HYBRID INFLATION IN SUPERGRAVITY WITHOUT INFLATON SUPERPOTENTIAL

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Abstract

We propose a new realisation of hybrid inflation in supergravity where the inflaton field does not appear in the superpotential but contributes only through the Kähler potential. The scalar potential derived from an $R$-invariant superpotential has the same form as that of the Linde’s original version. The correct magnitude of the density perturbations amplitude is found without any fine-tuning of the coupling parameter in the superpotential for an acceptable value of the fundamental energy scale of the theory. The $\eta$–problem was also resolved in this model.

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1 Introduction

It is now believed that it is necessary to invoke an inflationary era [1] in order to give a consistent description of the early Universe. Indeed, the standard hot big-bang model presents some conceptual problems related to the requirement of unnatural initial conditions, in particular the horizon and flatness problems. These problems have been resolved by assuming the existence of a sufficiently long period of fast inflation during which the energy density of the Universe is dominated by a non-vanishing vacuum energy. In such a scenario we can also explain the origin of the density perturbations which are responsible for the observed temperature anisotropies in the cosmic microwave background radiation (cmbr) and the large-scale structure of the observable Universe.

Supersymmetry which was initially motivated by the gauge hierarchy problem [2] seems to have interesting cosmological implications as well. On one side, supersymmetric theories often have non compact flat vacuum directions which remain flat to all order of perturbation theory [3]. They are then good candidates for inflation since a long period of inflation requires a sufficiently flat potential. The combination of inflation and supersymmetry naturally leads to consider inflation in supergravity. However, the single-field models of inflation present a common problem of naturalness. Indeed, to accomplish a successful inflationary scenario the corresponding constraints on the potential of the inflaton impose an unrealistic fine tuning of the parameters of the relevant theory of particle physics. This problem can be avoided in the hybrid inflation model proposed by Linde [4].

The hybrid inflation model is naturally realised in supersymmetric theories and its relevance to SUSY has been extensively investigated [5]. The non zero vacuum energy density during inflation can either be due to the vev of a $F$-term or that of a $D$-term. The scalar potential has two minima: a local one of value of the inflaton $S$ greater than some critical value $S_c$ (with a vanishing noninflaton field $\phi$), and a global supersymmetric one at $S = 0$. When $S \gg S_c$ the universe is dominated by a non vanishing vacuum energy, the slow rolling conditions are satisfied and inflation takes place until $S = S_c$ when a phase transition occurs causing the end of inflation.

The naturalness issue has been a great challenge for physicists. In this paper we want to contribute to the resolution of this problem in the case of a model of inflation in supergravity based on an $R$–invariant superpotential. Our approach consists of constructing a hybrid inflation model from the same superpotential as that of the model of reference [6] where the authors have obtained a flat potential as usually required for a successful inflation, but at the price of a very small coupling parameter. We have introduced a second scalar field that does not contribute
to the superpotential but appears only in the Kähler potential. The result is a hybrid inflation model where the non-vanishing energy is provided by the inflaton potential.

2 The initial model

In this section we give a brief review of the model presented in the reference [6]. This model is based on the discrete $Z_n$-R invariance which is given by the transformation:

$$
\phi(x, \theta) \rightarrow e^{-i\alpha_n} \phi(x, e^{i\alpha_n/2}\theta)
$$

(1)

on the inflaton field where $\alpha_n = \frac{2\pi k}{n} (k = \pm 1, \pm 2, \cdots)$.

The general form of the superpotential and the Kähler potential which have the $Z_n - R$ invariance is given by:

$$
W_n(\phi) = \phi \sum_{l=0}^{\infty} b_l \phi^{ln}
$$

(2)

$$
K(\phi, \phi^*) = \sum_{m=1}^{\infty} a_m (\phi \phi^*)^m
$$

(3)

The Kähler potential is taken to be of the minimal form:

$$
K(\phi, \phi^*) = \frac{\phi \phi^*}{M_p^2}
$$

(4)

The expression Eq.(2) is convergent only for $|\phi| \leq 0$, this corresponds also to a model of new inflation where the inflaton field begins its evolution near the origin. So, in order to find the approximate form of $V(\phi)$ near the origin $\phi \sim 0$, one can take only the terms:

$$
b_0 = \lambda v^2 \; ; \; b_1 = \frac{\lambda}{v^{n-2}} \frac{1}{n+1}
$$

the superpotential is then written as:

$$
W(\phi) = \left( \frac{\lambda}{v^{n-2}} \right) \left( v^n \phi - \frac{1}{n+1} \phi^{n+1} \right) + \cdots
$$

(5)

where $\lambda$ is a dimensionless coupling constant and $v$ is a constant of dimension one in mass unit$^1$.

