Target spectrum matrix definition for multiple-input-
multiple-output control strategies applied on direct-field-
acoustic-excitation tests

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Abstract. During the last two decades there have been several improvements on environmental
coustic qualification testing for launch and space vehicles. Direct field excitation (DFAX)
tests using Multiple-Input-Multiple-Output (MIMO) control strategies seems to become the
most cost-efficient way for component and subsystem acoustic testing. However there are still
some concerns about the uniformity and diffusivity of the acoustic field produced by direct
field testing. Lately, much of the documented progresses aimed to solve the non-uniformity of
the field by altering the sound pressure level requirement, limiting responses and adding or
modifying control microphones positions. However, the first two solutions imply modifying
the qualification criteria, which could lead to under-testing, potentially risking the mission.
Furthermore, adding or moving control microphones prematurely changes the system
configuration, even if it is an optimal geometric layout in terms of wave interference patterns
control. This research investigates the target definition as an initial condition for the acoustic
MIMO control. Through experiments it is shown that for a given system configuration the
performance of a DFAX test strongly depends on the target definition procedure. As output of
this research a set of descriptors are presented describing a phenomenon defined as “Energy-
sink”.

1. Introduction
Environmental testing is a necessity to ensure the survivability of hardware during launch of space
vehicles. During the ignition and liftoff launch stages the main source of dynamic loading is from a
not deterministic acoustic environment \cite{1} \cite{2}. Hence, the environmental acoustic test becomes a
fundamental qualification test to evaluate if an aerospace structure, system or component can
withstand the vibratory load it will encounter during its operational life.

Lately, the need of reducing the cost and risk of spacecraft acoustic qualification tests have made
many of the space contractors prefer to conduct Direct Field Acoustic eXcitation (DFAX) tests instead
of Reverberant Field Acoustic Tests (RFAT). The first approach consists in setting a loudspeaker array
around the test specimen. Then the acoustic control is performed by placing an array of control
microphones within the region between the loudspeakers array and the test specimen. Through experiments it is shown that for a given system configuration the performance of a DFAX test strongly depends on the target definition procedure. As output of this research a set of descriptors are presented describing a phenomenon defined as “Energy-sink”.

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However, reliably reproducing the characteristic diffuse field and the uniform sound pressure spatial distribution of RFAT still represents a challenge for DFAX.

1.1. Control systems

Close loop control methodologies have been proposed to achieve this goal. Previous publications have shown that Single-Input-Single-Output (SISO) control strategies are not appropriate for DFAX testing [3] [4]. That is because a single drive feeding phase-aligned loudspeakers located around a test specimen generates a wave interference pattern which leads to a non-uniform acoustic field in terms of sound pressure level (SPL). This non-uniformity exceeds the +/-3dB typically recommended tolerance levels [4]. It was also shown how DFAX with SISO controllers have induced vibrational responses reaching over-testing as well as under-testing levels. This could damage the test specimen during qualification tests or lead to spacecraft mission failure during operation [3]. For full-system testing, when the specimen occupies a large percentage of the volume in between loudspeakers, the lack of acoustic field uniformity is a particular issue. In that case different components of the system could be exposed to different SPL increasing the over and under-testing potential risks.

Multiple-Input-Multiple-Output (MIMO) control strategies have been proposed to enhance the diffusivity and uniformity of the acoustic field for DFAX [5] [6] [7]. MIMO was initially implemented for multi-shaker / multi-axis vibration control [8] [9] [10]. It has been recommended when a real-world vibration environment has to be applied, if all modes (in all directions) have to be excited simultaneously or if the test specimen is too heavy to be excited with a single shaker [11]. This control technique reproduces a more realistic excitation scenario in a much less time consuming test procedure (i.e. 6 degrees of freedom profiles are required to be tested). However, it demands a more complex test configuration (e.g. the test requirement must be defined as a target spectrum matrix instead of a single power spectrum density profile [11]) and computationally is much more expensive than a SISO implementation. For acoustic control applications, MIMO seems to give more flexibility than SISO to handle the acoustic wave interferences. Sensing multiple independent acoustic responses (MO) provides the controller with more information about the acoustic field (autocorrelation plus crosscorrelation) in order to generate/correct the sound sources contribution through multiple independent signals driving the loudspeakers (MI). Test results of existing MIMO controllers for DFAX applications have shown an improvement of the acoustic field uniformity compared with SISO. However, even with MIMO, the sound pressure levels have not been always in between the required tolerance limits [12]. Furthermore, there are still several open questions regarding: the energy efficiency of the control procedure, the optimal location for the transducers, the proper definition of the target spectrum matrix and its relationship with the required power spectrum density profile. Also there are open questions about: the best distribution of drives between loudspeakers sets, the best cross over configuration to derive the driving signal to the loudspeaker’s ways, the expected response of test specimens exposed to non-diffuse acoustic fields, and the most accurate simulations tools to avoid unnecessary pre-test instances.

