Research on the influence of different data distributions on the fitting results of subsonic compressible flow turbulence level and its uncertainty

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Abstract. On the basis of results of subsonic compressible flow turbulence level in high speed wind tunnels by hyperbolic fitting, the uncertainty of turbulence level is quantitatively studied by using Monte Carlo simulation method. In order to generate stochastic sample data for Monte Carlo simulation, two distribution types, uniform distribution and normal distribution, are used to generate the data points to be fitted. The effect of changing characteristic parameters of two distribution types on the turbulence fitting results and its uncertainty is studied. The results show that: (1) for uniform distribution, the larger the data distribution bandwidth is, the larger turbulence level and its uncertainty is; (2) for normal distribution, when the mean of the distribution is fixed and the standard deviation is changed, turbulence level results are basically the same, and the uncertainty and the standard deviation have the same trend; (3) comparing the two distribution types, turbulence level and its uncertainty can both be accurately solved, as long as the characteristic parameters are appropriate.

1. Introduction

Wind tunnel tests are the most effective means for aerodynamic research of aircrafts and indispensable parts of the study of complex aerodynamic characteristics in the development of aircrafts [1]. As the performance of aircrafts continues to increase, the accuracy of wind tunnel testing is becoming more and more demanding. The excellent wind tunnel flow quality is the premise for producing high fidelity data.

The wind tunnel flow field turbulence level is an important flow field quality. The turbulence levels in wind tunnel flow fields have not been investigated thoroughly. Scholars began to realize the importance of turbulence level measurements in subsonic compressible flows around 1930 and carried out research to understand wind tunnel disturbance mechanisms. Dryden [2-4] first applied a hot wire anemometer to measure the velocity fluctuation of low speed wind tunnel flow field. They corrected the problem of low frequency response of the hot wire anemometer at that time. From the literature above, the following two points can be summarized: first, hot wire anemometer is an appropriate instrument in the wind tunnel turbulence level measurement tests because of its high resolution and high response frequency. Second, the response equation of hot wire anemometer in incompressible flow is relatively simple [5]. As a result the difficulty of measuring turbulence level of low speed wind tunnels was firstly overcome in the early stage of the study of turbulence level measurement technique. Systematic and standardized measurement methods and processes were gradually formed [6]. However, the output voltage of hot wire anemometer in compressible flow is coupled by gas...
velocity, density and total temperature, and the response equation is not simple [7]. Therefore, the measurement technique of turbulence level based on hot wire anemometer in high speed wind tunnel is much more difficult than that in low speed wind tunnel, and the development time is longer than that in low speed wind tunnel. Horstman [8] studied the method of solving gas velocity, density and total temperature sensitivity coefficients of hot wire anemometer based on a large number of turbulence level measurement test data. A differential relationship between the coefficients and a series of dimensionless coefficients (such as Mach number $M$, Reynolds number $Re$ and Nusselt number $Nu$) was obtained. On the basis of the relationship, turbulence level of the transonic wind tunnel flow field was obtained. However, one may encounter the problem that the condition number of the equation coefficient matrix is too high in the solution process. The high condition number leads to the matrix approximately singular and difficulty in finding the inverse matrix. Under this circumstance, turbulence level cannot be accurately obtained. Similar to the literature above, some researchers [9-12] were working on solving the sensitivity coefficients in compressible flow through calibration test, and then solving the turbulence level. They achieved some results, but there are certain problems. Owen [13-14] and King [15] summarized turbulence level measurement technique using hot wire anemometry. They suggested that turbulence level can be estimated from measurement results using hot wire anemometer and pressure transducers simultaneously. However, the results they obtained here are different from what they obtained using hot wire anemometry only, which means the method is not exactly right [13]. Some modern optical techniques, like PIV [16] and LDV [17], can also be used to measure gas velocity. However, they cannot be used to obtain turbulence level because of the relatively low frequency response. Due to the stagnation of technological development, a relatively mature measurement method of turbulence level in compressible flow has not yet formed to the best of our knowledge. Therefore, no scholars have quantified the uncertainty of turbulence level, and the credibility of turbulence level measurement results in compressible flow in some studies cannot be estimated.

In this study, Monte Carlo simulation method is used for solving turbulence level and its uncertainty based on the previous turbulence level measurement results [18]. The effects of two different data distribution types, uniform distribution and normal distribution, on the Monte Carlo simulation results are explored. Characteristic parameters of two different distribution types are changed to see the effect on mean turbulence level and its uncertainty. The results show that under the premise of selecting appropriate characteristic parameters, the data points generated by the two distribution types can both be used to obtain the turbulence level and its uncertainty accurately.