In the above equation ... represent higher power part ($\phi^{kn+1}$ with $k \geq 2$).

In the minimal $N = 1$ supergravity a scalar potential $V$ is written as [2]

$$
V(\phi) = e^{\frac{|\phi|^2}{M_p^2}} \left\{ \frac{\partial W_n}{\partial \phi} \phi^* W_n^{M_p^2} \right\} - \frac{3|W_n|^2}{M_p^2}
$$

+ $D$-term

(6)

$^1$In fact, $v$ is a scale of the condensation of a superfield coupled to the inflaton. This condensation breaks a $U(1)$ symmetry down to the discrete $Z_n - R$ symmetry.
where $M_p$ is the reduced Planck mass $M_p = m_p / \sqrt{8\pi} = 2.4 \times 10^{18}$ GeV.

With the expressions (4) and (5), and the equation (6), the $Z_n - R$ invariant scalar potential becomes

$$V(\phi) = \left( \frac{\lambda}{v^{n-2}} \right)^2 \left( v^{2n} + \frac{1}{2} v^{2n} \left( \frac{\lambda}{M_p^2} \right)^2 - v^n (\phi^n + \phi^* n) \right)$$

(7)

Since $v \ll M_p$, the $\phi^2 / M_p^2$ term can be neglected for $n \geq 3$ which gives a very flat region in the inflaton potential near $\phi = 0$. Identifying the inflaton field with the real component of $\phi$ ($\varphi = \sqrt{2} Re \phi$), the relevant potential is now

$$V(\varphi) \simeq \tilde{\lambda}^2 \tilde{v}^4 \left[ 1 - 2 \left( \frac{\varphi}{\tilde{v}} \right)^n \right]$$

(8)

with $\tilde{\lambda} = \frac{1}{2} \lambda$ and $\tilde{v} = \sqrt{2} v$.

During the slow-rolling phase the inflationary dynamics is described by the equation of motion:

$$\dot{\varphi} \simeq \frac{2n \tilde{\lambda} M_p}{\sqrt{3} \tilde{v}^{n-1}} \varphi^{n-2}$$

(9)

The slow-rolling regime ends at:

$$\varphi_f^{n-2} \simeq \frac{1}{6n(n-1) M_p^2}$$

(10)

From the constraint imposed by the observed anisotropies on the amplitude of the density perturbations one deduces the equation:

$$\frac{\tilde{\lambda} \tilde{v}^2}{10\sqrt{3} \pi n M_p^3} \left\{ \frac{\tilde{v}^2}{2Nn(n-2) M_p^2} \right\}^{\frac{1-n}{n-2}} \sim 2 \times 10^{-5}$$

(11)

where $N$ is the total number of e-foldings of the inflationary phase.

The gravitino mass $m_{3/2}$ is given by:

$$m_{3/2} \simeq \frac{n}{\sqrt{2(n+1)}} \tilde{\lambda} \tilde{v} \left( \frac{\tilde{v}}{M_p} \right)^2$$

(12)

From Eqs.(11) and (12) one can determine $\lambda$ and $v$ for a given sets of $N$ and $m_{3/2}$. To illustrate the results we choose, for example, the case $v \sim 10^{15}$ GeV, $\lambda$ is then:

$$\lambda \sim 5 \times 10^{-7}$$

(13)

3 The hybrid inflation model

The authors have clearly succeeded to obtain a sufficiently flat potential for a successful inflationary model (as we can see from Eq.(8)). However, the price to pay was the fine-tuning
of the coupling parameter [Eq.(13)] as was always the case in one field models of inflation, in particular the fine tuning was the main problem of the new inflation scenario implemented in this model. Indeed, small parameters are inevitable in order to have sufficient inflation and the correct magnitude of density fluctuations. The resolution of this problem was one of the initial motivations of the hybrid inflation model proposed by Linde [4] and studied by Copeland et al. [7] where two scalar fields are relevant.

In the same way we try to construct a hybrid inflation model in supergravity. Our model is based on the superpotential given by Eq.(5) with the introduction of a second scalar field in such a way to preserve the initial $R-$invariance. The second field we will introduce in this model contributes only through the Kähler potential. The G-singlet fields which do not contribute to the superpotential have been considered in reference [8] to generate the mass-term of the inflaton field.

In this letter we show that such a field can play a more important role than simply contribute to the mass of the inflaton: it could even play the role of the inflaton. A simple realisation of such a model is provided by the superpotential [Eq.(5)] and the Kähler potential:

$$K = \frac{\phi^* \phi}{M_p^2} + \alpha \frac{S^* S}{M_p^2}$$  \hspace{1cm} (14)

where $\alpha$ is a small parameter ($\alpha \ll 1$).