This research was focused on the target spectrum matrix definition as an initial stage of the acoustic MIMO control. Through experiments it is shown that for a given rectangular system (with fixed location of sound sources and microphones), the performance of a DFAX test strongly depends on the target definition procedure. This procedure demands the MIMO controller user to define a target spectrum matrix consisting of reference power spectra terms as well as reference cross spectra terms [11]. Such a challenging task must conform some physical and algebraic constrains explained in the next section. As output of this research a set of descriptors are introduced to compare the performance of two types of target spectrum matrices. Besides, two additional descriptors are presented to describe a phenomenon defined as “Energy-sink”.

The following sections consist of: a brief review about MIMO system theory; the explanation of the energy-sink concept as a system-target compatibility topic; a DFAX experiment applying two different types of target spectrum matrices and finally a conclusion section resuming the acquired knowledge.
2. MIMO system theory

A MIMO system is characterized by its transfer function $H$ consisting of $m$ inputs $u_k \in \mathbb{R}^m$ and $l$ outputs $y_k \in \mathbb{R}^l$, where the subindex $k$ denotes a discrete time instant. Its discrete-time relation between input and output can be written as [13]:

$$y_k = H(q) u_k$$

where $H(q)$ is the transfer function operator which is usually a rational polynomial function of the backshift operator $q$. In frequency domain, the input-output relation becomes:

$$Y(\omega) = H(\omega) U(\omega)$$

where $Y(\omega) \in \mathbb{C}^l$ and $U(\omega) \in \mathbb{C}^m$ are the Fourier transforms of output and input; $H(\omega) \in \mathbb{C}^{l \times m}$ is the transfer function; and $\omega$ denotes the angular frequency.

The (power) spectrum is defined as the Fourier transform of the correlation sequence. An important property of the Fourier transform is that the correlation of two time series is equivalent to multiplication of the Fourier transform of the first series by the complex conjugate of the Fourier transform of the second series [12]. Multiplying $Y(\omega)$ by its complex conjugate transpose $Y(\omega)^*$ yields:

$$YY^H = HUH^H$$

Note that from equation (3) the frequency dependency is omitted for convenience of notation. The power spectrum matrix of the outputs, $S_{yy}(\omega) \in \mathbb{C}^{l \times l}$, is the expected value, or the infinite average [12], of equation (3). Finally, the output and input spectrum matrices are related as:

$$S_{yy} = HS_{uu}H^H$$

2.1. MIMO random control algorithm

The purpose of a MIMO random control algorithm for DFAX is the automatic computation of the driving signals required to reach the responses defined by the controller user through the target spectrum matrix ($S_{yy}^{REF}$). The computation is done through equation (5):

$$S_{uu} = H^H S_{yy}^{REF} (H^H)^H$$

where $\hat{H}$ denotes the Moore-Penrose pseudo-inverse of a matrix [14].

A feedback control implementation allows correcting the drives in case the system does not behave linearly and time invariant. This correction is performed sequentially based on the average of a fixed number of response time windows defining the loop time. The correction block is represented as $\Delta$ in the simplified MIMO random control algorithm scheme of figure 1.

![Figure 1. MIMO control algorithm simplified scheme.](image-url)

This scheme summarizes the control algorithm used for the experiment. The first step is the identification of the system. The FRFs obtained by this process allow the calculation of the initial set of drives $S_{uu}^{(0)}$ following equation (5). After the system is driven by first time, the difference between $S_{yy}^{(0)}$ and $S_{yy}^{REF(0)}$ is calculated ($\Delta$) [8]. This difference between matrices is used to modify $S_{yy}^{REF}$ keeping constant the estimated system ($\hat{H}$) in order to get an updated target spectrum matrix $S_{yy}^{REF(1)}$. This leads
to the computation of the second set of drives $S_{uw}^{(1)}$. The procedure is repeated according to the user configuration of the algorithm (e.g. amount of averages) and to the testing time specified by the qualification test.