2. A brief review of turbulence level solution and partial fitting results
In this section, the theoretical derivation results of solving subsonic compressible flow turbulence level using hyperbolic fitting method and partial solution results are briefly introduced (see Reference [18] for more details).

The turbulence level is defined as follows:

$$Tu = \sqrt{\frac{\Delta U}{U}},$$

(1)

where $Tu$ is the flow field turbulence level, and $U$ is the gas velocity.

For the constant temperature hot wire anemometer, according to its principles and a series of derivations, its response equation can be obtained:

$$\left(\sqrt{\theta^2}\right)^2 = \left(\frac{\Delta m}{m}\right)^2 r^2 - 2\left(\frac{\Delta m}{m}\right)\left(\frac{\Delta T_0}{T_0}\right) + \left(\frac{\Delta T_0}{T_0}\right)^2,$$

(2)

where $m$ is the mass flow rate of the gas. $T_0$ is the total temperature of the gas. $r$ is a variable related only to the temperature of the hot wire. $\sqrt{\theta^2}$ is a variable related to the temperature of the hot wire and the output voltage of the hot wire anemometer, and it is defined in detail in Reference [18].
Equation (2) conforms to the hyperbolic equation with the independent variable \( r \) and the dependent variable \( \sqrt{\theta^2} \). Therefore, the hyperbolic fitting can be performed on the turbulence level measurement test data using equation (2) and only the portion in first quadrant is taken. After the hyperbola fitting curve is obtained, two important parameters can be extracted from the curve: the intercept \( a \) of the hyperbolic curve on the vertical axis and the slope \( b \) of the hyperbolic asymptote. According to equation (2), \( a \) and \( b \) represent root mean square of the ratio of total temperature fluctuation and mass flow rate fluctuation to their mean values respectively:

\[
a = \sqrt{\left(\frac{\Delta T_0}{T_0}\right)^2}, \quad b = \sqrt{\left(\frac{\Delta m}{m}\right)^2}.
\]  

(3)

On this basis, the formula for solving the turbulence level can be derived:

\[
\left(\frac{\Delta U}{U}\right)^2 = H^2\left(\frac{\Delta m}{m}\right)^2 + J^2\left(\frac{\Delta T_0}{T_0}\right)^2 - 2HJ\left(\frac{\Delta m}{m}\right)\left(\frac{\Delta T_0}{T_0}\right),
\]  

(4)

\[
H = \frac{1}{M^2 - 1}, \quad J = \frac{1 + \frac{\gamma - 1}{2}M^2}{(\gamma - 1)(M^2 - 1)},
\]  

(5)

where \( \gamma \) is the gas specific heat ratio.

Partial turbulence level measurement test data from experiments are shown in Table 1.

**Table 1. Turbulence level measurement test data.**

| \( M = 0.330 \) | \( M = 0.420 \) | \( M = 0.525 \) | \( M = 0.627 \) | \( M = 0.719 \) |
|---|---|---|---|---|
| \( r \) | \( \sqrt{\theta^2} / 10^4 \) | \( r \) | \( \sqrt{\theta^2} / 10^4 \) | \( r \) | \( \sqrt{\theta^2} / 10^4 \) | \( r \) | \( \sqrt{\theta^2} / 10^4 \) |
| 0.034 | 7.53 | 0.034 | 5.95 | 0.034 | 6.23 | 0.034 | 5.59 | 0.033 | 7.56 |
| 0.053 | 6.57 | 0.053 | 5.43 | 0.052 | 5.27 | 0.052 | 4.72 | 0.051 | 4.55 |
| 0.073 | 4.49 | 0.073 | 4.64 | 0.073 | 3.61 | 0.073 | 3.81 | 0.071 | 3.68 |
| 0.095 | 4.25 | 0.096 | 3.66 | 0.119 | 4.13 | 0.119 | 3.68 | 0.116 | 3.97 |
| 0.119 | 5.60 | 0.121 | 5.04 | 0.146 | 4.37 | 0.146 | 3.57 | 0.142 | 4.21 |
| 0.146 | 5.19 | 0.148 | 4.81 | 0.176 | 5.17 | 0.177 | 3.84 | 0.171 | 4.83 |
| 0.177 | 5.88 | 0.179 | 5.78 | 0.210 | 5.90 | 0.212 | 4.36 | 0.205 | 5.32 |
| 0.211 | 6.33 | 0.215 | 6.73 | 0.250 | 7.07 | 0.252 | 5.16 | 0.243 | 6.27 |
| 0.251 | 7.66 | 0.257 | 7.65 | 0.638 | 15.3 | 0.659 | 12.5 | 0.603 | 12.3 |
| 0.640 | 22.5 | 0.681 | 21.2 | 1.290 | 32.4 | 1.399 | 27.5 | 1.165 | 24.5 |

The test data in Table 1 are subjected to hyperbolic fitting using equation (2), and the turbulence level is solved by using equations (4) and (5). The results are shown in Table 2.