This form of the Kähler potential clearly respects the initial discrete $R-$invariance Eq.(3) for the two fields.

The scalar potential is given by:

$$V(\phi, S) = \exp\left((|\phi|^2 + \alpha |S|^2)/M_p^2\right) \left(\frac{\lambda}{v^{n-2}}\right)^2 \left\{ v^n - \phi^n + \frac{\phi^*}{M_p^2} \left( v^n \phi - \frac{1}{n+1} \phi^{n+1} \right) \right\}^2 \right.$$  \hspace{1cm} (15)

By taking the leading terms in the exponential factor to be: $\lambda |S|^2/M_p^2$ the scalar potential takes the form:

$$V(\phi) = \left(\frac{\lambda}{v^{n-2}}\right)^2 \left\{ P_1(\phi^n) - 2|\phi|^2 P_2(\phi^n) + |\phi|^4 P_3(\phi^n) + \alpha |S|^2 |\phi|^2 P_2(\phi^n) \right.$$

$$\left. + \text{ higher powers of } |\phi|^n \right\} \hspace{1cm} (16)$$
where \( P_i(\phi^n) = v^{2n} - c(n)(\phi^n + \phi^*n) \) \((i = 1, 2, 2, 4)\) and \( c(n) \) is a function of \( n \).

The higher powers of \((\phi/v)^n\) do not contribute to the dynamics of the inflaton field and can be ignored. By appropriate transformation we can bring \( S \) and \( \phi \) to the real axis:

\[
S = \frac{\sigma}{\sqrt{2}} ; \quad \phi = \frac{\varphi}{\sqrt{2}}
\]

The relevant scalar potential is then:

\[
V(\sigma, \varphi) = \lambda^2 v^4 \left[ 1 - \frac{\varphi^2}{2M_p^2} \right]^2 + \frac{\sigma^2 \varphi^2}{4M_p^2} + \frac{\alpha \sigma^2}{2M_p^2}
\]

which can be written in the famous form:

\[
V(\sigma, \psi) = \frac{1}{4} \lambda' (M^2 - \psi^2)^2 + \frac{g^2}{2} \sigma^2 \psi^2 + \frac{1}{2} m_\sigma^2 \sigma^2
\]

by means of the following rescaling:

\[
M \equiv v
\]

\[
\lambda' \equiv (2\lambda)^2
\]

\[
g^2 \equiv \frac{\lambda^2 v^2}{M_p^2}
\]

\[
\psi \equiv \varphi \frac{v}{\sqrt{2}M_p}
\]

\[
m_\sigma^2 \equiv \frac{\alpha \lambda^2 v^4}{M_p^2}
\]

A detailed investigation of such a model has been presented by Copeland et al. [7]. For values of \( \sigma \) larger than:

\[
\sigma_c = 2M_p
\]

the minimum of \( V \) is at \( \varphi = 0 \), the energy density of the universe is dominated by the potential energy of the scalar field \( \sigma \):

\[
V(\sigma) = \frac{1}{4} \lambda' M^4 + \frac{1}{2} m_\sigma^2 \sigma^2
\]

Inflation ends when \( \sigma \) falls below \( \sigma_c \) and the fields rapidly adjust to their true vacuum values \((\varphi = M_p \text{ and } \sigma = 0)\) with a vanishing cosmological constant \( V = 0 \). However, inflation can end before \( \sigma \) reaches its critical value, when the potential becomes too steep to maintain the slow rolling. This happens when the slow rolling conditions [3]:

\[
\epsilon(\sigma) \equiv \frac{M_p^2}{2} \left( \frac{V'(\sigma)}{V(\sigma)} \right)^2 \ll 1
\]

\[
\eta(\sigma) \equiv M_p^2 \frac{V''(\sigma)}{V(\sigma)} \ll 1
\]

\[^2\text{The vanishing cosmological constant in the true vacuum state is a characteristic feature of hybrid inflation. This is in fact an advantage of our model since in reference [8] the cosmological constant was negative in the global minimum of the potential, the authors have introduced a U(1) gauge multiplet in the hidden sector and added a Fayet-Iliopoulos D-term so as to cancel the non vanishing cosmological constant, and this looks very artificial.}\]
cease to be valid. The corresponding value of \( \sigma \) \((\epsilon(\sigma_e) = 1)\) is given by \(^{[5]}\):

\[
\sigma_e = \frac{M_p}{\sqrt{16\pi}} \left( 1 + \sqrt{1 - \frac{8\pi M^4}{M_p^2 m_{\sigma^2}}} \right)
\]

which does not exist in our model. Hence, the end of inflation coincides with \( \sigma_e \). Note that the value \( \sigma_c \approx M_p \) corresponds also to the end of inflation in the Linde’s chaotic inflation \(^{[10]}\) which recalls the fact that hybrid inflation is initially a hybrid between chaotic inflation and phase transition based models of inflation. Furthermore, it is clear that during the inflationary phase the energy density of the universe is dominated by the constant false vacuum energy term in Eq.(26).