2.2. System dimension

Two kinds of systems can be configured for a MIMO DFAX test: square ($m = l$) or rectangular ($m \neq l$). Although an existing patent suggest the use of any of them [19], in this research a rectangular system has been investigated. A previous publication has shown the rectangular systems with larger amount of microphones than drives ($l > m$) have two benefits: reduced probability of producing ill-conditioned matrices [20]; and possibility to control more points which could improve the uniformity of the acoustic field.

2.3. Fundamental rank theorem

Recalling equation (4) an algebraic constraint should be mentioned. It should be taken into account when a target spectrum matrix must be defined by the MIMO control user. For any frequency line, the dimension of $S_{uu}$ is $m \times m$ while the dimension of $H$ is $l \times m$. According to the fundamental rank theorem [21]:

$$r(S_{uu}) \leq \min \{ r(H), r(S_{uu}) \}$$  \hspace{1cm} (6)

Meaning the maximum rank for $S_{yy}$, at the same frequency line than $S_{uu}$, will be at most $m$ (if $l > m$). This brings into question if it is meaningful for the controller user to set $S_{yy}^{REF}$ with higher rank than $m$ (when $l > m$).

2.4. Target achievability

From equation (4) and (5), and recalling the system is estimated:

$$S_{yy} = H \hat{H}^H S_{yy}^{REF} \left( \hat{H}^H H \right)^H$$  \hspace{1cm} (7)

Where $\hat{H}$ is the estimation of $H$. It means that $S_{yy}$ is going to be exactly $S_{yy}^{REF}$ when

$$H \hat{H}^H = \left( \hat{H}^H \right)^H H \hat{H}^H = I$$  \hspace{1cm} (8)

where $I$ is the identity matrix.

There are two requirements for equation (8) to be true: the estimation $\hat{H}$ must be equal to the true system $H$ and the system must have a right inverse matrix. The first can be assumed as explained in section 6.

Regarding the second requirement, if $H$ is square and has full rank it has a 2-sided inverse [22]. Then $\hat{H}^H = \hat{H}^H$ and $H \hat{H}^H = I = \hat{H}^H H$ accomplishing equation (8). However, in case of a rectangular system, $H$ would have a right inverse matrix only if its rank equals its number of rows ($l$) and the number of rows is less than the number of columns ($l < m$). This is the case of having more drives than microphones, an underdetermined system which is not useful for DFAX. For rectangular systems with larger amount of microphones than drives ($l > m$) the unique generalized inverse (Moore Penrose) is equal to the left inverse $\hat{H}^H = H_{left}^{+}$ [23]. In this case, if $H$ has full column rank, $P$ becomes the matrix which projects $C^l$ onto the column space of $H$ [22].

$$H H_{left}^{+} = H \hat{H}^H = P$$  \hspace{1cm} (9)

From equation (9), equation (7) can be rewritten as:

$$S_{yy} = P S_{yy}^{REF} P^H$$  \hspace{1cm} (10)

Combining equation (3) and (10) yields:

$$Y = \left(P Y^{REF} \right) \left(P Y^{REF} \right)^H$$  \hspace{1cm} (11)

Then, the output is expected to be the target only if:

$$\left(P Y^{REF} \right) = Y^{REF}$$  \hspace{1cm} (12)
This means the following statements are equivalent: the target \( Y^{\text{REF}} \) belongs to the column space of \( H \), the projection of \( Y^{\text{REF}} \) in the column space of \( H \) is \( Y^{\text{REF}} \); the target is achievable for the estimated system.

3. MIMO for DFAX: system-target relationship

3.1. The target spectrum matrix definition for DFAX

In MIMO control the target spectrum matrix \( S_{yy}^{\text{REF}} \) is composed by power spectral density (PSD) terms in its diagonal and by cross spectral density (CSD) terms. In other words, next to reference power spectra, also reference cross spectra need to be specified by the user. The last are the off diagonal terms representing the expected sound pressure relation between each controlled point in the acoustic field. For a couple of points \( a \) and \( b \), the CSD can be calculated through:

\[
\text{CSD}_{ab} = \left( \gamma_{ab}^2 \frac{PSD_{aa}PSD_{bb}}{\sqrt{PSD_{aa}PSD_{bb}}} \right)^{1/2} e^{j\phi_{ab}}
\]

where \( \gamma_{ab}^2 \) is the ordinary coherence value between \( a \) and \( b \), \( PSD_{aa} \) and \( PSD_{bb} \) are the power spectra and \( \phi_{ab} \) is the phase angle between \( a \) and \( b \).