**Table 2. Turbulence level results.**

| \( M \) | \( T_u(\%) \) |
|---|---|
| 0.330 | 0.355 |
| 0.420 | 0.357 |
| 0.525 | 0.329 |
| 0.627 | 0.306 |
| 0.719 | 0.425 |
3. Monte carlo simulation method for solving turbulence level and its uncertainty

In order to evaluate reasonably whether the results of a single turbulence level measurement test can accurately represent the true value of wind tunnel turbulence level, the uncertainty of turbulence level measurement test results needs to be evaluated. Since the turbulence level is solved by the fitting method, it is difficult to solve it using the traditional uncertainty transfer method as there is no explicit relationship. Monte Carlo simulation method is considered to avoid evaluating it through a large number of turbulence level measurement tests [19]. The Monte Carlo simulation method is a computational method based on probability and statistical theory. It links the problem to be solved with a certain probability distribution model. One can use computers to generate random or pseudo-random numbers to realize statistical simulation, and approximate solution to the problem is obtained through simulation [20]. The specific steps of the Monte Carlo simulation method are as follows:

(1) Construct a probability statistical model. When using hyperbolic fitting under each Mach number, a computer can be used for simulation test to generate a large number of scatter data to be fitted. The longitudinal distance between the scatter data to be fitted and that from the turbulence level measurement test results (hereinafter referred to as the known scatter data) conforms to a certain distribution law. A reasonable distribution model (such as uniform distribution and normal distribution) can be constructed to generate data.

(2) Random sampling of the model. Generate 1000 sets of random numbers that conform to a certain distribution law by using MATLAB. 1000 sets are proved to be enough for the simulation. The known scatter data is superimposed with the generated 1000 sets of random numbers, and we can obtain the scatter data to be fitted.

(3) Determine the evaluation value. Fit 1000 sets of scatter data using hyperbolic fitting method, and 1000 sets of turbulence level fitting results are obtained. The uncertainty of turbulence level is solved by the following equation:

$$u = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2},$$

where $u$ is the uncertainty, $i$ is the count variable, $n$ is the total number of turbulence level fitting results, and here $n$ is 1000. $x_i$ is turbulence level fitting result in group $i$, $\bar{x}$ is the mean of 1000 groups of turbulence level fitting results.

The uniform distribution and normal distribution are used to generate random numbers, and the influence of the characteristic parameters of two distribution types on the turbulence level and its uncertainty is investigated by adjusting the characteristic parameters. The longitudinal distance between the known scatter data and the fitted curve can be used as a reference value for the distribution characteristic parameters, which can be expressed as $D = [d_1 \ d_2 \ \ldots \ d_{10}]$.

3.1. Uniform distribution

The uniform distribution is used to generate longitudinal distance data. When the lower limit of the uniform distribution is set to 0, and the upper limit is set to max($D$) (the largest element in $D$) and mean($D$) (the average of the elements in $D$), Monte Carlo simulation is carried out to obtain 1000 sets of data of turbulence level and its uncertainty, which are shown in Tables 3 and 4, respectively.

Table 3. Mean turbulence level and its uncertainty (uniform distribution, upper limit is max(D)).

| $M$  | $Tu(\%)$ | $u(\%)$ |
|------|----------|---------|
| 0.330| 0.371    | 0.007   |
| 0.420| 0.373    | 0.007   |
| 0.525| 0.354    | 0.011   |
| 0.627| 0.331    | 0.012   |
| 0.719| 0.489    | 0.030   |
Table 4. Mean turbulence level and its uncertainty (uniform distribution, upper limit is mean(D)).

| M   | Tu(%) | u(%) |
|-----|-------|------|
| 0.330 | 0.361 | 0.002 |
| 0.420 | 0.363 | 0.002 |
| 0.525 | 0.339 | 0.005 |
| 0.627 | 0.315 | 0.005 |
| 0.719 | 0.446 | 0.010 |

3.2. Normal distribution
The normal distribution is used to generate longitudinal distance data. When the mean of the normal distribution is set to 0 and the standard deviation is set to mean(D), Monte Carlo simulation is carried out to obtain 1000 sets of data of turbulence level and its uncertainty shown in Table 5.

