispersion for the fermions (spinons) and a nearest neighbor tight binding dispersion

\[
\epsilon^b = -2t^b(\cos k_x + \cos k_y)
\]

for the bosonic degrees of freedom (holons) with \( t^b \) the hopping matrix element for the holons. It is important that we match the fermionic band structure with the ARPES measurements since the fermions have a bigger band mass and hence determine the dispersion relation seen in ARPES \[ \delta \].

The electrons measured in those experiments are thought of (within spin-charge separating models) as bound states of the heavy spin degrees of freedom and the light charge degrees of freedom. Since we are interested in the low energy effective theory the details of the broad dispersion for the holons (charge degrees of freedom) are not crucial and hence have been chosen as simple as possible. Furthermore to arrive at equations \[ \delta \] we assumed boson condensation of the holons.

With the above expressions for the spectral functions in the sample we can calculate the tunneling current using equation \[ \delta \] as

\[
\begin{align*}
\frac{dI}{dV} |_{V>0} &= 4e\pi \Gamma^2 N(E_F) \sum_{k,E(k)\leq V} \frac{x}{2} u^2_f(k) \\
\frac{dI}{dV} |_{V<0} &= -4e\pi \Gamma^2 N(E_F) \sum_{k,E(k)\leq |V|} \frac{x}{2} v^2_f(k) \\
&\quad+ \frac{1}{2N} \sum_{k,q} \left[ u_b(q-k)u_f(q) + v_b(q-k)v_f(q) \right]^2 \\
&\quad+ \frac{1}{2N} \sum_{k,q}^+ \left[ u_b(q-k)v_f(q) - v_b(q-k)u_f(q) \right]^2
\end{align*}
\]

where \( \sum^\pm_{k,q} = \sum_{k,q} [\Theta(E^f(q) + E^b_{\pm}(q-k)) - \Theta(E^f(q) + E^b_{\pm}(q-k) - |V|)] \).

When comparing the two \( \frac{dI}{dV} \) curves Fig. \[ \delta \] and \[ \delta \] one can see how the lowest energy physics is virtually identical. However on energy scales bigger than \( 4\Delta_0 \) a marked asymmetry in the background of the \( SU(2) \) model \( \frac{dI}{dV} \) shows up with an increase in the hole tunneling spectral weight.

The inset of Fig. \[ \delta \] depicts the incoherent contribution to the hole tunneling spectrum separately. Scaling the height of Fig. \[ \delta \] by \( \frac{1}{2} \) and adding the incoherent hole contribution results in Fig. \[ \delta \]. The increase in the hole tunneling spectral weight arises due to the fact that removing an electron from the sample requires the recombination of the spin and charge degrees of freedom into a single entity. This yields a mixing in of the higher energy holon dispersion whose detailed form is not known within the effective low energy theory.

Another feature so far not discussed is the scaling with the hole doping of the conductance peak corresponding to electrons tunneling into the sample. Comparing equations \[ \delta \] and \[ \delta \] we see that within the \( SU(2) \) model, the peak height scales linearly with \( x \) whereas there is no dependence of the peak height on doping within the \( d \)-wave approach. The doping dependence within \( SU(2) \) arises from the reduction of the overlap of the electron in the tip with the quasiparticle (as a bound state of holons and spinons) in the sample which crudely speaking means that an electron can only enter the sample on empty sites and then 'decay' into its constituent parts. The only doping dependence within the \( d \)-wave approach arises due to the chemical potential which dictates the separation of the double peak structure but not its height. The linear scaling with \( x \) within the \( SU(2) \) slave boson mean-field theory discussed here should be taken more as a
qualitative than exact quantitative prediction, since it is a mean-field result. Recent photoemission experiments \[ \text{by Ding et al.} \] observed a linear $x$ dependence of the quasiparticle peak, which fits the mean-field result of the SU(2) theory very well.

Thus, at the mean-field level we have found qualitative different behaviors with regards to the $x$ dependence of the hole tunneling background and the electron tunneling peak within $d$-wave BCS and the SU(2) slave boson theory. It is this difference in doping dependence, which should be experimentally testable and hence yield to a feature distinguishing between the two models.

Furthermore notice that the DOS contains singularities (for both BCS and the slave boson model) at the electron tunneling peak. The curvature of the measured $dI/dV$ curve at these peaks should give us an upper bound on the quasiparticle decay rate at the energy scale of the superconducting gap. Based on new experimental data by Pan et al. \[ \text{[2]} \], the quasiparticle decay rate can be as small as a few meV even for quasiparticles with energy as high as 40 meV. This is very different from the normal state where the quasiparticle decay rate is of the same order of magnitude as the quasiparticle energy. We would also like to remark that according to the photoemission and tunneling results for underdoped samples, the quasiparticle peak (with a width of order $T$) disappears completely above $T_c$ while the gap remains at $(0, \pi)$. Based on the slave-boson theory, the sharp electron-tunneling peak arises due to the condensation of the holons (whose weight is proportional to $x$ at $T = 0$). As $T$ approaches $T_c$, the fraction of the condensed holons vanishes. If we assume that the holons are very incoherent above $T_c$, we can conclude that the sharp electron tunneling peak should disappear above $T_c$. This picture from the slave-boson model is completely consistent with the observed results from photoemission experiments.

Another point to make here is about the van Hove singularity. Samples with small superconducting gaps ($\Delta \sim 25$ meV) show a double peak structure in the tunneling $dI/dV$ curve, and the double peak structure crosses over into a single peak for large superconducting gaps. At first sight, one might guess that the double peak structure is due to the van Hove singularity. However, after comparing the experimental lineshape with the theoretical lineshape, we conclude that the double peak cannot arise due to the van Hove singularity. This is because the experimental peaks at higher bias are quite symmetric, while the peaks from the van Hove singularity are very asymmetric (the peak at the hole side is much stronger than the peak on the electron side). Based on the dispersion obtained from the fitting of the ARPES measurements, the van Hove singularity should show up even for samples with larger gaps ($\Delta = 50$ meV). However, experimentally, one fails to see the van Hove singularity even when the gap is as small as 20 meV. \[ \text{[3]} \] This seems to suggest that quasiparticles have very short life-time (spin-charge separation) above the superconducting gap, and hence the van Hove singularity cannot be observed. This leaves us with the question of where the double-peak feature comes from if there are no well defined quasiparticles above the superconducting gap? We hope that the double-peak structure may give us some hints on how coherent quasiparticles emerge in the superconducting state from the incoherent normal state.

Finally we would like to point out that the results of this paper are obtained from a mean-field calculation within the slave-boson theory. One naturally questions the reliability of the mean-field result and how much of our result remains valid after the gauge and other fluctuations are included. One of our main results is the explanation of the asymmetric tunneling background. It can be traced back to the strong on-site repulsion. It is much easier to remove an electron than to add an electron and create a doubly occupied site. We believe this result is robust and will survive the fluctuations around the mean-field state. The second main result is that the weight of the coherent quasiparticle tunneling peak is proportional to the doping $x$. After including the fluctuations, we believe that the weight of the coherent peak should have a similar doping dependence. This is because the coherent peak comes from the quasiparticle which is a bound state of a spinon and a holon. However, the detailed dependence may be of a more general form $x^{1+\alpha}$ (i.e. the fluctuations may correct the exponent). \[ \text{[10]} \]

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