The number \( N \) of e-foldings of expansion which occur between two scalar field values is given by the expression:

\[
N(\sigma_1, \sigma_2) = -\frac{1}{M_p^2} \int_{\sigma_1}^{\sigma_2} \frac{V(\sigma)}{V'(\sigma)} d\sigma
\]

In our approximation this gives:

\[
N(\sigma_1, \sigma_2) = -\frac{1}{2M_p^2} \int_{\sigma_1}^{\sigma_2} \sigma d\sigma
\]

If we take \( \sigma_2 = \sigma_c = 2M_p \), the condition of sufficient inflation \( N \geq 70 \) translates to the constraint:

\[
\sigma_1 \geq 8M_p
\]

This constraint justifies the choice of the parameter \( \alpha \) as it was introduced in Eq.(14).

According to the analysis of reference \(^{[7]}\) the CMB constraint in this case is given by:

\[
\frac{1}{M_p} \lambda M \leq 5 \times 10^{-2}
\]

which translates in our model to the equation:

\[
\lambda \left( \frac{v}{M_p} \right)^4 \leq 5.9 \times 10^{-6}
\]

This can be achieved, for instance, by the choice \( \lambda \sim 10^{-2} \) and \( v \sim 10^{16} \) GeV, which is very acceptable.

Now let us comment on the value \( \sigma_c \) Eq.(25). The fact that \( \sigma > M_p \) during inflation is problematic if one considers the full expansion of \( K \) in the \( S \)–direction (Eq.(3)), since all the terms should be of the same order of magnitude and have important contributions in the scalar potential. Indeed, if we consider, for instance, the next term in Eq.(14) so that:

\[
K = \frac{\phi^* \phi}{M_p^2} + \alpha \frac{S^* S}{M_p^2} + \beta \frac{(S^* S)^2}{M_p^4}
\]
and demand that $\beta \ll \alpha$, we will have an additional term in the scalar potential:

$$\Delta V = \frac{\lambda''}{4}\sigma^4$$

where:

$$\lambda'' = \beta \frac{\lambda^2 v^4}{M_p^4}$$

(Note that the value of $\sigma_c$ remains unchanged.)

This contribution to the scalar potential can be smaller than that of the mass-term only if:

$$\sigma < \sqrt{\frac{2\alpha}{\beta} M_p}$$

which contradicts Eq.(25) when $\beta \ll \alpha$. However, we can always make an appropriate choice of the omitted terms in Eq.(3) of the Kähler potential ($a_1 = \alpha$ and other $a_i = 0$), it is possible to arrange for the potential Eq.(18).

However, the new term does not introduce any fine-tuning in the model. If we assume that $\Delta V > 1/2m_\sigma^2 \sigma^2$, the potential during inflation takes the form:

$$V(\sigma) = \frac{\lambda'}{4}(M^4 + B\sigma^4)$$

where:

$$B = \frac{\beta}{2M_p^2}$$

The investigation of such a potential has been the concern of reference [11] where several mechanisms of inflation have been studied. The CMB constraint in the case corresponding to our model gives:

$$\lambda^2 \beta \ll 10^{-4}$$

It is obvious that we can still have $\lambda \sim 10^{-2}$ for $\beta \ll 1$. This proves that even with higher powers in the expression Eq.(3) for the inflaton field there is no fine-tuning of the coupling parameter.

This model provides a solution to another problem: the so-called $\eta$–problem, which is a generic feature of minimal-supergravity models of inflation (see for example [1]). In such models there is a contribution of order $H^2$ to the mass square of the inflaton during inflation which gives a contribution of order unity to the slow-rolling parameter $\eta$ and then destroys the slow-rolling conditions. It is clear that in our model:

$$m_\sigma^2 \ll H^2 \simeq \frac{\lambda^2 v^4}{3M_p^2}$$
We have constructed a hybrid inflation model without inflaton superpotential, the inflaton field contributes only through the Kähler potential. This model presents the advantage of the original version of hybrid inflation model, namely the resolution of the fine-tuning problem. Another generic problem of the supergravity models which is the $\eta$–problem has also been overcome. On the other hand, while in the initial model the cosmological constant in the ground state was negative, in the present model it vanishes without need of any artificial mechanism to cancel it. Finally, since the higher powers of the non inflaton field $|\phi|^n$ do not contribute to the dynamics of the inflaton field, they can be neglected. The model is then independent of the integer $n$, and applies to all $Z_n - R$ symmetries without any restriction.

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