Table 5. Mean turbulence level and its uncertainty (normal distribution, standard deviation is mean(D)).

| M   | Tu(%) | u(%) |
|-----|-------|------|
| 0.330 | 0.356 | 0.008 |
| 0.420 | 0.357 | 0.006 |
| 0.525 | 0.329 | 0.016 |
| 0.627 | 0.306 | 0.017 |
| 0.719 | 0.426 | 0.033 |

According to the "3σ" principle, the probability that random numbers generated by the normal distribution fall in \((μ - 3σ, μ + 3σ)\) is as high as 99.74%. Therefore, the mean of the normal distribution can be set to 0, and the standard deviation can be set to mean(D)/3, Monte Carlo simulation is carried out and the results are shown in Table 6.

Table 6. Mean turbulence level and its uncertainty (normal distribution, standard deviation is mean(D)/3).

| M   | Tu(%) | u(%) |
|-----|-------|------|
| 0.330 | 0.355 | 0.003 |
| 0.420 | 0.357 | 0.002 |
| 0.525 | 0.329 | 0.005 |
| 0.627 | 0.306 | 0.006 |
| 0.719 | 0.426 | 0.012 |

3.3. Analysis of results

3.3.1. Uniform distribution. For the uniform distribution with two different upper limits, the curve of mean turbulence level with Mach number is shown in Figure 1, and the curve of turbulence level uncertainty with Mach number is shown in Figure 2.

Since \(\text{max}(D)\) is larger than mean(D), the larger the upper limit of uniform distribution is, the larger the mean turbulence level and its uncertainty is. When the bandwidth of the uniform distribution is wider, the distribution of the scatter data to be fitted is more dispersed, which leads to a relatively bad
fitting. As a result, turbulence level from Monte Carlo simulation is larger than that in Table 2. Moreover, the more dispersed the scatter data to be fitted is, the larger standard deviation of turbulence level obtained from Monte Carlo simulation is, which means the larger the uncertainty is.

![Figure 1. Contrast of mean turbulence level (uniform distribution with two upper limits).](image1)

![Figure 2. Contrast of turbulence level uncertainty (uniform distribution with two upper limits).](image2)

Comparing two characteristic parameters of the uniform distribution above, it can be known that the mean turbulence level is closer to the experimental value in Table 2 and mean turbulence level and its uncertainty are more reasonable when the upper limit of the uniform distribution is set to mean(D).

However, due to a large randomness of the value of max(D), it is not suitable as the upper limit of uniform distribution.

3.3.2. Normal distribution. For the normal distribution with two different standard deviations, the curve of mean turbulence level with Mach number is shown in Figure 3, and the curve of turbulence level uncertainty with Mach number is shown in Figure 4.

![Figure 3. Contrast of mean turbulence level (normal distribution with two standard deviations).](image3)

![Figure 4. Contrast of turbulence level uncertainty (normal distribution with two standard deviations).](image4)

It can be seen that the mean turbulence level obtained from Monte Carlo simulation is basically unchanged when the mean value of the normal distribution is constant and the standard deviation is changed. This is because although data from the normal distribution is distributed in the whole domain, the high probability distributed region is concentrated due to the "3 σ" principle. Therefore, the distribution of the scatter data to be fitted is less dispersed, which leads to a relatively good fitting. And though more dispersed scatter data to be fitted are obtained when the standard deviation is set to mean(D), there are nearly 70% of random numbers generated by the normal distribution fall in the region \((\mu + \text{mean}(D), \mu - \text{mean}(D))\). The region contains almost all the random numbers generated
by the normal distribution when the standard deviation is set to mean($D$)/3. The mean turbulence level results from the normal distribution with two standard deviations are nearly the same due to the overlap area mentioned before. In addition, the larger the standard deviation is, the more dispersed the scatter is relatively, and then the more dispersed the turbulence level results are, which means the larger the uncertainty is.

4. Conclusions
In this paper, the method for evaluating the uncertainty of subsonic compressible flow turbulence level in high speed wind tunnels is proposed. The mean turbulence level and its uncertainty results from the uniform distribution and the normal distribution with different characteristic parameters are discussed. The mean turbulence level results from the uniform distribution whose upper limit is mean($D$) and those from the normal distribution whose standard deviation is mean($D$)/3 are both close to the experimental value in Table 2. The uncertainty results from the two distributions mentioned above are nearly the same. The results means that the two distributions mentioned above can both be used to generate random numbers for Monte Carlo simulation method to solve the mean turbulence level and its uncertainty. In addition, the mean turbulence level result from the normal distribution whose standard deviation is mean($D$)/3 is much closer to the experimental value. Therefore, this exact distribution can be directly used to solve mean turbulence level and its uncertainty. The uniform distribution whose upper limit is mean($D$) can be used to validate the results.

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