The DFAX requirement profile is just a single PSD. Different aerospace contractors define different PSD requirements (PSD\text{req}) based on flight data. Usually they are expressed in normalized 1/1 octave band sound pressure levels in the frequency range between 31.5 Hz and 10 kHz. Each contractor also specifies the tolerance per band expressed in SPL and the testing acceptance time which usually is 60 seconds.

DFAX with MIMO random control requires the user to define the reference cross spectra which brings into question: what is the most appropriate criterion to do it?

3.1.1. The Ideal target. When aiming to reproduce a reverberant acoustic field in a DFAX test, sound pressure uniformity and diffusivity need to be achieved. This means that all the PSDs at all the \( l \) controlled points should be statistically equal to the PSD\text{req}. Moreover, as a diffuse acoustic field is aimed, the coherence between responses should be low. According to equation (13), low coherences translate to low CSDs. Then, Ideal target is defined as the target spectrum matrix with PSD\text{req} in its diagonal and CSDs=0. This kind of target spectrum matrix was already proposed by the patent US 2012/0300580 A1 [19] for DFAX testing.

3.1.2. The OL+PSD\text{req} target. Previous publications have suggested defining \( S_{yy}^{\text{REF}} \) from measured field data to perform a multi-exciters random vibration test in the laboratory [24] [12] [25] [26]. The benefit of using field data consists on its inherent achievability. Not just algebraically, but also as a physically realizable target. The OL+PSD\text{req} target is defined as the target spectrum matrix with PSD\text{req} in its diagonal and CSDs calculated through equation (13). The CSDs are calculated based on the phases and the coherences recorded at the control points during an Open Loop pre-test using \( m \) uncorrelated drives. Due to the fact that this target spectrum matrix is only partially built with measured field data, its achievability is not guaranteed.

3.2. Energy-sink

The energy-sink is the phenomena observed when a non-achievable target is imposed as target spectrum matrix of a MIMO random control process. It leads to under-testing responses at the controlled points and usually to over-testing at the monitoring points of the acoustic field.

The target non-achievability is strictly dependent of the system configuration and mostly defined by the loudspeakers and the microphones location (and their dynamics). As explained in the previous section, solving successfully a system implies the orthogonal projection of the target in the complex vector space generated by the linearly independent columns of \( H \). “Loss of energy” can be detected when a non-achievable target is projected in this subspace. It seems to be dependent of the Euclidean
distance (in the complex Euclidean space \( C^l \)) between the aimed target \( \gamma_{\text{REF}} \in C^l \) and its projection \( \text{Proj}_H(\gamma_{\text{REF}}) \in C^l \) following:

\[
\gamma_{\text{REF}} = \text{Proj}_H(\gamma_{\text{REF}}) + E
\]

where \( E \in C^l \) is the component of \( \gamma_{\text{REF}} \) perpendicular to the column space of the system. \( E \) belongs to the left nullspace of \( H \) and can be also written as a projection of the aimed target:

\[
E = (I - P) \gamma_{\text{REF}}
\]

where \( I \) is the identity matrix and \( P \) is the projection matrix from equation (9). Multiplying \( E \) by its complex conjugate transpose yields:

\[
E E^H = (I - P) \gamma_{\text{REF}} \gamma_{\text{REF}}^H (I - P)^H
\]

Finally, the energy-sink produced by certain target spectrum matrix can be predicted through the expected value, or the infinite average of equation (16):

\[
S_{ee} = (I - P) S_{yy} \gamma_{\text{REF}} (I - P)^H
\]

The infinite norm of \( S_{ee} \) can be considered an absolute descriptor allowing to evaluate, a priori, if a target spectrum matrix is going to be achievable. Geometric intuition leads to a second descriptor, a relative one defined as \( S_{ee,\phi} \). It is the ratio between the infinite norm of \( S_{ee} \) and the infinite norm of the projected target spectrum matrix \( S_{yy} \).

In the following sections are described the procedure and the results of DFAX experiments with MIMO random control. The performance of the test using the Ideal target and the \( OL + PSD_{\text{req}} \) target are compared in terms of uniformity, diffusivity and energy efficiency.

4. Procedure of DFAX experiments

4.1. Microphones and sound sources location

Four LMS Q Low/Mid-Frequency Volume Sources (A, B, C, D) were arranged in the semi-anechoic facility of Siemens Industry Software in Leuven, Belgium. Their location described a circumference, as shown with green squares in figure 2. Twenty microphones were randomly distributed inside the test volume in between the sources. Ten of them were G.R.A.S. Type 40PF (nominal sensitivity=50mV/Pa) and the rest G.R.A.S. Type 40BE (nominal sensitivity=3mV/Pa). From the twenty microphones, eight were randomly selected as Control microphones (Ctrl). They are represented in figure 2 with red triangles. The rest of the microphones (Mon) were used for the acoustic field monitoring and are represented as blue circles.

![Figure 2. Experimental transducers array.](image)

4.2. Test specimen

As this research focused on the properties of the acoustic field and the ability to control it with different kinds of Target spectrum matrices, it was not placed a test specimen in between the sources. Although it is taken into consideration how a structure in the test volume would modify the acoustic.
field due to reflection over its surface, diffraction on the edges and absorption of the body, it was not the purpose of this investigation to quantify these contributions.

4.3. Test requirement profile and frequency range
The narrow-band requirement profile used for this study was calculated from the original 1/1 octave band DFAX requirement shown in Table 1. As the four available Q sources for the experiment were not able to produce the necessary SPL, the requirement was attenuated 73dB per band. It leaded to the levels observed in figure 3a. This attenuation was necessary due to amplification constraints. It was considered appropriate because the background noise in the facility was below 50 dBSPL (equivalent continuous sound level).

| Table 1. Typical DFAX requirement profile in 1/1 octave band sound pressure levels. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Original Req (dB): 130.8 | 134.9 | 139 | 141 | 137.1 | 131 | 126.9 | 123.3 | 120.7 |
| Tolerance (dB): 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 2 |
| Freq (Hz): 31.5 | 63 | 125 | 250 | 500 | 1000 | 2000 | 4000 | 8000 | full range |

This study was also constrained in frequency range according to the sources frequency response. Based on these considerations figure 3b shows the PSD$_{req}$ used for this research. It was calculated from the SPL between 31.5 Hz and 400Hz, with 1.56 Hz frequency resolution, and according to recommendations [28].

4.4. Research descriptors
The uniformity and the diffusivity of the acoustic field as well as the energy efficiency were analysed through the following non-standardized descriptors. They allowed comparing the performance of the two types of target spectrum matrices.

4.4.1. Energy-sink. $|\text{See}|$ and $|\text{See} _\text{phi}|$, please refer to section 3.2.

4.4.2. Uniformity. #bands: Corresponds to the number of bands below or exceeding the tolerance level. As the amount of 1/3th octave bands between 31.5 and 400Hz is 12 and the number of mics was 20, the total number of bands is 240. The descriptor "#bands = x" indicates that x bands of 240 were outside the tolerance limits. RMSE: The root mean square error describes the sample standard deviation between the spatial SPL average and the required SPL profile. This descriptor can be understood as how good the spatial average response fits the requirement profile. The lower the RMSE the closer the response to the requirement.

4.4.3. Diffusivity. ABS$_\text{coh}$: The absolute coherence is the averaged coherence between all the combinations of microphones (upper off-diagonal elements of the output spectrum matrix).

4.4.4. Energy efficiency. AVG$_\text{Vrms}$: It is the average between the root mean square amplitude of the drives.
4.5. Experimental method

The sound sources and the microphones were connected to LMS SCADAS Mobile data acquisition hardware. The Q sources were driven with four independent driving signals. Besides the twenty microphones four additional inputs were used for the volume acceleration references provided by the Q sources.

The test procedure followed the scheme of figure 1. First the system was identified and then the target spectrum matrix was defined through an external script written in MATLAB. Both, the estimated system and the target spectrum matrix were input to LMS Test.Lab Environmental Testing Multi Axis Random Control software. This application was designed for vibration control. However, it allowed processing data in the audio frequency range of this study and also allowed processing “acoustical quantities” (e.g. pressure, volume acceleration). Hence, the software was considered suitable to conduct the close loop control, completing the sequence described for figure 1.

4.5.1. System identification strategy. The system FRFs were estimated through an experimental model of the dynamic system built from observed input-output data. In the control field, the techniques applied for this process are known under the term System Identification (SI) [31]. For vibration and acoustic control this process consists in performing a low-level pre-test applying non-parametric SI to estimate the system transfer function. Low-level excitation is recommended due to a safety measure and lack of knowledge about the system. However, as the SPL requirement for this research was attenuated and no device was exposed to the acoustic field, it was considered that there was no significant risk. Hence the SI was done at full-level. The benefit was that the signal to noise ratio (S/N) during the SI and during the DFAX test were the same. This might reduce the difference between the true system \(H\) and the estimated system \(\hat{H}\). The applied SI method was the \(H_1\) estimator. It involves an error signal, which is defined as the difference between the pressure responses and the estimated ones. Minimizing the mean square error with respect to the model system produces the multichannel Wiener-Hopf equations [32]. Then, in the frequency domain, the estimated transfer function is given by equation (18):

\[
H_f(\omega) = S_{yu}(\omega)S_{uu}(\omega)^{-1}
\]

Where \(S_{yu}\) is the cross spectrum matrix between outputs and inputs and \(S_{uu}\) is the input spectrum matrix calculated from a set of uncorrelated driving signals.

Recording the output signals during the SI (sound pressure time series at the control points) allowed calculating \(S_{xy}\). From \(S_{xy}\) was straightforward to obtain the realistic coherences and phases between the sensors in the acoustic field. As explained in section 3.1.2 they were necessary to define one of the evaluated target spectrum matrices.

5. DFAX experiment results

The results presented in the following section correspond to data acquired during DFAX tests. The data was processed in MATLAB for easier viewing. Unless stated otherwise in each subsection, the processed data below was not filtered or modified.

Figure 4 shows a submatrix of \(\hat{H}\). The plots represent the linear contribution of source A to control microphones 1 (\(h_{1A}\)) and 2 (\(h_{2A}\)) (left, top and bottom); and the contribution of source B to control microphones 1 (\(h_{1B}\)) and 2 (\(h_{2B}\)) (right, top and bottom) for the frequency range of this study.

The magnitude on the plots is not completely flat because of the acoustic wave ground reflection. A clear example of the ground effect is in case of source B \((x=2.79, y=1.43, z=0.95)\) and Ctrl 2 \((x=1.21, y=1.52, z=1.20)\). The path difference \(\Delta p\) between the direct sound and the reflected is 1.07m (assuming Snell’s law or specular reflection). According to the equation (19), at 20°C, the first cancelling frequency \(f_g\) is 160Hz.

\[
f_g = \frac{c}{2\Delta p}
\]
Figure 4. FRFs submatrix. a. $h_{1A}$ b. $h_{2A}$ c. $h_{1B}$ d. $h_{2B}$.

Figure 5 shows the condition number of the system for each frequency line. The condition number of a matrix is defined as the ratio between its maximum and its minimum singular values. It is an estimate of the error propagation in the solution of the linear system of equations. It was highlighted the importance of the system condition when an inversion should be calculated for closed-loop control [20] [27]. Therefore, to avoid instability of ill-conditioned systems it was set a singularity threshold equal to 0.1% for matrix inversion through truncated SVD. A continuous rank $= m = 4$ for all the frequency range is consistent with the threshold because the rank would be lower than $m$ only if the condition number is greater than or equal to 1000. Hence, no error is expected after matrix inversion.

5.1. Close loop control results: Ideal target

5.1.1. Energy-sink. The analysis of the target spectrum matrix and system in terms of conditioning and energy-sink is performed recalling equation (7) and the fundamental rank theorem [21]. Once the rank of the system per frequency line is known, the rank of the target spectrum matrix gives an initial idea if energy sink should be expected. In figure 6a it can be observed the rank of the Ideal Target is twice the one of the system for all the frequency range. It does not conform the algebraic constrains.

Figure 5. System condition.

Figure 6. a. Target spectrum matrix condition. b. Energy-sink.
Figure 6b shows the two proposed descriptors to quantify the energy sink. From these curves two global values can be calculated. $|\text{See}|$ is the absolute quantification of the Energy-sink and $|\text{See}_{\text{phi}}|$ is a relative metric between 0 and 1. The lower $\text{See}_{\text{phi}}$ the lower the expected Energy-sink (the aimed target is closer to the achievable one). In case of the Ideal target $|\text{See}|=2.51\times10^{-5}$ and $|\text{See}_{\text{phi}}|=0.44$. Then, a great loss of energy should be expected when the Ideal Target is aimed.

5.1.2. Uniformity. The DFAX requirement profile and tolerance are usually given in octave bands. Therefore the uniformity of the acoustic field is evaluated in 1/3 octave bands. However, as the controller operates in narrow-band the results can be analysed in both ways. Narrow-band results from figure 7a are particularly useful to observe how the control microphones responses (Ctrl) tend to be below the target requirement as a consequence of the Energy-sink. This is strictly evident over 125Hz where 14 bands of figure 7b are below the tolerance. Below this frequency the narrow band plot shows some frequency lines over the tolerance limit. This might be the result of the acoustic wave reflections on the ground of the facility. Their contribution is leading to a RMSE = 0.99dB.

![Figure 7. Control microphones responses. a. Narrow-band. b. 1/3 octave band.](image)

In case of the monitoring microphones (Mon) it can be observed how some of the narrow band results are exceeding the alarm and the abort limit on figure 8a. According to these power spectral density curves over the tolerance, #bands increased to 61. However RMSE decreased to 0.83 dB. The reason is that the bands exceeding the requirement level are compensating the control microphones levels below it. Due to this behaviour it is not advisable considering successful a DFAX test just based on the Avg level or the RMSE.

It is interesting to remark that Mon 11 and Mon 14 have the highest SPL. They are the closest non-controlled microphones to Source C (0.87m) and Source D (0.61m), respectively. It explains why the drives were not corrected during the close loop process to avoid overtesting in this points.

![Figure 8. Monitor microphones responses. a. Narrow-band. b. 1/3 octave band.](image)

5.1.3. Drive efficiency. The efficiency of a DFAX test with MIMO control depends of a chain of processes and devices properties. Although the overall efficiency might be critically affected by the ability of the speakers to transform electric energy in acoustic energy (commercial speaker’s efficiency is usually below 10%), the signal generation plays also an important role. According to the scheme of figure 1, the signal generation for a MIMO system depends directly of the aimed Target spectrum matrix. Then, in this research the drive efficiency analysis consists in evaluating how much driving
signal energy is required by different target spectrum matrices in order to accomplish the same DFAX SPL requirement. Figure 9a shows a segment of the volume acceleration time series. Figure 9b shows the estimated power spectral density (based on the whole time series length). As it can be observed some of the waveforms have opposite phase. This could explain the waste of driving energy when the controller tries to reach the requirement through destructive acoustic wave interference at the control points of the acoustic field. It also might explain the high SPL at monitor points due to constructive interference. In order to compare the driving signal amplitude between the Ideal and the OL+PSDreq targets, a global descriptor is calculated. AVG_Vrms is the average between the four root mean squares of the driving signals. For the Ideal target AVG_Vrms=1.55.

5.1.4. Diffusivity. A diffuse acoustic field is described as having:
- uniform spatially averaged (over a couple wavelengths) energy density;
- for any location in the field all directions of energy propagation are equally probable;
- and a random phase relation among propagating plane waves arriving at any location [3].

Due to the relatively short distance between sources, the first bullet could be evaluated only at mid/high frequencies. The second one would require of particle velocity measurements or P-P sound intensity measurements, although considerations should be taken concerning the distance between microphones’ capsules and the frequency measurement limitation. In this research the last feature was studied. It suggests that in a diffuse field the responses at the microphone locations should not be correlated.

The coherence measures the linear correlation between two time series at each frequency line and it is directly analogous to the squared correlation coefficient in linear regression [30]. Hence, it was considered as an appropriate metric to quantify the acoustic field diffusivity. Figure 10 shows the coherence between control microphones. The values were calculated from the averaged responses leading to AVG_coh= 0.19. The higher curves correspond to the cyan and black curves, which are the coherences between Ctrl 6/2 and between Ctrl 8/7.

![Figure 9. Driving signals. a. Time series. b. PSDs.](image)

![Figure 10. Coherence between control microphones.](image)

5.2. Close loop control results: OL+PSDreq target

5.2.1. Energy-sink. The first noticeable difference between the OL+PSDreq target spectrum matrix and the Ideal one is its rank. Figure 11a shows that for all the frequency range it is at most 4. It means that, although constant PSDreq were imposed for the target spectrum matrix diagonal, the maximum allowable range according to the fundamental rank theory was not exceeded. In other words, imposing realistic coherences and the phases during the OL+PSDreq target definition reduced the algebraically unrealistic rank of the Ideal target. However, a rank equal or below 4 is not a guarantee of target achievability. Figure 11b shows that the energy-sink is not null. Nevertheless, it is 4.88 times lower than the one for the Ideal target (See\textsubscript{phi}), and 8.26 times lower in absolute terms (|See|).
5.2. Uniformity. The narrow-band responses for the OL+PSD\textsubscript{req} target do not follow a systematic trend to be below the requirement profile as in case of the Ideal target. Figure 12b shows that all the 1/3 octave bands are in between the tolerance limits (Alarm). Then #bands=0 and RMSE= 0.22 dB for the control microphones.

Figure 12. Control microphones responses. a. Narrow-band. b. 1/3 octave band.

Figure 13a shows the narrow-band responses for the monitor microphones. Although they seem to be less dispersed than the produced by the Ideal target in figure 8a, they still show sharp peaks and dips exceeding the tolerance limits. Mon 13 and 14 show a particular dip at around 85Hz and Mon 15 a series of over-testing peaks between 50Hz and 125Hz. This is the frequency range where 10 of the 14 bands outside the tolerance limits are located.

Figure 13. Monitor microphones responses. a. Narrow-band. b. 1/3 octave band.

5.2.3. Drive efficiency. Figure 14a shows the drives have significantly lower amplitude than the ones of the Ideal target in figure 9a. The global descriptor is AVG_Vrms=0.57, which is 2.70 times lower than the same descriptor for the Ideal target. Figure 14b shows that most of the saved energy corresponds to low frequencies.

5.2.4. Diffusivity. Figure 15 shows that the main drawback of the OL+PSD\textsubscript{req} target is an increase of the spectral coherence between the microphones in the field. The ABS\textsubscript{coh} for the control microphones is 0.56, almost 3 times the coherence of the Ideal target. This could compromise the classification of the acoustic field as diffuse.
5.3. Comparison of Results

Table 2 shows the results of the experiments when both type of targets are set for the close loop control. In the next section the main conclusions are derived from the comparison of the research descriptors summarized in this table.

| Energy-sink | Uniformity | Energy efficiency | Diffusivity |
|-------------|------------|--------------------|-------------|
| Control mics. | Monitor mics. | Drives | Control mics. |
| See | See_ph | #band | RMSE | #band | RMSE | AVG_Vrms | ABS_coh |
| Ideal Target | 2.51e-5 | 0.44 | 14 | 0.99 | 61 | 0.22 | 1.55 | 0.19 |
| OL+PSD_{req} | 3.04e-6 | 0.09 | 0 | 0.83 | 14 | 0.51 | 0.57 | 0.56 |

6. Conclusions

DFAX experiments were performed in the semi-anechoic facility of Siemens Industry Software in Leuven, Belgium. A rectangular system was controlled with MIMO techniques. The results showed that the control performance depended of the aimed target spectrum matrix. The proposed energy-sink descriptors indicated accurately if loss of energy should be expected. However, it is not yet possible to predict precisely how much energy could be lost.

Regarding diffusivity, the Ideal target leaded to correlated drives and uncorrelated microphone responses. On the other hand the OL+PSD_{req} target leaded to highly correlated microphone responses and uncorrelated drives.

About drives efficiency, the Ideal target produced high amplitude drives which usually had opposite phase while the OL+PSD_{req} target produced drives with up to 2.70 times lower amplitudes. Moreover, the first did not guarantee control microphones responses achieving the required SPL and leaded to over-testing levels on the monitored locations. The OL+PSD_{req} target produced more uniform responses at the control points although some monitoring microphones responses were still exceeding the tolerance.

The analysis of the results suggests that the target definition procedure can still be improved. Novel techniques must be developed to achieve more efficient controllers leading to uniform acoustic fields. The uniformity should not be linked to highly coherent pressure responses in a trade-off sense.

Further measurements should be done in other kind of rooms (not semi anechoic) to study deeper the influence of the reflections and the diffraction in the direct field. Dedicated experiments should also be conducted to demonstrate if the lack of uniformity observed between 50Hz and 125Hz is due to ground reflections.

Higher OASPL requirements should also be studied to understand if concepts such as the critic distance could be applied in the DFAX test volume. Besides, higher OASPL would require more power. Electroacoustic transducers as electrodynamic loudspeakers could present nonlinearities due to changes in its nominal impedance because of higher temperatures. Assessing the capabilities of the controller to track the potential acoustic nonlinearities at higher level is a fundamental topic for further investigations.

Regarding the theoretical topics developed above could be concluded that although rectangular systems have advantages compared with the square ones, not all the targets are going to be achievable. In other words, not all the set of required RMS pressure amplitudes (Pa) at each microphone and the
phase relationships between the pressure waves are going to be realistic for certain rectangular system. Nevertheless, the same kind of remark must be done for ill-conditioned square systems. The columns of a rank deficient square matrix do not span any more the whole space because some of its columns are linearly dependent. Therefore, not all the targets are going to be achievable as happened with the rectangular systems. This analysis supports the decision to keep working with rectangular systems with larger amount of microphones than drives.

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